Detection of Damage Initiation and its Evolution in Insulating Brick from Non-linearity of Stress/Strain Curve

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Abstract. This paper deals with experimental verification of the present author's theory relating to estimation of damage accumulation in materials showing non-linear stress/strain curves. The theory is based on parallel elements model and gives that the second order derivative of a stress/strain curve is proportional to a probability density function of strain at which each element is damaged. Uni-axial compression test was conducted for an insulating brick and its probability density function is obtained from the non-linear stress/strain curve. The function coincides well with acoustic emission data simultaneously measured in the compression test. This fact shows that the theory is useful for quick and brief estimation of damage initiation and its evolution in insulating bricks.

1. Introduction
Ceramic materials are usually fractured in a catastrophic manner after linear elastic deformation, however, non-linear stress/strain relationship also can be seen in some materials such as polycrystalline graphite, refractories and porous materials [1-4]. Up to now, we have attributed the non-linearity to a local damage process in the materials, and therefore measured acoustic emission during a loading test in order to reveal its damage process [3,4]. Acoustic emission is certainly an excellent technique to detect damage events in materials, however we sometimes encounter the difficulties in fixing expensive measuring devices and adjusting optimal experimental conditions. Thus, for a long time, we have wanted a quick and brief technique to estimate damage initiation and its evolution in the materials.

In this paper, we develop a theory relating to estimation of damage accumulation in materials from non-linearity in stress/strain curves, and then verify the theory with an insulating brick under compression tests.

2. Theory
Consider that uni-axial loading is given to the parallel elements model, as shown in Figure 1. The loading is carried out under a constant displacement rate. Each element has its own damage strain depending on the probability density function \( f(\varepsilon) \) (viz. \( f(\varepsilon)\,d\varepsilon \) means the probability that an element is damaged at the strain from \( \varepsilon \) to \( \varepsilon + d\varepsilon \)). The initial Young's modulus of all elements is denoted by \( E \), and that after damaged is \( E' \). It follows that the constitutive equation of an element \( j \) can be expressed as below [5,6],

\[
\varepsilon_j = \int_{\varepsilon}^{\varepsilon'} \frac{1}{E} \, d\varepsilon + \int_{\varepsilon'}^{\varepsilon''} \frac{1}{E'} \, d\varepsilon''
\]
where \( \sigma_{dj} \) and \( \varepsilon_{dj} \) are the damage stress and strain of the element \( j \), respectively. For simplicity, we begin with finite number of element (viz. \( j = 1 \sim n \)). In advance, \( \varepsilon_{dj} \) may be renumbered in an ascendant order. When the applied strain \( \varepsilon \) is in the region of \( 0 < \varepsilon < \varepsilon_{d1} \), any element is not damaged, and the applied stress \( \sigma \) of the model can be expressed as below,

\[
\sigma = \sum_{j=1}^{n} V_{j} E \varepsilon = E \varepsilon
\]  

(2)

where \( V_{j} \) is the volume fraction of the element \( j \). When the applied strain \( \varepsilon \) reaches \( \varepsilon_{d1} \), the first element has just got damaged and then the applied stress is,

\[
\sigma = V_{1} \sigma_{d1} + \sum_{j=2}^{n} V_{j} E \varepsilon = V_{1} \sigma_{d1} + (1 - V_{1}) E \varepsilon
\]  

(3)

Furthermore, the applied strain \( \varepsilon \) is in the region of \( \varepsilon_{d1} < \varepsilon < \varepsilon_{d2} \), the applied stress is,

\[
\sigma = V_{1} \{ E \varepsilon + (E - E') \varepsilon_{d1} \} + (1 - V_{1}) E \varepsilon
\]  

(4)

With increasing the applied strain, each element is damaged one by one, and therefore the overall Young's modulus of the material gradually decreases according to the equations below,

\[
\sigma = \sum_{j=1}^{k-1} V_{j} E \varepsilon + \sum_{j=1}^{k-1} V_{j} (E - E') \varepsilon_{dj} + V_{j} \sigma_{dj} + \left( 1 - \sum_{j=1}^{k} V_{j} \right) E \varepsilon \quad (\varepsilon = \varepsilon_{dk})
\]  

(5)

\[
\sigma = \sum_{j=1}^{k} V_{j} E \varepsilon + \sum_{j=1}^{k} V_{j} (E - E') \varepsilon_{dj} + \left( 1 - \sum_{j=1}^{k} V_{j} \right) E \varepsilon \quad (\varepsilon_{dk} < \varepsilon < \varepsilon_{d(k+1)})
\]  

(6)

If the number \( n \) of element becomes infinite, \( V_{j} \) is approaching to infinitely small, and consequently the equation (5) is identical to the equation (6). By using the probability density function \( f(\varepsilon) \) of the damage strain (viz. \( \sum_{j=1}^{k} V_{j} \) is replaced by \( \int_{0}^{\varepsilon} f(t) dt \)), we obtain the non-linear stress/strain relationship by the following equation[5,6].

\[
\sigma = E' \int_{0}^{\varepsilon} f(t) dt + (E - E') \int_{0}^{\varepsilon} f(t) dt + E \varepsilon \left[ 1 - \int_{0}^{\varepsilon} f(t) dt \right]
\]  

(7)
After some differential operations of equation (7), we eventually obtain the following relation.

\[
f(\varepsilon) = \frac{1}{E - E'} \left[ -\frac{d^2 \sigma}{d\varepsilon^2} \right]
\]  

Equation (8) means that the second order derivative of a stress/strain curve is proportional to a probability density function of damage strain \( f(\varepsilon) \).

3. Experimental procedure

3.1. Specimen

To verify the equation (8), an insulating brick was used as a typical material showing non-linear stress/strain curve. Plate-like specimens were cut out from the insulating brick (Isolite Insulating Products Co., Ltd., type C1). Its dimensions were 50mm in width by 14mm in thickness by 65mm height. The porosity was estimated to be 57.8% from the bulk density.

3.2. Uni-axial compression test

As shown in Figure 2, uni-axial compression tests were carried out with a constant crosshead speed of 0.1mm/min. During the loading, load and displacement data were collected by a mechanical testing machine (JT-Tohshi, type SC-100H). At the same time, acoustic emission was detected by AE sensor (NF Electronic Instruments, AE-901S) attached to the surface of the specimen, and recorded by oscilloscope (Yokogawa Electric, DL7440) to obtain AE time series data.

In addition to single compression test, cyclic compression tests (viz. loading/unloading cycles) were also conducted to see how the damage accumulation was in the specimen.

4. Result and discussion

4.1. Single compression test

Figure 3(a) shows a stress/strain curve of the specimen. We see that non-linearity (convex upward) begins at the strain around 0.0025. Although we find another non-linearity (concave upward) at the beginning of loading, this is caused by unavoidable

![Mechanical testing machine](image_url)
clearance in the loading train. To eliminate this additional displacement, the stress/strain curve of the specimen is calibrated by that of large steel block. By using the equation (2), the probability density function of damage strain $f(\varepsilon)$ is obtained from the calibrated stress/strain curve, as shown in Figure 3(b). The function $f(\varepsilon)$ appears at the strain around 0.0023, and increases linearly, and finally has a maximum at 0.0030.

The acoustic emission (AE) data is also shown in Figure 3(b). The count rate of AE keeps constant at first; however, it abruptly increases at the strain around 0.0023, and then reaches the maximum at 0.0030. This change is almost the same as that in $f(\varepsilon)$, indicating that damage estimation from non-linearity in stress/strain curves is effective.

![Figure 3](image)

**Figure 3.** Stress/strain curves and AE data shown in Figure 3(b).

4.2. Cyclic compression tests

Figure 4(a) shows stress/strain curves of the specimen in cyclic compression tests. The loading/unloading cycles are three in this case.

Figure 4(b), (c), and (d) show comparison between $f(\varepsilon)$ and AE in the 1st, 2nd, and 3rd cycle, respectively. From these figures, we can see that the probability density function of damage strain $f(\varepsilon)$ coincides well with AE data in each cycle. This fact indicates that non-linear analysis of stress/strain curves provides quantitative information of damage initiation and subsequent change of the materials, equivalent to AE measurement.

![Figure 4](image)

**Figure 4.** Cyclic compression test, (a) stress/strain curves, (b) $f(\varepsilon)$ and AE in the 1st cycle, (c) $f(\varepsilon)$ and AE in the 2nd cycle, (d) $f(\varepsilon)$ and AE in the 3rd cycle.
It should be noted that damage initiation in Figure 3(b) is slightly different from Figure 4(b). This must be caused by the sampling problem because the thickness of specimens is not fully large enough to the size of microstructure. However, in this study, the authors dare to use thin specimen because clear AE data should be collected. If we use thicker specimens, we must obtain the same strain at initiation. So, we need more works to do in future, however, the fundamentals to detect damage initiation and its evolution from stress/strain curves of materials has just been shown in this study.

5. Conclusion
Uni-axial compression tests were carried out to an insulating brick to verify the present author’s theory relating to estimation of damage accumulation in materials showing non-linear stress/strain curves. Parallel elements model brings that the second order derivative of a stress/strain curve is proportional to a probability density function of strain at which each element is damaged. From the actual stress/strain curve of the insulating brick, its probability density function is gained. The function coincides well with acoustic emission data simultaneously measured in the compression test. This result shows that the theory is useful for quick and brief estimation of damage initiation and its evolution in insulating bricks.

Acknowledgement
The author greatly express their gratitude to Dr. T. Otsuka and Dr. K. Siono of Isolite Insulating Products Co., Ltd. for their materials supply.

References
[1] Gogotsi G A and Drozdov A V 1986 Refractories and Industrial Ceramics, 27 200
[2] Gogotsi G A 1988 Powder Metallurgy and Metal Ceramics, 27 908
[3] Ioka I, Yoda S and Konishi T 1990 Carbon 28 489
[4] Schmitt N, Berthaud Y and Poirier J 2000 J. Euro. Ceram. Soc. 20 2239
[5] Yasuda K, Furushima R, Matsuo Y and Shiota T 2006 Abs.Fall Meet.Ceram.Soc.Jpn. (Kofu, Sep.19-21, 2006) 19 127
[6] Yasuda K, Furushima R, Matsuo Y and Shiota T, 2007 Abs.Fall Meet.Ceram.Soc.Jpn. (Nagoya, Sep.12-14, 2007) 20 109