Understanding acceleration: An interplay between different mathematics and physics representations

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Abstract. Acceleration is often viewed as abstract – but when a person accelerates, e.g. in a car or on swings, trampolines, carousels or rollercoasters, the force required for the acceleration is felt throughout the body. If acceleration is introduced as force divided by mass rather than as a second derivative of displacement, the concept can be made accessible to much younger learners, such as 10-year olds taking part in supervised visits to an amusement park or playground. The force required for acceleration can be measured e.g. using smartphone sensors. The forces can also be illustrated in demonstrations that complement photos and video analysis. For a teacher this creates opportunities to discuss challenging concepts of force and motion, in enjoyable authentic situations.

1. Introduction
The force required to change motion is experienced throughout the accelerating body. By focusing on this force on the human body, the concept of acceleration as can be made accessible to young learners, such as 10-year olds taking part in supervised visits to an amusement park or playground [1,2]. However, traditionally the study of force and motion is introduced through non-motion, followed by situations where all forces cancel. Newton's first law of uniform rectilinear motion is counterintuitive, and seems to contradict everyday experience, where a cyclist needs to pedal to keep going, and a car comes to rest unless the engine continues to provide energy.

Although motion in everyday life is rarely restricted to one dimension, textbook discussions of acceleration often start with the mathematical description of one-dimensional motion, and remain within kinetics, where the motion is decribed without considering the forces that change it. If forces are introduced, they are rarely connected to the personal experience, but rather to inanimate objects that may be found in the physics classroom. In this paper we describe ways to increase student awareness of how the forces acting on them are related to the motion of their own body, by introducing a number of different representations of the motion to complement the mathematical description. The combination of several additional semiotic resources supports the development of a deeper understanding of force and motion.

The paper also gives examples of discussions among first-year physics and engineering students as they attempt to reconcile the mathematical descriptions of different physical situations with the experiences of their bodies, as well as electronic data and experimental demonstrations. For a teacher this creates opportunities to discuss force and motion in enjoyable authentic situations, where commonly held teaching-resistant misconceptions can be challenged.

2. Vertical motion
2.1 A Family amusement ride
A sequence of screen shots of a one-dimensional ride [3], as in figure 1, gives a graph-like description of the motion and offers a way to discuss the experience of the body in connection to the ride. During what parts of the ride do you expect to feel heavier than normal, and during what parts do you expect to feel lighter?

The elevation difference between the subsequent images is proportional to the velocity and can be marked with arrows. The difference between the length of these arrows is, in turn, proportional to acceleration, allowing an estimate of the force acting on the body during the different parts of the ride. During the ride itself, a little plastic spiral ("slinky") changes its length giving a visual measure of the acceleration or G force [1] and the forces may also be recorded using a smartphone [4,5] and presented graphically as in figure 2.

2.2 Bouncing on a Trampoline
Bouncing on a trampoline is another example of one-dimensional motion that may be more easily available than amusement rides. During the time without contact with the trampoline only the force of gravity acts and the bouncer experiences free fall – both on the way up and on the way down. For high bounces, the "flight time" is longer, and the feet leave the trampoline after a shorter contact time. This requires an upward acceleration considerably larger than $|g|$. The longer the flight time, the larger the force. Assuming that the force from the trampoline bed is proportional to the displacement (Hooke's law)
law), the whole motion can be analysed in terms of free fall and harmonic motion, and can be solved analytically [6].

In recent work [7] we describe a challenging student assignment, based on Rosannagh MacLennans Olympic gold medal routine of 10 jumps, in a total of 19 seconds, of which 16 seconds were flight time [8]. An empty graph was provided and the students were asked to draw elevation, velocity and acceleration during two full jumps. A partially completed graph is shown in figure 3.

If a trampoline is available, a small slinky, can be used to illustrate the relation between force and acceleration, as shown in figure 4 for the case of small bounces on a trampoline.

**Figure 3.** A partially filled worksheet to draw elevation, velocity and acceleration during two full jumps of Rosannagh MacLennans gold medal routine. The velocity is put to zero at the highest and lowest points. Combination with the known acceleration of -$g$ gives the centre-of-mass velocity during the flight times, which in turn gives the elevation of the centre of mass. The positive acceleration during the brief contact time must be sufficiently large to bring the velocity back to the highest positive value.

2.2.1. **Acceleration in the highest point.** The free-fall part of the trampoline motion is analogous to the motion of a ball that has left the hand as it is thrown up into the air. An interesting test question is to ask students about the acceleration in the highest point: The incorrect answer "zero" is very common, and may also be quite resistant to change. Below we summarise a couple of small-group discussions with first-year university students, following a question "If I throws this ball in the air – what is its acceleration at the highest point."

- In the first group, two students immediately exclaimed "zero", but when with a third student stating "9.81"one of the first students quickly changed his view to agree, noting "all the time" and a fourth commented "obviously".
- In the second group, the initial answer "zero", was met with an observation from the interviewer "I guessed you would say that". The next student's observation "The acceleration is g all the time" resulted in a surprised exclamation "Is it really? We learned that it was zero momentarily at the turning point" and the first student agreed: "Yes, I recognize that from school."
- In a third group, one student thinks it depends on how the ball is thrown, bringing in two dimensions as a possibility. When encouraged to keep in one dimension, the group starts to talk about relation between acceleration and the direction of motion. "There the velocity..."
decreases, but the acceleration is maximum" and another student adds "It is just because the acceleration is directed towards ... it is not in the direction of motion."

None of these group spontaneously invoked Newton's second law, relating force and acceleration.

![Image](image_url)

**Figure 4.** A slinky illustrating the change in acceleration during the different parts of the bouncing motion on a trampoline.

2.2.2 *Textbooks and acceleration.* In universities, mechanics is sometimes treated as part of mathematics, and mathematics is, indeed, an extremely useful tool to describe motions. Textbook presentations often follow the mathematical approach, making the common initial focus on one-dimensional motion a natural consequence, while obscuring important aspects of the concept: Acceleration as a change of direction is not an obvious possibility in a simplified treatment where motion is studied only in one dimension. However, already in one dimension, failure to distinguish between speed and velocity leads to confusion – if retardation is described as different from acceleration, neither applies to the situation where the body is at rest. Defining retardation as "negative acceleration" implies a definition of acceleration as derivative of speed, i.e. of $|v|$, corresponding to an acceleration $-|g|$ on the way up and $|g|$ on the way down, and an undefined acceleration for $v=0$. Many are the teachers who have been frustrated by the large number of students who state that the acceleration is zero in the highest point for a ball thrown up in the air - and are sometimes reluctant to give up their view, as seen also in the interviews above.

2.2.3 *Beyond velocity and acceleration: Jerk and higher derivatives.* In the theoretical model of trampoline bouncing, the acceleration is continuous, starting at zero when the feet reach the mat, but increasing rapidly. The derivative of acceleration, called jerk, thus becomes a step function in the moments when the feet reach (or leave) the mat. These discontinuities leads to singularities in "snap" which is the derivative of the jerk. The derivative of the step function can formally be described by a Dirac delta function. In processing authentic numeric data from a real trampoline bouncing, typically a running average is used, giving a more distributed form of the snap, and similarly for the higher derivatives which are often referred to as "snap, crackle and pop", as discussed in [9].
3. Acceleration in a horizontal plane
Motions in textbooks often start in the position $x=0$ at time $t=0$, with constant velocity, $v$, or with uniform acceleration, $a$, and initial velocity, $v_0=0$. Typical examples of horizontal motion include a bicycle, car or an occasional airplane. Horizontal acceleration can be measured with an object hanging from a string as shown in figure 5 for the case of take-off and landing of an airplane. The string counteracts the force of gravity, $mg$, while also providing the force required for acceleration, $ma$, and the acceleration can be expressed as $a=g \tan(\theta)$ in terms of the angle $\theta$ to the vertical.

The textbooks often spend long chapters on one-dimensional motion before discussing velocity and acceleration as vectors. Although the mathematical description of vectors may not be accessible for young learners, forces in many different types of three-dimensional motions are very concrete for the accelerating body. The classical teacup ride that is found in many different amusement parks is an example of two-dimensional motion in a horizontal plane. Another example is the waveswinger ride shown in figure 6 and discussed in earlier work [1, 10]. No mathematics is required for the initial observation that empty swings hang in the same angle as swings with riders [1,10].

![Figure 5. Illustration of horizontal acceleration during an airplane take-off and landing.](image)

![Figure 6. A wave swinger ride. Note how empty swings hang in the same angle to the vertical as swings with riders [1,10].](image)
the swings to the vertical provides a direct measure of the acceleration in circular motion. The observed angle can also be compared to a calculated value of acceleration expressed in terms of dimensions and period of rotation. The behaviour of water in a glass taken along on the ride can offer additional surprises to be discussed, leading to further investigations [10,11].

4. Discussion
This paper has shown several examples, with simple experiments offering a variety of representations of acceleration. We hope that the examples may inspire physics teaching that makes use of a wider set of semiotic resources, as a way to help students develop a deeper understanding of force and motion, in spite of its often counterintuitive character.

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