Generation of hybrid entanglement between a single-photon polarization qubit and a coherent state

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We propose a scheme to generate entanglement between a single-photon qubit in the polarization basis and a coherent state of light. The required resources are a superposition of coherent states, a polarization entangled photon pair, beam splitters, the displacement operation, and four photodetectors. Even when realistic detectors with a limited efficiency are used, an arbitrarily high fidelity can be obtained by adjusting a beam-splitter ratio and the displacement amplitude at the price of reducing the success probability.

I. INTRODUCTION

Entangled light fields have been extensively explored as tools for testing quantum mechanics and resources for quantum information processing. An intriguing challenge in this subject is to entangle different types of states of light such as microscopic and macroscopic states or wave-like and particledlike states [1] [14]. Some of those states have been found useful for quantum information applications [11] [14]. Recently, hybrid entanglement between a single photon in the polarization basis and a coherent state was found to be particularly useful for loophole-free Bell inequality tests [12], deterministic quantum teleportation, and resource-efficient quantum computation [18]. While single photons are regarded as nonclassical states as light quanta, coherent states are considered to be classical states as their P-functions are well defined [15] and they are robust against decoherence as “pointer states” [16]. In this regard, the hybrid entanglement is closely related to Schrödinger’s Gedanken experiment, where the fate of a classical object, the cat, is entangled with the state of a single atom [17].

Very recently, approximate implementations of hybrid entanglement between a qubit of the vacuum and single photon and a qubit of coherent states were demonstrated using the photon addition and subtraction techniques [3] [10]. The state explored in Ref. [3] was in the form of \(|0\rangle|\alpha\rangle + |1\rangle| -\alpha \rangle\) while a similar state of \((|0\rangle + |1\rangle)|\alpha\rangle + (|0\rangle - |1\rangle)| -\alpha \rangle\) was approximately demonstrated in Ref. [11], where \(|0\rangle\) is the vacuum, \(|1\rangle\) is the single photon and \(|\pm\alpha\rangle\) are coherent states of amplitudes \(\pm\alpha\). However, the state required to perform the aforementioned applications in Refs. [12] [13] was in fact in the form of \((|H\rangle|\alpha\rangle + |V\rangle| -\alpha \rangle\), i.e., the first mode should be in a definite single-photon state in the horizontal (\(H\)) or vertical (\(V\)) polarization. This type of hybrid entanglement, in spite of their usefulness, cannot be generated using the photon addition or subtraction as performed in Refs. [3] [10] because the first mode should be in a single photon state with definitely one photon. In principle, a cross-Kerr nonlinear interaction can be used to generate the required form of hybrid entanglement [15] [19], but it is a highly demanding task to achieve a clean nonlinear interaction using current technology [20] [22].

In this paper, we suggest a nondeterministic scheme to generate the desired form of hybrid entanglement between a single-photon polarization qubit and a coherent state field. Our scheme requires a superposition of coherent states (SCS), \(|\alpha\rangle + | -\alpha \rangle\) [24] [28], and a polarization entangled photon pair \(|H\rangle|V\rangle + |V\rangle|H\rangle\) as resources, in addition to beam splitters, the displacement operation and four photodetectors. We find that even when inefficient detectors are used, an arbitrarily high fidelity can be obtained by adjusting a beam-splitter ratio and the displacement amplitude. Our proposal is experimentally feasible using a squeezed single photon (or a squeezed vacuum state) as a good approximation of an ideal SCS [29]. Remarkably, high fidelities may still be obtained with these available resource states using current technology.

II. GENERATION SCHEME

We aim to generate the optical hybrid state

\[ |\Psi_\varphi(\alpha_f)\rangle_{AB} = \frac{1}{\sqrt{2}} (|H\rangle|\alpha_f\rangle_B + e^{i\varphi} |V\rangle | -\alpha_f \rangle_B), \]

where \(|\pm\alpha_f\rangle_B\) are coherent states in the field mode \(B\) and \(\varphi\) is a relative phase factor. As discussed in Ref. [30], this type of state shows obvious properties as macroscopic entanglement when \(\alpha\) is sufficiently large. For example, it is straightforward to show that the measure \(I\) as a macroscopic superposition [31] for this state has its maximum value \(I = \alpha_f^2 + 1\), i.e., the average photon number of the state. A classification of hybrid entanglement was attempted [32], according to which the state in Eq. (1) is categorized as a discrete-variable-like hybrid entanglement.

In order to generate the hybrid entanglement, as shown in Fig. 1 we first need to prepare a polarization entangled photon pair and a SCS as

\[ |\chi\rangle_{12} \otimes |\text{SCS}_\varphi(\alpha_i)\rangle_3, \]

where \(|\chi\rangle_{12} = (|H\rangle_1|V\rangle_2 + |V\rangle_1|H\rangle_2)/\sqrt{2}\) and \(|\text{SCS}_\varphi(\alpha_i)\rangle_3 = N_\varphi(|\alpha_i\rangle_3 + e^{i\varphi} | -\alpha_i \rangle_3\)

with \(N_\varphi = (2 + \cdots)\)
We consider effects of imperfect photodetectors that may lower the fidelity between the generated hybrid state and the ideal one. An imperfect photodetector with quantum efficiency $\eta$ can be expressed as a positive operator-valued measurement

$$\hat{E}_\eta^{(n)} = \sum_{m=0}^{\infty} \binom{n+m}{m} \eta^n (1-\eta)^m |n+m\rangle \langle n+m|$$  

(10)
in the photon number basis. The total measurement operator for our scheme described in Fig. 1 then becomes
\[
\Pi \eta = 1_A \otimes \hat{E}_{\eta,5R}^{(0)} \otimes \hat{E}_{\eta,5V}^{(1)} \otimes \hat{E}_{\eta,6R}^{(1)} \otimes \hat{E}_{\eta,6V}^{(0)} \otimes 1_B, \tag{11}
\]
and the heralded state is given by
\[
\rho_\eta = \frac{\text{Tr}_{56} [\Pi_\eta |\psi_\varphi\rangle \langle \psi_\varphi|]}{|\langle \psi_\varphi|\Pi_\eta|\psi_\varphi\rangle|. \tag{12}
\]
In the case of imperfect detection, the fidelity and the success probability can be calculated as
\[
\mathcal{F}_\eta^{AB} = AB |\langle \psi_\varphi|\rho_\eta|\psi_\varphi\rangle| = \frac{1}{2} \left( 1 + e^{-2(1-\eta)\left(\frac{1}{2} - 1\right)\alpha^2} \right) \tag{13}
\]
and
\[
P_{\eta,\text{tot}}^{\varphi} = 2(\psi_\varphi|\Pi_\eta|\psi_\varphi\rangle = 2N^2\eta^2\left(1 + \alpha^2\right)e^{-2(1-\eta)\alpha^2}, \tag{14}
\]
respectively. The fidelity and the success probability of the heralded state depend on \(\eta, \alpha_f\) and \(t\). We emphasize that as shown in Eq. (13), even if the detection efficiency \(\eta\) is limited, the hybrid state can be generated with an arbitrarily high fidelity by taking \(t \to 1\). The cost to obtain a high fidelity is to tolerate a low success probability which becomes zero as the fidelity reaches unity. Figures 2 and 3 show the fidelity and the success probability by changing various parameters.

In a real experiment, the polarization entangled photon pair \(|\chi\rangle_{12}\) used for our scheme may be mixed with the vacuum state \(|0\rangle_{12}\) for modes 1 and 2. The effective form of such a mixed state is
\[
\rho_\chi = z(|\chi\rangle\langle \chi|)_{12} + (1-z)(|0\rangle\langle 0|)_{12}, \tag{15}
\]
where \(0 < z \leq 1\). Remarkably, the vacuum component can be filtered by the conditioning measurement \(\Pi\). When states \(|0\rangle_{12} \otimes |\text{SCS}_{\varphi}(\alpha_f)\rangle_3\) are initially prepared, the states for modes 5 and 6 will become \(|\sqrt{2r}\alpha_i\rangle_5|0\rangle_6\) or \(|0\rangle_5|\sqrt{2r}\alpha_i\rangle_6\) before the heralding measurement (see Eq. (15)), and one of the modes will not contain any photon. Therefore, there is no chance to get the successful measurement event (i.e. single-photon measurement on the both modes 5 and 6). Meanwhile, the success probability decreases by factor \(z\) as the procedure starting with the vacuum state always fails.

B. Use of approximate resource states

The SCs required as resources for our scheme have been experimentally demonstrated while their fidelities and sizes are more or less limited [24–28]. As an example, it was shown that a photon-subtracted squeezed state (or equivalently, a squeezed single photon [33]) well approximates an ideal SCS, \(|\text{SCS}_{\alpha}(\alpha)\rangle\), for relatively small values of \(\alpha = 29, 34\), and its experimental demonstrations have been reported [24–27, 28]. A squeezed single-photon state in the Fock basis is
\[
\hat{S}(s)|1\rangle = \sum_{n=0}^{\infty} \frac{(\tanh s)^n}{(\cosh s)^{3/2}} \sqrt{(2n+1)!} |2n+1\rangle, \tag{16}
\]
where \(\hat{S}(s) = e^{-s/2}\sqrt{\alpha^2 - s^2}\) and \(s\) is the squeezing parameter. Its fidelity to an ideal state \(|\text{SCS}_{\alpha}(\alpha)\rangle\) is
\[
\mathcal{F}(s, \alpha) = |\langle \text{SCS}_{\alpha}(\alpha)|\hat{S}(s)|1\rangle|^2 = \frac{2\alpha^2 e^{\alpha^2(\tanh s - 1)}}{\left(\cosh s\right)^3(1 - e^{-2\alpha^2})}. \tag{17}
\]
For example, squeezing parameters \(s = 0.161\) and \(0.313\) approximate \(|\text{SCS}_{\alpha_f}(\alpha_f)\rangle\) with amplitude \(\alpha_f \approx 0.7\) and 1 with fidelities \(\mathcal{F} = 0.9998\) and 0.997, respectively [29]. We choose these two values for our investigation.

We note that for a small squeezing parameter \(s\), it is sufficient to reduce the state \(|\text{SCS}_{\alpha}(\alpha)\rangle\) in the number basis with an appropriate cut-off number \(n_{\text{cut}}\) for our numerical calculations. For example, the amplitude ratio of \(n = 7\) to \(n = 0\) of state [16] is less then 0.0005 for \(s = 0.313\) (and even smaller for \(s = 0.161\)), thus we take the cut-off number \(n_{\text{cut}} = 7\), where the actual
We can also model the beam splitter of transmittance $t$, where $\alpha$ is the parameter of resource states. Figure 4 shows that the squeezing parameter of state $\Psi_r(\alpha_f)$, $A_B$ using photon-subtracted squeezed states are approximate SCSs. The squeezing parameters used to obtain the photon-subtracted squeezed states are $s = 0.161$ (upper figures) and $s = 0.313$ (lower figures). The transmittance is $t = 0.9$ (dot-dashed lines), $t = 0.99$ (dashed lines) and $t = 0.999$ (solid lines), respectively. The vacuum portion of the polarization entangled pair is assumed to be $1 - z = 0.5$.

We can also model the beam splitter of transmittance $t$ $(r = \sqrt{1 - t^2})$ in the photon number basis, which transforms incoming modes $i$ and $j$ into outgoing modes $i'$ and $j'$ as

$$|n\rangle_i|m\rangle_j \rightarrow \sum_n \sum_m \sum_q \sum_p B_{pq}|p + m - q\rangle_i'|n - p + q\rangle_j', \quad (18)$$

where $B_{pq} = \left(\begin{array}{c} n \\ p \end{array}\right) \left(\begin{array}{c} m \\ q \end{array}\right) t^p q^{n - p} (-1)^{n - q}$.

Numerical calculations using $n_{\text{cut}}$ and the beam splitter model in the photon number basis is applied in order to calculate the fidelity and the success probability with approximate resource states. Figure 4 shows that the squeezing parameter of $s = 0.161$ (s = 0.313) and the vacuum portion of $z = 0.5$ result in the fidelity of the heralded hybrid entanglement with fidelity $F > 0.996$ ($F > 0.986$) and amplitude $\alpha_f \approx 0.7$ ($\alpha_f \approx 1.0$) by taking transmittance $t \geq 0.99$ and assuming realistic detector efficiency $\eta \geq 0.4$. We emphasize that the two chosen amplitudes here, $\alpha_f \approx 0.7$ and $\alpha_f \approx 1.0$, for hybrid entanglement were suggested as the best values for a loophole-free Bell test [12] and for the hybrid-qubit quantum computation [13], respectively. The success probability of the conditioning measurement with $t = 0.99$ varies from $P_{\text{tot}} \approx 10^{-4}$ to $P_{\text{tot}} \approx 10^{-3}$ by increasing the detection efficiency $\eta$ from 0.4 to 1.

In order to investigate a degree of entanglement for the heralded hybrid states, we evaluate negativity of the partial transpose \([5, 7]\), $E(\rho) = ||\rho^{\text{TP}}|| - 1 = -2 \sum_{\lambda_i} \lambda_i$, where $\rho^{\text{TP}}$ is the partial transpose of $\rho$ and $\lambda_i$ are its negative eigenvalues. The degree $E(\rho)$ ranges from 0 to 1, while an ideal hybrid state of $\alpha \gg 1$ results in $E(\rho) \approx 1$. The degree of entanglement are $E(\rho) = 0.922$ ($E(\rho) = 0.982$) for squeezing parameters $s = 0.161$ ($s = 0.313$) by taking $t = 0.99$, $z = 0.5$, and $\eta = 0.7$. The entanglement degrees can be compared with those of the ideal hybrid states with $\alpha_f = 0.7$ and $\alpha_f = 1.0$, i.e., $E(\rho) = 0.927$ and $E(\rho) = 0.991$, respectively.

IV. REMARKS

We have suggested a scheme to generate hybrid entanglement between a single photon qubit and a coherent state qubit. Unlike previous proposals \([8-10]\), our scheme enables one to generate the exact form of hybrid entanglement, without approximation, required for resource-efficient optical hybrid quantum computation \([13]\) and loophole-free Bell inequality tests \([12]\). The required resources are an SCS, an entangled photon pair, the displacement operation, four photodetectors, and beam splitters. Even when photodetectors with limited efficiencies are used, hybrid entanglement with an arbitrarily high fidelity can be generated at the price of a lower success probability. We have also analyzed fidelities of the generated states when approximate SCSs are used. According to our analysis, experimental implementation of our scheme seems feasible using current technology in spite of some expected experimental imperfections.

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\[1\] F. De Martini, Phys. Rev. Lett. 81, 2842 (1998).
\[2\] F. De Martini, F. Sciarrino, C. Vitelli, Phys. Rev. Lett. 100, 253601 (2008).
\[3\] P. Sekatski, B. Sanguinetti, E. Pomarico, N. Gisin, C. Simon, Phys. Rev. A 82, 053814 (2010).
\[4\] P. Sekatski, N. Sangouard, M. Stobińska, F. Bussieres, M. Afzelius, N. Gisin, Phys. Rev. A 86, 060301(R) (2012).
\[5\] R. Ghobadi, A. Lvovsky, C. Simon, Phys. Rev. Lett. 110, 170406 (2013).
\[6\] N. Bruno, A. Martin, P. Sekatski, N. Sangouard, R.T. Thew, N. Gisin, Nat. Phys. 9, 545 (2013).
[7] A. I. Lvovsky, R. Ghobadi, A. Chandra, A. S. Prasad, C. Simon, Nat. Phys. 9, 541 (2013).
[8] U. L. Andersen and J. S. Neergaard-Nielsen, Phys. Rev. A 88, 022337 (2013).
[9] H. Jeong, A. Zavatta, M. Kang, S.-W. Lee, L. S. Costanzo, S. Grandi, T. C. Ralph, and M. Bellini, Nat. Photon 8, 564 (2014).
[10] O. Morin, K. Haung, J. Liu, H. L. Jeannic, C. Fabre, and J. Laurat, Nat. Photon 8, 570 (2014).
[11] K. Park, S.-W. Lee, and H. Jeong, Phys. Rev. A 86, 062301 (2012).
[12] H. Kwon and H. Jeong, Phys. Rev. A 88, 052127 (2013).
[13] S.-W. Lee and H. Jeong, Phys. Rev. A 87, 022326 (2013).
[14] M. Stobińska, F. Töppel, P. Sekatski, A. Buraczewski, Phys. Rev. A 89, 022119 (2014).
[15] L. Mandel, Phys. Scrip T12, 34 (1986).
[16] W. H. Zurek, S. Habib, and J. P. Paz, Phys. Rev. Lett. 70, 1187 (1993).
[17] E. Schrödinger, Naturwissenschaften 23, 807 (1935).
[18] C. C. Gerry, Phys. Rev. A 59, 4095 (1999).
[19] H. Jeong, Phys. Rev. A 72, 034305 (2005).
[20] J. H. Shapiro, Phys. Rev. A 73, 062305 (2006).
[21] J. H. Shapiro, and M. Razavi, New J. Phys. 9, 16 (2007).
[22] J. Gea-Banacloche, Phys. Rev. A 81, 043823 (2010).
[23] B. He and A. Scherer, Phys. Rev. A 85 033814 (2012).
[24] A. Ourjoumtsev, R. Tualle-Brouri, J. Laurat, and P. Grangier, Science 312, 83 (2006).
[25] J. S. Neergaard-Nielsen, B. M. Nielsen, C. Hettich, K. Mølmer, and E. S. Polzik, Phys. Rev. Lett. 97 083604 (2006).
[26] A. Ourjoumtsev, H. Jeong, R. Tualle-Brouri, and P. Grangier, Nature 448, 784 (2007).
[27] H. Takahashi, K. Wakui, S. Suzuki, M. Takeoka, K. Hayasaka, A. Furusawa, and M. Sasaki, Phys. Rev. Lett. 101, 233605 (2008).
[28] T. Gerrits, S. Glancy, T. S. Clement, B. Calkins, A. E. Lita, A. J. Miller, A. L. Migdall, S. W. Nam, R. P. Mirin, and E. Knill, Phys. Rev. A 82, 031802(R) (2010).
[29] A. P. Lund, H. Jeong, T. C. Ralph, and M. S. Kim, Phys. Rev. A 70, 020101(R) (2004).
[30] H. Jeong, M. Kang, and H. Kwon, Opt. Comm. (2014), http://dx.doi.org/10.1016/j.optcom.2014.07.012.
[31] C.-W. Lee and H. Jeong, Phys. Rev. Lett. 106, 220401 (2011).
[32] K. Kreis and P. van Loock, Phys. Rev. A 85, 032307 (2012).
[33] A photon-subtracted squeezed state and a squeezed single photon are equivalent up to a normalization factor as $\hat{a}\hat{S}(s)|0\rangle = -\sinh(s)\hat{S}(s)|1\rangle$ [29, 34].
[34] H. Jeong, A. P. Lund, and T. C. Ralph, Phys. Rev. A 72, 013801 (2005).
[35] A. Peres, Phys. Rev. Lett. 77, 1413 (1996).
[36] M. Horodecki, P. Horodecki, and R. Horodecki, Phys. Lett. A 223, 1 (1996).
[37] J. Lee, M. S. Kim, Y. J. Park, and S. Lee, J. Mod. Opt., 47, 2151 (2000).