The Instantaneous Redshift Difference of Gravitationally Lensed Images: Theory and Observational Prospects

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Abstract

Due to the expansion of our universe, the redshift of distant objects changes with time. Although the amplitude of this redshift drift is small, it will be measurable with decade-long campaigns by the next generation of telescopes. Here we present an alternative view of the redshift drift which captures the expansion of the universe in single-epoch observations of the multiple images of gravitationally lensed sources. Considering a sufficiently massive lens, with an associated time delay of order decades, simultaneous photons arriving at a detector would have been emitted decades earlier in one image compared to another, leading to an instantaneous redshift difference between the images. We also investigated the peculiar velocity which may influence the redshift difference in observation. While still requiring the observational power of the next generation of telescopes and instruments, the advantage of such a single-epoch detection over other redshift drift measurements is that it will be less susceptible to systematic effects that result from requiring instrument stability over decade-long campaigns.

Unified Astronomy Thesaurus concepts: Gravitational lensing (1643); Cosmology (343)

1. Introduction

Modern cosmology is built on the concept of an expanding universe and, while there is overwhelming evidence for this expansion, direct detection of this phenomenon is challenging and will require a measurement of the evolving redshift of a cosmological source. This is known as the redshift drift and is of order of $10^{-18}$ s$^{-1}$ (Sandage 1961, 1962; McVittie 1962; Loeb 1998). However, such a direct detection would also provide a new cosmological probe for determining accelerating expansion (Uzan 2004, 2007), constraining dark energy (Balbi & Quercellini 2007; Corasaniti et al. 2007; Lake 2007; Zhang et al. 2007), and measuring the possible temporo-spatial variation of cosmological parameters (Molaro et al. 2005; Gordon et al. 2007; Geng et al. 2018; Amendola & Quartin 2021). In an extensive study, Liske et al. (2008) demonstrated that the redshift drift will be observable using next-generation telescopes, such as the Extreme Large Telescope (ELT), through a 20-year campaign of monitoring absorption lines of hundreds of quasars (Liske et al. 2008; Kim et al. 2015; Cooke 2019; Melia 2022).

Here we discuss the possibility of taking advantage of the gravitational lensing phenomenon to identify the redshift drift through single-epoch observations of gravitationally lensed sources. We take advantage of gravitational lensing time delays between the observed images (Blandford & Narayan 1986). This means that photons arriving at our detector today from different images were not emitted at the same instant (Refsdal & Bondi 1964; Press et al. 1992; Biggs et al. 1999). This implies different scale factors and therefore different redshifts at these different emission instants. Consequently, we expect a redshift difference, $\Delta z = \frac{D_t}{D_l} \Delta t$, between the images, where $\Delta t$ is the time interval at the source. Depending on the parameters of the lens (e.g., velocity dispersion, size of the core, and position concerning the source), these time delays range from several days to 100 years (Kochanek et al. 1989; Kormann et al. 1994). Thus, instead of a 20 year long campaign, we advocate targeting gravitationally lensed systems with long time delays to provide direct support for the expansion of the universe.

The structure of this paper is as follows: Section 2 discusses key concepts of the gravitational lensing; Section 3 introduces the redshift difference; Section 4 discusses the observability; and Section 5 concludes this paper.

2. Gravitational Lens Theory

Gravitational lensing, the deflection of light due to the presence of a gravitational field, can result in multiple paths connecting a source and an observer. Different paths, and also different gravitational potential along these paths, result in time delays. The time delay is given by Blandford & Narayan (1986) as

$$\Delta t = (1 + z_l) \frac{D_l D_q}{c D_{ls}} \frac{1}{2} \left( \frac{\beta}{\theta} - \phi(\theta) \right),$$

(1)

where $z_l$ and $z_s$ are the redshifts of the lens and source, $D_l$ and $D_s$ are the corresponding angular diameter distances, while $D_{ls}$ is the distance between the lens and source. The angle $\theta$ is the position of the image, $\beta$ is the undeflected location of the source (see Figure 1), and $\phi$ is the two-dimensional gravitational potential.

The difference in time delays between two images at the source is

$$\Delta t_2 = \frac{D_l D_q}{c D_{ls}} \frac{1}{1 + z_s} \left( \tau(\theta_2, \beta) - \tau(\theta_1, \beta) \right),$$

(2)

where $\tau$ is the time delay due to the gravitational potential.
where $\theta_1$ and $\theta_2$ refer to two individual images, and $\tau (\beta, \theta) \equiv \frac{1}{2} (\beta - \theta)^2 - \phi (\theta)$. For the purposes of this study we adopt the non-singular ellipsoidal lens model from Kochanek et al. (1989) to represent the gravitational potential:

$$\phi (\theta) = b \sqrt{s^2 + (1 - \varepsilon) \theta^2_s + (1 + \varepsilon) \theta^2_v},$$  \hspace{1cm} (3)

where $\theta' = (\theta'_x, \theta'_y)$ are coordinates centered at the lens and related to the sky coordinates $\theta = (\theta_x, \theta_y)$ by a translation $(\theta_{0x}, \theta_{0y})$ and a counterclockwise rotation $\theta_0$ in the lens plane. The quantity $s$ is the smoothing scale, $\varepsilon$ is the eccentricity component of the lens, and the $\sigma_v$ is the velocity dispersion of the lens. The value outside the root $b = 4 \pi \frac{D_s}{D_l} \frac{\Delta z}{1 + z}$, where $\sigma_v$ is velocity dispersion and $c$ is the speed of light. Throughout we assume a cosmological model with $H_0 = 67.4 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_M = 0.315$, and $\Omega_{\Lambda} = 0.685$ (Planck Collaboration et al. 2020). The time delay between two images is presented in Figure 2, and it shows that for large galaxy clusters ($\sigma_v > 1000 \text{ km/s}$) the time delay is larger than a decade.

3. Redshift Difference and Gravitational Lensing

3.1. The Redshift Difference in a Stationary System

In a flat universe with a cosmological constant the evolution of the scale factor has an analytical solution of the form (Bolejko et al. 2005)

$$a(t) = \left( \frac{\Omega_M}{\Omega_{\Lambda}} \right)^{1/3} \left[ \sinh \left( 3 \sqrt{\frac{2}{3} \Omega_{\Lambda} H_0 t} \right) \right]^{2/3}.$$ \hspace{1cm} (4)

As noted previously, the goal here is to identify the intrinsic difference in emission times for photons arriving simultaneously at the observer (see Figure 1). Assuming that the time difference at the source for a pair of images observed today is $\Delta t_{12}$, the expected redshift difference is

$$\Delta z_{12} = z_2 - z_1 = \frac{1}{a(t_1 + \Delta t_{12})} - \frac{1}{a(t_1)}$$

$$\approx -(1 + z_t) H(z_t) \frac{D_t D_s}{c D_s} [\tau (\theta_2, \beta) - \tau (\theta_1, \beta)],$$  \hspace{1cm} (5)

where $H(z_t)$ is the Hubble constant at redshift $z_t$, and other parameters are same as in Equation (2).

In deriving the above expression, we have assumed that the differing scale factors at emission are the dominant influence on the observed redshift of an image. Clearly, there will be higher-order effects in the journey of the photons, as they encounter the lensing mass at different times, which will influence other aspects of the gravitational lensing configuration, such as the angular diameter distances. However, these are second-order effects: as discussed below and presented in Figure 3, the redshift difference turns out to be very small. Even for a large cluster it is $\Delta z \sim 10^{-8}$, and substantially smaller for a typical galaxy. Consequently $\Delta z \Delta \tau \ll z \Delta \tau \Rightarrow (1 + z + \Delta z) \Delta \tau \approx (1 + z) \Delta \tau$. Similarly, changes in the distance $\Delta D \Delta \tau$ result in second-order
The redshift errors due to peculiar velocity changes with the number of sources in a cluster lensing system moving with velocity $v_L = 1000 \text{ km s}^{-1}$. The velocity dispersion of the lens is $\sigma_L = 1500 \text{ km s}^{-1}$ with $s = 0.1$, $\epsilon = 0.7$, and $z_L = 0.5$. The presented precision corresponds to the error with which the velocity of the lens can be inferred from a system with multiple lensed sources.

corrections. Consequently, we treat the product $H_0 D_L D_s / D_L$ to be the same for both images.

In addition, here we neglect the impact of the redshift difference due to the change in velocity and the peculiar velocity of the lens itself. The time difference, either at the lens or source, is less than 100 yr. Considering the result from Amendola et al. (2008) and Dam et al. (2021), the peculiar acceleration is less than 1 cm s$^{-1}$ per decade for a cosmic object, which means a redshift difference around $\sim 10^{-10}$.

A more important effect absent in Equation (5) is the Birkenshaw–Gull effect (Birkenshaw & Gull 1983). Due to the motion of the lens, the gravitationally lensed image will exhibit a frequency shift proportional to the lens peculiar velocity and angular separation between the image and the lens (Birkskeshaw & Gull 1983; Molnar & Birkinshaw 2003; Kiledar & Lewis 2010):

$$\Delta z_{pec} = \beta \gamma \sin \delta \cos \chi,$$

(6)

where $\beta = v_L / c$ is the ratio of lens velocity and speed of light, $\gamma = (1 - \beta^2)^{-1/2}$, $\alpha$ is the deflection angle, $\delta$ is the angle between lens velocity and line of sight, and $\chi$ is the direction of the velocity in the lens plane. This is the dominant effect. For example, for a massive cluster moving at 300 km s$^{-1}$ and producing gravitationally lensed images with an angular separation of 20″, Equation (6) yields $\Delta z_{pec} \sim 10^{-7}$, which is approximately one magnitude larger than the expected redshift difference inferred from Equation (5). However, for a massive cluster we expect to observed multiple sources. With each source having multiple images, one can use Equation (6) to infer the velocity of the lens and hence remove this effect from the data. To show that this can be done, we generated mock data that comprise randomly positioned sources around a lens with $s = 0.1$, $\epsilon = 0.7$, and $\sigma_L = 1500 \text{ km s}^{-1}$, located at located at $z_L = 0.5$, and moving with a peculiar velocity of amplitude $v_L = 1000 \text{ km s}^{-1}$. We then adopted the position precision of the ELT of $5 \times 10^{-3}$ arcsec (Marconi et al. 2021) and implemented the Markov chain Monte Carlo method using the code emcee (Akeret et al. 2013; Foreman-Mackey et al. 2013) to infer $v_L$, and its accompanying error. The results presented in Figure 4 show that at least 10 sources will be required to infer the velocity of the lens and subtract it from the signal.

In should be however noted that our model is based on a symmetrical potential of the form given by Equation (3). Realistic clusters comprise a large number of individual galaxies, making their potential less symmetrical. In addition the individual peculiar velocities of these galaxies will contribute to the redshift difference. Thus, in practical applications the number of required sources sufficient to minimize the noise due to the Birkenshaw–Gull effect will most likely be larger than 10. In conclusion, the effects of peculiar velocity are either negligible or can be subtracted with sufficient precision. Thus, in the following discussion, we only consider the stationary situation.

4. Observational Prospects

When addressing the question of observability we adopt a method based on Liske et al. (2008), which was pioneered by Connes (1985) and later developed by Bouchy et al. (2001). We use $\lambda_1$ and $\lambda_2$ to refer to the observed wavelength of the first and second images from which the redshift is determined. From Equation (16) and Figure 12 in Liske et al. (2008), we calculate the requirements we need to measure the redshift difference, with the main factors affecting the detectability being: (i) the amplitude of the effect, directly related to the mass of the object (hence the gravitational time delay), and (ii) the technical properties of the instrument itself.

First, for a galaxy lens, recently reported by Bettoni et al. (2019), where are $z_L = 0.56$, $z_s = 3.03$, $\sigma_L = 197.9 \text{ km s}^{-1}$, the maximal redshift difference is of order $\Delta z = 5 \times 10^{-12}$. Using a specific emission line, such as the C IV line, the largest wavelength difference in this system is $\Delta \lambda = 7.75 \times 10^{-9} \text{ Å}$. Thus, the redshift difference induced by a typical galaxy is beyond the detectability of current or even next-generation telescopes, whose accuracy is expected to be of the order of $\Delta \lambda / \lambda \approx 10^{-9}$ (Liske et al. 2008).

However, for a galaxy cluster with velocity dispersion $\sigma_L = 1500 \text{ km s}^{-1}$, eccentricity parameter $\epsilon = 0.6$ (see Equation (3)), $z_L = 0.5$, and source redshift $z_s = 4$, the expected redshift difference can be as large as $\Delta z = 2.67 \times 10^{-8}$. The largest wavelength difference in this system will be $\Delta \lambda = 4.14 \times 10^{-5} \text{ Å}$ for the C IV line. This substantial difference in wavelength should be within reach of not only the next generation of optical telescopes, but detectable also in the radio. Recently, it was reported that the Five-hundred-meter Aperture Spherical radio Telescope (FAST) will be, in principle, able to analyze the spectrum of observed objects at 1.4 GHz with a precision of the order of $10^{-7}$ (Lu et al. 2022). Although such measurements are challenging, and non-uniform matter distribution within absorbing clouds will contribute to the uncertainty of the redshift difference, in principle this opens a new possibility for detecting the expansion of the universe.

The main idea behind this method is that, instead of a decade-long campaign, we need to target systems where the time delay is at least of the order of a decade at the source. To maximize the redshift between gravitationally lensed images we need to maximize the interval between the emission instants of the photons that are detected today. This is affected by two features: (i) the cosmological configuration, i.e., the redshift of the lens $z_L$ and source $z_s$, which in turns affects distances, and (ii) the characteristics of a gravitational lens.
The impact of the cosmological configuration (i.e., redshift of lens $z_L$ and source $z_S$) for a cluster with a velocity dispersion $\sigma_v = 1000\ km\ s^{-1}$ presented in Figure 3 implies that the preferable redshift of the lens is $z_L = 0.5$ and a source $z_S > 3$. As for the characteristics of the lens, the main parameter influencing the amplitude of the redshift difference is the mass of the lens, encoded in the velocity dispersion of the adopted model. Other parameters such as $\epsilon$ and $s$ play a lesser role. For example, fixing $z_L = 0.5$ and $z_S = 4$, and changing the core radius $s$ by an order of magnitude, i.e., from $s = 0.1$ to $s = 11.9$ (see Wallington et al. 1995) results in a change of the redshift difference from approximately $\Delta z_\epsilon = 5.31 \times 10^{-9}$ to approximately $\Delta z_\epsilon = 5.65 \times 10^{-9}$. On the other hand the change in the velocity dispersion results in more significant changes, giving rise to a much larger effect. This is presented in Figure 5, where for example the maximal redshift difference changes from $\Delta z_\sigma = 4.7 \times 10^{-13}$ to $\Delta z_\sigma = 5.6 \times 10^{-9}$ when the velocity dispersion changes from $\sigma_v = 100\ km\ s^{-1}$ to $\sigma_v = 1000\ km\ s^{-1}$.

Thus, in order to observe the redshift difference we need to target cosmological objects with velocity dispersion larger than $\sigma_v = 1000\ km\ s^{-1}$, which puts us in the range of massive clusters with $M > 10^{15}\ M_\odot$. In order to maximize the effect of the redshift difference, the source redshift should be as large as possible, preferably $z_S \approx 4$, and the lens redshift should preferably be between $z_L = 0.4$ and $z_L = 0.7$. With such large clusters it is expected that there will be multiple background sources being lensed. With at least 10 lensed sources we will be able to remove the effect of the peculiar velocity of the lens. This implies that the phenomenon suggested in this paper should be observable with next-generation telescopes.

5. Summary and Perspective

Due to cosmic expansion, the observable properties of cosmological objects will change with time. The most studied effect is the redshift drift (Balbi & Quercellini 2007; Uzan et al. 2008), but other properties include position drift (Quartin & Amendola 2010; Krasiński & Bolejko 2011), flux drift (Bolejko et al. 2019), distance drift (Korzyński & Kopinski 2018; Grasso et al. 2019), as well as a drift of gravitationally lensed images (Piatella & Giani 2017; Covone & Sereno 2022). Here we have discussed a new phenomenon, the redshift difference between the images of gravitationally lensed systems. This method will allow us to measure the effect of the cosmological expansion in a single observation.

We find that the redshift difference is more sensitive to the cosmological locations of the source and the lens, as well as the lensing mass, with more massive lenses producing larger time delays and hence redshift difference. Hence, cluster lenses with time delays greater than decades make ideal targets for such a study.

It is important to emphasize that the redshift difference is a directly measurable variable of the expanding history of the universe. Although this will be a technically difficult observation, it has the potential to lead to a measurement akin to the redshift drift. The advantage of the redshift difference, as opposed to the classical measurement of the redshift drift, is that it does not require a decade-long observational campaign. By measuring the redshift difference in the gravitational lens, this long period of observation may shrink into a matter of days. This will eliminate problems related to keeping the instrument stable over such long periods. Thus, it will make the measurement of the redshift difference more accessible than the redshift drift. New instruments such as FAST, the Square Kilometer Array, and eventually the ELT will thus allow us to use this new method to study the evolution of the universe at high redshift.

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