WHITE DWARF SEISMOLOGY AND THE $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ RATE

TRAVIS S. METCALFE
Theoretical Astrophysics Center, Aarhus University
Ny Munkegade bldg. 520, 8000 Aarhus C, DENMARK

Abstract. Recent determinations of the internal composition and structure of two helium-atmosphere variable white dwarf stars, GD 358 and CBS 114, have led to conflicting implied rates for the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction. If we assume that both stars were formed through single-star evolution, then the initial analyses of their pulsation frequencies must have differed in some systematic way. I present improved fits to the two sets of pulsation data, helping to resolve the tension between the initial results.

1. Introduction

When a white dwarf is being formed in the core of a red giant star during helium burning, there are two nuclear reactions that compete for the available helium nuclei: the $3\alpha$ reaction, which combines three helium nuclei to form carbon, and the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction, which combines an additional helium nucleus with the carbon to form oxygen. At a given core temperature and density, the relative rates of these two reactions largely determines the C/O ratio in the resulting white dwarf star. The rate of the $3\alpha$ reaction is known to about 10% precision, but the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction is still uncertain by about 40%. So, if we can measure the C/O ratio in the core of a pulsating white dwarf, it is effectively a measurement of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate. The C/O ratio is interesting by itself, since the core composition in our models affect the derived cooling ages of white dwarfs by up to a few Gyr [1]. But we can also use it to provide an independent measurement of a nuclear reaction that is important to many areas of astrophysics, from the energetics of type Ia supernovae explosions to galactic chemical evolution.

The model-fitting method that I describe below has only been applied to DB white dwarfs, since they are structurally the simplest. But in principle it can be extended to the pulsating DA stars quite easily, and with a little
more work to the DOVs. Presently, the method requires that the spherical degree of the pulsation modes is known, and sufficient data exist for only two stars—though we have just finished a Whole Earth Telescope run on a third object. The first application was to GD 358, which showed 11 consecutive radial overtones with the same spherical degree during a WET run in 1990 [2, 3]. The second application was only recently finished, and came from single-site data on the star CBS 114, which showed 7 independent modes that all appear to be $\ell=1$ [4]. What we set out to do was search for a theoretical model that could reproduce, as closely as possible, the pulsation periods that we have observed in these stars.

2. Model-Fitting

We adjusted five different model parameters to try to match the observed periods. To make the final result as objective as possible, we wanted to explore the broadest range for each model parameter, defining the limits of the search based only on the physics of the model and on observational constraints. We allowed the mass to be anywhere from 0.45 to 0.95 M$_\odot$, which encompasses the vast majority of known white dwarf masses [5]. The temperature of our models was allowed to vary from 20,000 to 30,000 K, which easily includes the spectroscopic temperature determinations of all of the known DBV white dwarfs whether or not trace amounts of hydrogen are included in their atmospheres [6]. We looked at helium layer masses ranging from a fractional mass of $10^{-2}$ where helium burning will begin at the base of the envelope, down to a few times $10^{-8}$, close to the limit where our models no longer pulsate [7]. The final two parameters describe a simple C/O profile that has a constant ratio ($X_O$) out to some fractional mass point ($q$), where it then decreases linearly in mass to zero oxygen at the 95% mass point. The important features from the standpoint of pulsations are the central C/O ratio and the location and slope of the composition transition.

To get reasonable resolution, we allowed 100 values for each parameter inside these limits, so there are $10^{10}$ possible combinations of these five model parameters. Even if we had 1000 of today’s fastest processors, it would still take more than a year to calculate all of these models, so we used a slightly more clever method employing a genetic algorithm to explore the many possibilities. The way genetic algorithms work is initially like a Monte Carlo method, where we just take a random sample of parameter combinations. After this initial random sampling, a genetic algorithm explores new regions of the search space based on a sort of survival of the fittest scheme. By passing simulated data through this process, we can quantify how long we need to let it run to find the correct set of parameters most
of the time, and by running the entire process several times with different random initialization, we can ensure a very high probability of finding the globally optimal set of model parameters. In the end, the method requires a few million models to be calculated, concentrated mostly around the regions of the search space that yield better than average matches to the observations. So in addition to the optimal set of model parameters, we also end up with a fairly decent map of the search space, which gives us some sense of the uniqueness of the final answer.

To learn what the optimal values of the mass and central C/O ratio say about the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction, we need additional models that evolve a star from the main sequence through the red giant phase and into a white dwarf. For a given white dwarf mass, there are several things in the models that can be adjusted to change the central C/O ratio, but the ingredient that affects it the most is the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction. So, all we have to do is adjust this rate in the models until we end up with the mass and central C/O ratio that matches the fit from the genetic algorithm.

3. Results

In the initial application of this method to the data from the two stars, we found that GD 358 implied a reaction rate significantly higher than the extrapolations from laboratory measurements, while CBS 114 was right in line with the expectations. This led us to speculate that there might be some systematic error affecting our analysis of the two stars in different ways. There was another slight worry in the initial results: the masses and temperatures for both stars differed significantly from those inferred from spectroscopy. We thought this may have resulted from the use of slightly different mixing length parameters than the spectroscopic studies, so we repeated the fits using ML2/$\alpha=1.25$ to see if the discrepancy would disappear. In doing so, we also realized that there was a systematic difference in the way we were analyzing the two stars: for GD 358 the pulsation modes were consecutive radial overtones, so we were using both the periods and the spacings between the periods to judge which models provided the best match. For CBS 114 there was a gap in the sequence of observed modes, so we only used the periods themselves. When we repeated the fit for GD 358, we used only the periods to determine the best fit, just as we had for CBS 114.

The results of the new fits are shown in Table 1. Notice that both objects yield a rate for the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction that is consistent with laboratory extrapolations ($S_{300} = 200 \pm 80$, [10]), but GD 358 still seems to be a bit high compared to CBS 114, so this is only part of the answer. Also note that switching to the mixing length parameters used in the spectroscopic
TABLE 1. Optimal ML2/α=1.25 Models

| Object | $T_{\text{eff}}$ | $M/M_\odot$ | log($M_{He}/M_\odot$) | $X_O$ | $q$ | $\sigma_P(s)$ | $S_{300}$ |
|--------|-----------------|-------------|-------------------------|-------|----|---------------|----------|
| GD 358 | 21,300          | 0.695       | −2.95                   | 0.69  | 0.49| 1.11          | 215 ± 20 |
| CBS 114| 20,500          | 0.745       | −6.77                   | 0.58  | 0.51| 0.43          | 160 ± 20 |

analysis did not resolve the differences between the implied masses and temperatures: the two methods of inferring these parameters do not agree. Curiously, the optimal model of GD 358 has a thick helium layer, but for CBS 114 it is thin. Finally, both models show the transition point from a constant C/O ratio near the same fractional mass, and this location does not favor convective overshoot. Probably the decisive tests of all of these puzzles will come as we apply this method to additional DBV white dwarfs.

4. Conclusions

By measuring the interior composition of pulsating white dwarfs, we can get precise measurements of the important $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate, and as our models improve we can be more and more sure that they are not only precise, but also accurate. The model-fitting tool that we have used, involving a genetic algorithm, is a very powerful way to explore large ranges of interesting physical parameters and find the globally optimal model to match the observations. And since it evaluates so many models along the way, it produces some good maps of the search space in the regions where it is most interesting. True, this method is still computationally intensive, but there is no getting around that if we want the global solution, and Linux clusters are getting cheaper and cheaper. Finally, in the future we hope to be able to say something more about the detailed shape of the C/O profile all the way from the center to the surface.

References

1. Fontaine, G., Brassard, P., & Bergeron, P. 2001, PASP, 113, 409.
2. Winget, D. E. et al. 1994, ApJ, 430, 839.
3. Metcalfe, T. S., Winget, D. E., & Charbonneau, P. 2001, ApJ, 557, 1021.
4. Handler, G., Metcalfe, T. S., & Wood, M. A. 2002, MNRAS, in press.
5. Napiwotzki, R., Green, P. J. & Saffer, R. A. 1999, ApJ, 517, 399.
6. Beauchamp, A. et al. 1999, ApJ, 516, 887.
7. Bradley, P. A. & Winget, D. E. 1994a, ApJ, 421, 236.
8. Salaris, M., et al. 1997, ApJ, 486, 413.
9. Metcalfe, T. S., Salaris, M. & Winget, D. E. 2002, ApJ, 573, 803.
10. Angulo, C. et al. 1999, Nucl. Phys. A, 656, 3.