Numerical study on penetration mode of steel with prefabricated double cracks

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ABSTRACT. When a flat steel with double cracks fails, the stress fields between different cracks will interact. The area between the cracks, that is, the steel bridge area, will be penetrated. This paper embeds the strain strength criterion into the discrete element numerical simulation method, and uses the block discrete element software UDEC to simulate the crack propagation and penetration in the steel bridge region of the prefabricated double-crack flat steel sample. Numerical simulation results show that there are four basic penetration modes in the double-crack steel sample during compression: (1) Discontinuous mode, which is characterized by no penetration between cracks, and the two pre-crack tip wing cracks independently expand; (2) Shear penetration mode, which is characterized by shear cracks penetrating the steel bridge. The principal stress field and shear stress field are concentrated in the steel bridge area, but the shear stress plays a leading role in penetration; (3) Tensile penetration mode, its characteristics: In order to penetrate the steel bridge with tensile cracks, the principal stress field is highly concentrated accumulated and nucleated in the area of the steel bridge, and the steel bridge penetration is instantaneous; The process occurs after the peak intensity.

1. Introduction
Numerical simulation can verify the accuracy and applicability of the mechanism in the study of the crack initiation and propagation process of steel cracks. To some extent, it can simulate the growth of sample cracks and the way of crack evolution in real working conditions, which can provide guidance and basis for theory and practice. The introduction of Goodman joint element [1] greatly promoted the application of the finite element method in the simulation of crack growth. However, continuous numerical methods are based on the assumption of continuity, so they are not effective in simulating large deformations such as crack propagation, and they cannot reflect the initiation and nucleation of micro-cracks during crack propagation. Therefore, non-continuous numerical methods have been developed. At present, the widely used non-continuous types of numerical value methods include manifold element method (NMM), discontinuous deformation method (DDA) and discrete element method (DEM) [2].

In the field of crack propagation, DEM is widely used. DEM allows discrete bodies to produce limited displacement and rotation, or even complete separation; it can automatically identify new contacts generated by the calculation process [3]. The main programs include the block discrete element program UDEC [4] and the particle discrete element program PFC [5]. Potyondy et al. [6]
first used the PFC model to simulate the crack initiation behavior. Zhang et al. [7-8] simulated the cracking, propagation, and penetration process of single-crack specimens and double-crack specimens under uniaxial compression. Manouchehrian [9] studied the influence mechanism of confining pressure on crack growth through biaxial tests with controlled lateral restraints. Domestic scholars such as Jiang Mingjing [10], Yang Qing [11], Zhang Sherong [12] used PFC to simulate the propagation and penetration mechanism of double crack specimens with different inclination angles.

UDEC discretizes the calculation area into blocks through a finite number of intersecting discontinuous surfaces, and each block is divided into many grids by finite difference or finite element methods to calculate the stress, strain and displacement of the block. [13]. The mechanical contact behavior between blocks is expressed as normal stiffness and tensile strength criterion in the normal direction, and tangential stiffness and shear strength criterion in the tangential direction. Yan [14] used Voronoi to randomly divide the block and studied the crack propagation path and penetration mode of the pre-cracked specimen. Lan [15] simulated the anisotropy and nonlinearity of the specimen under the action of microcrack propagation. Kemeny [16] used a time-dependent crack cohesion model to simulate the strength weakening characteristics of the specimen when it is broken. Jiang [17] introduced a virtual crack and used the stress characteristics of the grid around the crack to represent the stress state of the virtual crack to simulate the growth of the crack. Kazerani [18] used a collection of irregular triangular blocks with viscous boundaries to characterize the characteristics of the sample and simulate the characteristics of crack growth.

The traditional UDEC simulation cracking strength criterion mostly adopts the Griffith strength criterion, the maximum tangential stress criterion, the minimum strain energy density criterion, and the Mohr Coulomb strength criterion. However, with the deepening of theoretical research and the needs of practical engineering, the criterion of initiation of tensile-shear cracks needs to be further explored. This paper embeds the strain strength criterion into the block discrete element method, establishes a numerical model with prefabricated double cracks, uses the discrete element software UDEC to simulate the growth of flat steel cracks, and aims at the crack initiation, propagation, and penetration process, and analysis of changes in stress field, strain field, displacement field and strength characteristics.

2. Strain strength criterion

In the traditional numerical simulation of crack growth, Griffith strength criterion and Mohr Coulomb criterion are widely used. The Griffith strength criterion is the expansion of the Griffith energy criterion in compressive fracture. The criterion assumes that the crack starts from the point of maximum tensile stress concentration, and the direction of crack propagation is consistent with the direction of maximum compressive stress. It is suitable for the judgment of tensile cracks. The Morcoullen strength criterion believes that the main form of failure of steel is shear failure, and the angle between the direction of fracture and the direction of maximum compressive stress is $45^\circ + \pi/2$, which is suitable for the judgment of shear cracks. Although there are some criteria such as the stress criterion that can simultaneously judge the tensile-shear failure, according to the numerical results, it is found that the ratio of the initiation strength to the peak strength obtained by applying this criterion is only 30% of the test results, which is not consistent with the real situation.

In this paper, the strain strength criterion is used as the basis for judging the initiation of tensile-shear cracks in steel. This strain strength criterion assumes: (1) Tensile cracks grow in the direction of the maximum principal strain. When the principal strain reaches a critical value, the tensile cracks begin to grow; (2) Shear cracks grow in the direction of the most dangerous stress state. When the Mohr circle exceeds the Mohr Coulomb failure line, the shear crack starts to expand. The critical state expression is:

\[
\begin{align*}
\varepsilon_3 &= -\varepsilon_t \\
\tau &= c + \sigma \tan \phi
\end{align*}
\]
In order to facilitate the judgment of tensile cracks and shear cracks and their application to numerical simulations, a crack judgment parameter system related to the strain intensity criterion is established and two crack judgment factors are defined:

\[ f_t = \sigma_3 - \nu \sigma_1 + E \varepsilon_t \]
\[ f_s = \sigma_1 - \frac{1 + \sin \phi}{1 - \sin \phi} \sigma_3 - 2c \frac{1 + \sin \phi}{1 - \sin \phi} \]

Among them, \( c \) is the cohesive force, \( \phi \) is the internal friction angle, \( \varepsilon_t \) is the critical linear strain, \( \sigma_1 \) and \( \sigma_3 \) is the principal stress of the crack tip area, and the pressure is positive. \( f_t \) is called the tensile crack judgment factor. When \( f_t > 0 \), tensile cracks are generated inside the steel; \( f_s \) is called the shear crack judgment factor. When \( f_s > 0 \), shear cracks are generated inside the steel.

### 3. Numerical model

In this paper, the numerical simulation adopts the extended discrete element model based on the traditional discrete element method (DEM) for secondary development [17]. This model sets up virtual cracks for all potential cracking areas to provide paths for crack propagation. The virtual crack has high strength and can simulate the mechanical properties of steel, such as stress and deformation, together with the block. The cracking of the high-strength virtual crack in the extended discrete element model is not caused by the failure of its contact characteristics, but the equivalent stress state at the position of the virtual crack satisfies the strength criterion, and the virtual crack is assigned the material parameters of the real crack. With the replacement of material parameters, the overall strength of the specimen will be weakened, effectively simulating the nonlinear behavior of the new cracks during the cracking process. The size of the numerical model is 100 mm×40 mm, and a virtual crack is used to cut it. The layout of the virtual cracks (direction, location) and the size of the unit block (virtual crack length) have a great influence on the simulation effect. Considering the diversity of cracking directions and calculation efficiency, a hexagonal block with a side length of 1.5 mm is finally selected, as shown in Figure 1.

![Figure 1. Extended discrete element model (left) and schematic diagram of network division (right).](image)

### 4. Analysis of numerical results

The numerical samples are loaded with displacement boundaries. During the loading process, the average stress and average displacement of the loaded version are monitored, and statistical functions are compiled in Fish language and embedded in the UDEC program to record the number of tensile cracks and shear cracks. Multiple sets of numerical simulation tests are performed according to the above methods. The crack length \( 2a \) is 14mm, the steel bridge length \( 2b \) is 21mm, and the pre-crack friction angle \( \phi \) is 35°. When the crack angle \( \alpha \) is 30°, the steel bridge angle \( \beta \) is 30, 90, 120, 180; when the crack angle \( \alpha \) is 60, the steel bridge angle \( \beta \) is 60, 90, 120, 180; when the crack inclination angle \( \alpha \) is 45°, the steel bridge angle \( \beta \) is 45, 90, 120, 180, respectively. Analyzing the results of the numerical simulation, four basic failure modes of the steel bridge area are summarized:
(1) The discontinuous mode of the steel bridge area. (2) The tensile penetration mode of the steel bridge area is mainly tensile crack penetration. (3) The shear penetration mode is dominated by the shear crack penetration in the steel bridge area. (4) The tensile-shear mixed penetration mode dominated by the tensile-shear mixed crack penetration in the steel bridge area. According to the numerical simulation results, four typical samples are selected to analyze the four penetration modes in detail.

4.1. Discontinuous mode

Take the numerical simulation results of the 30°-30° specimen as an example to describe the fracture process and mechanical response characteristics of the discontinuous mode in the steel bridge region. Figure 2 shows the stress-strain curve and the tensile-shear crack development curve, and uses 0, 1, 2, and 3 to identify the four characteristic moments of initial state, crack initiation, crack propagation, and specimen failure. Figure 3 shows the evolution of the crack growth morphology corresponding to the four marking points. From this, we can summarize the basic laws of the discontinuous pattern:

1. The tip of the pre-crack starts to crack (marking point 1), and the wing crack starts to expand at the tip of each pre-crack. There is no obvious crack connection between the two groups of wing cracks.

2. Wing crack propagation (marking point 2), the slope of the stress-strain curve is reduced, this is because the new crack surface weakens the overall force characteristics of the specimen.

3. The specimen is damaged (marking point 3), the shear crack curve rises, and part of the shear fracture occurs at the end of the new crack. The concentrated stress fields of the two pre-cracks in the double-crack specimen interact, and the crack growth mode is disturbed. The wing crack growth rate of the pre-crack near the loading end (the crack on the right) is greater than that of the pre-crack far from the loading end (the left side). Crack of the wing crack growth rate.

Figure 2. The stress-strain curve of 30°-30° specimen and the development curve of tensile/shear cracks.

Figure 3. Evolution of crack growth morphology of 30°-30° specimen.

Figure 4 and Figure 5 show the horizontal displacement field cloud diagram, principal stress field cloud diagram, and tensile stress distribution diagram of the whole process of specimen failure to
analyze the mechanism of discontinuity in the steel bridge area. It can be seen from the cloud diagram of the horizontal displacement field in Fig. 4 that at the time of crack initiation, there are two displacement areas (marking point 1) with the line connecting the tip of the pre-crack as the anti-symmetric axis on both sides of the steel bridge. This indicates that there is a tendency of sliding on both sides of the crack line in the steel bridge area. However, with the expansion of the wing crack (marking point 2), the antisymmetric axis of the displacement field gradually does not coincide with the connecting line of the steel bridge, and the horizontal displacement field of the steel bridge area gradually merges into one (marking point 3). The continuous propagation of the crack tip wing crack changes the distribution characteristics of the specimen displacement field, and to a certain extent inhibits the shear cracks in the steel bridge area through slipping. This is the main reason why there is no shear penetration in the steel bridge area.

Analyzing the principal stress field cloud diagram and the tensile stress distribution diagram in Figure 5, it can be found that in the steel bridge area, although the stress field between the two cracks interferes with each other, the steel bridge area is partially tensioned, but its density is not high. As the wing cracks crack, the steel bridge area becomes compressed. The wing crack propagates in the direction of the maximum compressive stress. When the angle of the steel bridge is small, the propagating direction of the wing crack differs greatly from the angle of the steel bridge. This is the main reason why the steel bridge area does not undergo tensile penetration.

![Figure 4](image)

**Figure 4.** Cloud diagram of the horizontal displacement field of the 30°-30° sample.

![Figure 5](image)

**Figure 5.** 30°-30° sample principal stress field cloud diagram (top) and tensile stress distribution evolution diagram (bottom).
4.2. Cut through mode

Taking the numerical simulation results of 60°-60° specimens as an example, the fracture process of the shear through mode of the steel bridge region and the macro- and micro-mechanical properties of the specimens are described. Figure 6 shows the stress-strain curve and tensile-shear crack development curve during the numerical simulation, and uses 0, 1, 2 and 3 to identify the four characteristic moments of initial state, crack initiation, steel bridge penetration, and sample failure. Figure 7 shows the evolution of the crack growth morphology corresponding to the four marking points. From this, the basic law of shear crack penetration mode can be summarized:

(1) The tip of the pre-crack is cracked (marking point 1), and the growth rate of the new crack at the inner tip of the pre-crack is greater than that of the new crack at the outer tip. Moreover, the new tensile-shear cracks propagate almost simultaneously.

(2) When the steel bridge penetrates (marking point 2), the shear cracks at the tip of the prefabricated crack are quickly connected, and the shear cracks in the steel bridge area penetrate. At this time, the new cracks at the outer tip of the pre-crack basically did not extend.

(3) After the steel bridge is penetrated, the strength of the sample continues to increase, the outer tip of the pre-crack expands, and the crack propagation shape is close to the wing crack mode. After that, the sample stress reaches the peak strength (marking point 3). The penetration stress is relatively close to the peak strength.

![Figure 6](image)

**Figure 6.** 60°-60° sample stress-strain curve and tensile/shear crack development curve.

![Figure 7](image)

**Figure 7.** Evolution of crack growth morphology of 60°-60° specimen.

Figures 8, 9, and 10 show the horizontal displacement field cloud diagram, principal stress field cloud diagram, tensile stress distribution diagram and shear strain field cloud diagram of the whole process of specimen failure to analyze the mechanism of shear penetration in the steel bridge area. It can be seen from the cloud diagram of the horizontal displacement field that, at the beginning of the test, the anti-symmetric axis of the horizontal displacement field in the steel bridge area is a broken line, which does not coincide with the line of the inner tip of the preform crack. However, when a shear crack occurs at the tip of the crack (marking point 1), the anti-symmetric axis gradually evolves into an inclined straight line and overlaps the line of the steel bridge. The two sides of the steel bridge
slip, which intensifies the propagation of the shear crack, leading Shear through phenomenon (marking points 2, 3).

Analyze the cloud diagram of the principal stress field and the shear stress cloud diagram: After the sample is cracked (marking point 1), the main stress and shear stress are concentrated in the steel bridge area due to the interference of the stress fields between the cracks. As the crack propagates, the main stress concentration gradually weakens and the shear stress concentration gradually increases (marking point 2), and finally, the tangential force takes a dominant position. Under the action of shear stress, the steel bridge slipped and failed on both sides, resulting in a shear crack penetration mode (marking point 3). Although there is a concentration of principal stress field in the steel bridge area, its concentration is smaller than that of the shear stress field, and tensile cracks are not enough to penetrate the steel bridge area.

Figure 8. Cloud diagram of the horizontal displacement field of the 60°-60° sample.

Figure 9. The main stress field cloud diagram of 60°-60° specimen (top) and the evolution diagram of tensile stress distribution (bottom).

Figure 10. The cloud diagram of the shear stress field of the 60°-60° sample.
4.3. Stretch through mode

Taking the numerical simulation results of 60°-90° specimens as an example, the fracture process of the tensile through mode in the steel bridge zone and the macro- and micro-mechanical properties of the specimens are described. Figure 11 shows the stress-strain curve and tensile-shear crack development curve of the sample during the numerical simulation process, and use 0, 1, 2 and 3 to identify the initial state, crack initiation, near penetration, steel bridge penetration, and sample failure. Feature moments. Figure 12 shows the evolution of the crack growth morphology corresponding to the four marking points. From this, the basic law of tensile crack penetration can be summarized:

(1) The pre-crack tip initiates cracking (marking point 1), which is different from the shear through mode. The growth rate of the new crack at the inner tip of the pre-crack is slower than the outer tip. The new cracks are mainly tensile cracks, and the shear cracks develop slowly.

(2) When near penetration (marking point 2), the length of the tensile crack in the steel bridge region is relatively shorter than the outer tip of the pre-crack.

(3) When the steel bridge penetrates (marking point 3), the tensile crack at the tip of the prefabricated crack buckles and expands, and it penetrates the entire steel bridge area instantly. At the same time, the sample reaches its peak strength and the stress-strain curve drops sharply. At this time, the wing crack at the outer tip of the pre-crack expanded, and a shear crack appeared.

Figure 13 shows the principal stress field cloud diagram and the evolution process of tensile stress distribution during the specimen fracture process to analyze the mechanism of tensile penetration and instantaneous failure of the double-crack specimen steel bridge. At the initial stage of loading, stress concentration occurred at the tip of the pre-crack (marking point 1). When approaching penetration (marking point 2), the distribution range of tensile stress and the concentration of principal stress have both increased compared to before. The high stress field in the steel bridge area is the first to merge. Then the steel bridge suddenly penetrated (marking point 3), the stress was released, and the cracks expanded unsteadily. When the peak strength is reached, the main stress concentration in the steel bridge area is very small. This is because the macroscopic cracks propagate and the specimen ruptures into the left and right sides, which bear the upper load respectively, and the steel bridge area forms a large-scale compressive stress area. In summary, the continuous accumulation of tensile stress during the crack propagation process is the accumulating stage of the tensile crack penetration and the main reason for the sudden occurrence of the penetration process.

Figure 11. 60°-90° sample stress and strain and tensile/shear crack development curve.
4.4. Pull-cut mixed through mode

Taking the numerical simulation results of the 45°-120° specimen as an example, the fracture process of the tensile-shear mixed penetration mode in the steel bridge region and the macro- and micro-mechanical properties of the specimen are described. Figure 14 shows the stress-strain curve and tensile-shear crack development curve of the sample during the numerical simulation process, and use 0, 1, 2 and 3 to identify the four characteristic moments of initial state, crack initiation, steel bridge penetration, and sample failure. The marking point 2 is after the marking point 3, indicating that the penetration phenomenon occurs after the sample reaches the peak intensity. Figure 15 shows the evolution of the crack growth morphology corresponding to the four marking points. From this, the basic law of tensile shear crack penetration can be summarized:

(1) The tip of the pre-crack starts to crack (marking point 1). The growth rate of the new cracks at both ends of the upper preform crack is faster, and the growth rate of the new cracks on both sides of the lower preform crack is slow. This is because the lower end of the sample is fixed and the upper end is loaded, the lower crack is basically in the vertical projection area of the upper crack, and the crack propagation is suppressed. The new cracks are mainly tensile cracks, and the shear cracks develop slowly.

(2) At the moment of peak strength (marking point 3), the new crack is about to penetrate the steel bridge, and the stress-strain curve drops sharply.

(3) When the steel bridge penetrates (marking point 2), an anti-wing crack with shear properties is formed at the outer tip of the pre-crack and the opposite side of the wing crack propagation direction.
The tensile crack and the anti-wing crack are connected to each other through the steel bridge area. The penetration of the steel bridge zone occurs after the sample reaches its peak strength.

Figure 14. 45°-120° sample stress and strain and tensile/shear crack development curve.

Figure 15. The evolution of the crack growth morphology of the 45°-120° sample.

Figures 16 and 17 show the main stress field cloud diagram, the tensile stress distribution change and the shear stress cloud diagram of the specimen fracture process to analyze the mechanism of the tensile-shear mixed penetration of the double-crack specimen steel bridge. At the initial stage of loading, the principal stress at the crack tip is concentrated, and the tensile stresses in the steel bridge region interfere with each other. However, the crack producing area of the anti-wing at the crack tip is always the compression zone (marking point 1). At the moment when the peak strength is reached (marking point 3), the upper area of the outer tip of the lower crack is still under pressure, so the crack arrest occurs when the tensile crack grows, and the stress-strain curve drops sharply. At the moment when the steel bridge penetrates (marking point 2), the wing crack at the upper tip of the lower preform crack propagates rapidly. Observing the shear strain cloud diagram, there is a concentration of shear stress at the crack initiation position of the anti-wing crack. The tensile cracks caused by the concentration of tensile stress and the shear cracks caused by the concentration of shear stress penetrate the steel bridge.
5. Conclusions
In this paper, the strain strength criterion is embedded in the second-developed extended discrete element model. The block discrete element program UDEC is used to simulate the extension of tensile cracks and shear cracks of prefabricated double-crack specimens, revealing the microscopic view of the crack interaction. Failure mechanism and macro-mechanical response. The main conclusions are as follows:

1) Numerical experiments have found that the basic penetration modes of the sample steel bridge with prefabricated double cracks are mainly divided into the following four categories: non-penetration mode, shear penetration mode, tensile penetration mode, and tension-shear mixed penetration mode.

2) The characteristic of the discontinuous mode is that the crack does not penetrate in the steel bridge area, the wing cracks generated at the tip of the prefabricated crack propagate independently, the horizontal displacement field of the steel bridge area merges, and the high strain localization is not formed in the steel bridge area.

3) The shear penetration mode is characterized by the shear cracks penetrating the steel bridge. The horizontal displacement field in the steel bridge area overlaps the anti-symmetric axis with the steel bridge connection line. The main stress field and the shear stress field concentration appears at the same time in the steel bridge area, but the shear stress is the steel bridge penetration plays a leading role.

4) Tensile penetration mode is characterized in that the tensile crack penetrating the steel bridge, the principal stress field at the crack tip is concentrated, and the high stress field is the first to fuse before the steel bridge penetrates, and the tensile crack penetrating the steel bridge is instantaneous.

5) The tensile-shear mixed penetration mode is characterized by the tensile cracks and the anti-wing cracks interconnecting through the steel bridge area. The new cracks are mainly tensile cracks, and the shear cracks develop slowly. The penetration process occurs after the specimen is destroyed.

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