A novel algorithm for optimal waveform design based on dual mutual information for radar systems

Fengming Xin¹, ⁴, Bin Wang¹, Zhiyong Xu³ and Xu Chen²

¹School of Computer and Communication Engineering, Northeastern University at Qinhuangdao, Qinhuangdao, China
²School of Computer Science and Engineering, Northeastern University, Shenyang, China
³Overseas Materials Management Department, Materials Branch of CNPC Greatwall Drilling Company, Tianjin, China
⁴Email: xfm_neu@126.com

Abstract. In this paper, the authors propose a waveform design algorithm based on dual mutual information criterion in clutter environment for cognitive radar. At the design stage, the authors determine the optimal waveform according to the criterion: maximizing the mutual information between the received signal and target impulse response while minimizing the mutual information between the received signal and clutter impulse response. Then, considering the transmitted waveform energy constraint, the optimization model for transmitted waveform is established. The maximum marginal allocation algorithm is used to obtain the final optimal waveform. Finally, the algorithm is simulated and its performance is analysed by Lagrange multiplier method. The simulation results demonstrate that the algorithm is validation.

1. Introduction

Traditional radar systems transmit the fix waveform, which is difficult to adapt to the complex environment. And cognitive radar (CR) is proposed by Haykin S [1], which is considered as a new generation radar system. CR is a closed-loop dynamic system, which is not only dependent on the adaptive signal processing in the radar receiver, but also through an adaptive radar transmitter in order to improve the performance. Waveform design is an important technology as the cognitive radar [1].

Optimal waveform design has been a topic of considerable research interest for several decades, and lots of works on waveform optimization and design have been done. Maximizing the signal-to-noise ratio (SNR) or signal-to-interference-plus-noise ratio (SINR) as optimization criterion can improve the performance of the target detection. For example, Guerci J R et. al. proposed to find signal/filter pairs to maximize SNR or the SINR to optimize the waveform [2]. Maio A D et. al. published a series of papers they considered the radar code design to achieve the best detection performance based on maximizing SNR criterion [3-5]. In [6], the authors made use of raising the SINR to derive an efficient algorithm for sequence design with the PAR and finite energy constraints to improve the ambiguity function.

A lot of approaches for waveform design based on information theory, especially mutual information (MI), have been proposed [7-14]. Maximizing mutual information between the received signal and target impulse response (TIR) to optimize the transmitted waveform was firstly proposed by Bell [7]. In [8], the authors optimized waveform for multiple target estimation, which used a weighted linear sum of the MI between target signatures and the corresponding received beams. In [9], the authors presented the
stochastic target model and used MI method to design the optimal waveform in signal-dependent interference, and derived the relationship between the MI and signal-to-noise ratio (SNR). In [10], the authors extended MI method to design transmitted waveform for MIMO radar systems and proposed another method that adopted the mean-square error (MSE) in estimating the TIR as the cost function and minimized MSE to optimize the waveform for target identification and classification. The proposed two methods have the same solution. In [11], the authors extended MI method to design transmitted waveform for MIMO radar systems and proposed another method that adopted the mean-square error (MSE) in estimating the TIR as the cost function and minimized MSE to optimize the waveform for target identification and classification. The proposed two methods have the same solution. In [11], the authors used TIR prediction to optimize the waveform based on MI criterion, the target detection performance of optimal waveform is closer to the real performance. In [12], Nijssure Y. et.al presented a novel system architecture for cognitive radar and optimized waveform based on MI minimization between successive target echoes. In [13], the authors also minimized MI between the radar target echoes to optimize waveform after minimizing the mean-square error of the estimates of target scattering coefficients based on Kalman filtering. In [14], the authors designed optimal waveform based on minimizing Kullback-Leibler divergence-locally most powerful (KLD-LMP) metric for the detection of the extended target in a colored noise environment, and derived the relationship between KLD, MI, and SNR.

In this paper, a new optimal waveform design method is proposed based on dual mutual information (DMI) criterion, which maximizes MI between the received signal and TIR while minimizes MI between the received signal and clutter impulse response (CIR) given a transmitted waveform. Then, the optimization model for waveform is established and the final solution is derived by maximum marginal allocation (MMA) algorithm.

The organization of this paper is as follows. In section 2, the signal model of radar system is formulated and the waveform design based on DMI criterion is presented. In section 3, the solution of the optimal waveform is derived by MMA method. Simulation results and analysis of the proposed algorithm are presented in section 4. The conclusion is given in section 5.

2. Signal model and waveform design

Figure 1 illustrates the radar signal model in signal-dependent interference. \( x(t) \) is a transmitted waveform. \( h(t) \) is the TIR, which is a zero-mean Gaussian random process with variance \( \sigma_h^2(f) \). \( c(t) \) is the CIR, which is a zero-mean Gaussian random process with variance \( \sigma_c^2(f) \), and \( n(t) \) is the additive Gaussian noise with zero mean and variance \( \sigma_n^2(f) \). The received signal \( y(t) \) could be expressed as

\[
y(t) = x(t) * h(t) + x(t) * c(t) + n(t)
\]

\[ (1) \]

Figure 1. Signal model in signal-dependent interference.

The received signal \( y(t) \) contains two components: the target echo \( x(t) * h(t) \) and clutter echo plus noise \( x(t) * c(t) + n(t) \). In order to get better performance, radar systems hope to receive signal that contains more target information and less clutter information. Hence, given a transmitted waveform \( x(t) \), the MI between the received signal \( y(t) \) and TIR \( h(t) \) should be maximum and the MI between the received signal \( y(t) \) and \( c(t) \) should be minimum. The optimized waveform should be satisfy the following
max \( I_T(y(t); h(t) \mid x(t)) \) and \( \min \{ I_C(y(t); c(t) \mid x(t)) \} \) \( \text{Eq. (2)} \)

where \( X(f) \) is the Fourier transform of \( x(t) \). \( I_T(y(t); h(t) \mid x(t)) \) is the MI between \( y(t) \) and \( h(t) \), \( I_C(y(t); c(t) \mid x(t)) \) is the MI between \( y(t) \) and \( c(t) \).

The mutual information \( I_T(y(t); h(t) \mid x(t)) \) can be expressed as [7]

\[
I_T(y(t); h(t) \mid x(t)) = T_y \int_W \ln \left[ 1 + \frac{|X(f)|^2 \sigma_h^2(f)}{T_y (\sigma_h^2(f) + |X(f)|^2 \sigma_h^2(f))} \right] df \tag{3}
\]

where the \( W \) is the transmitted waveform bandwidth, \( T_y \) is the duration of the received signal \( y(t) \).

Similar to (3), the mutual information \( I_C(y(t); c(t) \mid x(t)) \) can be expressed as

\[
I_C(y(t); c(t) \mid x(t)) = T_y \int_W \ln \left[ 1 + \frac{|X(f)|^2 \sigma_c^2(f)}{T_y (\sigma_c^2(f) + |X(f)|^2 \sigma_c^2(f))} \right] df \tag{4}
\]

Waveform design based on DMI criterion, according to (2), could be equivalent to

\[
\max \{ I_T - I_C \} \tag{5}
\]

Substitute (3) and (4) into (5), we have

\[
\max \{ I_T - I_C \} = T_y \int_W \ln \left[ 1 + \frac{|X(f)|^2 \sigma_h^2(f)}{T_y (\sigma_h^2(f) + |X(f)|^2 \sigma_h^2(f))} \right] df
\]

\[
- T_y \int_W \ln \left[ 1 + \frac{|X(f)|^2 \sigma_c^2(f)}{T_y (\sigma_c^2(f) + |X(f)|^2 \sigma_c^2(f))} \right] df
\]

\[
= T_y \int_W \left\{ \ln \left[ 1 + \frac{|X(f)|^2 \sigma_h^2(f)}{T_y (\sigma_h^2(f) + |X(f)|^2 \sigma_h^2(f))} \right] - \ln \left[ 1 + \frac{|X(f)|^2 \sigma_c^2(f)}{T_y (\sigma_c^2(f) + |X(f)|^2 \sigma_c^2(f))} \right] \right\} df
\]

\[
= T_y \int_W \left\{ \frac{A_1(f) |X(f)|^4 + B_1(f) |X(f)|^2 + D(f)}{A_2(f) |X(f)|^4 + B_2(f) |X(f)|^2 + D(f)} \right\} df \tag{6}
\]

where

\[
A_1(f) = \left[ T_y \sigma_h^2(f) + \sigma_h^2(f) \right] \sigma_h^2(f) \tag{7}
\]

\[
A_2(f) = \left[ T_y \sigma_c^2(f) + \sigma_c^2(f) \right] \sigma_c^2(f) \tag{8}
\]

\[
B_1(f) = \left[ T_y \sigma_h^2(f) + \sigma_h^2(f) \right] \sigma_h^2(f) + T_y \sigma_c^2(f) \sigma_c^2(f) \tag{9}
\]
\[ B_2(f) = \left[ T_y \sigma_n^2(f) + \sigma_c^2(f) \right] \sigma_n^2(f) + T_y \sigma_n^2(f) \sigma_c^2(f) \] (10)

\[ D(f) = T_y \sigma_n^2(f) \] (11)

Since the radar transmitter is constrained by the total transmitted energy, the optimization model for waveform design based on DMI criterion with transmitted energy constraint can be established as

\[ \max \int_{W} \left\{ \frac{A_1(f)}{X(f)^2} + B_1(f) \right\} + D(f) \] (12)

\[ \text{s.t. } \int_{W} |X(f)|^2 df = E_x \]

where, \( E_x \) is the total transmitted energy. If the clutter is free, \( \sigma_c^2(f) = 0 \), the optimization model (12) is equivalent to optimization model in reference [7].

3. MMA for optimal waveform solution

Since the objective function is not convex, it is difficult to solve the optimization model (12) with Lagrange multiplier method. However, we can take advantage of the MMA method to derive the final optimal waveform. Firstly, the optimization model (12) is discretized as

\[ \max T_y \sum_k \ln \left\{ \frac{A_1(f_k) X(f_k)}{A_2(f_k) X(f_k) + B_2(f_k) X(f_k)} + D(f_k) \right\} \Delta f_k \] (13)

\[ \text{s.t. } \sum_k |X(f_k)|^2 \Delta f_k = E_x \]

Let \( u(k) = |X(f_k)|^2 \), Substitute it into (13), the optimization model could be rewritten as

\[ \max T_y \sum_{k=1}^{N} \ln \left\{ \frac{A_1(f_k) u^2(k) + B_1(f_k) u(k) + D(f_k)}{A_2(f_k) u^2(k) + B_2(f_k) u(k) + D(f_k)} \right\} \Delta f_k \] (14)

\[ \text{s.t. } \sum_{k=1}^{N} u(k) = E_x \Delta f_k = u_{\text{max}} \]

The optimization problem of the model (14) is transformed to seek the maximum of

\[ D = \sum_{k=1}^{N} L(u(k), k) \] with the energy constraint, where

\[ L(u(k), k) = \ln \left( \frac{A_1(f_k) u^2(k) + B_1(f_k) u(k) + D(f_k)}{A_2(f_k) u^2(k) + B_2(f_k) u(k) + D(f_k)} \right) \] (15)

Since \( \sum_{k=1}^{N} u(k) = u_{\text{max}} \) and \( 0 \leq u(k) \leq u_{\text{max}} \), \( u(k) \) could take on values in the set \( \{0, \Delta, 2\Delta, \ldots, P\Delta\} \) for all \( k \), where \( \Delta \) is defined as minimum energy units and \( P\Delta = u_{\text{max}} \). Each step of the algorithm allocates units of energy until all units of energy are allocated. We then allocate the \( P\Delta \) units of “energy” by allocating \( \Delta \) units at each step of the algorithm. In the first step, Let \( u(j) = \Delta \) if \( L(u(j), j) > L(u(k), k) \) for all \( k \neq j \), then repeat the same procedure except the next \( \Delta \) to the value of \( k \) is chosen, which produces the maximum value of \( \{L(\Delta, 0), L(\Delta, 1), \ldots, L(\Delta, j-1), L(2\Delta, j)-L(\Delta, j), L(\Delta, j+1), \ldots, L(\Delta, N)\} \) or to the \( k \) that results in the maximum marginal increase. For example, let \( k = 3 \), \( P = 4 \), \( T_y = 1 \) and...
\[ u(1) + u(2) + u(3) = u_{\text{max}} = 4 \]. Assume that parameters \( \sigma_h^2(f_k), \sigma_c^2(f_k) \) and \( \sigma_n^2(f_k) \) are shown in Table 1.

**Table 1. Parameters.**

|   | \( f_1 \) | \( f_2 \) | \( f_3 \) |
|---|---|---|---|
| \( \sigma_h^2(f_k) \) | 2 | 2 | 3 |
| \( \sigma_c^2(f_k) \) | 1 | 1 | 2 |
| \( \sigma_n^2(f_k) \) | 1 | 2 | 2 |

We wish to maximize

\[
D = L(u(1), 1) + L(u(2), 2) + L(u(3), 3)
\]

\[
= \ln \left( \frac{6u^2(1) + 5u(1) + 1}{3u^2(2) + 4u(1) + 1} \right) + \ln \left( \frac{6u^2(2) + 5u(1) + 1}{3u^2(2) + 4u(1) + 1} \right) + \ln \left( \frac{6u^2(3) + 5u(1) + 1}{3u^2(2) + 4u(1) + 1} \right)
\]

(16)

Let \( \Delta = 1 \), we can allocate either 0, 1, 2, 3 or 4 units to \( u(1) \), \( u(2) \) and \( u(3) \). The possible values of \( L(u(k), k) \) are shown in Table 2, and correspond to the values of \( L(u(k), k) \) for \( u(k) = 1, 2, 3, 4 \).

When the energies are initially allocated \( \Delta = 1 \) units, the values of \( L(u(k), k) \), are 0.4055, 0.2877 and 0.2231, respectively. The maximum value, namely 0.4055, is chosen. Therefore, in the first step, 1 \( \Delta \) units are allocated to \( k = 1 \).

**Table 2. Values of \( L(u(k), k) \) for various values of \( u(k) \).**

| \( u(k) \) | \( k = 1 \) | \( k = 2 \) | \( k = 3 \) |
|---|---|---|---|
| 4 | 0.6698 | 0.5108 | 0.3365 |
| 3 | 0.5596 | 0.4700 | 0.3185 |
| 2 | 0.5108 | 0.4055 | 0.2877 |
| 1 | **0.4055** | 0.2877 | 0.2231 |

After the first step, 1 \( \Delta \) units are allocated completely, the new marginal energies are obtained in Table 3. Where \( k = 1 \) is the marginal energies, and the D values corresponding to \( k = 1, 2, 3 \) are 0.1053, 0.2877 and 0.2231, respectively. The maximum value, namely 0.2877, is chosen. Therefore, in the second step, 1 \( \Delta \) units are allocated to \( k = 2 \).

**Table 3. Marginal values of \( L(u(k), k) \) for various values of \( u(k) \) after first allocation.**

| \( u(k) \) | \( k = 1 \) | \( k = 2 \) | \( k = 3 \) |
|---|---|---|---|
| 4 | 0.5108 | 0.3365 |
| 3 | 0.2643 | 0.4700 | 0.3185 |
| 2 | 0.1541 | 0.4055 | 0.2877 |
| 1 | 0.1053 | **0.2877** | 0.2231 |

Table 4 shows the new marginal energies after the second step, which are 0.1542, 0.1178 and 0.2231, respectively. Therefore, 1 \( \Delta \) units are allocated to \( k = 3 \).
Table 4. Marginal values of $L(u(k), k)$ for various values of $u(k)$ after second allocation.

| $u(k)$ | $k = 1$ | $k = 2$ | $k = 3$ |
|--------|--------|--------|--------|
| 4      | 0.3365 |        |        |
| 3      | 0.2643 | 0.2231 | 0.3185 |
| 2      | 0.1541 | 0.1823 | 0.2877 |
| 1      | 0.1053 | 0.1178 | 0.2231 |

Similarly, the last energies allocation is shown in Table 5. The final energies allocation is summarized in Table 6. The final allocation is $\Delta$ for $k = 1$, $2\Delta$ for $k = 2$ and $\Delta$ for $k = 3$. More details on MMA method can be referred to reference [15].

Table 5. Marginal values of $L(u(k), k)$ for various values of $u(k)$ after third allocation.

| $u(k)$ | $k = 1$ | $k = 2$ | $k = 3$ |
|--------|--------|--------|--------|
| 4      |        |        |        |
| 3      | 0.2643 | 0.2231 | 0.1134 |
| 2      | 0.1541 | 0.1823 | 0.0954 |
| 1      | 0.1053 | 0.1178 | 0.0646 |

Table 6. Final allocation of energies.

| Step | $k = 1$ | $k = 2$ | $k = 3$ |
|------|--------|--------|--------|
| 1    | $\Delta$ |        |        |
| 2    |        | $\Delta$ |        |
| 3    |        |        | $\Delta$ |
| 4    |        |        | $\Delta$ |
| Final | $\Delta$ | $2\Delta$ | $\Delta$ |

4. Simulation results

We demonstrate the characteristics of the optimal waveform based on DMI criterion (named DMI-based waveform). The classic optimal waveform based on MI criterion is named as MI-based waveform [7]. The signal energy is $E = 10$ (energy unit). The noise PSD is $\sigma_n^2(f) = 0.1$. The clutter-to-noise ratio (CNR) is $CNR = 1.29dB$, the target-to-noise ratio (TNR) is $TNR = -5.39dB$.

Figure 2a shows the spectrum signatures of the target and clutter. Figure 2b shows the Energy spectral densities (ESD) of optimal waveforms based on DMI and MI criterions, respectively. Both two optimal waveforms allocate the most energy to some frequency bands in which the target spectrum is stronger. However, the most energy of DMI-based waveform is only allocated to the frequency bands in which the target spectrum is greater than that of the clutter spectrum. The reasons for this is that the optimization model (12), if using Lagrange multiplier to model (12), we have
\[ L \left( \| X(f) \|^2, \lambda \right) = T_s \int \ln \left[ \frac{S_m(f) + |X(f)|^2 \sigma_n^2(f)}{S_m(f) + |X(f)|^2 \sigma_n^2(f)} \right] df \]

\[ -\lambda \left[ E - \int |X(f)|^2 df \right] \]  

(17)

Figure 2. The spectrum signatures of the target and clutter and the optimal waveforms.

The (17) can equivalently maximize

\[ L \left( \| X(f) \|^2, \lambda \right) = T_s \ln \left[ \frac{S_m(f) + |X(f)|^2 \sigma_n^2(f)}{S_m(f) + |X(f)|^2 \sigma_n^2(f)} \right] - \lambda |X(f)|^2 \]  

(18)

The second derivatives of \( L \left( \| X(f) \|^2, \lambda \right) \) with respect to \( |X(f)|^2 \) is given by
\[ \frac{d^2 L \left( \left| X(f) \right|^2, \lambda \right)}{d \left( \left| X(f) \right|^2 \right)^2} = -T_\gamma \left[ \frac{\sigma_h^2(f) - \sigma_c^2(f)}{B(f) + 2A(f)\left| X(f) \right|^2} \right] \left( C(f) + B(f)\left| X(f) \right|^2 + A(f)\left[ \left| X(f) \right|^2 \right]^2 \right)^2 \]  

where \( A(f) = \sigma_h^2(f)\sigma_h^2(f) \), \( B(f) = \sigma_h^2(f)\left[ \sigma_h^2(f) + \sigma_c^2(f) \right] \) and \( C(f) = \sigma_h^2(f) \). If DMI-based waveform exists, the numerator of (19) should be greater than zero, that is \( \sigma_h^2(f) - \sigma_c^2(f) > 0 \). Thus, DMI-based waveform allocates the most energy to some frequency bands when \( \sigma_h^2(f) > \sigma_c^2(f) \).

In reference [4], the authors represented the conclusions of the relationship between the mutual information and SINR:

\[ I_T = T_f \int \ln \left( 1 + \text{SINR}(f) \right) df \]  

When mutual information increases, SINR will also increase and the detection performance will be better. The mutual information, given MI-based waveform (MIW) and DMI-based waveform (DMIW), are both calculated, respectively (see Table 7). In Table 7, since \( I_T(y(t); h(t) \mid \text{MIW}) > I_T(y(t); h(t) \mid \text{DMIW}) \), the detection performance of MI-based waveform should be better than that of DMI-based waveform according to (20). Although, \( I_T(y(t); h(t) \mid \text{MIW}) > I_T(y(t); h(t) \mid \text{DMIW}) \), the mutual information \( I_C \) given DMI-based waveform is less than that given MI-based waveform. We define the target-to-clutter information ratio (TCIR) as \( I_T / I_C \). The TCIR given DMI-based waveform is greater than that given MI-based waveform, it illustrates the target information takes more proportion than the clutter information in received signal.

| Waveform         | \( I_T \) | \( I_C \) | \( I_T / I_C \) |
|------------------|----------|----------|----------------|
| MI-based waveform| 0.0248   | 0.0211   | 1.175          |
| DMI-based waveform| 0.0223 | 0.0131   | 1.702          |

Figure 3. Detection probability curves corresponding to LFM and optimal waveforms.

In order to verify detection performance of DMI-based waveform, the linear frequency modulated (LFM) waveform is used as a benchmark. The energy spectrum density of LFM waveform is given by
The relationship between detection probability $p_d$ and the false alarm probability $p_{fa}$ is [16]

$$
ESD_{LFM}(f) = E_x / W, \ f \leq W / 2
$$

The detection probability curves are shown in Figure 3. The detection performance of DMI-based waveform is slightly worse than that of MI-based waveform, but better than that of the fix LFM waveform. It illustrates the simulation results in Figure 3 are consistent with the conclusions in reference [4].

5. Conclusions

A waveform design method based on dual mutual information criterion is proposed. Maximizing the MI between the received signal and TIR while minimizing the MI between the received signal and CIR as the objective function. Considering the transmitter energy limitations, the optimization model for waveform design is established. Since the optimization model is difficulty solved by Lagrangian multipliers, the MMA method is adopt to solve the final waveform (DMI-based waveform). DMI-based waveform can improve the target detection performance compared with the fixed waveform. Although detection performance of DMI-based waveform is slightly worse than the MI-based waveform, it can reduce the clutter information in received signal. Since it allocates the most energy in the frequency bands where the target spectrum is greater than clutter and decreases the clutter information in received signal.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (No. 61601109, 61473066), the Fundamental Research Funds for the Central Universities (No. N182304022), and the Natural Science Foundation of Hebei Province (No. F2018501051).

References

[1] Haykin S 2006 Cognitive radar: a way of the future J. IEEE Signal Process. Mag. 23(1) 30-40
[2] Pillai S U, Oh H S, Youla D C and Guerci J R 2000 Optimum transmit-receiver design in the presence of signal-dependent interference and channel noise J. IEEE Trans. Inf. Theory 46(2) 577-584
[3] Maio A D, Huang Y, Piezzo M, Zhang S and Farina A 2011 Design of optimized radar codes with a peak to average power ratio constraint J. IEEE Trans. Signal Process. 59(6) 2683-2697
[4] Maio A D, Nicola S D, Huang Y, Palomar D P, Zhang S and Farina A 2010 Code design for radar STAP via optimization theory J. IEEE Trans. on Signal Process. 58(2) 679-694.
[5] Miao A D, Aubry A and Farina A 2014 Radar waveform design in a spectrally crowded environment via nonconvex quadratic optimization J. IEEE Trans. on Aerosp. and Elect. Syst 50(2) 1138-1152
[6] Wu L, Babu P and Palomar D P 2017 Cognitive Radar-Based Sequence Design via SINR Maximization J. IEEE Trans. On SIGNAL Process. 65(3) 779-793
[7] Bell M R 1993 Information theory and radar waveform design J. IEEE Trans. Inf. Theory 39(5) 1578-1597
[8] Leshem A, Naparstek O and Nehorai A 2007 Information Theoretic Adaptive Radar Waveform J. IEEE J. Sel. Top. Signal Process. 1(1) 42-55
[9] Romero R A, Bae J and Goodman N A 2011 Theory and application of SNR and mutual information matched illumination waveform J. IEEE Trans. Aerosp. Electron. Syst. 47(2) 912-927
[10] Yang Y and Blum R S 2007 MIMO radar waveform design based on mutual information and minimum mean-square error estimation J. IEEE Transactions on the Aerospace and
Electronic Systems 43(1) 330-3436

[11] Xin F M, Wang J K, Wang B and Song X 2014 Waveform design for cognitive radar based on information theory C. Processing of 2014 International Conference on Multisensor Fusion and Information Integration for Intelligent Systems Beijing China 28-29

[12] Nijsure Y, Chen Y, Boussakta S, Yuen C, Chew Y H and Ding Z 2012 Novel system architecture and waveform design for cognitive radar radio networks J. IEEE Trans. Veh. Technol. 61(8) 3630-3642

[13] Yao Y, Zhao J and Wu L 2018 Cognitive radar waveform optimization based on mutual information and Kalman filtering J. Entropy 20 653 DOI 10.3390/e20090653

[14] Zhu Z, Kay S and Raghavan R 2017 Information-theoretic optimal radar waveform design J. IEEE Process. Letters 2017 24 274-278 DOI 10.1109/LSP.2017.2655879

[15] Kay S 2009 Waveform design for multistatic radar detection J. IEEE Trans. Aerosp. Electron. Syst 45 (3) 1153-1166

[16] Deng X, Qiu C and Cao Z 2012 Waveform design for enhanced detection of extended target in signal-dependent interference J. IET Radar Sonar Navig. 6(1) 30-38