Analytical study of the non-stationary temperature field of a thermally thin plate

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Abstract. The report analytically investigates the unsteady temperature field of a plate experiencing intense thermal action. The problem is reduced to solving a singularly perturbed boundary value problem of the nonstationary thermal conductivity equation with nonlinear boundary conditions on movable borders. The approximate solution of which is obtained in the form of an asymptotic decomposition of the solution in the sense of Poincaré in degrees of small parameters, depending on the proximity of the point under consideration to the borders.

1. Introduction
Modern development of equipment and technology is impossible without the use of high-intensity energy sources. These processes are fleeting and local in nature. Solid-state lasers are one of the common sources of energy. These laser systems are used in technological operations related to thermal effects on materials, and are also used for research and testing of various materials. In this case, both the impact materials and the installation themselves or its elements experience strong thermal loads [1-3,6]. Therefore, when studying the temperature fields arising in the material, determining the thermal mode of operation of energy sources, it is necessary to take into account thermal and optical distortions arising in the material or in structural elements.

The object of the study is a plate, which is subjected to intense thermal action in the form of a flat front, in the cross section of which is approximated by a Gaussian function. At the same time, heat exchange with the environment is carried out in the mode of protective heating, taking into account the radiation component. Thermal and optical distortions are taken into account by introducing into the mathematical model the terms described by the heat flux and the temperature dependence of the absorption capacity of the material. Thermoelastic deformations of a material are observed in the form of a change in its profile when the surface of the material is irradiated with laser radiation at intensities insufficient for melting and evaporation of the metal. In this case, the body experiences significant thermal stress, which is strongly manifested in the “intermediate” layer, i.e. in the layer between the surface and the inner regions of a given body, which greatly affects the performance and quality of mirrors of high-intensity lasers [2].

2. The problem Statement
The report poses the problem of determining the non-stationary temperature field of a plate irradiated by a powerful heat source in a short period of time, the thickness of which is much smaller than the other
dimensions. The above assumptions admit a one-dimensional coordinate formulation of the problem. Thus, the problem in the one-dimensional approximation is reduced to a nonstationary boundary value problem for the heat equation

\[
\frac{\partial^2 U(\xi, \tau)}{\partial \tau^2} = a^2 \frac{\partial^2 U(\xi, \tau)}{\partial \xi^2} + K(\tau) \frac{\partial U(\xi, \tau)}{\partial \xi} + A[U(\xi, \tau)], \quad (\xi, \tau) \in \Omega,
\]

\[
U(\xi, \tau) = U_0(\xi), \quad \tau \to 0,
\]

\[
\lambda \frac{\partial U(\xi, \tau)}{\partial \xi} = -q_1(\tau) + \sigma_1 [U^4(\xi, \tau) - U_1^4] + A_1 J(\tau), \quad \xi = 0,
\]

\[
\lambda \frac{\partial U(\xi, \tau)}{\partial \xi} = q_2(\tau) - \sigma_2 [U^4(\xi, \tau) - U_2^4], \quad \xi = h,
\]

\[
\Omega = \{ (\xi, \tau) : 0 < \xi < h, 0 < \tau \leq t_{00} \},
\]

\[
I(\tau) = I_0 \cdot \exp \left\{ -\frac{(\tau - t_0)^2}{2t_{10}} \right\}; \quad \sigma_i = c_i \cdot \sigma, \quad c_i, t_{00}, t_0, t_{10}, A_i, I_0 - \text{const}, \quad i = 1, 2.
\]

Where \(U(\xi, \tau)\) is the temperature, the required function; \(U_0(\xi)\) - initial temperature distribution; \(h\) - plate thickness; \(U_1, U_2, q_1(\tau), q_2(\tau)\) - the temperature of the medium and the heat flux on the corresponding edges of the plate; \(\lambda, \ a^2 = \lambda/c_\rho\) - coefficients of thermal conductivity and thermal diffusivity; \(\sigma, \ c_i\) - Stefan-Boltzmann constant and grayness coefficients of plate edges; \(I(\tau)\) is a function that approximates the cross section of a plane heat front. The response of a material to a thermal effect is described by two terms in the differential equation (1). Term \(K(\tau) \frac{\partial U(\xi, \tau)}{\partial \xi}\) takes into account the contribution of thermooptical distortions, where the coefficient determines the dynamics of distortions and has a very specific physical meaning [1, 12]. The second term characterizes the actual absorption capacity of the material by a dependence of the type \(A[U(\xi, \tau)] = A_0 + A_1 \cdot U(\xi, \tau), \quad A_i = \text{const}, \quad i = 0, 1 [1, 3, 4]\).

After changing variables, \(\mu = \xi + \frac{\tau}{t_0}\) \(K(z)dz\), \(\eta = \tau\), introducing dimensionless variables, \(\mu = \bar{x} \cdot x\), \(\eta = \bar{t} \cdot t\) and notations, \(T(\bar{x} \cdot x - \bar{x} \cdot \psi(\bar{t} \cdot t)) = T(x, t), \quad U_0(\bar{x} \cdot x) = T_0(x)\), we obtain a singularly perturbed boundary value problem of the heat equation with a small parameter \(Fo << 1\), which is a consequence of the transience of the process (small time scale \(\bar{t}\)) and a small thickness of the plate (smallness of the spatial scale \(\bar{x}\)), which has the following form

\[
\frac{\partial T(x, t)}{\partial t} = Fo \cdot \frac{\partial^2 T(x, t)}{\partial x^2} + A[T(x, t)],
\]

\[
T(x, t) = T_0(x), \quad t \to 0,
\]

\[
\frac{\partial T(x, t)}{\partial x} = -\varphi_1(t) + \gamma_1 \cdot [T^4(x, t) - U_1^4] + A_1 J(t), \quad x = \psi_1(t),
\]

\[
\frac{\partial T(x, t)}{\partial x} = \varphi_2(t) + \gamma_2 \cdot [T^4(x, t) - U_2^4], \quad x = \psi_2(t).
\]
\[(x, t) \in \Omega' = \{(x, t): \psi_1(t) < x < \psi_2(t), 0 < t \leq t_\infty\}, \quad (5)\]

where \( F_0 = \frac{a^2 T}{x^2}; 0 < F_0 << 1; \quad t_\infty = \frac{t_0}{t}, \quad H = \frac{h}{x}, \quad \varphi_i(t) = \frac{\bar{x}}{\lambda} q_i(\bar{t} \cdot t), \quad \bar{A}_0 = \frac{\bar{x}}{\lambda} A_0, \quad \gamma_i = (-1)^{i+1} \frac{\bar{x}}{\lambda} \sigma_i, \quad q_i(\bar{t} \cdot t) = q_0 = \text{const}, \quad \varphi_i(t) = \varphi_i(t_0) = \text{const}, \quad i = 1, 2; \quad \psi(t) = \frac{1}{x} \int K(z)dz, \quad \psi_i(t) = \psi(t), \quad \psi_2(t) = \psi(t) + H; \quad I(t) = I_0 \exp \left\{ - \left( \frac{t - t_0}{t_0} \right)^2 \right\}. \]

It is assumed that the irradiation time and thermophysical characteristics do not change significantly.

Thus, the original boundary value problem has been reduced to a singularly perturbed boundary value problem for the heat equation with nonlinear boundary conditions on moving boundaries.

For the asymptotic study of this problem, we use the method proposed by G.A. Nesenenko [7], which he called the "geometric-optical" asymptotic method. The main provisions of this method are reflected in the works [7-12].

Using the above scheme, the Green's function and the solution of the boundary value problem are obtained in the form of asymptotic expansions in powers of small parameters in the sense of Poincaré.

It should be noted that the form of the asymptotic expansion of both the Green's function and the solution of the boundary value problem depends on the "proximity" of the point under consideration to the boundary of the domain. In accordance with the introduced concept of "proximity" of a point to the boundary, the area is divided into "boundary", "intermediate" and "remote" from the boundaries of the zone, each of which uses its own asymptotic scale [9-12].

In each of these selected zones, the Laplace method (a special case of the saddle point method) is used for the asymptotic analysis of the corresponding integrals. Moreover, in the area of points remote from the boundaries, the classical version of the Laplace method is used, where the reference integral has the form: \( I = \int_0^\infty x^\alpha \exp \left\{ -\lambda x^2 \right\} dx, \quad \lambda > 0. \)

In the other zones, the Laplace method is used with some modification, which consists in the fact that an integral of the form is chosen as the reference integral:

\[ J = \int_0^\infty x^{\alpha-1} \exp \left\{ -\gamma x - \frac{\beta}{x} \right\} dx, \quad \gamma, \beta > 0. \]

This follows from the fact that integrals of the form are analyzed in these zones, where the value at which the smallest value is reached (at) and the condition of "proximity" of the point to the boundary is satisfied, merges with the end of the integration interval (that is), and therefore the classical Laplace method with a reference integral of the form cannot be applied. Therefore, in [7], it was proposed to use an integral of the form as a reference integral in the asymptotic analysis of integrals of the form.

3. Solution of the boundary value problem

Let us assume that the boundary changes according to a linear law, which is quite possible due to the physical meaning of the coefficient, i.e. then

\[ \psi(\bar{t} \cdot t) = \frac{\bar{x}}{\lambda} \bar{t} \cdot t = k_0 \cdot t = \psi_0(t), \quad k_0 = \frac{\bar{x}}{\lambda}, \quad k_0 = \frac{\bar{x}}{\lambda}, \quad \psi_1(t) = \psi_0(t), \quad \psi_2(t) = \psi_0(t) + H. \]

According to [3,7,10], we write down the integral representation of the solution to the problem (1)-(5)
\[ T(x,t) = \int_{\psi_1(t)}^{\psi_2(t)} T_0(y) G(x,t,y,0) dy + F_0 \cdot \phi_0(t) G(x,t;\psi_1(s),s) ds \]

\[ -F_0 \cdot \bar{A} \int_0^t \exp \left\{ -\frac{(s-t_0)^2}{T_0} \right\} G(x,t;\psi_1(s),s) ds \]

\[ -F_0 \cdot \phi_1(t) \int_0^t \psi_2(s) G(x,t;\psi_1(s),s) ds + F_0 \cdot \phi_0(t) G(x,t;\psi_2(s),s) ds \]

\[ + F_0 \gamma_2 \int_0^t \psi_2(s) G(x,t;\psi_2(s),s) ds + \int_{\psi_2(t)}^{\psi_1(t)} A[T(y,s)] G(x,y,t) dy ds \]

\[ = T_L(x,t) + T_L(x,t) + T_{NL}(x,t) + T_{NL}(x,t) + T_A(x,t) = T^*(x,t) + T_A(x,t). \quad (6) \]

Where \( G(x,t;y,s) \) - Green's function of the corresponding homogeneous boundary value problem

\[ \frac{\partial G(x,t;y,s)}{\partial t} = F_0 \cdot \frac{\partial^2 G(x,t;y,s)}{\partial x^2}, \]

\[ G(x,t;y,s) = \delta(x,y), \quad t \to s, \]

\[ \frac{\partial G(x,t;y,s)}{\partial x} = 0, \quad x = \psi_i(t), \quad i = 1,2, \]

\[ (x,t) \in \Omega' = \{(x,t): \psi_1(t) < x < \psi_2(t), \quad 0 < t \leq t_0 \}, \]

the solution of which can be found explicitly \([9,12,14]\):

\[ G(x,t;y,s) = \frac{1}{2\sqrt{\pi F_0(t-s)}} \exp \left\{ -\frac{(x-y)^2}{4F_0(t-s)} \right\} \]

\[ + \sum_{m=0}^{\infty} \frac{1}{2\sqrt{\pi F_0(t-s)}} \exp \left\{ -\frac{(x+y-2k_0s+2mH)^2}{4F_0(t-s)} + k_0 \frac{y-k_0s+mH}{F_0} \right\} \]

\[ + \exp \left\{ -\frac{[2k_0s-x-y+2(m+1)H]^2}{4F_0(t-s)} \right\} k_0 \left( \frac{y-k_0s+(m+1)H}{F_0} \right) \]

\[ + \exp \left\{ -\frac{(x+y+2(m+1)H)^2}{4F_0(t-s)} + k_0 \frac{(m+1)H}{F_0} \right\} \]

\[ + \frac{k_0}{2F_0} \left\{ -\frac{k_0 [y-k_0s+mH]}{F_0} \right\} \text{erfc} \left\{ \frac{x+y-2k_0s+2mH}{2\sqrt{F_0(t-s)}} \right\} \]

\[ + \exp \left\{ -\frac{k_0 [y-k_0s+(m+1)H]}{F_0} \right\} \text{erfc} \left\{ \frac{2k_0s-x-y+2(m+1)H}{2\sqrt{F_0(t-s)}} \right\} \].
To simplify calculations, assume that, then you can get the following expressions

\[ T_L(x,t) = T_0 + \frac{F_0}{2k_0} \sum_{i=1}^{\infty} q_{i0} \cdot A_i(x,t), \]  

where

\[ A_i(x,t) = e^{erfc} \left[ \frac{x - (i-1)H}{2\sqrt{F_0 t}} \right] + \exp \left[ (-1)^{i-1} \frac{k_i (x - k_i t -(i-1)H)}{F_0} \right] \times e^{erfc} \left[ \frac{x - 2k_i t -(i-1)H}{F_0} \right] \]

\[ + \sum_{m=0}^{\infty} \exp \left[ - \frac{k_i [x - k_i t +(m+i-1)H]}{F_0} \right] e^{erfc} \left[ \frac{2k_i t - x +(2m+i+1)H}{2\sqrt{F_0 t}} \right] \]

\[ - \exp \left[ \frac{k_i [x - k_i t -(m+i)H]}{F_0} \right] e^{erfc} \left[ \frac{2k_i t - x -(2m+i+1)H}{2\sqrt{F_0 t}} \right] \]

\[ + (-1)^{i-1} \exp \left[ (-1)^{i-1} \frac{k_i (m+1)H}{F_0} \right] e^{erfc} \left[ \frac{(-1)^{i-1} x + (2m+i+1)H}{2\sqrt{F_0 t}} \right] \]

\[ + (-1)^{-i-1} \left[ 1 + \frac{k_i \left[ (x - (i+1)(2m+3-i)H) \right]}{F_0} \right] \exp \left[ (-1)^{i-1} \frac{k_i m H}{F_0} - \frac{((-1)^{i-1} x + (2m+i-1)H)^2}{4F_0 t} \right] \]

\[ + (-1)^{-i-1} \exp \left[ (-1)^{i-1} \frac{k_i m H}{F_0} - \frac{((-1)^{i-1} x + (2m+i-1)H)^2}{4F_0 t} \right] \right\} \right\}, \quad \text{erfc}[z] = \frac{2}{\sqrt{\pi}} \int_{z}^{\infty} e^{-u^2} du. \]

\[ T_1(x,t) = -F_0 \cdot \overline{A}_i J_0 \int_{0}^{\infty} \exp \left[ \frac{(s - \overline{\tau}_0)^2}{\overline{\tau}_0^2} \right] G(x; \tau; \psi_1(s), s) ds. \]

Introducing notations

\[ G(x; \tau; \psi_1(s), s) = \frac{1}{2\sqrt{\pi F_0(t-s)}} \exp \left[ \frac{(x - \psi_1(s))^2}{4F_0(t-s)} \right] + \sum_{n=0}^{\infty} G_n(x; \tau; \psi_1(s), s), \]

\[ T_1(x,t) = -F_0 \cdot \overline{A}_i J_0 + \sum_{n=0}^{\infty} J_{1,m} = T_1^0(x,t) + \sum_{n=0}^{\infty} T_{1,m}, \]

let us carry out an asymptotic analysis of each of the integrals. 

Then for the first integral we have

\[ J_1^0(x,t) = \frac{1}{2\sqrt{\pi F_0}} \int_{0}^{\infty} \exp \left[ \frac{(t - \overline{\tau}_0)^2}{\overline{\tau}_0^2} \right] \exp \left[ \frac{(x - k_i s)^2}{4F_0(t-s)} \right] ds, \]

\[ J_1^0(x,t) = \frac{1}{\sqrt{\pi F_0}} \exp \left[ \frac{(t - \overline{\tau}_0)^2}{\overline{\tau}_0^2} - \frac{k_i [x - k_i t]}{2F_0} \right] \sum_{n=0}^{\infty} C_{n,0}(x,t). \]
\[ C_{1,0} = \sqrt{\pi} \cdot F_0 \left( \frac{2t_{10}}{A_0} \right) \left[ -\exp \left\{ \frac{x - k_0 t}{2Fo} \right\} \text{erfc} \left\{ A \sqrt{t} + \frac{x - k_0 t}{2Fo \cdot t} \right\} \right. \]

\[ + \exp \left\{ \frac{x - k_0 t}{2Fo} \right\} \frac{A_0}{A_0^2} \text{erfc} \left\{ -A \sqrt{t} + \frac{x - k_0 t}{2Fo \cdot t} \right\}, \ A_0^2 = k_0^2 t_{10}^2 - 4 \cdot Fo (t - \bar{t}), \ A^2 = \frac{A_0^2}{4Fo \cdot t_{10}^2}, \]

\[ C_{1,0} = 6 \sqrt{\pi} \cdot F_0 \left( \frac{2t_{10}}{A_0} \right) \left[ \frac{\bar{t}_{10}^2}{A_0^2} - \frac{t_{10}}{A_0} \right] \left[ \left( \frac{x - k_0 t}{2Fo} \right) + \frac{1}{3} F_0^2 \right] \text{erfc} \left\{ A \sqrt{t} + \frac{x - k_0 t}{2Fo \cdot t} \right\} \]

The other integrals are defined in a similar way.

To find the nonlinear component in expression (6), we introduce the notation \( T_L(x,t) = T_L(x,t) + T_J(x,t), T_{NL}(x,t) = T_{NL1}(x,t) + T_{NL2}(x,t), \) then \( T'(x,t) = T_{NL}(x,t) + T_{NL1}(x,t). \) We find the unknown functions \( v_1(t), v_2(t) \) by solving the system of integral equations written out from the boundary conditions (3) - (4)

\[ v_i(t) = \gamma_i \left[ F_i(v_1(t), v_2(t)) - \bar{u}_i \right], \ i = 1, 2, \]

where

\[ F_j(v_1(t), v_2(t)) = \left[ T_{ij}^{(j)}(t) + Fo \sum_{j=0}^{3} (-)^j \gamma_j \cdot K_i^{(j)}(v_1(t)) \right]^{1/4}, \ j = 1, 2; \]

\[ K_i^{(1)}(v_1(t)) = \int_0^t \left( v_1(s) \cdot G(v_1(t), t; v_1(s), s) ds \right), \ T_{ij}^{(j)}(t) = \left. T_{ij}(x,t) \right|_{x=\psi_1(t)}, \]

\[ K_i^{(2)}(v_1(t)) = \int_0^t v_1(s) \cdot G(v_2(t), t; v_2(s), s) ds, \bar{u}_i = (-1)^{i+1} \frac{\Theta}{\gamma_i} + U_i, \ i = 1, 2. \]

We seek the solution of this system of equations by the method of successive approximations according to the scheme \( v_i^{(n+1)}(t) = \gamma_i \left[ F_i(v_1^{(n)}(t), v_2^{(n)}(t)) - \bar{u}_i \right], \ i = 1, 2; \ n = 0, 1, 2, \ldots \)

Assuming \( v_1^{(0)}(t) = 0, v_2^{(0)}(t) = 0, \) for the first approximation we have \( v_1^{(1)}(t) = \gamma_1 \left[ F_1(0, t) - \bar{u}_1 \right], \ i = 1, 2. \) After carrying out an asymptotic analysis of the corresponding integrals for each approximation, we obtain asymptotic expansions in powers of the small parameter \( Fo \)

\[ v_i(t) = \sum_{j=0}^{\infty} c_i^{(j)}(t) \cdot Fo^j + O(Fo^{N+1}), \ i = 1, 2, \]

where the coefficients are calculated explicitly.

Substituting the obtained expansions into expression (6), we carry out an asymptotic analysis of the obtained integrals by the Laplace method [7-9], taking into account the conditions of “proximity” of the point \( (x,t) \) under consideration to the boundaries of the domain \( \Omega' \).
\[ T^s(x,t) = T_{t,0}(x,t) - Fo \cdot \gamma_i \left[ \sum_{i=0}^{N} c_i^{(1)}(s) \cdot Fo^i + O\left( Fo^{N+1} \right) \right] G(x,t;\psi_i(s),s)\,ds + Fo \cdot \gamma_j \left[ \sum_{i=0}^{N} c_j^{(2)}(s) \cdot Fo^i + O\left( Fo^{N+1} \right) \right] G(x,t;\psi_j(s),s)\,ds. \]

The solution to the required boundary value problem is determined by the method of successive approximations according to the scheme
\[ T^{(m+1)}(x,t) = T^s(x,t) + \int_0^{y(x,s)} A(T^{(m)}(y,s)) G(x,t;y,s)\,dyds, \quad T^{(0)} = T^s(x,t), \quad m = 0,1,2\ldots \quad (9) \]

After carrying out an asymptotic analysis of the integrals in expressions (9), using the Laplace method with reference integrals [7-9,15-16], we obtain an asymptotic expansion of the solution in scales of the form \[ \left\{ Fo^i \right\} \left\{ \left( |x-\psi_i(t)|/Fo \right)^{\alpha} \right\}, \left\{ \left( |x-\psi_j(t)|/Fo \right)^{\beta} Fo^i \right\}. \]

Then the following is true.

Statement. The asymptotics of the solution to the boundary value problem (7) - (11) has the following form:
1) In the “boundary layer” of the boundary \( x = k_i t \), where the condition is met \( x - k_i t = O(Fo^p) \), \( p > 1 \),
\[ T_{p\theta i}(x,t) = \sum_{i=0}^{N} \sum_{j=0}^{N} d_{i,j}^{(0)}(x,t) \left( x - k_i t / Fo \right)^{i} Fo^j + O\left( \frac{x - k_i t}{Fo} \right)^{N+1} Fo^{N+1}, \quad Fo \to 0. \]
2) In the “intermediate layer” of the boundary \( x = k_i t \), where \( x - k_i t = O(Fo) \), \( Fo \to 0 \),
\[ T_{p\theta i}(x,t) = \sum_{i=0}^{M} d_{i}^{(1)}(x,t) \cdot Fo^i + O(Fo^{M+1}). \]
3) In the “area remote from the borders \( x = k_i t = k_i t + H \) points”, (if \( p < 1 \)), \( x - k_i t = O(Fo^p) \), \( k_i t + H - x = O(Fo^p) \), \( Fo \to 0 \), \( T_{u\theta i}(x,t) \approx T_{0} + T_{s}(x,t). \)
4) In the “intermediate layer” of the boundary \( x = k_i t + H \), where \( k_i t + H - x = O(Fo) \), \( Fo \to 0 \),
\[ T_{p\theta i}(x,t) = \sum_{i=0}^{M} d_{i}^{(1)}(x,t) \cdot Fo^i + O(Fo^{M+1}). \]
5) In the “boundary layer” of the border \( x = k_i t + H \), \( k_i t + H - x = O(Fo^p) \), \( p > 1 \), \( Fo \to 0 \)
\[ T_{p\theta i}(x,t) = \sum_{i=0}^{N} \sum_{j=0}^{N} d_{i,j}^{(0)}(x,t) \left( \frac{k_i t + H - x}{Fo} \right)^{i} Fo^j + O\left( \frac{k_i t + H - x}{Fo} \right)^{N+1} Fo^{N+1}. \]

An approximate solution is obtained in the form of asymptotic decompositions in the sense of Poincaré, the coefficients of which are calculated explicitly [8,12-14]. The resulting decompositions satisfy the initial and boundary conditions of the problem posed.

Remark 1. In practice, they are often limited to the first few coefficients of the asymptotic scale \( \left\{ \left( |x-\psi_i(t)|/Fo \right)^{\alpha} \right\}, \quad i,j = 0,1,\ldots,N; \quad i + j \leq N \). For example, when simulating a high-current sliding discharge of a capacitor over the surface of a dielectric (fluoroplastic-4), where,
\[ a^2 = 1.03 \times 10^{-7} \text{ m}^2 / \text{cek}, \quad \bar{t} = 5 \times 10^{-9} \text{ cek}, \quad \bar{x} = 10^{-6} \text{ m}, \] parameter \( Fo \) takes on a value \( 5.15 \times 10^{-4} \), to obtain an approximate solution, it is enough to take \( N = 2 \).

Remark 2. The approximate solution \( T(x,t) \), obtained in the form of an asymptotic expansion in the sense of Poincaré allows differentiation and integration operations, therefore, we can find expressions for the heat flux \( T_i(x,t) \) and the rate of restructuring of the temperature field \( T_e(x,t) \).

Remark 3. The obtained analytical expressions for the approximate solution of a singularly perturbed boundary value problem allow us to perform a numerical-analytical analysis of the influence of material parameters (plate thickness, thermophysical properties, reflectivity and absorption capacity of the plate material, characteristics of the radiation source) on the dynamics of the temperature field in the plate.

Remark 4. The asymptotic method proposed and substantiated by G A Nesenenko [7,11], is applicable to find the asymptotics of solutions of both one-dimensional and multidimensional singularly perturbed linear and nonlinear boundary value problems of unsteady heat conduction in a domain with a complex (sufficiently smooth) movable geometry, with nonlinear boundary conditions.

4. Conclusion
An approximate solution of the posed singularly perturbed boundary value problem is obtained in the form of asymptotic decompositions in the sense of Poincaré, and the decomposition coefficients are calculated explicitly, which allows a parametric analysis of the problem posed: to reveal the influence of the irradiation regimes, the radiation component and the initial temperature, the absorption capacity of the material on the temperature distribution thermally thin plate.

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