Damage Identification in Polymer Composite Beams Based on Spatial Continuous Wavelet Transform

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Abstract. In present paper, the damage identification in two polymer composite beams is shown with two methods – spatial continuous wavelet transform (CWT) and mode shape curvature squares (MSCS) using experimental data from operational deflection shapes (ODS). Damage was introduced via low velocity impact drop tower. Statistical hypothesis approach was used to calculate standardized damage index (SDI) that served as a damage indicator for both methods. In order to truncate smaller magnitude SDI peaks, a threshold of 1.28, corresponding to the confidence level of 90 %, was applied. Damage estimate reliability (DER) vs wavelet scale test was performed in order to quantify the reliability of damage detection in terms of percentage at each scale parameter. Overall, 78 different wavelet functions were tested. Results indicate that both methods are capable to locate the area of damage.

1. Introduction

Composite materials possess superior mechanical properties as compared to the conventional materials. On the other hand, these properties of composites may be heavily deteriorated by the presence of such failure modes as matrix cracking, fibre pull-out, fibre fracture, fibre-matrix debonding and delamination between plies. Damage of any structure may cause failure, leading to tragic consequences, but it is especially true for engineering and civil structures. As a result, non-destructive structural health monitoring (SHM) methods have become an important research area in civil, mechanical and aerospace communities.

Over the years, researchers have proposed numerous vibration-based damage detection methods, that are based on the fact, that changes in dynamic characteristics, namely, natural frequencies, operation deflection shapes (ODS) and damping, are directly related to changes in stiffness of the structure. However, the major drawback of those methods is a need for the data of healthy structure [1], which sometimes can be difficult or even impossible to obtain. Possible solutions to this issue are employment of Gapped Smoothing Techniques to generate a smoothed surface of mode shape curvature, thus simulating a healthy state of the structure or using wavelet transform (WT) technique. WT technique, as we know it, originated in the 1990’s and was mainly used for signal singularity detection, signal denoising and image compression [2]. As an analyzing technique for damage detection, WT was first used in 1994 on vibration data by Newland [3, 4]. WT was successfully applied for damage detection in composite materials [5-8]. Katunin et al. [5-7] has been using 2D WT in numerical simulations of carbon-fibre reinforced polymer laminate rotor blades with nonlinear
geometry containing small notches, honey-comb composite sandwich structures, 10-layered cross-ply epoxy laminated composite plates, reinforced with E-glass cloth, subjected to low-velocity impact damage. Gallego et al. [8] was studying 2D WT ability to detect structural defects as delamination in carbon-fibre reinforced polymer plates.

In this paper, a method based on WT technique for damage identification in beam structure is described in terms of damage estimate reliability (DER) over dilation parameter values of 78 different wavelet functions. Out of these functions only best are chosen and respective results are compared to the well-known mode shape curvature squares (MSCS) method. Effectiveness and robustness of the proposed algorithms are demonstrated experimentally on two carbon/epoxy polymer composite beams subjected to single low-velocity impact damage.

2. Damage detection algorithms

2.1 Continuous wavelet transform

WT is a mathematical method to transform the original signal into a different domain where additional data analysis becomes possible, therefore damage-affected signal portion is revealed. WT employs wavelets, which are special functions, containing some oscillations, such that the mean of wavelet function is 0. Although WT is generally used to analyze signals in time domain, in spatial analysis time is replaced with a coordinate, say, $x$, giving rise to spatial signal $f(x)$ [2].

Continuous wavelet transform (CWT) is given by [9, 10]

$$W_{s,b} = \frac{1}{\sqrt{|s|}} \int_{-\infty}^{\infty} f(x) \cdot \psi^*_{s,b}\left(\frac{x-b}{s}\right)dx = \int_{-\infty}^{\infty} f(x) \cdot \psi^*_{s,b}(x)dx$$

(1),

where asterisk denotes complex conjugation and $\psi_{s,b}(x)$ is a set of wavelet family functions, derived from a mother wavelet function $\psi(x)$ by translating (parameter $b$) and dilating (parameter $s$) the $\psi(x)$ [9, 10]. Parameter $s$ is a real and positive number. If $0 < s < 1$, the function is expanded, if $s > 1$, it is compressed.

Equation (1) is used to calculate CWT coefficients, that are extremely sensitive to any discontinuities and singularities, present in the signal $f(x)$, therefore location of damage due to a sudden loss of stiffness can be detected in those ODS, that yield large amplitude wavelet coefficients.

Damage index (DI) for ODS is depicted as follows (recall equation (1))

$$DI_{i,CWT}^n = W_{i,s,b} = \int_{-\infty}^{\infty} w^n_{i,s,b}(x)dx$$

(2),

where $DI_{i,CWT}^n$ is a DI for each mode, $L$ is a length of the beam, $w_n$ is transverse displacement of the structure, $n$ is a mode number, $i$ is number of grid point in $x$ direction. The following wavelet families were analysed:

- complex Gauss wavelets of orders 1-5,
- complex Morlet wavelets of orders 1-4,
- Shannon wavelets of orders 1-5,
- Daubechies wavelets of orders 2-10,
- symlet wavelets of orders 2-8,
- coiflet wavelets of orders 1-5,
- Gauss wavelets of orders 1-8,
- biorhogonal wavelets,
- Haar, Meyer, discrete Meyer, Morlet and Mexican hat wavelets.

In practice, however, experimentally measured ODS are inevitably corrupted by measurement noise, which can lead to false peaks in DI profiles. These peaks could be mistakenly interpreted as damage or they could mask the peaks induced by real damage. Therefore the results for all ODS are summarized, giving rise to summarized DI, which is defined as the average summation of damage indices for all ODS $N$, normalized with respect to the largest value of each mode

$$DI_i = \frac{1}{N} \cdot \sum_{n=1}^{N} \frac{DI_{i,CWT}^n}{DI_{i,\text{max}}}$$

(3).
According to [11], the damage indices, determined for each element, are then standardized, according to the concept of statistical hypothesis testing – element \( i \) of the structure is classified as either healthy (null hypothesis \( H_0 \)) or damaged (alternate hypothesis \( H_1 \)). In order to test the hypothesis, the DI, given in equation (3), is standardized

\[
SDI_i = \frac{DI_i - \mu_{DI}}{\sigma_{DI}} \quad (4),
\]

where \( \mu_{DI} \) and \( \sigma_{DI} \) are mean value and standard deviation of damage indices in equation (3), respectively. The decision for the localization of damage is established, based on a pre-assigned classification criterion: choose \( H_0 \) if \( SDI_i < C_r \) or choose \( H_1 \) if \( SDI_i \geq C_r \), where \( C_r \) is a threshold value. Typical values of \( C_r \), widely used in literature, are 1.28, 2 and 3 for 90 %, 95 % and 99 % confidence levels for the presence of damage.

To quantify the reliability of wavelets to identify damage location, a new parameter, called “damage estimate reliability” (DER) is introduced and calculated as follows:

- The whole interval along the axis of the beam (x axis) is split into 3 parts – (a) before damage \( 0 \text{ mm} < x < 150 \text{ (320) mm} \), (b) damage \( 150 \text{ (320) mm} < x < 200 \text{ (370) mm} \) and (c) after damage \( 200 \text{ (370) mm} < x < 300 \text{ (550) mm} \) for Beam 2 (Beam 1).
- In each of these parts all values of SDI from equation (4) of a respective wavelet are summed up and divided by the number of data points in this particular interval, giving average amplitude \( (SDI_i) \) in a respective part.
- DER is equal to average SDI in zone of damage (part b), divided by the average SDI in all parts combined. It is expressed in percentage as:

\[
DER_i = \frac{SDI_i(b)}{SDI_i(a) + SDI_i(b) + SDI_i(c)} \times 100\% \quad (5).
\]

### 2.2 Mode shape curvature squares

Most of the mode shape curvature (MSC) damage detection methods require the baseline data of the healthy structure for inspection of the change in the modal parameters due to damage. Often these baseline modal parameters are not available.

In this paper, an interpolation technique with a Fourier series approximation is applied on a MSC data of the damaged structure, generating smooth MSC surfaces that are estimates of the healthy structure. The Fourier series is a sum of sine and cosine functions that describes a periodic signal:

\[
\kappa(x) = a_0 + \sum_{i=1}^{n} a_i \cdot \cos(n\omega x) + b_i \cdot \sin(n\omega x) \quad (6),
\]

where \( a_0 \) models a constant (intercept) term in the data and is associated with the \( i = 0 \) cosine term, \( \omega \) is the fundamental frequency of the signal, \( n \) is the number of terms (harmonics) in the series [12]. In this study, MSC data was approximated with Fourier series functions of orders 2-8. The obtained coefficient values \( a_0, a_i, b_i \) and \( \omega \) were used to reconstruct the approximation of MSC data using equation (6). The DI is defined as the absolute difference between squares of the measured curvature of the damaged structure and reconstruction of MSC approximation with Fourier series representing the healthy structure. The MSC are calculated from the ODS by the central difference approximation at grid point \( i \)

\[
DI_{i \text{MSCS}}^{n} = \left( \left( \frac{\partial^2 w^n}{\partial x^2} \right)_i - (\kappa^n_x)_i^2 \right) = \left( \left( \frac{w_{i+1}^n - 2w_i^n + w_{i-1}^n}{h^2} \right) - (\kappa^n_x)_i^2 \right) \quad (7).
\]
where \( w_\text{n} \) is a measured transverse displacement of the structure, \( \kappa_\text{xn} \) is reconstruction of Fourier series approximation of MSC data in \( x \) direction, \( n \) is a ODS number, \( i \) is number of grid point in \( x \) direction and \( h \) is the distance between two successive measured points. The maximum value indicates the location of damage. Afterwards, this DI is summarized, standardized and DER value is calculated, according to equation (3), equation (4) and equation (5), respectively.

3. Specimens and experimental set-up

Two laminated composite beams considered in this study are cut out from a carbon/epoxy composite plate with laminate lay-up \((0/90/+45/-45)\) s and ply thickness of \( t = 0.3 \) mm, thus thickness of the plate is 2.4 mm. Experimentally determined material properties of the plate are as follows: \( E_x = 54.5 \) GPa, \( E_y = 31.04 \) GPa, \( G_{xy} = 7.09 \) GPa, \( G_{yz} = 6.5 \) GPa, \( \nu_{xy} = 0.3 \), \( \rho = 1364.9 \) kg/m\(^3\). Dimensions of the beams: width \( B = 35 \) mm and thickness \( H = 2.4 \) mm for both beams, Beam 1 – length \( L = 550 \) mm, Beam 2 – \( L = 350 \) mm.

Impact tests were performed on INSTRON Dynatup 9250 HV drop tower in figure 1 (left). By varying the drop height, different impact energies were obtained. The beams were fixed by using pneumatic clamps. For the Beam 1 the impact energy of 15 J was selected, location of impact is set at the distance \( L_1 = 345 \) mm. For the Beam 2 – 10 J at the distance \( L_2 = 175 \) mm.

The resonant frequencies of the beams are measured by a POLYTEC PSV-400-B scanning laser vibrometer (figure 1 (right)). PSV system requires the definition of outer edges and scanning grid, which consists of points where vibrations are measured. 56 and 30 equally spaced scanning points are equally distributed to cover area of Beam 1 and Beam 2, respectively. Specimens were tested under clamped-clamped (two ends fixed) boundary conditions, where two vices were used to fix the ends of the beam (10 mm from both sides) with the clamped torque of 20 Nm. The beams are excited by a periodic input chirp signal, generated by the internal generator with a 4800 Hz bandwidth through a piezoelectric actuator discs. Afterwards, the beam is excited by a periodic sine wave signal with a frequency, equal to each resonant frequency to obtain the ODS by taking the Fast Fourier Transform of the response signal.

![Figure 1](image1.png)

**Figure 1.** Left: low-velocity impact testing, right: experimental set-up.

4. Results and discussion

SDI values were calculated for wavelets functions, mentioned in section 2.1 for scales \( s = \) from 1 to 128. For each of these 128 SDI values one DER value from equation (5) was calculated. DER values were also calculated, using MSCS method. Fourier series approximations are depicted with F2-F8. Respective results are shown in table 1.
Table 1. DER values for approximation functions and wavelets with highest maximum DER.

| Wavelets | CWT Beam 1 | CWT Beam 2 | MSCS Beam 1 | MSCS Beam 2 | Scale Beam 1 | Scale Beam 2 |
|----------|------------|------------|-------------|-------------|--------------|--------------|
| cMor2    | Beam 1     | Beam 2     | Beam 1      | Beam 2      |              |              |
| F4       | 90.2       | 88.8       | F3          | 78.8        | 86.7         |              |
| With threshold of 1.28 | 95.4 | 97.5 | 89.6 | 100 |

It was necessary to compute DER versus scale plots in order to determine the value of scale parameter at which damage identification was the most successful. As one can see in figure 2, DER peak values correspond to regions of finer scales. After the first peak, the rest of the DER(s) pattern is unpredictable – it resembles a noisy signal. Nevertheless, these plots DER(s) are unique to each wavelet and specimen under test. Only results for wavelets with highest maximum DER values are shown. Maximum DER values for Beam 1 are better by about 11 % (2 %) than respective results for Beam 2 in CWT (MSCS). The best wavelet function turned out to be complex Morlet function of order 2 for both beams. As for MSCS method, different Fourier functions performed differently – Fourier function of order 4 (F4) for Beam 1 was the best, whereas F3 for was the best for Beam 2.

Damage identification results for both beams are shown in figure 3 for CWT and MSCS methods. As one can see, the location of damage, which is enclosed by two red dashed lines, is successfully identified by both methods. There are some smaller peaks present in these plots, which are later filtered out by application of SDI threshold value of 1.28 (SDI1.28 in the plot).

Figure 2. DER (s) plots for both beams.

Figure 3. SDI (x) plots. Left – Beam 1, right – Beam 2. Up – no threshold, bottom – threshold of 1.28.
5. Conclusions
In the present paper, an experimental study on damaged location in two carbon/epoxy polymer composite beams, subjected to low-velocity impact damage, was conducted, exploiting two methods, namely, spatial CWT and MSCS. Success of damage detection was demonstrated using statistical hypothesis approach with a truncation of smaller magnitude peaks in SDI, thus obtaining damage localization with a confidence of 90%. The reliability of damage identification was determined via DER value, which was calculated for 78 different wavelet functions, each of which at scales from 1 to 128 and for Fourier approximation functions of orders 2 to 8 for MSCS method.

While both methods – CWT and MSCS offer an advantage that there is no need for ODS information of a healthy structure and their DER values are comparable, a significant benefit of MSCS method over CWT is that damage identification algorithm is less cumbersome – there is no need to analyse results at different parameter values (scales for wavelets) and there is a considerably smaller variation of available functions for MSCS method as opposed to large amount of wavelet functions. Also, note that only best results were shown, meaning that there is a considerable variation in DER values for wavelets. However, in real life situations, when location of damage in not known a priori, one must still perform SDI calculations for different functions and compare damage detection results, relying on the results that are similar among majority of functions, while dismissing the ones, which have located damage in a completely different position.

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