Decoherence in localized photon emission

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I. INTRODUCTION

The conventional method of describing optical emissions from quantum systems treats this process as a transition between non-localized eigenstates of the Hamiltonian. This represents a lack of resolution in time and space. While the advances in short time spectroscopy have already removed much of the limitation in temporal resolution, this must yet be achieved for the spatial resolution. However, advances in mesoscopic physics should eventually make such observations possible, opening the door to new phenomena of quantum coherence.

The calculations presented in sections 3 and 4 are based on a simple picture of near field spectroscopy, in which the local channels are only sensitive to emissions from one of two interacting two level systems. The non-local farfield modes are also included. The probability of photon detection in the localized channels is determined as a function of time and emission sequence, which mirrors the spatial evolution of the excitation in the observed system.

II. TIME RESOLVED PHOTON EMISSION

Our model for photon emission is based on a spatiotemporal interpretation of Wigner-Weisskopf theory \([1]\). If we describe the transition from an excited state \(| e \rangle\) to the ground state \(| g \rangle\), with an energy difference of \(\hbar \omega_0\) between these two states of the system, and choose the wavevector \(k\) to describe the states of the photon after emission, with \(kc = \omega\) as the dispersion of the evenly spaced field states near the resonance, then the wave function \(| \psi(t) \rangle\) for the emission process is described by the amplitudes

\[
\langle e; vac. | \psi(t) \rangle = e^{-i\omega_0 t - \Gamma t/2} \quad (1a)
\]

\[
\langle g; k | \psi(t) \rangle = \sqrt{\frac{c}{2\pi}} e^{-ikt} \frac{1 - e^{-\Gamma t/2 + ikc - i\omega_0 t}}{kc - \omega_0 + i\Gamma/2}. \quad (1b)
\]

This rather unintuitive result can be Fourier transformed to a one dimensional real space representation. In the case of the three dimensional light field, the one dimensional coordinate \(r\) represents the radius, while the angular dependence is defined by the symmetry of the transition, which is usually dipole radiation. For a more complete discussion of these questions, see \([1]\). The amplitudes of the single photon states now read

\[
\langle g; r | \psi(t) \rangle = -i \sqrt{\frac{\Gamma}{c}} e^{(\Gamma/2 + i\omega_0)(r/c - t)}
\]

for \(0 < x < ct\), else 0 \(\quad (2)\)

A decisive advantage of the real space representation is that it reveals the physical assumptions behind the approximations of Wigner Weisskopf theory. While the temporal evolution of the k-space result is difficult to interpret since it seems to include complicated reabsorption processes, the real space representation shows that the photon wavefunction moves with the constant velocity \(c\) away from the system, retaining the phase which the system had at the time of emission. There is no reabsorption in this base and the propagator of the field is a translation with constant velocity \(c\), without dispersion, so it is possible to assign to each real space state at \(r\) an emission time \(t-r/c\). This model of photon emission therefore provides a simple approach to questions of time resolved spectroscopy.

III. THE SYSTEM

We now apply this model to emissions from a system of two identical two state systems, which may exchange their excitations via a Förster interaction. The Hamiltonian of this system is given by

\[
\hat{H} = 2E_0 | ee \rangle \langle ee | + E_0 | eg \rangle \langle eg | + E_0 | ge \rangle \langle ge | + W | eg \rangle \langle eg | + W | eg \rangle \langle ge | \quad (3)
\]

where the base states have been chosen as the product states of the excited states e and the ground states g of the two identical two state systems. The Eigenstates of this Hamiltonian are given by

\[
| 2(+) \rangle = | ee \rangle
\]

\[
| 1(+) \rangle = \frac{1}{\sqrt{2}} (| eg \rangle + | ge \rangle)
\]

\[
| 1(-) \rangle = \frac{1}{\sqrt{2}} (| eg \rangle - | ge \rangle)
\]

\[
| 0(+) \rangle = | gg \rangle \quad (4)
\]

separated into symmetrical (\(\pm\)) and antisymmetrical (\(\mp\)) states with respect to an exchange of the two identical systems. The Hamiltonian in this base is

\[
\hat{H} = 2E_0 | 2(+) \rangle \langle 2(+) | + (E_0 + W) | 1(+) \rangle \langle 1(+) | + (E_0 - W) | 1(-) \rangle \langle 1(-) | \quad (5)
\]

The next step is to define the modes of the light field coupled to the system. In the conventional situation,
when no optical nanostructures are present and the size of the system is far less than the wavelength, only the nonlocal far field mode N couples to the system. However, in order to achieve spatial resolution, local modes which couple to the two identical parts of our system separately are necessary as well. In order to maintain the symmetry, we introduce the two local modes L1 and L2. L1 couples exclusively to transitions in the first system (from |ee⟩ to |ge⟩ and from |eg⟩ to |gg⟩), while L2 couples in the same manner to transitions in the second system (from |ee⟩ to |eg⟩ and from |ge⟩ to |gg⟩).

The base states of the light field in real space are therefore |r, N(+)⟩, |r, L1⟩ and |r, L2⟩. To reflect the symmetry of the system, it is convenient to transform the local fields to their symmetrical and antisymmetrical base states:

\[
| r, L(+)⟩ = \frac{1}{\sqrt{2}} (|r, L1⟩ + |r, L2⟩) \\
| r, L(−)⟩ = \frac{1}{\sqrt{2}} (|r, L1⟩ − |r, L2⟩)
\]

(6)

Since the unitary evolution of the system will preserve the parity, a transition between an antisymmetrical state and a symmetrical state requires the emission of an antisymmetrical photon, whereas all transitions within the symmetric system emit only symmetrical photons. Due to this property of the system, it is now very easy to determine the temporal evolution of the emission processes of the system.

IV. TEMPORAL EVOLUTION AND MEASUREMENTS

We are now ready to describe the emission process, starting from any excited state of the system. The most interesting case is the emission of two photons from the {ee} state, since the measurement of the two time correlation of the consecutive emissions should provide information on the coherent evolution of the system. To simplify our equations, however, it is sufficient to start from the states with a single excitation, since the measurement of the first photon emitted at time \( t = 0 \) effectively projects the system into a well defined state with a single excitation left. In the following we investigate the dynamics after the emission of a photon in L2 at \( t = 0 \). This is equivalent to the dynamics of \( |\psi(t = 0)⟩ = |eg⟩ \). The probabilities of emission calculated from this initial state represent the conditional probability of finding emissions in the L1, L2 and N channels, given that a photon was detected in L2 at \( t − r/c = 0 \). With \( \Gamma_N \) and \( \Gamma_L \) as the emission rates of the nonlocal and local modes respectively, and defining \( \omega_+ = (E_0 + W)/h \) and \( \omega_- = (E_0 − W)/h \), the evolution of the wavefunction is

\[
(1(−); vac. | ψ(t)) = \frac{1}{\sqrt{2}} e^{−(\Gamma_L + \Gamma_N)t/2−iω_−t} \quad (7a)
\]

\[
(0(+) ; r, N(+) | ψ(t)) = −i\sqrt{\frac{Γ_N}{2c}} e^{−(\Gamma_L/2+Γ_N/2+iω_+)(r/c−t)} \quad (7b)
\]

\[
(0(+) ; r, L(+) | ψ(t)) = −i\sqrt{\frac{Γ_L}{2c}} e^{−(Γ_L/2+Γ_N/2+iω_+)(r/c−t)} \quad (7c)
\]

\[
(0(+) ; r, L(−) | ψ(t)) = +i\sqrt{\frac{Γ_L}{2c}} e^{−(Γ_L/2+iω_-)(r/c−t)} \quad (7d)
\]

With these amplitudes, we can calculate all measurement probabilities. The effects of local coherence should be visible in the probabilities of measuring a photon in the L1 or the L2 channels. We therefore calculate the conditional probability of measuring a photon in L1 at a time \( t \) after the measurement of a photon in L2, assuming that both measurements occur at an equal distance \( r \) from the system:

\[
p_{L2,L1}(t) = | ⟨0(+) ; r, L1 | ψ(t + r/c)⟩ |^2
\]

\[
= 1/2(| ⟨0(+) ; r, L1 | ψ(t + r/c)⟩ |^2
\]

\[
+ | ⟨0(+) ; r, L(−) | ψ(t + r/c)⟩ |^2)
\]

\[
+Re(⟨0(+) ; r, L(+) | ψ(t + r/c)⟩
\]

\[
(ψ(t + r/c) | 0(+) ; r, L(−))⟩)
\]

\[
= \frac{Γ_L}{4c} e^{−Γ_Lt}[1 + 2e^{−Γ_Nt/2 cos(δω t)} + e^{−Γ_Nt}] \quad (8)
\]

with \( δω = 1/2(ω_+ − ω_-) = W/h \). The resulting time dependence describes the two time correlation between the emission events. Figure 1 illustrates this correlation for a typical choice of parameters, \( Γ_L ≪ Γ_N \) and \( δω = 8Γ_N \). In general, there are three timescales independently given by \( Γ_L \), \( Γ_N \) and \( δω \). Since nano-optical modes can be expected to have a much weaker coupling to the system than the far field modes, the assumption that \( Γ_L ≪ Γ_N \) is quite realistic, while the choice of \( δω \) remains somewhat arbitrary. The figure clearly shows the quantum beats of the excitation, which oscillates between the two systems at a frequency of \( δω \). These coherent beats decay on a timescale of \( Γ_N \). The quantitative expression for this decay is the ratio of the amplitude \( A \) and the mean value \( M \) of the beats:

\[
\frac{A}{M} = \frac{2e^{Γ_Nt/2}}{1 + e^{Γ_Nt}} = \frac{1}{\cosh(1/Γ_Nt/2)} \quad (9)
\]

Therefore, the effect of the far field on the observation of localized quantum beats is to cause decoherence. The decoherence is strongly non-exponential for \( Γ_Nt < 1 \).

V. INTERPRETATION

The temporal evolution of the wave function clearly shows that the fast transitions from |1(+)⟩ to |0(+)⟩
quickly reduce the amplitude of $|1(+)>$ relative to $|1(-)>$. Therefore the coherence between $|1(+)>$ and $|1(-)>$ is lost. This can be interpreted in terms of measurement theory as a potential measurement of $|1(+)>$ in the N channel, which necessarily disturbs the distinction between $|eg>\text{ and } |ge>$. The difference in the emission rates of $|1(+)>$ and $|1(-)>$ means that a photon detected in a local channel is most likely to be emitted by the $|1(-)>$ state, if $\Gamma_N t \gg 1$.

The presence of channels coupling both locally and nonlocally can be interpreted as a continuous weak measurement of non-commuting observables \[2\]. The decoherence in the local channels is then seen to arise from the quantum fluctuations of the field in the nonlocal channel. Note that the effect of the nonlocal channel is therefore seen even in those experiments, during which no photon is emitted into this channel (e.g. the L2-L1 emission discussed above). This possibility to observe the presence of the nonlocal mode even without the excitation of a photon in this channel is similar in nature to the possibility of an interaction free measurement discussed by Elitzur and Vaidmann in \[3\].

VI. CONCLUSIONS

The model presented here allows the description of simultaneous coupling to local and nonlocal modes by formulating the complete wave function of the system and the interacting fields. The results clearly show how quantum beats occurring between the two local subsystems decohere due to the influence of the nonlocal channel. Therefore, this model is a useful tool in the investigation of the relationship between coherent and dissipative phenomena observed with high spatial and temporal resolution.

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\[1\] H. F. Hofmann and G. Mahler, Quantum Semiclass. Optics 7, 489 (1995).

\[2\] Such a measurement of non-commuting observables can be explained by the use of the field to effectively increase the dimensions of the Hilbert space within which the projective measurement is performed. This method is often referred to as ancilla measurement. It was first introduced in C.W. Helstrom, Quantum Detection and Estimation Theory, Academic Press, New York 1974. For a comprehensive discussion, see A. Peres, Quantum Theory: Concepts and Methods, Kluwer Academic Publications, Dordrecht 1993.

\[3\] A. Elitzur and L. Vaidmann, Found. Phys. 23, 987 (1993).