On Patterns and Efficiency of Investment in Transport Infrastructure*

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Abstract

This paper develops a two-country model of intraindustry trade in which overseas shipping incurs transport costs. National governments make investment in the transport infrastructure to reduce the transport costs and enhance national welfare. This paper investigates what patterns of public investment can be derived as equilibrium outcomes and whether these equilibrium investment patterns are socially efficient. It is shown that, among others, if the public investment technology exhibits increasing returns at an international level, coordination problem of public investment may occur.

Key words: Trade; Transport costs; Infrastructure; Coordination failure

JEL Classification Number: F12; H54; N70

1 Introduction

High quality of transport infrastructure (e.g., airports and ports) plays a key role in enhancing more borderless trade between countries. As estimated by Anderson and van Wincoop (2004), transport costs are considerably high,¹ and Limão and Venables (2001) empirically show that a lack of transport infrastructure contributes extremely high transport costs and brings about the impediment for trade. Recent empirical studies show a richer set of evidence that infrastructure investment has positive effects on exporting firms’ productivity and on the facilitation of trade (e.g., Albarran et al., 2013; Coşar and Demir, 2016; Volpe Martincus and Blyde, 2013; Volpe Martincus et al., 2014; Volpe Martincus et al., 2017).

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¹ See also Hummels and Skiba (2004), Hummels (2007) and Hummels et al. (2009).
Existing empirical studies, however, have mainly focused on successful infrastructure investments, although some counterfactual analysis has been made. It is not clear from the data whether policy makers had an incentive to make investment in transport infrastructure in a certain region. It is also unclear whether the investment decision was efficient. In particular, because of the effects being transboundary or because of high investment costs, some investment projects are conducted by multiple countries and thus entails the issue of international cooperation. That is, when different countries participate in the public investment, each country has an incentive to maximize its own welfare, and without enforceable commitments, international cooperation is difficult to achieve. It is therefore of interest how the noncooperative outcome of the public investment game differs from the socially efficient outcome that can achieve under international cooperation. In this study, we address these issues by using a simple theoretical framework.

There have been a number of theoretical studies with endogenous transport costs, by incorporating private transport sectors (e.g., Takahashi, 2006; Behrens et al., 2009; Behrens and Picard, 2011; Ishikawa and Tarui, 2018) or public investment in transport infrastructure (e.g., Mun and Nakagawa, 2008; Tsubuku, 2014). Since our interest is in infrastructure investment that reduces transport costs, this study contributes to the latter class of studies. Specifically, we develop a two-country model of intraindustry trade in which overseas shipping incurs transport costs and national governments make invest in the transport infrastructure to reduce these costs. The reduction of transport costs improves national welfare, and taking this benefit and the cost of public investment into consideration, the governments make an investment decision noncooperatively. This paper investigates what patterns of public investment can be derived as equilibrium outcomes and whether these equilibrium investment patterns are socially efficient.

In contrast to the existing studies by Mun and Nakagawa (2008) and Tsubuku (2014), we shed light on the technology of public investment in international transport sector. Considering the public investment on an international transportation, its technology may exhibit not only decreasing returns to scale, but also increasing returns. Supposing a type of international transportation such as a railroad connecting different countries, a single country’s effort to develop the transport infrastructure is not sufficient to improve the quality of transportation. This means that the public investment can have a property of increasing returns to scale at an international level, which motivates us to discuss the implications of different technologies of public investment. Both Mun and Nakagawa (2008) and Tsubuku (2014) overlook such an increasing-returns technology in infrastructure investment; they focus on the case in which the technology of public investment exhibits decreasing returns. By analyzing the equilibrium outcomes both in the case of decreasing returns and in the case of increasing returns,

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2) There are anecdotes about cessation of infrastructure investment plans, such as China-led railway projects in Latin America and Southeast Asia that have witnessed cancellation.

3) Mun and Nakagawa (2008) incorporate the transport costs dependent on the level of public investment provided by import and export countries into the two-country model under perfect competition. Based on this model, they analyze the efficiency of public investment on transport sector and the welfare effects of foreign aid. Tsubuku (2014) treats international and domestic transport costs differently, and analyzes the economy where the national governments can influence the domestic transport costs through the public investment.

4) On the measurement of the returns to infrastructure investment, see, e.g., Ramey (2020).

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We discuss how the properties of equilibrium outcomes differ between these cases.

The findings of our paper are as follows. Assuming that investment decision is binary (i.e., “invest” or “not invest”), we find that if the public investment technology exhibits decreasing returns (DR) at an international level, the Nash equilibrium outcome of the public investment game is either both countries making infrastructure investment, neither country making the investment, or only one of the countries making the investment, depending on the level of fixed investment cost. If the public investment technology exhibits increasing returns (IR) at an international level, such equilibrium that only one country makes the investment does not emerge. Instead, there can be multiple Nash equilibria consisting of “both countries making the investment” and “neither country making the investment”. That is, in this case, coordination problems may exist.

We also show that the Nash equilibrium is inefficient, but the sources of inefficiency differ between DR and IR. In the DR case, we have a possibility that the Nash equilibrium outcome is no investment, while the cooperative solution is that one of countries makes investment. In the IR case, we have an inefficient noncooperative solution in the sense that “no investment” cannot be ruled out even though the socially optimal solution is definitely to make investment. In other words, in the DR case, inefficiency caused by noncooperative behavior implies insufficient levels of infrastructure investment, while in the IR case, inefficiency does not necessarily mean insufficient investment. The difference in the nature of inefficiency between the types of public investment technology can have an implication for evaluating the actual outcomes of infrastructure investments.

In order to understand the implications of the abovementioned difference between DR and IR, let us consider the following examples. As an example of DR type infrastructure, we may consider an investment in coast guard to achieve maritime safety, thereby reducing maritime shipping costs. Supposing that two countries being separated by a sea which is not too big, the investment in coast guard by one country may be enough to prevent almost all pirate attacks, and this is an efficient outcome. However, if the countries make the investment decision noncooperatively, neither of the countries may invest in piracy prevention even though it is socially optimal that one country invests. In that case, countries must rely on some third-party enforcement agencies to make efficient investments.

On the other hand, construction of terminal stations, rails, and cargo distribution centers for railways across national borders can be an example of IR type international infrastructure; if only one country makes large investment in these facilities but the other country does not, the latter country’s insufficient investment would become a bottleneck of smooth transportation. In this case, both countries find it optimal to make the large investment and thus, the

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5) Inefficiently low investment level in the absence of coordinated investment between countries has been demonstrated by Mun and Nakagawa (2008) in a different theoretical framework. This suggests that in the case of international decreasing returns in transport infrastructure, inefficiency comes with insufficient public investment. Note that Mun and Nakagawa’s (2008) research interest is how foreign aid can reduce such an inefficiency, and thus, their focus is different from ours. As for the efficiency perspective in Tsubuku (2014), since he focuses on public investment on domestic transport sector with constant costs of international transportation, no discussion is made about the inefficiency caused by a lack of international coordination of public investment.

6) Actually, the management of maritime security regarding pirate attacks is supported by the third-party such as the United Nations International Maritime Organization.
efficient outcome can occur as an equilibrium in which each country makes investment. In other words, the countries may not need the support by a third party to achieve the efficient level of transnational railway infrastructure. However, in the presence of increasing returns, we cannot exclude a possibility of coordination failure in which neither countries do not make investment in the railway construction. If that is the case, the railway construction should be encouraged by a third-party enforcement agency.\(^7\)

Section 2 sets up our two-country model, and the market equilibrium is derived in Section 3. Section 4 considers a noncooperative game of infrastructure investment, and derives Nash equilibrium configurations. The efficiency of the noncooperative outcomes, by comparing with cooperative solutions, is discussed in Section 5. Section 6 concludes.

2 The Model

The economy consists of two countries, Home and Foreign, in which there are two industries, agriculture and manufacturing. In the agricultural sector, a homogeneous good is produced under a constant-returns-to-scale technology and perfect competition. In the manufacturing sector, a continuum of firms produce horizontally differentiated goods under imperfect competition. Each differentiated good is produced by one firm, and there is no entry of firms into the manufacturing sector. Each firm is owned equally by all domestic consumers who receive equal shares of all firms’ profits. Labor is the only factor of production, and each consumer supplies one unit of labor inelastically.

The agricultural goods are traded within a country and between countries with no costs of transportation. In the manufacturing sector, by contrast, while no costs are incurred for domestically supplying the goods, overseas shipping of the goods requires a transport service, which causes additional costs. We assume that the consumers pay the transport costs. The transport costs depend on the transport infrastructure so that a larger stock of infrastructure reduces the transport cost per unit of overseas shipment. The investment in the infrastructure is made by national governments that seek to maximize the national welfare. The per-unit transport cost is denoted by \(\tau_r\), where \(r \in \{0, 1, 2\}\) is the number of countries that make an investment on the transport infrastructure. We assume that, without loss of generality, the transport costs become zero if both countries make the infrastructure investment; \(\tau_0 > \tau_1 > \tau_2 = 0\). The fixed cost of investment on the transport infrastructure per country is denoted by \(k > 0\).

The preference of a representative consumer in each country is represented by

\[
u(q(\omega), q_0; \omega \in \Omega) = \int_{\Omega} q(\omega) d\omega - \frac{1-\gamma}{2} \int_{\Omega} q(\omega)^2 d\omega - \frac{\gamma}{2} \left( \int_{\Omega} q(\omega) d\omega \right)^2 + q_0, \tag{1}
\]

where \(\Omega\) is the set of all differentiated goods in the world, \(q(\omega)\) is the consumption of a differentiated good produced by firm \(\omega\), and \(q_0\) is the consumption of the numeraire agricultural good. The parameter \(\gamma \in (0, 1)\) captures the degree of substitutability among manufacturing

7) For example, the Trans-Asian Railway is a project to create an railway network across Europe and Asia, under the leadership of the United Nations Economic and Social Commission for Asia and the Pacific.
8) This preference structure is similar to those in Ottaviano et al. (2002) and in Furusawa and Konishi (2007).
goods; the higher the parameter \( \gamma \), the higher the substitutability among these goods. The consumer in country \( i \) maximizes (1) subject to the following budget constraint:

\[
\int_{\omega} p(\omega) q(\omega) d\omega + \int_{\Omega_i} [p(\omega) + \tau_r] q(\omega) d\omega + q_0 = y_i, \quad i, j = H, F, \quad j \neq i,
\]

where \( \Omega_i \) is the set of differentiated goods produced in country \( i \), \( p(\omega) \) is the price of a differentiated good indexed by \( \omega \), and \( y_i \) denotes the consumer’s income in country \( i \). \( y_i \) consists of wage income \( w_i \), profit shares of the domestic firms minus the lump-sum tax collected by the government for public investment:

\[
y_i = w_i + \frac{S_i}{T} \pi_i - \frac{k}{T},
\]

where \( S_i = |\Omega_i| \) is the measure of firms that produce the differentiated goods in country \( i \) and \( l \) is the number of consumers in each country. Throughout this paper, we normalize the measure of total differentiated goods in the world to one; \( |\Omega| = 1 \).

The representative consumer maximizes (1) subject to (2). From the first-order conditions for utility maximization, we obtain the consumer’s demand function for the domestically produced good \( q_{ii}(\omega) \) and that for the imported good \( q_{ji}(\omega), j \neq i \), in country \( i (i, j = H, F) \) as follows:

\[
q_{ii}(\omega) = \frac{1}{1 - \gamma} \left[ 1 - p_{ii}(\omega) - \gamma (1 - P_i) \right], \quad \text{if } \omega \in \Omega_i, \tag{4}
\]

\[
q_{ji}(\omega) = \frac{1}{1 - \gamma} \left[ 1 - p_{ji}(\omega) - \tau_r - \gamma (1 - P_i) \right], \quad \text{if } \omega \in \Omega_j, \tag{5}
\]

where \( p_{ii}(\omega) \) and \( p_{ji}(\omega) \) are the prices of the domestically produced good and imported good, respectively, in country \( i \), and \( P_i \) is country \( i \)'s price index defined by

\[
P_i = \int_{\omega} p_{ii}(\omega) d\omega + \int_{\Omega_i} [p_{ji}(\omega) + \tau_r] d\omega. \tag{6}
\]

A manufacturing firm producing a good of variety \( \omega \) supplies to both domestic and foreign countries. We assume, without loss of generality, no marginal costs for production in the manufacturing sector. The operating profit of the firm located in country \( i \) is given by

\[
\pi_i(\omega) = p_{ii}(\omega) q_{ii}(\omega) l + p_{ji}(\omega) q_{ji}(\omega) l, \quad i, j = H, F, \quad j \neq i. \tag{7}
\]

In light of the demand functions (4) and (5), taking the price index \( P_i \) as given, each firm maximizes its profit by setting the prices of its product.\(^{10}\) From the first-order conditions for profit

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9) The parameter \( \gamma \) also indicates the degree of product differentiation; the higher value of \( \gamma \) means that the consumer recognizes manufacturing goods as less differentiated. If \( \gamma = 1 \), every manufacturing good is recognized as identical. If, by contrast, \( \gamma = 0 \), the manufacturing goods are perfectly different from one another.

10) The assumption of a continuum of manufacturing firms, implying that our model excludes strategic interaction among manufacturing firms, results in the same equilibrium outcome regardless of price or quantity competition.
maximization, the equilibrium prices of goods produced in country \(i\) are derived as follows:\(^{11})\)

\[
p_{ii} = \frac{1}{2} \left[ 1 - \gamma \left( 1 - \frac{1}{P} \right) \right], \quad p_{ij} = p_{ji} - \frac{\tau_r}{2}, \quad i, j = H, F, \ j \neq i. \tag{8}
\]

### 3 Market Equilibrium and National Welfare

From (6) and (8), the equilibrium prices of the manufacturing good supplied to the domestic market in country \(i\) and the manufacturing good exported to country \(j\), respectively, are derived, as a function of the per unit trade cost, as follows:

\[
p_{ii}(\tau_r) = \frac{1}{2-\gamma} \left( 1 - \gamma + \frac{\gamma S_i \tau_r}{2} \right), \tag{9}
\]

\[
p_{ij}(\tau_r) = \frac{1}{2-\gamma} \left( 1 - \gamma + \frac{\gamma S_i \tau_r}{2} \right) - \frac{\tau_r}{2}. \tag{10}
\]

As mentioned in the previous section, we assume that the homogeneous agricultural good is produced under constant returns and perfect competition. More specifically, we assume that one unit of the agricultural good is produced from one unit of labor. Assuming that the agricultural good produced in both countries and freely traded, wage rates in all countries is equal to one; \(w_H = w_F = 1\).

Solving the budget constraint (2) for \(q_0\) and rearranging terms by using (3) and (7), we obtain the demand for the homogeneous good in country \(i\) as follows:

\[
q_0 = y_i - \left( s_i p_{ij}(\tau_r) q_{ij}(\tau_r) + s_i \left[ p_{ji}(\tau_r) + \tau_r \right] q_{ji}(\tau_r) \right) \frac{s_i}{1-s_i} \frac{p_{ij}(\tau_r)}{(1-\gamma)}, \quad i, j = H, F, \ j \neq i. \tag{11}
\]

where \(q_{ji}(\tau_r) = p_{ji}(\tau_r) / (1-\gamma), \ i, j = H, F, \ j \neq i\). In light of (10), each manufacturing firm has an incentive to make international shipment if the per-unit transport cost is not too high, that is,

\[
\tau_0 < \min \left\{ \frac{2(1-\gamma)}{2-\gamma(1+s_H)}, \frac{2(1-\gamma)}{2-\gamma(1+s_F)} \right\}. \tag{12}
\]

In what follows, we assume that condition (12) is always satisfied.

Substituting (11) into (1), we obtain a representative consumer’s utility in country \(i\), which can be considered as country \(i\)’s national welfare, as a function of the per-unit transport cost \(\tau_r\) and the fixed cost of public investment \(k\):

\[
V_i(\tau_r, k) = U_i(\tau_r) + E X_i(\tau_r) - I M_i(\tau_r) - T C(\tau_r) - \frac{k}{I}, \tag{13}
\]

where

\(^{11})\) The first-order conditions for profit maximization derive the relationship between equilibrium quantities and prices as \(p_{ij} = (1-\gamma) q_{ij}, \ i, j = H, F\).
The functions \( U_i(\tau_r) \), \( EX_i(\tau_r) \), \( IM_i(\tau_r) \), and \( TC_i(\tau_r) \) represent a consumer’s gross utility, the export value of manufacturing goods, import payments, and the payment of transportation costs, respectively. Country \( i \)'s social welfare consists of a consumer’s gross utility \( U_i(\tau_r) \) and the per-capita manufacturing trade surplus \( EX_i(\tau_r) - IM_i(\tau_r) \) minus the transport-cost payments \( TC_i(\tau_r) \).

The effects of a change in the per-unit transport cost are derived as follows:

\[
\frac{\partial U_i(\tau_r)}{\partial \tau_r} = -\frac{s_j}{2-\gamma} \left\{ 1 - \frac{\gamma}{2} + \frac{(2-\gamma)^2 - \gamma^2 s_j \tau_r}{4(1-\gamma)(2-\gamma)} \right\} < 0, \tag{18}
\]

\[
\frac{\partial EX_i(\tau_r)}{\partial \tau_r} = -\frac{s_j}{2(1-\gamma)(2-\gamma)} \left\{ 2(1-\gamma) - [2-\gamma(1+s_i)] \tau_r \right\} < 0, \tag{19}
\]

\[
\frac{\partial IM_i(\tau_r)}{\partial \tau_r} = -\frac{s_j}{2(1-\gamma)(2-\gamma)} \left\{ 2(1-\gamma) - [2-\gamma(1+s_j)] \tau_r \right\} < 0, \tag{20}
\]

\[
\frac{\partial TC_i(\tau_r)}{\partial \tau_r} = \frac{s_j(1-\gamma) - [2-\gamma(1+s_i)] \tau_r}{(2-\gamma)(1-\gamma)}. \tag{21}
\]

Eq. (18) indicates that an increase in the per-unit transport cost reduces the gross utility. Eqs. (19) and (20) show that an increase in \( \tau_r \) also reduces both the export value and import payments.\(^{12}\) However, the sign of (21) is ambiguous because it depends on the sign of \( q_{ji} + \tau_r \frac{\partial q_{ji}}{\partial \tau_r} \) and \( q_{ji} \) is decreasing in \( \tau_r \).\(^{13}\)

4 A Noncooperative Game of Infrastructure Investment

Suppose that the national governments make investment decisions noncooperatively so as to maximize the national welfare (13). We assume that the governments move simultaneously, and each governments’ strategy set is \( \{k, 0\} \), that is, each government determines whether or not to make the infrastructure investment. Therefore, the payoff matrix of this noncooperative

\(^{12}\) Since \( 0 < \gamma < 1 \) and \( 0 \leq s_i \leq 1 \), it holds that \( 2 - (1 + \gamma) s_i > 0 \). Moreover, from condition (12), the expression in the curly braces has a positive sign. Therefore, we have the negative signs of (19) and (20).

\(^{13}\) There is an inverted-U shape relationship between the transport-cost payment the per-unit transport cost similarly to the Laffer curve in public finance.
Table 1: Payoff matrix of the noncooperative game of infrastructure investment

| Country H / F | Yes                  | No                  |
|---------------|----------------------|---------------------|
| Yes           | \( V_H (0, k), V_F (0, k) \) | \( V_H (\tau_1, k), V_F (\tau_1, 0) \) |
| No            | \( V_H (\tau_1, 0), V_F (\tau_1, k) \) | \( V_H (\tau_0, 0), V_F (\tau_0, 0) \) |

The policy game is illustrated by Table 1.

Let us define

\[
\Delta V_A^A (\tau_1) \equiv U_i (0) + EX_i (0) - IM_i (0) - \{ U_i (\tau_1) + EX_i (\tau_1) - IM_i (\tau_1) - TC_i (\tau_1) \},
\]

(22)

\[
\Delta V_B^B (\tau_0, \tau_1) \equiv U_i (\tau_1) + EX_i (\tau_1) - IM_i (\tau_1) - TC_i (\tau_1) - \{ U_i (\tau_0) + EX_i (\tau_0) - IM_i (\tau_0) - TC_i (\tau_0) \},
\]

(23)

\( i = H, F \). \( \Delta V_A^A (\tau_1) \) is country \( i \)'s gross benefit from public investment when country \( j (j \neq i) \) makes public investment, while \( \Delta V_B^B (\tau_0, \tau_1) \) is country \( i \)'s gross benefit from public investment when country \( j \) does not make public investment.

Let us denote “YY” as the pair of strategies when both countries make the public investment, “YN” (“NY”) as the pair of strategies when only country \( H (F) \) makes the investment, and “NN” as the pair of strategies when neither country makes the investment. In light of Table 1 and the expressions (22) and (23), we obtain the following lemma.

**Lemma 1** The Nash equilibrium configurations of the public investment game are characterized as follows.

1. **YY** is the unique Nash equilibrium if either

\[
\min \left\{ \Delta V_H^H (\tau_1), \Delta V_H^H (\tau_0, \tau_1), \Delta V_F^H (\tau_1), \Delta V_F^H (\tau_0, \tau_1) \right\} > k / l,
\]

\[
\min \left\{ \Delta V_H^H (\tau_1), \Delta V_H^H (\tau_0, \tau_1), \Delta V_F^H (\tau_1), \Delta V_F^H (\tau_0, \tau_1) \right\} > k / l > \Delta V_H^H (\tau_0, \tau_1), \text{ or}
\]

\[
\min \left\{ \Delta V_H^H (\tau_1), \Delta V_H^H (\tau_0, \tau_1), \Delta V_F^H (\tau_1), \Delta V_F^H (\tau_0, \tau_1) \right\} > k / l > \Delta V_H^H (\tau_0, \tau_1).
\]

2. **YN** is the unique Nash equilibrium if either

\[
\min \left\{ \Delta V_H^H (\tau_1), \Delta V_H^H (\tau_0, \tau_1), \Delta V_F^H (\tau_0, \tau_1) \right\} > k / l > \Delta V_H^H (\tau_1),
\]

\[
\min \left\{ \Delta V_H^H (\tau_1), \Delta V_H^H (\tau_0, \tau_1), \Delta V_F^H (\tau_1) \right\} > k / l > \max \left\{ \Delta V_H^H (\tau_1), \Delta V_F^H (\tau_0, \tau_1) \right\}, \text{ or}
\]

\[
\max \left\{ \Delta V_H^H (\tau_1), \Delta V_F^H (\tau_1), \Delta V_F^H (\tau_0, \tau_1) \right\} < k / l < \Delta V_H^H (\tau_0, \tau_1).
\]

3. **NY** is the unique Nash equilibrium if either
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\[
\begin{align*}
\text{min} \{\Delta V^H (\tau_0, \tau_1), \Delta V^B (\tau_0, \tau_1), \Delta V^B (\tau_0, \tau_1)\} & > k/l > \Delta V^F (\tau_1), \\
\text{max} \{\Delta V^H (\tau_1), \Delta V^B (\tau_0, \tau_1)\} & < k/l < \text{min} \{\Delta V^H (\tau_1), \Delta V^B (\tau_0, \tau_1)\}, \text{ or} \\
\text{max} \{\Delta V^H (\tau_1), \Delta V^B (\tau_0, \tau_1), \Delta V^B (\tau_0, \tau_1)\} & < k/l < \Delta V^B (\tau_0, \tau_1).
\end{align*}
\]

4. NN is the unique Nash equilibrium if either
\[
\text{max} \{\Delta V^H (\tau_1), \Delta V^B (\tau_0, \tau_1)\} < k/l < \Delta V^B (\tau_0, \tau_1),
\]
\[
\text{max} \{\Delta V^H (\tau_1), \Delta V^B (\tau_0, \tau_1), \Delta V^B (\tau_0, \tau_1)\} < k/l < \Delta V^B (\tau_0, \tau_1), \text{ or}
\]
\[
\text{max} \{\Delta V^H (\tau_1), \Delta V^B (\tau_0, \tau_1), \Delta V^B (\tau_0, \tau_1), \Delta V^B (\tau_0, \tau_1)\} < k/l.
\]

5. There are two Nash equilibria YY and NN if
\[
\text{min} \{\Delta V^H (\tau_1), \Delta V^B (\tau_1)\} > k/l > \text{max} \{\Delta V^B (\tau_0, \tau_1)\}.
\]

6. There are two Nash equilibria YN and NY if
\[
\text{max} \{\Delta V^H (\tau_1), \Delta V^B (\tau_1)\} < k/l < \text{min} \{\Delta V^B (\tau_0, \tau_1)\}.
\]

If either \(\Delta V^H (\tau_1) < k/l < \Delta V^B (\tau_0, \tau_1)\) and \(\Delta V^B (\tau_1) > k/l > \Delta V^B (\tau_0, \tau_1)\), or \(\Delta V^H (\tau_1) > k/l > \Delta V^B (\tau_0, \tau_1)\) and \(\Delta V^B (\tau_1) < k/l < \Delta V^B (\tau_0, \tau_1)\) holds, there is no Nash equilibrium.

\textbf{Proof.} See the Appendix. \(\square\)

In the present model, the strategic variable \(k\) is discrete, i.e., \(k\) is positive or zero, and the number of countries that make infrastructure investment determines the per-unit transport cost. We make the following specification on the technology of infrastructure investment. Let us assume \(\tau_i = \alpha \tau_o\), where \(0 < \alpha < 1\). The parameter \(\alpha\) indicates how the per-unit transport cost can be reduced from the situation where there is no public investment (i.e., \(\tau_0\)) to the situation where either country make infrastructure investment (i.e., \(\tau_i\)); the smaller the value of \(\alpha\), the larger the reduction in the per-unit transport cost. At the same time, \(\alpha\) also indicates the effectiveness of public investment from the situation where either country make infrastructure investment (i.e., \(\tau_i\)) to the situation where both country make investment (i.e., \(\tau_0 = 0\)); the larger the value of \(\alpha\), the greater the reduction in the per-unit transport cost. If \(\alpha < 1/2\), the effectiveness of infrastructure investment in reducing the transport cost becomes smaller as more investment will be made, as illustrated in Figure 1. This situation can be interpreted as the public investment technology exhibiting decreasing returns at an international level. By contrast, if \(\alpha > 1/2\), the more the public investment, the greater the reduction in the transport cost, as illustrated in Figure 2. This is the case where the public investment technology exhibits international increasing returns.

We henceforth assume that the two countries are completely symmetric, with equal measure of firms that produce differentiated goods \((s_H = s_F = 1/2)\). In this case, it holds that \(p_{ij} = p_{ji}\) and \(d_{ij} = d_{ji}\). Therefore, the manufacturing trade surplus is balanced; \(EX_i (\tau_i) = IM_i (\tau_i)\). This means that (13) can be simplified to \(V (\tau_i, k) = U (\tau_i) - TC (\tau_i) - k/l\) and therefore, (22)
Figure 1: International decreasing returns in infrastructure investment ($\alpha < 1/2$)

Figure 2: International increasing returns in infrastructure investment ($\alpha > 1/2$)
and (23) are rewritten as

\[ \Delta V^A (\tau_1) = U(0) - U(\tau_1) + TC(\tau_1), \]
\[ \Delta V^B (\tau_0, \tau_1) = U(\tau_1) - TC(\tau_1) - U(\tau_0) + TC(\tau_0). \]

Lemma 2. If the two countries have the equal share of manufacturing firms, the Nash equilibrium configurations of the public investment game are characterized as follows.

1. YY is the unique Nash equilibrium if \( \min \{ \Delta V^A (\tau_1), \Delta V^B (\tau_0, \tau_1) \} > k/l. \)

2. NN is the unique Nash equilibrium if \( \max \{ \Delta V^A (\tau_1), \Delta V^B (\tau_0, \tau_1) \} < k/l. \)

3. There are two Nash equilibria YY and NN if \( \Delta V^A (\tau_1) > k/l > \Delta V^B (\tau_0, \tau_1). \)

4. There are two Nash equilibria YN and NY if \( \Delta V^A (\tau_1) < k/l < \Delta V^B (\tau_0, \tau_1). \)

Proof. Obvious from Lemma 1. □

In light of (9), (10), (14), and (17), \( \Delta V^A (\tau_1) \) and \( \Delta V^B (\tau_0, \tau_1) \) are explicitly derived as

\[ \Delta V^A (\tau_1) = \frac{\tau_1 [16(1-\gamma)(3-2\gamma) - (11\gamma^2 - 32\gamma + 24)\tau_1]}{32(2-\gamma)^2(1-\gamma)}, \]
\[ \Delta V^B (\tau_0, \tau_1) = \frac{(\tau_0 - \tau_1)[16(1-\gamma)(3-2\gamma) - (11\gamma^2 - 32\gamma + 24)(\tau_0 + \tau_1)]}{32(2-\gamma)^2(1-\gamma)}. \]

From (26), we have

\[ \frac{d\Delta V^A (\tau_1)}{d\tau_1} > (\langle) 0 \iff \tau_1 < (\rangle) \frac{8(1-\gamma)(3-2\gamma)}{11\gamma^2 - 32\gamma + 24}. \]

That is, \( \Delta V^A (\tau_1) \) is hump-shaped in \( \tau_1. \) This is because an increase in \( \tau_r \) augments the transport-cost payment \( TC(\tau_r) \) if \( \tau_r \) is small, but \( TC(\tau_r) \) is decreasing in \( \tau_r \) if \( \tau_r \) is large, as demonstrated by (21). The gross utility \( U(\tau_r) \) is decreasing in \( \tau_r, \) as demonstrated by (18), which means that \( U(0) - U(\tau_1) \) is increasing in \( \tau_1. \) However, for larger rates of per-unit transport cost, \( TC(\tau_1) \) is decreasing in \( \tau_1, \) and so is \( \Delta V^A (\tau_1). \) Substituting \( \tau_0 = \tau_1 / \alpha \) into \( \Delta V^B (\tau_0, \tau_1) \), we obtain the similar relationship between \( \tau_1 \) and \( \Delta V^B (\tau_0, \tau_1 / \alpha, \tau_1), \) although the

14) Since \( U_H (\tau_r) = U_F (\tau_r) \) and \( TC_H (\tau_r) = TC_F (\tau_r) \), we omit the country subscript from these functions.
15) In the symmetric country case, the condition for international trade to be feasible (i.e., (12)) can be rewritten as \( \tau_0 \leq \frac{4(1-\gamma)}{(4-3\gamma)}. \) Even though this condition is satisfied, we cannot rule out the possibility that \( \tau_1 = \alpha \tau_0 > \frac{8(1-\gamma)(3-2\gamma)}{11\gamma^2 - 32\gamma + 24}. \)
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slope of $d\Delta V^B / dt_1$ depends on $\alpha$.

It is clear from (26) and (27) that $\Delta V^A(0) = \Delta V^B(0, 0) = 0$. In addition, it holds that

$$\frac{d\Delta V^A(t_1)}{dt_1} - \frac{d\Delta V^B(t_1 / \alpha, t_1)}{dt_1} = \frac{8\alpha(2\alpha - 1)(1 - \gamma)(3 - 2\gamma) - (2\alpha^2 - 1)(11\gamma^2 - 32\gamma + 24)\tau_1}{16\alpha^2(2 - \gamma)^2(1 - \gamma)}.$$  

(28)

Suppose that $\tau_1 = 0$. Then, the sign of (28) is negative (positive) if $\alpha < \frac{1}{2}(\alpha > \frac{1}{2})$. However, the coefficient of $\tau_1$ may have an opposite sign to the constant term, and if $\alpha < \frac{1}{2}$, the signs are unambiguously opposite. Since both $\Delta V^A(t_1)$ and $\Delta V^B(t_1 / \alpha, t_1)$ are hump-shaped, there can exist $\tilde{t}_1$ such that $\Delta V^A(\tilde{t}_1) = \Delta V^B(\tilde{t}_1 / \alpha, \tilde{t}_1)$. By solving $\Delta V^A(t_1) = \Delta V^B(t_1 / \alpha, t_1)$, we obtain

$$\tilde{t}_1 = \frac{16\alpha(1 - 2\alpha)(1 - \gamma)(3 - 2\gamma)}{(1 - 2\alpha^2)(11\gamma^2 - 32\gamma + 24)}.$$  

(29)

It holds that $\tilde{t}_1 > 0$ if $\alpha < \frac{1}{2}$.

Suppose that the public investment technology exhibits decreasing returns at an international level, i.e., $\alpha < \frac{1}{2}$. From Lemma 2 and the above discussion, the following proposition can be established (see also Figure 3).

**Proposition 1** Suppose that the two countries have the equal share of manufacturing firms. If the public investment technology exhibits international decreasing returns ($\alpha < \frac{1}{2}$), then $\Delta V^A(t_1) < \Delta V^B(t_1 / \alpha, t_1)$ holds for $t_1 < \tilde{t}_1$ and there is a range of the per-capita investment cost $k / l \in \left[\Delta V^A(t_1), \Delta V^B(t_1 / \alpha, t_1)\right]$ such that only one country makes the public investment in equilibrium. If $k / l < \Delta V^A(t_1)$, $YY$ is the unique Nash equilibrium; both countries make the public investment. If $k / l > \Delta V^B(t_1 / \alpha, t_1)$, $NN$ is the unique Nash equilibrium; neither country makes the public investment.

When per-capita investment costs $k / l$ are sufficiently large for given levels of initial trade costs $t_1$ and the technology parameter $\alpha$, the unique equilibrium outcome is $NN$, that is, neither country makes investment on the transport infrastructure. As investment costs become small, the public investment becomes effective in reducing the transport costs, which provides an incentive to make infrastructure investment for the government in each country. However, because the technology of transport infrastructure exhibits international decreasing returns, the advantage of making the infrastructure investment is not so large. Therefore, the incentive to make investment is not very strong unless the per-capita investment cost is sufficiently small. This leads the Nash equilibria $YN$ or $NY$ when $k / l$ is at moderate level.

16) If $\alpha \in \left(\frac{1}{2}, \sqrt{1/2}\right)$, the sign of the coefficient of $\tau_1$ and therefore that of (28) is unambiguously positive.
Next we consider the case where the public investment technology exhibits increasing returns at an international level, i.e., $\frac{\Delta V^A}{\Delta V^B} > \frac{1}{\alpha}$. For international trade to be feasible, $\frac{\Delta V^A}{\Delta V^B} > \frac{1}{\alpha}$ must hold. If this conditions is satisfied, $\Delta V^A(\tau_1) > \Delta V^B(\tau_1 / \alpha, \tau_1)$ holds, as illustrated in Figure 4. From Lemma 2, we obtain the following proposition.

**Proposition 2** Suppose that the two countries have the equal share of manufacturing firms. If the public investment technology exhibits international increasing returns ($\alpha > 1/2$), then $\Delta V^A(\tau_1) > \Delta V^B(\tau_1 / \alpha, \tau_1)$ holds for any given level of $\tau_1$ and there is a range of the per-capita investment cost $k/l \in [\Delta V^B(\tau_1 / \alpha, \tau_1), \Delta V^A(\tau_1)]$ such that there are two Nash equilibria YY and NN. If $k/l < \Delta V^B(\tau_1 / \alpha, \tau_1)$, YY is the unique Nash equilibrium; both countries make the public investment. If $k/l > \Delta V^A(\tau_1)$, NN is the unique Nash equilibrium; neither country makes the public investment.

17) We emphasize the difference in the technologies of infrastructure investment; that is, the difference between the case of decreasing returns and the case of increasing returns. As a matter of fact, if we assume constant returns in infrastructure investment (i.e., $\alpha = 1/2$), we can verify that Proposition 2 holds. In the case of constant returns, $\tau_0 = 2\tau_1$ holds, and substituting this into (27) yields

$$\Delta V^A(2\tau_1, \tau_1) = \tau_1 \left[ \frac{16(1-\gamma)(3-2\gamma)-3(11\gamma^2-32\gamma + 24)\tau_1}{32(2-\gamma)^3(1-\gamma)} \right]$$

and thus, $\Delta V^A(\tau_1) > \Delta V^B(2\tau_1, \tau_1)$ is always satisfied.
Contrary to the case of international decreasing returns, in the presence of international increasing returns of infrastructure investment, there is no such equilibrium that only one of the countries makes the public investment. Instead, there are multiple Nash equilibria, \( YY \) and \( NN \), if \( k/l \) is moderate for a given level of \( \tau_1 \). The mechanism is similar to that of “big push” models of industrialization introduced by Rosenstein-Rodan (1943) and elaborated by Murphy et al. (1989), where various sectors of the economy adopt increasing returns technologies simultaneously and they can each create income that becomes a source of demand for goods in other sectors, and therefore simultaneous industrialization of many sectors can enlarge their markets and make industrialization profitable even if no sector could break even industrializing alone. In a closed economy framework, Murphy et al. (1989, Section VI) associate the big push with multiple equilibria of an economy making large investments in a shared infrastructure. Our model can be interpreted as an open-economy counterpart of the big push model.

### 5 Efficiency of Policy Game Equilibria

Are the Nash equilibrium patterns of public investment efficient? To address this issue, we consider a cooperative solution in which the governments in both countries jointly make an investment decision to maximize the sum of national welfare \( V_H(\tau, k) + V_F(\tau, k) \).

Because the two countries are assumed to be symmetric, there are three possible outcomes. The first outcome is where both countries make public investment (\( YY \)) and the sum of their national welfare is \( V_H(0, k) + V_F(0, k) = 2V(0, k) \). The second outcome is where only one of these countries makes the investment (\( YN \) or \( NY \)) and the sum of welfare is \( V(\tau_1, k) + V(\tau_1, 0) \). The third outcome is where neither country makes the investment and the sum of welfare is \( V_H(\tau_0, 0) + V_F(\tau_0, 0) = 2V(\tau_0, 0) \).
Let us begin with a comparison between \( YY \) and \( YN \) (or \( NY \)). Since
\[
2V(0,k) - \{V(\tau_1,k) + V(\tau_1,0)\} = 2\left\{U(0) - \frac{k}{I}\right\} - \left\{2U(\tau_1) - 2TC(\tau_1) - \frac{k}{I}\right\}
\]
\[= 2\Delta V^A(\tau_1) - \frac{k}{I}, \tag{30}\]
the governments find it optimal to invest on infrastructure by both countries (one country) if
\[2\Delta V^A(\tau_1) > k/I \left(2\Delta V^A(\tau_1) < k/I\right).\]

Next we compare \( YY \) and \( NN \). Since
\[
2V(0,k) - 2V(\tau_0,0) = 2\left\{U(0) - \frac{k}{I}\right\} - 2\{U(\tau_0) - TC(\tau_0)\}
\]
\[= 2\left\{\Delta V^A(\tau_0) - \frac{k}{I}\right\}, \tag{31}\]
both countries will (will not) make investment if \( \Delta V^A(\tau_0) > k/I \left(\Delta V^A(\tau_0) < k/I\right).\)

Finally, let us compare \( YN \) (or \( NY \)) and \( NN \). Since
\[
V(\tau_1,k) + V(\tau_1,0) - 2V(\tau_0,0) = 2U(\tau_1) - 2TC(\tau_1) - \frac{k}{I} - 2\{U(\tau_0) - TC(\tau_0)\}
\]
\[= 2\Delta V^B(\tau_0,\tau_1) - \frac{k}{I}, \tag{32}\]
the governments choose infrastructure investment in one country if \( 2\Delta V^B(\tau_0,\tau_1) > k/I \) while they choose no investment if \( 2\Delta V^B(\tau_0,\tau_1) < k/I. \)

**Lemma 3** Suppose that the two countries have the equal share of manufacturing firms. Then, in the cooperative solution, the optimal investment pattern is always unique for a given parameter configuration. The optimal solutions are characterized as follows.

1. Both countries make infrastructure investment if \( \min\{\Delta V^A(\tau_0), 2\Delta V^A(\tau_1)\} > k/I. \)
2. Only one country makes infrastructure investment if \( 2\Delta V^B(\tau_0,\tau_1) > k/I > 2\Delta V^A(\tau_1) \).
3. Neither country makes infrastructure investment if \( \max\{\Delta V^A(\tau_0), 2\Delta V^B(\tau_0,\tau_1)\} < k/I. \)

**Proof.** Both countries will make infrastructure investment if (1i) \( YY \succ YN \) and (1ii) \( YY \succ NY \).

In light of (30) and (31), these conditions are met if \( \min\{\Delta V^A(\tau_0), 2\Delta V^A(\tau_1)\} > k/I. \)

Infrastructure investment will be conducted by only one country if (2i) \( YN \succ YY \) and (2ii) \( YN \succ NN \). In light of (30) and (32), these conditions hold if \( 2\Delta V^B(\tau_0,\tau_1) > k/I > 2\Delta V^A(\tau_1). \)

Both countries will not make investment if (3i) \( NN \succ YY \) and (3ii) \( NN \succ YN \). In light of (31) and (32), these conditions are satisfied if \( \max\{\Delta V^A(\tau_0), 2\Delta V^B(\tau_0,\tau_1)\} < k/I. \)

Let us further examine the properties of cooperative solution. We begin with the case
where the technology of infrastructure investment exhibits international decreasing returns, i.e.,
\( \alpha < 1/2 \). In this case, \( 2\Delta V^{A}(\tau_{1}) \Delta V^{A}(\tau_{0}) < 2\Delta V^{B}(\tau_{1} / \alpha, \tau_{1}) \) holds for \( \tau_{1} < \bar{\tau}_{1} \) (see Figure 5). Therefore, Lemma 3 can be simplified as follows.

**Proposition 3** Suppose that the two countries have the equal share of manufacturing firms and the two countries cooperatively determine their public investment decisions. In the case where the technology of infrastructure investment exhibits international decreasing returns \( (\alpha < 1/2) \) and where \( \tau_{1} < \bar{\tau}_{1} \), the optimal solutions are as follows.

1. **Both countries make infrastructure investment if** \( 2\Delta V^{A}(\tau_{1}) > k/l \).

2. **Only one country makes infrastructure investment if** \( 2\Delta V^{A}(\tau_{1}) < k/l < 2\Delta V^{B}(\tau_{1} / \alpha, \tau_{1}) \).

3. **Neither country makes infrastructure investment if** \( 2\Delta V^{B}(\tau_{1} / \alpha, \tau_{1}) < k/l \).

Comparing Proposition 3 with Proposition 1, we can verify that if \( \Delta V^{B}(\tau_{1} / \alpha, \tau_{1}) < k/l < 2\Delta V^{B}(\tau_{1} / \alpha, \tau_{1}) \), the Nash equilibrium outcome is no investment while the cooperative

![Figure 5: Cooperative solution in the case of international decreasing returns](image)

18) By definition, the \( 2\Delta V^{A}(\tau_{1}) \) curve and the \( 2\Delta V^{B}(\tau_{1} / \alpha, \tau_{1}) \) curve intersect at \( \tau_{1} = \bar{\tau}_{1} \). Moreover, by solving \( 2\Delta V^{A}(\tau_{1}) = \Delta V^{A}(\tau_{1} / \alpha) \) for \( \tau_{1} \), we obtain \( \tau_{1} = 16\alpha(1-2\alpha)(1-\gamma)(3-2\gamma) / \left[ \left( 1-2\alpha^{2} \right) \left( 11\gamma^{2}-32\gamma+24 \right) \right] \), which is equal to \( \bar{\tau}_{1} \). Thus, the \( \Delta V^{A}(\tau_{1} / \alpha) \) curve also passes through the intersection of the \( 2\Delta V^{A}(\tau_{1}) \) and \( 2\Delta V^{B}(\tau_{1} / \alpha, \tau_{1}) \) curves.
solution is that one of countries makes infrastructure investment. Therefore, the Nash equilibrium is inefficient in the sense that when the investment costs are moderate, the socially optimal public investment cannot be achieved in the Nash equilibrium.

We next consider the case where the technology of infrastructure investment exhibits international increasing returns ($\alpha > 1/2$). In this case, $2\Delta V^A(\tau_1) > 2\Delta V^B(\tau_1 / \alpha, \tau_1)$ holds (see Figure 6). Therefore, from Lemma 3 we obtain the following proposition.

**Proposition 4** Suppose that the two countries have the equal share of manufacturing firms and the two countries cooperatively determine their public investment decisions. In the case where the technology of infrastructure investment exhibits international increasing returns ($\alpha > 1/2$), it is optimal that both countries make infrastructure investment if $\Delta V^A(\tau_1 / \alpha) > k/l$ while neither country makes investment if $\Delta V^A(\tau_1 / \alpha) < k/l$.

Comparing Proposition 4 with Proposition 2, it follows that if $k/l \in \left[\Delta V^B(\tau_1 / \alpha, \tau_1), \Delta V^A(\tau_1)\right]$, there are two Nash equilibria $YY$ and $NN$, although $YY$ is the unique outcome when the two countries cooperate. In this case, the Nash equilibrium is inefficient because we cannot rule out the possibility of “no investment” despite it is definitely optimal to make investment. The inefficiency of Nash equilibrium in the case of international increasing returns is characterized as a “coordination failure”.

![Figure 6: Cooperative solution in the case of international increasing returns](image)
6 Concluding Remarks

This paper developed an intraindustry trade model where overseas shipping incurs transport costs and national governments invest in the transport infrastructure to reduce the transport costs. The findings from our analysis are as follows. If the public investment technology exhibits decreasing returns (DR) internationally, the public investment game has a unique Nash equilibrium, and depending on the level of fixed investment cost, the equilibrium outcome is either both countries making the infrastructure investment, no investment, or only one of the countries making the investment. By contrast, if the public investment technology exhibits increasing returns (IR) internationally, there can be multiple Nash equilibria consisting of both countries making the investment and no investment. That is, in this case, coordination problems may occur. The Nash equilibrium is also shown to be inefficient, but the sources of inefficiency differ between DR and IR. In the DR case, the Nash equilibrium is inefficient in the sense that no investment can be an equilibrium outcome even when the cooperative solution is one of countries making investment. In the IR case, the Nash equilibrium is inefficient in the sense that no-investment outcome cannot be ruled out even when it is socially optimal to make investment.

To highlight the properties of noncooperative equilibrium of infrastructure investment game between countries, we made some simplifying assumptions. For instance, we assumed that investment choice for each government was binary (i.e., to invest or not) and thus, the relationship between the per-unit transport cost and investment level was discrete. Assuming the transport costs as a continuous function of investment levels will enable us to conduct richer analysis. We also assumed a quasi-linear preference, which means that there is no income effect. However, income effects play an important role in infrastructure investment in the presence of industry agglomeration, as explored by Martin and Rogers (1995). In our model, the mass of firms in each country was exogenously given and thus, we did not consider firms’ location choice. Taking account of endogenous location choice will be an important topic for further research. Finally, we assumed free trade and focused on infrastructure investment in the absence of impediments to trade other than transport costs. By introducing tariffs or other forms of trade barriers, we will be able to discuss the relationship between trade and investment policies. These extensions are left for future research.19)

Appendix: Proof of Lemma 1

In light of Table 1, we can verify that there are three situations under which \( YY \) becomes the unique Nash equilibrium. The first is where \( V_H(0,k) > V_H(\tau_1,0) \), \( V_H(\tau_1,k) > V_H(\tau_0,0) \),

19) The actual form of infrastructure investments is more complex than the model considered in this paper. For example, the Greater Mekong Subregion (GMS) including Cambodia, Laos, Myanmar, Thailand, and Vietnam has attracted attention as the region with high growth potential, and a great amount of infrastructure investment has been made in the GMS. However, the infrastructure spendings in the GMS are largely financed in the form of Official Development Aid from bilateral donors such as Japan International Cooperation Agency (JICA) and multilateral donors such as the Asian Development Bank (ADB). The analysis of infrastructure investment game incorporating a variety of financing models is another direction of future research.
\( V_F(0,k) > V_F(\tau_1,0) \), and \( V_F(\tau_1,k) > V_F(\tau_0,0) \). These conditions are equivalent to \( \Delta V_H^A(\tau_1) > k/l \), \( \Delta V_H^B(\tau_0,\tau_1) > k/l \), \( \Delta V_P^A(\tau_1) > k/l \), and \( \Delta V_P^B(\tau_0,\tau_1) > k/l \), respectively. To sum up these conditions, we obtain the condition

\[
\min\left\{ \Delta V_H^A(\tau_1), \Delta V_H^B(\tau_0,\tau_1), \Delta V_P^A(\tau_1), \Delta V_P^B(\tau_0,\tau_1) \right\} > k/l.
\]

The second situation is where \( V_H(0,k) > V_H(\tau_1,0) \), \( V_H(\tau_1,k) > V_H(\tau_0,0) \), \( V_F(0,k) > V_F(\tau_1,0) \), and \( V_F(\tau_1,k) < V_F(\tau_0,0) \), that is, \( \Delta V_H^A(\tau_1) > k/l \), \( \Delta V_H^B(\tau_0,\tau_1) > k/l \), \( \Delta V_P^A(\tau_1) > k/l \), and \( \Delta V_P^B(\tau_0,\tau_1) < k/l \). These conditions are summarized to

\[
\min\left\{ \Delta V_H^A(\tau_1), \Delta V_H^B(\tau_0,\tau_1), \Delta V_P^A(\tau_1), \Delta V_P^B(\tau_0,\tau_1) \right\} > k/l > \Delta V_P^B(\tau_0,\tau_1).
\]

The last situation is where \( V_H(0,k) > V_H(\tau_1,0) \), \( V_H(\tau_1,k) < V_H(\tau_0,0) \), \( V_F(0,k) > V_F(\tau_1,0) \), and \( V_F(\tau_1,k) > V_F(\tau_0,0) \), that is, \( \Delta V_H^A(\tau_1) > k/l \), \( \Delta V_H^B(\tau_0,\tau_1) < k/l \), \( \Delta V_P^A(\tau_1) > k/l \), and \( \Delta V_P^B(\tau_0,\tau_1) > k/l \). These conditions are summarized to

\[
\min\left\{ \Delta V_H^A(\tau_1), \Delta V_H^B(\tau_0,\tau_1), \Delta V_P^A(\tau_0,\tau_1) \right\} > k/l > \Delta V_P^B(\tau_0,\tau_1).
\]

Next consider the situations under which \( NY \) becomes the unique Nash equilibrium, there are three situations, which are:

1. \( V_H(0,k) > V_H(\tau_1,0), V_H(\tau_1,k) > V_H(\tau_0,0), V_F(0,k) < V_F(\tau_1,0), \) and \( V_F(\tau_1,k) > V_F(\tau_0,0) \), each of which is equivalent to \( \Delta V_H^A(\tau_1) > k/l \), \( \Delta V_H^B(\tau_0,\tau_1) > k/l \), \( \Delta V_P^A(\tau_1) < k/l \), and \( \Delta V_P^B(\tau_0,\tau_1) > k/l \). These conditions are summarized as

\[
\min\left\{ \Delta V_H^A(\tau_1), \Delta V_H^B(\tau_0,\tau_1), \Delta V_P^A(\tau_0,\tau_1) \right\} > k/l > \Delta V_P^A(\tau_1).
\]

2. \( V_H(0,k) > V_H(\tau_1,0), V_H(\tau_1,k) > V_H(\tau_0,0), V_F(0,k) < V_F(\tau_1,0), \) and \( V_F(\tau_1,k) < V_F(\tau_0,0) \), each of which is equivalent to \( \Delta V_H^A(\tau_1) > k/l \), \( \Delta V_H^B(\tau_0,\tau_1) > k/l \), \( \Delta V_P^A(\tau_1) < k/l \), and \( \Delta V_P^B(\tau_0,\tau_1) < k/l \). These conditions are summarized as

\[
\min\left\{ \Delta V_H^A(\tau_1), \Delta V_H^B(\tau_0,\tau_1) \right\} > k/l > \max\left\{ \Delta V_P^A(\tau_1), \Delta V_P^B(\tau_0,\tau_1) \right\}.
\]

3. \( V_H(0,k) < V_H(\tau_1,0), V_H(\tau_1,k) > V_H(\tau_0,0), V_F(0,k) < V_F(\tau_1,0), \) and \( V_F(\tau_1,k) < V_F(\tau_0,0) \), each of which is equivalent to \( \Delta V_H^A(\tau_1) < k/l \), \( \Delta V_H^B(\tau_0,\tau_1) > k/l \), \( \Delta V_P^A(\tau_1) < k/l \), and \( \Delta V_P^B(\tau_0,\tau_1) < k/l \). These conditions are summarized as

\[
\max\left\{ \Delta V_H^A(\tau_1), \Delta V_H^B(\tau_0,\tau_1), \Delta V_P^B(\tau_0,\tau_1) \right\} < k/l < \Delta V_P^B(\tau_0,\tau_1).
\]

As for the situations under which \( N\!Y \) becomes the unique Nash equilibrium, there are
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three situations, which are:

1. \( V_H(0,k) < V_H(\tau_1,0), V_H(\tau_1,k) > V_H(\tau_0,0), V_F(0,k) > V_F(\tau_1,0), \) and \( V_F(\tau_1,k) > V_F(\tau_0,0), \) each of which is equivalent to \( \Delta V_H^A(\tau_1) < k/l, \Delta V_H^B(\tau_0,\tau_1) > k/l, \Delta V_F^A(\tau_1) > k/l, \) and \( \Delta V_F^B(\tau_0,\tau_1) > k/l. \) These conditions are summarized as

\[
\min\{\Delta V_H^B(\tau_0,\tau_1), \Delta V_F^A(\tau_1), \Delta V_F^B(\tau_0,\tau_1)\} > k/l > \Delta V_H^A(\tau_1).
\]

2. \( V_H(0,k) < V_H(\tau_1,0), V_H(\tau_1,k) < V_H(\tau_0,0), V_F(0,k) > V_F(\tau_1,0), \) and \( V_F(\tau_1,k) > V_F(\tau_0,0), \) each of which is equivalent to \( \Delta V_H^B(\tau_1) < k/l, \Delta V_H^B(\tau_0,\tau_1) < k/l, \Delta V_F^A(\tau_1) > k/l, \) and \( \Delta V_F^B(\tau_0,\tau_1) > k/l. \) These conditions are summarized as

\[
\max\{\Delta V_H^A(\tau_1), \Delta V_H^B(\tau_0,\tau_1), \Delta V_F^B(\tau_0,\tau_1)\} < k/l < \min\{\Delta V_F^A(\tau_1), \Delta V_F^B(\tau_0,\tau_1)\}.
\]

3. \( V_H(0,k) < V_H(\tau_1,0), V_H(\tau_1,k) < V_H(\tau_0,0), V_F(0,k) < V_F(\tau_1,0), \) and \( V_F(\tau_1,k) > V_F(\tau_0,0), \) each of which is equivalent to \( \Delta V_H^A(\tau_1) < k/l, \Delta V_H^B(\tau_0,\tau_1) < k/l, \Delta V_F^A(\tau_1) < k/l, \) and \( \Delta V_F^B(\tau_0,\tau_1) > k/l. \) These conditions are summarized as

\[
\max\{\Delta V_H^A(\tau_1), \Delta V_H^B(\tau_0,\tau_1), \Delta V_F^A(\tau_1)\} < k/l < \min\{\Delta V_F^A(\tau_1), \Delta V_F^B(\tau_0,\tau_1)\}.
\]

The last possibility for the unique Nash equilibrium is the outcome NN, which also has three situations:

1. \( V_H(0,k) < V_H(\tau_1,0), V_H(\tau_1,k) > V_H(\tau_0,0), V_F(0,k) < V_F(\tau_1,0), \) and \( V_F(\tau_1,k) < V_F(\tau_0,0), \) each of which is equivalent to \( \Delta V_H^B(\tau_1) < k/l, \Delta V_H^B(\tau_0,\tau_1) > k/l, \Delta V_F^A(\tau_1) < k/l, \) and \( \Delta V_F^B(\tau_0,\tau_1) < k/l. \) These conditions are summarized as

\[
\max\{\Delta V_H^A(\tau_1), \Delta V_F^A(\tau_1), \Delta V_F^B(\tau_0,\tau_1)\} < k/l < \min\{\Delta V_F^A(\tau_1), \Delta V_F^B(\tau_0,\tau_1)\}.
\]

2. \( V_H(0,k) < V_H(\tau_1,0), V_H(\tau_1,k) < V_H(\tau_0,0), V_F(0,k) > V_F(\tau_1,0), \) and \( V_F(\tau_1,k) < V_F(\tau_0,0), \) each of which is equivalent to \( \Delta V_H^A(\tau_1) < k/l, \Delta V_H^B(\tau_0,\tau_1) < k/l, \Delta V_F^A(\tau_1) > k/l, \) and \( \Delta V_F^B(\tau_0,\tau_1) < k/l. \) These conditions are summarized as

\[
\max\{\Delta V_H^A(\tau_1), \Delta V_H^B(\tau_0,\tau_1), \Delta V_F^B(\tau_0,\tau_1)\} < k/l < \min\{\Delta V_F^A(\tau_1), \Delta V_F^B(\tau_0,\tau_1)\}.
\]

3. \( V_H(0,k) < V_H(\tau_1,0), V_H(\tau_1,k) < V_H(\tau_0,0), V_F(0,k) < V_F(\tau_1,0), \) and \( V_F(\tau_1,k) < V_F(\tau_0,0), \) each of which is equivalent to \( \Delta V_H^A(\tau_1) < k/l, \Delta V_H^B(\tau_0,\tau_1) < k/l, \Delta V_F^A(\tau_1) < k/l, \) and \( \Delta V_F^B(\tau_0,\tau_1) < k/l. \) These conditions are summarized as
\[
\max \{\Delta V_H^A (\tau_1), \Delta V_H^B (\tau_0, \tau_1), \Delta V_F^A (\tau_1), \Delta V_F^B (\tau_0, \tau_1)\} < k / l.
\]

If \(V_H (0, k) > V_H (\tau_1, 0), V_H (\tau_1, k) < V_H (\tau_0, 0), V_F (0, k) > V_F (\tau_1, 0), \) and \(V_F (\tau_1, k) < V_F (\tau_0, 0),\) Table 1 reveals that \(YY\) and \(NN\) are Nash equilibrium outcomes. These conditions are equivalent to \(\Delta V_H^A (\tau_1) > k / l, \Delta V_H^B (\tau_0, \tau_1) < k / l, \Delta V_F^A (\tau_1) > k / l,\) and \(\Delta V_F^B (\tau_0, \tau_1) > k / l,\) respectively, and summarized as

\[
\min \{\Delta V_H^A (\tau_1), \Delta V_F^A (\tau_1)\} > k / l > \max \{\Delta V_H^B (\tau_0, \tau_1), \Delta V_F^B (\tau_0, \tau_1)\}.
\]

Finally, it can be verified that \(YN\) and \(NY\) are Nash equilibrium outcomes if \(V_H (0, k) < V_H (\tau_1, 0), V_H (\tau_1, k) > V_H (\tau_0, 0), V_F (0, k) < V_F (\tau_1, 0), \) and \(V_F (\tau_1, k) > V_F (\tau_0, 0),\) each of which is equivalent to \(\Delta V_H^A (\tau_1) < k / l, \Delta V_H^B (\tau_0, \tau_1) < k / l, \Delta V_F^A (\tau_1) < k / l,\) and \(\Delta V_F^B (\tau_0, \tau_1) > k / l.\) These conditions are summarized to

\[
\max \{\Delta V_H^A (\tau_1), \Delta V_F^A (\tau_1)\} < k / l < \min \{\Delta V_H^B (\tau_0, \tau_1), \Delta V_F^B (\tau_0, \tau_1)\}.
\]

For other sets of inequalities than those described above, there is no Nash equilibrium. □
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