Spacecraft reflector shape control

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Abstract. The paper considers a possible method of optimal controlling the mirror shape of a spacecraft (SC) large-sized reflector using the mechanisms affecting the reflector mirror shape in the course of its orbital operation. The authors give a theoretical justification for the application of the least squares problem and adapt the Gauss – Newton method used for solving it to the problem of the reflector mirror shape optimal control. Mathematical modeling confirmed the effectiveness of the method as applied to controlling the mirror shape of an umbrella type offset reflector with a diameter of more than 10 m, which includes a device for adjusting the reflector shape. The main issues of the method implementation are outlined in relation to the onboard system for monitoring the operational characteristics of a large-sized transformable antenna. The paper shows that the considered method can be used to control the operational characteristics of antennas with large-sized reflectors by affecting their geometric characteristics, namely, by bringing the reflector mirror shape to the desired profile. It is noted that in the context of meeting high requirements for the accuracy of the onboard antenna geometry, the use of the method will ensure the stability of the onboard antenna geometry and minimize the degradation of the antenna radio technical characteristics during orbital operation.

1. Introduction

In the development of modern communication spacecraft (SC) there are tendencies towards increasing the size of onboard antennas and transition to higher operating frequencies.

Inaccuracies unavoidable during the creation and operation of the antenna design cause a deviation of the field distribution from the calculated one on the emitting surface and corresponding distortions of the radiation pattern (RP). In particular, the difference of the reflector profile from a theoretical one leads to an increase in the radiation power contained in the side lobes and to a decrease in the antenna gain in the main direction. The permissible mean-square deviation (MSD) of the reflector mirror does not depend on the size of the reflector aperture; it is determined only by the antenna operating frequency and is estimated as $\lambda / 50$, where $\lambda$ is the wavelength of the operating frequency.

The fulfillment of the above requirement for folding reflectors with an aperture diameter of 10 m or more during the entire period of active existence, which is 15 years for modern SC, is an extremely difficult technical problem.

A promising direction in the development of space antenna technology is the creation of automatic controllable antenna structures. These structures through reconfiguration of the antenna geometry will
allow direct controlling its operational characteristics. For example, bringing the reflector mirror shape to the desired profile will allow forming a contour RP.

In the context of meeting the high requirements for the on-board antenna geometry accuracy, controlling the antenna structure will ensure the stability of the on-board antenna geometry and minimize the degradation of its radio technical characteristics under the influence of changing external (thermal loads, dynamic loads caused by the operation of on-board mechanisms, etc.) and internal (change in the properties of materials) disturbing factors.

To implement such antennas, their composition should include a performance monitoring system. Such a system should include non-contact sensor equipment for measuring the antenna geometric characteristics and mechanisms for controlling the antenna operational characteristics by influencing its geometric characteristics.

For the time being JSC “ISS” has patents for an umbrella reflector, which includes a device for adjusting the reflector shape [1, 2].

There is a number of foreign publications on the controlling the spacecraft reflectors shape [3–5].

In Russia the work on creation of a system for measuring the large-sized antennas geometric characteristics is being carried out at JSC “ISS” [6-8, 12]. This system is built on the basis of non-contact methods for measuring the coordinates of the reflector mirror points in the basic SC coordinate system with optical instruments.

The purpose of the research is to determine a possible method for optimal controlling the large-sized reflector mirror shape of the SC antenna using mechanisms acting on the reflector mirror shape according to information about the current state of the antenna geometry received from the measurement system (from the cloud of reflector mirror points) during its orbital operation (the method should allow bringing the reflector mirror shape to the shape required for full-fledged antenna operation for the intended purpose with maximum similarity) and to confirm the effectiveness of the proposed method using mathematical modeling.

The control object is an umbrella-type offset reflector model made in CAD ANSYS with a diameter of more than 10 m, which includes a device for adjusting the reflector shape. The modeling task is to simulate controlling the deformed reflector mirror shape in order to minimize deformations, that is, to bring it to the shape of a revolution paraboloid.

2. Description of the method

2.1. Statement of the problem of the reflector mirror shape optimal control

The problem of optimal control of the reflector mirror shape is formulated as bringing the antenna mirror shape to the required shape with maximum similarity. The surface having the desired shape will hereinafter be called the base surface.

We choose the distance from the points of the measured surface to the base surface in the normal direction as parameters for a quantitative assessment of the deviation degree of the mirror measured shape from the required one. This is due to the need to find the minimum distance between the base surface and the measured points of the antenna mirror surface when solving the optimal control problem. Further, we assume that for an arbitrary base surface the equation of which is given in a canonical form, there is a function of the shortest distance from the measured point to this surface (the analytical form of this function depends on the equation of the base surface and is derived in each case), then:

\[ r = f_{DIST}(x', y', z'), \]

where \( r \) is the shortest distance from the base surface to the point of the reflector mirror; \( x', y', z' \) are the coordinates of the reflector mirror point in the canonical coordinate system.

The conversion of the measured coordinates of the reflector mirror points from the basic coordinate system of the spacecraft (BCS SC) to the canonical coordinate system is written as:
\[ y' = A y + \tilde{y}, \quad z' = A z + \tilde{z} \tag{2} \]

where \( x, y, z \) are the measured coordinates of the reflector mirror point in the BCS SC; \( \tilde{x}, \tilde{y}, \tilde{z} \) are the coordinates of the canonical coordinate system center in the BCS SC;

\[
A = \begin{bmatrix}
\cos \phi \cos \theta & \cos \phi \sin \theta & -\sin \phi \\
\sin \psi \sin \phi \cos \theta - \cos \psi \sin \theta & \sin \psi \sin \phi \sin \theta + \cos \psi \cos \theta & \sin \psi \cos \phi \\
\cos \psi \sin \phi \cos \theta + \sin \psi \sin \theta & \cos \psi \sin \phi \sin \theta - \sin \psi \cos \theta & \cos \psi \cos \phi
\end{bmatrix}
\]

– coordinate transformation matrix written through Euler angles \( \psi, \phi, \theta \) [9].

The parameters \( \tilde{x}, \tilde{y}, \tilde{z}, \psi, \phi, \theta \) are the coordinates of the position of the base surface in the BCS SC.

The response of the reflector shape to the control signal of the \( n \)th number of mechanisms controlled through the coordinates of the reflector mirror points is recorded by measuring instruments. Moreover, in the general case each of the control signals \( v_j \) where \( j = 1, \ldots, n \), acts on the \( x, y, z \) coordinates of each point of the reflector mirror:

\[
\begin{align*}
    x &= f_1(v_1, v_2, \ldots, v_j), \\
y &= f_2(v_1, v_2, \ldots, v_j), \\
z &= f_3(v_1, v_2, \ldots, v_j),
\end{align*} \tag{3}
\]

where \( f_1(v_1, v_2, \ldots, v_j), f_2(v_1, v_2, \ldots, v_j), f_3(v_1, v_2, \ldots, v_j) \) are functions whose analytical form is unknown.

On the basis on the foregoing the substitution of (3) in (2) and (2) in (1) will provide a quantification function for the deviation of the reflector mirror shape from the base surface shape at one point \( r = R(\bar{x}) \) where \( \bar{x} = [\psi, \phi, \theta, \bar{x}, \bar{y}, \bar{z}, v_1, v_2, \ldots, v_j] \) is the state vector.

Let us suppose that the coordinates of the \( m \) points of the reflector mirror are measured, then bringing the reflector mirror shape to the base surface shape with maximum similarity will consist in selecting a state vector \( \bar{x} \) as a result of which the shortest distances \( r_i \) where \( i = 1, \ldots, m \), will be “close” to zero. One of the ways to formalize this problem is to pose it as the problem of finding the minimum sum of squares of \( R_i(\bar{x}) \). Thus, the problem is formulated as a search for a function unconstrained minimum of the form [10]:

\[
F(\bar{x}) = \frac{1}{2} \sum_{i=1}^{m} R_i(\bar{x})^2 = \frac{1}{2} \|R(\bar{x})\|^2, \tag{4}
\]

where \( \|R(\bar{x})\|^2 \) is the residual of the nonlinear vector function \( R(\bar{x}) \) with \( m \) components \( R_i(\bar{x}) \) at the point \( \bar{x} \).

2.2 Solution of the optimal control problem of the reflector mirror shape

To minimize function (4) it is advisable to use special algorithms developed specifically for least squares problems. The Gauss – Newton iterative method [10] is the simplest (requiring the least computational resources) method for solving this class of problems. The objective function \( F(\bar{x}) \) which in the argument contains control signals for \( n \) mechanisms affecting the reflector mirror shape is a peculiar feature of the use of this method in relation to the problem of optimal control of the reflector mir-
ror shape. Therefore, the transition at each iteration from one argument / root of the objective function to another (iterative process) means applying control signals and changing the configuration (stress-strain state) of the reflector structure. As a result of this, the minimized objective function \( F(\bar{x}) \) is modified at each iteration (the configuration of the reflector structure changes compared to its initial, zero configurations). In contrast to mathematical functions in order to return to the initial state of the reflector structure it is necessary to use control signals with the opposite sign. Thus, the use of the Gauss – Newton method in its “standard” form (finding the next more accurate approximation according to the approximate value of the argument / root of the objective function) will double the number of turns-on and the operating time of the mechanisms affecting the reflector shape. Due to the interdependence of such characteristics of drives as operating hours, number of turns-on, mass, consumption, overall dimensions, an increase in the service life of drives will inevitably lead to an increase in all these characteristics.

In the method under consideration the configuration of the reflector structure at each \( j^{th} \) control iteration is taken as the initial zero configuration. In this case the iterative control process is as follows: \( \bar{x}_{i+1} = \bar{x}_i + \bar{p}_i \), \( R_{i+1}(\bar{x}) = R_i((0 \times 6 \ v_{1,i+1} \ldots \ v_{n,i+1})) \), where

\[
\bar{x}_i = \begin{bmatrix} \psi_i \\ \vdots \\ \psi_n \\ 0 \times n \end{bmatrix} \quad ; \quad \bar{x}_{i+1} = \begin{bmatrix} \psi_{i+1} \\ \vdots \\ \psi_{n+1} \\ v_{1,i+1} \\ \vdots \\ v_{n,i+1} \end{bmatrix} \]

\[
\bar{p}_i = -(J_i^T J_i)^{-1} J_i R_i(\bar{x}_i) \]

is Gauss – Newton search direction for the function \( R_i(\bar{x}) \) at the point \( \bar{x}_i \); \( R_i((0 \times 6 \ v_{1,i+1} \ldots \ v_{n,i+1})) \) is the operation of applying control signals \( [v_{1,i+1}, \ldots, v_{n,i+1}] \); \( J_i \) is Jacobian matrix of the function \( R_i(\bar{x}) \) at the point \( \bar{x}_i \).

The values of the first derivatives in Jacobian matrix are found by a finite-difference approximation. Determination of the function \( R_i(\bar{x}) \) derivatives the with respect to a control signal means the determination of the reflector structure response to the application of a single control signal. In this case in order to build Jacobian matrix two turns on each of the mechanisms acting on the shape of the reflector mirror are required: turning on the first drive to apply a single control signal; measuring the coordinates of the mirror reflector points; turning on the first drive for applying the single control signal of the opposite sign; turning on the second drive for applying a single control signal; measuring the coordinates of the mirror reflector points; turning on the second drive for applying the single control signal of the opposite sign, etc.

3. Mathematical modeling

Mathematical modeling was to simulate controlling the shape of the deformed reflector mirror in order to minimize deformations, that is, to bring it to the shape of a rotation paraboloid (the base surface for this control case). The control object is an umbrella-type offset reflector model with a diameter of more than 10 m, which includes a reflector shape adjustment device (RSAD) [1, 2] made using the finite element method in CAD ANSYS (hereinafter referred to as the reflector).

The output elements of the RSAD are made in the form of screws and are located between the single center \( O_i \) and each guy, with the ends of which they are mechanically connected with the possibility of rotation and changing the effective distance from the connection point of each guy with a spoke to a single center independently of each other. The RSAD with 12 output elements each of which acts on one reflector spoke is used to control the reflector. Under the influence of the output element the spoke pivotally mounted on the reflector base rotates. As a result of the spoke movement two adjacent segments of the net cloth either rise or fall above the surface of the parent paraboloid. The first case
occurs when the distance between the single center of the RSAD output elements and the spoke increases, the second case - when this distance decreases.

The deformation of the reflector mirror was minimized as follows. Linear overload (inertial load) of 10 g in the positive direction of the OY axis was applied to the tuned reflector, which had the mirror MSD from the best fit paraboloid [11] of not more than 1 mm. Further, the mirror was brought to a parabolic surface with MSD relative to the best fit paraboloid of not more than 1 mm by redistributing the forces in the reflector cable-stayed structure. After that the external load was removed. In this case the reflector mirror MSD from the best fit paraboloid increased to 5.76 mm.

We present the MSD of the net cloth nodal points of the reflector mirror from the base parabolic surface at three control iterations: a deformed reflector (the coordinates of the best fit paraboloid were used as the first approximation of the coordinates of the base parabolic surface position) - 5.76 mm; the first control iteration is 4.05 mm; the second control iteration is 3.67 mm; the third control iteration is 3.66 mm.

The coordinates of 1812 nodal points of the net cloth were used in the modeling process when evaluating the reflector mirror shape and calculating the displacements of the RSAD output elements.

The antenna RP was calculated to evaluate the effectiveness of the method. A corrugated conical horn with a Gaussian field distribution was used as an irradiator in calculating the RP. The calculations were carried out for the operating frequency of the microwave antenna. When calculating the RP for a tuned and deformed reflector, the surface of the best-fit paraboloid was fully aligned with the surface of the parent paraboloid, which implies error-free control of the reflector position in six degrees of freedom. Similarly, the base parabolic surface was aligned with the parent paraboloid surface when calculating the RP for the reflector at three control iterations. The gain at the RP maximum and the position of the maximum are presented in table 1.

| Reflector structure state | Gain maximum position |
|---------------------------|-----------------------|
|                           | Gain, dB | Azimuth angle, ° | Elevation, ° |
| Tuned reflector           | 61.17    | 0              | 0           |
| Deformed reflector        | 53.12    | 1              | −12.75      |
| First control iteration   | 58.49    | 1              | −3.75       |
| Second control iteration  | 58.50    | 0              | −3.75       |
| Third control iteration   | 58.43    | 0              | −3.75       |

4. Conclusion
A possible method for controlling the mirror shape of a SCA large-sized reflector using mechanisms that affect the reflector mirror shape during its orbital operation is considered. The method allows bringing the reflector mirror shape to the shape required for full-fledged antenna operation according to the intended purpose with maximum similarity.

Mathematical modeling was carried out. It confirmed the effectiveness of the method in relation to controlling the shape of an umbrella type offset reflector with a diameter of more than 10 m, which includes RSAD. A 36% MSD decrease from the base parabolic surface in comparison with the initial MSD (from 5.76 mm to 3.66 mm) is achieved as a result of applying the method to the deformed reflector mirror surface. Antenna gain increase at the RP maximum by 5.38 dB (from 53.12 to 58.5 dB) and the approximation of the maximum to the desired position (the beginning of the base coordinate system): in azimuth from 1 ° to 0 °, in elevation from −12.75° to −3.75° (for microwave antenna operating frequency) are also achieved.

To apply the considered method which is an iterative process it is necessary to develop a justified criterion for stopping the control process (MSD, the number of iterations, radio technical characteristics, etc.) which allows meeting the antenna requirements in terms of operational characteristics.
It should also be noted that the used reflector model was not verified in terms of the response to RSAD control signals; therefore, it is advisable to test the considered method on a real reflector.

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