Optimization of FGM Composition for Better Environment Material

Mrinal Gautam¹ and Manish Chaturvedi¹

¹University Department, Mechanical Engineering, Rajasthan Technical University, Akelgarh, Kota, Rajasthan, 324010, India

E-mail: mrinal.phd18@rtu.ac.in

Abstract. In functionally graded heat-resisting material, the volume fraction of constituents varies functionally and continuously, and these are examined for high temperature engineering applications. By the spatial distribution of the volume fraction the thermal behaviour of FGMs is strongly influenced. Thus, the determination of volume fraction distribution becomes a critical part in the FGM design for a given loading condition and specification. In present work, averaging estimation approach viz, modified rule of mixture is considered. The present work is concerned with the Optimization of FGM Composition for better environment material. The investigation is carried out for uniform heating condition through the plate thickness. Classical laminate theory is used to determine residual stress. The code is developed on MATLAB for constrained minimization and the MATLAB function ‘fmincon’ is employed. The comparative study of three different thermal loading is presented. The effectiveness of residual stress minimization is explored for various boundary conditions.

1. Introduction

A novel set of materials named Functionally Graded Materials (FGMs) have newly been proposed that different material properties vary through the thickness and radial direction in a continuous behavior and, therefore free from interface weakness. Residual stresses, thermal stresses and stress concentrations are reduced by gradation of material properties. A functionally graded structure is described as, which to obtain a require function, two or more materials volume fractions are vary continuously as a function of location beside definite dimension (classically the thickness and radius) of the structure. Designers can get exceptional performance as well as modified response by the FGMs in thermal environment. In FGM the metal constituents provide stronger mechanical strength and reduce the risk of catastrophic fracture, even as in the ceramic constituents are resisting high-temperature environments for better thermal resistance characteristics. An Example, when Space Shuttle is entered into the Earth’s environment, ceramic tiles applies while thermal protection as of heat generated. An FGM composed of metal on the inside surface and ceramic on the outside surface Structure. Classify the FGMs into; (a) stepwise-graded materials, as well as (b) continuously graded. In case of stepwise-graded materials, give rise to a multi-layered form by an interface there between the adjacent discrete layers. In case of continuously graded form, the transform in microstructure, composition, and thus in the properties take places with position. [1, 2]

An extensive literature review is presented detailing the state of the art in functionally graded material. Birman and Byrd [3] very recently recognized a comprehensive FGM research covering comprehensively the period 1997-2007. Na and Kim [4] proposed by considering stress reduction and developing behavior of thermo-mechanical buckling to optimize the volume fraction of FGM composite panels. Sadollah and Bahreininejad [5] employed multiobjective optimum design, for functionally graded dental implant by simulated annealing and genetic algorithms. Kou.et.al [6] performed by using a particle swarm optimization and procedural model of functionally graded
materials. Noh et al. [7] has presented to optimize volume fraction distribution of Reliability-based design in functionally graded composites in optimal design. Ashjari and Khoshavan [8] performed Volume Fraction optimization in FGM plates, with two evolutionary methods: (a) particle swarm optimization method (b) Real-coded genetic method, and. Taheri et al. [9] proposed by an isogeometrical process for thermo-elastic optimization of material distribution of functionally graded structures, explaining the optimization complication by mathematical programming. Roque and Martins [10] developed the differential evolution to explain difficulties of natural frequencies to maximization, during optimization of functionally graded beams. Shi and Shemado [11] to optimize the interface shape, of functionally graded sandwich structures for designing. Mohammadiha and Ghariblu [12] presented two process; (a) geometrical average and (b) multidesign objective methods, for functionally graded foam to optimize crush behavior by multi tubes filled. Shabana et al. [13] presented the technique particle swarm optimization, in minimization the stresses in functionally graded cylinders under a pressure loading. Lieu and Lee [14] explained iso-geometric analysis, and adaptive hybrid evolutionary firefly method for modeling as well as optimization of functionally graded plates subjected to thermo-mechanical loading. Nguyen and Lee [15] analyzed lateral and, flexural-torsional buckling problems to optimize of thin-walled functionally graded beams. He and Sun [16] presented functionally graded material beam to optimize multi objective structural acoustic. Correia et al. [17] developed a method of multi objective optimization for ceramic-metal functionally graded plates by higher order form. Cheng et al. [18] employed the design of additive manufactured components, by stress constraints for topology optimization of functionally graded lattice structure. The objective of this work is to present the development of Optimization of FGM Composition for better environment material.

2. Formulation of FGM model

To study of FGMs, a model must be created that illustrates the function of composition throughout the material. In Figure 1, the volume fraction, $V_c$, describes the volume of ceramic at any point $z$ throughout the thickness $h$ according to a parameter $n$ which controls the shape of the function (as seen in Figure 1). $V_c$ is specified by

$$V_c(z) = \left( \frac{z}{h} + \frac{1}{2} \right)^n$$

(1)

![Figure1: Ceramic volume fractions across the FGM Layer](image)
3. Temperature profile and heat conduction modelling

In present work, thermal loading are considered i.e. uniform heating or cooling of FGM. The estimation of the thermal residual stresses through the thickness is discussed. Temperature is assumed to be uniform through the plate. The thermal residual stresses at any position through the thickness $z$, $\sigma$ are calculated using classical laminate theory [19].

$$\{\sigma\} = \left[\begin{array}{c} \sigma_{11} \\ \sigma_{22} \end{array}\right] = \left[\begin{array}{c} \epsilon_{11} \\ \epsilon_{22} \end{array}\right] - \{\Delta T\}$$

(2)

4. Problem definitions

Figure 3 shows a simply supported plate-like FGM at the uniform initial temperature, $T_0 = 300$ K, where boundary conditions and geometry dimensions are specified. For temperature loading, uniformly decrease or increase their temperature. The volume fraction in the graded layer varies through the thickness, in the $x$-direction as invariant. In this problem, a dominant thermal stress component is become by axial stress $\sigma_{xx}$. With initial volume fraction $(V_m)_i = 0.5$, we divide the graded layer into 8 uniform homogeneous sub-layers.

| Material Property               | Ni      | Al$_2$O$_3$ |
|---------------------------------|---------|-------------|
| Young’s modulus, $E$ (GPa)      | 199.5   | 393         |
| Poisson’s ratio, $\nu$         | 0.3     | 0.25        |
| Thermal expansion coefficient, $a$ ($10^{-6}$K) | 15.4    | 7.4         |
| Thermal conductivity, $k$ ($W/m\cdot K$) | 90.7    | 30.1        |
5. Residual Stress Minimization

5.1 Uniform Heating (or Cooling)

In this work, the design region is divided into 10 uniform homogeneous sub layers through initial volume fraction \( V/g2923(i) = 0.5 \). The total thickness of plate is 10 mm. The upper layer is pure ceramic and the lower layer is pure metal. The 8 middle layers are the FGM. Volume fraction in the FGM plate is being optimized towards the minimization of residual stresses (\( \sigma \)). Volume fraction optimization for minimizing thermal stress is assured as

$$\text{Find: } V/g2923(i), \quad i=1,2,...,n$$

$$\text{Minimize: } f = \max (\sigma_i)$$

$$\text{Subjected to: } \max (\sigma_i) - \sigma_{\text{per}} \leq 0$$

$$0 \leq V/g2923(i) \leq 1,$$

5.1.1 Validation of model

Considering modified rule of mixture to the estimation of the properties, the values of optimized metal volume fractions for the sub layers are presented in Table 2.

Table 2: Optimized Volume Fraction in all eight layers for Uniform heating (\( \Delta T = 200^\circ C \)), (using Modified rule of mixture)

| Layers | Volume Fraction of Metal, \( V_m \) |
|--------|----------------------------------|
| V1     | 0.8964                           |
| V2     | 0.7974                           |
| V3     | 0.7224                           |
| V4     | 0.5889                           |
| V5     | 0.4811                           |
| V6     | 0.3692                           |
| V7     | 0.2522                           |
| V8     | 0.1298                           |

The Table 2 shows the continuous volume fraction distributions, linearly interpolated through the centre points of sub layer volume fraction and the end points of Ni and Al2O3 – layer volume fractions. The optimized volume fraction, even as it varies almost linearly, presents steep gradient near Al2O3 – graded layer interface. It can also noted that the initial guess approaches 1 at Ni – graded layer interface and 0 at Al2O3 – graded layer interface. These results closely match with that of Cho [20].
5.1.2 Modified Rule of Mixture
The modified rule of mixture is considered for estimating the Young’s modulus and other properties in different sub-layers.

Table 3: Optimized volume fraction in all eight layers for uniform heating (using Modified Rule of Mixture)

| Layers | Volume Fraction of Metal, $V_m$ for $\Delta T = 200^\circ C$ | $\Delta T = 300^\circ C$ | $\Delta T = 400^\circ C$ |
|--------|---------------------------------------------------------------|--------------------------|--------------------------|
| $V_1$  | 0.8964                                                        | 0.8195                   | 0.8957                   |
| $V_2$  | 0.7974                                                        | 0.7232                   | 0.8247                   |
| $V_3$  | 0.7224                                                        | 0.7582                   | 0.6949                   |
| $V_4$  | 0.5889                                                        | 0.5247                   | 0.5902                   |
| $V_5$  | 0.4811                                                        | 0.4379                   | 0.4820                   |
| $V_6$  | 0.3692                                                        | 0.3976                   | 0.3698                   |
| $V_7$  | 0.2522                                                        | 0.2032                   | 0.2529                   |
| $V_8$  | 0.1298                                                        | 0.1007                   | 0.1301                   |

The results are presented in Table 3 and Figure 4. As seen from the Figure, the composition profile deviates from the linear. The profile also depends on the temperature difference to some extent.

![Figure 4: Distribution of Optimized volume fraction across the thickness for uniform heating (using Modified Rule of Mixture)](image)

The residual stress distribution obtained for estimation approach considered in the present work. For modified rule of mixture, the residual stress shows fluctuations across the thickness but maximum value is not very high.

6. CONCLUSION
In present work, the optimization of volume fraction is carried out of thermal loading i.e. uniform heating. The residual stresses are considered as objective functions and estimation approach viz modified rule of mixture is explored. For modified rule of mixture, temperature change has significant effect on volume fraction distribution particularly that obtained with curvature minimization. From
overall comparison, it can be said that modified rule of mixture estimation with residual minimization gives the best performance. In the above study a material considered is homogeneous having sufficient degree of thermal conductivity. However the analysis is presented in research article is not verified for heterogeneous material or for non thermal non conductive materials.

References

[1] Kayikci R, Sava S (2014), “Fabrication and properties of functionally graded Al/AlB2 composites”. J Compos Mater, 49(16):2029–2037. (https://doi.org/10.1177/0021998314541490)

[2] Mahamood RM, Akinlabi ET, Shukla M, Pityana S (2012), “Functionally graded material: an Overview”. In: Proceedings of the world congress on engineering, VIII, London (http://hdl.handle.net/10204/6548)

[3] Birman V, Byrd LW. (2007), “Modeling and analysis of functionally graded materials and structures”, ApplMech Rev; 60:195–216 (DOI: 10.1115/1.2777164)

[4] Na, K.S., Kim, J.H (2009),”Volume fraction optimization of functionally graded composite panels for stress reduction and critical temperature”, Finite Elements in Analysis and Design; 45 (11): 845-851. (https://doi.org/10.1016/j.finel.2009.06.023)

[5] Sadollah A, Bahreininejad A. (2011), “Optimum gradient material for a functionally graded dental implant using metaheuristic algorithms”. J Mech Behav Biomed Mater; 4(7):1384–95. (DOI: 10.1016/j.jmbbm.2011.05.009)

[6] Kou, X.Y., Parks, G.T., Tan, S.T. (2012), “Optimal design of functionally graded materials using a procedural model and a particle swarm optimization”, Computer-Aided Design; 44 (4): 300-310. (https://doi.org/10.1016/j.cad.2011.10.007)

[7] Noh, Y.J., Kang, Y.J., Youn, S.J., Cho, J.R., Lim, O.K.. (2013), “Reliability-based design optimization of volume fraction distribution in functionally graded composites” Computational Material Science; 69: 435-442. (https://doi.org/10.1016/j.commatsci.2012.12.003)

[8] Ashjari, M., Khoshravan, M.R. (2014), “Mass optimization of functionally graded plate for mechanical loading in the presence of deflection and stress constraints”, Composite Structures; 110: 118-132. (DOI: 10.1016/j.compstruct.2013.11.025)

[9] Taheri, A.H., Hassani, B., Moghaddam, N.Z.(2014), “Thermo-elastic optimization of distribution of functionally graded structures by an isogeometrical approach”, International J. f.solids and Structures; 51: 416-429. (https://doi.org/10.1016/j.ijsolstr.2013.10.014)

[10] Roque, C.M.C., Martins, P.A.L.S. (2015), “Differential evolution for optimization of
functionally graded beams”, *Composite Structures*; **133**:1191-1197.

(10.1016/j.compstruct.2015.08.041)

[11] Shi JX, Shimoda M. (2015), “Interface shape optimization of designing functionally graded sandwich structures”. *Compos Struct,* **125**:88–95.  
(https://doi.org/10.1016/j.compstruct.2015.01.045)

[12] Mohammadiha O, Ghariblu H. (2016), “Crush behavior optimization of multi-tubes filled by functionally graded foam”. *Thin-Walled Struct,* **98**:627–39.
(https://doi.org/10.1016/j.tws.2015.10.025)

[13] Shabana, Y.M., Elsawaf, A., Khalaf, H., Khalil, Y. (2017), “Stresses minimization in functionally graded cylinders using particle swarm optimization technique”, *Int. J. of Pressure Vessels and Piping,* **154**, 1-10.  
(https://doi.org/10.1016/j.ijpvp.2017.05.013)

[14] Lieu, Q.X., Lee, J. (2017), “Modeling and optimization of functionally graded plates under thermo-mechanical load using isogeometric analysis and adaptive hybrid evolutionary firefly algorithm”, *Composite Structures,* **179**:89-106.  
(https://doi.org/10.1016/j.compstruct.2017.07.016)

[15] Nguyen TT, Lee J. (2017), “Optimal design of thin-walled functionally graded beams for buckling problems”. *Compos Struct,* **179**: 459–67.
(https://doi.org/10.1016/j.compstruct.2017.07.024)

[16] He MX, Sun JQ. (2018) “Multi-objective structural-acoustic optimization of beams made of functionally graded materials”. *Compos Struct,* **185**:221–8
(https://doi.org/10.1016/j.compstruct.2017.11.004)

[17] Franco Correia VM, Aguilar Madeira JF, Araújo AL, Mota Soares CM. (2018), “Multiobjective optimization of ceramic-metal functionally graded plates using a higher order model”. *Composite Structures,* **183**: 146-160.
(https://doi.org/10.1016/j.compstruct.2017.02.013)

[18] Cheng L, Bai J, To AC. (2019), “Functionally graded lattice structure topology optimization for the design of additive manufactured components with stress constraints”. *Comput Methods Appl Mech Eng,* **344**:334–59.
(https://doi.org/10.1016/j.cma.2018.10.010)

[19] Shaw, L.L. (1998). “Thermal Residual Stresses in Plates and Coatings Composed of Multi Layered and Functionally Graded Materials”, *Composites Part B,* **29**: 199–210.
(https://doi.org/10.1016/S1359-8368 (97)00029-2)

[20] Cho JR, Ha DY (2002). “Volume fraction optimization for minimizing thermal stress in Ni-Al2O3 functionally graded materials”. *Mater Sci Eng A;* **334**:147–55.
(https://doi.org/10.1016/S0921-5093 (01)01791-9)