Investigation of the rare exclusive $B^*_c \rightarrow D_s \, \nu \bar{\nu}$ decays in the framework of the QCD sum rules

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Exclusive $B^*_c \rightarrow D_s \, \nu \bar{\nu}$ decay is studied in the framework of the three–point QCD sum rules approach. The two gluon condensate contributions to the correlation function are calculated and the form factors of this transition are found. The decay width and total branching ratio for this decay is also calculated.
I. INTRODUCTION

The standard model (SM) Higgs boson which is one of the most important components of the SM has been discovered by the ATLAS [1] and CMS [2] collaborations. Nowadays, we aim to find out the new physics beyond the SM. Heavy mesons with the different flavors like $B_c$ and $B^*_c$ mesons can provide a good testing benchmark not only for the predictions of the SM but also for searching the new physics beyond SM. The LHCb experiment has aimed to test the SM predictions and discover the possible new physics signals. In this regards, a lot of the experimental data released by the LHCb experiment [3].

The dominant decay mode of $B^*_c$ is $B^*_c \rightarrow B_c \gamma$ [4]. Rare $B^*_c \rightarrow D_s \nu \bar{\nu}$ proceeds FCNC transitions. This decay is roughly of the same order as that of the $B^*_c \rightarrow \eta_c \ell \bar{\nu}_\ell$ [5]. In the SM framework, the rare $B^*_c \rightarrow D_s \nu \bar{\nu}$ decay is dominated by the Z-penguin and box diagrams involving top quark exchanges. The theoretical uncertainties related to the renormalization scale dependence of running quark mass can be essentially neglected after the inclusion of next-to-leading order corrections [6]. This decay is theoretically very clean processes in compare with the semileptonic decays like the $B^*_c \rightarrow D_s \ell^+ \ell^-$ decay and is also sensitive to the new physics beyond the SM [7]. Moreover, this decay is complementary to the $B^*_c \rightarrow D_s \ell^+ \ell^-$ decay. Note that, the direct calculation of physical observables such as form factors suffer from sizable uncertainties. These can be greatly reduced through a combined analysis of the rare $B^*_c \rightarrow D_s \nu \bar{\nu}$ and $B^*_c \rightarrow D_s \ell^+ \ell^-$ [8] decays.

These decays have not yet been measured by the LHCb. There is no theoretical studies relevant to the form factors and branching ratios of $B^*_c \rightarrow D_s \nu \bar{\nu}$ decay. The form factors of these decays can be evaluated with the different approaches. Some of them are the light front, the constituent quark models [9] and the QCD sum rules. In this study the three–point QCD sum rules approach are used in the calculation of form factors. It is worth mentioning that the QCD sum rules have widely been utilized in calculation of the form factors (some of them can be found in Refs. [10]-[17]).

The paper has 3 sections: In section 2, the effective Hamiltonian and the three–point QCD sum rules approach are presented for completeness. In section 3, The numerical values of form factors are given and the sensitivity of the branching ratio is studied and conclusion is presented.

II. SUM RULES FOR THE $B^*_c \rightarrow D_s \nu \bar{\nu}$ TRANSITION FORM FACTORS

The FCNC $b \rightarrow s \nu \bar{\nu}$ decay is described within the framework of the SM at the quark level by the effective Hamiltonian [18]

$$\mathcal{H}_{\text{eff}} = \frac{G_F \alpha}{2\sqrt{2\pi} \sin^2 \theta_W} V_{tb} V_{ts}^* X(x) \bar{b} \gamma^\mu(1 - \gamma_5)s \bar{\nu}_\mu(1 - \gamma_5)\nu ,$$

where $G_F$ is the Fermi constant, $\theta_W$ is the Weinberg angle, $\alpha$ is the fine structure coupling constant and

$$X(x) = X_0(x) + \frac{\alpha_s}{4\pi} X_1(x) ,$$

(1)

(2)
The $X_0(x)$ is:

$$X_0 = \frac{x}{8} \left[ \frac{x+2}{x-1} + \frac{3(x-2)}{(x-1)^2} \ln x \right], \quad (3)$$

where $x = m_t^2/m_W^2$. The explicit form of $X_1(x)$ is given in Refs. 18 and 19. Note that, $X_1(x)$ gives about 3% contribution to the $X_0(x)$ term 20.

The Wilson coefficients (in our case $X_0(x)$ and $X_1(x)$) can be calculated in any gauge and they are gauge independent and the results should be gauge invariant. The Wilson coefficients are calculated in $R_\xi$ gauge. It is worth mentioning that local operators in the considered problem have anomalous dimensions. We have checked that taking into account anomalous dimensions can change numerical results at most 10%.

The matrix element of the exclusive $B_c^+ \to D_s \nu \bar{\nu}$ decays are found by inserting initial meson state $B_c^+$ and final meson state $D_s$ in Eq. (1).

$$M = \frac{G_F}{2\sqrt{2}\pi \sin^2 \theta_W} V_{tb} V_{ts}^* X(x) < D_s(p_D) | \bar{s} \gamma_\mu (1 - \gamma_5) b | B_c^+(p_B, \varepsilon) > \bar{\nu} \gamma_\mu (1 - \gamma_5) \nu, \quad (4)$$

where $\varepsilon$ is the polarization vector of $B_c^+$ meson, $p_B$ is the momentum of the $B_c^+$ and $p_D$ is the momentum of $D_s$ meson.

The matrix element of the Eq. (1) is written in terms of the form factors as follows:

$$< D_s(p_D) | \bar{s} \gamma_\mu (1 - \gamma_5) b | B_c^+(p_B, \varepsilon) > = \frac{A_{V}(q^2)}{m_{B_c^+}^2} \varepsilon_{\mu\nu\alpha\beta} \varepsilon^*_{\nu} p_B^\alpha p_D^\beta - i A_0(q^2) m_{B_c^+} \varepsilon^*_{\mu}$$

$$- i A_+ \frac{(q^2)}{m_{B_c^+}^2} (\varepsilon^* p_D^\mu) P_\mu - i A_- \frac{(q^2)}{m_{B_c^+}^2} (\varepsilon^* p_D^\mu) q_\mu, \quad (5)$$

here, Lorentz invariant and parity conservation are considered. Also, $A_i(q^2)$, where $i = V, +, -$ are the dimensionless transition form factors. $P_\mu = (p_B + p_D)_\mu$ and $q_\mu = (p_B - p_D)_\mu$ is the transfer momentum or the momentum of the $Z$ boson.

The matrix element in terms of the form factors is as:

$$M = \frac{G_F}{2\sqrt{2}\pi \sin^2 \theta_W} V_{tb} V_{ts}^* X(x) \left[ \frac{A_{V}(q^2)}{m_{B_c^+}^2} \varepsilon_{\mu\nu\alpha\beta} \varepsilon^*_{\nu} p_B^\alpha p_D^\beta - i A_0(q^2) m_{B_c^+} \varepsilon^*_{\mu} \right.$$

$$- i A_+ \frac{(q^2)}{m_{B_c^+}^2} (\varepsilon^* p_D^\mu) P_\mu - i A_- \frac{(q^2)}{m_{B_c^+}^2} (\varepsilon^* p_D^\mu) q_\mu \left. \right] \bar{s} \gamma_\mu (1 - \gamma_5) \nu, \quad (6)$$

where $A_1 = -i A_V$.

We try to calculate the aforementioned form factors by means of the QCD sum rules. The QCD sum rules begin with the following correlation functions:

$$\Pi_{\mu\nu}^{V-AV}(p_B^2, p_D^2, q^2) = i^2 \int d^4 x d^4 y e^{-ip_B x} e^{ip_D y} < 0 | T[J_{D_s}(y)J_{\mu}^{V-AV}(0)J_{\nu}^{AV}(x)] | 0 >, \quad (7)$$

where the interpolating currents are $J_{D_s}(y) = \bar{s} \gamma_\mu s$ and $J_{\nu}^{AV}(x) = \bar{b} \gamma_\nu c$ the $D_s$ and the $B_c^+$ meson states, respectively.

$J_{\mu}^{V-AV} = \bar{s} \gamma_\mu (1 - \gamma_5) b$ consists of the vector ($V$) and axial vector ($AV$) transition currents. After inserting the the two complete sets of the $B_c^+$ and $D_s$ meson, the correlation functions in Eq. (7) is written as follows:
\[\Pi_{\mu\nu}^{V-\mathrm{AV}}(p_B^2, p_D^2, q^2) = - \frac{<0 | J_{D_\gamma} | D_\gamma(p_D) \rangle < D_\gamma(p_D) | J_{\mu\nu}^{V-\mathrm{AV}} | B_c^*(p_B, \varepsilon) \rangle < B_c^*(p_B, \varepsilon) | J_{\nu B_\gamma}^c | 0 >}{(p_D^2 - m_{D_\gamma}^2)(p_B^2 - m_{B_c}^2)} + \cdots, \tag{8}\]

where \(\cdots\) shows the contributions come from higher states and continuum of the currents with the same quantum numbers.

The \(<0 | J_{D_\gamma} | D_\gamma(p_D) \rangle > and < B_c^*(p_B, \varepsilon) | J_{\nu B_\gamma}^c | 0 >\) matrix elements are defined as follows:

\[<0 | J_{D_\gamma} | D_\gamma(p_D) \rangle > = - \frac{f_{D_\gamma} m_{D_\gamma}^2}{m_s + m_c}, \quad < B_c^*(p_B, \varepsilon) | J_{\nu B_\gamma}^c | 0 > = f_{B_c} m_{B_c}^\varepsilon, \tag{9}\]

where \(f_{B_c}\) and \(f_{D_\gamma}\) are the leptonic decay constants of \(B_c^*\) and \(D_\gamma\) mesons, respectively. Using these equations and calculating the the summation over the polarization of the vector meson \(B_c^*\), the Eq.(8) is as follows:

\[\Pi_{\mu\nu}^{V-\mathrm{AV}}(p_B^2, p_D^2, q^2) = - \frac{f_{D_\gamma} m_{D_\gamma}^2}{m_c + m_s} \frac{f_{B_c} m_{B_c}^2}{m_D^2 - m_{D_\gamma}^2} \left[ A_0(q^2) m_{B_c}^\mu \epsilon_{\mu\nu\alpha\beta} P_B^{\alpha\beta} + A_1(q^2) m_{B_c}^\mu \epsilon_{\mu\nu\alpha\beta} P_B^{\alpha\beta} \right] + \text{excited states}, \tag{10}\]

This correlation function is calculated in terms of the quarks and gluons parameters by means of the the operator product expansion (OPE) as:

\[\Pi_{\mu\nu}^{V-\mathrm{AV}}(p_B^2, p_D^2, q^2) = \Pi_0^{V-\mathrm{AV}} m_{B_c}^\mu g_{\mu\nu} + \frac{\Pi_1^{V-\mathrm{AV}}}{m_{B_c}^\mu} P_{\mu B_B} + \frac{\Pi_2^{V-\mathrm{AV}}}{m_{B_c}^\mu} q_{\mu B_B} + i \frac{\Pi_3^{V-\mathrm{AV}}}{m_{B_c}^\mu} \epsilon_{\mu\nu\alpha\beta} P_B^{\alpha\beta}, \tag{11}\]

Each \(\Pi_i\) with \(i = 0, +, -\) and 1 contains of the perturbative and non-perturbative parts as in the following:

\[\Pi_i = \Pi_i^{\text{pert}} + \Pi_i^{\text{nonpert}}. \tag{12}\]

The bare-loop diagram given in Fig.1(a) is the contribution of the perturbative part. The non-perturbative part consists of the two gluon condensates diagrams \{see Fig.2(a-f)\}. Hence, contributions of the light quark condensates \{diagrams shown in Fig.1(b, c, d)\} vanish by applying the double Borel transformations \[16\].

The following double dispersion integrals are the contributions of the bare-loop diagrams in the correlation function:

\[\Pi_i^{\text{pert}} = - \frac{1}{(2\pi)^2} \int du \int ds \frac{\rho_i(s, u, q^2)}{(s - p_B^2)(u - p_D^2)} + \text{subtraction terms}, \tag{13}\]

One of the basic methods to solve the Feynman Integrals in order to calculate the spectral densities \(\rho_i(s, u, q^2)\) is Cutkosky rules where the quark propagators are replaced by Dirac Delta Functions: \(\frac{1}{p^2 - m^2} \rightarrow -2\pi i \delta(p^2 - m^2)\), which indicates that all quarks are on-shell.

Three delta functions appear as a result of the applying Cutkosky rules. These delta functions have to vanish at the same time. Therefore, we get the following inequality from the arguments of the delta functions:

\[-1 \leq \frac{2su + (s + u - q^2)(m_b^2 - s - m_c^2) + (m_c^2 - m_s^2)2s}{\lambda^{1/2}(m_b^2, s, m_c^2)\lambda^{1/2}(s, u, q^2)} \leq +1 \tag{14}\]
\[ \begin{align*}
\lambda(a, b, c) &= a^2 + b^2 + c^2 - 2ac - 2bc - 2ab.

\text{Following the standard calculations, the spectral densities are evaluated as:}

\rho_{1V-\text{AV}}^V &= N_c I_0(s, u, q^2) \left\{ C_1 (m_b - m_c) - (C_2 + 1) m_c + C_2 m_s \right\}
\rho_{0V-\text{AV}}^V &= N_c I_0(s, u, q^2) \left\{ -2m_c^2 + 2m_s m_c - [(C_1 + C_2 + 1)(-q^2 + s + u) + 2C_1 s + 2C_2 u] m_c + 2C_2 u \right\} + m_s [2C_1 s + C_2 (-q^2 + s + u)]
\rho_{+V-\text{AV}}^V &= N_c I_0(s, u, q^2) \left\{ C_1 (m_b - 2C_2 m_c - m_c + 2C_2 m_s) - (2C_2 + 1)(C_2 m_c + m_c - C_2 m_s) \right\}
\rho_{-V-\text{AV}}^V &= N_c I_0(s, u, q^2) \left\{ (2C_2 - 1)(C_2 m_c + m_c - C_2 m_s) + C_1 (m_b - 2C_2 m_c - m_c + 2C_2 m_s) \right\}
\end{align*}\]

where

\[ I_0(s, u, q^2) = \frac{1}{4\lambda^{1/2}(s, u, q^2)}. \]
Now, it is aimed to calculate the non-perturbative part of the Eq. (12) which consists of the gluon condensates diagrams shown in Fig. 2. The gluon condensate contributions are calculated in Fock-Schwinger gauge because in this gauge the gluon field is expressed in terms of gluon field strength tensor directly. The following type of the integrals has to be calculated in order to get the results of the gluon condensate diagrams [15, 21]:

\[
I_0[a, b, c] = \int \frac{d^4k}{(2\pi)^4} \frac{1}{[k^2 - m_b^2]^{\alpha} [(p_B + k)^2 - m_c^2\beta] ((p_D + k)^2 - m_s^2\gamma)},
\]

\[
C_1 = \frac{m_s^2(s - u - q^2) + u(2m_b^2 - s + u - q^2) - m_s^2(s + u - q^2)}{\lambda(s, u, q^2)}
\]
\[
C_2 = \frac{s(2m_s^2 + s - u - q^2) - m_s^2(s + u - q^2) - m_c^2(s - u + q^2)}{\lambda(s, u, q^2)}
\]
\[
N_c = 3.
\]

\[\text{(16)}\]
\[ I_\mu[a, b, c] = \int \frac{d^4k}{(2\pi)^4} \frac{k_\mu}{[k^2 - m_b^2]^{\alpha} [(p_B + k)^2 - m_c^2]^b [(p_D + k)^2 - m_s^2]^c}, \]

\[ I_{\mu\nu}[a, b, c] = \int \frac{d^4k}{(2\pi)^4} \frac{k_\mu k_\nu}{[k^2 - m_b^2]^{\alpha} [(p_B + k)^2 - m_c^2]^b [(p_D + k)^2 - m_s^2]^c}, \]

where \( k \) is the momentum of the spectator quark \( c \). The generic solutions for these integrals can be seen in Refs. [21]-[22]. Apart of our results for the contributions of the gluon condensate diagrams following the similar methods shown in Refs. [21]-[22] is given in Appendix.

The Borel transformations are applied for both phenomenological and QCD side (Eq. 11) in order to suppress the contributions of higher states and continuum. The QCD sum rules for the form factors (\( A_0, A_+ \), and \( A_- \)) are obtained by equalizing the Borel transformed forms of the physical side. The result is in the following formula:

\[
A_i(q^2) = \frac{(m_s + m_c)e^{m_s^2/M_i^2}m_B^2/M_i^2[D_{B^0}m_{D_s^+}^2]}{f_{B_s}m_{B_s}f_D^2} \left[ \frac{1}{(2\pi)^2} \int_{u_{\text{min}}}^{u_0} du \int_{s_{\text{min}}}^{s_0} ds \frac{\rho^{A_i}}{s - A_i} \frac{e^{-s/M_i^2 - u/M_i^2}}{s/M_i^2 - u/M_i^2} \right] + \frac{1}{24\pi^2}G_s < \frac{\alpha_s}{\pi} G^2 > \]

(18)

Note that, the contributions of the gluon condensates (\( C^{A_i} \)) are already considered in the numerical analysis. However, each of these explicit expressions are extremely long, it is found unnecessary to show all of them in this study. Therefore, one of these expressions (\( A^{AV} \)) is shown as a sample in Appendix. The \( s_0 \) and \( u_0 \) are the continuum thresholds in \( s \) and \( u \) channels, respectively. Also \( s_{\text{min}} = (m_b + m_c)^2 \) and \( u_{\text{min}} = (m_s + m_c)^2 \).

### III. NUMERICAL ANALYSIS

Having known the matrix element i.e., Eq. (6), the decay rate for \( B_c^+ \to D_s \nu \bar{\nu} \) decay is evaluated as follows:

\[
\frac{d\Gamma}{dq^2} = \frac{\alpha_s G_s^2 \lambda^{1/2}(m_{B_c}^2, m_{D_s^+}^2, q^2)}{3072\pi^5 m_{B_s}^2 \sin^2 \theta_W} \left\{ |A_0|^2 (m_{B_s}^4 - 2m_{B_s}^2(m_{D_s}^2 - 5q^2) + (m_{D_s}^2 - q^2)^2) \right. \\
- 2 \text{Re}[A_+ A_0^*] m_{B_s}^6 - (m_{D_s}^2 - q^2)^2 \left. \left( m_{B_s}^2 + 3m_{D_s}^2 + q^2 \right) + m_{B_s}^2 \left( 3m_{D_s}^2 - 2m_{D_s}^2q^2 - q^4 \right) \right\} m_{B_s}^2 \\
+ 2|A_1|^2 \frac{\lambda(m_{B_s}^2, m_{D_s}^2, q^2)}{m_{B_s}^6} + |A_+|^2 \frac{\lambda^2(m_{B_s}^2, m_{D_s}^2, q^2)}{m_{B_s}^6} \right\} 
\]

(19)

The expression for the decay rate shows that we need to know the input parameters shown in table I taken from Ref. [24].

Moreover, the values of the leptonic decay constants \( f_{B_c} = 0.415 \pm 0.031 \text{GeV} \) and the gluon condensate \( < \frac{\alpha_s}{\pi} G^2 > = 0.012 \text{ GeV}^4 \) are necessary for the evaluation of the form factors. In addition, the form factors contain four auxiliary parameters: the Borel mass squares \( M_1^2 \) and \( M_2^2 \) and the continuum threshold \( s_0 \) and \( u_0 \). The form factors are assumed to be independent or weakly dependent on these auxiliary parameters in the suitable chosen regions named as "working regions".

The contributions proportional to the highest power of \( 1/M_1^2 \) are supposed to be less than about 30% of the contributions proportional to the highest power of \( M_1^2 \). The lower bound of the \( M_1^2 \) can be determined...
by the above condition. In addition, the contributions of continuum must be less than that of the first resonance. This helps us to fix the upper bound of the $M_2^2$ and $M_2^2$. Therefore, we find the suitable region for the Borel mass parameters in the following intervals: $10 \text{ GeV}^2 \leq M_1^2 \leq 25 \text{ GeV}^2$ and $4 \text{ GeV}^2 \leq M_2^2 \leq 10 \text{ GeV}^2$.

The numerical value of the $s_0$ and $u_0$ are supposed to be less than the mass squared of the first excited state meson and excited state meson with the same quantum numbers. In other words, the $s_0$ and $u_0$ are between mass squared of the ground state meson and excited state meson with the same quantum numbers. The following regions for the $s_0$ and $u_0$ are chosen:

$$(m_{B^*_c} + 0.3)^2 \leq s_0 \leq (m_{B^*_c} + 0.7)^2 \quad \text{and} \quad (m_{D_s} + 0.3)^2 \leq u_0 \leq (m_{D_s} + 0.7)^2.$$ 

The form factors depend on the $q^2$. The detail of the dependence is complicated. We fit them to the following function:

$$F(q^2) = \frac{a}{1-q^2/m_{fit}^2} + \frac{b}{(1-q^2/m_{fit}^2)^2} \quad (20)$$

The $a$, $b$ and $m_{fit}$ are given in Table I:

| $m_{fit}$ | $A_1(q^2)$ | $A_0(q^2)$ | $A_+(q^2)$ |
|-----------|-------------|-------------|-------------|
| $A_1(q^2)$ | 5.01 ± 1.1  | 6.44 ± 1.4  | 5.00 ± 1.08 |
| $A_0(q^2)$ | 6.44 ± 1.4  | 1.11 ± 0.03 | 0.17 ± 0.06 |
| $A_+(q^2)$ | 5.00 ± 1.08 | 0.14 ± 0.04 | 0.28 ± 0.08 |
| $A_-(q^2)$ | 4.98 ± 1.07 | 0.14 ± 0.04 | 0.28 ± 0.08 |

TABLE II: Parameters appearing in the form factors of the $B^*_c \rightarrow D_s \nu \bar{\nu}$ decay in a four-parameter fit, for $M_1^2 = 15 \text{ GeV}^2$, $M_2^2 = 6 \text{ GeV}^2$, $s_0 = 46 \text{GeV}^2$ and $u_0 = 6 \text{GeV}^2$

The origin of the errors in Table II are the variation of $s_0$, $u_0$ and $M_{1,2}$ in the chosen intervals and the uncertainties of the input parameters.

In order to evaluate the branching ratio of the $B^*_c \rightarrow D_s \nu \bar{\nu}$ decay, the mean life time of the $B^*_c$ meson is needed. For the time being there is no experimental data on the mean life time of this meson. We follow the theoretical methods like Bethe-Salpeter model [26] and potential model [27], and estimate that the mean life time of the $B^*_c$ meson is in
the order of the mean life time of the $B_c$ meson. We assume that the total life-time $\tau_{B_c} \approx \tau_{B_c} = 0.452 \times 10^{-12}\text{s}$ \[23\]. Using the mean life time and the $q^2$ dependence of the form factors given by Eq.\[21\] in the kinematical allowed region $[0 \leq q^2 \leq (m_{B_c} - m_{D_s})^2]$ we study the branching ratios for $B_c^* \to D_s \nu\bar{\nu}$ decay. Our results for three different values of the $q^2 = (1, 6, 12) \text{GeV}^2$ are presented in Tables \[III\]. In addition, Fig. \[3\] depicts the dependence of the branching ratio on $q^2$ for full kinematical allowed region.

![Graph showing the dependence of the branching ratio on $q^2$ for $B_c^* \to D_s \mu^+\mu^-$ transitions.](image)

**FIG. 3:** The dependence of the branching ratio on $q^2$ for $B_c^* \to D_s \mu^+\mu^-$ transitions

Finally, we calculate the integrated branching ratio for $B_c^* \to D_s \nu\bar{\nu}$ decay as follows:

$$B_r = \int_0^{(m_{B_c} - m_{D_s})^2} B_r(q^2) dq^2 = (5.47 \pm 1.30) \times 10^{-8} \quad (21)$$

To sum up, we investigated the branching ratio and decay rate of the $B_c^* \to D_s \nu\bar{\nu}$ decay. The form factors of this decay were found in the framework of the QCD sum rules. In addition, the contributions of the two gluon condensates diagrams to the correlations function were obtained.
[1] G. Aad et al. (ATLAS Collaboration), Phys. Lett. B 716, 1 (2012).
[2] S. Chatrchyan et al. (CMS Collaboration), Phys. Lett. B 716, 30 (2012).
[3] R. Aaij et al. (LHCb Collaboration), Phys. Rev. Lett. 108, 251802 (2012); Phys.
[4] Z. G. Wang, Eur. Phys. J. C 73, 2559 (2013) [arXiv:1306.6160 [hep-ph]].
[5] Wang Zhi-Gang, Commun. Theor. Phys. 61, 81 (2014).
[6] G. Buchalla and A. J. Buras, Nucl. Phys. B 548, 309 (1999) [hep-ph/9901288].
[7] Z. -j. Xiao and L. -p. Yao, Commun. Theor. Phys. 38, 683 (2002) [hep-ph/0212008].
[8] G. Buchalla, Nucl. Phys. Proc. Suppl. 209, 137 (2010) [arXiv:1010.2674 [hep-ph]].
[9] C. Q. Geng, C. W. Hwang and C. C. Liu, Phys. Rev. D 65, 094037 (2002) [hep-ph/0110376].
[10] V. Bashiry, [arXiv:1305.6535 [hep-ph]].
[11] N. Ghahramany and A. R. Houshyar, Acta Phys. Polon. B 44, no. 9, 1857 (2013).
[12] L. -F. Gan, Y. -L. Liu, W. -B. Chen and M. -Q. Huang, Commun. Theor. Phys. 58, 872 (2012) [arXiv:1212.4671 [hep-ph]].
[13] Y. Sarac, K. Azizi and H. Sundu, Nucl. Phys. Proc. Suppl. 245, 164 (2013).
[14] A. Khodjamirian, C. Klein, T. Mannel and N. Offen, Phys. Rev. D 80, 114005 (2009) [arXiv:0907.2842 [hep-ph]].
[15] T. M. Aliev and M. Savci, Eur. Phys. J. C 47, 413 (2006) [hep-ph/0601287].
[16] K. Azizi, F. Falahati, V. Bashiry and S. M. Zebarjad, Phys. Rev. D 77, 114024 (2008) [arXiv:0806.0583 [hep-ph]].
[17] R. S. Marques de Carvalho, F. S. Navarra, M. Nielsen, E. Ferreira and H. G. Dosch, Phys. Rev. D 60, 034009 (1999) [hep-ph/9903326].
[18] G. Buchalla and A. Buras, Nucl. Phys. B 400 (1993) 225; Phys. Rev. D 54 (1996) 6782.
[19] T. Inami and C. S. Lim, Prog. Theor. Phys. 65 (1981) 287.
[20] T. M. Aliev, A. Ozpineci and M. Savci, Phys. Lett. B 506, 77 (2001) [hep-ph/0101066].
[21] V. V. Kiselev, A. K. Likhoded, A. I. Onishchenko, Nucl. Phys. B 569 (2000) 473.
[22] J. Schwinger, Phys. Rev. 82, 664 (1951).
[23] J. Beringer et al. (Particle Data Group), Phys. Rev. D86, 010001 (2012).
[24] Z. -G. Wang, Eur. Phys. J. A 49 (2013) 131 [arXiv:1203.6252 [hep-ph]].
[25] M. A. Shifman, A. I. Vainshtein, V. I. Zakharov, Nucl. Phys. B147 (1979) 385.
[26] A. Abd El-Hady, M. A. K. Lodhi and J. P. Vary, Phys. Rev. D 59, 094001 (1999) [hep-ph/9807225].
[27] V. V. Kiselev, A. E. Kovalsky and A. I. Onishchenko, Phys. Rev. D 64, 054009 (2001) [hep-ph/0005020].
In this section, we present the explicit expression for the coefficients $C^{Av}$ corresponding to the gluon condensates contributions of $g_{\mu\nu}$ structure entering to the expression for the form factors in Eq. (13).

\[
C^{Av} = (8m_b + 16m_c)(1, 1, 2) - (32m_b^3 + 16m_b^2m_c + 8m_c^2)I(1, 1, 3) - (16m_b^3 + 8m_b^2m_c^2 + 8m_c^3)
\]
\[
+ 8m_bq^2I(1, 2, 2) + (8m_b^3 + 16m_b^2m_c^2 - 8q^2m_c)I(1, 2, 3) - (24m_bm_c^2 - 24m_b^2m_c)I(1, 3, 1)
\]
\[
- 24m_b^2m_cI(1, 3, 2) - 8m_b^2m_cI(1, 3, 3) + 8m_bI(2, 1, 1) - (16m_b^3 - 8m_b^2m_c^2 - 8q^2m_c)I(2, 2, 1)
\]
\[
- 24m_b^2m_cI(2, 3, 1) + (32m_b^3 - 16m_b^2m_c^2 - 8q^2m_c)I(3, 1, 1) - (8m_b^5 + 16m_b^4m_c + 8q^2m_c^3)I(3, 2, 1)
\]
\[
- 8m_b^6m_cI(3, 3, 1) + (8m_bq^2 - 8q^2m_c)I(1, 1, 3) - 24m_bm_c^2q^2I(1, 1, 4) + 8m_bq^2I(1, 3, 1)
\]
\[
+ (8m_bq^2 - 8q^2m_c)I(1, 2, 2) + (8m_bq^2 + 8m_c^3 - 24m_bq^2 + 24m_b^2m_c^2 + 8m_c^3m_c^2)I(1, 2, 3)
\]
\[
+ 16q^2m_c^2I_1(2, 3, 1) - 16m_bq^2I_1(3, 1, 1) + (16m_b^3q^2 + 24m_b^2m_c^2)I(3, 1, 2)
\]
\[
+ (8m_bq^4 - 16m_b^3q^2 - 16m_b^2m_c^2 + 8m_b^2q^2 + 8m_b^3m_c^2)I(1, 2, 3, 1) + 16q^2m_c^2I_1(3, 2, 1)
\]
\[
+ 8q^2m_c^2I_1(3, 3, 1) + 72m_bq^2I_1(4, 3, 1) - 16m_bq^2I_2(1, 1, 3) - 24m_bq^2I_2(1, 1, 4) - 8m_bq^2I_2(2, 2, 2)
\]
\[
+ 16m_bq^2I_2(2, 2, 3) + 8m_bq^2I_2(1, 3, 1) + 16m_b^2q^2I_2(2, 3, 2) + 8m_b^2q^2I_2(3, 3, 3)
\]
\[
+ (8m_bq^2 - 8q^2m_c)I_2(2, 1, 2) + (40m_b^3q^2 - 16m_b^2q^2)I_2(2, 1, 3) + (8m_bq^4 - 40m_b^3q^2)I_2(3, 1, 1)
\]
\[
+ (8m_bq^4 - 16m_b^3q^2 + 16m_b^2m_c^2 - 8m_b^2q^2 + 8m_b^3m_c^2)I_2(1, 3, 2) + (8m_bq^4 - 16m_b^3q^2)
\]
\[
- 16m_b^2q^2 - 16m_b^2m_c^2 - 8m_b^3m_c^2)I_2(3, 1, 3) + 72m_b^2q^2I_2(4, 1, 1)
\]
\[
+ D_3 \left\{ (8m_b - 8m_b)I_1(3, 3, 1) \right\} + D_5 \left\{ 8m_bI_1(1, 3, 3) + 8m_bI_2(1, 3, 3) \right\}
\]
\[
+ D_0 \left\{ (-24m_b + 8m_b)I(1, 2, 3) + (8m_b - 24m_b)I(1, 3, 2) + (8m_b^3 - 24m_bm_c^2 - 8q^2m_c)I(1, 3, 3)
\]
\[
- 16m_bI_2(1, 2, 3) - 8m_bI_2(1, 3, 2) - (16m_b^3 + 8q^2m_c)I_2(1, 3, 3) \right\}
\]
\[
+ D_2 \left\{ D_0 \left[ 8m_bI(3, 3, 1) + (8m_b - 8m_b)I_1(3, 3, 1) + (16m_b - 16m_b)I_2(3, 3, 1) \right] + 8m_bI(2, 3, 1)
\]
\[
- (16m_b - 8m_b)I(3, 2, 1) + (8m_b - 16m_b)I_1(3, 2, 1)
\]
\[
+ (16m_b - 8m_b)I_2(3, 2, 1) + (16m_b^3 - 16m_d^2m_b + 8q^2m_b - 16m_b^3 - 8m_b^2m_c^3)I_1(3, 3, 1) \right\}
\]
\[
+ D_0 \left\{ D_0 \left[ 8m_bI_1(1, 3, 3) + 16m_bI_1(1, 3, 3) + 8m_bI_2(1, 3, 3) \right] + D_0 \left\{ -16m_bI(1, 2, 3)
\]
\[
- 16m_bI_1(1, 3, 2) - 16m_b^3I(1, 3, 3) - 16m_bI(2, 3, 1) + (8m_b - 16m_b)I(3, 2, 1)
\]
\[
+ (16m_b - 8m_b)I(3, 2, 1) + (16m_b - 8m_b)I_1(3, 2, 1) + (16m_b^3 - 16m_b^2m_b + 16m_b^2m_b)
\]
\[
- 16m_c^5I_1(3, 3, 1) - 16m_cI_2(1, 2, 3) - 8m_cI_2(1, 3, 2) - 16m_c^3I_2(1, 3, 3) + (32m_b - 16m_c)I_2(2, 3, 1) \\
+ (16m_b - 32m_c)I_2(3, 2, 1) + (32m_b^3 - 32m_cm_b^2 + 32m_b^2m_b - 32m_c^3)I_2(3, 3, 1) \\
+ 8m_cI_1(1, 1, 3) + 16m_c^3I_1(1, 2, 3) - 32m_cI_1(1, 3, 1) + 24m_c^5I_1(1, 3, 2) + 8m_c^5I_1(1, 3, 3) - 24m_c^5I_1(1, 4, 1) \\
+ (24m_b - 16m_c)I_2(2, 2, 1) + (-16m_c^3 + 32m_bm_c^2 - 32m_b^2m_c + 16q^2m_c)I_2(2, 3, 1) \\
+ (24m_b - 64m_c)I_2(3, 1, 1) + (24m_b^3 - 32m_cm_b^2 + 56m_b^2m_b + 8q^2m_b - 32m_c^3 + 16m_b^2q^2)I_2(3, 2, 1) \\
+ (-16m_b^5 + 32m_bm_c^4 - 32m_c^2m_b^3 + 16q^2m_b^3 + 32m_c^2m_b - 16m_b^4m_c + 16m_c^2q^2m_c)I_2(3, 3, 1) \\
+ 72m_b^5m_cI_2(4, 1, 1) + (-8m_b + 16m_c)I_1(1, 1, 3) - 16m_cI_1(1, 2, 2) + 32m_c^3I_1(1, 2, 3) \\
+ (-8m_b - 72m_c)I_1(1, 3, 1) + 32m_c^3I_1(1, 3, 2) + 16m_c^3I_1(1, 3, 3) + (-72m_c^5 + 24m_cm_c^2)I_1(1, 4, 1) \\
+ 8m_bI_2(1, 2, 2) + 16m_bq^2I_2(1, 2, 1) + (-16m_b^3 + 8m_cm_b^2 - 8m_b^2m_b - 16q^2m_b + 16m_c^3 \\
+ 8m_cq^2)I_2(1, 2, 3) + (32m_b - 136m_c)I_2(1, 3, 1) - 8m_cI_2(1, 3, 2) + (-8m_b^5 + 16q^2m_b^3 - 8q^4m_b)I_1(3, 1, 3) \\
+ (-16m_b^5 + 8m_cm_b^2 - 8m_c^2m_b - 8q^2m_b + 16m_bq^2 + 16m_cq^2)I_2(3, 2, 1) + (-8m_b^5 + 8m_cm_b^2 + 16m_c^2m_b^3 \\
- 16q^2m_b^3 + 16m_cq^2m_b^2 + 8m_c^2m_b - 16m_cq^2m_b + 8m_b^2 + 16m_c^3q^2)I_2(3, 3, 1) \\
+ (-72m_b^5 + 216m_b^2m_c^3)I_1(4, 1, 1) + 8m_cI_2(1, 1, 3) - 8m_cI_2(1, 2, 2) + 16m_c^2I_2(1, 3, 3) - 24m_c^3I_2(1, 4, 1) + 8m_bI_2(2, 1, 2) + (8m_b - 48m_c)I_2(3, 1, 1) \\
+ (8m_b^3 - 8q^2m_b)I_2(3, 1, 2) + 72m_b^5m_cI_2(4, 1, 1) \\
+ D_b \left\{ - 24m_b^4I_2(1, 4, 1) + 72m_b^5I_2(4, 1, 1)m_c + (24m_b - 16m_c)I_1(1, 1, 3) + (16m_b - 16m_c)I_1(1, 2, 2) \\
+ (-32m_b^3 + 48m_b^2m_c + 16q^2m_c)I_1(1, 2, 3) + (24m_b - 32m_c)I_1(1, 3, 1) + (-16m_b^3 + 40m_cm_b^2 \\
+ 16m_c^2m_c)I_1(1, 3, 2) + (-16m_b^5 + 24m_cm_b^4 + 16q^2m_b^3)I_1(1, 3, 3) + (8m_b + 8m_c)I_2(2, 2, 1) \\
+ (24m_b^3 + 16m_b^2m_c)I_2(2, 3, 1) + (8m_b - 40m_c)I_2(3, 1, 1) + (8m_b^3 + 8m_cm_b^2 + 8m_c^2m_b + 16m_c^3)I_2(3, 2, 1) \\
+ (8m_b^3 - 16m_b^2m_c^2 + 8m_cm_c^2)I_2(3, 3, 1) - 8m_bI_2(1, 1, 3) + (-8m_b - 24m_c)I_2(1, 1, 3) + (24m_b^4m_c^2 \\
- 24m_b^3I_1(1, 4, 1) - 8m_bI_2(1, 2, 2) + 16m_cm^2I_2(1, 2, 3) + (-16m_b^3 + 8m_cm_b^2 - 8m_b^2m_b \\
+ 16m_c^3I_1(2, 3, 1) + (16m_b - 40m_c)I_1(3, 1, 1) + (-24m_b^3 + 16q^2m_b)I_1(3, 1, 2) + (-8m_b^5 + 16q^2m_b^3 \\
- 8q^4m_b)I_2(1, 3, 1) + (-16m_b^5 + 8m_cm_b^2 - 8m_c^2m_b + 16m_b^3)I_2(3, 2, 1) + (-8m_b^5 + 8m_cm_b^2 + 16m_b^2m_b^3 \\
- 16m_b^3m_b^2 - 8m_b^2m_b + 8m_c^3)I_2(3, 3, 1) + (72m_b^5m_c - 72m_b^5I_1(1, 4, 1) + (-16m_b \\
+ 8m_c)I_2(1, 1, 3) - 8m_cI_2(1, 2, 2) + (16m_b^3 + 16q^2m_c)I_2(1, 2, 3) - (16m_b - 72m_c)I_2(1, 3, 1) + (16m^2 \\
+ 8q^2m_c)I_2(1, 3, 2) + (8m_b^3 + 16q^2m_b^3)I_2(1, 3, 3) + (-72m_b^5 + 48m_cm_b^2)I_2(1, 4, 1) - 8m_cI_2(2, 1, 2) \\
+ 32m_b^5q^2I_2(2, 2, 1) + (-32m_b^3 + 16m_cm_b^2 - 16m_b^2m_b + 32m_b^5)I_2(3, 1, 1) + (40m_b - 128m_c)I_2(3, 1, 1) \\
+ (-40m_b^3 + 24q^2m_b)I_2(3, 1, 2) + (-16m_b^5 + 32q^2m_b^3 - 16q^4m_b)I_2(3, 1, 3) + (-32m_b^3 + 16m_c^2m_b^2 \\
- 16m_b^2m_b + 32m_b^3)I_2(3, 2, 1) + (-16m_b^3 + 16m_cm_b^4 + 32m_b^2m_b^2 - 32m_b^2m_b^2 \right\}
\[-16m_6^4m_b + 16m_6^5I_2(3, 3, 1) + (-144m_6^3 + 216m_c m_6^2)I_2(4, 1, 1)\] 

(23)

where

\[
D^j_I \left[ I_n(M^2_1, M^2_2) \right] = (M^2_1)^i(M^2_2)^j \frac{\partial}{\partial(M^2_1)^i} \frac{\partial}{\partial(M^2_2)^j} \left[ (M^2_1)^i(M^2_2)^j I_n(M^2_1, M^2_2) \right].
\]