Resonator-Assisted Quantum Bath Engineering of a Flux Qubit

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We demonstrate quantum bath engineering for preparation of any orbital state with controllable phase factor of a superconducting flux qubit assisted by a microwave coplanar waveguide resonator. We have investigated the polarization efficiency of the arbitrary direction of the pseudo angular momentum space, and obtained an effective Rabi frequency by using the convergence condition of Markovian master equation. The processes of polarization can be implemented effectively in a dissipative environment created by resonator photon loss when the spectrum of the microwave resonator matches with the specially tailored Rabi and resonant frequencies of the drive. Our calculations indicate that state-preparation fidelities in excess of 99.9% and the required time on the order of magnitude of microsecond are in principle possible for experimentally reasonable sample parameters. Furthermore, our proposal is available to generalize to any other systems with spin-based qubits.

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One of the most promising achievements from the exploration of the hybrid quantum circuits is harnessing the advantages of the different quantum systems to discovery the new qualities that are not acquirable for either independent system [1, 2]. An exemplification is photon-participated initialization of atom, spin and superconducting qubits. Manipulation of genuine quantum systems requires that they should be effectively prepared into a well-defined quantum state, which is not only important for quantum error correcting of quantum information processors [3, 4] but is also of significance for the applications in enhancing quantum memories [5, 6].

In theory, any qubit can be prepared into its minimal energy state (say, ground state) when cooling to so low temperature that thermal excitation energy is much less than the energy splitting of the qubit. Consequently, low temperature environment is bound to slow down systems to reach the thermodynamic equilibrium, which retards the operations in quantum information processors [7]. More effective cooling schemes have been studied extensively in the context of Doppler and Sisyphus cooling [8, 9], algorithmic cooling [10, 11], cavity cooling [12–14], etc. Among these, cavity cooling which dissipates the kinetic energy in open environment created by cavity photon loss in a controlled manner has been investigated including atomic gases [14, 15], mechanical objects [16, 17] and spins [13, 18]. Currently, it was demonstrated that superconducting transmon qubit may be prepared in any pure state of the Bloch sphere with high fidelity assisted by a microwave cavity [19]. However, the phase factor of the prepared state is uncontrollable.

We present in this paper a scheme for preparation of any orbital state with controllable phase factor of a superconducting flux qubit including three mesoscopic Josephson junctions arranged in a superconducting loop assisted by single-mode coplanar waveguide (CPW) resonator. In particular, we investigate the polarization efficiency of the arbitrary direction of the pseudo angular momentum space, and obtain an effective Rabi frequency which depends on its polarization direction by using the convergence condition of Markovian master equation. The processes of polarization can be implemented rapidly enough in the direction where the spectrum of resonator matches with the specially tailored Rabi and resonant frequencies of the drive, which can be used for the state preparation of a superconducting flux qubit by adjusting system parameters. Our calculations indicate that state preparation fidelities in excess of 99.9% and the required time on the order of magnitude of microsecond are in principle possible with currently achievable sample parameters, which is significantly shorter than the thermal relaxation time for low-temperature superconducting flux qubit [20, 21]. Furthermore, our scheme is available to generalize to any other kinds of spins, such as electron spin and nuclear spin.

We here consider a superconducting flux qubit com-

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{(a) Superconducting flux qubit is labeled with the new eigenstates $|0\rangle$ and $|1\rangle$ with energy splitting $\omega_{sc}$. (b) Superconducting flux qubit is coupled to CPW resonators via the induced magnetic field. The blue sinusoidal curves describe the microwave drive.}
\end{figure}

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prising three mesoscopic Josephson junctions in a loop (depicted in FIG.1(b)) threaded by an induced magnetic field [22]. As shown in FIG. 1(a), two computational basis states of the superconducting flux qubit carry opposite macroscopic persistent currents. The superconducting flux qubit can be described by the effective Hamiltonian 
\[ H_{SC} = -(B_z \bar{\sigma}_z + B_x \bar{\sigma}_x)/2, \] where \( \bar{\sigma}_{x,z} \) are the Pauli matrices, \( B_x \) is the level repulsion, and \( B_z \) is the DC energy bias, and the rewritten qubit levels \( |0\rangle \) and \( |1\rangle \) have energies \( \pm \frac{1}{2} \omega_{dc} \) respectively, where \( \omega_{dc} = \sqrt{B_x^2 + B_z^2} \). In the presence of a microwave drive, Rabi oscillations between these levels are induced near resonance. And the superconducting flux qubit couples to CPW resonator via the induced magnetic field [23, 24]. The total Hamiltonian of joint system is taken as \( H = H_0 + H_d + H_r \) with
\[ H_0 = \omega_{dc} J_z + \omega_d a^+ a, \] (1)
\[ H_d = \Omega J_+ e^{-i\omega_r t} + \Omega J_- e^{i\omega_r t} + H.c., \] (2)
\[ H_r = 2g (a + a^+) J_z, \] (3)
where \( a (a^+) \) are the annihilation (creation) operators of CPW resonator with frequency \( \omega_d \) and linewidth \( \kappa \), \( \Omega \) and \( \omega_r \) are the Rabi and the counter-rotating frequencies of the drive with frequency \( \omega_r \) and \( g \) is the light-matter coupling constant in the unit of \( \hbar = 1 \). Here we have used the notation that \( J_\pm = J_x \pm i J_y \) are the pseudo angular momentum (PAM) operators for the superconducting flux qubit (where \( \sigma_i \) are Pauli matrices corresponding to the qubit states \( |0\rangle \) and \( |1\rangle \)).

In the interaction picture of \( H_1 = H_0 - \delta \omega_a a + \delta \omega_z J_z \), the Hamiltonian of the composite system with the standard rotating wave approximation (RWA) is
\[ \hat{H}_1 = \omega_d J_z + \omega_a a J_z + \delta \omega A J_z \]
\[ + \Re(\Omega) J_x - \Im(\Omega) J_y, \] (4)
with \( \delta \omega = \omega_{dc} - \omega_r L \) and \( \delta \omega = \omega_{dc} - \omega_r L \). This RWA is enforced in the parameters regimes \( \omega_{dc} L, \omega_r L \gg \kappa, \Omega, \bar{\Omega} \).

Assume that superconducting flux qubit should be prepared in any arbitrary superposition of ground and excited states on demand: \( \sin(\theta/2) |0\rangle + e^{i\phi} \cos(\theta/2) |1\rangle \) with \( \theta \in [0, \pi] \) and \( \phi \in [0, 2\pi] \), which must be the eigenstate of PAM operator
\[ J_x = -\sin \theta \cos \phi J_z - \sin \theta \sin \phi J_y + \cos \theta J_z \] (5)
with eigenvalue \( -\frac{1}{2} \).

To investigate the polarization efficiency of the arbitrary direction of the PAM space, we introduce a unitary transformation of PAM operators \( M \):\[
\begin{bmatrix}
J_x \\
J_y \\
J_z
\end{bmatrix} =
\begin{bmatrix}
\cos \theta \cos \phi & \cos \theta \sin \phi & \sin \theta \\
-\sin \phi & \cos \phi & 0 \\
-\sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta
\end{bmatrix}
\begin{bmatrix}
J_x \\
J_y \\
J_z
\end{bmatrix}.
\] (6)
After moving into the interaction frame of \( H_2 = \bar{\Omega} J_x + \delta \omega_A a \), the Hamiltonian (4) reduces to
\[ \hat{H}_2(t) = (A_x - \bar{\Omega}) J_x + \hat{H}_\Omega(t) + \hat{H}_z(t) + \hat{H}_-(t) + \hat{H}_+(t), \] (7)
\[ \hat{H}_\Omega(t) = (A_x - i A_y) e^{i\Omega t} J_z^\dagger + (A_x + i A_y) e^{-i \Omega t} J_z, \]
\[ \hat{H}_z(t) = \Theta_z e^{-i \delta \omega t} a J_z + \Theta_{2z} e^{-i \delta \omega^2 t} a J_z, \]
\[ \hat{H}_-(t) = \Theta_+ e^{i(\delta \omega - \bar{\Omega}) t} a J_z + \Theta_{1+} e^{-i(\delta \omega + \bar{\Omega}) t} a J_z, \]
\[ \hat{H}_+(t) = \Theta_- e^{i(\delta \omega + \bar{\Omega}) t} a J_z + \Theta_{1-} e^{-i(\delta \omega - \bar{\Omega}) t} a J_z, \]
with
\[ [2A_x, 2A_y, A_z]^T = M[\Re(\Omega), -\Im(\Omega), \delta \varpi]^T, \]
\[ [\Theta_x, \Theta_y, \Theta_z]^T = M[1, -i, 0]^T, \Theta_\pm = \frac{g}{2}(\Theta_x \pm i \Theta_y), \]
where \( \bar{\Omega} \) is the effective Rabi frequency that can be obtained by using the convergence condition of Markovian master equation, and \( J_z^\dagger = J_x \pm i J_y \) are the ladder operators in the \( z \)-basis.

There is no preference in the \( J_z \) direction for the dynamics of \( \hat{H}_2(t) \) and \( \hat{H}_\Omega(t) \) at the thermal equilibrium, while those of \( \hat{H}_{\pm}(t) \) would like to drive the superconducting flux qubit to \( \langle J_z \rangle = \pm \frac{1}{2} \frac{1}{T} \), respectively [13]. We may set \( \Delta = \delta \omega - \bar{\Omega} \) to be close to zero, so that the absolute value of \( \Delta \) is small compared to those of \( \delta \omega, \bar{\Omega}. \)

After making the second RWA in the interaction frame of \( H_2 \), the interaction Hamiltonian reduces to
\[ H_I(t) = (A_x - \bar{\Omega}) J_x + \Theta_+ e^{i \delta \omega_t} a J_z + \Theta_{1+} e^{-i \delta \omega_t} a J_z. \] (8)

The RWA used here is ensured when the absolute values of \( \delta \omega \) and \( \Omega \) are large compared to the time scale of interest \( (|\delta \omega|, |\Omega|) \gg \kappa, |A_x \pm i A_y|, |\Theta_x|, |\Theta_z| \).

To obtain the Markovian master equation for the driven flux qubit, we assume the bad resonator condition \( \kappa \gg g \). The reduced dynamics of the superconducting flux qubit in the interaction frame of the dissipator is given to the second order by the time-convolutionless (TCL) master equation [13]:
\[ \dot{\rho}(t) = \int_0^t dt \tau_c \left[ e^{iD_c^+} \{ L[H_I(t)] \} L[H_I(t - \tau)] \rho(t) \otimes \rho_{eq} \right], \] (9)
where \( L \) is the superoperator \( L[X] \rho(t) = -i[X, \rho(t)] \), \( \rho(t) = \tau_c \rho(t) \) \( \rho(t) \) is the reduced state of the superconducting flux qubit and \( \rho_{eq} \) is the equilibrium state of the resonator.

Using the algebraic transformation of the dissipator \( D_c [13] \): \[ e^{iD_c^+ [a]} = \mathbf{L} e^{iD_c^+ [a]} = e^{-i \kappa t^2 / 2} e^{i \bar{\Omega} t} [a^+], \] the master equation (9) reduces to
\[ \dot{\rho}(t) = \int_0^\infty \{ e^{-i \kappa t^2 / 2} \tau_c \{ L[H-(t)] L[H-(t - \tau)] \} \rho(t) \otimes \rho_{eq} \} \]
\[ + L[(A_x - \bar{\Omega}) J_x] L[(A_x - \bar{\Omega}) J_x] \rho(t) \} dt, \] (10)
where the cross terms for the 2nd order TCL master equation have been removed with the properties of our resonator equilibrium state: \( \tau_c [a \rho_{eq}] \tau_c [a^+ \rho_{eq}] = 0 \). We find the last term of the master equation (10) will not be convergent unless the constant of component Hamiltonian, \( A_x - \bar{\Omega} \), becomes zero. Therefore, we obtain the
The controllable phase factor of the prepared state originates from the dissimilarity of \( J_x \) and \( J_y \) components of PAM operator \( J_x \), while the state populations lie on \( J_z \) component. Considering the RWA condition \(|\Omega| \gg |A_x \pm i A_y|\), the phase and populations of the equilibrium state ultimately rely on the phase of Rabi frequency and the rate of \( \delta \omega/|\Omega| \), respectively. For the case \( \Delta J = -\sin \theta \cos \phi J_x - \sin \theta \sin \phi J_y + \cos \theta J_z \) with the eigenstate \( |\Omega,0\rangle + e^{i \phi} |\Omega,1\rangle \) that has the eigenvalue \((-\frac{1}{2})\), the corresponding parameters relationships are

\[
-\frac{\sin \theta \cos \phi}{\text{Re}(\Omega)} \approx \frac{\sin \theta \sin \phi}{\text{Im}(\Omega)} \approx \frac{\cos \theta}{\delta \omega},
\]

(12)

Therefore, the preparation of arbitrarily specified coherent superposition of the ground and excited states of a superconducting flux qubit can be implemented by adjusting system parameters \(|\text{Re}(\Omega)|, |\text{Im}(\Omega)|, |\delta \omega|\).

The most efficient cooling to the state \( \langle J_x \rangle = -\frac{1}{2} \) happens when the effective Rabi drive strength is matched to the spectrum of resonator \( \delta \omega = \Omega \), where the net cooling rate becomes

\[
\Gamma_x = \frac{4 |\Theta|^2}{\kappa^2 + 4 \Delta^2},
\]

(13)

and the master equation (10) reduces to a rate equation for the state populations:

\[
\frac{d}{dt} \tilde{P}_J(t) = \Gamma_x M_J \tilde{P}_J(t),
\]

(14)

with

\[
M_J = \begin{bmatrix}
-\bar{n} & \bar{n} + 1 \\
\bar{n} & -\bar{n} + 1
\end{bmatrix}.
\]

(15)

Here we finally consider the diagonal matrix elements \( P_{J, m}(t) = \langle J, m | g(t) | J, m \rangle \) of the reduced density operator \( g(t) \), which corresponds to the expectation value of the projection operator \( |J, m\rangle \langle J, m| \) at the arbitrary time \( t \), and define \( \tilde{P}_J(t) = (P_{J,-J}(t), P_{J,J}(t))^T \).

At the thermodynamic equilibrium, the state of the driven superconducting flux qubit satisfies \( \partial_t \tilde{P}_J(\infty) = 0 \) and can be given by \( \rho_{J,eq} = \sum_{m=-J}^{J} P_{J, m}(\infty) |J, m\rangle \langle J, m| \), where

\[
P_{J,-J}(\infty) = \frac{1}{e^{-\omega_J/\hbar T_c} + 1},
\]

(16)

\[
P_{J,J}(\infty) = \frac{e^{-\omega_J/\hbar T_c}}{e^{-\omega_J/\hbar T_c} + 1}.
\]

(17)

The expectation value of PAM operator \( J_z \) for the equilibrium state is

\[
\langle J_z \rangle_{eq} = \frac{1}{2} e^{-\omega_z/\hbar T_c} - \frac{1}{2} e^{-\omega_z/\hbar T_c} + 1
\]

(18)

In the ideal case where the resonator is cooled to its ground state \( (T_c \to 0) \), the probability of qubit being state \( \langle J_z \rangle = -\frac{1}{2} \) at equilibrium is given by \( P_{J,-J} \approx 1 \) and the final expectation value of PAM is approximately \( \langle J_z \rangle_{eq} \approx -1/2 \).

Assume that the superconducting flux qubit is taken to be maximally mixed in the basis of the PAM space \( |P_{J,m}(0) = 1/2, m = -J, J \rangle \). The simulated expectation value of \( \langle J_z(t) \rangle \) for the temperature of bath ranging from \( \bar{n} = 0 \) to \( \bar{n} = 0.5 \) is shown in FIG. 2(a), normalized by \(-J\) to obtain a maximum value of 1. When the processes of polarization are carried out at \( T_c = 100 \text{ mK} \), the corresponding expectation value of the number operator at equilibrium approximates null \((\bar{n} \approx 0)\) for \( \omega_z/2\pi = 6 \text{ GHz} \). Obviously, there is almost no effect of thermal relaxation being observed.

In analogy to the ensemble spin system, the expectation value \( \langle J_x(t) \rangle \) for the ideal case may be fitted to an exponential to derive an effective polarization time constant, \( T_x \) [13]

\[
-\langle J_x(t) \rangle = 1 - \exp \left(-\frac{t}{T_x}\right)
\]

(19)

with

\[
T_x \approx \frac{1}{\Gamma_x} = \frac{\kappa^2 + 4 \Delta^2}{g^2 \kappa (1 + \cos \theta)^2},
\]

(20)

showing that the most efficient polarization happens when the polarization is in \( J_z \) direction \((\cos \theta = 1)\). For the case \( \cos \theta < 0 \), we may change the matching to \( \delta \omega + \Omega = 0 \), so that polarization time is always less than \( \kappa/g^2 \). Effective dissipation rate in the units of \( g^2/\kappa = 1 \) versus the dimensionless parameters \( \Delta/\kappa \) and \( \theta \) is shown in FIG. 2(b). Apparently, the effective dissipation rate increase rapidly, when the Stokes photons are on resonance with the resonator.

In our paper, reasonable sample parameters require that they should be chosen to validate the Markov approximation \((\kappa \gg g)\), and adhere to the two RWAs, the first one made to remove the time-dependent terms of
interaction Hamiltonian \((\omega_c, \omega_L, \omega_{sc} \gg g, \kappa, \Omega, \Omega)\), the second used to isolate the exchange term of superconducting flux qubit and resonator of Eq. (8) \((|\delta\omega|, |\Omega| \gg \kappa, |A_x \pm iA_y|, |\Theta_0|, |\Theta_-|)\). Assuming that the superconducting flux qubit should be prepared in the ground states of \(J_x(J_y, J_z)\) eigenbasis. RWA condition \(|\Omega| \gg |A_x \pm iA_y|\) requires that \(|\text{Re}[\Omega]|^2 \gg |\delta\omega|^2 + |\text{Im}[\Omega]|^2 (|\text{Re}[\Omega]|^2 \gg |\delta\omega|^2 + |\text{Re}[\Omega]|^2, |\text{Re}[\Omega]|^2 \gg |\text{Re}[\Omega]|^2 + |\text{Im}[\Omega]|^2)\). Under the experimentally reasonable parameters listed in Table I, the polarization time is about \(1/\Gamma_x \simeq 1.6 \mu s / (1/\Gamma_y \simeq 1.6 \mu s, 1/\Gamma_z \simeq 0.4 \mu s)\) for the ideal case \((T_c = 0)\), which is significantly shorter than the thermal relaxation time for low-temperature superconducting flux qubit around \(20 \mu s\) [20, 21]. On the other hand, the effective Rabi frequency which depends on the polarization direction, the Rabi and resonant frequencies of the microwave drive, allows a fruitful adjustable range for experimental parameters.

Two main assumptions were made in the presented theoretical model. First, we have neglected the effects of thermal relaxation of the superconducting system. No effect of thermal relaxation was observed at \(T_c = 40\) mK (with \(\omega_T = T_c / h \kappa_{zh} \approx 0.13 \times 2\pi\) GHz \(\ll \omega_{sc} = 6 \times 2\pi\) GHz) [20, 21]. Second, the derivation of the Markovian master equation (10) assumes the bad resonator condition, which can be valid when the resonator dissipation rate, \(\kappa\), is much larger than the coupling strength between the superconducting flux qubit and resonator in the lowest excitation manifold i.e., \(\kappa \gg g\) [13].

In conclusion, we have demonstrated the initialization of a superconducting flux qubit assisted by a microwave resonator. The technique allows any orbital state of the Bloch sphere with arbitrary phase factor of the superconducting flux qubit to be prepared by adjusting Rabi frequency and the detunings of the drive and resonator. State preparation fidelities in excess of 99.9% and the required time on the order of magnitude of microsecond are in principle possible for experimentally reasonable parameters. Such a type of resonator-assisted qubit initialization method could find many applications in the future quantum technologies.

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