A wrinkling analysis and control of rectangular membrane structures with upscale modelling under on-orbit conditions

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Abstract. This paper presents detailed analysis of a novel concept on the formation and evolution of wrinkle pattern on a multilayer shell-membrane under the action of mechanical and thermal-mechanical loading. The double scale Fourier series is used to deduce the wrinkling equation of membrane structure. In this paper discusses the theoretical aspects of this process and carried out part of the numerical calculations. In particular: wrinkling analysis of square membrane with corner tensile loading. It contains: schematic loading condition on square membrane, membrane with shell elements and experimental analysis of square membrane with tensile loading. Also held stress analysis. In the next paper, the authors will consider a detailed analysis of the research.

1. Introduction

The large space structures are required for the deep space exploration, high speed communication, and accurate earth observation. But the weight of the payload increases when the size of structures increase in the conventional space structures and probability of the successes decreases. Also, the operation cost required to transport mass to the destination is high as well as the volume of payload is limited. Hence, achieving minimal launch mass and volume of space structures is always an important task for the space mission.\textsuperscript{[1]}–\textsuperscript{[5]} Therefore, ultra-lightweight and ultra-large membrane structures are the key element of inflatable reflector, solar sail, solar collector, sun-shields, deployable mirrors surface for the next generation of spacecraft. Many such membrane based space structures have been tested like IN-Step Inflatable antenna experiment\textsuperscript{[6]}, \textsuperscript{[7]}, IKAROS\textsuperscript{[8]}, CubeSat\textsuperscript{[9]}, \textsuperscript{[10]}.

Thin membranes are often subjected to stresses and strains when an external force applied. Membrane structure is very flexible and has strong geometric nonlinearity and can leads to an out of plane displacement due to their negligible flexural rigidity and presence of in-plane compressive stresses\textsuperscript{[2]}, \textsuperscript{[11]}–\textsuperscript{[14]}. Hence it is essential and desirable for a better understanding of the effects of wrinkles on the structural performance and its stability. Surface wrinkling may also be utilized to develop techniques for probing the surface characteristics of materials.\textsuperscript{[13]}, \textsuperscript{[15]}, \textsuperscript{[16]}

Many authors have discussed the various analytical, numerical and experimental methods to study the wrinkling behavior of thin membrane structures\textsuperscript{[16]}–\textsuperscript{[25]}. Reissner (1938) derived the tension field theory without initial tension under in-plane torsion to and analyzed the circular membrane\textsuperscript{[26]}. Stein and Hedgepeth (1961) to analyze partly wrinkled membranes using Stein’s theory\textsuperscript{[27]}. Using the Rayleigh-Ritz method, the circular stretched membrane is analyzed without bending rigidity, but
geometrical stiffness considered by [28]. Damil et al [29] have discussed a wrinkling phenomenon using a new multiscale approach taking one-dimensional beam model. Blandino et al [30] have discussed the wrinkling phenomena at the isothermal condition and prove that number of wrinkles increase with the tensile loads. Also, the amplitude of wrinkles increases with increase in thermal gradient. The full shell methods and the reduced resolution methods can be used to solve the wrinkle problems. For the thermo-mechanical load, Kodjo et al [31] have modified Roddeman’s model [18] to considering thermal effect in the model.

In this paper, thermal effect on flat membrane antenna has been discussed at orbital conditions. Also, authors discussed the wrinkling analysis of the membrane structures under different loading conditions, the effect of prestress, thickness variation, elastic properties, anchor points. Thermal stresses have been introduced in the classical equation of within the framework of the elastic shell theory. Here Asymptotic Numerical Method is used to solve the nonlinear equations and effective active flatness control method is used to minimize the RMS error of the surface wrinkles of the membrane structure. Also, the analysis of bi-layer structure has been performed to minimize the wrinkled area.

2. The mathematical formulation of nonlinear model of coupled thermal and elastic behavior

Current research discussion is focused on to develop a nonlinear model of thin membrane structures under mechanical and thermal loads. The Love–Kirchhoff plate theory is used to derive the equations of the wrinkling model within the von Karman approximation. The membrane model is considered as the elastic and isotropic. The thermal stress has been solving by applying the generalized Duhamel-Neumann form of the Hooke’s law. For the single element thermal strain, \( \varepsilon_{\text{therm}} \) under temperature change is defined as,

\[ \varepsilon_{\text{therm}} = \alpha \Delta T \]  

(1)

The relations of linear strain-displacement (Kinematical) are well known as,

\[ \varepsilon_{xx} = u_{,x} \]
\[ \varepsilon_{yy} = v_{,y} \]
\[ \gamma_{xy} = u_{,y} + v_{,x} \]  

(2)

Where, \( (\cdot)_{,i} = d(\cdot) / dx \) and \( (\cdot)_{,j} = d(\cdot) / dy \).

Let the displacement field be represented by

\[ u = \bar{\varphi}_i u_i \]
\[ v = \bar{\varphi}_j v_j \]  

(3)

Where ‘i’ is the subscript \( i (i=1,2,3,\ldots) \) applies for summation convention and \( u(x, y) \), \( v(x, y) \) and \( o(x, y) \) stand for the displacement functions. \( \bar{\varphi}_i(x, y) \) is the longitudinal variation in \( u(x, y) \), \( v(x, y) \) and \( w(x, y) \) along \( x \) and \( y \)-direction. From equation (2) and (3) we can obtain the strain components as

\[ \varepsilon_{xx} = u_{,i} \bar{\varphi}_i \]
\[ \varepsilon_{yy} = v_{,i} \bar{\varphi}_i \]
\[ \gamma_{xy} = u_{,y} \bar{\varphi}_i + v_{,x} \bar{\varphi}_i \]  

(4)

Now the constitutive (stress-strain) relations of the isotropic sheet, under plane stress can be written as,

\[ \sigma_{xx} = 3 \varepsilon_{xx} + 3 \varepsilon_{yy} \]
\[ \sigma_{yy} = 3 \varepsilon_{xx} + 3 \varepsilon_{yy} \]
\[ \sigma_{xy} = G \gamma_{xy} \]  

(5)
\[ E_{xx} = E_{yy} = \frac{E}{1 - \nu^2}, \quad E_{xy} = E_{yx} = \frac{\nu E}{1 - \nu^2} \]  

(6)

Where \( E \) is the Young’s Modulus, \( \nu \) is the Poisson’s ratio and \( G = E / 2(1 + \nu) \) is the shear modulus. On the basis of Fourier-related approach \[32\] the local unknown filed can be written as

\[ U(x, y) = \sum_{l=-\infty}^{\infty} U_l(x, y) \exp(iRx) \]  

(7)

Where \( R \) is the wave number and \( U_l(x, y) \) the unknown field slowly varies on the period \( [x, y + \frac{2\pi}{R}] \) of the oscillation. Using equation (7) the useful formula of membrane strain equation \[29\], \[32\] can be derived in as

\[ \{\gamma\} = \begin{bmatrix} \frac{\partial u}{\partial x} + \left( \frac{\partial w}{\partial x} \right)^2 + R^2w^2 \\ \frac{\partial v}{\partial y} + \left( \frac{\partial w}{\partial y} \right)^2 \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + 2 \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \end{bmatrix} \]  

(8)

The membrane strain formula Eq. (8) is similar to the Von Karman model. It contains two-part symmetric part of membrane strain and second part of the nonlinear part of wrinkling deformation and is given as

\[ \{\gamma^\prime\prime(w)\} = \begin{bmatrix} \left( \frac{\partial w}{\partial x} \right)^2 + R^2w^2 \\ \left( \frac{\partial w}{\partial y} \right)^2 \\ 2 \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \end{bmatrix} \]  

(9)

The wrinkling initiation in the bilayer structures can describe with the equations of the critical strain, wavelength, and wave number is sufficient and quite simple. The solution of bilayer structures should be extended to multilayer by following a simple intuitive procedure. The equation of wave number by relating the axial strain and to wave number for the bilayer wrinkling has been derived similarly to \[33\] as,

\[ \varepsilon_f \frac{t_f^4}{12} \ell^4 - \varepsilon_f t_f \gamma \ell^2 = -\frac{2\varepsilon_{f'}}{\xi} \ell \]  

(10)

Where \( \ell \) is the wave number, \( \varepsilon_f \) and \( \varepsilon_t \) refer to the plane strain of the film of upper layer lower layer. \( \nu_s \) is the Poison ratio of the substrate, \( t_f \) is the film thickness, and \( \xi = (3 - 4\nu_t) / (1 - \nu_t)^2 \).

**Material Properties.** Many materials with improved combinations of properties are used by space scientist/engineer to select their specific mission requirement \[2, 5, 7, 9\]. Membrane structures consist of two-dimensional pre-stress, elastic thin membrane as a major structural element. The membrane mentioned in this paper is different from the physical “true membrane”. Basically, bending and compressive rigidity does not found in the true membrane. Also, structures made from true membrane does not have structure stiffness. Shell-membrane has stiffness comes from initial pre-stress, not from the material and shape \[20\], \[22\]. In practice, any two-dimensional elastic continuum resists bending moment. However, if the tension is large and the curvatures are small, the effect of bending moment can be neglected. Thus, the membrane can be imagined as an extension of the string to two dimensions \[19\].
For the analysis of membrane structure following assumptions are made as; i) Effect of gravity on the membrane is negligible. ii) Displacement is only in vertical direction. iii) The membrane is thin enough to neglect its volume and only consider its area. iv) The magnitude of pre-stress remains constant and mass density assumed uniform throughout the membrane.

It is perceived that Kapton is one of the best-qualified materials for the space application, which has better mechanical and thermal properties like strength is to weight ratio, tensile strength, etc. The Material properties of Kapton®HN is as Density is 1420 kg/m³, Young’s Modulus is 2.50 x 10⁸ N/m², Poisson’s Ratio is 0.34, Tensile Strength is 231 x 10⁶ N/m², Coefficient of Thermal Expansion (α) is 20 ppm, Specific Heat is 1090 J/kg-K, Thermal Conductivity is 0.12 W/m K.

3. Numerical Examples

In this section several examples are presented for the nonlinear buckling element model is performed to wrinkling analysis of a thin membrane under mechanical and thermal loads. Characterization of the wrinkling behavior of the membrane structures has been carried out in these examples. The nonlinear buckling finite element analysis incorporating thin shell model is used to simulate the detailed wrinkles in the membrane structure. Elastic quasi-static shell analysis and parameter studies of the thin-film membrane are carried out using the geometrically nonlinear updated Lagrangian description and a displacement-based isoparametric finite element formulation. The nonlinear equation is solved by using the Newton-Raphson iteration and dichotomy method as [22], [34]. The macroscopic equation of reduced membrane model is implemented in Abaqus subroutine using Fortran compiler. In their simplest form, these schemes begin by assuming that the behavior of the membrane is linear elastic. Then, any compressive principal stresses are set to zero and the associated stiffness matrix coefficients are also set to zero. The principal stresses are recalculated at every iteration, to eschew history dependency in the results.

4. Wrinkling Analysis of Square Membrane with Corner Tensile Loading

In this section, wrinkling analysis of membrane under corner tensile loading is discussed as Fig. 1 (a). All corners are biaxial loaded by Kevlar tread. Kapton tape is used to hold the membrane under tension condition with Kevlar tread. Threads are directly attached to force sensors to measure the applied tension load as Fig. 2. The membrane and corner tabs are modeled using S4R thin shell elements of different thickness and corner beams are modeled using B21 beam elements with the general beam section. Mesh convergence shows that 2500 S4R shell elements are required for wrinkling analysis of square membrane (as Fig. 1 (b)). The *MPC, TIE function is used to connect each beam node to the corresponding shell element node. The membrane is constrained in both x and y-directions at the center node; all four side edges are left free; the z-translations and all rotational degrees of freedom of the corner beam nodes are restrained. The corner loads are distributed uniformly over the nodes of the beams (with only half of the forces applied at the end nodes). The analysis procedure is essentially identical for all of the simulations. First, a uniform pre-stretch of 0.1 mm is applied to provide some initial out-of-plane stiffness to the membrane. This is achieved by using the *INITIAL CONDITION, TYPE=STRESS parameter in ABAQUS. A nonlinear geometry analysis is then carried out, with the *NLGEOM option activated, to check the equilibrium of the system with this initial pre-stress. The chosen geometrical imperfections are then seeded onto the pristine mesh using the *IMPERFECTION command. The out-of-plane displacements following loading are obtained, showing the number of wrinkles, their amplitudes and their locations on the membrane.
5. Stress Analysis

The large out-of-plane deformations formed as local–post buckling pattern considered as wrinkles in membrane. Negligible bending stiffness and non-resistance of compressive stress in the membrane structures are main cause of the appearance of wrinkles [13]. Membrane structures will deflect immediately after occurrence of local in-plane compression within the structures and hence retain the in-plane principal stresses non-negative [15]. So, the discussion of the membrane mechanical has been distinct in two stages: State of stress before and after wrinkling.

The State of stress before wrinkling, the membrane is in flat conditions, i.e. state of plan stress. Here, membrane is considered as two-dimensional, isotropic elastic materials. Assuming that deformation is very small and body forces are zero before occurrence of wrinkle in the membrane.

Both membrane deformations and bending deformation can be taken in to account after the wrinkle formation in membrane. The FEM modelling of the wrinkling phenomenon has highly non-linear in nature (Lu & Tan, 2012; Nayyar, Ravi-Chandar, & Huang, 2014;). The prediction of wrinkling formation in membrane will play a great role in numerical simulation and design procedure. There is three criteria to predict wrinkle formation: principal stress, principal strain and principal stress-strain criterion (Wang, Tan, Du, & Wan, 2007). In all three criterion, principal stress criterion has been chosen for the analysis.

Let us consider $\sigma_1$ and $\sigma_2$ are the major principal stress and minor principal stress respectively. If
both major and minor principal stress are greater than zero, then membrane is in taut state. If major principal stress is greater than zero and minor principal stresses is non-positive then membrane is in wrinkled state. If both major and minor principal stresses are non-positive then membrane is in slack state.

Fig. 3 shows the distribution of major plane principal stresses at four locations of in the membrane. The stress limits are set at 0 to 30 N/mm$^2$ in order to better visualize the stress variation. The general trend is that the stress decreases as one moves away from the corners of the membrane. For T1/T2 =1 the higher stresses are localized near the four corners (see Fig. 4 (a)), but for increasing T1/T2 the major principal stresses tend to spread along the main diagonal (see Fig. 4 (b)). Because wrinkling is associated with the existence of (small) compressive stresses, it is instructive to consider also the distribution of the minor principal stresses.

![Figure 3. Von-Mises (minimal principal) Stress Value at different zone of the membrane structures](image)

(a) T$_1$/T$_2$=1  
(b) T$_1$/T$_2$=5

**Figure 4.** Stress distribution on membrane under different T$_1$/T$_2$ ration

**6. Conclusion**

In this paper, different structural and thermomechanical condition has been discussed to analyses wrinkling behavior of membrane structures. The following conclusions have been summarized.

The results confirmed that square membrane with corner tensile loading has four fans of wrinkles near the corners if T$_1$/T$_2$<1/(√2−1). But they go through the center if T$_1$/T$_2$ ≥ 1/(√2−1) and less number of wrinkles will be formed along diagonal [24]. In this, it is also observed that surface error of the membrane structures increases with increases with T$_1$/T$_2$. 

![Figure 4. Stress distribution on membrane under different T$_1$/T$_2$ ration](image)
A comprehensive study of this issue needs to be continued. In the next article, the authors will consider a detailed analysis of the research.

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