Detectability of Small-Scale Dark Matter Clumps with Pulsar Timing Arrays

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We examine the capability of pulsar timing arrays (PTAs) to detect very small-scale clumps of dark matter (DM), which are a natural outcome of the standard cold dark matter (CDM) paradigm. A clump streaming near the Earth or a pulsar induces an impulsive acceleration to encode residuals on pulsar timing data. We show that, assuming the standard abundance of DM clumps predicted by the CDM model, small-scale DM clumps with masses from $\sim 10^{-11} M_\odot$ to $\sim 10^{-8} M_\odot$ can be detectable by a PTA observation for a few decades with $\mathcal{O}(100)$ of pulsars with a timing noise of $\mathcal{O}(10)\text{ ns}$ located at $\gtrsim 3$ kpc away from the Galactic center, as long as these mass scales are larger than the cutoff scale of the halo mass function that is determined by the particle nature of DM. Our result suggests that PTAs can provide a unique opportunity for testing one of the most fundamental predictions of the CDM paradigm. In addition, the detections and non-detections can constrain the cutoff mass scale inherent to the DM model.

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I. INTRODUCTION

Variety of cosmological observations, e.g., cosmic microwave background and large-scale structure, can be consistently explained by the standard $\Lambda$ cold dark matter ($\Lambda$CDM) model with an initial condition set by inflation. One of the most fundamental predictions of the CDM paradigm is that cosmic structure grows hierarchically such that smaller structures form earlier. As a consequence, the CDM model predicts the mass function of DM clumps (halos) that extends to very small masses. The cutoff of the mass function at the small mass end is closely related to the nature of DM, e.g., it corresponds to the free streaming or kinetic decoupling scale and hence is related to e.g., the mass of DM particles for weakly interacting massive particle (WIMP) scenarios (e.g.,\cite{1–3}). Therefore detections of such small-scale DM clumps are crucial for the confirmation of the CDM paradigm, yet it is very challenging observationally.

Are there any ways to detect such small-scale DM clumps observationally? One possible way is to detect them indirectly via annihilations of DM particles (e.g.,\cite{4–9}). However, no DM annihilation signal was detected by the Fermi Gamma-ray Space Telescope, from which tight constraints on the DM annihilation cross section has been obtained\cite{10}. Another possible way is microlensing, although it appears that it would be difficult to probe very small ($M \lesssim 100 M_\odot$) mass clumps by microlensing (e.g.,\cite{11}).

In this paper, we propose to utilize pulsar timing arrays (PTAs) for searching small-scale DM clumps in the Galaxy. Previous studies considered the Shapiro time delay that is induced when an ultracompact DM halo crosses the line-of-sight between the Earth and a pulsar\cite{12–14}. Also PTA has been shown to be able to detect oscillations of gravitational potential induced by ultralight scalar DM with mass around $10^{-23} – 10^{-22}$ eV\cite{15}. Instead, we consider an impulsive acceleration of the Earth or one of the pulsars that is induced by a DM clump streaming near the Earth or the pulsar (Fig.\textsuperscript{1}), which was originally proposed to detect point mass objects such as primordial black holes (PBHs)\cite{16,17}. The idea of using PTAs to probe small-scale DM clumps has been briefly mentioned in\cite{9}. In this paper, we show that PTAs can provide independent and interesting constraints on the abundance of small-scale DM clumps with masses from $\sim 10^{-11} M_\odot$ to $\sim 10^{-8} M_\odot$.

This paper is organized as follows. In Sec.\textsuperscript{II} we estimate the mass distribution of small-scale DM clumps in the Galaxy. In Sec.\textsuperscript{III} we show the capability of PTAs for detecting them. Sec.\textsuperscript{IV} is devoted to discussions. Throughout the paper, we use cosmological parameter values obtained by Planck\cite{19}.

II. SMALL-SCALE CLUMPS IN THE GALAXY

We first consider the formation of small-scale DM clumps in Sec.\textsuperscript{IIA} and then the destruction processes and survival probability in Sec.\textsuperscript{IIB}.

A. Formation

In the concordance cosmology, seeds of structures are fluctuations generated during inflation. The primordial power spectrum is nearly scale-invariant with a small amplitude of $\mathcal{O}(10^{-5})$. The matter fluctuations start to grow linearly with the scale factor at around the matter-radiation equality, $z \sim z_{eq} = 2.35 \times 10^{4} \Omega_m h^2$. When
In this case, we may use an analytical fitting formula given in ]4(see their Eqs. 92-94) 
\[
\sigma_{\text{eq}}(M) \approx \frac{2 \times 10^{-4}}{\sqrt{f_s(\Omega_A)}} \left[ -\frac{1}{3} \ln \left( \frac{M}{M_{\text{eq}} \nu} \right) \right]^{3/2} \left( \frac{M}{M_{h,0}} \right)^{-(n_p-1)/6},
\]
where \(M_{h,\text{eq}} \approx 7.5 \times 10^{15} \Omega_m^{-2} h^{-4} M_\odot\) and \(M_{h,0} \approx 3.0 \times 10^{22} \Omega_m h^{-1} M_\odot\) are the mass inside the cosmological horizon at \(z = z_{\text{eq}}\) and \(z = 0\), respectively, and \(f_s(\Omega_A) = 1.04 - 0.82 \Omega_A + 2 \Omega_A^2\). The variance \(\sigma_{\text{eq}}(M)\) is a decreasing function of mass \(M\), indicating that structures with smaller masses form earlier.

We then estimate the density and radius of DM clumps at their formation. In the spherical collapse model (e.g., [21]), the average density can be given as
\[
\bar{\rho} \approx \kappa \rho_{\text{eq}} \left( \frac{\nu \sigma_{\text{eq}}(M)}{\delta_c} \right)^3,
\]
where \(\kappa = 18 \pi^2\), \(\rho_{\text{eq}} = \rho_0(1 + z_{\text{eq}}^3)\), and \(\rho_0 = 2.78 \times 10^{-7} \Omega_m h^2 M_\odot \text{pc}^{-3}\) is the mean matter density of the universe at \(z = 0\). The characteristics radius is
\[
\bar{R} \approx \left( \frac{3 M}{4 \pi \rho} \right)^{1/3}.
\]

Using Eq. [2], Eqs. [3] and [4] for a range of masses \(10^{-12} M_\odot \lesssim M \lesssim 10^{-6} M_\odot\) the average density can be approximated as
\[
\bar{\rho} \sim 0.41 M_\odot \text{pc}^{-3} \left( \frac{\nu}{2} \right)^3 \left( \frac{M}{10^{-10} M_\odot} \right)^{-3\alpha},
\]
and the radius of the clump is
\[
\bar{R} \sim 83 \text{ AU} \left( \frac{\nu}{2} \right)^{-1} \left( \frac{M}{10^{-10} M_\odot} \right)^{\alpha+1/3},
\]
where \(\alpha \approx 0.02\).

We assume that primordial fluctuations are nearly scale-invariant and follow the Gaussian distribution, which are consistent with observations at large scales. In this case, the clump formation probability is \(\propto e^{-\nu^2/2}\), indicating that clumps from low-sigma fluctuations dominate. Also, the average density of the clumps are almost scale invariant; larger mass clumps are only slightly denser (see Eq. [3]). However, a non-negligible amount of high-sigma fluctuations (\(\nu \gg 1\)) may be seeded for a certain range of clump mass, depending on inflation models. Such clumps, the so-called ultracompact mini-halos (UCMHs), can have larger survival probabilities and enhanced observational signatures (e.g., [1, 22, 23]). In this paper, we do not consider such possibility and focus on “normal” DM clumps from low-sigma fluctuations, which should represent a conservative estimate of the detectability of small-scale DM clumps.

Regardless of inflation models, there should be a lower cut-off mass scale of the clump formation. For instance, thermal relic DM models have free-streaming scales, below which clustering of DM particles is suppressed by...
their streaming motions. Damping of the power spectrum is also induced by acoustic oscillations after kinetic decoupling. In the case of 100 GeV WIMP model, the cutoff is located at around $M_{\text{cutoff}} \sim 10^{-6} M_{\odot}$ (e.g., [24]), although we note that the 100 GeV thermal relic WIMP model appears to be already excluded by Fermi $\gamma$-ray observations [10]. More massive (> TeV) WIMP models predict small-scale DM clumps as small as $\sim 10^{-10} M_{\odot}$ [3]. Furthermore, non-thermal DM models such as axions can have critical scales qualitatively similar to the above. For example, in the case of 10 $\mu$eV axion, the cut-off mass scale is at $M_{\text{cutoff}} \sim 10^{-12} M_{\odot}$ (e.g., [25]). One can strongly constraint the nature of DM by detecting these cutoff scales.

### B. Destruction and survival probability

Next we estimate the current abundance of small-scale DM clumps in the Galaxy. Since these DM clumps are fluffily seen as seen in Eqs. (5) and (6), they can be destructed due to gravitational interactions within the Galaxy. The most relevant process is the tidal destruction [26]: a clump will be destructed when the sum of the internal energy induced by tidal shocks,

$$\delta E = \frac{3 - \beta}{6(5 - 2\beta)} M \sum_i \delta v_i^2$$

becomes comparable to the binding energy of the clump

$$E_b = \frac{3 - \beta}{2(5 - 2\beta)} \frac{GM^2}{R},$$

where

$$\delta v_i = \frac{2b_i g_i}{V_i}$$

is the tidally-induced velocity perturbation in a clump experiencing an encounter with a gravitational source with a impact parameter $b_i$, the relative velocity $V_i$, and the tidal force per unit mass $g_i$, and $\beta$ represents the power-law index of the density profile of the clump at the characteristic radius, i.e.,

$$\rho = \frac{3 - \beta}{3 \bar{\rho}} \left( \frac{R}{\bar{R}} \right)^{-\beta}.$$  

In this paper, we set $\beta = 2$ with noting that our results are not sensitive to $\beta$ as long as $1 \lesssim \beta \lesssim 2$. As in the previous studies [4, 27, 28], we divide the destruction history into two parts; during the hierarchical structure formation of the Milky-Way halo and after the star formation in the Galaxy.

In the course of forming hierarchical structures, DM clumps can merge and be destructed by the tidal interaction between with each other. The DM clump mass density relative to the total DM mass density in the end is calculated as [4]

$$\xi(M, \nu) dM = \frac{y(\nu)}{\sqrt{2\pi}} \left( n + 3 \right) d\nu \frac{dM}{M}.$$  

where $y(\nu)$ is a monotonically increasing function of $\nu$ (see Fig. 2 of [4]), representing the fact that less dense clumps formed from a smaller fluctuation are selectively disrupted, and

$$n = -3 \left[ 1 + 2 \frac{\partial \log \sigma_{\text{eq}}(M)}{\partial \log M} \right].$$

Integrating Eq. (11) over $\nu$, the fraction of DM mass in clumps with a mass $M$ can be given as

$$\xi(M, \nu) \frac{dM}{M} \approx 0.02(n + 3) \frac{dM}{M}.$$  

It is worth noting that surviving clumps are mainly from low-sigma fluctuations, i.e., $\nu \lesssim 2$ [4].

The calculated $\xi_i$ from Eqs. (2), (12), and (13) is shown in Fig. 2 (dotted line). The clump mass density depends little on the clump mass even after taking account of the destruction process, $\xi_i \propto M^0$. This is due to the fact that the survival probability is sensitive to the average clump density, which is also almost scale invariant, $\bar{\rho} \propto M^0$ (see Eq. 5). At this point, $\gtrsim 0.1\%$ of the DM mass are in the form of small-scale clumps in each logarithmic mass interval $d\ln M$. Numerical simulations of the hierarchical structure formation predict clump abundances that are consistent with the estimate described above (e.g., [3, 30, 32]), although the uncertainties are still large. In Fig. 2, we also show the impact of the cutoff in the mass function, which is assumed to be an exponential cutoff ($\propto \exp[-(M_{\text{cutoff}}/M)^{2/3}]$) for simplicity.

When the Galaxy is formed at the center of the halo, DM clumps are further destructed by stars in the Galaxy. For example, in the case of a clump passing near a star with mass $M_*$, the exerted tidal force is on average $g \approx GM_* R/b^3$, and it is tidally destructed if $b \lesssim b_c$ where

$$b_c \sim 3000 \text{ AU} \left( \frac{V}{350 \text{ km s}^{-1}} \right)^{-1/2} \left( \frac{M_*}{M_{\odot}} \right)^{1/2}.$$  

We should note that the relative velocity is typically much larger than the tidally induced velocity, i.e., $\delta v/V \ll 1$, even for $b \lesssim b_c$. This means that the bulk of the destructed clump will not be captured by the star but fly away as a DM stream.

Here we estimate the surviving probability of clumps against the tidal destruction by Galactic stars following [28]. To this end, one has to assume the surface density of the stellar disk ($\sigma_d$) and the DM halo profile of the Milky Way ($\rho_h$). The former determines the survival probability of clumps against the tidal destruction by Galactic stars following [28]. To this end, one has to assume the surface density of the stellar disk ($\sigma_d$) and the DM halo profile of the Milky Way ($\rho_h$). The former determines the survival probability of clumps against the tidal destruction by Galactic stars following [28]. To this end, one has to assume the surface density of the stellar disk ($\sigma_d$) and the DM halo profile of the Milky Way ($\rho_h$).
probability of a clump per disk crossing, while the latter sets the orbits of clumps. Here we assume

\[ \sigma_d(r) = \frac{m_\ast}{2\pi r_d^2} e^{-r/r_d} \quad (15) \]

with \( m_\ast = 6 \times 10^9 \, M_\odot \) and \( r_d = 2.6 \, \text{kpc} \) and the NFW profile

\[ \rho_n(r) = \frac{\bar{\rho}_n}{(r/L)(1 + r/L)^2} \quad (16) \]

with \( \bar{\rho}_n = 1.4 \times 10^7 \, M_\odot \, \text{kpc}^{-3} \) and \( L = 16 \, \text{kpc} \). The resultant DM mass fraction in clumps in the Galaxy are shown in Fig. 2. The thick and thin solid black lines indicate the cases at the Galactic radii of 8 kpc and 3 kpc, respectively. Compared with \( \xi_i \) (Eq. 13), the survival fraction of DM clumps is decreased by \( > 10 \% \) at around the solar system. On the other hand, more than 99 % of the DM clumps have been destructed at an inner radius where stars are more densely distributed. Again, the survival probability does not depend strongly on the clump mass. Our results are broadly consistent with previous studies [4, 27–29].

### III. SEARCHING DARK MATTER CLUMPS WITH PULSAR TIMING ARRAYS

We now discuss how to detect small-scale DM clumps in the Galaxy with PTAs, following [18] in which the possibility of constraining the abundance of PBHs with PTAs was explored. The method is schematically described in Fig. [1]. We consider an impulsive acceleration driven by a flyby of a DM clump around a target mass, either the Earth or one of the pulsars. Residuals induced in the timing-of-arrival (TOA) data by the former and latter events are called Earth and pulsar terms, respectively.

Hereafter, we consider a PTA that consists of \( N_{\text{PSR}} \) pulsars with a comparable timing noise \( \sigma \). For simplicity, we assume the noises are white and have no correlation. The observation is also characterized by the total duration \( T_{\text{obs}} \) and sampling rate of the timing-of-arrival (TOA) data \( \nu_s \). For example, the Parkes PTA currently consists of \( N_{\text{PSR}} \approx 20 \) of pulsars with timing precisions of \( \sigma \sim 100 \, \text{ns} \), and the conventional data sampling rate is \( \nu_s \sim 0.5 \, \text{wk}^{-1} \) [34]. These parameters could be significantly improved (i.e., larger \( N_{\text{PSR}} \), smaller \( \sigma \), and larger \( \nu_s \)) in the era of Square Kilometre Array (but see e.g., [35, 36]).

We first describe the signal of a flyby event. The impulsive acceleration of the Earth or a pulsar occurs with a timescale of \( T = b/V \), or

\[ T \sim 30 \, \text{yr} \left( \frac{b}{2200 \, \text{AU}} \right) \left( \frac{V}{350 \, \text{km/s}} \right)^{-1}, \quad (17) \]

inducing timing residuals mainly at a Fourier frequency \( f = 1/T \). The mode amplitude is given as \( s_f \approx (aT^2/c) \times |\cos \theta| \), where \( a \approx GM/b^2 \) is the acceleration and \( \theta \) is the angle defined in Fig. [1]. Then, we can estimate the amplitude as

\[ s_f \sim 0.20 \, \text{ns} \left( \frac{M}{10^{-10} \, M_\odot} \right) \left( \frac{V}{350 \, \text{km/s}} \right)^{-2} \left( \frac{|\cos \theta|}{0.58} \right). \quad (18) \]

In the above estimate, we assume that the clump is a point mass object. As we will see below, the maximum detectable impact parameter \( b_{\text{max}} \) is typically comparable to \( b \), which means that detected clumps are being destructed by the Sun or one of the pulsars, leading the modification of the signal from the point mass approximation. However, this finite size effect is minor given that the tidally induced velocity is much smaller than the flyby velocity, i.e., \( \delta v/V < 1 \).

Neglecting the finite size effect, the signal of a DM clump is characterized by \( T \) and \( s_f \). On the other hand, we have four physical parameters, \( M, b, V, \) and \( \theta \), which

FIG. 2. The dark matter (DM) clump mass density relative to the total DM mass density in the Galaxy and the sensitivity of a pulsar timing array. The thick and thin black solid lines show the current mass density distribution of DM clumps at 8 and 3 kpc from the Galactic center, respectively. The dotted line indicates the mass density distribution without the effect of Galactic stars (see Eq. 13). The cutoffs depending on DM models are shown with dotted-dash lines. The red and blue lines indicate the parameter region in which DM clumps are detectable using the Earth and pulsar terms, respectively. For the pulsar term, the thick and thin lines show the cases that all the pulsars locate at 8 and 3 kpc from the Galactic center, respectively. We assume a future PTA observation with \( \sigma = 10 \, \text{ns} \) (solid), 50 ns (dashed), and \( \nu_s = 1 \, \text{wk}^{-1} \), \( N_{\text{PSR}} = 100 \), \( T_{\text{obs}} = 30 \, \text{yr} \), and set the detection threshold as \( \text{S/N} = 3 \).
reflect the abundance and dynamics of DM clumps in the Galaxy. In principle, the degeneracies in the parameters can be broken statistically. Also, the orbit of a DM clump approaching to the Earth can be estimated from the dipole pattern of the Earth term. Hereafter, we set fiducial values of \( V \) and \( \theta \) as follows. Assuming that scattering between DM clumps and the targets is isotropic, the ensemble average of \( \theta \) reduces to \( \sqrt{\cos^2 \theta} = 1/\sqrt{3} \sim 0.58 \). We assume that typical value of \( V \) is equal to the rms velocity of DM relative to the solar system, \( \sim 350 \) km s\(^{-1} \), which was estimated dynamically [37]. Give that the typical peculiar velocity of the observed millisecond pulsars are relatively small \( \lesssim 100 \) km s\(^{-1} \) with some exceptions [38], it is reasonable to adopt \( V = 350 \) km s\(^{-1} \) as a fiducial value for both the Earth and pulsar terms.

Next, we evaluate the timing noise of the detector. The Fourier mode of the noise at the frequency \( f = 1/T \) can be estimated as \( \approx \sigma/\sqrt{T/V_o} \) for a pulsar term. When using the Earth terms, the noise can be statistically reduced by a factor of \( 1/\sqrt{N_{\text{PSR}}} \). As a result, the effective noise for the DM clump search can be described as

\[
n_f \sim 0.25 \left( \frac{\sigma}{10 \text{ ns}} \right) \left( \frac{T}{30 \text{ yr}} \right)^{-1/2} \left( \frac{\nu_b}{1 \text{ wk}^{-1}} \right)^{-1/2} \times N_{\text{PSR}}^{-E/2}
\]

(19)

with \( E = 0 \) or 1 for the pulsar and Earth terms, respectively. From Eqs. (18) and (19), the signal-to-noise ratio is given as \( S/N = s_f/n_f \).

Finally, the event rate of DM clumps with a mass \( M \) passing near the Earth with an impact parameter \( b \) is \( \approx \pi b^2 V \xi(M,r)\rho_b(r)/M \). Such flybys occur in total \( N_{\text{PSR}} \) times more for the pulsars. Accordingly, the event rate can be estimated as

\[
\mathcal{R} \sim 0.014 \text{ yr}^{-1} \left( \frac{\xi}{10^{-3}} \right) \left( \frac{\rho_b}{0.011 M_\odot \text{ pc}^{-3}} \right) \times \left( \frac{M}{10^{-10} M_\odot} \right)^{-1} \left( \frac{b}{2200 \text{ AU}} \right)^2 \times \left( \frac{V}{350 \text{ km/s}} \right) N_{\text{PSR}}^{-1-E}.
\]

(20)

Detections of small-scale DM clumps by a PTA observation is possible if (i) the signal-to-noise ratios are sufficiently large, e.g., \( S/N > 3 \), (ii) the signal duration is shorter than the duration of the observation, \( T < T_{\text{obs}} \), and (iii) more than one events occur within the duration of the observation, \( \mathcal{R}T_{\text{obs}} > 1 \). From Eqs. (17) - (20), these conditions can be translated into

\[
\left( \frac{M}{10^{-10} M_\odot} \right) \left( \frac{b}{2200 \text{ AU}} \right)^{1/2} \gtrsim 1.1 \left( \frac{\sigma}{10 \text{ ns}} \right) \left( \frac{\nu_b}{1 \text{ wk}^{-1}} \right)^{-1/2} \times \left( \frac{V}{350 \text{ km/s}} \right)^{5/2} \times \left( \frac{S/N}{3} \right) N_{\text{PSR}}^{-E/2}.
\]

(21)

and

\[
\left( \frac{M}{10^{-10} M_\odot} \right) \left( \frac{b}{2200 \text{ AU}} \right)^{-2} \lesssim 0.43 \left( \frac{\xi}{10^{-3}} \right) \times \left( \frac{\rho_b}{0.011 M_\odot \text{ pc}^{-3}} \right) \times \left( \frac{V}{350 \text{ km/s}} \right) \left( \frac{T_{\text{obs}}}{30 \text{ yr}} \right) \times N_{\text{PSR}}^{1-E},
\]

(23)

respectively. We note that the conditions (i) and (iii) (Eqs. [21] and [22]) are more easily satisfied for larger \( b \), because both \( S/N \) and \( \mathcal{R} \) become larger. As a result, the minimum detectability is always set by cases with \( b \sim b_{\text{max}} \) in Eq. (22), which is at most a few 1000 AU for an observation within a human timescale. The detectable parameter range of DM clumps with a given PTA observation can be obtained by substituting \( b = b_{\text{max}} \) into Eqs. (21) and (22).

In Fig. 2, the red and blue lines show the detectable parameter regions for a PTA observation with \( \nu_b = 1 \text{ wk}^{-1} \), \( N_{\text{PSR}} = 100 \), and \( T_{\text{obs}} = 30 \text{ yr} \), setting the detection threshold to \( S/N = 3 \). For the solid and dashed lines, the average timing noises are assumed to be \( \sigma = 10 \) and 50 ns, respectively. The minimum clump mass is set by Eqs. (21) and (22), while the minimum mass density is set by Eqs. (22) and (23).

We find that the Earth and pulsar terms are highly complementary [18]. One can detect \( \sim 1/\sqrt{N_{\text{PSR}}} \) times lighter clumps with the Earth term and \( \sim N_{\text{PSR}} \) times less abundant clumps with the pulsar term, which is optimal for more massive clumps. Note that the effective sensitivity of using the pulsar term depends on the local DM density (see Eq. [23]) and thus the location of the pulsars in the Galaxy. The thick and thin blue lines correspond to the cases where all the pulsars are located at \( 8 \) kpc and \( 3 \) kpc from the Galactic center, respectively.

From Fig. 2, we conclude that small-scale DM clumps with masses ranging from \( \sim 10^{-11} M_\odot \) to \( \sim 10^{-8} M_\odot \) predicted by the standard CDM model can be detected by a PTA observation for a few decades with \( \mathcal{O}(100) \) of pulsars with a timing noise of \( \mathcal{O}(10) \) ns located at \( \gtrsim 3 \) kpc away from the Galactic center, if the cutoff mass scale is lower than these masses. Therefore the detection and non-detection of small-scale DM clumps by such PTA observations can provide interesting constraints on the particle nature of DM.

IV. DISCUSSION

Our work has demonstrated that small-scale DM clumps predicted in the standard framework of the CDM model can be detected by a PTA observation. Our estimate is based on a reasonable extrapolation of the well-constrained CDM model to small scales, and is based on
dipole. terms as GWs are quadrupole whereas flyby events are distinguished by the multipole patterns in the Earth amplitude and duration of the signal will be comparable to those of DM clumps. This suggests that the abundance of DM clumps derived in this paper, and therefore the event rate of the impulsive acceleration detectable by PTAs, may be significantly underestimated.

However, a caveat is its observational challenge. Given the unresolved systematic noises in timing data (e.g., [35, 36]), the proposed PTA observation to detect the DM clump density predicted by the standard CDM model will be challenging even in the era of Square Kilometre Array. Therefore development of new methodology to further reduce systematic noises in timing data is highly anticipated.

While we have considered the detectability of small-scale DM clumps in a standard CDM scenario, depending on inflationary models the primordial power spectrum can be more enhanced towards small scales, leading to more abundant DM clumps or UCMHs. For such cases, a more conservative PTA configuration still provides meaningful constraints. Although some search methods have been already proposed for UCMHs (e.g., [12–15]), our approach is complementary to these previous approaches. In particular, we note that another PTA method using Shapiro time delay is sensitive to larger DM clump masses of $M \gtrsim 10^{-4} M_\odot$ [12, 14].

We should also discuss competitive sources for a robust detection of DM clumps with PTAs. First of all, gravitational waves (GWs) from binary super-massive black holes are the primary target of PTAs (e.g., [41, 42]). The amplitude and duration of the signal will be comparable to those of DM clumps. In principle, the signals can be distinguished by the multipole patterns in the Earth term as GWs are quadrupole whereas flyby events are dipole.

A possible concern is that flyby events of non-DM objects, e.g., floating planets, may serve as false positives of DM clump signals. However, the expected event rate for these non-DM objects is significantly smaller than that of DM clumps. Also, in the case of using the Earth term, non-DM objects may be identified by other instruments such as optical telescopes.

Our method was originally proposed to detect PBHs with $\sim 10^{-11} - 10^{-8} M_\odot$ [17, 18]. The search for microlensing with the Subaru Hyper Suprime-Cam provides independent tight constraints on the abundance of PBHs in a similar mass range [43]. Since small-scale DM clumps are much less sensitive to microlensing due to their fluffy nature, the combination of microlensing searches and PTAs will allow us to statistically disentangle signals by PBHs and small-scale DM clumps in PTA observations. Furthermore, for high S/N events, flybys of DM clumps and PBHs might be distinguished by the details of the signals, because only the formers are destructed by the tidal shock.

Finally, if an ultralight scalar field with mass around $10^{-23} - 10^{-22}$ eV constitutes a good fraction of the DM, stochastic oscillations of gravitational potential can be detectable by future PTAs [14]. Although the amplitude and duration of the signal can be also comparable to those of DM clumps, it is expected to be monochromatic and induce monopole patterns in the Earth term. Such signals can be distinguished from flyby events of DM clumps.

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