Gravitation and regular Universe without dark energy and dark matter

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It is shown that isotropic cosmology in the Riemann-Cartan spacetime allows to solve the problem of cosmological singularity as well as the problems of invisible matter components – dark energy and dark matter. All cosmological models filled with usual gravitating matter satisfying energy dominance conditions are regular with respect to energy density, spacetime metrics and the Hubble parameter. At asymptotics cosmological solutions of spatially flat models describe accelerating Universe without dark energy and dark matter, and quantitatively their behaviour is identical to that of standard cosmological $\Lambda CDM$-model.

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INTRODUCTION

The general relativity theory (GR) is the base of gravitation theory, relativistic cosmology and astrophysics. At the same time GR leads to some principal problems and difficulties, which in particular appear in cosmology. The most principal problem of modern cosmology is connected with invisible matter components in the Universe – dark energy (DE) and dark matter (DM). The explanation of observational cosmological data in the frame of standard $\Lambda CDM$-model leads to conclusion that about 96% energy in the Universe is connected with DE and DM and only about 4% energy is related to usual barionic matter. As a result the present situation in cosmology and gravitation theory is similar to that in physics at the beginning of XX century, when the notion of "ether" was introduced with the purpose to explain various electrodynamic phenomena. Another principal problem of standard cosmology, which does not have acceptable solution still, is the problem of the beginning of the Universe in time in the past – the problem of cosmological singularity (PCS).

Many attempts were undertaken with the purpose to solve indicated problems in the frame of GR and existent candidates to quantum gravitation theory (string theory/M-theory, loop quantum gravity) as well as of different generalizations of GR (see for example [1]). Although a number of results were obtained, the search of the most satisfactory solutions of discussed problems is continued. As it was shown in a number of papers (see [2–9] and Refs herein) the gravitation theory in 4-dimensional Riemann-Cartan spacetime $U_4$ – the Poncaré gauge theory of gravity (PGTG) – offers opportunities to solve indicated cosmological problems. Note at first, that the PGTG is based on well known acceptable physical principles including gauge invariance principle. Indeed the PGTG is a necessary generalization of metric gravitation theory, if the Lorentz group is included to gravitation gauge group, and namely the PGTG but not metric gravitation theory corresponds to supergrav-
by spacetime torsion. However, the situation with DE- and DM-problems in the case of these HIM becomes the same as in GR. Isotropic cosmology based on HIM with two torsion functions allows to solve the PCS as well as the DE-problem [6, 8]. Moreover, the DM-problem together with DE-problem can be solved in the frame of such HIM [7]. However, simultaneous solution of PCS and DE- with DM-problems was not found still.

As it is shown in this paper all discussed problems can be solved in the frame of isotropic cosmology in the Riemann-Cartan spacetime by certain restrictions on indefinite parameters of gravitational Lagrangian \( \mathcal{L}_g \).

We will consider the PGTG based on sufficiently general following expression of gravitational Lagrangian (definitions and notations of [6, 9] are used below):

\[
\mathcal{L}_g = \left[ f_0 F + F^{\alpha \beta \mu \nu} (f_1 F_{\alpha \beta \mu \nu} + f_2 F_{\alpha \mu \beta \nu} + f_3 F_{\mu \nu \alpha \beta}) + F^{\alpha \mu \nu} (f_4 F_{\alpha \mu \nu} + f_5 F_{\mu \nu}) + f_6 F^2 + S^{\alpha \mu \nu} (a_1 S_{\alpha \mu \nu} + a_2 S_{\mu \nu}) + a_3 S^{\alpha \mu \nu} S_{\beta \mu \nu} \right],
\] (1)

The Lagrangian (1) includes the parameter \( f_0 = (16 \pi G)^{-1} \) (\( G \) is Newton’s gravitational constant, the light velocity \( c = 1 \)) and a number of indefinite parameters: \( f_i \) (\( i = 1, 2, \ldots 6 \)) and \( a_k \) (\( k = 1, 2, 3 \)). Physical consequences of PGTG depend essentially on restrictions on indefinite parameters \( f_i \) and \( a_k \). Some of such restrictions will be given below by investigation of HIM.

Gravitational equations for HIM with two torsion functions corresponding to gravitational Lagrangian (1) allow to obtain cosmological equations generalizing Friedmann cosmological equations of GR and equations for torsion functions given in general form in [9]. These equations contain five indefinite parameters:

\[
\begin{align*}
a &= 2a_1 + a_2 + 3a_3, & b &= a_2 - a_1, \\
f &= f_1 + \frac{f_2}{2} + f_3 + f_4 + f_5 + 3f_6, \\
q_1 &= f_2 - 2f_3 + f_4 + f_5 + 6f_6, & q_2 &= 2f_1 - f_3,
\end{align*}
\]

and their mathematical structure and physical consequences depend essentially on restrictions on these parameters. Unlike metric gravitation theory, quadratic in the curvature terms of \( \mathcal{L}_g \) do not lead to higher derivatives of \( R \) in cosmological equations; higher derivatives can appear because of terms of \( \mathcal{L}_g \) quadratic in the torsion tensor [9, 11]; in order to exclude higher derivatives of \( R \) from cosmological equations we have to put the restriction \( a = 0 \) [17]. The second restriction concerns the parameter \( q_2 \): if \( q_2 \neq 0 \), the equation for the torsion function \( S_2 \) is differential equation of the second order that leads to oscillating behaviour of the Hubble parameter [6, 8]; by putting \( q_2 = 0 \) we will obtain physically necessary consequences. Below we will analyze the main relations of isotropic cosmology given in [9] in general case without using any restrictions on indefinite parameters by putting the following restrictions: \( a = 0 \) and \( q_2 = 0 \).

Cosmological equations generalizing Friedmann cosmological equations of GR take the following form:

\[
\frac{k}{R^2} + (H - 2S_1)^2 - S_2^2 = \frac{1}{6f_0 Z} \left[ \rho - 6bS_2^2 + \frac{\alpha}{4}(\rho - 3p - 12bS_2^2)^2 \right],
\] (2)

\[
\dot{H} + H^2 - 2HS_1 - 2S_1 = -\frac{1}{12f_0 Z} \left[ \rho + 3p - \frac{\alpha}{2}(\rho - 3p - 12bS_2^2)^2 \right],
\] (3)

where \( \rho \) is the energy density, \( p \) is the pressure, \( H = \dot{R}/R \) is the Hubble parameter (a dot denotes the differentiation with respect to time), the parameter \( \alpha = \frac{f}{f_0} \) (\( f > 0 \)) has inverse dimension of energy density and \( Z = 1 + \alpha(\rho - 3p - 12bS_2^2) \). The torsion function \( S_1 \) is determined by the following way:

\[
S_1 = -\frac{\alpha}{4Z} \dot{\phi} - 3\dot{\phi} + 12f_0 \omega H S_2^2 - 12(2b - \omega f_0)S_2 S_2',
\] (4)

where dimensionless parameter \( \omega = 2f_0 - f_1 \neq 0 \) is introduced. The torsion function \( S_2 \) satisfies algebraic quadratic equation, which gives the following root

\[
S_2 = \frac{\rho - 3p}{12b} + \frac{1 - (b/2f_0)(1 + \sqrt{X})}{12b\alpha(1 - \omega/4)},
\] (5)

where \( X = 1 + \omega(f_0/b^2)(1 - b/f_0 - 2(1 - \omega/4)\alpha(\rho + 3p)) \) [18]. In order to reduce cosmological equations (2)-(3) to closed form we have to specify the content of HIM and its equation of state. In connection with this it should be noted that the matter content and its equation of state change during cosmological evolution and the form of equation of state depends on coupling of matter with gravitational field. In the case of usual gravitating matter with energy density \( \rho_m \) and pressure \( p_m \) coupled minimally with gravitation the equation of state can be written in usual form: \( p_m = \rho_m(\rho_m) \) [19] and the law of energy conservation takes the form as in GR:

\[
\dot{\rho}_m + 3H(\rho_m + p_m) = 0.
\] (6)

We introduce at early stage of cosmological expansion the scalar field \( \phi \) with potential \( V = V(\phi) \) as component of gravitating matter with the purpose to investigate inflationary HIM. By minimal coupling with gravitation the equation for scalar field takes the usual form as in GR:

\[
\ddot{\phi} + 3H\dot{\phi} = -\frac{\partial V}{\partial \phi}.
\] (7)

Then the total energy density \( \rho \) and pressure \( p \) are the following:

\[
\rho = \frac{1}{2}\dot{\phi}^2 + V + \rho_m \quad (\rho > 0), \quad p = \frac{1}{2}\dot{\phi}^2 - V + p_m.
\] (8)
Now by using the formula (5) for torsion function $S_2$ and eqs. (6)-(8) we transform the torsion function $S_1$ defined by (4) to the following form:

$$S_1 = -\frac{3f_0\omega}{4bZ}(HD + E),$$  \hfill (9)

where

$$D = \frac{1}{2} \left( \frac{3 dp_m}{dp_m - 1} \right) (\rho_m + p_m)$$

$$+ \frac{1}{3} (\rho_m - 3p_m) + \frac{2}{3} \Phi^2 + \frac{4}{3} \frac{b}{6f_0\omega(1 - \omega/4)} \sqrt{X}$$

$$+ \frac{1 - \omega(b/2f_0)}{2\sqrt{X}} \left[ (\frac{1}{2} dpm + 1) (p_m + p_m) + 4\Phi^2 \right]$$

$$- \frac{\omega f_0(1 - b/f_0)}{b\omega(1 - \omega/4)},$$

$$E = \left( 1 + \frac{1 - \omega(f_0/2b)}{\sqrt{X}} \right) \frac{\partial V}{\partial \Phi},$$

$$Z = \frac{-\omega/4 + (b/2f_0)(1 + \sqrt{X})}{1 - \omega/4}. \hfill (10)$$

By using the formulas for torsion functions we write the cosmological equations (2)-(3) in the following closed form:

$$\frac{k}{R^2} + H^2 = \frac{1}{6f_0Z} \left[ \rho + 6(f_0Z - b)S_2 + \frac{1 - (b/2f_0)(1 + \sqrt{X})^2}{4\alpha(1 - \omega/4)^2} \right], \hfill (11)$$

$$(\dot{H} + H^2) \left[ 1 + \frac{3f_0\omega}{2bZ} D \right] +$$

$$\frac{3f_0\omega}{2bZ} \left[ H (\dot{D} - \frac{Z}{Z} + E) + \dot{E} - \frac{Z}{E} \right]$$

$$= -\frac{1}{12f_0Z} \left[ \rho + 3p - \frac{1 - (b/2f_0)(1 + \sqrt{X})^2}{2\alpha(1 - \omega/4)^2} \right], \hfill (12)$$

where the quantities $S_2$, $D$, $E$, $Z$ have to be replaced according to (5) and (10).

By using obtained cosmological equations we will analyze properties of cosmological solutions at different stages of cosmological evolution. Simultaneously we will find by what restrictions on indefinite parameters $\alpha$, $b$ and $\omega$ physical consequences are the most satisfactory and correspond to observational cosmological data.

At first we will consider the evolution of cosmological models at asymptotics, when energy densities are small. If the value of dimensional parameter $\omega$ is sufficiently small $|\omega| \ll 1$, the following estimations are valid: $X \to 1$, $Z \to b/f_0$, $S_1 \to 0$ and the torsion function $S_2$ approximately is equal to:

$$S_2 = \frac{\rho - 3p}{12b} + \frac{1 - b/f_0}{12ab}. \hfill (13)$$

As a result cosmological equations (2)-(3) take the form of Friedmann cosmological equations with effective cosmological constant induced by pseudoscalar torsion function $S_2$:

$$\frac{k}{R^2} + H^2 = \frac{1}{6f_0} \left[ \rho(f_0/b) + \frac{1}{4}\alpha^{-1}(1 - b/f_0)^2(f_0/b) \right], \hfill (14)$$

$$\dot{H} + H^2 = -\frac{1}{12f_0} \left[ \rho + 3p)(f_0/b) - \frac{1}{2}\alpha^{-1}(1 - b/f_0)^2(f_0/b) \right]. \hfill (15)$$

By certain values of parameters $\alpha$ and $b$ physical consequences of eqs. (14)-(15) for flat model ($k = 0$) without introducing of DE and DM are identical to that of standard $\Lambda CDM$-model [7]. In fact, the energy density $\rho$ in eqs. (14)-(15) corresponds to total energy density of physical matter in the Universe, which consists of baryonic matter, relic radiation, neutrino etc. If dark matter does not exist, the value of $\rho$ is approximately equal to energy density of baryonic matter, because the contribution of other named components is sufficiently small, that leads to certain estimation of the parameter $b$ ($b \approx f_0/6$). Then by taking into account cosmological data concerning the dark energy we obtain the estimation of the parameter $\alpha$ [7] [20]. Because the cosmological equations (14)-(15) were obtained in zeroth approximation with respect to small parameter $\omega$, the following question appears: what are quantitative limits of applicability of these equations? In order to find estimation of parameter $\omega$, we will analyze the behaviour of cosmological solutions in the beginning of cosmological expansion.

First of all important physical consequences follow from formula (5) for $S_2$-function. If the parameter $\omega$ is positive ($0 < \omega \ll 1$), because the value of $X$ can not be negative we obtain principal constraint for admissible energy densities: $X = 1 + \omega(f_0^2/b^2)(1 - b/f_0) - 2(1 - \omega/4)(\alpha + 3p) \geq 0$ or by taking into account smallness of $\omega$ the following relation:

$$X = 1 - 2(f_0^2/b^2)\omega(\alpha + 3p) \geq 0. \hfill (16)$$

In the case of systems filled with usual matter with energy density $\rho_m$ without scalar fields the equality defined by (16) determines a limiting (maximum) energy density. When energy density $\rho_m$ is comparable with $\rho_{\text{max}}$, the gravitational interaction has the character of repulsion ensuring the regularity of such systems [21]. In the frame of HIM without dark matter the value of $\alpha^{-1}$ is comparable with average energy density in the Universe at present epoch. The order of limiting energy density $\rho_{\text{max}}$ is determined by the value of $\omega$ by $\alpha^{-1}$. If the value of $\rho_{\text{max}}$ is comparable or greater than energy density of quark-gluon plasma (but less than the Planckian one), we obtain that dimensionless parameter $\omega$ is extremely small. In connection with this the cosmological equations (14)-(15) are very good approximation at least for matter dominated
stage of cosmological evolution. In the case of systems including also scalar fields, for which energy density and pressure are defined by (8), the relation (16) determines in space of matter parameters \((\rho_m, \phi, \dot{\phi})\) domain of their admissible values. This domain is limited by surface \(L\) defined by \(X = 1 - 2(f_0/\beta^2)\omega^{\alpha}(\rho_m + 3p_m + 2\dot{\phi}^2 - 2V) = 0\). Moreover, it is necessary to take into account that additional restriction on admissible values of matter parameters \((\rho_m, \phi, \dot{\phi})\) follows from positivity of expression (5) for \(S_2^2\).

Now we will analyze the behaviour of cosmological solutions near the limiting energy density or limiting surface \(L\), where \(X \ll 1\). With this purpose we consider the expression of the Hubble parameter \(H\) following from cosmological equation (11):

\[
H_\pm = \left[ -\frac{3f_0\omega^{\alpha}}{2bZ}E \pm \left( \frac{1}{6f_0Z} \left[ \rho + 6(f_0Z - b)S^2_2 \right. \right. \right] \\
+ \left[ 1 - \frac{(b/2f_0)(1 + \sqrt{X})^2}{4\alpha(1 - \omega/4)^2} \right] - \frac{k}{R^2} \) \right)^{1/2} \left( 1 + \frac{3f_0\omega^{\alpha}}{2bZ}D \right)^{-1/2} .
\]

(17)

Similarly to isotropic cosmology based on HIM with the only torsion function \([2, 3]\), at asymptotics where energy densities are sufficiently small, \(H_-\) solutions correspond to cosmological compression and \(H_+\) solution – to cosmological expansion, and the transition from \(H_-\) to \(H_+\) solution take place by reaching the limiting energy density or limiting surface \(L\) (see below) [22]. In order to obtain some physical characteristics of such transitions the formula (17) for \(H_\pm\) can be simplified by taking into account the smallness of parameter \(\omega (\omega \ll 1)\) and also that at considering extreme conditions: \(\alpha^{-1} \ll \rho, X \ll 1\), \(\rho \sim (\omega\alpha)^{-1}\). As a result expressions of \(D, E, Z\) and \(S_2^2\) are simplified:

\[
D = \frac{1}{2} \left( \frac{3dp_m}{d\rho_m} - 1 \right) (\rho_m + p_m) + \frac{1}{3} (\rho_m - 3p_m) + \frac{2}{3} \dot{\phi}^2 \\
+ \frac{4}{3} V + \frac{1}{2\sqrt{X}} \left( \frac{3dp_m}{d\rho_m} + 1 \right) (\rho_m + p_m + 4\dot{\phi}^2),
\]

\[
E = \left( 1 + \frac{1}{\sqrt{X}} \right) \frac{\partial V}{\partial \phi}, Z = \frac{b}{2f_0}(1 + \sqrt{X}),
\]

\[
S_2^2 = \frac{\rho - 3p}{12b} + \frac{1}{12b\alpha} - \frac{(b/2f_0)(1 + \sqrt{X})}{12b\alpha} ,
\]

where the quantity \(X\) is defined by (16). By taking into account terms linear with respect to \(\sqrt{X}\) the expression (17) for \(H_\pm\) takes the following form:

\[
H_\pm = \left[ -\frac{2\partial V}{\partial \phi} + \sqrt{X} \left[ 2\partial V}{\partial \phi} \right] \right] \\
+ \frac{2b^2}{3f_0^2\omega^{\alpha}} \left[ \frac{1}{4b}(\rho_m + p_m + \dot{\phi}^2) - \frac{1}{kR^2} \right]^{1/2} \\
+ \sqrt{X} \left[ \frac{2b^2}{3f_0^2\omega^{\alpha}} \left[ \frac{1}{4b}(\rho_m + p_m - \dot{\phi}^2) - \frac{1}{kR^2} \right]^{1/2} \\
+ \frac{2}{3} (\rho_m - 3p_m) + \frac{4}{3} \dot{\phi}^2 + \frac{4}{3} V \right]^{-1} .
\]

(19)

In the case of models filled with usual matter without scalar fields in linear approximation with respect to \(\sqrt{X}\) the Hubble parameter and its time derivative take the following form:

\[
H_\pm = \frac{2b^2}{3f_0^2\omega^{\alpha}} \left[ \frac{1}{4b}(\rho_m + p_m) - \left( k/R^2 \right)^{1/2} \right] \\
\Xi = \frac{3f_0^2\omega^{\alpha}}{(3dp_m/d\rho_m + 1)(\rho_m + p_m)^{1/2}} ,
\]

(20)

By reaching a limiting energy density \((X = 0)\) the Hubble parameter vanishes and the value of its time derivative is the same for \(H_-\) and \(H_+\) solution and it is positive that corresponds to a bounce. By using obtained expression for the Hubble parameter it is easy to show that by given equation of state \(p_m = p_m(\rho_m)\) the evolution of scale factor \(R(t)\) near a bounce take the following form: \(R(t) = R_{\text{min}} + r_1t^2 + \ldots\), where \(t = 0\) corresponds to a bounce, \(R_{\text{min}}\) is minimum value of \(R\) depending on limiting energy density and given equation of state, the value of \(r_1 > 0\) is expressed by \(H\) at a bounce. In the case of models including also scalar fields the Hubble parameter does not vanish by reaching a limiting surface \(L\) and according to (19) its value on surface \(L\) is:

\[
H_L = \frac{-2\partial V}{\partial \phi} \\
(3dp_m/d\rho_m + 1)(\rho_m + p_m) + 4\dot{\phi}^2 .
\]

(21)

The bounce in this case takes place in points of extremum surface in space of matter parameters \((\rho_m, \phi, \dot{\phi})\), equation of which we obtain by setting \(H = 0\) in cosmological equation (11). Cosmological solutions can be found by numerical integration of eqs. (12), (6) and (7) by choosing initial conditions on extremum surface. Similarly to inflationary cosmological models built in the frame of isotropic cosmology with the only torsion function (see [2, 3]), if initial value of scalar field is sufficiently large we obtain regular inflationary solution containing transition stage from compression to expansion, inflationary
stage with slow-rolling behaviour of scalar field and post-inflationary stage with oscillating scalar field. Similarly to our works [2, 3] regular Big Bang scenario can be built on the base of such inflationary cosmological models. All cosmological solutions are regular with respect to energy density, the scale factor \( R \) and the Hubble parameter \( H \). Unlike HIM with the only torsion function \( S_1 \), in the case of considered HIM with two torsion functions the torsion does not diverge by reaching limiting surface \( L \) (or limiting energy density): the torsion function \( S_2 \) is regular and the torsion function \( S_1 \) undergoes a finite jump by reaching the surface \( L \) (or limiting energy density).

The solution of cosmological problems in the frame of PGTG presented above is achieved by virtue of the change of gravitational interaction in comparison with GR and Newton’s theory of gravity at cosmological scale. These changes are provoked by more complicated structure of physical spacetime, namely by spacetime torsion. In connection with this the following question appears: what situation takes place in the case of gravitating systems at other spatial scales (galaxies, stars, solar system), what possible role the torsion plays in these systems? First of all, the torsion should be important in gravitating systems at astrophysical scale (galaxies and their accumulation), by investigation of which the notion of dark matter was introduced more than 80 years ago. It should be noted that the investigation of such systems in the frame of PGTG is difficult mathematical problem, because the system of gravitational equations of PGTG in the case of non-homogeneous gravitating systems, including spherically symmetric and axially symmetric objects, is very complicated system of differential equations, and so far the DM-problem at physical scale is not solved. The conclusion obtained in this paper about possible existence of limiting energy density and gravitational repulsion effect at extreme conditions can be of principal meaning for theory of massive superdense stars, where gravitational repulsion effect has to prevent a collapse. Concerning the solar system, usual gravitational effects including relativistic corrections in this case can be obtained in the frame of PGTG, because the vacuum Schwarzschild solution for the metrics with vanishing torsion is exact solution and does not depend on values of indefinite parameters of gravitational Lagrangian (1) [16]. However, from physical point of view this solution has certain limits of its applicability, because the physical spacetime in the vacuum in the frame of accelerating Universe is de Sitter spacetime with non-vanishing torsion [9].

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