Data-driven Modeling of Chinese Gong Based on Sparse Identification of Nonlinear Dynamics

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Abstract. Gong is widely employed in Chinese folk bands, but its sound mechanism and mathematical model is still not fully resolved. This work employs MEMS gyroscope to collect the vibration angular rate of two types of Hand Gong including median pitch and high pitch Gongs. This paper further employs Sparse Nonlinear Dynamics Identification (SINDy) [1] to identify a reduced order model of Gong. The model is using angular velocity as state variable and candidate library of nonlinearity is chosen up to fifth-order polynomial. The results indicate that the nonlinearity is prominent up to the third order (cubic) and the cubic nonlinearity is not appearing for the rotation around the axis perpendicular to Gong surface. These results confirm previous observations that the oscillator with a cubic nonlinearity is able to well characterize the nonlinear vibration behavior of Gong [2]. One potential application of this work is on musical psychology.

1. Introduction

1.1. Background
Gong is a traditional Chinese percussion instrument, which plays a very important role in Chinese folk bands [3][4]. It is also widely used, not only in folk instrumental ensembles, but also Chinese Opera and dance accompaniment as shown in figure 1, figure 2 and figure 3. It is also an indispensable musical instrument in some daily celebration activities in traditional festivals like Dragon Boat Races and Lion Dances. Since the 1980s, with the general improvement of people’s life, the pursuit of cultural, artistic life is even more important. As the music of Gongs and drums are closely related to people’s daily lives, they had become more active in all parts of the country on an unprecedented scale, and the ways of performance developed a lot.

The Gong’s origin is likely from China’s Western Regions as early as sixth century. The term Gong originated in Java. Scientific and archaeological research has established that Burma, China, Java and Annam were the four main Gong manufacturing centres of the ancient world [5]. The Gong found its way into the Western World in the 18th century when it was also used in the percussion section of a Western-style symphony orchestra. A form of bronze cauldron Gong known as a resting bell was widely used in ancient Greece and Rome, for instance in the famous Oracle of Dodona, where disc Gongs were also used [3][4].
Figure 1. Pingxiang Chun Gong Performance Team participated in the 14th National Stars Award Trial

Figure 2. Pingxiang Chun Gong Performance Team participated in the 14th National Stars Award Trial
Nowadays, due to many factors, the artistic expression of national instrumental music becomes a minority art, and the relationship between them and the public is weakening gradually. How to hold the relationship between the traditional musical instruments and the modern aesthetics on the basis of maintaining the cultural tradition is a topic worth paying attention to nowadays. For example, it is worth finding a proper combination of western musical instruments and performing methods so as to make the traditional instruments like Gongs to keep and develop better. At the same time, as a fan of traditional culture and a Science and Engineering student, I try to discover more rules and principles of them in order to offer a reference for further application in life or some other fields. The next subsection will further introduce the background of sound mechanism of Gongs.

1.2. Research status review

Many researches have been done to study the sound phenomena and changing frequency of Chinese Gong. For example, Rossing & Fletcher [6] conducted experiments and found respectively falling and rising frequency for flat and curved plates, which indicate nonlinearity is sufficient to explain frequency shifting phenomenon. Fletcher [7] employed theoretical analysis to show that the fundamental modes in vibration decrease significantly when the amplitude is reaching half of the shell thickness. Legge & Fletcher [8] studied the nonlinearity, chaos and the sound of shallow Gongs and suggested two separate nonlinear mechanism contribute to the evolution of the sound. The first one is identified as an upward cascade of energy from low to high frequency modes and the second one is a transition from simple periodic to multiple fractional subharmonics [8]. Thomas et al. [9] studied the nonlinear equations of vibration of systems containing large deflection rods and and further employed this dynamics to identify and study physical source of nonlinear vibration. Chaigne et al. [10] reviewed the modeling of nonlinear vibration of Gong and cymbal including model based on nonlinear von K’arm’an equations, modal projection, and Nonlinear Normal Mode (NNM), as well as its prospective application in sound synthesis. Bader [11] studied mode coupling and pitch glide of Chinese Gong. Tsai et al. [12] studied vibration and sound property of a copper Chinese Gong and they showed that both fundamental frequency and mode shape are in a good agreement between theoretical and experimental modal analysis. Jossic et al. [13] employed mode damping control on
xiaoluogong to successfully change the sound property. Jossic et al. [2] experimentally showed the presence of 1:2 internal resonances suggesting that a single nonlinear mode modeling is not suitable for describing the pitch glide phenomenon of Gong. However, it is observed that simulation with two nonlinear modes in 1:2 internal resonance confirm qualitatively the experimental results [2]. As a result, a low-dimensional (reduced-order) but accurate model for describing the nonlinear vibration of Gong is still an open question.

Current mathematical models employed to describe this nonlinear effect include quadratic and cubic nonlinearities. Currently, there is not a widely accepted mathematical model for this. At the era of big data, it provides more possibility to identify the mathematical model directly from data. Sparse Identification of Nonlinear Dynamics (SINDy) was proposed in Brunton et al. [1], which becomes a popular manner to discover the governing equation in a data-driven manner. This is very suitable for physical problem where the mathematical model is not clear or a reduced order model is highly demanding. SINDy implement a sparse constraint which provides an accurate dynamical system model yet avoid over-fitting, which is shown to well reconstruct the model of Lorenz attractor and Hof bifurcation of flow behind cylinder [1]. This framework was further extended to identify model described by partial differential equations [14] and discovery of coordinates [15].

Therefore, this paper employs Sparse Identification of Nonlinear Dynamics (SINDy) to build the model of Gongs with the data-driven method. We employed MEMS gyroscope to collect the vibration angular rate of two types of Gong. Then, we employ SINDy to identify the nonlinear dynamics of Gong. The model is using angular velocity as state variable and candidate library of nonlinearity is chosen up to fifth order polynomial. The results indicates that the nonlinearity is prominent up to the third order, which confirm the previous observation that the oscillator with a cubic nonlinearity is able to well characterize the nonlinear vibration behavior of Gong [2].

The following paragraphs are organized as follows. Section 2 describes the experimental setup including Gong and sensors to measure vibration. We present measurement results in Section 3. Section 4 presents the basic formula of SINDy and the obtained model and results are presented in Section 5. We conclude this paper in Section 6.

2. Experimental setup
The experimental site is Chongqing Foreign Language School and the experimental time is August, 2019 - January, 2020.

This experiment employs Chinese Gong made up of ring copper (an alloy made by mixing copper, lead, and tin in certain proportions). Other parameters of Gong are listed in table 1. We choose high pitch and median pitch Hand Gong as experimental apparatus. These two Hand Gong have the same diameter but different side geometry, which makes difference in their sounds.

| Name     | Diameter (cm) | Thickness of side of Gong (cm) | Weight (kg) |
|----------|---------------|-------------------------------|-------------|
| High pitch | 21.5       | 0.1                           | 0.45        |
| Median pitch | 21.5       | 0.1                           | 0.45        |

Figure 4 shows the front view of Gong used in experiments and the left of figure 4 is the median pitch Gong and the right of figure 4 is the high pitch Gong. The left and right in figure 5 show the back view of the median pitch and high pitch Gongs used in experiments, respectively. Side views of these Gong are also shown in figure 6 that are employed in this experiments. The rope is held across the slide of Gongs and hung in mid-air.

We employ MEMS gyroscope (MPU6050) as shown in figure 7 to measure the angular velocity of Gong during vibration stage. Data measured from this MEMS sensor is transferred to the computer through bluetooth as shown in figure 8. Figure 12 depicts the sensor fixed on the experimental Gong.
The data measurement software and user interface is shown in figure 9 and figure 10. The sample frequency is set up as 200Hz. Figure 11 shows the example measurement results for (a) No. 6 and (b) No.11 for high pitch Gong.

Then, we hit the corresponding strike point using Gong hammer in a uniform strength on the Gong’s surface illustrated as red squares in Figure 13. In figure 13, the black circle represents the boundary of Gong, red squares represent strike position and blue square is the position occupied by sensor. The horizontal direction is aligned with X axis of MEMS gyroscope sensor and the vertical direction is alignen with Y axis of MEMS gyroscope sensor. We divide the X and Y axis as eight uniform length that are marked as the striking position shown in red circles in figure 13. We also draw lines that has 45° and 135° aligned with X axis, respectively. The cross points between these two lines and the boundary of Gong are also marked as the red circles in figure 13. We then fix the sensor at the center of the Gong using tape. In table 2, totally 16 strike points are listed, where six of them are in X axis and six of them are in Y axis. For each experiment, we firstly started collecting data and then hit the Gong after 10s, following which we stop collecting data after another 30s.

2.1. Error analysis

- Rust that remains on the Gongs after derusting may affects its sounds.
- Handmade Gongs may have size differences due to measurement error during the manufacturing process and Gongs surface is not completely flat.

![Figure 4. Front view of Gong. Left: Median pitch Gong Right: High pitch Gong.](image-url)
3. Sparse identification of nonlinear dynamics (SINDy)

Then, we employ sparse identification of nonlinear dynamics (SINDy) [1] to identify the nonlinear dynamics as a data-driven method. This framework SINDy has the benefit of obtaining a sparse dynamics that can avoid overfitting of the data, yet still obtained a good accuracy.

Here, we consider the nonlinear dynamics described as:

\[ x = f(x) \quad (1) \]

First, time-series data are collected and formed into a data matrix:

\[ X = [x(t_1), x(t_2), \ldots, x(t_m)]^T \quad (2) \]

Figure 5. Back view of Gong. Left: Median pitch Gong Right: High pitch Gong.

Figure 6. Side view of Gong. Left: Median pitch Gong Right: High pitch Gong.
Figure 7. MEMS Gyroscope used in measurement of Gong angular rate.

Figure 8. Photo of bluetooth connection employed to transfer data to the laptop.
Figure 9. Hitting the corresponding points of median pitch and high pitch Gongs

Figure 10. Hitting the corresponding points of median pitch and high pitch Gongs
Figure 11. Example measurement results for (a) No. 6 and (b) No. 11 for high pitch Gong.
Figure 12. Photos of sensor fixed on the Gong.

Figure 13. Illustration of experimental setup. Black circle represents the boundary of Gong; red squares represent strike position; blue square is the position occupied by sensor. Horizontal direction is aligned with X axis of MEMS gyroscope sensor and vertical direction is aligned with Y axis of MEMS gyroscope sensor.
Table 2. Horizontal (X) and vertical (Y) strike position (cm) in experiments for both high pitch and median pitch Gongs. Sensor is located at the center of each of Gongs; i.e., X = 0, Y = 0.

| No. | Strike position X (cm) | Strike position Y (cm) |
|-----|------------------------|------------------------|
| 1   | -10.8                  | 0                      |
| 2   | -8.1                   | 0                      |
| 3   | -5.4                   | 0                      |
| 4   | 5.4                    | 0                      |
| 5   | 8.1                    | 0                      |
| 6   | 10.8                   | 0                      |
| 7   | 0                      | -10.8                  |
| 8   | 0                      | -8.1                   |
| 9   | 0                      | -5.4                   |
| 10  | 0                      | 5.4                    |
| 11  | 0                      | 8.1                    |
| 12  | 0                      | 10.8                   |
| 13  | 7.6                    | 7.6                    |
| 14  | 7.6                    | -7.6                   |
| 15  | -7.6                   | -7.6                   |
| 16  | 7.6                    | -7.6                   |

Where $T$ denotes the matrix transpose. A similar matrix of time derivatives is formed:

$$
\dot{X} = [x(t_1), x(t_2), \ldots, x(t_m)]^T
$$

(3)

In practice, this can be directly computed from the data in $X$. Based on the data in $X$, a library of candidate of nonlinear functions $\Theta(X)$ is constructed:

$$
\Theta(X) = [1, X, X^2, \ldots, X^d]
$$

(4)

Here, the matrix $X^d$ denotes a matrix with column vectors given by all possible time series of the $d^{th}$ polynomials in the state $x$. The dynamical system may now be represented in terms of the data matrices as:

$$
\dot{X} = \Theta(X) \Xi
$$

(5)

Each column $\Xi_k$ in $\Xi$ is a vector of coefficients determining the active terms in the $k^{th}$ row equation. A parsimonious model will provide an accurate model fit in this equation with as few terms as possible in $\Xi$. Such a model may be identified using a convex $l_1$-regularized sparse regression:

$$
\Xi_k = \arg \min_{\Xi_k} \left\| \dot{X} - \Theta(X) \Xi_k \right\|_2 + \lambda \left\| \Xi_k \right\|_1
$$

(6)

Here, $\dot{X}_k$ is the $k^{th}$ column of $\dot{X}$. Spares regression (e.g., sequential thresholded least-squares method employed in SINDy) improve the numerical robustness of this identification for noisy overdetermined problems.

The sparse vectors $\Xi_k$ may be synthesized into a nonlinear dynamical system model:
\dot{x}_k = \Theta(x)\xi_k \tag{7}

Note that \(x_k\) is the \(k^{th}\) element of \(x\) and \(\Theta(x)\) is a row vector of symbolic function of \(x\), as opposed to the data matrix \(\Theta(X)\).

Identifying the most parsimonious nonlinear model by applying sparse regression in the library \(\Theta\) is a convex procedure. The alternative approach, which involves regression onto every possible sparse nonlinear structure, constitutes an intractable brute-force procedure. Note that, if \(\Theta(X)\) consists only of linear terms and if the sparsity promoting term is set to \(\lambda = 0\), this algorithm reduces to dynamic mode decomposition \[16\].

4. Results

Figure 14 shows the time history of three axis of angular velocity measured from the MEMS gyroscope with hitting positions (a): No. 1, (b): No. 3, (c): No. 4, (d): No. 6 in table 2, respectively. After hitting the Gong, we can see that the angular velocity has a significant change; e.g., after \(t = 5\) in 14 (a). The frequency of z axis angular velocity contains not only the high frequency mode that has the same frequency as x and y axis, but also the low frequency modes; e.g., in figure 14 (a). We can also see that the angular velocity with hitting position near the boundary in figure 14 (a) and (d) is much larger than that when we hit the inner position of the Gong in figure 14 (b) and (c).

Figure 15 shows the time history of three axis of angular velocity measured from the MEMS gyroscope with hitting positions (a): No. 7, (b): No. 9, (c): No. 10, (d): No. 12 in table 2, respectively. After hitting the Gong, we can see that the angular velocity has a significant change; e.g., after \(t = 5\) in 15 (a). The frequency of z axis angular velocity contains not only the high frequency mode that has the same frequency as x and y axis, but also the low frequency modes; e.g., in figure 15 (a). We can also see that the angular velocity with hitting position near the boundary figure 15 (a) and (d) is much larger than that when we hit the inner position of the Gong in figure 15 (b) and (c).

Figure 16 and 17 show the time history of measurements for the median pitch Gong. The results in figure 16 are associated with the same MEMS gyroscope with hitting positions (a): No. 1, (b): No. 3, (c): No. 4, (d): No. 6 in table 2. Measurements in figure 17 are associated with hitting positions (a): No. 7, (b): No. 9, (c): No. 10, (d): No. 12. Compared with measurement for high pitch hand Gong, the results in figures 16 and 17 show larger maximum value of angular velocity in Y axis. The results of angular velocity in Y axis can reach around 600 rad/s in figures 16 and 17 for median pitch Gong, while the maximum value is around 400 rad/s for high pitch hand Gong. Another main different feature for median hand Gong is that the vibration is decaying much faster than that for high pitch hand Gong; for example, comparing figure 14 (b) and 16 (b). These results indicates that the median pitch Gong has a different a larger stiffness than the high pitch Gong, which results in a faster decay rate and a larger response magnitude.

After conducting all of the data for different hitting position, we substitute these data into the SINDy algorithm described in equation (3.6). We are using angular velocity as state variable and candidate library of nonlinearity is chosen up to fifth order polynomial. The angular velocity is typically employed as the state variable for describing dynamics of a rigid body, where the nonlinear dynamics will involve only quadratic nonlinearity. Here, we also include the linear term and constant terms to represent a possible change of coordinates.

The higher order nonlinearity is chosen as candidate for the nonlinear dynamics so as to capture the underlying vibration behavior. For example, Jossic et al. \[2\] noted that a duffing oscillator with cubic nonlinearity can describe the nonlinear vibration behavior of Gong in a favorable manner. We also include higher order terms involving forth and fifth order nonlinearity, which may result in higher order nonlinear model.

We also make the assumption that the hitting force is a spatial dependent impulse forcing, and the dynamics of Gong will behave based on its intrinsic nonlinear dynamics and independent of the hitting
position. Then, we collect all of data for 16 different hitting positions to identify the nonlinear dynamics describing the behavior of Gong. The obtained dynamics of high pitch Gong is described as:

\[
\frac{dw_1}{dt} = 0.3534w_2 \\
+ 0.0038w_1w_1 + 0.0087w_1w_2 + 0.0349w_1w_3 + 0.0144w_2w_2 + 0.0713w_2w_3 + 0.0045w_3w_3 \\
+ 0.0002w_1w_3w_3 + 0.0007w_1w_5w_5 - 0.0002w_2w_5^2 - 0.0003w_3^3
\]  
(8)

\[
\frac{dw_2}{dt} = 0.0094w_2 \\
- 0.0005w_1w_1 - 0.0039w_1w_2 + 0.0057w_1w_3 + 0.0005w_2w_2 + 0.0067w_2w_3 + 0.0067w_3w_3 \\
- 0.0001w_3^3
\]  
(9)

\[
\frac{dw_3}{dt} = 0.1279w_2 \\
- 0.0033w_1w_1 - 0.0075w_1w_2 - 0.0084w_1w_3 - 0.0006w_2w_2 + 0.0042w_3^2
\]  
(10)

The obtained dynamics of median pitch Gong is described as:

\[
\frac{dw_1}{dt} = -0.0836w_2 \\
+ 0.0040w_1w_1 + 0.0113w_1w_2 + 0.0648w_1w_3 + 0.0020w_2w_2 + 0.0115w_2w_3 - 0.0130w_3w_3 \\
+ 0.0001w_2w_3^2 + 0.0003w_3^3
\]  
(11)

\[
\frac{dw_2}{dt} = -0.1223w_2 \\
- 0.0057w_1w_1 + 0.0117w_1w_3 \\
+ 0.0008w_2w_2 + 0.0112w_2w_3 + 0.0283w_3w_3 \\
- 0.0002w_1^2w_3 - 0.0001w_1w_2w_3 - 0.0003w_1w_3^2
\]  
(12)

\[
\frac{dw_3}{dt} = 0.0755w_2 \\
- 0.0016w_1w_1 - 0.0026w_1w_2 - 0.0016w_3^3
\]  
(13)

The dynamics identified from SINDy is shown in equation (8)(9)(10) for high pitch Gong and equation (11)(12)(13) for median pitch Gong. We can see that all nonlinearity are only discovered to be up to the third order. This results show that up to the third order nonlinear model will be enough for describing the nonlinear behavior of Gong. Moreover, in both the equation(8)(9)(10) and (11)(12)(13), we can see that the nonlinearity of $w_3$ is only up to the second order. This is likely due to the fact that our striking is in the direction of Z axis that is perpendicular to the surface of Gong. As a result, the main vibration and excited nonlinearity is focused in the vibration around the X and Y axes instead of Z axis.

Comparing the high pitch results in equation (8)(9)(10) and median pitch results in equation (11)(12)(13), we can see that the cubic nonlinear coupling is appearing in different places. For high pitch Gong, the cubic nonlinearity appearing in $\frac{dw_2}{dt}$ is only $w_3^3$ term, while more cubic nonlinear coupling is appearing in dynamics of $\frac{dw_1}{dt}$. On the other hand, for the median pitch Gong, only $w_3^3$
cubic nonlinearity is appearing in $\frac{dw_1}{dt}$, yet more cubic nonlinearity is appearing for the $\frac{dw_2}{dt}$. This is resulted from the feature difference between high pitch and median pitch Gong. Moreover, the coefficients and sign for each term in equations (8)(9)(10) and (11)(12)(13) are also different. Such a different dynamics behavior further implies the different mechanism of generating high pitch and median pitch sound.

Figure 14. Time history of three axis angular rate measured from MEMS Gyroscope for high pitch Hand Gong. Hiting positions are (a): No. 1, (b): No. 3, (c): No. 4 (d): No. 6 in table 2, respectively.
Figure 15. Time history of three axis angular rate measured from MEMS Gyroscope for high pitch Hand Gong. Hitting positions are (a): No. 7, (b): No. 9, (c): No. 10 (d): No. 12 in table 2, respectively.
Figure 16. Time history of three axis angular rate measured from MEMS Gyroscope for median pitch Hand Gong. Hitting positions are (a): No. 1, (b): No. 3, (c): No. 4 (d): No. 6 in table 2, respectively.
Figure 17. Time history of three axis angular rate measured from MEMS Gyroscope for median pitch Hand Gong. Hitting positions are (a): No. 7, (b): No. 9, (c): No. 10 (d): No. 12 in table 2, respectively.

5. Conclusions
This work employed MEMS Gyroscope to collect the vibration angular rate of two types of Hand Gong including median pitch and high pitch Gongs. This paper further employs Sparse Nonlinear Dynamics Identification (SINDy) [1] to identify a reduced order model of Gong. The model is using angular velocity as state variable and candidate library of nonlinearity is chosen up to fifth order polynomial.

- The results indicate that the nonlinearity is prominent up to the third order (cubic) and the cubic nonlinearity is not appearing for the rotation around the axis perpendicular to Gong surface;
- These results confirm previous observations that the oscillator with a cubic nonlinearity is able to well characterize the nonlinear vibration behavior of Gong [2]. This can be future applied to musical psychology [11].
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