Development of an empirical model of a variable speed vapor injection compressor used in a Modelica-based dynamic model of a residential air source heat pump

Bertrand Dechesne¹, Stephane Bertagnolio², Vincent Lemort¹

¹ University of Liege, Aerospace and Mechanical Engineering Department, Thermodynamics Laboratory, Chemin des chevreuils 7, Sart Tilman, Liege, Belgium
² Emerson Climate Technologies, Welkenraedt, Belgium
E-mail: bdechesne@ulg.ac.be

Abstract. This paper presents a steady-state model of a variable speed vapor injection scroll compressor. Two compressors were investigated. The developed empirical model is based on five dimensionless polynomials that were fitted using experimental data from a 2.7kW scroll compressor. A second set of data was used to show the prediction of the presented model for another device which exhibits a different swept volume. In both cases, the suction and injection mass flow rate were respectively predicted with a coefficient of determination equal to 99.9 and 94.3% and for the consumed power, 98.4% and 95.6%. A Modelica based dynamic model is then presented. The steady-state validation of the main components models is performed. Finally the control of the cycle using two PID controllers is presented and commented.

1. Introduction
Computational time for dynamic modeling of refrigeration systems is continuously decreasing thanks to the ever-increasing power of computational packages and processors. Dynamic modeling allows engineers to simulate more accurately the control strategies which are of prime importance for system performances.

In this field, Rasmussen et al. provides a deep literature review and very interesting simulation tutorial for the dynamic modeling of vapor compression cycles ([1] and [2]). Hongtao et al. ([3] and [4]) presented a model of flash tank injection heat pump developed in Modelica language. The studied compressor is a fixed speed scroll compressor. In [4], they presented results of the validation of their dynamic models for a system start-up and shut-down procedures. Jiazhen et al. [5] evaluated the transient performance of a heat pump working with different low-GWP refrigerants (R32 and D2Y60). In 2011, Li et al. [6] developed a dynamic model of a R134a automotive air-conditioning and showed the refrigerant mass migration during start-up and shut-down operations.

This paper presents a Modelica-based dynamic model of an air source heat pump (ASHP) using a variable speed vapor injection scroll compressor and R410A as working fluid. This work is based on the open-source and open-access ThermoCycle library [7] for the model components and CoolProp for the fluid thermodynamic properties [8]. An empirical model of the vapor
injection scroll compressor is developed using empirical correlations for the volumetric efficiency, isentropic efficiency and the ratio between the injection and suction mass flow rate. A simple control strategy is implemented where the needle position of the injection and heating valves is controlled via PID controllers to obtain a target superheat on the injection line and at the exhaust of the evaporator. By using the simulation model, the performance and the control of the ASHP in the heating mode is presented.

2. System overview
The studied system is represented on figure 1.

![Heat pump Modelica model in heating mode.](image)

It is a classical injection cycle with an economizer heat exchanger and two separated units. The outdoor unit is composed of the evaporator (8) and the heating electronic expansion valve (EEV, 7). Between these two units, split lines are present (vapour line 9 and liquid line 7). All the main components are presented in the following sections.

3. Compressor modeling
The proposed model uses a set of five dimensionless polynomials to predict the compressor behavior. The compressor model is treated as quasi steady-state, in fact, the time constants associated to the compressor dynamics are very small compared to those associated with heat exchangers and charge distribution [5]. The inputs of the compressor polynomials are the rotational speed \((rpm)\) and the three pressure levels, i.e. the suction pressure \((P_{su})\), injection pressure \((P_{inj})\) and discharge pressure \((P_{ex})\) or combinations of these pressures, i.e.

\[
\begin{align*}
  r_{p,\text{tot}} &= \frac{P_{ex}}{P_{su}} & \text{the total pressure ratio} \\
  r_{p,\text{inj}} &= \frac{P_{inj}}{P_{su}} & \text{the injection pressure ratio}
\end{align*}
\]
The five polynomials are then used to compute the overall electrical power consumption, the discharge thermodynamic state and both injection and suction mass flow rates. The only parameter of the model is the compressor swept volume \( V_s \) which makes it easy to use for different size of devices.

These five polynomials are namely the volumetric efficiency \( \varepsilon_v \), the isentropic efficiency \( \eta_s \), the variable speed drive efficiency \( \eta_{drive} \), the injection ratio \( X_{inj} \), i.e. the ratio between the injection and the suction mass flow and finally the losses ratio \( X_{loss} \), i.e. the ratio between the compressor ambient losses and the electrical consumption of the compressor.

The volumetric efficiency is defined as the ratio between the actual mass flow and the theoretical mass flow determined via the swept volume, i.e.

\[
\varepsilon_v = \frac{\dot{M}_{r,su}}{\rho_{r,su} \cdot V_s \cdot N_{rot}}
\]  

where \( \dot{M}_{r,su} \) is the suction mass flow rate \( (kg/s) \), \( \rho_{r,su} \) is the working fluid density defined at the suction port \( (kg/m^{3}) \) and \( N_{rot} \) is the rotational speed \( (rad/s) \).

The following formulation of the isentropic efficiency is implemented in order to take into account the injection process.

\[
\eta_s = \frac{\dot{M}_{r,su} \cdot (h_{r,ex,s} - h_{r,su}) + \dot{M}_{r,inj} \cdot (h_{r,ex,inj,s} - h_{r,inj})}{\dot{W}_{in}}
\]  

where

\[
\begin{align*}
\dot{M}_{r,inj} & \text{ is the injection mass flow rate, } kg/s \\
\dot{W}_{in} & \text{ is the electrical power delivered to the compressor motor, } W \\
h_{r,su} & \text{ is the refrigerant specific enthalpy at the suction port, } J/kg \\
h_{r,inj} & \text{ is the refrigerant specific enthalpy at the injection port, } J/kg \\
h_{r,ex,s} & \text{ is the specific enthalpy at the exhaust port for an isentropic compression from the suction to the exhaust pressure, } J/kg \\
h_{r,ex,inj,s} & \text{ is the specific enthalpy at the exhaust port for an isentropic compression from the injection to the exhaust pressure, } J/kg
\end{align*}
\]

The isentropic compression power for an injection compressor can be interpreted as two separate volumes, a suction and an injection volume, performing separately an isentropic compression process respectively from the suction to the discharge pressure and from the injection to the discharge pressure.

The ratio between the electrical power provided to the compressor motor \( (\dot{W}_{in}) \) and the total electrical power delivered to the drive \( (\dot{W}_{el}) \) is defined as the drive efficiency. The injection mass flow rate is determined via the injection ratio \( X_{inj} \).

By performing an energy balance on the compressor shell using the experimental data, a significant residual was calculated. Moreover, this residual had always the same sign, that is why it is generally considered as being linked to the compressor ambient losses. Thus, an additional dimensionless polynomial \( X_{loss} \) had to be added in order to predict the refrigerant temperature at the exhaust port accurately.

\[
X_{loss} = \frac{\dot{Q}_{loss}}{\dot{W}_{el}}
\]

Finally, the exhaust thermodynamic state can be computed via energy balance applied to the compressor shell, which is given by

\[
(\dot{M}_{r,su} + \dot{M}_{r,inj}) \cdot h_{r,ex} = \dot{W}_{in} + \dot{M}_{r,su} \cdot h_{r,su} + \dot{M}_{inj} \cdot h_{r,inj} - \dot{Q}_{loss}
\]
In conclusion, we have:

\[
\begin{pmatrix}
\eta_s \\
\varepsilon_v \\
\eta_{\text{drive}} \\
X_{\text{inj}} \\
X_{\text{loss}} \\
\end{pmatrix} = F(rpm, P_{su}, P_{inj}, P_{ex})
\]

and the outputs (\(\dot{W}_{el}, \dot{M}_{r,\text{su}}, \dot{M}_{\text{inj}}, h_{r,ex}\)) can all be computed using these polynomials and knowing the inlet and injection thermodynamic state as well as the rotational speed.

3.1. Mass flow rates determination

The validation process was made possible thanks to thirty experimental data points distributed all across the compressor operating map. As stated before, the rpm, the three pressure levels or the two pressure ratio were used as variables for the polynomials. The volumetric efficiency polynomial is a first order polynomial, i.e.

\[
\varepsilon_v = a_0 + a_1 \cdot rpm + a_2 \cdot P_{ex} + a_3 \cdot r_{p,tot} + a_4 \cdot r_{p,inj}
\]

Similarly, the injection ratio polynomial is given by:

\[
X_{\text{inj}} = b_0 + b_1 \cdot rpm + b_2 \cdot P_{ex} + b_3 \cdot P_{su} + b_4 \cdot r_{p,inj}
\]

Figure 2 presents the comparison between the predicted and measured values for the suction and injection mass flow rates. Both mass flow rates are given for two different compressors (CP1 and CP2). CP1 was used to fit the polynomials whereas the results presented for CP2 were obtained by using the same polynomials and by only updating the compressor swept volume.
3.2. Exhaust state and electrical consumption

Knowing the suction and injection states as well as both injection and suction mass flow rates and the discharge pressure, the isentropic efficiency polynomial gives the electrical power provided to the compressor motor. Via the losses ratio and the drive efficiency, it is then possible to determine the discharge temperature and the electrical power delivered to the variable speed drive. The drive efficiency is correlated to the rotational speed via a third order polynomials. A second order polynomial was chosen for the isentropic efficiency and a constant value for the losses ratio were selected, i.e.:

\[
\eta_{\text{drive}} = c_0 + c_1 \cdot \text{rpm} + c_2 \cdot \text{rpm}^2 + c_3 \cdot \text{rpm}^3
\] (7)

\[
\varepsilon_s = d_0 + \sum_{i=1}^{2}(d_i \cdot \text{rpm}^i + d_{i+2} \cdot P_{\text{ex}}^i + d_{i+4} \cdot P_{\text{inj}}^i + d_{i+6} \cdot r_{\text{p,tot}}^i)
\] (8)

\[
X_{\text{loss}} = e_0
\] (9)

In fact, the correlation between the losses and the electrical power is pretty strong as it can be seen on Figure 3.

![Figure 3. Compressor ambient losses versus electrical power for CP1 and CP2](image1)

![Figure 4. Predicted vs measured electrical power for CP1 and CP2](image2)

The discharge temperature is predicted with a RMSE of 2K for CP1 and 5K for CP2.

4. Heat exchangers

Dynamic heat exchanger models are based on the finite volume method where the energy and mass conservation equations are applied to several cells connected in series, more information can be found in [7]. The fluid cells are connected to a metal wall with no longitudinal conduction.

The main parameters of the dynamic heat exchangers models are the number of cells on each side, geometrical features such as the heat transfer area and the heat transfer coefficients (HTC) between both fluids and the metal wall. Different options are implemented to take into account the variation of the HTC (see [7]), in this case, a constant HTC in each zone (subcooled - two phase - superheated) was chosen.

Moving boundary models were developed in the ”EES” software using the \(\varepsilon\)-NTU method in each zone for the steady-state validation of the models.
4.1. Condenser
Reviews and comparisons of heat transfer and pressure drop correlations for plate heat exchangers are widely available in the literature, e.g. Garcia-Cascales et al. [16] and Ayub et al. [15]. In this study, Wanniarachchi [9] and Martin [12] correlations were used for single phase flows, Hsieh [10] correlations for evaporating and Yan [11] correlations for condensing flow were chosen.

The data set used to validate the models comes from a test rig similar to the cycle presented on Figure 1. In the condenser model, the secondary fluid inlet state is fixed, the working fluid inlet temperature and outlet subcooling are given as well. As the heat exchanger geometry is known, the main outputs of the models are the condensing temperature and the heat transfer rate once both mass flow rates are fixed. Results for the heat transfer rate are displayed on Figure 5. As the calculation of HTC correlations requires to compute transport properties of the fluid, which is particularly time consuming regardless of the properties calculation method, a constant averaged value per zone was chosen for the dynamic modeling of the heat exchangers. Moreover, a comparison using the HTC correlations and a constant averaged value of the HTC in each zone is performed. It can be concluded that using constant values in each zone does not affect greatly the results. This can be explained by the small pinch point that is present in this kind of heat exchanger.

The condensing temperature was determined with a RMSE of 0.5 K and the coefficient of determination for the heat transfer rate is equal to 98.9%.

![Figure 5. Predicted vs measured heat transfer rate at the condenser](image1)

![Figure 6. Predicted vs measured heat transfer rate at the economizer](image2)

4.2. Economizer
Similarly to the condenser analysis, the economizer results are given on figure 6. In this model, the considered inputs are both mass flow rates, on the high pressure side, the inlet state is fully determined and on the intermediate pressure side, the inlet temperature is considered as an input as well. No pressure drop is considered on the intermediate pressure side and the pressure on this line is therefore considered as the saturated liquid pressure at the inlet temperature. Hence, as the heat exchanger geometry is known, the main outputs of the models are the heat transfer rate, the high pressure side outlet temperature, the inlet quality and the injection superheat.

Once again, no noticeable difference is showed between the use of correlations or averaged values. However, the quality of the results is lower as the validation was performed on only seven experimental data points and the measurement of the injection mass flow rate is difficult.
4.3. Evaporator

The correlations used for the steady state validation of the evaporator model are developed by Wang ([13] and [14]). In order to determine the heat transfer resistance on the air side, an overall surface efficiency of 84% was determined (Nellis and Klein, [17]). Results are given on Figures 7 and 8.

![Figure 7. Evaporating temperature predictions vs measurements.](image1)

![Figure 8. Evaporator heat transfer rate predictions vs measurements.](image2)

No significant discrepancy is found between results using the averaged heat transfer coefficients or the correlations.

5. Electronic Expansion valves

Several electronic expansion valve (EEV) models are available in the literature. Jinghui Liu et al. [19] studied the thermodynamic behavior of choked flow in an EEV and gave the valve opening as a function of the pulse number. In another study, Bach et al. [18] developed an adimensional law (8 PI group) for the valve mass flow using the Buckingham PI-theorem. They proposed laws for two refrigerants (R410A and R404A) valid for two-phase inlet conditions. Park et al. [20] also presented empirical laws for an EEV working with either R22 or R410a. The correlation yielded satisfactory predictions within a relative deviation of 15.0%. They developed a model based on a dimensionless correction coefficient \( C_d \) (see Eq.10) which is a function of the working conditions through six pi-groups. In the frame of this work, a constant coefficient of correction was chosen as the EEV behavior is not of prime concern.

\[
\dot{m} = C_d \cdot A \cdot \sqrt{\rho_{su,EEV} \cdot (P_{su,EEV} - P_{su,EEV})} 
\]

6. Split Lines

The split lines are present between the inside and outside unit of the heat pump (c.f. Figure 1, component 6 and 9). They are modeled on the same principle than the heat exchangers. It is composed of several cells in series connected to a metal wall which can eventually exchange heat with the ambiance. In this case, the split lines have a maximum length of 25m and are under the surface of the ground, hence, the ambiance temperature was assumed to be constant at 10°C. It is understandable that the liquid line is particularly important in the dynamic behavior of the cycle as a great part of the refrigerant charge is present in this pipe and because of its length.
An upwind scheme is chosen for the propagation of the fluid properties from cell to cell. The issue of numerical diffusion of the heat in the pipe has to be managed by choosing an appropriate number of cells in the split line.

7. Drive cooler
A drive cooler (DC) is present in the studied heat pump (c.f. Figure 1, component 5). This heat exchanger is used to cool down the drive by rejecting the heat on the high pressure side after the economizer which lowers the subcooling at the heating EEV inlet. In the Modelica implementation, this heat exchanger has been modeled assuming that all the losses from de drive, i.e.

\[ \dot{Q}_{DC} = \dot{W}_{el} - \dot{W}_{in} = \dot{W}_{el} \cdot (1 - \eta_{drive}) \]  

are injected at this point neglecting any dynamics from the heat exchanger.

8. Liquid receiver and refrigerant charge
The liquid receiver is considered as one control volume and modeled using the energy and mass conservation principles assuming thermodynamic equilibrium at all times inside the control volume.

The overall refrigerant charge inside the cycle is calculated by adding the refrigerant mass from the different components. It has to be noted that no refrigerant mass is attributed to the compressor, drive cooler or EEVs as no control volume was considered for these components. The refrigerant charge is imposed by the initial conditions in the heat exchangers, split lines and tank. Hence, by varying the initial level in the liquid receiver, it is possible to adjust the total refrigerant charge in the cycle without changing the working conditions.

9. Control unit
The control unit is only composed of two PID controllers managing the injection and suction superheat by changing the orifice opening. Initialisation elements are added to disconnect the PIDs from the cycle during the first instants of the simulation which improves the robustness of the model without creating any time event.

10. Results
In this section, the cycle response to a compressor speed decrease from 7000 rpm to 3000 rpm is investigated. At the condenser, the water mass flow and inlet temperature are fixed to 0.6 kg/s and 50°C. At the evaporator, a dry air flow of 1.4 kg/s enters the coil at −4.2°C. Results are shown on Figures 9, 10, 11 and 12.

At time \( t = 50 \)s the compressor speed ramps down from 7000 to 3000 rpm in 30s. One can notice that the response of both EEV is not fast enough as the superheat drops significantly. As the EEVs openings decrease, both superheats increase until they stabilize at the set point values.

Figure 9 provides an interesting information on the refrigerant charge repartition, in fact, most of the fluid is present in the 15 meters long liquid split line, in the liquid receiver and in the condenser. Nevertheless, all the results presented in this section should be interpreted and used carefully as the dynamic model still has to be validated.
11. Conclusion and future work

In this study, a steady-state dimensionless model was developped for a variable speed injection scroll compressor. The only geometrical parameter of this compressor model is the swept volume. Five dimensionless polynomials ($\varepsilon_v$, $\eta_s$, $\eta_{drive}$, $X_{inj}$ and $X_{loss}$) are needed to compute both mass flow rates (injection and suction) and the electrical consumption of the compressor.

Constant heat transfer coefficients were chosen for the heat exchanger to improve the model robustness and computational efficiency without damaging significantly the accuracy of the models. In fact, no representative difference was noticed between results when using either averaged heat transfer coefficients in each zone or correlations from the literature.

Finally an example is given with the dynamic response of the cycle to a compressor speed decrease.
In the future, the frosting and defrosting processes occurring on the air side of the evaporator will be taken into account. The dynamic validation of the present cycle model is also needed and will surely lead to adaptation of the proposed models. The reverse of the cycle, start-up and shut down simulations will also be considered.

References

[1] Rasmussen, Bryan P. "Dynamic modeling for vapor compression systems - Part I: Literature review." HVAC&R Research 18, no. 5 (2012): 934-955.
[2] Rasmussen, Bryan P., and Bhaskar Shenoy. "Dynamic modeling for vapor compression systems Part II: Simulation tutorial." HVAC&R Research 18.5 (2012): 956-973.
[3] Qiao, Hongtao, Xing Xu, Vikrant Aute, and Reinhard Radermacher. "Transient modeling of a flash tank vapor injection heat pump system Part II: Simulation results and experimental validation." International Journal of Refrigeration 49 (2015): 183-194.
[4] Qiao, Hongtao, Xing Xu, and Reinhard Radermacher. "Transient modeling of a flash tank vapor injection heat pump system Part II: Simulation results and experimental validation." International Journal of Refrigeration 49 (2015): 183-194.
[5] Ling, Jiazhen; Qiao, Hongtao; Alabdulkarem, Abdullah; Aute, Vikrant; and Radermacher, Reinhard, "Modelica-based Heat Pump Model for Transient and Steady-State Simulation Using Low-GWP Refrigerants" (2014). International Refrigeration and Air Conditioning Conference. Paper 1421. http://docs.lib.purdue.edu/iracc/1421
[6] Li, Bin, Steffen Peuker, Predrag S. Hrnjak, and Andrew G. Alleyne. "Refrigerant mass migration modeling and simulation for air conditioning systems." Applied Thermal Engineering 31, no. 10 (2011): 1770-1779.
[7] Quoilin, Sylvain; Desideri, Adriano; Wronski, Jorrit; Bell, Ian; and Lemort, Vincent, "ThermoCycle: A Modelica library for the simulation of thermodynamic systems" (2014). Proceedings - 10th International Modelica Conference; doi:10.3384/ECPl4096683
[8] Bell, Ian H., Jorrit Wronski, Sylvain Quoilin, and Vincent Lemort. "Pure and pseudo-pure fluid thermophysical property evaluation and the open-source thermophysical property library Coolprop." Industrial & engineering chemistry research 53, no. 6 (2014): 2498-2508.
[9] Wanniarachchi, A. S., U. Ratnam, B. E. Tilton, and K. Dutta-Roy. Approach correlations for chevron-type plate heat exchangers. No. CONF-950828-. American Society of Mechanical Engineers, New York, NY (United States), 1995.
[10] Hsieh Y, Y., Lin T. F. Evaporation Heat Transfer and Pressure Drop of Refrigerant R-410A Flow in a Vertical Plate Heat Exchanger J. Heat Transfer 125(5), 852-857 (2003) (6 pages); doi:10.1115/1.1518498
[11] Yan, Yi-Yie, Hsiang-Chao Lio, and Tsing-Fa Lin. "Condensation heat transfer and pressure drop of refrigerant R-134a in a plate heat exchanger." International Journal of Heat and Mass Transfer 42, no. 6 (1999): 993-1006.
[12] Martin, Holger. "A theoretical approach to predict the performance of chevron-type plate heat exchangers." Chemical Engineering and Processing: Process Intensification 35, no. 4 (1996): 301-310.
[13] Wang, Chi-Chuan, and Chi, Kuan-Yu. "Heat transfer and friction characteristics of plain fin-and-tube heat exchangers, Part I : new experimental data". International Journal of Heat and Mass Transfer, 1999.
[14] Wang, Chi-Chuan, and Chi, Kuan-Yu. "Heat transfer and friction characteristics of plain fin-and-tube heat exchangers, Part I : new experimental data". International Journal of Heat and Mass Transfer, 1999.
[15] Ayub, Zahid H. (2003), "Plate Heat Exchanger Literature Survey and New Heat Transfer and Pressure Drop Correlations for Refrigerator Evaporators", Heat Transfer Engineering,24:5,3 16; doi: 10.1080/014576303040456
[16] Garcia-Cascales, J. R., F. Vera-Garcia, J. M. Corberan-Salvador, and J. Gonzalvez-Macia. "Assessment of boiling and condensation heat transfer correlations in the modelling of plate heat exchangers." International Journal of Refrigeration 30, no. 6 (2007): 1029-1041.
[17] Nellis, G. and Klein, S. Heat transfer. Madison: Cambridge University Press, 2009. Print.
[18] Bach, Christian K.; Braun, James E.; and Groll, Eckhard A., "A Virtual EXV Mass Flow Sensor for Applications With Two-Phase Flow Inlet Conditions" (2012). International Refrigeration and Air Conditioning Conference. Paper 2122.
[19] Liu, Jinghui, Jiangping Chen, and Zhijiu Chen. "Investigation on the choking flow characteristics in electronic expansion valves." International Journal of Thermal Sciences 47, no. 5 (2008): 648-658.
[20] Park, Chasik, Honghyun Cho, Yongtaek Lee, and Yongchan Kim. "Mass flow characteristics and empirical modeling of R22 and R410A flowing through electronic expansion valves." International Journal of Refrigeration 30, no. 8 (2007): 1401-1407.