Study of the single neutral top-pion production process at \(\gamma\gamma\) collider

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Abstract

\(\gamma\gamma \rightarrow \Pi_t^0\) is the major production mechanism of neutral top-pion at the linear colliders. In this paper, we calculate the cross section of the process \(\gamma\gamma \rightarrow \Pi_t^0\) and discuss the potential to observe the neutral top-pion via its various decay modes at the planned ILC. The study show that, among the various neutral top-pion production processes at the linear colliders, the cross section of \(\gamma\gamma \rightarrow \Pi_t^0\) is the largest one which can reach the level of \(10^1 \sim 10^2\) fb. Due to the existence of the tree-level flavor-changing coupling \(\Pi_t^0 t\bar{c}, \gamma\gamma \rightarrow \Pi_t^0 \rightarrow t\bar{c}\) can provide enough number of typical signals to identify the neutral top-pion with the clean SM background. Therefore, the process \(\gamma\gamma \rightarrow \Pi_t^0\) play an important role in searching for the neutral top-pion and test the TC2 model.

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1 Introduction

The mechanism that governs the electroweak symmetry breaking (EWSB) is at present the largest unknown in the Standard Model (SM). In order to solve the problem of EWSB, some new physics models beyond the SM have been proposed, such as: supersymmetry (SUSY) and dynamical EWSB mechanism concerning new strong interactions, etc. An important goal of studies at the next generation of positron-electron ($e^+e^-$) colliders is the proper understanding of the EWSB, and the discovery of new physics beyond the SM. If the new particles or interactions will be directly discovered at the Tevatron and LHC, the linear colliders will then play a crucial role in the detailed and thorough study of these new phenomena and in the reconstruction of the underlying fundamental theories. The International Linear Collider (ILC) [1] is a worldwide consensus linear accelerator $e^+e^-$ collisions in the energy range of 500 GeV to above TeV. By Compton backscattering of laser photons off the electron and positron beams, one can produce high luminosity $\gamma\gamma$ collisions with a wide spectrum of $\gamma\gamma$ center-of-mass energy. In addition, a high degree of circular polarization for each of the colliding photons can be achieved by polarizing the incoming electron and positron beams and the laser beams [2]. Photon colliders have distinct advantages in searches for and measurements of new physics objects. In general, phenomena in $e^+e^-$ and $\gamma\gamma, \gamma e$ collisions are similar because the same particles can be produced. However, the reactions are different and the photon colliders often give complementary information. Some phenomena can be studied better at photon colliders due to higher statistical accuracy. On the other hand, the cross sections of some processes in $\gamma\gamma$ collisions are larger than those in $e^+e^-$ collisions. In the hunt for physics beyond the SM, only small signs may be visible, therefore, the photon colliders provide optimal conditions for searching for the new physics.

Among the new physics models, technicolor (TC) model is a promising candidate of dynamical theories [3]. However, it is hard for technicolor to generate the fermion masses, specially, the heavy top quark mass. In order to overcome the shortcoming of the simple TC model and explain the large mass problem of the top quark, an interesting TC model, called the topcolor assisted technicolor (TC2) model, is proposed [4, 5, 6, 7] which gives a reason-
able explanation of the EWSB and heavy top quark mass. In the TC2 model, the topcolor interaction makes small contribution to the EWSB, and gives rise to the main part of the top quark mass $(1-\varepsilon)m_t$ with a model dependant parameter $0.03 \leq \varepsilon \leq 0.1$. The technicolor interaction plays a main role in the breaking of electroweak gauge symmetry. To account for the explicit breaking of quark and lepton flavor symmetries, the extended technicolor(ETC) was invented. The ETC interaction gives rise to the masses of the ordinary fermions including a very small portion of the top quark mass $\varepsilon m_t$. This kind of model predicts three CP odd top-pions ($\Pi^0_t, \Pi^\pm_t$) with large Yukawa couplings to the third family. The LEP-SLD precision measurement data of $R_b$ give a severe constraint on the mass of the charged top-pion. Even in an optimistic estimate the mass of the charged top-pion should be larger than 220 GeV. Because the top-pions are the typical physical particles of the TC2 model, the observation of top-pions can be regarded as the direct evidence of the TC2 model. The study of the various top-pion production mechanisms is well motivated which can offer the useful instruction to search for the top-pions. Via the $e^+e^-$ collisions, the neutral top-pion can be produced via the processes $e^+e^- \rightarrow f\bar{f}\Pi^0_t (f = u, d, s, t, b, c, \mu, \tau), e^+e^- \rightarrow t\bar{c}\Pi^0_t, e^+e^- \rightarrow \Pi^0_t Z, \Pi^0_t \gamma$. The main charged top-pion production mechanism are $e^+e^- \rightarrow t\bar{b}\Pi^-_t$ and $e^+e^- \rightarrow W^+\Pi^-_t$, these processes have been systematically studied. The photon colliders will offer another chance to search for the top-pions. We have studied the potential to discover the neutral top-pion via the processes $\gamma\gamma \rightarrow t\bar{t}\Pi^0_t, \gamma\gamma \rightarrow t\bar{c}\Pi^0_t, e^-\gamma \rightarrow e^-\Pi^0_t$. In general, the cross sections of above processes are at the level of $10^0 - 10^1$ fb, and it is promising to observe the top-pions at future linear colliders with high luminosity. Also, the charged top-pion production processes at the photon collider have been studies in the reference. On the other hand, the neutral top-pion can also be produced as s-channel resonances in the process $\gamma\gamma \rightarrow \Pi^0_t$ through top quark triangle loop and thus the full photon beam energy can be used to produce the heavy particle. The similar work has been done in the framework of a rescaled QCD models and the loe-scale TC model. In this paper, we calculate the cross section of $\gamma\gamma \rightarrow \Pi^0_t$ and study the possibility to observe the neutral top-pion via its various decay modes. We find that, with large cross section, $\gamma\gamma \rightarrow \Pi^0_t$ can provide a unique way to identify the neutral top-pion with high precision. Among the various decay modes of $\Pi^0_t, \Pi^0_t \rightarrow t\bar{c}$.
is a typical one due to the existence of the tree-level coupling $\bar{t}c\Pi^0_t$. Because there exists GIM(Glashow, Iliopoulos, and Maiani) mechanism in the SM, $\bar{t}c$ production rate in the SM is too small to observe $\bar{t}c$, the observable signal of $\bar{t}c$ might be a sound evidence of the new physics. So, $\Pi^0_t \to \bar{t}c$ is the ideal mode to observe $\Pi^0_t$. $\bar{t}c$ production processes via photon-photon collision have been studied in some new physics models(The Two Higgs Doublet Model(Model III), Minimal Supersymmistic Standard Model(MSSM) and TC2 model[16]. The study shows that the production rate of $\bar{t}c$ can reach the observable level in these new physics models. In our paper, the possibility to observe the neutral top-pion via other modes is also systematically studied.

The outline of the paper is as follows: In the next section, we describe the details of our calculation and give some analytic formulae. In the third section, we discuss the numerical results and give some conclusions.

2 The cross section of $\gamma\gamma \to \Pi^0_t$

As it is well known, the couplings of the top-pions to the three family fermions are non-universal. The top-pions have large Yukawa couplings to the third family and can induce large flavor changing couplings. The couplings of the neutral top-pion to quarks can be written as[14]:

$$i\frac{m_t\tan\beta}{v_w}[K^t_{UR}K^{\mathit{tt}}_{UL}tLtR\Pi^0_t + K^t_{UR}K^{\mathit{tt}}_{UL}\bar{t}LcR\Pi^0_t + \frac{m_b^*}{m_t}K^{bb}_{DL}\bar{b}LbR\Pi^0_t + h.c.].$$

(1)

Where, $\tan\beta = \sqrt{v_w^2/v_t^2 - 1}$, $v_w = 246$ GeV is the electroweak symmetry breaking scale, and $v_t = 60 \sim 100$ GeV is the top-pion decay constant. $m_b^*(m_b^* \approx \frac{3km_t}{8\pi^2} \sim 6.6k$ GeV) is the part of b-quark mass induced by the instanton and k ranges from 1 to $10^{-1}$ as in QCD, in our calculation, we take $k=0.5$ as a typical example. The factor $\tan\beta$ reflects the effect of the mixing between the top-pions and the would-be goldstone bosons. $K^t_{UL}$, $K^{bb}_{DL}$, $K^t_{UR}$ and $K^t_{UR}$ are the elements of the rotation matrices $K_{L,R}$ which are needed for diagonalizing the quark mass matrices. The matrix elements are given as

$$K^t_{UL} \approx K^{bb}_{DL} \approx 1, \quad K^t_{UR} = 1 - \varepsilon,$$

(2)
In this paper, we take $K_{UR}^{tc} = \sqrt{2\varepsilon - \varepsilon^2}$ and typically take $\varepsilon = 0.03, 0.06, 0.1$.

With the coupling $tt\Pi_0^0$, $\Pi_0^0$ couples to a pair of photon via top quark triangle loop (the contribution of bottom quark loop is much smaller than that of top quark loop, we ignore such contribution here). We explicitly calculate the top quark triangle loop and obtain the effective coupling of $\Pi_0^0 - \gamma - \gamma$ as:

$$
\frac{8m_t^2 \tan(1-\varepsilon)\alpha_e}{9v_w\pi}\varepsilon_{\mu\nu\rho\sigma}p_3^\mu p_2^\nu p_0^\rho C_0,
$$

where $C_0 = (-p_2, p_3, m_t, m_t, m_t)$ is the standard three-point Feynman integral with $p_3$ denoting the momentum of $\Pi_0^0$ and $p_2$ denoting the momentum of an incoming photon. Single $\Pi_0^0$ can be produced via photon-photon fusion process $\gamma\gamma \rightarrow \Pi_0^0$.

The lowest order parton cross section can be expressed by the decay width of $\Pi_0^0 \rightarrow \gamma\gamma$:

$$
\hat{\sigma}_{\gamma\gamma \rightarrow \Pi_0^0}(\hat{s}) = \sigma^0 M_\Pi^2 \delta(\hat{s} - M_\Pi^2),
$$

$$
\sigma^0 = \frac{\pi^2}{8M_\Pi^2} \Gamma(\Pi_0^0 \rightarrow \gamma\gamma),
$$

$$
\Gamma(\Pi_0^0 \rightarrow \gamma\gamma) = \frac{2m_t^4 \tan^2(1-\varepsilon)^2\alpha_e^2 M_\Pi^3}{81\pi^3 v_w^2} |C_0|^2,
$$

where $\hat{s}$ is the $\gamma\gamma$ center-of-mass (c.m.) energy squared and $M_\Pi$ denotes the mass of the neutral top-pion. The $\delta$ distribution can be approximately substituted by the Breit-Wigner form for zero-width $\delta$ distribution:

$$
\delta(\hat{s} - M_\Pi^2) \rightarrow \frac{1}{\pi} \cdot \frac{s\Gamma_\Pi / M_\Pi}{(\hat{s} - M_\Pi^2)^2 + (s\Gamma_\Pi / M_\Pi)^2},
$$

and changing kinematical factors $M_\Pi^2 \rightarrow \hat{s}$ appropriately. $\Gamma_\Pi$ is the total decay width of $\Pi_0^0$ which can be obtained by summing the decay widths of all the decay modes. The possible decay modes of $\Pi_0^0$ are $\Pi_0^0 \rightarrow t\bar{t}$, $t\bar{c}b\bar{b}$, $\gamma\gamma$, $gg$, $\gamma Z$ (if $M_\Pi < 2m_t$, $\Pi_0^0 \rightarrow t\bar{t}$ is forbidden). Their decay widths can be easily obtained as:

$$
\Gamma_{\Pi_0^0 \rightarrow t\bar{t}} = \frac{3(1-\varepsilon)^2 m_t^2 \tan^2(1-\varepsilon) M_\Pi^2}{8\pi v_w^2} \sqrt{1 - \frac{4m_t^2}{M_\Pi^2}},
$$
\[ \Gamma_{\Pi^0_{t} \rightarrow \ell \bar{\nu}} = \frac{3(2\varepsilon - \varepsilon^2)m_t^2 \tan^2 \beta (M_{\Pi}^2 - m_t^2)^2}{8\pi v_w^2 M_{\Pi}^2}, \]

\[ \Gamma_{\Pi^0_{t} \rightarrow b \bar{b}} = \frac{3m_b^2 \tan^2 \beta M_{\Pi}}{8\pi v_w^2} \sqrt{1 - \frac{4m_b^2}{M_{\Pi}^2}}, \]

\[ \Gamma_{\Pi^0_{t} \rightarrow \gamma \gamma} = \frac{(1 - \varepsilon)^2 m_t^4 M_{\Pi}^3 \tan^2 \beta \alpha_s^2}{4\pi^3 v_w^2} |C_0^*|^2, \]

\[ \Gamma_{\Pi^0_{t} \rightarrow \gamma Z} = \frac{\alpha_s^2 (1 - \varepsilon)^2 \tan^2 \beta M_{\Pi}^3}{9\pi^3 v_w^2} \tan^2 \theta_w (1 - \frac{m_t^2}{M_{\Pi}^2})^2 L^2(M_{\Pi}), \]

with

\[ C_0^* = C_0(-p_{\Pi}, p_g, m_t, m_t, m_t), \]

\[ L(M_{\Pi}) = \int_0^1 dx \int_0^1 dy [1 + (\frac{M_{\Pi}}{m_t})^2 x(x - 1)y^2 + (\frac{M_{\gamma}}{m_t})^2 yx(y - 1)]^{-1}, \]

here, \( p_{\Pi} \) and \( p_g \) denote the momenta of \( \Pi^0_{t} \) and one gluon, respectively.

After calculating the cross section \( \hat{\sigma}_{\gamma\gamma \rightarrow \Pi^0_{t}}(\hat{s}) \) for the subprocess \( \gamma\gamma \rightarrow \Pi^0_{t} \), we can obtain the total cross section at the \( e^+e^- \) linear collider by folding \( \hat{\sigma}_{\gamma\gamma \rightarrow \Pi^0_{t}}(\hat{s}) \) with the photon distribution function \( f_\gamma(x) \):

\[ \sigma_{\text{total}}(s) = \int_{\tau_{\text{min}}}^{\tau_{\text{max}}} d\tau \int_{x_{\text{max}}}^{x_{\text{max}}} dx f_\gamma(x) f_\gamma(\frac{\tau}{x}) \hat{\sigma}_{\gamma\gamma \rightarrow \Pi^0_{t}}(\hat{s}). \]  

Here, we denote \( \tau = \hat{s}/s \) with \( s \) being the \( e^+e^- \) c.m. energy squared. The photon distribution can be written as [18]:

\[ f_\gamma(x) = \frac{1}{D(\xi)} [1 - x + \frac{1}{1 - x} \frac{4x}{\xi(1 - x)} + \frac{4x^2}{\xi^2(1 - x)^2}], \]

and

\[ D(\xi) = (1 - \frac{4}{\xi} - \frac{8}{\xi^2}) \ln(1 + \xi) + \frac{1}{2} + \frac{8}{\xi} - \frac{1}{2(1 + \xi)^2}. \]

Where, \( \xi = \frac{4E_{\text{max}}}{m_e^2} \), \( E_e, \omega_0 \) are the incident electron energy and the laser-photon energy. \( x = \omega/E_e \) stands for the fraction of energy of the incident electron carried by the back-scattered photon. \( f_\gamma \) vanishes for \( x > x_{\text{max}} = \omega_{\text{max}}/E_e = \xi/(1 + \xi) \). In order to avoid the creation of \( e^+e^- \) pairs by the interaction of the incident and back-scattered photons, we
require $\omega_0 x_{\text{max}} \leq m_t^2/E_e$ which implies that $\xi \leq 2 + 2\sqrt{2} \approx 4.8$. Taking $\xi = 4.8$, we obtain $x_{\text{max}} \approx 0.83$, $D(\xi) \approx 1.8$. Therefore, we can obtain the up-limit and down-limit of integral as: $x_{\text{max}} = 0.83$, $x_{\text{min}} = \tau/x_{\text{max}}$, $\tau_{\text{max}} = x_{\text{max}}^2$, $\tau_{\text{min}} = M_{\Pi}^2/s$.

3 The numerical results and discussion

To obtain the numerical results, we take $m_t = 178$ GeV, $v_t = 60$ GeV. From the one-loop evaluation formula, we can obtain the value of $\alpha_e$ at energy scale of ILC. Therefore, there leave three free parameters: $\varepsilon$, $e^+e^-$ c.m. energy $\sqrt{s}$ and the mass of top-pion $M_{\Pi}$. To see the effect of $M_{\Pi}$, $\varepsilon$ and $\sqrt{s}$ on the cross section, we plot, in Fig.1-3, the cross section $\sigma_{\text{total}}$ as a function of $M_{\Pi}$ with $\varepsilon = 0.03, 0.06, 0.1$ and $\sqrt{s} = 500, 800, 1600$ GeV, respectively. We can see that the cross section is at the level of tens fb to a hundred fb. There exists a peak in the plot when $M_{\Pi}$ is near $2m_t$ which arises from the top quark triangle loop. On the other hand, it is shown that the increasing of $\sqrt{s}$ can depress the cross section.

The neutral top-pion should be detected via its decay modes. We know that the possible decay modes of $\Pi^0_t$ are $t\bar{t}$ (if $M_{\Pi} > 2m_t$), $t\bar{c}$, $b\bar{b}$, $\gamma\gamma$, $gg$, $\gamma Z$. The tree level decay modes $t\bar{t}$, $t\bar{c} b\bar{b}$ and the loop level decay mode $gg$ are the main decay modes of $\Pi^0_t$. The decaying branching ratios of the main decay modes are shown in Fig.4. We can see that $\Pi^0_t$ almost decays to $t\bar{t}$ when $M_{\Pi} > 2m_t$. The decay branching ratio of $\Pi^0_t \rightarrow t\bar{c}$ is the largest one when $t\bar{t}$ mode is forbidden. We focus on studying the potential to observe the $\Pi^0_t$ via these main decay modes. The event number of the signal via each mode can be obtained via the formula $N_X = L_{ee}\sigma_{\text{total}}Rb(\Pi^0_t \rightarrow X)$. The event number plot versus $M_{\Pi}$ is shown in Fig.5 with $\sqrt{s} = 800$ GeV, $\varepsilon = 0.06$ and the yearly luminosity $L_{ee} = 500fb^{-1}$. We can see that the event number of signal $t\bar{c}, b\bar{b}, gg$ is significantly depressed when $\Pi^0_t \rightarrow t\bar{t}$ is open. There are $10^3 - 10^4 t\bar{t}$ events can be produced for heavy $\Pi^0_t (M_{\Pi} > 2m_t)$. The total cross section of $\gamma\gamma \rightarrow t\bar{t}$ is at the level of $10^2 fb$ at TeV energy scale [19] in the SM. The number of $t\bar{t}$ event produced via heavy $\Pi^0_t$ decaying is comparable to that in the SM. Such $\Pi^0_t$ contribution to the $t\bar{t}$ production should be easily detected at the planned ILC. Therefore, the significant deviation of $t\bar{t}$ production from the SM prediction might provide the clue of $\Pi^0_t$ existence.
But it is difficult to identify $\Pi^0_t$ via $t\bar{t}$ mode. The reason is that the decay width of $\Pi^0_t \rightarrow t\bar{t}$ is very large and the peak of $t\bar{t}$ invariant distribution is too wide to be observed, we can not identify $\Pi^0_t$ via the $t\bar{t}$ invariant distribution. Therefore, we can conclude that $t\bar{t}$ is an ideal mode to search for the clue of TC2 model but it is not suitable to confirm the existence of heavy $\Pi^0_t$. Because topcolor is non-universal, there exists the flavor-changing coupling of the neutral top-pion to top and charm quarks. So, $\Pi^0_t$ can decay to $t\bar{c}$ at tree-level which is the typical feature of the TC2 model. Now, we discuss the potential to observe $\Pi^0_t$ via $t\bar{c}$ mode.

For the light $\Pi^0_t$, the branching ratio of $\Pi^0_t \rightarrow t\bar{c}$ is the largest one. Over $10^4$ $t\bar{c}$ events can be produced with the integral luminosity $500 fb^{-1}$. For heavy $\Pi^0_t$, the $t\bar{c}$ events significantly drop to $10^3$ with the opening of $t\bar{t}$ mode. In order to obtain a sound conclusion about the potential to observe the $\Pi^0_t$, we should consider the SM background and b-tagging and c-tagging efficiency. If we detect $\Pi^0_t$ via $t\bar{c}$, the only irreducible background arises from $\gamma\gamma \rightarrow t\bar{c}$ in the SM, the SM cross section of such process is much small (at the level of $10^{-8} fb^{[16]}$), therefore, the perfect identification of $t\bar{c}$ can make the background become very clean. To identify $t\bar{c}$, we should reconstruct top quark from its decay mode $W^+ b$. So, the b-tagging and c-tagging are need to identify $t\bar{c}$. We take b-tagging efficiency as 60% and c-tagging efficiency as 35%[20], there are also enough $t\bar{c}$ events which can be identified. For example, we typically take $t\bar{c}$ events as $10^4$, there are about $2.1 \times 10^3$ $t\bar{c}$ can be tagged, the corresponding statistical uncertainty at the 95% C.L. is 4.4%. So, with the clean background, the large number of $t\bar{c}$ events tagged and the observation of the peak of the $t\bar{c}$ invariant mass distribution, we can obtain the clear signal of neutral top-pion with high precision. As it is shown in Fig.5, a lot of $gg$ events can also be produced via $\Pi^0_t \rightarrow gg$ which leads to 2-jets. It is difficult to identify $\Pi^0_t$ via such 2-jets. So, $gg$ mode is not suitable to observe $\Pi^0_t$. For the $b\bar{b}$ mode, there are over $10^2 b\bar{b}$ events can be produced via light $\Pi^0_t$ decaying. The dominant background to $\gamma\gamma \rightarrow \Pi^0_t \rightarrow b\bar{b}$ comes from the SM process $\gamma\gamma \rightarrow b\bar{b}, c\bar{c}$. Various techniques can be used to suppress these background. The most effective technique appears to be to polarize the initial-state photons: $\Pi^0_t$ are only produced from an initial state with spin $J_z = 0$, whereas, the leading-order $b\bar{b}(c\bar{c})$ background are dominantly produced from the $J_z = \pm 2$ initial state. More precisely, production from the $J_z = 0$ state is suppressed by $m_q^2/s^{[21]}$. Hence, in the
region of the $\Pi_1^0$ resonance, the $b\bar{b}, c\bar{c}$ background are heavily suppressed if we choose initial state with $J_z = 0$. On the other hand, the processes $\gamma \gamma \rightarrow b\bar{b}g(c\bar{c}g)$ can mimic a two-jet event. So, such processes in the SM are also the important background. It is shown that the $m_q^2/s$ suppressions do not necessarily apply to the QCD corrected processes $\gamma \gamma \rightarrow b\bar{b}g(c\bar{c}g)$ and the largest background is from the $\gamma \gamma \rightarrow c\bar{c}g$. The existence of $\gamma \gamma \rightarrow b\bar{b}g(c\bar{c}g)$ background make the potential to observe $\Pi_1^0$ via $b\bar{b}$ mode become unclear, and a detail study is needed which is beyond this paper. Furthermore, in the SM or the MSSM, there exists the similar process $\gamma \gamma \rightarrow H \rightarrow b\bar{b}$. Therefore, to identify the $\Pi_1^0$ via $b\bar{b}$ mode, we should also find the different feature between $\gamma \gamma \rightarrow \Pi_1^0 \rightarrow b\bar{b}$ and $\gamma \gamma \rightarrow H \rightarrow b\bar{b}$. From above discussion, we can conclude that $\gamma \gamma \rightarrow \Pi_1^0 \rightarrow t\bar{c}$ is an ideal mode to detect neutral top-pion which can provide enough typical $\Pi_1^0$ signals.

In summary, the running of ILC with high energy and luminosity will provide a good chance to test the new physics models. With the realization of photon-photon collision, the neutral top-pion can be produced as s-channel resonances in the process $\gamma \gamma \rightarrow \Pi_1^0$ which is the most important production mechanism of neutral top-pion. In this paper, we calculate the cross section of the process $\gamma \gamma \rightarrow \Pi_1^0$ and discuss the potential to observe the neutral top-pion via its various decay modes. The result shows that the cross section is at the level of tens fb to a hundred fb. The possible decay modes of $\Pi_1^0$ are $tt$(if $M_{\Pi} > 2m_t$), $t\bar{c}$, $b\bar{b}$, $\gamma\gamma$, $gg$, $\gamma Z$. For the mode $\Pi_1^0 \rightarrow t\bar{c}$, there are $10^3 - 10^4$ $t\bar{c}$ events can be produced with the integral luminosity $L_{ee} = 500 fb^{-1}$ and the SM background is very clean. So, $\Pi_1^0 \rightarrow t\bar{c}$ is the most ideal mode to search for neutral top-pion.

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Figure 1: The cross section of $\gamma\gamma \rightarrow \Pi^0$ as functions of the top-pion mass $M_{\Pi}$ with $\sqrt{s} = 500$ GeV and $\varepsilon = 0.03, 0.06, 0.1$. 

Figure 2: The cross section of $\gamma\gamma \to \Pi^0$ as functions of the top-pion mass $M_{\Pi}$ with $\sqrt{s} = 800$ GeV and $\epsilon=0.03$, 0.06, 0.1.
Figure 3: The cross section of $\gamma\gamma \rightarrow \Pi^0_t$ as functions of the top-pion mass $M_{\Pi}$ with $\sqrt{s} = 1600$ GeV and $\varepsilon=0.03, 0.06, 0.1$. 
Figure 4: The decaying branching ratios of $\Pi^0_t \to t\bar{t}, t\bar{c}, b\bar{b}, gg$ as functions of the top-pion mass $M_{\Pi}$ with $\varepsilon = 0.06$. 
Figure 5: The production event numbers of signal via the decay modes: $\bar{t}t, \bar{t}c, \bar{b}b, gg$ as functions of $M_\Pi$ with $\sqrt{s} = 800$ GeV and $\varepsilon=0.06$. 