Chiral Morphing

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Abstract
Chiral symmetry undergoes a metamorphosis at $T_c$. For $T < T_c$, the usual Noether charge, $Q^a_5$, is dynamically broken by the vacuum. Above $T_c$, chiral symmetry undergoes a subtle change, and the Noether charge morphs into $Q^\beta_5$, with the thermal vacuum now becoming invariant under $Q^\beta_5$.

This vacuum is however not invariant under the old $Q^a_5$ transformations. As a result, the pion remains strictly massless at high $T$. The pion propagates in the early universe with a halo.

New order parameters are proposed to probe the structure of the new thermal vacuum.

1 Prologue

It is a pleasure for me to dedicate this to the memory of Marshak, whose life work has revolved around the pion and chirality. When he first came to City College as the eighth President in 1971, he not only plunged into his duties as president with characteristic energy and gusto, scheduling breakfast meetings with faculty and staff, but he also squeezed into his day an early (8 a.m.) theoretical physics seminar in the President’s Conference Room during his first year at City College.

In those days, it was with a sense of awe that we got to step into the halycon halls of administration for a round of physics seminars. And promptly at the hour of 8, Marshak would enter the Conference Room with the big oval table and the seminar would begin. Some of the invited speakers had come from as far as sixty miles away!

\footnote{Invited contribution to the Marshak Memorial volume, edited by E.C.G. Sudarshan, to be published by World Scientific Publishing Co., Singapore, 1994.}
After Sakita returned from France, the seminar was moved back to the more normal hour of 2 p.m. on Friday. By then, Marshak had gotten the hang of the presidency, and religiously found time to come to the seminar for the hour and half of physics with peace and tranquility.

His interest in quality and excellence did not flag through those days of NY City fiscal crisis, budget cuts and student unrest. In spite of it, he was able to set up the Sophie Davis biomedical program that has become one of the jewels of City College, the Levich institute of hydrodynamics, and many other initiatives that have had a lasting impact on City College.

For the setting up of the Levich institute, Marshak acted swiftly upon hearing of the impending emigration of Benjamin Levich from Russia and pulled all strings both domestically and internationally to get Benjamin Levich to come to City College. There were sensitive cold war constraints and other complications, and the deal involved Israel. And so on one Friday afternoon, Marshak entrusted Bunji and me with a hurried mission to intercept Yuval Ne’eman on one of his unannounced trips to New York.

So there we were, standing at the exit hall of the Kennedy airport waiting for El Al passengers to clear customs. We waited and waited. We checked with El Al to verify that Ne’eman was on the passenger list. The El Al security quietly alerted Ne’eman to the fact that ‘two unidentified oriental strangers were standing in the hallway’. And so Ne’eman was asked to come out by a side entrance, and we were finally accosted by Ne’eman only after he recognized who we were.

In those days, there was nothing that Marshak could not do as a President. We at City College owe a lot to him for his having assumed the mantle of office after open admissions plunged the college into a crisis of identity.

Amid all the turmoil, it was fitting that City College was the venue in 1977 for a 60th birthday celebration of Marshak as a truly distinguished physicist of international renown and stature, a scholar, humanitarian and administrator. The theme of the symposium, Five Decades of Weak Interactions[1], emphasized his abiding interest in chirality and $V - A$.[2]

2 The Pion at $T = 0$

The ubiquitous pion has played a central role in the early history of particle physics. Marshak’s two meson theory brilliantly resolved the early puzzle and confusion between the ($\mu$) meson observed at sea level and the strongly interacting $\pi$ meson proposed by Yukawa. The two meson theory was later confirmed by accelerator experiments that produced copious numbers of pions which decay to muons.

How does the pion couple to matter?

Theorists struggled with this. Early on, there was a dichotomy in the description, whether it should be a simple $\gamma_5$ coupling or a derivative coupling. One was a renormalizable coupling, while the other was manifestly not. And yet from the point of view of pion coupling

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to nuclear matter, there was an equivalence theorem that was valid in the nonrelativistic limit.

With the advent of current algebra and PCAC, the realization grew that the pion coupling is a special one. Rather than a simple $\gamma_5$ coupling, it is fundamentally a derivative coupling to matter.

The reason for this derivative coupling was clarified when the work of Nambu-Jona-Lasinio (NJL)\(^3\) led to a new understanding of the special role that the pion would play in the world at $T = 0$. For the fundamental Lagrangian, in the absence of a primordial mass, has the $U(2)_A$ chiral symmetry

$$\psi(x,t) \rightarrow e^{i \alpha \gamma_5} e^{i \vec{\tau} \cdot \vec{\alpha} \gamma_5} \psi(x,t)$$

This symmetry, so to speak, protects the fermion from acquiring a mass, just as gauge invariance in its simplistic form protects the photon from acquiring a mass. The Noether charge associated with this chiral symmetry has the form

$$Q^5_{ij} = \frac{1}{2} \int d^3x \psi^\dagger_j(x,t) \gamma_5 \psi^i(x,t)$$

$$= -\frac{1}{2} \sum_{p,s} s \left( a^i_{p,s} a^\dagger_{p,s} - b^\dagger_{-p,s} b_{-p,s} + \delta^i_j \right)$$

where $a_{p,s}$ and $b_{-p,s}$ are the massless quark and antiquark operators, and $s$ is defined to be $\pm 1$ for $R$ and $L$ helicities respectively.

$Q^5_{ij}$ may be decomposed into the Noether charges, $Q_5$ for the $U(1)_A$ symmetry, and $Q^a_5$ for the $SU(2)_A$ symmetry. $Q_5$ has, however, an instanton anomaly so that it is not a constant of motion but the isovector charge $Q^a_5$ commutes with the Hamiltonian

$$[H, Q^a_5] = 0.$$

Naively, we would expect the ground state to be invariant under $Q^a_5$ as well. As Nambu and Jona-Lasinio have already pointed out in 1961, however, the internal dynamics could result in a ground state that nevertheless is not invariant. This is much like in the ferromagnetic analogy, where even though the Hamiltonian is rotational invariant, the ground state is not the $J = 0$ state. It is instead a state where all the $N$ spins are lined up, so that it has maximum eigenvalue, $Nh/2$.

As a result of the dynamical symmetry breakdown, the fermions acquire mass, even though the fermions in the fundamental Lagrangian are massless. In addition, as Nambu and Goldstone\(^4,5\) have shown, a signature of this dynamical symmetry breakdown is the existence of a strictly massless excitation carrying the quantum numbers of $Q^a_5$.

But the observed pions are not massless, indicating an explicit breaking of the chiral symmetry. This is as a result of the electroweak breaking that generates a tree level running mass for the quarks through the Yukawa coupling. The pion is thus only an approximate Nambu-Goldstone boson, and we may use the (broken) QCD Noether charge as

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the interpolating field for the pion,

\[ \phi^a(\vec{x}, t) = \frac{i}{m_\pi^2 f_\pi} \partial^\mu J_\mu^5(\vec{x}, t) \] (5)

In the limit \( m_\pi \to 0 \), the axial current is conserved, and the interpolating field goes over into

\[ \phi^a(\vec{x}, t) = \frac{i a_\pi}{\Lambda_c} \bar{\psi} \gamma_5 \tau^a \psi \] (6)

The two forms of the interpolating fields are equivalent, although for low energy phenomenology, the former is more convenient. Because of the derivative in the interpolating field eq.(5), it is easy to see directly that the pion interaction with matter obeys the zero energy decoupling theorem. A soft pion with \( p_\mu \to 0 \) decouples from matter.

3 Probing the Vacuum

As Lee\[6\] emphasized in 1975, the physical vacuum instead of being an ‘empty’ state is rich with structure. In our case, the NJL vacuum is filled with massless quark-antiquark pairs of the same helicity. How may we hope to probe the vacuum and measure this rich structure?

One way is to create a mini big-bang at a heavy ion accelerator and hope that the violent collision creates in the center of mass frame a new vacuum in a small region of space which rapidly expands. Such a new vacuum will be a rich source of pion multiplicities which evaporate and may be observed at RHIC.

This new vacuum will not have translational invariance, since the center of mass is clearly the preferred frame. As suggested by Rubakov \textit{et al.}, \[7\], it is conceivable that the system could develop different domains within each of which is a different orientation of chiral condensates. These disoriented chiral condensates\[8\] lead to new charge correlations in the pion multiplicities.

The other way is to probe the physical vacuum through the measurement of order parameters. For chiral symmetry, the well-known order parameter is \( <\bar{\psi}\psi> \). There is a famous theorem which states that if \( <\bar{\psi}\psi> \neq 0 \), then

\[ Q^a|\text{vac}\rangle \neq 0. \] (7)

This is easily seen to be an immediate consequence of the the equal time commutation rule

\[ [Q^a, \ i \int d^3x \bar{\psi} \gamma^b \gamma_5 \psi] = -i \delta^{ab} \int d^3x \bar{\psi} \psi \] (8)

since by taking the vacuum expectation of the operators on both sides, a non-vanishing \( <\bar{\psi}\psi> \) implies that \( Q^a \) cannot annihilate the vacuum.

At issue in our later discussion is whether the converse of this theorem should also be true. I will show here by explicit construction that the converse is indeed false, and point
out how $\langle \bar{\psi} \psi \rangle$ is an incomplete indicator of dynamical chiral symmetry breaking in the ground state.

To fully probe the structure of the vacuum, other order parameters will have to be measured. As we shall see below, they are non-local order parameters, which explains why they have not been considered before by others.

4 NJL vacuum with Flavor

Nambu and Jona-Lasinio worked with the case of one flavor in writing down the paired state. They worked with a relativistic generalization of the BCS theory in superconductivity. The NJL ground state may be obtained as an $X_2$ rotation of the Fock space vacuum

$$|\text{vac}⟩ = \prod_{p,s} e^{i \theta_p X_{2p}} |0⟩ \quad (9)$$

where $X_{2p}$ is an element of the chirality algebra, represented by

$$X_{3p} = -\sum_s \frac{s}{2} \left( a_{p,s}^\dagger a_{p,s} + b_{-p,s}^\dagger b_{-p,s} \right) \quad (10)$$

$$X_{2p} = i \sum_s \frac{s}{2} \left( a_{p,s}^\dagger b_{-p,s}^\dagger - b_{-p,s} a_{p,s} \right) \quad (11)$$

$$X_{1p} = \sum_s \frac{1}{2} \left( a_{p,s}^\dagger b_{-p,s}^\dagger + b_{-p,s} a_{p,s} \right) \quad (12)$$

The $X$ operators generate the $SU(2)$ algebra, with $X_{3p}$ easily recognizable as the usual chirality charge. The $X_{2p}$ rotation populates the Fock vacuum with quark-antiquark pairs. The angle, $\theta_p$, of this rotation is related to the mass $m$ acquired by the fermion through the NJL gap equation

$$\tan 2\theta_p = \frac{m}{p} \quad (13)$$

Note that the limiting angle, $\theta_p = \pi/4$, corresponds to an infinite mass gap.

For strong interaction dynamics, we should in principle work with QCD. The ground state here involves not just quarks and antiquarks but also gluons. We imagine working with an effective theory where the gluon degrees have been integrated out, and so will continue to deal only with quarks and antiquarks.

For the case of two flavors, we shall consider the generalization of the NJL vacuum. We begin with the state

$$|\text{vac}⟩' = e^{i \theta_p X_{2p}} e^{i \theta_{\hat{n}} \hat{X}_{2p}} \hat{n} |Ω⟩ \quad (14)$$

where $\hat{X}_{2p}$ is an obvious isovector generalization of $X_{2p}$

$$\hat{X}_{2p} = \sum_s \frac{s}{2} \left( a_{p,s}^\dagger \sigma b_{-p,s}^\dagger - b_{-p,s} \sigma a_{p,s} \right) \quad (15)$$
and \( |\Omega > \) is the \( Q^a_5 \) chiral invariant background state. This background state is itself populated with *quartet* of quarks and antiquarks

\[
|\Omega > = \prod_{p,s} \left( \cos \alpha + \sin \alpha \ a_{1,p,s}^\dagger a_{2,p,s}^\dagger b_{+p,s}^{11} b_{-p,s}^{12} \right) |0 >
\]

(16)

In general, for \( \alpha \) not equal to the special fixed point, \( \alpha = \pi/4 \), the state \( |\text{vac}'> \) is dependent on \( \theta_p' \), and it is not invariant under \( Q^a_5 \) chirality. For the asymptotic case of \( \alpha = \pi/4 \), however, the background state is unchanged under the \( \vec{X}_2 \) rotation, and \( |\text{vac}'> \) becomes \( Q^a_5 \) invariant.

From this, we may construct the most general representation of a chiral non-invariant NJL vacuum by writing

\[
|\text{vac}> = \prod_{p,s} \left\{ f - s \bar{g} e^{i\xi} a_{p,s}^\dagger - s \bar{g}' e^{i\xi} \hat{n} \cdot (a_{p,s}^\dagger \bar{b}_{-p,s}^{11}) + h e^{i\xi} a_{1,p,s}^\dagger a_{2,p,s}^\dagger b_{-p,s}^{11} b_{+p,s}^{12} \right\} |0>
\]

(17)

where

\[
f = \frac{\cos \alpha}{2} \left( \cos 2\theta_p + \cos 2\theta'_p \right) + \frac{\sin \alpha}{2} \left( \cos 2\theta_p - \cos 2\theta'_p \right)
\]

(18)

\[
h = \frac{\cos \alpha}{2} \left( \cos 2\theta_p - \cos 2\theta'_p \right) + \frac{\sin \alpha}{2} \left( \cos 2\theta_p + \cos 2\theta'_p \right)
\]

(19)

\[
\bar{g} = (g \cos \xi' + ig' \sin \xi')
\]

(20)

\[
\bar{g}' = (g' \cos \xi' + ig \sin \xi')
\]

(21)

\[
g = \frac{(\cos \alpha + \sin \alpha)}{2} \sin 2\theta_p
\]

(22)

\[
g' = \frac{(\cos \alpha - \sin \alpha)}{2} \sin 2\theta'_p
\]

(23)

The generalization here is to introduce the \( \xi \) and \( \xi' \) phases. At \( T = 0 \), we may make use of the arbitrary phase in the \( b_{-p,s} \) operator to redefine it so that the \( f, g, g' \) and \( h \) functions are real. This comes about because the \( b_{-p,s} \) operator is defined in terms of the filled Dirac sea, and there is an arbitrary phase in the relation between \( b_{-p,s} \) and the negative energy operator, \( d_{p,s}^\dagger \). Once this has been fixed, however, the evolution of the functions for higher \( T \) may lead to new phases that cannot be absorbed into the definition of \( b_{-p,s} \). So, our eq.(17) is meant for the general thermal vacuum at higher \( T \).

Under a chiral transformation, the NJL vacuum changes to a new unitarily inequivalent ground state. Since \( Q^a_5 \) commutes with the Hamiltonian, this new ground state must have the same energy as the original one. It costs nothing in energy to chiral transform the vacuum. The excitations associated with this transformation are thus zero energy (Nambu-Goldstone) modes. For the case of two flavors, the Nambu-Goldstone modes have the same quantum numbers as the \( \vec{\pi} \) triplet.
5 Order Parameters Old & New

The standard signal for the breakdown of chiral symmetry is the nonvanishing of \( < \bar{\psi} \psi > \). As we shall see in this section it is a poor indicator of the state of chiral symmetry of the vacuum. A vanishing \( < \bar{\psi} \psi > \) does not imply that the vacuum is chiral symmetric.

To see this, we may evaluate explicitly the standard order parameter and find

\[
\frac{1}{2} < \bar{\psi} \psi > = \frac{1}{V} \sum_{p,s} \frac{s}{2} \langle \text{vac} | \left( a_{p,s}^\dagger b_{-p,s}^\dagger + b_{-p,s} a_{p,s} \right) | \text{vac} \rangle
\]

\[
= - \frac{\cos 2\alpha}{V} \sum_{p,s} \left( \sin 2\theta_p \cos 2\theta'_p \cos \xi \cos \xi' - \cos 2\theta_p \sin 2\theta'_p \sin \xi \sin \xi' \right) (25)
\]

Eq.(25) shows that, contrary to popular folklore, a vanishing \( < \bar{\psi} \psi > \) does not necessarily imply that the vacuum is chirally symmetric. For example, if the phases \((\xi, \xi') = (\pi/2, 0)\), say, the order parameter would vanish even though the vacuum has a non-zero chirality angle, \( \theta'_p \). Eq.(17) shows that the corresponding NJL vacuum is not \( Q^a \) invariant.

\( < \bar{\psi} \psi > \) by itself is thus a deficient indicator of the state of chiral symmetry at high temperatures. We need to measure other order parameters. Actually there is an \( SU(2N_f) \times SU(2N_f) \) chirality algebra\[18\] formed out of the bilinear quark and antiquark operators. The algebra includes the order parameters identified as

\[
Y_p = \sum_s \frac{s}{2} \left( a_{p,s}^\dagger b_{-p,s}^\dagger + b_{-p,s} a_{p,s} \right)
\]

\[
\vec{Y}_p = \sum_s \frac{s}{2} \left( a_{p,s}^\dagger \vec{\tau} b_{-p,s}^\dagger + b_{-p,s} \vec{\tau} a_{p,s} \right)
\]

\[
X_{2p} = i \sum_s \frac{s}{2} \left( a_{p,s}^\dagger b_{-p,s}^\dagger - b_{-p,s} a_{p,s} \right)
\]

\[
\vec{X}_{2p} = i \sum_s \frac{s}{2} \left( a_{p,s}^\dagger \vec{\tau} b_{-p,s}^\dagger - b_{-p,s} \vec{\tau} a_{p,s} \right)
\]

\[
\vec{N}_p = \sum_s \frac{1}{2} \left( a_{p,s}^\dagger \vec{\tau} a_{p,s} - b_{-p,s} \vec{\tau} b_{-p,s}^\dagger \right)
\]

where \( < \bar{\psi} \psi > \) may be obtained by a sum over all momenta of \( 2Y_p \).

By taking the vacuum expectation values of these order parameters in the generalized NJL vacuum, it can be shown that only when the vacuum expectation value of all five order parameters listed above vanish may we conclude that the ground state is chirally symmetric.\[18\]

While \( Y_p \) and \( \vec{Y}_p \), when summed over \( \vec{p} \), are related to the familiar local order parameters in space-time, the other order parameters are new and intrinsically non-local, with

\[
\sum_p X_{2p} = i \frac{1}{4} \int d^3 x \bar{\psi} \gamma_\mu \gamma_5 \vec{\nabla} \psi \quad + \text{h.c.}
\]
\[ \sum_p \bar{N}_p = \frac{1}{4} \int d^3 x \bar{\psi} \vec{\tau} \frac{\vec{\nabla} \cdot \vec{\nabla}}{\sqrt{-\nabla^2}} \psi + \text{h.c.} \]  

(32)

Their vacuum expectation values yield new insights into the structure of the chiral broken vacuum.

6 Signatures at high \( T \)

QCD studies have shown that \( < \bar{\psi} \psi > \) vanishes at \( T_c \), and stays zero above \( T_c \). The popular folklore is to conclude that chiral symmetry is restored above \( T_c \). This is supported by the work of Tomboulis and Yaffe \cite{10} who have shown rigorously in lattice gauge theories that the effective action has a strict global chiral invariance at high \( T \). This is also seen in the manifest chiral symmetry of the continuum QCD effective action at high \( T \) as derived by Braaten-Pisarski and Frenkel-Taylor-Wong. \cite{11}

And yet, the same continuum studies also show that a massless quark propagates in the hot QED and QCD environment as if it had a mass gap proportional to \( gT \), for \( p \) large. \cite{12, 13} At zero \( T \), the origin of fermion mass was attributed to chiral symmetry breakdown. Could this thermal mass at high \( T \) be due to the continued breakdown of chiral symmetry? If so, how is it possible that the effective action for hot QCD is nonetheless chiral invariant?

To resolve the apparent paradox in these signatures, it is best to follow the evolution of the NJL vacuum with temperature. In this picture, the key to chiral symmetry breaking is in the population of quark-antiquark pairs in the ground state. This population is controlled by the chirality angles, \( \theta_p \). In this language, the popular folklore asserts that \( \theta_p \) should decrease with temperature, so that the ground state becomes depopulated of these pairs. The pairs ‘heat up’ with temperature and proceed to break up. When the temperature reaches a critical point, \( \theta_p \) vanishes, and there are no more pairs in the new thermal ground state, and chiral symmetry is said to be restored. In the popular jargon, the quark-antiquark condensate dissolve above \( T > T_c \), and it very nicely explains away the vanishing of \( < \bar{\psi} \psi > \).

According to eq.(13), when \( \theta_p = 0 \), the gap parameter vanishes, and the quark should therefore propagate in a hot medium as if it were massless. For the NJL model, this is indeed what happens.

For QCD, however, the scenario with a vanishing mass gap poses a problem. For it conflicts with the well-known result that the massless quark propagates in a hot QCD medium as if it had a ‘thermal mass’ proportional to \( T \).

Could it be that QCD opts for an alternate scenario? As temperature increases, the chirality angle \( \theta_p \) increases rather than decreases, so that it approaches the critical angle \( \pi/4 \) as \( T \to \infty \). In this limit, according to eq.(13) the gap parameter approaches infinity proportional to \( T \), thus explaining the ‘thermal mass’.
At first sight, you might worry that a vanishing $< \bar{\psi} \psi >$ would pose a problem for this scenario. But as eq.(25) shows, a generalized NJL vacuum may very easily be populated with quark-antiquark pairs and still enjoy a vanishing $< \bar{\psi} \psi >$.

Such a thermal vacuum continues to break the old zero temperature chirality, and the pion should remain strictly massless at high $T$. This conclusion conforms to the earlier continuum study of the effective QCD action with external scalar $j$ and pseudoscalar $j_5$ sources which showed that the QCD pion is massless for all $T$.\[14\]

But in spite of the continued breaking of zero temperature chirality, the QCD effective Lagrangian at high $T$\[11\]

$$L_{\text{eff}} = -\bar{\psi} \gamma_\mu \partial^\mu \psi - \frac{T^2}{2} \bar{\psi} \left( \frac{\gamma_0 - \vec{\gamma} \cdot \hat{n}}{D_0 + \hat{n} \cdot \hat{D}} \right) \psi$$

shows a manifest global chiral symmetry invariance

$$\psi_\beta \rightarrow e^{i\beta} \gamma_5 \psi_\beta$$

How may we reconcile the two?

7 Chiral metamorphosis

The answer comes when you examine the Noether charge for this high $T$ chiral symmetry. It is a metamorphosis from the $T = 0$ Noether charge, $Q_5^a$. By carrying out a canonical quantization of this non-local effective action\[13\], I have found that it is given by

$$Q_5^{\alpha \beta} = -\frac{1}{2} \sum_{p,s} s \left( A^\dagger_{p,s} \tau^\alpha \sigma_{p,s} - B_{-p,s} \tau^\alpha B^\dagger_{-p,s} \right)$$

(35)

to be compared with the original chiral charge

$$Q_5^a = -\frac{1}{2} \sum_{p,s} s \left( a^\dagger_{p,s} \tau^a \sigma_{p,s} - b_{-p,s} \tau^a b^\dagger_{-p,s} \right)$$

(36)

The curious thing about this high temperature chirality is that the operators $A_{p,s}$ and $B_{-p,s}$ in eq.(35) here are massive operators. They are in contrast to eq.(34), given in terms of massless operators.

The new thermal vacuum is invariant under the high $T$ chirality. It is annihilated by $Q_5^{\alpha \beta}$ through the defining properties

$$A_{p,s} |\text{vac}\rangle_\beta = B_{-p,s} |\text{vac}\rangle_\beta = 0$$

(37)

As we will show below this thermal vacuum however is not annihilated by the massless operators, $a_{p,s}$ and $b_{-p,s}$. And by eq.(3), we may conclude that the zero temperature $Q_5^a$ symmetry is not restored for $T > T_c$. Instead, what happens is a metamorphosis of the chiral symmetry across $T_c$.

The morphing consists in replacing the massless with the massive operators in eq.(3).
8 Free vs Dressed Vacuum

How is this new thermal vacuum related to the vacuum of the massless free field?

At this point, a word about the notion of a thermal vacuum may be in order. The thermal average is taken over the entire spectrum of states with the usual Boltzmann weight. In thermofield dynamics\cite{16}, or real time field theory\cite{17}, we replace this thermal average with a thermal vacuum expectation value

$$<\bar{\psi}_H(0)\psi_H(0)>_{av} = \frac{1}{Z} \sum_n e^{-\beta E_n} <n|\bar{\psi}_H(0)\psi_H(0)|n> \quad (38)$$

\[\equiv \beta \langle vac|\bar{\psi}_\beta(0)\psi_\beta(0)|vac\rangle_\beta \quad (39)\]

where $|vac\rangle_\beta$ is a doubled Hilbert space, supplemented with the operators of the heat bath. The "heat bath degrees of freedom are a mirror image of the physical degrees of freedom, with the crucial difference that the time evolution operator is governed by $-\tilde{H}$.

Recall that in the original Hilbert space in which $\psi_H$ operates, at time $t = 0$, the Heisenberg field overlaps with the asymptotic field, $\psi_{in}$, as well as with the perturbative free field, $\psi_o$. From the local field identity

$$\bar{\psi}_o \psi_o(0) = \bar{\psi}_H \psi_H(0) \quad (40)$$

it follows that

$$<\bar{\psi}_o \psi_o>_{av} = <\bar{\psi}_H \psi_H>_{av} \quad (41)$$

and since at high $T$ the order parameter vanishes, we arrive at the condition in terms of the free massless operators

$$\frac{1}{V} \sum_{p,s} \beta \langle vac| s \left(a_{p,s}^\dagger \tilde{b}_{-p,s}^\dagger + b_{-p,s} a_{p,s}^\dagger \right) |vac\rangle_\beta = 0 \quad (42)$$

This equation provides the boundary condition that relates the thermal vacuum $|vac\rangle_\beta$, defined by eq.(37), to the perturbative free vacuum, defined by

$$a_{p,s}^\dagger |0> = b_{-p,s} |0> \quad (43)$$

This boundary condition is in fact what we have written down in eq.(25), with the added proviso that the mass gaps are proportional to $T$, since $A_{p,s}$ and $B_{-p,s}$ are massive operators.

9 Is there more than one $T_c$?

The BP-FTW effective action for hot QCD is valid at high $T$ when chirality has already morphed into the new $Q_5^\beta$ phase. Without studying the transition taking place at $T_c$, it is not possible to say definitively the precise phase that the vacuum is in. It is nevertheless interesting to speculate on it.
At zero temperature, the phase angles $\xi$ and $\xi'$ are vanishing, as per definition, while the chirality angles, $\theta_p$ and $\theta'_p$, corresponding to the isoscalar and isovector mass gaps, are non-zero. In the presence of the heat bath, the angles will become temperature dependent. For the chirality angles, they may range between 0 and $\pi/4$, corresponding to the physical range of the mass gap. At any finite temperature, the chirality angles must be less than $\pi/4$.

The new $Q^g_5$ phase will set in at the transition point due to any of the following possibilities:

1. For $T \geq T_c$, $(\xi, \xi') = (\pi/2, 0)$ or $(0, \pi/2)$

I like this possibility the most because of the mysterious phase of $\pi/2$ that it involves. The $\xi$ and $\xi'$ start at zero as temperature rises, until one of them reaches $\pi/2$ and freezes there for $T \geq T_c$. The chirality angles, $\theta_p$ and $\theta'_p$, begin at some non-vanishing values corresponding to the chiral broken ground state isoscalar and isovector mass gaps. These mass gaps grow with temperature and continue through the $T_c$ to become proportional to $T$ at high $T$. This scenario applies to any number of generations.

2. For $T \geq T_c$, $(\xi, \xi') = (0, 0)$, with $\theta_p = 0$

In this scenario, the phase angles $\xi$ and $\xi'$ remain zero throughout, while $\theta_p$ decreases with temperature until it reaches zero at $T_c$ and stays zero for higher temperatures. $\theta'_p$, on the other hand, grows with temperature. Above $T_c$, the $Q^g_5$ chiral symmetry is broken by the isovector mass gap which becomes proportional to $T$ at high $T$. This scenario is possible only for two or more generations. Actually, eq.(22) suggests also the alternative possibility of having $\theta'_p = \pi/4$ at $T_c$ in lieu of $\theta_p = 0$. This is ruled out, however, on physical grounds. For if $\theta'_p$ reaches $\pi/4$ at $T_c$, the mass gap will blow up at a finite temperature.

3. For $T \geq T_c$, $(\xi, \xi') = (\pi/2, \pi/2)$, with $\theta_p = 0$

In this scenario, both $\xi$ and $\xi'$ reach $\pi/2$ at the same temperature, $T_c$, when the isoscalar mass gap vanishes. The isovector mass gap increases with temperature as before.

With such a rich structure of phases, it would be surprising that they all undergo the phase transition at the same temperature, $T_c$. This raises the interesting question as to whether there are more than one $T_c$ in the so-called transition region, with $T_{c_1}$ as the
temperature where $\xi$ reaches $\pi/2$, and $T_{c2}$ the temperature when $\xi'$ reaches $\pi/2$. Studies of the new order parameters promise to shed more light on the chiral morphing transition region.

10 The Pion in the Early Universe

So far, our attention has been focused on the structure of the QCD vacuum at high temperatures. But the real messenger of the broken vacuum is the pion. At zero temperature, the properties of the pion have been well understood. At high temperatures, how does the pion interact with matter?

As mentioned earlier, the continued breaking of the old $Q_a^5$ chirality demands that the pion be a Nambu-Goldstone boson at high $T$. In the standard model, with a fundamental Higgs, electroweak symmetry is restored above $T_{ew}$. There is thus no explicit quark mass term in the effective Lagrangian at high $T$. The continued breaking of $Q_a^5$ chiral symmetry implies a strictly massless pion at high $T$.

The implications for this for the early universe are profound.

In the standard folklore, the early universe is an alphabet soup of quarks and gluons. At first people thought that the quarks and gluons are all massless. This has since been qualified with the realization that the quarks and gluons had thermal mass. It being of order $gT$, the thermal mass does not have a major impact on the equipartition of energy between quarks and gluons.

What is significant in the standard folklore is that the pions are missing from the alphabet soup.

In our understanding of the state of chiral symmetry at high $T$, the pion will have a very good reason to be in the alphabet soup in the early universe. Namely, the Nambu-Goldstone theorem is going to demand that the binding energy of the massless quark-antiquark state be so deep that the resulting bound state must be strictly massless. They cannot dissociate with temperature.

To be sure, the presence of pions in the alphabet soup is not going to change critically the entropy of the early universe, since they do not have that many degrees of freedom. There will presumably be other more subtle changes in the scenario of the cooling of the early universe. Not being a cosmology expert, I can only speculate that the ubiquitous pion will continue to hold the secret to the behavior of matter at high temperature and high densities.

I will end this discussion with a note on the properties of the pion at high temperature. For while the pion propagator has a massless pole in the complex momentum plane, lattice calculations also show that the pion has a screening mass that is proportional to $T$.

Is there a conflict between the two results?

What I will show is that in a hot environment, you can have a particle propagating through
the medium possessing both those properties. In the language of real time thermal field theory, it is easy to find an example of such a particle. For the physical massless pole is determined from the condition that the denominator of the propagator vanish \[ 19 \]

\[
\Gamma_\pi^{(2)}(p, p_0, T) = p^2 (1 + A)^2 - p_0^2 (1 + B)^2 = 0 \tag{44}
\]

where \( A \) and \( B \) are functions of \( p, p_0, T \). The screening mass on the other hand comes from integrating over the \( x, y, t \) coordinates (i.e. set \( p_x = p_y = p_0 = 0 \)) in the propagator, so that the pole for the correlation function in \( z \) occurs at \( p_z = im_{sc} \), where \( 1 + A(im_{sc}, 0, T) = 0 \)

In terms of a physical picture, when we receive light from a charged particle, we see it at its retarded position, and it is a sharp image. For the pion, the retarded function reads

\[
D_{ret}(\vec{x}, t) = \theta(-t) \left\{ \delta(t^2 - r^2) + \frac{T}{r} \theta(t^2 - r^2) \left[ e^{-T|t-r|} + e^{-T|t+r|} \right] \right\} \tag{45}
\]

so that the screening mass leads to an accompanying modulator signal that ‘hugs’ the light cone, with a screening length \( \propto 1/T \).

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