Roche-lobe Overflow in Eccentric Planet–Star Systems

Fani Dosopoulou1, Smadar Naoz2,3, and Vassiliki Kalogera1

1 Center for Interdisciplinary Exploration and Research in Astrophysics (CIERA) and Department of Physics and Astronomy, Northwestern University, 2145 Sheridan Road, Evanston, IL 60201, USA; FaniDosopoulou2012@u.northwestern.edu
2 Department of Physics and Astronomy, University of California, Los Angeles, CA 90095, USA
3 Mani L. Bhaumik Institute for Theoretical Physics, Department of Physics and Astronomy, UCLA, Los Angeles, CA 90095, USA

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Abstract

Many giant exoplanets are found near their Roche limit and in mildly eccentric orbits. In this study, we examine the fate of such planets through Roche-lobe overflow as a function of the physical properties of the binary components, including the eccentricity and the asynchronicity of the rotating planet. We use a direct three-body integrator to compute the trajectories of the lost mass in the ballistic limit and investigate the possible outcomes. We find three different outcomes for the mass transferred through the Lagrangian point $L_1$: (1) self-accretion by the planet, (2) direct impact on the stellar surface, and (3) disk formation around the star. We explore the parameter space of the three different regimes and find that at low eccentricities, $e \lesssim 0.2$, mass overflow leads to disk formation for most systems, while, for higher eccentricities or retrograde orbits, self-accretion is the only possible outcome. We conclude that the assumption often made in previous work that when a planet overflows its Roche lobe it is quickly disrupted and accreted by the star is not always valid.

Key words: binaries: close – binaries: general – planetary systems – planet–star interactions – planets and satellites: gaseous planets – stars: kinematics and dynamics

1. Introduction

Many of the shortest-period exoplanets, covering a large mass range, from Earth-size to Jupiter-size (hot Jupiters), are found in mildly eccentric orbits and nearly at or even interior to their Roche limit. This suggests that many planets are near Roche-lobe overflow (RLOF; e.g., Figure 1 in Jackson et al. 2017). For example, Li et al. (2010) suggested that WASP-12 b is in the process of RLOF (see also Patra et al. 2017). The presence of many gas giants in similar orbits suggests that RLOF may be common among exoplanets.

Even if a planet is not currently close enough to its host star to overflow, tidal interactions may eventually become important since planets are inside $\sim 0.1$ au and tidal orbital decay can drive the planet to the Roche limit (e.g., Levrad et al. 2009; Jackson et al. 2009; McQuillan et al. 2013; Poppenhaeger & Wolk 2014). This process tends to circularize the planet’s orbit. Furthermore, high eccentricity migration predicts that a large fraction of the planets may get disrupted. Specifically, the eccentric Lidov–Kozai mechanism (e.g., Naoz 2016) can result in disrupted Jupiters (e.g., Naoz et al. 2012; Petrovich 2015). These planets can plunge in, and cross the Roche limit, with extremely large eccentricity or with moderate ones that may result from planet–planet interactions (e.g., Antonini et al. 2016; Petrovich & Tremaine 2016; Hamers et al. 2017).

Most previous studies have assumed that whenever a planet crosses the Roche limit, it is quickly disintegrated and its material is accreted by the star (e.g., Jackson et al. 2009; Metzger et al. 2012; Schlafman & Winn 2013; Teitler & Königl 2014; Zhang & Penef 2014). However, recent studies showed that hot Jupiters might be only partially consumed, leaving behind lower-mass planets (Valsecchi et al. 2014, 2015; Jackson et al. 2016). These studies suggest the need for further investigation of the planetary system’s orbital evolution due to RLOF in eccentric systems, e.g., using secular evolution equations (e.g., Dosopoulou & Kalogera 2016a, 2016b). The accuracy of predictions for the fate of gas giants and the final orbits of their remnants depends on the trajectory that the mass lost through the Lagrangian point $L_1$ follows after its ejection.

In an eccentric system, the latter is determined by the eccentricity of the system as well as the masses, radii, and spins of the planet–star components. Depending on the formation history, giant exoplanets can be tidally locked or rotate asynchronously in their orbits around the star.

In this paper, we study short-period eccentric planet–star systems at the onset of RLOF. We assume that RLOF takes place at each subsequent periastron passage. We investigate the possible outcomes of mass overflow for a system with a generically asynchronous planet. We show that for a given initial eccentricity, depending on the binary mass ratio, the planet rotation rate, and the star radius, RLOF can lead to three different possible outcomes for the lost matter. These are (1) self-accretion by the planet, (2) direct impact on the stellar surface, and (3) disk formation around the star. These three different regimes do not uniquely lead to the disruption of the planet or the accretion of the lost matter by the star as is often assumed in previous studies. Here we explore the parameter space of these regimes calculating the trajectories of the lost particle in the ballistic limit.

This paper is organized as follows. In Section 2, we describe the system and the adopted methodology. In Section 3, we explore the system’s parameter space, identifying the regions that lead to different mass overflow outcomes. We conclude with Section 4.

2. Roche-lobe Overflow in Planet–Star Systems

2.1. System Set-up

We follow the formalism developed by Sepinsky et al. (2010) and consider a generically eccentric binary consisting of a planet with mass $M_p$ and radius $R_p$ and a star with mass $M_* \equiv M_*$ and radius $R_*$. We define the binary mass ratio as $q = M_p/M_*$ and, in what follows, we consider planet–star systems with
mass ratio \( q \) in the range of \(-6 \leq \log q \leq -2\). We assume that the planet rotates uniformly and with constant spin \( \Omega_p \) parallel or anti-parallel to the orbital angular velocity \( \Omega_{orb} \). We normalize \( \Omega_p \) by the orbital angular velocity at periastron \( \Omega_{orb,p} \), i.e.,

\[
\Omega_p = f_p \Omega_{orb,p},
\]

where now \( f_p \) defines the degree of pseudo-asynchronicity of the planet at periastron and (Equation (2.32) in Murray & Dermott 1999)

\[
\Omega_{orb,p} = \frac{2\pi (1 + e)^{1/2}}{P_{orb} (1 - e)^{3/2}},
\]

where \( R_{orb} \) is the binary orbital period and \( e \) is the binary eccentricity. For simplicity, from now on we refer to \( f_p \) as the planet’s degree of asynchronicity. We consider both prograde (\( f_p > 0 \)) and retrograde (\( f_p < 0 \)) planetary orbits. When \( f_p = 1 \), the planet is pseudo-synchronously rotating at periastron, while \( f_p < 1 \) denotes a sub-synchronously rotating planet.

We consider short-period systems where the planet undergoes RLOF and mass is lost through the Lagrangian point \( L_1 \). Sepinsky et al. (2007) investigated the existence and properties of equipotential surfaces and Lagrangian points in asynchronous, eccentric binary star and planetary systems. They showed that in an eccentric orbit the position of the Lagrangian point \( L_1 \) depends on the phase along the orbit and it is the smallest when the two bodies are at periastron (e.g., Figure 8 in Sepinsky et al. 2007). This implies that mass overflow is more likely to reoccur at each subsequent periastron passage. In what follows, we assume RLOF only at periastron and use the method developed by Sepinsky et al. (2007) to calculate the position of the Lagrangian point \( L_1 \) at periastron, \( x_{L_1} \). Sepinsky et al. (2007) also provided fitting formulae for the volume-equivalent Roche-lobe radius appropriate for asynchronous eccentric systems as a function of the binary mass ratio and the degree of asynchronicity of the overflowing body. Sepinsky et al. (2007) verified that for the low mass ratio systems considered here, the simpler formula given in Eggleton (1983) for the volume-equivalent Roche-lobe radius at periastron, \( R_{L_1} \), is still a good approximation for eccentric asynchronous systems (e.g., Figure 9 in Sepinsky et al. 2007). Thus, we use

\[
R_{L_1}(q) = r_p \frac{0.49 q^{2/3}}{0.6 q^{2/3} + \ln(1 + q^{1/3})} = r_p R_l(q),
\]

where \( r_p = a(1 - e) \) is the periastron distance and \( a \) is the binary semimajor axis. We assume the following planet mass \( M_p \) and radius \( R_p \) relation (Lissauer et al. 2011; Howard 2013)

\[
R_p = \begin{cases} 
R_l \left( \frac{M_p}{M_*} \right)^{1/2.06} & \text{if } M_p < M_*, \\
R_l & \text{if } M_p > M_*
\end{cases}
\]

where \( R_l, M_* \) are the Earth radius and mass and \( R_l, M_l \) are the Jupiter radius and mass. We also test two different mass–radius relations: (1) \( R_p = R_l(M_p/2.7M_*)^{1/3} \) (Wolfgang et al. 2016) and (2) \( R_p = (3M_p/4\pi\rho_p)^{1/3} \), where \( \rho_p \) is the planet density in the range of \( \rho_p = 0.1-10 \, \text{gr cm}^{-3} \). We find that our results presented in Section 3 are not affected by the mass–radius relation adopted.

Given the initial eccentricity of the system, \( e_0 \), the initial semimajor axis, \( a_0 \), is calculated such that the planet undergoes RLOF at periastron, i.e., for \( R_p = R_{L_1} \), we find \( a_0 = R_p/(r_L(q)(1 - e_0)) \). Notice that according to Equation (4), for \( M_p > M_* \), the initial periastron distance of the planet decreases with the mass ratio. For low initial eccentricities, \( 0.01 \leq e_0 \leq 0.2 \), the value of the initial semimajor axis lies in the range of \( 0.0047 \, \text{au} \leq a_0 \leq 0.012 \, \text{au} \).

### 2.2. Ballistic Limit

We assume that a particle with negligible mass \( M_{loss} \ll (M_p, M_*) \) is ejected from the planet at periastron from a point with relative distance \( x_{L_1} \) to the planet center of mass and with a velocity \( V_{loss} \) relative to an inertial reference frame. The planet and the star are treated as rigid spheres of uniform density and, as a first approximation, we can calculate the motion of the three bodies by modeling the system as three point masses moving only under the effect of gravity. In the ballistic limit and as long as \( M_{loss} \ll (M_p, M_*) \) the trajectories of the bodies are independent of the actual value of the lost matter.

The ejection velocity \( V_{e_j} \) of the mass transferred through the Lagrangian point \( L_1 \) is given by

\[
V_{loss} = V_p + \Omega_p \times x_{L_1,p} + V_{e_j},
\]

where \( V_p \) is the planet orbital velocity at periastron with magnitude \( V_p \approx \sqrt{GM_*(1 + e)/R_p} \), \( \Omega_p \times x_{L_1,p} \) is the planet rotational velocity at \( L_1 \), and \( V_{e_j} \) is the ejection velocity of the particle relative to the planet center of mass. We note here the important relation \( x_{L_1} > R_{L_1} = R_p \) (e.g., Sepinsky et al. 2007). This relation is key to our calculations since the ejection of the lost mass from the planet radius, \( R_p = R_{L_1} \), would lead in principle the system to self-accretion (described in Section 2.3).

The ejection velocity \( V_{e_j} \) depends on the type of mass loss. Overflow models often approximate the donor as having a discrete outer boundary at the photosphere. However, the upper atmospheres of close-in planets can be very hot and taper off into space. Here we make an estimate of the escape speed assuming an isothermal atmospheric mass loss through \( L_1 \). We compare this speed to the planet’s orbital velocity and show that it is reasonable to neglect the thermal speed contribution to the particle ejection velocity. The isothermal sound speed is defined as \( V_{th} = \sqrt{k_B T/\mu} \), where \( k_B \) is the Boltzmann constant, \( T \) is the atmospheric temperature, and \( \mu \) is the average mass of atmospheric particles. At the planet’s photosphere, one finds \( V_{e_j} \ll V_{th} \), while near \( L_1 \), the gas can be assumed to reach the isothermal sound speed \( V_{e_j} \approx V_{th} \). The planet photospheric temperature \( T_p \) can be estimated by assuming that the planet dayside emits as a blackbody at radiative equilibrium with its star, i.e., \( T_p = T_s \sqrt{(R_*/(2\pi^2a))} \), where \( T_s \) is the stellar effective temperature. For example, for a Sun-like star with \( R_s = 1 \, R_\odot \), \( M_\odot = 1 \, M_\odot \), and \( T_s = 6000 \, K \), this gives for \( a \approx 0.05 \) au a planet temperature \( T_p = 1500 \, K \). For an atmosphere composed entirely of molecular hydrogen, we have \( \mu = 2 \) amu when \( T_p < 2000 \, K \). Using these values, the isothermal sound speed is \( V_{th} \approx 3500 \, m \, s^{-1} \). The planet orbital velocity at periastron \( V_p \), assuming a small eccentricity, is \( V_p \approx \sqrt{GM_*/a} \approx 1.3 \times 10^5 \, m \, s^{-1} \), i.e., \( V_{e_j} \approx 0.01 V_p \). Note that, as long as \( V_{e_j}/V_p \approx 0.01 \), the angular momentum exchange between the
particle and the binary during transport is unaffected by changes in $V_\infty$. Given these considerations, here we assume no contribution to the particle ejection velocity from the thermal speed of the mass elements in the planet atmosphere and set $V_\infty = 0$.

We note that atmospheric mass loss is neither entirely adiabatic nor isothermal. In principle, there is also a transition between RLOF and evaporative mass loss. In the latter, the temperature in the upper atmosphere can greatly exceed $\sim 10^4 \text{ K}$ and the outflow becomes transonic (Murray-Clay et al. 2009; Trammell et al. 2011). However, recent studies have shown that for the vast majority of the systems considered here, if atmospheres are escaping at all, it is via RLOF (Jackson et al. 2017). A generalization of our calculations would account for the thermal speed in the planet atmosphere but this will add an extra dimension to the parameter space and it is beyond the scope of this paper.

In what follows, we use a direct three-body code (developed by Sepinsky et al. 2010) to integrate the equations of motion forward in time and calculate the ballistic trajectories of the lost particle.

### 2.3. Possible Outcomes of Roche-lobe Overflow

We evolve the planet–star–particle system for one orbit. We keep track of the particle distance to the planet and the star as a function of time and, as depicted in Figure 1, we find three possible outcomes for the particle lost through $L_1$ at periastron. These are that (1) the lost particle is self-accreted by the planet within one orbit (SA), (2) the lost particle directly impacts the stellar surface within one orbit (DI), (3) the lost particle undergoes no impact with the planet or the star and its ballistic trajectory intersects itself within one orbit leading possibly to the formation of a disk around the star through interactions with subsequently lost particles (DISK).

In Figure 1, we show characteristic examples of the three aforementioned mass overflow outcomes for different initial conditions $(q, f_p, e_0)$ and for star radii $R_\star = (1.0, 1.5) R_\odot$, and mass $M_\star = 1 M_\odot$. As shown in Figure 1, in the two systems in the left and middle panels, the lost particle follows the same ballistic trajectory. However, in these systems, a star with a radius $R_\star = 1 R_\odot$ leads to the formation of a DISK, while a star with a larger radius $R_\star = 1.5 R_\odot$ results in DI. In an initially more eccentric system, the lost particle has a larger ejection velocity, which leads to SA.

We also explore retrograde systems for which $f_p < 0$. In this case, we find that, as expected from Equation (5), the particle is always moving faster than the planet at the point of ejection. This leads always to SA independent of the eccentricity. In what follows, we focus on prograde orbits ($f_p > 0$).

### 3. Results

In this section, we explore the system parameter space and investigate the dependence of the mass overflow outcome on the initial conditions $(q, f_p, e_0)$ as well as the mass $M_\star$ and radius $R_\star$ of the star.

In Figure 2, we explore the mass ratio and asynchronicity parameter space of the system $(q, f_p)$ for two different initial eccentricities $e_0$ and three different types of stars: (1) an ultracool dwarf star with $M_\star = 0.08 M_\odot$ and radius $R_\star = 0.11 R_\odot$ similar to the one in the recently discovered TRAPPIST-1 planetary system (Gillon et al. 2017), (2) a Sun-like star with mass $M_\star = 1 M_\odot$ and radius $R_\star = 1.0 R_\odot$, and (3) a subgiant star with mass $M_\star = 1 M_\odot$ and radius $R_\star = 1.5 R_\odot$.

As shown in Figure 2 for a given eccentricity, $e_0$, the values of $f_p$ and $q$, which separate the SA and DISK/DI regimes do not depend on the mass $M_\star$ and radius $R_\star$ of the star. At low eccentricities, the most likely outcome is DISK/DI, while at higher eccentricities, $e \gtrsim 0.2$, SA takes over. Although increasing the star radius, $R_\star$, increases the parameter space for DI, for the cases we consider in Figure 2, the formation of a DISK appears to be a more common outcome than DI.

For the systems we consider in Figure 2, at a given $R_\star$, there are values of $q$ such that $R_\star > r_p - x_{pe}$, i.e., the planet comes into contact with the star. The parameter space for these types of systems is shown in Figure 2 as a gray area (CON). For planets less massive than Jupiter, the periastron distance increases with a decreasing mass ratio. This implies that for a small enough mass ratio the planet comes into contact with the stellar surface. This is depicted as the lower gray area in the right column in Figure 2, where, for the subgiant star, we estimate this lower limit to be $\log q \sim -5.25$. For planets more massive than Jupiter, we adopted an upper limit for the planet
radius \((R_p = R)\). This means that for these planets the periastron distance decreases with an increasing mass ratio. Thus, for large mass ratio above a threshold the planet and the star come into contact. This is depicted as the upper gray area in the right column in Figure 2 where, for the subgiant star, this upper threshold becomes \(\log q \sim -2.6\).

In Figure 3, we investigate the parameter space that separate the SA and DI/DISK regimes. As depicted in Figure 2, the values of \(f_p\) and \(q\) at which the transition to SA occurs do not depend on \(M_\ast\) and \(R_\ast\). Thus, for simplicity, we set in Figure 3 \(R_\ast = 0\) and \(M_\ast = 1\ M_\odot\), and explore the \((q, f_p)\) parameter space as a function of the initial eccentricity, \(e_0\). As Figure 3 indicates, for any initial eccentricity, DI/DISK can occur only for sub-synchronously rotating planets \(f_p < 1\). Increasing the initial eccentricity restricts the parameter space region for DI/DISK. We find that the DI/DISK regime dominates at low eccentricities, \(e_0 \lesssim 0.2\), while for higher eccentricities or retrograde orbits the only possible outcome is SA.

As shown in Figure 3, DI/DISK can occur only for \(e_0 < 0.2\). Within this restricted regime, we identify in Figure 4 two regions, depending on the star radius \(R_\ast\) and the mass ratio \(q\). As we mentioned before, for a given \(q\), there exists an upper limit to the radius of the star, \(R_{\ast,\text{max}}\), above which the planet and the star are in contact. Here we compute this upper limit setting \(R_{\ast,\text{max}} = r_p - R_{L, p}\) (notice that for low mass ratios \(x_{L, p} \approx R_{L, p}\)). This upper limit \(R_{\ast,\text{max}}\) is plotted in Figure 4 as a function of the mass ratio \(q\). If the star radius \(R_\ast\) falls within the region labeled as “DISK,” mass overflow always leads to the formation of a disk regardless of the values chosen for \(f_p\) and \(e_0\). This is because within the “DISK” regime the star radius \(R_\ast\) is always smaller than the minimum particle distance to the star. In the region labeled as “DI or DISK” the mass overflow outcome can be either DISK or DI depending on the specific values of \(f_p\) and \(e_0\). As depicted in Figure 4, as a star begins to expand without losing mass (e.g., as the star leaves the main sequence), a planet undergoing RLOF in a low-eccentricity orbit may lead to DI, before the planet gets in contact with the stellar surface. However, as shown in Figure 4, the region for which a DISK forms overall dominates the parameter space. This implies that at low eccentricities, RLOF at periastron leads to DISK formation for most systems.

4. Conclusions

We investigated the possible outcomes of RLOF at periastron for eccentric planet–star systems with a planet in an asynchronous orbit. We explore the system parameter space identifying the regimes that lead to different outcomes of the planet’s mass loss.

The main results of this paper are summarized below.

1. RLOF at periastron leads to one of three possible outcomes for the mass transferred through the Lagrangian
DISK dominates at low eccentricities, while for higher eccentricities, the only possible outcome is SA. The black region refers to the highest eccentricity that allows for DISK formation for most systems. The planet and the star are in contact. At low eccentricities RLOF at periastron leads to DISK formation for most systems.

We have considered as a proof of concept the ballistic approach to study Roche-lobe overflow in eccentric planet–star systems. We have shown that, at low eccentricities, RLOF leads to disk formation for most systems. For eccentric systems, we speculate that the dynamics may lead to the survival of giant planets near the Roche limit, as observed (Jackson et al. 2017). Using the formalism described in Dosopoulou & Kalogera (2016a, 2016b), the secular evolution of these systems can be investigated to test the speculation mentioned above and to compare to observations.

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