D-branes in the pp-wave background

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Abstract. We summarize the classification of the intersecting supersymmetric D-branes in the type IIB plane wave background based on the Green-Schwarz superstring formulation.

1. Introduction

Recently, the geometry corresponding to the Penrose limit of the $AdS_5 \times S^5$ background was shown to be a maximally supersymmetric type IIB string background, [1] - [4],

$$ds^2 = -2dx^+dx^- - \mu^2 x^2(dx^+)^2 + dx^I_2,$$

$$F_{+1234} = F_{+5678} = 2\mu.$$

We systematically classify static D-branes in the maximally supersymmetric type IIB plane wave background (1) [5] - [18] using the Green-Schwarz superstring theory based on the light-cone open string theory. [16, 17, 18] The light-cone worldvolume coordinates $X^\pm$ are to satisfy the Neumann boundary condition hence the instantonic branes and branes with only one light-cone coordinate along the worldvolume in [5] will be outside of our classification.

2. Flat D-branes in A Plane Wave Background

The Green-Schwarz light-cone action in the plane wave background (1) describes eight free massive bosons and fermions [2]. In the light-cone gauge, $X^+ = \tau$, the action is given by

$$S = \frac{1}{2\pi\alpha'} \int d\tau \int_0^{2\pi\alpha'|p^+|} d\sigma \left[ \frac{1}{2} \partial_+ X_I \partial_- X_I - \frac{1}{2} \mu^2 X^2_I - iS(\rho^A \partial_A - \mu\Pi)S \right]$$

where $\partial_\pm = \partial_\tau \pm \partial_\sigma$. The equations of motion following from the action (2) take the form

$$\partial_+ \partial_- X^I + \mu^2 X^I = 0,$$

$$\partial_+ S^1 - \mu \Pi S^2 = 0, \quad \partial_- S^2 + \mu \Pi S^1 = 0.$$

The open string action is just defined by the action (2) with string length $\alpha = 2\alpha'p^+$ imposed with appropriate boundary conditions on each end of the open string. We will use the notation and the convention in [16] with more refined indices. Neumann coordinates $X^r$ are decomposed into oblique directions $X^r$ and usual parallel directions $X^\tau$ : $r = (\hat{r}, \check{r})$. Similarly, Dirichlet coordinates $X^r'$ are also decomposed into oblique directions $X^{r'}$ and usual parallel directions $X^\tau'$.
\(X^{\prime} : r^{\prime} = (\hat{r}^{\prime}, \hat{v}^{\prime})\). For longitudinal coordinates \(X^r\) on D-branes without any worldvolume flux, we impose the Neumann boundary condition
\[
\partial_\sigma X^r|_{\partial \Sigma} = 0,
\]
while for transverse coordinates \(X^{r'}\) we have the Dirichlet boundary condition
\[
\partial_\tau X^{r'}|_{\partial \Sigma} = 0.
\]

In the case to include gauge field excitations considered later, some Neumann boundary conditions have to be modified as follow [5, 11, 13, 16, 19]
\[
(\partial_\sigma X^r \pm \mu X^r)|_{\partial \Sigma} = 0
\]
for some \(r \in N\). The fermionic coordinates also have to satisfy the following boundary condition at each end of the open string [20]
\[
(S^1 - \Omega S^2)|_{\partial \Sigma} = 0,
\]
where the matrix \(\Omega\) is the products of \(\gamma\)-matrices along worldvolume directions.

The boundary condition (8) has to be compatible with the fermionic equation of motion (4) and thus the possible type of D-branes shall be characterized by the matrix \(\Gamma\) defined by
\[
\Gamma \equiv \Pi \Omega \Pi \Omega.
\]

\(D_{\pm}\)-branes [5]-[10] are a specific class satisfying \(\Gamma = \pm 1\).

Table 1 shows possible flat D-branes with particular polarizations. Other flat D-branes with different polarizations can be generated by \(SO(4) \times SO(4)\)' rotations of those in Table 1.

For \(D_+\)-branes, there are the following possibilities [5, 8, 10, 11]:
\[
\begin{align*}
D3 : (m, n) &= (2, 0), (0, 2), \\
D5 : (m, n) &= (3, 1), (1, 3), \\
D7 : (m, n) &= (4, 2), (2, 4).
\end{align*}
\]

For \(D_-\)-branes, there are the following possibilities [5, 8, 11]:
\[
\begin{align*}
D1 : (m, n) &= (0, 0), \\
D3 : (m, n) &= (1, 1), \\
D5 : (m, n) &= (4, 0), (2, 2), (0, 4), \\
D7 : (m, n) &= (3, 3), \\
D9 : (m, n) &= (4, 4).
\end{align*}
\]
The D-branes discussed in [13, 14] correspond to the \( OD^3 \)-brane with \( \Gamma = -\gamma^{1256} \) and \( OD^-5 \)-brane in Table 1.

The mode expansions for the bosonic and fermionic fields can be done in a straightforward way [18].

3. Intersecting D-branes
In this section, we will generalize the previous analysis to the case of intersecting D-branes using the formalism in [17]. In particular, the fermionic coordinates have to satisfy the following boundary condition at each end of the open string

\[
(S^1 - \Omega_0 S^2)|_{\sigma=0} = 0, \quad (S^1 - \Omega_\pi S^2)|_{\sigma=\alpha} = 0,
\]

with the matrix \( \Omega_\theta = (\Omega_0, \Omega_\pi) \) satisfying

\[
\Pi_0 \Omega_0 \Pi_0 = \Gamma_0, \quad \Pi_\pi \Omega_\pi \Pi_\pi = \Gamma_\pi.
\]

Here the D-brane is either a \( D^\pm \)-brane or an OD-brane.

The coordinates \( X^I(\tau, \sigma) \) of a \( p-q \) string can be partitioned into four sets, NN, DD, ND, and DN, according to whether the coordinate \( X^I \) has Neumann (N) or Dirichlet (D) boundary condition at each end. For intersecting D-branes, we will use indices \((r, s, \cdots) = (\hat{r}, \hat{s}, \cdots; \check{r}, \check{s}, \cdots), (i, j, \cdots) = (\hat{i}, \hat{j}, \cdots; \check{i}, \check{j}, \cdots)\) and \((i', j', \cdots) = (\hat{i}', \hat{j}', \cdots; \check{i}', \check{j}', \cdots)\) for NN, DD, ND, and DN coordinates, respectively, with a distinction between hatted indices for oblique directions and dotted indices for parallel directions.

The mode expansion of the spinor field can be determined [18]. We take an appropriate combination of spinor fields \( \xi^A(\tau, \sigma) \) with integer modes and \( \eta^A(\tau, \sigma) \) with half-integer modes or with R-modes to be compatible with supersymmetry:

\[
S^1(\tau, \sigma) = \begin{cases} 
I_+ \xi^1(\tau, \sigma) + I_- \eta^1(\tau, \sigma), & \text{for A-type;} \\
- \xi^1(\tau, \sigma) + I_+ \eta^1(\tau, \sigma), & \text{for B-type;}
\end{cases}
\]

\[
S^2(\tau, \sigma) = I_+ \xi^2(\tau, \sigma) + I_- \eta^2(\tau, \sigma),
\]

where \( 16 \times 16 \) matrices \( I_+ \) and \( I_- \) are defined by

\[
I_+ = \frac{1}{2}(1 + \Omega_0^T \Omega_\pi), \quad I_- = \frac{1}{2}(1 - \Omega_0^T \Omega_\pi).
\]

The spinors \( \xi^A(\tau, \sigma) \) and \( \eta^A(\tau, \sigma) \) are taken as the solution of the equations of motion (4) satisfying the boundary condition (12) at 0.

We now require the spinors \( S^A(\tau, \sigma) \) in Eq. (14) to satisfy the equations of motion (4) and then we need the following condition on \( I_\pm \):

\[
\Pi I_\pm = \begin{cases} 
I_\pm \Pi, & \text{for A-type;} \\
I_\mp \Pi, & \text{for B-type.}
\end{cases}
\]

One can see that the condition (16) is equivalent to the following constraint

\[
\Gamma_0 \Gamma_\pi = \begin{cases} 
1 \text{ or } \gamma, & \text{for } \Gamma_\theta^T = \Gamma_\theta; \\
-1 \text{ or } -\gamma, & \text{for } \Gamma_\theta^T = -\Gamma_\theta.
\end{cases}
\]

The condition (17) clearly explains why a \( D^- \)-brane cannot have a supersymmetric intersection with a \( D_+ \)-brane, as was shown in [17], since \( \Gamma_0 = -1 \) and \( \Gamma_\pi = 1 \) for this kind of intersection. In addition, the condition (17) implies that there may be a supersymmetric intersection between
Table 2. Supersymmetry of flat D-branes. A D-brane with a gauge field condensate is denoted by the boldface. $q^\pm_D$ ($q'^\pm_D$) is the number of unbroken kinematical supersymmetry of $D_-$-type ($D_+$-type).

| D-brane type | $\Gamma$ | $\Omega$ | $q^\pm_D$ | $q'^\pm_D$ | $q^-$ |
|--------------|----------|----------|-----------|------------|-------|
| $D_-$5       | $-1$     | $(3,1), (1,3)$ | 0         | 0          | 8     |
| $D_-(2n+1)$ | $1$      | $(n,n)$, $n = 1, 2, 3, 4$ | 0         | 0          | 4     |
| $OD3$       | $\pm \gamma^{1,2,3}$ | $\frac{1}{2}(\gamma^1 - \gamma^8)(\gamma^2 + \gamma^5)$ | 4         | 4          | 4     |
| $OD5$       | $\pm \gamma^{1,2,3}$ | $\frac{1}{2}(\gamma^1 - \gamma^8)(\gamma^2 + \gamma^5)$ | 4         | 4          | 4     |
| $OD5$       | $\pm \gamma^{1,2,3}$ | $\frac{1}{2}(\gamma^1 - \gamma^8)(\gamma^2 + \gamma^5)$ | 4         | 4          | 4     |
| $OD7$       | $\pm \gamma^{1,2,3}$ | $\frac{1}{2}(\gamma^1 - \gamma^8)(\gamma^2 + \gamma^5)$ | 4         | 4          | 4     |
| $OD_{-5}$   | $\gamma$ | $\frac{1}{2}(\gamma^1 - \gamma^8)(\gamma^2 + \gamma^5)$ | 0         | 0          | 8     |
| $OD_{+5}$   | $-\gamma$ | $\frac{1}{2}(\gamma^1 - \gamma^8)(\gamma^2 + \gamma^5)$ | 8         | 0          | 8     |

**different classes of OD-brane or an $OD_{\pm 5}$-brane and a $D_{\pm p}$-brane only if they satisfy $\Gamma_0 \Gamma_\pi = \gamma$. This case preserves only kinematical supersymmetries.**

The case $\Omega_D^+ \Omega_\pi = 1$ corresponds to parallel $Dp$-branes while the case $\Omega_D^- \Omega_\pi = -1$ corresponds to $Dp$-anti-$Dp$ branes, but the cases $\Omega_D^0 \Omega_\pi = \pm \gamma$ and $\Omega_D^- \Omega_\pi = \pm \Xi$ correspond to $Dp$-$Dq$ or $Dp$-anti-$Dq$ branes with $\sharp_{ND} = 8$ and $\sharp_{ND} = 4$, respectively. Note that the B-type branes allow only the $\sharp_{ND} = 4$ case.

**4. Supersymmetry of Flat D-branes**

In a light-cone gauge, the 32 components of the supersymmetries for a closed string decompose into kinematical supercharges, $Q^+ \sigma$, and dynamical supercharges, $Q^- \tau$. [2]

An open string on a D-brane satisfying $\Gamma^T = \Gamma$ preserves 8 kinematical supersymmetries of which 4 supersymmetries are generated by $S_0^+$ and another 4 supersymmetries are generated by $S_0^-$. On the other hand, an open string on a D-brane satisfying $\Gamma^T = -\Gamma$ preserves no kinematical supersymmetry.

Now we investigate the dynamical supersymmetry preserved by an open string on a D-brane characterized by $\Gamma$ in Eq. (9). The dynamical supercharge of an open string is given by a combination of those of a closed string compatible with the open string boundary conditions. Due to the boundary condition (8), it turns out that the conserved dynamical supercharge is given by (a subset of)

$$q^- = Q^{-1} - \Omega Q^{-2}.$$  \hspace{1cm} (18)

Using the similar recipe used in the kinematical supersymmetry, it is not difficult to show that the dynamical supercharge density $q^-_\sigma$ in Eq. (18) also satisfies the conservation law

$$\frac{\partial q^-_\sigma}{\partial \tau} + \frac{\partial q^-_\sigma}{\partial \sigma} = 0,$$  \hspace{1cm} (19)

where

$$q^-_\sigma = \sqrt{\frac{1}{2p^+}} \left( \frac{\partial \gamma^\tau - \partial \gamma^\sigma}{\partial \gamma^\sigma} (S^1 - \Omega S^2) 
+ (\partial \gamma^\tau + \partial \gamma^\sigma)(S^1 + \Omega S^2) 
+ \mu \gamma^\tau \Omega \Pi(S^1 + \Gamma \Omega S^2) - \mu \gamma^\tau \gamma^\tau \Omega \Pi(S^1 - \Gamma \Omega S^2) \right).$$  \hspace{1cm} (20)
Table 3. Supersymmetry of intersecting D-branes. $\nu = n_{D_{-}} + n_{D_{+}}$ is the number of unbroken kinematical supersymmetry where $n_{D_{-}} = \frac{1}{2} \text{Tr} (1 + \gamma) P_{-} I_{\pm}$ ($D_{-}$-type) and $n_{D_{+}} = \frac{1}{2} \text{Tr} (1 - \gamma) P_{+} I_{\pm}$ ($D_{+}$-type). A D-brane with a gauge field condensate is denoted by the boldface.

| D-brane type | Intersection | $q^+$ | $q^-$ |
|-------------|-------------|-----|-----|
| $\Gamma_0 = \Gamma_\pi = -1$ | $D_{-}p - D_{-}q$ | $\nu$ | $\frac{1}{2} \text{Tr} (1 - \gamma) I_{\pm}$ |
| $\Gamma_0 = \Gamma_\pi = 1$ | $D_{+}p - D_{+}q$ | $\nu$ | $\frac{1}{2} \text{Tr} (1 - \gamma) P_{+} I_{\pm}$ |
| $\Gamma_0 = \Gamma_\pi = -\gamma$ | $OD_{-}5 - OD_{-}5$ | $\nu$ | $\frac{1}{2} \text{Tr} (1 - \gamma) I_{\pm}$ |
| $\Gamma_0 = \Gamma_\pi = \gamma$ | $OD_{+}5 - OD_{+}5$ | $\nu$ | 0 |
| $\Gamma_0 = \Gamma_\pi = \pm \gamma^{1256}$ | $OD_{p} - OD_{q}$ | $\nu$ | $\frac{1}{2} \text{Tr} (1 - \gamma) P^{-I_{\pm}} I_{\pm}$ |
| $\Gamma_0 = -1, \Gamma_\pi = -\gamma$ | $D_{-}p - OD_{-}5$ | $\nu$ | 0 |
| $\Gamma_0 = 1, \Gamma_\pi = \gamma$ | $D_{+}p - OD_{+}5$ | $\nu$ | 0 |
| $\Gamma_0 = \pm \gamma^{1256}, \Gamma_\pi = \pm \gamma^{3478}$ | $OD_{p} - OD_{q}$ | $\nu$ | 0 |

If $\Gamma = \pm 1$, we definitely recover the $D_{2}$-brane case [16]. It was shown in [5] that $D_{-}$-branes of type $(+, -, 3, 1)$ or $(+, -, 1, 3)$ with a constant worldvolume flux also preserve 16 supersymmetries which was not discussed in [16].

When $\Gamma = \Omega \Pi \Omega \Pi = 1$, there is also a new possibility for $D_{4}$-branes of type $(+, -, n, n)$ with $n = 1, 2, 3, 4$ to preserve dynamical supersymmetries by introducing a gauge field excitation, whose possibility was anticipated by Hikida and Yamaguchi [13] from general supersymmetry arguments.

$D_{4}$-branes of type $(+, -, n, n)$ with $n = 1, 2, 3, 4$ preserve 4 dynamical supersymmetries by introducing a gauge field excitation, consistent with the result in [13]. Note that the dynamical supersymmetry in this case is preserved regardless of transverse locations of D-brane.

When $\Gamma^T = -\Gamma$, Eq. (20) shows that there is no chance for $q_{\pi} |_{D_{4}}$ to vanish and thus an open string on this D-brane does not preserve any dynamical supersymmetry at all.

We summarized our results on the kinematical and dynamical supersymmetry of D-branes in Table 2. The results on the dynamical supersymmetry $D_{\pm}$-branes preserving 16 supersymmetries without flux have been identified in [16] and are omitted in the Table 2.

5. Supersymmetry of Intersecting D-branes

We now analyze the supersymmetry of intersecting D-branes. The supersymmetry of intersecting $D_{\pm}$-branes was completely identified in [17] using the Green-Schwarz worldsheet formulation which can also be applied to more general class of D-branes under consideration.

In general, the unbroken supersymmetry of intersecting D-branes is the ‘intersection’ of supersymmetries preserved by each brane. The intersection is characterized by the projection matrices $I_{\pm}$ in Eq. (15).

As for the dynamical supersymmetry of intersecting D-branes the (anti-)commutation relations among $\gamma^I = \{\gamma^r, \gamma'_{\pi}, \gamma^i, \gamma^i_{\pi}\}$, $\Omega_0$ and $\Omega_\pi$ are useful to find conserved dynamical supersymmetries:

$$\{\gamma^r, \Omega_0\} = \{\gamma^i, \Omega_0\} = [\gamma'_{\pi}, \Omega_0] = [\gamma^i, \Omega_0] = 0,$$

$$\{\gamma^r, \Omega_\pi\} = \{\gamma'_{\pi}, \Omega_\pi\} = [\gamma^i, \Omega_\pi] = [\gamma^i, \Omega_\pi] = 0.$$

We summarized the supersymmetry preserved by various configurations of intersecting D-branes in Table 3. The number of each type of kinematical supersymmetries depends on the
total number of ND and DN directions in a way determined by the projection matrix $I_\pm P_-$ ($D_-$-type) or $I_\pm P_+$ ($D_+$-type).

It was shown in [13, 14] that the plane wave background (1) admits supersymmetric curved D-branes as well as oblique D-branes. We also classified the supersymmetric curved D-branes.

6. Discussion

We presented the classification of supersymmetric D-branes in the type IIB plane wave background using the light-cone open string theory where only longitudinal D-branes are visible. We considered only static D-branes. However, one can generate new symmetry related D-branes which are in general time-dependent [8], using the symmetries of the action (2) and the target spacetime (1) broken by D-branes, e.g., the translation and the boost generators along the transverse directions, $P^r$ and $J_{\pm r}$. A rotating D-brane and a giant graviton in Penrose limit can be described by these symmetry related boundary conditions which preserve the same amount of supersymmetry [8].

Here, we studied parallel and orthogonally intersecting D-branes only. It will be straightforward to extend our analysis to D-branes intersecting at general angles [17]. Since the rotational symmetry is reduced to $SO(4) \times SO(4)'$, there are only two kinds of supersymmetric intersection at general angles, resulting in less supersymmetric D-brane configurations.

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