IMPROVED RESULTS ON EXPONENTIAL STABILITY OF DISCRETE-TIME SWITCHED DELAY SYSTEMS

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Abstract. In this paper, we study the exponential stability problem of discrete-time switched delay systems. Combining a multiple Lyapunov function method with a mode-dependent average dwell time technique, we develop novel sufficient conditions for exponential stability of the switched delay systems expressed by a set of numerically solvable linear matrix inequalities. Finally, numerical examples are presented to illustrate less conservativeness of the obtained results.

1. Introduction. In the past decades, systems analysis and control design for switched systems have attracted much attention among many researchers in a range of communities such as applied mathematics, systems theory and control engineering. There are two main reasons to cause it. Firstly, switched systems can be applied to model dynamical processes with switching characteristics in many practical applications such as mechanics [5] and automotive industry [9]. On the other hand, multi-controller switching can provide an effective mechanism to deal with dynamical systems with complex uncertainties and improve their system performance which cannot be made by a single control [5].

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Generally, stability analysis and control design of switched systems can be summarized into three basic problems [7]: (i) to find the conditions for stability under arbitrary switching, (ii) to identify asymptotically stabilizing switching signals, and (iii) to construct an asymptotically stabilizing switching signal. Many valuable results and efficient approaches have been devoted to answer these three basic problems.

Switching signals under consideration include arbitrary switching and constrained switching. As for the arbitrary switching case, the stability problem is firstly investigated using a common Lyapunov function method. For example, in [8], by constructing a common Lyapunov function (CLF), necessary and sufficient conditions for asymptotic stability of switched linear systems under arbitrary switching are explored. It should be noted that the existence of a CLF is only a sufficient condition for stability of switched systems under arbitrary switching signals, and sometimes even though the systems do not have a CLF, they can still be stable under certain switching laws [4]. To decrease the conservativeness of CLF results, a switched Lyapunov function (SLF) approach is proposed in [3]. However, sometimes it is not easy to find an appropriate CLF or SLF, and the switched system may still be unstable under certain switching signals even all the subsystems are exponentially stable [4]. Therefore, it is of great importance to restrict the switched signals in order to achieve stability performance of the switched systems. That is actually equivalent to the stability problem of switched systems under constrained switching. Moreover, it is well known that the multiple Lyapunov function (MLF) approach is an efficient method to study stability performance of the switched systems [1]. This approach is usually more amenable to applications than the CLF/SLF approach. Recently, more stability results based on SLF/MLF theory can be found in the literature such as [14, 15].

Except above, in [6], a Lie-algebraic sufficient condition for asymptotic stability is obtained for a class of systems consisting of a fixed family of asymptotically stable linear subsystems. By proving the existence of a quadratic common Lyapunov function for this family, it proves that if the associated Lie algebra matrix is solvable, the switched system is globally uniformly exponentially stable under arbitrary switching. In [12], piecewise quadratic Lyapunov functions are employed for two-mode switched linear time-invariant systems. Also based on piecewise quadratic Lyapunov functions, the synthesis problem is formulated as a bilinear matrix inequality problem and exponential stabilization for switched linear time-invariant systems is considered in [10]. For more details, see [18, 19] and the references therein.

Average dwell time (ADT) switchings, a class of typical constrained switching signals, mean that the number of switches in a finite interval is bounded and the average time between consecutive switching is not less than a constant [11]. However, the ADT switchings’ independence on system modes always lead to much conservativeness to stability criteria. In order to decrease the conservativeness, a new concept, a mode-dependent average dwell time (MDADT) technique, has been proposed in [22], where each mode in the underlying switched systems has its own ADT. This MDADT can guarantee the dwell time will not deteriorate the disturbance attenuation performance. Meanwhile, the increasing coefficient of the Lyapunov-like function at switching instants and the decay rate of the Lyapunov-like function during the running time of subsystems are set in a mode-dependent manner, which can reduce the conservativeness [22]. Therefore, how to extend the MDADT technique
to decrease the conservativeness of the existing stability results for the switched systems accordingly is a valuable problem to investigate. It motivates us to the study in this paper. Our main contribution in this paper can be stated as follows: By using a properly constructed decay-rate-dependent Lyapunov functional and the state variable transformation technique, sufficient conditions which explicitly characterize the switching signals specified by the MDADT are constructed to guarantee exponential stability of the switched delay systems.

The rest of the paper is structured as follows. In section 2, we present a class of discrete-time switched delay systems, necessary definitions on stability of switched systems are also provided. Section 3 is devoted to establish the results on exponential stability for the discrete-time switched delay system. In section 4, numerical examples are given to illustrate the obtained results. Finally, this paper is concluded in section 5.

2. Preliminaries and problem formulation. Consider the following discrete-time switched system with time-varying delay:

\[
L_{\sigma(t)}:\begin{cases}
x(t+1) = A_{\sigma(t)}x(t) + B_{\sigma(t)}x(t - d(t)) + C_{\sigma(t)}w(t) \\
x(l) = \psi(l), l = t_0 - d_M, t_0 - d_M + 1, t_0 - d_M + 2, \ldots, t_0 \\
z(t) = D_{\sigma(t)}x(t) + E_{\sigma(t)}w(t)
\end{cases}
\]

where \( x(t) \in \mathbb{R}^n \) is the system state, \( z(t) \in \mathbb{R}^m \) is the system output. \( \psi(l) \) is a vector-valued initial function, \( t_0 \) is the initial time point, and \( w(t) \) is the disturbance input which belongs to \( L_2[0, +\infty) \). \( d(t) \) is the time-varying delay and satisfies \( 0 < d_m < d(t) \leq d_M \), where \( d_m \) and \( d_M \) denote the lower and the upper bounds of the delays, respectively. \( \sigma(t) \) is the switching signal, which takes its values in the finite set \( S = \{1, \ldots, M\} \) (\( M \) is the number of the subsystems). \( A_{\sigma}(t) \), \( B_{\sigma}(t) \), \( C_{\sigma}(t) \), \( D_{\sigma}(t) \) and \( E_{\sigma}(t) \) are constant matrices. When \( t \in [t_i, t_{i+1}) \), \( i \in \mathbb{N} \), we call the \( \sigma(t_i) \)-th subsystem is active. When \( \sigma(t_i) = p = 1, 2, \ldots, m \), it means the \( p \)th model of \( [1] \).

Definition 2.1. [5] For any \( T_2 > T_1 \geq 0 \) and any switching signal \( \sigma(t) \), \( T_1 \leq t < T_2 \) let \( N_\sigma(T_1, T_2) \) denote the number of switchings of \( \sigma(t) \) over \( (T_1, T_2) \). If \( N_\sigma(T_1, T_2) \leq N_0 + (T_2 - T_1)/T_a \) holds for \( N_0 \geq 0 \) and \( T_a > 0 \), then \( T_a \) is called the average dwell time and \( N_0 \) is the chatter bound. As usual, we choose \( N_0 = 0 \) in this paper.

Remark 2.2. The concept of average dwell time was originally proposed for continuous-time switched systems in [5], and it has been modified to fit the discrete-time case in some papers such as [17, 20, 21, 16]. Definition 2.1 is borrowed from these existing results.

Definition 2.3. [22, 15] For the switching signal \( \sigma(t) \) and any \( T \geq t \geq 0 \), let \( N_{\sigma p}(T, t) \) be the switching number that the \( p \)th subsystem is activated over the interval \( [t, T] \) and \( T_p(T, t) \) denote the total running time of the \( p \)th subsystem over the interval \( [t, T]\), \( p \in S \). We say that \( \sigma(t) \) has a mode-dependent average dwell time (MDADT) \( \tau_{ap} \) if there exist positive numbers \( N_{0p} \) and \( \tau_{ap} \) such that

\[
N_{\sigma p}(T, t) \leq N_{0p} + \frac{T_p(T, t)}{\tau_{ap}}, \forall T \geq t \geq 0
\]

and we call \( N_{0p} \) the mode-dependent chatter bound. Here we choose \( N_{0p} = 0 \) in this paper.
Remark 2.4. As described in [22], Definition 2.3 constructs a class of different underlying systems to have its own ADT. The switching laws allow each mode in the underlying systems to have its own ADT. \( T_p(T_t) \) is not the total running time of the whole system, but the \( p \)th subsystem has its own entire running time which may disperse among the total running interval \([t, T]\). It is less restricted than the traditional ADT switching in practice.

Definition 2.5. The equilibrium point \( x^* = 0 \) of the switched delay system (1) is said to be exponentially stable under \( \sigma(t) \) if the solution \( x(t) \) of the switched delay system (1) satisfies
\[
x(t) \leq k \|x_{t_0}\| e^{-\mu(t-t_0)}, \forall t \geq t_0
\]
for \( k \geq 1 \) and \( \mu > 0 \), where \( \|\cdot\| \) denotes the Euclidean norm, and
\[
\|x_i\| = \sup_{-\tau \leq \theta \leq 0} \{x(t+\theta), \dot{x}(t+\theta)\}.
\]

Lemma 2.6. For any given matrices \( X, Y \in \mathbb{R}^{n \times n} \), the following inequality holds
\[
X^TY + Y^TX \leq \delta X^TX + \delta^{-1}Y^TY
\]
where \( \delta \) is any given positive real number.

Lemma 2.7. [13] Let \( A, D, E, F \) and \( P \) be real matrices of appropriate dimensions with \( P > 0 \) and \( F^TF \leq I \). Then, for any scalar \( \varepsilon > 0 \) satisfying \( P^{-1} - \varepsilon^{-1}DD^T > 0 \), we have
\[
(A + DFE)^TP(A + DFE) \leq A^T(P^{-1} - \varepsilon^{-1}DD^T)^{-1}A + \varepsilon E^TE
\]

Lemma 2.8. (Schur Complement) Let \( M, P, Q \) be given matrices such that \( Q > 0 \). Then,
\[
\begin{bmatrix}
P & M \\
* & -Q
\end{bmatrix} < 0 \iff P + MQ^{-1}M^T < 0
\]

Lemma 2.9. [2] Let \( \varphi(k) \in \mathbb{R}^n \) be a vector-valued function. If there exist any matrices \( R > 0, G_1, G_2 \) and a scalar \( d \geq 0 \), then the following inequality
\[
- \sum_{s=k-d}^{k-1} N^T(s)RN(s) \leq \eta^T(k) \begin{bmatrix}
G_1 + G_1^T & -G_1^T + G_2^T \\
* & -G_2 - G_2^T
\end{bmatrix} \eta(k) + \eta^T(k) \begin{bmatrix}
G_1^T \\
G_2^T
\end{bmatrix} dR^{-1} \begin{bmatrix}
G_1 \\
G_2
\end{bmatrix} \eta(k)
\]
holds, where \( N(s) = \varphi(s+1) - \varphi(s) \) and \( \eta(t) = \begin{bmatrix}
\varphi(t) \\
\varphi(t-d)
\end{bmatrix} \).

3. Stability analysis. We first consider the \( i \)-th subsystem of the switched delay system without disturbances:
\[
L_i : \begin{cases}
x(t+1) = Ax(t) + B_ix(t-d(t)) \\
x_{i_0}(l) = x(t_0 + l) = \psi(l), l = -d_M, -d_M + 1, -d_M + 2, \ldots, 0
\end{cases}
\]
Choose a Lyapunov functional candidate for \( L_i \) in the form of
\[
V_i(t) = V_{i_1}(t) + V_{i_2}(t) + V_{i_3}(t) + V_{i_4}(t)
\]
where
\[
V_{i_1}(t) = x^T(t)P_i x(t)
\]
and where 

$$V_{i2}(t) = \sum_{s=t-d(t)}^{t-1} \lambda_i^{2(s-t)} x^T(s) Q_i x(s)$$

$$V_{i3}(t) = \sum_{\theta=-d_M+2}^{-d_m+1} \sum_{s=t-1+\theta}^{t-1} \lambda_i^{2(s-t)} x^T(s) Q_i x(s)$$

$$V_{i4}(t) = \sum_{\theta=-d_M+1}^{0} \sum_{s=t-1+\theta}^{t-1} \lambda_i^{2(s-t)} y^T(s) R_i y(s)$$

where $P_i$, $Q_i$, $R_i$ are symmetric positive definite matrices, $\lambda_i > 1$ is a given constant, and

$$y(s) = \lambda_i x(s+1) - x(s).$$

Firstly, the decay estimation of the Lyapunov functional $V_i(t)$ along the trajectory of system (3) is derived in the Lemma 3.1.

**Lemma 3.1.** [16] Consider the switched delay system (3). For given positive integers $d_M$, $d_m$ and $\lambda_i$, suppose that there exist matrices $G_1$, $G_2$, $\Omega_1$, $\Omega_2$ and $\Omega_3$ such that

i) $\Omega_3 \leq 0$ (6)

ii) $\Omega_1 - \Omega_2 \Omega_3^{-1} \Omega_2^T \leq 0$ (7)

where

$$\Omega_1 = A_i^T P_i A_i - P_i + \lambda_i^{-2} Q_i + (d_M - d_m) \lambda_i^{-2} Q_i +$$

$$\lambda_i^{-2} d_M \left[ \lambda_i^2 A_i^T R_i A_i - \lambda_i R_i A_i - \lambda_i A_i^T R_i + R_i \right] + \lambda_i^{-2} (G_1 + G_1^T + d_M G_1^T R_i^{-1} G_1)$$

$$\Omega_2 = A_i^T P_i B_i + d_M (A_i^T R_i B_i - \lambda_i^{-1} R_i B_i) + \lambda_i^{-2} (-G_1^T + G_2 + d_M G_1^T R_i^{-1} G_2)$$

$$\Omega_3 = B_i^T P_i B_i - \lambda_i^{-2(1+d_m)} Q_i + d_M B_i^T R_i B_i + \lambda_i^{-2} (-G_2 - G_2^T + d_M G_1^T R_i^{-1} G_2)$$

with $P_i$, $Q_i$ and $R_i$ being symmetric positive definite matrices, then the Lyapunov functional $V_i(t)$ along the trajectory of the switched delay system (3) yields that

$$V_i(t) \leq \lambda_i^{-2(t-t_0)} V_i(t_0)$$

(8)

**Remark 3.2.** In this section, we have considered the $i$-th subsystem of (1) without uncertainties. The decay estimation [5] is derived by virtue of the properly constructed decay-rate-dependent Lyapunov functional [5] and LMI method. Note that if the disturbances in subsystem are considered, the decay estimation can still be carried out by referring to the standard techniques used in Lemma 3.1.

In what follows, the first goal is to use the MDADT method incorporated with the decay estimation [5] of the Lyapunov function $V_i(t)$ to derive exponential stability condition for the discrete-time switched time-delay system as follows:

$$\tilde{L_i} : \begin{cases} x(t + 1) = A_{\rho(t)} x(t) + B_{\rho(t)} x(t - d(t)) \\ x_i(t_0) = x(t_0 + l) = \psi(l), l = -d_M, -d_M + 1, -d_M + 2, \ldots, 0 \end{cases}$$

(9)

**Theorem 3.3.** For given constants $\beta > 0$, $\lambda_p > 1$, $\mu_p \geq 1$ and any delay $d_p(t)$ satisfying $0 < d_m < d_p(t) \leq d_M$, $p = 1, \ldots, m$, suppose that there exist appropriate matrices $G_1$, $G_2$, symmetric positive definite matrices $P_p > 0$, $Q_p > 0$, $R_p > 0$ satisfying

i) $\Omega_{3p} \leq 0$ (10)
where
\[ \Omega_1 = A^T P_P A_P - P_P - \lambda_p^{-2} Q_p + (d_M - d_m) \lambda_p^{-2} Q_p + \lambda_p^{-2} d_M \left[ \lambda_p^T R_p A_P - \lambda_p R_P A_P - \lambda_p A_P^T R_P + R_P \right] \]
\[ \Omega_2 = A^T P_P B_P + d_M (A^T R_P B_P - \lambda_p^{-1} R_P B_P) + \lambda_p^{-2} (-G_1^T + G_2 + d_M G_2^T R_P^{-1} G_2) \]
\[ \Omega_3 = B^T P_P B_P - \lambda_p^{-2(1+d_M)} Q_p + d_M B_P R_P B_P + \lambda_p^{-2} (-G_2 - G_2^T + d_M G_2^T R_P^{-1} G_2) \]
\[ \beta = \sum_{p=1}^{m} (2 \ln \lambda_p - \ln \mu_p / \tau_{ap}) \]

and
\[ P_p \leq \mu_p P_P, Q_p \leq \mu_p Q_q, R_p \leq \mu_p R_q, \forall p, q \in M \]

Then, the discrete-time switched delay system is exponentially stable with MDADT
\[ \tau_{ap} > \tau^* = \frac{\ln \mu_p}{2 \ln \lambda_p} \]

Proof. Choose the Lyapunov functional for system as follows
\[ V_{\sigma(t)}(t) = x^T(t) P_{\sigma(t)} x(t) + \sum_{s=t-d(t)}^{t-1} \lambda^{2(s-t)}_{\sigma(t)} x^T(s) Q_{\sigma(t)} x(s) + \sum_{\theta = -d_M + 1}^{-d_M + 2} \sum_{s = t-1+\theta}^{t-1} \lambda^{2(s-t)}_{\sigma(t)} x^T(s) Q_{\sigma(t)} x(s) + \sum_{\theta = -d_M + 1}^{-d_M + 2} \sum_{s = t-1+\theta}^{t-1} \lambda^{2(s-t)}_{\sigma(t)} y^T(s) R_{\sigma(t)} y(s) \]

where \( P_{\sigma(t)} > 0, Q_{\sigma(t)} > 0, R_{\sigma(t)} > 0 \) and \( y(s) = \lambda_{\sigma(t)} x(s + 1) - x(s) \); \( \lambda_{\sigma(t)} > 1 \) is a given constant. Based on Lemma 3.1, if (10) and (11) are satisfied, we have
\[ V_p(t) \leq \lambda_p^{-2(t-t_0)} V_p(t_0) \]
we can obtain by use of (12) and (14) that
\[ V_p(t) \leq \mu_p V_q(t) \]

Then, it follows from (14), (15) that
\[ V_{\sigma(t)}(t) \leq \lambda^{2(t-t_i)}_{\sigma(t_i)} V_{\sigma(t_i)}(t_i) \leq \mu_{\sigma(t)} \lambda^{2(t-t_i)}_{\sigma(t_i)} V_{\sigma(t_{i-1})}(t_i) \]
\[ \leq \mu_{\sigma(t)} \lambda^{2(t-t_i)}_{\sigma(t_i)} \lambda^{2(t_i-t_{i-1})}_{\sigma(t_{i-1})} V_{\sigma(t_{i-1})}(t_{i-1}) \]
\[ \leq \mu_{\sigma(t)} \mu_{\sigma(t_{i-1})} \lambda^{2(t-t_i)}_{\sigma(t_i)} \lambda^{2(t_i-t_{i-1})}_{\sigma(t_{i-1})} \lambda^{2(t_{i-1}-t_{i-2})}_{\sigma(t_{i-2})} V_{\sigma(t_{i-2})}(t_{i-2}) \]
\[ \leq \prod_{s=1}^{i} \mu_{\sigma(t_s)} \lambda^{2(t-t_s)}_{\sigma(t_i)} \lambda^{2(t_s-t_{s-1})}_{\sigma(t_i)} \lambda^{2(t_{s-1}-t_{s-2})}_{\sigma(t_{i-1})} \lambda^{2(t_{s-2}-t_{s-3})}_{\sigma(t_{i-2})} \cdots \lambda^{2(t_2-t_1)}_{\sigma(t_{i-1})} \lambda^{2(t_1-t_0)}_{\sigma(t_{i-2})} V_{\sigma(t_0)}(t_0) \]
\[ = \prod_{s=1}^{i} \mu_{\sigma(t_s)} \exp(-2 \sum_{s=1}^{i} \ln \lambda_{\sigma(t_{s-1})}(t_s - t_{s-1})) V_{\sigma(t_0)}(t_0) \]

which, combined with Definition 2.3 yields
\[ V_{\sigma(t)}(t) \leq \left( \prod_{p=1}^{m} \mu_p N_{\sigma_p(t,t_0)} \right) \exp(-2 \sum_{p=1}^{m} \ln \lambda_P(t, t_0)) V_{\sigma(t_0)}(t_0) \]
\[
\leq \exp \left\{ - \sum_{p=1}^{m} \left( 2 \ln \lambda_p - \frac{\ln \mu_p}{\tau_{ap}} \right) T_p(t,t_0) \right\} V_{\sigma(t_0)}(t_0)
\]

Due to (13), \(\tau_{ap} > \tau_{ap}^* = \frac{\ln \mu_p}{2 \ln \lambda_p}\). So,
\[
\sum_{p=1}^{m} \left( 2 \ln \lambda_p - \frac{\ln \mu_p}{\tau_{ap}} \right) > 0.
\]

Letting \(\beta = \sum_{p=1}^{m} \left( 2 \ln \lambda_p - \frac{\ln \mu_p}{\tau_{ap}} \right)\), we have
\[
ax(t)^2 \leq V_{\sigma(t)}(t) \leq e^{-\beta T_p(t,t_0)} b \sqrt{\phi_L^2}
\]
which satisfies
\[
x(t)^2 \leq \frac{b}{a} e^{-\beta T_p(t,t_0)} \phi_L^2
\]
where
\[
a = \min_{i \in M} \lambda_{\min}(P_i)
\]
\[
b = \max_{i \in M} \lambda_{\max}(Q_i) + 2(d_M - d_m)(d_M + 1) \max_{i \in M} \lambda_{\max}(R_i) +
2d_M(d_M + 1)(1 + \lambda)^2 \max_{i \in M} \lambda_{\max}(R_i)
\]
Therefore, by Definition 2.5, we can conclude that the switched delay system (9) is exponentially stable as long as the MDADT satisfies (13).

Remark 3.4. Theorem 3.3 extends the results obtained in [21], the delays being considered in this section is time-varying, which makes it more preferable and challenging in practice. In addition, this MDADT can guarantee the dwell will not deteriorate the disturbance attenuation performance. Meanwhile, the increase coefficient and the decay rate of the Lyapunov-like function are mode-dependent manner, making the conclusion less conservative than that in [21].

3.1. Illustrative examples. In this section, a numerical example will be given to demonstrate the validity of the results obtained above. Consider a switched delay system (9) consisting of four subsystems described by
\[
A_1 = A_2 = \begin{bmatrix} 0 & 0.3 \\ -0.2 & 0.1 \end{bmatrix}, \quad A_3 = A_4 = \begin{bmatrix} 0 & 0.3 \\ -0.2 & -0.1 \end{bmatrix}
\]
\[
B_1 = B_3 = \begin{bmatrix} 0 & 0.1 \\ 0 & 0.2 \end{bmatrix}, \quad B_2 = B_4 = \begin{bmatrix} 0 & 0.1 \\ 0 & 0 \end{bmatrix}
\]

Our purpose here is to design the admissible switching signals with MDADT such that the underlying system is exponentially stable. In order to illustrate the advantages of the proposed MDADT switching method, we will make a comparison between the design results of both switching signals for the systems with ADT switching [21] and MDADT switching. By different approaches and setting the relevant parameters appropriately, the computation results for the system with two different switching schemes are listed in Table 1. It can be seen that the lower bound of MDADT is obviously less than that in [21]. Note that one special case of MDADT switching is \(\tau_{a1}^* = \tau_{a2}^* = \tau_{a3}^* = \tau_{a4}^* = 0.1175\) by setting \(\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 1.5\), which is actually the ADT switching.

To further illustrate the advantages of MDADT switching, we construct two switching sequences with the ADT property and the MDADT property, respectively. The simulation performances of the resulting switched system, under the scheme of
Theorem 1 in [19] in practice to improve the system performances.

Therefore, the MDADT technique is more applicable and flexible enough. Then, the designed MDADT switching can achieve better state response and transient running time needed on those subsystems whose indices and transient behaviour are not good enough. The MDADT switching can remove the constraint of ADT switching, so there will be shorter running time on those subsystems.

It can be seen from the stairs that the state response of switched system under MDADT switching with \( \tau_a = 2 \) is more smooth and time-saving to achieve stability. If we use the ADT switching, based on the definition of ADT, every subsystem are required to share the same switching signals \( \tau_a^* = 0.1175 \), \( \mu = 1.1 \), and \( \lambda \leq 1.5 \), and the MDADT switching provides \( \tau_a^* = 0.0312, \tau_a^* = 0.0409, \tau_a^* = 0.0642, \tau_a^* = 0.1175 \), \( \mu_1 = \mu_2 = \mu_3 = \mu_4 = 1.1 \), \( \lambda_1 \leq 4.6, \lambda_2 \leq 3.2, \lambda_3 \leq 2.1, \lambda_4 \leq 1.5 \). The initial condition is \( \psi(l) = [1; 0.5] \) for all \( l = -2, -1, 0 \), and \( d(t) = \begin{cases} 2, & \text{if } t \text{ is odd}, \\ 1, & \text{if } t \text{ is even}. \end{cases} \)

**Table 1. Computation Results For The Switched Delay System Under Two Different Switching Schemes**

| Switching Schemes | ADT Switching | MDADT Switching |
|-------------------|---------------|-----------------|
| Criteria for controller design | Theorem 1 in [19] | Theorem 1 in this paper |
| Switching signals | \( \tau_a^* = 0.1175 \), \( \mu = 1.1 \), \( \lambda \leq 1.5 \) | \( \tau_a^* = 0.0312, \tau_a^* = 0.0409, \) \( \tau_a^* = 0.0642, \tau_a^* = 0.1175 \), \( \mu_1 = \mu_2 = \mu_3 = \mu_4 = 1.1 \), \( \lambda_1 \leq 4.6, \lambda_2 \leq 3.2, \lambda_3 \leq 2.1, \lambda_4 \leq 1.5 \) |

**Figure 1. State trajectories of switched system under ADT switching with \( \tau_a = 2 \)**

**Figure 2. State trajectories of switched system under MDADT switching with \( \tau_a^1 = 1, \tau_a^2 = 1, \tau_a^3 = 1, \tau_a^4 = 2 \)**

It can be seen that the state response of switched system under MDADT switching is more smooth and time-saving to achieve stability. If we use the ADT switching, based on the definition of ADT, every subsystem are required to share the some
Because the MDADT switching can remove the constraint of ADT switching, so there will be shorter running time needed on those subsystems whose indices and transient behaviour are not good enough. Then, the designed MDADT switching can achieve better state response and transient behaviour. Thus, the MDADT technique is more applicable and flexible in practice to improve the system performances for switched delay systems.

4. Conclusions. In this paper, the problem of stability analysis for discrete-time switched delay systems with a class of MDADT switching is investigated. By shortening the running time on those subsystems whose indices and transient behaviour are not good enough, it is demonstrated that the proposed MDADT switching is more flexible and efficient than the ADT switching. Combined with the multiple Lyapunov function method, sufficient conditions are derived to ensure exponential stability of discrete-time switched delay system in terms of a set of solvable linear matrix inequalities. Finally, a numerical example is given to demonstrate the practicability and less conservativeness of the developed results.

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