Verification of the Method for Constructing Computational Models of Hybrid Beams in Software and Computer Systems that Use the Finite Element Method as Their Basis

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Abstract. To analyze the operation of hybrid structures, it is necessary to build their models with subsequent calculation in special computational software systems that use the finite element method as their basis. One of the features of hybrid structures is the use of dissimilar materials in a single structure, arranged in a special way, ensuring their joint work. When creating a calculation model of a hybrid beam in PVC, it is required to take this feature into account and pay special attention to the contact areas of dissimilar materials. The article analyzes 2 methods for creating computational models in the SCAD, taking into account this feature, verification of the results obtained. Conclusions are drawn about the possibility of using these methods in design.

1. Introduction
Hybrid structures – a class of building structures made of dissimilar materials, arranged in cross-sections along the length of the structure in a special way that ensures effective use of the strength and deformation characteristics of the materials used.

To create models of hybrid construction, the SCAD software and computing system was selected. The chosen complex is based on the principle of the finite element method. Using this method, you can build models of hybrid structures from three-dimensional elements, with their subsequent calculation. Using the results obtained, it is possible to determine the stress-strain state of the structure and its components.

The main problem that arises when building models from three-dimensional elements is the correct setting of the boundary conditions of the model, the interface of dissimilar materials in the structure with each other. If the coupling of the component elements of the beam is performed incorrectly, the results obtained will incorrectly display the picture of the stress-strain state of the structure, which will lead to errors in the design.

The purpose of the analysis is to determine a method for constructing a computational model (SCAD) that provides adequate results of the stress-strain state of a hybrid structure in real operation.

2. Methods
For comparison, a hybrid design model was adopted, built in the SCAD in 2 ways:
Method №1. three-Dimensional elements of dissimilar materials at the points of contact are connected to each other by common nodes.

Method №2. Three-dimensional elements of dissimilar materials at the points of contact have separate nodes, with common joint deformations. At the point of contact of dissimilar materials, there are nodes whose coordinates are the same, and the nodes coincide in space. In the SCAD, which is based on the finite element method, three-dimensional elements of dissimilar materials defined in this way are considered not related to each other, that is, they work independently of each other. In this case, the implementation of joint work is provided by the "merge movements" function. This function combines three-dimensional elements of dissimilar materials, and their deformations at the nodes at the contact boundary are combined.

For analysis, a hybrid beam of rectangular cross-section with a span of 3 m. the Beam consists of a rectangular steel shell and a composite filling. The design characteristics of the beam and the materials it consists of are shown in table 1.

| Design parameter | h = 160 mm |
|------------------|------------|
| t₁ = 8 mm        | b₁ = 120 mm|
| t₂ = 10 mm       | b₂ = 120 mm|
| t₃ = 10 mm       | b₃ = 72 mm |
| E_steel = 206000 MPa (Rₚ=240 MPa) [3] | |
| E_composite ≈ 33000 MPa (Rₚ=872 MPa) [5] | |
| L_calculated = 3,0 m | |
| q_distributed = 32 kN /p. m. | |

Calculated cross-section Fig.1 for analysis – cross-section in the middle of the beam span (at a distance of 1.5 m from the supports).

The analysis criteria are the values of normal stresses, the nature of their distribution over the height and width of the section, and the amount of vertical deformation.

To verify the results of the calculation of models performed in different ways in the SCAD, an analytical calculation was previously performed - the stress-strain state of the hybrid beam was determined. For this purpose, the method of calculating beams of heterogeneous elasticity is used. A special feature of the method is the use of the given geometric parameters of the cross section, which take into account the properties of dissimilar materials that make up the hybrid structure. For a bar of heterogeneous elasticity, the main geometric characteristics given are calculated using the following formulas:
\[ F_0^{\text{quoted}} = F_0 + F_1 \frac{E_1}{E_0} + \cdots = \sum_{i=0}^{n-1} F_i \frac{E_i}{E_0}; \]
\[ S_0^{\text{quoted}} = S_0 + S_1 \frac{E_1}{E_0} + \cdots = \sum_{i=0}^{n-1} S_i \frac{E_i}{E_0}; \]
\[ I_0^{\text{quoted}} = I_0 + I_1 \frac{E_1}{E_0} + \cdots = \sum_{i=0}^{n-1} I_i \frac{E_i}{E_0}, \]
where:
\[ S_0^{\text{quoted}} \] – the given static moment of the section;
\[ F_0^{\text{quoted}} \] – reduced cross-sectional area;
\[ I_0^{\text{quoted}} \] - reduced moment of inertia of the section.

Determination of the values of operating voltages is carried out in accordance with the provisions of regulatory documents. The results of the analytical calculation are presented in table 2 and figure 2.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure2.png}
\caption{Distribution of normal stresses $\sigma_x$, [MPa] in a hybrid system over the cross-section height: a) composite filling; b) steel shell.}
\end{figure}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
Voltages & Calculated cross-sections (see figure 1) & \\
\hline
& I-I (steel) & II-II (steel) & II-II (composite) & III-III (steel) & III-III (composite) \\
\hline
$\sigma_x$, MPa & -121,35 & -107,87 & -17,00 & 0,00 & 0,00 \\
\hline
\end{tabular}
\caption{Table 2}
\end{table}

3. Method for building the model №1
Analysis of the stress-strain state of a model constructed using common nodes at the contact boundary of dissimilar materials.

A General view of the model constructed from three-dimensional elements in SCAD is shown in figure 3. The model is made of three-dimensional elements with dimensions of 8x10x20 mm ÷ 18x20x20 mm. Figures № 4, 5, 6 show plots of the distribution of normal stresses in the calculated cross-section of the hybrid beam.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure3.png}
\caption{General view of the hybrid beam.}
\end{figure}
Figure 4. Distribution of normal stresses $\sigma_x$ [MPa] along the length of a beam in a steel shell.

Figure 5. Plot of normal stresses $\sigma_x$ [MPa] in the calculated cross-section of the steel shell and composite filling.

Figure 6. Plot of normal stresses $\sigma_x$ [MPa] in the calculated cross-section for composite filling.
The calculation results are shown in Table 3 and Figure 7. The obtained values of normal stresses are compared with the analytically calculated results in Table 3. To analyse the distribution of normal stresses over the cross-section width, the IV-IV plane is selected, which is located 30 mm below the upper limit of the composite filling (see Figure 7.a).

![Figure 7](image)

**Figure 7.** Distribution of normal stresses $\sigma_x$, [MPa] in a hybrid system: a) along the cross-section height in the composite filling; b) in the IV-IV plane along the width of the entire cross-section.

### Table 3

| Voltages | Stress analysis planes in the design section (see Figure 1) |         |
|----------|----------------------------------------------------------|---------|
|          | I-I (steel)                  | II-II (steel)          | II-II (composite)       |
| $\sigma_{x \text{ max}}, \text{MPa}$ | -123.24 (increment – 1.56%) | -83.45 ÷ -107.46 | -85.61 (increment 403.6%) |

The deflection value is 6.22 mm (increment of the value is 1.37%).

### 4. Analysis of the results of the VAT model (construction method №1):

1) The nature of the distribution of normal stresses over the cross-section height in the model made according to the accepted method of constructing the model in the SCAD does not correspond to the nature of the stress distribution determined according to the analytical calculation. In this calculation case, a significant increase in the values of normal stresses in composite bulk elements bordering the steel belt was recorded (see Fig. 2.a), which differs from the results of the analytical calculation (see Fig. 7.a). The maximum values of the stresses acting in the composite are significantly higher than in the case of analytical calculation (see Table 3).

2) The nature of the distribution of normal stresses over the cross-section width in the model made according to the accepted method differs significantly from the nature of the stress distribution according to the analytical calculation. According to the analytical calculation, the value of normal stresses over the width of the composite should be constant. In this case, the distribution of normal stresses in the middle of the composite is constant, and in bulk composite elements connected to the elements of the steel shell by common nodes, a significant increase in the values of normal stresses is recorded (see Fig. 7.b).

3) The values of normal stresses acting in the steel shell received a slight increment – by 1.56% (see Table 3).

4) The deflection value increased by 1.37% compared to the calculated value (see Table 3).
It follows from the analysis that to create a computational model of three-dimensional elements that most adequately reflects the stress – strain state of a hybrid structure, you should not use method № 1- connecting dissimilar materials at the points of contact with common nodes. In this version of the model construction, a significant distortion of the structure is observed – a change in the nature of the distribution of normal stresses in the composite with a change in the stress values.

The most likely reason for this difference in the nature of the distribution of normal stresses in this method of construction is the peculiarity of the joint operation of finite elements with different elastic modules. Creating a joint operation of dissimilar elements at points of contact by connecting them at points of contact does not reflect the actual operation of a structure consisting of dissimilar materials. In reality, a hybrid structure consists of several heterogeneous elements and there is a real physical contact boundary between them. Setting this boundary in the calculation model through common nodes does not correspond to the actual operation of the design.

**Method for building the model №2.** Analysis of the stress-strain state of a model constructed using "combining displacements" of individual nodes of dissimilar materials at the boundary of their contact.

The model is made of three-dimensional elements with dimensions of 8x10x20 mm ÷ 18x20x20 mm. The difference between the models is only in the principle of connecting elements of dissimilar materials at the border of their contact. The other parameters are identical and accepted without changes. Figures 9 and 10 show plots of the normal stress distribution in the calculated cross-section of the hybrid beam.

![Figure 8. Distribution of normal stresses $\sigma_x$ [MPa] along the length of a beam in a steel shell.](image-url)
The calculation results are shown in table 4 and figure 11. The obtained values of normal stresses are compared with the analytically calculated results in table 4. To analyse the distribution of normal stresses over the cross-section width, the IV-IV plane is selected, which is located 30 mm below the upper limit of the composite filling (see figure 11.a).

**Figure 9.** Plot of normal stresses $\sigma_x$ [MPa] in the calculated cross-section of the steel shell and composite filling.

**Figure 10.** Plot of normal stresses $\sigma_x$ [MPa] in the calculated cross-section for composite filling.
**Figure 11.** Distribution of normal stresses $\sigma_x$, [MPa] in a hybrid system: a) over the cross-section height in the composite filling; b) in the IV-IV plane in the composite filling.

**Table 4**

| Volleys       | Расчётные сечения (см. рис. 1) | I-I (steel) | II-II (steel) | II-II (composite) |
|---------------|-------------------------------|-------------|---------------|-------------------|
| $\sigma_{x\text{ max}},$ MPa | - 123,24                      | - 108,69    | - 16,62       |
| (increment – 1.5%) | (increment – 0,7%) | (decline – 2,23%) |
| Amount of deflection – 6,224 mm (the increment is 1.4%) |

5. **Analysis of the results of the VAT model (construction method №2):**

1) the Nature of the distribution of normal stresses over the cross-section height in the model made according to the accepted method of constructing the model in the SCAD fully corresponds to the nature of the stress distribution determined according to the analytical calculation (see Fig. 2.a, 11.a). Differences in the values of the operating voltages are insignificant (see table 4).

2) the Nature of the distribution of normal stresses over the cross-section width in the model made according to the accepted method fully corresponds to the nature of the stress distribution according to the analytical calculation – the values of normal stresses are constant.

3) the Values of normal stresses acting in the steel shell received a slight increment – by 1.5 % (see table 3).

4) the deflection value Increased by 1.4% compared to the calculated value (see table 3).

To create a computational model of a hybrid structure from three-dimensional elements, use method №2-combining the movements of individual nodes of dissimilar materials at their points of contact. With this version of the model construction, it is possible to achieve a high degree of compliance of the obtained results of the stress-strain state with the results obtained according to the analytical calculation. The nature of the distribution of normal stresses over the height and width of the calculated cross-section fully corresponds to the nature of the stress distribution according to the analytical calculation.

Using the 2nd method of constructing computational models of hybrid structures, you can set the real physical boundary of the hybrid structure. The stress-strain state of the hybrid structure will, obtained from the calculated model, correspond to the real stress-strain state until the moment of structural failure – a situation when there will be a significant deformation of the structure, with the displacement or destruction of the physical boundary of dissimilar materials.
6. Conclusions
The analysis showed that in the case of constructing a hybrid beam model using method № 1, the SCAD smooths the stress distribution along the length and height of the model cross-section, and also does not display the real interaction of dissimilar materials at the boundary of their contact.

Assigning two nodes with the same coordinates to different finite elements (with different elastic modules) and then combining the deformations eliminates the problem described above. This is confirmed by the plot of the distribution of normal stresses over the height and width of the cross-section (reflected as a jump in the value of normal stresses at the interface of the interface of dissimilar materials), as well as by the joint relative deformations.

Thus, the option of constructing models No. 2 in the SCAD software and computing complex allows us to obtain an adequate reflection of the stress-strain state of the system with sufficient accuracy, which is confirmed by analytical calculation. Taking into account the properties of the software and computing complex, which use the finite element method, will allow us to adequately predict the stress-strain state of a hybrid structure described by various finite elements.

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