Study of Saros cycle and non-partial solar eclipse with Newton mechanics approach

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Abstract. We have derived Saros cycle period by Newton mechanics approach using a method that had been used by Chalub. We got the result approximately 6597,1 day; with the error compared to Saros cycle (6585,3 day) of approximately 0,1792%. For non-partial solar eclipse case, we too have derived the equation to relate between this sun eclipse and geometrical parameter of Earth, Moon, and Sun mathematically. This equation indicates that for total sun eclipse, the umbra's radius \( P \) on earth surface must have positive sign, whereas negative sign for annular sun eclipse. To test this equation, we compare it to a recorded (past or predicted) non-partial solar eclipse. The result indicates appropriateness for total and annular sun eclipse, but it does not appropriate for hybrid sun eclipse as we anticipated that is the magnitude of umbra's radius \( P \) for this eclipse must near zero.

1. Introduction

Scientists have analyzed the Earth's, Moon, and Sun motion system. Some results of the analysis provide astronomical data of Earth, Moon, and Sun which are referenced to explain the process of eclipse. However, the prediction of eclipses is still oriented to the period of Saros cycle which does not require such data. The Saros cycle is a periodicity of one exact type of eclipse within a certain time interval. The term "type" above refers to the occurrence of an eclipse and the next eclipse having the same geometry and characteristics. The Saros cycle periodicity has been calculated since the time of ancient Babylonia, which is about 6585.3 days

Chalub¹ has analyzed the calculation of Saros cycle periods based on Newton's mechanical theory, but his final results was indirectly intended to obtain the equation for Saros cycle period, but the period of the Moon that can be attributed to the Saros cycle. Based on that, in this paper, Saros cycle is calculated directly based on Newton's mechanical approach which refers to the Chalub’s method.

In addition to the Saros cycle calculations above, we also observed phenomena of non-partial solar eclipses on the Earth's surface in three circumstances; namely total, annular, and hybrid solar eclipses. These three possible circumstances indicate the dependence of eclipse events on the geometric state which includes Moon’s and Sun’s radius, and distance between Earth, Moon, and the Sun. If such geometric values and its relationship mathematically are known, then comparisons of the calculations with the occurrence of non-partial solar eclipses can be done to test the relation's validity.

2. The Actual Saros Cycle Formulation
The method referred to the calculation of Saros cycle periods is the averaging method in non-linear dynamical motion of system. The method is used by Chalub\cite{1} to calculate the average effect of the Sun's gravitational perturbation on the Moon's trajectory in one year. The result is obtained by the formulation of the angular velocity of Moon’s node (ascending or descending node), so it is subsequently used to obtain the formula of the period of the draconic month. The formulation can be used to calculate the period of the Saros cycle because it’s magnitude is equal to the time interval of the node evolves in one full rotation\cite{2}.

The angular velocity formula which mentioned above has been derived by Chalub\cite{1}:

$$\omega_s = 3GT_0m_S\left(1 - e_M^2\right)^3 \cos \beta \left(8\pi R_{ES}^3\right)^{-1} \quad (1)$$

where $\omega_s$ is the angular velocity of the node’s revolution, $G$ is the universal gravitational constant, $T_0$ is the period of sidereal month, $m_S$ is the mass of the Sun, $e_M$ is the eccentricity of the Moon's revolution path, $\beta$ is the angle of the orbit between the Earth and the Moon, and $R_{ES}$ is the average distance between the Earth and the Sun. Chalub\cite{1} derived the Equation (1) by assuming that $e_M$ is equal to zero. However, in this study, the value is still taken into account. The node evolves in the time interval of one full rotation or in the period interval of the Saros cycle $T_s$, \[ T_s = 2\pi \omega_s^{-1} = 16\pi^2 R_{ES}^3 \left(3GT_0m_S\left(1 - e_M^2\right)^3 \cos \beta \right)^{-1} \quad (2) \]

Based on Table 1, the calculation using Equation (2) results in 6597.3 days/cycle (6597.3 days for one cycle of eclipses) (error $\cong 0.1792\%$).

### Table 1. Several values of constants used in this study

| Symbol | Value |
|--------|-------|
| Universal gravitational constant | $G$ | $6.674 \times 10^{-11} \text{Nm}^2\text{kg}^{-2}\text{[1]}$ |
| Period of sidereal month | $T_0$ | $2.3606 \times 10^6\text{[1]}$ |
| Mass of the Sun | $m_S$ | $1.9884 \times 10^{30} \text{kg}\text{[1]}$ |
| Eccentricity of the Moon's revolution path | $e_M$ | $0.0549\text{[1]}$ |
| Eccentricity of the Earth's revolution path | $e_E$ | $0.0167086\text{[3]}$ |
| Angle of the orbit between the Earth and the Moon | $\beta$ | $5.145^\circ\text{[1]}$ |
| Average distance between the Earth and the Sun | $R_{ES}$ | $149,597,870,700 \text{km}\text{[4]}$ |
| Average distance between the Earth and the Moon | $R_{EM}$ | $384,402 \text{km}\text{[5]}$ |
| Moon’s radius | $R_M$ | $1,738.1 \text{km}\text{[6]}$ |
| Earth’s radius | $R_E$ | $6,378.137 \text{km}\text{[6]}$ |
| Sun’s radius | $R_S$ | $695,700 \text{km}\text{[7]}$ |

### 3. Relation Between Non-partial Solar Eclipse and Geometrical Value Of Earth, Moon, and Sun

Relation between non-partial solar eclipse and geometrical value of earth, moon and sun can be obtained by analyzing the radius of the umbra on the surface of the Earth. Based on Figure 1, the following mathematical formulas are obtained:

$$\tan \theta = \left(R_S - R_M\right) r_M^{-1} = R_S X^{-1} \quad (3)$$

$$X = R_S r_M \left(R_S - R_M\right)^{-1} \quad (4)$$
Figure 1. Illustration of position between the Earth, the Moon, and the Sun during eclipse. In this picture, $r_{MS}$ is the distance between the Moon and the Sun, $r_{EM}$ is the distance between the Earth and the Moon, $S$ is the distance between the point of intersection of the umbra and the surface of the Earth facing to the Moon, $X$ is the distance between the point of intersection of the umbra and the center of the Sun, $P$ is the umbra’s radius on the surface of the Earth (which in this case is positive upward).

\[
S = X - r_{MS} - r_{EM}
\]  

And

\[
P = S \tan \theta
\]  

The physical interpretation of Equation (3) to (6) can be seen in the Figure 1. Setting the Equation (3) to (6) in such way as to eventually obtain the following formula

\[
P = R_M - r_{EM} (R_S - R_M)(r_{ES} - r_{EM})^{-1}
\]  

where $r_{ES}$ is the distance between the center of the Earth and the Sun. The calculation results of the Equation (7) may contain positive or negative values. If the sign is positive, it means that total eclipse currently occurs; whereas according to Figure 1, the Earth's surface is subjected to the umbra. But if the sign is negative, it means an annular eclipse currently occurs; because the $P$'s lead downward so that the shadow intersection will not pass through the Earth's surface (in other words, Earth is a subject to the antumbra). For the case of a hybrid solar eclipse, the magnitude $P$ must be close to zero because it is the closeness between the total and annular solar eclipse phase.

Validity of Equation (7) can be tested by comparing it to the recorded non-partial solar eclipse previously. To do that, geometrical values are required when the eclipse that being referenced occurs. The value of $R_S$ and $R_M$ is fixed and can be referenced through the data that validity have been accepted, while $r_{EM}$ and $r_{ES}$ are variables (according to Kepler's law) which can be determined by finding their relation to the time of the eclipse that being referenced occurs. The relation is represented by eccentric anomaly $E$ as a function of time, which is derived by knowing the solution of Kepler's equation \[8\]

\[
E(t_g, T_{gs}) = 2 \pi t_g T_{gs}^{-1} + \sum_{n=1}^{\infty} 2 n^{-1} J_n(ne) \sin(n 2 \pi t_g T_{gs}^{-1})
\]  

where $t_g$ is the time of the solar eclipse which referred from the last perihelion or perigee, $T_{gs}$ is the sidereal period just when the solar eclipse occurs, and $J_n(ne)$ is Bessel function of the first kind.

Values of $t_g$ and $T_{gs}$ in this study were referenced from Espenak and Meeus’ calculations\[9\][10]. Relation between Equation (8) and distance $r$ (based on ellipse theory) is

\[
r(t_g, T_{gs}) = 1 - e \cos[E(t_g, T_{gs})]
\]
with \( e \) is the eccentricity of the orbit of the Moon or Earth depending on the distance observed, which values can be found in the table 1 sequentially denoted by \( e_M \) and \( e_E \). Equation (9) is used to determine the distance when the observed eclipse occurs, while \( r_{EM} \) and \( r_{ES} \) are determined by subtracting the result of the Equation (9) with the \( R_E \) Earth’s radius (since \( r_{EM} \) and \( r_{ES} \) are viewed from the surface of the Earth). The result of calculation \( P \) for some non-partial solar eclipse occurrence can be seen in Table 3.

### Table 2. Data on the value of \( P \) for each non-partial solar eclipse that referenced

| Date           | Eclipse Type | \( P \) (km) |
|----------------|--------------|--------------|
| 21 June 2001   | Total        | 94,685       |
| 14 December 2001 | Annular     | −77,013      |
| 10 June 2002   | Annular      | −41,979      |
| 4 December 2002 | Total        | 44,159       |
| 31 May 2003    | Annular      | −83,680      |
| 23 November 2003 | Total       | 57,261       |
| 8 April 2005   | Hybrid       | 34,781       |
| 3 October 2005 | Annular      | −96,294      |
| 29 March 2006  | Total        | 72,542       |
| 22 December 2006 | Annular     | −37,257      |
| 7 February 2008 | Annular     | −56,434      |
| 1 August 2008  | Total        | 81,545       |
| 26 January 2009 | Annular     | −137,868     |
| 22 July 2009   | Total        | 102,298      |
| 15 January 2010 | Annular     | −148,789     |
| 11 July 2010   | Total        | 95,962       |
| 20 May 2012    | Annular      | −96,735      |
| 13 November 2012 | Total       | 57,657       |
| 10 May 2013    | Annular      | −95,409      |
| 3 November 2013 | Hybrid      | 41,500       |
| 29 April 2014  | Annular      | −2,245       |
| 20 April 2015  | Total        | 64,224       |
| 9 March 2016   | Total        | 63,903       |
| 1 September 2016 | Annular    | −71,353      |
| 26 February 2017 | Annular    | −7,441       |
| 21 August 2017 | Total        | 72,837       |

### 4. Conclusion

The period of Saros cycle which is calculated through Newton's mechanical approach is 6596.8 days; compared to the experimental Saros cycle (6585.3 days) that produced an error of 0.1792%.

The results of comparison between non-partial solar eclipses and geometric values of the Earth, Moon, and Sun through the calculations of the \( P \) umbra’s radius shows suitability, except for hybrid solar eclipse who does not appropriate as we anticipated, that is the magnitude of umbra’s radius \( P \) for must near zero.
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