Lens binarity versus limb darkening in close-impact galactic microlensing events

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ABSTRACT

Although point caustics harbour a larger potential for measuring the brightness profile of stars during the course of a microlensing event than (line-shaped) fold caustics, the effect of lens binarity significantly limits the achievable accuracy. Therefore, corresponding close-impact events make a less favourable case for limb-darkening measurements than those events that involve fold-caustic passages, from which precision measurements can easily and routinely be obtained. Examples involving later Bulge giants indicate that a \( \sim 10 \) per cent misestimate on the linear limb-darkening coefficient can result with the assumption of a single-lens model that looks acceptable, unless the precision of the photometric measurements is pushed below the 1 per cent level even for these favourable targets. However, measurement uncertainties on the proper motion between lens and source are dominated by the assessment of the angular radius of the source star, and remain practically unaffected by lens binarity. Rather than judging the goodness of fit by means of a \( \chi^2 \) test only, run tests provide useful additional information that can lead to the rejection of models and the detection of lens binarity in close-impact microlensing events.

Key words: gravitational lensing – stars: atmospheres.

1 INTRODUCTION

In order to resolve the surface of the observed star during a microlensing event, the magnification pattern created by the lens needs to supply a large magnification gradient. Two such configurations meeting this requirement have been discussed extensively in the literature: a point caustic at the angular position of a single point-like lens (Gould 1994; Nemiroff & Wickramasinghe 1994; Witt & Mao 1994; Bogdanov & Cherepashchuk 1995, 1996; Witt 1995; Gould & Welch 1996; Gaudi & Gould 1999; Heyrovský, Sasselov & Loeb 2000; Heyrovský 2003) and a line-shaped fold caustic produced by a binary lens (Schneider & Wagoner 1987; Schneider & Weiβ 1987; Gaudi & Gould 1999; Rhie & Bennett 1999; Dominik 2004a,b,c). It has been pointed out by Gaudi & Gould (1999) that fold-caustic events are more common and their observation is easier to plan, whereas close-impact events, where the source transits a point caustic, can provide more information. However, I will argue that this apparent gain of information can usually not be realized due to potential lens binarity. In contrast to fold caustics, which form a generically stable singularity, point caustics are not stable and do not exist in reality. Instead, there is always a small diamond-shaped caustic containing four cusps.

In this paper, the influence of lens binarity on the measurement of stellar limb-darkening coefficients and proper motion is investigated and the arising limitations of the power of close-impact events where the source passes over a single closed caustic are discussed. Section 2 presents the basics of close-impact microlensing events with the effect of source size, the potential of measuring stellar proper motion and limb darkening and the effect of lens binarity. Section 3 demonstrates the influence of lens binarity on the extraction of information from such events. First, the influence of lens binarity on the light curves is demonstrated by means of two illustrative examples involving K and M Bulge giants. Subsequently, a simulation of data corresponding to these configurations is used to investigate potential misestimates of parameters if lens binarity is neglected. Section 4 presents the conclusions and a summary of the results.

2 CLOSE-IMPACT MICROLENSING EVENTS

2.1 Size of source star

As pointed out by Paczyński (1986), a point-like source star at a distance \( D_S \) from the observer exhibits a magnification due to the gravitational field of a lens star with mass \( M \) at \( D_L \) by a factor

\[
A(u) = \frac{u^2 + 2}{u \sqrt{u^2 + 4}},
\]

where source and lens are separated by the angle \( u \theta_E \) and

\[
\theta_E = \sqrt{\frac{4GM}{c^2}} \frac{D_S - D_L}{D_S D_L}
\]

denotes the angular Einstein radius.

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The proper motion $\mu$ of the source relative to the lens constitutes a microlensing event with the time-scale $t_E = \theta_E/\mu$, for which the lens–source separation becomes

$$u(t) = \sqrt{u_0^2 + [p(t)]^2},$$

where $p(t) = (t - t_0)/t_E$, so that $u_0$ is the closest approach that occurs at time $t_0$.

It was discussed by Witt & Mao (1994) as well as by Nemiroff & Wickramasinghe (1994) that the finite extent of the source star could cause observable deviations from the magnification factor $A(u)$ as given by equation (1), Gould (1994) has argued that for a source star with radius $R_s = D_s\theta_E/\rho_*$ and radial brightness profile $I(\rho) = \bar{I}\xi(\rho)$, where $\rho$ is the fractional radius and $\bar{I}$ is the average brightness, the finite-source magnification can be approximated as

$$A^{\text{fs}}(u, \rho_*, \xi) = A(u) B(u/\rho_*, \xi),$$

with

$$B(z; \xi) = \frac{1}{\pi} \int_0^{2\pi} \int_0^1 \frac{\xi(\rho)}{\sqrt{\rho^2 + 2\rho z \cos \varphi + z^2}} \rho \, d\rho \, d\varphi.$$  

In particular, $B(z) \approx 1$ for $z \gg 1$, so that for large angular separations (compared to $\rho_*$), the finite-source effects become negligible, whereas $B(z) \propto z$ for $z \ll 1$ implies strong effects for small angular separations. Therefore, microlensing events with small impact parameters $u_0 \ll \rho_*$ are the most likely to show prominent effects of finite source size. Stellar spectra can be used to derive radius $R_s$ and distance $D_s$ of the source star, yielding its angular radius $\theta_s = R_s/D_s$. On the other hand, microlensing observations yield $\rho_*$ and, therefore, the time-scale $t_* = t_E/\rho_*$, during which the source moves by its own angular radius on the sky. Therefore, the observation of finite-source effects in microlensing events provides a measurement of the proper motion between the lens and the source as

$$\mu = \frac{\theta_s}{t_*} = \frac{R_s}{D_s t_E \rho_*}.$$  

### 2.2 Limb darkening

As already indicated, the stellar surface is not uniformly bright, but a characteristic variation with the distance from the centre is observed, commonly known as limb darkening, which depends on wavelength and, therefore, on the filter used for the observations. A widely used model is the linear limb-darkening law (Milne 1921)

$$\xi(\rho) = 1 + \Gamma \left( \frac{3}{2} \sqrt{1 - \rho^2} - 1 \right),$$

with $0 \leq \Gamma \leq 1$, which is linear in $\cos \vartheta = \sqrt{1 - \rho^2}$, where $\vartheta$ is the emergent angle. If the point-source magnification $A(u)$ shows a strong variation over the face of the source star, dense and precise microlensing observations provide an opportunity for a measurement of the limb-darkening coefficient (Bogdanov & Cherepashchuk 1996; Gould & Welch 1996).

The strongest magnification gradients occur in the vicinity of caustics. While a single lens creates a point caustic at its angular position, binary lenses create finite caustic curves that contain cusps. As the angular separation between the binary-lens objects tends to zero, its diamond-shaped caustic with four cusps degenerates into the point caustic of a single lens, so that the impact parameter falls below the angular source radius, i.e. $u_0 < \rho_*$ (Bogdanov & Cherepashchuk 1996; Gould & Welch 1996; Gould & Gould 1999; Heyrovský et al. 2000; Heyrovský 2003), and passages of the source star over a (line-shaped) fold caustic created by a binary lens (Schneider & Wagoner 1987; Schneider & Weiß 1987; Gould & Gould 1999; Rieke & Bennett 1999; Dominik 2004a, c). In addition, the source might pass directly over a cusp, as for the event MACHO 1997-BLG-28 (Albrow et al. 1999b), for which the first limb-darkening measurement by microlensing has been obtained. As anticipated by Gaudi & Gould (1999), the vast majority of other limb-darkening measurements so far has arisen from fold-caustic passages (Afonso et al. 2000; Albrow et al. 2000, 2001; Fields et al. 2003; Cassan et al. 2004; Kubas et al. 2005), whereas two measurements from single-lens events have been reported so far (Alcock et al. 1997; Yoo et al. 2004). The remaining limb-darkening measurement by microlensing, on the solar-like star MOA 2002-BLG-33 (Abe et al. 2003), constituted a very special case, where the source simultaneously enclosed several cusps over the course of its passage.

### 2.3 Lens binarity

Since the majority of stars resides in some form of binary or multiple systems (e.g. Abt 1983), it seems at first sight a bit surprising that more than 85 per cent of the observed microlensing events appear to be consistent with the assumption of a single point-like lens. An important clue to this puzzle is that the separation of binaries covers a broad range of 6–7 orders of magnitude roughly from contact to typical stellar distances. Lens binarity however usually does not provide strong deviations to the light curve if the angular separation is much smaller or much larger than the angular Einstein radius $\theta_E$ (Mao & Paczyński 1991). Therefore, many binary lenses simply escape our attention by failing to provide an observable signal (Di Stefano 2000). While several binary-lens systems provide weak distortions, such as MACHO LMC-1 (Dominik & Hirshfeld 1994, 1996) or MACHO 1999-BLG-47 (Albrow et al. 2002), a characteristic signature is provided if the source passes over a fold or even a cusp caustic, OGLE-7 (Udalski et al. 1994b) being the first such observed event.

Thus, although a lens star is never a completely isolated object, it can be approximated as such in many cases. However, one needs to keep in mind that its caustic is a small diamond with four cusps rather than a single point. For a single lens, the computation of the light curve is a straightforward process, where a semi-analytical expression by means of elliptic integrals exists for uniformly bright sources (Witt & Mao 1994). For binary lenses, in contrast, the same computation becomes quite demanding, where even for the point-source magnification no closed expression exists, but in general a fifth-order polynomial needs to be solved (Witt & Mao 1995). For this paper, the algorithm of Dominik (1998) has been used, which is based on the contour plot technique by Schramm & Kayser (1987) and the application of Green’s theorem.

Compared to a single lens, a binary lens involves three additional parameters, which can be chosen as the mass ratio $q$, the angle $\alpha$ and the separation parameter $d$. Here, $q = M_2/M_1$ is the ratio between the masses of secondary and the primary component, which are separated on the sky by the angle $d\theta_u$, while $\alpha$ is measured from the vector pointing from the secondary to the primary towards the source trajectory, which therefore reads

$$u(t) = u_0 \left( -\frac{\sin \alpha}{\cos \alpha} + p(t) \frac{\cos \alpha}{\sin \alpha} \right).$$

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3 BINARY VERSUS LIMB DARKENING

3.1 Illustrative binary-lens configurations

In order to investigate the effect of lens binarity on the measurement of stellar limb darkening and proper motion, two different event configurations have been chosen as illustrative examples, where different angular separations between the lens objects have been considered for both of them. For the model referred to as ‘configuration I’, let us consider a K5 Bulge giant at $D_A \sim 8.5$ kpc, for which $M_V = -0.2$ and $V - I = +2.0$. One, therefore, obtains $I_0 = 12.45$, so that with an assumed extinction $A_V = 1.15$, the observed source magnitude becomes $I \sim 13.6$. Let us further assume a binary lens of total mass $M \sim 0.35 M_{\odot}$ at $D_B \sim 6.5$ kpc, so that the angular Einstein radius becomes $\theta_E \sim 320 \mu$as, the Einstein radius becomes $r_E = D_E \theta_E \sim 2.0$ au and its projection to the source distance becomes $r'_{\infty} = (D_{\odot}/D_B) r_E \sim 500 R_{\odot}$. With $R_* \sim 25 R_{\odot}$ for a K5 giant, a source size parameter $\rho_\star = 0.05$ is therefore adopted. In order to make an optimal case for observing, a rather low proper motion $\mu \sim 6 \mu$as d$^{-1}$, corresponding to a lens velocity of $v \sim 65$ km s$^{-1}$ relative to the source has been chosen, which yields an event time-scale $t_E = 55$ d. The impact parameter is chosen as $u_0 = 0.015$, corresponding to a peak magnification of a point source of $A_V \sim 70$. Configuration II has been chosen to be similar to the parameters of the observed event MACHO 1995-BLG-30 (Alcock et al. 1997), for which the source is an even larger star, namely, an M4 giant. According to the obtained model parameters, let us adopt $u_0 = 0.05$, $\rho_\star = 0.075$ and the event time-scale $t_E = 35$ d. For $D_B \sim 8.5$ kpc and $D_A \sim 6.5$ kpc, the appropriate stellar radius of $R_* \sim 60$ R$_{\odot}$ is obtained for $M \sim 0.7 M_{\odot}$, so that $r'_{\infty} \sim 800 R_{\odot}$. These choices yield $r_E \sim 2.9$ au and $\theta_E \sim 460 \mu$as, so that the proper motion becomes $\mu \sim 13 \mu$as d$^{-1}$ and the relative lens velocity is $v \sim 140$ km s$^{-1}$. With $M_V = +0.2$ and $V - I = +3.4$ for an M4 giant, an extinction of $A_I = 0.85$ yields the baseline magnitude $I_{\text{base}} \sim 12.3$. In general, the light received from the source is blended with additional light from other unresolved sources (that are not affected by microlensing) or from the lens star, which is quantified by the blend ratio $g = F_B/F_S$, where $F_B$ denotes the source flux and $F_S$ denotes the background (blend) flux. Since the choice of equal lens masses implies that the lens objects are M dwarfs of mass $M/2 \sim 0.18 M_{\odot}$ or $M/2 \sim 0.35 M_{\odot}$, their contribution to the total light falls well below the systematic error bars even at the observed $I$-baseline. Therefore, blending is neglected with the choice $g = 0$.

For the brightness of the source, a linear limb-darkening law according to equation (7) with a coefficient $\Gamma_I = 0.5$ has been adopted for both configurations.

Binary lenses are likely to cause an asymmetry to the light curve, which, however, can be arbitrarily small and can even vanish for some configurations. If the lens binarity itself remains unnoticed in an observed light curve, significant asymmetry effects will not show up either, because the latter produces the smaller deviations. Asymmetries are related to the choice of both the source trajectory angle $\alpha$ and the mass ratio $q$ for a given separation $d$. As discussed in great detail by Dominik (1999), the binary lens can be approximately treated as a quadrupole lens for sufficiently small $d$, where the caustic becomes symmetric. Within this limit, the light curve is symmetric not only if the source trajectory is parallel or perpendicular to the binary axis, but also if it is tilted by 45°. Since the single-lens assumption has to stand against any alternative, the discussion is simplified by assuming an equal-mass binary and a source trajectory parallel to the line connecting their angular positions, which preserves the symmetry of the light curve. By being able to show that binary-lens configurations exist that can mistakenly be identified as a single-lens case, it can be concluded that one only finds an upper limit to binarity, i.e. to the absolute eigenvalue of the quadrupole tensor $\bar{Q} = [q/(1 + q^2)] d^2$, rather than being able to claim that the lens is indeed a single object.

The model parameters and indicative corresponding physical properties of matching lens and source stars are summarized in Table 1. With the choice of bright source stars, a long caustic-passage duration of several days, and a large peak magnification, the chosen configurations constitute better-than-average cases for the measurement of the stellar brightness profile. Light curves that correspond to either of the adopted configurations are shown in Fig. 1 for a binary-lens separation parameter $d = 0.2$ along with light curves that correspond to single-lens models with otherwise identical parameters, while Fig. 2 shows the difference in magnitude between these lightcurves. Lens binarity decreases the magnification both around the peak and in the wing region, while an increase in magnification results in regions just before the lens centre is hit by the leading limb or just after it is hit by the trailing limb. For configuration I, these regions of increased magnification stretch over a much larger part of the light curve than the corresponding regions for configuration II.

3.2 The caustic passage

For a single lens, the corresponding point-like caustic transits the source between the epochs characterized by $p(t) = p_0^+(t)$, where

$$p_0^+(t) = \pm \sqrt{\rho_0^2 - u_0^2},$$

and these can be easily identified in the light curve.

In contrast, the central caustic of a binary lens contains four cusps, which are located at

| Table 1. Model parameters and corresponding physical properties for the two discussed event configurations. |
|------------------------------------------|-----------------|-----------------|
| Configuration I | Configuration II | Configuration I | Configuration II |
| $t_E$ (d) | 55 | 35 |
| $t_0$ (d) | 0 | 0 |
| $u_0$ | 0.015 | 0.05 |
| $\rho_\star$ | 0.05 | 0.075 |
| $\alpha$ | 0° | 0° |
| $q$ | 1 | 1 |
| $I_{\text{base}}$ | 13.6 | 12.3 |
| $g$ | 0 | 0 |
| $\Gamma_I$ | 0.5 | 0.5 |
| $D_S$ (kpc) | ~8.5 | ~8.5 |
| $D_L$ (kpc) | ~6.5 | ~6.5 |
| $M$ (M$_{\odot}$) | ~0.35 | ~0.7 |
| $\nu$ (km s$^{-1}$) | ~65 | ~140 |
| $\mu$ (\muas) | ~320 | ~460 |
| $r_E$ (au) | ~2.0 | ~2.9 |
| $r'_\infty$ (R$_{\odot}$) | ~500 | ~800 |
| $R_*$ (R$_{\odot}$) | ~25 | ~60 |
| Spectral type | K5III | M4III |
| $M_V$ | +0.2 | +0.2 |
| $V - I$ | +2.0 | +3.4 |
| $A_I$ | 1.15 | 0.85 |
| $m - M$ | 14.65 | 14.65 |

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Figure 1. Light curves corresponding to the binary-lens configuration I (left) and configuration II (right) with parameters as listed in Table 1 and a separation parameter $d = 0.2$ (solid line) along with those for a single-lens model with otherwise identical parameters (dashed line). In the lower panels, which show the full wing of the light curves, binary-lens and single-lens models are hardly distinguishable.

Figure 2. Difference between binary-lens light curves with the angular separation parameter $d = 0.2$ for configuration I (left) or configuration II (right) and their single-lens counterparts.
a point source, the four cusps degenerate into the point size, characterized by revealing the result given by equation (9). Depending on the source, while other cusps may or may not successively enter or exit the source. With this variety of characteristics, caustic entry and exit are not that well defined from the observed light curve as they are for a single lens.

For the two selected configurations and a separation parameter $d = 0.2$, Fig. 3 illustrates the passage of the leading stellar limb over the cusps, where snapshots of the source are shown whenever a cusp is first touched along with the light curve for $-4 < t < 0$ d. Dashed lines refer to the single-lens case.

With the source size parameter $\rho_\star$, being larger for configuration II, a smaller fraction of the source is subtended by the caustic for the same choice of the lens separation parameter $d$. A larger impact parameter $u_0$ places the effect of binarity more towards the limb of the source. Despite a larger source size parameter $\rho_\star$, the combination of a smaller impact parameter $u_0$ and a smaller event time-scale $t_0$ leads to a shorter caustic passage time, which, for a single lens, is $t_\star(p_0^+ - p_0^-) \sim 3.9$ d for configuration II compared to $t_\star(p_0^+ - p_0^-) \sim 5.2$ d for configuration I.

### 3.3 Simulated sampled light curves

In order to see what kind of information can be extracted from sampled light curves, simulated data sets have been created that correspond to the two chosen binary-lens configurations. The event-sampling parameters for this simulation have been chosen in analogy to an earlier investigation of fold-caustic passages (Dominik 2004c).

For data in the range $t_{\text{min}} \leq t \leq t_{\text{max}}$, the sampling is characterized by the sampling interval $\Delta t$, the fluctuation $f_\Delta t$ of the time at which the measurement is taken, and the relative sampling phase shift $f_{\text{phase}}$. 

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The sampling intervals $\Delta t$ for different regions of the light curve have been chosen to roughly comply with the observing strategy of the PLANET microlensing follow-up campaign (Albrow et al. 1998; Dominik et al. 2002) for the central region and with that of the OGLE-III survey (Udalski 2003) for the outer regions. These are listed in Tables 2 and 3.

For a reference magnitude of $m_{\text{ref}} = 16$ in $I$-band, an uncertainty of $\sigma_{\text{ref}} = 0.015$ in the observed magnitude has been assumed, corresponding to a $\sim 1.5$ per cent relative uncertainty in the measured flux. For other magnitudes, it is assumed that the photometric measurement follows Poisson statistics, so that its uncertainty is proportional to the square root of the measured flux. This uncertainty has been smeared with a relative standard deviation of $f_a = 0.125$. The resulting photometric error bar $\sigma_{\text{phot}}$ does not yet represent the total measurement uncertainty. Instead, a systematic error $\sigma_0$ is added in quadrature, which becomes dominant as the target gets bright. For the systematic error, three different values $\sigma_0 = 0.003$, $\sigma_0 = 0.005$ or $\sigma_0 = 0.01$ have been used.

All the above choices mean that the creation of synthetic data sets $(i^{(i)}, m^{(i)}, \sigma^{(i)})$ follows the rules

$$i^{(i)} = N[t_{\text{min}} + (i + f_{\text{phase}}) \Delta t, f_{\Delta t} \Delta t],$$

$$\sigma^{(i)} = \sigma_{\text{ref}} 10^{0.2[2(m^{(i)})-m_{\text{ref}}]},$$

$$\sigma_{\text{phot}}^{(i)} = N(\sigma^{(i)}, f_a \sigma^{(i)}),$$

$$\sigma^{(i)} = \sqrt{\left(\sigma_{\text{phot}}^{(i)}\right)^2 + \sigma_0^2}.$$

where $N(\mu, \sigma)$ denotes a value drawn randomly from a normal distribution with mean $\mu$ and standard deviation $\sigma$, $m^{(i)}$ denotes the magnitude at time $i^{(i)}$ for the adopted model parameters, and $i \in [0, (t_{\text{max}} - t_{\text{min}})/\Delta t]$. In order to simulate possible losses in the data acquisition due to observing conditions or technical problems, data points have been removed from the data set at random with a probability of $p_{\text{loss}} = 0.05$.

Figure 4. Binary-lens light curves (solid line) and simulated data for $\sigma_0 = 0.01$ for both configurations and a lens separation parameter $d = 0.2$. Also shown are light curves for corresponding single-lens models with otherwise identical parameters (dashed line). The time $t_{\text{ref}}$ at which the leading limb hits the single-lens point caustic is indicated by a vertical thin dashed line.

$m^{(i)} = N(m^{(i)}), \sigma^{(i)}$,

where $N(\mu, \sigma)$ denotes a value drawn randomly from a normal distribution with mean $\mu$ and standard deviation $\sigma$, $m^{(i)}$ denotes the magnitude at time $i^{(i)}$ for the adopted model parameters, and $i \in [0, (t_{\text{max}} - t_{\text{min}})/\Delta t]$.

In fact, a round-the-clock coverage (without daylight gaps) requires more than one telescope. Nevertheless, the effects of additional free parameters (mainly baselines magnitudes) and different telescope characteristics for multisite observations are neglected in favour of simplicity, and the data are treated such as resulting from a single telescope.

The quality of coverage of the data set is demonstrated in Fig. 4, which shows the simulated data points and the theoretical light curves for $d = 0.2$ and $\sigma_0 = 0.01$ for both binary-lens configurations around the contact of the leading limb with the lens centre. To indicate the influence of lens binarity, the corresponding single-lens light curves are also shown.

### 3.4 Best-matching single-lens models

#### 3.4.1 Full light curve sampled

Let us now assume a single lens and determine the corresponding model parameters by means of fits to the simulated data sets that correspond to either of the two binary-lens configurations. We will then see whether statistical tests suggest to accept the single-lens model and how well the true limb-darkening coefficient $\Gamma_l$ and the time-scale $t_\epsilon = t_{\text{ref}} \theta_\epsilon$, which yields the proper motion as $\mu = \theta_\epsilon / t_\epsilon$, are reproduced. Any significant offsets for acceptable single-lens models will limit the accuracy to which these parameters are determined on the assumption of such a model.

Table 2. Sampling intervals and number of data points for configuration I.

| $t_{\text{min}}$ (d) | $t_{\text{max}}$ (d) | $\Delta t$ | $N$ |
|----------------------|----------------------|------------|-----|
| 0                    | 3                    | 20 min     | 405 |
| 3                    | 6                    | 30 min     | 271 |
| 6                    | 20                   | 1 h        | 636 |
| 20                   | 30                   | 2 h        | 226 |
| 30                   | 60                   | 6 h        | 227 |
| 60                   | 100                  | 12 h       | 153 |
| 100                  | 150                  | 1 d        | 95  |
| 150                  | 300                  | 2 d        | 142 |

Notes. With $t_{\text{ref}} = 0$, a constant sampling rate has been adopted on both sides of the peak for intervals $-t_{\text{max}} \leq t \leq -t_{\text{min}}$ and $t_{\text{min}} \leq t \leq t_{\text{max}}$. Due to the applied phase shift $f_{\text{phase}}$ and the fluctuation of the time when the observations are taken, $f_{\Delta t}$, the actual times of observation may fall outside the originally designated interval. $N$ denotes the number of data points for each selected $t_{\text{min}}$ and $t_{\text{max}}$. In total, 2155 data points have been created.

Table 3. Sampling intervals and number of data points for configuration II.

| $t_{\text{min}}$ (d) | $t_{\text{max}}$ (d) | $\Delta t$ | $N$ |
|----------------------|----------------------|------------|-----|
| 0                    | 3                    | 20 min     | 405 |
| 3                    | 5                    | 30 min     | 181 |
| 5                    | 10                   | 1 h        | 227 |
| 10                   | 20                   | 2 h        | 227 |
| 20                   | 40                   | 4 h        | 227 |
| 40                   | 60                   | 6 h        | 152 |
| 60                   | 80                   | 12 h       | 79  |
| 80                   | 120                  | 1 d        | 75  |
| 120                  | 300                  | 2 d        | 168 |

Notes. Explanation of the table is same as that of Table 2 for configuration II, where a total of 1741 data points have been created.
The difference between the light curves for best-matching single-lens models and the ‘true’ binary-lens configuration for the three lens separations $d = 0.2, d = 0.15$ or $d = 0.1$ is shown in Fig. 5.

Table 4. Best-matching single-lens models to configuration I binary-lens models with different angular lens separations for different systematic errors.

| $d$   | $\sigma_0 = 0.003$ | $\sigma_0 = 0.015$ | $\sigma_0 = 0.015$ | $\sigma_0 = 0.005$ | $\sigma_0 = 0.15$ | $\sigma_0 = 0.015$ | $\sigma_0 = 0.005$ |
|-------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| $x^2_{\min}$ | 18054.9 | 4373.6 | 2235.6 | 8873.9 | 2969.9 | 2097.89 | 3895.1 | 2267.8 | 2037.6 |
| d.o.f.   | 2149   | 2149   | 2149   | 2149   | 2149   | 2149   | 2149   | 2149   | 2149   |
| $x^2_{\min}$/d.o.f. | 8.40  | 2.02   | 1.04   | 4.13   | 1.38   | 0.976  | 1.81   | 1.06   | 0.948  |
| $\chi(x^2 > x^2_{\min})$ | 124.5 | 27.6   | 13.2   | 67.7   | 11.5   | 0.78   | 22.7   | 1.80   | 1.72   |
| $P(x^2 > x^2_{\min})$ | $\leq 10^{-366}$ | $\leq 10^{-167}$ | $0.094$ | $\leq 10^{-996}$ | $\leq 10^{-30}$ | 0.781 | $\leq 10^{-113}$ | 0.037 | 0.957 |
| $t_e$ (d) | 55.44 | 54.99  | 54.93  | 55.28  | 54.94  | 54.91  | 55.18  | 54.91  | 54.91  |
| $\sigma_t$ | 0.0157 | 0.0149 | 0.0148 | 0.0155 | 0.0148 | 0.0147 | 0.0153 | 0.0148 | 0.0147 |
| $\rho_*$ | 0.0541 | 0.0518 | 0.0503 | 0.0542 | 0.0519 | 0.0503 | 0.0543 | 0.0520 | 0.0503 |
| $I_{\text{base}}$ | 13.609 | 13.604 | 13.601 | 13.608 | 13.604 | 13.602 | 13.607 | 13.603 | 13.602 |
| $\Gamma_I$ | 1.000 | 0.746  | 0.538  | 1.000  | 0.751  | 0.538  | 1.000  | 0.751  | 0.537  |

Notes. The complete data set has been included. The ‘true’ binary-lens parameters are shown in Table 1. The blending parameter has been fixed to $g = 0$, whereas $t_0$ has been allowed to vary, yielding $|t_0| \leq 90$ s in all cases. The quantity $\sqrt{2x^2_{\min} - 2n - 1}$ is a measure of the goodness of fit as a characteristic of a $\chi^2$ test yielding the equivalent deviation from the mean in units of the standard deviation for a Gaussian distribution, where $n$ is the number of degrees of freedom (d.o.f.), which is the number of data points reduced by the number of free model parameters, while $P(x^2 > x^2_{\min})$ denotes the associated probability.

Table 5. Best-matching single-lens models to configuration II binary-lens models with different angular lens separations for different systematic errors.

| $d$   | $\sigma_0 = 0.003$ | $\sigma_0 = 0.015$ | $\sigma_0 = 0.015$ | $\sigma_0 = 0.005$ | $\sigma_0 = 0.15$ | $\sigma_0 = 0.015$ | $\sigma_0 = 0.005$ |
|-------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| $x^2_{\min}$ | 5670.3 | 2371.2 | 1791.5 | 3161.4 | 1895.0 | 1666.8 | 2001.6 | 1668.7 | 1605.0 |
| d.o.f.   | 1735   | 1735   | 1735   | 1735   | 1735   | 1735   | 1735   | 1735   | 1735   |
| $x^2_{\min}$/d.o.f. | 3.27  | 1.37   | 1.03   | 1.82   | 1.09   | 0.961  | 1.10   | 0.962  | 0.925  |
| $\chi(x^2 > x^2_{\min})$ | 47.6 | 10.0   | 0.96   | 20.6   | 2.66   | 0.164  | 4.4    | 0.131  | 0.244  |
| $P(x^2 > x^2_{\min})$ | $\leq 10^{-494}$ | $\leq 10^{-23}$ | $0.168$ | $\leq 10^{-94}$ | $0.004$ | $0.877$ | $\leq 10^{-5}$ | 0.871 | 0.988 |
| $t_e$ (d) | 34.62 | 34.76  | 34.91  | 34.62  | 34.77  | 34.91  | 34.64  | 34.78  | 34.93  |
| $\sigma_t$ | 0.0489 | 0.0491 | 0.0497 | 0.0489 | 0.0491 | 0.0497 | 0.0489 | 0.0491 | 0.0497 |
| $\rho_*$ | 0.0817 | 0.0795 | 0.0758 | 0.0817 | 0.0796 | 0.0758 | 0.0816 | 0.0796 | 0.0757 |
| $I_{\text{base}}$ | 12.302 | 12.301 | 12.301 | 12.302 | 12.301 | 12.301 | 12.302 | 12.301 | 12.301 |
| $\Gamma_I$ | 1.000 | 0.984  | 0.607  | 1.000  | 0.997  | 0.605  | 1.000  | 0.997  | 0.605  |

Notes. The same quantities as in Table 4 are displayed. Again, all data points have been included in the fits and the ‘true’ binary-lens parameters are listed in Table 1. The blending parameter has been fixed to $g = 0$, while $t_0$ is allowed to vary, where $|t_0| \leq 90$ s resulted in all cases.
accuracy from close-impact microlensing events and be quite careful with results obtained under the assumption of a single lens.

An additional complication arises from the fact that photometric error bars reported by reduction algorithms such as DoPHOT (Schechter, Mateo & Saha 1993) tend to underestimate the true photometric uncertainty for microlensing observations, which are plagued by varying observing conditions during the course of an event. Apart from the systematic error that is taken into account for the simulated data set, underestimates of 10–20 per cent occur frequently (e.g. Udalski et al. 1994a; Albrow et al. 1998; Tsapras et al. 2003), so that the ratio \( \chi^2 / \text{d.o.f.} \) is not unlikely to exceed 1.4. This strongly limits the significance of the \( \chi^2 \) test as a measure of the goodness of fit.

If one accepts an underestimate by 20 per cent, the single-lens model for the configuration I binary-lens data with \( d = 0.15 \) and \( \sigma_0 = 0.005 \) becomes acceptable, while for configuration II, single-lens models look acceptable for \( d = 0.15 \) and all applied systematic errors, as well as for \( d = 0.2 \) and \( \sigma_0 = 0.01 \).

3.4.2 Only peak region observed

In addition to fits making use of the complete set of data points, best-matching single-lens models for the peak region of the lightcurve, defined as \(-3 \leq t \leq 3 \text{d}\), have been obtained for \( \sigma_0 = 0.003 \), for which the model parameters and the result of \( \chi^2 \) tests are displayed in Table 6. In contrast to the fits recognizing the full data set, the baseline \( I_{\text{base}} \) has been fixed to its ‘true’ value, while the blending parameter \( g \) has been allowed to vary. As expected from the larger discrepancies attributed to the wing regions of the light curves for configuration I, the model parameters for the fits restricted to the peak region are given in Table 6.

### Table 6. Best-matching single-lens models to the peak region of binary-lens light curves for both discussed configurations and different angular lens separations.

| \( \sigma_0 = 0.003 \) | \( d = 0.2 \) | \( d = 0.15 \) | \( d = 0.2 \) | \( d = 0.15 \) | \( d = 0.2 \) | \( d = 0.1 \) |
|------------------|------------|------------|------------|------------|------------|------------|
| \( \chi^2_{\text{min}} \) | 1121.4 | 926.8 | 461.9 | 3468.21 | 948.98 | 572.34 |
| \( \text{d.o.f.} \) | 398 | 398 | 398 | 398 | 398 | 398 |
| \( \chi^2_{\text{min}} / \text{d.o.f.} \) | 2.82 | 2.32 | 1.16 | 8.7 | 2.38 | 1.44 |
| \( \sqrt{2\chi^2_{\text{min}} - \sqrt{2n - 1}} \) | 19.2 | 14.9 | 2.20 | 55.1 | 15.4 | 5.6 |
| \( P(\chi^2 \geq \chi^2_{\text{min}}) \) | \( \lesssim 10^{-81} \) | \( \lesssim 10^{-49} \) | \( \leq 0.015 \) | \( \lesssim 10^{-161} \) | \( \lesssim 10^{-53} \) | \( \lesssim 10^{-68} \) |
| \( t_0 \) (d) | 90.68 | 58.51 | 56.01 | 34.02 | 34.02 | 34.92 |
| \( u_0 \) | 0.0140 | 0.0181 | 0.0159 | 0.0478 | 0.0487 | 0.0497 |
| \( \rho_* \) | 0.0332 | 0.0487 | 0.0495 | 0.0829 | 0.0802 | 0.0758 |
| \( g \) | 0.571 | 0.031 | 0.011 | 0.0034 | 0.0014 | 0.0005 |
| \( \Gamma \) | 1.000 | 0.663 | 0.531 | 1.000 | 1.000 | 0.605 |

Notes. Results of fits of single-lens models to the peak region \((-3 \leq t \leq 3 \text{d})\) of the simulated binary-lens data sets for both configurations for a systematic error \( \sigma_0 = 0.003 \) and different angular lens separations. The parameters of the underlying binary-lens models can be found in Table 1. In contrast to the fits that include the full data set, the baseline magnitude \( I_{\text{base}} \) has been fixed, whereas the blending parameter \( g \) has been allowed to vary. Again, the free parameter \( t_0 \) fulfilled \( |t_0| \lesssim 90 \) s in all cases.
peak region deviate more strongly from those obtained with the full data set. In order to adjust to the optimal model, the blending parameter $g$ has assumed a significant non-zero value. For all selected binary-lens separations $d$ for configuration II and for $d = 0.1$ for configuration I, the goodness of fit resulting from the $\chi^2$ test for the peak-region model and data is worse than for the models and data for the full light curve, so that a rejection of all considered single-lens models is indicated. However, if one accounts for a possible 20 per cent increase in the size of the photometric errors, the models with $d = 0.1$ for both configurations still survive. In any case, the $d = 0.1$ models are not recommended for rejection for the larger systematic errors $\sigma_0 = 0.005$ or $\sigma_0 = 0.01$ by means of a $\chi^2$ test over the peak region.

Fig. 6 shows a comparison of the differences between the binary-lens and the single-lens light curves of fits involving the full data set and those restricted to the peak region for a binary-lens separation parameter $d = 0.1$. While the differences between these two types of fits are negligible for configuration II, the apparent differences for configuration I demonstrate the amount of information contained in the region before the first and after the last contact of the stellar limb with the caustic. For configuration I, the maximal deviation is reduced from 1.2 per cent to 0.8 per cent for the peak-only fits in the peak region at the cost of a deviation of up to 1.4 per cent outside compared to 0.2 per cent when the full data set is considered.

### 3.4.3 Characteristic deviation patterns

Given the problems in rejecting single-lens models by means of a $\chi^2$ test, one is tempted to look for characteristic deviation patterns in order to decide on whether a model is in agreement with the observed data. An underestimate of error bars is not a problem for the assessment of a run test, which recognizes the sign of the residuals, but not their size. In this sense, it is complementary to the $\chi^2$ test, which is sensitive to the absolute value of the residuals, but blind to their signs. Although the obtained $\chi^2$ may look appropriate, a model is not acceptable if it fails a run test. Let a ‘run’ be defined as the longest contiguous sequence of residuals with the same sign, and let $N$ denote the total number of data points. $N_+$ the number of points with positive residuals, and $N_-$ the number of points with negative residuals. For $N > 10$, the distribution of the number of runs $n_r$ can be fairly approximated by a normal distribution with the expectation value

$$E(n_r) = 1 + \frac{2N_+N_-}{N}$$

and the standard deviation

$$\sigma_\text{er} = \sqrt{\frac{2N_+N_-N - N^2}{N^2(N-1)}}$$

The goodness of fit can be expressed by the probability $P_r = P(n_r \leq n_\text{er})$. In addition to $n_r$, other statistics such as the length of the longest run or the symmetry between $N_+$ and $N_-$ may be checked.

For the fits to the full data sets whose parameters are listed in Tables 4 and 5, Tables 7 and 8 show the result of the corresponding

### Table 7. Results of run tests for the best-matching single-lens models for configuration I.

| $d = 0.2$ | $d = 0.1$ | $d = 0.1$ | $d = 0.2$ | $d = 0.1$ | $d = 0.2$ | $d = 0.1$ | $d = 0.2$ | $d = 0.1$ | $d = 0.1$ |
|---|---|---|---|---|---|---|---|---|---|
| $N_+$ | 911 | 1013 | 1051 | 935 | 1034 | 1063 | 1016 | 1053 | 1071 |
| $N_-$ | 1244 | 1142 | 1104 | 1220 | 1121 | 1092 | 1139 | 1102 | 1084 |
| $E(n_r)$ | 1052.8 | 1074.6 | 1077.8 | 1059.7 | 1076.7 | 1078.3 | 1075.0 | 1077.9 | 1078.5 |
| $\sigma_\text{(er)}$ | 365 | 774 | 1064 | 523 | 928 | 1086 | 816 | 1032 | 1122 |
| $\delta$ | 30.4 | 13.0 | 0.60 | 23.5 | 6.4 | -0.33 | 11.2 | 1.98 | -1.88 |
| $P_r$ | $\leq 10^{-202}$ | $\leq 10^{-38}$ | 0.275 | $\leq 10^{-123}$ | $\leq 10^{-10}$ | 0.630 | $\leq 10^{-28}$ | 0.024 | 0.970 |

Notes. For the fits of single-lens models to the full data set corresponding to configuration I and different values of the lens separation $d$ and the systematic error $\sigma_\text{er}$ whose parameters are displayed in Table 4, run tests over the whole data set or the peak region ($-3 \leq \tau \leq 3$) revealed $N_+$ positive and $N_-$ negative residuals, where ‘positive’ means that the observed magnification exceeds the theoretical one. $E(n_r)$ denotes the expected number of runs and $\sigma_\text{(er)}$ denotes the corresponding standard deviation. For $n_\text{er}$ runs being found, the deviation in units of $\sigma(n_r)$ becomes $\delta = \sqrt{\frac{E(n_r) - n_\text{er}}{N}}$, which corresponds to a probability $P_r = P(n_r \leq n_\text{er})$. 

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run tests over the full light curve and over the peak region only. Requiring an associated probability \( P_t > 0.05 \) only lets the single-lens models for \( d = 0.1 \) and \( \sigma_0 = 0.01 \) or \( \sigma = 0.5 \) per cent for both configurations survive, while also the model for \( d = 0.15 \) and \( \sigma_0 = 0.01 \) for configuration I nearly makes it to the acceptable region if \( P_t > 0.01 \) has to be fulfilled. If one looks at the difference between single- and binary-lens light curves, one sees that it becomes undetectable as soon as it is overshadowed by the statistical spread of the data which appears to be the case for \( d = 0.1 \) and \( \sigma_0 = 0.01 \). In fact, the \( \chi^2 \) test is not limited by this effect, but in principle allows to detect signals below the noise level by means of a sufficiently large number of independent measurements.

3.4.4 Implications

The simulations show that single-lens models for the discussed configurations involving a K or M Bulge giant look acceptable for \( d = 0.1 \), unless the photometric uncertainties are pushed significantly below the 1 per cent level. The corresponding offset in the limb-darkening coefficient of \( \sim 10 \) per cent implies that the assumption of a single lens is incompatible with the desire of a precision measurement on \( \Gamma \). In contrast, the effect on the time-scale \( \tau \) is below 1 per cent, so that potential lens binarity does not have a significant effect on the measurement of the proper motion \( \mu = \theta_s / \tau \), given that the uncertainty in \( \theta_s \) is much larger. One may be tempted to argue that the binary-lens system is an unlikely configuration. However, \( d = 0.1 \) means a projected separation \( \bar{r} = 0.2 \) au for configuration I with two lens stars of mass \( M/2 \sim 0.18 \, M_{\odot} \) and \( \bar{r} = 0.3 \) au for configuration II with two lens stars of mass \( M/2 \sim 0.35 \, M_{\odot} \), which should not a priori be discarded.

## 4 CONCLUSIONS AND SUMMARY

Generally speaking, the potential binarity of the lens limits the accuracy of a limb-darkening measurement of the observed source star for close-impact microlensing events. The caustic near the centre of the lens star is never exactly point like, but always a small diamond with four cusps. If a point lens is assumed, the obtained limb-darkening coefficient is systematically offset, while the nature of the lens binarity may not be apparent. Although the inclusion of a binary lens in the modelling of the event will yield a proper limb-darkening measurement, the additional degrees of freedom still diminish the achievable precision. Unfortunately, such a computation is extremely demanding. Simulations for typical event configurations involving K or M Bulge giants show that single-lens models for binary-lens events that involve an offset of \( \sim 10 \) per cent on the limb-darkening coefficient \( \Gamma \) look acceptable both from \( \chi^2 \) and from run tests, unless photometry significantly below the 1 per cent level is possible. The measurement uncertainty of the proper motion \( \mu = \theta_s / \tau \), however, is dominated by the determination of the angular source radius \( \theta_s \), compared to which the accuracy limits on \( \tau \) caused by lens binarity for close-impact microlensing events are negligible.

In contrast, the measurement of limb darkening from fold-caustic passages does not suffer from any of the problems encountered for close-impact events. Limb-darkening coefficients can be obtained routinely and easily with precisions of a few per cent (e.g. Dominik 2004c). A binary lens is assumed a priori, but the measurement of limb darkening only depends on a smaller set of local properties rather than on the complete binary-lens parameter space. The use of the local approximation of the light curve in the vicinity of the fold-caustic passage (e.g. Albrow et al. 1999a; Dominik 2004b) makes the computation quite inexpensive and easy. Moreover, fold-caustic exits can be well predicted. Once a caustic entry has been observed, it is clear that a corresponding exit will occur, and the light curve on the rise to the caustic exit peak allows the prediction of the time of the caustic exit with sufficient precision, usually more than a day in advance. A fair coverage of the caustic entry as well as the determination of the spectral type of the source star and an early measurement of the event time-scale \( \tau_e \) even allows a rough guess on the passage duration and, therefore, a proper a priori assessment of the potential for measurements of the source brightness profile. While the characteristic properties of a fold-caustic exit, which provides a full scan of the source from the leading to the trailing limb, can therefore be estimated in advance, corresponding predictions for close-impact events are only possible after the leading limb has already passed the caustic, leaving only the second half of the caustic passage involving the trailing limb.

Hence, events involving fold-caustic passages turn out to be the clear favourite over close-impact events for measuring

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**Table 8.** Results of run tests for the best-matching single-lens models for configuration II.

| \( N_+ \) | \( N_- \) | \( E(n_1) \) | \( \sigma(n_1) \) | \( \sigma(n_1) \) | \( \sigma(n_1) \) | \( \sigma(n_1) \) | \( \sigma(n_1) \) | \( \sigma(n_1) \) | \( \sigma(n_1) \) | \( \sigma(n_1) \) | \( \sigma(n_1) \) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| \( d = 0.2 \) | \( d = 0.15 \) | \( d = 0.1 \) | \( d = 0.2 \) | \( d = 0.15 \) | \( d = 0.1 \) | \( d = 0.2 \) | \( d = 0.15 \) | \( d = 0.1 \) | \( d = 0.2 \) | \( d = 0.15 \) | \( d = 0.1 \) |
| 888 | 852 | 846 | 869 | 850 | 841 | 856 | 856 | 839 | 839 | 839 | 839 |
| 853 | 889 | 905 | 872 | 891 | 900 | 885 | 885 | 902 | 902 | 902 | 902 |
| 871.2 | 871.1 | 870.8 | 871.5 | 871.0 | 870.5 | 871.3 | 871.3 | 870.4 |

**Notes.** Results of run tests corresponding to fits of single-lens models to the full data set corresponding to configuration II and different values of the lens separation \( d \) and the systematic error \( \sigma_0 \) whose parameters are displayed in Table 5, where either the whole data set or the peak region \((-3 < t < 3 \, d)\) has been used. The quantities shown are the same as in Table 7.
limb-darkening coefficients apart from those where the source transits a cusp. The latter provide an opportunity to determine further limb-darkening coefficients beyond a linear law, which is quite challenging and usually impossible for fold-passage events (Dominik 2004a).

However, events with fold-caustic passages make the worse of the two cases for proper motion measurements. Fits to the light curve in the vicinity of the caustic passage only yield $t_{\perp}^{*} = t_{\star} / (\sin \phi)$ (e.g. Afonso et al. 1998; Albrow et al. 1999a; Dominik 2004b), where $\phi$ is the caustic-crossing angle, which needs to be determined from a model involving the full set of binary-lens parameters. While a precise measurement of $t_{\perp}^{*}$ is possible, the uncertainty in $t_{\star}$ is severely limited by degeneracies and apparent ambiguities for the caustic-crossing angle $\phi$ (Dominik 1999; Albrow et al. 1999c; Afonso et al. 2000), which can have a comparable or even larger influence on the determination of the proper motion $\mu$ than uncertainties in the angular stellar radius $\theta_{\star}$.

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REFERENCES
Abe F. et al., 2003, A&A, 411, L493
Abt H. A., 1983, ARA&A, 21, 343
Afonso C. et al., 1998, A&A, 337, L17
Afonso C. et al., 2000, ApJ, 532, 340
Albrow M. D. et al., 1998, ApJ, 509, 687
Albrow M. D. et al., 1999a, ApJ, 512, 672
Albrow M. D. et al., 1999b, ApJ, 522, 1011
Albrow M. D. et al., 1999c, ApJ, 522, 1022
Albrow M. D. et al., 2000, ApJ, 534, 894
Albrow M. D. et al., 2001, ApJ, 549, 759
Albrow M. D. et al., 2002, ApJ, 572, 1031
Alcock C. et al., 1997, ApJ, 491, 436
Bogdanov M. B., Cherepashchuk A. M., 1995, Astron. Rep., 39, 779
Bogdanov M. B., Cherepashchuk A. M., 1996, Astron. Rep., 40, 713
Cassan A. et al., 2004, A&A, 419, L1
Di Stefano R., 2000, ApJ, 541, 587
Dominik M., 1998, A&A, 333, L79
Dominik M., 1999, A&A, 349, 108
Dominik M., 2004a, MNRAS, 352, 1315
Dominik M., 2004b, MNRAS, 353, 69
Dominik M., 2004c, MNRAS, 353, 118
Dominik M., Hirshfeld A. C., 1994, A&A, 289, L31
Dominik M., Hirshfeld A. C., 1996, A&A, 313, 841
Dominik M. et al., 2002, P&SS, 50, 299
Fields D. L. et al., 2003, ApJ, 569, 1305
Gaudi B. S., Gould A., 1999, ApJ, 513, 619
Gould A., 1994, ApJ, 421, L71
Gould A., Welch D. L., 1996, ApJ, 464, 212
Heyrovský D., 2003, ApJ, 594, 464
Heyrovský D., Sasselov D., Loeb A., 2000, ApJ, 543, 406
Kubas D. et al., 2005, A&A, 455, 941
Mao S., Paczyński B., 1991, ApJ, 374, L37
Milne E. A., 1921, MNRAS, 81, 361
Nemiroff R. J., Wickramasinghe W. A. D. T., 1994, ApJ, 424, L21
Paczyński B., 1986, ApJ, 304, 1
Rhie S. H., Bennett D. P., 1999, preprint (astro-ph/9912050)
Schneider P., Wagoner R. V., 1987, ApJ, 314, 154
Schneider P., Weiß A., 1987, A&A, 171, 49
Schechter P. L., Mateo M., Saha A., 1993, PASP, 105, 1342
Schramm T., Kayser R., 1987, A&A, 174, 361
Tsapras Y., Horne K., Kane S., Carson R., 2003, MNRAS, 343, 1131
Udalski A., 2003, Acta Astron., 53, 291
Udalski A. et al., 1994a, Acta Astron., 44, 165
Udalski A., Szymański M., Mao S., Di Stefano R., Kaluzny J., Kubiač M., Mateo M., Krzeminski W., 1994b, ApJ, 436, L103
Witt H. J., 1995, ApJ, 449, 42
Witt H. J., Mao S., 1994, ApJ, 430, 505
Witt H. J., Mao S., 1995, ApJ, 447, L105
Yoo J. et al., 2004, ApJ, 603, 139

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