Gluon evolution at low $x$ and the longitudinal structure function

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Abstract

We obtain an approximate analytical form of the gluon distribution using the GLAP equation with a factorization ansatz, and test its validity by comparing it with that of Gluck, Reya and Vogt at low $x$ regime. We also present calculations of the longitudinal structure functions.

1 Introduction

In deep inelastic scattering we can directly study the structure of the proton, particularly the parton distributions $[1, 2, 3]$. The perturbative QCD gives the $Q^2$ evolution and asymptotic limits of the structure function. More recently, the study of structure functions at low $x$ $[4, 5, 6, 7, 8, 9, 10, 11]$ has become topical in view of the high energy collider like HERA $[12, 13]$ where previously unexplored small $x$ regime is being reached. In the small $x$ regime gluons are expected to be directly measurable. This expectation has led to several approximate phenomenological schemes $[14, 15, 16, 17, 18]$. Specifically measurement of longitudinal structure function $F_L$ has long been advocated $[1, 19, 20, 21, 22]$ as a direct probe of the gluon density at small $x$. There is even helpful suggestion $[14, 15, 21]$ that precise measurement of $F_L$ should indeed be possible at HERA regime $x \leq 10^{-2}$ and $Q^2 \sim 10 \sim 100 GeV^2$. 


The present paper aims at obtaining an approximate analytical form of gluon distribution using GLAP equations [23, 24, 25]. An additive assumption on the way to this is the factorization of the $x$ and $t(t = lnQ^2/\Lambda^2)$ dependence of the gluon density. We test the validity of the assumption by comparing with the leading gluon density of Gluck, Reya and Vogt (LO-GRV) [26]. We also use our results to calculate $F_L(x, Q^2)$ using its relation with gluons [14, 15] within the range of validity of our assumption and compare with those of collinear [20] and $K_T$-factorisation [22] approaches.

2 $x$ and $t$ evolution of the gluons

We start our derivation, taking only the leading term of the gluonic kernel of the GLAP equations [23, 24, 25]

$$\frac{\partial G(x, t)}{\partial t} = \frac{3\alpha_s(t)}{\pi} \left[ \left\{ \left( \frac{11}{12} - \frac{N_f}{18} \right) + \ln(1 - x) \right\} G(x, t) + \int_x^1 dx \left( \frac{zG(x, t)}{1 - z} - \frac{G(x, t)}{1 - z} \right) \right]$$

(1)

where $G(x, t) = xg(x, t), \alpha_s(Q^2) = \frac{12\pi}{33 - 2N_f} \log(Q^2/\Lambda^2)$ and $N_f =$ no. of flavours. Here we have neglected the contribution of the singlet structure function as it is expected to be small in the low $x$ regime. In order to facilitate our analytical solution, let us assume that the $x$ and $t$ dependence of the structure function are factorizable [27, 28]

$$G(x, t) = g(x)h(t)$$

(2)

with the condition

$$g(x) = G(x, t_0)$$

(3)

so that

$$g(x)\frac{\partial h(t)}{\partial t} = \frac{3\alpha_s(t)}{\pi} \left[ \left\{ \left( \frac{11}{12} - \frac{N_f}{18} \right) + \ln(1 - x) \right\} g(x)h(t) + \int_x^1 dz \left( \frac{zg(x/z) - g(x)}{1 - z} \right) h(t) \right]$$

(4)
Dividing by $g(x)$ throughout we have

\[
\frac{\partial h(t)}{\partial t} = \frac{3\alpha_s(t)h(t)}{\pi} \left\{ \left( \frac{11}{12} - \frac{N_f}{18} \right) + \ln(1-x) \right\} + \int_x^1 dz \left\{ z g(x/z) - g(x) \over (1-z) g(x) \right\} + \left( z (1-z) + \frac{1-z}{z} \right) g(x/z) g(x) \right\}
\]

or

\[
\frac{\partial h(t)}{h(t)} = \frac{3\alpha_s(t)\partial t}{\pi} \left\{ \left( \frac{11}{12} - \frac{N_f}{18} \right) + \ln(1-x) \right\} + I_g(x) \]

where

\[
I_g(x) = \int_x^1 dz \left\{ z g(x/z) - g(x) \over (1-z) g(x) \right\} + \left( z (1-z) + \frac{1-z}{z} \right) g(x/z) g(x) \right\}.
\]

using eq.(2) and solving eq.(6) we find

\[
\ln h(t) = \ln t \left[ \frac{36}{25} \left\{ \left( \frac{11}{12} - \frac{N_f}{18} \right) + \ln(1-x) + I_g(x) \right\} \right]
\]

or

\[
h(t) = t^{36/25} \left\{ \left( \frac{11}{12} - \frac{N_f}{18} \right) + \ln(1-x) + I_g(x) \right\}.
\]

Therefore,

\[
G(x, t) = G(x, t_0) \left( \frac{t}{t_0} \right)^{36/25} \left\{ \left( \frac{11}{12} - \frac{N_f}{18} \right) + \ln(1-x) + I_g(x) \right\}
\]

where

\[
I_g(x) = \int_x^1 dz \left[ \frac{z G(x/z, t_0) - G(x, t_0)}{(1-z) G(x, t_0)} + \left( z (1-z) + \frac{1-z}{z} \right) {G(x/z, t_0) \over G(x, t_0)} \right].
\]

Knowing the input parametrisation of the gluons and evaluating $I_g(x)$ numerically, we can find the gluon density for various $x$ and $t$. 

using eq.(10). We note that in the limit $x \to 0$, eq.(10) has the universal limiting behaviour

$$G(x, t) = G(x, t_0) \left( \frac{t}{t_0} \right)^{\frac{36}{25} \ln(1/x)}$$

(12)

to be compared with the standard double leading logarithmic expectations [29, 30]

$$G(x, t) \sim \exp \left[ \ln \frac{1}{x} \ln t \right]^\frac{1}{2}$$

(13)

which is not factorizable in $x$ and $t$, while log $G(x, t)$ is factorizable.

3 The longitudinal structure function $F_L(x, Q^2)$

Measurement of $F_L(x, Q^2)$ at low $x$ have been used to extract the gluon density [14, 15]

$$xG(x, Q^2) = \frac{3}{5} \times 5.8 \left[ \frac{3\pi}{4\alpha_s} F_L(0.417x, Q^2) - \frac{1}{1.97} F_2(0.75x, Q^2) \right]$$

(14)

for four active flavours. At low values of $x$, the gluon contribution dominates and to a fair approximation,

$$F_L(ax, Q^2) \approx \frac{2\alpha_s}{3\pi} \frac{1}{1.74} xG(x, Q^2)$$

(15)

Here $\alpha_s$ is the QCD coupling strength and $a$ is a parameter whose value is 0.417 for $F_L$ [13]. Using eq.(10) in eq.(15) we can thus obtain the longitudinal structure function. The behaviour of $F_L$ is known in $O(\alpha_s^2)$ [20] in collinear approach and was also studied in $O(\alpha_s)$ within $K_T$ factorization scheme in [22]. In our analysis, we compare our prediction for $F_L$ with those of [20] and [22] and study their differences.
4 Results and conclusions

The factorization assumption eq.(2) is in general not valid in theoretical framework describing the scaling violation in QCD i.e. in LO Altarelli-Parisi equations. Even in DLA only log $G(x, t)$ is factorizable in $x$ and $t$. We have therefore attempted to see how the predictions with this assumption compare with those of gluon distribution which does not have such an assumption, like LO-GRV [29]. This will enable us to find the kinematical region of its approximate validity.

In Fig. 1(a-l) we show the prediction of eq.(10)(curve marked 1) with factorization ansatz eq.(2) and compare with LO-GRV [26](curve marked 2) for representative $Q^2$ values $4, 5, 6, 8, 5, 10, 20, 40, 80, 100, 160, 1600, 10^4$, and $10^5$ GeV$^2$ and $10^{-4} < x < 10^{-1}$ starting with the evolution at $Q^2_0 = 4$ GeV$^2$. These figures show the following feature for smaller $x$ range ($x < 10^{-2}$) : at fixed $x$,the difference between the two increases as $Q^2$ is increased. As an illustration, at $x \sim 10^{-2}$ the difference increases from $\sim 0.1\%$ to 20% as $Q^2$ increases from 4.5 to 160 GeV$^2$. For each $Q^2$, there is a cross-over point for both the curves, where both the predictions are numerically equal. The cross-over point shifts to lower $x$ as $Q^2$ increases. Approximately, such cross-over occurs between $10^{-2} < x < 10^{-1}$ for $Q^2 \sim 4.5 - 160$ GeV$^2$ and between $10^{-3} < x < 10^{-2}$ for $Q^2 \sim 160 - 10^5$ GeV$^2$. We can therefore find the limited range of $x$ and $Q^2$ where our approximate expression for gluon density differs from LO-GRV by not more than 20% as shown in Fig.2.

In Fig.3 we compare our result for $F_L$ with those obtained with collinear [20] and $K_T$ factorization approach [22], at $Q^2 = 20$ GeV$^2$. Our result is found to be higher than those of [20] and [22]. As an illustration at $x \sim 10^{-2}$ our result differs from [20] by 33%,67%, and 66%, corresponding to full $F_L(O(\alpha_s^3))$, $O(\alpha_s)$ and $O(\alpha_s^2)$ respectively. On the other hand it differs by 98% with [22]. The difference increases as $x$ decreases. However, as the cross-over of the gluon distribution eq.(10) with LO-GRV occurs in the range $x \sim 10^{-1} - 10^{-2}$ for $Q^2 \sim 20$ GeV$^2$, the prediction may not be reliable for $x < 10^{-2}$. It however calls for quantitative study of $O(\alpha_s^2)$ and quark contributions within the present approach.

To conclude we have shown that for a limited range of $x$ and $Q^2$, the gluon density eq.(10) with factorization is numerically equivalent to LO-GRV.
We have then predicted the longitudinal structure function $F_L$ within that range and compared with those obtained in other approaches [20, 22]. Our result is found to be higher than those of [20, 22]. It will be interesting to see how our prediction for $F_L$ compares with the results of forthcoming experiments at HERA.

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References

[1] E.Reya: Phys. Rep. 69 (1981) 195 and references therein.
[2] G. Altarelli: Phys. Rep , 81 (1982) 1.
[3] R.G. Roberts: "The structure of the Proton". Cambridge University Press 1990.
[4] ZEUS Collab.: DESY 94-143; DESY 94-192.
[5] A.J. Askew, J. Kwiecinski, A.D. Martin, P.J. Sutton: Mod. Phys. Lett., A 8 (1993) 3813.
[6] J. Kwiecinski, A.D. Martin, P.J. Sutton: Phys. Rev. D 44 (1991) 2640.
[7] Violette Brisson: Glasgow, July 1994, DESY- H1 -09/94-385.
[8] A.D. Martin, R.G. Roberts, W.J. Stirling: Mod. Phys. Lett. A 20 & 21 (1995) 2885.
[9] M. Gluck, E. Reya, A. Vogt: Phys. Lett., B 306 (1993) 391.
[10] R.D. Ball, S. Forte: Phys. Lett. B 335 (1994) 77.
[11] R.D. Ball, S. Forte: Phys. Lett. B 336 (1994) 77.
[12] ZEUS Collab: Phys. Lett. B 316 (1993) 412.
[13] H1 Collab.: Nucl. Phys. B 407 (1993) 515, DESY 95-081/95-086.
[14] A.M Cooper-Sarkar et al.: Z. Phys. C39 (1988) 281.
[15] A.M. Cooper-Sarkar, R.C.E. Devenish, M. Lancaster: In Phys. at HERA vol.1, ed. Y.Buchmuller, G.Ingelman (DESY 1992) p. 155.
[16] K. Prytz: Phys. Lett. B 311 (1993) 286.
[17] K. Prytz: Phys. Lett. B 332 (1994) 393.
[18] M. Lancaster: Talk at the 27th International Conference on High Energy Physics, Glasgow, July 1994, DESY 94-204.
[19] L.H. Orr, W.J. Stirling: Phys. Rev. Lett. 66 (1991) 1673.
[20] E.B. Zijlstra, W.L van Neerven: Nucl. Phys. B 383 (1992) 552.
[21] E.L. Berger, R. Meng: Phys. Lett. B 304 (1993) 318.
[22] J. Blumlein: Nucl. Phys. B Proc.Suppl. 39 BC (1995) 22.
[23] G. Altarelli, G. Parisi: Nucl. Phys. B 126 (1977) 298.
[24] L.F. Abbott, W.B. Atwood, R.N. Barnett: Phys. Rev. D 22 (1980) 882.
[25] V.N. Gribov, L.N. Lipatov: Sov. J. Nucl. Phys. 15 (1972) 438.
[26] M. Gluck, E. Reya, A. Vogt: Z. Phys. C 53 (1992) 127.
[27] A. Saikia: Private Communication.
[28] D.K. Choudhury, A. Saikia: Pramana J. Phys. 33 (1989) 359.
[29] A.De. Rujula, S.L. Glashow, H.D. Politzer, S.B. Trieman, F. Wilczek, A. Zee, Phys. Rev. D 10 (1974) 1649.
[30] A.D. Martin, R.G. Roberts, W.J. Stirling: Phys. Rev. D 50 (1994) 6734.