Thermally assisted vortex motion in intrinsic Josephson junctions

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Abstract. The vortex dynamics in intrinsic Josephson junctions (IJJs) at finite temperatures has been investigated numerically by taking into account the thermal fluctuations. Our simulations based on the perturbed, coupled sine-Gordon model successfully reproduce the experimental results associated with the Josephson-vortex flow resistance (JVFR) at low bias currents. Depending on the junction length, bias current, and temperature, the JVFR oscillation is changed from the period of half flux quantum per junction to the period of one flux quantum per junction. It is shown that the oscillation is essentially due to the field dependence of the critical current. At currents slightly exceeding the critical current the stationary vortex lattice structure becomes unstable and an irregular vortex flow can be induced by thermal fluctuations in different junctions. Our simulation results strongly suggest that the triangular lattice of vorticies in the dynamical state is more stable rather than the rectangular one even in a submicrometer IJJ stack when IJJs are biased at a low current.

1. Introduction
It is well-known that layered high-\(T_c\) superconductors like \(\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+y}\) behave as vertical stacks of Josephson junctions, i.e., the so-called intrinsic Josephson junctions (IJJs) [1, 2, 3]. In such a system, a mutual coupling occurs between neighboring junctions when a magnetic field is applied parallel to the layers. Due to this property, the dynamics of Josephson vortices in IJJs has attracted considerable attention not only from the physical point of view but also due to the potential applications. In particular, the coherent motion of the rectangular Josephson-vortex lattice, i.e., \textit{in-phase} motion, which can result in an increase in the power of electromagnetic radiation, is an intriguing subject because of its practical application to devices such as the THz oscillator.

Recently, the measurement of the Josephson vortex-flow resistance (JVFR) oscillation at a low bias current is also expected to be one of the approaches for understanding the dynamical vortex structure in a highly dense vortex state. The first study of the vortex-flow resistance oscillation was carried out by Ooi et al.\[4\]. They observed a clear periodic oscillation of the vortex flow resistance in \(\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+y}\). Then, similar observations for samples smaller than 5 \(\mu\)m were reported by several researchers [5, 6, 7]. All these experiments showed a vortex-flow resistance oscillation with a period of \(H_0/2\) or \(H_0\), where \(H_0 = \Phi_0/(sL)\) is equal to the field corresponding to one extra flux quantum \(\Phi_0\) per junction of thickness \(s\) and length \(L\) in the stack. Further, such an oscillation was attributed to the flow of triangular or rectangular Josephson vortex lattices, respectively. There were also several theoretical \[8\] and numerical \[9, 10, 12\] studies on...
the relation between the oscillation period and the lattice structure. However, there remains a question concerning the effect of thermal fluctuations on the vortex-flow behavior. When IJJs are biased around critical current, the thermal fluctuations may play a significant role because the Lorentz force is not sufficiently large in comparison to the pinning force.

In this paper, we have studied the vortex-flow dynamics in IJJs at a finite temperature and discuss the lattice structure of the moving Josephson vorticies in them. We show that the thermal fluctuations become important even at low temperatures when a high magnetic field is applied to the IJJs.

2. Simulation model

We have simulated the statics and dynamics of the Josephson vortex in \( N \) stacked IJJs at a finite temperature, \( T \), using the standard perturbed sine-Gordon equations [13], which are given as

\[
\frac{\partial^2 \phi_i}{\partial \tau^2} + \alpha \frac{\partial \phi_i}{\partial \tau} + \sin \phi_i = \gamma + \sum_{j=1}^{N} A_{i,j} \frac{\partial^2 \phi_j}{\partial x^2} + \xi_i(x, \tau),
\]

where \( i = 1, 2, \ldots, N \) is the junction index, \( \phi_i(x, \tau) \) is the gauge-invariant phase difference for the \( i \)-th junction, \( \gamma \) is the bias current density normalized to the critical current density \( J_c \), and \( \alpha = \frac{p^2}{\pi I_c R^2 C J/\Phi_0} \) is the damping parameter (\( I_c \) : critical current, \( R \) : normal resistance per unit length, \( C J \) : junction capacitance per unit length). The spatial coordinate \( x \) and time \( \tau \) are normalized to the Josephson penetration depth \( \lambda_J = \frac{\Phi_0}{2 \pi \mu_0 d} \) and the inverse of the Josephson plasma frequency \( \omega_0 = \frac{\sqrt{2 \pi J_c/\Phi_0 C J}}{\Phi_0} \), respectively, where \( d' = d + 2 \lambda_L \coth(t/\lambda_L) \), \( t \) is the electrode thickness, \( d \) is the barrier thickness, \( \lambda_L \) is the London penetration depth, and \( \mu_0 \) is the permeability of vacuum. The matrix element \( A_{i,j} \) is the coupling coefficient and can be given as

\[
A = \begin{pmatrix}
1 & S & S & 0 \\
S & 1 & S & 0 \\
& & \ddots & \ddots \\
0 & S & 1 & S \\
S & 1 & & \\
\end{pmatrix}^{-1}
\]

with

\[
S = -\frac{\lambda_L}{d' \sinh(t/\lambda_L)}.
\]

The term \( \xi(x, \tau) \) represents the dimensionless noise current due to thermal fluctuations with the autocorrelation function

\[
\langle \xi_i(x_1, \tau_1) \xi_i(x_2, \tau_2) \rangle = 2 \Gamma_T \ell \alpha \delta(x_1 - x_2) \delta(\tau_1 - \tau_2), \\
\langle \xi_i(x, \tau) \rangle = 0.
\]

Here, \( \Gamma_T = \frac{2 \pi k_B T}{I_c \Phi_0} \) is the normalized temperature, \( \ell \) is the normalized junction length, \( \delta \) is a Dirac delta function, and \( k_B \) is the Boltzmann constant. In the present simulations the magnetic field \( H \) is applied by the boundary condition as

\[
\left. \frac{\partial \phi_i}{\partial x} \right|_{x=0, \ell} = -\frac{H}{J_c \lambda_J} (1 + 2S),
\]

where \( S \) for \( \text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta} \) is estimated as \(-0.49997\) by putting \( t = 0.3 \) nm, \( d = 1.2 \) nm and \( \lambda_L = 200 \) nm.
3. Results and discussion

We first show the temperature dependence of JVFR oscillations. Figure 1 shows the simulation results for the stack having parameters of $\ell = 10, N = 5, \alpha = 0.1$. In this figure, the vortex-flow resistance $R_{VF} = \nu/\gamma$ is normalized to a stack resistance $R_n = N/\alpha$ and $h = H/H_0$ is the reduced magnetic field. Assuming $I_c = 150 \mu A$, the $\Gamma_T$ values of 0.005, 0.01 and 0.02 correspond to the temperatures of 18, 36 and 74 K, respectively, which are close to the experimental situations. Figure 1(a) shows the field dependence of $R_{VF}/R_n$ at $\gamma = 0.01$ for $\Gamma_T = 0$. The $R_{VF}/R_n$ curve exhibits a periodic oscillation. From this, it is found that the static (zero-voltage) and dynamic (vortex flow) states appear alternately as $h$ increases and the oscillation has two characteristic periods at different ranges of $h$, i.e., the $H_0/2$ periodicity appears for $6 < h < 10.5$ and gradually becomes masked in the $H_0$ periodicity for $10.5 < h < 12.5$, further, only the $H_0$ periodicity can be distinguished for $h > 12.5$, where $R_{VF}$ shows peaks when $h$ is an integer. This is consistent with the previous work [12]. Moreover, we find that the peak values of $R_{VF}/R_n$ approach to unity with increasing $h$. For finite temperatures ($\Gamma_T \neq 0$), periodic oscillations can be also observed in the $R_{VF}/R_n$ curves, as shown in figures 1(b)-1(d). In contrast to the case of $\Gamma_T = 0$, however, the static state of $R_{VF} = 0$ appears only in the low field region. In figure 1, we see that both the field where the oscillation of $R_{VF}$ starts and the transient field where the oscillation period varies from $H_0/2$ to $H_0$ decrease as $\Gamma_T$ increases. The increase in $\Gamma_T$ also results in the suppression of the oscillation amplitude.

Figure 2 shows the $R_{VF} - h$ characteristics of 10-junction stacks with $\ell = 2, 5, 10$, and 50. As seen in the figure, for longer JJJs ($\ell = 50$) only the $H_0/2$ oscillation can be observed just like the experimental result measured by Ooi et al.[4]. In contrast, for small $\ell$ one can observe that $H_0/2$ oscillations smoothly transform into $H_0$ oscillations with increasing $h$. Furthermore, we find that the transition field from $H_0/2$ oscillations to $H_0$ ones decrease with decreasing $\ell$. These observations are well consistent with the experimental results [6, 7]

The oscillation of $R_{VF}$ observed in figures 1 and 2 is basically attributed to the field dependence of J-V-F oscillations. Figure 1 shows the field dependence of the vortex-flow resistance for five-fold stacks at $\gamma = 0.01$ for (a)$\Gamma_T=0$, (b)$\Gamma_T=0.005$, (c)$\Gamma_T=0.01$ and (d)$\Gamma_T=0.02$. The parameters of the simulations are : $\ell = 10$ and $\alpha = 0.1$.

Figure 2. Field dependence of $R_{VF}$ for ten-fold stacks with lengths of $\ell = 2$ (a), 5 (b), 10 (c), and 50 (d). The parameters of the simulations are : $\alpha = 0.05$, (a)$\Gamma_T = 0.03$, $\gamma = 0.005$, (b)$\Gamma_T = 0.01$, $\gamma = 0.01$, (c)$\Gamma_T = 0.01$, $\gamma = 0.005$, (d)$\Gamma_T = 0.002$, $\gamma = 0.002$. 

Figure 1. Field dependence of the vortex-flow resistance for five-fold stacks at $\gamma = 0.01$ for (a)$\Gamma_T=0$, (b)$\Gamma_T=0.005$, (c)$\Gamma_T=0.01$ and (d)$\Gamma_T=0.02$. The parameters of the simulations are : $\ell = 10$ and $\alpha = 0.1$. 


dependence of the critical currents of stacks. Figures 3(a) and 3(b) show the \( h \) dependence of the critical current \( \gamma_c \) for two stacks \( (\ell = 5,10) \) at \( \Gamma_T = 0 \). From this, we see that \( \gamma_c(h) \) oscillates with two characteristic periodicities similar to the recent experimental observation [6]. In low \( h \) region, \( \gamma_c(h) \) has local maximum at \( h = n/2 \) \( (n: \text{integer}) \), so that the oscillation of the \( H_0/2 \) periodicity is observed. On the other hand, in high \( h \) region the local maxima of \( \gamma_c(h) \) at \( h \) being an integer are significantly reduced in comparison to those at \( h \) being a half integer. As a result, \( \gamma_c(h) \) shows the \( H_0 \) oscillation. The crossover from \( H_0/2 \) to \( H_0 \) oscillations occurs around \( h \approx 4 \) for \( \ell = 5 \) and \( h \approx 15 \) for \( \ell = 10 \). According to the theoretical work by Koshelev [8], the normalized crossover field is given as \( h_{cr} = \ell^2/2\pi \). Using this equation we estimate \( h_{cr} \approx 4 \) for \( \ell = 5 \) and \( h_{cr} \approx 16 \) for \( \ell = 10 \). These values are consistent with those obtained from figure 3. Because the Josephson vortex flow occurs when the bias current exceeds the critical current and \( \gamma_c \) decreases with increasing temperature, for a larger bias current or in the case of a finite temperature the range of \( \gamma_c(h) \) falls below the bias current, so that the \( H_0 \) oscillation becomes dominant and also broadens the peaks of \( R_{VF} \), as shown in figures 1 and 2.

![Figure 3](image-url)

**Figure 3.** Field dependence of a critical current density \( \gamma_c \) for \( \Gamma_T = 0 \). The parameters of the simulations are : (a) \( \ell = 5, N = 5, \alpha = 0.05 \) and (b) \( \ell = 10, N = 5, \alpha = 0.1 \).

Next, we consider the effect of thermal fluctuations on the Josephson vortex flow. Figure 4(a) shows the \( \gamma - v \) curve of a five-fold stack with parameters of \( \ell = 5 \) and \( \alpha = 0.05 \) for \( h = 6.5 \) and \( \Gamma_T = 0.01 \). The \( \gamma - v \) characteristic of the same stack at \( \Gamma_T = 0 \) is shown in the inset. It is found that the noise, or fluctuations, rounds the characteristic and the maximum supercurrent is significantly suppressed in comparison with that for \( \Gamma_T = 0 \). Such a nonhysteretic characteristic also agrees with the experimental \( I - V \) characteristics observed in high magnetic field [4]. In figure 4(a), the second Fiske step corresponding to the lowest velocity mode is also observed around \( v = 4.5 \). The spatial-temporal vortex trajectories at different bias current are shown in figures 4(b)-4(d). At \( \gamma = 0 \) [point A in figure 4(a)], vortices are in a stationary state and form the square lattice though they vibrate to a small extent around the equilibrium position due to thermal fluctuations. In contrast, at bias point B \( (\gamma = 0.02) \) it is seen that there are two regimes of vortex motion: one is the region where vortices flow irregularly and the other is the region where vortices do not flow. Namely, the vortices in each junction move from \( x = \ell \) toward \( x = 0 \) with a repetition of the movement and the stopping at random. Furthermore, it is noteworthy that a rectangular-like lattice seems to be formed when the vortices stop but there is no correlation between the vortex motions in adjacent junctions when the vortex flow occurs in some of junctions. On the other hand, at point C \( (\gamma = 0.03) \) all junctions become vortex
Figure 4. Current-voltage characteristic of a five-fold stack with parameters of $\ell = 5$ and $\alpha = 0.05$ for $h = 6.5$ and $\Gamma_T = 0.01$. Spatial-temporal vortex trajectories through the stack in different bias points: (b) point A ($\gamma = 0$), (c) point B ($\gamma = 0.02$) and (d) point C ($\gamma = 0.03$).

flow state and moving Josephson vortices form triangular-like lattice as shown in figure 4(d). We also observed similar vortex flow phenomena at different $h$. From figure 4 we find that the dynamical lattice structure of the Josephson vortices is not directly related to the oscillation period of $R_{V_F}$.

The vortex motion shown in figure 4(c) can be understood by the thermal escape of Josephson vortices from spatially periodic potential associated with the free energy of each junction in the stack. In the absence of thermal fluctuations, when a sufficiently large magnetic field is applied to the stack, it becomes a multivortex state, and the Josephson vortices are arranged at the position of the local minima of energy in each junction if the bias current is smaller than the critical current $\gamma_c(T = 0)$. At $\gamma = \gamma_c$ the potential energy barrier disappears and the vortices start to move due to the Lorentz force. On the other hand, at a finite temperature, the thermal energy may allow the vortex to move from one pinning point to another in each junction even at $\gamma < \gamma_c(T = 0)$ if the thermal energy is comparable to the energy barrier in the potential. This induces a disordered vortex flow, i.e., a thermally assisted vortex flow, as shown in Figure 4(c). At higher bias currents, however, the Lorentz force becomes larger and the influence of the fluctuations on the vortex flow is almost negligible; in this situation, the vortex structure is changed depending on the bias current [13].

Finally, we consider the case of submicron IJJJs which are expected to generate the in-phase oscillation in the edge pinning model [8, 14]. Figure 5(a) shows the field dependence of $R_{V_F}$ of the stack with $\ell = 2$ and $N = 5$ for $\gamma = 0.02$ and $\Gamma_T = 0.02$. Assuming critical current densities of 1000 A/cm$^2$, the junction length corresponds to 0.62\mu m. In this case, only $H_0$ oscillation is observed. Figures 5(b) and 5(c) shows the vortex trajectories at $\gamma = 0.02$ and 0.04, respectively, for $h = 5.5$. One can see that at $\gamma = 0.02$ thermally assisted vortex flow is induced while at $\gamma = 0.04$ the stack is in a collective vortex-flow state where moving vortices form a triangular lattice. These results imply that in the low bias region the out-of-phase flow is stable rather than the in-phase flow even for submicron IJJJs. Therefore, to realize stable in-phase flow one may need to apply higher bias currents.
4. Conclusion

We have numerically studied the effect of thermal fluctuations on Josephson vortex dynamics for multivortex states in intrinsic Josephson junction stacks. We succeeded in reproducing the noise-rounded current-voltage characteristics in a high magnetic field and two different periodic oscillations of the Josephson vortex-flow resistance in the vicinity of the critical currents suppressed by a magnetic field. The oscillations of the vortex-flow resistance were observed to be essentially due to the modulations of the critical current of the stack. At a finite temperature we observed the disordered vortex-flow which is induced by thermal fluctuations.

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