Generic neutrino interactions with sterile neutrinos in light of neutrino-nucleus coherent scattering and meson invisible decays

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Abstract

In this work we study the current bounds from the CEνNS process and meson invisible decays on generic neutrino interactions with sterile neutrinos in effective field theories. The interactions between quarks and left-handed SM neutrinos and/or right-handed neutrinos are first described by the low-energy effective field theory (LNEFT) between the electroweak scale and the chiral symmetry breaking scale. We complete the independent operator basis for the LNEFT up to dimension-6 by including both the lepton-number-conserving (LNC) and lepton-number-violating (LNV) operators involving right-handed neutrinos. We translate the bounds on the LNEFT Wilson coefficients from the COHERENT observation and calculate the branching fractions of light meson invisible decays. The bounds on LNEFT are then mapped on the SM effective field theory with sterile neutrinos (SMNEFT) to constrain new physics above the electroweak scale. We find that the meson invisible decays can provide the only sensitive probe for τ neutrino flavor component and s quark component in the quark-neutrino interactions involving two (one) active neutrinos and for the effective operators without any active neutrino fields. The CEνNS process places the most stringent bound on all other Wilson coefficients. By assuming one dominant Wilson coefficient at a time in SMNEFT and negligible sterile neutrino mass, the most stringent limits on the new physics scale are 2.7 − 10 TeV from corresponding dipole operator in LNEFT and 0.5 − 1.5 TeV from neutrino-quark operator in LNEFT.

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I. INTRODUCTION

The coherent elastic neutrino-nucleus scattering (CE\textsubscript{ν}NS) process has been observed by the COHERENT experiment at the 6.7\ σ level\cite{1}. The Standard Model (SM) predicts the CE\textsubscript{ν}NS process through the Z boson exchange\cite{2} and the observation is consistent with the SM at the 1\ σ level. In the COHERENT experiment, the spallation neutron source produces prompt $\nu_\mu$ and delayed $\bar{\nu}_\mu$, $\nu_e$ which reach the low-background CsI detector. Besides the active neutrinos through the weak neutral current in the SM, any neutrino flavors including light right-handed (RH) neutrinos\footnote{RH neutrinos refer to sterile neutrinos which do not carry any SM gauge charges.} can be produced in the final state of the CE\textsubscript{ν}NS process. The COHERENT observation thus provides us an opportunity to explore the new physics (NP) associated with generic neutrino interactions in the presence of light RH neutrinos.

CE\textsubscript{ν}NS occurs when the transferred momentum during the neutrino scattering off a nucleus is smaller than the inverse of the nuclear radius. Thus, the relevant neutral currents can be well described by an effective field theory (EFT) below the electroweak (EW) scale. The low-energy effective field theory (LEFT) is an EFT defined below the electroweak scale $\Lambda_{\text{EW}} \sim 10^2$ GeV. In the LEFT, the dynamical degrees of freedom are the SM charged and neutral leptons and light quarks excluding the heavy top quark. They respect the unbroken gauge symmetries $SU(3)_c \times U(1)_{\text{em}}$ after integrating out the Higgs boson $h$, weak gauge bosons $W$, $Z$ and the top quark $t$ in the SM. The basis of LEFT operators up to dimension-6 (dim-6) has been written down in Ref.\cite{3}. If the LEFT is extended by RH neutrinos $N$, the corresponding effective field theory is named as LNEFT. An independent subset of lepton-number-conserving (LNC) operators with RH neutrinos $N$ at dim-6 in LNEFT was given in Ref.\cite{4}. In this paper we construct the additional lepton-number-violating (LNV) operators up to dim-6 involving $N$ which may or may not break the baryon number. Together with those in Refs.\cite{3,4}, they make up the complete and independent operator basis for the LNEFT up to dim-6. Moreover, to connect to NP above the electroweak scale, we match the LNEFT to the SM effective field theory extended by RH neutrinos $N$ (SMNEFT) at the electroweak scale\cite{5–8}. The SMNEFT respects the SM gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ and describes the physics above the electroweak scale up to the NP scale. In this paper we revise the SMNEFT operator basis involving $N$ in Ref.\cite{8} by changing some notations.

Recently, Refs.\cite{9–14} considered the bound from the CE\textsubscript{ν}NS on the LNC operators in LNEFT and SMNEFT. With the complete basis of LNEFT and SMNEFT, however, we can perform a comprehensive study of the constraints on both the LNC and LNV cases and investigate the implication for NP above the electroweak scale.

Besides the CE\textsubscript{ν}NS process, invisible decays of light mesons can give additional information for the nature of neutrinos. In the SM, the decay rates of $\pi^0, \eta, \eta' \rightarrow \nu\bar{\nu}$ are helicity suppressed and those of light vector mesons ($\phi$ and $\omega$) are also extremely small. Thus, the observation of any of these meson invisible decays would clearly indicate the existence of NP\cite{15–18}. Moreover, they can provide the only sensitive probe for some flavor components in the quark-neutrino in-
teractions [19] and for the effective operators without any active neutrino fields. In this work we study the correlation and complementarity of the CEνNS process and meson invisible decay for the bound on generic neutrino interactions with RH neutrinos in the frameworks of LNEFT and SMNEFT.

The paper is outlined as follows. In Sec. II, we describe the generic neutrino-photon/quark operators in the LNEFT basis. The LNEFT operators are then matched to the SMNEFT. We derive the general constraints on LNEFT Wilson coefficients (WCs) in both LNC and LNV cases from the CEνNS process in Sec. III. In Sec. IV, we give the analytical expressions for the invisible decay branching fractions of light mesons. Sec. V shows our numerical results and the lower bounds on the NP scales in the SMNEFT. Our conclusions are drawn in Sec. VI. Some calculational details are collected in the Appendixes.

II. GENERIC NEUTRINO INTERACTIONS WITH RH NEUTRINOS

The main focus of this work is on low-energy processes CEνNS and light meson invisible decays. Thus, we will start from the framework of LNEFT. The LNEFT is defined below the electroweak scale $\Lambda_{EW}$ and its dynamical degrees of freedom include the SM light particles excluding $h, W, Z, t$ and an arbitrary number of RH neutrinos $N$. The power counting of LNEFT is determined by both the NP scale $\Lambda_{NP}$ and the electroweak scale $\Lambda_{EW}$. The LNEFT consists of dim-3 fermion mass terms, dim-4 kinetic terms and higher dimensional operators $O_{i,L}^{(d)} (d \geq 5)$ (dim-$d$) built out of those light fields and satisfies the $SU(3)_c \times U(1)_{em}$ gauge symmetry. The LNEFT Lagrangian is

$$\mathcal{L}_{\text{LNEFT}} = \mathcal{L}_{d \leq 4} + \sum_i \sum_{d \geq 5} C_{i,L}^{(d)} O_{i,L}^{(d)},$$

where $C_{i,L}^{(d)}$ is the Wilson coefficient of operator $O_{i,L}^{(d)}$. Generally, the Wilson coefficient $C_{i,L}^{(d)}$ scales as $\Lambda_{EW}^{n+4-d}/\Lambda_{NP}^n$ with integer $n \geq 0$. In Appendix A we construct the complete and independent operator basis involving RH neutrinos $N$ up to dim-6 in the LNEFT for the study of generic neutrino interactions.

We assume the LNEFT is a low-energy version of the SMNEFT which is defined above the electroweak scale. In the SMNEFT, the renormalizable SM Lagrangian is extended by the RH neutrino sector and a tower of higher dimensional effective operators $O_{i}^{(d)}$ with increasing canonical dimension $d \geq 5$. The importance of these operators is measured by the Wilson coefficients $C_{i}^{(d)}$ with decreasing relevance

$$\mathcal{L}_{\text{SMNEFT}} = \mathcal{L}_{\text{SM}+N} + \sum_i \sum_{d \geq 5} C_{i}^{(d)} O_{i}^{(d)},$$

where $\mathcal{L}_{\text{SM}+N}$ is the renormalizable SM Lagrangian extended by RH neutrinos $N$. Generally, each Wilson coefficient $C_{i}^{(d)}$ is associated with a NP scale $\Lambda_{NP} = (C_{i}^{(d)})^{1/(4-d)}$. For a given NP model, it can be precisely expressed as the function of the parameters in the NP model through matching
and renormalization group running procedures. In Appendix B we collect the relevant SMNEFT operators used in our analysis for the generic neutrino interactions.

A. Generic neutrino operators in LNEFT basis

The generic neutrino operators entering the framework of LNEFT respect SU(3)$_c \times$U(1)$_{\text{em}}$ gauge symmetry and are constructed by a neutrino bilinear coupled to the photon field strength tensor or SM quark bilinear currents. In the basis of LNEFT for neutrinos, the dim-5 neutrino-photon and dim-6 neutrino-quark operators with lepton number conservation (LNC, $|\Delta L| = 0$) are given by [3]

\[ O_{\nu,NF} = (\overline{\nu}_\mu \sigma_{\mu \nu} N) F^{\mu \nu} + h.c. , \]
\[ O_{\nu2}^V = (\overline{\nu}_R \gamma_\mu q_R)(\overline{\nu}_\mu \gamma_\nu N) , \]
\[ O_{\nu2}^S = (\overline{\nu}_R \gamma_\mu q_R)(\overline{\nu}_R \gamma_\nu N) , \]
\[ O_{\nu2}^T = (\overline{\nu}_R \sigma_{\mu \nu} q_R)(\overline{\nu}_\mu \gamma_{\nu \alpha} N) + h.c. , \]
\[ O_{\nu2}^{V,V} = (\overline{\nu}_R \gamma_\mu q_R)(\overline{\nu}_R \gamma_\nu N) + h.c. , \]
\[ O_{\nu2}^{V,S} = (\overline{\nu}_R \gamma_\mu q_R)(\overline{\nu}_R \gamma_\nu N) + h.c. , \]
\[ O_{\nu2}^{V,T} = (\overline{\nu}_R \gamma_\mu q_R)(\overline{\nu}_R \gamma_\nu N) + h.c. , \]
\[ O_{\nu2}^{T,V} = (\overline{\nu}_R \gamma_\mu q_R)(\overline{\nu}_R \gamma_\nu N) + h.c. , \]
\[ O_{\nu2}^{T,S} = (\overline{\nu}_R \gamma_\mu q_R)(\overline{\nu}_R \gamma_\nu N) + h.c. , \]
\[ O_{\nu2}^{T,T} = (\overline{\nu}_R \gamma_\mu q_R)(\overline{\nu}_R \gamma_\nu N) + h.c. , \]

where $F^{\mu \nu}$ is the electromagnetic field strength tensor, $q$ can be either up-type quarks $u_i$ or down-type quarks $d_i$, $\nu_i$ are active left-handed neutrinos, and $N_i$ are RH neutrinos. The quark fields and the RH neutrino fields are in the mass basis, while the LH neutrino fields are in the flavor basis. Both $\nu_i$ and $N_i$ carry lepton number $L(\nu_i) = L(N_i) = +1$. The flavors of the two quarks and those of the two neutrinos in the above operators can be different although we do not specify their flavor indexes here. For the notation of the Wilson coefficients, we use the same subscripts as the operators, for instance $C_{V,pr\alpha\beta}$ together with $O_{\nu2}^{V,pr\alpha\beta}$, where $p, r$ denote the quark flavors and $\alpha, \beta$ are the neutrino flavors. We demand the vector operators to be hermitian, i.e. $C_X^{V,pr\alpha\beta} = C_X^{V,rp\beta\alpha*,}$ with $X = q\nu 1, q\nu 2, qN1, qN2$, to ignore the h.c. in Eqs. (4) and (5).

The relevant dim-5 and dim-6 operators which induce lepton number violation (LNV, $|\Delta L| = 2$) are

\[ O_{\nu2}^{N,NF} = (\overline{\nu}_R \gamma_\mu q_R)(\overline{\nu}_R \gamma_\nu N) + h.c. , \]
\[ O_{\nu2}^{N,V,V} = (\overline{\nu}_R \gamma_\mu q_R)(\overline{\nu}_R \gamma_\nu N) + h.c. , \]
\[ O_{\nu2}^{N,S,V} = (\overline{\nu}_R \gamma_\mu q_R)(\overline{\nu}_R \gamma_\nu N) + h.c. , \]
\[ O_{\nu2}^{N,T,V} = \overline{\nu}_R \sigma_{\mu \nu} q_R)(\overline{\nu}_R \gamma_\nu N) + h.c. , \]
\[ O_{\nu2}^{N,S,S} = (\overline{\nu}_R \gamma_\mu q_R)(\overline{\nu}_R \gamma_\nu N) + h.c. , \]
\[ O_{\nu2}^{N,T,S} = (\overline{\nu}_R \sigma_{\mu \nu} q_R)(\overline{\nu}_R \gamma_\nu N) + h.c. , \]
\[ O_{\nu2}^{N,T,T} = (\overline{\nu}_R \sigma_{\mu \nu} q_R)(\overline{\nu}_R \gamma_\nu N) + h.c. , \]

Note that the Wilson coefficients of the scalar operators are symmetric in the neutrino indices and the dipole and tensor operators are antisymmetric in the neutrino indices. Thus in particular

\[ \Delta \]

2 The operators involving charged leptons are not related to the processes of interest and thus we do not consider them here.
the operators with the tensor neutrino current $\tilde{\nu}_\alpha^{C} \sigma^{\mu\nu} \nu_\beta$ or $N^C_{\alpha} \sigma^{\mu\nu} N_\beta$ vanish for identical neutrino flavors ($\alpha = \beta$).

LNEFT is a valid description between the electroweak scale $\Lambda_{\text{EW}} = m_W$ and chiral symmetry breaking scale $\Lambda_\chi \simeq 1$ GeV for the low-energy neutrino-quark interactions. There are large logarithms produced by the ratio of the two scales in the perturbative expansion which can be resummed by solving the relevant renormalization group equations. In our case, the leading order contribution comes from the one-loop QCD and QED corrections. The vector (and axial-vector) current operators are not renormalized at one-loop level because of the QED and QCD Ward identities. However, the scalar and tensor current operators are renormalized and their one-loop renormalization group equations for the corresponding Wilson coefficients are \[20\]

\[
\mu \frac{d}{d\mu} C^S_q = -\left(\frac{\alpha_s}{2\pi} 3C_F + \frac{\alpha}{2\pi} 3Q_q^2\right) C^S_q, \quad C^S_q \in \{C^S_{q,v N_1}, C^S_{q,v N_2}, C^S_{q,v 1}, C^S_{q,v 2}, C^S_{q,N_1}, C^S_{q,N_1}\},
\]

\[
\mu \frac{d}{d\mu} C^T_q = \left(\frac{\alpha_s}{2\pi} C_F + \frac{\alpha}{2\pi} Q_q^2\right) C^T_q, \quad C^T_q \in \{C^T_{q,v N_1}, C^T_{q,v q_1}, C^T_{q,v q_1}\},
\]

where $C_F = (N_c^2 - 1)/2N_c = 4/3$ with $N_c = 3$ is the second Casimir invariant of the color group $SU(3)_c$, $Q_q$ is the electric charge of quark field $q$ in the corresponding operator in unit of positron’s charge $e$, and $\alpha = e^2/(4\pi)(\alpha_s = g_s^2/(4\pi))$ is the (strong) fine structure constant. The solutions for the above equations are straightforward and are given by

\[
C^S_q(\mu_1) = \left(\frac{\alpha_s(\mu_2)}{\alpha_s(\mu_1)}\right)^{3C_F/b} \left(\frac{\alpha(\mu_2)}{\alpha(\mu_1)}\right)^{3Q_q^2/b_e} C^S_q(\mu_2),
\]

\[
C^T_q(\mu_1) = \left(\frac{\alpha_s(\mu_2)}{\alpha_s(\mu_1)}\right)^{-C_F/b} \left(\frac{\alpha(\mu_2)}{\alpha(\mu_1)}\right)^{-Q_q^2/b_e} C^T_q(\mu_2),
\]

between two scales $\mu_1$ and $\mu_2$. Here $b = -11 + 2/3n_f$ with $n_f$ being the number of active quark flavors between scales $\mu_1$ and $\mu_2$, and $b_e = \sum_i \frac{\frac{1}{2} (N_c - 1) Q_i^2}{2(3n_\ell + 4n_u + 4n_d)/9}$ with $n_{\ell,u,d}$ being the active number of leptons/up-type quarks/down-type quarks between the two scales. If we take the scale $\mu_1$ ($\mu_2$) to be $\Lambda_\chi$ ($\Lambda_{\text{EW}}$), after including quark and lepton threshold effects, the numerical results are

\[
C^S_u(\Lambda_\chi) = 1.67 C^S_u(\Lambda_{\text{EW}}), \quad C^S_d(\Lambda_\chi) = 1.66 C^S_d(\Lambda_{\text{EW}}),
\]

\[
C^T_u(\Lambda_\chi) = 0.85 C^T_u(\Lambda_{\text{EW}}), \quad C^T_d(\Lambda_\chi) = 0.84 C^T_d(\Lambda_{\text{EW}}).
\]

Compared to the pure QCD running effect in [21], we find the QED correction is almost negligible. We can see the scalar-type operators are enhanced while the tensor-type operators are suppressed when evolving from the high scale $\Lambda_{\text{EW}}$ down to the low scale $\Lambda_\chi$.

For the neutrino dipole operators, the renormalization group evolution is

\[
\mu \frac{d}{d\mu} C_{\bar{\nu} \alpha \beta}^i_F = -\frac{\alpha}{2\pi} \frac{b_e}{2} C^{iF}_{\bar{\nu} \alpha \beta} - \frac{eN_c}{2\pi^2} \sum_r Q_q m_q C^{T,rr\alpha \beta}_{qr} \theta(\mu - m_q),
\]
where $C_{iF} \in \{ C_{\nu,N}, C_{\nu,F}, C_{NNF} \}$ and $C_{q_i}^T \in \{ C_{q_N}^{T}, C_{q_k}^{T}, C_{q_N}^{T} \}$ and $\theta$ is the Heaviside theta function. It includes the one-loop QED running of $C_{iF}$ \cite{20} which is the first term on the right-hand side of Eq. (16). Note that, at one-loop order, the renormalization group evolution of the Wilson coefficients of the tensor operators $O_{q_N}^{T}$, $O_{q_k}^{T}$ and $O_{q_N}^{T}$ induces a mixing into the dim-5 dipole operators $O_{\nu,NF}$, $O_{\nu,F}$ and $O_{NNF}$, that is the last term of Eq. (16)\textsuperscript{3}. The solution is

$$
C_{iF}(\Lambda_{EW}) = 1.03 C_{iF}(\Lambda_{EW}) + 3.0 \times 10^{-4} \text{ GeV} \, C_{u1}^{T}(\Lambda_{EW}) - 3.2 \times 10^{-4} \text{ GeV} \, C_{d1}^{T}(\Lambda_{EW}) - 6.4 \times 10^{-3} \text{ GeV} \, C_{s1}^{T}(\Lambda_{EW}) + 0.16 \text{ GeV} \, C_{u2}^{T}(\Lambda_{EW}) - 0.18 \text{ GeV} \, C_{b2}^{T}(\Lambda_{EW}) . \quad (17)
$$

We will numerically include the effect of the above renormalization group corrections when running up to the electroweak scale below. Due to the suppression by the light quark mass, the constraint on the tensor operators through the above mixing effect would be rather weak and thus we will not consider it in the following analysis. Note that, however, it can lead to a constraint on the WCs with heavy quark flavors which is beyond the scope of this work.

B. Matching to the SMNEFT

SMNEFT describes NP which enters at a sufficiently high scale above the electroweak scale. See Appendix B for a complete list of SMNEFT operators involving RH neutrinos $N$ up to dim-7 and the relevant dim-6 and dim-7 operators without $N$. LNEFT should be matched to SMNEFT at the electroweak scale $\mu = m_W$ in order to constrain NP. We list the relevant tree-level matching conditions for the LNC and LNV cases in Table I. Here $v = (\sqrt{2} G_F)^{-1/2} \approx 246 \text{ GeV}$ is the SM Higgs vacuum expectation value (vev), and $D$ is the unitary matrix transforming left-handed up-type quarks between flavor $u'_L$ and mass eigenstate $u_L$, i.e. $u'_L = D^\dagger u_L$. Under a chosen flavor basis, the flavor and mass eigenstates are identical for the left-handed down-type quarks and RH $u, d$ quarks, and $D$ is then the usual CKM matrix.

Note that SMNEFT operators modify the $Z$ boson couplings from their SM values $\left[Z_f\right]_{pr} = \delta_{pr} (T_3 - Q S_W^2)$ and the modified couplings are given by \cite{3}

$$
\begin{align*}
\left[Z'_\nu\right]_{pr} &= \frac{1}{2} \delta_{pr} - \frac{v^2}{2} \left( C_{H1}^{(1),pr} - C_{H1}^{(3),pr} \right), \\
\left[Z_{e_L}\right]_{pr} &= \left( -\frac{1}{2} + s_W^2 \right) \delta_{pr} - \frac{v^2}{2} \left( C_{H1}^{(1),pr} + C_{H1}^{(3),pr} \right), \\
\left[Z_{u_L}\right]_{pr} &= \left( \frac{1}{2} - \frac{2}{3} s_W^2 \right) \delta_{pr} - \frac{v^2}{2} \left( C_{Hq}^{(1),pr} - C_{Hq}^{(3),pr} \right), \\
\left[Z_{d_L}\right]_{pr} &= \left( -\frac{1}{2} + \frac{1}{3} s_W^2 \right) \delta_{pr} - \frac{v^2}{2} \left( C_{Hq}^{(1),pr} + C_{Hq}^{(3),pr} \right), \\
\left[Z_{\nu N}\right]_{pr} &= \frac{v^3}{4\sqrt{2}} \left( C_{NLL1}^{np} + 2 C_{NLL2}^{np} \right), \\
\left[Z_{\nu F}\right]_{pr} &= \frac{v^3}{4\sqrt{2}} \left( C_{FLL1}^{np} + 2 C_{FLL2}^{np} \right) .
\end{align*}
\quad (18)
$$

\textsuperscript{3} The dipole running also receives a similar contribution from the LNV tensor neutrino-charged lepton operators without the color factor $N_c$.  

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where \([Z_N]_{\nu\nu}\) is the modified neutral current coupling to the RH neutrinos defined via \(L_Z \supset -\tilde{\gamma}_2[Z_N]_{\nu\nu}\bar{N}_r\bar{N}_l\) and \([Z_{\nu N}]_{\nu\nu}\) is the modified neutral current coupling to the LNV current \(\tilde{\gamma}_3\gamma_\mu N\) defined via \(L_Z \supset -\tilde{\gamma}_2[Z_{\nu N}]_{\nu\nu}\gamma_\mu N_r + h.c.\). For simplicity, we do not consider the couplings \([Z_N]\) and \([Z_{\nu N}]\) modified by \(C_{HN}, C_{NL1}\) and \(C_{NL2}\) below. One has \(\tilde{g}_2 = \frac{e}{s_{\text{EW}}c_{\text{EW}}} \) with \(s_{\text{EW}}(c_{\text{EW}})\) being the sine (cosine) of the Weinberg angle as we neglect the contributions from SMEFT operators in the Higgs sector. In the following discussion we also do not consider contributions from the operators \(C_{H_1}^{(1)}, C_{H_1}^{(3)}, C_{R_{Hq}}^{(1)}, C_{H_{q1}}, C_{H_{q2}}\) and \(C_{H_{q3}}\), since they are strongly constrained from electroweak precision measurements.

As only the quark-flavor diagonal \(u, d\) or \(s\) quark bilinears contribute to the CE\(\nu\)NS process and the light unflavored meson invisible decays, we will simplify the notation for the LNEFT operators and the corresponding Wilson coefficients by dropping the superscripts for the quark fields and taking the subscript \(q\) to be either \(u\), or \(d\), or \(s\) to indicate the specific quark flavor. For instance, \(\mathcal{O}_{\nu_1}^V = (\bar{u}_l\gamma_\mu u_L)(\bar{\nu}_r\gamma^\mu\nu)\) and the corresponding Wilson coefficient is \(C_{\nu_1}^V\) for \(q = u\).

| Class | Matching of the Wilson coefficients at the electroweak scale \(\Lambda_{\text{EW}}\) |
|-------|--------------------------------------------------------------------------------------------------|
| LNC   | \(C_{\nu_1}^{V,\text{pro3}}\) = \(D_{\nu_1}D_{\nu_2}^*\left(\delta_{\nu_1\nu_2}\right)\) |
| \(\nu\nu\) case | \(C_{\nu_1}^{V,\text{pro3}}\) = \(\frac{1}{2} D_{\nu_1}D_{\nu_2}^*\left(\delta_{\nu_1\nu_2}\right)\) |
| LNV   | \(C_{\nu_1}^{V,\text{pro3}}\) = \(D_{\nu_1}D_{\nu_2}^*\left(\delta_{\nu_1\nu_2}\right)\) |
| \(\nu\nu\) case | \(C_{\nu_1}^{V,\text{pro3}}\) = \(\frac{1}{2} D_{\nu_1}D_{\nu_2}^*\left(\delta_{\nu_1\nu_2}\right)\) |

\(C_{\nu_1}^{V,\text{pro3}}\) is the modified neutral current coupling to the RH neutrinos defined via \(L_Z \supset -\tilde{\gamma}_2[Z_N]_{\nu\nu}\bar{N}_r\bar{N}_l\) and \([Z_{\nu N}]_{\nu\nu}\) is the modified neutral current coupling to the LNV current \(\tilde{\gamma}_3\gamma_\mu N\) defined via \(L_Z \supset -\tilde{\gamma}_2[Z_{\nu N}]_{\nu\nu}\gamma_\mu N_r + h.c.\). For simplicity, we do not consider the couplings \([Z_N]\) and \([Z_{\nu N}]\) modified by \(C_{HN}, C_{NL1}\) and \(C_{NL2}\) below. One has \(\tilde{g}_2 = \frac{e}{s_{\text{EW}}c_{\text{EW}}} \) with \(s_{\text{EW}}(c_{\text{EW}})\) being the sine (cosine) of the Weinberg angle as we neglect the contributions from SMEFT operators in the Higgs sector. In the following discussion we also do not consider contributions from the operators \(C_{H_1}^{(1)}, C_{H_1}^{(3)}, C_{R_{Hq}}^{(1)}, C_{H_{q1}}, C_{H_{q2}}\) and \(C_{H_{q3}}\), since they are strongly constrained from electroweak precision measurements.

As only the quark-flavor diagonal \(u, d\) or \(s\) quark bilinears contribute to the CE\(\nu\)NS process and the light unflavored meson invisible decays, we will simplify the notation for the LNEFT operators and the corresponding Wilson coefficients by dropping the superscripts for the quark fields and taking the subscript \(q\) to be either \(u\), or \(d\), or \(s\) to indicate the specific quark flavor. For instance, \(\mathcal{O}_{\nu_1}^V = (\bar{u}_l\gamma_\mu u_L)(\bar{\nu}_r\gamma^\mu\nu)\) and the corresponding Wilson coefficient is \(C_{\nu_1}^V\) for \(q = u\).
III. NEUTRINO-NUCLEUS COHERENT SCATTERING

In the COHERENT experiment, the Spallation Neutron Source produces $\nu_\mu$, $\bar{\nu}_\mu$ and $\nu_e$ from the decay of stopped $\pi^+$ and $\mu^+$. Each neutrino flavor reaches the CsI[Na] detector and contributes to the neutrino flux. The expected number of CE$\nu$NS events depends on the neutrino flux and the CE$\nu$NS differential cross section $d\sigma/dT$ with $T$ being the recoil energy of the nucleus. The differential cross section for $(\nu/N \rightarrow XN)$ coherent scattering, where $X \in \{\nu, \bar{\nu}, N, \bar{N}\}$ denotes a neutrino, is at leading order given by [13, 22]

$$d\sigma\over dT = \frac{G_F^2 M}{4\pi} \left[ \xi_S^2 \frac{T}{T_{\text{max}}} + \xi_V^2 \left(1 - \frac{T}{T_{\text{max}}}\right) + \xi_T^2 \left(1 - \frac{T}{2T_{\text{max}}}\right) + e^2 A_M^2 \left(\frac{1}{MT} - \frac{1}{ME_{\nu}}\right)\right]$$, \hspace{0.5cm} (19)

where $M$ is the nucleus mass, $E_\nu$ is the energy of the incoming neutrino and the maximal value of recoil energy $T$ is $T_{\text{max}} = \frac{2E^2_{\nu}}{M+2E_\nu} \simeq \frac{2E^2_{\nu}}{M}$. This cross section formula holds for negligible neutrino mass in final states, and thus applies for RH neutrino masses $m_N \lesssim 0.5$ MeV (see e.g. Ref. [13]) irrespective of the mixing between LH and RH neutrinos. The interference terms are suppressed by $T/E_\nu[13]$ and are thus not included here.

The $\xi_S, \xi_V, \xi_T$ and $A_M$ constants in Eq. (19) define the effective parameters describing the neutrino-nucleus interactions for scalar, vector, tensor and dipole currents, respectively. They depend on the Wilson coefficients of relevant currents, the number of protons $Z$ (and neutrons $N$) in the nucleus, the quantities connecting the quark-level matrix elements and the nucleon-level ones, and the nuclear form factor $F_p$ for protons (and $F_n$ for neutrons). By assuming that one single parameter is present at nuclear level at a time, the constraints on these effective parameters were studied through fitting the COHERENT data. The 90% CL bounds for the $\xi_S$ and $\xi_T$ parameters are [10]

$$\left|\frac{\xi_S}{NF(Q^2)}\right| < 0.62 \hspace{0.5cm} , \hspace{0.5cm} \left|\frac{\xi_T}{NF(Q^2)}\right| < 0.591 \hspace{0.5cm} ,$$ \hspace{0.5cm} (20)

where $F(Q^2)$ is the Helm form factor with $Q$ being the transferred energy. The 90% CL bound on the dipole operators is given by [13]

$$A_M = a_M v Z F_p(Q^2) \hspace{0.2cm} , \hspace{0.5cm} \frac{1}{2} a_M^2 \lesssim 7.2 \times 10^{-8} \hspace{0.5cm} ,$$ \hspace{0.5cm} (21)

where the factor of $1/2$ accounts for the missing projection operator in the cross section calculation in Ref. [13]. The scalar, tensor and dipole operators have no interference with the SM neutral current and the above bounds apply to both LNC and LNV cases.

For the vector currents the situation is more complicated and we have to distinguish between LNC and LNV operators. As listed in Table I, there is a SM contribution to the LNC vector operators with same-flavor quarks and same-flavor active neutrinos

$$C_{q\nu l(2), \nu}^{V,\nu} = -\frac{g_2}{2M_Z^2} (T_3 - Q_s^W) \delta_{\nu l} \delta_{\nu l}$$ \hspace{0.5cm} (22)
in terms of isospin $T_3$ and electric charge $Q_q$. Thus, the interference with the SM has to be taken into account for the NP part of these operators. There is no interference with the SM for the other vector operators. We thus discuss the constraints separately in the following subsections based on the recent studies of the COHERENT experiment for non-standard interactions (NSIs)\textsuperscript{4} \cite{23} and sterile neutrinos \cite{13}. Both of these studies provide constraints on the quark-level Wilson coefficients. Next we derive the matrix elements of scattering processes in terms of the LNEFT and sterile neutrinos \cite{13}. We thus discuss the constraints separately in the following subsections based on the short-distance contribution induced by the four-fermion operators. The matrix elements at the nucleus level for the scattering $\nu_\alpha(p_1)N(k_1) \to \nu_\beta/N_\beta(p_2)N(k_2)$ and $\bar{\nu}_\alpha(p_1)N(k_1) \to \bar{\nu}_\beta/N_\beta(p_2)N(k_2)$ are

$$\mathcal{M}(\nu_\alpha N \to \nu_\beta N) = \frac{1}{2} C^V_{\alpha\beta}(\bar{u}_\nu \gamma_\mu P_L u_\nu) N \gamma^\mu N,$$

$$-\mathcal{M}(\bar{\nu}_\alpha N \to \bar{\nu}_\beta N) = \frac{1}{2} C^V_{\alpha\beta}(\bar{u}_\nu \gamma_\mu P_R v_\nu) N \gamma^\mu N,$$

$$\mathcal{M}(\nu_\alpha N \to N_\beta N) = \frac{1}{2} C^S_{\alpha\beta}(\bar{u}_\nu P_L u_\nu) N N + C^T_{\alpha\beta}(\bar{u}_\nu \gamma_\mu P_R v_\nu) N \sigma^\mu N,$$

$$-\mathcal{M}(\bar{\nu}_\alpha N \to \bar{N}_\beta N) = \frac{1}{2} C^S_{\alpha\beta}(\bar{u}_\nu P_R v_\nu) N N + C^T_{\alpha\beta}(\bar{u}_\nu \gamma_\mu P_L u_\nu) N \sigma^\mu N,$$

where the spin-dependent terms are neglected as they are suppressed by $O(E/m_{p/n})$ with respect to spin-independent terms. One should note that the terms with tensor quark current have the property $\sigma^\mu P_{L/R} \otimes \sigma^\nu P_{L/R} = \sigma_{\mu\nu} P_{L/R} \otimes \sigma^\nu P_{L/R}$ and thus do not lead to spin-dependent terms. The matrix elements at the quark level are given in Appendix D as reference. In the above matrix elements for nucleus “i”, the coefficients

$$C^V_{\alpha\beta} = \sum_{q=u,d,s} \left[ \frac{1}{2} (C^V_{u\alpha} + C^V_{u\beta}) + C^V_{d\beta} + C^V_{d\alpha} \right] F_p(Q^2)$$

$$+ N_i \left[ C^V_{u\alpha} + C^V_{u\beta} + 2(C^V_{d\alpha} + C^V_{d\beta}) \right] F_n(Q^2),$$

$$C^S_{\alpha\beta} = \sum_{q=u,d,s} \left[ C^S_{q\nu N_1} N_1 + C^S_{q\nu N_2} \right] \left[ \sum_{m} \frac{m_p}{m_q} f_{T_q} F_p(Q^2) + \sum_{m} \frac{m_n}{m_q} f_{T_q} F_n(Q^2) \right],$$

\textsuperscript{4} The relationship between the chiral LNEFT operator basis and NSIs is discussed in Appendix C 1.
\[ C^{T,\alpha\beta}_{N\nu N} = \sum_{q=u,d,s} C^{T,\alpha\beta}_{q\nu N} \left[ Z_q \delta q F_p(Q^2) + N_i \delta n F_n(Q^2) \right] \]  

(25)

parameterize the vector, scalar and tensor contributions [12]. The number of neutrons and protons for Caesium and Iodine are \( N_{Cs} = 77.9, Z_{Cs} = 55 \) and \( N_{I} = 73.9, Z_{I} = 53 \), respectively. We assume the proton and neutron form factors are equal to the Helm form factor, i.e. \( F_p(Q^2) = F_n(Q^2) = F(Q^2) \). The connections between various quark currents and the nucleon-level ones can be found for instance in Refs. [24, 25]. \( f_{T,q}^{p/n} \) and \( \delta q^{p/n} \) are the nucleon form factors for scalar and tensor currents, respectively. For later numerical analysis, we take the following default values from micrOMEGAs 5.2 [26, 27]

\[
\begin{align*}
  f_{T,u}^p &= 0.0153, & f_{T,d}^p &= 0.0191, & f_{T,s}^p &= 0.0447, \\
  \delta_u^p &= 0.84, & \delta_d^p &= -0.23, & \delta_s^p &= -0.046 , \\
  f_{T,u}^n &= 0.0110, & f_{T,d}^n &= 0.0273, & f_{T,s}^n &= 0.0447, \\
  \delta_u^n &= -0.23, & \delta_d^n &= 0.84, & \delta_s^n &= -0.046 .
\end{align*}
\]  

(26)

Using the expressions for the matrix elements in Eq. (24) it is straightforward to compare them with those in Refs. [9, 10] and relate the LNEFT Wilson coefficients to the \( \xi \) parameterization. We obtain the following constraints on the scalar and tensor coefficients

\[
\begin{align*}
  \frac{\xi_S^2}{N^2 F^2} &= \sum_{\beta,i} \left| \frac{1}{2G_F} \sum_{q=u,d,s} \left( C^{S,\alpha\beta}_{q\nu N} + C^{S,\alpha\beta}_{q\nu N} \right) \left( \frac{Z_q}{N_i} \frac{m_p}{m_q} f_{T,q}^p + \frac{m_n}{m_q} f_{T,q}^n \right) \right|^2 < 0.62^2 , \\
  \frac{\xi_T^2}{N^2 F^2} &= 8 \sum_{\beta,i} \left| \frac{\sqrt{2}}{G_F} \sum_{q=u,d,s} C^{T,\alpha\beta}_{q\nu N} \left( \frac{Z_q}{N_i} \delta q^p + \delta q^n \right) \right|^2 < 0.591^2 .
\end{align*}
\]  

(27)  

(28)

These bounds apply for initial state neutrino flavor \( \alpha = e \) or \( \mu \). The 90% CL bounds on the quark-level vector Wilson coefficients can be read off from Fig. 12 in Ref. [23]. There is interference between the NP contribution and the SM contribution for LNC vector operators with same-flavor active neutrinos. The interference leads to the following constraints for the NP part of the Wilson coefficient for neutrino flavors \( ee \) and \( \mu\mu \)

\[
\begin{align*}
  C_{V,ee}^{NP} + C_{V,ee}^{NP} &= \frac{-0.45, 0.065}{} , \\
  \frac{C_{V,ee}^{NP} + C_{V,ee}^{NP}}{2\sqrt{2}G_F} &\in [-0.45, 0.060] , \\
  \frac{C_{V,\mu\mu}^{NP} + C_{V,\mu\mu}^{NP}}{2\sqrt{2}G_F} &\in [-0.45, -0.34] \cup [-0.049, 0.059] , \\
  \frac{C_{V,\mu\mu}^{NP} + C_{V,\mu\mu}^{NP}}{2\sqrt{2}G_F} &\in [-0.41, -0.31] \cup [-0.044, 0.054] .
\end{align*}
\]  

(29)

\footnote{The relationship between the chiral LNEFT operator basis and the quark-level parameterization in Ref. [10] is given in Appendix C 2}
Note that the allowed region for the operators $C_{qν1(2),NP}^{V,μμ}$ consists of two disjoint pieces. There is no interference for the other LNC vector operators

\[
\frac{C_{uv1,NP}^{V,μμ} + C_{uv1,NP}^{V,ττ}}{2\sqrt{2}G_F} < 0.13, \quad \frac{C_{dv1,NP}^{V,μμ} + C_{dv1,NP}^{V,ττ}}{2\sqrt{2}G_F} < 0.11, \quad (30)
\]

\[
\frac{C_{uv1,NP}^{V,ττ} + C_{uv1,NP}^{V,μμ}}{2\sqrt{2}G_F} < 0.18, \quad \frac{C_{dv2,NP}^{V,ττ} + C_{dv2,NP}^{V,μμ}}{2\sqrt{2}G_F} < 0.17, \quad (31)
\]

\[
\frac{C_{uv2,NP}^{V,ττ} + C_{uv2,NP}^{V,μμ}}{2\sqrt{2}G_F} < 0.16, \quad \frac{C_{dv1,NP}^{V,μμ} + C_{dv1,NP}^{V,ττ}}{2\sqrt{2}G_F} < 0.15, \quad (32)
\]

and thus their allowed regions are symmetric around zero. In the following numerical analysis, we will use the weakest bounds for the Wilson coefficients with diagonal neutrino flavors $ee$ and $μμ$ in Eq. (29) to obtain conservative constraints. Note that, if one adopts other bounds, a stronger limit for the relevant Wilson coefficient and a larger corresponding NP scale will be obtained.

For the long-distance contribution induced by the dipole operator $O_{νNF}$, the nucleon-level matrix elements are

\[
M(ν_αN → N_βN) = \frac{eG_F}{q^2} A_{LνNF}^{αβ}(ν_μσ_μPLu_ν)\bar{N}γ^μτ^νN, \quad (33)
\]

\[-M(\bar{ν}_αN → \bar{N}_βN) = \frac{eG_F}{q^2} A_{LνNF}^{αβ}(ν_μσ_μP_Rν_ν)\bar{N}γ^μτ^νN, \quad (33)\]

where the transferred 4-momentum is given by $q = p_1 - p_2 = k_2 - k_1$ and thus we find

\[
A_{LνNF}^2 = 2^2 \sum_β \left| \frac{2}{G_F} A_{LνNF}^{αβ} \right|^2 F_μ^2(Q^2). \quad (34)
\]

The constraint is given in Ref. [13]

\[
\frac{1}{2} A_{LνNF}^2 = \frac{1}{2} \sum_β \left| \frac{2}{G_F} A_{LνNF}^{αβ} \right|^2 < 7.2 \times 10^{-8}. \quad (35)
\]

B. LNV case

For LNV operators with at least one active neutrino $ν$, the relevant Lagrangian is

\[
\mathcal{L}_{LNV} \supset \sum_{q_ν1,q_ν2} \left[ C_{qν1}^{S,αβ} (qRqL)(v^C_ν ν_α) + C_{qν2}^{S,αβ} (qLqR)(\bar{v}^C_ν ν_α) + C_{qν}^{T,αβ} (qRσ^μνqL)(\bar{v}^C_ν σ_μνν_α) + C_{qν}^{T,αβ} (qRγ^μνqL)(\bar{v}^C_ν γ^μνν_α) + C_{qν}^{T,αβ} (qRγ^μνqL)(\bar{v}^C_ν σ^μνν_α) + C_{qν}^{T,αβ} (qRγ^μνqL)(\bar{v}^C_ν γ^μνν_α) + C_{qν}^{T,αβ} (qRγ^μνqL)(\bar{v}^C_ν σ^μνν_α) \right] + h.c. \quad (36)
\]

It leads to the following matrix elements at the nucleus level

\[
M(ν_αN → \bar{ν}_βN) = C_{N_ν}^{S,αβ}(v^C_ν PLu_ν)\bar{N}N - 2C_{N_ν}^{T,αβ}(v^C_ν σ_μPLu_ν)\bar{N}_σ^μνN, \quad M(\bar{ν}_αN → ν_βN) = C_{N_ν}^{S,αβ}(\bar{v}^C_ν P_Ru_ν)\bar{N}N - 2C_{N_ν}^{T,αβ}(\bar{v}^C_ν P_Rσ_μP_Ru_ν)\bar{N}_σ^μνN, \quad (37)
\]
\[ M(\nu_{\alpha}\bar{N} \rightarrow \bar{N}_\beta N) = -\frac{1}{2} C_{N\nu N}^{V,\alpha\beta}(\overline{\nu}_N^\gamma \gamma_{\mu} P_L u_{\nu}) \bar{N} \gamma^\mu N , \]
\[ M(\bar{\nu}_{\alpha}\bar{N} \rightarrow N_\beta \bar{N}) = -\frac{1}{2} C_{N\nu N}^{V,\alpha\beta*}(\overline{\nu}_N^\gamma \gamma_{\mu} P_L u_{\nu}) \bar{N} \gamma^\mu N , \]

with

\[ C_{N\nu N}^{V,\alpha\beta} = Z_i [2(C_{u\nu N1}^{V,\alpha\beta} + C_{u\nu N2}^{V,\alpha\beta}) + (C_{d\nu N1}^{V,\alpha\beta} + C_{d\nu N2}^{V,\alpha\beta})] F_p(Q^2) \]
\[ + N_i [(C_{u\nu N1}^{V,\alpha\beta} + C_{u\nu N2}^{V,\alpha\beta}) + 2(C_{d\nu N1}^{V,\alpha\beta} + C_{d\nu N2}^{V,\alpha\beta})] F_n(Q^2) , \]
\[ C_{N\nu N}^{S,\alpha\beta} = \sum_{q=u,d,s} (C_{q\nu 1}^{S,\alpha\beta} + C_{q\nu 2}^{S,\alpha\beta}) [Z_i m_p f_{n q}^p + N_i m_n f_{n q}^n] F_p(Q^2) \]
\[ + \sum_{q=u,d,s} C_{q\nu 2}^{T,\alpha\beta} [Z_i \delta_p^q f_{n q}^p + N_i \delta_n^q f_{n q}^n] \] (38)

The quark-level matrix elements in LNV case are also given in Appendix D.

For the scalar and tensor coefficients we again relate them to the \( \xi \) parameterization and get the following constraints

\[ \frac{\xi^2_S}{N^2 F^2} = \sum_{\alpha, \beta} \left| \frac{\sqrt{2}}{G_F} \sum_{q=u,d,s} (C_{q\nu 1}^{S,\alpha\beta} + C_{q\nu 2}^{S,\alpha\beta}) (Z_i m_p f_{n q}^p + N_i m_n f_{n q}^n) \right|^2 < 0.62^2 , \]
\[ \frac{\xi^2_T}{N^2 F^2} = 8 \sum_{\alpha, \beta} \left( \frac{2\sqrt{2}}{G_F} \sum_{q=u,d,s} C_{q\nu 1}^{T,\alpha\beta} (Z_i \delta_p^q + \delta_n^q) \right)^2 < 0.591^2 . \]

The above constraints apply for \( \alpha = e, \mu \) for initial neutrino flavors. The LNV vector currents lead to RH neutrinos in scattering final states and such process has been studied in Ref. [13] through fitting the COHERENT data for the \( \nu N \rightarrow \chi N \) scattering. After comparing the amplitudes and translating the bound developed in Ref. [13], we find the following constraint on the LNV vector Wilson coefficients

\[ \frac{1}{2} \sum_{\beta} \left| \frac{1}{\sqrt{2} G_F} (C_{q\nu 1}^{V,\alpha\beta} + C_{q\nu 2}^{V,\alpha\beta}) \right|^2 < 1.1 \times 10^{-2} , \]

where \( \alpha = e, \mu \) again and the factor of 1/2 accounts for the missing projection operator in the cross section calculation of Ref. [13].

For the long-distance contribution with the dipole operator \( \mathcal{O}_{\nu\nu F} \), the matrix elements are

\[ M(\nu_{\alpha}\bar{N} \rightarrow \bar{\nu}_{\beta} N) = -i \frac{e G_F}{q^2} A_{M\nu F}^{\alpha\beta}(\overline{\nu}_N^\gamma \sigma_{\mu\nu} P_L u_{\nu}) \bar{N} \gamma^\mu \nu^\nu N , \]
\[ M(\bar{\nu}_{\alpha}\bar{N} \rightarrow \nu_{\beta} \bar{N}) = -i \frac{e G_F}{q^2} A_{M\nu F}^{\alpha\beta*}(\overline{\nu}_N^\gamma \sigma_{\mu\nu} P_L u_{\nu}) \bar{N} \gamma^\mu \nu^\nu N , \]

with the transferred 4-momentum \( q = p_1 - p_2 = k_2 - k_1 \) and

\[ A_{M\nu F}^2 = Z^2 \sum_{\beta} \left| \frac{4}{G_F} C_{\nu\nu F}^{\alpha\beta} \right|^2 F_p(Q^2) . \]

The constraint is given in Ref. [13]

\[ \frac{1}{2} A_{M\nu F}^2 = \frac{1}{2} \sum_{\beta} \left| \frac{4}{G_F} C_{\nu\nu F}^{\alpha\beta} \right|^2 < 7.2 \times 10^{-8} . \]

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### IV. MESON INVISIBLE DECAY

Next we consider the constraints on the LNEFT Wilson coefficients from meson invisible decays as listed in Table II. For simplicity we focus on the case, where the mixing between LH and RH neutrinos can be neglected ($|\sin \theta|^2 \lesssim 0.01$).

#### A. Light pseudoscalar meson decays

For a pseudoscalar meson $P$, the transition matrix element to the vacuum state from the scalar, vector, and tensor quark currents are zero. The only non-vanishing matrix elements are for pseudoscalar currents, axial-vector currents and the anomaly matrix elements. They can be parameterized by the form factors $f^q_P, h^q_P$ and $a_P$ [29, 30]

$$
\langle 0 | \bar{q} \gamma^\mu \gamma_5 q | P(p) \rangle = i f^q_P p^\mu, \quad \langle 0 | \bar{q} \gamma_5 q | P(p) \rangle = - \frac{i h^q_P}{2m_q}, \quad \langle 0 | \alpha_\mu R G_{\mu\nu} \tilde{G}^\nu_{\alpha a} | P(p) \rangle = a_a P ,
$$

(45)

where $G_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} G^{\rho\sigma}$ and $\epsilon_{0123} = 1$ and the form factors satisfy $h^q_P = m^2_P f^q_P - a_P$. The form factors for the mesons $\pi^0, \eta, \eta'$ can be expressed in terms of the input form factors $f_q = 1.07 f_\pi$, $f_s = 1.34 f_\pi$, and $f_\pi = 130.2$ MeV [31]

$$
f^u_\pi = - f^d_\pi = \frac{1}{\sqrt{2}} f_\pi , \quad f^s_\pi = 0 ; \quad f^u_\eta = f^d_\eta = \frac{\sqrt{2}}{\sqrt{2}} f_\eta , \quad f^s_\eta = - s_\eta f_s ; \quad h^u_\eta = h^d_\eta = \frac{c_\phi}{\sqrt{2}} h_\eta, \quad h^s_\eta = - s_\eta h_s ,
$$

$$
f^u_{\eta'} = f^d_{\eta'} = \frac{\sqrt{2}}{\sqrt{2}} f_{\eta'} , \quad f^s_{\eta'} = c_\phi f_s ; \quad h^u_{\eta'} = h^d_{\eta'} = \frac{\sqrt{2}}{\sqrt{2}} h_{\eta'}, \quad h^s_{\eta'} = c_\phi h_s ,
$$

(46)

where $s_\phi = \sin \phi$ and $c_\phi = \cos \phi$ with $\phi = 39.3^\circ$ being the mixing angle between flavor $SU(3)$ octet $\eta_8$ and singlet $\eta_1$. We assume isospin symmetry following the FKS scheme [32–34] for the form factors of $\eta$ and $\eta'$ and take the numerical values from Ref. [29] unless otherwise stated. The pseudoscalar input form factor $h_\eta$ and $h_s$ can be expressed in terms of $f_q, f_s$ and $\phi$ as follows

$$
h_\eta = f_q (m^2_\eta c^2_\phi + m^2_\eta s^2_\phi) - \sqrt{2} f_s (m^2_{\eta'} - m^2_\eta) s_\phi c_\phi ,
$$

$$
h_s = f_s (m^2_\eta c^2_\phi + m^2_\eta s^2_\phi) - \frac{f_q}{\sqrt{2}} (m^2_{\eta'} - m^2_\eta) s_\phi c_\phi .
$$

(47)
Given the definition of the above-listed form factors, we can write the branching ratio for the invisible decay of a pseudoscalar meson to neutrinos as

$$B(P \to \text{inv.}) = \frac{\tau_P m_P}{16\pi} \sum_{\alpha,\beta} \left\{ 2 \left| \frac{m_N f_P^q}{2} \left( C_{qN1}^{\alpha\beta} - C_{qN2}^{\alpha\beta} \right) \right|^2 \left( 1 - \frac{m_N^2}{m_P^2} \right) \frac{1}{2} 
+ 2 \left| \frac{h_P^q}{4m_q} \left( C_{qN1}^{S,\alpha\beta} - C_{qN2}^{S,\alpha\beta} \right) \right|^2 \left( 1 - \frac{m_N^2}{m_P^2} \right) \right. 
+ \left. 2 \left| \frac{h_P^q}{2m_q} \left( C_{qN1}^{S,\alpha\beta} - C_{qN2}^{S,\alpha\beta} \right) \right|^2 \left( 1 - \frac{2m_N^2}{m_P^2} \right) \left( 1 - \frac{4m_N^2}{m_P^2} \right) \right\} $$

(48)

where we implicitly sum over light quark flavors u, d, s. The contributions in the first two lines describe LNC decays and the remaining lines LNV decays. In the SM, there is only a contribution to the invisible decay of a pseudoscalar meson to neutrinos as due to the tiny neutrino masses.

**B. Light vector meson decays**

The non-vanishing hadronic matrix element for an unflavored vector meson V with momentum p and polarization vector \( \epsilon^\nu \) can be parameterized as [30, 35]

$$\langle 0|\bar{q}\gamma^\mu q|V(p)\rangle = f^q_{V} m_{V} \epsilon^\nu, \quad \langle 0|\bar{q}\sigma^{\mu\nu} q|V(p)\rangle = i f^{T,\mu}_{V} \left( \epsilon^\nu - \epsilon^\nu_{T} \right).$$

(49)

In particular the form factors for the vector mesons \( \omega \sim \frac{u\bar{u} + d\bar{d}}{\sqrt{2}} \) and \( \phi \sim s\bar{s} \) are

$$f_{\omega}^u = f_{\omega}^d = \frac{1}{\sqrt{2}} f_{\omega}^s = 0, \quad f_{\phi}^d = f_{\phi}^s = \frac{1}{\sqrt{2}} f_{\phi}^t = 0,$$

$$f_{\omega}^u = f_{\omega}^s = 0, \quad f_{\phi}^u = f_{\phi}^s = 0, \quad f_{\phi}^{T,\mu} = f_{\phi}^{T,\nu} = 0,$$

(50)

with \( f_{\omega} = 187 \text{ MeV}, f_{\phi}^T = 151 \text{ MeV} \) [35], \( f_{\phi} = 233 \text{ MeV} \) and \( f_{\phi}^{T,\mu} = 177 \text{ MeV} \) [36].

Using the definition of these form factors, it is straightforward to derive an expression for the branching ratio of the vector meson invisible decay to neutrinos\(^6\)

$$B(V \to \text{inv.}) = \frac{\tau_V m_V^2}{48\pi} \sum_{\alpha,\beta} \left\{ 2 \left| \frac{f_{V}^q}{2} \left( C_{qN1}^{V,\alpha\beta} + C_{qN2}^{V,\alpha\beta} \right) \right|^2 \right\}$$

(51)

\(^6\)Calcutational details are collected in Appendix E.
\[ + 8 \left| f_{T,q}^T C_{\nu N}^{T,\alpha\beta} - e Q_q f_{V}^q C_{\nu N}^{\alpha\beta} \right|^2 \left( 1 + \frac{m_N^2}{m_V^2} - \frac{2 m_N^4}{m_V^4} \right) \left( 1 - \frac{m_N^2}{m_V^2} \right) \]
\[ + 16 \left| f_{V}^{\alpha\beta} C_{qN}^{\alpha\beta} - e Q_q f_{V}^q C_{\nu N}^{\alpha\beta} \right|^2 \]
\[ + 16 \left| f_{V}^{\alpha\beta} C_{qN}^{\alpha\beta} - e Q_q f_{V}^q C_{\nu N}^{\alpha\beta} \right|^2 \left( 1 + 2 \frac{m_N^2}{m_V^2} \right) \left( 1 - 4 \frac{m_N^2}{m_V^2} \right)^2 \]
\[ + 4 \left| \frac{f_{V}^q}{2} \left( C_{qN1}^{V,\alpha\beta} + C_{qN2}^{V,\alpha\beta} \right) \right|^2 \left( 1 - \frac{m_N^2}{2m_V^2} \right) \left( 1 - \frac{m_N^2}{2m_V^2} \right) \left( 1 - \frac{m_N^2}{m_V^2} \right) \right) , \]

where we implicitly sum over light quark flavors \( u, d, s \). The first three lines describe LNC decays and the latter three LNV decays. The dipole operators contribute to the vector meson invisible decays through a photon propagator and a QED vertex. Here we have split the contribution from the vector operators \( C_{qN1(2)}^{V} \) into the NP contribution and the SM part as

\[ C_{\text{SM}}^V = - \frac{g_2^2}{4 m_Z^2} \left[ \left( \frac{1}{2} - \frac{4}{3} s_W^2 \right) f_{V}^{\nu} - \left( \frac{1}{2} - \frac{2}{3} s_W^2 \right) (f_{V}^{\nu} + f_{V}^\phi) \right] , \]  

(52)

with

\[ C_{\text{SM}}^{\omega} = \frac{g_2^2 s_W f_{\omega}}{6 \sqrt{2} m_Z^2} , \quad C_{\text{SM}}^{\phi} = \left( \frac{1}{2} - \frac{2}{3} s_W^2 \right) \frac{g_2^2 f_{\phi}}{4 m_Z^2} . \]

(53)

The SM predictions for the vector meson invisible decays are

\[ B(\omega \rightarrow \text{inv.}) \approx 1.5 \times 10^{-13} , \quad B(\phi \rightarrow \text{inv.}) \approx 3.4 \times 10^{-10} , \]

(54)

and consequently negligible compared with the current experimental upper limits listed in Table II.

Generally, the scalar-type and tensor-type LNEFT operators can only be constrained by pseudoscalar and vector meson invisible decays, respectively. For vector-type operators, both pseudoscalar and vector meson decays are sensitive to LNC \( O_{qN1(2)}^{V} \) and LNV \( O_{qN1(2)}^{V,\nu} \). The LNC operators \( O_{qN1(2)}^{V} \) and all dipole operators only contribute to vector meson decay.

\section{V. NUMERICAL RESULTS}

In this section we present the numerical constraints on the Wilson coefficients of LNEFT and SMNEFT from the CE\( \nu \)NS process and meson invisible decays. We assume that one operator dominates at a time. We first show the upper bounds on the LNEFT Wilson coefficients from meson invisible decays as a function of \( m_N \) in Figs. 1 and 2. The different colored lines correspond to different mesons: \( \pi^0 \) (purple), \( \eta \) (red), \( \eta' \) (orange), \( \omega \) (dark green) and \( \phi \) (blue).

Fig. 1 shows the constraints for the dipole operators. Solid (dashed) [dotted] lines correspond to the Wilson coefficients \( C_{\nu N1}^{\alpha\beta} (C_{\nu N2}^{\alpha\beta}) \) [\( C_{\nu N1}^{\alpha\beta} \)]. The constraints on \( C_{\nu N1}^{\alpha\beta} \) are cutoff for smaller RH neutrino masses compared to the ones for \( C_{\nu N2}^{\alpha\beta} \) due to the smaller phase space with two massive RH neutrinos in the final state.

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FIG. 1. The upper bounds on the LNEFT Wilson coefficients of dipole operators from meson invisible decays as a function of the RH neutrino mass $m_N$.

In Fig. 2, solid (dashed) [dot-dashed] lines indicate vector (scalar) [tensor] Wilson coefficients. The horizontal dotted lines show the bounds on the Wilson coefficients without RH neutrino field for completeness. For $C^S_{\alpha\beta}$ and $C^S_{qN(2)}$ with symmetric neutrino flavors in the LNV case, which are shown in the bottom, we show the components with different flavors ($\alpha \neq \beta$). The bounds on the Wilson coefficients with identical flavors ($\alpha = \beta$) are enhanced by a factor of $\sqrt{2}$ with respect to those with $\alpha \neq \beta$. One can see that, from pseudoscalar meson decay, the upper limits on the $C^V_{qN(2)}$ in LNC case and the $C^V_{qN(1)}$ in LNV case both scale as $\sim 1/m_N$ and are thus less stringent than the constraints from vector meson decay in the small $m_N$ limit. The bounds on other coefficients turn out to be a constant if the decay is kinematically allowed.

Next, Tables III and IV show the constraints on the Wilson coefficients of LNEFT from the CE$\nu$NS process and meson invisible decays in the limit of massless RH neutrinos. The neutrino flavors $\alpha, \beta$ are arbitrary unless they are specified for $C^V_{\nu\nu(2)}$ or taken to be $\alpha = e, \mu$ in CE$\nu$NS process. In the LNV case, for the scalar-type operators with symmetric neutrino flavors, the numbers outside and inside the square bracket in Table IV indicate the case with different neutrino flavors $\alpha \neq \beta$ and identical flavors $\alpha = \beta$, respectively. The gray cell displays the strongest constraint for each Wilson coefficient. One can see that the vector meson decays provide the sole bound on the particular flavor components $C^V_{\tau\tau u(2)}$ and $C^V_{\nu\nu(2)}$ in the LNC case and $C^V_{\nu\nu(2)}$ in the LNV case. The coefficients without active neutrino degree of freedom, such as $C^V_{qN(2)}$ in the LNC case and $C^S_{qN(1)}, C^S_{qN}, C^T_{qN}$ in the LNV case, can only be constrained by meson decays. The CE$\nu$NS process places the most stringent bound on all remaining Wilson coefficients with $\alpha = e, \mu$. The remaining WCs with $\alpha = \tau$ can not be constrained by CE$\nu$NS and we highlight the strongest constraints by meson decays in light gray. In the last columns of Tables III and IV, we also show the effective scale derived from the strongest constraint for each Wilson coefficient. The effective scales shown in parentheses correspond to the WCs in light gray for $\alpha = \tau$. 

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FIG. 2. Upper bounds on the LNEFT Wilson coefficients of dim-6 neutrino-quark operators from meson invisible decays as a function of \( m_N \) for LNC (LNV) operators on the left (right). The top (middle) [bottom] row show WCs for up (down) [strange] quarks. For the LNV WCs \( C^{S,\alpha \beta}_{N1(2)} \), \( C^{S,\alpha \beta}_{q1(2)} \) \( C^{S,\alpha \beta}_{qN1(2)} \) we display the components with \( \alpha \neq \beta \). The bounds for the corresponding WCs with \( \alpha = \beta \) are stronger by a factor \( \sqrt{2} \).
| LNEFT WC | CE/NS | $\pi^0 \rightarrow$ inv. | $\eta \rightarrow$ inv. | $\eta' \rightarrow$ inv. | $\omega \rightarrow$ inv. | $\phi \rightarrow$ inv. | $\Lambda_{\text{LNEFT}} = |C_i|^{\frac{1}{\Gamma}}$ [GeV] |
|---|---|---|---|---|---|---|---|
| $C_{V,ee}^{\alpha} \text{[GeV]}$ | $1.5 \times 10^{-5}$ | - | - | - | $1.5 \times 10^{-1}$ | - | 260 |
| $C_{V,\mu\mu}^{\alpha} \text{[GeV]}$ | $4.3 \times 10^{-6}$ | - | - | - | $1.5 \times 10^{-1}$ | - | 480 |
| $C_{V,\tau\tau}^{\alpha} \text{[GeV]}$ | $5.9 \times 10^{-6}$ | - | - | - | $1.5 \times 10^{-1}$ | - | 410 |
| $C_{V,\mu\mu}^{\alpha} \text{[GeV]}$ | $1.5 \times 10^{-5}$ | - | - | - | $1.5 \times 10^{-1}$ | - | 260 |
| $C_{V,\tau\tau}^{\alpha} \text{[GeV]}$ | $5.3 \times 10^{-6}$ | - | - | - | $1.5 \times 10^{-1}$ | - | 440 |
| $C_{V,\nu L}^{\alpha} \text{[GeV]}$ | - | - | - | - | $1.5 \times 10^{-1}$ | - | 2.6 |
| $C_{V,\nu L}^{\alpha} \text{[GeV]}$ | $1.4 \times 10^{-5}$ | - | - | - | $1.5 \times 10^{-1}$ | - | 270 |
| $C_{V,\nu L}^{\alpha} \text{[GeV]}$ | $3.6 \times 10^{-6}$ | - | - | - | $1.5 \times 10^{-1}$ | - | 520 |
| $C_{V,\nu L}^{\alpha} \text{[GeV]}$ | $5.6 \times 10^{-6}$ | - | - | - | $1.5 \times 10^{-1}$ | - | 420 |
| $C_{V,\nu L}^{\alpha} \text{[GeV]}$ | $1.4 \times 10^{-5}$ | - | - | - | $1.5 \times 10^{-1}$ | - | 270 |
| $C_{V,\nu L}^{\alpha} \text{[GeV]}$ | $5.0 \times 10^{-6}$ | - | - | - | $1.5 \times 10^{-1}$ | - | 450 |
| $C_{V,\nu L}^{\alpha} \text{[GeV]}$ | - | - | - | - | $1.5 \times 10^{-1}$ | - | 2.6 |
| $C_{V,\alpha\beta} \text{[GeV]}$ | - | - | - | - | $6.2 \times 10^{-2}$ | - | 4.0 |

TABLE III. Constraints on the Wilson coefficients of the LNC operators in the LNEFT. The neutrino flavors for the vector type operators $C_{V,\nu L}^{\alpha} \text{[GeV]}$ are displayed explicitly. For CE/NS the initial neutrino flavor is $\alpha = e, \mu$. In other cases the neutrino flavors $\alpha, \beta$ are arbitrary. The gray cell displays the strongest constraint for each WC. In the last column we also show the effective scale derived from the strongest constraint for each WC. Note that in the last sector, the gray and light gray cells are for $\alpha = e, \mu$ and $\alpha = \tau$ flavors respectively. For the $\alpha = \tau$ case, the effective scale is shown in parentheses (...) in the last column.

We then include the one-loop QCD/QED running result for the LNEFT Wilson coefficients from the chiral symmetry breaking scale to the electroweak scale scale and match them to SM-NEFT at the electroweak scale in order to constrain new physics using the matching conditions from Table I. In Table V we display the constraints on the Wilson coefficients $C_i(\Lambda_{\text{EW}})$.
associated with the relevant dim-6 and dim-7 SMNEFT operators from the strongest limits of the corresponding LNEFT WCs in the gray sectors of Tables III and IV. By further assuming \( \Lambda_{\text{NP}} \equiv |C_1(\Lambda_{\text{EW}})|^{1/(4-d)} \) with \( d \) being the SMNEFT operator dimension, the constraints on the Wilson coefficients are also converted into the limits on the NP scale in units of the SM Higgs vev.

The most stringent bounds on the NP scale are

\[
\Lambda_{\text{NP}}^{\text{dim-6}} = (C_{NB}^{\alpha \beta} + C_{\text{NW}}^{\alpha \beta})^{-1/2} > 41 \, v \, (\alpha = e, \mu),
\]

\[
\Lambda_{\text{NP}}^{\text{dim-7}} = (2C_{LHB}^{\alpha \beta} + C_{\text{LHW}}^{\alpha \beta} - C_{\text{LHW}}^{\beta \alpha})^{-1/3} > 11 \, v \, (\alpha, \beta = e, \mu, \tau),
\]

from the corresponding dipole operators in LNEFT and

\[
\Lambda_{\text{NP}}^{\text{dim-6}} = (C_{\text{NNF}}^{\alpha \beta})^{-1/2} > 6.0 \, v \, (\alpha = e, \mu),
\]

\[
\Lambda_{\text{NP}}^{\text{dim-6}} = (C_{\text{NNF}}^{\alpha \beta})^{-1/2} > 2.9 \, v \, (\alpha = \tau),
\]

\[
\Lambda_{\text{NP}}^{\text{dim-7}} = (C_{\text{NNF}}^{\alpha \beta} + C_{\text{NNF}}^{\beta \alpha})^{-1/3} > 2.9 \, v \, (\alpha = e, \mu),
\]

TABLE IV. Constraints on the Wilson coefficients of the LNV operators in the LNEFT. For the scalar type operators, the numbers outside [inside] the square bracket indicate the case with the neutrino flavors \( \alpha \neq \beta [\alpha = \beta] \). Note that the Wilson coefficients \( C_{\text{NNF}}^{\alpha \beta} \) in the first sector and \( C_{\text{NNF}}^{\alpha \beta} \) in the last sector can not be constrained by CE\( \nu \)NS. The strongest constraints on them are from the light gray cells, and the corresponding effective scale is shown in parentheses (...).
| dim-6 SMNEFT WC | $|v^2|$ | $\Lambda_{\text{NP}} \equiv |C_i|^{-2} |v|$ | dim-6 SMNEFT WC | $|v^2|$ | $\Lambda_{\text{NP}} \equiv |C_i|^{-2} |v|$ |
|-----------------|--------|-----------------|-----------------|--------|-----------------|
| $C_{lq}^{[1]e_i e_j} + C_{lq}^{[3]e_i e_j}$ | 0.90 | 1.1 | $C_{l_q}^{[1]e_i e_j} - C_{l_q}^{[3]e_i e_j}$ | 0.82 | 1.1 |
| $C_{l_q}^{[1]e_i e_j} + C_{l_q}^{[3]e_i e_j}$ | 0.26 | 2.0 | $C_{l_q}^{[1]e_i e_j} - C_{l_q}^{[3]e_i e_j}$ | 0.22 | 2.1 |
| $C_{l_q}^{[1]e_i e_j} + C_{l_q}^{[3]e_i e_j}$ | 0.36 | 1.7 | $C_{l_q}^{[1]e_i e_j} - C_{l_q}^{[3]e_i e_j}$ | 0.34 | 1.7 |
| $C_{\chi}^{[1]e_i e_j} + C_{\chi}^{[3]e_i e_j}$ | 0.90 | 1.1 | $C_{\chi}^{[1]e_i e_j} - C_{\chi}^{[3]e_i e_j}$ | 0.82 | 1.1 |
| $C_{\chi}^{[1]e_i e_j} + C_{\chi}^{[3]e_i e_j}$ | 0.32 | 1.8 | $C_{\chi}^{[1]e_i e_j} - C_{\chi}^{[3]e_i e_j}$ | 0.30 | 1.8 |
| $C_{\chi}^{[3]e_i e_j} + C_{\chi}^{[3]e_i e_j}$ | 6.0 x 10^{-4} | 41 |

| dim-7 SMNEFT WC | $|v^3|$ | $\Lambda_{\text{NP}} \equiv |C_i|^{-2} |v|$ | dim-7 SMNEFT WC | $|v^3|$ | $\Lambda_{\text{NP}} \equiv |C_i|^{-2} |v|$ |
|-----------------|--------|-----------------|-----------------|--------|-----------------|
| $2C_{LHH} + 2C_{LHH} - C_{LHH}^{[3]}$ | 8.5 x 10^{-4} | 11 |
| $C_{LHH}^{[3]} + C_{LHH}^{[3]}$ | 0.09 | 2.2 | $C_{LHH}^{[3]} + C_{LHH}^{[3]}$ | 0.51 | 1.3 |
| $C_{LHH}^{[3]} + C_{LHH}^{[3]}$ | 0.97 | 1.0 | $C_{LHH}^{[3]} + C_{LHH}^{[3]}$ | 38 | 0.3 |
| $C_{LHH}^{[3]} + C_{LHH}^{[3]}$ | 0.04 | 2.9 | $C_{LHH}^{[3]} + C_{LHH}^{[3]}$ | 0.12 | 2.0 |
| $C_{LHH}^{[3]} + C_{LHH}^{[3]}$ | 1.5 | 0.88 | $C_{LHH}^{[3]} + C_{LHH}^{[3]}$ | 13 | 0.43 |
| $C_{LHH}^{[3]} + C_{LHH}^{[3]}$ | 0.33[0.48] | 1.5[1.3] | $C_{LHH}^{[3]} + C_{LHH}^{[3]}$ | 0.72[1.0] | 1.1[1.0] |
| $C_{LHH}^{[3]} + C_{LHH}^{[3]}$ | 0.08[0.12] | 2.3[2.0] | $C_{LHH}^{[3]} + C_{LHH}^{[3]}$ | 0.18[0.25] | 1.8[1.6] |
| $C_{LHH}^{[3]} + C_{LHH}^{[3]}$ | 0.21 | 1.7 | $C_{LHH}^{[3]} + C_{LHH}^{[3]}$ | 0.21 | 1.7 |

TABLE V. Constraints on the Wilson coefficients of the relevant dim-6 and dim-7 SMNEFT from the strongest limits for the corresponding LNEFT WC's in the gray sector of Table III and Table IV, where $v \simeq 246$ GeV is SM Higgs vacuum expectation value. For the dim-7 scalar type operators, the numbers outside [inside] the square bracket indicate the case with the neutrino flavors $\alpha \neq \beta [\alpha = \beta]$.

$$\Lambda_{\text{NP}}^{\text{dim-7}} = (C_{Q_{uLLH}}^{[1]})^{-1/3} > 2.0 \, v \, (\alpha = \beta = \tau),$$

from neutrino-quark operators in LNEFT.
VI. CONCLUSIONS

We investigate the complementarity of the CE\(\nu\)NS process and meson invisible decay in constraining generic neutrino interactions with RH neutrinos in effective field theories. The interactions between quarks and left-handed SM neutrinos and/or right-handed neutrinos are first described by the LNEFT between the electroweak scale and the chiral symmetry breaking scale. We complete the independent operator basis for the LNEFT up to dim-6 by including both the LNC and LNV operators. We translate the bounds on the LNEFT Wilson coefficients from the COHERENT observation and calculate the branching fractions of light meson invisible decays. Finally, we include the one-loop QCD/QED running for the LNEFT Wilson coefficients from chiral symmetry breaking scale to the electroweak scale. The bounds on the LNEFT Wilson coefficients are then matched up to the SMNEFT to constrain new physics above the electroweak scale.

We summarize our main conclusions in the following

• In the LNC case, the vector meson invisible decays provide the sole but weak constraint on \(C^{V,\tau\tau}_{q\nu N(1,2)}\), \(C^{V,\alpha\beta}_{q\nu N(1,2)}\) and \(C^{T,\alpha\beta}_{q\nu N}\) with \(\alpha = e, \mu\). The WCs \(C^{T,\alpha\beta}_{\nu\nu N}\), \(C^{S,\alpha\beta}_{q\nu N(1,2)}\) and \(C^{T,\alpha\beta}_{q\nu N}\) can only be constrained by meson decay.

• In the LNV case, the meson invisible decays provide the sole constraint on \(C^{\alpha\beta}_{NNF}\), \(C^{S,\alpha\beta}_{q\nu N(1,2)}\), \(C^{V,\alpha\beta}_{\nu\nu N}\) and \(C^{T,\alpha\beta}_{\nu\nu N}\). CE\(\nu\)NS gives the most stringent constraint on \(C^{\alpha\beta}_{\nu\nu N}\), \(C^{T,\alpha\beta}_{\nu\nu N}\) and the components with \(\alpha = e, \mu\) in \(C^{S,\alpha\beta}_{q\nu N(1,2)}\) and \(C^{V,\alpha\beta}_{\nu\nu N(1,2)}\). The WCs \(C^{S,\tau\beta}_{\nu\nu N}\) and \(C^{V,\tau\beta}_{\nu\nu N}\) can only be constrained by meson decay.

• The most stringent bounds on the NP scale in SMNEFT are

\[
\Lambda_{\text{NP}}^{\text{dim-6}} = \left( C^{\alpha\beta}_{NNB} + C^{\alpha\beta}_{NNW} \right)^{-\frac{1}{2}} > 41 v \simeq 10 \text{ TeV} \quad (\alpha = e, \mu), \\
\Lambda_{\text{NP}}^{\text{dim-7}} = \left( 2C^{\alpha\beta}_{LHB} + C^{\beta\alpha}_{LHW} - C^{\alpha\beta}_{LHW} \right)^{-\frac{1}{3}} > 11 v \simeq 2.7 \text{ TeV} \quad (\alpha, \beta = e, \mu, \tau),
\]

from the corresponding dipole operators in LNEFT and

\[
\Lambda_{\text{NP}}^{\text{dim-6}} = \left( C^{11\beta\alpha}_{QuNL} \right)^{-\frac{1}{2}} > 6.0 v \simeq 1.5 \text{ TeV} \quad (\alpha = e, \mu), \\
\Lambda_{\text{NP}}^{\text{dim-6}} = \left( C^{11\beta\alpha}_{QuNL} \right)^{-\frac{1}{2}} > 2.9 v \simeq 0.7 \text{ TeV} \quad (\alpha = \tau), \\
\Lambda_{\text{NP}}^{\text{dim-7}} = \left( C^{11\beta\alpha}_{QuLLH} + C^{11\beta\alpha}_{QuLLH} \right)^{-\frac{1}{3}} > 2.9 v \simeq 0.7 \text{ TeV} \quad (\alpha = e, \mu), \\
\Lambda_{\text{NP}}^{\text{dim-7}} = \left( C^{11\beta\alpha}_{QuLLH} \right)^{-\frac{1}{3}} > 2.0 v \simeq 0.5 \text{ TeV} \quad (\alpha = \beta = \tau),
\]

from neutrino-quark operators in LNEFT.
ACKNOWLEDGMENTS

TL would like to thank Yi Liao and Cen Zhang for very useful discussion and communication. TL is supported by the National Natural Science Foundation of China (Grant No. 11975129) and “the Fundamental Research Funds for the Central Universities”, Nankai University (Grants No. 63191522, 63196013). XDM is supported by the MOST (Grant No. MOST 106-2112-M-002-003-MY3). MS acknowledges support by the Australian Research Council via the Discovery Project DP200101470.

Appendix A: The complete operator basis involving RH neutrinos $N$ in the LNEFT

In this section we construct the complete and independent operator basis for the LNEFT involving RH neutrinos $N$ up to dim-6. We work in the chiral basis and collectively denote the left- and right-handed down-type quarks as $d_L$ and $d_R$, the up-type quarks as $u_L$ and $u_R$, charged leptons as $e_L$ and $e_R$, and the SM left-handed neutrino fields as $\nu$ and the RH neutrinos as $N$, respectively. We drop the flavor indices for all of these fields for simplicity. For a fermion field $\psi$, its charge conjugation is defined via $\psi^C = C \bar{\psi}^T$ where the matrix $C$ satisfies the relations $C^T = C^\dagger = -C$ and $C^2 = -1$. Except the up-type quarks with the total flavors $n_u = 2$, the remaining charged fermions have $n_f = 3$ flavors. We consider an arbitrary number $n_f$ of $N$ flavors.

At dim-5, it is easy to figure out that there are two independent non-hermitian operators

$$\mathcal{O}_{NNF} = (\bar{N} \sigma_{\mu\nu} N) F^{\mu\nu}, \quad \mathcal{O}_{\nu NF} = (\bar{\nu} \sigma_{\mu\nu} N) F^{\mu\nu}.$$  (A1)

The full list of independent LNEFT operators with at least one RH neutrino $N$ at dim-6 is listed in Tables VI and VII, where in the third and sixth columns in each table we also show the independent number of operators with flavors being considered. All those operators are classified in terms of the net number of the SM global baryon and lepton quantum numbers. An independent subset of lepton and baryon number conserving operators in LNEFT is given in Ref. [4].

Appendix B: The SMNEFT operator basis at dim-6 and dim-7

Besides the SMEFT operators at dim-6 [5] and dim-7 [6, 7], the SMNEFT also includes additional operators involving RH SM singlet fermions $N$. These operators with RH neutrino $N$ are classified in Ref. [8] and repeated in Table VIII at dim-6 and Table IX at dim-7. For the dim-7 operators, by using the Fierz transformations here, we have rearranged some of the four-fermion operators given in Ref. [8] to have clear flavor symmetry and quark-lepton current structure. In addition, for the operators involving gauge field strength tensors, we accompany a corresponding gauge coupling constant for each involved field strength tensor. Besides the operator basis involving RH neutrinos $N$ in Table VIII and Table IX, in our matching calculation we also need the
TABLE VI. Dim-6 operator basis involving RH neutrinos $N$ in LNEFT. Here all operators are non-hermitian expect those with a (H) in the first sector. The number of $\ast$ after each operator indicates the number of the RH neutrinos involved in the same operator.

following relevant SMEFT dim-6 operators

$$O_{lq}^{(1)} = (\overline{L} \gamma_{\mu} L)(\overline{Q} \gamma^\mu Q) ,$$
$$O_{lu} = (\overline{L} \gamma_{\mu} L)(\overline{\nu} \gamma^\mu u) ,$$
$$O_{ld} = (\overline{L} \gamma_{\mu} L)(\overline{\nu} \gamma^\mu d) ,$$

and also dim-7 operators

$$O_{LHB} = g_1 \epsilon_{ij} \epsilon_{mn} (\overline{L} C_i \sigma_{\mu \nu} L^m) H^j H^n B^{\mu \nu} ,$$
$$O_{dLQ LH1} = \epsilon_{ij} \epsilon_{mn} (\overline{d} L^i) (\overline{Q} C_j L^m) H^n ,$$
$$O_{LHW} = g_2 \epsilon_{ij} (\epsilon^{ij})_{mn} (\overline{L} C_i \sigma_{\mu \nu} L^m) H^j H^n W^{1\mu \nu} ,$$
$$O_{QuL LH} = \epsilon_{ij} (\overline{Q} u) (\overline{L} C_j L^i) H^j ,$$

where $g_{1,2}$ are the gauge coupling constants for the gauge groups $U(1)_Y$ and $SU(2)_L$, respectively.

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\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{Operator} & \text{Specific form} & \#(n_f, n_u) & \text{Operator} & \text{Specific form} & \#(n_f, n_u) \\
\hline
& (LR)(LR) & (RL)(LR) & (LR)(LR) & (RL)(LR) & (RL)(RL) \\
\hline
O_{eN1}^*(**)(\downarrow) & (\bar{e}_{LR}^c(L)(\overline{NC}N) & \frac{1}{2}n_f^2(n_f + 1) & O_{eN2}^*(**) & \bar{e}_{LR}^c(L)(\overline{NC}N) & \frac{1}{2}n_f^2(n_f + 1) \\
& \overline{\nu}_{1\sigma\nu_1}(N^{N_{\sigma\nu_1}}N) & \frac{1}{2}n_u^2(n_f - 1) & & & \\
\hline
O_{dN1}^*(**)(\downarrow) & (d_{LR}^c(R)(\overline{NC}N) & \frac{1}{2}n_f^2(n_f + 1) & O_{dN2}^*(**) & (d_{LR}^c(R)(\overline{NC}N) & \frac{1}{2}n_f^2(n_f + 1) \\
& \overline{\nu}_{1\sigma\nu_1}(N^{N_{\sigma\nu_1}}N) & \frac{1}{2}n_u^2(n_f - 1) & & & \\
\hline
O_{uN1}^*(**)(\downarrow) & (u_{LR}^c(R)(\overline{NC}N) & \frac{1}{2}n_f^2(n_f + 1) & O_{uN2}^*(**) & (u_{LR}^c(R)(\overline{NC}N) & \frac{1}{2}n_f^2(n_f + 1) \\
& \overline{\nu}_{1\sigma\nu_1}(N^{N_{\sigma\nu_1}}N) & \frac{1}{2}n_u^2(n_f - 1) & & & \\
\hline
O_{d_{dec}N1}^*(**)(\downarrow) & (d_{LR}^c(R)(\overline{NC}N) & n_f^2n_u & O_{d_{dec}N2}^*(**) & (d_{LR}^c(R)(\overline{NC}N) & n_f^2n_u \\
& \overline{\nu}_{1\sigma\nu_1}(N^{N_{\sigma\nu_1}}N) & n_f^2n_u & & & \\
\hline
O_{\nu NN}^*(**)(\downarrow) & (\overline{\nu}_{1\sigma\nu_1}(N^{N_{\sigma\nu_1}}N) & \frac{1}{2}n_f^2(n_f + 1) & O_{\nu_{\bar{\nu}N}}^*(**) & (\overline{\nu}_{1\sigma\nu_1}(N^{N_{\sigma\nu_1}}N) & \frac{1}{2}n_f^2(n_f + 1) \\
& (\overline{\nu}_{1\sigma\nu_1}(N^{N_{\sigma\nu_1}}N) & \frac{1}{2}n_f^2(n_f + 1) & & & \\
\hline
& (\Delta L, \Delta B) = (4, 0) & & & (\Delta L, \Delta B) = (1, -1) & \\
\hline
& (\Delta L, \Delta B) = (1, 1) & & & & \\
\hline
\end{array}
\]

**TABLE VII.** Continuation of Tab. VI.

**Appendix C: Relations to other operator bases**

In this appendix we briefly summarize how our operator basis relates to other bases used in papers which we refer to in the main part of the text.

1. **Non-Standard Interactions**

A commonly used operator basis are non-standard interactions (NSIs) \([37–39]\) (Recent progress on NSI can be seen in Ref. \([40]\) and the references therein.), which describe the interactions of active neutrinos at low energies. In particular, neutral-current interactions with quarks are described by

\[
\mathcal{L}_{\text{NSI}} = -\sqrt{2}G_F \bar{\nu}_\alpha \gamma_\mu P_L \nu_\beta (\bar{q} \gamma_\mu q),
\]

(C1)
TABLE VIII. The basis of dim-6 operators involving RH neutrino $N$ in SMNEFT [7], where $\alpha$, $\beta$, $\sigma$ and $i$, $j$ are $SU(3)_C$ and $SU(2)_L$ indices, respectively.

| $\psi^2 H^3 D (+h.c.)$ | $(\mathcal{L} R)(\mathcal{L} R)(+h.c.)$ | $(\mathcal{L} L)(\mathcal{R} R)$ |
|-------------------------|--------------------------------|---------------------|
| $O_{\text{LNH}}$ | $(\mathcal{L} N)(H^1 H)$ | $O_{\text{LQNN}}$ | $(\mathcal{E} N)(\psi)(\mathcal{Q} N)$ |
| $O_{\text{LNe}}$ | $\psi^2 H^2 D (+h.c.)$ | $O_{\text{LQNN}}$ | $(\mathcal{E} N)(\psi)(\mathcal{Q} N)$ |
| $O_{\text{HNe}}$ | $(\mathcal{N}^\gamma \psi N)(H^1 D^2_J H)$ | $O_{\text{LQNN4}}$ | $(\mathcal{E} N)(\psi)(\mathcal{Q} N)$ |
| $O_{\text{HNe}}$ | $(\mathcal{N}^\gamma \psi N)(H^1 D^2_J H)$ | $O_{\text{LQNN6}}$ | $(\mathcal{E} N)(\psi)(\mathcal{Q} N)$ |

TABLE IX. The basis of dim-7 operators involving RH neutrino $N$ in SMNEFT, where all of the operators are non-hermitian with the net global quantum number $|\Delta L - \Delta B| = 2$. Here $g_{1,2,3}$ are the gauge coupling constants for the gauge groups $U(1)_Y$, $SU(2)_L$, $SU(3)_C$, respectively, and $\alpha_i = g_i^2/(4\pi)$.

$$\psi^2 H^3 D (+h.c.)$$

with $\varepsilon^V_{\alpha \beta} = \varepsilon^V_{\beta \alpha}^*$. The $\varepsilon$ parameterization is related to the NP contribution to the vector Wilson coefficients in our operator basis via

$$-\sqrt{2} G_F \varepsilon^V_{\alpha \beta} = \frac{1}{2} \left( C_{\alpha \beta 1,\text{NP}} + C_{\alpha \beta 2,\text{NP}} \right),$$

and to the $\xi_V$ parameter in Eq. (19) via

$$\xi_V = 4 \left( (g_1^V + 2\varepsilon^V_{\alpha \alpha} + \varepsilon^V_{\alpha \alpha}) Z F_{\mu}(Q^2) + (g_2^V + \varepsilon^V_{\alpha \alpha} + 2\varepsilon^V_{\alpha \alpha}) N F_{\mu}(Q^2) \right)^2$$
with the SM couplings being
\[ g_V^p = \frac{1}{2} - 2 \sin^2 \theta_W, \quad g_V^\nu = -\frac{1}{2}. \]

2. CD parameterization

For the vector Wilson coefficients in LNV case, the relation to the quark-level \( C_V^q \) parameter in Ref. [13] is
\[ C_V^{q*} - D_A^{q*} = -\frac{1}{\sqrt{2} G_F} \left( C_{q\nu N1}^{V,\alpha\beta} + C_{q\nu N2}^{V,\alpha\beta} \right). \] \hspace{5cm} (C4)

For the scalar and tensor Wilson coefficients, we have the following relations to the quark-level parameters in Ref. [10]
\[ C_S^q + i D_P^q = \frac{1}{\sqrt{2} G_F} \left( C_{q\nu N1}^{S,\alpha\beta*} + C_{q\nu N2}^{S,\alpha\beta*} \right), \quad C_T^q - i D_T^q = \frac{\sqrt{2}}{G_F} C_{q\nu N}^{T,\alpha\beta}, \] \hspace{5cm} (C5)
in the LNC case and
\[ C_S^q + i D_P^q = \frac{\sqrt{2}}{G_F} \left( C_{q\nu N1}^{S,\alpha\beta} + C_{q\nu N2}^{S,\alpha\beta} \right), \quad C_T^q - i D_T^q = -\frac{2\sqrt{2}}{G_F} C_{q\nu N}^{T,\alpha\beta}, \] \hspace{5cm} (C6)
in the LNV case.

Appendix D: The quark-level matrix elements of neutrino scattering

In the LNC case, the quark-level amplitudes for the scattering \( \nu_\alpha(p_1)q(k_1) \to \nu_\beta/N_\beta(p_2)q(k_2) \) and \( \bar{\nu}_\alpha(p_1)q(k_1) \to \bar{\nu}_\beta/N_\beta(p_2)q(k_2) \) is given by
\[ \mathcal{M}(\nu_\alpha q \to \nu_\beta q) = \frac{1}{2} \left( C_{q\nu N1}^{V,\alpha\beta*} + C_{q\nu N2}^{V,\alpha\beta*} \right) (\overline{\nu}_\alpha \gamma_{\mu} P_L u_\nu)(\overline{q} \gamma_{\mu} q) + [\text{SD}], \]
\[ -\mathcal{M}(\bar{\nu}_\alpha q \to \bar{\nu}_\beta q) = \frac{1}{2} \left( C_{q\nu N1}^{V,\alpha\beta} + C_{q\nu N2}^{V,\alpha\beta} \right) (\overline{\nu}_\alpha \gamma_{\mu} P_R u_\nu)(\overline{q} \gamma_{\mu} q) + [\text{SD}], \]
\[ \mathcal{M}(\nu_\alpha q \to N_\beta q) = \frac{1}{2} \left( C_{q\nu N1}^{S,\alpha\beta*} + C_{q\nu N2}^{S,\alpha\beta*} \right) (\overline{\nu}_\alpha \gamma_{\mu} P_L u_\nu)(\overline{q} \gamma_{\mu} q) + C_{q\nu N}^{T,\alpha\beta*} (\overline{u}_N \sigma_{\mu\nu} P_L u_\nu)(\overline{q} \sigma_{\mu\nu} q)
+ \frac{2e Q_q}{q^2} C_{q\nu N}^{\mu\beta*} (\overline{u}_N \sigma_{\mu\nu} P_L u_\nu)(\overline{q} \gamma_{\mu} t_{\nu} q) + [\text{SD}], \]
\[ -\mathcal{M}(\bar{\nu}_\alpha q \to \bar{N}_\beta q) = \frac{1}{2} \left( C_{q\nu N1}^{S,\alpha\beta} + C_{q\nu N2}^{S,\alpha\beta} \right) (\overline{\nu}_\alpha \gamma_{\mu} P_R v_\nu)(\overline{q} q) + C_{q\nu N}^{T,\alpha\beta} (\overline{v}_N \sigma_{\mu\nu} v_\nu)(\overline{q} \sigma_{\mu\nu} q)
+ \frac{2e Q_q}{q^2} C_{q\nu N}^{\mu\beta} (\overline{v}_N \sigma_{\mu\nu} v_\nu)(\overline{q} \gamma_{\mu} t_{\nu} q) + [\text{SD}], \] \hspace{5cm} (D1)
where [SD] stands for spin-dependent terms and the exchanged 4-momentum \( q = p_1 - p_2 = k_2 - k_1. \)
In the LNV case, the quark-level amplitudes for the scattering \( \nu_\alpha(p_1)q(k_1) \to \bar{\nu}_\beta/N_\beta(p_2)q(k_2) \) and \( \bar{\nu}_\alpha(p_1)q(k_1) \to \nu_\beta/N_\beta(p_2)q(k_2) \) are

\[
M(\nu_\alpha q \to \bar{\nu}_\beta q) = (C_{q_1q_\nu}^{S,\alpha\beta} + C_{q_2q_\nu}^{S,\alpha\beta})(\overline{v}_\nu P_L u_\nu)(\overline{q} q) - 2C_{q_\nu}^{T,\alpha\beta} (\overline{v}_\nu \sigma_{\mu\nu} P_L u_\nu)(\overline{q} q) + \text{SD},
\]

\[
M(\bar{\nu}_\alpha q \to \nu_\beta q) = (C_{q_1q_\nu}^{S,\alpha\beta*} + C_{q_2q_\nu}^{S,\alpha\beta*})(\overline{v}_\nu P_R u_\nu)(\overline{q} q) - 2C_{q_\nu}^{T,\alpha\beta*} (\overline{v}_\nu \sigma_{\mu\nu} u_\nu)(\overline{q} q) + \text{SD},
\]

\[
M(\nu_\alpha q \to N_\beta q) = -\frac{1}{2} (C_{q_1q_\nu}^{V,\alpha\beta} + C_{q_2q_\nu}^{V,\alpha\beta})(\overline{v}_\nu \gamma_{\mu} P_L u_\nu)(\overline{q} q) + \text{SD},
\]

\[
M(\bar{\nu}_\alpha q \to N_\beta q) = -\frac{1}{2} (C_{q_1q_\nu}^{V,\alpha\beta*} + C_{q_2q_\nu}^{V,\alpha\beta*})(\overline{v}_\nu \gamma_{\mu} P_L u_\nu)(\overline{q} q) + \text{SD}. \tag{D2}
\]

**Appendix E: The matrix elements of meson invisible decays**

For the quark-level process of \( q\bar{q} \to \text{inv}_1(k_1)\text{inv}_2(k_2) \), the LNC amplitudes are

\[
M(q\bar{q} \to \nu_\alpha \bar{\nu}_\beta) = \left( C_{q_1q_\nu}^{V,\alpha\beta} \overline{q} \gamma_{\mu} P_L q + C_{q_2q_\nu}^{V,\alpha\beta} \overline{q} \gamma_{\mu} P_R q \right) \overline{u}_\nu \gamma_{\mu} P_L u_\nu,
\]

\[
M(q\bar{q} \to N_\alpha \bar{N}_\beta) = \left( C_{q_1q_N}^{S,\alpha\beta} \overline{q} \gamma_{\mu} P_L q + C_{q_2q_N}^{S,\alpha\beta} \overline{q} \gamma_{\mu} P_R q \right) \overline{u}_\nu \gamma_{\mu} P_L u_\nu,
\]

\[
M(q\bar{q} \to \nu_\alpha \bar{N}_\beta) = \left( C_{q_1q_\nu}^{S,\alpha\beta} \overline{q} \gamma_{\mu} P_L q + C_{q_2q_\nu}^{S,\alpha\beta} \overline{q} \gamma_{\mu} P_R q \right) \overline{u}_\nu \gamma_{\mu} P_L u_\nu,
\]

\[
M(q\bar{q} \to \bar{\nu}_\alpha N_\beta) = \left( C_{q_1q_N}^{S,\alpha\beta*} \overline{q} \gamma_{\mu} P_L q + C_{q_2q_N}^{S,\alpha\beta*} \overline{q} \gamma_{\mu} P_R q \right) \overline{u}_\nu \gamma_{\mu} P_L u_\nu,
\]

The LNV amplitudes with \( \Delta L = -2 \) are

\[
M(q\bar{q} \to \bar{\nu}_\alpha \nu_\beta) = 2 \left( C_{q_1q_\nu}^{S,\alpha\beta} \overline{q} P_L q + C_{q_2q_\nu}^{S,\alpha\beta} \overline{q} P_R q \right) \overline{v}_\nu \gamma_{\mu} P_L u_\nu,
\]

\[
M(q\bar{q} \to N_\alpha \bar{N}_\beta) = 2 \left( C_{q_1q_N}^{S,\alpha\beta} \overline{q} P_L q + C_{q_2q_N}^{S,\alpha\beta} \overline{q} P_R q \right) \overline{v}_\nu \gamma_{\mu} P_L u_\nu,
\]

\[
M(q\bar{q} \to \bar{\nu}_\alpha N_\beta) = - \left( C_{q_1q_N}^{V,\alpha\beta} \overline{q} \gamma_{\mu} P_L q + C_{q_2q_N}^{V,\alpha\beta} \overline{q} \gamma_{\mu} P_R q \right) \overline{v}_\nu \gamma_{\mu} P_L u_\nu, \tag{E2}
\]

where \( v_\nu(v_N) \) and \( v_\bar{\nu}(v_{N'}) \) are the spinors of anti-neutrinos \( \bar{\nu}_\alpha(N_\alpha) \) and \( \bar{\nu}_\beta(N_\beta) \), respectively. The amplitudes with \( \Delta L = 2 \) are

\[
M(q\bar{q} \to \nu_\alpha \nu_\beta) = 2 \left( C_{q_1q_\nu}^{S,\alpha\beta} \overline{q} P_R q + C_{q_2q_\nu}^{S,\alpha\beta} \overline{q} P_L q \right) \overline{v}_\nu \gamma_{\mu} P_L u_\nu,
\]

\[
M(q\bar{q} \to v_\nu(v_N)) = \frac{1}{2} \left( C_{q_1q_\nu}^{V,\alpha\beta} \overline{q} \gamma_{\mu} P_L q + C_{q_2q_\nu}^{V,\alpha\beta} \overline{q} \gamma_{\mu} P_R q \right) \overline{v}_\nu \gamma_{\mu} P_L u_\nu,
\]

\[
M(q\bar{q} \to v_{N'}(v_N)) = \frac{1}{2} \left( C_{q_1q_N}^{V,\alpha\beta} \overline{q} \gamma_{\mu} P_L q + C_{q_2q_N}^{V,\alpha\beta} \overline{q} \gamma_{\mu} P_R q \right) \overline{v}_{N'} \gamma_{\mu} P_L u_\nu.
\]
\[ + 2 \left( C_{\nu q}^{T,\alpha\beta} \bar{q} \gamma_\mu P R q - i 2 e Q q C_{\nu q}^{\alpha\beta} (k_1 + k_2) \gamma_\mu q \right) \bar{u}_\nu \sigma^{\mu\nu} P R u_N^C , \]

\[ \mathcal{M}(q\bar{q} \rightarrow N_\alpha N_\beta) = 2 \left( C_{\nu q N_1}^{S,\alpha\beta} \bar{q} P R q + C_{\nu q N_2}^{S,\alpha\beta} \bar{q} P L q \right) \bar{u}_N P_L u_N^C , \]

\[ + 2 \left( C_{\nu q N_1}^{T,\alpha\beta} \bar{q} \gamma_\mu P L q - i 2 e Q q C_{\nu q N_1}^{\alpha\beta} (k_1 + k_2) \gamma_\mu q \right) \bar{u}_N \sigma^{\mu\nu} P L u_N^C , \]

\[ \mathcal{M}(q\bar{q} \rightarrow \nu_\alpha N_\beta) = - \left( C_{\nu q N_1}^{V,\alpha\beta} \bar{q} \gamma_\mu P L q + C_{\nu q N_2}^{V,\alpha\beta} \bar{q} \gamma_\mu P R q \right) \bar{u}_\nu \gamma^\mu P L u_N^C , \] (E3)

where \( u_\nu(u_N) \) and \( u_{\nu'}(u_{N'}) \) are the spinors of neutrinos \( \nu_\alpha(N_\alpha) \) and \( \nu_\beta(N_\beta) \), respectively. We list the individual matrix elements for pseudoscalar invisible decays to neutrinos

\[ - \mathcal{M}(P \rightarrow \nu_\alpha \bar{\nu}_\beta) = \frac{i f_P}{2} \left( C_{\nu q 1}^{V,\alpha\beta} - C_{\nu q 2}^{V,\alpha\beta} \right) \bar{u}_\nu P_L u_{\nu'} = 0 , \]

\[ - \mathcal{M}(P \rightarrow N_\alpha \bar{N}_\beta) = \frac{i f_P}{2} \left( C_{\nu q N_1}^{V,\alpha\beta} - C_{\nu q N_2}^{V,\alpha\beta} \right) \bar{u}_N P_{L} u_{\nu'} , \]

\[ - \mathcal{M}(P \rightarrow \nu_\alpha \bar{N}_\beta) = \frac{h_P^2}{4m_q} \left( C_{\nu q N_1}^{S,\alpha\beta} - C_{\nu q N_2}^{S,\alpha\beta} \right) \bar{u}_\nu P_{L} u_{\nu'} , \]

\[ - \mathcal{M}(P \rightarrow N_\alpha \nu_\beta) = \frac{h_P^2}{4m_q} \left( C_{\nu q N_1}^{S,\alpha\beta} - C_{\nu q N_2}^{S,\alpha\beta} \right) \bar{u}_N P_{L} u_{\nu'} , \]

\[ - \mathcal{M}(P \rightarrow \bar{\nu}_\alpha \bar{\nu}_\beta) = \frac{h_P^2}{2m_q} \left( C_{\nu q 1}^{S,\alpha\beta} - C_{\nu q 2}^{S,\alpha\beta} \right) \bar{u}_{\nu'} P_{L} u_{\nu'} , \]

\[ - \mathcal{M}(P \rightarrow \bar{N}_\alpha \bar{N}_\beta) = \frac{h_P^2}{2m_q} \left( C_{\nu q N_1}^{S,\alpha\beta} - C_{\nu q N_2}^{S,\alpha\beta} \right) \bar{u}_{\nu'} P_{L} u_{\nu'} , \]

\[ - \mathcal{M}(P \rightarrow \bar{\nu}_\alpha \bar{N}_\beta) = if_P \left( C_{\nu q 1}^{V,\alpha\beta} - C_{\nu q 2}^{V,\alpha\beta} \right) \bar{u}_{\nu'} P_{L} u_{\nu'} , \]

\[ - \mathcal{M}(P \rightarrow \bar{N}_\alpha \nu_\beta) = if_P \left( C_{\nu q N_1}^{V,\alpha\beta} - C_{\nu q N_2}^{V,\alpha\beta} \right) \bar{u}_{\nu'} P_{L} u_{\nu'} , \] (E4)

where in each amplitude the quark label \( q \) is summed over the first three light quarks \((u, d, s)\) implicitly and \( u_\nu(u_N) \) and \( u_{\nu'}(u_{N'}) \) are the spinors of neutrinos \( \nu_\alpha(N_\alpha) \) and \( \nu_\beta(N_\beta) \), respectively. By evaluating the squared matrix elements, we find the following results for the vector and scalar currents

\[ |\bar{u}_\nu P_{L/R} u_{\nu'}|^2 = m_\nu^2 \left[ m_1^2 + m_2^2 - \frac{(m_2^2 - m_2^2)^2}{m_P^2} \right] , \]

\[ |\bar{u}_\nu P_{L/R} u_{\nu'}|^2 = m_\nu^2 \left[ 1 - \frac{m_1^2 + m_2^2}{m_P^2} \right] . \] (E5)

One can see that the above relation holds for any projection operator and it is true for either particles or antiparticles in the final states. It also applies for neutrino bilinears with charge-conjugate fields, since \( u(p, s) = C\bar{v}(p, s)^T \) and \( v(p, s) = C\bar{u}(p, s)^T \). Taking all this together, we obtain the branching ratio in Eq. (48).
Similarly the decay matrix elements of vector meson $V$ are given by

$$
\mathcal{M}(V \rightarrow \nu_\alpha \bar{\nu}_\beta) = m_V \left( \frac{f_V^q}{2} \left( C_{\nu q,1}^{V,\alpha\beta} + C_{\nu q,2}^{V,\alpha\beta} \right) \bar{u}_\beta \gamma_\mu P_L v_\nu \epsilon_\nu^\mu \right),
$$

$$
\mathcal{M}(V \rightarrow N_\alpha \bar{N}_\beta) = m_V \left( \frac{f_V^q}{2} \left( C_{\nu q,1}^{V,\alpha\beta} + C_{\nu q,2}^{V,\alpha\beta} \right) \bar{u}_\beta \gamma_\mu P_R v_\nu \epsilon_\nu^\mu \right),
$$

$$
\mathcal{M}(V \rightarrow \bar{\nu}_\alpha N_\beta) = i2 \left( f_V^q C_{\nu q,1}^{V,\alpha\beta*} - e_Q \frac{f_V^q}{m_V} C_{\nu q,2}^{V,\alpha\beta*} \right) \bar{u}_\beta \gamma_\mu P_R v_\nu \epsilon_\nu^\mu \right),
$$

$$
\mathcal{M}(V \rightarrow \bar{\nu}_\alpha \bar{N}_\beta) = i4 \left( f_V^q C_{\nu q,1}^{V,\alpha\beta} - e_Q \frac{f_V^q}{m_V} C_{\nu q,2}^{V,\alpha\beta} \right) \bar{u}_\beta \gamma_\mu P_R v_\nu \epsilon_\nu^\mu \right),
$$

$$
\mathcal{M}(V \rightarrow \bar{N}_\alpha \bar{N}_\beta) = i4 \left( f_V^q C_{\nu q,1}^{V,\alpha\beta} - e_Q \frac{f_V^q}{m_V} C_{\nu q,2}^{V,\alpha\beta} \right) \bar{u}_\beta \gamma_\mu P_R v_\nu \epsilon_\nu^\mu \right),
$$

$$
\mathcal{M}(V \rightarrow \nu_\alpha \bar{N}_\beta) = m_V \left( \frac{f_V^q}{2} \left( C_{\nu q,1}^{V,\alpha\beta*} + C_{\nu q,2}^{V,\alpha\beta*} \right) \bar{u}_\beta \gamma_\mu P_R v_\nu \epsilon_\nu^\mu \right),
$$

$$
\mathcal{M}(V \rightarrow \nu_\alpha \nu_\beta) = m_V \left( \frac{f_V^q}{2} \left( C_{\nu q,1}^{V,\alpha\beta} + C_{\nu q,2}^{V,\alpha\beta} \right) \bar{u}_\beta \gamma_\mu P_R v_\nu \epsilon_\nu^\mu \right),
$$

$$
\mathcal{M}(V \rightarrow N_\alpha N_\beta) = m_V \left( \frac{f_V^q}{2} \left( C_{\nu q,1}^{V,\alpha\beta} + C_{\nu q,2}^{V,\alpha\beta} \right) \bar{u}_\beta \gamma_\mu P_R v_\nu \epsilon_\nu^\mu \right),
$$

$$
\mathcal{M}(V \rightarrow \bar{\nu}_\alpha \bar{\nu}_\beta) = m_V \left( \frac{f_V^q}{2} \left( C_{\nu q,1}^{V,\alpha\beta} + C_{\nu q,2}^{V,\alpha\beta} \right) \bar{u}_\beta \gamma_\mu P_R v_\nu \epsilon_\nu^\mu \right),
$$

where we again sum over quark flavor $q = u, d, s$ implicitly. By evaluating the squared matrix elements, we find the following results for the vector and tensors currents

$$
\frac{1}{3} \sum_{\text{pol}} |\bar{u}_1 \gamma_\mu P_L u_2 \epsilon_\nu^\mu|^2 = \frac{2}{3} m_V^2 \left[ 1 - \frac{m_1^2 + m_2^2}{2m_V^2} \right],
$$

$$
\frac{1}{3} \sum_{\text{pol}} |\bar{u}_1 \gamma_\mu P_L u_2 \epsilon_\nu^\mu|^2 = \frac{2}{3} m_V^2 \left[ 1 + \frac{m_1^2 + m_2^2}{2m_V^2} \right],
$$

after averaging over the initial polarizations of the vector meson $V$. Combining the above results, we obtain the branching ratio given in the main part of the text in Eq. (51).

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