Antenna Pattern Optimization via Clustered Arrays

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Abstract—In this paper, two different architectures based on fully and partially clustered arrays are proposed to optimize the array patterns. In the fully clustered arrays, all the elements of the original array were divided into several equal subarrays, while in the partially clustered arrays, only the side elements were grouped into subarrays, and the central elements were left individually. The second architecture enjoys many advantages compared to the first one. The proposed clustered arrays use quantized amplitude distributions, thus, their corresponding patterns were associated with high side lobes. To overcome this problem, a constraint mask was included in the pattern optimization process. Simulation results show that the peak sidelobe level and the complexity of the feeding network in the partially clustered arrays can be reduced to more than $-28\,\text{dB}$ and $70.833\%$, respectively, for a total of 48 array elements, number of individual central elements = 24, number of clusters on both sides of the array $Q = 4$, and number of elements in each side cluster $M = 6$.

Finally, the principles of the proposed clustered arrays were extended and applied to the two dimensional planar arrays.

1. INTRODUCTION

Modern communication systems such as massive MIMO in the fifth generation (5G) wireless communication applications require a large number of array elements and consequently need a high number of transmit/receive (T/R) modules which are known as high cost manufacturing devices. In order to solve this problem, subarray architectures with fewer T/R modules have been proposed in the literature [1, 2] for the purpose of simplifying the implementation cost and maintaining satisfactory radiation characteristics. The general concept of subarrays involves the division of a large array of elements $N$ into small sub-groups each with $M$ elements so that $M \ll N$, and then controlling the weights of the sub-groups instead of each individual element in the large arrays [3].

Several methods have been proposed by researchers to jointly build efficient subarray architectures and obtain the optimum values of the subarray weights. These methods can be divided into two groups; the first group is based on the biological algorithms such as genetic algorithm and particle swarm optimization [4–8], and the second group is based on the sparseness constraints such as compressed sensing and the convex optimization [3, 9, 10].

Very recently, clustered-array architectures have also been suggested for the applications that require high directivity. Such a type of arrays enjoys a very simple feeding network by grouping more than one radiating elements into a single T/R module [11–13]. Thus, the amplitude excitations of grouped elements will be quantized to a certain value. This leads to a significant deterioration in the clustered array pattern. Also, the clustered arrays suffer from high quantization side lobes due to the amplitude quantization of the element excitations. In [14, 15], aperiodic or polyomino-based clustering strategies have been suggested to overcome the problem of quantization lobes.
In this paper, the array patterns are optimized using both regular and irregular clustered arrays architectures. In the regular clustered arrays, two or more elements are grouped into one subarray meaning that all the elements of the linear array are divided into subarrays with equal number of elements. This will be referred to as fully clustered array. By this way, the complexity of the feeding network in terms of the required number of T/R modules is greatly reduced. Moreover, the effect of the amplitude excitations errors which lead to the problem of high quantization lobes can be overcome during the optimization process of the elements excitations. The high side lobes are eliminated by incorporating a specific constraint mask in the array pattern optimization process which suppresses the excess side lobe power outside the mask.

In the irregular clustered arrays, another innovative architecture is proposed, where only the side elements of the array are divided into subarrays while the central elements of the original linear array are left without clustering, i.e., they are fed individually. By this way, more degrees of freedom will be available which contributes to better preservation of the constraint mask and eliminates the quantization lobes. Besides, some desired nulls at pre-specified unwanted directions can also be included in the optimization process. This architecture is able to provide excellent radiation characteristics with very simplified feeding network. This will be referred to as partially clustered array.

2. THE PROPOSED CLUSTERED ARCHITECTURES

In this section, the fully and partially clustered array architectures are illustrated, and their practical advantages are highlighted.

2.1. Fully Clustered Array

Consider a linear array antenna with even number of elements, \( N \). The elements are assumed to be symmetrically positioned and excited about the center of the array. This means that we have to deal with only half of the array elements instead of all of them. Consequently, the complexity will also be halved.

For fully clustered arrays, all the array elements, \( N \), are divided into several subarrays, say \( Q \), and each subarray, \( q \), contains \( M \) elements. Clearly, \( M \) is always less than \( N \), and \( N \) should be chosen such that the reminder of dividing \( N/M \) should be zero (i.e., \( Q \) should be integer). Figure 1 shows the architecture of the proposed fully clustered array.

From this figure and for example \( N = 48 \) elements, it can be shown that each subarray may contain any of the following number of elements, 2, 3, 4, 6, and 8. As the number of elements in the

Figure 1. Fully clustered array.
subarray increases, the complexity of the feeding network decreases. As mentioned, this reduction in the complexity comes at the cost of higher quantization side lobes in the clustered array pattern. To formulate a proper constraint mask (or the cost function) of the optimization process, first let us write the array factor of the clustered array shown in Figure 1 as:

\[ AF(u) = 2 \sum_{q=1}^{Q/2} A_q \sum_{n=1}^{N/2} \delta_{c_n,q} w_n \cos \left( \frac{(2n-1)}{2} k d u \right) \]  

where \( \delta_{c_n,q} \) is the delta function which is equal to 1 for \( c_n = q \) (i.e., the \( n \)th element belongs to the \( q \)th cluster) and 0 for \( c_n \neq q \). \( A_q \) is the amplitude of each clustered output; \( w_n \) is the complex excitations of each individual element in the original linear array which is equal to \( w_n = a_n e^{j p_n} \) where \( a_n \) and \( p_n \) represent the amplitude and phase excitations of the \( n \)th element, respectively, \( k = 2\pi/\lambda \) where \( \lambda \) is the wavelength in the free space, \( u = \sin(\theta) \) where \( \theta \) is the direction of the angle of the main lobe around the axis of the array; \( d \) is the separation distance between any two successive elements in the regular linear array. From Eq. (1), it can also be noted that the subarrays on each side of the array are considered to be symmetrically weighted. Thus, we have considered only the half values of \( Q \). For amplitude-only weighting, i.e., \( p_n = 0 \), Equation (1) can be normalized and further simplified to

\[ AF(u) = \sum_{q=1}^{Q/2} A_q \sum_{n=1}^{N/2} \delta_{c_n,q} a_n \cos \left( \frac{(2n-1)}{2} k d u \right) \]  

In order to find the values of the amplitude weighting at the subarray level, \( A_q \), first the amplitude excitations at the element level, \( a_n \), are optimized to obtain the desired array pattern with controlled sidelobes and a given beam width. Then, the value of each clustered weight \( A_q \) is computed by taking the mean value of the optimized element excitations \( a_1, a_m, \ldots, a_M \) that belong to \( q \)th cluster. As a result, the amplitude excitations of the array elements \( a_1, a_m, \ldots, a_M \) located within each subarray will be quantized to a new value equal to \( A_q \). Consequently, the side lobe pattern of the individually optimized array elements may change. To maintain the side lobe level of the clustered array pattern within the particular mask limit, we add an extra condition on the cost function which is defined as the excess sidelobe power outside the constraint mask. The lower the cost is, the better the solution is. For a given radiation pattern, each pattern point that lies outside the constraint mask contributes a value to the cost function equal to the power difference between the clustered array pattern due to the subarray and the desired pattern due to the regular linear array. Finally, the inputs to the cost function include an initial set of \( a_n \), the desired array pattern, and the constraint mask. Numerically, this cost function can be written as

\[ \text{cost} = \sum | AF(u) - \text{Mask limit} |^2 \]

Figure 2 shows the results of applying the fully clustered array for \( N = 48 \) elements, \( M = 3 \), number of subarrays, \( Q = 8 \), and the constraint mask is chosen to be at \(-30\) dB. Figure 3 shows the results of the clustered array under additional constraint of placing a wide null centered at \( 46^\circ \). It can be seen that the obtained results confirm the presented analysis. Also, the peak sidelobe level of the clustered array pattern does not exceed the constrained mask limit. More important, the complexity of the feeding network in the clustered array was found less than 16.6\% compared to that of the regular symmetrically excited linear array, 50\%.

2.2. Partially Clustered Array

Another new and innovative architecture is presented in this subsection where the side elements on both sides of the linear array are grouped into a number of subarrays followed by the weighting parameters \( A_q \) as shown in Figure 4. The central elements are connected and weighted individually. Then the overall array architecture becomes hybrid which has many advantages compared to the previous one. The major advantage of this architecture is its ability to provide more degrees of freedom than that of the fully clustered array. This, of course, relaxes the constraint mask of the clustered array pattern. In other words, the limit of the constraint mask can be lowered more than that of the fully clustered array which is very desirable in many applications.
Figure 2. (a) Fully clustered array pattern for $N = 48$, and $M = 3$, and (b) its corresponding amplitude distribution.

Figure 3. (a) Fully clustered array pattern with wide null for $N = 48$ and $M = 3$, and (b) it’s corresponding amplitude distribution.

For this architecture, Eq. (2) can be rewritten as follows:

$$ AF(u) = 2 \sum_{n=1}^{L/2} b_n \cos \left( \frac{(2n-1)k}{2} \right) u + 2 \sum_{q=1}^{Q/2} A_q \sum_{n=\frac{(N-L)+1}{2}}^{N/2} \delta_{Cnq} a_n \cos \left( \frac{(2n-1)k}{2} \right) u $$

(4)

where $b_n$ represents the optimized amplitudes of the individually feeding elements at the center of the array. Figures 5 and 6 show the results of the partially clustered array for $N = 48$. The number of the individually (or centrally) feeding elements was chosen to be $L = 18$, $M = 3$; the number of the subarrays on both sides of the array is $Q = 10$; and the constraint mask limit was chosen to be $-30 \text{ dB}$ as before. Again, these two figures confirm the effectiveness of the proposed partially clustered array.
Figure 4. Partially clustered array.

Figure 5. (a) Partially clustered array pattern for $N = 48$, $M = 3$, and $L = 18$, and (b) it’s corresponding amplitude distribution.

3. SIMULATION RESULTS

To illustrate the effectiveness of both described architectures (fully and partially clustered arrays), several scenarios were examined for each considered array configuration. In all scenarios, the genetic algorithm has been used as the optimization process with the following specifications: the number of population is 20; the rate of mutation is 0.15; and a single point crossover is used. Also, the amplitude-only weighting control is used. This means that the excitation phases of the array elements are set to 0. The total number of the array elements was chosen to be 48 (i.e., 24 elements on each side of the array with amplitude excitations being symmetric on both sides of the array). The number of the array elements in each subarray is ranged from $M = 1, 2, \ldots, 8$, where $M = 1$ corresponds to the regularly linear array without subarrays. The performances of both arrays in terms of peak side lobe level (PSLL), directivity, half power beam width (HPBW), and the percentage complexity of the feeding network as a function of the number of elements in each subarray, $M$, are depicted in Figures 7, 8, 9, and 10, respectively. Numerical values of these performance measurements are also shown in Tables 1 and 2.

The complexity percentage of the fully clustered array (FCA) may be defined by the ratio of the
Figure 6. (a) Partially clustered array pattern with wide null for $N = 48$, $M = 3$, $L = 18$, and (b) it’s corresponding amplitude distribution.

Table 1. Performance of the fully clustered array.

| $N = 24$ on each side of array | $M$ | 1  | 2  | 3  | 4  | 6  | 8  |
|-------------------------------|-----|----|----|----|----|----|----|
| PSLL (dB)                     |     | 31.3 | 31.1 | 30.4 | 29 | 27 | 24 |
| Complexity %                  |     | 50  | 25  | 16.6 | 12.5 | 8.33 | 6.25 |
| Directivity (dB)              |     | 14.13 | 14.11 | 14.0 | 14.24 | 14.34 | 14.4 |
| HPBW (deg.)                   |     | 2.69  | 2.7  | 2.72 | 2.63 | 2.62 | 2.53 |

Table 2. Performance of the partially clustered array.

| $N = 24$ on each side of the array | $M$ | 1  | 2  | 3  | 4  | 5  | 6  |
|-----------------------------------|-----|----|----|----|----|----|----|
| PSLL (dB)                         |     | 31.3 | 30.8 | 30.3 | 30.05 | 29.5 | 28 |
| Complexity %                      |     | 50  | 39.58 | 33.33 | 25 | 25 | 29.167 |
| Directivity (dB)                  |     | 14.13 | 14.07 | 14.1 | 14.17 | 14.24 | 14.33 |
| HPBW (deg.)                       |     | 2.75  | 2.76 | 2.755 | 2.74 | 2.7 | 2.68 |

number of the clusters to the total number of elements:

$$\text{complexity of the } FCA = \frac{Q}{2N} \times 100\% \quad (5)$$

whereas for partially clustered array (PCA) it is defined as

$$\text{complexity of fully clustered} = \frac{\text{No. of Individual Elements}}{2N} + \frac{Q}{2N} \times 100\% \quad (6)$$

From Figure 7, it can be observed that as the number of elements in the subarray increases, the PSLL will also rise. However, this effect becomes worse with the fully clustered array. From Figures 8 and 9, it is noticed that the directivity and HPBW change slightly with varying $M$. As expected, the complexity was inversely proportional to the value of $M$. 
Also, from Figure 10 it can be seen that there are slight changes in the required feeding network complexities of both clustered arrays. Again, this is an extra advantage for the partially clustered array. In the next scenario, the performance of the partially clustered array compared to that of the fully clustered array under large values of quantized parameter, $M$, is investigated. For both arrays, the total number of elements was $N = 48$, and the constraint limit was set to $-30$ dB. For a fully clustered array, the value of $M$ was set to 6 (i.e., there are 4 subarrays on each side of the array). Thus, the total number of degrees of freedom in such a case was 8. For a partially clustered array, the value of $M$ was also set to 6, two equal subarrays on each side of the array, and the number of the individually feeding elements was set to $L = 24$. In this case, the total number of the degrees of freedom was 28. Clearly, the partially clustered array pattern enjoys more relaxations on the constraint mask due to availability of higher number of degrees of freedom. This is evident from Figure 11, where the peak SLL in the partially clustered array pattern was $-33$ dB which is within the constraint limit, while the peak SLL value in the fully clustered array pattern was about $-25$ dB which is above the limit. Figure 11 also shows the cost function variations of the fully and partially clustered arrays versus the number of iterations. Moreover, the complexities of the fully and partially clustered arrays are calculated for the above two cases, and they are found to be 12.5% and 33.33%, respectively.

Next, the proposed idea was compared to the method presented in [11]. The results are shown
Figure 11. Comparisons between fully and partially clustered arrays.

Figure 12. Comparisons between partially clustered array and the methods of [11].

Figure 13. (a) Taylor array pattern, and (b) its distribution.
Figure 14. Results of applying the fully clustered array.

Figure 16. Results of the clustered planar array with $N \times N = 30 \times 30$ elements.
in Figure 12. In addition, the proposed idea was applied to the Taylor excited array [16], and the results are shown in Figures 14 and 15. For comparison, the original Taylor distribution along with its corresponding array pattern is also shown in Figure 13.

Finally, the idea was extended to the planar two-dimensional array with size $30 \times 30$, and the results are shown in Figure 16. The number of the array elements in each subarray is chosen to be $2 \times 2$. As can be seen from the three-dimensional array pattern, satisfactory radiation characteristics have been met. Thus, the proposed idea can be successfully applied to the large arrays such as massive MIMO arrays.

4. CONCLUSIONS

It is evident from current investigation that the desired radiation patterns can be obtained from both fully and partially clustered arrays providing efficient constraint mask. Consequences of using such clustered arrays were a great reduction in the complexity and cost of the array feeding network. Also, the proposed clustered array reduces the effects of the quantization amplitudes due to the unavoidable use of the available digital attenuators in practice.

Moreover, the proposed idea was extended and applied to the two dimensional planar arrays where the elements of each small subarray can be designed as tiles. Then, the array pattern was optimized subject to the optimal clustering configurations.

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