A Supersymmetric Enhancement of $\mathcal{N} = 1$ Holographic Minimal Model

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Abstract

By studying the $\mathcal{N} = 1$ holographic minimal model at the “critical” level, we obtain the lowest $\mathcal{N} = 2$ higher spin multiplet of spins $\left(\frac{3}{2}, 2, 2, \frac{5}{2}\right)$ in terms of two kinds of adjoint fermions for generic $N$. Their operator product expansions (OPEs) are determined. The OPEs between the above higher spin multiplet and the second $\mathcal{N} = 2$ higher spin multiplet of spins $\left(3, \frac{7}{2}, \frac{7}{2}, 4\right)$ are obtained. The corresponding Vasiliev’s oscillator formalism with a matrix generalization on $AdS_3$ higher spin theory, in the extension of $OSp(2|2)$ superconformal algebra, is obtained. Under the large $N$ (equivalently large central charge) limit, the OPEs in the extension of $\mathcal{N} = 2$ superconformal algebra in two dimensions provide the asymptotic symmetry algebra in the $AdS_3$ higher spin theory.
1 Introduction

It is known, in [1, 2], that WZW primary fields are also Virasoro primary fields. The Virasoro zeromode acting on the primary state is proportional to the quadratic Casimir operator of the finite Lie algebra. The conformal weight (or spin) of primary fields [1, 2] is given by the one half times the quadratic Casimir eigenvalues divided by the sum of the level and the dual Coxeter number of the finite Lie algebra. When the level is equal to the dual Coxeter number in the adjoint representation, the conformal weight becomes one half 1. For the diagonal coset model described in the subsection 7.3 of [1], the spin-$\frac{3}{2}$ current, which is $\mathcal{N} = 1$ supersymmetry generator and commutes with the diagonal spin-1 current, can be determined [3] and is given by the linear combination of two kinds of spin-1 currents and adjoint fermions. See, for example, (7.50) of [1]. For $SU(2)$ case, this leads to the coset construction of the $\mathcal{N} = 1$ superconformal algebra studied in [9]. For $SU(3)$ case, the coset construction of $\mathcal{N} = 1 W_3$ algebra is obtained in [7, 8, 9, 10]. Moreover, for $SU(N)$, the $\mathcal{N} = 1$ higher spin multiplets are found in [11]. Although the $\mathcal{N} = 1$ extension of the bosonic coset models is obtained from the particular level condition, we can also consider other cases where there are $\mathcal{N} = 2$ extension and $\mathcal{N} = 3$ extension respectively 2.

The Gaberdiel and Gopakumar proposal [17] is the duality between the higher spin gauge theory on $AdS_3$ space [18, 19] and the large $N$ 't Hooft limit of a family of $W_N$ minimal models. See also [20, 21, 22, 23]. This is the natural analogue of the Klebanov and Polyakov duality [24] relating the $O(N)$ vector model in three-dimensions to a higher spin theory on $AdS_4$ space. Then the obvious generalization of [17] is to consider the Klebanov and Polyakov duality in one dimension lower. By replacing the $SU(N)$ group by $SO(2N)$ or $SO(2N + 1)$, the relevant coset model is described in [25, 26]. The $\mathcal{N} = 1$ and $\mathcal{N} = 2$ extensions of the (bosonic) orthogonal coset model are obtained in [27]. See also the relevant work [28]. Moreover, we can start with the $\mathcal{N} = 1$ coset model and by putting the above level condition we obtain the $\mathcal{N} = 2$ extension of this supersymmetric coset model.

1 For example, the eigenvalue of quadratic Casimir operator is given by $2N$ and the dual Coxeter number is $N$ for $SU(N)$. We are left with the overall numerical factor $\frac{1}{2}$ which is equal to the conformal weight (or spin) of adjoint fermion.

2 By taking the adjoint spin-$\frac{1}{2}$ fermions in the second factor appearing in the numerator of the diagonal coset model [1], the coset construction of the $\mathcal{N} = 2$ superconformal algebra is obtained [12] and the $\mathcal{N} = 2$ higher spin currents are observed and determined in [13, 14]. The $\mathcal{N} = 3$ extension from $\mathcal{N} = 2$ coset model has been studied in [15, 16].
In this paper, we consider the following coset model \([29]\) at the “critical” level,
\[
\frac{G}{H} = \hat{SO}(2N + 1)_k \oplus \hat{SO}(2N)_1 \quad \text{with} \quad k = 2N - 1.
\] (1.1)
For \(SO(2N + 1)\), the quadratic Casimir eigenvalue for the adjoint representation is given by \((2N - 1)\) which is the same as the dual Coxeter number of \(SO(2N + 1)\). The central charge of the coset model at the critical level is given by
\[
c = \frac{3Nk}{k + 2N - 1} \bigg|_{k=2N-1} = \frac{3N}{2}.
\] (1.2)

The infinity limit of the central charge is equivalent to the infinity limit of \(N\). There exist additional adjoint fermions living in the first factor, as well as the adjoint fermions in the second factor, of the group \(G\) in (1.1).

We would like to construct the additional higher spin currents and their operator product expansions (OPEs) in the above coset model (1.1). We expect that the following “minimal” \(\mathcal{N} = 2\) extension of the \(\mathcal{N} = 1\) higher spin currents studied in [29, 30, 31]
\[
(1, \frac{3}{2}, \frac{3}{2}, 2); (\frac{3}{2}, 2, 2, \frac{5}{2}), (\frac{3}{2}, \frac{7}{2}, \frac{7}{2}, 4), (\frac{7}{2}, 4, 4, \frac{9}{2}), (5, \frac{11}{2}, \frac{11}{2}, 6), \cdots,
\]
\[
(n - \frac{1}{2}, n, n, n + \frac{1}{2}), (n + 1, n + \frac{3}{2}, n + \frac{3}{2}, n + 2), \cdots,
\] (1.3)
with \(n = 2, 4, 6, \cdots\). The first multiplet is the generators of well known \(\mathcal{N} = 2\) superconformal algebra [12]. The first two components of each \(\mathcal{N} = 2\) (higher spin) multiplet in (1.3) are new and superpartners of the last two components. The lowest \(\mathcal{N} = 2\) higher spin multiplet of spins \((\frac{3}{2}, 2, 2, \frac{5}{2})\) in terms of two kinds of adjoint fermions for generic \(N\) is obtained. Their operator product expansions (OPEs) are also determined. The OPEs between this \(\mathcal{N} = 2\) higher spin multiplet and the second \(\mathcal{N} = 2\) higher spin multiplet of spins \((3, \frac{7}{2}, \frac{7}{2}, 4)\) are described.

Furthermore, we construct the generators of the higher spin algebra in terms of oscillators corresponding to the first two higher spin multiplets in (1.3), from the OPEs realized in the coset model (1.1). Some of their (anti)commutators are given explicitly. Due to the presence of higher spin-\(\frac{3}{2}\) current in (1.3), the additional degree of freedoms is needed because the supersymmetry generator of spin-\(\frac{3}{2}\) has also the same spin-\(\frac{3}{2}\) and there is no room for the higher spin-\(\frac{3}{2}\) generator because they share a linear term in the oscillator and we cannot differentiate them each other. This requires a matrix generalization of the Vasiliev theory described in [15].

In section 2, the four currents of \(\mathcal{N} = 2\) superconformal algebra are realized in terms of the two kinds of adjoint fermions in the coset model (1.1).
In section 3, we analyze the first four $\mathcal{N} = 2$ higher spin multiplets in (1.3) for fixed $N = 4$.

In section 4, we determine the lowest $\mathcal{N} = 2$ higher spin multiplet in terms of two adjoint fermions for generic $N$, and obtain its OPE in $\mathcal{N} = 2$ superspace (and its component results are given also). By taking the large central charge limit (1.2), the asymptotic symmetry algebra $[32, 33, 34, 35, 36, 37]$ of the (matrix extension of) Vasiliev theory can be obtained. Moreover, the OPE between the lowest $\mathcal{N} = 2$ higher spin multiplet and the second $\mathcal{N} = 2$ higher spin multiplet is obtained using the Jacobi identity.

In section 5, after reviewing the “wedge” algebra of $\mathcal{N} = 2$ superconformal algebra, we construct the generators of the $OSp(2|2)$ higher spin algebra in terms of oscillators $[18, 19, 38, 39]$ and their algebra. The matching with the findings in the section 4 under the large $c$ limit together with wedge condition is analyzed.

In section 6, we summarize what we have found in this paper and some related open problems are given.

In Appendices A-F, we present some details described in the previous sections. The Thielemans package [40] is used.

2 The four currents of the $\mathcal{N} = 2$ superconformal algebra

We construct the four currents of the $\mathcal{N} = 2$ superconformal algebra in the coset model (1.1) along the line of [14].

2.1 Kac-Moody spin-1 currents

For the diagonal coset model in (1.1) [29], the spin-1 current $J^A(z)$ with level $k$ and the spin-$\frac{1}{2}$ current $\chi^i(z)$ (whose spin-1 current has the level 1) generate the affine Lie algebra $G = \widehat{SO}(2N+1)_k \oplus \widehat{SO}(2N)$. The adjoint indices $A, B, \cdots$ of $SO(2N+1)$ run over $1, 2, \cdots, N(2N+1)$ and the vector indices $i, j, \cdots$ of $SO(2N)$ run over $1, 2, \cdots, 2N$ which can be relabeled by adding the number $N(2N-1)$ respectively.\footnote{The adjoint indices $A, B, \cdots$ of $SO(2N+1)$ can be further decomposed into the $SO(2N)$ adjoint indices $a, b, \cdots = 1, 2, \cdots, N(2N-1)$ and the $SO(2N)$ vector indices $i, j, \cdots = 1 + N(2N-1), \cdots, 2N + N(2N-1) = N(2N+1)$. That is, the $SO(2N+1)$ adjoint indices run over $A, B, \cdots = 1, 2, \cdots, N(2N-1), 1 + N(2N-1), \cdots, 2N + N(2N-1)$.} The diagonal spin-1 current $(J^a + K^a)(z)$ with level $(k + 1)$, where $K^a(z)$ is the quadratic in the above spin-$\frac{1}{2}$ current (2.2), generates the affine Lie algebra $H = \widehat{SO}(2N)_{k+1}$.  

3
Due to the condition for the level $k = 2N - 1$, we can introduce the fermions $\psi^A(z)$ living in the first factor of $G$. Then we consider the two kinds of fermion fields $\psi^A(z)$ and $\chi^i(z)$ that satisfy the following OPEs

\[
\begin{align*}
\psi^A(z) \psi^B(w) &= -\frac{1}{(z-w)} \frac{1}{2} \delta^{AB} + \cdots, \quad A \equiv (a, i), \\
\chi^i(z) \chi^j(w) &= -\frac{1}{(z-w)} \frac{1}{2} \delta^{ij} + \cdots.
\end{align*}
\tag{2.1}
\]

The corresponding Kac-Moody spin-1 currents are obtained as follows:

\[
\begin{align*}
J^a(z) &\equiv f^{aBC} \psi^B \psi^C(z) = f^{abc} \psi^b \psi^c(z) + f^{aij} \psi^i \psi^j(z), \\
J^i(z) &\equiv f^{iBC} \psi^B \psi^C(z) = 2 f^{ija} \psi^j \psi^a(z), \\
K^a(z) &\equiv f^{aij} \chi^i \chi^j(z).
\end{align*}
\tag{2.2}
\]

Note that we have $f^{ijk} = 0 = f^{iab}$.

We can easily observe that they obey the following nontrivial OPEs from (2.1) and (2.2)

\[
\begin{align*}
J^a(z) J^b(w) &= -\frac{1}{(z-w)^2} (2N - 1) \delta^{ab} + \frac{1}{(z-w)} f^{abc} J^c(w) + \cdots, \\
J^a(z) J^i(w) &= \frac{1}{(z-w)} f^{aij} J^j(w) + \cdots, \\
J^i(z) J^j(w) &= -\frac{1}{(z-w)^2} (2N - 1) \delta^{ij} + \frac{1}{(z-w)} f^{ija} J^a(w) + \cdots, \\
K^a(z) K^b(w) &= -\frac{1}{(z-w)^2} \delta^{ab} + \frac{1}{(z-w)} f^{abc} K^c(w) + \cdots. 
\end{align*}
\tag{2.3}
\]

The spin-1 current $J^A(z)$ has the level $(2N - 1)$. By adding the first and last in (2.3), the diagonal spin-1 current $(J^a + K^a)(z)$ has the level $2N$.

### 2.2 Four currents in terms of fermions

We present the four currents of $\mathcal{N} = 2$ superconformal algebra.

- Coset spin-1 current

By using the $SO(2N)$ invariant tensor of rank 2, we can obtain the spin-1 current as follows:

\[
J(z) = i \delta^{ij} \psi^i \chi^j(z) = i \psi^i \chi^i(z).
\tag{2.4}
\]

It is obvious that the OPE between the spin-1 current and itself is given by

\[
J(z) J(w) = \frac{1}{(z-w)^2} c \frac{3}{3} + \cdots, \quad c = \frac{3}{2} N.
\tag{2.5}
\]
The overall factor in (2.4) is fixed by requiring the central term of the OPE $J(z)J(w)$ should behave as in (2.5). We easily check that the coset spin-1 current has no singular terms with the diagonal spin-1 current

$$(J^a + K^a)(z)J(w) = + \cdots. \quad (2.6)$$

In particular, the combination of the two fermions has the nonzero $U(1)$ charge, $\pm \frac{1}{2}$, associated with the spin-1 current

$$J(z)(\psi^i \pm i\chi^i)(w) = \pm \frac{1}{(z - w)} \frac{1}{2} (\psi^i \pm i\chi^i)(w) + \cdots. \quad (2.7)$$

Similarly, we have the regular term in the OPE between the spin-1 current $J(z)$ and the fermion $\psi^a(w)$

$$J(z)\psi^a(w) = + \cdots. \quad (2.8)$$

From the $U(1)$ charges of the fermions in (2.7) and (2.8), we can analyze the higher spin currents for fixed $U(1)$ charges in terms of fermions.

- **Coset spin-$\frac{3}{2}$ currents**

  The spin-$\frac{3}{2}$ currents should satisfy, as in (2.6),

  $$(J^a + K^a)(z)G^\pm(w) = + \cdots. \quad (2.9)$$

  Moreover, the defining equation with the above spin-1 current is given by

  $$J(z)G^\pm(w) = \pm \frac{1}{(z - w)} G^\pm(w) + \cdots. \quad (2.10)$$

  Then we obtain the following spin-$\frac{3}{2}$ currents of $U(1)$ charges $\pm 1$ (2.10)

  $$G^\pm(z) = \frac{1}{4\sqrt{2N - 1}} \left[ \mp i \psi^i J^i \pm 2i \psi^a K^a + 2\chi^i J^i \right](z). \quad (2.11)$$

  We can obtain the $\mathcal{N} = 1$ spin-$\frac{3}{2}$ current by adding these two spin-$\frac{3}{2}$ currents.

- **Coset spin-2 current**

  The spin-2 stress energy tensor (which satisfies the regular condition with the coset spin-1 current as in (2.6) and (2.9)) can be obtained from the difference between the ones in the group $G$ and the one in the subgroup $H$ with the correct coefficients

  $$T(z) = -\frac{1}{4(2N - 1)} (J^a J^a + J^i J^i)(z) - \frac{1}{2(2N - 1)} K^a K^a(z)$$

  $$+ \frac{1}{4(2N - 1)} (J^a + K^a)(J^a + K^a)(z). \quad (2.12)$$
The central charge is given by (2.5).

Therefore, the four currents of $\mathcal{N} = 2$ superconformal algebra in the coset model are summarized by (2.4), (2.11), and (2.12). In $\mathcal{N} = 2$ superspace, they can be organized by a single $\mathcal{N} = 2$ stress energy tensor as follows:

$$T(Z) = J(z) + \theta G^+(z) + \bar{\theta} G^-(z) + \theta \bar{\theta} T(z).$$

(2.13)

The defining OPEs between the four currents in (2.13) are given by (A.1).

3 The $\mathcal{N} = 2$ higher spin currents for fixed $N = 4$

In this section, we describe the $\mathcal{N} = 2$ higher spin multiplets in the coset model for $N = 4$. We show the existence of higher spin-$\frac{3}{2}$ (primary) current which will belong to the lowest $\mathcal{N} = 2$ higher spin multiplet. We observe the presence of other higher spin currents which live in other $\mathcal{N} = 2$ higher spin multiplets.

3.1 The first (lowest) $\mathcal{N} = 2$ higher spin multiplet

It is known in [29, 30, 31] that the lowest $\mathcal{N} = 1$ higher spin multiplet contains the higher spin-2 current and the higher spin-$\frac{5}{2}$ current. Due to the presence of additional fermions $\psi^a(z)$ and $\psi^i(z)$, there will be a possibility to have the additional lower $\mathcal{N} = 1$ higher spin multiplet of spins $\frac{3}{2}$ and 2 which is a superpartner of the above $\mathcal{N} = 1$ higher spin multiplet. Then it is natural to check whether there exists a higher spin-$\frac{3}{2}$ current or not in the coset model.

Let us consider the following most general spin-$\frac{3}{2}$ current with the unknown $U(1)$ charge $q$ and unknown coefficients as follows along the line of [16]:

$$W_{q}^{(\frac{3}{2})}(z) = C_1^{ABC} \psi^A \psi^B \psi^C + C_2^{ij} \chi^i \psi^B \psi^C + C_3^{Aij} \psi^A \chi^j \chi^k + C_4 \chi^i \chi^j \chi^k + C_5 \partial \psi^A + C_6 \partial \chi^i,$$

(3.1)

where the undetermined coefficients can be $SO(2N+1)$ or $SO(2N)$ (of $G$ in the coset model) invariant tensors.

To find this higher spin-$\frac{3}{2}$ current (3.1), the following conditions appearing in (B.1) together with the regular condition in the OPE with the diagonal spin-1 current (see [41, 1] for the GKO [6] coset construction) are used

$$T(z) \left. \frac{W_{q}^{(\frac{3}{2})}(w)}{(z-w)^2} \right|_{w} = \frac{3}{2} W_{q}^{(\frac{3}{2})}(w),$$

7
\[ T(z) W_q^{(\frac{3}{2})}(w) \bigg|_{z = w} = \partial W_q^{(\frac{3}{2})}(w), \]
\[ J(z) W_q^{(\frac{3}{2})}(w) \bigg|_{z = w} = q W_q^{(\frac{3}{2})}(w), \]
\[ (J^a + K^a)(z) W_q^{(\frac{3}{2})}(w) = + \cdots. \tag{3.2} \]

The first two conditions come from the primary field under the stress energy tensor, the third one is the \( U(1) \) charge \( q \) under the spin-1 current and the last one is the regular condition with the diagonal spin-1 current. See also the defining OPEs in (B.1).

It turns out that there exists only one higher spin-\( \frac{3}{2} \) primary field with \( U(1) \) charge \( q = 0 \) denoted by \( W_{q=0}^{(\frac{3}{2})}(z) \). The explicit expression of \( W_0^{(\frac{3}{2})}(z) \) for fixed \( N = 4 \) is given as follows:

\[ W_0^{(\frac{3}{2})}(z) = -\frac{5}{18} \left[ \psi^i J_i + \frac{9}{5} \psi^a K^a - \frac{1}{5} \psi^a J^a \right](z). \tag{3.3} \]

The previous relations in (2.2) are used. The last term of (3.3) does not appear in (2.11). The general expression for generic \( N \) will be given in next section.

In order to find the other primary currents that belong to the same \( N = 2 \) higher spin multiplet \( W_0^{(\frac{3}{2})}(w) \) belongs to, we can use the defining relations in (B.1). For given the lowest higher spin-\( \frac{3}{2} \) current (3.3), we can determine the following higher spin-2 currents of \( U(1) \) charge \( \pm 1 \) with the help of \( G^\pm(z) \) in (2.11) as follows:

\[ W_\pm^{(2)}(z) = -\frac{1}{12\sqrt{7}} \left[ -\frac{3i}{2} J^i J^i + \frac{i}{4} J^a J^a - \frac{7i}{2} J^a K^a + \frac{7i}{4} K^a K^a \mp 7 J^a L^a \pm 13 K^a L^a + 7 JJ \mp 70i \partial J \right](z), \tag{3.4} \]

where the spin-1 current \( L^a(z) \) by considering the product of two fermions is introduced as

\[ L^a(z) \equiv f^{aij} \psi^j \chi^j(z). \tag{3.5} \]

The relations (2.2) and (2.4) are used. The last four terms in (3.4) do not appear in (2.12).

Similarly, the last component higher spin-\( \frac{5}{2} \) current of vanishing \( U(1) \) charge can be determined by the OPE between the spin-\( \frac{3}{2} \) currents and the above higher spin-2 currents and it is given by

\[ W_0^{(\frac{5}{2})}(z) = -\frac{1}{12} \left[ 2i f^{abc} \psi^a K^b L^c - 3 f^{abc} \psi^a J^b L^c + 6 f^{abc} \psi^a \psi^b \psi^c J - 6 \psi^a J^a J - i d^{a j k b} \psi^a \psi^j \chi^k J^b + i d^{a j b c} \psi^a \psi^b \psi^c K^c \right](z), \tag{3.6} \]
where the last two terms contain the totally symmetric $d$ tensor of $SO(2N+1)$,

$$d^{ABCD} = \frac{1}{2} \text{Tr}[T^D T^A T^B T^C + T^D T^A T^C T^B + T^D T^B T^A T^C + T^D T^B T^C T^A], \quad A = (a, i). \quad (3.7)$$

Note that because the $SO(2N)$ adjoint index and the $SO(2N)$ vector index inside of $SO(2N+1)$ range over independently, there are nontrivial contributions in the last two terms of (3.6).

The relations (2.2), (2.4) and (3.5) are used. The three currents (3.4) and (3.6) satisfy the following regularity conditions with the diagonal spin-1 current as follows:

$$(J^a + K^a)(z) W^{(2)}_0(w) = + \cdots,$$

$$(J^a + K^a)(z) W^{(2)}_0(w) = + \cdots. \quad (3.8)$$

Therefore, the above four currents (3.3), (3.4) and (3.6) which satisfy (3.2) and (3.8), are components of the following lowest $N = 2$ higher spin multiplet

$$W^{(3)}_0 \equiv \left( W^{(3)}_0, W^{(2)}_+, W^{(2)}_-, W^{(2)}_0, W^{(2)}_0 \right). \quad (3.9)$$

We have also checked the defining OPEs in (3.1) for $h=\frac{3}{2}$ and $q=0$ completely.

The existence of $W^{(\frac{3}{2})}_0(Z)$ with the results of [30] strongly suggests that there would be the following $N = 2$ higher spin multiplets in addition to the lowest $N = 2$ higher spin multiplet $W^{(\frac{3}{2})}_0(Z)$ (3.9)

$$W^{(\frac{7}{2})}_0 \equiv \left( W^{(\frac{7}{2})}_0, W^{(4)}_+, W^{(4)}_-, W^{(2)}_0 \right),$$

$$W^{(4)}_0 \equiv \left( W^{(4)}_0, W^{(4)}_+, W^{(4)}_-, W^{(2)}_0 \right),$$

$$W^{(5)}_0 \equiv \left( W^{(5)}_0, W^{(4)}_+, W^{(4)}_- W^{(6)}_0 \right), \cdots. \quad (3.10)$$

The first multiplet of (3.10) is the $N = 2$ extension of the $N = 1$ higher spin multiplet of spins $\frac{7}{2}$ and $4$ [30]. The extra higher spin currents of spins $\frac{7}{2}$ and $4$ appear in the second multiplet of (3.10). Similarly, the first two components in the third multiplet of (3.10) are new additional ones. We will observe the presence of some of the higher spin currents for fixed $N = 4$ in next subsections.

### 3.2 The second $N = 2$ higher spin multiplet

In order to observe the existence of the second $N = 2$ higher spin multiplet $W^{(3)}_0(Z)$ in (3.10), we should show that there is no other primary higher spin-3 current besides $W^{(3)}_0(z)$
Similarly, the other component higher spin multiplet $W^{(3)}_7(z)$ with the unknown $U(1)$ charge $q$ and use the conditions (3.12) for this higher spin current as we did for $W^{(3)}_0(z)$. It turns out that there exists only one higher spin-3 (primary) current with $U(1)$ charge $q = 0$: the lowest higher spin current $W^{(3)}_0(z)$ of the second $\mathcal{N} = 2$ higher spin multiplet. The explicit expression of $W^{(3)}_0(z)$ for fixed $N = 4$ is given as follows:

$$W^{(3)}_0(z) = \left[ \frac{4}{847} (\psi^a K^a)(\psi^b L^b) + \frac{8}{847} (\psi^a L^a)(\psi^b J^b) + \frac{20}{7623} (\psi^i J^i)(\chi^j J^j) 
- \frac{799}{45738} J^a (d^{a}kj\chi^k K^b) + \frac{3995}{320166} K^a (d^{a}kj\chi^k K^b) + \frac{31}{6534} L^a (d^{ab}k\psi^b J^k) + 
+ \frac{1}{1089} J^a (d^{ab}k\psi^b J^k) + \frac{2327}{45738} J J J + \frac{19}{91476} i J^a J^a J - \frac{17387}{64032} i K^a K^a J + \frac{1949}{45738} i L^a L^a J 
+ \frac{106}{3267} i J^a K^a J - \frac{157}{45738} J^a \partial L^a - \frac{17}{7623} L^a \partial J^a - \frac{6947}{320166} K^a \partial L^a + \frac{646}{22869} i \partial^2 J \right](z). \quad (3.11)$$

The previous relations (2.2), (2.4) and (3.5) are used. In this case, the additional d tensor (3.7) with two adjoint indices and two vector indices appears in the second and third lines of (3.11).

The other components of the $\mathcal{N} = 2$ higher spin multiplet $W^{(3)}_0(Z)$ in (3.10) can be obtained easily from (3.11), in principle. Or they can appear in the OPEs between the components of the $\mathcal{N} = 2$ higher spin multiplet $W^{(3)}_0(Z)$ in (3.9). For example, the higher spin-3 current $W^{(3)}_0(z)$ in (3.11) appears in the first-order pole of the following OPE $W^{(2)}_0(z) W^{(3)}_0(w)$

$$W^{(2)}_0(z) W^{(3)}_0(w) = \frac{1}{(z-w)^3} \frac{5}{6} J(w) + \frac{1}{(z-w)^2} \frac{35}{6} \partial J(w) + \frac{1}{(z-w)} \left[ - \frac{2541i}{48} W^{(3)}_0 + \frac{35}{12} G^+ G^- 
+ \frac{49}{24} J T - \frac{91}{144} J J J - \frac{35}{24} \partial T + \frac{175}{72} \partial^2 J \right](w) + \cdots. \quad (3.12)$$

We can rearrange the first order pole in (3.12) in terms of various quasi primary fields. Similarly, the other component higher spin-$\frac{3}{2}$ currents $W^{(3)}_\pm(z)$ appear in the first-order pole of the following OPE $W^{(2)}_0(z) W^{(2)}_\pm(w)$ as follows:

$$W^{(2)}_0(z) W^{(2)}_\pm(w) = - \frac{1}{(z-w)^3} \frac{35}{3} G^+(w) + \frac{1}{(z-w)^2} \left[ \mp \frac{7}{12} J G^+ - \frac{91}{12} \partial G^+ \right](w) + \frac{1}{(z-w)} \left[ \frac{2541i}{48} W^{(2)}_\pm - \frac{119}{24} G^+ T \mp \frac{35}{24} G^+ \partial J + \frac{91}{48} J J G^+ \mp \frac{7}{8} J \partial G^+ 
- \frac{91}{32} \partial^2 G^+ \right](w) + \cdots. \quad (3.13)$$
In this case also, the rearrangement in terms of the quasi primary fields in the second and first order poles can be done. Lastly, the last component higher spin-4 current \( W_0^{(4)}(z) \) appears in the first-order pole of the following OPE \( W_0^{(5/2)}(z) W_0^{(7/2)}(w) \) as follows:

\[
W_0^{(5/2)}(z) W_0^{(7/2)}(w) = \frac{1}{(z-w)^5} \left[ \frac{70}{3} + \frac{1}{(z-w)^3} \left[ \frac{119}{6} T - \frac{7}{12} J J \right] \right] (w)
+ \frac{1}{(z-w)^2} \left[ -\frac{7}{12} J J + \frac{119}{12} \partial T \right] (w)
+ \frac{1}{(z-w)} \left[ -\frac{2541}{48} W_0^{(4)} - \frac{7}{12} G^- \partial G^+ - \frac{7}{12} G^+ \partial G^- + \frac{91}{24} J G^+ G^- \right.
- \frac{91}{48} J J T - \frac{91}{48} \partial G^+ G^- - \frac{7}{16} J \partial^2 J + \frac{119}{24} T T - \frac{35}{48} \partial J \partial J
+ \frac{91}{48} \partial^2 T \right] (w) + \cdots. \tag{3.14}
\]

Although the currents in the left hand side of (3.14) are equal to each other, the occurrence of the higher spin-4 current in the first order pole is expected, contrary to the two bosonic same currents (that is, the higher spin-5 current does not arise in the OPE of the higher spin-3 current and itself), because they in (3.14) are fermionic. For the OPEs in (3.13) and (3.14), we observe similar behavior in the \( \mathcal{N} = 1 \) coset model [30].

The results in (3.12), (3.13), and (3.14) show the existence of the second \( \mathcal{N} = 2 \) higher spin multiplet \( W_0^{(3)}(Z) \) from the OPE between the first \( \mathcal{N} = 2 \) higher spin multiplet \( W_0^{(5/2)}(Z) \) and itself, although the complete analysis is not done. In next section, we will see their relation explicitly from the Jacobi identity.

### 3.3 The third and fourth \( \mathcal{N} = 2 \) higher spin multiplets

To show the existence of the third \( \mathcal{N} = 2 \) higher spin multiplet \( W_0^{(7/2)}(Z) \) in (3.10), we should prove the presence of the first component of this multiplet \( W_0^{(7/2)}(z) \). We can check that the higher spin-\( \frac{7}{2} \) current \( W_0^{(7/2)}(z) \) appears in the second-order pole of the following OPE (coming from the OPE between the first and second higher spin multiplets),

\[
W_0^{(7/2)}(z) W_0^{(3)}(w) = \frac{1}{(z-w)^4} \frac{225i}{22} W_0^{(4)}(w) + \frac{1}{(z-w)^3} \frac{75i}{22} \partial W_0^{(4)}(w)
+ \frac{1}{(z-w)^2} \left[ W_0^{(4)} - \frac{32i}{33} G^- W_0^{(2)} + \frac{32i}{33} G^+ W_0^{(2)} + \frac{103i}{33} J W_0^{(4)} \right.
- \frac{85i}{22} J W_0^{(4)} + \frac{51i}{11} W_0^{(3)} T + \frac{i}{33} \partial^2 W_0^{(4)} \right]
+ \frac{1}{(z-w)} \left[ \frac{7}{3} \partial W_0^{(7/2)} + \frac{6i}{11} G^- \partial W_0^{(2)} - \frac{6i}{11} G^+ \partial W_0^{(2)} + \frac{8i}{33} J G^- W_0^{(2)} \right] (w)
+ \cdots. \tag{3.16}
\]
In (3.15), we can rearrange each singular term in terms of various quasi primary fields as explained before. The other components of this multiplet $W_0^{(\frac{5}{2})}(z)$ can be obtained from the defining OPEs in (B.1) with the explicit expression for the higher spin-$\frac{7}{2}$ current $W_0^{(\frac{7}{2})}(z)$ and spin-$\frac{3}{2}$ currents $G_\pm(w)$.

In order to observe the existence of the fourth $\mathcal{N} = 2$ higher spin multiplet $W_0^{(5)}(Z)$ in (3.10), we can see the existence of the first component of this multiplet $W_0^{(5)}(z)$. This higher spin-5 current $W_0^{(5)}(z)$ appears in the first-order pole of the following OPE (coming from the OPE between the first and third higher spin multiplets)

\[
W_0^{(\frac{7}{2})}(z) W_0^{(\frac{7}{2})}(w) = \frac{1}{(z-w)^3} \left[ \frac{196}{3} W_0^{(3)}(w) + \frac{1}{(z-w)^2} \left[ \frac{980i}{99} G^- \partial G^+ - \frac{980i}{99} G^+ \partial G^- + \frac{637i}{99} J J \partial J \\
- \frac{1274i}{99} J \partial T + \frac{490i}{99} \partial J T + \frac{112i}{11} W_0^{(\frac{7}{2})} W_0^{(\frac{7}{2})} \right] + \frac{49}{3} \partial W_0^{(3)} + \frac{245i}{99} \partial^2 J \right] (w) + \frac{1}{(z-w)} \left[ W_0^{(5)} + \cdots \right] (w) + \cdots \tag{3.16}
\]

Due to the complexity of the calculation of the first-order pole of (3.16), we could not find the explicit structure of the first-order pole. However, we have found that the first-order pole of this OPE cannot be expressed in terms of the descendant or known composite fields. This suggests the existence of the higher spin-5 current $W_0^{(5)}(z)$ belonging to the fourth higher spin multiplet. Therefore, we observe the existence of the fourth higher spin multiplet $W_0^{(5)}(Z)$ in (3.10). In next section, we will look for more algebraic structures for the general $\mathcal{N}$.

4. The OPEs between the $\mathcal{N} = 2$ higher spin multiplets

In the previous section, the presence of the following $\mathcal{N} = 2$ higher spin multiplets was found for $N = 4$,

\[
W_0^{(\frac{7}{2})} \equiv \left( W_0^{(\frac{7}{2})}, W_0^{(2)}, W_0^{(2)}, W_0^{(\frac{7}{2})} \right),
\]

4 Because the spin is 5, the order of higher spin-5 current looks like 10-th order in the two kinds of fermions. In fact, there are more than one million terms even for $N = 4$ case.
\[ W_0^{(3)} \equiv \left( W_0^{(3)}, W_0^{(5)}, W_0^{(7)}, W_0^{(4)} \right), \]
\[ W_0^{(2)} \equiv \left( W_0^{(2)}, W_0^{(4)}, W_0^{(6)}, W_0^{(5)} \right), \]
\[ W_0^{(5)} \equiv \left( W_0^{(5)}, W_0^{(6)}, W_0^{(4)}, W_0^{(5)} \right), \] 
\[ \cdots \tag{4.1} \]

Assuming that the super multiplets in (4.1) exist for \( N > 4 \), we move on to construct the (super) OPEs between the \( \mathcal{N} = 2 \) higher spin multiplets in (4.1) for \( N \).

### 4.1 The first \( \mathcal{N} = 2 \) higher spin multiplet for generic \( N \)

In order to find (super) OPE \( W_0^{(\pm)}(Z_1)W_0^{(\pm)}(Z_2) \), we should find the four component fields of \( W_0^{(\pm)}(Z) \) for \( N = 4, 5, 6, \) and 7 (or more \( N > 7 \)). Then from these results, the general forms of components field can be extracted.

From the first component field \( W_0^{(\pm)}(z) \) of the lowest \( \mathcal{N} = 2 \) higher spin multiplet \( W_0^{(\pm)}(Z) \) for \( N = 4, 5, 6, \) and 7, the general expression of \( W_0^{(\pm)}(z) \) for generic \( N \) can be obtained as follows:

\[
W_0^{(\pm)}(z) = \sqrt{\frac{3(N - 2)}{2(N - 1)(2N - 1)}} \left[ \frac{i(N - 1)}{2} J^i J_i + \frac{i}{4} J^a J^a - \frac{2(N - 1)i}{2} J^a K^a + \frac{(2N - 1)i}{4} K^a K^a \mp (2N - 1) J^a L^a \pm (4N - 3) K^a L^a \right] (z), \tag{4.2}
\]

which generalizes (3.3). Note that the order of \( N \) in the denominator of the inside of square root is quadratic. Then, the four different \( N \) values determine this \( N \) dependence completely. The numerator and denominator of relative coefficients behave as linearly. Therefore, we can determine the \( N \) dependence without any difficulty.

From the results of (4.1), the other component fields of \( W_0^{(\pm)}(Z) \) for \( N \) with the help of \( G^\pm \) are obtained as follows,

\[
W_0^{(2)}(z) = \frac{\sqrt{3}}{2(N - 1)(2N - 1)(3N - 2)} \left[ 2i f^{abc} \psi^{[a} K^{b} L^{c} - \frac{2(N - 1)i}{(N - 2)} f^{abc} \psi^{[a} J^{b} L^{c} \right.
\left. + 4(N - 1) f^{abc} \psi^{[a} \psi^{b} \psi^{c]} J - 4(N - 1) \psi^{a} J^{a} J - i \psi^{a} (d^{ajkb} \psi^{j} \chi^{k} J^{b}) \right.
\left. + i \chi^{i} (d^{ijkc} \psi^{j} \psi^{c} K^{e}) \right] (z), \tag{4.3}
\]

\[ \cdots \]
Note that the numerator and denominator of relative coefficients behave as linearly in $N$. Therefore, the lowest $\mathcal{N} = 2$ higher spin multiplet in terms of two fermions in the coset model is given by (4.2) and (4.3) for general $N$. In next subsection, the OPEs between them will be described. In order to obtain the next $\mathcal{N} = 2$ higher spin multiplet for generic $N$, we need to calculate the OPEs between (4.2) and (4.3), in principle, but we will not present them explicitly in this paper.

### 4.2 The OPE between the first $\mathcal{N} = 2$ higher spin multiplet

From the Jacobi identity between the two (higher spin) currents $(\mathbf{T}, W_0^{(3)}, W_0^{(\frac{3}{2})})$, the (super) OPE $W_0^{(3)}(Z_1) W_0^{(\frac{3}{2})}(Z_2)$ is completely determined (that is, all the structure constants in the right hand side of the OPE are fixed and written in terms of the central charge or $N$) for general $N$ with the help of [42] as follows:

$$W_0^{(3)}(Z_1) W_0^{(\frac{3}{2})}(Z_2) = \frac{1}{\bar{z}_{12}} \frac{2}{3} c + \frac{\theta_{12} \bar{\theta}_{12}}{\bar{z}_{12}} 3 \mathbf{T}(Z_2)$$

$$+ \frac{1}{z_{12}} \left[ -3 \theta_{12} DT + 3 \bar{\theta}_{12} \bar{D} \bar{T} + 3 \theta_{12} \bar{\theta}_{12} \partial T \right] (Z_2)$$

$$+ \frac{1}{z_{12}} \frac{c}{1 - c} \left[ c[D, \bar{D}] T + 3 T \bar{T} \right] (Z_2)$$

$$+ \frac{\theta_{12}}{z_{12}} \frac{1}{1 - c} \left[ (2 c - 3) \partial DT + 3 T \bar{D} T \right] (Z_2)$$

$$+ \frac{\bar{\theta}_{12}}{z_{12}} \frac{1}{1 - c} \left[ -(2 c - 3) \partial \bar{D} T + 3 T D T \right] (Z_2)$$

$$+ \frac{\theta_{12}}{z_{12}} \frac{1}{1 - c} \left[ \frac{C_{(3)}^{(3)}}{2(\frac{3}{2})^{\frac{3}{2}}} W_0^{(3)}(Z_2) \right]$$

Here the structure constant

$$(C_{(\frac{3}{2})}^{(3)})^2 = \frac{3(3 + 2c)(-9 + 4c)(-3 + 5c)}{2(-1 + c)(6 + c)(-3 + 2c)}$$

is fixed by requiring that the sixth-order pole of the OPE $W_0^{(3)}(z) W_0^{(3)}(w)$ should behave as $\frac{1}{(z-w)^6} z^\frac{c}{3}$. Using the relation (1.2), we can reexpress (4.5) in terms of $N$. Under the infinity
limit of \( c \), (1.5) is finite and is given by 30. We can rewrite the above OPE (4.4) in terms of various quasi primary fields as explained before in the component approach.

From (4.4), the OPEs between the component fields of \( W_0^{(2)}(Z) \) are extracted as follows

\[
W_0^{(2)}(z) W_0^{(2)}(w) = \frac{1}{(z-w)^3} \left( \frac{2c}{3} + \frac{1}{(z-w)} \right) \left( -2cT + 3JJ \right)(w) + \cdots,
\]

\[
\rightarrow \frac{1}{(z-w)^3} \frac{2c}{3} + \frac{1}{(z-w)} \left( 2T - \frac{3}{c}JJ \right)(w) + \cdots,
\]

\[
W_\pm^{(2)}(z) W_0^{(2)}(w) = \mp \frac{1}{(z-w)^2} 3G^\pm(w)
\]

\[
+ \frac{1}{(z-w)} \frac{1}{(1-c)} \left[ 3JG^\pm \pm (2c-3)\partial G^\pm \right](w) + \cdots,
\]

\[
\rightarrow \mp \frac{1}{(z-w)^2} 3G^\pm(w) + \frac{1}{(z-w)} \left[ -\frac{3}{c}JG^\pm \mp 2\partial G^\pm \right](w) + \cdots,
\]

\[
W_0^{(2)}(z) W_0^{(2)}(w) = \frac{1}{(z-w)^3} 3J(w) + \frac{1}{(z-w)^2} 3\partial J(w) + \frac{1}{(z-w)} \left[ C^{(3)}_{(2/3)(4/7)} W_0^{(3)} \right.
\]

\[
+ \frac{1}{(-1+c)(6+c)(-3+2c)} \left( -9c(-9+4c)G^-G^+ 
\right.
\]

\[
+ 9(9-3c+2c^2)JT - \frac{27}{2}(1+2c)JJJ
\]

\[
+ \frac{9}{2} c(-9+4c) \partial T + \frac{3}{2}(18-18c+3c^2+2c^3)\partial^2 J \right](w) + \cdots,
\]

\[
\rightarrow \frac{1}{(z-w)^3} 3J(w) + \frac{1}{(z-w)^2} 3\partial J(w)
\]

\[
+ \frac{1}{(z-w)} \left[ C^{(3)}_{(2/3)(4/7)} W_0^{(3)} - \frac{18}{c} G^-G^+ + \frac{9}{c} JT - \frac{27}{2c^2} JJJ + \frac{3}{2} \partial^2 J \right](w)
\]

\[
+ \cdots,
\]

\[
(4.6)
\]

where the large \( c \) limit is taken by keeping the \( \frac{1}{c} \) factor for the quadratic fields and the \( \frac{1}{c^2} \) factor for the cubic fields in the right hand sides of the OPEs in (1.5). See also [33, 44] for the large \( c \) limit. Then, this classical algebra will provide the asymptotic symmetry algebra of

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5Note that we have \( W_0^{(2)} = W_0^{(2)} \big|_{\bar{\theta} = \bar{\bar{\theta}} = 0} \), \( W^{(2)} = DW_0^{(2)} \big|_{\bar{\theta} = \bar{\bar{\theta}} = 0} \), \( W^{(2)} = \overline{2} W_0^{(2)} \big|_{\bar{\theta} = \bar{\bar{\theta}} = 0} \) and \( W_0^{(2)} = \overline{1} W_0^{(2)} \big|_{\bar{\theta} = \bar{\bar{\theta}} = 0} \). Similarly, the following relations \( J = T \big|_{\bar{\theta} = \bar{\bar{\theta}} = 0}, G^+ = DT \big|_{\bar{\theta} = \bar{\bar{\theta}} = 0}, G^- = \overline{2} T \big|_{\bar{\theta} = \bar{\bar{\theta}} = 0} \) and \( T = \overline{2} T \big|_{\bar{\theta} = \bar{\bar{\theta}} = 0} \) are satisfied. Then we can obtain (4.4) by selecting \( \bar{\theta}_{12} \) and \( \bar{\bar{\theta}}_{12} \) independent terms, \( \bar{\theta}_{12} \) dependent terms, \( \bar{\theta}_{12} \) dependent terms and the remaining terms respectively, acting \( D_1 \) or \( \overline{2} \) on both sides and putting \( \bar{\theta}_{12} = \bar{\bar{\theta}}_{12} = 0 \).

6In terms of mode expansions, we have

\[
\{ W_\nu^{(2)}, W_s^{(2)} \} = 2L_{r+s} + \frac{c}{3} (r^2 - \frac{1}{4}) \delta_{r,-s},
\]

\[
[ W_m^{(2)\pm}, W_r^{(2)} ] = \mp (m-2r) G_m^\pm,
\]
the (matrix extension of) AdS\textsubscript{3} bulk theory studied in [29]. The corresponding wedge algebra will be described in next section. In Appendix C, for convenience, other remaining OPEs are given explicitly.

### 4.3 The OPE between the first and the second $\mathcal{N} = 2$ higher spin multiplets

From the Jocobi identity between the three (higher spin) currents ($\mathbf{T}, \mathbf{W}_0^{(\frac{3}{2})}, \mathbf{W}_0^{(3)}$), and the Jocobi identity of the higher spin current ($\mathbf{W}_0^{(\frac{3}{2})}, \mathbf{W}_0^{(\frac{5}{2})}, \mathbf{W}_0^{(3)}$), it is straightforward to see that the (super) OPE $\mathbf{W}_0^{(\frac{3}{2})}(Z_1)\mathbf{W}_0^{(3)}(Z_2)$ for general $N$ is constructed completely as follows

\[
\mathbf{W}_0^{(\frac{3}{2})}(Z_1)\mathbf{W}_0^{(3)}(Z_2) = \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}} \left[ \frac{3(-9 + 4c)(-9 + 9c + 10c^2)}{4(-1 + c)(6 + c)(-3 + 2c)C_{(\frac{3}{2})}^{(3)\frac{3}{2}(\frac{3}{2})}} \right] \mathbf{W}_0^{(\frac{3}{2})}(Z_2) + \frac{1}{z_{12}} \left[ \frac{(-9 + 4c)(-9 + 9c + 10c^2)}{2(-1 + c)(6 + c)(-3 + 2c)C_{(\frac{3}{2})}^{(3)\frac{3}{2}(\frac{3}{2})}} \theta_{12}(-1)DW_0^{(\frac{3}{2})} + \bar{\theta}_{12}\bar{D}W_0^{(\frac{3}{2})} \right](Z_2) + \frac{1}{z_{12}} \left[ \frac{c(9 - 9c - 10c^2)}{2(-1 + c)(6 + c)(-3 + 2c)C_{(\frac{3}{2})}^{(3)\frac{3}{2}(\frac{3}{2})}} \right] [D, \bar{D}]W_0^{(\frac{3}{2})} + \frac{1}{z_{12}} \left[ \frac{9(-9 + 9c + 10c^2)}{2(-1 + c)(6 + c)(-3 + 2c)C_{(\frac{3}{2})}^{(3)\frac{3}{2}(\frac{3}{2})}} \theta_{12}\frac{1}{4}(\partial^2\text{pole-3})_{\theta = 0} + \mathbf{Q}_1 \right] \theta_{12}\theta_{12}\theta_{12} \left[ -\frac{7}{2}C^{(\frac{3}{2})}_{(3)\frac{3}{2}(\frac{3}{2})} \mathbf{W}_0^{(\frac{3}{2})} + \frac{1}{12}\theta^2(\partial^2\text{pole-4}) + \mathbf{Q}_3 \right](Z_2) + \frac{1}{z_{12}} \left[ \frac{1}{5}\theta^2(\partial^2\text{pole-2})_{\theta = \bar{\theta} = 0} + \mathbf{Q}_4 \right] + \theta_{12}\left[ C^{(\frac{3}{2})}_{(\frac{3}{2})\frac{3}{2}(\frac{3}{2})} \cdot \mathbf{W}_0^{(\frac{3}{2})} + \frac{1}{20}\theta^2(\partial^2\text{pole-3})_{\theta = 0} + \frac{1}{3}\partial\mathbf{Q}_1 + \mathbf{Q}_5 \right] + \bar{\theta}_{12}\left[ -C^{(\frac{3}{2})}_{(\frac{3}{2})\frac{3}{2}(\frac{3}{2})} \cdot \mathbf{W}_0^{(\frac{3}{2})} + \frac{1}{20}\theta^2(\partial^2\text{pole-3})_{\bar{\theta} = 0} + \frac{1}{3}\partial\mathbf{Q}_2 + \mathbf{Q}_6 \right] + \theta_{12}\bar{\theta}_{12}\left[ -\frac{3}{2}C^{(\frac{3}{2})}_{(\frac{3}{2})\frac{3}{2}(\frac{3}{2})} \cdot \mathbf{W}_0^{(\frac{3}{2})} + \frac{1}{60}\theta^3(\partial^2\text{pole-4}) + \frac{3}{7}\partial^2\mathbf{Q}_3 + \mathbf{Q}_7 \right](Z_2) + \cdots,
\]

\[
\{\mathbf{W}_0^{(\frac{3}{2})}, \mathbf{W}_0^{(\frac{3}{2})}\} = C^{(\frac{3}{2})}_{(\frac{3}{2})\frac{3}{2}(\frac{3}{2})} \mathbf{W}_r^{(3)} + \frac{3}{8}(2s + 1)(2s - 1)J_{r+s},
\]

up to the modes coming from the nonlinear terms (that is, infinity limit of $c$). The central term vanishes for $r = \pm \frac{3}{2}$. For $s = \pm \frac{3}{2}$, the second term of the last equation vanishes.
where the structure constant is determined as follows\footnote{We use simplified notations. For example, in the expression of (pole-3)\(\theta=0\) of (4.8), we take the third order pole of (4.8) by taking \(\theta=0\). Then we are left with the first term of the third order pole.}

\[
(C^{(7)}_{(4)(3)})^2 = \frac{(108 - 144c + 15c^2 + 7c^3)}{(39 - 53c + 13c^2 + c^3)}
\]  

(4.9)

by requiring that the seventh-order pole of the OPE \(W_0^{(7)}(z) W_0^{(7)}(w)\) should have \(\frac{1}{(z-w)^2} \frac{2c}{7}\). Under the infinity limit of \(c\), the above structure constant (4.9) is finite and is given by 7. The appearance of \(W_0^{(7)}(Z_2)\) in (4.8) has been observed in (3.15) in the component approach. There are seven quasi primary fields, \(Q_1(Z_2), \cdots, Q_7(Z_2)\) which will be presented in Appendix D. The large \(c\) limit can be taken in (4.8) as in previous subsection but we will not present them explicitly in this paper. The nonlinear term appearing in the second order pole in (4.8) vanishes in this limit and the quasi primary fields in Appendix D vanish under this limit. It is straightforward to obtain the (anti)commutators from (4.8), like as (4.7) and (C.2), which we do not present in this paper. They will provide us the correct relations between the higher spin generators corresponding to the first two \(\mathcal{N}=2\) higher spin multiplets.

The two important results in this section is summarized by (4.4) and (4.8). Other OPEs are given in Appendix E.

5 The \(AdS_3\) higher spin theory with matrix generalization

We construct the Vasiliev’s oscillator description corresponding to the first two \(\mathcal{N}=2\) higher spin multiplet discussed in previous sections.

5.1 The “wedge” algebra of \(\mathcal{N}=2\) superconformal algebra

The Lie algebra \(shs[\lambda = \frac{1}{2}]\) is generated by \(\hat{y}_\alpha (\alpha = 1, 2)\) whose fundamental commutator takes the form \([\hat{y}_\alpha, \hat{y}_\beta] = 2i\epsilon_{\alpha\beta}\) [18, 19, 38, 39, 45]. Note that there is no oscillator \(k\) dependence. The Chan-Paton factors are introduced and the generators of \(shs[\lambda = \frac{1}{2}]\) are given by the tensor product between the generators of \(shs(\lambda = \frac{1}{2})\) higher spin algebra and the \(GL(2)\) generators. Note that the ’t Hooft parameter \(\lambda = \frac{2N}{(2N+k-1)}\) in [29, 30] becomes \(\frac{N}{(2N-1)}\) at the “critical” level and this goes to \(\lambda = \frac{1}{2}\) under the infinity limit of \(N\).

The spin-\(\frac{3}{2}\) currents of \(\mathcal{N}=2\) superconformal algebra play the role of the four fermionic generators of the \(\mathcal{N}=2\) wedge algebra as follows [15] (the \(\mathcal{N}=2\) truncation of the \(\mathcal{N}=4\)
theory in [46]):

\[ \begin{align*}
G^\pm_{\frac{1}{2}} &= \left(-\frac{i}{4}\right)\hat{\gamma}_{1} \otimes \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \\
G^\pm_{-\frac{1}{2}} &= \left(-\frac{i}{4}\right)\hat{\gamma}_{2} \otimes \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}.
\end{align*} \tag{5.1} \]

By calculating the anticommutators between these operators in (5.1), we obtain

\[ \{G^+_r, G^-_s\} = L_{r+s} + \frac{1}{2}(r-s)J_{r+s}. \tag{5.2} \]

Here the spin-2 current of \( \mathcal{N} = 2 \) superconformal algebra plays the role of three bosonic generators, which are the matrix generalization of [46], of the \( \mathcal{N} = 2 \) wedge algebra

\[ \begin{align*}
L_1 &= \left(-\frac{i}{4}\right)\hat{\gamma}_{1} \hat{\gamma}_{1} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \\
L_{-1} &= \left(-\frac{i}{4}\right)\hat{\gamma}_{2} \hat{\gamma}_{2} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \\
L_0 &= \left(-\frac{i}{4}\right)\frac{1}{2}(\hat{\gamma}_{1} \hat{\gamma}_{2} + \hat{\gamma}_{2} \hat{\gamma}_{1}) \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},
\end{align*} \tag{5.3} \]

and the spin-1 has the following matrix form

\[ J_0 = \frac{1}{2} \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{5.4} \]

We can easily check the following commutators

\[ [J_0, G^\pm_r] = \pm G^\pm_r. \tag{5.5} \]

Then the \( \mathcal{N} = 2 \) wedge algebra [46] (generated by four bosonic and fermionic generators) is described by (5.2), (5.5) and the ones in the footnote 8 together with (5.1), (5.3) and (5.4). By restricting the mode indices in Appendix A to the wedge ones, we observe that the \( \mathcal{N} = 2 \) wedge algebra is reproduced.

### 5.2 The OSp(2|2) higher spin algebra

From the classical limit discussed in the section 4, we would like to construct the higher spin generators corresponding to the first two \( \mathcal{N} = 2 \) higher spin multiplets.

---

8 The remaining nontrivial nonzero commutators are given by \([L_m, L_n] = (m - n)L_{m+n}, [L_m, G^\pm_r] = (\frac{m}{2} - r)G^\pm_{m+r}\).
5.2.1 The twelve higher spin generators

By adding the matrix degree of freedoms to the bulk theory found in [29], we obtain the (lowest) two higher spin generators by requiring that we should have 
\[ [J_0, V_0^{(\frac{3}{2})}] = 0 \] and 
\[ [L_m, V_r^{(\frac{3}{2})}] = [(\frac{3}{2} - 1)m - r]V_{m+r}^{(\frac{3}{2})} \] given in Appendix B as follows:

\[ V_0^{(\frac{3}{2})} = \frac{1}{2}(\frac{-i}{4})\hat{y}_1 \otimes \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}, \quad V_{-\frac{1}{2}}^{(\frac{3}{2})} = \frac{1}{2}(\frac{-i}{4})\hat{y}_2 \otimes \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}. \] (5.6)

It is easy to check that we have the anticommutator

\[ \{V_r^{(\frac{3}{2})}, V_s^{(\frac{3}{2})}\} = \frac{1}{2}L_{r+s}. \] (5.7)

We observe that this anticommutator corresponds to the first one in the footnote 6 because the one half of \( W_r^{(\frac{3}{2})} \) satisfies (5.7) by restricting the mode \( r \) to \( \pm \frac{1}{2} \).

By calculating the anticommutators \( \{G_r^+, V_r^{(\frac{3}{2})}\} = -V_{r+s}^{(2)+} \) described in Appendix B by using (5.1) and (5.6), we obtain the following three higher spin generators

\[ V_{0}^{(2)+} = (\frac{-i}{4})\hat{y}_1 \otimes \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad V_{-1}^{(2)+} = (\frac{-i}{4})\hat{y}_2 \otimes \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \]

\[ V_{0}^{(2)} = (\frac{-i}{4})\frac{1}{2}(\hat{y}_1 \hat{y}_2 + \hat{y}_2 \hat{y}_1) \otimes \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}. \] (5.8)

Similarly, we can obtain the following three higher spin generators

\[ V_{0}^{(2)-} = (\frac{-i}{4})\hat{y}_1 \otimes \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad V_{-1}^{(2)-} = (\frac{-i}{4})\hat{y}_2 \otimes \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \]

\[ V_{0}^{(2)} = (\frac{-i}{4})\frac{1}{2}(\hat{y}_1 \hat{y}_2 + \hat{y}_2 \hat{y}_1) \otimes \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \] (5.9)

by calculating the anticommutators \( \{G_r^-, V_r^{(\frac{3}{2})}\} = V_{r+s}^{(2)-} \) explained in Appendix B with the help of (5.1) and (5.6). We obtain the following commutators

\[ [V_{m}^{(2)+}, V_{s}^{(\frac{3}{2})}] = \mp \frac{1}{4}(m - 2s)G_{m+s}^{\pm}, \] (5.10)

where (5.8), (5.9), (5.6) and (5.1) are used 9. It is easy to see that by multiplying \( \frac{1}{4} \) of the second relation (one half of \( W_{m}^{(2)+} \) and one half of \( W_{r}^{(\frac{3}{2})} \) of the footnote 6) we obtain (5.10) by restricting the mode indices to the wedge ones.

9We have the nontrivial relations \( [J_0, V_{m}^{(2)+}] = \pm V_{m}^{(2)+} \) (described in Appendix B) which is similar to (5.3).
Finally, from the relation \([G^\pm_r, V_{m}^{(2)\mp}] = V_{r+m}^{(\frac{3}{2})} \pm \frac{1}{2} (2r - m) V_{r+m}^{(\frac{5}{2})}\) given in Appendix B, we have four higher spin generators

\[
V_{\frac{3}{2}}^{(\frac{3}{2})} = (-\frac{i}{4})^3 \hat{y}_1 \hat{y}_1 \hat{y}_1 \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad V_{\frac{5}{2}}^{(\frac{3}{2})} = (-\frac{i}{4})^3 \hat{y}_2 \hat{y}_2 \hat{y}_2 \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},
\]

\[
V_{\frac{3}{2}}^{(\frac{5}{2})} = (-\frac{i}{4})^3 \frac{1}{3} (\hat{y}_1 \hat{y}_2 \hat{y}_2 + \hat{y}_1 \hat{y}_2 \hat{y}_1 + \hat{y}_2 \hat{y}_1 \hat{y}_1) \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},
\]

\[
V_{\frac{5}{2}}^{(\frac{5}{2})} = (-\frac{i}{4})^3 \frac{1}{3} (\hat{y}_1 \hat{y}_2 \hat{y}_2 + \hat{y}_2 \hat{y}_1 \hat{y}_1 + \hat{y}_2 \hat{y}_1 \hat{y}_1) \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.
\]

(5.11)

Note that the matrices appearing in the higher spin generators of spins \(\frac{3}{2}\) and \(\frac{5}{2}\) are diagonal in (5.6) and (5.11). The decomposition of these in the above commutator can be done by moving the oscillators appropriately. It is easy to check that we have the following nontrivial relations \([G^\pm_r, V_{s}^{(3)\mp}] = \frac{1}{2} (3r - s) V_{r+s}^{(2)\mp}\) which also appears in Appendix B. There are several ways to write down the last two higher spin generators of (5.11). We express them in symmetric way in the indices.

### 5.2.2 The next twenty four higher spin generators

For the second \(\mathcal{N} = 2\) higher spin multiplet, we can calculate the following anticommutators together with (5.6) and (5.11) by recalling that the higher spin-3 current appears in the OPE between the higher spin-\(\frac{3}{2}\) current and the higher spin-\(\frac{5}{2}\) current in the subsection 4.2,

\[
\{V_{r}^{(\frac{3}{2})}, V_{s}^{(\frac{5}{2})}\} = V_{r+s}^{(3)}.
\]

(5.12)

Compared to the previous relations (5.7) and (5.10), this result produces the new higher spin generators in the right hand side of (5.12). Here the five generators are given by

\[
V_{2}^{(3)} = (-\frac{i}{4})^2 \hat{y}_1 \hat{y}_1 \hat{y}_1 \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},
\]

\[
V_{1}^{(3)} = (-\frac{i}{4})^2 \frac{1}{3} (\hat{y}_1 \hat{y}_2 \hat{y}_2 + \hat{y}_1 \hat{y}_2 \hat{y}_1 + \hat{y}_2 \hat{y}_1 \hat{y}_1) \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},
\]

\[
V_{0}^{(3)} = (-\frac{i}{4})^2 \frac{1}{6} (\hat{y}_1 \hat{y}_2 \hat{y}_2 + \hat{y}_1 \hat{y}_2 \hat{y}_1 + \hat{y}_2 \hat{y}_1 \hat{y}_1 + \hat{y}_2 \hat{y}_1 \hat{y}_1 + \hat{y}_2 \hat{y}_1 \hat{y}_1) \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},
\]

\[
V_{-1}^{(3)} = (-\frac{i}{4})^2 \frac{1}{4} (\hat{y}_2 \hat{y}_2 \hat{y}_2 \hat{y}_1 + \hat{y}_2 \hat{y}_2 \hat{y}_1 \hat{y}_2 + \hat{y}_2 \hat{y}_2 \hat{y}_2 \hat{y}_2 + \hat{y}_2 \hat{y}_2 \hat{y}_1 \hat{y}_1) \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},
\]

\[
V_{-2}^{(3)} = (-\frac{i}{4})^2 \hat{y}_2 \hat{y}_2 \hat{y}_2 \hat{y}_2 \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.
\]

(5.13)

As before, we present the generators in symmetric way in the indices. Note that the anticommutators (5.12) can be seen from the third relation of the footnote 6 with the wedge condition
for the mode indices. We will present the remaining generators for the second \( \mathcal{N} = 2 \) higher spin multiplet in Appendix \( F \). We can continue to determine the next generators corresponding to the third \( \mathcal{N} = 2 \) higher spin multiplet by using the result of (4.8). We expect that they will take the form of tensor product of the generators which are symmetrized product of \( \hat{y}_\alpha \)'s and \( GL(2) \) generators. We can calculate their (anti)commutators straightforwardly although it will be rather tedious.

6 Conclusions and outlooks

We analyzed the \( \mathcal{N} = 1 \) holographic minimal model at the critical level, determined the lowest \( \mathcal{N} = 2 \) higher spin currents and obtained their OPEs and the OPEs between this higher spin currents and the second \( \mathcal{N} = 2 \) higher spin currents. We have studied the Vasiliev's oscillator construction with the matrix degree of freedoms which generalizes the \( OSp(2|2) \) superconformal algebra.

We list some relevant open problems which can be done in the near future.

- The next \( \mathcal{N} = 2 \) higher spin multiplet in terms of two adjoint fermions

So far, the expression for the higher spin currents for general \( N \) is obtained for the lowest ones. It is an open problem to determine the next \( \mathcal{N} = 2 \) higher spin currents for generic \( N \). Once the lowest component field of this multiplet is obtained, the remaining components can be determined by using the spin-\( \frac{3}{2} \) currents in (2.11) from Appendix \( B \).

- More general coset model

The generalization of the coset (1.1) is introduced in [47]. It would be interesting to observe the presence of the higher spin currents. See also [48, 49, 50] for recent relevant works in different context.

- \( \mathcal{N} = 2 \) superspace description for the two adjoint fermions

In the subsection 4.1, the lowest \( \mathcal{N} = 2 \) higher spin currents for general \( N \) is determined.

10 We obtain the remaining (anti)commutators using (5.6), (5.8), (5.9) and (5.11) as follows:

\[
[V_m^{(2)} \pm , V_n^{(2)+}] = V_{m+n}^{(3)} - \frac{1}{2} (m-n)L_{m+n} + \frac{1}{8} (m^2 - mn + n^2 - 1)J_{m+n},
\]

\[
[V_r^{(\frac{1}{2})} , V_m^{(2)+}] = -V_{r+m}^{(\frac{3}{2})+} + \frac{1}{32} (9 - 4r^2 + 8rm - 12m^2)G^+_m,
\]

\[
\{V_r^{(\frac{1}{2})} , V_s^{(\frac{1}{2})-}\} = V_{r+s}^{(4)} + \frac{1}{16} (-9 + 6r^2 - 8rs + 6s^2)L_{r+s}.
\]

(5.14)

Therefore, the \( OSp(2|2) \) higher spin algebra (extension of \( \mathcal{N} = 2 \) wedge algebra) contains (5.7), (5.10), (5.12) and (5.13). Furthermore, it is straightforward to calculate the (anti)commutators between the higher spin generators described in the subsection 5.2.1 and the ones in the subsection 5.2.2 and we expect they will appear in the corresponding (anti)commutators studied in the subsection 4.3 with an appropriate limit.
It is an open problem to observe whether we can write down the two adjoint fermions in $\mathcal{N} = 2$ superspace explicitly. This will make some complicated calculations easier.

- The general structure of the $\mathcal{N} = 2$ higher spin algebra

In subsection 5.2, some of the structure constants of the $\mathcal{N} = 2$ higher spin algebra corresponding to the coset model (11) were obtained. It is an open problem to check whether we can find the complete structure of the whole structure constants along the line of [38, 39]. Any (anti)commutators between the higher spin generators consists of a sum of other lower higher spin generators as well as the generators coming from $\mathcal{N} = 2$ wedge algebra. Once we further consider the $U(1)$ charges of these (anti)commutators, the right hand sides will be simpler. The main task is how to write down the structure constants in terms of arbitrary modes and spins explicitly.

- Vasiliev’s oscillator formalism with matrix generalization

It would be interesting to construct the Vasiliev’s oscillator formalism in different coset models [10, 11, 14, 16, 27]. We can analyze the present method in the general coset model. For example, there is a good exercise in the context of the large $\mathcal{N} = 4$ holography [46]. Sometimes it will be difficult to construct the OPEs as the spins increase in the coset model. However, the oscillator construction will give some hints in this difficulty and although the procedure to obtain the (anti)commutators in terms of oscillators is rather tedious, we can easily observe the presence of the higher spin generators by counting the number of oscillators.

- Asymptotic symmetry algebra at the quantum level

In section 4, we have found only two (super) OPEs between the first and second $\mathcal{N} = 2$ higher spin multiplets. For the remaining OPEs, some of the structure constants appearing in the right hand sides are not determined. See also Appendix E. In order to find them, we need to obtain the OPEs between the higher spin currents in terms of two adjoint fermions. Even $N = 4$ case leads to the very complicated singular terms. It would be interesting to obtain the complete OPEs appearing in Appendix E by determining all the unknown structure constants.

According to the behavior of higher spin algebra we described in the section 5, the anticommutators between the generator $V_0^{(4)}$ of higher spin-$\frac{5}{2}$ and the generators of the half integer higher spin provide the generators of lowest components corresponding to the second, fourth and sixth of (4.11) and so on (that is, $V_0^{(3)}, V_0^{(5)}, V_0^{(7)}, \cdots$). On the other hand, the commutators between the generator $V_0^{(4)}$ of higher spin-$\frac{5}{2}$ and the generators of the integer higher spin provide the generators of lowest components corresponding to the third, fifth and seventh and so on (that is, $V_0^{(5)}, V_0^{(11)}, V_0^{(15)}, \cdots$). By assuming the OPEs between the first three $\mathcal{N} = 2$ higher spin multiplets in (4.11) and the fourth and the fifth ones further (by writ-
ing down the possible terms in the right hand sides), the undetermined structure constants in Appendix E by applying Jacobi identities associated with these OPEs will be fixed.

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A The $\mathcal{N} = 2$ superconformal algebra

The nontrivial OPEs between the four currents of $\mathcal{N} = 2$ superconformal algebra (See [12] for example) are summarized by

\[
J(z) J(w) = \frac{1}{(z-w)^2} \frac{c}{3} + \cdots,
\]
\[
J(z) G^+(w) = \frac{1}{(z-w)} G^+(w) + \cdots,
\]
\[
J(z) G^-(w) = -\frac{1}{(z-w)} G^-(w) + \cdots,
\]
\[
J(z) T(w) = \frac{1}{(z-w)^2} J(w) + \cdots,
\]
\[
G^+(z) G^-(w) = \frac{1}{(z-w)^3} \frac{c}{3} + \frac{1}{(z-w)^2} J(w) + \frac{1}{(z-w)} \left[ T + \frac{1}{2} \partial J \right](w) + \cdots,
\]
\[
G^+(z) T(w) = \frac{1}{(z-w)^2} G^+(w) + \frac{1}{(z-w)} \frac{1}{2} \partial G^+(w) + \cdots,
\]
\[
G^-(z) T(w) = \frac{1}{(z-w)^2} G^-(w) + \frac{1}{(z-w)} \frac{1}{2} \partial G^-(w) + \cdots,
\]
\[
T(z) T(w) = \frac{1}{(z-w)^4} \frac{c}{2} + \frac{1}{(z-w)^2} 2T(w) + \frac{1}{(z-w)} \partial T(w) + \cdots. \tag{A.1}
\]

The $\mathcal{N} = 2$ superspace description can be found in [51]. For convenience, we present the (anti)commutators

\[
[J_m, J_n] = \frac{c}{3} m \delta_{m,-n},
\]
\[
[J_m, G^\pm_r] = \pm G^\pm_{m+r},
\]
\[
[J_m, L_n] = m J_{m+n},
\]
\[
\{ G^+_r, G^-_s \} = L_{r+s} + \frac{1}{2} (r-s) J_{r+s} + \frac{c}{6} (r^2 - \frac{1}{4}) \delta_{r,-s},
\]
\[
[G^+_r, L_m] = (r - \frac{m}{2}) G^+_r + m J_{r+m},
\]
\[
[L_m, L_n] = (m-n) L_{m+n} + \frac{c}{12} (m^3 - m) \delta_{m,-n}. \tag{A.2}
\]

In (A.2), the central terms vanish at $m = 0, r = \pm \frac{1}{2}$ or $m = 0, \pm 1$ respectively.

B The OPEs between the four currents and the higher spin currents

The nontrivial OPEs between the four currents of $\mathcal{N} = 2$ superconformal algebra and the four higher spin currents ($W_q^{(h)}, W_{q+1}^{(h+\frac{1}{2})}, W_{q-1}^{(h+\frac{1}{2})}, W_q^{(h+1)}$) with the spins and $U(1)$ charges are
given by

\[
\begin{align*}
J(z) W_q^{(h)}(w) &= \frac{1}{(z-w)} q W_q^{(h)}(w) + \cdots, \\
J(z) W_{q+1}^{(h+\frac{1}{2})}(w) &= \frac{1}{(z-w)} (q+1) W_{q+1}^{(h+\frac{1}{2})}(w) + \cdots, \\
J(z) W_{q-1}^{(h+\frac{1}{2})}(w) &= \frac{1}{(z-w)} (q-1) W_{q-1}^{(h+\frac{1}{2})}(w) + \cdots, \\
J(z) W_q^{(h+1)}(w) &= \frac{1}{(z-w)^2} h W_q^{(h)}(w) + \frac{1}{(z-w)} q W_q^{(h+1)}(w) + \cdots, \\
G^+(z) W_q^{(h+1)}(w) &= -\frac{1}{(z-w)} W_{q+1}^{(h+\frac{1}{2})}(w) + \cdots, \\
G^+(z) W_{q-1}^{(h+\frac{1}{2})}(w) &= \frac{1}{(z-w)^2} \left( h + \frac{q}{2} \right) W_q^{(h)} + \frac{1}{(z-w)} \left[ W_q^{(h+1)} + \frac{1}{2} \partial W_q^{(h)} \right] (w) + \cdots, \\
G^+(z) W_q^{(h+1)}(w) &= \frac{1}{(z-w)^2} \left[ h + \frac{1}{2} (q+1) \right] W_{q+1}^{(h+\frac{1}{2})}(w) + \frac{1}{(z-w)} \frac{1}{2} \partial W_{q+1}^{(h+\frac{1}{2})}(w) + \cdots, \\
G^+(z) W_q^{(h)}(w) &= \frac{1}{(z-w)} W_{q+1}^{(h+\frac{1}{2})}(w) + \cdots, \\
G^-(z) W_{q+1}^{(h+\frac{1}{2})}(w) &= \frac{1}{(z-w)^2} \left( h - \frac{q}{2} \right) W_q^{(h)} + \frac{1}{(z-w)} \left[ W_q^{(h+1)} - \frac{1}{2} \partial W_q^{(h)} \right] (w) + \cdots, \\
G^-(z) W_q^{(h+1)}(w) &= \frac{1}{(z-w)^2} \left[ h - \frac{1}{2} (q-1) \right] W_{q-1}^{(h+\frac{1}{2})}(w) + \frac{1}{(z-w)} \frac{1}{2} \partial W_{q-1}^{(h+\frac{1}{2})}(w) + \cdots, \\
T(z) W_q^{(h)}(w) &= \frac{1}{(z-w)^2} h W_q^{(h)}(w) + \frac{1}{(z-w)} \partial W_q^{(h)}(w) + \cdots, \\
T(z) W_{q+1}^{(h+\frac{1}{2})}(w) &= \frac{1}{(z-w)^2} \left( h + \frac{1}{2} \right) W_{q+1}^{(h+\frac{1}{2})}(w) + \frac{1}{(z-w)} \partial W_{q+1}^{(h+\frac{1}{2})}(w) + \cdots, \\
T(z) W_{q-1}^{(h+\frac{1}{2})}(w) &= \frac{1}{(z-w)^2} \left( h - \frac{1}{2} \right) W_{q-1}^{(h+\frac{1}{2})}(w) + \frac{1}{(z-w)} \partial W_{q-1}^{(h+\frac{1}{2})}(w) + \cdots, \\
T(z) W_q^{(h+1)}(w) &= \frac{1}{(z-w)^3} \frac{q}{2} W_q^{(h)} + \frac{1}{(z-w)^2} \left( h + 1 \right) W_q^{(h+1)}(w) + \frac{1}{(z-w)} \partial W_q^{(h+1)}(w) + \cdots. \\
\end{align*}
\]

The $\mathcal{N} = 2$ superspace description can be found in [4]. For convenience, we present the (anti)commutators as follows. The first four of (B.1) corresponds to

\[
\begin{align*}
[J_m, W_n^{(h,q)}] &= q W_{m+n}^{(h,q)}, \\
[J_m, W_n^{(h+\frac{1}{2},q\pm 1)}] &= (q \pm 1) W_{m+n}^{(h+\frac{1}{2}, q\pm 1)}, \\
[J_m, W_n^{(h+1,q)}] &= q W_{m+n}^{(h+1,q)} + mh W_{m+n}^{(h,q)}. \tag{B.2}
\end{align*}
\]

If $h$ is an integer, the next six of (B.1) is equivalent to

\[
\begin{align*}
[G^\pm_n, W_n^{(h,q)}] &= \mp W_n^{(h+\frac{1}{2}, q\pm 1)},
\end{align*}
\]
\[
\{G^\pm_r, W_s^{(h+\frac{1}{2}, q+1)}\} = W^{(h+1, q)}_s + \left[ \frac{q}{2} (r + \frac{1}{2}) \pm (h - \frac{1}{2})r \mp \frac{s}{2} \right] W^{(h, q)}_{r+s},
\]

\[
\left[ G^\pm_r, W^{(h+1, q)}_n \right] = \left[ (hr - \frac{n}{2}) \pm \frac{q}{2} (r + \frac{1}{2}) \right] W^{(h+\frac{1}{2}, q+1)}_{r+n}. 
\] (B.3)

If \( h \) is a half-integer, they correspond to

\[
\{G^\pm_r, W_n^{(h, q)}\} = \mp W^{(h+\frac{1}{2}, q+1)}_{r+n},
\]

\[
\left[ G^\pm_r, W_n^{(h+\frac{1}{2}, q+1)} \right] = W^{(h+1, q)}_n + \left[ \frac{q}{2} (r + \frac{1}{2}) \pm (h - \frac{1}{2})r \mp \frac{n}{2} \right] W^{(h, q)}_{r+n},
\]

\[
\{G^\pm_r, W_s^{(h+1, q)}\} = \left[ (hr - s) \pm \frac{q}{2} (r + \frac{1}{2}) \right] W^{(h+\frac{1}{2}, q+1)}_{r+s}. 
\] (B.4)

We have the following commutators for the last four of (B.1)

\[
\left[ L_m, W^{(h, q)}_n \right] = \left[ (h - 1)m - n \right] W^{(h, q)}_{m+n},
\]

\[
\left[ L_m, W^{{(h+\frac{1}{2}, q+1)}}_n \right] = \left[ (h - \frac{1}{2})m - n \right] W^{{(h+\frac{1}{2}, q+1)}}_{m+n},
\]

\[
\left[ L_m, W^{(h, q+1)}_n \right] = (hm - n) W^{(h+1, q)}_{m+n} + \frac{q}{4} (m + 1) m W^{(h, q)}_{m+n}. 
\] (B.5)

We can read off the \( q = 0 \) case from (B.2), (B.3), (B.4) and (B.5).

C The remaining OPEs in the subsection 4.2

One way to determine the remaining nontrivial OPEs in the subsection 4.2 is to start with the OPE in (4.4). Then we take the super derivatives in both sides and put the constraints \( \theta_{12} = 0 \) or/and \( \bar{\theta}_{12} = 0 \) properly in order to extract the OPEs in components. We present the remaining nontrivial OPEs (in addition to the two trivial ones) described in the subsection 4.2 as follows:

\[
W^{(2)}_-(z) W^{(2)}_+(w) = -\frac{1}{(z-w)^4} + \frac{1}{(z-w)^3} 3J + \frac{1}{(z-w)^2} \left[ -\frac{(3 - 4c)}{(-1 + c)} T \right.
\]

\[
- \frac{3}{2(-1 + c)} JJ + \frac{3}{2} \partial J \right](w) - \frac{1}{(z-w)} \left[ C^{(3)}_{\frac{1}{2}} 3W^{(3)}_0 \right.
\]

\[
+ \frac{1}{(-1 + c)(6 + c)(-3 + 2c)} \left( -6(c - 3)(5c - 3)G^-G^+ \right.
\]

\[
+ 3(9 + 8c^2)JT - \frac{27}{2}(1 + 2c)JJ
\]

\[
- \frac{3}{2}(6 + c)(-3 + 2c)\partial JJ - \frac{1}{4}(108 - 207c + 60c^2 + 8c^3)\partial T
\]
\[
\mathcal{W}_0(z) \mathcal{W}_+^2(w) = -\frac{1}{(z-w)^3} 6G^+(w) - \frac{1}{(z-w)^2} \frac{1}{2(-1+c)} \left[ 3JG^+ - (9-8c)\partial G^+ \right](w) \\
\mathcal{W}_0(z) \mathcal{W}_-^2(w) = -\frac{1}{(z-w)^3} 6G^-(w) + \frac{1}{(z-w)^2} \frac{1}{2(-1+c)} \left[ 3JG^- + (9-8c)\partial G^- \right](w) \\
\mathcal{W}_0^2(z) \mathcal{W}_0^2(w) = \frac{1}{(z-w)^5} 2c + \frac{1}{(z-w)^3} \frac{1}{(-1+c)} \left[ -(9-10c)T - \frac{3}{2}JJ \right](w)
\]
\[+ \frac{1}{(z-w)^2} \left( -1 + c \right) \left[ -\frac{1}{2} (9 - 10c) \partial T - \frac{3}{2} \partial JJ \right] (w)\]
\[+ \frac{1}{(z-w)} \left[ C^{(3)}_{(\frac{3}{2})} W^{(4)}_0 + \frac{1}{(1 + c)(6 + c)(-3 + 2c)} \left( 27(-5 + c)c \partial G^- G^+ \right. \right. \]
\[- 81(1 + 2c) J G^- G^+ - \frac{81}{2}(1 + 2c) J J T + \frac{81}{2}(1 + 2c) \partial T J \]
\[+ 27(3 - 4c + 2c^2) T T + 27(-5 + c)c \partial G^+ G^- + \frac{9}{4}(9 - 4c)c \partial J \partial J \]
\[+ \frac{9}{4}(21 + 2c)c \partial^2 J J + \frac{3}{4}(4c^2 - 18c + 45) \partial^2 T - \frac{27}{4}(1 + 2c) \partial^3 J \right] (w)\]
\[+ \ldots,\]
\[\rightarrow \frac{1}{(z-w)^3} 2c + \frac{1}{(z-w)^2} \left[ 10 T - \frac{3}{2c} J J \right] (w)\]
\[+ \frac{1}{(z-w)^2} \left[ 5 \partial T - \frac{3}{2c} \partial JJ \right] (w)\]
\[+ \frac{1}{(z-w)} \left[ C^{(3)}_{(\frac{3}{2})} W^{(4)}_0 + \frac{27}{2c} \partial G^- G^+ - \frac{81}{c^2} J G^- G^+ - \frac{81}{2c^2} J J T \right. \]
\[+ \frac{81}{2c^2} \partial T J + \frac{27}{c} T T + \frac{27}{2c} \partial G^+ G^- - \frac{9}{2c} \partial J \partial J + \frac{9}{4c} \partial^2 J J + \frac{3}{2} \partial^2 T \right] (w) + \ldots.\]

We take the large \( c \) limit.

In order to compare with the (anti)commutators from the oscillator description, we need to write down the commutators and anticommutators corresponding to (C.1) as follows:

\[
[W^{(2)}_m, W^{(2)}_n] = C^{(3)}_{(\frac{3}{2})} W^{(3)}_{m+n} - 2(m-n) L_{m+n} + \frac{1}{2} (m^2 - mn + n^2 - 1) J_{m+n} - \frac{c}{6} (m+1)m(m-1) \delta_{m,-n},
\]

\[
[W^{(2)}_r, W^{(2)}_n] = -C^{(3)}_{(\frac{3}{2})} W^{(7)}_{r+n} + \frac{1}{8} (9 - 4r^2 + 8rn - 12n^2) G^\pm_{r+n},
\]

\[
\{W^{(2)}_r, W^{(2)}_s\} = C^{(3)}_{(\frac{3}{2})} W^{(4)}_{r+s} + \frac{1}{4} (-9 + 6r^2 - 8rs + 6s^2) L_{r+s} + \frac{c}{12} \left( r - \frac{3}{2} \right) \left( r - \frac{1}{2} \right) \left( r + \frac{1}{2} \right) (r + \frac{3}{2}) \delta_{r,-s},\]

where the modes coming from the nonlinear terms are ignored (that is, infinity limit of \( c \)). It is easy to see that the central terms vanish for \( m = 0, \pm 1 \) or \( r = \pm \frac{1}{2}, \pm \frac{3}{2} \). As done before, the relations (C.2) contain the previous ones in the footnote [10] by restricting the mode indices to the wedge ones. Then we have the complete (anti)commutators given in (4.7) and (C.2) for the lowest \( \mathcal{N} = 2 \) higher spin multiplet.
D  The quasi primary operators appearing in the subsection 4.3

The various quasi primary fields appearing in the subsection 4.3 can be obtained as follows:

\[
Q_1 = \frac{1}{(-1 + c)(6 + c)(-3 + 2c)C^{(3)}_{(2)(4)}} \left[ \frac{27}{8} (15 - 31c + 10c^2) \partial D W_0^{(4)} 
- \frac{9}{2} (-3 + 5c)(-3 + 2c)T D W_0^{(2)} + 9(9 - 18c + 5c^2) D T W_0^{(2)} \right],
\]

\[
Q_2 = \frac{1}{(-1 + c)(6 + c)(-3 + 2c)C^{(3)}_{(2)(4)}} \left[ - \frac{27}{8} (15 - 31c + 10c^2) \partial D W_0^{(2)} 
- \frac{9}{2} (-3 + 5c)(-3 + 2c)T \partial D W_0^{(2)} + 9(9 - 18c + 5c^2) D T W_0^{(2)} \right],
\]

\[
Q_3 = \frac{1}{(-1 + c)(6 + c)(-3 + 2c)(-39 + 14c + c^2)C^{(3)}_{(2)(4)}} 
\times \left[ - \frac{9}{4} (27 - 387c + 585c^2 - 31c^3 + 10c^4) T[D, \overline{D}] W_0^{(2)} 
- \frac{9}{4} (-297 + 117c + 510c^2 + 200c^3) T T W_0^{(2)} - \frac{9}{2} c(54 - 15c - 143c^2 + 30c^3) \partial D T D W_0^{(2)} 
+ \frac{1}{4} (5103 - 10773c + 3861c^2 + 63c^3 - 330c^4)[D, \overline{D}] T W_0^{(2)} 
+ \frac{9}{2} c(54 - 15c - 143c^2 + 30c^3) D T D W_0^{(2)} 
- \frac{9}{16} (1701 - 3699c - 1817c^2 - 307c^3 + 170c^4) \partial^2 W_0^{(2)} \right],
\]

\[
Q_4 = \frac{1}{(-1 + c)(6 + c)(-3 + 2c)C^{(3)}_{(2)(4)}} \left[ \frac{18}{5} c(-9 + 4c) \partial[D, \overline{D}] W_0^{(2)} + \frac{27}{5} c(-9 + 4c) T \partial W_0^{(2)} 
+ 9c(-9 + 4c) \overline{D} T D W_0^{(2)} + 9c(-9 + 4c) D T \overline{D} W_0^{(2)} - \frac{81}{10} (-9 + 4c) \partial T W_0^{(2)} \right],
\]

\[
Q_5 = \frac{1}{(-1 + c)(6 + c)(-3 + 2c)(-39 + 14c + c^2)C^{(3)}_{(2)(4)}} 
\times \left[ \frac{9}{10} (-99 + 672c - 491c^2 + 22c^3 + 36c^4) \partial^2 D W_0^{(2)} 
+ 3(432 - 729c + 297c^2 + 4c^3 + 2c^4) T \partial D W_0^{(2)} 
+ \frac{9}{2} (-243 + 423c - 42c^2 + 20c^3) T T D W_0^{(2)} + 9(18 - 51c + 35c^2) T D T W_0^{(2)} 
+ \frac{3}{2} (-243 + 837c - 432c^2 - 24c^3 + 32c^4)[D, \overline{D}] T D W_0^{(2)} \right].
\]
\[
Q_6 = \frac{1}{(-1 + c)(6 + c)(-3 + 2c)(-39 + 14c + c^2)C_{(\frac{3}{2})(\frac{3}{2})}^{(3)}} \times 
\left[ -\frac{9}{10}(-99 + 672c - 491c^2 + 22c^3 + 36c^4)\partial^2TDW_0^{(\frac{3}{2})} \\
+ 3(432 - 729c + 297c^2 + 4c^3 + 2c^4)T\partial DW_0^{(\frac{3}{2})} \\
- \frac{9}{2}(-243 + 423c - 42c^2 + 20c^3)TTDW_0^{(\frac{3}{2})} - 9(18 - 51c + 35c^2)TDTW_0^{(\frac{3}{2})} \\
- \frac{9}{2}(54 - 216c + 123c^2 - 17c^3 + 6c^4)\overline{DT}[D, \overline{D}]W_0^{(\frac{3}{2})} \\
- \frac{3}{2}(-702 + 576c + 249c^2 - 193c^3 + 4c^4)TDTW_0^{(\frac{3}{2})} \\
+ \frac{3}{2}(-918 + 1152c - 69c^2 - 263c^3 + 4c^4)\partial DTW_0^{(\frac{3}{2})} \\
- \frac{3}{2}(-243 + 837c - 432c^2 - 24c^3 + 32c^4)[D, \overline{D}]TDDW_0^{(\frac{3}{2})} \\
- \frac{3}{2}(351 + 81c + 108c^2 - 134c^3 + 8c^4)\partial TDDW_0^{(\frac{3}{2})} \right],
\]

\[
Q_7 = \frac{1}{(-1 + c)(6 + c)(-3 + 2c)C_{(\frac{3}{2})(\frac{3}{2})}^{(3)}} \left[ \frac{9}{14}(45 + 6c + 8c^2)T\partial[D, \overline{D}]W_0^{(\frac{3}{2})} \\
+ \frac{9}{7}(15 + 52c)TT\partial DW_0^{(\frac{3}{2})} + 9(6 + c)TDTDW_0^{(\frac{3}{2})} + 9(6 + c)TDTTW_0^{(\frac{3}{2})} \\
+ \frac{9}{14}c(-93 + 34c)\overline{DT}\partial DW_0^{(\frac{3}{2})} + \frac{6}{7}(93 - 34c)c\partial DTDW_0^{(\frac{3}{2})} \\
+ \frac{3}{7}(63 - 42c + 16c^2)[D, \overline{D}]T\partial W_0^{(\frac{3}{2})} - \frac{9}{28}(63 - 42c + 16c^2)\partial[D, \overline{D}]TW_0^{(\frac{3}{2})} \\
+ \frac{9}{14}(93 - 34c)cDT\partial DW_0^{(\frac{3}{2})} + \frac{6}{7}c(-93 + 34c)\partial DTDW_0^{(\frac{3}{2})} \\
- \frac{9}{28}(57 + 2c + 40c^2)\partial[D, \overline{D}]W_0^{(\frac{3}{2})} - \frac{27}{14}(15 + 52c)\partial TTW_0^{(\frac{3}{2})} \\
+ \frac{9}{35}(42 - 59c + 22c^2)\partial^2W_0^{(\frac{3}{2})} \right].
\]

The component results of (4.8) can be read off by using the super derivatives on both sides of (4.8) and taking \(\theta_{12} = 0\) or/and \(\bar{\theta}_{12} = 0\) at the final stage. For the component expression
of the above quasi primary fields we can use (2.13) and the list of (4.1). We observe that the third order pole in the OPEs between the stress energy tensor and the components of (D.1) at \( \theta = \bar{\theta} = 0 \) vanish as usual.\[11\]

\[ E \] Other OPEs between the \( \mathcal{N} = 2 \) higher spin multiplets appearing in (4.1)

\[ E.1 \] The OPE between the first and the third \( \mathcal{N} = 2 \) higher spin multiplets

We present the OPE between the lowest and the third \( \mathcal{N} = 2 \) higher spin multiplets as follows:

\[
W_0^{(\frac{3}{2})}(Z_1) W_0^{(\frac{3}{2})}(Z_2) = \frac{\theta_{12} \bar{\theta}_{12}}{z_{12}^3} \left[ \frac{3(108 - 144c + 15c^2 + 7c^3)}{(1 - c)(-39 + 14c + c^2)C^{(\frac{7}{2})}(3)} \right] W_0^{(3)}(Z_2)
\]

\[ + \frac{\theta_{12}}{z_{12}^2} \left[ \frac{(432 - 990c + 645c^2 - 79c^3 - 14c^4)}{(-1 + c)(-3 + 2c)(-39 + 14c + c^2)C^{(\frac{7}{2})}(3)} D W_0^{(3)} + \hat{Q}_1 \right](Z_2)
\]

\[ + \frac{\bar{\theta}_{12}}{z_{12}^2} \left[ \frac{(432 - 990c + 645c^2 - 79c^3 - 14c^4)}{(-1 + c)(-3 + 2c)(-39 + 14c + c^2)C^{(\frac{7}{2})}(3)} \bar{D} W_0^{(3)} + \hat{Q}_2 \right](Z_2)
\]

\[ + \frac{\theta_{12} \bar{\theta}_{12}}{z_{12}^2} \left[ \frac{1}{3} \partial (\text{pole-3}) + \hat{Q}_3 \right](Z_2)
\]

\[ + \frac{1}{z_{12}^2} \left[ \frac{c}{(-1 + c)C^{(\frac{7}{2})}(3)} [D, \bar{D}] W_0^{(3)} + \hat{Q}_4 \right](Z_2)
\]

\[ + \frac{\theta_{12}}{z_{12}} \left[ \frac{2}{7} \partial (\text{pole-2})_{\theta=0} + \hat{Q}_5 \right](Z_2) + \frac{\bar{\theta}_{12}}{z_{12}} \left[ \frac{2}{7} \partial (\text{pole-2})_{\bar{\theta}=0} + \hat{Q}_6 \right](Z_2)
\]

\[ + \frac{\theta_{12} \bar{\theta}_{12}}{z_{12}^2} \left[ C^{(5)}(\frac{7}{2}) W_0^{(5)} + \frac{1}{14} \partial^2 (\text{pole-3}) + \frac{3}{8} \partial Q_3 + \hat{Q}_7 \right](Z_2) + \cdots
\]

The quasi primary fields, \( \hat{Q}_i(Z_2) \) which depends on \( T(Z_2), W_0^{(\frac{3}{2})}(Z_2) \) and \( W_0^{(3)}(Z_2) \) (we do not present them here), appear in the right hand side of the OPE. In particular, the complete expression for \( \hat{Q}_7(Z_2) \) is not determined because we do not use further OPEs between the first three \( \mathcal{N} = 2 \) higher spin multiplets and the fourth one \( W_0^{(5)}(Z_2) \).

\[ \text{\[11\]} \text{ Or in } \mathcal{N} = 2 \text{ superspace, this is equivalent to observe that the singular term of } \frac{\theta_{12} \bar{\theta}_{12}}{z_{12}^3} \text{ in the OPE } T(Z_1) Q_i(Z_2) \text{ where } i = 1, 2, \cdots, 7 \text{ vanishes. We have checked them explicitly.} \]
E.2 The OPE between the second $\mathcal{N} = 2$ higher spin multiplet

We have the OPE between the second $\mathcal{N} = 2$ higher spin multiplet as follows:

$$W_0^{(3)}(Z_1) W_0^{(3)}(Z_2) = \frac{1}{z_{12}^5} \left[ \frac{c}{3} + 3 \theta_{12} \bar{\theta}_{12} T \right] (Z_2)$$

$$+ \frac{1}{z_{12}^5} \left[ -3 \theta_{12} D T + 3 \bar{\theta}_{12} T + \theta_{12} \bar{\theta}_{12} \partial \text{(pole-6)} \right] (Z_2) + \frac{1}{z_{12}^4} \frac{1}{(1 - c)} \left[ c[D, \bar{D}] T + 3 TT \right] (Z_2)$$

$$+ \frac{1}{z_{12}^3} \left[ \theta_{12} \left( \frac{2}{3} \partial \text{(pole-5)}_{\theta = 0} + \tilde{Q}_1 \right) + \bar{\theta}_{12} \left( \frac{2}{3} \partial \text{(pole-5)}_{\theta = 0} + \tilde{Q}_2 \right) \right] (Z_2)$$

$$+ \frac{1}{z_{12}^2} \left[ \frac{1}{2} \partial^2 \text{(pole-6)} + C^{(3)}_{(3)} W_0^{(3)} + \tilde{Q}_3 \right] (Z_2) + \frac{1}{z_{12}^2} \left[ \frac{1}{2} \partial \text{(pole-4)}_{\theta = \bar{\theta} = 0} \right] (Z_2)$$

$$+ \frac{1}{z_{12}^3} \left[ \frac{1}{4} \partial^2 \text{(pole-5)}_{\theta = 0} + \frac{3}{5} \partial \tilde{Q}_1 - \frac{56c^3 - 147c^2 + 63c - 54}{6(c - 3)c(28c + 3)} C^{(3)}_{(3)} D W_0^{(3)} + \tilde{Q}_4 \right] (Z_2)$$

$$+ \frac{1}{z_{12}^3} \left[ \frac{1}{4} \partial^2 \text{(pole-5)}_{\theta = 0} + \frac{3}{5} \partial \tilde{Q}_2 + \frac{56c^3 - 147c^2 + 63c - 54}{6(c - 3)c(28c + 3)} C^{(3)}_{(3)} \bar{D} W_0^{(3)} + \tilde{Q}_5 \right] (Z_2)$$

$$+ \frac{1}{z_{12}^2} \left[ \frac{1}{6} \partial^3 \text{(pole-6)} + \frac{2}{3} \partial \left( C^{(3)}_{(3)} W_0^{(3)} + \tilde{Q}_3 \right) + \tilde{Q}_6 \right] (Z_2)$$

$$+ \frac{1}{z_{12}^4} \left[ \frac{3}{20} \partial^2 \text{(pole-4)}_{\theta = \bar{\theta} = 0} - \frac{(8c^2 + 9)}{6(c - 3)(28c + 3)} C^{(3)}_{(3)} [D, \bar{D}] W_0^{(3)} + \tilde{Q}_7 \right] (Z_2)$$

$$+ \frac{1}{z_{12}^2} \left[ \frac{1}{15} \partial^3 \text{(pole-5)}_{\theta = 0} + \frac{1}{5} \partial^2 \tilde{Q}_1 + \frac{4}{7} \partial \left( \text{last two terms in } \frac{\theta_{12}}{z_{12}^3} \right) + \tilde{Q}_8 \right] (Z_2)$$

$$+ \frac{1}{z_{12}^2} \left[ \frac{1}{15} \partial^3 \text{(pole-5)}_{\theta = 0} + \frac{1}{5} \partial^2 \tilde{Q}_2 + \frac{4}{7} \partial \left( \text{last two terms in } \frac{\bar{\theta}_{12}}{z_{12}^3} \right) + \tilde{Q}_9 \right] (Z_2)$$

$$+ \frac{1}{z_{12}^4} \left[ \frac{1}{24} \partial^4 \text{(pole-6)} + \frac{5}{21} \partial^2 \left( C^{(3)}_{(3)} W_0^{(3)} + \tilde{Q}_3 \right) + \frac{5}{8} \partial \tilde{Q}_6 \right] (Z_2)$$

$$- \frac{5 C^{(5)}_{\frac{1}{2}, \frac{1}{2}} C^{(3)}_{\frac{7}{2}, \frac{7}{2}}}{C^{(3)}_{\frac{1}{2}, \frac{1}{2}}} W_0^{(5)} + \tilde{Q}_{10} \right] (Z_2)$$

$$+ \frac{1}{z_{12}^2} \left[ \frac{1}{30} \partial^3 \text{(pole-4)}_{\theta = \bar{\theta} = 0} + \frac{1}{2} \partial \left( \text{last two terms in } \frac{1}{z_{12}^2} \right) \right] (Z_2)$$

$$+ \frac{1}{z_{12}^2} \left[ \frac{1}{72} \partial^4 \text{(pole-5)}_{\theta = 0} + \frac{1}{21} \partial^2 \tilde{Q}_1 + \frac{5}{28} \partial^2 \left( \text{last two terms in } \frac{\theta_{12}}{z_{12}^3} \right) \right]$$

$$+ \frac{5}{9} \partial \tilde{Q}_8 + \frac{C^{(5)}_{\frac{1}{2}, \frac{1}{2}} C^{(3)}_{\frac{7}{2}, \frac{7}{2}}}{C^{(3)}_{\frac{1}{2}, \frac{1}{2}}} D W_0^{(5)} + \tilde{Q}_{11} \right] (Z_2)$$

$$+ \frac{1}{z_{12}^2} \left[ \frac{1}{72} \partial^4 \text{(pole-5)}_{\theta = 0} + \frac{1}{21} \partial^2 \tilde{Q}_2 + \frac{5}{28} \partial^2 \left( \text{last two terms in } \frac{\bar{\theta}_{12}}{z_{12}^3} \right) \right]$$

$$+ \frac{1}{z_{12}^2} \left[ \frac{1}{21} \partial^2 \tilde{Q}_3 + \frac{5}{28} \partial^2 \left( \text{last two terms in } \frac{z_{12}^3}{\theta_{12}} \right) \right]$$

$$+ \frac{1}{z_{12}^2} \left[ \frac{1}{21} \partial^2 \tilde{Q}_4 + \frac{5}{28} \partial^2 \left( \text{last two terms in } \frac{z_{12}^3}{\bar{\theta}_{12}} \right) \right]$$

$$+ \frac{1}{z_{12}^2} \left[ \frac{1}{21} \partial^2 \tilde{Q}_5 + \frac{5}{28} \partial^2 \left( \text{last two terms in } \frac{z_{12}^3}{\theta_{12}} \right) \right]$$

$$+ \frac{1}{z_{12}^2} \left[ \frac{1}{21} \partial^2 \tilde{Q}_6 + \frac{5}{28} \partial^2 \left( \text{last two terms in } \frac{z_{12}^3}{\bar{\theta}_{12}} \right) \right]$$

$$+ \frac{1}{z_{12}^2} \left[ \frac{1}{21} \partial^2 \tilde{Q}_7 + \frac{5}{28} \partial^2 \left( \text{last two terms in } \frac{z_{12}^3}{\theta_{12}} \right) \right]$$

$$+ \frac{1}{z_{12}^2} \left[ \frac{1}{21} \partial^2 \tilde{Q}_8 + \frac{5}{28} \partial^2 \left( \text{last two terms in } \frac{z_{12}^3}{\bar{\theta}_{12}} \right) \right]$$

$$+ \frac{1}{z_{12}^2} \left[ \frac{1}{21} \partial^2 \tilde{Q}_9 + \frac{5}{28} \partial^2 \left( \text{last two terms in } \frac{z_{12}^3}{\theta_{12}} \right) \right]$$

$$+ \frac{1}{z_{12}^2} \left[ \frac{1}{21} \partial^2 \tilde{Q}_{10} + \frac{5}{28} \partial^2 \left( \text{last two terms in } \frac{z_{12}^3}{\bar{\theta}_{12}} \right) \right]$$
where the self coupling constant is determined by

$$(C_{(3)(3)}^{(3)})^2 = \frac{24(c-3)^2c^2(28c+3)^2}{(c-1)(c+6)(2c-3)(2c+3)(4c-9)(5c-3)}.$$ 

We do not present the quasi primary fields, $\bar{Q}_i(Z_2)$ ($i = 1, 2, \ldots, 13$) which depends on $T(Z_2), W_0^0(Z_2)$ and $W_0^{(3)}(Z_2)$. As before, the complete expressions for $\bar{Q}_{10}(Z_2), \bar{Q}_{11}(Z_2), \bar{Q}_{12}(Z_2)$, and $\bar{Q}_{13}(Z_2)$ are not determined.

### E.3 The OPE between the second and the third $\mathcal{N} = 2$ higher spin multiplets

We obtain the following OPE

$$W_0^{(3)}(Z_1) W_0^{(\tilde{z})}(Z_2) = \frac{\theta_{12} \bar{\theta}_{12}}{z_1^6} \left[ - \frac{3(6+c)(18 - 27c + 7c^2)}{2(-1 + c)(-39 + 14c + c^2)C_{(\tilde{z})/2}(\tilde{z})} \right] W_0^{(\tilde{z})}(Z_2)$$

$$+ \frac{1}{z_1^6} \left[ \frac{(6 + c)(18 - 27c + 7c^2)}{(-1 + c)(-39 + 14c + c^2)C_{(\tilde{z})/2}(\tilde{z})} \right] \left[ \theta_{12} D W_0^{(\tilde{z})} - \bar{\theta}_{12} \bar{D} W_0^{(\tilde{z})} \right] (Z_2) + \frac{\theta_{12} \bar{\theta}_{12}}{z_1^5} \left[ \frac{2}{3} \partial^2(\text{pole-6}) \right] (Z_2)$$

$$+ \frac{1}{z_1^4} \left[ \frac{(6 + c)(18 - 27c + 7c^2)}{(-1 + c)(-9 + 4c)(-39 + 14c + c^2)C_{(\tilde{z})/2}(\tilde{z})} \right] \left[ c[D, \bar{D}] W_0^{(\tilde{z})} + 9TW_0^{(\tilde{z})} \right] (Z_2)$$

$$+ \frac{\theta_{12} \bar{\theta}_{12}}{z_1^4} \left[ \frac{1}{2} \partial^2(\text{pole-6}) + \frac{21(675 - 531c + 3687c^2 - 2527c^3 + 220c^4 + 48c^5)}{4(-1 + c)(6 + c)(-3 + 2c)(-39 + 14c + c^2)C_{(\tilde{z})/2}(\tilde{z})} W_0^{(\tilde{z})} + \bar{Q}_3 \right] (Z_2)$$

$$+ \frac{\theta_{12} \bar{\theta}_{12}}{z_1^4} \left[ \frac{2}{5} \partial^2(\text{pole-4})_{\theta = \bar{\theta} = 0} + \bar{Q}_1 \right] (Z_2) + \frac{\theta_{12} \bar{\theta}_{12}}{z_1^4} \left[ \frac{3}{20} \partial^2(\text{pole-5})_{\theta = \bar{\theta} = 0} + \frac{1}{2} \partial \bar{Q}_1 \right]$$

$$- \frac{3(675 - 531c + 3687c^2 - 2527c^3 + 220c^4 + 48c^5)}{2(-1 + c)(6 + c)(-3 + 2c)(-39 + 14c + c^2)C_{(\tilde{z})/2}(\tilde{z})} D W_0^{(\tilde{z})} + \bar{Q}_5 ] (Z_2)$$

$$+ \frac{\theta_{12}}{z_1^4} \left[ \frac{3}{20} \partial^2(\text{pole-5})_{\theta = 0} + \frac{1}{2} \partial \bar{Q}_2 \right]$$
\[
+ \frac{3(675 - 531c + 3687c^2 - 2527c^3 + 220c^4 + 48c^5)}{2(-1 + c)(6 + c)(-3 + 2c)(-39 + 14c + c^2)C^{(3)}_{(\frac{1}{2})(\frac{1}{2})}} D W_0^{(\frac{1}{2})} + \bar{Q}_6 \right] (Z_2)
\]

\[
+ \frac{1}{15} \frac{\theta_{12} \bar{\theta}_{12}}{z_{12}^3} \left[ \dot{Q}_7 + \frac{1}{15} \partial^3(\text{pole-6}) + \frac{4}{7} \partial \left( \text{last two terms in } \frac{\theta_{12} \bar{\theta}_{12}}{z_{12}^3} \right) \right] (Z_2)
\]

\[
+ \frac{1}{7} \frac{\theta_{12} \bar{\theta}_{12}}{z_{12}^2} \left[ \frac{1}{2} \partial^4(\text{pole-6}) + \frac{5}{28} \partial^2 \left( \text{last two terms in } \frac{\theta_{12} \bar{\theta}_{12}}{z_{12}^3} \right) \right] (Z_2)
\]

\[
+ \frac{2}{105} \frac{\theta_{12} \bar{\theta}_{12}}{z_{12}} \left[ \frac{1}{2} \partial^3(\text{pole-5})_\theta = 0 + \frac{1}{3} \partial^2 \dot{Q}_1 + \frac{1}{2} \partial \left( \text{last two terms in } \frac{1}{z_{12}^3} \right) \right] \right] (Z_2)
\]

\[
+ \frac{2}{105} \frac{\theta_{12} \bar{\theta}_{12}}{z_{12}} \left[ \frac{1}{2} \partial^3(\text{pole-5})_\theta = 0 + \frac{1}{3} \partial^2 \dot{Q}_1 + \frac{1}{2} \partial \left( \text{last two terms in } \frac{1}{z_{12}^3} \right) \right] \right] (Z_2)
\]

We do not present the quasi primary fields, \( \bar{Q}_i(Z_2) \) \( (i = 1, 2, \ldots, 15) \) which depends on \( T(Z_2), W_0^{(\frac{1}{2})}(Z_2) \) and \( W_0^{(\frac{3}{2})}(Z_2) \). The complete expressions for \( \bar{Q}_{11}(Z_2), \bar{Q}_{13}(Z_2), \bar{Q}_{14}(Z_2), \) and \( \bar{Q}_{15}(Z_2) \) are not determined as before.

E.4 The OPE between the third \( \mathcal{N} = 2 \) higher spin multiplet

We present the following OPE

\[
W_0^{(\frac{1}{2})}(Z_1) W_0^{(\frac{1}{2})}(Z_2) = \frac{1}{z_{12}^2} \left[ \frac{2c}{7} + 3 \theta_{12} \bar{\theta}_{12} T \right] (Z_2)
\]
\[
\begin{align*}
&+ \frac{1}{z_{12}} \left[ 3(-\theta_{12}D\mathbf{T} + \bar{\theta}_{12}\overline{D}\mathbf{T}) + \theta_{12}\bar{\theta}_{12} \partial(\text{pole-7}) \right] (Z_2) + \frac{1}{z_{12}} \frac{1}{(1-c)} \left[ c[D,\overline{D}]\mathbf{T} + 3\mathbf{T}\mathbf{T} \right] (Z_2) \\
&+ \frac{1}{z_{12}} \left[ \theta_{12} \left( \frac{2}{3} \partial(\text{pole-6})_{\theta=0} + \bar{\theta}_1 \right) + \bar{\theta}_{12} \left( \frac{2}{3} \partial(\text{pole-6})_{\theta=0} + \bar{\theta}_2 \right) \right] (Z_2) \\
&+ \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}} \left[ \frac{1}{2} \partial^2(\text{pole-7}) + C^{(3)}_{(\overline{\mathcal{T}}(\mathcal{T}))} \mathbf{W}_0^{(3)} + \bar{\mathbf{Q}}_3 \right] (Z_2) + \frac{1}{z_{12}} \left[ \frac{1}{2} \partial(\text{pole-5})_{\theta=\theta=0} \right] (Z_2) \\
&+ \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}} \left[ \frac{1}{4} \partial^2(\text{pole-6})_{\theta=0} + \frac{3}{5} \partial \bar{\mathbf{Q}}_1 + C^{(7)}_{(\overline{\mathcal{T}}(\mathcal{T}))} \mathcal{D}\mathbf{W}_0^{(3)} + \mathbf{\bar{Q}}_4 \right] (Z_2) \\
&+ \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}} \left[ \frac{1}{4} \partial^2(\text{pole-6})_{\theta=0} + \frac{3}{5} \partial \bar{\mathbf{Q}}_2 - C^{(7)}_{(\overline{\mathcal{T}}(\mathcal{T}))} \mathcal{D}\mathbf{W}_0^{(3)} + \mathbf{\bar{Q}}_5 \right] (Z_2) \\
&+ \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}} \left[ \frac{1}{6} \partial^3(\text{pole-7}) + \frac{2}{3} \partial \left( C^{(3)}_{(\overline{\mathcal{T}}(\mathcal{T}))} \mathbf{W}_0^{(3)} + \bar{\mathbf{Q}}_3 \right) + \bar{\mathbf{Q}}_6 \right] (Z_2) \\
&+ \frac{1}{z_{12}} \left[ \frac{3}{20} \partial^2(\text{pole-5})_{\theta=\theta=0} + C^{(4)}_{(\overline{\mathcal{T}}(\mathcal{T}))}[D,\overline{D}]\mathbf{W}_0^{(3)} + \mathbf{\bar{Q}}_7 \right] (Z_2) \\
&+ \frac{\theta_{12}}{z_{12}} \left[ \frac{1}{15} \partial^3(\text{pole-6})_{\theta=0} + \frac{1}{5} \partial^2 \bar{\mathbf{Q}}_1 + \frac{4}{7} \partial \left( C^{(7)}_{(\overline{\mathcal{T}}(\mathcal{T}))} \mathcal{D}\mathbf{W}_0^{(3)} + \mathbf{\bar{Q}}_4 \right) + \mathbf{\bar{Q}}_8 \right] (Z_2) \\
&+ \frac{\theta_{12}}{z_{12}} \left[ \frac{1}{15} \partial^3(\text{pole-6})_{\theta=0} + \frac{1}{5} \partial^2 \bar{\mathbf{Q}}_2 + \frac{4}{7} \partial \left( -C^{(7)}_{(\overline{\mathcal{T}}(\mathcal{T}))} \mathcal{D}\mathbf{W}_0^{(3)} + \mathbf{\bar{Q}}_5 \right) + \mathbf{\bar{Q}}_9 \right] (Z_2) \\
&+ \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}} \left[ \frac{1}{24} \partial^4(\text{pole-7}) + \frac{5}{21} \partial^2 \left( C^{(3)}_{(\overline{\mathcal{T}}(\mathcal{T}))} \mathbf{W}_0^{(3)} + \bar{\mathbf{Q}}_3 \right) + \frac{5}{8} \partial \bar{\mathbf{Q}}_6 + C^{(5)}_{(\overline{\mathcal{T}}(\mathcal{T}))} \mathbf{W}_0^{(5)} + \mathbf{\bar{Q}}_{10} \right] (Z_2) \\
&+ \frac{1}{z_{12}} \left[ \frac{1}{30} \partial^3(\text{pole-5})_{\theta=\theta=0} + \frac{1}{2} \partial \left( C^{(4)}_{(\overline{\mathcal{T}}(\mathcal{T}))}[D,\overline{D}]\mathbf{W}_0^{(3)} + \mathbf{\bar{Q}}_7 \right) \right] (Z_2) \\
&+ \frac{\theta_{12}}{z_{12}} \left[ \frac{1}{72} \partial^4(\text{pole-6})_{\bar{\theta}=0} + \frac{1}{21} \partial^3 \bar{\mathbf{Q}}_1 + \frac{5}{28} \partial^2 \left( C^{(7)}_{(\overline{\mathcal{T}}(\mathcal{T}))} \mathcal{D}\mathbf{W}_0^{(3)} + \mathbf{\bar{Q}}_4 \right) + \frac{5}{9} \partial \bar{\mathbf{Q}}_8 \right] \\
&+ \frac{1}{z_{12}} \left[ C^{(4)}_{(\overline{\mathcal{T}}(\mathcal{T}))} \mathcal{D}\mathbf{W}_0^{(5)} + \mathbf{\bar{Q}}_{11} \right] (Z_2) \\
&+ \frac{\bar{\theta}_{12}}{z_{12}} \left[ \frac{1}{72} \partial^4(\text{pole-6})_{\bar{\theta}=0} + \frac{1}{21} \partial^3 \bar{\mathbf{Q}}_2 + \frac{5}{28} \partial^2 \left( -C^{(7)}_{(\overline{\mathcal{T}}(\mathcal{T}))} \mathcal{D}\mathbf{W}_0^{(3)} + \mathbf{\bar{Q}}_5 \right) + \frac{5}{9} \partial \bar{\mathbf{Q}}_9 \right] \\
&- \frac{C^{(11)}_{(\overline{\mathcal{T}}(\mathcal{T}))} \mathcal{D}\mathbf{W}_0^{(5)} + \mathbf{\bar{Q}}_{12} \right] (Z_2) \\
&+ \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}} \left[ \frac{1}{120} \partial^5(\text{pole-7}) + \frac{5}{84} \partial^3 \left( C^{(3)}_{(\overline{\mathcal{T}}(\mathcal{T}))} \mathbf{W}_0^{(3)} + \mathbf{\bar{Q}}_3 \right) + \frac{5}{24} \partial^2 \mathbf{\bar{Q}}_6 + \frac{3}{5} \partial \left( C^{(5)}_{(\overline{\mathcal{T}}(\mathcal{T}))} \mathbf{W}_0^{(5)} + \mathbf{\bar{Q}}_{10} \right) \right] (Z_2) \\
&+ \bar{\mathbf{Q}}_{13} \right] (Z_2) \\
&+ \frac{1}{z_{12}} \left[ \frac{1}{168} \partial^4(\text{pole-5})_{\theta=\bar{\theta}=0} + \frac{5}{36} \partial^2 \left( C^{(4)}_{(\overline{\mathcal{T}}(\mathcal{T}))}[D,\overline{D}]\mathbf{W}_0^{(3)} + \mathbf{\bar{Q}}_7 \right) + C^{(6)}_{(\overline{\mathcal{T}}(\mathcal{T}))}[D,\overline{D}]\mathbf{W}_0^{(5)} + \mathbf{\bar{Q}}_{14} \right] (Z_2)
\end{align*}
\]
where the structure constants are given by

\[
\begin{align*}
(C^{(3)}_{(\hat{7})^3(\hat{2})} \right)^2 &= \frac{27(675 - 531c + 3687c^2 - 2527c^3 + 220c^4 + 48c^5)^2}{2(-1 + c)(6 + c)(-3 + 2c)(3 + 2c)(-9 + 4c)(-3 + 5c)(-39 + 14c + c^2)^2}, \\
(C^{(4)}_{(\hat{7})^4(\hat{2})} \right)^2 &= \frac{6(-3942 + 9369c - 12222c^2 + 9646c^3 - 3371c^4 + 238c^5 + 48c^6)^2}{(-1 + c)(6 + c)(-3 + 2c)^3(3 + 2c)(-9 + 4c)(-3 + 5c)(-39 + 14c + c^2)^2}, \\
(C^{(5)}_{(\hat{7})^5(\hat{2})} \right)^2 &= \frac{54c^2(4968 - 14202c + 19152c^2 - 15215c^3 + 7000c^4 - 1593c^5 + 78c^6 + 16c^7)^2}{(-1 + c)(6 + c)(-3 + 2c)^3(3 + 2c)(-9 + 4c)(-3 + 5c)^3(-39 + 14c + c^2)^2}, \\
&\times \frac{1}{(-39 + 14c + c^2)^2(18 - 27c + 7c^2)^2}, \\
(C^{(6)}_{(\hat{7})^6(\hat{2})} \right)^2 &= \frac{50(216 - 459c + 114c^2 + 91c^3 - 22c^4)^2(C^{(5)}_{(\hat{7})^5(\hat{2})} \right)^2}{3(-3 + 2c)(3 + 2c)(-9 + 4c)(-3 + 5c)(-39 + 14c + c^2)(18 - 27c + 7c^2)}; \\
(C^{(5)}_{(\hat{7})^5(\hat{2})} \right)^2 &= \frac{2(-513 + 1269c - 855c^2 + 89c^3 + 22c^4)^2}{3(-3 + 2c)(3 + 2c)(-9 + 4c)(-3 + 5c)(-39 + 14c + c^2)(18 - 27c + 7c^2)}, \\
(C^{(6)}_{(\hat{7})^6(\hat{2})} \right)^2 &= \frac{2c^2(-3 + 2c)(-39 + 14c + c^2)(C^{(5)}_{(\hat{7})^5(\hat{2})} \right)^2}{3(3 + 2c)(-9 + 4c)(-3 + 5c)(18 - 27c + 7c^2)}. \\
\end{align*}
\]

Note that \(C^{(7)}_{(\hat{7})^7(\hat{2})}\) is undetermined. The quasi primary fields \(\tilde{Q}_i\) (\(i = 11, 12, \cdots, 17\)), are undetermined.

The analysis done in the section 5 can be described similarly in this Appendix.
The partners of (5.13) appearing in the subsection 5.2

In addition to (5.13), the remaining generators of the second $\mathcal{N} = 2$ higher spin multiplet, which can be obtained from the relations in the footnote [10] are given by

\[
V^{(\frac{1}{2})+}_{\frac{3}{2}} = 2(\frac{-i}{4})^{\frac{1}{2}}y_{1}y_{1}y_{1}y_{1} \otimes \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix},
\]

\[
V^{(\frac{1}{2})+}_{\frac{5}{2}} = (\frac{-i}{4})^{\frac{1}{2}}\frac{2}{5}(y_{1}y_{1}y_{1}y_{2} + y_{1}y_{2}y_{1}y_{1} + y_{1}y_{1}y_{2}y_{1} + y_{2}y_{1}y_{1}y_{1}) \otimes \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix},
\]

\[
V^{(\frac{1}{2})+}_{\frac{7}{2}} = (\frac{-i}{4})^{\frac{1}{2}}\frac{2}{10}(y_{1}y_{1}y_{1}y_{2} + y_{1}y_{2}y_{1}y_{2} + y_{1}y_{2}y_{2}y_{1} + y_{1}y_{2}y_{1}y_{1} + y_{1}y_{2}y_{2}y_{1} + y_{1}y_{2}y_{2}y_{1}) \otimes \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix},
\]

\[
V^{(\frac{1}{2})-}_{\frac{1}{2}} = 2(\frac{-i}{4})^{\frac{3}{2}}y_{2}y_{2}y_{2}y_{2} \otimes \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix},
\]

\[
V^{(\frac{1}{2})-}_{\frac{3}{2}} = 2(\frac{-i}{4})^{\frac{1}{2}}y_{1}y_{1}y_{1}y_{1} \otimes \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix},
\]

\[
V^{(\frac{1}{2})-}_{\frac{5}{2}} = (\frac{-i}{4})^{\frac{1}{2}}\frac{2}{5}(y_{1}y_{1}y_{1}y_{2} + y_{1}y_{2}y_{1}y_{1} + y_{1}y_{2}y_{2}y_{1} + y_{2}y_{1}y_{1}y_{1}) \otimes \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix},
\]

\[
V^{(\frac{1}{2})-}_{\frac{7}{2}} = (\frac{-i}{4})^{\frac{1}{2}}\frac{2}{10}(y_{1}y_{1}y_{1}y_{2} + y_{1}y_{2}y_{1}y_{2} + y_{1}y_{2}y_{2}y_{1} + y_{1}y_{2}y_{1}y_{1} + y_{1}y_{2}y_{2}y_{1} + y_{1}y_{2}y_{2}y_{1}) \otimes \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix},
\]

\[
V^{(\frac{1}{2})-}_{\frac{9}{2}} = 2(\frac{-i}{4})^{\frac{3}{2}}y_{2}y_{2}y_{2}y_{2} \otimes \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix},
\]

37
Then the higher spin generators corresponding to the second \( N = 2 \) higher spin multiplet are described by (5.13) and (F.1). We expect that the higher spin generators, in the extension of the \( OSp(2|2) \) higher spin algebra, in the list of (5.13) can be described by the tensor product between the generators (whose spins and the modes can be fixed by the number of oscillators \( \hat{g}_a \)) and the \( 2 \times 2 \) Pauli matrices (plus identity matrix) with the appropriate normalization factors.

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