Equilibrium shape of dry spot in isothermal liquid film on a horizontal substrate

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Abstract. Thin liquid films formed near dry spots make a big contribution to heat and mass transfer processes due to intensive evaporation. Based on the solution to the problem of determining the shape of the film free boundary in a vicinity of a stationary dry spot formed by gravitational and capillary forces at a given contact angle, a criterion for the development (or shrinkage) of a dry spot in an isothermal liquid film on a horizontal flat substrate was derived without using additional assumptions and postulating the film shape near the spot.

1. Introduction

Liquid films are widely used in industry, in particular for heat and mass transfer processes intensification. The decrease in film thickness allows the method of evaporative cooling to be more efficient. Submicron liquid film, formed on the borders of the dry spots due to the intensive evaporation, can contribute significantly to the total heat and mass transfer [1]. Thus, the authors [2], through the organization of film flow with small-scale wash-off dry spots, could obtain heat flux, one order of magnitude greater than those, taking place in a continuous flow of a film under the same fluid flow. However, when growing, dry spots dramatically reduce the efficiency of heat exchangers and can even lead to their failure [3,4]. Therefore, submicron liquid film formed in the cooling film near the border of dry spots, are of particular interest.

A number of experimental and theoretical works [5-14] is devoted to the analysis of the mechanisms of dry spots formation in the films and to their modeling. In theoretical works on modeling of dry spots rupture and formation of cross section of liquid film free surface in the area adjacent to the dry spot are usually approximated by circular arcs. Based on this approach, the authors of the article proposed a criterion for dry spots formation and development in isothermal liquid films on a horizontal substrate [14].

In this article, a new criterion for dry spot development in the liquid film is given without postulating the shape of the film near the spot. This is done on the basis of the rigorous solution to the problem of the shape of the film free boundary surrounding a stationary dry spot formed under gravity and surface tension forces at a given contact angle.

2. Derivation of the basic equations

Let a stationary film capillary fluid under gravity having a dry spot in the shape of a circle lie on a horizontal flat plate. The spot surroundings cross-sectioned with a plane passing through the center of the spot are illustrated in Figure 1.
At each point M on the generating line L a pressure drop occurs due to hydrostatic pressure $P$ in the liquid and to the presence of surface tension forces $F$. Write expressions for these pressures.

Hydrostatic pressure $P = \rho g (h - z)$
where $\rho$ is the fluid density and $g$ is acceleration of gravity.

The Laplace pressure
$F = \alpha (k_1 + k_2)$ where $k_1$ and $k_2$ are the main curvatures of the surface.

The rim will be stable under the condition when the total pressure at the points of the curve L is equal to zero
$F + P = 0$ or $\alpha (k_1 + k_2) + \rho g (h - z) = 0$.

According to Meusnier theorem [15], the curvature associated with the axial symmetry of the liquid film is $k_1 = \frac{\sin \theta}{r}$ where $r$ is the distance from the given point of the generatrix $L$ to the axis of symmetry of a dry spot. The second curvature related to the shape of the contour $L$ is $k_2 = \frac{d\theta}{dS}$ where $\theta$ is the angle between the substrate and the tangent to the contour $L$, and $S$ is the arc length along the contour $L$ counted from the point of contact of the contour $L$ with the substrate.

Thus, to find the form of the generating line $S = f (\theta )$ of the equilibrium curved surface of the film, we get the equation

$$\frac{d\theta}{dS} = -\frac{\rho g (h - z)}{\alpha} - \frac{\sin \theta}{r}, \text{ or } \frac{d\theta}{dS} = Bo (\bar{z} - 1) - \frac{\sin \theta}{\bar{r}}$$

(1)

where $\bar{z} = \frac{z}{h}$, $\bar{r} = \frac{r}{h}$, $-\text{ dimensionless coordinates}$, $Bo = \frac{\rho gh^2}{\sigma}$ $-\text{ Bond number}$, $\bar{S} = \frac{S}{h}$ $-\text{ dimensionless element of the curve length}$.

For the solution of differential equations it is necessary to specify boundary conditions. As such conditions we shall take:

$$\theta(0) = \theta_0, \bar{r}(0) = \bar{r}_0, \bar{z}(0) = 0$$

(2)

where $\theta_0$ is the contact wetting angle, and $\bar{r}_0$ is the dimensionless radius of the initial dry spot, the parameter unknown in advance.

3. The algorithm for solving the problem
Make sure first that the existence of dry spot for a given combination of $\theta_0$ and $Bo$ is possible. To find out this, it is sufficient to neglect the second term of the right side of equation (1) (assuming $\bar{r}_0 \rightarrow \infty$), and solve the equation

$$\frac{d\theta}{dS} = Bo (\bar{z} - 1).$$
Substituting the expression \( d\vec{S} = \frac{d\vec{x}}{\sin \theta} \) we get \( \sin \theta d\theta = Bo(z - 1)d\vec{x} \), and having integrated the latter we get \( \cos \theta - \cos \theta_0 = Bo(z - \frac{\vec{x}^2}{2}) \).

Assuming in this equation \( \theta = 0 \), \( \vec{x} = 1 \), we get:

\[
Bo^* = 2(1 - \cos \theta_0)
\]

Thus, for \( Bo < Bo^* \) a solution is possible (i.e., a film with a dry spot exists), otherwise, it is impossible for any \( \vec{r}_0 \). Conversely, for a given value of Bond number, there is a solution for \( \theta_0 > \theta_0^* \), where

\[
\theta_0^* = \arccos\left(1 - \frac{Bo}{2}\right).
\]

Now we supplement eqn 1 with two obvious equations and get a system

\[
\begin{align*}
\frac{d\theta}{d\vec{S}} &= Bo(z - 1) - \frac{\sin \theta}{\vec{r}} \\
\frac{d\vec{x}}{d\vec{S}} &= \sin \theta \\
\frac{d\vec{r}}{d\vec{S}} &= \cos \theta
\end{align*}
\]

Solving the system (4) together with boundary conditions (2), we obtain the form of the free boundary of the film. This task we shall solve using Runge-Kutta method.

In this case, since the boundary conditions have the unknown parameter, the problem is variational with respect to it. The variation is convenient to be made with the usage of the bisection method, thus, making a solution have a free boundary of the film on a horizontal asymptote at a height equal to the thickness of the liquid film, i.e. fulfilling the condition \( \vec{x} = 1 \) with \( \theta = 0 \) to the specified accuracy.

The value of the parameter \( \vec{r}_0 \), in which the latter condition is met, will give us the critical size of dry spot \( \vec{r}_0 \), while the solution of the system of equations (4) with this boundary condition will give the opportunity to obtain the desired form of the free boundary of dry spot surroundings.

4. The results of the calculations

In Fig. 2 the equilibrium form of the free boundary of the film adjacent to the spot, built by this method, and the form obtained on the basis of the approximation of the free boundary by an arc of a circle, with \( \theta_0 = 36^\circ \) for two values of Bond number are compared. As one can see, the form of the free boundary constructed by this method is markedly different from the arc of a circle: the usage of the approximation increases the critical radius of dry spot; the generating line of the surface of the film adjacent to dry spot achieves the horizontal asymptote faster.

![Figure 2](image.png)

**Figure 2.** Comparison of the equilibrium shape of the free boundary of the film adjacent to the spot, built by this method, and the form obtained on the basis of the approximation of the free boundary by an arc of a circle, with \( \theta_0 = 36^\circ \) for two values of the Bo.
Fig. 3 shows the dependence of the critical radius of the dry spot on Bond number and on the contact wetting angle for the free surface, defined by equations (4) in their exact interpretation (a), and in the case of its approximation by an arc of a circle (b).

With the values of Bo and $\theta_0$ below and to the left of the corresponding curve shown on the chart the dry spots tend to compression, while above and to the right of the curve their tendency is to expand.

As can be seen from the graphs, the critical size of dry spot is small at high angles of wetting, but with the decrease of $\theta_0$ it is growing rapidly, and a function $r_c$ of $\theta_0$ has a vertical asymptote at a particular value of $\theta_0 = \theta_0^*$. Thus, for $\theta_0 < \theta_0^*$ any dry spot tends to compression. Values of $\theta_0^*$ are defined by the value of Bond number (3') and are shown in Fig. 4.

In Fig. 5 one can see the correlation of the critical radii $K = \frac{r_c^o}{r_c}$ of dry spot obtained in different ways, where $r_c^o$ is the critical radius obtained with the usage of approximation and $r_c$ without it.
5. Conclusion

We obtained a solution to the problem of constructing the free boundary of the isothermal film of capillary gravity fluid lying on a horizontal plate and surrounding the dry spot it has within. The result we have achieved contributed, in its turn, to the solution of the problem of the critical radius value of a stationary dry spot, determining which spot, occurred in the film for whatever reasons, will disappear and which will develop.

The examples of calculations are given as well as the comparison of two methods of constructing the form of the free boundary of the film surrounding the dry spot and of determining the critical diameter of dry spot: a) on the basis of the obtained solution of the task and b) on the basis of the commonly used approximation of the cross section of the free boundary of the film by the arc of a circle. It is shown that in variant (a), as in option (b) [14], the defining parameters of the problem are Bond number and contact wetting angle. Qualitatively, the results obtained by the two approaches are close to each other, but quantitatively the critical values of dry spot in the case of its approximation by the arc of a circle are 2-3 times higher than those calculated more rigorously according to this method. We can also postulate a few changes of the form of the free boundary of the film adjacent to the dry spot. The boundary constructed on the basis of approximation lies above the corresponding curve constructed without the use of approximation and goes faster on the horizontal asymptote. This fact should be viewed as important, if to take into account the experiments showing that reduction of the thickness of the liquid film increases the efficiency of evaporative cooling method in heat and mass transfer exchangers and when analyzing the work of such apparatus it is necessary to know the thickness of the film, adjacent to dry spot as precisely as possible.

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