Stabilization and radion in de Sitter brane-world

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ABSTRACT

We consider the stabilization of de Sitter brane-world. The scalar field bulk-brane theory produces the non-trivial minimum of modulus potential where temporal radion is realized. The hierarchy problem (between Planck and electroweak scales) may be solved. However, the interpretation of radion is not so clear as in AdS brane-world. In particular, the introduction of two times physics or pair-creation of bulk spaces or identification of one of spatial coordinates with imaginary time (non-zero temperature) may be required.

PACS: 98.80.Hw, 04.50.+h, 11.10.Kk, 11.10.Wx
Keywords: radion, dS/CFT, brane world

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The investigation of five-dimensional brane-worlds naturally posed the problem of stabilization of extra dimension. In case of AdS bulk the corresponding mechanism of stabilization has been proposed by Goldberger-Wise [1] via the realization of the minimum of the corresponding modulus effective potential. However, the explicit stabilization of fifth dimension (without fine-tuning) may be presumably realized only by thermal quantum effects [2]. The physics of radion (which vacuum expectation value coincides with the fifth dimension radius) may play an important role in electroweak theory where, for example, S-parameter turns out to be modified.

The interest to AdS brane-worlds is motivated mainly by AdS/CFT correspondence. However, it has been recently proposed dS/CFT correspondence (see [3] for an introduction). As a result it is naturally to search for realistic dS brane-world scenarios. As one step in this direction we study how radion physics (or stabilization of extra dimension) may be realized in dS brane-world.

Let us start from the 5-dimensional de Sitter space as a bulk spacetime:

\[ ds^2 = G_{\mu\nu} dx^\mu dx^\nu = G_{tt} dt^2 + \sum_{i,j=1}^{4} g_{ij} dx^i dx^j = -dt^2 + e^{-2k|t|} \sum_{i=1}^{4} (dx^i)^2 . \]  

(1)

We also consider the scalar field, whose action is given by

\[ S_\phi = \int \sqrt{-G} \left( -\frac{1}{2} G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 \right) . \]  

(2)

One assumes that there are branes at \( t = 0 \) and \( t = T \), whose action is given by

\[ S_0 = \int \sqrt{-g} V_0(\phi) , \quad S_T = \int \sqrt{-g} V_T(\phi) . \]  

(3)

where \(-T \leq t \leq T\) and \( t \) is identified with \(-t\). Note that in the same way as in AdS bulk one can identify \( T \) with the vacuum expectation value of temporal radion field. In fact, \( T \) is a dynamical variable, which can be included in the \((tt)\)-component \( G_{tt} \) of the metric tensor in (1). If we change \( G_{tt} \) as \( C^2 G_{tt} \) by a positive constant \( C \), the last expression in (1) is changed as \( ds^2 = -C^2 dt^2 + e^{-2kC|t|} \sum_{i=1}^{4} (dx^i)^2 \). Further rescaling the time coordinate \( t \rightarrow \frac{t}{C} \), we obtain the last expression in (1), again, but the region where the coordinate \( t \) takes its value is changed as \(-\frac{T}{C} \leq t \leq \frac{T}{C}\). Repeating back this process, we find that \( T \) is a dynamical variable. In the following, we solve
the equation of motion for fixed $T$ and after that we substitute the obtained solution into the action (2), where all the dynamical variables except $T$ are integrated out.

With the assumption that the bulk de Sitter space is a background, by the variation over the scalar field $\phi$, we obtain the following equation:

$$0 = \partial_\mu \left( \sqrt{-G} G^{\mu\nu} \partial_\nu \phi \right) - m^2 \sqrt{-G} \phi - \sqrt{-g} V'_0(\phi) \delta(t) - \sqrt{-g} V'_2 \delta(t - T).$$

(4)

Considering that $\phi$ only depends on $t$ and using (1), one can rewrite (4) as

$$0 = -\partial_t \left( e^{-4k|t|} \partial_t \phi \right) - m^2 e^{-4k|t|} \phi - e^{-4k|t|} V'_0(\phi) \delta(t) - e^{-4k|t|} V'_T \delta(t - T).$$

(5)

In the bulk ($0 < t < T$), the solution is given by

$$\phi = e^{2kt} \left( ae^{\sqrt{4k^2-m^2}t} + be^{-\sqrt{4k^2-m^2}t} \right).$$

(6)

Here $a$ and $b$ are constants of the integration. The solution (6) is similar to that for the case that the bulk is AdS. Indeed, for the space-like radial coordinate as $r$, the solution in the AdS bulk is given by

$$\phi_{\text{AdS}} = e^{2kt} \left( ae^{\sqrt{4k^2+m^2}r} + be^{-\sqrt{4k^2+m^2}r} \right).$$

(7)

The main difference is the sign in front of $m^2$ in the root. Then in the dS bulk, the solution behaves as an exponential function if $4k^2 > m^2$ but if $4k^2 < m^2$ the solution behaves with vibration:

$$\phi = e^{2kt} \left( ae^{i\sqrt{m^2-4k^2}t} + be^{-i\sqrt{m^2-4k^2}t} \right).$$

(8)

Here $b$ should be a complex conjugate of $a$. If $4k^2 = m^2$, the solution has the following form:

$$\phi = (a_1 + a_2 t) e^{2kt}.$$  

(9)

The constants $a$ and $b$ in (6) can be determined to satisfy the boundary condition at $t = 0, T$, coming from the $\delta$-functions in (5):

$$\partial_t \phi|_{t\to+0} - \partial_t \phi|_{t\to-0} = -V'_0(\phi), \quad \partial_t \phi|_{t\to T+0} - \partial_t \phi|_{t\to T-0} = V'_T(\phi),$$

(10)

that is,

$$2 \left\{ (2k + \sqrt{4k^2-m^2}) a + (2k - \sqrt{4k^2-m^2}) b \right\} = V'_0(\phi),$$

$$2 \left\{ (2k + \sqrt{4k^2-m^2}) \left( ae^{(2k+\sqrt{4k^2-m^2})T} + be^{(2k-\sqrt{4k^2-m^2})T} \right) \right\} = -V'_T(\phi).$$

(11)
By substituting the solution (6) into the action (2), one has

\[
S_\phi = V_4 \left[ \frac{(2k + \sqrt{4k^2 - m^2})^2 - m^2}{4\sqrt{4k^2 - m^2}}a^2 \left( e^{2\sqrt{4k^2 - m^2}T} - 1 \right) \\
- \frac{(2k - \sqrt{4k^2 - m^2})^2 - m^2}{4\sqrt{4k^2 - m^2}}b^2 \left( e^{-2\sqrt{4k^2 - m^2}T} - 1 \right) - 2m^2abT \right]. \tag{12}
\]

Here \(V_4\) is the volume of the brane. By combining \(S_\phi\) (12) with \(S_0(\phi_t=0)\) and \(S_T(\phi_t=T)\) in (3), we might obtain the effective action for \(T\). As a special model, we consider the case that

\[
V_0' = V_T' = \alpha \text{ (constant)}. \tag{13}
\]

Then one can solve (11) with respect to \(a\) and \(b\):

\[
a = \frac{\alpha \left( 2k - \sqrt{4k^2 - m^2} \right) \left( e^{(2k-\sqrt{4k^2-m^2})T} + 1 \right)}{2m^2e^{2kT} \left( e^{-\sqrt{4k^2-m^2}T} - e^{\sqrt{4k^2-m^2}T} \right)}, \\
b = -\frac{\alpha \left( 2k + \sqrt{4k^2 - m^2} \right) \left( e^{(2k+\sqrt{4k^2-m^2})T} + 1 \right)}{2m^2e^{2kT} \left( e^{-\sqrt{4k^2-m^2}T} - e^{\sqrt{4k^2-m^2}T} \right)}. \tag{14}
\]

By substituting (14) into (12), we obtain

\[
S_\phi = \frac{\alpha^2e^{-4kT}V_4}{16m^2 \left( e^{-\sqrt{4k^2-m^2}T} - e^{\sqrt{4k^2-m^2}T} \right)^2} \times \left[ \left( m^2 - (2k - \sqrt{4k^2 - m^2})^2 \right) \left( e^{(2k-\sqrt{4k^2-m^2})T} + 1 \right)^2 \frac{1}{\sqrt{4k^2 - m^2}} \times \left( e^{2\sqrt{4k^2-m^2}T} - 1 \right) - \left( m^2 - (2k + \sqrt{4k^2 - m^2})^2 \right) \right] \times \frac{\left( e^{(2k+\sqrt{4k^2-m^2})T} + 1 \right)^2 \left( e^{-2\sqrt{4k^2-m^2}T} - 1 \right)}{\sqrt{4k^2 - m^2}} \times 8m^2T \left( e^{(2k-\sqrt{4k^2-m^2})T} + 1 \right) \left( e^{(2k+\sqrt{4k^2-m^2})T} + 1 \right) \right]. \tag{15}
\]
One can regard \(-S_\phi\) as an effective potential for radion \(T\):

\[
V(T) = -S_\phi .
\]  

(16)

Then when \(T \to 0\), \(V(T)\) behaves as

\[
V(T) \to \frac{\alpha^2 V_4 (2k^2 - m^2)}{m^2 (4k^2 - m^2)} \frac{1}{T} .
\]

(17)

On the other hand, when \(T \to +\infty\),

\[
V(T) \to \frac{\alpha^2 V_4}{16m^2} \left[ \frac{8k^2 - 2m^2 + 4k\sqrt{4k^2 - m^2}}{\sqrt{4k^2 - m^2}} - 8m^2 T e^{-2\sqrt{4k^2 - m^2}T} \right] .
\]

(18)

Note that \(0 < T < +\infty\). Eqs. (17) and (18) tell that the potential is bounded below and has non-trivial minimum if \(2k^2 > m^2\). In order to generate the hierarchy between the Planck scale \(10^{19}\) GeV and weak scale \(10^2\) GeV, if we assume \(kT \sim 50\) and also \(\sqrt{4k^2 - m^2}T \sim 1\), we can approximate the potential by (18), which has a minimum at

\[
T \sim \frac{1}{2\sqrt{4k^2 - m^2}} .
\]

(19)

Then in order to generate the hierarchy the fine-tuning is

\[
4 - \frac{k^2}{m^2} \sim 10^{-4} ,
\]

(20)

which might be natural if compare with the ratio of the weak and the Planck scale \(10^2/10^{19} = 10^{-17}\).

Now as the brane is Euclidean, in order to relate the above theory with the real electroweak physics, we need to Wick-rotate one more spatial coordinate on the brane. Then we have two time coordinates, which might cause the problem with the unitarity. The two time theory might not, at present, be so unnatural as the so-called F-theory is realization of two time theory. Even in the model here, the de Sitter time might be a rather formal one.

Another way to relate the above model with the real electroweak theory is that one regards a spatial coordinate, say \(x^4\) in (0) as an imaginary time with a period \(\beta = \frac{1}{T_m}\). Then the model under discussion describes the theory at finite temperature \(T_m\). Then the CFT at finite temperature may be naturally
Figure 1: The pair-creation and the pair-annihilation by the space-like branes at \( t = 0, T \).

described in the present framework. Furthermore if we put \( ds^2 = 0 \) in (1) by choosing \( x^1 = x^2 = x^3 = 0 \), we obtain \( dt = \pm e^{-kt} dx^4 \), that is,

\[
x^4 - x_0^4 = \pm \frac{e^{kt}}{k}.
\] (21)

Here \( x_0^4 \) is a constant of the integration. Since the temperature of the universe is proportional to the inverse of the scale of the universe, Eq.(21) indicates that the temperature decreases exponentially with time.

On the other hand, one can consider the case that the time coordinate is unique. Then if identify the time \( t \) with \( -t \), the obtained solution describes that the brane decays at \( t = 0 \) into two bulk spacetimes (the bulk spacetimes are pair-created) or the bulk spacetimes are pair-annihilated creating a brane at \( t = T \) as is shown in Fig.1.

If we do not do the identification, the universe has a periodicity of \( 2T \) as shown in Fig.4. When \( 0 < t < T \), the scale of the universe shrinks as \( a = e^{-kt} \) and when \( T < t < 2T \), the universe expands and \( a = e^{kt} \). When \( 2T < t < 3T \), as \( a = e^{-kt} \) and \( 3T < t < 4T \), as \( a = e^{kt} \), etc. In this scenario, in order to realize such 5-dimensional universe, which repeats shrinking and expansion, we need a matter to compensate the jump of the first derivative of the metric. The matter is is given by the space-like brane. Then our analysis indicates that the brane can decay by the effect of the scalar field but the brane could be created again, the created brane decays again. The period of
repeating the shrinking and expansion depends on the details of the scalar part of the action.

In the usual time-like branes in the AdS bulk spacetime, the branes can propagate in the time direction with a finite velocity, keeping the mutual distance $R$. The corresponding solution may exist in the dS bulk case with the role of $t$ and $x^5$ exchanged. Even if such a solution really exist, this would not change the above scenarios drastically.

Thus, our study indicates that if observable Universe is the brane embedded in five-dimensional de Sitter bulk then stabilization of fifth (temporal) dimension may occur via the minimum of modulus effective potential. The analog of radion may be introduced and hierarchy problem may be solved as well. However, as radion is related now with the time the interpretation of the radion physics and its relation with electroweak theory is not so clear as
in AdS bulk. In particular, even the evolution of the observable Universe may be drastically changed or second time coordinate should be introduced. Note finally that very recently (after the first version of this paper appeared), the localization of the graviton on the brane embedded in five-dimensional de Sitter bulk was shown in [5].

Acknowledgements

This research is supported in part by the Grant-in-Aid for Scientific Research on Priority Areas (B) of The Ministry of Education, Culture, Sports, Science and Technology (MEXT) (No. 13135101 and 13135208). One of authors (S.O.) acknowledges support of the Grant No. 13135101, which made possible to finish this work during his visit to Japan.

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