Pressure Drop of a Refrigerant Flowing Vertically Upward and Downward in Small Circular, Rectangular, and Triangular Tubes

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Abstract: In the present study, experiments were performed to examine the characteristics of the two-phase frictional pressure drop of an R410A refrigerant flowing vertically upward and downward for the development of a high-performance heat exchanger using small tubes or mini-channels for air-conditioning systems. The cross-sections of copper test tubes were 0.5, 0.7, 1.0, 1.5, and 2.0 mm circular tubes, and rectangular and triangular tubes with hydraulic diameters of 1.04 and 0.88 mm, respectively. The frictional pressure drops were measured in the range of mass fluxes of 30–400 kg m⁻² s⁻¹, with qualities from 0.05 to 0.9 and a saturation temperature of 10 °C. The characteristics of the measured pressure drops were compared in different inner diameters, cross-section shapes, and flow directions. In addition, Chisholm’s parameter and various modified Chisholm’s parameters for small tubes were examined to determine whether or not they reproduced our measurement data.

Keywords: heat exchanger; two-phase; mini-channel; pressure drop; correlation

1. Introduction

Recently, as a new high-performance heat exchanger for air-conditioning systems, a heat exchanger using small tubes or multi-port extruded tubes is being developed. To design such heat exchangers, it is necessary to accurately predict the characteristics of frictional pressure drop and heat transfer. It is supposed that the characteristics in small tubes are different from those in traditional tubes with relatively large diameters because surface tension becomes an important factor in small tubes. Therefore, two-phase evaporative flow in small tubes is attractive due to the higher heat transfer coefficient. On the other hand, the evaporation process occurs at a relatively lower pressure condition when compared to the condensation process in the heat pump cycle, so the frictional pressure drop is larger due to the high vapor velocity. Furthermore, the smaller the inner diameter, the larger the frictional pressure drop. Therefore, the frictional pressure drop may be a factor worth considering when designing a heat exchanger using small tubes. There are, however, few published studies relevant to the effect of tube cross-sectional shape or the flow direction in small tubes.

Ide and Matumura (1990) [1] ran experiments using 10 rectangular tubes with different aspect ratios (1–40) and arranged them in an inclination angle (from horizontally to vertically). Their experimental results were compared with the Lockhart–Martinelli correlation [2] and Akagawa’s correlation [3], which are widely used for conventional large circular tubes, but these correlations could not reproduce their data, especially those on low liquid superficial velocity and the high-inclination angle condition. They tried to relate the behavior of the pressure drop to the change in the flow pattern.

Wambsganss et al. (1991, 1992) [4,5] measured the frictional pressure drop of air–water mixtures flowing horizontally in a rectangular channel (19.05 x 3.18 mm). Their experimental results were compared with Chisholm’s correlation [6], and good agreement was observed.
for higher mass fluxes using Chisholm’s parameter $C = 21$. However, Chisholm’s parameter needs to improve accuracy at a lower mass fluxes condition, because most of the data lie between $C = 5–20$.

Mishima and co-workers (1993, 1996) [7,8] reported air–water mixtures flowing vertically in circular and rectangular tubes with diameters ranging from 1 to 5 mm. They reported that if the inner diameter is less than 5 mm, Chisholm’s parameter $C$ should consider the effect of the inner diameter, because they noted that Chisholm’s parameter changed from 21 to 0 as the hydraulic diameter decreased.

Wang et al. (2001) [9] reported the characteristics of the two-phase frictional pressure drop of R22, R407C, and R410A flowing horizontally in circular tubes. The inner diameter of test tubes was 3, 5, 7, and 9 mm, the frictional pressure drop could not be reproduced by Chisholm’s correlation, the mean deviation of the correlation was as high as 117.6%, and considerable over-predictions were observed at the low mass flux condition.

Lee and Lee (2007) [10] measured the water-air two-phase pressure drop flowing horizontal rectangular channel under the atmospheric pressure. The channels width was fixed to 20 mm, but the height ranges changed from 0.4 to 4 mm. They reported the mass flux and channel size affect the Chisholm parameter $C$.

Madrid et al. (2007) [11] performed an experiment of HFE-7100 flowing vertically upward in 40 parallel mini-channels with a rectangular cross-section of 0.5 2.6 mm and a hydraulic diameter of 0.84 mm. They found that a homogeneous model accurately predicted the frictional pressure drop, which depended on the vapor quality.

The purpose of this study is to comprehensively clarify the effects related to the cross-sectional shape and hydraulic diameter of the small tubes, and the effect of flow directionality using the alternative freon refrigerant R410A under the saturation pressure (temperature) at 1.09 MPa (10 °C), which is actually widely used and condition in the heat exchanger. In order to clarify these, the experiments were performed to examine the characteristics of the two-phase frictional pressure drop of an R410A refrigerant flowing vertically upward and downward. The cross-sections of copper test tubes were 0.5, 0.7, 1.0, 1.5, and 2.0 mm circular tubes, and rectangular and triangular tubes with hydraulic diameters of 1.04 and 0.88 mm, respectively. The characteristics of the measured pressure drops were clarified by different inner diameter tubes, cross-section shapes, and flow directions. In addition, comparing Chisholm’s parameter and various modified Chisholm’s parameters in Lockhart–Martinelli correlation for small tubes, I examined whether or not they reproduced our measurement data under the physical property conditions of the refrigerant in actual use.

2. Experimental Apparatus and Test Conditions

The schematic of the experimental apparatus is depicted in Figure 1. The R410A refrigerant was discharged by the plunger pump in a state of liquid single phase, and it flowed into the preheater through the mass flow meter. The refrigerant was heated by the preheater up to the desired enthalpy at the inlet of the test section. Then, the refrigerant returned to the pump through the test section and condenser. The saturation pressure at the outlet of the test section was controlled by cooling the refrigerant in the condenser with temperature-controlled brine.

The test section for upward flow is shown in Figure 2. The test section for downward flow was used upside down. The test tubes were made of copper; their cross-section shapes are shown in Figure 3. Table 1 shows the specifications of each tube. The inside diameters of circular tubes were 0.5, 0.7, 1.0, 1.5, and 2.0 mm, and the hydraulic diameters of the rectangular and triangular tubes were 1.04 and 0.88 mm, respectively. The corner of the flow channel had a radius of about 0.1 mm of curvature for both the rectangular and triangular tubes. The test tube was arranged vertically, and its length was 440 mm. The pressure drop between the inlet and the outlet pressure taps was measured by the differential pressure gauge.
The pressure drop between the inlet and the outlet pressure taps was measured by the differential pressure gauge. The temperature of the test-section surrounding air was adjusted very close to the test tube wall temperature in order to minimize the heat transfer between the test tube and the surroundings. The measurement was performed under the steady-state condition.

Figure 1. Schematic diagram of the experimental apparatus.

Figure 2. Test section for upward flow.
The frictional pressure drop were measured in the range of mass fluxes of 50 and 100. The qualities were in the range of 0.05 to 0.9 under the adiabatic condition, with 200 and 400 kg·m$^{-2}$·s$^{-1}$ for upward flow and 30, 50, 100, and 200 kg·m$^{-2}$·s$^{-1}$ for downward flow. The saturation temperature was 10°C. The measured total pressure drop consists of two components: the frictional pressure drop and the static pressure drop. Therefore, the frictional pressure drop $\Delta P_f$ (Pa) was obtained as

$$\Delta P_f = \Delta P_a - \Delta P_s$$

where $\Delta P_a$ (Pa) is the measured total pressure drop, a value obtained from differential pressure gauge, and $\Delta P_s$ (Pa) is the static pressure drop. The measurement error of the frictional pressure drop was ±0.2 kPa in general, while the maximum error was estimated to be ±0.7 kPa at a quality of about 0.1 due to the large estimation error of the static pressure drop.

The physical properties of R410A were evaluated using REFPROP Ver.10.0 [12].

3. Results

3.1. Measured Frictional Pressure Drop

The 351 data were gathered on the two-phase frictional pressure drop in small circular, rectangular, and triangular tubes. Table 2 shows the number of obtained data points in each tube, and these data are classified by the flow regimes following the method of Lockhart–Martinelli [2]. The definition of the limits of the liquid-phase or vapor-phase Reynolds number $Re_L$, $Re_G$ between laminar and turbulent is 2000.

$$Re_L = \frac{G(1-x)D}{\mu_L}$$

$$Re_G = \frac{GxD}{\mu_G}$$
Table 2. All measured frictional pressure drop data.

|         | N  | vv | vt | tv | tt |
|---------|----|----|----|----|----|
| Circular|    |    |    |    |    |
| Up      | 66 | 14 | 48 | 2  | 2  |
| Down    | 131| 41 | 90 | 0  | 0  |
| Rectangular|  |    |    |    |    |
| Up      | 40 | 8  | 28 | 1  | 3  |
| Down    | 36 | 15 | 21 | 0  | 0  |
| Triangular|  |    |    |    |    |
| Up      | 39 | 10 | 27 | 1  | 1  |
| Down    | 39 | 20 | 19 | 0  | 0  |
| Total   | 351| 108| 233| 4  | 6  |
| %       | 100.0 | 30.8 | 66.4 | 1.1 | 1.7 |

laminar: $Re_L, Re_G < 2000$, turbulent: $Re_L, Re_G > 2000$.

Figure 4 shows the distribution of all data in the plot of $Re_L$ vs. $Re_G$. Here, $G$ (kg·m$^{-2}$·s$^{-1}$) is the mass flux, $x$ (-) is quality, $D$ (m) is the hydraulic diameter, and $\mu$ (Pa·s$^{-1}$) is the viscosity. The subscripts L and G indicate liquid and gas, respectively. Although a wide range data from low mass flux to high mass flux were obtained, most of the data were located on the laminar liquid condition. The laminar liquid condition is characteristic of mini-channels, so the viscous force and the surface tension have a significant influence on the area between the fluid and channel. Hence, it is supposed that the frictional pressure drop in the small tube is larger than that of a conventional large tube.

3.1.1. Circular Tubes

This section examines the frictional pressure drop in circular tubes, whose data were obtained at wide ranges of the inside diameter.

Figures 5 and 6 show the relation between the measured frictional pressure drop gradients $\Delta P_f/\Delta L$ (Pa·m$^{-1}$) and quality $x$ (-) flowing vertically upward and downward, respectively. The parameter of these figures is the mass flux.
Figure 5. Frictional pressure drop vs. quality in upward flow. (Mass fluxes are parameters).

Figure 6. Cont.
As in the case of the conventional large circular tube, the measured frictional pressure drop in small circular tubes increased with increasing quality in the wide range with a maximum at around 0.8; after that, it mildly decreased with increasing quality to around quality 1.0. Moreover, increasing mass flux increased the frictional pressure drop.

To examine the effect of the different inner diameters, Figure 7 shows the relation between the measured frictional pressure drop gradients $\Delta P_f/\Delta L$ and quality $x$ flowing vertically downward; the inner diameter is the parameter. As in the case of the conventional large tubes, decreasing the inner diameter of tube increased the frictional pressure drop. However, the frictional pressure drop showed a different tendency with mass flux and the inner diameter. For the high mass flux condition (200 kg·m$^{-2}$·s$^{-1}$ and 400 kg·m$^{-2}$·s$^{-1}$), the frictional pressure drop monotonically increased with increasing quality in the range of from 0.05 to 0.8. On the other hand, for the low mass flux (50 kg·m$^{-2}$·s$^{-1}$ and 100 kg·m$^{-2}$·s$^{-1}$), the frictional pressure drop decreased with increasing quality in the low-quality region; its values were not affected by the different mass flux or inner diameter, and these are given in Figures 6 and 7. After that, the smaller the inner diameter, the higher the increase in frictional pressure drop at low quality. This physical phenomenon could also be confirmable in the upward flow, as shown in Figure 5. The phenomenon was confirmed by Inoue and Aoki (1965) [13], Ide and Matsumura (1990) [1] and Wambsganss et al. (1991, 1992) [4,5]. Inoue and Aoki performed an experiment on air–water mixtures flowing horizontally in a rectangular channel (19.05, 3.18 mm). The frictional pressure drop decreased with increasing quality in the low-quality region; its values were not affected by the different mass flux or inner diameter, and these are given in Figures 6 and 7. After that, the smaller the inner diameter, the higher the increase in frictional pressure drop at low quality. This physical phenomenon could also be confirmable in the upward flow, as shown in Figure 5. The phenomenon was confirmed by Inoue and Aoki (1965) [13], Ide and Matsumura (1990) [1] and Wambsganss et al. (1991, 1992) [4,5]. Inoue and Aoki performed an experiment on air–water mixtures flowing vertically upward in a circular tube with diameters ranging from 5 to 28.8 mm. They reported that the frictional pressure drop decreased with increasing quality in the low-quality region, and it became evident at a lower mass flux. Ide and Matsumura ran experiments using 10 rectangular tubes with different aspect ratios (1–40) and arranging them in an inclination angle (from horizontally to vertically). For the case of small channels with a large aspect ratio and hydraulic diameters smaller than about 10 mm, they noted that the frictional pressure drop made a bump in the vicinity of the transition from bubble to bubbly slug flow. Wambsganss et al. measured the frictional pressure drop of air–water mixtures flowing horizontally in a rectangular channel (19.05, 3.18 mm). The frictional pressure drop indicated a higher value around a quality of 0.002, the region that was transition area from plug or bubble flow to slug flow.
Figure 7. Frictional pressure drop vs. quality in downward flow. (Inner diameter is parameter).

3.1.2. Rectangular and Triangular Tubes

Figure 8 shows the relation between the measured frictional pressure drop gradients $\Delta P_f / \Delta L$ and quality $x$ flowing in rectangular and triangular tubes, comparing them with the data of a 1.00 mm circular tube that has a similar hydraulic diameter.

As in the case of the circular tube, the measured frictional pressure drop in both the small rectangular and triangular tubes increased with increasing mass flux and quality. Moreover, for the low mass flux condition (50 kg·m$^{-2}$·s$^{-1}$ and 100 kg·m$^{-2}$·s$^{-1}$), the frictional pressure drop indicated higher values in the low-quality region; after that, it decreased with increasing quality. In this region, the value of the frictional pressure drop was not affected by the different cross-section shape or mass flux.
Figure 8. Frictional pressure drop vs. quality flowing upward and downward in circular, rectangular, and triangular tubes.

The frictional pressure drop in the rectangular tube seems small compared to that of the triangular tube; it is supposed that there was no difference in the frictional pressure drop of both the tubes considering the hydraulic diameter that is 15% larger than that of the triangular tube. On the other hand, comparing the circular tube pressure drops, no differences were observed for the non-circular tubes pressure drop at low mass fluxes of 30 and 50 and high mass flux of 400 kg·m⁻²·s⁻¹. However, it became smaller at 50, 100, and 200 kg·m⁻²·s⁻¹, especially in downward flow.

As shown in Figure 9, I had conducted observation experiments using a high-speed camera for the circular, rectangular, and triangular channels with a hydraulic diameter of about 1 mm under the same refrigerant R410A and saturation temperature 10 °C condition of this study. Moreover, the pressure drop results will be discussed based on these images [14]. Figures 10–12 are the flow pictures in circular, rectangular, and triangular channels, respectively.

Figure 9. Test glass channels [14].
To summarize the differences between upward and downward flow, no effect of flow direction was observed on the frictional pressure drop of the circular tube. Thus, there was no difference in the flow patterns in circular channel. However, the rectangular and triangular non-circular tubes were clearly smaller in the downward flow. This is especially noticeable for the quality above 0.3. According to the visualization experiments, this region coincides with the transition from slug flow to annular flow in both tubes. It is especially interesting to note that the downward flow in Figures 11 and 12 with a mass flux of 50 kg·m$^{-2}$·s$^{-1}$ and a quality of 0.9 showed a completely different flow pattern than the upward flow under the same conditions. In other words, an annular flow without any disturbance was observed at low mass flux regardless of the quality. Since this flow pattern has not been reported before, I named it non-disturbance annular flow and refer to it as NA in the figures. This is because gravity (buoyancy) and surface tension have a great

![Figure 10. Circular channel visualization with plexiglass [14].](image)

![Figure 11. Rectangular channel visualization with plexiglass [14].](image)

![Figure 12. Triangular channel visualization with plexiglass [14].](image)
influence on the downward flow, and since the upward flow is in the opposite direction of gravity, the liquid held in the corner of the non-circular tube will always have a velocity distribution due to the shear force of the vapor and gravity, and the shear force from the tube surface will cause the velocity of the liquid to become unstable as it flows downstream. In the downward flow, the liquid is held at the corners by the surface tension and flows along the gravity, so the change in the liquid velocity becomes smaller, and it is thought that the disturbance is less likely to occur than in the upward flow. This is thought to be the physical mechanism that causes the difference in frictional pressure drop between the upward and downward flows.

These tendencies are verified in the discussion on the correlation below.

### 3.2. The Effect of Cross-Section Shapes

For the estimation of the frictional pressure drop for horizontal or vertical flow in conventional large circular tubes, the Lockhart–Martinelli correlation (1949) [2] is often used with the approximation of the two-phase frictional multiplier $\Phi^2_L(-)$ suggested by Chisholm (1969) [5]. This correlation especially agrees well in the annular flow. The Lockhart–Martinelli correlation is expressed as

$$\Phi^2_L = \frac{\Delta P_L / \Delta L}{\Delta P / \Delta L} \quad (4)$$

$$\frac{\Delta P_L}{\Delta L} = \frac{\lambda}{D} \frac{1}{2 \rho_L} G^2 (1 - x)^2 \quad (5)$$

where $\Delta P_L / \Delta L$ is the liquid single-phase flow frictional pressure drop gradient estimated by Equation (5), and $\lambda (-)$ is the coefficient of pipe friction.

For the laminar flow (Hagen–Poiseuille correlation):

$$\lambda = \frac{64}{Re_L} \quad (6)$$

For the turbulent flow (Blasius correlation):

$$\lambda = \frac{0.316}{Re_L^{0.25}} \quad (7)$$

The approximation of the two-phase frictional multiplier $\Phi (-)$ by Chisholm is expressed as

$$\Phi^2_{L, \text{Chisholm}} = 1 + \frac{C}{\chi} + \frac{1}{\chi^2} \quad (8)$$

where $\chi (-)$ denotes the Lockhart–Martinelli parameter, and $\chi (-)$ needs to be used properly by the flow regime situations. $C (-)$ indicates Chisholm’s parameter.

For the laminar (liquid)–laminar (vapor or gas) regime (vv):

$$\chi_{vv} = \left( \frac{1 - x}{x} \right)^{0.5} \left( \frac{\rho_G}{\rho_L} \right)^{0.5} \left( \frac{\mu_L}{\mu_G} \right)^{0.5} \quad (9)$$

For the laminar (liquid)–turbulent (vapor or gas) regime (vt):

$$\chi_{vt} = \left( \frac{16 \rho_G}{0.046 \rho_G^{0.2}} \right)^{0.5} \left( \frac{1 - x}{x} \right)^{0.5} \left( \frac{\rho_G}{\rho_L} \right)^{0.5} \quad (10)$$

For the turbulent (liquid)–laminar (vapor or gas) regime (tv):

$$\chi_{tv} = \left( \frac{0.046}{16} \right)^{0.5} \left( \frac{\rho_G}{\rho_L} \right)^{0.5} \left( \frac{1 - x}{x} \right)^{0.5} \left( \frac{\mu_L}{\mu_G} \right)^{0.5} \quad (11)$$
For the turbulent (liquid)–turbulent (vapor or gas) regime (tt):

\[ \chi_{tt} = \left( \frac{1 - x}{x} \right)^{0.9} \left( \frac{\rho_G}{\rho_L} \right)^{0.5} \left( \frac{\mu_L}{\mu_G} \right)^{0.1} \]  

Figures 13 and 14 show the relation between \( \Phi_L^2 \) and the inverse of \( \chi \) flowing vertically upward and downward with the calculation from Equation (8) using \( C = 5 \) denoted by the dash line and \( C = 12 \) denoted by the solid line. The error bars indicate the measurement errors. For \( C \) in Equation (8), the values of 5 and 12 are originally given by Chisholm for the laminar (liquid)–laminar (vapor or gas) and laminar (liquid)–turbulent (vapor or gas) regimes.

Figure 13. Two-phase frictional multiplier vs. inverse of Lockhart–Martinelli parameter flowing upward in circular, rectangular, and triangular tubes.
Figure 14. Two-phase frictional multiplier vs. inverse of Lockhart–Martinelli parameter flowing downward in circular, rectangular, and triangular tubes.

In the high $\chi^{-1}$ region, the characteristics were well correlated by the relation between $\Phi_L^2$ and $\chi^{-1}$ independently of the mass flux. Especially at 400 $\text{kg} \cdot \text{m}^{-2} \cdot \text{s}^{-1}$, Equation (8) with the parameter $C = 12$ well reproduced the measurements, and no difference among the tubes was observed, which indicates that the two-phase frictional pressure drop at high mass flux can correlate considering the effect of different hydraulic diameters only flowing in the liquid phase in the tube. Therefore, the cross-section shape is a minor effect at the high mass flux. Likewise, there was no difference in pressure drop at low mass fluxes of 30 and 50 $\text{kg} \cdot \text{m}^{-2} \cdot \text{s}^{-1}$. On the other hand, the pressure drop in the non-circular tubes became smaller at 100 and 200 $\text{kg} \cdot \text{m}^{-2} \cdot \text{s}^{-1}$, especially in downward flow. Hence, it is assumed that the effect of the cross-section shape becomes non-negligible.
In the low $\chi^{-1}$ region at low mass fluxes of 30, 50, and 100 kg m$^{-2}$ s$^{-1}$ in which the frictional pressure drop decreased with increasing quality, the measured $\Phi L^2$ was found to be significantly higher, and the values were approximately 10. Wambsganss et al. also noted that the value of $\Phi L^2$ was approximately 10 in the region where this phenomenon occurred. Figure 15 shows the relation between Chisholm’s parameter $C$ to replicate measurement values and the hydraulic diameter $D_h$. The parameter of this figure is the mass flux. The error bars indicate the measurement errors.

Figure 15. Optimum Chisholm parameter $C$ by the measurement vs. hydraulic diameter $D_h$.

In the case of circular tubes, the optimum Chisholm’s parameters $C$ lay between $C = 5$ and $C = 12$, and the $C$ increased with the increase in the mass flux with the same tube diameter. Moreover, the $C$ decreased with the decrease in the diameter with the same mass flux, especially below 1 mm. On the other hand, in the case of non-circular tubes, the optimum Chisholm’s parameters $C$ had the same value as that of the 1 mm circular tube at low mass flux of 50 kg m$^{-2}$ s$^{-1}$ and high mass flux of 400 kg m$^{-2}$ s$^{-1}$. However, the $C$ of non-circular tubes was smaller than that of the 1 mm circular tube at 100 and 200 kg m$^{-2}$ s$^{-1}$; notably, the $C$ for the downward flow was under 5.

3.3. The Effect of Flow Directions

Figure 16 shows a comparison of the measured frictional pressure drop between upward and downward flow in circular, rectangular, and triangular tubes.
In each tube, the frictional pressure drop of downward flow were higher than up. The pressure drop decreased with increasing quality in the low-quality and low mass flux region.

Regarding the circular tube, there were no adequate differences in the frictional pressure drop due to the different flow directions, where the pressure drop increased with increasing quality at the mid-quality region. No difference was also observed in the visualization of the circular tube shown in Figure 10. On the other hand, in regard to the non-circular tubes, the apparent discrepancies were observed at 100 and 200 kg·m$^{-2}·$s$^{-1}$. More specifically, maximum differences of 30% were observed at 100 kg·m$^{-2}·$s$^{-1}$. It is supposed that the difference was caused by the different flow property of the axial direction.

In brief, it is the same physical phenomenon described in the conclusion of Section 3.1.2.

3.4. Comparison of Various Modified Chisolm Parameters

In this section, the various modified Chisholm’s parameter in Lockhart–Martinelli correlation for small tubes are examined, and it is determined whether or not they reproduce the measurement data. As stated previously, Chisholm’s parameter in Lockhart–Martinelli correlation is often used for the estimation of frictional pressure drop in conventional large circular tubes. However, several researchers reported that the original Chisholm’s parameter showed a disagreement with the measurements in small channels. I also confirmed this fact in Figure 15. Therefore, the various modified Chisholm’s parameter were suggested. Thus, in this section, the calculated values using five modified Chisholm’s parameters are compared with the measurements data, except for the data of the decreasing frictional pressure drop with increasing quality, because the correlation including this tendency has not yet been proposed.

Wambsganss et al. (1991, 1992) [4,5] measured the frictional pressure drop of air–water mixtures flowing horizontally in a rectangular channel (19.05, 3.18 mm). Their experimental results were compared with Chisholm’s correlation. However, at the lower mass flux condition, the measurement data generally followed the form of Chisholm’s correlation, and the optimum Chisholm’s parameter increased with increasing mass flux.

Figure 16. Comparison of the measured frictional pressure drop between upward and downward flow in circular, rectangular, and triangular tubes.
As a consequence, they deemed Chisholm’s parameter to be not a constant but a function of the Lockhart–Martinelli parameter and mass flux. The modified parameter is as follows:

\[ C = f(\chi, Re_{L0}) = a\chi^b \quad (13) \]

\[ Re_{L0} = \frac{GD}{\mu_L} \quad (14) \]

where \( Re_{L0} \) is Reynolds number for mixture flowing as liquid, \( a = -2.44 + 0.00939 Re_{L0} \) and \( b = -0.938 + 0.000432 Re_{L0} \). This correlation is applicable for \( Re_{L0} < 2200 \) and \( <1.0 \). The reported average error of using this parameter was \( \pm 19\% \) for \( G \leq 400 \text{ kg m}^{-2} \cdot \text{s}^{-1} \).

Mishima and co-workers (1993, 1996) [7,8] reported on air–water mixtures flowing vertically in circular and rectangular tubes with a diameter range of from 1 to 5 mm. They reported that if the inner diameter is less than 5 mm, Chisholm’s parameter should consider the effect of the inner diameter, because they found Chisholm’s parameter to change from 21 to 0 as the hydraulic diameter decreased. Therefore, they suggested that the modified parameter is given by

\[ C = f(d) = 21\left(1 - e^{-0.333d}\right) \quad (15) \]

where \( d \) in mm is the hydraulic diameter of the channel, and this parameter is the dimension. The comparison of this parameter with the database demonstrated a good agreement, except for ammonia vapor flow within an error of \( \pm 12\% \).

Lee and Lee (2001) [10] proposed the modified Chisholm’s parameter based on 305 data points of horizontal rectangular channels with an air–water mixture flowing through them ((0.4, 1.0, 2.0, 4.0) 20 mm). They stated that Chisholm’s parameter depends on the flow regimes, and that the flow regimes depend on the mass flux and hydraulic diameter. Hence, the parameter \( C \) should consider the effects of the mass flux and hydraulic diameter. On the other hand, the surface tension is an important factor in the laminar (liquid)–laminar (gas or vapor) regime (vv); therefore, they proposed the non-dimensional parameter considering the effect of surface tension. They argued that the modified parameter is given by

\[ C = f(\beta, \psi, Re_{L0}) = A\beta^q\psi^rRe_{L0}^{s} \quad (16) \]

\[ \beta = \frac{\mu_L^2}{\rho_L \sigma D_h} \quad (17) \]

\[ \psi = \frac{\mu_L j}{\sigma} \quad (18) \]

where dimensionless parameters \( \beta \) and \( \psi \) are the effects of the surface tension \( \sigma \) (N·m\(^{-1}\)), the viscosity \( \mu \) and the velocity of the liquid slug \( j \) (m·s\(^{-1}\)), and constant \( A \) and exponents \( q, r, \) and \( s \) are determined by the flow regimes. Here, there are given numbers for exponents \( q \) and \( r \) in the laminar (liquid)–laminar (gas or vapor) regime (vv). On the other hand, in other flow regimes, the surface tension becomes insignificant, so exponents \( q \) and \( r \) are zero. Thus, Equation (16) is merely a function of the all-liquid Reynolds number, \( Re_{L0} \). A comparison of this parameter was reported with the measurements within an error of \( \pm 10\% \).

Zhang et al. (2010) [15] used an advanced information processing technique, which can be utilized to carry out the input “trial and error” analysis to select non-dimensional numbers that could well correlate with the two-phase frictional multiplier. When the hydraulic diameter of the channel and the Weber number \( We \) are inputted, the information processing techniques can significantly improve the prediction of the two-phase frictional multiplier. Therefore, Mishima and Hibiki’s correlation (Equation (15)) was modified using the non-dimensional Laplace constant. The non-dimensional Laplace constant \( Lo^* \) is defined as

\[ Lo^* \equiv \left[ \frac{\sigma}{\delta (\rho_f - \rho_g)} \right]^{0.5} / D_h \quad (19) \]
where \( g \) (m·s\(^{-2}\)) is gravity force.

Consequently, Mishima and Hibiki’s correlation was modified as shown below:

\[
C = f(Lo^*) = 21[1 - \exp(-0.358/Lo^*)]
\]  
(20)

This correlation is applicable for \( Re_L \leq 2000 \), \( Re_G \leq 2000 \), and \( 0.014 \leq D_h \leq 6.25 \) mm. The comparison of this parameter with seven items of reference data of the liquid–gas flow yielded 17.9\%, while that with three items of reference data of liquid–vapor flow yielded 21.7\%. For liquid–gas two-phase flow, it was recommended that Equation (20) may work better if the constant of \(-0.358\) is replaced with \(-0.674\), while for liquid–vapor flow, the constant of \(-0.142\) would work better.

For adiabatic liquid–gas two-phase flow:

\[
C = 21[1 - \exp(-0.674/Lo^*)]
\]  
(21)

For adiabatic liquid–vapor two-phase flow:

\[
C = 21[1 - \exp(-0.142/Lo^*)]
\]  
(22)

Li and Wu (2010) [16] collected the adiabatic two-phase pressure drop data in small channels from the literature. The collected database contains 12 different working fluids and a hydraulic diameter range of 0.148 to 3.25 mm. They noted that the Bond number \( Bo \) and Reynolds number \( Re \) may have the potential to relate into a general correlation, because there are four forces related to two-phase flow in channels: gravitational, inertia, viscous, and surface tension forces. The Bond number and Reynolds number include these forces. The Bond number \( Bo \) is defined as

\[
Bo = \frac{g(\rho_L - \rho_G)D_h^2}{\sigma}
\]  
(23)

where the Bond number has the same meaning in the equation as in the non-dimensional Laplace constant \( Lo^* \) equation (Equation (19)), with the difference of the exponent. The relationship between Chisholm’s parameter and the Bond number or liquid-phase Reynolds number \( Re_L \) was investigated. Chisholm’s parameter has a positive linear relationship with the Bond number when \( Bo \leq 1.5 \):

\[
Bo \leq 1.5, \quad C = f(Bo) = 11.9Bo^{0.45}
\]  
(24)

On the other hand, when \( 1.5 < Bo \leq 11 \), Chisholm’s parameter depends on

\[
1.5 < Bo \leq 11, \quad C = f(Bo, Re_L) = 109.4\left(Bo \cdot Re_L^{0.5}\right)^{-0.56}
\]  
(25)

It was reported that when \( Bo \leq 1.5 \), Equation (24) could predict 80.6\% of the data within ±30\%, and when \( 1.5 < Bo \leq 11 \), Equation (25) could predict 72.6\% of the data within ±30\%.

Figure 17 shows comparisons between the measured \( \Delta P_I/\Delta L \) and the calculated frictional pressure drop \( \Delta P_i \text{cal}/\Delta L \) using Chisholm and modified Chisholm’s parameters; the result is shown in Table 3. In Table 3, MD is the percentage mean deviation of the calculated frictional pressure drop from the experimental value. \( N \) is the number of data points. The mean deviation is defined as

\[
MD = \frac{1}{N} \sum \left| \frac{(\Delta P_I/\Delta L) - (\Delta P_i \text{cal}/\Delta L)}{\Delta P_i/\Delta L} \right| \times 100
\]  
(26)
Figure 17. Comparison of calculated and measured frictional pressure drop gradients.

The definition of the liquid limits or vapor Reynolds number $Re_L$, $Re_G$ between the laminar and turbulent is 2000. Of all 297 data points, 51 data points (17.2%) were laminar (liquid)–laminar (vapor); $N = 236$ data points (79.5%) were laminar (liquid)–turbulent (vapor); 4 data points (1.3%) were turbulent (liquid)–laminar (vapor); and 6 data points (2.0%) were turbulent (liquid)–turbulent (vapor).
Table 3. Percentage mean deviation MD of each modified Chisholm parameter. (upward and downward flow).

| Tube                  | N  | Chisholm | Wambsgans | Mishima | Zhang | Lee | Li |
|-----------------------|----|----------|-----------|---------|-------|-----|----|
| Cir. D0.5             | 26 | 53.5     | 68.1      | 24.6    | 16.0  | 37.2| 26.2|
| Cir. D0.7             | 36 | 28.3     | 50.6      | 29.5    | 18.9  | 37.4| 25.2|
| Cir. D1.0             | 69 | 34.2     | 51.4      | 22.4    | 14.3  | 33.3| 48.1|
| Cir. D1.5             | 41 | 36.1     | 60.5      | 18.0    | 22.1  | 35.8| 36.0|
| Cir. D2.0             | 4  | 25.2     | 48.1      | 12.1    | 25.5  | 24.4| 20.4|
| All Cir.              | 176| 36.1     | 55.7      | 22.9    | 17.5  | 35.1| 36.7|
| Rec.                  | 58 | 65.9     | 79.8      | 16.9    | 24.5  | 49.6| 119.4|
| Tri                   | 63 | 63.8     | 65.2      | 17.2    | 20.5  | 47.3| 74.6|
| Total                 | 297| 47.8     | 62.4      | 20.5    | 19.5  | 40.5| 60.9|

Bold number shows the best results.

Wambsgans et al. noted that Equation (13) is applicable for $Re_{L0} < 2200$ and <1.0; however, at this time, all 297 data points are adapted.

There are two applicable correlations suggested by Zhang et al., Equations (20) and (22). However, the calculation value from Equation (22) estimated less accurate 15% in whole comparing to Equation (20), so using the calculation from Equation (20) in Figure 13.

As several researchers have reported, Chisholm’s parameter, which assigns a constant value by the flow regime, could not consider the effect of the differences in tube diameters, so the calculated values had little correspondence with the measured values. The calculated values were almost higher than the measured values.

While Mishima, Zhang, and Li-Wu considered the effect of the differences in tube diameters, Mishima and Zhang had good prediction performances for the measured data. Notably, Mishima, and Zhang made good predictions of the non-circular and circular tubes, respectively. As mentioned above, to consider the effect of the differences in tube diameters, the use of the hydraulic diameter $D_h$ and the Bond number $Bo$ in modified Chisholm’s parameter is shown to be beneficial in mini-/micro-tubes.

4. Conclusions

In the present study, experiments were performed to examine the characteristics of the two-phase frictional pressure drop of an R410A refrigerant flowing vertically upward and downward. The cross-sections of copper test tubes were 0.5, 0.7, 1.0, 1.5, and 2.0 mm circular tubes, and rectangular and triangular tubes with hydraulic diameters of 1.04 and 0.88 mm, respectively. The characteristics of measured pressure drops were clarified by different inner diameter tubes, cross-section shapes, and flow directions. Some of the major conclusions from this paper are summarized below:

1. Although wide range data from low mass flux to high mass flux were obtained, most of data were located on the laminar liquid condition. Therefore, the viscous force and the surface tension have a significant influence on the area between the fluid and channel;
2. As in the case of the conventional large circular tube, the measured frictional pressure drop in small circular tubes increased with increasing quality in the wide range with a maximum at around 0.8; after that, it mildly decreased with increasing quality to a quality of around 1.0. Moreover, increasing mass flux increased the frictional pressure drop, and decreasing the inner diameter of tube increased the frictional pressure drop. In addition, for the low mass flux (50 kg·m$^{-2}$·s$^{-1}$ and 100 kg·m$^{-2}$·s$^{-1}$), the frictional pressure drop decreased with increasing quality in the low quality region, its values not being affected by the different mass flux or inner diameter;
3. As in the case of the circular tube, the measured frictional pressure drop in both the small rectangular and triangular tubes increased with increasing mass flux and quality. In addition, for the low mass flux condition (50 kg·m$^{-2}$·s$^{-1}$ and 100 kg·m$^{-2}$·s$^{-1}$), the
frictional pressure drop indicated higher values in the low-quality region, and that value of \( \Phi_L^2 \) was approximately 10; after that, it decreased with increasing quality. In this region, the value of frictional pressure drop was not affected by the different cross-section shape and mass flux;

4. The cross-section shape was slightly affected by the high mass flux. On the other hand, the pressure drop of the non-circular tubes became smaller at 100 and 200 kg m\(^{-2}\) s\(^{-1}\), especially in the downward flow. Hence, it was supposed that the effect of the cross-section shape became non-negligible;

5. In each tube, the frictional pressure drops of the downward flow were higher than those of the upward flow in which the pressure drop decreased with increasing quality in the low-quality region. Regarding the circular tube, there were no adequate differences in the frictional pressure drop due to the different flow directions, where the pressure drop increased with increasing quality at the mid-quality region. On the other hand, in regard to the non-circular tubes, the apparent discrepancies were observed at 100 and 200 kg m\(^{-2}\) s\(^{-1}\). More specifically, a maximum of 30% differences were observed at 100 kg m\(^{-2}\) s\(^{-1}\);

6. The various modified Chisholm’s parameters in Lockhart–Martinelli correlation for small tubes were examined regarding whether or not they could reproduce the measurement data. While Mishima and Hibiki, Zhang et al., and Li-Wu considered the effect of the differences in tube diameters, Mishima and Zhang had good prediction performances for the measured data. Therefore, to consider the effect of the differences in tube diameters, use of the hydraulic diameter \( D_h \) and bond number \( Bo \) in modified Chisholm’s parameter was shown to be beneficial in mini-/micro-tubes.

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