Variation Propagation in Multistage Machining Processes Using Dual Quaternions

To cite this article: F Yacob and D Semere 2019 IOP Conf. Ser.: Mater. Sci. Eng. 689 012019

View the article online for updates and enhancements.
Variation Propagation in Multistage Machining Processes Using Dual Quaternions

F Yacob and D Semere

1 Production Engineering, Royal Institute of Technology, Brinellvägen 68, 11428 Stockholm, Sweden

E-mail: filmona@kth.se

Abstract. The application of rigid transformations matrices in variation propagation has a long tradition in manufacturing community. However, the matrix-based modeling of variation propagation in multistage machining processes is complicated. Moreover, there is a need to improve the computational efficiency of manipulation of geometrical models for variation analysis purposes. This paper introduces the representation of rigid transformation by dual quaternions, which have a computational advantage and mathematical elegance compared to matrices. In comparison to a commercial tool, the implementation of the purposed method predicted parallelism with an average error of 4.3 % in a hypothetical two stage machining process. The proposed approach has a potential to be an alternative to matrix based rigid transformation practices in variation and tolerance analysis.

1. Introduction

Variation in manufactured parts is an expected characteristic due to variation in machining processes. The variation can be caused by many sources, including loose locator, wear, datum variation from upstream, and a tooltip deviation from its nominal path. The combined effect of these variation sources makes it non-trivial task to predict the resulting geometry of a part, and thereby harder to entertain what-if scenarios.

The existing methods of variation propagation methods apply matrices to capture the effect of the sources of error. One of the common methods, Stream of Variation, utilizes matrix representation of the rotational and translational transformation of geometrical models [1]. Such approaches become mathematically more complex when some of the assumptions relaxed. For instance, the position of the secondary locators is often considered at the same height. The generic setup model of parts and fixtures is very complicated, often reduced to orthogonal assembly cases [2].

The use of Skin Models Shapes has enabled inclusions of form errors in variation propagation related analysis [3]. Skin Model shapes have been applied in tolerance analysis [4], variation analysis [5], [6], and anomaly detection [7]. The underlying principle of such analysis is an assembly of Skin Model Shapes with help of transformation matrices. However, depending on the density of points per model and number of models, computational efficiency suffers on performing operations such as registration [5]. Since the continuous registration steps are required, any improvement in computational efficiency is desirable.

In light of these, quaternions and their extensions dual quaternions, have a potential in variation propagation analysis without the dependency on transformation matrices. Dual quaternions perform better in terms of computational efficiency, speed and robustness compared to matrices [8], [9]. Moreover, dual quaternions represent rotation and translation in a compact unified single variable of 8 dimensions. Rotation is computed based on axis of rotation, not constrained to the series of orthogonal
rotations, which makes it much easier to iteratively optimize over [10]. Dual quaternions have been successfully applied in object blending and skinning in computer graphics [11], hand-eye calibration in robotics [12] and in kinematics of industrial robots [9], [13] and pair-wise point cloud registration [14], to mention a few.

This paper introduces dual quaternions for the first time in performing variation propagation in multistage machining. Using the dual quaternions, the angle and the axis of rotation of the deviated assembly feature from corresponding locators’ plane is first computed for registration purpose. Following this, the distance between the feature’s and locators’ planes is used to obtain the translational part of dual quaternions. Once the registration is completed, the machining variation is induced by varying the depth cut. These steps are repeated at each station to predict the product geometry in subsequent stages.

2. Dual quaternions
Quaternions are compact mathematical representation of orientation of an object in space [15]. The fundamental concept is that 4 dimensional properties are required to represent 3D rotation. The quaternions has two parts, the real part \(q_0\) and the vector part \(q = q_1i + q_2j + q_3k\), which can be denoted by

\[
\tilde{q} = q_0 + q_1i + q_2j + q_3k = (q_0, \tilde{q})
\]

where \(q_0, q_1, q_2\) and \(q_3\) quaternion properties of the unit vectors and i, j, and k, such that \(i^2 = j^2 = k^2 = ijk = -1\).

The quaternion properties are computed using angle \(\theta\) and axis of rotation \(\mathbf{n}\) between an object and reference object.

\[
\tilde{q} = \left( \cos \left( \frac{\theta}{2} \right), \mathbf{n} \sin \left( \frac{\theta}{2} \right) \right) = (q_0, \tilde{q})
\]

Dual quaternions, as extension of quaternions, were derived from the field of dual algebra [16]. Dual quaternions embed the rotational and translation aspect of rigid motion simultaneously. Dual quaternions have two part: the real component \(\tilde{q}_r\) and dual components \(\tilde{q}_d\), which can be denoted by

\[
\tilde{q} = \tilde{q}_r + \epsilon \tilde{q}_d
\]

where \(\epsilon\) is a dual-operator such that \(\epsilon \neq 0, \epsilon^2 = 0\).

Equation (3) is equivalent to the homogenous transformation matrix of 4 by 4.

3. Assembly and feature representation

3.1 Assembling part to a fixture
In multistage machining, variation propagates due to use of machined feature as an assembly feature in succeeding stations. To assemble a part to a 3-2-1 layout fixture, for example, specific features should be registered to their corresponding locators one at a time. For planar surface, the normals to the planes of the locators and features can be used to determine the necessary angle and axis of rotation required to assemble the geometric model to fixture. Mathematically, for an angle \(\theta\) between two of the planes of two point clouds whose normal \(\mathbf{n}_a, \mathbf{n}_b \in \mathbb{R}^3\) and \(\mathbf{n}_b \in \mathbb{R}^3\), the axis of rotation \(\mathbf{n} \in \mathbb{R}^3\) is

\[
\mathbf{n} = \mathbf{n}_a \times \mathbf{n}_b
\]

The dual quaternion representation of pure rotation \(\tilde{\mathbf{R}}\) and pure translation \(\tilde{\mathbf{T}}\) are denoted by

\[
\tilde{\mathbf{R}} = \left( \cos \left( \frac{\theta}{2} \right), \mathbf{n} \sin \left( \frac{\theta}{2} \right) \right) + \epsilon (0_{1 \times 4})
\]

\[
\tilde{\mathbf{T}} = ([1, 0, 0, 0]) + \epsilon (0, \tilde{\mathbf{t}})
\]

where \(\tilde{\mathbf{t}} \in \mathbb{R}^3\) is the vector part of the quaternion representing the distance between corresponding points of the two planes. For rigid translation, the \(\tilde{\mathbf{t}}\) is the same for all points.
The combined transformation, rotation followed by a translation, becomes the product of the two dual quaternions.

$$\hat{q} = T \times R$$  \hspace{1cm} (7)

For a 3-2-1 fixture layout, a part is assembled in a sequence of primary, secondary and tertiary locators. To assemble the part model to secondary and tertiary locators, the model is translated in the direction where primary and tertiary planes, and primary and secondary planes, intersect, respectively. Thus, dual quaternion representation of the total transformation $\hat{A}_{\text{total}}$ becomes

$$\hat{A}_{\text{total}} = \hat{T}_t \times \left( \hat{T}_s \times \hat{R}_s \right) \times \left( \hat{T}_p \times \hat{R}_p \right)$$ \hspace{1cm} (8)

where the subscripts $p$, $s$ and $t$ denote the primary, secondary and tertiary datum/locators, respectively.

Following the definition transformation of point by dual quaternions, the point cloud of a feature with $v$ points $A = \{a_i \in \mathbb{R}^3 : i = 1 \ldots v\}$ is transformed to new position $A'$ by applying

$$A' = \hat{A}_{\text{total}} A \hat{A}_{\text{total}}^*$$ \hspace{1cm} (9)

3.2 Representation of machined surface

The dual quaternion representation of machined surface is based on the angular and translational distance between an actual and a nominal machined surface. These deviations can be computed by utilizing the landmarks (dummy) points as reference points that are not affected by the machining processes. The landmarks are easier to locate when placed at some distance from the point clouds of the model.

Without loss of generality, a 2D example of machining setup shown in Figure 1, the machined surface, which can be represented by coplanar points, remains horizontal despite the locator deviations. However, when the part is moved out of the setup by coinciding the landmarks, the machined surface results in orientation and position errors. Figure 1 shows an illustration the resulting machined surfaces before and after landmarks are coincided.

Mathematically, the normal of the displaced landmarks’ plane $\mathbf{n}_m$ and that of the reference landmarks $\mathbf{n}_n$, the axis of rotation is computed from $\mathbf{n}_{nm} = \mathbf{n}_n \times \mathbf{n}_m$. The dual quaternion representation of the machined feature $\hat{q}_{nm}$, derived from pure rotation $\hat{R}_{nm}$ and pure translation $\hat{T}_{nm}$ in the direction of $\mathbf{n}_n$ becomes

$$\hat{q}_{nm} = \hat{T}_{nm} \times \hat{R}_{nm}$$ \hspace{1cm} (10)

The transformation of the nominal feature $N = \{n_i \in \mathbb{R}^3 ; i = 1 \ldots w\}$ by $\hat{q}_{nm}$ gives the point cloud of the machined feature $M'$ with $w$ points.

$$M' = \hat{q}_{nm} N \hat{q}_{nm}^*$$ \hspace{1cm} (11)

In line with Skin Model Shape operations [4], the point clouds of the model and machined surface, obtained from Eq. 9 and Eq. 11 give

$$A_m = \{(A', M') \in \mathbb{R}^3\}$$ \hspace{1cm} (12)
3.3 Setup in multistage processes

In multistage machining, the machined feature becomes an assembly feature in succeeding stations. The datum feature in the station is assembled to the fixture of the next station. Thus, the shape model is first rotated 180°, say around x-axis, which can be achieved by multiplying with the dual quaternion \( \hat{R}_A \) using Eq. 13

\[
\hat{R}_A = \frac{\cos(180°)}{2}, [1, 0, 0] \sin\left(\frac{180°}{2}\right) + \epsilon(0_{1x4})
\]

Following the model transformation by Eq. 14, the same steps of assembling the model to locators using Eq. 8 and Eq. 9. are repeated at each station. The resulting machined surface becomes the subject of analysis.

4. Demonstration case

To demonstrate the approach, a hypothetical 2 stage machining process with 3-2-1 fixture layout was set as shown in Figure 2. In station 1 and 2, feature A and B were represented by coplanar points set at different depth of cuts per experiment, respectively.

![Figure 2. Part-fixture assembly. (a) side and front view of station 1 (b) side view of station 2](image)

**Table 1.** Inputs and predicted parallelism and normal of machined surface of station 2, based on nominal model and deviations of (0, 0, 0.5, 0, 0.5, 0, -0.1) in station 1.

| Deviations (mm) \(^a\) | CAM-Simulation (mm) \(^b\) | Dual quaternions (mm) \(^b\) |
|------------------------|---------------------------|---------------------------|
| (0.3, 0, 0, -0.4, 0, 0, 0.6) | 0.464, (0.0043, 0.0035, 0.9922) | 0.468, (0.0043, 0.0034, 0.9923) |
| (0, -0.1, 0, 0, -0.2, 0, 0) | 0.023, (0.0000, 0.0022, 0.9978) | 0.024, (0.0000, 0.0023, 0.9977) |
| (0, 0, -0.2, 0, 0, 0, -0.4) | 0.307, (0.0028, -0.0022, 0.9950) | 0.307, (0.0028, -0.0023, 0.9948) |
| (0, 0.2, 0, 0, -0.3, 0.4, 0.2) | 0.044, (0.0000, -0.0041, 0.9959) | 0.051, (0.0000, -0.0046, 0.9953) |
| (-0.3, 0, 0.1, -0.2, 0, 0.2, -0.5) | 0.593, (-0.0057 -0.0022, 0.9921) | 0.595, (-0.0057, -0.0023, 0.9920) |

\(^a\)(Locators\(_1,6,\) Tooltip), \(^b\)Parallelism of feature B relative to A, (normal of Feature B)

In each experiment, random locator deviations normal to fixtures were added that affect the length of the locators. With the help of Eq. 10, 3 dummy landmarks points were used transform the machined part from the fixture to the reference position. The experiment has been performed by inducing deviations to the setups. The prediction result of the second station, following the method proposed in this paper and the one acquired via Autodesk Inventor HSM CAM simulation 2016, is shown in Table 1. To performing machining simulation, this paper utilized the dual-quatierions’ operations implementation in Matlab reported in [17].

One of the weakness of dual quaternions is that the direction of angle of rotation is indeterministic. Thus, the rotation angle should be subtracted from 180 degree when the angle is more than 90 degree. Moreover, the approach proposed in this paper applies the sequence of rotation followed by translation. However, the approach can be extended to a method to that simultaneously applies rotation and translation steps.
5. Conclusion
Variation propagation modelling and analysis in multistage machining based on matrices is non-trivial task. As an alternative, this paper applied dual quaternions in variation propagation in multistage machining processes for the first time. The approach reduces the number of parameters in transformation matrices, which is an important step towards reducing the computational cost of manipulation of digital models. Towards this direction, this paper showed how variation propagation can be modelled using Skin Model Shapes and dual quaternions. The dual quaternions representation of orientation and position of a model was derived from the angles and positions between corresponding assembly features and fixtures. Following this, virtual machining was performed by positioning a point cloud on desired depth of cut per specific setup. The implementation of variation prediction based on dual quaternions provided parallelism accuracy of 4.3 % error compared to the one achieved by a commercial tool. In future, the approach will include the assembly of models with form errors using dual quaternions, and improvement to the overall accuracy of the predicted geometry.

Reference
[1] Abellán-nebot J V, Romero F, Julio S, and Mira S 2013 Manufacturing variation models in multi-station machining systems, Int. J. Adv. Manuf. Technol. pp 63–83
[2] Shi J 2006 Stream of Variation Modeling and Analysis for Multistage Manufacturing Processes CRC press
[3] Schleich B, Anwer N, Mathieu L, and Wartzack S 2014 Skin Model Shapes: A new paradigm shift for geometric variations modelling in mechanical engineering CAD Comput. Aided Des. 50 pp 1–15
[4] Schleich B and Wartzack S 2016 A Quantitative Comparison of Tolerance Analysis Approaches for Rigid Mechanical Assemblies, Procedia CIRP 43 pp 172–77
[5] Yacob F, Semere D, and Nordgren E 2018 Octree-Based Generation and Variation Analysis of Skin Model Shapes, J. Manuf. Mater. Process. 2 p 52
[6] Semere D, Yacob F, Hedlind M, and Bagge M 2018 Skin Model Based Tolerance and Variation Analysis, in Skin Model Based Tolerance and Variations Analysis pp 726-31
[7] Yacob F, Semere D, and Nordgren E 2019 Anomaly detection in Skin Model Shapes using machine learning classifiers Int. J. Adv. Manuf. Technol.
[8] Wang X and Zhu H 2014 On the Comparisons of Unit Dual Quaternion and Homogeneous Transformation Matrix 24 pp 213–229
[9] Sarıyıldız E, Cakiray E, and Temeltas H 2011 A comparative study of three inverse kinematic methods of serial industrial robot manipulators in the screw theory framework Int. J. Adv. Robot. Syst. 8 pp 9–24
[10] Kenwright B 2012 A Beginners Guide to Dual-Quaternions 2679 pp 190–194.
[11] Kavan L, Collins S, Žára J, and O’Sullivan C 2007 Skinning with dual quaternions Proc. symp. on Interactive 3D graphics and games pp 39-46
[12] Daniilidis K 1999 Hand-Eye Calibration Using Dual Quaternions Int. J. Rob. Res. 18 pp 286–98
[13] Sarıyıldız E and Temeltas H 2009 Solution of inverse kinematic problem for serial robot using dual quaternions and plücker coordinates IEEE/ASME Int. Conf. Adv. Intell. Mechatronics pp 338–43
[14] Wang Y, Wang Y, Wu K, Yang H, and Zhang H 2014 A dual quaternion-based, closed-form pairwise registration algorithm for point clouds ISPRS J. Photogramm. Remote Sens. pp 63–9
[15] Hamilton W R 1866 Elements of quaternions. Longmans, Green, & Company
[16] Clifford W K 1882 Mathematical papers
[17] Leclercq G, LeFèvre P, and Blohm G 2013 3D kinematics using dual quaternions: theory and applications in neuroscience Front. Behav. Neurosci. 7 pp 1–25