A dual Ginzburg-Landau model of the knotted chromo-electric flux-tube is revisited, in which the covariant decomposition of gluon field and the random phase approximation are used. It is shown that the SU(2) QCD vacuum is of type-II superconductor, with the Ginzburg-Landau parameter \( \kappa = \sqrt{3} \), being independent of the magnetic condensate and strong coupling used, and consistent with the lattice data. The mass spectrum of a number of \( f_J \) meson states with \( J \leq 2 \), which are taken to be of glue dominate, are computed with help of the energies of the knotted(linked) QCD fluxtubes. The low-lying \( f_J \) states (\( \leq 1.7 GeV \)) are shown to be associated with the string excitations of the types \( 2^T \) and \( K = 3_1 \) in knot topologies.

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Key Words QCD vacuum, Ginzburg-Landau model, Glueballs, \( f \) meson states, knotted fluxtube

1 Introduction

According to the theory of strong interaction, quantum chromodynamics(QCD), there will be pure-glue excitation known as glueball, and the \( q\bar{q} \) states with explicit glue(known as hybrid mesons), which can have quantum numbers forbidden to the \( q\bar{q} \) systems in the naive quark model. These gluonic excitations have been the subject of the extensive experimental and
theoretical studies (see, for instance, [1, 2]) as the identification and observation of these objects can be a good test of QCD at the low-energy limit. After the early work of the bag-model [3] of the glueballs, with mass prediction about 1.0-2.0 GeV/c^2, many glueball (or, glue-rich meson) states were explored with various approaches, including QCD on the lattice, the QCD sum rule, the constituent gluon models, and etc., leading to a deep relation between the properties of these states and the structure of the QCD vacuum [1, 4]. Owing to the mixing with the normal q \bar{q} states, to accommodate these gluonic excitations (except for the lightest tensor sector) remains to be an open question [1, 2, 4].

The present work revisits the low-lying f_J meson states (listed in the Particle Data Group [5]) using the dual Ginzburg-Landau (GL) model proposed in our previous work [6], which is based on the reformulated Yang-Mill (YM) theory in terms of the field decomposition [7, 8, 9] of gluon variables. We show that the vacuum of the SU(2) QCD is of a type-II superconductor type and the GL parameter is \( \kappa = \sqrt{3} \) for the vacuum condensate, being consistent with the lattice simulation [10] and independent of the magnetic condensate as well as the strong coupling that are used in the calculation. We further compute the mass spectrum for a number of the low-lying f_J meson states assuming that the these mesons are mainly formed by the tightly knotted (or linked) QCD fluxtubes, that is, the knotted (or linked) QCD strings (Hereafter, we will also use the term knots for the closed types of strings including links). The random vacuum approximation is used for the vacuum and main features of a few low-lying f_J states, the candidates for the glue-dominated states, are examined in associated with the knot topology of the chromo-electric fluxtubes for \( J \leq 2 \). Though the starting point for model construction is the two-color gluodynamics, the relevance of the present work to the real QCD can be inferred from the fact that \( N \)-color QCD can be expressed as a sum over copies of two-color QCD [11] as long as the Abelian dominance is valid for QCD [12], as shown in lattice QCD [13, 14, 15].

The idea of the gluonic fluxtubes [16, 17] can be traced back to the early days of QCD when the resonance string (i.e., the QCD string) was used to describe the confining force connecting the quarks [18]. The formation of a gluonic fluxtube between two widely separated quarks is widely accepted, and is supported by the lattice QCD simulations [13, 14, 15]. Meanwhile, it is expected that the glueballs, if exist, are intimately related with the closed fluxtubes that can be omitted, for instance, by a long linear string between the quarks. Using a flux-tube model, Isgur and Paton [19] predicted that the lightest glueball has the quantum number \( J^{PC} = 0^{++} \) and a mass of about 1.520 GeV/c^2. The further predictions made later by the flux-tube model [20] for the three lightest glueball masses are consistent with the lattice calculations [1, 21]. Assuming the closed (knotted) configurations for the gluonic fluxtubes, the glueballs were explored by using the various actions, such as the Nambu-Goto action in the case of the circular string [22], the nonlinear sigma actions [23] and its extensions [24] in the case of knotted strings. For the early attempts to model hadrons in terms of closed strings of quantized fluxtube, see Ref. [25].
Our study is motivated by the ideal representation \[26\] of the knot geometry, which provides a model-independent relationship between the length-to-diameter ratio and the average crossing number of knot with the given topology. This representation complements the nonlinear dynamics of the stable knot found in field theory\[27, 28\], and is greatly helpful in evaluating the glueball-like meson spectrum in terms of the knotted objects in a bag model of fluxtube\[29\]. The calculations in this work involve the Nielsen-Olesen(NO) vortices in the dual GL theory as an effective description of the gluonic fluxtubes, for which the knot geometry previously explored was utilized to calculate the energies of the low-lying glueball-like meson states, similar to that in \[29\].

2 The order parameters in long-distance gluodynamics

We begin with the reformulations of SU(2) gluodynamics via the covariant field decomposition of SU(2) gluon variables, known as Cho-Faddeev-Niemi(CFN) decomposition\[7, 8, 9\]. The gluon field $\vec{A}_\mu$ (the arrow denotes the three color indices $a = 1, 2, 3$, along the generators $\tau^a$) is decomposed into \[7\] $\vec{A}_\mu = A_\mu \hat{n} + \vec{C}_\mu + \vec{X}_\mu$, in which $A_\mu = \vec{A}_\mu \cdot \hat{n}$ is an Abelian gluonic potential, $\hat{n}(x)$ an unit isotriplet in color space, $\vec{C}_\mu := g^{-1} \partial_\mu \hat{n} \times \hat{n}$ the (non-Abelian) magnetic potential, and $\vec{X}_\mu$ (normal to $\hat{n}$) an covariant field, having to be constrained by two extra conditions\[30\].

We use the simplest choice of the condition\[9\], $\vec{X}_\mu = g^{-1} \phi_1 \partial_\mu \hat{n} + g^{-1} \phi_2 \partial_\mu \hat{n} \times \hat{n}$. With this change of variables, the YM Lagrangian becomes

$$L_Y = -\frac{1}{4} (F_{\mu\nu} - \frac{Z(\phi)}{g} H_{\mu\nu})^2 - \frac{1}{4g^2} \{(n_{\mu\nu} - iH_{\mu\nu})(\nabla^\mu \phi)^\dagger \nabla^\nu \phi + h.c\}, \quad (1)$$

where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$, $Z(\phi) \equiv 1 - |\phi|^2$, $\phi = \phi_1 + i\phi_2$, $n_{\mu\nu} \equiv \eta_{\mu\nu}(\partial \hat{n} \cdot \partial \hat{n})$, $\nabla^\mu \phi \equiv (\partial^\mu - igA^\mu)\phi$ is the U(1) covariant derivative induced by the gauge U(1) rotation $U(\alpha \hat{n}) = \exp(i\alpha n^a \tau^a)$ around direction $\hat{n}$. The chromo-magnetic field $H_{\mu\nu} \equiv \hat{n} \cdot (\partial_\mu \hat{n} \times \partial_\nu \hat{n})$ is partially dual to the chromo-electric field $F_{\mu\nu}$, as shown in \[8, 9\]. Such a change of variables in the reformulated dynamics \[1\] implements the Abelian projection \[12\] in a covariant way\[9\]. By choosing $\hat{n}(x)$ in \[1\] as an infrared order parameter and using renormalization group analysis, the Skyrme-Faddeev(SF) model\[27\], which supports the knotted solitons with nonzero Hopf charges, was proposed to model the low-energy dynamics of quantum YM theory\[9\].

To see the implications of order parameter $\hat{n}(x)$, it is helpful to re-examine the connection of the CFN field decomposition with the Abelian projection from the viewpoint of the basis change. As shown in \[30\] the choice of the constrains on $\vec{X}_\mu$ implies to choose a local gauge transformation which maps the generic gluon field $\vec{A}_\mu$ into a gauge-fixed surface in the space of the gluon fields, which yields that the map will necessarily be singular somewhere in spacetime. The magnetic degree enters through the topological variable $\hat{n}(x)$, which provides the knot dynamics\[27, 28\] and classifies the physical region $V$ considered (as a mapping from $V$ to $S^2$) according to the
homotopy \( \pi_2(V) \). We note that when the \( U(1) \) symmetry (rotation around \( \hat{n} \) by angle \( \alpha \)) is left unbroken, \( \hat{n} \) serves as the mapping transformation from the asymptotically free gluon \( \hat{A}_\mu \), which is represented in terms of the global basis \{\( \tau^{1-3} \)\}, to the infrared variables, which are represented in terms of the local basis \{\( \hat{n}, \partial_\mu \hat{n}, \partial_\mu \hat{n} \times \hat{n} \)\}, with nontrivial metric in the gauge group space. The QCD vacuum differs from the perturbative one owing to the nontrivial homotopic class of the map \( \hat{n}(x) \), or, equivalently, to the singularities (magnetic charges) in the magnetic potential \( \hat{C}_\mu \), namely, the zeroes of the map \( \hat{n}(x) \). The field decomposition in terms of the local basis collapses when \( \hat{n} \) becomes globally fixed \( \hat{n}(x) \to \hat{n}_0 \) (i.e., the norm of \( \partial_\mu \hat{n} \) vanishes) so that \( \{\hat{n}, \partial_\mu \hat{n}, \partial_\mu \hat{n} \times \hat{n} \} \) degenerates. In the latter situation, one has instead to go back, discontinuously in the field mapping, to the usual asymptotically free gluon variables \( A_\mu = A^a(x) \tau^a \) in the usual matrix representation of \( \hat{A}_\mu \). This discontinuous transition in the local gauge-fixing differs the asymptotically free phase of QCD from the confining one. Thus, the nonvanishing vacuum expectation value (VEV) \( \langle (\hat{n})^2 \rangle \) is required for the CFN field decomposition to be a true change of variables.

Another order parameter arises by looking the confining phase of the theory \( (1) \). Taking the pure Abelian gauge \( A_\mu = \partial_\mu \xi \) so that classically \( F_{\mu \nu} = 0 \), as should be in the presumed vacuum condensate, the theory \( (1) \) becomes

\[
\mathcal{L}^M = -\frac{Z^2(\rho)}{4g^2} H^2_{\mu \nu} - \frac{r^2}{4} \left[ \partial^\mu \rho \partial^\nu s + g \partial^\mu \xi \partial^\nu \rho \right] H_{\mu \nu} + (2g)^2 \left\{ [2g \partial^\mu \xi \partial^\nu s - g^2 \partial^\mu \xi \partial^\nu \xi - \partial^\mu s \partial^\nu s] \rho^2 - \rho^4 \rho \right\} n_{\mu \nu}
\]

in which \( \phi = \rho(x)e^{i\xi(x)} \) has been used. One sees here that \( Z(\rho = |\phi|) \) resembles the dia-electric factor in the dia-electric soliton model \( [31] \) and the gauge-invariant kernel in the effective model of confinement \( [32] \) if one interprets the background media associated with \( \phi \) as the QCD vacuum. Explicitly, \( Z(\phi \to 0) = 1 \) corresponds to the normal vacuum and \( Z(\rho \to v) \neq 0 \) (here, \( v = \langle \rho \rangle \) is positive constant) to the condensate. Indeed, taking the limit \( \phi \to ve^{i\xi_0} \), the Eq. \( (2) \) can be further reduced to

\[
\mathcal{L}^M = -\frac{(Z^2(\rho))_{\text{ren}}}{4g^2} (\hat{n} \cdot \partial_\mu \hat{n} \times \partial_\nu \hat{n})^2 - \frac{a^2(\partial \xi)^2}{2} (\partial_\mu \hat{n})^2 + \frac{1}{2} \partial^\mu \xi \partial^\nu \xi (\partial_\mu \hat{n} \cdot \partial_\nu \hat{n}) + V(n \cdot h, \xi) + \cdots.
\]

Here, the second term quadric in derivatives of \( \hat{n} \) and potential terms \( V(n \cdot h, \xi) \) are added based on the renormalization group analysis, as done in \( [9, 24] \). As we can see, \( (3) \) is an extended version of the SF model\( [27] \) (see also \( [24] \)). Given the nontrivial configuration \( \hat{n}(x) \), the knot is classified by non-zero Hopf charge \( Q = 1/(32\pi^2) \int d^3 x e^{ijk} C_i H_{jk} \), with \( C_i \) (having no local form in terms of \( \hat{n} \)) defined mathematically by \( dC = H \) (The notation \( H \equiv H_{jk} dx^j \wedge dx^k / 2 = (\hat{n}, d\hat{n} \wedge d\hat{n}) \)). Assuming there is a localized excitation \( G \) of \( \hat{n}(x) \) in subregion \( V_G \subset V \) and using the virial theorem, the energy density estimated by \( (3) \) is about \( H_G \propto 2(\langle \hat{n}_\mu \hat{n}^\mu \rangle^2) + V_{\text{min}}, \) which is in consistent with \( \langle (\hat{n}_\mu \hat{n}^\mu) \rangle \neq 0 \) in \( V_G \), while \( \langle \hat{n}_\mu \hat{n}^\mu \rangle = 0 \) far away from \( V_G \) due to the finite energy condition. The situation is opposite for \( \phi \), which tends to zero at the core of \( V_G \) and to the
nonzero condensate $v = \langle \rho \rangle$ outside $V_G$, as it should be according to the resemblance between $Z(\rho = |\phi|)$ in (2) and the dia-electric factor.

The quantitative behavior of $\hat{n}$ and $\phi$ in $G$ can be related to the chromo-magnetic symmetry breaking, where the chromo-magnetic symmetry $H_M$ is defined by [8]

$$D_\mu \hat{n}(x) \equiv (\partial_\mu + g\vec{A}_\mu \times \hat{n}(x)) = 0. \quad (4)$$

Observed that the CFN decomposition implies $D_\mu \hat{n}(x) = g(\vec{X}_\mu \times \hat{n})$, one sees that the breaking of $H_M$ is amount to the nonvanishing VEV. of $(D_\mu \hat{n})^2$, namely,

$$0 \neq g^2 \langle (\vec{X}_\mu \times \hat{n})^2 \rangle = \langle (\hat{n}_1 \partial_\mu \hat{n} - \hat{n}_2 \partial_\mu \hat{n})^2 \rangle = \langle (\hat{n}_1^2 + \hat{n}_2^2) (\partial_\mu \hat{n})^2 \rangle, \quad (5)$$

in which the reparameterization [9] for $\vec{X}_\mu$ is used. In the case of weak correlation between the off-diagonal variables $(\phi, \hat{n})$, (5) implies

$$0 \neq \langle |\phi|^2 \rangle, \text{ Outside core}(V_G),$$

$$0 \neq \langle (\partial_\mu \hat{n})^2 \rangle, \text{ in } V_G, \quad (6)$$

which agrees with the existence of the order parameters $\hat{n}$ and $\phi$. In short, the arising of the order parameter $\hat{n}$ and $\phi$ are associated with the breaking of $H_M$ defined by (4). Observed that $(\vec{X}_\mu \times \hat{n})^2 = \vec{X}_\mu^2$ in which $\vec{X}_\mu \cdot \hat{n} = 0$, the chromo-magnetic symmetry breaking (5) is amount to the $X$-field condensation

$$0 \neq \langle (\vec{X}_\mu)^2 \rangle = \langle |\phi|^2 (g^{-1} \partial_\mu \hat{n})^2 \rangle.$$

Such type of condensation, referred as the off-diagonal gluon condensation, was seen in the lattice simulations [33] for gluodynamics.

3 The random phase approximation

To have the effective dynamics of the Abelian-Higgs multiplets $(A_\mu, \phi)$, we need the further approximation associated with the vacuum of QCD. We will use the random phase approximation(RPA), which can be due to the residual $U(1)$ symmetry shared by the dual dynamics (1). To see this, it is convenient to use two normalized basis $\{\hat{e}_1, \hat{e}_2\}$, such that $\hat{e}_1 \times \hat{e}_2 = \hat{n}$, to describe the 2-dimensional plane $\Sigma(x)$ which is normal to $\hat{n}(x)$ at $x$. One can use a transformation from $\{\hat{e}_1, \hat{e}_2\}$ to $\{\hat{e}_+, \hat{e}_-\}$ explicitly for the isomorphic map: $SO(2) \rightarrow U(1)$.

Take $\hat{e}_\mu$ to be the direction vector of $\partial_\mu \hat{n}$, one can then write, as $\partial_\mu \hat{n}$ is in the plane $\Sigma(x),$

$$\hat{e}_\mu = a_1^1 \hat{e}_1 + a_2^2 \hat{e}_2,$$

$$= a_\mu \hat{e}_+ + \bar{a}_\mu \hat{e}_- \quad (7)$$
where \( a_{\mu}^{1,2} \) are the two real four-vectors such that \((a_{\mu}^{1})^2 + (a_{\mu}^{2})^2 = 1\), and

\[
\begin{align*}
a_{\mu} &= a_{\mu}^{1} - i a_{\mu}^{2}, \quad \bar{a}_{\mu} = a_{\mu}^{1} + i a_{\mu}^{2}, \\
\hat{e}_{+} &= \frac{1}{2} (\hat{e}_{1} + i \hat{e}_{2}), \quad \hat{e}_{-} = \frac{1}{2} (\hat{e}_{1} - i \hat{e}_{2}).
\end{align*}
\]

It is easy to show that

\[
\begin{align*}
\hat{e}_{+} \cdot \hat{e}_{-} &= 1/2, \\
\hat{e}_{+} \times \hat{e}_{-} &= -\frac{1}{2} i \hat{n} = -\hat{e}_{+} \times \hat{e}_{-}, \\
\hat{e}_{+} \times \hat{n} &= i \hat{e}_{+}, \quad \hat{e}_{-} \times \hat{n} = -i \hat{e}_{-}, \\
\hat{e}_{-}^2 &= |a_{\mu}|^2 = (a_{\mu}^{1})^2 + (a_{\mu}^{2})^2 = 1.
\end{align*}
\]

The last equation of (8) implies that \( a_{\mu}(x) = e^{i \theta_{\mu}(x)} \), with \( \theta_{\mu}(x) \) a real phase.

From the equations (8), one has

\[
\begin{align*}
\hat{e}_{\mu} \cdot \hat{e}_{\nu} &= \frac{1}{2} (a_{\mu} \bar{a}_{\nu} + \bar{a}_{\mu} a_{\nu}) \\
&= \cos(\theta_{\mu} - \theta_{\nu}), \\
(\hat{e}_{\mu} \times \hat{e}_{\nu}) \cdot \hat{n} &= \frac{1}{2} (\bar{a}_{\mu} a_{\nu} - a_{\mu} \bar{a}_{\nu}) \\
&= \sin(\theta_{\mu} - \theta_{\nu}), \\
\hat{e}_{\mu} \cdot \hat{e}_{\nu} + i (\hat{e}_{\mu} \times \hat{e}_{\nu}) \cdot \hat{n} &= e^{i(\theta_{\mu} - \theta_{\nu})},
\end{align*}
\]

Under the small gauge rotation \( U(\alpha \hat{n}) \), with \( \alpha \) a small angle, one can show that

\[
\begin{align*}
\hat{e}_{\mu} &\rightarrow \hat{e}_{\mu} + \delta \hat{e}_{\mu}, \\
&= \hat{e}_{\mu} + \hat{e}_{\mu} \times (\alpha \hat{n}), \\
&= (a_{\mu}^{1} + \alpha a_{\mu}^{2}) \hat{e}_{1} + (a_{\mu}^{2} - \alpha a_{\mu}^{1}) \hat{e}_{2}, \\
&= a'_{\mu} \hat{e}_{+} + \bar{a}'_{\mu} \hat{e}_{-},
\end{align*}
\]

where

\[
a'_{\mu} = a_{\mu} + i \alpha a_{\mu} = e^{i \alpha} a_{\mu}.
\]

This means that the gauge rotation \( U(\alpha \hat{n}) \) corresponds to

\[
\hat{e}_{\mu} \stackrel{U(\alpha \hat{n})}{\rightarrow} e^{i(\alpha + \theta_{\mu})} \hat{e}_{+} + e^{-i(\alpha + \theta_{\mu})} \hat{e}_{-},
\]

or equivalently, to the phase shift \( \theta_{\mu} \rightarrow \theta_{\mu} + \alpha \).

By writing \( \partial_{\mu} \hat{n} = M(x) \hat{e}_{\mu} \) and using (7) and (8), one has for the \( SU(2) \) gluon field

\[
\tilde{A}_{\mu} = A_{\mu} \hat{n} + \frac{M(x)}{g} [(i + \phi) a_{\mu} \hat{e}_{+} + h.c]
\]

one can show that under \( U(\alpha \hat{n}) \)

\[
\begin{align*}
\tilde{A}_{\mu} &\rightarrow \tilde{A}_{\mu} + \frac{1}{g} D_{\mu}(\tilde{A}_{\mu})(\alpha \hat{n}), \\
&= (A_{\mu} + \frac{\partial_{\alpha}}{g}) \hat{n} + \frac{M(x)}{g} [(i + \phi + i \alpha \phi) a_{\mu} \hat{e}_{+} + h.c], \\
&= (A_{\mu} + \frac{\partial_{\alpha}}{g}) \hat{n} + \frac{M(x)}{g} [(i + e^{i \alpha} \phi) a_{\mu} \hat{e}_{+} + h.c].
\end{align*}
\]
Comparison (10) with (11) shows that the rotation $U(\alpha \hat{n})$ leads to transformation in the variables $(A_\mu, \phi)$ as exactly as that for the Abelian Higgs multiplets: $A_\mu \to A_\mu + \partial_\mu \alpha / g, \phi \to e^{i\alpha} \phi$. This indicates that the unbroken $U(1)$ symmetry shared by the dual dynamics (11) is the local gauge rotation: $U(\alpha \hat{n})$.

We introduce the RPA such that $\langle \hat{e}_\mu \rangle = 0$, which, by noticing the residual symmetry (9), means

$$\langle e^{i\theta_\mu} \rangle = 0.$$  \hspace{1cm} (12)

Moreover, in the case that $\mu \neq \nu$ one has, from (12)

$$\langle e^{i(\theta_\mu - \theta_\nu)} \rangle \approx \langle e^{i\theta_\mu} \rangle \langle e^{-i\theta_\nu} \rangle = 0.$$  \hspace{1cm} (13)

The RPA introduced here assumes $\theta_\mu$ to be random distributed in the QCD vacuum. As will be explored in the following section, the RPA is very useful to find the effective $U(1)$ dynamics of the collective variables $(A_\mu, \phi)$.

In order to describe the knot-like excitations, the SF-like dynamics of the order parameter $\hat{n}$, similar to (3), are usually used, as done in [34, 24]. Owing to the difficulty for extracting the parameters in (3), an alternative approach is to use the dynamics of $(A_\mu, \phi)$ to describe the fine profile of the chromo-electric fluxtube, and to extract the energy of the closed fluxtube by utilizing the values of the universal invariants for knot geometry [26], which is main propose of this paper.

4 The dual Ginzburg-Landau model

From the section 2, we know that the order parameter $\phi(x)$ in (1) is suitable to play the role of soliton field interpolating in between the two vacua: $\phi(x) = 0$ and $\phi(x) = v(\neq 0)$. Writing $\phi(x) = \Phi(x) + \delta\phi$, where $\Phi(x)$ is the complex condensate and $\delta\phi$ its quantum fluctuation, one has a nonzero correlation

$$\langle \phi(x)\phi^\dagger(y) \rangle \approx \Phi(x)\Phi^*(y), \text{for } x^0 > y^0.$$  \hspace{1cm} (14)

To find an effective model for $(A_\mu, \phi)$ starting from (1), let us take the mean field approximation

$$\partial_\mu \hat{n}(x) = M \hat{e}_\mu(x),$$  \hspace{1cm} (15)

with $\hat{e}_\mu$ the unit vectors defined in (7), and $M = \langle (\partial_\mu \hat{n})^2 \rangle^{1/2}$ (no summing over $\mu$) a constant in spacetime. The equation (15) yields

$$\langle \partial_\mu \hat{n} \rangle^2 = M^2 \{ (\hat{e}_0)^2 - \sum_{i=1}^3 (\hat{e}_i)^2 \} = -2M^2$$

$$H_{\mu\nu} = M^2 h_{\mu\nu},$$  \hspace{1cm} (16)

$$h_{\mu\nu} = \hat{n} \cdot (\hat{e}_\mu \times \hat{e}_\nu) = \sin \theta_{\mu\nu}$$
with $\theta_{\mu\nu} = \theta_\mu - \theta_\nu$ the angle between the directions $\hat{e}_\mu$ and $\hat{e}_\nu$ in the internal space. In addition, one has
\[
\partial_\mu \hat{n} \cdot \partial_\nu \hat{n} = M^2 \cos \theta_{\mu\nu},
\]
\[
n_{\mu\nu} = -M^2(2\eta_{\mu\nu} - \cos \theta_{\mu\nu}),
\]
\[
H_{\mu\nu} H^{\mu\nu} = M^4 h_{\mu\nu} h^{\mu\nu} = \frac{M^4}{2} \sum_{\mu\nu}(1 - \cos 2\theta_{\mu\nu}).
\]

Defining the magnetic condensate $H$ by $H^2 \equiv \langle H_{\mu\nu} H^{\mu\nu} \rangle$ and using the RPA, one finds, by using (17) and (13),
\[
H^2 = 6M^4,
\]
(18)

namely, $M^2 = H/\sqrt{6}$. In deriving (18), we have used $\sum_{\mu\nu} 1 = 12$, and $\langle \cos 2\theta_{\mu\nu} \rangle \simeq 0$ according to the RPA. We see from (18) that the very existence of the nonvanishing VEV. $\langle (\partial \hat{n})^2 \rangle$ implies the chromo-magnetic condensation, $\langle H_{\mu\nu} H^{\mu\nu} \rangle \propto M^4$, which is due to the breaking $H_M$ of the magnetic symmetry.

It is important to note here that the RPA ignores the possible nontrivial bending (or, twisting) of the $\hat{n}(x)$ orientation, which is crucial to identify the knot-like excitations. In the case that $\hat{n}(x) = \hat{n}_{cl}(x) + \delta \hat{n}$, where the variation of the classic part $\hat{n}_{cl}(x)$ is small (namely, the contribution to $M$ arises mainly from the quantum fluctuation $\delta \hat{n}$), one can take $\hat{n}_{cl}$ to be approximately parallel. Then, upon using (12) and the mean field approximation (15) as well as (16), the gluodynamics (1) becomes
\[
\mathcal{L}^\gamma = -\frac{1}{4} F_{\mu\nu}^2 + \frac{M^2}{4g^2} Z(\phi) F_{\mu\nu}^{\mu\nu} + \frac{M^4}{4g^2} Z(\phi)^2 h_{\mu\nu}^2 + \frac{M^2}{4g^2} \{[2\eta_{\mu\nu} + e^{i\theta_{\mu\nu}}](\nabla^\mu \phi)^\dagger \nabla^\nu \phi + h.c\},
\]
which, after utilizing (18), becomes
\[
\mathcal{L}^{eff} \simeq -\frac{1}{4} F_{\mu\nu}^2 + \frac{H}{\sqrt{6g^2}} \langle (\nabla^\mu \phi)^\dagger \nabla^\nu \phi \rangle - \frac{H^2}{4g^2} \langle Z(\phi)^2 \rangle.
\]

Here, the following relations, which are due to the RPA, are used,
\[
\langle h_{\mu\nu} \rangle \simeq 0, \langle h^2_{\mu\nu} \rangle \simeq 6,
\]
\[
\langle e^{i\theta_{\mu\nu}} (\nabla^\mu \phi)^\dagger \nabla^\nu \phi \rangle \simeq 0.
\]

Using the Wick theorem and the Bose symmetry of the scalar field, one can show
\[
\langle (\phi^\dagger \phi)^2 \rangle = \langle \phi^\dagger \phi \rangle \langle \phi^\dagger \phi \rangle + \langle \phi^\dagger \phi \rangle \langle \phi^\dagger \phi \rangle + \langle \phi^\dagger \phi \rangle \langle \phi^\dagger \phi \rangle = 2\langle \phi^\dagger \phi \rangle^2,
\]
and
\[
\langle Z(\phi)^2 \rangle = \langle 1 + (\phi^\dagger \phi)^2 - 2\phi^\dagger \phi \rangle \
\approx 1 + 2(\Phi^* \Phi)^2 - 2\Phi^* \Phi \
= 2(\Phi^2 - 1/2)^2 + 1/4.
\]
where (14) is applied so that \( \langle (\nabla^\mu \phi )^\dagger \nabla_\mu \phi \rangle = (\nabla_\mu \Phi(x))^* \nabla^\mu \Phi(x) \). Rescaling the scalar \( \Phi \) to that with dimension of mass,
\[
\frac{\sqrt{H}}{\sqrt{6}g} \Phi(x) \to \Phi(x),
\]
we obtain, from (19), the effective dual GL model given by
\[
\mathcal{L}_{DGL}^{\text{DGL}} = -\frac{1}{4} F_{\mu \nu}^2 + |(\partial_\mu - igA_\mu)\Phi|^2 - V(\Phi) - \frac{H^2}{8g^2}.
\]
(20)

In terms of the re-scaled complex \( \Phi \), the potential in (20) is
\[
V(\Phi) = \frac{\lambda^2}{4} (|\Phi|^2 - v^2)^2,
\]
(21)
with the parameters given by
\[
\lambda = \sqrt{12}g, \\
v = \frac{\sqrt{H}}{\sqrt{6}g}.
\]
(22)

One sees that the effective model (20) for the gluodynamics takes the form of that for the dual superconductor\[34\]. It is remarkable that the scalar potential (21) assumes exactly the Mexico-hat form and ensures the dual Meissner effect for confining the chromo-electric field \( A_\mu \), since both of \( H \sim M^2 \) and \( g \) are positive. We also note that the ensuing dual superconductor picture with two vacua \( \Phi = 0 \) and \( \Phi = v \) in (20) agrees with the vacuum picture discussed in section 2.

It is known that the dual GL model (20) admits the NO vortex solution \[16\], with two length scales: the coherent length \( \xi = 1/m_\Phi \) and the penetrating length \( \lambda_L = 1/m_A \). The mass scales \( m_\Phi \) for the Higgs-like field \( \Phi \) and \( m_A \) for the chromo-electric field \( A_\mu \) are fixed by the explicit form of the potential (21). Writing in terms of \( \lambda \) and \( v \) in (22), or, equivalently of the magnetic condensate \( H \), they are
\[
m_\Phi = \frac{\lambda}{\sqrt{2}} v = \frac{\sqrt{\lambda}}{\sqrt{2}} \sqrt{H}, \\
m_A = \sqrt{2}gv = \frac{\sqrt{\lambda}}{\sqrt{6}}.
\]
(23)

which depend merely upon \( H = \sqrt{6}(\partial_\hat{n})^2 \) and are nonzero when the magnetic symmetry broken. Given the scales (23), one readily finds the GL parameter (defined by \( \kappa = \lambda_L/\xi \)) for the condensate vacuum to be
\[
\kappa = \frac{m_\Phi}{m_A} = \sqrt{3}, \text{ (type-II)},
\]
(24)
which is independent of the magnetic condensate \( H \). The GL parameter given by (24) predicts the vacuum of the SU(2) gluodynamics to be of the type of type-II superconductor, in nicely consistent with the lattice data \( \kappa = 1.702(= 0.16fm/0.094fm) \) for SU(2) gluodynamics\[10\].

The very fact that the potential (21) in (20) assumes the Mexico-hat form gives an independent argument for supporting the dual superconductor mechanism of the low-energy phase of QCD proposed by Nambu, ’t Hooft and others\[17, 12\]. The type-II superconductor was also confirmed in some of the lattice simulations, in which \( \kappa = 1.04(= 1.3614GeV/1.3123GeV) \)\[37\] and \( \kappa = 1.49(= 0.164fm/0.11fm) \)[38] were predicted.

9
5 Glueball-like mesons as knotted fluxtubes

Due to the $U(1)$ gauge symmetry for which $\pi_2(U(1) = 1)$, the dual GL model (20) does not allow the stable solution of closed fluxtube being against self-shrinking. To describe the knotted gluonic excitations, it entails to have a nonlinear dynamics of knotted configuration, such as SF-like model (3), which is difficult to solve for now. A way out is to use the Nambu-Goto action [22] to prevent the self-shrinking instability of the closed flux tubes in the GL model. In this section, we use (20) as a model of the gluonic flux-tube profile and calculate the glueball spectrum by taking into account both the string tension and the twisting of the knotted fluxtubes, given that the geometric ratios of the length to the diameter are known for a set of knot types [26].

Consider a slice of the vortex cross-section, with the cylindrical coordinates $(r, \theta)$ in it. We search for, as usual, the static NO(or, the Abrikosov) vortex solution to the model (20), in the Coulomb gauge ($\nabla \cdot \vec{A} = 0$), in the form

$$\vec{A}(\vec{x}) = \hat{\theta} A(r) = -\frac{n}{gr}[1 - F(r)], A_0 = 0,$$

$$\Phi(\vec{x}) = v\rho(r) \exp(in \theta),$$

with the boundary condition $F(r \to \infty) = 0, \rho(r \to \infty) = 1$. The chromo-electric field is given by $(\vec{E}, \vec{B})$, with $\vec{E} = 0$ in the present case, and

$$\vec{B} = \nabla \times \vec{A} = \hat{z} \frac{n}{gr} \left( \frac{dF}{dr} \right). \quad (25)$$

The static energy for (20) is

$$E^{DGL} = \int d^3x \left\{ \frac{\vec{B}^2}{2} + |(\partial_i - igA_i)\Phi|^2 + V(\Phi) + \frac{H^2}{8g^2} \right\}, \quad (26)$$

with $V(\Phi)$ given by (21). In terms of the vortex profiles $(F(r), \rho(r))$, the energy (26) becomes

$$E = \int d^3x \left[ \frac{1}{2} \left( \frac{n}{gr} \right)^2 \left( \frac{dF}{dr} \right)^2 + v^2 \left( \frac{d\rho}{dr} \right)^2 + \frac{n^2v^2}{r^2} F^2 \rho^2 + V(\rho) \right], \quad (27)$$

where the constant energy ($\varepsilon \propto \int H^2 d^3x$) in (26) is treated as the zero-point energy. Using (21) and (22), one has for the potential,

$$V(\rho) = \frac{H^2}{8g^2} (\rho^2 - 1)^2.$$

Introducing a dimensionless variable $x = r/a$, with $a$ a length scale, the static equations of motion for (27) are

$$\frac{d^2F}{dx^2} - \frac{1}{x} \frac{dF}{dx} = A\rho^2 F,$$

$$\frac{d^2\rho}{dx^2} + \frac{1}{x} \frac{d\rho}{dx} - \frac{n^2}{x^2} \rho = B\rho (\rho^2 - 1), \quad (28)$$

10
with the controlling parameters given by

\[ A = (\sqrt{2}gva)^2, \]
\[ B = (\lambda va/\sqrt{2})^2. \]

(29)

The asymptotic behavior of the vortex profiles \( F(x) \) and \( \rho(x) \) reads

\[ F(x \to 0) \approx 1 - c_n x^2, \]
\[ \rho(x \to 0) = c'_n K_n(\sqrt{B}x) \approx c'_n x^n, \]

(30)

and

\[ F(x \to \infty) = C_n K_1(\sqrt{A}x) \approx C_n \sqrt{\pi e} e^{-\sqrt{A}x}, \]
\[ \rho(x \to \infty) = 1 - C'_n K_0(\sqrt{2B}x) \approx 1 - C'_n \sqrt{\pi e} e^{-\sqrt{2B}x}, \]

(31)

with \( c_n, c'_n, C_n, C'_n \) the constants for given \( n \), and \( K_j(x) \) the Bessel function of second type. The expressions (30) and (31) are useful and will be incorporated in the specification of the boundary condition in the process of the numerical relaxation for solving (28) in finite (but large) interval.

Assuming that the vortex has a finite length \( L \) along \( z \) direction and integrating the angular(\( \theta \)) part, one has, from (27)

\[ E^L = 2\pi L \left[ \left(\frac{n/g}{a^2}\right)^2 I^{\text{glue}} + v^2 I^B + \frac{a^2 H^2}{g^2} I^N \right], \]

(32)

with three dimensionless integrals given by

\[ I^{\text{glue}} = \int_0^\infty dx \left( \frac{dF}{dx} \right)^2, \]
\[ I^B = \int_0^\infty dx \left[ x \left( \frac{d\rho}{dx} \right)^2 + \frac{n^2}{x} F^2 \rho^2 \right], \]
\[ I^N = \int_0^\infty dx \left[ \frac{x}{8} (\rho^2 - 1)^2 \right]. \]

(33)

The first term in (32) is the chromo-electric energy and roughly scales as \( \sim L(\text{tr} \Phi_E^2)/(\pi a^2) \), due to the flux conservation, with \( \Phi_E \) the flux of chromo-electric field (25) in the area \( \pi a^2 \) of the vortex cross-section. Since the three integrals in (33) are all positive for the NO vortex, the stable vortex exists, for which the first and third terms in (32) balance.

The minimization of (32) with respect to \( a \) yields

\[ a(=a_n) = \frac{k_n}{\sqrt{H}}, \]

(34)

which depends explicitly on the vortex quantum number \( n \). In deriving (31), (22) are used. On the other hand, using (22), (29) and (34), one has

\[ A = \frac{k_n^2}{\sqrt{6}}, B = \frac{\sqrt{6}k_n^2}{2}. \]

(35)

This means that the energy depends on \( a_n \) not only through (32) explicitly, but also implicitly through the integrals (33), which rely on \( a_n \) through the controlling parameters \( A \) and \( B \) (both of them \( \sim a_n^2 \)).
For given input \((\lambda, v)\), or equivalently, \((g, H)\), we numerically fix the length scale \(a\) in the case \(n = 1 \sim 7\), in two steps: (1) to find the vortex profile \((F, \rho)\) by solving the GL equation (28) using relaxation method with \(A\) and \(B\) given by (29) for a given \(a\); (2) to find the scale \(a = a_n\) for which the string tension \(\sigma = E^L/L\) is minimized by calculating \(E^L/L\) as a function of \(a\) through (32) and (33) for the profile given in (1). The optimal \(a\) and the corresponding \(k_n\), including three integrals \((I^{\text{glue}}, I^B, I^N)\), are listed in Table I. The numerical results are shown in FIG.1 for the profile \((F, \rho)\) and FIG.2 for the tension \(\sigma(a)\) as a function of \(a\), for \(n\) up to 4. The relation (35) are found to be fulfilled for calculated \(a_n\) in (54). In the process of the numerical relaxation, the asymptotic behavior (30) and (31) are incorporated in the boundary condition and the initial vortex profiles \(F_{\text{initial}}(x) = \text{sec}(\sqrt{Ax})\) and \(\rho_{\text{initial}}(x) = \text{tanh}(\sqrt{2Bx})\) are used.

| \(n\) | 1   | 2   | 3   | 4   | 5   | 6   | 7   |
|------|-----|-----|-----|-----|-----|-----|-----|
| \(k_n\) | 1.0019 | 1.2982 | 1.5204 | 1.5945 | 1.7426 | 1.9648 | 2.2611 |
| \(r_n(Gev^{-1})\) | 0.5012 | 1.0490 | 1.4235 | 1.6156 | 1.8551 | 2.1420 | 2.5520 |
| \(I^{\text{glue}}\) | 0.1941 | 0.1941 | 0.1942 | 0.1942 | 0.1936 | 0.1933 | 0.1944 |
| \(I^B\) | 0.6770 | 1.5893 | 2.8129 | 4.3547 | 5.7913 | 6.5896 | 7.3093 |
| \(I^N\) | 0.0370 | 0.0418 | 0.0461 | 0.0490 | 0.0525 | 0.0601 | 0.0687 |
| \(a_n(Gev^{-1})\) | 1.2850 | 1.6650 | 1.9500 | 2.0450 | 2.2350 | 2.5200 | 2.9000 |

Table I

Figure 1: The vortex profiles for \(n = 1 \sim 4\). \(F\) decreases with \(r\) and changes slightly for different \(n\), while \(\rho\) increases faster as \(n\) increases. The plot is also given for the chromo-electric field \(B\) (with unit of \(GeV^2\)), which decreases from 0.1866\(GeV^2\) (at \(r = 0\)) to 0 (at \(r = \infty\)). Here, the strong coupling and the magnetic condensation are taken to be \(g = 3.2198, H = 0.5246GeV^2\).

It can be seen that (32) describes the linear rising \((\sim \sigma L)\) of the confining potential for large
Figure 2: The string(vortex) tension $\sigma = E^T/L$ as a function of the length scale $a$ for $n = 1 \sim 4$. Here, $g = 3.2198$, $H = 0.5246 GeV^2$. 

$L$. When $L$ decreases, there will be corrections to (32) due to short-range gluonic interactions. In QCD we can expect, quite generally, the full energy for the low-lying spectrum of the confining fluxtube with the fixed ends at distance $L$ to be 

$$E(L) = \sigma L + \varepsilon + \frac{c_0}{L} + \frac{m\pi}{L} + O(\frac{1}{L^2}).$$  

(36)

Here, $\sigma$ is the string tension, $\varepsilon$ the the zero-point energy in (26), $m\pi/L$ ($m = 1, 2, \cdots$) the vibrationally-excited energies of the string flux, and $c_0 = -\pi/12$ is the Casimir energy of zero-point fluctuations of the string. Combining (36) with (32), one obtains for the energy of the knot-like gluonic fluxtube.

$$E_{n,m} = 4\pi\epsilon(K)r_n\left[\frac{(n/g)^2}{a_n}f_{\text{glue}} + a_n b^2 f_B + a_n^3 H^2 f_N\right] + \varepsilon + \frac{c_0 + m\pi}{2\epsilon(K)r_n a_n},$$  

(37)

where $r_n = R/a_n$ is the ratio of the core radius $R$ of the vortex to $a_n$ in (34), and $\epsilon(K) = L/(2R)$ a topological invariant which is universal for a given knot type. In principle, $\epsilon(K)$ can be evaluated by the dynamics of knot like (3). This is not available by now since we fail to fix the explicit form of the potential $V(n \cdot h, \xi)$ in (3). Fortunately, $\epsilon(K)$ were previously determined for a set of knot types ($K'$s) via the Monte-Carlo simulations, partial of which are listed in Table II and III for our references.

We use the fluxtube energy (37) for these knot types(denoted by $K = 2_1^1, 3_1$, etc. in topology) to model the spectrum of the glueball-like mesons. The computed spectra are shown and compared with the data of the partial $f_J$ states ($J \leq 2$) (also a few $\eta$ states) in the Table.
II in the case of $n = 1$ and of some low $m$'s. In the evaluation of the glueball-like meson energies using (37), the strong coupling and chromo-magnetic condensate are chosen as

$$g = 3.2198, H = 0.5246\text{GeV}^2$$

so that $4\alpha_s/3 = g^2/(3\pi) = 1.1$ (see Ref.[19]) and

$$\epsilon_{\text{vac}} = H^2/(8g^2) = (0.24\text{GeV})^4;$$
$$v = \sqrt{H}/(\sqrt{6}g) = 0.1016\text{GeV}$$

with $\epsilon_{\text{vac}}$ the vacuum energy density, given in the lattice computation[40]. $H$ is fixed based on (23) through the lattice data for the coherent length $\xi = 1/m\Phi$ and penetrating length $\lambda_L = 1/m_A$. The numeric result for $a_n$ is $a_1 = 1.2850$ for $n = 1$ with the tension

$$\sigma = 0.125\text{GeV}^2,$$

and the other cases $n \geq 2$ are excluded since they yield anomalously high string tension, such as $\sigma = 0.339, 0.695$, etc., see FIG.2.

Different with (38), we alternatively choose, by taking $v = 0.1094\text{GeV}$ and $\epsilon_{\text{vac}} = -0.7\text{GeV}$, the input values of the parameters which change $H$ slightly, and thereby the scalar condensate $v = \langle \Phi \rangle$ (see (22)) so that

$$g = 3.2198, H = 0.6080\text{GeV}.$$  

The numerical data corresponding to (40) are shown in Table II and III for the mass spectrum of a number of low-lying $f_J$ states with $J \leq 2$.

It can be seen from Table II and Table III that a remarkable agreement was reached between the experimental and predicted spectrum for the meson $f_J$ states, showing that the most of the meson $f_J$ states can be identified as the knotted string excitations of the types \((n, m) = (1, j), j = 0, 1, 2\). A little different suggestion for these identification are given in Table IV, with the prediction for glueballs with quantum number \(J^{CP} = 0^{++, 0^+, 0^-}, \) compared to the lattice data.

We account for the results in Table II and III as follows: a number of the meson $f_J$ states, when taken to be the glueball-like (i.e., the glueball dominate) states, can be viewed as a knot excitation of the chromo-electric fluxtubes with knot types $n^1_l (n = 1 \sim 5, l = 1, 2)$ in the topology terminology, in which the low-lying ($\leq 1.71\text{GeV}/c^2$) glueball-like meson states are best described by the fluxtube of the knot types $2^1_l$ and $3^1_l$, with \((n, m) = (1, j), j = 0, 1, 2\). It is unknown in our framework why some of vibrational modes lack the experimental counterparts, but it is known that the knot types $K = 2^2_k, 3^1_k$ corresponds to the low-lying $f_J$ states because they have the simplest topologies among the knot types $2^1_k, 3^1_k$, in the sense that they are non-shrinkable and they are shortest in the unit of the string diameter. While the minor variation, roughly $200\text{MeV}$
per unit of $m$, in the spectrum arises from the vibrational modes of the fluxtube, the spectrum of glueball-like mesons are mainly due to the knot geometry of fluxtubes.

| States  | Mass$^a$ | $m$ | $K^b$  | $e(K)^c$ | $E_{n,m}(G)$ |
|---------|----------|-----|--------|----------|--------------|
| $f_0(600)$ | 400-1200 | 1   | $2_1$  | 6.2832   | 674.6        |
| $f_0(980)$ | 980±10   | 0   | $2_1^*$ | 12.6     | 976.0        |
| $h_1(1170)$ | 2        | 2   | $2_1$  | 6.2832   | 1246.9       |
| $a_0(1260)$ |          | 1   | $2_1^*$ | 12.6     | 1262.2       |
| $f_0(1370)$ | 1200-1500| 1   | $2_1^*$ | 12.6     | 1262.2       |
| $f_0(1500)$ | 1507±5   | 0   | $3_1$  | 16.4     | 1500.1       |
| $f_0(1710)$ | 1718±6   | 1   | $3_1$  | 16.4     | 1719.4       |
| $\eta(1760)$ | 1760±11  | 3   | $2_1^*$ | 12.5664  | 1757.0       |
| $\eta(2225)$ | 2220±18  | 1   | $2_1^* \\ 0_1$ | 20.8496  | 2278.4       |
| $f_2(1910)$ | 1915±7   | 2   | $3_1$  | 16.4     | 1938.7       |
| $f_2(2010)$ | 2011±60  | 4   | $2_1^*$ | 12.5664  | 2015.5       |
| $f_0(2100)$ | 2103±7   | 3   | $3_1$  | 16.4000  | 2100.4       |
| $f_2(2150)$ | 2156±11  | 0   | $4_1$  | 21.2000  | 2153.6       |
| $f_0(2200)$ | 2189±13  | 0   | $4_1^*$ | 21.4000  | 2180.8       |
| $f_2(2300)$ | 2297±28  | 4   | $3_1$  | 16.4000  | 2298.5       |
| $f_2(2340)$ | 2339±60  | 1   | $4_1^*$ | 21.4000  | 2348.8       |

Table II
States & Mass & \(m\) & \(K\) & \(e(K)\) & \(E_{n,m}(G)\) \\
\hline
\(f_0(600)\) & 400-1200 & 1 & 2_1 & 6.2832 & 674.6 \\
\(f_0(980)\) & 980\(\pm 10\) & 0 & 2_1^2 & 12.6 & 976.0 \\
\(h_1(1170)\) & 2 & 2_1 & 6.2832 & 1246.9 \\
\(a_1(1260)\) & 1 & 2_1^2 & 12.6 & 1262.2 \\
\(f_0(1370)\) & 1200-1500 & 1 & 2_1^2 & 12.6 & 1262.2 \\
\(f_0(1500)\) & 1507\(\pm 5\) & 0 & 3_1 & 16.4 & 1500.1 \\
\(f_0(1710)\) & 1718\(\pm 6\) & 1 & 3_1 & 16.4 & 1719.4 \\
\(\eta(1760)\) & 1760\(\pm 11\) & 3 & 2_1^2 & 12.5664 & 1757.0 \\
\(\eta(2225)\) & 2220\(\pm 18\) & 1 & 2_1^2 \(* 0_1\) & 20.8496 & 2278.4 \\
\(f_2(1910)\) & 1915\(\pm 7\) & 2 & 3_1 & 16.4 & 1938.7 \\
\(f_2(2010)\) & 2011\(\pm 60\) & 4 & 2_1^2 & 12.5664 & 2015.5 \\
\(f_0(2100)\) & 2103\(\pm 7\) & 3 & 3_1 & 16.4000 & 2100.4 \\
\(f_2(2150)\) & 2156\(\pm 11\) & 0 & 4_1 & 21.2000 & 2153.6 \\
\(f_0(2200)\) & 2189\(\pm 13\) & 0 & 4_1^2 & 21.4000 & 2180.8 \\
\(f_2(2300)\) & 2297\(\pm 28\) & 4 & 3_1 & 16.4000 & 2298.5 \\
\(f_2(2340)\) & 2339\(\pm 60\) & 1 & 4_1^2 & 21.4000 & 2348.8 \\

Table III

\(^a\) The data from the PDG summary tables and [2]. \(^b\) The knot types in terms of notation \(n_l^k\), means a link of \(l\) components with \(n\) crossing, and occurring in the standard table of links() on the \(k\)th place. \(^c\) The data from the simulations[20] except for 2_1^2 and 2_1^2 \(* 0_1\). \(^d\) The data from Lattice simulations.

| States | Mass & \(m\) & \(K\) & \(e(K)\) & \(E_{n,m}(G)\) \\
\hline
\(0^{++}\) & 1710\(\pm 130\) & 1 & 3_1 & 16.4 & 1678.5 \\
\(0^{-+}\) & 2560\(\pm 155\) & 0 & 5_1 & 24.2000 & 2561.1 \\
\(0^{+-}\) & 4780\(\pm 290\) & 0 & 9_2 & 40 & 4703.3 \\

Table IV

6 Summary and discussions

The dual dynamics of the SU(2) QCD is revisited using the covariant (Cho-Faddeev-Niemi) decomposition of the gluon field. Assuming that the chromo-magnetic symmetry is broken and using the random phase approximation, we show by deriving a dual Ginzburg-Landau model for SU(2) gluodynamics that the QCD vacuum is of type-II superconductor. The Ginzburg-Landau
parameter is shown to be $\kappa = \sqrt{3}$, which is independent of the chromo-magnetic condensate $H$ and the strong coupling $g_s$ used, and agrees well with the lattice simulation. The mass spectrum of a number of low-lying $f_J$ states are calculated for the chromo-flux quantum $n = 1$ and found to be in a good agreement with the recent lattice data, by adding the energy correction arising from the twisting of the knotted(linked) QCD fluxtubes and the zero-point energy to the energy of the Nielsen-Olesen vortex in the dual Ginzburg-Landau model and applying the universal topological invariants of the knot geometry being the length-to-diameter ratios of the fluxtubes. A number of meson $f_J$ states are identified as the knot-like gluonic excitations in the form of closed chromo-electric fluxtubes with the chromo-flux quantum $n = 1$ and the vibrationally-excited string mode $m = 2$. The most of the low-lying $f_J$ states are found to be described by the knotted string excitations of the types $2^1_1$ and $K = 3_1$ topologies, having the quantum number $(n, m) = (1, j)$, $j = 0, 1, 2$.

Given that the glueballs can mix with the quark-antiquark states, as proposed in [41], our calculation present a support that some of the $q\bar{q}$-states(mainly $f_J$ states and a few $\eta$), listed in Table II and III,can have a dominate component of gluons, namely, the energy of the valence quarks is negligible, and as a result, these $f_J$ states can be well described by the knotted fluxtubes formed by the collective gluons. The origin of the uncertainty in the knot-type identifications for a few states, such as $f_0(980)$ and $f_0(1370)$, $f_2(2220)$, $\eta(2225)$ in Table II and III, remains unclear yet, and the further studies are needed for the origin though the quark hybrid component of the states is probable origin.

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