“Quasiparticle Charge” in Superconductors: Effect of Mott Physics or Hidden Order Parameter?

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The renormalization factor, dubbed “quasiparticle charge” \( Z_e \), of the coupling between the supercurrent and quasiparticles was analyzed in the context of high temperature cuprates in a recent paper by Ioffe and Millis.\(^1\) They observed that \( Z_e \) in cuprates deviates from the BCS value (\( Z_e = 1 \)), which was interpreted as the proximity effect near a Mott insulator. Here we show that the deviation from \( Z_e = 1 \) can occur, in general, even in the absence of quasiparticle interactions, when the superconducting order coexists with another order parameter with the same internal symmetry. As an example, we compute the coefficient of the linear temperature dependence of the superfluid density when the d-wave superconducting state coexists with the orbital antiferromagnetic state (d-density wave), and find that \( Z_e \) varies from \( 1/\sqrt{2} \) to 1.

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I. INTRODUCTION

The behavior of charge transport in both the normal and superconducting phases of cuprates is one of the puzzling issues in high temperature superconductors. In a recent paper, Ioffe and Millis discussed the effect of proximity to a Mott insulator on charge transport in the superconducting state of cuprates.\(^1\) They pointed out that the renormalization factor, dubbed “quasiparticle charge” \( Z_e \), of the the coupling between the supercurrent and quasiparticles is renormalized by quasiparticle interactions and contains useful information about strong correlations near a Mott insulator.\(^1\)\(^2\)\(^3\)

Even though the real quasiparticle charge is not conserved in superconductors and \( Z_e \) is not the conventionally defined charge, we will still use “quasiparticle charge” to call the renormalization factor mentioned above for convenience. As pointed out in the past\(^1\)\(^2\)\(^3\), \( Z_e \) appears in the linear temperature dependence of the superfluid density in d-wave superconductors (dSC). The low temperature superfluid density of d-wave superconductors is given by

\[
\rho_s(T) = \rho_s(0) - \alpha T, \quad (1)
\]

where

\[
\alpha = \frac{(\ln 2)Z_e^2 v_F}{2\pi v_\Delta}. \quad (2)
\]

Here \( v_F \) is the Fermi velocity and \( v_\Delta \) the gap velocity defined as \( v_\Delta = \Delta_0 a/\sqrt{2} \), where \( \Delta_0 \) is the maximum value of superconducting order parameter.

The behavior of the superfluid density in cuprates has been an important subject of debate. Lee and Wen observed that, in cuprates, \( \alpha \) does not strongly depend on the doping concentration, \( x \), while the zero temperature superfluid density is proportional to \( x \).\(^4\)\(^5\)\(^6\) This would be consistent with the phenomenology that the quasiparticles in the superconducting state of cuprates still couples to external electromagnetic field in the usual BCS manner even in the presence of strong correlation near a Mott insulator.\(^6\) On the other hand, two different strong coupling approaches to the t-J model lead to different behaviors of the superfluid density. The U(1) mean field theory predicts \( \alpha \propto x^2 \) as well as \( \rho_s(0) \propto x \) \(^7\)\(^8\), implying that the “quasiparticle” charge goes to zero as the Mott insulator is approached while the SU(2) gauge theory lead to \( \alpha \sim O(1) \) with the same \( \rho_s(0) \).\(^9\)\(^10\)

Later on, Millis et al\(^1\) pointed out that \( \alpha \) is in general renormalized by quasiparticle interactions even in the conventional d-wave superconductors. Millis et al further argued that the deviation from the standard BCS value \( Z_e = 1 \) should be regarded as an evidence of the strong correlation present near a Mott insulator.\(^1\)\(^2\)\(^3\) Recently, Ioffe and Millis extracted the values of \( v_F \) and \( v_\Delta \) from the angle resolved photoemission spectroscopy (ARPES) and the specific heat data of cuprates, and obtained the value of \( Z_e \) for several samples of cuprates. They found that \( Z_e \) varies from 0.6 to 1 in cuprates, and argued that the deviation from the BCS value (\( Z_e = 1 \)) is due to strong quasiparticle interactions near a Mott insulator.

In this paper, we show that the deviation from the BCS value of \( Z_e \) can also occur within the weak coupling approach without taking into account the quasiparticle interactions, when a superconducting state coexists with another ordered state with the same internal symmetry. In this case, the value of \( Z_e \) contains informations about the additional order parameter. As an example, we consider a system where the d-wave superconducting order coexists with the commensurate orbital antiferromagnetic order (d-density wave).\(^1\)\(^2\) The quasiparticles in this state have the Dirac spectrum near the nodes as the case of dSC and the superfluid density has the linear temperature dependence. By computing the coefficient of the linear temperature dependence of the superfluid density, we find that \( Z_e \) in Eq.\(^2\) varies from \( 1/\sqrt{2} \) to 1. The deviation from the BCS value (\( Z_e = 1 \)) in this case is not due to the quasiparticle interaction, but due to the existence of the additional order parameter. The super-
fluid density of the dSC coexisting with the dDW order was previously studied \cite{13, 14}, but the question of the renormalization factor \( Z_v \) was not addressed. Although we studied a specific system, our results can be applied to more general situation where a superconducting order coexists with another order parameter with the same internal symmetry.

In the next section, we compute the current-current correlation function in the coexisting dSC and dDW states. Those who are not interested in the details can skip the next section and jump to section III. In the third section, we provide the analytical expression of \( \alpha \) and extract the value of \( Z_v \). We compare our results with the consequences of other existing theories and discuss further implications in the final section.

II. SUPERFLUID DENSITY IN THE COEXISTING DDW AND DSC STATES

In the phase where the dSC and dDW coexist, the quasiparticles keep the Dirac spectrum as the case of dSC. In particular, when the chemical potential \( \mu \) is zero or \( \mu < T \ll \min(\Delta_0, W_0) \), the low energy nodal spectrum is well described by changing the gap velocity \( v_\Delta \) to \( \tilde{v}_\Delta = \sqrt{v_\Delta^2 + v_W^2} \) where \( v_W = W_0/\sqrt{2} \) and \( W_0 \) is the maximum value of the dDW order parameter. In this case, the gap velocity measured in ARPES will correspond to \( \tilde{v}_\Delta \). The density of states will be also modified, but in a trivial fashion: \( v_\Delta \) is replaced by \( \tilde{v}_\Delta \). Thus one may expect that the coefficient of the linear temperature dependence of the superfluid density is simply given by \( \alpha = (\ln 2)v_F/(2\pi \tilde{v}_\Delta) \). If this is the case, we will not be able to extract an independent information about \( v_W \) and practically \( Z_v = 1 \); experiments will not be able to tell the difference between the simple dSC state and the coexisting dSC/dDW states. We show below that this is not the case. \( Z_v \) has an independent information about \( v_W \) and in general it is not unity. The deviation from \( Z_v = 1 \) is in fact due to the existence of the dDW order parameter.

We derive the result described above by directly computing the current-current correlation function in a mean-field theory. We start from the mean-field Hamiltonian given by

\[
H = \sum_{k,\sigma} (c_{k,\sigma}^\dagger c_{k,\sigma} + iW_k c_{k,\sigma}^\dagger c_{k+Q,\sigma}) + \sum_{k,\sigma} \Delta_k c_{k,\sigma}^\dagger c_{-k,-\sigma} + h.c.,
\]

where

\[
\epsilon_k = -\frac{t}{2} (\cos k_x + \cos k_y),
\]

\[
\Delta_k = \frac{\Delta_0}{2} (\cos k_x - \cos k_y),
\]

\[
W_k = \frac{W_0}{2} (\cos k_x - \cos k_y).
\]

Here \( \Delta_k \) and \( iW_k \) are the order parameters of the dSC and dDW respectively. Using the Nambu basis defined as follows,

\[
\Psi_k = (c_{k,\uparrow}^\dagger, c_{k+Q,\uparrow}^\dagger, c_{-k,\downarrow}, c_{-k-Q,\downarrow}),
\]

the current operator can be written as

\[
j_\alpha(q) = t \sum_k \sin (k_\alpha + q_\alpha/2) \Psi_k^\dagger (\rho_3 I) \Psi_{k+q} + W_0 \sum_k \sin (k_\alpha + q_\alpha/2) \Psi_k^\dagger (i\rho_2 \sigma_3) \Psi_{k+q},
\]

where \( \alpha = x, y \). Here \( \rho_3 \)'s and \( \sigma_3 \)'s are the Pauli matrices and \( \rho_3 \sigma_3 \equiv \rho \otimes \sigma_3 \).

The superfluid density can be computed from the current-current correlation function, \cite{13}

\[
\rho_s(T) = \langle -K_x \rangle - \lim_{q \to 0} \lim_{\nu \to 0} \Lambda_{xx}(q, \nu),
\]

where \( \langle -K_x \rangle \) is the diamagnetic contribution and \( \Lambda_{xx}(q, \nu) \) is the paramagnetic current-current correlation function. The diamagnetic part can be computed as

\[
\langle -K_x \rangle = \sum_k \frac{\epsilon_k + W_k}{\Omega_k} \sin k_x \times \left( \frac{\Omega_k - \mu}{E_{1k}} \tanh \frac{E_{1k}}{2T} + \frac{\Omega_k + \mu}{E_{2k} \tanh \frac{E_{2k}}{2T}} \right)
\]

where

\[
E_{1k,2k} = \sqrt{(\Omega_k \pm \mu)^2 + \Delta_k^2},
\]

\[
\Omega_k = \sqrt{\epsilon_k^2 + W_k^2}.
\]

The paramagnetic contribution contains the information about \( Z_v \) and can be obtained from

\[
\Lambda_{xx}(q, i\nu_m) = i^2T \sum_{k} \sin k_x \sin (k_x + q_x) \text{Tr}[G(k, i\omega_n)(\rho_3 I)G(k + q, i\omega_n + i\nu_m)(\rho_3 I)]
\]

\[
+ W_0^2T \sum_k \sin k_x \sin (k_x + q_x) \text{Tr}[G(k, i\omega_n)(\rho_2 \sigma_3 I)G(k + q, i\omega_n + i\nu_m)(\rho_2 \sigma_3 I)],
\]

where

\[
G^{-1}(k, i\omega_n) = i\omega_n I + \epsilon_k \rho_3 \sigma_3 - \mu I \sigma_3 - W_k \rho_2 I + \Delta_k \rho_3 \sigma_1.
\]
Here $\nu_m = 2m\pi T$ and $\omega_n = (2n + 1)\pi T$ are the bosonic and fermionic Matsubara frequencies. Notice that the cross-terms proportional to $tW_0$ cancel out each other.

After the frequency sum, the correlation function has the following form. The first term, $\Lambda_{xx}^t$, of Eq. (14) which comes from the conventional current proportional to $t$ is obtained as follows by taking $\nu \to 0$ first and $\mathbf{q} \to 0$ later.

\[ \Lambda_{xx}^t = \sum_k i^2 \sin k_x \sin (k_x + q_x) \left[ \frac{1}{2} \left( 1 + \frac{A_1}{E_{1k}E_{1k+q}} - \frac{2W_k^2}{\Omega_k^2} \left( 1 + \frac{B_1}{E_{1k+q}} \right) \frac{f(E_{1k+q}) - f(E_{1k})}{E_{1k+q} - E_{1k}} \right) ight. 
+ \left. \frac{1}{2} \left( 1 - \frac{A_1}{E_{1k}E_{1k+q}} + \frac{2W_k^2}{\Omega_k^2} \left( 1 - \frac{B_1}{E_{1k+q}} \right) \frac{f(E_{1k+q}) + f(E_{1k}) - 1}{E_{1k+q} - E_{1k}} \right) \right] \],

where $f(x) = 1/(e^x/T + 1)$ is the Fermi function and

\[
A_{1,2} = \Delta_k^2 + \epsilon_k^2 - W_k^2 + \mu^2 + \frac{2\mu^2}{\Omega_k},
\]
\[
B_{1,2} = -\epsilon_k^2 - W_k^2 \pm \mu \Omega_k.
\]

On the other hand, the second term, $\Lambda_{xx}^W$, of Eq. (12) comes from the additional contribution to the current due to the existence of the dW order parameter. After taking $\nu \to 0$ first and $\mathbf{q} \to 0$ later, we get

\[ \Lambda_{xx}^W = \sum_k W_0^2 \sin (k_x) \sin (k_x + q_x) \left[ \frac{1}{2} \left( 1 + \frac{A_1}{E_{1k}E_{1k+q}} + \frac{2W_k^2}{\Omega_k^2} \left( 1 + \frac{B_1}{E_{1k+q}} \right) \frac{f(E_{1k+q}) - f(E_{1k})}{E_{1k+q} - E_{1k}} \right) \right. 
+ \left. \frac{1}{2} \left( 1 - \frac{A_1}{E_{1k}E_{1k+q}} - \frac{2W_k^2}{\Omega_k^2} \left( 1 - \frac{B_1}{E_{1k+q}} \right) \frac{f(E_{1k+q}) + f(E_{1k}) - 1}{E_{1k+q} - E_{1k}} \right) \right] \]

where

\[
A_{1,2} = \Delta_k^2 - \epsilon_k^2 + W_k^2 - \mu^2 \pm \frac{2\mu^2}{\Omega_k}.
\]

Therefore, the superfluid density can be obtained from Eq. (13) where $\Lambda_{xx} = \Lambda_{xx}^t + \Lambda_{xx}^W$.

### III. Value of the “Quasiparticle Charge” from Superfluid Density

At low temperatures, the leading paramagnetic contribution comes from the nodal quasiparticles and we can obtain the low energy quasiparticle dispersion by expanding $\epsilon_k$ near $\mu$, and $\Delta_k, W_k$ near the node as follows.

\[
\epsilon_k - \mu = v_F p_+,
\]
\[
\Delta_k = v_A p_-,
\]
\[
W_k = v_W p_-,
\]

where $p_+ = (p_x + p_y)/\sqrt{2}$ and $p_- = (p_x - p_y)/\sqrt{2}$ with the momentum, $\mathbf{p}$, measured from $\mathbf{k}_F$. Here, $v_F = ta/\sqrt{2}$, $v_A = \Delta_0/\sqrt{2}$, and $v_W = W_0/\sqrt{2}$. When $\mu = 0$ or $\mu < T$, the quasiparticle dispersion can be well described by

\[
E_{1k,2k}^2 = v_F^2 p_+^2 + (v_A^2 + v_W^2) p_-^2.
\]

Notice that the corresponding density of states has the following form.

\[
N(E) = \frac{E}{2\pi v_F \sqrt{v_A^2 + v_W^2}}.
\]

Therefore, the specific heat as well as the measured gap velocity in ARPES will be given by the combination of $v_W$ and $v_A$ through $\sqrt{v_A^2 + v_W^2}$. Now it is clear that the leading linear temperature dependence of the superfluid density comes from the terms with $df(E_{1k})/d(E_{1k})$ factor. The coefficient of the linear temperature dependence, $\alpha$, is obtained as

\[
\alpha = \ln 2 \frac{v_F}{2\pi} \left( 1 - \frac{v_F^2}{2(v_A^2 + v_W^2) + \mathcal{O}(W_0^2 t^2)} \right).
\]
Comparing Eq.\ref{eq:gap_velocity} and Eq.\ref{eq:gap_velocity2}, and replacing $v_\Delta$ by the measured gap velocity, $\sqrt{v_\Delta^2 + v_W^2}$, we find

$$Z_e^2 = \frac{1 - \frac{v_\Delta^2}{2(v_\Delta^2 + v_W^2)}}{1 - \frac{\mu^2}{2(v_\Delta^2 + v_W^2)}} = \frac{1}{Z_e^2}$$

where we neglect $O(W_0^2/t^2)$ terms. From the above result, one can easily see that $1/2 \leq Z_e^2 \leq 1$. If $W_0 \gg \Delta_0$, then $Z_e^2 \rightarrow 1/2$, while in the opposite case $\Delta_0 \gg W_0$ or $W_0 \rightarrow 0$, we get $Z_e^2 \rightarrow 1$. Therefore, $Z_e$ varies from $1/\sqrt{2} (= 0.71)$ to 1 which is curiously close to the values in cuprates obtained by Ioffe and Millis \cite{Ioffe}. In the discussion above, we assumed $\mu < T$. In the opposite limit, $\mu > T$, the low energy quasiparticle dispersion is better described by

$$E^2_{1k} = (v_F p^+)^2 + (v_\Delta p^-)^2$$

which does not have any information about $W_k$, the dDW order parameter. This is due to the fact that the quasiparticles recognize the presence of the dDW gap only when their energy scale becomes larger than the chemical potential. Carrying out the explicit computation of the current-current correlation function, we verified that the coefficient of the linear temperature dependence of the superfluid density is given by Eq.\ref{eq:gap_velocity2} with $Z_e = 1$.\cite{Ioffe}

Therefore, there is a cross-over in the value of $Z_e$ to unity when $\mu > T$ while $Z_e \neq 1$ for $\mu < T$. At the half-filling ($\mu = 0$), $Z_e$ is in general not unity. At any rate, if $\mu$ is smaller than $T$ in the experimentally relevant temperature range, the value of $Z_e$ is given by Eq.\ref{eq:gap_velocity2}.

\section{IV. CONCLUSION}

In the strong coupling regime, the coupling between the supercurrent and the quasiparticles can be strongly renormalized by quasiparticle interactions and lead to the deviation of $Z_e$ from the BCS value ($Z_e = 1$). Here we show that the deviation from $Z_e = 1$ can also occur in the weak coupling regime when the superconducting order parameter coexists with an additional order parameter with the same internal symmetry. We have shown that in the case of the coexisting dSC and dDW states, the “quasiparticle charge” is given by $Z_e = \left(1 - \frac{v_\Delta^2}{2(v_\Delta^2 + v_W^2)}\right)^{1/2}$ when the chemical potential $\mu$ is zero or $\mu < T$, where $v_W$ and $v_\Delta$ are the gap velocities determined by the maximum gaps of the dDW and dSC order parameters. Thus $Z_e$ varies from $1/\sqrt{2}$ to 1. The deviation of $Z_e$ from the unity is a good measure of the additional coexisting order parameter. Our result of $Z_e \neq 1$ is also applicable to the systems where a superconducting order parameter coexists with another order parameter with the same internal symmetry; e.g., $p$-wave superconductor coexisting with $p$-density wave, although the expression of $Z_e$ would be different for different systems.

Our results suggest that the determination of the value of $Z_e$ alone will not sharply distinguish two possibilities; Mott physics and the existence of a hidden or an additional order parameter. However, if there exists a superconductor far away from a Mott insulator while it exhibits $Z_e < 1$, one would strongly suspect that there might exist a hidden order parameter. The direct relevance of our results for the coexisting dSC and dDW states to cuprates has to discussed in conjunction with other experimental data and it is beyond the scope of this paper.

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