The $\Omega_c$-puzzle solved by means of spectrum and strong decay amplitude predictions

E. Santopinto,1 A. Giachino,1 J. Ferretti,2,3 H. García-Tecocoatzi,1 M. A. Bedolla,1,4 R. Bijker,5 and E. Ortiz-Pacheco5

1INFN, Sezione di Genova, via Dodecaneso 33, 16146 Genova, Italy
2CAS Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China
3Center for Theoretical Physics, Sloane Physics Laboratory, Yale University, New Haven, Connecticut 06520-8120, USA
4Instituto de Física y Matemáticas, Universidad Michoacana de San Nicolás de Hidalgo, Edificio C-3, Ciudad Universitaria, Morelia, Michoacán 58040, México
5Instituto de Ciencias Nucleares, UNAM, Mexico

Dated: November 6, 2018

The observation of new $\Omega_c = ssc$ states by LHCb [1] and the confirmation of four of them by Belle [2] may represent an important milestone in our understanding of the quark organization inside hadrons. By providing results for the spectrum of $\Omega_c(b)$ baryons and predictions for their $\Xi^{\pm}_c K^-$ decay channels, we suggest a possible solution to the $\Omega_c$ quantum number puzzle. We also discuss why the set of $\Omega_c(b)$ baryons are the most suitable environment to test the validity of three-quark and quark-diquark effective degrees of freedom. Finally, we calculate the masses and the partial decay widths of the $\Xi_c(6227)$ and $\Sigma_c(6097)$ states, just observed by LHCb [3,4]. Our results are in good agreement with LHCb experimental data.

PACS numbers: 14.20.Lq Charmed baryons, 4.20.Mr Bottom baryons, 13.30.-a Decays of baryons, 13.30.Eg Hadronic decays, 12.39.-x Phenomenological quark models.

I. INTRODUCTION

Recently, the LHCb Collaboration announced the observation of five narrow $\Omega_c$ states in the $\Xi^{\pm}_c K^-$ decay channel [1], $\Omega_c(3000)$, $\Omega_c(3050)$, $\Omega_c(3066)$, $\Omega_c(3090)$ and $\Omega_c(3119)$. They also reported the observation of another structure around 3188 MeV, the so-called $\Omega_c(3188)$, even though they do not have enough statistical significance to interpret it as a genuine resonance [1]. Later, Belle observed five resonant states in the $\Xi_c^{\pm} K^-$ invariant mass distribution and unambiguously confirmed four of the states announced by LHCb, $\Omega_c(3000)$, $\Omega_c(3050)$, $\Omega_c(3066)$, and $\Omega_c(3090)$, but no signal was found for the $\Omega_c(3119)$ [2]. Belle also measured a signal excess at 3188 MeV, corresponding to the $\Omega_c(3119)$ state reported by LHCb [2]. A comparison between the results reported by the two collaborations is displayed in Table I. Here, it is shown that the $\Omega_c(3188)$, even if not yet confirmed, was seen both by LHCb and Belle, while, on the contrary, the $\Omega_c(3119)$ was not observed by Belle. It is also worth to mention that the LHCb collaboration has just announced the observation of a new bottom baryon, $\Xi^0_b(6227)^-$, in both $\Lambda^0_b K^-$ and $\Xi^0_b \pi^-$ decay modes [3], and of two bottom resonances, $\Sigma^0_b(6097)^{\pm}$, in the $\Lambda^0_b \pi^{\pm}$ channels [4].

The discoveries of the new $\Omega_c$, $\Xi^0_b$ and $\Sigma^0_b$ resonances not only enrich the present experimental knowledge of the hadron zoo, but they also provide essential information to explain the fundamental forces that govern nature. As the hadron mass patterns carry information on the way the quarks interact one another, they provide a means of gaining insight into the fundamental binding mechanism of matter at an elementary level.

Both LHCb and Belle were not able to measure the $\Omega_c$ angular momenta and parities. For this reason, several authors tried to provide different quantum number assignments for these states. The current $\Omega_c$ puzzle consists of the discrepancy between the experimental results, reported by LHCb [1] and Belle [2], and the existing theoretical predictions [5–9]. Indeed, for a given $\Omega_c$ experimental state more than one quantum number assignment was suggested [5]. A particular case is that of the $\Omega_c(3119)$, which, according to Ref. [7], might correspond to a $J^P = \frac{1}{2}^+$ or $J^P = \frac{3}{2}^+$ state, while, according to Ref. [8], it is possibly a $J^P = \frac{5}{2}^-$ one. Alternatively, the authors of Refs. [10–12] suggested a molecular interpretation. From the previous discussion it comes out that, in the case of $\Omega_c(3119)$, not only the quantum number assignments are not univocal, but also the quark structure is still unclear. The issues we have to deal with are not restricted to the contrasts between the different interpretations provided in the previous studies, but are also related to the discrepancies on the quantum number assignments within a given study. For example, in Ref. [3] the authors provided different $J^P$ assignments for the $\Omega_c(3066)$ and $\Omega_c(3090)$ based on mass and decay width estimates. Moreover, the nature of the $\Omega_c(3188)$ state, observed both by LHCb and Belle, is not addressed in these studies [5–9]. These divergences between the theoretical interpretations created a puzzle which needs to be addressed urgently.

By estimating the contributions due to orbital, spin-spin and spin-orbit excitations to charmed baryon masses, we reproduce quantitatively the spectrum of the $\Omega_c$ states observed both by LHCb and Belle; see Table I. Based on our results, we describe these five states as $P$-wave $\lambda$-excitations of the $ssc$ system; we also calculate

\[ \text{PACS numbers: 14.20.Lq Charmed baryons, 4.20.Mr Bottom baryons, 13.30.-a Decays of baryons, 13.30.Eg Hadronic decays, 12.39.-x Phenomenological quark models.} \]
TABLE I: Measured masses in MeV of the six resonances observed in \( \Xi^+_c K^- \) decay channel (see text) according to the LHCb \(^1\) and the Belle \(^2\) collaborations in \( pp \) and \( e^+e^- \) collisions, respectively.

| \( \Omega_c \) excited state | Mass (LHCb \(^1\)) | Mass (Belle \(^2\)) |
|-----------------------------|------------------|------------------|
| 3000                        | 3000.4 ± 0.2 ± 0.1 | 3000.7 ± 1.0 ± 0.2 |
| 3050                        | 3050.2 ± 0.1 ± 0.1 | 3050.2 ± 0.4 ± 0.2 |
| 3066                        | 3065.6 ± 0.1 ± 0.3 | 3064.9 ± 0.6 ± 0.2 |
| 3090                        | 3090.2 ± 0.3 ± 0.5 | 3089.3 ± 1.2 ± 0.2 |
| 3119                        | 3119 ± 0.3 ± 0.9  | -                |
| 3188                        | 3188 ± 5 ± 13     | 3199 ± 9 ± 4     |

FIG. 1: Comparison between three-quark and quark-diquark baryon effective degrees of freedom. Upper panel: three-quark picture with two excitation modes. Lower panel: quark-diquark picture with one excitation mode.

II. RESULTS

A. S- and P-wave ssQ states.

The three-quark system Hamiltonian can be written in terms of two coordinates \(^13\), \( \rho \) and \( \lambda \), which encode the system spatial degrees of freedom (see Fig. \( \text{[I]} \)). Let \( m_\rho = m_q \) and \( m_\lambda = \frac{3m_\rho m_Q}{2m_\lambda} \) be the \( qqQ \) system reduced masses; then, the \( \rho \)- and \( \lambda \)-mode frequencies are \( \omega_{\rho,\lambda} = \sqrt{\frac{k}{m_{\rho,\lambda}}} \), which implies that in three equal-mass-quark baryons, in which \( m_\rho = m_\lambda \), the \( \lambda \)- and \( \rho \)-orbital excitation modes are completely mixed together. Instead, in heavy-light baryons, in which \( m_\rho \neq m_\lambda \), the two excitation modes can be disentangled from each other as long as the light-heavy quark mass difference increases, as explained in App. \( \text{[A]} \). Our predictions of the two excitation mode energies are the key ingredients in determining the correct degrees of freedom which have to be considered in order to obtain a reliable description of hadrons (as three-quark or quark-diquark states).

First of all, we construct the \( ssb \) and \( ssc \) ground and excited states to establish the quantum numbers of the five confirmed \( \Omega_c \) states. For simplicity, we use the compact notation \( ssQ \) to denote them (\( Q = c \) or \( b \)). A single quark is described by its spin, flavor, and color. As a fermion, its spin is \( S = \frac{1}{2} \), its flavor, spin-flavor, and color representations are \( 3_f, 6_{sf}, \) and \( 3_c, \) respectively. An \( ssQ \) (\( sqQ \)) state, \( |ssQ, S, \lambda; l_1, l_2, J\rangle \), is characterized by total angular momentum \( J = l_1 + l_2 + S_{tot} \), where \( S_{tot} = S_p + \frac{1}{2} \). In order to construct an \( ssQ \) color singlet state, the light quarks must transform under \( SU_c(3) \) as the anti-symmetric \( 3_c \) representation. The Pauli principle postulates that the wave function of identical fermions must be anti-symmetric for particle exchange. Thus, the \( ss \) spin-flavor and orbital wave functions have the same permutation symmetry: symmetric spin-flavor in \( S \)-wave, or antisymmetric spin-flavor in antisymmetric \( P \)-wave. Two equal flavour quarks are necessarily in the \( 6_f \) flavor-symmetric state. Thus, they are in an \( S \)-wave symmetric spin-triplet state, \( S_p = 1 \), or in a \( P \)-wave antisymmetric spin-singlet state, \( S_p = 0 \). If \( l_1 = l_2 = 0 \), then \( S_p = 1 \), and we find the two ground states, \( \Omega_{Q, \Sigma} \) and \( \Omega_{Q, \Sigma}' \), with \( J = S_{tot} = \frac{3}{2} \) or \( \frac{1}{2} \): \( |ssQ, 1, \frac{3}{2}, 0_p, 0_l, 0_\lambda, \frac{1}{2}\rangle \) and \( |ssQ, 1, \frac{3}{2}, 0_p, 0_l, 0_\lambda, \frac{1}{2}\rangle \), respectively. If \( l_1 = 0 \) and \( l_2 = 1 \), then \( S_p = 1 \) and, by coupling the spin and angular momentum, we find five excited states: \( |ssQ, 1, \frac{3}{2}, 0_p, 1_\lambda, \frac{1}{2}\rangle \), \( |ssQ, 1, \frac{3}{2}, 0_p, 1_\lambda, \frac{3}{2}\rangle \), \( |ssQ, 1, \frac{3}{2}, 0_p, 1_\lambda, \frac{1}{2}\rangle \), \( |ssQ, 1, \frac{3}{2}, 0_p, 1_\lambda, \frac{3}{2}\rangle \), and
In this section we explain how to estimate the mass splitting due to the \( \lambda \)-mode orbital excitation, the hyperfine and spin-orbit interaction, as well as the mass shift between \( \rho \) and \( \lambda \)-mode \( \Omega_{c(b),J^P} \) states. The mass splittings between \( \rho \) and \( \lambda \)-orbital excitations, which are calculated in App. A, are reported in Table IV. By means of these estimates, we predict in a parameter-free procedure the spectrum of the \( ssQ \) excited states constructed in the previous section. The predicted masses of the \( \lambda \) and \( \rho \)-orbital excitations of the \( \Omega \), and \( \Omega_{b} \) baryons are reported in Tables IV and VII, respectively. In particular, Table IV shows that we are able to reproduce quantitatively the mass spectra of the \( \Omega \) states observed both by LHCb and Belle; the latter are reported in Table I.

In order to estimate the mass spectra of the \( \rho \) and \( \lambda \)-orbital excitations of the \( ssQ \) states, we consider the mass splitting between the charmed baryon ground state and first \( P \)-wave \( \lambda \)-excitations. For example, the mass difference between \( \Lambda_{c,1/2} \) and its first orbital excited state, \( \Lambda_{c,3/2} \) (2592), is 305.79 ± 0.24 MeV; the average mass difference between \( \Xi_{c,1/2} \) and \( \Xi_{c,3/2} \) (2790) is 321.1 ± 6.5 MeV. Thus, we add their average value (305.8 + 321.1)/2 ≈ 313 MeV to \( \Omega \) in order to estimate its first orbital excitation. We obtain 2965.2 + 313.5 ≈ 3009 MeV. With this in mind, we identify this state, i.e., \( |ssc, 1, 1/2, 0, 0, 1, 1/2 \rangle \), with the lowest mass state observed by the LHCb collaboration, \( \Omega_{c}(3000) \). In the bottom sector, the mass splitting between \( \Lambda_{b}(5912) \) and its ground-state \( \Lambda_{b}(292) \) is 292 MeV. Thus, we expect a mass for the first excited \( \Omega_{b} \) state as its ground state mass, \( \Omega_{b}(6046) \), plus the orbital excitation contribution. This reads \( \Omega_{b}(6338) \).

We estimate the energy splitting due to the spin-spin interaction from the (isospin-averaged) mass difference between \( \Sigma_{c,1/2} \) (2520) and \( \Sigma_{c,3/2} \) (2453). This value (65 ± 8 MeV) agrees with the mass difference between \( \Omega_{c,1/2} \) (2695) and \( \Omega_{c,3/2} \) (2770), a value close to 71 ± 4 MeV. As a consequence, the hyperfine energy splitting between two states characterized by the same flavor configuration but different spins, specifically \( S_{tot} = 3/2 \) and \( S_{tot} = 5/2 \), is around 65 MeV. For instance, the \( |ssc, 1, 3/2, 0, 0, 1, 3/2 \rangle \) state mass, with \( S_{tot} = 3/2, 1 \lambda = 1, 1 p = 0 \) and \( J = 3/2 \), is expected to be \( M = 3008.7 + 64.6 \approx 3073 \) MeV, which is quite close to the experimental mass of \( \Omega_{c}(3006) \), namely \( M = 3065.6 \pm 0.1 \pm 0.3 \) MeV. Thus, we identify the \( |ssc, 1, 3/2, 0, 0, 1, 3/2 \rangle \) state with the observed \( \Omega_{c}(3006) \) resonance. In the bottom sector, the energy splitting due to the spin-spin interaction through the (isospin-averaged) mass difference between \( \Sigma_{b,1/2} \) and \( \Sigma_{b,3/2} \) is 20 ± 7 MeV. In such a way, we expect a mass difference between the two \( S \)-wave ground states, \( \Omega_{b,1/2} \) and \( \Omega_{b,3/2} \) close to 20 ± 7 MeV. Hence, we suggest to experimentalists to look for a \( \Omega_{b,3/2} \) (6066) resonance. Similarly to the charm sector, we also expect the \( |ssb, 1, 3/2, 0, 0, 1, 3/2 \rangle \) state to be related to a possible \( \Omega_{b}(6358) \) resonance.

According to the quark model, \( \Lambda_{c}(2959) \) and \( \Lambda_{c}(2625) \) are the charmed counterparts of \( \Lambda(1405) \) and \( \Lambda(1520) \), respectively. For this reason, their spin-parities are expected to be \( 1^- \) and \( 2^- \), respectively. The different values of spin and parity explain the mass difference between \( \Lambda_{c}(2625) \) and \( \Lambda_{c}(2595) \) (36 MeV) in terms of spin-orbit effects. We estimate that the mass of \( |ssc, 1, 1/2, 1, 0, 1, 3/2 \rangle \) is related to the previous spin-orbit splitting. We obtain a value of 3009 + 36 ≈ 3045 MeV, which is compatible with the mass of the \( \Omega_{c}(3050) \) within the theoretical uncertainty (of the order of a few tens of MeV) and experimental error. Thus, we identify the \( |ssc, 1, 1/2, 1, 0, 1, 3/2 \rangle \) state with the \( \Omega_{c}(3050) \) resonance. Through the estimation of orbital, hyperfine and spin-orbit interactions, we estimate the \( |ssc, 1, 3/2, 1, 0, 1, 3/2 \rangle \) and \( |ssc, 1, 1/2, 0, 0, 1, 3/2 \rangle \) mass values as 3109 ± 21 MeV and 3146 ± 24, respectively. These values are compatible with the masses of \( \Omega_{c}(3090) \) and \( \Omega_{c}(3188) \). Hence, we propose the following assignments: \( |ssc, 1, 3/2, 1, 0, 1, 3/2 \rangle \rightarrow \Omega_{c}(3090) \) and \( |ssc, 1, 1/2, 0, 0, 1, 3/2 \rangle \rightarrow \Omega_{c}(3188) \). In the bottom sector, the mass splitting due to the spin-orbit interaction between \( \Lambda_{b,3/2} \) (5912) and \( \Lambda_{b,1/2} \) (5920) is 8 MeV. Thus, we interpret the \( \Omega_{b}(6346) \), \( \Omega_{b}(6367) \) and \( \Omega_{b}(6375) \) resonances as the bottom counterparts of the \( \Omega_{c}(3050) \), \( \Omega_{c}(3090) \) and \( \Omega_{c}(3188) \), respectively. Recently, LHCb observed a new bottom baryon, \( \Xi_{b}(2727) \), in both \( L_{b}^{0}K^{-} \) and \( \Xi_{b}^{0}π^{-} \) decay modes \([3]\), and two bottom resonances, \( \Sigma_{b}(6097) \), in the \( L_{b}^{0}K^{-} \) and \( \Xi_{b}^{0}π^{-} \) channels \([3]\). In Figs. 4 and 5 we report the predictions for the mass spectrum of the \( \Xi_{b} \) and the \( \Sigma_{b} \) excited states according to the parameter reported in Table II. In the hypothesis that \( \Xi_{b}(6227) \) is compatible with our predictions, and also the mass of \( \Sigma_{b}(6097) \) is in good agreement with them. As displayed in Table VI the partial decay widths are also in agreement with the total decay widths reported by LHCb. This certifies the reliability of our predictions in the bottom sector.

We summarize all our proposed quantum number assignments for both \( \Omega_{c} \) and \( \Omega_{b} \) states in Figs. 2 and 3 respectively. In the charm sector, we find an excellent agreement between our predictions for the spectrum and the experimental data.
TABLE II: Ground and excited $\Delta M_{ssc}$ and $\Delta M_{ssb}$ states

| orbital | hyperfine spin-orbit $\Delta_{\rho-\lambda}$ |
|---------|--------------------------------------------|
| $\Delta M_{ssc}$ (MeV) | 313 ± 7 | 65 ± 8 | 36 ± 3 | 142 ± 7 |
| $\Delta M_{ssb}$ (MeV) | 293 ± 1 | 20 ± 8 | 8 ± 1 | 191 ± 1 |
| $\Delta M_{qqb}$ (MeV) | 293 ± 1 | 20 ± 8 | 8 ± 1 | 203 ± 1 |
| $\Delta M_{qsb}$ (MeV) | 293 ± 1 | 20 ± 8 | 8 ± 1 | 197 ± 1 |

FIG. 2: $\Omega_c$ mass spectra and tentative quantum number assignments. The theoretical predictions (red dots) are compared with the experimental results by LHCb [1] (blue line), Belle [2] (violet line) and Particle Data Group (black lines) [16]. Except the $\Omega_c(3188)$ case, the experimental error for the other states is too small to be appreciated in this energy scale. The spin-$\frac{1}{2}$ ground-state mass ($\Omega_c(2695)$) is an input value.

FIG. 3: $\Omega_b$ mass spectrum predictions (red dots) and $\Omega_b$ ground-state experimental mass (black line) according to Particle Data Group [16]. The theoretical uncertainties on the predicted masses and the experimental error on the $\Omega_b(6046)$ state are too small to be appreciated in this energy scale. The spin-$\frac{1}{2}$ ground-state mass ($\Omega_b(6046)$) is an input value.

FIG. 4: $\Xi_b$ mass spectra and tentative quantum number assignments for the recently observed $\Xi_b(6227)$ state [3]. The theoretical predictions (red dots) are compared with the experimental result by LHCb [3] and Particle Data Group (black lines) [16]. The spin-$\frac{1}{2}$ ground state masses, $\Xi_b(5793)$, and $\Xi_b'(5935)$ are input values, all the others are predictions.

C. Decay widths of $ssQ$ states

In the following, we compute the strong decays of $ssQ$ baryons in $sqQ-K$ ($q = u, d$) final states by means of the $^3P_0$ model [17-20] (see App. [3]). The $^3P_0$ model parameters, including a quark pair-creation strength, $\gamma_0$, the baryon, $\alpha_{\rho,\lambda}$, and meson, $\alpha_c$, harmonic-oscillator parameters, are reported in Table III.

TABLE III: $^3P_0$ model parameter values used to calculate the decay widths. The value of the $\pi$- and $K$-meson oscillator parameter, $\alpha_c$, is extracted from Ref. [21].

| parameter | $\alpha_{\rho}$ | $\alpha_{\lambda}$ | $\alpha_c$ | $\gamma_0$ |
|-----------|-----------------|-------------------|------------|----------|
| value     | 0.520 GeV       | 0.160 GeV         | 0.420 GeV  | 48.65    |

In order to compute the decay widths, we consider $\alpha_{\rho}$ and $\alpha_{\lambda}$ as independent parameters, with the constraint $\alpha_{\lambda} < \alpha_{\rho}$, which follows from the inequality $m_Q > m_s$. The $\lambda$-mode excitation energy diminution is a consequence of a $\lambda$-motion heavier inertia mass. We take the $\pi$- and $K$-meson oscillator parameter value ($\alpha_c$) from [21], and we set to zero the quark form-factor parameter, $\alpha_d$, which is related to the effective size of the created
The Ξ−c quark-antiquark pair. Tables IV and VII report our \( \Omega_c \) and Ξ−c states are observed by LHCb and Belle. Both the \( \Xi^+_cK^- \) branching ratios and the quantum numbers of the Ωc’s are unknown; we only have experimental informations on their total widths, \( \Gamma_{\text{tot}} \). Thus, our predictions have to satisfy the constraint: \( \Gamma(\Omega_c \rightarrow \Xi^+_cK^-) \leq \Gamma_{\text{tot}} \). In light of this, we state that our strong decay width results, based both on our mass estimates and quantum number assignments, are compatible with the present experimental data. In particular, the \( \lambda \)-mode decay widths of the \( \Omega_c \) states are in the order of 0.5 – 1 MeV, with the exception of \( \Omega_c(3000) \). The width of the previous state is much smaller, the \( \Omega_c(3000) \) being closer to the \( \Xi^+_cK^- \) threshold. The decay of the two \( \rho \)-excitations, |ssc, \( 0\frac{1}{2}, 1\rho, 0\lambda, \frac{3}{2} \rangle \rangle \) and |ssc, \( 0\frac{1}{2}, 1\rho, 0\lambda, \frac{3}{2} \rangle \rangle , \in \Xi^+_cK^- \) is forbidden by spin conservation. Similar considerations can be applied to the decay widths of \( \rho \)-mode \( \Omega_b \) states.

In conclusion, also the \( 3P_0 \) model results suggest that the five \( \Omega_c \) resonances, \( \Omega_c(3000), \Omega_c(3050), \Omega_c(3066), \Omega_c(3090), \) and \( \Omega_c(3188) \), could be interpreted as ssc ground-state \( P \)-wave \( \lambda \)-excitations. In principle, both the \( \Omega_c(3090) \) and \( \Omega_c(3119) \) resonances observed by LHCb are compatible with the properties (mass and decay width) of the |ssc, \( 0\frac{1}{2}, 1\lambda, \frac{3}{2} \rangle \rangle theoretical state. As Belle could neither confirm nor deny the existence of the \( \Omega_c(3119) \), given the low significance of its results for the previous state (0.4σ), we prefer to: 1) Assign |ssc, \( 0\frac{1}{2}, 1\lambda, \frac{3}{2} \rangle \rangle to the \( \Omega_c(3090); \) 2) Interpret the \( \Omega_c(3119) \) as a \( \Xi^+_cK^- \) bound state [10–12], the \( \Omega_c(3119) \) lying 22 MeV below the \( \Xi^+_cK^- \) threshold. See Fig. 5.

III. DISCUSSION

We calculated the \( \Omega_{c(b)} \)’s masses and \( \Xi^+_{c(b)} \) \( K^- \) strong decay amplitudes. By means of these mass and decay width predictions, we proposed an univocal assignment to the five \( \Omega_c \) states observed both by LHCb [1] and by Belle [2]: |ssc, \( 1\frac{1}{2}, 0\rho, 1\lambda, \frac{3}{2} \rangle \rangle \rightarrow \Omega_c(3000), \) |ssc, \( 1\frac{1}{2}, 0\rho, 1\lambda, \frac{3}{2} \rangle \rangle \rightarrow \Omega_c(3050), \) |ssc, \( 1\frac{3}{2}, 0\rho, 1\lambda, \frac{1}{2} \rangle \rangle \rightarrow \Omega_c(3066), \) \( \Omega_c(3090), \) and \( \Omega_c(3188) \). The latter was completely ignored in other studies [5,7]. In principle, both the \( \Omega_c(3119) \) and \( \Omega_c(3090) \) could be assigned to the |ssc, \( 0\frac{1}{2}, 0\rho, 1\lambda, \frac{3}{2} \rangle \rangle state. However, as Belle could neither confirm nor deny the existence of the \( \Omega_c(3119) \), we preferred the \( \Omega_c(3119) \) interpretation as a \( \Xi^+_cK^- \) meson-baryon molecule and assigned the \( \Omega_c(3090) \) to the |ssc, \( 0\frac{1}{2}, 0\rho, 1\lambda, \frac{3}{2} \rangle \rangle \rightarrow \Omega_c(3090) \) state. In conclusion, by providing ssc quantum number assignments for \( \Omega_c(3000), \Omega_c(3050), \Omega_c(3066), \Omega_c(3090), \) and \( \Omega_c(3188) \), and suggesting a molecular interpretation for \( \Omega_c(3119) \), we provided a self-consistent solution to the \( \Omega_c \) puzzle. The previous states were observed with high significance\(^1\), thus their existence is not questionable. The

\(^1\) The statistical significance of the \( \Omega_c \) states is computed as \( N_\sigma = \sqrt{\chi^2} \), where \( \chi^2 \) is the increase in \( \chi^2 \) when the resonance is excluded from the fit. The significance values reported by LHCb [1] and Belle [2] are the following

- \( \Omega_c(3000): N_\sigma(\text{LHCb}) = 20.4, N_\sigma(\text{Belle}) = 3.9; \)
the experimental data, the predicted partial decay amplitudes being lower than the total measured decay widths.

The only exception may be the \(\Omega_c\) states.

\[
\begin{align*}
|ssc, 1, \frac{1}{2}, 0, 1, \frac{1}{2} \rangle & \equiv \Omega_c(3000) \quad 3009 \pm 10 \quad 3000.4 \pm 0.2 \pm 0.1 \pm 0.3 \quad 0.013 \quad 4.6 \pm 0.6 \pm 0.3 \\
|ssc, 1, \frac{1}{2}, 0, 1, \frac{1}{2} \rangle & \equiv \Omega_c(3050) \quad 3045 \pm 13 \quad 3050.2 \pm 0.1 \pm 0.1 \pm 0.3 \quad 1.00 \quad 0.8 \pm 0.2 \pm 0.1 \\
|ssc, 1, \frac{1}{2}, 0, 1, \frac{1}{2} \rangle & \equiv \Omega_c(3090) \quad 3090 \pm 21 \quad 3090.2 \pm 0.3 \pm 0.5 \pm 0.3 \quad 0.34 \quad 8.7 \pm 1.0 \pm 0.8 \\
|ssc, 1, \frac{1}{2}, 0, 1, \frac{1}{2} \rangle & \equiv \Omega_c(3188) \quad 3146 \pm 24 \quad 3188 \pm 5 \pm 13 \quad 6.24 \quad 60 \pm 26 \\
|ssc, 0, \frac{1}{2}, 1, 0, \frac{1}{2} \rangle & \equiv \Omega_c(3119) \quad 4151 \pm 10 \quad – \quad 0.0 \quad – \\
|ssc, 0, \frac{1}{2}, 1, 0, \frac{1}{2} \rangle & \equiv \Omega_c(3188) \quad 4187 \pm 13 \quad – \quad 0.0 \quad –
\end{align*}
\]

In the previous case, there would be a dramatic impact on the description of hadrons at a quark level. Finally, as a test of our model in the bottom sector, we calculated the masses and partial decay widths of the \(\Sigma_b(6227)\) and \(\Sigma_b(6097)\) states just observed by LHCb [3] in the hypothesis that they are, respectively, the lowest mass excited state of \(\Xi_b(5935)\) and \(\Sigma_b(5813)\) ground states. The \(\Sigma_b(6227)\) and \(\Sigma_b(6097)\) masses and partial decay widths are compatible with our predictions.

\[
\begin{align*}
|qbb, 1, \frac{1}{2}, 0, 1, \frac{1}{2} \rangle & \equiv \Xi_b(6097) \quad 6106 \pm 5 \quad 6096.9 \pm 4.3 \\
|qbb, 1, \frac{1}{2}, 0, 1, \frac{1}{2} \rangle & \equiv \Xi_b(6227) \quad 6228 \pm 1 \quad 6226.9 \pm 2.3
\end{align*}
\]

\[
\begin{align*}
\Gamma(\Sigma_b^0(6097) \rightarrow \Lambda_b^0 + \pi^\pm) & = 2.78 \quad 30.0^{+7.2}_{-5.4} \\
\Gamma(\Xi_b^0(6227) \rightarrow \Lambda_b^0 + \pi^\pm) & = 8.18 \quad 18.1 \pm 5.4 \pm 1.8
\end{align*}
\]

\[
\begin{align*}
|sbb, 1, \frac{1}{2}, 0, 1, \frac{1}{2} \rangle & \equiv \Omega_c(3050) \quad 6338 \pm 3 \quad 0.0030 \\
|sbb, 1, \frac{1}{2}, 0, 1, \frac{1}{2} \rangle & \equiv \Omega_c(3090) \quad 6346 \pm 4 \quad 0.86 \\
|sbb, 1, \frac{1}{2}, 0, 1, \frac{1}{2} \rangle & \equiv \Omega_c(3119) \quad 6358 \pm 11 \quad 0.19 \\
|sbb, 1, \frac{1}{2}, 0, 1, \frac{1}{2} \rangle & \equiv \Omega_c(3188) \quad 6376 \pm 11 \quad 0.22 \\
|sbb, 0, \frac{1}{2}, 1, 0, \frac{1}{2} \rangle & \equiv \Omega_c(3119) \quad 6375 \pm 12 \quad 1.49 \\
|sbb, 0, \frac{1}{2}, 1, 0, \frac{1}{2} \rangle & \equiv \Omega_c(3188) \quad 6530 \pm 3 \quad 0.0
\end{align*}
\]

TABLE VI: Partial decay widths of the \(\Sigma_b(6097)\) and \(\Xi_b(6227)\) states obtained with the parameters reported in Table [11]. The predicted decay widths in \(\Lambda_b\pi\) channels, calculated with the experimental masses reported by LHCb, are compared with the total experimental widths [3].

only exception may be the \(\Omega_c(3119)\) and \(\Omega_c(3188)\).

We calculated the mass splitting between the \(\rho\) and \(\lambda\)-mode excitations of \(\Omega_c(b)\) resonances. The large mass difference between these two excitation modes could be the key to access the nature of the effective degrees of freedom involved in heavy-light baryon spectroscopy. Specifically, the non observation of the \(\rho\)-excitations in the predicted mass region would rule out the three quark model for the \(\Omega_c(b)\) baryons. In the previous case, there would be a dramatic impact on the description of hadrons at a quark level. Finally, as a test of our model in the bottom sector, we calculated the masses and partial decay widths of the \(\Xi_b(6227)\) and \(\Sigma_b(6097)\) states just observed by LHCb [3] in the hypothesis that they are, respectively, the lowest mass excited state of \(\Xi_b(5935)\) and \(\Sigma_b(5813)\) ground states. The \(\Xi_b(6227)\) and \(\Sigma_b(6097)\) masses and partial decay widths are compatible with our predictions.

- \(\Omega_c(3050)\): \(N_c(LHCb) = 20.4, N_c(Belle) = 4.6\);
- \(\Omega_c(3065)\): \(N_c(LHCb) = 23.9, N_c(Belle) = 7.2\);
- \(\Omega_c(3090)\): \(N_c(LHCb) = 21.1, N_c(Belle) = 5.7\);
- \(\Omega_c(3119)\): \(N_c(LHCb) = 10.4, N_c(Belle) = 0.4\);
- \(\Omega_c(3188)\): \(N_c(LHCb)\) is not provided, \(N_c(Belle) = 2.4\).
Appendix A: Mass splitting between $\lambda$- and $\rho$-excitations

The mass shift between $\rho$- and $\lambda$-modes is \[ \Delta_{\rho-\lambda} = E_\rho - E_\lambda = \omega \left[ 1 - \sqrt{\frac{2x + 1}{3}} \right], \]
where $x = \frac{m_l}{m_Q}$ with $m_l = m_\rho$ or $m_\lambda$ and $m_Q = m_c$ or $m_b$; $\hbar\omega$ is the harmonic oscillator spacing in the limit of equal mass quarks with mass $m_q$. For example, if we use the typical values of the quark masses, $m_\rho = 300$ MeV, $m_b = 510$ MeV, $m_c = 1750$ MeV and $m_b = 5112$ MeV \[24\], the mass shifts are $\Delta_{\rho-\lambda} \approx 142$ MeV for $\Omega_c$ states, and $\Delta_{\rho-\lambda} \approx 191$ MeV for the $\Omega_b$ states.

Appendix B: $^3P_0$ Decay model

The $^3P_0$ is an effective model to compute the open-flavor strong decays of hadrons in the quark model formalism \[17, 20\]. In this model, a hadron decay takes place in its rest frame and proceeds via the creation of an additional $q\bar{q}$ pair with vacuum quantum numbers, i.e. $J^{PC} = 0^+\pi$. We label the initial baryon- and final baryon- and meson-states as $A$, $B$ and $C$, respectively. The final baryon-meson state $BC$ is characterized by a relative orbital angular momentum $\ell$ between $B$ and $C$ and a total angular momentum $J = J_B + J_C + \ell$. The decay widths can be calculated as \[17, 18, 22\]
\[ \Gamma = \frac{2\pi\gamma_0^2}{2J_A + 1} \Phi_{A\rightarrow BC}(q_0) \sum_{M_{J_A},M_{J_B}} |M^{M_{J_A},M_{J_B}}| \]
Here, $M^{M_{J_A},M_{J_B}}$ is the $A \rightarrow BC$ amplitude which, for simplicity, is usually expressed in terms of hadron harmonic-oscillator wave functions, $\gamma_0$ is the dimension-less pair-creation strength, $q_0$ is the relative momentum between $B$ and $C$, and the coefficient $\Phi_{A\rightarrow BC}(q_0)$ is the relativistic phase space factor \[22\].

The parameters used to calculate the decay widths are reported in Table \[III\]. As a check of the parameter values, we calculate the $\Sigma_c^{++} \rightarrow \Lambda_c^+ \pi^+$ decay width; our result, $1.712$ MeV is in agreement with the experimental data from the PDG \[16\]: $\Gamma(\Sigma_c^{++} \rightarrow \Lambda_c^+ \pi^+) = 1.89^{+0.18}_{-0.09}$ MeV.

---

[1] Aaij, R. et al. [LHCb Collaboration]. Observation of five new narrow $\Omega_b^0$ states decaying to $\Xi_b^- K^-$. Phys. Rev. Lett. 118, 182001 (2017).
[2] Yelton, Y. et al. [Belle Collaboration], Observation of a new $\Sigma_c^0$ resonance. Phys. Rev. Lett. 121, 072002 (2018).
[3] Aaij, R. et al. [LHCb Collaboration], Observation of a new $\Xi_c^+$ resonance. Phys. Rev. Lett. 121, 072002 (2018).
[4] Aaij, R. et al. [LHCb Collaboration], Observation of two resonances in the $\Lambda_c^{0\pi^\mp}$ and $\Sigma_c^{0\pi^\pm}$ properties. arXiv:1809.07752
[5] Karliner, M. & Rosner, J. L. Very narrow excited $\Omega_c$ baryons. Phys. Rev. D 95, 114012 (2017).
[6] Zhao, Z., Ye, D. D., & Zhang, A. Hadronic decay properties of newly observed $\Omega_c$ baryons. Phys. Rev. D 95, 114024 (2017).
[7] Wang, K. L., Xiao, L. Y., Zhong, X. H., & Zhao Q. Understanding the newly observed $\Omega_c$ states through their decays. Phys. Rev. D 95, 116010 (2017).
[8] Padmanath, M. and Mathur, N. Quantum Numbers of Recently Discovered $\Omega_b^0$ Baryons from Lattice QCD. Phys. Rev. Lett. 119, 042001 (2017).
[9] Agaev, S. S., Azizi, K. & Sundu, H. Interpretation of the newly observed $\Omega_b^0$ states via mass and width. Eur. Phys. J. C 77, 386 (2017).
[10] Huang, Y., Xiao, C. J., Liu, Q. F., Wang, R., He, J. & Geng, L. Strong and radiative decays of $D\Xi$ molecular state and newly observed $\Omega_c$ states. Phys. Rev. D 97, 094013 (2018).
[11] Debastiani, V. R., Dias, J. M., Liang, W. H. & Oset, E. Molecular $\Omega_c$ states generated from coupled meson-baryon channels. Phys. Rev. D 97, 094035 (2018).
[12] Nieves, J., Pavao, R. & Tolos, L. $\Omega_c$ excited states within a SU(6)$_{\mu\tau}$ × HQSS model. Eur. Phys. J. C 78, 114 (2018).
[13] Isgur, N., & Karl, G. P Wave Baryons in the Quark Model. Phys. Rev. D 18, 4187 (1978).
[14] Copley, L. A., Isgur, N. & Karl, G. Charmed Baryons in a Quark Model with Hyperfine Interactions. Phys. Rev. D 20, 768 (1979).
[15] Capstick, S., & Isgur, N. Baryons in a Relativized Quark Model with Chromodynamics. Phys. Rev. D 34, 2809 (1986).
[16] Patrignani, C. et al. [Particle Data Group]. Review of Particle Physics. Chin. Phys. C 40, 100001 (2016).
[17] Micu, L. Decay rates of meson resonances in a quark model. Nucl. Phys. B 10, 521 (1969).
[18] Le Yaouanc, A., Oliver, L., Pene, O. & Raynal, J. C. Strong decays of baryons and missing resonances. Phys. Rev. D 75, 094017 (2007).
[19] Ferratti, J., Biček, R., Galatà, G., García-Tecocoati, H. & Santopinto, E. Strong decays of baryons and missing resonances. Phys. Rev. D 94, 074040 (2016).
[23] Yoshida, T., Hiyama, E., Hosaka, A., Oka, M. & Sadato, K. Spectrum of heavy baryons in the quark model. Phys. Rev. D 92, 114029 (2015).