Soft-gluon angular screening in heavy-quark fragmentation

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Abstract
A method to measure the suppression of soft-gluon radiation by a heavy quark ("dead cone") is discussed. We analyse this QCD phenomenon in the framework of the HERA experiment using Monte Carlo simulations.

1 Introduction

The suppression of bremsstrahlung off an accelerating massive particle is well known effect in classical electrodynamics [1]. The characteristic feature of such radiation is a large value of the photon emission angle with respect to the direction of motion of a charged particle. This angle is of order $m/E$, with $m$ and $E$ being the mass and the energy of the radiating particle.

Similarly, this effect is expected in QCD [2]. The gluon radiation from heavy quarks, c or b, is characterised by the angular screening, i.e. the soft gluon emission in the forward direction of a heavy quark is reduced within the angle $\Theta_0 = m_q/E_q$ ($m_q$ and $E_q$ are the mass and the energy of the heavy quark) [3,4]. The gluon-emission probability $d\sigma_{q\rightarrow gq}$ as a function of the angle $\Theta$ between the direction of motion of a soft gluon and an emitting quark is proportional to [4]

$$ \frac{C_F \alpha_s}{\pi} \frac{H^2(\Theta)dH^2(\Theta)}{(H^2(\Theta) + \Theta_0^2)^2}, \quad H(\Theta) = 2\sin\Theta/2. \quad (1) $$
Thus, $d\sigma_{q\to qg}$ is suppressed for $\Theta < \Theta_0 << 1$. This screening of the collinear singularity by the heavy-quark mass, known as the "dead cone", is the most important phenomenon which determines the shape of jets initiated by heavy quarks. For example, the form of $\ln(1/x)$ spectra can change drastically in the hard momenta region, compared with jets initiated by light quarks [3]. A recent theoretical overview of the dead cone effect is given in [5,6].

Although the angular screening has not been observed directly in QCD, unlike to electron radiation in QED, this effect has received an attention in Monte Carlo (MC) models simulating main aspects of high-energy interactions. An approximate method to include the dead cone was discussed in [7] and has been implemented in the parton shower of the HERWIG model [8]. The ARADNE model has different untested options to reproduce the effect in the Color Dipole Model [9]. The PYTHIA/JETSET model [10] contains the dead cone by construction of the parton shower. Note that the effect is not exactly implemented, but is rather a consequence of how the shower kinematics is set up [11].

There are a few obstacles to observe the screening effect experimentally. A large sample of high purity $c$ or $b$ events is necessary which then has to be used to determine the direction of the original heavy quarks. The dead cone can be rather small, thus it is important to have good two-track resolution. The decay products of heavy mesons should be removed from the events. The most serious concern, however, is to understand how strong high-order QCD and hadronisation can change hadronic angular distributions near the heavy-quark direction. For example, intensive color flows at hadronisation stage can produce a smearing effect both for the reconstruction of the heavy-quark direction as well as for the final-state hadrons originating from soft-gluon emissions. Resonance decay products can further mask the signal.

At LEP1 energies, heavy quarks are mostly coming from $Z \to c\bar{c}, b\bar{b}$ decay and carry about half of the beam energy. Therefore, the dead cone should exist at $\Theta_0 \sim 2^\circ$ for $c$ quarks and $\Theta_0 \sim 6^\circ$ for $b$ quarks [7]. The smallness of $\Theta_0$ at LEP makes it rather complicated to detect the angular screening due to the smearing effects described above. Some ideas on how to find this effect at LEP have been discussed in [12,7], however, since then no progress has been made to determine the size of the dead cone experimentally.

HERA provides an unique opportunity to observe the soft gluon depletion in the charm fragmentation since the energy for the $c\bar{c}$ production is small. For typical HERA kinematics and cuts used to reconstruct, say $D^*$ mesons, the energy of the $c$ ($\bar{c}$) quark is about 3-5 GeV. Thus the angular screening can be observed at an angle of order $20^\circ - 30^\circ$, which is by factor ten larger than that expected at LEP. As we will show in this paper, the high luminosity data for deep inelastic scattering (DIS) and photoproduction, which is already
delivered by HERA, allows a measurement of this phenomenon in details.

2 Angular distributions

In this paper we investigate the possibility to determine the size of the dead cone in DIS at HERA. For our study we use the AROMA 2.2 Monte Carlo program [13] unless otherwise stated. This model is the most suitable for charm study as it contains an exact matrix element for the heavy-quark production. The DIS events were generated at $Q^2 > 5 \text{ GeV}^2$ with positron and proton beams at energies $E_e = 27.5 \text{ GeV}$ and $920 \text{ GeV}$, respectively. The GRV94 structure function was used.

The AROMA generates the charm production only through the boson-gluon fusion (BGF) mechanism. This model contains the initial- and final-state QCD radiations. The parton shower, matched to the first-order matrix elements on the basis of the LEPTO model [14], is simulated with the PYTHIA/JETSET program. As mentioned above, the dead cone is not exactly implemented and comes out by construction of the shower. A typical opening angle $\Theta_q$ between gluon $g$ and quark $q$ in the splitting $q \rightarrow g q$ approximately equals to [10]

$$\Theta_q \approx \frac{1}{\sqrt{z_q(1 - z_q) E_q}},$$

(2)

where $z_q$ is the energy fraction carried by the gluon ($E_g = z_q E_q$). For light quarks, the opening angle is controlled by minimum (maximum) values of $z_q$, which in turn are determined by the QCD cut-off $Q_0$ ($Q_0 = 1 \text{ GeV}$ for JETSET default). For heavy quarks, this cut-off is less important due to the massive factor in (2).

Fig. 1 illustrates the angular distribution of the first gluon emission with respect to the BGF quark (antiquark). The MC samples contain at least one BGF quark at $P_T > 1.5 \text{ GeV}$ in the laboratory frame. The angle $\Theta_q$ is determined as $\Theta_q = \arctan(p_{\bar{q}} p_{\bar{g}} / |p_{\bar{q}}||p_{\bar{g}}|)$, with $\vec{p}_{\bar{q}}$ and $\vec{p}_{\bar{g}}$ being the 3-momenta of the BGF quark (antiquark) and of the parton-shower gluon originating from it. The difference between the angular distributions of $c$ and $u$ quarks illustrates the dead cone effect. As a cross-check[1], the angular distributions were obtained with a smaller mass of the charm quark (not shown). In this case the distribution shown with closed symbols exhibits at low $\Theta_q$ much the same

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[1] The study of the dead cone phenomenon is complicated by the fact that the most popular MC models, such as PYTHIA/JETSET, ARIADNE and HERWIG, do not contain a switch to exclude the dead cone from their parton showers.
Fig. 1. Distributions of the angle between the first parton-shower gluon emission and c (u) quark (or antiquarks) from the BGF charm production generated with AROMA for different cut-off values $Q_0$ in the parton shower. All histograms are normalized to unity.

Fig. 2. Angular distributions of all parton-shower gluons with respect to the initial c (u) quark (antiquarks) in the BGF for different values of the cut-off $Q_0$. 
rise as that for the light quark (open symbols). This trend reflects a reduction of the dead-cone size.

To demonstrate the sensitivity of this distribution to the cut-off value, the same distributions are shown for a smaller $Q_0$. The most important observation is that the cut-off does not play significant role for the gluon bremsstrahlung off the charm quarks, while a small "dead cone" in case of the light quark is due to kinematic constraints controlled by the value of $Q_0$.

The particle flows around the BGF quarks are shown in Figs. 2, 3 and 4. The first figure illustrates the distributions of the angle between all parton-shower gluons and the original quark. As for the single-gluon emission shown in Fig. 1, the dead cone is clearly seen at $\Theta < 0.6$ rad, although there exists a smearing effect caused by multi-gluon branchings.

The interpretation of Figs. 3 and 4 is more complicated due to the LUND string hadronisation involved and because only charged hadrons are counted. For the studies reported here only final-state hadrons within $|\eta| < 3$ are used to avoid counting particles close to the beam direction. Fig. 3 shows the angular distribution of charged hadrons without resonance decays. The dead cone is not seen anymore. One obvious reason for this is a double counting effect, i.e. a leading hadron associated with the initial BGF quark contributes to the distribution at small angular separations. For charm events, the leading particle is a charmed meson which closely follows to the direction of the initial $c$ ($\bar{c}$) quark and thus contributes to the angular distribution at $\Theta_{h-q} \sim 0$ as seen from Fig. 3. The resonance production smears the leading-particle effect as shown in Fig. 4. Another observation is that the hadron-level distributions are rather insensitive to the QCD cut-off.

Generally speaking, it is not meaningful to say that a given hadron originates from a particular quark in the string fragmentation model. However, to investigate the dead cone in more details, one can remove the leading particles containing flavours of the original BGF quark. Note that this can be done in a MC simulation, while the meaning of the leading particle is more problematic when one deals with the real experimental situations [15]. Fig. 5 shows the angular distribution of the charged hadrons after removing the leading particles. The figure demonstrates that, despite the smearing effects from the parton shower and hadronisation, the dead cone effect can be seen. Moreover, the hadron-level distributions for $c\bar{c}$ are rather close to the ones for parton-level.
Fig. 3. Distributions of the angle between the charged hadrons (without resonance decays) and c (u) BGF quark for different cut-off values in the parton shower.

Fig. 4. Distributions of the angle between the final-state charged hadrons and c (u) quark in the BGF for different cut-off values in the parton shower.
Fig. 5. Angular distributions for the final-state charged hadrons (without leading hadrons) with respect to c (u) quark in the BGF processes. We show the distributions before and after resonance decays (symbols and lines, respectively).

3 Initial-quark direction reconstruction

From the previous section it is clear that the most suitable Monte-Carlo independent method to detect the dead cone is to measure the angular distributions of final-state particles with respect to the original quark. To do this, the first step would be to understand how well the initial-quark direction can be reconstructed from the final-state hadrons.

The reconstruction of the light-quark direction can be performed using a jet clustering algorithm. The purity for light-quark initiated jets in an inclusive DIS event sample can be low, thus some cuts to reject heavy-flavour events can be used. For example, a selection of single-jet DIS events or the use of specific cuts [15] in the Breit frame to reduce the BGF type of events can be useful to suppress the contribution of heavy quarks.

One can measure the direction of the charm quark using a few methods. One can reconstruct, for instance, the four-momenta of $D^*$ mesons, or to perform the clustering of the final-state particles into jets similar to the light-quark sample.
We generated two separate DIS samples to study the reconstruction of the initial-quark direction. The first sample contains only uū quark topology, while the second sample consists of c̄c events from the BGF. We require to have at least one quark with a transverse momentum larger than 1.5 GeV. In addition, we accept final-state charged hadrons within |η| < 3.

To reconstruct jets, we use the inclusive KTCLUS jet algorithm [16] with $P_T$ recombination scheme in the laboratory frame. This algorithm is expected to be least affected by hadronisation effects and proton remnants. We select only a single jet with highest $E_T$.

The solid line in Fig. 6 shows the angular distribution for the uū sample. It is seen that the jet axis well reproduces the direction of the original quark. Other jet algorithms were found to be less reliable than the KTCLUS.

For the c̄c sample, we performed an analogous study (dashed line). In this case the jet algorithm cannot reproduce the direction of the initial quark as good as for the light quarks. The reason for this is decay products of charmed mesons as well as broadness of the heavy-flavour jets caused by the angular screening. Both effects are expected to lead to misreconstructions of the initial-quark direction when charged particle multiplicities are small.

To improve the initial-quark direction reconstruction for the c̄c sample, we selected a subsample with $D^*$ mesons and inhibited their decays (doted line in Fig. 6). The reconstruction of the quark direction for this method is as good as for the approach when the $D^*$ meson is used to determine the quark direction without the jet algorithm (dot-dashed lines).

This study illustrates that, for the given Monte Carlo simulation, the jet axis gives a measurement of the original-quark direction to within 130 mrad (7°) for the light-quark sample and to within 100 mrad (6°) for the charm sample with stable $D^*$ meson. The reconstructed $D^*$ meson itself gives the direction of the original c quark to within 80 mrad (5°). These values are small, compared to the size of the dead cone expected at HERA. Thus, the jet axis can be used to study the particle flows close to light or heavy quarks.

### 4 Dead-Cone reconstruction

From the above illustrations it can be seen that the experimental observation of the dead cone is possible by comparing the angular distributions of the final-state hadrons around the jet axis. The study of particle flows close to $D^*$ is the most reliable, however, it is difficult to find a proper way to compare this measurement with the gluon bremsstrahlung off a light-quark. Therefore,
Fig. 6. Distributions of the angle between the initial c (u) quark and the jet axis reconstructed using the KTCLUS algorithm for AROMA MC. The dotted line shows the distribution for events with stable $D^*$ mesons. Also shown is the angular distribution between the $D^*$ and charm quark (dot-dashed lines).

The jet reconstruction is required both for light- and heavy-quark samples.

The dead cone can be measured in a few steps:

1) To select inclusive events for a given phase-space region in $Q^2$ and $x$ and to reconstruct the jet axis using a jet algorithm, preferably the KTCLUS. Then the angular distribution of charged tracks with respect to the jet axis should be found. Note that additional cuts [15] to reject the BGF type of events can be useful to reduce contributions from heavy quarks.

2) To select a sample of events with reconstructed $D^*$ mesons for the same kinematic region as for the inclusive sample enriched with light quarks. To identify the charmed mesons, one can use the most popular exclusive decay channel $D^{*\pm}(2010) \to D^0\pi^\pm$ with $D^0 \to K^\mp\pi^\pm$ (+c.c.), where $\pi^\pm$ refers to a low momentum ("slow") pion. The decay products of the $D^*$ should be rejected and 4-momenta of the reconstructed $D^*$ mesons should be added to the same events.
3) To apply the jet cluster algorithm to the obtained sample using the same $P_T$ cut as for the inclusive events. The reconstructed jet axis can be used to obtain the angular distribution of all final-state hadrons with respect to the initial BGF quark.

4) To compare the angular distributions for the light-quark and charm samples. A difference between these two distributions at low $\Theta$ should give an estimate of the dead cone.

Fig. 7 shows the angular distributions using the prescription described above. We use $P_T > 3$ GeV cut to find jets both for the light-flavour and $c\bar{c}$ event samples. The latter contains the 4-momenta of stable $D^*$ mesons. The difference between the solid and the dashed lines gives an estimate of the dead-cone size ($\sim 25 - 30^\circ$). Note that this difference is bigger than that for the parton-level distributions due to the jet algorithm used to determine the initial-quark direction.

It should be noted that the $D$-meson fragmentation function in JETSET is a
function of the charm mass and is harder than that for light-quark mesons. Therefore, the dead cone effect can be influenced by non-perturbative effects related to the fragmentation [11]. As a final check on the above result, Fig. 8 shows the same distributions but using the HERWIG 6.1 [8] model which is based on the cluster hadronisation scheme. The model was used with default parameters and the DIS events were generated for the same kinematic range as for the AROMA. The HERWIG angular distributions are shifted to low $\Theta_h$ values, but are found to be a similar shape to those shown in Fig. 7. The difference between the solid and dashed lines seen from Fig. 8 again gives an estimate of the dead-cone size, which is smaller than that of the AROMA. This result may indicate that there exists a contribution to the observed depletion from fragmentation mechanism, however, in this paper we refrain from attempting a more detailed study of this issue.

In conclusion, a Monte-Carlo independent method to observe the dead cone for DIS or photoproduction is proposed. It is based on the measurement of particle flows close to the jet axis for heavy and light-quark samples. Using Monte Carlo simulations, it is shown that such a study worths experimental investigations, providing an estimate of the dead-cone size which is of importance for discriminating different parton-shower implementations, especially with respect to inclusion of heavy quarks.
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