Wave control through soft microstructural curling: bandgap shifting, reconfigurable anisotropy and switchable chirality

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Received 15 September 2016, revised 1 December 2016
Accepted for publication 19 December 2016
Published 3 February 2017

Abstract

In this work, we discuss and numerically validate a strategy to attain reversible macroscopic changes in the wave propagation characteristics of cellular metamaterials with soft microstructures. The proposed cellular architecture is characterized by unit cells featuring auxiliary populations of symmetrically-distributed smart cantilevers stemming from the nodal locations. Through an external stimulus (the application of an electric field), we induce extreme, localized, reversible curling deformation of the cantilevers—a shape modification which does not affect the overall shape, stiffness and load bearing capability of the structure. By carefully engineering the spatial pattern of straight (non activated) and curled (activated) cantilevers, we can induce several profound modifications of the phononic characteristics of the structure: generation and/or shifting of total and partial bandgaps, cell symmetry relaxation (which implies reconfigurable wave beaming), and chirality switching. While in this work we discuss the specific case of composite cantilevers with a PDMS core and active layers of electrostrictive terpolymer P(VDF-TrFE-CTFE), the strategy can be extended to other smart materials (such as dielectric elastomers or shape-memory polymers).

Supplementary material for this article is available online

Keywords: metamaterials, tunability, soft active materials, chirality, symmetry relaxation, cellular solids

(Some figures may appear in colour only in the online journal)

1. Introduction

Metamaterials are media whose internal architecture is carefully designed to elicit properties that are not attainable by conventional materials available in nature. Some of the most promising opportunities for metamaterials engineering are found in the control of acousto-elastic waves; acoustic metamaterials are typically (but not necessarily) periodic structures which display exotic wave manipulation effects, such as sub-wavelength filtering [1] and waveguiding [2], negative refraction [3] and cloaking [4]. One of the main drawbacks of metamaterials is their inherent passivity: a metamaterial is typically designed for a specific task and for a prescribed operational condition. Therefore, its geometry is determined and ‘frozen’ at the design stage. A modification of the excitation scenario (e.g., a variable frequency content) would most likely result in the need for a full re-design of the medium. To alleviate this issue, several strategies for active or...
adaptive control have been introduced over the past decade. The simplest approach involves a manual adjustment of the position of the constitutive elements of a periodic medium [5, 6]. More automated strategies revolve around the application of external non-mechanical stimuli to modify the mechanical properties of smart material inserts (e.g., shape memory alloys [7], shunted piezoelectric [8–15], magnetoeleastic [16–19], electrorheological [20] and magnetorheological [21] phases, thermally-stimulable polymeric materials [22], aerodynamic-loading-sensitive elements [23]), eventually resulting in a reversible modification of the wave propagation characteristics of the metamaterial. Tunable phononic characteristics are also attainable using nonlinear periodic structures. For example, by inducing buckling in some structural elements via the application of static loads, it is possible to trigger large deformations that can result in dramatic pattern reconfigurations [24–27]. Other avenues leverage the onset of geometric and material nonlinearities. For example, changing the level of precompression of a granular crystal results in a variation of its nonlinear characteristics [28]; this concept has been exploited to produce media with tunable bandgaps [29], wave directivity [30] and tunable wave focusing capabilities [31]. Finally, nonlinearly-induced higher-harmonics have recently been used to produce wavefields displaying modal mixing and augmented directivity patterns [32].

Recent advances in the field of soft active materials have opened new opportunities for tunability [33–39]. Pioneering, in this sense, are the works of Yang and Chen [33], who first proposed dielectric elastomers to tune the wave characteristics of periodic structures, of Bayat and Gordaninejad [35], who studied magnetorheological shape transforming lattices, and of Nouh et al [36], who investigated periodic plates with soft electrically-stiffened PVDF inclusions. While these works introduce soft active materials in the context of wave control devices, they only partially take advantage of the dramatic shape modifications enabled by soft active materials, such as dielectric elastomers [40, 41] and shape memory polymers [42, 43]. In this work, in light of these recent advances, we discuss the effects of localized and reversible shape modifications on the spectro-spatial wave control characteristics of soft cellular structures. The shape modifications are internal and localized, since they occur at the level of an auxiliary microstructure—here comprising non-structural cantilever elements. The reversibility comes from the fact that the cantilevers are equipped with smart actuators (here we consider layers of P(VDF-TrFE-CTFE) electrostrictive terpolymer [44] on a PDMS substrate). Excited with an external (electrical, in this case) stimulus, the actuators cause the auxiliary cantilevers to curl (i.e., experience extreme—yet reversible—rolling deformation), thus modifying their dynamic properties and their effect on the global wave manipulation capabilities of the medium. The idea introduced in this work is a logical continuation of a paradigm that has recently gained traction in the arena of phononic crystals/metamaterials [45–47], based on the idea of decoupling the static (load-bearing) properties from its dynamic (wave control) functionalities. Thanks to the dramatic shape modifications that can be achieved, the strategy presented herein is expected to produce more pronounced effects than previous implementations, which merely relied on material property correction. As a first step, we briefly discuss how global microstructural curling produces reversible shifts in the bandgaps landscape. We note that this initial part of our analysis is conceptually similar to a recent investigation independently pursued by Zhang et al [48] in the context of 3D printed shape memory polymers, in which the authors briefly touch on the appearance/disappearance of phononic bandgaps. In this respect, we hereby attempt to offer a complete rationale of the physical mechanisms that govern the evolution of the bandgap behavior induced by microstructural shape changes. We then proceed to show how selective curling can lead to symmetry relaxation of the unit cell, which in turn produces partial bandgaps which give way to focused wave patterns (reconfigurable anisotropy). Finally, we discuss how these internal shape modifications can introduce non-trivial chiral effects in the response of the periodic medium.

The paper is organized as follows. In section 2 we introduce our strategy for reconfigurable tunability of the lattice characteristics. In section 3 we discuss how we model the curling of a cantilever beam with smart material inserts in response to an applied electric field. In section 4 we report the results of the wave analysis, in terms of bandgap shifting, reconfigurable anisotropy and switchable chirality. Finally, the conclusions of our work are drawn in section 5.

2. Architecture and curling strategy

Given a cellular architecture, e.g. the regular hexagonal (RH) lattice sketched in figure 1(a), we introduce a symmetric population of microstructural elements, consisting of cantilevers located at the nodal locations, as shown in figure 1(b). These internal elements constitute an auxiliary microstructure, in that their addition (or removal) does not affect the connectivity of the primary lattice (which is still RH) and therefore does not affect its static properties nor its load bearing capabilities. The auxiliary cantilevers behave, to all intents and purposes, as resonators capable of enriching the dispersion relation (wave response) of the cellular medium by opening locally resonant bandgaps. Expanding on this passive architecture, we now let the cantilevers be instrumented with layers of active material and we apply external stimuli to produce reversible changes in their local shape, as sketched in figure 1(c). These shape transformations, which are localized at the microstructural level, cause a modification of the resonant frequency of the cantilevers and, in turn, a modification the global dynamic response (altering the frequency location of the locally resonant bandgaps). The net result is a medium with tunable wave characteristics where, remarkably, the tunability strategy does not interfere with other important structural functionalities.
3. Nonlinear deformation of soft, smart composite cantilevers

To obtain the levels of curling necessary to induce macroscopic changes in the response of the cantilevers, we are required to work with a compliant material substrate. To this end, we consider a cellular solid skeleton (main lattice plus cantilevers) made of polydimethylsiloxane (PDMS), a soft polymer. Each cantilever is instrumented with two thin patches of a soft active material, as shown in figure 2. We choose to work with P(VDF-TrFE-CTFE), an electrostrictive terpolymer capable of inducing ~4% electrostrictive strain. This high electrostrictive strain coupled with its high Young’s modulus makes this polymer uniquely suited for this internal cantilever application [44, 49]. Note that, due to the nature of the electrostriction phenomenon, the two patches are only capable of contracting in the thickness direction and expanding in the planar direction in response to a through-the-thickness electric field. For this reason, we only activate one patch at a time: activating the bottom patch allows for counterclockwise curling, while activating the top one causes the cantilever to curl clockwise. The mechanical properties for the PDMS substrate are selected within the range of properties that can be achieved with off-the-shelf PDMS kits: Young’s modulus $E = 2$ MPa, Poisson’s ratio $\nu = 0.5$, density $\rho = 965$ kg m$^{-3}$. The properties of P(VDF-TrFE-CTFE) are: Young’s modulus $E_p = 200$ MPa, Poisson’s ratio $\nu_p = 0.48$, density $\rho_p = 1300$ kg m$^{-3}$, coefficient of electrostriction $\beta = \beta_1 = 3 \times 10^{-18}$ m$^2$ V$^{-2}$, breakdown electric field $E_{b} = 350$ MV m$^{-1}$.

A single cantilever (before and after the application of an electric field to the bottom patch), with all its characteristic dimensions, is shown in figure 2. The two lines oriented at ±60° and stemming from the origin represent portions of the lattice links belonging to the hexagonal lattice structure. Throughout this analysis, we consider the following dimensions (which have been carefully selected to be compatible with fabrication methods currently available): the length of each lattice link is $L = 2.5$ cm, the thickness of a lattice link is $t = L/50 = 50$ $\mu$m, the length of a cantilever is $L_c = 0.8 \cdot L = 2$ cm, the thickness of a cantilever is $t_c = L_c/65 = 308$ $\mu$m, the distance from the base of the cantilever to the beginning of the patch is $d_p = 0.3 \cdot L_c = 6$ mm, the length of a patch is $L_p = 0.6 \cdot L_c = 12$ mm, the thickness of a patch is $t_p = 10 \mu$m.

To predict the curling of the terpolymer-PDMS sandwich cantilevers, we resort to an analytical model largely inspired by the work of Tajeddini and Muliana [50]—here adapted to the special case in which only one patch at a time is activated [51]. The key aspects of the model are summarized in the following (for a complete and detailed account of the formulation, refer to the supplementary data (SD) section). The kinematics are based on Reissner’s finite-strain beam theory [52], with the additional assumptions of initially straight configuration and shear indeterminacy [53, 54]; the latter assumption restricts the model to the treatment of slender beams. The nonlinear strain–displacement relations are:

$$\frac{du}{dx} = (1 + \epsilon_{\phi}) \cos \phi - 1,$$

$$\frac{dv}{dx} = (1 + \epsilon_{\phi}) \sin \phi,$$

where $u$ and $v$ are the beam’s axial and lateral displacements, $\epsilon_{\phi}$ is the axial strain at the beam’s neutral axis and $\phi$ is the rotational angle of the cross section of the deformed beam.
The composite cantilever is equipped with thin patches of a soft (compliant) active material, perfectly bonded to the substrate and symmetrically placed with respect to the neutral axis of the beam. In response to an external stimulus, a patch exerts axial forces on the substrate at their interface. Due to the fact that, for the considered geometry, the axial strain at the centroidal axis (\(\varepsilon_{c,0}\)) is typically negligible, the action of the patch is effectively akin to a constant bending moment applied to the span of the substrate sandwiched between the patches. In light of these observations, the following relations can be written:

\[
\varepsilon_{c,0} = 0, \quad \frac{d\phi}{dx} = \frac{M_{ac}}{EI},
\]

where \(I_c\) is the second moment of area of the substrate (\(I_c = b c^3/12\)), and \(M_{ac}\) is the moment due to the actuator’s action. Depending on whether we are activating the bottom (B) or top (T) patch, the moment can assume different values. If we only activate the bottom patch, the moment applied to the substrate is:

\[
M_{ac} = \frac{E_b b t_p t_p^2}{2(t_c + 6t_p)E_p} \epsilon_p \quad \text{if we only activate the bottom patch,}
\]

if we only activate the top patch, the resulting moment is:

\[
M_{ac} = \frac{E_b b t_p t_p^2}{2(t_c + 6t_p)E_p} \epsilon_p \quad \text{if we only activate the top patch,}
\]

where \(\epsilon_p\) is the free strain of the patch (strain undergone by an unconstrained patch under the action of the electric field); for our specific choice of soft active material, i.e., for an electrostrictive terpolymer, the free strain is:

\[
\epsilon_p = \beta (E^\epsilon)^2,
\]

where \(E^\epsilon\) is the applied electric field. Note that, to derive equations (5) and (6), we also assumed the patches to be much thinner than the substrate; as a consequence, we can safely assume the axial stress to be constant along the patch thickness. It is important to note that, as a result of the slenderness and of the pure-bending actuation, the established deformation field features small rotations and small strains.

The boundary value problem is solved in a piecewise fashion. Given the geometry considered in figure 2, we can identify three intervals: \(0 \leq x \leq d_p\), where \(M_{ac} = 0\) due to the absence of patches, \(d_p \leq x \leq L_p + d_p\), where \(M_{ac}\) is given by equation (5) or (6) (depending on whether we are activating the bottom or top patch, respectively), and \(L_p + d_p \leq x \leq L\) where again \(M_{ac} = 0\). The piecewise solution is given below. For \(0 \leq x \leq d_p\):

\[
\begin{align*}
    u(x) &= 0, \\
    v(x) &= 0.
\end{align*}
\]

For \(d_p \leq x \leq d_p + L_p\):

\[
\begin{align*}
    u(x) &= \frac{M_p}{M_{ac}} \sin \left( \frac{M_p(x-d_p)}{EI_p} \right) - x + d_p, \\
    v(x) &= \frac{M_p}{M_{ac}} \left[ 1 - \cos \left( \frac{M_p(x-d_p)}{EI_p} \right) \right].
\end{align*}
\]

For \(d_p + L_p \leq x \leq L\):

\[
\begin{align*}
    u(x) &= \cos \frac{M_p(x-d_p)}{EI_p} - x + d_p + \frac{EI_p}{M_{ac}} \sin \frac{M_p(x-d_p)}{EI_p}, \\
    v(x) &= \sin \frac{M_p(x-d_p)}{EI_p} + \frac{EI_p}{M_{ac}} \left( 1 - \cos \frac{M_p(x-d_p)}{EI_p} \right).
\end{align*}
\]

The curled shape shown in figure 2 is obtained by implementing equations (8)–(10) with the selected geometric and material parameters, for an imposed electric field \(E^\epsilon = 150 \text{ MV m}^{-1}\) applied to the bottom patch only.

4. Phononic analysis

The analysis of the phononic characteristics of the soft hexagonal lattice with straight/curled microstructure is carried out using a unit cell discretized with beam elements, and in-house MATLAB routines. Throughout this work, we refer to the cantilevers as straight or curled, according to whether they are in the undeformed or deformed configuration. The finite element model of the curled cantilevers is obtained by discretizing the curved profile predicted by the analytical model discussed in section 3 (shown in figure 2). In addition to accounting for the effect of the patches in the form of equivalent bending moment loads, we also explicitly incorporate the influence of the terpolymer patches on the mass and stiffness matrices by taking into account the multi-material structure of the cantilevers’ layered cross section according to the relative thickness of the layers. While a nonlinear model is employed to predict the curling deformation of the cantilevers, a linear small-on-large analysis suffices to describe the superimposed small deformations experienced during a transient wave event, which are a perturbation about the equilibrium deformed state reached after curling. For more details on this, including a numerical justification for the validity of the small-on-large model assumption, refer to the SD section. It is of course implied that the static moments that cause the cantilevers to curl are preserved throughout the dynamic analysis. Also note that, throughout our analysis, we neglect viscoelastic damping effects, which are outside the proof-of-concept scope of this work, although we recognize that they may significantly affect the predicted wave manipulation effects in future experimental settings.

4.1. Bandgap shifting

The most intuitive manifestation of microstructural shape changes is the onset of bandgap tunability. This effect is here illustrated by comparing the wave response of several unit cell configurations—with straight cantilevers, curled cantilevers, or a combination of both—at different frequencies. This result is achieved through a classical Bloch analysis which yields the dispersion relation for the lattice [55]. Dispersion relations are here portrayed in the form of band diagrams (figure 3), obtained by plotting the frequency values computed at points of the reciprocal wave vector space.
marked by a coordinate $s$ running along the contour of the first Brillouin zone (BZ) of the lattice \cite{56}. Details on the BZ, including the significance of points O, A, ..., G are reported in the SD section. Note that, in order to guarantee that the adopted representation is automatically compatible with the entire spectrum of symmetries (and lack thereof) that are established by arbitrarily curling the microstructural elements, we here work with the entire first BZ instead of the irreducible BZ of the hexagonal lattice, which would only be applicable in selected highly-symmetric cases. More information on this point is also reported in the SD section. This selection results in more complex (at times redundant, yet always complete) band diagrams containing a wealth of information on wave directivity. Details on how to navigate these plots and extract the necessary phononic characteristics are provided below.

We begin our analysis by considering the reference case of a RH lattice without cantilevers. The unit cell and its band diagram are shown in figures 3(a) and (b). In the frequency range of interest, we observe two modes of wave propagation: a slower S-mode, known to be dominated by beam bending deformation, and a very fast P-mode (note the almost vertical steep slope), dominated by longitudinal beam deformation mechanisms. In figures 3(c) and (d) we can see that adding a population of straight cantilevers yields a \textit{locally resonant bandgap} (highlighted in red and labeled BG1), along with the appearance of localized new modes which originate from the ‘splitting’ of the S-mode and cluster around the bandgap. Note that the onset of the bandgap ($f_1 \approx 6.19$ Hz) coincides with the first natural frequency of one of the straight composite cantilevers. When computing the band diagrams for the unit cells with cantilevers, we experienced some numerical issues when $s$ coincides with O (i.e., with the origin of the $k$-plane); details on this issue and on the reason why it does not affect our results are given in the SD section. Application of an electric field to the bottom patch of each cantilever ($E = -150$ MV m$^{-1}$) forces initially-straight cantilevers to curl counterclockwise, resulting in the architecture shown in figure 3(e). Intuition suggests that the curled cantilevers are stiffer than their straight counterparts, thus behaving as frequency-upshifted resonators; indeed, the locally resonant bandgap (now highlighted in blue and labeled BG2) shifts upwards. Also, the onset of the bandgap ($f_2 \approx 7.87$ Hz) coincides now with the first natural frequency of a curled cantilever. The shift can be quantified by evaluating the relative change between the onsets of the two bandgaps, and it amounts to 27\% for our choice of electric field. To understand
4.2. Reconfigurable anisotropy

In this section, we discuss how, through a strategy that allows individual resonator tuning and asymmetric microstructural reconfiguration, we can achieve profound spatial wave manipulation effects. To this end, we revisit and enhance the idea of relaxed cell symmetry [45, 47], which we had previously introduced in the context of piezo-shunt-controlled lattices. Recall that the anisotropic wave patterns that are established in periodic structures [57] usually reflect closely the symmetry (or lack thereof) of the unit cell. We can therefore expect that, by curling selected subsets of cantilevers, we would alter the symmetry landscape of the cell, thus inducing non-symmetric anisotropy patterns with pronounced wave beaming characteristics (in contrast with the symmetric behavior of the host lattice). This scenario is explored in figure 5. We can see that two of the cantilevers have been activated and curled, while the others are left in their undeformed state. From the band diagram in figure 5(a), we can see that the loss of symmetry of the unit cell has drastic repercussions on its response. This is especially visible for the 5th and 7th modes (highlighted in red and blue, respectively), and it can be best appreciated by looking at the corresponding iso-frequency contours in figures 5(b) and (c) (i.e., the dispersion surface of the selected mode is sliced at different frequencies; an increase in frequency is associated with a transition from light to dark contours). The hexagon bounds the region of the Cartesian wave plane corresponding to the BZ, and the points highlighted on the contour are the same points indicated on the abscissa of figure 5(a). We can see that both the 5th and the 7th mode lose the typical six-folded symmetry of hexagonal lattices. Taking a closer look at the 5th branch, we notice a partial bandgap manifesting as a pair of dips in the branch, spanning the BC and EF edges of the BZ. The presence of the partial bandgap and the morphology of the dispersion surface suggest that the wave response should be attenuated along directions characterized by wavevectors pointing from O to points on the BC and EF edges, and, conversely, be focused along directions OD and OG (and neighboring ones). Similar considerations can be made for the 7th mode, which features a partial bandgap spanning the CD and FG edges, with wave beaming expected along directions OE and OB.

To validate the predictions from the unit cell analysis, we test the response of a finite lattice comprising $11 \times 10$ unit cells. We can observe that the wave response is indeed affected by the reconfiguration of the cell, as evidenced by the shift in frequency and the modification of the wave propagation characteristics.
cells (figure 6) with the same architecture shown in figure 5(a) (simulations are performed with a Newmark-β time-integration algorithm). The bottom nodes of the lattice are clamped, and an in-plane excitation is applied to the mid-point of the upper boundary, as indicated by the arrow. Figure 6(a) represents the response to a 7-cycle burst with carrier frequency \( f = 6.7 \) Hz—belonging to the 5th mode. The wavefield depicts the total mechanical energy landscape in the lattice at a certain instant of propagation. We can see that the wave is mainly propagating along a direction which coincides with OD, while it is attenuated along the directions corresponding to the partial bandgap (left portion of the domain), in complete agreement with the iso-frequency contour of figure 5(b). Figure 6(b) represents the response of the same structure when the carrier frequency of the burst is \( f = 8.5 \) Hz—which belongs to the 7th mode. This wavefield displays an opposite pattern: the energy associated with the wave is now mainly propagating in the left portion of the domain, while the right portion remains de-energized. Again, this result is consistent with the unit cell analysis prediction of figure 5(c).

To summarize the results shown in this section, we can state that the selective curling of some cantilevers causes a profound modification of the wave anisotropy patterns. In particular, the availability of cantilevers resonating at different frequencies along different directions causes the appearance of partial bandgaps, which lead to spatially selective and beamed wave patterns. It is also interesting to point out that the same lattice presents opposite wave beaming characteristics at different frequencies. Due to the reversible nature of the curling, we can switch between different directivity patterns by simply curling other sets of cantilevers, ultimately enabling reconfigurable wave beaming.

4.3. Switchable chirality

In the previous section, we showed how relaxing the unit cell symmetry has drastic repercussions on the morphology of the wave patterns. To elucidate the rich opportunities for symmetry relaxation attainable with actively-curlable microstructures, we now proceed to provide a mechanistic rationale to link symmetry in the structure to symmetry in the response. For brevity, we restrict our analysis to the S-mode. This choice is motivated by two considerations. First, the deformation patterns associated with the S-mode are particularly easy to interpret, due to the fact that the mode mostly involves flexural deformation of the lattice links [45]. Secondly, since an S-like mode is observed at low frequencies for virtually every cell configuration, it offers a fair metric of comparison between different architectures.

Let us consider two cell configurations in which only two cantilevers (located at opposite nodes of an hexagonal cell) are curled: in the case of figure 7(a), both cantilevers are curled counterclockwise, while in figure 7(b) one is curled clockwise and the other counterclockwise. By inspecting the symmetry of the cell in figure 7(a), we note that the cell is characterized by geometric chirality—its shape cannot be recovered, after mirroring it about any axis, by resorting to simple translations and rotations (simply put, the architecture does not possess mirror symmetries). In contrast, the architecture in figure 7(b) is not chiral, since it features a mirror symmetry about the dashed axis. It is important to realize that the chirality is here introduced through shape modifications of the auxiliary microstructure, and can be switched on/off through the application of an electric stimulus, without modifying the primary lattice network. In this respect, it is qualitatively different from the chirality most commonly observed in lattice materials, which is associated with a special connectivity of the primary lattice [58–60]. To emphasize
its sole dependency on the microstructure, we refer to it as second-order chirality.

We consider the iso-frequency contour of the S-mode evaluated at $f$ is the maximum frequency of the S-mode for any specific architecture). In figure 7(c), we report the iso-frequency contour for the case in figure 7(a) (thick black line), compared to that of a reference case with all straight cantilevers (thin red line), which is characterized by the highest achievable degree of symmetry (6-fold rotational symmetry, 3 mirror axes and inversion symmetry). We observe that the geometrically chiral pattern of figure 7(a) induces chirality in the response—as highlighted by the lack of mirror symmetries in the black iso-frequency contour in figure 7(c). On the other hand, the response of the non-chiral geometry in figure 7(b), represented by the black contour in figure 7(d), is, as expected, non-chiral—as indicated by the existence of two mirror axes. Alone, these observations would lead to the partial (and overly simplistic) conclusion that geometric chirality in the unit cell implies chirality in the wave response. In the following, we will show that the connection between cell geometry and response, in terms of chirality, is significantly more subtle.

To better elucidate the onset of chirality in the response, we analyze how a wave impinging on the hexagonal cell along three characteristic directions interacts with differently-oriented cantilevers. To provide a more intuitive rationale, we replace the original cell with an analogous one in which the curled cantilevers are substituted with slanted (yet straight) ones, whose centers of mass, just like in the curled case, are off-centered with respect to lines connecting the hexagon’s

![Figure 7](image-url)
vertices (figure 7(e)). Note that this analogous model, albeit structurally different, is, for all intents and purposes, identical to the original one in terms of symmetry and geometric chirality, thus providing some useful qualitative information on the wave/cantilever interaction. The three directions of wave propagation we consider are marked as OA, OA’ and OA” in figure 7(c) and to the inflection points of the iso-frequency contours (where the response chirality manifests the most). When a shear wave impinges on the cell along a direction identified by one of those wave vectors, the unit cell will locally vibrate along a direction perpendicular to the wave vector. In figure 7(e), this is schematically denoted by sliding clamp constraints which, individually, only allow translation perpendicularly to the direction of the incoming wave. While the unit cell features a fixed set of internal beam-like resonators, they naturally display different vibrational characteristics according to the way in which they are excited. Our objective is to determine the landscape of effective internal resonating mechanisms that is available for waves traveling along different directions. For example, a shear wave impinging along OA (and shaking the cell along the direction perpendicular to OA), engages a cell characterized by two cantilevers parallel to OA (which are activated flexurally), two inclined by 30° and two inclined by 60° with respect to OA (these last four will undergo a blend of flexural and axial deformation). A wave along OA’, on the other hand, effectively sees a cell in which two are parallel to OA’ (thus activated axially), two are parallel to OA (thus activated flexurally) and two are inclined by 60° with respect to OA’ (mixed mode). Finally, with respect to a wave along OA”, two cantilevers are inclined by 30° and four are inclined by 60°. In light of these considerations, we can conclude that the establishment of

Figure 8. Effects of microstructural curling on the symmetry of the wave response: geometric chirality versus functional chirality. Comparison between two architectures differing in terms of curling orientation of the entire cantilever microstructure. (a), (b) Two geometrically-chiral unit cells in which all cantilevers are curled counterclockwise and clockwise, respectively. (c), (d) Responses for the architectures in (a) and (b), respectively. The dashed lines represent mirror axes of symmetry of the response and the arrows indicate wave vectors corresponding to selected directions of propagation. (e), (f) Analogous models for the cells in (a) and (b), respectively.
chirality in the response is linked to the availability of three distinct sets of resonating mechanisms along the three considered directions. In figure 7(f), we repeat the exercise for the architecture in figure 7(b). In this case, shear waves along OA and OA’ see a cell characterized by an identical set of effective resonators, consisting of two cantilevers parallel and one perpendicular to OA (or OA’, respectively), two inclined by 60° and one by 30° with respect to OA (or OA’, respectively). On the contrary, a wave along OA’ engages a cell with two cantilevers inclined by 30° and four inclined by 60° with respect to OA”. Consistently with this additional symmetry in the resonating mechanisms, the iso-frequency contour in figure 7(d) is identical along OA and OA’ and does not display chirality.

Let us now dig deeper into the role of the microstructural elements, to illustrate further implications of the second order chirality. First, we consider the unit cell configuration in figure 8(a), characterized by six counterclockwise-curled cantilevers and displaying geometric chirality, as highlighted by the absence of mirror symmetries. Interestingly, and counter-intuitively, its S-mode response, shown in figure 8(c) (thick black contour), is not chiral (all the dashed lines are axes of mirror symmetry). Identical considerations can be made for the configuration in figure 8(b), characterized by six clockwise-curled cantilevers (whose response is shown in figures 8(d)). These examples suggest that geometric chirality alone does not necessarily imply chirality of the response. To lift this apparent contradiction, we repeat the directional vibration analysis introduced above, here based on the analogous models of figures 8(e) and (f). It is easy to recognize that, in both cases, we have the same identical availability of resonating mechanisms along all directions. This explains why the responses of figures 8(e) and (d) are identical. We can argue that these architectures, despite being geometrically chiral, are functionally non-chiral—meaning that mirroring the cell about any axis fully preserves the effective functionality of the microstructure with respect to the S-mode.

Summarizing our findings, we can conclude that, for lattices with auxiliary microstructures that feature second order geometric chirality, functional chirality implies chirality in the response.

5. Conclusions

In this work, we have shown that we can resort to the localized shape modification of a population of soft auxiliary microstructural elements to attain a dramatic reconfiguration of the wave characteristics of soft cellular structures. In our structures, the microstructural elements are composite cantilever beams with soft active material inserts, that can curl upon the application of an electric field. This strategy allows for tunable wave control, since the localized curling deformations can be reversed by removing the electric fields. Another remarkable aspect of this strategy is that the wave control capabilities—enabled at the microstructural level—are completely independent from other functionalities and properties of the lattices (e.g., their load-bearing capability). The independent controllability of each cantilever allows considerable flexibility and allows for both spectral and spatial wave control. By curling all the cantilevers inside every unit cell in the same fashion, we can shift the location of the locally resonant bandgap. By curling selected sets of cantilevers in each cell, on the other hand, we relax the symmetry of the architecture, we introduce partial (directional) bandgaps and achieve pronounced wave beamforming. Due to the peculiar symmetry landscapes introduced by microstructural curling, we are also able to observe chirality effects of the ‘second-order’—i.e., independent from the lattice connectivity and only associated with the mechanical functionality of the microstructures.

Acknowledgments

SG and PC acknowledge the support of the National Science Foundation (grant CMMI-1266089). PC also acknowledges the support of the University of Minnesota through the Doctoral Dissertaton Fellowship. AM and VT acknowledge the support of the Air Force Office of Scientific Research (grant FA9550-14-1-0234). ZO and SA gratefully acknowledge the support of the National Science Foundation (EFRI grant 1240459) and the Air Force Office of Scientific Research.

References

[1] Liu Z, Zhang X, Mao Y, Zhu Y Y, Yang Z, Chan C T and Sheng P 2000 Science 289 1734–6
[2] Lemoîlt F, Kaina N, Fink M and Lerosey G 2013 Nat. Phys. 9 55–60
[3] Zhu R, Liu X N, Hu G K, Sun C T and Huang G L 2014 Nat. Commun. 5 5510
[4] Torrent D and Sánchez-Dehesa J 2008 New J. Phys. 10 063015
[5] Goffaux C and Vigneron J P 2001 Phys. Rev. B 64 075118
[6] Romero-García V, Lagarrique C, Groby J P, Richoux O and Tournat V 2013 J. Phys. D: Appl. Phys. 46 305108
[7] Ruzzene M and Baz A 2000 J. Vib. Acoust. 122 151–9
[8] Thorp O, Ruzzene M and Baz A 2001 Smart Mater. Struct. 10 979–89
[9] Airoldi L and Ruzzene M 2011 New J. Phys. 13 113010
[10] Wang G, Chen S and Wen J 2011 Smart Mater. Struct. 20 015026
[11] Casadei F, Delpero T, Bergamini A, Ermanni P and Ruzzene M 2012 J. Appl. Phys. 112 064902
[12] Bergamini A, Delpero T, De Simoni L, De Lillo L, Ruzzene M and Ermanni P 2014 Adv. Mater. 26 1343–7
[13] Zhu R, Chen Y Y, Barnhart M V, Hu G K, Sun C T and Huang G L 2016 Appl. Phys. Lett. 108 011905
[14] Yi K, Collet M, Ichchou M and Li L 2016 Smart Mater. Struct. 25 075007
[15] Cardella D, Celli P and Gonella S 2016 Smart Mater. Struct. 25 085017
[16] Robillard J F, Matar O B, Vasseur J O, Deymier P A, Stupinger M, Hladky-Hennion A C, Pennec Y and Djafari-Rouhani B 2009 Appl. Phys. Lett. 95 124104
[17] Schaeffer M and Ruzzene M 2015 J. Appl. Phys. 117 194903
[18] Allein F, Tournat V, Gusev V E and Theocharis G 2016 Appl. Phys. Lett. 108 161903
[19] Wang Z, Zhang Q, Zhang K and Hu G 2016 Adv. Mater. 28 9857–61
[20] Yeh J Y 2007 Physica B 400 137–44
[21] Xu Z, Wu F and Guo Z 2013 Solid State Commun. 154 43–5
[22] Walker E, Reyes D, Rojas M M, Krokerin A, Wang Z and Neogi A 2014 Appl. Phys. Lett. 105 143503
[23] Casadei F and Bertoldi K 2014 J. Sound Vib. 333 6532–47
[24] Bertoldi K and Boyce M C 2008 Phys. Rev. B 77 052105
[25] Wang P, Casadei F, Shan S, Weaver J C and Bertoldi K 2014 Phys. Rev. Lett. 113 014301
[26] Maurin F and Spadoni A 2014 J. Sound Vib. 333 4562–78
[27] Rudykh S and Boyce M C 2014 Phys. Rev. Lett. 112 034301
[28] Daraio C, Nesterenko V F, Herbold E B and Jin S 2006 Phys. Rev. E 73 026610
[29] Narisetti R K, Leanny M J and Ruzzene M 2010 J. Vib. Acoust. 132 031001
[30] Narisetti R K, Ruzzene M and Leanny M J 2011 J. Vib. Acoust. 133 061020
[31] Spadoni A and Daraio C 2010 Proc. Natl Acad. Sci. 107 7230–4
[32] Ganesh R and Gonella S 2015 Phys. Rev. Lett. 114 054302
[33] Yang W P and Chen L W 2008 Smart Mater. Struct. 17 051011
[34] Gei M, Roccabianca S and Bacca M 2011 IEEE/ASME Trans. Mechatronics 16 102–7
[35] Bayat A and Gordaninejad F 2015 J. Vib. Acoust. 137 011011
[36] Nouh M A, Aldraihem O J and Baz A 2016 J. Intell. Mater. Syst. Struct. 27 1829–45
[37] Jia K, Wang M, Lu T, Zhang J and Wang T 2016 Smart Mater. Struct. 25 055047
[38] Galich P I and Rudykh S 2016 Int. J. Solids Struct. 91 18–25
[39] Gertz R, Kochmann D M and Shmuel G 2016 Int. J. Solids Struct. (https://doi.org/10.1016/j.ijsolstr.2016.10.002)
[40] Ahmed S, Ounaies Z and Frecker M 2014 Smart Mater. Struct. 23 094003
[41] Shian S, Bertoldi K and Clarke D R 2015 Adv. Mater. 27 5814–9
[42] Felton S M, Tolley M T, Shin B, Onal C D, Demaine E D, Rus D and Wood R J 2013 Soft Matter 9 7688–94
[43] Ge Q, Dunn C K, Qi H J and Dunn M L 2014 Smart Mater. Struct. 23 094007
[44] Sigamani N S, Ahmed S and Ounaies Z 2014 Proc. ASME SMASIS 2014 vol 1, V001T01A030
[45] Celli P and Gonella S 2014 J. Appl. Phys. 115 103502
[46] Krödel S, Delpero T, Bergamini A, Ermanni P and Kochmann D M 2014 Adv. Eng. Mater. 16 357–63
[47] Celli P and Gonella S 2015 Appl. Phys. Lett. 106 091905
[48] Yang W P and Chen L W 2008 J. Appl. Phys. 103 014301
[49] Madden J D W, Vandesteeg N A, Anquetil P A, Madden P G A, Takshi A, Pytel R Z, Lafontaine S R, Wieringa P A and Hunter I W 2004 IEEE J. Ocean. Eng. 29 706–28
[50] Tajeddini V and Muliana A 2015 Compos. Struct. 132 1085–93
[51] Wang B T and Rogers C A 1991 J. Intell. Mater. Syst. Struct. 2 38–58
[52] Reissner E 1972 Z. Angew. Math. Phys. 23 795–804
[53] Irshik H and Gerstmayr J 2009 Actu. Mech. 206 1–21
[54] Muliana A 2015 Int. J. Non-Linear Mech. 71 152–64
[55] Phani A S, Woodhouse J and Fleck N A 2006 J. Acoust. Soc. Am. 119 1995–2005
[56] Brillouin L 1953 Wave Propagation in Periodic Structures 2nd edn (New York: Dover)
[57] Langley R S 1996 J. Sound. Vib. 197 447–69
[58] Spadoni A, Ruzzene M, Gonella S and Scarpa F 2009 Wave Motion 46 435–50
[59] Liebold-Ribeiro Y and Körner C 2014 Adv. Eng. Mater. 16 328–34
[60] Trainiti G, Rimoli J and Ruzzene M 2016 Int. J. Solids Struct. 97–98 431–44