ON ESTIMATING FORCE-FREEMESS BASED ON OBSERVED MAGNETOGRAMS

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ABSTRACT

It is a common practice in the solar physics community to test whether or not measured photospheric or chromospheric vector magnetograms are force-free, using the Maxwell stress as a measure. Some previous studies have suggested that magnetic fields of active regions in the solar chromosphere are close to being force-free whereas there is no consistency among previous studies on whether magnetic fields of active regions in the solar photosphere are force-free or not. Here we use three kinds of representative magnetic fields (analytical force-free solutions, modeled solar-like force-free fields, and observed non-force-free fields) to discuss how measurement issues such as limited field of view (FOV), instrument sensitivity, and measurement error could affect the estimation of force-freeness based on observed magnetograms. Unlike previous studies that focus on discussing the effect of limited FOV or instrument sensitivity, our calculation shows that just measurement error alone can significantly influence the results of estimates of force-freeness, due to the fact that measurement errors in horizontal magnetic fields are usually ten times larger than those in vertical fields. This property of measurement errors, interacting with the particular form of a formula for estimating force-freeness, would result in wrong judgments of the force-freeness: a truly force-free field may be mistakenly estimated as being non-force-free and a truly non-force-free field may be estimated as being force-free. Our analysis calls for caution when interpreting estimates of force-freeness based on measured magnetograms, and also suggests that the true photospheric magnetic field may be further away from being force-free than it currently appears to be.

\textit{Key words:} Sun: magnetic fields – Sun: photosphere – sunspots

1. INTRODUCTION

It is well known that solar eruptive activities, such as filament eruptions, flares, and coronal mass ejections, are closely related to the evolution of magnetic fields. That is, magnetic fields in the corona play a vital role in solar activities and provide a way to understand the nature of solar eruptions (e.g., Zhang & Low 2005). However, due to both intrinsic physical difficulties and observational limitations, direct measurement of the magnetic field in the corona is still difficult and only a limited number of examples have been given (Lin et al. 2004; Cargill 2009). At present, an accepted way to overcome this difficulty is to reconstruct a coronal magnetic field with the help of a force-free model, where the observed photospheric magnetic field is taken as a boundary condition (Wiegelmann & Sakurai 2012; Wiegelmann et al. 2015). With this approach, finding a force-free field suitable for use as the boundary condition for extrapolation becomes important. As a first step, one therefore estimates whether an observed photospheric vector magnetogram is force-free or not.

Several previous studies have estimated the degree of force-freeness of measured photospheric and chromospheric vector magnetic fields above active regions. Metcalf et al. (1995) estimated the force-freeness of active region NOAA 7216 and concluded that NOAA 7216 is not force-free in the photosphere but becomes force-free for heights above 400 km. This is the only work, to our knowledge, that has measured the height dependence of magnetic forces.

Later on, Moon et al. (2002) analyzed 12 vector magnetograms of three flare-eruptive active regions (NOAA 5747, 6233, and 6982) and concluded that photospheric magnetic fields are not far away from being force-free. They also showed that the degree of force-freeness depends on the character of an active region and its evolutionary stage. Tiwari (2012) studied vector magnetic fields at high spatial resolution and also concluded that sunspot magnetic fields are not far away from being force-free even though their force-freeness may change with time. However, Liu et al. (2013) carried out a statistical study of the force-freeness of photospheric magnetic fields in 925 active regions. They found that only about 25% of the magnetograms can be considered as being close to force-free, i.e., most photospheric magnetic fields (75%) are not force-free.

Whereas there is no consistency among previous studies on whether magnetic fields in the solar photosphere are force-free or not, our study here, unlike previous studies, is not to address whether a particular field is force-free or not, but to systematically study how the limitations of magnetic field measurement (e.g., field of view (FOV), instrument sensitivity, and measurement noise) could affect the judgment of force-freeness. We will use three kinds of magnetic field configurations as representative magnetic fields to show that the measurement limitations, and the noise level in particular, will significantly influence the judgment of force-freeness, and that previous studies may have suffered from this limitation. The paper is organized as follows. We describe the method in Section 2. In Section 3 we analyze how the measurement limitations could affect the estimation of force-freeness. Discussions are given in Section 4 and a brief summary is provided in Section 5.

2. THE METHOD

2.1. The Model

Generally, it is assumed that the magnetic field is force-free in the corona (Wiegelmann et al. 2014), because model results
suggest that the plasma $\beta$ (the ratio of the gas pressure to the magnetic pressure) is much less than unity ($\beta \ll 1$) in the corona (Gary 2001). In this case, the magnetic field satisfies the following equations (see the reviews by Wiegelmann & Sakurai 2012):

$$\nabla \times \mathbf{B} = \alpha \mathbf{B},$$  \hspace{1cm} (1)  

$$\nabla \cdot \mathbf{B} = 0,$$  \hspace{1cm} (2)

where $\mathbf{B}$ is the vector magnetic field and $\alpha$ is the force-free coefficient.

Among all possible force-free fields, if $\alpha = 0$, then there is no electric current in the space and the field is a potential field; if $\alpha$ is a constant, then the field is called a linear force-free field. In the real Sun, $\alpha$ usually varies spatially and the field is a general nonlinear force-free field (see Wiegelmann & Sakurai 2012 for details).

According to Low (1985), in an isolated magnetic structure, a necessary condition for a force-free field to exist above a measured layer is

$$F_x < F_p, \quad F_y < F_p, \quad F_z < F_p,$$  \hspace{1cm} (3)

where $F_x$, $F_y$, and $F_z$ are the components of the net Lorentz force and $F_p$ is the characteristic magnitude of the total Lorentz force that can be brought to bear on the atmosphere if the magnetic field is not force-free. Assuming that the magnetic field above the plane $z = 0$ (photosphere) vanishes as $z$ goes to infinity, the volume’s boundaries other than the lower one do not contribute in the half-space $z > 0$, so the Maxwell stress can be written as the following surface integrals:

$$F_x = -\frac{1}{4\pi} \int (B_x B_y) \, dx \, dy,$$

$$F_y = -\frac{1}{4\pi} \int (B_y B_z) \, dx \, dy,$$

$$F_z = -\frac{1}{8\pi} \int (B_z^2 - B_x^2 - B_y^2) \, dx \, dy,$$

$$F_p = \frac{1}{8\pi} \int (B_x^2 + B_y^2 + B_z^2) \, dx \, dy,$$  \hspace{1cm} (4)

where $B_x$, $B_y$, and $B_z$ are the three components of the vector magnetic field, $B_R$ is the vertical magnetic field, and $B_\theta$ and $B_z$ are the two components of the horizontal magnetic field $B_R$.

The above approach to estimating the force-freeness using measured vector magnetograms as the boundary condition at the plane $z = 0$ has become a common practice in the community. Whereas $F_x$, $F_y$, and $F_z$ must all be zero for an entirely force-free magnetic field, a magnetic field may be considered to be nearly force-free if the magnitudes of $F_x/F_p$, $F_y/F_p$, and $F_z/F_p$ are sufficiently small (Low 1985). Metcalf et al. (1995) suggested values of $|F_x|/|F_p|$, $|F_y|/|F_p|$, and $|F_z|/|F_p|$ less than 0.1 as a criterion for a measured magnetic field to be considered to be force-free. Following Metcalf et al. (1995) and others (e.g., Moon et al. 2002; Tiwari 2012; Liu et al. 2013), we will adopt this criterion in the following.

2.2. Representative Magnetic Fields

We will use three magnetic fields as representative fields to discuss the possible influences of measurement limitations on the estimate of force-freeness.

The first one is taken from analytical solutions provided in Low & Lou (1990). Under a few assumptions, such as the fields being axisymmetric and the solutions separable in $r$ and $\theta$ directions, Low & Lou (1990) reformulated Equations (1) and (2) into a second-order partial differential equation in spherical coordinates with constant $n$ and $m$, where the integer $n$ describes the power-law decreasing index in the $r$-direction and $m$ defines the number of node points in the $\theta$-direction. Finding eigenvalue solutions of this second-order partial differential equation generates a series of analytical nonlinear force-free fields. These fields can be transformed into Cartesian coordinates by arbitrarily positioning a plane, characterized by the parameters $l$ and $\Phi$, to produce 3D force-free magnetic fields that represent the magnetic fields over active regions with a striking geometric realism. Here $l$ is the distance between the surface plane and the location of a point source, and $\Phi$ is the angle between the axis of symmetry of the magnetic field and the $z$-axis in the Cartesian coordinate system. These analytical force-free solutions are quite useful in testing the properties of force-free fields. For example, they have been extensively used to test the reliability and accuracy of extrapolation algorithms for force-free fields (e.g., Schrijver et al. 2006).

We use one of these analytical force-free solutions as a representative field. The field is generated by $n = 1$, $m = 1$, $l = 0.3$, and $\Phi = \pi/2$ and is shown in Figure 1(a). Here the contours show the vertical field strength with solid contours indicating a field $B_z > 0$ and dashed contours showing $B_z < 0$. The contour interval is 180 G. Green squares outline the ranges of different FOVs to simulate different sizes of FOVs in real observations. The smaller the red number is, the larger the FOV is.

To make the field more comparable to real magnetograms, we have rescaled the field to make the maximum of $|B_z|$ be 2000 G. The value of 2000 G is not specifically chosen from among the possible values from 1000 to 5000 G of active regions. The rescaling is done merely to make our studied fields comparable. The particular value chosen would not influence our results qualitatively because the estimate of force-freeness is based on the ratio, not the absolute value, of $F_x$, $F_y$, or $F_z$ to $F_p$. Some properties of the field are listed in Table 1.

The second representative field is a modeled solar-like nonlinear force-free field that might exist on the Sun. In other words, we treat this modeled field as a more or less theoretical field but with a geometry more similar to a realistic one on the Sun, if such exists, than the analytical one given by Low & Lou (1990). We use a vector magnetogram of NOAA 11072 obtained by the Helioseismic and Magnetic Imager (HMI) on board the Solar Dynamics Observatory (SDO) (Schou et al. 2012) as the $z = 0$ boundary and carry out a 3D force-free field extrapolation. We select a time when the active region is near the center of the disk, at a position of S14W00. We use the magnetogram from the hmi.sharp_cea_720s series, which has been inverted by the HMI team using the Milne–Eddington (ME) inversion algorithm of Borrero et al. (2011), solved for the $180^\circ$ ambiguity using the minimum energy method (Metcalf 1994), and processed using a cylindrical equal-area projection. We then further preprocess the magnetogram using methods described in Wiegelmann et al. (2006, 2008) to make it suitable for force-free extrapolation. The extrapolation is done with the help of an optimization code described in Wiegelmann (2004).

We use the layer about 1 Mm above the $z = 0$ photosphere in the extrapolated field as the second representative field: the modeled solar-like nonlinear force-free field. Note that 1 Mm...
above the \( z = 0 \) photosphere already puts this field into the layer of the chromosphere. However, our purpose here is not to discuss what the true chromospheric magnetic field might be, but to get a modeled solar-like force-free field. So the particular height, 1 Mm or even 2 Mm above, makes no difference to the results of our study here. In addition, to make all our representative fields comparable, the maximum value of \( |B_z| \) of this field has also been normalized to be 2000 G. Some properties of this field are also shown in Table 1.

To quantify how good our force-free extrapolation is, we have calculated a few numbers like those in DeRosa et al. (2015). They are \( \langle \mathrm{CW} \sin \theta \rangle = 0.37 \) and \( \langle |f| \rangle = 7.9 \times 10^{-4} \). These numbers are of the same magnitudes as those in DeRosa et al. (2015). In particular, it shows the success of the force-free extrapolation when we get very small numbers of \( F_\text{c}/F_p = 0.00149, F_u/F_p = 0.00169 \), and \( F_d/F_p = -0.02141 \) for the modeled solar-like field. The original observed magnetogram has these numbers as \( F_\text{c}/F_p = -0.00207, F_u/F_p = 0.08495 \), and \( F_d/F_p = -0.03457 \). We see that \( F_\text{c}/F_p \) has reduced to be 2% of its original value.

Figures 1(b) shows the \( B_z \) map of this modeled solar-like force-free field. As before, colored rectangles in Figure 1(b), labeled with sequential numbers, outline ten different FOVs, to mimic limited FOVs in real observations. Again, the smaller the labeled number is, the larger the FOV is.

The third representative field is a non-force-free field. According to Tiwari (2012), the fields of active region NOAA 10960 are non-force-free in that \( F_\text{c}/F_p = 0.137, F_u/F_p = 0.093 \), and \( F_d/F_p = -0.482 \). We used a single vector magnetogram of this active region, obtained by the Solar Optical Telescope/Spectro-Polarimeter (SP) on Hinode (Kosugi et al. 2007). The SP magnetograms are inverted by the SP team from Stokes profiles using the MERLIN ME inversion algorithm (Skumanich & Lites 1987; Lites et al. 2007) and the inherent 180° azimuth ambiguity is resolved in the same way as for HMI data. Again, we choose a time, 03:04 UT on 2007 June 7, when the active region is near the center of the disk. Also, the maximum of \( |B_z| \) is rescaled to be 2000 G, to make the three representative fields comparable to each other. Information on this field can be found in Table 1 and a \( B_z \) map of it is shown in Figure 1(c), with the colored rectangles again representing different FOVs.

### 2.3. Mimic the Effect of FOV

As already stated in Canfield et al. (1991), a limited FOV may not be appropriate for Maxwell stress integration, for the use of which the flux balance is a prerequisite. To minimize this effect, Moon et al. (2002) considered only magnetograms whose magnetic imbalance (MI) is within 10%. Metcalf et al. (1995) as well as Tiwari (2012) used this approach too. However, the effect of limited FOV on the force-free measure has not been studied systematically. Aiming to discuss how the FOV could influence the judgment of force-freeness, we shrink the FOV from the original flux-balanced one to get a series of magnetograms with different FOVs, as indicated by the colored rectangles in Figure 1.

A discussion of the effect of a limited FOV is actually related to the effect of flux imbalance (MI). We follow Moon et al. (2002) to calculate the MI associated with different FOVs as the index to represent them. MI is defined as

\[
MI = \left( \frac{F^+ - F^-}{F^+ + F^-} \right) \times 100, \tag{5}
\]

where \( F^+ \) and \( F^- \) are upward (\( B_z > 0 \)) and downward (\( B_z < 0 \)) magnetic fluxes respectively.
2.4. Mimic the Effects of Sensitivity and Noise

Observed magnetograms also suffer from issues of instrument sensitivity and measurement noise. The random errors (noises) in the measurement of horizontal fields are particularly large, typically ten times those for vertical fields. For example, random errors in HMI are about 5 G in the line-of-sight component whereas the uncertainty in the transverse field is between 70 and 200 G (Wiegelmann et al. 2012; Hoeksema et al. 2014).

To deal with this situation, in previous studies usually only data points whose field strengths are larger than a certain value have been used. For example, Metcalf et al. (1995) only use data points with magnetic field strength greater than 150 G (1σ noise level in a transverse magnetic field). Moon et al. (2002) used a field strength larger than 100 G as a criterion. Liu et al. (2013) used data points with $|B_z| > 20$ G, $|B_y| > 150$ G, and $|B_x| > 150$ G. However, this “cutting” method is more equivalent to setting a low level of sensitivity; the large measurement error (noise) is still buried in the remaining data points.

We mimic the sensitivity and noise separately to divide the effects of these two issues. First, to simulate the different levels of sensitivity, set $B_x^0$ and $B_y^0$ as the horizontal and vertical field sensitivities respectively. If $\sqrt{B_x^2 + B_y^2} \leq B_z^0$ or $|B_z| \leq B_z^0$, then omit these pixels. Taking knowledge from previous studies, we have assumed $B_x^0 = B_y^0 = 10B_z^0$ and hence $B_x^0 = 10\sqrt{2}B_z^0$. We have studied the model for $B_z^0$ increasing from 0 G, with a constant step size of 1 G. Only the magnetogram with the largest FOV is studied. To quantify this undertaking, a number NP, defined as the percentage of the data points that have been omitted, is calculated for each $B_z^0$ level.

Similarly, given $\sigma_x$, $\sigma_y$, and $\sigma_z$ as the white noise in $B_x$, $B_y$, and $B_z$ respectively, to mimic the different levels of noise in the observed magnetograms, we have replaced the value of each pixel in the magnetogram using the following method: $B_\alpha$ is replaced by $B_\alpha + \sigma_\alpha$, $B_\beta$ by $B_\beta + \sigma_\gamma$, and $B_\gamma$ by $B_\gamma + \sigma_\gamma$. $\sigma_\gamma$ is created by multiplying $\sigma_\gamma$ by a normally distributed random number, and similarly for $\sigma_\alpha$ from $\sigma_\alpha^0$ and $\sigma_\beta$ from $\sigma_\beta^0$. Again, we have assumed $\sigma_\alpha^0 = \sigma_\beta^0 = 10\sigma_\gamma^0$, with $\sigma_\gamma^0$ increasing from 0 G with a constant step size of 1 G.

### Table 1

| FOV Number = 0 | $F_z/F_p$ | $F_y/F_p$ | $F_x/F_p$ | max($B_z$) (G) | min($B_z$) (G) |
|----------------|-----------|-----------|-----------|----------------|----------------|
| Low and Lou    | ...       | ...       | 1.57 × 10^{-8} | -0.00036 | -0.00468 |
| AR11072        | 2010 May 23: 0500 | S1W090 | 0.00149 | 0.00169 | -0.02141 |
| AR10960        | 2007 June 07: 0304 | S07W07 | 0.07035 | -0.03191 | -0.51746 |

3. ANALYSIS AND RESULTS

In this section, we quantify how different values for FOV (Section 3.1), instrument sensitivity (Section 3.2) and measurement noise (Section 3.3) could affect the estimation of force-freeness for analytical force-free solutions, extrapolated force-free fields, and observed non-force-free fields, respectively.

3.1. The Influence of FOV

Figures 2 shows how the different sizes of FOVs could influence the estimation of the force-freeness. Plotted in the left panels are the values of $F_z/F_p$, $F_y/F_p$, and $F_x/F_p$ can increase from the theoretical zero of force-free fields (FOV Number 0) to a magnitude larger than 0.1 (FOV Number 9). This suggests that a true force-free field (where $F_x/F_p = 0$, $F_y/F_p = 0$, and $F_z/F_p = 0$) may be mistakenly estimated as being non-force-free (where $|F_x/F_p| > 0.1$, $|F_y/F_p| > 0.1$, or $|F_z/F_p| > 0.1$) at FOV Number 9, where MI is larger than 90% (see panel (b)). However, before FOV Number 7, the changes in $F_x/F_p$, $F_y/F_p$, and $F_z/F_p$ are all small (with magnitude less than 0.1). This suggests that the wrong judgment may not happen even for an MI value as large as 43% (FOV Number 7) and a criterion setting as MI less than 10% is fairly safe.

As in panels (a) and (b), panels (c) and (d) show the results for the modeled solar-like force-free field based on the magnetogram of NOAA 11072. Before FOV Number 8, the changes in $F_z/F_p$, $F_y/F_p$, and $F_x/F_p$ are small and within a magnitude of 0.1, again suggesting that setting the MI within 10% is safe. Also, if the FOV is too small where the MI is too large, a wrong judgment can be made, as is the case for FOV Number 9, the severe flux imbalance (MI ~ 40%) does not seem to influence the measure, which implies that the MI is not the only factor that controls the estimate of force-freeness.

Panels (e) and (f) are for the observed non-force-free field (NOAA 10960). This field shows a large variation in MI with increasing FOV number. However, a fair judgment can still be made if we confine the MI to within 10%, as before FOV Number 3 for this field. So, again we see 10% of MI as a good criterion.

In summary, for the three representative fields, we see that a limited FOV with a large MI value indeed can have certain effects on the results of an estimation of force-freeness as previous studies have suggested (Canfield et al. 1991; Moon et al. 2002), and that setting a criterion such as using magnetograms whose MI is within 10% as suggested by many previous studies is a safe approach.

3.2. The Influence of Instrument Sensitivity

Figure 3 presents the results for the influence of instrument sensitivity on measurement of force-freeness. Variations of $F_z/F_p$, $F_y/F_p$, and $F_x/F_p$ are shown in the left panels. The right
panels show the variations of MI (lines with cross symbols) and of NP (lines with triangle symbols). The lower x-axis is $B_z^0$ in units of gauss and the upper x-axis is $B_x^0$ or $B_y^0$ also in units of gauss.

Again panels (a) and (b) are for the analytical force-free field, and panels (c) and (d) are for the modeled solar-like force-free field. Here we see that the variations in $F_x/F_p$, $F_y/F_p$, and $F_z/F_p$ are all within a magnitude of 0.1 for these two force-free fields. This suggests that the problem of instrument sensitivity does not influence the estimation of force-freeness seriously, at least for these two cases. For the analytical force-free field, even when MI has reached 52% and even when 90% of the data points have been omitted (case with $B_z^0 = 25$ G in Figure 3(b)), the changes in $F_x/F_p$, $F_y/F_p$, and $F_z/F_p$ are all within 0.05 in magnitude. The same is true for the modeled solar-like force-free field: even when MI has increased to 30% and more than 90% of data points have been omitted ($B_z^0 = 25$ G in Figure 3(d)), the changes in $F_x/$

Figure 2. Influence of different FOVs on estimating the force-freeness. (a) The variation of $F_x/F_p$ (blue line), $F_y/F_p$ (red line), and $F_z/F_p$ (black line) for the analytical force-free field. (b) The variation of MI for the analytical force-free field. (c), (d) Same as in panels (a) and (b), but for the modeled solar-like force-free field. (e), (f): Same as in panels (a) and (b), but for the observed non-force-free field.
For the observed non-force-free field as shown in panels (e) and (f), however, the problem of instrument sensitivity would not obviously influence the force-free measures only if $B_z^0$ is less than 12 G or MI is less than 10%. Therefore, we see that the instrument sensitivity would not influence the estimates of force-freeness too seriously as long as MI is controlled within the safe criterion of 10%.
3.3. The Influence of Measurement Noise

Figure 4 shows the effect of measurement noise for the three representative fields. Again panels (a) and (b) are for the analytical force-free field, panels (c) and (d) for the modeled solar-like force-free field, and panels (e) and (f) for the observed non-force-free field. Here the left panels are for $F_x/F_p$ and $F_y/F_p$, the right panels are for $F_z/F_p$. The lower $x$-axis is the added noise level of $\sigma^0_x$ and the upper $x$-axis is added noise level of $\sigma^0_y$ or $\sigma^0_z$, all in units of gauss. Since the noise we added is white noise, it would not change the total flux and

Figure 4. Influence of the measurement noise on estimating the force-freeness. (a) The variations of $F_x/F_p$ (blue line) and $F_y/F_p$ (red line) with different noise levels for the analytical force-free field. (b) The variation of $F_z/F_p$ (black line) for the analytical force-free field. (c), (d) Same as in panels (a) and (b), but for the modeled solar-like force-free field. (e), (f) Same as in panels (a) and (b), but for the non-force-free field. The vertical green dashed lines show the range of current measurement noise levels (70–200 G for transverse fields).
hence MI. A calculation of MI shows that the changes here are all less than 0.1%, so we do not plot it here.

In Figure 4, it can be seen that the measurement noises have little influence on \( F_z/F_p \) and \( F_z/F_p \) for the three representative fields. The variations in \( F_z/F_p \) and \( F_z/F_p \), with different noise levels, are all within 0.1 in magnitude. However, \( F_z/F_p \) increases monotonically with increasing noise level. Panel (b) shows that when \( \sigma_z^2 \) is larger than 10 G (100 G for \( \sigma_x^0 \) and \( \sigma_y^0 \)), \( F_z/F_p \) has increased to a value larger than 0.1 for the analytical force-free field. For the modeled solar-like force-free field (panel (d)), even when \( \sigma_z^2 = 5 \text{ G} \) (\( \sigma_x^0 = \sigma_y^0 = 50 \text{ G} \)), \( F_z/F_p \) has already increased to above 0.1. These suggest that a truly force-free field may be estimated as non-force-free by the calculation of \( F_z/F_p \).

For the non-force-free field (panel (f)), the value of \( F_z/F_p \) also increases monotonically with increasing noise level. This result is in a situation where, when \( \sigma_z^2 \) is between 10 and 15 G, the originally non-force-free field may appear to be force-free if we cut at the 2\( \sigma \) level, and the blue lines for cutting at the 2\( \sigma \) level, the noises in even the remaining 10% of data points can still lead to a wrong estimation. The real lines show the results of cutting at the 1\( \sigma \) level, and the blue lines for cutting at the 2\( \sigma \) level. Similarly to Figure 3, the values of MI and NP are plotted in the right panels, with the same color coding for the sensitivity cutting levels.

From the left panels of Figure 5, we see that the addition of the influence of instrument sensitivity to the influence of white noise does not change the results too much from what we have already seen in Figure 4. Still, a true force-free field may be estimated as being non-force-free, and a non-force-free one as force-free, if the noise level is high enough. Note that the changeover point, where a wrong judgment may be made, is high only compared to those results without any measurement noise. These changeover points of noise level are actually right within the range of current measurement noise levels.

If we take the non-force-free fields as an example, then when \( \sigma_z^2 \) or \( \sigma_x^0 \) or \( \sigma_y^0 \) is larger than 130 G, in the case of 2\( \sigma \) cutting, even though 95.6% data points have been omitted, the noise in the remaining data points can still lead to a wrong estimation.

These results call for serious caution in interpreting the \( F_z/F_p \) measures when using observed magnetograms. Even if cutting at a 2\( \sigma \) level, the noises in the remaining 10% of data points can still affect the estimation of force-freeness. In particular, note that most previous studies (Metcalf et al. 1995; Moon et al. 2002; Tiwari 2012; Liu et al. 2013) show that most active regions have their magnitudes of \( F_z/F_p \) larger than those of \( F_z/F_p \) and \( F_z/F_p \), and the judgment of force-free nature mainly depends on the former.

4. DISCUSSIONS

In this section, we discuss why \( F_z/F_p \) increases monotonically with increasing noise level as shown in Figures 4 and 5. Our explanation also leads to an interesting judgment of the nature of force-freeness on the photosphere.

This monotonous increase with the noise level is actually buried in the form of Equation (4). In real observations where measurement noise is unavoidable, the vector magnetogram was obtained, that is, \( \mathbf{B}' = (B'_x, B'_y, B'_z) \), is actually \( (B_x + \sigma_x, B_y + \sigma_y, B_z + \sigma_z) \), where \( \mathbf{B} = (B_x, B_y, B_z) \) denotes the true field and \( (\sigma_x, \sigma_y, \sigma_z) \) are corresponding noise levels. So, applying Equation (4) to \( \mathbf{B}' \), what we actually calculated are

\[
\begin{align*}
F'_x &= \frac{-1}{4\pi} \int \[(B_x + \sigma_x)(B_x + \sigma_z)\] \text{dx dy}, \\
F'_y &= \frac{-1}{4\pi} \int \[(B_x + \sigma_z)(B_y + \sigma_z)\] \text{dx dy}, \\
F'_z &= \frac{-1}{8\pi} \int \{(B_y + \sigma_y)^2 - (B_z + \sigma_z)^2\} \text{dx dy}, \\
&= \frac{1}{8\pi} \int \{(B_x^2 + 2B_x\sigma_x + \sigma_x^2 - B_z^2 - 2B_z\sigma_z - \sigma_z^2)\} \text{dx dy},
\end{align*}
\]

where \( B_x \) is the horizontal field (\( B_x^2 = B_x^2 + B_x^2 \)) and \( \sigma_x \) is the horizontal field noise (\( \sigma_x = \sqrt{\sigma_x^2 + \sigma_y^2} \)).

In this equation, the first-order terms, such as \( B_x\sigma_x, B_y\sigma_z, \) and \( B_z\sigma_z \), will cancel with each other in the integration because the added noise is a white noise. However, the second-order terms, that is, \( \sigma_x^2 - \sigma_z^2 \), will not cancel each other in the integration and will accumulate because \( \sigma_x \gg \sigma_z \). This is why the changes in \( F_z/F_p \) and \( F_z/F_p \) are almost negligible with increasing noise, whereas \( F_z/F_p \) increases dramatically with increasing noise, as shown in Figures 4 and 5.

We noticed that Moon et al. (2002) also estimated the resulting error of \( F_z/F_p \) caused by measurement noise. However, they only considered the first-order term, whose contribution is indeed small. Metcalf et al. (1995) realized this problem by stating that “since the \( F_z \) and \( F_0 \) integrals use the square of the field strengths, the noise in the measurements will be magnified in the results.” However, the approach they took is only to “cut” at a high level of 150 G, an approach that we have demonstrated would not remove the influence of the noise, as shown in Figure 5. Our analysis has shown that, even if we cut at the 2\( \sigma \) level, the noises in even the remaining 10% of data points will significantly influence the estimation of \( F_z/F_p \). To reduce this effect, we suggest that more accurate measurements are necessary, in order to better control the noise level to less than 40 G for transverse fields.

Having shown that the noises will increase the value of \( F_z/F_p \) implies that previous studies, based on observed magnetograms whose data contain noises, might have overestimated the value of \( F_z/F_p \). So, if the measured \( F_z/F_p \) is already negative, the true \( F_z/F_p \) may have an even more negative value. We
noticed that the $F_z/F_p$ values are negative for almost all the active regions in Moon et al. (2002) and the sunspots in Tiwari (2012), so we estimate that the true $F_z/F_p$ values may have even more negative values and the true magnitudes of $|F_z/F_p|$ may be larger than their current estimates. This implies that the photospheric magnetic fields may be non-force-free as Metcalf

Figure 5. Influence of the measurement noise and sensitivity on estimating the force-freeness. (a) The variation of $F_z/F_p$ for the analytical force-free field; black line: same as in Figure 4(b), adding noise without sensitivity cutting; red line: adding noise with 1σ cutting; blue line: adding noise with 2σ cutting. (b) The variation of MI and NP for the analytical force-free field, with the same color coding as in panel (a). (c) Same as in panels (a) and (b), but for the modeled solar-like force-free field. (d), (e) Same as in panels (a) and (b), but for the non-force-free field. The lower x-axis is $\sigma^0_z$ and the upper x-axis is $\sigma^0_x$ or $\sigma^0_y$, both in units of gauss. The vertical green dashed lines show the range of current measurement noise levels (70–200 G for transverse fields).
et al. (1995) and Liu et al. (2013) have stated, rather than “close” to force-free as Moon et al. (2002) and Tiwari (2012) have stated.

5. SUMMARY

In this investigation we have studied how measurement issues could influence the estimation and judgement of the force-freeness of magnetic fields. We have used three representative fields — analytical force-free solutions, modeled solar-like force-free fields, and observed non-force-free fields — and we mimic the effects of different FOVs, instrument sensitivities, and measurement noises to an extent similar to what current photospheric measurements suggest.

We find that the measurement issues can have certain effects on the results of an estimation of force-freeness. Among these factors, the problems of FOV and instrument sensitivity would not significantly influence the force-free measures if the vertical magnetic flux imbalance is less than 10%. However, the measurement error (white noise) has a significant impact. It may cause a true force-free field to be estimated as non-force-free and a non-force-free field to be estimated as force-free.

This is because the \( F_z \) and \( F_\rho \) integrals in the formula of Low (1985) use the square of the field strengths, so the noises in the measurement will be magnified by the integration instead of canceling with each other. Cutting the magnetogram at a high sensitivity level would not help. Our example shows that, even if cutting at the \( 2\sigma \) level, the noise in the remaining 10% of data points can still affect the estimation of force-freeness. To decrease this effect, the noise level of the measurement needs to be controlled at a level of less than 40 G for the transverse fields.

Taking account of current measurement noise levels, our results suggest that caution should be taken when using the observed magnetograms and Low’s formula to estimate the force-freeness and make a judgment. Our analysis also indicates that the true photospheric magnetic fields might be non-force-free as Metcalf et al. (1995) and Liu et al. (2013) have suggested, rather than “close” to force-free as Moon et al. (2002) and Tiwari (2012) have estimated.

A further note to add here is that different parts of sunspots, such as the umbra, inner penumbra, and outer penumbra, may have different properties of force-freeness. For example, Tiwari (2012) found that umbral fields are more force-free than penumbral fields and that the inner penumbra is more force-free than the middle and outer penumbra. However, this result was obtained by estimating the vertical tension force in different parts, a method not requiring magnetic flux balance (though requiring an estimate of the plasma density, which is a tough task in itself). So it would be difficult to use the method described in this paper to check this statement, although it is an interesting phenomenon that deserves further investigations.

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