A DSA-like digital signature protocol

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Abstract

In this paper we propose a new digital signature protocol inspired by the DSA algorithm. The security and the complexity are analyzed. Our method constitutes an alternative if the classical scheme DSA is broken.

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1. Introduction

Moderne data protection started with the work of Shannon[16] in 1949 on information theory. However modern public key cryptography appeared clearly when, in 1976 Diffie and Hellman[5] showed how any two network users can construct a common secret key even if they never meet. One year and few months later the RSA[14] method was published. It is considered as the most used cryptosystem in the daily life.

Digital signature is an important tool in cryptography. Its role in funds transfert, online business, electronic emails, users identification or documents integrity is essential. Let us recall its mechanism principle. A trusted authority prepares the keys for Alice who is a network user. There is a secret key \( k \) and a public key \( K \) depending on her identity parameters.

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If she wants to sign a document $D$, she must solve a hard problem $Pb(K, D)$ which is function of $K$ and $D$. She is able to find a solution as she possesses a supplementary information: her private key $k$. For anybody else, the problem, consisting generally of a difficult equation of a high mathematical level and based on the elements $K$ and $D$, is intractable even with the help of computers. No one can forge Alice personal digital signature on the document $D$. On the other hand, to validate and accept the signature, anyone and some time it is the judge, can verify if the answer furnished by Alice is correct or not.

One of the first concrete digital signature protocol was proposed by ElGamal[6] in 1985. It is based on the discrete logarithm problem computationally considered as intractable[7, p. 103] [17, p. 236] [1]. Provided that the signature system parameters are properly selected, the algorithm security of the scheme has never been threatened.

Many variants of this scheme have been created. In 1991, Schnorr [15] proposed a similar signature protocol inspired by ElGamal system. The digital signature algorithm or DSA[18], also deduced from the ElGamal signature equation, was definitively formulated by the National Institute of Standards and Technology (NIST) in 1994[9]. Some variants[4] of the DSA were published and several attacks were elaborated against it. In 1997 Bellare et al.[3] presented an attack where they showed that if the DSA algorithm uses linear congruential pseudorandom number generator then it is possible to recover the signer secret key. In 2002 Nguyen and Shparlinski[10] published a polynomial time algorithm that can totally break the DSA protocol if they get few bits from the nonces used to sign a certain number of documents. More recently Poulakis[13] used lattices theory to construct a system of linear equation that leads to the disclosure of the signer secret key. At last, in 2017, Angel et al.[2] with extensive experiments, elaborated a method where they exploit blocks in the ephemeral nonce and find the signer private key.

In this paper we propose a new digital signature scheme inspired by the DSA algorithm. The security and the complexity are analyzed. Our method constitutes an alternative if the classical protocol DSA is broken. The drawback of our algorithm is that the generation of the signature has three parameters instead of two for DSA and we have to execute one more modular exponentiation in the verification step. For a theoretical and a pedagogical interest, we present an extension of the method.

The paper is organized as follows: In the next section we briefly recall the description of the classical DSA digital signature. In section 3 we present our contribution and we conclude in section 4.
Classical notations will be adopted. So \( \mathbb{N} \) is the set of all natural integers. When \( a, b, n \in \mathbb{N} \), we write \( a = b \mod n \) if \( a \) is the remainder of the division of the integer \( b \) by \( n \), and \( a \equiv b \mod n \) if the number \( n \) divides the difference \( a - b \). If \( p \) is a prime integer then the set \( \mathbb{Z}_p = \left\{ \frac{\mathbb{Z}}{p\mathbb{Z}} \right\} \) is the multiplicative group of modular integers \( \{1, 2, \ldots, p-1\} \).

We begin by recalling the DSA signature method.

2. The standard DSA protocol[9, 18]

In this section, we describe the basic DSA scheme followed by the security analysis of the method.

2.1 Keys production

The signer selects two primes \( p \) and \( q \) such that \( q \) divides \( p - 1 \), \( 2^{t-1} < q < 2^t \) with: \( t \in \{160, 256, 384, 512\} \), \( 2^{t-1} < p < 2^t \), \( 768 < L < 1024 \) and \( L \) is a multiple of 64.

Then, he chooses a primitive root \( g \mod p \) and computes \( \alpha = g^{p-1} \mod p \). The signer selects also an integer \( x \) such that \( 1 \leq x \leq q-1 \) and calculates \( y = \alpha^x \mod p \). Finally, he publishes \((p, q, g, y)\) and keeps the parameter \( x \) secret as its private key.

2.2 The signature generation

Let \( h \) be a secure hash function[7, p. 33] that produces a 160-bit output.

To sign a message \( m \), the signer starts by selecting a random secret integer \( k \) smaller than \( q \) and called the nonce[7, p. 397]. Then, he computes successively \( r = (\alpha^k \mod p) \mod q \) and \( s = \frac{h(m) + a \cdot r}{k} \mod q \). Finally, the signature is the pair \((r, s)\).

2.3 The signature verification

The verifier of the signature calculates: \( u_1 = \frac{h(m)}{s} \mod q \) and \( u_2 = \frac{r}{s} \mod q \). Then, he computes: \( v = ((\alpha^{u_1} y^{u_2}) \mod p) \mod q \).

Depending on \( v = r \) or not, he accepts or rejects the signature.

2.4 Security of the method

To date the DSA system is considered as a secure digital signature scheme. Indeed to break it, attackers must first solve a famous hard
mathematical question: the discrete logarithm problem (DLP)[7, p. 103][17, p. 267]. Conversely, we ignore whether or not breaking the DSA scheme leads to an algorithm for solving the DLP. It’s a remarkable open problem. The introduction of the second large prime \( q \) in the DSA mechanism avoids Pohlig and Hellman attack[11] on the discrete logarithm problem.

Several attacks were mounted against DSA protocol. The reader is invited to see for instance references[2, 13, 10, 3]. It’s well known[8, p. 188] that, as for ElGamal signature scheme[6], using the same nonce \( k \) to sign two different documents reveals the system secret key. Therefore it’s mandatory to change the value of \( k \) at each signature.

The following section describes our main contribution.

3. A DSA-like digital signature protocol

In this section, we present a new DSA-like digital signature and we analyze its security and complexity.

3.1 Keys production

The fabrication of the keys is the same as for the DSA protocol. The signer selects two primes \( p \) and \( q \) such that \( q \) divides \( p - 1 \), \( 2^{t-1} < q < 2^t \) with: \( t \in \{160, 256, 384, 512\} \), \( 2^{L-1} < p < 2^{L} \), \( 768 < L < 1024 \) and \( L \) is a multiple of 64.

Then, he chooses a primitive root \( g \mod p \) and computes \( \alpha = g^{r^{-1}} \mod p \). The signer selects also an integer \( x \) such that \( 1 \leq x \leq q - 1 \) and calculates \( y = \alpha^x \mod p \). Finally, he publishes \( (p, q, g, y) \) and keeps the parameter \( x \) secret as its private key.

3.2 The signature generation

Let \( h \) be a collision resistant hush function[7, p. 323] such that the image of any message is belonging to the set \( \{1, 2, \ldots, q\} \).

To sign the document \( m \), Alice must solve the modular signature equation:

\[
\alpha^{h(m)} \equiv y^r s^t \mod p \equiv s \mod q
\]

where the unknown parameters \( r, s, t \) verify \( 0 < r < p \) et \( 0 < s, t, q \).

**Theorem 1**: Alice, with her secret key \( x \), is able to find a triplet \( (r, s, t) \) that verifies the signature equation (1).
Proof : Alice chooses two random numbers $k, l < q$ then computes $r = \alpha^k \mod p$ and $s = \alpha^l \mod p \mod q$. To get a solution of equation (1) it suffices to have $\frac{h(m)}{t} \equiv r^2 \mod q$. On the other hand the third parameter $t$ must verify $\frac{h(m) + xr + ks}{t} = \lambda[q]$ or

$$t = \frac{h(m) + xr + ks}{l} \mod q$$

(2)

3.3 The signature verification

To verify Alice signature $(r, s, t)$, Bob should do the following:

1. He first finds Alice public key $(p, q, \alpha, y)$.
2. He verifies that $0 < r < p$ and $0 < s, t < q$. If not he rejects the signature.
3. He computes $u_1 = \frac{h(m)}{t} \mod q$, $u_2 = \frac{r \mod q}{t} \mod q$ and $u_3 = \frac{s}{t} \mod q$.
4. He determines $v = ((\alpha^{u_1}y^{u_2}r^{u_3}) \mod p) \mod q$.
5. If $v = s$, Bob accepts the signature, otherwise he rejects it.

Before going on, we illustrate the procedure by an example. We took the same two large primes $p$ and $q$ and the generator $g$ from reference [12].

Example : Number $p$ is the 1024 bit-length prime:

$$p = 947722148354630050537346126889874207454007043676413224$$
$$022561203232011012018888701371707053734985713130303166$$
$$7987817480457498024477919590760609876896403173913477927$$
$$8848798229819934901324222106210711842549374102491417296$$
$$346772453897799554117544427007691684616643592277441939$$
$$13924495898621041399925210910234489,$$

and $q = 875964080856129786106302881659054003458244253873$.

The generator of the multiplicative group $\left( \frac{\mathbb{Z}}{p^2} \right)^*$ is
\[\begin{align*}
g &= 540101570024804126707242761888596856108217009274720 \\
&\quad 12898360235870708421296041390972417923715770599593251 \\
&\quad 42067778894704063133555201170818988618052933995304155 \\
&\quad 103003217921737377418069165608670923004369195350624645 \\
&\quad 066569726925118138080165144238960128731866130490259751 \\
&\quad 9067842079816229492516762912476306877. \\
\end{align*}\]

We find that:
\[\begin{align*}
a &= 527223567708677258197455859133222937206317701786932418 \\
&\quad 731523516024196571085174644862184845539033559830077347 \\
&\quad 165303013213561259060793012825365913429484950109971200 \\
&\quad 0119227222602799715625000222376432011868256652602811604 \\
&\quad 554081166154971036251524872300380557802833133742393048 \\
&\quad 3058768533820031165048898062091119992. \\
\end{align*}\]

Suppose that the secret key is:
\[\begin{align*}
x &= 371575259833906365510684947508061994685469500919, \text{ so :} \\
\end{align*}\]
\[\begin{align*}
y &= 6307754737188283687624666985220658584945893263294126314 \\
&\quad 383120993968845387965610104660428127774907934247973201 \\
&\quad 240651311501503440127132467171486473953854563208575553 \\
&\quad 814779236262480211822023628647029130388045798365258515 \\
&\quad 352888406082786247355168380930043030984132308743260982 \\
&\quad 09168959503180713095148123001718879430. \\
\end{align*}\]

Assume that Alice decides to sign the message \(m\) such that \(h(m) = 123456789123456789123456789\). She uses the random secret exponents \(k = 1250\) and \(l = 98561\). Therefore:
\[\begin{align*}
r &= 5781648655041794834001758922779428356186330897999304565 \\
&\quad 306500743940987577373931041772945246678159353311485596 \\
&\quad 586003732890945772134189937140959571809686022128752854 \\
&\quad 208088316685668654806317947603481040882949702784839302 \\
&\quad 99680824915315723510651254283784934213768940718951401 \\
&\quad 19901426026917705595060711695234702599; \text{ and} \\
s &= 54462109954698016824748794802717914623249907270. \\
\end{align*}\]
$t = 556119013460694353294511753174948468444082504155$.

To test the validity of Alice signature Bob first determines $u_1, u_2$ and $u_3$:

$u_1 = 141694501602616348876138369891969021089672807186$;

$u_2 = 662220963645670062957535725628437873682297068731$;

$u_3 = 68770021753905823557652121472773639225383494491$;

then Bob calculates $v = a^{u_1} y^{u_2} r^{u_3} \mod p \mod q$

$= 544621099954698016824748794802717914623249907270$

which is exactly $s$. In this case, Alice signature is accepted.

3.4 Security analysis

We discuss here some possible attacks. Suppose that Oscar, enemy of Alice, tries to impersonate her by signing the message $m$ without knowing her secret key $x$.

**Attack 1**: After receiving the signature parameters $(r, s, t)$ of a particular message $m$, Oscar may be wants to find Alice secret key $x$. If he uses equation(1), he will be confronted to the discrete logarithm problem $a^x \equiv b \mod q$ where $a = a^{\frac{r}{t}} \mod p$ and $b = s a^{-h(m)/t} r^{\frac{1}{t}} \mod p$. If Oscar prefers to exploit relation (2), he needs to know the two nonces $k$ and $l$. Their computation derives from the discrete logarithm problem.

**Attack 2**: Assume now that Oscar arbitrary fixes random values for two parameters and tries to find the third one.

(i) If he fixes $r$ and $s$ in the signature equation (1), and likes to determine the third unknown parameter $t$, he will be confronted to the discrete logarithm problem $a^{t'} \equiv s \mod p$ where $a = a^{\frac{h(m)}{t} y r^s} \mod p$ and $t' = \frac{1}{t} \mod q$.

(ii) If Oscar fixes $r$ and $t$ and wants to find the parameter $s$, he will be confronted to the equation $ab^x \equiv s \mod q$, where $a = a^{\frac{h(m)}{t} y^r} \mod p$ and $b = r^{\frac{1}{t}} \mod p$. In the mathematical literature, we don’t know any algorithm to solve this kind of modular equation.
(3) If Oscar fixes \( s \) and \( t \) then likes to calculate the parameter \( r \), he must solve the equation \( a^r r^b = c \ [q] \), where \( a = y^{1/t} \mod p \), \( b = \frac{t}{q} \mod q \) and \( c = x \alpha^{-h(m)/t} \mod p \). There is no known general method for solving this problem.

**Attack 3**: Assume that Alice used the same couple of exponents \((k, l)\) to sign two distinct message \( m_i \) and \( m_j \). Being aware of this fact, Oscar, from the first message signature obtains \( l_1 t_1 = h(m_1) + x r_1 + k s_1 \ [q] \) and from the second message \( l_2 t_2 = h(m_2) + x r_2 + k s_2 \ [q] \). As \( r_1 = r_2 \) and \( s_1 = s_2 \), Oscar is able to calculate the nonce \( l \). In contrast to the ElGamal and DSA schemes, it seems that there is no easy way to compute the exponent \( k \) and then to retrieve Alice secret key \( x \).

**Attack 4**: Let \( n \in \mathbb{N} \). Suppose that Oscar has collected \( n \) valid signatures \((r_i, s_i, t_i)\) for messages \( m_i \), \( i \in \{1, 2, \ldots, n\} \). Using (2), he will construct a system of \( n \) modular equations:

\[
\begin{cases}
  l_1 t_1 = h(m_1) + x r_1 + k s_1 \ [q] \\
  l_2 t_2 = h(m_2) + x r_2 + k s_2 \ [q] \\
  \vdots \\
  l_n t_n = h(m_n) + x r_n + k s_n \ [q]
\end{cases}
\]

where \( \forall i \in \{1, 2, \ldots, n\}, \ r_i = \alpha^k \mod p \) and \( s_i = \alpha^l \mod p \mod q \).

Since system \( (S) \) contains \( 2n + 1 \) unknown parameters \( x, r, s, i \in \{1, 2, \ldots, n\} \), it is not difficult for Oscar to propose a valid solution. But Alice secret key \( x \) has a unique possibility and therefore Oscar will never be sure what value of \( x \) is the right one. So this attack is not efficient.

**Attack 5**: Let us analyze the existential forgery[17, p. 285]. Suppose that the signature protocol is used without the hash function \( h \). Oscar can put \( r = \alpha^k y^{k'} \mod p \), \( s = \alpha^l y^{l'} \mod p \mod q \) for arbitrary numbers \( k, k', l, l' \). To solve the signature equation (1), it suffices to solve the system:

\[
\begin{cases}
  \frac{m + k s}{t} \equiv l \ [q] \\
  \frac{r + k' s}{t} \equiv l' \ [q]
\end{cases}
\]

\[
\begin{cases}
  m \equiv tl - ks \mod q \\
  t \equiv \frac{1}{q} [r + k's] \mod q
\end{cases}
\]

Hence \((r, s, t)\) is a valid signature for the message \( m \), but this attack is not realistic.

**Attack 6**: Suppose that Alice enemy Oscar is able to break the DSA scheme. In another words, given \( p, q, m, \alpha, y \), he can find integers \( r, s < q \) such
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that $\alpha^{h(m)} y^s \mod p = s[q]$. There is no evidence that Oscar is able to solve equation (1). It’s an advantage of our signature model: breaking the DSA scheme does not lead to breaking our protocol.

Remark 1: We end this security analysis by asking a question for which we have no answer: If someone is able to simultaneously break ElGamal, DSA and our own signature protocols, can he solve the general discrete logarithm problem?

3.5 Complexity

Productions of public and private keys in our protocol and in the DSA scheme are identical. So the number of operations to be executed is the same. In the generation of the signature parameters, we have one more parameter than in the DSA. To compute it, we use a supplementary modular exponentiation. For the verification step, we calculate three exponentiation instead of two for the DSA scheme.

Let $T_{\text{exp}}$ and $T_{\text{mult}}$ the times necessary to compute respectively an exponentiation and a multiplication. The total time to execute all operations using our method is as follows:

$$T_{\text{tot}} = 7T_{\text{exp}} + 8T_{\text{mult}}$$

As $T_{\text{exp}} = O(\log^3 n)$ and $T_{\text{mult}} = O(\log^2 n)$, (see [7, p. 72]), the final complexity of our signature scheme is

$$T_{\text{tot}} = O(\log^2 n + \log^3 n) = O(\log^3 n)$$

This proves that the execution of the protocol works in a polynomial time.

3.6 Theoretical generalization

For it’s pedagogical and mathematical interest, we end this paper by giving an extension of the signature equation (1). Let $h$ be a known and secure hash function as mentioned in section 2 and in the beginning of this section.

We fix an integer $n \in \mathbb{N}$ such that $n \geq 2$.

1. Alice begins by choosing her public key $(p, q, \alpha, y)$, where $p$ and $q$ primes such that $q$ divides $p - 1$.
Element \( \alpha \) is a generator of the subgroup of \( \mathbb{Z}/p\mathbb{Z} \) whose order is \( q \). \( y = \alpha^x \mod p \) where \( x \) is a secret parameter in \( \{1, 2, \ldots, q-1\} \). Integer \( x \) is Alice private key.

2. If Alice likes to produce a digital signature of a message \( m \), she must solve the congruence:

\[
\alpha^{x+1} \equiv y \alpha^{r_1} \alpha^{r_2} \alpha^{r_3} \ldots \alpha^{r_n} \mod p \equiv r_n \mod q
\]  

(5)

where the unknown parameters \( r_1, r_2, \ldots, r_n \) verify

\[
0 < r_1, r_2, \ldots, r_n < p \quad \text{and} \quad 0 < r_1, r_n < q.
\]  

(6)

**Theorem 2:** Alice, with her secret key \( x \) can determine an \((n+1)\) uplet \((r_1, r_2, \ldots, r_n, r_{n+1})\) that verifies the modular relation (5).

**Proof:** The signer Alice selects \( n-1 \) random numbers \( k_1, k_2, \ldots, k_{n-1} \in \mathbb{N} \) less than the prime \( q \) then computes \( r_i = \alpha^{k_i} \mod p \) for every \( i \in \{1, 2, \ldots, n-1\} \) and \( r_n = \alpha^{k_n} \mod p \mod q \). We have:

\[
\alpha^{x+1} \equiv y \alpha^{r_1} \alpha^{r_2} \alpha^{r_3} \ldots \alpha^{r_n} \mod p \equiv r_n \mod q.
\]

Equation (5) \( \leftrightarrow \alpha^{x+1} \equiv y \alpha^{r_1} \alpha^{r_2} \alpha^{r_3} \ldots \alpha^{r_n} \mod p \equiv r_n \mod q \).

It suffices to have \( \alpha^{x+1} \equiv y \alpha^{r_1} \alpha^{r_2} \alpha^{r_3} \ldots \alpha^{r_n} \mod p \equiv r_n \mod q \), which is equivalent to:

\[
\frac{h(m) + x r_1 + \sum_{i=1}^{n-1} k_i r_{i+1}}{r_{n+1}} \equiv k_n \mod q.
\]

So Alice determines the last unknown parameter \( r_{n+1} \) by calculating

\[
r_{n+1} = \frac{1}{k_n} [h(m) + x r_1 + \sum_{i=1}^{n-1} k_i r_{i+1}] \mod q
\]  

(7)

3. If Bob receives from Alice her signature proof \((r_1, r_2, \ldots, r_n, r_{n+1})\), he will be able to check whether the modular equation (5) is valid or not. He then deduces if he accepts or rejects this signature.

**Remark 2:** Let \( k_0 \) be Alice secret key \( x \), \( \vec{u} \) and \( \vec{v} \) respectively the vectors \((k_0, k_1, \ldots, k_{n-1})\) and \((r_1, r_2, \ldots, r_n)\). To easily memorize equality (7), observe
that \( r_{n+1} = \frac{h(m) + \overrightarrow{u}.\overrightarrow{v}}{k_n} \) where \( \overrightarrow{u}.\overrightarrow{v} \) denotes the classical inner product of the two vectors \( \overrightarrow{u} \) and \( \overrightarrow{v} \).

4. Conclusion

In this article, a new digital signature protocol was presented. We studied in details the security of the method and gave an analysis of its complexity. Our contribution can be seen as an alternative if the DSA algorithm is totally broken. For its purely mathematical and pedagogical interest, we furnished a general form of our proposed signature equation.

References

[1] Addepalli V. N. Krishna, Addepalli Hari Narayana & K. Madhura Vani Fully homomorphic encryption with matrix based digital signature standard, Journal of Discrete Mathematical Sciences and Cryptography, 20:2, 439-444, DOI: 10.1080/09720529.2015.1101882 (2017).

[2] J. Angel, R. Rahul, C. Ashokkumar, B. Menezes, DSA signing key recovery with noisy side channels and variable error rates, Progress in cryptology Indocrypt, pp147--165, Lecture Notes in Comput. Sci., 10698, Springer, Cham, (2017).

[3] M. Bellare, S. Goldwasser, and D. Micciancio, Pseudo-random number generation within cryptographic algorithms: the DSS case, In Proc. of Crypto’97, volume 1294 of LNCS. IACR, Palo Alto, CA, Springer-Verlag, Berlin, (1997).

[4] L. Chen-Yu, L. and L. Wei-Shen, Extended DSA. Journal of Discrete Mathematical Sciences and Cryptography Vol. 11, 5, pp545--550, (2008).

[5] W. Diffie and M. E. Hellman, New directions in cryptography. Information Theory, IEEE Transactions 22, N. 6, pp644--654, (1976).

[6] T. ElGamal, A public key cryptosystem and a signature scheme based on discrete logarithm problem. IEEE Trans. Info. Theory, IT-31, N. 4, pp469-472, (1985).

[7] J. A. Menezes, P. C. Van Oorschot, and S. A. Vanstone, Handbook of applied cryptography. CRC press, (1996).
[8] R. Mollin, *An Introduction to cryptography*, Second edition, Chapman & Hall/CRC, (2007).

[9] National institute of standard and technology (NIST). FIPS Publication 186, DSA, Department of commerce, (1994).

[10] P. Q. Nguyen and I. E. Shparlinski, *The insecurity of the Digital Signature Algorithm with partially known nonces*, J. of Cryptology, pp151--176, (2002).

[11] P. Pohlig and M. Hellman, *An Improved Algorithm for Computing Logarithms over GP(p) and Its Cryptographic Significance*, IEEE Transaction on information theory, Vol. IT 24, n. 1, (1978).

[12] T. Pornin, *Deterministic usage of the digital signature algorithm (DSA) and elliptic curve digital signature algorithm (ECDSA)*. RFC 6979, (2013).

[13] D. Poulakis, *New lattice attacks on DSA schemes*, J. Math. Cryptol. 10, no. 2, pp135--144, (2016).

[14] Rivest, R., Shamir, A., & Adeleman, L. *A method for obtaining digital signatures and public key cryptosystems*, Communication of the ACM Vol. no 21 (1978).

[15] C. P. Schnorr, *Efficient Signature Generation by Smart Cards*. Journal of Cryptology, pp161--174, (1991).

[16] C. E. Shannon, *Communication Theory of Secrecy Systems*, Bell System Technical Journal vol 28, pp656--715, (1949).

[17] D. R. Stinson, *Cryptography, Theory and Practice*, Third edition, Chapman & Hall/CRC (2006).

[18] http://www.umich.edu/x509/ssleay/fip186/fip186.htm.

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