The effect of three matters on KSS bound

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Abstract

In this paper we introduce the black brane solutions in AdS space in 4-dimensional (4D) Einstein-Gauss-Bonnet-Yang-Mills theory in the presence of string cloud and quintessence. Shear viscosity to entropy density ratio is computed via fluid-gravity duality, as a transport coefficient for this model.

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1 Introduction

The AdS/CFT duality [1]-[4] originates from string theory and provides a new perspective on quantum gravity. In its most general form, it is known as a gauge/gravity duality. This duality relates weakly coupled gravitational theories in $(D+1)$-dimensional AdS space-time with strongly coupled conformal field theories (CFTs) defined on the $D$-dimensional boundary of AdS. In particular, it gives support to study the transport coefficients from hydrodynamics to the quark-gluon plasma formed at relativistic heavy-ion collisions [5]-[8]. Shear viscosity $\eta$ [9]-[12] as one of these coefficients is the most well-known coefficient calculated by this duality, especially fluid-gravity duality [13]-[16].

The Kovtun-Son-Starinets (KSS) bound states that the ratio $\eta/s$ has a lower bound, $\frac{\eta}{s} \geq \frac{1}{4\pi}$, for all relativistic quantum field theories [11],[12] and can be interpreted as the Heisenberg uncertainty principle [13],[17] where $\eta$ and $s$ are the shear viscosity and entropy density, respectively. However, this conjecture violates for some theories like the Gauss-Bonnet gravity [18], the Gauss-Bonnet gravity in presence of $U(1)$ gauge field [19],[20], Horndeski theory [21], massive gravity for $c_i < 0$ [22], scalar-tensor

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gravity [23]-[25] and anisotropic black brane [26]. Observational cosmology shows the accelerated expansion of the Universe [27],[28]. Therefore, the Universe at present is becoming filled with a strange substance. This outcome is confirmed by the measurement of the Cosmic Microwave Background (CMB) using PLANCK space Satellite [29]. It is believed that this expansion is due to a substantial negative pressure arising from the dark energy that is up to 70% of the total energy of the Universe. There are two sources of this negative pressure: one is cosmological constant and the other is the so called quintessence [30],[31] which acts as a repulsive force against the gravity. Quintessence is described by a scalar field and a parameter $\omega$, defined as the ratio of the pressure to the energy density of the dark energy. Black hole solution surrounded by the quintessence introduced in [41]. String theory in which the fundamental ingredient of our universe is assumed to be one-dimensional strings, instead of particles, tries to merge the gravitational theory and quantum mechanics. An extension of this idea is to consider a cloud of strings [33]-[39] and study its possible measurable effects on long range gravitational fields of some sources, such as black holes. Letelier [33] has tried to extend the idea of a string to a black hole, possibly to reveal some properties of the black hole. Ghosh and et al. obtained a generalization for third-order Lovelock gravity [34], and Herscovich and et al. [35] obtained the solution for Einstein-Gauss-Bonnet theory in the Letelier spacetime.

General relativity (GR) provides the standard description of gravity and Gauss-Bonnet gravity is an extension gravity theory. Recently a new proposal has been offered wherein GB term is made in $4D$ dimensions [40]. In that, the GB coupling is scaled as $\alpha \rightarrow D^{-4}$ and thereby canceling out $(D - 4)$ factor in the equation, and then taking the limit $(D \rightarrow 4)$. This results into an effective equation in $4D$ which is in fact the Einstein-Gauss-Bonnet (EGB) equation written in $D = 4$. Then it could be solved in spacetime with some specific symmetries for different situations, black holes and cosmology.

In this paper, we consider $4D$ Einstein-Gauss-Bonnet-Yang-Mills theory (EGBYM) in the presence of string cloud and quintessence. Then, by calculation of $\frac{\alpha'}{D}$ via fluid-gravity duality we study the effects of quintessence, string cloud and Yang-Mills charge on the field theory dual side.

## 2 4D AdS Einstein-Gauss-Bonnet-Yang-Mills in presence of string cloud and quintessence black brane

The action of AdS Einstein-Gauss-Bonnet-Yang-Mills in the presence of string cloud and quintessence is:

$$I = \frac{1}{16\pi} \int d^Dx \sqrt{-g} \left[ R - 2\Lambda + \frac{\alpha'}{D-4} \mathcal{G} + F^{(a)}_{\mu\alpha} F^{(a)\mu\alpha} \right] + S_{cs} + S_{\text{quint}},$$

(1)
where $R$ is the scalar curvature, $\Lambda = -\frac{(D-1)(D-2)}{2l^2}$, $\alpha'$ is a (positive) Gauss-Bonnet coupling constant with dimension (length)$^{\frac{D-2}{2}}$, $G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$, $F^{(a)}_{\mu\nu} = \partial_{\mu}A_{\nu}^{(a)} - \partial_{\nu}A_{\mu}^{(a)} - i[A_{\mu}^{(a)}, A_{\nu}^{(a)}]$ is the Cartan subalgebra of $SU(2)$ Yang-Mills field strength tensor in which the gauge coupling constant is 1, $A_{\nu}$’s are the Cartan subalgebra of the $SU(2)$ gauge group Yang-Mills potentials.

By considering the following ansatz for the metric,

$$ds^2 = -f(r)N^2(r)dt^2 + \frac{dr^2}{f(r)} + r^2d\Omega^2_{D-2},$$

where $\kappa = -1, 0, 1$ and where $\Omega_{D-2}$ is the volume of unit $(D-2)$-sphere. $f(r)$ is defined by a new variable $\phi(r)$,

$$f(r) = \kappa - r^2\phi(r).$$

The action of cloud of string is as follows,

$$S_{\text{cs}} = \int_{\Sigma} P\sqrt{-g}d\tau d\sigma,$$

where $P$ is non-negative constant related to string tension, $(\tau, \sigma) = (\lambda^0, \lambda^1)$ is a coordinates of worldsheet of string with $\lambda^0$ and $\lambda^1$ are timelike and spacelike parameters respectively, $\gamma_{ab}$ is the induced metric on the world sheet is as follows,

$$\gamma_{ab} = g_{\mu\nu}(x)\frac{\partial x^\mu}{\partial \lambda^a}\frac{\partial x^\nu}{\partial \lambda^b},$$

and $\gamma = \det \gamma_{ab}$ is the determinant of the induced metric.

The bivector of string worldsheet $\Sigma$ is given by,

$$\Sigma^{\mu\nu} = \epsilon^{ab}\frac{\partial x^\mu}{\partial \lambda^a}\frac{\partial x^\nu}{\partial \lambda^b},$$

where $\epsilon^{ab}$ is the Levi-Civita tensor in two dimensions, $\epsilon^{01} = -\epsilon^{10} = 1$.

Therefore, the action of cloud of string [41] can be written as,

$$S_{\text{cs}} = P \int_{\Sigma} \sqrt{-\frac{1}{2} \Sigma^{\mu\nu}\Sigma_{\mu\nu}d\tau d\sigma},$$

The non-zero components of the bivector $\Sigma$ is $\Sigma^{tr} = -\Sigma^{rt} = -\frac{a}{P r^{D-2}}$. Where $a$ is an integration constant which is related to the cloud of string [33, 39].

The action of quintessence is introduce by Kiselev [41] and written in terms of scalar field as follows,

$$S_{\text{quint}} = \frac{1}{16\pi G} \int d^Dx \sqrt{-g} \left[ -\frac{1}{2}(\nabla\phi)^2 - V(\phi) \right].$$

The energy-momentum tensor of the quintessence dark energy in $D$ dimensions can be described by [42],

$$T^{t}_{t} = T^{r}_{r} = \rho = \frac{\omega\alpha(D-1)(D-2)}{4\mu(D-1)(\omega+1)},$$

$$T_{x_i}^{x_i} = \frac{\rho}{D-2}((D-1)\omega + 1), \quad i = (1, ..., D-2),$$

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where $\omega (-1 < \omega < -\frac{D-3}{D-1})$ and $\alpha$ are the quintessential state parameter and the positive normalization constant related to the density of quintessence, respectively. By variation of the action with respect to $A^{(a)}_\mu$ we have,

$$
\nabla_\mu F^{(a)\mu} = 0,
$$

(11)

For solving Eq.(11), we consider the ansatz for the gauge field as follows\cite{43},

$$
A^{(a)}(a) = \frac{i}{2} h(r) dt \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},
$$

(12)

so,

$$
h(r) = C_2 + Q \int_r^\infty \frac{1}{u^{D-2}} du,
$$

(13)

$$
h'(r) = \frac{Q}{r^{D-2}},
$$

(14)

By plugging the ansatz 2 into the action Eq.(1) yields,

$$
I = \frac{\Omega_{D-2}(D-2)}{16\pi} \int dt dr N(r) \left[ r^{D-1} \phi \left( 1 + \alpha' (D-3) \phi \right) + \frac{r^{D-1}}{l^2} + \frac{2Q^2 r^{D-D}}{(D-3)(D-2)} - \frac{2ar}{(D-2)} - \frac{\alpha}{r^{\omega(D-1)}} \right],
$$

(15)

where $'$ denotes derivative with respect to $r$. Equation of motion is given by variation of $\delta N(r)$ \cite{44}. The function $\phi$ is given by solving for the roots of a quadratic polynomial,

$$
\phi + \alpha' (D-3) \phi^2 = \frac{16\pi M}{(D-2)r^{D-1}\Omega_{D-2}} - \frac{1}{l^2} - \frac{2Q^2 r^{4-2D}}{(D-3)(D-2)} - \frac{2ar}{(D-2)r^{D-2}} + \frac{\alpha}{r^{\omega(D-1)}},
$$

(16)

where $M$ is a constant.

By solving Eq.(16) and plugging $\phi$ into Eq.(3) we will have,

$$
f(r) = \kappa - \frac{r^2}{2\alpha' (D-3)} \left[ -1 \pm \sqrt{1 - 4(D-3)\alpha' \left( \frac{1}{l^2} + \frac{2Q^2 r^{4-2D}}{(D-3)(D-2)} - \frac{16\pi M r^{1-D}}{\Omega_{D-2}} + \frac{2ar^{2-D}}{D-2} + \frac{\alpha}{r^{\omega(D-1)}(\omega+1)} \right)} \right].
$$

(17)

The line elements of planar black branes can be written as,

$$
ds^2 = -H(r) N(r)^2 dt^2 + \frac{dr^2}{H(r)} + \frac{r^2}{l^2} \sum_{i=1}^{D-2} dx_i^2.
$$

(18)
That’s easy to show $N(r)$ is constant by variation of $\phi(r)$ from Eq.(15).

$$H(r) = \frac{r^2}{2\alpha'(D-3)} \left[ 1 - \sqrt{1 - 4(D-3)\alpha'\left(\frac{1}{l^2} + \frac{2Q^2 r^{4-2D}}{(D-3)(D-2)} - \frac{16\pi Mr^{1-D}}{(D-2)\Omega_{D-2}} + \frac{2ar^{2-D}}{D-2} + \frac{\alpha}{r^{(D-1)(\omega+1)}}\right)} \right],$$

where $M$ and $Q$ are integration constants proportional to the mass and charge of the black hole respectively given by the following formulas:

$$M = \frac{(D-2)\Omega_{D-2}}{16\pi m},$$
$$Q^2 = \frac{(D-2)(D-3)}{2}q^2.$$

By defining new parameters as below,

$$\frac{(D-3)\alpha'}{l^2} = \lambda_{gb},$$
$$\frac{2a}{D-2} = A,$$

and substituting them into the Eq.(19) we will have,

$$H(r) = \frac{r^2}{2\lambda_{gb}l^2} \left[ 1 - \sqrt{1 - 4\lambda_{gb}\left(1 + \frac{q^2 l^2}{r^{2D-4}} - \frac{ml^2}{r^{D-1}} + \frac{Al^2}{r^{D-2}} + \frac{\alpha l^2}{r^{(D-1)(\omega+1)}}\right)} \right].$$

Black hole has an event horizon, so by applying the condition $H(r_+) = 0$ we can find $m$ as follows,

$$m = r_+^{D-1} + \frac{q^2}{r_+^{D-3}} + Ar_+ + \frac{\alpha l^2}{r_+^{(D-1)\omega}}.$$

By plugging $m$ in Eq.(22)

$$H(r) = \frac{r^2}{2\lambda_{gb}l^2} \left[ 1 - \sqrt{1 - 4\lambda_{gb}\left(1 - \frac{r_+^{D-1}}{r^{D-1}} + l^2 q^2(\frac{1}{r^{2D-4}} - \frac{1}{r_+^{D-1}} + \frac{1}{r_+^{2D-4}}) + P\right)} \right],$$

where,

$$P = Al^2\left(\frac{1}{r^{D-2}} - \frac{r_+^{D-1}}{r^{D-2}}\right) + \frac{\alpha l^2}{r^{(D-1)\omega}}\left(\frac{1}{r^{(D-1)\omega}} - \frac{1}{r_+^{(D-1)\omega}}\right).$$

In AdS/CFT correspondence, the speed of light in the boundary CFT is simply $c = 1$. So that, for the black brane solution in the asymptotic region we have $\lim_{r \to 0} N(r)^2 H(r) = 1$ to recover a causal boundary. By applying this criterion we will have,

$$N^2 = \frac{1 + \sqrt{1 - 4\lambda_{gb}}}{2}. \quad (26)$$
The temperature and the Hawking-Bekenstein entropy density follow as,

\[ T = \frac{1}{4\pi\sqrt{g_{tt}}} \partial_r g_{tt}|_{r=r_+} = \frac{N_{r_+}}{4\pi l^2} \left[ (D-1) + \frac{A l^2}{r_+^{D-2}} + \frac{(D-3)q^2}{r_+^{D-4}} - \frac{(D-1)\omega}{r_+^{(D-1)(\omega+1)}} \right], \]  

(27)

\[ s = \frac{4\pi}{\sqrt{-g}} \int d^3 x = 4\pi \left( \frac{r_+}{r_+} \right)^3. \]  

(28)

\section{3 \frac{\eta}{s} of this solution}

By introducing new variables \( z = \frac{r}{r_+} \), \( \omega = \frac{r}{r_+} \bar{\omega} \), \( k_3 = \frac{r}{r_+} \bar{k}_3 \), \( \bar{H}(z) = \frac{r^2}{r_+^2} H \), the line element in terms of new variables is as follows,

\[ ds^2 = -N^2 \bar{H} dt^2 + \frac{dz^2}{r_+^2 \bar{H}} + \frac{r^2 z^2}{l^2} \sum_{i=1}^{D-2} dx_i^2, \]  

(29)

where

\[ \bar{H}(z) = \frac{z^2}{2\lambda g} \left[ 1 - \sqrt{1 - 4\lambda g \left( 1 - \frac{1}{z^{D-1}} + \frac{q^2}{r_+^{2D-4}} \left( \frac{1}{z^{2D-4}} - \frac{1}{z^{D-1}} \right) + \mathcal{P} \right)} \right], \]  

(30)

\[ \mathcal{P} = \frac{A l^2}{r_+^{D-2}} \left( \frac{1}{z^{D-2}} - \frac{1}{z^{D-1}} \right) + \frac{\alpha l^2}{r_+^{(D-1)(\omega+1)}} \left( \frac{1}{z^{(D-1)\omega}} - \frac{1}{z^{(D-1)(\omega+1)}} \right). \]  

(31)

To calculate the shear viscosity, we perturb the background metric by \( \psi \). We call the perturbed part of metric \( \psi = h_y^{y} \) \cite{45,46}.

\[ ds^2 = -N^2 \bar{H} dt^2 + \frac{dz^2}{u^2 \bar{H}} + \frac{z^2}{u^2} \left( \sum_{i=1}^{D-2} dx_i^2 + 2\psi(t, x, z)dx_1 dx_2 \right), \]  

(32)

where \( u = \frac{1}{r_+} \).

Using Fourier decomposition,

\[ \psi(t, x, z) = \int \frac{d^{D-1}k}{(2\pi)^{D-1}} e^{-i\omega t + \bar{k}_i x_i} \psi(k, z). \]  

(33)

We substitute the perturbed metric Eq.(32) in Eq.(1) and expand the action up to second order of \( \psi \). Finally, the equation of motion for \( \psi(z) \) can be obtained by variation of the perturbed action with respect to \( \psi \) as follows,

\[ (K \psi')' + \omega^2 K_1 \psi - k_i^2 K_2 \psi = 0, \]  

(34)
where
\[ K = \frac{1}{16\pi} \sqrt{-g} g^{zz} g^{xi} K_3(z), \]
\[ K_1 = -\frac{1}{16\pi} \sqrt{-g} g^{tt} K_3(z), \]
\[ K_2 = \sqrt{-g^i x_i} K_4(z), \]
\[ K_3(z) = 1 - \frac{2\lambda gb}{D-3} \left[ z^{-1} \ddot{H}' + z^{-2}(D-5) \ddot{H} \right], \]
\[ K_4(z) = 1 - \frac{2\lambda gb}{(D-3)(D-4)} \left[ \dddot{H}'' + (D-5)(D-6)z^{-2} \dddot{H} + 2(D-5)z^{-1} \dddot{H}' \right], \]
\[ (35) \]
in which \( \dot{} \) denotes derivative with respect to \( z \). The factors \( (D-5) \) and \( (D-6) \) in the expression of \( K_4(z) \) is for \( D > 5 \) of the Gauss-Bonnet theory.

The Green function is,
\[ G_{xi}x_jx_i = \frac{K \dot{\psi}}{\psi}. \]
\[ (36) \]
The shear viscosity is calculated by Green-Kubo formula as the following,
\[ \eta_{xi}x_jx_i = \frac{-G_{xi}x_jx_i}{i\omega}, \]
\[ (37) \]
Now we write Eq. (34) in terms of the shear viscosity,
\[ \partial_z \eta_{xi}x_jx_i = \left( \frac{\eta_{xi}x_jx_i}{K} - K_1 \right) + \frac{i}{\omega} K_2 k_i^2, \]
\[ (38) \]
and then we can compute the shear viscosity by requiring horizon regularity as follows,
\[ \eta_{xi}x_jx_i = \sqrt{KK_1} \left|_{z=1} \right. = \frac{1}{16\pi} \frac{r_{+}^{D-2}}{1^{D-2}} \left( 1 - \frac{2\lambda}{D-3} \dddot{H}'(1) \right), \]
\[ (39) \]
The ratio of the shear viscosity to the entropy density for 4D charged black hole solutions in Gauss-Bonnet gravity in the presence of string cloud and quintessence is then
\[ \frac{\eta}{s} = \frac{1}{4\pi} \left[ 1 - \frac{2\lambda}{D-3} \dddot{H}'(1) \right], \]
\[ (40) \]
\[ \frac{\eta}{s} = \frac{1}{4\pi} \left[ 1 - \frac{2\lambda}{D-3} \left( (D-1) - (D-3)l^2 q^2 r_{+}^{4-D} + Al^2 r_{+}^{2-D} + Al^2 (D-1)r_{+}^{(1-D)(1+\omega)} \right) \right]. \]
\[ (41) \]
For \( \lambda \to 0 \) this value is \( \frac{\eta}{s} = \frac{1}{4\pi} \). It means that the KSS bound is saturated for Einstein-Hilbert gravity.

For \( q = A = \alpha = 0 \) we will have,
\[ \frac{\eta}{s} = \frac{1}{4\pi} \left[ 1 - 2 \frac{(D-1)}{(D-3)} \lambda \right], \]
\[ (42) \]
which is consistent with the literature[47].

In the limit $D \to 4$, we obtain

$$\frac{\eta}{s} = \frac{1}{4\pi} \left[ 1 - 2\lambda \left( 3 - l_2^2 q^2 r_+^{4-D} + A l_2^2 r_+^{(1-D)(1+\omega)} \right) \right].$$  \hspace{1cm} (43)

For $A = \alpha = 0$ the result is the same as [48],

$$\frac{\eta}{s} = \frac{1}{4\pi} \left[ 1 - 2\lambda \left( 3 - l_2^2 q^2 r_+^{4-D} \right) \right].$$  \hspace{1cm} (44)

4 Conclusion

In summary, we have investigated the effect of string cloud, quintessence and Yang-Mills charge in 4D Gauss-Bonnet gravity on the field theory dual side by the fluid-gravity duality. Our result shows that these three matters are affected in dual of Gauss-Bonnet gravity but they have no contribution in dual of Einstein-Hilbert gravity. Therefore, the KSS conjecture [12] is saturated in the limit $\lambda \to 0$. This conjecture tells us that the ratio $\eta/s$ has a lower bound, $\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B}$, for all relativistic quantum field theories at finite temperature without chemical potential [14] and can be interpreted as the Heisenberg uncertainty principle [11]. String cloud [49] and quintessence [22] in Einstein massive gravity does not contribute to the $\frac{\eta}{s}$. However, this conjecture violates for higher derivative theories of gravity like the AdS Einstein-Gauss-Bonnet-Yang-Mills in the presence of string cloud and quintessence as we confirmed in section 3. Our outcome shows that the coupling of this model is scaled as $\lambda \to \lambda \left( (D-1) - (D-3) l_2^2 q^2 r_+^{4-D} + A l_2^2 r_+^{(1-D)(1+\omega)} \right)$.

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