Exploring black holes as particle accelerators in realistic scenarios

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The possibility that rotating black holes could be natural particle accelerators has been subject of intense debate. While it appears that for extremal Kerr black holes arbitrarily high center of mass energies could be achieved, several works pointed out that both theoretical as well as astrophysical arguments would severely dampen the attainable energies. In this work we study particle collisions near Kerr–Newman black holes, by reviewing and extending previously proposed scenarios. Most importantly, we implement the hoop conjecture for all cases and we discuss the astrophysical relevance of these collisional Penrose processes. The outcome of this investigation is that scenarios involving near-horizon target particles are in principle able to attain, sub-Planckian, but still ultra high, center of mass energies of the order of $10^{21}$–$10^{23}$ eV. Thus, these target particle collisional Penrose processes could contribute to the observed spectrum of ultra high-energy cosmic rays, even if the hoop conjecture is taken into account, and as such deserve further scrutiny in realistic settings.

I. INTRODUCTION

Since Penrose’s original paper [1], pointing out the possibility to exploit rotating black holes’ ergoregions to extract energy, there have been several efforts in the literature aiming at developing and optimizing this idea. In [2], Bañados–Silk–West (BSW) observed that collisions of particles with specific angular momentum values produce arbitrarily high center of mass energies in extremal Kerr black hole spacetimes. This collisional Penrose process has been studied for different black hole scenarios, such as non-extremal Kerr [3–6], Kerr–(anti)de Sitter [7, 8], Reissner–Nordström [9, 10], and Kerr–Newman [11–13] geometries, reproducing the original BSW result. Also, taking into account the spin or the charge of the particles, one still obtains an arbitrarily high center of mass energy for collisions near to the horizon of rotating black holes [14–17]. For non-rotating black holes this feature is absent [18], as long as the colliding particles fall towards the black hole from large (infinite) radial distance.

More recently, an alternative mechanism, leading to infinite center of mass energies in collisions non-rotating black hole spacetimes, has been proposed [19]. The difference with respect to the BSW setup is that only one of the colliding particles comes from infinity while the other particle (the target particle) is created (or placed) close to the horizon. The latter feature means that the energy parameter of the particle trajectory of this second particle is small, which can be used to show that even in a non-rotating black hole spacetime, arbitrarily large center of mass energies can be achieved [19]. Following the same idea, a similar result can be derived in the case in which the spin of the particles is also taken into account [20]. For rotating black holes, it is even possible to consider the scenario of the collision of one infalling particle coming from infinity and a zero or negative energy particle, propagating inside the ergosphere [21].

Coming back to the above described collisional Penrose process, a common objection is that it is derived in an over idealized situation. Indeed, in the extant literature many arguments have been brought forward about why this infinite center of mass energy cannot emerge in a realistic physical situation. In particular by pointing out that in realistic situations several limiting mechanisms would not only prevent these unbounded energies but also lead to relatively low values.
The first of such mechanisms worth mentioning is the so-called "hoop conjecture". Proposed by Thorne in 1972 [22], it basically states that the presence of a particle with non-negligible energy in the vicinity of a black hole induces a backreaction which leads to a new, larger, horizon before the particle crosses the original one. This feature (in fact a weaker version of this statement [23]), was used in [24] to show that the above mentioned collisions cannot take place as close to the black hole horizon as needed for achieving ultra-high energies, but always for at a greater radius limited below by the hoop conjecture. The implementation of the hoop radius, the new event horizon of the enlarged black hole, avoids the prediction of infinite energy collisions outside an event horizon [24].

Other important limiting factors which one needs to take into account when searching for such high center of mass collisions near black holes are: the Thorne limit [25], which states that the spin of realistic astrophysical black holes is bounded from above [5, 26, 27]; the time which passes for an observer far away from the black hole, while the collision of two particles reaching the near horizon region of a black hole take place (this can be of order of the age of the universe) [18, 28]; multiple scatterings and collisions of the particle(s) emerging after the collision near the black hole [29–31]. All of these phenomena need to be taken into account and are basically screening the existence of a collisional Penrose process or damping its effectiveness as a particle accelerator.

In this work we begin by studying the most general scenario of a charged rotating black hole in Sec. II with the aim to clarify the general workings of the BSW mechanism and its variants. After that, we apply the hoop conjecture to the above discussed alternative scenarios (collision of two particles coming both from infinity, or one from infinity and one target particle with small energy), in Secs. III, and IV, respectively. Eventually, in Sec. V, we extend the ergosphere target particle scenario particle by implementing the hoop conjecture and considering a target particle with zero energy and small angular momentum.

We shall see that in all these scenarios the energy of the center of mass always stays finite (due to the hoop conjecture) but still can be very large. Also, in Sec. VI we show that the same energy-limiting arguments considered for the BSW mechanism are still effective in the aforementioned new scenarios. Nonetheless, we find that in the new class of collisional scenarios (those with a target particle placed close to the hoop horizon), very high center of mass energies are generically still achievable. Indeed, the order of these energies is higher than the highest energies detected in ultra high energy cosmic rays (UHECR). This seems to suggest that, at least in principle, these scenarios could contribute in a relevant way to the spectrum of these ultra energetic particles, and that a deeper investigation about the expected spectrum and rate of production (including some suggested astrophysical limiting mechanisms) is worth pursuing. We shall comment further about this in our conclusions Sec. VI.C.

II. PARTICLE MOTION IN KERR–NEWMAN BLACK HOLE GEOMETRY

In this section, we recall how the center of mass energy of particle collisions in the vicinity of a charged, rotating, Kerr–Newman black hole is derived. The metric defining this geometry is [11]

\[
g_{tt} = -\left(1 - \frac{2Mr - Q^2}{\Sigma}\right), \quad g_{t\varphi} = -\bar{a}\sin^2\theta \frac{2Mr - Q^2}{\Sigma}, \quad g_{rr} = \frac{\Sigma}{\Delta},
\]

\[
g_{\theta\theta} = \Sigma, \quad g_{\varphi\varphi} = \frac{\sin^2\theta}{\Sigma} \left((\bar{a}^2 + r^2)^2 - \bar{a}^2\Delta\sin^2\theta\right),
\]

where \(\Sigma = r^2 + \bar{a}^2\cos^2\theta\), \(\Delta = r^2 + \bar{a}^2 - 2rM + Q^2\), and \(\bar{a} = J/M\), with \(M\), \(J\), and \(Q\) being the mass, angular momentum and charge of the black hole. The horizon is characterized by the condition \(\Delta = 0\), while \(g_{tt} = 0\) defines the so-called ergosphere.

The geodesic motion of point particles in a Kerr-Newman spacetime can be obtained by solving the Hamilton equations of motion of the Hamilton function

\[
H(x, k_l) = g^{\mu\nu}k_{\mu}k_{\nu},
\]

which defines the dispersion relation \(H(x, k_l) = -m^2\) of massive \((m \neq 0)\) and massless \((m = 0)\) point particles. We will use this Hamiltonian approach to describe the particle motion in order to simplify the computations.

For the purpose of this article it suffice to restrict the particle trajectories to the the equatorial plane, i.e. \(\theta = \pi/2\). Due to the static and axial symmetry of the metric Kerr–Newman geometry we can characterize the trajectories of point particles in terms of the radial momentum \(k_r\). Using the dispersion relation, we can express the radial momentum as a function of the mass \(m\) and the constants of motion, i.e. the energy \(E = k_t\), and the angular momentum \(L = -k_\varphi\)

\[
k_r = \frac{1}{\Delta} \sqrt{(E(\bar{a}^2 + r^2) - \bar{a}L)^2 - (m^2r^2 + (\bar{a}E - L)^2)\Delta}.
\]
Consider two uncharged particles of equal mass \( m_1 = m_2 = m \) with four-momenta \( p_{1\nu} = (E_1, p_{1r}, 0, L_1) \) and \( p_{2\nu} = (E_2, p_{2r}, 0, L_2) \) respectively. The squared center of mass energy at the point of collision as function of \( r \) is defined as

\[
E_{cm}^2(r) = -g_{\mu\nu}(p_1^\mu + p_2^\nu)(p_1^\nu + p_2^\mu).
\]  

(4)

Using Eqs. (3) and (4) we find

\[
E_{cm}^2(r) = \frac{2}{r^2 \Delta} \left( (E_1(\bar{a}^2 + r^2) - \bar{a}L_1)(E_2(\bar{a}^2 + r^2) - \bar{a}L_2) + \Delta(m^2r^2 - (\bar{a}E_1 - L_1)(\bar{a}E_2 - L_2) \right.
\]

\[
- \sqrt{(E_1(\bar{a}^2 + r^2) - \bar{a}L_1)^2 - \Delta((\bar{a}E_1 - L_1)^2 + m^2r^2)}\sqrt{(E_2(\bar{a}^2 + r^2) - \bar{a}L_2)^2 - \Delta((\bar{a}E_2 - L_2)^2 + m^2r^2)} \right).
\]  

(5)

For collisions at the outer black hole horizon \( r_{\text{horizon}} = M(1 + \tau) \), with

\[
\tau = \sqrt{1 - \left( \frac{\bar{a}}{M} \right)^2 - \left( \frac{Q}{M} \right)^2},
\]  

(6)

one gets

\[
E_{cm}^2(r_{\text{horizon}}) = \frac{(\bar{a} (m^2 - E_1^2) + E_1 L_1)(E_2 L_1 - E_1 L_2)}{E_1 (\bar{a}^2 E_1 - \bar{a}L_1 + E_1 M^2(\tau + 1)^2)} + \frac{(\bar{a} (m^2 - E_2^2) + E_2 L_2)(E_1 L_2 - E_2 L_1)}{E_2 (\bar{a}^2 E_2 - \bar{a}L_2 + E_2 M^2(\tau + 1)^2)}
\]

\[+ \frac{m^2(E_1 + E_2)^2}{E_1 E_2}.
\]  

(7)

There are several scenarios for which this expression may become infinitely large.

As long as \( \bar{a} \neq 0 \), i.e. for rotating black holes, there exists a critical value of the angular momentum of the particles such that, if one of the colliding particles assumes this angular momentum, the denominator of one of the first two terms vanishes, as has been discussed in the literature [2]. A second possibility is that the energy of one of the particles is very small but non-vanishing and that its angular momentum is zero, which can be seen by applying a power series expansion for small \( E_1 \) or \( E_2 \) to (7). For non-rotating black holes, this applies without any constraint on the angular momentum of the particle, as has been pointed out in [19]. For rotating black holes, there exists the possibility of particle trajectories in the ergosphere with vanishing energy parameter, which would lead to a divergence in the last term of (7). This very last scenario has not yet been studied in the literature and we investigate it here for the first time in detail.

So far, the particles propagating through the black hole spacetime have been treated as test particles without backreaction onto the geometry. However, in case the energy of the individual particles or their center of mass energy becomes large, their backreaction cannot be neglected. A way to capture the influence of the particles on the geometry is the hoop conjecture [22]. It states that the event horizon of a black hole increases due to the presence of the energy-momentum of the particles. For Kerr-Newman black holes this hoop horizon radius is, see for example [23, 24],

\[
r_{\text{hoop}} = M + E_1 + E_2 + \sqrt{(M + E_1 + E_2)^2 - \frac{(J + L_1 + L_2)^2}{(M + E_1 + E_2)^2} - (Q + Q_1 + Q_2)^2},
\]  

(8)

where \( E_i, L_i, \) and \( Q_i \) the energies, angular momentum, and charges of the infalling particles. In the following, we will use the mass normalized quantities \( \bar{l}_i = L_i/(Mm) \), \( \bar{a} = \bar{a}/M \) and \( q = Q/M, q_i = Q_i/(Mm) \).

Thus, from the outside of the black hole, only collisions at \( r > r_{\text{hoop}} \) can, at least in principle, produce detectable signatures.

Having set the stage for our analysis of particle collisions, in the next section we slightly extend the results for two particles falling in towards the black hole from infinity and colliding near the horizon from the literature to Kerr-Newman black holes. Then we will present new insight about the collision between one particle that is infalling towards the black hole from infinity and one target particle being near the black hole with almost zero (small) energy parameter \( E \) and vanishing angular momentum \( L \) in section IV. Finally we study the novel scenario of a target particle with zero energy parameter in the ergoregion, colliding with an infalling particle from infinity in Section V.
III. PARTICLE COLLISIONS FOR INFALLING PARTICLES FROM INFINITY

In this section we consider the scenario proposed by BSW \[2\] in which two particles coming from infinity collide near the black hole. The non-extremal and extremal case are studied separately.

In general for particles coming from infinity, their energy parameter \(E\) is determined at \(r \to \infty\) by the dispersion relation (2)

\[k^2 + m^2 = E^2.\]  \hfill (9)

For our derivation, we assume particles which are initially at rest, i.e. \(E_1 = E_2 = m\), and neutral \(q_1 = q_2 = 0\), since the charge makes no qualitative difference in our findings.

A. Non-extremal case

We briefly discuss why the collision of particles, which are infalling from infinity, does not produce infinite or very large center of mass energies in non-extremal Kerr-Newman spacetimes, taking the hoop conjecture into account. This is in contrast to our later analysis of the collision between a target particle and an infalling particle from infinity in Secs. IV A and V.

In order to find the minimum radius for which the collision can take place we evaluate the hoop radius (8) for the colliding particles in consideration. For large black hole masses \(M\) and small particle masses \(m\), i.e. \(M \gg m\), we find, to first order in \(m\), and using \(\tau\) as in (6),

\[r_{\text{hoop}} = M(1 + \tau) + m \left(2 + \frac{2 + 2a^2 - a(l_1 + l_2)}{\tau}\right).\]  \hfill (10)

Note that there is an apparent divergence when the black hole is extremal, i.e. for \(\tau \to 0\). This is the reason why need to consider the extremal scenario separately.

Evaluating the squared center of mass energy (5) at the hoop radius (10) we find up to second order in \(m\)

\[\frac{E_{\text{cm}}^2(r_{\text{hoop}})}{m^2} = 4 + \frac{l_1(l_1 - l_2)}{a^2 - al_1 + (1 + \tau)^2} - \frac{l_2(l_1 - l_2)}{a^2 - al_2 + (1 + \tau)^2}.\]  \hfill (11)

Clearly there is an apparent divergence when one of the angular momentum of the particles have the critical value

\[l_i = \frac{a^2 + (1 + \tau)^2}{a}.\]  \hfill (12)

To see if this infinity can really be reached let us evaluate the hoop radius directly for one particle having this critical value of the angular momentum (10). The result is

\[r_{\text{hoop,crit}} = M(\tau + 1) + \frac{m(a^2 - al_2 - \tau^2 + 1)}{\tau},\]  \hfill (13)

and leads to a squared center of mass energy

\[\frac{E_{\text{cm}}^2(r_{\text{hoop,crit}})}{m^2} = \frac{\sqrt{2\sqrt{M\sqrt{a^2 + (\tau + 1)^2}(a^2 - al_2 + (\tau + 1)^2)}}}{\sqrt{a\sqrt{m(\tau + 1)\sqrt{a(l_2 - a) + \tau^2 - 1}}}} + \frac{2(-a^2l_2 + a^3 + a(\tau + 1)(3\tau + 1) - l_2\tau(\tau + 1))}{a\tau(\tau + 1)}.\]  \hfill (14)

This center of mass energy still may become infinite (very large) if either \(\tau \to 0\) (which we already said deserves a separated treatment) or if the second particle has the critical angular momentum value

\[l_2 = \frac{a^2 - \tau^2 + 1}{a}.\]  \hfill (15)

However, doing a power series expansion in \(m\) of the radial component of the momentum of the first particle (see Eq. (3)), at \(r_{\text{hoop,crit}}\) for any value of \(l_2\), one finds that the momentum becomes imaginary. Hence, particles with an angular momentum as in equation (12) do not exist.

Thus, with no surprise, for non-extremal Kerr-Newman black holes, infinite (or very high) center of mass energy collisions, with both particles infalling towards the black hole from infinity, are not allowed.
B. Extremal case

In the extremal case when \( q^2 + a^2 = 1 \) (or \( \tau = 0 \)), to lowest orders in \( m \), the hoop radius is

\[
r_{\text{hoop}} = M + \sqrt{2mM} \sqrt{2 + 2a^2 - a(l_1 + l_2) + 2m},
\]

and one easily finds

\[
E_{\text{cm}}^2(r_{\text{hoop}}) = \frac{(-4a^3(l_1 + l_2) + a^2 (l_1 + l_2)^2 + 8 + 4a^4 - 4a(l_1 + l_2) + (l_1 - l_2)^2 + 4)}{(a^2 - al_1 + 1)(a^2 - al_2 + 1)}.
\]

This expression potentially becomes infinite for a critical value

\[
l_1 = \frac{1 + a^2}{a},
\]

which is actually obtained in the \( \tau \to 0 \) limit of the critical value in the non-extremal case (12).

However, this infinity is not reached, since, when evaluating the hoop radius for one particle having the critical angular momentum (18), then

\[
r_{\text{hoop,crit}} = M + 2m + \sqrt{2(1 + a^2 - al_2)mM},
\]

and the leading order term in the squared center of mass energy becomes

\[
E_{\text{cm}}^2(r_{\text{hoop,crit}}) = \sqrt{2} \left( 2a - \sqrt{3a^2 - 1} \right) \sqrt{mM} \sqrt{a^2 - al_2 + 1},
\]

which is perfectly finite. However, it can possibly be very large due to the \( \sqrt{M} \) factor appearing. The result smoothly becomes the result for the extremal Kerr black hole for \( a \to 1 \)

\[
E_{\text{cm}}^2(r_{\text{hoop,crit}}) = 2 \left( \sqrt{2} - 1 \right) m\sqrt{mM} \sqrt{2 - l_2},
\]

which was already presented in [24].

IV. NEAR-HORIZON TARGET PARTICLE WITH SMALL ENERGY AND ZERO ANGULAR MOMENTUM

Now we are going to consider the scenario in which one of the particles comes from infinity, i.e. \( E_2 = m \), and the other particle, called the target particle, is near the horizon with small energy parameter \( E_1 \), i.e. \( M \gg m \gg E_1 \).

Again we assume that the particles are neutral \( q_1 = q_2 = 0 \). This scenario was originally suggested for non-rotating black holes in [19].

For this situation, to first order in \( E_1 \), the squared center of mass energy (7) becomes

\[
E_{\text{cm}}^2(r_{\text{horizon}}) = \frac{2}{m^2} \left( \frac{a^2 - al_2 + l_1^2 + (\tau + 1)^2}{a_1} + \frac{l_1 l_2}{a^2 - al_2 + (\tau + 1)^2} \right) + \frac{E_1}{m} \left( -\frac{(a^2 + (\tau + 1)^2)^2}{a_1^2 l_1^2} + \frac{l_1^2}{a^2 - al_2 + (\tau + 1)^2} \right) - \frac{(\tau + 1)^2}{a^2} + \frac{l_2(\tau + 1)^2}{al_1} + \frac{al_2}{l_1^2} + \frac{a + l_2}{a}.
\]

Thus a divergence may appear for \( l_1 \to 0 \).

As before, we will discuss in different sections the non-extremal and extremal scenarios and take the hoop conjecture into account.

A. Non-extremal case

Considering the hoop conjecture, the smallest radius at which the collision can take place is given by the hoop radius \( (8) \) with \( E_2 = m \), \( q_1 = q_2 = 0 \) and \( l_1 = 0 \). The first correction to the event horizon in the hoop radius is of order \( m \) and given by

\[
r_{\text{hoop}} = M (1 + \tau) + m \left( 1 + \frac{1 + a^2 - al_2}{\tau} \right).
\]
As in the previous scenario, this quantity seems to diverge when the black hole is extremal \((\tau = 0)\). However, as we will see in the next subsection, this is not the case.

Using (24) in the dispersion relation we find an expression for \(E_1(m, M, a, l_2)\), which becomes, when expanded in powers of \(M^{-1}\),

\[
E_1 = \frac{m(\tau + 1)\sqrt{2m}\sqrt{a^2 - al_2 + \tau + 1}}{\sqrt{M}\left(a^2 + (\tau + 1)^2\right)}.
\]  

(25)

Then, using (24) and (25) in (5) with \(E_2 = m, L_2 = l_2mM\) and \(L_1 = 0\) as well as \(Q = qM = \sqrt{1 - \tau^2 - a^2M}\), we find up to leading order in \(M\),

\[
E_{cm}^2(r_{\text{hoop}}) = \frac{m\sqrt{2mM}\left(a^2 - al_2 + (\tau + 1)^2\right)}{(\tau + 1)\sqrt{a^2 - al_2 + \tau + 1}}.
\]  

(26)

This center of mass energy may be very large (in contrast to the setup discussed in Section III A), but stays finite. One might think that tuning \(l_2 = \frac{a^2 + \tau + 1}{\tau}\) could still lead to an infinite value. For this choice however, the target particle is unphysical, since it does not satisfy the dispersion relation.

### B. Extremal case

Considering a collision between a target particle and a particle from infinity in the vicinity of a extremal Kerr-Newman black hole, \(\tau = 0\), the parameters \(E_2 = m, q_1 = q_2 = 0, l_1 = 0\) and \(m \geq E_1\) yield a leading order hoop radius, see (8), of

\[
r_{\text{hoop}} = M + m + \sqrt{2Mm\sqrt{1 + a^2 - al_2}}.
\]  

(27)

To ensure that the target particle is physically viable we again solve the dispersion relation at \(r = r_{\text{hoop}}\), (26), for \(E_1(m, M, a, l_2)\), expanded in powers of \(M^{-1}\). The leading order term is

\[
E_1 = \frac{m\sqrt{2m}\sqrt{a^2 - al_2 + 1}}{\sqrt{M}(1 + a^2)}.
\]  

(28)

Then, applying (27) and (28) in (5) with \(E_2 = m, L_2 = l_2mM\) and \(L_1 = 0\) as well as \(Q = qM = \sqrt{1 - \tau^2 - a^2M}\), we find, again up to leading order in \(M\),

\[
E_{cm}^2(r_{\text{hoop}}) = m\sqrt{2mM}\sqrt{a^2 - al_2 + 1}.
\]  

(29)

The results (28) and (29) can alternatively be obtained taking the limit \(\tau \to 0\) in (25) and (26), even though this limit cannot be taken directly for the hoop radius.

We can conclude that, also in the extremal scenario, the center of mass energy becomes large but stays finite.

### C. The hoop conjecture in Schwarzschild geometry

In [19] the possibility of an arbitrarily large center of mass energy was discussed for the non-rotating and non-charged black hole. From the results for the general Kerr-Newman spacetime, we find the influence of the hoop conjecture for this case by setting \(a = 0\) and \(\tau = 1\).

The hoop radius of a non-rotating Schwarzschild black hole in the presence of a target and incoming particle is

\[
r_{\text{hoop}} = 2(M + m).
\]  

(30)

The energy of the target particle follows from (25)

\[
E_1 = \frac{m\sqrt{m}}{\sqrt{M}},
\]  

(31)

and the squared center of mass energy can be read off from (26)

\[
E_{cm}^2(r_{\text{hoop}}) = 2m\sqrt{mM}.
\]  

(32)

Thus, by taking the hoop conjecture into account, the squared center of mass energy cannot be arbitrarily large. It is proportional to \(m\sqrt{mM}\), which is finite. The mass of the black hole determines the scale of the center of mass energy of the collision.
V. TARGET PARTICLE WITH ZERO ENERGY AND SMALL ANGULAR MOMENTUM

For rotating black holes in the ergosphere, it is possible that the target particle has zero energy, $E_1 = 0$ (see for example [21]), which leads to a possibly diverging center mass energy at the horizon according to (7). For small angular momentum $l_1$, and $E_2 = m$, we find

$$\frac{E_{cm}^2(r_{\text{horizon}})}{m^2} = \frac{E_1 l_1^2}{m (a^2 - al_2 + (\tau + 1)^2)} + \frac{a l_1 (E_1 - m) (E_1 + m)}{E_1 m (a^2 + (\tau + 1)^2)} + \frac{(E_1 + m)^2}{E_1 m}$$

$$+ l_1 \left( \frac{a^2 l_2 (E_1 - m) (E_1 + m)}{E_1^2 (a^2 + (\tau + 1)^2)^2} - \frac{a (E_1 - m) (E_1 + m)}{E_1^2 (a^2 + (\tau + 1)^2)} - \frac{l_2}{a^2 + (\tau + 1)^2} - \frac{l_2}{a^2 - al_2 + (\tau + 1)^2} \right).$$

(33)

(34)

This scenario can be treated analogously to the one for particles with small energy and zero angular momentum. To lowest order (neglecting terms that are products between $m$ and $l_1$), the hoop radius in the non-extremal case is again (24). The dispersion relation then fixes a small angular momentum $l_1$ (instead of the energy $E_1$ in the previous section), which becomes

$$l_1 = \frac{\sqrt{2m (\tau + 1)} \sqrt{\tau + a^2 - al_2 + 1}}{a \sqrt{M}}. \quad \text{(35)}$$

Noticeably, this quantity seems to diverge when $a = 0$, but this is just the manifestation that zero-energy particles do not exist in the non-rotating case. Combining these findings in the squared center of mass energy we obtain

$$E_{cm}^2(r_{\text{hoop}}) = m \frac{\sqrt{2mM (a^2 - al_2 + (1 + \tau)^2)}}{(\tau + 1) \sqrt{a^2 - al_2 + 1 + \tau}},$$

(36)

which is identical to (26). The limit to the extremal case can be treated by using the hoop radius (27) and setting $\tau \to 0$.

As in the small $E_1$ and $l_1 = 0$ case, for collisions at the hoop radius the center of mass energy stays finite. In contrast, when the hoop conjecture is not applied, one obtains the possibility of a diverging center of mass energy, as has been found in [3].

VI. DISCUSSION

We derived the center of mass energy of different types of particle collisions in Kerr-Newman spacetimes, by taking the hoop conjecture into account. We found that the hoop conjecture avoids infinite center of mass energies for all kinds of collisions: between infalling particles from infinity and between one infalling and one target particle.

Nevertheless, the hoop conjecture does not imply that the center of mass energies in particle collisions must be small, their value can still be large. To get an estimate how large these energies can be we calculate some orders of magnitude for realistic stellar black hole masses and angular momenta. Moreover there exist further mechanisms which prevent the detection of such high energetic collisions, which we briefly mention.

A. Realistic black holes as particle accelerators

Besides the hoop conjecture, one main argument against high center of mass collisions [5, 26, 27] is that black holes cannot become extremal [25]. Theoretical consideration limit their maximal angular momentum in astrophysical setting to $a = 0.998$.

Moreover, their charge is twelve orders of magnitude lower than their extremal value [32, 33]. Thus, realistic astrophysical black holes can safely considered as non-extremal Kerr black holes. This means that the BSW scenario does not apply for realistic black holes.

In contrast, as we pointed out in Sections IV and V, both target particle scenarios lead to a very high center of mass energies even for non extremal uncharged rotating black holes. Actually, the target particle scenario discussed in Section IV does so even for non-rotating black holes [19]. We saw that, even taking into account the hoop conjecture, the target particle scenarios lead to squared center of mass energies proportional to $m \sqrt{mM}$, as we derived in (26) (or (36)). Setting $l_2 = 0$, for the boundary values of the possible angular momentum parameter $a$, one finds

$$E_{cm}^2(r_{\text{hoop}}) = 1.98 \sqrt{mM} \quad \text{for} \quad a = 0.3,$$

$$E_{cm}^2(r_{\text{hoop}}) = 1.97 \sqrt{mM} \quad \text{for} \quad a = 0.998.$$

(37)
To get an insight of how high the center of mass energies become, for example in positron-electron pair collisions ($m = 0.511 \times 10^6$eV) we consider two type of black holes:

- stellar black holes of 10 solar masses ($M = 1.12 \times 10^7$eV) yield energies of the order
  \[ E_{\text{cm}}(r_{\text{hoop}}) = 1.5 \times 10^{21} \text{eV}, \]  
  \[ (38) \]
- the most massive black holes which have been detected with a Mass of $10^{10}$ solar masses [34, 35] would allow for energies of the order
  \[ E_{\text{cm}}(r_{\text{hoop}}) = 2.7 \times 10^{23} \text{eV}. \]  
  \[ (39) \]

Clearly, these energies do not reach the Planck energy $E_{\text{Pl}} = 1.22 \times 10^{28}$eV, but stay five orders of magnitude below that. Nevertheless, the energies which can be produced by a target particle-infalling particle collisional Penrose process in the ergosphere of a rotating non-extremal black hole are so large that they reach the energies of particles detected in ultra-high-energy cosmic rays (UHECR) [36–38]. Thus realistic rotating black holes indeed could be the physical objects accelerating particles to these enormous energies.

### B. Detection

A question, which needs to investigated in detail, is how much such collisional Penrose processes in the vicinity of rotating black holes contribute realistically to UHECRs, and how one can distinguish between particles accelerated by different processes.

It has been argued in the literature that several processes may damp the energies associated to these processes. The two main arguments are the following. First, that, for distant observers, the time until such a collision takes place close to the hoop radius of a black hole, can become arbitrarily large [18, 28]. Second, that, if such high energetic collisions happen near the a black hole, the secondary products produced in such a collision may loose energy very quickly, due to multiple scatterings taking place before the particles can escape towards the Earth and be detected [29–31].

Both effects have been investigated in some detail for the case of two infalling particles coming from infinity. A detailed discussion of these mechanisms for the target particle scenario is currently missing and deserve a detailed study in the near future.

### C. Conclusion

We found that, when the hoop conjecture is applied, infinite center of mass energy collisions are avoided outside of the event horizon of any type of Kerr–Newman black hole. The squared center of mass energy will always be of order $m \sqrt{mM}$, where $m$ is the mass of the particles and $M$ is the mass of the black hole. Thus for very heavy black holes, large center of mass energies are possible. The most massive black holes observed, $10^{10}$ solar masses, lead to energies of order $10^{23}$eV, which slightly higher than the highest energies detected in UHECRs.

While further investigation is needed in order to confront these findings with observations, they nonetheless seem to suggest that indeed non-extremal rotating black holes could be capable of accelerating particles up to and beyond the typical energies characterizing the observed UHECRs. A detailed analysis of how much particles produced in target particle scenarios could contribute to UHECRs and how often such collisions could happen in realistic settings is then the natural next step for the present investigation. In particular, arguments on the timescale of such collisions and multiple scatterings, which have been brought forward for the two infalling particles from infinity, will also have to be taken into account carefully. We hope to address these and other points in future investigations.

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