This paper investigates the theoretical Cramér-Rao bounds on estimation accuracy of longitudinal vehicle dynamics parameters. This analysis is motivated by the value of parameter estimation in various applications, including chassis model validation and active safety. Relevant literature addresses this demand through algorithms capable of estimating chassis parameters for diverse conditions. While the implementation of such algorithms has been studied, the question of fundamental limits on their accuracy remains largely unexplored. We address this question by presenting two contributions. First, this paper presents theoretical findings which reveal the prevailing effects underpinning vehicle chassis parameter identifiability. We then validate these findings with data from on-road experiments. Our results demonstrate, among a variety of effects, the strong relevance of road grade variability in determining parameter identifiability from a drive cycle. These findings can motivate improved experimental designs in the future.

1 Introduction

This paper explores the impact of drive cycle characteristics on the identifiability of longitudinal vehicle chassis parameters. From a conventional engineering perspective, vehicle chassis parameter estimation is an important exercise for identifying and controlling relevant vehicle models. For example, online mass estimation has significant importance to active vehicle safety systems [1]. Similarly, drag estimation can provide utility in quantifying the benefits associated with a heavy-duty vehicle’s participation in a vehicle platoon [2]. Furthermore, as connected and automated vehicle systems are studied in greater detail, online parameter estimation algorithms will become increasingly important, as the function of connected and automated vehicle (CAV) system optimization will likely depend on accurate knowledge of parameters including vehicle mass, drag, and rolling resistance coefficients. This consideration becomes even more important when considering vehicles with highly variable loads, including freight trucks [3-4].

The literature presents numerous vehicle parameter estimation algorithms. For example, research by Bae et al. [5] applies a recursive least-squares algorithm (RLS) to obtain online estimates of longitudinal parameters including vehicle mass from experimental data. Work by Fathy et al. explores online mass estimation using a supervisory algorithm which identifies predominantly longitudinal vehicle motion [6]. Vahidi et al. estimate both vehicle mass and road grade through RLS utilizing multiple forgetting factors. Rhode et al. [8] evaluate a generalized version of total RLS based on its experimental performance in estimating longitudinal parameters including rolling resistance coefficients and vehicle mass. The authors of this paper also explore offline nonlinear least-squares longitudinal chassis parameter estimation within the context of quantifying the impact of terrain variability on the identifiability of such parameters [9].

Vehicle chassis parameter estimation not limited to the use of longitudinal vehicle dynamics. For example, Raja-manl et al. [10] show that suspension dynamics can be utilized to effectively estimate vehicle chassis parameters including vehicle mass. Pence, Fathy, and Stein [11] demonstrate that vehicle mass can be estimated inexpensively from base excitation suspension dynamics. Reina et al. [12] utilize lateral vehicle dynamics with model-based estimation to
estimate vehicle states and mass.

Exploring factors which affect parameter identifiability dates back to a landmark paper by Bellman and Astrom [13]. Examining limitations related to the identifiability of model parameters provides an opportunity to observe new insights related to the nature of such estimation problems. Such insights can address fundamental questions like “What conditions of an on-road experiment impose the biggest limitations on chassis parameter estimation accuracy?”, or “In what ways does the evolution of the drive cycle affect the identifiability of vehicle chassis parameters?” We can also attempt to quantify the impact of the effects which are present in the scenarios posed by such questions, including for example quantifying the impact of terrain variability on vehicle chassis parameter identifiability. This is where our paper’s motivation lies.

The literature recognizes the impact of the design of a vehicle experiment on the accuracy of the resulting chassis parameter estimates. The SAE testing standard for longitudinal vehicle chassis parameter estimation outlines several basic experimental conditions which facilitate estimation accuracy [14]. Within another context, work by Muller et al. [15] demonstrates that the addition of speed bumps to a road allows relevant suspension parameters to be estimated with greater precision. While such work recognizes effects which facilitate estimation accuracy, there is a need to provide theoretical justification for such observations. Pursuing such theoretical justification could potentially reveal new insights which can improve experimental design. For instance, an understanding of the impact of terrain variability on longitudinal chassis parameter identifiability is fairly unexplored.

The Fisher information metric is an effective means for quantifying parameter identifiability [16]. The invertibility of the Fisher information matrix indicates whether the experiment in question can yield uniquely identifiable parameter estimates. Specifically, the Cramér-Rao theorem states that the best parameter estimation covariance achievable by an unbiased estimator is given by the inverse of the matrix-formulation of Fisher information. In this paper, we present a theoretical analysis which reveals the underlying effects and limitations on the best achievable estimation accuracy for longitudinal vehicle dynamics. For an appropriate estimation algorithm, our analysis leverages Fisher Information to mathematically show the form of the Cramér-Rao lower bounds on longitudinal parameter estimates. We use these expressions to show fundamental insights which can motivate improved experiment design.

The remaining organization of this paper is as follows. Section II details our formulation of the longitudinal vehicle dynamics. Section III describes the format of the Fisher information metric corresponding to the longitudinal dynamics. In Section IV, we derive an analytic Fisher matrix for a linear estimation case. We describe our methods for obtaining analytic expressions for Fisher information with nonlinear least-squares in Section V, including validation of our derivations through simulation and on-road experiments. Finally, after section V our conclusion summarizes and synthesizes insights from the paper’s findings.

### Table 1. Longitudinal Dynamics Model

| Value   | Description | Units |
|---------|-------------|-------|
| \(m\)  | vehicle mass | kg    |
| \(C_d\) | drag coefficient | -     |
| \(\mu\) | rolling resistance coefficient | -     |
| \(A_{ref}\) | frontal area | \(m^2\) |
| \(\rho\) | air density | \([kg/m^3]\) |
| \(g\)  | gravitational constant | \(\frac{m}{s^2}\) |

2 Longitudinal Vehicle Dynamics Model

The foundation of this paper’s analysis utilizes a nonlinear longitudinal vehicle dynamics model adopted from the literature [17]. This model formulation allows us to evaluate the identifiability of vehicle mass, drag coefficient, and rolling resistance coefficient parameters. The nonlinear state-space model is given by (12).

\[
\begin{align*}
\dot{x} &= v \\
\dot{v} &= \frac{1}{m}[F - \frac{1}{2}\rho C_d A_{ref} v^2] - \mu g \cos(\beta) - g \sin(\beta)
\end{align*}
\]

where \(x\) is the vehicle’s position, \(v\) is the vehicle velocity, \(\dot{v}\) is the vehicle acceleration, \(F\) is the propulsion force, and \(\beta\) is the road grade. Table 1 describes other relevant parameters of this model.

3 Formulation of Fisher Information

Fisher information is a metric which quantifies how much information a data set contains about a set of relevant model parameters [16]. In formulating our Fisher analysis, we first define relevant system output and longitudinal chassis parameters from the model described by (12). We specifically select vehicle mass \(m\), \(C_d\), and \(\mu\) to be the chassis parameters which we assume are being estimated.

\[
\theta = \begin{pmatrix}
m \\
C_d \\
\mu
\end{pmatrix}
\]

Fisher information analysis requires selection of a measured output variable. In this paper, we use longitudinal vehicle velocity as our measured output, as this aligns with our experimental setup.

The Fisher information matrix quantifies an expected curvature (i.e. Hessian) of the likelihood function specifically around the parameter estimate \(\theta\). In the case of linear
least-squares, the Fisher information matrix is computed as:

\[ I(\theta) = \frac{R^TR}{\sigma^2} \]  

(4)  

where \( R \) is the regressor matrix, and \( \sigma^2 \) is the sensor variance associated with output measurement noise for an assumed white, independent and identically distributed noise process. This indicates that estimation accuracy depends not only on how we excite the system, but on how well our sensors can accurately measure their intended signals.

For nonlinear least-squares problems, Fisher information can be computed as follows [18]:

\[ I(\theta) = \frac{S^T S}{\sigma^2} \]  

(5)  

We note that

\[ S_{ij} = \frac{\partial v(t_i)}{\partial \theta_j} \]  

(6)  

where \( i \) indicates the timestep and \( j \) indicates the parameter, is the sensitivity of the output with respect to the parameters. This sensitivity can be approximated via finite differences:

\[ \frac{\partial v(t_i)}{\partial \theta_j} = \frac{v(t_i, \theta_j + \epsilon_j) - v(t_i, \theta_j)}{\epsilon_j} \]  

(7)  

In section IV, we derive analytic continuous-time expressions for the output sensitivities given by (6). We therefore must approximate the summations in the Fisher information matrix with integrals when using these analytic expressions:

\[ \sum_{i=1}^{N} S_i S_j = f \int_0^T S_i(t) S_j(t) dt \]  

(8)  

where \( T \) is the final time and \( f \) is the sampling frequency.

If the Fisher information matrix is positive definite, the model parameters are said to be uniquely locally identifiable. Furthermore, the Cramér-Rao theorem states that the best achievable estimation covariance is obtained from the inverse of the Fisher information matrix. The diagonal terms of this covariance matrix are the Cramér-Rao lower bounds of the estimation variances for each parameter estimate in \( \theta \). These estimation error variances quantify the accuracy of the estimator. This paper's simulated and experimental identifiability analyses compare the error bounds obtained from Fisher analyses using both numeric and analytic methods.

4 Linear Estimation Case

We perform Fisher analysis for both the full nonlinear vehicle parameter estimation problem and a simplified linear case.

To perform the simplified linear analysis, we start with the basic longitudinal model formulation:

\[ m\ddot{v} = F - \frac{1}{2}(\beta^2 v^2 + mg\cos(\beta) - mg\sin(\beta)) \]  

(9)  

we assume the road grade \( \beta \) is sufficiently small such that

\[ m\ddot{v} \approx F - \frac{1}{2}(\beta^2 v^2 + mg - mg\beta) \]  

(10)  

Furthermore, we assume rolling resistance \( \mu \) is known. This is a fair assumption: vehicle mass can vary significantly (e.g. heavy-duty vehicles), and aerodynamic drag can likewise change due to inter-vehicle interactions (i.e. platooning), but rolling resistance tends to stay fairly constant. By assuming \( \mu \) and \( \beta \) are known, we can reformulate the longitudinal model to be conducive to application of ordinary least-squares as follows:

\[ m(\ddot{v} + \mu g + g\beta) + \frac{1}{2}\rho C_d A_f v^2 = F \]  

(11)  

where we now redefine \( x_1(t) = \dot{v} + \mu g + g\beta, x_2(t) = v^2, y = F, \theta_1 = m, \theta_2 = \frac{1}{2}\rho C_d A_f \). By using (8) to reformulate the Fisher matrix in continuous time, we obtain the following expression:

\[ I(\theta) \approx \frac{f}{\sigma^2} \left[ \int_0^T x_1^2(t) dt \int_0^T v(t) x_2(t) dt \int_0^T v(t)^2 dt \right] \]  

(12)  

From this point forward, we assume we are considering periodic conditions where \( v(t_0) = v(t_f) \) and elevation | \( t_0 | = \text{elevation} | t_f | \). Now, filling in the values for \( x_1(t) \) and \( x_2(t) \) as:

\[ \int x_1(t)x_2(t)dt = \int v^2 \dot{v} dt + \int v^2 \mu g dt + \int v^2 g \beta dt \]  

(13)  

Using integration by parts with \( u = v^2 \) and \( dv = \dot{v} dt \), we transform this expression into:

\[ \int v^2 \dot{v} dt = v^3 | t_0 | - \int 2v \dot{v} v dt \]  

(14)  

Since we assume periodic conditions, \( v^3 | t_0 | = 0 \). This means that

\[ \int v^2 \dot{v} dt = - \int 2v \dot{v} v dt = 0. \]  

(15)  

Analyzing the next term yields the following progression:

\[ \int v^2 \mu g dt = \mu g \int v^2 dt = \mu g | \frac{1}{3} v^3 | t_0 | = 0 \]  

(16)  

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due to the periodicity assumption.

So, these findings indicate that

$$\int x_1(t)x_2(t)dt = g\int v^2\beta dt$$  \hspace{1em} (17)

Finally, we assume the road grade and kinetic energy are sufficiently uncorrelated with each other. This assumptions renders the off-diagonal terms of the Fisher information matrix to equal zero. This leaves us with a diagonal Fisher information matrix:

$$I(\theta) = \frac{f}{\sigma^2} \left[ \begin{array}{cc} \int_0^T x_1^2(t)dt & 0 \\ 0 & \int_0^T x_2^2(t)dt \end{array} \right]$$  \hspace{1em} (18)

Now, we expand the remaining diagonal terms as follows:

$$\int x_1^2(t)dt = \int (\dot{v} + \mu g + g\beta)^2 dt$$  \hspace{1em} (19)

$$= \int (v^2 + \mu^2 g^2 + g^2 \beta^2 + 2\dot{v}\mu g + 2v\mu g + 2g^2 \mu \beta)dt$$  \hspace{1em} (20)

Given the periodicity assumption, this simplifies into:

$$\int x_1^2(t)dt = \int \dot{v}^2 dt + T\mu^2 g^2 + g^2 \int \beta^2 dt + 2g\int \dot{v}\beta dt$$  \hspace{1em} (21)

Finally, we expand the other diagonal term

$$\int x_2^2(t)dt = \int v^4 dt$$  \hspace{1em} (22)

resulting in the following Fisher information matrix:

$$I(\theta) = \frac{f}{\sigma^2} \left[ \begin{array}{cc} \int_0^T (\dot{v}(t) + g\beta(t))^2 dt + T\mu^2 g^2 & 0 \\ 0 & \int_0^T v^4(t)dt \end{array} \right]$$  \hspace{1em} (23)

This resulting expression allows us to infer several insights about chassis parameter identifiability.

- First, our ability to estimate the drag coefficient is largely dependent on the kinetic energy of the experiment squared. This supports the existing empirical testing standard, which dictates a vehicle be brought to high velocity and then coasted down slowly.
- Vehicle mass identifiability is subject to a host of more varied, interacting effects. A new insight from is that rolling resistance squared also affects the identifiability of the mass parameter. Specifically, the larger the rolling resistance coefficient the easier it will be for one to estimate mass from measurements of the vehicle’s propulsive force.
- Perhaps the most important factor which affects the mass parameter identifiability is the interplay between vehicle acceleration and gravitational acceleration. When expanding this integral out, it is clear the increased levels of road grade variability (as represented by the integral of $g^2\beta^2$) improve mass identifiability. However, this is only true insofar as the vehicle’s acceleration deviates from pure gravitational acceleration. In the worst case, vehicle acceleration $\dot{v} = -g\beta(t)$ or is purely dependent on gravity, which can lead to the mass estimation error to be considerably large. This makes intuitive sense. Consider that if one lets the vehicle accelerate and decelerate purely due to gravity up and down a series of hills, the vehicle will accelerate nearly the same way regardless of its mass. This makes it almost impossible to estimate vehicle mass unless the control input forces the vehicle to accelerate in ways which depart from pure gravitational acceleration. In this case, mean square road grade (the integral of road grade squared) will only improve estimation errors if the vehicle is controlled in such a way as to suppress the dependence of vehicle acceleration on road grade and gravity.

These insights provide theoretical support to existing empirical testing standards. In the remainder of this paper, we will explore these effects with more comprehensive theoretical and experimental analyses. Specifically, we will first derive analytic expressions for the Fisher information matrix when conducting full nonlinear least-squares to estimate vehicle mass, drag coefficient, and rolling resistance coefficient. From these derivations, we corroborate the results of this exploratory linear analysis.

5 Nonlinear Least-Squares Analysis

Now, we consider a full nonlinear least-squares problem where we estimate vehicle mass, drag, and rolling resistance coefficient. Simultaneous estimation of these three parameters is a more complicated process. By deriving analytic expressions for the Fisher information matrix, we can design simulation studies which replicate the conditions we found ideal in the previous section. In this section, we show that the effects which improve estimation accuracy for the full (i.e. 3 parameter) problem are similar to those we found in the previous section, where we only estimate vehicle mass and drag coefficient. In the following section, we also show experimental validation of our analytic Fisher information matrix which reveals strong correlation between $C_d$ and $-\mu$ sensitivities, which likely explains the agreement between our results in both cases.

As described in Section III, our format of Fisher information utilizes expressions for the sensitivity of vehicle velocity with respect perturbations in vehicle mass, drag coefficient, and rolling resistance coefficient parameters. Namely, we adopt equations (5) exactly. In this section, analytic expressions for the sensitivity of vehicle velocity with respect to perturbation in vehicle mass, $C_d$, and $\mu$ parameters are derived. Then, we assemble the full Fisher information matrix.
via \( \delta \).

Subsection 4.1 details our derivation of the nominal longitudinal velocity trajectory, and subsection 4.2 describes our analytic derivations for the sensitivity expressions. Subsection 4.3 demonstrates the accuracy of our analytic expressions through relevant simulation and experimentation.

### 5.1 Obtaining the Nominal Velocity Trajectory

To obtain the nominal velocity trajectory for use in our sensitivity derivations, we start with the full nonlinear longitudinal dynamics model with generalized, time-varying propulsion force and road grade:

\[
\dot{v}(t) = \frac{1}{m} [F(t) - \frac{1}{2} \rho C_d A_{ref} (v(t))^2] - \mu g \cos(\beta(t)) - g \sin(\beta(t)) \quad (24)
\]

In this paper, we treat propulsion force and road grade as time dependent quantities. In reality, they both depend on position which itself depends on time. We approximate them as time varying to simplify our derivations.

Writing (24) for a drive cycle subject to a flat terrain profile and constant propulsion force yields the following equation:

\[
\dot{v}_0 = \frac{1}{m} [F_0 - \frac{1}{2} \rho C_d A_{ref} v_0^2] - \mu g \quad (25)
\]

Here, we take a flat terrain profile as the operating point for a linearization of the model. If we take the following perturbations:

\[
F(t) = F_0 + \delta F(t) \quad (26)
\]

\[
\beta(t) = \beta_0 + \delta \beta(t) \quad (27)
\]

with \( \beta_0 = 0 \) and \( F_0 \) being the force required for our vehicle to maintain a constant predetermined speed \( v_0 \) across a flat terrain. Next, we can subtract (25) from the expression we get by plugging in (26) and (27) into (24) to obtain the following equation for \( \delta \dot{v} = \dot{v}_n - \dot{v}_0 \):

\[
\delta \dot{v}_n = \frac{\delta F(t)}{m} - \frac{1}{m} \rho C_d A_{ref} (v_n^2 - v_0^2) - \mu g \cos(\delta \beta(t)) - 1 - g \sin(\delta \beta(t)) \quad (28)
\]

Furthermore, the term \( v^2 - v_0^2 \) can be linearized into \( 2v_0 \delta v \) when neglecting higher order terms, where \( v_0 \) is the constant speed obtained by applying the constant propulsion force \( F_0 \) to the vehicle subject to a flat terrain:

\[
\delta \dot{v}_n = \frac{\delta F(t)}{m} - \frac{2}{m} \rho C_d A_{ref} v_0 \delta \dot{v}_n - \mu g \cos(\delta \beta(t)) - 1 - g \sin(\delta \beta(t)) \quad (29)
\]

Solving this ordinary differential equation with zero initial conditions for \( \delta v_n(t) \) yields the expression for perturbed vehicle velocity relative to steady state, with respect to an arbitrary time-varying terrain profile and propulsion force:

\[
\delta v_n(t) = e^{-\frac{2 \lambda A_{ref} \rho v_0 t}{m}} \int_0^t \sigma(\tau) \left[ \frac{\delta F(\tau)}{m} - g(\sin(\delta \beta(\tau))) - \mu + \mu \cos(\delta \beta(\tau)) \right] d\tau \quad (30)
\]

where

\[
\sigma(t) = e^{-\frac{2 \lambda A_{ref} \rho v_0 t}{m}} \quad (31)
\]

We can add this expression to \( v_0 \) to obtain the nominal linearized velocity trajectory for arbitrary time-varying terrain and propulsion force. This overall expression is essential to the analytic derivation of the sensitivity of velocity with respect to perturbations in each chassis parameter \( m, C_d, \) and \( \mu \).

### 5.2 Sensitivity Derivations

The following subsection details analytic calculations for the sensitivity of velocity with respect to perturbations in the longitudinal chassis parameter \( m \). The same procedure we use for \( m \) can be applied to the other chassis parameters, but for the purpose of brevity we omit those derivations.

Our first step is to rewrite the full longitudinal model with a percent perturbation applied to the mass parameter:

\[
\dot{v} = \frac{1}{m(1 + \varepsilon_m)} [F(t) - \frac{1}{2} \rho C_d A_{ref} v^2] - \mu g \cos(\delta \beta(t)) - g \sin(\delta \beta(t)) \quad (32)
\]

This added contribution can be effectively linearized to take the following form:

\[
\dot{v} = \frac{1 - \varepsilon_m}{m} [F(t) - \frac{1}{2} \rho C_d A_{ref} v^2] - \mu g \cos(\delta \beta(t)) - g \sin(\delta \beta(t)) \quad (33)
\]

The original longitudinal model in (24) can be subtracted from (33) to give the following expression for our new \( \delta \dot{v} \):

\[
\delta \dot{v}(t) = -\frac{\varepsilon_m}{m} [(F_0 + \delta F(t)) - \frac{1}{2} C_d \rho A_{ref} (v(t))^2 - \delta A(t) v(t) \delta v(t)] - \mu \rho A_{ref} (v(t))^2 \quad (34)
\]

where

\[
v(t) = v_0 + \delta v_n(t) \quad (35)
\]

\[
\dot{v}(t) = \dot{v}_0 + \delta \dot{v}(t) = \delta \dot{v}(t) \quad (36)
\]
and $\delta v_n(t)$ is represented by \([30,31]\) in Section 4.1. Upon dividing through by $\epsilon_m$, we obtain a simple time varying 1st order ordinary differential equation which we can solve directly for the sensitivity $\frac{\delta v(t)}{\epsilon_m}$:

$$
\frac{\delta v(t)}{\epsilon_m} = -\frac{1}{m} \left[ (F_0 + \delta F(t)) - \frac{1}{2} \rho A_{\text{ref}} v(t)^2 \right] - \frac{C_d}{m} \rho A_{\text{ref}} v(t) \frac{\delta v}{\epsilon_m} \tag{37}
$$

where $v(t)$ is sourced from \([35]\).

We use the MATLAB symbolic algebra toolbox to analytically solve this ODE for $\frac{\delta v(t)}{\epsilon_m}$. For the purpose of brevity we omit the final, lengthy, analytic expression for $\frac{\delta v(t)}{\epsilon_m}$.

### 5.3 Intuition from Derivations

We notice the commonalities the sensitivity expressions share with the simple derivation in the previous section. For instance, upon using a small angle approximation, the term

$$
m \int_0^T \alpha(t) \left( \frac{\delta F(t)}{m} - g \beta(t) \right) dt \tag{38}
$$

where $\alpha(t)$ includes additional effects, appears several times in the sensitivity expressions (and thus the resulting Fisher information matrix). This term originates from the perturbed velocity expression we solve for at the beginning of this section. Considering that $\frac{\delta F(t)}{m}$ is a propulsion induced acceleration term, this shows a commonality between the linear and nonlinear cases in terms of how road grade affects parameter identifiability. In fact, for the mass sensitivity equation, the interaction between propulsion force and gravitational acceleration is squared, similarly to the upper left hand entry of the linear Fisher information matrix \([23]\). This indicates that similar insights prevail in the nonlinear estimation case, namely that the interaction between acceleration and gravitational forces is important in determining parameter identifiability from an experiment.

The remainder of this paper presents simulation studies which validate our sensitivity expressions under various driving conditions.

### 5.4 Simulation and Experimental Validation of Sensitivity Expressions

To validate our analytic sensitivity expressions, we utilize both a simulation study and an experimental analysis. First, in our simulation study, we compute a representative driving segment with sufficient excitation. Then, we simulate longitudinal driving along that segment subject to the same longitudinal model parameterization given in \([9]\) corresponding to a Volvo VNL300 heavy-duty vehicle, namely $m = 8875$ kg, $C_d = 0.49$, and $\mu = 0.0056528$. Using the data from this simulated driving, we compute the parametric sensitivity signals using expressions given in Section 4.2 of this paper and those we obtain through \([9]\). For our analytic expressions, we take the operating points as (1) the average measured velocity and (2) the average measured wheel force. Figure 1 demonstrates this comparison for 20 minutes of simulated driving. The sensitivities plotted in Figure 1 are not normalized. As we can see, even with a relatively large range of velocities the requisite approximations we make in our analytic derivations end up having little effect on the accuracy of our final results. Furthermore, the Cramèr-Rao bounds we obtain from this simulation using numeric and analytic sensitivities are in strong agreement. Table 2 lists the estimation error bounds from both numerical and analytic sensitivities.

For our experimental study, we use data from an instrumented heavy-duty vehicle driving on-road to validate our analytic sensitivity expressions. The experimental vehicle is instrumented with a final-drive torque sensor, GPS, and a CAN interface which logs data through a Simulink real-time machine. Our use of GPS to measure vehicle velocity negates the requirement for consideration of wheel slip in our dynamical model. The precise details of this experimental setup can be referenced in \([9]\). This setup allows us to record values of wheel torque, road grade, velocity and acceleration, braking indicator, and other useful signals. To conduct our experimental comparison, we cut the data using the braking indicator to avoid unmodeled braking dynamics. Then, we compute the sensitivities of the vehicle velocity with respect to each nominal parameter value both numerically (via \([7]\) and using our analytic expressions from the previous section. With both sets of sensitivities, we compute Fisher information using \([5]\). Figure 2 demonstrates the results of this comparison. Overall, we observe strong agreement between numeric and analytic sensitivity signals for highway driving. Upon computing the Fisher information matrices for both the analytic and numeric cases, our resulting Cramèr-Rao estimation error lower bounds also agree. This validates the accuracy of our derivations, and shows the necessary approximations have minimal effect on the accuracy of our analytic expressions relative to a more computationally expensive and general approach for computing the sensitivities. Table 2 shows the Cramèr-Rao bounds obtained from the above data sample.

In both the simulated and experimental cases, the drag and rolling resistance sensitivities stabilize over time, whereas the mass sensitivity fluctuates throughout the experiment. This is due to the type of contributions each have directly on the vehicle velocity. Both drag and rolling resistance present consistently resistive effects on velocity, whereas vehicle mass can increase or decrease the vehicle velocity depending on the interactions between the propulsion force, road grade, and velocity itself. For instance, consider the case where the vehicle is traveling at constant velocity. If the drag coefficient increases, for a consistent experiment, we would expect the vehicle’s velocity to gradually decrease until stabilizing due to the quadratic drag effect. Conversely, if we increase the vehicle mass for a consistent experiment, the vehicle could speed up or slow down depending on whether it is traveling up or downhill. While the
Table 2. Cramér-Rao Bounds from Simulated and Experimental Validation of Sensitivity

|                     | Numerical (sim) | Analytic (sim) | Numerical (exp) | Analytic (exp) |
|---------------------|-----------------|----------------|-----------------|----------------|
| Mass \( m \) [kg]   | 47.30           | 48.00          | 26.91           | 28.89          |
| Drag Coefficient \( C_d \) [-] | 0.0043         | 0.0039         | 0.0108          | 0.0091         |
| Rolling Resistance \( \mu \) [-] | 0.00022         | 0.00020         | 0.00086         | 0.00078         |

![Fig. 1. Simulated validation of analytic sensitivity derivation](image)

pure gravitational acceleration will be the same with a scale in vehicle mass, the relative impact of the other forces (i.e. drag, rolling resistance, etc.) become smaller, which accentuates the gravitational effect. This can be seen most clearly in the mass sensitivity of the experiment in Figure 2, where the overall velocity does not change a great deal throughout the experiment. In this case, with consistent downhill driving at the beginning of the experiment the mass sensitivity gradually increases.

6 Conclusion

This paper presents two key contributions. These include (i) theoretical insights into the dynamics of chassis parameter identifiability, and (ii) two studies which illustrate the relationships between terrain variability, velocity, and propulsion force and longitudinal chassis parameter identifiability.

The first application of these insights pertains directly to the design of on-road experiments for longitudinal chassis parameter identifiability. Choosing routes for such experiments which possess the greatest degree of terrain variability could not only improve the accuracy of parameter estimates obtained from the experiment, but could also enable researchers to avoid unnecessarily costly instrumentation. Perhaps more significant than the application to experimental design is the relevance of this paper’s results to connected and automated vehicle (CAV) systems research. Much of this research relies on a priori knowledge of relevant vehicle parameters, including vehicle mass, drag, and rolling resistance coefficients. For actual implementation of CAV system optimization, these parameters may in fact need to be estimated in real time with online parameter estimation algorithms, and

![Fig. 1. Simulated validation of analytic sensitivity derivation](image)
such algorithms will need to interface with system-wide optimization. The effectiveness of CAV optimization algorithms will likely depend on the accuracy of these parameter estimates, and as a result choosing and weighting data from segments of road characterized by high terrain variability can improve the function of such algorithms.

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References
[1] Papelis, Y., Brown, T., Watson, G., Holz, D., and Pan, D., 2004, Study of ESC Assisted Driver Performance Using a Driving Simulator, Document ID: N04-033-PR, Published by University of Iowa, National Advanced Driving Simulator.
[2] Zabat, M., Farascaroli, S., Browand, F., Nestlerode, M., and Baez, J. (1994), Drag Measurements on a Platoon of Vehicles. Report from Institute of Transportation Studies at UC Berkeley, California PATH program, 1994.
[3] He, C., Ge, J., and Oroz, G. (2018), Data-based fuel-economy optimization of connected automated trucks in traffic. Proceedings of the 2018 American Control Conference. Milwaukee, WI USA, June 2018.
[4] Xu, C., Geyer, S., and Fathy, H. K. (2019), Formulation and Comparison of Two Real-Time Predictive Gear Shift Algorithms for Connected/Automated Heavy-Duty Vehicles. IEEE Transactions on Vehicular Technology, vol 68 number 8, pp. 7498-7510.
[5] Bae, H. S., Ryu, J. and Gerdes, J. C. (2001). Road Grade and Vehicle Parameter Estimation for Longitudinal Control Using GPS. 2001 IEEE Intelligent Transportation Systems Conference. Oakland, CA: 2001 IEEE Intelligent Transportation Systems Proceedings, pp.166-71.
[6] Fathy, H. K., Kang, D. and Stein, J. L., (2008). Online vehicle mass estimation using recursive least squares and supervisory data extraction. Proceedings of the 2008 American Control Conference (ACC). Seattle, WA USA, June 2008.
[7] A. Vahidi, A. Stefanopoulou and H. Peng (2005) Recursive least squares with forgetting for online estimation of vehicle mass and road grade: theory and experiments, Vehicle System Dynamics, 43:1, 31-55, DOI: 10.1080/00423110412331290446
[8] Rhode, S. and Gauterin, F. (2013). Online estimation of vehicle driving resistance parameters with recursive least squares and recursive total least squares. Proceedings of the 2013 IEEE Intelligent Vehicles Symposium (IV). Gold Coast, Australia, June 2013.
[9] Kandel, A. I., Wahba, M., Geyer, S. and Fathy, H. K. (2018). Impact of terrain variability on chassis parameter identifiability for a heavy-duty vehicle. Proceedings of the 2018 European Control Conference (ECC). Limassol, Cyprus, June 2018.
[10] Rajamani, R., and Hedrick, J. K., 1995, “Adaptive Observers for Active Automotive Suspensions: Theory and Experiment”, IEEE Transactions on Control System
[11] Pence, B., Fathy, H. K. and Stein, J. L. (2009). Sprung mass estimation for off-road vehicles via base-excitation suspension dynamics and recursive least squares. Proceedings of the 2009 American Control Conference. St. Louis: IEEE, pp. 5043-5048.

[12] Reina, G., Paiano, M., and Blanco-Claraco, J., Vehicle parameter estimation using a model-based estimator, Mechanical Systems and Signal Processing, Volume 87, Part B, 2017, Pages 227-241, ISSN 0888-3270, https://doi.org/10.1016/j.ymssp.2016.06.038.

[13] Bellman, R., and Astrom, K. J. (1970). On Structural Identifiability. Mathematical Biosciences, 7, 329-339. https://doi.org/10.1016/0025-5564(70)90132-X

[14] SAE International (2008). SAE standard J2263: Road Load Measurement Using Onboard Anemometry and Coastdown Techniques. Light Duty Vehicle Performance and Economy Measure Committee. SAE International, pp. 1-12.

[15] Muller, T., Ferris, J., Detweiler, Z. and Smith, H. (2009). Identifying vehicle model parameters using measured terrain excitations. SAE Technical Papers.

[16] Norton, J. P., An Introduction to Identification, 1st ed. Dover, 1986.

[17] Rajamani, R., Vehicle Dynamics and Control, 2nd ed. Springer US, 2012, vol. 14.

[18] Thomas, J. and Cover, T., Elements of Information Theory, 2nd ed. Wiley, 1991.