Influence of confined acoustic phonons on the Radioelectric field in a Quantum well

Do Tuan Long and Nguyen Quang Bau
Faculty of Physics, Hanoi University of Science, Vietnam National University
334 Nguyen Trai, Thanh Xuan, Hanoi, Vietnam
E-mail: dotuanlong@hus.edu.vn

Abstract. The influence of confined acoustic phonons on the Radioelectric field in a quantum well has been studied in the presence of a linearly polarized electromagnetic wave and a laser radiation. By using the quantum kinetic equation for electrons with confined electrons – confined acoustic phonons interaction, the analytical expression for the Radioelectric field is obtained. The formula of the Radioelectric field contains the quantum number \( m \) characterizing the phonons confinement and comes back to the case of unconfinned phonons when \( m \) reaches to zero. The dependence of the Radioelectric field on the frequency of the laser radiation, in case of confined acoustic phonons, is also achieved by numerical method for a specific quantum well AlGaAs/GaAs/AlGaAs. Results show that the Radioelectric field has a peak and reaches saturation as the frequency of the laser radiation increases.

1. Introduction
Recently, the study of the low-dimensional semiconductor systems (quantum wells, doping superlattices, compositional superlattices, quantum wires, quantum dots,...) has been increasingly interested. In these systems, the motion of electrons is restricted [7], and thus, leading to unusual kinetic properties of semiconductors such as: the absorption coefficient of electromagnetic waves [2], the Acoustoelectric effect [5], the Hall effect [6], the Radioelectric effect [8],... The Radioelectric effect, the effect of drag of charge carriers by electromagnetic waves, has been studied in [1, 4, 8]. However, these studies only considered the electrons - unconfined phonons interaction while recent works show the remarkable contribution of phonons confinement in these kinetic properties [2, 3]. So, in this work, we study the influence of confined acoustic phonons on the Radioelectric field in a quantum well.

In literature, there are some methods to solve this problem such as: the Boltzmann equation, the Kubo-Mori method, the quantum kinetic equation method. The last approach, because of its simplicity and accuracy, is chosen to use in this work.

The article is organized as follows: we briefly describe the Radioelectric effect, the Hamiltonian of electrons in quantum well and present basic formulae for the calculation. Numerical results and discussion are given in section 3. And the final section shows remarks and conclusions.
2. The Radioelectric field in a quantum well under the influence of confined acoustic phonons

We consider a quantum well with high infinite potential subjected to a linearly polarized electromagnetic field: \( \vec{E}(t) = \hat{E}(e^{-i\omega t} + e^{i\omega t}) \), \( \vec{H}(t) = \frac{\hbar}{2} [\vec{n}, \hat{E}(t)] \) (\( \hbar \omega \ll \varepsilon \), \( \varepsilon \) is an average carrier energy) and a laser radiation \( \vec{F}(t) = \vec{F} \sin(\Omega t) \) (\( \Omega \gg 1 \), \( \tau \) is the characteristic relaxation time). The Hamiltonian of the confined electrons - confined acoustic phonons system can be expressed as:

\[
\mathbf{H} = \sum_{n, \vec{p}_\perp} \varepsilon_n (\hat{p}_\perp - \frac{e}{\hbar c} \vec{A}(t)) a^+_n \vec{p}_\perp a_n \vec{p}_\perp + \sum_{m, \vec{q}_\perp} \hbar \omega_m \hat{q}_\perp b^+_m \vec{q}_\perp b_m \vec{q}_\perp + \sum_{n, n', \vec{p}_\perp, \vec{q}_\perp} C_m \hat{q}_\perp \int_0^\infty I_m^m(\varepsilon_n \vec{p}_\perp + \varepsilon_n \vec{q}_\perp) a^+_n \vec{p}_\perp a_{n'} \vec{q}_\perp b_m \vec{q}_\perp + b^+_m \vec{q}_\perp a_{n'} \vec{q}_\perp
\]

(1)

where \( a^+_n \vec{p}_\perp \) and \( a_n \vec{p}_\perp \) are the creation and annihilation operators of electrons (phonon) respectively; \( \varepsilon_n (\vec{p}_\perp) = \frac{\hbar^2}{2m^*} (\vec{p}_\perp^2 + \frac{n^2 \pi^2}{L^2}) \) is the electron energy; \( m^* \) is the effective mass of an electron; \( \hbar \omega_m \) is the confined acoustic phonon energy; \( \vec{A}(t) = \frac{\hbar}{\Omega} \vec{F} \cos(\Omega t) \) is the vector potential of the laser radiation.

\[
|C_m \hat{q}_\perp|^2 = \frac{\hbar^2 c^2}{2\pi^2} \sqrt{\frac{q_\perp^2 + \frac{m^2 \pi^2}{L^2}}{2}} \sin \left[ \frac{\pi}{L} \int_0^\infty \frac{\varepsilon_n \vec{p}_\perp + \varepsilon_n \vec{q}_\perp}{\varepsilon^2_n \vec{q}_\perp + \varepsilon_n \vec{p}_\perp + \hbar \omega_m \hat{q}_\perp - l\Omega} \right]
\]

(2)

Using the Heisenberg equation of motion, we establish the quantum kinetic equation for electrons distribution function \( f_n \vec{p}_\perp (t) \):

\[
\frac{\partial f_n \vec{p}_\perp (t)}{\partial t} + (e \vec{E}(t) + e \hat{E}_0 + \hbar \omega_m |\vec{p}_\perp, \vec{h}|, \frac{\partial f_n \vec{p}_\perp (t)}{\partial \vec{p}_\perp}) =
\]

\[
= \sum_{n', \vec{p}_\perp, \vec{q}_\perp} \frac{2\pi}{\hbar c} |C_m \hat{q}_\perp|^2 |I_m^m(\varepsilon_n \vec{p}_\perp + \varepsilon_n \vec{q}_\perp) f_{n'} \vec{p}_\perp + \vec{q}_\perp (t) - f_{n} \vec{p}_\perp (t)| \times
\]

\[
\int_0^\infty \left[ \delta (\varepsilon_n \vec{p}_\perp + \varepsilon_n \vec{q}_\perp - \varepsilon_{n'} \vec{p}_\perp - \hbar \omega_m \vec{q}_\perp - l\Omega) + \delta (\varepsilon_{n'} \vec{p}_\perp + \varepsilon_{n'} \vec{q}_\perp - \varepsilon_n \vec{p}_\perp + \hbar \omega_m \vec{q}_\perp - l\Omega) \right]
\]

(2)

here \( \vec{h}(t) = \frac{\hat{H}(t)}{\hbar} \), \( \omega_c = \frac{\hbar c}{m^*} \) is the cyclotron frequency.

We look for the electrons distribution function as:

\[
f_n \vec{p}_\perp (t) = f_0 (\varepsilon_n \vec{p}_\perp) - \vec{p}_\perp \chi_n \vec{p}_\perp (t) \frac{\partial f_0 (\varepsilon_n \vec{p}_\perp)}{\partial \varepsilon_n \vec{p}_\perp}
\]

(3)

where \( \chi_n \vec{p}_\perp (t) = \chi_0 (\varepsilon_n \vec{p}_\perp) + \chi'(\varepsilon_n \vec{p}_\perp) e^{-i\omega t} + \chi''(\varepsilon_n \vec{p}_\perp) e^{i\omega t} \) and \( f_0 (\varepsilon_n \vec{p}_\perp) \) is the equilibrium electron distribution function.

We substitute (3) into (2) and obtain:

\[
\chi_0 (\varepsilon_n \vec{p}_\perp) = \frac{\hbar c}{m^*} \tau (\varepsilon_n \vec{p}_\perp) \hat{E}_0
\]

\[
\chi'(\varepsilon_n \vec{p}_\perp) = \frac{\hbar c}{m^*} \frac{\tau (\varepsilon_n \vec{p}_\perp)}{1 - i\omega \tau (\varepsilon_n \vec{p}_\perp)} \hat{E}
\]

The total current density is presented in the form:

\[
\hat{J}_{tot} = \int_0^\infty \hat{R}(\varepsilon) d\varepsilon
\]

(4)
where

$$\vec{R}(\varepsilon) = \sum_{n, \vec{p}_\perp} \frac{e \hbar}{m} \vec{p}_\perp f_{n, \vec{p}_\perp}(t) \delta(\varepsilon - \varepsilon_{n, \vec{p}_\perp})$$ (5)

Multiplying both sides of the equation (2) by $\frac{e \hbar}{m} \vec{p}_\perp \delta(\varepsilon - \varepsilon_{n, \vec{p}_\perp})$ and sum over $n, \vec{p}_\perp$. Then, putting the root $\vec{R}(\varepsilon)$ of the resulting equation into (4) and assuming the sample in all direction is opened, regarding the electron gas is completely degenerate $f_0(\varepsilon_{n, \vec{p}_\perp}) = \theta(\varepsilon_F - \varepsilon_{n, \vec{p}_\perp})$. We obtain the expression of the Radioelectric field after some manipulation:

$$E_{0y} = \frac{2\omega_c \tau(\varepsilon_F)}{1 + \omega^2 \tau^2(\varepsilon_F)} \frac{a + c}{a + b} (E_x h_z - E_z h_x)$$ (6)

where: $\varepsilon_n = \frac{\hbar^2 \pi^2}{2m^* L^2} n^2$; $\varepsilon_{n'} = \frac{\hbar^2 \pi^2}{2m^* L^2} n'^2$

$$a = \sum_n \frac{\varepsilon^2}{\pi \hbar^2} (\varepsilon_F - \varepsilon_n)$$ (7)

$$b = \sum_{n, n', m} \frac{e^2 k T F^2}{4 \pi \hbar \Omega \rho c} |I_{nm'}|^2 \left\{ (\varepsilon_F - \varepsilon_n) (4\varepsilon_F - 3\varepsilon_{n'} - \varepsilon_n - \hbar \Omega) \tau(\varepsilon_F) - (\varepsilon_F - \varepsilon_{n'})(4\varepsilon_F - \varepsilon_n + \hbar \Omega) \tau(\varepsilon_F + \hbar \Omega) \right\}$$ (8)

$$c = \sum_{n, n', m} \frac{e^2 k T F^2}{4 \pi \hbar \Omega \rho c} |I_{nm'}|^2 \left\{ (\varepsilon_F - \varepsilon_n) (4\varepsilon_F - 3\varepsilon_{n'} - \varepsilon_n - \hbar \Omega) \tau(\varepsilon_F) \frac{1 - \omega^2 \tau^2(\varepsilon_F)}{1 + \omega^2 \tau^2(\varepsilon_F)} - (\varepsilon_F - \varepsilon_{n'})(4\varepsilon_F - \varepsilon_n + \hbar \Omega) \tau(\varepsilon_F) \frac{\tau^2(\varepsilon_F + \hbar \Omega)}{\tau(\varepsilon_F)} \frac{1 - \omega^2 \tau^2(\varepsilon_F + \hbar \Omega)}{1 + \omega^2 \tau^2(\varepsilon_F + \hbar \Omega)} \right\}$$ (9)

Formula (6) shows the dependence of the Radioelectric field on parameters of the system: the frequency $\Omega$ and amplitude $F$ of the laser radiation, the frequency $\omega$ of the linearly polarized electromagnetic wave, the temperature $T$ and and especially the quantum number $m$ characterizing the phonons confinement. When $m$ goes to zero, we obtain the results in the case of unconfined phonons.

3. Numerical results and discussions

In order to clarify the influence of confined acoustic phonons, we estimate numerical values of the Radioelectric field and graph in cases of confined phonons and unconfined phonons. The parameters used in this calculation are as follows: $m^* = 0.067 m_0$ ($m_0$ is the mass of a free electron), $\xi = 13.5$eV, $\rho = 5.32$gm$^{-1}$, $v_s = 5378$ms$^{-1}$, $\varepsilon_F = 30$meV, $\tau(\varepsilon_F) = 10^{-12}$s, $T = 35K$, $L = 30$nm.

Figure 1 shows the dependence of the Radioelectric field on the frequency of the laser radiation in cases of confined and unconfined phonons. It can be seen that nonlinear dependences of the Radioelectric field on the laser radiation frequency are similarly in both cases, the Radioelectric field has a peak and reaches saturation when the laser frequency increases. However, the maximum value of the Radioelectric field is much greater than that in the case of unconfined phonons. This behaviour of the Radioelectric field could be explain by the effect of confined acoustic phonons, the appear of the quantum number $m$ in the electron form factor $I_{nm}$, changes remarkably the intensity of the Radioelectric field. For more details, we numerically calculate the Radioelectric field for different values of the quantum number $m$ characterizing the phonons confinement. In figure 2, we can see that when $m$ increases, the Radioelectric field is also increasing.
Figure 1. The dependence of theRadioelectric field on the frequency of the laser radiation in
the case of confined phonons (solid line) and the case of unconfined phonons (dashed line). Here
\( E = 5 \times 10^4 \text{V/m} \) and \( \omega = 4.5 \times 10^{11} \text{s}^{-1} \).

Figure 2. The dependence of the Radioelectric field on the frequency of the laser radiation for
different values of the quantum number \( m \) characterizing the phonons confinement: \( m = 1 \)(solid
line), \( m = 2 \)(dashed line) and \( m = 5 \)(dotted line). Here \( E = 5 \times 10^4 \text{V/m} \) and \( \omega = 4.5 \times 10^{11} \text{s}^{-1} \).
4. Conclusions
In this paper, the influence of confined acoustic phonons on the Radioelectric field in a quantum well has theoretically studied. We have computed the analytical expression for the Radioelectric field, numerically calculated and graphed the theoretical result for the AlGaAs/GaAs/AlGaAs quantum well, and compared with the case of unconfined phonons. Results show that the formula of the Radioelectric field contains the quantum number $m$ characterizing the phonons confinement and comes back to the case of unconfined phonons when $m$ goes to zero. The Radioelectric field has a peak and reaches saturation when the laser frequency increases in both cases, but the Radioelectric field is much greater when phonons are confined. Hence, the phonons confinement is the causes of surprising changes to the Radioelectric field in the quantum well.

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