A Two-Loop Test of M(atrix) Theory

Katrin Becker♦

Institute for Theoretical Physics, University of California
Santa Barbara, CA 93106-4030

and

Melanie Becker⋆

Department of Physics, University of California
Santa Barbara, CA 93106-9530

We consider the scattering of two Dirichlet zero-branes in M(atrix) theory. Using the formulation of M(atrix) theory in terms of ten-dimensional super Yang-Mills theory dimensionally reduced to (0 + 1)-dimensions, we obtain the effective (velocity dependent) potential describing these particles. At one-loop we obtain the well known result for the leading order of the effective potential $V_{eff} \sim v^4/r^7$, where $v$ and $r$ are the relative velocity and distance between the two zero-branes respectively. A calculation of the effective potential at two-loops shows that no renormalizations of the $v^4$-term of the effective potential occur at this order.

May, 1997

♦ beckerk@itp.ucsb.edu

⋆ mbecker@denali.physics.ucsb.edu
1. Introduction

M-theory [1] is our strongest candidate to be a consistent quantum theory in eleven dimensions that includes gravity. At low energies and large distances M-theory is described by eleven-dimensional supergravity. However, very little was known about the degrees of freedom which describe its short distance behavior until last year, when Banks, Fischler, Shenker and Susskind [2] proposed that M-theory in the infinite momentum frame is described in terms of a supersymmetric matrix model. Furthermore, the only dynamical degrees of freedom or partons are Dirichlet zero-branes, so that the calculation of any physical quantity of M-theory can be reduced to a calculation in the matrix model quantum mechanics.

The quantum mechanical system describing these D0 branes was studied in connection with the eleven-dimensional supermembrane in [3] and [4] and in relation to short distance properties of D0 branes in [5] and [6]. A system of $N$ D0 branes is described in terms of nine $N \times N$ matrices $X_{a,b}^i$, $i = 1, \ldots, 9$ together with their sixteen fermionic superpartners $\psi$, which transform as spinors under the $SO(9)$ group of transverse rotations [7]. More concretely, the action describing this system can be regarded as ten-dimensional super Yang-Mills theory dimensionally reduced to $(0 + 1)$ space-time dimensions [3][5][6]:

$$S = \frac{1}{g} \int dt Tr \left( -D_t X^i D_t X_i + \frac{1}{2} [X_i, X_j]^2 + \text{(fermi)} \right).$$

(1.1)

This quantum mechanical problem has a $U(N)$ symmetry. While in the original formulation of [2] the large $N$ limit was implicit in the conjectured correspondence between M(atrix) theory and M-theory, a more recent formulation of the conjecture due to Susskind [8] is valid for finite $N$. The new conjecture states that the discrete light cone quantization of M-theory is exactly described by a $U(N)$ matrix theory. This shall be the framework we are interested in.

So far, the correspondence between M(atrix) theory and M-theory has been tested comparing scattering amplitudes of different p-branes with those of eleven-dimensional supergravity [9] [10] [11]. In all cases it was found a precise agreement for the long distance behavior of the potential between the branes. However, these computations were only at one-loop in the gauge coupling constant and it is rather possible that the correspondence
between M(atrix) theory and M-theory is spoiled by higher loop effects. It is the purpose of this paper to show that this correspondence is correct even at two loops!

We will be interested in the effective potential for two D0 branes. In \cite{2} it was suggested that terms with four space-time derivatives in the effective potential computed in M(atrix) theory should not be renormalized beyond one-loop for the correspondence with eleven-dimensional supergravity to be correct. Last week Dine and Seiberg \cite{12} found that this non-renormalization theorem is violated in similar three-dimensional theories by instanton effects in the spirit of \cite{13} and it was argued that possible corrections by perturbative loop effects may further violate the non-renormalization theorem. However we will find that two-loop effects do not violate the non-renormalization theorem of \cite{2} for this one-dimensional theory.

In section 2 we will introduce the background field method \cite{11} \cite{9} \cite{14}, which we will be using and give the explicit form of the dimensionally reduced gauge fixed super Yang-Mills action. In section 3 we derive the Feynman rules for bosonic and fermionic fields. The derivation of the one-loop effective action is presented in section 4. This can be easily derived from the results of \cite{11} and \cite{9} and we find agreement with eleven-dimensional supergravity. In section 5 we compute the two-loop effective action and show that the \( v^4 \) term is not renormalized at this order in perturbation theory. Our comments and conclusions are given in section 6.

2. Super Yang-Mills Action in (0+1) Dimensions

To compute the effective action for two zero-branes it will be convenient to work with the background field method as in \cite{11} and \cite{9}, since the explicit gauge invariance of the classical theory will not get lost when quantum corrections are included. A good introduction to the subject can be found in \cite{14}. Choosing units where \( 2\pi\alpha' = 1 \), the (0+1)-dimensional gauge theory obtained by dimensional reduction of the ten-dimensional \( N = 1 \) supersymmetric gauge theory is after gauge fixing\footnote{1 We will be using the conventions of \cite{3}.}

\[
S = \int dt \left( \frac{1}{2g} \text{Tr} F_{\mu\nu} F^{\mu\nu} - i \text{Tr} \bar{\psi} i D\psi + \frac{1}{g} \text{Tr} (\bar{D}^\mu A_\mu)^2 \right) + S_{\text{ghost}},
\]

(2.1)
Here $\psi$ is a real adjoint sixteen component fermion, $A_\mu$ is a $U(2)$ gauge field, $S_{\text{ghost}}$ is the ghost action whose explicit form will be written down later on and $\mu, \nu = 0, \ldots, 9$. We will be using the background field gauge condition

$$\bar{D}^\mu A_\mu = \partial^\mu A_\mu + [B^\mu, A_\mu], \quad (2.2)$$

where $B^\mu$ is the background field.

The action (1.1) can be obtained defining (2.3):

$$F_{0i} = \partial_t X_i + [A, X_i],$$
$$F_{ij} = [X_i, X_j],$$
$$D_t \psi = \partial_t \psi + [A, \psi],$$
$$D_i \psi = [X_i, \psi]. \quad (2.3)$$

Here $i = 1, \ldots, 9$, labels the bosonic fields $X_i$ and $A$ is the zero component of the gauge field appearing in (2.1).

We can now expand the action (2.1) around the classical background field $B^i$ by setting $X^i = B^i + \sqrt{g}Y^i$. We will choose $B_0 = 0$ and $B^i$ to satisfy the equations of motion.

The action is a sum of four terms

$$S = S_Y + S_A + S_{\text{fermi}} + S_{\text{ghost}}, \quad (2.4)$$

In Minkowski space the action for the fluctuations $Y_i$ involves cubic and quartic interactions

$$S_Y = \int dt \text{Tr} \left( - (\partial_t Y_i)^2 + [B_i, B_j][Y^i, Y^j] + [B_i, Y_j][Y^i, B^j] + [Y_i, B_j]^2 + [B_i, Y^i]^2 + 2\sqrt{g}[B_i, Y_j][Y^i, Y^j] + \frac{g}{2}[Y^i, Y_j]^2 \right). \quad (2.5)$$

Furthermore, the action for the gauge field $A$ involves interactions with derivatives as well as cubic and quartic interactions

$$S_A = \int dt \text{Tr} \left( (\partial_t A)^2 - 4\partial_t B_i[A, Y^i] - [A, B_i]^2 - 2\sqrt{g}\partial_t Y_i[A, Y^i] \right.$$
$$\left. - 2\sqrt{g}[A, B_i][A, Y^i] - g[A, Y_i]^2 \right). \quad (2.6)$$

---

2 By defining the fluctuations in this way the expansion of the effective action in powers of $g$ is an expansion in the number of loops.
We will be interested in performing our computations in Euclidean space so that we will transform \( t \rightarrow i \tau \) and \( A \rightarrow -i A \) later on.

Until now the explicit form of the background configuration has not been used. Since we are considering two zero-branes the Yang-Mills action can be expanded around a background corresponding to the motion on a straight line [11] [9]:

\[
B^1 = \frac{i}{2} \begin{pmatrix} vt & 0 \\ 0 & -vt \end{pmatrix} \quad \text{and} \quad B^2 = \frac{i}{2} \begin{pmatrix} b & 0 \\ 0 & -b \end{pmatrix},
\]

where \( v \) and \( b \) are the relative velocity and the distance between the two zero-branes. A convenient form of writing the action is in terms of \( U(2) \) generators [3]:

\[
A = \frac{i}{2} (A_0 \mathbb{1} + A_a \sigma^a), \\
X^i = \frac{i}{2} (X_0^i \mathbb{1} + X_a^i \sigma^a), \\
\psi = \frac{i}{2} (\psi_0 \mathbb{1} + \psi_a \sigma^a),
\]

where \( a = 1, 2, 3 \). The 0 components in this decomposition describe the free motion of the center of mass and will be ignored in the following. In terms of this notation, (2.7) takes the form

\[
B^1_3 = vt \quad \text{and} \quad B^2_3 = b.
\]

The Euclidean action for the fluctuations is then

\[
S_Y = i \int d\tau \left( \frac{1}{2} Y_1^i (\partial^2 - r^2) Y_1^i + \frac{1}{2} Y_2^i (\partial^2 - r^2) Y_2^i + \frac{1}{2} Y_3^i \partial^2 Y_3^i \\
- \sqrt{g} \epsilon^{i3x} \epsilon^{cbx} B_3^i \bar{Y}_a Y_b Y_c Y_d j - \frac{g}{4} \epsilon^{abx} \epsilon^{cdx} Y_a Y_b Y_c Y_d j \right).
\]

The Euclidean action for the gauge field takes the form:

\[
S_A = i \int d\tau \left( \frac{1}{2} A_1 (\partial^2 - r^2) A_1 + \frac{1}{2} A_2 (\partial^2 - r^2) A_2 + \frac{1}{2} A_3 \partial^2 A_3 \\
+ 2 \epsilon^{ab3} \partial_3 B^i_a A_b Y_c^i + \sqrt{g} \epsilon^{abc} \partial_3 Y_a Y_b^i \right.
\]

\[
\left. - \sqrt{g} \epsilon^{i3x} \epsilon^{cbx} B_3^i A_a A_b Y_c^i - \frac{g}{2} \epsilon^{abx} \epsilon^{cdx} A_a Y_b Y_c Y_d i \right).
\]

Diagonalizing the bosonic mass matrix in (2.10) and (2.11) we obtain 16 bosons with \( m^2 = r^2 = b^2 + (v\tau)^2 \), two bosons with \( m^2 = r^2 + 2v \), two bosons with \( m^2 = r^2 - 2v \) and 10 massless bosons. All these fields are real.
The Euclidean action for the fermionic fields is conveniently parametrized in terms of the decomposition
\[ \Gamma^0 = \sigma^3 \otimes 1_{16 \times 16}, \]
\[ \Gamma^i = i\sigma^1 \otimes \gamma^i, \]
where \( \sigma^i \) are Pauli matrices and \( \gamma^i \) are real and symmetric \[13\] and two new fermionic fields
\[ \psi_+ = \frac{1}{\sqrt{2}} (\psi_1 + i\psi_2), \]
\[ \psi_- = \frac{1}{\sqrt{2}} (\psi_1 - i\psi_2). \]
The action is then
\[ S_{\text{fermi}} = i \int d\tau \left( \psi^T_+ (\partial_\tau + v\tau \gamma_1 - b\gamma_2) \psi_- + \frac{1}{2} \psi^T_3 \partial_\tau \psi_3 ight. \]
\[ + \sqrt{\frac{g}{2}} (Y^i_1 - iY^i_2) \psi^T_+ \gamma^i \psi_3 + \sqrt{\frac{g}{2}} (Y^i_1 + iY^i_2) \psi^T_3 \gamma^i \psi_- \]
\[ - i \sqrt{\frac{g}{2}} (A_1 - iA_2) \psi^T_+ \psi_3 + i \sqrt{\frac{g}{2}} (A_1 + iA_2) \psi^T_3 \psi_3 \]
\[ \left. - \sqrt{g}Y^i_3 \psi^T_+ \gamma^i \psi_- + i\sqrt{g}A_3 \psi^T_+ \psi_- \right). \]
Here we see that \((\psi_+, \psi_-)\) are massive with mass matrix:
\[ m_f = v\tau \gamma_1 + b\gamma_2, \]
and \( \psi_3 \) is massless.

Performing a gauge transformation in the background field gauge fixing term in (2.1) one can derive the explicit form of the ghost action \[14\]:
\[ S_{\text{ghost}} = i \int d\tau \left( C^*_1 (-\partial_\tau^2 + r^2) C_1 + C^*_2 (-\partial_\tau^2 + r^2) C_2 - C^*_3 \partial_\tau C_3 ight. \]
\[ + \sqrt{g}e^{abc} \partial_\tau C^*_a C_b A_c - \sqrt{g}e^{a3x} \epsilon^{cbx} B^i_3 C^*_a C_b Y^i_3 \right). \]
This gives two complex bosons with mass \( r \) and one complex massless boson.

3. Feynman Rules

There are two possible approaches to compute the gauge invariant background field effective action \( \tilde{\Gamma}(0, B^i) \). The first one treats the background field \( B^i \) exactly, so that this
field enters in the propagators and vertices of the theory. To compute the effective action one has to sum over all 1PI diagrams without external lines. In the second approach one treats the background field perturbatively, so that it appears as external lines in the one particle irreducible graphs of the theory. We are following the first approach.

At this point it will be useful to describe the Feynman rules for bosonic and fermionic fields.

3.1. Feynman Rules for Bosonic Fields

To derive the Feynman rules one has to use the shifted action $S \left( B^i + \sqrt{g} Y^i \right)$, including the gauge fixing term and the ghost contribution that we have just derived. Vertices involving $Y^i$ and $B^i$ are only present inside diagrams and no external lines are present. The propagators of the bosonic fields involve the background field $B^i$. The explicit form of these propagators can be easily obtained once we realize that a relation to the one-dimensional harmonic oscillator can be found.

The Greens function $\Delta_B (\tau, \tau'|\mu^2 + (v\tau)^2)$ for the bosonic fields of our theory is the solution to the equation

$$(-\partial^2_\tau + \mu^2 + v^2\tau^2) \Delta_B (\tau, \tau'|\mu^2 + (v\tau)^2) = \delta(\tau - \tau').$$  \hspace{1cm} (3.1)

Here $\mu^2 = b^2$ or $\mu^2 = b^2 \pm 2v$ depending on the type of boson. Recall that for the one-dimensional harmonic oscillator with frequency $\omega^2 = v^2$ and $P$ and $Q$ are the usual operators satisfying $[Q, P] = i$ one has \cite{16}

$$\langle q_1 | \exp \left( -s \left( P^2 + v^2Q^2 \right) \right) | q_2 \rangle =$$

$$(\frac{v}{2\pi \sinh 2sv})^{1/2} \exp \left( -\frac{v}{2 \sinh 2sv} \left( (q_1^2 + q_2^2 + v^2) \cosh 2sv - 2q_1q_2 \right) \right).$$ \hspace{1cm} (3.2)

Using this result we obtain for the propagator of our bosonic fields the expression

$$\Delta_B (\tau, \tau'|\mu^2 + (v\tau)^2) =$$

$$\int_0^\infty ds e^{-u^2s} \sqrt{\frac{v}{2\pi \sinh 2sv}} \exp \left( -\frac{v}{2 \sinh 2sv} \left( (\tau^2 + \tau'^2) \cosh 2sv - 2\tau\tau' \right) \right).$$ \hspace{1cm} (3.3)

To leading order in $v$ this propagator reduces to

$$\Delta_B (\tau, \tau'|b^2) = \frac{1}{2b} e^{-b|\tau - \tau'|},$$ \hspace{1cm} (3.4)
which is the propagator of a free particle with mass $m^2 = b^2$. For $b^2 = 0$ this reduces to

$$\Delta_B(\tau, \tau' | 0) = -(\tau' - \tau)\theta(\tau' - \tau). \quad (3.5)$$

The two point functions of the bosonic fields are then

$$\langle Y_a^i(\tau)Y_b^j(\tau') \rangle = \delta_{ab}\delta^{ij}\Delta_B(\tau, \tau'| r^2), \quad (3.6)$$

for $a, b = 1, 2$ and $i, j = 2, \ldots, 9$ for particles with mass $m^2 = r^2$, and

$$\langle Y_3^i(\tau)Y_3^j(\tau') \rangle = \delta^{ij}\Delta_B(\tau, \tau'| 0) \quad \text{and} \quad \langle A_3(\tau)A_3(\tau') \rangle = \Delta_B(\tau, \tau'| 0), \quad (3.7)$$

for the massless fields. Further two point functions are linear combinations of two terms

$$\langle A_a(\tau)A_b(\tau') \rangle = \langle Y_a^1(\tau)Y_b^1(\tau') \rangle = \frac{1}{2}\delta_{ab}\left(\Delta_B(\tau, \tau'| r^2 + 2v) + \Delta_B(\tau, \tau'| r^2 - 2v)\right), \quad (3.8)$$

for $a, b = 1, 2$, and

$$\langle A_1(\tau)Y_1^2(\tau') \rangle = -\langle A_2(\tau)Y_1^1(\tau') \rangle = -\frac{1}{2}\left(\Delta_B(\tau, \tau'| r^2 + 2v) - \Delta_B(\tau, \tau'| r^2 - 2v)\right). \quad (3.9)$$

Finally, the explicit form of the vertices can be read off from formulas (2.10), (2.11) and (2.16) that we have found before.

3.2. Feynman Rules for Fermionic Fields

The fermionic fields $\psi_+$ and $\psi_-$ have a mass matrix given by (2.15), so that the fermionic propagator of these fields is a solution of the equation

$$(-\partial_\tau + v\tau\gamma_1 + b\gamma_2)\Delta_F(\tau, \tau'| v\tau\gamma_1 + b\gamma_2) = \delta(\tau - \tau'). \quad (3.10)$$

Using the gamma matrix algebra it is easy to see that the fermionic propagator can be expressed through the bosonic propagator:

$$\Delta_F(\tau, \tau'| v\tau\gamma_1 + b\gamma_2) = (\partial_\tau + v\tau\gamma_1 + b\gamma_2)\Delta_B(\tau, \tau'| r^2 - v\gamma_1), \quad (3.11)$$

while the propagator for massless fermions satisfies:

$$\Delta_F(\tau, \tau'| 0) = \partial_\tau\Delta_B(\tau, \tau'| 0). \quad (3.12)$$
Diagonalizing the mass matrix appearing in $\Delta_B$ in (3.11) one sees that we are left with 8 real fermions with mass $r^2 + v$ and 8 real fermions with mass $r^2 - v$.

We therefore have
\[
\langle \psi_+(\tau) \psi_-(\tau') \rangle = (\partial_\tau + v \tau \gamma_1 + b \gamma_2) \Delta_B (\tau, \tau'|r^2 - v \gamma_1),
\] (3.13)
and
\[
\langle \psi_3(\tau) \psi_3(\tau') \rangle = \partial_\tau \Delta_B (\tau, \tau'|0).
\] (3.14)

The explicit form of the fermionic vertices can be obtained from (2.14).

4. One-Loop Effective Action

In this section we compute the one-loop effective action for two D0 branes from the matrix model approach. This can be easily obtained from the results of [11] and [9].

In order to compute the one-loop effective potential, we are interested in the phase shift $\delta$ of one graviton scattered off the second one. This is related to the potential in the eikonal approximation as
\[
\delta = -\int d\tau V(b^2 + v^2 \tau^2),
\] (4.1)
where $b$ is the impact parameter and $v$ is the relative velocity of the D0 branes.

The phase shift can be obtained from the determinants of the operators $(-\partial_\tau^2 + M^2)$ that originate from integrating out the massive degrees of freedom at one-loop. The explicit expressions for the masses have been found in sections 2 and 3. The expressions for the determinants are then
\[
\text{det}^{-6}(-\partial_\tau^2 + r^2)\text{det}^{-1}(-\partial_\tau^2 + r^2 + 2v)\text{det}^{-1}(-\partial_\tau^2 + r^2 - 2v)
\text{det}^{4}(-\partial_\tau^2 + r^2 + v)\text{det}^{4}(-\partial_\tau^2 + r^2 - v).
\] (4.2)

The phase shift follows from a proper time representation of the determinants and it takes the form [11]:
\[
\delta = -\frac{1}{4} \int_0^\infty \frac{ds}{s} e^{-sb^2} \frac{1}{\sinh sv} (16 \cosh sv - 4 \cosh 2sv - 12).
\] (4.3)
If $b >> 0$ we can expand this equation in $s$ and find the long range potential
\[
V(r) = \frac{15}{16} \frac{v^4}{r^7}.
\] (4.4)

As argued in [2] this is precisely the result expected from a single (super) graviton exchange diagram in eleven dimensions.
5. The Two-Loop Effective Action

Our goal in this section is to compute the two-loop effective action of this system. It is given by the sum of all the graphs appearing in the figure below. We have indicated propagators for fluctuations $Y$ and gauge fields $A$ by wavy lines, ghost propagators by dashed lines and solid lines indicate fermionic propagators. The explicit expressions for the graphs are given by:

\[ \int d\tau \lambda_4 \Delta_1(\tau, \tau|m_1) \Delta_2(\tau, \tau|m_2), \tag{5.1} \]

for the diagram involving the quartic vertex $\lambda_4$, where $\Delta_1$ and $\Delta_2$ are the propagators of the corresponding particles and

\[ \int d\tau d\tau' \lambda_3^{(1)} \lambda_3^{(2)} \Delta_1(\tau, \tau'|m_1) \Delta_2(\tau, \tau'|m_2) \Delta_3(\tau, \tau'|m_3), \tag{5.2} \]

for the diagram involving the cubic vertices $\lambda_3^{(1)}$ and $\lambda_3^{(2)}$. From dimensional analysis we expect the two-loop effective action to be a series of the form\(^3\)

\[ \Gamma^{(2)} = \alpha_0 \frac{1}{r^2} + \alpha_2 \frac{v^2}{r^6} + \alpha_4 \frac{v^4}{r^{10}} + \ldots, \tag{5.3} \]

where $\alpha_i$ are numerical coefficients that we have to determine from the explicit computation of the Feynman diagrams. This follows from the fact that the loop expansion is an expansion in powers of $g$ and $g$ is a dimensionful parameter with $[g] = \ell^{-3}$.

Using the Feynman rules derived in section 3 it is straightforward though a bit lengthy to compute the 17 different Feynman diagrams at two loops. We will evaluate the integrals that appear by expanding the propagators in $s$, as we have done at one loop. We will now classify and give the results for the different contributions.

5.1. Diagrams Involving Only Bosonic Fields

These graphs correspond to figures (a), (b) and (c).

*Diagrams with Quartic Vertices*

\(^3\) Odd powers in $v$ in this expansion are vanishing.
Fig. 1: Diagrams contributing to the two-loop effective action. Propaga-
ators for fluctuations $Y$ and gauge fields $A$ are indicated by wavy lines, ghost
propagators by dashed lines and solid lines indicate fermionic propagators.

The first thing we should note is that massless particles do not contribute to graphs
involving quartic vertices. This is because in dimensional regularization

$$
\int \frac{d^d p}{p^2} = 0,
$$

for any dimension $d$. This can be interpreted as a cancellation between an UV and an IR
divergences

$$
\int \frac{d^d p}{p^2} = \frac{2\pi^{d/2}}{\Gamma(d/2)} \left( \int_{\infty}^{1} p^{d-3} dp + \int_{0}^{1} p^{d-3} dp \right).
$$

From the actions (2.10) and (2.11) we see that two types of quartic vertices involving
massive particles appear:

1) The vertex $-\frac{g}{2} \epsilon^{abx} \epsilon^{cdx} A_a Y_i^b A_c Y_i^d$, with the contribution:

$$
(a)_1 = -\frac{9}{4r^2} - \frac{123}{32} \frac{v^2}{r^6} - \frac{20799}{2560} \frac{v^4}{r^{10}} + \ldots
$$

2) The vertex $-\frac{g}{4} \epsilon^{abx} \epsilon^{cdx} Y_i^a Y_j^b Y_i^c Y_j^d$, with contribution:

$$
(a)_2 = -\frac{9}{r^2} - \frac{3}{8} \frac{v^2}{r^6} - \frac{4239}{640} \frac{v^4}{r^{10}} + \ldots
$$

Diagrams with Cubic Vertices

All diagrams with two cubic vertices involve one massless field. Diagrams involving
more massless fields vanish in dimensional regularization. First the diagrams involving the

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4 An overall factor $g$ is present in all the results below, which we have not written down for simplicity.
gauge field $A$ and the fluctuations $Y$, which we have denoted by (b) in our figure. First the diagrams with two vertices of the same type. We have to be careful with an additional factor $1/2$ that comes from expanding the actions to second order in $\sqrt{g}$ in all graphs involving two vertices of the same type.

3) Two vertices of type $\sqrt{g} \epsilon^{abc} \partial_\tau Y^i_a A_b Y^i_c$, give the contribution:

$$(b)_1 = \frac{81}{8r^2} + \frac{1379 v^2}{64 r^6} + \frac{188631 v^4}{5120 r^{10}} + \ldots \quad (5.8)$$

4) Two vertices of type $-\sqrt{g} \epsilon^{abc} B^i_3 A_a A_b Y^i_c$, give the contribution

$$(b)_2 = -\frac{3}{8r^2} - \frac{115 v^2}{64 r^6} - \frac{22893 v^4}{5120 r^{10}} + \ldots \quad (5.9)$$

5) Two vertices of type $-\sqrt{g} \epsilon^{abc} \epsilon^{cbx} B^i_3 Y^j_a Y^i_b Y^j_c$, give the contribution

$$(b)_3 = -\frac{3}{r^2} + \frac{107 v^2}{64 r^6} - \frac{17661 v^4}{1280 r^{10}} + \ldots \quad (5.10)$$

The graphs involving two different cubic vertices are

6) One vertex $-\sqrt{g} \epsilon^{abc} B^i_3 A_a A_b Y^i_c$, and one $-\sqrt{g} \epsilon^{abc} \epsilon^{cbx} B^i_3 Y^j_a Y^i_b Y^j_c$, give the contribution

$$(b)_4 = -\frac{15 v^2}{16 r^6} - \frac{243 v^4}{64 r^{10}} + \ldots \quad (5.11)$$

7) One vertex of type $-\sqrt{g} \epsilon^{abc} B^i_3 A_a A_b Y^i_c$, and one of type $\sqrt{g} \epsilon^{abc} \partial_\tau Y^i_a A_b Y^i_c$, give the contribution

$$(b)_5 = -\frac{3 v^2}{16 r^6} - \frac{63 v^4}{32 r^{10}} + \ldots \quad (5.12)$$

8) One vertex of type $-\sqrt{g} \epsilon^{abc} \epsilon^{cbx} B^i_3 Y^j_a Y^i_b Y^j_c$, and one of type $\sqrt{g} \epsilon^{abc} \partial_\tau Y^i_a A_b Y^i_c$, give the contribution

$$(b)_6 = \frac{15 v^2}{8 r^6} + \frac{99 v^4}{32 r^{10}} + \ldots \quad (5.13)$$

The diagrams involving the ghost fields are all of the form (c). There are two contributions where the two vertices are of the same type:

9) Two vertices of type $\sqrt{g} \epsilon^{abc} \partial_\tau C^*_a C_b A_c$, give the contribution

$$(c)_1 = -\frac{3}{8r^2} - \frac{73 v^2}{64 r^6} - \frac{7893 v^4}{5120 r^{10}} + \ldots \quad (5.14)$$
10) Two vertices of type $-\sqrt{g}e^{a3x}\epsilon^{cbx}B^i_a C^*_b Y^i_c$, give the contribution

$$
(c)_2 = \frac{1}{8 r^2} - \frac{17 v^2}{96 r^6} + \frac{2571 v^4}{5120 r^{10}} + \ldots
$$

Then there is a bosonic contribution involving ghost fields whose leading order is vanishing which involves two different vertices.

11) One vertex of type $-\sqrt{g}e^{a3x}\epsilon^{cbx}B^i_a C^*_b Y^i_c$, and one of type $\sqrt{g}e^{abc}\partial_\tau C^*_a C_b A_c$, give the contribution

$$
(c)_3 = \frac{v^2}{8 r^6} - \frac{9 v^4}{64 r^{10}} + \ldots
$$

The total contribution of diagrams involving bosonic fields is then

$$
(Total\ Contribution)_B = g \left( -\frac{8}{r^2} + \frac{155 v^2}{24} - \frac{1407 v^4}{160} \right)
$$

5.2. Diagrams Involving Bosonic and Fermionic Fields

There are 6 diagrams involving fermionic fields, which we have denoted by (d) in our figure. Diagrams involving two equal vertices are:

12) Two vertices of type $-\sqrt{g}Y^i_3 \psi^T_+ \gamma_i \psi_-$, give the contribution:

$$
(d)_1 = -\frac{16}{r^2} + \frac{5 v^2}{12 r^6} - \frac{417 v^4}{80 r^{10}} + \ldots
$$

13) Two vertices of type $i\sqrt{g}A_3 \psi^T_+ \psi_-$, give a vanishing contribution

$$
(d)_2 = 0 + \ldots
$$

Then there are 4 diagrams that involve two different vertices:

14) One vertex of type $-i\sqrt{2}(A_1 - i A_2) \psi^T_+ \psi_3$, and one of type $i\sqrt{2}(A_1 + i A_2) \psi^T_+ \psi_3$, give the contribution

$$
(d)_3 = \frac{2}{r^2} - \frac{25 v^2}{6 r^6} - \frac{237 v^4}{40 r^{10}} + \ldots
$$

15) One vertex of type $\sqrt{2}(Y^i_1 - i Y^i_2) \psi^T_+ \gamma^i \psi_3$, and one of type $\sqrt{2}(Y^i_1 + i Y^i_2) \psi^T_+ \gamma^i \psi_-$ give the contribution

$$
(d)_4 = \frac{18}{r^2} - \frac{5 v^2}{2 r^6} + \frac{693 v^4}{40 r^{10}} + \ldots
$$
Finally, there are two different diagrams that cancel each other. The first diagram contains the vertex $\sqrt{g/2}(Y_1^1 - iY_2^1)\psi_T^T \gamma^1 \psi_3$ and the vertex $i\sqrt{g/2}(A_1 + iA_2)\psi_T^T \psi_3$, and the second diagram contains the vertices $\sqrt{g/2}(Y_1^1 + iY_2^1)\psi_T^3 \gamma^1 \psi_-$ and $-i\sqrt{g/2}(A_1 - iA_2)\psi_T^T \psi_3$.

In total the contribution from the diagrams involving bosonic and fermionic fields is given by the sum of the previous results (again taking the factor $1/2$ into account for graphs involving two vertices of the same type):

$$\text{(Total Contribution)}_F = g \left( \frac{8}{r^2} - \frac{155 v^2}{24 r^6} + \frac{1407 v^4}{160 r^{10}} + \ldots \right).$$ (5.22)

This cancels (5.17) as promised!

6. Discussion and Conclusion

In this paper we have computed the effective action up to two loops for the scattering of two Dirichlet zero-branes in M(atrix) theory. At one loop we obtained the well known result for the leading order of the effective potential $V_{\text{eff}} \sim v^4/r^7$. A calculation of the effective potential at two loops showed that no renormalization of the $v^4$-term of the effective potential occur.

These results are in agreement with the predictions following from eleven-dimensional supergravity. The fact that the term with four space-time derivatives is not renormalized at two loops is in agreement with the non-renormalization theorem that was conjectured by Banks, Fischler, Shenker and Susskind [2].

In [12] it has been argued that possible renormalizations of the $v^4$-term may occur in the quantum mechanics problem since renormalizations in similar three-dimensional theories have been observed. This is not in contradiction with our result and it means that these renormalizations would occur in higher orders in perturbation theory. This is potentially allowed and would not be in contradiction with M-theory. Possible non-perturbative corrections due to instantons along the lines of [13] may also occur. These would be related to scattering amplitudes with M-momentum transfer [13], which have not been considered herein.
The renormalization of \((velocity)^4\)-terms has already been observed by Douglas, Ooguri and Shenker \cite{17} for M(atrix)-theory in curved spaces. It should be interesting to check if perturbative corrections at higher order in perturbation theory or non-perturbative corrections to \(v^4\)-terms appear in the theory we considered.

Acknowledgements
We would like to thank M. Dine, M. Douglas, D. Kabat, J. Polchinski and P. Pouliot for useful discussions. The work of K. B. was supported by NSF grant PHY89-04035 and the work of M. B. was supported by DOE grant DOE-91ER4061.
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