Magnetic tension and gravitational collapse

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Abstract

The gravitational collapse of a magnetized medium is investigated by studying qualitatively the convergence of a timelike family of non-geodesic worldlines in the presence of a magnetic field. Focusing on the field’s tension, we illustrate how the winding of the magnetic force lines due to the fluid’s rotation assists the collapse, while shear-like distortions in the distribution of the field’s gradients resist contraction. We also show that the relativistic coupling between magnetism and geometry, together with the tension properties of the field, lead to a magneto-curvature stress that opposes the collapse. This tension stress grows stronger with increasing curvature distortion, which means that it could potentially dominate over the gravitational pull of the matter. If this happens, a converging family of non-geodesic worldlines can be prevented from focusing without violating the standard energy conditions.

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1. Introduction

Magnetic fields are very common in astrophysical environments and stellar magnetism is a long established and very active branch of astrophysics. Nevertheless, the study of magnetic, and of electromagnetic, fields in strong gravity environments is less developed. Most of the available studies address the possible gravitational effects on the Maxwell field and relatively few look into the implications of magnetic fields, in particular, for gravitational collapse itself.

One of the most intriguing results so far has been obtained by Thorne in his analysis of Melvin’s cylindrical magnetic universe [1]. There, by developing the concept of ‘cylindrical energy’, the author reached the conclusion that ‘a strong magnetic field along the axis of symmetry may halt...
the cylindrical collapse of a finite cylinder before the singularity is reached’ [2]. The possible support of the Maxwell field against the gravitational collapse of massive bounded systems was also studied in [3]. That work led to solutions of the Einstein–Maxwell system where the gravitational attraction is solely balanced by magnetic stresses. Studies of contracting charged dust have also suggested that the fluid may ‘rebounce’, thus preventing black-hole formation [4]. It has been argued, on the other hand, that a collapsing spherically symmetric charged dust will produce naked singularities due to shell crossing [5]. Although these singularities are considered weak because the curvature invariants and the tidal forces remain finite [6], their appearance could also signal that a nonzero Lorentz force and spherically symmetric charged collapse may not be physically compatible [7].

In this paper we revisit the issue of the magnetic impact on gravitational collapse from an apparently entirely different viewpoint. We consider the non-spherical (but not necessarily cylindrical) collapse of a magnetized fluid, by studying the convergence of two neighbouring particle worldlines. As it turns out, we arrive at the same qualitative result as Thorne did, namely that the gravitational collapse of a magnetized fluid may stop before reaching the singularity. In our analysis, however, the reasons are seemingly unrelated to the cylindrical energy of Melvin’s universe or to the charge density of the magnetized matter. We find instead that it is the intricate coupling between magnetism and geometry and the tension properties of the magnetic force lines, namely their elasticity, which gives rise to ever increasing resisting stresses that may prevent the ultimate collapse from happening. Although these magneto-geometrical tension stresses are nothing more than the relativistic generalization of a rather well-known Newtonian effect, their existence remains largely unknown in the literature. Among the effects of the tension stresses we identify what is usually referred to as ‘magnetic braking’ and demonstrate how it assists the collapse of the magnetized fluid. We also find that shear-like distortions in the distributions of the field’s gradients resist contraction.

2. The worldlines of magnetized matter

We will study the magnetic implications for gravitational collapse qualitatively by testing the convergence of the particle worldlines. In doing so we will be using the covariant approach to general relativity [11], and in many respects our discussion will resemble that of [12]. Throughout this paper we assume conventional matter with positive gravitational mass and pressure, which means that the standard energy conditions are always fulfilled. High conductivity means that there is no electric field and that the magnetic field is ‘frozen in’ with the matter. This is the well-known MHD approximation [13].

When looking into the dynamics of gravitational collapse Raychaudhuri’s formula is the key equation, as it covariantly describes the volume evolution of a self-gravitating fluid element [14]. Consider a congruence of timelike worldlines tangent to the 4-velocity field $u^a$ (with
$u_au^a = -1$). These are the worldlines of the fundamental observers and follow the motion of the fluid. The Raychaudhuri equation determines the proper-time evolution of $\Theta = \nabla_a u^a$, the scalar measuring the average contraction (or expansion) between two neighbouring worldlines [11]. In a magnetized environment we have [15]

$$\Theta = -\frac{1}{2} \Theta^2 - \frac{1}{2} \kappa (\mu + 3p + B^2) - 2\sigma^2 + 2\omega^2 + D^a u_a + \dot{u}_a \ddot{u}^a,$$

where $\kappa = 8\pi G$, $\mu$ and $p$ are respectively the energy density and pressure of the fluid, $B^2 = B_a B^a$ measures the energy density and the isotropic pressure of the magnetic field ($B_a$), $\sigma^2$ and $\omega^2$ are the respective magnitudes of the shear and the vorticity associated with $u_a$ and $\dot{u}_a = u^b \nabla_b u_a$ is the 4-acceleration. The latter satisfies the momentum–density conservation law, which for a highly conductive perfect fluid takes the form [15]

$$(\mu + p + \frac{2}{3} B^2) \dot{u}_a = -D_a p - \epsilon_{abc} B^b \text{curl} B^c - \Pi_{ab} u^b,$$

where $D_a = h_{ab} \nabla_b$ is the covariant derivative operator orthogonal to $u_a$ and $\Pi_{ab} = -B_{[a} B_{b]}$ describes the magnetic anisotropic pressure. Note that we consider non-geodesic worldlines, since the motion of the particles is dictated by the combined Einstein–Maxwell field and not by gravity alone. Also, the fluid flow is not hypersurface orthogonal which explains the presence of the vorticity term in (1).

The right-hand side of equation (1) determines the dynamics of the average volume evolution. Terms that are positive definite lead to expansion, while negative definite terms cause contraction. Thus, when the standard energy conditions are satisfied, all the right-hand side terms have a clear-cut role with the exception of $D_a \dot{u}^a$, which in principle can go either way. For the rest of this study, we will focus on $D_a \dot{u}^a$ and examine its potential implications for the final fate of a collapsing magnetized fluid element.

3. Lorentz force and magnetic tension

The magnetic contribution to $D_a \dot{u}^a$ comes from the Lorentz force. The latter is always normal to the direction of the field lines and emerges whenever the magnetic pattern is distorted from the condition of local equilibrium. Here, the Lorentz force is determined by the acceleration vector

$$a_a = -\epsilon_{abc} B^b \text{curl} B^c = -\frac{1}{2} D_a B^2 + B^a D_b B_b,$$

with $a_a B^a = 0$. The first gradient in the right-hand side of the above is due to the magnetic pressure and the second comes from the field’s tension (e.g. see [13]). Insofar as the tension stress is not balanced by the gradients of the magnetic pressure, a net force is exerted on the fluid particles. Note that $\epsilon_{abc}$ is the totally antisymmetric alternating tensor orthogonal to $u_a$ and therefore $a_a u^a = 0$. Using the 3-Ricci identity (see equation (6) below), the projected gradient of $a_a$ decomposes as

$$D_b a_a = -\frac{1}{2} D_4 D_a B^2 + D_b B^c D_c B_a + B^c D_b D_c B_a + \mathcal{R}_{abcd} B^d B^e - 2\omega_{bc} B_{[a} B_{b]} B^e.$$

Here $\mathcal{R}_{abcd}$ is the Riemann tensor of the observer’s instantaneous three-dimensional rest space and $\omega_{ab}$ is the vorticity tensor associated with the fluid flow. The last four terms on the right-hand side of the above convey the magnetic tension effects. In particular, the magneto-curvature stress in equation (4) reflects the special status of vectors, as opposed to that of scalars, in general relativity. This special status stems from the geometrical nature of the

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1 Angled brackets indicate the symmetric, trace-free part of projected second-rank tensors (e.g. $B_{[a} B_{b]} = B_{[a} B_{b]} - (B^2/3) h_{ab}$, where $h_{ab} = g_{ab} + u_a u_b$ with $h_{ab} u^b = 0$).
theory and it is manifested in the Ricci identity
\[
2\nabla_{[a} \nabla_{b]} B_c = R_{dcba} B^d,
\]
(5)
applied here to the magnetic field vector, with \( R_{abcd} \) being the spacetime Riemann tensor. When projected orthogonal to \( u_a \), the above expression leads to what is commonly referred to as the 3-Ricci identity \([11]\)
\[
2 D_{[a} D_{b]} B_c = R_{dcba} B^d - 2 \omega_{ab} h_{cd} B^d.
\]
(6)

The Ricci identities argue for a direct interaction between vector sources and spacetime curvature, which adds to the standard interlay between matter and geometry as we know it from the Einstein field equations. In the magnetic case this direct coupling also brings into play the tension properties of the field, namely the elasticity of the magnetic force lines, and couples it in an intricate way with the geometry of the space. This unique feature, the magnetic tension, manifests the well-known fact that the field lines ‘do not like to bend’ and react to any attempt that distorts them. Indeed, within the Newtonian theory the magnetic tension is known to trigger restoring stresses which depend on the strength of the field and on the deformation of the magnetic force lines (measured by their curvature radius) \([13]\). What the general relativistic expression (4) shows is that deviations from Euclidean geometry will lead to analogous tension stresses, which are also proportional to the magnetic strength and to the amount of curvature distortion. This time, however, it is the curvature of the space itself that causes part of the magnetic deformation. In other words, the magnetic field ‘feels’ the curvature of the space in a way which is dictated by the Ricci identities and this is demonstrated by the magneto-curvature stress in the right-hand side of equation (4). The effects of this tension stress are generally counter-intuitive because of the nature of the magnetic property itself and its subtle coupling with the geometry of the space. The latter means that even weak magnetic fields can lead to a strong overall effect under favourable circumstances. The potentially pivotal implications of magnetism, and of the magnetic tension in particular, for cosmology were originally discussed in \([16]\). These results have been confirmed and extended in \([17]\), although an oversight prevented the authors of the latter paper from recognizing the key role of the magnetic tension. Here, we will consider the implications of the same tension stresses for the gravitational collapse of a magnetized fluid.

Suppose that both the fluid and the magnetic field have a nearly homogeneous energy density distribution. In practise this means that any inhomogeneities that might be present are relatively small and that we may ignore spatial gradients in the energy density of the two sources. For the magnetic field our assumption implies that the Lorentz force term in the right-hand side of (2) is dominated by the tension stresses. This is exactly what we need, since our aim is to investigate the effect of these particular magnetic stresses. Then, equation (2) gives
\[
(\mu + p + B^2) \ddot{u}_a = -\epsilon_{abc} B^b \text{curl} B^c,
\]
(7)
Taking the projected divergence of the above, using the trace of (4), with \( D_a B^a = 0 \) due to the absence of electric fields, and substituting into equation (1) we arrive at
\[
\dot{\Theta} + \frac{1}{2} \dot{\Theta}^2 = -\frac{1}{2} \kappa (\mu + 3 p + B^2) - 2 (\sigma^2 - \Sigma^2) + 2 (\omega^2 - W^2) + \epsilon^{-1} R_{ab} B^a B^b + \dot{u}_a \dot{u}^a,
\]
(8)
where \( \epsilon = \mu + p + B^2, \Sigma^2 = (D_a B_b)^2 / 2 \epsilon, \omega^2 = (D_a B_b)^2 / 2 \epsilon \) and \( R_{ab} = R^c_{a bh} \) is the 3-Ricci tensor given by the Gauss–Codacci equation \([11, 18]\)
\[
R_{ab} = h_{a}^{c} h_{b}^{d} R_{cd} + R_{acbd} u^c u^d - \kappa_{c}^{c} k_{ab} + k_{ac} k_{cb},
\]
(9)
where \( k_{ab} = D_a u_b \) is the extrinsic curvature tensor. The second term in the right-hand side of the above ensures that, in addition to the usual matter fields, the Weyl curvature (i.e. tidal
forces and gravitational waves) also contributes to the geometry of the 3-space. Also note that in deriving equation (8) we have assumed magnetic flux conservation, which guarantees that the rotation term in the trace of (4) vanishes.

4. The magnetic tension stresses

We will first turn our attention to the quantities \( \Sigma^2 \) and \( W^2 \) in the right-hand side of (8), which are the magnitudes of \( \Sigma_{ab} = D_a B_b / \sqrt{\epsilon} \) and \( W_{ab} = D_a B_b / \sqrt{\epsilon} \), respectively. The former of these tensors describes distortions in the magnetic field distribution and the latter is the magnetic twist tensor. Note that although \( \Sigma_{ab} \) and \( W_{ab} \) have the shear and the vorticity as their respective kinematical analogues, their effect on \( \Theta \) is exactly the opposite of the one normally associated with the shear and the vorticity proper (see also [19]). This counterintuitive behaviour reflects the fact that both \( \Sigma \) and \( W \) carry the tension properties of the field and manifests the tendency of the magnetic force lines to remain straight. Thus, while a nonzero shear assists the collapse, the corresponding magnetic stress tries to balance this effect out. Consider also the tensor \( W_{ab} \), which has magnitude \( W^2 = W_{ab} W^{ab} / 2 = (\text{curl} B_a)^2 / 4 \epsilon \) and is triggered by the winding of the magnetic field lines around a rotating fluid element. Following (8), a nonzero \( W \) will always reduce the gravitational effect of the vorticity. This manifestation of the field’s tension is also known as ‘magnetic braking’ and can accelerate the gravitational collapse of a rotating star [10].

For our purposes the key quantity is the magneto-curvature tension stress \( R_{ab} B^a B^b \) in the right-hand side of equation (8), which measures the curvature of the 3-space along the direction of the magnetic force lines. Starting from (9) one can show that, when only the magnetic field is present, \( R_{ab} B^a B^b = 0 \) [20]. Therefore, despite the magnetic energy input, the curvature of the 3-space in the direction of the field lines is zero; a result independent of the magnetic strength. Technically speaking, it is the negative pressure of the field along its own direction which cancels out the positive contribution of the magnetic energy density. More intuitively, it is the inherent tendency of the magnetic force lines to remain ‘straight’ which is responsible for the aforementioned null result. Clearly, in the presence of other sources \( R_{ab} B^a B^b \neq 0 \). Then, the magneto-curvature effect on \( \Theta \) is rather unexpected. For \( R_{ab} B^a B^b < 0 \) the magneto-geometrical stress in (8) brings the particle worldlines closer, but pushes them apart when the field lines are ‘positively curved’, that is for \( R_{ab} B^a B^b > 0 \) (see [16] and also [17]). This is against the common perception, which always associates positive curvature with gravitational contraction. As with the magnetic shear and the magnetic vorticity stresses discussed earlier, the reason for the counter-intuitive behaviour of the magneto-curvature term is the tension properties of the field lines.

Let us assume, mainly for illustration purposes, that the effect of \( \Sigma \) and \( W \) is cancelled out by that of their kinematic counterparts. In other words, we will consider the case where \( \omega^2 + \Sigma^2 = W^2 + \sigma^2 \). Although it may not appear so initially, we will later show that this assumption is much less restrictive that it looks. For the moment we note that, in addition to counteracting each other, the pairs \( \omega^2, W^2 \) and \( \sigma^2, \Sigma^2 \) are of the same nature and ‘perturbative order’ (i.e. quadratic in \( D_a u_b \) and \( D_a B_b \)). The former of these properties supports the assumption that the aforementioned opposing pairs are very likely to balance each other out. The latter ensures that these terms, unlike \( R_{ab} B^a B^b \) for example, become appreciable only in highly inhomogeneous configurations. Under such conditions, the Raychaudhuri equation reads

\[
\dot{\Theta} + \frac{1}{3} \Theta^2 = -R_{ab} u^a u^b + c_2^2 R_{ab} n^a n^b + \dot{u}_a \dot{u}^a,
\]

(10)
where \( n_a = B_a/\sqrt{B^2} \), \( c_a^2 = B^2/\epsilon \) is the Alfvén speed and \( R_{ab} \) is the Ricci tensor of the spacetime with \( R_{ab}u^au^b = \kappa(\mu + 3p + B^2)/2 > 0 \). The latter means that the strong energy condition is satisfied.

It should be noted that, in addition to the aforementioned arguments regarding the relative gravitational input of the shear, the vorticity and of their magnetic counterparts, bypassing these terms also helps to isolate the two resisting stresses in equation (8), namely \( c_a^2R_{ab}n^an^b \) and \( u_au^a \). The latter has been analysed within a collapsing perturbed Tolman–Bondi spacetime in [7], where it was found to grow faster than all the other terms in the right-hand side of the Raychaudhuri equation. Here we will focus on the magneto-curvature tension stress instead. In any physically realistic scenario of stellar collapse this term is always positive and therefore it always resists against further contraction. Moreover, the strength of the gravito-magnetic stress is essentially proportional to the amount of the curvature distortion. This feature distinguishes the tension stress from the rest and makes it a very promising candidate for outbalancing the gravitational pull of the matter.

5. Gravitational pull versus gravito-magnetic tension

The assumption of a spatially homogeneous energy density distribution for the sources means that \( D_{ap} = 0 \), which in turn guarantees that the 4-acceleration vector \( u_au^a \) depends entirely on the magnetic field (see equation (7)). Therefore, when the field is absent all the positive definite terms in the right-hand side of (10) vanish and an initially converging congruence (i.e. one with \( \Theta_0 < 0 \)) will focus (i.e. \( \Theta \rightarrow -\infty \)) within a finite amount of time, unless the energy conditions are violated. This is a fundamental and well-known result about gravitational collapse (e.g. see [18]). In the magnetic presence, however, the two positive definite terms in the right-hand side of (10) will resist the collapse. Thus, ignoring the supporting effect of \( u_au^a \), we argue that the magneto-curvature effects can prevent a converging congruence of non-geodesic worldlines from focusing, without violating the standard energy conditions, if

\[
c^2_aR_{ab}n^an^b \geq R_{ab}u^au^b. \tag{11}
\]

Obviously, the above also holds when the earlier imposed constraint \( \omega^2 + \Sigma^2 = W^2 + \sigma^2 \) is replaced by \( \omega^2 + \Sigma^2 \geq W^2 + \sigma^2 \). It still holds when \( \omega^2 + \Sigma^2 < W^2 + \sigma^2 \), provided that \( u_au^a/2 > W^2 + \sigma^2 - \omega^2 - \Sigma^2 \). So, condition (11) holds for a variety of combinations between \( \omega^2, \Sigma^2, W^2 \) and \( \sigma^2 \). This means that the earlier restriction placed on these quantities is not essential for the validity of our argument. The same can also be said about the fluid inhomogeneities, given that pressure gradients resist the collapse (through the last term in the right-hand side of (10)). Our approximations, however, have helped to isolate and demonstrate the role of the curvature terms in equation (8), which should decide the ultimate fate of the collapse. Indeed, the main reason for focusing on the two geometrical quantities in the right-hand side of (8) is that, as the collapse proceeds, we expect the curvature to dominate. Then, the fate of the collapsing magnetized fluid should be decided by the balance between the two quantities in equation (11). Therefore, the curvature terms in (8) have not been unduly favoured at the expense of the rest. If the gravitational pull of the matter, which is the driving force behind the collapse, is outbalanced, there is a realistic possibility of avoiding worldline focusing. The magneto-curvature tension stresses open this possibility. This is so because, while only the usual matter fields contribute to \( R_{ab}u^au^b \), the tension stress \( R_{ab}n^an^b \) has additional contributions from other sources (see equation (9)). Probably the most important among these extra sources is the Weyl curvature. This monitors the long-range gravitational field and contributes to \( R_{ab} \) directly via the electric Weyl component [11]. The latter describes the tidal forces which are expected to increase dramatically as
the collapse proceeds. These additional contributions to the 3-Ricci curvature mean that, in principle, the magneto-curvature tension stresses can outbalance the gravitational pull of the matter. Although the final outcome depends on the precise characteristics of the collapse, our analysis points towards the conjectural but very intriguing possibility that if the magnetic line deformation due to spatial curvature distortions is strong enough, the resulting tension stresses may just be able to avert the formation of caustics and eventually the ultimate collapse of the magnetized fluid.

6. Discussion

The immediate implication of the above study is that violating the standard energy conditions to prevent an initially converging congruence from focusing may not be always necessary when magnetic fields are present. In particular, nonspherical magnetized collapse (the magnetic presence will inevitably distort spherical symmetry to a larger or lesser degree) may not end up in a $\Theta \to -\infty$ singularity because of the field’s tension. Whether our analysis applies to physical situations, such as the gravitational implosion of a massive star, depends on what the exact properties of a realistic collapse are. It might be, for example, that stellar magnetic fields do not survive to the later stages of the collapse, or that the MHD limit is no longer a good approximation. We have no reason to believe that either of these possibilities may be true however. In fact, the opposite appears more likely given the presence of very strong magnetic fields in compact stellar objects like neutron stars. It might also be that the gravitational pull of the matter always prevails and condition (11) is never satisfied. Very recent numerical simulations of magnetized hypermassive neutron-star collapse indicate that, at least when axial symmetry holds, this seems to be the case [10]. If condition (11) is met, however, the magnetic tension could stop the convergence of the fluid worldlines and the same could also happen to the contraction of a magnetized star. Similarly, if one evolves a magnetized universe backward in time, they may find a highly curved state that could potentially re-expand into the past. It should be noted, however, that avoiding worldline convergence only means avoiding a singularity in the congruence and does not guarantee a singularity-free spacetime; at least under the current consensus of what a singularity is [18]. For example, the spacetime can still be geodesically incomplete. Nevertheless, any such singularities should be of limited influence if most of the matter can successfully avoid them.

Stresses that support against gravitational collapse are not exclusively particular to a magnetic presence. It is well known that whenever the worldlines are not hypersurface orthogonal or geodesics, supporting stresses always appear due to rotation or pressure gradients. Vorticity, for example, has been considered in the past as a possible way of preventing the ultimate collapse [21]. Therefore, it is not so much the existence of magnetic related stresses that resist gravitational collapse, as the nature of the stresses themselves. In this respect, our key result is that, when magnetic fields are involved, one of the supporting stresses depends on (in fact it is proportional to) the distortion of the curvature itself. The presence of this stress is a direct and inevitable consequence of the vector nature of magnetic fields and of the geometrical nature of general relativity, while their counter-intuitive effects result from the tension properties of the field. It is the elasticity of the magnetic force lines, their inherit tendency to remain ‘straight’, which manifests itself as a reaction to curvature distortions that is proportional to the distortion itself. In a sense, it appears as though the elastic properties of the field have been injected into the fabric of the space.

Finally, when considering the unconventional magnetic behaviour described so far, the reader should keep in mind that magnetic fields are rather unusual sources themselves. The Maxwell field is the only vector source that we know that exists in the universe today. Within
the geometrical framework of general relativity, the status of vector fields is different from that of the ordinary matter. This is so because, in addition to the Einstein field equations, vector sources interact with the spacetime geometry through the Ricci identities as well. This purely geometrical coupling, which has been at the centre of our discussion, is already known to trigger some other rather nontrivial effects. The best-known example is probably the ‘scattering’ of electromagnetic radiation by the gravitational field, which leads to what is commonly referred to in the literature as ‘wave tales’ [22]. Here, we have outlined the potential implications of effectively the same Einstein–Maxwell coupling for gravitational collapse.

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