The conductance of quantum point contacts (QPCs) is quantized in units of $2e^2/h$ \[1, 2\]. In addition to these integer conductance steps, an extra conductance plateau around $0.7(2e^2/h)$ has been experimentally observed \[3, 4, 5, 6\]. Recently a generalized single-impurity Anderson model has been invoked to describe transport through QPCs \[5\]. According to this model, motivated by density-functional calculations that reveal the formation of a quasi-bound state at the QPC \[5\], the tunneling of a second electron through that state is suppressed by Coulomb interactions, and is enhanced at low temperatures by the Kondo effect \[10\]. Thus at temperatures larger than the Kondo temperature $T_K$, the conductance will be dominated by transport through the singly occupied level ($G \sim e^2/h$), growing at lower temperature towards the unitarity limit, $G = 2e^2/h$. Kondo physics has indeed been observed at low temperature and voltage bias $V$ \[7\]. The fact that there are effectively two conductance channels affects not only the conductance but also the current shot noise. Around conductance of $G \sim e^2/h$, the model predicts one highly transmitting channel ($T_1 \simeq 1$) and one poorly transmitting channel ($T_2 \simeq 0$). Thus, as the noise is expected to be proportional to the sum of $T_i(1 - T_i)$ over all channels, it should exhibit a dip near that value of the conductance \[11\], in contrast with the traditional view which associates a conductance of $G \sim e^2/h$ with $T_1 \simeq T_2 \simeq 1/2$ and maximal noise. A reduction in the noise through a QPC near $G \sim e^2/h$ has indeed been observed experimentally \[12, 13, 14\]. The dip was observed to be quite sensitive to magnetic fields. In this letter we present a detailed calculation of the noise based on the above model and demonstrate that it reproduces the experimental data. The magnetic field dependence arises from two factors: the dependence of the splitting of the two channels on the field, and the quenching of the Kondo effect. Specific predictions on the disappearance of the dip in the current noise at low temperature, voltage bias and magnetic field, due to the unitarity limit of the Anderson model are made.

The main theoretical difficulty with calculating the noise is that the limit of perfect conductance through a given channel is not accessible via traditional perturbation theory for this interacting problem. Thus an earlier calculation of the noise through a Kondo impurity \[16\] had to rely on more elaborate methods in order to be extended to lower temperatures. Because of the additional complexity of the generalized Anderson model, employed to describe QPCs (see below), these methods are not directly applicable. In this work we employ a new type of perturbation theory, starting from the high magnetic field $B$ limit. In this limit spin-flip processes are suppressed, and the current and noise can be exactly (and trivially) calculated, to all orders in the tunneling, giving rise to two separate channels. Perturbation in $1/B$ allows us to follow the contributions and mixing of the two channels. By comparing to the traditional perturbation theory, around zero $B$, we are able to interpolate the noise between the two regimes (see Eq. \[9\] below). This formula, which reduces in the known limits to the obtained perturbative results, allows us to compare to experiment in the whole magnetic field regime, yielding excellent agreement with experiment (Fig. 1) and allowing specific predictions.

**Model Hamiltonian:** The extended Anderson Hamiltonian, invoked in \[8\] to model the QPC differs from the usual single-impurity Anderson model in two aspects: (1) the tunneling amplitude of the first electron into the quasi-bound state $V^{(1)}$ is larger than that of the second electron $V^{(2)}$ (see also \[17\]), and (2) both couplings increase exponentially as the energy of the incoming electron rises above the QPC barrier, $E_{qpc}$, defined to be the zero of energy. This Anderson model can be transformed into a Kondo Hamiltonian by performing a Schrieffer-Wolff transformation \[14\].

\[
H = \sum_{k\sigma \in \mathcal{L}, \mathcal{R}} \varepsilon_{k\sigma} c_{k\sigma}^\dagger c_{k\sigma} + \sum_{k, k', \sigma \in \mathcal{L}, \mathcal{R}} (J_{k\sigma, k', \sigma}^{(1)} - J_{k\sigma, k', \sigma}^{(2)}) c_{k\sigma}^\dagger c_{k'\sigma}
\]
Green functions which can be expressed in the terms of the retarded, advanced and the "Keldysh" Green function, \( G^K(\omega) \). For the two leads, the unperturbed Keldysh Green functions are \( g^K_{k\sigma \ell, R, \sigma}(\omega) = -2\pi i (1 - 2f_{L, R}(\omega)) \), where \( f_{L, R}(\omega) = f_{FD}(\omega \pm eV/2) \) are the respective distribution functions in the leads, which depend on the voltage difference \( V \). It is more convenient to work in the symmetric and anti-symmetric combinations of the two leads \( g^K_{\pm} = g^K_{\ell, R} \pm g^K_{R, L} \).

When the magnetic field is large the exchange part of the Kondo Hamiltonian can be neglected. Therefore \( S_{z} \) can be treated as a conserved "classical" parameter. In this case, averaging over \( S_{z} \), one can calculate the conductance and noise exactly,

\[
G_{\infty} = \frac{e^2}{h} (T_{1} + T_{2})
\]

\[
S_{\infty} = \frac{e^2}{h} \sum_{\ell} [eV \coth(eV/2T_{1}(1 - T_{1}) + 2TT_{1}^2]
\]

Where \( T_{1,2} \), the transmission probabilities for the two channels, are expressed in terms of the coupling constants of Kondo Hamiltonian \( g_{i} = 4\pi \nu J^{(i)} \),

\[
T_{i} = \frac{g_{i}^{2}}{1 + g_{i}^{2}}
\]  

In the large coupling limit the transmission probabilities go to unity. Since, as function of energy, \( g_{1} \) first increases to a large value, while \( g_{2} \) becomes large only when \( \epsilon_{F} = \epsilon_{0} + U \), then, for large magnetic fields, as a function of gate voltage, the conductance, in units of \( 2e^2/h \), will first rise to \( 1/2 \) and then to unity. Concurrently, the shot noise, the first part of \( S_{\infty} \), will have a dip at the first conductance plateau, in agreement with experiments (Fig. 1).

As the magnetic field decreases, the exchange terms in the Hamiltonian have to be taken into account, influencing and mixing the contributions of \( J^{(1)} \) and \( J^{(2)} \) to the conductance and the noise. As the magnetic field is still large, we can expand the conductance and noise to second order in the spin-flip processes, still allowing infinite order in \( J^{(1)} \) in the non spin-flip processes. The resulting non-equilibrium noise is a function of applied voltage and also depends on the non-equilibrium magnetization \( M(B, T, V) \). The latter is reduced to its equilibrium value \( M_{eq} = < S_{z} > = (-1/2) \tanh(B/2T) \) if \( B > V \). The resulting additional contributions to the noise \( S_{B} \) and the linear response conductance \( G_{B} \) (obtained from the noise via the fluctuation-dissipation theorem, \( G = S(V \to 0)/(2T) \)) in this limit are

\[
S_{B} = \frac{\epsilon_{F}^{2}}{2h} (g_{1} + g_{2})^2 \left[ \frac{m_{1} + m_{2}}{2} (A_{\pm} + 4BM) - m_{1}m_{2} (g_{1} + g_{2}) A_{\pm} \right]
\]

\[
G_{B} = \frac{B}{2T \sinh^{2} \frac{eV}{2T}} (m_{1} + m_{2}) (g_{1} + g_{2})^2
\]
where  
\[ A_{\pm} = B \coth(B/2T) \pm \frac{1}{2} [B_+ \coth(B_+/2T) + B_- \coth(B_-/2T)] \]  
(7)

Here \( m_{1,2} = 1/(1 + g_2^2/\delta^2) \) and \( B_{\pm} = B \pm eV \). In equation \( \hat{g}_2 \) was considered small. The nonequilibrium magnetization is given by \( \hat{M} = -B/A_+ \). In the limit of small \( g_{1,2} \), equation \( (6) \) reduces to the zero frequency current-current correlation function obtained in \( \delta^2 \).

Note that the corrections to the infinite field limit, due to spin flips, depend on \( \coth(B/2T) \), and thus decrease exponentially with increasing the ratio \( B/T \).

We note that the conductance, to this order, can be written as the expansion of an expression similar to that of Eq. (4), with \( \hat{g}_2^2 \) in Eq. (6) replaced by \( g_2^2 \), with  
\[ g_2^2 = g_1^2 + \frac{B}{T \sinh \frac{B}{2}} \frac{(g_1 + g_2)^2}{1 + (g_1 + g_2)^2}. \]  
(8)

Note that even though \( g_2^2 \) is small, \( g_2^2 \) can become substantial at smaller magnetic field due to higher order processes involving \( g_2^2 \). Thus the second channel will also contribute to transport, raising the conductance plateau from its value of \( 0.5 \times 2e^2/h \) at large magnetic field. This is consistent with the observation that the value of "0.7" plateau usually does not drop experimentally below \( 0.6 \times 2e^2/h \).

The second order spin-flip processes do not give rise to Kondo physics. For this one has to go to third order in the J’s. As argued in \( \delta^2 \), the Kondo effect will be dominated by the \( J^{(2)} \) term. Due to the step-like increase of the couplings, the bottom of the band, \( E_{qpc} \), is effectively very close to the Fermi energy, and thus only virtual processes that involve the empty states will renormalize the couplings. This will be manifested in the logarithmically divergent terms, arising from the third order processes. The \( J^{(1)} \) terms will involve integrals over the small region between \( E_{qpc} \) and \( \varepsilon_F \). The \( J^{(2)} \) terms, on the other hand, will involve integrals from \( \varepsilon_F \) to the upper band edge \( D \) (or to \( U \)), and these give rise to the Kondo effect.

These logarithmic contributions only appear in the Kondo regime \( \varepsilon_0 + U > \varepsilon_F > \varepsilon_0 \). A lengthy calculation yields the Kondo contribution to the noise \( S_K \) and conductance \( G_K \),

\[ S_K = \frac{e^2 g_0^2}{h \pi} \left\{ \coth\left( \frac{B_+}{2T} \right)[F(B) + F(B_+) + F(eV)] + \coth\left( \frac{B_-}{2T} \right)[F(B) + F(B_-) - F(eV)] + \coth\left( \frac{eV}{2T} \right)[F(B_+) - F(B_-)] + 2M[2F(B) + F(B_+) + F(B_-)] \right\} \]  
(9)

\[ G_K = 2 \frac{g_0^3}{\pi} (1 + \frac{2B}{T \sinh \frac{B}{2T}}) \ln \frac{D}{\sqrt{B^2 + T^2}} \]  
(10)

where \( F(x) = x \ln(D/\sqrt{x^2 + T^2}) \). For \( B > eV \) we can apply the expression for the equilibrium magnetization \( M_{eq} \).

At low temperature (and zero magnetic field) the logarithm contributions to the noise and conductance will diverge, signalling the onset of the Kondo effect below the Kondo temperature \( T_K \approx U \exp(-\pi/g_2) \). Using the renormalization-group approach, one can sum up the most divergent logarithms in the higher order Kondo contributions (Eq.10). Separating the contribution to this Kondo series from the leading terms and summing up the series leads to the final expression for the total conductance,

\[ G_{tot} = \frac{e^2}{h} (\hat{T}_1 + \hat{T}_2) \]  
(11)

with  
\[ \hat{T}_1 = \frac{g_0^2}{1 + g_0^2}, \]  
\[ \hat{T}_2 = \frac{g_0^2}{1 + g_0^2} - \frac{B}{T \sinh \frac{B}{T}} + G_{2RG}, \]  
(12)

and  
\[ G_{2RG} = \frac{1}{(\ln \frac{\sqrt{B^2 + T^2}}{T_K})^2} \frac{\pi^2}{8} (1 + \frac{2B}{T \sinh \frac{B}{T}}) \]  
(13)

with the Kondo temperature \( T_K \approx U \exp(-\pi/g_2) \). The Kondo contribution enhances the contribution of the second channel, and gives rise to the merging of the "0.7" feature with the first \( 2e^2/h \) conductance step. As pointed out in \( \delta^2 \), the resulting \( T_K \) increases exponentially with \( \varepsilon_F \), in agreement with the experimental observation that \( T_K \) increases exponentially with the gate voltage \( \delta \). A similar expression can be obtained for the total noise, but in the following we will use the noise expression similar to that appearing in Eq. (11), with \( g_0^2 \) given by Eq. (12). We also note that if one replaces \( \sqrt{B^2 + T^2} \) by \( \sqrt{B^2 + T^2 + V^2} \) in Eq. (13), the formula (11) for the conductance not only agrees with the expansion around large magnetic fields, but also with the expansion (in \( J_1 \)) at small fields \( \delta^2 \). Thus this interpolation formula should be reliable at the whole range of magnetic fields.

Comparison with experiment and conclusions. Fig. 1 compares our calculation to the experimental results of Ref. 14 and of Ref. 15. In (a) and (b) we compare the Fano factor, which is obtained, following Ref. 14, by subtracting from the full noise the thermal contribution (the last term in (12) plus \( 2T[G - (e^2/h)(T_1 + T_2)] \)), and dividing this difference by the current. Plotting the Fano factor against conductance, makes the theoretical plot practically independent of the values of \( \varepsilon_0, U \) and \( \delta \), which determine the dependence of the conductance on gate
voltage. The ratio of $g_2^2/g_1^2$ was assumed small (= 0.01) in the spirit of the model, and the curves for 3 values of magnetic field, in the ratio 0 : 3 : 8 as those used in the experiment, are depicted with good agreement with experiment. The data of Ref. [15] allow an even more quantitative comparison with experiment, as we used the actual values of magnetic field, voltage and temperature reported to the experiment. To get the best fit with experiment we used a $g$-factor of 0.35, indicating either the inaccuracy of the theory or the estimate of temperature in the experiment. Interestingly, the zero-field dip in the noise is quite small, even though the bare contribution of the second channel to the conductance is negligible. This is due to the contributions of higher order processes, discussed below Eq. (8).

While the experiments were carried out outside the Kondo regime, due to the relative high voltage applied, the theory predicts that, for temperatures and voltages smaller than the Kondo temperature, the dip in the noise will disappear at zero field, due to the unitary limit of the Kondo effect.

It is interesting to note that a perhaps related dip appears in the measurement of dephasing in a quantum dot [23], as measured by a nearby quantum point contact, when the point contact is in the "0.7" regime. The present theory suggests a simple explanation of this effect: as the dephasing in the quantum dot is by the current noise in the point contact [24], a dip in the noise will be associated with a dip in the dephasing rate in the quantum dot. A detailed calculation of this effect will be presented elsewhere [25].

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FIG. 1: (a) The Fano factor, calculated from the theory, versus zero-bias conductance at different magnetic fields, $g \mu_B B/k_B T = 0, 4.5, 12$, compared to the experimental results of Ref. [14] (b), for B=0, 3 and 8 Tesla. The parameters used in the theory were $eV = k_B T, V^{(1)/2}/2\pi = 1, V^{(2)/2}/2\pi = 0.01$. In (c) the noise is calculated for the same parameters as those corresponding to the data of Ref. [14], depicted at (d), with the magnetic field values denoted in the legend, $k_B T = 280 mK$ and $V = 240 \mu V$. The values of $V^{(1)}$ are the same as in (a). In order to get the best comparison to the experiment a value of $g$-factor of 0.35 was used.

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