Novel electric field effects on Landau levels in multi-Weyl semimetals

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Multi-Weyl semimetals (WSMs) have an anisotropic non-linear dispersion along a 2-D plane and a linear dispersion in an orthogonal direction. They have topological charge ‘n’ and are created when two or multiple Weyl points or nodes with nonzero net monopole charge are brought together onto a high-symmetry point. We study the perturbation corrections up to second order of such multi-WSMs in crossed electric (E) and magnetic (B) fields in the low electric field approximation. As a result, the first order correction lifts only n-fold degeneracy of the lowest Landau levels (LLs) while the higher Landau levels are modified due to the second order perturbation. We study the signatures of these corrections due to electric fields on the density of states (DOS) subjected to a magnetic field.

I. INTRODUCTION

After graphene, Weyl semimetal (WSM) is another gapless system which is going to study at a rapid pace. WSM is a three dimensional analog of graphene where the low energy Hamiltonian has isotropic relativistic linear dispersion in k space (which obey the 3D Weyl equation) from accidental degenerate band touching points referred to as Weyl points. The electronic states around the Weyl points possess a nonzero Berry curvature, which gives rise to topological charge ±1 [1]. As a consequence, WSMs host topologically protected surface states in the form of open Fermi arcs terminating at the projections of bulk Weyl points (WPs) of opposite chirality. Other fascinating physical consequences of these Berry phases are exotic transport phenomena such as a large negative magnetoresistance due to chiral anomaly [2–4]. In recent angle-resolved photoemission spectroscopy (ARPES) and scanning tunneling microscopy (STM) experiments, several materials such as Cd3As2 [5–11], Na3Bi [12], NbAs [13], TaP [14], TaAs [15, 16], ZrTe5 [18–24] have been identified as Weyl semimetals. Also, several attempts have been made on the realization of WSMs in artificial systems such as photonic crystals [25–29].

In addition to above isotropic or single WSM, a new three-dimensional topological semimetals have been proposed in materials with certain point-group characterized by $C_{4,6}$ symmetries [30–32] e.g. the double(triple)-Weyl semimetals have band touching points with quadratic(cubic) dispersions along $k_x - k_y$ plane and linear dispersion along $k_z$ direction. The double-Weyl nodes are protected by $C_4$ or $C_6$ rotation symmetry and its half-metallicity has been realized in the three-dimensional semimetal HgCr2Se4 in the ferromagnetic phase, with a single pair of double-Weyl nodes along the z-direction [33]. However, the material realization of triple-WSMs remains elusive. The double-Weyl node possesses a monopole (anti monopole) charge of +2 (-2) and it shows double-Fermi arcs on the surface Brillouin zone (BZ) [30–32]. Similarly, triple-Weyl node possesses a monopole (anti-monopole) charge of +3 (-3) and it shows triple-Fermi arcs on the surface BZ.

The single or isotropic-WSMs are predicted to have a fascinating response similar to graphene and 2D-WSM in crossed electric (E) and magnetic fields (B) [34]. These systems have topologically protected gapless Dirac or Weyl nodes with relativistic dispersion. This naturally induces Lorentz boosts in crossed electric (E) and magnetic fields (B) [35]. As a consequence of Lorentz invariance of the Dirac equation, these problems can be solved exactly by choosing a reference frame in which the electric field vanishes as long as the drift velocity $v_d = E/B$ is smaller than the Fermi velocity ($v_F$), which plays the role of an upper bound for the velocity as the speed of light c in relativity. This lifts the Landau levels (LLs) degeneracy. As a consequence, the LLs spacing is reduced and at a particular value of $E/v_FB$, the LLs get collapse [34–35].

Our main contribution is to extend this mechanism for multi-WSMs(double and triple-WSMs). The double and triple-WSMs have non-Lorentz invariant physics due to quadratic and cubic dispersions respectively in $k_x - k_y$ plane and therefore their crossed electric and magnetic fields response cannot be solved exactly due to the absence of a reference frame in which the electric field vanishes. We, therefore, study this problem by perturbation theory up to second order corrections in electric fields as it has been discussed for multilayers graphene [36]. The rest of the paper is organized as follows. In Section II we study the Landau level spectrum of the multi-Weyl semimetals in the presence of an in-plane uniform electric field. These problems have been well studied in 2-dimensional single layer graphene, 2-D WSMs and 3-dimensional single WSMs and type-II WSMs [35, 37–40]. In Section III, we study the response of multi-WSMs in...
crossed electric and magnetic fields by perturbation theory. We also calculate the density of states in crossed fields in Section III. Finally, we make some concluding remarks in Section IV.

II. LANDAU LEVELS FORMATION IN MULTI-WEYL SEMIMETALS

The non-interacting low energy effective Hamiltonian for a single multi-Weyl semimetals is given by [41–44],

\[ H = \alpha_n[(\hat{p}_-)^n \sigma_+ + (\hat{p}_+)^n \sigma_-] + \chi v_z \hat{p}_z \sigma_z, \]  \tag{1}

where \( \sigma_{\pm} = \frac{1}{2}(\sigma_x \pm i\sigma_y), \hat{p}_\pm = \hat{p}_x \pm i\hat{p}_y, \) and \( \chi = \pm 1 \) is the chirality, ‘\( n \)’ represents monopole charge , \( v_z \) is the Fermi velocity along \( \hat{z} \) direction and \( \alpha_n \) is the material dependent parameter, e.g. \( \alpha_1 \) and \( \alpha_2 \) are the Fermi velocity and inverse of the mass respectively for single and double WSMs. The energy spectrum of Eq.(1),

\[ \epsilon_n(k) = s\sqrt{\alpha_n^2 (\hbar k)^2 + (\hbar k v_z)^2}, \]  \tag{2}

where \( s = \pm 1 \) and \( k_{\parallel} = \sqrt{k_x^2 + k_y^2} \) is the momentum along \( \hat{x}-\hat{y} \) plane. The density of states of such an anisotropic Hamiltonian is given by \( g(e) \sim e^{2/n} \) [41]. In the presence of large external magnetic field, the Hamiltonian Eq.(1) form the Landau level spectrum [15, 47]. Under an external magnetic field \( B \) directed along \( z \)-axis, we make the usual Peierls substitution \( p \rightarrow p + eA \) \( (e > 0) \) in Hamiltonian Eq. (1) with the vector potential \( A \). We choose \( A \) in the Landau gauge \( (A_y = Bx) \). Therefore, the Hamiltonian Eqn.(1) transforms to

\[ H = \begin{pmatrix} \alpha_n(\hat{p}_x - i(\hat{p}_y + eBx))^n & v_z \hat{p}_z \\ v_z \hat{p}_z & -\alpha_n(\hat{p}_x + i(\hat{p}_y + eBx))^n \end{pmatrix} \]  \tag{3}

where \( \hat{p}_{x,y,z} = -i\hbar \partial_{x,y,z} \). The time-independent Schrödinger equation \( H\Psi = E\Psi \). Since the above Hamiltonian contain explicitly only \( x \), we will look for the solutions of the usual form

\[ \Psi = \Psi(x)e^{i\hbar(k_ny + k_z z)} \]  \tag{4}

so that Eqn.(3) transforms to

\[ \Psi(x) = E\Psi(x) \]  \tag{5}

have n- degenerate chiral modes i.e. the lowest LLs for double and triple WSM have two and three fold degeneracy, respectively.

The normalized solutions for \( m \geq n \) are

\[ \Psi_{m,s=+} = \frac{1}{\sqrt{2}} \begin{pmatrix} (-i)^n b_m \psi_{m-n} \\ a_m \psi_m \end{pmatrix} \]  \tag{9}

\[ \Psi_{m,s=-1} = \frac{1}{\sqrt{2}} \begin{pmatrix} -(-i)^n a_m \psi_{m-n} \\ b_m \psi_m \end{pmatrix} \]  \tag{10}

whereas for \( m < n \)

\[ \Psi_n = \begin{pmatrix} 0 \\ \psi_n \end{pmatrix} \]  \tag{11}

where \( a_m = \left( 1 + \frac{\nu \hbar k}{E_m} \right)^{1/2}, b_m = \left( 1 - \frac{\nu \hbar k}{E_m} \right)^{1/2} \) and \( \psi_m \) are the usual normalized eigenfunctions of a free electron in a magnetic field

\[ \psi_m = \frac{\sqrt{1}}{\sqrt{2^{m+n} \sqrt{\pi}}} e^{-u^2/2} H_m(u) \]  \tag{12}

Here \( H_m \) are Hermite polynomials.
III. ENERGY LEVELS IN THE PRESENCE OF MAGNETIC FIELD AND ELECTRIC FIELD

Let us assume that in addition to the magnetic field, one has an uniform electric field along the x direction. This add to the Hamiltonian a term of the form $-eE\mathbf{1}$ where $V$ is the electric potential associated with the applied electric field $E = (E, 0, 0)$ and $\mathbf{1}$ is the $2 \times 2$ unit matrix. Then the single particle Hamiltonian is given by

$$H = \begin{pmatrix}
  v_x \hat{p}_z - eE x & \alpha_n \left( \hat{p}_x - i(\hat{p}_y + eBx) \right)^n \\
  \alpha_n \left( \hat{p}_x + i(\hat{p}_y + eBx) \right)^n & -v_x \hat{p}_z - eE x
\end{pmatrix}$$

(13)

We try the wave function as $\Psi_{1,2} = \Psi_{1,2}(x)e^{i\hbar(k_y y + k_z z)}$. For $n=1$, the above $2 \times 2$ Hamiltonian is an equivalent to a tilted WSMs and can be exactly solved for $E/v_F B < 1$ due to Lorentz invariant physics of the Hamiltonian. The corresponding eigenvalues are given by

$$\Psi_{1,2}(x) = \tilde{E}_m \Psi_{1,2}(x) = (E_m - \frac{\varepsilon}{B} \hbar k_y)\Psi_{1,2}(x)$$

(16)

When $\varepsilon$ is small, in the zeroth order approximation over $\lambda$ we have:

$$\hbar v_z k_z \psi_1 + (-i)^n 2^{n/2} \frac{h^n}{\gamma B} \alpha_n \hat{a} \psi_2 = (E_m - \frac{\varepsilon}{B} \hbar k_y)\psi_1$$

(17)

We cannot neglect here $\frac{\varepsilon}{B} \hbar k_y$ because $\hbar k_y$ may be large.) Thus, we have the same Landau levels as without electric field just shifted by $\frac{\varepsilon}{B} \hbar k_y$.

B. The first order correction

In the first order approximation, we have

$$\Psi_{1,2}(x) = \tilde{E}_m \Psi_{1,2}(x) = (E_m - \frac{\varepsilon}{B} \hbar k_y)\Psi_{1,2}(x)$$

(18)

One can easily see that the first-order term due to $\lambda$ vanishes for $m \geq n$. For $m < n$, we use degenerate first order perturbation theory and finds that there is the first order correction to the energy for $m \geq 2$. This correction is given by

$$E_m = \frac{1}{\gamma} \sqrt{v_x^2 \hbar^2 k_z^2 + \frac{2eBh}{\gamma} m \alpha_n^2 + \frac{\varepsilon}{B} \hbar k_y}$$

(14)

where $\gamma = 1/\sqrt{1 - \beta^2}$, $\beta = \varepsilon/\alpha_1 B$. Therefore, when $E/\alpha_1 B = 1$, there is a complete collapse of the Landau Levels(LLs). The above Hamiltonian is not exactly solvable for topological charge $n > 1$ due to the absence of a reference frame in which the electric field vanishes and therefore we solve the above problem for low electric field and high magnetic field such that Landau levels remain intact.

A. The zero order approximation

Let us introduce an another parameter $\lambda$

$$\lambda = \frac{e\varepsilon l_B}{\sqrt{2}}$$

(15)

such that $\lambda$ is always small with respect to the leading energy scale $2^{n/2} h^n \alpha_n / l_B^2$ which holds at low electric field $E$. This validates the perturbation theory. Now, the Schrödinger equation (13) reads

$$\Psi_{1,2}(x) = \tilde{E}_m \Psi_{1,2}(x) = (E_m - \frac{\varepsilon}{B} \hbar k_y)\Psi_{1,2}(x)$$

(16)
levels basis $\Psi_m$ by unitary transformation

$$
\Phi_m = \sum_{m'=0}^{n-1} a_{m'} \Psi_{m'}
$$

(20)

where $a_{m'}$ is the amplitude of $\Psi_{m'}$ and summation is over lowest Landau levels (i.e. $m' < n$). The matrices $\Omega$ formed from $\Psi_m$ and $\Psi'_m$ for double and triple-WSMs reduce to

$$
\begin{pmatrix}
0 & -\lambda \\
-\lambda & 0
\end{pmatrix} = \Omega
$$

(21)

and

$$
\begin{pmatrix}
0 & -\lambda & 0 \\
-\lambda & 0 & -\sqrt{2}\lambda \\
0 & -\sqrt{2}\lambda & 0
\end{pmatrix} = \Omega
$$

(22)

respectively which have eigenvalues

$$
\begin{align*}
\hat{E}_m^{(1)} &= \pm \lambda; \\
\hat{E}_m^{(2)} &= 0, \pm \sqrt{3}\lambda
\end{align*}
$$

(23)

for double and triple-WSMs respectively. Therefore, in the presence of electric field $\mathcal{E}$, the $n$-fold degeneracy of chiral lowest LLs is lifted.

C. The second order approximation

We have already seen that the first-order term in $\mathcal{E}$ in powers of $\lambda$ vanishes for $m \geq n$. Therefore, we look at the second-order term

$$
\hat{E}_m^{(2)} = \lambda^2 \sum_{k \neq m, s'} \frac{1}{E_{m,s}^{(0)} - E_{k,s'}^{(0)}} |\Psi_{m,s}^\dagger (\hat{a}^\dagger + \hat{a}) \Psi_{k,s'}|^2
$$

(24)

where $s, s'$ denotes the band index.

First, we consider the case $m \geq n$ for which Eq. (24) can be rewritten as

$$
\hat{E}_m^{(2)} = \lambda^2 \frac{1}{4} \left( [a_{m+1} a_m \sqrt{m+1} + b_{m+1} b_m \sqrt{m-n+1}]^2 \\
+ \frac{1}{E_m^{(0)} - E_{m+1}^{(0)}} [a_{m-1} a_m \sqrt{m+1} + b_{m-1} b_m \sqrt{m-n}]^2 \\
+ \frac{1}{E_m^{(0)} + E_{m+1}^{(0)}} [a_{m+1} b_m \sqrt{m+1} - a_{m+1} b_m \sqrt{m+n}]^2 \\
+ \frac{1}{E_m^{(0)} + E_{m-1}^{(0)}} [a_{m-1} b_m \sqrt{m-n+1} - a_{m-1} b_m \sqrt{m-n-1}]^2 \right)
$$

$$
= \frac{\lambda^2}{4} \zeta_m
$$

(25)

with

$$
\zeta_m = \frac{2}{\theta_1} \left[ (2n + 1 - n) E_m^0 + 2n \lambda v_z k_z + (2n + 1 - n) (\lambda v_z k_z)^2 \right. \\
\left. + \frac{2}{\theta_2} (m+1) m (m-1) \cdots (m-n+1) \omega^2 \alpha_n^2 \right]
$$

$$
\frac{2}{\theta_1} \left[ (2m - n) E_m^0 + 2n \lambda v_z k_z + (2m - n) (\lambda v_z k_z)^2 \right. \\
\left. + \frac{2}{\theta_2} m (m-1) \cdots (m-n) \omega^2 \alpha_n^2 \right]
$$

(26)

where $\theta_1 = (E_m^0)^2 - (E_{m+1}^0)^2 = -n \omega^2 \alpha_n^2$ and $\theta_2 = (E_m^0)^2 - (E_{m-1}^0)^2 = n \omega^2 \alpha_n^2$.

In particular, for a single WSMs case ($n = 1$), the second order energy correction

$$
\hat{E}_m^{(2)} = -\frac{1}{4} \beta^2 [2 E_m^2 + \frac{m}{E_m^2} (\alpha_1 \alpha_2)]
$$

$$
= -\frac{1}{2} \beta^2 E_m^2 - \frac{1}{2} m \frac{E_m^2}{E_m^2} (heB) \alpha_1^2 \beta^2
$$

(27)

Therefore, the modified energy

$$
E_m = E_m^0 - \frac{1}{2} \beta^2 E_m^0 - \frac{1}{2} m \frac{E_m^2}{E_m^2} (heB) \alpha_1^2 \beta^2 + \frac{\mathcal{E}}{B} h ky
$$

$$
\approx \frac{1}{\gamma} E_m^0 - \frac{1}{2} m \frac{E_m^2}{E_m^2} (heB) \alpha_1^2 \beta^2 + \frac{\mathcal{E}}{B} h ky
$$

(28)
where we have taken $\gamma = 1/\sqrt{1 - \beta^2} \approx 1/(1 - \beta^2)$ at low $\beta$ in above equation. This energy agrees with exact results for type-II WSMs or tilted single WSMs in ref. [40].

$$E^2_m(k_z) = \frac{1}{\gamma} \sqrt{v_z^2 \hbar^2 k_z^2 + \frac{2eB\hbar}{\gamma} \frac{m\alpha^2}{\omega^4 \alpha^2_2 m(m-1)} + \frac{E}{B} \hbar k_y} \tag{29}$$

Similarly, we can show that Landau levels spectrum of other multi WSMs gets modified in the presence of electric field. For double-WSMs, the second order energy correction

$$\tilde{E}^{(2)}_m(k_z) = \lambda^2 \left[ -\frac{2(m-1)}{2E_m^0} + \frac{h v_z k_z}{\omega^4 \alpha^2_2 m(m-1)} \right] \tag{30}$$

and for triple-WSMs, we have second order energy correction

$$\tilde{E}^{(2)}_m(k_z) = \lambda^2 \left[ \frac{(E_m^0)^2}{3E_m^0 \omega^6 \alpha^2_3} m(m-2) - \frac{2(m-1)}{3E_m^0} \frac{2h v_z k_z}{m(m-1)(m-2)\omega^6 \alpha^2_3} \right] + \frac{E}{B} \hbar k_y \tag{31}$$

Therefore, the corresponding modified energies for double WSMs is

$$E_m = E_m^0 + \lambda^2 \left[ -\frac{(2m-1)}{2E_m^0} + \frac{h v_z k_z}{\omega^4 \alpha^2_2 m(m-1)} \right] + \frac{E}{B} \hbar k_y \tag{32}$$

and for triple-WSMs, the modified energy dispersion

$$E_m = E_m^0 + \lambda^2 \left[ \frac{(E_m^0)^2}{3E_m^0 \omega^6 \alpha^2_3} m(m-2) - \frac{2(m-1)}{3E_m^0} \frac{2h v_z k_z}{m(m-1)(m-2)\omega^6 \alpha^2_3} \right] + \frac{E}{B} \hbar k_y \tag{33}$$

For $m<n$, we use degenerate perturbation theory (Rayleigh-Schrödinger solution) and show it easily that the second order correction vanishes due to the cancellation from positive and negative energy bands of higher Landau levels. Therefore, the modified lowest LLs energy dispersion due to the first and second order corrections.

$$E_m = -h k_z v_z + \frac{E}{B} \hbar k_y - \lambda \text{EV}[^\Omega] \tag{34}$$

where $\text{EV}(\Omega)$ represents the eigenvalues of $\Omega$.

Thus, energy spectrum of lowest LLs for double and triple-WSMs are $-h k_z v_z + \frac{E}{B} \hbar k_y \pm \lambda$ and $-h k_z v_z + \frac{E}{B} \hbar k_y$, respectively.
The density of states (DOS) governs the pattern of quantum oscillation measurements (through Shubnikov-de Haas effect or De Haas-van Alphen effect) of all transport and thermodynamic quantities \[43, 49, 50\]. In the presence of high magnetic fields, the energy dispersion of multi-WSMs in the plane perpendicular to \( \mathbf{B} \) is completely suppressed while it is still dispersive along the direction of the applied magnetic field. Thus the magnetic field step down the dimension of the system and the WSM in the external magnetic field can be visualized as a collection of one-dimensional systems with multiple sub-bands due to Landau levels. The DOS of multi-WSMs can be worked out by the following definition,

\[
D(\varepsilon) = \frac{1}{2\pi l_B} \sum_m \int_{-\infty}^{\infty} \frac{dk_z}{2\pi} \delta[E^0_m(k_z) - \varepsilon],
\]

where \( m \) labels Landau level index, \( k_z \) is the conserved momentum of the one-dimensional conducting channel along the \( \mathbf{B} \) direction and \( l_B \) is the magnetic length.

The DOS for multi-WSM in the absence of in-plane electric field \( \mathcal{E} \) can be obtained analytically. Let us first consider the dispersion for a single WSM

\[
E^0_m(k_z) = \sqrt{\hbar^2 v_F^2 k_z^2 + m\omega^2\alpha_1^2},
\]

In such a system, each \( m \geq 1 \) LLs crosses the Fermi energy \( \varepsilon \) twice at critical momentum \( k_{zc} = \pm \sqrt{\varepsilon^2 - m\omega^2\alpha_1^2}/(\hbar v_F) \), whereas the \( m = 0 \) LL only cuts the Fermi energy once, at \( k_z = -\varepsilon/(\hbar v_F) \). Therefore, the DOS for this single WSM is given by

\[
D(\varepsilon) = D_0 \left[ 1 + \sum_{m \geq 1} \frac{2\varepsilon \Theta(\varepsilon - \sqrt{m\omega\alpha_1})}{\sqrt{\varepsilon^2 - m\omega^2\alpha_1^2}} \right],
\]

where \( D_0 = 1/(4\pi^2\hbar^2 v_F) \) and \( \Theta(x) \) is the Heaviside Theta function.

Similarly, the DOS for double WSM and triple WSMs can be obtained analytically by the above same arguments. The DOS for double-WSM is given by

\[
D(\varepsilon) = D_0 \left[ 2 + \sum_{m \geq 2} \frac{2\varepsilon \Theta(\varepsilon - \sqrt{m(m-1)\omega^2\alpha_2})}{\sqrt{\varepsilon^2 - m(m-1)\omega^2\alpha_2^2}} \right],
\]

and for triple-WSM

\[
D(\varepsilon) = D_0 \left[ 3 + \sum_{m \geq 3} \frac{2\varepsilon \Theta(\varepsilon - \sqrt{m(m-1)(m-2)\omega^3\alpha_3})}{\sqrt{\varepsilon^2 - m(m-1)(m-2)\omega^3\alpha_3^2}} \right],
\]

where the numbers 2 and 3 in Eqs. (38) and (39) accounted for the two-fold and three-fold degeneracy of the lowest LLs respectively.

In the presence of an in-plane electric field \( \mathcal{E} \) along the \( x \)-direction, the DOS cannot be calculated analytically. Therefore, we compute it numerically using Eq.(35) and display in Fig. 4, Fig. 5 and Fig. 6 with increasing values of the electric field for single, double and triple-WSMs respectively. The magnetic oscillations have a sawtooth appearance originating from the \( k_z \) dispersion of higher Landau levels (i.e. \( m \geq n \)). The peaks correspond to the spacing of the singularities which go like \( B^m/2 \) where \( B_{eff} \) is the effective or reduced effective magnetic field due to the electric field \( \mathcal{E} \). In the case of single-WSMs, \( B_{eff} \) is reduced strongly compared to the bare applied magnetic field \( B \) along \( z \)-direction whereas it has a minor modifications for the case of double and triple-WSMs. As a result, a considerable shift of peaks towards low \( \varepsilon \) are observed with increasing electric field \( \mathcal{E} \) in the case for single-WSMs whereas there is a minor change in shift of peaks towards low \( \varepsilon \) are observed for the double and triple-WSMs case. In the case of single-WSM, the LLs becomes closer and closer with increasing \( \mathcal{E} \) and at the critical value \( \mathcal{E}_c = v_F B \), it collapses in the plane perpendicular direction to \( B \) while it is still dispersive along \( B \). The corresponding DOS squeezes with the electric field and at a critical value \( \mathcal{E}_c \), it reaches a constant value which corresponds to DOS of the one-dimension dispersion along the \( z \)-direction. For double and triple-WSMs, there are no such modifications in their DOS with
The DOS for Landau levels in a single-WSM, renormalized by $D_0$ when (a) $\mathcal{E} = 1$ (b) $\mathcal{E} = 2$ and (c) $\mathcal{E} = 3.5$ respectively. Dark and dotted plots shows the DOS without and with electric fields. Other parameters: $B=5$, $v_z=1$, $\alpha_1 = 1$.

$\mathcal{E}$ due to the non-collapse of the LLs and therefore, their DOS show small changes with $\mathcal{E}$. Further for lowest LLs (i.e. $m < n$), we observe that there is no change of the plateau in DOS at low energy $\epsilon$ due to the lifting of the degeneracy of their lowest LLs in the case of double and triple-WSMs. Since the degeneracy of lowest Landau level in the presence of in-plane electric field are symmetrically shifted about its lowest Landau levels energy $-hv_z k_z$ in case of double and triple-WSMs. Therefore, when we add contributions from these shifted lowest Landau levels, it density of states (DOS) remains constants. These changes of DOS of multi-WSMs with $\mathcal{E}$ could be detected in angle-resolved quantum oscillations. e.g. the above features of DOS could be reflected in the specific heat and magnetization.

FIG. 5. DOS for Landau levels in a double-WSM, renormalized by $D_0$ when (a) $\mathcal{E} = 3$ (b) $\mathcal{E} = 4$ and (c) $\mathcal{E} = 5$ respectively. Dark and dotted plots shows the DOS without and with electric fields. Other parameters: $B=5$, $v_z=1$, $\alpha_1 = 1$.

V. CONCLUSION

In conclusion, we summarize the main findings of the present manuscript. We have analyzed a perturbative study of a multi-WSMs in crossed electric and magnetic fields in low electric field approximation. This problem cannot be exactly solved for monopole charge $n > 1$ due to the absence of a reference frame in which the electric field vanishes. Therefore, we have calculated energy corrections up to second order. The main consequences of this electric field $\mathcal{E}$ are the n-fold degeneracy of the lowest Landau levels is lifted while the higher one remain unaffected due to the first order correction in electric field $\mathcal{E}$. The higher Landau levels have corrections due to the second order perturbation in electric field $\mathcal{E}$ while lowest Landau levels remain unaffected. We have compared the density of states of multi-WSMs system for both the absence and presence of the electric field. The lowest LLs (i.e. $m < n$) have no change of plateau in DOS at low energy $\epsilon$ in the case of double and triple-WSMs even with the lifting of the degeneracy.
FIG. 6. DOS for Landau levels in a triple-WSM, renormalized by $D_0$ when (a) $E = 5$ (b) $E = 10$ and (c) $E = 25$ respectively. Dark and dotted plots shows the DOS without and with electric fields. Other parameters: $B=5$, $v_z = 1$, $\alpha_1 = 1$.

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