The Economic and Long-Term Health Consequences of Canadian COVID-19 Lockdowns

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Introduction
During the COVID-19 pandemic, policy-makers worldwide faced the challenge of balancing reduced disease transmission from economic lockdowns against the costs of these restrictions. These costs are not small: in Canada, total hours worked fell 27.6% during the initial lockdown between February and April 2020. Even 11 months later, many sectors remained shuttered: wholesale and retail hours worked in January 2021 were 10% lower than in February 2020, and restaurant and accommodation jobs were 32.8% below their prepandemic levels. Further, job loss is not merely an economic problem. Long-term health consequences result from spells of unemployment and reduced income (e.g., Laliotis, Ioannidis, and Stavropoulou 2016). Appropriately balancing the costs and benefits of lockdowns is particularly relevant during possible future COVID-19 waves or other public health emergencies, since economic and health data gathered in early 2020 can help better target restrictions.

We examine lockdown policies by focusing on effects other than disease transmission. In particular, we combine...
an economic model with a public health model modeling the effect of income loss on medium-run excess morbidity and mortality. The standard lockdown trade-off weighs disease transmission from keeping sectors open against the direct harm to that sector from a closure. However, the economic consequences in each sector are not contained there, and the health consequences of income loss vary across sectors both due to the demographics of workers in that sector itself and in other sectors which suffer indirect harm. Both the qualitative and quantitative nature of these harms are not obvious ex ante. It is important to understand the relative magnitudes of these effects compared with the benefits of reduced disease transmission. Even more basically, it is important to know whether the best sectors to shut down are those on equilibrium economic, long-term health, and disease transmission grounds are similar.

Our economic model estimates the direct productivity shock that lockdowns impose on each sector and traces the effect of these restrictions on the rest of the economy using the “production network” approach. The fundamental idea is that productivity gains or losses in one sector affect firms that use that sector as an input to their own production. For example, if rail transportation were prohibited, and oil extraction otherwise unrestricted, there would nonetheless be serious harms to the oil industry, since sending their product to refineries would become more expensive. On the other hand, a restriction on rail transportation is likely to have little impact on the television broadcasting industry.

The public health model uses well-calibrated estimates of excess mortality and health care utilization from reduced income to map these sector-by-sector economic projections into negative health outcomes (Rosella et al. 2018; Rosella, O’Neill, and Fisher 2020). Income loss harms mental health, cardiovascular health, food security, and continuity of medical care, which in turn result in higher rates of chronic disease, multimorbidity, and early death. These risks vary substantially according to sociodemographics, and hence the long-term health consequences of lockdowns vary substantially depending on which sectors face equilibrium job loss (Fitzpatrick et al. 2015).

Empirically, we begin with estimated productivity shocks that are based on third-party calculations of what fraction of Canadian jobs in each sector are possible to do at home and an estimate of what percentage of jobs are “essential” according to a March 2020 Italian classification and a more disaggregated Canadian government classification. We trace through the indirect equilibrium effects of these direct productivity shocks. The model-estimated reduction in employment, as measured by hours worked by sector, is highly correlated to the actual reduction in employment by sector in April 2020 at the peak of the lockdown. Finally, we use the labour force demographics of each sector and estimated unemployment to project excess deaths and excess high-resource health system users. The latter measure captures non-mortality outcomes that nonetheless represent major adverse changes in long-term health.

Our primary findings are the following. First, there are large and heterogeneous economic harms from lockdowns by sector. Nearly half of the equilibrium economic consequences of the March 2020 Canadian COVID-19 lockdown were indirect. That is, about half of the economic decline from lockdowns results from productivity shocks spreading downstream to less restricted industries that use the focal industry products as an input. Second, a single month of economic restrictions similar to the peak April restrictions causes nearly 5,000 medium-run excess deaths if the resultant income shocks are not mitigated through fiscal policy. Third, when reopening, or when choosing sectors to reclose during subsequent waves or future epidemics, closing retail, restaurants, and bars first is optimal on both economic and long-term health grounds. This is true even though our model does not account for differences in disease transmission across sectors.

Why? On economic grounds, these public-facing sectors generally sell directly to end consumers rather than to other industries, so productivity shocks remain “contained” compared with, for instance, closing a manufacturing plant. On health grounds, keeping upstream industries open rather than retail and restaurants is preferable even though the latter employ many low-wage workers and many workers in demographic categories susceptible to adverse health reactions from income loss. The reason is that direct productivity shocks in these sectors do not lead to many job losses elsewhere and hence have few long-term health consequences for workers in other industries.

Note that our estimates do not account for fiscal transfers to affected workers or policy changes to attempt to mitigate long-run health outcomes. That is, they should be read as the cost of unmitigated lockdowns, and hence as a baseline with which to weigh various policy responses. Note, however, that the cost of fiscal mitigation is additional government debt. A full reckoning of the economic and public health consequences of mitigation via fiscal transfers would need to account for the restricted future government fiscal position due to this debt, including its ability to provide public health spending.

Although we do not consider the impact of shutdowns in non-economic sectors such as schools or public transportation, we note that these can be incorporated into the present model by adjusting the estimated sector-specific productivity shocks. For instance, if online-only schools were estimated to reduce productivity by workers with young children by 20%, and a sector during the March lockdown has 80% of its workers deemed either essential
or able to work from home, then keeping schools closed shifts the overall direct productivity shock for young parents from 20% to 36%. We can then use that modified productivity shock estimate to trace through the equilibrium economic and health consequences, just as we could with productivity shocks driven by other factors. As long-term shutdowns of in-person schooling are rare historically, we do not have a strong empirical basis to perform this extension in the present article (though see Green et al. 2021) for a derivation of the centrality of schools, properly measured, in the Canadian input–output matrix).

The models most similar to our work are Baqee et al. (2020) and Rio-Chanona et al. (2020), which consider a production-network approach to the American second wave and the reopening of the British economy, respectively. They attempt to model $R_p$ explicitly by industry, which we do not. The former article explicitly models transmission in an SIR-type model to investigate the efficacy of economic restrictions as opposed to demographic segregation. It also permits shifts in demand and direct supply shocks rather than only productivity shocks. We attempt to account for the long-run health consequences of economic dislocation by industry, which they do not. All three articles use input–output tables at the national level to examine how shutdowns in one industry affect the broader economy.

Methods
Let us begin by describing the mechanics of the economic and public health models. We will then describe the relevant data inputs and discuss the model outputs in terms of both the March 2020 lockdown and alternative “second wave” lockdowns that differ in which industries they target.

The Fundamentals of the Economic Model
The economic model has three fundamental features. We provide full mathematical details of the model in Appendix A.

First, the model takes as input a productivity shock faced by 54 sectors of the Canadian economy. This productivity shock is an estimate of the direct decline in output per worker in each sector as a result of COVID-19–related restrictions, assuming demand and all input prices were held constant. Our baseline productivity shock is based on existing “essential worker” classifications and on estimates of the potential to work from home across Canadian industries as estimated by Blit, Skuterud, and Veall (2020). Pre–COVID-19 estimates of the fraction of jobs that can be done from home by industry are highly correlated with actually observed changes in remote work, as Dingel and Neiman (2020) first observed in the US context.

Second, the output of the model is the change in economic activity and employment by sector, as well as an estimate of the fall in overall GDP and employment, once we account for the interactions across sectors. Productivity shocks propagate downstream: a productivity shock to rail transportation makes shipping vegetables more expensive, and hence lowers productivity indirectly in the restaurant industry. The mathematical technique showing how to derive these changes originates with Leontief (1951), with important theorems provided by Acemoglu et al. (2012), Baqee and Farhi (2019), Gabaix (2011), and Hulten (1978).

Third, the model is in equilibrium with wages that are fixed in the short run. That is, it assumes that all firms in all sectors optimally recalibrate production, prices, and employment in response to a shock in productivity in other sectors, and all consumers recalibrate purchases based on implied price changes. Following these recalibrations, supply equals demand in all productive sectors. A drop in productivity leads to a fall in employment rather than a fall in wages. This is a widely observed empirical phenomenon during short-run economic declines (e.g., Barattieri, Basu, and Gottschalk 2014).

A brief explanation is needed as to why we model lockdowns as a productivity shock. The effects of COVID-19 lockdowns most generally can be seen as affecting demand, labour supply, and productivity. Demand shifts as consumers reallocate consumption toward products whose relative value rises as a result of lockdown; housing outside urban centers during the pandemic is an example. Labour supply falls as it retrenches in response both to formal limits on the ability to work and to informal attempts to avoid disease. Productivity falls because lockdown restrictions reduce the quantity of output that can be produced with any given set of labour and capital at a firm. All three factors have been investigated in modern production network economics, including in the context of the 2020 pandemic (Baqee and Farhi 2020). We model lockdown restrictions as a productivity shock alone. A labour supply restriction implies that, for example, a car factory can still produce as efficiently, but merely must do so at a lower level of output. Limits on how much labour the factory can use then affect other sectors only via the reduced income of their workers (again, see Baqee and Farhi 2020 for a clean derivation). A productivity shock, as in our model, implies that the factory is less efficient at producing cars in proportion to the fraction of labour it is unable to use. These shocks will therefore also affect industries that rely on cars as an input to their own production.

Modeling the shock of COVID-19 as a general retrenchment in homogenous labour used by all sectors is reasonable in studying the consequences of the negative shock to national income. It is less compelling when the goal is to understand the differential production environments of industries.
Figure 1 displays the economic and public health links we draw out. Lockdowns lead to sector-specific productivity shocks. These shocks lower GDP and employment directly, but also indirectly, as shocks in one sector lower productivity in downstream sectors. The equilibrium productivity consequences imply unemployment in each sector as a function of the initial lockdown policy. Unemployed workers, in the absence of fiscal intervention, see their household income fall. This reduction in household income, combined with the workers’ demographics, implies a probabilistic increase in morbidity and mortality.

The Fundamentals of the Public Health Model
The public health model begins with Canadian survey evidence on personal and household income by sector, age, health characteristics, and demographics. A reconstruction of the Canadian income distribution by age and demographics is then generated by simulating random job loss, and hence personal income loss, for workers in each sector according to the probabilities estimated by the economic model for a given lockdown scenario.

This estimated income distribution is fed into two well-validated population-level risk tools. The first estimates the change in early mortality, defined as death before age 75, over the next five years (Rosella et al. 2020). The second estimates the change in high health care resource utilization over the next five years, defined as an individual on whom health system spending is in the top 5% of historic Canadian data (Rosella et al. 2018). These mortality and morbidity estimates are a function of pre-existing health, age, immigration history, sex, and income. In particular, lower household income is predictive of negative health consequences.3

Figure 1: A Summary of the Connection between Economic and Long-Term Health Models

Data Sources and Definitions
The economic and public health models together require three sets of data. First, we use an estimate of the productivity shock to each sector at the peak of the Canadian “first wave” lockdown in April 2020, drawing on prior research on work-from-home potential in Canada and calculations of the fraction of essential workers by industry. Second, we use Canadian government data on the economic input–output linkages across sectors to estimate how those shocks propagate. Third, we use nationally representative panel survey data on health covariates, demographic characteristics, income, and sector of employment to link health-relevant demographics to the workforce of each sector.

In each dataset, we focus on 3-digit NAICS industries. This level of disaggregation includes sectors such as “Fishing, Hunting and Trapping” and “Support Activities for Transportation.” Because many of these industries are small and have similar industrial links across the economy, as well as similar effects due to COVID-19, we combine some industries so that 54 remain in the final sample.4

Estimated Productivity Shock
Blit et al. (2020) use O*NET survey data to compute the fraction of Canadian workers who could work from home; the overall estimate of 41% of workers matches closely a
similar Canadian exercise by Gallacher and Hossain (2020) and the US calculation by Dingel and Neiman (2020). Rio-Chanona et al. (2020) estimate the consequences of a severe lockdown using highly disaggregated estimates of jobs that are “essential” based on scoring from the Italian government. Occupation-level essential worker lists that are comparable with NAICS industry classifications are not available in the Canadian context; thus, for nearly all industries, we take the Italian classification as representative of the Canadian lockdown experience. However, there are important broad sectoral differences: for example, the mining industry was allowed to operate in Canada, but not in Italy. We therefore modify the essential worker classification using Public Safety Canada’s list of essential workers by industry, as described below. Combining these two measures gives an estimate of the fraction of workers in each industry whose productivity temporarily falls to zero during a COVID-19 lockdown, assuming that in the most severe lockdown, all non-essential workplaces are shuttered and only work-from-home tasks are permitted in those sectors.

We modify the Italian essential worker list in only four ways, each based on Canadian policy. First, in Canada, mining, mining services, food and beverage retail, gas stations, and a number of manufacturing sectors, primarily linked to the resource and food production sectors, were deemed essential even when these sectors were not fully open in Italy. We therefore assume that they suffer no direct productivity shock from the lockdown. Second, in the Italian classification, no restaurants were considered essential, but in Canada, “restaurant employees necessary to support take-out and food delivery operations” were permitted to work. At the lockdown peak, only 56% of food and beverage establishments, including bars, were closed. We therefore assume that one-fourth of restaurant workers are essential, roughly half the figure estimated for hotel accommodations in the Italian classification. Third, while education and health care were deemed essential even in the Italian data, broad swathes of these sectors were not permitted to operate in Canada, including executive education, dentists, optometrists, and massage therapists. We are unaware of precise estimates of what fraction of workers in these industries were essential, but we use a rough estimate that only 80% of workers in these industries are essential in the Canadian classification.

Fourth, we apply a technical correction to the real estate sector, which in Canadian input–output tables includes the consumption value of owner-occupied housing. We assume that the fraction of gross output coming from owner-occupied housing is unaffected by the pandemic, and for the remaining third of real estate gross output use the external work-from-home and Italian essential worker estimates as given.

We assume that industries suffer a productivity shock in April 2020 due to the COVID-19 lockdown equal to precisely the fraction of workers in that industry who neither can work from home nor are deemed essential. Table 1 shows the fraction of workers who can work from home, and the fraction deemed essential, by industry. Using these estimates, column 2 of Table 3 in Empirical Estimates gives the estimated peak Canadian lockdown “Shock” in percentage terms. For example, the construction sector saw a 28% direct shock to productivity, while manufacturing faced a 13% shock, and government and non-profit services no direct shock due to the fact that nearly all jobs in that sector can be done at home or were deemed essential.

Input–Output Tables
In the Online Appendix, the mathematical derivation of the link between a vector of productivity shocks by sector and a vector of implied employment and GDP declines, we note that the only empirical data required to trace shocks across industries are the “Leontief inverse.” This matrix depends entirely on the so-called technical coefficients linking each sector to every other. Essentially, the technical coefficients tell you how much of the production of industry A is used in production by industry B. The Leontief inverse captures the fact that it is not only productivity shocks to A directly that affect B, but also productivity shocks to industry C that produces some good used as an input by A, and industry D that produces an input for industry C, and so on.

The relative use of each industry as inputs into other industries is given by “supply and use” tables produced by most countries as a standard government statistic. We use the most recent Canadian version, the 2015 detail level of the symmetric input-output table. Although the data are five years old, the general patterns of, for instance, the importance of refined petroleum to the fishing industry as compared to retail mattress stores to the fishing industry is not likely to have changed substantially. We construct final use using producer-paid prices. We use 2018 employment and payroll data from the Labour Force Survey to construct employment by sector. There are substantial differences in the amount of gross production by industry per worker: oil and gas extraction has 24 times the gross output per worker of eating and drinking establishments. Therefore, a 1% decline in GDP completely driven by the restaurant sector has vastly different consequences for unemployment than a 1% decline in GDP completely due to oil and gas extraction.

Income Distribution by Demographics
Household income by demographics, sector of work, and employment status is constructed using the 2013–2014 cycle of the Canadian Community Health Survey (CCHS). The CCHS is a nationally representative cross-sectional survey conducted by Statistics Canada on an ongoing basis.
Table 1: Computation of Productivity Shocks

| Code | NAICS Category                                | WFH Index | % Essential | Lockdown Shock |
|------|-----------------------------------------------|-----------|-------------|----------------|
| 23   | Construction                                  | 0.172     | 0.66        | 0.71848        |
| 32   | Manufacturing                                 | 0.37      | 0.792       | 0.86896        |
| 41   | Wholesale Trade                               | 0.52      | 0.31        | 0.6688         |
| 91   | Government and Non-profit Services            | 0.66      |             |                |
| 111  | Crop Production                               | 0.455     |             |                |
| 112  | Animal Production and Aquaculture             | 0.455     |             |                |
| 113  | Forestry and Logging                          | 0.152     |             |                |
| 114  | Fishing, Hunting, and Trapping                | 0.048     |             |                |
| 211  | Oil and Gas Extraction                        | 0.509     |             |                |
| 212  | Mining (except Oil and Gas)                   | 0.239     |             |                |
| 213  | Support Activities for Mining                 | 0.321     |             |                |
| 221  | Utilities                                     | 0.497     |             |                |
| 441  | Motor Vehicle and Parts Dealers               | 0.375     | 0.6         | 0.75           |
| 442  | Furniture and Home Furnishings Stores         | 0.377     | 0.17        | 0.48291        |
| 443  | Electronics and Appliance Stores             | 0.405     | 0.17        | 0.50615        |
| 444  | Building Material and Garden Equipment        | 0.316     | 0.17        | 0.43228        |
| 445  | Food and Beverage Stores                      | 0.267     |             |                |
| 446  | Health and Personal Care Stores               | 0.304     | 0.17        | 0.42232        |
| 447  | Gasoline Stations                             | 0.211     |             |                |
| 448  | Clothing and Clothing Accessories Stores      | 0.294     | 0.17        | 0.41402        |
| 451  | Sporting Goods, Hobby, Music, and Book Stores | 0.319     | 0.17        | 0.43477        |
| 452  | General Merchandise Stores                    | 0.293     | 0.25        | 0.46975        |
| 453  | Miscellaneous Store Retailers                 | 0.359     | 0.17        | 0.46797        |
| 454  | Nonstore Retailers                            | 0.545     | 0.17        | 0.62235        |
| 481  | Air Transportation                            | 0.267     |             |                |
| 482  | Rail Transportation                           | 0.208     |             |                |
| 483  | Water Transportation                           | 0.23      |             |                |
| 484  | Truck Transportation                           | 0.193     |             |                |
| 485  | Transit and Ground Passenger Transport        | 0.116     |             |                |
| 488  | Support Activities for Transportation         | 0.425     |             |                |
| 492  | Couriers and Messengers                       | 0.315     |             |                |
| 493  | Warehousing and Storage                       | 0.402     |             |                |
| 511  | Publishing Industries (except Internet)       | 0.865     | 0.46        | 0.9271         |
| 512  | Motion Picture and Sound Recording            | 0.478     | 0           | 0.478          |
| 517  | Telecommunications                            | 0.701     |             |                |
| 518  | Data Processing and Hosting                   | 0.937     |             |                |
| 519  | Other Information Services                    | 0.589     |             |                |
| 522  | Credit Intermediation and Related Activities  | 0.961     |             |                |
| 523  | Securities, Commodity Contracts, and Related  | 0.96      |             |                |
| 524  | Insurance Carriers and Related Activities     | 0.97      |             |                |
| 531  | Real Estate                                   | 0.831     | 0           | 0.831          |
| 541  | Professional, Scientific, and Technical Services | 0.851  | 0.94        | 0.99106        |
| 561  | Administrative and Support Services           | 0.361     | 0.44        | 0.64216        |
| 562  | Waste Management and Remediation Services     | 0.216     |             |                |
| 611  | Educational Services                          | 0.814     |             | 0.814          |
| 621  | Ambulatory Health Care Services               | 0.218     | 0.8         | 0.8436         |
| 623  | Nursing and Residential Care Facilities       | 0.225     | 0.8         | 0.845          |
| 624  | Social Assistance                             | 0.213     | 0.8         | 0.8426         |

(Continued)
Table 1: Continued

| Code | NAICS Category                      | WFH Index | % Essential | Lockdown Shock |
|------|------------------------------------|-----------|-------------|----------------|
| 711  | Performing Arts and Spectator Sports | 0.526     | 0           | 0.526          |
| 713  | Amusement, Gambling, and Recreation | 0.254     | 0           | 0.254          |
| 721  | Accommodation                      | 0.118     | 0.5         | 0.559          |
| 722  | Food Services and Drinking Places  | 0.057     | 0.44        | 0.47192        |
| 811  | Repair and Maintenance             | 0.157     | 0.96        | 0.96628        |
| 812  | Personal and Laundry Services      | 0.136     | 0.28        | 0.37792        |
| 814  | Private Households                 | 0.31      | 1           | 1              |

Note: The productivity shock in each industry is estimated to be equal to the fraction of workers who can work at home \(W\) or who are essential \(E\). That is, the shock is equal to \(W + E(1 - W)\).

Sources: Authors’ compilations. Work-from-home fractions are based on the calculations in Blit et al. (2020) using O*NET survey data. Essential worker classifications are derived from highly disaggregated Italian occupational restrictions reported in Rio-Chanona et al. (2020), with modifications based on idiosyncrasies in Canadian data as described in the main text.

We restrict to CCHS populations aged 18–74, as we are interested in excess morbidity and mortality driven by income loss. All employed CCHS respondents are asked to describe the kind of business, industry, or service of their workplace, with answers coded into the 2007 NAICS classification. Each respondent also reported total household income in the 12 months prior to the interview. Respondents were ranked into national household income quintiles according to the ratio of their self-reported income to Statistics Canada’s low income cut off (LICO). LICO is assigned according to geography of residence and household size and accounts for regional differences in cost of living and per-person equivalents.

We focus on two health outcomes, one concerning health system impacts from high resource users (HRUs) and one concerning mortality (death before age 75). HRUs are broadly defined as individuals who account for a large portion of health spending. For instance, in Ontario, the top 5% of health system users account for over half of all system costs (Wodchis, Austin, and Henry 2016). High resource utilization is also seen as a measure of overall population health, since HRUs tend to have multiple chronic conditions and report poor general health (Reid et al. 2003). For this study, HRUs were defined as the costliest 5% of health care system users in a given year, in the pre–COVID-19 distribution. Premature deaths are defined as any death between the ages of 18 and 74, a common definition (Thomas, Dorling, and Smith 2010). Premature mortality is a robust measure of population health, as many deaths before the age of 75 are preventable through appropriate intervention.

From Data to Lockdown Scenarios

For each lockdown scenario, we estimate GDP, employment, and mortality and morbidity changes as follows.

From the baseline April Canadian lockdown productivity shocks, we consider the costs and benefits of opening various sectors by adjusting the extent of the shock in each industry. The model is analytically tractable for any user-defined lockdown scenario, but we focus on three in particular. First, we calculate changes in the baseline scenario relative to no COVID-19 restrictions. Second, we consider a plan that keeps all industries at their April lockdown level except for “public facing sectors”: the retail, accommodations, and food and beverage sectors. Third, we keep all industries at their April lockdown level except for a series of “upstream” sectors: manufacturing and wholesale. The direct economic consequences of the two lockdown reductions are almost identical, but the impact on economic and health outcomes will be very different once we trace indirect economic effects of loosening productivity shocks on each set of industries.

Using these shocks and the Leontief inverse matrix, we compute the estimated decline in production in each of our 54 sectors once equilibrium shock propagation is taken into account. In particular, we can decompose this shock into the decline in production in the focal industry and in every other industry individually (see the Online Appendix for details). Since wages are assumed to be fixed in the short run, a 10% decline in equilibrium productivity in a given sector leads to a 10% decline in employment. Overall GDP and employment declines are not identical. Which is larger depends on whether productivity declines are concentrated in sectors with relatively low wages and low capital use, or sectors with high wages and high capital use. In the latter, it takes a larger decline in the dollar value of production to cause a single layoff.

Finally, we use the public health model to estimate excess mortality and morbidity across the economy caused by a given decline in productivity in each sector. For instance, a 1% productivity shock to manufacturing leads to equilibrium declines in production in manufacturing as well as in industries using manufacturing as an input, industries using those industries as an input, and
so on. For each shocked industry–affected industry pair, the estimated percentage decline in production is equal to the estimated percentage decline in the workforce. Therefore, using CCHS survey data, we simulate this decline by choosing survey respondents in each industry at random and setting their personal income to zero. Using the resultant household income distribution, we consider the set of workers estimated to fall into a different quintile of the income distribution and compute the predicted change in mortality and morbidity given that income change, holding all other demographic and pre-existing health covariates constant. Summing these estimates, we compute the excess mortality and morbidity caused by each dollar of productivity shocks and by each lost job due to direct productivity shocks for each industry relative to a baseline with no COVID-19–related job loss. Multiplying these estimates by the productivity shock in a given scenario produces the aggregate excess early death and high utilization of the health system.

Empirical Estimates

Table 2 states the aggregate consequences of our three lockdown scenarios. In particular, note that the retail and upstream sectors are of similar economic size. However, the propagation of restrictions in sectors like manufacturing to the rest of the economy means that keeping upstream sectors open has 3.0 times the benefit to GDP, 2.3 times the benefit in prevented mortality, and 2.5 times the benefit in prevented morbidity.

To understand why the different lockdown scenarios have such radically different impacts, and why the economic benefits and public health benefits generally go in the same direction, let us examine each scenario in detail.

Consider first the April lockdown, described in Table 3. The four data columns give the assumed productivity shock, the share of GDP and employment made up by each sector, and the estimated equilibrium change in productivity given propagated shocks from other industries. The estimated productivity shock takes a value of 0 if the industry is unrestricted, and a value of -1 under a complete legal restriction on any production in the sector.

With these productivity shocks, based on work-from-home ability and essential worker status as described in the previous section, we estimate a 24.6% decline in GDP and a 30.8% decline in employment. The indirect shocks are enormous. If the assumed productivity shocks affected only the direct industry they hit, and not other industries through the input–output links, there would only be a 14.5% decline in GDP, rather than 24.6%, and a 20.1% decline in employment, rather than 30.8%. The heterogeneity of these indirect effects within sectors is also clear. For example, although the direct productivity shock on manufacturing is only 13%, the overall effect of the lockdown on manufacturing is a 29.7% decline as their intermediate good suppliers face restrictions. Likewise, the utilities sector sees a 6.5% decline in output and employment even though they face no formal direct restrictions. A single month of lockdown at the April level is estimated to cause nearly 5,000 excess deaths, and a similar increase in high-cost health care system users. Recall again that these estimates are based on unmitigated income shocks, so are potentially amenable to being avoided with fiscal policy replacing that lost income.

Table 4 shows how model-estimated sectoral declines in employment (“ModelHoursChange”) compare to the observed decline in hours worked from the Canadian Labour Force Survey (“TrueHoursChange”). This survey is by industry at roughly the 2-digit NAICS level, so we aggregate our estimated employment changes in all 54 sectors into 16 2-digit sectors for which monthly seasonally adjusted Labour Force Survey data are available. The overall fall in hours worked in the data is 27.7% between February and April 2020, very close to the 30.8% we estimate in the model. Note that data on the magnitude of the lockdown were not calibrated to match this figure. Even at the sectoral level, most sectors in Table 4 see declines in the model very similar to what we actually observed. However, public administration, retail, building support services, and cultural sectors perform worse in

| Lockdown Scenario       | GDP Decline | Employment Decline | Excess Deaths | Excess HRU |
|-------------------------|-------------|--------------------|---------------|------------|
| April Lockdown          | -24.6%      | -30.8%             | 4,723         | 5,110      |
| Keep Retail Open        | -20.8%      | -22.0%             | 3,857         | 4,274      |
| Keep Upstream Open      | -16.3%      | -22.8%             | 2,787         | 2,948      |

Notes: GDP and Employment Decline are model estimates. Excess Deaths are excess Canadian deaths before age 75 in the next five years per month of the assumed lockdown, if unmitigated with fiscal policy. Excess HRU is the number of additional individuals who will transition to health system spending in the pre–COVID-19 top 5% within five years, also per month under the assumed lockdown scenario. “April Lockdown” assumes productivity shocks across the real economy in all sectors who cannot work from home and are not deemed essential. “Keep Retail Open” assumes the April Lockdown, except that retail, accommodations, restaurants and bars are allowed to open. “Keep Upstream Open” assumes the April lockdown, except that manufacturing and wholesale have no COVID-19–related restrictions.

Source: Authors’ calculations.
### Table 3: Canadian Peak Lockdown Scenario: 24.6% GDP and 30.8% Employment Decline

| Sector                                      | Shock  | GDPShare | EmployShare | ChangeFromShock |
|---------------------------------------------|--------|----------|-------------|-----------------|
| Real Estate                                 | −0.17  | 12.9     | 2.1         | −23.6           |
| Manufacturing                               | −0.13  | 10.8     | 8.5         | −29.7           |
| Government and Non-profit Services          | 0.00   | 9.5      | 7.9         | −11.0           |
| Construction                                | −0.28  | 8.1      | 7.6         | −41.5           |
| Professional and Technical Services         | −0.01  | 6.6      | 7.2         | −10.0           |
| Health Care Services                        | −0.16  | 5.7      | 7.0         | −24.5           |
| Educational Services                        | −0.19  | 5.5      | 7.5         | −23.7           |
| Wholesale Trade                             | −0.33  | 5.3      | 4.3         | −42.0           |
| Credit Intermediation and Related           | 0.00   | 3.9      | 2.4         | −5.6            |
| Oil and Gas Extraction                      | 0.00   | 3.2      | 0.4         | −9.6            |
| Administrative and Support Services         | −0.36  | 2.6      | 4.9         | −45.5           |
| Utilities                                   | 0.00   | 2.1      | 0.7         | −6.5            |
| Telecommunications                          | 0.00   | 2.0      | 0.8         | −12.7           |
| Insurance Carriers                          | 0.00   | 1.6      | 1.3         | −6.3            |
| Food Services and Drinking Places           | −0.53  | 1.6      | 6.1         | −67.8           |
| Mining (except Oil and Gas)                 | 0.00   | 1.3      | 0.4         | −9.5            |
| Truck Transportation                        | 0.00   | 1.1      | 1.4         | −13.2           |
| Securities and Investments                  | 0.00   | 1.1      | 1.4         | −7.7            |
| Crop Production                             | 0.00   | 1.1      | 1.0         | −12.4           |
| Nursing and Residential Care Facilities     | −0.15  | 1.0      | 2.3         | −21.6           |
| Food and Beverage Stores                    | 0.00   | 1.0      | 2.6         | −8.1            |
| Support Activities for Mining               | 0.00   | 0.8      | 0.5         | −9.9            |
| Motor Vehicle and Parts Dealers             | −0.25  | 0.8      | 1.2         | −32.4           |
| Support Activities for Transportation       | 0.00   | 0.8      | 0.8         | −13.6           |
| Repair and Maintenance                      | −0.03  | 0.8      | 1.3         | −14.0           |
| Accommodation                               | −0.44  | 0.6      | 1.1         | −54.8           |
| Health and Personal Care Stores             | −0.58  | 0.6      | 1.2         | −66.2           |
| Publishing Industries (except Internet)     | −0.07  | 0.6      | 0.4         | −18.8           |
| Social Assistance                           | −0.16  | 0.5      | 2.1         | −24.8           |
| General Merchandise Stores                  | −0.53  | 0.5      | 1.2         | −61.3           |
| Air Transportation                          | 0.00   | 0.5      | 0.4         | −15.3           |
| Clothing and Clothing Accessories Stores    | −0.59  | 0.5      | 1.4         | −69.7           |
| Rail Transportation                         | 0.00   | 0.5      | 0.2         | −10.4           |
| Personal and Laundry Services               | −0.62  | 0.4      | 1.5         | −74.3           |
| Animal Production and Aquaculture           | 0.00   | 0.4      | 0.8         | −14.6           |
| Building and Garden Material Dealers        | −0.57  | 0.4      | 0.8         | −65.2           |
| Amusement and Gambling and Recreation       | −0.75  | 0.4      | 1.0         | −91.6           |
| Gasoline Stations                           | 0.00   | 0.4      | 0.4         | −6.8            |
| Waste Management                            | 0.00   | 0.3      | 0.3         | −11.0           |
| Transit and Ground Transportation           | 0.00   | 0.3      | 0.9         | −14.8           |
| Performing Arts and Spectator Sports        | −0.47  | 0.2      | 0.9         | −66.1           |
| Couriers and Messengers                     | 0.00   | 0.2      | 0.3         | −12.7           |
| Forestry and Logging                        | 0.00   | 0.2      | 0.2         | −13.0           |
| Furniture and Home Furnishings Stores       | −0.52  | 0.2      | 0.5         | −63.7           |
| Miscellaneous Store Retailers               | −0.53  | 0.2      | 0.6         | −62.6           |
| Electronics and Appliance Stores            | −0.49  | 0.2      | 0.3         | −57.7           |
| Motion Picture and Sound Recording          | −0.52  | 0.2      | 0.4         | −91.7           |
| Warehousing and Storage                     | 0.00   | 0.2      | 0.3         | −7.1            |
| Nonstore Retailers                          | −0.38  | 0.2      | 0.3         | −47.1           |
| Sports, Music, Hobby and Book Stores        | −0.57  | 0.2      | 0.5         | −65.5           |
| Data Processing and Hosting                 | 0.00   | 0.2      | 0.1         | −9.5            |
| Fishing, Hunting and Trapping               | 0.00   | 0.1      | 0.1         | −11.5           |
| Other Information Services                  | 0.00   | 0.1      | 0.2         | −10.0           |
| Water Transportation                        | 0.00   | 0.1      | 0.1         | −15.8           |

Source: Authors’ calculations.
the model than in true data, while transportation and the resource sectors perform better in the model. The former may be due in part to the unionization of the public sector and the potential to shift more sales work to an online model than projected, and in the latter to demand shocks generated by COVID-19 (a feature we do not attempt to model). Nonetheless, the relative rankings of sectors by their equilibrium COVID-19 shock is fairly accurate; note that this is true even though half of the total COVID-19 shock is coming from indirect industry linkages rather than direct restrictions.

Consider now the two scenarios with partial reopening. The public-facing opening leaves a direct shock to productivity, across the economy, of 11.6% of GDP and 12.4% of employment, compared with the direct cumulative shocks of 14.5% and 20.1% in the full Canadian lockdown baseline. Recall that these shocks are relative to February: that is, opening the public-facing sectors recovers 14.5 minus 11.6, or 2.9 percentage points of GDP, and likewise 20.1 minus 12.4, or 7.7 percentage points of employment. The upstream opening leaves a direct shock to productivity across the economy of 11.3% to GDP and 17.6% to employment. That is, these two reopening plans lead to roughly similar cumulative improvements in GDP, looking only at the direct legal ability to do business, and the “public-facing” opening improves the employment picture more due to the many low-wage employees in those sectors. However, the story is very different once we consider indirect shocks.

Tables 5 and 6 give the assumed shocks and overall change in output and employment by sector in these scenarios. Summing up over sectors and accounting for indirect links across industries, the public-facing reopening certainly helps the economy: GDP is down 20.8% and employment down 22.0% from their pre-COVID-19 levels, versus 24.6% and 30.8% in the full lockdown. However, opening the upstream sectors only leaves GDP down 16.3% and employment down 22.8%. Recall that the size of the economy directly affected by these two reopening plans was quite similar. Nonetheless, since public-facing firms use inputs produced upstream, whereas sectors such as mining do not have their productivity affected when, say, restaurants are closed, the overall bang for the buck of reopening upstream sectors is much greater. Indeed, overall unemployment winds up at a similar level in both reopening plans even though the public-facing sectors such as hotels and restaurants are much more labor-intensive. Again, the reason is the indirect benefits to partially reopened labor-intensive sectors from fully opening upstream sectors.

Why do restrictions on upstream sectors lead to so much more indirect harm to the economy? Further, why does their closure drive so many deaths and so much excess morbidity in Table 2? Table 7 provides an answer. This table orders sectors by their “jobs multiplier.” The jobs multiplier is the estimated number of jobs across the economy lost for every job lost in the focal industry due to a direct productivity shock. Oil and gas extraction, for instance, has many high-wage workers and produces a product used in many downstream industries. Since the workers are high-wage, each lost job is the result of a substantial productivity decline. This decline means higher prices in every industry using oil and gas, or using products that use oil and gas in their production, and so
### Table 5: Effect of Opening Public-Facing Sectors: 20.8% GDP and 22.0% Employment Decline

| Sector                                      | Shock  | GDP Share | Employ Share | Change From Shock |
|---------------------------------------------|--------|-----------|--------------|-------------------|
| Real Estate                                 | -0.17  | 12.9      | 2.1          | -23.0             |
| Manufacturing                               | -0.13  | 10.8      | 8.5          | -28.8             |
| Government and Non-profit Services          | 0.00   | 9.5       | 7.9          | -10.1             |
| Construction                                | -0.28  | 8.1       | 7.6          | -40.9             |
| Professional and Technical Services         | -0.01  | 6.6       | 7.2          | -8.6              |
| Health Care Services                        | -0.16  | 5.7       | 7.0          | -23.6             |
| Educational Services                        | -0.19  | 5.5       | 7.5          | -23.1             |
| Wholesale Trade                             | -0.33  | 5.3       | 4.3          | -40.7             |
| Credit Intermediation and Related           | 0.00   | 3.9       | 2.4          | -4.9              |
| Oil and Gas Extraction                      | 0.00   | 3.2       | 0.4          | -8.8              |
| Administrative and Support Services         | -0.36  | 2.6       | 4.9          | -44.1             |
| Utilities                                   | 0.00   | 2.1       | 0.7          | -5.9              |
| Telecommunications                          | 0.00   | 2.0       | 0.8          | -12.1             |
| Insurance Carriers                          | 0.00   | 1.6       | 1.3          | -5.5              |
| Food Services and Drinking Places           | 0.00   | 1.6       | 6.1          | -14.1             |
| Mining (except Oil and Gas)                 | 0.00   | 1.3       | 0.4          | -8.8              |
| Truck Transportation                        | 0.00   | 1.1       | 1.4          | -12.2             |
| Securities and Investments                  | 0.00   | 1.1       | 1.4          | -6.7              |
| Crop Production                             | 0.00   | 1.1       | 1.0          | -11.9             |
| Nursing and Residential Care Facilities     | -0.15  | 1.0       | 2.3          | -21.1             |
| Food and Beverage Stores                    | 0.00   | 1.0       | 2.6          | -7.2              |
| Support Activities for Mining               | 0.00   | 0.8       | 0.5          | -9.2              |
| Motor Vehicle and Parts Dealers             | 0.00   | 0.8       | 1.2          | -6.3              |
| Support Activities for Transportation       | 0.00   | 0.8       | 0.8          | -11.7             |
| Repair and Maintenance                      | -0.03  | 0.8       | 1.3          | -12.6             |
| Accommodation                               | 0.00   | 0.6       | 1.1          | -9.9              |
| Health and Personal Care Stores             | 0.00   | 0.6       | 1.2          | -7.2              |
| Publishing Industries (except Internet)     | -0.07  | 0.6       | 0.4          | -17.0             |
| Social Assistance                           | -0.16  | 0.5       | 2.1          | -23.6             |
| General Merchandise Stores                  | 0.00   | 0.5       | 1.2          | -6.9              |
| Air Transportation                          | 0.00   | 0.5       | 0.4          | -12.8             |
| Clothing and Clothing Accessories Stores    | 0.00   | 0.5       | 1.4          | -9.4              |
| Rail Transportation                         | 0.00   | 0.5       | 0.2          | -10.0             |
| Personal and Laundry Services               | -0.62  | 0.4       | 1.5          | -73.0             |
| Animal Production and Aquaculture           | 0.00   | 0.4       | 0.8          | -14.0             |
| Building and Garden Material Dealers        | 0.00   | 0.4       | 0.8          | -7.0              |
| Amusement and Gambling and Recreation       | -0.75  | 0.4       | 1.0          | -86.3             |
| Gasoline Stations                           | 0.00   | 0.4       | 0.4          | -6.1              |
| Waste Management                            | 0.00   | 0.3       | 0.3          | -9.3              |
| Transit and Ground Transportation           | 0.00   | 0.3       | 0.9          | -13.6             |
| Performing Arts and Spectator Sports        | -0.47  | 0.2       | 0.9          | -63.8             |
| Couriers and Messengers                     | 0.00   | 0.2       | 0.3          | -11.1             |
| Forestry and Logging                        | 0.00   | 0.2       | 0.2          | -12.2             |
| Furniture and Home Furnishings Stores       | 0.00   | 0.2       | 0.5          | -10.0             |
| Miscellaneous Store Retailers               | 0.00   | 0.2       | 0.6          | -8.2              |
| Electronics and Appliance Stores            | 0.00   | 0.2       | 0.3          | -7.0              |
| Motion Picture and Sound Recording          | -0.52  | 0.2       | 0.4          | -90.2             |
| Warehousing and Storage                     | 0.00   | 0.2       | 0.3          | -6.4              |
Table 5: Continued

| Sector                                      | Shock | GDPShare | EmployShare | ChangeFromShock |
|---------------------------------------------|-------|----------|-------------|-----------------|
| Nonstore Retailers                          | 0.00  | 0.2      | 0.3         | −7.9            |
| Sports, Music, Hobby, and Book Stores       | 0.00  | 0.2      | 0.5         | −7.6            |
| Data Processing and Hosting                 | 0.00  | 0.2      | 0.1         | −8.3            |
| Fishing, Hunting, and Trapping              | 0.00  | 0.1      | 0.1         | −10.8           |
| Other Information Services                  | 0.00  | 0.1      | 0.2         | −8.1            |
| Water Transportation                        | 0.00  | 0.1      | 0.1         | −14.8           |

Source: Authors’ calculations.

Table 6: Effect of Opening Upstream Sectors: 16.3% GDP and 22.8% Employment Decline

| Sector                                      | Shock | GDPShare | EmployShare | ChangeFromShock |
|---------------------------------------------|-------|----------|-------------|-----------------|
| Real Estate                                 | −0.17 | 12.9     | 2.1         | −21.8           |
| Manufacturing                               | 0.00  | 10.8     | 8.5         | −3.5            |
| Government and Non-profit Services          | 0.00  | 9.5      | 7.9         | −6.9            |
| Construction                                | −0.28 | 8.1      | 7.6         | −31.4           |
| Professional and Technical Services         | −0.01 | 6.6      | 7.2         | −7.1            |
| Health Care Services                        | −0.16 | 5.7      | 7.0         | −20.2           |
| Educational Services                        | −0.19 | 5.5      | 7.5         | −21.6           |
| Wholesale Trade                             | 0.00  | 5.3      | 4.3         | −5.5            |
| Credit Intermediation and Related Services  | 0.00  | 3.9      | 2.4         | −4.1            |
| Oil and Gas Extraction                      | 0.00  | 3.2      | 0.4         | −4.4            |
| Administrative and Support Services         | −0.36 | 2.6      | 4.9         | −42.0           |
| Utilities                                   | 0.00  | 2.1      | 0.7         | −3.6            |
| Telecommunications                          | 0.00  | 2.0      | 0.8         | −8.1            |
| Insurance Carriers                          | 0.00  | 1.6      | 1.3         | −4.5            |
| Food Services and Drinking Places           | −0.53 | 1.6      | 6.1         | −57.7           |
| Mining (except Oil and Gas)                 | 0.00  | 1.3      | 0.4         | −3.3            |
| Truck Transportation                        | 0.00  | 1.1      | 1.4         | −4.8            |
| Securities and Investments                  | 0.00  | 1.1      | 1.4         | −5.3            |
| Crop Production                             | 0.00  | 1.1      | 1.0         | −2.7            |
| Nursing and Residential Care Facilities     | −0.15 | 1.0      | 2.3         | −18.7           |
| Food and Beverage Stores                    | 0.00  | 1.0      | 2.6         | −5.6            |
| Support Activities for Mining               | 0.00  | 0.8      | 0.5         | −3.3            |
| Motor Vehicle and Parts Dealers             | −0.25 | 0.8      | 1.2         | −30.0           |
| Support Activities for Transportation       | 0.00  | 0.8      | 0.8         | −7.2            |
| Repair and Maintenance                      | −0.03 | 0.8      | 1.3         | −9.1            |
| Accommodation                               | −0.44 | 0.6      | 1.1         | −49.0           |
| Health and Personal Care Stores             | −0.58 | 0.6      | 1.2         | −63.7           |
| Publishing Industries (except Internet)     | −0.07 | 0.6      | 0.4         | −14.5           |
| Social Assistance                           | −0.16 | 0.5      | 2.1         | −20.6           |
| General Merchandise Stores                  | −0.53 | 0.5      | 1.2         | −58.8           |
| Air Transportation                          | 0.00  | 0.5      | 0.4         | −5.8            |
| Clothing and Clothing Accessories Stores    | −0.59 | 0.5      | 1.4         | −66.6           |
| Rail Transportation                         | 0.00  | 0.5      | 0.2         | −5.2            |
| Personal and Laundry Services               | −0.62 | 0.4      | 1.5         | −68.1           |
| Animal Production and Aquaculture           | 0.00  | 0.4      | 0.8         | −3.2            |
| Building and Garden Material Dealers        | −0.57 | 0.4      | 0.8         | −62.0           |

(Continued)
Table 6: Continued

| Sector                                | Shock | GDPShare | EmployShare | ChangeFromShock |
|---------------------------------------|-------|----------|-------------|-----------------|
| Amusement and Gambling and Recreation | −0.75 | 0.4      | 1.0         | −86.1           |
| Gasoline Stations                     | 0.00  | 0.4      | 0.4         | −4.5            |
| Waste Management                      | 0.00  | 0.3      | 0.3         | −6.6            |
| Transit and Ground Transportation     | 0.00  | 0.3      | 0.9         | −6.9            |
| Performing Arts and Spectator Sports  | −0.47 | 0.2      | 0.9         | −61.9           |
| Couriers and Messengers               | 0.00  | 0.2      | 0.3         | −6.4            |
| Forestry and Logging                  | 0.00  | 0.2      | 0.2         | −3.3            |
| Furniture and Home Furnishings Stores | −0.52 | 0.2      | 0.5         | −59.8           |
| Miscellaneous Store Retailers         | −0.53 | 0.2      | 0.6         | −59.7           |
| Electronics and Appliance Stores      | −0.49 | 0.2      | 0.3         | −55.1           |
| Motion Picture and Sound Recording    | −0.52 | 0.2      | 0.4         | −84.6           |
| Warehousing and Storage               | 0.00  | 0.2      | 0.3         | −4.1            |
| Nonstore Retailers                    | −0.38 | 0.2      | 0.3         | −43.5           |
| Sports, Music, Hobby, and Book Stores | −0.57 | 0.2      | 0.5         | −63.0           |
| Data Processing and Hosting           | 0.00  | 0.2      | 0.1         | −6.1            |
| Fishing, Hunting, and Trapping        | 0.00  | 0.1      | 0.1         | −2.5            |
| Other Information Services            | 0.00  | 0.1      | 0.2         | −7.4            |
| Water Transportation                  | 0.00  | 0.1      | 0.1         | −5.7            |

Source: Authors’ calculations.

on. As a result, oil and gas extraction has a jobs multiplier of 10: each lost job due to a direct productivity shock in oil and gas extraction causes ten lost jobs overall. On the other hand, furniture stores have a jobs multiplier of 1.1. The furniture stores are largely used by end consumers and not other productive businesses, and the low wages of workers mean that the size of the productivity shock necessary to create one layoff is fairly small in any case.

Deaths per job and HRU per job, in the rightmost two columns, estimate the normalized excess mortality and morbidity caused by a single job loss in the focal sector. For instance, the sector where a job loss causes the smallest equilibrium increase in morbidity as measured by HRUs is food services and drinking places, which we normalize to 1. Manufacturing having an HRU per job of 10.4 means that each job loss in manufacturing causes 10.4 times the morbidity increase of a single job lost from a restaurant.

Deaths per job and HRU per job, in the rightmost two columns, estimate the normalized excess mortality and morbidity caused by a single job loss in the focal sector. For instance, the sector where a job loss causes the smallest equilibrium increase in morbidity as measured by HRUs is food services and drinking places, which we normalize to 1. Manufacturing having an HRU per job of 10.4 means that each job loss in manufacturing causes 10.4 times the morbidity increase of a single job lost from a restaurant.

Table 6 is similar in structure. Instead of investigating how one job lost in a focal sector affects total jobs lost, deaths, and HRU in equilibrium, we examine how a productivity shock affects GDP, deaths, and HRU. “GDP multiplier,” also known as the Domar weight and described in more detail in Appendix A, states the overall decline in GDP over the decline in GDP caused by the direct productivity shock. For instance, a productivity shock that directly reduces crop production by one million dollars will generate an overall GDP decline in equilibrium three times that size. Deaths per GDP and HRU per GDP are again normalized relative to the sector for which a decline in productivity caused the lowest equilibrium excess mortality and morbidity. In both cases, the real estate sector has the smallest indirect negative effects. Compared with real estate, an equivalent direct shock to productivity in animal production and aquaculture leads to 14.8 more excess deaths and 14.0 times higher morbidity. These multipliers differ from the jobs-based multiplier because sectors with low wages, and that are inputs to other low wage sectors, see more workers unemployed for a given productivity shock. Nonetheless, shocks to the high-wage manufacturing sector still cause high excess mortality and morbidity because of its GDP multiplier of 4.4: 77% of the decline in GDP from a shock to manufacturing occurs in other sectors that use manufactured goods or their derivatives. This includes sectors with many low-wage, high-risk employees.

Finally, return to the final two columns of Table 1 at the beginning of this section. Recall that judiciously selecting which sectors of the economy should be opened, given the network linkages connecting the economy, can lead to much quicker economic recovery. Reopening policies focused on upstream sectors also have the benefit of saving lives caused by economic shocks: targeted relatively high-wage manufacturing and wholesale sectors will at equilibrium protect many jobs in low-wage sectors as well. We estimate that opening upstream rather than consumer-facing sectors will more than halve the deaths related to income loss that would otherwise be faced with unmitigated economic shocks caused by a lockdown.
| Sector                                | GDPShare | EmployShare | JobsMultiplier | DeathsPerJob | HRUPerJob |
|---------------------------------------|----------|-------------|----------------|--------------|-----------|
| Oil and Gas Extraction                | 3.2      | 0.4         | 10.0           | 17.0         | 19.3      |
| Mining (except Oil and Gas)           | 1.3      | 0.4         | 6.5            | 11.7         | 12.6      |
| Manufacturing                         | 10.8     | 8.5         | 5.6            | 9.9          | 10.4      |
| Forestry and Logging                  | 0.2      | 0.2         | 5.2            | 9.9          | 11.0      |
| Utilities                             | 2.1      | 0.7         | 3.6            | 5.9          | 7.1       |
| Animal Production and Aquaculture     | 0.4      | 0.8         | 3.5            | 7.1          | 7.5       |
| Telecommunications                    | 2.0      | 0.8         | 3.4            | 5.7          | 6.3       |
| Data Processing and Hosting           | 0.2      | 0.1         | 3.4            | 4.5          | 5.1       |
| Real Estate                           | 12.9     | 2.1         | 3.3            | 5.7          | 6.4       |
| Crop Production                       | 1.1      | 1.0         | 3.2            | 5.8          | 6.6       |
| Fishing, Hunting, and Trapping        | 0.1      | 0.1         | 3.1            | 7.9          | 8.4       |
| Publishing Industries (except Internet) | 0.6    | 0.4         | 3.0            | 4.5          | 5.2       |
| Air Transportation                    | 0.5      | 0.4         | 2.8            | 4.5          | 5.0       |
| Insurance Carriers                    | 1.6      | 1.3         | 2.8            | 4.5          | 5.3       |
| Support Activities for Mining         | 0.8      | 0.5         | 2.7            | 5.0          | 6.5       |
| Waste Management                      | 0.3      | 0.3         | 2.7            | 4.9          | 5.1       |
| Couriers and Messengers               | 0.2      | 0.3         | 2.6            | 5.7          | 5.5       |
| Water Transportation                  | 0.1      | 0.1         | 2.5            | 4.6          | 5.4       |
| Support Activities for Transportation | 0.8      | 0.8         | 2.4            | 4.2          | 4.5       |
| Credit Intermediation and Related     | 3.9      | 2.4         | 2.3            | 3.5          | 3.8       |
| Motion Picture and Sound Recording    | 0.2      | 0.4         | 2.3            | 3.0          | 2.7       |
| Securities and Investments            | 1.1      | 1.4         | 2.2            | 2.7          | 3.4       |
| Professional and Technical Services   | 6.6      | 7.2         | 2.2            | 3.7          | 4.4       |
| Truck Transportation                  | 1.1      | 1.4         | 2.1            | 4.6          | 5.4       |
| Other Information Services            | 0.1      | 0.2         | 1.9            | 2.8          | 3.2       |
| Administrative and Support Services   | 2.6      | 4.9         | 1.9            | 3.1          | 3.0       |
| Rail Transportation                   | 0.5      | 0.2         | 1.7            | 3.4          | 4.3       |
| Repair and Maintenance                | 0.8      | 1.3         | 1.7            | 4.1          | 3.7       |
| Warehousing and Storage               | 0.2      | 0.3         | 1.6            | 3.7          | 3.4       |
| Accommodation                         | 0.6      | 1.1         | 1.5            | 2.1          | 2.3       |
| Performing Arts and Spectator Sports  | 0.2      | 0.9         | 1.4            | 3.4          | 2.9       |
| Transit and Ground Transportation     | 0.3      | 0.9         | 1.3            | 2.7          | 3.5       |
| Construction                          | 8.1      | 7.6         | 1.3            | 2.2          | 2.6       |
| Wholesale Trade                       | 5.3      | 4.3         | 1.3            | 1.7          | 1.9       |
| Motor Vehicle and Parts Dealers       | 0.8      | 1.2         | 1.2            | 2.8          | 2.6       |
| Health Care Services                  | 5.7      | 7.0         | 1.2            | 1.0          | 1.3       |
| Government and Non-profit Services    | 9.5      | 7.9         | 1.2            | 2.6          | 3.0       |
| Amusement and Gambling and Recreation | 0.4     | 1.0         | 1.2            | 3.3          | 2.3       |
| Electronics and Appliance Stores      | 0.2      | 0.3         | 1.2            | 2.3          | 1.6       |
| Food Services and Drinking Places     | 1.6      | 6.1         | 1.2            | 1.5          | 1.0       |
| Personal and Laundry Services         | 0.4      | 1.5         | 1.1            | 1.4          | 1.4       |
| Gasoline Stations                     | 0.4      | 0.4         | 1.1            | 1.5          | 1.2       |
| Nonstore Retailers                    | 0.2      | 0.3         | 1.1            | 1.8          | 2.6       |
| General Merchandise Stores            | 0.5      | 1.2         | 1.1            | 2.0          | 1.8       |
| Building and Garden Material Dealers  | 0.4      | 0.8         | 1.1            | 2.2          | 2.2       |
| Furniture and Home Furnishings Stores | 0.2      | 0.5         | 1.1            | 1.9          | 2.1       |
| Nursing and Residential Care Facilities | 1.0    | 2.3         | 1.1            | 1.7          | 2.0       |
| Social Assistance                     | 0.5      | 2.1         | 1.1            | 1.8          | 2.0       |

(Continued)
Table 7: Continued

| Sector                                      | GDPShare | EmployShare | JobsMultiplier | DeathsPerJob | HRUPerJob |
|---------------------------------------------|----------|-------------|----------------|--------------|-----------|
| Educational Services                        | 5.5      | 7.5         | 1.1            | 1.7          | 2.0       |
| Clothing and Clothing Accessories Stores    | 0.5      | 1.4         | 1.0            | 1.2          | 1.3       |
| Health and Personal Care Stores             | 0.6      | 1.2         | 1.0            | 1.3          | 1.3       |
| Sports, Music, Hobby, and Book Stores       | 0.2      | 0.5         | 1.0            | 1.6          | 1.1       |
| Food and Beverage Stores                    | 1.0      | 2.6         | 1.0            | 1.9          | 1.5       |
| Miscellaneous Store Retailers               | 0.2      | 0.6         | 1.0            | 2.3          | 2.6       |

Source: Authors’ calculations.

Table 8: Relationship of Long-Term Health to Productivity Depends on Three Factors

| Sector                                      | GDPShare | EmployShare | GDPMultiplier | DeathsPerGDP | HRUPerGDP |
|---------------------------------------------|----------|-------------|---------------|--------------|-----------|
| Animal Production and Aquaculture           | 0.4      | 0.8         | 5.6           | 14.8         | 14.0      |
| Forestry and Logging                        | 0.2      | 0.2         | 4.7           | 9.3          | 9.2       |
| Manufacturing                               | 10.8     | 8.5         | 4.4           | 8.6          | 8.0       |
| Motion Picture and Sound Recording          | 0.2      | 0.4         | 3.6           | 7.9          | 6.2       |
| Couriers and Messengers                     | 0.2      | 0.3         | 3.3           | 9.9          | 8.4       |
| Other Information Services                  | 0.1      | 0.2         | 3.0           | 8.3          | 8.4       |
| Crop Production                             | 1.1      | 1.0         | 3.0           | 6.3          | 6.3       |
| Mining (except Oil and Gas)                 | 1.3      | 0.4         | 2.7           | 3.7          | 3.6       |
| Administrative and Support Services         | 2.6      | 4.9         | 2.6           | 6.3          | 5.4       |
| Water Transportation                         | 0.1      | 0.1         | 2.6           | 3.9          | 4.3       |
| Securities and Investments                  | 1.1      | 1.4         | 2.6           | 3.9          | 4.3       |
| Data Processing and Hosting                 | 0.2      | 0.1         | 2.6           | 3.1          | 3.2       |
| Support Activities for Transportation       | 0.8      | 0.8         | 2.5           | 5.1          | 4.8       |
| Air Transportation                          | 0.5      | 0.4         | 2.5           | 4.3          | 4.3       |
| Publishing Industries (except Internet)     | 0.6      | 0.4         | 2.4           | 4.0          | 4.0       |
| Waste Management                            | 0.3      | 0.3         | 2.4           | 4.7          | 4.4       |
| Support Activities for Mining               | 0.8      | 0.5         | 2.4           | 3.1          | 3.6       |
| Insurance Carriers                          | 1.6      | 1.3         | 2.3           | 3.8          | 4.0       |
| Professional and Technical Services         | 6.6      | 7.2         | 2.3           | 4.4          | 4.7       |
| Fishing, Hunting, and Trapping              | 0.1      | 0.1         | 2.3           | 6.0          | 5.6       |
| Oil and Gas Extraction                      | 3.2      | 0.4         | 2.3           | 2.5          | 2.6       |
| Truck Transportation                         | 1.1      | 1.4         | 2.3           | 6.4          | 6.7       |
| Repair and Maintenance                      | 0.8      | 1.3         | 2.1           | 7.7          | 6.1       |
| Performing Arts and Spectator Sports        | 0.2      | 0.9         | 2.0           | 13.6         | 10.2      |
| Warehousing and Storage                     | 0.2      | 0.3         | 2.0           | 6.8          | 5.6       |
| Telecommunications                          | 2.0      | 0.8         | 2.0           | 2.5          | 2.5       |
| Accommodation                               | 0.6      | 1.1         | 1.9           | 4.2          | 4.1       |
| Transit and Ground Transportation           | 0.3      | 0.9         | 1.8           | 9.9          | 11.1      |
| Credit Intermediation and Related           | 3.9      | 2.4         | 1.8           | 2.3          | 2.2       |
| Utilities                                   | 2.1      | 0.7         | 1.8           | 2.0          | 2.2       |
| Food Services and Drinking Places           | 1.6      | 6.1         | 1.5           | 6.2          | 3.7       |
| Amusement and Gambling and Recreation       | 0.4      | 1.0         | 1.4           | 9.1          | 5.6       |
| Rail Transportation                         | 0.5      | 0.2         | 1.3           | 1.6          | 1.8       |
| Construction                                | 8.1      | 7.6         | 1.3           | 2.3          | 2.3       |
| Health Care Services                        | 5.7      | 7.0         | 1.3           | 1.4          | 1.5       |

(Continued)
Table 8: Continued

| Sector                                      | GDPShare | EmployShare | GDPMultiplier | DeathsPerGDP | HRUPerGDP |
|---------------------------------------------|----------|-------------|---------------|--------------|-----------|
| Real Estate                                 | 12.9     | 2.1         | 1.3           | 1.0          | 1.0       |
| Motor Vehicle and Parts Dealers             | 0.8      | 1.2         | 1.3           | 4.4          | 3.6       |
| Personal and Laundry Services               | 0.4      | 1.5         | 1.3           | 4.9          | 4.4       |
| Electronics and Appliance Stores            | 0.2      | 0.3         | 1.2           | 4.9          | 3.1       |
| Social Assistance                           | 0.5      | 2.1         | 1.2           | 7.7          | 7.5       |
| Wholesale Trade                             | 5.3      | 4.3         | 1.2           | 1.6          | 1.5       |
| Government and Non-profit Services          | 9.5      | 7.9         | 1.2           | 2.3          | 2.4       |
| Furniture and Home Furnishings Stores       | 0.2      | 0.5         | 1.1           | 4.7          | 4.5       |
| Building and Garden Material Dealers        | 0.4      | 0.8         | 1.1           | 4.9          | 4.3       |
| General Merchandise Stores                  | 0.5      | 1.2         | 1.1           | 4.6          | 3.7       |
| Clothing and Clothing Accessories Stores    | 0.5      | 1.4         | 1.1           | 3.8          | 3.5       |
| Nursing and Residential Care Facilities     | 1.0      | 2.3         | 1.1           | 4.4          | 4.6       |
| Nonstore Retailers                          | 0.2      | 0.3         | 1.1           | 3.8          | 5.1       |
| Sports, Music, Hobby, and Book Stores       | 0.2      | 0.5         | 1.1           | 5.2          | 3.2       |
| Gasoline Stations                           | 0.4      | 0.4         | 1.1           | 2.0          | 1.4       |
| Health and Personal Care Stores             | 0.6      | 1.2         | 1.1           | 2.9          | 2.7       |
| Miscellaneous Store Retailers               | 0.2      | 0.6         | 1.1           | 8.5          | 8.5       |
| Educational Services                        | 5.5      | 7.5         | 1.1           | 2.6          | 2.7       |
| Food and Beverage Stores                    | 1.0      | 2.6         | 1.1           | 5.7          | 4.0       |

Source: Authors’ calculations.

Discussion and Limitations
What are our key findings? First, we need to be particularly concerned about COVID-19–related restrictions in sectors that supply many other industries (have a high “Domar weight”) and hence propagate shocks to their industry across the economy. These shocks affect not only GDP and employment, but also the long-term health of workers who lose their jobs both in the directly affected industry and in those indirectly harmed as the shock spreads downstream.

In principle, a policy-maker should also be concerned about industries that are largely consumer-facing if they also have many low-wage workers. These industries see disproportionate unemployment, and hence long-term health outcomes, for a given shock to GDP. Quantitatively, however, sectors such as manufacturing are important to so many other industries that avoiding economic restrictions on that sector is better on overall GDP, unemployment, and long-term population health grounds than avoiding restrictions on sectors such as restaurants. This result is driven by the fact that 77% of the effect on GDP of a productivity restriction in manufacturing occurs indirectly as targeted plants raise prices on inputs used in other industries, whereas less than 10% of the overall harm from restrictions on restaurants occurs elsewhere. That is, whether you want to maintain overall GDP, limit unemployment, or prevent long-run health consequences that result from income loss, there is no trade-off: it is most important on all three grounds to keep upstream sectors open relative to retail, restaurants, and bars.

Properly targeting economic restrictions during a pandemic has large economic and health consequences. A three-month lockdown at the severity of April 2020 lowers quarterly GDP by nearly 25%, puts 30% of the country out of work, and leads to over 15,000 early deaths and 15,000 transitions to high morbidity if this income loss is not ameliorated through policy. Although fiscal policy can mitigate these long-term health costs, it is not free: CERB and work subsidies in 2020 drove Canada’s public deficit to wartime levels. Whether the long-term health consequences of lockdowns are mitigated through income supports or shortened with a rapid vaccination schedule, their magnitudes are substantial.

How seriously should we take these estimates? The Leontief approach pursued here is a linearized approximation. When productivity shocks become large—air travel in the United States and China fell more than 95% at the crisis peak—error due to this linearization will begin to bind. Further, there are no highly credible estimates of the productivity shock of the existing Canadian lockdown; hence we are only approximating the size of the sector-by-sector shocks using third-party assumptions about work-from-home potential and essential worker classification. These estimates should therefore be treated by policy-makers as rough. Their value lies more in displaying the relative trade-offs on
output and employment across sectors rather than on the absolute magnitudes.

It is important to keep in mind three limitations of our empirical results. First, we do not model any government countermeasures, such as direct payments to unemployed workers. In general, incorporating demand-side policies requires a model more complex than the present one, and it is difficult to do in an analytically tractable way (to keep this present model usable by policy-makers, we restricted ourselves to models that can be computed analytically in real time in a “dashboard” format).

Second, a production network model can only trace the indirect economic consequences of a shutdown in sectors that are connected via economic transactions. For instance, shutdowns of schools or public transit surely reduce productivity of some industries. We also do not consider the health consequences of delayed medical treatment, vaccination, or surgery due to restrictions on the medical sector. Rio-Chanona et al. (2020) argue, in the UK context, that school and transit-related shutdowns are likely to have minor economic consequences relative to the full lockdown, while Green et al. (2021) argue that Canadian school shutdowns function similarly to a highly central industry in the production network sense.

Third, we do not explicitly incorporate disease transmission probability by sector. To solve for the optimal COVID-19-response policy, a model of transmission dynamics based on workplace social interactions, perhaps incorporating the structure of the network each worker interacts with, is needed. A number of researchers have incorporated these transmission dynamics into equilibrium economic models like ours (e.g., Baqae et al. 2020). However, this literature tends not to discuss the non-COVID-19 health consequences of lockdowns we focus on.

Notes

1. On general health consequences of lockdowns, food insecurity, and mental health, see Lippi et al. (2020), Carroll et al. (2020), Rossi et al. (2020).
2. We assume that production in each industry is Cobb-Douglas. This production function is constant-returns-to-scale and has the well-known property that income and substitution effects exactly cancel. This means that shocks do not go upstream. For example, a shock to the restaurant industry causes total industry production to fall, and also causes the amount of inputs it uses for a given amount of output to rise. These two effects exactly cancel out, meaning that the shock to restaurants does not harm productivity upstream. The constant elasticity of substitution across inputs implicit in the Cobb-Douglas form is, of course, a nontrivial assumption. See, e.g., Baqae and Farhi (2019).
3. The causal link between unemployment spells and mortality is well established. See, e.g., Gerdtham and Johannesson (2003) or the meta-analysis in Roelofs et al. (2011).
4. In particular, standard production network theory assumes that all final consumption is done by households. In practice, especially for small, open economies like Canada, final use includes domestic consumption plus net taxation plus net exports. Industries that are largely made up of imports will therefore have negative final use. By aggregating similar industries where some are net exporters and some are net importers, we ensure that all industries considered have positive final use, and hence that a lockdown in any sector lowers measured GDP.
5. The Italian classification maps nearly to NAICS industries, unlike many provincial or other national essential worker lists.
6. See Public Safety Canada (2021).
7. In particular, the following manufacturing sectors were deemed essential: food, beverage, wood products, paper, petroleum and coal products, chemical manufacturing, plastics, nonmetallic mineral products, primary metals, and transportation equipment.
8. The education and health sectors are largely unlinked in the production network to other industries, so this assumption has little effect on any of our primary policy results.
9. This is Table 3610001 in the Stats Canada database naming scheme.
10. The simulations are run Monte Carlo 100 times to estimate average health consequences by sector.
11. Real estate has a higher share of GDP than employment, since in national accounting, the consumption value of owner-occupied housing in included in GDP even though it uses no labour. In general, high-wage and high-capital industries such as oil and gas production have higher GDP shares than employment shares, and low-wage, low-capital industries such as food services have higher employment shares than GDP shares.
12. Both in a legal sense, and in terms of surveyed employment changes, April 2020 was the peak of the Canadian lockdown.
13. Baqae and Farhi (2019) discuss how broad the general production network approach is when we relax these assumptions, while Carvalho and Tahbaz-Salehi (2019) provide a recent update on the state of the production networks literature.
14. “Perfect” competition means that no firm has market power: any economic profit is competed down to zero by other firms offering similar products, and firms therefore take the prices their products receive as given when maximizing profits.
15. Wassily Leontief won the 1973 Nobel Prize in Economics largely for his work on production networks.
16. Note that I - T is invertible, since T is nonnegative with a spectral radius less than 1: the dollar value of inputs used must be less than the dollar value of output if the firm is maximizing.
17. The relative scale of A, can be chosen so that Ω = 0 without changing any comparative statics.

References

Acemoglu, D., V.M. Carvalho, A. Ozdaglar, and A. Tahbaz-Salehi. 2012. “The Network Origins of Aggregate Fluctuations.” Econometrica 80(5):1977–2016. https://doi.org/10.3982/ECTA9623.
Baqae, D.R., and E. Farhi. 2019. “The Macroeconomic Impact of Microeconomic Shocks: Beyond Hulten’s Theorem.”
Let an economy consist of \( N \) sectors, each made up of perfectly competitive firms.\(^{14}\) Output is industry \( i \) is denoted \( y_i \) and calculated in the Cobb–Douglas form:

\[
y_i = A_i l_i^{\alpha_i} \prod_{j=1}^{N} x_j^{\alpha_{ij}}.
\]

In this equation, \( y_i \) is the dollar value of output from industry \( i \), \( A_i \) is the “total factor productivity” of that industry, \( l_i \) is the amount of labour used, and \( x_j \) is the dollar-value of goods from industry \( j \) used as intermediate goods in industry \( i \). The exponents \( \alpha_i \) denote the relative importance of labour and various intermediate inputs in production. We assume that \( \sum_{j=1}^{N} \alpha_{ij} + \alpha_{il} = 1, \forall i \). That assumption ensures that production has constant returns to scale: doubling labour and all inputs used in any given industry doubles output. Note that \( \alpha_i \) can be positive: the manufacturing industry, for instance, uses some types of manufactured goods in producing other manufactured goods. Note also that \( \alpha_i \) can differ from \( \alpha_j \): the petroleum refining industry supplies gasoline to the fishing industry, but the fishing industry does not provide salmon as an input into petroleum refining.

We assume that there is a unit mass of labour coming from a representative household with the utility function over consumption \( c \) for each of the \( N \) goods

\[
u(c) = \sum_{i=1}^{N} \beta_i \ln \left( \frac{c_i}{\beta_i} \right),
\]

where \( \sum_{i=1}^{N} \beta_i = 1 \). The parameters \( \beta \) represent the relative weight of each sector’s goods in the household’s utility function. This form is often called “log-utility.”

In this setup, we will now look for a “competitive equilibrium.” In this equilibrium, we look for a vector of prices in each industry \( p \), and a wage \( w \), such that if the household maximizes utility and firms maximize profits

\[
\pi_i = p_i y_i - w l_i - \sum_{j=1}^{N} p_j x_{ij} \text{ demand in each industry exactly equals supply } (y_i = c_i + \sum_{j=1}^{N} x_{ij}).
\]

We will then “shock” the economy by reducing productivity \( A_i \), find the new price vector such that demand equals supply, and investigating therefore what happens to output and demand for workers in each industry. We will assume sticky downward wages, so that changes in productivity affect the level of unemployment while keeping the wage unaffected: this is a standard short-run assumption. Note finally that GDP is gross output minus intermediate outputs, which is equal to total consumption \( \sum_{i=1}^{N} c_i = w \), where the equality comes from the fact that only households consume final goods in the model, and their income is exactly \( w \).

This appears at first glance to be a challenging mathematical problem. However, the “input-output” or “production network” approach shows that there is a simple link between productivity changes and output/employment changes. This technique originates with Leontief (1951), with important theorems provided by Acemoglu et al. (2012), Baqaee and Farhi (2019), Gabaix (2011), and Hulten (1978).\(^{15}\) The critical mathematical idea will be that, due to the Envelope Theorem, to a linear approximation the effect of productivity shocks on gross output and employment will depend on an analytically tractable matrix of links between industries.

The dollar value of output of industry \( i \) is equal to the dollar value of all the intermediate uses of \( i \) in other industries, plus the final consumption of consumers. That is, for each \( i \),

\[
p_i y_i = p_i x_{i1} + p_i x_{i2} + \ldots + p_i x_{iN} + p_i c_i.
\]

In matrix form, letting gross output \( q \) be the vector where \( q = p y \), final consumption \( c \) the vector where \( c = p c \), and the “technical coefficients” matrix \( T \) where \( t_{ij} = \frac{p_j y_j}{p_i y_i} \), we have that

\[
q = T q + c \rightarrow q = (I - T)^{-1} c.
\]

The technical coefficients matrix \( T \) has as its \( ij \)th entry the fraction of the dollar value of \( j \)’s output that is paid to \( i \) for intermediate goods. We refer to \( L = (I - T)^{-1} \) as the Leontief inverse.\(^ {16} \)

We can now use the first-order conditions from the firm maximization problem to link the Leontief inverse \( L \) to the change in output and employment when productivity falls. Note that firm \( i \) chooses labour \( l_i \) and intermediate goods \( x_{ij} \) to maximize profits,

\[
\pi_i = p_i y_i - w l_i - \sum_{j=1}^{N} p_j x_{ij} = p_i A_i l_i^{\alpha_i} \prod_{j=1}^{N} x_j^{\alpha_{ij}} - w l_i - \sum_{j=1}^{N} p_j x_{ij}.
\]

The first-order conditions with respect to \( L \) and \( x \) give

\[
\frac{\alpha_{ij} p_j y_j}{w} \text{ and } \frac{x_{ij}}{p_j}.
\]

Therefore,

\[
p_j x_{ij} = \frac{\alpha_{ij} p_j y_j}{w} \rightarrow \frac{t_{ij} x_{ij}}{y_i} = \alpha_{ij} = \frac{t_{ij}}{y_i}.
\]

That is, the technical coefficients matrix \( T \) is the transpose of the matrix made up of the production function coefficients. We will recall this when we link theory to data. Plugging the optimal choices of \( L \) and \( x \) into the production function and taking the natural logarithm, we have

\[
y_i = A_i \left( \frac{\alpha_{ij} p_j y_j}{w} \right)^{\alpha_{ij}} \prod_{j=1}^{N} \left( \frac{\alpha_{ij} p_j y_j}{p_j} \right)^{\alpha_{ij}},
\]

therefore, \( \forall i \),

\[
\ln p_i w = \sum_{j=1}^{N} \alpha_{ij} \ln p_j w \ln A_i - \phi = \sum_{j=1}^{N} t_{ij} p_j w - \ln A_i - \phi.
\]

\(^{14}\) This assumes that \( N \) sectors is large enough that the analysis is valid.\(^{15}\) The critical mathematical idea will be that, due to the Envelope Theorem, to a linear approximation the effect of productivity shocks on gross output and employment will depend on an analytically tractable matrix of links between industries.\(^{16}\) The Leontief inverse.

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where \( \mathcal{O} \) is a constant.\(^{17} \) Letting \( \bar{p} = \left( \ln \frac{p_1}{w}, \ln \frac{p_2}{w}, \ldots \right) \) and \( \bar{a} = (\ln A_1, \ln A_2, \ldots) \), the vector of prices relative to the wage for labour \( \bar{p} = T \bar{p} - \bar{a} \), hence \( \bar{p} = -(1 - T)^{-1} \bar{a} \).

We now have mapped from the productivity shock to changes in relative prices. How can we relate changes in prices \( \bar{a} \) to changes in output and employment? Recall from the firm profit maximization first-order condition that \( x_{ij} = \frac{\alpha_{ij} p_j y_i}{p_j} \), and note that the first-order condition from the household utility function is \( c_j = \beta_j \frac{w}{p_j} \). Plugging these into the condition that supply equals demand,

\[
y_i = c_i + \sum_{j=1}^{N} x_{ij},
\]

we have that

\[
p_j y_j = \beta_j w + \sum_{i=1}^{N} \alpha_{ij} p_j y_i = \beta_j w + \sum_{i=1}^{N} l_{ji} y_i.\]

Note that GDP in this setting is final goods production in dollar terms minus intermediate goods production in dollar terms, which is just equal to what is paid to the households for their labour, \( w \). Let \( \lambda_j \) be the gross output of industry \( j \) over GDP, or \( \frac{p_j y_j}{w} \): we will refer to \( \lambda \) as the Domar weights. We therefore have that

\[
\lambda_j = \beta_j + \sum_{i=1}^{N} l_{ji} \lambda_j
\]

and in matrix form

\[
\lambda = (1 - T)^{-1} \beta;
\]

hence \( \lambda_j = \frac{p_j y_j}{w} = \sum_{j=1}^{N} \beta_j L_{ji} \).

Expanding terms, we have that \( \bar{p}_j = \ln \frac{p_j}{w} = -\sum_{j=1}^{N} L_{ij} \ln A_j \). We therefore have that

\[
\ln y_j = \sum_{j=1}^{N} L_{ij} \ln A_j + \theta,
\]

where \( \theta \) is a constant that does not depend on the shock \( A \). We have, then, that the effect of a vector of productivity shocks, in the first-order approximation from the logarithm, causes a percentage decline in output in each industry depending on the Leontief inverse parameters alone. Note also that since the production function is constant-returns-to-scale and wages are assumed to be sticky at \( w \), a fall in output of \( x \% \) will also reduce employment in that sector by \( x \% \). The formula mapping from productivity shocks on \( A \) to output and employment declines therefore can be computed analytically.

Finally, to see how these changes in gross output by industry map into overall GDP, from \( \ln \frac{p_j}{w} = -\sum_{j=1}^{N} L_{ij} \ln A_j \), multiply each side by \( \beta_j \), sum over \( i \), and use \( w = GDP \) to get

\[
\ln GDP = \sum_{i=1}^{N} \beta_i \ln p_i + \sum_{i=1}^{N} \sum_{j=1}^{N} \beta_j L_{ij} \ln A_j.
\]

By choosing a numeraire price such that \( \sum_{i=1}^{N} \beta_i \ln p_i = 0 \), without loss of generality, we have that

\[
\ln \left( \frac{GDP}{\mathcal{O}} \right) = \sum_{i=1}^{N} \sum_{j=1}^{N} \beta_j L_{ij} \ln A_j = \sum_{i=1}^{N} \lambda_i \ln A_i.
\]

That is, if a productivity shock of 1 percent hits each industry, the total fall in GDP is equal to 1 percent times the sum of Domar weights, which always sum to at least 1.