Determination of the Dzyaloshinskii-Moriya interactions

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Using in-plane field dependence of the precessional flow of chiral domain walls (DWs) to simultaneously determine bulk and interfacial Dzyaloshinskii-Moriya interactions (DMIs) is proposed. It is found that effective fields of bulk and interfacial DMIs have respectively transverse and longitudinal components that affect differently the motion of chiral DWs in magnetic narrow heterostructure strips. The in-plane field dependence of DW velocity has a dome-shape or a canyon-shape, depending on whether the driving force is an in-plane current or an out-of-plane magnetic field. The responses of their center shifts to the reversal of topological wall charge and current/field direction uniquely determine the nature and strength of DMI therein. Operable procedures are proposed and applied to explain existing experimental data.

I. INTRODUCTION

Dzyaloshinskii-Moriya interaction (DMI), the antisymmetric exchange coupling, was originally proposed to explain the weak ferromagnetism in antiferromagnets [1, 2], and is now known as a general interaction that widely exists in magnetic systems, especially the magnetic heterostructures. The importance of DMI in manipulating magnetic structures and dynamics has been recognized and an upsurge of research was witnessed in the passing decades after a successful explanation of huge remanent magnetization enhancement due to the DMI induced by Au or Pt impurities in metallic spin glasses [3] and distinct features of chiral DW dynamics in ultrathin magnetic films [4]. The main consensuses of the community are: (i) DMI comes from the spin-orbit coupling in magnetic systems with broken inversion symmetry either in a bulk or at an interface; (ii) DMI is crucial for stabilizing chiral magnetic solitons, such as skyrmions [5, 6] and chiral DWs [7, 8]; (iii) DMI plays an important role in the dynamics of both magnetic chiral solitons [9, 10] and spin waves [11]. Therefore precise determination of the nature and strength of DMI is not only of a fundamental issue, but also practically important.

Existing schemes for measuring the DMI strength in magnetic heterostructures all presuppose that only interfacial DMI (i-DMI) exists in the underlying systems. Generally, they belong to two groups. Schemes in group I are based on magnetization switching (DW propagation) processes [12–20], while those in group II are based on spin-wave excitation and propagation [21–25]. However in real heterostructures, bulk and interfacial DMI can coexist, therefore it is urgent to distinguish and measure them appropriately. In this work, we propose two parallel schemes (current-driven and field-driven) by which both DMIs can be simultaneously probed via precessional flow of chiral DWs in ferromagnetic (FM) layers of narrow-strip shaped heterostructures under in-plane magnetic fields. The averaged wall velocities are functions of in-plane fields and the resulting curves are domes (canyons) when walls are driven by in-plane currents (out-of-plane fields). The responses of their center shifts to the reversal of topological wall charge and current/field direction uniquely determine the nature and strength of DMI therein.

II. MODEL

The magnetic energy density $\mathcal{E}_0(M)$ of the FM layer in a heterostructure consists of four parts: the exchange part $\mathcal{E}_{ex} = A(\nabla \mathbf{m})^2$ with $A$ being the exchange stiffness, and $\mathbf{m} = \mathbf{M}/M_s$ ($M_s$ represents the saturation magnetization), the Zeeman part $\mathcal{E}_Z = -\mu_0 \mathbf{M} \cdot \mathbf{H}$ with the external applied field $H$, the anisotropy part $\mathcal{E}_{ani} = (\mu_0 M_s^2/2)(-k_E m_z^2 + k_M m_z^2)$ where $k_E$ ($k_M$) is the total (crystalline plus shape) anisotropy coefficient in easy (hard) axis, and the DMI contribution. For i-DMI, $\mathcal{E}_i = D_i (m_z(r) \nabla \cdot \mathbf{m}(r) - \mathbf{m}(r) \cdot \nabla \mathbf{m}(r))$ [26]. While for bulk DMI (b-DMI), $\mathcal{E}_b = D_b (\mathbf{m}(r) \cdot \nabla \times \mathbf{m}(r))$ [27]. Here $D_{i(b)}$ is the i(b)-DMI strength. The corresponding DMI-induced effective fields are $\mathbf{H}_i(r) = 2D_i (\nabla m_z - (\nabla \cdot \mathbf{m})/\mu_0 M_s)$ and $\mathbf{H}_b(r) = -2D_b (\nabla \times \mathbf{m})/\mu_0 M_s$, respectively.

Under external field and currents, the Lagrangian $\mathcal{L}$ of this FM layer (with external normal $\mathbf{n} = e_z$, see Fig. 1) is

$$\frac{\mathcal{L}}{\mu_0 M_s^2} = -\frac{\cos \theta \partial \phi}{\gamma M_s} \frac{\partial \mathbf{m}}{\partial t} - \frac{B_J \phi \partial \cos \theta}{\gamma M_s} \frac{\partial \mathbf{m}}{\partial (\mathbf{j} \cdot \mathbf{r})} + \frac{H_{2\mu L}}{M_s} \mathbf{p} - \frac{\mathcal{E}_0}{\mu_0 M_s^2},$$

with the dissipative functional

$$\mathcal{F} = \frac{\alpha}{2\gamma M_s} \left( \frac{\partial \mathbf{m}}{\partial t} - \frac{\beta B_J}{\alpha} \frac{\partial \mathbf{m}}{\partial (\mathbf{j} \cdot \mathbf{r})} \right)^2 \mathbf{m}^2 - \frac{H_{2\mu L}}{M_s} (\mathbf{m} \times \mathbf{m}_b) \cdot \frac{\partial \mathbf{m}}{\partial t},$$

describing the Gilbert damping and spin-orbit antidamping processes [28–30]. Here $\mathbf{r}(t)$ and $\mathbf{j}(t)$ are the polar and azimuthal angles of $\mathbf{m}(t)$, respectively. $\alpha$ is the damping constant and $\beta$ is the nonadiabatic spin-transfer torque (STT) coefficient. $\gamma = \mu_0 c$ with $\mu_0$ and $c$ being the vacuum permeability and electron gyromagnetic ratio, respectively.
where an overdot means the Lagrangian-Rayleigh equation,

\[ \dot{\phi} = -\frac{(1 + \alpha \beta) B_J}{(1 + \alpha^2)} - \frac{\eta \gamma \pi \Delta / 2}{1 + \alpha^2} \left[ (H_{FL} - \alpha H_{ADL}) \cos \phi \right. \\
- \left. H_{\perp} \sin(\phi - \phi_\perp) + \eta H_{\|} \sin \frac{H_K}{\pi} \sin 2\phi \right], \]

\[ \phi = \frac{(\alpha - \beta) B_J}{(1 + \alpha^2) \Delta} + \frac{\eta H_{\|} \sin \phi}{(1 + \alpha^2) \Delta} \left[ (H_{FL} + \frac{H_{ADL}}{\alpha}) \cos \phi \right. \\
- \left. H_{\perp} \sin(\phi - \phi_\perp) + \eta H_{\|} \sin \frac{H_K}{\pi} \sin 2\phi \right]. \]

The magnetization dynamics is then fully described by the Lagrangian-Rayleigh equation,

\[ \frac{d}{dt} \left( \frac{\delta \mathcal{L}}{\delta X} \right) - \frac{\delta \mathcal{L}}{\delta X} + \frac{\delta \mathcal{F}}{\delta X} = 0, \]

where an overdot means \( \dot{X} \) and \( X \) is a related coordinate. When \( X = \theta(\phi) \), the familiar Landau-Lifshitz-Gilbert equation[10] is recovered. In principle, \( \theta(r,t) \) and \( \phi(r,t) \) vary from point to point, thus generates huge number of degrees of freedom. To obtain collective behaviors, Lagrangian-based collective coordinate models are adopted which need pre-set ansatz. For narrow heterostructures, the Walker ansatz[31, 32]

\[ \ln \tan \frac{\bar{\phi}}{2} = \frac{X - g(t)}{\Delta}, \quad \bar{\phi} = \phi(t) \]

provides pretty good description of real wall configuration. In this ansatz, \( g, \Delta \) and \( \phi \) are wall center position, wall width and in-plane magnetization angle, respectively. \( \eta = +1(-1) \) corresponds to “\( \uparrow \downarrow (\downarrow \uparrow) \)” wall and is the topological wall charge. For narrow-strip geometry as shown in Fig. 1, the \( e_x \) and \( e_y \) axes respectively indicate the “longitudinal (L)” and “transverse (T)” directions. Accordingly, the in-plane components of effective fields from i-DMI and b-DMI are \( \mathbf{H}_{i}(x) = 2D_1(y) e_x / (\mu_0 M_s) \) and \( \mathbf{H}_{b}(x) = 2D_0(y) e_y / (\mu_0 M_s) \), respectively. Clearly, \( \mathbf{H}_{i(b)} \) has longitudinal (transverse) component proportional to \( V_{i(b)} \), which is reversed under wall charge reversal \( \eta \to -\eta \). This leads to the totally different responses of chiral DWs under longitudinal and transverse in-plane fields.

**III. CURRENT-DRIVEN \( v_{DW} \sim H_{\perp} \) DOMES**

In this section, we present the current-driven scheme. As an example, we focus on DW dynamics under \( J_a = j_a e_x \) in narrow heterostructure strips with pure i-DMI. Now \( H_{\perp} = 0 \) and an in-plane field \( H_{\perp} = H_{\perp} \left( \cos \phi, e_x + \sin \phi, e_y \right) \) is exerted. By viewing \( q, \phi \) and \( \Delta \) as three collective coordinates and integrating the resulting dynamical equations along longitudinal direction, the following closed equation set is obtained

\[ \dot{q} = -\left( 1 + \alpha \beta \right) B_J - \frac{\eta \gamma \pi \Delta / 2}{1 + \alpha^2} \left[ (H_{FL} - \alpha H_{ADL}) \cos \phi \right. \\
- \left. H_{\perp} \sin(\phi - \phi_\perp) + \eta H_{\|} \sin \frac{H_K}{\pi} \sin 2\phi \right], \]

\[ \dot{\phi} = \frac{(\alpha - \beta) B_J}{(1 + \alpha^2) \Delta} + \frac{\eta H_{\|} \sin \phi}{(1 + \alpha^2) \Delta} \left[ (H_{FL} + \frac{H_{ADL}}{\alpha}) \cos \phi \right. \\
- \left. H_{\perp} \sin(\phi - \phi_\perp) + \eta H_{\|} \sin \frac{H_K}{\pi} \sin 2\phi \right]. \]

with \( H_K \equiv k_H M_s \) and \( H_L \equiv D_1 / (\mu_0 M_s) \). The total magnetic energy \( E_0 = \int_{\infty}^{+\infty} \xi_0(\mathbf{M})d\mathbf{x} = 2A / (\mu_0 M_s^2 \Delta) \left( k_H + k_H \sin^2 \phi \right) - \pi \mu_0 M_s H_L \Delta \cos(\phi - \phi_\perp) + \eta \pi D_1 \sin \phi \). When \( j_a = 0 \), the wall keeps static. Without \( H_{\perp} \), the minimization of \( E_0 \) provides the static wall width \( A_0 = \sqrt{2A / (\mu_0 k_H M_s^2)} \) and \( \cos x = -\eta \text{sgn}(D_1) \), leading to a typical Néel wall with definite chirality selected by the i-DMI. Under finite \( j_a \), the wall starts to move. In principle, the explicit Walker limit is complicated under the coexistence of in-plane field, STT and SOT. Nevertheless, for large enough \( j_a \) the traveling-wave mode collapses and the wall falls into the precessional-flow mode. The time average of \( \dot{q} \) gives the wall’s drifiting velocity.

First we consider longitudinal in-plane fields \( H_{\perp} = H_{\perp} e_x \). For large enough currents, after performing linearization of \( \sin \phi \) and \( \sin 2\phi \) for \( |\phi| < 1 \), the second equation in (5) turns to (1 + \alpha^2) \dot{\phi} = \eta f_{iL} - g_{iL} \cdot \phi, \) with \( f_{iL} = (\alpha - \beta) B_J + \eta \pi \gamma \pi \Delta / \Delta + H_{ADL}, \) \( \alpha / \Delta \) and \( g_{iL} = \eta \pi \gamma \pi \Delta / \Delta + H_{ADL} / \alpha / \Delta \) and \( g_{iL} = \alpha / \Delta + H_{ADL} / \alpha / \Delta \). When \( g_{iL} = 0 \) (i.e. \( H_{\perp} = \Delta H_{\perp} \equiv \eta H_{\perp} - 2H_K / \pi \)), the wall rotates evenly \( \phi = \text{const} \) thus leading to constant velocity \( v_{iL} = \frac{\Delta}{(1 + \alpha)^2} B_J + \eta \pi \gamma \pi \Delta / \Delta + H_{ADL} / \alpha / \Delta \) \( \sim (1 + \alpha^2) v_{iL} = \frac{\Delta}{(1 + \alpha^2) B_J + \eta \pi \gamma \pi \Delta / \Delta + H_{ADL} / \alpha / \Delta \} \cdot \left( 1 + \exp[\phi_{iL} / (1 + \alpha^2)] \right) \). When \( g_{iL} = 0 \), \( \phi (t) = (\eta f_{iL} / g_{iL}) \cdot \left( 1 - \exp[\phi_{iL} / (1 + \alpha^2)] \right) - \eta \gamma \pi \Delta / \Delta + H_{ADL} / (\alpha / \Delta) \exp[\phi_{iL} / (1 + \alpha^2)] \) \( \sim (1 + \alpha^2) v_{iL} = \frac{\Delta}{(1 + \alpha^2) B_J + \eta \pi \gamma \pi \Delta / \Delta + H_{ADL} / \alpha / \Delta \} \cdot \left( 1 + \exp[\phi_{iL} / (1 + \alpha^2)] \right) \). The time needed for \( \phi \) changing from 0 to \( \pi \gamma \pi \Delta / \Delta + H_{ADL} / \alpha / \Delta \) is \( \delta t = (1 + \alpha^2) / |\phi_{iL} | \sin \left( 1 - |\phi_{iL} / f_{iL} / \phi_{iL} | \right) \). By defining \( \tau_{iL} = \phi_{iL} / f_{iL} / \phi_{iL} \), the average wall velocity then reads

\[ v_{iL} = \frac{\eta \pi \gamma \pi \Delta / \Delta + H_{ADL} / (\alpha / \Delta) \cdot |\tau_{iL}|}{2\alpha} \]
with the constraint $|\tau_{i,L}| < 1$. Clearly it achieves its extremum $v_{i,L}^m = \eta\pi\Delta\gamma H_{ADL}/(2\alpha) - \beta B_J/\alpha$ when $\ln(1 - |\tau_{i,L}|) \rightarrow 0$, that is $H_L = \delta H_{i,L} \pm \Delta H_{i,L}$ with $\Delta H_{i,L} = 2|f_{i,L}|/(1 + \alpha\gamma)$. In real magnetic heterostructures, the effective damping in FM strips is enhanced from 0.001 to 0.01 to 0.2 to 0.9[33,34]. Meanwhile $\beta$ remains the order of 0.01. Consequently, the $v_{i,L} \sim H_L$ curve is a symmetric dome with respect to $H_L = \delta H_{i,L}$. In “weak SOT” limit, the two $\eta = \pm 1$ domes for $\eta_J > 0$ (thus $B_J > 0$) locates in $v < 0$ half plane. This corresponds to “20 Å Mn$_3$Sb” case in Ref.[35]. In addition, all four $v_{i,L} \sim H_L$ domes ($\eta = \pm 1$, $J_a \parallel \pm e_z$) are fully nondegenerate due to the presence of i-DMI effective field in longitudinal direction [see Fig. 2(a)]. As SOT increases, for appropriate combination of $(H_{FL}, H_{ADL})$, the $v_{i,L} \sim H_L$ domes for $\eta_J > 0$ can be reversed up to $v > 0$ half plane. The latest examples are “20 Å Mn$_3$Ge” and “10 Å Mn$_3$Sn” cases in Ref.[35].

For transverse in-plane filed ($H_{L} = H_T e_x$), the resulting $v_{i,T} \sim H_T$ curve is also dome-shaped with its center locating at $\delta H_{i,T} = 2H_K/\pi - H_{FL} - H_{ADL}/\alpha$ hosting a maximum wall velocity $-[(1 + \alpha\beta)B_J + \pi\Delta\gamma H_{i,T}]/(1 + \alpha^2)$. Following similar procedure, the average wall velocity is

$$v_{i,T} = -\frac{\beta B_J}{\alpha} - \frac{\pi\Delta\gamma H_{ADL}}{2\alpha} \left[ \frac{1}{\tau_{i,T}} + \frac{\sgn(\tau_{i,T})}{\ln(1 - |\tau_{i,T}|)} \right] + \frac{\Delta f_{i,T}}{\alpha(1 + \alpha^2)} \frac{|\tau_{i,T}|}{\ln(1 - |\tau_{i,T}|)}, \quad |\tau_{i,T}| < 1$$

with $\tau_{i,T} \equiv g_{i,T}/f_{i,T}$, $f_{i,T} = (\alpha - \beta)B_J/\Delta + \alpha\pi\gamma H_{i,T}/2$ and $g_{i,T} = \alpha\pi\gamma(H_T + H_{FL} + H_{ADL}/\alpha - 2H_K/\pi)/2$. The minimum
velocity, $-\beta B_i / \alpha - \pi \Delta H_{\text{ADL}} \text{sgn}(\eta_{i,T})/(2 \alpha)$, is achieved at $H_T = \delta H_{\text{AT}} \pm \Delta H_{\text{AT}}$ with $\Delta H_{\text{AT}} = 2 f_{\text{i,T}}/(\alpha \gamma)$. As shown in Fig. 2(b), now the center shifts of two $v_{\text{i,T}} \sim H_T$ domes ($\eta = \pm 1$) under the same current coincide due to the absence of $\mathbf{H}_1$ component in transverse direction. Physically, the center shifts of all these domes come from the total internal effective fields in the corresponding direction. When completely balanced by external in-plane fields, the wall rotates almost evenly thus reaches its extremum velocity.

Parallel analytics can be done when b-DMI dominates. The resulting dynamical equation set is the same as Eq. (5), except for the substitution “$D_i \sin \varphi = -D_b \cos \varphi$”. Accordingly, the $v_{\text{b,L(T)}} \sim H_{\text{T}}$ curves are also dome-shaped, however reverse differently to the reversal of $\eta$ and $J_{\text{b}}$ due to the different definitions of $f_{\text{b,L(T)}}$ and $g_{\text{b,L(T)}}$ [see Figs. 2(c)-2(d) and the corresponding columns in Table I]. Similarly, the center shifts of these domes stem from the total internal effective fields in the corresponding axes.

Given the results above, we propose the following procedure to simultaneously probe both DMIs in a narrow-strip shaped heterostructure:

1. Prepare quasi 1D DWs in FM layer with different topological charge ($\eta = \pm 1$).
2. Apply a strong enough (exceeding Walker limit) in-plane current $J_a$ along $\mathbf{e}_z$. For each $\eta$, the dependence of wall drifting velocity on $H_L = H_{\text{T}}$ is measured. Reverse the current direction with unchanged strength and repeat the measurements. Then four $v \sim H_{\text{T}}$ domes ($\eta = \pm 1, J_{\text{a}} \parallel \pm e_z$) are obtained.
3. Repeat the measurements in steps (C2) for $H_L = H_{\text{T}}$ to obtain another four $v \sim H_{\text{T}}$ domes.
4. For fixed $J_a$, if the center shifts $\delta H_{\text{L}}$ of $v \sim H_{\text{T}}$ domes split when $\eta = +1 \rightarrow -1$, then $\text{i-DMI}$ exists with strength $D_i = \mu_0 M_s \delta_{\text{L}} \cdot [(\delta H_{\text{L}})_{\eta=+1} - (\delta H_{\text{L}})_{\eta=-1}] / 2$. The justification of using static DW width $\Delta_0$ instead of $\Delta$ is similar to Appendix B of Ref. [36].
5. For fixed $J_a$, if the center shifts $\delta H_{\text{L}}$ of $v \sim H_{\text{T}}$ domes split when $\eta = +1 \rightarrow -1$, then b-DMI exists with strength $D_b = \mu_0 M_s \delta_{\text{L}} \cdot [(\delta H_{\text{L}})_{\eta=+1} - (\delta H_{\text{L}})_{\eta=-1}] / 2$.

The above procedure and related physics can be perfectly applied to the newly released experimental data in unit-cell-thick perpendicularly magnetized Heusler films [35]. First, for all three materials ($\text{Mn}_3 \text{Ge}$, $\text{Mn}_3 \text{Sn}$ and $\text{Mn}_2 \text{Sn}$) therein, center shifts of $v \sim H_{\text{T}}$ domes split when $\eta = +1 \rightarrow -1$, declaring the existence of finite i-DMI. Meantimes, all center shifts of $v \sim H_{\text{T}}$ domes coincide for $\eta = \pm 1$ under the fixed current, thus excludes the possibility of finite b-DMI. In addition, from the center shifts of $v \sim H_{\text{T}}$ domes and wall widths obtained already, i-DMI strengths for 20 Å $\text{Mn}_3 \text{Ge}$, 10 Å $\text{Mn}_3 \text{Sn}$ and 20 Å $\text{Mn}_2 \text{Sn}$ films are estimated as $-0.5 \text{ mJ/m}^2$, $12.5 \text{ mJ/m}^2$, and $-2.88 \text{ mJ/m}^2$, respectively. Second, for $J_a \parallel e_z$ and $\eta = +1$, the $v \sim H_{\text{T}}$ domes for 20 Å $\text{Mn}_3 \text{Sn}$ lie in $v < 0$ half plane, while those for 20 Å $\text{Mn}_3 \text{Ge}$ and 10 Å $\text{Mn}_3 \text{Sn}$ lie in $v > 0$ half plane. This indicates that SOTs in the latter two materials are stronger than that in the former, so that the $v \sim H_{\text{T}}$ domes are reversed up. Third, the original exclusion of i-DMI in that work by the unchanged center shifts of $v \sim H_{\text{T}}$ domes for 10 Å $\text{Mn}_3 \text{Sn}$ with additional CoGa overlayer capped is questionable. Indeed, i-DMI describes the exchange interaction between magnetization in Heusler films intermediated by heavy-metal atoms in Ta substrates. Thus it should not be affected too much when the CoGa overlayer is added. On the other hand, we cautiously assume that the wall width does not vary much after CoGa is capped. Therefore the nearly unchanged $\delta H_{\text{L}}$ is understandable. At last, the shrink of wall velocity can be attributed to further shunting of total current by additional layers.

IV. FIELD-DRIVEN $v_{\text{DW}} \sim H_{\text{L}}$ CANYONS

Except for the current-driven scheme in the above section, the field-driven counterpart can also be proposed. Now the motion of chiral DWs is induced by pure out-of-plane field $H_{\text{L}}$, thus $B_j, H_{\text{IL}}$ and $H_{\text{ADL}}$ are all absent. For i-DMI, the closed equation set turns to

$$\dot{q} = \eta \gamma \beta \Delta / 2 \frac{2}{1 + \alpha^2} \left[ 2 \alpha \pi H_{\text{L}} \sin(\varphi - \varphi_{\text{L}}) - \eta H_{\text{L}} \sin \varphi + H_{\text{K}} \sin 2 \varphi \right]$$

$$\dot{\varphi} = \alpha \gamma \beta / 2 \frac{2}{1 + \alpha^2} \left[ \frac{2}{\alpha \pi H_{\text{L}} \sin(\varphi - \varphi_{\text{L}})} + \eta H_{\text{L}} \sin \varphi - H_{\text{K}} \sin 2 \varphi \right], \quad (8)$$

$$\Delta = \frac{6 \gamma_0}{\alpha \pi} \left( 2 \alpha M_s \Delta_2 - \frac{M_s}{\pi} (k_{\text{L}} + k_{\text{H}} \sin^2 \varphi) \right)$$

$$+ H_{\text{L}}(\cos(\varphi - \varphi_{\text{L}})).$$

For longitudinal in-plane fields ($H_{\text{L}} = H_{\text{T}} e_z$), the second equation in (8) turns to $(1 + \alpha^2) \dot{\phi} = \tilde{f}_{\text{L}} - \tilde{g}_{\text{L}} \cdot \varphi$, with $\tilde{f}_{\text{L}} = \gamma H_{\text{L}}$ and $\tilde{g}_{\text{L}} = \alpha \pi \gamma (H_{\text{L}} - \eta H_{\text{T}} - 2 H_{\text{K}} / \pi) / 2$. The average wall velocity then reads

$$v_{\text{L}} = \eta \frac{\Delta}{\alpha} \gamma H_{\text{L}} + \eta \frac{\Delta \tilde{f}_{\text{L}}}{\alpha(1 + \alpha^2)} \ln \left( \frac{|\tilde{g}_{\text{L}}|}{|\tilde{f}_{\text{L}}|} \right). \quad (9)$$

Generally $\alpha < 1$, thus the $v_{\text{L}} \sim H_{\text{L}}$ curve is canyon-shaped, as shown in Fig. 3(a). Its minimum, $\eta \alpha \gamma \beta H_{\text{L}} / (1 + \alpha^2)$, locates at $H_{\text{L}} = \delta H_{\text{L}} = \eta H_{\text{T}} - 2 H_{\text{K}} / \pi$. While at $H_{\text{L}} = \delta H_{\text{L}} \pm \Delta H_{\text{L}}$ with $\Delta H_{\text{L}} = 2 |\tilde{f}_{\text{L}}| / (\alpha \gamma)$ the velocity reaches its maximum $\eta \Delta \gamma H_{\text{L}} / \alpha$. Parallel deductions are performed for other three cases and the resulting $v_{\text{L}} \sim H_{\text{T}}$ and $v_{\text{b,L(T)}} \sim H_{\text{L}}(\text{T})$ curves take the similar form as in Eq. (9) thus are also canyon-shaped [see Figs. 3(b)-3(d)]. The corresponding center shifts and half width are listed in Table II.

Based on these results, similar procedure of probing DMIs in narrow heterostructure strips using out-of-plane fields can be proposed:

1. Prepare quasi 1D DWs in FM layer with different topological charge ($\eta = \pm 1$).
2. Apply a strong enough (exceeding Walker limit) out-of-plane field $H_{\text{T}}$. For each $\eta$, the dependence of wall drifting
velocity on $H_L$ is measured. Reverse the direction of $H_L$ with unchanged strength and repeat the measurements. Then four $v \sim H_L$ canyons ($\eta = \pm 1$, $H_L \parallel \pm e_z$) are obtained.

(F3) Repeat the measurements in steps (F2) for $H_T$ to obtain another four $v \sim H_T$ canyons.

(F4) For fixed $H_L$, if the center shifts $\delta H_L$ of $v \sim H_L$ canyons split when $\eta = +1 \rightarrow -1$, then i-DMI exists with strength $D_i = \mu_0 M_s \Delta \eta = [\delta H_L]_{\eta = +1} - (\delta H_L)_{\eta = -1}/2$.

(F5) For fixed $H_L$, if the center shifts $\delta H_T$ of $v \sim H_T$ canyons split when $\eta = +1 \rightarrow -1$, then b-DMI exists with strength $D_b = \mu_0 M_s \Delta \eta = [\delta H_T]_{\eta = +1} - (\delta H_T)_{\eta = -1}/2$.

The above discussion lays the foundation of the extraction operations of i-DMI coefficient from precessional-flow canyons under longitudinal in-plane fields, for example in Pt/Co/AlOx heterostructures by Thiaville and Pizzini et al. in 2016[34, 36]. However since they did not provide wall velocities under transverse in-plane fields, the existence of b-DMI can not be determined.

V. DISCUSSION

Before the end of this paper, several points need to be clarified. First, in the “current-driven” scheme, the dome center shifts are independent on the real current density $j_T$ flowing through the FM layers of magnetic heterostructures which is generally hard to directly measure. This provide the universality of this scheme in determining the nature and strength of DMIs, since it does not mix the intrinsic properties and external stimuli together.

Second, the strong in-plane current density and/or out-of-plane magnetic fields overcomes the pinning process and makes the precessional-flow mode of chiral DWs in longitudinal direction hardly affected by the stochastic fields originated from impurities and disorder in magnetic heterostructures. Also the relatively large wall velocity makes the experimental observation easier thus improve the data accuracy. These are the extra advantages of our schemes except for their intrinsic universality.

Third, our theory holds under the assumption that $|g_{(b),L(T)}/f_{(b),L(T)}| < 1$, or equivalently not too far away from the dome summits or canyon bottoms. Therefore, it can not explain the further evolution of wall velocity when in-plane fields go further beyond the half width $\Delta H_{(b),L(T)}$. Fortunately, the probing procedures of both DMIs [(C1)-(C5) or (F1)-(F5)] only depend on the position of dome summits or canyon bottoms, which makes our scheme universal. Also, our theory holds for large enough in-plane currents or out-of-plane fields since now DWs precess almost evenly thus our theory holds for large enough in-plane currents or out-of-plane fields.

At last, in our theory “$q - \varphi - \Delta$” model[29, 30] is adopted. For ideal narrow-strip shaped heterostructures when considering the DMI-induced wall tiling $\chi[28]$ and canting $\theta_c[37]$ in domains from in-plane fields, a more complicated wall ansatz

$$\tan \phi = \frac{e^R + \tan(\theta_c/2)}{1 + e^R \tan(\theta_c/2)}, \quad \phi = \varphi(t)$$

\[\text{(10)}\]
can be proposed with $R \equiv \eta (x - q) \cos \chi + y \sin \chi / \Delta$. By integrating the resulting dynamical equations over strip surface in $xy-$plane, alternative Lagrangian-based collective coordinate models, such as the “$q - \varphi - \chi$”[28] or “$q - \varphi - \chi - \Delta$”[38, 39] models, emerge. However they are too complicated to provide clear criteria in constructing operable procedures and explaining experimental data. Generally in analyzing the position and shape of $v \sim H_\perp$ curves, the “$q - \varphi - \chi - \Delta$” model is enough. In addition, for real wider heterostructures with disorder, the walls take complex meander shape with its magnetization vector rotating several times along the wall and thus show unconscious tilting $\chi$[33, 34, 36]. This leads to negligible longitudinal (transverse) component of $\mathbf{H}_{\text{m}}(z)$ which is proportional to $\nabla m_z$, hence explains the feasibility of procedures (C1)-(C5) and (F1)-(F5) for extracting both DMIs in not-too-thin heterostructures.

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