SEARCHING FOR COLD DARK MATTER

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Abstract

The differential cross-section for the elastic scattering of the lightest supersymmetric particle (LSP) with nuclear targets is calculated in the context of currently fashionable supersymmetric theories (SUSY). An effective four fermion interaction is constructed by considering i) $Z^0$ exchange ii) $s$-quark exchange and iii) Higgs exchange. It is expressed in terms of the form factors $f^0_V, f^0_A, f^0_S$ (isoscalar) and $f^1_V, f^1_A$ and $f^1_S$ (isovector) which contain all the information of the underlying theory. Numerical values were obtained using representative input parameters in the constrained parameter space of SUSY phenomenology. Both the coherent and for odd-$A$ nuclei the incoherent (spin) nuclear matrix elements were evaluated for nuclei of experimental interest. The spin matrix elements tend to dominate for odd nuclei but the coherent matrix elements become more important in all other cases. For the coherent part the Higgs contribution competes with the $Z$- and $s$-quark contributions. Cross-sections as high as $10^{-37} \text{cm}^2$ have been obtained.

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1. Introduction.

In recent years the phenomenological implications of supersymmetry (SUSY) are being taken seriously [1-3]. Pretty accurate predictions at low energies are now feasible in terms of few input parameters in the context of unified minimal SUSY without commitment to specific gauge groups[4-8]. More or less such predictions do not seem to depend on arbitrary choices of some parameter or untested assumptions [1-3,9-40]. One may not have to wait, however, till supersymmetry is discovered in high energy colliders. Instead one may look now at phenomena which supersymmetry might affect, e.g. proton decay, lepton flavor violation [10] ($\mu \rightarrow e\gamma$ etc) and dark matter. In the present paper we will concentrate on the implications of supersymmetry on dark matter [11-16].

There is ample evidence [17-28] that more than 90% of the mass of our galaxy, or even of the whole universe, is made up of matter of unknown nature. If one goes beyond the standard model of weak and electromagnetic interactions one has a number of choices for the dark matter candidates. The most obvious choice is, of course, particles which exist, like neutrinos if they have a mass of $\sim 10^{eV}$. Such light particles are expected to be relativistic and constitute the Hot Dark Matter Component (HDM). Another possibility are the axions, which were introduced to account for the strong CP problem. Even though they have not been found in any of the experimental searches, this does not mean that they should be ruled out entirely. The third and most appealing possibility, linked closely with supersymmetry, is the lightest supersymmetric particle (LSP) which is expected to be neutral (see section 2). This particle, which will be denoted by $\chi_1$, is expected to be massive [1-3] ($10-100 GeV$) moving with non relativistic velocities (kinetic energy $10-100 KeV$). In the absence of R-parity violating interactions such a particle is absolutely stable. It constitutes the Cold Dark Matter (CDM) component. The CDM component is needed for the large scale structure formation in the universe. As a matter of fact the ratio of CDM to HDM [23] is 2:1. Since one expects a baryonic component of about 10%, one is lead to the scenario

$$\Omega_{CDM} = 0.6, \quad \Omega_{HDM} = 0.3, \quad \Omega_B = 0.1$$

(1)
where
\[ \Omega_i = \frac{\rho_i}{\rho_c}, \quad \rho_c = \frac{3H^2}{8\pi G_N} \simeq 10^{10} \text{nucleons} m^3 h_0^2 \]  
(2)

where \( H \) is the Hubble constant, \( G_N \) is Newton’s gravitational constant and \( h_0 \) lies between 0.5 and 1.

The detection of LSP can in principle be achieved by devices which are able to detect particles which are interacting very weakly [24-25]. This can be achieved by detecting the recoiling nucleus in the reaction
\[ \chi_1 + (A, Z) \rightarrow \chi_1 + (A, Z) \]  
(3)

The recoiling energy can be converted into phonon energy and detected as temperature rise. This requires a crystal at low temperatures with sufficiently high Debye temperature. The detector should be small enough to permit anticoincidence shielding to reduce background and large enough to allow a sufficient number of counts. A compromise of about 1kg is achieved. Another possibility is to use superconducting granules suspended in a magnetic field. The produced heat will destroy the superconductor and the resulting magnetic flux will trigger a signal in the pick-up coil. Again a mass of about 1 kg is optimum.

The background of such detectors is composed of cosmic rays and natural radioactivity. It can be tackled by utilising the so-called modulation effect, caused by the change in the relative velocity of LSP and the detector due to the diurnal [26] and annual [27] motion of the Earth.

The indirect detection is another possibility. The LPS’s trapped in the gravitational field of the sun will pair annihilate producing high energy particles. Of particular interest are high energy neutrinos originating from the sun, which can be detected by the neutrino telescopes.

In the present paper we will calculate the cross section for the scattering of the LSP by a nucleus. We will utilize the recent developments in supersymmetric theories which have yielded a substantially constrained parameter space. In the first step, along the lines of ref [11-16], we will construct from the elementary couplings the effective four fermion interaction which couples the LSP to the quarks. The next step consists in writing the relevant four fermion interaction at the quark level. The final step consists in transforming...
this interaction at the nucleon level. In the present work we will consider Z, s-quark and Higgs exchange. For the first two exchanges the basic interaction can be read off from the appendix of ref. [5]. The transformation to the nucleon level is straightforward [28]. One can thus construct the 4 needed form factors, i.e. the vector and axial vector isoscalar \( f_0^V, f_0^A \) and isovector \( f_1^V, f_1^A \) form factors. Since, however, the Higgs exchange contribution is important due to the coherent effect of all nucleons we will also provide the model dependent scalar form factors \( f_0^S \) (isoscalar) and \( f_1^S \) (isovector). The diagrams which involve Higgs exchange are a bit more model dependent [5]. Also in this case the transition to the nucleon level is not so straightforward [29]. We will see that the spin dependent nuclear matrix elements arising from the Axial currents are important especially for light nuclei. We will estimate them in this work and provide more accurate calculations in a future publication.

2. Effective Lagrangian.

Before proceeding with the construction of the effective Lagrangian we will briefly discuss the nature of the lightest supersymmetric particle (LSP) focusing on those ingredients which are of interest to dark matter.

2.1. The nature of LSP

In currently favorable supergravity models the LSP is a linear combination [1-3,5] of the neutral four fermions \( \tilde{B}, \tilde{W}_3, \tilde{H}_1 \) and \( \tilde{H}_2 \) which are the supersymmetric partners of the gauge Bosons \( B_\mu \) and \( W_3^\mu \) and the Higgs scalars \( H_1 \) and \( H_2 \). Admixtures of s-neutrinos are expected to be negligible.

In the above basis the mass-matrix takes the form [1]

\[
\begin{pmatrix}
  M_1 & 0 & -m_Z c_\beta s_w & m_Z s_\beta s_w \\
  0 & M_2 & m_Z c_\beta c_w & -m_Z s_\beta c_w \\
  -m_Z c_\beta s_w & m_Z c_\beta c_w & 0 & -\mu \\
  m_Z s_\beta s_w & -m_Z c_\beta c_w & -\mu & 0 \\
\end{pmatrix}
\] (4)
In the above expressions $c_W = \cos \theta_W$, $s_W = \sin \theta_W$, $c_\beta = \cos \beta$, $s_\beta = \sin \beta$ where $\tan \beta = <v_2>/<v_1>$ is the ratio of the vacuum expectation values of the Higgs scalars $H_2$ and $H_1$. $\mu$ is a dimensionful coupling constant which is not specified by the theory (not even its sign). The parameters $\tan \beta, M_1, M_2, \mu$ are determined by a phenomenological fit to the data. In our numerical treatment we used the three possibilities of table VII of ref.1 which we considered to be representative samples in the allowed by the data SUSY parameter space (see table I).

By diagonalizing the above matrix we obtain a set of eigenvalues $m_j$ and the diagonalizing matrix $C_{ij}$ as follows

$$
\begin{pmatrix}
\tilde{B}_R \\
\tilde{W}_{3R} \\
\tilde{H}_{1R} \\
\tilde{H}_{2R}
\end{pmatrix} = (C_{ij}) \begin{pmatrix}
\chi_{1R} \\
\chi_{2R} \\
\chi_{3R} \\
\chi_{4R}
\end{pmatrix};
\begin{pmatrix}
\tilde{B}_L \\
\tilde{W}_{2L} \\
\tilde{H}_{1L} \\
\tilde{H}_{2L}
\end{pmatrix} = (C^{*}_{ij}) \begin{pmatrix}
\chi_{1L} \\
\chi_{2L} \\
\chi_{3L} \\
\chi_{4L}
\end{pmatrix}
$$

Another possibility to express the above results in photino-zino basis $\tilde{\gamma}, \tilde{Z}$ via

$$
\tilde{W}_3 = \sin \theta_W \tilde{\gamma} - \cos \theta_W \tilde{Z} \\
\tilde{B}_0 = \cos \theta_W \tilde{\gamma} + \sin \theta_W \tilde{Z}
$$

In the absence of supersymmetry breaking ($M_1 = M_2 = M$ and $\mu = 0$) the photino is one of the eigenstates with mass $M$. One of the remaining eigenstates has a zero eigenvalue and is a linear combination of $\tilde{H}_1$ and $\tilde{H}_2$ with mixing angle $\sin \beta$. In the presence of SUSY breaking terms the $\tilde{B}, \tilde{W}_3$ basis is superior since the lowest eigenstate $\chi_1$ or LSM is primarily $\tilde{B}$. From our point of view the most important parameters are the mass $m_1$ of LSP and the mixings $C_{j1}, j = 1, 2, 3, 4$ which yield the $\chi_1$ content of the initial basis states. These parameters are given in table II.

We are now in a position to find the interaction of $\chi_1$ with matter. We distinguish three possibilities involving $Z$-exchange, $s$-quark exchange and Higgs exchange.

2.2.1. The $Z$-exchange contribution

4
This can arise from the interaction of Higgsinos with $Z$ which can be read from eq.C86 of ref.[5].

\[
L = \frac{g}{\cos\theta_W} \frac{1}{4} [\tilde{H}_{1R}\gamma_{\mu}\tilde{H}_{1R} - \tilde{H}_{1L}\gamma_{\mu}\tilde{H}_{1L} - (\tilde{H}_{2R}\gamma_{\mu}\tilde{H}_{2R} - \tilde{H}_{2L}\gamma_{\mu}\tilde{H}_{2L})] Z^\mu
\]  

(7)

Using eq. (5) and the fact that for Majorana particles $\bar{\chi}\gamma_{\mu}\chi = 0$, we obtain

\[
L = \frac{g}{\cos\theta_W} \frac{1}{4} (|C_{31}|^2 - |C_{41}|^2)(\bar{\chi}_{1}\gamma_{\mu}\gamma_{5}\chi_{1}) Z^\mu
\]  

(8)

which leads to the effective 4-fermion interaction (see fig. 1a)

\[
L_{eff} = \frac{g}{\cos\theta_W} \frac{1}{4} 2(|C_{31}|^2 - |C_{41}|^2)(-\frac{g}{2\cos\theta_W} \frac{1}{q^2}\bar{\chi}_{1}\gamma_{\mu}\gamma_{5}\chi_{1}) J_{\mu}^Z
\]  

(9)

where the extra factor [5] of 2 comes from the Majorana nature of $\chi_1$. The neutral hadronic current $J_{\mu}^Z$ is given by

\[
J_{\lambda}^Z = -\bar{q}\gamma_{\lambda}(\frac{1}{3}\sin^2\theta_W - (\frac{1}{2}(1 - \sin^2\theta_W))\tau_3 q)
\]  

(10)

at the nucleon level it can be written as

\[
\bar{J}_{\lambda}^Z = -\bar{N}\gamma_{\lambda}(\sin^2\theta_W - g_{V}(\frac{1}{2} - \sin^2\theta_W)\tau_3 + \frac{1}{2}g_{A}\gamma_5\tau_3) N
\]  

(11)

Thus we can write

\[
L_{eff} = -\frac{G_F}{\sqrt{2}} (\bar{\chi}_{1}\gamma_{\lambda}\gamma_{5}\chi_{1}) J_{\lambda}(Z)
\]  

(12)

where

\[
J_{\lambda}(Z) = \bar{N}\gamma_{\lambda}[f_{V}^0(Z) + f_{V}^1(Z)\tau_3 + f_{A}^0(Z)\gamma_5 + f_{A}^1(Z)\gamma_5\tau_3] N
\]  

(13)

and

\[
f_{V}^0(Z) = 2(|C_{31}|^2 - |C_{41}|^2)\frac{m_{Z}^2}{m_{Z}^2 - q^2}\sin^2\theta_W
\]  

(14)

\[
f_{V}^1(Z) = -2(|C_{31}|^2 - |C_{41}|^2)\frac{m_{\mu}^2}{m_{\mu}^2 - q^2}g_{V}(\frac{1}{2} - \sin^2\theta_W)
\]  

(15)

\[
f_{A}^0(Z) = 0
\]  

(16)

\[
f_{A}^1(Z) = 2(|C_{31}|^2 - |C_{41}|^2)\frac{m_{Z}^2}{m_{Z}^2 - q^2}\frac{1}{2}g_{A}
\]  

(17)
with $g_V = 1.0, g_A = 1.24$. We can easily see that

$$f_V^1(Z)/f_V^0(Z) = -g_V\left(\frac{1}{2\sin^2\theta_W} - 1\right) \simeq -1.15$$

Note that the suppression of this $Z$-exchange interaction compared to the ordinary neutral current interactions arises from the smallness of the mixings $C_{31}$ and $C_{41}$, a consequence of the fact that the Higgsinos are normally quite a bit heavier than the gauginos. Furthermore, the two Higgsinos tend to cancel each other.

2.2.2 The $s$-quark mediated interaction

The other interesting possibility arises from the other two components of $\chi_1$, namely $\tilde{B}$ and $\tilde{W}_3$. Their corresponding couplings to $s$-quarks can be read from the appendix C4 of ref.\[5\]. They are

$$L_{\text{eff}} = -g\sqrt{2}\{\bar{q}_L[T_3\tilde{W}_{3R} - \tan\theta_W(T_3 - Q)\tilde{B}_R]q_L + \tan\theta_W C_{11}\tilde{B}_Lq_L + \tan\theta_W C_{11}Q\tilde{B}_Lq_L\} + HC$$

where $\tilde{q}$ are the scalar quarks (SUSY partners of quarks). A summation over all quark flavors is understood. Of interest to us are, of course, the flavors $u$ and $d$. The above interaction is to high accuracy diagonal in quark flavor. Using eq. (15) we can write the above equation in the $\chi_i$ basis. Of interest to us here is the part

$$L_{\text{eff}} = g\sqrt{2}\{(\tan\theta_W(T_3 - Q)C_{11} - C_{21})\bar{q}_{1R}\chi_1q_L + \tan\theta_W C_{11}Q\chi_{1L}\bar{q}_R\}$$

where $Q$ is the charge and $T_3$ the third component of the isospin operator. The effective four fermion interaction (fig. 1b) takes the form

$$L_{\text{eff}} = \frac{(g\sqrt{2})^2}{2}\{[C_{11}\tan\theta_W(T_3 - Q) - C_{21}T_3]^2}{q^2 - m_{q_L}^2}(\bar{q}_{1R}\chi_1q_L) + \frac{|\tan\theta_W C_{11}Q|^2}{q^2 - m_{q_R}^2}(\bar{q}_{1L}\chi_{1L})\}$$

The factor of two arises from the Majorana nature of $\chi_1$. Employing a Fierz transformation \[30\], it can be cast in a more convenient form

$$L_{\text{eff}} = (g\sqrt{2})2(-1)\frac{1}{2}\{[C_{11}\tan\theta(T_3 - Q) - C_{21}T_3]^2}{q^2 - m_{q_L}^2}(\bar{q}_{1R}\gamma\chi_{1L})\}$$
\[ + \frac{|\tan \theta_W C_{11} Q|^2}{q^2 - m_{q_{\bar{r}}}^2} (\bar{q}_{R} \gamma_{\lambda} q_{R}) (\bar{\chi}_{1L} \gamma^\lambda \chi_{1L}) \]  

This can be written compactly as

\[ L_{\text{eff}} = \frac{G_F}{\sqrt{2}} 2 \{ \bar{q}_\gamma (\beta_{0R} + \beta_{3R} \tau_3) (1 + \gamma_5) q 
- \bar{q} \gamma (\beta_{0L} + \beta_{3L} \tau_3) (1 - \gamma_5) q \} (\bar{\chi}_{1} \gamma^\gamma \gamma^5 \chi_{1}) \]

with

\[ \beta_{0R} = \left( \frac{4}{9} \chi_{5_{R}}^2 + \frac{1}{9} \chi_{d_{R}}^2 \right) |C_{11} \tan \theta_W|^2 \]

\[ \beta_{3R} = \left( \frac{4}{9} \chi_{5_{R}}^2 - \frac{1}{9} \chi_{d_{R}}^2 \right) |C_{11} \tan \theta_W|^2 \]

\[ \beta_{0L} = \frac{1}{6} C_{11} \tan \theta_W + \frac{1}{2} C_{21} |q_{\bar{u}_{L}}^2| + \frac{1}{6} C_{11} \tan \theta_W - \frac{1}{2} C_{21} |q_{\bar{d}_{L}}^2| \]

\[ \beta_{3L} = \frac{1}{6} C_{11} \tan \theta_W + \frac{1}{2} C_{21} |q_{\bar{u}_{L}}^2| - \frac{1}{6} C_{11} \tan \theta_W - \frac{1}{2} C_{21} |q_{\bar{d}_{L}}^2| \]

with

\[ \chi_{q_{\bar{L}}}^2 = \frac{m_W^2}{m_{q_{\bar{L}}}^2 - q^2}, \quad \chi_{q_{\bar{R}}}^2 = \frac{m_W^2}{m_{q_{\bar{R}}}^2 - q^2}, \quad \bar{q} = \bar{u}, \bar{d} \]

The above parameters are functions of the four-momentum transfer which in our case is negligible. Proceeding as above we can obtain the effective Lagrangian at the nucleon level as

\[ L_{\text{eff}} = -\frac{G_F}{\sqrt{2}} (\bar{\chi}_{1} \gamma^\gamma \gamma^5 \chi_{1}) J_\lambda(\bar{q}) \]

\[ J_\lambda(\bar{q}) = \bar{N} \gamma_{\lambda} \{ f_0^0(\bar{q}) + f_0^1(\bar{q}) \tau_3 + f_A^0(\bar{q}) \gamma_5 + f_A^1(\bar{q}) \gamma_5 \tau_3 \} N \]

with

\[ f_0^0 = 6(\beta_{0R} - \beta_{0L}), \quad f_0^1 = 2(\beta_{3R} - \beta_{3L}), \quad f_A^0 = 2g_V(\beta_{0R} + \beta_{0L}), \quad f_A^1 = 2g_A(\beta_{3R} + \beta_{3L}) \]

We should note that this interaction is more suppressed than the ordinary weak interaction by the fact that the masses of the s-quarks are usually larger
than that of the gauge boson $Z^0$. In the limit in which the LSP is a pure bino ($C_{11} = 1, C_{21} = 0$) we obtain

$$\beta_{0R} = \tan^2 \theta_W \left( \frac{4}{9} \chi_{uR}^2 + \frac{1}{9} \chi_{dR}^2 \right)$$
$$\beta_{3R} = \tan^2 \theta_W \left( \frac{4}{9} \chi_{uR}^2 - \frac{1}{9} \chi_{dR}^2 \right)$$
$$\beta_{0L} = \frac{\tan^2 \theta_W}{36} (\chi_{uL}^2 + \chi_{dL}^2)$$
$$\beta_{3L} = \frac{\tan^2 \theta_W}{36} (\chi_{uL}^2 - \chi_{dL}^2)$$

(29)

Assuming further that $\chi_{uR} = \chi_{dR} = \chi_{\tilde{u}_L} = \chi_{\tilde{d}_L}$ we obtain

$$f_V^1(\tilde{q})/f_V^0(\tilde{q}) \simeq + \frac{2}{9}$$
$$f_A^1(\tilde{q})/f_A^0(\tilde{q}) \simeq + \frac{6}{11}$$

(30)

If, on the other hand, the LSP were the photino ($C_{11} = \cos \theta_W, C_{21} = \sin \theta_W, C_{31} = C_{41} = 0$) and the s-quarks were degenerate there would be no coherent contribution ($f_V^0 = 0$ if $\beta_{0L} = \beta_{0R}$).

2.2.3. The intermediate Higgs contribution

The process (3) can be mediated via the physical intermediate Higgs particles (see fig. 2). The relevant interaction can arise out of the Higgs-Higgsino-gaugino interaction which takes the form

$$L_{HXX} = \frac{g}{\sqrt{2}} \left( \bar{W}_R \tilde{H}_{2L} H_2^{0*} - \bar{W}_R \tilde{H}_{1L} H_1^{0*} \right. - \left. \tan \theta_w (\tilde{B}_R \tilde{H}_{2L} H_2^{0*} - \tilde{B}_R \tilde{H}_{1L} H_1^{0*}) \right) + H.C.$$  

(31)

Proceeding as above we can express $\bar{W}$ an $\tilde{B}$ in terms of the appropriate eigenstates and retain the LSP to obtain

$$L = \frac{g}{\sqrt{2}} \left( (C_{21} - \tan \theta_w C_{11}) C_{41} \bar{\chi}_{1R} \chi_{1L} H_2^{0*} \right. - \left. (C_{21} - \tan \theta_w C_{11}) C_{31} \bar{\chi}_{1R} \chi_{1L} H_1^{0*} \right) + H.C.$$  

(32)
We can now proceed further and express the fields $H_0^*$ and $H_2^*$ in terms of the fields $\varphi_1 = \chi_1, \varphi_2 = \chi_2, \varphi_3 = -i \chi_3$. The fourth field $\varphi_4 = -i \chi_4$ has been eaten up the gauge boson (Goldstone boson). $\chi_1$ and $\chi_2$ are eigenfields of the real parts and $\chi_3$ and $\chi_4$ are those of imaginary parts. They are normally designated as $h, H, A$ and $G$. The results are, of course, model dependent. At the tree level we obtain the mixings $\theta_r$ and $\theta_i$ between the real and imaginary parts as follows

\[
\tan^2 \theta_r = \tan^2 \beta \frac{2m_0^2 + m_Z^2}{2m_0^2 - m_Z^2} \tag{33}
\]

\[
\tan^2 \theta_i = \tan^2 \beta \tag{34}
\]

The masses are

\[
m_1^2 = m_h^2 = \frac{1}{2} [(m_0^2 + \frac{1}{2} m_Z^2) - \sqrt{(m_0^2 + \frac{1}{2} m_Z^2)^2 - 2m_0^2m_Z^2 \cos^2 2\beta}] \tag{35}
\]

\[
m_2^2 = m_H^2 = \frac{1}{2} [(m_0^2 + \frac{1}{2} m_Z^2) + \sqrt{(m_0^2 + \frac{1}{2} m_Z^2)^2 - 2m_0^2m_Z^2 \cos^2 2\beta}] \tag{36}
\]

\[
m_3^2 = m_A^2 = m_0^2 \quad \text{with} \quad m_0^2 = -\mu B / \sin 2\beta \tag{37}
\]

We thus can write

\[
H_0^* = \sum_{j=1}^{3} \xi_j^{(3)} \varphi_j, \quad H_2^* = \sum_{j=1}^{3} \xi_j^{(4)} \varphi_j \tag{38}
\]

Even though one can now include radiative corrections [31] in our work we found it adequate to use the above expressions and take $m_0$ and $\tan \beta$ from ref.[1]. The results are presented for the reader’s convenience in table III.

We thus obtain

\[
L = \frac{g}{\sqrt{2}} (C_{11} \tan \theta_w - C_{21}) \sum_{j=1}^{3} (C_{3j} \xi_j^{(3)} - C_{4j} \xi_j^{(4)}) \tag{39}
\]

For the quark vertex we need the Yukawa interactions

\[
L_Y = f_{ij} u_{il}^0 u_{jl}^0 H_2^0 + f_{ij} d_{il}^0 d_{jl}^0 H_1^0 + H.C. \tag{40}
\]
In terms of the physical fields we obtain

\[
L_Y = m_f^u \bar{u}_i L u_{iR} + \frac{g}{2\sqrt{2} m_W \sin \beta} \bar{u}_i L u_{iR} \varphi_j + m_d^d \bar{d}_i L d_{iR} + \frac{g}{2\sqrt{2} m_W \sin \beta} \bar{d}_i L d_{iR} \varphi_j
\]  

(41)

(sumation over \(i\) and \(j\) is understood). Combining the above results we obtain the four-fermion interaction

\[
L_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \bar{\chi}_1 \chi_1 \bar{q} \left[ f_s^+ + f_s^- \right] q
\]  

(42)

with

\[
f_s^\pm = 2((C_{11} \tan \theta_w - C_{21}) \sum_{j=1}^{3} (C_{4j} \xi_j^{(4)} - C_{3j} \xi_j^{(3)}))(\frac{m_u}{m_w \sin \beta} \xi_j^{(4)} \pm \frac{m_d}{m_w \cos \beta} \xi_j^{(3)})
\]  

(43)

In order to reduce this to the nucleon level we follow the work of Addler [30] which leads to

\[
L_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \bar{\chi}_1 \chi_1 \bar{N} \left[ f_s^0 + f_s^1 \tau_3 \right] N
\]  

(44)

with

\[
f_s^0 = 1.86 f_s^+ \quad f_s^1 = 0.48 f_s^-
\]  

(45)

3. Evaluation of the nuclear matrix elements.

Combining for results of the previous section we can write

\[
L_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \left\{ (\bar{\chi}_1 \gamma^\lambda \gamma_5 \chi_1) J_\lambda + (\bar{\chi}_1 \chi_1) J \right\}
\]  

(46)

where

\[
J_\lambda = \bar{N} \gamma_\lambda (f_V^0 + f_V^1 \tau_3 + f_A^0 \gamma_5 + f_A^1 \gamma_5 \tau_3) N
\]  

(47)

with

\[
\begin{align*}
 f_V^0 &= f_V^0(Z) + f_V^0(\bar{q}), & f_V^1 &= f_V^1(Z) + f_V^1(\bar{q}) \\
 f_A^0 &= f_A^0(Z) + f_A^0(\bar{q}), & f_A^1 &= f_A^1(Z) + f_A^1(\bar{q})
\end{align*}
\]  

(48)
and
\[ J = \bar{N}(f_s^0 + f_s^1 \tau_3)N \] (49)

By performing a straightforward trace calculation we obtain
\[ |m|^2 = \frac{1}{m_1^2} \left\{ (p_\lambda^J J_\lambda)(p_\mu^J J_\mu^*) + (p_\lambda^J J_\lambda^*)(p_\mu^J J_\mu) - J_\lambda J_\mu^* p_\mu(p_\lambda) \right\} - m_1^2 J_\lambda J_\mu^* + p_\mu p_\lambda |J|^2 \] (50)

By noting that the LSP is an extremely non relativistic particle \((\beta \leq 10^{-3})\) we retain the leading term for each type of matrix element to get
\[ |m|^2 = \frac{1}{m_1^2} \left\{ \frac{1}{2}(E_f E_i - m_1^2 + p_i \cdot p_f)|J_0|^2 + \frac{1}{2}(E_i E_f + m_1^2)|J|^2 + E_i E_f |J|^2 \right\} \]
\[ \simeq \frac{(E_f E_i - m_1^2 + p_i \cdot p_f)}{m_1^2} |J_0|^2 + |J|^2 + |J|^2 \] (51)

where \(E_i, p_i, E_f, p_f\) and \(m_1\) are the kinematical variables of LSP (in the laboratory frame).

The first and the last matrix elements \(J_0\) and \(J\) are non zero even for \(0^+ \rightarrow 0^+\) transitions. Furthermore, all nucleons participate coherently and we expect the matrix elements of \(J_0\) and \(J\) to be proportional to the mass number \(A\). The matrix element of \(J\) is expected to be smaller than that of \(J_0\) due to the smallness of the quark masses (see previous section). The coefficient of \(J_0\), however, is quite small for non relativistic LSP’s (the opposite sign of \(m_1^2\) is a consequence of the Majorana nature of LSP). It is proportional to \(\beta = \nu/c\). Finally the matrix element of \(J\) vanishes for \(0^+ \rightarrow 0^+\) transitions (to leading order). Even for \(J^x \neq 0\) it is expected to be smaller than that of \(J_0\) especially for heavy nuclei, since not all nucleons participate (non coherence).

From the above discussion we conclude that, due to the Majorana nature of LSP, the matrix element \(|m|^2\) is suppressed. Therefore, all three matrix elements need be considered. One can easily find
\[ |J_0|^2 = A^2 |F(q^2)|^2 [f_{V}^0 \cdot f_{V}^1 \frac{N-Z}{A}]^2 \] (52)
\[ J^2 = A^2 |F(q^2)|^2 [f_{s}^0 \cdot f_{s}^1 \frac{N-Z}{A}]^2 \] (53)
where \( F(q^2) \approx 1 \) is the nucleon form factor for momentum transfer \( q \). Also

\[
|J|^2 = \frac{1}{2J_i + 1} |\langle J_i |\bar{\sigma}| J_i \rangle|^2 [g_A^0 \pm g_A^1]^2
\]  

(54)

where the \( +(-) \) sign is associated for protons or neutron holes (neutrons or proton holes).

The nuclear matrix element \( \langle J_i |\bar{\sigma}| J_i \rangle \) vanishes for \( J_i = 0^+ \). For \( J_i \neq 0^+ \) it depends on the details of the structure of the nucleus. For \(^{207}\text{Pb}\), however, it can easily be evaluated, since it is a single particle configuration (one \( 2p_{1/2} \) neutron hole outside the closed shell). For a single particle configuration we get

\[
\frac{1}{2j + 1} |\langle \ell j |\bar{\sigma}| \ell j \rangle|^2 = \begin{cases} j/(j + 1), & j = \ell - 1/2 \\ (j + 1)/j, & j = \ell + 1/2 \end{cases}
\]

(55)

Thus for \(^{207}\text{Pb}\) we obtain

\[
|J| = \frac{1}{3} (f_A^0 + f_A^1)^2
\]  

(56)

For the other odd nuclei which are relevant as targets in searching for dark matter the situation is not so simple. Detailed calculations are under way. For the time being we will present estimates for the three light nuclei \((^{3}_2\text{He}, ^{19}_9\text{F} \text{ and } ^{23}_{11}\text{Na})\). We will assume \([32]\) that the space wave function has symmetry characterized by a Young Tableaux \([f]\) which is as symmetric as possible. This is due to the fact that the two nucleon interaction is attractive and short-ranged. It thus favors nucleon pairs in which the nucleons are as close as possible. The spin-isospin wave function is characterized by symmetry \([\tilde{f}]\) (\([\tilde{f}]\) is obtained from \([f]\) by interchanging rows and columns \([32]\)). This guarantees that the total wave function is antisymmetric. The orbital angular momentum is assumed to be the lowest allowed.

1. The \(^{3}_2\text{He}\) target. The wave function is assumed to be spatially symmetric, \([f] = [3]\), i.e. of the form

\[
\psi(gs) = [3]L = 0; [1^3]s = \frac{1}{2}; I = \frac{1}{2}, \quad I_3 = -\frac{1}{2}; J = \frac{1}{2}
\]  

(57)

The isoscalar matrix element vanishes while the isovector matrix element is

\[
|J|^2 = 27 |f_A^1|^2
\]  

(58)
2. The $^{19}_9F$ target. The wave function is described in terms of three nucleons outside the closed shell $^{16}_8O$ nucleus, i.e.

$$\psi(gs) = [3](60)L = 0; [1^3]s = \frac{1}{2}, \quad I = \frac{1}{2}, \quad I_3 = -\frac{1}{2}; \quad J = \frac{1}{2} \quad (59)$$

where SU(3) representation (60) has the largest value of the Casimir invariant [33]. The obtained matrix element is the same as above, i.e.

$$|J|^2 = 27|f_A^1|^2 \quad (60)$$

3. The $^{23}_{11}Na$ system. It is described as 7 particles outside the closed shell. The spatial symmetry is assumed to be $[f] = [43]$, i.e.

$$\psi(gs) = [43](83)L = 1; [2221]s = \frac{1}{2}, \quad I = \frac{1}{2}, \quad I_3 = -\frac{1}{2}; \quad J = \frac{3}{2} \quad (61)$$

The SU(4) spin - isospin symmetry is equivalent to $[1^3]$. The matrix element is suppressed by the factor dim $[4,2]/$ dim $[4,3]$, $[42]$ and $[4,3]$ viewed as representation of the symmetric groups $[32]$ $S_6$ and $S_7$. This ratio is 9/14. Furthermore, an angular momentum reduction coefficient of 1/9 enters yielding

$$|J|^2 = \frac{9}{14} \frac{1}{9} 27 |f_A^1|^2 = \frac{27}{14} |f_A^1|^2 \quad (62)$$

At this point we should mention that for the extra non-relativistic process (3) traditional nuclear physics techniques should be more reliable than attempts to extract $|J|^2$ [15] from the EMC data [34].

4. Cross-Sections.

With the above ingredients the differential cross section can be easily calculated. For the benefit of the experimentalists we prefer to present our results in the laboratory frame. The scattering is in the forward direction $\xi = \hat{p}_i \cdot \hat{q} \geq 0$, ($p_i$ is the initial LSP momentum, $q$ the momentum transferred to the nucleus). After making the non-relativistic approximation one finds that

$$d\sigma \over d\Omega = \frac{\sigma_0}{\pi} \frac{(m_1)^2}{m_p} \frac{1}{(1 + \eta)^2} \xi \theta(\xi) \{ A^2 [\beta^2 (f_0^V - f_A^0 N - Z/A)^2 \times \}$$

13
\[
\times \left( 1 - \frac{2\eta + 1}{(1 + \eta)^2} \right) + \left( f_s^0 - f_s^1 \frac{N - Z}{A} \right)^2 \right] \\
+ \frac{1}{2J_i + 1} < J_i || \tilde{\sigma}(f_A^0 + \tau_3 f_A^1) || J_i >^2 \} \right) \quad (63)
\]

where \( m_p \) is the proton mass, \( \eta = m_1/m_A \) (\( m_A \) is the mass of the nucleus), \( \beta = \upsilon/c \) (\( \upsilon \) is the velocity of LSP) and

\[
\sigma_0 = \frac{1}{2\pi}(G_F m_p)^2 = 0.77 \times 10^{-38} \text{cm}^2 \quad (64)
\]

The total cross-section becomes

\[
\sigma = \sigma_0 \left( \frac{m_1}{m_p} \right)^2 \times \frac{1}{(1 + \eta)^2} \left\{ A^2 \left[ \beta^2 \left( f_V^0 - f_V^1 \frac{N - Z}{A} \right)^2 \frac{2\eta^2 + 2\eta + 1}{2(1 + \eta)^2} \right] \\
+ \left. \left( f_s^0 - f_s^1 \frac{N - Z}{A} \right)^2 \right] + \frac{1}{2J_i + 1} \right| < J_i || (f_A^0 + \tau_3 f_A^1) \sigma || J_i >^2 \} \right) \quad (65)
\]

We notice that all exchanges (Z, s - quark and Higgs) allow for a coherent contribution of all nucleons yielding a matrix element proportional to the nuclear mass \( A \). The Z and s-quark contribution is suppressed by a factor \( \beta^2 \) for a non-relativistic LSP which, as we have already mentioned, is due to the majorana nature of LSP [11,12]. The parameters \( f_V^0, f_A^0 \) and \( f_A^1 \) as well as the ratios of the isovector to the isoscalar coefficients, \( f_V^1/f_V^0 \) and \( f_s^1/f_s^0 \), are presented in table IV.

From the data of this table we can draw the following general conclusions:

1. For \( 0^+ \rightarrow 0^+ \) transitions the Higgs contribution becomes dominant for solutions 1 and 2 in spite of the smallness of the quark masses (\( m_u = 5MeV, m_d = 10MeV \)). For solution 3 the Higgs contribution becomes comparable to the combined effect of the Z and s-quark exchange.

2. The isovector contribution is small in all cases and additive to the isoscalar contribution. The isovector contribution of the Z-exchange is partly cancelled by that of the s-quark contribution.

3. For odd nuclear targets (\( J_i \neq 0 \)) the spin contribution becomes important. This contribution, which arises from the Z and s-quark exchanges, is not suppressed by the majorana nature of the LSP. It will dominate unless
the matrix element of the axial current is accidentally suppressed. Since this
does not scale with $A^2$, it is somewhat less important in the case of heavy
nuclear targets.

4. The coherent Z and s-quark contribution has an extra angular de-
pendence, which, given a sufficient number of events, could be used for its
experimental discrimination.

5. Discussion of the results.

Using the formulas given above and the data of tables I-IV we can com-
pute the total cross-sections for the LSP scattering with nuclei which can
be used as targets. We see that the cross-section for $0^+ \to 0^+$ transitions, as
well as and the coherent part of the cross-section for odd-mass nuclei, tend
to increase with square of the mass number $A$. The spin matrix element
does not show such an increase and depends on the details of the nuclear
structure. It has been reliably calculated only in the case of $^{207}\text{Pb}$ which is a
single nucleon hole outside the closed shell. Our numerical results are shown
in tables Va (for Z and s-quark exchange) and Vb (for Higgs exchange). From
these tables, we verify that for the coherent part the Higgs contribution, even
though the quarks are very light, becomes dominant for solutions 1 and 2).
For solution 3 the Z and s-quark exchange competes with the Higgs contri-
bution. The largest cross-section is obtained for $^{207}\text{Pb}$. Indeed for $^{207}\text{Pb}$ we get
Solution 1 : $8.1 \times 10^{-38} \text{cm}^2$
Solution 2 : $1.9 \times 10^{-37} \text{cm}^2$
Solution 3 : $2.6 \times 10^{-38} \text{cm}^2$
Unfortunately this cross section is extremely small. This makes the detection
of LSP extremely difficult. Indeed the event rate is given by [16]

$$ \frac{dN}{dt} = \frac{\rho_x}{m_1} <v> \frac{m}{Am_p} $$

(66)

where $\rho_x$ is the density of LSM in our solar system, $m, A$ are the mass and
the mass number of the target, $m_1$ is the mass of LSP and $<v>$ its average
velocity. We find

$$ \frac{dN}{dt} = 5.0 \text{day}^{-1} \frac{\rho_x}{0.3 \text{GeV/cm}^3} \frac{100}{m_1 \text{GeV}} \frac{m}{1 \text{Kg}} \frac{V}{320 \text{Km/s}} \frac{\sigma}{10^{-34} \text{cm}^2} $$

(67)
Using $\rho_x = 0.3\text{GeV/cm}^3$, $m = 1\text{kg}$ and $V = 320\text{Km}$ [15,16], and our results $m_1 = 27\text{GeV}$ and $\sigma = 1.9 \times 10^{-37}\text{cm}^2$ (solution 2) for $^{207}\text{Pb}$ we obtain

$$\frac{dN}{dt} = 6.6 \times 10^{-3}\text{events/day} \simeq 2.5\text{events/year}$$  \hspace{1cm} (68)

Finally we should remark that, even though, as we have mentioned earlier, the predictions of SUSY have become quite constrained and reliable in recent years, the calculated cross-section for process (3) has some uncertainties in it. It depends on the inverse fourth power of the s-quarks and Higgs particles. It also depends on the small mixings $C_{41}$ and $C_{31}$ (eqs. (14)-(17) for Z-exchange and eq. (43) for Higgs exchange). In spite of this, within the allowed parameter space with rather wide variations, e.g. $\tan\beta$ ranging from 1.5 to 10, the variation in the calculated cross section is not very large. The nuclear matrix elements for the coherent process, which is all there is for $0^+$ targets, is very accurate. The evaluation of the spin matrix element for odd targets is less accurate but it can be reliably estimated, at least for $^{207}\text{Pb}$. So our estimate for the cross section should be viewed as quite reliable. Thus, barring completely unforeseen circumstances, the event rate is expected to be small.
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**Figure Captions**

Fig. 1. Two diagrams which contribute to the elastic scattering of LSP with Nuclei. Z-exchange (fig. 1a) and s-quark exchange (fig. 1b). Due to the Majorana nature of LSP only its pseudovector coupling contributes. $J_{\lambda}$ can be parametrized in terms of four form factors $f_{0}^{V}, f_{1}^{V}, f_{0}^{A}, f_{1}^{A}$.

Fig. 2. The same as in fig. 1, except that the intermediate Higgs exchange is considered. This leads to an effective scalar interaction with two form factors $f_{S}^{0}$ (isoscalar) and $f_{S}^{1}$ (isovector)
Table I: SUSY parameters which are relevant for the scattering of LSP with nuclei. They were taken from ref. [1].

| Solution | $\tan\beta$ | $\mu$ (GeV) | $M_1$ (GeV) | $M_2$ (GeV) | $m_{\tilde{u}_R}$ (GeV) | $m_{\tilde{d}_R}$ (GeV) | $m_{\tilde{u}_L}$ (GeV) | $m_{\tilde{d}_L}$ (GeV) | $m_1$ (GeV) |
|----------|--------------|--------------|-------------|-------------|------------------|------------------|------------------|------------------|-------------|
| 1        | 10           | 450          | 126         | 245         | 677              | 676              | 700              | 705              | 126         |
| 2        | 1.5          | -218         | 45          | 90          | 276              | 276              | 283              | 288              | 27          |
| 3        | 5            | 304          | 102         | 200         | 551              | 550              | 570              | 575              | 102         |

Table II: The relevant components $C_{j1}, j = 1, 2, 3, 4$ of LSP or $\chi_1$ (see eq. (10)) and its masses $m_1$ obtained from the data of table I.

| Variable | Solution 1 | Solution 2 | Solution 3 |
|----------|------------|------------|------------|
| $C_{11}$ | .9945      | .8225      | .9891      |
| $C_{21}$ | -.5779 x 10^{-2} | -.4343 x 10^{-2} | -.6258 x 10^{-2} |
| $C_{31}$ | .1029      | .2968      | -.1458     |
| $C_{41}$ | -.1897 x 10^{-1} | -.2164 | -.2136 x 10^{-1} |

Table III: The mass of the physical Higgs particles $\varphi_i$ and the coefficients $\xi_j^{(3)}$ and $\xi_j^{(4)}$ in the decomposition of the neutral Higgs particles $H_1^{0*}$ and $H_2^{0*}$ i.e. $H_1^{0*} = \sum_j \xi_j^{(3)} \varphi_j$ and $H_2^{0*} = \sum_j \xi_j^{(4)} \varphi_j$ (j = 1,2 correspond to the real parts and j = 3 to the imaginary part).

| Variable       | Solution 1 | Solution 2 | Solution 3 |
|----------------|------------|------------|------------|
| $m_1$(GeV)     | 68.7       | 34.5       | 85.7       |
| $m_2$(GeV)     | 119        | 221        | 197        |
| $m_3$(GeV)     | 130        | 217        | 201        |
| $\xi_1^{(3)}$  | 0.5149     | 0.6100     | 0.4581     |
| $\xi_2^{(3)}$  | -0.4847    | -0.3877    | -0.5385    |
| $\xi_3^{(3)}$  | 0.7036     | 0.5883     | 0.6934     |
| $\xi_1^{(4)}$  | 0.4847     | 0.3877     | 0.5385     |
| $\xi_2^{(4)}$  | 0.5149     | 0.6100     | 0.4581     |
| $\xi_3^{(4)}$  | 0.0704     | 0.3922     | 0.0139     |
Table IV: The parameters $\beta f_V^0, f_S^0, f_A^1, f_A^1$ and $f_V^1/f_V^0, f_S^1/f_S^0$ for three SUSY solutions (see text). The value of $\beta = 10^{-3}$ was used. Also in the evaluation of $f_S^0$ and $f_A^1$ we used $m_u = 5MeV$ and $m_d = 10MeV$ for the quark masses.

| quantity                          | Solution 1                          | Solution 2                          | Solution 3                          |
|-----------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|
| $\beta f_V^0(Z)$                  | $0.475 \times 10^{-5}$               | $1.916 \times 10^{-5}$               | $0.966 \times 10^{-5}$               |
| $f_V^1(Z)/f_V^0(Z)$               | -1.153                               | -1.153                               | -1.153                               |
| $\beta f_V^1(\tilde{q})$         | $1.271 \times 10^{-5}$               | $0.798 \times 10^{-5}$               | $1.898 \times 10^{-5}$               |
| $f_V^1(\tilde{q})/f_V^0(\tilde{q})$ | 0.222                               | 2.727                                | 0.217                                |
| $\beta f_V^0$                     | $1.746 \times 10^{-5}$               | $2.617 \times 10^{-5}$               | $2.864 \times 10^{-5}$               |
| $f_V^0/f_V^1$                     | -0.153                               | -0.113                               | -0.251                               |
| $f_S^0$                           | $1.71 \times 10^{-5}$                | $8.02 \times 10^{-4}$                | $-5.51 \times 10^{-5}$               |
| $f_S^1/f_S^0$                     | -0.24                                | -0.15                                | -0.25                                |
| $f_A^0(Z)$                        | -                                    | -                                    | -                                    |
| $f_A^1(Z)$                        | $1.27 \times 10^{-2}$                | $5.17 \times 10^{-2}$                | $2.58 \times 10^{-2}$                |
| $f_A^0(\tilde{q})$               | $0.510 \times 10^{-2}$               | $3.55 \times 10^{-2}$                | $0.704 \times 10^{-2}$               |
| $f_A^1(\tilde{q})$               | $0.277 \times 10^{-2}$               | $0.144 \times 10^{-2}$               | $0.423 \times 10^{-2}$               |
| $f_A^0$                           | $0.510 \times 10^{-2}$               | $3.55 \times 10^{-2}$                | $0.704 \times 10^{-2}$               |
| $f_A^1$                           | $1.55 \times 10^{-2}$                | $5.31 \times 10^{-2}$                | $3.00 \times 10^{-2}$                |
Table Va: The isospin correction $IC = |1 - \frac{f_1}{f_0} \frac{N-Z}{A}|^2$ and the total coherent correction associated with Z-boson and s-quark exchange for various nuclei of interest. For some odd mass nuclei we also present in parenthesis the cross-section associated with the spin matrix elements $|J|^2$.

| Nucleus | Solution 1 | Solution 2 | Solution 3 |
|---------|------------|------------|------------|
|         | IC         | $\sigma(cm^2)$ | IC         | $\sigma(cm^2)$ | IC         | $\sigma(cm^2)$ |
| $^4_2He$ | 0.90       | $1.8 \times 10^{-46}$ | 0.93       | $3.1 \times 10^{-46}$ | 0.84       | $4.6 \times 10^{-45}$ |
|         |            | (1.5 $\times 10^{-38}$) |            | (7.9 $\times 10^{-38}$) |            | (5.4 $\times 10^{-38}$) |
| $^{19}_9Fe$ | 1.02       | $2.3 \times 10^{-43}$ | 1.01       | $1.8 \times 10^{-43}$ | 1.03       | $5.8 \times 10^{-43}$ |
|         |            | (1.5 $\times 10^{-38}$) |            | (7.9 $\times 10^{-38}$) |            | (5.4 $\times 10^{-38}$) |
| $^{23}_{11}Na$ | 1.01       | $4.6 \times 10^{-43}$ | 1.01       | $3.2 \times 10^{-43}$ | 1.02       | $1.1 \times 10^{-42}$ |
|         |            | (1.1 $\times 10^{-39}$) |            | (5.6 $\times 10^{-39}$) |            | (3.9 $\times 10^{-39}$) |
| $^{40}_{20}Ca$ | 1.00       | $3.1 \times 10^{-42}$ | 1.00       | $1.5 \times 10^{-42}$ | 1.00       | $7.0 \times 10^{-42}$ |
| $^{64}_{31}Ga$ | 1.03       | $2.1 \times 10^{-41}$ | 1.03       | $6.4 \times 10^{-42}$ | 1.07       | $4.5 \times 10^{-41}$ |
| $^{72}_{32}Ge$ | 1.03       | $2.1 \times 10^{-41}$ | 1.03       | $6.4 \times 10^{-42}$ | 1.06       | $4.6 \times 10^{-41}$ |
| $^{75}_{35}As$ | 1.04       | $2.4 \times 10^{-41}$ | 1.03       | $7.0 \times 10^{-42}$ | 1.06       | $5.2 \times 10^{-41}$ |
| $^{76}_{33}Ge$ | 1.05       | $2.5 \times 10^{-41}$ | 1.04       | $7.5 \times 10^{-42}$ | 1.08       | $5.7 \times 10^{-41}$ |
| $^{127}_{53}I$ | 1.05       | $1.1 \times 10^{-40}$ | 1.04       | $2.5 \times 10^{-41}$ | 1.08       | $2.4 \times 10^{-40}$ |
| $^{134}_{54}Xe$ | 1.06       | $1.4 \times 10^{-40}$ | 1.04       | $2.8 \times 10^{-40}$ | 1.09       | $2.8 \times 10^{-40}$ |
| $^{207}_{82}Pb$ | 1.07       | $4.2 \times 10^{-40}$ | 1.05       | $7.6 \times 10^{-41}$ | 1.11       | $8.9 \times 10^{-40}$ |
|         |            | (7.6 $\times 10^{-39}$) |            | (1.3 $\times 10^{-38}$) |            | (1.9 $\times 10^{-38}$) |
Table Vb: The isospin correction $IC = |1 - \frac{f_1 N-Z}{f_S A}|^2$ and the total cross-section associated with Higgs exchange in the LSP scattering with nuclei.

| Nucleus | Solution 1 | | | Solution 2 | | | Solution 3 | |
|---|---|---|---|---|---|---|---|---|
| | IC | $\sigma (cm^2)$ | | | IC | $\sigma (cm^2)$ | | | IC | $\sigma (cm^2)$ | |
| $^3_{\text{He}}$ | 0.846 | $1.9 \times 10^{-44}$ | | | 0.903 | $3.4 \times 10^{-43}$ | | | 0.840 | $1.8 \times 10^{-45}$ | |
| $^{19}_{9}\text{Fe}$ | 1.03 | $2.6 \times 10^{-41}$ | | | 1.02 | $2.6 \times 10^{-40}$ | | | 1.03 | $2.5 \times 10^{-42}$ | |
| $^{23}_{11}\text{Na}$ | 1.02 | $3.3 \times 10^{-41}$ | | | 1.01 | $4.8 \times 10^{-40}$ | | | 1.02 | $3.4 \times 10^{-42}$ | |
| $^{40}_{20}\text{Ca}$ | 1.00 | $3.5 \times 10^{-40}$ | | | 1.00 | $2.4 \times 10^{-39}$ | | | 1.00 | $3.4 \times 10^{-41}$ | |
| $^{71}_{31}\text{Ga}$ | 1.06 | $2.9 \times 10^{-39}$ | | | 1.03 | $1.2 \times 10^{-38}$ | | | 1.06 | $2.6 \times 10^{-40}$ | |
| $^{72}_{32}\text{Ge}$ | 1.05 | $3.0 \times 10^{-39}$ | | | 1.04 | $1.2 \times 10^{-38}$ | | | 1.05 | $2.6 \times 10^{-40}$ | |
| $^{75}_{33}\text{As}$ | 1.06 | $3.2 \times 10^{-39}$ | | | 1.03 | $1.3 \times 10^{-38}$ | | | 1.06 | $2.8 \times 10^{-40}$ | |
| $^{76}_{32}\text{Ge}$ | 1.08 | $3.2 \times 10^{-39}$ | | | 1.05 | $1.3 \times 10^{-38}$ | | | 1.08 | $2.9 \times 10^{-40}$ | |
| $^{127}_{53}\text{I}$ | 1.08 | $1.8 \times 10^{-38}$ | | | 1.05 | $4.7 \times 10^{-38}$ | | | 1.08 | $1.5 \times 10^{-39}$ | |
| $^{134}_{54}\text{Xe}$ | 1.10 | $1.9 \times 10^{-38}$ | | | 1.06 | $5.3 \times 10^{-38}$ | | | 1.10 | $1.7 \times 10^{-39}$ | |
| $^{207}_{82}\text{Pb}$ | 1.10 | $7.3 \times 10^{-38}$ | | | 1.06 | $1.8 \times 10^{-37}$ | | | 1.10 | $5.8 \times 10^{-39}$ | |
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