Complete One-Loop Corrections to $e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^0h^0$ for Different Scenarios

S. M. Seif$^1$, T.A. Azim$^1$

$^1$Faculty of Science, Physics Department, Cairo University, Giza, Egypt.

(Dated: November 20, 2014)

Abstract

In the present work, the full one-loop corrections to the production of a light neutral minimal supersymmetric standard model Higgs boson ($h^0$) with a pair of lightest neutralinos ($\tilde{\chi}_1^0$) in $e^+e^-$ collisions within the Minimal Supersymmetric Standard Model (MSSM) are presented. The details of the renormalization scheme used are presented. Our results also include the QED corrections as well as the weak corrections. It is found that the contribution from the weak and QED corrections is significant and needs to be taken into account in the future linear collider experiments. Numerical results for two different SUSY scenarios—Higgsino and Gaugino scenarios—for $e^+e^-$ are given.

PACS numbers:...

1 Introduction

One of the main goals of the Tevatron and the LHC experimental programs was to detect a Higgs boson. On the 4th of July 2012, the CMS and the ATLAS experimental teams at the LHC, announced independently, that they both discovered a previously unknown boson of mass between 125 and 127 GeV [1, 2, 3], whose behavior so far is "consistent with" a Higgs boson, and it is confirmed likely, on March 2013, to be a Higgs boson, although yet it is unclear which model best supports the particle or whether multiple Higgs bosons exist. This discovery has impact on the search for particles such as the standard model Higgs boson ($h^0$) with a pair of lightest neutralinos ($\tilde{\chi}_1^0$) in $e^+e^-$ collisions within the Minimal Supersymmetric Standard Model (MSSM) are presented. The details of the renormalization scheme used are presented. Our results also include the QED corrections as well as the weak corrections. It is found that the contribution from the weak and QED corrections is significant and needs to be taken into account in the future linear collider experiments. Numerical results for two different SUSY scenarios—Higgsino and Gaugino scenarios—for $e^+e^-$ are given.

In view of the experimental prospects, it is inevitable to include higher-order terms in the calculation of the measured quantities in order to achieve theoretical predictions matching the experimental accuracy. Former studies on chargino-pair production [9, 10, 11] and scalar-quark decays [12] have revealed that the Born-level predictions can be influenced significantly by one-loop radiative corrections.

In this paper, we use on-shell renormalization scheme in the loop calculations of the Higgs and neutralino sectors of the CP-conserving MSSM. The calculation was performed using the FeynArts and FormCalc computer packages. All the renormalization constants, required to determine the various counterterms for the Higgs, neutralino and other sectors, being implemented in the MSSM version of FeynArts [13] for completion at the one-loop level. The resulting amplitudes were algebraically simplified using FormCalc and then converted to a FORTRAN program. The LoopTools package was used to evaluate the one-loop scalar and tensor integrals [14].

The paper is arranged as follows: The analytical calculations of the Born cross section to the $e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^0h^0$ process is given in section 2, where some numerical results are shown. The virtual, the electroweak, and the soft photonic corrections are studied in section 3. The numerical results are presented in section 4. Finally, the conclusions are given in section 5.
The integration is performed over the three-body phase space of the final state. The three-body phase space element $d\Phi_3$ is read as

$$d\Phi_3 = \delta^4(p_1 + p_2) - \sum_{i=3}^5 \frac{d^3 p_i}{(2\pi)^3 2E_i},$$

from which we get

$$\sigma^0 = \frac{|M_0|^2}{16(2\pi)^3} \left\{ \delta^4(p_1 + p_2 - p_3 - p_4 - p_5) \right\} + \sum_{s_{\text{spins}}} |M_0|^2 d\Phi_3. \quad (1)$$

where $p_1, p_2, p_3, p_4$ and $p_5$ are the four-momenta of the incoming and the outgoing particles, respectively. $|M_0|^2$ is the square of amplitudes corresponding to the Feynman diagrams in Fig. 1 and $s = (p_1 + p_2)^2$ is the square of the total energy in the centre of mass system. The Feynman diagrams contributing to the production of the lightest CP-even Higgs boson in association with neutralino pairs is shown in Fig. 1. The diagrams where the $h^0$ boson is emitted from the electron and positron lines give negligible contributions. A first class of contributions (a) is formed by diagrams where the Higgs boson is emitted from the neutralino states, the latter being produced through $s$-channel $Z$ boson exchange and left- and right-handed selectron exchange. A second class (b) is formed by the Higgs-strahlung production process, where the $Z$ boson is virtual and splits into two neutralinos. Finally, a third class (c) consists of the diagrams where the Higgs boson is emitted from the internal selectron lines. The cross section will therefore depend on the $h^0$ boson couplings to both the neutralinos and sleptons [15]. In the MSSM, the Higgs boson couplings to the neutralinos $\tilde{\chi}^0_i$ depend on the parameter $\mu$ and the gaugino ---bino and wino--- mass parameters $M_1$ and $M_2$.

**Figure 1:** The lowest order (LO) Feynman diagrams for the $e^+e^- \rightarrow \tilde{\chi}^0_1\tilde{\chi}^0_1 h^0$ process.

As clarified in [16, 17], all Higgs bosons in the MSSM, except the lightest CP-even one, are too heavy to play an important role in both the current and the near future experiments. Therefore, the present study concentrates on the lightest Higgs boson $h^0$ only. The LO cross-section is studied for the process $e^+e^- \rightarrow \tilde{\chi}^0_1\tilde{\chi}^0_1 h^0$ as a function of the light neutral Higgs boson mass $m_{h^0}$. From the interaction Lagrangian [18, 19]
suitable set of counterterms for the renormalization of
The UV divergences are renormalized by introducing a
used. Self-energy and vertex diagrams contain IR and
inside loops, the complete supersymmetric spectrum is
self-energy contributions Fig. 4. For the virtual particles
diagrams: The virtual vertex corrections Fig. 2, the box
contributions to the propagators. Fig. 3, and the
higher-order terms in the calculations of mass spectra,
experimental accuracy require the proper inclusion of
breaking. Adequate theoretical predictions matching the
measurements of masses and cross sections, the funda-
metric models, especially for the MSSM. From precise
the best environment for precise studies of supersym-
The linear electron-positron collider is considered to be
the coupling constants and the renormalization of
the external wave functions.
As a calculational frame, the on-shell renormalization
scheme has been chosen such that all particle masses are
defined as pole masses, i.e., on-shell quantities. Thus, the
cross sections are directly related to the physical masses
of the external particles and the other particles entering
the loops. The theoretical basis of the calculation is out-
lined in the following subsections, where the presentation
is based on refs.[21, 22].
In this work, the complete set of Feynman graphs is
calculated with help of the packages FeynArts and
FormCalc. We implemented our renormalization proce-
dure into these packages. The virtual correction suffers
from both ultraviolet (UV) and infrared (IR) singular-
ities. For a proper treatment of the appearing UV di-
vergencies, counter terms are introduced in the on-shell
renormalization scheme where all particle masses are de-
defined as pole masses, i.e., on-shell quantities. To preserve
supersymmetry, the used regularization scheme is dimen-
sional reduction (DR). The loop graphs with virtual pho-
ton exchange also introduce IR singularities. Therefore,
real photon emission has to be included to obtain a finite
result.

\[
\sigma^{corr}(e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^0 h_0) = \sigma^{ren}(e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^0 h_0) + \sigma(e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^0 h_0\gamma).
\]

3 One-Loop Calculations

The linear electron-positron collider is considered to be
the best environment for precise studies of supersym-
metric models, especially for the MSSM. From precise
measurements of masses and cross sections, the funda-
mental parameters of the MSSM Lagrangian can be re-
constructed [20] to shed light on the mechanism of SUSY
breaking. Adequate theoretical predictions matching the
experimental accuracy require the proper inclusion of
higher-order terms in the calculations of mass spectra,
cross sections and decay rates.
The radiative corrections to the neutralino pair pro-
duction with light neutral Higgs boson can be classified as
the following generic structure of one-loop Feynman
diagrams: The virtual vertex corrections Fig. 2, the box
diagrams contribute to the propagators. Fig. 3, and the
self-energy contributions Fig. 4. For the virtual particles
inside loops, the complete supersymmetric spectrum is
used. Self-energy and vertex diagrams contain IR and
UV divergences. Box diagrams are IR and UV finite.
The UV divergences are renormalized by introducing a
suitable set of counterterms for the renormalization of

3.1 Renormalization of Neutralino

The bilinear part of the Lagrangian describing the neu-
tralino sector of the MSSM involves the \( \mu \) parameter, the
soft–breaking gaugino-mass parameters \( M_1 \) and \( M_2 \), and
the Higgs vacua \( v_i \), which are related to \( \tan \beta = v_2/v_1 \) and
to the W mass \( M_W = gv/2 \) with \((v_1^2+v_2^2)^{1/2}\) [21, 22].

Renormalization constants are introduced for the
neutralino mass matrix \( Y \)

\[
Y = \begin{pmatrix}
M_1 & 0 & -M_Z s_W \cos \beta & M_Z s_W \sin \beta \\
0 & M_2 & -M_Z c_W \cos \beta & M_Z c_W \sin \beta \\
-M_Z s_W \cos \beta & M_Z c_W \sin \beta & 0 & -\mu \\
M_Z s_W \cos \beta & -M_Z c_W \sin \beta & -\mu & 0
\end{pmatrix}
\]

and for the neutralino fields \( \tilde{\chi}_i^0 (i = 1, \ldots, 4) \) by the following transformation

\[
Y \rightarrow Y + \delta Y,
\]

\[
\omega_L \tilde{\chi}_i^0 \rightarrow (\delta_{ij} + \frac{1}{2} [\delta Z_{\chi_0^0}]_{ij}) \omega_L \tilde{\chi}_j^0,
\]

\[
\omega_R \tilde{\chi}_i^0 \rightarrow (\delta_{ij} + \frac{1}{2} [\delta Z_{\chi_0^0}]_{ij}) \omega_R \tilde{\chi}_j^0. \tag{3}
\]

The matrix \( \delta Y \) consists of the counterterms for the
following parameters in the mass matrix \( Y \): the soft-
breaking gaugino-mass parameters $M_1$ and $M_2$, the Higgsino mass parameter $\mu$, $\tan \beta$, the $Z$ mass and the electroweak mixing angle $s_W = \sin \theta_W$, $c_W = \cos \theta_W$. The matrix-valued renormalization constant $\delta Z_{\tilde{\psi}}$ is a general complex $4 \times 4$ matrix.

Using the on-shell approach [22, 23], the pole mass of the neutralino, $\chi_1^0$, is considered as an input parameters to specify the neutralino Lagrangian in terms of physical quantities. This is equivalent to the specification of the parameters $\mu, M_1$ and $M_2$, which are related to the input masses in the same way as in LO, as a consequence of the on-shell renormalization conditions, and to the definition of the respective counterterms. In this way, the tree-level masses $m_{\chi_1^0}$ as well as the counterterm matrix $\delta Z_{\tilde{\psi}}$, are fixed. Furthermore, one requires that the matrix of the renormalized one-particle-irreducible two-point vertex functions $\Gamma_{ij}^{(2)}$ becomes diagonal on the on-shell external momenta. This fixes the non-diagonal entries of the field-renormalization matrix $\delta Z_{\tilde{\psi}}$; their diagonal entries are determined by normalizing the residues of the propagators to be unity.

### 3.2 Renormalization of Higgs Sector

The Higgs sector of the MSSM [23, 24] consists of two scalar doublets

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} (v_1 + \phi_1^0 - i \chi_{11}^0)/\sqrt{2} \\ -\phi_1 \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^0 + i \chi_{22}^0/\sqrt{2} \\ (v_2 + \phi_2^0 - i \chi_{22}^0)/\sqrt{2} \end{pmatrix}$$

with opposite hypercharge $Y_1 = -Y_2 = -1$ and vacuum expectation values $v_1, v_2$. The quadratic part of the potential

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 + m_{12}^2 \epsilon_{ab} H_1^a H_2^b + h.c. + \frac{1}{8} (g_1^2 + g_2^2) (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 - \frac{g_2^2}{2} |H_1 \bar{H}_2|^2,$$  \hspace{1cm} (4)

where $m_{12}^2$ is defined to be negative and $\epsilon_{12} = -\epsilon_{21} = -1$, with soft breaking parameters $m_1^2, m_2^2, m_{12}^2$ and the gauge couplings $g_1, g_2$ is diagonalized by the rotations

$$\left( \begin{array}{c} H^0 \\ h^0 \end{array} \right) = \left( \begin{array}{cc} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{array} \right) \left( \begin{array}{c} \phi_1^0 \\ \chi_{11}^0 \end{array} \right)$$

$$\left( \begin{array}{c} C^0 \\ A^0 \end{array} \right) = \left( \begin{array}{cc} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{array} \right) \left( \begin{array}{c} \chi_2^0 \\ \chi_2^0 \end{array} \right)$$

$$\left( \begin{array}{c} G^+ \\ H^+ \end{array} \right) = \left( \begin{array}{cc} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{array} \right) \left( \begin{array}{c} \phi_2^+ \\ \phi_2^+ \end{array} \right)$$

$G^0, G^+$ describe the unphysical Goldstone modes. The spectrum of physical states consists of

2 neutral bosons with CP = 1: $h^0, H^0$ (scalars)
1 neutral boson with CP = −1: $A^0$ (pseudoscalar)
2 charged bosons: $H^\pm$.

The masses of the gauge bosons and the electromagnetic charge are determined by

$$M_Z^2 = \frac{1}{4} (g_1^2 + g_2^2) (v_1^2 + v_2^2),$$

$$M_W^2 = g_2^2 (v_1^2 + v_2^2),$$

$$v^2 = \frac{g_1^2 g_2^2}{g_1^2 + g_2^2}.$$  \hspace{1cm} (7)

Thus, the potential (5) contains two independent free parameters, which can conveniently be chosen as

$$\tan \beta = \frac{v_2}{v_1}, M_A^2 = -m_{12}^2 (\tan \beta + \cot \beta)$$  \hspace{1cm} (8)

where $M_A$ is the mass of the $A^0$ boson.

Expressed in terms of (8), the masses of the other physical states read:

$$m_{h^0, h^0}^2 = \frac{1}{2} \left[ M_A^2 + M_Z^2 \pm \sqrt{(M_A^2 + M_Z^2)^2 - 4 M_A^2 M_Z^2 \cos^2 2 \beta} \right]$$

$$m_{h^+}^2 = M_A^2 + M_W^2,$$  \hspace{1cm} (9)

and the mixing angle $\alpha$ in the $(H^0, h^0)$-system is derived from

$$\tan 2\alpha = \tan 2\beta \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2}, -\frac{\pi}{2} < \alpha \leq 0.$$  \hspace{1cm} (10)

Hence, masses and couplings are determined by only a single parameter more than in the standard model.

The dependence on $M_A$ is symmetric under $\tan \beta \leftrightarrow 1/\tan \beta$, and $m_{h^0}$ is constrained by:

$$m_{h^0} < M_Z \cos 2\beta < M_Z.$$  \hspace{1cm} (11)

This simple scenario, however, is changed when radiative corrections are taken into account.

The tree-level mass matrix $m_0$ of the neutral scalar system that represents bare mass system is diagonalized by (6). Loop contributions to the quadratic part of the potential (neglecting the $q^2$-dependence of the diagrams) modify the mass matrix

$$m_0 \rightarrow m_0 + \delta m = m$$  \hspace{1cm} (12)
Re-diagonalizing the one-loop matrix \( m_{H^\pm, h^0} \) yields the corrected mass eigenvalues \( m_{H^\pm, h^0} \), replacing (9), and an effective mixing angle \( \alpha_{eff} \) instead of (10). The renormalization constants [23] are defined as follows:

\[
\begin{align*}
B_\mu &\rightarrow (Z_{B_\mu}^2)^{1/2}B_\mu, \\
W^a_\mu &\rightarrow (Z_{W^a}^2)^{1/2}W^a_\mu, \\
H_1 &\rightarrow Z_{H_1}^{1/2}H_1, \\
\psi^R \rightarrow (Z_{\psi}^R)^{1/2} \psi^R, \\
g_2 &\rightarrow Z_1^W(Z_2^W)^{-3/2}g_2, \\
g_1 &\rightarrow Z_1^B(Z_2^B)^{-3/2}g_1, \\
v_1 &\rightarrow Z_{H_1}^{1/2}(v_1 - \delta v_1), \\
m_2^2 &\rightarrow Z_{H_2}^{-1}(m_1^2 + \delta m_1^2), \\
m_{12}^2 &\rightarrow Z_{H_1}^{-1/2}Z_{H_2}^{-1/2}(m_{12}^2 + \delta m_{12}^2).
\end{align*}
\] (13)

The complete definitions and the explicit expressions of the renormalization constants of the other sectors: sfermion sector, MSSM parameters and fields including those of SM as the electric charge and the masses of W, Z, and the fermions and their counterterms in addition to \( \tan \beta \), all these are treated as described in [23, 26, 27], and can have either soft or collinear nature. The collinear singularity is regularized by keeping electron (positron) mass. The general phase-space-slicing method (PSS) is adopted to separate the soft photon emission singularity from the real photon emission processes. By using this method, the bremsstrahlung phase space is divided into singular and non-singular regions by the soft photon cut-off, \( \Delta E \), and the cross section of the real photon emission process is decomposed into soft and hard terms [30],

\[
\sigma^{\text{real}} = \sigma^{\text{soft}}(\Delta E) + \sigma^{\text{hard}}(\Delta E) \\
= \sigma^0(\Delta_{\text{soft}} + \Delta_{\text{hard}}).
\] (17)

where \( \Delta_{\text{soft}} = \Delta_{\text{weak}} \) and \( \Delta_{\text{hard}} = \Delta_{\text{QED}} \).

The energy of the radiated photon in the center of mass system frame is considered as a soft term, \( \Delta_{\text{soft}} \), with radiated photon energy \( k_\gamma^0 \leq \Delta E \), and a hard term, \( \Delta_{\text{hard}} \), with \( k_\gamma^0 > \Delta E \), where \( k_\gamma^0 = \sqrt{|k_\gamma|^2 + m_\gamma^2} \) and \( m_\gamma \) is the photon mass, which is used to regulate the (IR) divergences existing in the soft term. For practical calculations, \( \sigma^{\text{hard}} \) is divided into a collinear part, where the photon is within an angle smaller than \( \Delta \theta \) with respect to the radiating particles, and the complementary non-collinear part [23],

\[
\sigma^{\text{hard}} = \sigma^{\text{coll}}(\Delta \theta) + \sigma^{\text{non-coll}}(\Delta \theta).
\] (18)

\( \sigma^{\text{non-coll}} \) is calculated numerically with the help of multidimensional numerical integration routines DIVONNE that based on Monte Carlo, and CUHRE, which are both part of the CUBA-library [31], while \( \sigma^{\text{soft}} \) and \( \sigma^{\text{coll}} \) are calculated analytically.

The weakness of the definition in Eq.(17) is the large \( \Delta E \) dependence of the weak and QED components \( \propto \log(\frac{\Delta E^2}{\mu^2}) \). However, the sum of both is cutoff independent. Therefore, we extract the \( \Delta E \) terms and the leading logarithms \( \log(\frac{\Delta E}{m^2}) \), caused by collinear soft photon emission, from the weak corrections and add them.
to the QED corrections \cite{32}. According to this definition, both corrections are now \( \Delta E \) independent. The main part of the QED corrections arises from these leading logarithms \( L_e \), originating from photons in the beam direction. This leads to a large dependence on the experimental cuts and detector specifications. Therefore, we use the structure function formalism \cite{33} and subtract the leading logarithmic \( O(\alpha) \) terms of the initial state radiation, \( \sigma^{ISR, LL} \). After subtraction of these process-independent terms, only the non-universal QED corrections remain. The total one-loop (renormalized) cross section \( \sigma^{1-loop} \) is expressed as:

\[
\sigma^{1-loop} = \sigma^0 + \sigma^{virt} + \sigma^{weak} + \sigma^{QED},
\]

where

\[
\sigma^{weak} = \sigma^{soft} - \tilde{\sigma},
\]

\[
\sigma^{QED} = \sigma^{hard} + \tilde{\sigma} - \sigma^{ISR, LL},
\]

with

\[
\tilde{\sigma} = \frac{\alpha}{\pi} \left[ (L_e - 1) \log \frac{4\Delta E^2}{s} + \frac{3}{2} L_e \right] \sigma^0,
\]

\[
\sigma^{ISR, LL} = \frac{\alpha}{\pi} L_e \int_0^1 dx \Phi(x) \sigma^0(sx),
\]

\[
\Phi(x) = \lim_{\epsilon \to 0} \left\{ \delta(1-x) \left[ \frac{3}{2} + 2 \log(\epsilon) \right] + \theta(1-x-\epsilon) \frac{1 + x^2}{1-x} \right\}.
\]

The integrated cross section at the one–loop level, can be written in the following way:

\[
\sigma^{1-loop} = \sigma^0 + \sigma^0 \Delta,
\]

pointing out the relative correction

\[
\Delta = (\sigma^{1-loop} - \sigma^0)/\sigma^0,
\]

with respect to the Born cross section. The relative correction \( \Delta \) can be decomposed into the following parts, indicating their origin,

\[
\Delta = \Delta_{self} + \Delta_{vertex} + \Delta_{box} + \Delta_{QED} + \Delta_{weak}.
\]

\section{Numerical Results}

In present work, two different scenarios are studied. In the higgsino scenario the neutralinos are both nearly pure higgsinos and therefore the process is dominated by the schannel \( Z_0 \) exchange. In the gaugino scenario with binos as \( \tilde{\chi}_1^0 \) states, the selectron exchange diagrams play the most important role. In the following, we distinguish between the tree-level, the 1-loop corrections, and the conventional weak and QED corrections to the improved tree-level as discussed in the last section. For the SM input parameters the following values have been used \( \alpha(m_Z) = 1/127.922, M_W = 80.399 \) GeV, \( M_Z = 91.1876 \) GeV, \( \sin^2 \theta_W = 1 - m_W^2/m_Z^2 \), \( m_t = 173.4 \) GeV, \( m_b = 4.7 \) GeV. \( M_2 \) is fixed by the gaugino unification relation

\[
M_2 = \frac{3}{\tan^2 \theta_W M_1},
\]

and the gluino mass is related \( M_1 \) by

\[
m_{\tilde{g}} = (\alpha_s(m_{\tilde{g}})/\alpha) \sin^2 \theta_W M_1.
\]

The following numerical examples, the mass spectrum of the SUSY particles are set as shown in Tables 1 and 2 for Higgsino and Gaugino scenarios respectively. The free parameters that have been used in our calculations are specified as follows:

\begin{itemize}
  \item For simplicity, in the sfermions sector, all soft-SUSY breaking parameters are assumed equal and all trilinear couplings are set to a common value \( A \). The mixing between sfermion generations is neglected, \( M_{SU3} = M_L \simeq M_R \).
  \item The MSSM Higgs sector is parametrized by the CP-odd mass, \( m_A \), and tan \( \beta \), taking into account radiative corrections with the help of FormCalc.
  \item The chargino-neutralino sector is fixed by choosing a value for the gaugino-mass terms \( M_1, M_2 \) and for the Higgsino-mass term \( \mu \).
\end{itemize}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
Particle & Mass/[GeV] \\
\hline\hline
\( h^0 \) & 105.341 \\
\hline
\( H^0 \) & 700.275 \\
\hline
\( A^0 \) & 200.218 \\
\hline
\( H^\pm \) & 416.025 \\
\hline
\( \tilde{g} \) & 1063.46 \\
\hline
\( \tilde{\chi}_1^\pm \) & 99.2230 \\
\hline
\( \tilde{\chi}_2^\pm \) & 414.032 \\
\hline
\( \tilde{e}_L \) & 352.582 \\
\hline
\( \tilde{\mu}_L \) & 352.519 \\
\hline
\( \tilde{\tau}_L \) & 349.346 \\
\hline
\( \tilde{u}_L \) & 345.881 \\
\hline
\( \tilde{d}_L \) & 350.860 \\
\hline
\( \tilde{c}_L \) & 345.667 \\
\hline
\( \tilde{s}_L \) & 350.853 \\
\hline
\( \tilde{L}_L \) & 281.995 \\
\hline
\( \tilde{b}_L \) & 343.316 \\
\hline
\hline
\end{tabular}
\caption{The mass spectrum of the SUSY particles for Higgsino scenario}
\end{table}
Table 2: The mass spectrum of the SUSY particles for Gaugino scenario

| Particle | Mass/[GeV] | Particle | Mass/[GeV] |
|----------|------------|----------|------------|
| $h^0$    | 110.985    | $\tilde{\chi}_1^0$ | 91.5340    |
| $H^0$    | 393.987    | $\tilde{\chi}_2^0$ | 181.009    |
| $A^0$    | 393.600    | $\tilde{\chi}_3^0$ | 359.502    |
| $H^\pm$  | 401.727    | $\tilde{\chi}_4^0$ | 378.874    |
| $\tilde{g}$ | 525.351   | $\tilde{\nu}_e$ | 495.905    |
| $\tilde{\chi}_1^\pm$ | 180.516   | $\tilde{\nu}_\mu$ | 495.905    |
| $\tilde{\chi}_2^\pm$ | 379.562   | $\tilde{\nu}_\tau$ | 493.614    |
| $\tilde{t}_L$ | 501.812   | $\tilde{t}_R$ | 502.257    |
| $\tilde{t}_R$ | 495.440   | $\tilde{\tau}_L$ | 508.551    |
| $\tilde{\nu}_L$ | 497.123   | $\tilde{\nu}_R$ | 498.787    |
| $\tilde{d}_L$ | 500.591   | $\tilde{d}_R$ | 503.475    |
| $\tilde{c}_L$ | 497.107   | $\tilde{c}_R$ | 498.807    |
| $\tilde{s}_L$ | 500.557   | $\tilde{s}_R$ | 503.508    |
| $\tilde{b}_L$ | 484.474   | $\tilde{b}_R$ | 519.044    |

The parameters for the Higgsino Scenario are set as: 
$\{M_2, \mu, A, \tan \beta, m_A, M_{SUSY}\} = \{400 \text{ GeV}, -100 \text{ GeV}, 400 \text{ GeV}, 10, 700 \text{ GeV}, 350 \text{ GeV}\}$, with SUSY mass spectrum shown in table 1.

The parameters for the Gaugino scenario are set as:  
$\{M_2, \mu, A, \tan \beta, m_A, M_{SUSY}\} = \{197.6 \text{ GeV}, 353.1 \text{ GeV}, -100 \text{ GeV}, 10.2, 393.6 \text{ GeV}, 500 \text{ GeV}\}$, with SUSY mass spectrum shown in table 2.
5 Conclusions

The full one-loop corrections to the lightest neutralino pair production with light neutral Higgs boson in $e^+e^-$ collisions have been calculated. The calculation was performed in an analytical way with an independent check using the FeynArts and FormCalc computer packages. The chosen renormalization scheme can be used for the complete MSSM parameter space. A particular attention is paid to an appropriate definition of the weak and QED corrections. We extracted the non-universal QED corrections by subtracting the initial state radiation (ISR). The full one-loop corrections are in the range of 12-20% for Higgsino scenario, and of 40-70% for Gaugino scenario, thus they have to be taken into account in future linear collider experiments. The maximum cross sections are presented in Tables 3 and 4 for both scenarios. The full one-loop corrections for the Gaugino scenario are in the range of 20-30% for the same reaction according to ref. [34], showing the effect of the chosen parameters on the calculations.

Acknowledgments

We would like to express our sincere gratitude to our advisor Pro. Dr. M. Khaled Hegab, and our thanks to Pro. Dr. Samiha Abou Stiet for revising the paper.

References

[1] A. Cho, Science 337, 141 (2012)
[2] The ATLAS Collaboration, Phys. Lett. B 716, 1 (2012)
[3] The ATLAS Collaboration, Phys. Rev. D 86, 032003 (2012)
[4] C. Beskidt, W. de Boer, D. I. Kazakov. arXiv:1402.4650 [hep-ph]
[5] H. P. Nilles, Phys. Rep. 110, 1 (1984); H. E. Haber and G. L. Kane, Phys. Rep. 117, 75 (1985); R. Barberi, Riv. Nuovo. Cim. 11, 1 (1988)
[6] The ATLAS Collaboration. arXiv:1403.5294 [hep-ex]
[7] J.A. Aguilar et al., in TESLA Technical Design Report, Part III: Physics at an $e^+e^-$ Linear Collider, 2001, ed. by R. D. Heuer, D. Miller, F. Richard, and P. M. Zerwas. arXiv:hep-ph/0106315
[8] C. Adolphsen et al. (International Study Group Collaboration), in International study group progress report on linear collider development, 2000, SLAC-R-559 and KEKREPORT-2000-7
[9] M. A. Diaz, S. F. King, D. A. Ross, Nucl. Phys. B 529, 23 (1998)
[10] S. Kiyoura, M. M. Nojiri, D. M. Pierce, Y. Yamada, Phys. Rev. D 58, 075002 (1998)
[11] T. Blank, W. Hollik, Eur. Phys. J. C 24, 619 (2002). arXiv:hep-ph/0011092
[12] J. Guasch, W. Hollik, J. Solà, Phys. Lett. B 510, 211 (2001)
[13] T. Hahn, C. Schappacher, Comp. Phys. Commun. 143, 54 (2002)
[14] T. Hahn, Nucl. Phys. Proc. Suppl. B 89, 231 (2000); G. J. van Oldenborgh, J. A. M. Vermaseren, Z. Phys. C 46, 425 (1990)
[15] A. Datta, A. Djouadi, J. L. Kneur, Phys.Lett. B 509, 299 (2001). arXiv:hep-ph/0101353
[16] K. Nakamura, J. Phys. G: Nucl. Part. Phys. 37, 075021 (2010)
[17] G. K. Egüyan, M. Jurcisin, D. I. Kazakov, Mod. Phys. Lett. A 14, 601 (1999). arXiv:hep-ph/9807411
[18] J. F. Gunion, H. E. Haber, Nucl. Phys. B 272, 1 (1986)
[19] H. E. Haber, G. L. Kane, Phys. Rep. B 117, 75 (1985)
[20] M. Jurcisin, D. I. Kazakov, Mod., Phys. Lett. A 14, 671 (1999). arXiv:hep-ph/9902290
[21] S. Y. Choi et al., Eur. Phys. J. C 14, 535 (2000). arXiv:hep-ph/0002033; G. Moortgat-Pick et al., Eur. Phys. J. C 22, 563 (2001). arXiv:hep-ph/0108117
[22] T. Fritzsche, W. Hollik, Eur. Phys. J. C 24, 619 (2002). arXiv:hep-ph/0203159
[23] T. Fritzsche and W. Hollik, Nucl. Phys. Proc. Suppl. B 135, 102 (2004). arXiv:hep-ph/0407095; T. Fritzsche, in The International Conference on Linear Colliders, Paris, 2004. arXiv:hep-ph/0408307
[24] A. Dabelstein, Z. Phys. C 67, 495 (1995). arXiv:hep-ph/9409375
[25] A. Farzinnia, H. He, J. Ren, Phys. Lett. B 727, 141 (2013). arXiv:hep-ph/1308.0295
[26] W. Hollik, H. Rzehak, Eur. Phys. J. C 32, 127 (2003). arXiv:hep-ph/0305328
[27] A. Denner, Fortschr. Phys. 41, 307 (1993)
[28] G. ’t Hooft and M. Veltman, Nucl. Phys. B 153, 365 (1979); L. Jing-Jing, M. Wen-Gan, Z. Ren-You, G. Lei, J. Yi, H. Liang, Phys. Rev. D 75, 053007 (2007)
[29] G. W. T. Giele, E. W. N. Glover, Phys. Rev. D 46, 1980 (1992); W. T. Giele, E. W. Glover and D. A. Kosower, Nucl. Phys. B 403, 633 (1993)
[30] B. Y. Al-Negashi, T. A. El-Azim, I. A. M. Abdul-Mageed, IREPHY 7, 259 (2013)
[31] T. Hahn, Comput. Phys. Commun. 168, 78 (2005). arXiv:hep-ph/0404043
[32] A. Denner, S. Dittmaier, Nucl. Phys B 398, 239 (1993); M. Böhm, S. Dittmaier, Nucl. Phys B 409, 3 (1993)
[33] W. Beenakker, A. Denner, Int. J. Mod. Phys. A 9, 4837 (1994)
[34] S. M. Seif, T. A. Azim, I. A. Abdul-Mageed, AJBAS 8, 52 (2014)