Two references modulation of polarization for phase-shifting holographic interferometry

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Abstract. Phase-shifting interferometry requires of a modulator able to introduce the phase steps needed to produce N+1 interferograms, where N is the number of shifts. The phase mod 2\pi can be found by solving the (N+1)·(N+1) equation system. Recently, it has been shown that modulation of polarization can be adapted to these techniques with some advantages, as its capability to capture several interferograms simultaneously. In this report, it is shown that this modulation can also be adapted to holographic imagery. To show that, the storage of two slightly different wavefronts is first made with a double exposure heterodyne technique. This means that we use a different reference wave in each recording, thus obtaining a Dnliler-type hologram. This hologram is later reconstructed with both references at the same time, each reference modulated in polarization independently in order to obtain circular polarizations of opposite sign for each reference prior to the respective reconstruction. Observing interference fringes through a linear polarizing, a given shift is determined by the angle \psi of the polarizing filter transmission axis. Capturing the N+1 interferograms generated as described, usual phase-shifting algorithms allow phase extraction. Experimental results for the case N = 3 are shown.

1. Holographic interferometry with modulation of polarization
Double-exposure holography using two references has been discussed for heterodyne holographic interferometry since long [1]. It has been described particularly in detail when using piezoelectric stacks as phase modulators (or acoustic-optical light modulators, as described earlier elsewhere [1].) Because such a technique can easily adapt phase-shifting techniques, it is possible to imagine the use of piezoelectric elements as phase shifters. But modulation of polarization represents another phase-shifting modulator scheme with possible advantages. In this communication, the basics of the method are described, along with some corresponding experimental results.

1.1. Hologram recording with two references
Following the prescriptions in ref.[1], Fig.1 shows an experimental set-up to achieve two exposures of an object O (a fish figure on a round base) subject to a small change between exposures (due to a steel ball on the base at the left of the fish that was placed in only one exposure). The object O was illuminated with two beams to avoid shadows, thus providing an even illumination. The paths of the illuminating beams are ABCO₁OP and AB'O₂OP, where O₁ and O₂ are
Figure 1. Experimental set-up for two-exposures two-object beams holographic recording. A, B, B’: beam splitters. C, D, C’, D’: plane mirrors. O1, O2, R1, R2 are microscope objectives. P, holographic film. Switches are drawn as rectangles with a diagonal within. The object on a round base is a figure representing a fish (which is present in both expositions), together with a ball at its left (present in only one exposure).

microscope objectives used as beam expanders. These beams generate the object wave to be recorded in the recording medium P (photographic film) after their respective reflections in O. One exposure was taken with switch 2 closed and switch 1 open, so the first reference wave has traveled along the path $\text{ABR_1DP}$, where R$_1$ is a microscope objective. The second exposure was taken with switch 1 closed and switch 2 open, so the second reference wave has traveled along the path $\text{AB’C'R_2D'P}$, where R$_2$ is a microscope objective. Care was taken in adjusting these four paths to bring them equal as much as possible. The four beams were generated from a He-Ne laser operating at 632.8 nm.

2. Wavefront reconstruction: modulation of polarization

Once the film was developed and fixed, image reconstruction was achieved by changing the polarization of each reference beam so that one beam results left-circularly polarized and the other beam results right-circularly polarized, as sketched in Fig.2. A linear polarizer is placed between the hologram and a CCD camera to detect interference fringes. The resulting interference pattern shifts the fringes in dependence on the angle of the transmission axes of the polarizer with respect to the horizontal plane [2,3].

The basics of these observations are as follows. Two elliptically polarized beams of opposite hands can be written as the following Jones vectors

$$
\overrightarrow{J}_L(x, y) = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 1 \\ e^{i\alpha'} \end{array} \right), \quad \overrightarrow{J}_R(x, y) = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 1 \\ e^{-i\alpha'} \end{array} \right) e^{-i\phi(x, y)},
$$

where $\alpha'$ is the effective phase shift between orthogonal components and $\phi(x, y)$ is the phase difference between beams in the plane of detection. After a linear polarizing $J_\psi^l$ with axis at angle $\psi$, two emerging beams $\overrightarrow{J}', \overrightarrow{J}''$ are obtained according to
Figure 2. Experimental set-up for capture of the interference fringes generated by the superposition of two wavefronts recorded at different times and reconstructed with different reconstructing beams. Reconstructing beam of path $ABR_1DP$ is left-handed circular polarized, while reconstructing beam of path $AB'C'R_2DP$ is right-handed circular polarized. To detect fringes, a linear polarizer has to be placed between hologram P and camera.

$$J'_\psi = \begin{pmatrix} \cos^2 \psi & \sin \psi \cos \psi \\ \sin \psi \cos \psi & \sin^2 \psi \end{pmatrix}, \quad \vec{J} = J'_\psi \vec{J}_L \quad \vec{J}^\theta = J'_\psi \vec{J}_R$$

(2)

The total irradiance of the superposition of both beams can be calculated to be

$$\| \vec{J}_T \|^2 = \| \vec{J} \| = 1 + \sin 2\psi \cos \alpha' + A(\psi, \alpha') \cos [\xi(\psi, \alpha') - \phi(x, y)]$$

(3)

with the phase shift $\xi(\psi, \alpha')$ given by

$$\xi(\psi, \alpha) = \tan^{-1} \left( \frac{\sin 2\psi \sin \alpha' + \sin^2 \psi \sin 2\alpha'}{\cos^2 \psi + \sin^2 \psi \cos 2\alpha' + \sin 2\psi \cos \alpha'} \right)$$

(4)

and

$$A(\psi, \alpha') = 1 + \sin 2\psi \cos \alpha'$$

(5)

so, the value of $\alpha'$ defines the angle $\psi$ needed to obtain a desired $\xi$ value. In order to use phase-shifting techniques, the $\xi$–values of $0$, $\pi/2$, and $3\pi/2$ rads are chosen. For the case of $\alpha' = \pi/2$, $A(\psi, \pi/2) = 1$, $\xi(\psi, \pi/2) = 2\psi$ and the $\psi$–values are $0$, $\pi/4$, and $3\pi/4$ rads Otherwise, a different set of $\psi$ values has to be employed. This result can be applied when using wavelengths not lying within the tolerance range of the wave retarders employed [2,3].

3. Experimental results

After using basically the set-up of Fig.2, the resulting images are shown in Fig.3 for four interferograms (Fig.3.b). The fringes shown are approximately on the object (Fig.3.a). After each 90 phase shift, four interferograms are obtained (Fig.3.b). Opposite fringe contrast patterns can be recognized for 180 shifts. Each interferogram was displayed as an image with $640 \times 512$
pixels and 8-bit gray levels. A normalization procedure was next used for each image taking into account the particular extreme values of irradiance. Then, a conventional algorithm for phase-shifting interferometry for 4 interferograms was used. The retrieved, unwrapped phase [2-4] shows no fringing (Fig.3.c). This fact might be an indication that each shift was the proper one.

Figure 3. Experimental results. a) Reconstruction of one wavefront (left) and simultaneous reconstruction of two wavefronts by employing the set-up of Fig.2 (right). b) Interference patterns with introduction of phase shifts of 0, 90, 180 and 270 degs, the polarizing filter angles in degrees were \( \psi_0 = 0, \psi_1 = 45, \psi_2 = 90 \) and \( \psi_3 = 135 \) degs (case of \( \alpha' = \pi/2 \)). c) Phase extraction by using a conventional phase-shifting routine and unwrapping (gray levels and 3D plots).

Additional similar results with fringes out of the object’s surface can be seen in Figs.4 and 5 for the same subject at different angle views. To achieve circular polarization as nearly as possible and due to the design of the polarization components available, a wavelength of 532.5 nm was used for reconstruction. Because the main interest in this communication is to show the feasibility of the method, no further scale or depth corrections were carried out.

Figure 4. Experimental results using fringes out of the object’s surface. a) Two sets of four interferograms, 90 degs phase-shift. b) Unwrapped phase for each set (gray levels and 3D plots).

4. Discussion
Holographic interferometry with two references [6] can be performed with modulation of polarization and phase-shifting techniques. Care must be taken in order to maintain each beam in its polarization state as prescribed. This requires the use of tension-free lenses among other features. As an advantage of this technique (as compared with piezoelectric stacks, for instance) it is worthwhile to mention that the calibration procedure must be done only once (determination
of the linear polarizer transmission angle used for detecting fringes.) This means that no further calibration has to be done either for different angles of view or even if an adjustment in the fringes must be carried out.

![Figure 5](image)

**Figure 5.** Experimental results using fringes out of the object’s surface. a) Two sets of four interferograms, 90 degs phase-shift. b) Unwrapped phase for each set (gray levels and 3D plots).

In holographic interferometry with two references with modulation of polarization, on the other hand, more than four interferograms could be used with appropriate algorithms (for example, 5, 7 or 9) [4]. A simple image processing deals with the problem of having several interferograms, each with possibly different intensities and/or fringe contrasts. The procedure could be employed for several viewing angles. Wavelengths other than the used for polarization components designs can be employed using the values of $\psi$ properly as described in the previous section ($\alpha' \neq \pi/2$). Change of wavelength between stages of recording and reconstruction could be also done for practical reasons, as when fitting the tolerances of the optical components available (wave retarders, for example.) When changing wavelengths, of course, quantitative evaluation of the results in proper units needs of well-known corrections related with corresponding changes in dimensional scales and positions [6, 7]. Other kind of corrections must be done when the fringes do not appear over the object’s surface [6, 7].

When using a film as the recording medium, it tends to act as a membrane, so it is very sensitive particularly to acoustical vibrations. For the experimental results shown in this communication, the shifts were carried out in sequence because the phase to inspect remains fixed. However, a one shot scheme would be possible by multiplexing the field of the two circular polarized beams with a phase grating and performing detection around each diffraction order with a linear polarizer at the adequate angle $\psi$ [5]. In principle, each diffraction order would need of a linear polarizing at a given different value of $\psi$. Besides, in this scheme, no special components (as another hologram [8] or micropolarizer arrays [9], for example) other than the standard ones described would be needed.

In conclusion, a new method for phase extraction in two references holographic interferometry based on modulation of polarization was described. Although we show experimental results with four interferograms, it becomes possible to extend the number of interferograms to more than four, if desired. The proposed technique is feasible and can be implemented with only standard optical elements for recording media such as photographic films. With a given wavelength, the calibration of polarizing filters must be made only once even for different viewing angles or for different optical path adjustments. Thus, applications of heterodyne holographic interferometry [1] can be adapted to phase-shifting techniques in a rather simpler fashion. After due considerations, other approaches could also receive benefits from modulation of polarization,
as phase-shifting digital holography [10], digital holographic interferometry [11], or dynamical holography [12].

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