Heating a quantum dipolar fluid into a solid

J. Sánchez-Baena,1,2 Claudia Politi,3,4 F. Maucher,5,1 Francesca Ferlaino,3,4 and T. Pohl1

1Center for Complex Quantum Systems, Department of Physics and Astronomy, 
Aarhus University, DK-8000 Aarhus C, Denmark
2Departament de Física, Universitat Politècnica de Catalunya, Campus Nord B4-B5, 08034 Barcelona, Spain
3Institut für Quantenoptik und Quanteninformation, 
Österreichische Akademie der Wissenschaften, Innsbruck, Austria
4Institut für Experimentalphysik, Universität Innsbruck, Austria
5Departament de Física, Universitat de les Illes Balears & IAC-3, 
Campus UIB, E-07122 Palma de Mallorca, Spain

(Dated: September 2, 2022)

Raising the temperature of a material enhances the thermal motion of particles. Such an increase in thermal energy commonly leads to the melting of a solid into a fluid and eventually vaporises the liquid into a gaseous phase of matter. Here, we study the finite-temperature physics of dipolar quantum fluids and find surprising deviations from this general phenomenology. In particular, we describe how heating a dipolar superfluid from near-zero temperatures can induce a phase transition to a supersolid state with a broken translational symmetry. The predicted effect agrees with experimental measurements on ultracold dysprosium atoms, which opens the door for exploring the unusual thermodynamics of dipolar quantum fluids.

A supersolid is an exotic phase of matter in which particles develop regular spatial order and simultaneously support the frictionless flow of a superfluid. Having evaded experimental verification for several decades [1], supersolidity can now be observed in Bose-Einstein condensates of ultracold atoms with finite-range interactions [2–6]. Spontaneous symmetry breaking in these systems occurs in the form of regular periodic patterns of the condensate density as first predicted by Gross in 1957 [7]. One would thus expect the lowest possible temperatures to provide optimal conditions for supersolidity by ensuring a high degree of phase coherence and maximal population of the Bose-Einstein condensate. On the contrary, we demonstrate here that thermal fluctuations in dipolar condensates do not merely diminish global phase coherence but can instead facilitate the formation of periodic modulations of the condensate density. This finding sheds light on recent experimental observations and reveals an unusual fluid-solid phase transition, whereby a supersolid state of matter emerges upon increasing the temperature.

As we shall see below, this surprising behaviour arises from the anisotropic nature of the dipole-dipole interaction

\[ V_{dd}(r) = \frac{C_3}{4\pi} \left( 1 - 3 \cos^2 \frac{\theta}{r^3} \right), \tag{1} \]

which has repulsive as well as attractive contributions, depending on the angle \( \theta \) between the atomic dipoles and the distance vector \( r \) of two interacting atoms. The interaction strength \( C_3 \) and the atomic mass \( m \) define a length scale \( a_d = \frac{mC_3}{(12\pi\hbar^2)} \) that competes with the scattering length \( a \) of the short-range interaction between the atoms. This competition between \( a_d \) and \( a > 0 \) can cause the condensate to collapse when the stabilizing short-range repulsion is not sufficient to overcome the

\[ \frac{\hbar^2 k^2}{2m} \geq \epsilon_d, \]

where \( \epsilon_d = \frac{\hbar^2}{(12\pi^2)^2} a_d^2 \) parametrizes the characteristic energy scale of the dipole-dipole interactions. Our experiments in an ultracold gas of dysprosium atoms demonstrate the temperature-driven emergence of supersolidity. The measured contrast of axial density modulations is shown by the colored dots in panel (b). The observations show that supersolidity at higher temperatures indeed occurs for smaller atom numbers, \( N_e \), in the condensate, in agreement with the theoretical transition line (purple line).

FIG. 1. Heating a dipolar quantum fluid can lead to the emergence of a supersolid phase of matter. (a) This is demonstrated in the thermodynamic phase diagram for an infinitely elongated Bose-Einstein condensate in a radial harmonic trap with no axial confinement, as illustrated in panel (c). In between the superfluid (blue) and supersolid (red) region, both phases coexist (purple region) as characteristic for a first-order phase transition. The calculations were performed for a fixed chemical potential \( \mu/\epsilon_d = 1 \), where \( \epsilon_d = \frac{\hbar^2}{(12\pi^2)^2} a_d^2 \) parametrizes the characteristic energy scale of the dipole-dipole interactions. Our experiments in an ultracold gas of dysprosium atoms demonstrate the temperature-driven emergence of supersolidity. The measured contrast of axial density modulations is shown by the colored dots in panel (b). The observations show that supersolidity at higher temperatures indeed occurs for smaller atom numbers, \( N_e \), in the condensate, in agreement with the theoretical transition line (purple line).

* jsbaena@phys.au.dk
attractive part of the dipole-dipole interaction between the atoms \[8^{10}\]. Subsequent experiments \[11^{13}\] have however found a higher level of stability, which arises from quantum fluctuations \[14^{15}\] that prevent the otherwise inevitable collapse of the condensate \[16^{18}\]. In fact, the balance of attraction and repulsion effectively enhances the role of quantum fluctuations \[17\] beyond the semiclassical mean-field physics of weakly interacting quantum gases. This yields a unique setting that has revealed rich physics and a host of new quantum states, from self-bound quantum droplets \[12^{13} 19\] and supersolid phases \[3\, 5\, 20, 21\] to complex patterns in two-dimensional fluids \[22^{23}\].

Given this striking role of quantum fluctuations in dipolar Bose-Einstein condensates, one may also anticipate significant effects of thermal fluctuations despite the ultralow temperatures that are required to reach quantum degeneracy. To address this question we start from the grand canonical potential \(\Omega\) of the system at a finite temperature \(T\). For a weakly interacting gas with a high fraction of atoms in the condensate, one can use Bogoliubov theory to determine \(\Omega\). This yields simple expressions for infinitely extended homogeneous systems \[24\] that can be applied to describe trapped inhomogeneous gases within a local density approximation. Hereby, one determines the Bogoliubov excitation spectrum and all relevant observables for a homogeneous particle density \(\rho\), which is then identified as \(\rho \equiv |\psi(r)|^2\) with the local condensate wave function \(\psi(r)\) at a given position \(r\). This permits to express the grand canonical potential as

\[
\Omega = E_0 + \frac{k_B T}{(2\pi)^3} \int \! dr \int \! dk \; \ln \left( 1 - e^{-\frac{\epsilon_k}{k_B T}} \right),
\]

where \(k_B\) denotes the Boltzmann constant and \(E_0\) is the zero-temperature grand canonical energy that contains the mean-field interaction energy and leading order corrections due to quantum fluctuations \[14\], i.e. small occupations of excited states above the formed Bose-Einstein condensate. The dispersion \(\epsilon_k = \sqrt{\tau_k (\tau_k + 2 |\psi(r)|^2 \tilde{V}(k))}\) of these excitations is determined by the kinetic energy \(\tau_k = \hbar^2 k^2/(2m)\) of the atoms and the Fourier transform \(\tilde{V}(k) = \frac{4\pi \hbar^2 a}{m} + V_{dd}(k)\) of their total interaction potential.

Minimizing \(\Omega\) with respect to \(\psi(r)\) then yields a nonlinear wave equation that accounts for quantum as well as thermal fluctuations (see Methods Section). At zero temperature, it describes the mean-field physics of the condensate and captures leading-order effects of quantum fluctuations through an effective density-dependent potential \(H_{\text{qu}}\) \[17\] that increases the energy of the system. The second term in Eq. (2) yields an additional potential

\[
H_{\text{th}}(r) = \int \! \frac{dk}{(2\pi)^3} \tilde{V}(k) f_k(r) \tau_k \frac{\tau_k}{\epsilon_k(r)},
\]

that accounts for finite-temperature effects. It describes the interaction between the condensate and thermally created excitations that populate Bogoliubov modes according to the Bose distribution \(f_k = 1/(e^{\epsilon_k/k_B T} - 1)\). The resulting form of the finite-temperature extended Gross-Pitaevskii equation (TeGPE) agrees with the result of Hartree-Fock Bogoliubov theory \[23^{25}\], and includes relevant fluctuation terms that are commonly neglected within the Popov approximation \[26\] (see Methods Section).

Let us first use this framework to study an elongated atomic gas that is confined harmonically in the \(x - y\) plane and extends infinitely in the \(z\)-direction without confinement along the \(z\)-axis. Figure \(1a\) shows the thermodynamic phase diagram obtained by simulating the imaginary time evolution of the TeGPE at a fixed chemical potential \(\mu\) (see Methods Section). At zero temperature, we find a superfluid-supersolid quantum phase transition, with a co-existence region that is expected for a first-order phase transition \[27\]. While increasing the temperature may generally be expected to melt the

FIG. 2. Raising the temperature of a dipolar quantum fluid can induce a pronounced roton-maxon spectrum of its collective excitations, as shown in panel (a) for an infinitely elongated condensate along the \(z\)-axis [see Fig.1(c)]. Heating the fluid tends to lower the energy of the roton minimum and eventually softens the roton excitation as the temperature increases. This effect can be traced back to the density dependence of the energy correction caused by fluctuations, shown in panel (b). While quantum fluctuations yield an energy \(H_{\text{qu}}\) (dashed line) that increases with a rising condensate density \(\rho = |\psi|^2\), the contribution \(H_{\text{th}}\) from thermal fluctuations decreases (solid lines). The thermal energy correction \(H_{\text{th}}(r)\), therefore, acts as a focusing nonlinearity that supports the formation of regular density modulations. This is illustrated in panel (c,d), where we show the axial density \(\rho_{\parallel}(z) = \int dx dy \rho(x,y)\) along with the axial potential \(\tilde{H}_{\text{th}} = \rho_{\parallel} \int dx dy \rho(x,y) H_{\text{th}}(r)\), respectively. The calculations are performed for \(a/a_d = 0.7\) and \(\mu = \varepsilon_d\).
supersolid phase [28], we find instead that it shifts the transition towards weaker dipole-dipole interactions. As a result, heating the system effectively drives a phase transition from a fluid into a solid phase.

We can understand this effect from the excitation spectrum of the condensate in the superfluid phase. To this end, we solve the time-dependent TeGPE within linear response theory to find the excitation spectrum \( \omega_k \) for periodic plane-wave excitations along the \( z \)-direction. As shown in Fig. 2, the obtained dispersion exhibits the expected roton-maxon form [29–32], known from low-temperature helium [33] and Bose-Einstein condensates with finite-range interactions [34–37]. The local minimum at finite momenta supports the formation of roton quasiparticles, which were introduced by Landau as elementary vortices to describe superfluidity in \(^4\)He [33]. Experiments show that the roton minimum in helium decreases with increasing temperature [38] due to roton-roton scattering [39]. Yet, the roton energy remains sizable at the transition to a normal-fluid phase [35], beyond which it only varies weakly with temperature. The presence of a Bose-Einstein condensate in dilute dipolar superfluids, however, enhances the effect of thermal fluctuations due to the larger energy scale of the interaction between Bogoliubov excitations and the condensate. A similar effect is found for atoms with light-induced interactions and predicted to lower the roton minimum and cause enhanced condensate depletion [40]. In the present case, we find a thermal softening of the roton mode that can drive an instability of the superfluid and thereby cause the formation of a supersolid phase with increasing temperature.

We can gain further intuition about the underlying mechanism by closer inspection of the two fluctuation energies \( H_{\text{qu}} \) and \( H_{\text{th}} \) that both contribute a local nonlinearity to the wave equation for \( \psi(r) \). \( H_{\text{qu}} > 0 \) is the Lee-Huang-Yang correction to the equation of state [14,15], and raises the ground state energy due to the small condensate depletion caused by the atomic interactions. It therefore increases for higher particle densities and stronger interactions, as shown in Fig. 2(b). Consequently, \( H_{\text{qu}} \) generates an effective repulsion that stabilizes the condensate against collapse [17], and shifts the roton instability towards higher densities and stronger dipole-dipole interactions. On the contrary, \( H_{\text{th}} \) increases as we lower the density of the condensate [see Fig. 2(b)]. This behaviour is readily understood as follows. Decreasing the condensate density increases the fraction of thermally excited, non-condensed atoms [25]. In the limit where this fraction remains small, such an increase implies a larger potential energy due to interactions with the thermal atoms. It therefore contributes a positive energy correction that decreases upon increasing the density \( \rho = |\psi|^2 \) of the condensate. As a result, thermal fluctuations energetically favour higher condensate densities, such that \( H_{\text{th}} \) acts as a focusing nonlinearity which lowers the roton energy and facilitates the formation of a density-modulated phase, as illustrated in Fig. 2.

We recently observed experimental signatures of this effect by studying the cooling-heating lifecycle of bosonic dysprosium atoms at ultralow temperatures [11]. The experiment starts from a thermal cloud of \( 10^5 \) atoms in a cigar-shaped optical dipole trap and traces the time evolution of the gas as it is cooled evaporatively to quantum degeneracy by lowering the depth of the trap. During the continual cooling and thermalization we observed the expected emergence of supersolidity, and studied the equilibrium states of the quantum fluid across the supersolid phase transition. The measured density profiles indicate a higher degree of modulation at higher temperatures. While this has cast mystery on the origin of the observations, they can now be used to corroborate and benchmark our theoretical understanding. Figure 1 shows our measured contrast of the axial density modulations for different temperatures and condensed-atom numbers (see Methods Section). The results confirm the formation of a supersolid phase with increasing temperature in good agreement with the theoretical transition line obtained numerically by the TeGPE. Moreover, Fig. 3 compares our measured axial density, \( \rho(z) \), to the theoretical prediction. The predicted zero-temperature ground state corresponds to an unstructured superfluid and deviates qualitatively from the observed supersolid state. The result of our finite-temperature TeGPE simulation, however, agrees with the experiment and reproduces quantitatively the period and amplitude of the measured density modulations. This remarkable level of agreement offers strong indication that the observed supersolid has indeed been generated by the finite temperature of the atoms.

The possibility to make detailed comparisons between theory and experiments opens up several directions for exploring the surprising thermodynamic behaviour of quantum ferrofluids. Already, the ground state phase diagram exhibits a rich structure, including first-order as
well as second-order quantum phase transitions in one and two-dimensional systems [27, 42, 43]. This offers a promising starting point for investigating how thermal fluctuations influence the nature of the fluid-solid phase transition and may affect the physics of higher dimensional supersolids [44, 45], which can come in a diverse range of complex patterns [22, 23, 42]. 

Our present findings motivate future experiments and first-principle simulations [46, 47] to expand the phase diagram of Fig. 1 into the high-temperature domain and draw a direct connection to the more familiar physics of liquid-solid phase transitions in the absence of superfluidity. Such numerical approaches may also reveal how the present phenomenology extends into the regime of strong interactions, which is becoming accessible in experiments with ultracold polar molecules [48–50]. Equally important, an improved understanding of finite-temperature effects in dipolar quantum fluids could help resolving current questions about quantitative discrepancies between measurements and theory [3, 31].

We thank Yongchang Zhang, Georg Bruun, and Jordi Boronat for valuable discussions and the Er-Dy team in Innsbruck for experimental support. This work was supported by the DNRF through the Center of Excellence "CCQ" (Grant agreement no.: DNRF156) and the Carlsberg Foundation through the ‘Semper Ardens’ Research Project QCoLo. JB acknowledges funding by the European Union, the Spanish Ministry of Universities and the Recovery, Transformation and Resilience Plan through a grant from Universitat Politècnica de Catalunya. FF and CP acknowledge support through an ERC Consolidator Grant (RARE, No. 681432), the QuantERA grant MAQS (No I4391-N) and the FOR grant (2247/PI2790) by the Austrian Science Fund FWF.

Appendix A: The nonlinear wave equation

The grand canonical potential is minimal in equilibrium such that we can minimize Eq. (2) with respect to the condensate wave function \( \psi(\mathbf{r}) \). This yields the nonlinear wave equation:

\[
\mu \psi(\mathbf{r}) = \left( -\frac{\hbar^2 \nabla^2}{2m} + U(\mathbf{r}) + \frac{4\pi \hbar^2 a}{m} |\psi(\mathbf{r})|^2 \right) + \int d\mathbf{r}' V_{dd}(\mathbf{r} - \mathbf{r}') |\psi(\mathbf{r}')|^2 + H_{qu}(\mathbf{r}) + H_{ch}(\mathbf{r}) \psi(\mathbf{r}),
\]

which determines the equilibrium state of the condensate for a given chemical potential \( \mu \). We consider a harmonic trapping potential \( U(\mathbf{r}) = \frac{\hbar^2}{2m}(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2) \), with trapping frequencies \( \omega_x, \omega_y, \omega_z \) along the three cartesian axes. The first four terms correspond to the Gross-Pitaevskii equation that describes the mean-field physics of the condensate at zero temperature. The next term is given by \( H_{qu}(\mathbf{r}) = \frac{\hbar^2}{2m} \gamma_{qu} |\psi(\mathbf{r})|^2 \) and accounts for leading order effects of quantum fluctuations with a strength \( \gamma_{qu} \) that increases with \( a \) and \( a_d \) [14, 15, 17]. Finite-temperature effects are captured by the last term as given in Eq. (3). We note here that the applied local-density approximation can cause an infrared divergence of the momentum integral in Eq. (3). However, the finite system size of trapped systems yields a natural momentum cutoff that ensures converged results. Indeed, we find that our calculated condensate wave functions are not sensitive to the precise choice of the momentum cutoff for relevant trap geometries.

Appendix B: Finite-temperature simulations

We have calculated the condensate wave function at finite temperatures by simulating the imaginary time evolution of the wave equation (A1). More concretely, we replace \( \mu \psi \) by \( -\partial_t \psi \) in Eq. (A1) and simulate the time evolution of the \( \psi(\mathbf{r},t) \) reaches a steady state for a given norm \( N_c = \int d\mathbf{r} |\psi(\mathbf{r},t)|^2 \). \( N_c \) corresponds to the number of condensate atoms under 3D confinement as considered in Figs. 1(b) and 3 and yields the axial density \( N_c/L = L^{-1} \int d\mathbf{r} \int_0^L dz |\psi(\mathbf{r},t)|^2 \) for a given length \( L \) of the periodic simulation box as considered in Figs. 1(a) and 2. Finally, we determine the chemical potential from Eq. (A1) in order to construct the thermodynamic phase diagram shown in Fig. 1(a). The results shown in Figs. 1(b) and 3 are obtained for the experimental trap parameters \( \omega_z/2\pi = 88 \text{Hz}, \omega_y/2\pi = 141 \text{Hz}, \) and \( \omega_z/2\pi = 36 \text{Hz} \), and a scattering length of \( 89a_0 \), where \( a_0 \) is the Bohr radius. The simulations of Figs. 1(a) and 2 have been performed for \( w_x = 0.0717 \varepsilon_{dd}, w_y = 0.142 \varepsilon_{dd} \), and \( w_z = 0 \). In all cases, the dipoles are considered to be polarized along the y-axis.

Appendix C: Experimental determination of the average axial density and the density contrast

We probe the emergence of a supersolid state via time-of-flight measurements and in-situ Faraday phase contrast imaging [41]. The former provides information on the global phase coherence and the latter allows us to extract the modulation contrast from the in-situ atomic density. In particular, to obtain the modulation contrast of the experimental points in Fig. 1, we proceed as follows. For each \( \{N_c, T\} \), we record the in-trap density distribution and integrate along the direction orthogonal to the droplet chain to get a 1D density profile \( \rho_1(z) \). We repeat the measurement under the same experimental conditions for 10-20 times. For each profile, we then calculate its Fourier Transform, \( \tilde{\rho}_1(k) = \int e^{-ikz} \rho_1(z) \)dz. Finally, we determine the average of \( |\tilde{\rho}_1(k)| \) and obtain the modulation contrast as the ratio between the Fourier component at the modulation wavelength and the density \( |\tilde{\rho}_1(0)| \) at \( k_z = 0 \).

The experimental profile shown in Fig. 3 is obtained by averaging the density distributions from the repeated measurements. The central maximum in each profile is
shifted to the origin, \( z = 0 \), to correct for unavoidable center-of-mass fluctuations in the experiment. Moreover, we remove negative density contributions in each density image, which are caused by small but inevitable misalignments of the imaging objective.

[1] M. H. W. Chan, R. B. Hallock, and Reatto L., “Overview on solid \(^4\)He and the issue of supersolidity,” Journal of Low Temperature Physics 172, 317 (2013).
[2] J. Léonard, A. Morales, P. Zupancic, T. Esslinger, and T. Donner, “Supersolid formation in a quantum gas breaking a continuous translational symmetry,” Nature 543, 87 (2017).
[3] F. Böttcher, J.-N. Schmidt, M. Wenzel, J. Hertkorn, M. Guo, T. Langen, and T. Pfau, “Transient supersolid properties in an array of dipolar quantum droplets,” Phys. Rev. X 9, 011051 (2019).
[4] L. Chomaz, D. Petter, P. Ilzhöfer, G. Natale, A. Trautmann, C. Politi, G. Durastante, R. M. W. van Bijnen, A. Patscheider, M. Sohmen, M. J. Mark, and F. Ferlaino, “Long-lived and transient supersolid behaviors in dipolar quantum gases,” Phys. Rev. X 9, 021012 (2019).
[5] L. Tanzi, E. Lucioni, F. Fàmà, J. Catani, A. Fioretti, C. Gabbanini, R. N. Bisset, L. Santos, and G. Modugno, “Observation of a dipolar quantum gas with metastable supersolid properties,” Phys. Rev. Lett. 122, 130405 (2019).
[6] S. C. Schuster, P. Wolf, S. Ostermann, S. Slama, and C. Zimmermann, “Supersolid properties of a Bose-Einstein condensate in a ring resonator,” Phys. Rev. Lett. 124, 143602 (2020).
[7] Eugene P. Gross, “Unified theory of interacting bosons,” Phys. Rev. 106, 161–162 (1957).
[8] L. Santos, G. V. Shlyapnikov, P. Zoller, and M. Lewenstein, “Bose-Einstein condensation in trapped dipolar gases,” Phys. Rev. Lett. 85, 1791 (2000).
[9] D. H. J. O’Dell, S. Giovannazzi, and C. Eberlein, “Exact hydrodynamics of a trapped dipolar Bose-Einstein condensate,” Phys. Rev. Lett. 92, 250401 (2004).
[10] D. C. E. Bortolotti, S. Ronen, J. L. Bohn, and D. Blume, “Scattering length instability in dipolar Bose-Einstein condensates,” Phys. Rev. Lett. 97, 160402 (2006).
[11] H. Kadaw, M. Schmitt, M. Wenzel, C. Wink, T. Maier, I. Ferrier-Barbut, and T. Pfau, “Observing the Rosensweig instability of a quantum ferrofluid,” Nature 530, 194 (2016).
[12] M. Schmitt, M. Wenzel, F. Böttcher, I. Ferrier-Barbut, and T. Pfau, “Self-bound droplets of a dilute magnetic quantum liquid,” Nature 539, 259 (2016).
[13] L. Chomaz, S. Baier, D. Petter, M. J. Mark, F. Wächter, L. Santos, and F. Ferlaino, “Quantum-fluctuation-driven crossover from a dilute Bose-Einstein condensate to a macrodroplet in a dipolar quantum fluid,” Phys. Rev. X 6, 041039 (2016).
[14] Ariston R. P. Lima and Axel Pelster, “Quantum fluctuations in dipolar Bose gases,” Phys. Rev. A 84, 041604 (2011).
[15] A. R. P. Lima and A. Pelster, “Beyond mean-field low-lying excitations of dipolar Bose gases,” Phys. Rev. A 86, 063609 (2012).
[16] D. S. Petrov, “Quantum mechanical stabilization of a collapsing Bose-Bose mixture,” Phys. Rev. Lett. 115, 155302 (2015).
[17] F. Wächter and L. Santos, “Quantum filaments in dipolar Bose-Einstein condensates,” Phys. Rev. A 93, 061603 (2016).
[18] C. R. Cabrera, L. Tanzi, J. Sanz, B. Naylor, P. Thomas, P. Cheiney, and L. Tarruell, “Quantum liquid droplets in a mixture of Bose-Einstein condensates,” Science 359, 301 (2018).
[19] F. Böttcher, M. Wenzel, J.-N. Schmidt, M. Guo, T. Langen, I. Ferrier-Barbut, T. Pfau, R. Bombín, J. Sánchez-Baena, J. Boronat, and F. Mazzanti, “Dilute dipolar quantum droplets beyond the extended Gross-Pitaevskii equation,” Phys. Rev. Research 1, 033088 (2019).
[20] L. Tanzi, S. M. Roccozzu, E. Lucioni, F. Fàmà, A. Fioretti, C. Gabbanini, G. Modugno, A. Recati, and S. Stringari, “Supersolid symmetry breaking from compressional oscillations in a dipolar quantum gas,” Nature 574, 382 (2019).
[21] M. Guo, F. Böttcher, J. Hertkorn, J.-N. Schmidt, M. Wenzel, H. P. Büchler, T. Langen, and T. Pfau, “The low-energy goldstone mode in a trapped dipolar supersolid,” Nature 574, 386 (2019).
[22] Y.-C. Zhang, T. Pohl, and F. Maucher, “Phases of supersolids in confined dipolar Bose-Einstein condensates,” Phys. Rev. A 104, 013310 (2021).
[23] J. Hertkorn, J.-N. Schmidt, M. Guo, F. Böttcher, K. S. H. Ng, S. D. Graham, P. Uerlings, T. Langen, M. Zwierlein, and T. Pfau, “Pattern formation in quantum ferrofluids: From supersolids to superglasses,” Phys. Rev. Research 3, 033125 (2021).
[24] S. Giorgini, L. P. Pitaevskii, and S. Stringari, “Thermodynamics of a trapped Bose-condensed gas,” Journal of Low Temperature Physics 109, 309 (1997).
[25] E. Aybar and M. O. Öktem, “Temperature-dependent density profiles of dipolar droplets,” Phys. Rev. A 99, 013620 (2019).
[26] A. Griffin, “Conserving and gapless approximations for an inhomogeneous Bose gas at finite temperatures,” Phys. Rev. B 53, 9341 (1996).
[27] P. B. Blake, D. Baillie, L. Chomaz, and F. Ferlaino, “Supersolidity in an elongated dipolar condensate,” Phys. Rev. Research 2, 043318 (2020).
[28] F. Cinti, M. Boninsegni, and T. Pohl, “Exchange-induced crystallization of soft-core Bosons,” New Journal of Physics 16, 033038 (2014).
[29] L. Santos, G. V. Shlyapnikov, and M. Lewenstein, “Roton-maxon spectrum and stability of trapped dipolar Bose-Einstein condensates,” Phys. Rev. Lett. 90, 250403 (2003).
[30] R. M. Wilson, S. Ronen, J. L. Bohn, and H. Pu, “Manifestations of the roton mode in dipolar Bose-Einstein condensates,” Phys. Rev. Lett. 100, 245302 (2008).
[31] D. Petter, G. Natale, R. M. W. van Bijnen, A. Patscheider, M. J. Mark, L. Chomaz, and F. Ferlaino, “Probing the roton excitation spectrum of a stable dipolar Bose gas,” Phys. Rev. Lett. 122, 183401 (2019).
[32] J.-N. Schmidt, J. Hertkorn, M. Guo, F. Böttcher, M. Schmidt, K. S. H. Ng, S. D. Graham, T. Langen, M. Zwierlein, and T. Pfau, “Roton excitations in an oblate dipolar quantum gas,” Phys. Rev. Lett. 126, 193002 (2021).

[33] L. Landau, “On the theory of superfluidity,” Phys. Rev. 75, 884 (1949).

[34] D. H. J. O’Dell, S. Giovanazzi, and G. Kurizki, “Rotonics in gaseous Bose-Einstein condensates irradiated by a laser,” Phys. Rev. Lett. 90, 110402 (2003).

[35] N. Henkel, R. Nath, and T. Pohl, “Three-dimensional roton excitations and supersolid formation in Rydberg-excited Bose-Einstein condensates,” Phys. Rev. Lett. 104, 195302 (2010).

[36] R. Motíl, F. Brennecke, K. Baumann, R. Landig, T. Donner, and T. Esslinger, “Roton-type mode softening in a quantum gas with cavity-mediated long-range interactions,” Science 336, 1570 (2012).

[37] Y.-C. Zhang, V. Walther, and T. Pohl, “Long-range interactions and symmetry breaking in quantum gases through optical feedback,” Phys. Rev. Lett. 121, 073604 (2018).

[38] O. W. Dietrich, E. H. Graf, C. H. Huang, and L. Passell, “Neutron scattering by rotons in liquid helium,” Phys. Rev. A 5, 1377 (1972).

[39] T. Kebukawa, “The Temperature Dependence of Phonon Velocity and Roton Minimum in Liquid He II,” Progress of Theoretical Physics 49, 388. (1973).

[40] I. E. Mazets, D. H. J. O’Dell, G. Kurizki, Davidson N., and W. P. Schleich, “Depletion of a Bose-Einstein condensate by laser-induced dipole-dipole interactions,” Journal of Physics B: Atomic, Molecular and Optical Physics 37, S155 (2004).

[41] M. Sohmen, C. Politi, L. Klaus, L. Chomaz, M. J. Mark, M. A. Norcia, and F. Ferlaino, “Birth, life, and death of a dipolar supersolid,” Phys. Rev. Lett. 126, 233401 (2021).

[42] Y.-C. Zhang, F. Maucher, and T. Pohl, “Supersolidity around a critical point in dipolar bose-einstein condensates,” Phys. Rev. Lett. 123, 015301 (2019).

[43] G. Biagioni, N. Antolini, A. Alañna, M. Modugno, A. Fioretti, C. Gabbanini, L. Tanzi, and G. Modugno, “Dimensional crossover in the superfluid-supersolid quantum phase transition,” Phys. Rev. X 12, 021019 (2022).

[44] M. A. Norcia, C. Politi, L. Klaus, E. Poli, M. Sohmen, M. J. Mark, R. N. Bisset, L. Santos, and F. Ferlaino, “Two-dimensional supersolidity in a dipolar quantum gas,” Nature 596, 357 (2021).

[45] L. Tanzi, J. G. Maloberti, G. Biagioni, A. Fioretti, C. Gabbanini, and G. Modugno, “Evidence of superfluidity in a dipolar supersolid from nonclassical rotational inertia,” Science 371, 1162 (2021).

[46] A. Macia, J. Sánchez-Baena, J. Boronat, and F. Mazzanti, “Droplets of trapped quantum dipolar bosons,” Phys. Rev. Lett. 117, 205301 (2016).

[47] F. Cinti, A. Cappellaro, L. Salasnich, and T. Macrì, “Superfluid filaments of dipolar bosons in free space,” Phys. Rev. Lett. 119, 215302 (2017).

[48] L. Anderegg, S. Burchesky, Y. Bao, S. S. Yu, T. Karman, E. Chae, K.-K. Ni, W. Ketterle, and J. M. Doyle, “Observation of microwave shielding of ultracold molecules,” Science 373, 779 (2021).

[49] A. Schindewolf, R. Bause, X.-Y. Chen, M. Duda, T. Karman, I. Bloch, and X.-Y. Luo, “Evaporation of microwave-shielded polar molecules to quantum degeneracy,” (2022), arXiv:2201.05143 [cond-mat.quant-gas].

[50] M. Schmidt, L. Lassablière, G. Quéméner, and T. Langen, “Self-bound dipolar droplets and supersolids in molecular Bose-Einstein condensates,” Phys. Rev. Research 4, 013235 (2022).