PLASMA EFFECTS ON FAST PAIR BEAMS. II. REACTIVE VERSUS KINETIC
INSTABILITY OF PARALLEL ELECTROSTATIC WAVES

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ABSTRACT

The interaction of TeV gamma-rays from distant blazars with the extragalactic background light produces relativistic electron–positron pair beams by the photon–photon annihilation process. Using the linear instability analysis in the kinetic limit, which properly accounts for the longitudinal and the small but finite perpendicular momentum spread in the pair momentum distribution function, the growth rate of parallel propagating electrostatic oscillations in the intergalactic medium is calculated. Contrary to the claims of Miniati and Elyiv, we find that neither the longitudinal nor the perpendicular spread in the relativistic pair distribution function significantly affect the electrostatic growth rates. The maximum kinetic growth rate for no perpendicular spread is even about an order of magnitude greater than the corresponding reactive maximum growth rate. The reduction factors in the maximum growth rate due to the finite perpendicular spread in the pair distribution function are tiny and always less than $10^{-4}$. We confirm earlier conclusions by Broderick et al. and our group that the created pair beam distribution function is quickly unstable in the unmagnetized intergalactic medium. Therefore, there is no need to require the existence of small intergalactic magnetic fields to scatter the produced pairs, so that the explanation (made by several authors) for the Fermi non-detection of the inverse Compton scattered GeV gamma-rays by a finite deflection intergalactic magnetic field is not necessary. In particular, the various derived lower bounds for the intergalactic magnetic fields are invalid due to the pair beam instability argument.

Key words: cosmic rays – diffuse radiation – gamma rays: galaxies – instabilities – plasmas

Online-only material: color figures

1. INTRODUCTION

The new generation of air Cherenkov TeV γ-ray telescopes (HESS, MAGIC, VERITAS) have detected about 30 cosmological blazars with strong TeV photon emission; the most distant blazars are 3C279 (redshift $z_r = 0.536$), 3C66A ($z_r = 0.444$), and PKS 1510−089 ($z_r = 0.361$). Any of these are more distant than $z_r = 0.16$ produce energetic $e^\pm$ particle beams in double photon collisions with the extragalactic background light (EBL). These pairs with typical Lorentz factors $\gamma = 10^5$ $\Gamma_0$ (with scaling factor $\Gamma_0$) are expected to inverse Compton (IC) scatter on the cosmic microwave background radiation, on a typical scale length $l_{\text{IC}} \sim 0.75 \Gamma_0^{-1}$ Mpc, thus producing gamma-rays with energy of order 100 GeV, which have not been detected by the Fermi satellite. Given the still relatively short distance $l_{\text{IC}}$, both pair production and IC emission occur primarily in cosmic voids of the intergalactic medium (IGM), which fill most of the cosmic volume. It has been argued that the IC scattered gamma-rays then are still energetic enough for further pair-production interactions, giving rise to a full electromagnetic cascade as in a vacuum.

However, the pair beam is subject to two stream-like instabilities that are both electrostatic and electromagnetic in nature (Broderick et al. 2012; Schlickeiser et al. 2012b). In this case, the electromagnetic pair cascade does not contribute to the multi-GeV flux, as most of the pair beam energy is transferred to the IGM with important consequences for its thermal history. Moreover, there is no need to require the existence of small intergalactic magnetic fields to scatter the produced pairs, so the explanation for the Fermi non-detection of the IC-scattered GeV gamma-rays by a finite deflecting intergalactic magnetic field (Neronov & Vovk 2010; Tavecchio et al. 2011; Dolaj et al. 2011; Taylor et al. 2011; Dermer et al. 2011; Takahashi et al. 2012; Vovk et al. 2012) is not necessary.

In their instability analysis, Schlickeiser et al. (2012b, hereafter referred to as Paper I) and Broderick et al. (2012) approximated the pair parallel momentum distribution function $g(x) = \delta(x - x_c)$ by a sharp delta function, where $x = p_\parallel / (m c)$ denotes the parallel pair momentum $p_\parallel$ in units of $m c = 5.11 \times 10^{15}$ eV/c (c: speed of light), which is commonly referred to as reactive linear instability analysis. This approximation has been recently criticized by Miniati & Elyiv (2013), who noted that the finite momentum spread of the pair distribution function (referred to as a kinetic instability study) will significantly reduce the maximum electrostatic growth rate to a level that the full electromagnetic pair cascade as in a vacuum is not modified. The study of Cairns (1989), based on nonrelativistic kinetic plasma equations, indicated that the kinetic/reactive instability character depends strongly on the plasma beam and plasma background parameters, such as beam density $n_b$, beam speed $\beta c$ and background particle density $N_c$, and temperature $T_c$. Significant differences between reactive and kinetic instability rates occur, particularly for beam-to-background particle density ratios exceeding $n_b / N_c > 10^{-5}$. However, as argued below, in our case of pair beams in the IGM medium, this ratio is of the order of $n_b / N_c \simeq 10^{-15}$, much below the critical value $10^{-5}$, so that we are in a regime where reactive and kinetic instability studies should differ significantly according to Cairns (1989).

However, as noted, the work of Cairns (1989) is based on nonrelativistic kinetic plasma equations. The purpose of this work is to investigate the claim of Miniati & Elyiv (2013) for parallel propagating electrostatic fluctuations using the correct relativistic kinetic plasma equations. Relativistic kinetic instability studies are notoriously difficult and complicated.
due to plasma particle velocities close to the speed of light. Therefore, extreme care is necessary in order to include all relevant relativistic effects. We therefore will repeat in detail the linear instability analysis in the kinetic limit using the realistic pair momentum distribution function. For mathematical simplicity, we will restrict our analysis to parallel wave vector orientations with respect to the direction of the TeV gamma-rays generating the relativistic pairs. In our analysis, we will also use a more realistic modeling of the fully ionized IGM plasma as isotropic thermal distributions.

2. DISTRIBUTION FUNCTIONS AND EARLIER REACTIVE INSTABILITY RESULTS

2.1. Intergalactic Medium

The unmagnetized IGM consists of protons and electrons of density \( N_e = 10^{-7} N_p \) cm\(^{-3} \) (with scaling factor \( N_\gamma \)). Any neutral atoms or molecules do not participate in the electromagnetic interaction with the pairs. In Paper I, we modeled the IGM plasma with the cold isotropic particle distribution functions \((a = e, p)\)

\[
F_a(p, p_\perp) = \frac{N_a}{2\pi p_\perp} \delta(p) \delta(p_\perp),
\]

where \( p \) and \( p_\perp \) denote the momentum components parallel and perpendicular to the incoming \( \gamma \)-ray direction in the photon–photon collisions, respectively. Here, we take into account the finite temperature \( T_{\gamma} \) of the IGM plasma particles, adopting the isotropic Maxwellian distribution function

\[
F_a(p) = \frac{N_a \mu_a}{4\pi (m_a c^2) K_2(\mu_a)} e^{-\mu_a \sqrt{1 + \frac{p^2}{m_a c^2}}},
\]

where \( p = \sqrt{p_\parallel^2 + p_\perp^2} \) and \( \mu_a = m_e c^2 / (k_B T_a) = 2 / \beta_a^2 \), where \( \beta_a = \sqrt{2 k_B T_a / (m_e c^2)} \) is the thermal IGM velocity in units of the speed of light. Photoionization models of the IGM (Hui & Gnedin 1997; Hui & Haiman 2003) indicate nonrelativistic electron temperatures \( T_e = 10^4 T_4 \) K, implying very small values of \( \beta_e = 1.8 \times 10^{-3} T_4^{1/2} \ll 1 \) and large values of \( \mu_e \gg 1 \). If we scale the proton temperature \( T_p = \chi T_e \), we obtain \( \beta_p = \sqrt{\frac{\chi}{\chi - 1}} \beta_e \) with the electron–proton mass ratio \( \chi = m_e / m_p = 1/1836 \). For proton to electron temperature ratios \( \chi \ll \xi \sim 1836 \), we find that \( \beta_p \ll \beta_e \).

2.2. Intergalactic Pairs from Photon–Photon Annihilation

Schlickeiser et al. (2012a) analytically calculated the pair production spectrum from a power law distribution of the gamma-ray beam up to the maximum energy \( M \) (all energies in units of \( m_e c^2 \)), interacting with the isotropically soft photon Wien differential energy distribution \( N(k_\gamma) \propto k_\gamma^2 \exp(-k_\gamma/\Theta) \) representing the EBL with \( \Theta \simeq 2 \times 10^{-7} \) corresponding to 0.1 eV. They found that the pair production spectrum is highly beamed into the direction of the initial gamma-ray photons, so that a highly anisotropic, ultra-relativistic velocity distribution of the pairs results. With respect to the parallel momentum \( x = p_\parallel / (m_e c) \), the pair momentum distribution function is strongly peaked at \( M_e = \Theta^{-1} \) for the case of effective pair production \( M \gg M_e \). The differential parallel momentum spectrum of the generated pairs can be well approximated as

\[
n(x) = A_1 e^{-\frac{x}{x_c}} \left[ 1 + \left( \frac{x}{x_c} \right)^{3/2} \right] H(x),
\]

with the step function \( H(x) = [1 + (x/|x|)]/2 \) and the two characteristic normalized momenta

\[
x_c = \frac{M_e}{\ln r_0}, \quad x_b = M_e \frac{t_0^2/3}{2/3} = 0.2 M_e \frac{t_0^2/3}{2/3},
\]

where \( r_0 = \sigma_T N_0 R \), with the total number density of EBL photons \( N_0 \simeq 1 \) cm\(^{-3} \), denotes the transversal optical depth of gamma-rays. Both characteristic momenta \( x_b > x_c \gg 1 \) are very large compared with unity since \( M_e \simeq 2 \times 10^6 \). As noted in Schlickeiser et al. (2012a), the analytical approximation (Equation (3)) agrees rather well with the numerically calculated production spectrum using the code of Elyiv et al. (2009). The parallel momentum spectrum of pairs (Equation (3)) exhibits a strong peak at \( x_c \), is exponentially reduced \( \propto \exp(-x_c/x) \) at smaller momenta, and exhibits a broken power law at higher momenta (see Figure 7 in Schlickeiser et al. 2012a).

During this analysis here, we will simplify the parallel momentum spectrum (Equation (3)) slightly to the form

\[
n(x) = A_0 g(x), \quad g(x) = x^{-s} e^{-\frac{x}{x_c}} H(x),
\]

where we keep the essential features of the spectrum (Equation (3)), namely the exponential reduction below \( x_c \) and the power-law behavior at high parallel momentum values. But instead of allowing for the broken power-law behavior above and below \( x_b \), we represent this part only as a single power law with a spectral index \( s = p - 1/2 \). As we will see later, this simplification only affects the damping rate of plasma fluctuations, whereas the growth rate is caused by the exponential reduction below \( x_c \).

The associated pair phase space density is then given by

\[
f_b(p_\perp, x) = \frac{n_b}{2\pi p_\perp m_e c} A_0 g(x) G(p_\perp, b)
\]

with the normalization factor \( A_0 \) determined by the total beam density

\[
n_b = 10^{-22} n_{22} = \int d^3 p_b \text{ cm}^{-3}.
\]

In Paper I, we ignored any finite spread of the pair distribution function in perpendicular momentum \( p_\perp \), i.e.,

\[
G(p_\perp) = \delta(p_\perp).
\]

Here, we allow for such a perpendicular spread by adopting

\[
G(p_\perp, b) = \frac{H[bm_e c - p_\perp]}{bm_e c}
\]

with finite values of \( b \). The special form (Equation (9)) of the perpendicular momentum distribution function is chosen because of the limit

\[
\lim_{b \to 0} G(p_\perp, b) = \delta(p_\perp),
\]

which can be readily proven by inspecting with an arbitrary function \( W(p_\perp) \) the expression

\[
Y = \lim_{b \to 0} \int_0^\infty dp_\perp \int_0^\infty \frac{b m_e c}{d p_\perp} \left[ \frac{d W(p_\perp) G(p_\perp, b)}{dp_\perp} \right] d p_\perp.
\]
Using the Taylor expansion of the function $W$ near $p_\perp = 0$

$$W(p_\perp) \simeq W(p_\perp = 0) + p_\perp \left[ \frac{dW(p_\perp)}{dp_\perp} \right]_{p_\perp = 0} + \cdots \quad (12)$$

readily yields

$$Y = \lim_{b \to 0} \left[ W(p_\perp = 0) + \frac{m_e c b}{2} \left[ \frac{dW(p_\perp)}{dp_\perp} \right]_{p_\perp = 0} + \cdots \right] = W(p_\perp = 0). \quad (13)$$

Therefore, in the limit $b \to 0$, the broadened perpendicular distribution function (Equation (9)) reduces to the distribution function (Equation (8)) with no perpendicular spread.

Using the phase space density (Equation (6)) with Equations (5) and (9) in the normalization condition, (Equation (7)) then yields

$$1 = A_0 \int_0^\infty dx \, g(x) = A_0 \Gamma(s - 1) U(s - 1, s, x_c) \simeq A_0 \Gamma(s - 1) x_c^{s-1}, \quad (14)$$

where $\Gamma(a)$ is the gamma function and $U(a, b, z)$ denotes the confluent hypergeometric function of the second kind. Its argument $x_c$ is very large, so that we have approximated $U(s - 1, s, x_c) \simeq x_c^{s-1}$ for values of $s > 1$. Therefore, the normalization factor has to be

$$A_0 = \frac{x_c^{s-1}}{\Gamma(s - 1)}. \quad (15)$$

Now, we estimate the value of the maximum normalized perpendicular momentum $b$. With extensive Monte Carlo simulations, Miniati & Elyiv (2013) determined the maximum angular spread of the beamed pairs to be $\Delta \phi = 10^{-5}$, in agreement with the kinematic estimate (see Equation (5) of Miniati & Elyiv 2013)

$$10^{-5} = \Delta \phi = \frac{m_e c^2 \sigma_{0}(s_0 - 1)}{2 E_{\gamma}} < \frac{m_e c^2 s_0}{2 E_{\gamma}} = \frac{\Theta}{2}, \quad (16)$$

where we use the invariant maximum center of mass energy square $s_0 = E_{\gamma}/\Theta/m_e c^2$. This maximum angular spread determines

$$\frac{p_{\perp,\text{max}}}{p_1} = \frac{b}{x} = \tan (\Delta \phi) = \frac{\tan(\Theta/2)}{2} \simeq \Theta \quad \frac{1}{2}, \quad (17)$$

so that with Equation (4)

$$b = \frac{x \Theta}{2} \simeq \frac{x_c \Theta}{2} \simeq \frac{1}{2 \ln \tau_0} = \frac{7.2 \times 10^{-2}}{1 + \frac{\ln \tau_0}{3 \ln 10}}, \quad (18)$$

which, for $\tau_0 = 10^3 \tau_3$, is well below unity. The maximum perpendicular momenta of the generated pair distribution are less than 40 keV/c.

### 2.3. Reactive Instability Results

As noted before, in Paper I we approximated the parallel pair distribution function (Equation (11)) by a sharp delta function $m_e c g(x) = \delta(x - x_c)$ and ignored any finite spread i.e., $G(p_\parallel) = \delta(p_\parallel)$. Moreover, we modeled the unmagnetized IGM as a fully ionized cold electron–proton plasma. In agreement with the earlier reactive instability study of Broderick et al. (2012), we found that very quickly oblique (at propagation angle $\theta$) electrostatic fluctuations are excited. The growth rate $(\Im \omega_{\text{max}}$ and the real part of the frequency $(\Re \omega_{\text{max}}$ at maximum growth are given by

$$(\Im \omega_{\text{max}} \simeq \frac{3^{1/2}}{2} \omega_{p,e} \alpha(\theta) = 1.5 \times 10^{-6} N_{17}^{1/6} n_{22}^{1/3} x_{c,6}^{-1/3} \times [1 - \beta_1^2 \cos^2 \theta]^{1/3} \quad \text{Hz} \quad (19)$$

and

$$(\Re \omega_{\text{max}} \simeq \omega_{p,e} \left( 1 - \frac{\alpha(\theta)}{2} \right) \quad = \omega_{p,e} \left[ 1 - 5 \times 10^{-8} \left( \frac{n_{22}}{N_{17} x_{c,6}} \right)^{1/3} \left[ 1 - \beta_1^2 \cos^2 \theta \right]^{1/3} \right], \quad (20)$$

respectively, with the electron plasma frequency $\omega_{p,e} = 17.8 N_{17}^{1/2}$. Note that we have corrected a mistake in Paper I in the numerical factor in the growth rate (Equation (12)). $n_b = 10^{-22} n_{22} \text{ cm}^{-3}$ represent typical pair densities in cosmic voids, $x_e = 10^3 x_{c,6}$ and

$$\alpha(\theta) = 10^{-7} \left( 1 - \beta_1^2 \cos^2 \theta \right)^{1/3} n_{22}^{1/3} N_{17}^{-1/3} x_{c,6} \ll 1 \quad (21)$$

with $\beta_1 = x_c / \sqrt{1 + x_c^2}$.

The maximum growth rate occurs at the oblique angle $\theta_E = 39.2^\circ$ and provides the shortest electrostatic growth time:

$$\tau_{\text{e}}^{-1} = \gamma_{E,\text{max}} = 1.1 \times 10^{-6} n_{22}^{1/3} N_{17}^{1/6} x_{c,6} \quad \text{Hz}. \quad (22)$$

Even if nonlinear plasma effects are taken into account, we concluded in Paper I that most of the pair beam energy is dissipated generating electrostatic plasma turbulence, which prevents the development of a full electromagnetic pair cascade as in a vacuum.

For later comparison, we note that for parallel wave vector orientations $\theta = 0$ Equation (14) reduce to

$$\alpha(0) = 10^{-11} \frac{n_{22}^{1/3}}{N_{17}^{1/3} x_{c,6}} \quad (23)$$

implying for the real and imaginary frequency parts at maximum growth (Equations (19) and (20))

$$\overline{(\Re \omega_{\text{max}}(\theta = 0) \simeq \omega_{p,e} \left( 1 - \frac{\alpha(0)}{2} \right) \quad = \omega_{p,e} \left[ 1 - 5 \times 10^{-12} \left( \frac{n_{22}}{N_{17} x_{c,6}} \right)^{1/3} \right] \simeq \omega_{p,e} \quad (24)$$

and

$$\overline{(\Im \omega_{\text{max}} \simeq \frac{3^{1/2}}{2} \omega_{p,e} \alpha(0) = 1.5 \times 10^{-10} N_{17}^{1/6} n_{22}^{1/3} x_{c,6}^{-1}} \quad \text{Hz}. \quad (25)$$
3. ELECTROSTATIC DISPERSION RELATION

The dispersion relation of weakly damped or amplified \(|\gamma| \ll \omega_R\) parallel electrostatic fluctuations with wavenumber \(k\) and frequency \(\omega = \omega_R + \gamma\) in an unmagnetized plasma with gyrotropic distribution functions is given by (Schlickeiser 2010)

\[
0 = \Lambda(\omega, k) = 1 + \sum_a \int_0^\infty dp_\perp \frac{\rho_a}{\omega_a} \frac{\partial f_a}{\partial p_\perp} + \int_0^\infty dp_\perp \frac{\rho_\perp}{\Gamma_a(\omega - k v_\parallel)} \frac{\partial f_a}{\partial p_\perp},
\]

(26)

The dispersion function \(\Lambda(k, \omega)\) is symmetric \(\Lambda(\omega, -k) = \Lambda(\omega, k)\) with respect to the wavenumber \(k\), so that it suffices to discuss positive values of \(k > 0\). Inserting the distribution functions of Equations (2), (6), and (9), using nonrelativistic values of \(\beta_a \ll 1\), then provides

\[
0 = \Lambda(R, I) = 1 - \frac{2 \omega^2 p_e b}{N_e} \lim_{t \to 0} D_p(R, I, b) - \sum_a \frac{\omega^2 p_a}{k^2 c^2} Z'(\frac{z}{\beta_a}),
\]

(27)

where \(Z'(t)\) denotes the first derivative of the plasma dispersion function (Fried & Conte 1961; Schlickeiser & Yoon 2012, Appendix A) with a complex argument as \(z = \omega/(k c) = R + i I\) with \(R = \omega_R/(k c)\) and \(I = \gamma/(k c)\). For weakly damped/amplified fluctuations, we use the approximations

\[
Z(t) \simeq -2 i \sqrt{t} e^{-i \omega_H t} H[1 - |R|] - 2(1 - 2t^2), \quad \text{for } |t| \ll 1,
\]

(28)

\[
Z'(t) \simeq -2i \sqrt{t} e^{-i \omega_H t} H[1 - |R|] + \frac{1}{t^2} \left[1 + \frac{3}{2t^2}\right], \quad \text{for } |t| \gg 1.
\]

(29)

We note that the imaginary part is the same in both approximations. The expression

\[
D_p(R, I, b) = \frac{1}{b} \int_0^b dq \int_0^\infty dx \frac{\chi \frac{d\chi}{dx}}{x - z + \sqrt{1 + q^2 + x^2}}
\]

\[
= \frac{1}{z} \int_0^\infty dx \frac{d\chi}{dx} + \frac{1}{b} \int_0^b dq \int_0^\infty dx \frac{\chi \frac{d\chi}{dx}}{\sqrt{1 + q^2 + x^2}} - \frac{1}{2},
\]

(30)

with \(q = p_\perp/(m_e c)\), represents the pair beam contribution to the electrostatic dispersion relation.

The first \(x\) integral in Equation (29) vanishes because \(g(0) = g(\infty) = 0\), leaving

\[
D_p(R, I, b) = \frac{1}{b} \int_0^b dq \int_0^\infty dx \frac{\chi \frac{d\chi}{dx}}{\sqrt{1 + q^2 + x^2}} - R - i I.
\]

(31)

With Dirac’s formula

\[
\lim_{t \to 0} \frac{1}{a - i t} = \mathcal{P} \frac{1}{a} + i \pi \delta(a),
\]

(32)

where \(\mathcal{P}\) denotes the principal value, we obtain for the limit

\[
\lim_{t \to 0} D_p(R, I, b) = \frac{1}{b} \int_0^b dq \int_0^\infty dx \frac{z \frac{d\chi}{dx}}{\sqrt{1 + q^2 + x^2}} - R - i I.
\]

(33)

The last integral has a nonvanishing value provided that \(x_0(R, q) \in [0, \infty]\), which requires subluminal real phase speeds \(\omega \ll c\).

Because of the small factor \((2n_e/N_e) \ll 1\), we ignore the contribution of the real principal part of Equation (32) to the dispersion relation (Equation (27)), but we keep the imaginary part with the result

\[
0 = \Lambda(R, I) \simeq 1 - \sum_a \frac{\omega^2 p_a}{k^2 c^2} Z\left(\frac{R + i I}{\beta_a}\right)
\]

\[
- \frac{2 \omega^2 p_e b}{N_e k^2 c^2(1 - R^2)^{3/2}} H[1 - |R|] \times \int_0^b dq \sqrt{1 + q^2} \frac{d\chi}{dx} \frac{d\chi}{dx} x_0(R, q)
\]

\[
= 1 - \frac{1}{k^2 \beta_e^2} \left[Z\left(\frac{R + i I}{\beta_e}\right) + \frac{1}{\chi} Z\left(\frac{R + i I}{\sqrt{\chi \xi \beta_e}}\right)\right]
\]

\[
- \frac{2 \pi p_b}{N_e} H[1 - |R|] \frac{x_0^{\gamma-1}}{k^2 \Gamma(s - 1)(1 - R^2)^{3/2}} J(b),
\]

(34)

where we have introduced the integral

\[
J(b) = \frac{1}{b} \int_0^b dq \sqrt{1 + q^2} \frac{d\chi}{dx} \frac{d\chi}{dx} x_0(R, q),
\]

(35)

the normalized wavenumber

\[
\kappa = \frac{k c}{\omega_{pe}},
\]

(36)

and the normalization constant (Equation (15)).

Separating the dispersion function into real and imaginary parts \(\Im \Lambda + \Re \Lambda\), we find

\[
\Re \Lambda(R, I) = 1 - \frac{1}{k^2 \beta^2} \left[\Im Z\left(\frac{R + i I}{\beta_e}\right) + \frac{1}{\chi} \Im Z\left(\frac{R + i I}{\sqrt{\chi \xi \beta_e}}\right)\right]
\]

(37)

and

\[
\Im \Lambda(R, I) = -\frac{1}{k^2 \beta^2} \left[\Im Z\left(\frac{R + i I}{\beta_e}\right) + \frac{1}{\chi} \Im Z\left(\frac{R + i I}{\sqrt{\chi \xi \beta_e}}\right)\right]
\]

\[
- \frac{2 \pi p_b}{N_e} H[1 - |R|] \frac{x_0^{\gamma-1}}{k^2 \Gamma(s - 1)(1 - R^2)^{3/2}} J(b).
\]

(38)
respectively, where $R = \omega_R/(k c) = \omega_R/(\omega_{p,e} \kappa)$. We then find that
\[
\gamma(\kappa) = -\omega_{p,e} \kappa \frac{\partial \mathcal{A}(R, I = 0)}{\partial R} \bigg|_{R=0} = \gamma_b(\kappa) - \gamma_L(\kappa)
\] (49)

is given by the difference of the growth rate $\gamma_b(\kappa)$ from the anisotropic relativistic pair distribution and the positively counted Landau damping rate $\gamma_L(\kappa)$ from the thermal IGM plasma with
\[
\gamma_p(\kappa, b) = \frac{2\pi \omega_{p,e} n_b}{\gamma_R(\kappa, b)} H[1 - R] x e^{1 - \chi_b/R} J(b)
\] (50)

and
\[
\gamma_L(\kappa) = \frac{2\pi^{1/2} \omega_{p,e} R H[1 - R]}{\gamma_R(\kappa, b)} \left[ e^{-\chi_b/R} + \frac{1}{\pi^{1/2} \kappa^{3/2}} e^{-\chi_b/R} \right]
\] (51)

3.2. Electrostatic Modes

In Appendix A, we derive approximations of the integral in Equation (40), valid for values of $b \leq b_0$, where $b_0 = 7.2 \times 10^{-2}$. According to the estimate from Equation (18), $b$ is significantly smaller than unity. In terms of the value of Equation (42) at $b = 0$, we obtain
\[
J(b) \simeq J(0) B(X)
\] (43)

with
\[
X(b, A) = \sqrt{\frac{A}{2b}},
\] (44)

where the correction function
\[
B(X) = \frac{e^{X^2}}{X} [F(X) + h(A, s)(F(X) - X)],
\] (45)

with
\[
h(A, s) = \frac{(s - 1)A - s(s - 2)}{2A(A - s)}
\] (46)
can be expressed in terms of Dawson’s integral $F(X)$ (see definition (A8)). If the correction function (Equation (45)) is smaller than unity, the perpendicular spread will reduce the growth rate $\gamma_0$ of the fluctuations. If the correction function is greater than unity, it will enhance the growth rate $\gamma_0$, each case compared with the case of no perpendicular spread $b = 0$.

3.1. General Kinetic Instability Analysis

For weakly damped or amplified ($|\gamma| \ll \omega_R$) fluctuations, the real and imaginary phase speed (or frequency) parts of the fluctuations are given by (Schlickeiser 2002, p. 263)
\[
\mathcal{A}(R, I = 0) = 0
\] (47)

and
\[
I = \frac{\gamma}{k c} = -\frac{\partial \mathcal{A}(R, I = 0)}{\partial R} \bigg|_{R=0}
\] (48)
which is positive for values of $A > s$ corresponding to

$$R < \frac{1}{\sqrt{1 + (s/x_c)^2}} \simeq 1 - \frac{s^2}{2x_c^2},$$

(57)

given the very large value of $x_c$ (see Equation (4)). As long as $R \leq 1 - \epsilon$ with

$$\epsilon = \frac{s^2}{2x_c^2} = \frac{1}{2}[\epsilon \ln \tau_0]^2 < O(10^{-12}),$$

(58)

the pair parallel momentum distribution provides a positive growth rate $\gamma_p$.

At wavenumbers $\kappa_L < \kappa \ll \beta^{-1}$, the dispersion relation (Equation (B7)) of Langmuir oscillations readily yields

$$\frac{\partial \mathcal{H}(R, \kappa)}{\partial R} = 2\delta_c^2 R^3 + 6\beta_c^2 (1 + \chi_c^2)$$

$$\simeq 2 \frac{\kappa \kappa_L^2}{\kappa^2 R^3} [R^2 + 3\beta_c^2] \simeq \frac{2}{\kappa^2 R^3}$$

(59)

because Langmuir oscillations occur at phase speeds $R \gg \beta_c$.

Inserted into Equations (56) and (51), the growth rate as a function of the variable $A$ in Equation (41) becomes

$$\gamma_p(A, b = 0) = \gamma_p^0 \kappa x_c C(A, s)$$

(60)

with

$$C(A, s) = \frac{A^{s-2}(A - s)}{\Gamma(s - 1)} e^{-A}$$

(61)

and the constant

$$\gamma_p^0 = \frac{\pi \omega_p \epsilon N_b}{N_c} H[1 - R],$$

(62)

whereas the Landau damping rate is

$$\gamma_L = \pi^{1/2} \omega_p \epsilon N_c H[1 - R] \left( \frac{R}{\beta_c} \right)^3 e^{-x_c^2},$$

(63)

### 4.2. Maximum Growth Rate

The function $C(A, s)$, defined in Equation (61), is plotted in Figure 2 for three values of $s = 1.5, 2$, and $2.5$. It has one zero at $A_N(s) = s$, is negative for $A < s$, and positive for $A > s$, in agreement with Equation (57). Extrema are located at values of $A$ satisfying

$$A^2 - (2s - 1)A + s(s - 2) = 0.$$  

(66)

For values of $1 < s \leq 2$, the function $C(A, s)$ attains its maximum value at

$$A_0(1 < s \leq 2) = \frac{2s - 1}{2} \left[ 1 + \sqrt{1 + \frac{s(2 - s)}{(s - 1) s}} \right].$$

(67)

For the special case $s = 2$, we find $A_N(2) = 2$ and $A_0(2) = 3$ and the maximum value

$$C_{\text{max}}(s = 2) = e^{-3}.$$
For values of \( s > 2 \), the function \( C(A, s) \) has a negative minimum at

\[
A_{\text{min}}(s > 2) = \frac{2s - 1}{2} \left[ 1 - \sqrt{1 - \frac{s(s - 2)}{(s - \frac{1}{2})^2}} \right]
\]

and a positive maximum at

\[
A_0(s > 2) = \frac{2s - 1}{2} \left[ 1 + \sqrt{1 - \frac{s(s - 2)}{(s - \frac{1}{2})^2}} \right].
\]

It is straightforward to show that the location of the maximum \( A_0(s) < A_x(s) \) is always above the location of the zero \( A_x(s) \), in agreement with Figure 2. In Table 1, we calculate the locations \( A_0(s) \) and values of \( C_{\text{max}}(s) \) for different values of \( s \).

| \( s \) | \( A_0 \) | \( C_{\text{max}}(s) \) | \( B(A_0(s), b = 0.1, s) - 1 \) |
|---|---|---|---|
| 1.5 | 2.32 | 1.17 \times 10^{-2} | -1.35 \times 10^{-5} |
| 2.0 | 3.00 | 4.98 \times 10^{-2} | -2.25 \times 10^{-5} |
| 2.5 | 3.66 | 6.44 \times 10^{-2} | -3.35 \times 10^{-5} |
| 3.0 | 4.30 | 7.58 \times 10^{-2} | -4.62 \times 10^{-5} |
| 4.0 | 5.56 | 9.28 \times 10^{-2} | -7.62 \times 10^{-5} |

For ease of exposition, we continue with the simplest case \( s = 2 \). From Equation (60), we then obtain for the maximum kinetic growth rate

\[
\gamma_p^{\text{max}}(b = 0) = \frac{\gamma_p^0 K_0 x_c}{\varepsilon^3},
\]

which occurs at \( A_0 = 3 \), corresponding to values of \( K_0(R) = x_c/3 \) and values of \( R_0^2 = \frac{1}{1 + \frac{9}{4 \beta_e^2 \gamma_e^2}} \simeq 1 - \frac{9}{\beta_x^2 \gamma_x^2} \).

slightly below unity. In Figure 3, we show the growth rate from Figure 1 now as a function of the variable \( A \). We note that the location of the maximum and the zero in the case \( s = 2 \) agree exactly with the analytical values.

With the dispersion relation (Equation (52)) and the definition (Equation (53)), we find for the corresponding wavenumber

\[
\kappa_0 = \frac{1}{\sqrt{1 - \frac{3 \beta_e^2}{4} - \frac{9}{4 \beta_x^2 \gamma_x^2}}} \approx 1 + \frac{3 \beta_e^2}{4} + \frac{9}{2 \beta_x^2} \approx 1.
\]

Maximum growth occurs at frequencies

\[
\omega_{R, 0} = \omega_{p, e} K_0 R_0 \approx \omega_{p, e},
\]

in perfect agreement with the reactive result (Equation (24)).

Moreover, the maximum growth rate (Equation (71)) is given by

\[
\gamma_p^{\text{max}}(b = 0) = 2.8 \times 10^{-9} \frac{N_{\text{eff}} x_c^6}{N_{\gamma}^{1/2}} \text{ Hz},
\]

which is about an order of magnitude larger than the maximum reactive growth rate (Equation (25)). Apparently, the spread in parallel momentum of the pair distribution function does not reduce the maximum growth rate of parallel Langmuir oscillations, in disagreement with the result of Miniati & Elyiv (2013).

At the same values of \( R_0 \) and \( \kappa_0 \), because of the exponential factor, the Landau damping rate (Equation (63)) of Langmuir oscillations is negligibly small:

\[
\gamma_L(R_0) = \pi^{1/2} \omega_{p, e} \kappa_0 R_0 \left( \frac{R_0}{\beta_e} \right)^3 e^{-\frac{k_0^2}{\beta_e^2}} \simeq \frac{\pi^{1/2} \omega_{p, e}}{\beta_e^4} e^{-\frac{1}{\beta_e^2}} < 10^{-10}.
\]

5. KINETIC INSTABILITY ANALYSIS OF LANGMUIR OSCILLATIONS FOR A FINITE PERPENDICULAR SPREAD

With the correction function (Equation (45)) for finite perpendicular spreads below the limit \( b_\text{h} \), the growth rate in this case

\[
\gamma_p(b) = B(X) \gamma_p(b = 0)
\]

is simply related to the growth rate \( \gamma_p(b = 0) \). The growth rate \( \gamma_p(b) \) with finite spread as compared with the growth rate \( \gamma_p(b = 0) \) with no finite spread is enhanced (reduced) if the correction function (Equation (45)) is greater (smaller) than unity. The correction function reads

\[
B(X) = B(A, b, s) = \frac{e^{x^2}}{X} [(1 + h)F(X) - hX]
\]

with the function

\[
h(A, s) = \frac{(s - 1)A - s(s - 2)}{2A(A - s)}.
\]

We noted before that the growth rate \( \gamma_p(b = 0) \) is positive only for values of \( A > s \), so we restrict our analysis to this range. For
A > s, Equation (79) is positive for all values of $A > s > 1$. With $A = s + t$, Equation (79) reads
\[
 h(t, s) = \frac{t + (s - 1)t}{2t + s} - \frac{s - 1}{2(t + s)} + \frac{s}{2t + s} \tag{80}
\]
with $t \in (0, \infty]$. The function is strictly decreasing since
\[
 \frac{dh(t, s)}{dt} = -\frac{(s - 1)t^2 + 2st + s^2}{2t^2(t + s)^2} \tag{81}
\]
is always negative. No extreme values occur in the interval $(0, \infty]$. For later use, we note that the condition $h(A, s) = 1/2$ leads to the equation
\[
 A^2 - (2s - 1)A + s(s - 2) = 0, \tag{82}
\]
which is identical to Equation (66), determining the maximum growth rate $\gamma_p^\text{max}(b = 0)$ through the function $C(A, s)$. Hence, at the maximum $A_0(s)$, the function
\[
 h(A_0(s), s) = \frac{1}{2} \tag{83}
\]
for all values of $s$. Moreover, for larger values of $A > A_0(s)$, the function $h(A, s) < 1/2$.

5.1. Correction Function for the Maximum Growth Rate

The maximum growth rate $\gamma_p^\text{max}(b = 0)$ occurs at $A_0(s)$ listed in Table 1. For values of $b < 0.1$, the variable in Equation (44)
\[
 X = \sqrt{\frac{A_0(s)}{2}} b < 0.071 \sqrt{A_0(s)} < 0.17 \tag{84}
\]
is smaller than unity for all values of $b < 0.1$, because for $s \leq 4$ we calculated $A_0(s) \leq A_0(4) = 5.56$. We therefore use the series expansion (Equation (A11)) for Dawson’s integral in Equation (78) to find
\[
 B(X \ll 1) \simeq e^{X^2} \left[ 1 - \frac{2}{3}(1 + h)X^2 \left( 1 - \frac{2}{5}X^2 \right) \right] \simeq 1 - \frac{2h - 1}{3} X^2 - \frac{4h - 1}{10} X^4. \tag{85}
\]

With the value of Equation (83), the quadratic terms vanish and we obtain the correction
\[
 B(A_0(s), b, s) \simeq 1 - \frac{X^4}{10} = 1 - \frac{A_0^2(s)b^4}{40}. \tag{86}
\]

For the maximum value of $b = 0.1$, we calculate the reduction factor $B(A_0(s), b, s) \simeq 1$ for different values of $s$. The results are listed in Table 1. As can be seen, the reduction factors due to the finite spread in the pair distribution function are tiny, always less than $-10^{-4}$. Contrary to the statement of Miniati & Elyiv (2013), we find that the finite perpendicular spread does not significantly reduce the maximum growth rate.

5.2. General Behavior of the Correction Function

Dawson’s integral satisfies the linear differential equation
\[
 \frac{dF(X)}{dX} = 1 - 2XF(X), \tag{87}
\]
so that the first derivative of the correction function (Equation (78)) is given by
\[
 \frac{d}{dX} B(X) = \frac{e^{X^2}}{X^2} \left[ (1 + h)X - (1 + h)F(X) - 2hX^3 \right]. \tag{88}
\]
The extreme value of the correction function $B(X_E)$ occurs at $X_E$ given by the solution of the transcendental equation
\[
 X_E - F(X_E) = \frac{2h}{1 + h} X_E^2. \tag{89}
\]
Inserting this condition into Equation (78), we obtain for the extreme value of the correction function
\[
 B_E = e^{X_E^2} \left[ 1 - 2hX_E^2 \right]. \tag{90}
\]
We recall that for values of $A > A_0(s)$, corresponding to $X_E > b\sqrt{A_0(s)}/2$, the function $h \ll 1/2$. The first and second derivative of Equation (90) are given by
\[
 \frac{dB_E}{dX_E} = 2X_E e^{X_E^2} \left[ (1 - 2h) - 2hX_E^2 \right] \tag{91}
\]
and
\[
 \frac{d^2B_E}{dX_E^2} = 2e^{X_E^2} \left[ (1 - 2h) + 2(1 - 5h)X_E^2 - 4hX_E^4 \right]. \tag{92}
\]
The function $B_E$ has a single maximum at $X_E^\text{max} = (1 - 2h)/2h$ given by
\[
 B_E^\text{max} = 2he^{\frac{h}{2} - 1}. \tag{93}
\]
For the given $b$, Equation (90) corresponds to the extreme value
\[
 B_E(A_E) = e^{\frac{i\pi}{2}s} \left[ 1 - h^2 A_E \right]
 = e^{\frac{i\pi}{2}s} \left[ 1 - \frac{b^2}{2} \left( s - 1 - \frac{s}{A_E - s} \right) \right], \tag{94}
\]
where we inserted the function $h(A_E, s)$ from Equation (79). Even without knowing the value of $A_E$, we can draw some interesting conclusions from Equation (94).

For values of $s \ll A_E \ll (2/b^2)$, Equation (94) approaches
\[
 B_E(s \ll A_E \ll \frac{2}{b^2}) \simeq 1 - \frac{b^2(s - 1)}{2}, \tag{95}
\]
producing at most a tiny correction over a wide range of $s \ll A_e \ll 200$, in agreement with our earlier discussion of the maximum growth rate.

6. SUMMARY AND CONCLUSIONS

The interaction of TeV gamma-rays from distant blazars with the EBL produces relativistic electron–positron pair beams by the photon–photon annihilation process. The created pair beam distribution is unstable to linear two-stream instabilities of both electrostatic and electromagnetic nature in the unmagnetized IGM. Based on a linear reactive instability analysis, Broderick et al. (2012) and Schlickeiser et al. (2012b) concluded that the created pair beam distribution function is quickly unstable to the excitation of electrostatic oscillations in the unmagnetized IGM, so that the generation of IC scattered GeV gamma-ray photons by the pair beam is significantly suppressed. Because most of the pair kinetic energy is transferred to electrostatic fluctuations,
less kinetic pair energy is available for IC interactions with the microwave background radiation fields. Therefore, there is no need to require the existence of small intergalactic magnetic fields to scatter the produced pairs, so that the explanation (made by several authors) for the Fermi non-detection of the IC scattered GeV gamma-rays by a finite deflecting intergalactic magnetic field is not necessary. In particular, the various derived lower bounds for the intergalactic magnetic fields are invalid due to the pair beam instability argument.

Miniati & Elyiv (2013) argued that the more appropriate linear kinetic instability analysis, accounting for the longitudinal and the small but finite perpendicular momentum spread in the pair momentum distribution function with longitudinal and perpendicular spread. Contrary to the claims of Miniati & Elyiv (2013), we find that neither the pairwise kinetic instability analysis in the kinetic limit for parallel propagating pairs nor the reactive instability analysis, accounting for the longitudinal spread in the pair distribution function are tiny, and always factors in the maximum growth rate due to the finite perpendicular spread.

Because $b$ is significantly smaller than unity, we approximate

$$\cos t \simeq 1 - \frac{t^2}{2},$$

so that with $\arctan b \simeq b$

$$Y(b, A, p) \simeq \int_0^b dq \left[ 1 - pq^2 \right] e^{\frac{2t^2}{A}}.$$

We restrict our analysis to values of $\beta_c \lesssim R \ll R_0$, where $R_0$ denotes the real phase speed (Equation (72)), where the maximum growth rate $\gamma_{\text{max}} (b = 0)$ for no angular spread occurs (see Section 4.2). In this case, the variable from Equation (41)

$$A(R \lesssim R_0) \gtrsim A(R_0) > 3$$

is always larger than 3. The main contribution to the integral in Equation (A6) is then indeed provided by small values of $q \ll 1$, so that the approximation of Equation (A5) is justified.

The integral in Equation (A6) can be expressed in terms of Dawson’s integral (Abramowitz & Stegun 1972, chap. 7.1; Lebedev 1972, chap. 2.3) and the error function of the imaginary argument

$$F(x) = e^{-x^2} \int_0^x dt \ e^{t^2},$$

as

$$Y(b, A, p) = \sqrt{\frac{2}{A}} e^{x^2} \left[ F(X) + \frac{p}{A} [F(X) - X] \right],$$

with

$$X(b, A) = \sqrt{\frac{A}{2}} b.$$

Dawson’s integral (Equation (A8)) has a maximum $F_m = 0.541$ at $x_m = 0.924$, where the series expansion is

$$F(x) = \sum_{n=0}^{\infty} \frac{(-1)^n 2^n x^{2n+1}}{(2n + 1)!} = x \left[ 1 - \frac{2}{3} x^2 + \frac{4}{15} x^4 + \cdots \right].$$

and the asymptotic expansion is

$$F(x \gg 1) \simeq \frac{1}{2x} \left[ 1 + \frac{1}{2x^2} + \frac{3}{4x^4} \right].$$

In Figures 4 and 5, we compare the numerically evaluated exact integral (Equation (A4)) with its approximation (Equation (A9)) for $p = 2$ and two values of $A = 3$ and $A = 100$. In both cases, the agreement is excellent for values of $b < 0.1$.

According to Equations (A1) and (A3), we obtain the approximations

$$J(b) \simeq \frac{A e^{-A}}{K^{s+1}(R)} \frac{e^{x^2}}{X} \left[ (1 - \frac{5}{A}) F(X) + (s - 1)A - s(2 - 2) \frac{F(X) - X}{2A^2} \right].$$

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### APPENDIX A

#### APPROXIMATIONS OF THE INTEGRAL $J(b)$

We introduce

$$T(b, A, s) = \frac{K^{s+1}(R)}{A} e^{A b} J(b),$$

so that, according to Equation (40),

$$T(b, A, s) = e^A \int_0^b dq \frac{e^{-\sqrt{q^2 + t^2}}}{(1 + q^2 t^2)^{3/2}} \left[ 1 - \frac{s}{A} \sqrt{1 + q^2} \right].$$

The substitution $q = \tan t$ provides

$$T(b, A, s) = e^A \left[ \int_0^{\arctan b} dt \cos^{-1} t e^{-A \cos t} - \frac{s}{A} \int_0^{\arctan b} dt \cos^{-2} t e^{-A \cos t} \right].$$

with

$$Y(b, A, p) = e^A \int_0^{\arctan b} dt \cos^{2p} t e^{-A \cos t}. \quad (A4)$$

We confirm the earlier conclusions by Broderick et al. (2012) and Paper I that the created pair beam distribution function is quickly unstable in the unmagnetized IGM.

As our analysis has shown, relativistic kinetic instability studies are notoriously difficult and complicated due to plasma particle velocities close to the speed of light. Therefore, extreme care is necessary in order to include all relevant relativistic effects, as was done in the present study.

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Figure 4. Comparison of the numerically evaluated exact integral (Equation (A4)) with its approximation (Equation (A9)) for \( p = 2 \) and \( A = 3 \).

The small argument expansion of Equation (A11) readily yields

\[ J(0) = J(b = 0) = \frac{A - s}{K^{s+1}R}e^{-A}, \quad (A14) \]

which agrees with Equation (42), so that the correction function (Equation (45)) becomes

\[ B(X) = \frac{e^{X^2}}{X} [F(X) + h(A, s)(F(X) - X)], \quad (A15) \]

with

\[ h(A, s) = \frac{(s - 1)A - s(s - 2)}{2A(A - s)}. \quad (A16) \]

APPENDIX B
COLLECTIVE ELECTROSTATIC MODES

Equation (47), together with the real part of the dispersion relation (Equation (37)), reads

\[ 0 = \Re(\Lambda(R, I = 0)) = 1 - \frac{1}{\kappa^2 \beta_c^2} \left[ \Re Z' \left( \frac{R}{\beta_c} \right) + \frac{1}{\chi} \Re \left( \frac{R}{\sqrt{\chi} \beta_c} \right) \right]. \quad (B1) \]

In order to use the asymptotic expansions (Equation (28)) for proton–electron temperature ratios \( \chi \ll \xi^{-1} = 1836 \), we have to consider three cases.

1. In the case of phase speeds larger than \( \beta_c \),

\[ R \gg \beta_c, \quad (B2) \]

both arguments of the \( Z' \) function are large compared with unity, so that we can use the asymptotic expansion

\[ \Re Z'(t \gg 1) \simeq \frac{1}{t^2} \left[ 1 + \frac{3}{2t^2} \right]. \quad (B3) \]

2. In the case of intermediate phase speeds,

\[ \frac{R}{\beta_c} \ll 1 \ll \frac{R}{\sqrt{\chi} \beta_c}, \quad (B4) \]

we use the expansion (Equation (B3)) in the third term of Equation (B1) and the asymptotic expansion for small arguments

\[ \Re Z(t \ll 1) \simeq -2[1 - 2t^2] \quad (B5) \]

in the second term of Equation (B1).

3. In the case of very small phase speeds,

\[ R \ll \sqrt{\chi \xi \beta_c}, \quad (B6) \]

we use the expansion (Equation (B5)) in the second and third term of Equation (B1).

We consider each case in turn.

B.1. Large Phase Speed \( R \gg \beta_c \)

Here, we readily obtain for Equation (B1)

\[ \Re(\Lambda(R, \kappa)) = 1 - \frac{1 + \frac{\xi}{\kappa^2}}{2R^2} - \frac{3\beta_c^2(1 + \chi \xi^2)}{2\kappa^2 R^4} = 0, \quad (B7) \]

yielding the dispersion relation

\[ R^4 - \frac{1 + \frac{\xi}{\kappa^2}}{\kappa^2} R^2 - \frac{3\beta_c^2}{2\kappa^2} = 0, \quad (B8) \]

with the solution

\[ R^2 = \frac{1 + \frac{\xi}{\kappa^2}}{2\kappa^2} \left[ 1 + \sqrt{1 + \frac{6\beta_c^2 \kappa^2}{(1 + \xi^2)}} \right] \simeq \frac{1}{2\kappa^2} \left[ 1 + \sqrt{1 + 6\beta_c^2 \kappa^2} \right]. \quad (B9) \]

The requirement \( R \gg \beta_c \) implies the wavenumber restriction \( \beta_c^2 \kappa^2 \ll 2.5 \). Likewise, the subluminality requirement \( R \ll 1 \) demands

\[ \kappa^2 > \kappa_L^2 = 1 + \frac{3\beta_c^2}{2}. \quad (B10) \]
In this wavenumber range, the solution (Equation (B8)) reduces to
\[ R^2 \simeq 1 + \frac{3 \beta_c^2}{2} = \frac{1 + \frac{3}{2} \beta_c^2 k^2}{\kappa^2}, \] 
(B11)
corresponding to Langmuir oscillations
\[ \omega_n^2 = \omega_{p,e}^2 [1 + 3 \kappa^2 \lambda_{De}] \] 
(B12)

for \( 2^{-1/2} \beta_e \lesssim k \lambda_{De} \ll 1 \) with the electron Debye length \( \lambda_{De} = \beta_e c / \sqrt{2 \omega_{p,e}} \).

**B.2. Intermediate Phase Speed** \( \sqrt{\chi \xi} \beta_e = \beta_p \ll \beta_c \)

In this case, we derive for Equation (B1)
\[ \Re \Lambda(R, \kappa) \simeq 1 + \frac{2}{\beta_c^2 k^2} - \frac{\xi}{\kappa^2 R^2} - \frac{4 R^2}{\beta_c^2 k^2} = 0, \]
(B13)
yielding the dispersion relation
\[ R^4 - \frac{\beta_c^2}{2} \left( 1 + \frac{\beta_c^2 k^2}{2} \right) R^2 + \frac{\xi \beta_c^4}{4} = 0, \]
(B14)

with the two formal solutions
\[ R_{1,2}^2 = \frac{\beta_c^2}{4} \left( 1 + \frac{\beta_c^2 k^2}{2} \right) \left[ 1 \pm \sqrt{\frac{1 - \frac{4 \xi}{(1 + \frac{\beta_c^2 k^2}{2})^2}}{2}} \right] \]
(B15)
The first solution
\[ R_1^2 \simeq \beta_c^4 \left( 1 + \frac{\beta_c^2 k^2}{2} \right) \]
(B16)
violates the restriction \( R^2 \ll \beta_c^2 \), leaving
\[ R^2 = R_2^2 \simeq \frac{\xi \beta_c^4}{2} \left( 1 + \frac{\beta_c^2 k^2}{2} \right) \]
(B17)
as the only solution. This ion sound wave solution has to fulfill the second restriction \( R^2 \gg \chi \xi \beta_c^4 \), corresponding to the condition
\[ 1 + \frac{\beta_c^2 k^2}{2} \ll \frac{1}{2 \chi}, \]
(B18)
which is only possible for values of \( \chi \ll 1 \) or \( T_p \ll T_e \). In this case, the solution (B17) holds for wavenumbers \( \kappa^2 \beta_c^2 \ll \chi^{-1} \).

Therefore, the ion sound wave solution only exists for \( T_p \ll T_e \) at wavenumbers \( (\lambda_{De} k)^2 \ll (2 \chi)^{-1} \) with frequencies
\[ \omega_R^2 = \frac{\beta_c^2 \xi^2 k^2}{2(1 + \lambda_{De} k^2)}, \quad \lambda_{De} k^2 \ll \frac{1}{2} = \frac{T_e}{2 T_p}. \]
(B19)

**B.3. Very Small Phase Speed** \( R \ll \sqrt{\chi \xi} \beta_e = \beta_p \)

In this case, we derive for Equation (B1)
\[ \Re \Lambda(R, \kappa) \simeq 1 + \frac{2(1 + \chi)}{\chi \beta_e^2 k^2} - \frac{4 \chi^2}{\chi \beta_e^2 k^2} = 0, \]
(B20)
yielding the dispersion relation
\[ R^2 = \frac{(1 + \chi) \chi \beta_p^2}{2(1 + \chi \beta_p^2)} + \frac{\chi \beta_e^2 k^2}{4(1 + \chi \beta_p^2)} \]
(B21)

which cannot be fulfilled. Therefore, no electrostatic modes with very small phase speeds exist.

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