LOWER ESTIMATES ON EIGENVALUES OF QUANTUM GRAPHS

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Abstract. A method for estimating the spectral gap along with higher eigenvalues of quantum graphs has been introduced by Amini and Cohen-Steiner in [1] recently: it is based on a new transference principle between discrete and continuous models of a graph. We elaborate on it by developing a more general transference principle and by proposing alternative ways of applying it. To illustrate our findings, we present several spectral estimates on planar metric graphs that are oftentimes sharper than those obtained by isoperimetric inequalities and further previously known methods.

Mathematics subject classification (2010): 34B45 (05C50, 35P15, 81Q35).

Keywords and phrases: Spectral geometry of quantum graphs, planar graphs, double cover conjecture, normalized Laplacians.

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