The current status of rare kaon decay experiments is reviewed. New limits in the search for Lepton Flavor Violation are discussed, as are new measurements of the CKM matrix.

1 Introduction

The field of rare kaon decays has a long and rich history. As the field has evolved so has the definition of ‘rare’. In this review it refers to branching ratios less than $\sim 10^{-10}$. The two areas of greatest interest have been the very sensitive searches for physics beyond the Standard Model (SM) through lepton flavor violating (LFV) decays and the studies of the SM picture of CKM mixing and CP violation that have recently begun to come to fruition.

2 Lepton Flavor Violating Decays

There is solid experimental evidence for an additive quantum number associated with each family of charged leptons. While there is no SM mechanism for LFV (even if $m_\nu\neq0$, LFV is too small to observe in the charged lepton sector), there is no underlying gauge symmetry preserving lepton flavor. Observation of LFV would be unambiguous evidence for physics beyond the SM, and is predicted by many extensions to the SM.

Due to excellent sensitivity, the mass scale probed by rare kaon decay experiments is quite large. This can be seen by comparing the $K^0_L \to \mu e$ decay, through a hypothetical LFV vector boson with coupling $g_X$ and mass $M_X$ to the conventional $K_{\mu2}$ decay ($g$ and $M_W$); a lower limit on $M_X$ can be derived:

$$M_X \approx \frac{g_X}{g} \left(\sin^2 \theta_c\right)^{1/4} M_W \left[\frac{\Gamma(K^+ \to \mu^+ \nu)}{\Gamma(K^0_L \to \mu e)}\right]^{1/4}$$

$$\approx 200\text{TeV}/c^2 \times \frac{g_X}{g} \times \left[\frac{10^{-12}}{B(K^0_L \to \mu e)}\right]^{1/4}$$

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2.1 $K^o_L \rightarrow \mu e$

The E871 experiment at BNL has finished, with two long runs in 1995–96. No events were seen in the signal region, with an expected background of 0.1 events (see Fig. 1), and the 90% CL limit is $B(K^o_L \rightarrow \mu e) < 4.7 \times 10^{-12}$.

Figure 1. Final E871 data sample after all cuts, with no events in the signal region. The exclusion box was used to set cuts in an unbiased way on data far from the signal region. The shape of the signal box was optimized to maximize signal/background.

There are no plans to pursue this decay further; it is expected to be limited by background from $K^o_L \rightarrow \pi^\pm e^\mp \nu_e$ with a $\pi \rightarrow \mu$ decay and an electron scattering in the vacuum window or first tracking chamber at $\sim 10^{-13}$.

2.2 $K^+ \rightarrow \pi^+ \mu^+ e^-$

The E865 experiment at BNL has collected data for the decay $K^+ \rightarrow \pi^+ \mu^+ e^-$ during the 1995, 1996, and 1998 runs of the AGS. The 90% CL limit on this mode from the 1995 run, similar in sensitivity to the predecessor experiment E777, was $B(K^+ \rightarrow \pi^+ \mu^+ e^-) < 2 \times 10^{-10}$. From the 1996 run, with no events above a $\pi\mu e$ likelihood $> 20\%$, the limit is $B(K^+ \rightarrow \pi^+ \mu^+ e^-) < 4.8 \times 10^{-11}$ (see Fig. 2). Combining the results from E777 and the E865 runs in 1995 and 1996 a limit of $B(K^+ \rightarrow \pi^+ \mu^+ e^-) < 3.2 \times 10^{-11}$ is obtained. The final sensitivity, including 1998 data, is expected to be $\sim 3$ times better. There are no plans to continue with this search.
Figure 2. Final 1996 data sample after all cuts, with no events above $\pi\mu e$ likelihood of 20%. Also shown are $\pi\mu e$ Monte Carlo events passing all cuts.

2.3 $K_L^0 \to \pi^0 \mu^+\mu^-$

The current limit on $K_L^0 \to \pi^0 \mu^+\mu^-$ from E799-I at FNAL, is $B(K_L^0 \to \pi^0 \mu^+\mu^-) < 3.1 \times 10^{-9}$ (90% CL). This measurement has very small background levels, so E799-II (KTeV) will be able to substantially improve upon this limit.

3 $K_L^0 \to \mu^+\mu^-$ and other Semi-Rare Decays

Two modes for which it may be possible to extract fundamental CKM parameters, from measurement of the short distance contributions, are $K_L^0 \to \mu^+\mu^-$ and $K_L^0 \to \pi^0 \ell^+\ell^-$. The $K_L^0 \to \pi^0 \ell^+\ell^-$ modes are covered in detail by Taku Yamanaka elsewhere in this volume. The current 90% CL limit for $K_L^0 \to \pi^0 e^+e^-$ is $B(K_L^0 \to \pi^0 e^+e^-) < 5.6 \times 10^{-10}$ and the limit for $K_L^0 \to \pi^0 \mu^+\mu^-$ is $B(K_L^0 \to \pi^0 \mu^+\mu^-) < 3.4 \times 10^{-10}$. Both of these measurements have background which will slow improvement in these modes.

3.1 $K_L^0 \to \mu^+\mu^-$

The mode $K_L^0 \to \mu^+\mu^-$, which has played such an important role in the development of the SM (e.g. the GIM mechanism and the prediction of the charm quark), has now been measured to the unprecedented precision of 1.5%,
with 6200 events, by E871.

The decay $K^0_L \rightarrow \mu^+ \mu^-$ is dominated by $K^0_L \rightarrow \gamma \gamma$ with the two real photons converting to a $\mu^+ \mu^-$ pair. This contribution is precisely calculated using QED from a measurement of the $K^0_L \rightarrow \gamma \gamma$ branching ratio. There is also a long distance dispersive contribution, through off-shell photons. Most interesting is the short distance contribution, through internal quark loops, dominated by the top quark. A measurement of this short distance contribution is sensitive to the real part of the elusive $V_{td}$, and will determine $\rho$:

$$B_{SD}(K_L^0 \rightarrow \mu^+ \mu^-) = 1.51 \times 10^{-9} A^4 (\rho_0 - \bar{\rho})^2$$

with $\rho_0 = 1.2$, $\bar{\rho} = \rho (1 - \lambda^2/2)$ and $\lambda$, $A$, $\rho$ from the Wolfenstein parameterization of the CKM matrix. The current measurement of the branching ratio $B(K_L^0 \rightarrow \mu^+ \mu^-) = (7.18 \pm 0.17) \times 10^{-9}$ by the E871 collaboration represents a factor of three improvement in the uncertainty and the error on $B(K_L^0 \rightarrow \mu^+ \mu^-)$ no longer dominates the ratio:

$$\frac{\Gamma(K^0_L \rightarrow \mu \mu)}{\Gamma(K^0_L \rightarrow \gamma \gamma)} = \frac{B(K^0_L \rightarrow \mu^+ \mu^-)}{B(K^0_L \rightarrow \pi^+ \pi^-)} \frac{B(K^0_L \rightarrow \pi^0 \pi^0)}{B(K^0_L \rightarrow \gamma \gamma)} = [\frac{3.474 \pm .054}{1.004 \pm .002}[2.19 \pm .03][632 \pm .009][1.55\%][0.23\%][1.28\%][1.42\%]$$

$$= (1.213 \pm 0.030) \times 10^{-5}$$

This value is only slightly above the unitarity bound from the on-shell two photon contribution

$$\frac{\Gamma(K^0_L \rightarrow \mu^+ \mu^-)}{\Gamma(K^0_L \rightarrow \gamma \gamma)} > 1.195 \times 10^{-5}$$

and leaves very little room for the short distance contribution. Currently, with conservative estimates of the long distance dispersive contribution, a limit on $\rho$ can be extracted $\rho > -0.33$ (90% CL).

Unlike $K^0_L \rightarrow \mu^+ \mu^-$, the decay $K^0_L \rightarrow e^+ e^-$ is predominantly through two off-shell photons, so this decay is less interesting for extracting SM parameters. However, the recent observation of four events by E871 with $B(K^0_L \rightarrow e^+ e^-)$

$$= (8.7^{+5.7}_{-4.1}) \times 10^{-12}$$

is consistent with Chiral Perturbation Theory (ChPT) predictions and is the smallest branching ratio ever measured.

### 3.2 $K^0_L \rightarrow e^+ e^- \gamma$, $K^0_L \rightarrow \mu^+ \mu^- e^+ e^-$

Many new measurements of radiative kaon decays have been reported recently. In particular, new measurements of $K^0_L \rightarrow e^+ e^- \gamma$ and $K^0_L \rightarrow e^+ e^- e^+ e^-$ have been made by NA48 and KTeV. These measurements
are substantially improved over previous values and even larger improvements will be obtained when the complete data sets are analyzed. For example, the new measurement from NA48 [3] \( B(K^0_{L} \to e^+ e^- \gamma) = (1.06 \pm 0.02 \pm 0.02 \pm 0.04) \times 10^{-5} \) is \( \sim 70 \) times more sensitive than the previous measurement and KTeV will improve this by an additional factor of \( \sim 20 \) beyond NA48. These modes, which are not rare, with branching ratios from \( 10^{-8} - 10^{-5} \), may help to determine the long distance dispersive contribution to \( K^0_L \to \mu^+ \mu^- \). Additional work in ChPT is needed to extract the short distance physics.

4 \( K \to \pi \nu \bar{\nu} \)

The \( K \to \pi \nu \bar{\nu} \) modes — \( K^+ \to \pi^+ \nu \bar{\nu} \) and \( K^0_L \to \pi^0 \nu \bar{\nu} \) — are the 'golden modes' for determining the CKM parameters \( \rho \) and \( \eta \); and, along with the other golden mode \( B^0_d \to \psi K^0 \) and perhaps \( \Delta m_{B_s}/\Delta m_{B_d} \), provide the best opportunity to over-constrain the unitary triangle and to search for new physics.

The unitarity of the CKM matrix can be expressed as

\[
V_{us}^* V_{ud} + V_{cs}^* V_{cd} + V_{ts}^* V_{td} = 0
\]

with the three vectors \( V_{us}^* V_{ud} \) converging to form an elongated triangle in the complex plane. The first vector \( V_{us}^* V_{ud} \) is well determined from the decay \( K^+ \to \pi^+ e^+ \nu_e \). The height will be measured by \( K^0_L \to \pi^0 \nu \bar{\nu} \) and the third side \( V_{ts}^* V_{td} \) will be measured by \( K^+ \to \pi^+ \nu \bar{\nu} \). The \( K \to \pi \nu \bar{\nu} \) decays are sensitive to the magnitude and imaginary part of \( V_{td} \). From these two modes the unitarity triangle can be completely determined.

The theoretical uncertainty in \( K^+ \to \pi^+ \nu \bar{\nu} \) (~7\%) is small and even smaller in \( K^0_L \to \pi^0 \nu \bar{\nu} \) (~1\%); the hadronic matrix element can be extracted from the well measured \( K^+ \to \pi^+ e^+ \nu_e \) (\( K_{e3} \)) decay. The branching ratios have been calculated to next-to-leading-log approximation, complete with isospin violation corrections and two-loop-electroweak effects. Based on current understandings of SM parameters, the branching ratios can be expressed as [8]

\[
B(K^+ \to \pi^+ \nu \bar{\nu}) = \frac{\kappa_{e3} \alpha^2 B(K_{e3})}{2 \pi^2 s^2 W |V_{us}|^2} \sum_t |X_t^e(x_t) V_{ts}^* V_{td} + X_t^e(x_t) V_{cs}^* V_{cd}|^2
\]

\[
= 8.88 \times 10^{-11} A^4 \left( \langle \overline{\rho}_0 - \overline{\rho} \rangle^2 + (\sigma \overline{\eta})^2 \right)
\]

\[
= (0.82 \pm 0.32) \times 10^{-10}
\]

\[
B(K^0_L \to \pi^0 \nu \bar{\nu}) = \frac{\gamma_{e3} \alpha^2 B(K_{e3})}{2 \pi^2 s^2 W |V_{us}|^2} \sum_t |Im(V_{ts}^* V_{td}) X_t(x_t)|^2
\]

\[
= 4.08 \times 10^{-10} A^2 \eta^2
\]

\[
= (3.1 \pm 1.3) \times 10^{-11}
\]
The decay $K_L^0 \rightarrow \pi^0 \nu \bar{\nu}$ is direct CP violating, and offers the best opportunity for measuring the Jarlskog invariant $J_{CP}$, the fundamental measure of CP violation in the SM.

4.1 $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

In E787’s analysis of the 1995–97 data sample, one clean $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ event lies in the signal box (see Fig. 3), with a measured background of $0.08 \pm 0.02$ events. Based on this one event, the branching ratio is $B(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 1.5^{+3.4}_{-1.2} \times 10^{-10}$. From this measurement, a limit of $0.002 < |V_{td}| < 0.04$ is determined; in addition, the following limits on $\lambda_t \equiv V_{ts}^* V_{td}$ can be set: $|Im\lambda_t| < 1.22 \times 10^{-3}$, $-1.10 \times 10^{-3} < Re\lambda_t < 1.39 \times 10^{-3}$, and $1.07 \times 10^{-3} < |\lambda_t| < 1.39 \times 10^{-3}$. The final sensitivity of the E787 experiment, based on data from 1995–98, should reach a factor of two further, to the SM expectation for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$.

A new experiment, E949, is under construction and will run in 2001–03. Taking advantage of the entire AGS proton flux and the experience gained with the E787 detector, E949 with modest upgrades should observe 10 SM events in a two year run. The background is well-understood and is $\sim 10\%$ of the SM signal.

A proposal for a further factor of 10 improvement has been initiated at FNAL. The CKM experiment (E905) plans to collect 100 SM events, with B/S
∼ 10%, in a two year run starting in ∼2005. This experiment will use a new technique, with K$^+$ decay-in-flight and momentum/velocity spectrometers.

4.2 $K^0_L \rightarrow \pi^0 \nu \bar{\nu}$

The current best direct limit on $K^0_L \rightarrow \pi^0 \nu \bar{\nu}$ comes from the KTeV run in 1997: $B(K^0_L \rightarrow \pi^0 \nu \bar{\nu}) < 5.9 \times 10^{-7}$ (90% CL).

An even more stringent limit can be derived in a model independent way from the E787 measurement of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$:

$$B(K^0_L \rightarrow \pi^0 \nu \bar{\nu}) < 4.4 \times B(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 2.9 \times 10^{-9} \ (90\%CL) \quad (8)$$

The next generation of $K^0_L \rightarrow \pi^0 \nu \bar{\nu}$ experiments will start with E391a at KEK, which hopes to reach a sensitivity of ∼ $10^{-10}$. While this does not reach the SM level, the experiment will make a large step forward and learn more about how to do this difficult experiment. The plan would then be to move the detector to the JHF and push to a sensitivity of $O(10^{-13})$.

Two other experiments propose to reach sensitivities of $O(10^{-13})$ — E926 (KOPIO/RSVP) at BNL and E804 (KAMI) at FNAL. KAMI plans to reuse the excellent CsI calorimeter from KTeV and to operate at high kaon momentum to achieve good photon energy resolution and efficiency. The flux will increase substantially over KTeV due to the Main Injector. KOPIO follows a different strategy. The kaon center of mass will be reconstructed using a bunched proton beam and a very low momentum $K_L$ beam. This gives 2 independent criteria to reject background: photon veto and kinematics — allowing background levels to be directly measured from the data — and gives further confidence in the signal by measuring the spectra. The necessary flux will be obtained using the entire AGS proton current. The low energy beam also substantially reduces backgrounds from neutrons and other sources. After three years of running, 65 SM events will be observed with a S/B ≥ 2:1.

5 Conclusions and Future Prospects

The unprecedented sensitivities of the rare kaon decay experiments in setting limits on LFV have constrained many extensions of the SM. The discovery of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ has opened the doors to measurements of the unitarity triangle completely within the kaon system. Significant progress in the determination of the fundamental CKM parameters will come from the generation of experiments that is starting now. Comparison with the B-system will then over-constrain the triangle and test the SM explanation of CP violation.
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