Violation of Lorentz invariance and dynamical effects in high energy gamma rays

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The relation between the violation of Lorentz invariance and the dynamical effects in high energy gamma rays production is discussed. By using the framework of noncommutative classical electrodynamics, it is shown that full dynamical calculations are required to put bounds on the Lorentz violating scale by the phenomenological analysis of these processes as, for example, the synchrotron radiation from the CRAB nebula. It is observed that an improvement of the present bound on the scale of noncommutativity can be obtained only by astrophysical observations of gamma ray spectra in strong magnetic fields such as pulsars.

1. INTRODUCTION

Quantum Gravity (QG) is a work in progress \cite{1}. Nevertheless the analysis of possible phenomenological effects at low energy with respect to its natural scale, $M_{\text{QG}}$, is an active field. Two of the aspects currently under the most intense investigation, the effects of noncommuting space-time coordinates \cite{2} and the violation of Lorentz invariance \cite{3,4}, have clearly deep connections, although in the literature this is not always emphasized.

The simplest way to express noncommutativity of space-time coordinates,

$$[x^\mu, x^\nu] = i\theta^{\mu\nu}, \quad (1)$$

can actually be related to QG \cite{3}, while the breaking of Lorentz invariance is often presented by introducing modified dispersion relations, e.g.

$$E^2 = p^2 + m^2 + p^3/M_{\text{QG}}, \quad (2)$$

again motivated by QG or effective field theories \cite{3,6}.

The role of possible Lorentz violating (LV) effects in ultra high energy cosmic rays was used to put bounds on the QG scale $M_{\text{QG}}$ by combining the former dispersion relations together with the energy momentum conservation. In this simple and well defined “kinematic scheme”, it is easy to modify the standard thresholds for decay processes and particle production in collisions to avoid, for example, the GZK cutoff and to put limits on $M_{\text{QG}}$ from the experimental constraints on the not observed electrodynamic processes, as vacuum Cherenkov radiation or $\gamma \to e^+e^-$ (forbidden in the standard case but allowed by the violation of Lorentz invariance \cite{4}).

2. DYNAMICAL AND KINEMATIC ANALYSIS

For a more comprehensive phenomenological analysis, the previous kinematic scheme is not enough and one needs a full dynamical calculation of some processes involving emission or absorption of radiation. To this purpose one can consider, for example, the effective field theory introduced in \cite{6}, where the Lorentz violating operators of canonical dimension $\leq 5$ have been introduced in the Lagrangian. This produces, on the one hand deformed dispersion relations (such as the one in Eq. (2)), on the other hand, a modification of the standard electrodynamics. However, even though some dynamical assumptions
were introduced in order to put bounds on $M_{QG}$ from high energy astrophysical gamma rays processes, a complete calculation of such kind of phenomena in this framework is still missing. This approach, which gives tight bounds on $M_{QG}$ \[7\], leaves open the question on the consistency of the dynamical assumptions with the deformed dispersion relations.

To clarify this point, let us consider the investigated limit on the QG scale obtained by the analysis of the synchrotron radiation from the CRAB nebula. \[7\]. In this case to constrain $M_{QG}$ the following modified dispersion relations for photons

$$E^2 = \bar{p}^2 + \xi p^3 / M$$

(3)

and electrons

$$E^2 = \bar{p}^2 + m^2 + \eta p^3 / M,$$

(4)

are used (where $1/M_{QG}$ in Eq. \[2\] is replaced by $\xi / M$ or $\eta / M$ depending on the particle species, and $M = 10^{19}$ GeV), while the validity of the standard synchrotron radiation formulas (as the one for the angle of the emitted radiation and the critical frequency) are still assumed to hold.

The use of the modified dispersion relations within the un-modified dynamics has been already criticized in \[8\] and supported by heuristic arguments in \[9\] (see also the more recent analysis in \[10\]). However there is no explicit calculation of the synchrotron radiation in the effective field theory which gives the relations in Eqs. \[3\] and \[4\], and, on the other hand, it is possible to give other heuristic arguments that show how the deformed dispersion relation for photons in Eq. \[3\] produces strong modifications in the synchrotron radiation formulas.

The simplest one is discussed below. Let us assume, in a classical framework, the following dispersion relation for photons

$$E^2 = \bar{p}^2 + \alpha p^n ,$$

(5)

with $\alpha > 0$ and $n \geq 2$, related to a new wave equation in vacuum. Then, the translation-invariant retarded Green function can be evaluated \[11\]

$$G(x - x') = \int d^4 p \frac{e^{-ip(x-x')} \alpha}{\omega^2 - \bar{p}^2(1 + \alpha p^{n-2})}$$

(6)

and the electromagnetic potential generated by a source $J_\mu$ is given by

$$A_\mu(x) = \int d^4 x' G(x - x') J_\mu(x') .$$

(7)

Due to the shift of the poles in Eq. \[3\], the standard retarded Green function \[11\] has the following corrections

$$G_{corr}(x - x') \sim \alpha (t - t')^{n-1} \times \delta^{n-1} \left( t - t' - \frac{|\vec{x} - \vec{x}'|}{c} \right),$$

(8)

where $\delta^n$ is the $n-$derivative of the $\delta-$function. In turn, the derivatives of the $\delta-$function introduce corrections to the electric and magnetic fields which depend on the derivatives of the acceleration $d^n \beta / dt^n$. In the standard case the electric field is proportional to the source acceleration, $E \sim \dot{\beta}$, for $n = 2$ the correction is proportional to $\ddot{\beta}$ and so on, and the final result is a strong modification of the synchrotron radiation spectrum due to the relation in Eq. \[7\].

3. NONCOMMUTATIVE ELECTRODYNAMICS

The previous heuristic argument suggests that the correlation between LV terms in the photon dispersion relation and dynamical effects should be treated in a well defined framework which takes consistently into account both these crucial ingredients.

An example of such a dynamical scheme is the noncommutative electrodynamics (NCED) \[12\] where the violation of Lorentz invariance and the dynamical corrections to the standard processes are controlled by the same parameters. The introduction of noncommuting space-time coordinates implies a deformed product between noncommuting fields, called Moyal *-product \[2\]. The Seiberg-Witten map \[13\] allows to write the action of NCED in terms of the standard product of usual commutative field. At first order in $\theta$ and in the vacuum, one has

$$I = -\frac{1}{4} \int d^4 x [F_{\mu\nu} F_{\mu\nu} - \frac{1}{2} \theta^{\alpha\beta} F_{\alpha\beta} F_{\mu\nu} F_{\mu\nu} + 2 \theta^{\alpha\beta} F_{\alpha\mu} F_{\beta\nu} F_{\mu\nu}] ,$$

(9)
where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. In the presence of a background magnetic field $\vec{b}$ the plane wave solutions exist. Waves propagating transversely to $\vec{b}$ enjoy a modified dispersion relation

$$\omega/c = k(1 - \vec{\theta}_T \cdot \vec{b}_T)$$

while waves propagate along the $\vec{b}$ direction at the standard speed of light $c$. One can also introduce an external source $J_\mu = (pc, \vec{J})$ and study the modified Maxwell equations, as done in [12].

In the calculation of the synchrotron radiation (with the standard setting of a charged particle moving in the plane $(x, y)$ with speed $\vec{v}_0 = (0, 0, b), \vec{b} = (0, 0, b), \vec{\theta}_0^k = e^{i k \theta} \vec{b}_k$ and $\theta = (0, 0, \theta)$, $\lambda = 2\theta b$, due to the shift of the poles in the modified dispersion relation Eq. (10), the retarded Green function turns out to be

$$G(\vec{R}, \tau) \sim \frac{1}{R} \delta(\tau - R/c) - \lambda \left( \frac{1 - c\tau/R}{R} \delta(\tau - R/c) + \frac{\tau}{R} \delta'(\tau - R/c) \right)$$

where $\tau = t - t'$. The first term is the standard result. The correction to the electric field is due to the second term in Eq. (11).

$$E_{corr}^2 \sim \lambda \left[ \frac{1}{c(1 - \vec{n} \cdot \vec{\beta})} \right] \frac{d}{dt'} \left( \frac{1}{c(1 - \vec{n} \cdot \vec{\beta})} \frac{d}{dt'} \frac{\vec{n}c(t - t')}{(1 - \vec{n} \cdot \vec{\beta}R)} \right)_{ret}$$

and contains a term proportional to the derivative of the acceleration. Let us note that in the previous formulas $\lambda$ is the parameter which describes both the violation of Lorentz invariance and the modified dynamics.

For the synchrotron radiation observed far from the source, in the limit $\beta \rightarrow 1$, and for frequencies in the region $\omega_0 << \omega << \omega_c = 3\omega_0\gamma^3$ (where $\omega_0$ is the cyclotron frequency), the correction to the spectrum $I(\omega)$ at fixed emission angle, is [13]

$$X \equiv \frac{dI(\omega)/d\Omega}{dI(\omega)/d\Omega }_{\lambda=0} \sim 1 + 10 \left( \frac{\omega_0}{\omega} \right)^{2/3} \frac{\lambda\gamma^4}{\rho}$$

and it is potentially large since the coefficient of the parameter $\lambda$ is proportional to $\gamma^4$ (with $\gamma = 1/\sqrt{1 - \beta^2}$) and depends on the frequency. Moreover there is a $O(\lambda)$ correction to the emission angle.

In NCED one can also evaluate the modification to the Cherenkov radiation (for a medium with magnetic permeability $\mu = 1$, and electric permeability $\epsilon = \epsilon(\omega)$) and, also in this case, the energy radiated per unit distance along the path of the charged particle at fixed frequency, i.e. $d^2E/(dx d\omega)$, which turns out as

$$\frac{d^2E}{dx d\omega} \sim \frac{\omega \epsilon(\lambda - 1 + \beta^2 \epsilon)}{c^2 (1 + \epsilon^2)}$$

has a quite different form with respect to the standard case [15]. Moreover one can show that, due to noncommutative effects, the Cherenkov radiation in vacuum ($\epsilon = 1$) is possible if $\lambda > (1 - \beta^2)/\beta^2$ and, in this case, the emission angle is fixed by

$$\cos^2 \theta = \frac{\beta^2(1 + \lambda) - 1}{\beta^2(\lambda - 1 + \beta^2)}$$

From the previous discussion it seems clear to us that, as in NCED, one is able to put limits on the LV parameters by dynamical processes involving radiation, only by consistently considering the modified dispersion relations and the modified dynamics.

4. BOUNDS ON THE NONCOMMUTATION PARAMETER

From the discussion in the previous Section it seems natural to ask if it is possible to put bounds directly on the noncommutativity parameter $\theta$ by the modification of processes involving high energy gamma rays in NCED, but one need to be careful as NCED is affected with serious problems in the quantum phase. These problems are related to a peculiar correspondence between the ultraviolet and infrared perturbative regimes (see e.g. [16], [17], [18], [19]). It is still unclear whether this correspondence is an artifact of the perturbative calculations or a more fundamental (hence more serious) problem. For instance in [20] it is shown that there are noncommutative scalar field theories where the connection is actually absent. These facts evidently mean that the noncommutative quantum theory is still a “work in
progress”, and in the above calculations we used the (more safe) classical approach which, in the limit \( \theta \to 0 \), reproduces the standard results [12].

With this warning one can go beyond the use of NCED just as an interesting theoretical laboratory, and analyze the possibility that the \( O(\theta) \) corrections to the processes involved in high energy gamma rays astrophysics, may improve the present bound on the noncommutativity parameter \( \theta < (10 \text{TeV})^{-2} \) [21].

The parameter \( \lambda = 2b\theta \), which controls the dynamical effects and the LV terms, depends also on the background magnetic field \( b \). Since galactic and extragalactic magnetic fields are weak, there is no improvement on the present bound by the kinematic modification of the thresholds for the (not forbidden) processes: \( \gamma \to e^+e^- \), \( \gamma\gamma \to e^+e^- \), \( e^- \to \gamma e^- \) [22].

On the other hand, with the present limit on \( \theta \) the correction to the synchrotron spectrum is

\[
X = \frac{dI(\omega)/d\Omega}{dI(\omega)/d\Omega|_{\theta=0}} < 1 + \left( \frac{\omega_0}{\omega} \right)^{2/3} b \times 10^{-21} \times (E_{\text{electron}}(\text{MeV})/(MeV))^4
\]

where \( b \) is the magnitude of the magnetic field expressed in Tesla. For a 20 TeV electron the correction \( X_{\text{corr}} \) is

\[
X_{\text{corr}} = \left( \frac{\omega_0}{\omega} \right)^{2/3} b \times 10^8
\]

and the improvement on the present bound requires strong magnetic fields.

5. CONCLUSIONS

According to the brief present analysis one can conclude that:

1) The bounds on the Lorentz violating scale based on the purely “kinematic scheme” reviewed in Stecker’s talk [4] are robust because they are independent from the underlying dynamics.

2) The bounds from the “cocktail analysis”, which mix kinematic and dynamical effects, are model dependent and rely on dynamical assumptions. Such bounds require full dynamical calculations in the effective field theories consistently with the modified dispersion relations.

3) In order to obtain new limits on the scale of noncommutativity from electrodynamic processes one needs to consider the gamma ray spectra in strong magnetic fields as for instance in a pulsar. For this type of analysis one has to take into account the whole noncommutative effects in the standard chain structure of the radiation process [23].

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