New formulae for the $(-2)$ moment of the photo-absorption cross section, $\sigma_{-2}$

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Two new formulae for the $(-2)$ moment of the photo-absorption cross section, $\sigma_{-2}$, have been determined, respectively, from the 1988 compilation of Dietrich and Berman and a mass-dependent symmetry energy coefficient, $a_{sym}(A)$. The data for $A \gtrsim 50$ follow, with a RMS deviation of 6%, the power-law $\sigma_{-2} = 2.4A^{5/3}\text{pb}/\text{MeV}$, which is in agreement with Migdal’s calculation of $\sigma_{-2} = 2.25A^{5/3}\text{pb}/\text{MeV}$ based on the hydrodynamic model and the $\sigma_{-2}$ sum rule. The additional inclusion of $a_{sym}(A)$ provides a deeper insight to the nuclear polarization of $A \gtrsim 10$ nuclei.

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The ratio of the induced dipole moment to an applied constant electric field yields the static nuclear polarizability, $\alpha$. On using the hydrodynamic model and assuming inter-penetrating proton and neutron fluids with a well-defined nuclear surface of radius $R = r_0 A^{1/3}$ fm, Migdal [1, 2] obtains,

$$\alpha = \frac{e^2 R^2 A}{40 a_{sym}} = 2.25 \times 10^{-3} A^{5/3} \text{fm}^3,$$

(1)

where $a_{sym} = 23$ MeV is the symmetry energy coefficient in the Bethe-Weizsäcker semi-empirical mass formula [3, 4] and $r_0 = 1.2$ fm. This semiclassical treatment considers the nuclear symmetry energy, $a_{sym}(N-Z)^2/A$, to be spread uniformly throughout the nucleus as a symmetry energy density $a_{sym}(\rho_n - \rho_p)^2/\rho$.

Alternatively, $\alpha$ can be calculated from the $(-2)$ moment of the total electric-dipole photo-absorption cross section, $\sigma_{-2}$,

$$\sigma_{-2} = \int_0^\infty \frac{\sigma_{\text{tot}}(E_{\gamma})}{E_{\gamma}^2} dE_{\gamma},$$

(2)

using second-order perturbation theory [5, 6]. It follows from the sum rule [7, 8].

$$\alpha = 2e^2 \sum_n \frac{\langle |i||E_1||n\rangle \langle n||E_1||i\rangle}{E_{\gamma}},$$

(3)

where $E_{\gamma}$ is the $\gamma$-ray energy corresponding to a transition connecting the ground state $|i\rangle$ and an excited state $|n\rangle$, $M$ the nucleon mass, and $\sigma_{\text{tot}}(E_{\gamma})$ the total photo-absorption cross section. The $\sigma_{\text{tot}}(E_{\gamma})$ cross section generally includes the $(\gamma, n) + (\gamma, p) + (\gamma, np) + (\gamma, 2n) + (\gamma, 3n) + (\gamma, F)$ channels, which are in competition in the giant dipole resonance (GDR) region [9, 10].

On comparing Eqs. (3) and (6), Migdal extracted $\sigma_{-2}$ as [1]

$$\sigma_{-2} = 2.25A^{5/3}\text{pb}/\text{MeV}.$$

(4)

This power-law relationship was empirically confirmed by Levinger in 1957 from a fit to the available $\sigma_{-2}$ data [2],

$$\sigma_{-2} = 3.5k A^{5/3}\text{pb}/\text{MeV}.$$

(5)

Levinger’s fit is shown in Fig. 1 and included eleven $\sigma_{-2}$ data points (squares) with approximate estimations for the high-energy, neutron multiplicity and $\sigma(\gamma, p)$ contributions. The polarizability parameter $k$ is the ratio of the observed GDR effect to that predicted by the hydrodynamic model [2], as determined by comparing the measured $\sigma_{-2}$ values and Eq. (5). This comparison yields $k = 1$ for the ground state of nuclei with $A \gtrsim 20$ [2]. Lighter nuclei require larger values of $k$ to reproduce the data. Using Eqs. (6) and (7) the nuclear polarizability is given by

$$\alpha = 3.5k \times 10^{-3} A^{5/3} \text{fm}^3,$$

(6)

which depends on the nuclear size and $k$.

In 1988, Dietrich and Berman re-compiled the photo-neutron cross-section data [11]. This compilation included $(\gamma, n) + (\gamma, np) + (\gamma, 2n) + (\gamma, 3n) + (\gamma, F)$ data from studies at Livermore, Giessen, Saclay and other laboratories which used monochromatic photon beams generated in-flight annihilation of positrons [12].
Figure 1 shows the \( \sigma_{-2} \) data (in \( \mu b/MeV \)) from the Dietrich and Berman compilation (circles) \(^{11}\), by integrating Eq. 2 between the \((\gamma, n)\) threshold and an upper limit of \( E_{\gamma_{\text{max}}} = 20–50 \) MeV. These integration limits include the GDR but do not take into consideration \( \sigma(\gamma, p) \) contributions and the rise of \( \sigma(E_{\gamma}) \) at around 140 MeV due to pion exchange currents \(^{12}\). Because of the \( 1/E_\gamma^2 \) factor, \( \sigma_{-2} \) is less sensitive to these high-energy contributions, which account for less than 10% of the total \( \sigma_{-2} \) value \(^{2, 12-14}\). This plot uses the mean value when several measurements were available for the same isotope and excluded data from natural samples unless one single isotope dominated the isotopic abundance.

These data follow a power-law,

\[
\sigma_{-2} = 2.4 \kappa A^{5/3} \mu b/MeV,
\]

with a RMS deviation of 30% for \( \kappa = 1 \). For \( A \geq 50 \), on excluding the \(^{58}\text{Ni}\) data point which has a large \( \sigma(\gamma, p) \) contribution \(^{13, 16}\), the agreement is even better, as shown in Fig. 1 with a RMS deviation of 6%. This formula agrees with the one published by Berman and Fultz in their 1975 review paper for \( A \geq 60 \): \( \sigma_{-2} = 2.39(20)A^{5/3} \mu b/MeV \) \(^{13}\). For \( A < 50 \), Fig. 1 presents large deviations from \( \kappa = 1 \) for \( A = 4n \), \( T_\gamma = 0 \) nuclei (\( \kappa < 1 \)) and loosely-bound light nuclei with \( A < 20 \) (\( \kappa > 1 \)). To emphasize this point, Fig. 2 shows a similar plot of the polarizability parameter \( \kappa \) vs. \( A \) by comparing Eq. 11 and the empirical \( \sigma_{-2} \) values \(^{11}\).

The missing \( \sigma(\gamma, p) \) contribution in the Dietrich and Berman compilation is the reason for the \( \kappa < 1 \) values observed \(^{11}\) for many \( A < 50 \) nuclei and \(^{58}\text{Ni}\). For heavier nuclei, neutron emission is the favorable decay mode due to the strong suppression of proton emission by the Coulomb barrier. Proton emission is, however, the predominant decay mode for \( A = 4n \) self-conjugate nuclei with \( A \lesssim 50 \) \(^{17, 18}\). For example, \( \sigma(\gamma, p) \approx 7 \times \sigma(\gamma, n) \) in \(^{40}\text{Ca}\) \(^{19}\). This is because of the isospin selection rule \( \Delta T = \pm 1 \) for \( E1 \) excitations in a \( T_\gamma = 0 \) self-conjugate nucleus \(^{4, 21}\). For a nucleus with a ground state of isospin \( T \), there is an isospin splitting of the GDR \(^{21}\) which corresponds to excited proton \((T + 1)\) and neutron \((T)\) resonances, with the \( T + 1 \) resonance generally lying at a higher excitation energy. The isospin selection rule \(^{21}\) favors the excitation of \( T + 1 \) states \(^{20}\), which predominantly decay by proton emission \(^{17, 23}\). Although the \( \sigma(\gamma, p) \) data are scarce, the \( \sigma_{-2} \) sum rule \(^{20}\) seems to be exhausted once the \( \sigma(\gamma, p) \) contributions are included \(^{17, 18, 27}\).

\(^3\)The total photoneutron cross section, \( \sigma(\gamma, n) \), for \(^{58}\text{Ni}\) is relatively small because of the \( \sigma(\gamma, p)/\sigma(\gamma, n) \) ratio is also controlled by the relative level densities in the residual nuclei, i.e., the ratio of the number of open channels, \( N_e/N_n \). For \(^{58}\text{Ni}\), \( N_e/N_n \approx \sigma(\gamma, p)/\sigma(\gamma, n) \approx 2 \) \(^{12}\).

\(^4\)\( \Delta T = \pm 1 \) isovector transitions are isospin forbidden as the Wigner coefficient \( \left( T_f \ 1 \ T_i \right) \) vanishes for \( T_\gamma = 0 \) \(^{20}\).

\(^5\)With only small admixtures \(^{22, 23}\), isospin is a good approximation in photonuclear reactions for light nuclei involving photons in the range of the electric dipole absorption \(^{24}\).

\(^6\)Most neutron emission from excited states with isospin \( T + 1 \) is forbidden, whereas neutron emission from excited states with isospin \( T \) is allowed \(^{25}\). These selection rules follow from the respective Clebsch-Gordan coefficients in the transition probabilities.
The larger GDR effect ($\kappa > 1$) observed in Fig. 2 for light nuclei with $A \lesssim 20$ may be explained from the mass dependence of the symmetry energy coefficient, $a_{\text{sym}}(A)$, of relevance to test 3N forces \cite{25} and describe neutron stars and supernova cores \cite{29,30}. As mentioned above, Migdal utilized a constant value of $a_{\text{sym}} = 23$ MeV to determine $\sigma_{-2}$ in Eq. 5. Nevertheless, the mass dependence of $a_{\text{sym}}(A)$ has long been established in the liquid droplet model \cite{31} and recognized as the fundamental parameter describing the GDR \cite{13}. Its form has since been refined, despite its current model dependency \cite{32}, with the advent of high-precision mass measurements.

From a global fit to the binding energies of isobaric nuclei with $A \geq 10$ \cite{32}, extracted from the 2012 atomic mass evaluation \cite{33}, Tian and co-workers determined $a_{\text{sym}}(A)$ as,

$$a_{\text{sym}}(A) = S_{\text{v}} \left( 1 - \frac{S_{\text{s}}}{S_{\text{v}} A^{1/3}} \right), \quad (12)$$

with $S_{\text{v}} \approx 28.32$ MeV being the bulk symmetry energy coefficient and $S_{\text{s}} / S_{\text{v}} \approx 1.27$ the surface-to-volume ratio. Within this approach, the extraction of $a_{\text{sym}}(A)$ only depends on the Coulomb energy term in the Bethe-Weizsäcker semi-empirical mass formula and shell effects \cite{32}, which are both included in Eq. 12 \cite{32}. Figure 3 illustrates the mass dependency of $a_{\text{sym}}(A)$ and clearly prevents the use of a constant $a_{\text{sym}}$ value.

\[\text{FIG. 3. (Color online) Symmetry energy coefficient, } a_{\text{sym}}(A), \text{ of finite nuclei as a function of mass number } A \text{ using Eq. 12} \text{.}\]

After introducing this mass dependence in Eqs. 10 and 11, $\alpha$ and $\sigma_{-2}$ are given by,

$$\alpha = \frac{1.8 \times 10^{-3} A^2}{A^{1/3} - 1.27} \text{ fm}^3, \quad (13)$$

$$\sigma_{-2} = \frac{1.8 A^2}{A^{1/3} - 1.27} \text{ mb/MeV}. \quad (14)$$

Equation 14 is plotted in Fig. 4 for $A \geq 10$ nuclides (solid line). Encouragingly, the increasing upbend observed as $A$ decreases provides an explanation for the large GDR effects observed in light nuclei. However, the validity of the hydrodynamic model remains to be tested for the lightest $A < 10$ nuclei.

More generally, Eq. 14 provides a means to evaluate nuclear polarizability without invoking a polarizability parameter. As shown in Fig. 4, most of the data points either fall below the predicted curve ($A < 70$) or merge with it where neutron emission is favorable ($A \geq 70$). These facts indicate that Eq. 14 could exhaust the $\sigma_{-2}$ sum rule for both photoneutron and photoproton cross sections and, hence, incorporate the actual GDR effect to the nuclear polarizability. Consequently, the mass-dependent $\sigma_{-2}$ curve may provide an estimate for the missing $\sigma(\gamma, p)$ contribution. For example, the predicted value of $\sigma_{-2}$ for $^{40}\text{Ca}$ is in agreement with the experimentally determined $\sigma(\gamma, p)/\sigma(\gamma, n)$ ratio. Additional experimental and theoretical work are needed to test the generality of these findings and evaluate deviations from the hydrodynamic model.

\[\text{FIG. 4. (Color online) The } ( -2 ) \text{ moment of the total photo-absorption cross section } \sigma_{-2} \text{ vs } A \text{ on a log-log scale using Eq. 14 (solid line). For comparison purposes, Eq. 11 (dashed line) and the data from the 1988 compilation are also plotted.}\]

In conclusion, a new empirical formula (Eq. 11) for the $( -2 )$ moment of the photo-absorption cross section, $\sigma_{-2}(A)$, has been determined from the latest photoneutron cross-section compilation with monoenergetic photons. The $\sigma_{-2}$ data include most of the photoneutron channels but excludes relevant $\sigma(\gamma, p)$ contributions for $A \lesssim 50$ nuclides. This new empirical formula presents a...
A RMS deviation of 6% for $A \gtrsim 50$ and is in better agreement with Migdal’s calculation of $\sigma_{-2}$ (Eq. 8) on combining the hydrodynamic model and second-order perturbation theory.

Additionally, $\sigma_{-2}$ has been inferred (Eq. [14] using a mass-dependent symmetry energy coefficient, $a_{\text{sym}}(A)$, determined by Tian and collaborators for $A \geq 10$ nuclei, which includes Coulomb energy and shell corrections. The resulting curve seems to account for the actual GDR effects as it exhausts the $\sigma_{-2}$ sum rule for most $A \geq 10$ nuclei in the Dietrich and Berman compilation. Moreover, it provides an explanation for the larger polarization effects found in light nuclei with $10 \lesssim A \lesssim 20$. Additional work is needed to test this new equation and evaluate deviations from the hydrodynamic model. It is encouraging, though, that the curve nicely merges with the $\sigma_{-2}$ data for $A \gtrsim 70$, in agreement with the dominant photoneutron cross sections for heavy nuclei.

Data compilations of currently available photoproton and photoneutron cross sections remain to be done. The $\sigma(\gamma, p)$ data are scarce compared to the $\sigma(\gamma, n)$ data and extensive work is desirable throughout the nuclear chart. These new data are crucial to test the $\sigma_{-2}$ sum rule and provide a means to remove the model dependency of $a_{\text{sym}}(A)$, which, in turn, may lead to a better understanding of 3N forces, neutron stars and supernova cores.

Furthermore, this work has direct implications in: 1) broadly used Coulomb-excitation codes such as GOSIA [30], where the polarization potential has to be modified with either Eq. [11] which requires a determination of $\kappa$ for $A \lesssim 50$ nuclei, or Eq. [14] once its generality has been fully tested; and 2) shell model calculations of $\kappa$ [37, 39]. To date, both approaches have broadly regarded Levinger’s empirical formula (Eq. [9]). For clarity purposes, these implications will be presented in a separate manuscript.

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