What is the Criterion for a Strong First Order Electroweak Phase Transition in Singlet Models?

Amine Ahriche

Faculty of Physics, University of Bielefeld, Postfach 100131, D-33501 Bielefeld, Germany.
Department of Physics, University of Jijel, BP 98, Ouled Aissa, DZ-18000 Jijel, Algeria.

It is widely believed that the existence of singlet scalars in some Standard Model extensions can easily make the electroweak phase transition strongly first order, which is needed for the electroweak baryogenesis scenario. In this paper, we will examine the strength of the electroweak phase transition in the simplest extension of the Standard Model with a real singlet using the sphaleron energy at the critical temperature. We find that the phase transition is stronger by adding a singlet; and also that the criterion for a strong phase transition $\Omega(T_c)/T_c \gtrsim 1$, where $\Omega = (\nu^2 + (x - x_0)^2)^{\frac{3}{2}}$ and $x (x_0)$ is the singlet vev in the broken (symmetric) phase, is not valid for models containing singlets, even though often used in the literature. The usual condition $v_L/T_c \gtrsim 1$ is more meaningful, and it is satisfied for a large part of the parameter space for physically allowed Higgs masses.

PACS numbers: 98.80.Cq 11.10.Wx 11.15.Ha

I. INTRODUCTION

The Standard Cosmological Model has been successful in describing the early universe, it is supported by a number of important observations: the expansion of the universe, the abundance of the light elements and the cosmic microwave background (CMB) radiation. These three measurable signatures strongly support the notion that our universe evolved from a dense, nearly featureless hot gas, just as the Big Bang model predicts.

However it fails to explain some serious problems like the nature of dark matter and dark energy; and the dominance of matter over antimatter.

From a theoretical point of view, there is no justification to assume that the universe started its evolution with a defined baryon asymmetry: $n_b(t = 0) > n_\bar{b}(t = 0)$. The natural assumption is that the universe was initially neutral. Direct observations show that the universe contains no appreciable primordial antimatter. In addition, the success of big bang nucleosynthesis requires that the ratio of the effective baryon number ($n_b - n_\bar{b}$) to the entropy density should be between 1

$$2.6 \times 10^{-10} < \eta \equiv \frac{n_b - n_\bar{b}}{s} < 6.2 \times 10^{-10}.$$  

This number has been independently determined to be $\eta = (8.7 \pm 0.3) \times 10^{-11}$ from precise measurements of the relative heights of the first two microwave background (CMB) acoustic peaks by the WMAP satellite. Thus how can one understand the origin of this asymmetry? This is what is called the problem of baryogenesis (for a review see [3]). In 1967, Sakharov forwarded his three conditions for baryogenesis [4], which are summarized in the existence of processes which: (1) violate $B$ number, (2) violate $C$ and $CP$ symmetries; and (3) take place out of equilibrium.

One of the most interesting scenarios for baryogenesis is the electroweak baryogenesis (For a review, see [5]), where the third Sakharov condition is realized via a strong first order phase transition at the electroweak scale.

In gauge theories, a first order phase transition takes place if the vacuum of the theory does not correspond to the global minimum of the potential. Since it is energetically unfavored, the field changes its value to the true vacuum (i.e. the absolute minimum of the potential). Because of the existence of a barrier between the two minima, this mechanism can happen by tunneling or thermal fluctuations via bubble nucleation. The electroweak baryogenesis scenario is realized when the $B$ and $CP$ violating interactions pass through the bubble wall. These interactions are very fast outside the bubbles but suppressed inside. Then a net baryon asymmetry results inside the bubbles which are expanding and filling the universe at the end.

In order to compute the net baryon number, the rate of $B$ violating processes in the broken phase is needed. In the Standard Model, $B$ number is violated at the quantum level [6], where the transition between two topologically distinct $SU(2)_L$ ground states is possible.

The transition between two neighboring ground states breaks both lepton and baryon numbers by $\Delta L = \Delta B = 3$. To find the rate of $B$ violating processes, one needs to know the sphaleron solution, which is a static field configuration that interpolates between the two distinct ground states. The sphaleron configuration was found in [7] for the $SU(2)_L$ model.

A model-independent condition in order that the phase transition be strong enough was derived in [8]:

$$E_{sp} (T_c) / T_c > 45,$$  

where $E_{sp}$ and $T_c$ are the sphaleron energy and the critical temperature, respectively. Since it was shown in [9]...
that \( E_{cp}(T) \propto v(T) \), the condition (2) can be translated for the case of Standard Model to (11)

\[ v_c/T_c > 1, \]  

(3)

where \( v_c \) is the field value at the critical temperature. However this condition (3) is not fulfilled in the case of Standard Model, because the thermal induced cubic term is2 not large enough; also this leads to an unacceptable upper bound on the Higgs mass (12)

\[ m_h \leq 42 \text{ GeV}. \]  

(4)

The constraint gets even stronger when the two-loops effect and a proper treatment of the top quark are included (13). It is clear that this bound is in contradiction with the lower bound coming from LEP \( m_h > 114 \text{ GeV} \).

But this severe bound can be avoided by adding a complex scalar singlet that couples only to the Higgs doublet with an appropriate choice of the theory parameters in a way that the singlet does not develop a vev (15).

If a new scalar (or many scalars which can be singlets or in doublet w.r.t. \( SU(2)_L \)) acquiring a vacuum expectation value \( x \) are added to the Standard Model, the term \( v_c \) in (3) should perhaps be replaced by \( \Omega_c \) which equals \( (v_c^2 + x^2) \frac{3}{2} \); or \( (v_c^2 + (x - x_0)^2) \frac{3}{2} \) when the false vacuum is \( (0, x_0) \) instead of \( (0, 0) \) (16). Then (3) becomes

\[ \Omega_c/T_c > 1. \]  

(5)

If the new particle(s) is a singlet(s), cubic terms can exist in the potential at tree-level, and therefore the phase transition gets stronger without the need of the thermally induced one (16, 17).

In this work, we want to check for a model with a singlet whether the passage from (2) to (3) is true or not? We will consider the simplest extension of the Standard Model with a real singlet. This paper is organized as follows: In the second section, we introduce briefly this model, and find the sphaleron solution in the third section. In the fourth section, we discuss the strength of the first order electroweak phase transition (EWPT). And finally we give our conclusion.

II. THE STANDARD MODEL WITH A SINGLET ‘SM+S’

Let us consider an extension of the Standard Model by a singlet real scalar \( S \) coupled only to the standard Higgs.

We concentrate here on the scalar sector (SM Higgs and the added singlet) and the \( SU(2)_L \) gauge sector.3

The effective Lagrangian

The Lagrangian is given by

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + (D_\mu \phi) (D^\mu \phi) + \frac{1}{2} (\partial_\mu S) (\partial^\mu S) - V_{eff}(\phi, S), \]  

(6)

where \( \phi \) is the Higgs doublet

\[ \phi^T = 1/\sqrt{2} (\chi_1 + i \chi_2, h + i \chi_3) \]  

(7)

where \( h \) is the scalar standard Higgs, \( \chi\)'s are the three Goldstone bosons, and \( F_{\mu\nu}^a \) is the \( SU(2)_L \) field strength

\[ F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g e^{abc} A_\mu^b A_\nu^c. \]  

(8)

\( D_\mu \) is the covariant derivative; when neglecting the \( U_Y(1) \) gauge group, it is given by

\[ D_\mu = \partial_\mu - \frac{i}{2} g a^a A_\mu^a. \]  

(9)

Finally, \( V_{eff}(\phi, S) \) is the effective potential, which is at tree-level given by

\[ V_0(\phi, S) = \lambda |\phi|^4 - \mu_1^2 |\phi|^2 + \mu_2 |\phi|^2 S^2 + \rho |\phi|^2 S + \lambda^\prime \phi^4 - \frac{\lambda^\prime}{2} S^4 - \frac{\lambda^\prime}{3} S^3 - \frac{\rho^2}{2} S^2. \]  

(10)

We can eliminate \( \mu_2^2 \) and \( \lambda^\prime \) by making \((v, s)\) as the absolute minimum of the one-loop effective potential at zero temperature, where \( v = 246.22 \text{ GeV} \) is the standard Higgs vev.

Now, we write the explicit formula of the one-loop effective potential. We will consider the contributions of the gauge bosons, the standard Higgs \( h \), the singlet \( S \), the Goldstone bosons \( \chi_{1,2,3} \) and the top quark. The field-dependent masses at zero temperature are given by

\[ m_t^2 = \frac{1}{4} y_t^2 h^2, \quad m_2^2 = \frac{3}{4} y_s^2 g^2 h^2, \quad m_W^2 = \frac{3}{4} g^2 h^2, \]  

\[ m_W^2 = \lambda h^2 - \mu_1^2 + \mu_2^2, \]  

\[ m_{h, S}^2 \rightarrow m_{h, S}^2 = \frac{1}{2} \left( (3 \lambda + \omega) h^2 + (3 \lambda S + \omega) S^2 + (\rho + 2 \alpha) S - \mu_1^2 - \mu_2^2 \right) \]  

\[ + 4 \left( 2 \omega S + \rho \right) S \]  

\[ \pm \sqrt{\left( (3 \lambda - \omega) h^2 - (3 \lambda S - \omega) S^2 + (\rho + 2 \alpha) S - \mu_1^2 + \mu_2^2 \right)^2 + 4 \left( 2 \omega S + \rho \right) S} \]  

(11)

where \( y_t \) is the Yukawa coupling for the top quark, and \( g^2 = g^2 + g_t^2, \) however we neglected the \( U_Y(1) \) gauge group and therefore \( g = 0 \) and \( m_Z = m_W \); and \( m_{1,2}^2 \) are

---

1 This was also checked for the Minimal Supersymmetric Standard Model (MSSM) in (16), then (3) is valid also for the MSSM.

2 This term is forbidden by symmetry at tree level, however it appears as \( T(m^2)^2 \) from the thermal bosonic contributions to the effective potential at one-loop.

3 Since we are interested here in the Sphaleron solution, we assume that the \( U_Y(1) \) contribution to the sphaleron energy to be negligible as in the case of Standard Model (18).
the Higgs-singlet eigenmasses. Then the one-loop correction to the effective potential at zero temperature is given by

\[ V_1^{T=0}(h, S) = \sum_{i=W,Z,t,h,S,\chi} n_i G \left( m_i^2(h, S) \right) \]

(12)

and

\[ G(x) = \frac{x^2}{64\pi^2} \left\{ \log \left( \frac{x}{Q^2} \right) - \frac{3}{2} \right\}. \]

Here Q is the renormalization scale, which we take to be the standard Higgs vev \( Q = v \); and \( n_i \) are the particle degree of freedom; which are

\[ n_W = 6, n_Z = 3, n_h = 1, n_\chi = 3, n_S = 1, n_t = -12. \]

The temperature-dependent part at one loop is given by [19]

\[ V_1^{T\neq0}(h, S, T) = \frac{T^4}{2\pi^2} \sum_{i=W,Z,t,h,S,\chi} n_i J_{B,F}(m_i^2(h, S)/T^2) \]

and

\[ J_B(\theta) = \int_0^\infty dx\ x^2 \log \left\{ 1 - \exp \left[ -\sqrt{x^2 + \theta} \right] \right\} \]

\[ J_F(\theta) = \int_0^\infty dx\ x^2 \log \left\{ 1 + \exp \left[ -\sqrt{x^2 + \theta} \right] \right\} \]

(13)

where \( a_6 = 16\pi^2 \exp(3/2 - 2\gamma_E) \), \( a_f = \pi^2 \exp(3/2 - 2\gamma_E) \) and \( \gamma_E = 0.5772156649 \) is the Euler constant. There is also another part of the effective potential which is the ring (or daisy) contribution [20]

\[ V_{\text{ring}}(h, S, T) = -\frac{T^4}{12\pi} \sum_{i=W,Z,t,h,S,\chi} n_i \left\{ (M_i^2(h, S, T))^2 - (m_i^2(h, S))^2 \right\}, \]

(17)

where \( M_i^2(h, S, T) \)’s are the thermal masses of the bosons, which are given by

\[ M_i^2(h, S, T) = m_i^2(h, S) + \Pi_i \]

(18)

and \( \Pi_i \) is the thermal correction to the mass, its values for the bosons in our model are:

\[ \Pi_W^L = \frac{11}{6} g^2 T^2, \quad \Pi_W^T = 0 \]

\[ \Pi_Z^L = \frac{11}{6} g^2 T^2, \quad \Pi_Z^T = 0 \]

\[ \Pi_h = \left( g^2/4 + \lambda/2 + y^2/4 + \omega/6 \right) T^2 \]

\[ \Pi_{hh} = \left( g^2/4 + 3\lambda/2 + y^2/4 + \omega/6 \right) T^2 \]

\[ \Pi_{SS} = \left( \lambda_S/4 + 2\omega/3 \right) T^2, \quad \Pi_{hS} \approx 0 \]

(19)

where the script \( L \) (\( T \) ) denotes the longitudinal (transversal) mode for \( W \) and \( Z \). Then the full one-loop effective potential at finite temperature is the sum of [10], [12], [16] and [17]:

\[ V_{\text{1-loop}}^{\text{eff}}(h, S, T) = V_0(h, S) + \sum_{i=W,Z,t,h,S,\chi} n_i G \left( m_i^2(h, S) \right) + \frac{T^4}{2\pi^2} \sum_{i=W,Z,t,h,S,\chi} n_i J_{B,F}(m_i^2(h, S)/T^2) \]

\[ -\frac{T^4}{12\pi} \sum_{i=W,Z,h,S,\chi} n_i \left\{ \left(M_i^2(h, S, T)\right)^2 - \left(m_i^2(h, S)\right)^2 \right\}. \]

(20)

The mass-squared values of the Goldstone bosons or the Higgs-singlet eigenstates can be negative. In the case where a mass value (or more) is negative, the cubic term in [13] becomes non analytic, that’s not a problem since it is already replaced by the thermal mass in [17], where it will be compensated by the thermal correction. If the thermal correction is not enough to compensate the negative value, this term should be omitted since it is imaginary and does not belong to the effective potential.

**The space of parameters**

In our theory, we have quite a few parameters,

\[ \lambda, \lambda_S, \omega, \rho, \alpha, \mu_h, \mu_S, \]

in addition to the singlet vev \( x \). As mentioned above, \( \mu_h \) and \( \mu_S \) can be eliminated as

\[ \mu_h^2 = \lambda v^2 + \omega x^2 + \rho x + \frac{1}{2} \frac{\partial V_{\text{eff}}^{\text{tree}}(h, S)}{\partial h} \bigg|_{h=S=x} \]

\[ \mu_S^2 = \omega v^2 + \frac{\mu_r^2}{2x} + \lambda_S x^2 - \alpha x + \frac{1}{2} \frac{\partial V_{\text{eff}}^{\text{tree}}(h, S)}{\partial S} \bigg|_{h=S=x} \]

(21)

(22)

after which our free parameters are \( \lambda, \lambda_S, \omega, \rho, \alpha \) and \( x \). Since the theory is invariant under the discrete symmetry \( (x, \rho, \alpha) \to (x, -\rho, -\alpha) \), we will assume only positive values for the singlet vev \( x \). We want also to keep the perturbativity of theory by imposing \( \lambda, \lambda_S, |\omega| < 1 \). We choose the parameters, \( \lambda, \lambda_S, \omega, \rho, \alpha \) and \( x \), lying in the ranges:

\[ 0.001 \leq \lambda, \lambda_S \leq 0.6 \]

\[ -0.6 \leq \omega \leq 0.6 \]

\[ 100 \leq x/\text{GeV} \leq 350 \]

\[ -350 \leq \alpha/\text{GeV} \leq 350 \]

\[ -350 \leq \rho/\text{GeV} \leq 350 \]

(23)

The stability of the theory implies that the potential goes to infinity when the field goes to the infinity in any direction, which implies \( \omega^2 < \lambda \times \lambda_S \). Moreover, we need that any minimum or extremum of the potential should be in the range of the electroweak theory; let us say that all the minima and extrema must be inside the circle \( h^2 + \)}
$S^2 = \{600 \text{ GeV}\}^2$ in the $h - S$ plane; and therefore the potential is monotonically increasing outside this circle in any direction.

In the Standard Model, the Higgs mass lower bound is given by $m_h^{SM} > 114 \text{ GeV}$ [14]. The mixing between the standard Higgs and the singlet changes the couplings of the standard Higgs to all the SM sector (gauge bosons and leptons), and therefore this bound is not viable. In our work, we will not derive the new lower bound for the Higgs mass, but we will restrict ourselves only with masses $m_{1,2}$ in the range 65 GeV to 450 GeV.

### III. SPHALERON IN THE 'SM+S'

In order to find the sphaleron solution for this model, we follow the same steps as in the $SU(2)_L$ model. Applying Euler-Lagrange conditions on the effective Lagrangian, (12) or (20), we find the field equations

\[
\frac{\partial}{\partial \zeta} F^{\mu \nu \tau} - g e^{\mu \nu \sigma} A^{\sigma}_{\mu} F^{\alpha \tau} + \frac{1}{4} g^2 h^2 A^{\tau} = 0
\]

\[
\frac{\partial^2 h}{\partial \zeta^2} - \frac{1}{4} g^2 h A_{\mu} A^{\mu} + \frac{1}{4} \frac{\partial}{\partial \zeta} V_{eff} (\phi, S, T) = 0
\]

\[
\frac{\partial^2 S}{\partial \zeta^2} + \frac{2}{\Omega} V_{eff} (\phi, S, T) = 0.
\]

(24)

We will work in the orthogonal gauge where

\[
A_0 = 0, \quad x_i A_i = 0.
\]

(25)

We will not use the spherically symmetric ansatz for \{\phi, A_i\} in [7], but another equivalent one [21],

\[
A_0^a (x) = 2 (1 - f (r)) \frac{e^{aij} x_j}{g r^2}
\]

\[
H (x) = \frac{h}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad h = v L (r)
\]

\[
S (x) = x R (r).
\]

(26)

Here $v$ and $x$ are the Higgs and singlet vevs in the general case (zero or nonzero temperature). Then one can rewrite the field equations (23) as 4

\[
\zeta^2 \frac{\partial^2 f}{\partial \zeta^2} = 2 f (1 - f) (1 - 2 f) - \frac{1}{4} \frac{v^2}{\Omega^2} L^2 (1 - f)
\]

\[
\frac{\partial}{\partial \zeta} \left( \frac{\zeta^2}{g^2 v^2 \Omega^2} \frac{\partial V_{eff} (h, S, T)}{\partial h} \right) \bigg|_{h = v L, S = x R} = 2 L (1 - f)^2 + \frac{\zeta^2}{g^2 v^2 \Omega^2} \frac{\partial V_{eff} (h, S, T)}{\partial S} \bigg|_{h = v L, S = x R}
\]

\[
\frac{\partial}{\partial \zeta} \left( \frac{\zeta^2}{g^2 v^2 \Omega^2} \frac{\partial V_{eff} (h, S, T)}{\partial S} \right) \bigg|_{h = v L, S = x R}
\]

(27)

where $\zeta = g \Omega r$; the parameter $\Omega$ can take any non-vanishing value of mass dimension one (for example, $v$, $x$ or $\sqrt{v^2 + x^2}$); and the energy functional is given by

\[
E_{Sp} (T) = \frac{4 \pi \Omega}{g} \int_0^{+\infty} d \zeta \left\{ \frac{4}{\zeta^2} F^2 (1 - f)^2 + \frac{8}{\zeta^4} f^2 (1 - f)^2\right. \]

\[
+ \frac{v^2}{\Omega^2} \zeta^2 \left( \frac{\partial}{\partial \zeta} L \right)^2 + \frac{v^2}{\Omega^2} L^2 (1 - f)^2 \]

\[
+ \frac{x^2}{\Omega^2} \zeta^2 \left( \frac{\partial}{\partial \zeta} R \right)^2 + \frac{\zeta^2}{g^2 \Omega^2} \times \}

\{

\{ V_{eff} (v L, x R, T) - V_{eff} (v, x, T) \}\},

(28)

with the boundary conditions (See Appendix A)

\[
\begin{align*}
\zeta & \sim 0 \quad f \sim \zeta^2 & \quad \zeta \to \infty \quad f \to 1 \\
L & \sim \zeta & \quad R \sim a + b \zeta^2; \\
& & \quad L \to 1. \\
& & \quad R \to 1. \\
\end{align*}
\]

(29)

Let us now compare the energy functional (28) to that of the minimal Standard Model (eq. (10) in [7]). The difference between these quantities is of course the contribution of the singlet, which contains the kinetic term, the mixing with the standard Higgs; and a contribution to the potential term. However if we compare (28) with the same quantity in the MSSM case (Eq. (2.22) in [10]), we find that in the MSSM both Higgs fields, $h_1$ and $h_2$, have similar contributions to the sphaleron energy, and its general form remains invariant under $h_1 \leftrightarrow h_2$. However this is not the case for (28) if $h \leftrightarrow S$, because of a missing term like $R^2 (1 - f)^2$. 5

For the MSSM sphaleron energy, its form is invariant under $h_1 \leftrightarrow h_2$, and it scales like $\{v^2 + x^2 \}^2$; and for our model 'SM+S', a similar invariance is absent. Could it nevertheless be that $E_{Sp} \propto \{v^2 + (x - x_0)^2 \}^2$? We will check this in the next section.

But when comparing (28) with the same quantity for the Next-to-Supersymmetric Standard Model (NMSSM); (eq. (2.20) in [23]; after eliminating explicit CP phases), we find no large difference expect for what comes from the fact that the NMSSM contains a doublet more than the 'SM+S'; and we remark also similar equations of motion and also similar boundary conditions.

The analytic solution of the system (27) is not possible, this should be done numerically. To solve this system numerically, we need to transform it into a system of 6 first-order differential equations, and therefore we have a first order two-point boundary problem, then we use the so-called relaxation method to solve it. This method is well explained in section 17.3 of [24].

As an example, we solve the system (27) for four chosen sets of parameters (A, B, C and D); and then we can

---

4 There is a similar work done in [22], however there is a difference in the definition in the theory parameters, and also there is an error on the r.h.s. of the first equation in (19) in this paper, where the term $u^2/v^2$ should be corrected as $u^2/v^2$ according to his notation. In our notation it is the term $v^2/\Omega^2$ in the first equation in [27].

5 To be more precise, the absence of a mixing between the singlet and the gauge field is not the only reason to spoil this invariance, but this invariance is absent also in the tree-level potential.
compute the sphaleron energy \(E_{sp}\) at any temperature \(T \leq T_c\). All the results for the sets A, B, C and D are summarized in Table I.

| \(\lambda\)  | \(\lambda_s\) | \(\omega\) | \(x/\text{GeV}\) | \(\alpha/\text{GeV}\) | \(p/\text{GeV}\) | \(m_1/\text{GeV}\) | \(m_2/\text{GeV}\) | \(T_c/\text{GeV}\) | \(E_{sp}(0)/\text{GeV}\) | \(v_c/T_c\) | \(\Omega_c/T_c\) | \(E_{sp}(T_c)/T_c\) |
|----------------|----------------|----------|----------------|----------------|----------------|----------------|----------------|----------------|-----------------|----------|----------|----------------|
| 0.4000         | 0.4000         | 0.5000   | 0.4150         | 0.3818         | 0.2818         | 0.3818         | 0.3000         | 0.838           | 0.5000           | 0.386       | 1.232    | 0.3000           |
| 0.4000         | 0.4200         | 0.4100   | 0.5500         | 0.2818         | 0.2818         | 0.3818         | 0.3000         | 0.838           | 0.5000           | 0.386       | 1.232    | 0.3000           |
| 0.4000         | 0.4200         | 0.4100   | 0.5500         | 0.2818         | 0.2818         | 0.3818         | 0.3000         | 0.838           | 0.5000           | 0.386       | 1.232    | 0.3000           |

Table I: Representative parameter values and the corresponding values of the scalar masses, critical temperature and different ratios needed for the criterion of a strong first order phase transition.

From table I, the set (A) satisfies both conditions (5) and (2), (D) does not satisfy either of them, and both (B) and (C) satisfy (5) but not (2).

The profiles of the functions \(f\), \(L\) and \(R\) are shown in Figure 1.

![Figure 1](image)

Figure 1: A, B, C and D represent the profiles of the functions \(f\), \(L\) and \(R\) for the sets of parameters A, B, C and D in table I respectively. The continuous lines represent the profiles at zero temperature and the dashed ones represent the profiles of the functions at finite temperature.

From all cases in Fig. 1 we remark that the singlet profile is not much different than the unity, due to Neumann type boundary at \(r = 0\). This comes from the fact that the singlet couples to the Higgs doublet and not the gauge fields. Then we claim that the singlet contribution to the sphaleron energy \(E_{sp}\) should be small compared to doublet and gauge field contributions.

IV. THE PHASE TRANSITION IN THE 'SM+S'

In Ref [10], the authors have studied the EWPT strength using the same tree-level potential as [10] with some differences in the parameter definitions. They easily got a strong first order phase transition even for Higgs masses much larger than \(T\). And of course they used the criterion (5) instead of (3), where the quantity \(v_c\) is replaced by \(\Omega_c = \frac{\text{v}^2 + (x_c - x_0 c)^2}{\pi}$$. Since \(\Omega_c/T_c \geq v_c/T_c\) is always fulfilled, the phase transition gets stronger for a larger parameter space compared with the minimal Standard Model case.

Let us take a random choice of about 3000 parameters in the ranges [23], and make a comparison between the two different criteria of the strong first order phase transition [5] and [2]. We show the plots of the quantities \(\Omega_c/T_c\) and \(E_{sp}(T_c)/T_c\) as functions of the lightest Higgs mass \(m_1\) in Fig. 2.

Comparing the number of points above and below the dash-dotted line in both cases (a) and (b) in Fig. 2, we remark that the first order phase transition is stronger than that of the Standard Model with both criteria. However according to the large number of points below the dash-dotted in (a), there are a lot of points which satisfy [5] but they do not really give a strong first order phase transition according to [2].

When comparing the points in Fig. 2 with the curve which represents the Standard Model case, we remark that the addition of a singlet increases, in general, the quantity \(E_{sp}(T_c)/T_c\) which is relevant to the phase transition strength; that there are even a large number of points above the line \(E_{sp}(T_c)/T_c = 45\).

The passage from the criterion (2), which is model-independent, to [5], was based on two assumptions [11]:

(I) The sphaleron energy \(E_{sp}(T)\) scales like the vev \(v(T)\).\(^6\)

(II) The sphaleron energy at \(T = 0\), is taken to be 1.87 in units of \(4\pi v/g\).

If the assumption (I) is satisfied in our model 'SM+S', i.e. \(E_{sp}(T) \propto \Omega(T)\); and \(E_{sp}(0) \simeq 1.87 \times 4\pi \Omega(0)/g\), then [5] is the condition of a strong first order phase transition, but this not the case as mentioned above.

In general, the value of the sphaleron energy at zero temperature is significantly different from 1.87 in units of \(4\pi \Omega(0)/g\), thus if the assumption (I) is fulfilled, then the criterion [5] is still viable but should be relaxed as

---

\(^6\) As mentioned above, this was verified for the Standard Model [5]; and the Minimal Supersymmetric Standard Model [11].
\[ \frac{\Omega_e}{T_c} \gtrsim 1 + \delta, \] where \( \delta \) describes the deviation from the assumption (II).

In order to probe the assumption (I) for our case, i.e.

\[ E_{Sp}(T) \propto \Omega(T), \] (30)

we take the sets (A), (B), (C) and (D) used in table I in the previous section, and plot the ratios \( \frac{\nu(T)}{\nu(0)} \), \( \frac{\Omega(T)}{\Omega(0)} \) and \( \frac{E_{Sp}(T)}{E_{Sp}(0)} \); as functions of temperature, which lies between the critical temperature and another value. The results are shown in Fig. 3.

Let us here comment on Fig. 3. For the case of (A), the ratio \( E_{Sp}(T)/E_{Sp}(0) \) is close to both \( \nu(T)/\nu(0) \) and \( \Omega(T)/\Omega(0) \), which is almost 1 at \( T_c \). For the case of (B), the ratio \( E_{Sp}(T)/E_{Sp}(0) \) is very close to \( \nu(T)/\nu(0) \); however it is different a bit from 1 at \( T_c \). At the temperature \( T \approx 204.5 \text{ GeV} \), there exists a secondary first order phase transition, it happens on the axis \( h = 0 \), where the false vacuum \( (0, x_0) \) is changed suddenly. In the case (C), the ratio \( E_{Sp}(T)/E_{Sp}(0) \) is closer to \( \nu(T)/\nu(0) \) than to \( \Omega(T)/\Omega(0) \); it is also different from 1 at \( T_c \). In the last case (D), there is also a secondary first order phase transition around \( T \approx 256 \text{ GeV} \); where the true vacuum \( (\nu, x) \)

changes discontinuously. We cannot call this an EWPT because the scalar \( h \) has already developed its vev. The ratio \( E_{Sp}(T)/E_{Sp}(0) \) is still scaling like \( \nu(T)/\nu(0) \), but significantly different from 1.

It is clear that \( E_{Sp}(T) \) does not scale like \( \Omega(T) \), but roughly speaking it scales like \( \nu(T) \); with a little deviation in some cases.

We claimed previously that the contribution of the singlet \( S \) to the sphaleron energy is small, and therefore this may be the reason why \( E_{Sp}(T) \) does not behave like \( \Omega(T) \); and also does not behave exactly like \( \nu(T) \). In order to estimate the effect of the singlet field \( S \) on the sphaleron energy [25], we compute the sphaleron energy [25] with replacing the singlet \( S \) by its vev \( \nu \), which we denote \( E_{Sp}(T) \). Then the fifth term in (28) disappears and the third equation in (24) and (27) disappears also; the problem is reduced to a \( SU(2) \), Higgs-gauge system [5]; but with a modified potential \( V_{eff}(h) = V_{eff}(h, x, T) \).

We find that the singlet \( S \) gives a negative contribution to the sphaleron energy which is larger at higher temperatures. But the contribution size is, generally, negligible. The maximum contribution in the case (A) is less or equals 2.1 \%, less than 1.1 \% for in case (B), and almost zero in the case (C): it is less than 0.08 \%; and this is expected because the function \( R \) in Fig. 1 C is very close to 1. In the case (D), there are two different phases: at the first one before the secondary phase transition i.e, \( T_c > T > 256 \text{ GeV} \); the singlet contribution is significant (between 8-9 \%), this may be due to the smallness of the Higgs doublet vev at this range. While in the second phase \( T < 256 \text{ GeV} \); the singlet contribution, as in the other cases, is less than 2 \%. Then in the absence of secondary first order phase transitions, we can neglect the singlet contribution, but in its presence the singlet...
contribution can be sizeable but not as large as that of the Higgs doublet or gauge fields.

To justify this picture, we take again 3000 random sets of parameters and plot $E_{Sp}(T_c)/T_c$ as a function of $\Omega_c/T_c$ in Fig. 4.

![Figure 4: $E_{Sp}(T_c)/T_c$ vs $\Omega_c/T_c$ for 3000 randomly chosen sets of parameters.](image)

Since there exist too many points in the region $(E_{Sp}(T_c)/T_c \leq 45 \cap \Omega(T_c)/T_c \geq 1)$, the criterion [6] is not the definition of a strong first order EWPT. However, it is satisfied for all points that give really a strong first order EWPT except for 10 points due to the existence of secondary first order phase transitions. Then we are now sure that [6] does not describe a strong first order EWPT.

In the sphaleron transitions, the singlet $S$ has no relation to lepton or baryon number breaking phenomena. It does not couple to fermions or gauge bosons; it is just a compensating field in the field equations; (24) and (27); and its effect on the sphaleron transition is negligible as shown above. Then we claim that only the Higgs doublet vev is relevant for the phase transition strength.

We take 3000 random sets of parameters used previously, and plot $E_{Sp}(T_c)/T_c$ as a function of $v_c/T_c$ in Fig. 5.

![Figure 5: $E_{Sp}(T_c)/T_c$ vs $v_c/T_c$ for 3000 randomly chosen sets of parameters.](image)

It is clear that $E_{Sp}(T_c)/T_c$ scales almost exactly to $v_c/T_c$ except for some points, and [3] can describe the strong first order EWPT criterion for most of the points. Then when studying the EWPT in models with a gauge singlet, one should treat the problem as the SM case (in case of one doublet) with replacing the singlet by its vev; and look for the Higgs vev in the path $\partial V_{eff}(h,S)/\partial S|_{S=x} = 0$; whether it is larger than the critical temperature i.e., $v_c/T_c \geq 1$?

\footnote{Except some points due to the existence of secondary first order phase transitions; or due to the significant singlet contribution to the sphaleron energy; especially for smaller Higgs vev values.}

V. CONCLUSION

In this paper, the electroweak phase transition for the Standard Model with a singlet is studied using the known criteria in the literature in addition to the model-independent criterion found in [5]. The authors [16, 17] found that the EWPT gets stronger even for Higgs masses above the bound [4]. They modified the simple criterion [3] into [6], where they replaced the Higgs vev by the distance between the two degenerate minima in the $h - S$ plan.

In our work, we checked whether this criterion is viable for this kind of models or not. We took the Standard Model with a real singlet, then we studied the EWPT using the sphaleron configuration at the critical temperature, then we checked whether all the steps of the passage from the model independent criterion (2) to (3) in the Standard Model case, are respected for our model (i.e. the passage from [2] to [3]) or not?

We found that the EWPT gets stronger even for Higgs masses larger than 100 GeV; and this model does not suffer from the severe Higgs mass bound [4]. However, we remarked that a sizeable number of the parameters satisfy the modified criterion but do not really give a strong first order EWPT, this allowed us to conclude that the 'modified criterion' is not the criterion that describes a strong first order EWPT.

In order to understand why this modified criterion is not viable in this case, we returned back to the SM to see how the passage from the model-independent criterion to the simpler one proceeds? We found that the two assumptions needed for the passage to the simpler criterion are not fulfilled, in general, in our model 'SM+S':

- The sphaleron energy at zero temperature is different...
from the value 1.87 in units of \(4\pi\Omega/g\).

• The sphaleron energy at finite temperature does not scale like \(\Omega(T)\).

We guess that the reason for this is that the singlet does not couple to the gauge field, then the missing of some contributions to the sphaleron energy like \(R^2(1-f)^2\) in \(\Omega(T)\), can spoil the scaling law, \(\Omega(T) \propto \Omega(T)\). This can be inspired if we compare this situation with the case of the MSSM, where this scaling law does work; and the general form of the sphaleron energy is invariant under \(h_1 \leftrightarrow h_2\). The fact that the singlet does couple only to the Higgs doublet leads to a singlet profile in the sphaleron configuration in a Neumann type at \(r = 0\), which makes the singlet contribution too small. Another important remark is that the possibility of secondary first order phase transitions can, sometimes, spoil this scaling law.

As a conclusion, we can say that the condition \(\Omega_c/T_c \geq 1\) is not valid as a strongly first order phase transition criterion. But the usual condition \(\nu_c/T_c \geq 1\), is still the viable one, which can describe the strong first order phase transition for the majority of the physically allowed parameters as stated in Fig. 3. Moreover, this can be satisfied even for Higgs masses in excess of 100 GeV unlike in the Standard Model.

Then in such a model where the singlets couple only to the Higgs doublets, it is convenient to study the EWPT within an effective model that contains only doublets, where the singlets are replaced by their vev’s. We expect similar conclusion for models like the Next-to-Minimal Supersymmetric Standard Model (NMSSM), where in this model the singlet couples only to the two Higgs doublets; and its profile is a Neumann type in the sphaleron configuration. Then the criterion for a strong first order EWPT is \(\{v_1^2 + v_2^2\}^{1/2}/T \geq 1\) at the critical temperature, instead of \(\{v_1^2 + v_2^2 + (x - x_0)^2\}^{1/2}/T \geq 1\).

Acknowledgments

I want to thank Mikko Laine for useful remarks and corrections in the manuscript. The author is supported by DFG.

Appendix A: THE BOUNDARY CONDITIONS

To find the boundary conditions of \(\Omega(T)\), one should take into account that the energy functional \(\Omega(T)\) should be finite. It is clear that in order for the contributions of the second and fourth terms in \(\Omega(T)\) to be finite, \(f\) must go to unity at the limit \(\zeta \rightarrow \infty\). According to the sphaleron definition, scalars go to their vacuum at the infinity, i.e. \(L, R \rightarrow 1\) when \(\zeta \rightarrow \infty\), which makes the last term contribution to \(\Omega(T)\) finite. Thus one can write all the functions as \(1 - c_i \exp\{-d_i\zeta\}\), and find the values of \(c_i\) and \(d_i\) by inserting this behavior into the differential equations \(\Omega(T)\).

In the limit \(\zeta \sim 0\), let us assume that the functions \(f, L, R\) have the profiles

\[
\begin{align*}
  f(\zeta) & \sim \zeta^{n_f}, \\
  L(\zeta) & \sim c_1 + \zeta^{n_L}, \\
  R(\zeta) & \sim c_2 + \zeta^{n_R},
\end{align*}
\]

(A1)

where \(n_f, n_L, n_R\) are some positive constants. In this limit, \(\Omega(T)\) can be approximated as

\[
\begin{align*}
  \frac{\partial^2}{\partial \zeta^2} f & \approx \frac{2}{\zeta^2} f - \frac{1}{4\Omega^2} f L^2 \\
  \frac{\partial^2}{\partial \zeta^2} L & \approx -\frac{2}{\zeta} \frac{\partial}{\partial \zeta} L + \frac{2}{\zeta^2} L \\
  \frac{\partial^2}{\partial \zeta^2} R & \approx -\frac{2}{\zeta} \frac{\partial}{\partial \zeta} R + \frac{1}{g^2\pi T^2} \frac{\partial V_{eff}(h,S,T)}{\partial S} \bigg|_{h=vL, S=xR}
\end{align*}
\]

(A2)

From the second equation in \(\Omega(T)\), one can easily conclude that \(L \sim \zeta\) or \(\zeta^{-2}\), however the second choice makes the energy functional integral \(\Omega(T)\) divergent, thus \(L \sim \zeta\) or \(\{c_1 = 0, n_L = 1\}\). Using this result, one can conclude from first equation in \(\Omega(T)\) that \(f \sim \zeta^2\). However the situation is different for the last equation \(\Omega(T)\), then one can make

\[
\frac{1}{g^2\pi T^2} \frac{\partial V_{eff}(h,S,T)}{\partial S} \bigg|_{h=vL, S=xR} \sim a\zeta^2 + \{A + b\zeta^2\} R(\zeta) + BR^2(\zeta) + CR^4(\zeta)
\]

(A3)

then inserting \(A1\) in \(\Omega(T)\), one finds that the only possibilities are \(n_R = -1\) and \(n_R = 2\), where the first choice is excluded in order that the energy functional integral \(\Omega(T)\) to be convergent, thus \(R \sim a + b\zeta^2\). Therefore at \(\zeta = 0\), \(R\) satisfies the boundary condition of Neumann type, while \(f\) and \(L\) satisfy those of Dirichlet type. The boundary conditions are summarized in \(\Omega(T)\).

[1] B. Fields and S. Sarkar, astro-ph/0601514
[2] D. N. Spergel et al., astro-ph/0603449
[3] A. Riotto, hep-ph/9804754
[4] A.D. Sakharov, JETP Lett. 5, 24 (1967).
[5] V.A. Rubakov and M.E. Shaposhnikov, Usp. Fiz. Nauk 166, 493 (1996); Phys. Usp. 39, 461 (1996).
[6] G. 't Hooft, Phys. Rev. Lett. 37, 8 (1976); Phys. Rev. D 14, 3432 (1976); 18, 2199(E) (1978).
[7] R.F. Klinkhamer and N.S. Manton, Phys. Rev. D 30, 2212 (1984).
[8] A.I. Bochkarev, S.V. Kuzmin and M.E. Shaposhnikov, Phys. Rev. D 43, 369 (1991).
[9] S. Braibant, Y. Brihaye and J. Kunz, Int. J. Mod. Phys. A8, 5563 (1993).
[10] J.M. Moreno, D.H. Oaknin, M. Quiros, Nucl. Phys. B\textbf{483}, 267 (1997).
[11] M.E. Shaposhnikov, Nucl. Phys. B\textbf{287}, 757 (1987); B\textbf{299}, 797 (1988).
[12] A.I. Bochkarev and M.E. Shaposhnikov, Mod. Phys. Lett A\textbf{2}, 417 (1987).
[13] Z. Fodor and A. Hebecker, Nucl. Phys. B\textbf{432}, 127 (1994).
[14] Particle Data Group (W-M Yao et al.), J. Phys. G: Nucl. Part. Phys. \textbf{33}, 1 (2006).
[15] G.W. Anderson and L.J. Hall, Phys. Rev. D \textbf{45}, 2685 (1992); K.E.C. Benson, Phys. Rev. D \textbf{48}, 2456 (1993); J.R. Espinosa and M. Quiros, Phys. Lett B\textbf{305}, 98 (1993).
[16] J. Choi and R.R. Volkas, Phys. Lett. B\textbf{317}, 385 (1993); S.W. Ham, Y.S. Jeong and S.K. Oh, J. Phys. G\textbf{31}, 857 (2005).
[17] Y. Kondo, I. Umemura, K. Yamamoto, Phys. Lett. B\textbf{263}, 93 (1991); N. Sei, I. Umemura and K. Yamamoto, Phys. Lett. B\textbf{299}, 286 (1993).
[18] J. Kunz, B. Kleihaus and Y. Brihaye, Phys. Rev. D \textbf{46}, 3587 (1992); B. Kleihaus, J. Kunz and Y. Brihaye, Phys. Lett. B\textbf{273}, 100 (1991).
[19] L. Dolan and R. Jackiw, Phys. Rev. D \textbf{9}, 3320 (1974).
[20] M.E. Carrington, Phys. Rev. D \textbf{45}, 2933 (1992).
[21] T. Akiba, H. Kikuchi and T. Yanagida, Phys. Rev. D \textbf{38}, 1937 (1988); 40, 588 (1989).
[22] J. Choi, Phys. Lett. B\textbf{345}, 253 (1995).
[23] K. Funakubo, A. Kakuto, S. Tao and F. Toyoda, Prog. Theor. Phys. \textbf{114}, 1069 (2005).
[24] W.H. Press, S.A. Teukolsky, W.T. Vetterling and B.P. Flannery, \textit{Numerical Recipes in Fortran 77: The Art of Scientific Computing} (Cambridge University Press, 1992).
[25] K. Funakubo, S. Tao and F. Toyoda, Prog. Theor. Phys. \textbf{114}, 369 (2005); M. Pietroni, Nucl. Phys. B\textbf{402}, 27 (1993); A.T. Davies, C.D. Froggatt and R.G. Moorehouse, Phys. Lett. B\textbf{372}, 88 (1996); S.J. Huber and M.G. Schmidt, Nucl. Phys. B\textbf{606}, 183 (2001); A. Menon, D.E. Morrissey and C.E.M. Wagner, Phys. Rev. D \textbf{70}, 035005 (2004).