Intrinsic Josephson effect and nonequilibrium soliton structures in two-gap superconductors

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We predict a new dynamic state in current-carrying superconductors with multicomponent order parameter. If the current density \(J\) exceeds a critical value \(J_\text{c}\), an interband breakdown caused by charge imbalance of nonequilibrium quasiparticles occurs. For \(J > J_\text{c}\), the electric field penetrating from current leads gives rise to various static and dynamic soliton phase textures, and voltage oscillations similar to the nonstationary Josephson effect. We propose experiments to observe these effects which would probe the multicomponent nature of the superconducting order parameter.

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There are experimental evidences that several superconductors, including \(MgB_2\), \(NbSe_2\), \(\kappa-BEDT \) compounds have a multicomponent order parameter \(\psi\) with internal degrees of freedom. For instance, for two weakly coupled s-wave order parameters \(\psi_1 = \Delta_1 e^{i\theta_1}\) and \(\psi_2 = \Delta_2 e^{i\theta_2}\) corresponding to two different parts of the Fermi surface (like in \(MgB_2\)), the internal degree of freedom is the interband phase difference \(\theta(x, t) = \theta_1 - \theta_2\). In this case, besides the usual phase-locked states \((\theta = 0 \text{ or } \pi)\), intrinsic phase textures \(\theta(x, t)\) can occur. The soft modes associated with the fluctuations of \(\theta(x, t)\) are nearly decoupled from the gap fluctuations and behave like the Anderson plasmons in Josephson junctions. These modes may manifest themselves as resonances in the ac Josephson effect, or static \(2\pi\) phase kinks in \(\theta(x)\).

In this Letter we show that, although static metastable textures \(\theta(x)\) do not manifest themselves in the equilibrium magnetic response, the situation radically changes for nonequilibrium current states where the charge imbalance near normal leads gives rise to dynamic phase slip structures \(\theta(x, t)\) propagating into a superconductor (Fig. 1). We predict a new dynamic state above the critical current density \(J > J_\text{c}\) which marks the onset of the current-induced breakdown of the superconducting state. For the multicomponent \(\psi\), such breakdown is a two-stage process. First, at \(J = J_\text{c}\), an interband phase breakdown occurs, resulting in spontaneous phase solitons in \(\theta(x, t)\) and ac voltage oscillations. At higher currents, \(J > J_\text{c}\), both gaps \(\Delta_{1,2}\) are suppressed by the pairbreaking effects. For weak interband coupling, \(J_\text{c}\) is much smaller than the depairing current density \(J_\text{d}\).

We obtain the equations of motion for \(\theta\) and the electric field \(E\) near the critical temperature \(T \approx T_\text{c}\), using the time-dependent Ginzburg-Landau (TDGL) equations generalized to a two-gap superconductor:

\[
\Gamma_\mu(\partial_t \psi_\mu - 2\pi c i \varphi/\phi_0) = -\delta F/\delta \psi_\mu, \tag{1}
\]

where \(\mu\) runs from 1 to 2, \(\varphi\) is the electric potential, \(\phi_0\) is the flux quantum, \(c\) is the speed of light, \(\Gamma_\mu\) are damping constants, and the free energy \(F = \int d^3 r \left(f_1 + f_2 + f_m + f_{\text{int}}\right)\) contains the magnetic part \(f_m = |\nabla \times \mathbf{A}|^2/8\pi\), the GL inband part \(f_\mu\), and the interband interaction \(f_{\text{int}}\)

\[
f_\mu = a_\mu |\psi_\mu|^2 + \frac{\beta_\mu}{2} |\psi_\mu|^4 + g_\mu \left(\nabla + \frac{2\pi i}{\phi_0} \mathbf{A}\right) \psi_\mu^2, \tag{2}
\]

\[
f_{\text{int}} = \gamma (\psi_1 \psi_2^* + \psi_1^* \psi_2) = 2\gamma \Delta_1 \Delta_2 \cos \theta. \tag{3}
\]

Here \(\mathbf{A}\) is the vector potential, \(\gamma\) is a coupling constant, the asterisk means complex conjugation. The qualitative results of this work remain valid for any other periodic dependencies of \(f_{\text{int}}(\theta)\) on \(\theta\), for example, \(f_{\text{int}} = 2\Delta_1^2 \Delta_2^2 (\gamma_1 \cos 2\theta + \gamma_2)\), suggested for heavy-fermion superconductors. We focus here on two-gap superconductors for which two cases are possible: (i) The Cooper instability occurs at the same temperature in both bands, hence \(\alpha_\mu(T) \propto T - T_\text{c}\), \(\gamma \propto T - T_\text{c}\), while \(g_\mu\) and \(\beta_\mu\) are constants. (ii) The critical temperature \(T_\text{c1}\) in the band 1 is higher than \(T_\text{c2}\), in the band 2, so \(\psi_1\) appears spontaneously below \(T_\text{c}\), while \(\psi_2\) at \(T_\text{c1} < T < T_\text{c}\) is induced by the interband interaction, because \(\alpha_1 \propto T - T_\text{c1}\), \(\alpha_2 \propto T - T_\text{c2}\), and \(\gamma\), \(g_\mu\) and \(\beta_\mu\) are constants. In this paper we consider the first scenario, assuming that \(\gamma \ll \alpha_1, \alpha_2\), which is likely to occur in \(MgB_2\).

Eqs. (1) yield the following gap equations

\[
\Gamma_\mu \Delta_\mu = g_\mu \nabla^2 \Delta_\mu - \alpha_\mu \Delta_\mu - \beta_\mu \Delta_\mu^3 - \gamma \Delta_\mu \cos \theta, \tag{4}
\]

where \(\tau = 2\) if \(\mu = 1\), and \(\tau = 1\) if \(\mu = 2\), \(\theta = \theta_1 - \theta_2\), \(\alpha_\mu = \alpha_\mu + (2\pi Q_\mu/\phi_0)^2 g_\mu\), and \(Q_\mu = \mathbf{A} - \phi_0 \nabla \theta/2\pi\). The imaginary part of Eq. (4) gives

\[
\Gamma_\mu \left(\frac{2\pi c}{\phi_0} \varphi\right) \Delta^2_\mu = -\frac{2\pi c}{\phi_0} \nabla (\Delta_\mu^2 Q_\mu) \pm \gamma \Delta_1 \Delta_2 \sin \theta. \tag{5}
\]

\[
\nabla \times \nabla \times \mathbf{A} = (4\pi/e)(\sigma \mathbf{E} + \mathbf{J}_\text{s}) \tag{6}
\]

with the plus sign corresponding to \(\mu = 1\). The current density \(\mathbf{J} = \sigma \mathbf{E} + \mathbf{J}_\text{s}\) has the ohmic component \(\mathbf{J}_\text{n} = \sigma \mathbf{E}\).
proportional to the electric field \( \mathbf{E} = -\nabla \varphi - \mathbf{A}/c \), and the normal conductivity \( \sigma \). The supercurrent

\[
J_s = -8\pi^2 c(g_1\Delta_\mu^2 Q_1 + g_2\Delta_\mu^2 Q_2)/\phi_0^2
\]

is a sum of independent intraband contributions. The boundary conditions, \((\partial_s + 2\pi A_s/\phi_0)\psi_s = \psi_s/\mu_s\), between a superconductor and a normal metal imply zero perpendicular supercurrents for both \( \psi_1 \) and \( \psi_2 \). Excluding \( Q_{1,2} = -(\phi_0 J_s + 4\pi c g_2 (\Delta_1^2 + \Delta_2^2) \nabla \varphi / [8\pi^2 c(g_1\Delta_1^2 + g_2\Delta_2^2)]) \) from Eqs. (2) and (3), we obtain the energy density \( J = f_0 + f_\theta + \sum_\mu (\alpha_\mu^2 \Delta_\mu^2 + \gamma_\mu \nabla \Delta_\mu^2 + \beta_\mu \Delta_\mu^2)/2 \), where the magnetic and phase dependent parts, \( f_m \) and \( f_\theta \) turn out to be decoupled from each other:

\[
f_m = \frac{H^2}{8\pi} + \frac{\phi_0^2 J_s^2}{(g_1\Delta_1^2 + g_2\Delta_2^2)}
\]

\[
f_\theta = \frac{g_1 g_2 \Delta_1^2 \Delta_2^2 (\nabla \varphi)^2}{(g_1\Delta_1^2 + g_2\Delta_2^2)} + 2\gamma_1 \Delta_1 \Delta_2 \cos \theta
\]

For constant gaps \( \Delta_\mu \), variation of Eq. (8) with respect to \( \mathbf{H} \) yields the London equation \( \mathbf{H} \nabla^2 \mathbf{H} = \pm \sin \theta \) which has a single-solution \( \mathbf{H} \), or periodic solutions analogous to the Josephson vortices in a magnetic field \( \mathbf{H} \). However, the interband phase difference \( \theta \) is decoupled from static magnetic fields, while the Gibbs free energy of a Josephson contact contains the interaction term \( \propto (\mathbf{H} \nabla \varphi) \). Thus, equilibrium nonuniform solutions \( \theta(x) \) are energetically unfavorable as compared to the uniform phase-locked state \( \theta = 0 \) for \( \gamma < 0 \) or \( \theta = \pi \) for \( \gamma > 0 \). By contrast, various dynamic or quenched phase textures can be generated during current-induced interband breakdown.

We consider the weak coupling limit \( \gamma \ll \alpha_\mu \), for which the uniform gaps \( \Delta_{1,2} \) are unaffected by \( \theta(r,t) \), so Eqs. (3)-(8) give a time-dependent London equation with the account of dissipation and charge imbalance effects [14]. Expressing \( Q_{1,2} \) in Eqs. (3) in terms of \( \mathbf{J} \) and \( \nabla \varphi \), and then subtracting the equations for \( \theta_1 \) and \( \theta_2 \) from each other, we obtain the following equation for \( \theta \)

\[
\tau_\theta \dot{\theta} = L_\theta^2 \nabla^2 \varphi \pm \sin \theta + \alpha_\theta \text{div} \mathbf{J}_s,
\]

where the relaxation time \( \tau_\theta \), the decay length \( L_\theta \), and the charge coupling parameter \( \alpha_\theta \) are given by

\[
\tau_\theta = \Gamma_1 \Gamma_2 \Delta_1 \Delta_2/|\gamma(g_1\Delta_1^2 + \Gamma_2 \Delta_2^2)|,
\]

\[
L_\theta^2 = g_1 g_2 \Delta_1 \Delta_2 / |\gamma(g_1\Delta_1^2 + g_2\Delta_2^2)|
\]

\[
\alpha_\theta = \frac{4\pi c |\gamma|(g_1\Delta_1^2 + g_2\Delta_2^2)}{(4\pi c |\gamma| g_1\Delta_1^2 + g_2\Delta_2^2)}
\]

The signs in Eq. (10) correspond to the sign of \( \gamma \). As seen from Eq. (10), the phase mode is unaffected by any distribution of bulk supercurrents, unless there is a nonequilibrium charge imbalance \( \text{div} \mathbf{J}_s = -\sigma \text{div} \mathbf{E} \) caused by the electric field \( \mathbf{E} \) penetrating from normal current leads. The excess charge density provides a driving term in the sine-Gordon equation for \( \theta \), where the coupling constant \( \alpha_\theta \) is nonzero if \( g_1 \Gamma_2 \neq g_2 \Gamma_1 \), which implies different electron diffusivities in the bands 1 and 2.

To obtain the equation for \( \mathbf{E} \), we add Eqs. (2) for \( \theta_1 \) and \( \theta_2 \), then take the gradient of the sum and express \( Q_{1,2} \) in terms of \( \mathbf{J} \) and \( \nabla \varphi \). This yields

\[
\tau_\varphi \dot{\varphi} + \mathbf{E} - L_e \text{grad} \varphi + \alpha_\varphi \nabla \theta = \tau_\varphi \mathbf{J}/\sigma,
\]

where \( \mathbf{J}(t) \) is the driving current density, \( L_e \) is the electric field penetration depth, \( \tau_\varphi \) is the charging time constant, and the coupling term \( \alpha_\varphi \nabla \theta \) describes an electric field caused by moving phase textures:

\[
\tau_e \dot{\varphi} + \mathbf{E} - L_e \text{grad} \varphi + \alpha_e \nabla \theta = \tau_e \mathbf{J}/\sigma,
\]

We first use Eqs. (10) and (14) to calculate the phase textures in a microbridge of length \( 2a \) (Fig. 1a) for which Eq. (10) at \( \gamma < 0 \) takes the form

\[
L_\theta^2 \theta'' - \sin \theta + \alpha_\theta J_s' = 0
\]

Here \( \theta'(x) \) and \( J_s(x) \) vanishes at the bridge edges, \( x = \pm a \), where \( E(x,t) = E_0 \cos(x/L_e)/\cos(a/L_e) \) equals the value \( E_0 = J/\sigma \) provided by an external power source, \( \theta'(0) = E'(0) = 0 \), and the prime denotes differentiation over \( x \). Now we obtain the condition under which Eq. (14) has a stable solution \( \theta(x_0) \) localized at the edge of a long (\( a > L_e \)) bridge, where \( \theta_0 = \theta(a) \). We first consider the limit \( L_e \approx L_e \), for which \( J_s(x) = -\sigma E'(x) \) is essential only in a narrow region \( a-L_e < x < a \). Multiplying Eq. (14) by \( \theta' \) and integrating from \( 0 \) to \( a-L_e \), we obtain \( \theta'(a-L_e) = -\sigma E/L_e^2 \) obtained by integrating Eq. (14) over \( a-L_e < x < a \), where \( \theta(a-L_e) = \theta_0 \), and \( \sin \theta \) can be neglected. Excluding \( \theta'(x) \), we arrive at the equation \( \alpha_\theta J_t = 2L_\theta/|\sin \theta_0/2| \), which has solutions for \( \theta_0 \) only below the critical current density \( J_t = 2L_\theta/\alpha_\theta \):

\[
J_t = 8\pi c(g_1\Delta_1^2 + \Gamma_2 \Delta_2^2) \sqrt{|\gamma| g_1 g_2 (g_1\Delta_1^2 + g_2\Delta_2^2)} \phi_0/\sqrt{2\Delta_1 \Delta_2 (g_1\Gamma_1^2 - g_2\Gamma_2)}
\]

In the opposite limit \( L_e \approx L_e \), the Laplacian in Eq. (10) can be neglected. Then the steady-state solutions \( \sin \theta(x) = -\sigma \alpha_e E'(x) \), and \( E(x) = E_0 \cos(x/L_e)/\cos(a/L_e) \), exist only if \( \sigma \alpha_e E'(a) \approx 1 \), or \( J < J_t = L_e/\alpha_e \tan(a/L_e) \), where

\[
J_t = \frac{|\gamma| (g_1\Delta_1^2 + \Gamma_2 \Delta_2^2) \sin \theta_0 (g_1\Delta_1^2 + g_2\Delta_2^2)}{\sqrt{2\Delta_1 \Delta_2 (g_1\Gamma_1^2 - g_2\Gamma_2)} \tan(a/L_e)}
\]

If \( \gamma(T) \) and \( \alpha_\mu(T) \) linearly pass through zero at \( T_c \), Eqs. (19) and (20) yield \( J_t \propto (T_c - T)^{3/2} \).
For $J > J_t$, the charge-induced interband breakdown gives rise to a striking dynamic phase texture in which solitons periodically appear near the current leads and then propagate to the bulk. The actual soliton dynamics depends on the particular boundary conditions, so we performed numerical simulations of Eqs. \([10, 17]\) for the different geometries shown in Fig. 1. We consider the limit $\alpha \approx \alpha_0 \ll \min(\tau_0 \Delta^2, \tau_L^2 \Delta^3)$ of weak coupling between $\theta$ and $E$, for which the driving term $\text{div} \mathbf{E}$ in Eq. \([10]\) is mostly determined by the static $E(x)$ near the current lead, while the last term in Eq. \([17]\) gives only a small ac correction to $E$ of the second order in $\alpha_0 \Delta^2$.

Fig. 2 shows the soliton formation near the current lead in the thin microbridge (Fig 1a) as $J$ was instantaneously turned on from zero to a value below $J_t$. Such current step produces a phase soliton localized near the edge, while the bulk of the bridge remains in the phase-locked state with either $\theta = 0$ or $\pi$, depending on the sign of $\gamma$. For $J < J_t$, this behavior is characteristic of all geometries shown in Fig. 1, whereas the dynamic phase texture for $J > J_t$ can be very different.

We start with the bridge geometry (Fig. 1a), for which $E(x, t)$ and $\theta(x, t)$ are even and odd functions of $x$, respectively, so the boundary conditions are: $E(\pm a, t) = E_0$, $E'(0, t) = 0$, $\theta(0) = 0$, and $\theta'(\pm a, t) = 0$. The latter condition ensures that supercurrents in both bands vanish at the normal electrodes, where $J = \sigma E_0$. In this case a soliton first appears at the bridge edge, but for $J > J_t$, it is pushed to the bulk by the strong gradient of $E(x)$. Then the next soliton forms near the edge and the process repeats periodically, resulting in the propagation of the soliton chain into the bulk, as shown in Fig. 3. After the first soliton reaches the center of the bridge, it stops (because $\theta(0) = 0$), while new solitons keep entering the bridge from the current lead.

During this soliton pileup, the mean slope $\dot{\theta}$ increases, reaching a critical value $\dot{\theta}$ at which the soliton generation at the edge stops and a static phase texture forms. For $J \gg J_t$, the maximum soliton density $n = \dot{\theta}/2\pi$ can be estimated from the static Eq. \([11]\) in which the rapidly oscillating sin $\theta$ can be neglected, and $E(x) = J \sinh(x/L_c)/\sigma \sinh(a/L_c)$. This yields $\dot{\theta}(x) = \alpha_0 J L_c [\cosh(x/L_c) - \cosh(x/L_c)] / L_0^2 \sinh(a/L_c)$, giving for the long bridge ($a \gg L_c$):

$$\dot{\theta}_e = J L_e \phi_0 (\Gamma_1 g_2 - \Gamma_2 g_1)/4 \pi c g_1 g_2 (\Gamma_1 \Delta^2 + \Gamma_2 \Delta^2)$$

(21)

During the formation of the soliton chain, $t < t_c \sim \tau_0 \dot{\theta}_e/2\pi$, voltage oscillations are generated on the bridge. A similar transient behavior occurs at the point contact (Fig. 1b), for which $\text{div} \mathbf{E} = 2I \exp(-r/L_0) / \sqrt{\pi \tau_0} (r_0 + L_c) r$, where $I$ is the current through the semi-spherical contact of radius $r_0$. In this case the increase of $I$ above $I_c$ causes propagation of concentric soliton shells until the static structure forms as the critical phase gradient $\sim \dot{\theta}_e$ is reached.

A very different soliton dynamics occurs in the 4-terminal geometry (Fig. 1c), for which the supercurrents make 90° turns around the central stagnation point $(x = 0)$ where $\nabla \theta = 0$. As a result, $E(x)$ and $\theta(x, t)$ becomes odd and even functions of $x$, respectively, which radically changes the dynamics of the phase textures as compared to the bridge, since now the solitons do not stop as they reach the center, but keep moving until they reach the opposite current lead. This is due to the fact that the driving charge density $\text{div} \mathbf{E}$ does not change sign all the way along the horizontal leg of the cross in Fig. 1c. The corresponding total extra charge (which plays the same role here as the transport current in a Josephson junction) is exactly compensated by the opposite charge distributed along the vertical leg. Thus, the mean phase gradient $\dot{\theta}$ remains constant, while the phase slippage at the normal leads results in a continuous soliton shuttle along the vertical leg. This behavior is evident from the drive charge density $\text{div} \mathbf{E}$ in Eq. \([10]\) and $\text{div} \mathbf{E}$ in Eq. \([17]\). The latter condition ensures that supercurrents in both bands are killed as they reach the center, but keep moving until they reach the opposite current lead. This is due to the fact that the driving charge density $\text{div} \mathbf{E}$ does not change sign all the way along the horizontal leg of the cross in Fig. 1c. The corresponding total extra charge (which plays the same role here as the transport current in a Josephson junction) is exactly compensated by the opposite charge distributed along the vertical leg. Thus, the mean phase gradient $\dot{\theta}$ remains constant, while the phase slippage at the normal leads results in a continuous soliton shuttle along the vertical leg.

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FIG. 1: Geometries in which the interband phase breakdown could occur. Here N labels normal electrodes, gray domains show phase solitons moving along thin arrows, and block arrows indicate current directions. Static phase textures form in microbridges (a) and point contacts (b), while in the four-terminal geometry (c) the solitons continuously move from one current lead to another.

FIG. 2: Formation of a static phase soliton in $\theta(x)$ near the bridge edge after the current density was instantaneously turned on from 0 to $J = 0.99J_1$ at $t = 0$. Times and distances from the center ($x = 0$) are taken in the units of $\tau_\theta$ and $a$, respectively, $L_e = a/10$, $L_\theta = 0.1L_e$.

FIG. 3: Dynamics of formation of a static soliton chain in the bridge of length $2a$ after $J(t)$ was instantaneously turned on from 0 to $1.025J_1$ at $t = 0$. Only the right half $(0 < x < a)$ is shown, and the rest is the same as in Fig. 2.

FIG. 4: Moving soliton shuttle along the right half of the horizontal leg $(0 < x < a)$ in the four-terminal geometry shown in Fig. 1c. $J(t)$ was instantaneously turned on from 0 to $1.012J_1$ at $t = 0$, and the rest is the same as in Fig. 2.

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[14] If $\Delta_1$ appears at a higher $T_{c1}$, the induced gap $\Delta_2 = \gamma_1\Delta_1 \cos \theta/\alpha_2$ at $T_{c2} < T < T_{c1}$ depends on $\theta$. In that case $\theta$ is a poor variable, because Eq. (4) turns into a single TDGL equation for $\Delta_1$ with $\alpha_1 \to \alpha_1 - \gamma_2 \cos^2 \theta/\alpha_2$, which describes only locked states with $\theta = 0$ or $\pi$.

[15] For $\tau_e \gg \tau_\theta$ and $L_e^2 \gg L_\theta^2$, the ac component of $E$ obeys $\tau_e \dot{E} = L_e^2 E'' + \alpha_e \theta'$, whence $E' \simeq -\alpha_e \theta/L_e^2$ for small soliton velocities $v \ll L_e/\tau_e$, and $E' \simeq \alpha_e \theta/\tau_e$ for $v \gg L_e/\tau_e$. This just renormalizes the length and time scales in Eq. (10): $\tau_\theta \to \tau_\theta + \alpha_e \alpha_0/L_e^2$, $L_\theta \to L_\theta$ for $v \ll L_e/\tau_e$, and $\tau_\theta \to \tau_\theta$, $L_\theta^2 \to L_\theta^2 + \alpha_e \alpha_0/\tau_\theta$ for $v \gg L_e/\tau_e$, but does not change the behavior of $\theta(x,t)$.

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