An Orientifold with Fluxes and Branes via T-duality

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Abstract

String compactifications with non-abelian gauge fields localized on D-branes, with background NSNS and RR 3-form fluxes, and with non-trivial warp factors, can naturally exist within T-dual versions of type I string theory. We develop a systematic procedure to construct the effective bosonic Lagrangian of type I T-dualized along a six-torus, including the coupling to gauge multiplets on D3-branes and the modifications due to 3-form fluxes. Looking for solutions to the ten-dimensional equations of motion, we find warped products of Minkowski space and Ricci-flat internal manifolds. Once the warp factor is neglected, the resulting no-scale scalar potential of the effective four-dimensional theory combines those known for 3-form fluxes and for internal Yang-Mills fields and stabilizes many of the moduli. We perform an explicit comparison of our expressions to those obtained from $\mathcal{N} = 4$ gauged supergravity and find agreement. We also comment on the possibility to include D9-branes with world-volume gauge fluxes in the background with 3-form fluxes.

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1 Introduction

Orientifolds provide a natural framework for string compactifications that can accommodate space-time filling D-branes, internal fluxes for the various tensor field strengths and non-trivial warp factors at the same time.\(^1\) Since these are the main ingredients in many of the recent phenomenological investigations concerning low string scale models \([2, 3]\), moduli stabilization through flux-induced potentials \([4, 18]\), Randall-Sundrum-like warped compactifications \([10, 51]\) or any kind of brane world scenarios, they are of central interest among the classes of string compactifications relevant for four-dimensional particle physics. They evade the powerful no-go theorems prohibiting warped compactifications with fluxes to four-dimensional Minkowski space \([51, 52, 53, 54]\) within the context of Kaluza-Klein reduction of supergravity, since in contrast to traditional supergravity theories they contain objects of negative energy density like orientifold planes. However, type I models, or their T-dual descriptions usually called type I’\(^2\), are notoriously difficult to treat: no explicit description of the effective action of type I’ models has been given.\(^2\) The main purpose of the present paper is to remedy this latter point and provide the effective bosonic action for a certain theory T-dual to type I string theory, including the coupling of the supergravity fields to the non-abelian gauge theory sector localized on D3-branes, as well as the relevant modifications due to background 3-form fluxes. More precisely, we construct the type I’ model with 3-form fluxes, which in the absence of 3-form fluxes is dual to type I via six T-dualities. We consider this the simplest version of an orientifold with 3-form fluxes and D-branes and a starting point for phenomenologically more sophisticated constructions.

The motivation to combine models with D-branes and orientifold planes (O-planes) with background 3-form fluxes comes from the fact that the former provide interesting non-abelian gauge fields, potentially with chiral charged matter,\(^3\) while the latter add to the scalar potential of the effective theory, such that at least some scalar fields get massive and decouple. This removes some of the vacuum degeneracy, a necessary step on the way to realistic string models. The total scalar potential gets a contribution not only from the ten-dimensional kinetic term of the type IIB 3-form, projected to type I’ and including a Chern-Simons (CS) correction, but also from the Dirac-Born-Infeld (DBI) action of the Yang-Mills (YM) gauge fields supported by the D-branes. Both the 3-form flux and the gauge fields

\(^1\)See \([11]\) for a comprehensive introduction to orientifolds.

\(^2\)But for some information on \(\mathcal{N} = 1\) effective four-dimensional type I supergravity in a T-duality invariant formulation see \([55]\).

\(^3\)For the most part of this paper we only consider D3-branes at smooth points of the background, which do not lead to chiral spectra. Generalizations of the present model would need to include higher-dimensional branes with non-trivial gauge fluxes, singular geometries or with point-like intersections to support chiral fermionic zero modes (see e.g. \([56, 57, 58]\)), a possibility which we also comment on in section \([5]\).
add positive energy densities and the constraints that follow from the equations of motion require this energy density to cancel against the negative contribution of orientifold planes in a Minkowski vacuum.

Within supergravity, scalar potentials can be generated by gauging isometries of the scalar manifold (for a recent review see [59]). A systematic procedure can be applied to derive the form of the effective action. The effective actions obtained from string compactifications with fluxes have so far been understood as corresponding to gaugings of a particular type, namely abelian gaugings of Peccei-Quinn (PQ) isometries, i.e. gaugings of axionic shifts. The \( \mathcal{N} = 4 \) gauged supergravity describing the type I' model at hand was studied in a series of papers [60, 61, 34, 35, 38, 62, 47]. Here we derive the bosonic part of the effective action by explicitly performing the six T-dualities of type I and make contact with the formulas of [47], that contains the most explicit formulation of the model, thus elucidating the correspondence between the field variables used in the supergravity literature and those naturally arising in string theory. A deeper understanding of the relation between string compactifications in non-trivial backgrounds and the gaugings of the relevant supergravity is certainly worthwhile.

It is important to note that for our present purposes T-duality always manifests itself as a set of field redefinitions in the action. We are not going to transform a given vacuum of type I, but instead we transform the action for the dynamical degrees of freedom of type I string theory into a formulation in terms of the type I' fields after six T-dualities. In performing the six T-dualities we have to assume the internal space has six isometries (i.e. it is a six-torus). However, after performing the transformations one can reinterpret the resulting four-dimensional action as coming from a non-covariant ten-dimensional action of the type I' theory. In the ten-dimensional Lagrangian, the closed string fields are allowed to depend also on all the internal coordinates, while the open string fields only depend on world-volume coordinates (compare the discussion in [63]).

In deriving the full ten-dimensional action and the correct equations of motion, we have to make use of an important difference between type I theory and its T-dual version type I'. This difference permits us, for the type I' theory, to include complex 3-form fluxes and other non-dynamical backgrounds even though their fluctuations are projected out. This comes about by noting that the world sheet parity \( \Omega \) which is “divided out” in getting from type IIB to type I string theory is mapped to \( \Omega \Theta(-1)^{F_L} \) under T-duality. Here \( \Theta \) is the reflection of all internal directions \( x^i \) and \( F_L \) the left-moving space-time fermion number operator. A field of type IIB that was odd under \( \Omega \) has to vanish at any point in type I theory, whereas a field odd (even) under \( \Omega \Theta(-1)^{F_L} \) is only required to be antisymmetric (symmetric) under \( x^i \rightarrow -x^i \) in type I' theory. The zero-mode fluctuations of the bosonic fields are constant on a torus and therefore have to be even under \( \Omega \Theta(-1)^{F_L} \). On
the other hand, one can keep all kinds of background terms, which naively do not appear in the action for the fluctuations obtained from the type I action by T-duality. Thus, even though the fluctuations of the internal components of the two antisymmetric 2-tensors of IIB are projected out, one can include a background flux for the two 3-form field strengths (which are even under \( \Omega \Theta(-1)^{F_L} \)), such that their corresponding potentials are odd), generating a potential for the closed string moduli. This is the reason why one is interested in the T-dual version of type I in the first place. In addition also the odd component of the field strength \( F_5 \) with one or five internal indices can take on a background value as long as it is antisymmetric under \( x^i \mapsto -x^i \). This is important to find explicit solutions to the ten-dimensional equations of motion as a non-trivial profile for \( C_4 \) is needed for D3-branes or O3-planes to exist.

In order to derive the action of type I' including the non-dynamical background terms in the sense explained above, we combine two different strategies. First we employ six T-dualities of the type I action. This produces an effective action equivalent to the type I action and therefore including all the consistent couplings of bulk supergravity and non-abelian gauge fields. However, as we already mentioned, this requires the fields to be independent of the internal coordinates. In particular we cannot include any torsion for the metric and therefore it is impossible to directly derive the effects of NSNS 3-form flux in the T-dual theory this way.\(^4\) We then add in modifications due to fluxes for the NSNS 3-form and \( F_5 \). The fact that this adding in by hand is not arbitrary and can be performed in a well defined and systematic manner rests on the twofold definition of type I':

\[
\text{T-duality of Type I} \leftrightarrow \text{Type IIB} \quad \frac{\Omega \Theta(-1)^{F_L}}{\Omega \Theta(-1)^{F_L}}.
\]  

This means the closed string sector of the theory can be deduced from the fact that it is given by projecting the effective action of type IIB by the T-dual worldsheet parity. This second definition does not give any prescription how to incorporate the gauge fields, but combined with the knowledge of the T-dual action of type I, it provides enough additional information to include the closed string background fields. Thus, the T-duality of type I and the T-dual projection of type IIB are complementary regarding terms involving the open string fields and the non-dynamical background fields such as the NSNS 3-form flux, respectively.\(^5\) The resulting action then serves as a four-dimensional effective action for a comparison to the Lagrangian of the gauged \( N = 4 \) supergravity as mentioned already. Rein-

\(^4\)T-duality in the presence of NSNS flux and the corresponding non-trivial metric configurations have been discussed in [64, 65, 66, 67] (see also [68, 69, 70, 71, 72, 48] for analogous works on the non-Kähler vacua of the heterotic string). However, there T-duality is not performed along all the internal coordinates.

\(^5\)In [72] the effect of fluxes in the world volume theory of branes has been analyzed from a different point of view, resting on anomaly constraints.
terpreted as ten-dimensional, we can use it to study more general vacua than torus compactifications as well.

The solutions for the ten-dimensional equations of motion generalize the situation without gauge fields considered e.g. in [51, 21]. As in that case, one can find an explicit solution in the form of a warped product of four-dimensional Minkowski space and a Ricci-flat internal metric involving also a non-trivial profile for the components of $F_5$ that respect four-dimensional Poincaré invariance. The appearance of the warp factor has several implications. On the one hand, warp factors have been argued [49, 21, 36] to be able to generate exponential hierarchies between the effective energy scales at different locations on the internal space, such that gauge fields localized on different D-branes may experience suppression or enhancement of gravitational and gauge-theoretical effects. On the other hand, the appearance of the warp factor implies that our actual starting point, a direct product $\mathbb{R}^4 \times \mathbb{T}^6$, no longer solves the ten-dimensional equations of motion. Thus, in principle, the warp factor should be taken into account in a dimensional reduction. However, in the large volume limit the warp factor scales to a constant away from singular sources, and it has been argued that one may completely neglect it in this limit [21]. At the classical level, the overall volume is a free parameter, as is manifest from the no-scale structure of the effective potential (see [73, 74] for its phenomenological implications). Thus one can choose an arbitrarily large value and consider the direct-product ansatz as an approximate solution. However, as soon as quantum corrections to the effective four-dimensional action are taken into account, this line of reasoning should be modified, as they seem to spoil the no-scale structure [32, 43]. The same is true if one modifies the model by including higher-dimensional branes [45, 46]. These are in principle capable of fixing the overall volume, but in any attempt to do so one has to make sure that this is done at a value sufficiently large that one can still neglect the warp factor, if the four-dimensional effective action derived via dimensional reduction is to remain reliable. In this context we also include some comments on the perspectives of fixing the overall volume by adding world-volume gauge fluxes on calibrated D9-branes. It appears that a fixing of the volume at the string scale would be difficult to avoid, which could invalidate the effective approach.

The paper is organized as follows. In section 2 we develop a systematic procedure to apply T-duality to type I string theory on the level of the effective Lagrangian including the coupling to gauge fields, and show how to add modifications due to 3-form fluxes in the background. In section 3 we discuss the effective potential that arises through this procedure in some detail, its relation to the formulas known from gauged supergravity, and the truncation to $\mathcal{N} = 1$ supersymmetry. In section 4 the equations of motions are derived and various forms of constraints produced. We also discuss briefly how the four-dimensional effective action of the previous sections can be justified by the large volume scaling argument. In sec-
tion we finally add a couple of comments on the addition of higher-dimensional D-branes subject to certain calibration conditions. The appendix collects some technical material and additions to the main body of the paper.

Before getting started, let us mention the following caveat. Unfortunately, there is no standard definition of the Hodge-star in the literature about the class of models under consideration. Our definition, given in appendix A.1 differs from the one used e.g. in \cite{75, 21, 27} but coincides with the one used in \cite{31, 33, 38, 47}. Thus, what is called an imaginary self-dual (ISD) 3-form flux in \cite{75, 21, 27} is imaginary anti-self-dual (IASD) in our conventions and vice versa.

## 2 Construction of the action via T-duality

In this section we perform six T-dualities of type I theory. This operation defines the effective action of type I’ on the dual six-torus. The main objective, then, is to find the modifications in the action when complex 3-form fluxes are added to the background, following the philosophy outlined in the introduction. We first go through the procedure ignoring the coupling to the non-abelian vector fields of the open string sector, and only then reconsider the full model.

Our starting point is type I supergravity, which, including the coupling to non-abelian vectors of the gauge group $SO(32)$, is described by the action

$$S_I = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left(e^{-2\Phi}(R + 4\partial_{\mu}\Phi\partial^{\mu}\Phi) - \frac{1}{2}|\tilde{F}_3|^2\right) - \frac{1}{2g_{10}^2} \int d^{10}x \sqrt{-g} e^{-\Phi} \text{tr} |\mathcal{F}|^2,$$

where $\mathcal{F} = \frac{1}{2} \mathcal{F}_{MN}^a T^a dx^M \wedge dx^N$ is given by

$$\mathcal{F}_{MN}^a = 2\partial_{[M}A_{N]}^a + f^{abc}A_{M}^b A_{N}^c,$$

and we use the definition

$$\tilde{F}_3 = dC_2 - \frac{\kappa_{10}^2}{g_{10}^2} \omega_3,$$

with $\omega_3$ the gauge CS 3-form

$$\omega_3 = \text{tr} \left(A \wedge dA - i \frac{2}{3} A \wedge A \wedge A \right).$$

In general one defines the Chern Simons forms $\omega_{2j-1}$ by

$$d\omega_{2j-1} = \text{tr} \mathcal{F}^j.$$
Except for all the terms involving the vector fields, the type I action (2) is obtained by quotienting world sheet parity $\Omega$ out of the type IIB action\footnote{The issue of the self-duality constraint on $F_5$ will be dealt with later.} 

$$S_{\text{IIB}} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left( e^{-2\Phi} \left( R + 4\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}|H_3|^2 \right) - \frac{1}{2} \left( |F_1|^2 + |F_3|^2 + \frac{1}{2}|F_5|^2 \right) \right) - \frac{1}{4\kappa_{10}^2} \int C_4 \wedge dB_2 \wedge dC_2 \ ,$$

with

$$F_3 = dC_2 + C_0 dB_2 \ ,$$
$$F_5 = dC_4 + \frac{1}{2} C_2 \wedge dB_2 - \frac{1}{2} B_2 \wedge dC_2$$

and $H_3 = dB_2$, $F_1 = dC_0$. After modding out $\Omega$, only the RR 2-form enters \cite{2} via $F_3 = dC_2$, while the NSNS 2-form $B_2$ is projected out. Similarly, the T-dual version is a truncation of the type IIB theory (modding out the T-dual $\Omega$-projection \cite{11}) coupled to vectors. The duality operation then replaces the degrees of freedom \{g_{IJ}, C_2, \Phi\} of type I by those of type $I'$, which are

- 1 graviton : $g_{\mu\nu}$ ,
- 12 gauge bosons : $B_{i\mu}$, $C_{i\mu}$ ,
- 38 scalars : $g_{ij}$, $C_{ijkl}$, $\tau = C_0 + ie^\Phi$ .

In terms of $\mathcal{N} = 4$ supersymmetry, these make up a spin-2 and six abelian vector multiplets. In addition, there are vector multiplets with bosonic field content $(A_\mu^a, A_i^a)$ in the adjoint of the gauge group, a subgroup of $SO(32)$. These fields are referred to as open string fields and arise from the presence of D3-branes, the T-dual images of the D9-branes of type I. The O9-plane of type I theory splits into 64 O3-planes, but in the presence of fluxes the number $N_{D3}$ of D3-branes needed to cancel their charge is no longer fixed. Rather, turning on 3-form flux modifies the tadpole condition to \cite{21}

$$\frac{1}{2} N_{\text{flux}} + N_{D3} = 16 \ ,$$

thus effectively replacing some of the D3-branes by flux. The precise form of $N_{\text{flux}}$ will be given later, cf. \cite{11}.

### 2.1 T-duality rules

For some of the conventions used in the following we refer the reader to the appendix. Using the standard Kaluza-Klein (KK) ansatz for the metric

$$ds_{10}^2 = g_{IJ} dx^I dx^J = g_{\mu\nu} dx^\mu dx^\nu + G_{ij}(dy^i + A_{i\mu}^a dx^\mu)(dy^j + A_{j\nu}^a dx^\nu) \ ,$$

the issue of the self-duality constraint on $F_5$ will be dealt with later.
the transformations of the NSNS sector can be deduced from e.g. \[76\]. The formulas for the replacements according to dualizing all six circles simplify for type I to

\[ G_{ij} \mapsto G_{ij}, \quad g_{\mu\nu} \mapsto g_{\mu\nu}, \quad (12) \]

and

\[ g_{\mu k} \mapsto G_{ki} B_{\mu i}. \quad (13) \]

This last operation amounts to replacing the KK vectors by \( A_{\mu}^i \mapsto B_{\mu i} \), since \( g_{\mu k} = G_{ik} A_{\mu}^i \). Finally, the dilaton transforms according to

\[ e^{2\Phi} \mapsto \frac{e^{2\Phi}}{\det(G_{ij})}. \quad (14) \]

Note that we do not distinguish the fields of type I or its T-dual from their ancestor fields in type IIB, since we identify the effective action that is obtained by projection from type IIB with that of type I (respectively its T-dual) when the open string vector fields are set to zero.

The transformation properties of the RR fields on a higher-dimensional torus can be found e.g. in \[77, 64, 78\]. We will stick to the formalism of \[77\] here, which leads to the same results as the one of \[64, 78\]. It was shown there that the type IIB equations of motion for the action (7) can alternatively be derived from an action that is manifestly invariant under T-duality, where the second line of (7), the kinetic terms for the RR forms plus the CS term, is replaced by

\[ S_{RR+CS} \rightarrow -\frac{1}{8\kappa_{10}^2} \sum_{q=1}^{5} |F_{2q-1}|^2, \quad (15) \]

and \( F_p \) is now defined more generally as

\[ \sum_{q=1}^{5} F_{2q-1} = e^{-B_2} \wedge \sum_{q=0}^{4} dD_{2q}. \quad (16) \]

It is important that no CS term needs to be included explicitly in (15). It would appear automatically if one dualizes the forms of high degree, see \[77\] for more details. The \( D_p \) transform in the spinor representation of \( O(6,6,\mathbb{Z}) \) \[77, 64, 78\].

To complete the definitions note

\[ D_0 = C_0, \quad D_2 = C_2 + C_0 B_2, \quad D_4 = C_4 + \frac{1}{2} B_2 \wedge C_2 + \frac{1}{2} C_0 B_2^2, \quad (17) \]

and

\[ * F_1 = F_9, \quad * F_3 = -F_7, \quad * F_5 = F_5, \quad (18) \]

\[ ^7 \text{A similar democratic version of type II supergravity was discussed in \[79\].} \]
the latter to be imposed after deriving the equations of motions from the action.

One can actually give a more explicit definition for the field strengths of higher
degree and we will use this later in (89), see e.g. [79].

We are now interested only in the particular element of the whole $O(6,6,\mathbb{Z})$
symmetry group that corresponds to (12). This element is given by

$$D_{\mu_1...\mu_q\iota_1...\iota_n} \mapsto (-1)^{n(n-1)/2} \hat{\epsilon}^{\iota_1...\iota_n\iota_{n+1}...\iota_6} D_{\mu_1...\mu_q\iota_{n+1}...\iota_6} ,$$

where the hat on $\hat{\epsilon}$ means that here the epsilon-symbol takes values $\hat{\epsilon}^1...6 = \hat{\epsilon}_{1...6} = 1$ and the indices are contracted with Kronecker-deltas instead of the metric. To write (19) in terms of tensor quantities, one has to factor out the determinant $G = \det(G_{ij})$ from the internal metric to cancel the factor $G$ appearing in the contractions, that is

$$D_{\mu_1...\mu_q\iota_1...\iota_n} \mapsto (-1)^{n(n-1)/2} \sqrt{G} \hat{\epsilon}^{\iota_1...\iota_n\iota_{n+1}...\iota_6} D_{\mu_1...\mu_q\iota_{n+1}...\iota_6} .$$

Now $\epsilon_{1...6} = \sqrt{G}$, $\epsilon^{1...6} = 1/\sqrt{G}$ and indices are contracted with $G_{ij}$. The field strengths $F_p$ can be transformed in a similar vein, by simply replacing $D_p$ with $dD_p$ in (20) [64]. Thus the components of the RR 3-form field strength have the following transformation properties

$$
\begin{align*}
(dC_2)^{\{0,3\}}_{ijk} &\mapsto -\frac{\sqrt{G}}{3!} \epsilon^{ijklmn} (dD_2)^{\{0,3\}}_{lmn} , \\
(dC_2)^{\{1,2\}}_{\mu ij} &\mapsto -\frac{\sqrt{G}}{4!} \epsilon^{ijklmn} (dD_4)^{\{1,4\}}_{\mu klmn} , \\
(dC_2)^{\{2,1\}}_{\mu \nu i} &\mapsto +\frac{\sqrt{G}}{5!} \epsilon^{ijklmn} (dD_6)^{\{2,5\}}_{\mu \nu jklmn} , \\
(dC_2)^{\{3,0\}}_{\mu \nu \rho} &\mapsto +\frac{\sqrt{G}}{6!} \epsilon^{ijklmn} (dD_8)^{\{3,6\}}_{\mu \nu \rhoijklmn} .
\end{align*}
$$

It is evident that the T-dual of the type I RR 2-form also includes RR forms of
degree six and eight, which we later dualize to forms of lower degree. Finally,
according to [63] the internal components of the non-abelian vectors are mapped via

$$A^a_i \mapsto A^{ai} ,$$

which is easily applied to $\mathcal{F}$ and $\omega_3$. It is important to note that the $A^{ai}$ are actually the independent dual open string moduli, in other words, the dual scalars $A^a_i$ depend implicitly on the metric. The same is then true e.g. for $\mathcal{F}_{ij}$ and $\omega_{ijk}$.

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8Note that the overall sign is arbitrary. The derivation of the transformation rules from the rules given in [79] is performed in the appendix (but see also [78]).

9Once more, we would like to refer to the appendix for the definitions of the notations used here. The upper indices $\{p, q\}$ refer to the bi-degree as forms on the non-compact and internal part of the ten-dimensional space-time.
2.2 T-duality without vectors

We now apply the rules assembled in the previous section, first to the pure supergravity part of the type I bosonic action. Our starting point is \( S[A=0] = \frac{1}{2\kappa_{10}^2} \int d^{10} x \sqrt{-g} \left( e^{-2\Phi} \left( R + 4\partial_{\mu} \Phi \partial^{\mu} \Phi \right) - \frac{1}{2} |F_3|^2 \right) \),

which we split into

\[
S[A=0] = S_{NSNS} + S_{RR}.
\]

Let us consider the NSNS part first. Using the rules of the last section and the usual form of the NSNS action on a torus (see e.g. (2.17) of [80]), it is straightforward to verify that it is mapped as

\[
S_{NSNS} \mapsto \frac{1}{2\kappa_{10}^2} \int d^{10} x \sqrt{-g} e^{-2\Phi} \left( R + 4\partial_{\mu} \Phi \partial^{\mu} \Phi - \frac{1}{4} G^{ij} H_{ij\nu} H^{\nu\mu} \right),
\]

where we defined

\[
H_{ij\nu} = 2\partial_{[\nu} B_{\sigma]} j.
\]

Note that after T-duality the metric in (9) has no off-diagonal components and that the original Kaluza-Klein vectors have been mapped to components of the NSNS \( B \)-field (26).

Let us now consider the RR part. There is a helpful trick to make the appearance of the KK vectors explicit [77]. Using (11) one can rewrite the kinetic terms for the RR forms as

\[
|F_p|^2 = |F'_p|^2_{G_{ij}, g_{\mu\nu}}
\]

by redefining

\[
F'_p = F_p|_{dy' \rightarrow dy' - A'_{\mu} dx'}. \tag{28}
\]

On the left-hand side of (27) the contractions are performed with the full metric, whereas on the right-hand side, the off-diagonal part is omitted and absorbed into a redefinition of the field strength. In other words, defining new forms as in (28), one can perform all contractions using the internal or external components of the metric only. In the dual theory \( F'_p = F_p \) and we can omit the prime. Using (21), we obtain

\[
(F'_3)_{\mu_1 \ldots \mu_n i_1 \ldots i_3} \mapsto (-1)^{(3-n)(2-n)/2} \sqrt{G} (3 + n)! e^{i_1 \ldots i_6} (F_{3+2n})_{\mu_1 \ldots \mu_n i_4 \ldots i_{3+n}}. \tag{29}
\]

The right-hand side exactly takes the form of (16) subject to the projection in the T-dual model. This is because the KK vectors of the metric, that appear in the
definition \((23)\) of \(F_3\), are mapped to those of the NSNS \(B\)-field exactly in a way required by \((16)\). Applying \((29)\) in \((23)\) maps the RR kinetic term according to

\[
- \frac{1}{4\kappa_{10}^2} \int d^{10}x \sqrt{-g} |F_3|^2 \quad \mapsto \quad - \frac{1}{4\kappa_{10}^2} \int \left( F_9^{(3,6)} \wedge *F_9^{(3,6)} + F_7^{(2,5)} \wedge *F_7^{(2,5)} \right. \\
+ F_5^{(1,4)} \wedge *F_5^{(1,4)} + F_3^{(0,3)} \wedge *F_3^{(0,3)} \left. \right),
\]

where the Hodge star \(*\) is with respect to the ten-dimensional T-dual metric. In applying the T-duality rules of section \(2.1\) along all six internal directions, we have to assume that none of the fields depend on the internal coordinates. This is in particular true for the metric components \((11)\). Thus we do not consider any non-trivial spin connection for the metric before T-duality. From the results of \([64, 65, 66, 67]\) this implies that we are not able to produce any non-trivial NSNS flux directly via T-duality and in \((30)\) we implicitly assume \((dB_2)^{(0,3)} = 0\).

In principle \((30)\) gives all the terms in the T-dual theory that come from the RR 3-form field strength of type I, including a purely internal part \(F_3^{(0,3)}\). In order to make contact with the standard form that would be obtained from type IIB by the T-dual projection, which involves only field strengths of degree five and lower, we next remove the RR forms of unconventionally high degree from the action in favor of their dual forms. According to the standard procedure (see e.g. \([81]\)) this would amount to imposing the Bianchi identity for \(F_9^{(3,6)}\) and \(F_7^{(2,5)}\), respectively, via Lagrange multipliers and then integrating out \(F_9^{(3,6)}\) and \(F_7^{(2,5)}\), leaving the Lagrange multipliers as the dual degrees of freedom. However, in the presence of 3-form flux \(F_3^{(0,3)}\) this method does not seem to be applicable in a straightforward way.\(^\text{10}\) We therefore follow a different strategy, first setting also the RR 3-form flux in \((30)\) to zero. For this case we perform the dualization of \(F_9^{(3,6)}\) and \(F_7^{(2,5)}\) and only afterwards infer the effects of non-vanishing flux in the effective action.

So let us for the moment consider \((30)\) without the 3-form flux. In order to dualize \(F_9^{(3,6)}\) we impose its Bianchi identity by adding a Lagrange multiplier term

\[
\delta S = - \frac{1}{2\kappa_{10}^2} \int C_0 d^{(1,0)} \left( F_9^{(3,6)} + B_2^{(1,1)} \wedge (dD_6)^{(2,5)} - \frac{1}{2} (B_2^{(1,1)})^2 \wedge (dD_4)^{(1,4)} \right).
\]

We called the Lagrange multiplier \(C_0\), anticipating that it will be identified with the RR scalar in a moment. This becomes clear by inspection of its kinetic term

\(^\text{10}\)The problem derives from the fact that if one naively imposes the Bianchi identities of \(F_9^{(3,6)}\) and \(F_7^{(2,5)}\) one generates terms involving different components of the RR field strengths as appearing in \((30)\), like e.g. \(F_7^{(4,3)}\). Their proper treatment requires one to use a democratic version of type I supergravity, involving \(C_2\) and \(C_6\) on the same footing before T-duality, cf. appendix \(C\). After T-duality this would also include kinetic terms for \(F_7^{(4,3)}\) etc., which make it possible to properly dualize these components.
below and by comparison with the (truncated) type IIB action. A partial integration in the first term of (31) leads to

\[ \delta S = \frac{1}{2\kappa_{10}^2} \int (dC_0)^{(1,0)} \wedge F_9^{(3,6)} - \frac{1}{2\kappa_{10}^2} \int C_0(dB_2)^{(2,1)} \wedge F_7^{(2,5)} . \quad (32) \]

Now integrating out \( F_9^{(3,6)} \) through

\[ * F_9^{(3,6)} = (dC_0)^{(1,0)} \quad (33) \]

and plugging this back into the action produces a kinetic term for \( C_0 \):

\[ \frac{1}{4\kappa_{10}^2} \int (dC_0)^{(1,0)} \wedge * (dC_0)^{(1,0)} = -\frac{1}{4\kappa_{10}^2} \int d^{10}x \sqrt{-g} |F_1^{(1,0)}|^2 . \quad (34) \]

If we next want to integrate out \( F_7^{(2,5)} \) we impose its Bianchi identity by adding a Lagrange multiplier term

\[ \delta S = \frac{1}{2\kappa_{10}^2} \int C_2^{(1,1)} \wedge d^{(1,0)} \left( F_7^{(2,5)} + B_2^{(1,1)} \wedge dD_4^{(1,4)} \right) . \quad (35) \]

Again we anticipate that the Lagrange multiplier is identified with the RR 2-form, as can be read off from (38) below. Then it is also clear that the only relevant component of \( C_2 \) is \( C_2^{(1,1)} \). Performing a partial integration for the first term of (35) leads to

\[ \delta S = -\frac{1}{2\kappa_{10}^2} \int (dC_2)^{(2,1)} \wedge F_7^{(2,5)} + \frac{1}{2\kappa_{10}^2} \int (C_2)^{(1,1)} \wedge (dB_2)^{(2,1)} \wedge (dC_4)^{(1,4)} . \quad (36) \]

Together with (32), integrating out \( F_7^{(2,5)} \) now gives

\[ * F_7^{(2,5)} = - (dC_2)^{(2,1)} - C_0(dB_2)^{(2,1)} . \quad (37) \]

Inserting this into the action we obtain the kinetic term

\[ \frac{1}{4\kappa_{10}^2} \int ( (dC_2)^{(2,1)} + C_0(dB_2)^{(2,1)} ) \wedge * ( (dC_2)^{(2,1)} + C_0(dB_2)^{(2,1)} ) \quad (38) \]

\[ = -\frac{1}{4\kappa_{10}^2} \int d^{10}x \sqrt{-g} |F_3^{(2,1)}|^2 , \]

and a Chern-Simons term

\[ S_{CS} = \frac{1}{2\kappa_{10}^2} \int (dC_2)^{(1,1)} \wedge (dB_2)^{(2,1)} \wedge C_4^{(0,4)} . \quad (39) \]

In this way, rewriting the T-dual of the type I action in terms of potentials with degree up to four produces the correct CS term that one expects from truncating
type IIB, even though type I does not possess such a term on its own. In all, this action is exactly what one would get from a reduction of type IIB subject to imposing the self-duality constraint on $F_5$, as we verify in appendix B.

Let us now study what changes should occur due to inclusion of 3-form flux. As was explained in the introduction, we make use of the fact that type $I'$ theory with all non-abelian vector fields set to zero is a truncation of type IIB by modding out the T-dual projection $[1]$. It is obvious from the last term of (30) that a scalar potential appears in the presence of flux. From (30) we would infer a term

$$\frac{1}{4\kappa_{10}^2} \int d^{10}x \sqrt{-g} |F_3^{(0,3)}|^2 ,$$  
(40)

where, as mentioned below (30), only a $F_3^{(0,3)} = (dC_2)^{0,3}$ term arises directly from T-duality. However, comparing to the type IIB action truncated by $[1]$, the expression (40) for the potential receives an additional contribution from the NSNS sector of the type IIB action, i.e. from the term

$$\frac{1}{4\kappa_{10}^2} \int d^{10}x \sqrt{-g} e^{-2\Phi} |H_3^{(0,3)}|^2 ,$$  
(41)

and another modification due to the fact that the RR 3-form field strength actually appears in the combination $[13]$. Then the total potential can be expressed via the complex combination

$$G_3 = dC_2 + \tau dB_2 = F_3 + ie^{-\Phi} H_3 ,$$  
(42)

where $\tau = C_0 + ie^{-\Phi}$, in the form

$$S_{\text{pot}} = -\frac{1}{4\kappa_{10}^2} \int d^{10}x \sqrt{-g} |G_3^{(0,3)}|^2 .$$  
(43)

Coupling the theory to the D3-branes will, besides other modifications, give rise to further contributions to the potential, coming from the world-volume scalars and from the tension of the branes. From the point of view of the truncated type IIB theory, (43) is the obvious part of the potential, while from the point of view of type I' it is the other way around and (43) cannot be derived directly via T-duality. Since the internal components $C_2^{(0,2)}$ and $B_2^{(0,2)}$ are projected out of the spectrum, $G_3^{(0,3)}$ is not a flux for any (dynamical) field strength in type I', but just some antisymmetric background parameter. It is then not obvious that a potential term (43) can appear as part of a consistent modification of the Lagrangian that

\footnote{Calling this term a potential is slightly imprecise, as (40) is still a term in the T-dual ten-dimensional action. What we mean is of course that this term leads to a potential in the effective theory - after dimensional reduction.}
allows supersymmetry to be preserved on the level of the action. It can be deduced from a comparison with type IIB, but the systematic approach to determine such potentials is through gauged supergravity \[34, 38, 47\].

Furthermore, the kinetic term for the scalars \(C_4^{(0,4)}\), i.e. the penultimate term in (30), is modified in the presence of fluxes. Again, via T-duality one can only infer a correction \(F_5^{(1,4)} = (dC_4)^{(1,4)} - \frac{1}{2} B_2^{(1,1)} \wedge (dC_2)^{(0,3)}\), but comparison with type IIB \(8\) shows that actually the combination

\[
F_5^{(1,4)} = (dC_4)^{(1,4)} - \frac{1}{2} B_2^{(1,1)} \wedge (dC_2)^{(0,3)} + \frac{1}{2} C_2^{(1,1)} \wedge (dB_2)^{(0,3)}
\]

appears. Finally, when dualizing \(F_9^{(3,6)}\) and \(F_7^{(2,5)}\) one would in principle have to adapt their Bianchi identities in (31) and (35). Fortunately, this would just influence the resulting Chern-Simons term and not the kinetic terms for \(C_0, C_2^{(1,1)}\) and \(B_2^{(1,1)}\). Hence, the kinetic terms in the RR sector after T-duality are

\[
S_{\text{kin}} = - \frac{1}{4\kappa_{10}^2} \int d^1 u \sqrt{-g} \left( |(dC_0)^{(1,0)}|^2 + |(dC_2)^{(2,1)} + C_0(dB_2)^{(2,1)}|^2 + |(dC_4)^{(1,4)} - \frac{1}{2} B_2^{(1,1)} \wedge (dC_2)^{(0,3)} + \frac{1}{2} C_2^{(1,1)} \wedge (dB_2)^{(0,3)}|^2 \right) 
\]

\[
= - \frac{1}{4\kappa_{10}^2} \int d^1 u \sqrt{-g} \left( |F_1^{(1,0)}|^2 + |F_3^{(2,1)}|^2 + |F_5^{(1,4)}|^2 \right)
\]

We see that under a gauge transformation of the Kaluza-Klein vectors \(C_2^{(1,1)}\) and \(B_2^{(1,1)}\), the scalars \(C_4^{(0,4)}\) have to transform in order to render their kinetic term gauge invariant. This complication can be turned to our advantage in that it reveals the necessary modification of the CS terms as in \(60\). The Chern-Simons term must be modified to

\[
2\kappa_{10}^2 S_{\text{CS}} = \int (dC_2)^{(2,1)} \wedge (dB_2)^{(2,1)} \wedge C_4^{(0,4)} \\
- \frac{1}{2} \int C_2^{(1,1)} \wedge (dB_2)^{(2,1)} \wedge B_2^{(1,1)} \wedge (dC_2)^{(0,3)} \\
- \frac{1}{2} \int B_2^{(1,1)} \wedge (dC_2)^{(2,1)} \wedge C_2^{(1,1)} \wedge (dB_2)^{(0,3)}
\]

In summary, the RR part of the action (without vectors) is of the form

\[
S_{\text{RR}} = S_{\text{pot}} + S_{\text{kin}} + S_{\text{CS}}
\]

where \(S_{\text{pot}}, S_{\text{kin}}\) and \(S_{\text{CS}}\) are given by \(43\), \(45\) and \(46\).
To illustrate the notation let us write out the covariant derivative for the axions descending from $C_4$. Since the only dynamical components of $B_2$ and $C_2$ are $B_2^{(1,1)}$ and $C_2^{(1,1)}$, the last term in (45) is proportional to the square of 

$$\partial_\mu (C_4)_{ijkl} - 2(B_2)_\mu[i](dC_2)_{jkl} + 2(C_2)_\mu[i](dB_2)_{jkl}.$$  (48)

In order to make contact with the standard conventions in supergravity, as used in [28, 31, 47] in particular, let us introduce a different parameterization of the axionic scalars

$$\partial_\mu (C_4)_{ijkl} = \frac{1}{2} \frac{1}{\sqrt{G}} \epsilon_{ijklmn} \partial_\mu \beta^{mn},$$

$$(B_2)_\mu[i](dC_2)_{jkl} = -\frac{1}{8} \epsilon_{ijklmn} (\star dC_2)^{mnp} (B_2)_{\mu p},$$

and similarly for the third term, with $\star$ denoting the six-dimensional internal Hodge operator. This leads to a covariant derivative

$$D_\mu \beta^{ij} = \partial_\mu \beta^{ij} + \frac{1}{2} \sqrt{G} (\star dC_2)^{ijk} (B_2)_{\mu k} - \frac{1}{2} \sqrt{G} (\star dB_2)^{ijk} (C_2)_{\mu k}$$  (50)

for $\beta^{ij}$. Note that the combinations $\sqrt{G} (\star dC_2)^{ijk}$ and $\sqrt{G} (\star dB_2)^{ijk}$ are actually constant, independent of the metric, for constant background fluxes $(dC_2)_{ijk}$ and $(dB_2)_{ijk}$. They correspond to the flux parameters $f_{\alpha \Sigma \Gamma}^{\Delta}$ of [34], while the $\beta^{ij}$ serve as axionic moduli also independent of the metric. Later we will come back to the precise relation between our notation and the one used in [34, 47]. In the light of (21), the reparameterization (49) just reintroduces the T-dual variables, which reflects the fact that the dependence of mass parameters on the radii of the torus was found to be inverted in [34].

### 2.3 Coupling to vector fields

In the previous section we have performed the T-duality of the type I action without vector fields, reorganized the RR forms to be able to compare to the truncated type IIB action, and then deduced the modification due to 3-form fluxes. We now want to add in the CS correction appearing in (4) and the Yang-Mills action for the vectors. Thus, our starting point is now (2). In order to T-dualize (2) we need to make the appearance of the KK vectors in the terms involving $F$ and $\omega_3$ explicit.

Let us take $F$ first. On a torus we have

$$\mathcal{F}^{a}_{\mu \nu} = 2 \partial_{[\mu} A^a_{\nu]} + f^{abc} A^b_{\mu} A^c_{\nu},$$

$$\mathcal{F}^{a}_{\mu i} = D_\mu A^a_i = \partial_\mu A^a_i + f^{abc} A^b_{\mu} A^c_i,$$

$$\mathcal{F}^{a}_{ij} = f^{abc} A^b_i A^c_j.$$  (51)

Our notation does not distinguish scalars $A_i$ from vector fields $A_\mu$ and we do not use a background gauge flux $f^{a}_{ij}$. This is related to the fact that we intend to stick
to D3-branes after the T-duality. We shall come back to this point later. As in (28) we introduce the components of $F'$ as

$$F'_{\mu \nu} = F_{\mu \nu} + 2A_\nu A_\mu, \quad F'_{\mu i} = F_{\mu i} + A_\mu A_i, \quad F'_{ij} = F_{ij},$$

where $F_{MN}$ is defined as in (3). Using new vector fields, invariant under KK gauge transformations,

$$\tilde{A}_\mu^a = A_\mu^a - A^a_i A_i,$$

it is straightforward to verify the relations

$$F'_{\mu i} = \tilde{D}_\mu A_i^a, \quad F'_{\mu \nu} = \tilde{F}^a_{\mu \nu} + 2\partial_{\mu A_\nu} A_i^a,$$

where we used

$$\tilde{D}_\mu A_i^a = \partial_\mu A_i^a + f^{abc} \tilde{A}^b_\mu A_i^c, \quad \tilde{F}^a_{\mu \nu} = 2\partial_{\mu A_\nu} A_i^a \tilde{A}_i^a.$$

Thus, using (22), the kinetic term for the vectors is mapped as follows under T-duality:

$$\int d^{10}x \sqrt{-g} e^{-\Phi} \text{tr} |F|^2 \mapsto \int d^{10}x \sqrt{-g} e^{-\Phi} \left( G_{ij} g^{\mu \nu} \tilde{D}_\mu A_{ai} \tilde{D}_\nu A_{aj} + \frac{1}{2} g^{\mu \rho} g^{\sigma \nu} (\tilde{F}_{\mu \rho} + H_{i \rho \sigma} A_i) (\tilde{F}_{\nu \sigma} + H_{j \nu \sigma} A_j) + \frac{1}{2} G_{ij} G_{kl} f^{abc} f^{ade} A_{bi} A_{ck} A_{di} A_{el} \right),$$

where now

$$\tilde{A}_\mu^a = A_\mu^a - A^a_i B_\mu i,$$

and $H_{i \nu}$ is given by (20). The term in the last line of (56) represents a contribution to the potential for the open string fields and can also be written as $\text{tr} |F|_{(0,2)}^2$ up to a constant. Notice that (56) is separately invariant under the gauge transformations

$$B_{\mu i} \to B_{\mu i} + \partial_\mu \epsilon_i,$$

with $\tilde{A}_\mu^a$ and $A^a_i$ inert, and

$$\tilde{A}_\mu^a \to \tilde{A}_\mu^a + \partial_\mu \epsilon^a + f^{abc} \tilde{A}_\mu^b \epsilon^c, \quad A^a_i \to A^a_i + f^{abc} A^b_i \epsilon^c,$$

with $B_{\mu i}$ inert.
Let us now turn to the mapping of $\omega_3$ under the duality transformation, shifting to $\omega'_3$ as before. It is given by

\[
\begin{align*}
(\omega'_3)_{ij \ell}^{\{0,3\}} & \mapsto \frac{1}{3!} \epsilon_{ijklmn} (\ast \omega'_3)_{lmn}^{\{0,3\}} , \\
(\omega'_3)^{(1,2)}_{\mu ij} & \mapsto \epsilon_{ijklmn} \left( \frac{1}{4!} (\ast \omega'_3)^{(1,4)}_{\mu klmn} - \frac{1}{3!} (B_2)^{(1,1)}_{\mu ij} (\ast \omega'_3)_{lmn}^{\{0,3\}} \right) , \\
\frac{1}{2} (\omega'_3)^{(2,1)}_{\mu \nu i} & \mapsto \epsilon_{ijklmn} \left( \frac{1}{2} \frac{1}{5!} (\ast \omega'_3)^{(2,5)}_{\mu \nu jklmn} - \frac{1}{4!} (B_2)^{(1,1)}_{\mu ij} (\ast \omega'_3)_{klmn}^{\{1,4\}} + \frac{1}{2} \frac{1}{5!} (B_2)^{(1,1)}_{\mu ij} (B_2)^{(1,1)}_{\nu kl} (\ast \omega'_3)_{lmn}^{\{0,3\}} \right) , \\
\frac{1}{3!} (\omega'_3)^{(3,0)}_{\mu \nu \rho} & \mapsto \epsilon_{ijklmn} \left( \frac{1}{3!} \frac{1}{6!} (\ast \omega'_3)^{(3,6)}_{\mu \nu \rho jklmn} - \frac{1}{2} \frac{1}{5!} (B_2)^{(1,1)}_{\mu ij} (B_2)^{(1,1)}_{\nu kl} (\ast \omega'_3)_{jkmn}^{\{1,4\}} + \frac{1}{2} \frac{1}{5!} (B_2)^{(1,1)}_{\mu ij} (B_2)^{(1,1)}_{\nu k} (\ast \omega'_3)_{jlmn}^{\{0,3\}} \right) .
\end{align*}
\]

Since $\ast$ denotes the six-dimensional internal Hodge operation, $\ast \omega_3$ is a formal sum of forms of degree 3, 5, 7 and 9:

\[
\ast \omega_3 = (\ast \omega_3)^{\{0,3\}} + (\ast \omega_3)^{\{1,4\}} + (\ast \omega_3)^{\{2,5\}} + (\ast \omega_3)^{\{3,6\}} .
\]

Together with (21) we can then write

\[
(F_3^{(p,3-p)})_{\mu_1 \ldots \mu_p \ldots \i_3 \ldots \i_{3-p}} \mapsto - \frac{(-1)^{p(p-1)/2} \sqrt{G}}{(p + 3)!} \epsilon^{i_1 \ldots i_{3-p} j_1 \ldots j_{3+p}} (F_3^{(p,p+3)})_{\mu_1 \ldots \mu_p j_1 \ldots j_{3+p}} ,
\]

where we have defined

\[
\hat{F}_3^{(p,p+3)} = \left[ e^{-B} \wedge \sum_{q=0}^{p} (dD + \gamma(-1)^{(q-1)/2} \ast \omega_3)^{\{q,q+3\}} \right]^{(p,p+3)} .
\]

with the abbreviation

\[
\gamma = \frac{\kappa_1^2}{g_s^2 \sqrt{G}} .
\]

Using this rule, (30) now becomes

\[
- \frac{1}{4\kappa_1^2} \int d^{10} \sqrt{-g} |F_3|^2 \mapsto \frac{1}{4\kappa_1^2} \int \left( \hat{F}_9^{(3,6)} \wedge \ast \hat{F}_9^{(3,6)} + \hat{F}_7^{(2,5)} \wedge \ast \hat{F}_7^{(2,5)} + \hat{F}_5^{(1,4)} \wedge \ast \hat{F}_5^{(1,4)} + \hat{F}_3^{(0,3)} \wedge \ast \hat{F}_3^{(0,3)} \right) .
\]

The potential term $|\hat{F}_3^{(0,3)}|^2$ is now modified due to the open string scalars. For vanishing 3-form flux and taken together with the potential term of (56), this is
the potential known also for the heterotic string [82, 83, 84].

To eliminate the RR forms of high degree, we now follow the same procedure as before and assume vanishing 3-form flux while dualizing \( \hat{F}_9^{[3,6]} \) and \( \hat{F}_7^{[2,5]} \). In order to dualize \( \hat{F}_9^{[3,6]} \), we impose its Bianchi identity by adding the Lagrange multiplier term (anticipating that the Lagrange multiplier will be identified with the RR scalar as in (31))

\[
\delta S = -\frac{1}{2\kappa_1^2} \int C_0 d^{(1,0)} \left[ \hat{F}_9^{[3,6]} + B_2^{[1,1]} \wedge (dD_6)^{[2,5]} - \frac{1}{2} (B_2^{[1,1]})^2 \wedge (dD_4)^{[1,4]} \right. \\
+ \gamma \left( (\ast \omega_3)^{[3,6]} - B_2^{[1,1]} \wedge (\ast \omega_3)^{[2,5]} \right) - \frac{1}{2} (B_2^{[1,1]})^2 \wedge (\ast \omega_3)^{[1,4]} + \frac{1}{3!} (B_2^{[1,1]})^3 \wedge (\ast \omega_3)^{[0,3]} \right] \\
= \frac{1}{2\kappa_1^2} \int (dC_0)^{[1,0]} \wedge \hat{F}_9^{[3,6]} - \frac{1}{2\kappa_1^2} \int C_0 \left[ (dB_2)^{[2,1]} \wedge \hat{F}_7^{[2,5]} \\
+ d^{[1,0]} \left( \gamma (\ast \omega_3)^{[3,6]} - B_2^{[1,1]} \wedge d^{[1,0]} (\gamma (\ast \omega_3)^{[2,5]} \right) \\
- \frac{1}{2} (B_2^{[1,1]})^2 \wedge d^{[1,0]} (\gamma (\ast \omega_3)^{[1,4]} + \frac{1}{3!} (B_2^{[1,1]})^3 \wedge d^{[1,0]} (\gamma (\ast \omega_3)^{[0,3]} \right] .
\]

Integrating out \( \hat{F}_9^{[3,6]} \) leads to the same result as in (31) and the only difference, as compared to the case without vectors, appears in the structure of the Chern-Simons terms. To replace \( \hat{F}_7^{[2,5]} \), we add

\[
\delta S = \frac{1}{2\kappa_1^2} \int C_2^{[1,1]} \wedge d^{[1,0]} \left( \hat{F}_7^{[2,5]} + B_2^{[1,1]} \wedge dC_4^{[1,4]} \right. \\
+ \gamma \left[ - (\ast \omega_3)^{[2,5]} - B_2^{[1,1]} \wedge (\ast \omega_3)^{[1,4]} + \frac{1}{2} (B_2^{[1,1]})^2 \wedge (\ast \omega_3)^{[0,3]} \right] .
\]

Again, integrating out \( \hat{F}_7^{[2,5]} \) leads to the old result (33). Further, apart from the kinetic and potential terms given in (34), (38) and the second line of (65), we obtain the following Chern-Simons terms

\[
S_{CS} = \frac{1}{2\kappa_1^2} \int \gamma \left( (dC_2)^{[2,1]} - C_0 (dB_2)^{[2,1]} \right) \\
\wedge \left( (\ast \omega_3)^{[2,5]} + B_2^{[1,1]} \wedge (\ast \omega_3)^{[1,4]} - \frac{1}{2} (B_2^{[1,1]})^2 \wedge (\ast \omega_3)^{[0,3]} \right) \\
+ (dC_0)^{[1,0]} \wedge (\ast \omega_3)^{[3,6]} - B_2^{[1,1]} \wedge (\ast \omega_3)^{[2,5]} \\
- \frac{1}{2} (B_2^{[1,1]})^2 \wedge (\ast \omega_3)^{[1,4]} + \frac{1}{3!} (B_2^{[1,1]})^3 \wedge (\ast \omega_3)^{[0,3]} \right) \\
+ \frac{1}{2\kappa_1^2} \int (dC_2)^{[2,1]} \wedge (dB_2)^{[2,1]} \wedge C_4^{[0,4]} .
\]
Using (51), the definition
\[ F_{j\mu
u} = 2\partial_{[\mu}C_{\nu]j} \quad (69) \]
and the expressions
\[
(\omega_3)_{\mu\rho} = 6A_\mu^\alpha \partial_\rho A_\alpha^i + 2f^{abc} A_\mu^a A_\rho^b A_\alpha^c,
\]
\[
(\omega_3)^i_{\mu\rho} = 2A_\mu^i \partial_\rho A^{ai} + 2A_\mu^{ai} \partial_\rho A_\nu^{aj} + 2f^{abc} A_\mu^a A_\rho^b A^{aij},
\]
\[
(\omega_3)^{ij}_{\mu} = -2A_\mu^{aij} \partial_\rho A_\rho^i + 2f^{abc} A_\mu^a A_\rho^b A_\sigma^{cj},
\]
\[
(\omega_3)^{ijk} = 2f^{abc} A_\mu^a A_\rho^b A_\sigma^c,
\]
one finds
\[
\frac{1}{2} (\omega_3)^i_{\mu\rho} + B_{[\mu j]} (\omega_3)^{ij}_{\nu] \rho} - \frac{1}{2} B_{[\mu j]} B_{\nu] k} (\omega_3)^{ijk} = A_\rho^i \tilde{F}_{\mu\nu} + A_\rho^{aij} \partial_\mu B_\nu^{ij} - 2\partial_\mu \left( A_\rho^{ai} \tilde{A}_\rho^i \right),
\]
\[
(\omega_3)^i_{\mu\rho} - 3B_{[\mu i]} (\omega_3)^{ij}_{\nu] \rho} + 3B_{[\mu i]} B_{\nu] j} (\omega_3)^{ijk} - B_{[\mu i]} B_{\nu] j} B_{\rho] k} (\omega_3)^{ijk} = (\tilde{\omega}_3)^i_{\mu\rho} + 6A_\mu^a \tilde{A}_\rho^a \partial_\nu B_\rho^i,
\]
where we also used the definitions (55) and (57). Now one can express the Chern-Simons terms as
\[
S_{CS} = - \frac{1}{4\kappa_3^2} \int d^{10} x \sqrt{-g} \epsilon^{\mu\nu\rho} \left( \frac{1}{2} C_0 \tilde{F}_{\mu\nu} \tilde{F}_\rho^{\sigma} \right) (72)
\]
\[
= \left( F_{j\mu\nu} - C_0 H_{j\mu\nu} \right) \left( A_\rho^{ai} \tilde{F}_\rho^{ij} + \frac{1}{2} A_\rho^{aij} A_\sigma^{ai} H_\rho^{ij} \right) + \frac{1}{2\kappa_3^2} \int (dC_2)^{(2,1)} \wedge (dB_2)^{(2,1)} \wedge C_4^{(0,4)}.
\]
As will be pointed out in section 2.6, this expression matches the results obtained from supergravity (up to some numerical factors).

Let us again see which changes occur in the presence of 3-form flux. As in the case without vectors, (55) shows that the kinetic term for the scalars \(C_4^{(0,4)}\) changes and additional terms in the potential appear. The covariant derivative of the axions \((C_4)_{ijkl}\), which was (48) when setting all open string fields to zero, now reads
\[
\partial_\mu (C_4)^{ijkl} - 2(B_2)^{[\mu ij} (dC_2)_{\nu]kl} + 2\gamma (\ast \omega_3)_{ijkl} + 2(C_2)^{[\mu ij} (dB_2)_{\nu]kl} + \gamma (\ast \omega_3)_{\muijkl}.
\]
(73)
Introducing alternative variables as in (56) one can rewrite this up to an overall factor as
\[
D^{ij}_{\mu} = \partial_\mu \tilde{g}^{ij} + \frac{1}{2} \sqrt{G} (\ast dC_2)^{ij}(B_2)^{\mu k} - \frac{1}{2} \sqrt{G} (\ast dB_2)^{ijk} (C_2)^{\mu k} + \gamma \sqrt{G} \left( (\omega_3)^{ij}_{\mu} - (\omega_3)^{ijk} B_{\mu k} \right).
\]
(74)
Using \((\omega_3)^{ij}_\mu - (\omega_3)^{ijk} B_{\mu k} = -2 A^{a[i} \tilde{D}_{\mu} A^{a j]}\). \(\tag{75}\)

Thus \(\tag{74}\) can alternatively be written as

\[
D_{\mu} \beta^{ij} = \partial_{\mu} \beta^{ij} + \frac{1}{2} \sqrt{G} (\ast dC_2)^{ijkl} (B_2)_{\mu k} - \frac{1}{2} \sqrt{G} (\ast dB_2)^{ijkl} (C_2)_{\mu k} - 2 \sqrt{G} \gamma A^{a[i} \tilde{D}_{\mu} A^{a j]} . \tag{76}\]

Now \(\tag{76}\) is invariant under the gauge transformation \(\tag{59}\), whereas invariance under \(\tag{58}\) requires \(\beta^{ij}\) to transform according to

\[
\beta^{ij} \rightarrow \beta^{ij} - \frac{1}{2} \sqrt{G} (\ast dC_2)^{ijk} \epsilon_k . \tag{77}\]

Similarly, under

\[
(C_2)_{\mu k} \rightarrow (C_2)_{\mu k} + \partial_{\mu} \epsilon_k \tag{78}\]

\(\beta^{ij}\) has to transform by

\[
\beta^{ij} \rightarrow \beta^{ij} + \frac{1}{2} \sqrt{G} (\ast dB_2)^{ijk} \epsilon_k . \tag{79}\]

In the framework of gauged supergravity this implies that the same translational isometries of the RR scalars are gauged as in the case without vectors \[34, 47\]. Demanding invariance of the action under gauge-transformations of the Kaluza-Klein vectors requires that the Chern-Simons term in the last line of \(\tag{72}\) is replaced by \(\tag{46}\), analogously to the case without vectors. Apart from this modification we do not expect any further changes to \(\tag{72}\) due to the fluxes, because the open string Chern-Simons terms do not involve the axions \((C_4)_{ijkl}\) and are already by themselves invariant under gauge-transformations of the Kaluza-Klein vectors.\(\tag{12}\)

On the other hand, the 3-form potential does receive additional contributions as argued above in the case without vectors. Comparison with type IIB theory implies that there is an additional potential term of the form \(|H_3^{[0,3]}|^2\) and that instead of \((dC_2 + \gamma \ast \omega_3)^{[0,3]}\) it is the full \((dD_2 + \gamma \ast \omega_3)^{[0,3]}\) that should enter into \(|\tilde{F}_3^{[0,3]}|^2\). The two terms can again be combined to give

\[
|(F_3 + \gamma \ast \omega_3)^{[0,3]}|^2 = |(G_3 + \gamma \ast \omega_3)^{[0,3]}|^2 = |\tilde{G}_3^{[0,3]}|^2 . \tag{80}\]

Actually, there is a loop-hole in our strategy to combine the T-dual data and the truncated type IIB action in this case. The expression \(\tag{80}\) contains the cross-term \(2 \gamma C_{0} (dB_2)^{ijk} (\ast \omega_3)^{ijk}\). This term contains NSNS 3-form flux and open-string fields at the same time and thus vanishes in both limits. Furthermore, its presence or

\(\text{\textsuperscript{12}}\)Ultimately, this is justified by comparison to the gauged supergravity of \[47\] and, as mentioned already, our results match the expressions given in \[47\] without further modifications.
absence is not restricted by any symmetry argument. This is different from the corresponding cross-term in the kinetic term for $\beta^{ij}$, which is required by KK gauge invariance, cf. \cite{1403.4471}. Also the shift-symmetry of $C_0$ is broken by the presence of the D3-branes and can not help to fix the coefficient of $\gamma C_0 (dB_2)_{ijk} (\star \omega_3)^{ijk}$. Nevertheless, we believe that \cite{1403.4471} is the correct combination, because it leads to the same potential as found in the supergravity approach \cite{1307.5098,1405.0911}, as we will show in a moment.

Putting all the pieces together, the bosonic action derived by T-duality of the type I string contains four different parts

$$S = S_{\text{EH}} + S_{\text{kin}} + S_{\text{pot}} + S_{\text{CS}}.$$ \hfill (81)

In the string frame, they are given in turn by

$$S_{\text{EH}} = \frac{1}{2\kappa_i^{10}} \int d^{10} x \sqrt{-g} e^{-2\phi} R,$$

$$S_{\text{kin}} = \frac{1}{2\kappa_i^{10}} \int d^{10} x \sqrt{-g} e^{-2\phi} \left( 4 \partial_\mu \phi \partial^\mu \phi - \frac{1}{4} G^{ij} H_{i\mu\nu} H_{j}^{\mu\nu} \right)$$

$$- \frac{1}{4\kappa_i^{10}} \int d^{10} x \sqrt{-g} \left( |F_1^{(1,0)}|^2 + |F_3^{(2,1)}|^2 + |F_5^{(1,4)}|^2 \right)$$

$$- \frac{1}{2\kappa_i^{10}} \int d^{10} x \sqrt{-g} \gamma e^{-\Phi} \left( G_{ij} g^{\mu\nu} \tilde{D}_\mu A^{ai} \tilde{D}_\nu A^{aj} \right.$$

$$\left. + \frac{1}{2} g^{\mu\nu} g^{\rho\sigma} (\tilde{F}_a^{\mu\rho} + H_{i\mu\nu} A^{ai}) (\tilde{F}_a^{\nu\sigma} + H_{j\nu\sigma} A^{aj}) \right), \hfill (82)$$

$$S_{\text{pot}} = - \frac{1}{4\kappa_i^{10}} \int d^{10} x \sqrt{-g} \left( |G_3^{(0,3)}|^2 + 2 \gamma e^{-\Phi} \text{tr} |F^{(0,2)}|^2 \right), \hfill (83)$$

$$S_{\text{CS}} = - \frac{1}{4\kappa_i^{10}} \int d^{10} x \sqrt{-g} \gamma e^{\nu\rho\sigma} \left[ \frac{1}{2} C_0 \tilde{F}_a^{\mu\nu} \tilde{F}_a^{\mu\rho} \right.$$

$$\left. - (F_{j\mu\nu} - C_0 H_{j\mu\nu}) \left( A_{aj} \tilde{F}_a^{\rho\sigma} + \frac{1}{2} A_{aj} A^{ai} H_{i\rho\sigma} \right) \right]$$

$$+ \frac{1}{2\kappa_i^{10}} \int (dC_2)^{(2,1)} \wedge (dB)^{(2,1)} \wedge C_4^{(0,4)}$$

$$- \frac{1}{4\kappa_i^{10}} \int C_2^{(1,1)} \wedge (dB_2)^{(2,1)} \wedge B_2^{(1,1)} \wedge (dC_2)^{(0,3)}$$

$$- \frac{1}{4\kappa_i^{10}} \int B_2^{(1,1)} \wedge (dC_2)^{(2,1)} \wedge C_2^{(1,1)} \wedge (dB_2)^{(0,3)} . \hfill (84)$$

### 2.4 Additional modifications from the non-abelian DBI

To identify candidate couplings in the effective action that involve 3-form flux and open string fields at the same time, one can study the non-abelian D-brane world volume action in the formulation given in \cite{1307.5098}. It turns out that only the DBI part
contains a term that has to be added to the effective action derived via T-duality. The CS part in principle has the potential to modify the tadpole condition, but we will argue that actually it does not.

From the expansion of the DBI action for a D3-brane in the presence of NSNS 3-form flux one deduces a correction to the effective action \[63\] that in our conventions reads

\[ i \frac{3}{g_{10}^2 \sqrt{G}} e^{-\Phi} \text{tr}(A^i A^j A^k) H_{ijk} = -\frac{1}{2\kappa_{10}^2} \frac{1}{3!} \gamma e^{-\Phi} \omega_{ijk} H_{ijk} . \]  

(85)

This term represents an extra contribution to the naive abelian term for the tension of a D3-brane in the presence of NSNS 3-form flux.

By analyzing the non-abelian CS action of the D3-branes one may have expected a modification of the relevant component of the equation of motion for \( C_4 \), since one finds a further coupling linear in \( C_4 \left\{ A^j B_2 \right\} \), i.e.

\[ \int \text{tr} \left( C_4 \left\{ A^j B_2 \right\} \right) \sim \int d^4 x \sqrt{-g_4} \epsilon^{\mu \nu \rho \sigma} (dC_6)_{\mu \nu \rho \sigma \rho i j k} \text{tr}(A^i A^j A^k) , \]  

(86)

using the symbolic notation of \[63\], i.e. \( i A_i A^i B_2 = A^j A^i B_{ij} \). This term could potentially modify the tadpole cancellation condition for D3-brane charge. However, one has to take into account that the CS action involves RR-fields of all degrees and \( 87 \) is accompanied by another contribution, the direct analogue of the term that lead to dielectric D0-branes in \[63\],

\[ \int \text{tr} \left( P [i A_i A C_6] \right) \sim \int d^4 x \sqrt{-g_4} \epsilon^{\mu \nu \rho \sigma} (dC_6)_{\mu \nu \rho \sigma \rho i j k} \text{tr}(A^i A^j A^k) , \]  

(87)

where \( P \) denotes the (non-abelian) pull-back. The component of \( dC_6 \) that occurs here is related to the 3-form flux via duality. To make this more precise, one has to use the democratic version of the ten-dimensional type I action (already mentioned in footnote \[10\]), that involves \( C_6 \) on the same footing as \( C_2 \) even before applying the T-duality, cf. appendix \[C\]. This formulation also involves a kinetic term for the field strength \( F_7^{(4,3)} \) after T-duality. In general, \( F_7 \) is given by

\[ F_7 = dC_6 + H_3 \wedge (C_4 - \frac{1}{2} C_2 \wedge B_2) , \]  

(88)

where we made use of the general formula (see e.g. \[79\])

\[ \sum_{q=1}^{5} F_{2q-1} = \sum_{q=1}^{5} d\tilde{C}_{2q-2} + H_3 \wedge \sum_{q=2}^{5} \tilde{C}_{2q-4} , \]  

(89)

\[ ^{13}\text{Note that our NSNS } B\text{-field differs by a sign from the one used there.} \]
and took into account that our $C_4$ differs from $\tilde{C}_4$ used in [79] by $\tilde{C}_4 = C_4 - \frac{1}{2} C_2 \wedge B_2$, whereas all other $C_p$ coincide, i.e. $\tilde{C}_p = C_p$ for $p \neq 4$. In [80] the difference between $\tilde{C}_4$ and $C_4$ does not matter because $(B_2)_{\mu\nu}$ and $(C_2)_{\mu\nu}$ are projected out and their backgrounds vanish in the vacuum. Now we notice that the two terms [86] and [87] nicely combine into

$$\int d^4x \sqrt{-g} \varepsilon^{\mu\nu\rho\sigma} (F_7)_{\mu\nu\rho\sigma ij k} \text{tr}(A_i A_j A_k) + \cdots,$$

(90)

where the dots stand for further terms, involving also $C_4^{(2,2)}$, which is dualized in favor of $C_4^{(0,4)}$ in the end. We do not want to go into further details here, but just observe that upon dualizing $F_7^{(4,3)}$ in a similar vein as done in appendix $\mathbf{C}$ the term [86] disappears from the action and there is no additional contribution to the tadpole condition for $C_4^{(4,0)}$ after eliminating the superfluous degrees of freedom. This means that in the tadpole condition (10) the contribution of the fluxes to the effective 3-brane charge is

$$N_{\text{flux}} = \frac{1}{2 \kappa_{10}^2 \mu_3} \int_{T_6} F_3^{(0,3)} \wedge H_3^{(0,3)},$$

(91)

exactly as in the case without the open string fields, because the CS correction from the non-abelian D-brane action drops out. Here $\mu_3$ denotes the 3-brane string frame tension which for general $p$-branes is given by\textsuperscript{14}

$$\mu_p = \frac{1}{\sqrt{2}} \frac{2 \pi (4 \pi^2 \alpha')^{-(p+1)/2}}.$$

(92)

Moreover, notice that in the self-dual action the kinetic term for $C_2$ appears with a factor $1/2$ as compared to the usual type I action. Thus also e.g. the 3-form potential after T-duality would first come with the same additional factor. It is only after dualizing $F_7^{(4,3)}$ that the full 3-form potential, as given in (83), is obtained.

The fact that the tadpole condition is not changed as compared to the case without open string fields is important for the form of the effective four-dimensional potential. First note that this tadpole constraint descends from the equation of motion of $(C_4)_{\mu\nu\rho\sigma}$ in the ten-dimensional theory and does not arise as a dynamical equation in four dimensions. Instead it needs to be imposed as a constraint that determines the number of D3-branes. The dual theory constructed in the previous section is only consistent in the presence of 3-form fluxes if the fluxes do not contribute any 3-brane charges, i.e. one necessarily has $N_{\text{flux}} = 0$. This is because T-dualizing pure type I naturally leads to a theory with 16 D3-branes\textsuperscript{15} that already

\textsuperscript{14}Note that the tension includes a factor $\frac{1}{\sqrt{2}}$ as compared to the corresponding formula of type IIB, as is required in type I and its T-duals, see [85] for instance.

\textsuperscript{15}There is a well-known ambiguity of how to count the individual branes. In a microscopic description of the T-dual model, there are 64 O3-planes located at the fixed points of $\Theta$, such that local charge cancellation as in type I is only achieved in the effective action if all sources are completely smeared out. The number 16 simply refers to the rank of the gauge group.
fully cancel the charge (and tension) of the O3-planes. A non-vanishing $N_{\text{flux}}$ would then lead to a surplus of 3-brane charge. This will be discussed in more detail in section 4.1, but let us go slightly ahead of things and already mention that in general the potential receives a further extra contribution from the tension of the O3-planes and D3-branes, when $N_{\text{flux}} \neq 0$. To accommodate this, the first term of (83) can be rewritten using the splitting of $\hat{G}_3^{0,3}$ under $\star$ into imaginary self-dual and anti-self-dual components, i.e. $\star \hat{G}_3^{\text{ISD}} = i \hat{G}_3^{\text{ISD}}$, $\star \hat{G}_3^{\text{IASD}} = -i \hat{G}_3^{\text{IASD}}$. One then verifies, similarly to [21], that the 3-form flux potential term combines with the extra non-abelian correction to the brane tension into

$$
\left( |\hat{G}_3^{0,3}|^2 - \frac{4i}{3} \gamma e^{-\Phi} \text{tr}(A^i A^j A^k) H_{ijk} \right) d\text{vol} = 2 |\hat{G}_3^{\text{ISD}}|^2 d\text{vol} + 2 e^{-\Phi} F_3^{0,3} \wedge H_3^{0,3} \quad (93)
$$

with $d\text{vol} = \sqrt{G} d^6 x$. Due to the unmodified tadpole condition we see that the second term on the right-hand side of (93) is cancelled by the tension of the localized objects.

Note that the absence of the CS correction to the tadpole condition is indispensable for producing a positive definite scalar potential as is required by matching the results of gauged supergravity [38, 47]. In particular, couplings that could drive a dielectric Myers’ effect, which would lead to non-commutative brane solutions, now appear as the cross terms in $|\hat{G}_3^{\text{ISD}}|^2$. At a global minimum of the potential, when $G_3$ is IASD, they therefore cancel out [27, 38].

Let us make a further comment here. The vector couplings in the kinetic terms (82) are asymmetric in that Chern-Simons corrections do not occur in the kinetic terms for $C_0$, $C_2^{1,1}$ and $B_2^{1,1}$. This is due to the fact that we did not include the Chern-Simons term

$$
- \mu_9 \int dC_2 \wedge \omega_7
$$

as part of

$$
\mu_9 \sum_p \int C_p \wedge \chi(F)
$$

for the D9-branes in our starting point type I action (2). Under T-duality such a term would be mapped according to

$$
(dC_2)^{q,3-q} \wedge (\omega_7)^{4-q, q+3} \mapsto - \frac{(-1)^{g(q-1)/2}}{\sqrt{G}} (dD_{2+2q})^{q,3+q} \wedge (\star \omega_7)^{4-q,3-q}, \quad (96)
$$

[16] Let us remind the reader that we are using a different definition of the Hodge star than the one used e.g. in [25, 21, 27].

[17] Moreover, this form of the potential is necessary to be consistent with the constraints that derive from the equations of motion, as discussed in section 4.
leading, among other things, to terms involving \((dD_8)^{3,6}\) and \((dD_6)^{2,5}\) together with the appropriate components of \(*\omega_7\). Clearly this would modify the dualizing process described above, leading to Chern-Simons corrections involving \(*\omega_7\) in the kinetic terms for \(C_0, C_2\) and \(B_2\) and to new Chern-Simons terms similar in nature to \((\mathfrak{R})\) but lengthier. In that case the kinetic terms appearing in \((\mathfrak{8})\) involve the field strengths

\[
\hat{F}_5^{1,4} = (dD_4 + \gamma \ast \omega_3)^{1,4} - B^{1,1} \wedge (dD_2 + \gamma \ast \omega_3)^{0,3},
\]

\[
\hat{F}_3^{2,1} = (dD_2 - 2\gamma \ast \omega_7)^{2,1} - B^{1,1} \wedge (dD_0 + 2\gamma \ast \omega_7)^{1,0},
\]

\[
\hat{F}_1^{1,0} = (dD_0 + 2\gamma \ast \omega_7)^{1,0},
\]

(97)

instead of \(F_3^{2,1}\) and \(F_1^{1,0}\). In this symmetrized version, the coupling of the truncated type IIB action to the gauge fields, at least for the kinetic terms, can be summarized by adding to the \(dD_p\) the appropriate component of \(\gamma \ast \omega_3\) or \(2\gamma \ast \omega_7\) depending on the bi-degree of the form. However, as \((\mathfrak{9})\) is a higher order correction in the open string fields and we restrict ourselves to the lowest order of the DBI action, we preferred not to include the corrections due to \(\omega_7\) in \((\mathfrak{8})\).

### 2.5 The action in the Einstein frame

Finally, in order to make contact to the supergravity literature, we have to transform \((\mathfrak{8})\) into the Einstein frame. Transforming to the ten-dimensional Einstein-frame by rescaling the metric with the string coupling, \(g_{IJ} \to e^{\Phi/2} g_{IJ}\), leads to

\[
2\kappa_{10}^2 S = \int d^{10}x \sqrt{-g} \left[ R - \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - \frac{1}{4} e^{-\Phi} G_{\mu\nu} H_{ij} H^i_j + \frac{1}{4} e^\Phi F_{\mu\nu} C_{0\mu\nu} + \frac{1}{4} e^\Phi F_{\mu\nu} H_{ij} (F_{\mu\nu} + C_0 H_{ij}) + \frac{1}{4} G^{-1} G_{ik} G_{jl} D_\mu \beta^{ij} D^\mu \beta_{kl} - \gamma G_{ij} \tilde{D}_\mu A^{ai} \tilde{D}^\mu A^{aj}
\]

\[
- \frac{1}{4} e^\Phi \left( |\tilde{F}_{3}^{0,3}|^2 - \frac{4i}{3} \gamma e^{-\Phi} \text{tr}(A^i A^j A^k) H_{ijk} \right)
\]

\[
- \frac{1}{4} e^\Phi G_{ij} G_{kl} f^{a b e} f^{c d e} A^{bi} A^{ck} A^{dj} A^{el}
\]

\[
- \frac{1}{4} \int d^{10}x \sqrt{-g} \epsilon^{\mu\nu\rho\sigma} \left[ \gamma \sqrt{G}(C_0 F_{\mu\nu} F_{\rho\sigma} - 2 \left( F_{\mu\nu} - C_0 H_{\mu\nu} \right) \left( A^{ai} \tilde{F}_{\rho\sigma} + \frac{1}{2} A^{aj} A^{ai} H_{\rho\sigma} \right) - \beta^{ij} F_{\mu\nu} H_{\rho\sigma} - C_{i\mu} B_{j\nu} H_{k\rho\sigma} \sqrt{G} \ast dC_2 \ast j^i k - B_{ij} C_{j\nu} F_{k\rho\sigma} \sqrt{G} \ast dB_2 \ast j^i k \right),
\]

(98)
where we introduced the variables $\beta^{ij}$ and used (76). This form of the action will be relevant in the section when we allow the bulk fields to vary over the whole internal space, and the open-string fields over the world-volume of the D-branes, looking for solutions to the ten-dimensional equations of motion.

Dimensional reduction of (98) and a Weyl rescaling $g_{\mu\nu} \rightarrow G^{-1/2} g_{\mu\nu}$ to go to the four-dimensional Einstein-frame produces the following effective action

$$
2\kappa_4^2 S = \int d^4x \sqrt{-g_4} \left[ R - \frac{1}{4} G_{ij} G^{ij} \partial \mu \partial \nu \tilde{G}_{kl} - \frac{1}{2} \partial \mu \Phi \partial \nu \Phi - \frac{1}{4} e^{-\Phi} \tilde{G}^{ij} H_{i\mu\nu} H_{j}^{\mu\nu}
\right.
$$

$$
- \frac{1}{2} e^{2\Phi} \partial \mu C_0 \partial \nu C_0 - \frac{1}{4} e^\Phi \tilde{G}^{ij} (F_{ij\mu\nu} + C_0 H_{i\mu\nu})(F_{ij\mu\nu} + C_0 H_{j}^{\mu\nu})
$$

$$
- \frac{1}{4} \tilde{G}_{ik} \tilde{G}_{jl} B_{ij} \beta^{kl} - \tilde{\gamma} \tilde{G}_{ij} \delta_{\mu} \partial \nu A^{ai} \delta_{\mu} \partial \nu A^{aj}
$$

$$
- \frac{1}{2} e^{-\Phi} (\tilde{F}_{\mu\nu} + H_{i\mu\nu} A^{ai}) (\tilde{F}_{\mu\nu} + H_{j}^{\mu\nu} A^{aj})
$$

$$
- \sqrt{G^{-1}} e^{-\Phi} |G_3^{\text{ISD}}|^2 - \frac{1}{2} \tilde{\gamma} e^\Phi \tilde{G}_{ij} \tilde{G}_{kl} f^{abc} f^{ade} A^{bi} A^{ck} A^{dj} A^{el}
$$

$$
\left. - \frac{1}{4} \int d^4x \sqrt{-g_4} e^{\mu\nu\rho\sigma} \left[ \tilde{\gamma} \left( \tilde{C}_0 \tilde{F}_{\mu\nu} \tilde{F}_{\rho\sigma}^a \right)
\right.
$$

$$
- 2 (F_{i\mu\nu} - C_0 H_{i\mu\nu}) (A^{ai} \tilde{F}_{\rho\sigma}^a + \frac{1}{2} A^{aj} A^{ai} H_{i\rho\sigma})
$$

$$
- \beta^{ij} F_{i\mu\nu} H_{j}^{\mu\nu}
$$

$$
- C_{ij} B_{j\nu} H_{k\rho\sigma} \sqrt{G} (\ast dC_2)^{ijk} - B_{i\mu} C_{j\nu} F_{k\rho\sigma} \sqrt{G} (\ast dB_2)^{ij\kappa} \right] .
$$

Here we have already taken into account the tension of the localized objects to get the form of the potential (fifth line) and introduced the following notation

$$
\tilde{G}_{ij} = \frac{1}{\sqrt{G}} G_{ij}, \quad \tilde{\gamma} = \gamma \sqrt{G} = \frac{\kappa_2^2}{g_{10}^2}.
$$

The action is now in the form that should allow for a direct comparison to the gauged supergravity theory that captures the effective dynamics of type I strings with background 3-form fluxes.

### 2.6 Comparison to gauged supergravity

In this section, we would like to make explicit the comparison of our results in (99) to those found in the gauged supergravity approach, by explaining the translation of notation, parameters and fields. We view this as strong independent confirmation that the effective theory, obtained by modifying the T-dual action in the manner described above to capture the effects of 3-form fluxes in type I, is a sensible approximation of the string dynamics in the supergravity regime. To what

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18We take the background volume to be $(2\pi\alpha'^{1/2})^6$ and therefore have $\kappa_4^2 = \kappa_1^2 (4\pi^2\alpha')^{-3}$. 

26
extent it is approximate will be subject of section 4.

The expressions we are going to compare are the covariant derivative of the axionic scalars, already discussed in (76), and the gauge kinetic and Chern-Simons part (84) of the action (99). Finding basic agreement (up to two factors of 2 and a sign, see below) with the supergravity results of [47], see also [34, 35, 38] for partial results, we conclude that the effective models are identical. A third object of great interest is, of course, the scalar potential, to which the entire section 3 will be devoted. The expression given in equations (4.97), (4.98) and (5.132) of [47] for the covariant derivative of the axion is

\[ D_{\mu} B_{\Lambda\Sigma} = \partial_{\mu} B_{\Lambda\Sigma} + f_{\alpha}^{\Lambda\Sigma\Gamma} A_{\mu\Gamma}^{\alpha} - a^{[\Lambda}\nabla_{\mu} a^{\alpha\Sigma]}, \]  

(101)

where the \( f_{\alpha}^{\Lambda\Sigma\Gamma}, \alpha = 1, 2, \) are the numerical parameters for the 3-form fluxes, \( A_{\mu\Gamma}^{\alpha} \) the abelian KK vector fields and the \( a^{\alpha\Lambda} \) are scalars charged under the non-abelian gauge group. Finally, \( \nabla_{\mu} \) denotes their gauge covariant derivative. In view of (76) this suggests the following mapping of fields

\[ B_{\Lambda\Sigma} \leftrightarrow \beta^{ij}, \quad A_{\mu\Lambda}^{1} \leftrightarrow (B_{2})_{\mu i}, \quad A_{\mu\Lambda}^{2} \leftrightarrow (C_{2})_{\mu i}, \quad a^{\alpha\Lambda} \leftrightarrow -A_{\alpha i}, \]  

(102)

and flux parameters

\[ f_{1}^{\Lambda\Sigma\Gamma} \leftrightarrow \frac{1}{2} \sqrt{G}(dC_{2})^{ijk}, \quad f_{2}^{\Lambda\Sigma\Gamma} \leftrightarrow -\frac{1}{2} \sqrt{G}(dB_{2})^{ijk}, \]  

(103)

where we have set \( \tilde{\gamma} = 1 \) throughout. The signs are going to become clearer below. The non-abelian vectors should map to the \( \tilde{A}_{a}^{\mu} \) of (57). Using this map leads to agreement between our result (76) and (101) (up to a factor of 2 for the last term).

From the gauge kinetic and CS part of (99) we can read off the \( \theta \)-angles and coupling constants of the different gauge fields, which are already present in the ungauged theory and which we define as the coefficients in front of \( F_{\mu\nu} F_{\mu\nu} \) or \( \epsilon_{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \), respectively. \( F_{\mu\nu} \) now stands for any kind of gauge field strength, \( F_{\mu\nu}^{a}, F_{\mu\nu}^{1}, \) or \( H_{\mu\nu} \). We put them into a matrix labelled by \((a, \alpha, i)\) and read off

\[ \theta^{ab} = -\frac{1}{2} C_{0}\delta^{ab}, \quad \theta^{i1a} = A^{ai}, \quad \theta^{i2a} = -C_{0}A^{ai}, \quad \theta^{i1j1} = 0, \]

\[ \theta^{i2j2} = -\frac{1}{2} C_{0}A^{ai}A^{aj}, \quad \theta^{i1j2} = \theta^{i2j1} = \frac{1}{4} \beta^{ij} + \frac{1}{4} A^{ai}A^{aj}, \]  

(104)

and

\[ (g^{-2})^{ab} = -e^{-\Phi} \delta^{ab}, \quad (g^{-2})^{i1a} = 0, \quad (g^{-2})^{i2a} = -2e^{-\Phi} A^{ai}, \]

\[ (g^{-2})^{i1j1} = -\frac{1}{2} e^{\Phi} \tilde{G}^{ij}, \quad (g^{-2})^{i2j2} = -\frac{1}{2} (e^{-\Phi} + e^{\Phi} C_{0}^{2}) \tilde{G}^{ij} - A^{ai} A^{aj} e^{-\Phi}, \]

\[ (g^{-2})^{i1j2} = (g^{-2})^{i2j1} = -\frac{1}{2} C_{0} e^{\Phi} \tilde{G}^{ij}, \]  

(105)

\footnote{We are using gauge group indices \( a, b, \ldots \) instead of \( i, j, \ldots \) and \( A, B, \ldots \) instead of \( I, J, \ldots \) to avoid confusing them with our space-time indices.}
leaving a factor $1/(4\kappa^2)$ in front of the action. The corresponding supergravity results can be found in (5.130) together with (3.90)\textsuperscript{20} of \[47\] in the form of a matrix $\mathcal{N}$ of complex coupling constants. Using

$$F^\pm_{\mu\nu} = \frac{1}{2}(F_{\mu\nu} \pm \hat{i} \epsilon_{\rho\sigma\mu\nu} F^{\rho\sigma}) ,$$

(106)

and suppressing indices one has

$$-i(\mathcal{N} F^+_{\mu\nu} F^{+\mu\nu} - \mathcal{N} F^-_{\mu\nu} F^{-\mu\nu}) = \text{Im}(\mathcal{N}) F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \text{Re}(\mathcal{N}) F_{\mu\nu} F^{\rho\sigma} \epsilon^{\mu\nu\rho\sigma} .$$

Thus the $\theta$-angles and couplings are given by\textsuperscript{21}

$$\begin{align*}
\mathcal{N}^{ab} &\leftrightarrow 2\theta^{ab} + i(g^{-2})^{ab} , \\
\mathcal{N}^{i\alpha\alpha} &\leftrightarrow \theta^{i\alpha\alpha} + \frac{i}{2}(g^{-2})^{i\alpha\alpha} , \\
\mathcal{N}^{i\alpha j\beta} &\leftrightarrow 2\theta^{i\alpha j\beta} + i(g^{-2})^{i\alpha j\beta} .
\end{align*}$$

(107)

The expressions one finds for the entries of $\mathcal{N}$ are identical to those in (104) and (105) upon identifying

$$C \leftrightarrow C_0 , \quad \varphi \leftrightarrow -\Phi , \quad g^{\Lambda \Sigma} \leftrightarrow \frac{1}{2} \tilde{G}^{ij} ,$$

(108)

in addition to (102), up to the overall factor of $1/(4\kappa^2)$, the sign in the last line of (105) and a factor of 2 in the $\beta^{ij}$-dependent term in (104). To complete the translation of fields, the $L_\alpha$ in the $SU(1,1)/U(1)$ coset are translated into our $SL(2,\mathbb{R})/U(1)$ scalars $C_0, \Phi$ by

$$L^1 \leftrightarrow -\frac{i}{\sqrt{2}} e^{\Phi/2} , \quad L^2 \leftrightarrow \frac{i}{\sqrt{2}} e^{\Phi/2} (C_0 + ie^{-\Phi})$$

(109)

with $L_1 = L^2, L_2 = -L^1$.

### 3 The potential

From the fifth line of (99) we can read off the effective four-dimensional potential

$$V_{\text{eff}} = \frac{e^{\Phi}}{2\kappa^2 \sqrt{G}} |(G_3 + \gamma * \omega_3)|^{12} + \frac{\gamma e^{\Phi}}{4\kappa^4} \tilde{G}_{ij} \tilde{G}_{kl} f^{abc} f^{ade} A^{bi} A^{ck} A^{dj} A^{el} .$$

(110)

Before discussing the implications of this potential let us compare it to the expression derived with the formalism of $\mathcal{N} = 4$ gauged supergravity. In the conventions of \[47\] the scalar potential was written\textsuperscript{22}

$$V_{\text{SUGRA}} = \frac{1}{2} |F^{ABC-}|^2 + C^{ABC-}|^2 + \frac{1}{4} |L_2 f^{abc} q^{Aq} q^{CB}|^2 ,$$

(111)

\textsuperscript{20}Beware a missprint in an older version, where in the first line of (3.91) the last term should contain a factor of $i$ instead of $\hat{i}$.

\textsuperscript{21}Note the extra factor of 2 in the second line of (5.130) in \[47\].

\textsuperscript{22}We absorb factors of $p!$ into our definition of the norm (161).
where $F$ is the 3-form flux

$$ F^{ABC} = L^\alpha f^{ABC}_\alpha = L^\alpha f^{\Lambda\Sigma\Gamma}_\alpha E^A_\Lambda E^B_\Sigma E^C_\Gamma, \quad (112) $$

the upper index $F^-$ referring to the ISD part, $C^{ABC}$ is the CS correction

$$ C^{ABC} = L_2 f^{abc} a^A q^{AB} q^{BC} = L_2 f^{abc} a^A \delta^B_\Sigma a^C E^A_\Lambda E^B_\Sigma E^C_\Gamma, \quad (113) $$

and indices are pulled back with the vielbein $E^A_\Lambda$ which is related to the metric

$$ g^{\Lambda\Sigma} = \delta^{AB} E^A_\Lambda E^B_\Sigma. \quad (114) $$

The metric moduli together with the axions from $B^{\Lambda\Sigma}$ and the $a^\alpha\Lambda$ parametrize the scalar manifold

$$ \frac{SO(6, 6 + N)}{SO(6) \times SO(6 + N)}. \quad (115) $$

The identification of the two expressions is then accomplished by applying (102), (103) and (108), where one has to take care that the extra signs from $\star\star = -1$ and (109) cancel out. We find agreement $V_{\text{SUGRA}} = V_{\text{eff}}$ after setting $\kappa_4^2 = \tilde{\gamma} = 1$ in (110). Strictly speaking, this choice of parameters is allowed only within supergravity. String theory relates the two in a way that is not consistent with setting them equal.

An important phenomenological feature of (110) is that it involves the dilaton not just as a prefactor, so one can hope to stabilize the string coupling. Concentrating on Minkowski vacua we require

$$ |(G_3 + \gamma \star \omega_3)^{\text{ISD}}|^2 = 0, \quad \text{tr}|F^{(0,2)}|^2 = 0. \quad (116) $$

As $\text{tr}|F^{(0,2)}|^2 = 0$ also implies vanishing of $\omega_3$, the relevant term for fixing $\Phi$ is given by $|G_3^{\text{ISD}}|^2 = 0, \quad (38)$. The condition for $G_3$ to be IASD can be written

$$ \star F_3 - e^{-\Phi} H_3 = 0. \quad (117) $$

Rewriting this via (103) one finds a relation

$$ f_1^{\Lambda\Sigma\Gamma} - C_0 f_2^{\Lambda\Sigma\Gamma} + e^{-\Phi}(\star f_2)^{\Lambda\Sigma\Gamma} = 0, \quad (118) $$

where $\star f_2$ is defined with respect to the metric $g_{\Lambda\Sigma}$. Setting $C_0 = 0$ for simplicity, we see that

$$ f_1^{\Lambda\Sigma\Gamma} + e^{-\Phi} \frac{1}{3!} \sqrt{\det(g_{\Lambda\Sigma})} g^{\Lambda\Lambda'} g^{\Sigma\Sigma'} g^{\Gamma\Gamma'} \dot{\epsilon}_{\Lambda\Gamma\Sigma\Gamma\Delta\Pi\Omega} f_2^{\Delta\Pi\Omega} = 0, \quad (119) $$

where $\dot{\epsilon}$ is just again the antisymmetric symbol with value $\pm 1$. It appears that in general the dilaton becomes a function of the metric moduli that also depends on the choice of fluxes. It was argued in (17) that the 3-form fluxes allowed by the gauging, i.e. consistent with a supersymmetric Lagrangian, can be put into a form such that, in complex coordinates, all components $f^{n\bar{m}n}_\alpha$ and $f^{\alpha n\bar{m}}$ vanish. We will see in section (32) that this ensures a fixing of the dilaton at least in vacua in which supersymmetry has been broken to $N = 1$ via a super-Higgs-mechanism.
3.1 The role of the superpotential

In compactifications of type IIB theory on Calabi-Yau spaces with fluxes, the scalar potential that descends from the kinetic term of $G_3$, after breaking supersymmetry to $\mathcal{N} = 1$ either by orientifolding \[21\] or by taking a certain decompactification limit as in \[5, 13\], can be expressed in terms of the superpotential

$$ W_{\text{flux}} = \int_{\mathcal{M}_6} G_3 \wedge \Omega_3 , $$

proposed by independent arguments in \[11, 12\], with $\Omega_3$ the holomorphic 3-form. In that case, the scalar potential only depends on the complex structure moduli of the Calabi-Yau and the dilaton. As is well known, in the heterotic or type I string the superpotential gets a further contribution involving the scalars descending from the ten-dimensional vectors \[82, 83, 84\]

$$ W_{\text{het/1}} = \int_{\mathcal{M}_6} \omega_3 \wedge \Omega_3 . $$

As in the $\mathcal{N} = 1$ case, also for $\mathcal{N} = 4$ the potential descends from the ten-dimensional kinetic terms and can thus be written in the form of $|G_{3}^{\text{ISD}}|^2$ plus $\text{tr}[F^{(0,2)}]^2$, up to constants. Therefore, one might wonder whether it is still possible to express the potential in terms of a “superpotential”. In view of the effective scalar potential (110) we have found in type I' with extra 3-form flux, it seems more appropriate to consider the “superpotential”

$$ W_{I'} = \int_{\mathcal{M}_6} (G_3 + \gamma \ast \omega_3) \wedge \Omega_3 . $$

However, in the toroidal compactification the potential also depends on the Kähler form, which reflects the fact that the moduli space of a torus does not split into a direct product of complex structure and Kähler moduli as on a Calabi-Yau. On a Calabi-Yau manifold, the ISD 3-forms are the $(1,2)$- and $(3,0)$-forms, see e.g. \[21\]. Moreover, all $(1,2)$-forms are primitive \[86\]. On the torus, on the other hand, there are non-primitive $(1,2)$-forms of the form $J \wedge dz^m$ where $J$ is the Kähler form, and these are also IASD. Moreover, there are further ISD $(2,1)$-forms of the form $J \wedge dz^m$ \[27\] which all enter into the scalar potential (110). These facts make it very cumbersome to express the potential in terms of the superpotential (122).23

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23In order to follow the calculation in the Calabi-Yau case \[5, 13, 21, 24\], one would have to know what should replace the formula $D_2 \Omega_3 = \chi_\alpha$, that gives a basis for the $(2,1)$-forms $\chi_\alpha$ in terms of covariant derivatives of $\Omega_3$ with respect to the complex structure moduli $Z^a$. For the torus, one would need a corresponding formula giving a basis of only the primitive $(2,1)$-forms. However, the split of the $(2,1)$-forms into primitive and non-primitive ones depends on the moduli.
Still, the superpotential (120) has been used in the literature to encode the conditions for unbroken supersymmetry in a toroidal (or K3 × T^2) background with 3-form flux \(27, 41\). There it was shown that demanding supersymmetry is strong enough to fix the period matrix of the torus completely, which implies fixing its complex structure. The conditions for supersymmetry are stronger than demanding extremality of the potential, but are equivalent to the extremality conditions of the superpotential (120). As only ISD 3-form flux enters the potential, there are possibilities to turn on fluxes without generating a vacuum energy. The IASD 3-forms consist in \(G_3\) being a \((0,3)\)-, or a primitive \((2,1)\)-form or of the type \(J \wedge d\bar{z}^m\). On the other hand, in order to preserve supersymmetry the flux has to be a primitive \((2,1)\)-form \(75, 87\) for at least one complex structure. The number of unbroken supersymmetries depends on the number of complex structures for which this condition is fulfilled \(27, 28\). Moreover, it is obvious from the formulas of the gravitino masses given in \(34\) that turning on any non-trivial 3-form flux breaks supersymmetry at least to \(\mathcal{N} = 3\). Since the coupling to the open string fields is manifest in the supervariation of the gravitinos, dilatinos and gauginos by replacing \(F^{ABC}\) with \(F^{ABC} + C^{ABC}\) as used in \(111\) \(17\), it is to be expected that the supersymmetry conditions in the coupled system are also captured by the modified superpotential (122) (in addition to the "D-term" appearing in the second contribution to the potential (111)), although the scalar potential (110) cannot be expressed through \(W_I\) in an obvious way.

### 3.2 Truncating to \(\mathcal{N} = 1\)

Regarding the discussion of the superpotential (122) in the previous section, it appears interesting to study the breaking of supersymmetry in the present model from \(\mathcal{N} = 4\) to smaller numbers of supercharges, especially to \(\mathcal{N} = 1\), in which case the potential should be expressible in terms of a superpotential (and a D-term). This has been done to some extent in the framework of supergravity \(88, 89, 34, 35, 38\) and in this section we would simply like to make contact to \(35\).

By turning on more and more components of the 3-form flux one can successively break supersymmetry from \(\mathcal{N} = 4\) to \(\mathcal{N} = 1\), as has been described in \(34, 35\). In the super-Higgs effect, the decoupling of massive modes is restricted by the requirement that they must fill massive representations of the surviving supersymmetry algebra. If one breaks supersymmetry from \(\mathcal{N} = 4 \rightarrow 3 \rightarrow 2 \rightarrow 1\) successively, 6, 10 or all 12 (Kaluza-Klein) vector fields get massive via their St"uckelberg couplings \(74\). This is precisely as required by a successive decoupling of massive 3/2 multiplets under \(\mathcal{N} = 3, 2, 1\) supersymmetry, which eat just the 6, 4 and 2 vectors, respectively, those that disappear from the massless sector. In this successive su-

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24The case of K3 × T^2 has also been analyzed from the point of view of gauged supergravity in \(44\).
persymmetry breaking the matter content of (9), a spin-2 and six spin-1 multiplets of $\mathcal{N} = 4$, leads to a spin-2 and three chiral multiplets of $\mathcal{N} = 1$ \[35\]. Therefore, 3 gravitini, all 12 vectors, 18 out of 21 scalars $\tilde{G}_{ij}$, 12 out of 15 axions $\beta^{ij}$ and $\tau$ get masses with suitable degeneracies to fill massive $\mathcal{N} = 1$ multiplets. The remaining three complex scalars are given by the diagonal components $\tilde{G}_{1\bar{1}}$, $\tilde{G}_{2\bar{2}}$, $\tilde{G}_{3\bar{3}}$ of the now hermitian metric and the appropriate components of $\beta^{ij}$. Together with the open string moduli they parametrize the remnant scalar manifold

$$\frac{SU(1,1)}{U(1)} \times \frac{SO(6,6+N)}{SO(6) \times SO(6+N)} \rightarrow \left( \frac{SU(1,1+N)}{U(1)} \right)^3. \quad (123)$$

In this particular situation one can convince oneself that the condition (117) is really sufficient to fix the dilaton. The square root of the determinant of the metric is the product $\tilde{G}_{1\bar{1}}\tilde{G}_{2\bar{2}}\tilde{G}_{3\bar{3}}$ of the real parts of the only remaining three moduli, which drop out from (119) if the fluxes are restricted as explained below (119). Thus (117) becomes independent of the metric and $\Phi$ is fixed to some rational value once the fluxes $F_3$ and $H_3$ are subjected to Dirac quantization. A natural value for the string coupling would appear to be of order one, but moderately small numbers are also easily accessible. In \[38\] it was suspected that in the process of integrating out the dilaton, the first term of the potential (110) vanishes, such that the potential of the effective $\mathcal{N} = 1$ theory used in \[35\] is solely given by the second term with a constant value for the dilaton. We were not able to perform this integrating out explicitly, but assume in the following that the $\mathcal{N} = 1$ potential is only given by the second term of (110).

It is well known in the literature how to obtain the $\mathcal{N} = 1$ potential from a dimensional reduction of the kinetic term for the vector fields \[82, 83\]. From (110) (and setting $2\pi \alpha'^{1/2} = 1$) we extract

$$4g_{10}^2 V_{\mathcal{N} = 1} = e^{\Phi} \tilde{G}_{ik} \tilde{G}_{jl} F^{aij} F^{akl} = e^{\Phi} \tilde{G}_{ik} \tilde{G}_{jl} f^{abc} f^{ade} A^b A^c A^d A^e, \quad (124)$$

and it is understood that the dilaton and all metric moduli except the diagonal ones are frozen in the $\mathcal{N} = 1$ vacuum. The potential (124) can be split into an F-term and a D-term by identifying the metric components with the (real parts of the) Kähler coordinates $t_m$ in the following way

$$t_m + \bar{t}_m - C^{am} \bar{C}^{am} = (\tilde{G}_{mm})^{-1}. \quad (125)$$

Here we have introduced the complex fields

$$C^{am} = \frac{1}{\sqrt{2}} (A^{a(2m-1)} + i A^{a(2m)}) \quad (126)$$

Using the Bianchi identity (which implies $F^a \wedge F^a = 0$), one can rearrange terms to arrive at

$$\tilde{G}_{ik} \tilde{G}_{jl} F^{aij} F^{akl} = 2 \tilde{G}_{mn} \tilde{G}_{op} (F^{amo} F^{anp} + F^{amn} F^{apo}) \quad (127)$$
The $(2, 0)$ part can be written
\[ \tilde{G}_{m\bar{n}} \tilde{G}_{\alpha\bar{\beta}} F^{a m} F^{\alpha\bar{\beta}} = e^K \sum_{m=1}^{3} (t_m + \bar{t}_m - C^a_m \bar{C}^a_m) \frac{\partial W}{\partial C^a_m} \frac{\partial \bar{W}}{\partial \bar{C}^a_m}, \tag{128} \]
with superpotential
\[ W = \frac{1}{3!} f^{abc} C^a_m C^b_m C^c_m \hat{\epsilon}_{mno}, \tag{129} \]
where the hat on the epsilon symbol was explained below (19), and Kähler potential
\[ \mathcal{K} = - \sum_{m=1}^{3} \log (t_m + \bar{t}_m - C^a_m \bar{C}^a_m). \tag{130} \]
That the potential-term (128) is of the usual form $e^K (K^{a\bar{a}} D_a W \bar{D}_{\bar{a}} \bar{W} - 3|W|^2)$, where $\alpha$ denotes all chiral fields, $t^m$ and $\bar{C}^m$, and $K^{a\bar{a}}$ the inverse of the Kähler metric, was shown in [35]. The fact that the $-3|W|^2$ drops out in (128) is due to the no-scale structure of the potential. The choice of complex coordinates implies a diagonal period matrix, and the holomorphic 3-form can be written $\Omega_3 = dz_1 \wedge dz_2 \wedge dz_3$, so that (129) can be expressed as in (121). In a similar way one can identify the $(1, 1)$ component of (127) with an $\mathcal{N} = 1$ D-term
\[ \tilde{G}_{m\bar{n}} \tilde{G}_{\alpha\bar{\beta}} F^{a m} F^{\alpha\bar{\beta}} = \left( \sum_{m=1}^{3} \frac{1}{t_m + \bar{t}_m - C^a_m \bar{C}^a_m} f^{bcd} C^c_m \bar{C}^d_m \right)^2. \tag{131} \]
Interestingly, adding a world-volume gauge flux by hand, i.e. adding a constant $f^{a m} \bar{n}$ to $F^{a m} = f^{abc} C^b m C^c \bar{m}$, would lead to a Fayet-Iliopolous term in (131).

## 4 Vacua with fluxes and warped metric

In the previous sections we have employed T-duality to transform the effective action obtained by compactifying type I strings on a torus and neglecting all dependence of the fields on internal directions. The additional terms in the effective action with NSNS 3-form flux on the internal space were added in via comparison with type IIB. However, constant fields will in general no longer solve the ten-dimensional equations of motion once fluxes are turned on. In this section we shall look for solutions to these equations, based on the action (98), with more general background configurations and non-trivial profiles admitting four-dimensional

\[ \text{Note the misprint in formula (12) of [35].} \]

\[ \text{Note that one could have defined the variables $C^a m$ of (126) alternatively as $C^a m = \frac{1}{\sqrt{2}} (A^{a(2m-1)} - i A^{a(2m)})$. In that case, the superpotential (129) would have been of the form (122), with vanishing $G_3$.} \]
Minkowski vacua. As explained in the introduction, in order to derive the equations of motion we allow for a dependence of the bulk fields on the internal coordinates and further modify the action (98) by introducing a background for the 5-form field strength commensurate with the T-dual world sheet parity projection [I] of type IIB. Moreover, we take into account that the D3-branes and O3-planes T-dual to the D9-branes and O9-planes are actually localized objects. We consider the modified equations of motion as describing the coupling of the bulk fields to the tension of the branes (respectively O-planes) and their world-volume fields. We find that the gauge fields only cause minor corrections to the known solutions for a background with vanishing world-volume fields [75, 21]. For these solutions it has been argued that constant internal fields may still be considered approximate solutions in the large volume limit where the warp factor may be taken to be approximately constant, thus a posteriori justifying the effective action (99) — at least in this limit.

4.1 Generalized ansatz and modified action

Our starting point is the ten-dimensional action (98) in which we now allow the bulk fields to depend on the internal coordinates. Furthermore, we have to implement some modifications, that we discuss in the following.

As was already explained, since the modified world sheet projection \( \Omega \Theta(-1)^F \) does not project out fields locally, but only demands symmetry or antisymmetry of the background, we may also allow non-trivial backgrounds for non-dynamical fields, as long as they respect Poincaré invariance. This refers to the 3-form fluxes \( G_{3}^{(0,3)} \), which are not field strengths of any dynamical potentials but do survive the projection, and also to the components \( F_{5}^{(0,5)} \) and \( F_{5}^{(4,1)} \) of the 5-form. The latter is also subject to the self-duality constraint, such that both 5-form components can be parametrized through a single function:

\[
(F_5)_{\mu_1 \ldots \mu_4 i} = \frac{1}{\sqrt{g_4}} \epsilon_{\mu_1 \ldots \mu_4 \partial_i \alpha}, \quad (F_5)_{i_1 \ldots i_5} = \frac{1}{\sqrt{g_4}} \epsilon_{i_1 \ldots i_5 j} (\partial_k \alpha) g^{jk},
\]

where \( \alpha = \alpha(x^i) \) is antisymmetric under \( \Theta \), since \( \Omega(-1)^F \) leaves \( C_{4}^{(4,0)} \) invariant. In the equations of motion, this background enters in the same way as in type IIB. For the metric we use the general warped product ansatz

\[
ds_{10}^2 = g_{IJ} dx^I dx^J = \Delta(x^k)^{-1} \hat{g}_{\mu \nu} dx^\mu dx^{\nu} + \Delta(x^k)^{b} \hat{g}_{ij} dx^i dx^j.
\]

The factor in front of the internal part is purely conventional, but if one intends to identify \( \hat{g}_{ij} \) with a Ricci-flat or even constant metric eventually, it can be helpful to

\footnote{Note that due to our different definition of the Hodge star the internal components have the opposite sign as in [75].}
make the warping explicit. In general, one may also want to consider a non-trivial internal profile for the axion $F^{(0,1)}_1$, as was done in \[90\] to analyze the effects of 7-branes, but we shall not do so here. Furthermore, in looking for a solution to the coupled system of equations we restrict our ansatz to the case of vanishing field strengths $F^{(2,1)}_3$ and $F^{(1,0)}_1$.

Second, the open string fields should be allowed to vary only over the world-volume of the D-branes present in the background. As long as all fields were constant on the internal space, there was no distinction between fields localized on some brane and fields propagating throughout the bulk. T-duality of type I string theory produced (56), where the kinetic terms for the gauge fields only involve the determinant of the four-dimensional metric, as for gauge fields localized on a D3-brane, but there is no delta-function to localize the fields. Thus, (56) refers to “smeared out” D3-branes. In the same way, there was no localization of CS interactions involving open string fields. Further, once we have introduced localization, the tensions of D-branes and O-planes do not cancel locally anymore, and we need to make the tension terms explicit. O3-planes and D3-branes appear naturally in the toroidal model, since they are just the images of the O9-planes and D9-branes of type I under T-duality. In fact, one could also try to introduce higher-dimensional branes, say D7-branes, and have non-trivial gauge fluxes on their world-volume, which induces effective D3-brane charges. We refrain from doing so here, and come back to this option later. One of the main reasons is that adding world-volume gauge fluxes would require us to take higher terms in the DBI effective action into account, which would restrict us to abelian gauge fields, since the non-abelian DBI action is not known to higher order.

From expanding the DBI action to second order in the gauge field strength one infers the relative factor between the tension term and the terms of the world-volume action already present in (58) to be $\frac{1}{2}(2\pi \alpha')^2$. Setting $2\pi \alpha' = 1$ we thus use for the D3-brane action\(^{28}\)

\[
S_{D3} = -\mu_3 \int d^{10}x \sqrt{-g_4} \left( \sum_{\text{branes}} \left( s_m - \frac{i}{3} \text{tr}(A^i A^j A^k) H_{ijk} \right) \delta^6(x_m - \bar{x}_m) + \frac{1}{2} \sum_{\text{branes}} s_m \delta^6(x_m - \bar{x}_m) \left( g_{ij} g^{\mu\nu} \tilde{D}_\mu A^{ai} \tilde{D}_\nu A^{aj} + \frac{1}{2} e^{-\Phi} g^{\mu\nu} g^{\rho\sigma} (\tilde{F}_a + H_{i\rho\sigma} A^{ai})(\tilde{F}_b + H_{j\rho\sigma} A^{aj}) \right. \\
+ \frac{1}{2} e^{-\Phi} g_{ij} g_{kl} f^{abc} f^{ade} A^{bi} A^{ck} A^{dj} A^{el} \right) \right). \tag{134}
\]

The additional term from the non-abelian correction to the DBI action has also

\(^{28}\)Note that this is consistent with (59) as $\frac{1}{2}(2\pi \alpha')^2 \mu_3 = \hat{\gamma}(2\kappa_4^2)^{-1}$, where $\kappa_4$ is given in footnote 18 and we used (52) and $g^{10}_4 = \sqrt{2}(2\pi)^2 \alpha'^4$, cf. \[91\].
been included above. Just to keep track of the overall sign we have introduced coefficients $s_m = \pm 1$ for the tension of the branes. In order to avoid kinetic terms with the wrong sign for the world volume fields, one would choose $s_m = 1$. In the background, the only non-vanishing terms are the tension in the first line and the scalar potential $\text{tr} |\mathcal{F}^{(0,2)}|^2$ in the last. The sum over branes now also implies that the gauge fields are labelled individually for any single stack of branes, but we do not want to encumber the notation with another index. Since we have made the D-brane tension explicit we also have to add the O3-plane tension

$$S_{O3} = -\mu_3 Q_3 \int d^{10}x \sqrt{-g_4} \sum_{k=1}^{64} \delta^6(x_k - \bar{x}_k)$$

(135)

for the 64 planes localized at the fixed points $\bar{x}_k$ of $\Theta$ and $Q_3 = -1/4$. Here we assume that the O3-planes have standard negative tension and negative charge. The total action for the open string sector including the O3-planes is then given by

$$S_{op} = S_{D3} + S_{O3} .$$

(136)

In the same fashion, the CS interactions with open string fields have to be localized. For instance, when we write $|\hat{F}_3|^2$, we now understand

$$|\hat{F}_3|^2 d\text{vol} = |dD_2|^2 d\text{vol} + \sum_{\text{branes}} q_m (2\pi)^6 \delta^6(x_m - \bar{x}_m) \left(2\gamma (\star \omega_3) \wedge dD_2 + \gamma^2 |\star \omega_3|^2 d\text{vol} \right),$$

(137)

with $d\text{vol} = d^{10}x \sqrt{-g}$. The $q_m = \pm 1$ are parameters for the RR charge of the D3-branes and distinguish branes from anti-branes and enter in all the topological terms. Since the CS forms $\star \omega_3$ will mostly only appear inside $\hat{F}_p$ in the following, we will not make this kind of localization explicit, to keep the notation compact.

### 4.2 Equations of motion and constraints

The most interesting equations of motion to consider are the two sets of Einstein equations for internal and external indices, the equation for the dilaton, and the charge conservation constraint coming from the equation of motion for $C_4$. Einstein’s equations with the general warped ansatz (133) are abbreviated, using the notation of (170) (see the appendices A.2 and A.3),

$$R_{\mu\nu} = \hat{R}_{\mu\nu} + \frac{1}{2} \hat{g}_{\mu\nu} \Delta^{1-3b} \hat{g}^{ij} \nabla_i (\Delta^{2(b-1)} \partial_j \ln(\Delta)) = \kappa_{10}^2 S_{\mu\nu} ,$$

$$R_{ij} = \kappa_{10}^2 S_{ij} .$$

(138)

The explicit dependence of $R_{ij}$ on the warp factor is given in the appendix in (168). Furthermore, as in the appendix, $\nabla_i$ is the covariant derivative of $\hat{g}_{ij}$.
The first set of equations allows us to determine the warp factor, which then has to be checked with the second set. We specialize to maximally symmetric four-dimensional space-times, i.e. $\hat{g}^{\mu \nu} \hat{R}_{\mu \nu} = m_4^2$ where $m_4^2$ is zero, positive or negative for Minkowski, de Sitter or anti-de Sitter space respectively. Then, upon taking the trace, one arrives at

$$\hat{g}^{ij} \nabla_i \left( \Delta^{2(b-1)} \partial_j \ln(\Delta) \right) = -\frac{\Delta}{4} \left| \hat{G}_3^{(0,3)} \right|^2 \hat{g} - \frac{1}{2} \Delta^{3b-1} m_4^2$$

$$+ \frac{\Delta^{2+2b}}{2\hat{g}_4} \left( \partial_i \alpha \right) \left( \partial_j \alpha \right) \hat{g}^{ij} - \frac{\Delta^{-2} \kappa_{10}^2 \kappa_{13} Q_3}{\sqrt{\hat{g}_6}} \sum_{k=1}^{64} \delta^6(x_k - \bar{x}_k)$$

$$- \frac{\Delta^{-2} \kappa_{10}^2 \kappa_{13} Q_3}{\sqrt{\hat{g}_6}} \left( \sum_{\text{branes}} \left( s_m - \frac{i}{3} \text{tr}(A_i^i A_j^j A_k^k) H_{ijk} \right) \delta^6(x_m - \bar{x}_m) \right)$$

$$+ e^\Phi \Delta^{-2b} \sum_{\text{branes}} \text{tr} \left| \mathcal{F}^{(0,2)} \right|^2 s_m \delta^6(x_m - \bar{x}_m),$$

where $\hat{g}_4$ denotes the absolute value of the determinant of $\hat{g}_{\mu \nu}$ and the subscript on $\left| \hat{G}_3^{(0,3)} \right|^2 \hat{g}$ implies that the contractions are to be performed with $\hat{g}^{ij}$. Furthermore, $\text{tr} \left| \mathcal{F}^{(0,2)} \right|^2 = \frac{1}{2} \hat{g}^{ij} \hat{g}^{kl} f^{iabc} f^{jade} A_i^a A_j^d A_k^c A_l^e$ is given by $\Delta^{41} \frac{1}{2} \hat{g}_{ij} \hat{g}_{kl} f^{iabc} f^{jade} A_i^a A_j^d A_k^c A_l^e$, when expressed through the metric independent fields $A^a_i$. The integral of the right-hand side over the internal space is now forced to vanish. The D-brane tension and the contributions of the YM sector add up with the 3-form flux, while the O3-plane tensions serve as positive contributions on the right-hand side, which may be employed to balance the negative contributions from the fluxes.\(^{29}\) The cosmological constant and the correction term of the form $\text{tr}(A_i^i A_j^j A_k^k) H_{ijk}$ can add to either the positive or negative contributions.

Upon specializing to $b = 1$ the second set of equations becomes tractable. Combining the two sets of Einstein equations one can eliminate the term with the Laplacian acting on the warp factor and obtains

$$\frac{\Delta}{4} \hat{R}_{\mu \nu} \hat{g}^{\mu \nu} + \frac{1}{6} \hat{R}_{ij} \hat{g}^{ij} = \frac{2}{3} \hat{g}^{ij} \left( \partial_i \ln(\Delta) \right) \left( \partial_j \ln(\Delta) \right) - \frac{\Delta}{12 \hat{g}_4} \hat{g}^{ij} \left( \partial_i \alpha \right) \left( \partial_j \alpha \right)$$

$$- \frac{\kappa_{10}^2}{12} \left( \mathcal{T}_{ij} - \mathcal{T}_{ij}^{\text{flux}} \right) \hat{g}^{ij}. \quad (140)$$

An obvious solution with $\hat{R}_{\mu \nu} = \hat{R}_{ij} = 0$ consists in $\alpha = \pm \Delta^{-2}$ and $(\mathcal{T}_{ij} - \mathcal{T}_{ij}^{\text{flux}}) \hat{g}^{ij} = 0$. This can be satisfied by $\text{tr} | \mathcal{F} |^2 = 0$. There is, however, no constraint on $\hat{G}_3$ from (140) since $\mathcal{T}_{ij}^{\text{flux}} \hat{g}^{ij} = 0$, cf. (172).

\(^{29}\)The sign conventions are such that negative contributions on the right-hand side of (139) are related to positive energy density.
On the other hand, combining (139) with the equation of motion for the external components of $C_4$ leads to a further constraint. The latter has been discussed in some detail in section 2.4, where it was pointed out that no corrections due to the non-abelian CS action occur. It reads

\[
(dF_5)^{(0,0)} = F_3^{(0,3)} \wedge H_3^{(0,3)} + 2\kappa_{10}^2 \mu_3 \sum_{\text{branes}} q_m \pi_m + 2\kappa_{10}^2 \mu_3 Q_3 \sum_{k=1}^{64} \pi_k ,
\]

(141)

and explicitly we have

\[
\frac{1}{\sqrt{g_4}} \left( \dot{g}^{ij} \nabla_i \alpha + \frac{4}{\Delta} \dot{g}^{ij} (\partial_j \Delta)(\partial_j \alpha) \right) = -\frac{1}{(3!)^2 \Delta} F_{ijk} H_{lmn} \epsilon^{ijklmn} - \frac{2\kappa_{10}^2 \mu_3}{\sqrt{g_6} \Delta^4} \left( \sum_{\text{branes}} q_m \delta^6(x_m - \bar{x}_m) + Q_3 \sum_{k=1}^{64} \delta^6(x_k - \bar{x}_k) \right).
\]

(142)

The $\pi_m$ and $\pi_k$ stand for the delta-function-like 6-forms with support at the location of the 3-branes and 3-planes. Using the splitting (93) of $\hat{G}_3$ in (139) and by adding $\Delta^2$ times (139) and $-\frac{1}{2} \Delta^4$ times (142), one can derive the powerful constraint

\[
-\frac{\Delta^4}{2} \dot{g}^{ij} \nabla_i \alpha + \frac{4}{\Delta^2} \dot{g}^{ij} (\partial_j \Delta)(\partial_j \alpha) = -\frac{\Delta^6}{2} \dot{g}^{ij} \partial_i \left( \frac{\alpha}{\sqrt{g_4}} + \Delta^{-2} \right) \partial_j \left( \frac{\alpha}{\sqrt{g_4}} + \Delta^{-2} \right) - \frac{1}{2} e^\phi |\hat{G}_3|_g^2 \sum_{\text{branes}} \frac{\kappa_{10}^2 \mu_3}{\sqrt{g_6}} \delta^6(x_m - \bar{x}_m) - \frac{1}{2} \Delta^4 m_4^2 .
\]

(143)

This generalizes the result of [21] by including the world-volume fields of the D3-branes and allowing for $m_4^2 \neq 0$. Integrating the total derivative, a number of restrictions follow. The four-dimensional cosmological constant has to satisfy $m_4^2 \leq 0$, excluding de Sitter space within the chosen ansatz. In addition, $s_m = q_m = 1$ follows, so there are no anti-branes. For vanishing cosmological constant, i.e. $\dot{g}_{\mu \nu} = \eta_{\mu \nu}$, all terms on the right-hand side are negative semi-definite and therefore have to vanish individually. Thus,

\[
\alpha = -\Delta^{-2} , \quad |\hat{G}_3|_g^2 = \text{tr} |\mathcal{F}^{(0,2)}|^2 = 0.
\]

(144)

These conditions are equivalent to asking for a global minimum of the effective potential (110) obtained by neglecting the warp factor

\[
\mathcal{V}_{\text{eff}} = 0 \quad \iff \quad \left( \hat{G}_3 = 0 \text{ and } |\mathcal{F}|^2 = 0 \right) .
\]

(145)

Since this potential is non-negative, global minima are precisely the Minkowski vacua. These conclusions are clearly not valid for four-dimensional anti-de Sitter
space, which seems to demand a choice for \( \alpha \) such that \( \alpha / \sqrt{\hat{g}_4} \) is independent of the external coordinates.

Using the formulas from appendix A.2 and A.3 it is possible to show that the conditions (144) ensure that the full set of Einstein equations are satisfied if

\[
\hat{g}^{ij} \hat{\nabla}_i \partial_j \Delta^2 = -\frac{1}{2} e^\Phi |\hat{g}_{3}|_{\hat{g}}^2 - \frac{2 \kappa_{10} \mu_3}{\sqrt{\hat{g}_6}} \left( \sum_{\text{branes}} \left( s_m - \frac{i}{3} \text{tr}(A^i A^j A^k) H_{ijk} \right) \delta^6(x_m - \bar{x}_m) + Q_3 \sum_{k=1}^{64} \delta^6(x_k - \bar{x}_k) \right).
\]

To show this one has to use the fact that \( T_3 - \text{flux} = 0 \) for purely IASD (or ISD) \( \hat{G}_3 \), as was noticed in [75]. Imposing (144), the equation determining the warp factor (146) is actually equivalent to (142).

The other equations of motions are not too difficult to find. The equation for \( \Phi \) is

\[
\frac{\Delta^3}{\sqrt{-g}} \partial_I \left( g^{IJ} \sqrt{-g} \partial_J \Phi \right) = -\frac{1}{2} \left( e^\Phi |\hat{F}_{3}^{(0,3)}|_{\hat{g}}^2 - e^{-\Phi} |H_{3}^{(0,3)}|_{\hat{g}}^2 \right) + \frac{\kappa_{10} \mu_3}{\sqrt{g_6} \Delta^2} e^\Phi \sum_{\text{branes}} \text{tr} |\mathcal{F}|_{\hat{g}}^2 s_m \delta^6(x_m - \bar{x}_m).
\]

This equation imposes a vanishing of the dilaton tadpole. By four-dimensional Poincaré invariance the right-hand side of the equation has to integrate to zero. An IASD 3-form fulfills \( |\hat{F}_3|^2 = e^{-2\Phi} |H_3|^2 \) and therefore the YM potential must vanish, \( \text{tr} |\mathcal{F}|^2 = 0 \). Thus, the solutions of the constraint coming from the Einstein equations are compatible with the vanishing of the dilaton tadpole.

To summarize the logic: Asking for a Minkowski vacuum leads to (139) with \( m_4^2 = 0 \), combining with (141) implies (144) and is compatible with \( \hat{R}_{ij} = 0 \) at constant dilaton. We are then dealing with a warped product of a Calabi-Yau or torus and Minkowski space, where the warp factor is determined via (146), which is equivalent to (142). The situation described above is a variant of the known no-go theorems of [51, 52, 53, 54]. Several escape routes are at least in principle known: In an anti-de Sitter vacuum, a positive term would appear on the right-hand side of (139) that could balance the vacuum energy of the potential terms. If higher curvature corrections to the action were considered the semi-definiteness could also be spoiled and de Sitter solutions may exist as well. Perturbative [32] or non-perturbative [43] corrections to the effective potential also appear to modify the conclusions drawn above. Finally, one may also want to allow an explicitly time-dependent background [92].
4.3 Large volume scaling limit: separation of mass scales

The solution given by the conditions (144) is too simple to fix all the metric moduli; e.g., it always leaves the overall volume of the internal space as a free parameter. We will later come back to the possibility to fix also this by adding world-volume gauge fluxes on higher-dimensional branes. For the moment, one can observe that, like in [21], the relations that determine the solution are all invariant under constant rescaling of the metric. This is in accord with the form of the effective potential despite the fact that both terms in (110) scale differently under $g_{ij} \rightarrow t g_{ij}$. The reason is that for a Minkowski vacuum both terms have to vanish separately. This agrees with the results of [34, 47] where the no-scale structure of the gauged four-dimensional $\mathcal{N} = 4$ supergravity theory, that is the effective description of the present scenario, was demonstrated explicitly.

While the scale-independence of (144) is unfortunate in that it does not lead to a fixing of the volume modulus, it has been argued to allow for a limit of parametrically large volume where the warping becomes insignificant [21, 28]. In this limit, one can perform a standard dimensional reduction of the action to four dimensions. Then (110) really takes the role of an effective potential of a theory obtained by expanding around a given solution to the equations of motion. This is justified by inspection of (142) (or alternatively of (139)): The metric factors on the left-hand side of (142) scale like $t^{-1}$, on the right-hand side like $t^{-3}$. As $F_{ijk}$ and $H_{ijk}$ are just constants, the warp factor itself has to go as $1 + t^{-2}$ at large $t$. This argument is, of course, only valid away from the positions of the branes and planes. Ignoring the contributions from these regions one can then set $\Delta$ to a constant in the large $t$ limit and derive the effective potential (110) by dimensional reduction. In this situation, it appears challenging to find ways to stabilize the volume modulus in the effective theory that is only valid at very large volume. Given a correction to (110) that fixes the volume, one would need to make sure that this is done at large enough values that the effects of the warping are still negligible. In principle, it would of course be much nicer if one could do a reduction without using the scaling argument by explicitly including the warp factor, similar to [36].

The scaling argument can also be used for branes of higher dimensions. In order to neglect the warp factor at large $t$ the contributions of all relevant types of matter on the right-hand side of the Einstein equations have to fall off faster than $t^{-1}$. The tension of an arbitrary D$p$-brane (in Einstein-frame) leads to

$$S_{\mu\nu}^{Dp} = \frac{p - 7}{8} \mu_p e^{(p-3)\Phi/4} \sqrt{g_\perp} g_{\mu\nu} \delta^{9-p}(x_m - \bar{x}_m).$$

(148)

Here $g_\perp$ indicates the determinant of the metric restricted to the normal bundle of the brane. If the metric is constant this is the transverse volume. $S_{\mu\nu}$ then scales
like $t^{(p-9)/2}$ and $p < 7$ is required for the scaling argument to work.

The validity of the approach in addition requires a separation of the flux-induced masses and the masses of KK modes that have been neglected throughout. It has been argued in [9, 27] that the ratio of masses induced by the 3-form flux and the KK masses scales like

$$\left( m_{3\text{-flux}} : m_{\text{KK}} \right) = \left( R^{-3} : R^{-1} \right), \quad (149)$$

where $R$ denotes the dimensionless (average) radius of the background torus. Thus also from here we see that we need a large volume, this time to ensure a decoupling of the KK states. The same reasoning is still true in the case at hand since the additional second term in the potential (110) does not introduce any new mass terms for the geometric moduli. This is due to the fact that $F^{(0,2)}$ vanishes in the background, independently of the geometric moduli. Thus the masses only depend on the 3-form flux, as is also apparent from the mass formulas in [17]. This would be different if one considered higher-dimensional branes with internal world-volume fluxes, to which we come back in the next section. In that case one would expect an additional mass scale from the world-volume fluxes, similar to the situation in the heterotic string discussed in [9] and an intermediate scale appears, below the KK but still above the 3-form flux scale:

$$\left( m_{3\text{-flux}} : m_{2\text{-flux}} : m_{\text{KK}} \right) = \left( R^{-3} : R^{-2} : R^{-1} \right). \quad (150)$$

5 Generalization of the open string sector

In this final section we want to go beyond the well-defined framework of the model obtained from type I by T-duality, and allow higher-dimensional D-branes of even dimension and with internal world-volume gauge fluxes. This means we use

$$F_{ij}^a = f_{ij}^a + f^{abc} A^b A^c, \quad (151)$$

instead of (51). The constant flux parameters $f_{ij} = f_{ij}^a T^a$ take values in the Cartan subalgebra of the gauge group. They characterize a non-trivial gauge bundle on the world-volume. It is well known that the open string boundary conditions with constant fluxes change from Dirichlet to mixed Dirichlet-Neumann conditions. This implies that performing two T-dualities on a Dp-brane wrapping a two-dimensional torus with flux $f_{12} \neq 0$ turns it into another Dp-brane of identical dimension, but with the flux inverted. Therefore, introducing such gauge fluxes into the original type I string theory, assuming a factorization of the background into $T^6 = (T^2)^3$ for simplicity, could have taken us to a T-dual theory with D3, D5, D7 or D9-branes (depending on the number of internal directions with world-volume flux) wrapping 0, 2, 4 or all directions of the $T^6$ after the duality, and with non-trivial
gauge fluxes on their world-volume. At the same time, the O9-planes would still map to O3-planes. The D5, D7 and D9-brane charges then have to cancel among the branes, so that some of them need to carry negative charge as well and behave as anti-branes. This leads to the conclusion that there are no supersymmetric vacua with non-trivial world-volume flux in a flat background on a torus, which is in accord with the known fact that the toroidal type I compactification with world-volume gauge fluxes on D9-branes, that has been of some phenomenological interest recently, does not have a supersymmetric ground state [93]. To achieve a better behaved model one may consider orbifold compactifications which also have O7-planes in addition to the O3-planes. In this case supersymmetric configurations of D9-branes with world-volume fluxes do exist [94, 95, 96, 97] 30. Here we content ourselves with studying a few general properties of D9-brane flux vacua to obtain a rough impression of the effects of warping and perspectives for moduli stabilization.

In the presence of internal gauge fluxes, we have to use the DBI effective action including terms with higher powers of $F$, since in the presence of world-volume fluxes the higher powers in $F$ contribute to leading order in a derivative expansion of the effective action. Furthermore, some of the CS corrections in $\omega_7$ are no longer higher order and need to be incorporated into the action as in (97). On the other hand, the full DBI effective action can only reliably be used for an abelian gauge group. Therefore we now pass to a $U(1)^N$ gauge group by turning on Higgs fields. In this way we lose the possibility to produce chiral matter in various representations, which is one of the main motivations to be interested in this variant of the model [98, 99, 100]. However, as said above, here we are mainly interested in the prospects for moduli stabilization. To simplify the notation we also restrict ourselves to D9-branes, using

$$S_{\text{DBI}} = -\mu_9 \int_{R^4 \times M_6} d^{10}x \sqrt{-g_4} e^{3\Phi/2} \sqrt{\det(g_{ij} + F_{ij} e^{-\Phi/2})}$$

as effective world-volume action. Considering D5- or D7-branes should pose no further difficulties and would lead to similar conclusions.

### 5.1 Supersymmetry and $\kappa$-symmetry

In this section we discuss the conditions for preserving world-volume supersymmetry on D9-branes in the simultaneous presence of 3-form fluxes and O3-planes in the background. In their absence they are given by the calibration conditions

\footnote{This type of models, combined with 3-form fluxes, has been considered recently in [15, 16].}
found in [101].

The background bulk fields are not only required to satisfy the conditions (154) in order to solve the equations of motion, but we also impose $G^{3}\text{ASD}$ to be of type (2,1) and primitive \[75] \[87] to preserve supersymmetry. One then finds that in this background there is a Killing spinor that can be expressed as a covariantly constant spinor rescaled by a scalar function only \[75],

$$\epsilon = \Delta^{-1/4} \eta .$$

The background is 1/2 BPS, since $\eta$ also satisfies a chirality projection. In general, world-volume supersymmetry of D-branes in non-trivial backgrounds is preserved if the supervariations can be absorbed by $\kappa$-symmetry variations. This is translated into the projection

$$\Gamma \epsilon = \epsilon ,$$

where $\Gamma = \Gamma(P[g],F)$ is the $\kappa$-projector, an operator depending on the pull-back of the (warped) metric and the gauge field strength $F$, and $\epsilon$ is a Killing spinor of the background bulk theory. Explicitly, the $\kappa$-symmetry projector is defined by

$$\Gamma = e^{-a/2} (\Gamma_0 \ldots \Gamma_p \otimes i\sigma_2) e^{a/2}$$

with

$$a = \frac{1}{2} Y_{ij} E_i^A E_j^B \Gamma^{AB} \otimes \sigma_3 = \frac{1}{2} \Delta Y_{ij} \hat{E}_i^A \hat{E}_j^B \Gamma^{AB} \otimes \sigma_3 ,$$

$$Y_{ij} = \text{"arctan"}(F_{ij}) , \quad \hat{g}_{ij} = \delta_{AB} \hat{E}_i^A \hat{E}_j^B ,$$

where "arctan" is explained in [102], to which we refer the reader for further information on the notation and techniques. The details do not play a prominent role in our discussion here. The important point for us is the existence of a Killing spinor of the type (153) in a background of supersymmetric 3-form fluxes and O3-planes, a rescaled (covariantly) constant Killing spinor. Neglecting the backreaction of the D9-branes themselves, one can use this Killing spinor to derive analogous calibration conditions as found in [101] by just modifying the background metric via inclusion of the warp factor. A simple example with a flat background $\hat{g}_{ij} = \delta_{ij}$ has been studied in [103].

5.2 Calibrated branes

We are now ready to apply the results of [101] to find the supersymmetry conditions for D9-branes with world-volume gauge fluxes in a given background of O3-planes.
and 3-form fluxes. It is then required that the purely holomorphic components of $\mathcal{F}$ vanish and that
\[
e^{i\theta} \sqrt{\det(g_{ij} + \mathcal{F}_{ij})} \, dx_1 \wedge ... \wedge dx_6 = \frac{1}{3!} (J + i\mathcal{F})^3, \tag{157}
\]
\[
\cos(\theta) \left( \frac{1}{2!} J \wedge J \wedge \mathcal{F} - \frac{1}{3!} \mathcal{F} \wedge \mathcal{F} \wedge \mathcal{F} \right) = \sin(\theta) \left( \frac{1}{3!} J \wedge J \wedge J - \frac{1}{2!} \mathcal{F} \wedge \mathcal{F} \wedge J \right)
\]
hold for some phase $\theta$ on all the D9-branes. $J = \Delta \hat{J}$ now stands for a rescaling of the Kähler form $\hat{J}$ of the Calabi-Yau. For non-trivial $\Delta$ this $J$ is not closed. The background O3-planes (and any possible O7-planes) are calibrated with an angle $\theta = 3\pi/2$. If one neglects the warp factor, i.e. uses $\Delta = 1$, an overall world-volume supersymmetry can be preserved by adopting the same angle for all the D9-branes as well. It is important to note that fixing $\theta$, induces an implicit dependence of the Kähler moduli on $\mathcal{F}$. The scalar potential for the Kähler moduli in the Einstein frame reads
\[
V_{\text{DBI}} \sim \int_{\mathcal{M}_6} \sqrt{\det(g_{ij} + \mathcal{F}_{ij}e^{-\Phi/2})} = \int_{\mathcal{M}_6} \Re \left( \frac{e^{-i\theta}}{3!} (J + i\mathcal{F}e^{-\Phi/2})^3 \right). \tag{158}
\]
One further needs to impose the charge conservation constraint, which now includes the open string CS interactions $\mu_9 C_4 \wedge \text{ch}(\mathcal{F})$, such that (141) is modified to
\[
dF_5 = F_3 \wedge H_3 + 2\kappa_4^2 \mu_3 Q_3 \sum_{k=1}^{64} \pi_k + 2\kappa_4^2 \mu_9 \sum_{\text{branes}} \mathcal{F} \wedge \mathcal{F} \wedge \mathcal{F},
\]
which leads to the condition
\[
\frac{1}{3(4\pi^2\alpha')^3} \sum_{\text{branes}} \int_{\mathcal{M}_6} \mathcal{F} \wedge \mathcal{F} \wedge \mathcal{F} = 32 - N_{\text{flux}}. \tag{159}
\]
Similarly, the couplings of RR 8-form potentials in the open string CS action leads to a tadpole constraint involving $\mathcal{F}$, which states that the sums of the effective RR 7-brane charge has to be zero. As already said above, in order to fulfill (157) for all branes with the same angle $\theta$ one would have to depart from the flat torus example or to include O7-planes via an orbifold construction as in [45, 46]. Then (157) together with $G_3^{\text{ISD}} = 0$ ensures the vanishing of the total potential energy and (157) degenerates into
\[
J \wedge J \wedge J = 3J \wedge \mathcal{F} \wedge \mathcal{F} \tag{160}
\]
for any individual stack of branes. This constraint ensures the balancing of the 9-brane and 5-brane tensions, while 3- and 7-brane charges and tensions are unconstrained and only cancel upon adding the O-planes. It fixes the overall radius in an obvious manner. The natural scale for its value is however of the order of
\(\sqrt{\alpha'}\), with a constant of proportionality given by a combination of flux numbers. As one should maintain the splitting of scales \((150)\), it would be required to have hierarchically large flux quantum numbers, which by \((159)\) would demand some “integer fine tuning” to cancel the tadpoles. Furthermore, once the volume modulus gets fixed at a finite value, the warp factor is not trivial anymore. Writing \(J = \Delta \hat{J}\) for closed \(\hat{J}\) one sees that \((160)\) can not be fulfilled anymore. It would actually be equivalent to \(\hat{J}^3 = \Delta^{-2} \hat{J} \wedge \mathcal{F}^2\), where the left-hand side is closed and the right-hand side not.

This probably does not mean that the volume modulus cannot be stabilized in this way, since the qualitative stabilization mechanism due to the balancing of the 9-brane and 5-brane tensions should still take place. On the other hand, it seems that the non-trivial warp factor leads to a breaking of supersymmetry, once the volume is fixed. A similar result has been found in \([103]\). However, for a complete analysis one would also have to take into account the backreaction of the background towards the presence of the D9-branes and possible O7-planes. This would probably lead to a significant modification of the calibration condition \((157)\) but one should expect the above results on the stabilization of the volume to be qualitatively true also in the full story \([45, 46]\).

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A Technicalities

A.1 Conventions and notation

Tangent-frame indices are generally denoted $I, J, \ldots = 0, \ldots, 9$, which decompose into $i, j, \ldots = 4, \ldots, 9$ and $\mu, \nu, \ldots = 0, \ldots, 3$. Once we complexify coordinates, we use $m, n, \ldots$. We use the following standard conventions for differential forms: For $\Omega_p \in \bigwedge^p T^* M_{10}$, $M_{10} = \mathbb{R}^4 \times T^6$, write

$$\Omega_p = \frac{1}{p!} \Omega_{I_1 \ldots I_p} dx^{I_1} \wedge \cdots \wedge dx^{I_p},$$

$$|\Omega_p|^2 = \frac{1}{p!} \Omega_{I_1 \ldots I_p} \bar{\Omega}^{I_1 \ldots I_p},$$

(161)

where complex forms obviously refer to the complexified cotangent bundle, for $\Omega_p^{(n,p-n)} \in \bigwedge^n T^* \mathbb{R}^4 \times \bigwedge^{p-n} T^* T^6$ analogously

$$\Omega_p^{(n,p-n)} = \frac{1}{n!(p-n)!} \Omega_{\mu_1 \ldots \mu_{n+1} \ldots i_{p-n}} dx^{\mu_1} \wedge \cdots \wedge dx^{\mu_n} \wedge dx^{i_1} \wedge \cdots \wedge dx^{i_{p-n}},$$

$$|\Omega_p^{(n,p-n)}|^2 = \frac{1}{n!(p-n)!} \Omega_{\mu_1 \ldots \mu_{n+1} \ldots i_{p-n}} \bar{\Omega}^{\mu_1 \ldots \mu_{n+1} \ldots i_{p-n}}.$$  

(162)

Ten-dimensional Hodge duality is defined by

$$\ast \Omega_p = \frac{1}{p!(10-p)!} \epsilon^{I_1 \ldots I_p I_{p+1} \ldots I_{10}} \Omega_{I_1 \ldots I_p} dx^{I_{p+1}} \wedge \cdots \wedge dx^{I_{10}},$$

(163)

and the six-dimensional Hodge star is given by

$$(\ast \Omega_p)^{(n,6-p+n)}_{\mu_1 \ldots \mu_{n+1} \ldots j_{6-p+n}} = \frac{1}{(p-n)!} \epsilon^{i_1 \ldots i_{p-n} j_{1+} \ldots j_{6-p+n}} \Omega_{\mu_1 \ldots \mu_{n+1} \ldots i_{p-n}}.$$  

(164)

We also define derivative operators

$$d_{[1,0]} \Omega_p^{(n,p-n)} = (d \Omega_p^{(n,p-n)})^{(n+1,p-n)},$$

$$d_{[0,1]} \Omega_p^{(n,p-n)} = (d \Omega_p^{(n,p-n)})^{(n,p+1-n)}.$$  

(165)

For the totally antisymmetric tensor $\epsilon$ we use the convention

$$\epsilon_{1 \ldots D} = \pm \sqrt{g_D}, \quad \epsilon^{1 \ldots D} = \frac{1}{\sqrt{g_D}},$$

(166)

with $g_D$ being the (absolute value of the) determinant of the metric and the sign depends on the signature of the metric.
A.2 Einstein equations with warp factors

In this appendix we give the results for the Ricci tensor of a general warped metric ansatz (133). With that ansatz the Christoffel symbols are found

\[ \Gamma_{\mu \nu \rho} = \hat{\Gamma}_{\mu \nu \rho} , \quad \Gamma_{i j}^i = \Gamma_{j i}^i = 0 , \]

\[ \Gamma_{\mu i}^i = -\frac{1}{2} \delta_{\mu}^i \partial_i \ln(\Delta) , \quad \Gamma_{\mu \nu}^i = \frac{1}{2} \hat{g}_{\mu \nu} \Delta^{-1-b} \hat{g}_{ij} \partial_j \ln(\Delta) , \]

\[ \Gamma_{i j}^k = \hat{\Gamma}_{i j}^k + \frac{b}{2} (\delta_j^i \partial_k \ln(\Delta) + \delta_i^j \partial_k \ln(\Delta) - \hat{g}_{jk} \hat{g}_{il} \partial_l \ln(\Delta)) . \] (167)

This leads to

\[ R_{\mu \nu} = \hat{R}_{\mu \nu} + \frac{1}{2} \hat{g}_{\mu \nu} \Delta^{-1-b} \left( \hat{g}^{ij} \hat{\nabla}_i \partial_j \ln(\Delta) - 2(1 - b) \hat{g}^{ij} \partial_i \ln(\Delta) \partial_j \ln(\Delta) \right) \]

\[ = \hat{R}_{\mu \nu} + \frac{1}{2} \hat{g}_{\mu \nu} \Delta^{-3b} \hat{g}^{ij} \hat{\nabla}_i \left( \Delta^{2(b-1)} \partial_j \ln(\Delta) \right) , \]

\[ R_{ij} = \hat{R}_{ij} + 2(1 - b) \hat{\nabla}_i \partial_j \ln(\Delta) + (b^2 - 2b - 1) (\partial_i \ln(\Delta)) (\partial_j \ln(\Delta)) + b(1 - b) \hat{g}_{ij} \hat{g}^{kl} \partial_k \ln(\Delta) \partial_l \ln(\Delta) - \frac{b}{2} \hat{g}_{ij} \hat{g}^{kl} \hat{\nabla}_k \partial_l \ln(\Delta) , \] (168)

where \( \hat{\nabla}_i \) involves the Christoffel symbols \( \hat{\Gamma}_{i j}^k \). We define the energy momentum tensor for some action \( S[g] \) by

\[ T_{IJ} = -\frac{2}{\sqrt{-g}} \delta S \delta g_{IJ} , \] (169)

\( T \) denoting its trace \( T_{IJ} g^{IJ} \), and the Einstein equations are then written

\[ \frac{1}{\kappa_{10}^2} R_{IJ} = S_{IJ} , \quad S_{IJ} = T_{IJ} - \frac{1}{8} g_{IJ} T . \] (170)

In a flat Minkowski vacuum \( (\hat{R}_{\mu \nu} = 0) \) the trace of the space-time components then always takes the form

\[ \frac{1}{\kappa_{10}^2} \hat{g}^{ij} \hat{\nabla}_i \left( \Delta^{2(b-1)} \partial_j \ln(\Delta) \right) = \frac{1}{4} \Delta^{3b-2} \left( T_{\mu \nu} g^{\mu \nu} - T_{ij} g^{ij} \right) . \] (171)

From this follows the famous constraint [51] that the right-hand side, with the given warped ansatz and without higher derivative terms in the action, has to integrate to zero. Thus, whenever it is positive or negative definite, it has to vanish, which then implies the absence of fluxes and warp factors.
A.3 Energy momentum tensors

In this section we give the explicit forms of the energy momentum tensors of the various background contributions referring to the ten-dimensional action in the Einstein-frame. The only Poincaré invariant background form fields we turn on are the internal components of the 3-form, i.e. the fluxes \( \hat{G}_3^{(0,3)} \), and the components \( F_5^{[4,1]} \) and \( F_5^{[0,5]} \) (i.e. we do not consider any background for the RR axion that would be needed in discussing D7-brane solutions, cf. [90]). The 5-form ansatz has to be self-dual and was given in (132). It satisfies \( F_5 = (1 + \ast)\alpha \wedge dx^0 \wedge ... \wedge dx^5 \), such that we have in particular \( |F_5|^2 = 0 \). For these fluxes one finds the following contributions to the energy momentum tensor,

\[
T_{\mu\nu}^{3\text{-flux}} = - \frac{\Delta^{-3b}}{4\kappa_4^2} e^\Phi g_{\mu\nu} |\hat{G}_3^{(0,3)}|_g^2, \\
T_{ij}^{3\text{-flux}} = - \frac{1}{4\kappa_4^2} e^\Phi \left( g_{ij} \Delta^{-3b}|\hat{G}_3^{(0,3)}|^2_\hat{g} - (\hat{G}_3^{(0,3)})(\hat{G}_3^{(0,3)})_{ij}mn \hat{g}^{km} \hat{g}^{ln} \right), \\
T_{\mu\nu}^{5\text{-flux}} = \frac{1}{4\kappa_4^2} \left( \frac{1}{3!} (F_5)_{\mu\rho\sigma\tau\nu}(F_5)_{\nu\alpha\beta\gamma\jmath} g^{\rho\alpha} g^{\sigma\beta} g^{\tau\gamma} g^{ij} \right), \\
T_{ij}^{5\text{-flux}} = \frac{1}{4\kappa_4^2} \left( \frac{1}{4!} (F_5)_{iKLMN}(F_5)_{jOPQR} g^{KO} g^{LP} g^{MQ} g^{NR} \right), \\
T_{ij}^{5\text{-flux}} = \frac{1}{2\kappa_4^2} g_{ij}(\partial_k \alpha)(\partial_\jmath \alpha)g^{kl} - (\partial_k \alpha)(\partial_\jmath \alpha). 
\]

Note \( T_{ij}^{3\text{-flux}} g^{ij} = T_{iJ}^{5\text{-flux}} g^{IJ} = 0 \). If one specializes to \( b = 1, \alpha = -\Delta^{-2} \) and \( \hat{g}_{\mu\nu} = \eta_{\mu\nu} \) one can rewrite (168) into (173).

\[
R_{\mu\nu} - \kappa_4^2 T_{\mu\nu}^{5\text{-flux}} = \frac{1}{4} \eta_{\mu\nu} \Delta^{-4} \hat{g}^{ij} \hat{\nabla}_i \hat{\nabla}_j (\Delta^2), \\
R_{ij} - \kappa_4^2 T_{ij}^{5\text{-flux}} = \hat{R}_{ij} - \frac{1}{4} \hat{g}_{ij} \Delta^{-2} \hat{g}^{kl} \hat{\nabla}_k \hat{\nabla}_l (\Delta^2). 
\]

This is the starting point for the self-dual 3-brane solution. In the background the only allowed components of the open string gauge fields are \( F^{(0,2)} \). From (134) we then get for the open string sector

\[
T_{ij}^{op} = T_{ij}^{ten} + T_{ij}^{YM}, \\
T_{\mu\nu}^{ten} = - \frac{\mu_3}{\sqrt{g_6}} g_{\mu\nu} \sum_{\text{branes}} \left( s_m - \frac{i}{3} \text{tr}(A^i A^j A^k) H_{ijk} \right) \delta^6(x_m - \bar{x}_m), \\
T_{\mu\nu}^{YM} = - \frac{\mu_3 \Delta^{-2b}}{2\sqrt{g_6}} g_{\mu\nu} e^\Phi \sum_{\text{branes}} \text{tr}[F^{(0,2)}]_g^2 s_m \delta^6(x_m - \bar{x}_m), \\
T_{ij}^{YM} = \frac{\mu_3}{\sqrt{g_6}} e^\Phi g^{kl} \sum_{\text{branes}} \text{tr}(F_{ik} F_{jl}) s_m \delta^6(x_m - \bar{x}_m), 
\]

(174)
where we added the correction from the non-abelian DBI action to the tension. For orientifold planes we write

\[ T^{O3}_{\mu\nu} = \frac{-\mu_3 Q_3}{\sqrt{g_6}} g_{\mu\nu} \sum_{k=1}^{64} \delta^6(x_k - \bar{x}_k), \quad T^{O3}_{ij} = 0. \tag{175} \]

One then also has \( T^{YM}_{IJ} g^{IJ} = 0 \).

## A.4 T-duality of RR forms

In this section we specialize the rules for dualizing RR forms given in [77] to the case relevant in this paper. The essential step is to replace a RR \( p \)-form \( \Omega_p \) with a state

\[ \Omega^{q,p-q} \rightarrow \frac{1}{(p-q)!} \left( \frac{1}{q!} \Omega^{q,p-q} \delta_{\mu_1...\mu_{p-q}} dx^{\mu_1} \wedge ... \wedge dx^{\mu_{p-q}} \right) \psi^{i_1}\dagger ... \psi^{i_{p-q}}\dagger |0\rangle, \tag{176} \]

where \( \psi^{i\dagger} \) and \( \psi_i \) are raising and lowering operators

\[ \{ \psi^{i\dagger}, \psi_j \} = \delta^i_j, \quad \{ \psi_i, \psi_j \} = 0 = \{ \psi^{i\dagger}, \psi^{j\dagger} \}, \tag{177} \]

acting on a vacuum with \( \psi_i|0\rangle = 0 \) and \( \langle 0|0 \rangle = 1 \). The elements of the duality group, \( \Lambda \in O(6,6) \), then act on the space of states via operators \( \Lambda \), whose action on \( (\psi^{i\dagger}, \psi_j)_{i,j=1,...,6} \) is given by

\[ \Lambda = \begin{pmatrix} A & B \\ C & D \end{pmatrix}: (\Lambda \psi^{i\dagger} \Lambda^{-1}, \Lambda \psi \Lambda^{-1}) = (\psi^{i\dagger}, \psi) \begin{pmatrix} A & B \\ C & D \end{pmatrix}. \tag{178} \]

At the same time, the internal components of the metric (in our case of vanishing internal NSNS \( B \)-field) transform according to

\[ G \mapsto (AG + B)(CG + D)^{-1}. \tag{179} \]

We are interested in just one particular element that corresponds to inverting the radii of all \( d = 6 \) circles. This is in fact not a completely well specified operation, since the signs that can appear in the mapping of the RR forms depend on the order in which the circles are dualized. In view of (179) we choose the element \( \Lambda \) with \( A = D = 0 \) and \( B = C = 1_6 \), where \( 1_6 \) denotes the six-dimensional unity matrix. This element is given by\(^{31}\)

\[ \Lambda = \Lambda_1^{-1} ... \Lambda_6^{-1}, \quad \Lambda_i^{-1} = \psi^{i\dagger} - \psi_i, \tag{180} \]

which satisfies \( \Lambda^2 = -1 \) and

\[ \Lambda \psi_i \Lambda^{-1} = \psi^{i\dagger}, \quad \Lambda \psi^{i\dagger} \Lambda^{-1} = \psi_i. \tag{181} \]

\(^{31}\)In terms of the notation used in [77] this corresponds to \( \Lambda = C^- \).
The action of $\Lambda$ on a given RR form (176) can then be obtained from

$$
\Lambda \psi^i \ldots \psi^n |0\rangle = \psi^i \ldots \psi^n \Lambda |0\rangle = \frac{(-1)^{n(n-1)/2}}{(6-n)!} \hat{\epsilon}_{i_1 \ldots i_n} \psi^{i_{n+1}} \ldots \psi^{i_6} |0\rangle , (182)
$$

where $\hat{\epsilon}$ has been introduced below (19).

B Kinetic and CS terms for the scalars $(C_4)_{ijkl}$

The kinetic terms for the scalars $(C_4)_{ijkl}$ given in (45) (however without 3-form flux) and the Chern-Simons terms of (39) are also derivable directly by a reduction of the truncated type IIB action. In this section we calculate them in a similar fashion as is done e.g. in [29, 81]. We start with the pseudoaction of type IIB string theory without imposing the self-duality of the 5-form field strength, reduce it to four dimensions and afterwards impose the self-duality by adding a suitable Lagrange multiplier. The relevant part of the ten-dimensional action is

$$
S = \frac{1}{8 \kappa_{10}^2} \int F_5 \wedge \ast F_5 - \frac{1}{4 \kappa_{10}^2} \int C_4 \wedge dB_2 \wedge dC_2 + \cdots . (183)
$$

The field strength $F_5$ is defined as in (39), i.e. after the truncation to the T-dual type I$'$ theory

$$
F_5 = (dC_4)^{(1,4)} + (dC_4)^{(3,2)} - \frac{1}{2} B_2^{(1,1)} \wedge (dC_2)^{(2,1)} + \frac{1}{2} C_2^{(1,1)} \wedge (dB_2)^{(2,1)} . (184)
$$

This corresponds to expanding $C_4$ as

$$
C_4 = C_4^{(2,2)} + C_4^{(0,4)} . (185)
$$

The self-duality condition for $F_5$ gives

$$
(dC_4)^{(1,4)} = \ast \left( (dC_4)^{(3,2)} - \frac{1}{2} B_2^{(1,1)} \wedge (dC_2)^{(2,1)} + \frac{1}{2} C_2^{(1,1)} \wedge (dB_2)^{(2,1)} \right) . (186)
$$

Plugging (184) and (185) into (183) we obtain

$$
S = \frac{1}{8 \kappa_{10}^2} \int \left[ (dC_4)^{(3,2)} - \frac{1}{2} B_2^{(1,1)} \wedge (dC_2)^{(2,1)} + \frac{1}{2} C_2^{(1,1)} \wedge (dB_2)^{(2,1)} \right] \wedge \ast \left[ (dC_4)^{(3,2)} - \frac{1}{2} B_2^{(1,1)} \wedge (dC_2)^{(2,1)} + \frac{1}{2} C_2^{(1,1)} \wedge (dB_2)^{(2,1)} \right] + (dC_4)^{(1,4)} \wedge \ast (dC_4)^{(1,4)}
$$

$$
- \frac{1}{4 \kappa_{10}^2} \int C_4^{(0,4)} \wedge (dB_2)^{(2,1)} \wedge (dC_2)^{(2,1)} . (187)
$$

50
In order to implement the self-duality condition (186) we add the Lagrange multiplier $\tilde{C}_4^{0,4}$
\[
\delta S = -\frac{1}{4\kappa_{10}^2} \int \left( F_5^{[3,2]} + \frac{1}{2} B_2^{[1,1]} \wedge (dC_2)^{[2,1]} - \frac{1}{2} C_2^{[1,1]} \wedge (dB_2)^{[2,1]} \right) \wedge (d\tilde{C}_4)^{[1,4]} .
\]
(188)

The equation of motion for $C_4^{0,4}$ implies
\[
F_5^{[3,2]} = (dC_4)^{[3,2]} - \frac{1}{2} B_2^{[1,1]} \wedge (dC_2)^{[2,1]} + \frac{1}{2} C_2^{[1,1]} \wedge (dB_2)^{[2,1]} .
\]
(189)

On the other hand, the equation of motion for $F_5^{[3,2]}$ in combination with the self-duality condition (186) leads to the identification of $(dC_4)^{[1,4]}$ and $(d\tilde{C}_4)^{[1,4]}$
\[
(d\tilde{C}_4)^{[1,4]} = (dC_4)^{[1,4]} .
\]
(190)

Using this in $S + \delta S$ gives (after a partial integration)
\[
S = \frac{1}{4\kappa_{10}^2} \int (dC_4)^{[1,4]} \wedge * (dC_4)^{[1,4]} + \frac{1}{2\kappa_{10}^2} \int (dC_2)^{[2,1]} \wedge (dC_2)^{[2,1]} \wedge C_4^{0,4} ,
\]
(191)

which obviously coincides with (39) and the kinetic term for $C_4^{0,4}$ in (155) in the absence of fluxes.

Let us also mention here that it is not straightforward to modify this procedure to include also the 3-form fluxes. If one just naively plugs the ansatz with 3-form fluxes into the action (183), one has to replace
\[
(dC_4)^{[1,4]} \rightarrow (dC_4)^{[1,4]} - \frac{1}{2} B_2^{[1,1]} \wedge (dC_2)^{[0,3]} + \frac{1}{2} C_2^{[1,1]} \wedge (dB_2)^{[0,3]}
\]
(192)
in (183), (186) and (187). In addition one gets a further Chern-Simons term
\[
-\frac{1}{4\kappa_{10}^2} \int C_4^{[2,2]} \wedge \left( (dB_2)^{[2,1]} \wedge (dC_2)^{[0,3]} - (dC_2)^{[2,1]} \wedge (dB_2)^{[0,3]} \right) .
\]
(193)

One immediately runs into trouble now when one considers the equation of motion for $C^{[2,2]}$. It is given by
\[
d^{[1,0]} * F_5^{[3,2]} = -(dB_2)^{[2,1]} \wedge (dC_2)^{[0,3]} + (dC_2)^{[2,1]} \wedge (dB_2)^{[0,3]} ,
\]
(194)

which, however, is not consistent with the self-duality condition (186) after performing the substitution (192). This would require a factor $1/2$ on the right side of (194), as
\[
d^{[1,0]} F_5^{[1,4]} = -\frac{1}{2} (dB_2)^{[2,1]} \wedge (dC_2)^{[0,3]} + \frac{1}{2} (dC_2)^{[2,1]} \wedge (dB_2)^{[0,3]} .
\]
(195)

This formula is consistent with the $\{2,4\}$-component of the Bianchi identity
\[
(dF_5)^{[2,4]} = -(dB_2)^{[2,1]} \wedge (dC_2)^{[0,3]} + (dC_2)^{[2,1]} \wedge (dB_2)^{[0,3]} ,
\]
(196)
as the latter actually gets two contributions. Only one is from $d^{[1,0]} F_5^{[1,4]}$. The other one comes from $d^{[0,1]} F_5^{[2,3]}$ and is identical to the first one.
C The self-dual type I action

Due to the fact that the conventional formulation of the CS action of D-branes involves RR potentials of all (even) degrees (in type IIB) \[63\], it might seem more appropriate to start with a formulation of type I supergravity that is democratic in the appearance of the RR fields \(C_2\) and \(C_6\). Its closed string part could be obtained by quotienting world-sheet parity out of the democratic version of type IIB, whose RR sector was given in \([15]\), and which is also much better adapted to make the invariance of the type IIB action under T-duality explicit. We did not follow this approach because the introduction of the superfluous degrees of freedom is only needed in a discussion of the correct treatment of the CS-terms appearing in the D3-brane action, cf. section 2.4. For the derivation of the T-dual action performed in chapter 2 it is actually not necessary to work with this democratic formulation. Nevertheless, we give it here for completeness and because of its relevance for the discussion in section 2.4.

The self-dual type I action which contains \(C_2\) and \(C_6\) democratically is given by

\[
S_{\text{self-dual}} = \frac{1}{2 \kappa_{10}^2} \int d^{10} x \sqrt{-g} \left( e^{-2\Phi} \left( R + 4 \partial_\mu \Phi \partial^\mu \Phi \right) - \frac{1}{4} |\tilde{F}_3|^2 - \frac{1}{4} |\tilde{F}_7|^2 \right) - \frac{1}{2 g_{10}^2} \int d^{10} x \sqrt{-g} e^{-\Phi} \text{tr} |\mathcal{F}|^2
\]

\[\] \[\] \[\text{(197)}\]

\[
- \frac{1}{4 g_{10}^2} \int dC_2 \wedge \omega_7 - \frac{1}{4 g_{10}^2} \int dC_6 \wedge \omega_3 + \frac{\kappa_{10}^2}{4 g_{10}^2} \int \omega_7 \wedge \omega_3 .
\]

We have included terms \(dC_p \wedge \omega_{9-p}\) stemming from the Chern-Simons action \(C_p \wedge \text{ch}(\mathcal{F})\) of the 9-branes. In analogy to \(\tilde{F}_3\) we then define

\[
\tilde{F}_7 = dC_6 - \frac{\kappa_{10}^2}{g_{10}^2} \omega_7 .
\]

The normalization of the D9-brane CS terms is exactly one half of the ordinary. In addition to this action the duality condition

\[
* \tilde{F}_3 = -\tilde{F}_7
\]

has to be imposed after deriving the equations of motion. This procedure is justified primarily by checking that the equations of motion and Bianchi identities that result from the self-dual action become identical to the standard equations derived from \([2]\) plus the D9-brane interaction \(dC_2 \wedge \omega_7\), after imposing the constraint \([199]\). This requirement lead us to introduce the slightly exotic interaction \(\omega_7 \wedge \omega_3\).

\[32\]Actually, as already mentioned in the main text, in the presence of the \(\omega_7\)-terms it is not consistent to just keep the quadratic term \(|\mathcal{F}|^2\) in an expansion of the DBI action. It is understood that further terms would have to be added in a rigorous treatment.

\[33\]Such terms are, however, familiar from anomaly cancellation, see e.g. formula (A.14) in \([104]\).
Let us also check that the action (197) really reproduces the old action (2) plus D-brane Chern-Simons term after the duality constraint (199) has been eliminated. Thus we add the Lagrange multiplier term

$$\delta S = -\frac{1}{4\kappa^2_{10}} \int d\tilde{C}_2 \wedge dC_6 = -\frac{1}{4\kappa^2_{10}} \int d\tilde{C}_2 \wedge (\tilde{F}_7 + \frac{\kappa^2_{10}}{g^2_{10}} \omega_7) .$$

(200)

The equation of motion for $\tilde{F}_7$ is just

$$\ast \tilde{F}_7 = -\tilde{F}_3 = -d\tilde{C}_2 + \frac{\kappa^2_{10}}{g^2_{10}} \omega_3 ,$$

(201)
i.e. $\tilde{F}_3$ has the same dependence on $\tilde{C}_2$ as $\tilde{F}_3$ has on $C_2$. Comparing with (199) and using $\ast \ast = 1$ for a form of odd degree in an even-dimensional space-time of Lorentzian signature, we see that

$$\tilde{F}_3 = \tilde{F}_3 .$$

(202)

Using this and (201) to eliminate $\tilde{F}_7$ leads to the original action

$$S'_1 = \frac{1}{2\kappa^2_{10}} \int d^{10}x \sqrt{-g} \left( e^{-2\Phi} (R + 4\partial_\mu \Phi \partial^\mu \Phi) - \frac{1}{2} |\tilde{F}_3|^2 \right)$$

$$- \frac{1}{2g^2_{10}} \int d^{10}x \sqrt{-g} e^{-\Phi} \mathrm{tr} |\mathcal{F}|^2 - \frac{1}{2g^2_{10}} \int dC_2 \wedge \omega_7$$

(203)

involving only the RR 2-form $C_2$ including the properly normalized Chern-Simons action of a D9-brane.

On the other hand, one may want to integrate out $C_2$ in place of $C_6$. Instead of (200) we add

$$\delta S = -\frac{1}{4\kappa^2_{10}} \int dC_2 \wedge d\tilde{C}_6 = -\frac{1}{4\kappa^2_{10}} \int (\tilde{F}_3 + \frac{\kappa^2_{10}}{g^2_{10}} \omega_3) \wedge d\tilde{C}_6 .$$

(204)

The equation of motion of $\tilde{F}_3$ is given by

$$\ast \tilde{F}_3 = d\tilde{C}_6 + \frac{\kappa^2_{10}}{g^2_{10}} \omega_7 .$$

(205)

Comparing this with (199) now leads to the relation

$$d\tilde{C}_6 = -dC_6 .$$

(206)

With the help of (206) and (205) we can eliminate $\tilde{F}_3$ and get

$$S''_1 = \frac{1}{2\kappa^2_{10}} \int d^{10}x \sqrt{-g} \left( e^{-2\Phi} (R + 4\partial_\mu \Phi \partial^\mu \Phi) - \frac{1}{2} |\tilde{F}_7|^2 \right)$$

$$- \frac{1}{2g^2_{10}} \int d^{10}x \sqrt{-g} e^{-\Phi} \mathrm{tr} |\mathcal{F}|^2 - \frac{1}{2g^2_{10}} \int \tilde{F}_7 \wedge \omega_3$$

(207)
In particular there is now an extra Chern-Simons term of the form

$$\frac{1}{2g_{10}^2} \int \omega_3 \wedge \omega_7 .$$

(208)
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