Active extension portfolio optimization with non-convex risk measures using metaheuristics

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Abstract

We consider the optimization of active extension portfolios. For this purpose, the optimization problem is rewritten as a stochastic programming model and solved using a clever multi-start local search heuristic, which turns out to provide stable solutions. The heuristic solutions are compared to optimization results of convex optimization solvers where applicable. Furthermore, the approach is applied to solve problems with non-convex risk measures, most notably to minimize Value-at-Risk. Numerical results using data from both the Dow Jones Industrial Average as well as the DAX 30 are shown.

1 Introduction

In this paper we consider the optimization of active extension portfolios, which are also known as 1x0/x0 (most commonly 130/30) portfolios, see e.g. [Lo and Patel] 2008, [Gastineau] 2008, and [Thomas] 2007. The idea is to extend long-only portfolios to contain a certain additional percentage $x$ of the investors budget on the long-side of the portfolio and additionally short $-x$ percent of the assets in the portfolio. Classical approaches to create active extension portfolios are often based on sorting returns and applying a momentum approach. Historically well-performing assets are used for the additional long part and bad-performing assets for the short part. Commonly, no real optimization is conducted. The problem is that this methodology does not fit well into the classical Modern Portfolio Theory optimization framework, i.e. the Markowitz approach to calculate risk-optimal financial portfolios as shown by [Markowitz] 1952. This approach is defined by calculating a risk-optimal portfolio $x$ given a set of $a$ financial assets for which a vector of expected returns $M$ and a co-variance matrix $C$ exists. Further constraints $X$ may be added (e.g. long and short restrictions, ...), i.e.

$$\begin{align*}
\text{minimize} \quad & x \quad C \quad x^T \\
\text{subject to} \quad & x \times M \geq \mu \\
& x \in X.
\end{align*}$$

The issue with this approach is that uncertainty is implicitly modeled, only the first and second moments of the loss distribution are used. This is especially problematic in times of some financial crisis. Furthermore, from an financial point of view the Variance is probably not the most useful risk measure, because is is penalizing the upside. From an optimization point of view, the quadratic programming framework is too rigid to implement additional extensions. The optimization of expected shortfall objectives and constraints cannot easily be put on top of this rather specific base model. More importantly, the quadratic programming objective does not allow for a simple active extension constraint, because such an approach is based on the
solution structure of linear programs. For this reason, we will apply the technology of stochastic programming, see [Ruszczynski and Shapiro 2003], [Wallace and Ziemba 2005], and [King and Wallace 2013] for more details about this technique.

While there are convex optimization reformulations for many important risk measures (e.g. the Mean Absolute Deviation as proposed by Konno and Yamazaki [1991] or the LP-based Conditional Value-at-Risk (Expected Shortfall) approach by Rockafellar and Uryasev [2000] and Rockafellar and Uryasev [2002], respectively), problems arise when an investor needs to integrate non-convex risk measures like the Value-at-Risk [Jorion 1997]. For this purpose metaheuristics can prove to be useful. Many metaheuristics have been shown to solve portfolio optimization problems with a varying degree of success. A good overview can be found in the three volumes on Natural Computing in Computational Finance, see [Brabazon and O’Neill 2008], [Brabazon and O’Neill 2009], and [Brabazon et al. 2010]. Most of the presented metaheuristics are complex and don’t scale well given the fact that the underlying optimization problem is rather simple from a heuristic optimization point of view. Therefore we propose a simple, yet powerful multi-start local search heuristic, which integrates structural information of the portfolio optimization process into its heuristic framework.

This paper is organized as follows. Section 2 presents a short overview of scenario-based portfolio optimization, Section 3 describes the proposed heuristic, while Section 4 provides numerical results using data from both the Dow Jones Industrial Average as well as the DAX 30 index. Section 5 concludes the paper.

2 Scenario-based Portfolio Optimization

Stochastic programming is well suited to model optimization problems under uncertainty, because of its inherent feature to split a model into an objective and a subjective part explicitly within the optimization modeling process. In terms of financial portfolio optimization, the constraint set $X$ contains e.g. regulatory and organizational constraints, i.e. the objective part. The subjective views on the underlying uncertainty is expressed by a scenario set $S$, which in this specific application contains a discrete asset return probability (uncertainty) model, i.e. a set of different probable returns for each asset. Using this scenario set and a heuristic approach, any risk measure (VaR, Omega, . . . ) can be integrated, because the evaluation of the respective loss distribution $\ell_x$ for some portfolio $x$, i.e. $\ell_x = \langle x, S \rangle$ can be used to evaluate with any functional – independent of its underlying mathematical structure.

An investor usually faces a bi-criteria optimization problem, i.e. she wants to maximize the expected return while also minimizing the risk. The meta-model for this scenario-based stochastic portfolio optimization problem is thus given by:

$$\begin{align*}
\text{maximize} & \quad \text{Return}(\ell_x) \\
\text{minimize} & \quad \text{Risk}(\ell_x) \\
\text{subject to} & \quad x \in X
\end{align*}$$

While the multi-criteria optimization problem is interesting from a research point of view, in most practical applications we will reformulate the above optimization problem to a single objective model, where the risk is minimized in the objective and the return is controlled via a constraint and a given minimum acceptable lower bound $\mu$ on the expected return, i.e.

$$\begin{align*}
\text{minimize} & \quad \text{Risk}(\ell_x) \\
\text{subject to} & \quad \text{Return}(\ell_x) \geq \mu \\
& \quad x \in X
\end{align*}$$
3 Multi-start local search heuristics

We implemented a simple but powerful multi-start local search heuristic to solve the active extension portfolio optimization problem. It contains of three parts:

1. Sample a number $n_1$ of random portfolios using a special sampling algorithm.
2. Improve the best $n_2$ random portfolios using an iterative $\varepsilon$-improvement procedure.
3. Pick the improvement with the best objective (risk-ratio).

Finally, if the $n_2$ portfolios differ, then the mean of these portfolios is taken and the iterative $\varepsilon$-improvement is applied to this resulting portfolio again.

3.1 Portfolio Sampling

Drawing random numbers and creating portfolios out of these numbers can be tricky. This is especially true when one needs good starting points because of the danger to get stuck in local optima based on the underlying heuristic technique. This is valid in our case for the subsequent portfolio $\varepsilon$-improvement. Hence, the general structure of optimal portfolios needs to be taken into consideration before sampling portfolios. The main structure is that many real-world assets in risk-optimal portfolios are not chosen at all, i.e. only a small subset is selected. The applied sampling method in this paper requires the following parameters:

- The amount of long (default: 0.3) and short (default: 0.1) assets in percent.
- The upper and lower bound on each asset (default: 0.5 long and $-0.1$ short).
- The sum of the long (default: 1.30) and short (default: $-0.3$) part of the portfolio.

Randomly sampled portfolios with these default values exhibit the well-known sparsity of real-life risk-optimal (active extension) portfolios.

3.2 Iterated portfolio $\varepsilon$-improvement

The $\varepsilon$-improvement is a multi-start local search heuristic based on the $n_2$ best sampled portfolios from the above described sampling procedure. Each asset is modified by $\pm \varepsilon$, i.e. out of the initial portfolio, $a \times 2$ new portfolios are created. Thereby the lower and upper bounds are easily satisfiable. The resulting portfolios are normalized to the given sum of both the long and the short side. Finally, the (locally) best improvement is chosen to be the next portfolio until no local improvement can be accomplished.

Depending on the structure of the underlying scenario set as well as the given constraints, a different set of $\varepsilon$ might be applicable. Simple solutions can be computed with e.g. $\varepsilon = (0.05, 0.01, 0.001)$, i.e. the local search will be repeated three times with a smaller $\varepsilon$ in each run. A broader set of $\varepsilon$ can be used too.

4 Numerical results

We have tested the algorithm with assets of two mayor financial stock indices – both the Dow Jones Industrial Average DJIA (containing the assets AA, AXP, BA, BAC, CAT, CSCO, CVX, DD, DIS, GE, HD, HPQ, IBM, INTC, JNJ, JPM, KO, MCD, MMM, MRK, MSFT, PFE, PG, T,
TRV, UNH, UTX, VZ, WMT, XOM) with weekly returns from the year 2012 as well as the DAX 30 (containing the assets ADS.DE, ALV.DE, BAS.DE, BAYN.DE, BEI.DE, BMW.DE, CBK.DE, CON.DE, DAI.DE, DBK.DE, DB1.DE, LHA.DE, DPW.DE, DTE.DE, EOAN.DE, FRE.DE, FME.DE, HEL.DE, HEN3.DE, IFX.DE, SDF.DE, LXS.DE, LIN.DE, MRK.DE, MUV2.DE, RWE.DE, SAP.DE, SIE.DE, TKA.DE, VOW3.DE) with weekly returns from the year 2013. The above mentioned ticker symbols are taken from Yahoo! Finance, which also served as the data source.

The algorithm was implemented using R [R Core Team, 2013]. The optimization of the convex linear programs has been conducted with the GNU Linear Programming Kit 4.54. The convex quadratic optimization problems have been solved with the quadprog R package, which implements the optimization algorithm proposed by Goldfarb and Idnani [1983].

4.1 Long-only Markowitz Portfolios

Consider the DJIA weekly returns from the year 2012. In Fig. 1 you can see the global solution computed with a quadratic solver and two randomly sampled portfolios, which do exhibit the general structure of a portfolio, but are far off the optimal solution. In Fig. 2 you can see the global solution again as well as two heuristic solutions with \( n_1 = 10000 \) random samples and an \( \varepsilon \)-improvement with \( \varepsilon = (0.05, 0.01, 0.001) \). After (25, 21, 28) as well as (29, 16, 24) iterations, we reach the global solution. In this simple long-only Markowitz problem, the process does not need to be iterated.
4.2 Active-extension CVaR Portfolios

In this second example we use DAX 30 weekly returns from the year 2013. Here we try to compute the CVaR-optimal portfolio, but with active extension constraints (130/30) and with certain upper and lower bounds on the assets, i.e. a maximum of 0.5 long and −0.1 short. This turns out to be more tricky. We start with 10000 random samples and apply ε-improvement with ε = (0.05, 0.02, 0.01) to the n = 10 best sampled portfolios. The results differ slightly such that we compute the average and approximate again. The result is shown in Fig. 3. While the heuristic solution is not completely similar to the global solution, it is valid from a portfolio managers point of view, as the optimization result is used as a guideline and will not be implemented exactly as optimized – partly because this is not even possible on real markets.

The portfolio compositions can also be seen in Table 1. In this table only assets which do exhibit a non-zero allocation in one of the solutions are shown. In addition, the main risk parameters of the assets are shown, i.e. both the standard deviation as well as the 95%-Value at Risk. Here we can see that e.g. the assets RWE.DE and TKA.DE show exactly the same risk structure in these two risk parameters, such that the misassignment of the heuristic is not too problematic from an investors point of view.

4.3 Active-extension VaR Portfolios

With the knowledge that our CVaR portfolios are almost spot on from an investors perspective, we can now apply the same procedure to compute an active-extension portfolio using non-convex risk-measures. We use data from the DJIA and apply the same setup as in Section [active-extension-cvar-portfolios], but change the objective function to minimizing the Value-at-Risk.
Table 1: Different portfolio allocations in the 130/30 CVaR case.

|       | Std.Dev | VaR (95%) | Global Solution | Heuristic Solution |
|-------|---------|-----------|-----------------|-------------------|
| BEI.DE| 0.02    | −0.03     | 0.04            | 0.06              |
| DAL.DE| 0.03    | −0.04     | 0.08            | 0.05              |
| DBK.DE| 0.04    | −0.06     | −0.07           | 0.00              |
| DB1.DE| 0.03    | −0.04     | 0.32            | 0.26              |
| LHA.DE| 0.04    | −0.05     | −0.06           | −0.03             |
| DPW.DE| 0.03    | −0.03     | 0.20            | 0.22              |
| DTE.DE| 0.03    | −0.04     | 0.20            | 0.22              |
| FRE.DE| 0.02    | −0.03     | 0.21            | 0.23              |
| FME.DE| 0.03    | −0.03     | 0.13            | 0.12              |
| HE1.DE| 0.04    | −0.06     | −0.05           | −0.04             |
| HEN3.DE| 0.03   | −0.03     | 0.00            | −0.05             |
| MRK.DE| 0.02    | −0.03     | 0.11            | 0.10              |
| RWE.DE| 0.04    | −0.07     | −0.04           | 0.00              |
| TKA.DE| 0.04    | −0.07     | 0.00            | −0.04             |

Figure 3: Active extension 130/30 CVaR-optimal portfolio.
Figure 4: Active extension 130/30 Value at Risk-optimal portfolio.

The result is shown in Fig. 4.

5 Conclusion

In this paper we have presented a clever multi-start local search heuristic for active extension portfolios with convex but also non-convex risk measures. Despite its simplicity, the metaheuristic works very well. Future extensions include an automatic tuning of the set of $\varepsilon$ and further empirical tests, especially with a larger set of assets.

References

A. Brabazon and M. O’Neill, editors. Natural Computing in Computational Finance, volume 100 of Studies in Computational Intelligence. Springer, 2008.

A. Brabazon and M. O’Neill, editors. Natural Computing in Computational Finance, Volume 2, volume 185 of Studies in Computational Intelligence. Springer, 2009.

A. Brabazon, M. O’Neill, and D. Maringer, editors. Natural Computing in Computational Finance, Volume 3, volume 293 of Studies in Computational Intelligence. Springer, 2010.

G. L. Gastineau. The short side of 130/30 investing: For the conservative portfolio manager. The Journal of Portfolio Management, 34(2):39–52, 2008.
D. Goldfarb and A. Idnani. A numerically stable dual method for solving strictly convex quadratic programs. *Mathematical Programming*, 27(1):1–33, 1983.

P. Jorion. *Value at Risk: the new benchmark for controlling market risk*, volume 2. McGraw-Hill New York, 1997.

A.J. King and S.W. Wallace. *Modeling with Stochastic Programming*. Springer Series in Operations Research and Financial Engineering. Springer, 2013.

H. Konno and H. Yamazaki. Mean-absolute deviation portfolio optimization model and its applications to tokyo stock market. *Management Science*, 37(5):519–531, 1991.

A.W. Lo and P.N. Patel. 130/30: The new long-only. *The Journal of Portfolio Management*, 34(2):12–38, 2008.

H. Markowitz. Portfolio selection. *The Journal of Finance*, 7(1):77–91, 1952.

R Core Team. *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria, 2013. URL [http://www.R-project.org](http://www.R-project.org).

R. T. Rockafellar and S. Uryasev. Optimization of Conditional Value-at-Risk. *Journal of Risk*, 2:21–42, 2000.

R. T. Rockafellar and S. Uryasev. Conditional Value-at-Risk for general loss distributions. *Journal of Banking & Finance*, 26(7):1443–1471, 2002.

A. Ruszczyński and A. Shapiro, editors. *Stochastic programming*, volume 10 of *Handbooks in Operations Research and Management Science*. Elsevier Science B.V., Amsterdam, 2003.

R. Thomas. The alpha and beta of 130/30 strategies. *The Journal of Investing*, 16(4):25–32, 2007.

S. W. Wallace and W. T. Ziemba, editors. *Applications of stochastic programming*, volume 5 of *MPS/SIAM Series on Optimization*. Society for Industrial and Applied Mathematics (SIAM), 2005.