Performance Analysis Of Two Heterogeneous Server Queueing Model with Intermittently Obtainable Server Using Matrix Geometric Method

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Abstract. We consider two heterogeneous server, unit arrived according to Poisson distribution with rate $\lambda$. The customers must enter into the service facility first, and if it is not possible they join the queueing facility to form the queue in the order of their arrival. The two servers have heterogeneous service rate $\mu_1$ for server 1 and $\mu_2$ for server 2 ($\mu_1 \neq \mu_2$). Server 1 is always obtainable and server 2 is intermittently obtainable. Server 2 goes to service in some important dissimilar/ strange jobs when the number of the customer is greater than or equal to zero. But prior going to execute these strange jobs, the server 2 must first entire his in hand service. Otherwise server 2 is either working state or idle. By introducing the bivariate process $\{M(t), S(t); t \geq 0\}$ the stationary analysis has been carried out using Matrix Geometric Method. Some numerical results are also presented.

Keywords. Intermittently obtainable, Idle, Working state, Bivariate Processes, Matrix Geometric method

1. Introduction
The research work in queueing theory led to many extensions in basic queueing theory due to its significance in fields like telecommunication, aviation, manufacturing and production, transportation and many more. Mostly, interference occur in working period. Interference occur in the form of server breakdown or due to the unobtainability of the server. Queueing systems with service interference have been wide range of applications in manufacturing system, computer system and telecommunication system over more than two decades. In many applied queueing system, environment often occur where server are subject to service interference. For example, in a machine processing center, machine breakdown may occur due to feature such as electrical failure.

In the present study, an increasing enthusiasm in m/m/2 queueing model with poisson arrival and two heterogeneous server. Server 1 is always obtainable and server 2 is intermittently obtainable or idle. Interference in service occur for a random length of time but occur only after the service in hand is completed. This service is termed as intermittently obtainable service. Server goes to working state from idle state in order to provide service to customers or server goes to intermittently obtainable state.
from idle state in order to provide service to customers. The intermittently obtainable server goes to assist some very swift odd/dissimilar jobs when the queue length is greater than or equal to zero (i.e. non-exhaustive service). Due to the presence of intermittently obtainable server in computer network and communication network.

In the important concept of Steady state Probabilities have been found using Matrix geometric technique.

Queueing model with intermittently obtainable server have been studied by many researchers in the past. Transient/steady state solution of a single channel queue with arrival and intermittently available server was analyzed by Chaudhry (1974). Federgrune and Green (1986) proposed by queueing system with service interruptions. Queueing analysis of markovian queue having two heterogeneous servers with catastrophes using matrix geometric technique was analyzed by Indra and Vijay Rajan (2017). A multi server retial queue with break down and geometric loss was studied by Kalyanaraman and Seenivasan (2011). A multi-server retrial queueing system with unreliable server was presented by Kalyanaraman and Seenivasan (2010). A many-server queue with service interruptions was introduced by Mitran and Avi-Itzhak (1968). Neuts (1981) Proposed by Matrix-Geometric solutions in stochastic models. A queueing problem with intermittently available server and arrivals and departures in batches of variable size was studied by Sharda (1968). Time dependent solution of a queueing system with intermittently available server was analyzed by Sharda and Garg (1986).

Patrons who find all server active on arrival await for service in order of arrival. On remaining full busy periods of GI/ G/c queues and their relation to stationary point processes was studied by Gharamansi (1990). On the waiting-time and busy period distributions for a general birth-and-death queuing model was introduced by Natvig (1975). On the busy period distribution of the M/G/2 queueing system was analyzed by Wiens (1989). If no customer in the system, then server goes to idle. This period of time is called idle period. Utilization of idle time in an M/G/1 queueing system was presented by Levy and Yechiali (2007).

Queueing systems with service interruptions have been studied by Naïn (1983). A single server queue with mixed types of interruptions was introduced by Nicola (1986). Nunez-Queija (2000) proposed by Sojourn times in a processor sharing queue with service interruptions. A heavy-traffic limit for many-server queues with service interruptions was analyzed by Pang and Whitt (2009). Sengupta (1990) presented by Queue with service interruptions in an alternating random environment. A single server queue with service interruptions has been studied by Takine and Sengupta (1997).

The rest of the paper is organized as follows: In section 2, We describe of the queueing model. In section 3, the model analyzed using a numerical example. The last section contains a brief conclusion. For a detailed review of main results and the literature of multi server queues one may refer the monograph by Falin and Templeton (1997).

2. Model Description

We consider queueing system with two heterogeneous sever, server 1 and server 2. Patrons arrived according to Poisson distribution with rate λ. These arriving patrons form a single queue based on the order of their arrivals. The two server have heterogeneous service parameter μ₁ for server 1 and μ₂ for server 2 where μ₁ ≠ μ₂. The service time at both servers follow exponential distribution. Server 1 is always obtainable and server 2 is intermittently obtainable or idle. Server 2 goes to lead some crucial dissimilar/ odd jobs when the queue length is greater than or equal to zero. The intermittently obtainable server 2 is must first complete his in hand service. Obtain ability time of server 2 follows exponential distribution with rate ν. Idle server 2 is server goes to working period from idle period in order to issue service to patrons or server goes to intermittently obtainable period from idle period in.
order to issue service to patrons. Duration of idle time of server 2 follows exponential distribution with rate $\theta_1$ for working or $\theta_0$ for intermittently obtainable. The system structure is shown in figure 1.

![Figure 1: The system structure](image)

Let $P(t) = (M(t),S(t))$ be the state of the process at time $t$, where $M(t)$ is the number of patron in the system and

$S(t) = \begin{cases} 
0 & \text{if server 2 is Intermittently obtainable} \\
1 & \text{if server 2 is Working} \\
2 & \text{if server 2 is Idle}
\end{cases}$

Then $\{M(t),S(t); t \geq 0\}$, a Markov process with the state space set arranged in lexicographical order is as follows, $\{(i,j); i \geq 0; j = 0,1,2\}$

Infinitesimally generator matrix $Q$ is as follows:

$$
Q = \begin{pmatrix}
A_0 & B_0 & 0 & 0 & \cdots \\
B_2 & B_1 & B_0 & 0 & \cdots \\
0 & B_2 & B_1 & B_0 & \cdots \\
0 & 0 & B_2 & B_1 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}
$$

Where

$$
A_0 = \begin{pmatrix}
- (\lambda + \nu) & \nu & 0 & \cdots \\
0 & -\lambda & 0 & \cdots \\
\theta_0 & \theta_1 & - (\lambda + \theta_0 + \theta_1) & \cdots \\
\end{pmatrix};
B_0 = \begin{pmatrix}
\lambda & 0 & 0 \\
0 & \lambda & 0 \\
0 & 0 & \lambda
\end{pmatrix}
$$
\[ B_1 = \begin{pmatrix} -\lambda + \nu + \mu_1 & \nu & 0 \\ 0 & -\lambda + \mu_1 + 2\mu_2 & 0 \\ \theta_0 & \theta_1 & -\lambda + \mu_1 + \theta_0 + \theta_1 \end{pmatrix}; \quad B_2 = \begin{pmatrix} \mu_1 & 0 & 0 \\ \mu_2 & \mu_1 + \mu_2 & 0 \\ 0 & 0 & \mu_j \end{pmatrix} \]

Let M and S be the stationary random variable for the number of patrons in the system and the status of the server 2.

We define \( P_{ij} = \{ M = i, S = j \} = \lim_{t \to \infty} P\{ M(t) = i, S(t) = j \} \) where \((i, j) \in \text{state space set} \).

The stationary probability matrix \( P \) is given by \( P = (P_0, P_1, P_2, \ldots) \) where \( P_i = (P_{i0}, P_{i1}, P_{i2}) \) for \( i \geq 0 \).

The stationary probability matrix \( P \) is solved by using \( PQ = 0 \).

\[ P_0A_0 + P_1B_2 + \ldots = 0 \quad (1) \]
\[ P_0B_0 + P_1B_1 + P_2B_2 + \ldots = 0 \]
\[ P_1B_0 + P_2B_1 + P_3B_2 + \ldots = 0 \]
\[ \vdots \]
\[ P_iB_0 + P_{i+1}B_1 + P_{i+2}B_2 = 0 \quad (2) \]

And
\[ P_i = P_0R^i, \quad i \geq 1. \quad (3) \]

From equation \((1)\)
\[ P_0A_0 + P_1B_2 = 0 \]
\[ P_0(A_0 + RB_2) = 0 \quad (4) \]

and \( P_iB_0 + P_{i+1}B_1 + P_{i+2}B_2 = 0 \), using Eq.\((3)\), we have

\[ P_0R^i(B_0 + RB_1 + R^2B_2) = 0, \quad i \geq 1. \quad (5) \]

The normalizing equation is given by
\[ P_0[I - R]^{-1}e = 1 \quad (6) \]

Here \( 'e' \) is column vector of appropriate length of 1’s.

The matrix \( R \) is the minimal solution to the matrix non-linear equation
\[ B_0 + RB_1 + R^2B_2 = 0, \quad (7) \]

Where \( R \geq 0 \) and it is an irreducible non-negative matrix of spectral radius less one.

An iterative method can be used to compute \( R \) as follows.
\[ R_0 = 0 \quad (8) \]
\[ R_{n+1} = -B_0B_1^{-1} - R_n^2B_1^{-1}, \quad n \geq 0. \quad (9) \]

For a Markov process with such generators, Neuts [12] has obtained the stability condition as
\[ PB_0e \leq PB_2e \quad (10) \]

Where the row vector \( P = (P_0, P_1, P_2) \) is obtained from the infinitesimal generator \( B = B_0 + B_1 + B_2 \). \( B \) is given by
\[ B = \begin{pmatrix} -\nu & \nu & 0 \\ \mu_2 & -\mu_2 & 0 \\ \theta_0 & \theta_1 & -(\theta_0 + \theta_1) \end{pmatrix} \quad (11) \]

It can be shown that \( B \) is irreducible and that the row vector \( P \) is unique such that
\[ PB = 0 \quad \text{and} \quad Pe = 1 \quad (12) \]
From Eq.(12), we have
\[ P_2 = 0 \]
\[ P_1 = \frac{\nu}{\mu_2} P_0 \quad \text{and} \quad (13) \]
\[ P_0 = \left[ 1 + \frac{\nu}{\mu_2} \right]^{-1} \]

The stability condition takes the form
\[ \lambda [P_0 + P_1 + P_2] < \mu_1 [P_0 + P_1 + P_2] + 2\mu_2 P_1. \quad (14) \]

Equation (13) gives the steady state probability B.

3. Numerical Example

Now we present numerical results to the model discussed in the above section. Our objective is to demonstrate the effect of the parameter on the system characteristics. By varying \( \lambda, \mu_1 \) and \( \mu_2 \), Totally nine examples are presented in the section.

The changes in the value of one parameter \( \lambda \) at time keeping the other parameter values constant

Example 3.1, Example 3.2, Example 3.3 are presented in below.

3.1. Example.

For \( \lambda = 0.4, \mu_1=0.4, \mu_2 = 0.5, \nu = 0.3, \theta_0 = 0.1, \theta_1 = 0.2 \), and the R matrix is given by

\[ R = \begin{pmatrix} 0.5106 & 0.1397 & 0.0000 \\ 0.0512 & 0.2710 & 0.0000 \\ 0.1363 & 0.1235 & 0.4312 \end{pmatrix} \]

Table 1: Probability Vectors

|   | Total       |
|---|-------------|
| P_0 | 0.1491 0.4069 0.0001 | 0.5561 |
| P_1 | 0.0970 0.1311 0.0000 | 0.2281 |
| P_2 | 0.0562 0.0491 0.0000 | 0.1053 |
| P_3 | 0.0312 0.0212 0.0000 | 0.0524 |
| P_4 | 0.0170 0.0101 0.0000 | 0.0271 |
| P_5 | 0.0092 0.0051 0.0000 | 0.0143 |
| P_6 | 0.0050 0.0027 0.0000 | 0.0077 |
| P_7 | 0.0027 0.0014 0.0000 | 0.0041 |
| Total | | 0.9954 |

By using the above R matrix, the probability vectors \( P_i = P_0 R^i \), \( i = 1,2,3,\ldots \), where \( P_0 \) is calculated from the relation \( P_0[ A_0 + RB_2] = 0 \) and normalization condition \( P_0 [I - R]^T v = 1 \) for the numerical parameter chosen above, the row vector \( P_0 \) is given by \( P_0 = (0.1491, 0.4069, 0.0001) \). Further the remaining vectors \( P_i \)'s are obtained from \( P_i = P_0 R^i \), \( i = 1, 2, 3,\ldots \) and are presented in Table 1. In the
table column 2, 3 and 4 represent the three components of $P_i$, $i = 0, 1, 2, \ldots$ the last column represent the sum of the three components. It is verified that the total probability is $0.9954 \approx 1$.

3.2. Example.
For $\lambda = 0.5$, $\mu_1 = 0.4$, $\mu_2 = 0.5$, $\nu = 0.3$, $\theta_0 = 0.1$, $\theta_1 = 0.2$, and the R matrix is given by

$$R = \begin{pmatrix}
0.6218 & 0.1796 & 0.0000 \\
0.0772 & 0.3352 & 0.0000 \\
0.1849 & 0.1614 & 0.5000
\end{pmatrix}$$

| Table 2: Probability Vectors |
|------------------------------|
| $P_0$ | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ | $P_6$ | $P_7$ | $P_8$ | $P_9$ | $P_{10}$ | $P_{11}$ | Total |
| 0.1247 | 0.1000 | 0.0714 | 0.0489 | 0.0329 | 0.0220 | 0.0146 | 0.0097 | 0.0065 | 0.0043 | 0.0028 | 0.0019 | 0.4151 |
| 0.2904 | 0.1197 | 0.0581 | 0.0323 | 0.0196 | 0.0125 | 0.0081 | 0.0053 | 0.0035 | 0.0023 | 0.0016 | 0.0010 | 0.2197 |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0358 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0812 |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0525 |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0345 |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0227 |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0150 |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0100 |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0044 |
| 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0029 |
| Total | 0.9941 |

By using the above R matrix, the probability vectors $P_i = P_0 R^i$, $i = 1, 2, 3, \ldots$, where $P_0$ is calculated from the relation $P_0[\varphi_0 + \varphi B_1] = 0$ and normalization condition $P_0[I - R] \varphi = 1$ for the numerical parameter chosen above, the row vector $P_0$ is given by $P_0 = (0.1247, 0.2904, 0.0000)$. Further the remaining vectors $P_i$’s are obtained from $P_i = P_0 R^i$, $i = 1, 2, 3, \ldots$ and are presented in Table 2. In the table column 2, 3 and 4 represent the three components of $P_i$, $i = 0, 1, 2, \ldots$ the last column represent the sum of the three components. It is verified that the total probability is $0.9941 \approx 1$.

3.3. Example.
For $\lambda = 0.6$, $\mu_1 = 0.4$, $\mu_2 = 0.5$, $\nu = 0.3$, $\theta_0 = 0.1$, $\theta_1 = 0.2$, and the R matrix is given by

$$R = \begin{pmatrix}
0.7254 & 0.2196 & 0.0000 \\
0.1067 & 0.3977 & 0.0000 \\
0.2355 & 0.2001 & 0.5569
\end{pmatrix}$$
Table 3: Probability Vectors

| P_i  | Total     |
|------|-----------|
| P_0  | 0.0880    | 0.1816 | 0.0004 | 0.2700 |
| P_1  | 0.0833    | 0.0916 | 0.0002 | 0.1751 |
| P_2  | 0.0703    | 0.0548 | 0.0001 | 0.1252 |
| P_3  | 0.0568    | 0.0372 | 0.0000 | 0.0941 |
| P_4  | 0.0452    | 0.0273 | 0.0000 | 0.0725 |
| P_5  | 0.0357    | 0.0208 | 0.0000 | 0.0565 |
| P_6  | 0.0281    | 0.0161 | 0.0000 | 0.0442 |
| P_7  | 0.0221    | 0.0126 | 0.0000 | 0.0347 |
| P_8  | 0.0174    | 0.0099 | 0.0000 | 0.0273 |
| P_9  | 0.0137    | 0.0077 | 0.0000 | 0.0214 |
| P_10 | 0.0107    | 0.0061 | 0.0000 | 0.0168 |
| P_11 | 0.0084    | 0.0048 | 0.0000 | 0.0132 |
| P_12 | 0.0066    | 0.0038 | 0.0000 | 0.0104 |
| P_13 | 0.0052    | 0.0030 | 0.0000 | 0.0082 |
| P_14 | 0.0041    | 0.0023 | 0.0000 | 0.0064 |
| P_15 | 0.0032    | 0.0018 | 0.0000 | 0.0050 |
| P_16 | 0.0025    | 0.0014 | 0.0000 | 0.0039 |
| P_17 | 0.0020    | 0.0011 | 0.0000 | 0.0031 |
| P_18 | 0.0016    | 0.0009 | 0.0000 | 0.0025 |
| P_19 | 0.0012    | 0.0007 | 0.0000 | 0.0019 |
| P_20 | 0.0010    | 0.0005 | 0.0000 | 0.0015 |
| P_21 | 0.0008    | 0.0004 | 0.0000 | 0.0012 |

By using the above R matrix, the probability vectors \( P_i = P_0 R^i \), \( i = 1, 2, 3, \ldots \), where \( P_0 \) is calculated from the relation \( P_0 [A_0 + R B_2] = 0 \) and normalization condition \( P_0 [I - R]^{-1} e = 1 \) for the numerical parameter chosen above, the row vector \( P_0 \) is given by \( P_0 = (0.0880, 0.1816, 0.0004) \). Further the remaining vectors \( P_i \)'s are obtained from \( P_i = P_0 R^i, i = 1, 2, 3, \ldots \) and are presented in Table 3. In the table column 2, 3 and 4 represent the three components of \( P_i, i = 0, 1, 2, \ldots \) the last column represents the sum of the three components. It is verified that the total probability is 0.9951 \( \approx 1 \).

The changes in the value of one parameter \( \mu_1 \) at time keeping the other parameter values constant. Example 3.4, Example 3.5, Example 3.6 are presented in below.

3.4. Example.
For \( \lambda = 0.4, \mu_1 = 0.5, \mu_2 = 0.5, v = 0.3, \theta_0 = 0.1, \theta_1 = 0.2 \), and the R matrix is given by

\[
R = \begin{pmatrix}
0.4555 & 0.1147 & 0.0000 \\
0.0406 & 0.2531 & 0.0000 \\
0.1038 & 0.0987 & 0.3999
\end{pmatrix}
\]
Table 4: Probability Vectors

|   | Total       |
|---|-------------|
| $P_0$ | 0.1598 0.4516 0.0001 0.6115 |
| $P_1$ | 0.0911 0.1326 0.0000 0.2237 |
| $2P_2$ | 0.0469 0.0440 0.0000 0.0909 |
| $P_3$ | 0.0232 0.0165 0.0000 0.0397 |
| $P_4$ | 0.0112 0.0068 0.0000 0.0180 |
| $P_5$ | 0.0054 0.0030 0.0000 0.0084 |
| $P_6$ | 0.0026 0.0014 0.0000 0.0040 |
| $P_7$ | 0.0012 0.0006 0.0000 0.0018 |
| Total | 0.9980 |

By using the above R matrix, the probability vectors $P_i = P_0 R^i$, $i = 1,2,3,...$, where $P_0$ is calculated from the relation $P_0[A_0+R B_2] = 0$ and normalization condition $P_0[I - R]^{-1} e = 1$ for the numerical parameter chosen above, the row vector $P_0$ is given by $P_0 = (0.1598, 0.4516, 0.0001)$. Further the remaining vectors $P_i$'s are obtained from $P_i = P_0 R^i$, $i = 1, 2, 3,...$ and are presented in Table 4. In the table column 2, 3 and 4 represent the three components of $P_i$; $i = 0, 1, 2, ...$ the last column represent the sum of the three components. It is verified that the total probability is 0.9980 ≈ 1.

3.5. Example.
For $\lambda = 0.4$, $\mu_1 = 0.6$, $\mu_2 = 0.5$, $\nu = 0.3$, $\theta_0 = 0.1$, $\theta_1 = 0.2$, and the R matrix is given by

$$
R = \begin{pmatrix}
0.4110 & 0.0959 & 0.0000 \\
0.0327 & 0.2377 & 0.0000 \\
0.0632 & 0.0773 & 0.3713 \\
\end{pmatrix}
$$

Table 5: Probability Vectors

|   | Total       |
|---|-------------|
| $P_0$ | 0.1665 0.4867 0.0004 0.6537 |
| $P_1$ | 0.0844 0.1317 0.0001 0.2162 |
| $P_2$ | 0.0390 0.0394 0.0001 0.0785 |
| $P_3$ | 0.0173 0.0131 0.0000 0.0304 |
| $P_4$ | 0.0075 0.0048 0.0000 0.0123 |
| $P_5$ | 0.0033 0.0019 0.0000 0.0052 |
| $P_6$ | 0.0014 0.0008 0.0000 0.0022 |
| Total | 0.9985 |

By using the above R matrix, the probability vectors $P_i = P_0 R^i$, $i = 1,2,3,...$, where $P_0$ is calculated from the relation $P_0[A_0+R B_2] = 0$ and normalization condition $P_0[I - R]^{-1} e = 1$ for the numerical parameter chosen above, the row vector $P_0$ is given by $P_0 = (0.1665, 0.4867, 0.0004)$. Further the remaining vectors $P_i$'s are obtained from $P_i = P_0 R^i$, $i = 1, 2, 3,...$ and are presented in Table 5. In the table column 2, 3 and 4 represent the three components of $P_i$; $i = 0, 1, 2, ...$ the last column represent the sum of the three components. It is verified that the total probability is 0.9985 ≈ 1.
3.6. Example.
For $\lambda = 0.4$, $\mu_1=0.7$, $\mu_2= 0.5$, $\nu = 0.3$, $\theta_0 = 0.1$, $\theta_1 = 0.2$, and the R matrix is given by

$$R = \begin{pmatrix} 0.3741 & 0.0812 & 0.0000 \\ 0.0267 & 0.2243 & 0.0000 \\ 0.0640 & 0.0667 & 0.3453 \end{pmatrix}$$

| Table 6: Probability Vectors | Total |
|-------------------------------|-------|
| P_0                          | 0.1487 | 0.4512 | 0.0002 | 0.6001 |
| P_1                          | 0.1041 | 0.1360 | 0.0000 | 0.2401 |
| P_2                          | 0.0493 | 0.0467 | 0.0000 | 0.0960 |
| P_3                          | 0.0210 | 0.0174 | 0.0000 | 0.0384 |
| P_4                          | 0.0086 | 0.0067 | 0.0000 | 0.0153 |
| P_5                          | 0.0035 | 0.0027 | 0.0000 | 0.0062 |
| P_6                          | 0.0014 | 0.0011 | 0.0000 | 0.0025 |
| Total                        |        |        |        | 0.9986 |

By using the above R matrix, the probability vectors $P_i = P_0 R^i$, $i = 1, 2, 3, …$, where $P_0$ is calculated from the relation $P_0[A_0+RB_0] = 0$ and normalization condition $P_0[I-R]^1 e = 1$ for the numerical parameter chosen above, the row vector $P_0$ is given by $P_0 = (0.1487, 0.4512, 0.0002)$. Further the remaining vectors $P_i$’s are obtained from $P_i = P_0 R^i$, $i = 1, 2, 3, …$ and are presented in Table 6. In the table column 2, 3 and 4 represent the three components of $P_i$, $i = 0, 1, 2, …$ the last column represent the sum of the three components. It is verified that the total probability is $0.9986 = 1$.

The changes in the value of one parameter $\mu_2$ at time keeping the other parameter values constant. Example 3.7, Example 3.8, Example 3.9 are presented in below.

3.7. Example.
For $\lambda = 0.4$, $\mu_1=0.4$, $\mu_2= 0.6$, $\nu = 0.3$, $\theta_0 = 0.1$, $\theta_1 = 0.2$, and the R matrix is given by

$$R = \begin{pmatrix} 0.5102 & 0.1222 & 0.0000 \\ 0.0468 & 0.2383 & 0.0000 \\ 0.1362 & 0.1080 & 0.4312 \end{pmatrix}$$

| Table 7: Probability Vectors | Total |
|-------------------------------|-------|
| P_0                          | 0.1607 | 0.4197 | 0.0001 | 0.5805 |
| P_1                          | 0.1016 | 0.1197 | 0.0000 | 0.2213 |
| P_2                          | 0.0575 | 0.0409 | 0.0000 | 0.0984 |
| P_3                          | 0.0312 | 0.0168 | 0.0000 | 0.0480 |
| P_4                          | 0.0167 | 0.0078 | 0.0000 | 0.0245 |
| P_5                          | 0.0089 | 0.0039 | 0.0000 | 0.0128 |
| P_6                          | 0.0047 | 0.0020 | 0.0000 | 0.0067 |
| P_7                          | 0.0025 | 0.0011 | 0.0000 | 0.0036 |
| P_8                          | 0.0013 | 0.0006 | 0.0000 | 0.0019 |
| Total                        |        |        |        | 0.9977 |
By using the above R matrix, the probability vectors $P_i = P_0^i$, $i = 1, 2, 3, \ldots$, where $P_0$ is calculated from the relation $P_0[A_0+RB_2] = 0$ and normalization condition $P_0[I-R]^{-1}e = 1$ for the numerical parameter chosen above, the row vector $P_0$ is given by $P_0 = (0.1607, 0.4197, 0.0001)$. Further the remaining vectors $P_i$'s are obtained from $P_i = P_0^i$, $i = 1, 2, 3, \ldots$ and are presented in Table 7. In the table column 2, 3 and 4 represent the three components of $P_i$, $i = 0, 1, 2, \ldots$ the last column represent the sum of the three components. It is verified that the total probability is $0.9977 \approx 1$.

3.8. Example.
For $\lambda = 0.4$, $\mu_1 = 0.4$, $\mu_2 = 0.7$, $\nu = 0.3$, $\theta_0 = 0.1$, $\theta_1 = 0.2$, and the R matrix is given by

$$R = \begin{pmatrix} 0.5099 & 0.1089 & 0.0000 \\ 0.0431 & 0.2126 & 0.0000 \\ 0.1362 & 0.0962 & 0.4312 \end{pmatrix}$$

| P_i | P_0 | P_1 | P_2 | P_3 | P_4 | P_5 | P_6 | P_7 | Total |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-------|
| P_0 | 0.1699 | 0.4294 | 0.0000 | 0.5993 |
| P_1 | 0.1051 | 0.1098 | 0.0000 | 0.2149 |
| P_2 | 0.0583 | 0.0348 | 0.0000 | 0.0931 |
| P_3 | 0.0312 | 0.0138 | 0.0000 | 0.0450 |
| P_4 | 0.0165 | 0.0063 | 0.0000 | 0.0228 |
| P_5 | 0.0087 | 0.0031 | 0.0000 | 0.0118 |
| P_6 | 0.0046 | 0.0016 | 0.0000 | 0.0062 |
| P_7 | 0.0024 | 0.0008 | 0.0000 | 0.0032 |
| Total | | | | 0.9957 |

By using the above R matrix, the probability vectors $P_i = P_0^i$, $i = 1, 2, 3, \ldots$, where $P_0$ is calculated from the relation $P_0[A_0+RB_2] = 0$ and normalization condition $P_0[I-R]^{-1}e = 1$ for the numerical parameter chosen above, the row vector $P_0$ is given by $P_0 = (0.1699, 0.4294, 0.0000)$. Further the remaining vectors $P_i$'s are obtained from $P_i = P_0^i$, $i = 1, 2, 3, \ldots$ and are presented in Table 8. In the table column 2, 3 and 4 represent the three components of $P_i$, $i = 0, 1, 2, \ldots$ the last column represent the sum of the three components. It is verified that the total probability is $0.9957 \approx 1$.

3.9. Example.
For $\lambda = 0.4$, $\mu_1 = 0.4$, $\mu_2 = 0.8$, $\nu = 0.3$, $\theta_0 = 0.1$, $\theta_1 = 0.2$, and the R matrix is given by

$$R = \begin{pmatrix} 0.5095 & 0.0982 & 0.0000 \\ 0.0398 & 0.1921 & 0.0000 \\ 0.1359 & 0.0865 & 0.4312 \end{pmatrix}$$
Table 9: Probability Vectors

|   | Total |
|---|-------|
| P₀ | 0.1550 | 0.1545 | 0.1723 | 0.4818 |
| P₁ | 0.1085 | 0.0598 | 0.0743 | 0.2426 |
| P₂ | 0.0678 | 0.0286 | 0.0320 | 0.1284 |
| P₃ | 0.0400 | 0.0149 | 0.0138 | 0.0687 |
| P₄ | 0.0229 | 0.0080 | 0.0060 | 0.0369 |
| P₅ | 0.0128 | 0.0043 | 0.0026 | 0.0197 |
| P₆ | 0.0070 | 0.0023 | 0.0011 | 0.0104 |
| P₇ | 0.0038 | 0.0012 | 0.0005 | 0.0055 |
| P₈ | 0.0021 | 0.0007 | 0.0002 | 0.0030 |

Total 0.9970

By using the above R matrix, the probability vectors \( P_i = P_0 R^i \), \( i = 1, 2, 3, \ldots \), where \( P_0 \) is calculated from the relation \( P_0[A_0+RB_2] = 0 \) and normalization condition \( P_0[I - R]^{-1} e = 1 \) for the numerical parameter chosen above, the row vector \( P_0 \) is given by \( P_0 = (0.1550, 0.1545, 0.1723) \). Further the remaining vectors \( P_i \)'s are obtained from \( P_i = P_0 R^i \), \( i = 1, 2, 3, \ldots \) and are presented in Table 9. In the table column 2, 3 and 4 represent the three components of \( P_i \), \( i = 0, 1, 2, \ldots \) the last column represent the sum of the three components. It is verified that the total probability is 0.9970 ≈ 1.

4. Conclusion

In this paper we have considered a two server queueing system with intermittently obtainable server. We have obtain the steady state probability vector by applying matrix geometric method. Furthermore, we have performed numerical analysis by assuming particular values to the parameter.

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