Modelling of Maximum Annual Flood for Regional Watersheds Using Markov Model

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Abstract

A study was undertaken to develop and apply the first-order Markov model in generating the synthetic stream flow for Adada, Ivo, Otamiri, Imo and Ajali rivers located in South East Nigeria. The best-fit probability distributions for the maximum annual stream flows were log-normal, Weibull, normal and lognormal for Adada, Ajali, Ivo, Otamiri and Imo (Umowpaga) rivers respectively were used in simulating the associated random deviates. The stream flow data for 10 years (1980 to 1989) were used to develop the model and to predict the stream flow. The model performance was evaluated with the aid of the coefficient of determination (R²). The results indicated a satisfactory performance as evidenced by R² values of 51.69%, 31.3%, 21.4%, 33% and 21% for Adada, Ajali, Ivo, Otamiri and Imo rivers respectively. The developed models were used to generate 50-year synthetic stream flows of these rivers. The study showed that the annual Markov models have proved to be a powerful tool to extend 10-year record of maximum annual runoffs.

Keywords: Markov Model, Synthetic Stream Flow, South East Nigeria.

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INTRODUCTION

There has been a severe increase in water scarcity in several regions around the globe owing to anthropogenic activities and climate change [1-3]. One of the major problems being faced by water resources engineers is the issue of determining the most applicable type of extreme value model, approximating the variables in the distribution and obtaining the required probability for flood distribution [4]. Real-time approach to flood forecasting together with the construction of gigantic hydraulic structures can aid in mitigating drought and flood hazards, as well as minimizing their associated losses [5]. However, owing to the extreme probability of their events, the danger arising from these natural disasters cannot be totally avoided, and thereby constituting much challenge to the water engineers in trying to estimate the frequency of their occurrences for a particular region [6]. In recent years, various studies have focused on the generation of synthetic stream flows across the world. To evaluate the anticipated water resources performance, simulation studies are usually performed using stochastic models for annual stream flow [7]. The generated synthetic stream flows are of equivalent significance as the historic stream flow in studying diverse practicable alternatives in the planning, design as well as in the operation of water resources schemes [8]. A reliable generation of synthetic stream flow is, therefore, of importance for territorial water resources implementations such as water transfer, maximum water resources distribution and reservoir management programmes [9].

Basically, the two major approaches in hydrological modelling are stochastic and deterministic modeling. The deterministic model takes into account the effect of biological, chemical and physical processes which take place within the hydrological system. The stochastic model on the other hand, generates the randomness existing between the hydrological outputs and inputs. A limited number of parameters exhibit a unique relationship involving the experimental curves or the functional descriptions for representing a hydrological system within the system [10]. This type of technique is applicable in conceptual- based models like the soil moisture computational model [11].

The regular hydrological processes can be stochastic and deterministic or only stochastic models.
Deterministic models can be used in the appreciable description of the actuality of a hydrological system particularly in cases when the intensity of uncertainty is appreciably small. Owing to an increase in the awareness on the varieties of sources for uncertainty in basically, all facets of environmental modelling, developing modelling approaches which take into account the stochastic nature of environmental processes is imperative [12].

The time series models are the most commonly used, and they are used for simulation or hydrological forecasting [11] and they include the Nearest Neighbour Method, the ARMA-Markov model and the Auto-regressive Moving Average Model (ARMA) [13]. In a related study [14], suggested a non-parametric approach for the generation of long span of streamflow prediction. This approach is based mainly on daily use of non-parametric-based regression analysis and streamflow data for relating the covariate to the forecast variables.

The theory and application of Markov process are well documented in literature. Markov chains have been used for streamflow generation in long, medium and short-range hydrological prediction. The application of Markov chains technique in run-off rainfall modeling has been in use for ages. A first order Markov chain was proposed by Gabriel in 1957 for simulating the event of dry and wet seasons [11]. In another research [15], adopted a Markov-based mixture model in generating the annual synthetic streamflows. The model is made up of a two-state-based Markov chain in which the states considered were the normal together with the low streamflows, whereas the values of the streamflows in each of the states were generated with the aid of two normal distributions. The model was found important in the sizing of the reservoir owing to its capacity in generating consistent stochastically synthetic streamflow series for dry seasons. Jackson, B. B [16] equally suggested a continuous time Markov chain death-birth process with the aid of discrete streamflow levels as state, in the modelling of the phenomenological monitoring showing that low values of streamflow are more persistent than those of high ones. The model was successful in the generation of annual stream flows used in the optimization of the reservoir emission. Haan, C. T et al., [17] gave a Markov-based chain model of first order which was tested using seven rainfall stations with the aid of 7 x 7 transition matrices. The geometric progression was adopted in the determination of the class boundaries across the various states present in the Markov chain. A comparison was done between the actual and the simulated rainfall in different scenarios, and it was found out that the model exhibited satisfactory performance. Yakowitz, S. T [18] presented a non-parametric based Markov model for use in short-tern forecasting. The model entails the inference on the distribution of conditional probability for the subsequent streamflow which is assigned to its current value. In comparison with the ARMA modelling approach, the non-parametric based model showed a better performance. Yakowitz, S. T [18] described the N-th order class of Markov chain models as a daily streamflow model, and he proposed statistical based approaches for inference within the class. He highlighted the characteristics of the N-th order Markov chain models as being unique amidst the non-parametric streamflow models. With the use of this device, ignoring traditional but unreliable assumptions such as prior assumptions on time distribution between extreme occurrences, recessions and gamma distribution of flows is possible. Katz, R. W [19] in his study, carried out an analysis on the approach suggested by Tong in 1975, in estimating the order of Markov chain using Bayesian and Akaike's Information Criteria. He inferred that models of higher magnitude than the first-order chains should not be regarded. Yakowitz, S. T [20] adopted the non-parametric approach in the modelling of data from streamflow on the issue of flood alerts. He described a non-parametric inference technique which approaches the optimum decision function in the case of flood warning, when there is an increase in the span of the historical record for any immobile Markov procedure in accordance with [20]. In another study [14], obtained a non-parametric technique for generating distributive forecasts involving long-span streamflow variables with the aid of conceptual hydrological models.

Although, some researches have been carried out on the prediction of annual rainfall Markov model, to the best of our knowledge no study has been reported on modelling of maximum annual flood for South East, Nigeria using Markov model. Given 10-year data of maximum annual runoffs, this study is focused on the generation of 30-year synthetic data of five rivers drawn from each of the five states of South East, Nigeria using Markov model.

METHOD
Study Area

The south-eastern part of Nigeria is an area which covers about 76,358km² south of the Benue valley and east of the lower Niger. In terms of geophysical zones, the region consists of five states out of 36 states in Nigeria, and they include Abia, Anambra, Ebonyi, Enugu and Imo states. The region is situated between longitudes 7°E and 9°E and latitudes 4°N and 7°N. In terms of relief, the land surface of eastern part of Nigeria can be classified as plains, highlands and the lowlands [21]. These are the plains and Climate-wise, eastern Nigeria is characterized by seasonal distribution of rainfall which depends on the interaction of the Equatorial Easterlies, Tropical Maritime air mass and Tropical Continental air mass. The rainfall pattern is characterized by long rainy season as well as short dry season, and it is controlled by the movement involving inter-tropical convergence. The long rainy season starts...
from April to July, while the short dry season occurs in August, and it is followed by a short rainy season which occurs between September to October [21]. The map of the study area is shown in Fig-1 while the monthly discharge records that were used covered over 10 years for the selected Rivers Adada in Enugu, Otamiri in Owerri, Ivo in Ebonyi and Imo (umuokpara) in Umuahia (Table-1).

![Fig-1: Major Towns of South-eastern Nigeria](image)

**Table-1: Geographical locations for the Rivers**

| STATE | RIVER | STATION     | LAT.  | LONG.  | CATCHMENT AREA (Km$^2$) | ZERO LEVEL OF GAUGE (m) | GB M LOCAL (m) |
|-------|-------|-------------|-------|--------|-------------------------|-------------------------|----------------|
| IMO   | Otamiri| Nekede    | 05°26’ | 07°02’E | 100                      | 97.71                    | 100            |
| Abia  | Abia   | Umuopara   | 05°33’N | 07°25’E | 1450                    | 86.50                    | 100            |
| Anambra| Adada | Umulokpa | 06°38’N | 07°11’E | 890                     | NA                      | NA             |
| Enugu | Ivo   | Aguobu-Umumba | 06°19’N | 07°13’E | 900                     | NA                      | NA             |
| Ebonyi| Ivo   | Imezi-Olo  | 06°28’N | 06°11’E | 125                     | NA                      | NA             |

**Source:** Anambra-Imo River Basin Authority, Owerri

NA=Not applicable.

The discharge data were obtained from the Anambra-Imo River Basin Authorities. The maximum monthly runoffs of the selected basins were collated from the available discharge records to form a time series of maximum annual runoff for the selected catchment of Rivers Adada in Enugu, Otamiri in Owerri, Ivo in Ebonyi and Imo (umuokpara) in Umuahia (Table-2).

**Identification and Distribution of Annual Runoff**

The historical ten (10)-year stream flow data were extended using the Markov Model for the five rivers under study by initially ascertaining the probability distribution of the random deviates associated with the annual runoff data of the rivers under study. Initially, Log normal, Normal, Generalized Extreme Value, Log Pearson, Gumbel, Log-Pearson type 3 and Pearson type 3 statistical distributions were fitted to the maximum monthly runoff for 10 years of data (1980–1989) for the stream gauging stations in each of the five states of southeastern Nigeria.

To ascertain the probability distribution of the random deviates associated with the annual runoff data the rivers under study, the best fitting distribution for the annual discharge was obtained using the Anderson-Darling test as expressed in Equations 1 and 2. The probability distribution of the annual runoff was calculated using the Minitab software application. The distribution having the highest value of p was accepted as the probability distribution of the random variable. This study used the Anderson-Darling (A-D) test to confirm the statistical hypothesis showing whether a particular probability distribution gives an adequate fit to the measured annual maximum flood series data and can be approved as the best fit distribution. The goodness of fit tests were performed to measure the distance existing between the distribution and the data being tested as well as to compare the distance obtained with some threshold value. The distance obtained is known as the test statistic, while the threshold value is known as the critical value.
Table-2: Maximum Annual Discharge of Selected Rivers in south east Nigeria

| Year     | Maximum Annual Discharge (m³/s) | River Ivo | River Ajali | River Imo (umuokpara) | River Otamiri | River Adada |
|----------|---------------------------------|-----------|-------------|-----------------------|---------------|-------------|
| 1978/79  | 0.00                            | -         | 270.00      | 12.10                 | -             | -           |
| 1979/80  | 11.33                           | 12.15     | 158.40      | 12.90                 | 50.28         | -           |
| 1980/81  | -                               | 12.65     | -           | 10.55                 | 42.51         | -           |
| 1981/82  | -                               | 12.24     | -           | 11.48                 | 46.95         | -           |
| 1982/83  | 11.33                           | 12.61     | 158.37      | 7.20                  | 46.71         | -           |
| 1983/84  | 11.18                           | 12.55     | 156.04      | 10.70                 | 57.13         | -           |
| 1984/85  | 10.91                           | 12.61     | 126.20      | 9.38                  | 50.53         | -           |
| 1985/86  | 10.91                           | 12.55     | 194.40      | 11.78                 | 45.67         | -           |
| 1986/87  | 10.91                           | 11.8      | 115.00      | 13.54                 | 47.86         | -           |
| 1987/88  | 10.90                           | 12.27     | 223.20      | -                     | 47.99         | -           |
| 1988/89  | 10.91                           | 12.44     | 42.85       | -                     | 50.15         | -           |

Source: Anambra-Imo River Basin Authority, Owerri

If the value of test statistic is less than the critical value, then the fit is regarded as a good one. The test was performed at 5% significant levels. The symbol X signifies the random variable for the annual maximum flows, while n denotes the sample size. To compare the fit of an observed cumulative distribution function with an expected cumulative distribution function, the Anderson-Darling procedure was used. The Anderson-Darling test statistic (AD) is given in Equation 1.

\[ AD = -n - \frac{1}{n} \sum_{i=1}^{n} (2i-1) \left[ \ln F(X_i) - \ln(1 - X_n - i - 1) \right] \] .............. (1)

Where \( F(X) \) and \( i \) are the cumulative distribution function in the specified distribution and the \( i \)th sample with the data arranged in ascending order respectively. After obtaining AD, the value was adjusted for small sample sizes using Equation 2.

\[ AD^* = AD \left( 1 + \frac{0.95}{n} + \frac{2.23}{n^2} \right) \] .............................................. (2)

Synthetic Generation of Annual Discharge

Markov model was used in this study to extend the length of the historical data obtained from the stream flow. The sample parameters of the annual flow distribution such as the mean, variance and lag-one serial correlation coefficient were initially estimated using the following relations:

The arithmetic mean \( \mu \) of the historical 10 years data of runoff was obtained using Equation 3.

\[ \mu = \frac{1}{N} \sum_{t=1}^{N} X_t \] ......................................................... (3)

Serial covariance zero, \( C_0 \), was used to obtain the variance as shown in Equation 4.

\[ C_0 = \frac{1}{N} \sum_{t=1}^{N} (X_t - \mu)^2 \] ......................................................... (4)

The lag-one serial correlation coefficient was used in describing the strength of the relationship between a value in the sequence and that preceding it by one-time interval; i.e lag-one. The data is denoted by \( r_1 \) where, \( t = 1,2,3...10 \) years. To obtain lag-one serial correlation coefficient \( r_1 \), the following steps were followed: the serial covariance one, \( C_1 \) was first evaluated using Equation 5.

\[ C_1 = \sum_{t=1}^{N} \frac{(X_t - \mu)(X_{t-1} - \mu)}{N} \] ......................................................... (5)

The lag-one serial correlation coefficient \( r_1 \) was calculated using Equation 6.

\[ r_1 = \frac{C_1}{C_0} \] ......................................................... (6)

For generating annual flows \( Q \), the first-order Markov model is given in Equation 7.

\[ Q = \bar{Q} + r_1 (Q_1 - \bar{Q}) + z \sigma \sqrt{T} - r_1^2 \] ......................................................... (7)
Where $\sigma$= standard deviation of $Q$, $z_i$ is the random deviate whose distribution was established. $Q$= mean of $Q$, $i$= 1year to n year (flows in series), $r_i$= 1-lag autocorrelation coefficient. Markov The model assumed generally that the $z_i$ in the generation equation are distributed independently with constant variance and mean zero and as the annual runoff data. The model also assumed that the whole impact the previous flow had on the current one is reflected in the value of the previous flow. These computations were done using Microsoft Excel software application.

**RESULTS**

**Probability Distributions of Maximum Annual Runoff**

The distribution having the highest value of $p$ was accepted as the probability distribution of the random deviates as shown in bold font in Table-3 for Rivers Adada, Ajali, Ivo, Otamiri and Imo (Umuokpara).

| Table-3: Results of Anderson-Darling Test for Best Fitting Distribution for Annual Runoff |
|--------------------------------------|--------|--------|
| **Adada River**                     |        |        |
| Distribution                        | AD     | P      | LRT P |
| Normal                              | 0.408  | 0.279  |        |
| Lognormal                           | 0.354  | 0.385  |        |
| 3-Parameter Lognormal               | 0.308  | *      | 0.534 |
| Exponential                         | 3.987  | <0.003 |        |
| 2-Parameter Exponential             | 1.140  | 0.023  | 0.000 |
| Weibull                             | 0.673  | 0.067  |        |
| 3-Parameter Weibull                 | 0.368  | 0.446  | 0.065 |
| Smallest Extreme Value              | 0.781  | 0.035  |        |
| Largest Extreme Value               | 0.324  | >0.250 |        |
| Gamma                               | 0.365  | >0.250 |        |
| 3-Parameter Gamma                   | 0.770  | *      | 1.000 |
| Logistic                            | 0.300  | >0.250 |        |
| Loglogistic                         | 0.273  | >0.250 |        |
| 3-Parameter Loglogistic             | 0.250  | *      | 0.607 |
| **Ajali River**                     |        |        |        |
| Distribution                        | AD     | P      | LRT P |
| Normal                              | 0.556  | 0.112  |        |
| Lognormal                           | 0.570  | 0.103  |        |
| 3-Parameter Lognormal               | 0.584  | *      | 0.641 |
| Exponential                         | 4.413  | <0.003 |        |
| 2-Parameter Exponential             | 1.785  | <0.010 | 0.000 |
| Weibull                             | 0.492  | 0.205  |        |
| 3-Parameter Weibull                 | 0.485  | 0.133  | 0.746 |
| Smallest Extreme Value              | 0.485  | 0.211  |        |
| Largest Extreme Value               | 0.786  | 0.034  |        |
| Gamma                               | 0.592  | 0.136  |        |
| 3-Parameter Gamma                   | 5.809  | *      | 1.000 |
| Logistic                            | 0.516  | 0.138  |        |
| Loglogistic                         | 0.523  | 0.129  |        |
| 3-Parameter Loglogistic             | 0.516  | *      | 0.694 |
| **Ivo River**                       |        |        |        |
| Distribution                        | AD     | P      | LRT P |
| Normal                              | 1.137  | <0.005 |        |
| Lognormal                           | 1.137  | <0.005 |        |
| 3-Parameter Lognormal               | 1.034  | *      | 0.000 |
| Exponential                         | 3.565  | <0.003 |        |
| 2-Parameter Exponential             | 1.961  | <0.010 | 0.000 |
| Weibull                             | 1.146  | <0.010 |        |
| 3-Parameter Weibull                 | 1.061  | 0.009  | 0.000 |
| Smallest Extreme Value              | 1.146  | <0.010 |        |
| Largest Extreme Value               | 1.438  | <0.010 |        |
| Gamma                               | 1.256  | <0.005 |        |
| 3-Parameter Gamma                   | 0.949  | *      | 0.000 |
| Logistic                            | 1.163  | <0.005 |        |
| Loglogistic                         | 1.164  | <0.005 |        |
| 3-Parameter Loglogistic             | 0.948  | *      | 0.000 |
| **Otamiri River**                   |        |        |        |
| Distribution                        | AD     | P      | LRT P |
| Normal                              | 0.167  | 0.910  |        |
| Lognormal                           | 0.311  | 0.495  |        |
| 3-Parameter Lognormal               | 0.170  | *      | 0.252 |
| Exponential                         | 3.146  | <0.003 |        |
| 2-Parameter Exponential             | 1.467  | <0.010 | 0.000 |
| Weibull                             | 0.129  | >0.250 |        |
| 3-Parameter Weibull                 | 0.127  | >0.500 | 0.938 |
| Smallest Extreme Value              | 0.143  | >0.250 |        |
| Largest Extreme Value               | 0.426  | >0.250 |        |
| Gamma                               | 0.254  | >0.250 |        |
| 3-Parameter Gamma                   | 3.229  | *      | 1.000 |
| Logistic                            | 0.135  | >0.250 |        |
| Loglogistic                         | 0.212  | >0.250 |        |
| 3-Parameter Loglogistic             | 0.135  | *      | 0.355 |
| **Imo (Umuokpara) River**           |        |        |        |
| Distribution                        | AD     | P      |        |
| Normal                              | 0.317  | 0.486  |        |
| 3-Parameter Lognormal               | 0.656  | *      |        |
| 2-Parameter Exponential             | 0.970  | 0.047  |        |
| 3-Parameter Weibull                 | 0.316  | 0.480  |        |
| Smallest Extreme Value              | 0.261  | >0.250 |        |
| Largest Extreme Value               | 0.548  | 0.151  |        |
| 3-Parameter Gamma                   | 0.635  | *      |        |
| Logistic                            | 0.327  | >0.250 |        |
| 3-Parameter Loglogistic             | 0.391  | *      |        |
Table 4 shows the random variables associated with Rivers Adada, Ajali, Ivo, Otamiri and Imo (Umuokpara). The random deviates associated with River Adada had a lognormal distribution with the highest p value of 0.385. The random deviates associated with River Ajali had a Weibull distribution with the highest p value of 0.205. The random deviates associated with River Ivo had a Weibull distribution with the highest p value of 0.010. The random deviates associated with River Otamiri had a normal distribution with the highest p value of 0.910. The random deviates associated with River Imo (Umuokpara) had a lognormal distribution with the highest p value of 0.486.

Model Formulation
To obtain the equation for generating runoff data beyond the available 10-year data, the mean discharge, $\mu$, correlation coefficient, $r_1$ and standard deviation, $\sigma$ respectively as given in Tab. 4 were substituted in Equation 7 to yield the annual Markov model for forecasting of annual discharge in Rivers Otamiri, Imo, Adada, Ajali and Ivo as shown in Table 5.

Table 4: Parameters of Annual Discharge for Substituting into Annual Markov Model

| RIVER | STATION     | Mean  | St. Deviation | 1-Lag Autocorrelation ($r_1$) | Probability Distribution | Skewness       | Kurtosis     |
|-------|-------------|-------|---------------|--------------------------------|--------------------------|----------------|--------------|
| Adada | Umulokpa    | 48.5771 | 3.86757       | -0.006134                      | Lognormal                | 0.918875       | 2.32332      |
| Ajali | Aguobu-umumba | 12.3867 | 0.271846      | -0.086016                      | Weibull                  | -1.21317       | 1.1451       |
| Ivo   | Imezi-Olo   | 11.0486 | 0.197558      | 0.645272                       | Weibull                  | 0.847137       | -1.51071     |
| Otamiri | Nekede    | 11.4236 | 2.12483       | 0.263082                       | Normal                   | -0.576458      | 0.62177      |
| Imo   | Umuopara    | 131.315 | 87.372        | 0.045242                       | Normal                   | -0.270547      | -0.660512    |

Model Calibration
The computation of Markov Model for predicting discharge of Adada River is

$$Q = 48.5771 + (-0.006134)Q_{t-1} - (-0.06134)48.5771 + z_i\left(3.86757 \left(1 + (-0.006134)^2\right)\right)$$

$$Q = 48.8750719314 - 0.006134Q_{t-1} + 3.8674972388iz_i$$

The computation of Markov Model for predicting discharge of Ajali River:

$$Q = 12.3867 + (-0.086016)Q_{t-1} - (-0.086016)12.3867 + z_i\left(0.271846 \left(1 + (-0.086016)^2\right)\right)$$

$$Q = 13.45215438 - 0.086016Q_{t-1} + 0.27084iz_i$$

The computation of Markov Model for predicting discharge of Ivo River

$$Q = 11.0486 + (0.645272)Q_{t-1} - (0.645272)11.0486 + z_i\left(0.197558 \left(1 + (0.645272)^2\right)\right)$$

$$Q = 3.9192477808 + 0.645272Q_{t-1} + 0.150925z_i$$

The computation of Markov Model for predicting discharge of Otamiri River

$$Q = 11.4236 + (0.263082)Q_{t-1} - (0.263082)11.4236 + z_i\left(2.12483 \left(1 + (0.263082)^2\right)\right)$$

$$Q = 8.41825664648 + 0.263082Q_{t-1} + 2.0499796iz_i$$

The computation of Markov Model for predicting discharge of Imo (Umuopara) River

$$Q = 131.315 + (0.045242)Q_{t-1} - (0.045242)131.315 + z_i\left(87.372 \left(1 + (0.045242)^2\right)\right)$$

$$Q = 125.3740404677 + 0.045242Q_{t-1} + 87.282536z_i$$

Table 5: Annual Markov Model for Forecasting of Maximum Annual Discharge

| Rivers    | Annual Markov | $R^2$ (%) |
|-----------|---------------|-----------|
| Adada river | $Q = 48.8750719314 - 0.006134Q_{t-1} + 3.8674972388iz_i$ | 51.69     |
| Ajali river | $Q = 13.45215438 - 0.086016Q_{t-1} + 0.27084iz_i$ | 31.3      |
| Ivo River   | $Q = 3.9192477808 + 0.645272Q_{t-1} + 0.150925z_i$ | 21.38     |
| Otamiri river | $Q = 8.41825664648 + 0.263082Q_{t-1} + 2.0499796iz_i$ | 32.97     |
| Imo river   | $Q = 125.3740404677 + 0.045242Q_{t-1} + 87.282536z_i$ | 21.01     |
Simulation of Maximum Annual Runoff

Table-6: Computations for Simulating Maximum Annual Runoff in Adada River

| YEAR | Random Deviates, $z_i$ | $Q = 48.8750719314 - 0.006134Q_{i-1} + 3.86749723881z_i$ |
|------|------------------------|--------------------------------------------------|
| 1980 | 0.777501              | $Q_2 = 48.8750719314 - 0.006134 \times 0 + 3.86749723881z_i = 50.27546$ |
| 1981 | 3.645211              | 48.8750719314 - 0.006134 \times 50.27546 + 3.86749723881z_i = 42.51 |
| 1982 | 12.9971               | 48.8750719314 - 0.006134 \times 42.51 + 3.86749723881z_i = 46.95 |
| 1983 | 4.163395              | 48.8750719314 - 0.006134 \times 46.95 + 3.86749723881z_i = 46.70538 |
| 1984 | 3.489922              | 48.8750719314 - 0.006134 \times 46.70538 + 3.86749723881z_i = 57.13145 |
| 1985 | 2.783882              | 48.8750719314 - 0.006134 \times 57.13145 + 3.86749723881z_i = 50.52656 |
| 1986 | 1.824315              | 48.8750719314 - 0.006134 \times 50.52656 + 3.86749723881z_i = 45.6654 |
| 1987 | 8.616544              | 48.8750719314 - 0.006134 \times 45.6654 + 3.86749723881z_i = 47.86443 |
| 1988 | 0.629635              | 48.8750719314 - 0.006134 \times 47.86443 + 3.86749723881z_i = 47.99252 |
| 1989 | 2.071504              | 48.8750719314 - 0.006134 \times 47.99252 + 3.86749723881z_i = 50.14973 |

As an illustration, after generating random deviates $z_i$ from lognormal distribution, 10-year synthetically generated maximum annual runoff of Adada river is computed in Table-6. This procedure is repeated for Rivers Ajali, Ivo, Otamiri and Imo (Umuokpara) using the best fitting distribution of the random deviate and assuming that the maximum annual discharge in the year preceding the first discharge record is $Q_1 = 0$

Model Validation

The scatter plot between the predicted runoff of the corresponding years for Rivers Adada, Ajali, Ivo, Otamiri and Imo (Umuokpara) and the observed 10-year runoff is shown in Fig-2. The validity of the annual Markov models was verified using the values obtained for the coefficient of determination ($R^2$) of the scatter plot.
It can be observed that the scatter plots confirmed the values of the coefficients of determination for the Markov models. In which Rivers Adada, Ajali, Ivo, Otamiri and Imo had values of 51.69%, 31.3%, 21.4%, 33% and 21% respectively (Fig-2). This implied that Markov model of River Adada would predict annual runoff better than those of the other rivers simply because the data of maximum annual runoff of River Adada fulfilled the major core assumption of the Markov model which is based on the fact that the whole impact the previous flow had on the current one is reflected in the value of the previous one.

CONCLUSION

The annual Markov models were fitted to the maximum annual runoffs of the selected catchment areas. The random deviates associated with Rivers Adada, Ajali, Ivo, Otamiri and Imo (Umuokpara) had lognormal (p = 0.385), Weibull (p = 0.205), Weibull (p = 0.010), normal (p = 0.910) and lognormal (p = 0.486). Markov model of River Adada has the highest coefficient of determination of 51.69% and is the best to predict annual runoff among other rivers simply because the whole impact the previous flow had on the current one is reflected in the value of the previous one.

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