Sivers, Boer-Mulders and transversity distributions in the difference cross sections in SIDIS

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Abstract. A major experimental program is presently underway to determine the Sivers, Boer-Mulders and transversity distributions, vital for understanding the internal structure of the nucleon. To this end we consider the Sivers, Boer-Mulders and transversity azimuthal asymmetries of the difference cross sections of hadrons with opposite charges in SIDIS reactions with unpolarized and transversely polarized target $l + N \rightarrow l' + h + X$, $h = \pi^{\pm}, K^{\pm}, h^{\pm}$. We show that on deuteron target these asymmetries are particularly simple and determine the sum of the valence-quark $Q_V = u_V + d_V$ transverse momentum dependent distributions without any contributions from the strange or other sea-quark functions.

At present, data on these asymmetries are presented for the integrated asymmetries i.e. the $x_B$- and $z_h$-dependent asymmetries. If data are available in small bins in $Q^2$, so that $Q^2$-dependence can be neglected, these expressions simplify dramatically leading to remarkably simple and powerful tests of the simplifying assumptions used in extracting these functions from the data.

1. Introduction
We consider the transverse momentum dependent (TMD) Sivers, Boer-Mulders (BM) and transversity functions, measured in transversely polarized or unpolarized semi-inclusive deep inelastic scattering (SIDIS) $l + N \rightarrow l' + h + X$.

At present, the extraction of these transverse momentum dependent (TMD) functions has been relatively simplistic, involving two key conventions:

a) The analysis is in leading order in perturbative QCD

b) Both the parton distributions (PDFs) and fragmentation functions (FFs) which depend on parton intrinsic momentum $k_L$, are typically parametrized under the following conventions:

1) a factorized form for the $x_B/z_h$ and $k_L$-dependences:

$$\Delta f(x_B \text{ or } z_h, k^2_L) = \Delta f(x_B \text{ or } z_h) e^{-k^2_L/\langle k^2_L \rangle}.$$  

2) the $x_B(z_h)$-dependence is proportional to the collinear PDFs (FFs), 3) the $Q^2$-evolution is in the collinear PDFs (FFs) and 4) the $k_L$-dependence is Gaussian type and flavour independent.
The functional form of the TMD functions is still under discussion and these are the simplest, commonly used at present standard parametrizations. Additional assumptions about the sea-quark contributions are made as well. For BM functions it is assumed that they are proportional to Sivers functions.

The goal of our studies is to show that if one uses the so called ”difference” asymmetries one can 1) obtain information about the valence-quark TMDs without contributions from the sea-quarks, 2) test the standard parametrization and the assumption about BM function using relations between measurable quantities only. These tests are crucial as they would verify to what extend the used standard parametrizations provide a reliable information on the TMD distributions for the present set of data.

The ”difference” cross section asymmetries are combinations of the type:

\[ A^{h-h} = \frac{\Delta \sigma^h - \Delta \sigma^{\bar{h}}}{\sigma^h - \sigma^{\bar{h}}} \]  

(2)

where \( \sigma^h \) and \( \Delta \sigma^h = \sigma^{h\uparrow} - \sigma^{h\downarrow} \) are the unpolarized and polarized cross sections respectively. The arrows indicate the polarization of the target, and \( h \) and \( \bar{h} \) are hadrons with opposite charges.

Previously the difference cross section asymmetries [1, 2] have appeared rather useful in the simple collinear picture and the valence quark helicity parton densities and fragmentation functions were determined directly [3]. Here we extend our studies to the non-collinear picture when transverse momentum dependence is included.

Presently, data on Sivers, Boer-Mulders (BM) and transversity asymmetries are presented as functions of only one of the kinematic variables \( x_B \) or \( z_h \), sometimes with, in addition, the \( Q^2 \) and \( P_T \) dependence, with integration over the measured intervals of the other variables. We obtain analytic expressions for the \( x_B \) and \( z_h \) dependence of the asymmetries. These expression strongly simplify if in the measured kinematic intervals the binning in \( Q^2 \) is small enough to allow the neglect of the \( Q^2 \)-dependence of the parton densities and FFs. Here we shall present the general formulae for Sivers asymmetries only, and the formulae when this simplification is valid for all asymmetries. Most of the presented results can be found in more details in ref. [4].

We present the asymmetries on deuteron target, when the asymmetries provide information on the sum of the valence-quark TMDs \( \Delta f_{QV} \), \( Q_V = u_V + d_V \) and the results appear especially simple. The results on proton targets are obtained with the simple replacement \( \Delta f_{QV} \rightarrow e_q^2 \Delta f_{uu} - e_d^2 \Delta f_{dv} \) for final pions and \( \Delta f_{QV} \rightarrow e_q^2 \Delta f_{uv} \) for final kaons. The explicit expressions for them are given in ref. [4].

2. Sivers functions

The Sivers distribution function \( \Delta^N f_{q/S_T} (x_B, k_{\perp}) \) appears in the expression for the number density of unpolarized quarks \( q \) with intrinsic transverse momentum \( k_{\perp} \) in a transversely polarized proton \( p^T \) with 3-momentum \( P \) and transverse spin \( S_T \) [5]:

\[ f_{q/p^T} (x_B, k_{\perp}) = f_{q/p} (x_B, k_{\perp}) + \frac{1}{2} \Delta^N f_{q/S_T} (x_B, k_{\perp}) \cdot S_T \cdot (\hat{P} \times \hat{k}_{\perp}) \]  

(3)

where \( f_{q/p} (x_B, k_{\perp}) \) are the unpolarized \( x_B \) and \( k_{\perp} \)-dependent parton densities and the triple product induces a definite azimuthal \( \sin(\phi_h - \phi_S) \)-dependence, \( \phi_h \) and \( \phi_S \) are the azimuthal angles of the final hadron and the spin of the target, \( x_B = Q^2/2(P.q) \) is the usual Bjorken variable.

To access the Sivers TMDs one considers the \( \sin(\phi_h - \phi_S) \) azimuthal moment \( A^{sin(\phi_h - \phi_S)}_{UT} \) of the transverse single-spin asymmetry in SIDIS [6]. In accordance with this we define Sivers functions...
asymmetry for the difference cross section analogously:

\[ A_{UT}^{h, k}(x_B, Q^2, z_h, P_T^2) = \frac{1}{S_P} \int d\phi_h d\phi_S \left[ \frac{d^6 \sigma^+ - d^6 \sigma^-}{d^6 \sigma^+ + d^6 \sigma^-} \right]^{h, k} \sin(\phi_h - \phi_S) \]  

where \( d^6 \sigma^\pm = d\sigma^\pm e^{\pm \epsilon h X} / \int (dx_B dQ^2 dz_h dP_T^2 d\phi_h d\phi_S) \) stands for the differential cross section of SIDIS with an unpolarized lepton beam on a transversely polarized target in the kinematic region \( P_T \leq k_\perp \ll Q \) at order \( (k_\perp/Q) \), \( P_T \) is the transverse momentum of the final hadron in the \( \gamma^* p \) c.m. frame, and \( z_h, Q^2 \) and \( y \) are the usual measurable SIDIS quantities:

\[ z_h = \frac{(P.P_h)}{(P.q)}, \quad Q^2 = -q^2, \quad q = l - l', \quad y = \frac{(P.q)}{(P.l)}, \quad Q^2 = 2ME_{x_B}y \]  

with \( l \) and \( l' \), \( P \) and \( P_h \) the 4-momenta of the initial and final leptons, and hadrons. Throughout the paper we follow the notation and kinematics of ref. [6].

In general, only the valence-quark TMD functions enter the difference cross sections. When a deuteron target is used, a further simplification occurs – independently of the final hadrons, only one parton density enters – the sum of the valence quarks:

\[ \Delta f_{QV/S_T}(x_B, k_\perp) = \Delta f_{uV/S_T}(x_B, k_\perp) + \Delta f_{dV/S_T}(x_B, k_\perp) \]  

Following the convention for the standard parametrizations, we adopt the following form for valence-quark Sivers function:

\[ \Delta f_{QV/S_T}(x_B, k_\perp) = \Delta f_{QV/S_T}(x_B) \sqrt{2e} \frac{k_\perp}{M_s} \frac{e^{-k_\perp^2/(k_\perp^2)_s}}{\pi(k_\perp^2)_s} \]  

where

\[ \Delta f_{QV/S_T}(x_B, Q^2) = 2N_{QV}^{Siv}(x_B) QV(x_B, Q^2) \]  

and \( QV = uV + dV \) is the sum of the collinear valence PDFs, and

\[ \langle k_\perp^2 \rangle_s = \frac{\langle k_\perp^2 \rangle}{\langle k_\perp^2 \rangle + M_s^2} \]  

The unknowns are \( M_s \) (involved in the definition of \( \langle k_\perp^2 \rangle_s \)) and \( N_{QV}^{Siv}(x_B) \). They are to be determined in the integrated \( z_h \) - and \( x_B \) - Sivers difference asymmetries, for which we obtain:

\[ A_{UT}^{h, k}(z_h) = B_{Siv}^h(z_h) \frac{z_h}{\sqrt{(p_+^2) + z_h^2 \langle k_\perp^2 \rangle_s}}, \quad h = \pi^+, K^+, h^+ \]  

\[ B_{Siv}^h(z_h) = A_{Siv} \int dx_B \int dQ^2 \frac{1 + (1 - y^2)}{Q^2} \Delta f_{QV/S_T}(x_B, Q^2) \frac{D_{uV}^h(z_h, Q^2)}{2 \int dx_B \int dQ^2 \frac{1 + (1 - y^2)}{Q^2} QV(x_B, Q^2) \frac{D_{uV}^h(z_h, Q^2)}{2} \} \]  

and

\[ A_{UT}^{h, k}(x_B) = C_{Siv}^h(x_B) N_{QV}^{Siv}(x_B), \quad h = \pi^+, K^+, h^+ \]  

\[ C_{Siv}^h(x_B) = A_{Siv} \int dQ^2 \frac{1 + (1 - y^2)}{Q^2} QV(x_B, Q^2) \int dz_h \frac{D_{uV}^h(z_h, Q^2)}{\sqrt{(P_T^2)_s}} \]  

\[ \int dQ^2 \frac{1 + (1 - y^2)}{Q^2} QV(x_B, Q^2) \int dz_h D_{uV}^h(z_h, Q^2) \]  

\[ \int dQ^2 \frac{1 + (1 - y^2)}{Q^2} QV(x_B, Q^2) \int dz_h D_{uV}^h(z_h, Q^2) \]
Eqs. (10) and (12) determine $N^{Siv}(x_B)$ and $\langle k_T^2 \rangle_s$ without any contributions from the sea quarks.

For bins corresponding to a reasonably small interval $\Delta Q^2$ in $Q^2$, we replace the integral over $Q^2$ by $Q^2$ times the $Q^2$-dependent functions evaluated at the mean value $\bar{Q}^2$ for the bin. Then (10) and (12) become particularly simple:

$$A_{UT}^{Siv/h-h}(z_h, \bar{Q}^2) = B_{Siv}(\bar{Q}^2) \frac{z_h}{\sqrt{\langle p_T^2 \rangle_s + z_h^2 \langle k_T^2 \rangle_s}}, \quad (14)$$

$$A_{UT}^{Siv/h-h}(x_B, \bar{Q}^2) = C_{Siv}^{h}(\bar{Q}^2) N^{Siv}(x_B), \quad (15)$$

as $B_{Siv}$ and $C_{Siv}^{h}$ are constant factors:

$$B_{Siv}(\bar{Q}^2) = A_{Siv} \int dx_B [1 + (1 - \bar{y})^2] \Delta f_{QV/s_T}(x_B, \bar{Q}^2), \quad \forall h \quad (16)$$

$$C_{Siv}^{h}(\bar{Q}^2) = A_{Siv} \int dz_h z_h \frac{D_{hV}(z_h, \bar{Q}^2)}{\int dz_h D_{hV}(z_h, \bar{Q}^2)} \sqrt{\langle P_T^2 \rangle_s}, \quad h = \pi^+, K^+ \quad (17)$$

Here

$$\bar{y} = \frac{\bar{Q}^2}{2 M E x_B}, \quad A_{Siv} = \frac{\sqrt{2\pi}}{2\sqrt{2}} \frac{\langle k_T^2 \rangle_s^2}{M_s \langle k_T^2 \rangle_s}, \quad \langle P_T^2 \rangle_s = \langle k_T^2 \rangle + z_h^2 \langle k_T^2 \rangle_s \quad (18)$$

Eq. (14) is remarkably strong prediction both for its explicit $z_h$ behaviour, determined solely by the Gaussian dependence on $k_T$, and for its independence of $h$. Eq. (16) implies that Sivers $x_B$-asymmetries should have the same $x_B$-behaviour for all final hadrons.

3. Boer-Mulders distributions

The extraction of Boer-Mulders and transversity distributions is more complicated as compared to Sivers parton densities. The reason is that Sivers functions enter the cross section in convolution with the unpolarized TMD fragmentation functions, known from multiplicities, while the BM and transversity functions enter the cross sections in convolution with the transversely polarized TMD FFs, the so-called Collins functions $\Delta^N h/q \langle \tilde{s}_h, p_T \rangle$. The latter can, in principle, be extracted from $e^+e^- \rightarrow h_1 h_2 X$, but at present are rather poorly known.

The Boer-Mulders function determines the distribution of transversely polarized quarks $q^\uparrow$ in an unpolarized proton $p$ [7]:

$$\Delta^N f_{q^\uparrow/p}(x_B, k_T) \equiv \Delta f_{q^\uparrow/p}(x_B, k_T) = -\frac{k_T}{m_p} h_1^\uparrow(x_B, k_T). \quad (19)$$

It is accessed measuring the $\cos 2\phi_h$-momentum in unpolarized SIDIS differential cross section. The difference BM asymmetry is defined respectively:

$$A_{BM}^{h-h} = \frac{\int d\phi_h \cos 2\phi_h d^2\sigma^{h-h}}{\int d\phi_h d^2\sigma^{h-h}}. \quad (20)$$

On deuteron target, independently of the final hadrons $h - h$, it determines only the sum of the valence quark BM distribution:

$$\Delta f_{q^\uparrow/p}(x_B, k_T) \equiv \Delta f_{q^\uparrow/p}(x_B, k_T) = \Delta f_{q^\uparrow/p}(x_B, k_T) + \Delta f_{q^\uparrow/p}(x_B, k_T) \quad (21)$$
that is parametrized accordingly:

\[
\Delta f^{QV}_{s/u/p}(x_B, k_\perp, Q^2) = \Delta f^{QV}_{s/u/p}(x_B, Q^2) \sqrt{2k_\perp} \frac{k_\perp}{M_{BM}} \frac{e^{-k_\perp^2/(k_\perp^2)_{BM}}}{\pi(k_\perp^2)_{BM}}
\]

\[
\Delta f^{QV}_{s/u/p}(x_B, Q^2) = 2N_{BM}^{QV}(x_B) Q_V(x_B, Q^2),
\]

(22)

where \(N_{BM}^{QV}(x_B)\) and \(M_{BM}\) are to be determined from the integrated BM asymmetries. Here we present only the \(x_\mu\)-dependent asymmetry. For all final \(h\) it determines the same \(N_{BM}^{QV}\):

\[
A_{BM}(x_B)^{h-h} = C_{BM}^{h}(x_B) N_{BM}^{QV}(x_B), \forall h
\]

(23)

where for \(h = \pi^+, K^+\) we have:

\[
C_{BM}^{h}(x_B) = -4e A_{BM} A_{Coll} \frac{\int \int dQ^2 dz h_{1-y} Q_V(x_B, z_h) \Delta N D_{h/u/V}(z_h) / \langle P_2^y \rangle_{BM}}{\int \int dQ^2 dz h_{1+(1-y)^2} Q_V(x_B) D_{h/u/V}(z_h)}.
\]

(24)

For \(h = h^\pm\) the relative expression is given in [4].

If \(Q^2\)-evolution can be neglected, then \(A_{BM}(x_B)^{h-h}\) is determined solely by \(N_{BM}^{QV}\):

\[
A_{BM}^{h-h}(x_B, Q^2) = \frac{1 - \bar{y}}{1 + (1 - \bar{y})^2} C_{BM}^{h}(Q^2) N_{BM}^{QV}(x_B), \quad h = \pi^+, K^+, h^+
\]

(25)

where \(C_{BM}^{h}\) are constant factors. This implies, in particular, that \(A_{BM}(x_B)^{h-h}\) have the same \(x_\mu\)-behaviour:

\[
A_{BM}^{\pi^+ - \pi^-}(x_B) \simeq A_{BM}^{K^+ - K^-}(x_B) \simeq A_{BM}^{h^+ - h^-}(x_B) \quad \text{or} \quad \frac{A_{BM}^{h^+ - h^-}(x_B)}{A_{BM}^{h^+ - h^-}(x_B)} = \ldots = \text{const}
\]

(26)

4. Relations between BM and Sivers asymmetries

In current analysis an additional simplifying assumption is made, namely the BM function is assumed proportional to its chiral-even partner – the Sivers function [8]. In our case this reads:

\[
\Delta f^{QV}_{s/u/p}(x, k_\perp) = \frac{\lambda_{QV}}{2} \Delta f^{QV}_{S/v}(x, k_\perp),
\]

(27)

where \(\lambda_{QV}\) is a fitting parameter. This implies that BM and Sivers \(x_B\)-asymmetries measure the same \(N_{Siv}^{QV}\) and they are related:

\[
\frac{A_{BM}^{h-h}(x_B)}{A_{Siv}^{h-h}(x_B)} = \frac{\lambda_{QV}}{2} C_h(x_B), \quad h = \pi^+, K^+, h^+
\]

(28)

where \(C_h(x_B)\) is independent of \(N_{Siv}\):

\[
C_h(x_B) = -4\sqrt{2} e \frac{\int \int dQ^2 dz h_{1-y} Q_V(x_B, Q^2, z_h) \Delta N D_{h/u/V}(z_h, Q^2) / \langle P_2^y \rangle_{BM}}{\int \int dQ^2 dz h_{1+(1-y)^2} Q_V(x_B, Q^2, z_h) D_{h/u/V}(z_h, Q^2) / \langle P_2^y \rangle_{Siv}}, \quad h = \pi^+, K^+
\]

(29)

If \(Q^2\)-evolution can be neglected the ratio of the asymmetries is completely fixed:

\[
\frac{A_{BM}^{h-h}(x_B)}{A_{Siv}^{h-h}(x_B)} = \frac{\lambda_{QV}}{2} \frac{1 - \bar{y}}{1 + (1 - \bar{y})^2} C^{h}(Q^2),
\]

(30)

where \(C_h\) is a constant factor.

Thus, if data exist for a range of \(x_B\) values at the same \(Q\), eq. (30) should allow a test of the used connection between BM and Sivers functions without requiring knowledge of BM, Sivers or even Collins functions, involving only measurable quantities.
5. Relations between BM and Collins asymmetries

The distribution of transversely polarized quarks $q^\uparrow$ in a transversely polarized proton $p^\uparrow$ defines the transversity distributions $h_{1q}(x_q)$ or $\Delta_T q(x_q)$:

$$\Delta_T q(x_q) = h_{1q}(x_q) = \int d^2k_\perp h_{1q}(x_q, k_\perp),$$

where $h_{1q}(x_q, k_\perp)$ is the transversity distribution depending on the parton transverse momentum. It is selected by the Collins asymmetry, which for the difference cross sections is:

$$A_{UT}^{h_{1q} \sin(\phi_h + \phi_\perp), h_{1q}} = \frac{1}{S_T} \int d\phi_h d\phi_S \left[ d^6\sigma^\uparrow - d^6\sigma^\perp \right]^{h_{1q} \sin(\phi_h + \phi_\perp)}.$$

(31)

Again, on deuterium target it measures the sum of the valence-quarks transverse distributions:

$$h_{1QV}(x_q, k_\perp) \equiv h_{1uv}(x_q, k_\perp) + h_{1dv}(x_q, k_\perp)$$

(33)

and $h_{1QV}(x_q, k_\perp)$ and $\Lambda_{QV}(x_q)$ are the unknown quantities.

Common for BM and Collins difference asymmetries is that they are convoluted with the same Collins FF, $\Delta^N D_{h^+/uv\uparrow}(z_h, p_\perp)$, and that the $x_q$ and $z_h$ dependencies factorize. This allows to relate the $z_h$-dependent BM and Collins asymmetries. This relation is especially simple for small enough bins in $Q^2$ with a completely fixed $z_h$-dependence and the same for all final hadrons:

$$\frac{A_{BM}^{h_{1q} \sin(\phi_h + \phi_\perp), h_{1q}}(z_h)}{A_{UT}^{h_{1q} \sin(\phi_h + \phi_\perp), h_{1q}}(z_h)} = \lambda_{QV} \frac{z_h \sqrt{\langle P^2 \rangle_{BM}}}{\langle P^2 \rangle_{UT}} B, \quad h = p^+, K^+$$

(35)

where $\langle P^2 \rangle_{BM}$ and $\langle P^2 \rangle_{UT}$ are fixed by the Gaussian form:

$$\langle P^2 \rangle_{BM} = \langle P^2 \rangle_{C} + z_h^2 \langle k^2 \rangle_{B}, \quad \langle P^2 \rangle_{UT} = \langle P^2 \rangle_{C} + z_h^2 \langle k^2 \rangle_{U}$$

(36)

and $B$ is independent of both $z_h$ and the final hadron. Eq. (35) will hold only if $\Delta^N D_{h^+/uv\uparrow}$ enters both $A_{BM}^{h_{1q} \sin(\phi_h + \phi_\perp), h_{1q}}$ and $A_{Coll}^{h_{1q} \sin(\phi_h + \phi_\perp), h_{1q}}$ and thus, it presents a remarkably simple test, involving only measurable quantities, not only of the standard parametrization, but of the whole QCD picture.

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