INSTANTONS AND STRUCTURE OF PENTAQUARK

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Abstract

We are discussing the influence of the complex structure of the QCD vacuum on the properties of the exotic multiquark states, specially the possibility for the existence of a deeply bound pentaquark. We show that the specific spin-flavor properties of the instanton induced interaction between the quarks leads to the existence of light tri- and di-quark clusters inside the pentaquark. This strong quark correlations might be behind the anomalous properties of the pentaquark.

1 Introduction

The status of the exotic $\Theta^+$ baryon still very controversial both in theory and experiment (see reviews [1] and [2]). The instantons, strong fluctuations of gluon fields in the vacuum, play a crucial role in the realization of spontaneous chiral symmetry breaking in Quantum Chromodynamics and in the effective description of the spectroscopy for conventional hadrons. The instantons induce the ’t Hooft interaction between the quarks which has strong flavor and spin dependence, a behavior which explains many features observed in the hadron spectrum and in hadronic reactions (see reviews [3, 4, 5] and references therein).

In a recent papers [6], we have suggested a triquark-diquark model for the pentaquark based on instanton induced interaction. This interaction produces a strong attraction in flavor antisymmetric states. As a result of this dynamics quasi-bound light $ud$ and $uds$– states can be formed. Furthermore the instanton induced interaction governs the dynamics between quarks at intermediate distances, i.e. $r \approx \rho_c \approx 0.3$ fm, where $\rho_c$ is the average instanton size in the QCD vacuum. This scale is much smaller than the confinement size $R \approx 1$ fm and therefore it favors that the clusters inside the large confinement region exist.

2 Pentaquark structure in a constituent quark model with an instanton induced interaction

The most important instanton induced interaction in quark systems is the multiquark ’t Hooft interaction, which arises from the quark zero modes in the instanton field (see Fig. 1).
Figure 1: The instanton induced a) three-quark $uds$ interaction and b) two-quark $ud$, $us$, $ds$ interactions. In the figure $I$ denotes the instanton, $i, j = u, d, s, i \neq j$.

For $N_f = 3$ (Fig. 1a) and $N_c = 3$ this interaction is given by [7]:

$$L_{eff}^{(3)} = \int d\rho \ n(\rho) \left\{ \prod_{i=u,d,s} \left( m_i^{cur} \rho - \frac{4\pi}{3} \rho^3 \bar{q}_i R q_i L \right) + \frac{3}{32} \left( \frac{4}{3} \pi^2 \rho^3 \right)^2 \left[ \left( j^a_{u} j^a_{d} - \frac{3}{4} j^a_{uu} j^a_{dd} \right) \left( m_i^{cur} \rho - \frac{4\pi}{3} \rho^3 \bar{q}_s R q_s L \right) \right] + \frac{9}{40} \left( \frac{4}{3} \pi^2 \rho^3 \right)^2 d^{abc} j^a_{uu} j^b_{dd} j^c_{ss} + \text{perm.} \right\} + \frac{9}{400} \left( \frac{4}{3} \pi^2 \rho^3 \right)^3 d^{abc} j^a_{uu} j^b_{dd} j^c_{ss} + \text{perm.},$$

where, $m_i^{cur}$ is the quark current mass, $q_{R,L} = (1 \pm \gamma_5)q(x)/2$, $j_i^a = \bar{q}_i R \lambda^a q_i L$, $j_{\mu\nu}^a = \bar{q}_i R \sigma_{\mu\nu} \lambda^a q_i L$, $\rho$ is the instanton size and $n(\rho)$ is the density of instantons.

One can obtain an effective two-quark interaction induced by instantons from the three-quark interaction (1) by connecting two quark legs through the quark condensate (Fig. 1b). In the limit of small instanton size one obtains simpler formulas for effective two- and three-body point-like interactions [8, 9, 10]:

$$H_{eff}^{(2)}(r) = -V_2 \sum_{i \neq j} \frac{1}{m_i m_j} \bar{q}_i R(r) q_i L(r) \bar{q}_j R(r) q_j L(r) \left[ 1 + \frac{3}{32} (\lambda^a u \lambda^a d + \text{perm.}) \right] + \frac{9}{32} (\vec{\sigma}_u \cdot \vec{\sigma}_d \lambda^a u \lambda^a d + \text{perm.}) \right] + (R \leftrightarrow L),$$

and

$$H_{eff}^{(3)}(r) = -V_3 \prod_{i=u,d,s} \bar{q}_i R(r) q_i L(r) \left[ 1 + \frac{3}{32} (\lambda^a u \lambda^a d + \text{perm.}) \right] + \frac{9}{32} (\vec{\sigma}_u \cdot \vec{\sigma}_d \lambda^a u \lambda^a d + \text{perm.}) - \frac{9}{320} d^{abc} \lambda^a \lambda^b \lambda^c (1 - 3(\vec{\sigma}_u \cdot \vec{\sigma}_d + \text{perm.})) - \frac{9 f_{abc}}{64} \lambda^a \lambda^b \lambda^c (\vec{\sigma}_u \times \vec{\sigma}_d) \cdot \vec{\sigma}_s \right] + (R \leftrightarrow L),$$

where $m_i = m_i^{cur} + m^*$ is the effective quark mass in the instanton liquid. These forms are suitable for calculating the instanton induced contributions within a constituent quark picture.
In addition to the instanton interaction, we will take into account the perturbative one-gluon hyperfine interaction

\[ V_{OGE}^{qq} = - \sum_{i>j} \frac{b}{m_i m_j} \vec{\sigma}_i \cdot \vec{\sigma}_j \lambda^a_i \lambda^a_j, \]  

between quarks.

We use the following mass formula for the colorless ground hadronic states and color triquark and diquark states

\[ M_h = E_0^{B,M} + \sum_i N_i m_i + E_{I2} + E_{I3} + E_{OGE}, \]

where \( N_i \) is number of the quarks with flavor \( i \) in the state. In Eq. (5)

\[ E_{OGE} = \langle h|V_{OGE}|h\rangle = -\sum_{i>j} \frac{b}{m_i m_j} M_{i,j}^{OGE}, \]

\[ E_{I2} = \langle h|V_{I2}|h\rangle = -\sum_{i\neq j} \frac{a}{m_i m_j} M_{i,j}^{I2}, \]

and \( E_{I3} \) are the matrix elements of the OGE and two- and three-body instanton interactions, respectively.

After fit of the baryon and vector meson masses we have got the following values for the parameters [6]

\[ m_0 = 263 \text{ MeV}, \quad m_s = 407 \text{ MeV}, \quad E_0^M = 214 \text{ MeV}, \]

\[ E_0^{BR} = 429 \text{ MeV}, \quad a = 0.0039 \text{ GeV}^3, \quad b = 0.00025 \text{ GeV}^3. \]  

Now we estimate the mass of \( \Theta^+ \) \( udud\bar{s} \) in the model with instanton induced correlations between the quarks. One of the peculiarities of the instanton induced interaction is its strong flavor dependence, i.e., it is not vanishing only for the interaction among quarks of different flavor. For the \( ud \) diquark system the strong instanton attraction is possible only in the isospin \( I = 0 \) channel. Thus, preferably the configuration in the \( udud \) subsystem will be two separated isoscalar \( ud \) diquarks. The remaining antiquark \( \bar{s} \) can join one of the diquarks to create a triquark \( uds \) configuration in the instanton field. In this triquark state all quarks have different flavors, therefore the instanton interaction is expected to be maximal. Another peculiarity of the instanton interaction is that it is maximal in the system with the minimal spin. Thus, a pentaquark configuration with \( S = 1/2 \) \( uds \) triquark and \( ud S = 0 \) diquark should be preferable. Therefore our final triquark–diquark picture for the pentaquark with instanton forces between quarks arises as shown in Fig. 2a, where the triquark is a quasi-bound state in the field of the instanton (anti–instanton) and the diquark is a quasi-bound state in the anti–instanton (instanton) field. To avoid the coalescence of the triquark–diquark state into single \( udu\bar{d}s \) cluster configuration, where the instanton interaction is expected to be much weaker, due to the Pauli principle for the same flavor quarks in instanton field, we assume a non-zero orbital momentum \( L = 1 \) in the triquark–diquark system. The centrifugal barrier protects the clusters from getting close and prohibits the formation of the much less bound five quark cluster.
It should be mentioned, that, from our point of view, the possibility of a pentaquark configuration formed by two \textit{ud}-diquark clusters and a single antiquark \(\bar{s}\), shown in (Fig. 2b), as implied by the Jaffe–Wilczek \cite{11} and the Shuryak–Zahed \cite{12} models, is suppressed by extra powers of the instanton density, \(f = n_{\text{eff}}\pi^2\rho^4_c \approx 1/10\) in the instanton model as compared with the triquark–diquark configuration of Fig. 2a.

![Figure 2](image-url)

\(\text{Figure 2: (a) Our instanton model for the pentaquark, (b) is the instanton picture for JW and SZ models. I (A) denotes instanton (anti–instanton) configurations. Dashed lines indicate gluon lines.}\)

According to the Pauli statistics in the \textit{uds} \(I = 0\) triquark state the \textit{ud} diquark can be in \(S = 0\) spin and \(\bar{s}c\) color state (A state) or in \(S = 1\), \(6_c\) color state (B state). In KL \cite{13} only B has been considered. In fact, there is a strong mixing between the two states due to both the one-gluon and the instanton interactions, and one cannot neglect either.

Finally we have for the \textit{ud}–diquark and the \textit{uds}–triquark states the following masses (see for details \cite{6})

- diquark : \(M_{\text{di}} = 442\) MeV, \(M_{0\text{di}} = 740\) MeV,
  \(\Delta M_{\text{OGE}} = -24\) MeV, \(\Delta M_{I2} = -274\) MeV;
- triquark A : \(M_{\text{tri}} = 955\) Mev, \(M_{0\text{tri}} = 1362\) MeV,
  \(\Delta M_{\text{OGE}} = -40\) MeV, \(\Delta M_{I2} = -407\) MeV, \(\Delta M_{I3} = 40\) MeV;
- triquark B : \(M_{\text{tri}} = 859\) MeV, \(M_{0\text{tri}} = 1362\) MeV,
  \(\Delta M_{\text{OGE}} = -50\) MeV, \(\Delta M_{I2} = -513\) MeV, \(\Delta M_{I3} = 60\) MeV;
- off–diagonal AB : \(\Delta M_{\text{OGE}} = 32\) MeV, \(\Delta M_{I2} = 164\) MeV,
  \(\Delta M_{I3} = -49\) MeV, \(\Delta M_{I3} = 40\) MeV; \(\Delta M_{I3} = -49\) MeV,

where \(M_0\) is the mass of the state without the one-gluon and instanton contributions. From (8) it follows that the two-body instanton interaction gives a very large and negative contribution to the masses for all diquark and triquark states. At the same time, the one-gluon contribution is rather small. After diagonalization of the mass matrix for the A and B states, we obtain for the two mixed triquark states

\[M^{\text{tri}}_{\text{light}} = 753\text{ MeV and }M^{\text{tri}}_{\text{heavy}} = 1061\text{ MeV}.\]
The mass of light triquark cluster is smaller than the sum of the masses of the $K$ meson and the constituent $u$ and $d$ quarks. Therefore, the pentaquark cannot dissociate to the $Ku(d)$ system. Thus, the $\Theta^+$, as a system of light triquark and diquark clusters, can decay only by rearrangement of the quarks between these clusters. However, this rearrangement is suppressed by the orbital momentum $L = 1$ barrier between the clusters. As a consequence, the centrifugal barrier, provides the mechanism for a very small width in the case of the $\Theta^+$.

Let us estimate the total mass of $\Theta^+$ if built as a system of a triquark cluster with mass 753 MeV, a diquark cluster with mass 442 MeV bound together in relative $L = 1$ orbital momentum state. The reduced mass for such triquark–diquark system is $M_{\text{red}}^{\text{tri–di}} = 279$ MeV. This mass is approximately equal to the “effective” reduced mass of the strange quarks in the $\Phi$ meson, $M_{\text{red}}^{\Phi} \approx M_{\Phi}/4 = 255$ MeV. For two strange quarks, the $L = 1$ energy of orbital excitation, can be estimated from the experimental mass shift between $\Phi$ meson and the $L = 1 f_1(1420)$ state

$$\Delta E(L = 1) \approx M_{f_1(1420)} - M_{\Phi} = 400 \text{ MeV.}$$

By neglecting the small difference between the reduced mass in the strange–anti–strange quark system and the triquark–diquark system, we estimate the mass of the light pentaquark in our model as

$$M_{\Theta^+} = M_{\text{tri}}^{\text{light}} + M_{\text{di}} + \Delta E(L = 1) \approx 1595 \text{ MeV},$$

which is close to the data.

## 3 Conclusion

We have suggested in papers [6], as reported here, a triquark–diquark model for the pentaquark based on instanton induced interaction. It is shown, within the constituent quark model, that this strong interaction leads to the very light $uds\bar{s}$ triquark and $ud$ diquark color states. In order to check our suggestion we have done a sum rule calculation which incorporates the direct instanton effects [16]. We have shown that instantons lead to a large stability for the correlator of the color triquark current as a function of the Borel parameter. We observe the formation of two negative parity $uds\bar{s}$ states with spin one-half and isospin zero. These triquark states might be behind of the unusual properties of the observed pentaquark state presented here as in [6].

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