Laboratory test on maximum and minimum void ratio of tropical sand matrix soils

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Abstract. Sand is generally known as loose granular material which has a grain size finer than gravel and coarser than silt and can be very angular to well-rounded in shape. The present of various amount of fines which also influence the loosest and densest state of sand in natural condition have been well known to contribute to the deformation and loss of shear strength of soil. This paper presents the effect of various range of fines content on minimum void ratio $e_{\text{min}}$ and maximum void ratio $e_{\text{max}}$ of sand matrix soils. Laboratory tests to determine $e_{\text{min}}$ and $e_{\text{max}}$ of sand matrix soil were conducted using non-standard method introduced by previous researcher. Clean sand was obtained from natural mining site at Johor, Malaysia. A set of 3 different sizes of sand (fine sand, medium sand, and coarse sand) were mixed with 0% to 40% by weight of low plasticity fine (kaolin). Results showed that generally $e_{\text{min}}$ and $e_{\text{max}}$ decreased with the increase of fines content up to a minimal value of 0% to 30%, and then increased back thereafter.

1. Introduction

In natural condition of soil, the grains itself may be rounded, spherical, sub-angular, angular, well-rounded, and other complex geometric characteristics. The composition of soil is solid (mineralogy of grain, grain size and grain distribution, shapes and others), water, and air. The appearance of different composition of solid, water, and air in soil influences soil properties such as shear strength, permeability, and settlement. In seismic condition, granular materials such as sand are considered as susceptible material to liquefy wherever there are tendencies of the materials to re-arrange and may result in loss of shear strength (Perlea, 2000).

Das (2010) stated that the denseness of packing is influenced by particle size distribution, shape of soil particles, and relative density. The structures of the natural cohesionless soil in loose and dense conditions are shown in Figure 1.

![Figure 1. Structures of cohesionless soil (Das, 2010)](image)

Lade et al. (1998) stated the characteristics and properties of granular materials are most often related to relative density, $D_r$, $D_s$ is commonly used to indicate the in-situ denseness or looseness of granular
soil (Das, 2010). Polito and Martin (2001) has shown that the liquefaction resistance of soil is controlled by $D_r$ of soil. The relative density of the field void ratio, $e$ between maximum void ratio, $e_{\text{max}}$ and minimum void ratio, $e_{\text{min}}$ (Lade et al., 1998) can be defined as:

\[ D_r = \frac{e_{\text{max}} - e}{e_{\text{max}} - e_{\text{min}}} \cdot 100 \]  

(1)

To date, research on the effect of fines content in sand mixtures in term of soil behavior is still the interest by numerous researchers due to the contradictory results obtained. From Lade et al. (1998), using Nevada sand mixing with Nevada fines, it can be seen the $e_{\text{min}}$ and $e_{\text{max}}$ of both mixtures decreased with increase of fines content, $F_c$ until about 25% of fines content, and started to increase with further increase of fines. Belkhatir et al. (2011) performed an experiment on sand-silt mixtures and shows the $e_{\text{min}}$ and $e_{\text{max}}$ decreased with the increase of fines content until 30% of fines content, then the $e_{\text{min}}$ and $e_{\text{max}}$ increased with further increase of fines content. The same pattern has also been plotted by Monkul et al. (2016), Cubrinovski and Ishihara (2002), Lade and Yamamuro (1997), and Yamamuro and Lade (1997); but the transition from decrease to increase of void ratio occurred at different percentage of fines between 20% to 40%. However, Xenaki and Athamasopoulus (2003) shows that the $e_{\text{min}}$ in sand-fines mixtures increased with increase of fines while $e_{\text{max}}$ decreased until about 45% of fines content and started to increase with further increase of fines content. Cubrinovski and Ishihara (2002) shows that there is a clear trend among $e_{\text{min}}$, $e_{\text{max}}$, and $F_c$ where both $e_{\text{min}}$ and $e_{\text{max}}$ increase with $F_c$. From Figure 2, they produced the relationship between $e_{\text{min}}$ and $e_{\text{max}}$ for four different types of soils such as the followings:

![Figure 2](image)

**Figure 2.** Effect of fines content on $e_{\text{min}}$ and $e_{\text{max}}$ (Cubrinovski and Ishihara, 2002)

a) Clean sands ($F_c = 0-5\%$):

\[ e_{\text{max}} = 0.072 + 1.53e_{\text{min}} \]  

(2)

b) Sands with fines ($5 < F_c \leq 15\%$):

\[ e_{\text{max}} = 0.25 + 1.37e_{\text{min}} \]  

(3)

c) Sands with fines and clay ($15 < F_c \leq 30\%$, $P_c = 5-20\%$):

\[ e_{\text{max}} = 0.44 + 1.21e_{\text{min}} \]  

(4)

d) Silty soils ($30 < F_c \leq 70\%$, $P_c = 5-20\%$):

\[ e_{\text{max}} = 0.44 + 1.32e_{\text{min}} \]  

(5)
Where, $F_c = \text{fine fraction smaller than 0.075mm}$
$P_c = \text{clay-size < 0.005mm}$

From their research, Cubrinovski and Ishihara (2002) concluded that for sand with more than 30% fines, the fines are the controlling grain fraction in the particle structure and deformational behaviour of the soils. While for fines content less than 20%, the packing structure is obviously dominated by the sand matrix. Lade et al. (1998) comprehensively discussed the behaviour of fines in sand in binary packing. In their analysis, initially (0% fines) the large particles formed a structure, then the small particles dropped into primary fabric of large particles (increase of fines content) until voids of primary fabrics had been filled by the fines. Continuous increasing of fines beyond the minimum void ratio caused the large particles to be pushed apart until reaching 100% fines. The diagram of theoretical variation of minimum and maximum void ratio in binary packing is shown in Figure 3.

![Figure 3. Theoretical variation of void ratio with fines in binary packing (Lade et al., 1998)](image)

Although there are intensive studies done by Cubrinovski and Ishihara (2002), Lade et al. (1998), Yamamuro and Lade (1997), and Belkhatir et al. (2011) on the behaviour of sand mixtures, there is a lack of study on the effect of fines content on the $e_{\text{max}}$ and $e_{\text{min}}$ of various sand sizes for tropical sand (fine sand, medium sand, and coarse sand) mixed with tropical kaolin. Hence, laboratory experimental test had been conducted for tropical sands of various sizes mixed with 0% - 40% by weight of kaolin (fines). The aim of this study is to investigate the effect of fines content ranged from 0% - 40% on $e_{\text{min}}$ and $e_{\text{max}}$ of sand matrix soils of tropical origin. The experimental results were obtained from experimental procedure introduced by Yamamuro and Lade (1997) for the determination of maximum and minimum density of sand matrix soils, while BS 1377-2:1990 was used for grain size distribution and hydrometer analysis.

2. Testing materials

2.1 Sand matrix soil
Sand that always contain little or significant amount of fines is known as sand matrix soils (Tan et al., 2015 and Marto et al., 2016). The selected clean sand used for testing was obtained from a mining site at Johor area located 350 km from Kuala Lumpur, Malaysia. This sand has a white colour and is widely used for construction. From particle size distribution graph, the sand sample has mean diameter, $D_{50} = 0.47 \text{ mm}$, a coefficient of curvature, $C_c = 0.86$ and coefficient of uniformity, $C_u = 2.14$. According to Unified Soil Classification System (USCS), the sand is classified as SP (Poorly graded sands). White kaolin as low plasticity fine was used to reconstitute sand matrix soil by mixing with different
percentage of different sizes of sand. Figure 4 shows the materials used to reconstitute sand matrix soil at different size of sand while Figure 5 shows the particle size distribution for clean sand and kaolin used in this study.

**Figure 4.** Sample to reconstitute samples of sand matrix soil. a) Coarse sand; b) Medium sand; c) Fine sand; d) Kaolin

**Figure 5.** Grain size distribution graph of clean sand and kaolin

### 2.2 Methodology

To date, there are several procedures available for determining maximum void ratios and minimum void ratios such as BS 1377-4:1990 (clause 4.2: maximum density of sands, clause 4.4: minimum density of sands), ASTM D4253-00 Standard Test for Maximum Index Density, and ASTM D4254-00 Standard Test for Minimum Index Density, but both standards are limited to small amount for fines (passing 63µm) and up to 15% soil passing 75µm respectively. Lade et al. (1998), Chubrinovski and Ishihara (2002), and Tsukamoto et al. (2014) presented results using a new procedure on test procedure introduced by Yamamuro and Lade (1997) for minimum and maximum densities of sands which applied...
for fines content from 0% to 100%, and it shows a good correlation between maximum and minimum void ratios. Although there are no appropriate standards for soil content up to 40% fines (passing 63µm), but the studies done by Tan et al. (2015), Marto et al. (2016), Chang et al. (2015), Yamamuro and Lade (1997), Lade et al. (1998), Chubrinovski and Ishihara (2002), and Tsukamoto et al. (2014) show the trends do not vary and show a good correlation between the void ratios. Hence this research used test procedure introduced by Yamamuro and Lade (1997) on maximum density and minimum density of sands to determine maximum void ratio and minimum void ratio of sand matrix soils. The grain size distribution of sand and kaolin were carried out by wet sieve and hydrometer analyses based on BS 1377-2:1990.

Clean sands were separated into three grain sizes ranges: coarse sands (retain 2.0 mm to 0.6 mm); medium sands (retain 0.425 mm to 0.212 mm); and fine sands (retain 0.150 mm to 0.063 mm). These three grain sizes of sand were reconstituted by mixing with low plasticity fines (kaolin) at different percentage (0% to 40%) by weight as listed in Table 1. The kaolin is manufactured by Kaolin (Malaysia) Sdn. Bhd.

Table 1. Experimental program for determination of minimum void ratio and maximum void ratio of sand matrix soils

| Sand Matrix Soils       | Sample Code | Percentage by weight (%) | Sand | Kaolin |
|-------------------------|-------------|---------------------------|------|--------|
| Clean Sand              | S100K0      | 100                       |      | 0      |
| Coarse Sand             | S100K0 - C  | 100                       |      | 0      |
| Coarse Sand + Kaolin    | S90K10 - C  | 90                        | 10   |        |
|                         | S80K20 - C  | 80                        | 20   |        |
|                         | S70K30 - C  | 70                        | 30   |        |
|                         | S60K40 - C  | 60                        | 40   |        |
| Medium Sand             | S100K0 - M  | 100                       |      | 0      |
| Medium Sand + Kaolin    | S90K10 - M  | 90                        | 10   |        |
|                         | S80K20 - M  | 80                        | 20   |        |
|                         | S70K30 - M  | 70                        | 30   |        |
|                         | S60K40 - M  | 60                        | 40   |        |
| Fine Sand               | S100K0 - F  | 100                       |      | 0      |
| Fine Sand + Kaolin      | S90K10 - F  | 90                        | 10   |        |
|                         | S80K20 - F  | 80                        | 20   |        |
|                         | S70K30 - F  | 70                        | 30   |        |
|                         | S60K40 - F  | 60                        | 40   |        |

Note: S = sand, K = kaolin, C = coarse, M = medium, F = Fine

3. Test Results
As can be seen from Figure 6, $e_{\text{max}}$ of sand matrix soil decreased with the increase of fines content but reached minimum value at different percentage of fines. Coarse sand + kaolin reached minimal value of $e_{\text{max}}$ at 30%, while medium sand + kaolin and fine sand + kaolin reached minimal value of $e_{\text{max}}$ at 20% and 10% respectively. The $e_{\text{max}}$ rapidly increased after reaching the minimum value which shows similar trends addressed by Lade et al. (1998), Monkul et al. (2016), Chubrinovski and Ishihara (2002), Yamamuro and Lade (1997), and Lade and Yamamuro (1997) and Belkhatir et al. (2011). However, for fine sand + kaolin, a different pattern from past researches of the behavior of the sand matrix soils had been noticed. For the fine sand matrix soils, the minimal $e_{\text{max}}$ was achieved at 10% fines content. $e_{\text{max}}$ was seen to increase sharply when fines content was 20% and the value remained constant thereafter.
Figure 6. Maximum void ratio-fines content relationship of sand matrix soil

For \( e_{\text{min}} \), the trend of the relationship with percentage of fines is similar with \( e_{\text{max}} \). As can be seen in Figure 7, medium sand matrix soils achieved minimal \( e_{\text{min}} \) at 30% fines content unlike for \( e_{\text{max}} \) which was 20%. \( e_{\text{min}} \) reached a minimum value approximately at 30% of fines content for both coarse sand + kaolin and medium sand + kaolin; while for fine sand + kaolin, \( e_{\text{min}} \) reached a minimum value approximately at 10% of fines content.

Figure 7. Minimum void ratio-fine content relationship of sand matrix soil

4. Conclusions
Reconstituted samples of sand matrix soils have been tested to evaluate the effects of various ranges of fines content (kaolin) on the maximum void ratio, \( e_{\text{max}} \) and minimum void ratio, \( e_{\text{min}} \) of sand matrix soils. Based on the results, the following conclusion can be drawn:

a) \( e_{\text{min}} \) and \( e_{\text{max}} \) decreased with increase of fines (kaolin) while it was 30% in sand matrix soils for coarse sands + kaolin. The same trend was observed for \( e_{\text{min}} \) and \( e_{\text{max}} \) for medium sand + kaolin and fine sand + kaolin but at different percentage of fines ranges of 10% to 30%. Fine sand + kaolin shows a small percentage of fines, ie. 10% when it reached minimal value of \( e_{\text{min}} \) and \( e_{\text{max}} \).

b) The effect of fines content in sand matrix soils on \( e_{\text{max}} \) and \( e_{\text{min}} \), observed in coarse sand + kaolin and medium sand + kaolin, is essentially related to the role of fines. From the results, it shows the transition of the effect of fines content in sand matrix soils from sand (0% fines content) dominated by sand particle to sand dominated by 100% fines content.
c) Fines sand + kaolin shows the fines (kaolin) have already filled up the fine sand void at the small amount of fines content, and resulted in constant value of $\varepsilon_{\text{max}}$ and $\varepsilon_{\text{min}}$ after reaching a minimal value of $\varepsilon_{\text{max}}$ and $\varepsilon_{\text{min}}$.

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