PAPR Reduction in All-optical OFDM Systems Based on Phase Pre-emphasis

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Abstract. This paper investigates the peak-to-average power ratio (PAPR) theory in all-optical orthogonal frequency division multiplexing (OFDM) optical fibre communication systems. We find out that phase pre-emphasis could effectively reduce PAPR in all-optical OFDM communication systems which employ intensity modulation-direct detection (IM-DD) method. An equation is developed and proposed to calculate suitable phasing values for pre-emphasis. Furthermore, we find out that phase pre-emphasis cannot reduce PAPR effectively in all-optical OFDM systems that employ Phase Shift Keying (PSK) or Quadrature Amplitude Modulation (QAM) method.

1. Introduction
Optical OFDM has become a promising technique in long-haul and high-speed optical transmission systems, for its high spectral efficiency, relatively low bit rate and advanced robustness against chromatic dispersion and polarization mode dispersion [1-3]. All-optical OFDM has attracted much attention recently, for its advantage of eliminating processing speed limitation set by electronics [4-7].

High PAPR is a serious intrinsic defect in all-optical OFDM systems, deteriorating nonlinear impairment in optical fibers. However, hardly any investigations are focusing on the PAPR characteristic in all-optical OFDM systems. In conventional optical OFDM systems, PAPR reduction is implemented electronically by introducing phase pre-emphasis before modulation and post-processed by DSP at the receiver [9,10]. We have recently proposed a PAPR reduction scheme based on phase pre-emphasis in all-optical IM-DD OFDM systems, where we used numerical method rather than equations to obtain suitable phasing values [8].
This paper studies the PAPR theory in all-optical OFDM systems. We find out that phase pre-emphasis can effectively reduce PAPR in all-optical IM-DD OFDM systems; while in PSK or QAM OFDM systems, it is not as effective any more. An equation is developed to calculate suitable phasing values for all-optical IM-DD OFDM systems. The PAPR theory in all-optical OFDM systems is analyzed in Section 2. Section 3 and Section 4 evaluate the effectiveness of PAPR reduction based on phase pre-emphasis in all-optical IM-DD OFDM systems and in PSK or QAM OFDM systems, respectively. Section 5 is the conclusions.

2. PAPR theory in all-optical OFDM systems

Figure 1 depicts the configuration of an all-optical OFDM system. At the transmitter, a serial data stream is converted into multiple parallel data streams by the serial to parallel convertor (S/P). An electro-absorption modulator (EAM) generates a phase-locked optical pulse train, which is then split into multiple identical pulse trains. The subcarriers are modulated before entering an optical inverse discrete Fourier transformer (OIDFT), which implements the optical inverse Fourier transformation to generate all optical OFDM symbols. The modulation here can be Intensity modulation or PSK modulation. After transmission, the optical discrete Fourier transformer (ODFT) rebuilds optical signals in every sub-carrier at the receiver. Detectors demodulate data in every sub-carrier after EAM sampling. The EAMs work as optical time gates, whose on-state is 1/N of an OFDM symbol’s duration and off-state is the rest of an OFDM symbol’s duration [4]. The parallel to serial processor (P/S) combines multiple parallel electronic data streams into a serial data stream. Because all optical OFDM systems need no electronic FFT or D/A circuits, the limitation of processing speed and high cost of electronic circuits are eliminated. The main difference between the all-optical OFDM system shown in Fig.1 and the optical coherent-OFDM systems [1,2] is that in the all-optical OFDM system, subcarriers are modulated before entering an OFDM multiplexer (the OIDFT), while in coherent-OFDM systems, OFDM symbols are generated by electronics and then modulated on optical light.

Figure 1 Configuration of an all-optical OFDM system
In optical OFDM systems, data stream is typically modulated by PSK or QAM, before entering an inverse fast Fourier transformation (IFFT) processor. The base band equivalent time-domain signal $X_n$ can be expressed by Eq. (1).

$$X_n = \text{IFFT} \{ X_k \} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k \exp(j \frac{2\pi}{N} nk) \quad (n = 0, 1, \ldots, N - 1)$$

where $N$ is the number of sub-carriers, and $X_k$ denotes the kth modulated PSK or QAM symbol. $X_k$ can be written as $X_k = a_k + jb_k$, where $X_k$ and $b_k$ indicate the real component and imaginary component of $X_k$, respectively. The PAPR in discrete time OFDM systems is generally defined in Eq. (2).

$$\text{PAPR}_{db} = 10\log \left( \frac{\max_{0 \leq n < N-1} |X_n|^2}{\text{mean}|X_n|^2} \right)$$

The complementary cumulative distribution function (CCDF) of PAPR can be written as Eq.(3):

$$P(\text{PAPR} > \text{PAPR}_0) = 1 - \left( 1 - e^{-\text{PAPR}_0} \right)^N$$

Eq.(3) depicts the PAPR characteristics in conventional optical OFDM systems. Nevertheless, Eq.(3) is only applicable to all-optical OFDM systems with PSK or QAM mapping. Eq.(3) will be unestablished in all-optical IM-DD OFDM systems so that modifications are needed for PAPR investigation. Unfortunately, there are no analytical probability distribution functions so far. Numerical simulation is the only way to investigate PAPR characteristic under this condition [8].

### 3. Phase pre-emphasis in all-optical IM-DD OFDM systems

Eq. (1) implies that every OFDM symbol contains $N$ components ($X_0, X_1, \ldots, X_{N-1}$), lining up in sequence in time domain. In binary IM-DD systems shown in fig. 1, $X_k$ follows 0-1 distribution, it’s clear that $|X_n| = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} |X_k|$ and $|X_n|$ reaches a maximum when $n=0$. Because the value of $|X_n|$ always reaches its maximum when $n=0$, PAPR will be reduced if the in-phase condition (when $n=0$) is eliminated. In our PAPR reduction scheme, different phase pre-emphasises are introduced and Eq.(1) is converted to Eq.(4) and (5).

$$x_n = \text{IFFT} \{ x_k \} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} x_k \exp(j \frac{2\pi}{N} nk) \quad (n = 0, 1, \ldots, N - 1)$$

$$x'_k = x_k \exp(j \phi_k) \quad (k = 0, 1, \ldots, N - 1)$$

where $\phi_k$ is the kth phase pre-emphasis. Chosen suitable values for $\phi_k$, the in-phase condition (when $n=0$) can be eliminated and PAPR will be reduced.

From the PAPR definition Eq. (2), it is clear that the distribution of $|X_n|^2$ determines PAPR characteristics. Using equation (4), $|X_n|^2$ can be written as:

$$|x_n|^2 = \frac{1}{N} \left\{ \sum_{k=0}^{N-1} x_k^2 + 2 \sum_{k=0}^{N-2} \sum_{l=k+1}^{N-1} x_k x_l \exp \{ j \frac{2\pi}{N} n (1 - k) + \phi_l - \phi_k \} \right\}$$

Set

$$Q_n = \sum_{k=0}^{N-2} \sum_{l=k+1}^{N-1} x_k x_l \exp \{ j \frac{2\pi}{N} n (1 - k) + \phi_l - \phi_k \}$$

To reduce PAPR, suitable values of $\phi_k$ should be chosen to weaken $Q_n$ fluctuation.
Q_n can be seen as the summation of vectors with a length of zero or one, because X_k follows 0-1 distribution. The vectors can be separated into N parts, N lines in Eq.(8). X_k is a random variable, this means that the vectors vary randomly. So that each line of Eq.(8) represents the summation of vectors and the number of the vectors vary randomly according to the values of X_k. Therefore, the summation will be small if all the vectors are symmetrically distributed on a circle. It is effective to choose suitable values of ϕ_k to make the vectors in the first or second line in Eq.(8) arranged symmetrically, in order to reduce PAPR. It should be mentioned that it is hard to calculate ϕ_k when more lines in Eq.(8) are considered. Considering the first line in Eq.(8), suitable values of ϕ_k following:

\[ (\phi_{k+1} - \phi_k) - (\phi_k - \phi_{k-1}) = \frac{2\pi}{N} \]  

A solution of Eq.(9) is:

\[ \phi_k = k(k - 1)\pi \quad \text{mod} \quad N - 1 \]  

Similarly, when considering the second line in Eq.(8), another solution is:

\[ \phi_k = \begin{cases} 
\frac{k(k - 2)\pi}{2(N - 2)} & k = 0, 2, 4, ..., N - 2 \\
\frac{(k - 1)\pi + (k - 1)^2\pi}{N - 1} & k = 1, 3, 5, ..., N - 1 
\end{cases} \]  

We calculate the average of Eq.(18) and (19) to get the values of ϕ_k.

\[ \phi_k = \begin{cases} 
\frac{k(k - 2)\pi}{4(N - 2)} & k = 0, 2, 4, ..., N - 2 \\
\frac{k(k - 1)\pi}{4(N - 1)} & k = 1, 3, 5, ..., N - 1
\end{cases} \]  

Phasing values calculated by Eq.(12) are used to simulate the PAPR reduction effectiveness in a binary IM-DD OFDM system with 16 subcarriers (N=16). We develop a Matlab package to calculate PAPR. During each simulation, 40000 sample points are used. Figure 2 depicts the comparison of the probability distributions of PAPR between all-optical IM-DD OFDM systems with and without phase pre-emphasis, where X-axis and Y-axis represent the PAPR value and the corresponding possibility, respectively.

It can be seen that the possibility of high PAPRs become much smaller while phase pre-emphases are used. And the most possible PAPRs reduce for several dBs. Figure 3 depicts the differences between CCDFs, where X-axis and Y-axis represent PAPR0 value and the possibility when PAPR is larger than PAPR0, respectively. It is obvious that PAPRs have been reduced effectively. While the possibility that
PAPR is larger than PAPR0 is 0.0032, PAPR has been reduced by 3.46dB (N=16), 5.28dB (N=32) and 7.41dB (N=64). This means that phase pre-emphasis will be even more effective to reduce PAPR in OFDM systems with a large number of subcarriers. For the effectual reduce of PAPR, it will reduce the nonlinear effects in optical fibers greatly [8].

Figure 2 Probability distribution of PAPR

Figure 3 The CCDF of PAPR
4. Phase pre-emphasis in all-optical PSK or QAM OFDM systems
In all-optical PSK or QAM OFDM systems, phase pre-emphasis is ineffective for PAPR reduction. First of all, phase pre-emphases are a kind of phase noise. In order not to deteriorate the detection seriously, only small values of $\phi_k$ are acceptable. Second, because the aforementioned in-phase condition (when $n=0$) is not satisfied any longer, phase pre-emphasis which is aimed to destroy the in-phase condition cannot reduce PAPR noticeably. To illustrate the point clearly, a simple example will be discussed.

Assume a 4-subcarrier all-optical OFDM system that employs binary PSK modulation scheme, so $X_k$ can only be 1 or -1. Eq.(1) can be written in a matrix form, as shown in Eq.(13). The operation can be seen as summations of vectors. Figure 4 depicts that $X_k (k=0,1,2,3)$ is the summation of four vectors, indicated by the red arrows.

$$\begin{bmatrix}
X_0 \\
X_1 \\
X_2 \\
X_3
\end{bmatrix} = \begin{bmatrix}
1 & 1 & 1 & 1 \\
1 & i & -1 & -i \\
1 & -1 & 1 & -1 \\
1 & -i & -1 & i
\end{bmatrix}\begin{bmatrix}
X_0 \\
X_1 \\
X_2 \\
X_3
\end{bmatrix}$$

(13)

As an example, assume the input values are [-1,-1,1,1]. Using the vector summation, 1 indicates the vector itself, while -1 indicates a vector with an opposite pointing direction. $[X_0,X_1,X_2,X_3]$ can be calculated out as [0,-2+2i, 0,-2-2i]. Figure 5 illustrates that $|X_0|^2$ and $|X_3|^2$ reach the maximum and PAPR is 3dB, where the blue arrow represents the vectors’ summation. Because $X_1$ is vertical to $X_3$, when phase pre-emphasis is introduced to reduce $|X_3|^2$ then $|X_1|^2$ will be enhanced. For example, if $\phi_k$ are chosen as $[-\pi/8, -\pi/8, \pi/8, -\pi/8]$, $|X_1|^2$ will reduce from 2 to 0.5858, while $|X_3|^2$ increase from 2 to 3.4142, with the PAPR increases from 3dB to 5.33dB. Therefore, phase pre-emphasis cannot reduce PAPR here. What is more, when PSK or QAM are used, the phase of $X_k$ is variable according to different input data, making the calculation of $\phi_k$ complicated.

Figure 4  IFFT in a matrix form
5. Conclusions
This paper investigates the PAPR theory in all-optical OFDM systems. We find out that phase pre-emphasis can effectively reduce PAPR in all-optical IM-DD OFDM systems. An equation is developed to calculate suitable phasing values. Phase pre-emphasis can reduce PAPR effectively in all-optical IM-DD OFDM systems, especially in those systems with a large number of subcarriers. Therefore, it will reduce nonlinear effects in optical fibers greatly. We also find out that phase pre-emphasis cannot reduce PAPR noticeably in all-optical PSK or QAM OFDM systems.

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