Magnetoresistance in semiconductor structures with hopping conductivity: effects of random potential and generalization for the case of acceptor states

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Abstract

We reconsider the theory of magnetoresistance in hopping semiconductors. First, we have shown that the random potential of the background impurities affects significantly preexponential factor of the tunneling amplitude which becomes to be a short-range one in contrast to the long-range one for purely Coulomb hopping centers. This factor to some extent suppresses the negative interference magnetoresistance and can lead to its decrease with temperature decrease which is in agreement with earlier experimental observations. We have also extended the theoretical models of positive spin magnetoresistance, in particular, related to a presence of doubly occupied states (corresponding to the upper Hubbard band) to the case of acceptor states in 2D structures. We have shown that this mechanism can dominate over classical wave-shrinkage magnetoresistance at low temperatures. Our results are in semi-quantitative agreement with experimental data.

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I. INTRODUCTION

The problem of magnetoresistance in the hoping transport was addressed decades ago. In particular, an interest to this topic was related to important additional information provided by corresponding experiments (including estimates of the localization length). The most general and natural mechanism of positive magnetoresistance of orbital nature is related to shrinkage of the localized wave function by magnetic field; it was extensively reviewed in [1]. Then another important mechanism of orbital magnetoresistance was considered by Nguyen, Shklovskii and Spivak (for the review see [2]). It is related to a presence of under-barrier scattering of hopping electrons by intermediate hopping sites and to interference between different hopping trajectories. Note that for the effective interference the difference of lengths of different trajectories should not exceed the localization length which restrict the location of the trajectories to so-called ”cigar region”. A significance of this mechanism was emphasized by the factor of exponentially-broad scatter of hopping probabilities corresponding to different ”hopping resistors”. As a result of ”logarithmic averaging” over different configurations the most important role is played by those interference patterns where the total hopping probability almost vanishes as a result of the destructive interference. The magnetic field suppresses the interference and thus the average effect is negative magnetoresistance which appears to be linear at weak magnetic field (although becoming quadratic at $H \rightarrow 0$).

An important features of the approach discussed in [2] were as follows. First, the authors exploited an assumption of a presence of many intermediate scatterers. Second, following the theory [3], the authors assumed the preexponential factor to be equal to $\mu/r$ where $\mu$ is scattering amplitude, $r$ is a distance between the hopping site and the scattering center. The picture of interference magnetoresistance considered in [2] was very rich including a change of the sign of magnetoresistance, effects of spin glass etc.

Somewhat later the problem was also discussed in [4], [5] where it was noticed that in realistic situations the number of intermediate scatterers is small and most probably equal to one or (in average) even less. Another important ingredient of the paper [5] was a usage of wave functions typical for Coulomb centers which have not contained preexponential depending on $r$. In contrast to the ”scattering states” of [2] which contained preexponential factors decaying with $r$, this situation can be specified as ”strong scattering case”. Note that, although in [5] the authors considered 2D hopping, they addressed to the case of
delta-doped layer and thus the asymptotic of the wave functions was similar to 3D. The important result of theory suggested in [5] was the following: the patterns of the negative magnetoresistance were almost universal predicting the maximum value of $\sim 0.6$ of the total resistance, and even the combination of the negative magnetoresistance and positive wave-shrinkage magnetoresistance gave the maximum value of negative peak (with respect to average resistance) of around 40 percents.

Unfortunately, these predictions for "strong scattering case" were not in a good agreement with experiment. First, in most of experimental studies the effect of negative magnetoresistance have not exceeded 10 percents and typically was around several percents. Then, it was shown [6] that in 3D semiconductors the negative magnetoresistance is suppressed with a decrease of temperature after the crossover from Mott-type hopping (at higher temperatures) to Efros-Shklovskii hopping over the states within the Coulomb gap. In the paper [6] we explained such a behavior as a result of a decrease of concentration of the scattering centers within the Coulomb gap. However our calculations were based on the assumption that the preexponential factor of the wave functions asymptotic corresponded to scattering states of [2] ("weak scattering case") rather than to hydrogen-like asymptotics exploited in [5]. Later [7] we have also demonstrated that to fit the experimental data one should also take into account spin mechanisms of magnetoresistance. The first one, considered in [8], is based on the fact that the intermediate scatterer should be occupied to produce a negative scattering amplitude. Thus the interference depends on the mutual orientation of the spin of the hopping electron and of the spin of scattering center. Without external magnetic field only one half of the configurations gives an interference. In magnetic field all localized spins are aligned which increases the role of interference and, correspondingly, leads to an increase of resistance.

Another spin mechanism of positive magnetoresistance was first considered in Ref. [9] and then was studied in detail in [10]. It is related to a presence of doubly-occupied hopping sites (corresponding to the upper Hubbard band). Due to spin correlations on these sites requiring $s$-pairing of the spins (recall that we consider here electron rather than hole hopping) some hopping transitions are suppressed in magnetic field (like ones from single-occupied site to single-occupied site).

As it was mentioned above, the incorporation of all of the relevant factors allowed us to reach a quantitative agreement between the theoretical model and experimental data.
However basing the scattering state asymptotic we exploited an assumption of correlated impurity configurations which had no solid theoretical prove.

Another important request to the theory of hopping magnetoresistance was related to 2D hopping. As we have mentioned above, the theoretical model of [5] exploited 3D localized wave functions which do not hold for typical experiments for doped quantum wells where the wave functions have 2D character. Then, we should mention a new important experimental results [11],[12] obtained for selectively-doped quantum well structures where both centers of the wells and centers of the barriers were doped ensuring a formation of the upper Hubbard band. These structures demonstrated a suppression of negative magnetoresistance with a decrease of temperature for the samples with higher degree of doping. Although we attempted to explain this behavior in a similar way as for 3D structures in [6], it hardly works because of an important difference between 2D and 3D physics.

In what follows we will give a consistent description of magnetoresistance in both 3D and 2D structures including different orbital and spin mechanisms. An important conclusion of ours is that in most occasions one deals with a "weak scattering case" rather than with "strong scattering case". If we are restricted to the lower Hubbard band, the decisive factor is related to the presence of charged centers outside of the "cigar region" not involved into interference. The random potential imposed by these centers restricts the extension of the hydrogen-like asymptotics of the scattering centers up to the distance to the closest charged center while outside this region the preexponential of the asymptotics appears to be similar to the one for the potential well case ("weak scattering limit"). For the case of the states within the upper Hubbard band an additional factor is related to the non-Coulombic potential of the scattering center which is also of a short-range character. The resulting picture of hopping magnetoresistance appears to be different from the one suggested in [5] (based on the pure Coulomb wave functions) and from the one of [2] (exploiting the assumption of large number of intermediate scatterers). We also emphasize a role of spin mechanisms of positive magnetoresistance which can dominate over wave-shrinkage magnetoresistance at low temperature. In this concern a special analysis is given to spin mechanisms for acceptor centers which have an important differences with respect to the earlier discussed case of donor impurities.
II. NEGATIVE MAGNETORESISTANCE IN 3D CASE.

Let us consider negative magnetoresistance in 3D case. As it was mentioned above, an important ingredient to be included with respect to the previous studies is a random potential imposed by the intermediate charged centers (including both donors and acceptors).

We shall start from a solution of a Schrodinger equation

\[
-h^2 \frac{\Delta \Psi}{2m} + U_0(r)\Psi + U(r)\Psi = E\Psi. \tag{1}
\]

Here \(U_0(r)\) is the potential of impurity (\(U_0 = -\alpha/r\) in the case of hydrogen-like impurity level) and \(U(r)\) is random potential that comes from the charged centers mentioned above, \(m\) is the electron mass in the conduction band (or a hole mass in valence band) and \(E\) is exact electron energy (we consider \(|E| \gg U(r)\)).

Because of the fact that typical hopping lengths are much larger than characteristic localization length \(a\), we can solve (1) at \(r \gg a\). Moreover, the typical hopping length \(r_h\) appears to be much larger than typical distance between charged centers which can be roughly estimated as \(n^{-1/3}\) where \(n\) is a dopant concentration. Indeed, for 3D variable range hopping of the Mott type

\[
r_h = \xi a, \quad \xi = \left(\frac{T_0}{T}\right)^{1/4}, \tag{2}
\]

where

\[
T_0 \simeq \frac{21}{g\alpha^3}, \tag{3}
\]

where \(g \sim n/E_B\) is the density of states, \(E_B\) being Bohr energy. Thus one obtains

\[
r_h n^{1/3} \sim \left(\frac{21an^{1/3}E_B}{T}\right)^{1/4} \tag{4}
\]

that is even for the Mott law \(r_h n^{1/3} \gg 1\). The more so it holds for the Coulomb gap regime where \(r_h\) strongly exceeds the corresponding values for the Mott regime.

Thus we are interested in asymptotics of the wave functions at distances much larger than \(n^{1/3}\) which for moderately compensated material can be considered as the correlation length of the random potential imposed by the charged centers. If so, we can make an important conclusion. Namely, the random potential \(U\) is formed by the long-range Coulomb centers and in this sense the potential produced by the "parent" (for the considered wave function) impurity at distances larger than \(n^{-1/3}\) makes no difference with respect to potential produced by other charged centers. In other words, for \(r > n^{-1/3}\) one should not discriminate
between $U$ and $U_0$ and should assume that the resulting potential $U$ has a spatial average equal to zero.

Having in mind that $r >> a$ we approach the problem of the asymptotics of the wave function by means of the WKB method. We introduce the function $\phi$ with an account of the normalization factor for the function $\Psi$ as: $\Psi = (\pi a^3)^{-1/2} \exp(-\phi/\hbar)$. The Schroedinger equation in 3D case leads to

$$\phi'^2 - \hbar (\phi'' + 2\phi'/r) = 2m(U(r) + |E|). \quad (5)$$

We will expand this equation into series with respect to $\hbar \to 0$. In zero order we have

$$\phi_0 = \sqrt{2m(U + |E|)} \quad (6)$$

This order gives us the exponent. To get the pre-exponent factor we should use the first order of perturbation theory. Here we have in mind that the function $\phi_0'$ at large $r$ is actually a constant - the more so that its linear expansion in $U(r)$ is averaged out.

$$\phi_1 = \frac{\hbar}{r} \quad (7)$$

Accordingly the expression for $\phi$ up to the first order is

$$\phi = \int \sqrt{2m(U + |E|)} dr + \hbar \ln r/r_{min}. \quad (8)$$

where $r_{min} \sim n^{-1/3}$. And finally the wavefunction $\Psi$ is

$$\Psi = \exp \left( -\frac{1}{\hbar} \int \sqrt{2m(U + |E|)} dr \right) \frac{r_{min}}{r(\pi a^3)^{1/2}} \quad (9)$$

Having in mind the considerations given above, we can average

$$\frac{1}{\hbar} \int \sqrt{2m(U + |E|)} dr = kr$$

where

$$k = (1/\hbar) \left\langle \sqrt{2m(U + |E|)} \right\rangle.$$

Note that $k$ differs from $k_0 = \sqrt{2m|E|}/\hbar$ only in second order of $U/|E|$ $(* \propto U^2/E^2)$, as the mean value $\langle U \rangle$ is zero. Also we neglect $U$ in the pre-exponent factor.

So at distances from the scattering center larger than the correlation length of the random potential (assumed to be equal to average distance between the charged centers) the
wave function asymptotics has a preexponential factor $\propto r^{-1}$ which agrees with the scheme exploited in [2], [6], [7] for 3D hopping.

Now, following approaches [2], [6] let us estimate the hopping probability between the sites 1 and 2 in a presence of intermediate "scattering center" with an account that the energies of the centers obey a relation $|E_3| \gg |E_1|, |E_2|$ as

$$P \propto |J_1 + J_2|^2, \quad J_1 = I_{12}, \quad J_2 = \frac{I_{13}I_{32}}{|E_3|}. \quad (10)$$

Here $J_1$ and $J_2$ are hoping amplitudes related to direct and scattered path correspondingly. Note that the destructive interference (leading to negative magnetoresistance) implies that $E_3 < 0$ which means that in the equilibrium the scattering site is occupied.

The energy overlapping integrals are given as

$$I_{ij} = \mathcal{E}_B \frac{r_{\text{min}}}{r_{ij}} \exp(-r_{ij}/a) \quad (11)$$

where we have assumed that $r_{ij} > r_{\text{min}} = n^{-1/3}$; $\mathcal{E}_B$ being the Bohr energy. Without a magnetic field this amplitudes are real. Though in the magnetic field their phases are different and hoping probability is

$$P \propto |J_1 + J_2 e^{i\varphi}|^2. \quad (12)$$

Here phase difference $\varphi$ is equal to $\varphi = 2\pi \Phi/\Phi_0$, where $\Phi$ is the magnetic flux through the surface bounded by hoping paths. $\Phi_0$ is the elementary magnetic flux. Accordingly, the interference magnetoresistance for the situation $\varphi < 1$ can be given as

$$\ln \frac{r(H)}{r(0)} \propto -\left\langle \int dE_3 g(E_3) \int \ln \left[ 1 + J_1(J_1 - J)\frac{\varphi^2}{J_2^2} \right] d^3r_3 \right\rangle, \quad (13)$$

Here $J = J_1 + J_2$, $g$ is the density of states and $r_3$ is scatterer position. Angle brackets corresponds to the ensemble average. We consider magnetoresistance to be determined over hops with small $J$, so we neglect the term $J_1/J$ in (13) and get

$$\ln \frac{r(H)}{r(0)} \propto -\left\langle \int dE_3 g(E_3) \int \ln \left[ 1 + \frac{J_1^2\varphi^2}{J_2^2(r_3, E_3)} \right] d^3r_3 \right\rangle. \quad (14)$$

To obey $J_1 \simeq J_2$ one, first, should have $r_{12} \simeq r_{13} + r_{23}$ with an accuracy of the order of the localization length $a$. Then, having in mind the preexponential factors one notes that for small $r_{\text{min}}$ the only possibility to obey the relation is to have one of the distances, $r_{13}$ or $r_{23}$ to be small. We will assume that it holds for $r_{23}$ which is estimated as

$$r_{23} \sim r_{\text{min}} \frac{\mathcal{E}_B}{E_3}. \quad (15)$$
Let us choose the surface at which \( J = 0 \) and transform an integration over \( r_3 \) in a way
\[
d^3r_3 \rightarrow d^2RdR_{\perp}
\]
where \( R \) is the coordinate on the surface in question while \( R_{\perp} \) is a coordinate along the normal to the surface where we assume that \( R_{\perp} = 0 \) corresponds to \( J = 0 \). In the lowest order in \( R_{\perp} \) we have \( J = (dJ/dR_{\perp})R_{\perp} \). As it is seen, the integration of the logarithm term over \( R_{\perp} \) gives
\[
\frac{\varphi J_1}{dJ/dR_1}
\]
Finally, the integration of the factor
\[
\frac{J_1}{dJ/dR_{\perp}}
\]
over \( d^2R \) approximately gives a volume accessible for the site 3. Note that we have \( r_{13} + r_{23} \leq r_{12} + a \), thus the projection of \( r_{23} \) to the plane normal to \( r_{12} \) should be less than \( (r_{23}a)^{1/2} \). As a result, the integration over the spatial coordinate \( r_3 \) gives
\[
\sim \frac{r_{23}^2E_{3}^{2}}{E_{3}^{2}}a\varphi
\]
In its turn, the area of the interference loop (entering the estimate of \( \varphi \)) is
\[
r_{12}\left(\frac{r_{23}E_{3}}{E_{3}a}\right)^{1/2}
\]
Note that these estimates actually hold for all accessible values of \( r_{23} \) up to \( r_{23} \sim r_{12}/2 \).

The final result depends on the behavior of \( g(\varepsilon) \). For \( g = const \) (Mott-type hopping) the integration over \( E_3 \) is naturally controlled by the lower limit which accordingly to Eq. \ref{eq:15} corresponds to the larger possible value of \( r_{23} \sim r_{12}/2 \). In this case the r.h.s. of Eq.\ref{eq:13} is \( \propto r_{h}^{5/2} \) where \( r_{h} \sim r_{12} \).

In contrast, for the Coulomb gap hopping the integration over \( E_3 \) is controlled by the upper level, \( E_C \), corresponding to the edge of the Coulomb gap. In this case r.h.s. of Eq.\ref{eq:13} is \( \propto r_{h} \) since the value of \( r_{23} \) does not depend on \( r_{h} \).

Now let us consider a combination of NMR with a positive magnetoresistance related to wave function shrinkage which can be estimated as
\[
\ln \frac{\rho(H)}{\rho(0)} = \left(\frac{H}{B}\right)^2
\]
where
\[
B^2 = \frac{\alpha e^3}{r^3_h a e^2}
\]
Here \( \alpha \) is a numerical parameter resulting from the percolation theory; for Mott type hopping \( \alpha \sim 400 \) while for the Coulomb gap hopping different sources give \( \alpha \sim 300 \) and \( \alpha \sim 700 \).

In its turn, NMR can be rewritten as
\[
\ln \frac{\rho(H)}{\rho(0)} = k \frac{H}{B} \tag{20}
\]
where
\[
k = g_M E_B r_{min} r_h 2a \alpha^{1/2} \quad \text{Mott law}
\]
\[
k = \frac{k^3 r_\Delta^{5/2}}{e^6 r_h^{1/2} a E_C^2 \alpha^{1/2}} \quad \text{ES law.} \tag{21}
\]
Here \( \kappa \) is the dielectric constant, \( r_\Delta \) is the typical hopping length for the states corresponding to the edge of the Coulomb gap while \( E_C \) is a width of the Coulomb gap. One sees that as a result we have minimum of resistance,
\[
H_{min} = k \frac{2}{B}, \quad \ln \frac{\rho(H_{min})}{\rho(0)} = -\frac{k^2}{4} \tag{22}
\]
It is seen that the value of \( H_{min} \) decreases with a temperature decrease irrespectively to the type of the variable range hopping. At the same time for samples corresponding to Mott law the absolute value of resistance in minimum increases with a temperature decrease while for the case of the Coulomb gap hopping it decreases with temperature decrease.

III. NEGATIVE MAGNETORESISTANCE IN 2D.

Let us now approach the problem of negative magnetoresistance in the 2D structure where impurity wave functions are quantized in the orthogonal to impurity plane direction. First we will consider the case when we deal only with single occupied or empty impurity centers (as it was done above for 3D case).

Let us start with approximation of 2D impurity wave function in the \( r \gg a \) limit. Analogously to previous case we neglect \( U_0(r) \) and introduce function \( \phi \) as \( \Psi = (\pi a^2)^{-1/2} \exp(-\phi/h) \). The corresponding WKB equation is
\[
\phi'^2 - \hbar (\phi'' + \phi'/r) = 2m(U(r) + |E|). \tag{23}
\]
Following the same procedure as was applied for 3D case we obtain
\[
\Psi = \exp \left( -\frac{1}{\hbar} \int \sqrt{2m(U + |E|)} dr \right) \left( \frac{r_{min}}{r \pi a^2} \right)^{1/2}. \tag{24}
\]
Analogously to 3D case this wave function is nearly equal to the potential well wave function
\( \propto \exp(-kr)/\sqrt{r} \) where
\( k = \left( \frac{\sqrt{2m|E| + U}}{\hbar} \right) \)
which differs from
\( k_0 = \frac{\sqrt{2m|E|}}{\hbar} \)
only in the second order of \( U/E \).

Now let us consider negative magnetoresistance related to the interference contribution to the hopping probability. Following the same lines as for 3D case we obtain the equation similar to [14] except that the integration is over \( d^2r_3 \) and the density of states \( g \) also corresponds to 2D. An important difference is related to the fact that now the value of \( J \) vanishes at

\[ r_{23} = r_{\text{min}} \left( \frac{E_B}{E_3} \right)^2 \]  

(25)

With a similar transformation of the variables the integration of the logarithmic term over the coordinates gives

\[ \left( \frac{r_{\text{min}} E_B^2}{E_3^2} \right)^{3/2} a^{1/2} \varphi \]  

(26)
while the effective loop area is

\[ r_{12} \left( r_{\text{min}} \left( \frac{E_B}{E_3} \right)^2 a \right)^{1/2} \]  

(27)

Since in 2D in the Coulomb gap regime \( g \propto \varepsilon \) one notes that irrespectively to the hopping law the integration over \( E_3 \) is controlled by the lower possible values of \( E_3 \) leading finally to the estimates of \( r_{23} \sim r_{12} \). Thus one obtains

\[ k_2 = g_M E_B r_{\text{min}}^{1/2} r_h 2a^{1/2} \alpha^{1/2} \quad \text{Mott law} \]
\[ k_2 = \frac{k^2}{e^4} E_B r_{\text{min}}^{1/2} r_h 2a^{1/2} \alpha^{1/2} \quad \text{ES law} \]  

(28)

Thus, at is seen, for the situation considered above in 2D the only combination of interference NMR and wave-shrinkage PMR can not lead to a suppression of NMR with a decrease of temperature (at least for low temperature limit of linear NMR) since for both laws the temperature derivative of \( k \) stays to be negative.

An important feature of the 2D quantum well structure is an easy possibility to have an occupation of the upper Hubbard band. Namely, if we dope not only the well regions, but also the barrier regions, the carriers from the barriers are captured by the wells and can form doubly occupied states. It was this situation which was realized in our experiments described in [11], [12]. Since in these experiments we dealt with GaAs/AlGaAs structures of quantum wells with p-doping by Be, here we will also imply acceptor centers.
In our experiments the central regions of both wells and barriers were nearly equally doped by acceptor impurity Be. Thus the holes from the barriers have a possibility to occupy the second position for the acceptor in the wells forming the upper Hubbard band. However for the hole there was another possibility - to stay around its native acceptor in the barrier forming single-occupied center which we will denote as $\tilde{A}^0$. The corresponding scenario was first discussed in [13]. As a result, at the Fermi level we have centers with different occupation numbers - at least, $A^+$ (doubly occupied), $A^0$ (single occupied) $\tilde{A}^0$ (holes bound to the barrier acceptor) and $A^-$ (empty barrier acceptor with no hole around).

The possibility for the hole to form $A^+$ or $\tilde{A}^0$ center depends on relation between the binding energies of these centers, $U_b$ and $\tilde{U}_b$. In particular, if $U_b > \tilde{U}_b$, then all the barrier acceptors form $A^-$ centers while all the acceptors in the well form $A^+$ centers. However for our experiments of the distance between the barrier acceptor and the interface between the barrier and well was not large and we expect $\tilde{U}_b > U_b$. In this case the probability to form $A^+$ center depends on the distance between the barrier acceptor and the closest acceptor in the well. Indeed, the formation of $A^+$ center profit from the interaction between $A^+$ center and $A^0$ center [13].

Here we assume that some holes from the barrier are still coupled to their parent acceptors ($\tilde{A}^0$ centers) and some are localized on the acceptors in the well ($A^+$ centers). According to charge conservation the number of $\tilde{A}^-$ centers (that are free $\tilde{A}^0$ centers) is equal to the number of $A^+$ centers.

$$N(A^+) = N(\tilde{A}^-).$$ (29)

In addition, we believe that there exists a random potential that overlap the energies of different types of centers. If the variances of $\tilde{A}^0$ and $\tilde{A}^-$ energies are equal, (29) leads to equal densities of states for $\tilde{A}^0$ and $\tilde{A}^-$ at the Fermi level. For our purpose we assume that this densities of states are at least comparable.

As for the negative magnetoresistance for the upper Hubbard band, it can be considered in the same way as for the lower Hubbard band discussed above. Note that the scattering potential strongly decays with distance $U_0 \propto r^{-4}$ and thus the corresponding asymptotics of the wave functions are similar to the one given by Eq.24 but one should take $r_{\text{min}} = a$. 

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IV. SPIN MECHANISMS OF MAGNETORESISTANCE FOR ACCEPTOR STATES

We shall start from the mechanism of spin magnetoresistance first suggested in [8] which seems to be especially important for acceptor dopants. It is related to the fact that interference can occur only if spin states of the final states for both tunneling paths coincide. For 3-site configuration we discuss it means that the initial and intermediate centers should have the same spin projections (we remind that for destructive interference in question the energy of intermediate center should be negative, i.e. at the equilibrium this center should be occupied). For the case of acceptor states corresponding to the lower Hubbard band the corresponding configuration is in our case $\tilde{A}^0 - \tilde{A}^0 - A^-$ where the role of intermediate center is played by the site $\tilde{A}^0$. Since the hole has spin $3/2$, we have 4 projections of the spin and thus the probability for two sites $\tilde{A}^0$ to have the same spin projections is $P(H = 0) = 1/4$. However in strong magnetic field the site spins are aligned and spin does not affect the (destructive) interference, that is in this case $P(H \to \infty) = 1$. Thus an increase of the magnetic field leads to an enhancement of destructive interference which means positive magnetoresistance which was noted in [8].

In a presence of the states representing the upper Hubbard band (in our case of $A^+$ and $A^0$ states) the situation is somewhat more complicated. In particular, it is related to the fact that the spin structure of doubly occupied $A^+$ center is more complex than for single occupied site. In particular, the total spin of $A^+$ center is 2 (see [14]) and the possible spin states of $A^+$ center are the following:

\[
|J = 2, J_z = -2\rangle = \frac{1}{2}\Psi^{(1)}_{3/2}\Psi^{(2)}_{-3/2} - \frac{1}{2}\Psi^{(1)}_{-3/2}\Psi^{(2)}_{1/2} \]
\[
|J = 2, J_z = -1\rangle = \frac{1}{2}\Psi^{(1)}_{1/2}\Psi^{(2)}_{-3/2} - \frac{1}{2}\Psi^{(1)}_{-3/2}\Psi^{(2)}_{1/2} \]
\[
|J = 2, J_z = 0\rangle = \frac{1}{4}\Psi^{(1)}_{3/2}\Psi^{(2)}_{-3/2} + \frac{1}{4}\Psi^{(1)}_{-3/2}\Psi^{(2)}_{-1/2} - \frac{1}{4}\Psi^{(1)}_{-3/2}\Psi^{(2)}_{3/2} - \frac{1}{4}\Psi^{(1)}_{1/2}\Psi^{(2)}_{1/2} \]
\[
|J = 2, J_z = 1\rangle = \frac{1}{2}\Psi^{(1)}_{3/2}\Psi^{(2)}_{-1/2} - \frac{1}{2}\Psi^{(1)}_{1/2}\Psi^{(2)}_{3/2} \]
\[
|J = 2, J_z = 2\rangle = \frac{1}{2}\Psi^{(1)}_{3/2}\Psi^{(2)}_{1/2} - \frac{1}{2}\Psi^{(1)}_{1/2}\Psi^{(2)}_{3/2} \]

where $\Psi^{(1,2)}$ - are the wave functions characterized by given spin projections of the two holes.

Basing on these considerations one can show that for the destructive interference involving
purely the states of the upper Hubbard band, that is for configurations $A^+ - A^+ - A^0$, $P(H = 0) \sim 1/4$ while $P(H \to \infty) = 1$.

If the states of both of the Hubbard bands coexist at the Fermi level, it can be estimated that the average statistical factor $P(H = 0)$ is still of the order of $1/4$, although its value at strong fields, $P(H \to \infty)$ appears to be somewhat smaller than unity.

At weak magnetic fields one expects a degree of spin alignment to be $\propto (\mu g H)^2/T^2$ and thus the statistical factor is equal

$$P(H) \approx P(H = 0) + \alpha \left(\frac{\mu g H}{T}\right)^2$$

(30)

Here the coefficient $\alpha$ according to more detailed statistical calculations which we are going to present elsewhere can be estimated to be of the order of $1/2$.

Since $P$ describes probability of the destructive interference, one concludes that at weak fields the positive magnetoresistance resulting from statistical factor $P$ is quadratic in terms of magnetic field. It can be estimated as follows:

$$\ln \frac{R(H)}{R(0)} = \alpha \left(\frac{\mu g H}{T}\right)^2 \left|\frac{\Delta R_{sat}}{R(0)}\right|$$

(31)

where and $\Delta R_{sat}$ is the saturation value for the interference contribution to resistance with no account of spin degrees of freedom which is achieved when the phase $\varphi$ in Eq.(12) exceeds $2\pi$. As it can be estimated, the ratio $\Delta R_{sat}/R(0)$ is $\propto r_h$ for Mott hopping and $\propto r_h^{1/2}$ for the Coulomb gap hopping.

In its turn, it coexists with linear negative magnetoresistance (of orbital nature) which at relatively weak fields can be estimated as

$$P(H = 0)\Delta R_{sat} \frac{H}{H_{sat}}$$

As it is known, if at the Fermi level the states of the lower and the upper Hubbard bands coexist, there also exists a specific spin mechanism of positive magnetoresistance first considered in [9] (and later discussed in detail in [10]) for n-type 3D structures. In such structures one deals with $D^0$ (occupied donors), $D^-$ (doubly occupied donors) and $D^+$ (empty donors). Without external magnetic field the following configurations of hops are possible: $D^- \to D^0$, $D^0 \to D^0$, $D^0 \to D^+$, $D^- \to D^+$. In the magnetic field the spins of $D^0$ centers are polarized and thus the hops $D^0 \to D^0$ are forbidden (since in the final state of the second site corresponding to $D^-$ the spins should be in opposite directions). In the
same way the transitions $D^- \rightarrow D^+$ are also suppressed. Thus the resistance increases as a result of application of external magnetic field.

In our case of p-type structures the situation appears, again, more complicated due to more complex structure of $A^+$ centers. However, in general, the considerations given in [9, 10] still hold. Basing on the calculations similar to given in [10] one obtains for weak field limit $\mu g H < T$ the following estimate:

$$\ln \frac{R(H)}{R(0)} = CF \left( \frac{g \mu_B H}{T} \right)^2$$

(32)

where $C \sim 1/3$,

$$F = \frac{2 g_l g_u}{(g_l + g_u)^2}$$

(33)

while $g_l, g_u$ are the densities of states of the lower and upper Hubbard bands. Note that for the low concentration of dopants $g_u$ is controlled by the concentration of $A^+$ centers while $g_l$ - by the concentration of $A^-$ centers and thus $g_l = g_u$. At stronger magnetic fields when $\mu g H > T$, the corresponding contribution to magnetoresistance still increases with magnetic field increase until $\mu g H$ reaches the value $\xi T$ and then saturates [9], [10].

One notes that at low enough temperatures the positive magnetoresistance of the spin nature suggested in [9] can exceed the wave shrinkage magnetoresistance. At the same time this contribution at relatively weak fields when $\mu g H < T$ is expected to be comparable to the spin magnetoresistance resulting from interference term discussed above. Summarizing the both spin contributions to quadratic magnetoresistance we estimate the coefficient $k$ resulting from the similar parametrization of the positive quadratic and linear negative magnetoresistance as was done above:

$$k_2 = g_M E_B r_{\text{min}}^{1/2} r_h 2 a^{1/2} \beta$$ \quad Mott law

$$k_2 = \frac{k^2}{e^2} E_B^2 r_{\text{min}} r_h^{1/2} 2 a^{1/2} \beta$$ \quad ES law

$$\beta = P(H = 0) \frac{T}{g \mu_B} \left( \frac{r_h^{3/2} a^{1/2} e}{ch} \right) \left( CF + \alpha \frac{\Delta R_{\text{sat}}}{R(0)} \right)^{-1/2}$$

(34)

Thus, as it is seen, for the Mott case at $T \rightarrow 0$ $k \propto T^{1/3}$ while for the ES case it is $\propto T^{1/4}$.

Note that in our calculations we assumed that the value of $H_{\text{min}}$ still corresponds to linear behavior of negative magnetoresistance which means that the magnetic flux through the interference area is much less than magnetic flux quantum $\Phi_0$. The critical field $H_{\text{sat}}$
corresponding to a crossover from the linear behavior to saturation of the negative magnetoresistance is given as

\[ H_{\text{sat}} \simeq \frac{\Phi_0}{2\pi r_h^{3/2} a^{1/2}} \] (35)

Correspondingly, if \( H_{\text{min}} \) given by Eq.22 appears to be larger than \( H_{\text{sat}} \) our calculations given above are invalid and one should compare positive magnetoresistance with saturated negative magnetoresistance rather than with linear negative magnetoresistance. One notes that in contrast to linear magnetoresistance which is proportional to the area of the interference loop for the saturation magnetoresistance this proportionality is omitted. As a result, as it was noted above, the temperature dependence of the saturation value of negative magnetoresistance \( \Delta R_{\text{sat}}/R(0) \) results from factors \( \propto r_h \) for Mott hopping and \( \propto r_h^{1/2} \) for the Coulomb gap hopping. It is seen that the corresponding increase of the saturation magnetoresistance with temperature decrease is much weaker than increase of the positive magnetoresistance. Then, in the case \( H_{\text{min}} > H_{\text{sat}} \) it is the value of \( H_{\text{sat}} \) which corresponds to minimal resistance since it separates a region of resistance decrease due to negative magnetoresistance and resistance increase due to positive magnetoresistance. However at this situation spin magnetoresistance (31) is also saturated so the temperature behavior of positive magnetoresistance is related to (18) and (or) to (32).

In its turn let us consider the temperature behavior of the relation between \( H_{\text{min}} \) and \( H_{\text{sat}} \). According to Eq.22 and Eq.34

\[ H_{\text{min}} = 2g_M E_B r_{\text{min}}^{1/2} \frac{T^2}{(g\mu_B)^2 CF} \frac{r_h^{5/2} e}{c\hbar} \propto T^{7/6} \quad \text{(Mott law)} \]

\[ H_{\text{min}} = 2\kappa^2 e^4 r_{\text{min}} a \frac{T^2}{(g\mu_B)^2 CF} \frac{r_h^2 e}{c\hbar} \propto T \quad \text{(ES law)} \] (36)

At the same time \( H_{\text{sat}} \propto T^{1/2} \) for Mott law and \( H_s \propto T^{3/4} \) for ES law. Thus the ratio \( H_{\text{min}}/H_{\text{sat}} \) decreases with temperature decrease and this decrease is more pronounced for Mott law.

V. DISCUSSION

At Fig.1 we present our experimental results from Ref. 6 for 3D hopping concerning temperature behavior of magnetoresistance for regimes of Coulomb gap hopping (Fig. 1a) and of Mott-type hopping (Fig. 1b). It is seen that these results are in a qualitative
agreement with predictions of Sec.2. In particular, the minimal value of resistance increases with temperature decrease for Mott type hopping and decreases for the Coulomb gap hopping. As it was noted in the Introduction, the agreement was strongly improved when we had taken into account more subtle spin effects \[7\], however here we will not go into these details discussed earlier.

FIG. 1: Temperature behavior of magnetoresistance for bulk CdTe crystals doped by donor impurities (Cl). Fig. 1a - the curves for the sample in the Coulomb gap regime, Fig. 1b - for the sample in the Mott regime.

At Figs. 2,3 we present experimental results described in \[11\], \[12\] for p-GaAs/AlGaAs:Be
multiple quantum well structures with different dopant concentration $n$. It is seen that for the sample with smaller concentration (Fig. 2) the negative magnetoresistance is strongly enhanced with temperature decrease while for the sample with larger dopant concentration (Fig. 3), in contrast, it is suppressed with a temperature decrease.

FIG. 2: Temperature behavior of resistance for the structures of 10 GaAs wells of thickness 15 nm, separated by AlGaAs barriers with thickness 15 nm. The central parts of both wells and barriers were doped by p-type impurity Be with concentration $1 \cdot 10^{17} \text{cm}^{-3}$.

To our opinion, the difference of magnetoresistance curves for samples with different $n$ is related to the following fact. The sample with smaller concentration is far from the metal-insulator transition and the localization length is relatively small, $a \sim 10 \text{nm}$. Thus, in a view of small $n$ and small $a$ the 3-cite approximation for interference contribution holds, $nr_h(ar_{\text{min}})^{1/2} \leq 1$. For heavily doped sample $a \sim 20 \text{nm}$ and $n \sim 10^{12} \text{cm}^{-2}$, correspondingly, $nr_h(ar_{\text{min}})^{1/2} > 1$. As a result, the 3-cite approximation for this sample does not hold and the interference loop includes large number of scatterers. As it was noted above, the spin statistical factor for each additional site with non-zero spin at $H = 0$ for acceptor impurities is $\sim 1/4$. Correspondingly, the interference contribution for loops involving many intermediate scatterers vanishes at $H = 0$. As a result, the linear contribution to negative magnetoresistance is, in any case, much smaller than for weakly doped samples. In contrast, the quadratic positive magnetoresistance resulting from the statistical factor given by Eq.
FIG. 3: Temperature behavior of magnetoresistance for the structures similar to described at Fig. 2 but with concentration of Be $9 \cdot 10^{17}$ cm$^{-3}$.

strongly increases with temperature decrease,

$$\propto T^{-7/3} \quad (Mott \ law), \quad \propto T^{-9/4} \quad (ES \ law)$$ (37)

In addition, we can expect that for the sample with large $n H_{sat} < H_{min}$, thus it is $H_{sat}$ which plays a role of $H_{min}$. Due to weak temperature dependence of $R_{sat}$ the temperature behavior at the fields larger than $H_{sat}$, that is corresponding to the minimum of $\rho(H)$, is completely controlled by the spin PMR which gives $\rho(H) \propto T^{-\alpha}$ with $\alpha > 2$. Indeed, an increase of resistance by a factor of 4-5 is observed at the fields larger than 0.3 T for the temperature variation from 0.9 to 0.4 K.

VI. CONCLUSIONS

To conclude, we have reconsidered existing theory of hopping magnetoresistance. We have shown that the random potential induced by the background impurities can affect the asymptotics of the localized states and, as a result, suppress to some extent the negative magnetoresistance related to interference effects. We have also generalized the theory for the case of acceptor states in 2D structures including the effects of the upper Hubbard band. The results obtained are in agreement with existing experimental data. In particular, we
explain the suppression of negative magnetoresistance with temperature decrease observed earlier for both 3D and 2D structures.

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