Study of the Urban Road Networks of Le Mans

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Abstract

An urban road network of Le Mans in France is analyzed. Some topological properties of network are investigated, such as degree distribution, clustering coefficient, diameter, and characteristic path length. These results suggest that our network is a "small-world" network with short average shortest path and large clustering coefficient. Furthermore, double power-law distribution is found in degree distribution which is distinct from the single power-law and a novel degree distribution function is also given. Some analysis on this function extend the comprehension of the origination of the double power-law distribution widely dispersed in nature.

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1 Introduction

During the last decade, the growing availability of large databases, the increasing computing powers, as well as the development of reliable data analysis tools, have constituted a better machinery to explore the topological properties of complex networks [1,2,3,4]. Most of studies has revealed that, despite the inherent differences, main real networks are characterized by the same topological properties, such as relatively small path lengths, high clustering coefficients, scale-free degree distributions, degree correlations, motifs, and community structures. As a particular class of complex networks, spatial networks are different from other networks, which are those embedded in the real space, i.e. networks whose nodes occupy a precise position in two or three dimensional Euclidean space, and whose edges are real physical connections. It is not surprising that the topology of spatial networks is strongly constrained by their geographical embedding, such as airport networks, urban street networks. In this paper we focus on an urban road networks.

Urban road networks, in one way or another, are portrayals of the history of the country’s development. Hence the topology studies of various spatial networks could show significantly different degree distributions. For example, the degree distribution of internet network (presupposing that the network nodes are routers) has the form $P(k) \sim k^{-\gamma}$ where $\gamma_{out} = \gamma_{in} \approx 2.48$ [1]. The power grid of the western US is described by a complex network with exponential form degree distribution [5]. In addition, the analysis of the spatial distribution of the network nodes shows that both indicated networks are fractals with the method of box counting [3,6]. But, here, we show other distinct degree distribution, double power-law, in urban road networks of Le Mans in France (Le Mans is one small city in France). Urban road networks with links and nodes representing road segments and junctions respectively which is a primal graph, exhibit unique features different from other networks. As road networks are almost planar, they show a very limited range of node degrees and could never be scale-free like airport networks or the internet [6]. Nevertheless, there is an interesting connection between those scale-free spatial networks and road networks, since both are extreme cases of an optimization process minimizing average travel costs along all shortest paths, given a set of nodes and a total link length.

In the present paper, the organization is as follows. Section 2 presents the results of degree distribution and cumulative degree distribution of urban road network of Le Mans. In section 3, we analyze the relationship between the double power-law distribution function and its parameters. In section 4 and 5, some topological properties of network are investigated, such as clustering coefficients, diameter and average shortest path length. Conclusions and discussions are given in the last part, section 6.
We investigate the urban road network of Le Mans in France. Previous works on urban traffic network usually were based on a primal graph, where intersections were turned into nodes and roads into edges. Here, we look at them with other paradigm, a dual graph, where a city is transformed into a topological graph by mapping the roads into nodes and the intersections into edges between the nodes. The advantage of the dual graph is that it does not exhibit any geographic constrain compared to primal graph. Hence, in structure of our network, there are 1585 nodes and 5066 edges.

During the research of complex network science, degree distribution \( p(x) \) is always one of the most important characteristics. In Figure 1, we plot the degree distribution in double-log plot and fit the data with black solid line with function as follows, which is called “double power-law” function by us:

\[
p(x) = \frac{a}{(x^b + c \cdot x^d)},
\]

where \( a = 46.7, b = 3.1, c = 1999, d = -1.5 \). The slopes of two straight dashed lines are 3.1 and -1.5 respectively. The reason for choosing this kind of function was explained in [7]. Many similar distributions were found in other researches including the highway networks of Korean [8] and urban public transport networks of Los Angeles in American [9]. Nevertheless, they were named “truncated power-law distributions” by authors. In fact, abundant works with “double power-law” were analyzed as the truncated power law [10,11,12,13,14,15,16] in which more attention was paid to large degree value in one power-law region and small degree value in another one was neglected with various reasons. In addition, it is worth to notice that in Figure 1, the node with degree 5 has the largest probability, in other words, crossover point has maximum value of probability, which is similar to the Moscow region road network in which the node with degree 3 had largest probability in degree distribution [17]. It is very different from non-road spatial network’s degree distributions in which the node with smallest degree, not crossover point, has the largest probability generally. This is decided by the feature of spatial networks such as transportation networks, since the topology of spatial networks is strongly constrained by their geographical embedding. Furthermore, both belonging to transportation networks, road network still has distinct degree distribution form from airport networks [7] in which crossover point does not have maximum value of probability. Since there is a strong restriction to the growth of the degree in road networks which is induced by the geographical arrangement of the edges, however, there is no so strong geographical restriction to the edges existence in the airport networks.

Towards degree distribution function, combined Eq. (1) and Figure 1, we could
simply find that parameter $a$ is the normalization constant, $b$ and $d$ are two scale exponents in two regions of power-law respectively. So, in principle, from the standpoint of mathematics, this kind of function could produce any form of “double power-law” distribution, which could be seen having some relationships with the origin of this phenomenon.

An alternative way of presenting degree data is to make a plot of the cumulative distribution function

$$P(x) = \int_x^{40} \frac{a}{(x^b + c * x^d)}dx,$$

which is the probability that the degree is greater than or equal to $x$ and 40 is the maximum degree in our network. The advantage of cumulative distribution is that all the original data are represented and it reduces the noise in the tail. After calculation of Eq. (2), we get

$$P(x) = \frac{-42.5 + 0.4 * x^{2.5}}{x - 40} \text{Hypergeometric2F1}[0.54, 1, 1.54, -0.0005 * x^{4.6}].$$

where $\text{Hypergeometric2F1}[a, b, c, z]$ is the hypergeometric function defined by:

$$2F_1(a, b; c; z) = \frac{\Gamma(c)}{\Gamma(b) \Gamma(c - b)} \int_0^1 t^{b-1}(1 - t)^{c-b-1}(1 - tz)^{-a}dt.$$ (4)

From Figure 2, we also find that there is double power-law distribution in log-log plot with two exponents -0.017 and -3.35 respectively.

![Fig. 1. Degree distribution of urban road network of Le Mans city in France. $p(x)$ is the degree distribution function, and $x$ is the degree of each node. “double power-law” phenomenon is obviously showed in log-log plot and our fitting curve with black solid line matches the data well. Two straight dashed lines define two power laws with scale exponents 3.1 and -1.5 respectively. Our fitted function is $p(x) = 46.7/(x^{3.1} + 1999 * x^{-1.5}).$]
Fig. 2. Cumulative distribution function of degree of urban road network of Le Mans city in France. $P(x)$ is the cumulative degree distribution function, and $x$ is the degree of each node. “double power-law” phenomenon is also showed in log-log plot. Two straight dashed lines define two power laws with scale exponents -3.35 and -0.017 respectively.

3 Relationship between double power-law function and its parameters

We make an elaborate analysis on the expression of $p(x)$. Firstly, we do the normalization for the probability function by using equation $\sum_{x=1}^{100} p(x) = 1$ since the degree of each node is discrete variable. As parameter $a$ is the normalization constant, so we pay more attention to the influence on the expression of $p(x)$ caused by the varying of parameters $b, c, d$. From the Figure 1, we could find that $b$ and $d$ have the directly relationship with the exponents of two power-law regions. Hence, our discussions mainly focus on the varying of $b$ and $d$. Based on the different features of degree distribution in airport networks and urban road networks, our discussions are divided into two classes: one is that degree at crossover point has non-largest probability, behaved as both of $b, d$ are positive like airport networks; another is that degree at crossover point has largest probability, behaved as either $b$ or $d$ is negative, the rest is positive, like urban road networks. The case of $b = d$ reduces to familiar single power-law distribution. In each class, we attempt to analyze the role of each parameter played. At the same time, we make sure that $p(x)$ is always normalized. We define the intersection of two power-law in all figures as “crossover” and define the scaling behavior in the region of $1 < x < x_{\text{crossover}}$ as “the first power-law” and define that in the region of $x_{\text{crossover}} < x < 100$ as “the second power-law”.

In the first case $b > d > 0$, we choose the probability function in the form of $p(x) = a/(x^b + 1293139 \times x^{0.5})$ which is a monotonical function. In the panel $a$ of Figure 3, it is found that the exponents of the first power-law are the same
with fixed $d$. As $b$ increases, the exponents of the second power-law become bigger and the value of crossover decreases. In the panel $b$, the expression of function is $p(x) = a/(x^4 + 1293139 * x^d)$. Due to the identity of parameter $b$, three curves have the same exponents of the second power-law. As $d$ increases, the exponents of the first power-law become bigger and the value of crossover decreases. In the panel $c$, we have the function $p(x) = a/(x^{20} + c * x^3)$. As the result of equality of both $b$ and $d$, the overlap of curves of the first power-law and the parallelity of curves of the second power-law are showed evidently. The effect of parameter $c$ is to change the position of crossover. The value of crossover increases with parameter $c$ increasing. So, based on above results, we draw a conclusion that when $b > d > 0$, parameter $b$ and $d$ are directly in charge of the exponents of the second and the first power-law respectively. Parameter $c$ is in charge of the position of crossover when $b$ and $d$ are fixed.

Fig. 3. Variation tendency of probability function $p(x)$ against its parameters $b, c, d$ when $b > d > 0$ in log-log plot. In the panel $a$, the exponents of the first power-law are identical since parameter $d$ is fixed. Those of the second power-law become bigger with $b$ increasing. While the value of crossover becomes smaller. In the panel $b$, the exponents of the second power-law are identical because of invariance of parameter $b$, those of the first power-law behave the same as panel $a$. But the value of crossover increases along with parameter $d$. In the panel $c$, the exponents of each power-law are identical respectively by reason of unchanged $b$ and $d$. Parameter $c$ directly influences the position of the crossover.

In the second case $0 < b < d$, we choose $p(x) = a/(x^b + 10^{-80} * x^{150})$. In Figure 4, we make a comparison with the Figure 3. It is observed that the variation tendency in the panel $a$ and $b$ of Figure 4 is quite the reverse of them in Figure 3. The variation tendency in the panel $c$ of both figures are mostly the same. So, from discussions of the first class, it could be seen that in networks such as airport networks, the probability function is monotonical decreasing. In addition, the first power-law with smaller $b$ or $d$ has larger probability than the second power-law with the bigger $b$ or $d$.

Next, we will discuss the second class which means the function exhibits
Fig. 4. Variation tendency of probability function $p(x)$ against its parameters $b, c, d$ when $0 < b < d$ in log-log plot. In the panel $a$, the exponents of the second power-law are identical since parameter $d$ is fixed. Those of the first power-law become bigger along with parameter $b$. In the meantime, the value of crossover varies bigger. In the panel $b$, the exponents of the first power-law are identical because of unchanged parameter $b$. Those of the second power-law become bigger along with $d$. The value of crossover varies smaller when $b$ increases. In the panel $c$, the exponents of each power-law are the same respectively. The value of crossover decreases when parameter $c$ increases, which behaves opposite to that in Figure 3.

nonmonotonicity like Figure 1. In Figure 5, we take the function as $p(x) = a/(x^b + 1999 * x^{-1.5})$ in the panels $a (b > |d|)$ and $b (b < |d|)$ with negative $d$. In the panels $c (|b| > d)$ and $d (|b| < d)$ with negative $b$, the function is $p(x) = a/(x^b + 10^{-4} * x^{2.5})$. We could find the variation tendencies are both the same for panel $a$ and $b$, and for panel $c$ and $d$. For the former, as $b > d$, $b$ is in charge of the second power-law. Meanwhile, because of $|b| > |d|$ in panel $a$, probability function has small value of crossover. However, in panel $b$, because of $|b| < |d|$, probability function has big value of crossover. For the latter, as $b < d$, $d$ is in charge of the second power-law. In panel $c$, since $|b| > |d|$, probability function has small value of crossover. It is vice versa in panel $d$. So, we draw a conclusion that the second power-law is charged by bigger parameter out of $b$ and $d$. And, probability function has big value of crossover when $|d| > |b|$.

4 Clustering coefficient

A common property of social networks is that cliques form, representing circles of friends or acquaintances in which every member knows every other member. This inherent tendency in cluster is quantified by the clustering coefficient. For each node $i$ having $x_i$ edges which connect it to $x_i$ other nodes, if the nearest neighbors of this node are part of a clique, there would be $x_i(x_i - 1)/2$ edges between them. The ratio between the number $E_i$ of edges that actually exist
Fig. 5. Variation tendency of probability function $p(x)$ against parameter $b$ when one of $b, d$ is negative. From each panel, we could find the exponents of the first power-law are negative and those of the second power-law are positive. With the increasing of the absolute value of parameter $b$, the exponent of power-law depended on $b$ is increasing. The value of crossover becomes smaller when the absolute value of parameter $b$ increases.

between these $x_i$ nodes and the total number $x_i(x_i - 1)/2$ gives the value of the clustering coefficient of node $i$:

$$C_i = \frac{2E_i}{x_i(x_i - 1)}. \quad (5)$$

The clustering coefficient of the whole network is the average of all individual $C_i$’s. By calculation, $C$ of the entire urban road network is 0.042. To check whether our network is a small-world network or not, we construct a random network with same number of nodes and edges in our network and get $C_{rand} = 0.004$ which is much smaller than that of urban road network.

5 Characteristic path length

Shortest paths play an important role in the transport and communication within a network. It is useful to represent all the shortest path lengths of a graph $G$ as a matrix $D$ in which the entry $d_{ij}$ is the length of the geodesic from node $i$ to node $j$. The maximum value of $d_{ij}$ is called the diameter of the network. A measure of the typical separation between two nodes in the network is given by the average shortest path length, also known as characteristic path length, defined as the mean of geodesic lengths over all couples of nodes [18]:

$$L = \frac{1}{N(N-1)} \sum_{i,j \in G, i \neq j} d_{ij}, \quad (6)$$
From above equation, we get the average shortest path of urban road network $L = 4.183$ and $L_{rand} = 4.000$ from artificial random network we constructed. $L \approx L_{rand}$ and $C \gg C_{rand}$ suggest that urban road network of Le Mans has the small-world network’s properties. In the meanwhile, we calculate the diameter of this network and its value is equal to 8.

6 Concluding remarks

In the present paper, we have done a empirical study on urban road network of Le Mans in France and investigated some structure properties of it. From its degree distribution, double power-law, we find that urban road network as one of spatial networks has special degree distribution form from other networks, since there are many constrains from its geographical embedding. The phenomenon that the node with smallest degree has largest probability in most of single power law distribution is not true here. The node at the crossover point has the largest probability in network we studied.

In order to make a deeply understanding of double power-law distribution, we elaborately analyze the structure of the probability function. Our discussions are divided into two classes based on the feature of degree distribution of two transportation networks, road network and airport network. One class is that both of $b$ and $d$ are positive; another is that either of two parameters is negative. Through our analysis, the conclusion is that the bigger parameter of $b$ and $d$ is in charge of the behavior in the second power-law with bigger degree. The smaller one is in charge of that in the first power-law with smaller degree. Absolute value of $b$ is more bigger than that of $d$, the smaller the value of crossover. In addition, parameter $c$ just controls the position of crossover in the limit of both $b$ and $d$ unchanged.

At last, through the calculation of clustering coefficient and average shortest path, small world properties have been found. Though these topological properties are investigated and a novel double power-law distribution function is given, the mechanism of the organization of double power-law structure and that of the transition from single power-law to double power-law are still open questions in complex network study. It is necessary to research more other features of double power-law distribution in next stage, such as the behavior of its entropy. Perhaps a model producing such distribution is worthwhile to consider.
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