Spin and Parity of $\Xi_{3/2}$ Exotic Baryon from Kaon Scattering on the Nucleon

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Abstract

We calculate total cross section for production of the $\Xi_{3/2}(1862)$ exotic baryon in $\bar{K}N \rightarrow K\Xi_{3/2}$ reaction assuming the following spin-parity values of the $\Xi_{3/2}$ baryon $J^\pi = \frac{1}{2}^+, \frac{1}{2}^-, \frac{3}{2}^+$ and $\frac{3}{2}^-$. We demonstrate that the reaction total cross section strongly depends on the spin of the $\Xi_{3/2}$ baryon.

Key words: exotic baryon, spin, parity

1 Introduction

Experimental observation [1,2,3,4,5,6,7,8,9,10,11,12,13] of a narrow baryon, $\Theta^+$, which, due to its strangeness $S = +1$, cannot be a three-quark bound system pushes great interest to physics of exotic hadrons, see, e.g., [14,15] and further references therein. The $\Theta^+$ mass is close to 1540 MeV/$c^2$ and width is much smaller than a typical hadron width. It was found no evidence for $\Theta^{++}$ [4,6,9,13] which leads to the conclusion that $\Theta^+$ should be isoscalar. The $\Theta^+$ baryon has been included as a three-star resonance in 2004 PDG listings.

Somewhat later two another candidates for the exotic baryons, $\Xi^{−−}_{3/2}$ and $\Xi^0_{3/2}$, with strangeness $S = −2$, mass near 1860 MeV/$c^2$ and narrow width < 18 MeV were reported by the NA49 collaboration [16]. According to its strangeness and electric charge the minimal value of the $\Xi_{3/2}$ isospin is $I = \frac{3}{2}$. It is natural to assume that the isospin

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singlet $\Theta^+$ and isospin quartet $\Xi^\pm, \Xi^0, \Xi^+_3$ and $\Xi^-_3$ should be members of the same flavor multiplet ($\bar{10}_f$).

Besides that, a narrow anti-charmed baryon with a minimal constituent quark composition $uudd\bar{c}$ was observed by the H1 collaboration [17].

Finally, a narrow peak at 1734 MeV/$c^2$ in $\Lambda K^0$ invariant mass observed in preliminary results from the STAR experiment at RHIC was interpreted as a pentaquark state with the isospin $I = \frac{1}{2}$ [18].

Despite these impressive results, the negative results of a search for $\Theta^+$ [19,20,21,22], as well as for $\Xi^\pm_3$ [23,24], were also reported very recently. So it is evident that a new kind of experimental study is necessary to clarify the situation. For example,

- Experiments with high statistics and different beams and targets to confirm or reject the observed exotic baryons and to search, if any, for new exotic states.
- Measurement of spin and parity of the observed pentaquarks.

A lot of theoretical models were proposed to interpret the obtained experimental results for the exotic baryons, chiral-skyrmion models, constituent quark models, QCD sum rules, lattice QCD, etc., see discussion in [14,15]. Here it is important to stress, that all such calculations, been “turned to” the experimental mass and width of the $\Theta^+$, give very different predictions for the spectroscopy of exited exotic baryons, as well as predict different spin-parity quantum numbers for the $\Theta^+$ and the $\Xi^\pm_3$. For example, in the chiral-skyrmion model the lowest exotic states are members of the $\bar{10}_f$-plet with $J^\pi = \frac{1}{2}^+$ [25,26,27]. The next states belong to the $27_f$-plet with $J^\pi = \frac{3}{2}^+$ [28,29,30,31]. Some of them are very close to appropriate states from the $\bar{10}_f$-plet, but could have different flavor quantum numbers. In turn, the constituent quark model predicts two partners of ideally mixed $10_f$ and $8_f$ multiplets with $J^\pi = \frac{1}{2}^+$ and $\frac{3}{2}^+$ splitting within tens of MeV [32].

The aim of this paper is to estimate the production cross section of the $\Xi^\pm_3$ baryon with spin $\frac{3}{2}$. We are concentrated on the simplest strong interaction reaction, $\Xi^\pm_3$ production in $\bar{K}N$ scattering

$$\bar{K}N \rightarrow K\Xi^\pm_3,$$

and demonstrate that the total cross section for the exotic baryon with spin $\frac{3}{2}$ is at least 50 times larger than that for the exotic baryon with spin $\frac{1}{2}$.

The paper is organized as follows. In Section 2 we formulate the model. Then in Section 3 we determine the parameters of the model and provide numerical calculations. Conclusions are given in Section 4.
2 Model and effective Lagrangians

We estimate the reaction cross section considering Born diagrams with Σ baryon pole in the $s$ and $u$ channels, Figure 1. We use $k$ and $k'$ for the kaon momentum in the initial and final state, $p$ and $q$ for the proton and $\Xi_{3/2}$ momentum, respectively.

The cross sections for different channels of the reaction (1) are connected by isospin Clebsh-Gordan coefficients

$$
\sigma(\bar{K}^0 p \rightarrow K^0\Xi_{3/2}^+) = 3\sigma(\bar{K}^0 p \rightarrow K^+\Xi_{3/2}^0) = 3\sigma(K^- p \rightarrow K^0\Xi_{3/2}^-) = \\
= 3\sigma(\bar{K}^0 n \rightarrow K^+\Xi_{3/2}^-) = 3\sigma(\bar{K}^0 n \rightarrow K^0\Xi_{3/2}^0) = 3\sigma(K^- p \rightarrow K^+\Xi_{3/2}^-) = \\
= 3\sigma(K^- n \rightarrow K^0\Xi_{3/2}^-) = \sigma(K^- n \rightarrow K^+\Xi_{3/2}^-). 
$$

(2)

The $KN\Sigma$ effective Lagrangian is well known

$$
\mathcal{L}_{KN\Sigma} = ig_{KN\Sigma}\Sigma\gamma_5 K N + \text{h.c.} \quad (3)
$$

with the coupling constant $g_{KN\Sigma} = 3.54$ [33]. From Jülich-Bonn potential $g_{KN\Sigma} = 5.38$ [34]. We use the first value. Because spin and parity of the $\Xi_{3/2}$ baryon are unknown we use one of the following $K\Sigma\Xi_{3/2}$ Lagrangians depending on the spin-parity of the $\Xi_{3/2}$ baryon

$$
\begin{align*}
\mathcal{L}_{K\Sigma\Xi_{3/2}} &= ig_{K\Sigma\Xi_{3/2}}\Sigma\gamma_5 K \Xi + \text{h.c.}, \quad \text{for } J^p(\Xi_{3/2}) = \frac{1}{2}^+ \\
\mathcal{L}_{K\Sigma\Xi_{3/2}} &= g_{K\Sigma\Xi_{3/2}}\Sigma K \Xi_{3/2} + \text{h.c.}, \quad J^p(\Xi_{3/2}) = \frac{1}{2}^- \\
\mathcal{L}_{K\Sigma\Xi_{3/2}} &= \frac{g_{K\Sigma\Xi_{3/2}}}{m_{\Xi_{3/2}}} \Sigma \gamma_5 \partial_\mu K \Xi_{3/2}^\mu + \text{h.c.}, \quad J^p(\Xi_{3/2}) = \frac{3}{2}^+ \\
\mathcal{L}_{K\Sigma\Xi_{3/2}} &= \frac{g_{K\Sigma\Xi_{3/2}}}{m_{\Xi_{3/2}}} \Sigma \partial_\mu K \Xi_{3/2}^\mu + \text{h.c.}, \quad J^p(\Xi_{3/2}) = \frac{3}{2}^- \quad (4)
\end{align*}
$$
In (4) we use Rarita-Schwinger field $\Xi_{3/2}(x)$ for a particle with spin $\frac{3}{2}$. An additional factor $1/m_{\Xi_{3/2}}$ (where $m_{\Xi_{3/2}}$ is the $\Xi_{3/2}$ mass) is introduced to make coupling constant $g_{K\Sigma\Xi_{3/2}}$ dimensionless for the spin-$\frac{3}{2}$ $\Xi_{3/2}$ baryon.

A spinor $U^\mu(q)$ for a free spin-$\frac{3}{2}$ particle satisfies the following equation

$$(\not{q} - m_{\Xi_{3/2}})U^\mu(q) = 0,$$  \hspace{1cm} (5)

with constraints

$$\gamma_\mu U^\mu(q) = 0$$

where $\not{q} \equiv \gamma_\nu q^\nu$ and $\gamma_\nu$ are Dirac $4 \times 4$ matrices. The normalization condition reads

$$U^\mu(q)U_\mu(q) = -2m_{\Xi_{3/2}}.$$  \hspace{1cm} (6)

The spin summation formula for the Rarita-Schwinger spinor reads

$$\sum_{\text{spin}} U_\mu(q)U_\nu(q) = -\frac{1}{3m_{\Xi_{3/2}}} (\not{q} + m_{\Xi_{3/2}}) \left( g_{\mu\nu} - \frac{q_\mu q_\nu}{m_{\Xi_{3/2}}^2} - \frac{1}{4} [\gamma_\mu, \gamma_\nu] \right) (\not{q} + m_{\Xi_{3/2}}) \equiv P_{\mu\nu}(q).$$  \hspace{1cm} (7)

Reaction scattering amplitude squared, summed over spin states of the $\Xi_{3/2}$ and averaged over nucleon spin states reads

$$|\mathcal{M}|^2 = |\mathcal{M}_s|^2 + |\mathcal{M}_u|^2 + \mathcal{M}_s \mathcal{M}_u^* + |\mathcal{M}_u|^2,$$  \hspace{1cm} (8)

where $\mathcal{M}_s$ and $\mathcal{M}_u$ are the $s$ and $u$ pole terms corresponding to left and right diagrams of Figure 1. For the $\Xi_{3/2}$ baryon with $J^\pi = \frac{3}{2}^+$ appropriate terms of (9) are

$$\mathcal{M}_x \mathcal{M}_y^* = \frac{1}{2} g_{K\Sigma\Xi_{3/2}} g_{KN\Sigma} \frac{F^2(\kappa_x^2)}{\kappa_x^2 - m_{\Sigma}^2} \frac{F^2(\kappa_y^2)}{\kappa_y^2 - m_{\Sigma}^2} \frac{\kappa_x^\mu \kappa_y^\nu}{3m_{\Xi_{3/2}}^2} \times \text{Tr} \left\{ (\not{k}_x - m_{\Sigma}) P_{\mu\nu}(q) (\not{k}_y - m_{\Sigma}) (\not{p} + m_N) \right\},$$

where $x, y$ labels either $s$- or $u$-channel, $\kappa_x (\kappa_u)$ is the momentum of the intermediate $\Sigma$ baryon, $\kappa_x = k + p$, $\kappa_u = q - k$, and $F(\kappa^2)$ is form factor. We use a relativistically invariant parameterization for the form factor from Ref. [36]

$$F(\kappa^2) = \frac{\Lambda^2}{\sqrt{\Lambda^4 + (\kappa^2 - m_{\Xi_{3/2}}^2)^2}}.$$  \hspace{1cm} (10)

extracted from the cross section $\gamma p \rightarrow K^+ \Lambda$. The cut-off parameter $\Lambda = 0.85$ GeV [36]. The above formula (10) is for positive parity $\Xi_{3/2}$ baryon, in the case of negative parity one has to change the sign of $\Xi_{3/2}$ mass entering $P_{\mu\nu}$.

The trace in the expression (10) was calculated on computer analytically.
Table 1

|       | $J = 1/2$ | $J = 3/2$ |
|-------|-----------|-----------|
| $\pi = +1$ | 3.84     | 38.4     |
| $\pi = -1$ | 0.53     | 5.34     |

3 Numerical calculations and discussion of the results

We estimate the coupling constant $g_{K\Sigma\Xi^{3/2}}$ by the same procedure, which was used in Ref. [35]. Both exotic baryons, $\Theta^+$ and $\Xi^{3/2}$, are assumed to belong to the same $SU(3)_f$ multiplet. Assuming $SU(3)_f$ symmetry for the interaction one gets

$$g_{K\Theta} = g_{K\Sigma\Xi^{3/2}}.$$  \hspace{1cm} (12)

Since the $\Theta^+$ has only one decay channel, $\Theta^+ \to KN$, one can simply calculate the coupling constant $g_{K\Theta}$ from the total $\Theta$ width

$$\Gamma_{\Theta} = g_{K\Theta}^2 \frac{(m_\Theta \mp m_N)^2 - m_K^2}{4\pi m_\Theta^2} \times \begin{cases} Q & \text{for } J^\pi = \frac{1\pm}{2} \\ \frac{Q^3}{3m_\Theta^3} & \text{for } J^\pi = \frac{3\pm}{2} \end{cases}$$  \hspace{1cm} (13)

where $m_\Theta$, $m_N$ and $m_K$ are masses of the $\Theta^+$, the nucleon and the kaon and

$$Q = \frac{1}{2m_\Theta} \sqrt{m_\Theta^4 + m_N^4 + m_K^4 - 2m_\Theta^2 m_N^2 - 2m_\Theta^2 m_K^2 - 2m_N^2 m_K^2}$$  \hspace{1cm} (14)

is the kaon momentum in the $\Theta^+$ rest frame. Taking $\Gamma_{\Theta} = 15$ MeV as an upper limit for the $\Theta^+$ decay width one obtains values summarized in Table 1.

The estimated total cross section of the $\Xi_{3/2}$ production with spin $3/2$ are displayed on Figure 2. We also compare our results with the results of Ref. [35] for the $\Xi_{3/2}$ with spin $1/2$. One concludes that

• The total cross section for the same parity but different spin of the $\Xi_{3/2}$ is approximately 50–100 times larger for the spin $3/2$ than for the spin $1/2$.
• Similarly to the case of spin $1/2$ [35] the cross section $\sigma(J^\pi = \frac{3^+}{2})$ is approximately two orders larger than $\sigma(J^\pi = \frac{3^-}{2})$.

It must be also stressed that according to (13) the cross section is proportional to the $\Theta^+$ width, $\Gamma_{\Theta^+}$, which is unknown from experiment. The coupling constant $g_{K\Sigma\Xi^{3/2}}$ in Table 1 was estimated from the “average” upper limit of the width, $\Gamma_{\Theta^+} < 15$ MeV. Further restrictions on the width come from the $K^+d$ total cross section, $\Gamma_{\Theta^+} < 6$ MeV, [37], and from PWA of $K^+N$ scattering in the $I = 0$ channel, $\Gamma_{\Theta^+} < 1$ MeV, [38,39].
4 Conclusions

We estimate the upper limit of the total cross section for the $\Xi_{3/2}^-$ production in $\bar{K}N$ scattering employing $\Xi_{3/2}$ spin-parity $J^\pi = \frac{3}{2}^+$ and $J^\pi = \frac{3}{2}^-$. The estimate was done using the $s$ and $u$ pole diagrams with $\Sigma$ hyperon in the intermediate state, Figure 1. We compare our results with the results of [35] for the $\Xi_{3/2}$ with $J^\pi = \frac{1}{2}^+$ and $J^\pi = \frac{1}{2}^-$. It is shown that from the two cross sections with the same parity and different spins, the cross section for the spin $\frac{3}{2}$ of the $\Xi_{3/2}$ is 50–100 times larger than that for the $\Xi_{3/2}$ with spin $\frac{1}{2}$.

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