Continuous and discontinuous transitions in the depinning of two-dimensional dusty plasmas on a one-dimensional periodic substrate

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Langevin dynamical simulations are performed to study the depinning dynamics of two-dimensional dusty plasmas on a one-dimensional periodic substrate. From the diagnostics of the sixfold coordinated particles $P_b$ and the collective drift velocity $V_d$, three different states appear, which are the pinning, disordered plastic flow, and moving ordered states. It is found that the depth of the substrate is able to modulate the properties of the depinning phase transition, based on the results of $P_b$ and $V_d$, as well as the observation of hysteresis of $V_d$ while increasing and decreasing the driving force monotonically. When the depth of the substrate is shallow, there are two continuous phase transitions. When the potential well depth slightly increases, the phase transition from the pinned to the disordered plastic flow states is continuous, however, the phase transition from the disordered plastic flow to the moving ordered states is discontinuous. When the substrate is even deeper, the phase transition from the pinned to the disordered plastic flow states changes to discontinuous. When the substrate further increases, as the driving force increases, the pinned state changes to the moving ordered state directly, so that the disordered plastic flow state disappears completely.

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I. INTRODUCTION

Many driven systems can be characterized by a collection of interacting point particles that passes through disordered or ordered substrates under a uniform force [1, 8]. Examples of these systems include vortex lattices in superconductors with periodic arrays of pinning sites [2], arrays of nanostructured pinning sites [3], colloidal monolayers driven across ordered surfaces [4], Wigner crystals [5], and pattern-forming systems [6, 7]. It was discovered that these systems could exhibit critical depinning transitions [8] when an applied uniform force is combined with a substrate. When the external driving force, $F_d$, is too small to overcome the confinement by the substrate, the system is trapped in one of many possible metastable configurations. As the external driving force, $F_d$, gradually increases, the initial configuration becomes unstable and moves, and may be stopped frequently by the elastic forces. As $F_d$ increases further, the system can “avalanche” and move at higher speeds [8]. The transitions between these different states can be characterized by the critical depinning thresholds [1, 8].

Dusty plasma, a collection of highly charged micron-sized dust particles in a partially ionized gas [9–17], can also be coupled to a substrate, as studied in [18–20]. Under typical laboratory conditions, these dust particles are charged to a high negative charge of $\sim -10^4 e$, and they can self-organize into a single layer plane, i.e., forming a two-dimensional dusty plasma (2DDP) [21, 22]. In experiments, these highly charged dust particles are strongly coupled, exhibiting collective solid-like [23–25] or liquid-like behaviors [26–28]. Substrates have been experimentally realized in 2DDPs using a striped electrode, as demonstrated in [23, 30]. Recently, the coupling of 2DDP with a one-dimensional periodic substrate (1DPS) has been studied using simulations, which focused on the phonon spectra [18], the structural transitions [19], and also the diffusion [19, 20]. If a uniform force is applied on all particles of 2DDP with 1DPS, for example using the laser radiation force in experiments [31], the depinning dynamics can be investigated. The depinning dynamics of one row of dust particles in each potential well of 1DPS was investigated in [32] using simulations, and it is found that, for a certain range of the substrate depth, three different states appear as the external force increases gradually from zero, which are the pinned, disordered plastic flow and moving ordered states. However, for different configurations of 2DDP under 1DPS, such as for two rows of dust particles in each potential well of 1DPS, the depinning dynamics would be more complicated, as studied
so that the force from the 1DPS is $F$. Assume that the 1DPS has the form of and the Langevin random kicks \[40, 41\], respectively. We where $\lambda$ is the binary Yukawa interaction \[39\] between dust particles, $\phi_{ij} = Q^2\exp(-r_{ij}/\lambda_D)/4\pi\epsilon_0r_{ij}$, where $r_{ij}$ is the distance between dust particles $i$ and $j$. The terms of $-\nu m\dot{r}_i$ and $\xi_i(t)$ are the frictional drag and the Langevin random kicks \[40, 41\], respectively. We assume that the 1DPS has the form of

$$U(x) = U_0 \cos(2\pi x/w),$$

(2)

so that the force from the 1DPS is $F_s = -\partial U(x)/\partial x = (2\pi U_0/w)\sin(2\pi x/w)\hat{x}$, which is in the $x$ direction. Here, $U_0$ and $w$ are the depth and width of the potential well, in units of $E_0 = Q^2/4\pi\epsilon_0a$ and $b$, respectively. The last term on the right of Eq. (1), $F_d$, is the external driving force, in units of $F_0 = Q^2/4\pi\epsilon_0a^2$. Note that, we use the inverse nominal 2D dusty plasma frequency, $\omega_{pd}^{-1} = (Q^2/2\pi\epsilon_0ma^3)^{-1/2}$, to normalize the time scale, and use either the Wigner-Seitz radius $a$ or the lattice constant $b$ to normalize the length scale \[33, 34\].

We simulate $N = 1024$ particles constrained within a $61.1a \times 52.9a$ 2D plane with periodic boundary conditions. Since the size in the $x$ direction is $61.1a = 32.07b$, to satisfy the periodic boundary conditions, we specify the width of the potential well as $w/b = 2.004$, which corresponds to 16 full potential wells. For the depth of the potential well, we consider four different values, $U_0/E_0 = 0$, 0.01, 0.05, 0.10 and 0.25. To reduce the temperature effect on the depinning dynamics, we fix the conditions of the simulated 2DDP at $\Gamma = 1000$ and $\kappa = 2$, corresponding to the typical solid/crystal state in the absence of substrates or external forces \[12\]. The gas damping rate is chosen to be comparable to the typical experimental value of $\nu/\omega_{pd}^{-1} = 0.027$. For each simulation run, we integrate $\geq 10^7$ steps of Eq. (1) with a time step of $0.0028\omega_{pd}^{-1}$ (or $0.0007\omega_0^{-1}$ only for $U_0/E_0 = 0.25$) to obtain the positions and velocities of all particles. We also performed a few test runs with 4096 particles to verify that all results reported here are not affected by the total particle number. Other simulation details are the same as in \[32\].

II. SIMULATION METHODS

Without substrates, traditionally, 2DDP can be characterized by two dimensionless parameters \[32, 39\], the coupling parameter $\Gamma = Q^2/(4\pi\epsilon_0ak_BT)$ and the screening parameter $\kappa \equiv a/\lambda_D$. Here, $a = (\pi n)^{-\frac{1}{2}}$ is the Wigner-Seitz radius \[37\] with areal number density $n$, $T$ is the particle kinetic temperature, $Q$ is the charge of each dust particle, and $\lambda_D$ is the screening length.

We use Langevin dynamical simulations to investigate the depinning dynamics of 2DDP on a 1DPS, using the equation of motion \[18, 19, 32, 38\]

$$m\ddot{r}_i = -\nabla U(Q) - \nu m\dot{r}_i + \xi_i(t) + F_s + F_d,$$

(1)

for the dust particle $i$. Here, the first term on the right of Eq. (1), $-\nabla U(Q)$ is the binary Yukawa interaction \[39\] between dust particles, $\phi_{ij} = Q^2\exp(-r_{ij}/\lambda_D)/4\pi\epsilon_0r_{ij}$, where $r_{ij}$ is the distance between dust particles $i$ and $j$. The terms of $-\nu m\dot{r}_i$ and $\xi_i(t)$ are the frictional drag and the Langevin random kicks \[40, 41\], respectively. We assume that the 1DPS has the form of

$$U(x) = U_0 \cos(2\pi x/w),$$

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so that the force from the 1DPS is $F_s = -\partial U(x)/\partial x = (2\pi U_0/w)\sin(2\pi x/w)\hat{x}$, which is in the $x$ direction. Here, $U_0$ and $w$ are the depth and width of the potential well, in units of $E_0 = Q^2/4\pi\epsilon_0a$ and $b$, respectively. The last term on the right of Eq. (1), $F_d$, is the external driving force, in units of $F_0 = Q^2/4\pi\epsilon_0a^2$. Note that, we use the inverse nominal 2D dusty plasma frequency, $\omega_{pd}^{-1} = (Q^2/2\pi\epsilon_0ma^3)^{-1/2}$, to normalize the time scale, and use either the Wigner-Seitz radius $a$ or the lattice constant $b$ to normalize the length scale \[33, 34\].

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III. RESULTS

In the depinning procedure of 2DDP on 1DPS, when the driving force increases gradually, three typical dynamical states appear, which are the pinned state, the disordered plastic state and the moving ordered state, respectively, as shown in Fig. 1. When the driving force is very small, all of the particles are pinned around their equilibrium locations due to the 1DPS, so that the particles are neatly arranged in two rows within one potential well of the substrate. When the driving force is larger, some particles can escape from the 1DPS and the cages formed by their neighbors, forming a disordered plastic state. When the driving force on each particle is high enough to overcome the 1DPS, all particles move with a constant rate of increase in the velocity along the direction of the driving force, and these particles are distributed in an ordered triangular lattice, independent of the potential wells. Note that these three states are similar to the states observed for superconducting vortices \[43\], a defective flux-line lattice \[44\], vortex lattices \[45\], Skyrmions \[46\], and the depinning of 2DDP with only one row of particles within one potential well \[32\].
FIG. 1: Snapshots of particle positions (dots) for a 2D Yukawa crystal with $\Gamma = 1000$ and $\kappa = 2$ under the 1DPS (curve) of $U(x) = U_0 \cos(2\pi x/w)$ ($U_0/E_0 = 0.10$ and period $w = 2.0048$) while experiencing different external driving forces. When $F_d/F_0 = 0$ in (a), the system is in the pinned state, so that the particles are neatly arranged in two rows within one potential well of the substrate. When $F_d/F_0 = 0.04$ in (b), the system is in the disordered plastic flow state. When $F_d/F_0 = 0.08$ in (c), the system is in the moving ordered state, so that the particles are distributed in an ordered triangular lattice, independent of the locations of the potential wells.

a. Continuous and discontinuous phase transitions

We calculate the static structural measure of the six-fold coordinated particles $P_6$ as the driving force $F_d$ increases for five different values of $U_0/E_0$, as shown in Fig. 2. Here, $P_6$ is defined as $P_6 = \langle \sum_{i=1}^{N_d} \delta(6 - z_i) \rangle / N_d$ [34], where $z_i$ is the coordination number of particle $i$ obtained from the Voronoi construction. For a perfect triangular lattice, $P_6 = 1.0$, while, for a more disordered system, the value of $P_6$ is reduced.

We find that in our simulated system, the depinning dynamic state depends on not only the magnitude of the driving force, but also the depth of the substrate, as shown in Fig. 2. When $U_0/E_0 = 0.05$ and 0.10, from Fig. 2, as the driving force increases from zero, the value of $P_6$ varies over three distinctive ranges, which correspond to the pinned, disordered plastic flow and moving ordered states in Fig. 1. When the driving force is very small, for the two depths of the substrate $U_0/E_0 = 0.05$ and 0.10, $P_6 \approx 0.65$, corresponding to the pinned state. Here, within each potential well, the particles are pinned around the bottom to form two rows, as shown in Fig. 1(a). As the driving force increases gradually, the value of $P_6$ decreases substantially to a lower value of around 0.35, which is a typical value for a disordered plastic flow state. Thus, the lower value of $P_6$ of around 0.35 for $U_0/E_0 = 0.05$ and 0.10 in the middle range of the driving force in Fig. 2 correspond to the disordered plastic flow state in Fig. 1(b). When the driving force increases enough to completely overcome the constraint from the 1DPS, $P_6$ suddenly increases to a higher value of around 0.9, corresponding to the moving ordered state in Fig. 1(c).

When the substrate depth is fairly deep, for example $U_0/E_0 = 0.25$ as shown in Fig. 2, we find that the second disordered plastic flow state disappears completely. As the driving force $F_d$ gradually increases from 0, the value of $P_6$ stays around the initial low value of about 0.45 until $F_d/F_0 > 0.10$, then suddenly jumps directly to around 0.9. We do not find a decrease in the $P_6$ from our data analysis, suggesting that the second disordered plastic flow state is not present. Note that the initial value of $P_6$ is lower than what is found for shallower substrate depths because the 1DPS can more strongly distort the arrangement of the particles.

When the substrate depth is very shallow, for example $U_0/E_0 = 0.01$ as shown in Fig. 2, the process is slightly different. In the initial state, we find a fairly high $P_6 \approx 0.85$. This is completely different from what we observe in the pinned state for other substrates because the shallow potential well can only exert a weak constraint on the particles. When the external force increases, the stability of the system is destroyed and $P_6$ decreases to a low value of around 0.45, corresponding to the disordered plastic flow state. As the external force further increases, $P_6$ increases back to about 0.85, which suggests that the particles have rearranged into an ordered triangular lattice, independent of the potential well locations. Note that in the initial and final moving ordered states, the value of $P_6$ is nearly unchanged and is given by $P_6 \approx 0.85$.

From Fig. 2, the continuous/discontinuous property of the phase transition is visible from the variation of the value of $P_6$. For $U_0/E_0 = 0.01$, as $F_d$ increases from 0, $P_6$ diminishes continuously to about 0.45 and then continuously returns to its initial high value, suggesting that these two phase transitions are both continuous. Simi-
For a perfect triangular lattice, $P_b$ is reduced for a more disordered system, since $P_b$ is defined as $\langle \sum_{i=1}^{N_d} d(6 - z_i) \rangle / N_d$, where $z_i$ is the coordination number of particle $i$ obtained from a Voronoi construction. When $U_o/E_0 = 0.01$, 0.05 and 0.10, as $F_d$ increases from 0, three different states can be clearly observed, which are the initial high value of $P_b$ (well above 0.6), then a low value of around 0.45 or even lower, and finally the high value again (above 0.8). For $U_o/E_0 = 0.05$ and 0.10, these three values of $P_b$ correspond to the pinned, disordered plastic flow and moving ordered states observed in Fig. 1. However, when $U_o/E_0 = 0.25$, as $F_d$ increases from 0, the value of $P_b$ jumps from the initial value of 0.45 directly to around 0.9. This structure measure also reflects the property of the phase transition. For $U_o/E_0 = 0.01$, as $F_d$ increases from 0, $P_b$ drops continuously to about 0.45 and then continuously returns to its initial high value, suggesting that the two phase transitions are both continuous. Similarly, for $U_o/E_0 = 0.05$, as $F_d$ increases from 0, $P_b$ drops continuously to about 0.35 and returns suddenly to a high value of $> 0.8$, suggesting that the first phase transition is continuous, while the second is discontinuous. For $U_o/E_0 = 0.10$ and 0.25, all of the phase transitions are discontinuous.

Our results on the collective drift velocity $V_x$ for various substrates as a function of the external driving force are presented in Fig. 3. Here, we calculate the collective drift velocity using $V_x = N_d^{-1} \langle \sum_{i=1}^{N_d} v_i \cdot \hat{x} \rangle$. The unit of $V_x$ is $V_0 = (Q^2 / 4\pi \epsilon_0 ma)^{1/2}$. For the typical values of $U_o/E_0 = 0.05$ and 0.10, when the external force is small, the collective drift velocity is almost zero, corresponding to the pinned state. As the external force gradually increases, the collective drift velocity increases relatively steeply, corresponding to the disordered plastic flow state. Finally, when the external force is very large, the collective drift velocity increases linearly with $F_d$, corresponding to the moving ordered state. Note that, as found in [32], for the final moving ordered state, the collective drift velocity $V_x$ also increases linearly with the external driving force $F_d$ at a fixed slope of $\nu m$, in-
dependent of the 1DPS.

Our previous conclusion about the continuous/discontinuous property of the phase transition observed from the static structural measure of $P_b$ above is further verified by the collective drift velocity $V_x$ results in Fig. 3. There are three types of dynamical states in Fig. 3. Two of them can be easily identified as the initial pinned state where the collective drift velocity is zero, and the final moving ordered state where the collective drift velocity increases linearly with $F_d$. Other data points between these two lines belong to the plastic flow state.

For the substrate with $U_0/E_0 = 0.05$, as the driving force increases gradually, the increase of $V_x$ from the initial pinned state to the second disordered plastic flow state is continuous, while the later increase of $V_x$ from the disordered plastic flow phase to the final moving ordered state is discontinuous or abrupt. For the substrate with $U_0/E_0 = 0.10$, the two-step increases of $V_x$ from the initial pinned state to the disordered plastic flow state, and then to the final moving ordered state, are both discontinuous as a function of increasing driving force. For the deep substrate of $U_0/E_0 = 0.25$, $V_x$ remains zero until the driving force increases to more than 0.1, then $V_x$ suddenly jumps directly from 0 to the final linear range with increasing $F_d$, i.e., directly from the initial pinned to the final moving ordered state, without passing through the disordered plastic flow state. Note that in addition to the static structural measures and the collective drift velocity, we provide the trajectories of our simulated 2DDP under this 1DPS, as presented in the Supplementary Materials of [47].

For the shallow substrate of $U_0/E_0 = 0.01$, we find that the collective drift velocity always increases continuously and almost overlaps with that of the zero-substrate case, with only a small deviation when $F_d/F_0 \approx 0.18$, as magnified in the inset of Fig. 3. This feature suggests that for this sample the depinning process involves collective motion of all of the particles particles, which is quite different from the typical plastic depinning process from the initial pinned state for the other cases studied here. Based on the combination of the linear increase of $V_x$ with $F_d$ in the initial state in Fig. 3 with the corresponding $P_b$ result in Fig. 2 for $U_0/E_0 = 0.01$, we determine that all of the particles begin to move in the direction of the driving force as a rigid object, as the final moving ordered state. This initial state is transient, however, since its structure and collective drift velocity are partially modified when $F_d/F_0 \approx 0.18$.

Clearly, the continuous/discontinuous property of $V_x$ at each transition observed from Fig. 3 is consistent with that presented in $P_b$ in Fig. 2. Our conclusion is based on both the static structural and dynamical measures from our simulations of 2DDP on the 1DPS.

For a physical procedure, the hysteresis generally results from the lagging of the system response to an external modification. Typically, a process with hysteresis shows an overshoot during its evolution [48]. As described in [1, 8, 10], when an overshoot is present, the critical depinning threshold is reduced, so that an originally non-hysteretic depinning transition becomes increasingly hysteretic. Here, we investigate whether the hysteresis feature exists in the depinning of 2DDP with 1DPS.

From [2, 8, 13], the hysteresis feature is directly related to the property of the depinning phase transition. For the first-order, or discontinuous, phase transition, when the static structural or dynamical measures both show abrupt jumps, hysteresis would in principle be ex-
pected to appear, whereas for a second-order, or continuous, phase transition, there should not be any hysteresis. We next present the overall drift velocity $V_z$, as the driving force $F_d$ increases and decreases monotonically in our simulations.

Our results on the hysteresis of the collective drift velocity as the driving force increases and decreases monotonically from our simulations, are presented in Fig. 4. Here, $F_d^+$ represents the increase of $F_d/F_0$ from 0 to 0.125, while $F_d^-$ represents the decrease of $F_d/F_0$ from 0.125 back to 0. When $F_d$ increases and decreases monotonically, our focus is on whether the phase transition would exhibit hysteresis. For $U_0/E_0 = 0.01$ in Fig. 4(a), as $F_d$ either increases or decreases monotonically, $V_z$ always follows the same trace without any hysteresis. For $U_0/E_0 = 0.05$ in Fig. 4(b), there is a single hysteresis loop when $F_d/F_0$ is around 0.03, corresponding to the transition between the disordered plastic flow and the moving ordered state. For $U_0/E_0 = 0.10$ in Fig. 4(c), there are two obvious hysteresis loops. A smaller loop is centered at $F_d/F_0 \approx 0.02$, while the larger loop is near $F_d/F_0 \approx 0.04$. For $U_0/E_0 = 0.25$ in Fig. 4(d), there is a single huge hysteresis loop when $0.4 \leq F_d \leq 0.10$, corresponding to the transition between the pinned state and the moving ordered state.

As our chief conclusion in this paper, we discover both first-order and second-order depinning phase transitions in the 2DDP under 1DPS from our simulations. The first-order depinning transition exhibits a discontinuity in the structural/dynamical measures when the external driving force increases, and hysteresis appears when the driving force decreases/increases monotonically. However, for the second-order depinning transition, the structural/dynamical measures are always continuous, and there is no hysteresis.

Clearly, both the hysteresis in Fig. 4 and the discontinuities in $P_6$ and $V_z$ in Figs. 2 and 3 indicate that the transition from the plastic flow to the moving ordered state for the substrate with $U_0/E_0 = 0.05$, the two transitions for the substrate with $U_0/E_0 = 0.10$, and the single transition for the substrate with $U_0/E_0 = 0.25$ are first-order. The lack of hysteresis in Fig. 4 and the continuity of $P_6$ and $V_z$ in Figs. 2 and 3 indicate that the two transitions for the substrate with $U_0/E_0 = 0.01$ and the transition from the pinned state to the plastic flow state for the substrate with $U_0/E_0 = 0.05$ might be second-order.

c. The property of kinetic temperature

To explore the underlying dynamical characteristics of the depinning phase transition of 2DDP under 1DPS, we also calculate the kinetic temperature in Fig. 5 using $k_B T = m \langle \sum_{i=1}^{N} (v_i - \langle v_i \rangle)^2 \rangle / 2$. Here, we use only the fluctuation of the velocity, while the collective drift motion during the depinning process is removed. We express the kinetic temperature in units of $T_0 = Q^2/(4\pi \varepsilon_0 a)$.

The kinetic temperature at the depinning of our simulated 2DDP also reflects the continuity and discontinuity of the phase transitions. For the substrate with $U_0/E_0 = 0.01$, as the driving force increases from zero, the kinetic temperature increases gradually to its maximum value when $F_d/F_0 \approx 0.018$, then decreases gradually, as shown in Fig. 5(a). This gradual variation of the kinetic temperature probably suggests that the two transitions from the pinned to the disordered plastic flow phase, then to the final moving ordered phase, are both continuous. For $U_0/E_0 = 0.05$ in Fig. 5(b), as
the driving force increases from zero, the kinetic temperature increases gradually to its maximum value when \( F_d/F_0 \approx 0.033 \), then decreases abruptly, suggesting that the transition from the pinned state to the plastic flow phase is continuous, while the transition from the plastic flow to the moving ordered phase is first-order. For \( U_0/E_0 = 0.10 \) in Fig. 5(c), as the driving force increases to \( F_d/F_0 \approx 0.021 \), the kinetic temperature jumps suddenly to a higher nonzero value, increases smoothly to its maximum when \( F_d/F_0 \approx 0.049 \), then decreases abruptly to a value of nearly zero. This variation of the kinetic temperature suggests that the two transitions from the pinned to the disordered plastic flow state is continuous; however, the phase transition from the disordered plastic flow state to the moving ordered state becomes discontinuous. When the substrate is even deeper, the phase transition from the pinned to the disordered plastic flow state also changes to discontinuous. When the substrate is further deepened, as the driving force increases, the pinned state jumps directly to the moving ordered state, and the disordered plastic flow state completely disappears.

IV. SUMMARY

We investigate the depinning dynamics of 2DDP under 1DPS using Langevin dynamical simulations. Note, the depinning dynamics have been investigated in many over-damped systems, however, far less is known about what happens when the mass or inertial effects play an important role, whereas dusty plasma is an ideal system to study such effects. In our study here, various diagnostics are calculated, such as the static structural measures of the sixfold coordinated particles \( P_0 \), the collective drift velocity \( V_x \), the kinetic temperature, and the hysteresis of \( V_x \) while the driving force increases and decreases monotonically. Similar to the depinning dynamics in other physical systems, we find that there are typically three different states, which are the pinned, disordered plastic flow, and moving ordered states.

From our simulation results, we find that the depth of the substrate can change the properties of the depinning phase transitions. When the depth of the substrate is shallow, there are two continuous phase transitions. When the depth of the potential well is slightly higher, the phase transition from the pinned to the disordered plastic flow state is continuous; however, the phase transition from the disordered plastic flow state to the moving ordered state becomes discontinuous. When the substrate is even deeper, the phase transition from the pinned to the disordered plastic flow state also changes to discontinuous. When the substrate is further deepened, as the driving force increases, the pinned state jumps directly to the moving ordered state, and the disordered plastic flow state completely disappears.

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