Discharge estimation for trapezoidal open channels applying fuzzy transformation method to a flow equation

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ABSTRACT

The aim of this paper is to implement fuzzy logic theory in order to estimate the discharge for open channels, which is a well-known physical problem affected by many factors. The problem can be solved by the Manning equation but the parameters present uncertainties as to their true-real values. Especially, the Manning $n$ roughness coefficient, which is an empirically derived coefficient, presents quite a high variation for different substrates. With the help of fuzzy logic and utilizing a fuzzy transformation method, it is possible to include the uncertainties of the problem in the calculation process. In this case, it is feasible to estimate the discharge, putting more emphasis on different uncertainty rates of the Manning roughness coefficient, while the rest of the parameters remain with constant or zero uncertainty level. By taking different $\alpha$-cut levels, it is shown that the methodology gives realistic and reliable results, presenting with great accuracy the variations of the water discharge for trapezoidal open channels. In this way, a possible underestimation or overestimation of the actual physical condition is avoided, by helping engineers and researchers to obtain a more comprehensive view of the real physical conditions, and thus make better management plans.

Key words: discharge, fuzzy logic, Manning equation, open channel, transformation method

HIGHLIGHTS

• Novel use of fuzzy logic in the Manning equation.
• A new approach to estimate the discharge in open natural channels.
• Includes Manning’s roughness coefficient uncertainties in the calculation process.
• Comparison between the uncertainty parameters of the Manning equation.
• Provides accurate information for better management plans and decisions.

INTRODUCTION

It is widely known that ambiguities and uncertainties in the values of parameters appear in many problems of mechanical and hydraulic engineering which are introduced in the corresponding mathematical models. Normally, these uncertainties arise from incomplete or imprecise information or from identification problems. They mainly refer to uncertain model parameters, sometimes also uncertain system inputs or uncertain initial or boundary conditions. These uncertainties greatly influence the numerical results, and they should be taken seriously into consideration in calculations.

The discharge estimation of water into open channels is a well-known and complicated problem for artificial channels such as irrigation systems, water supply systems etc., but even more complicated for natural open channels such as rivers, streams etc., due to their high variability. As yet, the majority of researchers are still using Manning's roughness coefficient in terms of hydrodynamics, hydraulics, hydrology and water resource engineering (Choo et al. 2018).

The Manning $n$ is a coefficient which represents the roughness or friction applied to the flow by the channel and is used in the Manning equation for estimating the discharge in open channel flow. Manning's equation has a big advantage because it requires only two mathematical parameters, the water surface slope $S$ and the roughness $n$ from the bed load to estimate the discharge. However, it presents a certain drawback and does not work well due to a high sensitivity of the coefficient $n$ which is discharge-dependent and applies only to a specific flow condition (Geem & Kim 2018). Furthermore, the Manning coefficient is influenced by channel slope, flow velocity, Reynolds number, Froude number and sediment load. Zhang et al. (2010)

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investigated the potential effects of sediment load on the Manning coefficient on a fixed flume bed under wide ranges of hydraulics and sediment loads. Their work showed that the Manning coefficient decreased with increases in Reynolds number and Froude number. Also, although a qualitative description of movable bed roughness was clearly defined a long time ago (Yen 2002), its precise value in an unsteady sediment transport process is still difficult to determine (Huang 2007). In general, the procedure for selecting $n$ values is quite complex and usually requires advanced judgment and expertise (Bahramifar et al. 2013). Moreover, one of the best methods is to have field observations from an experienced hydraulic engineer (Choo et al. 2018), but such a method is both time-consuming and costly. In this regard, and because it is an empirically derived coefficient, which is dependent on and affected by many factors (Chow 1959), the scientific community has shown in the past great interest in various methods for selecting appropriate values of roughness coefficient for discharge estimation in an open channel (Cowan 1956; Limerinos 1970; Shiono et al. 1999; Mailapalli et al. 2008; Khatua et al. 2012; Dash & Khatua 2016; Pradhan & Khatua 2018). Additionally, and because it is difficult to obtain a general equation to provide accurate estimation of the Manning coefficient, due to a lack of knowledge of some physical processes associated with channel formation and maintenance, new procedures for the estimation of Manning’s $n$ coefficient have been developed using innovative methods such as gene expression programming (Azamathulla et al. 2013), adaptive neuro-fuzzy inference system (ANFIS) (Samandar 2011; Bahramifar et al. 2013; Moharana & Khatua 2014; Al-Husseini 2015) and machine learning techniques (Mohanta et al. 2018).

More specifically, the fuzzy set theory proposed by Zadeh (1965) proved to be a powerful tool for modelling uncertainties and for processing subjective information in mathematical models, giving a big boost to simulation procedures dealing with fuzzy parameterized models. In recent years, fuzzy parameterized models have become quite popular in engineering sciences and help in analyzing systems in view of the uncertainty of the parameters. Discharge estimation in open channels is a hydraulic problem characterized by ambiguities in all its parameters, but in this work greater attention is given to the Manning roughness coefficient for different substrates, of which due to complexities of physical processes the exact value is often more sensitive and presents the greatest ambiguity of all factors. In this regard, it is easily understood that the ambiguity of the roughness coefficient greatly affects the discharge results.

Understanding the need for accurate prediction of the Manning roughness coefficient and including its uncertainties, in this article, the reduced transformation method proposed by Hanss (2002, 2005) is implemented in order to transform the Manning equation in a fuzzy environment giving the final discharge estimation in a fuzzy form. The method can generally be considered as an advanced and extended version of the so-called vertex method proposed by Dong & Shah (1987) but with certain advantages. In contrast to classic fuzzy arithmetic, fuzzy arithmetic based on the transformation method does not exhibit the effect of overestimation, which is a critical point when we deal with the Manning equation. The method uses fuzzy numbers to solve the model and is generally used to simulate and analyze systems with uncertain parameters. In this way, the Manning equation parameters can be treated as fuzzy numbers with the potential to give different uncertainty rates to parameters, thus including the uncertainties in the results. The fuzzy transformation method works well by using triangular fuzzy numbers which are simple to apply, with easy and intuitive interpretation, based on numerical results as well as graphic representation. The method is applied in four different open channel types of trapezoidal profile. Also, two different scenarios were considered. In the first scenario all the parameters of the problem present uncertainties and in the second the roughness coefficient is the only uncertain parameter, accepting that all other parameters are accurate as to their values with zero uncertainty rate.

**METHODS**

**Preliminaries**

When the surface of a flow is open to the atmosphere, in other words when there is only atmospheric pressure on the surface, the flow is named open channel flow. Water flows in rivers and streams are obvious examples of open channel flow in natural channels.

Classified according to its origin, a channel may be either natural or artificial. Geomorphology of natural channels concerns their shape and structure. Natural channel sections are in general very irregular, usually varying from approximately parabolic to approximately trapezoidal (Chow 1959).
On the other hand, artificial open channels are built for some specific purpose, such as irrigation, water-power development, etc., and are usually designed with regular geometric shapes such as rectangular or trapezoidal. Figure 1 presents the construction of a trapezoidal channel and a side view.

The Manning equation is the most-used equation to analyze open channel flows. It is a semi-empirical equation for simulating water flows in channels and culverts where the water is open to the atmosphere, i.e. not flowing under pressure. It was introduced by the Irish engineer Robert Manning in 1891 as an alternative to the Chezy equation and can be evaluating as follows (Manning 1891):

\[ V = \frac{1}{n} R^{2/3} S_0^{1/2} \]  

where \( V \) is the average velocity (m/s), \( R \) is the hydraulic radius (m), \( S_0 \) is the channel slope, and \( n \) is the Manning coefficient. Also,

\[ R = \frac{A}{P} \quad \text{with} \quad P = b + 2y\sqrt{1 + z^2} \quad \text{and} \quad A = (b + y)z \]

where \( P \) is the wetted perimeter (m), \( A \) is the channel cross-sectional area (m²), \( b \) is the bottom width (m), \( y \) is the water width (m) and \( z \) is the side slope.

The final discharge \( Q \) (m³s⁻¹) is calculated from the equation:

\[ Q = V \cdot A \]

To estimate the discharge in a reliable way, first it is important to understand and analyze the problem in individual steps to accurately identify the gaps appearing in the problem that are creating uncertainties. Then, the fuzzy methodology seems to be the most appropriate method of approaching the problem by enabling the uncertainties to be included in the calculations in a simple way. Figure 2 presents the problem approach and its solution.

**Fuzzy methodology**

**Definition 1.** In fuzzy logic we consider a set \( X \) and a subset \( P \) of \( X \) which consists of a set of ordered pairs of the form:

\[ P = \{(x, \mu_P(x)); x \in [0, 1]\} \]

where \( \mu_P(x) \) is the value of the membership function of \( x \) in \( P \).

**Definition 2.** A domain of a fuzzy set \( P \) is the set of elements \( x \) whose membership function has positive values:

\[ \text{supp}(P) = \{x; \mu_P(x) > 0\} \]

**Definition 3.** Consider a fuzzy set \( \tilde{P} \in F(R) \). Then the \( \alpha \)-cuts of \( \tilde{P} \) are:

\[ [\tilde{P}]^\alpha = [P^\alpha_\cap(x), P^\alpha_\cup(x)] \]
According to the theorem of Negoită & Ralescu (1975) and Goetschel & Voxman (1986) the membership function and the \( \alpha \)-cut form of a fuzzy number \( P \) are equivalent and more specifically the \( \alpha \)-cuts (Equation (6)) represent \( \tilde{P} \), provided that the two functions are monotonous (\( P_{\alpha}(x) \) increasing, \( P_{\alpha}^+(x) \) decreasing) and \( P_{\alpha} \leq P_{\alpha}^+ \), for \( \alpha = 1 \).

**Definition 4.** Among the large number of existing fuzzy numbers, some types are of particular importance and are used extensively (Zimmermann 1992). Triangular, trapezoidal, and Gaussian fuzzy numbers are in the foreground.

A triangular fuzzy number has the form (Hanss 2005):

\[
\tilde{p} = \text{tfn}(\bar{x}, a_l, a_r)
\]  

where the parameter \( \bar{x} \), denotes the modal value of the fuzzy number, and \( a_l \) and \( a_r \) are the left-hand and right-hand worst-case deviations from the modal value.

In Figure 3(a) the straight section \( (a_l(\alpha) - a_r(\alpha)) \) is called the interval of confidence of the fuzzy number, while the ordinate \( \alpha \) is called the level of presumption in accordance with Kaufmann & Gupta (1991). Due to its rather simple membership function of the linear type, the triangular fuzzy number is one of the most frequently used fuzzy numbers (Hanss 2005).

A trapezoidal fuzzy number (Figure 3(b)), which is a generalization of the triangular fuzzy number, has the form:

\[
\tilde{p} = \text{tpfn}(a_l, b_l, b_r, a_r)
\]

A Gaussian fuzzy number (Figure 3(c)) has the form:

\[
\tilde{p} = \text{gfn}(\bar{x}, \sigma_l, \sigma_r)
\]

![Figure 2](image1.png) **Figure 2** | Methodology for identifying the problem and its solution.

![Figure 3](image2.png) **Figure 3** | Fuzzy numbers \( \tilde{p} \): (a) triangular form, (b) trapezoidal form and (c) Gaussian form.
As in the triangular fuzzy number again here, the modal value is denoted by the parameter $\tilde{x}$, and $\sigma_l$ and $\sigma_r$ denote the left-hand and right-hand spreads, corresponding to the standard deviations of the Gaussian distribution.

**Reduced transformation method**

According to Hanss (2002), given a problem with a number of $n$ independent parameters which are assumed to be uncertain, the parameters can be represented by fuzzy numbers $\tilde{p}_i$, $i = 1, 2, \ldots, n$, each of them decomposed (Figure 4) into a set $P_i$ of $m+1$ intervals $X^{(i)}_j$, $i = 1, 2, \ldots, n$, of the form:

$$P_i = \{X^{(0)}_i, X^{(1)}_i, \ldots, X^{(m)}_i\}, \quad i = 1, 2, \ldots, n$$

with

$$X^{(0)}_i = [a^{(0)}_i, b^{(0)}_i], \quad a^{(0)}_i \leq b^{(0)}_i, \quad i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, m$$

For the decomposition of a parameter into $\alpha$-cuts, the $\mu$-axis is subdivided into $m$ segments, equally spaced by $\Delta\mu = 1/m$. The $m+1$ levels of membership $\mu_i$ are given by:

$$\mu_j = \frac{j}{m}, \quad j = 1, 2, \ldots, m$$

The intervals of each level of membership $\mu_j$, $j = 0, 1, \ldots, m$, can now be transformed into arrays $\tilde{X}^{(j)}_i$, of the following form:

$$\tilde{X}^{(j)}_i = \{(a^{(0)}_i, \beta^{(0)}_i), (a^{(1)}_i, \beta^{(1)}_i), \ldots, (a^{(m)}_i, \beta^{(m)}_i)\}$$

with

$$a^{(i)}_i = (a^{(0)}_i, \ldots, a^{(i)}_i), \quad \beta^{(i)}_i = (\beta^{(0)}_i, \ldots, \beta^{(i)}_i)$$

Note that $a^{(i)}_i$ and $\beta^{(i)}_i$ are the lower and upper bounds of the interval $\tilde{X}^{(j)}_i$ at the membership level $\mu_i$ for the $i$th uncertain model parameter. Each of the transformed arrays $\tilde{X}^{(j)}_i$ has a total number of $(m+1-j)^n$ entries (Klimke 2003). Assuming the

![Figure 4](attachment:Figure_4.png) | Decomposition of the $i$th uncertain parameter into intervals.
system to be simulated is expressed by an arithmetical expression $F$ of the form:

$$\tilde{q} = F(\tilde{p}_1, \tilde{p}_2, \ldots, \tilde{p}_n)$$  \hspace{1cm} (13)

its evaluation is then carried out by evaluating the expression separately at the positions of the arrays using the conventional arithmetic for crisp numbers. Thus, if the output $\tilde{q}$ of the system can be expressed in its decomposed and transformed form by the arrays $Z = [a, b]$, $j = 1, 2, \ldots, m$, the $k$th element $kZ$ of the array $Z$ is given by:

$$kZ = F(kX_1, kX_2, \ldots, kX_m)$$  \hspace{1cm} (14)

where $kZ$ denotes the $k$th element of the array $X$. Finally, the fuzzy-valued result $\tilde{q}$ of the problem can be obtained in its decomposed form:

$$Z = [a, b], \quad j = 1, 2, \ldots, m$$  \hspace{1cm} (15)

by re-transforming the arrays $Z$ according to the recursive formulae:

$$a = \min_k (a^{i+1}, kZ) \quad \text{and} \quad b = \max_k (b^{i+1}, kZ),$$

$$j = 1, 2, \ldots, m - 1$$

and

$$a = \min_k (kZ) = \min_k (b) = b(m)$$

The reduction of fuzzy arithmetic to multiple crisp-number operations means that the transformation method can be implemented quite easily in an appropriate software environment.

**Fuzzy mathematical model**

Moving from classical logic to fuzzy set theory, the five (four for rectangular profile, $z = 0$) above parameters lead to the following rational expressions:

$$P(\bar{b}, \bar{y}, \bar{z}) = \bar{b} + 2\bar{y} \sqrt{1 + \bar{z}^2}$$

$$A(\bar{b}, \bar{y}, \bar{z}) = (\bar{b} + \bar{z})\bar{y}$$

$$R(\bar{b}, \bar{y}, \bar{z}) = \frac{(\bar{b} + \bar{z})\bar{y}}{\bar{b} + 2\bar{y} \sqrt{1 + \bar{z}^2}}$$

$$V(\bar{b}, \bar{y}, \bar{z}, \bar{S}_0, \bar{n}) = \left[ \frac{1}{\bar{n}} \left( \frac{(\bar{b} + \bar{z})\bar{y}}{\bar{b} + 2\bar{y} \sqrt{1 + \bar{z}^2}} \right)^{2/3} \cdot \bar{S}_0^{1/2} \right]$$

Therefore, the final expression of the discharge equation (Equation (3)) in analytical form will be:

$$Q(\bar{b}, \bar{y}, \bar{z}, \bar{S}_0, \bar{n}) = \left[ \frac{1}{\bar{n}} \left( \frac{(\bar{b} + \bar{z})\bar{y}}{\bar{b} + 2\bar{y} \sqrt{1 + \bar{z}^2}} \right)^{2/3} \cdot \bar{S}_0^{1/2} \right] \cdot (\bar{b} + \bar{z})\bar{y}$$  \hspace{1cm} (18)

**RESULTS AND DISCUSSION**

**Definition of problem scenarios**

In order to implement the methodology we will use two scenarios which refer to different open channel types with different uncertainty rates in the parameters of the problem. A necessary step first is to define the open channel substrates as well as the values and corresponding uncertainty rates of the Manning equation parameters.
In this regard, recommended Manning’s $n$ values for four different types of natural channels are given in Table 1. Chow (1959) suggested the three values of minimum, normal and maximum of $n$ for each kind of channel.

In Table 2 the cases taken into consideration and the uncertainty rates ($Ur$) that are accepted for the Manning roughness coefficient which were obtained according to Table 1 are listed.

As can be easily observed, all the parameters of the problem remain with constant values for all cases, except for the $n$ coefficient. This way comparison of the different scenarios is possible, showing the importance of the uncertainty of the $n$ factor in the calculations. As is easily understood, the computational process is quite complex and time-consuming. However, the transformation method can be implemented quite easily in an appropriate software environment. In this regard, computer algorithms were developed in the MATLAB environment and specifically in this work the algorithm as presented by Klimke (2003) for the reduced transformation method was used.

The two different scenarios taken into consideration are as follows:

- **Scenario 1**: This scenario applies the transformation method to estimate the discharge for the case 1a (as a computational example) of Table 2 and for $\alpha$-cut level 0. In this scenario all parameters present uncertainties.
- **Scenario 2**: This scenario follows the same procedure as in scenario 1 but takes into consideration only the uncertainty rate of Manning’s roughness coefficient. Therefore, the parameters $b$, $y$, $z$, $S_0$ have zero uncertainty rate ($Ur$ 0%) and the parameter $n$ has the uncertainty rate as shown in Table 2. Similarly in this case the transformation method was applied keeping the same values for all parameters.

**Scenario implementation**

The main purpose of the following calculations is the presentation of the methodology in the physical problem with real values in the parameters. The transformation method is applied in order to estimate the discharge for case 1a of Table 2, as an example, and for $\alpha$-cut level 0.

Starting from the initial and simple calculations and according to Equations (1)–(3) the following is obtained:

$$A = 122.16 \, \text{m}^2, \; P = 104.32 \, \text{m}, \; R = 1.17 \, \text{m},$$
$$V = 0.7 \, \text{m/s}^{-1}, \; Q = 86.70 \, \text{m}^3\text{s}^{-1}$$

| Case | $b$ (m) | %  | $y$ (m) | %  | $z$ | %  | $n$ | %  | $S_0$ | %  |
|------|--------|----|---------|----|-----|----|-----|----|-------|----|
| 1a   | 100    | 5  | 1.20    | 5  | 1.5 | 5  | 0.035 | 10 | 0.0005 | 4  |
| 1b   | 100    | 5  | 1.20    | 5  | 1.5 | 5  | 0.07  | 30 | 0.0005 | 4  |
| 2a   | 100    | 5  | 1.20    | 5  | 1.5 | 5  | 0.04  | 20 | 0.0005 | 4  |
| 2b   | 100    | 5  | 1.20    | 5  | 1.5 | 5  | 0.05  | 30 | 0.0005 | 4  |
The next step involves the transformation of the parameters of the problem into fuzzy triangular numbers. Thus, according to the uncertainty rates, presented in Table 2, of parameters and from Equation (4), the following triangular fuzzy numbers arise:

\[ \begin{align*}
\hat{b} &= \text{tfn}(100, 5, 5), \quad \hat{y} = \text{tfn}(1.2, 1.14, 1.26), \quad \hat{n} = \text{tfn}(0.035, 0.0315, 0.0385), \\
\hat{z} &= \text{tfn}(1.5, 1.425, 0.0385, 1.575), \quad S_0 = \text{tfn}(0.0005, 0.00048, 0.00052) 
\end{align*} \]

Having now transformed the parameters into triangular fuzzy numbers, it is possible to create the intervals of confidence for the fuzzy numbers \( \hat{b}, \hat{y}, \hat{z}, \hat{n} \) and \( S_0 \) for the \( \alpha \)-cut = 0 as follows:

\[ X_1 = [95, 105], \quad X_2 = [1.14, 1.26], \quad X_3 = [1.425, 1.575], \quad X_4 = [0.0315, 0.0385] \text{ and } X_5 = [0.00048, 0.00052] \]

The transformation will convert those intervals into arrays \( \hat{X}_i \) (with \( i = 1, \ldots, n \)) each of length \( 2^{n-5} = 32 \) for \( \alpha \)-cut = 0 using Equations (11) and (12).

The final results are given in Table 3, replacing the appropriate values in Equation (18). From these results and considering the minimum and maximum values of \( \hat{Q} \), the final interval \( Q = [67.32, 111.95] \) is obtained.

A similar approach is utilized for all \( \alpha \)-cuts of case 1a and for all cases in general as well as for both scenarios. The results for all cases and scenarios are listed in Table 4.

Figure 5 shows the final discharge estimations for both scenarios and for all cases at all \( \alpha \)-cut levels. It is very useful that the graphs are presented in this form as the variation of the final discharge estimation is more obvious for the same case (i.e. 1a, 1b, etc.) in the two different scenarios.

Observing the final intervals of Table 4 as well as their graphical representation in Figure 5, first it is important to highlight that applying scenario 1, where all the parameters have a degree of uncertainty, we would expect to see a big difference in the discharge results in relation to scenario 2, where only the \( n \) coefficient is uncertain. However, it was clear that although the variation of the discharge decreases with scenario 2, the overall problem uncertainty remains at high levels for all cases. According to the results, the differences in terms of the variation are approximately in the range of 5% to 15% for the

### Table 3 | Final calculations of \( \hat{Q} \) for case 1a

| \( Q(x_1, x_2, x_3, x_4, x_5) \) | Value | \( Q(x_1, x_2, x_3, x_4) \) | Value |
|---|---|---|---|
| 1 | \( Q(95, 1.14, 1.425, 0.0315, 0.00048) \) | 82.28 | 17 | \( Q(105, 1.14, 1.425, 0.0315, 0.00048) \) | 85.64 |
| 2 | \( Q(95, 1.14, 1.425, 0.0315, 0.00052) \) | 90.93 | 18 | \( Q(105, 1.14, 1.425, 0.0315, 0.00052) \) | 94.64 |
| 3 | \( Q(95, 1.14, 1.425, 0.0385, 0.00048) \) | 97.23 | 19 | \( Q(105, 1.14, 1.425, 0.0385, 0.00048) \) | 101.20 |
| 4 | \( Q(95, 1.14, 1.425, 0.0385, 0.00052) \) | 107.45 | 20 | \( Q(105, 1.14, 1.425, 0.0385, 0.00052) \) | 111.83 |
| 5 | \( Q(95, 1.14, 1.575, 0.0315, 0.00048) \) | 82.36 | 21 | \( Q(105, 1.14, 1.575, 0.0315, 0.00048) \) | 85.73 |
| 6 | \( Q(95, 1.14, 1.575, 0.0315, 0.00052) \) | 91.01 | 22 | \( Q(105, 1.14, 1.575, 0.0315, 0.00052) \) | 94.73 |
| 7 | \( Q(95, 1.14, 1.575, 0.0385, 0.00048) \) | 97.34 | 23 | \( Q(105, 1.14, 1.575, 0.0385, 0.00048) \) | 101.31 |
| 8 | \( Q(95, 1.14, 1.575, 0.0385, 0.00052) \) | 107.56 | 24 | \( Q(105, 1.14, 1.575, 0.0385, 0.00052) \) | 111.95 |
| 9 | \( Q(95, 1.26, 1.425, 0.0315, 0.00048) \) | 67.32 | 25 | \( Q(105, 1.26, 1.425, 0.0315, 0.00048) \) | 70.06 |
| 10 | \( Q(95, 1.26, 1.425, 0.0315, 0.00052) \) | 74.39 | 26 | \( Q(105, 1.26, 1.425, 0.0315, 0.00052) \) | 77.43 |
| 11 | \( Q(95, 1.26, 1.425, 0.0385, 0.00048) \) | 79.55 | 27 | \( Q(105, 1.26, 1.425, 0.0385, 0.00048) \) | 82.80 |
| 12 | \( Q(95, 1.26, 1.425, 0.0385, 0.00052) \) | 87.91 | 28 | \( Q(105, 1.26, 1.425, 0.0385, 0.00052) \) | 91.50 |
| 13 | \( Q(95, 1.26, 1.575, 0.0315, 0.00048) \) | 67.39 | 29 | \( Q(105, 1.26, 1.575, 0.0315, 0.00048) \) | 70.14 |
| 14 | \( Q(95, 1.26, 1.575, 0.0315, 0.00052) \) | 74.46 | 30 | \( Q(105, 1.26, 1.575, 0.0315, 0.00052) \) | 77.50 |
| 15 | \( Q(95, 1.26, 1.575, 0.0385, 0.00048) \) | 79.64 | 31 | \( Q(105, 1.26, 1.575, 0.0385, 0.00048) \) | 82.89 |
| 16 | \( Q(95, 1.26, 1.575, 0.0385, 0.00052) \) | 88.00 | 32 | \( Q(105, 1.26, 1.575, 0.0385, 0.00052) \) | 91.59 |
### Table 4 | Final discharge $Q$ ranges for all cases and for all $\alpha$-cuts

| $\alpha$-cut | Scenario 1 |  |  |  |
|----|---|---|---|---|
| | 1a | 1b | 2a | 2b |
| 0 | $[67.32, 111.95]$ | $[28.48, 71.97]$ | $[54.00, 110.20]$ | $[39.87, 100.75]$ |
| 0.25 | $[71.70, 104.98]$ | $[31.46, 62.65]$ | $[58.65, 99.96]$ | $[44.05, 87.71]$ |
| 0.5 | $[76.38, 98.47]$ | $[34.84, 55.03]$ | $[63.80, 90.95]$ | $[48.82, 77.04]$ |
| 0.75 | $[81.37, 92.39]$ | $[38.79, 48.69]$ | $[69.51, 82.97]$ | $[54.31, 68.17]$ |
| 1 | $[86.70, 86.70]$ | $[43.35, 43.35]$ | $[75.86, 75.86]$ | $[60.69, 60.69]$ |

| $\alpha$-cut | Scenario 2 |  |  |  |
|----|---|---|---|---|
| | 1a | 1b | 2a | 2b |
| 0 | $[78.82, 96.33]$ | $[33.34, 61.93]$ | $[63.22, 94.83]$ | $[46.68, 86.70]$ |
| 0.25 | $[80.65, 93.73]$ | $[35.38, 55.93]$ | $[65.97, 89.25]$ | $[49.54, 78.31]$ |
| 0.5 | $[82.57, 91.26]$ | $[37.69, 51.00]$ | $[68.96, 84.29]$ | $[52.77, 71.40]$ |
| 0.75 | $[84.58, 88.92]$ | $[40.32, 46.86]$ | $[72.25, 79.85]$ | $[56.45, 65.61]$ |
| 1 | $[86.70, 86.70]$ | $[43.35, 43.35]$ | $[75.86, 75.86]$ | $[60.69, 60.69]$ |

![Figure 5](image_url)  
*Figure 5 | Discharge estimations for both scenarios for cases (a) 1a and 1b, (b) 2a and 2b.*
four different substrates. This fact shows the great influence that the $n$ coefficient has on the Manning equation and consequently on the final estimation of the discharge. It is also noticeable that having all the parameters of the problem in uncertain mode, the problem becomes quite complex as the number of parameters for the case of a trapezoidal channel profile is large ($n = 5$) and the total degree of uncertainty of the problem reaches the point where the results are difficult to interpret. Even though the transformation method was applied to the problem, which gives the opportunity to avoid overestimation errors and gives satisfactory results, it turned out that a complex problem consisting of many uncertain parameters requires advanced judgment in terms of understanding the results and making final decisions.

In a more flexible context, and considering the results, it is suggested in the first stage of the study to take into consideration only the $n$ coefficient as an uncertain parameter. However, if it is absolutely necessary to treat other parameters of the problem in a fuzzy way, then it is proposed that the uncertainty rate given to each parameter should be based on the engineer’s skills and expertise. The uncertainty rate should be wisely defined without extreme definition values.

Nonetheless, we must admit that the presented methodology was applied to an ideal problem of trapezoidal cross-section open channel flow and its theoretical approach in the investigated problem. This methodology has a limitation in its application to natural open channels (i.e. streams, rivers) in which the cross-section is irregular and variable. In these cases, field measurements and reliable cross-sectional data of slopes, depth, substrates etc. along the flow are considered necessary for the accurate application of the methodology and the extraction of correct conclusions.

CONCLUSIONS
The main goal of this work was to implement an already known fuzzy method, namely the reduced transformation method, to a practical problem of hydraulic engineering described by the Manning equation. The variation of Manning’s roughness coefficient is quite high and accurate values cannot usually be obtained. It should be noted that the purpose of this work was not to find an optimal value of the Manning’s roughness coefficient for each type of channel, as has been done so far in the literature, but to include mainly the uncertainties of the $n$ coefficient in the calculations. Using the transformation method in its reduced form, the normal value of the coefficient for each case is introduced and variations are considered, thus achieving satisfactory accuracy in estimating the discharge. Finally, as the results show, the uncertainty of the roughness coefficient cannot be neglected since its influence on the results is quite high. Fuzzy methodology and more specifically the transformation method is suitable for engineering problems, described by uncertain parameters, because it is simple to apply and avoids overestimation errors. This way, better management plans and decisions can be carried out.

DATA AVAILABILITY STATEMENT
All relevant data are included in the paper or its Supplementary Information.

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