Radiation of RR states from NS Fivebranes

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Abstract

We show that the recent observation that near extremal NS fivebranes decay due to Hawking radiation, can be understood in terms of their coupling to RR states which does not vanish even as the string coupling goes to zero.
In a recent paper [1] it has been argued that systems of NS fivebranes in type IIA and type IIB string theory, which are expected to decouple from the bulk ten dimensional theory in the limit when the string coupling goes to zero [2], in fact emit Hawking radiation into the bulk [2]. In this note the source of this radiation is identified. It is argued that while the NSNS states of the bulk theory do decouple in the relevant limit, the RR states do not, provided we work self-consistently by using the induced metric from the bulk on the brane. On the other hand if we take the flat metric on the brane then it decouples from all bulk modes. The latter is perhaps to be identified with the decoupled theories of [2] but they should not be considered to be the source of the supergravity fivebrane. The latter needs to be treated self-consistently. We also take the matrix model limit [4],[7],[6] and show that the system of NS fivebranes that is thus obtained, does in fact decouple and there is no Hawking radiation into the bulk supergravity coming from string theory. However there is radiation into the space defined by classical configurations of the moduli space of the matrix model.

The bosonic part of the effective theory on the M-theory fivebrane is given by the following Lagrangian.

\[ I = \frac{1}{(2\pi)^{3/2}} \int_{W_6} d^6\xi \sqrt{\det g_{\mu\nu} \partial_\mu X^\mu \partial_\nu X^\nu} \]

\[ + \frac{1}{4} \int_{W_6} (db_2 - P(C_3) \wedge \ast (db_2 - P(C_3))) + \frac{1}{2 \cdot 3!} \int_{W_6} dP(C_3) \wedge b_2 \] (0.1)

In the above \( b_2 \) is the chiral two form living on the world volume of the fivebrane and strictly speaking there is no action for it. We are however using the above only to determine the couplings to bulk (weak) supergravity fields. So for our purposes it is sufficient to impose the self duality constraint at the level of the equations of motion. \( C_3 \) is the three form field of 11-D supergravity and \( P() \) is the pullback map from the eleven manifold to the world volume \( W_6 \) of the fivebrane. Its presence is required by the fact that M2-branes can end on these 5-branes [2,3].

1There is a hint of this phenomenon in the works cited in [3].

2See the discussion in [3].

3It should be noted that although the above action is not gauge invariant by itself under the gauge
The NS fivebrane of type IIA string theory is obtained from the M-theory fivebrane when the circle on which M-theory is compactified is taken to be transverse to the fivebrane.  The relation between the metrics and the anti-symmetric tensor fields of the two supergravities are given by,

\[ ds_M^2 = e^{-2\phi/3} ds_{IIA}^2 + e^{4\phi/3} (dy - C_1)^2. \]

\[ C_3^{(M)} = C_3^{(IIA)} + 3B_2 \wedge dy. \] (0.2)

So we have for the IIA fivebrane

\[ I = \frac{1}{(2\pi)^{5/6}s} \left[ \int_{W_6} d^6\xi e^{-2\phi} \sqrt{\det g_{\mu\nu} \partial_\mu X^\mu \partial_\nu X^\nu} 
+ \frac{1}{4} \int_{W_6} (db_2 - P(C_3) \wedge *(db_2 - P(C_3))) + \frac{1}{23!} \int_{W_6} dP(C_3) \wedge b_2 \right] \] (0.3)

It should be noted that in the above (as well as in all equations below) \( l_s = l_p \) is the string scale determined in the string metric while \( l_p \) is the M-theory Planck scale determined in the M-metric.

The bulk low energy supergravity action of IIA string theory has the action

\[ I_{IIA} = -\frac{1}{2\kappa^2} \int_{M_{10}} \sqrt{-G} e^{-2\phi} \left\{ R - 4(\nabla \phi)^2 + \frac{1}{12} H^2 \right\} 
- \frac{1}{2\kappa^2} \int_{M_{10}} \left( \frac{1}{2} F_2 \wedge *F_2 + \frac{1}{2} F_4 \wedge *F_4 \right) - \frac{1}{2\kappa^2} \int_{M_{10}} F_4' \wedge F_4' \wedge B_2, \] (0.4)

where

\[ 2\kappa^2 = (2\pi)^7 l_s^8, \ H_3 = dB_2, \ F_2 = dC_1, \ F_4' = dC_3, \ F_4 = F_4' - H_3 \wedge C_1. \] (0.5)

Corresponding to the fivebrane whose effective action was written down above there is a solution of the equations of motion of this supergravity [12], i.e.

\[ ds^2 = - \tanh^2 \sigma dt^2 + (g^2 l_s^2 \mu \cosh^2 \sigma + kl_s^2)(d\sigma^2 + d\Omega_3^2) + dy_5^2 \]

\[ e^{2\phi} = g^2 + \frac{k}{\mu \cosh^2 \sigma} \]

\[ H = ke_3 l_s \] (0.6)

variation \( \delta C_3 = d\Lambda, \delta b_2 = \Lambda \) this ‘anomaly’ is cancelled by anomaly inflow from the bulk supergravity action[9][10]. For the complete action including the coupling of the M2-branes and references to earlier work see [10].

\(^4\text{If this circle is in a longitudinal direction then we obtain the D4-brane of type IIA.}\)
In the above the coordinates $y_5$ represent the directions along the fivebrane and $\epsilon_3$ is the volume element on the unit three sphere. The coordinate $\sigma$ parametrizes only the region outside the horizon which is at $\sigma = 0$. The usual Schwarzschild coordinate is related to $\sigma$ by the equation

$$r = r_0 \cosh \sigma, \quad r_0^2 = \frac{g^2 \mu l_s^2}{s^2}.$$

(0.7)

Note that in these coordinates the horizon is at $r = r_0$. The ADM mass of the fivebrane is

$$M = \frac{V_5}{(2\pi)^5 l_s^5} \left( \frac{k}{g^2} + \mu \right),$$

(0.8)

and we may identify $\mu$ as the dimensionless energy density on the brane \cite{1}.

We identify the source of the Hawking radiation by following the procedure of Das and Mathur \cite{11}. To get canonically normalized kinetic terms for fluctuations around these background fields we need to put for the NSNS sector,

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + g\kappa h_{\mu\nu}, \quad B_{\mu\nu} = B_{\mu\nu}^{(0)} + g\kappa b_{\mu\nu}^{(1)}, \quad \phi = \phi^{(0)} + g\kappa \varphi.$$

(0.9)

where the superscript $(0)$ refers to the background solution given by (0.6). It should be noted here that the relevant normalization is that of the asymptotic fields (i.e. for $\sigma \to \infty$) where the effective $e^{2\phi} = g^2$. This is what would go into an S-matrix calculation of the emission process as in \cite{11}.

For the RR fields however the properly normalized fluctuations are (the background RR fields are zero for this NSNS brane)

$$C_1 = \kappa c_1, \quad C_3 = \kappa c_3.$$  

(0.10)

Note that in this limit the background dilaton term (at the horizon) that goes into the first term of (0.3) is $e^{-2\phi(0)} = \frac{k}{s} \kappa$ though we should actually put the brane at a ‘stretched horizon’ where $\sigma \sim O(1)$ since there is a coordinate singularity at $\sigma = 0$.

These scalings show us that in the $g \to 0$ limit with the energy density $\mu$ on the brane fixed, NSNS fluctuations will decouple. However the RR fluctuations do not decouple.

\footnote{It should be remembered that our definition of $\kappa$ does not involve factors of $g$.}
and one has the following surviving couplings.

\[
\Delta I = \frac{1}{(2\pi)^5 s} \left[ \frac{1}{4} \int_{W_6} (db_2 - P(C_3) \wedge *(db_2 - P(C_3))) + \frac{1}{2.3!} \int_{W_6} dP(C_3) \wedge b_2 \right],
\]

(0.11)

In the above the pull back of \(C_3\) is explicitly,

\[
P(C_3)_{[bcdf]} = C_{\nu\lambda\sigma} \partial_b X^\nu \partial_c X^\lambda \partial_d X^\sigma.
\]

(0.12)

Thus we see that excitations of the moduli fields \(X\) of fivebrane can annihilate to produce Hawking radiation of the \(C_3\) field.

A comment on the fact that by itself the (second integral of the) five-brane Lagrangian is not gauge invariant under \(\delta b_2 = \Lambda_2, \delta C_3 = d\Lambda_2\) is in order here. As in the M-theory case the problem stems from the fact that the five-brane couples naturally to the six-form gauge field \(C_6\) whose field strength is dual to that of \(C_3\). However the bulk supergravity action contains only the \(C_3\) form field (and there is no dual formulation of the supergravity action). In the M-theory case [10] the resolution involved introducing a particular solution to the modified Bianchi identity. The present case is simply the dimensionally reduced one coming from that argument. Thus we solve \(dH_3 = 2\kappa^2 \delta(M_{10} \rightarrow W_6)\) in the presence of the five-brane by writing \(H_3 = H'_3 + \theta_3\) where \(dH'_3 = 0, d\theta = \delta, d \ast \theta = 0\). The Chern-Simons like term in the bulk action (0.4) is then modified as follows

\[
- \frac{1}{2\kappa^2} \int_{M_{10}} [dC_3 \wedge dC_3 \wedge B_2 + \frac{1}{2} C_3 \wedge dC_3 \wedge (dB_2 + \theta_3)].
\]

(0.13)

It can be seen explicitly that the gauge variation of this bulk term cancels the variation of the world volume action.

The invariance under the transformations \(\delta b_2 \Lambda_2, etc\) implies that the \(b_2\) field can be gauged away. Thus we conclude that in such a gauge the entire coupling of the five-brane to the RR field of the bulk comes from a term of the form

\[
\int P(C_3) \wedge *P(C_3).
\]

(0.14)

Similar arguments can be made for the (1,1) gauge theory on the type IIB NS fivebrane. In this case there is for instance a coupling of the four form \(C_4\) whose field strength
is self-dual. This may be obtained by S-duality from the corresponding D5-brane action, namely
\[
\frac{1}{(2\pi)^5 l_s^5} \int (F_2 - P(C_2)) \wedge P(C_4). \tag{0.15}
\]
(Note that the field \(C_2\) is the dual of \(B_2\) and \(C_4\) is self-dual under \(SL(2, Z)\).)

The relevant bulk term is
\[
\frac{1}{2\kappa^2} \int_{M_{10}} \left( \frac{1}{2} F_3 \wedge *F_3 + \frac{1}{2} F_5 \wedge F_5 + C_4 \wedge H_3 \wedge F_3 \right). \tag{0.16}
\]

The comments about actions for self-dual fields made after (0.3) apply in the above expression too. Also as in the IIA case an anomaly under the gauge transformation \(\delta C_4 = d\Lambda_3\) is cancelled by anomaly inflow from the bulk, and the rescaling to get properly normalized bulk fields does not involve factors of \(g\), so that the coupling of these bulk RR fields to the NS fivebrane survives the limit \(g \to 0\).

Let us now verify that these arguments are consistent with the observations of [1],[13], on the supergravity side. The Hawking temperature is identified in these papers as the period (in Euclidean time) of the Euclidean section of the five-brane metric. In the \(g \to 0\) case this gave
\[
T_H = \frac{1}{2\pi l_s \sqrt{k}} \tag{0.17}
\]
This result appears to be rather puzzling at first sight. The Hawking temperature is a semi-classical effect and should vanish in the limit that the effective \(\hbar\) i.e. the loop counting parameter is zero. In string theory (in natural units) this parameter is the asymptotic value of the dilaton namely \(g^2\) and one should expect the temperature to vanish when this parameter is zero. To clarify the issue let us review the usual argument.

Consider the propagation of quantum fields with a Lagrangian \(L(\phi, \partial \phi)\) in some non-trivial background metric. The partition function may be represented as a Euclidean functional integral with periodic boundary conditions; i.e. we may write
\[
\text{tr} e^{-\beta H} = \int d\phi e^{-\frac{1}{\beta} \int_0^{\Theta_H} L}. \tag{0.18}
\]
Here \(\Theta_H\) is the period with which the Euclidean time is identified in order to avoid a conical singularity at the origin (which is the horizon). This enables us to identify the
Hawking temperature as

\[ T_H = \beta^{-1} = \frac{\hbar}{\Theta_H}. \] (0.19)

Now in IIA string theory the effective action (for NSNS states) is given by the terms in the first integral in (0.4) and these have an explicit factor of \( e^{-2\phi} \) and hence one should take the effective \( \hbar \) to be \( g^2 \) and therefore the temperature should go to zero for the emission of these states. However the RR field couplings are given by the second integral which does not have a factor of the dilaton and so the effective \( \hbar \) is one. This explains why the correct result for the temperature is obtained by just calculating the periodicity of the Euclidean section. Similar arguments can clearly be made for type IIB as well.

It should be stressed that in the above discussion we have used the metric induced from the bulk as the metric on the brane. This is the consistent choice if we want the brane to be the source of the corresponding supergravity metric. i.e. this corresponds to considering the coupled action \( S[g] + I[g] \) where the two actions are the supergravity and brane actions, and taking derivatives with respect to \( g \) etc to get equations with a source term. On the other hand in much of the literature on brane dynamics leading to Hawking radiation etc. uses the flat metric on the brane. However if one were to do this in this case it can be immediately seen that all modes will decouple.[6] For in this case the kinetic term for the moduli looks like \( g^{-2} \int \sqrt{\text{det} \eta_{\mu\nu} \partial_a X^\mu \partial_b X^\nu} \). Then we need to rescale the moduli by writing \( U = X / g^{2\frac{3}{4}} \) and hold \( U \) fixed as we take the limit \( g \to 0 \). Now it is clear that all couplings to the bulk disappear; for instance the RR coupling \( \int (c_3(\partial X)^3)^2 \) goes to zero like \( g^4 \). This brane clearly cannot radiate and presumably should be identified with the isolated NS 5-brane theory of Seiberg[2]. On the other hand if we wish to make a connection to the supergravity solution corresponding to the fivebrane then we should be looking at a self-consistent solution as discussed earlier.

Let us now discuss what happens in the matrix model limit [1], [7], [6]. In this case the limit to be taken is not \( g \to 0 \) with \( l_s \) fixed, but

\[ g, l_s \to 0; \quad \frac{g}{l_s^3} = g_{YM}^2 \equiv l_m^{-3}, x_m = \frac{p_m^2}{l_s^2} x = l_m^2 U \text{ fixed}. \] (0.20)

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Where $x$ is any spatial coordinate. The last condition is just the statement that the gauge field is kept fixed as one goes to the limit. $l_m$ is effectively the Planck length of the supergravity that is generated by the matrix model. Since we are rescaling spatial coordinates we should also rescale the time (in order to keep the velocity of light fixed at $c = 1$) and we write

$$t_m = \frac{l_m^2}{l_s^2} t.$$  

(0.21)

In the above as well as below the subscript $m$ will denote quantities that have been rescaled in accordance with the matrix model requirements. By compactifying on a five torus and T-dualizing we get an effective string coupling. The ADM mass per unit volume in the string coordinates is given in (0.3). In the limit considered in [1] (i.e. $g \to 0$, $l_s, \mu$ fixed) the extremality parameter $r_0 \to 0$. In the matrix model limit the appropriate mass per unit volume is

$$\frac{M_m}{V_{5m}} = \frac{l_m^{12}}{l_s^{12}} \frac{M}{V_5} = \frac{k}{(2\pi)^{5/6}} + \frac{l_s^4}{(2\pi)^{5/12}} \mu.$$  

(0.22)

Note that in terms of the rescaled extremality parameter

$$\mu = \frac{l_m^4}{l_s^4} r_0.$$  

(0.23)

Thus taking the limit with $\mu$ fixed implies that the rescaled extremality parameter goes to zero. Thus the physical situation is the analog of that considered in [1] but with the rescaled variables. However now the expression for the temperature gets rescaled as follows (note that this is simply the inverse of the rescaling of the time).

$$T_m = \frac{l_m^2}{l_s^2} T = \frac{l_s}{2\pi \sqrt{k l_m^2}}.$$  

(0.24)

This goes to zero as $l_s$ goes to zero. It is easy to check that this is consistent with the microscopic picture. The RR coupling to the five brane is (schematically)

$$\frac{1}{l_s^{10}} \int (e^{\dots} (\partial X)^{3 \dots})^2 = \frac{l_s^{8} \times l_s^{12}}{l_s^{10} l_m^{12}} \int (e^{\dots} (\partial X_m)^{3 \dots})^2.$$  

(0.25)

This goes to zero in the limit.

The above discussion is of course for (transverse) five branes which are to be regarded as composites of zero-branes. Unlike the case of the membrane and the longitudinal
fivebrane there is no clear evidence that this in fact exists. On the other hand we may compactify on a five torus and take the matrix model limit. Using S and T duality it has been shown in [4], [7], [6] that this gives the (limit of) the NS fivebrane in type IIB. The decoupling argument will follow on the same lines as above.

Thus we see that the matrix model limit gives decoupled five-brane theories which do not radiate even into the infinite tube and are strictly six dimensional. This rather than the original limit \((g \to 0, l_s \text{ fixed})\) appears to be the proper limit to get decoupled theories.

One final comment is that although the matrix model is decoupled from the original bulk string theory it is supposed to regenerate the supergravity metric that we are studying as a classical configuration of its moduli space. In this space one should then see the Hawking radiation etc that was suppressed in the original (string) theory. It is easily seen that the corresponding situation is obtained by using \((0.20), (0.21)\) to rewrite the metric as

\[
\begin{align*}
    ds^2_m &\equiv \frac{l_m^4}{l_s^4}ds^2 = -(1 - \frac{r^2_{m0}}{r^2_m})dt^2_m \\
    &\quad + (1 + \frac{k_m l^2_m}{r^2_m})\left(\frac{dr^2_m}{1 - \frac{r^2_{m0}}{r^2_m}} + r^2_m d\Omega_3^2\right) + dy^2_{m6}
\end{align*}
\]

where we have defined \(y_m = \frac{l^2_s}{l^2_m} y\) and \(k_m = \frac{k}{l^2_m}\). Effectively now the original string length (which goes to zero) is now replaced by \(l_m\) which remains finite in the limit. Thus there is now a finite temperature \((0.17)\) and a corresponding non-vanishing microscopic coupling to the bulk fields which are now the bulk fields of the regenerated supergravity coming from the matrix model. All this of course assumes that the matrix model moduli space correctly reproduces supergravity.

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