Null Energy Condition and Dark Energy Models

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Null Energy Condition (NEC) requires the equation of state (EoS) of the universe \( w_u \) satisfy
\[ w_u \geq -1, \]
which implies, for instance in a universe with matter and dark energy dominating \( w_u = w_m \Omega_m + w_{de} \Omega_{de} = w_{de} \Omega_{de} \geq -1 \). In this paper we study constraints on the dark energy models from the requirement of the NEC. We will show that with \( \Omega_{de} \sim 0.7 \), \( w_{de} < -1 \) at present epoch is possible. However, NEC excludes the possibility of \( w_{de} < -1 \) forever as happened in the Phantom model, but if \( w_{de} < -1 \) stays for a short period of time as predicted in the Quintom theory NEC can be satisfied. We take three examples of Quintom models of dark energy, namely the phenomenological EoS, the two-scalar-field model and the single scalar model with a modified Dirac-Born-Infeld (DBI) lagrangian to show how this happens.

I. INTRODUCTION

It is well known that energy conditions play an important role in classical theory of general relativity and thermodynamics[1]. In classical general relativity it is usually convenient and efficient to restrict a physical system to satisfy one or some of energy conditions for study, for example, in the proof of Hawking-Penrose singularity theorem[2, 3], the positive mass theorem[4] and so on, while in thermodynamics energy conditions are the bases for obtaining entropy bounds[5, 6]. Among those energy conditions, the null energy condition is the weakest one which states that for any null vector \( n^\mu \) the stress energy tensor \( T_{\mu\nu} \) should satisfy the relation
\[ T_{\mu\nu} n^\mu n^\nu \geq 0. \]

In general, the violation of NEC leads to the breakdown of causality in general relativity and the violation of the second law of thermodynamics[7]. These pathologies require that the total stress tensor in a physical spacetime manifold should obey the NEC. In the framework of the standard 4-dimensional Friedmann-Robertson-Walker (FRW) cosmology the NEC implies \( \rho + p \geq 0 \), which in turn gives rise to a constraint on the equation of state of the universe (EoS) \( w_u \) defined as the ratio of pressure to energy density, \( w_u \geq -1 \). In this paper we study the constraints on the dark energy models from the requirement of \( w_u \geq -1 \).

In the early Universe with radiation dominant the EoS of the universe \( w_u \) is approximately equal to \( \frac{1}{3} \) and in the matter dominant period \( w_u \) is nearly zero, so NEC is satisfied well. However when the dark energy component is not negligible we have
\[ w_u = w_m \Omega_m + w_{de} \Omega_{de} \geq -1, \]
where the subscripts ‘\( m \)’ and ‘\( de \)’ stand for matter and dark energy, respectively. With \( w_m = 0 \), inequality (2) becomes
\[ w_{de} \Omega_{de} \geq -1. \]

From the inequality above, we can see that models of dark energy with \( w_{de} \geq -1 \) such as the Cosmological Constant and the Quintessence satisfy the NEC, but the models with \( w_{de} < -1 \) predicted for instance by the Phantom theory where the kinetic term of the scalar field has a wrong sign does not. Interestingly we can see that NEC might be satisfied in models if \( w_{de} < -1 \) stays for a short period of time during the evolution of the universe. In this paper we will show this happens in the Quintom models of dark energy.

The paper is organized as follows: in section II we will present three examples of the Quintom models to show how the NEC is satisfied and the section III is the summary of the paper.

II. NULL ENERGY CONDITION AND THE QUINTOM DARK ENERGY

Quintom is a dynamical model of dark energy[8]. It differs from the Cosmological Constant, Quintessence[9], Phantom[10], K-essence[11] and so on in the determination of the cosmological evolution. Although the current data
in combination with the 3-year WMAP [12], the recently released 182 SNIa Gold sample [13] and also other cosmological observational data show the consistence of the Cosmological Constant, it is worth noting that the dynamical dark energy models are not excluded and Quintom dark energy is mildly favored (for recent references see e.g. [14, 15, 16]). The most salient feature of the Quintom model is that its EoS can smoothly cross $-1$. In this section we will study the implications of NEC on the Quintom models. Working with three specific examples we will show the NEC can indeed be satisfied.

A. A phenomenological model with parameterized EoS across $-1$

With a simple calculation, the inequality (3) can be rewritten as

\[(1 + w_{de}(a)) \Omega_{de0} \exp \left\{ \int_1^a [-3(1 + w_{de}(a'))]d\ln a' \right\} + \frac{\Omega_{m0}}{a^3} \geq 0 , \tag{4}\]

where the subscript ‘0’ represents today’s value.

We firstly start with a phenomenological model with a parameterized EoS which will be able to cross over $-1$:

\[w_{de} = w_0 + w_1(1 - a) , \tag{5}\]

where $a$ is the scale factor which we normalize to be $a_0 = 1$ at present. One can see that when $a$ is equal to $(1 + w_0 + w_1)/w_1$, the EoS of dark energy crosses $-1$. This type of parametrization for the EoS has been widely used in the literature [17, 18] for the fitting of constraining the EoS to the observational data.

Inserting Eq. (5) into the inequality (4), the NEC puts a constraint on the parameters $w_0$ and $w_1$. In Fig. 1 we take $w_0 = -0.9$ and $w_1 = -0.3$, and then plot the evolution of the dark energy EoS and also the EoS of the universe respectively. One can see from Fig. 1 that the EoS of dark energy $w_{de}$ crosses $-1$ from below to above. In this case the EoS of the universe $w_u$ evolves from zero which corresponds to the matter dominant epoch, and then reaches its minimal value during which the universe enters the dark energy dominant period, and finally returns to zero in the future. We can read directly from this figure that the minimal value of $w_u$ stays above $-1$ which satisfies the NEC.

![Figure 1: Plot of the evolution of the EoS $w_{de}$ with $w_{de} = w_0 + w_1(1 - a)$, and the EoS $w_u$ for the universe as a function of $\ln a$. Here in the numerical calculation we have taken $w_0 = -0.9$ and $w_1 = -0.3$.](image)

One could take another example with a different parametrization of the EoS:

\[w_{de} = w_0 + w_1\left(1 - a + \frac{3}{16}a^2 \right) . \tag{6}\]
This example is different from the previous one since we introduce a square term of the scale factor which makes the EoS $w_{de}$ crosses $-1$ twice. Taking proper values of those parameters, this model can give a scenario that dark energy stays in the Phantom-like state only for a while and then returns to be Quintessence-like. After taking the similar calculation, we plot the evolution of the EoS of the model and the universe respectively in Fig. 2. In this example the EoS of dark energy $w_{de}$ starts to evolve from Quintessence-like in the past, and then enters the Phantom-like state for a short period of time, and finally returns to above $-1$. We also find that $w_u$ behaves similar to that in Fig. 1. Its value resides on zero in the past and in the future which means that matter have dominated and will dominate again the evolution of the universe. Although $w_u$ runs away from zero during the evolution, its value keeps being larger than $-1$ as shown in Fig. 2. Therefore, this model can be consistent with the NEC as well.

![Figure 2](image-url)

Figure 2: Plot of the evolution of the EoS $w_{de}$ with $w_{de} = w_0 + w_1 (1 - a + \frac{3}{16} a^2)$, and the EoS $w_u$ for the universe as a function of $\ln a$. Here in the numerical calculation we have taken $w_0 = -0.93$ and $w_1 = 0.23$.

### B. The Two-scalar-field Quintom model

Building the field model of Quintom dark energy is a challenge due to the No-Go theorem which has been proved in Ref. [19](also see Ref. [8, 20, 21, 22, 23, 24]). This No-Go theorem forbids a traditional scalar field model with a lagrangian of general form $L = L(\phi, \nabla \phi)$ from having its EoS cross over the cosmological constant boundary. According to this theorem, dynamical models like Quintessence, Phantom and K-essence are unable to realize their EoS cross $-1$. Therefore, to realize a viable Quintom field model in the framework of Einstein’s gravity theory, one needs to introduce extra degrees of freedom to the conventional theory with a single scalar field. The simplest Quintom model is constructed by two scalars with one being Quintessence-like and another Phantom-like proposed firstly in Ref. [8], and this model has been widely studied in detail later on. In recent years there have been a lot of activities in the theoretical study on building Quintom models, such as a single scalar with high-derivative [25, 26], vector field[27], extended theory of gravity[28], Lorentz-violating dark energy models[29] and so on, see e.g. [30].

In this section we investigate a two-scalar-field Quintom model of dark energy in flat FRW cosmology which is described by the action

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} + \mathcal{L}_m + \mathcal{L}_{de} \right],$$  \hspace{1cm} (7)
with

\[ \mathcal{L}_{dc} = \frac{1}{2} \partial_\mu \phi_1 \partial^\mu \phi_1 - \frac{1}{2} \partial_\mu \phi_2 \partial^\mu \phi_2 - V(\phi_1, \phi_2), \]

where \( R \) is the Ricci scalar of the universe, \( \mathcal{L}_m \) is the lagrangian of matter, \( \mathcal{L}_{dc} \) is the lagrangian of dark energy, and the metric is in form of \((+, - , - , -)\). Here the field \( \phi_1 \) has a canonical kinetic term, but \( \phi_2 \) is a ghost field. With \( \mathcal{L}_{dc} \) in (8), we can easily obtain the energy density and the pressure of this model,

\[ \rho_{dc} = \frac{1}{2} \dot{\phi}_1^2 - \frac{1}{2} \dot{\phi}_2^2 + V, \]

\[ p_{dc} = \frac{1}{2} \dot{\phi}_1^2 - \frac{1}{2} \dot{\phi}_2^2 - V, \]

where the dot denotes the derivative with respect to the cosmic time, and by the variational principle the Einstein equations are given by

\[ H^2 = \frac{8\pi G}{3} \left( \frac{1}{2} \dot{\phi}_1^2 - \frac{1}{2} \dot{\phi}_2^2 + V + \rho_m \right), \]

\[ \ddot{\phi}_1 + 3H \dot{\phi}_1 + \frac{dV}{d\phi_1} = 0, \]

\[ \ddot{\phi}_2 + 3H \dot{\phi}_2 - \frac{dV}{d\phi_2} = 0. \]

Since it is required that the total EoS of the universe satisfies the NEC, it is clear that the Phantom component of this kind of model can only be dominant for a short while and hence it gives a constraint on choosing the potential of the Phantom field. In the following numerical calculations, we will choose an appropriate potential of Phantom field and different potentials of Quintessence field, which can satisfy the NEC and give various Quintom scenarios.

In Fig. 3 we choose the potentials of Quintessence and Phantom field to be \( V_{\phi_1} = V_1 \exp(\lambda_1 \phi_1^2/M^2) \) and \( V_{\phi_2} = V_2 \exp(\lambda_2 \phi_2^2/M^2) \), respectively. Here we normalize the fields with Planck scale and choose \( M \) to be \( 0.01M_{pl} \). During the evolution, the Phantom field will climb up along its potential and become dominant because its energy density will increase while that of Quintessence field will decrease. However, since there is a maximum of the potential, after reaching that point, the Phantom field will stay on the top and stop affecting the evolvement of the universe. From the figure, we can see that after the EoS of dark energy being less than \(-1\) today for a short while, it can exit in the future and actually approach to \(-1\). In this case we read from the figure that the EoS of the universe can be always larger than \(-1\), and thus the NEC can be easily satisfied.

In Fig. 4 we choose another potential form of the Quintessence field to be \( V_{\phi_1} = \frac{1}{2} m^2 \phi_1^2 \) with the Phantom’s potential unchanged. We can see from the figure that, similar to the reason above, Phantom can only dominate the universe for a while, and due to different potential of Quintessence, the EoS of dark energy model behaves very differently. The figure shows that after the universe exit the Phantom dominating phase, the Quintessence will dominate the universe again, and the EoS of dark energy will cross \(-1\) twice. In the future, however, this EoS will also become de-sitter like. In the whole process, the EoS of the universe will always be above the cosmological constant boundary, and this model of Quintom dark energy also satisfies the NEC.

### C. A single scalar field with a modified DBI lagrangian

Having presented the examples of two-scalar-field Quintom models in consistent with the NEC, we in this section consider a class of Quintom models described by an effective lagrangian involving higher derivative operators.

Due to the contribution of higher derivative terms, this kind of models can give rise to an EoS across \(-1\) as pointed out in Ref. 23. A connection of this type of Quintom theory to the string theory has been considered in Ref. 31 and 26. Here we take the string-inspired model in [31] for a detailed study to check whether it satisfies the requirement of the NEC. The action of this Quintom dark energy is given by

\[ S_{dc} = \int d^4x \sqrt{-g} \left[ -V(\phi) \sqrt{1 - \alpha^2 \nabla_\mu \phi \nabla^\mu \phi + \beta \phi \Box \phi} \right]. \]

1 Here and in the following, we would like to redefine the parameter \( \tilde{\lambda}_1 \) and \( \tilde{\lambda}_2 \) to be \( \lambda_1/M^2 \) and \( \lambda_2/M^2 \).
Figure 3: Plot of the evolution of the EoS $w_{de}$ for the two-scalar-field Quintom dark energy with the potential $V_{\phi_1} = V_1 \exp(\lambda_1 \phi_1^2)$, $V_{\phi_2} = V_2 \exp(\lambda_2 \phi_2^2)$, and the EoS $w_u$ for the universe as a function of $\ln a$. Here in the numerical calculation we have taken $V_1 = (3.1623eV)^4$, $V_2 = (1.8612eV)^4$, $\lambda_1 = -0.8 \times 10^4$, $\lambda_2 = -0.5 \times 10^4$ and the initial values are $\phi_{i1} = 0.02$, $\dot{\phi}_{i1} = 5.0 \times 10^{-62}$, $\phi_{i2} = -0.01$ and $\dot{\phi}_{i2} = 2.0 \times 10^{-62}$.

Figure 4: Plot of the evolution of the EoS $w_{de}$ for the two-scalar-field Quintom dark energy with the potential $V_{\phi_1} = \frac{1}{2}m^2\phi_1^2$, $V_{\phi_2} = V_2 \exp(\lambda_2 \phi_2^2)$, and the EoS $w_u$ for the universe as a function of $\ln a$. Here in the numerical calculation we have taken $m = 1.0 \times 10^{-29}eV$, $V_2 = (1.8612eV)^4$, $\lambda_2 = -0.5 \times 10^4$ and the initial values are $\phi_{i1} = 0.02$, $\dot{\phi}_{i1} = 5.0 \times 10^{-62}$, $\phi_{i2} = -0.01$ and $\dot{\phi}_{i2} = 2.0 \times 10^{-62}$.
This is a generalized version of “Born-Infeld” action with the introduction of the $\beta'$ term (see, for example, for derivation of the higher derivative term from string theory). To the lowest order, the Box-operator term $\phi \Box \phi$ is equivalent to the term $\nabla_\mu \phi \nabla^\mu \phi$ when the tachyon is on the top of its potential. However when the tachyon rolls down from the top of the potential, these two terms exhibit different dynamical behavior. The two parameters $\alpha'$ and $\beta'$ in could be arbitrary in the case of the background flux being turned on. One interesting feature of this model is that it provides the possibility of its EoS $w_{de}$ running across the cosmological constant boundary. In the following studies to make two parameters ($\alpha'$, $\beta'$) dimensionless, it is convenient to redefine $\alpha = \alpha' M^4$ and $\beta = \beta' M^4$ where $M$ is an energy scale of the effective theory of the field.

From (14) we obtain the equation of motion for the scalar field $\phi$:

$$\frac{\beta}{2} \Box (V f) + \alpha \nabla_\mu \left( \frac{V \nabla^\mu \phi}{f} \right) + M^4 \phi V f + \frac{\beta V}{2f} \Box \phi = 0 ,$$

(15)

where $f = \sqrt{1 - \alpha' \nabla_\mu \phi \nabla^\mu \phi + \beta' \Box \phi}$ and $V = dV/d\phi$. Correspondingly, the energy stress tensor of Quintom dark energy is given by

$$T_{\mu \nu} = g_{\mu \nu} \left[ V f - \frac{\beta}{2M^4} \nabla_\rho \left( \frac{\phi V}{f} \nabla^\rho \phi \right) \right] + \frac{\alpha}{M^4} \frac{V}{f} \nabla_\mu \phi \nabla^\mu V \phi + \frac{\beta}{2M^4} \nabla_\mu \left( \frac{\phi V}{f} \right) \nabla^\mu \phi ,$$

(16)

In order to simplify the calculation, we technically define another parameter $\psi \equiv \frac{\alpha c}{\alpha \phi} = - \frac{\phi V}{2M^4 f}$ to solve (15) and (10). In the framework of a flat FRW universe filled with a homogeneous scalar field $\phi$, we have the equations of motion in forms of

$$\ddot{\phi} + 3H \dot{\phi} = \frac{\beta \phi}{4M^4 \psi^2} V^2 - \frac{M^4 \phi}{\beta \phi} + \frac{\alpha}{\beta \phi} \dot{\phi}^2 ,$$

(17)

$$\ddot{\psi} + 3H \dot{\psi} = (2\alpha + \beta) \left( \frac{M^4 \psi^2}{\beta^2 \phi^2} - \frac{V^2}{4M^4 \psi} \right) - \frac{\beta \phi}{2M^4 \psi} V \phi - \frac{2\alpha}{\beta \phi} \dot{\phi} \psi - (2\alpha - \beta) \frac{\alpha \psi}{\beta^2 \phi^2} \dot{\phi}^2 ,$$

(18)

and the energy density and the pressure of this field can be written as

$$\rho_{de} = - \frac{\alpha \psi}{\beta \phi} \dot{\phi}^2 - \psi \dot{\phi} - \frac{\psi \dot{\phi}^2}{4M^4 \psi} V^2 - \frac{M^4 \psi}{\beta \phi} ,$$

(19)

$$p_{de} = - \frac{\alpha \psi}{\beta \phi} \dot{\phi}^2 - \psi \dot{\phi} - \frac{\psi \dot{\phi}^2}{4M^4 \psi} V^2 + \frac{M^4 \psi}{\beta \phi} ,$$

(20)

According to the restriction of the NEC, we need this Quintom model to satisfy the inequality (14). Although in this case the phase space of Quintom dark energy is constrained, we will show below that the inequality (14) can be satisfied easily. In the numerical study, we constrain the parameters $\alpha$ and $\beta$ so that when expanding the derivative terms in the square root to the lowest order the model in (14) gives rise to a canonical kinetic term for the scalar field $\phi$, i.e., $\alpha + \beta > 0$.

Through calculating Eqs. (17), (18) and the Friedmann equations numerically and then comparing the results in the inequality (14), we can judge whether a model is consistent with the NEC. We first consider a model with the potential $V(\phi) = \exp(-\lambda \phi) \exp(\lambda \phi)$. In the calculation we normalize the fields $\phi$ and $\psi$ with the Planck scale and choose the energy scale $M = 1.2211 \times 10^{-4} eV$. For a given set of the model parameters, we make the numerical calculations and plot the evolutions of the EoS of the Quintom dark energy and the universe, which are shown in Fig. 6. We can read that the EoS of dark energy $w_{de}$ starts from a value larger than $-1$, then evolves to less than $-1$, and soon exits the Phantom-like state, and eventually approaches the cosmological constant asymptotically. During the evolution the period of time for dark energy to be Phantom-like is very short. Therefore, the corresponding EoS of the universe evolves from the matter dominant period with $w_u = 0$ to the dark energy dominant period smoothly without violating the NEC as shown in Fig. 5.

For another example we take a different form of potential $V(\phi) = \frac{1}{2} m^2 \phi^2$. Similar to what have been done above, we obtain another example consistent with the NEC. The normalization in this case is the same as that in the above one. In Fig. 6, we give the evolutions of the EoS of this Quintom model and the universe respectively. One can see from this figure the $w_u$ satisfies the NEC, but the detailed evolution of the universe differs from the one shown in
Figure 5: Plot of the evolution of the EoS $w_{de}$ for the string-inspired Quintom dark energy with the potential $V(\phi) = \frac{V_0}{\exp(-\lambda \phi)} + \exp(\lambda \phi)$ and the EoS $w_u$ for the universe as a function of $\ln a$. Here in the numerical calculation we have taken $\lambda = 10^{-3}$, $V_0 = (1.3183 \times 10^{-3} \text{eV})^4$, $\alpha = 0.6$, $\beta = 0.4$ and the initial values are $\phi_i = 12$, $\phi'_i = -2.3 \times 10^{-5}$, $\psi_i = -3.84 \times 10^2$ and $\psi'_i = -40$. In choosing the initial values the prime represents the derivative with respect to $\ln a$.

In Fig. 5, the EoS of Quintom dark energy $w_{de}$ crosses the cosmological constant boundary from a fixed value below $-1$ to above and then evolves close to $-1$ in the future. In this case the EoS of the universe $w_u$ runs from zero when the universe is dominated by the matter, and then approaches the de-Sitter phase asymptotically.

III. CONCLUSION

In this paper we have studied the implications of NEC in the models of dark energy. We show that NEC excludes the models with $w_{de} < -1$ forever as predicted by the Phantom dark energy, however allows the possibility of having $w_{de} < -1$ for a short period of time as it happens in the Quintom models. We have shown explicitly in this paper three examples of Quintom models where NEC is satisfied.

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Figure 6: Plot of the evolution of the EoS $w_{de}$ for the string-inspired Quintom dark energy with the potential $V(\phi) = \frac{1}{2} m^2 \phi^2$ and the EoS $w_u$ for the universe as a function of $\ln a$. Here in the numerical calculation we have taken $m = 1.435 \times 10^{-35} \text{eV}$, $\alpha = 1.3$, $\beta = 0.5$ and the initial values are $\phi_i = 10$, $\phi_i' = -4.3 \times 10^{-3}$, $\psi_i = -4 \times 10^2$ and $\psi_i' = -1.9$. In choosing the initial values the prime represents the derivative with respect to $\ln a$.

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