Numerical simulation of breaking regular and irregular wave propagation above a sloping bottom

Didit Adytiap, Semeidi Husrins, and Adiwijaya1

1School of Computing, Telkom University, Jl. Telekomunikasi 1 Bandung 40257, Indonesia
2Research & Development Centre for Marine and Coastal Resources, Agency for Marine & Fisheries Research & Development, Ministry of Marine Affairs & Fisheries - The Rep. of Indonesia, Jl. Pasir Putih II Ancol Timur, Jakarta 14430, Indonesia

Corresponding author: adytia@telkomuniversity.ac.id

Abstract. An accurate water wave model with breaking capability is important for accurate prediction of wave evolution especially when the wave approaching shallower water. Not only accuracy in the dispersion relation of the wave model that is important, the nonlinearity of the model also plays significant role in representing effect of wave shoaling. In this paper, we simulate propagation of breaking regular and irregular wave above a sloping bottom by using a dispersive wave model so-called variational Boussinesq model. Results of simulation are compared with experimental data from a hydrodynamic laboratory. We discuss effects of wave energy transfer from main wave frequencies to higher order frequencies when wave reaching shallower water and breaking.

1. Introduction

A good understanding of water wave behavior is essential for design activities related with water wave in coastal and offshore engineering, such as design of harbour, coastal defense, offshore platform, etc. With an increase of computer performances in the several last decades, it has been a long aim of coastal engineers to create an accurate and efficient wave model and its numerical implementation to help their work, especially to understand water wave behavior for design process. The hardest challenge for the wave model is to simulate wave propagation accurately from deep water to shallower water. Physical aspects such as dispersion, nonlinearity, shoaling, refraction, diffraction, breaking should be modelled accurately in the wave model and its numerical implementation.

The most popular type of water wave model among coastal engineering community is Boussinesq-type models (BTMs). Its popularity started when Peregrine in 1967 [1] improved the original wave equations by J.V. Boussinesq in 1887 from originally only for flat bottom to be depth dependence equations. Most noticeable BTMs introduced in 90’s are the Boussinesq of Madsen & Sorensen [2] and the Boussinesq of Nwogu [3]. At that time, these BTMs were limited only for weakly dispersive and weakly nonlinear wave. The ideas of these two BTMs were soon very popular among wave modelers. Many researchers extend the capabilities of these BTMs to be applicable for wide range of waves such as for deep water (very dispersive wave) and also fully nonlinear waves. Nevertheless, the extension of these models into a strong dispersion and fully nonlinear wave model results in a long and a complex set of equations, i.e. number of terms in the equations are significantly increased and also introduce higher order derivatives (up to 6th order spatial derivatives). These complexities may
increase difficulties for numerical implementation. Review related to the development of these BTMs can be read in [4,5].

In this paper, we use a rather different Boussinesq type of model, so-called the Variational Boussinesq (VB) model that is derived from a variational approach (see [6, 7]). Instead of introducing higher order derivative terms in the equations, the VBM only introduced 2nd order spatial derivatives and has no mixed temporal-spatial derivative. The model has been implemented numerically using pseudo-spectral method in [8], and using Finite Element Method (FEM) in [9, 10, 11]. The model is capable for simulating long wave such as tsunami [12, 13] and short wave such as wind waves [7, 11]. Nevertheless, the capability of the VB model [7, 11] was limited for simulating non-breaking waves.

Wave breaking process is a crucial physical phenomenon that has to be included in a wave model, especially for simulating wave propagation from deep to shallower water, where the wave become steeper and eventually breaks. In this paper, we extend the capability of the model especially for simulating breaking waves. To that end, we use an eddy-viscosity breaking mechanism as proposed by [14] to be incorporated in the Pseudo-Spectral (PS) implementation of the VB model.

The content of this paper is as follows. In the next section, we describe the basic derivation of the VB model. The section is followed by a description of the eddy-viscosity breaking mechanism as proposed [14]. To show the performance of the implementation of the breaking mechanism in the PS implementation of the VB model, we simulate propagation of breaking regular and irregular waves above a sloping bottom. Experimental data is available from a hydrodynamic laboratory to compare with the results of simulation. Finally, some conclusions are described in the last section.

2. Variational Boussinesq Model
The variational Boussinesq (VB) model is derived via so-called variational principle [15]. Here the water is assumed to be incompressible, inviscid and the flow is irrotational. Zakharov [16] and Broer [17] show independently that water wave phenomena can be exactly described as a Hamiltonian system, as $\partial_t \eta = \delta \phi \mathcal{H}$, and $\partial_t \phi = -\delta \eta \mathcal{H}$, where $\mathcal{H}$ is so called the Hamiltonian or the total energy (sum of kinetic and potential energy), $\eta$ and $\phi$ are the canonical variables of the Hamiltonian system denoting surface elevation and surface potential, respectively. $\delta \phi \mathcal{H}$ and $\delta \eta \mathcal{H}$ are the variational derivatives of $\mathcal{H}$ with respect to $\eta$ and $\phi$, respectively.

In this paper we restrict the model to be 1D problem. The horizontal and vertical axes are denoted by $x$ and $z$, and the time is denoted by $t$. As derived in [6], in the VB model, the fluid potential $\Phi(x,z,t)$ is approximated by the following expression: $\Phi(x,z,t) = \phi(x,t) + \Sigma_m F_m(z) \psi_m(x,t)$, where $F_m$ is a priori chosen vertical function and $\psi_m$ is an horizontally dependent function that should satisfy an elliptic equation. By substituting the approximation of $\Phi(x,z,t)$ for the VB model above, we can obtain the equations of VB model as follows

$$\partial_t \eta = -\partial_z ((h+\eta)u) - \partial_x (\beta \partial_x \psi)$$  (1)

$$\partial_t u = -g\partial_z \eta - u \partial_u$$  (2)

$$-\partial_x (\alpha \partial_x \psi) + \gamma \psi = \partial_x (\beta u)$$  (3)

The first and second equations are the continuity and momentum equation, whereas the third equation is an elliptic equation for searching function $\psi_m$ that has to be solved every time step. Here $\alpha, \beta$ and $\gamma$ are coefficients that depend on vertical profile $F_m$. Note that the vertical profile $F_m$ denote completely the dispersion quality of the VB model. The optimization of vertical profile $F_m$ for broad band wave can be read in [6, 7]. For a parabolic vertical profile as proposed by [7], the expression for $\alpha, \beta$ and $\gamma$ are as follows $\alpha = 2H^2 / 15$, $\beta = -H^2 / 3$, $\gamma = H / 3$, where $H = h + \eta$ is the total depth.

In this paper, the systems (1-2) are implemented numerically by using the Pseudo-Spectral (PS) method, while the (linear) elliptic system (3) is solved by using Finite Element Method or FEM [6]. The combination of PS method and FEM is investigated in [8]. The main idea of the PS method is to
calculate spatial derivation in the Fourier space by exploiting Fourier and Invers Fourier transform, while the multiplication operation in the nonlinear terms are performed in the real space [8].

3. Breaking Mechanism

In order to incorporate wave breaking mechanism in the wave model, we use the eddy-viscosity breaking mechanism of Kennedy et al 2000 [14]. To that aim, an eddy viscosity term \( R_b \) is added in the momentum equation of the VB model, such that the equation (5) become \( \partial_t u = -g \partial_z \eta - u \partial_x u + R_b \). where \( R_b \) is defined as \( R_b = \frac{1}{H} \partial_x F \), with \( H = \eta + h \) is the total depth. Based on [14] the \( F \) is defined as \( F = \nu \partial_z (Hu) \), where \( \nu \) is the so-called eddy-viscosity coefficient, i.e. \( \nu = bH \partial_z (Hu) \). The eddy viscosity model of Kennedy is actually derived based on the Shallow Water Equations (SWE) where the term \( \partial_x (Hu) \) is the normal velocity \( N \), i.e. from the continuity equation of the SWE: \( \partial_x \eta = -\partial_z (Hu) = -N \). The coefficient \( b \) in the expression of \( \nu \) contains a mixing length coefficient \( \delta_b \) and a smooth function \( B \) that determine the cessation of the wave breaking [14] for the details). In this paper, we use \( \delta_B = 1.5 \) for two simulations in the next section.

4. Test Cases

For testing the PS-implementation and the Kennedy’s eddy viscosity breaking mechanism in the VB model, we perform two test cases, i.e. breaking regular and irregular waves propagating above a sloping bottom. These two cases are based on a physical experiment that was performed in the twin flumes of Leichtweiss - Institute (LWI), TU Braunschweig, Germany. The flume consists of 1m and 2m wide parallel flumes, with approximately 90m long and 1.2m high. The original aim of the experiment is to investigate energy wave dissipation by an artificial parameterized mangrove forest that is placed in the 2m-wide flume. For this paper, we only use the experiment results in the 1m-wide flume, where there is no obstacle in the basin. In the middle of the flume, it is installed a sloping bottom with gradient 1:20 that is started from depth \( h_0=0.415 \) m up to \( h_1=0.2 \) m. The layout of the experiment is shown in Figure 1. There are 6 wave gauges (WG) installed in the basin as shown in Figure 1. To perform numerical simulations, instead of using the signal at the wave flap at \( x=0m \), we use the signal at \( WG1a=21.415m \) as the influx signal for the wave model. The signal is influxed into the domain of computation by using a method so-called embedded influxing as proposed by [18].

![Figure 1. Layout of the experiment setup.](image)

4.1. Breaking Regular Wave

The first case that is simulated is a breaking regular wave (with case number 2009051403). In the experiment, monochromatic (regular) waves are influxed by the wave flap with wave height of 0.2m (above 0.615m depth) and with wave period of 2.5s (or frequency of 0.4Hz). As stated previously, to perform the simulation, the signal at \( WG1a \) is used as an influx signal for the simulation. For the numerical simulation we use a uniform grid with grid size \( dx=0.08m \). The simulation is performed until time \( t=200s \).
Figure 2. Signal comparison between the simulation (solid red lines) and the measurement (dashed blue lines) at positions WG-4a, WG-5a, and WG-6a for the regular wave case 2009051403.

Figure 3. Comparison of amplitude spectrum between simulation (solid red lines) and measurement (dashed blue lines) at positions WG-4a, WG-5a, and WG-6a for the regular wave case 2009051403.

In Figure 2, results of numerical simulation are compared with signals from the experimental data at WG4a (at the foot of the slope), WG5a (at the top of the slope) and WG6a (at the shallower area). The comparison in signal shows a good agreement with the experimental data. The signal plots in WG4a and WG5a show that the wave become steeper as the wave propagate from deep water into shallower water. The signal in WG6a shows that a significant amount of energy is released as the wave breaks between WG5a and WG6a that is shown by an amplitude reduction, i.e. from 0.15 m in WG5a to 0.09 m in WG6a.

Since the wave propagates from deep area (WG4a) into shallower area (WG5a), there are effects of wave shoaling and wave nonlinearity. These effects can be shown from plots of amplitude spectrum of signal in WG4a to WG6a in Figure 3. In the WG4a, the signal already contains relatively small bound waves (low and high frequencies waves that are formed by the free wave, i.e. wave with frequency $f_0 = 0.4\text{Hz}$). As the wave propagates into shallower area, in WG5a, the energy of bound waves become larger. In other words, there are transfer of energy from the free wave ($f_0 = 0.4\text{Hz}$) into the bound waves, i.e. wave with frequencies of $f = 2f_0, 3f_0, 4f_0$, etc (or higher order waves). In the
WG6a, the amplitude spectrum plot shows that there are a significant amount of energy is released for the free wave as well as the bound waves.

4.2. Breaking Irregular Wave

The second test is the breaking irregular wave that is propagate in the same domain setting as in the previous case. The case number of the experiment is 2009051509. In this experiment, an irregular wave is influxed in the wave flap with significant wave height $H_s$ of 0.2 m and with peak wave period $T_p$ of 2.5 s (or corresponds with $f_p = 0.4 Hz$). Similar to the regular case, we use a uniform grid with grid size of $dx=0.08 m$. The simulation is performed until time $t=400s$.

Just as in the previous case, results of the numerical simulation are compared with the experimental data at WG4a, WG5a and WG6a, which are shown in Figure 4 for the signal and in Figure 5 for the amplitude spectrum. Different than the previous case, in this irregular wave case, the wave propagate in a more complex behaviour, where dispersion plays significant role besides the nonlinearity, i.e. lower wave frequency (long wave) propagates faster than higher wave frequency (short wave). Nevertheless, the comparison in Figure 4 and 5 show a good agreement between the simulation and the experimental data.

![Figure 4](image)

**Figure 4.** Signal comparison between the simulation (solid red lines) and the measurement (dashed blue lines) at positions WG-4a, WG-5a, and WG-6a for the irregular wave case 2009051509.

Similar to the regular wave case, the irregular wave breaking position is between WG5a and WG6a. This can be seen by the waveheight reduction in the WG6a in comparison to the waveheight in WG5a (Figure 4). An energy transfer from the main (free) wave frequencies into higher and lower frequencies can also be observed in the amplitude spectrum plots of signal in WG-4a to WG-5a in Figure 5. The amplitude spectrum plot in the WG6a shows a significant wave energy reduction that is caused by wave breaking.

5. Conclusion and discussion

An extension of a Pseudo-Spectral implementation of the Variational Boussinesq (VB) model with Kennedy’s eddy-viscosity breaking mechanism is presented. The accuracy of the implementation is tested by comparing results of simulations with laboratory experiment of breaking regular and irregular wave propagation above a sloping bottom. The comparison in signal and amplitude spectrum of the simulation show a good agreement with the experimental data. The simulation can reconstruct the experiment accurately, for effects of dispersion, nonlinearity and breaking process. Effects of wave breaking, i.e. wave energy reduction, is clearly seen in the plot of amplitude spectrum. Besides that, in
the regular wave simulation, a transfer of energy from main frequency to higher frequencies can be seen clearly from the plot of amplitude spectrum.

![Figure 5. Comparison of amplitude spectrum between simulation (solid red lines) and measurement (dashed blue lines) at positions WG-4a, WG-5a, and WG-6a for the irregular wave case 2009051509.](image)

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