Extended Axion Electrodynamics: Anomalous Dynamo-Optical Response Induced by Gravitational pp-Waves

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Abstract—We extend the Einstein–Maxwell-axion theory, including into the Lagrangian cross-terms of dynamo-optical type, which are quadratic in the Maxwell tensor, linear in the covariant derivative of the macroscopic velocity four-vector $U^i$, and linear in the pseudoscalar (axion) field $\phi$ or its gradient four-vector. We classify the new terms with respect to irreducible elements of the covariant derivative of $U^i$ of the electromagnetically active medium: the expansion scalar, acceleration four-vector, shear and vorticity tensors. Master equations of the extended axion electrodynamics are used for the description of the response of an axionically active electrodynamic system, induced by a pp-wave gravitational background. We show that this response has a critical nature, i.e., the electric and magnetic fields, dynamo–optically coupled to the axions, anomalously grow under the influence of the external pp-wave gravitational field.

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1. INTRODUCTION

Axion electrodynamics as an extension of the Faraday–Maxwell electromagnetic theory is based on the prediction of axions, massive pseudo-Goldstone bosons [1–3]. The first discussion concerning the pseudoscalar-photon interaction appeared in [4]; however, the interest in axion electrodynamics has significantly grown later, after publication of the paper [5]. Important aspects of the axion theory and of its astrophysical and cosmological applications can be found, e.g., in the reviews and book [6–10]. Experimental results concerning the axion-photon interactions are published, e.g., in [11–16].

Axion electrodynamics can be indicated as the standard one, if the Lagrangian contains only one cross-term $\frac{1}{4} F_{mn} F^{mn}$, in which the product of the pseudoscalar (axion) field $\phi$ and of dual Maxwell (pseudo)tensor behaves as a true tensor. In that theory axion-induced phenomena can be visualized when the axion field $\phi$ has a non-vanishing gradient four-vector $\nabla k \phi \neq 0$; accordingly, the invariant $I = g^{ik} \nabla i \phi \nabla k \phi$ can be positive, negative or equal to zero. The last case relates to models with pp-wave symmetry, for which the axion field depends on retarded time only. Here we focus on precisely this model and consider the pp-wave gravitational background as a scene for the electromagnetic field evolution in a non-uniformly moving axionically active medium.

In [17–23] we considered extensions of axion electrodynamics, keeping in mind that even if the axion field is initially constant, external gravitational and electromagnetic fields are able to activate frozen axion-photon couplings in the course of evolution of the corresponding physical system. In particular, we considered a nonminimal axion–photon coupling excited by gravitational waves [17]; nonstationary optical activity and gradient-type models of the axion-photon coupling in a cosmological context [18, 19]; fingerprints of dark matter axions in the terrestrial electric and magnetic field variations [20]; electromagnetic waves in an axion-active plasma [21, 22]; axionically induced anomalous behavior of the electromagnetic response to gravitational waves [23].

In this paper we consider the so-called dynamo-optical extension of axion electrodynamics. This term was introduced in [24] to describe the influence of a non-uniform (irregular) motion of a physical system on its electromagnetic response. A generalization of dynamo-optical type of the Einstein–Maxwell theory was carried out in [25]. In this paper we add a new element into the theory, namely, the pseudoscalar (axion) field, thus providing a dynamo-optical extension of axion electrodynamics, which is a distinctive part of the Einstein–Maxwell-axion theory.

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2. THE MODEL

2.1. The Action and Standard Definitions

Master equations of extended axion electrodynamics form a subset of the total set of equations of the Einstein–Maxwell-axion model (see, e.g., [17, 23]); they can be obtained by variation of the action functional

\[ S = \int d^4x \sqrt{-g} \left[ \frac{R}{2\kappa} + L_{(m)} + L_{(EM)} + L_{(A)} \right] \]

with respect to the electromagnetic potential four-vector \( A_i \), and the dimensionless pseudoscalar (axion) field \( \phi \), respectively. Equations of the gravity field, as a result of variation of this action functional with respect to the space-time metric \( g_{ik} \), are presented in [22], and we do not consider them in this paper. As usual, \( g \) is the determinant of the metric; \( R \) is the Ricci scalar; \( \kappa = 8\pi G/c^4 \) is the Einstein constant. The Lagrangian of the axion field has the form

\[ L_{(A)} = \frac{1}{2} \Psi_0^2 \left[ m_{(A)}^2 \phi^2 + V(\phi^2) - g^{mn} \nabla_m \phi \nabla_n \phi \right], \]

where \( \nabla_m \) is a covariant derivative; the parameter \( 1/\Psi_0 \) is a coupling constant of the axion-photon interaction; the term \( m_{(A)} = \frac{\epsilon}{\hbar} m_{(a)} \) is a rescaled axion mass \( m_{(a)} \); \( \hbar \) is the Planck constant. The total Lagrangian of the electromagnetic field in an axionically active medium, \( L_{(EM)} \), is considered to be quadratic with respect to the Maxwell tensor \( F_{ik} \equiv \nabla_i A_k - \nabla_k A_i \). This Lagrangian includes the dual tensor \( F^{*mn} = \frac{1}{2} \epsilon^{mnpq} F_{pq} \), where, as usual, \( \epsilon^{mnpq} = (-g)^{-1/2} E^{mnpq} \) is the Levi-Civita tensor, \( E^{mnpq} \) is the skew-symmetric Levi-Civita symbol with \( E^{0123} = 1 \). The dual Maxwell tensor satisfies the condition \( \nabla_k F^{*ik} = 0 \), which is treated as a subset of the electromagnetic equations.

Keeping in mind the necessity of a phenomenological decomposition of the extended Lagrangian \( L_{(EM)} \) into irreducible parts, we use the following standard representation of the tensor \( F^{ik} \):

\[ F^{ik} = E^i U^k - E^k U^i - \epsilon^{ikmn} B_m B_n. \]

Here \( U^i \) is the macroscopic velocity four-vector; we use the Landau–Lifshitz definition of \( U^i \) considering it as a timelike eigenvector of the stress–energy tensor of matter (see, e.g., [23] for details). The electric field four-vector, \( E^i \equiv F^{ik} U_k \), and the magnetic induction four-vector, \( B_i \equiv F_{ik} U^k \), are, clearly, orthogonal to the velocity four-vector \( U^i \).

To classify the irreducible terms describing interactions of dynamo–optical type, we use in the decomposition of the Lagrangian \( L_{(EM)} \) the following standard representation of the covariant derivative of the velocity four-vector:

\[ \nabla_i U_k = U_i D U_k + \sigma_{ik} + \omega_{ik} + \frac{1}{3} \Delta_{ik} \Theta. \]

Here \( DU_k \equiv U^i \nabla_i U_k \) is the medium acceleration four-vector; \( \Theta \equiv \nabla_m U^m \) is the expansion scalar; the traceless symmetric shear tensor \( \sigma_{ik} \), the skew-symmetric vorticity tensor \( \omega_{ik} \), and the projector \( \Delta_{ik} \) are

\[ \sigma_{ik} \equiv \Delta_{i}^{P} \Delta_{k}^{q} \nabla_{p} U_{q} - \frac{1}{3} \Delta_{ik} \Theta, \]

\[ \omega_{ik} \equiv \Delta_{i}^{q} \Delta_{k}^{p} \nabla_{p} U_{q}, \quad \Delta_{ik} = g_{ik} - U_{i} U_{k}. \]

The symbols \((ik)\) and \([ik]\) denote symmetrization and skew-symmetrization, respectively. Also, we use two auxiliary tensors: the angular velocity (pseudo) four-vector \( \omega_i \), and the skew-symmetric tensor \( \Omega_{pq} \) given by

\[ \omega_i \equiv -\omega_{ik} U^k, \quad \Omega_{pq} \equiv U^r D_{[pq]} U^r. \]

We consider the quantities \( \phi, \nabla_i \phi, E^i, B^k, U_i, DU_i, L \), standard representation of the covariant derivative of the velocity four-vector:

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\[ \omega_i \equiv -\omega_{ik} U^k, \quad \Omega_{pq} \equiv U^r D_{[pq]} U^r. \]
The terms linear in the pseudoscalar field $\phi$ and in the convective derivative $D\phi$ contain the pseudovector $B^i$, providing that $L_2$ and $L_3$ are pure scalars:

\[
L_2 = \frac{\phi}{4} E_m B_n (\lambda_{21} \Theta g^{mn} + \lambda_{22} \sigma^{mn} + \lambda_{23} \alpha^{mn}), \tag{10}
\]

\[
L_3 = \frac{D\phi}{4} E_m B_n (\lambda_{31} \Theta g^{mn} + \lambda_{32} \sigma^{mn} + \lambda_{33} \alpha^{mn}). \tag{11}
\]

All terms linear in the spatial gradient of $\phi$, $\Delta^i \nabla_k \phi$, can be reduced to a set of three scalars $L_4$, $L_5$, $L_6$. We first construct scalars linear in the vorticity tensor; it is convenient to use in this case the pseudovector $\omega_i$ guaranteeing that the product $\omega_m \nabla_n \phi$ is a true tensor:

\[
L_4 = \frac{1}{4} \omega_{(m} \nabla_{n)} \phi \left[ \Delta^{mn} \left( \lambda_{41} E_k E^k + \lambda_{42} B_k B^k \right) + (\lambda_{43} E^m E^n + \lambda_{44} B^m B^n) \right]. \tag{12}
\]

Second, we list the terms linear in the acceleration four-vector $D\dot{U}_k$ (terms with $\dot{\phi}$ cannot appear):

\[
L_5 = \frac{1}{4} \nabla_n \phi \ D\dot{U}_m \left( \lambda_{51} \Delta^{mk} E^k B_k + \lambda_{52} E^m B^k + \lambda_{53} E^m B^n \right). \tag{13}
\]

Third, there are two terms linear in the shear tensor:

\[
L_6 = \frac{1}{4} \rho^{mnp} \sigma_{mk} \nabla_n \phi \times \left( \lambda_{61} E^k E_p + \lambda_{62} B^k B_p \right). \tag{14}
\]

Our statement is that the decomposition presented above is irreducible, and the number of independent coupling constants $\lambda_{11}, \ldots, \lambda_{62}$, appearing in front of the listed terms, is twenty one. Keeping in mind symmetry arguments, further we reduce the number of these parameters using the following relations:

\[
\lambda_{11} = -\lambda_{12}, \quad \lambda_{13} = \lambda_{14},
\]

\[
\lambda_{21} = \frac{1}{3} \lambda_{22}, \quad \lambda_{31} = \frac{1}{3} \lambda_{32},
\]

\[
\lambda_{42} = -\lambda_{41}, \quad \lambda_{44} = \lambda_{33}, \quad \lambda_{62} = \lambda_{61}. \tag{15}
\]

Also, we put $\lambda_{16} = 0$, since the corresponding term does not enter into the electrodynamic equations due to their specific structure.

2.3. Master Equations

2.3.1. A pp-wave gravitational background.

We consider test electromagnetic and pseudoscalar fields in the pp-wave gravitational background with the line element

\[
ds^2 = 2\text{d}udv - L^2 \left[ e^{2\beta}(dx)^2 + e^{-2\beta}(dy)^2 \right], \tag{16}
\]

where $u = (ct - x^1)/\sqrt{2}$ is the retarded time and $v = (ct + x^1)/\sqrt{2}$ is the advanced time. For such a model, the background factor $L(u)$ satisfies the requirements

\[
L'' + (\beta')^2 L = 0, \quad L(0) = 1, \quad L'(0) = 0, \tag{17}
\]

where the prime denotes a derivative with respect to the retarded time $u$, and $\beta(u)$ is an arbitrary function of $u$ with the initial value $\beta(0) = 0$. We consider the medium to be at rest, and the velocity four-vector has the form

\[
U_u = U_v = 1/\sqrt{2}, \quad U_2 = U_3 = 0. \tag{18}
\]

In this case $D\dot{U}_i = 0$, $\omega_{ik} = 0$, $\Theta = \sqrt{2}L'/L$,

\[
\sigma^k_i = \frac{\Theta}{2} \left( \frac{1}{3} \Delta^k_i - C^k_i C^i_k \right) + \frac{\beta'}{\sqrt{2}} \left( \delta^2_{jk} \delta^3_{kl} - \delta^3_{ik} \delta^3_{jk} \right). \tag{19}
\]

We assume that the electromagnetic and axion fields inherit the pp-wave symmetry of the gravitational background, thus the unknown functions depend on $u$ only: $E^i(u), B^k(u), \phi(u)$ (see [23]).

2.3.2. Electrodynamic equations. The electrodynamic equations have the standard form

\[
\nabla_k H^{ik} = 0, \quad \nabla_k E^{*ik} = 0, \tag{20}
\]

where the excitation tensor can now be written as

\[
H^{ik} = U^n \left[ \delta^{ik}_{mn} \frac{\partial L_{(EM)}}{\partial E_m} + \epsilon^{ik}_{mn} \frac{\partial L_{(EM)}}{\partial B_m} \right], \tag{21}
\]

using the explicit $(E, B)$ representation of the scalar $L_{(EM)}$ (7)–(14). This procedure is routine, and we omit the details of $H^{ik}$ decomposition. If the unknown functions depend on retarded time only, integration of (19) gives six integrals:

\[
L^2 H^{iu}(u) = H^{iu}(0), \quad L^2 F^{*iu}(u) = F^{*iu}(0), \tag{22}
\]

(i = $v, x^2, x^3$). Three of them do not contain $\phi(u)$:

\[
B_v(u) = \frac{B_v(0)}{L^2(u)}, \quad B_2(u) = e^{2\beta}[B_2(0) + E_3(0) - E_3(u)],
\]

\[
B_3 = e^{-2\beta} [B_3(0) + E_2(u) - E_2(0)]. \tag{23}
\]

The longitudinal integral (for $i = v$) yields

\[
E_v(u) = \frac{\varepsilon E_v(0) - B_v(0) [\phi(u) - \phi(0)]}{L^2 \left[ \varepsilon + 4\Theta(u)[3\lambda_{11} - \lambda_{13}] \right]} \tag{24}
\]
Two remaining integrals contain both \( \phi(u) \) and \( \phi'(u) \); using \( (22) \), they can be written in the form
\[
J(u) = 2A(u)e^{2\beta}E_2(u) + 3\beta'B(u)E_3(u) = J_2(u),
\]
\[
3\beta'B(u)E_2(u) + 2A(u)e^{2\beta}E_3(u) = J_3(u),
\]
where the auxiliary functions are defined as follows:
\[
A(u) = \lambda_{13}\Theta + 6(\varepsilon - 1/\mu) + 6\Theta\lambda_{11},
B(u) = \lambda_{32}\phi' + \sqrt{2}\lambda_{22}\phi.
\]
The source term \( J_2(u) \) in \( (24) \) is of the form
\[
J_2(u) = \frac{3}{2\sqrt{2}}\left[ E_3(0) + B_2(0) \right] \left\{ 6\lambda_{22}\phi' + \Theta B - \sqrt{2}\phi'\lambda_{32} + 48\sqrt{2}(\phi - \phi(0)) \right\}
+ [B_3(0) - E_2(0)] \left\{ e^{-2\beta}(\Theta(\lambda_{13} - 6\lambda_{11}) - \frac{12}{\mu}(1 - e^{-2\beta}) \right\}
- 12E_2(0)(\varepsilon - 1/\mu).
\]
The term \( J_3 \) can be obtained from \( (26) \) by replacing \( \beta \rightarrow -\beta, E_2(0) \rightarrow E_3(0), B_2(0) \rightarrow -B_3(0) \).

2.3.3. Axion field evolution equations. Variation of the action \( (1) \) with respect to the axion field \( \phi \) gives the equation
\[
\left[ \Box + m^2(\lambda) + V'(\phi^2) \right] \phi = -\frac{1}{\Psi^2_0} \left( B_kE^k - J \right).
\]
If \( J = 0 \), we deal with standard pseudoscalar field equation; using explicit representations of \( L_2, ..., L_6 \) (see \( (10) - (14) \)), we can write the dynamo-optical source \( J \) in the following compact form:
\[
J = 2\left\{ -\frac{L_2}{\phi} + (\Theta + D) \left( \frac{L_4}{D\phi} \right) \right\}
+ \nabla_n \left[ \frac{\partial}{\partial(\nabla_n \phi)} (L_4 + L_5 + L_6) \right].
\]
Clearly, this term is quadratic in the electromagnetic field components, \( E^i \) and \( B^k \); it is up to second order in the irreducible decomposition elements of the covariant derivative of the velocity four-vector, \( \Theta, Du_k, \sigma_{mn}, \omega_{mn} \), and contains the second covariant derivatives \( \nabla_n \nabla_m U_n \).

2.3.4. The initial state. If the gravitational wave is absent \( (u < 0) \), the pseudoscalar, electric and magnetic fields are assumed to be constant \( (\phi(0), E_i(0), B_i(0), \lambda_{13}, \lambda_{11}) \), and the system as a whole to be at rest. Clearly, at \( u = 0 \) Eqs. \( (22) - (24) \) with \( (25) \) and \( (26) \) are converted into identities for arbitrary \( \phi(0), E_i(0), B_i(0). \) The initial value \( \phi(0) \) can be found from the reduced equation \( (27) \) with \( J = 0 \):
\[
\left[ m^2(\lambda) + V'(\phi^2) \right] \phi(0) = -\frac{1}{\Psi^2_0} B_k(0)E^k(0),
\]
where \( E_i(0) \) and \( B_i(0) \) are arbitrary constants.

3. ANOMALOUS BEHAVIOR OF THE ELECTROMAGNETIC RESPONSE

The set of master equations of the axion electrodynamics in the pp-wave gravitational background presented above (see \( (22) - (28) \)) is a coupled set of nonlinear equations; a search for exact solution of this system is a sophisticated but very interesting problem. In this sense, the exact solutions discussed in \( [23] \) correspond to the case of absence of dynamo-optic phenomena, nevertheless, those results let us expect the formulated dynamo-optical problem to be solved in the nearest future with some special conditions for the coupling constants. Here we restrict ourselves to analyzing a truncated model, which nevertheless displays the main new feature: the anomalous nature of the axion-photon coupling in a non-uniformly moving medium.

3.1. Longitudinal Electromagnetic Fields

We use the term longitudinal if the initial electric and/or magnetic fields are directed along the gravitational wave propagation axis. In this case, transversal components cannot be produced, and thus we deal with magnetic and electric fields given by \( (22), (23) \) and with the axion field of the form
\[
\phi(u) = \frac{\phi(0)}{L^4m^2(\lambda)\Psi^2_0 \left[ \varepsilon + \frac{1}{\Theta}(3\lambda_{11} - \lambda_{13}) \right] + 2B^2(0) \Psi^2_0}.
\]
The denominator of \( E_v(u) \) in \( (23) \) can tend to zero as \( u \rightarrow u^* \), when \( \Theta(u^*) = 6\varepsilon(\lambda_{13} - 3\lambda_{11})^{-1} \), thus providing an anomalous growth of the longitudinal electric response. The axion field \( \phi(u^*) \) at this moment remains finite if \( B_v(0) \neq 0 \).

3.2. Transversal Electromagnetic Fields

If \( E_v(0) = B_v(0) = 0 \) but \( E_2(0), ..., E_5(0) \neq 0 \), we deal with a three-dimensional nonlinear set of coupled evolutionary equations. Let us illustrate the appearance of a response anomaly for the case of relic dark matter axion domination (see, e.g., \( [19, 20] \). This term means that the density of axions produced by the electromagnetic field is much smaller that the
density of relic cosmological axions. In such a case, we consider the function $\phi(u)$ to be fixed and face the linear algebraic system (24). The corresponding Cramer determinant $\Delta(u)$

$$
\Delta(u) = 4A^2(u) + 9\beta'^2B^2(u)
$$

(31)

turns to zero at the moment $u_*$ at which $A(u_*) = 0$, $B(u_*) = 0$, or in more detail

$$
\Theta(u_*) = \frac{6}{\lambda_{13}} \left( \frac{1}{\mu - \varepsilon} \right),
\lambda_{32} = \lambda_{22} = 0.
$$

(32)

This anomaly, appearing in the response of transversal electric and magnetic fields, is mixed and can be identified as an axionic-dynamo-optical anomaly, in contrast to the pure dynamo-optical longitudinal anomaly in (23).

4. CONCLUSIONS

1. The dynamo-optical extension of the Einstein-Maxwell-axion theory is shown to have twenty one coupling constants as a maximum; the number of essential parameters can be reduced to thirteen by physical assumptions.

2. Longitudinal electric and magnetic fields in an axionically active medium, evolving in the pp-wave gravitational background, can possess a dynamo-optical anomaly, which describes an amplification of the electromagnetic response if $3\lambda_{11} - \lambda_{13} \neq 0$.

3. Transversal electric and magnetic fields can display an anomaly of a new type, which differs from the dynamo-optical one by the dependence on the axion field.

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