We discuss the effective photonic potential for TM waves in inhomogeneous isotropic media. The model provides an easy and intuitive comprehension of form birefringence, paving the way for a new approach on the design of graded-index optical waveguides on nanometric scales. We investigate the application to nanophotonic devices, including integrated nanoscale wave plates and slot waveguides. © 2018 Optical Society of America

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In several situations light can be described as a scalar field, including e.g. the propagation of wider-than-wavelength beams in isotropic materials. Under this condition, a Schrödinger equation can be used in lieu of the Maxwell’s equations [1]. Nonetheless, the scalar approximation fails even in the presence of discontinuous interfaces between two different isotropic materials, causing for example form birefringence in step-index waveguides [2]. Currently, there is a continuous effort towards the miniaturization of sub-wavelength optical waveguides [3–9]. In fact, one of the main aims of nanophotonics is to shrink optical geometries. We verify the validity of our results by direct comparison with finite-difference time domain (FDTD) simulations, establishing a dielectric constant \( \epsilon \) dependent on \( x \). It is well known that Maxwell’s equations in two dimensions read [2]

\[
\begin{align*}
\partial_z^2 E_y + \partial_x^2 E_y + k_0^2 n^2(x) E_y &= 0, \\
\partial_z^2 H_y + \partial_x^2 H_y + k_0^2 n^2(x) H_y - \partial_x \log \epsilon \partial_x H_y &= 0,
\end{align*}
\]

where \( k_0 = \omega/c \) is the vacuum wavenumber. According to Eqs. (1-2), the electromagnetic field is always polarization-dependent in an inhomogeneous material (even in the simplest case of a slab waveguide), that is, form birefringence is an intrinsic property of the system. In modern terminology, optical waves are subject to an intrinsic spin-orbit interaction [21] proportional to the geometry-dependent term \( \partial_x \log \epsilon \), the latter becoming relevant when the refractive index \( n(x) \) varies appreciably on distances comparable with the wavelength \( \lambda \).

To enlighten the formal analogy with respect to the Schrödinger equation, we write Eq. (1) in the form

\[
\partial_z^2 E_y - V_{TE}(x) E_y = 0,
\]

where \( V_{TE} = k_0^2 n^2(x) \). In the paraxial limit, Eq. (2) closely recalls the Schrödinger equation (setting the equivalent Hamiltonian \( \hat{H}_{TM} = (\hat{p} - \hat{A})^2 + \hat{V}_{TM} \), with \( \hat{p} = -i \hbar \hat{\partial}_x \)) for a massive particle subject to a scalar (V) and a vector potential (A). Indeed, Eq. (2) can be recast as

\[
\partial_z^2 H_y = -\left( \partial_x - \frac{1}{2} \partial_x \log \epsilon \right)^2 H_y - \left\{ k_0^2 n^2(x) - \frac{1}{2} \left( \partial_x \log \epsilon \right)^2 - \frac{1}{2} \left( \partial_x \log \epsilon \right)^2 \right\} H_y.
\]

Hence, the effective vector potential is \( \hat{A} = -i \hbar \partial_x \log n \) [22] and the effective photonic potential is given by [16]

\[
V_{TM} = V_{TE} + \frac{\partial_x^2 n}{n} - 2 \left( \frac{\partial_x n}{n} \right)^2.
\]

Finally, the effective vector potential vanishes if the Weyl-like gauge transformation \( \psi = H_y e^{-i \int A dx} = H_y e^{-\frac{1}{2} \log \epsilon / \epsilon_o} = \frac{H_y}{n(x)} \) is applied [12, 15]. In fact, the z-component of the Poynting vector for the TM wave is \( S = -\frac{\epsilon \mu \hat{H} \cdot \hat{E}}{2} \) thus the derived scalar field \( \psi \) conserves the integral \( P = \int |\psi|^2 dx \), a property fulfilled by any solution of the Schrödinger equation.

Let us start by considering a bell-shaped index well, for example a Gaussian shape \( n = n_0 + \Delta n_0 e^{-x^2/\omega^2} \). The top row in Fig. 1 [23] shows the potential for the TM polarization. Up to \( \omega \approx \lambda \), \( V_{TM} \) is very similar to \( V_{TE} \) and the form birefringence is negligible. For \( \omega / \lambda = 0.1 \), a dip in the center of the effective potential
The position of the maximum form birefringence depends on \( \Delta n_0 \). For example, these results find applications in the investigation of fs-written waveguides [18].

Equation (4) can also be used to design a refractive index distribution \( n(x) \) to provide a desired photonic potential \( V_{TM}(x) \), let us call it \( V_{design} \). For example, \( V_{TM}(x) \) can be designed to minimize the bend losses related with the evanescent tails. Then, Eq. (4) turns into a nonlinear boundary value problem for the profile \( n(x) \), where \( V_{design} \) plays the role of a forcing term. Standard techniques, such as relaxation algorithms and shooting method, can then be applied to find the solution. In Fig. 4, we show the results when the target is a Gaussian potential, i.e., \( V_{design} = V_0 \exp\left(-x^2/\lambda^2\right) + \Delta n_0 \). The corresponding refractive index, computed via a standard shooting method with \( \Delta n_0 = 0.5 \) and \( w/\lambda = 0.1 \).

For wide guides, the form birefringence goes as \( \Delta n_0 \) and \( w/\lambda \), respectively. All the results are computed for \( n_0 = 1.5 \).

\( \Delta n_0 \) and \( w/\lambda \). In the inset the same curves are replotted in log-log scale. In the legend the corresponding \( \Delta n_0 \) are reported. All the results are computed for \( n_0 = 1.5 \).
The solution for $\psi$ plotted in Fig. 4 panel (c) and (d), respectively. For $V_0 = 2$ the overall variation in $n(x)$ is circa 1.5, in turn yielding a form birefringence $n_{TE} - n_{TM} \approx 0.15$, to be compared with the value of 0.05 obtained for $\Delta n_0 = 0.5$ for a TE Gaussian nanoguide (see Fig. 3). FDTD simulations (Fig. 5) confirm that the confinement occurs for both the polarizations, the field profile matching the theoretical predictions. The TE wave undergoes larger coupling losses than the TM polarization, see the solid lines in Fig. 5.

We now use Eq. (4) to investigate the light behavior in the presence of a slot waveguide [3, 19] encompassing a graded-index (GRIN) profile. For the refractive index distribution we make the ansatz $n = n_0 + \Delta n_0 e^{-x^2/\omega^2} - \Delta n_0 e^{-x^2/\omega'^2}$, that is, a Gaussian waveguide (as the one used in Figs. 1-3) with a Gaussian-shaped dip in the center. Thus, we implicitly set $n(x = 0) = n_0$. The corresponding photonic potentials for TE and TM polarizations are shown in Fig. 6(a). The largest differences between TE and TM modes arise around the central hole, due to the significant gradient in the refractive index. Around $x = 0$, the potential for the TM component is dominated by the hole contribution, thus it is fully analogous to Fig. 1, but inverted in sign. The effect of the central spikes in $V_{TM}$ on the eigenmode can be ascertained by comparing $E_y$ [TE case, solid blue line in Fig. 6(b)] with $\psi$ [TM case, dash-dotted red line in Fig. 6(b)]. The solution for $\psi$ shows a sharp sub-wavelength peak of width $\approx \lambda/50$ around $x = 0$. Recalling that $\text{Re}(S \cdot \hat{z}) = |\psi|^2$, the TM mode supports a strong local amplification of the carried energy inside the low refractive index core, an important property for optical tweezers [27, 28], for example. On the other hand, the magnetic field $H_y = n(x) \psi$ features a dip in $x = 0$ [dotted green curve in Fig. 6(b)], the latter being deeper than for the TE mode [solid blue line in Fig. 6(b)]. Finally, the shape of the transverse electric field in the TM case $E_x = H_y/n^2(x)$ [dashed orange line in Fig. 6(b)] is similar to $\psi$, but the prominence of the peak is even stronger than for $\psi$. Noteworthy, the electric field is the quantity to maximize when light-matter interaction needs to be enhanced (supposing a dipolar electric interaction) [29]. The prominence of the electric field spike depends strongly on the lateral extension of the central defect, i.e., $\omega$ in our case. Narrower defects [e.g. $\omega / \lambda = 0.01$ in Fig. 6(c)] yield more prominent peaks [compare with Fig. 6(d) where $\omega / \lambda = 0.1$] owing to the deepest potential $V_{TM}$ (directly determining $\psi$) and the largest jump in the refractive index (through the relationship between $\psi$ and $E_x$). We verified our predictions simulating the light behavior in the slot waveguide by means of FDTD simulations, the comparison being plotted in Fig. 7. For any value of $\Delta n_0$, the electric fields in the TE and TM case encompass an opposite trend: for TM polarizations the electric field is larger in the low-index core than in the larger index adjacent regions, whereas a dip is observed for TE waves.

In conclusion, we used the effective photonic potential for TM waves as a new method to design and analyze nanometric optical waveguides, fully accounting for the intrinsic spin-orbit interaction in the sub-wavelength regime. We applied our findings to the design of waveplates and slot waveguides. Our method finds direct application to the investigation of bend losses via transformation optics [30]. Future generalizations include nonlinear effects [29] and the extension to the 3D case. Possible implementation in effective medium theories for nano-
Fig. 6. Graded-index slot waveguide. (a) Photonic potential $V/k^2 - n_0^2$ and (b) electromagnetic field (peak normalized to unity) versus the transverse coordinate $x/\lambda$ for the two polarizations when $\Delta n_0 = 2$ (corresponding e.g. to Si-on-insulator waveguides). Inset in panel (a): magnification of $V_{TM}$ around the symmetry axis $x = 0$. (c-d) Normalized transverse electric field $E_x$ for the TM case versus the maximum refractive index, for a defect of width $w/\lambda = 0.01$ and $w/\lambda = 0.1$. All the results are computed for $n_0 = 1.5$ and $w/\lambda = 0.4$.

patterned metamaterials, including nanogratings [31, 32] and dielectric metasurfaces [33], can be envisaged as well.

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Fig. 7. Graded-index slot waveguide. The different panels show the comparison between the electromagnetic field (peak normalized to unity) obtained analytically (dashed lines) and the FDTD result at 45 $\alpha$ (solid lines) for $\Delta n_0$ as marked. Here $n_0 = 1.5$, $w/\lambda = 0.1$ and $w/\lambda = 0.4$.

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