Thawing quintessence from the inflationary epoch to today

Gaveshma Gupta,1 Raghavan Rangarajan,1‡ and Anjan A. Sen2†

1Physical Research Laboratory, Navrangpura, Ahmedabad, 380009, India
2Centre for Theoretical Physics, Jamia Millia Islamia, New Delhi 110025, India

(Dated: December 23, 2014)

By considering observational constraints from the recent Union2.1 Supernova type Ia data, the baryon acoustic oscillations data, the cosmic microwave background shift parameter measurement by Planck and the observational Hubble parameter $H(z)$ data we obtain a lower bound on the initial value of the quintessence field in thawing quintessence models of dark energy. For potentials of the form $V(\phi) \sim \phi^{2N}$ we find that the initial value $\phi_i > 7 \times 10^{18}$ GeV. We then relate $\phi_i$ to the duration of inflation by assuming that the initial value of the quintessence field is determined by quantum fluctuations of the quintessence field during inflation. From the lower bound on $\phi_i$ we obtain a lower bound on the number of e-foldings of inflation, namely, $N > 2 \times 10^{11}$.

PACS numbers:

I. INTRODUCTION

Cosmological observations1 2 3 of the past one and a half decades indicate that the Universe is undergoing accelerated expansion. Although a non-zero cosmological constant can explain the current acceleration of the Universe, one still has to explain why it is so small and why only at recent times it has started to dominate the energy density of the Universe4. These issues have motivated the exploration of alternative theories to explain the late time acceleration as due to a source of energy referred to as dark energy5 6. A quintessence model is one amongst such theories where the dark energy arises from a scalar field $\phi$ rolling slowly down a potential.

The equation of state parameter $w$ can be defined as the ratio of the pressure to the energy density

$$w = p/\rho.$$  (1)

A cosmological constant is equivalent to $w = -1$ whereas a quintessence field generates a time dependent equation of state $w(t) > -1$. Caldwell and Linder7 showed that the quintessence models in which the scalar field rolls down its potential towards a minimum can be classified into two categories, namely freezing and thawing models, with quite different behavior. In thawing models at early times the field gets locked at a value away from the minimum of the potential due to large Hubble damping. At late times when Hubble damping diminishes, the field starts to roll down towards the minimum. These models have a value of $w$ which begins near $-1$ and gradually increases with time. In freezing models the field rolls towards its potential minimum initially and slows down at late times as it comes to dominate the Universe. These models have a value of $w$ which decreases with time. In both cases $w \approx -1$ around the present epoch. Thawing models with a nearly flat potential provide a natural way to produce a value of $w$ that stays close to, but not exactly equal to $-1$. The field begins with $w \approx -1$ at high redshifts, and $w$ increases only slightly by low redshifts. These models depend on initial field values (in contrast with freezing models of quintessence).

In the present work we evaluate the cosmological consequences of the evolving quintessence field by considering various observational datasets and obtain plausible initial values of the scalar field $\phi_i$ in the context of thawing models. The bound on the the allowed values of $\phi_i$ has been previously obtained in Ref. 8. Our current numerical analysis provides stronger constraints on $\phi_i$. We then relate the initial value to quantum fluctuations of $\phi$ during inflation and thereby to the duration of inflation. Our work in this article is organised as follows. In section II we describe the thawing quintessence model in the standard minimal framework. In section III we provide a detailed description of the datasets used to obtain the observational constraints on the parameters of the model. In section IV we use the results obtained in our investigation to obtain a lower bound on the initial value of $\phi$. We then discuss the generation of the initial value by quantum fluctuations during inflation and use the lower bound on $\phi_i$ to obtain a lower bound on the number of e-foldings of inflation, $N$. We end with our conclusions in section V.

II. THE THAWING QUINTESSENCE SCENARIO

We will assume that the dark energy is provided by a minimally coupled scalar field $\phi$ with the equation of motion for the homogeneous component given by

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0,$$  (2)

where the Hubble parameter $H$ is given by

$$H = \left( \frac{\dot{a}}{a} \right) = \sqrt{\rho/3M_p^2}.$$  (3)
The pressure and density of the scalar field are given by
\[ p = \frac{\dot{\phi}^2}{2} - V(\phi) \]
and
\[ \rho = \frac{\dot{\phi}^2}{2} + V(\phi) \]
respectively, and the equation of state parameter \( w \) is given by Eq. (1). At late times, the Universe is dominated by dark energy due to \( \phi \), and non-relativistic matter; we can neglect the radiation component.\(^1\) We assume a flat Universe so that \( \Omega_\phi + \Omega_m = 1 \). Then Eqs. (2) and (3), in a Universe containing only matter and the potential energy, so that \( \Omega_\phi = x^2 + y^2 \), while the equation of state is rewritten as
\[ \gamma = 1 + w = \frac{2x^2}{x^2 + y^2}. \]
It is convenient to work in terms of \( \gamma \), since we are interested in models for which \( w \) is near \(-1\), so \( \gamma \) is near zero. Eqs. (2) and (3), in a Universe containing only matter and a scalar field, become\(^2\)
\[ x' = -3x + \lambda \sqrt{\frac{3}{2}} y^2 + \frac{3}{2} x [1 + x^2 - y^2], \]
\[ y' = -\lambda \sqrt{\frac{3}{2}} xy + \frac{3}{2} y [1 + x^2 - y^2], \]
\[ \lambda' = -\sqrt{6} \lambda^2 (\Gamma - 1)x, \]
where
\[ \Gamma \equiv V \frac{d^2 V}{d\phi^2} / \left( \frac{dV}{d\phi} \right)^2. \]
We numerically solve the system of Eqs. (11) - (13) for various power law potentials from an initial time at decoupling \( t_i \) till \( t_0 \) today. We choose \( \gamma_i \sim 0 \), and for different values of \( \lambda_i \) and \( \Omega_{m0} = 1 - \Omega_\phi \) (using the flatness condition) we obtain \( x \), \( y \) and \( \lambda \) as functions of \( a \). The information about the potential is encoded in the parameter \( \Gamma \) given above. We use our solutions for \( x \), \( y \) and \( \lambda \) and the relation \( a^{-1} = 1 + z \) (with \( \delta_0 = 1 \)) to express the normalized Hubble parameter as
\[ H^2(z, \lambda_i, \Omega_{m0}) = \frac{H^2_0(z, \lambda_i, \Omega_{m0})}{H^2_0 \left( 1 - \Omega_\phi (z, \lambda_i, \Omega_{m0}) \right)} \]
We then utilise this expression of the Hubble parameter to calculate the luminosity distance and angular diameter distance and relate them and \( H \) to the observations of type Ia supernovae (SN) data\(^10\), the baryonic acoustic oscillations (BAO) data\(^11\), the cosmic microwave background (CMB) shift parameter\(^12\) and the observational Hubble parameter (HUB) data\(^13\) and constrain the parameter space for each model defined by the values of \( \lambda_i \) and \( \Omega_{m0} \). The allowed values of \( \lambda_i \) from our numerical analysis give constraints on \( \phi_i \), the value of \( \phi \) at decoupling. Based on our arguments below, \( \phi_i \sim \phi_f \), where \( \phi_f \) is the value of \( \phi \) at the end of inflation. We then study the conditions on inflation to obtain \( \phi_f \).

III. OBSERVATIONAL DATASETS

We use the \( \chi^2 \) analysis to constrain the parameters of our assumed parameterization. We will use the maximum likelihood method and obtain the total likelihood function for the parameters \( \lambda_i \) and \( \Omega_{m0} \) in a model as the product of independent likelihood functions for each of the datasets being used. The total likelihood function is defined as
\[ L_{\text{tot}}(\lambda_i, \Omega_{m0}) = e^{-\frac{\chi^2_{\text{tot}}(\lambda_i, \Omega_{m0})}{2}}, \]
where
\[ \chi^2_{\text{tot}} = \chi^2_{\text{SN}} + \chi^2_{\text{BAO}} + \chi^2_{\text{CMB}} + \chi^2_{\text{HUB}} \]
is associated with the four datasets mentioned above. The best fit value of parameters is obtained by minimising \( \chi^2 \) with respect to \( \lambda_i \) and \( \Omega_{m0} \). In a two dimensional

\(^1\) In our numerical analysis we will go back in time till decoupling. At that epoch the energy density in radiation will be more than that in \( \phi \). The radiation component \( \Omega_r \) is included in our numerical analysis.

\(^2\) Our code solves Eqs. (11) - (13) for a given \( \gamma_0 \), \( \lambda_i \) and \( \Omega_{m0} \). We choose a small value of \( \gamma_i \sim 10^{-3} \) and do not vary it for our analysis. To get the physical solution, for a certain \( \lambda_i \), we vary \( \Omega_{m0} \) till we get a solution that satisfies \( \Omega_{m0} = 1 - \Omega_\phi \) for a given \( \Omega_{m0} \). With this \( \Omega_{m0} \), we re-solve the differential equations to get \( x \), \( y \) and \( \lambda \) as functions of \( a \). This process is repeated for different values of \( \lambda_i \) and \( \Omega_{m0} \). We also set \( \Omega_{m0} = 9.22 \times 10^{-5} \) from Planck\(^13\).
parametric space, the likelihood contours in 1σ and 2σ confidence region are given by $\Delta \chi^2 = \chi^2 - \chi^2_{\text{min}} = 2.3$ and 6.17 respectively.

A. Type Ia Supernovae

Type Ia supernovae are very bright and can be observed at redshifts up to $z \sim 1.4$. They have nearly the same luminosity which is redshift independent and well calibrated by the light curves. Hence they are very good standard candles.

The distance modulus of each supernova is defined as

$$
\mu_{th}(z) = 5 \log_{10} D_L^{th}(z) + \mu_0 ,
$$

where the theoretical Hubble free luminosity distance $D_L^{th}$ in a flat universe for a model is given by

$$
D_L^{th}(z, \lambda_i, \Omega_m) = H_0 d_L^{th} = (1 + z) \int_0^z \frac{dz'}{H(z', \lambda_i, \Omega_m)} .
$$

Above, $d_L^{th}$ is the luminosity distance, $H$ is obtained from Eq. (15), and $\mu_0$ is the zero point offset.

We construct the $\chi^2$ for the supernova analysis after marginalising over the nuisance parameter $\mu_0$ as

$$
\chi^2_{SN}(\lambda_i, \Omega_m) = A(\lambda_i, \Omega_m) - \frac{B^2(\lambda_i, \Omega_m)}{C} ,
$$

where

$$
A(\lambda_i, \Omega_m) = \sum_i \left( \frac{\mu_{obs}(z_i) - \mu_{th}(z_i, \lambda_i, \Omega_m)}{\sigma_{\mu_{obs}}(z_i)} \right)^2
$$

$$
B(\lambda_i, \Omega_m) = \sum_i \left( \frac{\mu_{obs}(z_i) - \mu_{th}(z_i, \lambda_i, \Omega_m)}{\sigma_{\mu_{obs}}^2(z_i)} \right)
$$

$$
C = \sum_i \frac{1}{\sigma_{\mu_{obs}}^2(z_i)} .
$$

(B should not be confused with the coefficient of the $V \sim \phi^{-2}$ potential.) $\mu_{obs}(z_i)$ is the observed distance modulus at a redshift $z_i$ and $\sigma_{\mu_{obs}}(z_i)$ is the error in the measurement of $\mu_{obs}(z_i)$. The latest Union2.1 compilation [10] of supernovae Type Ia data consists of the measurement of distance moduli $\mu_{obs}(z_i)$ at 580 redshifts $z_i$ over the range $0.015 \leq z \leq 1.414$ with corresponding $\sigma_{\mu_{obs}}(z_i)$. In our analysis we include the $\chi^2_{SN}$ given by Eq. (20) in Eq. (17).

B. Baryon Acoustic Oscillations

Baryon acoustic oscillations (BAO) refers to oscillations at sub-horizon length scales in the photon-baryon fluid before decoupling due to collapsing baryonic matter and counteracting radiation pressure. The acoustic oscillations freeze at decoupling, and imprint their signatures on both the CMB (the acoustic peaks in the CMB angular power spectrum) and the matter distribution (the baryon acoustic oscillations in the galaxy power spectrum). To obtain the constraints on our models from the BAO we use the comoving angular diameter distance

$$
d_A^{th}(z, \lambda_i, \Omega_m) = \frac{1}{H_0} \int_0^{z} \frac{dz'}{H(z', \lambda_i, \Omega_m)} .
$$

The redshift at decoupling $z_*$ is obtained from the fitting formula in Ref. [15] as 1090.29. The dilation scale $D_V$ is given by

$$
D_V^{th}(z_{BAO}, \lambda, \Omega_m) = H_0 \left( \frac{z_{BAO}}{H(z_{BAO}, \lambda_i, \Omega_m)} \right)^{1/3}
$$

$$
\times \left( \int_0^{z_{BAO}} \frac{dz}{H(z, \lambda_i, \Omega_m)} \right)^{2/3},
$$

where $H(z, \lambda_i, \Omega_m)$ is obtained from Eq. (15). We construct the $\chi^2$ for the BAO analysis as

$$
\chi^2_{BAO}(\lambda_i, \Omega_m) = \sum_{ij} \left( x_i - d_i \right) (C_{ij})^{-1} \left( x_j - d_j \right) ,
$$

where $x_i = d_A^{th}(z_{BAO}, \lambda_i, \Omega_m)$ are the values predicted by a model, $d_i$ is the mean value of $d_A^{th}(z_{BAO}, \lambda_i, \Omega_m)$ from observations given in Table 1 of Ref. [11], and $(C_{ij})^{-1}$ is the inverse covariance matrix in Eq. (3.24) of Ref. [11]. We include $\chi^2_{BAO}$ in Eq. (17).

C. Cosmic Microwave Background

The locations of peaks and troughs of the acoustic oscillations in the CMB angular spectra are sensitive to the distance to the decoupling epoch. To obtain the constraints from CMB we utilise the shift parameter $R$ from Planck [12]. The CMB shift parameter $R$ is given by

$$
R^th(z_*, \lambda_i, \Omega_m) \equiv \sqrt{\Omega_m H_0^2 d_A^{th}(z_*, \lambda_i, \Omega_m)}
$$

where $d_A^{th}(z, \lambda_i, \Omega_m)$ is given by Eq. (22).

The shift parameter $R(z_*)$ from Planck observations is $1.7499 \pm 0.0088$ [12]. Thus we obtain $\chi^2_{CMB}$ as

$$
\chi^2_{CMB}(\lambda_i, \Omega_m) = \left( \frac{1.7499 - R^th(z_*, \lambda_i, \Omega_m)}{0.0088} \right)^2 ,
$$

(25)

[3] In calculating $z_*$ we have used the mean values of $\Omega_{b0}h^2$ and $\Omega_{c0}h^2$ from Planck+WP [13].
D. The observational Hubble parameter $H(z)$

The parameter $H(z)$ describes the expansion history of the Universe and plays a central role in connecting dark energy theories and observations. An independent approach, regarding the measurement of the expansion rate is provided by ‘cosmic clocks’. The best cosmic clocks are galaxies. The observational Hubble parameter data can be obtained based on differential ages of galaxies.

Recently, Farooq et al. have compiled a set of 28 datapoints for $H(z)$ data, listed in Table 1 of Ref. [13]. We use the measurements from Ref. [13] of the Hubble parameter $H_{obs}(z_i)$ at redshifts $z_i$, with corresponding one standard deviation uncertainties $\sigma_i$, and the current value of the Hubble parameter $H_0 = 67.3 \pm 1.2$ km s$^{-1}$ Mpc$^{-1}$ from Planck [12]. Then for $H = H/H_0$

$$\chi^2_{HUB}(\lambda_i, \Omega_{m0}) = \sum_i \left( \frac{H_{obs}(z_i) - H_{th}(z_i, \lambda_i, \Omega_{m0})}{\sigma_{H_{obs}}(z_i)} \right)^2$$

with $H_{th}(z, \lambda, \Omega_{m0})$ obtained from Eq. [15], and $\sigma_{H_{obs}}$ obtained from $\sigma_i$ and the uncertainty in $H_0$.

IV. RESULTS AND DISCUSSION

In Fig. 1 we present the likelihood contours for models in the $\lambda_i$ and $\Omega_{m0}$ phase space for fixed $\gamma_i$ separately for each dataset discussed in the previous section (for the quadratic potential). In Figs. 2 and 3 we present the likelihood contours after combining all the datasets for each potential. We find that for $V \sim \phi^2$ the values of $|\lambda_i|$ greater than 0.67 are excluded at $2\sigma$ confidence level whereas for $V \sim \phi^{-2}$ the values of $|\lambda_i|$ greater than 0.72 are excluded at $2\sigma$ confidence level. The best fit value for $\Omega_{m0}$ in both cases turns out to be 0.30 which is well within the bounds on $\Omega_{m0}$ by Planck [12]. The best fit value for $\lambda_i$ is 0.0008 and 0.0009 respectively.

From Eq. [8],

$$\lambda_i = \mp \frac{2M_P}{\phi_i}$$

where $\mp$ refers to $V = \frac{1}{2}m^2\phi^2, A\phi^{-2}$. For $V = \frac{1}{2}m^2\phi^2$, $|\lambda_i| < 0.67$ implies we need $|\phi_i| > 7.2 \times 10^{18}$ GeV. For $V = A\phi^{-2}$, $|\lambda_i| < 0.72$ implies we need $|\phi_i| > 6.7 \times 10^{18}$ GeV. Hereafter we take $\phi_i$ positive and

$$\phi_i > 7 \times 10^{18} \text{ GeV}$$

for both potentials. 4

---

4 One may obtain a similar bound, for the quadratic potential, by equating the energy density in $\phi$ today, $\rho_{\phi0} \approx (1/2)m^2\phi^2$, with $\Omega_{\phi0} H_0^2/(8\pi G)$, where one has assumed that the light quintessence field has not evolved much since decoupling. Then for $\Omega_{\phi0} = 0.7$ [12] and $m \lesssim H_0$ one gets $\phi_i \gtrsim 2.04 \times 10^{18}$ GeV, similar to the bound in Eq. [28]. However, a priori one would not necessarily expect similar results.

We note that the likelihood contours for both the potentials look very similar. This indicates that the datasets that we compare with are not able to distinguish between the different potentials under consideration. We have confirmed that the plots of $x$, $y$, $\lambda_i$, and $\Omega_{\phi}$ as functions of $a$, for $\Omega_{m0} = 0.3$ show variations between the potentials, but the key parameter that is relevant for
comparison of the models with the different datasets is $H(z)$. In Fig. 4 we plot $H$ against $z$ for both potentials for different values of $\lambda_i$ with $\Omega_{m0} = 0.3$. The curves for the two potentials for the same $\lambda_i$ are degenerate. While the curves in $H$ do not appear to be different for different $\lambda_i$ for the same potential the data does indicate a relative difference in the likelihoods for different points in the $\lambda_i - \Omega_{m0}$ plane. This is because the SN and CMB datasets disfavor large value of $\lambda_i$, as seen in Fig. 4. The SN data has a large number of datapoints while the CMB shift parameter is very precisely measured and hence these datasets are more sensitive to the variations in $\lambda_i$.

We now consider plausible values of $\phi_i$ that one may obtain in an inflationary universe by considering a condensate of $\phi$ being generated due to quantum fluctuations during inflation. A light scalar field of mass $m$ ($m \ll H_I$, where $H_I$ is the Hubble parameter during inflation) will undergo quantum fluctuations during inflation. The fluctuations are given by Eq. (7) of Ref. [17] (we presume $H_I$ does not vary during inflation)

$$\langle \delta \phi^2 \rangle = \int_{a_i H_I}^{a H_I} \frac{d^3 k}{2 \pi^3} |\delta \phi_k|^2 = \frac{3 H_I^4}{8 \pi^2 m^2} \left[ 1 - \exp \left( -\frac{2 m^2 N}{3 H_I^2} \right) \right]$$

(29)

FIG. 3: The 1σ and 2σ confidence regions in the $\lambda_i - \Omega_{m0}$ plane for $V = A\phi^{-2}$, or $\Gamma = 1.5$, constrained by the SN+BAO+CMB+HUB data. The two vertical lines represent the Planck bounds on $\Omega_{m0}$. The thick dot represents the best fit values $\lambda_i = 0.0009, \Omega_{m0} = 0.3003$.

FIG. 4: The Hubble parameter as a function of redshift $z$. The solid and dashed lines correspond to $\Gamma = 0.5$ ($V = m^2 \phi^2/2$) and $\Gamma = 1.5$ ($V = A\phi^{-2}$) respectively for $\lambda_i = 0.2, 0.5$ and 1, and $\Omega_{m0} = 0.3$. The inset shows the Hubble parameter for $\lambda_i = 1$ for both the potentials up to a redshift 0.6.

FIG. 5: The evolution of field $\phi$ as a function of scale factor $a$ from $a = 0.001$ at decoupling to $a = 1$ today. The solid lines represent $\Gamma = 0.5$ ($V = m^2 \phi^2/2$) whereas the dashed lines represent $\Gamma = 1.5$ ($V = A\phi^{-2}$) for $\lambda_i = 0.2, 0.5$ and 1. We have taken $\Omega_{m0} = 0.3$ here.
We consider the case where \( m^2N \ll H_I^2 \). Then

\[
\langle \delta \phi^2 \rangle = \left( \frac{H_I}{2\pi} \right)^2 N. \tag{30}
\]

Ignoring an initial value of \( \phi \) at the beginning of inflation, and any potential driven classical evolution, the value of \( \phi \) at the end of inflation is

\[
\phi_I = \sqrt{\langle \delta \phi^2 \rangle} = \frac{H_I}{2\pi} \sqrt{N} \tag{31}
\]

The above discussion is in the context of a quadratic potential, \( V = \frac{1}{2}m^2\phi^2 \). For \( V = A\phi^{-2} \) we presume that the potential is very flat during inflation and so Eq. \( \ref{30} \) again describes the evolution of \( \phi \).

After inflation \( \phi \) will evolve in two ways.

- \( \phi \) will evolve classically in its potential \( V(\phi) \). Our numerical analysis in Fig. 5 shows that the field does not evolve much between decoupling and the present epoch. We expect its evolution is slower at earlier times. Therefore we may ignore the evolution of \( \phi \) from the end of inflation till decoupling. (For a quadratic potential, \( m \ll H_0 \ll H_I \) and therefore the assumption \( \phi_i \sim \phi_I \) is obviously justified.)

- After inflation, modes of the \( \phi \) field re-enter the horizon (during the radiation and matter dominated eras). These should no longer be considered part of the homogeneous \( \phi \) condensate at late time. This can affect \( \phi \) by removing \( \phi \) fluctuations generated over the last 30-60 e-foldings of inflation. But our constraint on \( N \) below will be many orders of magnitude larger so we can ignore this too in our use of Eq. \( \ref{31} \).

Therefore we may take \( \phi_i \approx \phi_I \), i.e., we presume the field \( \phi \) has not evolved much from inflation till decoupling. From our analysis above we have \( \phi_i > 7 \times 10^{16} \text{ GeV} \). Then, from Eq. \( \ref{31} \), we get a lower bound on \( N \) as

\[
N > \frac{2 \times 10^{39} \text{ GeV}^2}{H_I^2}. \tag{32}
\]

From the Planck bound on the tensor-to-scalar ratio, \( H_I < 9 \times 10^{13} \text{ GeV} \). Therefore we finally obtain

\[
N > 2 \times 10^{11}. \tag{33}
\]

Combining Eq. \( \ref{32} \) and the Planck bound on \( H_I \) we get \( H_I^2/N < 3 \times 10^{10} \text{ GeV} \). For a quadratic quintessence potential with \( m \lesssim H_0 \approx 10^{-42} \text{ GeV} \) one sees that our assumption that \( m^2 \ll H_I^2/N \) is highly feasible.

Ref. \[20\] also considers a dark energy condensate from quantum fluctuations of the quintessence field during inflation. Their analysis is primarily in the asymptotic limit \( m^2N \ll H_I^2 \) of Eq. \( \ref{29} \). They also consider \( m^2N \ll H_I^2 \) with an argument similar to that in the footnote following Eq. \( \ref{28} \).

\section*{V. CONCLUSIONS}

In this article we have numerically solved for the evolution of the quintessence field \( \phi \) in thawing models of dark energy for potentials of the form \( V \sim \phi^\pm 2 \) and different values of \( \Omega_{m0} \) and \( \lambda_i = -(M_P/V)dV/d\phi = \mp M_P/\phi_i \), where \( \phi_i \) is the field value at decoupling. From this we obtain the Hubble parameter \( H(z) \), and the luminosity distance and angular diameter distance, as given in Eqs. \( \ref{13} \) \( \ref{14} \) and \( \ref{22} \) respectively. We then relate them to the observations of type Ia supernovae data, the baryon acoustic oscillations data, the cosmic microwave background shift parameter and the observational Hubble parameter data to constrain the values of \( \lambda_i \) and \( \Omega_{m0} \). The likelihood contours for the two potentials looks very similar (due to similar \( H(z) \) behaviour) and in both the cases we obtain \( \phi_i > 7 \times 10^{18} \text{ GeV} \). We have argued that \( \phi \) does not evolve much between the end of inflation and decoupling and then considered a scenario where the field value at the end of inflation \( \phi_I \) is due to quantum fluctuations of \( \phi \) during inflation. This allows us to use the lower bound on \( \phi_i \approx \phi_I \) to constrain the duration of inflation – the number of e-foldings \( N \) is constrained to be greater than \( 2 \times 10^{11} \), for \( H_I < 9 \times 10^{13} \text{ GeV} \).

The inflationary paradigm does not stipulate an upper bound on \( N \) and so large values of \( N \) such as that required above are plausible. Large values of \( N \) are more likely in large field inflation models as discussed in Ref. \[20\]. However very large values of \( N \) can imply a large variation \( (\gg M_P) \) in the inflaton field during inflation which can be problematic if the inflationary scenario constitutes a low energy effective field theory derivable from some Planck scale theory \[21\]. On the other hand, one can get a large value of \( N \) in eternal inflation scenarios without a large net variation in the inflaton field \[22\].

\section*{Acknowledgments}

RR would like to acknowledge Gaurav Goswami for useful discussions.

\begin{thebibliography}{99}
\bibitem{1} A. G. Riess \textit{et al.} [Supernova Search Team Collaboration], Astron. J. \textbf{116}, 1009 (1998) \texttt{astro-ph/9805201}.
\bibitem{2} S. Perlmutter \textit{et al.} [Supernova Cosmology Project Collaboration], Astrophys. J. \textbf{517}, 565 (1999)
\end{thebibliography}
[3] J. L. Tonry et al. [Supernova Search Team Collaboration], Astrophys. J. 594, 1 (2003) [astro-ph/0305008].
[4] I. Zlatev, L. M. Wang and P. J. Steinhardt, Phys. Rev. Lett. 82, 896 (1999) [astro-ph/9807002].
[5] E. J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys. D 15, 1753 (2006) [hep-th/0603057].
[6] M. Li, X. D. Li, S. Wang and Y. Wang, Commun. Theor. Phys. 56, 525 (2011) [arXiv:1103.5870].
[7] R. R. Caldwell and E. V. Linder, Phys. Rev. Lett. 95, 141301 (2005) [astro-ph/0505494].
[8] G. Gupta, S. Majumdar and A. A. Sen, Mon. Not. Roy. Astron. Soc. 420, 1309 (2012) [arXiv:1109.4112].
[9] R. J. Scherrer and A. A. Sen, Phys. Rev. D 77, 083515 (2008) [arXiv:0712.3450].
[10] N. Suzuki, D. Rubin, C. Lidman, G. Aldering, R. Amanullah, K. Barbary, L. F. Barrientos and J. Botyanszki et al., Astrophys. J. 746, 85 (2012) [arXiv:1105.3470].
[11] R. Giostri, M. V. d. Santos, I. Waga, R. R. Reis, M. O. Calvao and B. L. Lago, JCAP 1203, 027 (2012) [arXiv:1203.3213].
[12] P. A. R. Ade et al. [Planck Collaboration], Astron. Astrophys. (2014) [arXiv:1303.5076].
[13] O. Farooq and B. Ratra, Astrophys. J. 766, L7 (2013) [arXiv:1301.5243].
[14] S. Nesseris and L. Perivolaropoulos, Phys. Rev. D 72, 123519 (2005) [astro-ph/0511040].
[15] W. Hu and N. Sugiyama, Astrophys. J. 471, 542 (1996) [astro-ph/9510117].
[16] D. J. Eisenstein et al. [SDSS Collaboration], Astrophys. J. 633, 560 (2005) [astro-ph/0501171].
[17] A. D. Linde, Phys. Lett. B 116, 335 (1982).
[18] P. A. R. Ade et al. [Planck Collaboration], Astron. Astrophys. 571, A22 (2014) [arXiv:1303.5082 [astro-ph.CO]].
[19] C. Ringeval, T. Suyama, T. Takahashi, M. Yamaguchi and S. Yokoyama, Phys. Rev. Lett. 105, 121301 (2010) [arXiv:1006.0368].
[20] G. N. Remmen and S. M. Carroll, Phys. Rev. D 90, 063517 (2014) [arXiv:1405.5538].
[21] D. Baumann and L. McAllister, arXiv:1404.2601 [hep-th], see Sec. 4.3.
[22] A. H. Guth, J. Phys. A 40, 6811 (2007) [hep-th/0702178].