The dual control algorithm of nonlinear dynamic processes in the conditions of incomplete a priori information

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Abstract. The problem of controlling nonlinear dynamic systems under conditions of incomplete a priori information about the model of the differential equation of the process is considered. Nonparametric models are given in which the order of the difference equation of the dynamic process is refined based on the rule for distinguishing significant variables. According to this rule, only those variables are included in the nonparametric model for which the minimum bandwidth is minimal. Nonparametric algorithms of dual control of nonlinear objects are given. Control devices built on the basis of these algorithms perform not only the direct function of controlling the object, but also its study. The process of training the dual control system with active accumulation of information is analyzed. The results of a numerical study of the application of the nonparametric adaptive dual control algorithm are presented.

1. Introduction
Most of the developed control algorithms, including control algorithms for nonlinear dynamic objects, belong to the class of parametric ones [1-3]. This means that at the stage of setting the control problem it is assumed that the parametric structure of the model of the object under study is known, accurate to the parameters from the available a priori information. This approach has received significant development in the framework of the theory of adaptive control systems [3], which involves preliminary parameterization of the object. Adaptive systems in the formation of control actions use current information about the object of study [4, 5]. It should be noted that in most cases the available a priori information is not enough for a reasonable choice of the parametric structure of the model of a dynamic object. Also, one of the characteristic situations is the lack of current information due to the lack of appropriate controls.

Of significant interest is the development of the theory of dual control. The lack of a priori information about the object of study leads to the need to combine the study of the object and control. The control actions are dual in nature. They serve as a means of studying, cognizing an object, but also as a means of bringing the object to the required state. Such control, in which control actions are dual in nature, is called dual control. Dual control was discovered by A.A. Feldbaum in 1960 and developed on the basis of the theory of statistical solutions [6]. It should be noted that learning control systems are not only capable of studying the characteristics of an object, but also, storing them in memory, develop rational control actions. The problem of dual control was also considered by Ya.Z. Tsypkin in [7]. In this statement of the control problem, information is needed on the parametric structure of the model of the object of study. The theory of dual control was further developed in the studies of various authors, in particular B. Wittenmark [8]. In the paper, attention is drawn to the cumbersomeness of analytical results on the synthesis of an optimal control algorithm, and control
algorithms are replaced in the form of non-optimal dual controllers. Note that in the formulation of the problem, the probability distribution of the interference, as well as the parametric model of the object, are assumed to be known. Such an approach is applicable only to the control of well-studied processes whose parametric structure is known. Some issues related to dual control, namely, the synthesis of dual control systems related to the method of maximum likelihood in the absence of a priori information about unknown parameters of an object, were developed by V.P. Zhivoglyadov in [9]. Later, dual adaptive control algorithms were developed for linear stochastic systems with constant but unknown parameters [10, 11], and various control methods for a linear discrete system with non-stationary random parameters in the presence of interference were studied [12].

In the above studies, control problems were considered under conditions of parametric uncertainty. In the case when the structure of the dynamic process is not defined accurate to the vector of parameters, one of the possible ways to solve the control problem is to use nonparametric adaptive control systems that preserve the components of dualism [13, 14]. This article proposes a nonparametric dual control algorithm, the construction of which is based on the use of a nonparametric regression estimate, which takes into account information about the order of the difference equation of the dynamic process model.

2. Problem set-up
The task of controlling a dynamic object is to form such control actions $u(t)$ that ensure that the output variable of the object $x(t)$ is reduced to the master action $x^*(t)$. In this case, the state of the dynamic system is determined by the values of the controlled input variables $u(t)$, as well as the values of the outputs of the object at previous times.

Let us consider in detail the idea of automatic control of various objects, including dynamic ones, using devices synthesized by the inverse operator method. We introduce the operator of object $A$, which describes the process

$$
\begin{align*}
    x(t) &= A < u(t) > \\
    A^{-1} x(t) &= A^{-1} A < u(t) >, \\
    u(t) &= A^{-1} x(t).
\end{align*}
$$

(1)

By setting the desired trajectory of the output variable $x(t) = x^*(t)$, we find from (1) the ideal value of the control action $u^*(t)$. Thus, (1) can be classified as ideal regulators. However, the problem is that in most cases it cannot be built, especially since the operator $A$ is unknown. In the absence of a priori information about the operator $A$, one of the ways to solve this problem is to use nonparametric control algorithms, which will be shown in detail below.

Figure 2 shows the control block diagram of the process. In figure 2, the following notation is used: $(t)$ is continuous time, index $t$ is discrete time, random interference measurements $h_t^x, h_t^e$ of the process variables, $\xi(t)$ is a vector random noise. Variables are monitored over a time interval $\Delta t$.

The paper considers classes of control objects that can be described by difference equations of the form:
\[ x_t = F(x_{t-1}, \ldots, x_{t-k}, u_t, \zeta_t). \]  

(2)

where \( F \) is an unknown functional, \( k \) is the order of the difference equation, which is bounded by \( \leq k_{\text{max}} \). The input and output of a dynamic object is represented by measurements forming a sample of the form \( \{u_i, x_i\}, i = 1, s \), where \( s \) is the sample size, \( u_i, x_i \) measures the input and output of the object at time \( t_i \).

The following nonparametric estimate of the regression function based on observational data \( \{x_i, u_i, i = 1, s\} \) can be accepted as a nonparametric model of the object \([13]\):

\[
    x_s^k = \sum_{i=1}^{s} \frac{x_i}{\sum_{i=1}^{s} \phi \left( \frac{u_s - u_i}{c_s^{x_j}} \right)} \prod_{j=1}^{k} \phi \left( \frac{x_{s-j} - x_{i-j}}{c_s^{x_j}} \right).
\]

(3)

where \( \phi(\cdot) \) is a kernel function, \( c_s^{x_j}, c_s^{x[j]} \) is a bandwidth. The optimal bandwidths are found by minimizing a quadratic error function (4) by using the sliding exam method

\[
    R(c_s^u, c_s^{x[1]}, \ldots, c_s^{x[k]}) = \sum_{q=1}^{s} \left( x_q(u_q, x_{q-1}, \ldots, x_{q-k}) - x_q \right)^2 = \min_{c_s^u, c_s^{x[1]}, \ldots, c_s^{x[k]}} \sum_{q=1}^{s} (x_q - x_q)^2, \quad q \neq i,
\]

(4)

where \( i \) is the index in the equation (3).

Figure 2. Dynamic object control scheme.
3. Definition of significant variables

It is significant in the estimate (3) that, in accordance with each output delayed variable \(x_{s-1}, \ldots, x_{s-k}\), some values of the bandwidth \(c_{s}^{x_{1}}, \ldots, c_{s}^{x_{2}}\). Let us give an example, let \(c_{s}^{x_{1}}\) be the bandwidth for \(x_{s-1}\), and \(c_{s}^{x_{2}}\) for \(x_{s-2}\). If \(c_{s}^{x_{1}} < c_{s}^{x_{2}}\) then \(\Phi\left(\frac{x_{s-1}-x_{t-1}}{c_{s}^{x_{1}}}\right) < \Phi\left(\frac{x_{s-2}-x_{t-1}}{c_{s}^{x_{2}}}\right)\), and then the output variable \(x_{s-1}\) has a greater effect on the output quantity than \(x_{s-2}\). Graphically, this is expressed as follows (figure 3).

![Figure 3. The dependence of the value of nuclear function \(\Phi(\cdot)\) of the bandwidths \(c_{s}\).](image)

In the figure 3: \(\Phi_{1} = \Phi\left(\frac{x_{s-1}-x_{t-1}}{c_{s}^{x_{1}}}\right)\), \(\Phi_{2} = \Phi\left(\frac{x_{s-2}-x_{t-1}}{c_{s}^{x_{2}}}\right)\).

The algorithm for calculating significant variables \(x_{s-j}\) is based on the following scheme. First, the initial value of \(k\) is given. The model is constructed by equation (4) and the relative error \(W_{0}\) is calculated:

\[
W = \sqrt{\frac{1}{s} \sum_{i=1}^{s} (x_i - x_{i}^{s})^2 / \sum_{i=1}^{s} 1 \left( \frac{m_{x} - x_{i}}{s - 1} \right)^2}
\]

where \(m_{x}\) is an expected value.

For each \(i\)-th iteration, the following set of actions is performed:

1. For each coefficient the optimal value is found: \(c_{s}^{x_{1}} = c_{s}^{x_{1}},\ldots, c_{s}^{x_{k}} = c_{s}^{x_{k}}\)
2. The maximum of all the values obtained is found: \(c_{\text{max},s}^{x_{j}}\)
3. The model is constructed by the equation (4). The multiplier \(\Phi\left(\frac{x_{s-j}-x_{t-j}}{c_{s}^{x_{j}}}\right)\) is excluded, taking into account that \(j\) is a number for \(c_{\text{max},s}^{x_{j}}\)
4. A relative error \(W_{i}\) is calculated.

These actions will be repeated until \(W_{i} \geq W_{i-1}\).

4. Nonparametric dual control algorithm

The nonparametric dual control algorithm has the form [13,14]:

\[
u_{s+1} = u_{s}^{*} + \Delta u_{s+1},
\]
where $u^*_s$ is the component that accumulates information about the object of study, and $\Delta u_{s+1} = \varepsilon(x^*_{s+1} - x_s)$ are the learning search steps.

The dual control scheme is shown in figure 4.

The dualism of algorithm (5) is as follows. At the first control steps, the main role in the formation of control actions is played by the term $\Delta u_{s+1}$ from formula (5). But already with the accumulation of information about the object, the role of the term $u^*_s$ increases.

In this case, as the term $u^*_s$ from equation (5), we can take the expression (6)

$$u^*_s = \sum_{i=1}^{s} \phi(x^*_{s+1} - x_i) \prod_{j=1}^{k} \phi(x_{s-j} - x_{i-j}) \sum_{i=1}^{s} \phi(x_{s+1} - x_i) \prod_{j=1}^{k} \phi(x_{s-j} - x_{i-j}).$$

The control algorithm of nonlinear dynamic systems is constructed as follows. Based on the rule for distinguishing essential variables, the order of the spread equation of the dynamic process model $k$ is determined, which is further used in the calculation of control actions in (5), where only those variables that were selected by the algorithm are present.

In this experiment, the quality of control was evaluated according to two characteristics:

- The regulation time $t_p$ is the time from the start of control until the moment when the output value differs from the task no more than some given value $\alpha$. Usually taken $\alpha = 0.05x$.
- The relative control error $W_p$ is equal to the total deviation of the actual process output from the setpoint during the entire control time with respect to the setpoint, expressed in relative values in %.

$$W_p = \frac{1}{s} \sum_{i=1}^{s} |x_i - x^*_i| / x^*$$

Shows the degree to which the output value deviates from the reference, expressed as a percentage.

Figure 5 shows the results of a computational experiment for controlling a nonlinear dynamic system $x_t = 0,2x_{t-1}^{1.2} - 0,3x_{t-2}^{1.2} + 0,4\cos(x_{t-3}) + 1,5u_t$.

The relative control error for the case shown in Figure 5 is $W_p = 0.216$. Figure 6 shows an experiment on controlling a dynamic object described by the difference equation $x_t = 0,2x_{t-1}^{1.2} - 0,3x_{t-2} + 0,4x_{t-3} + 1,5u_t$ under external noise $\xi(t) = 5\%$.

In Fig. 6, the relative control error is $W_p = 0.162$.

The developed nonparametric algorithm for controlling a nonlinear dynamic system allows you to effectively control processes, even in the presence of external noise.
5. Conclusion

The article provides a solution to the control problem for the case of nonlinear dynamic systems. Nonparametric algorithms, as was previously known, can be effectively applied to solve identification and control problems for processes from a class or category of linear ones. Naturally of interest is the situation when the process under investigation is non-linear, to one degree or another. In the article, a nonlinear dynamic system was considered as a system at the input of which output variables were delayed by the corresponding number of clock cycles. Moreover, the model of a dynamic system was a nonparametric estimate of the regression function from observations. To improve this model, an algorithm for determining the order of the difference equation of the dynamic process model was proposed. The algorithm reduces to determining the delayed components of the output variable of the model of the difference equation of a dynamic object. One of the main advantages of the proposed method in comparison with the currently dominant approaches is the fact that the developed nonparametric algorithm is more applicable to practical tasks, as it is able to work in conditions of small a priori information about the object.

The computational experiments presented above showed that a nonparametric control algorithm can also be used for objects for which the principle of superposition is no longer fulfilled.
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