Refined dependences of power-law form for calculation of hydraulic resistance and velocity distribution

Valery Borovkov¹, Julia Bryanskaya¹ and Aleksandra Ostyakova¹,²

¹Moscow State University of Civil Engineering, Yaroslavskoye Shosse, 26, Moscow, 129337, Russia
²Laboratory of the dynamics of channel flows and ice heat, Water problem Institute RAS, Gubkina str., 3, Moscow, 119333, Russia

E-mail: ¹mg-su-hydraulic@yandex.ru; ²alex-ost2006@yandex.ru

Abstract. To solve the engineering problems of hydraulic engineering, water supply and sanitation, prevent emergency situations at hydraulic structures, regulate channel processes, efficient ecological monitoring of water bodies, and develop measures that exclude crisis ecological situations, more and more accurate estimates are required for the hydraulic characteristics of water flows. Inaccurate assumptions and approximate methods of integration can lead to significant errors that significantly affect the results of predicting flooding of riverine areas; this requires further refinement of the calculation methods. Until now, theoretical and experimental substantiations of the basic propositions of the theory of turbulence, hydraulic resistances, channel processes cannot be considered sufficiently complete. The compositions of problems associated with flows in wide channels, problems of turbulence, ax symmetric flows in smooth and rough pipes, are of scientific and practical interest. The article investigates the dependencies for the calculation of kinematic and dynamic flow characteristics in deformed and undeformable boundaries. New formulas are given for a refined description of the distribution of flow velocities and hydraulic resistance of turbulent flows in pipes and channels. Based on the known fundamental research, the calculation-theoretical methods are refined. The results obtained for flows in non-deformable channels can be useful for estimating not only flow in pipes, but also river flows in hard, undeformable boundaries and with minor channel deformations (in channels and rivers). For the first time, the analysis made it possible to obtain power-law approximations of the resistance laws for smooth and rough tubes and to establish the actual relationship between the exponents in the velocity profile and the law of resistance.

1. Introduction
The fundamental problem of designing structures is the reliability of their work, to ensure that reliable methods of engineering calculations should be used for the structures in question. The solution of problems related to hydraulic engineering, forecasting and regulation of channel processes, prevention of emergency situations in various hydraulic structures (pressure and non-pressure pipes, canals, etc.), effective environmental monitoring of the status of rivers and banks of rivers, reservoirs and other water bodies and the development of measures that exclude crisis ecological situations, requires the most accurate hydraulic characteristics of water flows.

Inaccurate assumptions and approximate methods of integration can lead to significant errors that significantly affect the results of predicting flooding of riverine areas; this requires further refinement
of the computational methods. Until now, theoretical and experimental substantiations of the basic propositions of the theory of turbulence, hydraulic resistances, channel processes cannot be considered sufficiently complete.

Until now, questions are being discussed about the corrected, correct calculation of the hydraulic resistance for various flow conditions.

In most cases, hydraulic calculations are carried out using conventional, traditional formulas, often without questioning their applicability to conditions of water flow in a particular watercourse. The composition of tasks related to flows of pressure and non-pressure pipes, wide channels, problems of turbulence, axisymmetric flows in smooth and rough pipes are of scientific and practical interest.

This paper suggests more precise relationships for the hydraulic calculations of velocity distribution and hydraulic resistance in deformable and undeformable boundaries based on the power and logarithmic distribution of velocities and the laws of resistance for pipes and for open flows. Thus, theoretical and theoretical methods are being refined on the basis of known fundamental research.

2. Materials and methods

Recently, the dependencies of the power-law form for the velocity distribution in the stream have acquired the status of universal ones. In the well-known classical study of the power profile of the velocity with an exponent varying depending on the Karman parameter and the hydraulic resistance coefficient \( \lambda \) [1], [2], it is concluded that the profile of a power-law type is exactly the same as the logarithmic profile. Comparison of the velocity profiles of the power law and the logarithmic form provides additional useful information on the parameters of these profiles and the degree of their correspondence to known resistance laws. Using the relationship between the mean and maximum velocity in an axisymmetric tube, obtained by using the logarithmic velocity profile

\[
\frac{u_{\text{max}}}{u_c} = \frac{2.3}{\kappa} \log \frac{r}{k_\varepsilon} + 8.48
\]  

(1)

where \( u_{\text{max}} \) – maximum flow velocity in the pipe, \( u_c \) - dynamic velocity, \( \kappa \) - Karman parameter, \( r \) - pipe radius, \( k_\varepsilon \) - coefficient of equivalent roughness, and by the power-law profile of velocity [1]:

\[
\frac{u}{u_{\text{max}}} = \left( \frac{z}{h} \right)^n
\]  

(2)

where \( u \) – the flow velocity at a distance \( z \) at a depth \( h \), \( n \) – exponent, and also using the power profile

\[
\frac{u}{u_{\text{max}}} = \left( \frac{z}{r} \right)^n
\]

and the relationship between the mean \( V \) and the maximum velocity \( u_{\text{max}} = 1 + 1.5n \), we find for \( \kappa = 0.4 \) and \( n = 0.9\sqrt{\lambda} \):

\[
\frac{1}{\sqrt{\lambda}} = 2.03\log \frac{r}{k_\varepsilon} + 1.65
\]

that practically does not differ from the law of resistance of Nikuradze for rough pipes. Similarly, for an open flow, we obtain using the logarithmic distribution of the velocity (1) with allowance for

\[
\frac{V}{u_c} = \frac{\sqrt{8}}{\sqrt{\lambda}} \quad \text{and} \quad n = A\sqrt{\lambda}
\]

we have:

\[
\frac{1}{\sqrt{\lambda}} = 2.03\log \frac{2h}{k_\varepsilon} + 2.386 - A
\]  

(3)
Taking into account for open flows the law of resistance of A. P. Zegzda [3] \( \frac{1}{\sqrt{\lambda}} = 2.03 \log \frac{2r}{k_s} + 1.52 \), we find from (3) that the value of \( A = 0.87 \), which practically coincides with the value of \( A \) in the ratio \( n = 0.9 \sqrt{\lambda} \), which obtained for pressure pipes by A D Altshul and V Nunner [4].

Since the exponent \( n \) and the parameter \( \kappa \) do not remain constant with the distance from the water flow boundary [5], the comparison of the power and logarithmic velocity profiles provides useful information in this regard too. Thus, from the conditions \( u_{\text{max}} = \text{idem} \) and \( \frac{du}{dz} = \text{idem} \), the following relationship between the values of \( \kappa \) and \( n \) is obtained:

\[
\frac{u_{\text{max}}}{V} = \frac{\sqrt{8}}{\sqrt{\lambda}} \kappa n \left( \frac{z}{n} \right)^n = 1
\]  

Expressing \( u_{\text{max}} \) as \( \lambda \) in the form \( 1 + A \sqrt{\lambda} \), we obtain a relation for calculating the change of \( n = f_1(z) \) for the known \( \kappa = f_2(z) \). Since these functions are not known to us, we attempt to calculate \( n = f_1(z) \) for \( \kappa = \text{const} \). The results of such a calculation, which should be considered only as the first approximation of the function \( n = f_1(z) \) (figure 1), are agrees with the processing data of Nikuradze's experiments [5], [6].

![Figure 1](image.png)

**Figure 1.** Change the exponent in the velocity profile along the flow cross-section. Calculation for \( \lambda \) equal to: (1) – 0.015; (2) – 0.025; (3) – 0.030; (4) – 0.40; (5) – 0.50; (6) – averaging curve

From the condition of the coincidence of the values of the velocity, determined from the logarithmic and power-law velocity profile, one can obtain the following dependence, indicating that the Karman parameter not only varies along the vertical coordinate \( z \), but also depends on the value of \( \lambda \):

\[
\kappa = \frac{1.15 \left( 2 \log \frac{z}{h} + \frac{1}{\sqrt{\lambda}} - 2.34 \right)}{\left( 1 + 0.9 \sqrt{\lambda} \right) \sqrt{8} \left( \frac{z}{h} \right)^n - 8.48}
\]
Along with the logarithmic form of the law of resistances, formulas are widely used to calculate the resistance of a power type. For example, the Blasius formula \( \lambda \sim \left( \frac{1}{\text{Re}} \right)^{1/7} \) is obtained theoretically from the power-law profile of the velocity at \( n = \frac{1}{7} \) [7]. However, as shown by Nikuradze, the Blasius dependence agrees with the experimental data only in a limited range of Reynolds numbers.

An analysis of the Nikuradze formula for rough pipes (figure 2) shows that it cannot be approximated by a power law with a constant exponent over the whole range of variation \( \frac{d}{k_s} \).

\[
\text{Figure 2. Power approximation of the logarithmic formula of the Nikuradze resistance for rough pipes (design points)}
\]

It is generally accepted [8] that the exponent \( m \) in the resistance formula and \( n \) in the velocity profile are related by:

\[
m = \frac{2n}{n + 1} \quad (5)
\]

The relationship is obtained by comparing the power-law resistance formula:

\[
\lambda = \frac{a}{\text{Re}^m} \quad (6)
\]

and the expression for the coefficient of resistance obtained by integrating the power law of the speed:

\[
\lambda = 2^{n+1} \cdot N^{2n} \cdot \left[ \frac{2(n+1)(n+2)}{2^n} \right]^{n+1} \quad (7)
\]

where \( N = \frac{u \cdot \delta_v}{\nu} \) - the dimensionless value of the viscous sublayer, up to which the validity of the power-law velocity profile, \( \delta_v \) – thickness of viscous sublayer.

As a result of comparing the dependences (6) and (7), we conclude that the exponents of the degree over the number Re are equal. This conclusion, first made long ago [9], when the relationship between the exponent \( n \) and \( \lambda \) was not yet established, should now be considered very inaccurate or completely erroneous. Indeed, the numerator of expression (7) is a function of coefficient of hydraulic resistance \( \lambda \), and therefore depends on the Reynolds number. The analysis of expression (7) showed that the
numerator for \( n = 0.9\sqrt{\lambda} \) and \( N = 11.6 \) really depends on the value of value \( \lambda \), and the entire expression can be written with sufficient accuracy in the form:

\[
\lambda = 3.3 \left( \frac{1}{\text{Re}} \right)^{0.41^{n+1}}
\]  

(8)

According to this relationship, the relationship between \( m \) and \( n \) turns out to be significantly different from that given in [8], however, the presence of the dependence of \( m \) on \( \lambda \) also in this case is beyond doubt. Note that for values of \( \lambda \) in the range 0.02-0.025, the dependence (8) is very close to the Blasius formula [7].

Since the dependence \( n = 0.9\sqrt{\lambda} \) is sufficiently reliable and justified by experimental data, we investigate the connection between \( m \) and \( \lambda \) using the experimental dependences of Nikuradze [10], [11].

Based on Nikuradze’s data on the resistance of roughened pipes, the following power-law resistance formula:

\[
\lambda = 2.12 \left( \frac{k_s}{d} \right)^{3.25^{0.4}}
\]  

(9)

where \( d \) – pipe diameter.

The obtained power dependence (9) corresponds to a logarithmic formula with an accuracy of 1.5 \%. Given the coincidence of the elements of the exponent in the laws of resistance (8) and (9) for smooth and rough pipes, the resistance formula for smooth pipes can be transformed to the form:

\[
\lambda = 3.45 \left( \frac{28.5}{\text{Re}} \right)^{3.25^{0.4}}
\]  

(10)

this formula corresponds to the dependence (8) with an accuracy of 0.5 \%.

It is of interest to compare the found exponent \( m = \frac{5.5\sqrt{\lambda}}{1+0.9\sqrt{\lambda}} \) in the resistance laws with the exponent \( n = 0.9\sqrt{\lambda} \) in the velocity profile.

The calculation data \( m \) from the obtained relation, shown in figure 3, are approximated with sufficient accuracy by the expression:

\[
m = \frac{6n}{n+1}
\]  

(11)

Figure 3. The exponent in the law of resistance, 1 - calculated points
The last relation (11) differs significantly from the known relation (5). It should also be noted that
the numerical coefficients in (9) and (10) differ from the analogous coefficients in the well-known
Altshul's formula [1].

3. Results
The article analyzes kinematic and dynamic dependencies for the flow of water in pressure pipes and
open channels. It is concluded that the comparison of the logarithmic and power-law velocity profiles
gives similar accuracy in the calculations. In the course of the above considerations, the point is
stressed that the parameters of the velocity profile and the coefficient of hydraulic resistance are not
identical along the vertical section in the flow of the liquid.

A comparison is made of the exponent n in the velocity profile with the exponent in the law of
resistance at different values of the coefficient of hydraulic resistance.

A functional dependence of the value of n and the Karman parameter on the distance from the
bottom of the water flow is given.

A universal formula for the coefficient of resistance and exponent in the velocity profile is
given; this formula operates at any distance from the bottom and is well compared with the Blasius formula.

The analysis of theoretical and experimental data made it possible to obtain power-law
approximations of the resistance laws for smooth and rough boundaries and to establish the actual
relationship between the exponents in the velocity profile and the law of resistance.

4. Conclusions
Obtained dependencies for hydraulic calculations of velocity distribution and hydraulic resistance in
deformable and undeformable boundaries, based on the power and logarithmic distribution of
velocities and the laws of resistance for pipes and for open flows.

Thus, theoretical and theoretical methods are being refined on the basis of known fundamental
research.

The results obtained for flows in non-deformable channels can be useful for estimating not only
flow in pipes, but also river flows in hard, undeformable boundaries and with minor channel
deformations (in canals and rivers).

References
[1] Bogomolov A I, Borovkov V S and Mayranovsky F G 1979 High-speed flows with a free
surface (Moscow: Stroyizdat) p 347
[2] Borovkov V S, Baykov V N and Fomin A A 2005 Hydraulic engineering. Natural studies of
the kinematic structure of river streams of various water content No 4 p 26-28
[3] Zegzhda A P 1957 Hydraulic losses on friction in channels and pipelines (Moscow-
Leningrad: Stroyizdat) p 277
[4] Altshul A D 1982 Hydraulic resistance (Moscow: Nedra) p 222
[5] Bryanskaya Yu V 2003 Improvement of hydraulic calculation methods of characteristics of
flow and resistance in pipes. Dissertation for the degree of candidate of technical sciences.
(Moscow: Moscow Civil Engineering University) p 165
[6] Bryanskaya Yu V, Markova I M and Ostyakova A V 2009 Hydraulics of water and weighted
flows in hard and deformable boundaries (Moscow: ASV) p 264
[7] Schlichting G 1969 The boundary layer theory (Moscow: Nauka) p 742
[8] Povh I L 1969 Technical hydromechanics (Leningrad: Machine building) p 524
[9] Richter G 1936 Hydraulics of Pipelines (Moscow-Leningrad: ONTI) p 324
[10] Nikuradse I 1932 Forschungs-heft (Forschungs auf dem Gebiete des Ingenieur-wesens).
Gesetzmessigkeiten der turbulenten Stroemung in glatten Rohren 356 p 1-36
[11] Nikuradse I 1933 Forschungs-Heft (Forschungs auf dem Gebiete des Ingenieur-wesens).
Stroemungsgesetze in rauhen Rohren 361 p 1-22