Conductance through itinerant geometrically frustrated electronic systems

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We study a two terminal electronic conductance through an AB$_2$ ring which is an example of the family of itinerant geometrically frustrated electronic systems. These systems are characterized by the existence of localized states with nodes in the probability density. We show that such states lead to distinct features in the conductance. For zero magnetic flux, the localized states act as a filter of the zero frequency conductance peak, if the contact sites have hopping probability to sites which are not nodes of the localized states. For finite flux, and in a chosen orthonormal basis, the localized states have extensions ranging from two unit cells to the complete ring, except for very particular values of magnetic flux. The conductance exhibits a zero frequency peak with a dip which is a distinct fingerprint of the variable extension of these localized states.

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I. INTRODUCTION

The conductance through molecular devices, nanowires and other nano systems has been extensively studied both theoretically and experimentally. Nano transport phenomena such as Coulomb blockade, conductance quantization, resonant tunneling, quantum interference, Aharonov-Bohm oscillations in the conductance are now well understood.

The conductance fingerprints of localized states, however, induced by the topology of a nanocluster has never been addressed as far as we know. Do these localized states inhibit the electronic transport through the cluster or is the conductance indifferent to their existence? The answer is rather complex and unexpected. In this paper we show that, in the case of the AB$_2$ ring (which is an example of the family of itinerant geometrically frustrated electronic systems), these localized states act as zero frequency conductance absorbers for zero magnetic flux, but surprisingly generate a dipped zero frequency conductance peak when magnetic flux is applied. Similar features should be observed in the conductance through other elements of the family of the itinerant geometrically frustrated electronic systems of the Lieb lattice kind, that is, systems which display localized states with nodes in their probability density.

This paper is organized in the following way: First we recall recent exact results about the eigenstates of the AB$_2$ tight-binding ring, and in particular we discuss the form of the localized states when magnetic flux is present. Next we discuss the conductance through an AB$_2$ ring for several different scenarios including different placements of the conducting leads and different values of the threading flux.

II. EXACT DIAGONALIZATION OF THE AB$_2$ CHAIN AND LOCALIZED STATES

In order to address the phenomena of coherent transport through an AB$_2$ ring, we consider a two terminal set up of one-dimensional (1D) tight-binding leads coupled to the AB$_2$ ring, as depicted in Fig. 1. Our results are easily generalized to the case of 3D leads as long as only one site of each lead contacts the cluster. We shall often focus on the case where the number of cells of the AB$_2$ ring, $N_c$, is equal to 4, and assume that each plaquette is threaded by an identical magnetic flux, $\phi$.

FIG. 1: The AB$_2$ ring is connected at sites $L$ and $R$, to semi-infinite tight binding leads via a hopping amplitude $t'$. Except where otherwise stated, the hopping amplitude of the leads is taken to be the same to that of the star, $t$. Here it is shown the situation for $N_c = 4$, a particular case we will study in detail. The magnetic fluxes threading the plaquettes and the inner ring are respectively $\phi$ and $\phi_i$.

A. Exact diagonalization

The Hamiltonian of the full system is given by

$$H_{\text{ring}} + H_{\text{leads}} + H_{LR},$$

(1)
FIG. 2: (a) In the absence of magnetic flux, rewriting the Hamiltonian in the basis of antibonding $B^{-}$, bonding $B^{+}$ and A states, one obtains a tight-binding ring of sites A and bonding BC sites (with hopping constant $\sqrt{2}t$) and a ring of decoupled anti-bonding BC states. (b) The hopping term from the left lead to a B site of the AB$_2$ ring, in the basis of antibonding BC, bonding BC and A states, becomes $t_L\sqrt{2}$. (c) For a incident particle with energy $\omega = -2t\cos(k)$, an extra transverse hopping $t_a$ to a dangling site effectively modifies the on-site energy of site $j$ to $\epsilon_j = t_a^2/\omega$.

where $H_{\text{leads}}$ is the Hamiltonian of the isolated leads, assumed to be semi-infinite,

$$H_{\text{leads}} = -t \sum_{j=1}^{\infty} \sum_{\sigma=\uparrow,\downarrow} |a_{j,\sigma}\rangle \langle a_{j+1,\sigma}|$$

$$+ |a_{j-1,\sigma}\rangle \langle a_{j,\sigma}| + \text{H.c.},$$  \hspace{1cm} (2)

where $|a_{j,\sigma}\rangle$ is a lead Wannier state at site $j$ and with spin $\sigma$. $j \in (-\infty, -1]$ correspond to left lead states while $j \in [1, \infty)$ correspond to right lead states. $H_{\text{star}}$ is the Hamiltonian for an AB$_2$ chain with $N_c$ unit cells,

$$H_{\text{ring}} = -t \sum_{j=1}^{N_c} \sum_{\sigma=\uparrow,\downarrow} e^{i\phi_{j}/2N_c} (|A_{j,\sigma}\rangle \langle B_{j,\sigma}| + |B_{j,\sigma}\rangle \langle A_{j+1,\sigma}|)$$

$$+ e^{-i\phi_{j}/2N_c} (|C_{j,\sigma}\rangle \langle A_{j,\sigma}| + |A_{j+1,\sigma}\rangle \langle C_{j,\sigma}|) + \text{H.c.},$$

where $|A_{j,\sigma}\rangle$, $|B_{j,\sigma}\rangle$, $|C_{j,\sigma}\rangle$ correspond to states on A, B and C sites, respectively, of the $j$th cell/plaquette, with spin $\sigma$. Here we have chosen a gauge such that the Peierls phases are equally distributed in the inner ring and in the outer ring. In Fig. 2 an AB$_2$ ring is shown with a magnetic flux $\phi$ threading each plaquette and a magnetic flux $\phi'$ threading the inner ring. The magnetic flux enclosed by the outer ring is $\phi_o = \phi + 4N_c\phi/4$ and we introduce an auxiliary flux $\phi'$ such that $\phi_o = \phi' + 2N_c\phi/4$, $\phi_i = \phi' - 2N_c\phi/4$. The inner sites in the AB$_2$ ring of Fig. 2 are denoted as C sites and the outer sites as B sites. Spinal sites are denoted as A sites. The hybridization between the AB$_2$ ring and the leads is given by

$$H_{LR} = - \sum_{\sigma=\uparrow,\downarrow} t_L |a_{-1,\sigma}\rangle \langle X_{L,\sigma}| + t_R |a_{1,\sigma}\rangle \langle X_{R,\sigma}| + \text{H.c.},$$

$$\hspace{1cm} (3)$$

where $t_{L,R}$ are the hopping amplitudes coupling the leads and the star and $X$ stands for an $A$, $B$ or $C$ site depending on where the left (L) and right (R) contacts are. Since

we don’t consider spin-spin interactions, each spin channel is independent and we disregard spin in the rest of the paper, without any loss of generality.

Without interactions, the tight-binding AB$_2$ chain has a flat band even in the presence of magnetic flux (Fig. 3 displays the dispersion relation of the nearest neighbor AB$_2$ chain for several values of the plaquette threading flux). The eigenvalues for an arbitrary value of flux are given by

$$\epsilon_{\text{flat}} = 0,$$

$$\epsilon_{\pm} = \pm 2t \sqrt{1 + \cos(\phi/2)\cos(\phi'/N_c + k)},$$

$$\hspace{1cm} (4)$$

where $k$ is the momentum.
Local states associated with the flat band can be written in the most compact form as standing waves in one (in the absence of magnetic flux) or two consecutive plaquettes (in the presence of magnetic flux) [22] in the particular case of zero flux, localized states are simply the anti-bonding combination of the B and C states, 

$$\text{BC}^-_j = |B_j\rangle - |C_j\rangle|/\sqrt{2},$$

and itinerant states in the AB$_2$ ring are linear combinations of A and bonding BC$_{}^+$ states, 

$$\text{BC}^+_j = |B_j\rangle + |C_j\rangle|/\sqrt{2}. $$

Rewriting the Hamiltonian in the basis of antibonding BC$_{}^-$, bonding BC$_{}^+$ and A states, one obtains a tight-binding ring of sites A and bonding BC sites (with hopping constant $\sqrt{2}t$) and a ring of decoupled anti-bonding BC states, as shown in Fig. 2a.

The number of localized states is equal to the number of rhombi and in the presence of flux, if written in the most compact form (each localized state taking place in two consecutive plaquettes) they form a non-orthogonal set of states. Orthogonalization of these set of states implies that the extension of the localized states ranges from two consecutive plaquettes to the complete ring (see Fig. 4e), except for $\phi = 0$ and for $\phi = \pi$ (in this case, the orthogonal localized states occupy only two consecutive plaquettes). This will imply a clear difference in the conductance when compared with the zero flux case. Note that a gap opens between the localized band and the itinerant bands when flux is present.

Assuming $\phi' = 0$ to simplify, the non-orthogonal localized states are of the form 

$$\langle B_j\rangle - \epsilon^{\frac{\jmath}{2}} |C_j\rangle + (\epsilon^\frac{\jmath}{2} |B_{j+1}\rangle - |C_{j+1}\rangle)$$

where the sites have been numbered clockwise in the AB$_2$ ring, that is, indices $j$ indexes the plaquettes in the AB$_2$ ring.

Since the localized states have nodes at the A sites, we can write these localized states indicating only the one-particle state amplitudes at the pairs of B and C sites is the AB$_2$ ring, that is, we can write the localized state as a list with $2N_c$ entries $(b_1, c_1, ..., b_n, c_n)$, where $b_j$ and $c_j$ are, respectively, the value of the wavefunction on site $B_j$ and $C_j$. Then our localized states are,

$$|\psi_j\rangle = \frac{1}{\sqrt{4}} \begin{pmatrix} 0, \ldots, 0, 1, b_j, -e^{-i\phi/2}, c_j, e^{-i\phi/2}, b_{j+1}, c_{j+1}, \ldots, 0 \end{pmatrix}.$$  

Note that for $\phi = 0$, we have for $|\psi_j\rangle$ that $b_j = b_{j+1}$ and $c_j = c_{j+1}$, and for $\phi = \pi$, $\langle \psi_j \psi_{j+1} \rangle = 0$. There are many possible ways of constructing an orthogonal basis for the subspace of localized states. Our results for the conductance are obviously independent of this choice. We simply use the Gram-Schmidt orthogonalization, starting with the basis

$$|\psi_1\rangle = \frac{1}{\sqrt{4}} \begin{pmatrix} 1, -e^{-i\phi/2}, -e^{-i\phi/2}, -1, 0, \ldots, 0 \end{pmatrix},$$

$$|\psi_2\rangle = \frac{1}{\sqrt{4}} \begin{pmatrix} 0, 1, -e^{-i\phi/2}, e^{-i\phi/2}, -1, 0, \ldots, 0 \end{pmatrix},$$

$$|\psi_{N_c}\rangle = \frac{1}{\sqrt{4}} \begin{pmatrix} e^{-i\phi/2}, -1, 0, \ldots, 0, 1, -e^{-i\phi/2} \end{pmatrix}.$$  

For such a basis we have

$$\langle \psi_i | \psi_j \rangle = \delta_{i,j} + \frac{\cos \phi/2}{2} (\delta_{i-1,j} + \delta_{i+1,j}).$$

For simplicity, let us define the support of a wavefunction, denoted by supp, to be those site where the wavefunction is non-zero. Then we have supp $|\psi_j\rangle = \{B_j, C_j, B_{j+1}, C_{j+1}\}$.

Let $\{|\phi_j\rangle\}_{j=1}^{N_c}$ denote the orthonormalized basis after the G-S procedure, defined by the recursive expression,
We focus on $\phi \neq \pi$ (for in that case, the basis is already orthonormalized) and begin by making $|\phi_1\rangle = |\psi_1\rangle$, which implies supp $\phi_1 = \{B_1, C_1, B_2, C_2\}$. Then $|\phi_2\rangle = |\psi_2\rangle = (\phi_1|\psi_2\rangle|\phi_1\rangle$. If $\phi_1 \neq 0$, supp $\phi_2 = \{B_1, C_1, B_2, C_2, B_3, C_3\}$. We then have $|\phi_1\rangle = |\psi_3\rangle - |\phi_2|\psi_3\rangle|\phi_2\rangle$. Note that $|\phi_1\rangle$ and $|\psi_3\rangle$ have disjoint support, hence $\phi_1|\psi_3\rangle = 0$. Also, $\phi_2|\psi_3\rangle \propto |\psi_2\rangle \neq 0$. Since supp $|\psi_3\rangle = \{B_3, C_3, B_4, C_4\}$ and supp $\phi_2 = \{B_1, C_1, \ldots, B_3, C_3\}$, and since, there is no destructive interference on sites $B_3$ and $C_3$ (it is a simple exercise to show this), $\phi_3 = \{B_1, C_1, \ldots, B_4, C_4\}$. Continuing the above procedure we finally arrive at

$$\text{supp } |\phi_j\rangle = \{B_1, C_1, \ldots, B_{j+1}, C_{j+1}\}.$$  

and therefore the extension of the orthogonalized localized states (constructed this way) ranges between two consecutive plaquettes and the full AB$_2$ ring. This is illustrated schematically in Fig. 4c.

However, as we have already mentioned, one has two exceptions: i) for $\phi = 0$, the states $|\alpha_j\rangle = (0, \ldots, 0, 1, -1, 0, \ldots, 0)$ already constitute an orthogonal set of localized states for $\phi = 0$ as stated in the previous paragraph; ii) for $\phi = \pi$, $|\psi_j|\psi_{j+1}\rangle = 0$ are orthogonal and in this case the range of the localized states in their most compact form is just two plaquettes.

There are many possible ways of constructing an orthogonal basis for the subspace of localized states. It must be stressed that our results for the conductance are obviously independent of this choice.

Using the construction for localized states $\alpha_j$ it is easy to extend some of the results presented in this paper to geometries other than the AB$_2$ geometry. To make this more concrete let us give some examples. Let us start by considering an AB$_2$ chain with an arbitrary number of cells. Assume, for now, that the flux through each cell has the same value, a situation we call ferromagnetic (shown in Fig. 4b). Then, localized states occupying only two cells can be found for an arbitrary value of flux (albeit non orthogonal, except when $\phi = \pi$) while for zero flux one can find localized states occupying only one cell as shown in Fig. 4a. Now consider a situation where the magnetic flux through each plaquette is symmetric to the one threading its neighboring cells, a situation we call anti-ferromagnetic. Then a similar state to the situation above can be found as is shown in Fig. 4e. For this particular case, using this construction, we can find localized states that occupy two cells. However, these states form an orthonormal basis only for $\phi = \pi$ and for the ferromagnetic flux situation, as can be readily seen calculating the overlap between neighboring states. Let $|\psi_j\rangle$ be the state localized in the $j$th and $(j+1)$th cells. For the ferromagnetic situation the overlap between neighboring states is $\langle \psi_{j+1}|\psi_j\rangle = \frac{\cos(\phi/2)}{2}$ while for the anti-ferromagnetic situation one has $\langle \psi_{j+1}|\psi_j\rangle = -\frac{1}{2}$.

Note that this flux threading each cell is not necessarily an external flux, since it may be generated by the spin of an atom/molecule, embedded into the chain as is the case of some copper oxide systems, namely CuO$_2$ chains. 26,29

If the flux through each plaquette is distinct, but repeats every $N$ cells (Fig. 4d shows the situation for $N = 2$), one can also use the same construction to find localized states. In this case however, one must consider $2N$ adjacent cells instead of 2, as before, and we will find a localized states that extend through $2N$ cells. As before, these are not necessarily orthonormal, but can be made so by using Gram-Schmidt orthonormalization. In the extreme case, where there is no periodicity, translational invariance is obviously broken and our procedure will give us an extended state instead. A particular case of the $N = 2$ situation, with $\phi_1 = 2\phi_2$ has been studied in 30.

### III. CONDUCTANCE THROUGH THE AB$_2$ RING

In this section we discuss the conductance through the AB$_2$ ring. We will begin by addressing the case without magnetic flux. Since no two-particle interactions are considered in this paper, the transmission probability $|t(\omega)|^2$ for an incident particle with momentum $k$ and energy $\omega = -2\cos(k)$ can be calculated using quantum scattering theory and is given by the following expression:

$$|t(\omega)|^2 = 4t_l^2 t_R^2 \sin^2 k \langle R|e_1\hat{I}_s - H_s + e^{ik}(t_L^2|L\rangle\langle L| + t_R^2|R\rangle\langle R|)|^{-1}|L\rangle|^2, \quad (10)$$

where the inverse is to be found within the subspace of the cluster sites (in our case, the AB$_2$ ring) positions and $\hat{I}_s$ is the identity operator in that subspace. This equation includes the effect of the coupling of the ring to the leads as modifications of the on-site energies of sites $L$ and $R$. If the conductance is normalized by the conductance of an ideal one dimensional system, $G_0 = e^2/\pi \hbar$, then the conductance is given by the transmission probability at the chemical potential. In what follows, we will always deal with this normalized conductance, i.e., transmission probability.

In Fig. 5, we show several profiles of the conductance through the AB$_2$ ring with four unit cells as function of the energy of the incident electron (or chemical potential of the leads) or as function of a potential $V_{\text{gate}}$ applied...
FIG. 5: Normalized conductance through the AB$_2$ ring as function of the energy of the incident electron (or chemical potential of the leads) and as function of $V_{\text{gate}}$ for several positions of the leads. The positions of the leads are shown on the top figures. We show beside the AB$_2$ circuits, figures of equivalent systems which have exactly the same conductance profiles. Parameters: $t_L = t_R = 0.3t$.

to the AB$_2$ ring. These profiles correspond to certain positions of the leads which are shown at the top of the Fig. 5a, Fig. 5b and Fig. 5c. In these figures we also include diagrams of equivalent systems, that is, systems that exhibit exactly the same conductance profiles as the AB$_2$ ring.

In the case of Fig. 5b, the leads are connected to sites A, therefore the anti-bonding BC "sites" can be ignored since they are completely decoupled from the leads. The remaining "ring" of sites A and bonding BC sites form a tight binding ring. Therefore, if the contacts are sites A, the conductance is exactly the same as that of the equivalent tight-binding ring (with hopping constant $\sqrt{2}t$).

For small coupling between the leads and the AB$_2$ ring, the conductance has peaks when the chemical potential coincides with any of the system eigenvalues of the AB$_2$ ring, due to resonant tunnelling. These peaks have the Breit-Wigner shape. In Fig. 5b, three peaks A, B and C are observed in $G(V_{\text{gate}})$ in a potential interval corresponding to the bandwidth of the leads (the chemical potential of the leads is equal to zero). The same peaks should also be observed in the $G(\omega)$-plot of Fig. 5, where $V_{\text{gate}} = 0$ and the chemical potential (or equivalently the energy $\omega$ of the incident particle) is varied from the bottom of the leads band to the top. However peaks A and B are absent because they correspond to the bottom and top energies of the leads bands and the particle velocity is zero for these energies.

If the left contact is a site B or C (let us assume it is a site B) and the right contact is a site A, as in the case of Fig. 5b, the conductance profile is the same as that of a tight-binding ring but with the $\omega = 0$ peak absent. This absence reflects the fact that the hopping term from the left lead to a B site of the AB$_2$ ring, in the basis of
antibonding BC\(^-\), bonding BC\(^+\) and A states, becomes a hopping between the left lead and a bonding state bonding BC\(^+\) and a hopping to a localized state BC\(^-\), both with a smaller hopping constant \(t_L/\sqrt{2}\). Since this localized state is decoupled from all other states of the ring, it only leads to a reflected wave back into the left lead. For \(\omega = 0\), this reflected wave interferes destructively with the incident wave and one can say the localized state BC\(^-\) acts as a conductance absorber for frequencies close to \(\omega = 0\) (in close analogy with \(\lambda/4\) sound absorbers). The absence of the BC\(^-\) acts as a conductance absorber for frequencies close to \(\omega = 0\) (in close analogy with \(\lambda/4\) sound absorbers).

If both the left and right contacts are sites B (or C) as shown in Fig. 2c, an analogous reasoning applies and the system is equivalent to a linear chain with a dangling site connected to leads but with two dangling sites, one at the end of each lead. Again, localized states act as a filter of the incident wave, and one can say the localized state is decoupled from all other states of the ring, it only leads to a reflected wave back into the left lead. For \(\omega = 0\), this reflected wave interferes destructively with the incident wave and one can say the localized state BC\(^-\) acts as a conductance absorber for frequencies close to \(\omega = 0\) (in close analogy with \(\lambda/4\) sound absorbers).

This result can be explained with an analogy with a quarter wavelength anti-reflection coating (see the labeling of the sites in the inset of Fig. 7a), the maximum starts at one, for several choices of contacts positions. When the peak is absent (otherwise the peak is absent) and shows distinct behavior as function of the positions of the contact sites and as function of the magnetic flux. As shown in the inset of Fig. 7a, the peak maximum decays quasi-exponentially as a function of the contact sites distance, except for the first two distances, where the conductance peak maximum remains the same. Such behavior is also visible in Fig. 8 where the logarithm of the conductance for small frequencies oscillates for small \(\phi\) and goes smoothly to zero as \(\phi\) approaches zero.
The oscillations near $\phi = 0$ reflect the contribution to the conductance of the itinerant states which oscillates as a consequence of the Aharonov-Bohm effect due to the varying flux threading the inner region of the $AB_2$ ring (an uniform field was applied to the $AB_2$ cluster). These oscillations disappear as $\phi$ grows due to the larger gap between the itinerant bands and the $\omega = 0$ energy. Note that localized states do not "feel" this inner flux, that is, their energy is independent of this field and therefore do not contribute to an Aharonov-Bohm effect.

When the contacts are the sites $B_1$ and $C_2$, contrasting behavior occurs and the maximum approaches zero when $\phi$ is small and tends to 0.25 when $\phi$ goes to $\pi$. For larger distance between contacts, the graphs of the maximum of the $\omega = 0$ conductance peak exhibit dome-like profiles (see Fig. 7D(b)), with the peak maximum growing from near zero when $\phi$ is small, reaching a maximum value and decreasing to zero when $\phi$ approaches $\pi$.

These results can explained recalling our previous discussion of the extension of localized states when the flux is finite. Since this extension ranges from two unit cells to the full ring, and ignoring the itinerant states of the ring which are energetically far from the $\omega = 0$ energy region, the conductance is only finite if one has localized states that extend from the left contact to the right contact in the $AB_2$ ring. More precisely, we can divide the localized states in the following way: states I that extend from the left contact to the right contact, that is, that have finite wavefunction amplitudes at the sites L and R of the $AB_2$ ring; states II that have finite wavefunction amplitudes at the site L but not at the site R of the $AB_2$ ring; states III that have finite wavefunction amplitudes at the site R but not at the site L of the $AB_2$ ring; states IV that have zero wavefunction amplitudes at both the sites L and R of the $AB_2$ ring. Note that the choice of basis for the subspace of localized states influences the number of states in each of these groups, but the explanation for the conductance results remains the same. The larger the extension of the localized states, the smaller the wavefunction amplitude at the contact sites, and consequently the smaller the effective hopping between the extremities of the leads and the localized state. So, our system is equivalent to that displayed in the top diagram of Fig. 9a. The hopping constants between the leads (smaller, red spheres) and the localized states (larger, black spheres) are in general different but all these hopping can be simplified and the system can be reduced to that shown in the bottom diagram of Fig. 9a.

In fact, several dangling sites (states II) contribute to the on-site energy of the site at the end of the left lead and
One should recall that the non-orthogonal localized states can be found of the form \( |\psi_j\rangle = 1/2 (|B_j\rangle - i |C_j\rangle) + (i |B_{j+1}\rangle - |C_{j+1}\rangle) \). The overlap between consecutive localized states is equal to \( \cos(\phi/2)/2 \), so it is zero whenever \( \phi = \pi \), and 1/2 when \( \phi = 0 \) (the latter value implies that shorter and orthogonal localized states can be found of the form \( BC_j = 1/\sqrt{2}(|B_j\rangle - |C_j\rangle) \)). We consider only the mean evolution of the conductance, that is, the dependence of the conductance remaining if the oscillations due to the Aharonov-Bohm effect are removed. This behavior consists of the following: if the contacts are the sites \( B_1 \) and \( C_1 \), the maximum of the conductance is one for zero flux and with increasing magnetic flux, the conductance decreases and becomes zero for flux equal to \( \pi \). Note that for \( \phi = \pi \), the localized states, \( |\psi_j\rangle = 1/2 (|B_j\rangle - i |C_j\rangle) + (i |B_{j+1}\rangle - |C_{j+1}\rangle) \), are orthogonal and both leads couple to only two of these states, \( |\psi_1\rangle \) and \( |\psi_N\rangle \). That is, we have only two states of type I and all other localized states are of type IV. The transport through the cluster is given by the transfer terms to these localized states of the form \( \langle \psi_1 | H | 0 \rangle \) which collected (omitting the hopping terms in the leads) give rise to \( (-t_1/2)|\psi_1| + i (\psi_{N+1}) + (-t_R/2) |N+1 \rangle \). But \( (|\psi_1| + i |\psi_{N+1}\rangle + |\psi_{N+1}|) \) and \( |\psi_1| + i |\psi_{N+1}\rangle + |\psi_{N+1}| \) are orthogonal bras, therefore the left and right leads are effectively decoupled and the transmittance is zero. A similar reasoning can be followed when \( \phi \) approaches zero. In this case the leads couple to only one localized state, \( BC_1^* = 1/\sqrt{2}(B_1 - C_1) \), that is, we have one state of type I and no dangling sites, so the transmittance approaches one.

If the contacts are the sites \( B_1 \) and \( C_2 \), the maximum of the conductance is zero for zero flux and, with increasing magnetic flux, the conductance increases and becomes \( 1/4 \) for flux equal to \( \pi \). The fact that the conductance maximum approaches zero as \( \phi \) goes to zero is common to all other contact possibilities with exception of the previous one and reflects a similar argument, that is, the left lead couples to only one localized state, \( BC_1^* = 1/\sqrt{2}(B_1 - C_1) \) and the right lead couples only to one other localized state which is orthogonal to the former, and consequently the transmittance is zero. The fact the conductance goes to \( 1/4 \) when the flux goes to \( \pi \) can also be justified as before, collecting the trans-
fer integrals and one has \((-t_L/2) |0\rangle \langle \psi_1 | + i \langle \psi_{N+1} | - (t_R/2) |N+1\rangle \langle \psi_1 | + (\psi_2 | + H.c., and this corresponds to the bottom diagram displayed in Fig. 9d with \(t_3 = it_L/2, t_4 = -it_L/2\), \(t_2 = t_L/2\), and \(t_3 = t_R/2\). Since we considered \(t_L = t_R\), all these hopping constants are equal in absolute value and therefore the conductance is equal to 1/4 in agreement with what is shown in Fig. 9b. Note that the phase terms are irrelevant at the dangling sites.

If one of the contacts is the site B and the other is a C site with \(j \neq N_c, 1, 2\), the maximum of the conductance goes to zero as the flux goes to zero and with increasing magnetic flux, the conductance increases, reaches a maximum (this maximum becomes smaller as the distance between contacts increases) and goes again to zero when the flux approaches \(\pi\), reflecting the fact that the orthogonal localized states are all two unit cells long.

IV. CONCLUSIONS

We have shown that localized states in itinerant geometrically frustrated electronic systems generate rather striking behavior in the two terminal electronic conductance. In the absence of magnetic flux, the localized states act as a filter of the zero frequency conductance peak (we suggested an analogy with \(\lambda/4\) sound absorbers), if there is a finite hopping probability between the leads contact sites and the localized states. In contrast, when magnetic flux is present, some localized states contribute to the appearance of a zero frequency conductance peak while other localized states act as a conductance absorber, and as a consequence, the conductance exhibits a zero frequency peak with a dip.

We have shown that such different roles of the localized states are due to the fact that the presence of magnetic flux implies that any orthogonal basis of the subspace of localized states is composed of localized states with variable extensions (ranging from two unit cells to the complete ring, in the case of the \(AB_2\) ring studied in this paper). Such peculiar dipped peak fixed at the localized states energy, even when magnetic flux is varied, is a distinct fingerprint of the existence of localized states in itinerant geometrically frustrated electronic systems. Furthermore, depending on the distance between contact sites, different profiles for the maximum of the dipped conductance peak as a function of the magnetic flux have been obtained, and this implies that the two terminal conductance can be used as a probe of the localized states spatial dependence.

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