A Smart Sensor for the Heat Flow of an Extruder

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Abstract: Two mathematical models are derived to describe the thermal dynamics of a plastics production machine. The Smart Sensor design is based on the finite volume model. The accuracy of the derived finite volume model is validated by a finite element model. Those models are validated by measurements. A Smart Sensor for the heat flow between the extruder cylinder and screw conveyor is designed. Such a configuration has the advantage to decouple the thermal control problem from the specific granulate.

Keywords: extruder, plastic extrusion, thermal modelling, finite volume, finite element, observer

1. INTRODUCTION

Extruders represent essential parts in the plastic production process as they convert the granulate to melt. Its control performance has a great impact on the work machines and therefore on the quality of the products. There exists several contributions which are devoted to mathematical modelling, simulation and optimization of the production process, see, e.g., Kochhar and Parnaby (1977), Tadmor and Gogos (2013), Acur and Vlachopoulos (1982), Zheng and Gao (2012) and Ravi and Balakrishnan (2010).

Here, a precise mathematical model is constructed, so that the system order is as small as possible. The model is derived following the method of finite volumes (FV). Its simulation outcome is confronted to the numeric results from a precise finite element (FE) model, generated by the simulation package FEniCS (described in Logg et al. (2012) and Alnæs et al. (2015)). A similar FE approach is described by Belav˘ y et al. (2018). These models are the basis for the design of a Smart Sensor which estimates the heat flow between the screw conveyor and the cylinder. The cylinder is actuated by electrical heating tapes and equipped with thermocouples.

The paper is organized as follows: The extruder setup is discussed in Section 2. In Section 3 the FV as well as the FE approach are described and the simulation results of both models are compared with measurements from an industrial device. The design of the Smart Sensor is shown in Section 4 together with experimental results by different extrusion experiments.

2. THE EXTRUDER SETUP

The setup of a typical extruder is shown in Fig. 1, it consists of a cylinder with length \( L \), a inner diameter \( d_1 \) and an outer diameter \( d_2 \). The screw conveyor is driven by a motor. The granulate is admixed by a dosing device in the filling zone. \( N_{th} \) heating tapes are mounted around the extruder cylinder. A common extruder setup comprises \( N_h \) heating zones, which are formed by several heating tapes. The heating tapes are connected in parallel. The number of heating tapes per heating zone is given by \( N_{th,h} = N_{th}/N_h \). The state of the extruder is given by the temperature distribution \( T(t,x) \) at time \( t \) and the location coordinate \( x \). The thermocouples (\( S_0 - S_{13} \) in Fig. 1) provide us with the temperature at the place of installation.

![Fig. 1. A typical extruder configuration.](image-url)
coupling of the heating zones. This should not be mixed up
with the approach described above where the sensors are
mounted far from the heating zones. One should keep in
mind, that this configuration does not allow active cooling,
but the measured \( T_h \) can be considered as input to the
mathematical model of the cylinder. Even if the system
equations are linear, the control problem is highly non-
linear because of the restriction (no active cooling) in the
control input.

3. MATHEMATICAL MODELLING / VALIDATION

3.1 Finite Volume Approach

The extruder to be modelled consists of rotationally
symmetrical heating tapes installed around the cylinder.
Therefore, the model can be described as a rotationally
symmetrical problem. Due to the rotational symmetry
assumption, the position is determined by two coordinates
\( x = (x \ y) \). The governing equations for our problem are
given by the Fourier heat equation in cylindrical coordi-
ates
\[
\rho c_p \frac{\partial T}{\partial t} = \frac{1}{\rho c_p} \left( \frac{\partial}{\partial r} \left( \lambda \frac{\partial T}{\partial r} \right) \right) + \frac{\partial}{\partial x} \left( \lambda \frac{\partial T}{\partial x} \right).
\]

The parameters \( \rho \), \( c_p \) and \( \lambda \) (see Tab. 1) are assumed to
be constant inside each individual FV element.

Table 1. Notations for the model.

| Name       | Symbol | Unit |
|------------|--------|------|
| time       | \( t \) | s    |
| temperature| \( T \) | K    |
| density    | \( \rho \) | kg/m³ |
| specific heat capacity | \( c_p \) | J/kgK |
| thermal conductivity | \( \lambda \) | W/mK |

Due to the rotationally symmetry assumptions, a FV
element is described by a ring element with length \( \Delta x_i \)
and the radial thickness \( \Delta r_i \).

Fig. 2. Equivalent circuit for a typical extruder configura-
tion.

The FV elements are connected by ports, so that a number
of elements \( n_x \) in the axial and \( n_r \) in the radial direc-
tion arises, resulting in a total number of \( n = n_x n_r \) elements.
The number of elements correspond to the resulting system
order. The surfaces of the \( i \)-th FV are named \( A_i \) and
the volume \( V_i \). According to Fig. 2 the following abbreviations
\( T(t, x, r - \Delta x) = T_{s,i} \), \( T(t, x, r + \Delta r) = T_{h,i} \), \( T(t, x, r -
\Delta r) = T_{3,i} \) and \( T(t, x, r + \Delta r) = T_{4,i} \) are used. The size
of the elements are denoted by \( \Delta r_i \) and \( \Delta x_i \), this allows
us to combine FV elements of different sizes. Different
volumes sizes enables a simple adaption of the elements to
the cylinder including heating tapes and thermocouples.
The following equations
\[
C_i \frac{\partial T_{s,i}}{\partial t} = \frac{1}{R_{1,i}}(T_{1,i} - T_{s,i}) + \frac{1}{R_{2,i}}(T_{2,i} - T_{s,i}) \]
\[
+ \frac{1}{R_{3,i}}(T_{3,i} - T_{s,i}) + \frac{1}{R_{4,i}}(T_{4,i} - T_{s,i}),
\]
with the quantities
\[
C_i = 2r_i \Delta r_i \pi \Delta x_i \rho c_p,i = V_i \rho c_p,i \]
\[
R_{1[1,2],i} = \frac{\Delta x_i}{2r_i \Delta r_i \pi \lambda_i} = \frac{\Delta x_i}{A_{1[1,2],i} \lambda_i}
\]
\[
R_{3,i} = \frac{\Delta r_i}{2\pi(r_i + \frac{\Delta r_i}{2}) \Delta x_i \lambda_i} = \frac{\Delta r_i}{4A_{3,i} \lambda_i}
\]
\[
R_{4,i} = \frac{\Delta r_i}{2\pi(r_i - \frac{\Delta r_i}{2}) \Delta x_i \lambda_i} = \frac{\Delta r_i}{4A_{4,i} \lambda_i}
\]
represent the spatially discretised model according to Fig.
2. It is straightforward by combining (2) and (3) to derive
\[
\rho_c c_p,i \frac{\partial T_{s,i}}{\partial t} = \lambda_i \left( \Delta x_i \frac{\partial^2 T_{s,i}}{\partial x^2} + \frac{1}{r_i} \frac{\partial T_{s,i}}{\partial r} + \frac{\partial^2 T_{s,i}}{\partial r^2} \right) \]
which coincided with (1), confirming that the heat equa-
tion approximately corresponds to the model of an electri-
cal network which consists of a capacitor and resistors.

The input of the network are temperature sources or heat
flow sources. Each FV element has four ports. The port
variables are \( T_{s,i} \) and \( \dot{q}_{s,i} \), which can be combined with
the neighbour or with the environment. In addition, the
port variables are connected by one linear equation, see,
Fig. 2, for the port variables and the temperature \( T_i \) of
the FV element. In particular the interconnection with a
neighbour element \( i \), where index \( s \) and \( \hat{s} \) mark nodes of
the \( i \)-th and \( \hat{s} \)-th FV element, respectively, is given by
\[
T_{s,i} = T_{s,\hat{i}} \quad \dot{q}_{s,i} = -\dot{q}_{s,\hat{i}}.
\]
This method has to be applied iteratively to generate a set
of additional equations. To complete the model construc-
tion, boundary conditions (BC) have to be defined. Simple
connections to the environment are given by
\[
T_{x,i}(t, x) = T_{\chi}(t, x) \quad q_{x,i}(t, x) = \dot{q}_{\chi}(t, x),
\]
where the quantities \( \dot{q}_{x,i} \) and \( T_{x,i} \) follow from the network
equations. These are the well known Dirichlet (7a) and
Neumann (7b) boundary conditions. Other boundary con-
ditions are
\[ \dot{q}_{x,i}(t, \mathbf{x}) = \alpha_i(\mathbf{x})(T_{x,i}(t, \mathbf{x}) - T_0(t, \mathbf{x})) \quad (8a) \]
\[ \dot{q}_{x,i}(t, \mathbf{x}) = \alpha_i(\mathbf{x})(T_{x,i}(t, \mathbf{x}) - T_0(t, \mathbf{x})) + \alpha_{ht,i}(\mathbf{x})(T_{ht,i}(t, \mathbf{x}) - T_h(t, \mathbf{x})) \quad (8b) \]
\[ \dot{q}_{x,i}(t, \mathbf{x}) = \alpha T_{ht,i}(t, \mathbf{x}) \quad (8c) \]

The connection with the environment is established by (8a), whereas \( T_0 \) is the ambient temperature and \( \alpha_i \) is the heat transfer coefficient between cylinder and environment. The heating tapes are defined by (8b), whereas \( \alpha_{ht,i} \) is the heat transfer coefficient between cylinder and the heating tape and can be interpreted as depicted in Fig. 3 on the left. The presented discretization allows us to take all types of boundary conditions into account.

If we combine all the equations of our network (2) with all types of boundary conditions into account.

\[ \dot{x} = Fx + Hd + Gu \]
\[ 0 = \dot{F}x + Hd + Gu \]

with an invertible block diagonal matrix \( \dot{H} \) and the eliminable quantities from the network equations \( d \). Therefore, it is straightforward to derive the linear state space model

\[ \dot{x} = Ax + Bu = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & A_{23} \\ 0 & A_{32} & 0 \end{bmatrix} x_c + \begin{bmatrix} B_s \\ 0 \\ B_c \end{bmatrix} u \]
\[ y = Cx = [C_1 \ 0 \ 0]x \]

The state is given by the temperatures \( T_i \) of the FE elements and input by the temperature sources

\[ u = [T_{h,1} \ \cdots \ \ T_{h,N_h} \ T_0] \]
\[ x = [x_T^T \ x_T^T \ x_{sc}] = [T_1 \ \cdots \ \ T_{n,m}] \]

The state vector \( x \) consists of all FE temperatures \( T_i \) and can be divided into the temperatures belonging to the cylinder zone \( x_c \), to the screw conveyor zone \( x_s \), and the screw core zone \( x_{sc} \). A description of the introduced zones follows below. The input vector \( u \) contains the ambient temperature \( T_0 \) and all temperatures sources \( T_{h,j} \). The output \( y \) includes the temperatures of the points where the sensors are placed (the points \( S_i \) in Fig. 1).

![Fig. 3. BC realisation for nodes \( P \in \Gamma_2 \) on the left, boundary decomposition in sets \( \Gamma_1 \) to \( \Gamma_2 \) and the zone definition at the right.](image)

The right side of Fig. 3 illustrates the considered modelling area, which is described by an outer diameter \( d_{2} \) and the extruder length \( L \). Three radially limited material related areas are defined: from the centre axis to the core of the screw conveyor \( r_c = \text{screw core zone} \) \((c_{p,s}, \lambda_s \text{ and } \rho_s)\), the area form the core of the screw conveyor to the inner radius of the extruder cylinder \( d_1/2 \) - \text{screw conveyor zone} \((c_{p,s}, \lambda_s \text{ and } \rho_s)\) and from \( d_1/2 \) to the outer radius of the extruder \( d_2/2 \) - \text{cylinder zone} \((c_{p,c}, \lambda_c \text{ and } \rho_c)\). In Fig. 3 two sets \( \Gamma_1 \) to \( \Gamma_2 \), consisting of all points that are located on the border area \( \Gamma = \Gamma_1 \cup \Gamma_2 \), can be seen. The set \( \Gamma_2 \) includes open nodes that are directly located under the heating tapes and \( \Gamma_1 \) includes nodes that are in direct contact with the environment. The most inner FV elements are disks not rings.

To monitor the heat flow across the border of the extruder and the heat flow injected by the heating tapes, we can extend the output with additional equations. A big advantage this model offers is that the operator gets information about the temperature distribution as well as the heat flows inside or out of the extruder. The mutual influence of the heating tapes is also taken into account.

### 3.2 Finite Element Approach

A model created using the finite element (FE) approach is used to derive an accurate reference solution to find an FE model (10) with the lowest possible system order that is comparably accurate. The FE model comes with a much higher system order resulting in a higher resolution and accuracy of the computed temperature distribution. In order to derive a FE model, a variational formulation of the partial differential equations is needed. The locational functions describing \( c_p(\mathbf{x}), \lambda(\mathbf{x}) \text{ and } \rho(\mathbf{x}) \) have to be defined so that the different zones, illustrated in Fig. 3, are taken into account. A FE program like FEniCS , see Logg et al. (2012), Alnus et al. (2015) can only solve static problems. By help of the Backward Euler formula

\[ \frac{\partial T(t, \mathbf{x})}{\partial t} = \frac{T(k) - T(k-1)}{\Delta t} \]

the dynamic problem is replaced by a sequence of static ones with the time step \( \Delta t \). The variational formulation with the test functions \( v \in V \) is given by

\[ \int_{\Omega} \rho c_p T(k - 1) v \, dA = -\Delta t \lambda \int_{\Gamma} \frac{\partial T(k)}{\partial r} v \, dA + \int_{\Gamma} \left( \rho c_p T(k) v + \Delta t \lambda \frac{\partial T(k)}{\partial r} v + \frac{\partial T(k)}{\partial x} \frac{\partial v}{\partial x} \right) \, dA \]

\[ -\Delta t \lambda \int_{\Gamma} \alpha_{s1}(T(k) - T_0) v \, ds \]

\[ -\Delta t \lambda \int_{\Gamma} (\alpha_{s2}(T(k) - T_0) + \alpha_{ht,i}(T(k) - T_{h,i})) v \, ds, \]

where \( \alpha_{s1} \) and \( \alpha_{s2} \) denote the surface independent heat transfer coefficients between the extruder and its surrounding and \( \alpha_{ht,i} \) describe the surface independent heat transfer coefficients between the \( i \)-th heating tape and the extruder. The space of allowed test functions \( V = \{ v(\mathbf{x}) \in H^1(\Omega) : v(\mathbf{x}) = 0, \forall \mathbf{x} \in \Gamma_{in} \} \) is the Sobolev space \( H^1 \) where \( \Omega \) is the area of integration and \( \Gamma_{in} \) its boundaries on which a BC in the form of (7a) is going to be implemented. Because of space limitations the derivation of (13) is not discussed in detail.

### 3.3 Validation Through Heat Up Experiments

To verify the accuracy of the created models, the results are compared with measurements. For this purpose, the heating tapes of an industrial extruder in form of
Fig. 4. Comparison between the measurement and the FV- and FE-model simulation results.

Fig. 5. Connection between percentage power $P$ and the heating tape temperature $T_h$. 

4. SMART SENSOR

Section 3 proves that the derived FV model reflects the dynamic of the extruder very well. In this Section we design the Smart Sensor based on this model with the aim of observing the thermic behaviour of the material during an extrusion process by estimating the heat flow crossing the surface between the granulate (inside the cylinder) and the extruder cylinder. The FV model is cut along $\Gamma_3$ (Fig. 3), resulting in a new model that considers only the cylinder zone, as shown in Fig. 7 depicted as a gray area.
The disturbance model reflects the fact that the change of the heat flows is sufficiently small, resulting in a high accuracy for the operating states near equilibrium positions. The heat flows \( q \) are interpreted as disturbances acting onto the model and estimate them by the use of the Smart Sensor, which is based on a discretised state space representation of (17).

\[
\begin{align*}
\dot{x}_{c,k+1} &= [A \ B_q] \begin{bmatrix} x_{c,k} \\ q_{k+1} \end{bmatrix} + B_u u_k \\
y_k &= [C_1 \ 0]' x_{c,k} + q_k
\end{align*}
\]

(18)

The discretised quantities regarding system (17) are marked with a bar. Now a LQG-disturbance observer can be designed based on system (18). The input for the observer system consists of the heating tape, the ambient and the inner sensors temperatures \( u_{1}^{i} = [u^T \ T_{S_{0}} \cdots \ T_{S_{13}}]' \). The output is an estimation of the inner temperature distribution \( \hat{x} \) and the heat flows \( \hat{q} \).

The stationary observation result coming from an extrusion experiment on a real extruder is depicted in Fig. 6. The illustrated results are snapshots taken about one hour after extrusion at consistent conditions. Negative heat flows represent a cooling impact due to the extruder cylinder or can be interpreted as an energy consumption effect on the granulate side. The picture on the right side shows the heat flows across the surface between cylinder and granulate, where the filling zone is located around element \( i = 35 \) and the extruder end at \( i = 1 \). The granulate, mainly existing in a solid state in the area of the filling zone, forces the heating flow \( i = 30, \ldots, 35 \) to be positive, because of high friction. The direction of the heat flow changes over the next elements \( i = 20, \ldots, 30 \), reflecting a high thermal energy consumption from the granulate. The heat flow decreases to approximately zero at the end of the extruder \( i = 0, \ldots, 20 \). The three different colours represent for three different rotational speeds at constant pressure as listed in the legend. The picture on the left shows the observation of the impact of the actively heated work machine mounted to the extruder during the extrusion.

The thermal behaviours of different granulate types are illustrates in Fig. 8. The first material (green) has a strong
cooling effect on the cylinder and therefore requires a high heating power to maintain a steady state. The second granulate (black) exhibits a minor cooling effect, the red one even shows a slight heating effect. The observer is capable of observing the thermal impact of the granulates and characterizes them without the knowledge about process parameters of the material. The extruder endowed with a Smart Sensor can be used for granulate behaviour investigations, for observing mechanical wear regarding the screw conveyor or even for optimizing the mechanical extruder build-up itself. The extruder cylinder, combined with the installed temperature sensors, acts like a sensor observing the material behaviour at any time. For this reason, the non-linear material behaviour gets characterized by this sensor without the information of material related parameters and therefore no model of the processed granulate is needed. We are now capable of taking a look inside the extruder, through the help of the developed Smart Sensor. We can use the Smart Sensor as basis for different control concepts to optimize the extrusion process.

5. CONCLUSIONS

Two thermal models of a plastic extruder based on the FE and FV approach are derived. We have used the FV model to design a material independent Smart Sensor, which is capable of estimating the energy exchange between the extruder cylinder and the granulate. It changes the descriptive model equations of the respected model, which would be highly non-linear if we add a model of the processed material, to a set of linear system equations. The models are validated by comparing their simulation outcomes to measurements generated by heating experiments on a real extruder. The Smart Sensor has been tested on the real extruder by extrusion experiments. The Smart Sensor can be used as a tool for analysing the granulate behaviour, for checking the work quality from the extruder setup or even for optimizing the mechanical extruder build-up itself. We aim to design a controller, based on the proposed modelling and Smart Sensor strategy, with the goal to control the extruder as fast as possible into a stationary desired extrusion state.

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