On 1/2-BPS membrane instantons in $\text{AdS}_4 \times S^7/\mathbb{Z}_k$

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Abstract

According to the covariant open superstring description of D-branes in the $\text{AdS}_4 \times \mathbb{C}P^3$ background, 1/2-BPS D2-branes are purely instantonic. Based on this and by taking the eleven dimensional viewpoint, we identify the 1/2-BPS instantonic M2-brane configurations in the $\text{AdS}_4 \times S^7/\mathbb{Z}_k$ background, which reduces to the $\text{AdS}_4 \times \mathbb{C}P^3$ under the large $k$ limit, and evaluate their action values. We also consider the previously known 1/2-BPS instantonic objects in ten dimensions from the M2-brane viewpoint to compare with our results.

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The covariant open superstring description of D-branes [1,2] is a useful tool in classifying supersymmetric D-branes, especially 1/2-BPS D-branes, in a given supersymmetric background. It has been successfully applied to some important backgrounds in superstring theories such as the flat spacetime [1], IIB plane wave [2], IIA plane wave [3], AdS$_5 \times S^5$ [4–7], and AdS$_4 \times \mathbb{CP}^3$ [8] backgrounds. The data obtained after the classification of supersymmetric D-branes are however ‘primitive’ in a sense that they do not tell us about which configuration of a given D-brane is really supersymmetric or which part of the background supersymmetry is preserved on the D-brane worldvolume. Nevertheless, the classification provides us a good guideline for further exploration of supersymmetric D-branes. Indeed this has been illustrated for the AdS$_5 \times S^5$ background in [9].

In our previous work [8], we have obtained the data about 1/2-BPS D-branes in the AdS$_4 \times \mathbb{CP}^3$ background. One interesting point from the data is that 1/2-BPS D2-brane is purely instantonic and there is no 1/2-BPS Lorentzian D2-brane in contrast to other D-branes of different dimensionalities.

As is well known, the Type IIA superstring theory in the AdS$_4 \times \mathbb{CP}^3$ background is dual to the Aharony-Bergman-Jafferis-Maldacena (ABJM) theory [10]. Under this correspondence, the study on the nonperturbative aspects of the ABJM theory has been a fascinating research subject, in which some instantonic objects have played important roles.\footnote{For comprehensive review, see [11].} Basically, there are two types of instantonic objects called worldsheet and membrane instantons. Both of them are known to be 1/2-BPS and, in the bulk side, are identified as string [12] and D2-brane [13] instantons wrapping certain subspaces of $\mathbb{CP}^3$. By the way, the result of [8]
indicates that, in addition to these instantons, there is a possibility to have other 1/2-BPS instantonic D2-brane configurations.

In the present work, we try to identify such 1/2-BPS D2-brane configurations purely based on the data obtained in [8] and evaluate their action values. These 1/2-BPS D2 brane configurations can be lifted to membrane (M2-brane) configurations. Then one can extrapolate 1/2 BPS M2-brane configurations in the \( \text{AdS}_4 \times S^7/Z_k \) background for all finite \( k \), which reduces to the \( \text{AdS}_4 \times \text{CP}^3 \) in the large \( k \) limit. We also consider the ten dimensional instantons of [12] and [13] and comment on the corresponding the M2-brane configurations.

In the next section, we give a brief description of the \( \text{AdS}_4 \times S^7/Z_k \) background. The possible 1/2-BPS instantonic M2-brane configurations are identified in Sec. 3, and their action values are evaluated in Sec. 4. Finally, the conclusion follows in Sec. 5.

## 2 \( \text{AdS}_4 \times S^7/Z_k \) background

The \( \text{AdS}_4 \times S^7/Z_k \) background originated from the near horizon limit of M2-brane supergravity solution is composed of the \( \text{AdS}_4 \times S^7/Z_k \) geometry

\[
ds^2 = \frac{R^2}{4} ds^2_{\text{AdS}_4} + R^2 ds^2_{S^7/Z_k}
\]

and the four-form field strength

\[
F_4 = \frac{3}{8} R^3 \epsilon_{\text{AdS}_4}
\]

where \( R \) is the radius of \( S^7 \) given by

\[
R = \ell_p (2^5 \pi^2 Nk)^{1/6}
\]

with the eleven dimensional Planck length \( \ell_p \) and the number of M2-branes \( Nk \), and \( \epsilon_{\text{AdS}_4} \) is the volume form of \( \text{AdS}_4 \) space. The metric of \( \text{AdS}_4 \) is, in the global coordinates,

\[
ds^2_{\text{AdS}_4} = -\cosh^2 \rho dt^2 + d\rho^2 + \sinh^2 \rho (d\theta^2 + \sin^2 \theta d\psi^2).
\]

As for the geometry of \( S^7/Z_k \), it is natural to express it through the \( U(1) \) Hopf fibration over \( \text{CP}^3 \) for its obvious connection with the Type IIA background [14]:

\[
ds^2_{S^7/Z_k} = ds^2_{\text{CP}^3} + \frac{1}{k^2} (dy + A)^2,
\]

where \( y \) is the \( U(1) \) fiber coordinate with the period \( y \sim y + 2\pi \), \( A \) is a one-form given by

\[
A = \frac{k}{2} \left( \cos \alpha d\chi + \cos^2 \frac{\alpha}{2} \cos \theta_1 d\phi_1 + \sin^2 \frac{\alpha}{2} \cos \theta_2 d\phi_2 \right),
\]
and the \( \mathbb{CP}^3 \) geometry is as follows:

\[
d s_{\mathbb{CP}^3}^2 = \frac{1}{4} d\alpha^2 + \frac{1}{4} \cos^2 \frac{\alpha}{2} (d\theta_1^2 + \sin^2 \theta_1 d\phi_1^2) + \frac{1}{4} \sin^2 \frac{\alpha}{2} (d\theta_2^2 + \sin^2 \theta_2 d\phi_2^2) \\
+ \frac{1}{4} \sin^2 \frac{\alpha}{2} \cos^2 \frac{\alpha}{2} (2d\chi + \cos \theta_1 d\phi_1 - \cos \theta_2 d\phi_2)^2,
\]

with the ranges of angles, \( 0 \leq \alpha, \theta_1, \theta_2 \leq \pi \) and \( 0 \leq \chi, \phi_1, \phi_2 \leq 2\pi \).

From the metric of (5), we choose the elfbeine for \( S^7/\mathbb{Z}_k \) as

\[
e^4 = \frac{1}{2} d\alpha, \quad e^5 = \frac{1}{2} \sin \frac{\alpha}{2} \cos \frac{\alpha}{2} (2d\chi + \cos \theta_1 d\phi_1 - \cos \theta_2 d\phi_2), \\
e^6 = \frac{1}{2} \cos \frac{\alpha}{2} d\theta_1, \quad e^7 = \frac{1}{2} \cos \frac{\alpha}{2} \sin \theta_1 d\phi_1, \\
e^8 = \frac{1}{2} \sin \frac{\alpha}{2} d\theta_2, \quad e^9 = \frac{1}{2} \sin \frac{\alpha}{2} \sin \theta_2 d\phi_2, \\
e^\flat = -\frac{1}{k} (dy + A).
\]

The reason for this choice is to make the Kähler structure of \( \mathbb{CP}^3 \) in a canonical form. The Kähler structure itself can be read off from one property of one-form \( A \), \( dA = 2kJ \), where \( J \) is the Kähler two-form of \( \mathbb{CP}^3 \). If we compute \( dA \) using (6), we obtain

\[
dA = -2k \left( e^4 \wedge e^5 + e^6 \wedge e^7 + e^8 \wedge e^9 \right),
\]

which indeed shows that \( J \) has the block diagonal structure, that is, the canonical form and thus the above choice for the elfbeine is a proper one.

### 3 1/2-BPS M2-brane instantons

For the investigation of 1/2-BPS M2-brane configurations, we need the Killing spinor \( \epsilon \) of the \( \text{AdS}_4 \times S^7/\mathbb{Z}_k \) background. It has been obtained in \([13,15,16]\) by solving the Killing spinor equation and has the following form.

\[
\epsilon = \mathcal{M} \epsilon_0,
\]

where \( \epsilon_0 \) is a 32 component constant spinor\(^3\) and \( \mathcal{M} \) is given by

\[
\mathcal{M} = e^{\frac{2}{3} \gamma_{71}} e^{\frac{2}{3} \gamma_7} e^{\frac{2}{3} (\gamma_{76} - \gamma_{75})} e^{\frac{2}{3} (\gamma_{70} + \gamma_{78})} e^{-\frac{2}{3} \gamma_{72}} e^{-\frac{2}{3} \gamma_{76}} e^{-\frac{2}{3} \gamma_{78}} e^{-\frac{2}{3} \gamma_{79}} \times e^{\frac{2}{3} \gamma_{71}} e^{\frac{2}{3} \gamma_{70}} e^{\frac{2}{3} \gamma_{72}} e^{\frac{2}{3} \gamma_{73}}
\]

\(^2\epsilon_{0,1,2,3}\) are for the \( \text{AdS}_4 \) space. Although their explicit form is not necessary in this work, if we follow the choice of \([13]\), they are \( \epsilon^0 = \frac{1}{2} \cosh \rho dt, \epsilon^1 = \frac{1}{2} d\rho, \epsilon^2 = \frac{1}{2} \sinh \rho d\theta, \) and \( \epsilon^3 = \frac{1}{2} \sinh \rho \sin \theta d\psi \).

\(^3\)The constant spinor \( \epsilon_0 \) has 24 free components for \( k > 2 \). Otherwise, that is, for \( k = 1, 2 \), it has 32 free components. For more detailed discussion, see Refs. \([13,15,16]\).
with \( \dot{\gamma} = \gamma^{0123} \) and a set of coordinate combinations,

\[
\begin{align*}
\xi_1 &= \frac{1}{2}(\phi_1 + \chi + 2y), \\
\xi_2 &= \frac{1}{2}(-\phi_1 + \chi + 2y), \\
\xi_3 &= \frac{1}{2}(\phi_2 - \chi + 2y), \\
\xi_4 &= \frac{1}{2}(-\phi_2 - \chi + 2y).
\end{align*}
\] (12)

The Killing spinor \( \epsilon \) is obtained in the Lorentzian signature. Since we are concerned about the instantonic configuration for which the spacetime is taken to be Euclidean, we should recast \( \epsilon \) in a way to respect the Euclidean nature. However, we will not care much about it because the time coordinate will be set to zero and our interest is the consistent projection operators acting on \( \epsilon \) (strictly speaking \( \epsilon_0 \)) which identify the 1/2-BPS configurations.

Having the Killing spinor, the 1/2-BPS instantonic M2-brane configurations can be considered by using the usual equation

\[
\Gamma \epsilon = \epsilon ,
\] (13)

which is obtained by combining the spacetime supersymmetry and \( \kappa \)-symmetry transformation. The symbol \( \Gamma \) is the spinorial matrix appearing in the \( \kappa \)-symmetry projector and satisfies \( \Gamma^2 = 1 \) and \( \text{Tr} \Gamma = 0 \). The explicit expression of \( \Gamma \) for M2-brane is

\[
\Gamma = \frac{i}{3! \sqrt{g}} \epsilon^{ijk} \Pi^a_i \Pi^b_j \Pi^c_k \Gamma_{abc}
\] (14)

where \( \Pi^a_i = \partial_i X^\mu e^a_\mu \) and \( g \) is the determinant of the induced metric on M2-brane, \( g_{ij} = \Pi^a_i \Pi^b_j \eta_{ab} \). The indices \( i, j, k \) are those of the M2-brane worldvolume, and \( \mu, \nu, \ldots \) \((a, b, \ldots)\) are the curved (tangential) spacetime indices. Now, by using the Killing spinor (10), the Eq. (13) can be rewritten as

\[
\tilde{\Gamma} \epsilon_0 = \epsilon_0 ,
\] (15)

where we have defined

\[
\tilde{\Gamma} \equiv \mathcal{M}^{-1} \Gamma \mathcal{M} .
\] (16)

Here \( \tilde{\Gamma}^2 = 1 \) is guaranteed because \( \Gamma^2 = 1 \). For a given M2-brane configuration, \( \Gamma \) and \( \mathcal{M} \) have the corresponding expressions by which \( \tilde{\Gamma} \) is determined. If the resulting \( \tilde{\Gamma} \) does not depend on any worldvolume coordinate, then the M2-brane configuration is confirmed to be 1/2-BPS.

For specifying a M2-brane configuration, let us introduce a notation

\[
[X, Y, Z] ,
\] (17)
which means that the M2 worldvolume coordinates \((\zeta^1, \zeta^2, \zeta^3)\) are identified as \(\zeta^1 = X, \zeta^2 = Y, \) and \(\zeta^3 = Z\). In other words, the notation represents a static gauge for the worldvolume reparametrization. Except for the coordinates along which M2-brane spans, all other coordinates in \(\mathcal{M}\) which are transverse to M2-brane are set to zero. These are the natural generalizations of the previous results of 1/2-BPS D2-brane configurations, which can be easily verified in M theory setting. However, if a polar coordinate among \(\alpha, \theta_1, \theta_2\) is taken to be transverse one, it is kept to be an arbitrary constant.

Before investigating the instantonic M2-brane configurations based on the covariant open string description of 1/2-BPS D-branes, we briefly reconsider the previously explored string worldsheet instanton [12] and D2-brane instanton [13] from the viewpoint of eleven dimensional M2-brane. Both of them have been studied in the context of ten dimensional IIA string theory and turned out to be 1/2-BPS. As for the string worldsheet instanton, it wraps \(\mathbb{CP}^1 \subset \mathbb{CP}^3\) parametrized by \(\alpha\) and \(\chi\). Since a string is nothing but the M2-brane wrapping the M-theory circle direction \(y\), the corresponding M2-brane configuration is given by \([\alpha, \chi, y]\). The expression of \(\Gamma(14)\) becomes simply \(\Gamma = -i\hat{\gamma}_{45}\) and \(\tilde{\Gamma}\) of Eq. (16) is evaluated as

\[
\tilde{\Gamma} = -i\hat{\gamma}_{45}. \tag{18}
\]

By the way, since there are two more two spheres within \(\mathbb{CP}^3\) parametrized by \((\theta_1, \phi_1)\) and \((\theta_2, \phi_2)\), one may be curious about whether the configurations \([\theta_1, \phi_1, y]\) and \([\theta_2, \phi_2, y]\) are also 1/2-BPS. Actually, they are but in a restricted sense. As for \([\theta_1, \phi_1, y]\), it is not difficult to check that the configuration is 1/2-BPS only when its transverse position in \(\alpha\) is zero, \(\alpha = 0\).\(^4\) In this case, \(\Gamma = -i\gamma_{67}\) and \(\tilde{\Gamma}\) simply becomes \(-i\gamma_{67}\). As for another configuration \([\theta_2, \phi_2, y]\), it is 1/2-BPS only when \(\alpha = \pi\). For this, we get \(\Gamma = -i\gamma_{89}\) and the corresponding \(\tilde{\Gamma}\) as \(i\hat{\gamma}_{45}{89} = -i\gamma_{67}\).\(^5\)

On the other hand, the D2-brane instanton spans the Lagrangian submanifold \(\mathbb{RP}^3\) within \(\mathbb{CP}^3\) which is parametrized by the coordinates \((\chi, \vartheta, \varphi)\) with the identifications \(\vartheta = \theta_1 = \theta_2\) and \(\varphi = \phi_1 = -\phi_2\). Since a D2-brane in ten dimensions is just an M2-brane in eleven dimensions, the M2-brane configuration corresponding to the D2-brane instanton is simply \([\chi, \vartheta, \varphi]\). Then the \(\Gamma\) of (14) for this configuration becomes \(\Gamma = -i\gamma_{67}\)\(e^{\frac{i}{2}(2\gamma_{58}+\gamma_{79})}\) and the evaluation of \(\tilde{\Gamma}\) of (16) leads to

\[
\tilde{\Gamma} = -i\gamma_{67}\,e^{\frac{i}{2}(\hat{\gamma}_{45}+\gamma_{58}+\gamma_{79})}. \tag{19}
\]

\(^4\)One can check that the resulting two sphere has maximal radius \(R/2\) when \(\alpha = 0\).

\(^5\)An identity \(\hat{\gamma}_{45}{67}{89} = \gamma_{3}\) has been used.
Obviously, the above two $\hat{\Gamma}$'s, (18) and (19), are independent of any of the worldvolume coordinates. Thus, from Eq. (15), it is confirmed that the previously known M2-brane configurations are 1/2-BPS as they should be.

Now let us turn to our main concern and investigate the 1/2-BPS M2-brane instantonic configurations based on the covariant open string description of D-branes. According to the open string description [8], purely instantonic D2-brane can be 1/2-BPS under the following condition: only one worldvolume direction is allowed to extend in each of three two dimensional subspaces of $\mathbb{C}P^3$. The three subspaces are realized by looking at the Kähler structure of $\mathbb{C}P^3$ (9). Following this condition, we see that there are eight candidates for the 1/2-BPS M2-brane instantonic configurations which are $[\alpha, \theta_1, \theta_2], [\alpha, \theta_1, \phi_2], [\alpha, \phi_1, \theta_2], [\alpha, \phi_1, \phi_2], [\chi, \theta_1, \theta_2], [\chi, \theta_1, \phi_2], [\chi, \phi_1, \theta_2]$, and $[\chi, \phi_1, \phi_2]$.

We have investigated all the candidates and found that only four configurations in which $\alpha$ is a worldvolume direction are 1/2-BPS. In what follows, we describe the 1/2-BPS configur-ations in sequence. First of all, for the configuration $[\alpha, \theta_1, \theta_2]$, the $\Gamma$ of (14) becomes
\[ \Gamma = i\gamma_{468}, \]
and the corresponding $\tilde{\Gamma}$ of (16) is evaluated as
\[ \tilde{\Gamma} = i\gamma_{468}. \] (20)

Secondly, the configuration $[\alpha, \theta_1, \phi_2]$ with fixed $\theta_2$ leads to $\Gamma = i\gamma_{456}e^{-\frac{1}{4}\gamma_{79}}e^{-\theta_2\gamma_{59}}e^{-\frac{1}{4}\gamma_{75}}$, and we obtain the corresponding $\tilde{\Gamma}$ as
\[ \tilde{\Gamma} = i\gamma_{456}e^{\frac{1}{4}(\gamma_{48} - \gamma_{59})}. \] (21)

The third configuration $[\alpha, \phi_1, \theta_2]$ is equivalent to $[\alpha, \theta_1, \phi_2]$. So we just move on to the last configuration $[\alpha, \phi_1, \phi_2]$ with fixed $\theta_1$ and $\theta_2$. For this configuration, we have $\Gamma = -i\gamma_{456}e^{-\frac{1}{4}\gamma_{59}}e^{\theta_1\gamma_7}e^{-\theta_2\gamma_{59}}e^{\frac{1}{4}\gamma_{45}}$, and the corresponding $\tilde{\Gamma}$ is obtained as
\[ \tilde{\Gamma} = -i\gamma_{456}e^{\frac{1}{4}(\gamma_{48} + \gamma_7)}e^{\frac{1}{4}(\gamma_{48} - \gamma_{59})}. \] (22)

We see that all the resulting $\tilde{\Gamma}$'s in (20), (21), and (22) do not depend on any of the worldvolume coordinates. Therefore Eq. (15) shows that half of the spacetime supersymmetry $(1 + \tilde{\Gamma})\epsilon_0$ is preserved on the M2-brane worldvolume.

At this point, one may ask why the other four configurations in which $\chi$ is a worldvolume direction are not 1/2-BPS. Although we have checked all of them, we just take one of them as an example and briefly illustrate the reason. Consider $[\chi, \theta_1, \theta_2]$. For this configuration, $\Gamma$ of (14) is simply $\Gamma = -i\gamma_{682}$ and $\mathcal{M}$ of (11) becomes $\mathcal{M} = e^{\frac{1}{4}(\gamma_{79} - \gamma_{75})}e^{\frac{1}{4}(\gamma_{75} + \gamma_{48})}e^{-\frac{1}{4}(\gamma_{72} + \gamma_{67} - \gamma_{45} - \gamma_{89})}$. The evaluation of $\tilde{\Gamma}$ of (16) is the process of pushing $\Gamma$ to the left of $\mathcal{M}^{-1}$ (or to the right...
of $\mathcal{M}$). But, unlike the previous cases, this process makes us face with a problem from the beginning. That is, what we get for the $\theta_1$ dependent part is

$$e^{-\theta_1^4(\gamma_6 - \gamma_7)\gamma_6\gamma_7} = \gamma_6\gamma_7,$$

which means that $\tilde{\Gamma}$ depends on the worldvolume coordinate $\theta_1$. There is no way to eliminate the $\theta_1$ dependence. This is also the case for other worldvolume coordinates, $\theta_2$ and $\chi$. Thus $\tilde{\Gamma}$ depends on the worldvolume coordinates implying that we have different projection operators at different worldvolume positions. This inconsistency explicitly shows that the M2-brane configuration $[\chi, \theta_1, \theta_2]$ is not 1/2-BPS. Of course, the configuration may be less supersymmetric by imposing other projection conditions. However, since here we are interested in 1/2-BPS objects, it will not be considered further.

### 4 Instanton action

For 1/2-BPS M2-brane instanton configurations considered in the last section, let us evaluate their classical action values. The basic purpose is to compare the resulting values with those for the string worldsheet instanton and D2-brane instanton obtained in [12] and [13] respectively.

The bosonic part of the Euclidean M2-brane action is given by

$$S = T_2 \int_{\mathcal{M}_3} d^3\sigma \sqrt{g} + iT_2 \int_{\mathcal{M}_4(M_3=\partial\mathcal{M}_4)} F_4,$$

where $T_2$ is the M2-brane tension,

$$T_2 = \frac{1}{4\pi^2 g_p^3},$$

and $g$ is the determinant of the induced metric on the M2-brane worldvolume. Since the M2-brane instantons we have considered are placed within $S^7/\mathbb{Z}_k$ while the background four-form field strength $F_4$ is turned on in the AdS$_4$ space as one can see from Eq. (2), the Wess-Zumino term including $F_4$ does not contribute to the action. Thus we only need to evaluate the Nambu-Goto type kinetic term.

We first consider the configuration $[\alpha, \phi_1, \phi_2]$ with fixed $\theta_1$ and $\theta_2$. For this, $\sqrt{g}$ is computed as $\frac{R^3}{8} \cos \frac{\alpha}{2} \sin \frac{\alpha}{2}$. We see that there is no dependence on the transverse coordinates $\theta_1$ and $\theta_2$, and hence the action value will always be the same regardless of the transverse position of the M2-brane configuration. Having the expression for $\sqrt{g}$, it is straightforward to evaluate the action, which is obtained as follows:

$$S = 2T_2 \int_0^\pi d\alpha \int_0^{2\pi} d\phi_1 \int_0^{2\pi} d\phi_2 \frac{R^3}{8} \cos \frac{\alpha}{2} \sin \frac{\alpha}{2}$$
\[ = \pi^2 T_2 R^3 \]
\[ = \pi k \sqrt{\frac{2N}{k}}, \]  
(26)

where Eqs. (3) and (25) have been used in the last step. We note that the \( \alpha \) integration has doubled in the first line as \( \int_0^\pi d\alpha \rightarrow 2 \int_0^\pi d\alpha \) basically because the nature of \( \alpha \) is a polar angle. The two angles \( \alpha \) and \( \chi \) parametrize a sphere as one notices from Eq. (7). One worldvolume direction of the present M2-brane wraps this sphere along \( \alpha \) with fixed \( \chi \). But, since \( \alpha \) is a polar angle, the worldvolume direction covers not a circle but half the circle geometrically. Thus, in order to get the correct action value, we should double the integration over \( \alpha \).

By following the same way, the action values for other configurations \([\alpha, \theta_1, \theta_2], [\alpha, \theta_1, \phi_2], \) and \([\alpha, \phi_1, \theta_2] \) can be evaluated. The resulting values are exactly the same with that of \([\alpha, \phi_1, \phi_2] \) obtained in Eq. (26). We note that the action value \( \pi k \sqrt{2N/k} \) agrees with that for the D2-brane instanton wrapping the Lagrangian submanifold \( \mathbb{RP}^3 \) within \( \mathbb{CP}^3 \) computed in [13]. Although the D2-brane result of [13] has been obtained in ten dimensional IIA theory, it is not difficult to check that it holds even in eleven dimensions without any modification by considering the M2-brane configuration corresponding to the D2-brane wrapping the \( \mathbb{RP}^3 \). Thus it seems plausible that all the 1/2-BPS M2-brane instantons which become D2-brane instantons in ten dimensions are characterized by the action value \( \pi k \sqrt{2N/k} \).

A noteworthy point of the above result is that the action value is valid even at finite \( k \) in contrast to the ten dimensional case where \( k \gg 1 \) is assumed for the validity of Type IIA \( \text{AdS}_4 \times \mathbb{CP}^3 \) background. Actually, this situation continues to hold even for the case of worldsheet instanton for which the action value has been obtained as [12]

\[ S = 2\pi \sqrt{\frac{2N}{k}}. \]  
(27)

If we evaluate the corresponding M2-brane configuration for the worldsheet instanton, that is, \([\alpha, \chi, y] \), then we get the same result \( 2\pi \sqrt{2N/k} \) yet valid at finite \( k \).\(^6\) Thus we see that the action values of the ten dimensional 1/2-BPS instantons are not modified at finite \( k \).

5 Conclusion

Based on the data [8] obtained from the covariant open superstring description of 1/2-BPS D-branes in Type IIA \( \text{AdS}_4 \times \mathbb{CP}^3 \) background, we have explored the possible 1/2-BPS M2-brane instanton configurations from the eleven dimensional viewpoint. It has been shown

\(^6\)We note that the configurations \([\theta_1, \phi_1, y] \) at \( \alpha = 0 \) and \([\theta_2, \phi_2, y] \) at \( \alpha = \pi \) considered in the last section also have the same action value.
that there exist four additional 1/2-BPS M2-brane instanton configurations in addition to the previously known ones. All of them, which are interpreted as D2-brane instantons in ten dimensions, have the same action value $\pi k \sqrt{2N/k}$ which is identical to that of D2-brane instanton studied in [13]. One important result is that the action value is valid even at finite $k$. Actually, this is also the case for the 1/2-BPS M2-brane instanton configuration corresponding to the worldsheet instanton [12]. We speculate that the validity of action values at finite $k$ is due to large amount of supersymmetry.

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