Modeling and simulation of fractional order COVID-19 model with quarantined-isolated people

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Communicated by: M. Efendiev

The dynamics of diseases and effectiveness of control policies play important role in the prevention of epidemic diseases. To this end, this paper is concerned with the design of fractional order coronavirus disease (COVID-19) model with Caputo Fabrizio fractional derivative operator of order $\Omega \in (0, 1]$ for the COVID-19. Verify the nonnegative special solution and convergence of the scheme with in the domain. Caputo-Fabrizio technique apply with Sumudu transformation method is used to solve the fractional order COVID-19 model. Fixed point theory and Picard Lindelof approach are used to provide the stability and uniqueness of the results. Numerical simulations conspicuously demonstrate that by applying the proposed fractional order model, governments could find useful and practical ways for control of the disease.

KEYWORDS
Banach space, fractional order COVID-19 model, Picard Lindelof approach, uniqueness

MSC CLASSIFICATION
17C65; 26A33

1 | INTRODUCTION

Epidemiological study plays an important role to understand the impact of infectious disease in a community. In mathematical modeling, we check models, estimation of parameters, measure sensitivity through different parameters, and compute their numerical simulations with the help of building models. This type of research is used to control parameters and ratio spread of disease.1 These types of diseased models are often called infectious diseases (i.e., the disease which transferred from one person to another person). Measles, rubella, chicken pox, mumps, aids, and gonorrhea syphilis are the examples of infectious disease.2

Severe acute respiratory syndrome (SARS) is caused by a coronavirus and plays important role in the investigation of coronavirus.3 According to the group of investigators that has been working since 30 years on the coronavirus family, they investigated that SARS and coronavirus have many similar features like biology and pathogenesis.4 RNA enveloped viruses known as coronavirus are spread among humans, mammals, and birds. Many respiratory, enteric, hepatic, and neurological diseases are caused by coronavirus.5 Human disease is caused by six types of coronavirus.6 In 2019, China faced a major outbreak of coronavirus disease 2019 (COVID-19), and this outbreak has the potential to become a worldwide pandemic. Interventions and real-time data are needed for the control on this outbreak of coronavirus.7 In previous studies, the transfer of the virus from one person to another person, its severity, and history of the pathogen in the first week of the outbreak have been explained with the help of real-time analyses. In December 2019, a group of people in Wuhan admitted to the hospital, and they all were suffering from pneumonia, and the cause of pneumonia was idiopathic. Most of the people believed that eating of wet markets meet and seafood was caused of pneumonia. Investigation on etiology and epidemiology of disease was conducted on December 31, 2019, by Chinese Center for Disease Control and Prevention
ASLAM ET AL. (China CDC) with the help of Wuhan city health authorities. Epidemically changing was measured by time-delay distributions including date of admission to hospital and death. According to the clinical study on the COVID-19, symptoms of coronavirus appear after 7 days of onset of illness. The time from hospital admission to death is also critical to the avoidance of underestimation when calculating case fatality risk.

The fractional order that involves integration and transects differentiation with the help of fractional calculus is used to help in better understanding of explanation of real-world problems than ordinary integer order also in the modeling of real phenomena due to a characterization of the memory and hereditary properties. The idea of fractional derivative is introduced by Riemann Liouville which is based on power law. The new fractional derivative which is utilizing the exponential kernel is proposed in Caputo and Fabrizio after Caputo-Frabrizio. A new fractional derivative with a nonsingular kernel involving exponential and trigonometric functions is proposed in previous studies, and some related new approaches for epidemic models are illustrated here. Important results related to this new operator are established and some examples are provided in Baleanu et al.

The rest of the paper is organized as follows: in Section 2, we give the analysis of the mathematical model of COVID-19 with quarantine and isolation period. In Section 3, Caupto Fabrizio techniques with Sumudu Transform are used for simulation and numerical solution. In Section 4, fixed point theorems are used for stability analysis and uniqueness of model. In Section 5, we discuss on the solution obtained by purposed scheme. Finally, in Section 6, we give the conclusion of this study.

2 | PRELIMINARIES

Let \( H^1(a, b) = \{ f \mid f \in L^2(a, b) and f' \in L^2(a, b) \} \), where \( L^2(a, b) \) is the space of square integrable function on the interval \((a, b)\).

**Definition 1.** Let \( \Theta \in H^1(c, d) \), \( d > c \), \( \Omega \in (0, 1) \), then the new fractional order in Caputo derivative sense is as follows:

\[
\mathcal{D}_t^\Omega \Theta(t) = \frac{N(\Omega)}{1 - \Omega} \int_c^t \Theta'(y) \exp \left[ \frac{-\Omega t - y}{1 - \Omega} \right] dy,
\]

where \( N(\Omega) \) represents the normalization function with \( N(0) = N(1) = 1 \).

But, if the function does not belong to \( H^1(c, d) \), then the derivative can be write as

\[
\mathcal{D}_t^\Omega \Theta(t) = \frac{\Omega N(\Omega)}{1 - \Omega} \int_c^t (\Theta(t) - \Theta(y)) \exp \left[ -\Omega \frac{t - y}{1 - \Omega} \right] dy.
\]

**Remark 1.** If we take \( \sigma = \frac{1 - \Omega}{\Omega} \in [0, \infty) \), then the new Caputo derivative having fractional order as

\[
\mathcal{D}_t^\Omega \Theta(t) = \frac{N(\sigma)}{\sigma} \int_c^t \Theta'(y) \exp \left[ -\frac{t - y}{1 - \sigma} \right] dy, N(0) = N(\infty) = 1.
\]

Further, \( \lim_{\sigma \to 0} \frac{1}{\sigma} \exp \left[ -\frac{t - y}{\sigma} \right] = \delta(y - t) \).

**Definition 2.** Let \( \Omega \in (0, 1) \), then the fractional integral of order \( \Omega \) of a function \( \Theta(t) \) is given by

\[
\mathcal{I}_t^{\Omega} \Theta(t) = \frac{2(1 - \Omega)}{2 - \Omega} \frac{\Theta(t)}{N(\Omega)} + \frac{2\Omega}{(2 - \Omega)N(\Omega)} \int_0^t \Theta(s) ds, t \geq 0.
\]

**Remark 2.** This definition indicates that the fractional integral of the Caputo form of function of order \( 0 < \Omega \leq 1 \) is an average of \( \Theta \) function and its integral of order one. Hence, we get

\[
\frac{2(1 - \Omega)}{(2 - \Omega)N(\Omega)} + \frac{2\Omega}{(2 - \Omega)N(\Omega)} = 1.
\]
This equation gives an explicit formula for

\[ N(\Omega) = \frac{2}{2 - \Omega}, \quad 0 \leq \Omega \leq 1. \]

Assuming this derivative, the new Caputo derivative of order \(0 < \Omega < 1\) has been reformulated as

\[ CD^\Omega_t(\Theta(t)) = \frac{1}{1 - \Omega} \int_0^t \Theta'(y) \exp \left[-\Omega \frac{t - y}{1 - \Omega} \right] dy. \]

**Definition 3.** Let \(\Theta(t)\) be a function for which the Caputo-Fabrizio exists, then the Sumudu transform of the Caputo-Fabrizio fractional derivative of \(\Theta(t)\) is given as\(^{10}\)

\[ S\left(\frac{CD^\Omega_t(\Theta(t))}{\Theta(t)}\right) = M(\Omega) \frac{S(\Theta(t)) - \Theta(0)}{1 - \Omega + \Omega \Delta}, \]

where the normalization function is denoted by \(M(\Omega)\) with \(M(0) = M(1) = 1\).

## 3 | MATHEMATICAL MODEL

The populations are classified into eight compartments: susceptible population \((S)\), exposed people \((E)\), asymptomatic people \((A)\), infected people with symptoms \((I)\), hospitalise \((H)\), and recovered \((R)\) people. Also, in this model, quarantined susceptible people and isolated exposed people are represented by \(S_q\) and \(E_q\), respectively.\(^{27,28}\)

\[
\begin{align*}
\frac{dS}{dt} &= -\left(\beta c + cq(1 - \beta)\right)S(I + \theta A) + \lambda S_q, \\
\frac{dE}{dt} &= \beta c(1 - q)S(I + \theta A) - \sigma E, \\
\frac{dI}{dt} &= \sigma \phi E - (\delta_1 + \alpha + \gamma_1)I, \\
\frac{dA}{dt} &= \sigma (1 - \phi)E - \gamma AA, \\
\frac{dS_q}{dt} &= (1 - \beta)cqS(I + \theta A) - \lambda S_q, \\
\frac{dE_q}{dt} &= \beta cqS(I + \theta A) - S_q E_q, \\
\frac{dH}{dt} &= \delta_1 I + \delta_2 E_q - (\alpha + \gamma_H)H, \\
\frac{dR}{dt} &= \gamma_1 I + \gamma_A A + \gamma_H H,
\end{align*}
\]

where \(c\) is contact rate, \(\beta\) is the probability of transmission per contact, \(q\) quarantined rate of exposed individuals, and \(\theta\) is transition rate of exposed individuals to the presymptomatic. Moreover, \(\sigma\) denotes the transition rate of exposed individuals to the infected class. \(\lambda\) indicates the rate at which the quarantined uninfected contacts were released into the wider community. \(\phi\) stands to the probability of having symptoms among infected individuals. \(\alpha\) is the disease-induced death rate. \(\delta_1\) is the transition rate of symptomatic infected individuals to the quarantined infected class. \(\delta_2\) is the transition rate of quarantined exposed individuals to the quarantined infected class. Also \(\gamma_1, \gamma_A,\) and \(\gamma_H\) denote the recovery rate of infected individuals, asymptomatic infected individuals, and quarantined infected individuals.\(^{28}\) The fractional order nonlinear COVID-19 model is given as
If we want to get the classical model then we will take \( \Omega = 1 \). By Sumudu transform, some results are obtained. By applying fixed point theorem, existence of solution is discussed. We have also proved the uniqueness of solution.

### 4 | CAPUTO-FABRIZIO DERIVATIVE FOR COVID-19 MODEL

By using the definition of Caputo Fabrizio technique with Sumudu Transform is given as

\[
\begin{align*}
S(t) &= S(0) + \frac{1}{M(\Omega)} \left[ -\lambda S_q \right], \\
E(t) &= E(0) + \frac{1}{M(\Omega)} \left[ -\beta c q (1 - \beta) S(I + \theta A) - \gamma H T - \gamma A A \right], \\
I(t) &= I(0) + \frac{1}{M(\Omega)} \left[ -\delta_1 I - \gamma H H - \lambda S_q \right], \\
A(t) &= A(0) + \frac{1}{M(\Omega)} \left[ -\sigma q E - \gamma H H - \gamma A A \right], \\
S_q(t) &= S_q(0) + \frac{1}{M(\Omega)} \left[ -\lambda S_q \right], \\
E_q(t) &= E_q(0) + \frac{1}{M(\Omega)} \left[ -\beta c q (1 - \beta) S(I + \theta A) - \gamma H T - \gamma A A \right], \\
H(t) &= H(0) + \frac{1}{M(\Omega)} \left[ -\delta_1 I - \gamma H H - \lambda S_q \right], \\
R(t) &= R(0) + \frac{1}{M(\Omega)} \left[ -\gamma H H - \gamma A A \right].
\end{align*}
\]
Using inverse Sumudu transform on system (3), we get

\[
S(t) = S(0) + S^{-1} \left[ \frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)} S[-(\beta c + cq(1 - \beta))S(I + \theta A) + \lambda S_q] \right],
\]

\[
E(t) = E(0) + S^{-1} \left[ \frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)} S[\beta c(1 - q)S(I + \theta A) - \sigma E] \right],
\]

\[
I(t) = I(0) + S^{-1} \left[ \frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)} S[\sigma \rho E - (\delta_1 + \alpha + \gamma_1)I] \right],
\]

\[
A(t) = A(0) + S^{-1} \left[ \frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)} S[\sigma (1 - \rho)E - \gamma_A A] \right],
\]

\[
S_q(t) = S_q(0) + S^{-1} \left[ \frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)} S[(1 - \beta)cqS(I + \theta A) - \lambda S_q] \right],
\]

\[
E_q(t) = E_q(0) + S^{-1} \left[ \frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)} S[\beta cqS(I + \theta A) - S_q E_q] \right],
\]

\[
H(t) = H(0) + S^{-1} \left[ \frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)} S[\delta_1 I + \delta_q E_q - (\alpha + \gamma_H)H] \right],
\]

\[
R(t) = R(0) + S^{-1} \left[ \frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)} S[\gamma_1 I + \gamma_A A + \gamma_H H] \right].
\]

The following recursive formula is given

\[
S_{n+1}(t) = S_n(0) + S^{-1} \left[ \frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)} S[-(\beta c + cq(1 - \beta))S(I + \theta A) + \lambda S_q] \right],
\]

\[
E_{n+1}(t) = E_n(0) + S^{-1} \left[ \frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)} S[\beta c(1 - q)S(I + \theta A) - \sigma E] \right],
\]

\[
I_{n+1}(t) = I_n(0) + S^{-1} \left[ \frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)} S[\sigma \rho E - (\delta_1 + \alpha + \gamma_1)I] \right],
\]

\[
A_{n+1}(t) = A_n(0) + S^{-1} \left[ \frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)} S[\sigma (1 - \rho)E - \gamma_A A] \right],
\]

\[
S_{q(n+1)}(t) = S_{q(n)}(0) + S^{-1} \left[ \frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)} S[(1 - \beta)cqS(I + \theta A) - \lambda S_q] \right],
\]

\[
E_{q(n+1)}(t) = E_{q(n)}(0) + S^{-1} \left[ \frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)} S[\beta cqS(I + \theta A) - S_q E_q] \right],
\]

\[
H_{n+1}(t) = H_n(0) + S^{-1} \left[ \frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)} S[\delta_1 I + \delta_q E_q - (\alpha + \gamma_H)H] \right],
\]

\[
R_{n+1}(t) = R_q(0) + S^{-1} \left[ \frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)} S[\gamma_1 I + \gamma_A A + \gamma_H H] \right].
\]

and the solution of (4) is given by

\[
S(t) = \lim_{n \to \infty} S_n(t), E(t) = \lim_{n \to \infty} E_n(t),
I(t) = \lim_{n \to \infty} I_n(t), A(t) = \lim_{n \to \infty} A_n(t),
S_q(t) = \lim_{n \to \infty} S_{q(n)}(t), E_q(t) = \lim_{n \to \infty} E_{q(n)}(t),
H(t) = \lim_{n \to \infty} H_n(t), R(t) = \lim_{n \to \infty} R_n(t).
\]

### 4.1 Stability analysis of iteration method with fixed point theorem

Let suppose \( X_{11,||} \) as a Banach space and \( P \) as self map of \( X_1 \). Let \( y_{n+1} = g(P, y_n) \) be particular recursive procedure. Suppose that \( F(P) \) the fixed point set of \( P \) has at least one element that \( y_n \) converges to a point \( P \in F(P) \). Let \( \{x_n \subseteq X_1\} \) and define
If \( \lim_{n \to \infty} e^n = 0 \) implies that \( \lim_{n \to \infty} x^n = P \), then the iteration technique \( y_{n+1} = g(P, y_n) \) is said to be P-stable. The iteration will be P-stable, if all these conditions are fulfilled for \( y_{n+1} = P y_n \) which is also known as Picard's iteration.

**Theorem 1.** Let \( (Z_1, \| \|) \) be a Banach space and \( P \) be a self-map of \( Z_1 \) satisfying \( \| Pz - P \| \leq C \| z - Pz \| + c \| z - y \| \) for all \( z, y \in Z_1 \) where \( 0 \leq C, 0 \leq c < 1 \). Suppose that \( P \) is picard P-stable. Let the following recursive formula from (4) connected to (2).

\[
\begin{align*}
S_{n+1}(t) &= S_n(0) + S^{-1} \left[ \frac{1 - \Omega + \Omega \Delta}{M(\Omega)} S[ - (\beta c + cq(1 - \beta)) S(I + \theta A) + \lambda S_q] \right], \\
E_{n+1}(t) &= E_n(0) + S^{-1} \left[ \frac{1 - \Omega + \Omega \Delta}{M(\Omega)} S[\beta c(1 - q) S(I + \theta A) - \sigma E] \right], \\
I_{n+1}(t) &= I_n(0) + S^{-1} \left[ \frac{1 - \Omega + \Omega \Delta}{M(\Omega)} S[\sigma \theta E - (\delta_1 + \alpha + \gamma_1) I] \right], \\
A_{n+1}(t) &= A_n(0) + S^{-1} \left[ \frac{1 - \Omega + \Omega \Delta}{M(\Omega)} S[\sigma (1 - \varphi) E - \gamma_A A] \right], \\
S_{q(n+1)}(t) &= S_{q(n)}(0) + S^{-1} \left[ \frac{1 - \Omega + \Omega \Delta}{M(\Omega)} S[1 - \beta) cq S(I + \theta A) - \lambda S_q] \right], \\
E_{q(n+1)}(t) &= E_{q(n)}(0) + S^{-1} \left[ \frac{1 - \Omega + \Omega \Delta}{M(\Omega)} S[\beta cq S(I + \theta A) - S_q E_q] \right], \\
H_{n+1}(t) &= H_n(0) + S^{-1} \left[ \frac{1 - \Omega + \Omega \Delta}{M(\Omega)} S[\delta I + \delta_q E_q - (\alpha + \gamma_H) H] \right], \\
R_{n+1}(t) &= R_n(0) + S^{-1} \left[ \frac{1 - \Omega + \Omega \Delta}{M(\Omega)} S[\gamma_1 I + \gamma_A A + \gamma_H H] \right],
\end{align*}
\]

where \( \frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)} \) is the fractional Lagrange multiplier.

**Theorem 2.** Let us define a self-map \( P \) as

\[
\begin{align*}
P(S_n(t)) &= S_{n+1}(t) = S_n(0) + S^{-1} \left[ \frac{1 - \Omega + \Omega \Delta}{M(\Omega)} S[ - (\beta c + cq(1 - \beta)) S(I + \theta A) + \lambda S_q] \right], \\
P(E_n(t)) &= E_{n+1}(t) = E_n(0) + S^{-1} \left[ \frac{1 - \Omega + \Omega \Delta}{M(\Omega)} S[\beta c(1 - q) S(I + \theta A) - \sigma E] \right], \\
P(I_n(t)) &= I_{n+1}(t) = I_n(0) + S^{-1} \left[ \frac{1 - \Omega + \Omega \Delta}{M(\Omega)} S[\sigma \theta E - (\delta_1 + \alpha + \gamma_1) I] \right], \\
P(A_n(t)) &= A_{n+1}(t) = A_n(0) + S^{-1} \left[ \frac{1 - \Omega + \Omega \Delta}{M(\Omega)} S[\sigma (1 - \varphi) E - \gamma_A A] \right], \\
P(S_{q(n)}(t)) &= S_{q(n+1)}(t) = S_{q(n)}(0) + S^{-1} \left[ \frac{1 - \Omega + \Omega \Delta}{M(\Omega)} S[1 - \beta) cq S(I + \theta A) - \lambda S_q] \right], \\
P(E_{q(n)}(t)) &= E_{q(n+1)}(t) = E_{q(n)}(0) + S^{-1} \left[ \frac{1 - \Omega + \Omega \Delta}{M(\Omega)} S[\beta cq S(I + \theta A) - S_q E_q] \right], \\
P(H_n(t)) &= H_{n+1}(t) = H_n(0) + S^{-1} \left[ \frac{1 - \Omega + \Omega \Delta}{M(\Omega)} S[\delta I + \delta_q E_q - (\alpha + \gamma_H) H] \right], \\
P(R_n(t)) &= R_{n+1}(t) = R_n(0) + S^{-1} \left[ \frac{1 - \Omega + \Omega \Delta}{M(\Omega)} S[\gamma_1 I + \gamma_A A + \gamma_H H] \right].
\end{align*}
\]
The system is P-stable in $L^1(a, b)$ if

\[
(1 - (M_1 + M_2) \frac{\beta_i}{N} f(\gamma) - (M_2 + M_3) \frac{\beta_k}{N} g(\gamma) - (M_2 + M_4) \frac{\beta_d}{N} h(\gamma) \\
- (M_2 + M_3) \frac{\beta_i}{N} I(\gamma) - \mu j(\gamma)) < 1,
\]

\[
(1 + (M_1 + M_2) \frac{\beta_i}{N} f(\gamma) + (M_2 + M_3) \frac{\beta_k}{N} g(\gamma) + (M_2 + M_4) \frac{\beta_d}{N} h(\gamma) \\
+ (M_2 + M_3) \frac{\beta_i}{N} I(\gamma) - \sigma j(\gamma) - \mu j(\gamma)) < 1,
\]

\[
\{1 + \sigma f_1(\gamma) - \nu_1 g_1(\gamma) - c h_1(\gamma) - \tau I_1(\gamma) - \mu j(\gamma)\} < 1,
\]

\[
\{1 + \nu_1 g_1(\gamma) + \nu_2 g_2(\gamma) - \nu_3 g_3(\gamma) - \mu j(\gamma)\} < 1,
\]

\[
\{1 + \nu_1 g_1(\gamma) + \nu_2 g_2(\gamma) - \xi k(\gamma)\} < 1,
\]

\[
\{1 + \nu_3 g_3(\gamma)\} < 1.
\]

**Proof.** Firstly, we have to show that $P$ has a fixed point. For this, we evaluate the following. $\forall (m, n) \in N \times N$

\[
P(S_n(t)) - P(S_m(t)) = S_n(t) - S_m(t) + S^{-1} \left[ \frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)} S[-(\beta c + cq(1 - \beta))S_n(I_n + \theta A_n) + \lambda S_{q_m(n)}] \right]
\]

\[
- S^{-1} \left[ \frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)} S[-(\beta c + cq(1 - \beta))S_m(I_m + \theta A_m) + \lambda S_{q_m(n)}] \right],
\]

\[
P(E_n(t)) - P(E_m(t)) = E_n(t) - E_m(t) + S^{-1} \left[ \frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)} S[\beta c(1 - q)S_n(I_n + \theta A_n) - \sigma E_n] \right]
\]

\[
- S^{-1} \left[ \frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)} S[\beta c(1 - q)S_m(I_m + \theta A_m) - \sigma E_m] \right],
\]

\[
P(I_n(t)) - P(I_m(t)) = I_n(t) - I_m(t) + S^{-1} \left[ \frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)} S[\sigma q E_n - (\delta_1 + \alpha + \gamma_1)I_n] \right]
\]

\[
- S^{-1} \left[ \frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)} S[\sigma q E_m - (\delta_1 + \alpha + \gamma_1)I_m] \right],
\]

\[
P(A_n(t)) - P(A_m(t)) = A_n(t) - A_m(t) + S^{-1} \left[ \frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)} S[\sigma(1 - \phi)E_n - \gamma A_n] \right]
\]

\[
- S^{-1} \left[ \frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)} S[\sigma(1 - \phi)E_m - \gamma A_m] \right],
\]

\[
P(S_{q_m(n)}(t)) - P(S_{q_m(m)}(t)) = S_{q_m(n)}(t) - S_{q_m(m)}(t) + S^{-1} \left[ \frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)} S[(1 - \beta)cq S_n(I_n + \theta A_n) - \lambda S_{q_m(n)}] \right]
\]

\[
- S^{-1} \left[ \frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)} S[(1 - \beta)cq S_m(I_m + \theta A_m) - \lambda S_{q_m(m)}] \right].
\]

**Proof.** Firstly, we have to show that $P$ has a fixed point. For this, we evaluate the following. $\forall (m, n) \in N \times N$
By taking norm on both sides of the first equation of (6), we get

\[
P(E_{q(n)}(t)) - P(E_{q(m)}(t)) = E_{q(n)}(t) - E_{q(m)}(t) + S^{-1} \left[ \frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)} S[\beta c q S_n(I_n + \theta A_n) - S_{q(n)}E_{q(n)}] \right]
\]

\[
- S^{-1} \left[ \frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)} S[\beta c q S_m(I_m + \theta A_m) - S_{q(m)}E_{q(m)}] \right].
\]

\[
P(H_n(t)) - P(H_m(t)) = H_n(t) - H_m(t) + S^{-1} \left[ \frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)} S[\delta_1 I + \delta_q E_{q(n)} - (\alpha + \gamma_I)H_n] \right]
\]

\[
- S^{-1} \left[ \frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)} S[\delta_1 I + \delta_q E_{q(m)} - (\alpha + \gamma_I)H_m] \right].
\]

\[
P(R_n(t)) - P(R_m(t)) = R_n(t) - R_m(t) + S^{-1} \left[ \frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)} S[\gamma_I I_n + \gamma_A A_n + \gamma_I H_n] \right]
\]

\[
- S^{-1} \left[ \frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)} S[\gamma_I I_m + \gamma_A A_m + \gamma_I H_m] \right].
\]

By taking norm on both sides of the first equation of (6), we get

\[
\|P(S_n(t)) - P(S_m(t))\| = \|S_n(t) - S_m(t)\|
\]

\[
+ S^{-1} \left[ \frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)} S[\beta c + cq(1 - \beta)]S_n(I_n + \theta A_n) + \lambda S_{q(n)} \right]
\]

\[
- S^{-1} \left[ \frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)} S[\beta c + cq(1 - \beta)]S_m(I_m + \theta A_m) + \lambda S_{q(m)} \right].
\]

By triangular inequality, this equation becomes

\[
\|P(S_n(t)) - P(S_m(t))\| \leq \|S_n(t) - S_m(t)\|
\]

\[
+ \|S^{-1} \left[ \frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)} S[\beta c + cq(1 - \beta)]S_n(I_n + \theta A_n) + \lambda S_{q(n)} \right]
\]

\[
- S^{-1} \left[ \frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)} S[\beta c + cq(1 - \beta)]S_m(I_m + \theta A_m) + \lambda S_{q(m)} \right]\|.
\]

By more simplification, we get

\[
\|P(S_n(t)) - P(S_m(t))\| \leq \|S_n(t) - S_m(t)\|
\]

\[
+ S^{-1}[S\left(\frac{(1 - \Omega + \Omega \Delta)}{M(\Omega)}\right) || - (\beta c + cq(1 - \beta))S_n(I_n - I_m)||
\]

\[
+ || - (\beta c + cq(1 - \beta))S_n(S_n - S_m)|| + || - (\beta c + cq(1 - \beta))S_m(S_n - S_m)||
\]

\[
+ || - (\beta c + cq(1 - \beta))A_n(S_n - S_m)|| + ||(S_{q(n)} - S_{q(m)})||.
\]

Because both solutions have the same role, we can assume that

\[
\|S_n(t) - S_m(t)\| \cong ||I_n(t) - I_m(t)||.
\]

\[
||S_n(t) - S_m(t)|| \cong ||A_n(t) - A_m(t)||.
\]

\[
||S_n(t) - S_m(t)|| \cong ||S_{q(n)}(t) - S_{q(m)}(t)||.
\]
By putting this in Equation (9), we have

\[
\|P(S_n(t)) - P(S_m(t))\| \leq \|S_n(t) - S_m(t)\| \\
+ S^{-1}[S\left(1 - \Omega + \Omega\Delta\right)\|1 - (\beta c + cq(1 - \beta))S_n - S_m\|] \\
+ \| - (\beta c + cq(1 - \beta))I_m(S_n - S_m)\| + \| - \theta(\beta c + cq(1 - \beta))A_m(S_n - S_m)\| \\
+ \| \lambda(\beta c + cq(1 - \beta))/(S_n - S_m)\|. 
\]

(11)

Since \(S_n, I_m, AND A_m\) are bounded, therefore, we have three positive constants \(M_1, M_2, M_3\) \(\forall t\) such that \(\|S_n\| < M_1, \|I_m\| < M_2, \|A_m\| < M_3\) \(\forall (m, n) \in N \times N\).

Substitute in Equation (10), we get

\[
\|P(S_n(t)) - P(S_m(t))\| \leq (1 - (M_1 + M_2)(\beta c + cq(1 - q))g(\gamma) \\
- (M_1 + M_2)\theta(\beta c + cq(1 - q))h(\gamma))\|S_n - S_m\|. 
\]

(12)

Here \(f, g, \) and \(h\) are functions from \(S^{-1} \left[ S(1 - \Omega + \Omega\Delta) \right] \).

Similarly, we can get

\[
\|P(E_n(t)) - P(E_m(t))\| \leq (1 - \sigma f(\gamma) + (M_1 + M_2)(\beta c(1 - q))g(\gamma) \\
- (M_1 + M_3)\theta(\beta c(1 - q))h(\gamma))\|S_n - S_m\|, 
\]

\[
\|P(I_n(t)) - P(I_m(t))\| \leq \{1 + \sigma \phi f_1(\gamma) - (\sigma_1 + \alpha + \gamma_1)g_1(\gamma)\} \|S_n - S_m\|, 
\]

\[
\|P(A_n(t)) - P(A_m(t))\| \leq \{1 + \sigma(1 - \phi)g_1(\gamma) - \gamma A g_2\} \|S_n - S_m\|, 
\]

\[
\|P(S_{\bar{q}(in)}(t)) - P(S_{\bar{q}(im)}(t))\| \leq (1 - \lambda f_2(\gamma) + (M_1 + M_2)(cq(1 - \beta))g_2(\gamma) \\
+ (M_1 + M_3)\theta(cq(1 - \beta))h_2(\gamma))\|S_n - S_m\|, 
\]

\[
\|P(E_{\bar{q}(in)}(t)) - P(E_{\bar{q}(im)}(t))\| \leq (1 + \beta cq(M_1 + M_2)f(\gamma) + \beta cq(M_1 + M_3)\theta(\beta c(1 - q))g(\gamma) \\
- M_4f(\gamma) - M_5k(\gamma))\|S_n - S_m\|, 
\]

\[
\|P(H_n(t)) - P(H_m(t))\| \leq (1 + \sigma_1 f(\gamma) + \sigma g(\gamma) - (\alpha + \gamma H)h(\gamma))\|S_n - S_m\|, 
\]

\[
\|P(R_n(t)) - P(R_m(t))\| \leq (1 + \gamma_1 f(\gamma) + \gamma A g(\gamma) + \gamma H h(\gamma))\|S_n - S_m\|, 
\]

(13)

where

\[
(1 - (M_1 + M_2)(\beta c + cq(1 - \beta))f(\gamma) - (M_1 + M_3)\theta(\beta c + cq(1 - \beta)) < 1, 
\]

\[
(1 - \sigma f(\gamma) + (M_1 + M_2)(\beta c(1 - q))g(\gamma) + (M_1 + M_3)\theta(\beta c(1 - q))h(\gamma)) < 1, 
\]

\[
\{1 + \sigma \phi f_1(\gamma) - (\sigma_1 + \alpha + \gamma_1)g_1(\gamma)\} < 1, 
\]

(14)

\[
\{1 + \sigma(1 - \phi)g_1(\gamma) - \gamma A g_2\} < 1, 
\]

\[
(1 - \lambda f_2(\gamma) + (M_1 + M_2)(cq(1 - \beta))g_2(\gamma) + (M_1 + M_3)\theta(cq(1 - \beta))h_2(\gamma)) < 1, 
\]
Uniqueness of the special solution

The system (2) has an exact solution by which the special solution converges for a large number. Here, by using iteration method, we will show the uniqueness of special solution of Equation (2). We suppose that

\[ (1 + \beta c)(M_1 + M_2)f(y) + \beta cq(M_1 + M_3)g(y) - M_4 j(y) - M_5 k(y) < 1, \]

\[ (1 + \sigma_1 f(y) + \sigma_2 g(y) - (\alpha + \gamma_H)h(y)) < 1, \]

\[ (1 + \gamma_1 f(y) + \gamma_2 g(y) + \gamma_H h(y)) < 1. \]

Thus, the nonlinear P-self mapping has a fixed point. Now, we will show that P satisfies the condition in Theorem 4.2. Let (11) and (12) hold and therefore using

\[ C = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0) \]

\[ P(S, E, I, A, S_q, E_q, H, R) = \begin{cases} 
-(\beta c + cq(1 - \beta))S(I + \theta A) + \lambda S_q, \\
\beta c(1 - q)S(I + \theta A) - \sigma E, \\
\sigma A - (\sigma_1 + \alpha + \gamma_1)I, \\
(1 - \sigma)E - \gamma_A A, \\
(1 - \beta)cqS(I + \theta A) - \lambda S_q, \\
\beta c q(I + \theta A) - S_q E_q, \\
\delta_1 I + \delta_q E_q - (\alpha + \gamma_H)H, \\
\gamma_1 I + \gamma_2 A + \gamma_H H. 
\end{cases} \]

The above condition shows that the condition of Theorem 4.1 exist for the nonlinear mapping P. Thus, all the condition in Theorem 4.2 exist for the nonlinear mapping P. Hence, P is Picard P-stable.

4.2 | Uniqueness of the special solution

Here, by using iteration method, we will show the uniqueness of special solution of Equation (2). We suppose that system (2) has an exact solution by which the special solution converges for a large number \( m \). Hilbert space \( H = L(a, b) \times (0, T) \) is defined by \( y : (a, b) \times (0, T) \rightarrow R \) such that \( \int \int \int y \, dx \, dy < \infty \). We have the following operator

\[ P((Z_{11} - Z_{12}, Z_{21} - Z_{22}, Z_{31} - Z_{32}, Z_{41} - Z_{42}, Z_{51} - Z_{52}, \ldots), (W_1, W_2, W_3, \ldots, W_8)) \]

\[ Z_{61} - Z_{62}, Z_{71} - Z_{72}, Z_{81} - Z_{82}, (W_1, W_2, W_3, \ldots, W_8), \]

where \((Z_{11} - Z_{12}), (Z_{21} - Z_{22}), \ldots, (Z_{31} - Z_{32})\) are special solution for subpopulation \( S, E, I, A, S_q, E_q, H, R \) respectively of the system. However,

\[ P((Z_{11} - Z_{12}, Z_{21} - Z_{22}, Z_{31} - Z_{32}, \ldots, Z_{81} - Z_{82}), (W_1, W_2, W_3, \ldots, W_8)) \]

\[ P((Z_{11} - Z_{12}, Z_{21} - Z_{22}, Z_{31} - Z_{32}, \ldots, Z_{81} - Z_{82}), (W_1, W_2, W_3, \ldots, W_8)) \]

\[ = \begin{cases} 
(\lambda(Z_{51} - Z_{52}) - (\beta c + cq(1 - \beta))(Z_{11} - Z_{12})(Z_{31} - Z_{32}) + \theta((Z_{11} - Z_{12})(Z_{41} - Z_{42})), W_1), \\
(-\sigma(Z_{21} - Z_{22}) + (\beta c(1 - q))(Z_{11} - Z_{12})(Z_{31} - Z_{32}) + \theta((Z_{11} - Z_{12})(Z_{41} - Z_{42})), W_2), \\
(\sigma(X_{21} - Z_{22}) - (\sigma_1 + \alpha + \gamma_1)(Z_{11} - Z_{12}), W_3), \\
(\sigma(1 - \phi)(Z_{21} - Z_{22}) + \gamma_A(Z_{41} - Z_{42}), W_4), \\
(-\beta c q(Z_{11} - Z_{12})(Z_{31} - Z_{32}) + \theta((Z_{11} - Z_{12})(Z_{41} - Z_{42})), W_5), \\
(\beta c q((Z_{11} - Z_{12})(Z_{31} - Z_{32}) + \theta((Z_{11} - Z_{12})(Z_{41} - Z_{42})), W_6), \\
(\sigma_1(Z_{31} - Z_{32}) + \gamma_A(Z_{41} - Z_{42}) - (\alpha + \gamma_H)(Z_{71} - Z_{72}), W_7), \\
(\gamma_1(Z_{31} - Z_{32}) - \gamma_A(Z_{41} - Z_{42}) + \gamma_H(Z_{71} - Z_{72}), W_8). 
\end{cases} \]
By evaluating first equation of (15)

\[
(\lambda(Z_{51} - Z_{52}) - (\beta c + cq(1 - \beta))(Z_{11} - Z_{12})(Z_{31} - Z_{32}) + \theta((Z_{11} - Z_{12})(Z_{41} - Z_{42})), W_1) \\
\equiv (\lambda(Z_{51} - Z_{52}), W_1) + (-(\beta c + cq(1 - \beta))(Z_{11} - Z_{12})(Z_{31} - Z_{32}), W_1) \\
+ (\theta((Z_{11} - Z_{12})(Z_{41} - Z_{42})), W_1).
\]

(16)

Since both the solution play same role, we can suppose that

\[
(Z_{11} - Z_{12}) \equiv (Z_{21} - Z_{22}) \equiv (Z_{31} - Z_{32}) \equiv \ldots \equiv (Z_{81} - Z_{82}).
\]

Then Equation (16) becomes

\[
(\lambda(Z_{11} - Z_{12}) - (\beta c + cq(1 - \beta))(Z_{11} - Z_{12})^2 + \theta((Z_{11} - Z_{12})^2)), W_1).
\]

By relationship between norm and inner product, we will get

\[
(\lambda(Z_{11} - Z_{12}) - (\beta c + cq(1 - \beta))(Z_{11} - Z_{12})^2 + \theta((Z_{11} - Z_{12})^2)), W_1) \\
\equiv (\lambda(Z_{11} - Z_{12}), W_1) + ((-\beta c + cq(1 - \beta))(Z_{11} - Z_{12})^2, W_1) + (\theta(Z_{11} - Z_{12})^2), W_1) \\
\leq \lambda\|(Z_{11} - Z_{12})\|W_1\| + ((\beta c + cq(1 - \beta))(Z_{11} - Z_{12})^2\|W_1\| + \theta((Z_{11} - Z_{12})^2\|W_1\| \\
= (\lambda + (\beta c + cq(1 - \beta))\|\sigma_1 + \theta \sigma_1\|(Z_{11} - Z_{12})\|W_1\|.
\]

Repeating the same pattern from second to eighth equation of (15), we find

\[
(-\sigma(Z_{21} - Z_{22}) + (\beta c(1 - q))(Z_{11} - Z_{12})^2 + \theta((Z_{11} - Z_{12})^2)), W_2) \\
\leq (\sigma + (\beta c(1 - q))\|\sigma_2 + \theta \sigma_2\|(Z_{21} - Z_{22})\|W_2\|.
\]

(17)

\[
(\sigma_0(Z_{21} - Z_{22}) - (\sigma_0 + (\sigma_0 + \gamma_1)(Z_{31} - Z_{32})), W_3) \leq (\sigma_0 + (\sigma_0 + \gamma_1))(Z_{31} - Z_{32})\|W_3\|,
\]

\[
(\sigma(1 - \sigma)(Z_{21} - Z_{22}) - \gamma_A(Z_{41} - Z_{42}), W_4) \leq (\sigma(1 - \sigma) + \gamma_A)(Z_{41} - Z_{42})\|W_4\|,
\]

\[
\|((1 - \beta)\|\sigma_5 + \theta \sigma_5\|Z_{51} - Z_{52})\|W_5\|,\]

\[
(\beta cq((Z_{11} - Z_{12})^2 + \theta(Z_{11} - Z_{12})(Z_{41} - Z_{42})) - \lambda(Z_{51} - Z_{52}), W_5) \\
\leq ((1 - \beta)\|cq\sigma_5 + \theta \sigma_5\|, W_5),
\]

\[
(\|\|cq\sigma_5 + \theta \sigma_5\|, W_5) \leq (\beta cq\|\sigma_6 + \theta \sigma_6\|)(Z_{61} - Z_{62})\|W_6\|,
\]

\[
(\sigma_1(Z_{31} - Z_{32}) + \sigma_4(Z_{61} - Z_{62}) - (\alpha + \gamma_l)(Z_{71} - Z_{72}), W_7) \leq (\sigma + \sigma_4 + \alpha + \gamma_l)(Z_{71} - Z_{72})\|W_7\|,
\]

(18)

Putting Equations (17) and (18) in (15), we get

\[
P((Z_{11} - Z_{12}, Z_{21} - Z_{22}, Z_{31} - Z_{32}, \ldots, Z_{81} - Z_{82}), (W_1, W_2, W_3, \ldots, W_8)) \leq
\]

\[
\left\{
\begin{array}{c}
(\lambda + (\beta c + cq(1 - \beta))\|\sigma_1 + \theta \sigma_1\|)(Z_{11} - Z_{12})\|W_1\|, \\
(\sigma + (\beta c(1 - q))\|\sigma_2 + \theta \sigma_2\|)(Z_{21} - Z_{22})\|W_2\|, \\
(\sigma_0 + (\sigma_0 + \gamma_1))(Z_{31} - Z_{32})\|W_3\|, \\
(\sigma(1 - \sigma) + \gamma_A)(Z_{41} - Z_{42})\|W_4\|, \\
((1 - \beta)\|cq\sigma_5 + \theta \sigma_5\|)(Z_{51} - Z_{52})\|W_5\|, \\
(\beta cq\|\sigma_6 + \theta \sigma_6\|)(Z_{61} - Z_{62})\|W_6\|, \\
(\sigma + \sigma_4 + \alpha + \gamma_l)(Z_{71} - Z_{72})\|W_7\|, \\
(\gamma + \gamma_A + \gamma_l)(Z_{81} - Z_{82})\|W_8\|.
\end{array}
\right.
\]

But for sufficiently large values of \( m_i \), with \( i = 1, 2, 3, \ldots, 8 \) both the solution converge to exact solution, using the topological concept, there exist eight very small positive parameters \( l_{m_1}, l_{m_2}, \ldots, l_{m_8} \) such that
\[\begin{align*}
\|S - Z_1\|, \|S - Z_2\| &< \frac{l_{m_1}}{3(\lambda + \beta(c + cq(1 - \beta)) \sigma_1 + \theta \sigma_2)} \|Z_{11} - Z_{12}\| \|W_1\|, \\
\|E - Z_1\|, \|E - Z_2\| &< \frac{l_{m_2}}{3(\lambda + \beta(c + cq(1 - \beta)) \sigma_1 + \theta \sigma_2)} \|Z_{21} - Z_{22}\| \|W_2\|, \\
\|I - Z_1\|, \|I - Z_2\| &< \frac{l_{m_3}}{3(\lambda + \beta(c + cq(1 - \beta)) \sigma_1 + \theta \sigma_2)} \|Z_{31} - Z_{32}\| \|W_3\|, \\
\|A - Z_1\|, \|A - Z_2\| &< \frac{l_{m_4}}{3(\lambda + \beta(c + cq(1 - \beta)) \sigma_1 + \theta \sigma_2)} \|Z_{41} - Z_{42}\| \|W_4\|, \\
\|S_q - Z_1\|, \|S_q - Z_2\| &< \frac{l_{m_5}}{3(\lambda + \beta(c + cq(1 - \beta)) \sigma_1 + \theta \sigma_2)} \|Z_{51} - Z_{52}\| \|W_5\|, \\
\|E_q - Z_1\|, \|E_q - Z_2\| &< \frac{l_{m_6}}{3(\lambda + \beta(c + cq(1 - \beta)) \sigma_1 + \theta \sigma_2)} \|Z_{61} - Z_{62}\| \|W_6\|, \\
\|H - Z_1\|, \|H - Z_2\| &< \frac{l_{m_7}}{3(\lambda + \beta(c + cq(1 - \beta)) \sigma_1 + \theta \sigma_2)} \|Z_{71} - Z_{72}\| \|W_7\|, \\
\|R - Z_1\|, \|R - Z_2\| &< \frac{l_{m_8}}{3(\lambda + \beta(c + cq(1 - \beta)) \sigma_1 + \theta \sigma_2)} \|Z_{81} - Z_{82}\| \|W_8\|. 
\end{align*}\]

By putting the exact solution in the right side of above equation and implementing the triangular inequality by taking \(M = \max(m_1, m_2, m_3, \ldots, m_8)\), \(l = \max(l_{m_1}, l_{m_2}, l_{m_3}, \ldots, l_{m_8})\), we get

\[
\begin{align*}
\begin{cases}
(\lambda + \beta(c + cq(1 - \beta)) \sigma_1 + \theta \sigma_1)\|Z_{11} - Z_{12}\| \|W_1\|, \\
(\sigma + \beta(c(1 - q)) \sigma_2 + \theta \sigma_2)\|Z_{21} - Z_{22}\| \|W_2\|, \\
(\sigma \phi + \sigma_1 + \alpha + \gamma_1)\|Z_{31} - Z_{32}\| \|W_3\|, \\
(\sigma(1 - \phi) + \gamma_A)\|Z_{41} - Z_{42}\| \|W_4\|, \\
((1 - \beta) cq \sigma_5 + \theta \sigma_{5+1})\|Z_{51} - Z_{52}\| \|W_5\|, \\
(\beta cq \sigma_6 + \theta \sigma_6)\|Z_{61} - Z_{62}\| \|W_6\|, \\
(\sigma + \sigma q + \alpha + \gamma_H)\|Z_{71} - Z_{72}\| \|W_7\|, \\
(\gamma + \gamma_A + \gamma_H)\|Z_{81} - Z_{82}\| \|W_8\|.
\end{cases}
\end{align*}
\]

As \(l\) is a very small positive parameter, with the help of topological idea, we get

\[
\begin{align*}
\begin{cases}
(\lambda + \beta(c + cq(1 - \beta)) \sigma_1 + \theta \sigma_1)\|Z_{11} - Z_{12}\| \|W_1\|, \\
(\sigma + \beta(c(1 - q)) \sigma_2 + \theta \sigma_2)\|Z_{21} - Z_{22}\| \|W_2\|, \\
(\sigma \phi + \sigma_1 + \alpha + \gamma_1)\|Z_{31} - Z_{32}\| \|W_3\|, \\
(\sigma(1 - \phi) + \gamma_A)\|Z_{41} - Z_{42}\| \|W_4\|, \\
((1 - \beta) cq \sigma_5 + \theta \sigma_{5+1})\|Z_{51} - Z_{52}\| \|W_5\|, \\
(\beta cq \sigma_6 + \theta \sigma_6)\|Z_{61} - Z_{62}\| \|W_6\|, \\
(\sigma + \sigma q + \alpha + \gamma_H)\|Z_{71} - Z_{72}\| \|W_7\|, \\
(\gamma + \gamma_A + \gamma_H)\|Z_{81} - Z_{82}\| \|W_8\|.
\end{cases}
\end{align*}
\]

But it is clear that

\[
\left(\frac{\beta_i}{N} \sigma_1 + \frac{\beta_h}{N} \sigma_1 + \frac{\beta_q}{N} \sigma_1 + \frac{\beta_r}{N} \sigma_1 + \mu\right) \neq 0\left(\frac{\beta_i}{N} \sigma_2 + \frac{\beta_h}{N} \sigma_2 + \frac{\beta_q}{N} \sigma_2 + \frac{\beta_r}{N} \sigma_2 + (\mu + \sigma)\right) \neq 0,
\]

\((\sigma \phi + \sigma_1 + \alpha + \gamma_1) \neq 0, (\sigma(1 - \phi) + \gamma_A) \neq 0, ((1 - \beta) cq \sigma_5 + \theta \sigma_{5+1}) \neq 0(\beta cq \sigma_6 + \theta \sigma_6) \neq 0\)

\((\sigma + \sigma q + \alpha + \gamma_H) \neq 0, (\gamma + \gamma_A + \gamma_H) \neq 0.\)

Therefore, we get

\[\|Z_{11} - Z_{12}\| = 0, \|Z_{21} - Z_{22}\| = 0, \|Z_{31} - Z_{32}\| = 0, \|Z_{41} - Z_{42}\| = 0, \|Z_{51} - Z_{52}\| = 0, \|Z_{61} - Z_{62}\| = 0, \|Z_{71} - Z_{72}\| = 0, \|Z_{81} - Z_{82}\| = 0\]

which shows that
\[ Z_{11} = Z_{12}, Z_{21} = Z_{22}, Z_{31} = Z_{32}, Z_{41} = Z_{42}, \]
\[ Z_{51} = Z_{52}, Z_{61} = Z_{62}, Z_{71} = Z_{72}, Z_{81} = Z_{82}. \]

Hence, result proved.

5 | RESULTS AND DISCUSSION

Numerical simulation was performed to analyze the control with Caputo Fabrizio fractional technique of fractional COVID-19 virus model. Nonlinear occurrence with mathematical analysis of outbreak of COVID-19 has been presented. Model consists of eight subcompartments according to population and values of parameter are given in other studies.\(^{27,28}\)

By using Caputo Fabrizio fractional derivative, the numerical outcomes of the model for different fractional values of \( \Omega \) are obtained according to steady state point. To observe the effects of factors on the mechanics of the fractional order model, it can be observed at different numerical ways having the value of given parameter with finite time. These simulations reveal that a value change affects the model’s dynamics. Here, we can also observe that the fractional value results are more reliable as compared to classical derivative. It gives better approach to desired value to control the disease. The graphs of the approximate solutions against different fractional order \( \Omega \) are provided as in Figures 1–8. \( S \) and \( E \) start increasing by decreasing the fractional values while infected \( I \) and \( A \) start decreasing by decreasing the fractional values. Similarly, the \( S_q \) and \( E_q \) start increasing by decreasing the fractional values while as \( H \) and \( A \) starts decreasing by decreasing the fractional values. The behavior approaches in all figures to steady state by decreasing the fractional values which predict that the solution will be more efficient by decreasing the fractional values. In the terms of the values of the travel restriction, the decrease contact rate and high level of quarantine is only possible when number of cases from epicenter

**FIGURE 1** Simulation of \( S(t) \) in a time at different fractional parameter values [Colour figure can be viewed at wileyonlinelibrary.com]

**FIGURE 2** Simulation of \( E(t) \) in a time at different fractional parameter values [Colour figure can be viewed at wileyonlinelibrary.com]
is low. Some new results are treated with the support of Sumudu transform and Picard's successive approximation methods. Furthermore, numerical solution of the model represents different noninteger values. The main approach to slowing down the transmission of COVID-19 is to remain home and to put sick individuals in a distant location or a protected place as far as possible. Ultimately, efficient and appropriate care of sick patients must be given, and medicines, tones, and nutrients must be distributed to noninfected individuals to prevent them.
6 | CONCLUSIONS

In this paper, the treatment for COVID-19 is discussed with the combination of nonlinear fractional order and Caputo-Fabrizio. On the bases of this model, nonsingular exponentially decreasing kernels which was appeared in the Caputo-Fabrizio derivation is made. The biomedical COVID-19 model is represented on the basis of theoretical and numerical investigation. The control of COVID-19 disease in the society is represented through this model in the form
of stability, uniqueness, and applicability. The solutions are derived through approaches of fixed point theory and Picard Lindelof. Numerical results are used to analyze effects of different values of fractional order. We have found that fractional order systems show richer dynamics than the classical version of the integer order. In the future, the above research can be applied to more complex physics models. Convergence analysis shows the efficiency and precision of the proposed method. Our results shows that fractional-order models can provide better data turnarounds than integer-order models in some situations.

CONFLICT OF INTEREST
This work does not have any conflicts of interest.

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**How to cite this article:** Aslam M, Farman M, Akgül A, Sun M. Modeling and simulation of fractional order COVID-19 model with quarantined-isolated people. *Math Meth Appl Sci.* 2021;44:6389–6405. [https://doi.org/10.1002/mma.7191](https://doi.org/10.1002/mma.7191)