Analysis of axial crushing of fiber reinforced metal tube with an eccentricity factor

L Z Liu*, Y Tang, C Q Wang and F X Zheng

1 School of Materials Science and Engineering, Northeastern University, Shenyang 110819, China
2 Key Laboratory of Lightweight Structural Materials, Liaoning Province, China
liulizhong93@163.com

Abstract. The fiber reinforced metal tubes have been proved to exhibit an excellent crashworthy performance and a high ratio of strength to weight in many experiments. Some theoretical models are promoted to predict the mean crushing force, which benefits the design of these structures. Based on the solution of Wang and Lu, the theoretical analysis is presented in this paper to consider the eccentricity factor, which represents the ratio of the outward part to the total folding length. The agreement between the calculated and experimental data reveals that the analytical model is reasonable.

1. Introduction
Thin-walled tubes are often used as energy absorbers in moving vehicles to protect the people and structures. Many theoretical, experimental and numerical studies related to this topic have appeared in the literatures. Among different type of thin-walled structures, the circular tubes are widely employed because of their low cost and excellent energy absorption capacity. The theoretical studies on the axial crushing of thin-walled circular tubes could be traced back to the works of Alexander[1] and Pugsley[2] around 1960s. Andrews[3] classified the deformation modes of the axial collapse of cylindrical tubes. Many other researchers made their efforts to further improve the theoretical models for the axial crushing of circular tubes. In addition to the circular metal tube, a combination of metal and composite material has been used to improve crashworthiness properties. These fiber reinforced metal tubes have been proved to exhibit an excellent crashworthy performance and a high ratio of strength to weight[4-7]. A simplified analytical model for the static crushing of externally reinforced metal tubes was presented by Hanefi[8]. In the paper, the classical Alexander’s solution was modified to take into account the contribution of the metal and composite wall. Mamalis et al[9] developed the theoretical analysis of the failure mechanism of thin-walled fiberglass composite tubes under static axial compression. Wang and Lu[10] presented a theoretical model to predict the mean crushing force of arbitrarily fiber reinforced metal tubes with a ring collapse mode. This paper presented a more reasonable predictive model of axial crushing of fiber reinforced metal tubes. The crushing model took into account both the radial inward and outward folds.

In this paper, the theoretical analysis is modified to predict the mean crushing force based on the works from Wang and Lu[10]. In their works, the fold develops to the inward and outward direction, and the two displacements are assumed to be equal, as shown in figure 1(b). The experiments of axial crushing of carbon fiber reinforced steel tubes have been carried out in our laboratory. The results revealed that the inward and outward displacements were slightly different. Singace[11,12] presented
the axial crushing analysis considering the eccentricity factor, which was used to represent the ratio of the outward part to the total folding length. From this point of view, the eccentricity factor for the Wang and Lu’s works is 0.5, which means that the length of the inward and outward part is equal. According to the Singace’s works and our experiments, the eccentricity factor of 0.65 should be a more appropriate choice for the analysis, as shown in figure 1(c). The following sections will describe the analysis with the eccentricity factor of 0.65 based on the works from Wang and Lu. The theoretical prediction results are compared to the experimental data from our laboratory.

![Figure 1. Crushing model of a fiber reinforced metal wall section.](image)

2. Theoretical analytical model

In the collapse of fiber reinforced metal tubes, the formation of plastic hinges leads to the developing of folds. Two folds are selected to deduce the formula, as shown in figure 1(c). Some assumptions are made to simplify the derivation. The hinges are considered as stationary plastic hinges. The metal wall obeys the Von Mises yield criterion. The work hardening of the metal is ignored. The work of the external force is dissipated by the bending at the five hinges, the circumferential stretching and compression of the tube wall between the hinges. These energy dissipation mechanisms are described in the following sections.

2.1. Energy dissipated by the bending at the hinges

With the increase of the external axial force, the bending moment of the tube wall increases. When the bending moment increases to the plastic limit bending moment \( M \), the plastic hinges appear. For the assumption of no work hardening, the hinges continuously deform to the angle \( \alpha \) until the adjacent folds touch together. In the five plastic hinges of two folds, hinges 2 and 4 are at outward of the tube wall, while hinges 1, 3 and 5 inward. Therefore, the hinges are restricted by fiber wall.

The plastic limit bending moment \( M_0 \) at outward convolution hinges 2 and 4 is

\[
M_0 = \frac{1}{4} \sigma_0 h_m^2 C_0
\]

where

\[
\sigma_0 = \frac{2}{\sqrt{3}} \sigma_s
\]

\[
C_0 = 1 + 2 \frac{\sigma_{czt}}{\sigma_s} \frac{h_f}{h_m} + 2 \frac{\sigma_{czt}}{\sigma_s} \left( \frac{h_f}{h_m} \right)^2 - \left( \frac{\sigma_{czt}}{\sigma_s} \right)^2 \left( \frac{h_f}{h_m} \right)^2
\]

\( \sigma_s \) is the yielding strength of the metal, \( \sigma_{czt} \) is the axial tensile strength of the fiber reinforced layer. \( h_m \) and \( h_f \) are the thickness of metal wall and that of fiber reinforced layer, respectively. Considering the eccentricity factor of 0.65, during the increment \( d\alpha \), the increment of work done by bending deformation at hinges 2 and 4 is

\[
dW_0 = 4\cdot 2\pi \cdot (r + 0.325L \sin \alpha) M_0 d\alpha
\]

where, \( r \) is the inner radius of the tube. \( L \) is the half length of a fold.

The plastic limit bending moment \( M_1 \) at inward convolution hinges 1, 3 and 5 is
\[ M_1 = \frac{1}{4} \sigma_0 h_m^2 C_1 \]  
\[ C_1 = 1 + 2 \frac{\sigma_{czt} h_c}{\sigma_s h_m} + 2 \left( \frac{\sigma_{czt}}{\sigma_s h_m} \right)^2 - \left( \frac{\sigma_{czt}}{\sigma_s h_m} \right)^2 \]

where, \( \sigma_{czt} \) is the axial compressive strength of the fiber reinforced layer. Considering the eccentricity factor of 0.65, during the increment \( da \), the increment of work done by bending deformation at hinges 1, 3 and 5 is

\[ \text{d} W'_1 = 4 \cdot 2 \pi (r \cdot 0.175L \sin \alpha) M_1 \text{d} \alpha \]  

The total bending energy dissipated in the collapsing convolution of the four element, when \( \alpha \) increases from 0° to 90°, is

\[ W_b = W_0 + W'_1 \]  

\[ W_0 = \int_0^{\pi} 4 \cdot 2 \pi (r \cdot 0.325L \sin \alpha) M_0 \text{d} \alpha + \int_0^{\pi} 4 \cdot 2 \pi (r \cdot 0.175L \sin \alpha) M_1 \text{d} \alpha \]  

\[ W'_0 = \frac{4}{\sqrt{3}} \pi \left( r \pi^2 \sigma_s h_m^2 C_x + \frac{1.4}{\sqrt{3}} \ln \sigma_s h_m^2 C_y + 0.6 \frac{\pi}{\sqrt{3}} \sigma_s h_m^2 C_0 \right) \]

2.2. Energy dissipated by the circumferential tensile and compressive strain of the metal wall

The circumferential tensile strain appears when hinges 2 and 4 moves outward. Considering the eccentricity factor of 0.65, which means that the circumferential tensile strain exists in the 65 percent of the length \( L \), as shown in figure 1(c). During the increment \( da \), the mean circumferential tensile strain between hinge 2 and 4 is

\[ \text{d} \epsilon_0 = \frac{2 \pi [r \cdot 0.325L \sin (\alpha + da)] - 2 \pi (r \cdot 0.325L \sin \alpha)}{2 \pi (r \cdot 0.325L \sin \alpha)} \]  

the increment of work done within the metal wall in circumferential stretching is

\[ \text{d} W_2 = 4 \sigma_0 L h_m \cdot 2 \pi (r \cdot 0.325L \sin \alpha) \text{d} \epsilon_0 \]  

The mean circumferential compressive strain between hinges 1, 3 and 5 is

\[ \text{d} \epsilon_1 = \frac{2 \pi [r \cdot 0.175L \sin (\alpha + da)] - 2 \pi (r \cdot 0.175L \sin \alpha)}{2 \pi (r \cdot 0.175L \sin \alpha)} \]  

and the increment of work done within the metal wall under circumferential compression is

\[ \text{d} W_3 = 4 \sigma_0 L h_m \cdot 2 \pi (r \cdot 0.175L \sin \alpha) \text{d} \epsilon_1 \]  

The total work done by the metal wall under circumferential strain is

\[ W_m = \int_0^{\pi} 2.6 \pi \sigma_0 L^2 \cos \alpha h_m \text{d} \alpha + \int_0^{\pi} 1.4 \pi \sigma_0 L^2 \cos \alpha h_m \text{d} \alpha \]

and can be simplified as

\[ W_m = 4 \pi \sigma_0 L^2 h_m \]  

2.3. Energy dissipated by the circumferential tensile and compressive strain of fiber reinforced layer

The circumferential compression of fiber reinforced layer between hinge 1, 3 and 5 is the same as that of the metal layer. The fiber reinforced layer in circumferential compression can be considered as a perfectly plastic material, so the strain is

\[ \text{d} \epsilon_2 = \frac{2 \pi [r \cdot 0.175L \sin (\alpha + da)] - 2 \pi (r \cdot 0.175L \sin \alpha)}{2 \pi (r \cdot 0.175L \sin \alpha)} \]

the increment of work done is
\[ dW_4 = 4 \sigma_{chc} L h_c \cdot 2\pi (r - 0.175 L \sin \alpha) \, d\epsilon_2 \]

\[ W_5 = 1.4\pi \sigma_{chc} h_c \frac{L^2}{3} \]

where, \( \sigma_{chc} \) is the circumferential critical stress of the fiber reinforced layer subjected to compression.

The fiber reinforced layer between hinges 2 and 4 is under circumferential tension. The fiber layer is elastic until it fractures when the circumferential strain reaches a critical value of nearly 1.77%. Thereafter, the fiber reinforced layer does not exert any stretching resistance circumferentially. The work done in circumferentially stretching the fiber reinforced layer is concluded as follows [10]

\[ W_6 = \frac{2}{3} \pi r^2 h_c \sigma_{cht} \epsilon_s^2 \]

where, \( \sigma_{cht} \) and \( \epsilon_s \) are the critical stress and strain of the fiber reinforced layer subjected to circumferential tension, respectively.

The total work dissipated by the circumferential strain of the fiber reinforced layer is

\[ W_c = 1.4\pi \sigma_{chc} L \frac{h_c^2}{2} + \frac{2}{3} \pi r^2 h_c \sigma_{cht} \epsilon_s^2 \]

2.4. Mean external crushing force

The total work done by the mean external crushing force \( F \) acting over the effective crushing length of the four compressive elements is \( F\delta \) [10]. The mean crushing force calculated from the balance of energy can be expressed as

\[ F \delta = W = W_b + W_m + W_c \]

where, \( \delta = 3.44L - 2h \)

The effective crushing length is [10]

\[ \delta = 3.44L - 2h \]

where, \( h \) is the total thickness of the metal and the fiber reinforced layer.

Therefore, the expression for the mean crushing force \( F \) is

\[ F = \frac{A + (D + E)L^2 + (B + C)h_c^2}{3.44L - 2h} \]

where

\[ A = 22.79 \pi r \sigma_{m} h_m^2 C_y + 2.09 \pi r^2 h_c \sigma_{cht} \epsilon_s^2 \]

\[ B = 4.40 \pi \sigma_{chc} h_c \]

\[ C = 14.51 \pi \sigma_{m} h_m \]

\[ D = 2.54 \pi \sigma_{zt} h_m C_y \]

\[ E = 1.09 \pi \sigma_{cht} C_0 \]

\[ L = 0.58h + 0.58[ h_c^2 + \frac{3.44(2hD + 2hE + 3.44A)}{4(B + C)} ]^{1/2} \]

3. Experimental verification

Some crushing tests on the carbon fiber reinforced steel circular tubes are performed in our laboratory to verify the solutions in this paper. The steel tubes with thickness of 1 mm and diameter of 50 mm are wrapped by the carbon fiber reinforced layers. The number of plies of the carbon fiber reinforced layers is from 1 to 5. In all samples, the fiber direction is always perpendicular to the axis of tubes. Other parameters related to the solutions of equation (21) are shown in table 1. The comparisons are presented in table 2.

| \( \sigma_s \) (MPa) | \( \sigma_{cht} \) (MPa) | \( \sigma_{chc} \) (MPa) | \( \sigma_{zt} \) (MPa) | \( \sigma_{czc} \) (MPa) | \( \epsilon_s \) |
|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| 340                 | 1218                | 884                 | 64                  | 84                  | 0.018               |

From table 2, it can be seen that the percentage errors between the experimental results and the present calculations based on equation (21) are less than 13.0%, and the average value is 5.9%. The agreement between the analysis and experiments indicates that the analytical model is reasonable.


Table 2. Comparisons between the theoretical prediction and experimental data.

| Samples | h_m(mm) | h_c(mm) | r(mm) | Experimental data, F(kN) | Predicting data, F(kN) | Error (%) |
|---------|---------|---------|-------|--------------------------|------------------------|-----------|
| CY1     | 1.00    | 0.51    | 25.02 | 32482                    | 28431                  | 12.5      |
| CY2     | 0.99    | 1.02    | 25.00 | 39649                    | 39280                  | 0.9       |
| CY3     | 1.01    | 1.50    | 25.01 | 54417                    | 52955                  | 2.7       |
| CY4     | 1.00    | 1.98    | 25.00 | 67753                    | 67475                  | 0.4       |
| CY5     | 1.00    | 2.51    | 24.98 | 76238                    | 86176                  | -13.0     |

4. Conclusions

The axial crushing of a metal tube with external fiber reinforced layers is studied in this paper with theoretical analysis method. An analytical model is modified and presented to calculate the mean crushing force based on the works of Wang and Lu[10]. In the presented model, the eccentricity factor of 0.65 is adopted to represent the ratio of the outward part to the total folding length. The theoretical results agree with the experimental data from our laboratory, which indicates the analytical model is reasonable.

Acknowledgments

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References

[1] Alexander J M 1960 An approximate analysis of the collapse of thin cylindrical shells under axial loading Q. J. Mech. Appl. Math. XIII 10-15
[2] Pugsley A and Macaulay M 1960 The large-scale crumpling of thin cylindrical columns Q. J. Mech. Appl. Math. XIII 1-9
[3] Andrews K R F, England G L and Ghani E 1983 Classification of the axial collapse of cylindrical tubes under quasi-static loading Int. J. Mech. Sci. 25 687-696
[4] Lima R M, Ismarrubie Z N, Zainudin E S and Tang S H 2012 Energy absorption capability of hybrid tube made by mild steel and GFRP under quasi-static loading Adv. Mater. Res. 383-390 2741-6
[5] Abbas T, Ya H H and Abdullah M Z 2017 Comparison of energy absorption of aluminium-composite tubes subjected to axial loading IOP Conf. Ser.: Mater. Sci. Eng. 205 012020
[6] Zhu G H, Sun G Y, Liu Q, Li G Y and Li Q 2017 On crushing characteristics of different configurations of metal-composites hybrid tubes Compos. Struct. 175 58-69
[7] Shi P L, Yu Q, Huang R, Zhao X and Zhu G H 2019 Crashworthy and performance-cost characteristics of aluminum-CFRP hybrid tubes under quasi-static axial loading Fiber. Polym. 20 384-397
[8] Hanefi E H and Wierzbicki T 1996 Axial resistance and energy absorption of externally reinforced metal tubes Compos.Part B 27B 387-394
[9] Mamalis A G, Manolakos D E, Demosthenous G A and Ioannidis M B 1996 Analysis of failure mechanisms observed in axial collapse of thin-walled circular fiberglass composite tubes Thin.Wall. Struct. 24 335-352
[10] Wang X and Lu G 2002 Axial crushing force of externally fibre-reinforced metal tubes Proc. Inst.Mech.Eng. Part C.J.Mech.Eng.Sci. 216 863-874
[11] Singace A A, Elsobky H and Reddy T Y 1995 On the eccentricity factor in the progressive crushing of tubes Int. J. Solids Struct. 32 3589-3602
[12] Singace A A 1999 Axial crushing analysis of tubes deforming in the multi-lobe mode Int. J. Mech. Sci. 41 865-890