Quantum state transfer through a qubit network with energy shifts and fluctuations

Andrea Casaccino  
Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, MA, 02139, USA  
Information Engineering Department, University of Siena, I-53100 Siena, Italy

Seth Lloyd  
Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, MA, 02139, USA

Stefano Mancini  
Department of Physics, University of Camerino, I-62032 Camerino, Italy

Simone Severini  
Institute for Quantum Computing and Department of Combinatorics & Optimization,  
University of Waterloo, Waterloo N2L 3G1, ON Canada

We study quantum state transfer through a qubit network modeled by spins with XY interaction, when relying on a single excitation. We show that it is possible to achieve perfect transfer by shifting (adding) energy to specific vertices. This technique appears to be a potentially powerful tool to change, and in some cases improve, transfer capabilities of quantum networks. Analytical results are presented for all-to-all networks and all-to-all networks with a missing link. Moreover, we evaluate the effect of random fluctuations on the transmission fidelity.

I. INTRODUCTION

The rapid growth of the area of quantum information has led to consider the idea of multi users quantum networks with the final goal of realizing a number of nano-scale devices and communication protocols. The study of networks of interacting qubits (spins) constitutes a good testing ground for this purpose. In the last few years, this kind of networks have been specifically considered to be good candidates for engineering perfect quantum channels and allowing information transfer between distant locations (see also [4], for a review). Such networks appear to be useful for the implementation of data buses in quantum mechanical devices, in particular because they undergo a free dynamics after an initial set-up.

In this perspective, the possibility of having perfect state transfer (for short, PST) comes from suitable quantum interference effects in the network dynamics. However, one of the problems arising in such a scenario is given by natural dispersion effects and destructive interference, which determine a loss of information between communicating sites. In the worst cases, information can even remain totally localized, due to Anderson localization effects. While this situation may still be useful, this is not the case when designing protocols for distant communication.

In a number of recent papers, PST has been related to the combinatorial properties of networks (see, e.g., [2], and the references contained therein). In particular, in the XY model (respectively, the XYZ model), when considering a single excitation, it has been shown that PST essentially depends on the eigensystem of the adjacency matrix of the graph (respectively, the Laplacian matrix), because certain invariant eigenspaces of the total Hilbert space evolve independently.

Here we discuss the problem of how to improve the fidelity of excitation transfer for a fixed interaction (XY model) and network. In particular, we show that, by a suitable energy shift corresponding to some vertices in the network, it is possible to achieve perfect transfer in cases where this does not usually happen. We conjecture that this is possible in many networks, whenever we add a suitable amount of energy. Moreover, we evaluate the effect of random fluctuations on the transmission fidelity. We separately consider noise affecting qubits’ frequencies and qubits’ couplings and we show signatures of Anderson localization as well as of stochastic resonance.

The structure of the paper is as follows. In Section II, we describe the model considered here. In Section III, we give rigorous results for all-to-all networks and all-to-all networks with a missing link, therefore extending the cases studied in [2]. For these networks, we show that a certain energy shift allows PST. Indeed, it is well-known that there is no PST for an all-to-all network without energy shift. For the case of an all-to-all network with a missing link, the energy shift changes the periodicity of the evolution. In Section IV, we discuss how to enhance the transfer fidelity for a linear spin chain. It is known that a spin chain with constant couplings allows PST between its end-vertices only when it has length two or three. Evidence given by numerics show that PST can be achieved in chains of any length by an appropriate energy shift independent of the number of nodes. The drawback is a rapid increase of the transfer time. Furthermore, the number of geodesics between the input and output vertex seems to play a role in determining the transfer time. Finally, in Section V, we show how noise affects the transfer. In particular, we show that disordered couplings are more deleterious than disordered frequencies when
optimal energy shift is used. In the absence of such a shift, the noise may enhance the transmission fidelity. Conclusions are drawn in Section VI, where we briefly summarize the results and outline potential applications.

II. SET-UP

Let \( G = (V, E) \) be a simple undirected graph (that is, without loops or parallel edges), with set of vertices \( V(G) \) (such that \( |V(G)| = n \)) and set of edges \( E(G) \). The adjacency matrix of \( G \) is denoted by \( A(G) \) and defined by \( [A(G)]_{ij} = 1 \), if \( ij \in E(G) \); \( [A(G)]_{ij} = 0 \) if \( ij \notin E(G) \). The adjacency matrix is a useful tool to describe a network of \( n \) spin-1/2 quantum particles. If one considers the \( XY \) interaction model then \( \{i,j\} \in E(G) \) means that the particles \( i \) and \( j \) interact by the Hamiltonian \( H_{XY}(G)_{ij} = (X_iX_j + Y_iY_j) \). Throughout the paper, \( X_i \) and \( Y_i \) denote the usual Pauli operators of the \( i \)-th particle.

Here we consider unit coupling constant. Thus, the Hamiltonian of the whole network reads

\[
H_{XY}(G) = \frac{1}{2} \sum_{i \neq j=1}^{n} [A(G)]_{ij} (X_iX_j + Y_iY_j) \quad (1)
\]

and it acts on the Hilbert space \( (C^2)^{\otimes n} \). Let us now restrict our attention to the single excitation subspace \( C^n \), i.e., the subspace of dimension \( n \) spanned by the vectors \( \{|1\rangle, \ldots, |n\rangle\} \). A vector \( |j\rangle \) indicates the presence of the excitation on the \( j \)-th site and the absence on all the others. This is equivalent to the following tensor product of the \( Z \)-eigenstates \( \{0 \ldots 010 \ldots 0\} \), being 1 in the \( j \)-th position. In the basis \( \{|1\rangle, \ldots, |n\rangle\} \), the Hamiltonian coming from Eq. (1) has entries \( [H_{XY}(G)]_{ij} = 2 [A(G)]_{ij} \). This will be called the \( XY \) adjacency matrix of the graph \( G \). Hereafter, we shall consider the possibility of adding an amount \( \Delta E \) of free energy to desired sites. In this case, the \( XY \) Hamiltonian reads

\[
[H_{XY}(G, E_t)]_{ij} = \begin{cases} 
\Delta_E(i), & \text{if } i = j; \\
2, & \text{if } i, j \in E(G); \\
0, & \text{otherwise.} 
\end{cases} \quad (2)
\]

We simply write \( \Delta_E \) instead of \( \Delta_E(i) \) when \( i \) is clear from the context. Finally, let us recall the definition of the \emph{fidelity} at time \( t \) between vertex \( i \) and vertex \( j \) as \( f_G(i, j; t) := |\langle i | e^{-iH_G t} | j \rangle |^2 \), where \( i \) represents the input vertex and \( j \) the output vertex (in short I/O).

III. FIDELITY

In this section, we present rigorous results about the effects of an energy shift only in the input/output vertices for two specific networks: we consider the case of the \emph{complete graph}, \( K_n \), and of the \emph{complete graph with a missing link}, \( K_n^- \). In these two cases, given the Hamiltonian \( H_{XY} \), we express analytically the fidelity and the transfer time as a function of \( n \) and \( \Delta_E \).

A. Complete graph

Every two vertices of the complete graph \( K_n \) are adjacent. For this graph, we can prove the next result:

\textbf{Theorem 1} Let \( \alpha = \sqrt{4n^2 - 4(n - 4)\Delta_E + \Delta_E^2} \) with \( n \geq 4 \) and \( k \in \mathbb{N} \). For an energy shift \( \Delta_E(i, j) \) on the vertices \( i, j \in I/O \), we have the following observations:

- \( \max f_{K_n}(i, i; t) = \max f_{K_n}(j, j; t) = 1 \), for \( \Delta_E(i, j) = 2n \) and \( t = 2k\pi/\alpha \);
- \( \max f_{K_n}(k, k; t) = 1 \), for every \( k \notin I/O \) and \( t = 4k\pi/\alpha \).

When \( i \neq j \),

- \( \max f_{K_n}(i, j; t) = 1 \), for \( \Delta_E(i, j) = 2n \) and \( t = (2\pi + 4\pi k)/\alpha \);
- \( \max f_{K_n}(k, l; t) = [(\alpha(n - 2) - 2)^2 / 4\alpha^2(n - 2)^2] \)
  for \( k, l \notin I/O \) and \( t = 2k\pi/\alpha \).

\textbf{Proof.} The \( XY \) adjacency matrix of \( K_n \) has the form

\[
[H_{XY}(K_n)]_{ij} = \begin{cases} 
\Delta_E, & \text{if } i = j \in I/O; \\
0, & \text{if } i = j \notin I/O; \\
2, & \text{otherwise.}
\end{cases} \]

The characteristic polynomial \( P(\lambda) \) can be obtained as a function of \( n \) and \( \Delta_E \):

\[
P(\lambda) = (\lambda + 2)^{n-3}(\Delta_E - 2 - \lambda) \times (4(n - 1) - 2(n - 3)\Delta_E + 2(n - 2)\lambda + \Delta_E \lambda - \lambda^2).
\]

The roots of \( P(\lambda) \) are as follows: \( \lambda_1 = \Delta_E - 2, \lambda_2^{n-3} = -2, \lambda_3^{n-4} = (2(n - 2) + \Delta_E \pm \sqrt{\Delta_E^2 - \Delta_E}) \). A corresponding (unnormalized) orthogonal basis of eigenvectors can be written as

\[
|\lambda_1\rangle = (-1, 0, \ldots, 0, 1)
\]

and

\[
|\lambda_2^{\leq l \leq n-3}\rangle_u = \begin{cases} 
-\frac{1}{2} u, & \text{if } u \in \{2, n - r : 1 \leq r \leq l - 1\}; \\
1, & \text{if } u = n - l; \\
0, & \text{otherwise},
\end{cases}
\]

\[
|\lambda_3^{\pm \leq k \leq \pm 1}\rangle_u = (1, \omega^\pm, \ldots, \omega^\pm, 1),
\]

where \( \omega = \frac{1}{4(n - 2)}(2(n - 4) - \Delta_E \pm \alpha) \). Thus, from the spectral decomposition of the unitary matrix in the canonical basis, \( U(t)(K_n) = e^{-iH_{(K_n)} t} \), we have the following diagonal entries:
• if \( i \in I/O \) then
\[
[U_t(K_n)]_{ii} = \frac{1}{4\alpha} (\alpha - 2n + \Delta_E + 8) e^{-i\lambda_3t} + \frac{1}{4\alpha} (\alpha - 2n - \Delta_E + 8) e^{-i\lambda_3t} + \frac{1}{2} e^{-i\lambda_3t};
\]
• if \( i \notin I/O \) then
\[
[U_t(K_n)]_{ii} = \frac{1}{n-2} (n-3) e^{-i\lambda_3t} + \frac{1}{2(n-4)\alpha} (\alpha - 2n + \Delta_E + 8) e^{-i\lambda_3t} + \frac{1}{2(n-4)\alpha} (\alpha - 2n - \Delta_E - 8) e^{-i\lambda_3t}.
\]

The off-diagonal entries of \( U_t(K_n) \) are as follows:
• if \( i \neq j \) and \( i, j \in I/O \) then
\[
[U_t(K_n)]_{ij} = \frac{\Delta_E - 2(n-4) + \alpha}{4\alpha} e^{-i\lambda_3t} + \frac{2(n-4) - \Delta_E + \alpha}{4\alpha} e^{-i\lambda_3t} - \frac{1}{(n-2)\alpha} e^{-i\lambda_3t};
\]
• if \( i \neq j \) and \( i \in I/O \) and \( j \notin I/O \) or \( j \notin I/O \) and \( i \notin I/O \), then
\[
[U_t(K_n)]_{ij} = 2(e^{-i\lambda_3t} - e^{-i\lambda_4t})/\alpha.
\]

This gives us the tools to evaluate the fidelity for generic situations. For instance, if we take \( i, j \in I/O \), the fidelity \( f(i, j; t) = |\langle j | U_t(K_n) | i \rangle|^2 \) reads
\[
f(i, j; t) = \frac{\Delta_E^2 + 3\alpha^2 - 4\Delta_E(n-4) + 4(n-4)^2}{8\alpha^2} + \frac{(8 + \Delta_E + \alpha - 2n)(\alpha + 2n - 8 - \Delta_E)}{8\alpha^2} \cos(\alpha t) - \frac{\Delta_E - 2(n-4) + \alpha}{4\alpha} \cos\left(\frac{2n - \Delta_E + \alpha}{2}\right) + \frac{\Delta_E - 2(n-4) - \alpha}{4\alpha} \cos\left(\frac{2n - \Delta_E - \alpha}{2}\right).
\]

Imposing \( \Delta_E = 2n \), PST is achieved for \( t = \frac{1}{\alpha}(2\pi + 4\pi k) \) with \( k \in \mathbb{N} \).

The main results are visualized in Fig. 1 where the fidelity \( f(i, j; t) \) between any two vertices \( i \) and \( j \) of a complete graph is plotted as a function of time \( t \). It is well known that, in this case, there is no PST without energy shift as it is shown by the dashed line. On the contrary, optimal energy shift \( \Delta_E = 2n \) allows PST (fidelity equal to one) at times \( t = (2\pi + 4\pi k)/\alpha \) with \( k \in \mathbb{N} \) as shown by the solid line.

B. Complete graph with a missing link

The graph \( K^{-}_n \) is obtained from \( K_n \) by deleting an edge, specifically the one between the input and the output vertex. The next result describes the behavior of the system in this case:

**Theorem 2** Let \( \beta = \sqrt{4(n^2 + 2n - 7) - 4(n-3)\Delta_E + \Delta^2} \) with \( n \geq 4 \) and \( k \in \mathbb{N} \). For an energy shift \( \Delta_E(i, j) \) on the vertices \( i, j \in I/O \), we have the following observations:
- \( \max f_{K_n^{-}}(i, j; t) = \max f_{K_n}(i, j; t) = 1 \), for \( \Delta_E(i, j) = 2n - 6 \) and \( t = 2k\pi/\beta \);
- \( \max f_{K_n^{-}}(k, k; t) = 1 \), for every \( k \notin I/O \) and \( t = 4k\pi/\beta \).

When \( i \neq j \),
- \( \max f_{K_n^{-}}(i, j; t) = 1 \), for \( \Delta_E(i, j) = 2n - 6 \) and \( t = (2\pi + 4\pi k)/\alpha \);
- \( \max f_{K_n^{-}}(i, k; t) = 16/\beta^2 \), for \( \Delta_E(i) = 2n - 6 \), \( k \notin I/O \) and \( t = (2\pi + 4\pi k)/\beta \);
- \( \max f_{K_n^{-}}(k, l; t) = [(\beta(n-2) - 2)^2/4\beta^2(n-2)^2] \), for \( k, l \notin I/O \) and \( t = 2k\pi/\beta \).

**Proof.** The proof is very similar to the one of Theorem 1. The characteristic polynomial \( P(\lambda) \) of the \( XY \) adjacency matrix of \( K^{-}_n \) can be obtained as function of \( n \) and \( \Delta_E \):
\[
P(\lambda) = (\lambda + 2)^{n-3}(\Delta_E - 2 - \lambda) \times \left(4(n-1) - 2(n-3)\Delta_E + 2(n-2)\lambda + \Delta_E\lambda - \lambda^2 \right).
\]
The roots of $P(\lambda)$ are as follows: $\lambda_1 = \Delta_E$, $\lambda_2^{\pm3} = -2$, $\lambda_3^{\pm4} = (2(n - 3) + \Delta_E \pm \beta)/2$. A corresponding (unnormalized) orthogonal basis of eigenvectors can be written as

$$|\lambda_1\rangle = (-1, 0, \ldots, 0, 1),$$

$$[|\lambda_2^{\pm1}\rangle]_u = \begin{cases} -\frac{1}{2}, & \text{if } u \in \{2, n - r : 1 \leq r \leq l - 1\}; \\ 1, & \text{if } u = n - l; \\ 0, & \text{otherwise,} \end{cases}$$

$$[|\lambda_3^{\pm}\rangle] = (1, \omega^{\pm}, \ldots, \omega^{\pm}, 1),$$

where $\omega^{\pm} = \frac{1}{\sqrt{4(n-3)}}(2(n-3) - \Delta_E \pm \beta)$. The diagonal entries of $U_t(K_n) \equiv e^{-iH(K_n)t}$ are given in terms of its spectral decomposition:

- if $i \in I/O$ then
  $$[U_t(K_n)]_{ii} = \frac{1}{4\beta} (\beta - 2n + \Delta_E + 6) e^{-i|\lambda_3|t} + \frac{1}{4\beta} (\beta - 2n - \Delta_E + 6) e^{-i|\lambda_4|t} + \frac{1}{2} e^{-i|\lambda_1|t};$$

- if $i \notin I/O$ then
  $$[U_t(K_n)]_{ii} = \frac{1}{n - 2} (n - 3) e^{-i|\lambda_1|t} + \frac{1}{2n\beta - 4\beta} (\beta - 2n + \Delta_E + 6) e^{-i|\lambda_3|t} + \frac{1}{2n\beta - 4\beta} (\beta + 2n - \Delta_E - 6) e^{-i|\lambda_4|t}.$$  

The off-diagonal entries of $U_t(K_n)$ are as follows:

- if $i \neq j$ and $i, j \in I/O$ then
  $$[U_t(K_n)]_{ij} = \frac{\Delta_E - 2(n - 3) + \beta}{4\beta} e^{-i|\lambda_3|t} + \frac{2(n - 3) - \Delta_E + \alpha}{4\beta} e^{-i|\lambda_4|t} - \frac{1}{2} e^{-i|\lambda_1|t};$$

- if $i \neq j$, $i \in I/O$ and $j \notin I/O$ or viz., then
  $$[U_t(K_n)]_{ij} = 2(\beta - 2n + \Delta_E + 6) e^{-i|\lambda_3|t} - e^{-i|\lambda_4|t}/\beta;$$

- if $i \neq j$ and $i, j \notin I/O$ then
  $$[U_t(K_n)]_{ij} = \frac{\Delta_E - 2(n - 3) + \beta}{2(n - 2)\beta} e^{-i|\lambda_3|t} + \frac{2(n - 3) - \Delta_E + \alpha}{2(n - 2)\beta} e^{-i|\lambda_4|t} - \frac{1}{(n - 2)\beta} e^{-i|\lambda_1|t}.$$

If we take $i, j \in I/O$, the fidelity $f(i, j; t) = |\langle j|U_t(K_n)|i\rangle|^2$ reads

$$f(i, j; t) = \frac{\Delta_E^2 + 3\beta^2 - 4\Delta_E(n - 3) + 4(n - 3)^2}{8\beta^2} + \frac{(6 + \Delta_E + \beta - 2n)(\beta + 2n - 6 - \Delta_E)\cos(\beta t)}{4\beta}$$

$$+ \frac{6 - 2n + \Delta_E - \beta}{4\beta} \cos \left[ t \frac{2n - 6 - \Delta_E + \beta}{2} \right] + \frac{6 - 2n + \Delta_E - \beta}{4\beta} \cos \left[ t \frac{2n - 6 - \Delta_E - \beta}{2} \right].$$

If we assume that $\Delta_E = 2n - 6$ then PST is achieved for $t = \frac{1}{\beta} (2\pi + 4\pi k)$ with $k \in \mathbb{N}$. ■

Notice that for $K_n^-$ we have $\Delta_E = 2n - 6$, while for $K_n$ we have $\Delta_E = 2n$. This fact alone does not provide enough information to conjecture that the energy shift required for PST in a graph with $m$ edges is proportional to $m$. Indeed, the energy shift appears to be a nonlinear function of the eigensystem of the matrix $H_{XY}(G)$. Also, notice that the matrices $H_{XY}(K_n^-)$ and $H_{XY}(K_n^-)$ satisfy the relation $(H_{XY}(K_n^-) : H_{XY}(K_n^-))^T = H_{XY}(K_n^-) \cdot H_{XY}(K_n^-)$.

The main results are visualized in Fig. 2. Here, the fidelity $f(i, j; t)$ between the two nonadjacent vertices $i$ and $j$ of a complete graph with a missing link with $n = 5$ as a function of time $t$ in two settings: in the absence of energy shift (dashed line) and in the presence of optimal energy shift (solid line).

![FIG. 2: Fidelity $f(i, j; t)$ between the two nonadjacent vertices $i$ and $j$ of a complete graph with a missing link with $n = 5$ as a function of time $t$ in two settings: in the absence of energy shift (dashed line) and in the presence of optimal energy shift (solid line).](image-url)
when the fidelity reaches its maximum. However, the use of optimal energy shift ($\Delta E = 2n - 6$) allows PST at shorter times $t = \frac{1}{2} (2\pi + 4\pi k)$ with $k \in \mathbb{N}$ as shown by the solid line, thus facilitating the information transfer.

### IV. SPIN CHAINS

A special case of a network is represented by a linear spin chain, where the vertices at the extremities are considered as input and output. By adding an appropriate free energy shift $\Delta E$ (independent of the number of nodes $n$) to these vertices, numerical results involving relatively large chains point out that we can always achieve PST. This is remarkable because usual spin chains with more than three vertices do not allow PST. As a counterpart of this fact, the transfer time generally grows rapidly with the number of vertices and with the amount of energy $\Delta E$. However, in some special cases like the three vertices chain, where PST is achievable without adding energy, the energy shift only causes a larger transfer time. Table I shows accordingly some numerical results for chains of small length and energy shift on the end-vertices. Apart from $n = 2$, for the sake of clarity, we take the closest integer to the real values obtained.

It is plausible that the transfer time in a generic network decreases as the number of paths between the input and the output vertex increases. The minimum transfer time is clearly achieved when the two vertices are adjacent. Thus, for a fixed amount of energy $\Delta E$, numerical results show that the transfer time for the maximum fidelity, $t_{ij}(G)$, between vertices $i$ and $j$ of a network $G$ on $n$ vertices, is

$$t_{ij}(G) \approx O \left( \frac{t_{\Delta E, k}}{p_{\min}(i, j)} \right).$$

Here $t_{\Delta E, k}$ is the transfer time of the spin chain with $n$ vertices and $p_{\min}(i, j)$ is the number of different geodesics between $i$ and $j$. Table II shows the transfer time required to obtain a fidelity close to one, when we consider antipodal vertices in graphs of a family constructed as follows: only two vertices, which are then said to be antipodal, have degree 1; all other vertices have degree 2 and belong to paths connecting the antipodal vertices. Such paths are disjoint and have only the antipodal vertices in common. The number of vertices in a graph with $l$ paths of length $n$ is $n + (n - 2)l$, for $n \geq 3$. Table II gives evidence that we can gradually cut the transfer time by increasing the number of paths. Intuitively, an equivalent result should be also obtained by modifying the couplings in the original chain.

### V. FLUCTUATIONS

In this section, we analyze the problem of transferring an energy excitation in the presence of noise. We keep working with $K_n$ and $K_n^2$. In practice, we consider a gaussian stochastic process $\xi_{ij}$ of zero mean and $\sigma^2$ variance, affecting the energy of the particles (qubits’ frequencies) or the interaction energies (qubits’ couplings). Under this assumption, the Hamiltonian entries become

$$[H_{XY}(G, \xi)]_{ij} = \begin{cases} 
\Delta E + \xi_{ii}, & \text{if } i = j \in I/O; \\
2 + \xi_{ij}, & \text{if } ij \in E(G); \\
0 + \xi_{ij}, & \text{otherwise.}
\end{cases}$$

We then distinguish two cases: noise affecting the vertices and noise affecting the edges. Formally,

1. $\xi_{ii} \neq 0$, forevery $i \in V(G)$ and $\xi_{ij} = 0$, when $i \neq j$;
2. $\xi_{ij} \neq 0$, foreveryriand$ij \in E(G)$ and $\xi_{ii} = 0$.

We are interested in evaluating the average fidelity as a function of the variance of the independent gaussian random variables. The chosen energy shift $\Delta E$ is the optimal one, according to the results of Section III.

The results are reported in Fig. III. By comparing top and bottom graphics we can see that disordered couplings are more deleterious than disordered frequencies with an optimal energy shift. This fact has been already pointed out in a different context by Gammaitoni et al. in [7]. The decay of fidelity over $\sigma^2$ comes from the fact that the noise causes localization phenomena for the excitation transfer [4]. This is more evident for $K_n$ where the “degree of disorder” is higher (compare top-left and top-right plots). Furthermore, in the absence of energy shift the noise may enhances the transmission fidelity (top-left and bottom-right plots). This is reminiscent of stochastic resonance effects [3].

| $\Delta E \setminus n$ | 2 | 3 | 4 | 5 |
|------------------------|---|---|---|---|
| 10                     | 0.7 | 5 | 19 | 99 |
| 20                     | 0.7 | 8 | 1 | 8010 |
| 30                     | 0.7 | 12 | 178 | 2665 |
| 40                     | 0.7 | 16 | 313 | 6260 |
| 50                     | 0.7 | 20 | 494 | 12294 |

**TABLE I:** Numerical results of the transfer time for chains of small length and varying energy shift on the end-vertices.

| $l \setminus n$ | 2 | 3 | 4 | 5 |
|----------------|---|---|---|---|
| 1             | 5 | 19 | 99 |
| 2             | 3 | 11 | 62 |
| 3             | 2 | 9 | 42 |
| 4             | 1 | 6 | 36 |

**TABLE II:** Numerical results for the decrease of the transfer time with a fixed energy shift on the end-vertices and an increasing number of paths.
FIG. 3: On the left, average fidelity between any two vertices of a complete graph as a function of the variance \( \sigma^2 \) (at optimal time). At the bottom (resp. top) is represented the case 1. (resp. 2.). The solid line refers to the presence of optimal energy shift at input/output vertices while the dashed line refers to the absence of such shift. On the right, average fidelity between the two nonadjacent vertices of a complete graph with a missing link as a function of the variance \( \sigma^2 \) (at optimal time). At the bottom (resp. top) is represented the case 1. (resp. 2.). The solid line refers to the presence of optimal energy shift at input/output vertices while the dashed line refers to the absence of such a shift.

VI. CONCLUSIONS

We have shown how to enhance the fidelity of excitation transfer in a quantum spin network with a fixed interaction (XY model) and network. It turns out that it is possible to achieve perfect transfer with the use of suitable energy shifts in all-to-all networks and in all-to-all networks with a missing link. We conjecture that this is possible in any network. This technique is promising for future applications as recent works with superconducting qubits suggest ([10] and references therein). Finally, we have shown how different kinds of noise affect the transfer fidelity. We believe that our results could open up new perspectives for communication or information processing in quantum networks.

Acknowledgments. The authors would like to thank Stefano Pirandola, Masoud Mohseni and Yasser Omar for useful discussions. The work of S. M. is supported by the European Commission, under the FET-Open grant agreement HIP, number FP7-ICT-221889. Research at IQC is supported in part by DTOARO, ORDCF, CFI, CIFAR, and MITACS.

[1] Anderson P. W., Phys. Rev. 109, 1492 (1958).
[2] Bernasconi A., Godsil C., Severini S., Phys. Rev. A 78, 052320 (2008). arXiv:0808.0510v1 [quant-ph].
[3] Bose S., Phys. Rev. Lett. 91, 207901 (2003). arXiv:quant-ph/0212041v2.
[4] Bose S., Contemporary Physics, Vol. 48 (1), pp. 13-30, 2007. arXiv:0802.1224v1 [cond-mat.other].
[5] Bose S., Casaccino A., Mancini S., Severini S., Communication in XYZ All-to-All Quantum Networks with a Missing Link, Int. J. Quantum Inf., 7, no 4, 2009. arXiv:0808.0748v2 [quant-ph].
[6] Christandl M., Datta N., Ekert A. and Landahl A.J., Phys. Rev. Lett. 92, 187902 (2004). arXiv:quant-ph/0309131v2.
[7] De Chiara G., Rossini D., Montangero S. and Fazio S., Phys. Rev. A 72, 012323 (2005). arXiv:quant-ph/0502148v2.
[8] Gammaitoni L., Hanggi P., Jung P. and Marchesoni F., Rev. Mod. Phys. 70, 223 (1998).
[9] Kimble H.J., Nature 453, 1023 (2008). arXiv:0806.4195v1 [quant-ph].
[10] Strauch F. W. and Williams C. J., Phys. Rev. B 78, 094516 (2008). arXiv:0708.0577v3 [quant-ph].
[11] Subrahmanyam V., Phys. Rev. A 69, 034304 (2004). arXiv:quant-ph/0307135v2.