Nuclear processes in magnetic fusion reactors with polarized fuel

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Abstract

We consider the processes $d + d \rightarrow n + ^3He$, $d + ^3He \rightarrow p + ^4He$, $d + ^3H \rightarrow n + ^4He$, $^3He + ^3He \rightarrow p + p + ^4He$, $^3H + ^3He \rightarrow d + ^4He$, with particular attention for applications in fusion reactors. After a model independent parametrization of the spin structure of the matrix elements for these processes at thermal colliding energies, in terms of partial amplitudes, we study polarization phenomena in the framework of a formalism of helicity amplitudes. The strong angular dependence of the final nuclei and of the polarization observables on the polarizations of the fuel components can be helpful in the design of the reactor shielding, blanket arrangement etc. We analyze also the angular dependence of the neutron polarization for the processes $\vec{d} + \vec{d} \rightarrow n + ^3He$ and $\vec{d} + ^3\bar{H} \rightarrow n + ^4He$.
I. INTRODUCTION

Nuclear fusion reactions, like $d + d \rightarrow n + ^3He$, or $d + ^3H \rightarrow n + ^4He$, are characterized by a large dependence on the spins of the colliding particles. It has been suggested \[\text{[1]}\] to use this property in magnetic fusion reactors with polarized nuclear fuel. A magnetic field of about 1 kG can keep the necessary direction of the polarization of the interacting nuclei, during a time which is longer in comparison with the reaction time. Different technical solutions might be used: injection of polarized frozen pellets, or polarized targets for inertial fusion.

The strong dependence of the fusion reaction rates on the polarization states results in an increasing or a decreasing of the cross section (with respect to the unpolarized case), depending on the colliding nuclei polarization directions. These characteristics can be used to optimize a fusion reactor in different ways:

- The possible enhancement of the fusion rates for $\vec{d} + ^3\vec{H}e$ and the suppression of $\vec{d} + \vec{d}$-collisions would make this fuel competitive with $d + ^3H$, as it would, in particular, result in a clean reactor.
- The strong anistropy of the neutron angular dependence in $\vec{d} + ^3\vec{H}$-collision helps in optimizing the reactor shielding and the blanket design.
- $\vec{d} + ^3\vec{H}$-collisions can be source of intensive monochromatic polarized neutrons, with the choice of the polarization direction.

A precise knowledge of the spin structure of the threshold matrix elements for the processes induced by: $d + ^3He$, $d + ^3H$, $^3He + ^3He$, $^3H + ^3He$-collisions is required. At energies up to 10 keV, which are typical for fusion reactors, the S-state interaction of the colliding particles dominates and the general analysis of polarization phenomena is essentially simplified.

For this particular low energy domain, the general analysis of polarization phenomena \[\text{[2]}\] (in terms of scalar or helicity amplitudes) does not seem adequate, as it does not include the simplified characteristics of threshold regime. The main reason is that, at threshold,
there is only one physical direction: the 3-momentum of the produced particles. Therefore, the parametrization of the corresponding matrix elements has to be done on the basis of one direction. It can not be derived as a limiting case of a general approach based on two independent directions, the 3-momenta of the initial and final particles, which define the reaction plane. For S-state collisions we can not define this plane, having only one direction. The analysis of threshold polarization phenomena is also simpler and requires a dedicated parametrization, which is not a limiting case of a general parametrization.

Note that, in this connection, the spin structure for the threshold kinematics is equivalent to (and sometimes simpler than) the case of collinear kinematics.

Our aim is to analyze here in the most general and complete form the reactions relevant to magnetic fusion reactors, with polarized fuel. Following the line of our previous paper \[3\], where we considered the reaction \( d + ^3He \rightarrow p + ^4He \), we will give here the general parametrization of the threshold amplitudes for \( d + ^3He \), \( d + d \) and \( ^3He + ^3He \)-collisions, with special attention to the angular distribution of the reaction products for different possible polarization states of the colliding particles, without any particular assumption about the reaction mechanism. For this aim we develop a formalism for the parametrization of the spin structure of the threshold matrix elements in terms of the small number of allowed partial amplitudes, taking into account the most general properties of the strong interaction.

In a fusion reactor the reaction rates and the angular distributions depend on the direction of the magnetic field. We use in this analysis a particular set of helicity amplitudes, with quantization axis along the direction of the magnetic field. (The effects of the magnetic field were not discussed in our previous paper \[3\]).

The paper is organized as follows. In section 2 on the basis of some of our previous results concerning the reaction \( d + ^3H( ^3He) \rightarrow n(p) + ^4He \), we derive the angular dependence of the differential cross sections for different polarization states of the colliding particles, and the angular dependence of the polarization of the produced neutrons (protons).

The process \( d + d \rightarrow n + ^3He \), discussed in Section 3, is characterized by a set of three independent partial amplitudes (for S-state \( dd \)-collisions). We derive the limits of the integral
coefficients for polarized particles collisions, which quantify the change of the cross section due to the polarizations of the colliding particles. These coefficients depend only on the ratio of the square of the partial amplitudes.

Section 4 contains the discussion of the properties of some processes induced by $^3He + ^3He$, $^3H + ^3H$, and $^3H + ^3He$-collisions. The Pauli principle (for colliding or produced identical fermions), essentially simplifies the spin structure of these reactions, and it is possible in some cases to give model-independent predictions for polarization phenomena.

II. THE COMPLETE EXPERIMENT FOR THE REACTION

\[ d + ^3H(\bar{He}) \rightarrow n(p) + ^4He \]

A. Introductory remarks

The reaction $d + ^3H \rightarrow n + ^4He$ in the near threshold region is very interesting for the production of thermonuclear energy and plays an important role in primordial nucleosynthesis. The lowest $^3_2^+ \text{ level of } ^5He$ has excitation energy $E_x = 16.75 \text{ MeV}$ (only 50 keV above $d + ^3H$-threshold) and has a width of 76 keV.

The microscopic explanation of the nature and the properties of this resonance is very complicated and still under debate in the physics of light nuclei. The interpretation [4] of this resonance as a shadow pole [5] introduces a new concept in nuclear physics, after atomic and particle physics. The possibility that the corresponding shadow poles for the two charge symmetric systems $d + ^3He$ and $d + ^3H$ (or $p + ^4He$ and $n + ^4He$) occupy different Riemann sheets, due to the difference in electric charges of the participating particles, can not be presently ruled out. Such phenomena can be considered as a new mechanism of violation of isotopic invariance of the strong interaction [6].

Due to the close connection of the three processes $d + ^3He \rightarrow d + ^3He$, $n + ^4He \rightarrow n + ^4He$ and $d + ^3H \rightarrow n + ^4He$ through the unitarity condition, the partial wave analysis [7,8] can not be performed independently for each reaction. The corresponding amplitudes are complex.
functions of the excitation energy. The multilevel \( R \)-matrix approach allows to parametrize this dependence in terms of few parameters as shift, penetration factors and hard-sphere phase shift \[9\]. All characteristics of the \( J^P = \frac{3}{2}^+ \)-resonance, like the position, the width and particularly the Riemann sheet, can be found using an \( S \)-matrix approach \[4,10–12\].

The polarization phenomena are very important in the near threshold region, even for the S-state interaction. In this respect the reaction \( d + ^3H \rightarrow n + ^4He \) plays a special role, because the presence of a D-wave in the final state results in nonzero one-spin polarization observables, such as, for example, the tensor analyzing power. In order to fully determine the two possible threshold (complex) amplitudes, two-spin polarization observables have to be measured, for example in collisions of polarized deuteron with polarized \( ^3He \)-target. Here we will generalize our previous analysis \[9\], taking into account the presence of a magnetic field, which is necessary in order to conserve the polarization of the fuel constituents in a magnetic fusion reactor \[4\].

For very small colliding energies the analysis of polarization phenomena for the reaction \( d + ^3H \rightarrow n + ^4He \) can be carried out in a general form. In the framework of a formalism, based on the polarized structure functions, we will point out the observables which have to be measured in order to have a full reconstruction of the spin structure of the threshold amplitudes. Data on cross section and tensor analyzing power exist, at threshold \[13\] (for a review see \[14\]). Among the two-spin observables, the measurement of a spin correlation coefficient, together with the cross section and the tensor analyzing power, allows to realize the complete experiment.

B. Spin structure of the matrix element

Let us first establish the spin structure of the matrix element. From the P-invariance of the strong interaction and the conservation of the total angular momentum, two partial transitions, for \( d + ^3He \rightarrow p + ^4He \) (as well as for \( d + ^3H \rightarrow n + ^4He \)) are allowed:

\[
S_i = \frac{1}{2} \rightarrow J^P = \frac{1}{2}^+ \rightarrow \ell_f = 0,
\]
where $S_i$ is the total spin of the $d + ^3\text{He}$-system and $\ell_f$ is the orbital angular momentum of the final proton. The spin structure of the threshold matrix element can be parametrized in the form:

\[ \mathcal{M} = \chi_2^\dagger \mathcal{F}_{th} \chi_1, \]

\[ \mathcal{F}_{th} = g_s \vec{\sigma} \cdot \vec{D} + g_d (3 \vec{k} \cdot \vec{D} \vec{\sigma} \cdot \vec{k} - \vec{\sigma} \cdot \vec{D}), \]

where $\chi_1$ and $\chi_2$ are the two component spinors of the initial $^3\text{He}$ and final $p$, $\vec{D}$ is the 3-vector of the deuteron polarization (more exactly, $\vec{D}$ is the axial vector due to the positive parity of the deuteron), $\vec{k}$ is the unit vector along the 3-momenta of the proton (in the CMS of the considered reaction) and $g_s$ and $g_d$ are the amplitudes of the $S$- and $D$- production of the final particles, and they are complex functions of the excitation energy. Note that, in the general case, the spin structure of the matrix element, for the considered processes, contains six different contributions and the corresponding amplitudes are functions of two variables.

The general parametrization of the differential cross section in terms of the polarizations of the colliding particles (in S-state) is given by:

\[
\frac{d\sigma}{d\Omega}(d + ^3\text{He}) = \left( \frac{d\sigma}{d\Omega} \right)_0 [ 1 + A_1 (Q_{ab}k_a k_b) + A_2 \vec{S} \cdot \vec{P} + A_3 \vec{k} \cdot \vec{P} \vec{k} \cdot \vec{S} + A_4 \vec{k} \cdot \vec{P} \times \vec{Q} ], \quad Q_a = Q_{ab} k_b, \]

where $(d\sigma/d\Omega)_0$ is the differential cross section with unpolarized particles, $\vec{P}$ is the axial vector of the target ($^3\text{He}$) polarization, $\vec{S}$ and $Q_{ab}$ are the vector and tensor deuteron polarizations. The density matrix of the deuteron can be written as:

\[
D_a D_b^\dagger = \frac{1}{3} (\delta_{ab} - \frac{3}{2} i \epsilon_{abc} S_c - Q_{ab}), \quad Q_{aa} = 0, \quad Q_{ab} = Q_{ba}. \]

After summing over the final proton polarizations one can find the following expressions:

\[
A_1 \left( \frac{d\sigma}{d\Omega} \right)_0 = -2 \Re g_s g_d^* - |g_d|^2, \quad A_2 \left( \frac{d\sigma}{d\Omega} \right)_0 = -|g_s|^2 - \Re g_s g_d^* + 2|g_d|^2, \]

\[
A_3 \left( \frac{d\sigma}{d\Omega} \right)_0 = 3 \Re g_s g_d^* - 3|g_d|^2, \quad A_4 \left( \frac{d\sigma}{d\Omega} \right)_0 = -2 \Im g_s g_d^*. \]

\[ S_i = \frac{3}{2} \rightarrow J^e = \frac{3}{2} \rightarrow \ell_f = 2, \quad (1) \]

The coefficients $A_i$ are related by the following linear relation: $A_1 + A_2 + A_3 = -1$ for any choice of amplitudes $g_s$ and $g_d$. The integration of the differential cross section over the $\vec{k}$-directions gives:

$$\sigma(\vec{d} + \vec{3H}) = \sigma_0(1 + A \vec{S} \cdot \vec{P}), \quad A = A_2 + \frac{1}{3} A_3 = \frac{-|g_s|^2 + |g_d|^2}{|g_s|^2 + 2|g_d|^2},$$

and it is independent from the tensor deuteron polarization.

The presence of S-wave contribution (the amplitude $g_s$), decreases the value of the integral coefficient $A$ whereas, in the fusion resonance region, where the D-wave dominates, the maximum value, $A = 1/2$, is reached. In the complete experiment (which gives $|g_s|^2$, $|g_d|^2$ and $Re g_s g_d^*$), the amplitudes $|g_s|$ and $|g_d|$ can be found in a model independent way, with the help of the following formulas:

$$9|g_s|^2 = (5 + 2A_1 - 4A_2) \left( \frac{d\sigma}{d\Omega} \right)_0,$$

$$9|g_d|^2 = (2 - A_1 + 2A_2) \left( \frac{d\sigma}{d\Omega} \right)_0,$$

$$-9Re g_s g_d^* = (1 + 4A_1 + A_2) \left( \frac{d\sigma}{d\Omega} \right)_0.$$

One can see that the integral coefficient $A$ can be determined from polarized nuclei collisions by measuring:

- the tensor analyzing power $A_1$ in $\vec{d} + \vec{3H}e \rightarrow p + \vec{4He}$,

- the spin correlation coefficient $C_{xx} = C_{yy} = A_2$ (if the $z$-axis is along $\vec{k}$-direction.)

Let us study now the polarization properties of the outgoing nucleons. We will show that can be predicted only from the tensor analyzing power, $A_1$. The polarization $\vec{P}_f$ of the produced nucleon depends on the polarization $\vec{P}$ of the initial $\vec{3He}$ (or $\vec{3H}$) as follows: $\vec{P}_f = p_1 \vec{P} + p_2 \vec{k} \cdot \vec{P}$, where the real coefficients $p_i, \ i = 1, 2$, characterize the spin transfer coefficients (from the initial $\vec{3He}$ or $\vec{3H}$ to the final nucleon): $K'_x = p_1 + p_2 \cos^2 \theta, \ K'_x = \ldots$
$p_2 \sin \theta \cos \theta$, where $\theta$ is the angle between $\vec{k}$ and $\vec{P}$. Averaging over the polarizations of the initial deuteron, we can find:

$$p_1 \left( \frac{d\sigma}{d\Omega} \right)_0 = -\frac{1}{3} \left( |g_s|^2 + 4Re \ g_s g_d^* + 4|g_d|^2 \right),$$

$$p_2 \frac{d\sigma}{d\Omega_0} = 4Re \ g_s g_d^* + 2|g_d|^2,$$

$$3p_1 = -1 + 2A_1, \quad p_2 = -2A_1, \quad 3p_1 + p_2 = -1.$$

This analysis holds in the presence of S-state only, in the entrance channel. The validity of this assumption can be experimentally verified with the measurement of T-odd one-spin polarization observables, as the analyzing powers in $\vec{d} + ^3He \rightarrow p + ^4He$ induced by vector deuteron polarization or $d + ^3\vec{He} \rightarrow p + ^4He$. This observable is very sensitive to the presence of even a small P-wave contribution, due to its interference with the main amplitude.

C. Helicity amplitudes

We calculate here the helicity amplitudes $F_{\lambda_1 \lambda_2 \lambda_3}$, with $\lambda_1 = \lambda_d$, $\lambda_2 = \lambda_{^3He}$, $\lambda_3 = \lambda_p$ (or $\lambda_n$), in terms of the partial amplitudes $g_s$ and $g_d$. This formalism is very well adapted for the analysis of angular distributions of the reaction products, in conditions of fusion reactors (with polarized fuel) and to the description of polarization phenomena. The direction of magnetic field $\vec{B}$ can be chosen as the most preferable quantization axis ($z-$axis). The formalism of the helicity amplitudes allows to study the angular dependence of the polarization observables, relative to $\vec{B}$. For example, the polarization properties of the neutron in $\vec{d} + ^3\vec{He} \rightarrow n + ^4He$ can be easily calculated in terms of these amplitudes.

The peculiar strong angular dependence of all observables is due to the presence (in conditions of fusion polarized reactor) of two independent physical directions, $\vec{k}$ and $\vec{B}$. So even for the S-state interaction, a non trivial angular dependence of the reaction products appears, i.e. some angular anisotropy, related to the initial polarizations. As all the polarizations of both colliding particles depend on the same magnetic field $\vec{B}$, the results for these observables depend only on the angle $\theta$, between $\vec{k}$ and $\vec{B}$. The case of the collision
of polarized beam with polarized target, where the beam and the target may have different
directions of polarization is more complicated, but it can also be treated in the framework
of the helicity formalism.

The deuteron polarization vector $\vec{D}^{(\lambda)}$ (with a definite helicity $\lambda$), can be chosen as:
$\vec{D}^{(0)} = (0, 0, 1)$ and $\vec{D}^{(\pm)} = 1/\sqrt{2}(\pm 1, i, 0)$. So the following expressions for the six possible
helicity amplitudes can be found:

$$
F_{0+,+} = g_s - (1 - 3 \cos^2 \theta)g_d, \quad F_{++,+} = \frac{3}{\sqrt{2}} \sin^2 \theta g_d,
$$
$$
F_{0+,-} = \frac{3}{2} \sin 2\theta g_d, \quad F_{++,} = \frac{3}{2\sqrt{2}} \sin 2\theta g_d,
$$
$$
F_{++,+} = -\frac{3}{2\sqrt{2}} \sin 2\theta g_d, \quad F_{-,,-} = -\frac{1}{\sqrt{2}} [2g_s + (1 - 3 \cos^2 \theta)g_d],
$$

(7)

where $\theta$ is the nucleon production angle, relative to the $\vec{B}$ direction. Other possible helicity
amplitudes, with reversed helicities of all particles, can be obtained from (7), by parity
reversion.

One can see, that for collinear kinematics, i.e. for $\theta = 0^\circ$, only two helicity amplitudes
are nonzero: $F_{0+,+} = g_s + 2g_d, \quad F_{-,,-} = -\sqrt{2}(g_s - g_d)$. So for the corresponding differential
cross section (for collisions of unpolarized particles) one can find:

$$
\left( \frac{d\sigma}{d\Omega} \right)_{\theta=0} = \frac{1}{3} \left( |F_{0+,+}|^2 + |F_{-,,-}|^2 \right) = |g_s|^2 + 2|g_d|^2.
$$

The same formula is also correct in the general case (i.e., for $\theta \neq 0$) for collisions of unpo-
larized particles:

$$
\left( \frac{d\sigma}{d\Omega} \right)_0 = \frac{1}{6} \sum_{\lambda} |F_{\lambda_1,\lambda_2,\lambda_3}|^2 = \left( \frac{d\sigma}{d\Omega} \right)_{\theta=0}.
$$

This relation between the differential cross section for collinear kinematics and the cross
section at any angle $\theta$ is valid for S-state interaction, only.

### D. Collision of polarized particles

The angular dependence of the reaction products in $d + ^3H \rightarrow n + ^4He$ for different
polarization states of the colliding particles can be derived from (7).
• Collisions of longitudinally polarized deuterons ($\lambda_d = 0$), with polarized $^3H$ or $^3He$:

$$\sigma_{0+}(\theta) = |F_{0+,+}|^2 + |F_{0+,-}|^2 = |g_s|^2 + 2Re \ g_s g_d^*(-1 + 3 \cos^2 \theta) + |g_d|^2(1 + 3 \cos^2 \theta). \quad (8)$$

• $\vec{d} + \vec{3He}$ collisions with parallel polarizations (relative to $\vec{B}$):

$$\sigma_{++}(\theta) = |F_{++,+}|^2 + |F_{++,-}|^2 = \frac{9}{2}|g_d|^2 \sin^2 \theta. \quad (9)$$

• $\vec{d} + \vec{3He}$ collisions with antiparallel polarizations:

$$\sigma_{+-}(\theta) = |F_{+-,+}|^2 + |F_{+-,-}|^2 = 2|g_s|^2 + 2Re \ g_s g_d^*(1 - 3 \cos^2 \theta) + \frac{1}{2}(1 + 3 \cos^2 \theta)|g_d|^2. \quad (10)$$

The sum of all these polarized cross sections is independent from polar angle $\theta$: the unpolarized cross section is isotropic, as expected for $S$-state interaction.

For the pure fusion resonance (with $g_s = 0$), the angular distribution of the reaction products depends specifically on the direction of the polarizations of the colliding particles: the $\sin^2 \theta$-dependence for parallel ($++$) collisions, becomes a dependence in $(1 + 3 \cos^2 \theta)$ for (+-) and (0+) collisions, to be compared with the isotropic behavior of the unpolarized collisions. Such definite and strong anisotropy can play a very important role in the design of the neutron shield of a reactor and of the blanket, where energetic neutrons (from $d + ^3H \rightarrow n + ^4He$) can produce $^3H$ through the reaction $n + ^6Li \rightarrow ^3H + ^4He$. Once a $d + ^3H$-reactor is beginning to operate, $^3H$-fuel can be produced in $^6Li$-blanket. In principle, this blanket can contain polarized $^6Li$, for a more efficient $^3H$-production in $\vec{n} + ^6Li$-collisions.

From Figs. 1 and 2, one can see that the angular dependence of the cross sections for polarized collisions, is essentially influenced by the presence of the S-wave amplitude and its relative phase.

Let us calculate now the following ratios:

$$R_{\lambda_1 \lambda_2} = \frac{\int_{-1}^{+1} \sigma_{\lambda_1 \lambda_2}(\theta)d \cos \theta}{\int_{-1}^{+1} d \cos \theta(d\sigma/d\Omega)_0},$$

from Eqs. (8-10) for $\sigma_{\lambda_1 \lambda_2}(\theta)$:
\[ R_{0+} = 1, \quad R_{++} = \frac{3}{2} f, \quad R_{+-} = \frac{1}{2} (4 - 3f). \]  

(11)

So we can write the following limits:

\[ 0 \leq R_{++} \leq \frac{3}{2} (g_s = 0), \quad \frac{1}{2} \leq R_{+-} \leq 2 (g_d = 0). \]

In the fusion resonance region, \((f = 1)\) the \((++)\)-collisions increase the reaction yield (in comparison with collisions of unpolarized particles) with a maximum coefficient \(\leq 3/2\), for pure D-wave fusion resonance. Using the notations of \([\Pi]\) one can obtain the following general formula for the differential cross section of \(\vec{d} + \vec{3H}\) (or \(\vec{d} + \vec{3He}\))-collisions:

\[
\frac{d\sigma}{d\Omega} (\vec{d} + \vec{3H}) = 6|g_d|^2 \left\{ \frac{3}{4} a \sin^2 \theta + \frac{b}{6} \left[ \frac{2}{f} - (1 - 3 \cos^2 \theta) \left( 1 + \frac{2 \Re g_s g_d^*}{|g_d|^2} \right) \right] \right. \\
\left. + \frac{c}{12} \left[ \frac{8}{f} - 6 - (1 - 3 \cos^2 \theta) \left( 1 - \frac{4 \Re g_s g_d^*}{|g_d|^2} \right) \right] \right\}.
\]

(12)

Here \(a = d_+ t_+ + d_- t_-\), \(b = d_0\), \(c = d_+ t_+ + d_- t_+\) and \(d_+, d_0, d_-\) are the fractions of deuterons with polarization respectively parallel, transverse, antiparallel to \(\vec{B}\), while \(t_+\) and \(t_-\) are the corresponding fractions for \(\vec{3H}\). The relations \(d_+ + d_0 + d_- = 1\) and \(t_+ + t_- = 1\) hold. The case \(a = b = c = 1/3\) corresponds to unpolarized collisions.

Note that the predicted angular dependence for \(b\) and \(c\) contributions, Eq. (12), differs essentially from the corresponding expression of \([\Pi]\). It coincides only for the special case \(f = 1, g_s = 0\). The denominator in Eq. (2) from ref. \([\Pi]\) must be also different.

From (12) one can find the following expression for the differential cross section of collisions of polarized deuterons with unpolarized \(\vec{3H}\):

\[
\frac{d\sigma}{d\Omega} (\vec{d} + \vec{3H}) = 2|g_d|^2 \left[ \frac{1}{f} + P_{zz} \frac{1 - 3 \cos^2 \theta}{4} \left( 1 + \frac{2 \Re g_s g_d^*}{|g_d|^2} \right) \right],
\]

i.e. it depends on the tensor deuteron polarization only. We used above the standard definition: \(P_{zz} = d_+ - 2d_0 + d_-\). Due to the \((1 - 3 \cos^2 \theta)\) dependence, after integration over \(\theta\), the cross section, again, does not depend on the deuteron polarization.

\(^{1}\)In particular the ratio of amplitudes \(f = 2|g_d|^2/(|g_s|^2 + 2|g_d|^2)\) was firstly defined in \([\Pi]\).
E. Polarization of neutrons in $\vec{d} + \vec{3H}$ collisions

Using the helicity amplitudes (7) it is possible to predict also the angular dependence of the neutron polarization in $\vec{d} + \vec{3H} \rightarrow n + ^4{He}$, in the general case of polarized particle collisions:

$$\left(n_+ - n_- \right) \frac{d\sigma}{d\Omega}(d + \vec{3H}) = \frac{9}{2} (d_- t_- - d_+ t_+) \sin^2 \theta (1 - 2 \cos^2 \theta) |g_d|^2 +$$

$$+ d_0 (t_+ - t_-) \left[ |g_s|^2 - 2(1 - 3 \cos^2 \theta) \text{Re} \ g_s g^*_d + (1 - 15 \cos^2 \theta + 18 \cos^4 \theta) |g_d|^2 \right] +$$

$$+ (d_+ t_- - d_- t_+) \frac{1}{2} \left[ 4 |g_s|^2 + 4(1 - 3 \cos^2 \theta) \text{Re} \ g_s g^*_d + (1 - 15 \cos^2 \theta + 18 \cos^4 \theta) |g_d|^2 \right],$$

where $n_{\pm}$ is the fraction of neutrons, polarized parallel and antiparallel to the direction of the magnetic field.

Let us write some limiting cases of this general formula:

(a) Collisions of polarized deuterons with unpolarized $^3{H}$-nuclei:

$$\left(n_+ - n_- \right) \frac{d\sigma}{d\Omega}(d + \vec{3H}) = (d_- t_- - d_+ t_+) \left[ |g_s|^2 + (1 - 3 \cos^2 \theta) \text{Re} \ g_s g^*_d -$$

$$- (2 - 3 \cos^2 \theta) |g_d|^2 \right]. \quad (13)$$

(b) Collisions of unpolarized deuterons with polarized $^3{H}$-nuclei:

$$\left(n_+ - n_- \right) \frac{d\sigma}{d\Omega}(d + \vec{3H}) = \frac{t_- - t_+}{3} \left[ |g_s|^2 + 4(1 - 3 \cos^2 \theta) \text{Re} \ g_s g^*_d -$$

$$- 2(2 - 3 \cos^2 \theta)|g_d|^2 \right]. \quad (14)$$

In the case of fusion resonance ($g_s = 0$), these formulas reduce to:

$$\left(n_+ - n_- \right) \frac{d\sigma}{d\Omega}(d + \vec{3H}) = \frac{9}{4} \sin^2 \theta (1 - 2 \cos^2 \theta)(d_- t_- - d_+ t_+) +$$

$$\frac{1}{2} \left[ d_0 (t_+ - t_-) + \frac{1}{2} (d_+ t_- - d_- t_+)(1 - 15 \cos^2 \theta + 18 \cos^4 \theta) \right]. \quad (15)$$

Averaging over the polarizations of $d$ (or $^3{H}$) one can find particular expressions:

$$\left(n_+ - n_- \right) \frac{d\sigma}{d\Omega}(d + \vec{3H}) = \frac{1}{2} (t_- - t_+)(2 - 3 \cos^2 \theta)$$
and
\[(n_+ - n_-) \frac{d\sigma}{d\Omega}(\vec{d} + 3^3H) = -\frac{1}{3}(d_- - d_+)(2 - 3\cos^2\theta).\]

The angular dependence of most of these polarization observables is sensitive to the relative value of the \(g_s\) and \(g_d\) amplitudes, due to the \(g_sg_d^*\)-interference contributions. Of course, in the region of the fusion resonance the \(g_d\) amplitude is dominant. However the temperature conditions, typical for a fusion reactor, correspond to collision energies lower than the energy of the fusion resonance. Even a small \(g_s/g_d\) ratio can change the angular behaviour of the polarization observables. In Figs. 3-4 we show, in a 3-dimensional plot, the dependence of the neutron polarization on the ratio \(x = |g_s|/g_d|\) and on the production angle \(\theta\) for three values of the relative phase \(\delta\), \(\delta = 0, \pi/2, \pi\), for \(\vec{d} + 3^3H\) and \(d + 3^3\bar{H}\)-collisions.

The exact determination of the parameters \(x\) and \(\delta\), is crucial for thermonuclear processes. This is a reason to perform a complete experiment for this reaction as discussed earlier [3].

The important point is that even at very low energies, where the spin structure is simplified, a complete experiment must include the scattering of polarized beam on polarized target. The full reconstruction of the threshold matrix elements requires this type of experiment.

III. PROCESSES \(d + d \rightarrow n + 3^3He\) AND \(d + d \rightarrow p + 3^3H\)

A. Introductory remarks

The \(d + d \rightarrow n + 3^3He\) and \(d + d \rightarrow p + 3^3H\) reactions at low energy have a very wide spectrum of fundamental and practical applications, from the discovery of tritium and helium isotopes [15], to the important role for primordial nucleosynthesis in the early Universe and fusion energy production with polarized and unpolarized fuel [1,16]. These processes are of large interest in nuclear theory: for example, in a four nucleon system, contrary to three nucleon system, broad resonant states can be excited [19]. The angular dependence of the differential cross sections [20,21] and the polarization observables [11-14] for these charge symmetric reactions constitutes a good test of the isotopic invariance for the low energy
nuclear interaction. The $dd-$ interaction is also connected to muon catalyzed processes $(\mu dd) \rightarrow \mu + p + ^3H \text{ or } (\mu dd) \rightarrow \mu + n + ^3He$ [26], where only the P-state of the $dd$-system is present, at low energy.

In the general case the spin structure of the matrix element for $d + d \rightarrow n + ^3He \text{ or } p + ^3H$ is quite complicated, with 18 independent spin combinations, and therefore with 18 complex scalar amplitudes, which are functions of the excitation energy and the scattering angle. However, at very small collision energies, where the $S-$state deuteron interaction has to dominate, this structure is largely simplified. The identity of the colliding deuterons, which are bosons, is an important guide for the partial amplitude analysis in order to determine the spin structure of the reaction amplitude. The determination of the polarization observables is indispensable, for this purpose. The four possible analyzing powers for $\vec{d} + d$-collisions, $A_y, A_{zz}, A_{xz} \text{ and } A_{xx} - A_{yy}$ were measured at $E_d \leq 100 \text{ keV}$, as well as the angular dependence of the differential cross section [20,21,25].

The knowledge of the relative role of different orbital angular momenta (and the corresponding partial amplitudes) is essential for the solution of different fundamental problems concerning these processes, like the possibility to build a thermonuclear ”clean” reactor with polarized $d + ^3He$-fuel. The main reaction $d + ^3He \rightarrow p + ^4He$ does not produce radioactive nuclei, and the possibility to decrease the cross section of $\vec{d} + d$-collisions (which produces $n + ^3He \text{ or } p + ^3H$) with parallel polarizations, will decrease the production of neutrons and the tritium. Direct experimental data about $\vec{d} + d$- low energy collisions are absent, so the dependence of the cross section on the polarization states of the colliding particles can be calculated only from theoretical predictions or from different multipole analysis.

The theoretical predictions and the results of multipole analysis seem very controversial now, even at very low energy. In the first partial wave analysis [27,28] it was found that the S-state $dd-$interaction in the quintet state (i.e. with total spin $S_i = 2$) is smaller in comparison with the $S_i = 0$ interaction. This was consistent with the conclusion of ref. [1], that in a polarized reactor it is possible to suppress $\vec{d} + d$-collisions. Later [29], it was pointed out that strong central forces with $D-$state in $^3He$ can induce a large $dd-$ interaction in the
quintet state and resonating-group calculations [29] found that polarized collisions are not suppressed. On the other hand, DWBA calculations give a large suppression for the ratio of polarized on unpolarized cross section, \( \sigma_{++}/\sigma_0 \simeq 0.08 \) in the range \( E_d = 20 - 150 \) keV, even after inclusion of the \(^3\text{He}\) D-state. A more recent analysis [31,32] based on R–matrix approach, concludes that this ratio does not decrease with energy. Note that in principle, it can be energy dependent [33].

Again, a direct measurement of polarized \( dd\)-collisions would greatly help in solving these problems and the complete experiment will allow to reconstruct the spin structure of the reaction amplitude. Therefore, the considerations based on \( S\)–wave only, have to be considered as the first necessary step which can illustrate the possible strategy of the complete experiment for this case.

**B. Partial amplitudes**

We establish here the spin structure of the threshold matrix element for the \( d + d \rightarrow n + ^3\text{He} (p + ^3\text{H}) \) process. For \( S\)-state \( dd\)–interaction the following partial transitions are allowed:

\[
\begin{align*}
S_i = 0 & \rightarrow J^\pi = 0^+ \rightarrow S_f = 0, \quad \ell_f = 2, \\
S_i = 2 & \rightarrow J^\pi = 2^+ \rightarrow S_f = 0, \quad \ell_f = 2, \\
S_i = 2 & \rightarrow J^\pi = 2^+ \rightarrow S_f = 1, \quad \ell_f = 2,
\end{align*}
\]

where \( S_i \) is the total spin of the colliding deuterons, \( \ell_f \) is the orbital angular momentum of the final nucleon. Note that the Bose statistics for identical deuterons allows only even values of initial spin, that is \( S_i = 0 \) and \( S_i = 2 \) for the \( S\)-state. The resulting spin structure of the threshold matrix element can be written as:

\[
\mathcal{M} = i(\chi_3^\dagger \sigma_2 \bar{\chi}_1^\dagger) \left[ g_1 \vec{D}_1 \cdot \vec{D}_2 + g_2 (3\vec{k} \cdot \vec{D}_1 \vec{k} \cdot \vec{D}_2 - \vec{D}_1 \cdot \vec{D}_2) \right. \\
\left. + g_3 (\vec{\sigma} \cdot \vec{k} \times \vec{D}_1 \vec{k} \cdot \vec{D}_2 + \vec{\sigma} \cdot \vec{k} \times \vec{D}_2 \vec{k} \cdot \vec{D}_1) \right],
\]

(16)
where $\chi_1$ and $\chi_3$ are the 2-component spinors of the produced nucleon and $^3He$ (or $^3H$), $\vec{D}_1$ and $\vec{D}_2$ are the 3-vectors of the deuteron polarization, $\vec{k}$ is the unit vector along the 3-momenta of the nucleon (in the CMS of the considered reaction). The amplitudes $g_1$ and $g_2$ describe the production of the singlet $n + ^3He$-state, and the amplitude $g_3$ - the triplet state. The complete experiment in $S$-state $dd$-interaction implies the measurement of 5 different observables, to determine 3 moduli and two relative phases of partial amplitudes.

The validity of the $S$-state approximation in the near threshold region can be checked by measuring any T-odd polarization observable, the simplest of which are the one-spin observables as the vector analyzing power in the reaction $\vec{d} + d \rightarrow n + ^3He$ \cite{25}. Note that Eq. (16) is correct also for the threshold matrix elements of the inverse process: $n + ^3He \rightarrow d + d$ (or $p + ^3H \rightarrow d + d$).

C. Helicity amplitudes

In order to establish the angular dependence of the reaction products, for collisions of polarized particles, in the presence of magnetic field, let us derive the helicity amplitudes. The spin structure of the $d + d$ reactions is more complex in comparison to $d + ^3He$. The analysis of polarization phenomena is also more complicated. It was mentioned in \cite{1}, that an enhancement factor, equal to 2 can be obtained in a polarized plasma \cite{4} for the reaction $d + d \rightarrow n + ^3He$, if the deuterons are polarized transversally to the direction of the magnetic field, i.e. for (00)-collisions, in an ordinary thermal ion distribution. Alternatively, if colliding beams or beam and target methods are used (inertial fusion), the two ions should be polarized in opposite direction, relatively to the field. In case of collisions of deuterons with parallel polarizations i.e (++) or (−−), a large suppression of the reaction rate is expected.

It is then interesting to analyze all possible configurations of the polarization of the col-

\footnote{Note that this holds only for the partial wave analysis \cite{27,28}.}
liding deuterons. We can classify the helicity amplitudes according to the following scheme:

**I)** 00 collisions: the polarization is transverse to the magnetic field → 2 independent amplitudes;

**II)** ++ collisions: the polarization parallel to the magnetic field → 4 independent amplitudes;

**III)** +- collisions: collisions with deuterons with antiparallel polarization, in the same direction as the magnetic field → 4 independent amplitudes;

**IV)** 0+ collisions: collisions of one deuteron with polarization transverse to the magnetic field with the other deuteron polarized along the magnetic field → 4 independent amplitudes;

The corresponding helicity amplitudes $F_{\lambda_1\lambda_2,\lambda_3\lambda_4}$, (with $\lambda_1 \equiv \lambda_d$, $\lambda_2 \equiv \lambda_d$, $\lambda_3 \equiv \lambda_{3He}$, $\lambda_4 \equiv \lambda_N$) are given in terms of partial amplitudes:

(I) $F_{00,++} = -\sin 2\theta g_3$,  
$I_{00,+-} = g_1 - (1 - 3\cos^2 \theta)g_2$,

(II) $F_{++,++} = \sin 2\theta g_3$,  
$I_{++,-+} = \sin^2 \theta(\frac{3}{2}g_2 + g_3)$,

$I_{++,--} = 0$,  
$I_{+-,+-} = \sin^2 \theta(\frac{3}{2}g_2 + g_3)$,

(III) $F_{+-,++} = F_{+-,-+} = -\frac{1}{2}\sin 2\theta g_3$,

(IV) $F_{0+,++} = \frac{1}{\sqrt{2}}(-1 + 3\cos^2 \theta)g_3$,  
$I_{0+,-+} = \frac{1}{2\sqrt{2}}\sin 2\theta(3g_2 + g_3)$,

$I_{0+,--} = \frac{1}{\sqrt{2}}\sin^2 \theta g_3$,  
$I_{0+,-+} = \frac{1}{2\sqrt{2}}\sin 2\theta(-3g_2 + g_3)$.

where $\theta$ is the nucleon production angle relative to $\vec{B}$ direction.
D. Angular dependence for collisions of polarized deuterons

After summing over the polarization states of the produced particles, the cross section of the process $\vec{d} + \vec{d} \rightarrow n + ^3He$, for definite deuteron polarizations, can be written as:

$$\sigma_{00}(\theta) = 2 \left(|F^{0+}_{00,00}|^2 + |F^{0+}_{00,-1}|^2\right) = 2|g_1 - g_2(1 - 3 \cos^2 \theta)|^2 + 8 \sin^2 \theta \cos^2 \theta|g_3|^2,$$

$$\sigma_{++}(\theta) = \sum_{\lambda_3, \lambda_4} |F^{++}_{+\lambda_3 \lambda_4}|^2 = \sin^2 \theta \left[\frac{9}{2} \sin^2 \theta|g_2|^2 + 2(1 + \cos^2 \theta)|g_3|^2\right],$$

$$\sigma_{+-}(\theta) = \sum_{\lambda_3, \lambda_4} |F^{-+}_{-\lambda_3 \lambda_4}|^2 = 2|g_1|^2 + 2\text{Re} \ g_1 g_2^*(1 - 3 \cos^2 \theta + \frac{1}{2}(1 - 3 \cos^2 \theta)^2|g_2|^2 + 2 \sin^2 \theta \cos^2 \theta |g_3|^2,$$

$$\sigma_{0+}(\theta) = \sum_{\lambda_3, \lambda_4} |F^{0+}_{0,\lambda_3 \lambda_4}|^2 = 9 \sin^2 \theta \cos^2 \theta |g_2|^2 + (1 - 3 \cos^2 \theta + 4 \cos^4 \theta)|g_3|^2.$$

With the help of these formulas we can estimate the corresponding integral ratios:

$$R_{\lambda_1 \lambda_2} = \frac{\int_{-1}^{1} \sigma_{\lambda_1 \lambda_2}(\theta)d \cos \theta}{\int_{-1}^{1} d \cos \theta (d\sigma/d \cos \theta)_0},$$

which characterize the relative role of polarized collisions with respect to unpolarized ones:

$$R_{00} = \frac{315 + 4r}{53 + 2r}, \quad R_{++} = \frac{36 + r}{53 + 2r}, \quad R_{+-} = \frac{1215 + r}{53 + 2r}, \quad R_{0+} = \frac{9 + r}{53 + 2r},$$

where $r = (3|g_2|^2 + 2|g_3|^2)/|g_1|^2$. It is interesting that all these ratios depend on a single contribution of the moduli of the partial amplitudes, the ratio $r \geq 0$. The ratios $R_{\lambda_1 \lambda_2}$ are limited by:

$$1.2 \leq R_{00} \leq 3, \quad 0 \leq R_{++} \leq 3.6, \quad 1.2 \leq R_{+-} \leq 12, \quad 0 \leq R_{0+} \leq 0.9,$$

where the upper limits correspond to $g_2 = g_3 = 0$, (when only the $g_1$ amplitude is present), and the lower limits correspond to $g_1 = 0$ (for any amplitudes $g_2$ and $g_3$). But the exact values of $R_{\lambda_1 \lambda_2}$ depend on the relative value of the partial amplitudes, through one parameter, $r$.

The general dependence of the differential cross section for $\vec{d} + \vec{d}$-collisions, can be written in terms of partial cross sections $\sigma_{\lambda_1 \lambda_2}$ as follows:

$$\frac{d\sigma}{d\Omega}(\vec{d} + \vec{d}) = (d_+^2 + d_-^2)\sigma_{++}(\theta) + d_0^2\sigma_{00}(\theta) + 2d_+d_-\sigma_{+-}(\theta) + 2d_0(d_+ + d_-)\sigma_{0+}(\theta),$$
where we used the evident relations between $\sigma_{\lambda_1 \lambda_2}$: $\sigma_{++}(\theta) = \sigma_{--}(\theta)$, $\sigma_{0+}(\theta) = \sigma_{0-}(\theta)$, $\sigma_{+-}(\theta) = \sigma_{-+}(\theta)$, due to the P-invariance of the strong interaction, and the standard notation: $d_+$, $d_0$ and $d_-$ for different deuteron fractions in polarized plasma.

Using Eq. (20) one can find some interesting limiting cases. Setting for example, $d_+ = d_-$ (deuterons with tensor polarization only: $P_{zz} = 1 - 3d_0$, $P_z = 0$), one can obtain the following dependence of the differential cross section on $P_{zz}$:

$$\frac{d\sigma}{d\Omega}(\vec{d} + \vec{d}) = a_0(\theta) + 2P_{zz}a_1(\theta) + \frac{1}{2}P_{zz}^2a_2(\theta),$$  \hspace{1cm} (21)$$

where the coefficients $a_i(\theta)$, $i = 0 - 2$, are linear combinations of the helicity cross sections $\sigma_{\lambda_1 \lambda_2}$:

$$9a_0(\theta) = 2[\sigma_{++}(\theta) + \sigma_{+-}(\theta)] + \sigma_{00}(\theta) + 4\sigma_{0+}(\theta),$$

$$9a_1(\theta) = \sigma_{++}(\theta) + \sigma_{+-}(\theta) - \sigma_{00}(\theta) - \sigma_{0+}(\theta),$$  \hspace{1cm} (22)

$$9a_2(\theta) = \sigma_{++}(\theta) + \sigma_{+-}(\theta) + 2\sigma_{00}(\theta) - 4\sigma_{0+}(\theta).$$

So, measuring the $P_{zz}$-dependence of the cross section for $\vec{d} + \vec{d}$ collisions, one can determine all 3 coefficients $a_i(\theta)$ (at each angle $\theta$). This allows to determine the individual helicity partial cross sections $\sigma_{\lambda_1 \lambda_2}(\theta)$:

$$\sigma_{00}(\theta) = a_0(\theta) - 4a_1(\theta) + 2a_2(\theta),$$

$$\sigma_{0+}(\theta) = a_0(\theta) - a_1(\theta) - a_2(\theta),$$  \hspace{1cm} (23)

$$\sigma_{++}(\theta) + \sigma_{+-}(\theta) = 2a_0(\theta) + 4a_1(\theta) + a_2(\theta).$$

In order to disentangle the $\sigma_{++}(\theta)$ and $\sigma_{+-}(\theta)$ contributions, an additional polarization observable has to be measured, from the collisions of vector polarized deuterons ($d_\pm = \frac{1}{3} \pm \frac{1}{2}P_z$, $d_0 = \frac{1}{3}$):

$$\frac{d\sigma}{d\Omega}(\vec{d} + \vec{d}) = a_0(\theta) + \frac{P_z^2}{2}(\sigma_{++}(\theta) - \sigma_{+-}(\theta)).$$  \hspace{1cm} (24)$$
The linear $P_z$ contribution is forbidden by the P-invariance of the strong interaction. Only the measurement of the $P_z^2$ contribution allows to separate the cross sections $\sigma_{++}(\theta)$ and $\sigma_{+-}(\theta)$.

This analysis is equivalent to the discussion of the complete experiment (in terms of helicity cross sections $\sigma_{\lambda_1 \lambda_2}(\theta)$).

Finally let us derive the polarization properties of the neutrons in the process $\vec{d} + \vec{d} \rightarrow n + ^3He$. Using eqs. (17) for the helicity amplitudes, one can find for the $\theta$ dependence of the neutron polarization (for the different spin configurations of the colliding deuterons):

\begin{equation}
(n_+ - n_-)\sigma_{++}(\theta) = 2 \sin^2 \theta d_+^2 \left[ 3 \text{Re} g_2 g_3^* + 2 \cos^2 \theta |g_3|^2 \right],
\end{equation}

\begin{equation}
(n_+ - n_-)\sigma_{0+}(\theta) = 2 d_0 d_+ \cos^2 \theta \left[ -(1 - 2 \cos^2 \theta) |g_3|^2 + 3 \sin^2 \theta \text{Re} \ g_2 g_3^3 \right],
\end{equation}

\begin{equation}
(n_+ - n_-)\sigma_{00}(\theta) = (n_+ - n_-)\sigma_{+-}(\theta) = 0,
\end{equation}

where $n_+$ and $n_-$ are the fractions of polarized neutrons with spin parallel and antiparallel relative to the $\vec{B}$ direction.

The production of unpolarized neutrons for 00-collisions of deuterons results from P-invariance, and for $-+$ collisions results from the identity of colliding deuterons and from the P-invariance.

**E. Complete experiment for $d + d \rightarrow n + ^3He$**

Due to three complex partial amplitudes for the S-wave $dd$–interaction for the process $d + d \rightarrow n + ^3He$, the measurement of a large number of observables is necessary, in order to perform the complete experiment. This study will be based on the formalism of the polarized structure functions, previously used in [3] for the process $d + ^3H \rightarrow n + ^4He$.

Let us consider the collisions of polarized deuterons $\vec{d} + \vec{d} \rightarrow n + ^3He$. The differential cross section can be parametrized in the following general form:
\[
\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_0 \left[ 1 + A_1(\vec{k} \cdot \vec{Q}_1 + \vec{k} \cdot \vec{Q}_2) + A_2 \vec{S}_1 \cdot \vec{S}_2 + A_3 \vec{k} \cdot \vec{S}_1 \vec{k} \cdot \vec{S}_2 \\
+ A_4 \vec{k} \cdot \vec{Q}_1 \vec{k} \cdot \vec{Q}_2 + A_5 \vec{Q}_1 \cdot \vec{Q}_2 + A_6 Q_{1ab} Q_{2ab} \\
+ A_7(\vec{k} \cdot \vec{S}_1 \times \vec{k} \cdot \vec{S}_2 + \vec{k} \cdot \vec{S}_1 \times \vec{Q}_1) \right),
\]

where \( \vec{S}_1 \) and \( \vec{S}_2 \) (\( Q_{1ab} \) and \( Q_{2ab} \)) are the vector (tensor) polarizations of the colliding deuterons. The real coefficient \( A_1 \) describes the tensor analyzing power in \( \vec{d} + \vec{d} \rightarrow n + ^3\text{He} \), \( A_2 - A_7 \) are the spin correlation coefficients in \( \vec{d} + \vec{d} \rightarrow n + ^3\text{He} \). The coefficients \( A_1 - A_6 \) are T-even polarization observables and \( A_7 \) is the T-odd one (due to the specific correlation of the vector polarization of one deuteron and the tensor polarization of the other deuteron).

Note that these coefficients \( A_i \) cannot fix the relative phases of the singlet amplitudes \( g_1 \) and \( g_2 \) (from one side) and the triplet amplitude \( g_3 \) (from the other side). The complete experiment has to be more complex than the determination of the polarization observables \( A_i \). The polarization transfer coefficients from the initial deuteron to the produced fermion (\( n \) or \(^3H \)) have to be measured, too.

After summing over the polarizations of the produced particles in \( \vec{d} + \vec{d} \rightarrow n + ^3\text{He} \), the following expressions can be found, for the coefficients \( A_i, i = 1 - 7 \), in terms of the partial amplitudes \( g_k \), \( k = 1 - 3 \):

\[
-\frac{9}{2} A_1 \left( \frac{d\sigma}{d\Omega} \right)_0 = 3|g_2|^2 + |g_3|^2 + 6\text{Re } g_1 g_2^*,
\]

\[
A_2 \left( \frac{d\sigma}{d\Omega} \right)_0 = -|g_1|^2 + 2|g_2|^2 + |g_3|^2 - \text{Re } g_1 g_2^*,
\]

\[
A_3 \left( \frac{d\sigma}{d\Omega} \right)_0 = -3|g_2|^2 - |g_3|^2 + 3\text{Re } g_1 g_2^*,
\]

\[
\frac{9}{4} A_4 \left( \frac{d\sigma}{d\Omega} \right)_0 = 9|g_2|^2 - 4|g_3|^2
\]

\[
\frac{9}{2} A_5 \left( \frac{d\sigma}{d\Omega} \right)_0 = -6|g_2|^2 + 6\text{Re } g_1 g_2^* + 2|g_3|^2,
\]

\[
\frac{9}{2} A_6 \left( \frac{d\sigma}{d\Omega} \right)_0 = |g_1|^2 + |g_2|^2 - 2\text{Re } g_1 g_2^*.
\]
\[ A_7 \left( \frac{d\sigma}{d\Omega} \right)_0 = -2 \text{Im} \ g_1 g_2^* , \]

where \((d\sigma/d\Omega)_0\) is the differential cross section with unpolarized particles:

\[ \left( \frac{d\sigma}{d\Omega} \right)_0 = \frac{2}{9} \left[ 3|g_1|^2 + 6|g_2|^2 + 4|g_3|^2 \right] = \frac{2}{9} |g_1|^2 (3 + 2r). \]

Using these expressions, the following relations can be found between the coefficients \(A_i\):

(a) linear: between T-even polarization observables,

\[ A_2 + A_3 + \frac{9}{2} A_6 = A_1 + A_4 - \frac{1}{3} A_3 + \frac{7}{4} A_5 = 0 \]

(b) quadratic, relating the T-odd asymmetry \(A_7\) with the T-even coefficients \(A_i, i = 1-6; \)

\[ \frac{9}{4} (1 + A_1^2 - A_7^2) = A_2^2 + (A_2 + A_3)^2 + 6(A_1 A_2 + A_1 A_3 + A_2 A_3) \]

Therefore, the measurements of \((d\sigma/d\Omega)_0\) and 3 coefficients \(A_i, i = 1-3\), allow to find the moduli of all S-wave partial amplitudes \(g_k, k = 1-3\), and the relative phase of the singlet amplitudes \(g_1\) and \(g_2\):

\[ 18|g_1|^2 = (9 - 12A_2 - 4A_3) \left( \frac{d\sigma}{d\Omega} \right)_0 , \]

\[ -18|g_2|^2 = (9 + 18A_1 + 6A_2 + 10A_3) \left( \frac{d\sigma}{d\Omega} \right)_0 , \]

\[ 2|g_3|^2 = (3 + 3A_1 + 2A_2 + 2A_3) \left( \frac{d\sigma}{d\Omega} \right)_0 , \]

\[ 18 \text{Re} \ g_1 g_2^* = (-9A_1 + 2A_3) \left( \frac{d\sigma}{d\Omega} \right)_0 . \]

So these measurements can be considered as the first step of the complete experiment for the process \(d + d \rightarrow n + ^3He\) in the near threshold conditions.

Using these expressions, one can find the following expression for the ratio \(r\):

\[ r = 3 \frac{1 + a}{1 - 2a}, \quad a = \frac{2}{9} (3A_2 + A_3). \]

Let us discuss, for completeness, the simplest polarization phenomena for the inverse reaction, \(n + ^3He \rightarrow d + d\), when the deuterons are produced in S-state. In this case this
process can be described by the same set of partial amplitudes. The cross section for the collisions of polarized particles, $\vec{n} + ^3\text{He} \rightarrow d + d$, can be parametrized in the following way:

$$\frac{d\sigma}{d\Omega}(\vec{n} + ^3\text{He}) = \left(\frac{d\sigma}{d\Omega}\right)_0 \left[1 + B_1 \vec{P}_1 \cdot \vec{P}_2 + B_2 \vec{k} \cdot \vec{P}_1 \vec{k} \cdot \vec{P}_2\right],$$

where $\vec{P}_1$ and $\vec{P}_2$ are the polarizations for $n$ and $^3\text{He}$ and $B_i$, i=1,2, are the spin correlation coefficients for the inverse reaction, which can be expressed as functions of the partial amplitudes:

$$B_1 = -3 \frac{|g_1|^2 + 2|g_2|^2}{3|g_1|^2 + 6|g_2|^2 + 4|g_3|^2}, \quad B_2 = \frac{4|g_3|^2}{3|g_1|^2 + 6|g_2|^2 + 4|g_3|^2}.\quad (29)$$

These coefficients are not independent, $-B_1 + B_2 = 1$, as a result of the peculiar spin structure of the matrix element for the $n + ^3\text{He}$–collision. Comparing Eq. (27) and Eq. (29), one can find the following relation:

$$\frac{9}{4}B_2 = 3(1 + A_1) + 2(A_2 + A_3).$$

In order to find the relative phases of the singlet and triplet amplitudes in $d+d \rightarrow n+^3\text{He}$, it is necessary to measure the polarization transfer coefficients (from the initial deuteron to the final neutron). The most general parametrization of the neutron polarization can be written in the following form:

$$\vec{P} = p_1 \vec{S} + p_2 \vec{k} (\vec{k} \cdot \vec{S}) + p_3 \vec{k} \times \vec{Q},$$

where $p_i$, i=1-3, are the real structure functions, characterizing the corresponding coefficients of polarization transfer:

$$p_1 = p_2 = 0, \quad p_3 = -\frac{4\text{Im}(g_1 - g_2)g_3^*}{3|g_1|^2 + 6|g_2|^2 + 4|g_3|^2}.$$ 

Both T-even coefficients $p_1$ and $p_2$ are identically zero. This results from the specific spin structure of the threshold amplitude, due to the identity of the colliding deuterons and the S-wave interaction. The T-odd coefficient $p_3$ is sensitive to the relative phases of singlet and triplet amplitudes.
IV. $^3He$ AND $^3H$ COLLISIONS

A. Three-particle production

We discuss here the spin structure and the polarization observables for the following processes (at low energy for colliding particles):

$$^3H + ^3H \rightarrow 2n + ^4He, \quad ^3H + ^3He \rightarrow n + p + ^4He$$

$$^3He + ^3He \rightarrow 2p + ^4He, \quad ^3H + ^3He \rightarrow d + ^4He$$

Due to the presence of identical fermions in initial and final states, the spin structure of the symmetric processes, $^3He + ^3He \rightarrow 2p + ^4He$ and $^3H + ^3H \rightarrow 2n + ^4He$, for the S-state interaction of colliding nuclei is built on a single transition: $S_i = S_f = 0$, with the following parametrization:

$$g_0(\chi_2^\dagger \chi_1)(\bar{\phi}_1 \sigma_2 \phi_2),$$

where $\chi_1$ and $\chi_2$ ($\phi_1$ and $\phi_2$) are the 2-component spinors of the produced nucleons (colliding nuclei), $g_0$ is the singlet-singlet amplitude, which dependence on the energies of the produced particles has a dynamical character and can be, in principle, complicated. However a model independent expression of the cross section as a function of the polarizations $\vec{P}_1$ and $\vec{P}_2$ of the colliding nuclei can be given as:

$$\frac{d\sigma}{d\omega}(\vec{P}_1, \vec{P}_2) = \left(\frac{d\sigma}{d\omega}\right)_0 (1 - \vec{P}_1 \cdot \vec{P}_2).$$

This dependence is correct for any 3-particle phase-space element $d\omega$, and then, also for the total cross section.

Therefore the polarization of colliding nuclei decreases the reaction rate, in comparison with unpolarized collisions. In a fusion reactor, this property will favour the plasma production through the main $d + ^3H$-reaction, which has a larger Q-value and uses $d$–fuel, preventing the waste of the very expensive $^3H$ in 'non effective' $^3H + ^3H$ collision.

Due to the non-identity of the colliding and produced fermions, the process $^3H + ^3He \rightarrow n + p + ^4He$ is characterized by two independent transitions, a singlet one, (with amplitude equal to half of the $^3H + ^3H$ amplitude, due to the isotopic invariance of the nuclear interaction) and a triplet amplitude. then the matrix element can be written as:
\[ \mathcal{M}(^{3}H + ^{3}He) = \frac{1}{2}g_{0}(\chi_{2}^{\dagger}\sigma_{2}\chi_{1}^{\dagger})(\bar{\phi}_{1}\sigma_{2}\phi_{2}) + \frac{1}{2}g_{1}(\chi_{2}^{\dagger}\sigma_{a}\sigma_{2}\chi_{1}^{\dagger})(\bar{\phi}_{1}\sigma_{2}\sigma_{a}\phi_{2}). \] (30)

Therefore the dependence of the cross section for \(^{3}H + ^{3}He\)-collisions on the polarizations of the colliding particles is:

\[ \frac{d\sigma}{d\omega}(\vec{P}_{1}, \vec{P}_{2}) = \left( \frac{d\sigma}{d\Omega} \right)_{0}(1 + A\vec{P}_{1} \cdot \vec{P}_{2}), \quad A = \frac{|g_{0}|^{2} + |g_{1}|^{2}}{|g_{0}|^{2} + 3|g_{1}|^{2}}, \quad -1 \leq A \leq \frac{1}{3}. \]

The coefficient \( A \) is determined by the ratio \( R \) of the cross sections for \(^{3}H + ^{3}H\) and \(^{3}H + ^{3}He\) collisions (with unpolarized particles):

\[ R = \frac{\sigma(^{3}H + ^{3}H)}{\sigma(^{3}H + ^{3}He)} = \frac{4|g_{0}|^{2}}{|g_{0}|^{2} + 3|g_{1}|^{2}}, \quad 6A = 2 + R. \]

This result, of course, is valid only for the S-state interaction in the considered reactions.

The relative phase of the partial amplitudes \( g_{0} \) and \( g_{1} \) determines the polarization transfer from the initial \(^{3}He\) to the final \( n\):

\[ \vec{P}_{n} = p\vec{P}_{i}, \quad p = 2\frac{|g_{1}|^{2} + Re \ g_{0}g_{1}^{*}}{|g_{0}|^{2} + 3|g_{1}|^{2}}. \]

Note that in the process \(^{3}H + ^{3}He\) we neglected the production of final particles in the D-state, which is allowed in the general case by the conservation of the total angular momentum and P-parity: \( S_i = 1 \rightarrow S_f = 1, \ \ell_{1,2} = 2 \), where \( \ell_{1} \) is the orbital angular momentum of the \( np\)-system and \( \ell_{2} \) is the orbital angular momentum of the \(^{4}He\), relative to the center of mass of the \( np\)-system. The spin structure for these transitions can be written as:

\[ \chi_{2}^{\dagger}(k_{a}\vec{\sigma} \cdot \vec{k} - \frac{1}{3}\sigma_{a})\sigma_{2}\chi_{1}^{\dagger} (\bar{\phi}_{1}\sigma_{2}\sigma_{a}\phi_{2}), \]

where \( \vec{k} \) is the unit vector along the 3-momentum of the \(^{4}He\) (if \( \ell_{2} = 2, \ \ell_{1} = 0 \)) or along the proton 3-momentum (if \( \ell_{1} = 2, \ \ell_{2} = 0 \)). The matrix element for the production of two P-waves, \( \ell_{1} = \ell_{2} = 1 \) is comparable, in principle, to the D-wave production.

**B. The deuteron production**

Let us finally discuss the process \(^{3}H + ^{3}He \rightarrow d + ^{4}He\), (Q-value=14.3 MeV), which is also occurring in a \( d + d \) or \( d + ^{3}He\)-fusion reactor. The spin structure of the threshold
amplitude can be established using the generalized Pauli principle (which is valid at the level of the isotopic invariance). Due to the isospin value $I = 0$ for the entrance channel, the sum $S_i + \ell_i$ must be odd, and for S-interaction ($\ell_i = 0$), we have $S_i = 1$, with the following two transitions allowed: $S_i = 1 \rightarrow J^\pi = 1^+ \rightarrow \ell_f = 0$ and $\ell_f = 2$, where $\ell_f$ is the orbital angular momentum of the produced deuteron. Therefore, the matrix element can be written as:

$$
\mathcal{M} = \bar{\phi}_1\sigma_2 \left[ h_0 \vec{\sigma} \cdot \vec{D}^* + h_2 (3\vec{\sigma} \cdot \vec{k} \vec{k} \cdot \vec{D}^* - \vec{\sigma} \cdot \vec{D}^*) \right] \phi_2,
$$

where $\vec{D}$ is the deuteron polarization 3-vector, $h_0$ and $h_2$ are the partial amplitudes, corresponding to $\ell_f = 0$ and 2. These amplitudes are complex functions of the excitation energy. The complete experiment, for this process, must contain the measurement of at least three different observables.

The dependence of the differential cross section on the polarizations of the colliding nuclei, have the same structure as eq. (28), with:

$$
B_1 \left( \frac{d\sigma}{d\Omega} \right)_0 = \frac{1}{2} |h_0|^2 + 2Re h_0 h_2^* + 2|h_2|^2, \quad B_2 \left( \frac{d\sigma}{d\Omega} \right)_0 = -3(|h_2|^2 + 2Re h_0 h_2^*),
$$

$$
\left( \frac{d\sigma}{d\Omega} \right)_0 = \frac{3}{2} (|h_0|^2 + 2|h_2|^2), \quad 3B_1 + B_2 = 1.
$$

This results from the absence of a singlet state of the colliding nuclei, in the discussed process.

After $\vec{k}$- integration, one can find:

$$
\sigma(\vec{P}_1, \vec{P}_2) = \sigma_0 (1 + B \vec{P}_1 \cdot \vec{P}_2), \quad B = B_1 + \frac{1}{3}B_2 = \frac{1}{3},
$$

independently on the relative value of the partial amplitudes. Therefore the dependence of the total cross section from the polarizations can be predicted exactly, due to the specific spin structure of the threshold matrix element.

The information that can be obtained from the tensor analyzing power in the inverse reaction, $\vec{d} + ^4He \rightarrow ^3H + ^3He$, does not give any new constraint on the partial amplitudes.
The cross section is written, in terms of the tensor analyzing power $A_d$:

$$
\frac{d\sigma}{d\Omega}(d + 4He) = \left( \frac{d\sigma}{d\Omega} \right)_0 [1 + A_d Q_{ab} k_a k_b]
$$

and

$$
A_d = \frac{|h_2|^2 + 2Re h_0 h_2^*}{|h_0|^2 + 2|h_2|^2},
$$

i.e. $A_d = \frac{1}{3} B_2$. This means that the measurement of the tensor analyzing power $A_d$ allows to define both correlation coefficients, for the direct reaction $^3H + ^3He \rightarrow d + 4He$.

The complete experiment has to include the measurement of polarization transfer coefficients in $^3H + ^3He \rightarrow d + 4He$, where the vector deuteron polarization $\vec{S}$ can be written in the following general form:

$$
\vec{S} = s_1 \vec{P} + s_2 \vec{k} \times \vec{P},
$$

with the following expressions for the real coefficients $s_i$, $i = 1, 2$:

$$
s_1 = \frac{|h_0|^2 - Re h_0 h_2^*}{|h_0|^2 + 2|h_2|^2}, \quad s_2 = \frac{-Re h_0 h_2^* + |h_2|^2}{|h_0|^2 + 2|h_2|^2}.
$$

So, finally, the complete experiment for $^3H + ^3He \rightarrow d + 4He$ has to include the measurements of $(d\sigma/d\Omega)_0$, $B_1$ and $s_1$:

$$
9|h_0|^2 = 2(3B_1 + 4s_1 - 2) \left( \frac{d\sigma}{d\Omega} \right)_0,
$$

$$
-9|h_2|^2 = (3B_1 + 4s_1 - 5) \left( \frac{d\sigma}{d\Omega} \right)_0,
$$

$$
-9Re h_0 h_2^* = 2(-3B_1 - s_1 + 2) \left( \frac{d\sigma}{d\Omega} \right)_0.
$$

The results of all other polarization experiments can be expressed in terms of these quantities. As an example, let us consider the tensor deuteron polarization from $^3H + ^3He \rightarrow d + 4He$:

$$
Q_{ab} = Q \left[ k_a (\vec{k} \times \vec{P})_b + k_b (\vec{k} \times \vec{P})_a \right], \quad Q = \frac{Im h_0 h_2^*}{|h_0|^2 + 2|h_2|^2}.
$$

This T-odd polarization observable can be connected with the T-even coefficients $s_1$ and $B_1$ through the quadratic relation:

$$
\frac{9}{2} Q^2 = -B_1(B_1 - 1) - 2(1 - s_1 - B_1)^2.
$$
In order to analyze the angular dependence of the reaction products for $^3\text{H} + ^3\text{He} \rightarrow d + ^4\text{He}$ in a magnetic field $\vec{B}$, one can extend the formalism of helicity amplitudes, as for the previously studied reactions.

V. CONCLUSIONS

We have studied nuclear reactions, at low energies, involved in fusion reactors, with special emphasis on their strong dependence on the polarization states of the colliding nuclei. The results obtained here on the angular dependence and the reaction rate dependence on the polarizations, can be used as a guideline in the conception of magnetic fusion reactors. The polarization of the produced particles is also important, as it can help the fusion process in a working reactor. For example, in a reactor based on $d + ^3\text{H}$-fuel, the intensive flux of 14 MeV neutrons can be used in the $\text{Li}$–blanket, not only for its heating, with consequent production of electric power, but also to produce extra $^3\text{H}$-fuel, through the processes: $n + ^6\text{Li} \rightarrow ^3\text{H} + ^4\text{He}$ and $n + ^7\text{Li} \rightarrow n + ^3\text{H} + ^4\text{He}$. Due to the definite polarization properties of these reactions, one can increase, in principle, the yield of $^3\text{H}$.

We showed that the polarization and the angular distribution of the neutrons, produced in the process $d + ^3\text{H} \rightarrow n + ^4\text{He}$ depends strongly on the relative value of the two possible partial amplitudes. The presence of a contribution (even relatively small) of the $J^{\pi} = 1/2^+$ amplitude is very important for polarization phenomena.

For the reaction $d + d \rightarrow n + ^3\text{He}$ (with three independent threshold partial amplitudes) the situation is more complicated. The $d + d$-reactions produce energetic neutrons and tritium, and should be suppressed in a $d + ^3\text{He}$ reactor.

The detailed information about partial amplitudes of different reactions can be obtained, in a model independent way, through the realization of the complete experiment. Even at low energy, where the spin structure of all matrix elements is highly simplified, the complete experiment includes the scattering of a polarized beam on a polarized target. These experiments, which are absent up to now, allow the full reconstruction of the spin
structure of the threshold amplitudes.

The main results contained in this paper can be summarized as follows:

• We give a model independent parametrization of the spin structure of the threshold matrix elements for the following reactions: \(d + d \rightarrow n + ^3He\), \(d + ^3H \rightarrow n + ^4He\), \(^3H + ^3H \rightarrow n + n + ^4He\), \(^3He + ^3He \rightarrow p + p + ^4He\), \(^3H + ^3He \rightarrow p + n + ^4He\), and \(^3H + ^3He \rightarrow d + ^4He\).

• The helicity amplitudes for the processes \(d + d \rightarrow n + ^3He\) and \(d + ^3H \rightarrow n + ^4He\) are calculated in terms of partial threshold amplitudes, choosing the direction of the polarized magnetic field as the quantization axis.

• The angular distribution of the reaction products for \(\vec{d} + \vec{d}\) and \(\vec{d} + \vec{^3He}\)-collisions shows a strong dependence on the polarization of the colliding particles, and it can be very important to optimize the blanket and the shielding of a reactor.

• The polarization properties of neutrons, produced in the processes \(d + ^3H \rightarrow n + ^4He\) and \(d + d \rightarrow n + ^3He\) are derived for collisions of polarized particles.

• The spin structure of the matrix elements and polarization properties are derived for some \(A=3\) induced processes.

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Figure Caption

**Figure 1.** Ratio $\sigma_{0+}/\sigma_{00}$, as a function of $x = |g_s|/|g_d|$ and $y = \cos \theta$ for the reaction $\vec{d} + ^3\text{He} \rightarrow n + ^4\text{He}$, for different values of the phase $\delta$: (a) $\delta = 0$, (b) $\delta = \frac{\pi}{2}$ and (c) $\delta = \pi$, from Eq. (8).

**Figure 2.** Ratio $\sigma_{+-}/\sigma_{00}$ as a function of $x$ and $y$ for the reaction $\vec{d} + ^3\text{He} \rightarrow n + ^4\text{He}$, for different values of the phase $\delta$: (a) $\delta = 0$, (b) $\delta = \frac{\pi}{2}$ and (c) $\delta = \pi$, from Eq. (10).

**Figure 3.** Neutron polarization $P_n = \left( -2 + 3y^2 + x \cos \delta(1 - 3y^2) + x^2 \right) / \left( 2 + x^2 \right)$, as a function of $x$ and $y$ in $\vec{d} + ^3\text{He}$-collisions, for different values of the phase $\delta$: (a) $\delta = 0$, (b) $\delta = \frac{\pi}{2}$ and (c) $\delta = \pi$.

**Figure 4.** Neutron polarization $P_n = \frac{1}{3} \left[ 2(2 - 3y^2) + 4x \cos \delta(1 - 3y^2) + x^2 \right] / \left( 2 + x^2 \right)$, as a function of $x$ and $y$ in unpolarized $d + ^3\text{He}$-collisions, for different values of the phase $\delta$: (a) $\delta = 0$, (b) $\delta = \frac{\pi}{2}$ and (c) $\delta = \pi$. 
