Population transfer under local dephasing

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Abstract
Stimulated Raman adiabatic passage is a well-known technique for quantum population transfer due to its robustness against various sources of noises. Here we consider quantum population transfer from one spin to another via an intermediate spin which is under dephasing noise. We obtain an analytic expression for the transfer efficiency under a specific driving protocol, showing that dephasing could reduce the transfer efficiency, but the effect of dephasing could also be suppressed with a stronger laser coupling or a longer laser duration. We also consider another commonly used driving protocol, which shows that this analytic picture is still qualitatively correct.

Keywords: Stimulated Raman adiabatic passage; Population transfer; Coherent quantum control

1 Introduction
Complete population transfer from an initial state to a final state has profound importance in both quantum and classical physics. On the quantum part, it has long been an active research area in quantum optics [1–3], and it is a fundamental technique for the physical realization of quantum information processing [4–9]. On the classical part, it has been used as a technique to achieve power or intensity inversion in classical systems [10], such as waveguide couplers [11], wireless energy transfer [12], and graphene systems [13, 14].

Stimulated Raman adiabatic passage (STIRAP) has been one of the most important techniques for complete population transfer. In its standard implementation, an initial state $|1\rangle$ and a final state $|3\rangle$ are coupled to a common intermediate state $|2\rangle$, by a pump laser and a Stokes laser respectively [15]. Complete population transfer between states $|1\rangle$ and $|3\rangle$ can then be achieved if the laser pulses are applied adiabatically and in a counter-intuitive order (the Stokes laser applies first), with the adiabatic condition

$$\dot{\theta}(t) \ll \Omega(t).$$

Here $\Omega(t) = \sqrt{\Omega_P^2(t) + \Omega_S^2(t)}$ with $\Omega_P(t)$ and $\Omega_S(t)$ the Rabi frequencies of the pump laser and Stokes laser respectively, and $\tan(\theta(t)) = \Omega_P(t)/\Omega_S(t)$. STIRAP has important advantages such as being robust against the variations of the experimental conditions [16], and against the decaying of the intermediate state [17–19]. STIRAP via multiple intermediate states has also been considered [20], as well as generalizations to intermediate state...
as a continuum [19, 21–23] and a lossy continuum [24], where it is shown that significant partial population transfer can still be achieved. Recently, STIRAP via a thermal state or a thermal continuum has been studied, showing that the efficiency of population transfer will be reduced significantly in this case [25].

In this work, we focus on the effect of dephasing on the efficiency of population transfer via STIRAP. Dephasing is a common type of noise in quantum systems which induces decay of the off-diagonal terms in the density operator \( \hat{\rho} \). The standard three-level STIRAP with dephasing for all energy levels has already been considered in Ref. [26], where it is shown that the population of the final state \( \rho_{33}(t) \) approximately satisfies

\[
\rho_{33} = \frac{1}{3} + \frac{2}{3} e^{-\gamma_{13} \eta},
\]

with \( \eta = \frac{2}{3} \int_{-\infty}^{\infty} dt \sin^2(2\theta(t)) \). Here \( \rho_{ij} \) denotes the element of \( \langle i | \hat{\rho} | j \rangle \) for \( i, j \in \{1, 2, 3\} \), and \( \gamma_{ij} \) denotes the decay rate of the element \( \rho_{ij} \) for \( i \neq j \). We note that in deriving Eq. (2) two assumptions have been made: 1) weak dephasing (\( \gamma_{ij} \ll \Omega_1(t) \)) and 2) adiabatic evolution such that the terms proportional to \( \dot{\theta}(t) \) can be neglected, and one is left with an expression which is independent of \( \gamma_{12} \) or \( \gamma_{23} \) or the laser strength \( \Omega(t) \).

Here we consider a slightly different physical setup. Concretely, we study population transfer from a spin \( q_1 \) to another spin \( q_3 \), via an intermediate spin \( q_2 \). We assume that \( q_2 \) is under dephasing noise, while \( q_1 \) and \( q_3 \) are noise-free. The relevant states, namely \( \{|100\}, |010\}, |001\rangle \), form a three-level system which has a one-to-one correspondence with the standard three-level STIRAP. A possible physical setup of our model is the information transfer between two well-protected cavities via a dephasing channel. We derive an analytic expression for transfer efficiency under a specific driving protocol. Based on this expression we then obtain an additional adiabatic condition on top of Eq. (1), which is related to the dephasing strength. We show that dephasing reduces the transfer efficiency. However, the effect of dephasing could be suppressed by a stronger laser coupling or a longer laser duration. The paper is organized as follows. We introduce our model in Sect. 2. Then in Sect. 3, we derive the analytic expression for the transfer efficiency as well as the additional adiabatic condition under which complete population transfer can still be achieved. We show that the analytic expression agrees well with the predictions from the exact quantum master equation in a wide parameter range with numerical simulations. We conclude in Sect. 4.

### 2 Model

Our model consists of three spins in which the intermediate spin acts as a bus for population transfer and is under dephasing noise, as shown in Fig. 1. The quantum Lindblad master equation describes the equation of motion [27, 28], which is

\[
\frac{d\hat{\rho}(t)}{dt} = -i[\hat{H}(t), \hat{\rho}] + \mathcal{D}(\hat{\rho}),
\]

where the Hamiltonian \( \hat{H}(t) \) takes the form

\[
\hat{H}(t) = \Delta \sum_{j=1}^{3} \hat{\sigma}_j^z + \Omega_{\sigma}(t)(\hat{\sigma}_1^+ \hat{\sigma}_2^- + \hat{\sigma}_2^+ \hat{\sigma}_1^-) + \Omega_\delta(t)(\hat{\sigma}_2^+ \hat{\sigma}_3^- + \hat{\sigma}_3^+ \hat{\sigma}_2^-),
\]
with $\Delta$ the energy difference for all spins ($\Delta = 0$ will be assumed throughout this work as a standard setting for STIRAP). The pump laser $\Omega_p(t)$ couples the 1-th spin to the 2-th spin, while the Stokes laser couples the 3-th spin to the 2-th spin. The dissipator $D$ takes the form

$$D(\hat{\rho}) = \gamma \left( \hat{\sigma}_z \hat{\rho} \hat{\sigma}_z - \hat{\rho} \right),$$

with $\gamma$ the dephasing strength. The Hamiltonian as well as the dissipation conserves the total number of excitations. Since the initial state in the context of STIRAP is chosen as $|100\rangle$, we are restricted to the single-excitation subspace spanned by three states $\{|100\rangle, |010\rangle, |001\rangle\}$ only. We can see that our model remains the same if the intermediate spin is replaced by a dephasing bosonic mode and rotating wave approximation is applied to the spin-boson couplings, where the intermediate bosonic mode may be physically implemented using a cavity. It is also possible to generalize the intermediate state in our model as a chain of spins as in straddle STIRAP \cite{29,30}, or a bosonic continuum \cite{24,25} under dephasing.

Now using the mapping

$$|100\rangle \leftrightarrow |1\rangle, \quad |010\rangle \leftrightarrow |2\rangle, \quad |001\rangle \leftrightarrow |3\rangle,$$

the Hamiltonian in Eq. (4) can be rewritten in the basis of $\{|1\rangle, |2\rangle, |3\rangle\}$ as

$$\hat{H}(t) = \begin{bmatrix} 0 & \Omega_p(t) & 0 \\ \Omega_p(t) & 0 & \Omega_s(t) \\ 0 & \Omega_s(t) & 0 \end{bmatrix},$$

and the dissipator $D$ can be written as

$$D(\hat{\rho}) = \gamma \left( \hat{F} \hat{\rho} \hat{F}^\dagger - \hat{\rho} \right) = -2\gamma \begin{bmatrix} 0 & \rho_{12} & 0 \\ \rho_{21} & 0 & \rho_{23} \\ 0 & \rho_{32} & 0 \end{bmatrix},$$

with

$$\begin{align*} F = \begin{bmatrix} \hat{a} & \hat{b} \\ \hat{b}^\dagger & \hat{a}^\dagger \end{bmatrix}, \\
\hat{F} = \begin{bmatrix} \hat{a}^\dagger & \hat{b}^\dagger \\ \hat{b} & \hat{a} \end{bmatrix}. \end{align*}$$
with
\[
\hat{F} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}
\]
being the operator \( \hat{\sigma}_z \) in the new basis. Compared to the model studied in Ref. [26], we can see from Eq. (8) that we have \( \gamma_{13} = 0 \). In this case Eq. (2) predicts complete population transfer irrespective of the value of \( \gamma \). However as will be clear later, \( \gamma \) would significantly suppress the transfer efficiency in this case if it is comparable to other parameters such as \( \Omega(t) \). Therefore in the following we perform a more refined calculation for the population transfer efficiency which would allow us to see more clearly the role of dephasing.

3 Results and discussions
The Hamiltonian in Eq. (7) can be diagonalized with three instantaneous eigenstates
\[
|+\rangle = \frac{\sqrt{2}}{2}(\sin(\theta)|1\rangle + |2\rangle + \cos(\theta)|3\rangle);
\]
\[
|d\rangle = \cos(\theta)|1\rangle - \sin(\theta)|3\rangle;
\]
\[
|\rangle = \frac{\sqrt{2}}{2}(\sin(\theta)|1\rangle - |2\rangle + \cos(\theta)|3\rangle),
\]
corresponding to eigenvalues \( \Omega(t) \), 0, and \( -\Omega(t) \) respectively. In the ideal three-level STIRAP, one adiabatically changes \( \theta(t) \) from 0 to \( \pi/2 \) by tuning the ratio \( \Omega_\rho(t)/\Omega_\varphi(t) \) such that once the initial state is chosen to be \( |1\rangle \), it will always remain in \( |d\rangle \) (therefore it will eventually be in \( |3\rangle \)). The unitary matrix \( W \) to diagonalize \( \hat{H}(t) \) can be written as
\[
W(\theta) = \begin{bmatrix} \frac{\sqrt{2}}{2} \sin(\theta) & \cos(\theta) & \frac{\sqrt{2}}{2} \sin(\theta) \\ \frac{\sqrt{2}}{2} \sin(\theta) & 0 & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \cos(\theta) & -\sin(\theta) & \frac{\sqrt{2}}{2} \cos(\theta) \end{bmatrix}.
\]
To gain better insight into the time evolution, we go to the adiabatic picture, in which the Eq. (3) becomes [26]
\[
\frac{d\hat{\rho}_a}{dt} = -i[\hat{H}_a, \hat{\rho}_a] - [M, \hat{\rho}_a] + D_a(\hat{\rho}_a).
\]
Here \( \hat{\rho}_a, \hat{H}_a, D_a \) are the corresponding operators to \( \hat{\rho}, \hat{H}, D \) respectively in the adiabatic basis \( \{|+, |d\rangle, |-\rangle\} \), which can be written as
\[
\hat{\rho}_a = W^\dagger \hat{\rho} W = \begin{bmatrix} \rho_{++} & \rho_{+d} & \rho_{+-} \\ \rho_{d+} & \rho_{dd} & \rho_{d-} \\ \rho_{-+} & \rho_{-d} & \rho_{--} \end{bmatrix},
\]
\[
\hat{H}_a(t) = W^\dagger \hat{H} W = \Omega(t) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix},
\]
\[
D_a(\hat{\rho}_a) = \gamma (\hat{F}_a \hat{\rho}_a \hat{F}_a^\dagger - \hat{\rho}_a),
\]
with
\[
\hat{F}_a = W^\dagger \hat{F} W = \begin{bmatrix}
0 & 0 & -1 \\
0 & -1 & 0 \\
-1 & 0 & 0
\end{bmatrix}.
\]
(18)

Here we have used \( \rho_{uv} \) with \( u, v \in \{+, d, -\} \) to denote the element \( \langle u|\hat{\rho}_a|v \rangle \). The gauge matrix \( \mathbf{M} \) satisfies
\[
\mathbf{M} = W^\dagger \hat{W} = \frac{\sqrt{2}}{2} \hat{\theta} \begin{bmatrix}
0 & -1 & 0 \\
1 & 0 & 1 \\
0 & -1 & 0
\end{bmatrix},
\]
(19)

which results from the time dependence of the adiabatic basis. Now substituting Eqs. (16), (17), (19) into Eq. (14), we get the following set of equations

\[
\begin{align*}
\dot{\rho}_{++} &= \frac{\sqrt{2}}{2} \hat{\theta} (\rho_{dd} + \rho_{+d} + \rho_{-d}) + \gamma (\rho_{+d} - \rho_{++}); \\
\dot{\rho}_{--} &= \frac{\sqrt{2}}{2} \hat{\theta} (\rho_{-d} + \rho_{-+} + \rho_{-d}) + \gamma (\rho_{++} - \rho_{--}); \\
\dot{\rho}_{dd} &= -\frac{\sqrt{2}}{2} \hat{\theta} (\rho_{+d} + \rho_{+d} + \rho_{-d} + \rho_{-d}); \\
\dot{\rho}_{+d} &= -i \Omega \rho_{+d} - \frac{\sqrt{2}}{2} \hat{\theta} (-\rho_{dd} + \rho_{+d} + \rho_{+d}) + \gamma (\rho_{++} - \rho_{+d}); \\
\dot{\rho}_{d+} &= -i \Omega \rho_{d+} - \frac{\sqrt{2}}{2} \hat{\theta} (\rho_{dd} + \rho_{++} + \rho_{++}) + \gamma (\rho_{d+} - \rho_{d+}); \\
\dot{\rho}_{++} &= -2i \Omega \rho_{++} + \frac{\sqrt{2}}{2} \hat{\theta} (\rho_{+d} + \rho_{+d}) + \gamma (\rho_{++} - \rho_{++}).
\end{align*}
\]
(20a)–(20f)

We note that in Ref. [26], the second term in Eq. (14) is neglected since it depends on \( \dot{\theta} \) which is assumed to be small. However in our case if this term is neglected, we will arrive at a solution where the population is trapped in \( |d\rangle \), since it is a dark state of the dissipator \( \mathcal{D}_a \).

The set of Eqs. (20a)–(20f) are difficult to solve analytically in general. However, they can be significantly simplified with several reasonable assumptions. First, in the context of STIRAP, the adiabatic condition in Eq. (1) requires \( \dot{\theta} \) to be smaller compared to other relevant parameters. Second, we assume that in the adiabatic basis the off-diagonal terms of \( \hat{\rho}_a \) are small, that is \( \rho_{uv} \ll 1 \) if \( u \neq v \). Now we subtract Eq. (20b) from Eq. (20a) and get an equation for \( g = \rho_{++} - \rho_{--} \) as
\[
\dot{g} = \frac{\sqrt{2}}{2} \hat{\theta} (\rho_{+d} + \rho_{+d} - \rho_{-d} - \rho_{-d}) - 2\gamma g.
\]
(21)

The first term on the right-hand side of Eq. (21) contains \( \dot{\theta} \) and \( \rho_{+d} + \rho_{+d} - \rho_{-d} - \rho_{-d} \) which are both small numbers. Thus we neglect this term and get \( \dot{g} = -2\gamma g \). Since \( g(t) \) is initially 0, and get \( g(t) = 0 \) for all \( t \), namely
\[
\rho_{++}(t) = \rho_{--}(t).
\]
(22)
Similarly, subtracting Eq. (20e) from Eq. (20d), we get an equation for \( h = \rho_{+d} - \rho_{d-} \) as

\[
\dot{h} = -i\Omega h - \gamma (h + h^\ast),
\]  

(23)

where we have used \( \rho_{++,}(t) = \rho_{--}(t) \). Now since \( h(t) \) is initially 0, from Eq. (23) we have \( h(t) = 0 \) for all \( t \), namely

\[
\rho_{+d}(t) = \rho_{d-}(t).
\]

(24)

Finally, the second term on the right-hand side of Eq. (20f) can be neglected for the same reason, and we get

\[
\dot{\rho}_{+-} = -2i\Omega \rho_{+-} + \gamma (\rho_{--} - \rho_{+-}).
\]

(25)

Since \( \rho_{+-} \) is initially 0, from Eq. (25) we get

\[
\rho_{+-}(t) = 0
\]

(26)

for all \( t \). Substituting Eqs. (22), (24), (26) back into Eqs. (20a)–(20f), and assuming \( \rho_{+d} = a + ib \) with \( a(t) \) and \( b(t) \) real functions, we get the following closed set of equations for \( \rho_{dd}, a, b \)

\[
\begin{align*}
\dot{\rho}_{dd} &= -2\sqrt{2} \dot{\theta} a; \\
\dot{a} &= \Omega b - \frac{\sqrt{2}}{4} \dot{\theta} (1 - 3\rho_{dd}); \\
\dot{b} &= -\Omega a - 2\gamma b,
\end{align*}
\]

(27a)

(27b)

(27c)

where we have used \( \text{tr}(\dot{\rho}_a) = 1 \). Equations (27a)–(27c) are still difficult to solve analytically since their coefficients are time-dependent in the general case.

In the following, we consider a specific driving protocol as follows:

\[
\begin{align*}
\Omega_P(t) &= \Omega_0 \sin \left( \frac{\pi t}{2T_0} \right); \\
\Omega_S(t) &= \Omega_0 \cos \left( \frac{\pi t}{2T_0} \right),
\end{align*}
\]

(28a)

(28b)

where \( \Omega_0 \) denotes the strength of the laser coupling and \( T_0 \) is the total duration of it. Such a choice of driving protocol is more of theoretical convenience than of experimental relevance. Later we will also consider the more commonly used Gaussian driving protocol.

We can see that \( \Omega_P(0)/\Omega_S(0) = 0 \) and \( \Omega_P(T_0)/\Omega_S(T_0) = \infty \). The advantage of the protocol in Eq. (28a)–(28b) is that we have \( \Omega(t) = \Omega_0, \dot{\theta}(t) = \frac{\pi t}{2T_0} \) and \( \dot{\theta} = \frac{\pi}{2T_0} \). We further assume that in Eqs. (27a)–(27c) \( a(t) \) and \( b(t) \) are slowly varying variables compared to \( \rho_{dd}(t) \). (This approximation is partially due to the fact that in the adiabatic limit \( \rho_{dd}(t) \approx 1 \) while the off diagonal terms \( a(t) \approx b(t) \approx 0 \).) As a result we can set \( \dot{a} = \dot{b} = 0 \) and then Eqs. (27a) can be solved as

\[
\rho_{dd}(t) = \frac{1}{3} + \frac{2}{3} e^{-3\gamma t},
\]

(29)
with
\[ \chi = \frac{2\gamma \dot{\theta}^2}{\Omega^2}. \] (30)

To check the validity of Eq. (29), we compared it with the exact numerical solutions from Eq. (14). In Fig. 2(a) we show an instance of the driving protocol in Eq. (28a)–(28b). In Fig. 2(b), (c), (d) we show \( \rho_{dd} \) predicted by our analytical solution in Eq. (29) and the exact solution by Eq. (14) as functions of time \( t \) versus different values of \( \gamma, \Omega_0, T_0 \) respectively.

We can see that our analytic prediction agrees very well with the exact solution in the wide parameter range we have considered.

It would be more clear to directly look at the dynamics in the original basis \( \{|1\rangle, |2\rangle, |3\rangle\} \). Here we show the exact time evolution of the occupations \( \rho_{11}, \rho_{22} \) and \( \rho_{33} \) in Fig. 3 by directly solving the Lindblad master equation (Eq. (3)) in the original picture. From Fig. 3(a), (c), (e), we can see that for smaller \( \gamma \), and larger \( \Omega_0 \) or \( T_0 \), the intermediate state \( |2\rangle \) is indeed seldomly occupied during the time evolution, and from Fig. 3(b), (d), (f) we can see that most of the population indeed transfers from state \( |1\rangle \) to state \( |3\rangle \) when these conditions are satisfied.

To this end we discuss the implications of our analytic solution in Eq. (29). From Eq. (11) we have \( \rho_{33}(T_0) = \rho_{dd}(T_0) \). Therefore the final occupation of \( \rho_{dd}(T_0) \) represents the population transfer efficiency. Then from Eq. (29) we have
\[ \rho_{33}(T_0) = \frac{1}{3} + \frac{2}{3} e^{-\frac{\gamma \dot{\theta}}{\Omega^2}}, \] (31)
where we have used $\dot{\theta}T_0 = \pi/2$. We can see that $\rho_{33}(T_0)$ decreases exponentially with $\gamma$. However, the effect of dephasing can be made arbitrarily small if we increase laser coupling strength $\Omega$ or increase the laser duration (such that $\dot{\theta}$ will be smaller). Interestingly, based on Eq. (31) we can define an additional adiabatic condition on top of Eq. (1) which takes the dephasing strength into account. The additional adiabatic condition is simply $-3\chi T_0 \ll 1$, which is

$$\dot{\theta} \ll \frac{\Omega^2}{3\pi \gamma}. \quad (32)$$

Complete population transfer can still be achieved as long as Eqs. (1), (32) are both satisfied. Now for comparison, when using the same driving protocol as in Eq. (28a)–(28b), we will obtain $\eta = 3T_0/8$ in Eq. (2), that is, the population transfer efficiency is independent of $\Omega$, but decreases exponentially both with $\gamma_{13}$ and $T_0$. Therefore complete population transfer can never be achieved as long as $\gamma_{13} \neq 0$, since we can neither tune $\dot{\theta}$ to be very

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**Figure 3** The time evolution of occupations on states $|1\rangle$, $|2\rangle$ and $|3\rangle$ under the driving protocol in Eq. (28a)–(28b). (a), (c), (e) The occupation on state $|2\rangle$, namely $\rho_{22}$, as a function of time $t$, where the gray solid lines from darker to lighter correspond to $\gamma = 0, 2, 4$ in (a), to $\Omega_0 = 2, 4, 6$ in (c), and to $T_0 = 40, 120, 200$ in (e). (b), (d), (f) The green solid lines corresponding to the left axis show $\rho_{11}$ as a function of $t$ with $\gamma = 0, 2, 4$ in (b), $\Omega_0 = 2, 4, 6$ in (d) and $T_0 = 40, 120, 200$ in (f), while blue solid lines from darker to lighter corresponding to the right axis show the evolution of $\rho_{33}$ under the same conditions. The other parameters used are $\gamma = 2$, $\Omega_0 = 2$, $T_0 = 40$ if not particularly specified.
small ($T_0$ will be very large and increase the damping) or very large (which breaks the adiabatic condition in Eq. (1)). The reason for the qualitatively different predictions between our result and that of Ref. [26] can be seen from Eq. (14). In our case, the absence of dephasing on the first and third spin results in the missing of the term $\gamma_{13}$ as defined in Ref. [26]. As a result, the state $|d\rangle$ forms a dark space that is decoupled from the states $|\pm\rangle$ in the adiabatic limit (which means that $\dot{\theta} \to 0$ and the second term on the right hand side of Eq. (14) can be neglected) and perfect population transfer can still be achieved.

Our analytic solution in Eq. (29) does not hold for general laser driving protocols. To show the validity of the physical picture we obtained based on the specific driving protocol in Eq. (28a)–(28b), we numerically study the effect of local dephasing under the commonly used Gaussian driving protocol as follows

$$\Omega_1(t) = \Omega_0 \exp\left(-\frac{(t - \tau/2)^2}{T^2}\right); \quad \text{(33)}$$

$$\Omega_3(t) = \Omega_0 \exp\left(-\frac{(t + \tau/2)^2}{T^2}\right). \quad \text{(34)}$$

Here $T$ denotes the width of the Gaussian laser coupling, $\tau$ is the delay between the two lasers. We choose the duration of the lasers, denoted as $T_0$, to be $T_0 = 8T$, namely the time-dependent driving will start from $t = -T_0/2$ and end at $t = T_0/2$. The population $\rho_{dd}$ on the dark state $|d\rangle$ as functions of $\gamma$, $\Omega_0$ and $T_0$, are shown in Fig. 4. From Fig. 4(a), we can clearly see that dephasing can strongly suppress the population transfer efficiency. While from Fig. 4(c), (e), we can see that near-perfect population transfer can be restored by increasing $\Omega_0$ or $T_0$. This demonstrates that the physical picture obtained from our analytic solution is still valid for other laser driving protocols. Additionally, from Fig. 4(b), (d), (f), we can see that the simplified set of equations in Eqs. (27a)–(27c) agree very well with the exact solutions in the wide parameter range considered in Fig. 4, which again validates our approximations made to derive it.

To this end, we discuss about possible physical implementation for our theoretical model. The coupled cavities system could be a good candidate for us, for example it has already been shown that coupled cavities system can be effectively described as coupled spins [31,32], whose Hamiltonian is almost identical to the one we used in Eq. (4) and the coupling strength can be tuned up to 100 MHz [33]. Moreover, dephasing is a common source of noise for cavities, with the typical decoherence time (inverse of the dephasing strength) $T_c$ ranging from hundreds of nano seconds to a few micro seconds [34, 35]. Therefore, it is possible to realize our theoretical model using two perfect cavities which are both coupled to an intermediate bad cavity under dephasing noise. We believe this work could significantly enhance the theoretical understanding of population transfer through noisy quantum channels and could be useful for practical quantum information processing.

4 Conclusion

To summarize, we have considered population transfer using STIRAP between two spins via an intermediate spin under dephasing noise, while these two spins themselves are dephasing-free. We derive an analytic expression for the population transfer efficiency under a specific laser driving protocol. Based on the analytical expression, we obtain an
additional adiabatic condition which is related to the strength of dephasing, under which complete population transfer could still be achieved. We show that population transfer efficiency would be reduced by dephasing, but could be restored by using a stronger laser coupling or a longer laser duration. We have also shown that this physical picture is still qualitatively correct for the commonly used Gaussian type of laser driving. Our result is helpful for a better understanding the effect of dephasing on the quantum population transfer based on STIRAP.

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Abbreviations
STIRAP, Stimulated Raman adiabatic passage.
Availability of data and materials
The data sets supporting the results of this article are included within the article. The code for solving the exact Lindblad master equation is implemented by one of the author (Chu Guo) and is available upon reasonable request.

Declarations

Ethics approval and consent to participate
Not applicable.

Consent for publication
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Competing interests
The authors declare no competing interests.

Author contributions
W.H. and C.G. wrote the first version of manuscript. W.H. and C.G. contributed to conception, modelling. W.Z. and X.D. re-wrote the manuscript. X.D. and C.G. contributed to supervision. All authors contributed to manuscript revision, read, and approved the submitted version.

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