Contrarian Deterministic Effect: the “Hung Elections Scenario”

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A contrarian is someone who deliberately decides to oppose the prevailing choice of others. The Galam model of two state opinion dynamics incorporates agent updates by a single step random grouping in which all participants adopt the opinion of their respective local majority group. The process is repeated until a stable collective state is reached; the associated dynamics is fast. Here we show that the introduction of contrarians may give rise to interesting dynamics generated phases and even to a critical behavior at a contrarian concentration $a_c$. For $a < a_c$ an ordered phase is generated with a clear cut majority-minority splitting. By contrast when $a > a_c$ the resulting disordered phase has no majority: agents keep shifting opinions but no symmetry breaking (i.e., the appearance of a majority) takes place. Our results are employed to explain the outcome of the 2000 American presidential elections and that of the 2002 German parliamentary elections. Those events are found to be inevitable. On this basis the “hung elections scenario” is predicted to become a common occurrence in modern democracies.

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In this letter, we study the effects of contrarian choices on the dynamics of opinion forming. A contrarian is someone who deliberately decides to oppose the prevailing choice of others whatever this choice is [1]. Contrarian strategy is becoming a growing new trend of modern democracies most studied in finance [1,2].

The Galam model of two state opinion dynamics incorporates agent updates by a series of single steps. In each step random groups are formed in which all participants adopt the opinion of their respective local majority group [3–5]. The process is repeated until a stable collective state is reached. The associated dynamics is fast and leads to a total polarization along either one of the two competing states A and B. The direction of the opinion flow is monitored by an unstable separator at some critical density $p_c$ of agent supporting the A opinion.

In the case of odd size groups, $p_c = \frac{1}{2}$. By contrast even sizes make $p_c \neq \frac{1}{2}$. The corresponding asymmetry in the dynamics of respectively opinion A and B arises from the existing of local collective doubts at a tie. The unstable separator may then be simultaneously at a value of 23% for one state and at 77% for the other [4]. Large size group accelerates reaching the final state with a drastic reduction in the number of required updates. In the limit of a single grouping which includes the entire population one update is enough to complete the full polarization. Recently a generalization to any distribution of group sizes was achieved yielding a very rich and complex phase diagram [3]. The model was subsequently applied to rumor phenomena [6].

Earlier version of this approach is found in the study of voting in democratic hierarchical systems [5]. There, groups of agents vote for a representative to the higher level using a local majority rule. In the mean field limit, going up the hierarchy turns out to be exactly identical to an opinion forming process in terms of equations and dynamics. Instead of voting, agents update their opinions. The probability of electing an A representative at some hierarchy level $n$ is equal to the proportion of A opinions after $n$ updates [5,4]. Recent studies by Krapivsky and Redner further explored the dynamical properties of the Galam model, restricted to one group of size 3 [7].

This work contributes to the now growing field of applications of Statistical Physics to social and political behaviors [8–15]. First denoted “Sociophysics” in a founding paper [8] we extend the label to “Global Physics”. At this stage it is worth stressing we are not aiming at an exact description of the real social and political life, but rather, doing some crude approximations, to enlighten essential features of an otherwise very complex and multiple phenomena.

Here, the dynamics of Contrarian behavior is studied using the Galam model of two state opinion dynamics restricted to odd sizes. Introduction of contrarians at a low density $a$ is found to unfold the total polarization dynamics. The corresponding fully ordered state with one unique opinion becomes mixed with a stable majority-minority splitting. But the symmetry breaking is preserved with a clear cut majority along the initial global majority. The unstable separator is also left unchanged at $p_c = \frac{1}{2}$.

However, contrarians are found to give rise to a critical behavior at a contrarian concentration $a_c$. When $a > a_c$ a new disordered stable phase with no majority appears. There agents keep shifting opinions but no symmetry breaking (i.e., the appearance of a majority) takes place. Contrarians have turned the unstable separator $p_c$ into the unique stable attractor of the dynamics. Opinion flows have been reversed. The value of $a_c$ depends on the size distribution of update groups.

Our results are employed to explain the outcome of the 2000 American presidential elections and that of the
2002 German parliamentary elections. Those events are found to be inevitable. On this basis the “hung elections scenario” is predicted to become a common occurrence in modern democracies.

We start with a very simple model of opinion forming [5,3]. Considering an ideal society before a major election, people start discussing the issue during the election campaign. Groups are formed randomly in which all participants adopt the local majority state. Focusing first on the group size 3, an initial 2 A (B) with one B (A) ends up with 3 A (B). To follow the time evolution of the vote intentions we need an estimate of the numbers of respective vote intentions \( N_+ (t) \) for A and \( N_+ (t) \) for B at some time t from a N person population. It can be evaluated using polls. Each person is assumed to have a probability to adopt the local majority state. Focusing first on the group size 3. A number of \( m \) discussion cycles gives the series \( P_t \) (2). A number of \( m \) discussion cycles gives the series \( P_t \) (2) with, 

\[
\begin{align*}
\frac{n_+}{N} &= p_+(t) = \frac{N_+ (t)}{N}, \quad (1)
\end{align*}
\]

with,

\[
\begin{align*}
p_+ (t) + p_- (t) &= 1. \quad (2)
\end{align*}
\]

Accordingly, one cycle of local opinion updates via three persons grouping leads to a new distribution of vote intention as,

\[
\begin{align*}
p_+ (t+1) &= p_+ (t)^3 + 3p_+ (t)^2 p_- (t), \quad (3)
\end{align*}
\]

where \( p_+ (t+1) > p_+ (t) \) if \( p_+ (t+1) > \frac{1}{2} \) and \( p_+ (t+1) < p_+ (t) \) if \( p_+ (t+1) < \frac{1}{2} \). Indeed from Eq. (2) vote intention \( p_+ (t) \) flows monotonically toward either one of two stable point attractors at \( P_+ = 1 \) and \( P_+ = 0 \). An unstable point separator attractor is located at \( p_c = \frac{1}{2} \). It separates the two basins of attraction associated respectively to the point attractors.

During an election campaign people go through several successive different local discussions. To follow the associated vote intention evolution we iterate Eq. (2). A number of \( m \) discussion cycles gives the series \( p_+ (t+1), p_+ (t+2), \ldots, p_+ (t+m) \). For instance starting at \( p_+ (t) = 0.45 \) leads successively after 5 intention updates to the series \( p_+ (t+1) = 0.43, p_+ (t+2) = 0.39, p_+ (t+3) = 0.34, p_+ (t+4) = 0.26, p_+ (t+5) = 0.17 \) with a continuous decline in A vote intentions. Adding 3 more cycles would result in zero A vote intention with \( p_+ (t+6) = 0.08, p_+ (t+7) = 0.02 \) and \( p_+ (t+8) = 0.00 \). Given any initial intention vote distribution, the random local opinion update distribution the random local opinion update distribution leads toward a total polarization of the collective opinion. Individual and collective opinions stabilize simultaneously along the same and unique vote intention either A or B.

The update cycle number to reach either one of the two stable attractors can be evaluated from Eq. (2). It depends on the distance of the initial densities from the unstable point attractor. An analytic formula is derived below (see Eq. (6)). However, every update cycle takes some time length, which may correspond in real terms to some number of days. Therefore, in practical terms the required time to eventually complete the polarization process is much larger than the campaign duration, thus preventing it to occur. Accordingly, associate elections never take place at the stable attractors. From above example at \( p_+ (t) = 0.45 \), two cycles yield a result of 39% in favor of A and 61% in favor of B. One additional update cycle makes 34% in favor of A and 66% in favor of B.

At this stage we are in a position to insert in the model the existence of contrarians. A contrarian is defined as follows [1]. Once a local group reaches a consensus driven by the majority rule, there exists some people, which once they left the group, shift to the opposite vote intention. The shift is independent of the choice itself. Setting contrarian choices at a density \( a \) with \( 0 < a < 1 \), the density of A opinion given by Eq. (2) becomes,

\[
\begin{align*}
p_+ (t+1) &= (1-a)[p_+ (t)^3 + 3p_+ (t)^2 p_- (t)] + a[p_- (t)^3 + 3p_- (t)^2 p_+ (t)], \quad (4)
\end{align*}
\]

where first term corresponds to the regular update process and second term to contrarian contribution from local groups where the local majority was in favor of B. From Eq. (4), the effect of low-density contrarian choices is readily seen as illustrated in Figure (1) in the case \( a = 0.10 \), i.e., with 10% contrarian choices as compared to the pure case \( a = 0 \).

FIG. 1. Equation (4) with \( P_+ (t+1) \) as function of \( P_+ (t) \) at respectively \( a = 0 \) and \( a = 0.10 \). In the second case the two stable point attractors have moved from total polarization towards coexistence of mixed vote intentions with a clear cut majority-minority splitting.
Main effects are twofold. First both stable point attractors are shift toward coexistence vote intention values. Total polarization is averted with,

\[ P_{+A(B)} = \frac{(2a - 1) \pm \sqrt{12a^2 - 8a + 1}}{2(a-1)}, \]  

which are defined only in the range \( a \leq \frac{1}{6} \). For instance a value of \( a = 0.10 \) yields \( P_{+A} = 0.85 \) and \( P_{+B} = 0.15 \). At \( P_{+A} = 0.85 \) exists a stable coexistence of vote intentions at respectively 0.85 in A favor with 0.15 in B favor. The reverse holds at \( P_{+B} = 0.15 \). At contrast contrarian choices keep unchanged the unstable point separator at \( \frac{1}{2} \).

The second effect from contrarian choices is an increase in the number of cycle updates in reaching the stable attractors. For instance starting as above at \( p_+(t) = 0.45 \) with \( a = 0.10 \) leads now to the series \( p_+(t+1) = 0.44, p_+(t+2) = 0.43, p_+(t+3) = 0.42, p_+(t+4) = 0.40, p_+(t+5) = 0.38 \). Additional 12 updates are required to reach the stable attractor at 0.15. All cycles score to 17 against only 8 without contrarian choices. A vote at two update cycles from above same example would give a voting result of 43% in favor of A and 57% in favor of B instead of respectively 39% and 61% at \( a = 0 \).

An approximate formula can be derived from Eq. (4) to evaluate the update cycle number required to reach either one of the two stable attractors. It writes,

\[ n \simeq \frac{1}{\ln[\frac{1}{2}(2a-1)]} \ln[\frac{p_c - P_S}{p_c - p_+(t)}] + \frac{1.85}{(2a-1)^{\frac{3}{2}}}, \]  

where last term is a fitting correction. \( P_S = P_{+B} \) if \( p_+(t) < p_c \) while \( P_S = P_{+A} \) when \( p_+(t) > p_c \). The number of cycles being an integer, its value is obtained from Eq. (6) rounding to an integer. At \( a = 0 \), i.e., no contrarian choices, \( n \) is always a small number as shown in Figure (2). Eq. (6) gives 8 at an initial value \( p_+(t) = 0.45 \) and 4 at \( p_+(t) = 0.30 \), which are the exact values obtained by successive iterations from Eq. (3). At \( a = 0.10 \) we found also the exact values of 17 and 9 as from Eq. (4).

FIG. 2. Approximate number of cycles of vote intention updates to reach a total polarization of opinion as function of an initial support \( P_+(t) \).

Both Eq. (6) and Figure (2) show explicitly the contrarian choice drastic effect in increasing the number of required levels to reach the stable point attractors. That means much longer real time. In practical terms it implies a quasi-stable coexistence of both vote intentions not too far from fifty percent but yet with a clear-cut majority in one direction, which is determined by the initial majority.

However contrarian choices may lead to a radical qualitative change in the whole vote intention dynamics. Eq. (5) shows that at a density of \( a = \frac{1}{6} \approx 0.17 \), contrarian choices make both point attractors to merge simultaneously at the unstable point separator \( p_c = \frac{1}{2} \) turning it to a stable point attractor. Consequences on the vote intention dynamics are drastic. The flow direction is reversed making any initial densities to converge toward a perfect equality between vote intention for A and B. In physical terms, contrarians produce a phase transition from a majority-minority phase into a fifty percent balance phase with no majority-minority splitting. In the ordered phase elections always yield a clear-cut majority. At contrast in the disordered phase elections lead to a random outcome driven by statistical fluctuations. An illustration is shown in Figure (3) for 20% of contrarians.

FIG. 3. \( P_+(t+1) \) as function of \( P_+(t) \) at \( a = 0 \) and \( a = 0.20 \). In the first case the vote intention flows away from the unstable point attractor at \( \frac{1}{2} \) toward either one of the stable point attractors at zero or one. In the second case, contrarian choices have reversed the flow directions making any initial densities to flow toward \( \frac{1}{2} \), the now stable and unique point attractor.

In real social life people don’t meet only by group of 3. However, generalizing above approach to larger sizes is straightforward and does not change the qualitative feature of the model. Dynamics reversal driven by contrarians towards the disorder phase with no majority-minority...
splitting is preserved. The main effect is an increase in the value of the contrarian critical density at which the phase transition occurs. In the case of an odd size $k$, Eq. (4) becomes,

$$p_+ (t+1) = (2a - 1) \sum_{i=0}^{k} C_i^k p_+ (t)^i p_- (t)^{k-i} + a,$$  \hspace{0.5cm} (7)

where $C_i^k \equiv \frac{k!}{(k-i)!i!}$. The instrumental parameter in determining the flow direction and the associate phase transition is the eigenvalue at the point attractor $p_c = \frac{1}{2}$. It is given by,

$$\lambda = (2a - 1) \left[\frac{1}{2} \right]^{k-1} \sum_{i=\frac{k-1}{2}}^{k} (2i - k) C_i^k.$$  \hspace{0.5cm} (8)

The range $\lambda > 1$ determines an unstable point attractor with an ordered phase characterized by the existence of a majority-minority splitting. At contrast, $\lambda < 1$ makes the point attractor stable. The case $\lambda = 1$ determines the critical value of the contrarian choice density $a_c$ at which the phase transition occurs. From Eq. (8), we get,

$$a_c = \frac{1}{2} \left( 1 - \left[ \frac{1}{2} \right]^{k-1} \sum_{i=\frac{k-1}{2}}^{k} (2i - k) C_i^k \right)^{-1}.$$  \hspace{0.5cm} (9)

In the case $k = 3$ we recover the above result $a = \frac{1}{8} \simeq 0.125$. From Eq. (9) it is seen that $a_c \rightarrow \frac{1}{2}$, $k \rightarrow +\infty$ with $0.33$ at $k = 5$ and $0.30$ at $k = 9$.

We have presented a simple model to study the effect of contrarian choices on opinion forming. At low densities $a$ the opinion dynamics leads to a mixed phase with a clear cut majority-minority splitting. However, beyond some critical density $a_c$, contrarians make all the attractors to merge at the separator $p_c$. It becomes the unique attractor of the opinion dynamics. When $a > a_c$, vote intentions flow deterministically with time towards an exact equality between A and B opinions. In this new disordered stable phase no majority appears. Agents keep shifting opinions but no symmetry breaking (i.e., the appearance of a majority) takes place. There an election would result in effect in a random winner due to statistical fluctuations. The value of $a_c$ depends on the size distribution of update groups.

Accordingly, our results shed a totally new light on recent elections in America (2000) and Germany (2002). It suggests those “hung elections” were not chance driven. On the opposite, they are a deterministic outcome of contrarians. As a consequence, since contrarian thinking is becoming a growing trend of modern societies, the subsequent “hanging chad elections” syndrome is predicted to become both inevitable and of a common occurrence.

While finalizing this manuscript we have notice Ref. [16] by Mobilia and Redner in which a phase transition in a disordered opinion phase is also obtained via an interesting extension of Galam model (restricted to one group of size 3) which combines locally majority and minority rules. However the microscopic rules used as well as the socio-political interpretation and the critical values are different from those of the present work.

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