Forecasting of Bandar Lampung's Population Using Growth Mathematical Models

A Nuryaman¹, M A A F Pamungkas¹ and S Saidi¹

¹Department of Mathematics, Faculty of Mathematics and Natural Science, Lampung University. Jl. Prof. Soemantri Brojonegoro No. 1 Gedong Meneng Bandarlampung, Indonesia

E-mail: aang.nuryaman@fmipa.unila.ac.id

Abstract. This paper investigates the projection on the population of Bandar Lampung City in 2017 - 2030 through a mathematical modeling approach. Here, we use four growth mathematical models particularly simple linear regression model, exponential model, logistic model and demographic transition model. The best approximation is chosen based on the smallest Mean Absolute Percentage Error (MAPE) value of the four models compared to the actual data. By using population data from 1995 - 2011, the results show that the model which gives the smallest MAPE is exponential model and the largest MAPE is demographic transition model.

1. Introduction

The number of population in a city, province or country is one of factors that are the center of attention of government. Bandar lampung is the capital of Lampung province with an area of ±169.21 km². The population density of Bandar lampung in 2016 is 5896 people per km².

Based on data from Central Bureau of Statistics (Badan Pusat Statistik/BPS) of Bandar lampung, the population density of Bandar lampung in each year shows an increasing trend. The increasing trend of population density leads to social issues such as the availability of jobs, poverty and others. These problems must be the concern of the Bandar Lampung government. Therefore, to anticipate those problems, the government have to project the future population number as reference in determining of development policies. Mathematical modeling can be used as a tool to forecast the number of population in the future.

Earlier studies [1-4] have studied and used mathematical modeling to predict the population number in the future. Junior [1] used exponential (Malthusian) model to study the population growth of Sergipe. He determined the rate of population growth \( r \) with the following formula:

\[
r = \left( \frac{\delta_t}{\delta_0} \right)^{1/t} - 1
\]

Where \( \delta_t \) denote population number at time \( t \). In [2-4], in addition to the exponential model, the authors also used logistic model to predict the population number.

In this paper, we will discuss the population number forecasting for Bandar Lampung city using four models namely simple linear regression model, exponential model, logistic model and demographic
transition model. The accuracy of each model is evaluated by the mean absolute percentage errors (MAPE) with the criteria as specified in [5].

This paper is organized as follow. In Section 2, secondary data, methods and mathematical model reviewes are described. In Section 3, the prediction of population for each model is presented. The conclusions are written in the last section.

2. Materials and Methods
The secondary data that we used in this paper are obtained from Central Bureau on Statistics (Badan Pusat Statistik/BPS) of Indonesia for the period 1995 to 2016. The data are divided into two parts, 80% of data used to determine the parameter values in the related mathematical model and the rest to validate the mathematical model that was formed. Based on the available data, let \( t = 0,1,2,...,16 \) correspond to the year 1995,1996,1997,...,2011 and \( P(t) \) denotes population number at time \( t \).

Here, estimation of population in the future year will be investigated using four mathematical models those are simple linear regression model, exponential model, logistic model and demographic transition model. The prediction accuracy of each mathematical model is determined by considering the MAPE value which is defined by the formula [5]:

\[
MAPE = \frac{(100\% / n) \sum_{i=0}^{n} |A_i - F_i|}{A_i}
\]

where \( A_i \) is the actual value and \( F_i \) is the forecast value.

The model that will be used to forecast population number is the model which gives the smallest MAPE’s value.

2.1 Simple linear regression model
Let we have data in the ordered pair \((t_0, P_0), (t_1, P_1), (t_2, P_2), \cdots, (t_i, P_i), \cdots, (t_n, P_n)\). Assume that relationship between time \((t)\) and population number \(P(t)\) is linear that is can be written as follow

\[
P = a + bt + \varepsilon
\]

where \( P = P(t) \). Least square method [6] gives

\[
a = \frac{(\sum_{i=1}^{n} t_i \sum_{i=1}^{n} t_i^2 - \sum_{i=1}^{n} t_i \sum_{i=1}^{n} t_i P_i)}{(n \sum_{i=1}^{n} t_i - (\sum_{i=1}^{n} t_i)^2) \text{ and}}
\]

\[
b = \frac{(n \sum_{i=1}^{n} t_i P_i - \sum_{i=1}^{n} t_i \sum_{i=1}^{n} t_i P_i)}{(n \sum_{i=1}^{n} t_i^2 - (\sum_{i=1}^{n} t_i)^2)}
\]

with \( P_i = P(t_i) \).

2.2 Exponential model
The exponential model was proposed by Malthus [7] based on the assumption that the population grows at a rate proportional to the population size. This model can be written as an initial value problem

\[
\frac{dP}{dt} = rP, \ 0 \leq t; \ P(0) = P_0
\]

where the solution is

\[
P(t) = P_0 e^{rt}
\]

In our research, grow rate \( r \) determined by choosing two data \( P_0 \) and \( P(t_k) \) with

\[
r = \frac{\ln\left(\frac{P(t_k)}{P_0}\right)}{t_k}
\]

such that MAPE value of the forecast data using equation (5) is minimum.

2.3 Logistic model
The logistic growth model is a modification of the exponential model which proposed by Verhulst [8]. This model assumes that the population growth not only depends on the population size but also on how far this size is from its upper limit i.e. its carrying capacity (maximum supportable population). The governing equation of the logistic model is

\[
\frac{dP}{dt} = r \left(1 - \frac{P}{\kappa}\right) P, \ 0 \leq t; \ P(0) = P_0
\]
where \( K \) is called carrying capacity. The solution of the initial value problem (7) is given by

\[
P(t) = \frac{P_0 K}{P_0 + (K - P_0)e^{-\tau t}}
\]  

(8)

In [4], carrying capacity is determined using formula

\[
K = \frac{P(t_1)P(t_3) - 2P(t_0)P(t_2) + P(t_1)P(t_3)}{P^2(t_1) - P(t_0)P(t_3)}
\]

In this paper, the \( K \) and \( r \) parameters are determined by selecting three data \( P(t_k), P(t_{k+1}) \) and \( P(t_{k+2}) \) with

\[
K = \frac{P(t_k)(2P(t_k)P(t_{k+2}) - P(t_{k+1})P(t_{k+2}) - P(t_k)P(t_{k+2}))}{P(t_k)P(t_{k+2}) - P(t_{k+1})}
\]

and

\[
r = -\frac{1}{\tau} \frac{P(t_k)(2P(t_k)P(t_{k+1}) - P(t_{k+2})P(t_k))}{P(t_{k+1})(P(t_k) - P(t_{k+2}))}
\]

such that MAPE’s value of the forecast data using equation (8) is minimum. Here \( \tau \) denotes time period with \( \tau = 1, 2, ..., 8 \) and \( k = 0, 1, 2, ..., 16 \).

2.4 Demographic transition model

Here, we adopt the model which was proposed by Adeng and Marroquin in [9], and described as follow

\[
\frac{dP}{dt} = (r - fW)P \left( 1 - \frac{P}{aN} \right)
\]

\[
\frac{dN}{dt} = N_0 - \delta P
\]

\[
\frac{dW}{dt} = \lambda P - \gamma PW
\]

The variables \( P, N \) and \( W \) denote population, natural resources and pollution, respectively. Initial values of each dependent variable are \( P(0) = P_0, N(0) = N_0 \) and \( W(0) = W_0 \). Based on available data, in this article, natural resources refer to total area of agricultural land, while pollution corresponds to landfill. The value of landfill rate use year 2000’s version of SNI 19-3983-1995 data where Bandar Lampung was included as medium city.

3. Results and discussion

Based on available data, we get mathematical model of Bandar Lampung’s population growth using simple linear regression is \( P(t) = 684440.14 + 12199.15t \) with correlation coefficient \( \rho = 0.98 \) and coefficient of determination \( R^2 = 96.19\% \). Furthermore, for the exponential model, we get a growth rate is \( r = 1.64\% \) based on 1995 and 2011 data. While the logistic model gives a growth rate about \( r = 1.6\% \) based on data in 1995, 1997 and 1999. The comparison between actual and estimation data of Bandar lampung population for each model is presented in table 1 below.

| Year | Population actual data | Linear Regression | Exponential | Logistic | Demographic transition |
|------|------------------------|------------------|-------------|----------|------------------------|
| 1995 | 685862                 | 684440          | 685862      | 685862   | 685862                 |
| 1996 | 696949                 | 696639          | 697189      | 696949   | 695988                 |
| 1997 | 708212                 | 708838          | 708704      | 708212   | 706540                 |
| 1998 | 719659                 | 721038          | 720408      | 719654   | 717435                 |
| 1999 | 731290                 | 733237          | 732306      | 731277   | 728419                 |
The other side, table 2 illustrates the validation step for each model. From table 1 and table 2, modelling step and validation give an information that the exponential model is the best model to forecast Bandar Lampung population number. Table 3 presents the projection of Bandar Lampung population number from 2017 until 2030.

| Year | Population actual data | Linear Regression | Exponential | Logistic | Demographic transition |
|------|------------------------|-------------------|-------------|----------|------------------------|
| 2000 | 743109                 | 745436            | 744400      | 743084   | 739595                 |
| 2001 | 754892                 | 757635            | 756694      | 755078   | 750959                 |
| 2002 | 767036                 | 769834            | 769191      | 767262   | 762510                 |
| 2003 | 790895                 | 782033            | 781895      | 779639   | 774249                 |
| 2004 | 800490                 | 794233            | 794808      | 792212   | 786305                 |
| 2005 | 809860                 | 806432            | 807935      | 804983   | 798425                 |
| 2006 | 844608                 | 818631            | 821278      | 817955   | 810737                 |
| 2007 | 812133                 | 830830            | 834842      | 831133   | 823243                 |
| 2008 | 822880                 | 843029            | 848629      | 844518   | 835946                 |
| 2009 | 833517                 | 855228            | 862645      | 858113   | 848849                 |
| 2010 | 881801                 | 867427            | 876892      | 871923   | 862095                 |
| 2011 | 891374                 | 879627            | 891374      | 885950   | 875407                 |

MAPE 1,032% 0,933% 0,936% 1,222%

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| Year | Population actual data | Linear Regression | Exponential | Logistic | Demographic transition |
|------|------------------------|-------------------|-------------|----------|------------------------|
| 2012 | 902885                 | 891826            | 906095      | 900198   | 892268                 |
| 2013 | 942039                 | 904025            | 921060      | 914669   | 906199                 |
| 2014 | 960695                 | 916224            | 936272      | 929367   | 920499                 |
| 2015 | 979287                 | 928423            | 951734      | 944296   | 934875                 |
| 2016 | 997728                 | 940622            | 967453      | 959459   | 949632                 |

MAPE 4,161% 2,194% 2,697% 3,704%

Table 3: Projection of population

| Year | Projection of population number |
|------|---------------------------------|
| 2017 | 983430                          |
| 2018 | 999672                          |
| 2019 | 1016182                         |
| 2020 | 1032965                         |
| 2021 | 1050025                         |
| 2022 | 1067366                         |
| 2023 | 1084994                         |
| 2024 | 1102913                         |
| Year | Projection of population number |
|------|---------------------------------|
| 2025 | 1121128                         |
| 2026 | 1139644                         |
| 2027 | 1158466                         |
| 2028 | 1177598                         |
| 2029 | 1197047                         |
| 2030 | 1216816                         |

The demographic transition model provides the biggest MAPE’s value possible due to the application of limited assumptions on natural resources and pollution where we assume only the total of agricultural land and volume of landfill. In addition, limited information on some parameter values also affect the estimation accuracy based on this model.

In the other side, based on table 1. Table 2 and table 3, every year the Bandar Lampung population grows up with growth rate around 1.6%. In addition to natural birth and death factors, the existence of three state higher education institutions, job opportunities and more complete facilities become the factor of increasing trend of Bandar Lampung’s population. In addition to being economically beneficial, the population migration to Bandar Lampung can also create new problems such as social problems, availability of jobs, housing problems, health problems and so on. The government must be pay attention to these problems when making public policies.

4. Conclusions
We have presented some mathematical models to forecast the population of Bandar lampung from 2017 until 2030. Based on available data, we conclude that the exponential model is the best model to estimate population number of Bandar lampung city with MAPE’s value is 0,933%. By using the exponential model, the population projection in 2030 is 1216816 people.

Acknowledgment
The authors would like to thanks BPS’s leader and Dinas Pertanian’s leader of Bandar Lampung city for their valuable helping to acces data that we need.

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