HIERARCHICAL SYSTEMS IN OPEN CLUSTERS

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Abstract. In this paper we study the formation, evolution and disruption of hierarchical systems in open clusters. With this purpose, N-body simulations of star clusters containing an initial population of binaries have been carried out using Aarseth’s NBODY4 and NBODY5 codes.

Stable triples may form from strong interactions of two binaries in which the widest pair is disrupted. The most frequent type of hierarchical systems found in the cluster models are triples in which the outer star is single, but in some cases the outer body is also a binary, giving a hierarchical quadruple. The formation of hierarchical systems of even higher multiplicity is also possible. Many triple systems are non-coplanar and the presence of even a very distant and small outer companion may affect the orbital parameters of the inner binary, including a possible mechanism of significant shrinkage if the binary experiences a weak tidal dissipation.

The main features of these systems are analyzed in order to derive general properties which can be checked by observations. The inner binaries have periods in the range $1 - 10^3$ days, although rich clusters may have even smaller periods following common envelope evolution. For triple systems, the outer body usually has a mass less than $1/3$ of the binary, but is sometimes a collapsed object with even smaller mass. The formation of exotic objects, such as blue stragglers and white dwarf binaries, inside hierarchical triple systems is particularly interesting. An efficient mechanism for generating such objects is the previous formation of a hierarchical system in which the inner binary may develop a very short period during a common envelope phase, which finally results in a stellar collision.
1. Introduction

In the past few years significant evidence for a large number of binary and multiple stellar systems in galactic clusters has been obtained from observational surveys (Mathieu et al. 1990; Mermilliod et al. 1992; Ghez et al. 1993; Mason et al. 1993; Mayer et al. 1994; Mermilliod et al. 1994; Simon et al. 1995), both for zero age main sequence and pre-main-sequence stars. For multiple systems the frequency estimated for star forming regions is up to 35% and for the field is up to 20%. However, the fraction of these systems detected in open clusters is smaller, although the improvement in observational techniques has increased in the past few years. Currently, the majority of multiple systems discovered in open clusters are triples and quadruples. These systems are usually highly hierarchical. Triple (or even higher multiplicity) systems are found in the Pleiades (Mermilliod et al. 1992), the Hyades (Griffin & Gunn 1981, Griffin et al. 1985, Mason et al. 1993), Praesepe (Mermilliod et al. 1994), M67 (Mathieu et al. 1990), and NGC 1502 (Mayer et al. 1994).

The majority of binary systems observed in open clusters are thought to be primordial, but there is no preferred formation mechanism for multiple systems (dynamical or primordial) at present. Most of the multiple systems studied are hierarchical because of the intrinsic stability of these systems. The origin of observed multiple systems has not been clear since the beginning of the study of these systems. Duquennoy (1988) analyzed a sample of 17 systems (14 triples and 3 quadruples) in the solar neighbourhood. He obtained a linear correlation between the logarithm of the inner and outer binary period. This was interpreted as an indication of preferential primordial origin for these systems. From a theoretical point of view, Boss (1991) has suggested that the formation of hierarchical systems occurs during the collapse of protostellar cores. Recently, Mermilliod et al. (1994) have found significant period ratios ($X = P_{\text{out}}/P_{\text{in}} \simeq 250$) in clusters which suggest a dynamical origin. Although the question of the origin of these hierarchical systems is far from being answered, we assume here that all the hierarchical systems formed in open clusters have a purely dynamical origin.

The formation and dynamical evolution of hierarchical systems in open clusters can be studied within the context of $N$-body simulations because complex interactions between stars can readily be followed in detail by numerical methods. This approach has recently been adopted (Aarseth 1996a, Kiseleva et al. 1996, Eggleton & Kiseleva 1996, Kiseleva 1996). In this paper the results of almost a hundred cluster models are analyzed with the purpose of studying the formation, evolution and final destinies of hierarchical systems in clusters. These models have been obtained using direct $N$-body integration by the standard workstation code NBODY5 (Aarseth...
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1985, 1994) and the more recent NBODY4 (Aarseth 1996b).

NBODY5 consists of a fourth-order predictor corrector scheme with individual time steps. In order to account for stellar evolution and mass loss (stellar winds and supernova events), we use the fast fitting functions of Eggleton, Fitchett and Tout (1989) and Tout (1990) for population I stars.

NBODY4 is based on the so-called Hermite scheme (Makino 1991) which forms the basis of a new generation of special-purpose computers (Makino, Kokubo & Taiji 1993). Mass loss by stellar winds is now treated according to a modified Reimers (1975) expression (Tout 1990). Chaotic tidal motion (Mardling 1995), tidal circularization (Mardling & Aarseth 1996), exchange of mass (Roche overflow) in binaries and magnetic braking are also included.

The models with NBODY5 have been studied on a DEC 2100 4/275 AXP system. All the calculations performed with NBODY4 were made at Cambridge on the HARP-2 computer.

2. Stability of hierarchical systems

The main aim of this paper is to study the formation and evolution of hierarchical systems in open clusters. Considered as isolated from field stars, such systems can survive over a long time. Small non-hierarchical systems, except in a few special cases, are always unstable in the long term (Marchal 1990). Even for hierarchical triple systems, stability is not an easy question. There are a number of criteria to identify triple systems as stable or unstable, obtained analytically or numerically (see e.g. Kiseleva 1996). For historical reasons, the models described here employ two criteria which are in fact fairly similar for most mass ratios involving stars. Thus the older NBODY5 code employs the Harrington (1975) criterion in the form

\[ F^{\text{min}} = A \left( 1 + B \log \left( \frac{1 + m_3/(m_1 + m_2)}{3/2} \right) \right) \]

with \( A = (2.65 + e)(1 + m_3/(m_1 + m_2))^{1/3} \) modified by the inner eccentricity, \( e \), according to Bailyn (1984) and \( B = 0.7 \). Here \( F^{\text{min}} \) denotes the critical ratio of the outer periastron distance of the mass \( m_3 \) to the inner apastron distance of \( m_1 + m_2 \).

In the NBODY4 code, we adopt the stability criterion of Eggleton and Kiseleva (1995, hereafter EK). In either case, we use their definition of stability: that a hierarchical triple system is stable if it persists continuously in the same configuration (which excludes exchange and disintegration).

The stability criterion of EK used for the NBODY4 models is based on the critical period ratio for stability, \( X_0^{\text{min}} = P_{\text{out}}/P_{\text{in}} \). It was derived from a numerical study which examined a wide range of parameters: (a) eccen-
tricities of both inner and outer orbits, $e_{in}$ and $e_{out}$; (b) relative inclinations of inner and outer orbits, from prograde to retrograde; (c) initial relative phase; (d) both mass ratios, $q_{in} = m_1/m_2 \geq 1$ and $q_{out} = (m_1 + m_2)/m_3$. This criterion can be written in two forms. Let $Y_{0}^{\text{min}}$ again be the critical ratio $F_{\text{min}}$. For computational purposes $Y_{0}^{\text{min}}$ is a more relevant parameter, since it depends only (to a certain level of approximation) on the two mass ratios. The period ratio $X_{0}^{\text{min}}$ also depends on the two eccentricities and is more useful for observed triples, since their periods and eccentricities, rather than semi-major axes, are normally determined by observation. Note that the inclination and phase are not included explicitly in the criterion. The two forms of the stability criterion are given by:

\[
(X_{0}^{\text{min}})^{2/3} = \left(\frac{q_{out}}{1 + q_{out}}\right)^{1/3} \frac{1 + e_{in}}{1 - e_{out}} Y_{0}^{\text{min}}, \tag{2}
\]

\[
Y_{0}^{\text{min}} \approx 1 + \frac{3.7}{q_{out}^{1/3}} - \frac{2.2}{1 + q_{out}^{1/3}} + \frac{1.4}{q_{in}^{1/3} q_{out}^{1/3}} - 1. \tag{3}
\]

This criterion discriminates between those triples that are likely to last a long time in the absence of strong external perturbations and those which may break up rapidly. Comparison of the two criteria used shows reasonable agreement, except for the less probable case of a massive outer component.

There are two main conditions for identifying the ‘birth’ of a hierarchical subsystem in our simulations:

- The inner binary must be hard and the outer component must also form a hard binary with the inner binary.
- The criterion (3) for hierarchical stability must be satisfied.

If both the above conditions are satisfied simultaneously the subsystem is considered as a hierarchical triple (or higher multiplicity) system and a number of parameters of this system are recorded. The subsequent treatment consists of combining the inner binary into one body to permit a two-body treatment (recursively if double hierarchy).

A hierarchical system is terminated when one of the following situations occurs:

- The stability criterion (3) is violated because, for example, of significant changes of $e_{out}$ due to small secular external perturbations.
- The external perturbation exceeds a critical value.
- Effects of stellar evolution become important or the predicted pericentre distance becomes too small.

In the last case, however, the same hierarchical system usually appears again, with slightly different parameters (e.g. during gentle mass loss) and we take this situation into account in the statistical analysis of results.
In the present calculations it is possible to produce triple systems, quadruple systems and double hierarchies. Triple systems consist of an inner binary and an outer single star, quadruple systems also have a binary as an outer body and in double hierarchies we have a nested system (binary + outer body + another outer body) with up to six stars. Hence, we can study even hierarchical sextuple systems. Recently Mardling and Aarseth (1996) have developed a new stability criterion based on a chaos description (Mardling 1995) which contains the inner and outer eccentricities explicitly as well as the mass ratio $m_3/(m_1 + m_2)$ for coplanar orbits. Except for large $e_{\text{out}}$, it gives values that are somewhat smaller than EK.

3. Cluster models

In order to obtain realistic results it is important to choose a general mass function (hereafter IMF). In all the models the IMF used is described by:

$$f(m) = \frac{0.3 X}{(1 - X)^{0.55}},$$

where $X$ is a random variable in $[0, 1]$. This IMF is a fit to Scalo’s (1986) results. For binaries we introduce a correlation $(m_1/m_2)' = (m_1/m_2)^{0.4}$, subject to the sum being constant, which yields mass ratios closer to unity than for random samples.

The initial NBODY5 models have a mass range of 0.1-15.0 $m_\odot$ for the single stars. Spherical symmetry and constant density are assumed, with the ratio of total kinetic and potential energy fixed at 0.25. All these models have random and isotropic initial velocities. Stars outside twice the classical tidal radius are assumed to escape and are removed from the calculations.

The models with NBODY4 have a mass range of 0.2-10.0 $m_\odot$ for the single stars. Initial coordinates and velocities are generated from an equilibrium King model in an external galactic field (Heggie and Ramamani 1992). In each code the cluster is assumed to be in a circular orbit in the solar neighbourhood, with a linearized tidal force added to the equations of motion (Aarseth 1985, 1994).

The probability of formation of hierarchical systems in a cluster with only single stars is very small; hence in order to produce a relatively large number of such systems, a significant initial binary population is needed (de la Fuente Marcos 1996). NBODY4 models can have an arbitrary number of binaries; for example models with 5100 stars have 100 primordial binaries. However, for NBODY5 models ($N = 100, 500$ and $1000$) two binary fractions, 10 % and 50 %, have been studied. Apart from the binary fraction some other parameters must be specified for the initial binary population; in particular, the semi-major axis and eccentricity must be chosen. For an initial population of hard binaries the semi-major axis is about $-GM^2/4EN$, 

where $G$ is the gravitational constant, $M$ is the total mass, $E$ is the energy. The semi-major axis is taken from a uniform distribution: $a_b = a^0_b 10^{-q}$, where $a^0_b$ is a parameter whose value is $1/N$ in units of the virial radius, and $q$ is equal to $X \log R$. $X$ is a random number uniformly distributed in the interval $[0, 1]$ and $R$ is a parameter. The latter gives the spread in semi-major axes and thus the spread in energies and periods. For example, in the $N = 5100$ models with NBODY4 the binary semi-major axis is in the range $0.1 - 100$ AU. For NBODY5 models, all the runs have been repeated with $R = 5, 10, 50, 100$, with maximum values of the semi-major axes in the range $230 - 1000$ AU. Moreover, all the runs have been computed twice: one time with the chain regularization excluded and another time with this option included. Chain regularization (Mikkola & Aarseth 1990, 1993) is a numerical treatment of close encounters in compact subsystems in which the external perturbation due to nearby stars is taken into account.

4. Results

In this section we discuss some representative results. We are mainly concerned with results which can be checked directly with observational ones.

4.1. OVERALL EVOLUTION

Although there are three sets of different simulations, we find some common features. As expected, the majority of the systems formed are triple, but some NBODY4 models show almost the same tendency to form quadruples. Hierarchies do not form at a preferred cluster evolution stage. Usually in clusters with primordial binaries the first hierarchies appear at about $0.025 - 0.05 \; T_{cl}$, where $T_{cl}$ is the total life-time of the cluster, and in the cluster remnant (when $N$ is a few tens) there are sometimes long-lived hierarchies. The distribution of inclinations for hierarchies is fairly symmetrical, with a pronounced peak centred on $90^\circ$ (Kiseleva 1996). In large clusters at a late stage of the evolution, many short-lived systems form via repetitive triple-binary and triple-triple exchanges. Sometimes a hierarchical system leaves the cluster, escaping before the disruption of the system takes place. Poor clusters ($N = 100$) with wide binaries do not show any tendency to form hierarchical systems since we require the outer orbit to be a hard binary. Sometimes the outer body is a collapsed object (white dwarf). The largest number of hierarchies observed at the same time is five.

First we present some results for NBODY5 models. In the triple systems which form, the inner binary is usually not primordial. The quadruple systems found in poor clusters are more eccentric in models with no chain treatment. These systems form typically at the late stages of cluster evolution in rich clusters and inside the cluster core. For systems which include
the chain treatment the quadruple systems formed in rich clusters are more eccentric. They also form preferentially inside the core. For triple systems the outer star usually has a mass less than one-third of the binary mass and sometimes 1/15, when the outer body is a collapsed object. The typical life-time for the systems formed is a few million years, or $10^3$ outer periods. The inner binaries of the hierarchies have periods from a few days to $10^3$ days.

NBODY4 models also show preferential formation of triples but now quadruples are relatively abundant (up to 40% of all hierarchical systems in $N = 5100$ models). Quadruples tend to form at late stages of the cluster evolution. On the other hand, the corresponding percentage of systems with primordial inner binaries is significantly greater than for the NBODY5 models (about 70% vs 40%) because of the shorter periods. Recent models include a new technical feature of so-called double hierarchies. In such configurations an outer body (single or binary) is added to an existing stable triple or quadruple, provided the stability criterion is satisfied. These systems tend to occur during very late stages when the core has expanded significantly; however, they tend to be short lived in the models studied so far with termination due to significant external perturbation.

The formation of hierarchical systems may have interesting observational consequences (Aarseth 1992). In such models, after the formation of a hierarchical triple system the pericentre of the outer body is usually close enough to the inner binary to perturb its eccentricity. This systematic perturbation often promotes a stellar collision of the inner binary. This process has been observed in several NBODY5 models with $N = 1000$ and 50% primordial binaries, and is quite common in NBODY4 models which contain binaries with periods of days.

4.2. THE PERIOD – ECCENTRICITY PLANE

One of the most interesting diagrams for displaying the results of binary observations is the plot of the eccentricity versus the logarithm of the period because it provides more insight into binary astrophysics than other distributions of orbital elements and is expected to be equally useful in the study of hierarchies. We have plotted the eccentricity of the outer body (single or binary) against the logarithm of its period (in days). The results depend on the type of model and also on the membership and initial orbital elements of the binary population. As remarked above, our models can be classified into three different groups but several features are common to all:

- A lower cut-off period below which no orbits are observed.
- An upper cut-off period above which no orbits are observed.
• A zone of forbidden eccentricities for systems in which the inner binary is not primordial.

Figure 1. Eccentricity – period diagram of the outer binary in days. Left panel is for an NBODY5 model, the right one is for an NBODY4 model. High eccentricity systems are short-lived in all the models.

The short period cut-off. Its value is dependent on the initial values of the initial primordial binary parameters. For the NBODY4 models common envelope evolution is included so a few systems are observed below the cut-off period; most of these suffer a physical collision which sometimes produces a blue straggler. For models without Roche overflow and common envelope evolution wide inner binaries implies longer outer periods; i.e. models with very hard binaries form hierarchical systems which permit shorter periods for the outer body. Moreover, the cut-off is larger if the inner binary is not primordial. In fact, even for the NBODY4 models, we observe a clear cut-off in the period ratio with no exceptions, due to common envelope evolution or collisions. Another interesting feature is the dependence on the nature of the outer body (single or binary). The cut-off is larger for systems in which the outer body is a single star.

The long period cut-off. This is mainly due to the stability limit against perturbations from the neighbouring stars. Models with wider binaries show higher values of the upper period cut-off. The width of the period distribution depends strongly on the initial distribution of the semi-major axes of the binaries; small binaries generate sharp distributions of periods for the outer body. The high cut-off also depends on the nature of the inner binary. Systems with an inner primordial binary show slightly higher periods and the same is found for systems in which the outer body is also a binary.
The forbidden region. A dependence of the eccentricity distribution on the character of the inner binary is observed. When the inner binary is not primordial no systems are observed with eccentricities smaller than 0.20. Compared with real systems, this can be considered as a limit below which all the systems observed should contain a primordial binary. This must reflect the fact that non-primordial binaries, although usually formed by exchanges or disruptions of primordial systems, are wider and less energetic than primordial ones. The existence of real systems below this limit with a non-primordial inner binary may be explained as an effect of tidal interaction. There is also a correlation between the membership of the cluster and the lower eccentricity observed. Finally, systems formed in poor clusters are more eccentric on average.

4.3. THE (LOG P_B, LOG P_O) PLANE

In order to compare our results with those from the observational literature, we consider the plane log P_O vs log P_B, where P_O is the outer orbital period and P_B the inner period. We compare mainly with a sample of triples (fig. 2) in the solar neighbourhood by Duquennoy (1988). He finds a linear correlation for a sample of 13 triples with components of solar type. The slope of the straight line is 0.68 with a correlation coefficient of 0.91. From the relation between the periods he concludes that triple systems are rarely formed by capture, but rather by fragmentation processes, because capture would produce random combinations of periods. He suggests dynamical evolution as another possible cause in order to reproduce the observed distribution of periods. However, triples in our models have a purely dynamical origin and we find a similar behaviour.

For an NBODY5 model with 1500 stars and 500 primordial binaries (fig. 3) we obtain a slope of 0.68 ± 0.09. For an NBODY4 model with 5100 stars and 100 primordial binaries we obtain a slope of 0.53 ± 0.03. Both results include systems in which the outer body is either a single or a binary. However, the number of quadruple systems in the first case is about 12 % (4:34) and about 45 % in the second. We analyze the two subsamples (triples and quadruples) in the latter model. For triple systems we obtain a slope of 0.63574 ± 0.00008 and for quadruple systems we find 0.4219 ± 0.0001. The results suggest that there are two kinds of correlations depending on the nature of the outer body.

Figure 3 shows the plot for an NBODY4 model; the upper contour is clearly linear and is connected with the stability criterion. Our results seem to be compatible with those from the observations in spite of our theoretical sample of hierarchical systems containing a wide range of masses and different stages of evolution. The lack of systems with large outer periods
but small inner period is of theoretical and observational interest. This is probably connected with the formation mechanism in which other stars act as perturbers during wide encounters. Thus it has been noted from the calculations that hierarchical formation mainly takes place when two or more binaries suffer a close encounter. It would be desirable to compare our re-

Figure 2. Period diagram for the observational sample of multiple systems listed in Duquennoy (1988). Note the larger outer periods in comparison with fig. 3 because most of them are estimated parameters in the original paper.

Figure 3. The left figure shows the plane $\log P_O$ vs $\log P_B$ for a model studied with NBODY5. The right figure is for a model performed with NBODY4. The linear upper contour is due to the criterion for stability. Note that inner binaries in NBODY4 have smaller periods due to the astrophysical phenomena included (see the text).
there are only three such systems with orbital elements for both the inner binary and the outer body. Two are in the Hyades, vB 75 and $\mu$ Orionis; the latter is a quadruple. The other system is a quadruple in NGC 1502, SZ Cam.

4.4. THE (LOG $P_O$, $M_B/M_O$) PLANE

Although this plane (fig. 4) also contains much astrophysical information, it is not easy to obtain the relevant data from observations. Here $M_B$ is the mass of the inner binary and $M_O$ is the mass of the outer body. For the NBODY4 models the majority of the systems formed fall in the mass ratio range $1–2.2$. There are a few systems with higher ratios and the upper cut-off is about 14. The number of systems with mass ratios below 1 is very small. As for the other correlations there is a clear distinction between quadruple systems and triples. The range of mass ratios for quadruples is $0.4–2.2$, which is expected because in most cases both binaries are primordial. For triple systems there are several systems with ratios greater than 2.2 but none below 0.8. As regard the nature of the inner binary, systems with no primordial binaries have a range $0.8–3.0$ with only one system above (14). For systems with a primordial inner binary the range is wider.

![Figure 4](image)

*Figure 4.* Mass ratio versus logarithm of the period of the outer body for an NBODY5 model (left) and an NBODY4 model. Note the sharp mass ratio for quadruple systems in the latter.

5. Tidal dissipation in triple systems

Neither primordial binaries, nor those that are formed in clusters later (for example by exchange), have exactly circular orbits to start with. However,
tidal friction will circularise binary orbits on a rather short time-scale compared with the nuclear time-scale, provided that at least one star of the binary has a radius comparable to the separation between binary components and the dynamical influence of other stars on the binary orbit is negligible. The situation can be very different for binaries within hierarchical triple systems, especially those where the inner and outer orbits are nearly perpendicular (as our numerical simulations shows, not a rare situation). It can be shown analytically and numerically (Marchal 1990, Heggie 1996, Kiseleva 1996) that for non-coplanar triple systems there is a quasi-periodic change of the inner eccentricity (on a time-scale $\sim P_{\text{out}}^2/P_{\text{in}}$) during which it reaches a maximum value $e_{\text{in}}^{\text{max}}$. This value only depends on the inclination $i$ between the two orbital planes; other parameters affect only the time-scale. If $i \approx 90^\circ$, $e_{\text{in}}^{\text{max}} \approx 1$ and the two stars may collide or suffer a strong tidal interaction. This effect cannot be neglected in numerical studies of triple stars in clusters. The combined influence of tidal friction and of the third component on the binary orbit may produce interesting and even dramatic results, such as for example a severe shrinking of the orbit.

In order to investigate the interaction between tidal friction and the gravitational dynamics of point masses in a hierarchical triple system we consider two well-known isolated triples $\beta$ Per (Algol) and $\lambda$ Tau, which have well-defined orbital parameters. The influence of the distant third body induces eccentricity in the orbit of the close pair. Even if the third body is distant, as in $\beta$ Per ($P_{\text{out}}/P_{\text{in}} \approx 237$), its effect need not be small. Because the observed $i \approx 100^\circ$ (Lestrade et al. 1993), in the absence of a dissipative process like tidal friction the eccentricity of the inner pair should cycle between 0 and $\sim 1$, on a time-scale of $\sim 10^3$ years (fig 5, left panel). Since the observed eccentricity is $\sim 0$, and presumably has been small throughout its current phase of Roche overflow, tidal friction must act successfully in this system. The right panel of fig. 5 shows how tidal friction (included in the equations of motion of the three-body problem as described by Eggleton 1996) can reduce the effect of the distant companion on the inner eccentricity. The recipe for tidal friction dissipates orbital energy but conserves angular momentum; it decreases the semi-major axis and orbital period of the inner binary, and hence increases the period ratio. In many cases this effect can be rather insignificant. For $\beta$ Per (and other similar systems with high inclination) we find a rather narrow range of tidal friction values which is not strong enough to prevent significant quasi-periodic variations of the inner eccentricity, and yet is strong enough to decrease sharply the binary semi-major axis every time the eccentricity reaches its local maximum. Figure 6 shows this effect. This is a possible mechanism for the production of close binaries and/or other exotic objects, particularly in clusters.
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Figure 5. The effect of tidal friction on long-term modulations of the inner eccentricity in β Per triple systems. The parameter $\lambda$ defines the strength of tidal friction and is described in Eggleton (1996).

Figure 6. The possible shrinking of the binary orbit in a triple system like β Per under the influence of rather weak tidal friction.

6. Conclusions

This work has allowed us to discuss various aspects of hierarchical systems in open clusters. We have found that it is, indeed, possible to reproduce some observational properties (such as the linear correlation of periods) of hierarchical systems as well as to predict some characteristics of these systems for observations. Although the results described in this paper are encouraging, it is still not clear how the fraction of primordial binaries influences the formation rate of hierarchical systems and how the tidal effects, which were only discussed briefly here, affect the stability of systems with short periods. These questions will be left for future developments.
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