Revised Energy Levels of Atomic Lanthanum Considering Hyperfine Structure

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Abstract

The experimental wavenumbers of 2118 spectral lines from calibrated Fourier transform (FT) spectra have been used to determine revised energy values of 405 fine structure levels of atomic lanthanum, 264 levels of even and 141 levels of odd parity, respectively. For the determination of the level energies a weighted global fit of the center-of-gravity (cg) wavenumbers of all 2118 spectral lines has been done. All lines have been classified previously by different spectroscopic methods and by different authors. In order to achieve high accuracy, the hyperfine (hf) structure was taken into account when determining the cg wavenumbers from the FT spectra. The total uncertainty for all revised energy levels lies below 0.007 cm⁻¹. A compilation of all revised energy levels along with their hf structure data is given, as well as a list of the spectral lines used in this investigation.

Unified Astronomy Thesaurus concepts: Atomic spectroscopy (2009); Interferometers (805)

Supporting material: machine-readable tables

1. Introduction

A comprehensive atomic database including data on lanthanum (La), in particular laboratory-based atomic data with precise values of atomic and ionic fine structure energies, is very important for defining appropriate spectral lines in astrophysical data. Therefore, the studies are of interest for numerous astronomical applications.

In order to derive a detailed abundance analysis of elements and to determine the age of newly discovered stars (Yong et al. 2006, 2012; Cescutti et al. 2007; Shaviv 2012; Jacobson & Friel 2013; Hollek et al. 2015; Hansen et al. 2018; Hill et al. 2019) experimental studies on new laboratory-based atomic data of various elements are urgently needed. In such investigations, rare-earth elements are of particular interest, and so also La.

As an example of the importance of the La spectrum, the Cepheids in the outer Galactic disk were found to have an overabundance of La compared to other metals (Yong et al. 2006). The prediction of elemental abundances in the Milky Way disk contributes mainly to the understanding of star formation (Cescutti et al. 2007). In order to understand the nucleosynthesis process of astrophysical objects and to precisely estimate the age of different stars, knowledge of the elemental abundances is required (Shaviv 2012).

In the last few decades great advances have been made in astronomy and astrophysics, particularly in relation to the performance of the related instruments. Larger telescopes and space-based observatories with better resolutions have opened up new possibilities for observing weaker objects. More accurate laboratory data are required to analyze this astrophysical data. A further example is the study of stellar abundances of s-process elements in open clusters (Yong et al. 2012; Jacobson & Friel 2013). Precise knowledge of the level energies is important for the choice of suitable spectral lines, in particular to exclude situations with blended lines.

With the atomic number of 57, La is the first element of the rare-earth metals. In its natural abundance La has two isotopes, ¹³⁹La and ¹⁳⁸La, with natural abundances of 99.910% and 0.090%, respectively. The atomic spectrum of La is characterized by a broad hyperfine (hf) structure of the predominant isotope ¹³⁹La, which has a nuclear spin of 7/2, a quite large nuclear magnetic dipole moment of μ_N = 2.7830455(9) μ_N (Raghavan 1989), and a relatively small nuclear electric quadrupole moment of Q = 0.20(1)μ (Raghavan 1989).

Many experimental and theoretical studies have already focused on the spectrum of La, in particular atomic La (La I). In early scientific studies, tables of spectral lines (Harrison et al. 1945; Meggers et al. 1961; Harrison 1969) were published. Analysis of La I lines, performed by several authors, lead to the energies of fine structure levels, summarized by Martin et al. (1978). The energy values available in the National Institute of Standards and Technology (NIST) atomic database (Kramida et al. 2021) are based on the values given there.

Measurements of the hf structure have been published by Anderson (1934), Murakawa & Kamei (1953), Murakawa (1961), Ting (1957), Childs & Goodman (1971, 1977, 1978), and Childs & Nielsen (1988), and investigations of the nuclear quadrupole moment of La isotopes by Anderson (1934), Murakawa (1954), Lührs (1955), Ting (1957), and Fischer et al. (1972).

Later, laser spectroscopic measurements of hf structure were undertaken by several research groups (Govindarajan & Pramila 1989; Shaw et al. 1990; Caiyan et al. 1990; Kajoch et al. 1996; Jin et al. 2001, Pramila 1990; Gangskry et al. 1997; Başar et al. 2007; Furmann et al. 2007; Başar et al. 2009; Furmann et al. 2009, 2010). In two works of the group in Poznan (Furmann et al. 2009, 2010), new energy levels of La were also discovered.

Subsequently, a research group in Graz, in collaboration with researchers from Pakistan, Turkey, Poland, and Germany presented a huge number of additional levels (Nighat et al. 2010; Güzelçimen et al. 2012; Siddiqui et al. 2013;
Gamper et al. 2014; Windholz et al. 2014; Başar et al. 2017; Windholz 2016; Sobolewski et al. 2017a). In the most recent papers, the energy values of newly discovered levels are based on the preliminary results of the present study (Windholz & Gamper 2020; Windholz & Binder 2020b; Öztürk et al. 2020; Faisal et al. 2021; Windholz & Binder 2020a; Windholz et al. 2021).

In addition, theoretical and semiempirical studies have been carried out by Ben Ahmed et al. (1976, 1974), Biêmont & Quinet (2003), Dembczynski et al. (2010), Furmann et al. (2007), and Childs & Nielsen (1988). In recent years, measurements of the Zeeman effect have come back into focus (Sobolewski et al. 2017b, 2017c, 2018, 2019). In several of these papers a great uncertainty of the energy values of the levels was stated, and it was noted that a revision of the level energies was pending. We should mention that the knowledge of the line classifications and hf constants obtained in numerous previous studies enabled this study.

A corresponding investigation for singly ionized La based on the same experimental spectra was previously carried out by the authors of our group (Güzelçimen et al. 2018). Therefore, the experimental details are only briefly summarized here. For more detailed information, we refer to this reference.

### 2. Experimental Data

Calibrated Fourier transform (FT) spectra of a hollow cathode discharge lamp in a spectral range from 330–1450 nm (6900–30,000 cm$^{-1}$) serve as experimental data for the present study. The high-resolution FT spectra, recorded with the Bruker IFS 125 HR FT-IR spectrometer at the Laser Centre at the University of Latvia, have been previously evaluated in Güzelçimen et al. (2013) and Güzelçimen et al. (2018). In the VIS and UV wavelength range a Hamamatsu R928 photomultiplier was used to detect the spectra, and in the IR wavelength range an InGaAs diode.

The cathode consisted of La in its natural abundance with a purity of 99.9%. The pressure of argon chosen as a buffer gas and the discharge current were approximately 1 mbar and 80 mA, respectively. By cooling the hollow cathode discharge lamp with liquid nitrogen, the Doppler line broadening could be reduced. For more explanation of the experimental details of the setup, see Güzelçimen et al. (2013).

For the calibration of the FT spectra we used Ar II lines, as recommended in the paper of Learner & Thorne (1988). These lines were selected due to their low wavelength sensitivity against discharge conditions, but span only the wavelength range 514–423 nm. To extend this range, we calculated further lines with the same upper levels as given by Learner & Thorne. We then calibrated the FT spectra in order to obtain the differences between the calculated and observed wavenumbers smaller than 0.003 cm$^{-1}$; for most of the selected Ar II lines this difference was about 0.001 cm$^{-1}$. Thus the absolute accuracy of the wavenumber is examined as being better than 0.003 cm$^{-1}$ in the entire wavelength range from 330–1450 nm.

### 3. Hyperfine Structure and Global Fit

For the determination of the level energies a regression analysis based on the Ritz combination principle with a weighted global fit of the wavenumbers of all well-classified lines has been done. The Ritz combination principle says

$$\tilde{\nu} = \frac{1}{\lambda} = E_{\text{upper}} - E_{\text{lower}},$$

where $\tilde{\nu}$ is the wavenumber of a spectral line, $\lambda$ is the corresponding wavelength in vacuum, and $E_{\text{upper}}$ and $E_{\text{lower}}$ are the values of the two fine structure energy levels in a spectral transition. Here, both the wavelength and the energy are the center of gravity (cg) of the hf splitting. Because most atomic La lines have a broad hf pattern, the hf splitting of the energy levels has a major impact and the hf effect must be considered when determining the cg wavelength. Due to the numerous previous works, this was possible.

We have used the Elements program (Windholz & Guthöhrlein 2003; Windholz 2016), which allows us to classify the spectral lines based on a list of all of the known energy levels and on the corresponding hf data. The program also allows the simulation of the hf pattern of a line using a Gaussian profile with an adjustable value of the FWHM. A graphical comparison of the simulated hf pattern and experimental curve can be displayed. The cg of the line can be precisely determined by best matching to the experimental one, through shifting the simulated curve along the wavenumber axis. With this method, the cg wavenumber can be achieved with high accuracy. For lines with a signal-to-noise ratio (S/N) higher than 10 in the normalized FT spectra, we have noticed that the accuracy of the cg is almost independent of S/N. In these cases, an uncertainty better than 0.003 cm$^{-1}$ is estimated.

For lines with a S/N less than 10, the lower accuracy was taken into account by a weighting factor in the global fit (see below).

Using Equation (1) for each line, an overdetermined system of equations was established in which each line is represented by one equation and each level energy by an unknown variable. Of course many more equations than unknown variables are required to solve the system, i.e., many more lines than levels. The cg of the ground level was fixed to zero during the fit.

Because not all lines have the same accuracy and quality, a weighting factor (wf) was introduced by performing a weighted fit. The criteria based on the resolution and S/N values applied for assignment of La I spectral lines have been established as mentioned in Table 1.

Some examples of line profiles are given in Figure 1. In Figure 1(a) a part of the spectrum without lines is shown to explain the definition of the noise level. The main part of the noise is between the intensity level $\pm 1$. The classification of the lines in Figures 1(b)–(l) is given in Table 2. The experimental profiles are normalized for comparison with simulated hf patterns. For the simulations the hf constants given in

Table 1

| wf   | Criteria                                                                 | Number Lines |
|------|---------------------------------------------------------------------------|--------------|
| 10   | Strong lines with S/N $\geq$ 20                                          | 1072         |
| 5    | Lines with $20 > S/N \geq 10$, if their cg can be especially good determined | 96           |
| 3    | Lines with $20 > S/N \geq 10$ lines with S/N $\leq$ 10, if their cg can be especially good determined | 407          |
| 1    | Lines with S/N $\leq$ 10                                                 | 543          |
Tables 3 and 4 have been used. All shown lines except the last one (Figure 1(l)) have been simulated with Gaussian profiles for each hf component.

In example (e) despite of the blend situation (on the left side an Ar II line and on the right side the line at 417.825 nm) the cg wavelength can be well determined. Therefore, even with a value of S/N \( \leq 10 \), the value 3 was assigned for wf.

The line shown in example (f) is a transition to ground level and appears to be slightly self-absorbed. Due to the fact that both J values are the same (3/2) and the hf constants are very similar, the profile is nearly symmetric and thus the cg wavelength can be easy determined. Some other lines show slight self-absorption, but the determination of cg is not influenced by this either.

![Figure 1. Some examples of line profiles. (a) x-axis in nanometers, (b)–(d) x-axis in MHz. Wavelengths for labeling the figures (top-right corner) in nanometers. (a) Definition of noise level, here shown in the wavelength range between 391.8 and 391.7 nm (corresponding to a wavenumber interval of 6.5 cm\(^{-1}\)). (b) Line at 371.429 nm, S/N = 157, FWHM = 2500 MHz, wf = 10. (c) Line at 391.675 nm, S/N = 3, FWHM = 2200 MHz, wf = 1. (d) Line at 392.716 nm, S/N = 5, FWHM = 2200 MHz, wf = 1. (e) Line at 417.829 nm, S/N = 15, FWHM = 2200 MHz, wf = 3 (see explanation in the text). (f) Line at 550.134 nm, S/N = 7100, FWHM = 2200 MHz, wf = 10 (see explanation in the text). (g) Line at 577.616 nm, S/N = 7, FWHM = 1200 MHz, wf = 3. (h) Line at 578.091 nm, S/N = 4, FWHM = 1200 MHz, wf = 1. (i) Line at 579.132 nm, S/N = 3200, FWHM = 1400 MHz, wf = 10 (see explanation in the text). (j) Line at 705.586 nm, S/N = 73, FWHM = 1800 MHz, wf = 10. (k) Line at 848.061 nm, S/N = 24, FWHM = 2400 MHz, wf = 10. (l) Line at 943.832 nm, S/N = 23100, FWHM = 2400 MHz, wf = 10 (see explanation in the text).]
Due to the high intensity in example (i), a small saturation effect arises, and the smaller hf components seem to be enlarged in intensity. Nevertheless, the cg wavelength can be determined precisely, and \( \nu_f = 10 \) has been chosen.

For line (l) at 943.8321 nm with $S/N = 23,100$, the simulation with a sum of 80% Gauss profile + 20% Lorentz profile, both with FWHM 2400 MHz, fits well. A value of \( \nu_f = 10 \) was chosen. Example (l) shows the strongest line in our investigation. However, this line is not self-reversed and can be well simulated.

By using the GlobalFit program (Güzelçimen et al. 2018), written by one of the authors (L.W.), the overdetermined equation system using a least-squares method was solved for a total of 2118 spectral lines of La I selected from the FT spectra.

4. Results

Table 2 contains a list of all 2118 lines together with their classification. The wavelength, \( \lambda_{air} \), in the first column is the cg of the hf splitted lines determined from the FT spectra using the program Elements (as discussed above). The wavenumbers, \( \nu \), in the second column were calculated from these wavelengths using the dispersion of standard air, following Peck & Reeder (1972). The value \( \Delta \nu \) given in the third column is the difference between the wavenumber \( \nu \) and the calculated difference between the best-fitted energies of the upper and lower levels. In the fourth column, the $S/N$ value is given; in the fifth column the $\nu_f$ as discussed above. In the sixth and seventh columns, the energy, \( E_n \), and the total angular

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### Table 2

| $\lambda$ (nm) | $\nu$ (cm$^{-1}$) | $\Delta \nu$ (cm$^{-1}$) | S/N | $\nu_f$ | $E_n$ (cm$^{-1}$) | $2 J_n$ | $E_e$ (cm$^{-1}$) | $2 J_e$ |
|----------------|------------------|------------------------|-----|---------|-----------------|-------|----------------|-------|
| 334.22288      | 29,911.552       | -0.007                 | 5   | 1       | 30,964.718      | 7     | 1053.159       | 5     |
| 338.14393      | 29,564.715       | -0.003                 | 11  | 3       | 29,564.718      | 1     | 1000           | 3     |
| 338.86055      | 29,502.185       | 0.007                  | 10  | 3       | 29,502.178      | 5     | 0.000          | 3     |
| 345.06392      | 28,971.836       | 0.003                  | 18  | 5       | 28,971.833      | 3     | 0.000          | 3     |
| 346.11828      | 28,883.853       | -0.001                 | 23  | 10      | 29,936.743      | 3     | 1053.159       | 5     |
| 348.06041      | 28,722.421       | -0.005                 | 12  | 3       | 29,775.585      | 5     | 1053.159       | 5     |
| 350.69789      | 28,506.416       | 0.004                  | 5   | 1       | 28,506.412      | 5     | 0.000          | 3     |
| 351.40550      | 28,449.016       | -0.003                 | 150 | 10      | 29,502.178      | 5     | 1053.159       | 5     |
| 351.84447      | 28,413.523       | -0.003                 | 9   | 1       | 29,466.684      | 7     | 1053.159       | 5     |
| 356.41392      | 28,049.254       | 0.006                  | 4   | 1       | 36,495.288      | 1     | 8446.040       | 3     |

Note: \( \lambda \): center-of-gravity wavelength in air from the FT spectrum; \( \nu \): center-of-gravity wavenumber column was calculated from the wavelength, \( \Delta \nu = \nu - [E_n - E_e] \); S/N: signal-to-noise ratio in normalized FT spectra; \( \nu_f \): weighting factor for global fit (see text); \( E_n, J_n, E_e, J_e \): energy and total angular momentum quantum number of the electrons for the levels of odd parity or even parity, respectively. This table contains 2118 La I lines.

(This table is available in its entirety in machine-readable form.)

### Table 3

| $E_{n,ref}$ (cm$^{-1}$) | $2 J_n$ | Nol | Stat. Unc. (cm$^{-1}$) | Total Unc. (cm$^{-1}$) | $E_{e,ref}$ (cm$^{-1}$) | $\Delta E$ (cm$^{-1}$) | Ref. Level | $A$ (MHz) | $\Delta A$ (MHz) | $B$ (MHz) | $\Delta B$ (MHz) | Ref. A, B |
|------------------------|-------|-----|-----------------------|------------------------|------------------------|----------------------|------------|------------|-----------------|------------|-----------------|---------|
| 0.000                  | 3     | 45  | 0.000                 | 0.000                  | 0.000                  | 0.000                | [10]       | 141.1959    | 16              | 44.781        | 14     | [14]            |
| 1053.159               | 5     | 56  | 0.0000                | 0.0000                 | 0.000                  | -0.005              | [10]       | 182.1706    | 6               | 54.213        | 14    | [14]            |
| 2668.175               | 3     | 44  | 0.0000                | 0.0000                 | 0.000                  | -0.013              | [10]       | -480.292    | 15.188         | 25             | 22  | [22]            |
| 3000.993               | 5     | 53  | 0.0000                | 0.0000                 | 0.000                  | -0.009              | [10]       | 300.563     | 1               | 10.873        | 25    | [22]            |
| 3494.522               | 7     | 38  | 0.0000                | 0.0000                 | 0.000                  | -0.004              | [10]       | 462.868     | 1               | 17.925        | 24    | [24]            |
| 4121.569               | 9     | 27  | 0.0000                | 0.0000                 | 0.000                  | -0.003              | [10]       | 489.534     | 1               | 32.180        | 34    | [34]            |
| 7011.904               | 5     | 59  | 0.0000                | 0.0000                 | 0.000                  | -0.005              | [10]       | 304.372     | 2               | 28.091        | 30    | [30]            |
| 7231.415               | 1     | 29  | 0.0000                | 0.0000                 | 0.000                  | -0.002              | [10]       | 7231.407    | 0.008          | 2460.161      | 0      | [3]             |
| 7490.521               | 3     | 42  | 0.0000                | 0.0000                 | 0.000                  | -0.003              | [10]       | 7490.521    | 0.000          | 929.618       | 37.221 | [3]             |
| 7679.944               | 5     | 52  | 0.0000                | 0.0000                 | 0.000                  | -0.003              | [10]       | 7679.939    | 0.005          | 802.172       | -34.186 | [3]             |
Table 4
Revised La I Levels of Odd Parity

| Ei (cm⁻¹) | 2J₀ | Nol | Stat. Unc. (cm⁻¹) | Total Unc. (cm⁻¹) | ΔEref (cm⁻¹) | Ref. Levelᵃ | A (MHz) | ΔA (MHz) | B (MHz) | ΔB (MHz) | Ref. A, B |
|-----------|-----|-----|------------------|------------------|--------------|-------------|--------|----------|--------|---------|----------|
| 13,260.368 | 3   | 40  | 0.0002           | 0.003            | 13,260.38    | −0.012      | [13]   | −352.1   | 0.2    | 39.3    | 1.5       | [2]      |
| 13,631.031 | 5   | 46  | 0.0002           | 0.003            | 13,631.04    | −0.009      | [13]   | 959.35   | 0.05   | −14.5   | 4.5       | [2]      |
| 14,095.675 | 1   | 21  | 0.0005           | 0.004            | 14,095.69    | −0.015      | [13]   | −581.4   | 1.3    | 0       |           | [1]      |
| 14,708.912 | 3   | 46  | 0.0003           | 0.003            | 14,708.92    | −0.008      | [13]   | 586.2    | 0.4    | 41      | 3         | [2]      |
| 14,804.066 | 5   | 61  | 0.0003           | 0.003            | 14,804.08    | −0.014      | [13]   | 335.01   | 0.74   | 23.64   | 0.95      | [11]     |
| 15,019.493 | 7   | 48  | 0.0003           | 0.003            | 15,019.51    | −0.017      | [13]   | 673.9    | 0.3    | 72      | 3         | [2]      |
| 15,031.632 | 3   | 33  | 0.0003           | 0.003            | 15,031.64    | −0.008      | [13]   | −672.6   | 0.4    | −16     | 3         | [2]      |
| 15,196.821 | 5   | 40  | 0.0003           | 0.003            | 15,196.83    | −0.009      | [13]   | 410.4    | 0.4    | 50      | 2         | [2]      |
| 15,219.891 | 1   | 21  | 0.0004           | 0.003            | 15,219.89    | 0.001       | [13]   | −257.8   | 0.3    | 0       |           | [2]      |
| 15,503.628 | 5   | 48  | 0.0004           | 0.003            | 15,503.64    | −0.012      | [13]   | 586.5    | 0.1    | 11      | 2         | [2]      |

Notes: Nol: number of spectral lines which has been calculated for this level (see text); ΔEref: revised energy values determined in this work; stat. unc.: statistical uncertainty; total unc.: total uncertainty; Ei: uncorrected energy values from their original reference; ΔE = Ei,ref − Ei,ref; A, B, ΔA, ΔB: magnetic dipole and electric quadrupole hf constants and corresponding error bars. a: A = 441 (7) MHz published in [7]; A = 367(2) MHz published in [15]; new A value: tw. b: A = 464 (5) MHz published in [7]; new A value: tw. c: A = −29.5 (3.0) MHz published in [7]; new A value: tw. d: A = −29,232.7 (6.0) MHz published in [7]; new A value: tw. e: A = 229 (10) MHz published in [8]; new A value: tw. f: A = 124 (5) MHz published in [8]; new A value: tw. g: A = 785 (9) MHz published in [8]; new A value: tw. h: A = 97.2 (7.0) MHz published in [7]; new A value: tw. i: A = 126.5 (900) MHz published in [7]; new A value: tw. ⁴ In [13] given with J = 7/2; ² changed to 5/2;  Nov A value: this work.

Table references: tw: A determined in this work for the first time; [1] Başar et al. (2007); [2] Başar et al. (2009); [3] Başar et al. (2017); [4] Caiyan et al. (1990); [5] Childs & Goodman (1978); [6] Faisal et al. (2021); [7] Furmann et al. (2007); [8] Furmann et al. (2009); [9] Gamper et al. (2014); [10] Güzelçimen et al. (2012); [11] Jian et al. (2011); [12] Kajoch et al. (1996); [13] Martin et al. (1978); [14] Nighat et al. (2010); [15] Siddiqui et al. (2013); [16] Windholz et al. (2014); [17] Windholz & Binder (2020a); [18] Windholz & Gamper (2020); [19] Windholz et al. (2021). This table contains 141 La I levels of even parity.

(This table is available in its entirety in machine-readable form.)

momenentum quantum number, J₀, of the levels of odd parity and in the eighth and ninth columns Ei and Ji for the levels of even parity are represented, respectively.

The resulting revised energy values of even and odd parity are given in Table 3 and in Table 4, respectively. The description of the columns in the tables is as follows: the revised energy determined in this work and J values are shown in the first two columns, the number of spectral lines (nol) which has been used to deduce the revised energy value is indicated in the third column, the statistical uncertainty, resulting from the fit, and the total uncertainty, which additionally takes into account the calibration uncertainty of 0.003 cm⁻¹ in the FT spectra, are presented in the fourth and fifth columns, respectively. Columns six to eight contain the uncorrected energy values from their original source, the difference to the newly determined energy values, and the reference for the level energy. The hf constants A and B together with their error bars ΔA and ΔB, as well as corresponding references for the hf data, are given in the following columns.

The statistical uncertainties of the level energies fluctuate between 0.000 and 0.004 cm⁻¹, having an average value of 0.0009 cm⁻¹. Accordingly, the values for the total uncertainty are between 0.003 and 0.007 cm⁻¹. Total uncertainty and energy values are consequently given with three post-decimal positions. Although the statistical and calibration uncertainty are not correlated, we decided to use the approach to simply add both errors in order to give proportionally more weight to the statistical error, which is smaller than the systematic error.

For energy values that were only calculated from a single line, no meaningful value can be calculated for the statistical uncertainty. Therefore, no value is given in the third column. In order to provide a value for the total uncertainty in spite of the lack of statistical uncertainty, the maximum value of 0.004 cm⁻¹ for the statistical uncertainty was added to the calibration uncertainty of 0.003 cm⁻¹. This leads to an estimated value of 0.007 cm⁻¹ for all these levels.

The hf constants A and B are values that allow accurate determination of the cg wavenumbers of all La I lines in the Elements program. For many levels, hf data are available from various sources. Based on many years of experience, we have selected the most suitable values of the hf constants for each level. Results from Doppler-free measurements were chosen with preference, if available. All hf constants were tested for reliability by matching experimental and simulated hf patterns. In some cases, we were able to improve the cg values, using the improved hf constants proposed in this study. For some other levels, no hf constants were previously known, and we give the corresponding values in Tables 3 and 4 for the first time. Such constants were determined either from laser spectroscopic investigations or directly from lines in the FT spectra. The uncertainties given for values determined in our work are standard uncertainties, determined from results of different lines, or estimated due to our experience.

Four-hundred and five energy levels have gone into our investigation. Of this amount of levels, 146 levels having even parity and 136 having odd parity, respectively, are mentioned in the standard work of Martin et al. (1978). One-hundred and eighteen even-parity and five odd-parity levels, discovered by the groups in Poznan and Graz, are also contained in Tables 3 and 4, respectively. For comparison, the original energy values from the literature are also listed in Tables 3 and 4.

Levels mentioned in the standard work (Martin et al. 1978) but not otherwise displayed in Tables 3 and 4 are listed in Table 5. For these levels, either no lines were found in our FT spectrum, or their classification was not sure. Nevertheless, for...
some of these levels magnetic dipole hf constants and improved energy values could be received from laser spectroscopic measurements. Levels below the ionization limit of 44981 cm$^{-1}$ (Garton & Wilson 1966), for which lines were found neither in our FT spectrum nor with optogalvanic laser spectroscopic measurements, are most likely nonexistent. These levels can be recognized by the fact that no information is given in the columns of Table 5. Other levels, listed in Martin et al. (1978) are far above the ionization limit (see Table 5).

Roughly 200 further energy levels, found by the groups in Poznan, Graz, and Istanbul (see corresponding references in Section 1), are not treated in the present study, since we could not classify any lines in our FT spectra as transitions in which these levels are involved. Nevertheless, these levels exist without any doubt. Most of them were extracted from laser spectroscopic experiments with optogalvanic detection, which is more sensitive than FT emission spectra in some ranges of the level energies.

### Table 5
Levels given in Martin et al. (1978) but not Contained in Tables 3 and 4

| $E$ (cm$^{-1}$) | 2J | Parity | Revised $E$ (cm$^{-1}$) | A (MHz) [1] | A (MHz) [2] | A (MHz) tw | Nol FT | Nol OG |
|---|---|---|---|---|---|---|---|---|
| [3] | [3] | | tw | | | | |
| 36,792.36 | 7 | e | | 34,015.781 | | 345.1(2.0) | 1* | 6 |
| 32,415.73 | 9 | o | | 34,213.531 | | 334(3) | 0 | 4 |
| 34,015.76 | 3 | o | | 34,015.781 | | 345.1(2.0) | 1* | 6 |
| 34,213.53 | 5 | o | | 258(9) | | 345.1(2.0) | 1* | 6 |
| 34,358.60 | 5 | o | | 34,213.531 | | 334(3) | 0 | 4 |
| 34,850.38 | 3 | o | | −244.1(9.0) | | −478(2) | 0 | 2 |
| 34,982.43 | 5 | o | | 34,982.438 | | 669(5) | 0 | 2 |
| 34,988.17 | 7 | o | | | 334(20) | 0 | 2 |
| 35,232.81 | 5 | o | | | | | |
| 35,236.06 | 9 | o | | | 188.4(2.5) | 1* | 2 |
| 35,292.53 | 5 | o | | 35,287.211 | | 103(9) | 0 | 2 |
| 35,482.70 | 1 | o | | 35,482.74 | | 2311(5) | 0 | 2 |
| 35,860.64 | 1 | o | | 35,860.658 | | 563(9) | 0 | 2 |
| 35,888.45 | 9 | o | | 35,888.556 | | −6(3) | 1* | 2 |
| 35,999.06 | 7 | o | | 98.1(9.0) | | | 0 | 0 |
| 36,074.74 | 11 | o | | | | | |
| 36,081.09 | 5 | o | | | | | |
| 36,333.08 | 3 | o | | 36,333.105 | | 430.5(1.5) | 0 | 0 |
| 36,454.46 | 7 | o | | −109.9(0.9) | | 144(5) | 1* | 3 |
| 37,731.59 | 5 | o | | 37,731.610 | | 158.5(10) | 144(5) | 1* | 3 |
| 38,061.57 | 3 | o | | 38,061.553 | | 777(5) | 787(2) | 1* | 2 |
| 39,350.50 | 7 | o | | | | | |
| 40,910.11 | 7 | o | | | | | |
| 44,978.60 | 3 | o | | | | | |
| 44,978.90 | 1 | o | | | | | |

| Ionization limit (44,981 cm$^{-1}$) | [4] |
|---|---|
| 47,675.00 | 3 | o |
| 49,176.50 | 3 | o |
| 50,030.30 | 3 | o |
| 50,591.40 | 3 | o |
| 50,968.30 | 3 | o |
| 51,242.20 | 3 | o |
| 51,441.60 | 3 | o |
| 51,591.20 | 3 | o |
| 51,591.20 | 1 | o |
| 51,708.50 | 3 | o |
| 51,800.30 | 3 | o |
| 51,874.10 | 3 | o |
| 51,874.10 | 1 | o |
| 51,939.20 | 3 | o |
| 51,939.20 | 1 | o |
| 51,989.00 | 3 | o |
| 51,989.00 | 1 | o |
| 52,030.40 | 3 | o |
| 52,030.40 | 1 | o |
| 52,069.80 | 3 | o |

**Notes.** A: magnetic dipole hf constant, number in parenthesis: error bar; Nol FT: number of lines in the FT spectrum; Nol OG: number of lines measured with optogalvanic spectroscopy.

* Classification not sure.

**Table references:** tw: this work; [1] Furmann et al. (2007); [2] Furmann et al. (2009); [3] Martin et al. (1978); [4] Garton & Wilson (1966).
5. Conclusion

In summary, 405 La I fine structure energy levels, of which 264 are even parity and 141 odd parity, are accurately revised on 2118 spectral lines from wavenumber-calibrated FT spectra taken into account the hf structure. Both lists of all improved energy levels together with hf data, as well as of all the lines investigated, are exhibited. The value of total uncertainty for all revised energy levels lies below 0.007 cm$^{-1}$ by accurate determination of the cg wavenumbers of all lines. We would like to inform the reader that the present study is a continuation of previous scientific studies (Akhtar & Windholz 2012; Windholz et al. 2016; Güzelçimen et al. 2018; Windholz et al. 2019), and significantly emphasizes the particular role of the hf identification of spectral lines in astrophysical objects (e.g., stellar sources) of unknown element composition.

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References

Akhtar, N., & Windholz, L. 2012, JPhB, 45, 095001
Anderson, O. E. 1934, PhRv, 45, 685
Başar, G., Başar, G., Er, A., & Kröger, S. 2007, PhysS, 75, 572
Başar, G., Başar, G., & Kröger, S. 2009, OptCo, 282, 562
Başar, G., Gamper, B., Güzelçimen, F., et al. 2017, JQSRT, 187, 505
Ben Ahmed, Z., Bauche-Arnoult, C., & Wyatt, J.-F. 1974, Phys, 77, 148
Ben Ahmed, Z., Verges, J., Wilson, M., & Giacchetti, A. 1976, PhyBC, 37, 6
Biémont, E., & Quinet, P. 2003, PhST, 105, 38
Caiyan, L., Jianan, Q., Lizhou, Z., & Fucheng, L. 1990, JPhD, 23, 1327
Cescutti, G., Matteucci, F., François, P., & Chiappini, C. 2007, A&A, 84, 275
Childs, W. J., & Goodman, L. S. 1971, PhRvA, 3, 25
Childs, W. J., & Goodman, L. S. 1977, JOSA, 67, 1230
Childs, W. J., & Goodman, L. S. 1978, JOSA, 68, 1348
Childs, W. J., & Nielsen, U. 1988, PhRvA, 37, 6
Demchyzëski, J., Elantokowska, M., Furmann, B., Ruczkowski, J., & Stařeksa, D. 2010, JPhB, 43, 065001
Faisal, M., Siddiqui, I., & Windholz, L. 2021, JQSRT, 260, 107452
Fischer, W., Hühnernann, H., Mandraek, K., & Ihle, H. 1972, PhLB, 40, 87
Furmann, B., Stařeksa, D., & Demchyzëski, J. 2007, Phys, 76, 264
Furmann, B., Stařeksa, D., & Demchyzëski, J. 2009, JPhB, 42, 175005
Furmann, B., Stařeksa, D., & Demchyzëski, J. 2010, JPhB, 43, 015001
Gamper, B., Glowacki, P., Siddiqui, I., Demchyzëski, J., & Windholz, L. 2014, JPhB, 47, 165001
Gangursky, Y. P., Karaiyavan, D. V., Marinova, K. P., Markov, B. N., & Zemlyanoi, S. G. 1997, ZPhyD, 41, 251
Garton, W. R. S., & Wilson, M. 1966, ApJ, 145, 333
Goswindaajan, J., & Pramila, T. 1988, JOSA, 6, 1275
Güzelçimen, F., Başar, G., Tamanis, M., et al. 2013, ApJS, 208, 18
Güzelçimen, F., Siddiqui, I., Başar, G., Kröger, S., & Windholz, L. 2012, JPhB, 45, 135005
Güzelçimen, F., Tonka, M., Uddin, Z., et al. 2018, JQSRT, 211, 188
Hansen, C. J., El-Souri, M., Monaco, L., et al. 2018, ApJ, 855, 83
Harrison, G. R. 1969, Massachusetts Institute of Technology Wavelength Tables (Cambridge, MA: MIT Press)
Harrison, G. R., Rosen, N., & McNally, J. R. 1945, JOSA, 35, 658
Hill, V., Skuladottir, Á, Tolstoy, E., et al. 2019, A&A, 626, A15
Holleke, J. K., Frebel, A., Placco, V. M., et al. 2015, ApJ, 814, 121
Jacobson, H. R., & Friel, E. D. 2013, AJ, 145, 107
Jin, W.-G., Endo, T., Uematsu, H., Minowa, T., & Katsuragawa, H. 2001, PhRvA, 63, 064501
Kajoch, A., Krzykowski, A., Stařeksa, D., Furmann, B., & Jarosz, A. 1996, AcPPA, 89, 517
Kramida, A., Balchenko, Yu., Reader, J. & NIST ASD Team 2021, NIST Atomic Spectra Database v5.9, 78, (Gaithersburg, MD: National Institute of Standards and Technology), https://physics.nist.gov/asd
Leamer, R. C. M., & Thorne, A. P. 1988, JOSA, 5, 2045
Lührs, G. 1955, ZPhy, 141, 486
Martin, W. C., Zalubas, R., & Hagan, L. 1978, Atomic Energy Levels: The Rare-Earth Elements NSRDS-NBS 60, (Washington, DC: National Bureau of Standards), https://www.osti.gov/biblio/6507735
Meggers, W. F., Corliss, C. H., & Scribner, B. F. 1961, AcSpe, 17, 1137
Murakawa, K. 1954, JPSJ, 9, 391
Murakawa, K. 1961, JPSJ, 16, 2533
Murakawa, K., & Kamei, T. 1953, PhRv, 92, 325
Nighat, Y., Raith, M., Hussain, M., & Windholz, L. 2010, JPhB, 43, 125001
Özürt, I. K., Başar, G., Başar, G., et al. 2020, JQSRT, 253, 107100
Peck, E. R., & Reeder, K. 1972, JOSA, 62, 958
Pramila, T. 1990, PhysS, 42, 189
Rainis, M., Arcimowicz, B., & Uddin, Z. 2016, JQSRT, 227, 185
Sobolewski, L. M., Binder, T., Uddin, Z., et al. 2020, JQSRT, 256, 107340
Windholz, L., Arcimowicz, B., & Uddin, Z. 2016, JQSRT, 176, 97
Windholz, L., & Binder, T. 2020a, Atoms, 8, 88
Windholz, L., & Binder, T. 2020b, Atoms, 8, 23
Windholz, L., Banfield, T., & Glovacki, P., et al. 2021, JQSRT, 273, 107843
Windholz, L., Gamper, B. 2020, JQSRT, 256, 107340
Windholz, L., Gamper, B., Glowacki, P., & Demchyzëski, J. 2014, SAR, 2, 10
Windholz, L., & Guthöhrlein, G. H. 2003, PhST, 105, 55
Sobolewski, L. M., Binder, T., Guény, C., et al. 2017a, JQSRT, 200, 108
Sobolewski, L. M., Windholz, L., & Kwela, J. 2017b, JQSRT, 201, 30
Sobolewski, L. M., Windholz, L., & Kwela, J. 2017c, JQSRT, 189, 221
Sobolewski, L. M., Windholz, L., & Kwela, J. 2017d, JQSRT, 201, 180
Sobolewski, L. M., Windholz, L., & Kwela, J. 2019, JQSRT, 227, 185
Yong, D., Carney, B. W., Almeida, M. L. T., & de; Pohl, B. L. 2006, AJ, 131, 2256
Yong, D., Carney, B. W., & Friel, E. D. 2012, AJ, 144, 95