Many-body Forces in Magnetic Neutron Stars

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Abstract

In this work, we study in detail the effects of many-body-forces on the equation of state and the structure of magnetic neutron stars. The stellar matter is described within a relativistic mean field formalism that takes into account many-body-forces by means of a nonlinear meson field dependence on the nuclear interaction coupling constants. We assume that matter is at zero temperature, charge neutral, in beta equilibrium, and populated by the baryon octet, electrons, and muons. In order to study the effects of different degrees of stiffness in the equation of state, we explore the parameter space of the model, which reproduces nuclear matter properties at saturation, as well as massive neutron stars. Magnetic field effects are introduced both in the equation of state and in the macroscopic structure of stars by the self-consistent solution of the Einstein–Maxwell equations. In addition, the effects of poloidal magnetic fields on the global properties of stars, as well as density and magnetic field profiles, are investigated. We find that not only different macroscopic magnetic field distributions but also different parameterizations of the model for a fixed magnetic field distribution impact the gravitational mass, deformation, and internal density profiles of stars. Finally, we show that strong magnetic fields significantly affect the particle populations of stars.

Key words: dense matter – equation of state – magnetic fields – stars: magnetars

1. Introduction

Neutron stars are one of the possible outcomes of the evolution of heavy stars. The transition from a main-sequence star to a compact remnant happens through gravitational collapse, after depletion of nuclear fuel. During the collapse, matter in the core of these stars is compressed beyond the point of neutron drip, reaching densities of many times nuclear saturation density. The collapse is then stopped by repulsive nuclear interactions that are dominant in this regime. Because of angular momentum and magnetic flux conservation, the rotation rates and magnetic fields of these stars are exceptionally amplified, reaching the typical values of $P \sim 1 \text{ s}$ and $B_c \sim 10^{12} \text{ G}$, respectively, during the collapse. Considered the densest stars known, neutron stars have about the mass of the Sun compressed in a small radius of roughly $10 \text{ km}$, harboring ultra-dense nuclear matter in their interiors. These objects provide a unique environment for investigating fundamental questions in physics and astrophysics, including matter under extreme conditions, such as the effects of gravity, magnetic fields, and nuclear interactions in the strong regime.

Certain classes of neutrons stars known as soft gamma repeaters and anomalous X-ray pulsars present surface magnetic fields of the order of $B_s \sim 10^{15} \text{ G}$, which is much higher than those present in normal pulsars. These objects are called magnetars. Although the internal magnetic fields cannot be constrained by observations, the virial theorem predicts that the internal magnetic fields of magnetars can reach up to $B_c \sim 10^{18} - 10^{20} \text{ G}$ in the stellar center (Lai & Shapiro 1991; Cardall et al. 2001; Ferrer et al. 2010). It is estimated that approximately 10% of neutron stars are magnetars (Kouveliotou et al. 1998; Olausen & Kaspi 2014).4

The origin of such high magnetic fields is still under debate. Note, for example, that the magnetic flux conservation during the gravitational collapse does not suffice as an explanation of the extreme magnetic fields present in magnetars. In this case, a $1.4 M_\odot$ star would need to have a radius smaller than its Schwarzschild radius in order to generate a magnetic field of $B \sim 10^{15} \text{ G}$ (Tatsumi 2000). The magnetohydrodynamic (MHD) dynamo mechanism is currently the most accepted theory to describe the surface magnetic fields in magnetars. This theory, developed by Duncan & Thompson in the 1990s (Duncan & Thompson 1992; Thompson & Duncan 1993), is based on the amplification of the stellar magnetic fields through the combination of rapid rotation and convective processes in the plasma during the proto–neutron star phase. Hence, such an effective dynamo mechanism can build up strong magnetic fields compatible with those present in magnetars. After about $\sim 10 \text{ s}$ (end of proto–neutron star phase), thermal effects become irrelevant, stopping convection processes and leaving behind highly magnetized neutron stars. The MHD mechanism successfully explains several phenomena that occur on the surface of magnetars, such as their steady X-ray emission, gamma-ray explosions, and giant flare events (Thompson & Duncan 1995, 1996).

In order to consistently describe magnetars in a general relativistic framework, it is necessary to take magnetic fields into account to model both the equation of state (EoS) of matter inside stars and their global structure. The first studies on strong magnetic field effects in a low-density Fermi gas were performed by Canuto & Chiu et al. (1968) and Canuto & Chiu (1968a, 1968b, 1968c), followed by calculations using

4 http://www.physics.mcgill.ca/~pulsar/magnetar/main.html
more realistic hadronic models with magnetic field effects (Chakrabarty et al. 1997; Broderick et al. 2000). Over the years, it has been shown that the presence of strong magnetic fields significantly affects the energy levels of charged particles due to Landau quantization. These effects have been taken into account in the EoS of models in order to describe hyperon stars (Chakrabarty et al. 1997; Broderick et al. 2000, 2002; Lopes & Menezes 2012; Sinha et al. 2013; Casali et al. 2014; Gomes et al. 2014; Gao et al. 2015), quark stars (Perez Martinez et al. 2008; Felipe et al. 2011; Orsaria et al. 2011; Paulucci et al. 2011; Denke & Pinto 2013; Dexheimer et al. 2014; Isayev 2015), hybrid stars (Rabhi et al. 2009; Dexheimer et al. 2012, 2013), and stars with meson condensates Schramm et al. (2015). The effects of the anomalous magnetic moment (AMM) were also investigated through the corresponding coupling of the baryons to the electromagnetic field tensor (see Canuto & Chiu 1968a; Broderick et al. 2000, 2002; Strickland et al. 2012). Moreover, Landau quantization effects on the EoS give rise to an anisotropy in the matter energy–momentum tensor components (Perez Martinez et al. 2008; Strickland et al. 2012), indicating that deformation effects can be important in the microscopic description of these objects and, therefore, that spherical symmetry is not the best approximation to describe the macroscopic structure of magnetized neutron stars.

The inclusion of magnetic field effects in the calculation of the stellar macroscopic structure involves solving the coupled Einstein–Maxwell equations using a metric able to describe deformed objects, i.e., at least a two-dimensional metric. Simplified solutions for the problem were proposed by Mallick & Schramm (2014), Manreza Paret et al. (2015), and Zubairi et al. (2015), where a perturbation of the metric was carried out. Numerically, the formalism developed by Bonazzola et al. (1993), implemented in the LORENE library, takes into account both rotation and magnetic fields in a full calculation of the stellar structure of neutrons stars. This formalism was initially applied to a one-parameter EoS (Bocquet et al. 1995; Cardall et al. 2001). Only recently, self-consistent calculations including magnetic fields in both the EoS and stellar structure were implemented to describe quark stars in Chatterjee et al. (2015) and hybrid stars in Franzon et al. (2015).

Although recent self-consistent calculations for magnetic neutron stars have shown that the Landau quantization effects barely impact the global properties of such objects (Chatterjee et al. 2015; Franzon et al. 2015), as a side product of this work, we cross-check these results for the case of hyperon stars complementing previous results for quark and hybrid stars. Another important analysis, which is lacking in the literature, regards the impact of the EoS stiffness on strongly magnetized stars. Calculations of the structure of magnetic neutron stars were performed using different hadronic models (Bocquet et al. 1995; Cardall et al. 2001; Dexheimer et al. 2016), but the focus of the analysis has always been on the impact of different magnetic field configurations on the global properties of stars using one specific model and parameterization.

The determination of the nuclear matter EoS at high densities is still an open question and also one of the main goals of current nuclear astrophysics research. Since current lattice QCD calculations cannot reach the regime of high densities due to the highly oscillatory behavior in the functional integral, it is not possible to describe the EoS of dense, strongly interacting matter on a fundamental level. However, assuming that only baryonic degrees of freedom are relevant for the energy scales present in neutron stars, the nuclear interaction can be reasonably approximated by effective relativistic mean field hadronic models. In such approaches, the baryon–baryon interaction is described by the exchange of scalar and vector mesons, which simulate the attractive and repulsive features of the nuclear interaction. In the past, different couplings were suggested, including models with nonlinear contributions Typel & Wolter (1999), and meson field dependence Zimanyi & Moszkowski (1990), Taurines et al. (2001), Dexheimer et al. (2008), Gomes et al. (2015), among others. In particular, the many-body forces (MBF) model Taurines et al. (2001), Dexheimer et al. (2008), Gomes et al. (2015) introduces a field dependence of the couplings, making them indirectly density-dependent. The nonlinear terms that arise from the coupling expansion are an important feature of the MBF model, as these contributions can be interpreted as contributions from MBF due to meson–meson interactions.

The observation of massive neutron stars Demorest et al. (2010), Antoniadis et al. (2013) indicates that the EoS of nuclear matter must be very stiff in the regime of high densities and low temperatures. The degree of stiffness in the nuclear matter EoS is directly related to the repulsive interaction among particles at high densities, as well as to the particle content in the core of the stars. In particular, it has been extensively discussed in the literature whether exotic degrees of freedom might populate the core of neutron stars. On the one hand, it is more energetically favorable for the system to populate new degrees of freedom, such as hyperons (Dexheimer & Schramm 2008; Ishizuka et al. 2008; Bednarek et al. 2012; Fukukawa et al. 2015; Gomes et al. 2015; Maslov et al. 2015; Oertel et al. 2015; Lonardoni et al. 2015, 2016; Biswal et al. 2016; Burgio & Zappalà 2016; Chatterjee & Vidana 2016; Mishra et al. 2016; Vidana 2016; Yamamoto et al. 2016; Tolos et al. 2017); Torres et al. 2017), delta isobars (Fong et al. 2010; Schurhoff et al. 2010; Drago et al. 2014, 2016; Cai et al. 2015; Zhu et al. 2016), and meson condensates (Ellis et al. 1995; Menezes et al. 2005; Takahashi 2007; Ohnishi et al. 2009; Alford et al. 2010; Fernandez et al. 2010; Mesquita et al. 2010; Mishra et al. 2010; Lim et al. 2014; Muto et al. 2015), in order to lower its Fermi energy (starting at about two times the saturation density). On the other hand, the EoS softening due to the appearance of exotica might turn some nuclear models incompatible with observational data, in particular with the recently measured massive neutron stars. One possible way to overcome this puzzle is the introduction of an extra repulsion in the YY interaction Schaffner & Mishustin (1996), Bombaci (2016), allowing models with hyperons to be able to reproduce massive stars (Dexheimer & Schramm 2008; Bednarek et al. 2012; Weissenborn et al. 2012; Banik et al. 2014; Bhowmick et al. 2014; Gusakov et al. 2014; Lopes & Menezes 2014; van Dalen et al. 2014; Yamamoto et al. 2014; Gomes et al. 2015). Another possible solution is the introduction of a deconfinement phase transition at high densities Bombaci (2016), with a stiff EoS for quark matter, usually associated with quark vector interactions (see Klähn et al. 2013 and references therein).

In this work, we investigate the impact of different parameterizations of the MBF model and different magnetic field configurations on the global properties and the density and magnetic profiles of magnetic neutron stars. First, we calculate an error estimate for using spherical Tolman–Oppenheimer–Volkoff (TOV) solutions for magnetic stars
(instead of a two-dimensional deformed solution). In order to uniquely identify the dependence of global stellar properties on the EoS, we fix the baryonic mass of the stars. Then, we study different stellar properties, such as gravitational mass, deformation, density, and magnetic field profiles, and particle population.

This paper is organized as follows. In Section 2, the MBF model with the inclusion of magnetic fields is presented. In Section 3, we discuss the main features of the Einstein–Maxwell equations. Section 4 estimates the error in neglecting stellar deformation when describing magnetic neutron stars. In Section 5, we show the impact of different degrees of EoS stiffness and magnetic field distributions on the global properties and internal magnetic and density profiles of a fixed baryon mass star. Section 6 is dedicated to discussing the particle populations of the stars. Finally, in Section 7, we present our conclusions.

2. The MBF Formalism

In this section, we present the model used for describing microscopic matter in this work. We also describe how magnetic fields modify the model through the introduction of Landau quantization.

The Lagrangian density of the MBF model presented in Gomes et al. (2015) includes for the first time a coupling dependence on all scalar fields. It reads:

\[
\mathcal{L} = \sum_b \bar{\psi}_b \left( \gamma_0 \left( i \partial^\mu - g_{sb} \omega^\mu - g_{sb} \phi^\mu - g_{sb} I_{2b} \delta^\mu_3 \right) - m_b^* \right) \psi_b + \frac{1}{2} \partial_{\mu} \sigma \partial^\mu \sigma - m_{2b}^2 \sigma^2 + \frac{1}{2} \left( \frac{1}{2} \omega_{ij} \omega_{\mu \nu} + m_{2ij}^2 \omega_{ij} \right) \partial^\mu _i \partial^\nu _j \sigma \\
+ \frac{1}{2} \left( \frac{1}{2} g_{\nu \mu} \mathcal{Q}^{\mu \nu} + m_{2i}^2 \mathcal{Q}_i \right) + \frac{1}{2} \partial_{\mu} \delta \partial^\mu \delta - m_{2b}^2 \delta^2 \\
+ \frac{1}{2} \left( \frac{1}{2} \phi_{\mu \nu} \phi^{\mu \nu} + m_{2i}^2 \phi \phi_i \right) + \sum_l \psi_l \gamma_5 (i \partial^\mu - m_l) \psi_l + \sum_b \bar{\psi}_b q_{b \ell} A^\mu \psi_b + \sum_l \bar{\psi}_l q_{\ell} \lambda^\mu \psi_l.
\]

(1)

The subscripts \(b\) and \(l\) correspond to baryon \((p^+, n^0, \Lambda^0, \Sigma^+, \Sigma^0, \Xi^-, \Xi^0, \Xi^-)\) and lepton \((e^+, \mu^+)\) degrees of freedom, respectively. The first line and the second term in the fourth line of Equation (1) represent the Dirac Lagrangians for baryons and leptons. The electromagnetic interaction is introduced by the coupling to the photon field \(A^\mu\) field in the last line, and other terms represent the Lagrangian densities of scalar mesons \((\sigma, \delta)\) and vector mesons \((\omega, \varphi, \phi)\). The meson–baryon coupling appears in the first term for the vector mesons \((g_{sb}, g_{sb}, g_{sb})\), and the scalar ones are contained in the baryon effective masses \((m_{2b}^*)\). The introduction of the \(\delta\) and \(\varphi\) isovector fields is important to extrapolate the model to isospin asymmetric systems, such as neutron stars, and the \(\phi\) vector meson adds a repulsive component in the YY interaction.

The effects of the MBF contribution are introduced in the effective couplings of the scalar mesons,

\[
g_{ib} = \left( 1 + g_{sb} \mathcal{Q} + g_{sb} I_{2b} \delta_3 \right) \frac{\zeta}{m_b}.
\]

(2)

for \(i = \sigma, \delta\) and, consequently, in the baryon masses as

\[
m_b^* = m_b - (g_{ib} \sigma + g_{ib} I_{2b} \delta_3).
\]

(3)

If we were to expand the scalar couplings around the \(\zeta\) parameter, we would generate nonlinear contributions from self and crossed terms of the scalar fields \((\sigma, \delta)\). These couplings simulate the effects of MBF in the nuclear interaction, which are controlled by the \(\zeta\) parameter. In this way, each value of the parameter generates a different EoS and, hence, a different set of nuclear saturation properties.

In this work, we vary the \(\zeta\) values, obeying the following constraints for symmetric matter: binding energy per nucleon \(B/A = -15.75\,\text{MeV}\) and saturation density \(\rho_0 = 0.15\,\text{fm}^{-3}\). The values of the nuclear asymmetric matter properties at saturation are fixed as symmetry energy \(a_{\text{sym}} = 32\,\text{MeV}\) and its slope \(L_0 = 97\,\text{MeV}\). We choose the lowest value of the slope that is common to all parameterizations of the MBF model, corresponding to the lowest value for the parameterization \(\zeta = 0.040\) Gomes et al. (2015). The corresponding remaining nuclear properties at saturation, associated with the sets of parameters used in this work (in agreement with experimental data), are collected in Table 1.

When the hyperon degrees of freedom are taken into account throughout this paper, we describe their vector couplings using the SU(6) spin-flavor symmetry Dover & Gal (1985), Schaffner et al. (1994). We fit the hyperon-sigma coupling in order to reproduce the following values of the hyperon potential depths Schaffner-Bielich & Gal (2000): \(U_A^N = -28\,\text{MeV}\), \(U_B^N = +30\,\text{MeV}\), and \(U_C^N = -18\,\text{MeV}\). However, it is important to mention that only two \((\zeta = 0.040, 0.059)\) of the parameterizations in Table 1 describe hyperon stars in agreement with the observational data for massive stars Gomes et al. (2015). Also, although some parameterizations of the MBF model are very stiff, note that the formalism is relativistically invariant and, consequently, obeys causality.

The introduction of magnetic fields alters the energy levels of the charged particles, which become Landau quantized as follows:

\[
e_F = \begin{cases} 
\sqrt{(m_e^2 + k^2)}, & q = 0 \\
\sqrt{(m_e^2 + k_{Fz}^2 + 2q|B|)}, & q \neq 0
\end{cases}
\]

(4)

Table 1: Basic Nuclear Properties at Saturation and Couplings for Different Parameterizations of the MBF Model

| \(\zeta\) | \(m_b^*/m_n\) | \(K_0\,\text{(MeV)}\) | \((g_{sb}/m_n)^2\) | \((g_{sb}/m_n)^2\) | \((g_{sb}/m_n)^2\) | \((g_{sb}/m_n)^2\) |
|---|---|---|---|---|---|---|
| 0.040 | 0.66 | 297 | 14.51 | 8.74 | 4.466 | 0.383 |
| 0.059 | 0.70 | 253 | 13.44 | 7.55 | 5.571 | 1.820 |
| 0.085 | 0.74 | 225 | 12.21 | 6.37 | 6.467 | 3.103 |
| 0.129 | 0.78 | 211 | 10.84 | 5.16 | 7.057 | 4.038 |

Note. The columns are as follows: the MBF parameter \(\zeta\), nucleon effective mass \(m_b^*\), compressibility modulus \(K_0\), and meson couplings \((g_{sb}, g_{sb}, g_{sb})\).

This table includes the following parameters:

- \(m_b^*/m_n\): Nucleon effective mass
- \(K_0\): Compressibility modulus
- \((g_{sb}/m_n)^2\): Coupling strength

The table provides a comparison of different parameterizations of the MBF model, with \(\zeta\) values ranging from 0.040 to 0.129.
for uncharged and charged fermions, respectively, with \( m_i = m_i^p \) in the case of baryons. The Landau quantum number \( \nu \) is given in terms of the orbital and spin quantum numbers as

\[
\nu \equiv l + \frac{1}{2} - \frac{s q}{2 |q|},
\]

running within the range \( 0 \leq \nu \leq \nu_{\text{max}} \), denoting the highest Landau orbit with nonvanishing particle occupation at zero temperature (see Strickland et al. 2012 for more details).

3. The Einstein–Maxwell Solutions

In what follows, we introduce the formalism used to calculate the macroscopic structure of magnetic neutron stars. The spacetime in general relativity is described by the Einstein equations,

\[
R_{\mu \nu} - \frac{1}{2} R g_{\mu \nu} = 8 \pi G T_{\mu \nu},
\]

where \( R_{\mu \nu} \) is the Ricci tensor, \( R \) is the Ricci scalar, \( T_{\mu \nu} \) is the energy–momentum tensor of matter and electromagnetic fields, and \( G \) is the Newton’s gravitational constant.

With the assumption of a stationary, axisymmetric spacetime and maximum-slice quasi-isotropic coordinates, the line element in the form \( 3 + 1 \) decomposition of spacetime can be cast in the form

\[
ds^2 = -N^2dt^2 + A^2(dr^2 + r^2d\theta^2) + B^2r^2\sin^2(\theta) (d\phi - \omega dt)^2,
\]

with \( N(r, \theta) \), \( A(r, \theta) \), \( B(r, \theta) \), and \( \omega(r, \theta) \) being functions only of the coordinates \( r, \theta \). In this case, the final system of equations for each metric potential is given by

\[
\Delta_2[(NB - 1)r \sin \theta] = 8 \pi N A^2 Br \sin \theta (S'_\phi + S'_\theta),
\]

\[
\Delta_2[\ln A + \nu] = 8 \pi N A^2 S'_\phi + \frac{3B^2r^2 \sin^2 \theta}{4N^2} \partial_\omega \partial_\omega - \partial_\nu \partial_\nu, \]

\[
\Delta_3 \nu = 4 \pi N A^2 (E + S) + \frac{B^2r^2 \sin^2 \theta}{2N^2} \partial_\omega \partial_\omega - \partial_\nu \partial(\nu + \ln B),
\]

and

\[
\left[ \Delta_3 - \frac{1}{r^2 \sin^2 \theta} \right] (\omega \sin \theta) = -16 \pi N A^2 \frac{p_\nu}{B^2} \frac{1}{r \sin \theta} + r \sin \theta \omega \partial(\nu - 3 \ln B),
\]

where the short notation was introduced,

\[
\Delta_2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2},
\]

\[
\Delta_3 = \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{1}{r^2 \tan \theta} \frac{\partial}{\partial \theta},
\]

\[
\nu = \ln N,
\]

with the total energy of the system \( E = E^{\text{PF}} + E^{\text{EM}} \), the total momentum density flux \( j_\nu = J_\nu^{\text{PF}} + J_\nu^{\text{EM}} \), and the total stress tensor \( S = S^{\text{PF}} + S^{\text{EM}} \), where PF and EM are the perfect fluid and electromagnetic field contributions, respectively. In addition, in the final system of gravitational field equations (Equations (8)–(11)), terms such as \( \partial_\omega \partial_\omega \) are defined as

\[
\partial_\omega \partial_\omega = \frac{\partial_\omega}{\partial \omega} \frac{\partial_\omega}{\partial \omega} + \frac{1}{r^2} \frac{\partial_\omega}{\partial \theta} \frac{\partial_\omega}{\partial \theta}.
\]

The Faraday tensor \( F_{\mu \nu} \) is derived from the magnetic four-vector potential \( A_\mu \) as

\[
F_{\mu \nu} = A_{\nu, \mu} - A_{\mu, \nu},
\]

such that the Maxwell equation,

\[
F_{\alpha \beta \gamma} + F_{\beta \gamma \alpha} + F_{\gamma \alpha \beta} = 0,
\]

is automatically satisfied. The remaining Maxwell equations can be expressed in terms of the two nonvanishing components of the magnetic vector potential \( A_\mu \) as

\[
F^{\alpha \beta}_{\beta} = 4 \pi j^\alpha.
\]

For example, the Maxwell–Gauss equation for \( A_t \) reads

\[
\Delta_3 A_t = - \mu_0 A^2 (g_{\phi \phi} j^\phi + g_{\theta \theta} j^\theta) - \frac{B^2}{N^2} \nu r^2 \sin^2 \theta \partial A_\nu \partial N^\phi
\]

\[- \left( 1 + \frac{B^2}{N^2} \nu r^2 \sin^2 \theta \partial^2 \right) \partial A_\phi \partial \omega
\]

\[- (\partial A_t + 2 \omega \partial A_\phi) \partial (\ln B - \nu)
\]

\[- 2 \frac{\omega}{r} \frac{\partial A_\phi}{\partial r} + \frac{1}{r \tan \theta} \frac{\partial A_\phi}{\partial \theta}.
\]

and, from the Maxwell–Ampere equation, we have an equation for \( A_\phi \):

\[
\left[ \Delta_3 - \frac{1}{r^2 \sin^2 \theta} \right] \left( \frac{A_\phi}{r \sin \theta} \right) = - \mu_0 A^2 B^2 (j^\phi - \omega j^\theta) r \sin \theta
\]

\[- \frac{B^2}{N^2} r \sin \theta \partial_\omega \partial (\partial A_t + \omega \partial A_\phi) + \frac{1}{r} \partial A_\phi \partial (\ln B - \nu).
\]

In this approach, the equation of motion reads

\[
H(r, \phi) + \nu(r, \phi) + M(r, \phi) = \text{const},
\]

where \( H(r, \phi) \) is the heat function defined in terms of the baryon number density \( n \) and the magnetic potential \( M(r, \phi) \) is given by

\[
H = \int_{n_0}^{n} \frac{1}{e(n) + p(n)} \frac{dp}{dn} (n) dn,
\]

\[
M(r, \phi) = M(A_\phi(r, \phi)) \equiv - \int_{A_\phi(r, \phi)}^{0} f(x) dx,
\]

respectively, where \( f(x) \) is the current function, which we choose to be constant in this work. According to Bocquet et al. (1995), other choices for \( f(x) \) are possible, but they do not change the results qualitatively in the polar direction. The quantitative differences are explored by using different electric amplitudes \( j_\nu \). From the current function and the EoS, one obtains the electric current,

\[
j^\phi - \Omega j^\theta = (e + p) f(x),
\]

where \( e \) and \( p \) are the energy density and pressure of matter, respectively.
The stress–energy tensor of the magnetic field is calculated using the standard expression,

$$\tau^{EM}_{\mu\nu} = \frac{1}{4\pi} \left( F_{\mu\nu} F^{\nu\mu} - \frac{1}{4} F_{\rho\sigma} F^{\rho\sigma} g_{\mu\nu} \right),$$  \hspace{1cm} (24)

from which one can obtain the sources of the gravitational fields. The electromagnetic contributions to the total energy and momentum density of the system are, respectively,

$$E^{EM} = \frac{1}{2\mu_0} B^r B_t,$$  \hspace{1cm} (25)

$$J^{EM}_\phi = \frac{1}{\mu_0} A^2 (B^r E^\theta - E^r B^\theta).$$  \hspace{1cm} (26)

The stress three-tensor components are given by

$$S^{EMr}_{\mu} = \frac{1}{2\mu_0} (E^r E_\theta - E^\theta E_r + B^\theta B_\theta - B^r B_r),$$  \hspace{1cm} (27)

$$S^{EM\theta}_{\mu} = \frac{1}{2\mu_0} (E^r E_\theta - E^\theta E_r + B^\theta B_\theta - B^r B_r),$$  \hspace{1cm} (28)

$$S^{EM\phi}_{\mu} = \frac{1}{2\mu_0} (E^r E_\phi + B^r B_\phi),$$  \hspace{1cm} (29)

being the electric field components, as measured by the Eulerian observer $O_0$, written as Lichnerowicz (1967)

$$E_\theta = \left( 0, \frac{1}{N} \left[ \frac{\partial A_t}{\partial r} + \omega \frac{\partial A_\phi}{\partial r} \right], \frac{1}{N} \left[ \frac{\partial A_t}{\partial \theta} + \omega \frac{\partial A_\phi}{\partial \theta} \right], 0 \right),$$  \hspace{1cm} (30)

and the magnetic field given by

$$B_\theta = \left( 0, \frac{1}{Br^2 \sin \theta} \frac{\partial A_{\phi}}{\partial \theta}, - \frac{1}{Br \sin \theta} \frac{\partial A_{r}}{\partial r}, 0 \right),$$  \hspace{1cm} (31)

where $A_t$ and $A_\phi$ are the two nonzero components (for a poloidal magnetic field) of the electromagnetic four-potential $A_\mu = (A_t, 0, 0, A_\phi)$.

A coordinate-independent characterization of the stellar equator can be done by calculating the circumferential equatorial radius ($\theta = \pi/2$),

$$R_{circ} := \frac{1}{2\pi} C,$$  \hspace{1cm} (32)

where $C$ is the proper length of the circumference of the star in the equatorial plane. So, one obtains

$$R_{circ} = \frac{1}{2\pi} \int_0^{2\pi} \sqrt{g_{\phi\phi}} d\phi = \int_0^{2\pi} B(r, \theta) r \sin \theta d\phi = r_{eq} B, \frac{r_{eq}}{\pi/2},$$  \hspace{1cm} (33)

where $r_{eq}$ is the equatorial coordinate radius.

4. Spherical versus Deformed Solutions

We start by using the MBF model to calculate the EoSs of charge-neutral beta-equilibrated hadronic matter at zero temperature including the effects of Landau quantization. This formalism is then applied to describe (nonrotating) magnetic neutron stars by solving the Einstein–Maxwell equations using the LORENE C++ library.

First, it is important to point out that as the magnetic field limit for stable solutions within this approach is of the order of $\sim 10^{18}$ G, the inclusion of strong magnetic fields in the calculations of the EoS of hadronic matter does not play a significant role for the global properties of the stars, as was already discussed in Chatterjee et al. (2015), Franzon et al. (2015). This stems from the fact that the effects of strong magnetic fields in the EoS become very significant only for magnetic fields of about $\sim 5 \times 10^{18}$ G. The horizontal axis shows the gravitational mass, and the vertical axis shows the circular radius.

That being said, we have calculated the mass–radius diagram using the MBF model with and without magnetic field effects on the EoS of nucleonic matter only and for matter including hyperons. We have confirmed that the global properties of stars are essentially the same in both cases (including hyperons or not). This can be seen in Figure 1, which shows the mass–radius diagram for only a specific parameterization of the MBF model (with nucleons and leptons) and a specific current function used in the solution of the Einstein–Maxwell equations (2D solutions). Our results are in agreement with the Nambu-Jona-Lasinio (NJL) model calculations used to describe quark stars Chatterjee et al. (2015). In the case of Franzon et al. (2015), where a chiral model was used to describe hybrid stars, this is not exactly the case, because the baryonic AMMs enhance magnetic field effects and, therefore, have some influence on the macroscopic properties of stars.

Since, in this work, AMM effects are not included, all subsequent analysis of macroscopic stellar properties is done without taking into account the effects of magnetic fields on the EoS of the MBF model. However, it is important to stress that, as magnetic field effects modify the population of stars, cooling processes must also be altered Dexheimer et al. (2012), Sinha & Sedrakian (2015), Tolos et al. (2017).

In the past, several authors have described magnetic neutron stars by including magnetic field effects only on the EoS and computing the macroscopic structure of stars by solving the spherical isotropic TOV equations, under the assumption that the deformation of magnetic stars would be small. However, recent self-consistent calculations for magnetic stars Franzon et al. (2015) have proven this assumption wrong, since the stars can be deformed by more than 50% for a poloidal magnetic field distribution that reproduces strong central magnetic fields up to $\sim 10^{18}$ G.
To exemplify this, we compare the mass–radius relations for the 2D solutions described in the previous section and the TOV solutions for the highest magnetic field configuration considered in this work, using an electric current amplitude $j_0 = 3.5 \times 10^{15}$ A m$^{-2}$ in the first case. For the TOV solution, we make use of the magnetic field profile calculated from the 2D solutions (given by the maximum mass configuration) in the pure magnetic field contribution, which is added isotropically to the EoS. Note that this is not correct, since the pure magnetic field contribution enters with different signs in different directions in the energy–momentum tensor; however, this is a frequently used assumption in the literature.

From Figure 2, one can check that not taking magnetic field effects into account for the macroscopic structure of stars leads to an overestimation of the maximum mass allowed by a specific EoS, as well as to an underestimation of the equatorial radius of stars. In particular, for the parameterization $\zeta = 0.040$ of the MBF model with hyperons, we obtain maximum baryon masses of $M_{\text{b}}^{\text{max}} = 2.57 M_\odot$ and $M_{\text{g}}^{\text{max}} = 2.98 M_\odot$ for 2D and TOV solutions with magnetic fields, respectively. A similar comparison can be done for the radii of a $M_\odot = 1.4 M_\odot$ star, from which we calculate $R_{1.4M_\odot}^{\text{TOV}} = 14.95$ km and $R_{1.4M_\odot}^{\text{2D}} = 11.88$ km, again for deformed and TOV solutions with magnetic fields, respectively. Here it is important to stress that the radial comparison is done between the equatorial radius of the 2D solution and the isotropic radius of the magnetic TOV result.

An analogous analysis can be performed for the gravitational mass, as shown in Figure 3. For the same parameterization of the nuclear model, maximum gravitational masses of $M_{\text{b}}^{\text{max}} = 2.23 M_\odot$ and $M_{\text{g}}^{\text{max}} = 2.50 M_\odot$ are estimated for 2D and TOV solutions with magnetic fields, respectively. The radii of the $M_\odot = 1.4 M_\odot$ stars are $R_{1.4M_\odot}^{\text{TOV}} = 15.11$ km and $R_{1.4M_\odot}^{\text{2D}} = 12.05$ km, again, for the 2D and magnetic TOV cases, respectively.

We now focus on the maximum baryon mass star obtained from the 2D LORENE solution, with a maximum baryon mass $M_{\text{b}}^{\text{max}} = 2.57 M_\odot$. One can check that the corresponding gravitational mass for the magnetic TOV solution is $M_{\text{g}} = 2.21 M_\odot$, which is a much smaller value than the maximum solution $M_{\text{g}}^{\text{max}} = 2.50 M_\odot$ obtained from the magnetic TOV solution but similar to the correct value obtained for the full LORENE solution $M_{\text{g}}^{\text{max}} = 2.23 M_\odot$. Similarly, we can take the $M_\odot = 2.2 M_\odot$ star and check the circular radius estimation with both the magnetic TOV and 2D solutions, obtaining $R_{2.2M_\odot}^{\text{TOV}} = 12.64$ km and $R_{2.2M_\odot}^{\text{2D}} = 15.29$ km, respectively. From these results, we can conclude that there is an overestimation of the maximum gravitational mass (in the magnetic TOV case) when comparing both maximum solutions but only a small change for a fixed baryon mass. However, for the equatorial radius of stars, the difference is much more pronounced (even for the same baryon mass), with a difference of 2.65 km.

More generally, the results show that there is an overestimation of 15.95% for the maximum baryon mass and 12.11% for the gravitational mass by the magnetic TOV approach. The equatorial radius is underestimated by 20.54% for the $1.4 M_\odot$ baryon mass star and 20.25% for the $1.4 M_\odot$ gravitational mass star. Some of the errors in the estimate of the global properties of magnetic neutron stars come from the inappropriate metric used to solve the TOV equations. A spherical metric with equal-sign pressure contributions incorrectly describes the magnetic field in all directions, allowing for stable, more massive stars. When one follows an axisymmetric approach, some of the magnetic pressure components generate the stellar deformation, and only some of the field effects lead to an increase in the mass of stars.

Figure 4 shows the impact of different magnetic field configurations on the mass–radius relation for the parameterization $\zeta = 0.040$ of the MBF model with hyperons. The red curve is the nonmagnetic TOV solution, which gives a maximum gravitational mass $M_{\text{g}} = 2.15 M_\odot$. The other curves correspond to different choices for the electric current amplitude $j_0$ in the LORENE code that generate a different magnetic field configuration throughout the star. In particular, the black curve for $j_0 = 3.7 \times 10^{15}$ A m$^{-2}$ corresponds to the maximum central magnetic field configuration $B_c = 1.1 \times 10^{18}$ G. The results show that magnetic stars present a gravitational mass increase from 2.15 $M_\odot$ to 2.22 $M_\odot$ for the maximum mass configuration and an increase of respective radii from $R_{1.4M_\odot}^{\text{2D}} = 12.88$ km to $R_{1.4M_\odot}^{\text{2D}} = 15.98$ km from the nonmagnetic to the highest magnetic field configuration. This is due to the fact that the Lorentz force...
The same is true if we fix magnetic dipole moment dependant on both the EoS and the electric current amplitude. The current that gives rise to a magnetic field, which is determined by the Lorentz force, which acts against gravity, enhancing the gravitational mass and equatorial radius of stars, which become more oblate (a $r_p/r_{eq}$ ratio farther from 1).

From the EoS point of view, smaller values of the MBF parameter have a shielding effect on the scalar couplings, maximizing the vector meson contributions and, thus, generating stiffer EoSs (see Gomes et al. 2015 for more details). Consequently, they allow for more massive and larger stars (see Figure 5). Therefore, for a fixed magnetic field distribution, small values of $\zeta$ generate higher gravitational masses. Nevertheless, for larger values of the magnetic dipole moment, the magnetic field contribution dominates, and the EoS effects cannot be seen. For the highest magnetic field configuration reported in this work, the associated gravitational mass for the different parameterizations ranges from $1.98 M_\odot$ (for $\zeta = 0.040$) to $1.94 M_\odot$ (for $\zeta = 0.129$) for $j_0 = (1.0–3.5) \times 10^{15}$ A m$^{-2}$ keeping $M_b = 2.2 M_\odot$.

As one can see in Figure 7, the smaller the $\zeta$, the larger the radius of the stars. Note that, although the mass in this case is larger, the stellar volume scales as $R^3$. As a consequence, the central density is lower for smaller values of the MBF parameter (as shown in Table 2). In this way, we can identify relations between radius, central density, and central magnetic field with the deformation of stars. In particular, the most deformed configuration (for $\zeta = 0.040$ and $j_0 = 3.5 \times 10^{15}$ A m$^{-2}$) generates a radius ratio $r_p/r_{eq} = 0.6$, as one can see in Figure 6.

This is not a straightforward result, since the 2D calculation of all star properties depends both on the EoS and on the magnetic field configuration given by the electric current amplitude. A stiffer EoS allows for higher radii and, consequently, lower central densities and central magnetic fields. However, these stars are less compact and, consequently, more easily deformed. In principle, one could expect the softer EoS to be the one that generates more deformed stars due to the higher central magnetic field. Nevertheless, as softer EoS stars have smaller radii, they are more compact and, hence, more difficult to deform.

Figure 6 also shows the isocontours for the fixed values of the magnetic moment of the stars as a function of the many-body parameter and the electric current amplitude. The results show that it is necessary to increase the electric current amplitude in order to reproduce the same dipole moment for a fixed baryon mass star if this star is described by a soft EoS. Here, again, the determination of the magnetic moment depends both on the EoS (which is determinant for the radius) and on the magnetic field distribution, which comes from $j_0$. In order to have the same magnetic moment, the softer EoS must compensate for its smaller radius by increasing the magnetic field, generating slightly more deformed stars for the case of a fixed magnetic moment.
In the following, we focus our attention on the interior of a star with $M_\odot = 2.2 M_\odot$ for the highest magnetic field configuration $\mu = 3.5 \times 10^{15} \text{ A m}^2$ in order to check the impact of different MBF contributions on their magnetic field and density profiles. The baryon density distribution as a function of the polar radius is shown in Figure 8 for different choices of the $\zeta$ parameter. As already mentioned, the stiffest EoS generates larger radii and lower central densities. Because of the larger stellar radii produced by the stiffer EoS, it is possible to see a crossing for the density curves. A similar but enhanced result is found for the density profile in the equatorial direction, although it is important to stress that the density distribution is anisotropic for magnetic stars. The poloidal magnetic field distribution makes stars oblate, i.e., more flattened in the polar direction and expanding in the equatorial direction (Franzon et al. 2015).

Another important point to make regarding the stellar baryon density distribution is that the Lorentz force reverses its direction along the equatorial plane of magnetized stars at some distance from the center but still inside the star, as already pointed out in Cardall et al. (2001) and Franzon & Schramm (2015). Note that, depending on the magnetic field distribution (which in turn depends on the EoS), this might even lead to an off-center maximum baryon density (Franzon et al. 2015, 2016).

As already mentioned, the intensity of the central magnetic field is directly related to the central density reached inside the

| $\zeta$ | $j_0 (10^{15} \text{ A m}^{-2})$ | $\mu (10^{15} \text{ Am}^2)$ | $B_r (10^{17} \text{ G})$ | $B_c (10^{17} \text{ G})$ | $\rho_c (\text{fm}^{-3})$ |
|--------|-------------------------------|------------------|------------------|------------------|------------------|
| 0.040  | 0                             | n.a.             | n.a.             | n.a.             | 0.376            |
| 0.040  | 1.0                           | 0.57             | 1.03             | 3.48             | 0.375            |
| 0.040  | 2.0                           | 1.21             | 1.47             | 4.66             | 0.364            |
| 0.040  | 3.0                           | 2.09             | 2.70             | 7.03             | 0.335            |
| 0.040  | 3.5                           | 2.88             | 3.80             | 8.09             | 0.299            |
| 0.059  | 0                             | n.a.             | n.a.             | n.a.             | 0.432            |
| 0.059  | 1.0                           | 0.52             | 0.67             | 2.44             | 0.432            |
| 0.059  | 2.0                           | 1.09             | 1.47             | 4.91             | 0.420            |
| 0.059  | 3.0                           | 1.86             | 2.64             | 7.39             | 0.389            |
| 0.059  | 3.5                           | 2.47             | 3.60             | 8.52             | 0.354            |
| 0.085  | 0                             | n.a.             | n.a.             | n.a.             | 0.519            |
| 0.085  | 1.0                           | 0.46             | 0.68             | 2.62             | 0.517            |
| 0.085  | 2.0                           | 0.96             | 1.48             | 5.26             | 0.503            |
| 0.085  | 3.0                           | 1.60             | 2.59             | 7.88             | 0.469            |
| 0.085  | 3.5                           | 2.07             | 3.42             | 9.11             | 0.434            |
| 0.129  | 0                             | n.a.             | n.a.             | n.a.             | 0.676            |
| 0.129  | 1.0                           | 0.38             | 0.70             | 2.92             | 0.668            |
| 0.129  | 2.0                           | 0.80             | 1.50             | 5.84             | 0.651            |
| 0.129  | 3.0                           | 1.30             | 2.55             | 8.72             | 0.610            |
| 0.129  | 3.5                           | 1.63             | 3.27             | 10.1             | 0.573            |

**Note.** The columns are as follows: the MBF parameter $\zeta$, electric current amplitude $j_0$, dipole moment of the stars $\mu$, surface and central magnetic fields, and central density for each choice of parameter.
stars; hence, higher $\zeta$ values allow for more (centrally) magnetized stars (see Figure 9). In particular, for a fixed electric current amplitude configuration, the central magnetic field varies from $8 \times 10^{17}$ G for the $\zeta = 0.040$ parameterization to $1.01 \times 10^{18}$ G for the $\zeta = 0.129$ case. For a detailed discussion of poloidal magnetic field profiles in magnetic stars, see Dexheimer et al. (2016).

6. Particle Population

As already discussed in the introduction, the global properties of nonmagnetic stars are strongly dependent on the particle population and, in particular, on the potential appearance of exotic degrees of freedom at high densities. In Figure 10, we show the mass–radius diagram for the four parameterizations of the MBF model used in this work for both nucleonic (solid lines) and stars that also contain hyperons (dashed lines), not including any magnetic field effects. As one can see, all the parameterizations for nucleonic stars are in agreement with the observational data, but only the first two parameterizations ($\zeta = 0.040, 0.059$) are able to reproduce hyperon stars with masses above $1.97 M_\odot$. This is the current lower bound, including the error in the measurement of massive neutron star observations (Antoniadis et al. 2013). In particular, for nucleonic and hyperon stars, respectively, the maximum masses estimated for each parameterization (for $B = 0$) are $2.57 M_\odot$ and $2.15 M_\odot$ for $\zeta = 0.040$, $2.42 M_\odot$ and $1.99 M_\odot$ for $\zeta = 0.059$, $2.25 M_\odot$ and $1.83 M_\odot$ for $\zeta = 0.085$, and $2.07 M_\odot$ and $1.65 M_\odot$ for $\zeta = 0.129$ (Gomes et al. 2015), indicating a mass decrease of $\sim 0.42 M_\odot$ due to the appearance of hyperons.

Figure 11 shows the particle population as a function of baryon density for the parameterization $\zeta = 0.040$ (the stiffest one), which is the one where hyperons appear at lower densities. For this parameterization, the density threshold for $\Lambda$ hyperon appearance is $0.305 \text{ fm}^{-3}$, while for the other parameterizations the order in which the particles appear is the same but shifted to higher densities: $0.319 \text{ fm}^{-3}$ for
The Astrophysical Journal, 850:20 (13pp), 2017 November 20

Gomes et al.

\[ j_0 = 3.0 \times 10^{15} \text{ A m}^{-2} \] and \[ \rho_b/\rho_n = 0.142 \] for \[ j_0 = 2.0 \times 10^{15} \text{ A m}^{-2} \] for \( \zeta = 0.040 \). In all magnetic field configurations analyzed for this parameterization, only the case of \( j_0 = 2 \times 10^{15} \text{ A m}^{-2} \) allows for the appearance of \( \Xi^- \) particles with a density threshold \( \rho_b = 0.377 \text{ fm}^{-3} \). The largest interval of densities in which \( \Lambda \) particles appear has a size of 0.127 fm\(^{-3} \) for the same value of the electric current amplitude \( j_0 \). Summarizing, it is necessary to have surface and central magnetic fields higher than \( B_1 = 3.8 \times 10^{17} \text{ G} \) and \( B_2 = 8.01 \times 10^{17} \text{ G} \) for the total disappearance of hyperons (all assuming \( \zeta = 0.040 \)).

Still regarding the hyperon population content as a function of the radius \((r_f/r_f)_{eq} \), one can also see from Table 3 that this quantity increases when comparing the nonmagnetic case and the case of \( j_0 = 2 \times 10^{15} \text{ A m}^{-2} \) but then decreases for higher values of electric current amplitude. Such behavior comes from both the central density decrease and the increase of the stars’ radius toward the equatorial direction. From Figure 11, one can see that the decrease of the central density from \( \rho_b = 0.452 \text{ fm}^{-3} \) for \( j_0 = 0 \) to \( \rho_b = 0.415 \text{ fm}^{-3} \) for \( j_0 = 2.0 \times 10^{15} \text{ A m}^{-2} \) does not significantly affect the hyperon fraction, and, moreover, the number of hyperon particles \((\Lambda, \Xi^-)\) is the same, whereas the (equatorial) radius varies from \( r_f^{eq} = 7.50 \text{ km} \) to \( r_f^{eq} = 7.65 \text{ km} \), respectively. This means that the star gets larger in the equatorial direction, keeping roughly the same fraction of hyperons and, consequently, increasing the values of \((r_f/r_f)_{eq}\). However, when higher magnetic field configurations are introduced, the central density decreases, not reaching the \( \Xi^- \) hyperon threshold and substantially lowering the hyperon and \((r_f/r_f)_{eq}\) value fractions.

Finally, we can also look at the \( \zeta = 0.059 \) parameterization in Table 3. As this parameterization has an EoS that is softer than the previous one, higher central densities are reached and at least a small fraction of hyperons is present for all magnetic field configurations analyzed. For this parameterization, higher densities are reached (compared to \( \zeta = 0.040 \)) for higher magnetic field configurations, and, in particular, for \( j_0 = 3.0 \times 10^{15} \text{ A m}^{-2} \), both \( \Lambda \) and \( \Xi^- \) hyperon thresholds are reached \((\rho_b, \Lambda) = 0.319 \text{ fm}^{-3}, \rho_b, \Xi^- = 0.040 \text{ fm}^{-3}\). The \( j_0 = 2.0 \times 10^{15} \text{ A m}^{-2} \) case generates a larger interval of densities populated by hyperons, 0.261 fm\(^{-3} \). The percentage of the stars populated by hyperons is also larger than for the stiffer EoS parameterization, reaching ~65% of the lowest magnetic field configuration star in this analysis. For this parameterization, \( \Lambda \) hyperons reach the central fraction \( \rho_b/\rho_n = 0.09 \) for \( j_0 = 3.5 \times 10^{15} \text{ A m}^{-2} \), \( \rho_b/\rho_n = 0.167 \) for \( j_0 = 3.0 \times 10^{15} \text{ A m}^{-2} \), and \( \rho_b/\rho_n = 0.218 \) for \( j_0 = 2.0 \times 10^{15} \text{ A m}^{-2} \). However, we stress that such a density interval containing hyperons is much smaller than the one covered by nonmagnetic neutron stars described by the MBF model, which is more than ~0.5 fm\(^{-3} \) (Gomes et al. 2015).

7. Final Discussion

We have investigated the impact of different parameterizations of the MBF model with different degree of stiffness on the properties of highly magnetized nucleon and hyperon stars. Four parameterizations of the model were used that were all able to reproduce nuclear properties at saturation. We calculated the global properties of neutron stars self-consistently by including Landau quantization effects on the EoS and solving the Einstein–Maxwell field equations for a

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**Figure 11.** Particle population for the parameterization \( \zeta = 0.040 \) of the MBF model for nonmagnetic stars. The vertical axis is the particle fraction normalized by the baryon density, and the horizontal axis is the baryon density.

\( \zeta = 0.059, 0.333 \text{ fm}^{-3} \) for \( \zeta = 0.085 \), and 0.349 fm\(^{-3} \) for \( \zeta = 0.129 \). In other words, for a given central density, presented in Tables 2 and 3, one can track the particle population from Figure 11 to find out which degrees of freedom are present inside the star.

It was already shown in the previous section that strong magnetic fields decrease the central density of stars. From a microscopic point of view, it is also important to emphasize that Landau quantization has the effect of shifting the threshold of particle appearance (Gomes et al. 2014). Nevertheless, the latter is a small effect compared to the former, and it was, therefore, not taken into account in the calculations presented in this section. A thorough analysis of the former effect can be found for the MBF model in Dexheimer et al. (2013) and Gomes et al. (2014).

We now focus only on the two parameterizations that can describe massive hyperon stars, \( \zeta = 0.040, 0.059 \). From Table 3, one can see that for \( \zeta = 0.040 \), the highest magnetic field configuration \( j_0 = 3.5 \times 10^{15} \text{ A m}^{-2} \) generates an \( M_b = 2.2 \, M_\odot \) star with a central density of 0.299 fm\(^{-3} \), which is lower than the \( \Lambda \) threshold. This means that, for such a choice of parameters, the star will be entirely nucleonic. The dramatic change in this specific star population is illustrated in Figure 12, which shows the particle population as a function of the radius for a nonmagnetic star (top panel) and as a function of the polar radius for the highest magnetic field configuration (bottom panel), when the hyperon population vanishes entirely. Analogous results are found for the equatorial direction.

For the second-highest magnetic field configuration, \( j_0 = 3.0 \times 10^{15} \text{ A m}^{-2} \), \( \Lambda \) hyperons are allowed in a very small density interval of 0.064 fm\(^{-3} \) (difference between the central density and the \( \Lambda \) hyperon threshold for this parameterization). The last column in Table 3 shows the stellar radii fraction that contain hyperons in the equatorial direction \((r_f/r_f)_{eq} \), which in this case is larger than 0.5 (50% of the star). This is the case because the baryon density increases slowly toward the center of magnetic stars described by stiff EoSs. This behavior can be seen in Figure 8 in the polar direction, but it is even more drastic in the equatorial direction. In particular, \( \Lambda \) hyperons reach the central fraction \( \rho_h/\rho_n = 0.07 \) for
stars Chatterjee et al. (2015), Franzon et al. (2015). A comparison for magnetic neutron stars done by solving the TOV equations (including the pure magnetic field contribution isotropically) and the 2D self-consistent calculations was carried out for the first time. Our results show that neglecting the deformation of stars leads to an overestimate of more than 12\% for both gravitational and baryon maximum masses and an underestimate of more than 20\% for the equatorial radius of 1.4\ $M_{\odot}$ stars. From these results, we concluded that using the TOV equations to describe magnetic neutron stars is not the correct approach, since it generates large errors in the calculation of the global properties of stars already for central magnetic fields lower than \(\sim 10^{18}\) G.

In order to focus on the effects of MBF on magnetic stars, the baryon mass of the stars was fixed to 2.2\ $M_{\odot}$, and we varied both the magnetic field configuration (through the electric current amplitude $j_0$) and the MBF parameter $\zeta$, responsible for the different stiffness of the EoS. The MBF parameter has the effect of shielding the scalar couplings of the model, enhancing the repulsion among particles for low values of $\zeta$. As a consequence, more massive, as well as larger, stars are generated for the stiffest parameterization of the MBF model. These results hold for both nonmagnetic and magnetic stars.

In addition, by applying different parameterizations of the model to describe highly magnetic stars, it was shown that the softer parameterizations allow for higher central densities and, consequently, higher central magnetic fields. Also, because of the larger radii described by stiff parameterizations of the model, these stars are less compact and, hence, more easily deformed. It is important to emphasize that, because of the choice of poloidal magnetic field distributions, oblate shapes are favored for highly magnetic stars. Note, however, that several studies suggest that toroidal contributions might play an important role in the stability of magnetic stars Braithwaite & Spruit (2004), Marchant et al. (2011), Lasky et al. (2011), Ciolfi & Rezzolla (2013), Akgun et al. (2013), Mitchell et al. (2015), Armaza et al. (2015), Mastrano et al. (2015). Still, even in this case, we expect our qualitative results to hold.

It was also shown in this work that strong magnetic field distributions decrease the central densities of neutron stars, which has a very large impact on the particle population of these objects. In particular, for the parameterizations in agreement with the observational data for nonmagnetic hyperon stars ($\zeta = 0.040.0.059$), hyperons populate only a small interval of densities but a significant portion of the stellar volume. As already shown in Franzon et al. (2016) for one EoS, the strangeness is overall lower in a star with a larger

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Table 3

| $\zeta$ | $j_0 (10^{15} \text{ A m}^{-2})$ | $\mu (10^{12} \text{ Am}^{-2})$ | $R_\text{p} (10^{17} \text{ G})$ | $R_\text{eq} (10^{17} \text{ G})$ | $\rho (\text{fm}^{-3})$ | $(r_\text{Y}/r_\text{eq})_\text{in}$ |
|--------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| 0.040  | 0  | n.a.  | n.a.  | n.a.  | 0.452  | 0.534  |
| 0.040  | 2.0 | 1.17  | 1.45  | 4.77  | 0.415  | 0.542  |
| 0.040  | 3.0 | 2.06  | 2.67  | 7.08  | 0.353  | 0.507  |
| 0.040  | 3.5 | 2.88  | 3.78  | 8.09  | 0.299  | 0.597  |
| 0.059  | 0  | n.a.  | n.a.  | n.a.  | 0.649  | 0.631  |
| 0.059  | 2.0 | 0.95  | 1.45  | 5.40  | 0.589  | 0.647  |
| 0.059  | 3.0 | 1.70  | 2.57  | 7.77  | 0.485  | 0.629  |
| 0.059  | 3.5 | 2.38  | 3.53  | 8.65  | 0.385  | 0.597  |

Note. The table includes a new column with the fraction of the star’s equatorial radius that contains hyperons (where $r_\text{Y}$ is the radius in which the hyperon population appears and $r_\text{eq}$ is the radius at the surface).

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Figure 12. Particle population of the $M_\odot = 2.2\ M_\odot$ star for nonmagnetic stars (top panel) and magnetized stars with $j_0 = 3.5 \times 10^{15} \text{ A m}^{-2}$ (bottom panel) for the parameterization $\zeta = 0.040$ of the MBF model. The vertical axis is the particle fraction normalized by the baryon density, and the horizontal axis is the (polar) radius. In the top panel, hyperons appear at $r_\text{Y} = 7.50\ \text{km}$, and the total radius is $r_\text{eq} = 14.03\ \text{km}$. In the bottom panel, hyperons never appear ($r_\text{Y} = 0$), and the total polar radius is $r_\text{p} = 7.27\ \text{km}$.

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Poloidal magnetic field distribution in an axisymmetric metric (2D solutions).

First, we confirmed that microscopic magnetic field effects on the EoS are not significant for the description of the global properties of nucleonic stars using our EoS, in agreement with previous works that found similar results for quark and hybrid
magnetic field, but it stays steady for a larger portion of its radius.

Although we have used a specific nuclear model to describe neutron stars in this work, our results are general and can (and should) be tested with different models. Nevertheless, it would be interesting to compare them quantitatively with results from other models. As a future perspective of this work, studies of the thermal evolution of such objects will be important for the search for potential observational signals of hyperon stars and their creation during the decay of the stellar magnetic field over time. Finally, studies including both strong magnetic fields and fast rotation are important to describe the initial stages of the lives of highly magnetized neutron stars and are already being carried out.

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