Solar Dynamo Model with Diamagnetic Pumping and Nonlocal Alpha-Effect

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Abstract A combination of diamagnetic pumping and a nonlocal $\alpha$-effect of the Babcock-Leighton type in a solar dynamo model helps to reproduce observations of solar magnetic activity. The period of the solar cycle can be reproduced without reducing magnetic diffusivity in the bulk of the convection zone below the standard mixing-length value of $10^{13}$ cm$^2$s$^{-1}$. The simulated global fields are antisymmetric about the equator and the toroidal-to-poloidal field ratio is about a thousand. The time-latitude diagrams of magnetic fields in the model without meridional flow, however, differ from observations. Only when the meridional flow is included and the $\alpha$-effect profile peaking at mid latitudes is applied, can the observational butterfly diagrams be reproduced.

Keywords: Solar Cycle: models · Magnetohydrodynamics (MHD) · Convection Zone · Turbulence

1. Introduction

The aim of this paper is to draw attention to the possibility of resolving several problems of dynamo theory for solar activity by combining diamagnetic turbulent pumping and a nonlocal $\alpha$-effect in a solar dynamo model.

In the absence of reliable data on magnetic fields in the deep solar interior, modeling the solar dynamo remains a controversial issue (for a recent review, see Tobias, 2009). Consensus has developed that the dynamo is driven by two basic effects. The $\Omega$-effect of nonuniform rotation produces a strong toroidal field from a poloidal one and the $\alpha$-effect of cyclonic motion (Parker, 1955) regenerates the poloidal field. Already the first models of an $\alpha\Omega$-dynamo produced oscillatory solutions resembling the solar cycle (Leighton, 1969; Steenbeck and Krause, 1969). If discrepancies between computed and observed parameters within two orders of magnitude are tolerable, these pioneering models are quite satisfactory. A closer agreement is difficult and may even seem impossible to obtain in view of the basic physics of the dynamo process and decades-long practice of dynamo simulations.

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already noticed that turbulent magnetic diffusivity should be reduced much below the standard mixing-length value of $\eta \approx 10^{13}\text{cm}^2\text{s}^{-1}$ in order to reproduce the observed period of the solar cycle. The time of diffusive decay of a magnetic field can be estimated as $T_d \approx d^2 \eta^{-1}$, where $d$ is the depth of the convection zone. A cyclic dynamo has to regenerate fields in a shorter time in order to overpower the diffusive decay, so that $P_{\text{cyc}} < T_d$ ($P_{\text{cyc}}$ is the period of the magnetic cycle). The observed period can be reproduced with a diffusivity value much smaller than $10^{13}\text{cm}^2\text{s}^{-1}$, this case, however, is difficult to justify. Eddy viscosity or thermal diffusivity below $10^{13}\text{cm}^2\text{s}^{-1}$ cannot be a correct parameterization for solar convection because with such small eddy diffusion the external layers of the Sun are still unstable (Trommelen et al., 1994; Kitchatinov and Mazur, 2000). The assumption of small magnetic diffusion needs to explain why the same turbulent mixing transports momentum and heat much more efficiently than a magnetic field. Theories of turbulent transport coefficients do not support the assumption of large magnetic Prandtl number. Direct numerical simulations of Yousef, Brandenburg, and Rüdiger (2003) also give this number of the order of unity.

Observations show a clear predominance of the equator-antisymmetric (dipolar) component in the global magnetic field of the Sun (Stenflo, 1988). The symmetric part is relatively small and does not show an 11-year cycle. The critical dynamo numbers for the excitation of the global modes of these two types of symmetry are usually very close together. The threshold dynamo number for the antisymmetric modes may be relatively small, leading to the preference for this type of symmetry, but the situation can usually be changed to the preference for the symmetric modes by small variations of parameters in a dynamo model. In other words, the equatorial symmetry is unstable to small changes in the design of dynamo models. Another problem is related to the ratio of toroidal- to poloidal-field amplitudes. The ratio is not smaller than a thousand if the poloidal field is estimated by its polar value and the toroidal field - by the field strength in sunspots. Solar differential rotation of about 30% can produce in the 11 years of the solar cycle a toroidal field that is at most 40 times stronger than the poloidal field. Strong radial gradient of rotation in the tachocline does not change this estimation because the radial field should be as much weaker there compared to the meridional field as the radial shear is larger than the latitudinal shear (except for in cases of the relic field penetrating from the radiation zone, or the field having a structure with multiple latitudinal belts, both cases present new problems).

Another problem of the so-called catastrophic quenching of the $\alpha$-effect (Gruzinov and Diamond, 1994; Brandenburg and Subramanian, 2005) is not directly related to observations but is “internal” for theory. It can be outlined as follows. The large-scale fields generated by $\alpha$-effect dynamos are helical. As the magnetic helicity is conserved, small-scale magnetic fields attain helicity equal in amount and opposite in sign to that of large-scale fields. Helical small-scale fields produce their own magnetic $\alpha$-effect that counteracts the $\alpha$-effect of whatever origin so that the total $\alpha$-effect strongly diminishes.

In a recent paper (Kitchatinov and Olemskoy, 2011a), we demonstrated that the catastrophic quenching is alleviated in a solar dynamo model that combines
a nonlocal $\alpha$-effect and diamagnetic pumping of large-scale fields by turbulent convection. In this new publication it is shown that with this model other above-mentioned problems also fade. The diamagnetic pumping, which will be discussed in the next section, concentrates the magnetic field at the bottom of the convection zone. The turbulent diffusion in this region is small. As a result, the solar cycle period can be reproduced. The poloidal field in the near-base region is much stronger than on the surface and sufficiently strong for the differential rotation to generate kilogauss toroidal fields over a solar cycle. The critical dynamo number for dipolar dynamo modes is considerably smaller compared to quadrupolar modes, so that the model always results in a global field of dipolar parity. However, the model still has difficulties in reproducing time-latitude diagrams of solar magnetism. We include the meridional flow and vary the latitudinal profile of the $\alpha$-effect to reproduce observational butterfly diagrams.

2. Diamagnetic Pumping

Diamagnetic pumping of large-scale fields was predicted theoretically long time ago (Zeldovich, 1957; Rädler, 1968) but is scarcely known in the dynamo community and is usually ignored in dynamo models (see, however, Rüdiger and Brandenburg, 1995; Kapyla, Korpi, and Tuominen, 2006; Guerrero and de Gouveia Dal Pino, 2008).

The nature of diamagnetic pumping is well illustrated by the case of inhomogeneous 2D turbulence with the fluctuating velocity $u'$ uniform along a direction defined, say, by the unit vector $e$ and inhomogeneous in the direction perpendicular to $e$: $\nabla \langle u'^2 \rangle \neq 0, (e \cdot \nabla) u' = 0$. For this case, Zeldovich (1957) found that the mean electromotive force, $E = \langle u' \times B' \rangle$ ($B'$ is a fluctuating magnetic field), can be written as

$$E = -\nabla \times (\eta_r B), \tag{1}$$

if the large-scale field $B$ is also perpendicular to $e$; $\eta_r$ is the turbulent magnetic diffusivity. This means that the large-scale field is transported with the effective velocity

$$U_{dia} = -\nabla \eta_r. \tag{2}$$

The diamagnetic pumping expels the field from the regions of relatively high turbulence intensity. However, for the case of the mean field parallel to the direction $e$, the mean electromotive force reads

$$E = -\eta_r \nabla \times B \tag{3}$$

and there is no diamagnetic pumping. The magnetic field parallel to the $e$-direction, $e \parallel B$, behaves like a scalar field. The diamagnetic pumping, therefore, is related to the vectorial nature of magnetic fields, and there is no counterpart of diamagnetic pumping for scalar fields.
Figure 1. Pictorial explanation of diamagnetic pumping (see text). Turbulent intensity increases from left to right. Undisturbed magnetic lines are shown by dashed lines. When a flux tube is displaced by motion, its right boundary is moving faster on average. Displacements to the left compress the flux tube and the field strength in the displaced region increases. Material is evacuated from the displaced region as shown by arrows. Displacements to the right result on average in a decrease in field strength in the displaced region.

The pumping effect can be interpreted as follows (Figure 1). Imagine that there is a background field perpendicular to both the direction $\mathbf{e}$ and the gradient of turbulence intensity. If the turbulent motion displaces a flux tube of such a field as the one on the left in Figure 1, i.e., in the direction of decreasing turbulence intensity, the left boundary of the tube is on average moving slower than the right boundary. The tube is compressed, and the field strength in the displaced region increases. Similarly, displacement to the right decreases the field strength in the displaced region. Turbulent mixing produces on average the field transport in the direction of decreasing turbulent intensity, i.e., turbulent conducting fluids behave diamagnetic.

In the 3D case, the expression for the effective velocity of diamagnetic transport changes to (Krause and Rädler, 1980)

$$U_{\text{dia}} = -\frac{1}{2} \nabla \eta_T.$$  \hspace{1cm} (4)

The diamagnetic pumping can be very efficient near the bottom of the convection zone where the intensity of turbulent convection changes sharply with depth (Kitchatinov and Rüdiger, 2008). Doubts have been expressed concerning the efficiency of the pumping in a nonlinear regime (Vainshtein and Kitchatinov, 1983). However, turbulent diamagnetism was recently detected in laboratory experiments with liquid sodium (Spence et al., 2007). Direct numerical simulations also show downward pumping of large-scale fields near the base of convectively unstable layers (Tobias et al., 1998; Tobias et al.; Dorch and Nordlund; Ossendrijver et al., 1998, 2001, 2002; Zigler and Rüdiger, 2003). Whether the simulations indeed show the turbulent diamagnetism is not perfectly clear. Their results were interpreted in terms of the Drobyshevski and Yuferev (1974) effect of topological pumping (Tobias et al., 2001; Dorch and Nordlund, 2001). Anyway, the diamagnetic and topological
pumpings are of the same sense (downward) near the base of the convection zone. We apply the Equation 4 for the effective transport velocity in our dynamo model.

It may be noted that turbulent diffusion and pumping are both reduced by rotation and/or a magnetic field. Tobias et al. (2001) and Ziegler and Rüdiger (2003) observed the rotational quenching effect in numerical simulations. An analytical theory of turbulent transport predicts that though the turbulent diffusion and pumping are modified, the relation 4 between their isotropic parts survives under the influence of rotation or a magnetic field (Kitchatinov, 1988).

3. The Model
3.1. Dynamo Equations

Our dynamo model is based on the mean-field induction equation

\[ \frac{\partial B}{\partial t} = \nabla \times (V \times B + E). \tag{5} \]

The large-scale flow is a superposition of rotation and meridional circulation,

\[ V = e_{\phi}r \sin \theta \Omega f(r, \theta) + \frac{1}{\rho} \nabla \times \left( e_{\phi} \frac{\psi}{r \sin \theta} \right), \tag{6} \]

where the usual spherical coordinates are used, \( \Omega \) is the characteristic value of angular velocity, \( f \) is the normalized frequency of differential rotation, \( \psi \) is the meridional flow stream function, \( \rho \) is density, and \( e_{\phi} \) is the azimuthal unit vector.

The mean electromotive force is written as

\[ E = -\sqrt{\eta T} \nabla \times \left( \sqrt{\eta T} B \right) + A, \tag{7} \]

where the first term on the right stands for both turbulent diffusion and the diamagnetic pumping of Equation 4. The second term accounts for the \( \alpha \)-effect in its nonlocal formulation (Brandenburg, Rädler, and Schrinner, 2008)

\[ A = \int \alpha(r, r') B(r') \, d^3r'. \tag{8} \]

The local \( \alpha \)-effect is subject to the catastrophic quenching mechanism (Brandenburg and Subramanian, 2005) while the nonlocal \( \alpha \)-effect is not (Kitchatinov and Olemskoy, 2011a), and for this reason it may be dominating in the dynamo process. Another possibility to avoid the catastrophic quenching is to invoke sufficiently efficient fluxes of magnetic helicity (Guerrero, Chatterjee, and Brandenburg, 2010).

Similar to the mean flow of Equation 6, the magnetic field can be written as a superposition of its toroidal and poloidal parts,

\[ B = e_{\phi}B + \nabla \times \left( e_{\phi} \frac{A}{r \sin \theta} \right), \tag{9} \]
where $A$ is the poloidal field potential.

Here we introduce normalized variables. The time is measured in units of $R^2_\odot/\eta_0$; $\eta_0$ is the characteristic value of the eddy diffusivity. The magnetic field is normalized to the field strength $B_0$ for which nonlinear effects become essential, and the $\alpha$-parameter is normalized to its characteristic value $\alpha_0$. The poloidal field potential is measured in units of $\alpha_0 B_0 R^3_\odot/\eta_0$. The density is normalized to its surface value $\rho_0$, and the stream function of meridional flow is measured in units of $\rho_0 R^2_\odot V_0$; $V_0$ is the amplitude of the surface meridional flow. From now on, the same notations are kept for the normalized variables as used before for their unnormalized counterparts, except for the fractional radius $x = r/R_\odot$ and normalized diffusivity $\eta = \eta_r/\eta_0$. The normalized equation for the toroidal field reads

\[
\frac{\partial B}{\partial t} = \frac{\eta}{x^2} \frac{\partial}{\partial \theta} \left( \frac{1}{\sin \theta} \frac{\partial (\sin \theta B)}{\partial \theta} \right) + \frac{1}{x} \frac{\partial}{\partial x} \left( \sqrt{\eta} \frac{\partial (\sqrt{\eta} x B)}{\partial x} \right) + \frac{R_m}{x} \frac{\partial}{\partial \theta} \left( \frac{B}{\rho x \sin \theta} \frac{\partial \psi}{\partial x} \right) - \frac{R_m}{x} \frac{\partial}{\partial x} \left( \frac{B}{\rho x \sin \theta} \frac{\partial \psi}{\partial \theta} \right)
\]

\[
+ \frac{D}{x} \left( \frac{\partial f}{\partial x} \frac{\partial A}{\partial \theta} - \frac{\partial f}{\partial \theta} \frac{\partial A}{\partial x} \right), \tag{10}
\]

where

\[
D = \frac{\alpha_0 \Omega R^3_\odot}{\eta_0}; \tag{11}
\]

is the dynamo number and

\[
R_m = \frac{V_0 R_\odot}{\eta_0}; \tag{12}
\]

is the magnetic Reynolds number for the meridional flow. The $\alpha \Omega$-approximation is applied to neglect the alpha-effect in the toroidal field Equation (10). This equation describes the toroidal-field production by differential rotation, its advection by the meridional flow, diamagnetic pumping, and turbulent diffusion.

The poloidal-field equation with nonlocal $\alpha$-effect is written as

\[
\frac{\partial A}{\partial t} = \frac{\eta}{x^2} \sin \theta \frac{\partial}{\partial \theta} \left( \frac{1}{\sin \theta} \frac{\partial A}{\partial \theta} \right) + \sqrt{\eta} \frac{\partial}{\partial x} \left( \sqrt{\eta} \frac{\partial A}{\partial x} \right) + \frac{R_m}{\rho x^2 \sin \theta} \frac{\partial \psi}{\partial x} \frac{\partial A}{\partial \theta} - \frac{\partial \psi}{\partial \theta} \frac{\partial A}{\partial x}
\]

\[
+ x \sin \theta \cos \theta \int_{x_i} \hat{\alpha}(x, x') B(x', \theta) \, dx', \tag{13}
\]

where $x_i$ is the radius of the inner boundary. The integration in this equation is only in the radius with the upper limit $x$. This qualitatively reflects the fact that the nonlocal $\alpha$-effect at some point $x$ is contributed by the buoyant magnetic loops rising from deeper layers ($x' < x$) and that buoyant velocities are almost vertical. Our dynamo Equations (10) and (13) are very similar to those in our
previous publication \cite{Kitchatinov and Olemskoy, 2011a}, but now we include meridional flow and neglect the (local) magnetic $\alpha$-effect which was shown to be insignificant when nonlocal $\alpha$-effect is allowed for.

Our boundary conditions assume a perfect conductor beneath the inner boundary of radius $x_i$ and pseudo-vacuum conditions on the top,

$$\frac{\partial (\sqrt{\eta}B)}{\partial x} = 0, \quad A = 0 \quad \text{for} \quad x = x_i,$$

$$\frac{\partial A}{\partial x} = 0, \quad B = 0, \quad \text{for} \quad x = 1.$$ (14)

The bottom boundary in our model is at $x_i = 0$.

7. The initial-value problem for the dynamo Equations (10) and (13) was solved numerically by the grid-point method and explicit time-stepping. The diamagnetic pumping leads to a high concentration of the magnetic field near the bottom. To resolve fine structures near the bottom, a nonuniform grid was applied over the radius with the grid spacing $\Delta x \sim \eta^{1/2}$. The grid over the latitude was uniform.

The equatorial symmetry was usually not prescribed. The field was evolved in time starting from a mixed-parity initial field and the solution relaxed eventually to a certain equatorial symmetry. In order to determine the critical dynamo numbers for the excitation of the dipolar ($B(\theta) = -B(\pi - \theta)$) and quadrupolar ($B(\theta) = B(\pi - \theta)$) dynamo modes, additional boundary conditions selecting the field mode of certain equatorial symmetry were imposed on the equator.

3.2. Model Design

3.2.1. Differential Rotation

For the differential rotation, we use the approximation of helioseismological data suggested by Belvedere, Kuzanyan, and Sokoloff \cite{Belvedere, Kuzanyan, and Sokoloff, 2000},

$$f(x, \theta) = \frac{1}{461} \sum_{m=0}^{2} \cos \left( 2m \left( \frac{\pi}{2} - \theta \right) \right) \sum_{n=0}^{4} C_{nm} x^n. \quad (15)$$

The coefficients $C_{nm}$ of this equation are given in Table 1 of Belvedere, Kuzanyan, and Sokoloff (2000). Figure 2 shows the angular velocity contours.

3.2.2. Profiles of the $\alpha$-Effect and Diffusivity

The kernel function of the nonlocal $\alpha$-effect in the poloidal field Equation (13) was prescribed as follows,

$$\hat{\alpha}(x, x') = \frac{\phi_b(x') \phi_\alpha(x)}{1 + B^2(x', \theta)},$$

$$\phi_b(x') = \frac{1}{2} \left( 1 - \text{erf} \left( \frac{(x' - x_b)}{h_b} \right) \right),$$

$$\phi_\alpha(x) = \frac{1}{2} \left( 1 + \text{erf} \left( \frac{(x - x_\alpha)}{h_\alpha} \right) \right). \quad (16)$$
where erf is the error function and $B^2$ in the denominator of the first equation accounts for the usual algebraic quenching of the $\alpha$-effect. We always use $x_b = x_i + 2.5 h_b$ and $x_\alpha = 1 - 2.5 h_\alpha$ to ensure smoothness of the kernel functions in the simulation domain. The parameter $h_b$, parameter represents the thickness of the near-bottom region of toroidal magnetic fields producing the $\alpha$-effect. The parameter $h_\alpha$ represents the thickness of the near-surface layer where this effect is produced. The $\alpha$-effect with the kernel function of Equation (16) is very close to the nonlocal model of [Brandenburg and Käpylä (2007)]. It is also similar to the Babcock-Leighton mechanism for the poloidal field production used in the dynamo models of [Durney (1995)] and [Dikpati and Charbonneau (1999)].

The turbulent diffusivity in the bulk of the convection zone varies slightly with depth. Near the base of the convection zone, it drops sharply with increasing depth. The diffusivity profile of our model reads

$$\eta(x) = \eta_n + \frac{1}{2}(1 - \eta_n) \left(1 + \text{erf}\left(\frac{x - x_\eta}{h_\eta}\right)\right),$$

(17)

where $\eta_n$ is the ratio of diffusivity near the inner boundary to its value in the bulk of the convection zone. Computations were performed for the smallest value of $\eta_n = 10^{-4}$ we were able to apply.

The major part of our computations was performed with the following values of parameters: $x_\eta = 0.74$, $h_\eta = 0.01$, $h_\alpha = 0.02$, and $h_b = 0.002$. The profiles of diffusivity and kernel functions of the $\alpha$-effect of Equation (16) for this set of parameters are shown in Figure 3. Dependence of the results on the model parameters $h_b$ and $h_\alpha$ will be discussed at the beginning of Section 4 to explain this choice.
3.2.3. Meridional Flow

A poleward meridional flow of the order of 10 m s$^{-1}$ is observed on the solar surface (Komm, Howard, and Harvey, 1993). Helioseismology confirms that the flow persists up to a depth of about 12 Mm (Zhao and Kosovichev, 2004). Theoretical modeling remains the only source of knowledge about the flow in deeper regions. Recent simulations predict that one cell of meridional circulation occupies the entire thickness of the convection zone and the return flow at the bottom is not small compared to the surface (Kitchatinov and Olemskoy, 2011b). Beneath the convection zone, the flow is small (Gilman and Miesch, 2004; Kitchatinov and Rüdiger, 2006). All these findings are qualitatively reflected by the following representation for the stream function of the meridional flow

$$\psi = -\cos \theta \sin^2 \theta \phi(x),$$

$$\phi(x) = \begin{cases} 
\frac{1}{1-x_s} \int_{x_s}^x \rho(x')\eta(x')(x'-x_s) \, dx' & \text{for } x \geq x_s \\
C \int_{x_s}^x \rho(x')\eta(x')(x_s-x') \, dx' & \text{for } x \leq x_s.
\end{cases} \tag{18}$$

In this equation, $x_s$ is the radius of the stagnation point where the meridional velocity changes sign, and $C$ is a parameter whose value is adjusted to ensure continuity of the stream function at the stagnation point.

The stratification of the convection zone is almost adiabatic. An adiabatic profile for ideal gas was used for the normalized density,

$$\rho(x) = \left(1 + C \rho \left(\frac{1}{x} - 1\right)\right)^{3/2}, \tag{19}$$
where $C_\rho = 10^3$.

The profile of the normalized meridional velocity of our model is shown in Figure 4. The radius of the stagnation point, $x_s \approx 0.84$, was adjusted so that the amplitudes of the flow above and below this point are equal. We set the parameter $p = 0.25$ in Equation (15), so that the bottom flow is about ten times slower than the maximum velocity of the deep equator-ward flow.

4. Results and Discussion

4.1. Models without Meridional Flow

We will discuss the models without meridional flow ($R_m = 0$) first. Then, we will include the flow to see what effect it produces.

![Figure 5](image.png)

**Figure 5.** Dependence of the critical dynamo number (a) and the magnetic cycle period (b) on the thickness $h_b$ of the near-bottom layer producing the $\alpha$-effect. Full and dotted lines show the results for dipolar and quadrupolar dynamo modes, respectively.
Figure 5 shows the dependencies of the critical dynamo number and magnetic cycle period on the thickness $h_b$ of the near-bottom layer producing the $\alpha$-effect (for constant $h_t = 0.02$). The cycle period $P_{cyc}$ is defined as half the complete period of magnetic oscillations in order to compare with the 11-year solar cycle. The dipolar parity and the observed period of magnetic oscillations can be reproduced only with a sufficiently thin bottom layer, $h_b < 4 \times 10^{-3}$. Only in this case the equator-antisymmetric modes of the magnetic field are preferred and the cycle period is sufficiently long. For the value $\eta_0 = 10^{13}\text{cm}^2\text{s}^{-1}$ of magnetic diffusivity, the diffusion time $R_\odot^2/\eta_0 \approx 15$ years and the cycle period in physical units, $P_{cyc}R_\odot^2/\eta_0$, is close to 11 years for small $h_b$. Also for a thin bottom layer only, the critical dynamo number for the excitation of magnetic fields of dipolar parity is about 30% smaller compared to quadrupolar parity modes so that the model produces equator-antisymmetric global fields similar to the Sun. A strong decrease of magnetic diffusivity towards the bottom boundary is also important for preference of dipolar parity \cite{Chatterjee2004, Hotta2010}. We conclude that our model can reproduce a solar cycle only if the $\alpha$-effect is produced by sufficiently deep-seated toroidal fields.

The dynamo numbers shown in Figure 5 have a minimum at about $h_b = 2 \times 10^{-3}$, and all the results to follow were produced with this value. For smaller $h_b$, the critical dynamo numbers are larger because of the decrease in the toroidal magnetic flux producing the $\alpha$-effect. For larger $h_b$, the bottom layer is thicker than the vertical scale of the toroidal field and the fields of opposite signs partly cancel each other’s contribution to the $\alpha$-effect. The dependence on the thickness $h_t$ of the top layer where the $\alpha$-effect is produced is simpler. The cycle period shown in Figure 6 does not depend on $h_t$ until the thickness becomes so large that the top and bottom layers overlap. There is also a smooth decrease in the critical dynamo numbers with increasing $h_t$. We fix $h_t = 0.02$ for further computations.

Figure 7 shows the magnetic field patterns in the meridional cross-section for several instances of a cycle. The magnetic field is highly concentrated at
Figure 7. Contours of toroidal-field (top row) and poloidal field lines (bottom row) for several instances of a magnetic cycle. The time of the run in units of $R_\odot^2/\eta_0$ is shown at the top. Full (dotted) lines show positive (negative) levels and clockwise (anticlockwise) circulation. The pictures of the upper row are re-scaled so that the upper (dashed) boundary shows the radius of $r = 0.74R_\odot$, below which the toroidal fields are localized. The dynamo number $D = 2.2 \times 10^4$ is slightly above the critical value of $1.9 \times 10^4$.

The concentration is provided by the diamagnetic pumping. The pumping, however, does not affect the (radial) component of the field parallel to the pumping direction and the poloidal field lines can come to the surface. The toroidal field is confined below the radius of $x_\eta$ where the magnetic diffusivity drops sharply with depth. The poloidal field near the bottom is also much stronger than at the top. As a result, the toroidal field amplitude in our model is about a thousand times stronger than the amplitude of the surface polar field. If the poloidal field at the bottom were of the same order as at the top, the differential rotation would not be able to produce a strong toroidal field over a cycle. Note that the deep toroidal field of several kilogauss in our model is the mean field. The total field can include strong fluctuations on the background of the mean field.

Figure 8 shows butterfly diagrams for the surface radial field and deep toroidal field. The toroidal field diagram shows the contours of the quantity

$$B = \sin \theta \int_{x_1}^{1} \phi_1(x) B(x) \, dx,$$

(20)
Figure 8. Butterfly diagram of the depth-integrated toroidal field $B$ of Equation (20) (top panel) and the surface radial field (bottom) for the model without meridional flow. Time is shown in units of $R_\odot^2/\eta_0$. $D = 2.2 \times 10^4$.

to which the surface $\alpha$-effect of our model is proportional. The factor $\sin \theta$ in Equation (20) accounts for the dependence of the length of toroidal flux tubes on latitude (it is supposed that the probability of sunspot production is proportional to the length of the tube).

Figure 9. Time-latitude diagrams for the model with meridional flow. A standard latitudinal profile of the $\alpha$-effect, $\alpha \sim \cos \theta$, was used. The dynamo number $D = 2.6 \times 10^4$ is slightly above the critical value of $D_c = 2.36 \times 10^4$. The top and bottom panels show the deep toroidal field and the surface radial field respectively.
The radial field diagram of Figure 8 is similar to observational diagrams of Stenflo (1988) and Obridko et al. (2006). The toroidal field diagram, however, has too broad ‘wings’ and shows too slow an equatorial drift compared to the butterfly diagram of sunspots. There is a possibility to improve the agreement with allowance for meridional flow.

4.2. Models with Meridional Flow

All computations with meridional flow were performed for the magnetic Reynolds number $R_m = 10$ that is a plausible solar value for the magnetic diffusivity $\eta_0 = 10^{13}$ cm$^2$ s$^{-1}$. The ‘local’ Reynolds number in the near-bottom region of low diffusivity is, of course, much larger. It may be expected, therefore, that the meridional flow influences primarily the field migration near the bottom. Indeed, the toroidal field diagram of Figure 9 shows much faster equatorial drift compared to Figure 8.

![Figure 10](image.png)

**Figure 10.** Time-latitude diagrams for the model with meridional flow and $\sin^2 \theta \cos \theta$-profile of the $\alpha$-effect. Computations were performed for the dynamo number $D = 4.2 \times 10^4$, slightly above the critical value of $D_c = 3.96 \times 10^4$. The top and bottom panels show the deep toroidal field and the surface radial field respectively.

Now, however, the poloidal field diagram becomes unsatisfactory. It shows equatorial drift also at high latitudes where the poleward migration is actually observed (Stenflo, 1988). The magnetic diffusion near the surface is high and the meridional flow does not influence the surface fields directly. However, the surface poloidal fields are related to the deep toroidal fields by the $\alpha$-effect and follow the equatorial migration of the deep fields.

The polar drift of the high-latitude poloidal field can be re-established by a change in the latitudinal profile of the $\alpha$-effect. Observations indicate that the current helicity of solar active regions (the $\alpha$-effect proxy) does not increase steadily with latitude but has a maximum at mid latitudes (Sokoloff et al.,...
Theoretical tilt-angles of bipolar magnetic regions show also humps in the latitudinal profiles (D’Silva and Choudhuri, 1993). Figure 10 shows the model results for the $\sin^2 \theta \cos \theta$-profile of the $\alpha$-effect, i.e., $\sin \theta$ in the last term of Equation (13) was changed to $\sin^3 \theta$. With this profile, the poloidal field diagram is in closer agreement with observations.

Simulations that take into account the meridional flow also result in magnetic fields of dipolar parity. The critical dynamo numbers for the excitation of dipolar and quadrupolar dynamo modes in the model with the $\alpha$-effect peaking at mid latitudes ($\alpha \sim \sin^2 \theta \cos \theta$) are $D_{c}^d = 3.96 \times 10^4$ and $D_{c}^q = 4.64 \times 10^4$, respectively. In the model where the $\alpha$-effect peaks at poles ($\alpha \sim \cos \theta$), the critical dynamo numbers are closer to each other, $D_{c}^d = 2.36 \times 10^4$ and $D_{c}^q = 2.49 \times 10^4$. Nevertheless, this model also shows the preferred excitation of the equator-antisymmetric magnetic fields.

Though the meridional flow is important for the latitudinal drift of the magnetic fields in our model, the model does not belong to the so-called advection-dominated dynamos. Figure 10 does not show high concentration of surface fields towards the poles typical of advection-dominated dynamo models. This is because of relatively large magnetic diffusivity near the surface. The cycle period in our model is not controlled by the meridional flow. A sufficiently long cycle is realized mainly due to the low magnetic diffusivity in the thin layer near the bottom boundary and partly due to the diamagnetic pumping to this layer.

To probe for the effect of diamagnetic pumping, the computations were repeated with the pumping switched off (for the model with meridional flow and mid-latitude peaking $\alpha$-effect). The cycle period at the critical dynamo number reduced to $P_{cyc} = 0.47$, i.e., a bit less than twice. Other model parameters are influenced much more. Without diamagnetic pumping, the amount of toroidal magnetic flux in the near-bottom region, producing the $\alpha$-effect, decreases strongly. Accordingly, the critical dynamo numbers increase by almost two orders of magnitude, to $D_{c}^d = 2.47 \times 10^6$ and $D_{c}^q = 2.49 \times 10^6$ with no clear preference for dipolar parity. Without the pumping, the poloidal field at all depths is of the same order as at the surface. Accordingly, the ratio of amplitudes of the toroidal field to the surface polar field decreased to about 43 from 920 in the model with diamagnetic pumping.

5. Summary

Joint application of diamagnetic pumping and a nonlocal $\alpha$-effect strongly influences the solar-type dynamo model to bring it generally closer to observations. The pumping concentrates magnetic fields near the base of the convection zone where diffusivity is relatively small. As a result, the solar cycle period can be reproduced with the standard mixing-length value of eddy diffusivity, $\eta_T \approx 10^{13} \text{cm}^2\text{s}^{-1}$, in the bulk of the convection zone. The near-base concentration of the poloidal field makes it possible for differential rotation to wind a strong toroidal field over a solar cycle so that the toroidal field amplitude in the dynamo model is about a thousand times stronger than the surface polar field. The model produces equator-antisymmetric global fields.
The combination of diamagnetic pumping and nonlocal $\alpha$-effect also resolves the theoretical problem of the so-called catastrophic quenching of the alpha-effect \citep{Kitchatinov.2011b}.

The model without meridional flow does not, however, reproduce the butterfly diagram of sunspot activity. The time-latitude diagram of the toroidal field shows too slow equatorial drift and too broad latitudinal distribution in this case (Figure 8). This can be corrected by allowance for the meridional circulation and the change of the latitudinal distribution of the $\alpha$-effect from a pole-peaked to a mid-latitude-peaked profile (Figure 10).

The prescription for the $\alpha$-effect remains the main uncertainty of the model. Operation of the nonlocal $\alpha$-effect of the Babcock-Leighton type on the Sun is plausible from both observational \citep{2010A&A...518A...7D} and theoretical \citep{1995ApJ...441..886C} evidences. The mathematical formulation of this effect remains, however, uncertain. If our model is considered as an inverse problem, it predicts that the $\alpha$-effect is produced by deep-seated toroidal fields and it is largest at mid latitudes.

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References

Belvedere, G., Kuzanyan, K.M., Sokoloff, D.D.: 2000, Mon. Not. Roy. Astron. Soc. 315, 778.
Brandenburg, A., Käpylä, P.J.: 2007, New J. Phys. 9, 305.
Brandenburg, A., Subramanian, K.: 2005, Phys. Rep. 417, 1.
Brandenburg, A., Rädler, K.-H., Schrinner, M.: 2008, Astron. Astrophys. 482, 739.
Caligari, P., Moreno-Insertis, F., Schüssler, M.: 1995, Astrophys. J. 441, 886.
Chatterjee, P., Nandy, D., Choudhuri, A.R.: 2004, Astron. Astrophys. 427, 1019.
Choudhuri, A.R., Schüssler, M., Dikpati, M.: 1995, Astron. Astrophys. 303, L29.
Dasi-Espuig, M., Solanki, S.K., Krivova, N.A., Cameron, R., Peñauela, T.: 2010, Astron. Astrophys. 518, A7.
Dikpati, M., Charbonneau, P.: 1999, Astrophys. J. 518, 508.
Dorch, S.B.F., Nordlund, Å.: 2001, Astron. Astrophys. 365, 562.
Drobyshevski, E.M., Yuferev, V.S.: 1974, J. Fluid Mech. 65, 33.
D'Silva, S., Choudhuri, A.R.: 1993, Astron. Astrophys. 272, 621.
Durney, B.R.: 1995, Solar Phys. 160, 213.
Gilman, P.A., Miesch, M.S.: 2004, Astrophys. J. 611, 568.
Gruzinov, A.V., Diamond, P.H.: 1994, Phys. Rev. Lett. 72, 1651.
Guerrero, G., de Gouveia Dal Pino, E.M.: 2008, Astron. Astrophys. 485, 267.
Guerrero, G., Chatterjee, P., Brandenburg, A.: 2010, Mon. Not. Roy. Astron. Soc. 409, 1619.
Hotta, H., Yokoyama, T.: 2010, Astrophys. J. Lett. 714, 308.
Käpylä, P.J., Korpi, M.J., Tuominen, I.: 2006, Astron. Nachr. 327, 884.
Kitchatinov, L.L.: 1988, Astron. Nachr. 309, 197.
Kitchatinov, L.L., Mazur, M.V.: 2000, Solar Phys. 191, 325.
Kitchatinov, L.L., Olemskoy, S.V.: 2011a, Astron. Nachr. 332, 496.
Kitchatinov, L.L., Olemskoy, S.V.: 2011b, Mon. Not. Roy. Astron. Soc. 411, 1059.
Kitchatinov, L.L., Rüdiger, G.: 2006, Astron. Astrophys. 453, 329.
Kitchatinov, L.L., Rüdiger, G. 2008, Astron. Nachr. 329, 372.
Köhler, H.: 1973, Astron. Astrophys. 25, 467.
Komm, R.W., Howard, R.F., Harvey, J.W.: 1993, Solar Phys. 147, 207.
Krause, F., Rädler, K.-H.: 1980, Mean-Field Magnetohydrodynamics and Dynamo Theory, Akademie-Verlag, Berlin.
Leighton, R.B.: 1969, Astrophys. J. 156, 1.
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Obridko, V.N., Sokoloff, D.D., Kuzanyan, K.M., Shelting, B.D., Zakharov, V.G.: 2006, Mon. Not. Roy. Astron. Soc. 365, 827.
Ossendrijver, M., Stix, M., Brandenburg, A., Rüdiger, G.: 2002, Astron. Astrophys. 394, 735.
Parker, E.N.: 1955, Astrophys. J. 122, 293.
Rädler, K.-H.: 1969, Z. Naturforch. 23а, 1851.
Rüdiger, G., Brandenburg, A.: 1995, Astron. Astrophys. 296, 557.
Sokoloff, D., Zhang, H., Kuzanyan, K.M., Obridko, V.N., Tomin, D.N., Tutubalin, V.N.: 2008, Solar Phys. 248, 17.
Spence, E.J., Nornberg, M.D., Jacobson, C.M., Parada, C.A., Taylor, N.Z., Kendrick, R.D., Forest, C.B.: 2007, Phys. Rev. Lett. 98, 164503.
Steenbeck, M., Krause, F.: 1969, Astron. Nachr. 291, 271.
Stenflo, J.O.: 1988, Astrophys. Space Sci. 144, 321.
Tobias, S.M.: 2009, Space Sci. Rev. 144, 77.
Tobias, S.M., Brumell, N.H., Clune, T.L., Toomre, J.: 1998, Astrophys. J. Lett. 502, 177.
Tobias, S.M., Brumell, N.H., Clune, T.L., Toomre, J.: 2001, Astrophys. J. 549, 1183.
Tuominen, I., Brandenburg, A., Moss, D., Rieutord, M.: 1994, Astron. Astrophys. 284, 259.
Vainshtein, S.I., Kitchatinov, L.L.: 1983, Geophys. Astrophys. Fluid Dyn. 24, 273.
Yousef, T.A., Brandenburg, A., Rüdiger, G.: 2003, Astron. Astrophys. 411, 321.
Zhao, J., Kosovichev, A.G.: 2004, Astrophys. J. 603, 776.
Zeldovich, Ya.B.: 1957, JETP 4, 460.
Ziegler, U., Rüdiger, G.: 2003, Astron. Astrophys. 401, 433.
