Sensitivity of tensor analyzing power in the process $d + p \rightarrow d + X$ to the longitudinal isoscalar form factor of the Roper resonance electroexcitation

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Abstract

The tensor analyzing power of the process $d + p \rightarrow d + X$, for forward deuteron scattering in the momentum interval 3.7 to 9 GeV/c, is studied in the framework of $\omega$ exchange in an algebraic collective model for the electroexcitation of nucleon resonances. We point out a special sensitivity of the tensor analyzing power to the isoscalar longitudinal form factor of the Roper resonance excitation. The main argument is that the $S_{11}(1535)$, $D_{13}(1520)$ and $S_{11}(1650)$ resonances have only isovector longitudinal form factors. It is the longitudinal form factor of the Roper excitation, which plays an important role in the $t$–dependence of the tensor analyzing power. We discuss possible evidence of swelling of hadrons with increasing excitation energy.

25.40.Ny, 25.10+s, 13.40.Gp, 14.20.Gk

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I. INTRODUCTION

In a previous paper [1] it was shown that the polarization observables in inclusive scattering of high energy deuterons by protons at zero scattering angle, are sensitive to the ratio

\[ r = \frac{\sigma_L}{\sigma_T}, \]

where \( \sigma_L \) and \( \sigma_T \) are the cross sections of absorption of virtual isoscalar photons with longitudinal and transversal polarizations by nucleons. In the framework of the \( \omega \)-exchange mechanism for the considered reaction, it was found that this sensitivity is especially large in the region of \( N^* \)-excitations with masses 1.4 – 1.6 GeV, for \( r \leq 0.5 \). This interval is especially interesting when compared with the data obtained by inclusive \( eN \)-scattering. This sensitivity is due, in particular, to the properties of the deuteron electromagnetic form factors. In [1] a simplified assumption was used, namely, the ratio \( r \) was taken to be a free parameter independent of the four momentum transfer square, \( t = -q^2 \). The value that best fitted the data was \( r = 0.1 \). It is then interesting to compare this value with that of a realistic model for the nucleon resonances electroexcitation. In this work we present the results of the analysis of polarization phenomena in \( d + p \rightarrow d + X \), using the predictions of the model [2-4] for the \( t \)-dependence of the isoscalar form factors of electromagnetic \( N \rightarrow N^* \) transitions. We show that hadronic probes of nucleon structure, in particular, using polarized particles, may give interesting and important information concerning form factors of \( N^* \)-excitations. The selectivity of reactions such as \( p(d, d')X \) or \( p(\alpha, \alpha')X \) to the isoscalar part of the \( N^* \)-electroexcitation makes these processes complementary to electron-nucleon inelastic scattering, for the study of the \( N^* \)-structure. Note that in the framework of the \( \omega \)-exchange model [1], all polarization phenomena for \( d + p \rightarrow d + X \) can be predicted without any free parameters, using only existing information about the deuteron electromagnetic form factors and about the ratio \( r \).

This paper is organized as follows. In section II we give the \( t \)-dependence of the tensor analyzing power \( T_{20} \) and \( r \) in the model [1]. Formulas for transverse and longitudinal amplitudes of the algebraic collective model [2-4] are recalled in section III. Results and discussions are presented in section IV and the conclusions are drawn in section V.

II. \( t \)-DEPENDENCE OF \( T_{20} \) AND \( r \)

We will analyze here the polarization phenomena in the process \( d + p \rightarrow d + X \), for forward deuteron scattering, in the framework of the model [1] based on \( \omega \)-exchange (Fig. 1). The \( \omega \)-meson is preferred, among the isoscalar mesons as \( \sigma \) or \( \eta \), for several reasons. The \( \omega NN \)– coupling is large; the \( \omega \)-meson, being a spin 1 particle, can induce strong polarization effects and an energy-independent cross section. When considered as an isoscalar photon, then the cross sections and the polarization observables can be calculated from the known electromagnetic properties of the deuteron and \( N^* \), through the vector dominance model. These special properties of the \( \omega \)-exchange mechanism allow an experimental test of the
validity of this model, similar to the Rosenbluth test of the one-photon mechanism, in case of elastic and inelastic electron-hadron scattering. The details of the model are described in [1]. We will recall here only the final expressions, necessary for the present analysis.

The tensor analyzing power in \( d + p \rightarrow d + X \), \( T_{20} \), can be written in terms of the electromagnetic form factors as:

\[
T_{20} = -\sqrt{2} \frac{V_1^2 + (2V_0V_2 + V_2^2)r}{4V_1^2 + (3V_0^2 + V_2^2 + 2V_0V_2)r},
\]

where \( V_0(t), V_1(t) \) and \( V_2(t) \) are related to the standard electromagnetic deuteron form factors: \( G_c(t) \) (electric), \( G_m(t) \) (magnetic) and \( G_q(t) \) (quadrupole) by:

\[
V_0 = \sqrt{1 + \frac{2\tau}{3}} G_c - 2G_q \),
V_1 = \sqrt{\tau} G_m, \quad V_2 = \frac{\tau}{\sqrt{1 + \tau}} \left[ -G_c + 2 \left( 1 - \frac{1}{3\tau} \right) G_q \right],
\]

and \( \tau = -t/4M_d^2 \), where \( M_d \) is the deuteron mass. The ratio \( r \) characterizes the relative role of longitudinal and transversal isoscalar excitations in the transition \( \omega + N \rightarrow X \). In case of the Roper excitation we can write:

\[
r_R(t) = \frac{|A_1^p + A_1^n|^2}{|A_{1/2}^p + A_{1/2}^n|^2},
\]

where \( A_1^N \) (\( A_{1/2}^N \)) is the longitudinal (transversal) form factor of the \( P_{11}(1440) \)-excitation on proton (\( N = p \)) or neutron (\( N = n \)) targets. This formula can be generalized to the excitation of any nucleon resonance \( N^* \) as follows:

\[
r_{N^*}(t) = \frac{|A_1^p + A_1^n|^2}{|A_{1/2}^p + A_{1/2}^n|^2 + |A_{3/2}^p + A_{3/2}^n|^2} \equiv \sigma_L(t)/\sigma_T(t),
\]

where \( A_{1/2}^N \) and \( A_{3/2}^N \) are the two possible transversal form factors, corresponding to total \( \gamma^* + N^- \) helicity equal to 1/2 and 3/2 respectively.

In case of overlapping resonances, taking into account the finite values of the resonance widths, Eq. (3) can be generalized to

\[
r \rightarrow r(t, W) = \frac{\sum_i \sigma_{L,i}(t)B_i(W)C_i}{\sum_i \sigma_{T,i}(t)B_i(W)C_i},
\]

where \( B_i(W) \) is a Breit-Wigner function for the \( i \) - th \( N^* \)-resonance with a definite normalization:

\[
C_i^{-1} = \int_{M + M_\pi}^{\infty} dW B_i(W),
\]

\( M \) is the nucleon mass, \( M_\pi \) is the pion mass and \( W \) is the effective invariant mass of the \( X^- \) system in \( d + p \rightarrow d + X \) (i.e. the mass of the resonance). Let us mention that for forward deuteron scattering in \( d + p \rightarrow d + X \) the variables \( t \) and \( W \) are not independent, for a fixed energy of the incoming deuteron there is a definite correspondence between \( t \) and
The following observations, based on Eq. (1), can be made. All information about the $\omega NN^*$-vertex is contained in the function $r$ only. $T_{20}$ is especially sensitive to the small value of $r(t, W)$ in the interval $0 \leq r \leq 0.5$. A zero value of $r$ results in a $t$-independent value for $T_{20}$, namely $T_{20} = -1/\sqrt{8}$, for any value of the deuteron electromagnetic form factors. The position of the points at $q \simeq 1.85$ fm$^{-1}$ and $q \simeq 5$ fm$^{-1}$, (with $q = \sqrt{-t}$), at which all theoretical curves for different $r$ intersect, is determined by the models used for the deuteron form factors. In Fig. 2 we show the $t$-dependence of $r$, calculated on the basis of Eq. (3), for two different cases: (i) considering the contribution of only the Roper resonance (dotted line), (ii) considering the sum of the contributions of the following resonances

\begin{equation}
P_{11}(1440), \ S_{11}(1535), \ D_{13}(1520), \ S_{11}(1650), \end{equation}

which are overlapping in this mass region (solid line). The form factors of the $N^*$-excitations on proton and neutron targets were derived using an algebraic collective model [2,3], some details of which will be discussed in the next section. This will then allow to understand the behaviour of the ratio $r$ as shown in Fig. 2.

### III. ALGEBRAIC COLLECTIVE MODEL

The formulas for transverse and longitudinal helicity amplitudes used in the present work, are based on the collective string model of baryons [2,4], assuming $SU_{sf}(6)$ symmetry. In this model the nucleon resonances are interpreted in terms of rotations and vibrations of a Y-shaped string configuration with a prescribed distribution of charges and magnetization. The underlying algebraic structure of the model enables a derivation of closed expressions for masses, electromagnetic and strong couplings of baryon resonances. Electromagnetic transverse and longitudinal form factors on proton and neutron targets for nucleon resonances below 2 GeV are shown in Tables I-II as a function of the photon momentum $k$. The parameters relevant to these observables are the constituent mass, $m = 0.336$ GeV, magnetic moment, $\mu = 0.127$ GeV$^{-1}$ (g-factor $g=1$), and a scale parameter of the distribution, $a = 0.232$ fm. The state of the resonance is recalled in Table II only.

As seen from Table II, of the four resonances mentioned in (6), only the Roper resonance has a nonzero isoscalar longitudinal form factor. All other three resonances cannot be excited by isoscalar longitudinal virtual photons. The isoscalar longitudinal amplitudes of $S_{11}(1535)$ and $D_{13}(1520)$ vanish because of spin-flavor symmetry, while both isoscalar and isovector longitudinal couplings of $S_{11}(1650), D_{15}(1675)$ and $D_{13}(1700)$ vanish identically. This behavior of the isoscalar form factors is essential for the correct description of the existing experimental data on the $t$-dependence of $T_{20}$ for the process $d + p \rightarrow d + X$.

The longitudinal isoscalar ($A_S = A_S^p + A_S^n$) form factor of the Roper resonance in the collective-string model of [2,4] is, apart from an overall constant,

\begin{equation}
A_S = \frac{4ka(1 - 2k^2a^2)}{(1 + k^2a^2)^4}.
\end{equation}
This form factor has a zero at $k^2 = \frac{1}{2a^2}$. This is also the zero of $r$ seen in Fig. 2. The location of the zero depends on the value of $a$, which in turn characterizes the size of the string configuration. An accurate determination of the location of the zero for the transition form factor from the $t$-dependence of $T_{20}$ could therefore provide an independent measure of the transition radius of the Roper resonance. In particular, it could shed some light on the question whether or not hadrons swell with increasing excitation energy. The latter can be studied within the collective-string model [2,3] by introducing the stretchability of the string $\xi$ via the ansatz

$$a = a_0 \left(1 + \xi \frac{W - M}{M}\right)$$

with $a_0 = 0.232$ fm.

The calculations reported below are performed in the Breit frame for which

$$k^2 = -t + \frac{(W^2 - M^2)^2}{2(W^2 + M^2) - t}.$$  \hfill (9)

From Eq. (9), one can see that $k$ depends not only on $t$, but on $W$ too.

**IV. RESULTS AND DISCUSSION**

In Fig. 3 we report the theoretical predictions, using Eq. (3), together with the existing experimental data. In such an approximation, $T_{20}$ is a universal function of $t$ only, without any dependence on the initial deuteron momentum. The experimental values of $T_{20}$ for $p(d, d)X$ [5,6], for different momenta of the incident beam are shown as open symbols. These data show a scaling as a function of $t$, with a small dependence on the incident momentum in the interval 3.7–9 GeV/c. On the same plot the data for the elastic scattering process $e^- + d \rightarrow e^- + d$ [7] are shown (filled stars). All these data show a very similar behavior: negative values, with a minimum in the region $|t| \approx 0.35 \text{ GeV}^2$ and an increase towards zero at larger $|t|$. The lines are the result of the $\omega$-exchange model calculation for the $d+p \rightarrow d+X$ process. The dashed-dotted line correspond to $r = 0$, i.e. to $T_{20} = -1/\sqrt{8}$ as mentioned previously. Calculations based on the algebraic collective model [2,3] are shown for the case when only the Roper resonance is considered (dotted line) and for the case when all the four resonances (6) are considered (solid line). The required deuteron form factors, $G_c$, $G_q$, $G_m$, were taken from [8] (calculated in a relativistic impulse approximation) and they reproduce well the $T_{20}$-data for $ed$ elastic scattering [4]. When $r \gg 0$ or if the contribution of the deuteron magnetic form factor $V_1$ is neglected, then $T_{20}$ does not depend on the ratio $r$, and coincides with $t_{20}$ for the elastic $ed$-scattering (with the same approximation).

From Fig. 3 it appears that the $t$-behavior of $T_{20}$ is very sensitive to the value of $r$, at relatively small $r$, $r \leq 0.5$. The values of $r$, predicted by the collective model [2,3] give a good description of the data, when taking into account the contribution of all four resonances (6). These data, in any case, exclude a very small value of $r$, $r \ll 0.1$ as well.
as very large values of $r$. Such sensitivity of $T_{20}$ for $d + p \to d + X$ to the ratio of the corresponding isoscalar form factors of the $N^*$-excitation clearly indicates the presence of the Roper resonance in this process. Such an indication was hardly found in the differential cross section for inclusive scattering with unpolarized particles [1].

In the framework of the $\omega$-exchange mechanism, electromagnetic isovector components of the $N \to N^*$ transition cannot contribute. This is important as the other resonances in this mass region, $S_{11}(1535), D_{13}(1520)$ and $S_{11}(1650)$ are essentially isovector in the collective model [2,3] (as well as in other quark models), so the isoscalar longitudinal form factors for $N \to N^*$ are identically zero for any value of $t$. The ratio $r$ contains (in the numerator) the contribution of only the Roper resonance. It is this specific property of the Roper resonance (combined with the $t-$dependence of the deuteron form factors) that induces the specific $t-$behavior of the isoscalar ratio $r$ and of the analyzing power $T_{20}$ as shown in Figs. 2 and 3, respectively.

In this connection, we mention that the $\omega$-exchange model predicts the general features of the polarization observables. For example, the crossing of all the theoretical curves for $T_{20}$ at two points, is determined by the relative value of the deuteron electromagnetic form factors. For any model of $r$ we will have $T_{20} \leq -1/\sqrt{8}$ in the region $2 \leq q \leq 5$ fm$^{-1}$. Future data from Jefferson Lab [10], concerning $T_{20}$ in $e^- + d \to e^- + d$ will help in defining the exact position of these points. For $q \leq 6$ fm$^{-1}$, $T_{20}$ cannot be positive.

Using the generalized formula, Eq. (4), the $t-$behavior of the ratio $r(t, W)$ depends on the initial deuteron momentum. From Fig. 4 one can see that, in the interval 3.7–9 GeV/c this dependence is not so large, in agreement with experimental data. This is also true for the momentum dependence of $T_{20}$, (Fig. 5). The agreement between theoretical predictions and experimental data is generally good, at least for $q \leq 2$ fm$^{-1}$.

Of course, this model for $d + p \to d + p$ can be improved, taking into account for example, other meson exchanges, or the effects of the strong interaction in initial and final states. However these corrections are strongly model- and parameter- dependent; the existing experimental data are not good enough to constrain the additional parameters which have to be added. In this case we lose the predictive power of our ”parameter free” model. The successful description of the polarization observable $T_{20}$ can be considered as a strong indication that the $\omega$- exchange is the main mechanism for the considered process.

We analyzed also the sensitivity to a possible stretching mechanism [8], leading to the swelling of hadrons with increasing excitation energy. We use the parameterization of Eq. (8) for the scale parameter $a$, with $\xi = 0.5$ and $\xi = 1$ (the last value is consistent with the analysis of the experimental mass spectra, Regge trajectories). The results are reported in Figs. 6 and 8 for $r(t, W)$ and Figs. 7 and 9 for $T_{20}$, for the different values of initial deuteron momentum. The behavior of $r(t, W)$ is seen to be very sensitive to $\xi$. Introduction of swelling gives a more negative slope to $T_{20}$ in better agreement with experiment although the position of the minimum at $p_d = 3.7$ and 9 GeV/c is still shifted to higher $q$ values compared to that measured by the data.

Similar results can be obtained for other polarization observables. In Fig. 10 we show
the $t-$dependence of the vector polarization transfer coefficient, $K'_{y}$, from the initial to the scattered deuteron. It is characterized by a strong sensitivity to the ratio $r$ for $q \geq 3$ fm$^{-1}$. This observable is especially interesting in this region, because $T_{20}$ vanishes around $q \simeq 5$ fm$^{-1}$ (for any value of $r$). Our calculations predict quite a large absolute value of this observable and a strong dependence on the variable $t$.

Let us note in this connection, that, all T-even polarization observables are nonzero and large in absolute value. This is an intrinsic property of $\omega$-exchange. In contrast, all T-odd polarization effects cancel, because we neglected the effects of strong interaction in initial and final states. However, for collinear kinematics, all spin-one T-odd polarization observables must be zero, in any model. The most simple T-odd polarization observable, which exists in the general case for the collinear kinematics, corresponds to the correlation coefficient $\bar{u} \cdot P \times Q$, $Q_a = Q_{ab} u_b$, where $\bar{u}$ is the unit vector along the initial 3-momentum, $P$ is the proton polarization and $Q_{ab}$ is the deuteron tensor polarization. A measurement of these observables will give a direct information on the presence and intensity of the final or initial strong interaction. The tensor analyzing power $T_{20}$, being a T-even observable, is less sensitive to the effects of strong interaction in initial and final states, as these effects induce ‘quadratic corrections’ to any T-even polarization observable. We can schematically write: $P^{(+)} = P^{(+)}_0 (1 + \delta^2)$, where $P^{(+)}$ is any T-even polarization observable and $\delta$ represents the correction due to rescattering effects. Any T-odd polarization observable, $P^{(-)}$, is directly proportional to $\delta$.

V. CONCLUSIONS

We have shown the sensitivity of the tensor analyzing power in the process $d + p \rightarrow d + X$ (for forward deuteron scattering) to the relative value of the cross sections for the absorption of virtual isoscalar photon ($\gamma^* + N \rightarrow N^*$) with longitudinal and transversal polarizations. The main point is that only the Roper resonance excitation is characterized by a nonzero isoscalar longitudinal form factor, whose $t-$behavior drives the $t-$dependence of the tensor analyzing power. The other resonances, lying in this mass region, such as $S_{11}(1535)$, $D_{13}(1520)$ and $S_{11}(1650)$ are characterized, due to the specific quark structure, by a pure isovector nature of longitudinal virtual photons absorbed by nucleons. Without excitation of the Roper resonance, $r = 0$, and the value for $T_{20}$ becomes $t-$independent: $T_{20} = -1/2\sqrt{2}$, in evident disagreement with existing data. The specific behavior of $r$ obtained in the framework of the collective model [4] for the Roper resonance, and in the presence of other resonances, with a definite isotopic structure, are very important for obtaining a good description of the data. In the framework of the $\omega-$exchange mechanism, the $t-$behavior of $T_{20}$ for the reaction $p(d, d')X$, is affected, on one side, from the specific $t-$ dependence of all three deuteron electromagnetic form factors, and on the other side, from the values of $r$ when all four overlapping resonances $P_{11}(1440)$, $S_{11}(1535)$, $D_{13}(1520)$ and $S_{11}(1650)$ contribute. The main property is the role of the Roper resonance excitation, with nonzero longitudinal isoscalar form factor.
Also to be noticed is that the sensitivity of $T_{20}$ to $r$ is especially large for $r$ in the interval $r \leq 0.5$, which is in agreement with the data on inclusive $eN$-scattering at these values of excitation energy. Our model predicts a small momentum dependence for $T_{20}$, consistent with the experimental data. The results are also stable with respect to the parameter $\xi$, the stretchability of the nucleon string, in the range $\xi = 0.5 - 1$.

The study of polarization observables in the reaction $d + p \rightarrow d + X$ can be considered as an additional method to measure the $t-$dependence of the ratio $r$. Moreover, the process $p(d, d')X$ is sensitive to the isoscalar contributions to $r$. The vector polarization transfer coefficient $K^y_{y'}$ is also sensitive to this ratio, in a different region of $t$.

We can consider the existing data about $T_{20}$ not only as an evidence for the Roper resonance excitation, but also as a tool to study properties of isoscalar form factors for the excitation of $N^*$ resonances, complementary to the inelastic electron-nucleon scattering, $e^- + N \rightarrow e^- + N^*$. The possibility to unify in a common picture such different processes, as $e^- + d \rightarrow e^- + d$ and $e^- + N \rightarrow e^- + N^*$, from one side, and a hadronic process as $d + p \rightarrow d + X$ from another side, suggests a new perspective to study nucleon structure through electromagnetic and hadron excitation of nucleonic resonances.

At large energies of colliding particles, instead of $\omega-$exchange it is necessary to consider Pomeron ($P$) exchange, i.e. the mechanism of diffractive excitation of $N^*$, in $dp$-forward scattering. Concerning polarization phenomena, the properties of $\omega$ and $P$- are similar. But in the general case, the vertexes of $PNN^*$ and $Pdd$-interactions are different from $\omega NN^*$ and $\omega dd$-vertexes: only in the framework of the hypothesis about Pomeron-photon analogy, the prediction of these models coincide, at least concerning polarization phenomena.

VI. ACKNOWLEDGMENTS

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FIGURES

FIG. 1. $t$–channel meson exchange for $d + p \rightarrow d + X$

FIG. 2. The ratio $r$ for the case when only the Roper excitation is considered (dotted line) and for the case when all four resonances (6) are considered, (solid line), from Eq. (3).

FIG. 3. Experimental data for $T_{20}$ for $e^- + d \rightarrow e^- + d$ elastic scattering (filled stars) [7] and $d + p \rightarrow d + X$ at incident momenta of 3.75 GeV/c (open diamonds) [5], 5.5 GeV/c (open circles), 4.5 GeV/c (open squares), 9 GeV/c (open triangles) [6]. Prediction of the $\omega$–exchange model for $r = 0$ (dashed-dotted line). Calculations with $r$ using collective form factors (Tables I-II) are shown for the case when only the Roper resonance is considered (dotted line) and for the case when all the four resonances (6) are considered (solid line).

FIG. 4. $q$–dependence of $r = \sigma_L/\sigma_T$ for excitation of only the Roper resonance (dashed line), for excitation of all resonances (6) with (solid line) and without (dotted line) width effects, for different deuteron momenta $p_d$: (a) $p_d=3.7$ GeV/c, (b) $p_d=4.5$ GeV/c, (c) $p_d=5.5$ GeV/c, (d) $p_d=9$ GeV/c.

FIG. 5. $q$–dependence of $T_{20}$ for different deuteron momenta $p_d$: (a) $p_d=3.7$ GeV/c, (b) $p_d=4.5$ GeV/c, (c) $p_d=5.5$ GeV/c, (d) $p_d=9$ GeV/c.

FIG. 6. $q$–dependence of $r = \sigma_L/\sigma_T$ for $\xi = 0.5$ (same notations as in Fig. 4).

FIG. 7. $q$–dependence of $T_{20}$ for $\xi = 0.5$ (same notations as in Fig. 5).

FIG. 8. $q$–dependence of $r = \sigma_L/\sigma_T$ for $\xi = 1$ (same notations as in Fig. 4).

FIG. 9. $q$–dependence of $T_{20}$ for $\xi = 1$ (same notations as in Fig. 5).

FIG. 10. Vector polarization transfer coefficient $K_y$ as a function of $q$, for $d + p \rightarrow d + X$. Prediction of the $\omega$–exchange model for $r = 0$ (dashed-dotted line). Calculations using the collective form factors (Tables I-II) are shown for the case when only the Roper resonance is considered (dotted line) and for the case when all the four resonances (6) are considered (solid line).
TABLE I. Transverse proton ($A^p_\nu$) and neutron ($A^n_\nu$) helicity amplitudes ($\nu = 1/2, 3/2$) of nucleon resonances below 2 GeV in the collective model [2,3]. $Z(x) = -\frac{1}{(1+x^2)^2} + \frac{3}{2x} H(x)$ with $H(x) = \arctan x - \frac{x}{(1+x^2)}$. $\chi_1$ and $\chi_2$ are parameters [3].

| Resonance     | $\nu$ | $A^p_\nu$                                                                 | $A^n_\nu$                                                                 |
|---------------|-------|---------------------------------------------------------------------------|---------------------------------------------------------------------------|
| $N(1440)P_{11}$ | 1/2   | $-2\chi_1 \sqrt{\frac{\pi}{k_0}} \mu k \frac{2k^2a^2}{(1+k^2a^2)^3}$ | $\frac{4}{3} \chi_1 \sqrt{\frac{\pi}{k_0}} \mu k \frac{2k^2a^2}{(1+k^2a^2)^3}$ |
| $N(1520)D_{13}$ | 1/2   | $2i \sqrt{\frac{\pi}{k_0}} \mu \frac{1}{(1+k^2a^2)^2} \frac{mk_0a}{g} - k^2a$ | $-2i \sqrt{\frac{\pi}{k_0}} \mu \frac{1}{(1+k^2a^2)^2} \frac{mk_0a}{g} - \frac{1}{3} k^2a$ |
|               | 3/2   | $2i \sqrt{\frac{3}{2}} \sqrt{\frac{\pi}{k_0}} \mu k \frac{1}{(1+k^2a^2)^2} \frac{mk_0a}{g}$ | $-2i \sqrt{\frac{3}{2}} \sqrt{\frac{\pi}{k_0}} \mu k \frac{1}{(1+k^2a^2)^2} \frac{mk_0a}{g}$ |
| $N(1535)S_{11}$ | 1/2   | $i \sqrt{2} \sqrt{\frac{\pi}{k_0}} \mu \frac{1}{(1+k^2a^2)^2} \frac{2mk_0a}{g} + k^2a$ | $-i \sqrt{2} \sqrt{\frac{\pi}{k_0}} \mu \frac{1}{(1+k^2a^2)^2} \frac{2mk_0a}{g} + \frac{1}{3} k^2a$ |
| $N(1650)S_{11}$ | 1/2   | 0                                                                         | $i \chi_2 \sqrt{\frac{2}{3}} \sqrt{\frac{\pi}{k_0}} \mu k \frac{ka}{(1+k^2a^2)^2}$ |
| $N(1675)D_{15}$ | 1/2   | 0                                                                         | $-i \sqrt{\frac{3}{5}} \sqrt{\frac{\pi}{k_0}} \mu k \frac{ka}{(1+k^2a^2)^2}$ |
|               | 3/2   | 0                                                                         | $-i \chi_2 \sqrt{\frac{2}{3}} \sqrt{\frac{\pi}{k_0}} \mu k \frac{ka}{(1+k^2a^2)^2}$ |
| $N(1680)F_{15}$ | 1/2   | $-\sqrt{3} \sqrt{\frac{\pi}{k_0}} \mu \frac{2mk_0a}{g} - k^2a \frac{1}{ka}Z(ka)$ | $-\frac{2}{\sqrt{3}} \sqrt{\frac{\pi}{k_0}} \mu k Z(ka)$ |
|               | 3/2   | $-2\sqrt{6} \sqrt{\frac{\pi}{k_0}} \mu \frac{mk_0a}{g} \frac{1}{ka}Z(ka)$ | $0$                                                                         |
| $N(1700)D_{13}$ | 1/2   | 0                                                                         | $i \chi_2 \sqrt{\frac{2}{3}} \sqrt{\frac{\pi}{k_0}} \mu k \frac{ka}{(1+k^2a^2)^2}$ |
|               | 3/2   | 0                                                                         | $i \chi_2 \sqrt{\frac{2}{3}} \sqrt{\frac{\pi}{k_0}} \mu k \frac{ka}{(1+k^2a^2)^2}$ |
| $N(1710)P_{11}$ | 1/2   | $\sqrt{2} \chi_2 \sqrt{\frac{\pi}{k_0}} \mu \frac{2k^2a^2}{(1+k^2a^2)^3}$ | $-\frac{2}{\sqrt{3}} \chi_2 \sqrt{\frac{\pi}{k_0}} \mu \frac{2k^2a^2}{(1+k^2a^2)^3}$ |
| $N(1720)P_{13}$ | 1/2   | $-\sqrt{2} \sqrt{\frac{\pi}{k_0}} \mu \frac{3mk_0a}{g} + k^2a \frac{1}{ka}Z(ka)$ | $\frac{2\sqrt{2}}{3} \sqrt{\frac{\pi}{k_0}} \mu k Z(ka)$ |
|               | 3/2   | $\sqrt{6} \sqrt{\frac{\pi}{k_0}} \mu \frac{mk_0a}{g} \frac{1}{ka}Z(ka)$ | $0$                                                                         |
TABLE II. Longitudinal proton \( A^p_\ell \) and neutron \( A^n_\ell \) helicity amplitudes of nucleon resonances below 2 GeV in the collective model \([2,3]\). Notation as in Table I.

| Resonance       | State                           | \( A^p_\ell \)                                                                 | \( A^n_\ell \)         |
|-----------------|---------------------------------|--------------------------------------------------------------------------------|------------------------|
| \( N(1440)P_{11} \) | \( 2^8_{1/2}[56,0^+](1,0);0 \) | \( 2\chi_1 \sqrt{\frac{m}{k_0} \mu \frac{mk_n a}{g} \frac{4ka(1-2k^2a^2)}{(1+k^2a^2)^4}} \) | 0                      |
| \( N(1520)D_{13} \) | \( 2^8_{3/2}[70,1^-](0,0);1 \) | \( 2i \sqrt{\frac{m}{k_0} \mu \frac{mk_n a}{g} \frac{-3k^2a^2}{(1+k^2a^2)^4}} \) | \( A^n_\ell = -A^p_\ell \) |
| \( N(1535)S_{11} \) | \( 2^8_{1/2}[70,1^-](0,0);1 \) | \( -i \sqrt{2} \sqrt{\frac{m}{k_0} \mu \frac{mk_n a}{g} \frac{1-3k^2a^2}{(1+k^2a^2)^4}} \) | \( A^n_\ell = -A^p_\ell \) |
| \( N(1650)S_{11} \) | \( 4^8_{1/2}[70,1^-](0,0);1 \) | 0                                                                              | 0                      |
| \( N(1675)D_{15} \) | \( 4^8_{5/2}[70,1^-](0,0);1 \) | 0                                                                              | 0                      |
| \( N(1680)F_{15} \) | \( 2^8_{5/2}[56,2^+](0,0);0 \) | \( -\sqrt{3} \sqrt{\frac{m}{k_0} \mu \frac{mk_n a}{g} \left( \frac{4ka}{(1+k^2a^2)^4} - \frac{3}{ka} Z(ka) \right)} \) | 0                      |
| \( N(1700)D_{13} \) | \( 4^8_{3/2}[70,1^-](0,0);1 \) | 0                                                                              | 0                      |
| \( N(1710)P_{11} \) | \( 2^8_{1/2}[70,0^+](0,1);0 \) | \( -\sqrt{2} \chi_2 \sqrt{\frac{m}{k_0} \mu \frac{mk_n a}{g} \frac{4ka(1-2k^2a^2)}{(1+k^2a^2)^4}} \) | \( A^n_\ell = -A^p_\ell \) |
| \( N(1720)P_{13} \) | \( 2^8_{3/2}[56,2^+](0,0);0 \) | \( \sqrt{2} \sqrt{\frac{m}{k_0} \mu \frac{mk_n a}{g} \left( \frac{4ka}{(1+k^2a^2)^4} - \frac{3}{ka} Z(ka) \right)} \) | 0                      |
Fig. 1
