Haines jumps simulation in X-ray CT image of natural sandstone

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Abstract. In this paper, we investigate the effects which are manifested at two-phase drainage flows in porous media at domination of capillary forces – Haines jumps. This pore-scale event is a rapid or burst-like invasion process and is reflected in sharp pressure drop during drainage displacement. This phenomenon was explored in three-dimensional digital model of natural sandstone which was extracted using X-ray computed tomography. For two-phase flow simulation the lattice Boltzmann equations (LBE) were used. The interface interactions between fluids and wetting effects were modeled with color-gradient method in framework of the LBE. We performed an association between pressure drop at Haines jump and direct imaging of corresponding rapid invasion event in 3D model. It was shown that Haines jump occurs at meniscus migration from narrow pore throat to its wide body. The velocity of meniscus displacement exceeds bulk flow rate in more than 100 times and it looks like burst-like invasion. Also, we studied the effects of flow rate and surface tension on Haines jumps statistics. It was revealed that Haines jumps at high flow rates occur simultaneously and individual jumps can’t be distinguished on pressure signal. At regimes, where only individual Haines jumps take place, the flow rate influence is insignificant and the distribution of pressure drops can be fitted using lognormal law. Increasing of surface tension leads to growth of mean value of pressure drop and amount of Haines jumps.

1. Introduction
The problems of two-phase fluid flows through porous media are topical in many branches of science and techniques; hydrogeology, pedology and soil sciences, underground water dynamics, and oil- and gas-field developments. According to studies [1-3], two-phase flow in porous media at displacement of wetting phase by injection of non-wetting phase (drainage flow) is controlled by viscous and capillary flow, and can be determined by capillary number \( Ca \) and viscous ratio between non-wetting and wetting fluids \( M \). The capillary number characterizes the relation between viscous and capillary forces and is defined as \( Ca = \frac{\mu_{nw} u_{nw}}{\sigma \cos \theta} \), where \( \mu_{nw}, u_{nw}, \sigma \) and \( \theta \) are the dynamic viscosity, velocity of injected non-wetting fluid, surface tension and contact wetting angle, respectively.

In this paper, we focus on investigation of phenomena which occur in porous media at domination of capillary forces. Such flow pattern at drainage flow is accompanied with rapid or burst-like invasion events, which are reflected in sharp pressure drops. This phenomenon is called Haines jumps. Several works studied Haines jumps in 2D micromodels [4-6]. These events were also observed experimentally in 3D models of porous media using fast-synchrotron X-ray computed tomography [7]. Unfortunately, temporal resolution of this method is limited and about 10-30 seconds, whereas the
pore-scale events occur in millisecond time-scale. Thus, the numerical simulation is the only method for investigation of pore-scale events in 3D.

The aim of this study is to make an association between pressure drop and direct imaging of Haines jump in 3D digital model of natural sandstone, which was extracted using X-ray computed tomography. Also, we investigate the effect of flow rate and surface tension on statistical distribution of pressure drops.

2. Materials and methods

2.1. Mathematical model. Lattice Boltzmann equations

The paper considers a two-phase isothermal flow of immiscible incompressible liquids in porous medium. The lattice Boltzmann equations are used to describe the flow of liquids. This mathematical model is described in many works [8, 9], so we will not give a detailed mathematical description of it, but dwell only on the basic assumptions.

In the framework of LBE, the flow of a medium is considered as dynamics of a particles ensemble with a given finite number of possible velocities. The flow domain is represented by grid with a square or cubic cell shape. The set of these form a lattice. During a time step Δt particles without interaction with each other can make one act of transition between adjacent lattice sites. One-particle distribution functions \( f(r, u, t) \) are used to describe the state of the system at each grid node. This function shows the part of particles at time \( t \) located in the vicinity of point \( r(x,y,z) \) with coordinates from \( x \) to \( x+\Delta x \), from \( y \) to \( y+\Delta y \), from \( z \) to \( z+\Delta z \) and with velocity range from \( u(u_s, u_y, u_z) \) do \( u(u_s+\Delta u_s, u_y+\Delta u_y, u_z+\Delta u_z) \).

For a two-dimensional flow domain, a discrete set of velocities D2Q9 is given as follows: \( e_1 = c(0,0,0); e_2 = c(1,0,0); e_3 = c(-1,0,0); e_4 = c(0,1,0); e_5 = c(0,-1,0); e_6 = c(0,0,1); e_7 = c(0,0,-1); e_8 = c(1,1,0); e_9 = c(-1,1,0), e_{10} = c(1,-1,0); e_{11} = c(-1,-1,0), e_{12} = c(1,0,1); e_{13} = c(-1,0,1), e_{14} = c(1, 0, -1); e_{15} = c(-1,0,-1), e_{16} = c(0,1,1), e_{17} = c(0,-1, 1); e_{18} = c(0,1,-1), e_{19} = c(0, -1,-1), \) where \( c = ∆l / ∆t \) – the lattice speed, \( ∆l \) – the grid spacing. Each velocity vector from a given set \( e_i \) (i=1..19) corresponds to the distribution function \( f_i(r,t) \), depending only from \( t \) and \( r \).

The dynamics of the particles ensemble of each fluid is described in several stages. The first is called streaming step. At this stage, the particles move to neighboring nodes during time \( ∆t \), and the direction of the motion speed does not change. The second stage deals with the collision process of particles, as a result of which the distribution function tends to the equilibrium state. The third stage describes the interface interaction between two fluids and between fluids and the solid phase. The distribution function evolution for each fluid in time and space is described by equation (1):

\[
f_i^k (r + e_i ∆t, t + ∆t) = f_i^k (r, t) + (Ω_i^k (r, t) + 1) - 2\mu^k (r, t)
\]

(1) 

\( k = 1, 2 \) – indicates fluid type. \( (Ω_i^k)^4 \) is a collision operator. There are two main models, which describe collision of particles – Single Relaxation Time (SRT) [8, 9] and Multi Relaxation Time (MRT) [10]. The results obtained with MRT model use have a more exact match when comparing with known analytical solutions [10] (Couette flow) than SRT model. So, that is why in our work we use MRT approach.

The relaxation coefficient \( τ^k \) is the determining parameter in the SRT and MRT models and controls kinematic viscosity according to the equation (2):

\[
μ^k = \left( 2τ^k - 1 \right) ∆t^2 / 6
\]

(2) 

In contrast to the classical equations of the fluid and gas mechanics (for example, the Navier-Stokes equations), where the solution is sought in the "pressure-velocity" variables, the LBE are
solved in the "density-velocity" variables. The macroscopic density and the velocity components of fluids mixture in the cell are calculated using (3) and (4) formulas, respectively:

\[ \rho^k(r,t) = \sum_{i=1}^{19} f_i^k(r,t) \]  (3)

\[ u(r,t) = \frac{1}{\rho} \sum_{k=1}^{2} \sum_{i=1}^{19} e_i f_i^k(r,t) \]  (4)

The pressure \( p^k \) in LBE, which is produced by each fluid, is related to its density by the following relation: \( p^k = \rho^k c^2 / 3 \) [8].

The phenomena at the interface of fluids are described using the color-gradient method [11]. It consists of several types:
1) calculation of the color field gradient \( g \), which components are calculated by the formula:

\[ g(r,t) = \sum_{i=1}^{19} e_i (f_i^1(r + e_i \Delta t, t) - f_i^2(r + e_i \Delta t, t)) \]  (5)

Traditionally, one fluid is red, and the second is blue;
2) description of the surface tension effects on the interface between fluids:

\[ (\Omega^k)^2 = \frac{A}{2} |g| (2 \cdot \cos^2 (\alpha_i) - 1) \]  (6)

where parameter \( A \) controls surface tension \( \sigma \), \( \alpha_i \) is an angle between \( g \) and \( e_i \) direction.
3) «recoloring» step – modification of \( f_i^k \) after equation (1) solving:

\[ (f_i^1)^* = \frac{\rho^1}{\rho} f_i + \beta \frac{\rho^1 \cdot \rho^2}{\rho^1} f_i^{eq} \cdot \cos(\alpha_i), \quad (f_i^2)^* = \frac{\rho^2}{\rho} f_i - \beta \frac{\rho^1 \cdot \rho^2}{\rho} f_i^{eq} \cdot \cos(\alpha_i), \]

where \( \rho = \rho^1 + \rho^2 \), \( f_i = f_i^1 + f_i^2 \), \( f_i^{eq} \) - equilibrium function distribution [8], calculated at a density \( \rho \) and zero velocity. The parameter \( \beta \) influences on thickness of the fluid interface. In this paper, its value is 0.8 and can not exceed 1.

On the solid and external impermeable boundaries of the flow domain we use "bounce back" conditions [8, 9]. All numerical simulations are carried out at creating of a pressure gradient along OX axis. The non-wetting fluid is injected through the input boundary and wetting fluid flows through the output section. On the input boundary section the normal component of the velocity of injected fluid is known, and on the output – pressure of fluids mixture. Tangential to the input and output boundaries components of fluids velocities are equal to zero. In LBE, such conditions are given by Zou and He equations [12].

2.2. Image processing and sample description

The digital model of pore space was formed on the base of X-ray computed tomography. The scanning of the sample performed on a micro-/nanofocus X-ray monitoring system for computed tomography and 2D inspection GE Phoenix v|tome|x s 240. All measurements were performed by using a microfocus X-ray tube. Optimal values of the X-ray tube amperage and voltage were selected, which ensure the best contrast of the image. 3D distribution of the X-ray radiation attenuation linear coefficient in the volume of investigated sample was computer reconstructed after two-dimensional shadow projections processing obtained at various rotation angles of the sample \( (0^\circ < \phi < 360^\circ) \) during scan procedure. Preliminary computer processing, segmentation and analysis of the digital...
cores geometric characteristics were made in the Avizo Fire Edition program (Visualization Sciences Group).

In this paper, sandstone from Vostochno-Birlinskoe oil field (Ulyanovsk region, Russia) was used. The rock component of samples is polymineral and consists, in the main, of quartz, and also contains feldspars, dolomite, siderite, calcite, illite, kaolinite, clay minerals and hematite. We selected a cubic core with size of 3.7 millimeters for which X-ray tomography scanning was performed. The resolution of digital model was 4.0 μm. All numerical simulations were carried out in fragments with dimensions of 300×150×150 cells. The porosity of this fragment is 0.273 and the absolute permeability coefficient \( k_{XX} = 0.645 \) μm². Figure 1a shows the X-ray CT image in gray scale. Figure 1a illustrates a porous structure of model which was extracted after binarization. The gray color scale in figure 1a characterizes the X-ray radiation attenuation intensity by different areas of the sample: light gray regions are the granules of sandstone, and black represents the pore space. The binarized image in our method is viewed as a text file with (x, y, z) coordinates of the voxels, and with a pointer for each cell, for example, "1" for the pores and "0" for the skeleton.

![Figure 1](image1.png)

**Figure 1.** Digital X-ray CT model of sandstone with resolution of 4 μm and size of 300×150×150 cells: a – 3D-image in gray scale; b – porous space extracted after image binarization.

3. Results and discussion

3.1. Haines jumps observation

In this section, we report results of computational experiment which was carried out at \( u = 0.2\times10^{-3} \) m/s, \( \sigma = 20 \) mN/m. The viscosities of both fluids were \( 1\times10^{-6} \) m²/s. Figure 2a illustrates the dynamics of pressure difference between inlet and outlet sections during drainage. According to shown data, events of sharp pressure drops and growth are observed. The invasion event which occurs at pressure drop is a Haines jump. Let’s consider in detail a period between 0.248 and 0.252 injected pore volume (figure 2a), which corresponds to sharp pressure drop and make an association with pore-scale invasion event.
Figure 2. The dynamics of pressure difference at drainage displacement: a – total simulation time; b – period between 0.248 and 0.252 injected pore volumes is focused.

Figure 3a and 3b illustrate a non-wetting phase distribution in «event 1» and «event 2» (figure 2b). The region occupied by non-wetting phase between «event 1» and «event 2» is shown in yellow in red box in figure 3b. Figures 3c and 3f are focused on invasion event shown in red box. The localization of liquids interface in narrow pore throat, which will move in subsequent time, is shown in figure 3c («event 1» in figure 2b). The reason of drainage event in this region is in satisfying of two conditions: the pressure difference between input and output sections of the sample exceeds meniscus capillary pressure in current pore; the interface is located in narrow pore throat and moves to pore body at sharp increasing of pore channel size. Such character of interface migration is reflected and associated with quick drop of capillary pressure.

Figure 3d illustrates the snapshot at «event 2». It can be noted that observed pore invasion event at a drainage front displacement occurs rapidly and looks like a burst-like flow. At measurement of medium interface velocity at Haines jump, shown in figure 3, it was revealed that its magnitude is nearly 0.1 m/s. This value exceeds bulk flow rate about 500 times. Simultaneously, there is no violation of mass and momentum conservation laws, because all invasion events occur only in single pore channel.

Increase of pressure difference after Haines jumps is associated with interface movement in pores with narrowing sizes. Also, at imaging of invasion events it was noted that increasing of pressure difference leads to “switching” of mobile interface. The displacement will occur in new regions, where capillary pressure is minimal. According to figure 2a, such changing of displacement regions is the distinguishing feature of two-phase flows at capillary forces domination.
Figure 3. Distributions of non-wetting phase during pore-scale drainage events: a, b – displacement process between «event 1» and «event 2» (see figure 2); c, d – focusing on these events shown in red box.

3.2. Haines jumps statistics
In this section, we investigate the influence of injected fluid velocity and surface tension on statistical distribution of pressure drops at Haines jumps.

Firstly, a series of numerical simulations at various fluid velocities was carried out. Their magnitudes were 0.1, 0.2, 0.4, 0.8 and $1.5 \times 10^{-3}$ m/s. All simulations were conducted before the breakthrough of injected fluid through the outlet section. The surface tension was 20 mN/m. The viscosities of both fluids were $1 \times 10^{-6}$ m$^2$/s. It was found that amount of pressure drop events significantly decreases at flow rate more than $0.2 \times 10^{-3}$ m/s. At lowest flow rates, 110 and 100 pressure drop events were observed, whereas at the highest fluid velocity their amount was only 37. At association with invasion events at direct imaging of non-wetting-phase distribution it was revealed that at flow rates less than $0.2 \times 10^{-3}$ m/s drainage displacement occurs at migration of only one interface, whereas at higher flow rates invasion events are observed simultaneously in several pores, and the amount of pore channels increases with the growth of fluid velocity. Haines jumps at high flow rate occur simultaneously and individual jumps can’t be distinguished on pressure signal. Figure 4 illustrates the distribution of individual Haines jumps at fluid velocities 0.1 and $0.2 \times 10^{-3}$ m/s. According to shown data, the distributions obey the lognormal law. The mean values are 0.160 kPa
and 0.164 kPa. It can be concluded that at flow regimes at which only individual jumps occur, the magnitude of flow rate affects pressure drops distributions slightly. Also it was observed that increasing of flow rates leads to decreasing of mean value of pressure drops. Its value changes from 0.160 kPa for lowest flow rates to 0.116 kPa for the highest fluid velocity. On the other hand, the interpretation of experiments at high flow rates is difficult due to simultaneous Haines jumps.

![Figure 4](image.png)

**Figure 4.** Distribution of pressure drops at Hanes jumps calculated at flow rates 0.1 and 0.2×10⁻³ m/s. Solid lines are the result of fitting using lognormal law.

Secondly, a numerical experiment at \( \sigma = 10 \) mN/m and \( u = 0.2 \times 10^{-3} \) m/s was carried out. Unfortunately, in framework of the LBE surface tension is accompanied with spurious currents on interface. Further increasing of surface tension (more than 20 mN/m) leads to unstable work of the program. The distributions of pressure drops at Haines jumps calculated at 10 and 20 mN/m are illustrated at figure 5. According to results, the amount of individual Haines jumps at surface tension decreasing changes from 100 to 83 events. Weakening of interfacial interaction leads to decreasing of mean pressure jump from 0.164 kPa to 0.063 kPa which is also reflected in figure 5.

![Figure 5](image.png)

**Figure 5.** Distribution of pressure drops at Hanes jumps calculated at surface tensions 10 and 20 mN/m. Solid lines are the result of fitting using lognormal law.
4. Conclusions
In this paper, the Haines jumps events were studied. At association between pressure drops and direct imaging of invasion event in 3D model of natural sandstone it was revealed that Haines occur at meniscus migration from narrow pore throat to its wide body. The velocity of interface movement exceeds bulk flow rate in more than 100 times. That is why, Haines jump looks like burst-like displacement. It was shown that increasing of flow rates leads to simultaneous Haines jumps which are not distinguished at controlling of pressure difference. At regimes, where only individual Haines jumps occur, flow rate affects insignificantly on pressure drop statistics. At investigation of surface tension effect on Haines jump statistics it was revealed that growth of interface interaction magnitude leads to increase of the main value of pressure drop. It was shown that distributions of pressure drops can be approximately fitted with lognormal law.

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