Improved Sufficient Conditions for Exact Convex Relaxation of Storage-Concerned ED

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Abstract—To avoid simultaneous charging and discharging of storages, complementarity constraints are introduced to storage-concerned economic dispatch (ED), which makes the problem non-convex. This letter concerns the conditions under which the convex relaxation of storage-concerned ED with complementarity constraints is exact. Two new sufficient conditions are proposed, proved and verified to significantly reduce the conservatism of recent results [3], [4].

Index Terms—Energy Storage, Economic Dispatch, Complementarity Constraint, Convex Relaxation

I. INTRODUCTION

One difficulty appears in storage-concerned economic dispatch (ED) calculation is how to avoid simultaneous charging and discharging, which is unrealistic for most energy storage technologies [1]. The introduction of auxiliary binary variables [1], [2] as well as complementarity constraints [3] are widely considered in the literature. The former leads to mixed integer programming (MIP) and the latter results in non-convex nonlinear programming (NLP). Both are NP-hard. By simply dropping the complementarity constraints in the latter approach, the non-convex problem can be relaxed to a convex problem which is polynomial time solvable by interior point method. This letter concerns the conditions under which above relaxation is exact in the sense that the convex problem attains the same optimal solution as the original non-convex one. We improve the results in recent papers [3], [4]. First, we propose a local marginal price (LMP) related sufficient condition which is weaker than those given in [3] and [4]. Second, we present an even weaker condition concerning the sizes of the storages where and when the first condition is violated.

II. STORAGE-CENTERED ED

Consider a power network with bus set \( N \) and branch set \( L \). \( T = \{1, \ldots, T\} \) denotes the set of time slots. The storage-concerned ED problem minimize the objective function

\[
 v(\Omega) = \sum_{t \in T} \sum_{i \in N} \left( g_i(p_i^c(t)) - f_i(p_i^d(t)) + h_i(p_i^l(t)) \right)
\]

where \( \Omega = (p_i^c(t), p_i^d(t), p_i^l(t))_{t \in T, i \in N} \), subject to:

\[
\begin{align*}
0 & \leq p_i^c(t) \leq \overline{p}_i^c, & \alpha_{i,1}(t), \alpha_{i,2}(t) \\
0 & \leq p_i^d(t) \leq \overline{p}_i^d, & \alpha_{i,3}(t), \alpha_{i,4}(t)
\end{align*}
\]

This work was supported in part by Engineering and Physical Sciences Research Council (EP/L014351/1).

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\[
p_i^d(t) + \overline{p}_i^d(t) = 0 \quad (4)
\]

\[
P_i^d \leq p_i^c(t) \leq \overline{p}_i^c \quad (5)
\]

\[
\sum_{i \in N} s_i(t) \leq \overline{s}_i, \quad \beta_{i,1}(t), \beta_{i,2}(t) \quad (6)
\]

\[
R_i^d \Delta t \leq p_i^d(t + 1) - p_i^d(t) \leq R_i^c \Delta t \quad (7)
\]

\[
\sum_{i \in N} \left( p_i^c(t) + p_i^d(t) - p_i^d(t) \right) = \sum_{i \in N} D_i(t), \quad \lambda(t) \quad (8)
\]

\[
P_i^d \leq \sum_{i \in N} GSF_{j-i} \left( p_i^c(t) + p_i^d(t) - p_i^d(t) - D_i(t) \right) \quad (9)
\]

where

\[
s_i(t) = (1 - \epsilon_i)^t s_i^0 + \sum_{\tau=1}^{t} (1 - \epsilon_i)^{t-\tau} \left( \eta_i p_i^c(\tau) - p_i^d(\tau) / \eta_i \right) \Delta t. \quad (10)
\]

The decision variables include the grid-side energy storage charging power \( p_i^c(t) \) and discharging power \( p_i^d(t) \) and the generator active power output \( p_i^l(t) \). Convex quadratic discharging cost \( g_i \), linear storage charging fee \( f_i \) and convex quadratic generation cost \( h_i \) form the objective function \( v(\Omega) \). Inequalities (2) and (3) set the limits for storage charging and discharging power. Complementarity constraint \( (4) \) ensures storages operate either in the charge or discharge mode. The upper and lower bounds of generator output and storage energy are enforced by \( (5) \) and \( (6) \). \( (7) \) represents the generator ramp rate constraint. \( (8) \) is the power balance equation of the whole system, and \( (9) \) represents the bidirectional transmission capacity limits. The self-discharging effect has been considered by the self-discharge rate \( \epsilon_i \), \( \alpha_{i,1}(t) \), \( \alpha_{i,2}(t) \), \( \alpha_{i,3}(t) \), \( \alpha_{i,4}(t) \), \( \beta_{i,1}(t) \), \( \beta_{i,2}(t) \), \( \lambda(t) \), \( \mu_{j,1}(t) \), \( \mu_{j,2}(t) \) are multipliers of corresponding constraints. For brevity, we do not include net charging requirements constraints and time-varying limits \( (3) \) in our formulation. But the propositions in this paper also hold when those constraints are considered.

III. CONVEX RELAXATION AND EXACTNESS

Two problems are considered. The first problem is the non-convex original problem formally stated as OP: \( \min_{\Omega} v(\Omega) \) s.t. \((2) \sim (9)\) whose feasible set, optimum and optimal solution are denoted as \( F_0, v_0^* \) and \( \Omega_0^* \). We assume OP has unique global solution. The second problem is the convex relaxation problem RP: \( \min_{\Omega} v(\Omega) \) s.t. \((2) \sim (3), (5) \sim (9)\) with feasible set, optimum and optimal solution denoted as \( F_\Omega, v_\Omega^* \) and \( \Omega_\Omega^* \). Obviously, \( F_0 \subseteq F_\Omega \) and \( v_0^* \leq v_\Omega^* \).

At first, we present an improved LMP related condition and the exactness of RP under this condition.

[1] C. Duan, W. Fang and J. Liu, “Improved Sufficient Conditions for Exact Convex Relaxation of Storage-Concerned ED,” in IEEE Transactions on Power Systems, vol. 32, no. 3, pp. 1811-1820, May 2017.

[2] C. Duan and J. Liu, “A Local Marginal Price-Based Local Exact Relaxation of Storage-Concerned Economic Dispatch,” in IEEE Transactions on Smart Grid, vol. 10, no. 6, pp. 6432-6442, Nov. 2019.

[3] C. Duan, W. Fang and J. Liu, “A Mixed Integer Programming Formulation for Storage-Concerned Economic Dispatch,” in IEEE Transactions on Power Systems, vol. 36, no. 1, pp. 131-139, Jan. 2021.

[4] C. Duan and J. Liu, “A Local Marginal Price-Based Local Exact Relaxation of Storage-Concerned Economic Dispatch,” in IEEE Transactions on Smart Grid, vol. 10, no. 6, pp. 6432-6442, Nov. 2019.
TABLE I  
NUMERICAL VALIDATION OF EXACTNESS CONDITIONS

| Cond. 1 | Cond. 2 | Conds in [3] |
|---------|---------|--------------|
| 1.5-2.5 | 2.0     | yes          |
| 1.5-2.5 | 1.2     | yes          |
| 1.5-2.5 | -1      | yes          |
| 1.5-2.5 | -3      | yes          |

Considered Asmp. 1 and noticing the positivity of $\alpha_{1,2}(t)$, $\alpha_{4,1}(t) and $1/\eta_i^c - \eta_i^g$, we have $\Gamma(t) < 0$. So $\beta_{1,2}(t) > 0$, i.e. $s_i(t) = S_i$. This contradicts to Cond. 2. So $p_i(t)^d(t) = 0, \forall i \in \mathcal{N}, t \in \mathcal{T}$, i.e. $\Omega_i^c \in \mathcal{F}_0$ and $v_i^0 \leq v_i^*$. Therefore $v_i^0 = v_i^*$. Similarly, we have $\Omega_i^g = \Omega_i^g$. 

Lemma 2 states that if the LMPs at some buses go below the bound given in Cond. 1, the exactness of RP can still be guaranteed provided the storage installed at those buses has large enough energy capacity. The maximal energy capacity needed can also be estimated based on the forecasted LMPs. It can be analyzed from (12) and (13) that, by assuming $\Gamma(t) \geq 0$, the storage charges with $\eta_i^c \Gamma(t)$ when $LMP_i < f_i^c$, and discharge with $\min\left(\frac{T_i^c}{\Delta t}, \frac{1-v_i^0}{s_i(t) - 1} \right) \eta_i^g$ when $LMP_i > f_i^g$. 

The following procedure can be used to check the exactness of RP. 1) Forecast the LMPs by using historical data and check Cond.1; if satisfied, RP is exact; 2) If Cond.1 does not hold, find out those $i$ and $t$ at which Cond.1 is violated; 3) Estimate the maximum energy capacity needed at bus $i$. If $S_i$ is larger than the maximal energy capacity needed, RP is still exact.

IV. NUMERICAL VALIDATION

Numerical tests are conducted on IEEE 30-bus system with 3 wind farms and 5 storages. The maximal load is 189 MW, and the maximal wind generation is 60MW. Both load and wind generation vary according to daily forecasted curves. A time horizon of 24h in time steps of 0.5h is considered. The relaxed problem is solved by SDPT3 with YALMIP.

Results are shown in Table I. When fix $(f_i^c,g_i^g) = (1.5,2.5)$, the lower bound of LMP for the exactness of RP is 1.5 provided by [3] versus -2.76 by Cond. 1 in this paper. Row 1 to row 3 of Table I demonstrate the validity and superiority of Cond. 1. When LMPs decrease to -3, Cond.1 and Cond.2 are violated thus simultaneous charging and discharging happens, shown in row 4. But after we enlarge the energy capacity of storages from 2 MWh to 10 MWh, the exactness of RP is recovered, shown in row 5. The proposed Cond. 1 and Cond. 2 significantly reduce the conservatism of previous results.

REFERENCES

[1] R. Jahr, S. Karaki, and J. Korbane, “Robust multi-period OPF with storage and renewables,” IEEE Trans. Power Syst., vol. PP, no. 99, pp. 1–10, 2014.
[2] P. Malysz, S. Sirouspour, and A. Emadi, “An optimal energy storage control strategy for grid-connected microgrids,” IEEE Trans. Power Syst., vol. 5, pp. 1785–1796, July 2014.
[3] Z. Li, Q. Guo, H. Sun, and J. Wang, “Sufficient conditions for exact relaxation of complementarity constraints for storage-concerned economic dispatch,” IEEE Trans. Power Syst., vol. PP, no. 99, pp. 1–12, 2015.
[4] Z. Li, Q. Guo, H. Sun, and J. Wang, “Further discussions on sufficient conditions for exact relaxation of complementarity constraints for storage-concerned economic dispatch,” arXiv preprint arXiv:1505.02493 2015.