LETTER TO THE EDITOR

A Fundamental Problem in Quantizing General Relativity

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Abstract
We point out a fundamental problem that hinders the quantization of general relativity: quantum mechanics is formulated in terms of systems, typically limited in space but infinitely extended in time, while general relativity is formulated in terms of events, limited both in space and in time. Many of the problems faced while connecting the two theories stem from the difficulty in shoe-horning one formulation into the other. A solution is not presented, but a list of desiderata for a quantum theory based on events is laid out.

Keywords Foundations of quantum mechanics · Quantum information · Quantum mechanics · General relativity · Quantum gravity

Consider the axiomatic formalization of the two main theories in modern physics, quantum theory and general relativity (GR). Strikingly, one finds that they refer to different objects, a fact that, perhaps, has not been widely appreciated. Indeed, quantum mechanics deals with systems whereas general relativity deals with events. These are substantially different objects: a system has finite spatial extent typically, but infinite temporal extent, whereas an event has finite spatial and temporal extent. Clearly one may define an event as “something that happens to a system”, but that still considers a system as the fundamental object, whereas a theory of spacetime should be only formulated in terms of events and the system should be a derivative notion: a “succession of events”. In other words, there is a difference between a general-relativistic theory of quantum mechanics (e.g. [1,2]) and a quantum theory of general relativity. The first is formulated in terms of systems, the second in terms of events. It seems that only the first avenue has been explored so far, without yet achieving a definitive theory. This issue is related to the “problem of time” (e.g. [3–8]).

In this paper we detail this problem and present a list of characteristics that a quantum theory for events could have. This is not a solution to the problem but, hopefully, a small first step.
The formulation of quantum mechanics can be summarized by its four axioms (e.g. [9,10]): (a) The state of a system is described by a unit vector $|\psi\rangle$ in a complex Hilbert space, and the system’s observable properties are described by self-adjoint operators on that space; (b) The state space of a composite system is given by the tensor product of the spaces of the component systems; (c) The time evolution of an isolated system is described by a unitary operator acting on the system state, $|\psi(t)\rangle = U_t |\psi(t=0)\rangle$ or, equivalently, by the Schrödinger equation; (d) The probability that a measurement of a property $A$, described by the operator with spectral decomposition $\sum_a a \Pi_a$, returns a value $a$ is given by $P(a) = \langle \psi | \Pi_a | \psi \rangle$ (Born rule), where $|\psi\rangle$ is the system’s state. The rest of quantum theory can be derived from these axioms.

The centrality of the concept of “system” is clear, it appears in all axioms. Its (loose) definition is “system≡a quantum degree of freedom”. A few examples: (1) a point particle without spin which is defined solely by its position degree of freedom; (2) a particle with spin, which is a composite system composed by the position degree of freedom and its spin degree of freedom; (3) a quantum field, which is defined by the probability amplitude of having an excitation at each point in spacetime; etc. It is clear that a system is eternal, namely infinitely extended in time, since there is no prescription for any time evolution of the Hilbert spaces [11]: only the state evolves in time (in the Schrödinger picture used here). While infinitely extended in time, the system is typically not infinitely extended in space. There is a further asymmetry in the spacetime description of quantum systems: time appears in the evolution postulate (c) as a (classical) parameter $t$ which is external to the theory, not a dynamical variable. In contrast, the spatial degrees of freedom can typically be described as quantum properties. In other words, time is a post-selected (classically conditioned) quantity [12]: $|\psi(t)\rangle$ describes the state conditioned on the time being $t$ (Working in the Heisenberg picture\(^1\) does not change the substance: the conditioning is shifted to the observables.) Summarizing, a quantum system is spacetime-asymmetric in two ways: both in its spacetime extension and in the fact that its spatial properties are described as quantum-numbers (operators), while its temporal properties are described as physical parameters described by real numbers.

Incidentally, both problems are bypassed\(^2\) in quantum field theory. QFT considers systems that are infinitely extended also in space (fields), thus recovering the symmetry for the spacetime extension: fields are infinitely extended both in space and in time. It also considers space and time both as real numbers and not dynamical variables. Nonetheless, these “tricks” are not sufficient to fully quantize general relativity [17] (except in the special cases in which it is possible to give a Hamiltonian formulation

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1 The equivalence between the Heisenberg and Schrödinger pictures assumes the existence of a Hamiltonian with some regularity conditions [13] and assumes that the Hamiltonian does not have nontrivial dependence on the system’s parameters [14].

2 Despite appearances, explicit covariance can be attained in quantum field theory in general. In the Heisenberg picture, where the dynamical equations can be written in covariant form, the state implicitly refers to a specific reference frame [15]: a Heisenberg-picture state encodes the system preparation [16] which physically happens at a specific time in some reference frame. This requires a specific timeslice of spacetime to write the Heisenberg state of a field [15] (even though this state is time-independent). Nonetheless, if one considers the preparation procedure as something that can happen on a spacelike surface (a la Tomonaga-Schwinger), then a state transformed through a unitary representation of the Lorentz group encodes the preparation in another reference frame and, in this sense, relativistic covariance is maintained.
arguably, this is because properties (at a specific time) of infinitely-extended systems are ill defined in general relativity and because field operators are evolved by a global time coordinate\(^3\) which is meaningful only for few types of spacetimes. Essentially, quantum field theory recovers a symmetry between space and time by using infinitely extended systems (the fields), but this approach has not been too fruitful to achieve a full quantization of GR, arguably because QFT is (usually) constructed in the Heisenberg picture, where field operators are defined at a certain (global) time: what Dirac calls “Instant form” of QM \([19]\). Global time is, of course, not defined in GR, where the coordinate time does not in general have any physical meaning.

We will not remind here the axiomatic formulation of classical general relativity (e.g. \([20]\), Chap. 17): for our aims it is sufficient to remark that the theory relates the spacetime geometry to its energy-momentum content. The object of the theory is then spacetime, namely, events.

The tension between the two formulations is evident, and its origin is clear: quantum theory is formulated in terms of systems, general relativity in terms of events. General relativity describes spacetime, quantum mechanics describes systems. One may argue that, in light of the spectacular experimental successes of quantum theory, which has been tested to better precision than general relativity, one should try to retain the former and modify the latter. This has (loosely) been the road followed up to now by the most promising ways to quantize general relativity. Here we argue, instead, that a modification of quantum theory to accommodate events might also be a viable route. This does not necessarily entail a modification of the formulation (a–d) introduced above. For example, it has been shown that quantum theory can be extended to treat time as a dynamical variable \([12,21–33]\) in such a way that the conventional formulation is recovered by conditioning the states and observables on a measurement of time. A quantum theory formulated in terms of events, instead of systems, might be obtained analogously.

A truly quantum description of events should be able to assign a probability amplitude for an event to happen in some location at some time. In such a theory, one would have to provide predictions for events such as “at time \(t\), in position \(x\) a spin was found in the state \(|\uparrow\rangle\)”. The current quantum physics (excluding quantum field theory) is only able to provide predictions for properties of a system (e.g. a particle with spin), such as the probability that “at time \(t\), in position \(x\), the spin was found in the state \(|\uparrow\rangle\)”, a small but crucial difference. In the first case, the starting point would be the event that happens at spacetime position \(t, x\), in the second case the starting point is the definition of the Hilbert space for a system (the particle with spin). The current theory is not able to introduce a Hilbert space for events, but only for systems (although approaches such as relational quantum mechanics suggest ways to overcome this problem \([1]\)). This denotes a schizophrenic attitude: the spatial position of a system is a quantum property (“finding the particle at position \(x\) given that time is \(t\)”), but the temporal position of an event is not (“finding the particle at time \(t\) given that the position is \(x\)”). Indeed, if a system (e.g. a particle) is prepared at position \(A\) at time \(t_A\) and detected later at position \(B\) at time \(t_B\) the statement “the particle crosses

\(^3\) “There is no way of defining a relativistic proper time for a quantum system which is spread all over space” \([18]\).
intermediate positions” leads to all kinds of double-slit-type paradoxes [34] and is carefully avoided in the conventional theory. However, the conventional theory tells us that the “particles crosses all intermediate times” between $t_A$ and $t_B$. Technically, this is implicit in the wavefunction normalization at all times $\int dx |\langle x|\psi(t)\rangle|^2 = 1 \forall t$, with $|x\rangle$ position eigenstate: we cannot assign an intermediate position, but we know that at all intermediate times the particle “exists”, i.e. if one were to look, it would be found somewhere with certainty. This observation is just a restatement of the fact (noted above) that positional degrees of freedom $|x\rangle$ are treated dynamically, but temporal degrees of freedom $t$ are treated parametrically. But this is not just a problem of how the evolution postulate (c) is formulated, it is a fundamental problem of how the theory is formulated in terms of eternal systems that “exist” at all times (Fig. 1).

An immediate difficulty is encountered in devising a quantum theory for events: the identification of what is a ‘quantum event’. We should not consider it as a primitive intuitive notion: it is far from intuitive. We also cannot appeal to the textbook definitions of ‘event’ since they are useless (in my opinion). Basically two definitions appear in the literature. The first defines ‘event’ as a point in spacetime, where spacetime is the set of events, e.g. [35,36]. The obvious circularity of such definition implies that it is taken as a primitive, undefined notion. The second defines ‘event’ as an intersection of world-lines, e.g. [20, Sect. 1.2], which provides a physical content to the definition in order to bypass the ‘hole argument’. Unfortunately, we cannot use it in the quantum realm, where world-lines (trajectories) are meaningless. Nonetheless, a closely related concept, spacetime coincidences, has been proposed [31, Sect. 5.1.5], in the quantum gravity context. The notion of ‘quantum event’ connected to it is a ‘joint eigenstate of
a complete set of kinematical observables’ [31, Sect. 5.2.1]. While a good definition for the aims of canonical quantizations of gravity [1], it is still based on properties of quantum systems and it does not reduce to a classical event in the classical limit [37,38] of coherent states. 4

We do not have a good definition for a quantum event, but for the sake of clarification we can introduce a definition “quantum event ≡ something that happens to a quantum system”. As specified above, this is not entirely acceptable, because it retains the centrality of the system which we want to abandon, but at this stage we are unfortunately unable to provide a better definition.

We now list the properties that a quantum theory for events could possess. We emphasize again that we do not advocate abandoning the conventional quantum formulation (a–d) given above: that formulation must be retained in the appropriate limits, e.g. the case in which experiments are timed with an external (classical) clock. The quest for a quantum theory for events is closely connected to the time problem [3–8].

1. Quantum events axiom: the ‘states and observable axiom’ (a) should be replaced by an axiom referring to events rather than systems, presumably by introducing a ‘Hilbert space for events’. This will hopefully permit a covariant formulation of the theory in which space and time are treated symmetrically.

2. Multiple events axiom: the tensor product structure of the ‘multiple systems axiom’ (b) should be replaced by some mathematical construction that is able to describe multiple events. A quantum system should be introduced as a derivative notion, as a succession of quantum events.

3. Dynamics axiom: the privileged role of “time” in the ‘evolution axiom’ (c) must be avoided. This might require a shift in the philosophy of the theory [31,39], since quantum theory is formulated as a dynamical theory: given the initial conditions and given the dynamics (the Hamiltonian), the theory makes predictions or retrodictions in time. General relativity in its covariant formulation 5 is not a dynamical theory in the same sense [31]. At the very least, quantum theory should be modified to accommodate a multi-fingered time, so that an operational observable meaning (proper time) can be attached to ‘time’.

Comments on the above desiderata:
(1) A quantum theory for events should be able to describe events that happen at a quantum superposition of different times, and even quantum superpositions of different causal orderings of events [32,33,44–51]. Conventional quantum theory is unable to describe events in superposition [52–54], although it seems possible to create them experimentally [55–57], namely to create situations where we have a coherent superposition of events such as “the atom emitted the photon at time $t_1$ + the atom emitted the photon at time $t_2$”. The possibility of superposing events has the important implication

4 Indeed its interpretation becomes unclear if one uses, as a complete set of observables, the momenta of all the particles. Even the limitation to position observables will not help in the case of multiple systems: it would identify multiple points in spacetime, the positions of particles conditioned on what a clock shows.
5 Of course, general relativity can also be given a non covariant formulation (geometrodynamics), and also a Hamiltonian formulation [20,35].
6 Multi-fingered time refers to the fact that different observers, following different world lines, experience different proper times, e.g. [40–43].
that properties incompatible to ‘spacetime position’ exist. For example, the conjugate property will be the event’s energy-momentum, namely the event’s generator of spacetime translations. This is a property that must be attached to a quantum event because of quantum superposition and of complementarity. In the specific case where an event refers to a transformation of a system’s state, we can connect the event’s time extent to the system’s energy through the energy-time [58–60] uncertainty relations. In contrast, the position-momentum [61,62] uncertainty relation relates a system’s spread in momentum to its position uncertainty and not to its spatial extent (this asymmetry prevents the formulation of a covariant uncertainty relation [63]). However, in the case of more general notions of events, the physical interpretation of the event’s energy-momentum is unclear: in the classical case, there is no energy or momentum necessarily associated to an event, but one could assign to it the total energy-momentum of the systems whose trajectories intersection defines the event. Similarly, in the quantum case, in the spirit of relationalism of “spacetime coincidences” [31, Sect. 5.6], an intriguing possibility is to connect the energy-momentum of the event to the energy and momentum of the (quantum) reference frame system employed to give a physical significance to the quantum event. In this respect, it has been shown that defining time as “what is shown on a clock” and taking into account the clock’s energy, one can introduce a dynamical time variable [12,21,22]. A similar procedure can be implemented for space [64,65], but it is not clear whether these methods can be applied to spacetime [66]. In the context of quantum field theory, a quantum state for a spacetime region was proposed in the general boundary formulation as an extension of the quantum state of the field at a certain time [32,33] and a way to build spacetime from the concept of separability was explored in [67].

(2) The tensor product can be adapted to describe multiple events [49,50,68–70], but arguably it is not the natural way to do it: the main reason for tensor products in quantum theory is that they give the correct law of composition of probability amplitudes for measurements on multiple systems, but they will not give the correct composition law for measurements on multiple events, because later events may be influenced by earlier ones. In particular, a description of a quantum system as a “succession of quantum events” would require properties that cannot be easily captured through a tensor product because properties of a system at different times cannot be accessed independently [68] (a measurement at one time influences successive measurements). Even when there is no causal connection, different observers can assign different temporal ordering to spacelike separated events, so they will give a completely different quantum account of their spacetime description [18]. The tensor product gives the correct statistics for causally unrelated events, but not for causally related ones, where tricks such as post-selected teleportation (also called Choi–Jamiolkowski extension) [68,70,71] or procedures based on path integrals [32,33] must be used instead to obtain the correct statistics.7

(3) One may expect that the measurement problem is a major drawback in the change of paradigm [39] for quantum theory in going from a dynamical theory that makes predictions based on an initial state and on a dynamics to a theory that describes the

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7 Incidentally, it appears that post-selected teleportation and the path integral formulation are equivalent procedures [72].
whole history of the system. In truth, this problem can be sidestepped by carefully avoiding any interpretation of the “wavefunction collapse” as a dynamical transformation \[18,31\] and by using von Neumann’s description of a measurement apparatus as a dynamical coupling of some memory degree of freedom with the system being measured \((73, \text{Chap. VI})\). In this way, a complete “history state” can be described, and the correct measurement statistics and multiple-measurement correlations can be recovered by simply using the Born rule as shown in \([12]\), without having to invoke changes to the measurement axiom (d). [This approach bypasses most of the interpretation-dependent issues that relate to the collapse of the wavefunction.]

A successful quantum theory for events would presumably be one that satisfies the above requirements 1–3 and, concurrently, recovers the conventional axiomatic formulation (a–d) in the appropriate limits, e.g. when defining a system as a succession of events. Promising and clever theories that describe events in quantum mechanics have appeared in the literature, for example \([46,49–51,68,69,74–78]\), but none of them satisfy all these requirements.

A motivation that it is quantum theory rather than general relativity that may need reformulating comes from the universe’s expansion (more generically, from FLRW metrics). Indeed, the number of events in today’s universe is vastly larger than immediately after the big bang. Nonetheless our current narrative in terms of systems rather than events forces us to say that the system (i.e. the number of quantum degrees of freedom in the universe) is unchanged, since Hilbert spaces cannot evolve So (i) a larger number of events ‘happen’ to the same number of systems. Similarly problematic is black hole evaporation: since most of the Hawking radiation is composed of low-energy photons, whether or not the initial information is preserved, the process of creating a black hole from matter and waiting for its evaporation “creates” degrees of freedom. With our current theory, this can be described only by saying that (ii) these degrees of freedom are only “activated” and were present from the start in some previously existing system. While the statements (i) and (ii) may be enforced technically by appealing to the continuous nature and to the infinite dimensionality of the Hilbert space of quantum field theory, it is a rather silly narrative. It would be much better to describe these phenomena in terms of a variable number of events rather than in terms of fixed, eternal systems (otherwise most systems would be “inactive” and useless for large intervals of time). A further motivation for an event-based quantum theory is the existence of vacuum solutions to Einstein’s equations, namely universes made only of spacetime, with no “systems”. Without a prescription for quantum events, those universes cannot have a quantum description (admittedly, this impossibility might be a feature: in such universes no observers can presumably exist). The definition of event used here as “a quantum degree of freedom” implies the presence of an external spacetime because quantum states are only defined at a certain time (in the Schroedinger picture) or quantum observables are only defined at a certain time (in the Heisenberg picture). Then, there is no way that a vacuum solution of Einstein’s equations that ‘contains’ only spacetime and no ‘internal’ systems can be described in quantum theory, unless one uses a theory that quantizes space itself. In other words, when using Dirac’s Instant form \([19]\) the time degree of freedom is saturated (post-selected, classically conditioned) and cannot be considered as a quantum degree of freedom: so a
quantization of pure spacetime in the Instant form (or in any other forms) is hopeless
and one can, at most, quantize only space.

If one were successful in quantizing general relativity, arguably a good testbed for
it would be to provide a description of highly pathological spacetimes, such as the
Gödel one [79]. Indeed, quantum theory is unable to deal with closed timelike curves
without a profound modification [71,72,80] and some proposed approaches encounter
difficulties if they are present, since in this case the Hawking-Malament theorem [81]
does not hold [48]. Since these pathological spacetimes are well described within the
theory of general relativity, then we should expect that this description also carries
over to the quantized version of general relativity, since one expects that the latter
theory is more general than its classical limit.

In conclusion, general relativity and quantum theory could be merged more easily
if one were to go beyond their current formulation in terms of events and systems
respectively. We have suggested a possible roadmap 1–3 for a new quantum theory to
overcome this problem while still retaining the formulation (a–d) that is commonly
accepted.

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