Shear Viscosity of \( \text{Sr}_2\text{RuO}_4 \) in the Normal state

A. Karimi and M. A. Shahzamanian

Department of Physics, Faculty of Science, University of Isfahan, 81744, Isfahan, Iran.

E-mail: a_karimi90@yahoo.com

Abstract. The shear viscosity tensor of \( \text{Sr}_2\text{RuO}_4 \) is calculated. It is assumed that the low temperature metallic state of it is a two dimensional Fermi liquid. The Boltzmann equation has been solved in the normal state by using new method of calculation which is completely different from that of Abrikosov-Khalatnikov approximation. We show that the normal state viscosity coefficients varies with temperature as \( T^{-1} \) which in agreement with experimental results.

1. Introduction

\( \text{Sr}_2\text{RuO}_4 \) is the first layered perovskite superconductor without comprising coppers which is isostructural with high-\( T_c \) cuprate components\(^1\). Shortly after the discovery of the superconductivity \(^1\) in this compound, the possibility of spin triplet p-wave superconductivity was proposed in \( \text{Sr}_2\text{RuO}_4 \) \(^2\), in contrast to single d-wave pairing high-\( T_c \) cuprate. Strong support for triplet (p-wave) pairing is given by the results of Ishida et al.\(^3\) who have measured the Knight shift with a field parallel to the \( \text{RuO}_2 \) planes; the spin susceptibility measured by the Knight shift is not suppressed below \( T_c \), unlike a singlet superconductor. Also, muon spin rotation \( (\mu SR) \) measurements in the Meissner state in zero field\(^4\) have revealed spontaneous fields, which can be generated by domain boundaries, surfaces, and impurities in a superconductor which breaks time-reversal symmetry\(^5\). This results evidenced that the spin triplet p-wave superconductivity is realized in \( \text{Sr}_2\text{RuO}_4 \). The simplest \( \Delta \) consistent with this two properties possesses an orbital wave function \( \hat{d} = \Delta \hat{z}(k_x + i k_y) \)[6,7,8].

As usual, the metallic state exists above \( T_c \) or above the upper critical field \( (H_{c2}) \) in the presence of an externally applied magnetic field. \( \text{Sr}_2\text{RuO}_4 \) has the \( K_2\text{NiF}_4 \) structure with body centered tetragonal symmetry and two dimensional layered perovskite structural. Oguchi\(^9\) performed a calculation of the electronic energy band structure within the local density approximation\( (\text{LDA}) \), which predicted that the Fermi surface consistent of three strongly two dimensional sheets as \( \alpha, \beta, \) and \( \gamma \). It is worth mentioning that band structural calculation are in a fairly good agreement with results of de Hass-van Alphen experiments\(^8\).

Measurements of the bulk heat capacity, Pauli spin susceptibility, resistivity, and Hall number in \( \text{Sr}_2\text{RuO}_4 \) provided early indications that \( \text{Sr}_2\text{RuO}_4 \) is an ordinary metallic Fermi liquid, albeit a very anisotropic one and with strongly renormalized electron mass \( (m^* = 4m) \)[10].

If we apply to the system a static disorder such as the velocity gradient which create a momentum flow that is limited only by the quasiparticle collision. The induced flow is proportional to the applied gradient. The coefficient of proportionality is the shear viscosity.
By carrying out ultrasound attenuation experiment, Lupien et al. [11] and Matsui et al. [12] showed that shear viscosity at temperature above the transition temperature decrease with increasing temperature. By extrapolation of experimental data, we found that reduced viscosity varies as $T^{-1}$ which is in excellent agreement with the results of our calculations.

2. Collision integral

In the absence of a magnetic field the kinetic equation for the distribution function has the usual form[13]

\[
\frac{\partial n}{\partial t} + \frac{\partial n}{\partial \mathbf{v}} + \frac{\partial n}{\partial \mathbf{p}} \cdot \nabla \mathbf{v} - \frac{\partial n}{\partial \mathbf{p}} \cdot \mathbf{u} = I(n)
\]

where $I(n)$ is the collision integral and $n = n(\mathbf{p}, \sigma, \mathbf{r})$ is the distribution function for quasiparticles. Under non-equilibrium conditions, the energy is $\varepsilon = \varepsilon_0 + \delta\varepsilon$, and the particle distribution is

\[
\psi(\mathbf{p}, \sigma, \mathbf{r}) = n_0(\varepsilon_0) + \frac{\partial n_0}{\partial \varepsilon_0} \psi(\mathbf{p}, \sigma, \mathbf{r})
\]

Where $\psi$ gives a measure of the departure from equilibrium. If there is in the liquid a motion with a spatially slightly inhomogeneous velocity $\mathbf{\bar{u}}$, the distribution function will be only slightly different from its equilibrium value $n = n_0 + \delta n$ where $n_0 = \left[ \exp \left( \frac{\varepsilon_0 - \mu - \mathbf{p} \cdot \mathbf{u}}{K_B T} \right) \right]^{-1}$, $\delta n \ll n_0$.

With keeping the terms which contribute to the shear viscosity, and supposing $\mathbf{\bar{u}}, \mathbf{\bar{v}}, \mathbf{\bar{u}}$ are zero at the point considered, to first order in $\delta\nu_{\sigma}(p)$ we have

\[
-\frac{1}{2} \frac{\partial n_0}{\partial \varepsilon_0} \mathbf{p}_i \mathbf{p}_k \left( \frac{\partial \mathbf{u}_i}{\partial \mathbf{r}_k} \mathbf{p}_k + \frac{\partial \mathbf{u}_k}{\partial \mathbf{r}_i} \mathbf{p}_i + \frac{2}{3} \delta_{ik} \frac{\partial \mathbf{u}_i}{\partial \mathbf{r}_i} \right) = I(n)
\]

By substituting Eq.(2) and keeping the terms to first order in $\nu_{\sigma}(p)$ and using the fact that in the normal state quqsiparticle number is conserved, one can show that at low temperatures, only the binary processes are dominated. On the right-hand side of Eq.(1) we have the collision integral as

\[
I(n) = \frac{4n^2}{(2\pi h)^4} \int d\theta_1 d\theta_2 \psi(\theta) d\omega_0(x+y-t) \left[ 1-n_0(x) \right] \left[ 1-n_0(y) \right] \left[ \sum_{m=0}^{\infty} \left( \psi_{2m}^m(x) p_2^m(\cos \theta_2) + \psi_{2m}^m(y) p_2^m(\cos \theta_2) \right) \right] - 2 \sum_{m=0}^{\infty} \psi_{m}^m(x) p_2^m(\cos \theta_2) \sum_{m=0}^{\infty} \psi_{m}^m(y) p_2^m(\cos \theta_2) \right]
\]

Where $x = \frac{\varepsilon_0 - \mu}{K_B T}$, $y = \frac{\varepsilon_1 - \mu}{K_B T}$ and $t = \frac{\varepsilon_2 - \mu}{K_B T}$. $\Theta$ is the angle between $p_1$ and $p_2$, $\Theta$ is the angle between $p_1$ and $p_3$, $\Theta_4$ is the angle between $p_1$ and $p_4$. In the above equation $\psi$ has been written in terms of spherical harmonics, i.e., $\psi(p_1) = \psi(p_1, \Theta) = \sum_{m=0}^{\infty} \psi_{m}^m(x) p_2^m(\cos \Theta_4)$.

To solve the linearized Boltzmann equation it is suitable to define $q(t)$ as

\[
\psi = \frac{1}{2} q(t) \left( \frac{\partial \mathbf{u}_i}{\partial \mathbf{r}_k} + \frac{\partial \mathbf{u}_k}{\partial \mathbf{r}_i} + \frac{2}{3} \delta_{ik} \frac{\partial \mathbf{u}_i}{\partial \mathbf{r}_i} \right)
\]

By using the relation $\delta n = -\frac{n_0(t) (1-n_0(t))}{K_B T} \psi = \frac{\partial n_0}{\partial \varepsilon_0} \psi = \frac{\partial n_0}{\partial \varepsilon_0} \psi$ [13], and substituting Eq.(5) we get
\[ n_0(t)[1-n_0(t)] = \frac{4\pi^2 K_B T}{(2\pi^2)^4} \int d\theta d\phi \omega(\theta) \int d\omega_0(t)n_0(x + y - t)[1-n_0(x)][1-n_0(y)] \]
\[ \times \left[ 2 \sum_{m=0}^2 q(t) p^M_m(\cos \theta) + 2 \sum_{m=0}^2 q(x + y - t) p^M_m(\cos \theta) - 2 \sum_{m=0}^2 q(x) p^M_m(\cos \theta) - 2 \sum_{m=0}^2 q(y) p^M_m(\cos \theta) \right] \]

A conclusion that from the results obtained by Abrikosov-Khalatnikov can be seen that \( q(t) \) is a symmetry function, i.e., \( q(x + y - t) = q(x) = q(y) \) [13], according to that are in the normal state, with performing integration on \( x \), we get

\[ q(t) = \frac{4\pi^3 h^4}{m^2 T} \langle \omega(t) \rangle \]

### 3. Shear viscosity

The shear viscosity in general is a forth-rank tensor, which is defined by the relation

\[ \Pi_{ik} = -\sum_{i,m,a} \eta_{muk} \left( \frac{\partial u_i}{\partial \theta_k} + \frac{\partial u_i}{\partial \theta_i} - \frac{2}{3} \delta_{ik} \frac{\partial u_i}{\partial \theta_i} \right) \]

Where \( \Pi_{ik} \), the momentum flux tensor is

\[ \Pi_{ik} = \int p_i \frac{\hat{\varepsilon}(p)}{\hat{\rho}(p)} \frac{\partial n_0}{\partial \theta_0} \omega(r) d\tau \]

By substituting Eq.(5) in Eq.(9) and compared with Eq.(8) we have

\[ \eta_{muk} = \int p_i \frac{\hat{\varepsilon}(p)}{\hat{\rho}(p)} \frac{\partial n_0}{\partial \theta_0} \frac{1}{2} q(t) p_i \frac{\hat{\varepsilon}(p)}{\partial \theta_k} d\tau \]

Where we put \( p_i = p_0 \hat{\rho} \). so

\[ \eta_{muk} = \frac{p_0^2 \pi h}{2m^2 T} \langle \omega(\theta) \rangle \int \hat{\rho}_m \hat{\rho}_k d\theta \]

By calculating the angular integration, components of shear viscosity is obtained as follows

\[ \eta_{xx} = \mu_{yy} = \frac{3\pi^2 h^4}{8n^3 k_B T} \langle \omega(\theta) \rangle \]
\[ \eta_{xy} = \mu_{yx} = \frac{p_0^2 \pi^2 h^2}{8n^3 k_B T} \langle \omega(\theta) \rangle \]

Now, we write the viscosity components in terms of shear viscosity at \( T_c \), \( \eta(T_c) \).

\[ \eta_{xx} = \eta_{yy} = \frac{3\pi^2 h^4}{8n^3 k_B T} \left( \frac{T_c}{T} \right) \]
\[ \eta_{xy} = \frac{1}{3} \eta_{xx} \left( \frac{T_c}{T} \right) \]

Therefore, the temperature dependence of shear viscosity components were obtained as \( T^{-1} \). By carrying out ultrasound attenuation experiment, Lupein et al.[11] were analyzed shear viscosity. The results of their experiment showed that shear viscosity at temperature above the transition temperature decreases with increasing temperature. By extrapolation of experimental data, we found that reduced viscosity varies as \( T^{-1} \) which is in agreement with the results of our calculations.

### 4. Conclusion

The shear viscosity coefficient components of \( Sr_2RuO_4 \) are calculated in the Boltzmann approach. We generalized the approach to the two dimensional and obtained the temperature dependence of \( \eta \) as \( T^{-1} \).
Lupien et al. [11] and Matsui et al. [12] obtained a large unusual anisotropy in their experimental data. Walker et al. [14] have showed that strong anisotropy of the ultrasound attenuation in Sr$_2$RuO$_4$ is intimately connected with square lattice structure. Walker et al. [14] by taking into account the electron-phonon coupling showed that the relation between the phonon life time and electron life time is as follows [14]

$$\frac{1}{\tau_{q,J}} = \frac{8!}{\rho \omega_{j}^2} \tau_{q,J} F_{J} (k,q) - \left\langle F_{J} (k,q) \right\rangle^2_{FS}$$

(16)

Where $N_F = \frac{m_p}{\pi^2}$ is the density of states at the Fermi level in the normal state and $F_{J} (k,q)$ is the electron-phonon matrix element for each sheet.

If we renormalized the electron life time in Eq.(7), the results in Eq.(12) and Eq.(13) may be rewritten as

$$\bar{\eta}_{T00} = \frac{8! m_p}{\pi^2} \frac{3! \pi^2 V_F e^2}{8! (v(o))} \left\langle F_{J}^2 (k,q) \right\rangle_{FS} - \left\langle F_{J} (k,q) \right\rangle^2_{FS}$$

(17)

$$\bar{\eta}_{L100} = \frac{8! m_p}{\pi^2} \frac{9! \pi^2 V_F e^2}{8! (v(o))} \left\langle F_{J}^2 (k,q) \right\rangle_{FS} - \left\langle F_{J} (k,q) \right\rangle^2_{FS}$$

(18)

Similarly, calculations are done for other modes. Relations (17) and (18) show that the electron-phonon interaction do not change the dependence of the shear viscosity components.

As we mention the temperature dependence of the shear viscosity coefficient components is proportional to $T^{-1}$ which is in agreement with the results of Lupien et al. [11] and Matsui et al. [12].

The calculation of these components in the superconducting state is under our constraction and will be published elsewhere.

Acknowledgments
This study has been financially supported by the research council of the University of Isfahan, Iran.

References
[1] Maeno Y, Hashimoto H, Yoshida K, Nishizaki S, Fujita T, Bednorz J G and Lichtenberg F 1994 Nature (Lonon) 372 532
[2] Rice T M and Sigrist M 1995 J.Phys.Condens.Matter 7 L643
[3] Ishida K, Mukuda H, Kitaoka Y, Asayama K, Mao ZQ, Mori Y and Maeno Y 1998 Nature (London) 396 65
[4] Luke G M et al. 1998 Nature (Lonon) 394 558
[5] Sigrist M and Ueda K1991 Rev.Mod.Phys. 63 239
[6] Kealey P G et al. 2000 Phys. Rev. Lett. 84 6094
[7] Mukuda H, Ishida K, Kitaoka Y, Mori Y and Maeno Y 1995 J.Low Temp. Phys. 117 1587
[8] Mackenzie A P and Maeno Y 2003 Rev. Mod. Phys. 75 657
[9] Oguchi T 1995 Phys. Rev.B 511 385-88
[10] Begemann C, Mackenzie A P, Julian S R, Forsythe D and Ohmichi E 2003 Adv.Phys. 52 639
[11] Lupien C, Macfarlane W A, Proust C, Taillefer L, Mao Z Q and Maeno Y 2001 Phys. Rev.Lett. 86 5986
[12] Matsui H, Yoshida Y, Mukai A, Settai R, Onuki Y, Takei H, Kimura N, Aoki H and Toyota N 2001 Phys.Rev. B 63 060505(R)
[13] Abrikosov A A and Khalatnikov I M 1959 Rep.Porg. Phys 22 329
[14] Walker M B, Smith M F and Samokhin K V 2001 Phys.Rev. B 65 014517