QCD Analysis of the H dibaryon

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Abstract

The status of theoretical studies of the $H$ dibaryon is reviewed. Some recent developments including the effect of the instanton induced interaction and the QCD sum rule results are discussed in detail.

1. Introduction

$H$ is a $J = 0$ dibaryon predicted in the flavor $SU(3)_f$ singlet representation (figure 1). It has strangeness $-2$, isospin 0 and hypercharge $Y = B + S = 0$. In the valence quark model it consists of two $u$, two $d$ and two $s$ quarks. $H$ is stable against strong decays if its mass is below the two baryon thresholds, $\Lambda\Lambda$ (2231MeV) – $N\Xi$ ($n\Xi^0$: 2254MeV, $p\Xi^-$: 2260MeV) – $\Sigma\Sigma$ ($\Sigma^0\Sigma^0$: 2385 MeV, $\Sigma^+\Sigma^-$: 2387 MeV). The binding energy of $H$ is measured from the lowest two baryon threshold, $\Lambda\Lambda$. As a six-quark object, this is truly an EXOTIC hadron, whose existence alone is of great significance in hadron physics.

Experimental searches of $H$ have continued for some time and yet no evidence of deeply bound states is found[1]. On the contrary, recent ($K^−, K^+$) reaction experiments on emulsion targets identified a few candidates of double-hypernuclei with binding energy of less than 20 MeV[2]. Because such a double hypernucleus would make a strong-interaction transition into $H$ and an ordinary nucleus, its existence kills the possibility of deeply bound $H$.

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In this report, I am going to review the status of theoretical analysis of $H$. After a brief review of the history, some recent progresses are discussed in detail.

2. Brief History

In 1977, Jaffe predicted the $H$ dibaryon in the MIT bag model\cite{3} and since then almost all possible models of hadron dynamics have been applied to this problem. Yet the conclusion is not reached yet. The mechanism for the $H$ binding in the bag model is simple. It is due to a strong attraction of color-magnetic gluon exchanges between quarks. The $n$-quark state in the flavor $SU(3)$ representation $[f]$ acquires the color-magnetic gluon interaction energy given by

$$E_{cm}(S, [f]) = E_0 \left[ n(n - 10) + C_2[f] + \frac{4}{3} S(S + 1) \right]$$

where $S$ is the total spin and $C_2[f]$ denotes the value of the quadratic Casimir operator, such as $C_2 = 0$ for the singlet, 12 for the octet and 24 for the decuplet. The overall constant $E_0$ is given by a spatial integration of the quark wave functions. Instead we can estimate $E_0$ from the $N - \Delta$ mass splitting, assuming that the same interaction gives the full splitting and that the bag radius is independent of $n$. (The latter is not valid for the bag model, while it is justified for the potential model of confinement. The mechanism of confinement gives an ambiguity here.) As the $N - \Delta$ energy difference due to the color-magnetic gluon is $16 \times E_0$, we find $E_0 = (M_\Delta - M_N)/16 \simeq 18$ MeV. The $E_{cm}$ gives the minimum, $E_{cm} = -24E_0$, at $S = 0$ and $C_2 = 0$, that corresponds to the $H$ dibaryon (flavor singlet, spin zero). Thus our estimation $E_{cm}(H) \approx -450$ MeV shows that the color-magnetic gluon exchange interaction strongly favors the $H$ dibaryon.

Soon after the first prediction of $H$ was made, it was noticed that two other effects are important in determining the binding energy of the $H$ dibaryon. One is the effect of confinement. The difference of the bag volume energy between the hyperon $\Lambda$ and $H$. This, in fact, causes the most serious ambiguity in predicting the $H$ dibaryon mass. In the bag model, we have never tested the bag volume energy term in the system with more than three quarks. Because the bag (surface) is not treated dynamically, one cannot
describe the hadron-hadron interaction in the bag model properly.

The other important effect is the Pauli exclusion principle among the valence quarks. For the nucleon-nucleon interaction, one finds that the Pauli exclusion gives repulsion, while introduction of the strange quarks reduces the “exclusiveness” and thus the repulsion is expected to be reduced in the \( \Lambda - \Lambda \) system.

In order to study the effect of confinement and the Pauli exclusion principle, the quark cluster model\(^\text{[4]}\) based on the nonrelativistic quark model with potential confinement was applied to the two baryon systems\(^\text{[5]}\). This approach has a strong advantage that the coupling of \( H \) to the two-baryon systems, \( \Lambda - \Lambda \), \( N \Xi \) and \( \Sigma \Sigma \), can be taken into account consistently with the two baryon dynamics. The quark cluster model is successful in describing the short distance part of the nuclear force and thus application to the two hyperon systems is a straightforward extension.

The six-quark structure of \( H \) is represented by a resonating group method wave function as

\[
\Psi_H(1-6) = \sum_{(BB')=(\Lambda\Lambda),(N\Xi),(\Sigma\Sigma)} A [\phi_B(123) \phi_B'(456) \chi_{BB'}] \tag{2}
\]

where \( \phi_B \)'s are the internal quark wave functions of the baryon and \( \chi_{BB'} \) describes the relative motion of the \( (BB') \) channel. \( A \) is the antisymmetrization operator for the six quarks. The resonating group method is employed to solve the Schrödinger equation. The full antisymmetrization is taken into account and induces the quark exchange interaction between the baryons.

In 1983, Oka-Shimizu-Yazaki\(^\text{[5]}\) found that the quark exchange diagrams (figure 2) associated with the color-magnetic gluon exchange yield a strong attraction to the flavor-singlet two-baryon state.

\[
|H\rangle \approx \frac{1}{\sqrt{8}} \left[ A|\Lambda\Lambda\rangle + \sqrt{2}A|N\Xi\rangle - \sqrt{3}A|\Sigma\Sigma\rangle \right] \tag{3}
\]

We, however, found that the quark exchange interaction alone cannot make \( H \) bound. It is, however, found that \( H \) couples most strongly to \( N\Xi \) and that a \( N\Xi \) bound state will appear in the \( \Lambda\Lambda \) continuum spectrum as a sharp resonance state. The wave function at the resonance shows the flavor singlet structure, eq.(3). Further studies in the quark
cluster model have suggested that the long-range meson exchange interactions may give
enough attraction to make a bound $H$, while it was pointed out that the choice of the
confinement is crucial in predicting the binding energy\textsuperscript{[3,7]}.

The Skyrme model of baryons, which is based on the chiral symmetry in the mesonic
effective lagrangian, was also applied to the $H$ problem\textsuperscript{[8]}. A new type of the topological
soliton configuration was proposed to describe a compact $B = 2$ state, which is called
the $SO(3)$ Skyrmion\textsuperscript{[9]}. This configuration has the right properties for the $H$ dibaryon
and predicts a deeply bound $H$ state. It is however, not well understood how quantum
corrections are treated. Especially, couplings of two baryon states seem important but it
is not included in the $H$ mass calculation.

The lattice QCD is the most promising approach to the exact solution of QCD at
low energies. The time-like correlator of $H$ on the lattice was calculated but several
inconsistent results were presented so far\textsuperscript{[10]}. We suspect that the lattice size is not large
enough to contain the whole $H$, especially when the binding energy is small and two
baryon states couple to $H$ strongly.

Besides the existence and the binding energy of $H$, the most important question
regarding $H$ is how compact it is. Once $H$ is identified, then we must determine whether
$H$ is like a compact 6-quark object or just like a $\Lambda\Lambda$ bound state. Theory predictions are
again quite diverse from a compact object to a loosely bound two baryons.

3. Effects of Axial U(1) Anomaly

The prediction of the $H$ mass relies on the validity of the quark model description
of hadrons (and hadronic interactions). A simple hamiltonian with a quark confinement
and a one-gluon exchange interaction made a great success in the meson and baryon
spectroscopy. A few exceptional cases include the lowest pseudoscalar mesons, such as
$\pi$, $K$, $\eta$ and $\eta'$ mesons. The octet mesons, $\pi$, $K$ and $\eta$ are generally considered as the
Nambu-Goldstone (NG) bosons of the spontaneous breaking of the chiral $SU(3) \times SU(3)$
symmetry. The nonrelativistic quark model description of these NG mesons are reasonably
good, maybe except for the pion, which is so deeply bound that the one-gluon exchange force may not explain the full binding energy.

Weinberg pointed out that the ninth member of the pseudoscalar nonet, $\eta'$, is too heavy to be regarded as a NG boson[11]. Indeed, the axial $U(1)_A$ symmetry is explicitly broken in the quantum theory (due to an anomaly) and thus the ninth NG boson does not exist. Therefore the large mass of $\eta'$ should also be explained by an interaction that causes the $U(1)_A$ breaking.

In the quark model description, the effect of the $U(1)_A$ breaking can be represented by the so-called instanton induced interaction (III), which is derived by 't Hooft considering a coupling of light quarks to instanton field configurations[12,13]. The quark-instanton coupling induces an effective 6-quark vertex shown in the figure 3(a). This interaction changes the chirality of each light flavor from L to R and is antisymmetric in the flavor indices. (This is the reason why this interaction is often called the determinant interaction.) The strength of III can be determined by the $\eta - \eta'$ mass difference, which comes partly from the flavor $SU(3)$ breaking and mixing and partly from the effect of the $U_A(1)$ breaking term. We find that the two effects are of the same order[14]. Recent analysis of the $\eta \to \gamma\gamma$ decay also suggests a significant strength of III[15].

In applying this interaction to the baryon problem, we employ the nonrelativistic valence quark model, and reduce III into a nonrelativistic form[14]. We obtain a two-body $\text{III}$ (figure 3(b)) given by

$$V_{\text{III}}^{(2)} = V_0^{(2)} \sum_{i<j} \frac{15}{16} A_f^{(2)} \delta(\vec{r}_{ij}) \left[ 1 - \frac{1}{5} (\vec{\sigma}_i \cdot \vec{\sigma}_j) \right]$$

and a three-body interaction (figure 3(a)),

$$V_{\text{III}}^{(3)} = V_0^{(3)} \sum_{(i,j,k)} \frac{189}{160} A_f^{(3)} \delta(\vec{r}_{ij})\delta(\vec{r}_{jk}) \left[ 1 - \frac{1}{7} \{ (\vec{\sigma}_i \cdot \vec{\sigma}_j) + \text{permutations} \} \right]$$

where $A_f^{(n)}$ stands for the antisymmetrization of $n$ quarks in the flavor space. Thus this interaction is nonzero only for the flavor antisymmetric combination of quarks. The strength of the two-body III is determined by that of the three body III by $V_0^{(2)} = V_0^{(3)} \langle \bar{q}q \rangle / 2$. 

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It is easy to see that the three-body instanton induced interaction \((5)\) vanishes in the ground state baryons because they are not in the flavor singlet representation. The two-body interaction \((4)\) gives a contribution for the flavor antisymmetric pairs of quarks. We find that the spin structure of the two-body III is identical to the color-magnetic gluon exchange and therefore it also explains the hyperfine splittings of the baryon spectrum, such as the \(N - \Delta\) splitting\([16,17]\). The III strength determined by the \(\eta - \eta'\) spectrum, indeed, explains 30-40\% of the hyperfine splitting.

The three-body III plays a significant role in the \(H\) system. \(H\) contains two sets of antisymmetric \(u - d - s\) quarks. We performed the quark cluster model analysis of \(H\) including the III term in the quark hamiltonian. We find that the contribution of the three-body III to the \(H\) is strongly repulsive, while the two-body III gives moderate attraction. The net result amounts to pushing the \(H\) mass up by about 40 – 50 MeV. It is easy to understand that the three-body III is repulsive. The ratio of the two-body III and three-body III is proportional to the quark condensate in the vacuum that is negative. The two-body III is attractive and induces the quark condensate.

We performed the quark cluster model calculation for \(H\)\([14]\) including the quark exchange interaction, the effective meson exchange interaction and the instanton induced interaction. We found that the attraction due to the two-body III is mostly absorbed into the meson exchange interaction when it is adjusted to the \(NN\) scattering data. Thus the effect of the three-body III gives a net repulsion. Our final conclusion is that \(H\) is either barely bound or unbound, depending on how strong the III is. Even if it is not bound, it is still possible to have a \(\Lambda\Lambda\) resonance state below the \(N\Xi\) threshold.

4. QCD Sum Rule analysis

The QCD sum rule is a novel way of studying hadron spectrum and properties directly from QCD\([18]\). The sum rule takes advantage of analyticity of current correlators and relates the asymptotic free region of QCD to the nonperturbative physical region. On one side (theoretical side) the correlator is calculated perturbatively for a large Euclidean momentum carried by the current and then the result is analytically continued to the
physical spectrum region. It requires some matrices of quark-gluon local operators in the vacuum, which are determined either by other sum rules or by imposing consistency of the sum rule. On the phenomenological side, the physical spectral density is parameterized in a form with discrete poles and continuum parts, whose parameters (position of the poles, coupling strengths, thresholds for the continuum, etc.) are determined so as to coincide with the theoretical side extrapolated analytically from the deep Euclidean region. The process of the analytical continuation may often be subtle because the physical parameters are not so sensitive to the short distance behavior of the correlator. The Borel sum and the finite-energy sum rule are two popular techniques employed so that one can find the most appropriate weight function in comparing the phenomenological and theoretical sides.

Recently we have applied the QCD sum rule to the $H$ dibaryon problem. We construct the interpolating current for the $H$ dibaryon as a product of two currents representing the flavor-octet baryons,

$$J_H(x) = \sum_{\alpha=1}^{8} J^{\alpha}_B C\gamma_5 J^{\alpha}_B$$

where $C$ is the charge conjugation operator and $\alpha$ labels the flavor octet members. We apply the QCD sum rule for the current correlator (figure 4) defined by

$$\Pi_H(x) = \langle T[J_H(x)J_H^\dagger(0)] \rangle$$

Details of this calculation should be referred to ref.\[19\]

It happens that the sum rule cannot effectively fix the continuum threshold, and thus the prediction has a large ambiguity. We therefore compare the $H$ sum rule with a similar sum rule for the “dinucleon” $D$, which is a hypothetical bound state of two nucleons (protons) in the spin singlet state. Comparison of those two states is easy because they have the same spin and thus similar current structures. To our surprise, we have found that those two sum rules are almost identical in the $SU(3)$ limit and therefore predict nearly the same masses for $H$ as the $^1S_0$ di-nucleon $D$, which is unbound experimentally albeit close to be bound. This degeneracy of $H$ and $D$ has its origin in the similarity of the theoretical sides in the $SU(3)$ limit.

Effects of $SU(3)$ breaking are taken into account as terms proportional to the strange quark mass and the difference of $\langle \bar{s}s \rangle$ and $\langle \bar{u}u \rangle$. Then we find that the mass of $H$ is about
(2.19 ± 0.07) GeV, the central value of which is about 40 MeV below the ΛΛ threshold. Because the result is extremely sensitive to the choice of the continuum threshold energy, the number should not be taken too seriously. Yet, from the comparison with the “D sum rule” we conclude that the binding of \( H \) is not as large as that expected in the original quark models (without the instanton effects). It should also be noted that the quark model generally predicts the largest binding energy in the \( SU(3) \) limit, i.e., the symmetry breaking reduces the binding energy. The sum rule predicts the opposite tendency.

5. Conclusion

The conclusion here is very short. The theory predictions of the \( H \) dibaryon mass have climbed from a deeply-bound “6-quark exotic object” to an unbound “two baryons”, while the experimental lower limit has increased. In fact, no definite prediction is yet given. In a sense this is frustrating, but it can also be interpreted that the \( H \) dibaryon physics contains an essential part of QCD. The hadron physics so far made a lot of predictions based on the symmetry. The best example would be the soft pion theorems for pion dynamics. They are quite robust because the chiral symmetry protects them. Situation seems entirely different in baryon physics. Most predictions there are model dependent, while few systematic expansion methods are successful. The fact that the \( H \) predictions have big disparity among the models indicates that it contains an interesting physics.

Experimental searches are still strongly encouraged.

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Figure 1: $H$ dibaryon with $J^\pi = 0^+$, $I = 0$, $B = 2$, $S = -2$ and $Y = 0$. 
Figure 2: A quark exchange diagram associated with a gluon exchange.
Figure 3: (a) Instanton induced interaction for the flavor antisymmetric $u - d - s$ system.
(b) Two-body III for the $I = 0$ $u - d$ system.
Figure 4: QCD sum rule for the $H$ current correlator.