Iron line profiles and self-shadowing from relativistic thick accretion discs

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ABSTRACT
We present Fe Kα line profiles from and images of relativistic discs with finite thickness around a rotating black hole using a novel code. The line is thought to be produced by iron fluorescence of a relatively cold X-ray-illuminated material in the innermost parts of the accretion disc and provides an excellent diagnostic of accretion flows in the vicinity of black holes. Previous studies have concentrated on the case of a thin, Keplerian accretion disc. This disc must become thicker and sub-Keplerian with increasing accretion rates. These can affect the line profiles and in turn can influence the estimation of the accretion disc and black hole parameters from the observed line profiles. We here embark on, for the first time, a fully relativistic computation which offers key insights into the effects of geometrical thickness and the sub-Keplerian orbital velocity on the line profiles. We include all relativistic effects such as frame-dragging, Doppler boost, time dilation, gravitational redshift and light bending. We find that the separation and the relative height between the blue and red peaks of the line profile diminish as the thickness of the disc increases. This code is also well suited to produce accretion disc images. We calculate the redshift and flux images of the accretion disc and find that the observed image of the disc strongly depends on the inclination angle. The self-shadowing effect appears remarkable for a high inclination angle, and leads to the black hole shadow being in this case, completely hidden by the disc itself.

Key words: accretion, accretion discs – black hole physics – line: profiles – galaxies: active – X-rays: galaxies.

1 INTRODUCTION
The fluorescent Kα iron emission line has been observed in several active galactic nuclei (AGNs) with a broad and skewed line profile. The line is thought to be produced by iron fluorescence of a relatively cold X-ray-illuminated material in the innermost parts of the accretion disc. Bearing in mind that the line is intrinsically narrow in the local rest frame of the emitting material, and is transformed into broad, skewed profile by Doppler shifts and gravitational redshift effects, thus the line profile encodes the nature of the structure, geometry and dynamics of the accretion flow in the immediate vicinity of the central black hole, as well as the geometry of the space–time, thereby providing key information on the location and kinematics of the cold material. Investigating these spectral features in X-ray luminous black hole systems opens a unique window and allows us to probe the physics that occurs in the vicinity of a black hole, and provides one way to test theory of strong field gravity.

Calculations of the line profiles emitted from an accretion disc around a black hole have been performed by several authors. Theoretical Fe Kα line profiles from a thin disc around a Schwarzschild black hole were calculated by Fabian et al. (1989). Laor (1991) extended those to the extreme Kerr metric. These calculations are based on a geometrically thin, optically thick accretion disc [hereafter SSD, following Shakura & Sunyaev (1973)], on which the accreting material is considered to be in Keplerian orbit around a central black hole. Further, the line emissivity is assumed to vary with $r$ in a power-law form. Efforts have been made later on to include various physically plausible processes in the accretion flow, such as spiral wave, disc warp and gravitational redshift effects (Pariev & Bromley 1998; Fukue 2000; Hartnoll & Blackman 2000, 2001, 2002; Fukumura & Tsuruta 2004), as well as taking into consideration of the geometry and the relative motion of the primary X-ray source (Ruszkowski 2000; Dabrowski & Lasenby 2001; Lu & Yu 2001; Nayakshin & Kazanas 2001) towards a more realistic emissivity distribution. Some authors considered also the ionization effect, the emission from plunging region on the Fe K line and reflection spectrum (Matt, Fabian & Ross 1993; Ross & Fabian 1993; Matt, Fabian & Ross 1996; Reynolds & Begelman 1997).

In the calculations, two basic approaches have been used to map the disc into the sky plane. The first method follows photon trajectories starting from a given initial locus of emission region in the local rest frame of the disc to the observer at infinity. In this case...
a transfer function (Cunningham 1975; Laor 1991; Speith, Riffert & Ruder 1995; Wilms, Speith & Reynolds 1998) is usually introduced as an integration kernel which includes all relativistic effects in line profile calculation. The integration for the line flux is then performed directly on the surface of the accretion disc. The transfer function was first introduced by Cunningham (1975), who presented the numerical results for a grid of parameters aiming at estimating the relativistic effect on the continuum emission from SSD, and was refined and discussed in great detail by Speith et al. (1995).

The second method adopts a ray-tracing approach (Dabrowski et al. 1997; Fanton et al. 1997; Cadez, Fanton & Calvani 1998; Müller & Camenzind 2004; Cadez & Calvani 2005). Following the trajectories of photons from the sky plane to the accretion disc, in this method the image of the disc on the observer’s sky is derived first and then the line flux is obtained by integrating over this image, weighted by the redshift factor and the radial disc emissivity profile. Recently, Beckwith & Done (2004) developed a fast, accurate, high-resolution code which can be used to generate high-resolution line profiles numerically. Beckwith & Done (2005) extended it to include the contribution of higher order photons to the line profiles. But all of these approaches are restricted to SSD.

On the other hand, direct imaging of accretion discs around a black hole is one of the most exciting areas of study to be considered in the future. The fact that a black hole possesses an event horizon makes a black hole cast a shadow upon the background light with a size of roughly 10 gravitational radii that is due to the bending of light by the black hole, and this shadow is nearly independent of the spin or orientation (Falcke, Melia & Agol 2000; Zakharov et al. 2005). However, for a black hole embedded in an optically thick accretion flow the shape and position of the shadow will be altered regardless of the black hole spin (Takahashi 2004; Watarai et al. 2005). From an observational point of view, the highest angular resolution is obtained with Very Long Baseline Interferometry (VLBI) at millimetre wavelengths (so-called mm-VLBI) with an angular resolution of a few tens of microarcseconds. This corresponds to a spatial resolution of only a few tens of gravitational radii for the nearby galaxies. Future Global mm-VLBI Array at short millimetre wavelengths therefore should allow to map the direct vicinity of the supermassive black holes (SMBHs) such as Sgr A* and M87, and offers new possibilities to study the immediate environment of SMBHs (Krichbaum et al. 2004; Shen et al. 2005; Broderick & Loeb 2006; Yuan, Shen & Huang 2006). In the X-ray band, the proposed Microarcsecond X-ray Interferometry Mission (MAXIM) aims to obtain submicroarcsecond resolution X-ray images of nearby galactic nuclei (Reynolds & Nowak 2003; Cash 2005). At this resolution, one can capture the image of an event horizon of the central massive black hole in a nearby AGN. The combination of high-resolution radio interferometry and interferometric X-ray spectroscopy would form a powerful tool to study SMBHs and their environment with high accuracy and provide unprecedented and direct constraints on the dynamics and geometry of the disc, as well as the geometry of the space–time.

With the development of the observational techniques, high-quality observational data will eventually become available. By fitting the data, one can in principle constrain the parameters of the accretion disc system; this will provide both a direct evidence for the existence of a black hole and a way of quantitative test of general relativity in strong gravity. However, accurate quantitative analysis of observational data requires a sophisticated model that treats all relativistic effects with a realistic accretion disc structure. At present, such a complete model is still not available. To our knowledge, SSD breaks down when the accretion rate approaches the Eddington rate. At this limit the disc must become geometrically thick and be sub-Keplerian (Abramowicz et al. 1988; Wang & Zhou 1999; Shadmehri & Khajenabi 2005), that is the so-called slim disc. For a thick disc, Nandra et al. (1995) pointed out it would be of lower density than a standard α-disc which would increase the ionization parameter (flux divided by density), thus leads to iron in the inner parts of the disc becoming fully ionized and no iron lines at all. For slim disc, this may not be the case, the broad, ionized Fe Kα line was discovered in some narrow-line Seyfert 1 (NLS1) galaxies (Ballantyne, Iwasawa & Fabian 2001a; Boller et al. 2002, 2003; Fabian et al. 2004; Gallo et al. 2007), which have been thought to work with high accretion rates. So, slim disc has received much more attention because it can be basically used to account for spectral features in NLS1 (Mineshige et al. 2000; Wang & Netzer 2003; Chen & Wang 2004). With the increasing evidence for ionized accretion discs in NLS1, the spectra and emission lines of slim discs need to be studied in more details. Motivated by the above considerations, a geometrically and optically thick accretion disc model is presented making an attempt at gaining an insight into the effects of disc geometry and dynamics on the line profiles and disc images. Following the idea presented by Speith et al. (1995), we extend their method to the finite thick disc, and adopt elliptic integrals to improve the performance of the code which is much faster than the direct integral and widely used by many authors.

The paper is organized as follows. In Section 2 we summarize the assumptions behind our model, and present the basic equations relevant to our problem, while some more technical aspects like the formulae of the integration for photon trajectories expressed in terms of the inverse Jacobian elliptic functions are given in Appendix A. We present our results in Section 3, and summarize the conclusions and discussion in Section 4.

2 ASSUMPTIONS AND METHOD OF CALCULATION

The aim of this paper is to consider how the accretion disc geometry and dynamics affect the Fe Kα line profiles and disc images. To this end, the disc shape and structure must be determined first. To obtain a rigorous model, one should solve the disc structure equations numerically. However, this is beyond the scope of the current work. For simplicity, we adopt a conical surface for the disc geometry. The thickness of the disc can be described by the half subtending angle δ (0 < δ < π/4). When δ = 0, the disc reduces to SSD. The complementary angle of δ is denoted by δc which is the angle between the symmetric axis of the system and the radial direction of the disc surface. The parameters of this model include: the radii of the emitting disc zone Rin, Rout, the spin of the black hole a, the inclination angle of the disc (δi) and the disc surface angle (δc), the radial emissivity index p, the angular velocity index (see below) n, respectively. In addition, the angular dependence of the emissivity also shall be given.

2.1 Assumptions and basic equations

The propagation of radiation from the disc around a Kerr black hole and the particle kinematics in the disc were studied by many authors. We review properties of the Kerr metric and formulae for its particle orbits, and summarize here the basic equations relevant to this work. Throughout the paper we use units in which G = c = 1, where G is the Gravitational constant, c the speed of light. The background space–time geometry is described by Kerr metric. In

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Boyer–Lindquist coordinates, the Kerr metric is given by
\[ ds^2 = -e^{2\psi} dt^2 + e^{2\phi} (d\phi - \omega dt)^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \Sigma \sin^2 \theta d\phi^2, \]
where
\[ e^{2\psi} = \frac{\Sigma \Delta}{A}, \quad e^{2\phi} = \frac{\sin^2 \theta A}{\Sigma}, \quad \omega = \frac{2Mar}{A}. \]
\[ \Sigma = r^2 + a^2 \cos^2 \theta, \quad \Delta = r^2 + a^2 - 2Mr, \]
\[ A = (r^2 + a^2)^2 - a^2 \Delta \sin^2 \theta. \]
Here \( M, a \) are the black hole mass and specific angular momentum, respectively.

The general orbits of photons in the Kerr geometry can be expressed by a set of three constants of motion (Cartier 1968). Those are the energy at infinity \( E \), the axial component of angular momentum \( L_\phi \) and Cartier’s constant \( Q = (\gamma q^2 E^2) \). The 4-momentum of a geodesic has components
\[ p_\mu = (p_t, p_r, p_\phi, p_\theta) = (-E, \pm E \sqrt{\mathcal{R}/\Delta}, \pm E \sqrt{\mathcal{R}/\Delta}, E\lambda), \]
with
\[ R = r^4 + (a^2 - \lambda^2 - q^2)r^2 + 2M[q^2 + (\lambda - a^2)r - a^2q^2], \]
\[ \Theta = q^2 + a^2 \cos^2 \theta - \lambda^2 \cot^2 \theta. \]

From this, the equations of motion governing the orbital trajectory can be obtained. The technical details are given in Appendix A.

We assume that the disc is of a cone-shaped surface, axisymmetric, and lies on the equatorial plane of the black hole. Photons are emitted or reflected from the gas on the conical disc surface which moves along circular orbits. The radial drift of the gas on the disc surface is neglected. Thus, the 4-velocity is chosen to be of the form
\[ u^\mu = u^t(\partial_t, 0, 0, \Omega_\partial_\theta) = (u^t, 0, 0, u^\phi), \]
where \( \Omega = u^\phi/u^t \) is the angular velocity of the emitting gas. The choice of \( \Omega \) must satisfy the causality condition. For sub-Keplerian velocity, we adopt the modification of \( \Omega \) firstly introduced by Ruszkowski & Fabian (2000):
\[ \Omega = \left( \frac{\partial}{\partial \pi/2} \right)^{1/6} \Omega_k + \left[ 1 - \left( \frac{\partial}{\partial \pi/2} \right)^{1/6} \right] \omega, \]
where \( \partial \) is the poloidal Boyer–Lindquist coordinate, \( \Omega_k = M^{1/2}/(r^{2/3} + aM^{1/2}) \) is the Keplerian angular velocity and \( \omega \) is the angular velocity of the gravitational drag. It is easy to verify that \( \Omega \leq \Omega_k \).

For describing physical processes near a Kerr black hole, Boyer–Lindquist coordinates, which are unphysical in the ergosphere, are inconvenient. In order to make physics appear simple in their expressions, the locally non-rotating frames (LNRF) were introduced by Bardeen, Press & Teukolsky (1972). The relation between the local rest frame attached to the disc fluid and LNRF is given by a Lorentz transformation. In the LNRF, the azimuthal component of 3-velocity reads
\[ v = e^{\psi-\chi}(\Omega - \omega) = \frac{A \sin \theta}{\Sigma \sqrt{\Delta}} (\Omega - \omega). \]
The corresponding Lorentz factor \( \gamma \) as measured by LNRF is defined as \( \gamma = (1 - v^2)^{-1/2} \).

Due to relativistic effects, the photon frequency will shift from the emitted frequency \( v_e \) to the observed one \( v_o \) received by a rest observer with the hole at infinity. We introduce a \( g \) factor to describe the shift which is the ratio of observed frequency to emitted one:
\[ g = \frac{v_o}{v_e} = \frac{p \cdot u_o}{p \cdot u_e} = \frac{e^\gamma (1 - e^\gamma)^{1/2}}{1 - \Omega \lambda}, \]
where \( p, u_o, u_e \) are the 4-momentum of the photon, the 4-velocity of the observer and the emitter, respectively.

The specific flux density \( F_o(v_o) \) at frequency \( v_o \) as observed by an observer at infinity is defined as the sum of the observed specific intensities \( I_o(v_o) \) from all parts of the accretion disc surface,
\[ F_o(v_o) = \int I_o(v_o) d\Omega_{obs}. \]

In order to calculate the integration over \( d\Omega_{obs} \), we must first obtain the disc image or find the relation between the element of the solid angle and the disc linked by the null geodesic. The apparent position of the disc image as seen by an observer is conveniently represented by two impact parameters \( \alpha \) and \( \beta \), measured relative to the direction to the centre of the black hole. The impact parameters \( \alpha \) and \( \beta \) are, respectively, the displacements of the image in the directions perpendicular and parallel to the projection of the black hole spin. They are related to two constants of motion \( \lambda \) and \( q \) by (Cunningham & Bardeen 1973; Cunningham 1975)
\[ \alpha = -\lambda \sin \theta_o, \quad \beta = \pm \left( \frac{q^2 + a^2 \cos^2 \theta_o - \lambda^2 \cot^2 \theta_o}{1/2} \right), \]
where \( \theta_o \) is the angle between the observer and the rotation axis of the black hole (i.e. the inclination angle). The element of solid angle seen by the observer is then
\[ d\Omega_{obs} = \frac{dr \cdot d\beta}{r_o^2} = \frac{1}{r_o^2} \frac{\partial (\lambda, \beta)}{\partial (\lambda, q)} \frac{\partial (\lambda, q)}{\partial (r, g)} \frac{dr}{dg} \]
\[ = \frac{q}{r_o^2 \sin \theta_o} \frac{\partial (\lambda, q)}{\partial \lambda} \frac{dr}{dg}, \]
where \( r_o \) is the distance from the observer to the black hole.

Substituting equation (11) into equation (9) gives the desired result:
\[ F_o(v_o) = \frac{q}{r_o^2 \sin \theta_o} \int e^{\gamma^2} \delta(v_o - g v_e) \frac{\partial (\lambda, q)}{\partial \lambda} \frac{dr}{dg} \]

To perform the integration, the form of the disc emissivity in the integrand also needs to be given. In general, it can be a function
of the radius, $r_e$, and polar angle, $\theta_e$, of an emitted photon with the surface normal of the disc in the rest frame of the emitting gas. This angle is determined by taking the dot products of the photon 4-momentum $p$ with the surface normal $n$. The surface normal in the rest frame is

$$n = \Sigma^{-1/2} \delta / \partial \theta.$$  

(13)

By definition, we get

$$\cos(n_e) = \frac{p \cdot n}{p \cdot u_e} = \frac{p \cdot n}{p \cdot u_e},$$

$$= g \sqrt{\Omega / \Sigma},$$

$$= e^e(1 - v^2) \Omega^{1/2} \Sigma^{-1/2} / (1 - \Omega \lambda).$$  

(14)

If the emission is isotropic in the rest frame, we do not need to know $n_e$. More generally, we take the form

$$\epsilon(r_e, \mu_e) = \epsilon(r_e) f(\mu_e),$$

(15)

where $\mu_e$ is the cosine of the emitted angle (cos $n_e$). And the radial emissivity is assumed to vary as a power law with emissivity index $\nu$:

$$\epsilon(r_e) \propto r_e^{-\nu}.$$  

(16)

We consider three possible cases for the angular dependence of the emissivity (Beckwith & Done 2005): (1) isotropic emission, $f(\mu_e) = 1$; (2) limb-darkening law $f(\mu_e) \propto (1 + 2.06 \mu_e)$; (3) limb-brightening law $f(\mu_e) \propto 1/\mu_e$.

### 2.2 Method of calculation

With all of the preparation described in the previous section, we now turn to how to calculate the line profiles and the disc images numerically. We divide the disc into a number of arbitrarily narrow rings, and emission from each ring is calculated by considering its axisymmetry. We shall denote by $r_f$ the radius of each such emitting ring. For each ring there is a family of null geodesic along which radiation flows to a distant observer at polar angle $\theta_o$ from the disc’s axis. As far as the Fe Kα emission line is concerned, for a given observed frequency $v_o$ the null geodesic in this family can be picked out if it exists. So, the weighted contribution of this ring to the line flux can be determined. The total observed flux can be obtained by summing over all emitting rings. Changing the observed frequency, the line profiles will be obtained.

This family of null geodesic for each ring can be used to map the accretion disc on to sky plane, that is, disc imaging. A geodesic in this family connects an emitting region in the ring to the distant observer. The constants of motion $\lambda$ and $q$ of this geodesic can be used to determine the apparent position of the emitting point on the sky plane using the corresponding two impact parameters $\alpha$ and $\beta$. Different geodesic is associated with different point. Using geometric optics, one determines the appearance of the ring from this family of geodesic, then in this way images (at infinity) of the accretion disc are obtained.

The main numerical procedures for computing the line profiles are as follows.

(i) Specify the relevant disc system parameters: $r_{in}$, $r_{out}$, $a$, $p$, $n$, $\alpha$, $\beta$, and the angular emissivity.

(ii) The disc surface is modelled as a series of rings with the radii $r_f$ and weights $w_f$ which are calculated using an algorithm given by G. B. Rybicki (Press et al. 1992).

(iii) For a given couple $(r_f, g)$ of a ring, the two constants of motion $\lambda$ and $q$ are determined if they exist. This is done in the following way: the value of $\lambda$ is obtained by another form of equation (6),

$$\lambda = \frac{1}{\Omega} \left[ \frac{1}{1 - \frac{e^\lambda}{\gamma g}} \right] = \frac{1}{\Omega} \left[ \frac{1 - \frac{e^\lambda(1 - v^2)^{1/2}}{g}}{g} \right],$$

(17)

the value of $q$ is determined by solving photon trajectory equation (A1). Then the contribution of this ring on the flux for given frequency $v_o$ with respect to $g$ is estimated.

(iv) For a given $g$, the integration over $r$ of equation (12) can be replaced by a sum over all the emitting rings

$$F_o(v_o) = \sum_{i=1}^{n} \frac{q_e v_o^3}{r_i^2 v_i^3} \sin \theta_o \frac{\partial (\lambda, q)}{\partial (r, g)} |_{r=r_i} w_i.$$  

(18)

The Jacobian $\partial (\lambda, q)/\partial (r, g)$ in the above formula was evaluated by the finite difference scheme. From the above formula, one determines the line flux at frequency $v_o$ from the disc.

(v) Varying $g$, the above steps are repeated.

The observed line profile as a function of frequency $v_o$ is finally obtained in this way.

### 3 RESULTS

The model and the computer code described in this paper are suitable for disc inner edge located at any $r_m > r_{ms}$, where $r_{ms}$ is the radius of the marginally stable orbit. For simplicity, in all plots presented in this paper, we assume that $r_m = r_{ms}$. We have taken a disc from $r_{ms}$ to $r_{max} = 20r_g$ (focus on strong gravitational effects) for Kerr metric case and treated spin of the black hole as a free parameter for different observed inclinations and disc thickness. Due to its astrophysical importance, we choose the iron fluorescence line at 6.4 keV in what follows.

#### 3.1 Relativistic emission-line profiles

The numerical code discussed in the previous section was used to model emission-line profiles for different model parameters. To test the performance of our code, we first compared the line profiles generated by our code when the disc is reduced to SSD to those generated by the code described in Beckwith & Done (2004), and found that the overall match is fairly good, especially for the Schwarzschild metric case. Fig. 1 shows the results for parameters identical to those presented by Beckwith & Done (2004).

The dependence of the line profiles on the disc thickness is shown in Fig. 2. The angular velocity and the emissivity law take the forms: $\Omega = \left( \frac{\lambda}{\Omega_e} \right)^{1/3} \Omega_e + \left[ 1 - \left( \frac{\lambda}{\Omega_e} \right)^{1/3} \right] \nu_0, \epsilon(r_e) \propto r_e^{-3}$ and $f(\mu_e) \propto (1 + 2.06 \mu_e)$. Two cases for $n = 2$ (top panel) and $n = 3$ (bottom panel) are presented in this figure. It is explicit that the separation and relative height between the blue and red peaks diminish as disc thickness increases because the disc becomes more sub-Keplerian. This effect is also clearly illustrated in Fig. 3. The index $n$ in $\Omega$ describes the deviation of the angular velocity from Keplerian one. As the deviation from Keplerian velocity increases the height of the blue peak of the line decreases significantly.

Fig. 4 compares the line profiles at different viewing angles $\theta_{oa} = 15^\circ, 30^\circ, 45^\circ, 60^\circ$ and $75^\circ$ for a maximal Kerr black hole with the disc extending from $r_{ms}$ to $20r_g$ and $\theta_e = 60^\circ, 70^\circ$. At high inclinations the self-shadowing effect has been taken into account. Due to gravitational lensing (light-bending) effect, there is still a substantial fraction of light that can reach the observer at infinity.
Figure 1. The relativistic line profiles computed by our code both for the Schwarzschild ($a = 0$) and maximal Kerr ($a = 0.998$) cases for $\vartheta_o = 30^\circ$ (left-hand side) and $85^\circ$ (right-hand side). The disc zone is from $r_{in}$ to $r_{out} = 20 r_g$ and located at equatorial plane, where $r_g$ is the gravitational radius. Upper panel: The Schwarzschild metric for $\epsilon (r_e) \propto r_e^{-3}$ and $f(\mu_e) = 1$. Lower panel: The maximal Kerr metric for $\epsilon (r_e) \propto r_e^{-3}$ and $f(\mu_e) \propto (1 + 2.06 \mu_e)$. The flux in all cases is given using the same arbitrary units, and all our results are unsmoothed.

Figure 2. The relativistic line profiles as a function of the disc thickness for (from bottom to top at the red peak) $\vartheta_e = 50^\circ, 60^\circ, 70^\circ, 80^\circ$ and $90^\circ$ for a maximal Kerr black hole with the disc extending from 1.235 to 20 $r_g$. The observer inclination equals $30^\circ$ and angular velocity takes the form $/Omega_1 = (\vartheta / \pi)^{1/n} \Omega_K + [1 - (\vartheta / \pi)^{1/n}] \omega$, here $n$ is set to 2 (upper panel) and 3 (lower panel). The emissivity law takes the forms $\epsilon (r_e) \propto r_e^{-3}$ and $f(\mu_e) \propto (1 + 2.06 \mu_e)$.

We also calculated the effects of emissivity on the form of the relativistic line profile. The radial emissivity is taken as a power law with the index $p$, which determines the relative contribution from different radii of the disc. Here we focus on the influence of anisotropic emission on the line profile. Different angular emissivity laws have striking effects on the line profile, which we illustrate in Fig. 3 for a maximal Kerr geometry with the disc extending from $r_{in}$ to 20 $r_g$ at $\vartheta_o = 30^\circ$, $\vartheta_e = 60^\circ$ (upper panel), $\vartheta_e = 70^\circ$ (lower panel). The index $n$ is set to 3 and the emissivity law is the same as in Fig. 2.

Figure 3. The relativistic line profiles as a function of the angular velocity represented by the parameter $n$ in equation (4): $/Omega_1 = (\vartheta / \pi)^{1/n} \Omega_K + [1 - (\vartheta / \pi)^{1/n}] \omega$ for $n = 1, 2, 3, 4$ (from top to bottom at the redshift peak) for a maximal Kerr black hole with the disc extending from 1.235 to 20 $r_g$ and $\vartheta_o = 30^\circ$, $\vartheta_e = 60^\circ$ (upper panel), $\vartheta_e = 70^\circ$ (lower panel). The emissivity law is the same as in Fig. 2.

Figure 4. The relativistic line profiles as a function of the observed inclinations for $\vartheta_o = 15^\circ, 30^\circ, 45^\circ, 60^\circ$ and $75^\circ$ for a maximal Kerr black hole with the disc extending from 1.235 to 20 $r_g$ at $\vartheta_e = 60^\circ$ (upper panel), $\vartheta_e = 70^\circ$ (lower panel). The index $n$ is set to 3 and the emissivity law is the same as in Fig. 2.

We also calculated the effects of emissivity on the form of the relativistic line profile. The radial emissivity is taken as a power law with the index $p$, which determines the relative contribution from different radii of the disc. Here we focus on the influence of anisotropic emission on the line profile. Different angular emissivity laws have striking effects on the line profile, which we illustrate in Fig. 5 for a maximal Kerr geometry with the disc extending from $r_{in}$ to 20 $r_g$ at $\vartheta_o = 30^\circ$, $\vartheta_e = 70^\circ$. The angular emissivity takes one of the three forms: (i) $\epsilon (r_e) \propto r_e^{-3}$, $f(\mu_e) \propto (1 + 2.06 \mu_e)$, (ii) $\epsilon (r_e) \propto r_e^{-3}$, $f(\mu_e) = 1$, (iii) $\epsilon (r_e) \propto r_e^{-3}$, $f(\mu_e) \propto \mu_e^{-1}$. From
Figure 5. The relativistic line profiles generated by our model with (a) $\epsilon(r_e) \propto r_e^{-3}$, $f(\mu_e) \propto (1 + 2.06\mu_e)$ (black line), (b) $\epsilon(r_e) \propto r_e^{-3}$, $f(\mu_e) = 1$ (red line), (c) $\epsilon(r_e) \propto r_e^{-3}$, $f(\mu_e) \propto \mu_e^{-1}$ (green line), for a maximal Kerr black hole with the disc extending from $1.235$ to $20r_g$ and $\theta_o = 30^\circ$, $\theta_e = 70^\circ$. The sub-Keplerian angular velocity is the same as in Fig. 2. All profiles are scaled to unity for better comparison in this case.

Figure 6. Comparison of the relativistic line profiles generated by our model with different spins $a = 0, 0.5, 0.998$. The emission-line region is from $r_{ms}$ to $20r_g$ and the angular velocity and the emissivity law are the same as in Fig. 4. The angles are marked in each figure.

the figure one can see the relative height of the blue wing changes a lot for different angular emissivity laws, anticorrelated with the slope of the red wing.

The line profiles as a function of the black hole spin are also demonstrated. For a low or intermediate inclination angle the line profiles are shown in Figs 6 and 7. Note that the red wings change significantly whereas the blue peaks almost are not affected by the spin. At high inclinations, the effect of the self-shadowing dramatically alters the line profile for a thick disc. The results are illustrated in Figs 8 and 9 with angular emissivity $f(\mu_e) \propto (1 + 2.06\mu_e)$ and $f(\mu_e) = 1$, respectively. For $\theta_e = 60^\circ$, the line profiles are almost the same; this implies that the line emission from the inner parts of the disc is completely obscured by the outer parts of the disc. At high viewing angles, the impact of angular emissivity law on the relativistic line profiles is also striking.

3.2 Accretion disc images

We present in Figs 10 and 11 the redshift and flux images of the accretion disc and black hole shadows on the ($\alpha, \beta$) plane for an
extreme Kerr black hole, for $\theta_o = 5^\circ, 30^\circ, 55^\circ, 80^\circ$, and $\theta_e = 60^\circ, 70^\circ$. Redshift images are coloured by the associated values of $g$ as measured by the infinity observer, which is defined by the scale at the top of each image. Flux images are coloured by $10^4 \varepsilon g^4$, again with the scale defined at the top of each image. The parameters $n$ and $p$ are both set to 3 and $f(\mu_e) \propto (1 + 2.06 \mu_e)$. The images are distorted by the combined action of Doppler effects, gravitational redshift and light bending in the vicinity of the black hole. Note that at small inclination angle ($\theta_o = 5^\circ$), the observed radiation is all redshifted, and therefore the emission-line profiles will have a net redshift. On the other hand, at an intermediate inclination angle ($\theta_o = 30^\circ, 55^\circ$), the innermost part of the disc is notably redshifted, whereas the observed radiation from the approaching side is remarkably enhanced by the Doppler boost. Moreover, the light ray emitted by the far side of the disc is bent by the gravity of the black hole, resulting in the vertical asymmetry of the image, as if it were bent towards the observer. Note also that the self-shadowing effect is remarkable at a high inclination angle ($\theta_o = 80^\circ$), and therefore the black hole shadow in this case does not appear at all. The shape, size and position of the black hole shadows are also affected by the self-shadowing, which is different from those of SSD (Takahashi 2004).

4 SUMMARY

We have developed a computer code both to calculate the line profiles of a relativistic thick accretion disc around a black hole and to generate the images of accretion discs. The code includes all relativistic effects. It also includes the effect of self-shadowing of the disc, that is, the outer disc blocks the emission from the inner region. The code can handle any value of the black hole spin, the different viewing angles, the disc inner radius ($r_{in} \geq r_{in}^* = r_{in}$) and the disc thickness ($\delta \leq \pi/4$). It also allows the user to choose one of the three types of angular emissivity laws: isotropic emission, limb-darkening or limb-brightening laws.

We show that the separation and the relative height between the blue and red peaks of the line profiles diminish as the thickness of the disc increases because of the sub-Keplerian motion. The angular emissivity form has also a significant influence on the line profile. The results of one peak line profile present in intermediate viewing angle in our model is different from those in SSD for low viewing angle. To see the self-shadowing effect more clearly, images of the disc and the black hole shadows are also presented in this paper. The
self-shadowing effect is very important for high inclination angle. Future X-ray observations of high state accreting systems such as NLS1 galaxies will be important to test whether the disc in these systems are indeed thick.

Here we just present a simple disc model with a conical surface aimed at getting an insight into the effects of geometric and dynamic influence on the line profiles and disc images. For a non-equatorial disc, we consider the self-shadowing and sub-Keplerian effects on them, as well as the contribution of the light with Carter’s constant $Q < 0$ which is different from those for equatorial disc. For simplicity, in this paper we neglected the influence of radial drift of a flow on the line profile. Other limitations include: the thickness of the disc may vary with radius, it probably also has a substantial warp and the effects of photoionization of the surface layers of the accretion disc on the emission lines are not taken into account. X-ray reflection by photoionized accretion discs has been investigated in some detail (Ross & Fabian 1993; Ross, Fabian & Young 1999; Ballantyne, Ross & Fabian 2001b; Ross & Fabian 2005). The ionization parameter has clearly a large effect on emission lines. Evidence for reflection by ionized accretion discs in NLS1 has been accumulated in the literature in recent years (see e.g. Ballantyne et al. 2001a; Boller et al. 2002, 2003; Fabian et al. 2004; Gallo et al. 2007). Furthermore, the radial drift of the flow for a sub-Keplerian disc may have significant influence on the line profiles. A more realistic disc model should take into account both the sub-Keplerian and radial velocity effects on the line profiles. This effect will be investigated in the near future.

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Then, the equations governing the orbital trajectories are separable. Since the system is stationary and axisymmetric, only the motions in $r$ and $\theta$ are required in the calculation of the radiation spectrum from the disc. The motion in the $r$–$\theta$ plane is governed by (Bardeen et al. 1972; Chandrasekhar 1983)

$$\int_{r_1}^{r_2} \frac{dr}{\sqrt{R(r)}} = \pm \int_{\theta_\mu}^{\theta_\epsilon} \frac{d\theta}{\sqrt{\Theta(\theta)}}, \quad (A1)$$

where

$$R(r) = r^4 + (a^2 - \lambda^2 - Q^2)r^2 + 2(Q + (\lambda - a^2)r - a^2 Q), \quad (A2)$$

$$\Theta(\theta) = Q + a^2 \cos^2 \theta - \lambda^2 \cos^2 \phi, \quad (A3)$$

and $r_0$ and $\theta_0$ are the starting values of $r$ and $\theta$.

Define $\mu = \cos \theta$, then equation (A1) becomes

$$\int_{\mu_0}^{\mu} \frac{d\mu}{\sqrt{\Theta(\mu)}} = \pm \int_{\mu_0}^{\mu_+} \frac{d\mu}{\sqrt{\Theta(\mu)}}, \quad (A4)$$

where $\mu_0 = \cos \theta_0$ and

$$\Theta(\mu) = Q + (a^2 - \lambda^2 - Q) \mu^2 - a^2 \mu^2 = a^2 (\mu^2_+ + \mu^2_-)(\mu^2_+ - \mu^2_-), \quad (A5)$$

and $\mu^2_\pm$ are defined by

$$\mu^2_\pm = \frac{1}{2a^2} \left\{ \left[ (\lambda^2 + Q - a^2) + 4a^2 Q \right]^{1/2} \mp (\lambda^2 + Q - a^2) \right\}. \quad (A6)$$

For $Q > 0$, both $\mu^2_+$ and $\mu^2_-$ are non-negative. When $Q < 0$, $\mu^2_-$ is less than zero. Note, $\mu^2_+ - \mu^2_- = Q/a^2$.

For a photon emitted by the disc, the integral over $\mu$ can be worked out with the inverse Jacobian elliptic integral

$$\int_{\mu}^{\mu_+} \frac{d\mu}{\sqrt{\Theta(\mu)}} = \int_{\mu}^{\mu_+} \frac{d\mu}{\sqrt{a^2 (\mu^2_+ + \mu^2_-)(\mu^2_+ - \mu^2_-)}}, \quad (A7)$$

$$= \begin{cases} \frac{1}{a \mu_+} \sin^{-1} \left( \frac{\mu^2_+ - \mu^2_-}{\mu^2_+ + \mu^2_-} \right), & (\mu^2_+ > 0), \\ \frac{1}{a \mu_-} \sin^{-1} \left( \frac{\mu^2_+ - \mu^2_-}{\mu^2_+ + \mu^2_-} \right), & (\mu^2_- < 0), \end{cases}$$

where $0 < \mu < \mu_+$ for $\mu^2_+ > 0$ and $\sqrt{-\mu^2_-} < \mu < \mu_+$ for $\mu^2_- < 0$.
The integral over $r$ can also be worked out with inverse Jacobian elliptic integrals. To do so, we need to find out the four roots of $R(r) = 0$. $R(r) = 0$ may have four real roots, two real roots and two complex roots or four complex roots. We consider the three cases separately.

**Case A:** $R(r) = 0$ has four real roots. Let us denote the four roots by $r_1, r_2, r_3$ and $r_4$ in the ascending order.

The integral over $r$ can be worked out by the following integration:

$$
\int_1^r \frac{dr}{\sqrt{R}} = \int_1^{r_1} \frac{dr}{\sqrt{(r - r_1)(r - r_2)(r - r_3)(r - r_4)}} = \frac{2}{\sqrt{(r_1 - r_2)(r_2 - r_3)}} \text{sn}^{-1} \left( \sqrt{\frac{(r_2 - r_4)(r_1 - r_2)}{(r_1 - r_4)(r_1 - r_2)}} \frac{m_4}{m_4} \right),
$$

where

$$
m_4 = \frac{(r_1 - r_3)(r_2 - r_3)}{(r_1 - r_3)(r_2 - r_4)}. \tag{A8}
$$

When $r_1 = r_2$, the integral over $r$ can be expressed in terms of a logarithm, which is of no practical interest for this paper.

**Case B:** $R(r) = 0$ has two complex roots and two real roots. Let us assume that $r_1$ and $r_2$ are complex, $r_3$ and $r_4$ are real and $r_3 > r_4$. The physically allowed region for photons is given by $r > r_3$. If we write $r_1$ and $r_2$ in the form $r_1 = u + iv$ and $r_1 = u - iv$, the integral over $r$ can be worked out by the following integration:

$$
\int_1^r \frac{dr}{\sqrt{R}} = \int_1^{r_1} \frac{dr}{\sqrt{(r - u)^2 + v^2}(r - r_3)(r - r_4)} = \begin{cases} 
\frac{1}{\sqrt{pq}} \text{sn}^{-1} \left( \sqrt{\frac{pq(r - r_3)(r - r_4)}{(p + q)r - r_3q - r_4p}} \frac{m_2}{m_2} \right), & (r < r_3), \\
\frac{1}{\sqrt{pq}} \left[ 2K(m_2) - \text{sn}^{-1} \left( \sqrt{\frac{pq(r - r_3)(r - r_4)}{(p + q)r - r_3q - r_4p}} \frac{m_2}{m_2} \right) \right], & (r > r_3),
\end{cases} \tag{A10}
$$

where

$$
p^2 = (r_3 - u)^2 + v^2, \quad q^2 = (r_4 - u)^2 + v^2, \\
r_c = \frac{r_3q - r_4p}{p - q}, \quad m_2 = \frac{(p + q)^2 - (r_3 - r_4)^2}{4pq}
$$

and

$$
K(m_2) = \text{sn}^{-1}(1|m_2). \tag{A11}
$$

$K(m_2)$ is the complete elliptic integral of the first kind. When $r = \infty$, equation (A10) is equivalent to equation (28) of Čadež et al. (1998). It can be directly verified that

$$
K(m_2) - \text{sn}^{-1} \left( \frac{2\sqrt{pq}}{p + q} \frac{m_2}{m_2} \right) = \text{sn}^{-1}(\sqrt{1 - 1/\lambda_1}|m_2). \tag{A12}
$$

**Case C:** $R = 0$ has four complex roots. Let us denote the four roots by $r_1, r_2, r_3$ and $r_4$, $r_1 = r_2^*, r_3 = r_4^*$, where $^*$ stands for complex conjugate. If we set $B = |r_1 + r_2|, C = r_1r_2, D = r_3r_4$, it leads to $R(r) = (r^2 - Br + C)(r^2 + Br + D)$. In this case, in order to express the quartic in terms of $r^2$, we make the substitution

$$
s = \frac{r - \lambda_2}{r - \lambda_1},
$$

where $\lambda_1, \lambda_2 (\lambda_1 < \lambda_2)$ are the two real roots of equation

$$
2B\lambda^2 + 2(D - C)\lambda + B(D + C) = 0.
$$

This yields

$$
R(r) = \frac{(p_1s^2 + q_1)(p_2s^2 + q_2)}{(s - 1)^2} = \frac{p_1p_2(s^2 + q^2)(s^2 + q^2)}{(s - 1)^2}, \tag{A13}
$$

where

$$
p_1 = \lambda_1^2 - B\lambda_1 + C, \quad q_1 = \lambda_2^2 - B\lambda_2 + C, \\
p_2 = \lambda_1^2 + B\lambda_1 + D, \quad q_2 = \lambda_2^2 + B\lambda_2 + D, \\
p^2 = \max(q_1/p_1, q_2/p_2), \quad q^2 = \min(q_1/p_1, q_2/p_2).
$$
From equation (A13) the integral over $r$ can be calculated by means of the substitution $s = \frac{r - \lambda_2}{r - \lambda_1}$ through the following formula

$$\int_{r_c}^{\infty} \frac{dr}{\sqrt{R}} = \int_{r_c}^{\infty} \frac{dr}{\sqrt{(r^2 - Br + C)(r^2 + Br + D)}}$$

$$= \begin{cases} \frac{\lambda_2 - \lambda_1}{\sqrt{p^2 p_1 p_2}} \left[ \text{sn}^{-1} \left( \frac{1}{\sqrt{1 + q^2}} m_c \right) - \text{sn}^{-1} \left( \frac{s^2}{s^2 + q^2} m_c \right) \right], & (r_e > \lambda_2), \\ \frac{\lambda_2 - \lambda_1}{\sqrt{p^2 p_1 p_2}} \left[ \text{sn}^{-1} \left( \frac{1}{\sqrt{1 + q^2}} m_c \right) + \text{sn}^{-1} \left( \frac{s^2}{s^2 + q^2} m_c \right) \right], & (r_e < \lambda_2), \end{cases}$$

(A14)

where

$$m_c = \frac{p^2 - q^2}{p^2}.$$  

(A15)

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