Correlation between lasing and transport properties in a quantum dot-resonator system

Pei-Qing Jin$^{1}$, Michael Marthaler$^{1}$, Jared H. Cole$^{1,2}$, Maximilian Köpke$^{3}$, Jürgen Weis$^{3}$, Alexander Shnirman$^{4,5}$, and Gerd Schön$^{1,5}$

$^{1}$ Institut für Theoretische Festkörperphysik, Karlsruhe Institute of Technology, 76128 Karlsruhe, Germany
$^{2}$ Applied Physics, School of Applied Sciences, RMIT University, Melbourne 3001, Australia
$^{3}$ Max-Planck-Institut für Festkörperforschung, D-70569 Stuttgart, Germany
$^{4}$ Institut für Theorie der Kondensierten Materie, Karlsruhe Institute of Technology, 76128 Karlsruhe, Germany
$^{5}$ DFG Center for Functional Nanostructures (CFN), Karlsruhe Institute of Technology, 76128 Karlsruhe, Germany
E-mail: pei-quing.jin@kit.edu

Abstract. We study a double quantum dot system coherently coupled to an electromagnetic resonator. By suitably biasing the system, a population inversion can be created between the dot levels. The resulting lasing state exists within a narrow resonance window, where the transport current correlates with the lasing state. It allows probing the lasing state via a current measurement. Moreover, the resulting narrow current peak opens perspective for applications of the setup for high resolution measurements.

Circuit quantum electrodynamics (CQED) setups with superconducting qubits coupled to a superconducting resonator provide possibilities to explore quantum optics effects in new parameter regimes [1]. Recently lasing and cooling effects were demonstrated in such systems [2, 3, 4]. The strong coupling regime achieved in these setups revealed qualitatively novel phenomena. An example is the non-monotonous behavior of the linewidth of the emission spectrum in a single qubit maser, which is influenced by quantum noise in a characteristic way [5, 6].

Here we propose a different CQED setup where a double quantum dot is coupled to a superconducting resonator in a geometry indicated in Fig. 1. The double quantum dot is biased such that a single electron occupies the lowest empty orbital in the left or right dot. These two relevant states are denoted as pure charge states $|1,0\rangle$ and $|0,1\rangle$ with an energy difference $\epsilon$. A coherent interdot tunneling with strength $t$ is assumed to couple the two pure charge states. The resulting eigenstates are mixtures of pure charge states with mixing angle $\theta = \arctan(t/\epsilon)$. Transitions between these eigenstates are allowed due to an electrical dipole interaction with the resonator. Under the rotating wave approximation, the Hamiltonian for the coupled system in the eigenbasis of the two-level system is given by the standard Jaynes-Cumming Hamiltonian

$$H_{\text{sys}} = \frac{\hbar \omega_0}{2} \sigma_z + \hbar \omega_r a^\dagger a + \hbar g (a^\dagger \sigma_- + a \sigma_+). \quad (1)$$

Here $\omega_0 = \sqrt{\epsilon^2 + t^2}/\hbar$ and $\omega_r$ denote the frequencies of the two-level system and the resonator,
respectively, where a detuning $\Delta = \omega_0 - \omega_r$ is allowed, and $a$ ($a^\dagger$) represents the annihilation (creation) operator of photons in the resonator.

The dynamics of the coupled dot-resonator system is studied in the frame of a master equation for the reduced density matrix $\rho$. Within the conventional Born-Markovian approximation, the master equation is given by

$$\dot{\rho} = -\frac{i}{\hbar} [H_{\text{sys}}, \rho] + \sum_i \frac{\Gamma_i}{2} \left( 2L_i\rho L_i^\dagger - L_i^\dagger L_i\rho - \rho L_i^\dagger L_i \right).$$

The dissipative dynamics is described in Lindblad form with operators $L_i$. For the decay of resonator, we adopt the standard form $L_r = a$ with rate $\Gamma_r = \kappa$. For relaxation and decoherence of the two-level system, the corresponding Lindblad operators are given by $L_\downarrow = \sigma_-$ with rate $\Gamma_\downarrow$ and $L_\varphi = \sigma_z$ with rate $\Gamma_\varphi$. Throughout the paper we consider low temperatures with vanishing thermal photon number and excitation rates. The incoherent tunneling between electrodes and quantum dots are also included (see below).

To achieve the population inversion for lasing we assume the system to be biased such that only the electro-chemical potentials of the states $|1,0\rangle$ and $|0,1\rangle$ lie within the bias window. At low temperatures compared to the charging energy, an electron can only tunnel into the dot system from the left electrode to the left dot. It leads to a transition from the state $|0,0\rangle$ to $|1,0\rangle$. This process is described by a Lindblad operator $L_L = |1,0\rangle\langle 0,0|$. Similarly, an electron can tunnel out into the right lead, creating a transition from $|0,1\rangle$ to $|0,0\rangle$, which is described by $L_R = |0,0\rangle\langle 0,1|$. For simplicity, we assume these two processes having the same tunneling rate $\Gamma$. A non-equilibrium state is achieved with enhanced population in the state $|1,0\rangle$ with higher energy as compared to the state $|0,1\rangle$. This effect persists in the eigenbasis. The resulting population inversion in the absence of relaxation between dot levels, becomes

$$\tau_0 = \frac{4 \cos \theta}{3 + \cos(2\theta)}.$$  

As the system approaches the degeneracy point with $\theta = \pi/2$, the pure charge states are coupled strongly by the interdot tunneling, and the population inversion decreases.

A current flows through the double dot when the tunneling cycle $|0,0\rangle \rightarrow |1,0\rangle \rightarrow |0,1\rangle \rightarrow |0,0\rangle$ is completed. We evaluate the current using $I = e \sum_{i,j} \Gamma_{i,j} \langle i | \rho_{\text{eq}} | j \rangle$, where the index $i$ refers to the eigenstates of the two-level system as well as the state $|0,0\rangle$, and $\Gamma_{i,j}$ denotes
Figure 2. (a) Current as a function of the energy difference $\epsilon$. (b) Average photon number $\langle n \rangle$, Fano factor $F$ and transport current $I$ as functions of the detuning. In both panels, we choose the interdot tunneling strength $t = 0.3 \hbar \omega_1$, incoherent tunneling rate $\Gamma = 10^{-5} \omega_1$, decay rate $\kappa = 10^{-5} \omega_1$, relaxation and decoherence rates $\Gamma_i = \Gamma^o = 10^{-4} \omega_1$, and coupling strength $g = 3 \times 10^{-4} \omega_1$.

the transition rate from state $|i\rangle$ to $|j\rangle$. The transport current at low temperature (vanishing thermal photons) is plotted in Fig. 2 as a function of the energy difference $\epsilon$. At the degeneracy point $\epsilon = 0$, a broad peak shows up due to the coherent interdot tunneling, which width is given by the tunneling rate $t$ (here $t \gg \hbar \Gamma$) [7, 8]. With respect to the degeneracy point, the current exhibits asymmetric behavior. On the absorption side ($\epsilon < 0$), an electron that tunnels into the left dot is blocked in the state $|1, 0\rangle$ since no photon is present to provide the energy for tunneling to the right dot. Therefore on the absorption side, only the residue from the broad current peak remains. On the emission side ($\epsilon > 0$), a second current peak arises when the two-level system becomes resonant with the oscillator. For realistic values of the parameters, this current peak is much sharper. It correlates with a lasing state within a narrow resonance window with width $W \approx 2g\sqrt{\Gamma/\kappa - \Gamma}$ for small $\theta$ [9], since a photon is generated in the resonator when an electron tunnels between the two dots. Besides, the relaxation of the two-level system, which opens up an incoherent channel, increases the overall current on the emission side.

The lasing state is characterized by the average photon number $\langle n \rangle$ as well as the Fano factor $F \equiv (\langle n^2 \rangle - \langle n \rangle^2)/\langle n \rangle$. As indicated in Fig. 2, at low temperature, the photon number vanishes for large detuning, since the quantum dot system does not interact effectively with the resonator. The system is in the non-lasing regime and the Fano factor can be approximated by $F \approx 1 + 1$. When moving towards the resonance, the system undergoes a lasing transition, accompanied by a sharp increase in the photon number, and an enhanced Fano factor since the amplitude fluctuations increase. At resonance, the photon number reaches a maximum, and the Fano factor is near 1, indicating the system is in a coherent state.

To study experimentally the predicted lasing effect, it is most suitable to process a double-quantum-dot system with in-situ tunable parameters which allow controlling the electron numbers and energy levels within the quantum dots but also the tunnel couplings towards the leads and/or between the quantum dots. A double quantum dot system based on a 2D electron system (2DES) which is embedded in a GaAs/(AlGa)As heterostructure close to the heterostructure surface offers such properties. The 2DES is structured by either applying negative voltages to structured metal electrodes on top of the heterostructure surface, or by etching grooves into the heterostructure to the depth of the 2DES and thereby dividing the 2DES into regions acting as leads, gates, and quantum dots. The second approach is more suitable in our case as the CPW resonator can be directly deposited on the heterostructure without being affected by normal conducting metal electrodes which might cause energy loss for the resonator. By removing the 2DES in most areas of the resonators’s mode volume – except
for the region of the quantum dot system in the center of maximum field of the resonator – the interaction of the electrically dissipative 2DES with the resonator field is diminished.

In such an approach, the typical geometrical diameter of a quantum dot is 0.5 to 0.7 µm, leading – due to electrostatic depletion from the etched grooves – to an effective quantum dot diameter of about 0.2 to 0.4 µm. The respective single-electron charging energies \( (e^2/2C) \) are then in the range of 0.5 to 1 meV. Excitations in such quantum dots can be characterized by electrical transport spectroscopy and are found at energies which are typically 1/10 to 1/3 of the single-electron charging energy. A disadvantage of such structures might be that the tunable tunnel barriers are energetically shallow and spatially broad.

The proposal here relies on a double-quantum dot where a single electron jumps from one dot to the other. The respective energy difference \( \epsilon \) for the electron sitting on either quantum dot is tunable by applying gate voltages. The respective electrical dipole change \( d \) which couples to the resonator field is about the distance between both quantum dot centers times the elementary charge, i.e. here about \( d = 0.6 \mu m \times e \). For an electric field strength of \( E = 0.2 V \cdot m^{-1} \) - typical of such a CPW resonator, we find a maximum coupling factor of \( g = d \cdot E/\hbar \approx 20 MHz \).

Achieving the lasing action puts constraints on parameters, namely, a relatively strong coupling compared to dissipations [6],

\[
g > \frac{\sqrt{\Gamma \phi}}{2\tau_0},
\]

where \( \Gamma \phi \) denotes the total rate accounting for all sources of dissipation of the two-level system. For small \( \theta \) it reduces to \( \Gamma \phi \approx \Gamma_{\downarrow}/2 + \Gamma_{\uparrow} + \Gamma/4 \). To present date, CPW resonators on GaAs substrates with \( \kappa = 10^{-4} \omega_r \) have been reported [10], two orders of magnitude larger than achieved on other substrates. This value of \( \kappa \) would be sufficient to obtain lasing, if \( \Gamma \phi \) can be kept in the range of 10 MHz.

On the other hand, the lifetime of an electron inside the double-quantum dot system has to be long enough to allow for stimulated photon emission while making the transition from one dot to the other. The respective tunnel rates can be tuned to low value. However, we want to detect currents, which under optimum conditions requires current above 1 pA, i.e. the tunneling rates have a lower limit given by \( 1/\tau < I/e \approx 0.6 MHz \). This would suffice to detect the lasing peak of magnitude about \( 2e\Gamma/3 \) [9]. Given sufficiently high coupling it should also be possible [11] to observe the change in photon number characteristic of lasing.

**Acknowledgments**

We acknowledge helpful discussion with S. André and A. Romito, as well as the financial support from the Baden-Württemberg Stiftung via the Kompetenznetz Funktionelle Nanostrukturen.

**References**

[1] Schoelkopf R and Girvin S 2008 Nature 451 664
[2] Astafiev O, Inomata K, Niskanen A O, Yamamoto T, Pashkin Yu A, Nakamura Y and Tsai J S 2007 Nature 449, 588
[3] Grajcar M, van der Ploeg S H W, Izmalkov A, Il’ichev E, Meyer A G, Fedorov A, Shnirman A and Schön G 2008 Nature Physics 4, 612
[4] Martzhaler M, Schön G and Shnirman A 2008 Phys. Rev. Lett. 101, 147001
[5] André S, Brosco V, Shnirman A and Schön G 2009 Phys. Rev. A 79, 053848
[6] André S, Jin P Q, Brosco V, Cole J H, Romito A, Shnirman A and Schön G 2010 Phys. Rev. A 82, 053802
[7] van der Vaart N C, Godijn S F, Nazarov Yu V, Harmans C J P M, Mooij J E, Molenkamp L W and Foxon C T 1995 Phys. Rev. Lett. 74, 4702
[8] Stoof T H and Nazarov Yu V 1996 Phys. Rev. B 53, 1050
[9] Jin P Q, Martzhaler M, Cole J H, Shnirman A and Schön G arXiv:1103.5051 [cond-mat.mes-hall]
[10] Frey T, Leek P J, Beck M, Ensslin K, Wallraff A and Ihn T arXiv:1104.3535 [cond-mat.mes-hall]
[11] Schuster D I, Houck A A, Schreier J A, Wallraff A and Gambetta J M 2007 Nature 445, 515