ACCRETION MODES IN COLLAPSARS - PROSPECTS FOR GRB PRODUCTION

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ABSTRACT

We explore low angular momentum accretion flows onto black holes formed after the collapse of massive stellar cores. In particular, we consider the state of the gas falling quasi-spherically onto stellar-mass black holes in the hypercritical regime, where the accretion rates are in the range $10^{-3} \lesssim \dot{M} \lesssim 0.5 M_{\odot}$ s$^{-1}$ and neutrinos dominate the cooling. Previous studies have assumed that in order to have a black hole switch to a luminous state, the condition $l \gtrsim r_g c$, where $l$ is the specific orbital angular momentum of the infalling gas and $r_g$ is the Schwarzschild radius, needs to be fulfilled. We argue that flows in hyperaccreting, stellar mass disks around black holes are likely to transition to a highly radiative state when their angular momentum is just above the threshold for disk formation, $l \sim 2 r_g c$. In a range $r_g c < l < 2 r_g c$, a dwarf disk forms in which gas spirals rapidly into the black hole due to general relativistic effects, without any help from horizontal viscous stresses. For high rotation rates $l \gtrsim 2 r_g c$, the luminosity is supplied by large, hot equatorial bubbles around the black hole. The highest neutrino luminosities are obtained for $l \approx 2 r_g c$, and this value of angular momentum also produces the most energetic neutrinos, and thus also the highest energy deposition rates. Given the range of $l$ explored in this work, we argue that, as long as $l \gtrsim 2 r_g c$, low angular momentum cores may in fact be better suited for producing neutrino-driven explosions following core collapse in supernovae and γ-ray bursts.

Subject headings: accretion, accretion disks — dense matter — hydrodynamics — gamma rays: bursts — supernovae: general

1. INTRODUCTION

The collapse of massive cores in evolved stars is clearly one of the most energetic events in astrophysics, producing observable electromagnetic, neutrino, and, in all likelihood, gravitational signatures. When it was discovered that the accretion shock launched from the proto–neutron star after core bounce would stall under a wide range of conditions, energy transfer from the proto–NS to the outer regions through neutrinos was invoked as a possible mechanism to re–energize the shock wave and explode the star (Bethe & Wilson 1985). One–dimensional (spherically symmetric) models including the relevant microphysics and detailed neutrino processes have consistently failed to produce successful supernova explosions (Rampp & Janka 2000; Liebendörfer et al. 2001; Thompson, Burrows & Pinto 2003; Burrows & Thompson 2003). This has led to the exploration of other possible channels and physical mechanisms to transfer the energy to the shock and power the explosion.

The rotation and magnetic field of the progenitor and the newborn compact object are important ingredients for a global understanding the explosion mechanism (LeBlanc & Wilson 1970; Wheeler, Meier & Wilson 2002; Akiyama et al. 2003; Ardavan, Bisnovatyi–Kogan & Moiseenko 2003; Thompson, Quataert & Burrows 2005; Wilson, Mathews & Dallid 2005) and the subsequent evolution of the system, as is convection (Janka & Müller 1990). Recent studies have addressed rotation in two (Fryer & Heger 2000) and three dimensions (Fryer & Warren 2004; Janka et al. 2003), although a definitive conclusion is still not available. Any of these effects may enhance the effective neutrino luminosity in the inner regions and thus power the explosion. Implications range from the success or failure of the explosion itself, the possible production of a classical γ–ray burst, the imparted kick to the newborn neutron star or black hole, and the generation of a strong gravitational wave signal.

Unfortunately, determining the precise manner in which a star is rotating at core collapse is extremely complicated, depending upon evolutionary details such as mass loss from the main-sequence star and the configuration of the magnetic field, to name but two issues (Spruit 2002; Heger et al. 2004) which lead to substantial core spin–down in late evolutionary phases. Binary interactions may also clearly affect the angular momentum of a star prior to core collapse, through spin–up torques by tidal interactions.

In the context of GRBs, the collapsar model (Woosley 1993) invokes the formation of a massive accretion disk around a new–born black hole through fall–back accretion after core collapse. The energy release associated with the huge accretion rates (of order 0.1 solar masses per second) may be sufficient, if focused along the rotation axis of the star, to

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produce a GRB. The angular momentum of the infalling gas is an important parameter in this case, since it determines if a disk will form (if there is no rotation only Bondi–like accretion will ensue), as well as its dimensions. The accretion disk is likely to be small enough that general relativity will play an important role in its inner regions, allowing accretion even for finite values of angular momentum, as will be discussed below. A critical concern when assessing the viability of progenitors for the production of GRBs has thus been their rotation rate, as can best be determined from stellar evolution considerations. We note here that Beloborodov & Illarionov (2001) and Yoon & Langer (2005) have recently pointed out a new evolutionary channel for very massive stars, in which mass and associated angular momentum losses are greatly reduced by thorough mixing on the main sequence and the subsequent avoidance of the giant phase. Previous work in collapsar models and the associated neutrino–cooled accretion flows (MacFadyen & Woosley 1999; Poplam, Woosley & Fryer 1999; Narayan, Piran & Kumar 2001) has generally considered relatively high rotation rates and associated angular momentum values, typically with \( l \approx 6r_c^2c \). With these values large, centrifugally supported disks several hundred kilometers across promptly form around the newborn black hole.\(^1\) In this paper we consider a wide range of rotation rates, covering cores with near radial inflow to values slightly below those usually considered in collapsar calculations. In the low-angular momentum limit, we find that considerable energy release is still possible, and could contribute significantly to the energy release available for a GRB. Some elements of this picture have been considered in a quite different context before, namely High–Mass X–ray Binaries, and we draw some analogy and useful comparisons from these studies. In §2 we detail the consequences of angular momentum on the flow of collisionless matter, and note the regime where hydrodynamics comes into play. A presentation of the different accretion modes that occur, depending on whether angular momentum transport plays a role or not is made in §3. We conclude in §4 with a discussion on the prospects for GRB production from such systems.

2. THE CONSEQUENCES OF ANGULAR MOMENTUM

Before proceeding to the particular results of this set of calculations, relevant for collapsars, this section is devoted to more general considerations. The reader may wish to also consult the clear introductory exposition by Beloborodov & Illarionov (2001).

2.1. Flow lines for ballistic parabolic motion

Consider purely ballistic motion of collisionless matter with zero energy in the gravitational potential well of a central mass, \( M \). In Newtonian theory, a test mass will follow a parabolic trajectory, uniquely determined by its angular momentum, \( l \). If the central mass is a mathematical point, this will occur for any value of \( l \). In a realistic astrophysical situation, the mass \( M \) has a finite radius \( R \), so capture orbits exist whenever the periastron distance is smaller than \( R \). One may term accretion in such a situation “direct accretion”, or accretion by capture. The effective cross section of the mass \( M \) for this capture, \( \sigma_{\text{Newt}} = \pi R^2(1 + 2GM/v^2_{\infty}R) \), where \( v_{\infty} \) is the particle’s velocity far from the mass \( M \), is larger than the physical cross section, \( \sigma = \pi R^2 \), because of gravitational focusing (Shapiro & Teukolsky 1983).

In general relativity (GR), there is an additional effect, because the centrifugal barrier disappears even for finite \( l \). Any particle with enough energy at a given angular momentum will inevitably fall onto the central mass. For definiteness, if \( l = 2r_c^2c \), the effective potential exhibits a local maximum at \( r = 4GM/c^2 \) (where an unstable, marginally bound circular orbit is possible). A particle in this situation with energy \( E > 0 \) may thus directly accrete. Capture orbits are thus of a different nature than in Newtonian theory, and the effective cross section for this to occur may be written as \( \sigma_{\text{GR}} = 4\pi(2M)^2/v^2_{\infty} \), which corresponds to a critical angular momentum \( \ell_{\text{crit}} = 4GM/c \).

Consider now a situation in which a rotating cloud of particles, with angular momentum about the \( z \)-axis, enshrouds a central black hole of mass \( M \). For vanishing particle energy, these will be in near-free-fall and follow approximately parabolic orbits. For a monotonically increasing distribution of \( l \) with the polar angle \( \theta \), particles closer to the poles will fall more easily than those along the equator. We may divide the trajectories into three categories, shown in Figure 1: (a) those that accrete directly; (b) those that would accrete onto the black hole directly, but cross the equatorial plane, \( z = 0 \), before they are able to do so; and (c) those that encounter their circularization radius (where the centrifugal force balances gravity) on the equatorial plane and will not accrete directly if the equatorial angular momentum is high enough. Lines of type (b) coming from one hemisphere in fact encounter an approaching line from the opposite direction. If the energy in vertical motion is dissipated efficiently (through radiation) a thin disk will form. The material in this disk does not have sufficient angular momentum to remain in orbit, and will thus fall onto the central mass in a short timescale, even in the absence of any mechanism that transports angular momentum. This particular scenario, with Compton cooling in mind, was applied by Beloborodov & Illarionov (2001) in the context of high–mass X–ray binaries (HMXBs). That the flow lines corresponding to ballistic motion do indeed cross obviously indicates that a shock may form, depending on the local sound speed and fluid velocity, and that a full analysis requires the inclusion of hydrodynamical effects.

2.2. Hydrodynamical effects

Initially, a shock may form in the equatorial plane, \( z = 0 \), and the energy in vertical motion will be transformed into internal energy. If cooling occurs, some of this energy will be lost from the system (through photons or neutrinos, depending on the physical conditions). If the cooling rate \( \dot{q} \) is able to balance the energy input behind the shock front,

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\(^1\) The production of magnetically driven outflows has been considered as well, see Proga et al. (2003).
the disk will remain geometrically thin, with a scale height \( H \ll r \). The fluid will then orbit the central mass until it reaches a circular orbit corresponding to its angular momentum. In Newtonian theory, this is the circularization radius \( r_\text{c} = l^2/GM \). Thereafter, if angular momentum is removed, it may slowly accrete onto the central mass through a sequence of quasi–Keplerian orbits. In the case of a relativistic potential, as considered here, only the orbits of type (c) described in \( \S 2.1 \) will be like this. Type (b) trajectories will remain in the equatorial plane after passing through the shock front, but the fluid along them will never find a proper circularization radius, because of its low value of angular momentum. This will form the fast, inviscid and near free–fall disk envisaged by Beloborodov & Ilarionov (2001).

If the cooling mechanism is not so efficient, and is unable to dissipate all the internal energy generated at the shock, a hot toroidal bubble will form and grow in the equatorial plane. Its size and stability depend on several factors. The first of these is simply the total energy input, fixed by the accretion rate at the outer boundary. The second is the cooling rate, given by the relevant electromagnetic and neutrino processes (in the context of collapsars only neutrino cooling plays a role). If the cooling rate does not drop precipitously with decreasing density or temperature, the bubble may eventually stabilize and reach a stationary configuration. A larger bubble offers a larger volume within which to dissipate the internal energy produced at the shock. The third is the geometry of injection at the outer boundary, which directly affects the flow. For example, if the injection is restricted to the equatorial plane (Chen et al. 1997), polar outflows may appear, which transport energy from the equatorial regions to higher latitudes. Finally, the efficiency of angular momentum transport may cause enough advection of matter and its accompanying internal energy so as to stabilize the flow, or at least keep the bubble below a certain maximum size, even if it is variable on short timescales. For the case of inviscid hydrodynamic accretion, Proga & Begelman (2003a) have showed that the formation of a hot torus directly affects the accretion rate onto the central object. The fluid that would otherwise reach the equator at large radii is deflected by the torus into a narrower funnel close to the rotation axis.

Considering these further complexities, namely: (i) “partial” cooling, in which not all the internal energy generated at the shock is removed from the system and (ii) a finite optical depth, which acts in a similar way, it is clear that a full analytic solution cannot be obtained, and that numerical calculations are necessary to study this scenario.

### 2.3. Input physics and initial conditions

We believe the following assumptions will not alter the significance of our overall results. We assume azimuthal symmetry and perform our calculations in cylindrical \((r,z)\) coordinates. This allows for greater spatial resolution than a 3D calculation and a solid discussion of angular momentum effects. We do not assume reflection symmetry with respect to the equatorial plane. We take the fall back fluid to be in free–fall (i.e. on parabolic orbits) in the central potential well of the black hole and superimpose on it an angular momentum distribution corresponding to rigid–body rotation, \( l = l_0 \sin^2 \theta \), where \( \theta \) is the polar angle, measured from the rotation axis. \( l_0 \) is small compared to the local Keplerian value needed to maintain a circular orbit, so it is nearly in free–fall. Capture orbits are an essential ingredient in this model, so we use the formula of Paczynski & Wiita (1980) for the gravitational potential of the black hole, \( \Phi(r) = -GM_{BH}/(r-r_g) \), where \( r_g = 2GM_{BH}/c^2 \) is the Schwarzschild radius.

This choice of matter and angular momentum distributions simplifies the study of the problem, and its application to accretion following core collapse, detailed below in \( \S 3 \). In a more detailed treatment one could consider realistic stellar profiles in pre–supernova cores (although these are derived in one dimension, and mapping them to two or three dimensional dynamical studies is not a trivial matter). The details of energy release, however, and the driving of potential outflows are determined by processes in the innermost, shocked regions of the flow, and are thus relatively insensitive to the imposed external density profile. Furthermore, during the relatively short timescales explored here, the variation of these profiles may be assumed to remain constant with reasonable accuracy.

We consider an equation of state with contributions from radiation, non–degenerate relativistic electron–positron pairs and non–degenerate \( \alpha \) particles and free nucleons. For the latter, we assume nuclear statistical equilibrium and calculate the mass fraction of photodisintegrated nuclei as (Qian & Woosley 1996):

\[
X_{\text{nuc}} = 22.4 \left( \frac{\rho [\text{g cm}^{-3}]}{10^{10}} \right)^{-3/4} \left( \frac{T[K]}{10^{10}} \right)^{9/8} \exp(-8.2 \times 10^{10}/T[K]).
\]

If this expression results in \( X_{\text{nuc}} \geq 1 \) we set \( X_{\text{nuc}} = 1 \). The pressure is given by:

\[
P = \frac{11}{12} a T^4 + \frac{1 + 3X_{\text{nuc}}}{4} \frac{\rho kT}{m_\alpha},
\]

where all the symbols have their usual meanings. We have assumed for simplicity that the electron fraction is constant throughout, with \( Y_e = 0.5 \), i.e., there is one neutron for every proton. Clearly for very high densities this is not the case, as neutronization occurs and lowers \( Y_e \), reaching \( Y_e \approx 0.3 \) for \( \rho \approx 10^{10} \text{g cm}^{-3} \). This has two direct effects: (i) the contribution to the pressure from electrons is altered, and (ii) cooling through \( \epsilon_\nu \) capture onto free nucleons is modified. In the range of densities and temperatures explored here, however, both of these effects are quite small and neglecting them does not alter the essence of our results (see, however, Lee, Ramirez Ruiz & Page 2002 for a detailed dynamical calculation which does consider these effects in the context of post–merger accretion disks).

The gas is opaque to electromagnetic radiation, and the main source of cooling (other than advection into the black hole) is neutrino emission. Given the high temperatures and the degree of photodisintegration, the dominant terms...
arise from $e^\pm$ annihilation and $e^\pm$ capture by free nucleons. The corresponding cooling rates (consistently with the equation of state) are then (Kohri & Mineshige 2002, Itoh et al. 1988):

$$\dot{q}_{\text{cap}} = X_{\text{nuc}} 9.2 \times 10^{33} \left(\frac{T[K]}{10^{11}}\right)^6 \frac{\rho[g cm^{-3}]}{10^{10}} \text{erg cm}^{-3} \text{s}^{-1},$$

and

$$\dot{q}_{\text{pair}} = 4.8 \times 10^{33} \left(\frac{T[K]}{10^{11}}\right)^9 \text{erg cm}^{-3} \text{s}^{-1}.$$

Nucleon–nucleon bremsstrahlung neutrino emission is also included in the code, but is insignificant compared with the other two. Photodisintegration of $\alpha$ particles is also taken into account in the energy equation (see Lee, Ramirez–Ruiz & Page 2005). The densities and temperatures do not rise enough for the gas to be optically thick to neutrinos, i.e. $\tau_\nu < 1$ always (this is estimated by considering coherent scattering off free nucleons and $\alpha$ particles, as well as absorption by free nucleons). Thus all the energy in the form of neutrinos is lost immediately, and the cooling is extremely efficient in this respect. The hydrodynamics is followed using a 2D Smooth Particle Hydrodynamics code (Lee & Ramirez–Ruiz 2002; Lee, Ramirez–Ruiz & Page 2004), which includes all the terms from the viscous stress tensor and uses an $\alpha$–prescription for the magnitude of the viscosity, $\nu = c_s^2 \alpha / \Omega_k$ ($c_s$ is the local sound speed and $\Omega_k$ is the Keplerian angular velocity).

In reality, neutrinos, once emitted, are capable of transporting energy in the flow and depositing it through the inverse of the processes just described. At the low optical depths in fact encountered in our calculations, this will alter the internal energy only in the innermost regions of the flow, and by less than approximately one per cent. This energy deposition may be quite relevant in the production and driving of outflows, but its effect on the overall accretion flow is small enough that we neglect it in a first approximation.

3. ACCRETION MORPHOLOGIES AND LUMINOSITIES

We now address the particular case of accretion following core collapse in massive stars within the context outlined in §2. The relevant parameters for a calculation are the equatorial angular momentum, $l_0$, the viscosity $\alpha$, the accretion rate at the outer boundary, $M$, and the location of the boundary itself, typically at 50$r_g$ (although for the highest values of angular momentum we explored it was placed at 80$r_g$). We have concentrated on variations in the $l_0 - \alpha$ plane for various values of $M$. For all the calculations presented here, the central black hole is assumed to contain $M_{\text{BH}} = 4M_\odot$. At a radius $r = r_{\text{in}}$, the fluid is removed from the calculation and its mass is added to that of the black hole. Since we wish to resolve the accretion flow in the region where the effects of General Relativity become important, this inner boundary is necessarily very close to the Schwarzschild radius, and typically $r_{\text{in}} = 1.5 - 2r_g$. This is much less than the value employed by Mackay & Woosley (1995) ($r_{\text{in}} = 50$ km, or about 6$r_g$ for $M_{\text{BH}} \approx 3M_\odot$), which allowed them to follow the calculations for over 10 seconds, while we have computed the evolution for a few tenths of one second. Given these timescales and the characteristics of the late–time launching of winds from the accretion disks in their calculations (speeds of order 20,000 km s$^{-1}$ over distances of $\approx 10,000$ km), we do not expect such outflows to occur in the present set of simulations.

3.1. Low angular momentum – Dwarf Disks

If the angular momentum of the infalling gas is low, a substantial fraction of the fluid is accreted directly onto the black hole, and the rest produces an equatorial shock, where the energy in vertical motion goes into thermal energy. It is efficiently radiated away in neutrinos (recall that the densities are not high enough for neutrino opacities to play any role), and a thin, dwarf disk forms. The actual simulation looks very much like the analytical streamlines for collisionless matter plotted in Figure 1. After an initial transient lasting $\approx 200r_g/c \approx 6$ ms (corresponding to the free–fall time of the material that is close to the black hole), the system reaches a stationary state, in which as much energy is dissipated in the shock as is radiated in neutrinos.

The steady, low–angular momentum flow configuration is insensitive to the actual value of the viscosity parameter that is used. Calculations with and without viscosity ($\alpha = 0.1$ and $\alpha = 0$) for $l_0 = 1.9r_g c$ result in an identical structure, shown in Figure 2a. The system is in the inviscid regime, in which accretion onto the central mass is driven by general relativistic dynamical effects, and not by the transport of angular momentum. In the Newtonian regime this solution is non–existent because of the lack of capture orbits for finite $l$. Results of a test calculation with the same initial conditions as just described, but in a background Newtonian potential, where $\Phi(r) = \Phi_N(r) = -GM_{\text{BH}}/r$ are displayed in Figure 2b. As expected, the infalling gas is perfectly able to settle at the circularization radius and the dwarf disk never forms. Instead a shocked, hot toroidal bubble is created, akin to those found in high–angular momentum calculations described below in §3.2.

The neutrino luminosity as a function of time is fairly constant, although fast variations at small amplitude are apparent. Part of this variability is of numerical origin, because the equatorial dwarf disk is so thin that it is difficult to resolve adequately (the noise level decreases if the resolution of the calculation is increased). The density in this type of flow is essentially given by the free–fall type conditions and mass conservation through the continuity equation, as

$$\rho \approx \frac{\dot{M}}{4\pi r^2 v_r} = \frac{\dot{M}}{4\pi r_g^2 c} \left(\frac{r}{r_g}\right)^{-3/2} = 2 \times 10^9 \left(\frac{\dot{M}}{0.5M_\odot \text{s}^{-1}}\right) \left(\frac{r}{r_g}\right)^{-3/2} \text{g cm}^{-3}$$

(5)
for $M_{\text{BH}} = 4M_\odot$. The crucial difference from spheroidal Bondi-type accretion is in the formation of the equatorial shock because of the finite, albeit small, value of angular momentum, which raises the temperature to $\approx 3 - 4$ MeV. Thermal effects dominate, and the mean neutrino energy is accordingly of order $E_\nu \approx 4kT$.

### 3.2. High angular momentum - Toroidal Bubbles

For high values of $l_0$, the material closest to the equator has enough angular momentum to remain in stable circular orbit around the black hole. We first consider $\dot{M} = 0.01M_\odot$ s$^{-1}$, and defer higher accretion rates to §3.3. A shock initially forms close to the circularization radius, and the energy in free–fall is transformed into internal energy. The temperature and density in the gas rise rapidly behind the shock front, to a few $\times 10^{10}$ K, and $\approx 10^{-7} - 10^{-9}$ g cm$^{-3}$, depending on the assumed value of $\dot{M}$. A hot toroidal bubble is promptly formed and intense neutrino emission takes place within it (see Figure 4). The material is rapidly photodisintegrated into nucleons and protons after passing through the shock, and $e^\pm$ annihilation and capture onto free nucleons contribute to the total neutrino luminosity. The global behavior of the accretion flow depends essentially on the efficiency of angular momentum transport, and also on the accretion rate. For high viscosities ($\alpha = 0.1$), the gas is accreted efficiently so that the growth of the bubble is slower, and the total neutrino luminosity is lower by about a factor of 2 with respect to the inviscid ($\alpha = 0$) case. This comes about because: (i) a large fraction of the bubble is hot enough to radiate copious amounts of neutrinos, and high viscosity implies a smaller radiating volume due to the efficient transport of angular momentum (see Figure 4); and (ii) more efficient angular momentum transport means that less matter accumulates in the equatorial region inside the bubble, and thus less energy is released in neutrinos. Note that the fact that a higher viscosity also implies a greater amount of dissipated energy does not reverse this trend.

The neutrino luminosity for $l_0 = 2.1r_g c$ and $\dot{M} = 0.01M_\odot$ s$^{-1}$ as a function of time is shown in Figure 5. There are large–amplitude fluctuations, due to variations in the size, shape and structure of the growing toroidal bubble. Changes by up to a factor of two occur over a background emission rate that is fairly steady, and on a timescale $\Delta t \approx r_h/c_s$, where $r_h \approx 20r_g$ is the size of the hot bubble, and $c_s$ is the sound speed. The temperature is typically $T \approx 10^{10}$ K, so this gives $\Delta t \approx 2$ ms, in rough agreement with the observed variability timescale of $\approx 150 r_g/c = 4 - 5$ ms. Most of the emission arises from the innermost equatorial regions, and it is changes in this volume that affect the total luminosity.

In the inviscid limit there is no angular momentum transport, and so a substantial fraction of the matter cannot accrete. The toroidal bubble thus grows, fluctuating on short timescales and eventually becoming as large as the computational domain (at which point the calculation is stopped). We note that outflows are ubiquitous in this case, as they are a natural way of transporting the stored energy to larger radii. A detailed analysis of their characteristics is beyond the scope of this paper and is left for future work. If the viscosity is finite, however, the bubble eventually reaches a steady state, in which the accretion by angular momentum transport is balanced by the mass and energy inflow at the outer boundary.

For a given magnitude of the viscosity, the size of the bubble depends on the equatorial angular momentum, $l_0$. The circularization radius $R_c$ for matter falling along $z = 0$ in the pseudo–Newtonian potential is determined by the condition $R_c^2/[R_c - r_g]^2 = r_0^2/GM_{\text{BH}}$, and gives a first estimate of the shock location. All other things being equal, at large angular momentum the torus will be larger, and of lower density and temperature. Despite the larger neutrino–emitting volume and due to the steep dependence of the cooling rates on the temperature, for a fixed magnitude of the viscosity, larger tori have lower luminosities.

### 3.3. Varying the accretion rate

We have considered variations in the accretion rate by computing models with $\dot{M} = 10^{-3}, 10^{-2}, 10^{-1}, 5 \times 10^{-1}$ M$_\odot$ s$^{-1}$. An increase by one order of magnitude affects the density by about the same amount, simply because of mass conservation. The temperature also rises, but only about $10^{1/4}$ per order of magnitude increase in density. Since the neutrino emission rates from pair capture and annihilation scale as $T^9$ and $\rho T^6$ respectively, the total luminosities should increase by $\approx 10^{2.5}$ and $10^{2.25}$. So the total change is by a factor $\approx 10^{5}$. Indeed this is what is seen when the actual values for $L_\nu$ are computed, so that at a fixed $l_0$, $L_\nu \propto M^{2.5}$. At very high accretion rates the pre–shock region is dense and cold enough that degeneracy effects start to become important. The post–shock region, however, is always hot enough that our assumptions concerning the equation of state are still valid.

For the highest accretion rates, $\dot{M} \approx 0.1 - 0.5$ M$_\odot$ s$^{-1}$, likely to occur in the initial stages of fallback following the formation of the black hole, the solution changes quantitatively from that described above in §3.2 because of the increased density. The annihilation of $e^\pm$ is essentially a thermal process, and as such its emissivity depends only on the temperature, while pair capture onto free nucleons involves interactions with the background fluid, making the emissivity proportional to the density. As the density increases, pair capture becomes increasingly important until it eventually dominates the total cooling rate by a factor of four at $\dot{M} = 0.5$ M$_\odot$ s$^{-1}$. This is illustrated in Figure 6, where the differential and integrated neutrino luminosities are shown for two different accretion rates at the same equatorial angular momentum (separated into the two main processes contributing to the cooling rate). The net result is that calculations without efficient transport of angular momentum reach higher densities and emissivities, and thus produce smaller equatorial bubbles than if viscosity is present (see Figure 6).

As for lower accretion rates, with finite viscosity the hot torus eventually stabilizes, with a total volume determined by the equatorial angular momentum. Figure 7 shows the radial run of several variables along $z = 0$ for such a stable configuration. The accretion shock is clearly seen as a large jump in density and temperature. Within the torus, the rotation curve is nearly Keplerian, and the radial velocity shows variations due to the stable, large scale circulation.
pattern that is established and maintained (see Figures 3 and 7). These solutions are similar to those obtained analytically by [Popham, Woosley & Fryer (1999)] and numerically by [MacFadyen & Woosley (1999)] and [Proga et al. (2003)]. In the region exterior to the shock front they differ because we have assumed pure free fall, whereas their models consider either an extended Keplerian disk or an actual infalling stellar envelope with substantial pressure support. This effective boundary condition has little effect on the structure and evolution of the inner accretion flow. Further insight can be gained about the structure of the accretion flow by plotting the density as a function of the temperature (see Figure 4). The location of the shock is clearly seen as a rapid rise in temperature and density, separating two regions in the $\rho - T$ plane. It is evident as well that most of the high-density material within the hot toroidal bubbles at high angular momentum, $l_0 \geq 2.1 r_g c$, and the entire fluid in the dwarf disks in the opposite limit, is entirely photodisintegrated (the corresponding threshold is indicated by the black continuous line in each $\rho - T$ frame of Figure 4).

The physical conditions in the dwarf disks are largely insensitive to the adopted value of $l_0$ at a fixed accretion rate, and we typically find $\rho \approx 5 \times 10^9$ g cm$^{-3}$ and $T \approx 3 - 4$ MeV for $M = 0.5$ $M_\odot$ s$^{-1}$. The total amount of mass under these conditions is, however, affected by $l_0$, and impacts upon the absolute neutrino luminosity $L_\nu$.

The hot toroidal bubbles are evident in the panels of Figure 4 in the larger area of the $\rho - T$ plane that is occupied by the fluid, since there is a considerable gradient in both temperature and density as one moves from the immediate post–shock region to the inner equatorial disk. Note how the maximum density slowly decreases as $l_0$ rises (at roughly constant temperature) as the bulk of the equatorial inflow has a larger circularization radius. This also affects the neutrino luminosity, although less dramatically than in the case of dwarf disks for a comparable fractional change in $l_0$.

Regardless of the accretion rate, when the system enters the inviscid regime in the low angular momentum limit, $L_\nu$ becomes independent of $\alpha$ and the luminosity drops rapidly. This can be seen in the large panel in Figure 4 where we plot the luminosity as a function of the equatorial angular momentum $l_0$ for inviscid and viscous calculations at high and low accretion rates. The joining of the two classes of solutions at $l_0 \approx 2 r_g c$ marks the transition to inviscid accretion and the appearance of the dwarf disk in near free–fall. For $l_0 < 2 r_g c$, $L_\nu \propto l_0^{16}$, reflecting the fact that a large fraction of the infalling material is directly accreted by the black hole, and thus the rapid shutoff as the angular momentum is decreased. In the opposite case we find $L_\nu \propto l_0^{-12}$, because larger toroidal bubbles are less dense and have lower temperatures than smaller ones. Combining all of these results, we find that the neutrino luminosity can be fitted piecewise as:

$$L_\nu \approx \begin{cases} 
8 \times 10^{30} \left( \frac{l_0}{1.9 r_g c} \right)^{16} \left( \frac{\dot{M}}{0.5 M_\odot s^{-1}} \right)^{2.5} \text{erg s}^{-1} & \text{for } l_0 \leq 2 r_g c \\
10^{52} \left( \frac{l_0}{2.2 r_g c} \right)^{-2} \left( \frac{\dot{M}}{0.5 M_\odot s^{-1}} \right)^{2.5} \text{erg s}^{-1} & \text{for } l_0 \geq 2 r_g c.
\end{cases}$$

(6)

3.4. Neutrino spectrum and energy deposition

The shifting contributions to the cooling rate from different processes as the accretion rate increases will affect the emergent neutrino spectrum. Neutrinos produced by pair annihilation have energies $E_\nu \approx 4 kT$. If degeneracy effects are important, those arising from pair capture will have energies comparable with the electron Fermi energy, and one can show that $E_\nu \approx 9 (\rho_{10} Y_e)^{1/3}$ MeV. For the models we have computed with the highest accretion rates this is indeed the case. With these two expressions we have computed the neutrino energies for models with $\dot{M} = 0.5 M_\odot$ s$^{-1}$ (see Table 1) as a function of the equatorial angular momentum, $l_0$. Altering $l_0$ clearly has an effect on the energies of the emitted neutrinos.

For GRBs, the relevant deposition process is neutrino annihilation $\nu_e + \bar{\nu}_e \rightarrow e^- + e^+$ with a corresponding deposition rate given by $Q_{\nu, \nu} = \bar{\sigma}_0 \nu_e^2 \langle E_\nu \rangle \zeta$ where $\bar{\sigma}_0 = 3K G^2 F / 4$ is the characteristic weak interaction cross section per unit energy squared, $G^2 F = 5.29 \times 10^{-44} \text{cm}^2 \text{MeV}^{-2}$ is the Fermi constant, $K$ is a phenomenological electro–weak parameter usually taken to be $0.1 - 0.2$, $A$ is the surface area of the absorbing region, and the multiplicative factor $\zeta$ takes into account the geometry of the emitting region ([Ramirez-Ruiz & Socrates, 2001]). For a fixed amount of energy release i.e., $L_\nu$, the efficiency for producing an explosion increases with $\langle E_\nu \rangle$.

3.5. Accompanying winds, supernovae and $\gamma$–ray bursts

As mentioned above (§2.3), we have not included the effects of neutrino heating in our calculations, and believe that it will not affect the behavior of the inner accretion flow. The neutrinos may, however, ablate baryonic material from the surface of the disk. The properties of such winds, arising from proto–neutron stars following core–collapse, have been investigated before in spherical symmetry ([Qian & Woosley, 1990]). This is a different environment, so we can consider it only as a very rough approximation for our case. With the typical numbers from our simulations, we find that the mass outflow rate will be approximately given by

$$M_b \sim 5 \times 10^{-4} \left( \frac{L_\nu}{10^{52} \text{erg s}^{-1}} \right)^{5/3} M_\odot \text{ s}^{-1}.$$  

Most of the ablation is due to the absorption on nucleons. These outflows are relatively slow. Even the gas that is ejected from the innermost region of the disk has a speed of only $\sim 0.1 c$; the gas that comes out from larger
If successfully shock breakout depends on delayed neutrino heating from the proto–neutron star. So perhaps one are almost certainly needed. The prompt formation of a black hole can provoke the failure a core collapse supernova, and emitted spectrum.

Such a disk is different from its standard high-

we may estimate their power as (Blandford & Payne 1982):

\[ L_{\text{MHD}} = \frac{B^2\Omega^2}{2} \approx 10^{52} \text{erg s}^{-1}, \]  

where we have assumed that the field energy is at 10% of equipartition with the internal energy, \( \rho c^2 \), and typical values at the marginally stable orbit for calculations with \( \dot{M} = 0.5 M_\odot \text{ s}^{-1} \) and \( \rho_0 \approx 2 r_c c \) (\( \rho \approx 10^{10} \text{g cm}^{-3}, kT \approx 5 \text{ MeV} \)). However, if the more massive central black hole is rapidly spinning, its larger energy reservoir is in principle extractable through MHD coupling to the disk by the Blandford & Znajek (1977) mechanism. These are clearly very rough estimates as none of these effects is explicitly included in our calculations, but it is clear that the large densities obtained even with modest values of the equatorial angular momentum allow in principle substantial magnetic fields to be anchored in the flow. Recent numerical MHD calculations assuming both adiabatic and neutrino–cooled flows in collapsing envelopes do show these general features (Proga & Begelman 2003; Mizuno et al. 2004; Proga 2005; McKinney 2005; De Villers, Staff & Ouyed 2005).

Strong winds from hypercritical accretion flows may play an important role in the production and morphology of supernovae and \( \gamma \)-ray bursts. In the first place, the corresponding energy, however transferred to the outer layers, may actually explode the star and power the supernova explosion itself (MacFadgen & Woosley 1999, MacFadgen 2003; Thompson, Quataert & Burrows 2003). Second, the toroidal geometry inherent in flows endowed with angular momentum will break the spherical symmetry, and allow for higher efficiency than purely radial inflow in terms of liberating the gravitational energy in the system. Asymmetries may be reflected in the explosion itself and its remnant or in the beaming of a possible \( \gamma \)-ray burst, and can be quantified through line profiles (Mazzali et al. 2003) and collimation of the outflow (Panaitescu & Kumar 2001). Third, if the accretion flow cools insufficiently because it becomes so dense that the internal energy is simply advected into the black hole (Lee, Ramirez–Ruiz & Page 2004; Janiuk et al. 2004; Lee, Ramirez–Ruiz & Page 2005), it is possible that strong winds may actually revive the stalled accretion shock and make it reach the stellar envelope (Kohri, Narayan & Piran 2004). The argument is analogous to that invoked for classical Advection Dominated Accretion Flows, or ADAFs (Narayan & Yi 1994), except that the processes involve neutrino, instead of radiative cooling. Finally, the nucleosynthesis in such winds has been the focus of a number of recent studies, particularly in what concerns the production of \( r \)-process elements (Pruet, Thompson & Hoffman 2004).

If any strong winds or outflows, by whatever mechanism, are eventually driven from the vicinity of the black hole or the surface of the accretion disk, these can affect the incoming flow itself, by removing both mass and angular momentum. While such feedback almost certainly occurs, and could under certain circumstances play an important role in the overall dynamics, it is outside the scope of the present paper and is left as future work.

The observable consequences are thus potentially far–reaching, and will be intimately connected with the processes occurring in the innermost regions of the flow. As we have shown here, a crucial ingredient in this respect is the angular momentum of the accreting gas, even at values that are usually considered to be too low in terms of the dynamics and the associated energy release.

4. DISCUSSION

Spherical accretion flows on to black holes have in general low radiative efficiencies, and this applies regardless of the cooling mechanism. If the flow is slightly rotating, the situation changes dramatically. Much of our effort in this work is therefore dedicated to determining the state of the gas falling quasi–spherically onto stellar–mass black holes with accretion rates in the range \( 10^{-3} \lesssim \dot{M} < 0.5 M_\odot \text{ s}^{-1} \). In this hypercritical regime, the gravitational accretion energy is carried away by neutrinos (Chevalier 1989).

Whether a disk forms or not depends on the precise value of the angular momentum, which is difficult to calculate as it depends on the physics of angular momentum transport inside the progenitor star. An approximate condition for disk formation is that the specific angular momentum of the infalling gas \( l \) exceeds \( r_c \). In most previous discussions, it has been assumed that \( l \gg r_c \) in order for a black hole to switch to a luminous state. We argue here that flows in hyperaccreting, stellar mass disks around black holes are likely to transition to a highly radiative state when their angular momentum is just above the threshold for disk formation \( l \sim 2r_c \). In this regime, a dwarf disk forms in which gas rapidly spirals into the black hole because of general relativistic effects without any help from horizontal viscous stresses. Such a disk is different from its standard high-\( l \) counterpart as regards to its dynamics, energy dissipation and emitted spectrum.

It is extremely unlikely that the progenitors of GRBs are just very massive, single WR stars. Special circumstances are almost certainly needed. The prompt formation of a black hole can provoke the failure a core collapse supernova, if successfully shock breakout depends on delayed neutrino heating from the proto–neutron star. So perhaps one
important distinction between a GRB and an ordinary supernova is whether a black hole or a neutron star is formed in the aftermath. However, not all black hole formation events can lead to a GRB: if the minimum mass of a single star that leads to the formation of a black hole is as low as $25M_{\odot}$, this would overproduce GRBs by a large factor (see Izzard et al. 2004; Podsiadlowski et al. 2004).

The most widely discussed additional element for GRB production is angular momentum: a rapidly rotating core is thought to be an essential ingredient in the collapsar model (Woosley 1993; MacFadyen & Woosley 1999). Massive stars are generally rapid rotators on the main sequence. However, there are many well-established mechanisms, such as mass loss and magnetic torques, by which they can lose a substantial amount of angular momentum during their evolution. Therefore, it is not at all clear whether the cores of massive single stars will ever be rotating rapidly at the time of explosion (e.g. Heger et al. 2003). Relaxing the need for rapid rotation may indeed increase the estimated rate for the single star channel. The specific angular momentum $l$ of the accreting core is thus a key factor. Evolutionary models (Heger et al. 2005) indicate that at $M = 4M_{\odot}$, the equatorial angular momentum of the pre-supernova core inside a star with $M = 15M_{\odot}$ is $l_{0} \approx 3 \times 10^{16} \text{ cm}^{2} \text{ s}^{-1} = 0.8r_{g}c$, if one considers the effects of magnetic torques, and $l_{0} \approx 30 \times 10^{16} \text{ cm}^{2} \text{ s}^{-1} = 8r_{g}c$ if these are ignored. For more massive stars the angular momentum is reduced even further (by approximately one order of magnitude for a star with $M = 25M_{\odot}$). Clearly, relatively low values of angular momentum may be quite common in the interior of these stars.

The simplest way to generate a high-$l$ disk ($l \gg r_{g}c$) is for the core to reside in a tight binary and be in corotation with the binary. The requirement that the total core angular momentum exceed the maximum angular momentum of a Kerr black hole places interesting limits on the binary period (Izzard et al. 2004; Podsiadlowski et al. 2004). If we make the reasonable assumption that the binary is circular, and model the core as an $n = 3$ polytrope, the binary period must be smaller than

$$P_{\text{orb}} \sim 4(M_{\text{core}}/2M_{\odot})^{-1/2}(R_{\text{core}}/10^{10} \text{ cm})^{2} \text{ h}.$$  

This orbit could be tight enough that the core may in fact have been stripped of its helium in a common envelope to form a CO core. By contrast, in most cases, the core of a massive, single star is unlikely to retain the required angular momentum as its outer hydrogen layers are blown off in a stellar wind. In this case, the accretion flows would be quasi–spherical with the binary. The requirement that the total core angular momentum exceed the maximum angular momentum places interesting limits on the binary period (Izzard et al. 2004; Podsiadlowski et al. 2004). If we make the reasonable assumption that the binary is circular, and model the core as an $n = 3$ polytrope, the binary period must be smaller than

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Fig. 1.— Flow lines for ballistic, nearly parabolic motion of particles surrounding a black hole of mass $M$ and rotating about a given axis, $z$, with angular frequency $\Omega$. It is assumed that the specific angular momentum, $l$, increases monotonically with the polar angle $\theta$, and that flow lines do not intersect before reaching the equator. The particular lines drawn here are for rigid body rotation, with $l = l_0 \sin^2 \theta$. Lines of type (a) directly impact the black hole, those of type (b) would do so as well if they did not cross the equator first, while those of type (c) have enough angular momentum to remain in equatorial orbit at a finite radius $r$. If the energy in vertical motion is dissipated efficiently, a thin inviscid disk will form in the equatorial plane, denoted by a thick line.
Fig. 2.— Logarithmic density contours and velocity field in the meridional plane for low angular momentum calculations ($l_0 = 1.9 r_g c$). The accretion rate is $\dot{M} = 0.5 \, M_{\odot} \, s^{-1}$. (a) In the pseudo–Newtonian potential of Paczyński & Wiita, the flow is steady and the character of the solution is independent of the adopted value of the viscosity parameter $\alpha$ and of the accretion rate. Only a thin dwarf equatorial disk is present at small radii. (b) In a Newtonian potential an accretion shock is formed as the gas encounters the centrifugal barrier and a hot toroidal bubble is created. The contours are evenly spaced every 0.5 dex and labeled in units of $g \, cm^{-3}$. 

(a) Paczynski–Wiita Potential

(b) Newtonian Potential
Fig. 3.— Logarithmic density contours and velocity field in the meridional plane for high angular momentum calculations ($l_0 = 2.1 r_g c$), with $\alpha = 0$ (left column) and $\alpha = 0.1$ (right column), at $t = 500 r_g / c$ (top row) and $t = 1500 r_g / c$ (bottom row). The accretion rate is $\dot{M} = 0.01 M_\odot \, \text{s}^{-1}$. In both cases the hot toroidal bubble grows continuously as the shock front moves outward, but it is clearly smaller in the high-viscosity case. The contours are evenly spaced every dex and labeled in units of $g \, \text{cm}^{-3}$. 
Fig. 4.— Neutrino luminosity per radial interval, $dL_\nu/dr$ (lower lines), in units of erg cm$^{-1}$ s$^{-1}$, as a function of radius at (a) $t = 500 r_g/c$, (b) $t = 1000 r_g/c$, and (c) $t = 1500 r_g/c$ for high-angular momentum calculations ($l = 2.1 r_g c$) without ($\alpha = 0$, solid lines) and with ($\alpha = 0.1$, dashed lines) viscosity, for $M = 0.01 M_\odot$ s$^{-1}$. The corresponding solid and dashed lines above show the integrated luminosity $L_\nu$ in erg s$^{-1}$. The outward motion of the shock is clearly visible in the sharp drop in emissivity as the calculation progresses.
Fig. 5.— Neutrino luminosity $L_{\nu}$ in erg s$^{-1}$ as a function of time for inviscid ($\alpha = 0$) runs with (a) $l_0 = 1.9 r_g c$, $\dot{M} = 0.01$ $M_\odot$ s$^{-1}$; (b) $l_0 = 2.1 r_g c$, $\dot{M} = 0.01$ $M_\odot$ s$^{-1}$; (c) $l_0 = 2.1 r_g c$, $\dot{M} = 0.5$ $M_\odot$ s$^{-1}$. For low angular momentum it is essentially steady, although strong variability around a steady average is apparent. For high angular momentum, high amplitude, quasi-periodic variations are seen as a result of large-scale variations in the size and shape of the hot toroidal bubble.
Fig. 6.— Same as Figure 4, but for $l = 2.2r_gc$ and $\alpha = 0$. Calculations with $\dot{M} = 0.01 \, M_\odot \, s^{-1}$ (top) and $\dot{M} = 0.5 \, M_\odot \, s^{-1}$ (bottom) are shown at $t = 1500 r_g c$. The solid (dashed) lines show the contribution from pair annihilation (pair capture onto free nucleons) to the total luminosity. At high accretion rates pair capture dominates the cooling and makes the hot toroidal bubble smaller.
Fig. 7.— Same as Figure 3 but for $\dot{M} = 0.5 \, M_\odot \, \text{s}^{-1}$ and $l_0 = 2.1 r_g c$. As before, the hot toroidal bubble grows as the shock front moves outward, but this time, as opposed to the case of low accretion rate, it is clearly smaller in the inviscid case. The contours are evenly spaced every dex and labeled in units of $g \, \text{cm}^{-3}$. 
Fig. 8.— Density, temperature and velocities along the equator, $z = 0$, for the calculation with $l_0 = 2.2r_g c$, $\alpha = 0.1$ and $\dot{M} = 0.5 M_\odot \, \text{s}^{-1}$ at late times, when the toroidal bubble has stabilized. In the bottom left panel, the solid (dashed) line is for the radial (azimuthal) component of the velocity. The bottom right panel shows the computed angular velocity (solid line) as well as the Keplerian solution in the Paczyński-Wiita potential for reference (dashed line).
Fig. 9.— Flow structure in the density–temperature plane for calculations with varying equatorial angular momentum $l_0$ (labeled) at $M = 0.5 \, M_\odot \, s^{-1}$ and color coded according to the volume cooling rate (in units of erg cm$^{-3}$ s$^{-1}$). The black and grey solid lines across each plot show the degeneracy ($kT = 7.7 \rho^{1/3} \, \text{MeV}$) and photodisintegration (50% of $\alpha$ particles into free nucleons by mass) thresholds respectively. The transition from a dwarf disk to a hot toroidal bubble as $l_0$ increases beyond $\approx 2.1 r_g c$ is clearly seen, as is the slight decrease in maximum density at high angular momentum. The characteristic neutrino luminosity is plotted in the large panel as a function of $l_0$ for two different mass accretion rates, and viscous ($\alpha = 0.1$, black lines) and inviscid calculations ($\alpha = 0$, grey lines). The qualitative change in the dependence of $L_\nu$ on $l_0$ occurs around $2.1 r_g c$. Note that at high accretion rates, even low angular momentum configurations ($l_0 \approx 1.8 r_g c$) are capable of releasing up to $10^{54}$ erg s$^{-1}$. 
TABLE 1

| $l_0/(r_gc)$ | $E_{\nu}^{\text{pair}}$(MeV) | $E_{\nu}^{\text{ann.}}$(MeV) | $\rho_{\text{max}}$ (g cm$^{-3}$) | $kT$(MeV) |
|--------------|-------------------------------|-------------------------------|----------------------------------|------------|
| 2.1          | 11                            | 12                            | $3 \times 10^{10}$               | 3          |
| 2.2          | 11                            | 12                            | $3 \times 10^{10}$               | 3          |
| 2.5          | 5                             | 12                            | $3 \times 10^{9}$                | 3          |
| 3.0          | 4                             | 12                            | $2 \times 10^{9}$                | 3          |

*For all the runs shown here, $\alpha = 0$ and $M = 0.5 M_\odot$ s$^{-1}$. 