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The impact of the coronavirus crisis on the market price of risk

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**ABSTRACT**

We study an equilibrium risk and return model to explore the effects of the coronavirus crisis and associated skewness on the market price of risk. We derive the moment and equilibrium equations, specifying skewness price of risk as an additive component of the effect of variance on mean expected return. We estimate our model using the flexible skewed generalized error distribution, for which we derive the distribution of returns and the likelihood function. Using S&P 500 Index returns from January 1980 to mid-October 2020, our results show that the coronavirus crisis generated a deeply negative reaction in the skewness and total market price of risk, more negative even than the subprime and the October 1987 crises.

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1. Introduction

Severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) and the disease it causes (COVID-19) were first identified after a cluster of cases was observed in December 2019 in Wuhan, China. In response to reports of new cases surging daily in China and subsequently in Europe, particularly Italy, the market’s negative reaction to the spread of the coronavirus began in mid-February 2020, and the first large crash occurred on February 21. The U.S. equity markets hit a low on March 23, following the World Health Organization’s declaration of a pandemic on March 11, the exponential increase in daily new COVID-19 cases worldwide and government decisions to enact lockdowns (Chan and Marsh, 2020; Nikolopoulos et al., 2020).

In this paper, we include the coronavirus crisis in an intertemporal asset pricing model, to study its impact on the market price of risk. Our results have important implications for estimating the depth of the coronavirus financial crisis and its comparison with the October 1987 crisis and the 2008 subprime mortgage lending crisis. Our analysis highlights the role of the skewness price of risk in the depth of financial crises as a means to analyze investors’ reaction to bad news and to provide insights into the management of such crises.

The economic case for including skewness in models of risk and return is that the mean and variance are not the only factors driving the returns distribution (unconditional in the cross-section or conditional in the time series), because, ceteris paribus, risk-averse investors must prefer right-skewed portfolios to left-skewed ones. For example, adding assets that decrease a portfolio’s skewness (and thus making them more left-skewed) should result in higher expected returns. Skewness is consequently a key element in models of risk and return, particularly for short- and medium-term investors (Adcock, 2014). During periods of excess volatility, investors realize that their expected returns will not materialize and switch to less risky assets, thereby introducing increased negative skewness to the price of risk.

Because of the downturn’s exogenous nature, the coronavirus crisis presents a natural experiment to examine the role of skewness in models of risk and return. Chan and Marsh (2020) show
that the market's trajectory in the coronavirus crisis resembles that of the Great Depression and the Lehman Brothers collapse, but newer data for the coronavirus era show a quicker market rebound since the U.S. market low on March 23, 2020. This phenomenon might be the result of the different and exogenous nature of the coronavirus crisis, with markets perceiving that the economy will rebound more easily and, perhaps more importantly, because of the quickly enhanced liquidity by policy makers. We note, however, that the coronavirus market crash was much deeper than the 2008 crash, reflecting a very strong investor reaction as well as associated changes in risk and investment horizons.

On this basis, our paper focuses on two interrelated questions. First, we examine whether skewness is significantly more negative during crisis periods while maintaining the prediction of a positive risk–return nexus. The latter is important for consistency with key relevant theory in management science and financial economics (e.g., French et al., 1987; Adcock, 2014; Theodossiou and Savva, 2016). Second, we examine whether the sharpness and exogeneity of the coronavirus crisis implies that skewness was more negative during this period than in the subprime crisis.

We incorporate skewness into an intertemporal asset pricing model. For a positive risk-return relation, investors require a higher (or lower) risk premium during periods that are more (or less) volatile. Technically, this dynamic implies that a portfolio’s excess return is a positive function of the conditional variance of returns (e.g., Engle et al., 1987; Pedersen, 2000). In this respect, we build on the work of Theodossiou and Savva (2016), who explicitly model skewness in the price of risk and augment their model by including time-varying conditional skewness and kurtosis, as well as crisis periods.

Specifically, we examine the joint effects of price shocks reflected in the October 87, subprime and coronavirus crises on the distribution of returns using a skewed extension of the generalized autoregressive conditional heteroskedasticity in the mean (GARCH-M) model from Glosten et al. (1993). Two important underlying assumptions in the empirical risk–return trade-off literature are the constant market price of risk and the symmetric conditional distribution for returns (Theodossiou and Savva, 2016). First, our model specification relaxes the above assumptions and allows for time-varying skewness and kurtosis under a very flexible distribution (skewed generalized error distribution, or SGED), with upside and downside volatility effects incorporated into the 3rd and 4th moments. The SGED distribution has the appealing property of allowing us to develop exponential asset pricing equations, thereby significantly improving flexibility. Second, these effects are incorporated into the risk-return trade-off, which is also time varying and is decomposed into two components: the pure price of risk and the skewness/kurtosis price of risk. The skewness/kurtosis equation depends on the conditional asymmetry parameter, and these factors together determine how the pure and skewness prices of risk affect expected returns. Importantly, we derive the risk-neutral equation of returns, which shows the effects of crises, skewness and other parameters on equilibrium returns. Our model’s key prediction is that a negative crisis parameter value implies a negative unconditional asymmetric parameter and yields a negatively skewed distribution for returns.

We estimate our model using data from the S&P 500 Index for the period January 2, 1980 to October 14, 2020 (10,286 daily continuously compounded returns). We use maximum likelihood estimation with robust standard errors and obtain confidence intervals with the procedure of Rapach and Wohar (2009). Our main results show the importance of modeling crisis periods and associated skewness in asset pricing models. Aside from finding large volatility increases in the crisis periods, and consistently with our theoretical priors, we show that the conditional skewness price of risk is indeed negative in our full sample. Importantly, the conditional skewness price of risk drops (becomes more negative) by approximately 3.8 and 2.5 times during the October 87 and subprime crises compared with the “regular” period, whereas the equivalent decrease during the coronavirus crisis is approximately 4.4 times larger than that in the regular period. These decreases show the immense reaction of investors during crisis periods, as well as the rapid changes in sentiment regarding portfolio risk and the investment horizon. Evidently, the negative reactions were deeper during the coronavirus crisis than in the subprime crisis. Finally, we show that accounting for skewness in our model allows for the theoretically positive risk-return relation to be maintained.

The remainder of the paper proceeds as follows. The next section develops the moment equations for prices of financial assets, derives the risk-neutral equilibrium equation for their returns, and explores the basic properties of their distributions. Section 3 specifies the key equations for the conditional moments in our model, as well as the risk-neutral equilibrium equation for returns. Sections 4 discusses the estimation method and the empirical findings. Section 5 concludes the paper.

2. Distribution of equilibrium returns

2.1. Moments for returns and prices

An asset’s price at time \( t + 1 \) is

\[
P_{t+1} = P_t e^{r_{t+1}} = P_t e^{|\Phi_t^t| + \sigma_{t+1}^2 \Phi_t^t \cdot z_{t+1,1}},
\]

where \( P_t \) is the price at time \( t \), \( r_{t+1} \) and \( z_{t+1,1} \) are continuously compounded returns, and their standardized values associated with the period \( t \) to \( t + 1 \).

\[
\mu_{t+1} = E \left( \Phi_t^t \right)
\]

and

\[
\sigma_{t+1}^2 = \text{var} \left( \Phi_t^t \right).
\]

are the conditional mean and conditional variance of \( r_{t+1} \) according to the information set at time \( t \), denoted by \( \Phi_t \), which includes past relevant information used in forecasting the parameters of the distribution of returns and prices. Let

\[
dF^{zt}_{zt} = f_{zt} \left( z_{t+1,1} \right) dz_{t+1,1}
\]

denote the conditional probability mass function (pmf) of standardized returns. This and its associated probability density function (pdf) \( f_{zt} \) are assumed to be continuous and unimodal with a moment generating function. The existence of a moment generating function is a requirement for the formation of price expectations.

The moment function for asset prices is computed with

\[
E \left( P_{t+1}^s \left| \Phi_t \right. \right) = S^s_t e^{\mu_{t+1}^s} \int_{-\infty}^{\infty} e^{\sigma_{t+1}^s z_{t+1,1}} dF^{zt}_{zt} = P_t e^{\mu_{t+1}^s + \text{ln} E^{zt}_{zt} ,}
\]

where

\[
E^{zt}_{zt} = \int_{-\infty}^{\infty} e^{\sigma_{t+1}^s z_{t+1,1}} dF^{zt}_{zt},
\]

for \( s = 1, 2, \ldots \). The conditional mean and conditional variance of \( S^s_{t+1} \) are, respectively,

\[
E \left( P_{t+1}^s \left| \Phi_t \right. \right) = P_t e^{\mu_{t+1}^s + \text{ln} E^{zt}_{zt} ,}
\]

and

\[
\text{var} \left( P_{t+1}^s \left| \Phi_t \right. \right) = P_t^2 e^{2\mu_{t+1}^s} \left( e^{\text{ln} E^{zt}_{zt}} - e^{2 \text{ln} E^{zt}_{zt}} \right).
\]
2.2. Equilibrium returns

The sum of the price and accrued dividend at the end of period $t$ to $t + 1$ is

$$P_{t+1} + D_{t+1} = P_t e^{q_{t+1} + r_{t+1} t} + q_{t+1} r_{t+1},$$  

(7)

where

$$D_{t+1} = P_t (e^{r_{t+1} - 1})$$

is the accrued dividend and $q_{t+1}$ is a continuous dividend payment rate associated with the period.

In liquid and frictionless markets, the absence of arbitrage opportunities requires the expected value of the price and accrued dividends discounted at the asset’s required rate of return to be equal to its price at the beginning of the period. That is:

$$E \left( P_{t+1} + D_{t+1} | \Phi_t \right) e^{-r_{t+1} t - q_{t+1} t} = P_t e^{-r_{t+1} t - q_{t+1} t + q_{t+1} + \ln E_{t+1}} = P_t,$$

where $r_{t+1}$ is the risk-free interest rate and $\rho_{t+1}$ is the asset’s risk premium for the period $t$ to $t + 1$. The risk premium $\rho_{t+1}$ represents the minimum return in excess of the risk-free rate $r_{t+1}$ that induces investors to buy or hold the risky asset. The above equality yields the following equilibrium equation return rate

$$\tilde{r}_{t+1} = r_{t+1} + \rho_{t+1} = q_{t+1} + \ln E_{t+1} | \Phi_t + \mu_{t+1} t,$$

where $\mu_{t+1}$, $q_{t+1}$ and $\ln E_{t+1} | \Phi_t$ are as defined previously.

2.3. Distribution of standardized returns

We examine the impact of the distributional asymmetry of equilibrium returns with the following transformation:

$$z_{t+1} = -\tilde{r}_{t+1} - \left(1 + \frac{\ln(w_{t+1}) \lambda_{t+1} z_{t+1}}{\gamma_{t+1}} \right) \theta_{t+1} t + W_{t+1},$$  

(9)

where $\lambda_{t+1}$ and $\theta_{t+1}$ are suitably constructed scaling constants, $\lambda_{t+1}$ is an asymmetry parameter defined over the closed interval $[-1, 1]$, and $W_{t+1}$ is a symmetric zero-mode random variable with pdf $dF_{w_{t+1}} = dw_{t+1}$. The parameter $\lambda_{t+1}$ controls the shape of $dF_{w_{t+1}}$ to the left and right of its mode. That is, $dF_{w_{t+1}}$ will be left-skewed for $\lambda_{t+1} < 0$, right-skewed for $\lambda_{t+1} > 0$ and symmetric for $\lambda_{t+1} = 0$.

From the above, it follows that

$$\text{mode} \left( z_{t+1} | \Phi_t \right) = \tilde{r}_{t+1} \left(1 + \frac{\ln(w_{t+1}) \lambda_{t+1}}{\gamma_{t+1}} \right) \theta_{t+1} t,$$

and the conditional mean of returns

$$\mu_{t+1} | \Phi_t = \frac{\mu_{t+1} t + \mu_{t+1} t + \sigma_{t+1} \tilde{r}_{t+1} \theta_{t+1}}{\tilde{r}_{t+1} t + \sigma_{t+1} \tilde{r}_{t+1} t},$$

(10)

and the conditional mode of returns is

$$\mu_{t+1} | \Phi_t = \mu_{t+1} + \tilde{r}_{t+1} t + \sigma_{t+1} \tilde{r}_{t+1} t.$$

(11)

The measure

$$\delta_{t+1} = \left( \frac{\mu_{t+1} t - \mu_{t+1} t}{\sigma_{t+1} \tilde{r}_{t+1} t} \right)$$

is Pearson’s well-known mode of skewness. As shown in Theodossiou and Savva (2016), in the intertemporal asset pricing model of Engle et al. (1987), the parameter $\delta_{t+1}$ represents the component of the market price risk due to skewness (skewness market price of risk).

The distribution for standardized returns is expressed in terms of $w_{t+1}$ as follows:

$$dF_{w_{t+1}} = \left| \frac{dz_{t+1}}{dw_{t+1}} \right| f_{z_{t+1}}(z(w_{t+1})) dw_{t+1}$$

$$= \left(1 + \frac{\ln(w_{t+1}) \lambda_{t+1} z_{t+1}}{\gamma_{t+1}} \right) \theta_{t+1} t + f_{w_{t+1}} \left( \frac{z(w_{t+1})}{\kappa} \right) dw_{t+1}$$

$$= \left(1 + \frac{\ln(w_{t+1}) \lambda_{t+1} z_{t+1}}{\gamma_{t+1}} \right) \theta_{t+1} t + f_{w_{t+1}} \left( \frac{z(w_{t+1})}{\kappa} \right) dw_{t+1},$$

(12)

As shown in the Appendix, the equations for the mode $\delta_{t+1}$, the scaling parameter $\theta_{t+1}$, the variances of upside and downside returns, and Pearson’s moment coefficients of skewness and kurtosis are, respectively:

$$\delta_{t+1} = 2 \lambda_{t+1} t \theta_{t+1} t G_{t+1},$$

$$\theta_{t+1} = \frac{1}{\sqrt{\left(1 + 3 \lambda_{t+1}^2 \right) G_{end} - 4 \lambda_{t+1}^2 \left(1 + \gamma_{t+1}^2 \right) G_{wall} t}},$$

$$\text{var} \left( \delta_{t+1} \right) = \left( \lambda_{t+1}^2 + 2(1 - \lambda_{t+1}^2) \delta_{t+1} t \theta_{t+1} t G_{wall} + (1 + \lambda_{t+1})^2 \theta_{t+1} t G_{wall} \right) \sigma_{t+1}^2,$$

$$\text{var} \left( \delta_{t+1} \right) = \left( \delta_{t+1}^2 + 2(1 - \lambda_{t+1}^2) \delta_{t+1} t \theta_{t+1} t G_{wall} + (1 + \lambda_{t+1})^2 \theta_{t+1} t G_{wall} \right) \sigma_{t+1}^2,$$

$$\text{SK}_{t+1} = 4 \lambda_{t+1} t \left(1 + 2 \lambda_{t+1}^2 \right) \theta_{t+1} t + 3 \lambda_{t+1}^2 \theta_{t+1} t G_{wall} - 3 \delta_{t+1} t - \delta_{t+1}^2,$$

$$\text{KU}_{t+1} = \left(1 + 10 \lambda_{t+1}^2 + 5 \lambda_{t+1}^4 \right) G_{wall} \theta_{t+1} t - 4 \delta_{t+1} t, \text{SK}_{t+1} - 6 \delta_{t+1} t - \delta_{t+1}^2,$$

(13)

(14)

(15)

(16)

(17)

(18)

where $G_{wall}$ is defined as

$$G_{wall} = E \left( \left| W_{t+1} \right| | \Phi_t \right) = 2 \int_{0}^{\infty} w_{t+1} \left| F_{w_{t+1}} \left( k \right) \right| dw_{t+1}$$

$$= \left(1 + \frac{\ln(w_{t+1}) \lambda_{t+1} z_{t+1}}{\gamma_{t+1}} \right) \theta_{t+1} t + f_{w_{t+1}} \left( \frac{z(w_{t+1})}{\kappa} \right) dw_{t+1},$$

(12)

(13)

(14)

(15)

(16)

(17)

(18)

and

$$\text{KU}_{t+1} = \left(1 + 10 \lambda_{t+1}^2 + 5 \lambda_{t+1}^4 \right) G_{wall} \theta_{t+1} t - 4 \delta_{t+1} t, \text{SK}_{t+1} - 6 \delta_{t+1} t - \delta_{t+1}^2,$$

(19)

(20)
2.4. Competing frameworks used in the literature

The distribution for standardized returns, given by Eq. (12), can be obtained from a two-sided general distribution by setting its left and right tail parameters with

\[ \theta_{t,1} = \left( 1 - \lambda_{t,1} \right) \theta_{t,1} \text{ for } w_{t,1} < 0 \text{ or } z_{t,1} < -\delta_{t,1} \]

and

\[ \theta_{t,1} = \left( 1 + \lambda_{t,1} \right) \theta_{t,1} \text{ for } w_{t,1} > 0 \text{ or } z_{t,1} > -\delta_{t,1} \]

When shocks below the mode are scaled differently from shocks above the mode, the resulting distribution will be asymmetric. Specifically, the distribution will be left-skewed for \( \theta_{t,1} < \theta_{t,1} \) (i.e., \( \lambda_{t,1} < 0 \)), right-skewed for \( \theta_{t,1} > \theta_{t,1} \) (i.e., \( \lambda_{t,1} > 0 \)) and symmetric for \( \theta_{t,1} = \theta_{t,1} \) (i.e., \( \lambda_{t,1} = 0 \)); see Savva and Theodossiou (2018) for details. The distance between the tail parameters determines the extent of skewness.

Eq. (12) constitutes a generalization of the binormal distribution used in Feunou et al. (2012) to model downside and upside conditional heteroskedasticity and assess the risk-return trade-off in financial markets; see also Roon and Karehike (2007), Eqs. (11), (13), (15) and (16) are fully consistent with the consumption-based reduced form equilibrium relationship between the conditional risk and mean returns of an investor with disappointment aversion preferences as in Gul (1991) and Feunou et al. (2012). In Gul’s utility framework, investors value downside risk and upside uncertainty differently. Setting the conditional shape parameter \( \kappa_{t,1} = 2 \), with some algebraic manipulations, the relevant equations in Feunou et al. (2012) can be obtained as special cases.

The two-sided distribution is also in line with the bad environment-good environment framework used by Bekaert et al. (2015) and Bekaert and Engstrom (2017) in the formulation of a consumption-based asset pricing model with external habit formation. In this framework, upside and downside shocks are assumed to follow a centered gamma distribution with different tail parameters. In contrast, we derive the risk-return measures using the basic assumptions that the distribution of returns is continuous and unimodal, and has a moment generating function. In this general framework, any of the known distributions described earlier can be used in the estimation and testing of the models.

3. Specification of conditional moments

We examine the joint impact of price shocks on the distribution of financial returns by using an SGED extension of the GARCH-M model of Glosten et al. (1993), as also recently used by Bekaert et al. (2015) and Bekaert and Engstrom (2017). We extend the model’s structure to account for time-varying shape and asymmetry parameters and the impact of the October 1987, subprime and COVID-19 crises.

3.1. Conditional variance

The conditional variance of returns is specified as

\[ \sigma^2_{t,1} = \text{var} \left( r_{t,1} \mid \Phi_t \right) = \sum_{j=1}^{3} \epsilon_{p,t} D_{j,t} + \epsilon_0 + \left( \alpha + \alpha_N N_t \right) \epsilon^2_{t} + \beta \sigma^2_{t-1} \]

where \( D_{j,t} \) is an indicator variable that takes the value of 1 during the period of crisis \( j \) and 0 otherwise. \( j = 1, 2, 3 \); \( \epsilon_{t} = r_{t} - \mu_{t,1} \) is a regression innovation used as a proxy for market shocks, \( N_t = 1 \) for \( \epsilon_{t} \), and \( N_t = 0 \) otherwise. Specifically, \( j = 1, 2, 3 \) denote, respectively, the October 87 crash, the subprime mortgage crisis and the coronavirus crisis. The parameter \( \nu_{D,j} \) captures the impact of crisis \( j \) on the conditional variance of returns. The second and third terms account for asymmetric volatility and volatility clustering. Larger positive values for \( \alpha \) and \( \beta \) are indicative of higher volatility persistence. The parameter \( \eta_t \) measures the impact of past negative shocks on current market volatility. In fact, the persistence of volatility is

\[ \nu = \alpha + \alpha N \left( \bar{\epsilon}_t < 0 \right) < 1 \]

which \( P(\bar{\epsilon}_t < 0) \) is the probability of negative shocks. For left-skewed distributions, the latter probability is greater than 0.5; for symmetric distributions, it is equal to 0.5; and for right-skewed distributions, it is greater than 0.5. The stationarity of volatility requires that \( \nu < 1 \).

3.2. Conditional mode and mean

The conditional mode for equilibrium returns is

\[ m_{t,1} = \text{mode} \left( r_{t,1} \mid \Phi_t \right) = m_0 + \sum_{j=1}^{3} m_{D,j} D_{j,t} + b r_t + \left( c + \sum_{j=1}^{3} c_{D,j} D_{j,t} + \delta_{t,1} \right) \sigma_{t,1} \]

where \( D_{j,t} \) as defined previously, \( r_t \) is a past return, \( \sigma_{t,1} = \text{std}(r_{t,1} \mid \Phi_t) \) is used as a proxy for conditional risk, \( c \) is the GARCH-in-mean coefficient (Engle et al., 1987), and \( c_{D,j} \) is the deviation from \( c \) due to crisis \( j \). In a symmetric world, the conditional mode is identical to the conditional mean of equilibrium returns, \( m_{t,1} = m_{t,1} \). Consequently, the parameter \( c \) may be interpreted as the market price of risk in a symmetric world. For the crisis periods \( j = 1, 2 \) and 3, the market price of risk will be \( c + c_{D,j} \).

The relationship between conditional risk and mean returns (see Eq. (11)) is given by

\[ \mu_{t,1} = E \left( r_{t,1} \mid \Phi_t \right) = m_0 + \sum_{j=1}^{3} m_{D,j} D_{j,t} + b r_t + \left( c + \sum_{j=1}^{3} c_{D,j} D_{j,t} + \delta_{t,1} \right) \sigma_{t,1} \]

where \( \delta_{t,1} \) is Pearson’s mode of skewness. Eqs. (13) and (14) imply that

\[ \delta_{t,1} = \frac{2 \lambda_{t,1} G_{1,1}}{\sqrt{\left( 3 \lambda_{t,1}^2 + 1 \right) G_{2,1} + 4 \lambda_{t,1}^2 G_{3,1}^2}} \]

Of note, \( \delta_{t,1} \) has the same sign as the conditional asymmetry parameter \( \lambda_{t,1} \). It is a function of the absolute moments \( G_{1,1} \) and \( G_{2,1} \), which depend on the time-varying shape parameter \( \kappa_{t,1} \).

The substitution of \( \mu_{t,1} \) into Eq. (8) yields

\[ \hat{r}_{t,1} = m_0 + \sum_{j=1}^{3} m_{D,j} D_{j,t} + b r_t + q_{t,1} + \ln E_{t,1} \]

which is a risk-neutral equilibrium equation for returns. The equation shows how crises, volatility and other parameters affect...
equilibrium returns. Of note, \( \delta_{t+1|t} \) is a function of the asymmetry and shape parameters that indirectly accounts for variations during the three crises periods.

The conditional risk premium is given by

\[
\rho_{t+1|t} = \left( c + \sum_{j=1}^{3} c_{0j} D_{j,t} \right) \sigma_{t+1|t} + \delta_{t+1|t} \sigma_{t+1|t} \sigma_{t+1|t} \left( \text{skewness risk premia} \right) \tag{26}
\]

where the first component represents the risk premia during the regular and crisis periods (pure risk premia), and the second component represents the risk premia associated with skewness (skewness risk premia). For financial assets, the skewness premia have been found to be negative (e.g., Savva and Theodossiou, 2018). This finding is a result of stronger investor reactions in downside than upside markets resulting in larger downside volatility and consequently in a negatively skewed distribution for equilibrium returns. In turn, negative skewness implies a negative asymmetry parameter \( \lambda_{t+1|t} < 0 \) and negative skewness premia \( \delta_{t+1|t} \sigma_{t+1|t} < 0 \). One possible explanation for this phenomenon is that large increases in volatility induce investors to move out of stocks and into bonds, thereby driving current prices down and resulting in larger future returns for stocks; e.g., Campbell and Hentschel (1992) and Harrison and Zhang (1999).

The risk premium decomposition implied by Eq. (26) resembles that of Bollerslev et al. (2018), wherein the market risk premium is decomposed into its “normal” sized price fluctuations risk and jump tail risk. The left jump tail risk, as well as the negative skewness premia, may be seen as proxies for market fear; see also Bollerslev and Todorov (2011).

### 3.3. Conditional asymmetry

The conditional asymmetry parameter, which controls the shape of the distribution of returns to the left and right of its conditional mode, is specified as

\[
\lambda_{t+1|t} = \text{asym} \left( r_{t+1|t} \mid \Phi_t \right) = 1 - \frac{2}{1 + e^h_{t+1|t}}, \tag{27}
\]

where

\[
h_{t+1|t} = \gamma_0 + \sum_{j=1}^{3} \gamma_{0j} D_{j,t} + \gamma_N u_t^t + \gamma_P u_t^p + \gamma_T h_{t|t-1} \tag{28}
\]

and

\[
u_t = \frac{r_t - \mu_{t|t-1}}{\sigma_{t|t-1}} \tag{29}
\]

are standardized returns in excess of their conditional mode. The measures \( u_t^t = |u_t| \) for \( u_t > 0 \) and zero otherwise, and \( u_t^p = |u_t| \) for \( u_t > 0 \) and zero otherwise are proxies for downside and upside shocks, respectively. This specification of the asymmetry coefficient closely follows that in Feunou et al. (2012).

The asymmetry index \( h_{t+1|t} \) is directly associated with the asymmetry parameter of the distribution of returns \( \lambda_{t+1|t} \). That is, negative values of \( h_{t+1|t} \) yield negative \( \lambda_{t+1|t} \) values and vice versa. The intercept \( \gamma_0 \), a measure of unconditional asymmetry, can be negative, zero, or positive. For a negatively skewed distribution, the intercept is expected to be negative (and vice versa for a positive value). The crisis parameter \( \gamma_{0j} \) measures the impact of crises on the intercept; thus, a negative value indicates a higher degree of skewness during the crisis period. The coefficient \( \gamma_N \) measures the marginal impact of downside price shocks on the asymmetry index \( h_{t+1|t} \) and the asymmetry parameter \( \lambda_{t+1|t} \). A positive value indicates that past downside price shocks have a positive impact on both \( h_{t+1|t} \) and \( \lambda_{t+1|t} \) (and vice versa). A large downside shock triggered by a disastrous decline in portfolio values will result in lower conditional asymmetry parameter \( \lambda_{t+1|t} \) in the next period and thus a larger probability for downside markets. Consequently, the skewness risk premium in Eq. (23) will decline, thereby resulting in higher future expected returns; see Savva and Theodossiou (2018) for further discussion of this effect. Higher future expected returns are associated with larger equity premia, as in Wachter (2013). This association is also consistent with Farago and Tedongap (2018), who show that downside risk is priced in cross section asset returns. In contrast, the coefficient \( \gamma_P \) measures the marginal impact of past price shocks on \( h_{t+1|t} \) and \( \lambda_{t+1|t} \). Positive values indicate that past upside price shocks have a positive impact on the parameters (and vice versa). The coefficient \( \gamma_T \) measures the persistence of past upside and downside shocks on the conditional values of \( h_{t+1|t} \) and \( \lambda_{t+1|t} \).

As shown in the Appendix, the conditional probabilities for downside and upside markets are

\[
P \left( r_{t+1} \leq m_{t+1|t} \right) = \frac{1}{2} \left( 1 - \lambda_{t+1|t} \right) = \frac{1}{1 + e^{h_{t+1|t}}} \tag{30}
\]

and

\[
P \left( r_{t+1} > m_{t+1|t} \right) = \frac{1}{2} \left( 1 + \lambda_{t+1|t} \right) = \frac{1}{1 + e^{h_{t+1|t}}} \tag{31}
\]

### 3.4. Conditional shape parameter

As in Mazur and Pipieh (2018), we examine the dynamic behavior of the shape parameter \( k_{t+1|t} \) by using

\[
k_{t+1|t} = k_N - \frac{q_T}{1 + e^{g_{t+1|t}}} \tag{32}
\]

where

\[
g_{t+1|t} = \sum_{j=1}^{3} d_{0j} D_{j,t} + d_0 + d_N u_t^t + d_P u_t^p + d_T h_{t|t-1}. \tag{33}
\]

\( u_t^t \) and \( u_t^p \) are as defined previously, and \( k_N \) and \( k_T \) are the predetermined lower and upper limits for the time varying shape parameter \( k_{t+1|t} \). For the estimation, \( k_N \) and \( k_T \) are set to 0.3 and 1.8, respectively. The parameters \( d_{0j} \) and \( d_P \) control the shape of the distribution to the left and right of the conditional mode \( m_{t|t-1} \). Zero values for \( d_N \) and \( d_T \) indicate a time invariant shape parameter \( k_{t+1|t} \).

### 4. Empirical findings

This section presents the estimated models and discusses the empirical findings.

#### 4.1. SGED-GARCH model estimation

The distribution of returns is

\[
f_t \left( r_{t+1} \mid \Phi_t \right) = \frac{1}{2 \sqrt{2 \pi} \sigma_{t+1}} \exp \left( \frac{1}{2 \sigma_{t+1}^2} \left( r_{t+1} - \mu_{t+1|t} - \delta_{t+1|t} \sigma_{t+1} \right)^2 \right), \tag{34}
\]

where \( \mu_{t+1|t}, \sigma_{t+1|t}, \delta_{t+1|t}, \theta_{t+1|t}, \lambda_{t+1|t}, k_{t+1|t} \) and \( h_{t+1|t} \) are as defined previously. This is the conditional version of the SGED developed in
Table 1

| Data                  | Obs.       | Mean       | St. dev.  | Min.       | Max.       | Skewness | Kurtosis |
|-----------------------|------------|------------|-----------|------------|------------|----------|----------|
| Full Sample           | 10,286     | 0.016      | 1.138     | –22.927    | 10.953     | –1.146   | 25.972   |
| Crash 87              | 60         | –0.369     | 4.086     | –22.927    | 8.682      | –2.907   | 18.444   |
| Subprime crisis       | 525        | –0.023     | 2.122     | –9.474     | 10.953     | –0.181   | 8.071    |
| Coronavirus crisis    | 167        | 0.016      | 2.608     | –12.771    | 8.962      | –0.715   | 5.727    |

The table reports detailed summary statistics (number of observations, mean, standard deviation, minimum, maximum, skewness, and kurtosis) for returns for the full sample, the sub-sample corresponding to the October 87 crash, subprime crisis, and the coronavirus crisis. The data includes daily returns and covers the period January 2, 1980 to October 14, 2020.

**Theodossiou (2015).** Since its inception, the SGED has been used in the literature for the measurement of risk, pricing of options, and modeling of the time-series behavior of returns of stock indices, currencies, oil, precious metals and so forth. In contrast to the skewed generalized t that is often used in empirical work, the SGED enables the development of asset exponential asset pricing equations. It yields for $k_{t+1} = 1$ the skewed Laplace or double exponential distribution, for $k_{t+1} = 2$ the skewed normal distribution used in Feuon et al. (2012) and for $k_{t+1} = \infty$ the uniform distribution; see also Feuon et al. (2016).

We obtain maximum likelihood estimates (MLE) for the parameters of the conditional mean, variance, asymmetry and shape equations of the distribution of returns via the optimization procedure of the sample log-likelihood from Berndt et al. (1974):

$$L(\theta) = \sum_{t=1}^{T} \log f_{t}(\theta | r_{t+1}, \Phi_{t}) = \sum_{t=1}^{T} l_{t+1}(\theta),$$

where $f_{t}(\theta | r_{t+1}, \Phi_{t})$ is the conditional likelihood function of returns associated with Eq. (34), and $\theta$ is a column vector of parameters for the conditional mean, variance, asymmetry, and shape equations specified previously. We obtain estimates for the time-varying skewness price of risk $\delta_{t+1} \Phi_{t}$ via the substitution of the MLE for $k_{t+1}$ and $\lambda_{t+1}$ into Eq. (24). Finally, we obtain robust standard errors for the MLE estimates denoted by $\hat{\theta}$ from the equation

$$\text{var}(\hat{\theta}) = \left( \sum_{t=1}^{T} \frac{\partial^{2} l_{t+1}}{\partial \theta \partial \theta'} \right)^{-1} \sum_{t=1}^{T} \frac{\partial l_{t+1}}{\partial \theta} \frac{\partial l_{t+1}}{\partial \theta'} \left( \sum_{t=1}^{T} \frac{\partial^{2} l_{t+1}}{\partial \theta \partial \theta'} \right)^{-1}.$$

These standard errors are more appropriate in the case of misspecified sample likelihood functions (e.g., Engle and Gonzalez-Rivera, 1991; Bollerslev and Wooldridge, 1992).

Moreover, we calculate confidence intervals following the bootstrap procedure of Rapach and Wohar (2009). Specifically, we generate a series of innovations to construct a pseudo-sample of observations, making $T + 100$ independent draws from the SGED distribution. Using the randomly drawn innovations, we create the series according to the equations of the model (in which the parameters of the model are set to their MLEs). To randomize the initial observations of the pseudo-series, we drop the first 100 transient start-up, thus leaving us with a pseudo-sample of $T$ observations matching the original sample. Then, we estimate the model using the pseudo-sample and repeat the process 1000 times. We construct the 90% confidence intervals for each parameter using the percentile method in Davidson and MacKinnon (1993).  

4.2. Baseline empirical findings

Our sample covers the period January 2, 1980 to October 14, 2020, corresponding to 10,286 daily continuously compounded returns for the S&P 500. We compute the returns using the equation

$$r_{t+1} = 100 \ln \left( \frac{S_{t+1}}{S_{t}} \right),$$

where $S_{t}$ and $S_{t+1}$ are the values of the index on two consecutive trading days. We differentiate among four periods: the October 1987 crash (October 15, 1987 to January 11, 1988), the subprime crisis (June 1, 2008 to February 1, 2010), the coronavirus crisis (February 21, 2020 to the end of our sample), and the “regular” period (the remainder of our sample).  

We report summary statistics in Table 1.

Panel A of Table 2 reports the estimates for the conditional variance of daily returns along with its structural terms for the two crisis periods. The results indicate intense volatility clustering throughout the period. The volatility persistence

$$VP = \alpha + \beta \gamma \text{E}(\varepsilon_{t} < 0) + \beta \gamma (0.0269 + 0.1309 \cdot 0.4851 + 0.8876 \cdot 0.978)$$

is very high but stationary (given that $VP < 1$). The asymmetry coefficient is positive, thus supporting previous findings that negative stock market shocks have a larger impact than positive shocks on future volatility. In general, these results are typical in studies of financial markets, particularly the S&P 500 (e.g., Sun and Yu, 2020).

The intercepts of the conditional variance of returns appear to be larger in the October 87 crash and the coronavirus periods.

Panel A of Table 3 reports the monthly means of the conditional standard deviations, $\sigma_{t+1}$, in the regular, October 87 crash, subprime crisis, and coronavirus crisis periods. Notably, the mean of $\sigma_{t+1}$ is approximately 4.1 times larger during the October 87 crash period, 2 times larger during the subprime crisis and 2.4 times larger during the coronavirus crisis period than the regular period. The pairwise t-test statistics and Fig. 1 confirm these large differences, and the figure shows large spikes in the three crisis periods (the largest spike in the coronavirus crisis).

Panel B of Table 2 presents the estimated parameters of the conditional mode equation. Negative serial correlation is present in the return series. The pure market price of risk during the regular period is 0.1467 and statistically significant. The bootstrap intervals also confirm statistically significant positive deviations of the pure market price of risk during the October 87, subprime and coronavirus crisis periods. Specifically, the pure market price of risk is 0.3146 (0.1467 + 0.1679) during the October 87 crash period, 0.2628 (0.1467 + 0.1611) during the subprime crisis and 0.4521 (0.1467 + 0.3054) during the coronavirus crisis. In addition, Panel B of Table 3 reports the means of the conditional mode of daily returns, $m_{t+1}$, for the three crisis and regular periods.

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1. To estimate the model, all functions were coded in GAUSS and are available from the authors on request.
The table reports the estimation results from our asset pricing model. The data includes daily returns and covers the period January 2, 1980 to October 14, 2020. * indicates statistical significance at the 5% level. The conditional variance, mode, asymmetry, and shape parameters for daily returns are respectively given by Eqs. (21), (22), (27) and (30). The coefficients with subscripts 87, SubP and Covid represent deviations in October 87 market crash, subprime and coronavirus crisis periods. The periods for the October 87 market crash is October 15, 1987 to January 11, 1988, the subprime crisis April 1, 2008 to April 30, 2010 and the coronavirus crisis February 21, 2020 to October 14, 2020. The coefficient c measures the impact of conditional standard deviation of returns on their conditional mode and is interpreted as the “pure price of risk”. The conditional mean of returns $\mu_{t,87}$ is given by Eq. (23), where $h_{t,87}$ is the skewness price of risk. The lower part of the table reports the arithmetic averages of the conditional Pearson’s conditional coefficients of skewness and kurtosis, computed using the Eqs. (15) and (16).

The means are approximately 9 times larger during the October 87 crash period, 4 times larger during the subprime period and 8 times larger during the coronavirus period than the regular period. The t-test statistics for testing these differences are highly statistically significant, as illustrated in Fig. 2.

Panel C of Table 2 reports the estimated parameters for the asymmetry index $h_{t,87}$ and the asymmetry parameter $\lambda_{t,87}$ of the conditional distribution of daily returns. The intercept of the asymmetry index is negative and statistically significant, thus indicating a negatively skewed distribution of returns. The bootstrap intervals for its deviations in the October 87, subprime and coronavirus periods are statistically significant and negative, thereby implying larger negative values during the three crisis periods and thus more-negatively skewed distribution for returns. The coefficient for past downside market shocks is also negative, thus indicating a negative impact of such shocks on current values of the asymmetry index and parameter. However, the coefficient for past upside market shocks is positive, thereby indicating a positive impact on current values of the asymmetry index and parameter. Finally, the coefficient for past values of the asymmetry index is negative, thus indicating strong mean reversion of the asymmetry index over time.

Panel C of Table 3 reports the monthly means of the conditional asymmetry values, $\lambda_{t,87}$, during the October 87, subprime, coronavirus and regular periods. During the regular period, their mean value is $-0.0555$, showing that on average, 5.55 % more returns positioned to the left of the conditional mode of the distribution of returns (negative skewness). The respective percentages during the October 87, subprime and coronavirus crisis periods are 28.54 %, 13.36 % and 25.95 %. Fig. 3 confirms the significant decrease in the asymmetry parameter $\lambda_{t,87}$ during the three crisis periods. The decrease in the monthly averages is indeed considerably deeper for the October 87 and coronavirus periods, showing more negative skewness than that in the subprime and regular periods.

Panel D of Table 2 reports estimates for the conditional shape index $g_{t,87}$ and parameter $k_{t,87}$ of the distribution of daily returns. As expected, the intercept of the shape index is statistically significant and positive. The bootstrap intervals for its deviations in the October 87 are quite large and negative, depicting more leptokurtic tails than those in the regular period. During the subprime and coronavirus periods, the deviations are mildly negative and positive, respectively, depicting slight deviations of the tails. The coefficients for past downside and upside market shocks are negative, thus indicating a negative impact on the shape parameter (more lep-
Table 3  
Testing Differences in the Means of Conditional Parameters of the Distribution of Daily Returns in the Regular, October 87, Subprime and Coronavirus Periods.

| Period                  | Mean   | STD    |
|-------------------------|--------|--------|
| Regular                 | 0.9203 | 0.3680 |
| October 87              | 3.7285 | 2.1590 |
| Subprime                | 1.7966 | 1.1172 |
| COVID-19                | 2.1696 | 1.4730 |
| Difference in the means tests | Diff  | T-Value|
| October 87 vs. Regular  | 2.8082 | 10.0739|
| Subprime vs. Regular    | 0.8763 | 17.9343|
| COVID-19 vs. Regular    | 1.2493 | 10.9548|

| Period                  | Mean   | STD    |
|-------------------------|--------|--------|
| Regular                 | 0.1141 | 0.1201 |
| October 87              | 1.0106 | 0.8607 |
| Subprime                | 0.4522 | 0.3910 |
| COVID-19                | 0.9108 | 0.7490 |
| Difference in the means tests | Diff  | T-Value|
| October 87 vs. Regular  | 0.8966 | 8.0683 |
| Subprime vs. Regular    | 0.3381 | 15.7806|
| COVID-19 vs. Regular    | 0.7968 | 13.7441|

| Period                  | Mean   | STD    |
|-------------------------|--------|--------|
| Regular                 | – 0.0555 | 0.0693 |
| October 87              | – 0.2854 | 0.0750 |
| Subprime                | – 0.1336 | 0.0665 |
| COVID-19                | – 0.2595 | 0.0618 |
| Difference in the means tests | Diff  | T-Value|
| October 87 vs. Regular  | – 0.2299 | – 23.6951 |
| Subprime vs. Regular    | – 0.0781 | – 26.1835 |
| COVID-19 vs. Regular    | – 0.2040 | – 42.2109 |

| Period                  | Mean   | STD    |
|-------------------------|--------|--------|
| Regular                 | 1.3236 | 0.1224 |
| October 87              | 0.8871 | 0.1189 |
| Subprime                | 1.2869 | 0.0991 |
| COVID-19                | 1.3774 | 0.1081 |
| Difference in the means tests | Diff  | T-Value|
| October 87 vs. Regular  | – 0.4366 | – 28.3518 |
| Subprime vs. Regular    | – 0.0368 | – 8.1683 |
| COVID-19 vs. Regular    | 0.0538 | 6.8558|

| Period                  | Mean   | STD    |
|-------------------------|--------|--------|
| Regular                 | – 0.0839 | 0.1004 |
| October 87              | – 0.3662 | 0.0874 |
| Subprime                | – 0.1977 | 0.0958 |
| COVID-19                | – 0.3810 | 0.0811 |
| Difference in the means tests | Diff  | T-Value|
| October 87 vs. Regular  | – 0.2822 | – 32.2037 |
| Subprime vs. Regular    | – 0.1138 | – 26.4541 |
| COVID-19 vs. Regular    | – 0.2971 | – 46.7070 |

| Period                  | Mean   | STD    |
|-------------------------|--------|--------|
| Regular                 | 0.0533 | 0.1079 |
| October 87              | – 0.1978 | 0.2843 |
| Subprime                | 0.1049 | 0.2227 |
| COVID-19                | 0.1393 | 0.2424 |
| Difference in the means tests | Diff  | T-Value|
| October 87 vs. Regular  | – 0.2511 | – 6.8402 |
| Subprime vs. Regular    | 0.0516 | 5.2795|
| COVID-19 vs. Regular    | 0.0860 | 4.5751|

Fig. 1. Conditional Standard Deviation of Daily Returns, Monthly Averages.

Fig. 2. Conditional Mode for Daily Returns, Monthly Averages.

Fig. 3. Conditional Asymmetry Parameter for Distribution of Daily Returns.

tokurtic distributions). Finally, the coefficient for past values of the shape index is positive and less than one, thereby indicating mean reversion of the shape index and parameter over time.

Panel D of Table 3 reports the respective means of the conditional shape parameter, $k_{t+1|T}$, for the regular, subprime and coronavirus periods. The values during these three periods are approximately 1.3, showing a large deviation of the distribution of returns from normality (the shape parameter for the normal distribution is 2). The mean during the October 87 crash period is
approximately 0.9, showing extreme leptokurtosis. Fig. 4 confirms the latter results.

Panel E of Table 3 presents the statistics for the skewness price of risk $\delta_{p+1|t}$, given by Eq. (24) for the four periods. Its mean values during the October 87, subprime and COVID-19 periods are –0.3662, –0.1977 and –0.381, respectively. These number are statistically larger in absolute values than its mean of –0.0839 during the regular period. The documented decreases during the three crisis periods relative to the regular period are 336 %, 136 % and 454 %, respectively. Figs. 5–7 illustrate the relevant distributions (with lines also drawn for the respective normal distributions) and show the shift in the distributions to the left of those of regular periods. Most notably, Fig. 8 illustrates the immense decreases in $\delta_{p+1|t}$ during the crisis periods. Interestingly, the decreases during the October 87 and COVID-19 periods are deeper than those in the subprime period.

Finally, in Panel F of Table 3, we report the means for conditional daily risk premia computed with Eq. (26). Interestingly the risk premia during the October 87 crash period are negative. These are, however, larger and positive during the subprime and COVID-19 periods. The difference in the means of the risk premia during the regular period are statistically smaller than the premia during the subprime and COVID-19 periods. In fact, the risk premia during the subprime period are approximately 1.9 times larger and during the COVID-19 period and 2.1 times than those in the regular period. Fig. 9 illustrates the time series behavior of their monthly averages.

4.3. Additional results

We conduct several robustness tests on our baseline results. The most important of these tests is that our model somehow places unequal weight on the COVID-19 crisis than on the other crises included in our baseline model or crises not included in our baseline model (e.g., the Dot-com bubble and the 09/11 crash). To this end, we rerun our model with different sets of crises. For expositional brevity, in Table 4, for each model, we report only the skewness price of risk parameters $\delta$ for the regular and crises periods, as well as the tests for the differences in the means (equivalent of Panel E, Table 3). Specifically, we report results from models including the (i) October 87, 1990 and COVID-19 (Panel A); (ii) October 87, 09/11 and COVID-19 (Panel B); (iii) October 87, Dot-com and COVID-19 (Panel C); (iv) SARS/H1N1, subprime and COVID-19 (Panel D); and (v) October 87, extended subprime (up to the end of 2010) and COVID-19 (Panel E) crises.

The notable results are as follows. First, the mean and standard deviation for the COVID-19 period remain very similar to our baseline results, thus implying that including different crises in our model does not significantly affect the skewness price of risk of the
COVID-19 crisis. Second, the additional crises included (Recession 90, 09/11, Dot-com and SARS/H1N1) have more negative skewness price of risk than that in the regular period. The 09/11 crisis has the highest mean-standard deviation ratio (significant at the 1% level), followed by the Dot-com crisis (significant at the 5% level), the Recession 90 crisis (significant at the 5% level) and SARS/H1N1 crisis (significant at the 10% level). Thus, our model captures the relative importance of any crisis included in the empirical analysis. Third, extending the period of the subprime crisis (to cover all of 2010) does not significantly affect our baseline results.

Moreover, using the S&P 500 might mask important sectoral differences in our results. Indeed, the results on the S&P indicate the macro consequences of crises (e.g., Barro 2006; Gabaix 2008), whereas specific sectors (particularly those hurt most by a collapse in consumer confidence) might be hurt more intensely (Salgado et al., 2020). Thus, we also separately examine nine important sectors in the S&P 500. Table 5 provides summary statistics of sectoral returns. The sectoral data are available only after 1990 for most sectors, and thus we exclude the analysis on the 1987 crash. Further, the energy and real estate sectors are not included, owing to a lack of daily observations for the main period of study.

For expositional brevity, we report the results for the pure price of risk and the skewness price of risk in Fig. 10. Evidently, skewness is most negative in the consumer discretionary goods services, as expected, because consumers consider these non-essential, and their consumption declines the most during a collapse in confidence. In addition, as expected, materials and financials come second, whereas technology and industrials follow. The price-inelastic health care services and utilities are least affected. Overall, the sectoral analysis shows that the sectors with high price elasticity of demand have the most negative skewness price of risk. In

Table 4
Testing Differences in the Means of Conditional Skewness Price of Risk Parameters ($\beta_{3,t+1}$) of the Distribution of Daily Returns in Various Alternative Specifications.

| A. October 87 crash, Recession 90, and COVID-19 | Period | Mean | STD |
|-----------------------------------------------|-------|------|-----|
| Regular                                       | – 0.0957 | 0.1086 |
| October 87                                    | – 0.3333 | 0.0677 |
| Recession 90                                   | – 0.2202 | 0.1013 |
| COVID-19                                       | – 0.3853 | 0.0798 |
| Difference in the means tests                 | Diff T-Value |
| October 87 vs. Regular                         | – 0.2377 | – 26.9989 |
| Recession 90 vs. Regular                       | – 0.1245 | – 16.6551 |
| COVID-19 vs. Regular                           | – 0.2897 | – 48.8971 |

| B. October 87 crash, 09/11, and COVID-19      | Period | Mean | STD |
|-----------------------------------------------|-------|------|-----|
| Regular                                       | – 0.1012 | 0.1112 |
| October 87                                    | – 0.3391 | 0.0694 |
| 09/11                                         | – 0.3278 | 0.1030 |
| COVID-19                                       | – 0.3912 | 0.0823 |
| Difference in the means tests                 | Diff T-Value |
| October 87 vs. Regular                         | – 0.2378 | – 26.3511 |
| 09/11 vs. Regular                              | – 0.2266 | – 4.9172 |
| COVID-19 vs. Regular                           | – 0.2900 | – 7.8745 |

| C. October 87 crash, Dot-com, and COVID-19    | Period | Mean | STD |
|-----------------------------------------------|-------|------|-----|
| Regular                                       | – 0.0952 | 0.1073 |
| Dot-com                                       | – 0.3347 | 0.0674 |
| COVID-19                                       | – 0.2253 | 0.0996 |
| Difference in the means tests                 | Diff T-Value |
| October 87 vs. Regular                         | – 0.2395 | – 27.3138 |
| Dot-com vs. Regular                            | – 0.1300 | – 19.3660 |
| COVID-19 vs. Regular                           | – 0.2899 | – 53.4496 |

| D. SARS/H1N1, Subprime, and COVID-19          | Period | Mean | STD |
|-----------------------------------------------|-------|------|-----|
| Regular                                       | – 0.0972 | 0.1220 |
| SARS/H1N1                                      | – 0.1842 | 0.1022 |
| Subprime                                      | – 0.1984 | 0.1008 |
| COVID-19                                       | – 0.4114 | 0.08319 |
| Difference in the means tests                 | Diff T-Value |
| SARS/H1N1 vs. Regular                          | – 0.0870 | – 20.0341 |
| Subprime vs. Regular                           | – 0.1012 | – 22.1389 |
| COVID-19 vs. Regular                           | – 0.3142 | – 81.2841 |

| E. October 87 crash, extended Subprime and COVID-19 | Period | Mean | STD |
|------------------------------------------------------|-------|------|-----|
| Regular                                              | – 0.0866 | 0.1057 |
| October 87                                           | – 0.3246 | 0.0636 |
| Subprime ext.                                        | – 0.2119 | 0.1014 |
| COVID-19                                              | – 0.3754 | 0.0738 |
| Difference in the means tests                        | Diff T-Value |
| October 87 vs. Subprime ext.                        | – 0.2379 | – 28.7463 |
| Subprime vs. Subprime ext.                          | – 0.1253 | – 27.5188 |
| COVID-19 ext. vs. Subprime ext.                     | – 0.2887 | – 85.0465 |
Table 5
Summary statistics of sectoral returns.

| Sector            | Mean | St. dev. | Skewness | Kurtosis |
|-------------------|------|----------|----------|----------|
| Communication     | 0.008| 1.328    | –0.047   | 10.446   |
| Services          |      |          |          |          |
| Consumer          | 0.036| 1.289    | –0.261   | 11.560   |
| Discretionary     |      |          |          |          |
| Consumer staples  | 0.030| 0.963    | –0.164   | 12.867   |
| Technology        | 0.044| 1.678    | 0.029    | 9.222    |
| Financials        | 0.021| 1.751    | –0.181   | 20.141   |
| Health care       | 0.036| 1.180    | –0.184   | 9.491    |
| Industrials       | 0.028| 1.264    | –0.426   | 11.799   |
| Materials         | 0.021| 1.400    | –0.326   | 11.306   |
| Utilities         | 0.014| 1.129    | –0.106   | 17.859   |

The table reports summary statistics (mean, standard deviation, skewness, and kurtosis) for sectoral returns. The data includes daily returns and covers the period January 2, 1990 to October 14, 2020 (7833 observations).

![Figure 10](image)

Fact, this observation is most prevalent in the coronavirus crisis, which additionally, as compared with previous crises, shows the significant impact of this crisis on the skewness price of risk.

5. Conclusions

We model the COVID-19 crisis in an equilibrium model of asset pricing with skewness in the price of risk. We define the conditional moment equations as functions of the subprime and COVID-19 crises, and we show theoretically that skewness becomes more negative in crisis periods. We further define the risk-neutral equilibrium equation for returns as a function of crisis periods. Notably, in equilibrium, skewness is an additive parameter affecting the variance on returns (the pure price of risk). This result is important theoretically, because the model allows the effect of the pure price of risk on returns to be positive (consistent with a positive risk and return relation).

We estimate our model using S&P 500 data for January 2, 1980 to October 14, 2020. We use the skewed generalized error distribution, for which we derive the distribution of returns, the likelihood function, robust standard errors and bootstrapped confidence intervals. Our results show extreme skewness during the October 87, subprime and COVID-19 crises, whereas the skewness during other turmoil periods is significantly milder. In fact, the skewness during the COVID-19 crisis overshadows that during the subprime crisis. Our findings open new research pathways in financial economics for understanding the effects of sharp financial crises on the market price of risk.

Appendix A. Supplementary data

Supplementary material related to this article can be found, in the online version, at doi:https://doi.org/10.1016/j.jfs.2020.100840.

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