High Frequency Edge Information-based Sampling Algorithm for Multi-scale Block Compressive Sensing

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Abstract. The same sampling rate is applied to all subblocks of the same layer, which results in the deviation of the information contained in the sampling observation in the multi-scale block compressive sensing algorithm from the real information of the original image, thus affecting the accuracy of image reconstruction. For this reason, the multi-scale block compressive sensing adaptive sampling method (HEM) using HF edge information is proposed. In this method, the edge information is extracted by using the high frequency coefficients of wavelet transform, and then the edge information of the subblock of wavelet decomposition graph is calculated, and then converted into the adaptive sampling rate of each subblock. The influence of different decomposition layers and block sizes on HEM method is discussed through testing in 6 standard images. The experimental results show that the proposed algorithm can improve the PSNR of the reconstructed image at the same sampling rate on standard test images and cable trench inspection images. Among them, Stanwick image(S=0.1 and S=0.2), Russland image(S=0.3) and Stomach CT image(S=0.4) are partial exceptions.

Keywords: Block compressive sensing; Edge information; Adaptive sampling; Multi-scale.

1. Introduction

In the application of image and video processing, Compressive Sensing (CS) technology will encounter the problems of occupying storage space and high complexity of reconstruction algorithms. For example, for a grayscale image of 256×256 size, when the sampling rate is 0.2, if it takes 2 bytes to store an integer, the measurement matrix will occupy about 1.6G of memory during CS calculation, and an image of 512×512 takes up about 25.6G of storage space. The technology of Block Compressive Sensing (BCS) developed in recent years provides a new way to solve this problem. Tsaig et al. proposed the BCS reconstruction algorithm combining CS and wavelet transform[1], and realized the reconstruction of the whole image by rebuilding and merging the image subblocks separately. Gan et al. used the same operator to compress and sample images in the space-time domain, and proposed the BCS reconstruction algorithm for natural images[2]. The above idea of block segmentation was quickly applied in the field of image fusion[3,4], but block effect was easy to occur in the reconstruction process. Therefore, scholars proposed a sparse reconstruction algorithm (BCS-SPL) combined with the smooth projection Landweber method, which effectively suppressed the block effect and improved the reconstruction performance of BCS [5-7]. Fowler et al proposed the multi-scale sampling(MS), and use the SPL reconstruction method of BCS (MS-BCS-SPL) in 2011[8]. This method reached the block effect and good effect to improve the reconstruction quality. However, the multi-scale BCS algorithm still has the following problems: 1) The smaller the image block size is, although the storage space required for the operation is reduced,
the larger the number of blocks also makes the calculation amount increase; 2) When processing complex images with texture details, the reconstructed image still has rough block edges. In order to further improve the performance of the reconstruction algorithm, scholars proposed a variety of improvement measures based on MS-BCS-SPL to improve the image reconstruction quality [9-10]. In order to improve the reconstruction accuracy of BCS algorithm, on the basis of MS-BCS-SPL, combined with priori information of high frequency subband and weighted points scale thought, this paper proposes a adaptive multi-scale edge based on HF information BCS algorithm, which realizes more efficient adaptive sampling. The proposed method can obtain higher quality reconstructed images at a lower sampling rate.

The structure of this paper is as follows: Next, the theory and algorithm of BCS are introduced. Then briefly summarize the idea of multi-scale BCS. Section 4 presents an adaptive multi-scale BCS algorithm based on high frequency edge information. In Section 5, the proposed method is tested and analyzed on test images and cable trench inspection video images. Finally, the conclusion is given.

2. Block Compressive Sensing Theory

CS theory shows that if an image has sparse representation in a transformation domain, the image can be linearly mapped to a low-dimensional measurement vector through a measurement matrix. The measurement vector preserves enough useful information of the original image, and the reconstruction algorithm can be used to recover the original signal. For a video image $f \in \mathbb{R}^{N_f \times 1}$ represented by column vectors, the measurement matrix $\Phi \in \mathbb{R}^{M \times N}$ ($M \ll N$) is used for compressed sampling, which can be expressed as

$$y = \Phi f$$

(1)

BCS divides the $M \times N$ video images into non-overlapping $B \times B$ image subblocks, and the $k$ subblock is represented by $f_k$ ($k = 1, 2, \cdots, K$), where $K$ represents the total number of all subblocks corresponding to each frame of video. Image blocks are expressed as column vectors, and all image blocks $f_k$ are measured with the orthogonal measurement matrix $\Phi_b \in \mathbb{R}^{M_b \times B^2}$ ($M_b \ll B^2$), and the observation vector of image blocks is obtained as follows:

$$y_k = \Phi_b f_k, \quad k = 1, 2, \cdots, K$$

(2)

Where, $y_k \in \mathbb{R}^{M_b \times 1}$ represents the compressive measurement result of the $k$'th video image block, and $M_b$ represents the observed number of this image block. Accordingly, the global measurement matrix $\Phi$ of images can be expressed as a block diagonal matrix $\Phi = \text{diag}(\Phi_b)$.

Although BCS is a non-global compressive observation method, on the whole, it is equivalent to using a global measurement operator with block diagonal structure for each frame of video in the compressive observation process. To store $\Phi$, $M \times N$ elements need to be stored, while the size of $\Phi_b$ is $M_b \times B^2$, which greatly reduces the storage space requirement of the measurement matrix. It can be seen that block processing can greatly reduce the storage cost, observation computation and algorithm complexity, and enable BCS to be applied to large image processing without the restriction of computer hardware conditions. In addition, during the image transmission, as long as an image subblock is encoded, it can be sent immediately, which greatly improves the real-time performance of the processing.

3. Multi-scale Block Compressive Sensing

Combined with multi-scale idea, Fowler et al proposed the multi-scale wavelet domain weighted block compressive sensing (MS-BCS-SPL) algorithm [8], the algorithm will change after the HF wavelet coefficients of block, according to the target sampling rate to determine the stratified sampling rate, namely the subband at the same sampling rate, sampling rate may be different in different layers of subband, measurement for each subblock to get the observation vector.
Let the observation matrix of the image $f$ in BCS be $\Phi$, decompose $\Phi$ into a multi-scale transformation matrix $\Omega$ and a multi-scale observation matrix $\Phi'$, then the observed quantity is rewritten as:

$$y = \Phi f = \Phi' \Omega f$$  \hspace{1cm} (3)

If $\Omega$ represents the wavelet decomposition of $L$ layer, then $\Phi'$ consists of $L$ different observation operators, the same observation operator of the same layer. Let the discrete wavelet transform (DWT) of $f$ be expressed as:

$$\hat{f} = \Omega f$$  \hspace{1cm} (4)

$\hat{f}$ is divided into subblocks with the size of $B_s \times B_t$, and the corresponding size of $\Phi$ is used for measurement sampling. $\hat{f}_{i,s,j}$ is used to represent the $j$'th subblock of the $s$'th subband of the $L$'th layer, and its observed quantity can be expressed as:

$$y_{i,s,j} = \Phi \hat{f}_{i,s,j}, \quad s \in \{H,V,D\}, \, 1 \leq l \leq L$$  \hspace{1cm} (5)

Equation (5) refers to sampling only the HF coefficients in blocks and retaining the LF directly, where H,V and D respectively represent the horizontal, vertical and diagonal subbands.

### 4. Adaptive Sampling Method Using High Frequency Edge Information

In BCS, it has become a consensus to determine the subblock sampling rate adaptively based on the amount of information. Multi-scale adaptive sampling method at present, basically is to use LF information of wavelet domain, such as edge, texture, direction, to guide the partitioning of HF sampling [8-10], but BCS measurement sampling was conducted in HF subband of the algorithm, and the LF subband retained (i.e., sampling rate is 1) directly, the indirect method to determine the sampling rate, and HF subband truth there is a deviation of the distribution of information resulting in the amount of sampling observation deviated from the original image information contained in real information, will ultimately affect the accuracy of image reconstruction.

Based on the above analysis, in order to obtain the observation quantities containing more true information of the original images through observation sampling, a method of HF edge based adaptive measurement(abbreviated as HEM) is proposed in this paper. HEM method first calculates the information of each subblock in the edge information graph of HF coefficient, and then adaptively determines the sampling rate of subblocks according to the proportion of subblock information in the total information. The sampling process is divided into three steps. Firstly, the edge information of the HF coefficient is extracted, then the adaptive sampling rate is calculated, and finally the HF coefficient is sampled. Specific implementation steps for HEM are as follows:

1. Extracting edge information of HF coefficients

   **Step1.1** Take the original image $f$ of $A \times A$ for $L$ layer discrete wavelet transform to obtain the wavelet decomposition image $\hat{W} f$, set the LF coefficient of $\hat{W} f$ to zero, and then make inverse wavelet transform to obtain the HF estimated image $\hat{f}_s$.

   **Step1.2** To detect $\hat{f}_s$ Canny edge, get edge information image $E f$;

2. Calculate the subblock adaptive sampling rate

   **Step2.1** Divide $\hat{f}_s$ and $\hat{W} f$ into non-overlapping $B \times B$ subblocks in the same way; Calculate the information of each subblock in $\hat{f}_s$ according to $H_E (m,n) = \frac{\sum_{p=(m-1)B+1}^{mB} \sum_{q=(n-1)B+1}^{nB} \varepsilon_{p,q} |e_{p,q}|}{\sum_{p=(m-1)B+1}^{mB} \sum_{q=(n-1)B+1}^{nB} |e_{p,q}|}$, where $H_E (m,n)$ represents the information of subblock $(m, n)$ (i.e. the row $m$ and column $n$) of edge information image $\hat{f}_s$, and $\varepsilon_{p,q}$ represents the pixel value (1 or 0) of $(p, q)$ in $\hat{f}_s$.

   **Step2.2** Calculate the information of each wavelet decomposition subblock $H_w (i, j)$ in $\hat{W} f$ according to all the subblocks $(u,v)$ corresponding to $(i, j)$ in $H_E$. 

3
Step 2.3 Set the target sampling rate $S$, calculate the normalized information $\bar{W}_H$ on the three subbands of layer $l$ respectively, equation (6) is used to calculate the sampling rate $S_{ij}$ of the subblock $(i, j)$ of $f_w$:

$$S_{ij} = \begin{cases} 
0.1S, & \beta \bar{W}_H(i, j) \leq 0.1S \\
\beta \bar{W}_H(i, j), & 0.1S < \beta \bar{W}_H(i, j) < 1 \\
1, & \beta \bar{W}_H(i, j) > 1
\end{cases}$$

Equation (6)

Where, $\beta$ represents the sampling ratio parameter, and the calculation equation is as follows:

$$\beta = \left( A^2 S - B^2 \frac{1}{4^{l-1}} \right) \left( B^2 \left\| \bar{W}_H \right\| \right)^{-1}$$

Equation (7)

3. Block adaptive sampling

Step 3.1 Rearrange the sampling rate of LF coefficient and HF coefficient calculated by equation (6) to form a vector according to the sequence of columns in $(S_{ij})$, and write it as $(S_1, S_2, \ldots, S_6)$;

Step 3.2 Randomly select $M_k = \left\lfloor S_k B^2 \right\rfloor$ row vectors from the random Gaussian matrix $\Phi_b$ to form the measurement matrix $\Phi_{b_k}$ ($M_k \times B^2$) of the $k$’th subblock, and observe each subblock separately, that is, get the adaptive observation vector $\{y_k = \Phi_{b_k} f_{w_k}, k = 1, 2, \ldots, b\}$.

The above is the multi-scale block adaptive sampling observation method proposed in this paper by using edge information of HF coefficient (HEM).

5. Experimental Results and Analysis

5.1. Experiment Setting

Sampling and reconstruction experiments were conducted on six 256×256 test images to illustrate the effectiveness of the HEM method, including two medical images, natural and SAR image each. The reconstruction process is obtained by directly replacing the observation steps in MS-BCS-SPL algorithm with HEM method[8]. Biorthogonal wavelet transform $\Omega$ is adopted for transform of the algorithm, while dual-tree DWT $\Phi'$ is adopted for sparse transformation. In the comparison algorithm, in addition to MS-BCS-SPL algorithm, there is also the EAM-BCS-SPL algorithm obtained by using the adaptive sampling method of LF edge information combined with BCS-SPL[9]. For the purpose of distinction, the algorithm in this paper is abbreviated as HEM-BCS-SPL. All the LF coefficients are retained in the measurement sampling of all algorithms, and a random Gaussian orthogonal matrix $\Phi_b$ of $B^2 \times B^2$ is generated. According to the sampling methods in the three algorithms, the sampling rate $S_b$ of blocks is calculated, and the measurement matrix $\Phi_{b_k}$ of each subblock is formed by randomly taking $M_k$ rows from $\Phi_b$.

5.2. Comparison of Experimental Results of Different Decomposition Layers and Block Sizes

Next, through experimental tests, suggested values are given for setting the parameters (number of wavelet decomposition layers and image block size) of the HEM-BCS-SPL algorithm presented in this paper. Set the decomposition layer number of wavelet $\Omega$ as $L = 2, 3, 4$, and image block size as $B = 16, 32, 64$, respectively. This paper adopts the HEM-BCS-SPL algorithm to conduct experiments on the image House.

Table 1 shows the PSNR value of the reconstructed image and the running time of the algorithm under different decomposition layers and different image block sizes of HEM-BSC-SPL. As can be seen from the table, with the increase of the number of layers $L$ of wavelet decomposition, the calculation time of the algorithm becomes longer, and the PSNR of the reconstructed image obtained by $L+1$ layer decomposition is slightly lower than that of the reconstructed image obtained by $L$ layer decomposition. In contrast, the PSNR value of the decomposed layer with 4 layers decreased slightly.
than that of the decomposed layer with 3 layers. The reason is that the LF coefficients are all retained, the decomposition scale increases, and the energy of the LF image containing the original image decreases, which affects the quality of the reconstructed image.

Table 1. Experimental results of HEM-BSC-SPL algorithm on House image

| Scale | Experimental results | S=0.1 | S=0.3 | S=0.5 |
|-------|-----------------------|-------|-------|-------|
|       | PSNR(dB)              | 16    | 32    | 64    | 16    | 32    | 64    | 16    | 32    | 64    |
| L=2   | PSNR(dB)              | 30.55 | 30.87 | 30.94 | 34.42 | 34.87 | 35.12 | 38.39 | 38.58 | 38.75 |
|       | Time(s)               | 19.47 | 22.51 | 163.18| 19.26 | 22.13 | 162.83| 19.69 | 24.37 | 171.56|
| L=3   | PSNR(dB)              | 29.15 | 29.67 | 29.85 | 33.21 | 33.75 | 34.43 | 36.82 | 36.98 | 37.16 |
|       | Time(s)               | 20.23 | 22.49 | 165.37| 20.33 | 24.92 | 164.82| 20.53 | 25.14 | 172.27|
| L=4   | PSNR(dB)              | 28.56 | 28.43 | 29.14 | 31.43 | 31.47 | 31.76 | 35.11 | 35.35 | 36.04 |
|       | Time(s)               | 22.15 | 23.15 | 167.28| 21.61 | 26.13 | 169.39| 20.78 | 27.26 | 174.13|

As shown in table 1, at the same scale, block size B has an impact on PSNR and calculation time of reconstructed images. In the case of $B=32$ and $B=64$, the calculation time value varies greatly with a difference of at least 130s. In the following experiments, HEM-BSC-SPL algorithm takes $L=3$ and $B=32$.

5.3. Comparison of Experimental Results of Reconstructed Performance

This paper used MS-BCS-SPL algorithm[8-9], EAM-BCS-SPL algorithm[10-11] and the HEM-BCS-SPL algorithm to carry out the reconstruction experiment of the test images. Table 2 shows the PSNR values of the reconstruction results of 4 test images with different total sampling rates. Through comparison, Table 2 for medical, natural and SAR images, according to the energy in all sampling rate under the HEM-BCS-SPL algorithm results of PSNR better than MS-BCS-SPL, especially for difficult to deal with complex image Barbara, HEM-BCS-SPL algorithm has achieved the best results of the three algorithms.

Table 2. Comparison of the reconstruction performance (PSNR) of the three algorithms

| Image     | Algorithm | Target sampling rate |
|-----------|-----------|----------------------|
|           | $S=0.1$   | $S=0.2$   | $S=0.3$   | $S=0.4$   | $S=0.5$   |
| Stomach CT| MS-BCS-SPL| 26.73     | 29.92     | 32.49     | 35.31     | 36.83     |
|           | EAM-BCS-SPL| 26.99     | 30.51     | 32.78     | 35.72     | 37.48     |
|           | HEM-BCS-SPL| 27.02     | 30.64     | 32.90     | 35.69     | 37.54     |
|           | MS-BCS-SPL| 30.35     | 33.13     | 33.89     | 36.79     | 38.09     |
| House     | EAM-BCS-SPL| 30.53     | 33.71     | 34.51     | 37.01     | 38.38     |
|           | HEM-BCS-SPL| 30.71     | 34.06     | 34.95     | 37.48     | 38.78     |
|           | MS-BCS-SPL| 23.56     | 24.65     | 26.95     | 28.26     | 29.74     |
| Barbara   | EAM-BCS-SPL| 23.61     | 24.93     | 26.65     | 27.87     | 29.11     |
|           | HEM-BCS-SPL| 23.85     | 25.37     | 27.02     | 28.28     | 29.85     |
|           | MS-BCS-SPL| 22.84     | 25.22     | 26.93     | 28.68     | 29.68     |
| Russland  | EAM-BCS-SPL| 22.99     | 25.49     | 27.06     | 28.99     | 30.11     |
|           | HEM-BCS-SPL| 23.10     | 25.83     | 27.04     | 29.05     | 30.91     |

As shown in table 2, when the sampling rate increases from 0.1 to 0.5, the reconstruction results of HEM-BCS-SPL algorithm on the six test images increase in turn. The best result is obtained on the House image. When $S=0.5$, the reconstructed image reaches 38.78. Compared with EAM-BCS-SPL algorithm, the reconstruction results of the HEM-BCS-SPL algorithm on Stanwick image($S=0.1$ and $S=0.2$), Russland image($S=0.3$) and Stomach CT image($S=0.4$) are slightly worse, and the reconstruction results of other situations are better. According to the comprehensive comparison of all experimental results, the reconstruction performance of HEM-BCS-SPL algorithm is better than that of MS-BCS-SPL and EAM-BCS-SPL.

Figure 1 compares the reconstruction results of 480×272 cable trench inspection image using three algorithms. It can be intuitively seen from figure 1 that the visual effect of HEM-BCS-SPL algorithm in image reconstruction is better than that of MS-BCS-SPL algorithm and EAM-BCS-SPL algorithm, and the PSNR value of reconstructed image is improved compared to that of the latter two.
6. Conclusion
In this paper, the edge characteristics of the image are used to block the HF coefficients between different scales and within the same scale after wavelet transform, and then the sampling rate is adjusted adaptively according to the energy of the blocks, thus realizing the adaptive sampling of multi-scale compressive sensing in the wavelet domain. Compared with the sampling method of MS-BCS-SPL and using LF edge information, the reconstruction quality and visual effect of the proposed algorithm are improved for standard test images and cable trench inspection images. The algorithm proposed in this paper is designed only from the perspective of sparse effect, without considering the subsequent application of the image. In some applications, special requirements are sometimes put forward for image sparsity, such as preserving local details. The design of efficient BCS algorithm under specific application background will be considered as the next step.

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