Modified trilinear stress-strain diagram of concrete designed for calculation of beams with fiberglass rebar

Ilshat Mirsayapov and George Apkhadze

1Kazan State University of Architecture and Engineering, Kazan, Russia
E-mail: georgevt@yandex.ru

Abstract. An improved technique for determining the strength of reinforced beams with polymer composite reinforcement based on a nonlinear deformation model using a three-line diagram of concrete deformation under uniaxial compression with an elastic, elastic-plastic zone and a concrete softening zone is considered and proposed. The features of the strength analysis of beams with fiberglass reinforcement are revealed. The calculation method presented in this article allows us to minimize the use of empirical coefficients to determine the height of the compressed zone of concrete and at the same time does not lead to labor-intensive calculated dependencies. The method is based on the use of the Euler – Bernoulli beam theory and the use of physical stress-strain relations for concrete. This calculation method allows you to set accurately the height of the compressed zone of concrete, and accordingly the strength of the beam elements in the normal section. Good results of comparison with experimental data were obtained.

Key words: buildings, structures, construction, concrete, fibre-reinforced concrete, beams.

1 Introduction

The increasing production and use in the design of composite polymer reinforcement dictates new requirements for design methods, which often require taking into account additional factors that are not inherent in well-studied reinforced concrete structures [1].

These factors include increased deformability of flexible concrete elements reinforced with polymer composite reinforcement, which in turn determines their fracture pattern [2–4]. With sufficiently small diameters of the reinforcement, due to a sufficiently large design resistance and a relatively small deformation modulus, the destruction of composite concrete bending elements occurs from crushing of concrete in the compressed zone when the deformation in extreme compressed fibers reaches the cross section of the ultimate relative concrete deformation to compression [5]. It is worth noting that the value of ultimate compressive deformations of concrete according to the results of experimental studies [6] does not have a definite exact value and varies over a rather large range.

The modern regulatory approach to the design of reinforced concrete bending elements based on the method of ultimate efforts SP63.13330.2018 does not consider the case of clarifying the height of the compressed zone of concrete when the condition $\xi = x/h_0 \leq \xi_R$ is not met, which in turn leads to large reserves when using this approach in the design of composite ton bending elements. In this regard, [7] provides an improved calculation method for determining the height of the compressed zone of concrete when determining the bearing capacity of reinforced beams, borrowing some of the position of a nonlinear deformation model and minimizing the difference between the average ratio of the experimental and calculated bearing capacity in non-reinforced and reinforced flexible composite elements. However, the disadvantage of such a simplified method of the limit force method is the experimental coefficient for taking into account the completeness of the diagram of the compressed zone...
(0.8 – for low-strength heavy concrete, 0.7 – for high-strength heavy and fine-grained concrete), which was not constant in the normative method SniP 2.03.01-84, but depending on the compressive strength of concrete, and on which the strength of the reinforced bending element depends to a large extent.

It is worth noting that all models require refinement in the case of a design operating under conditions of prolonged load taking into account creep deformations [8-13].

The proposed calculation method involves a complete analytical solution to the problem of determining the strength of reinforced composite concrete bending elements using some of the prerequisites:

- the work of concrete in the tensile section zone is not taken into account;
- the work of composite reinforcement in compression is not taken into account;
- the distribution of the relative deformations of concrete and reinforcement in a normal section up to failure with some simplification obeys the Euler – Bernoulli beam theory, i.e. the normal section remains flat and perpendicular to the normal axis of the element;
- ultimate relative compression deformations of concrete are accepted not constant, but depending on the mechanical characteristics of concrete;
- the diagram of the compressed zone of concrete in the normal section of the element at the time of reaching the limit state is 3 sections (conditionally elastic, elastic-plastic and plastic);
- the reduction of compressive stresses at the extreme points of the cross section is taken into account when the limiting state is reached, i.e. the descending branch of the deformation diagram of compressed concrete is taken into account (section 2-3 in figure 1).

![Figure 1. Modified three-line diagram of concrete deformation with softening under uniaxial compression.](image)

**2 Materials and methods**

Accounting for the descending branch of the concrete deformation diagram is based on the assumption that the internal energy in the concrete element is equal before the softening begins and the internal energy in the concrete element from the moment the concrete softening process begins until it collapses, i.e. equality of areas ($\Omega_1 = \Omega_2$) limited by the axis of relative deformations and the curve of deformation of concrete under uniaxial stress state (figure 1). In this case, the conditionally elastic zone of the concrete deformation diagram is limited to $0.6R_b$, and the end of the descending branch, based on the data of many experimental studies [14-17] is taken at a stress level of $0.85R_b$. 
The calculation model of the normal section of a bent reinforced composite concrete element when determining the strength by the proposed method is presented in figure 2.

For this computational model, it is possible to compose the calculated dependencies taking into account the assumptions presented above.

From figure 1 for the appropriate class and type of concrete, we specify the ultimate deformations of concrete under uniaxial stress state:

\[
\Omega_1 = \Omega_2; \quad (1)
\]

\[
\varepsilon_{b2} = \frac{3.45\varepsilon_{b0} - 0.4\varepsilon_{p1}}{1.85} = \frac{0.0069 \cdot 0.24\varepsilon_{b0}}{1.85}. \quad (2)
\]

The heights of each section of the compressed zone of concrete are determined on the basis of the Euler – Bernoulli beam theory:

- for conditionally elastic zone of compressed concrete:
  \[
  x_1 = \frac{\varepsilon_{b1}}{\varepsilon_{b2}}; \quad (3)
  \]

- for elastic-plastic zone of compressed concrete:
  \[
  x_2 = \frac{\varepsilon_{b3} - \varepsilon_{b4}}{\varepsilon_{b2}}; \quad (4)
  \]

- for plastic zone of compressed concrete:
  \[
  x_3 = \frac{\varepsilon_{b5} - \varepsilon_{b6}}{\varepsilon_{b2}}. \quad (5)
  \]

For a bent element in a normal section, the equation of equality of internal forces in a compressed and stretched zone can be drawn up taking into account ultimate deformations of concrete in a compressed zone and stresses in composite reinforcement less than ultimate stresses, what describes the case of failure in a compressed zone of concrete:

\[
0.5R_b(x_1 + 1.6x_3 + 1.85x_3) = \sigma_f \cdot A_f. \quad (6)
\]

In this case, the stresses in the tensile rebar are taken in view of its absolutely elastic operation a short-term load action, what is confirmed by experimental studies [18-19]:

\[
\sigma_f = \varepsilon_f \cdot E_f, \quad (7)
\]

where the relative deformations of the composite reinforcement are taken from the following ratio according to the calculation model (figure 2):

\[
\frac{\varepsilon_{f}}{\varepsilon_{b2}} = \frac{h_0 - x}{x}. \quad (8)
\]
The ultimate bending moment, perceived by the normal section, is determined from the equation of equality of the external and internal moment relative to the axis passing at the level of the center of gravity of the tensile rebar:

$$M_{\text{ult}} = 0.5R_b b \left[ 0.6x_1(h_0 - a_1) - 1.6x_2(h_0 - a_2) + 1.85x_3(h_0 - a_3) \right], \quad (9)$$

where are the distances from the center of gravity of the stretched reinforcement to the center of gravity of the corresponding section of the compressed zone of concrete, determined from the following relations (figure 2):

- \(a_1 = \frac{v_3}{v_b} \cdot \frac{2}{3} \cdot x_1^*\); \((10)\)
- \(a_2 = x_1^* + 0.4583 \cdot x_2^*\); \((11)\)
- \(a_3 = 0.5135 \cdot x_3^*\). \((12)\)

After substitution in Eq. (6), the general form of the expression for determining the height of the compressed zone of concrete will look like this:

$$x = - \frac{A_f E_f \varepsilon_{b2} + \sqrt{(A_f E_f \varepsilon_{b2})^2 + \left( 1.85 - \frac{\varepsilon_{b1}}{\varepsilon_{b2}} - 0.25 \frac{\varepsilon_{b0}}{\varepsilon_{b2}} \right) \cdot A_f E_f \varepsilon_{b2} R_b h_0 b}}{R_b b \cdot \left( 1.85 - \frac{\varepsilon_{b1}}{\varepsilon_{b2}} - 0.25 \frac{\varepsilon_{b0}}{\varepsilon_{b2}} \right)}. \quad (13)$$

Thus, this technique allows you to do without introducing into the expressions for determining the height of the compressed zone of concrete coefficient of accounting for the completeness of the plot.

3 Results and discussions

Strength of bent elements with a single lower fiberglass reinforcement is compared for different classes of heavy (table 1) and fine-grained (table 2) concrete using the ultimate efforts method (MUE) SP295.1325800.2017 and SP63.13330.2018, the refined ultimate efforts method [7] and according to the proposed methodology based on a nonlinear deformation model.

Table 1. Comparison of the strength \((M_{\text{ult}})\) according to various methods with a percentage of reinforcement of \(\mu = 0.7\%\) and heavy concrete of classes B15-B80 (SP 63.13330.2018).

| Parameter | Method | Class of concrete (SP 63.13330.2018) |
|-----------|--------|-------------------------------------|
|           |        | B15       | B20       | B25       | B30       | B40       | B50       | B60       | B80       |
| \(M_{\text{ult}}\), kNm | 1       | 4.26      | 5.77      | 7.27      | 8.52      | 11.03     | 13.79     | 16.55     | 18.14     |
|           | 2       | 8.50      | 10.38     | 12.05     | 13.32     | 15.63     | 17.91     | 19.97     | 21.20     |
|           | 3       | 8.63      | 10.53     | 12.21     | 13.48     | 15.77     | 17.95     | 19.88     | 22.40     |

Table 2. Comparison of the strength \((M_{\text{ult}})\) according to various methods with a percentage of reinforcement of \(\mu = 0.7\%\) and fine-grained concrete of classes B15-B80 (SP 63.13330.2018).

| Parameter | Method | Class of concrete (SP 63.13330.2018) |
|-----------|--------|-------------------------------------|
|           |        | B15       | B20       | B25       | B30       | B40       |
| \(M_{\text{ult}}\), kNm | 1       | 3.761     | 5.089     | 6.417     | 7.523     | 8.629     | 9.736     |
|           | 2       | 7.931     | 9.688     | 11.249    | 12.440    | 13.553    | 14.600    |
|           | 3       | 8.554     | 10.418    | 12.050    | 13.292    | 14.434    | 15.486    |

Note: 1 – ultimate efforts method (MUE) SP 63.13330.2018; 2 – an updated methodology of the method of ultimate efforts SP 295.1325800.2017; 3 – the proposed calculation method.

*the values in this line are calculated for fine-grained concrete under conditions of natural hardening with a decreasing coefficient of 0.89 to the initial modulus of elasticity.
When calculating the strength of reinforced samples according to Eqs. (1)-(12), the results are closest to the experimental values of destructive moments. A comparison of the values of the strength of the samples according to the updated method [7] and the proposed method with the data of experimental studies [20-21] are shown in figure 3.

![Figure 3. Comparison of experimental strength capacity with theoretical values for different classes of concrete (SP 63.13330.2018) and percent reinforcement section of beams.](image)

For the presented sample, the average deviation in percent of the ultimate bending moment calculated by the proposed calculation method, taking into account the above assumptions, from the experimental destructive bending moment is 2.54 %.

4 Conclusion
1. Strength calculations of normal sections of reinforced bending elements with polymer composite reinforcement should be made using a three-line deformation diagram of concrete taking into account the softening, taking various ultimate compressive strains of concrete depending on the class and type of concrete, which is in good agreement with data from experimental studies.

2. The calculation method presented in this article allows minimizing the use of empirical coefficients for determining the height of the compressed zone of concrete and at the same time does not lead to laborious design dependencies.

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