One-legged hop of compliance control based on minimum-jerk

J X Zhao, Q Ch Yao, B Y Xing, P Xu, Zh G Liang, Ch X Jiang and B Su*

Unmanned Ground Vehicle R&D Center, China North Vehicle Research Institute, Beijing, 100072, China

Email:bosu@noveri.com.cn

Abstract. Aiming at the trajectory planning and compliance of the one-legged hop of a quadruped robot, a method for designing the optimal trajectory of the bouncing by minimizing the jerk index is proposed, and active compliance control is used to reduce the impact of the end of the foot and improve the compliance of the one-legged. First, design a one-legged bounce strategy. The bounce is divided into three phases: take-off phase, vacant phase, and buffer phase, and trajectory optimization is performed for each phase. Secondly, a compliance control method is designed. During the bouncing process, the impact of the foot end is large, which will affect the stability of the robot and even destroy the mechanical structure of the leg. The joint level adopts force-position mixed control, and the one-legged level adopts impedance control to eliminate the impact of ground impact on one-legged. Finally, a one-legged model of a quadruped robot was established in the Vortex multi-body dynamics system. Simulation experiments have proved the feasibility of trajectory optimization and compliance control.

1. Introduction
The foot robot has a strong ability to adapt to unstructured terrain, and can walk on grass, gravel roads, sandy soil, up and down slopes, steps, trenches and other terrains. Bounce is a kind of motion method of foot robot, which can provide richer gait for foot robot. With the continuous development of foot robots, there is also a higher requirement for the character of a one-legged.

In 1986, Raibert [1] began research on one-legged bounce robots, abstracting the robot's legs into a spring-loaded inverted pendulum (SLIP) to achieve stable jumping motion. MIT's Cheetah3 [2] proposed a method for quadruped robots to achieve the best jumping behaviour, including trajectory optimization, accurate high-frequency tracking and stable landing control. MIT's minicheetah [3] has achieved backflip.

Wang Shenjiang [4] and others used parametric optimization methods to achieve joint space trajectory planning, and the optimization goal was to minimize the robotic joint control potential; Zhong Jianfeng [5] and others proposed a jumping method based on cubic curve trajectory tracking to meet the performance requirements of one-legged jumping and reduce the impact of the sole of the foot; Chen Jianwen [6] and others divided the one-legged bounce into four stages. Using the analysis of one-legged motion, the foot end trajectory was planned, and a one-legged control system was designed. The inner loop is a position controller with speed feedforward, and the outer loop is force feedback type impedance controller. Yin Peng [7] and others adopted the method of foot contact force...
compensation control and actively controlled the expansion and contraction of one leg to eliminate the interference of the ground stiffness change on the body movement.

This article focuses on the one-legged bounce strategy and active compliance control during the bounce process. The one-legged bounce strategy considers energy consumption to minimize the jerk index to plan the movement path of the centroid. The bounce trajectory planning problem is transformed into a second optimal planning problem. During the robot jumping process, there will be a large impact on the foot and poor compliance. Ground contact will produce a large ground impact force, which will affect the stability of the robot. In severe cases, it can damage the mechanical structure of the leg. The robot's legs are not equipped with compliance cushioning elements, such as springs, so active compliance control is particularly important. In this paper, joint level control is used at the joint level, while impedance control is used at the one-legged level to achieve compliance control of the one-legged.

Active compliance is mainly achieved through control algorithms. Among them, impedance control and force-position mixed control are typical active compliance controls. Impedance control was proposed by Hogan [8] in the 1980s, and force-level hybrid control was proposed by Mason and Raibert et al. [9] Both are currently widely used in active compliance control of robots.

2. One-legged Jumping Strategy Design

Virtualize one leg into a spring, compress the leg, and then increase the amount of deformation of the spring to make the foot end have an upward force to achieve a one-legged bounce. In order to achieve efficient and reliable one-legged bounce, a reasonable bounce strategy is essential. The strategy is divided into three phases, the take-off phase, the empty phase and the buffer phase.

\[ \begin{align*}
  h &= h_0 \\
  v &= v_0 \\
  a &= a_0
\end{align*} \] (1)

Wherein h is the centroid position.

2.1. Take-off stage

The trajectory of the take-off phase is designed based on the position of the center of mass, the speed of movement, and the energy consumption. Three points were selected as the boundary conditions during the take-off phase, corresponding to (a), (b), and (c) in figure 1.

2.1.1. Boundary conditions in the take-off phase.
A. Take-off start point \( P_1 \),

B. Take-off energy storage lowest point \( P_2 \)
C. Take-off end point $P_t$

\[
\begin{aligned}
  h &= h_t \\
  v &= 0 \\
  a &= 0
\end{aligned}
\]  

(2)

The take-off trajectory is planned after the take-off start point, and the proposed algorithm uses the minimum energy consumption as an index to dynamically plan the trajectory. The Jerk value of the candidate track is integrated as its energy consumption, and the track with the least energy consumption is selected as the take-off track.

2.1.2. Design of jumping trajectory planning algorithm. In summary, the selected bounce end point is a one-legged real-time online dynamic planning, and the take-off trajectory requires not only the setting of the take-off point but also the minimum energy consumption. The bounce trajectory planning problem is a quadratic optimal planning problem.

The z-axis desire trajectory can be represented by an n-th polynomial of order, and its polynomial parameter vector is , then the objective function of the minimized trajectory Jerk is as follows,

\[
\min_{p} f(p) = \min \sum_{i=1}^{k} \int_{t_{i-1}}^{t_i} \left[ \left(0, 0, 0, 6, 24\tau, \ldots, \frac{n!}{(n-3)!} \tau^{n-3} \right) \cdot p \right]^{T} \left[ \left(0, 0, 0, 6, 24\tau, \ldots, \frac{n!}{(n-3)!} \tau^{n-3} \right) \cdot p \right] d\tau
\]

(4)

Where $k$ is the number of trajectory segments and $t_i (i=1,2,\ldots,k)$ is the start and end time of each foot end trajectory. By further planning the parameters of each point, the following equation constraints can be constructed, where the starting point of the trajectory is the position of the initialized one-legged, the midpoint is $P_2$, and the end of the trajectory is $P_n$. In addition, the position, velocity and acceleration of the two foot trajectories before and after $P_2$ should be continuous.

2.2. Vacant stage

As shown in (d) of figure. 1, because it is a one-legged bouncing in place, the leg in the vacant phase maintains the leg length at the end of the take-off phase, and the posture remains unchanged. The height of the bounce is estimated from the law of conservation of energy, ignoring energy loss.

\[
h = \sqrt{\frac{v^2}{2g}}
\]

(5)

2.3. Landing buffer stage

During the same take-off phase, point $P_t$ and $P_f$, as shown in states (e) and (f) in figure. 1, are selected as the boundary conditions of the landing buffer phase, and the trajectory of the landing buffer is planned. The compliance control is used to achieve the buffering, and the impact of the ground force on the legs when the bounce is dropped is reduced.

A. Landing start point $P_t$

When the legs first started to land, the leg length was the leg length when taking off the ground. Assuming there is no loss, the speed of the landing and the speed of the ground are the same, the directions are opposite, and the acceleration is gravity acceleration.
\begin{align*}
\begin{aligned}
\dot{h} &= h_2 \\
\dot{v} &= -v_i \\
\dot{a} &= g
\end{aligned}
\end{align*}
\tag{6}

B. Landing end point \( P_l \)
The landing end point is the starting point of the take-off phase, and the leg length is the initial leg length, which is the default height before take-off.

\begin{align*}
\begin{aligned}
\dot{h} &= h_0 \\
\dot{v} &= 0 \\
\dot{a} &= 0
\end{aligned}
\end{align*}
\tag{7}

In the same take-off phase, the trajectory of landing buffer is planned using the minimized jerk index.

3. One-legged compliance control

In the process of one-legged bouncing, the rigid contact between the foot end and the ground will produce a large impact force. The impact force will not only affect the mechanical structure of the legs, but also affect the stability of the robot. Since passive flexible elements are not added to the legs, active compliance control is essential. In this paper, joint hybrid position and force control is applied at the joint level, and impedance control is applied to one leg.

3.1. Joint hybrid position and force control

Hybrid position and force control, respectively constructing a force control loop and a position control loop, and combining the output torques of the two as the final output control torque.

Most of the force control loops directly output the desired torque. The motor response may appear after the phenomenon, and the deviation signal of the actual torque and the desire torque of the joint motor is introduced to quickly eliminate the torque deviation. The output torque of the force control loop is:

\[
\tau_f = \tau_d + k_{pf}(\tau - \tau_d)
\]
\tag{8}

Wherein, \( \tau_f \) is the control torque output by the force control loop, \( \tau \) is the actual output torque of the motor, \( \tau_d \) is the desire joint torque, and \( k_{pf} \) is the proportionality factor.

The output according to the position control is determined by the actual angle and the desire angular deviation of the actual output and the current speed (position differential).

\[
\tau_q = k_{pq}(q_d - q) - k_{dq}\dot{q} + \dot{\tau}_q
\]
\tag{9}

Wherein, \( \tau_q \) is the control torque output by the position control loop, \( q \) is the actual angle of the joint, \( q_d \) is the desired angle of the joint, \( \dot{q} \) is the actual velocity of the joint, \( k_{pq} \) is the stiffness of the joint system, and \( k_{dq} \) is the joint system damping.

The actual output control torque is:

\[
\tau = \tau_f + \tau_q
\]
\tag{10}

The principle block diagram of the force-bit hybrid control is as follows:
Figure 2. The principle block diagram of the force-bit hybrid control.

3.2. One-legged impedance control.
In the previous section, the motion trajectory of a one-legged bounce was planned, and the desire foot end trajectory can be obtained.

First, the desired position $q_d$ of each joint can be calculated from the desire foot end position $P_d$ through inverse kinematics; Secondly, a one-legged is virtualized as a spring damper, and the virtual end of the foot is generated by the deviation of the end of the foot $(P_e - P_d)$, the current speed $v$ of the foot, and the gravity feedforward $mg$, and calculated by the one-legged Jacobi matrix $J$ to obtain the expectations of each joint Moment. After that, the desired torque and the desired position of the joint are sent to the joint motor for execution. Finally, the actual foot position is calculated from the actual position of the shutdown by the positive kinematics of the one-legged. The Jacobian matrix of the legs is calculated. The control block diagram is shown in figure 3.

The virtual toe force $F$ is

$$F = k_p (P_e - P_d) + k_d v + mg$$

(11)

Wherein, $k_p$ is the stiffness of the joint system, $k_d$ is the damping joints.

Figure 3. Impedance Control Based on Foot Position Planning.

4. Simulation
This paper studies the one-legged bounce control of a foot-type robot, and proposes a one-legged bounce control strategy and a compliant control method for the legs. In order to verify the control strategy, a one-leg simulation model of a four-legged robot was first built in the multi-body dynamics system Vortex, and then the one-leg bounce control was performed to verify the effect of active compliance control.

The one-legged simulation model is established as shown in figure 4 below. The fuselage is locked in the x and y directions, and only the z-direction control is performed, that is, the control in the height direction of the fuselage.
The one-legged simulation model is established as shown in figure 4 below. The fuselage is locked in the x and y directions, and only the z-direction control is performed, that is, the control in the height direction of the fuselage. The four-legged robot has a three-joint configuration, which locks when one leg is bouncing, and only the hip joint and knee joint move. Joint motion is driven by a motor model, which simulates the motion performance of a real motor.

![Figure 4. The one-legged simulation model.](image)

During the one-legged bounce control, the position and moment information of each joint, as well as the centroid position, speed, and acceleration of the one-legged model, were recorded every 0.005s to verify the control strategies proposed in the previous two sections.

### 4.1. Active compliance control strategy verification

![Closed-loop curve of torque and angle of each joint.](image)
In figures 5, all graphs show joint positions and moments during a one-legged bounce simulation. The hip joint and knee joint are tracked according to the desired position and force. As can be seen from the figure, the joint force position control of the joint is better, and the position and force tracking is more accurate. The force control can track the desire force in time, while the position tracking shows some lag, but it has little effect on the stability of the whole one-legged bounce. During the bouncing process, the moment of the knee joint has reached 680N \cdot m, and the moment of the knee joint is significantly greater than the moment of the forward pendulum joint during contact.

In the landing buffer phase, the compliance control strategy is obviously effective, the ground impact force is significantly reduced, the joint torque output is reduced rapidly, and the force control effect is obvious.

4.2. Verification of One-Legged Bounce Strategy

All graphs in figure 6 show the position, velocity, and acceleration of the center of mass during the one-legged bounce. The trajectory is planned by minimizing the Jerk index as the objective function and the constraints of each stage in the bounce process. The one-legged bounce went through three stages of compression, take-off, flying, and landing buffer.

![Figure 6](image)

It is obvious that the state of each stage can be switched. In the initial state of take-off, the height of the fuselage from the ground is 0.68m; when the energy is compressed and stored, the height of the fuselage and the ground is 0.46m; when it is off the ground, the leg length is 0.9m and the speed is 2m/s. At this time, the foot end trajectory is not planned for the vacated segment. The speed of the flying phase remains unchanged in the data in the figure because the trajectory of the flying phase is not planned, the state before leaving the ground is maintained, and the airframe does not perform other controls. However, in practice, the acceleration of gravity decreases to the highest bounce, and then increases to the speed above the ground. This is not shown in the figure, and it is hereby explained.
Figure 7 shows that the foot position follows well, the bounce strategy is effective, and the continuous bounce is also very regular.

5. Conclusion
This article takes a four-legged robot's one-leg system as the research object, and studies the one-leg bounce strategy and compliance control during the bounce process. The one-leg bounce phase is divided into three phases: the take-off phase, the vacant phase, and the buffer phase. The jerk index was used to plan the trajectory of the centroid, and the bounce trajectory planning problem was transformed into a quadratic optimal planning problem. During the robot's foot-to-ground contact, the impedance control with gravity feedforward is used to fully reduce the damage of the ground impact force to the robot and improve the flexibility and stability of the robot. The joint layer adopts force-position hybrid control, and the force control loop adds torque deviation compensation to make the actual torque of the joint quickly follow the desire torque; the position control loop uses the impedance control model to build the control loop of the joint, so that the joint has good compliance performance. A one-leg simulation model of a four-legged robot was built on the multi-body dynamics system Vortex. The one-leg bounce strategy and compliance control were simulated and verified. The results show that the control method can control one-leg bounce and has good compliance.

Acknowledgments
This work was supported by the National Natural Science Foundation of China(Grant No.91748211).

References
[1] Raibert M H 1986 Legged Robots that Balance (MIT press)
[2] Nguyen Q, Powell M J, Katz B, Di Carlo J and Kim S 2019 Int. Conf. on Robotics and Automation(Montreal) (IEEE) 7448-54
[3] Katz B, Di Carlo J and Kim S 2019 Int. Conf. on Robotics and Automation(Montreal) (IEEE) 6295-301
[4] Wang Shenjiang and Guo Zuhua 2015 China Mechanical Engineering 26 (17) 2330-35
[5] Zhong Jianfeng, Luo Xin and Yu Yijun 2014 Machinery & Electronics 4 60-4
[6] Chen Jianwen, Tan Yuegang, Lei Chenglin, et al. 2019 Digital Manufacturing Science 1 7
[7] Yin Peng, Li Mantian, Wang Jun, et al. 2015 Machinery & Electronics 4 3-7
[8] Hogan N 1985 Jou. Dyn. Sys., Meas., Control 107(1) 1-7
[9] Cheah C C, Kawamura S and Arimoto S 2003 Automatica 39(5) 847-55