Integrations of Continuous Hesitant Fuzzy Information in Group Decision Making With a Case Study of Water Resources Emergency Management

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This work was supported by the National Natural Science Foundation of China under Grant 71771155.

ABSTRACT With the increasing application of (probabilistic) hesitant fuzzy sets in decision-making, the existing integrated methods of hesitant fuzzy information have become too complicated to meet the needs of increasingly complex practical decision-making problems. Therefore, this paper combines the related knowledge of probability theory to firstly introduce the concept of continuous hesitant fuzzy element (C-HFE). Subsequently, the concept of uniform hesitant fuzzy element (U-HFE) is proposed, and discrete (probabilistic) hesitant fuzzy information is transferred to continuous one, benefited from the connection between U-HFEs and C-HFEs with uniform distribution. After then, integration methods of C-HFEs based on mathematical derivation are developed, which lays a theoretical foundation for the continuity of hesitant fuzzy information. Further, facing the problem that the method of mathematical derivation is too tedious, based on computer simulation, this paper proposes another integration method of C-HFEs, which is more concise and easier to apply. Finally, an example of the evaluation of water resources emergency management plans is given to apply the above method to practical decision-making problems.

INDEX TERMS Continuous hesitant fuzzy element, uniform hesitant fuzzy element, information aggregation, group decision-making, water resources emergency management.

I. INTRODUCTION

Since the advent of fuzzy sets in 1965 [1], various extended forms have been proposed [2]–[6]. These expressions of uncertain information have made the presentation of decision-making information both accurate and complete. However, at the same time, as the expressions of information becomes more and more complex, especially after hesitant fuzzy sets (HFSs) have been widely used in decision-making problems, our requirements for the calculation and integration of decision-making information become higher and higher.

Existing calculation and integration methods for hesitant fuzzy information are mainly derived from the work of Xia and Xu [7]. After that research, several hesitant fuzzy inform-
However, with the rapid development of the society and economy, the practical decision-making problems have become more and more complicated. The most significant change is the increase in the number of decision makers. In fact, in today’s society, a large decision-making problem often involves many fields. A single decision maker or a small number of decision makers can no longer cope with such complicated problems. Therefore, group decision-making (especially large-scale group decision-making) is more and more widely used in practical decision-making processes. The substantial increase in the number of decision makers puts forward higher and higher requirements for the integration of decision-making information. Under this condition, in addition to the accuracy and completeness, the simplicity of the calculation and integration process has become increasingly important. As a result, the disadvantages of the existing methods gradually become apparent. As the number of decision makers increases, the number of memberships in the calculation process increases geometrically. If probability information is added, the calculation process will become much more tedious. These problems can cause serious difficulty to our decision-making process. Therefore, simpler calculation methods are needed to deal with increasingly complex practical decision-making (especially large-scale group decision-making) problems. To address such an issue, some scholars have proposed their own solutions from different perspectives [20], [21]. However, few studies have focused on solving the problem from a continuous perspective.

In 2017, Zhang et al. [14] first proposed the concept of P-HFE with continuous form, but did not investigate its operation and integration. By definition, P-HFE with continuous form uses a function to express membership and probability information, which can effectively avoid the trouble caused by too many memberships in the calculation process. Therefore, hesitant fuzzy information can be more conveniently applied to complex practical decision-making problems. On the other hand, in many decision-making problems, we often face knowledge and information in the form of interval-values to express the opinions of experts. Therefore, the aggregation of these kinds of information is also greatly needed for us to solve decision-making problems. However, most of the existing integration methods [22]–[28] for various interval values only deal with the upper and lower bounds of the interval, which leads significant information loss and inaccurate conclusion. This is because these methods completely ignore the distribution information, the calculation process is lack of mathematical rigor, and as a consequence the results obtained are not accurate. Thus, more accurate and correct ways are needed to process the uncertain information in the form of interval-values.

Based on the above motivations, this paper investigates further on P-HFE in continuous form. First, we modify the definition of P-HFE with continuous form and unify the continuous form of HFE and P-HFE to form continuous hesitant fuzzy element (C-HFE). Subsequently, we proposed the concept of uniform hesitant fuzzy element (U-HFE) and used its connection with C-HFE with uniform distribution to link discrete HFEs with C-HFEs. Next, we develop an integration method of C-HFEs with normal distribution and uniform distribution based on mathematical derivation, which lays a theoretical foundation for the operation and integration of C-HFEs. After that, a simpler aggregation method of C-HFEs based on computer simulation is introduced to make the C-HFEs easier and more effective when applying to solve decision-making problems. Finally, an example of the evaluation of water resources emergency management plans is given to illustrate the application of the above method in practical decision-making problems.

The rest structure of this paper is organized as follows: Some related preliminaries are introduced in Section 2. In Section 3, the definition of U-HFE is given, and the two different forms of HFEs, discrete HFEs and C-HFEs are linked. Also, in this section, we make some further elaborations on C-HFEs. And then in Section 4, the operation and aggregation methods of C-HFEs with normal distribution and uniform distribution, which are based on mathematical derivation, is given. Further, an integration method of general C-HFEs based on computer simulation is proposed After that, in Section 5, we give an example to apply the proposed approach to the evaluation of water resources emergency management plans and provide a comparative analysis to expand the applicability of the above methods in practical decision-making problems. Finally, the conclusions and outlooks are given in Section 6.

II. PRELIMINARIES

A. HESITANT FUZZY SETS

At first, we introduce and review some basic concepts about hesitant fuzzy sets.

In 2010, Torra [4] proposed the concept of hesitant fuzzy set (HFS) on the basis of fuzzy set (FS) and intuitionistic fuzzy set (IFS). Subsequently, Xia and Xu [7] proposed the mathematical expression of HFS. The basic component of the HFS is the hesitant fuzzy element (HFE) [7]. Each HFE can contain multiple membership degrees. This means that HFSs have the following two advantages over previous expressions of uncertain information: (1) when a single expert gives decision-making information with HFEs, he has more room for hesitation; (2) in group decision-making problems, HFEs can gather the opinions of multiple experts into one element for subsequent processing. Therefore, HFSs can contain more initial decision-making information than those forms of fuzzy information before. Then, some calculation rules of HFEs are also defined as follows:

Definition 1 [7]: Let $h$, $h_1$ and $h_2$ be three HFEs, then

\begin{enumerate}
  \item $h^\lambda = \bigcup_{y \in h} \{y^\lambda \}, \lambda > 0$
  \item $\lambda h = \bigcup_{y \in h} \{1 - (1 - y)^\lambda \}, \lambda > 0$
  \item $h_1 \oplus h_2 = \bigcup_{y_1 \in h_1, y_2 \in h_2} \{\frac{1+y_1+y_2}{2}\}$
  \item $h_1 \otimes h_2 = \bigcup_{y_1 \in h_1, y_2 \in h_2} \{y_1 y_2\}$
\end{enumerate}
B. PROBABILITY HESITANT FUZZY SETS

Although HFS can more completely and accurately express the initial decision-making information given by decision makers, it still has the problem of information loss. To solve this problem, Zhu [5] added the probability information into HFS and defined the concept of the probabilistic hesitant fuzzy set (P-HFS). He added probability information to HFEs, so that each membership degree in HFEs has a corresponding probability to express its importance. Later, Zhang et al. [14] improved this definition and proposed the P-HFE with the continuous form. In this paper, this definition is refined as follows:

Definition 2 [14]: Let x be a continuous variable and change from 0 to 1. \( p(x) \) is a continuous function with the domain \( D(x) \), and satisfies the following three properties:

1. \( p(x) \geq 0 \), when \( x \in [0, 1] \cap D(x) \)
2. \( \int_{x}^{\bar{x}} p(x)dx \leq 1 \)
3. \( P(a < x \leq b) = \int_{a}^{b} p(x)dx \), when \( \underline{x} \leq a \leq b \leq \bar{x} \)

where \( \underline{x} \) and \( \bar{x} \) are the lower and upper bounds of the value of \( x \) respectively. Let \( d(x) = [0, 1] \cap D(x) \), then \( h(p) = \left\{ d(x) \left( \int p(x)dx \right) \right\} \) is also a P-HFE, and we call it a P-HFE with the continuous form.

Remark: If we extend HFEs to continuous form, we can get the same definition. Thus, we use continuous hesitant fuzzy element instead of P-HFE with the continuous form, and abbreviate it as C-HFE.

In order to compare the size of different P-HFEs with the continuous form, we need the following definition:

Definition 3 [14]: If a P-HFE is given as follows: P-HFE \( h(p) = \left\{ d(x) \left( \int p(x)dx \right) \right\} \). Then, the score of the P-HFE can be defined as the following form:

\[
\text{score}(h(p)) = \frac{\int_{\underline{x}}^{\bar{x}} x \cdot p(x)dx}{\int_{\underline{x}}^{\bar{x}} p(x)dx}.
\]

Then, we can give a partial order of P-HFEs with the continuous form with the following formulas:

1. If \( s(h_1(p)) > s(h_2(p)) \), then \( h_1(p) > h_2(p) \);
2. If \( s(h_1(p)) = s(h_2(p)) \), then \( h_1(p) \) is equivalent to \( h_2(p) \), denoted as \( h_1(p) \sim h_2(p) \).

III. DETAILED EXPLANATION OF C-HFES

A. UNIFORM HESITANT FUZZY ELEMENTS AND C-HFES WITH UNIFORM DISTRIBUTION

Because of the advantages of HFSs, in practical decision-making problems, DMs are often asked to give HFEs as initial assessment information. We consider the following HFE \( \{0.8, 0.7, 0.6, 0.5\} \).

For the HFE, in group decision-making, it is normal to have such an evaluation value. However, if there is only one expert, obviously, such an evaluation value is unreasonable. It’s hard to explain why the expert skips 0.6 after he gave 0.5 and 0.7 directly. Therefore, when a single expert gives an evaluation value, a form like \( \{0.8, 0.7, 0.6, 0.5\} \) is more logical and reasonable.

HFEs like \{0.8, 0.7, 0.6, 0.5\} have an important feature: if we arrange all of its membership degrees in order of magnitude, an arithmetic progression can be obtained. We call such HFEs uniform HFEs.

Definition 4: For an HFE \( h = \{\gamma_l | l = 1, 2, \ldots, |h|\} \), if \( \{\gamma_{(l)} | l = 1, 2, \ldots, |h|\} \) is an arithmetic progression, where \( \gamma_{(l)} \) is the \( l \)-th largest value of all \( \gamma_l \). Then, we call HFE \( h \) a uniform HFE (U-HFE).

Through our previous analysis, we can know that, when a single expert gives evaluation values with HFEs, he usually gives U-HFEs. Obviously, compared with general HFEs, U-HFEs have better properties. For example, for the U-HFE \( \{0.5, 0.7\} \), suppose that an expert gives the two membership degrees. Then, we have reason to believe that 0.6 is also acceptable in his opinion. Furthermore, we can find that adding a membership 0.6 has little effect on the properties of the original HFE \( \{0.5, 0.7\} \). For example, its score has not changed and it is still a U-HFE. Based on this idea, we can also consider it in reverse. For the uniform HFE \( \{0.5, 0.6, 0.7\} \), its membership degree 0.6 can be deleted actually. The resulting new U-HFE has the same score as the original U-HFE. In this way, the original U-HFE is simplified.

It is worth noting that, after adding or deleting the corresponding membership degrees, the deviation degree of the original HFE will change accordingly. In some cases, this does have some impact on the outcome of the decision-making process. In particular, the removal of membership degrees is likely to result in larger deviations. However, in some simple decision-making problems, the decision-making results may only depend on the score values. In such cases, properly deleting a part of the membership degrees can greatly simplify the operations in the decision-making process.

Following the above discussion, there is another point which is worthy of attention. We can add 0.6 to the HFE \( \{0.5, 0.7\} \). After that, we can also insert two membership degrees 0.55 and 0.65 in it. And, this kind of operation can go on...If we keep doing this, then the membership degrees in the U-HFE will be infinitely close to a uniform distribution. And this just can be represented by the P-HFE with the continuous form mentioned in Definition 4. Obviously, the membership in the corresponding P-HFE obeys \( U(0.5, 0.7) \). Thus, we can get a C-HFE

\[
h(p) = \{[0.5, 0.7] \cdot U(0.5, 0.7)\}
\]

In this way, we can convert each uniform HFE into a corresponding C-HFE with a uniform distribution.

It can be seen that C-HFEs with uniform distribution and the information expressions based on interval-values are somewhat similar in form. However, they are fundamentally different. The differences between them are mainly reflected in the way of calculation. The operation and integration of C-HFEs with uniform distribution are based on probability theory. It is more reasonable than the calculation method of interval values. We will elaborate on this point in Section 4.
B. FURTHER DISCUSSION OF C-HFES

In this subsection, we will introduce and analyze C-HFES in more detail.

The definition of C-HFE is derived from the density function and P-HFE. But there are two fundamental differences between the probability distribution and C-HFE: (1) for a probability distribution, the integral value of its density function over the full domain is required to be 1, but that value of C-HFE just needs to be not greater than 1. (2) The domain of the probability density function could be unlimited, but the domain of C-HFE is always limited to [0,1]. However, it does not mean that the probability density function cannot be applied to hesitant fuzzy decision-making problems. Firstly, some probability distributions, such as uniform distributions, can be limited to some subsets of [0,1]. Thus, we can use C-HFES with this kind of probability distribution to replace some probability distributions, such as uniform distributions, to applied to hesitant fuzzy decision-making problems. Firstly, not mean that the probability density function cannot be unlimited, but the domain of C-HFE just needs to be not greater than 1. (2) The domain of the probability density function over the full domain is required to be 1, but that value of C-HFE is not necessarily. However, it does not mean that the research results obtained using mathematical derivation are the most accurate. Calculations by other methods usually only give approximate results. Further, the calculation of the two most commonly used C-HFES (C-HFES with normal distribution and C-HFES with uniform distribution) can be solved to some extent with mathematical derivation. In this section, we try to use mathematical derivation to obtain a method for the calculation and integration of C-HFES with normal distribution and uniform distribution.

As we all know, if two random variables \(X \sim N(\mu_1, \sigma_1^2)\) and \(Y \sim N(\mu_2, \sigma_2^2)\) are independent with each other, then \(X + Y \sim N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)\). Based on that and Definition 1, the “addition” of two C-HFES with normal distribution can be obtained as follows:

\[
\text{Definition 5: Suppose that } h_1(p) = \{[a_1, b_1](N(\mu_1, \sigma_1^2))\}, \quad h_2(p) = \{[a_2, b_2](N(\mu_2, \sigma_2^2))\} \text{ are two C-HFES which are independent to each other. Then,}
\]

\[
\left[\begin{array}{c}
\mu_1 + \mu_2 \\
\frac{\sigma_1^2 + \sigma_2^2}{4}
\end{array}\right] \times [a_1, b_1, a_2, b_2]
\]

Similarly, we can generalize the operation law to the case of \(n\) C-HFES.

\[
\text{Definition 6: Let } h_i(p) = \{[a_i, b_i](N(\mu_i, \sigma_i^2))\} \text{ are } n \text{ C-HFES which are independent to each other. Then,}
\]

\[
\bigoplus_{i=1}^{n} h_i(p) = \left[\begin{array}{c}
\sum_{i=1}^{n} a_i \\
\sum_{i=1}^{n} b_i
\end{array}\right] \times \left[\begin{array}{c}
\frac{\sum_{i=1}^{n} \mu_i}{n} \\
\frac{\sum_{i=1}^{n} \sigma_i^2}{n}
\end{array}\right]
\]

In this way, we can get the “addition” of \(n\) C-HFES with normal distribution. Obviously, it satisfies the law of exchange but not the law of association. This does not prevent its application in the integration of C-HFES. Below we introduce an aggregation operator of C-HFES:

\[
\text{Definition 7: Let } h_i(p) = \{[a_i, b_i](N(\mu_i, \sigma_i^2))\} (i = 1, 2, \cdots, n) \text{ be } n \text{ C-HFES which are independent to each other. Then, the continuous hesitant fuzzy weighted averaging operator of C-HFES with normal distribution is defined as follows:}
\]

\[
\text{CHFWA}_\lambda(h_1(p), h_2(p), \cdots, h_n(p)) = \bigoplus_{i=1}^{n} \omega_i h_i(p)
\]

\[
= \left[\begin{array}{c}
\sum_{i=1}^{n} \omega_i a_i \\
\sum_{i=1}^{n} \omega_i b_i
\end{array}\right] \times \left[\begin{array}{c}
\lambda \sum_{i=1}^{n} \omega_i \mu_i, \\
\lambda \sum_{i=1}^{n} \omega_i \sigma_i^2
\end{array}\right]
\]

where \(\lambda > 0\), and \(\sum_{i=1}^{n} \omega_i = 1\).

Here, we need to make two explanations for the above calculation and integration method. 1) The meanings of the independence between C-HFES and the independence between random variables are basically the same. In most practical decision-making problems, the C-HFES involved in the integration process are independent to each other. (Such as information from different experts) When those C-HFES
are not independent, the above method is not applicable.

2) Unlike traditional calculation methods, in the proposed approach, we first calculate the distribution of the integration of all C-HFEs over the entire space. Then we intercept the part of the distribution in the feasible region as the final operation result. This is determined by the nature of the normal distribution. For C-HFEs, the above method is obviously more concise and has a solid theoretical foundation as it is built on the probability theory.

The above is a calculation and integration method of C-HFEs with uniform distribution and is based on mathematical derivation. Next, we will introduce a method to deal with the calculation and integration of C-HFEs with uniform distribution.

According to the result given by Sadooghi-Alvandi et al. [29], assume that \( X_1, X_2, \ldots, X_n \) are \( n \) independent random variables, where \( X_i, i = 1, 2, \ldots, n \) are uniformly distributed on \( (a_i, b_i) \). Further let \( S_n = \sum_{i=1}^{n} X_i \). Then, for \( n \geq 2 \), the density function of \( S_n \) is given as the following form:

\[
f_n(x) = \frac{1}{A_n(n-1)!} \times \left\{ x^{n-1} + \sum_{k=1}^{n} (-1)^k \sum_{J_k} \left[ \left( x - \sum_{i=1}^{k} a_{ij} \right) \right]^{n-k+1} \right\}
\]

for \( 0 \leq x \leq \sum_{i=1}^{n} a_i \). Where \( A_n = \prod_{k=1}^{n} a_k \), \( J_k = \{ (j_1, j_2, \ldots, j_k) | 1 \leq j_1 < j_2 < \cdots < j_k \leq n \} \).

From the analysis above, we can see that the problem of calculation and integration of C-HFEs is basically solved.

In this section, we try to solve the calculation and integration of C-HFEs with normal distribution and uniform distribution from the perspective of mathematical derivation, as these two distributions are most important and widely used in applications. Although the calculation and integration problems of C-HFEs based on other distributions are left to be the future research topics, the research lays the theoretical foundation and provides an effective method for the calculation and integration of C-HFEs.

### B. OPERATION AND AGGREGATION METHODS OF C-HFES BASED ON COMPUTER SIMULATION

The mathematical derivation-based information integration method proposed in the previous subsection laid a good foundation for the integration of C-HFEs. However, in some practical decision-making problems, it is difficult to apply it to the decision-making processes directly due to the computation complicated. To address such a difficulty, this subsection will develop a different and more effective method to solve the information integration problems of C-HFEs. In particular, we will focus on another calculation and integration method of C-HFEs with the case based on C-HFEs with uniform distribution, as other kinds of C-HFEs can be processed in a similar way.

Assume that \( h_1(p) = \{[a_1, b_1](U(a_1, b_1)) \} \) and \( h_2(p) = \{[a_2, b_2](U(a_2, b_2)) \} \) are two C-HFEs with uniform distribution. If we want to calculate their sum, we must know the distribution of the sum. To solve the problem, we can use computer simulation to fit the distribution of the sum. For the case where multiple C-HFEs are added, the processing method is the same. Moreover, in real decision-making problems, different elements often have different weights.

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**Definition 9:** Let \( h_i(p) = \{[a_i, b_i](U(a_i, b_i))\} \) \( i = 1, 2, \ldots, n \) be \( n \) independent C-HFEs. Then, the continuous hesitant fuzzy weighted averaging operator of C-HFEs with uniform distribution has the following form:

\[
\text{CHFWA}_n(h_1(p), h_2(p), \ldots, h_n(p)) = \bigoplus_{i=1}^{n} \omega_i h_i(p)
\]

\[
= \left\{ \left[ \sum_{i=1}^{n} \omega_i a_i, \sum_{i=1}^{n} \omega_i b_i \right] \left( \int p(x) \, dx \right) \right\}
\]

where \( \omega_i > 0 \), \( \sum_{i=1}^{n} \omega_i = 1 \), and the density function

\[
p(x) = \left( \frac{1}{ \left( \prod_{i=1}^{n} (\omega_i b_i - \omega_i a_i) \right) (n-1)! } \right)
\times \left( \left( x - \sum_{i=1}^{n} \omega_i a_i \right)^{n-1} + \sum_{k=1}^{n} (-1)^k \right)
\times \sum_{J_k} \left( \left( x - \sum_{i=1}^{n} \omega_i a_i \right) \right)_{+}^{n-1}.
\]
Thus, we only need to add the weight information to the fitting process. As a result, the overall calculation process is relatively simple.

The benefits of this simulation method can be outlined as follows: At first, it is very concise and does not need to make complicated mathematical derivations repeatedly. Further, we can choose any form of function to fit the results. The advantage here is that, if we choose the appropriate form of function, the subsequent processing of the results will become very convenient. In this paper, polynomial functions are chosen in this method [30]. There are two main reasons behind this choice. First, the polynomial function is flexible in form, and the fitting result based on it is more accurate. Second, the subsequent processing of polynomial functions, such as finding the scoring function and calculating convolutions of distributions, is relatively simple.

Below we use an example to explain this simulation method in more detail:

**Example 1:** Given three C-HFEs

\[
\begin{align*}
h_1(p) &= \{[0.5, 0.7] \cup (0.5, 0.7)\}, \\
h_2(p) &= \{[0.5, 0.8] \cup (0.5, 0.8)\}, \\
h_3(p) &= \{[0.4, 0.7] \cup (0.4, 0.7)\}
\end{align*}
\]

Below we use the simulation method to calculate the result of \(h(p) = h_1(p) \cup h_2(p) \cup h_3(p)\).

Assume that \(X_1 \sim U(0.5, 0.7), X_2 \sim U(0.5, 0.8), X_3 \sim U(0.4, 0.7)\). Let \(X = X_1 + X_2 + X_3\) and denote the density function of \(X\) as \(p(x)\). In addition, let the simulation result of \(X\) and \(p(x)\) be \(X'\) and \(p'(x)\), respectively. Based on the computer simulation, we can get \(p'(x)\) at first:

\[
p'(x) = -1604600x^6 + 1174600x^5 - 278190x^4 + 18662x^3 + 467.08x^2 - 3.3626x + 0.16154
\]

Then, according to the simulation result, we can obtain the lower and upper bounds of \(x'0.47442\) and \(0.72627\), respectively. Thus, the simulation result of \(h(p)\) can be obtained as follows:

\[
h(p) = \left\{0.47442, 0.72627\right\} \left(\int p(x) \, dx\right)
\]

where \(p(x) = -1604600x^6 + 1174600x^5 - 278190x^4 + 18662x^3 + 467.08x^2 - 3.3626x + 0.16154\).

![Figure 1. Comparison of the results obtained by the three methods.](image)

The figure above shows a comparison of the results obtained by using mathematical derivation, plotting points, and fitting with a polynomial function. The three curves from top to bottom are the images of the density function derived mathematically, the density function fitted by the polynomial function, and the empirical density function of the simulated data of the real distribution, respectively. As you can see, the results we obtained using the polynomial function fitting are very accurate.

It is mentioned earlier in the paper that C-HFEs with uniform distribution and information expressions based on interval-values are somewhat similar in form. In fact, an interval-value is equivalent to a uniform distribution. In the existing integration methods for interval values, usually only the upper and lower bounds of the interval-values are dealt with and processed, and the final calculation results are still interval-values. Unfortunately, such methods are not reasonable, as they completely ignore the distribution information and the results obtained are not accurate. The integration methods of the two C-HFEs mentioned in this paper consider the distribution information, and the calculation results are more accurate and reliable. In fact, they also provide a new approach to the aggregation of uncertain information expressed in the form of interval-values.

According to our discussion in Subsection 3.1, U-HFEs are valuable in practical decision-making problems. Benefited from the above work, an operation and integration method of U-HFEs can be summarized as follows:

**Step 1:** Convert all U-HFE involved in the integration process into C-HFEs with uniform distribution;

**Step 2:** Obtain the distribution of the integration result of all C-HFEs by using computer simulation;

**Step 3:** Determine the upper and lower bounds of the integration result according to the simulation result. And then obtain the final result.

In the next section, we will introduce a case study of a decision-making problem based on information integration of U-HFEs and C-HFEs to show the applicability of the above method in the practical decision-making processes.

V. A CASE STUDY ABOUT THE EVALUATION OF WATER RESOURCES EMERGENCY MANAGEMENT PLANS

A. THE PROPOSED METHOD FOR THE EVALUATION OF WATER RESOURCES EMERGENCY MANAGEMENT PLANS

In recent years, major global natural disasters and man-made accidents have occurred frequently. It poses extremely high challenges to the emergency management of water resources in countries and governments around the world. On March 11, 2011, a magnitude 9 earthquake struck Japan. Subsequently, explosions occurred at Units 1, 3, 2, and 4 of the Fukushima Daiichi Nuclear Power Plant in Fukushima Prefecture, causing the worst nuclear accident after the Chernobyl nuclear accident in the former Soviet Union. The accident caused high-level radioactive wastewater to enter surface water, groundwater, and seawater. The presence of radioactive iodine and radioactive cesium was detected in residential drinking water and nearby seawater, which
affected the local and global water safety. On November 8, 2018, a mountain fire broke out in Paradise Town, Butte County, northern California. The mountain fire became the most destructive fire in the history of California, destroying more than 60,000 hectares, causing damage to nearly 19,000 homes and other buildings, killing more than 80 people, and displacing large numbers of residents. The paradise town, which is closest to the source of fire and only has a population of 27,000, had almost been burned into ruins. In 2019 and 2020, wildfires in Brazil’s Amazon rainforest and Australia also occurred for several months, causing huge losses to the affected countries and people, as well as the local natural environment. In fact, all these severe natural disasters are testing the water resources emergency management and deployment capabilities of governments. The improper decision-making when facing disasters has also caused the governments of the aforementioned countries to face great doubts from the international community.

As far as China’s situation is concerned, since the beginning of the 21st century, global climate change has led to frequent climate anomalies and extreme weather phenomena. Large-scale water supply crises caused by extreme floods and droughts have occurred from time to time. At the same time, natural disasters such as floods, earthquakes, fires, typhoons, mudslides, and man-made damage such as industrial pollution may damage water sources and water supply pipelines, resulting in water difficulties in local areas during and even after the disaster. China has a large population and a high population density in some areas. When disasters occur in densely populated areas such as large cities, improper handling or inappropriate methods will cause huge losses to local residents’ production and lives. The above situation places extremely high demands on water resources emergency management level in China. With the rapid development of society and economy, emergency management of water resources is no longer the responding and managing to a water resource incident by a water conservancy department alone, but a systematic project that requires multi-sector and multi-agency coordination.

Water resource emergencies occur quickly and cause great harm. Therefore, it is an important issue for emergency management of water resources in order to quickly respond to emergencies of water resources and ensure that residents’ domestic water, and ensure that social stability can be protected in an emergency and that limited water resources can be reasonably allocated. In response to this situation, local governments usually make some response plans for possible water resource emergencies. When an accident does occur, it is important to accurately evaluate these options in order to make the best choice from them.

Assume that a typhoon suddenly attacks a city and damages its urban water supply system. The local government has three emergency plans $P_1$, $P_2$, and $P_3$ to deal with the disaster. They invited three experts and asked them to evaluate the three plans from the following three perspectives: protection of industrial water ($A_1$), guarantee of residents’ domestic water ($A_2$), post-mortem recovery ($A_3$). Three experts evaluated the three solutions from three perspectives, and the results are shown in the following three tables:

**TABLE 1.** The Information Given by the First Expert $E_1$.

|       | $P_1$   | $P_2$   | $P_3$   |
|-------|---------|---------|---------|
| $A_1$ | [0.5, 0.6, 0.7] | [0.7, 0.75, 0.8] | [0.55, 0.6] |
| $A_2$ | [0.8, 0.85, 0.9] | [0.4, 0.5, 0.6] | [0.5, 0.55, 0.6, 0.65] |
| $A_3$ | [0.7, 0.75, 0.8] | [0.5, 0.55, 0.6] | [0.55, 0.6, 0.65] |

**TABLE 2.** The Information Given by the First Expert $E_2$.

|       | $P_1$   | $P_2$   | $P_3$   |
|-------|---------|---------|---------|
| $A_1$ | [0.4, 0.5, 0.6] | [0.6, 0.7, 0.8] | [0.55, 0.6, 0.65] |
| $A_2$ | [0.45, 0.55] | [0.75, 0.76, 0.77, 0.78] | [0.6, 0.65] |
| $A_3$ | [0.75, 0.78, 0.81] | [0.3, 0.4, 0.5] | [0.5, 0.6] |

**TABLE 3.** The Information Given by the First Expert $E_3$.

|       | $P_1$   | $P_2$   | $P_3$   |
|-------|---------|---------|---------|
| $A_1$ | [0.4, 0.5, 0.6] | [0.5, 0.6, 0.7] | [0.55, 0.65, 0.7] |
| $A_2$ | [0.7, 0.8] | [0.5, 0.6, 0.7, 0.8] | [0.65, 0.68] |
| $A_3$ | [0.75, 0.8] | [0.4, 0.5, 0.6, 0.7] | [0.45, 0.55, 0.65] |

According to the method mentioned in the previous section, we first need to convert these U-HFEs into C-HFEs with uniform distribution. Thus, the following three tables can be obtained:

**TABLE 4.** Decision-Making Matrix of C-HFEs With Uniform Distribution $E_1$.

|       | $P_1$          | $P_2$          | $P_3$          |
|-------|----------------|----------------|----------------|
| $A_1$ | $\left[ \begin{array}{c} 0.5, 0.7 \\ U(0.5, 0.7) \end{array} \right]$ | $\left[ \begin{array}{c} 0.7, 0.8 \\ U(0.7, 0.8) \end{array} \right]$ | $\left[ \begin{array}{c} 0.55, 0.6 \\ U(0.55, 0.6) \end{array} \right]$ |
| $A_2$ | $\left[ \begin{array}{c} 0.8, 0.9 \\ U(0.8, 0.9) \end{array} \right]$ | $\left[ \begin{array}{c} 0.4, 0.6 \\ U(0.4, 0.6) \end{array} \right]$ | $\left[ \begin{array}{c} 0.5, 0.65 \\ U(0.5, 0.65) \end{array} \right]$ |
| $A_3$ | $\left[ \begin{array}{c} 0.7, 0.8 \\ U(0.7, 0.8) \end{array} \right]$ | $\left[ \begin{array}{c} 0.5, 0.6 \\ U(0.5, 0.6) \end{array} \right]$ | $\left[ \begin{array}{c} 0.55, 0.65 \\ U(0.55, 0.65) \end{array} \right]$ |
First, we integrated the evaluation values of the three plans ($E_i$, $i = 1, 2, 3$) by each of the three experts ($P_i$, $i = 1, 2, 3$) on each attribute, and obtained their comprehensive evaluation values of the three plans as follows:

\[ h_{P_1,E_1}(p) = \left\{ \int (p(x))dx \right\}, \]
\[ p(x) = 41281000x^6 - 147350000x^5 + 2973000x^4 - 380610x^3 + 20775x^2 - 118.83x + 0.34714 \]
\[ h_{P_1,E_2}(p) = 0.5349, 0.65095 \quad \left( \int (p(x))dx \right), \]
\[ p(x) = 693970000x^6 - 241150000x^5 + 32798000x^4 - 2207800x^3 + 70011x^2 - 516.69x + 1.138 \]
\[ h_{P_1,E_3}(p) = 0.48809, 0.69391 \quad \left( \int (p(x))dx \right), \]
\[ p(x) = 7346700x^6 - 4587900x^5 + 1106200x^4 - 128640x^3 + 6475.8x^2 - 8.7251x + 0.31143 \]

\[ h_{P_2,E_1}(p) = \left\{ \int (p(x))dx \right\}, \]
\[ p(x) = 510800x^6 - 2415200x^5 + 1339700x^4 - 282130x^3 + 18334x^2 - 109.54x + 0.4334 \]
\[ h_{P_2,E_2}(p) = 0.55143, 0.69144 \quad \left( \int (p(x))dx \right), \]
\[ p(x) = -159570000x^6 + 67091000x^5 - 10119000x^4 + 640410x^3 - 16693x^2 + 363.17x - 0.78449 \]

\[ h_{P_2,E_3}(p) = \left\{ \int (p(x))dx \right\}, \]
\[ p(x) = -1604600x^6 + 1174600x^5 - 278190x^4 + 18662x^3 + 647.08x^2 - 3.3626x + 0.16154 \]
\[ h_{P_3,E_1}(p) = \left\{ \int (p(x))dx \right\}, \]
\[ p(x) = 599910000x^6 - 174710000x^5 + 23480000x^4 - 1789700x^3 + 61921x^2 - 276.33x + 0.79682 \]
\[ h_{P_3,E_2}(p) = 0.55164, 0.63178 \quad \left( \int (p(x))dx \right), \]
\[ p(x) = -589560000x^6 + 144260000x^5 - 119420000x^4 + 3508000x^2 - 15657x^2 + 224.59x + 0.085998 \]
\[ h_{P_3,E_3}(p) = \left\{ \int (p(x))dx \right\}, \]
\[ p(x) = -264380000x^6 + 97354000x^5 - 12590000x^4 + 645750x^3 - 12366x^2 + 335.66x - 0.23987 \]

Next, we integrated the opinions of all three experts. Based on the familiarity and authority of the three experts in the field, three experts are given weights of 0.4, 0.4, 0.2, respectively. Thus, the final evaluation values of the three plans ($P_i$, $i = 1, 2, 3$) can be obtained:

\[ h_{P_1}(p) = \left\{ \int (p(x))dx \right\}, \]
\[ p(x) = -3944400000x^6 + 1143200000x^5 - 1171900000x^4 + 4847300x^3 - 66056x^2 + 410.29x - 0.37833 \]
\[ h_{P_2}(p) = \left\{ \int (p(x))dx \right\}, \]
\[ p(x) = -1582600000x^6 + 54186000x^5 - 71297000x^4 + 3970400x^3 - 84215x^2 + 676.32x - 1.1528 \]
\[ h_{P_3}(p) = \left\{ \int (p(x))dx \right\}, \]
\[ p(x) = -5781900000x^6 + 1222100000x^5 - 926830000x^4 + 294910000x^3 - 349690x^2 + 1639.7x - 1.6831 \]

Calculating the scores of the three C-HFEs, we can get:

\[ s(h_{P_1}(p)) = \frac{\int x \cdot p_1(x)dx}{\int p_1(x)dx} = 0.66 \]

where $x = 0.59977$, $\bar{x} = 0.69667$, and $p_1(x) = -3944400000x^6 + 1143200000x^5 - 1171900000x^4 + 4847300x^3 - 66056x^2 + 410.29x - 0.37833$

\[ s(h_{P_2}(p)) = \frac{\int x \cdot p_2(x)dx}{\int p_2(x)dx} = 0.62 \]
where \( x = 0.54795 \), \( \bar{x} = 0.67113 \) and \( p_2(x) = -1582600000x^6 + 541860000x^5 - 712970000x^4 + 3970400x^3 - 84215x^2 + 676.32x - 1.1528 \).

\[
\begin{align*}
\int_{\underline{x}}^{\bar{x}} x \cdot p_3(x) dx &= 0.6 \\
\int_{\underline{x}}^{\bar{x}} p_3(x) dx &= 0.6 
\end{align*}
\]

where \( x = 0.56077 \), \( \bar{x} = 0.63161 \) and \( p_3(x) = -5781900000x^6 + 1222100000x^5 - 926830000x^4 + 294910000x^3 - 349690x^2 + 1639.7x - 1.6831 \).

Obviously, according to the calculation results above and Definition 3, we can obtain that

\[ P_1 > P_2 > P_3 \]

Since the core content of this paper is the theory and method of operation and integration of continuous hesitant fuzzy information, here we use a relatively simple example to illustrate the operation and integration method of C-HFEs and U-HFEs proposed above. In fact, the above method has greater advantages in more complex decision-making problems. We will apply this method to more complex practical decision-making problems in our future research.

### B. COMPARATIVE ANALYSIS AND DISCUSSIONS

In order to verify the effectiveness and simplicity of this method, we use the original information integration method of HFEs to solve the same problem and make a comparative analysis and discussion in this section.

We use Definition 1 and its extensions to aggregate the evaluation values of the three plans \((P_i, i = 1, 2, 3)\) by each of the three experts \((E_i, i = 1, 2, 3)\) on each attribute. P-HFEs are using here to express the integration results. Then, the experts’ total evaluation values of the three plans can be shown in the following table:

Next, we integrate the decision-making information of the three experts according to the weights of 0.4, 0.4, and 0.2. And then, the final evaluation values of three plans \(h_{P_1}(p), h_{P_2}(p)\) and \(h_{P_3}(p)\) can be obtained, where \(h_{P_i}(p) = h_{P,E_1}(p) \oplus h_{P,E_2}(p) \oplus h_{P,E_3}(p)\), \(i = 1, 2, 3\). Each of these 3 P-HFEs contains more than 100 memberships and corresponding probabilities. Due to space limitations, we do not show them here. Below, we give the scores of three P-HFEs:

\[
s(h_{P_1}(p)) = 0.66, s(h_{P_2}(p)) = 0.62, s(h_{P_3}(p)) = 0.59
\]

Obviously, \(s(h_{P_1}(p)) > s(h_{P_2}(p)) > s(h_{P_3}(p))\). Then, the following result can be obtained:

\[ P_1 > P_2 > P_3 \]

Comparing the calculation process and results of the two methods, the following two conclusions can be obtained:

(1) Judging from the results, the ranking results obtained by the two methods are the same, and the difference between the scores of the three plans obtained by the two methods is very small. This proves the effectiveness of the proposed method.

(2) From the perspective of the calculation process, the method proposed in this paper is much simpler than the existing methods, which effectively reduces the negative impact of too many membership degrees on the calculation process.

In fact, in this paper, the discrete hesitant fuzzy information is continuously processed, which is a new attempt to improve the integration skills of the hesitant fuzzy information.
We believe that similar methods will become more mature and have wider applications in practical decision-making problems in future.

VI. CONCLUSION AND OUTLOOKS
The main work of this paper is to try to use the concept of density function to process the originally discrete hesitant fuzzy information continuously based on the combining of probability theory and fuzzy decision-making theory. The main innovations are summarized as follows:

1. This paper proposes the concept of U-HFE, establishes the connection between discrete HFEs and continuous HFEs, and lays the foundation for the continuity of HFSs, so that the continuous processing method can be more widely used in practical decision-making problems.

2. This paper develops the integration methods of C-HFEs with normal distribution and C-HFEs with uniform distribution from the perspective of mathematical derivation, which provides a theoretical foundation for the integration of C-HFEs.

3. In this paper, a further integrated method of C-HFEs is given based on a computer simulation algorithm. This method and the method given in (2) also provide new ideas for the integration of various expression forms of decision-making information which is based on interval-values. Further, a case study and the corresponding comparative analysis are used to illustrate its applicability in practical decision-making problems. Compared with the method based on mathematical derivation, this method provides a simpler calculation and has the potential to be used to solve a wider range of problems.

In the future, at first, the integration methods of C-HFEs based on mathematical derivation needs further research. At this stage, such methods have some weaknesses such as tedious calculation and inconvenient to be used. This leads to their limitation for wide applications. However, mathematical derivation can provide a theoretical basis for the integration of C-HFEs, and therefore is also an integral part of the study of C-HFEs. Therefore, for future research, we should consider using some methods to simplify and improve the calculation process. Next, as this paper focuses on the methodology development, the calculation examples provided are relatively simple. We will improve these methods in future research and apply them to more complex and practical decision-making problems. Finally, we should continue to study the continuity of HFSs, and even the continuity of language information such as HFLTSs. The continuity here includes two aspects, namely the continuity of definition and the continuity of processing. If such a problem can be better solved, then in the future, when we face decision-making problems based on uncertain information such as HFSs, HFLTSs, P-HFSs and PLTSs, the processing method will be simpler and more flexible.

ACKNOWLEDGMENT
The authors thank the Editor-in-Chief, the Associate Editor and the anonymous reviewers for their helpful comments and suggestions, which have led to an improved version of this paper.

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