Radiation effects on MHD free convection flow along vertical flat plate in presence of Joule heating and heat generation

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Abstract

In this paper, the effect of radiation on magnetohydrodynamic (MHD) free convection flow along a vertical flat plate in presence of Joule heating and heat generation have been investigated. The governing equations associated with the conduction based boundary conditions are transformed into dimensionless form employing the appropriate transformations and then solved numerically using the implicit finite difference method with Keller box scheme. The numerical solutions are obtained in terms of velocity profiles, temperature distributions, skin friction coefficient and heat transfer rate and then presented graphically and discussed. A complete parametric analysis is done on these numerical results to show the effects of the magnetic parameter, radiation parameter, Joule heating parameter and heat generation parameter. It is found that radiation, Joule heating and heat generation play significant role on MHD natural convection flow during heat transfer. To illustrate the accuracy of the results, the present results for the local skin friction and surface temperature distribution excluding the effects of magnetic field parameter, radiation parameter, Joule heating parameter and heat generation parameter are compared with the results of Merkin and Pop designed for the fixed value of Prandtl number and an excellent agreement were found.

Keywords: Radiation; MHD; Joule heating; heat generation; vertical flat plate; finite difference method
1. Introduction

Heat transfer in presence of magnetic field has received much attention by many researchers due to its potential used in science, engineering and industrial applications such as nuclear power plants, cooling of transmission lines and electric transformer etc. Accordingly, effect of radiation on MHD is of considerable interest because of its wider applications in space technology and others. Many researchers studied the effect of radiation on magnetohydrodynamic free convection flows under diverse surface boundary conditions using different mathematical technique. The problem of natural convection-radiation interaction on boundary layer flow with Rosseland diffusion approximation along a vertical thin cylinder has been investigated by Hossain and Alim [1]. Hossain and Takhar [2] employed finite difference approximation to analyze the effects of conduction-radiation interaction on natural convection boundary layer flow of a viscous incompressible fluid along an isothermal horizontal plate. Abdel-naby et al. [3] studied the radiation effects on MHD unsteady free convection flow over a vertical plate with variable surface temperature. Hossain [4] analyzed the effect of viscous and Joule heating effects on MHD free convection flow with variable plate temperature. The heat generation effect on MHD natural convection flow along a vertical flat plate was studied by Mamun et al. [5] employing finite difference techniques. Palani and Kim [6] applied implicit finite difference scheme of Crank-Nicolson method to analyze the importance of Joule heating and viscous dissipation effects on MHD flow along inclined plate subject to variable surface temperature.

In the above studies some have considered only the effects of radiation or the effect of Joule heating and heat generation on free convection flow including or excluding magnetohydrodynamic effect for various geometries. However, as no work has been conducted combinedly along a vertical plate, so we attempted to find out the effect of radiation on MHD free convection flow along a vertical flat plate in presence of Joule heating and heat generation in this analysis and then a detailed derivations of the governing equations and solution procedure are described in the following sections.

2. Governing equations of the flow

Let us consider a steady free convection boundary layer flow of an incompressible and electrically conducting fluid along a vertical flat plate. Let $l$ and $b$ are the length and thickness of the plate, respectively. $T_b$ is the temperature at the outer surface of the plate and $T_\infty$ is the temperature of the ambient fluid where $T_b < T_\infty$. A uniform magnetic field of strength $H_0$ is acted along the $\vec{y}$ -axis. The $\vec{x}$ -axis is taken along the vertical flat plate in upward direction and the $\vec{y}$ -axis is normal to the plate. The flow configuration of physical model and coordinates system as:
The equations governing the flow under these assumptions with the Boussinesq approximation can be expressed as follows:

\begin{equation}
\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0
\end{equation}

\begin{equation}
\bar{v} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{u} \frac{\partial \bar{u}}{\partial \bar{y}} = \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + g \beta (T_f - T_\infty) - \frac{\sigma H_0^2 \bar{u}}{\rho}
\end{equation}

\begin{equation}
\bar{v} \frac{\partial T}{\partial \bar{x}} + \bar{u} \frac{\partial T}{\partial \bar{y}} = \frac{k_f \rho C_p}{\rho} \frac{\partial^2 T}{\partial \bar{y}^2} - 4 \Gamma (T_f - T_b) + \frac{\sigma H_0^2 \bar{u}^2}{\rho} \bar{C}_p + \frac{Q_0}{\rho} \bar{C}_p (T_f - T_\infty)
\end{equation}

where \( \Gamma = \int_{\lambda}^\infty K_{\lambda} (\theta \lambda d \lambda \cdot K_{\lambda} = K_{\lambda} (T_b) \) is the mean absorption coefficient, \( E_{\lambda} \) is Plank’s function and \( T_f \) is the temperature of the fluid in the boundary layer. The following boundary conditions based on conduction are considered to solve the governing equations:

\begin{equation}
\bar{v} = \bar{u} = 0, \ T_f = T (\bar{x},0), \ \frac{\partial T}{\partial \bar{y}} = \frac{k_f}{b k_f} (T_f - T_b) \quad \text{at} \quad \bar{y} = 0, \ \bar{x} > 0 \quad \text{and} \quad \bar{u} \rightarrow 0, \ T_f \rightarrow T_\infty \quad \text{at} \quad \bar{y} \rightarrow \infty, \ \bar{x} > 0
\end{equation}

The governing equations and the boundary conditions (1)-(4) are made non-dimensional, using the following non-dimensional variables:

\begin{equation}
x = \frac{\bar{x}}{l}, \ y = \frac{\bar{y}}{l} \ Gr^{-1/4}, \ u = \frac{\bar{u}}{l} \ Gr^{-1/2}, \ v = \frac{\bar{v}}{l} \ Gr^{-1/4}, \ \theta = \frac{T_f - T_b}{T_b - T_\infty}, \ \text{Gr} = \frac{g \beta l^3 (T_f - T_\infty)}{\nu^2}
\end{equation}

where \( \theta \) is the dimensionless temperature. The non dimensional forms of the governing equations are:

\begin{equation}
\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0
\end{equation}

\begin{equation}
u \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{u} \frac{\partial \bar{u}}{\partial \bar{y}} + M u = \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \theta
\end{equation}

\begin{equation}
u \frac{\partial \theta}{\partial \bar{x}} + \bar{u} \frac{\partial \theta}{\partial \bar{y}} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial \bar{y}^2} - R (\theta - 1) + J u^2 + Q \theta
\end{equation}

and the corresponding boundary condition (4) can be written as in the following dimensionless form:

\begin{equation}
u = 0, \ \theta - 1 = 0 \rightarrow 0, \ \theta \rightarrow 0 \quad \text{at} \quad \bar{y} \rightarrow 0, \ \bar{x} > 0 \quad \text{and} \quad u \rightarrow 0, \ \theta \rightarrow 0 \quad \text{at} \quad \bar{y} \rightarrow \infty, \ \bar{x} > 0
\end{equation}

here \( M = (\sigma H_0^2 l^2 / \mu) \ Gr^{-1/2} \) is the magnetic parameter, \( R = (4 \Gamma l^2 / \nu) \ Gr^{-1/2} \) is the radiation parameter, \( P r = \mu C_p / k_f \) is the Prandtl number, \( Q = Q_0 l^2 \ Gr^{-1/2} / \mu C_p \) is the heat generation parameter, \( J = \sigma H_0^2 \nu C_p \ Gr^{1/2} / \rho C_p (T_b - T_\infty) \) is the Joule heating parameter and \( p = (k_f / k_s) (b/l) \ Gr^{1/4} \) is the conjugate conduction parameter. The value of the conjugate conduction parameter \( p \) depends on \( b/l \), \( k_f/k_s \) and...
but each of which depends on the types of considered fluid and the solid. In the present analysis we have taken \( p = 1 \).

The momentum and the energy equations (6) and (7) are transformed into the following nonlinear partial differential equations using similarity transformations

\[
\psi = x^{4/5}(1 + x)^{-1/20} f(x, \eta), \quad \theta = x^{1/5}(1 + x)^{-1/20} h(x, \eta)
\]

are

\[
\begin{align*}
    f'' &= \frac{16 + 15x}{20(1 + x)} f f' - \frac{6 + 5x}{10(1 + x)} f'^2 - M x^{2/5} (1 + x)^{1/10} f' + h = x(f'^2 \frac{\partial f'}{\partial x} - f f' f'^2 \frac{\partial f}{\partial x}), \\
    \frac{1}{Pr} h'' &= \frac{16 + 15x}{20(1 + x)} f h' - \frac{6 + 5x}{10(1 + x)} f'^2 - R x^{2/5} (1 + x)^{1/10} h + R x^{1/5} (1 + x)^{3/10} \\
    &+ J x^{7/5} (1 + x)^{1/10} f'^2 + Q x^{2/5} (1 + x)^{1/10} h = x(f'^2 \frac{\partial h}{\partial x} - h' \frac{\partial f'}{\partial x}).
\end{align*}
\]

Here the primes denote partial derivative with respect to \( \eta \). The boundary conditions (8) take the following form:

\[
\begin{align*}
    f(x, 0) = f'(x, 0) = 0, \quad h(x, 0) = -(1 + x)^{1/4} + x^{1/5} (1 + x)^{1/20} h(x, 0) \quad \text{at} \quad y = 0 \bigg\}\bigg. \\
    f'(x, \infty) \rightarrow 0, \quad h(x, \infty) \rightarrow 0 \quad \text{at} \quad y \rightarrow \infty.
\end{align*}
\]

2.1 Comparison of the results

The comparison of the skin friction coefficients and the surface temperature between the present work and the work of Merkin and Pop [8] are shown in the following table. We observed in this table, the present analysis is an excellent agreement with the published work of Merkin and Pop [8].

| \( \frac{1}{x^5} = \xi \) | Merkin and Pop [8] \( C_{f0} \) | Merkin and Pop [8] \( \theta(x, 0) \) | Present work \( C_{f0} \) | Present work \( \theta(x, 0) \) |
|---|---|---|---|---|
| 0.7 | 0.430 | 0.651 | 0.424 | 0.651 |
| 0.8 | 0.530 | 0.686 | 0.529 | 0.687 |
| 0.9 | 0.635 | 0.715 | 0.635 | 0.716 |
| 1.0 | 0.745 | 0.741 | 0.744 | 0.741 |
| 1.1 | 0.859 | 0.762 | 0.860 | 0.763 |
| 1.2 | 0.972 | 0.781 | 0.975 | 0.781 |

3. Results and discussion

The numerical results are calculated from the solutions of the governing equations (9) and (10) subject to the boundary conditions represented in the equation (11). The numerical values of the velocity, temperature, local skin friction coefficient and rate of heat transfer are obtained for different values of magnetic parameter \( M \), radiation parameter \( R \), Joule heating parameter \( J \) and heat generation parameter \( Q \) for a Prandtl number of 0.73. Detailed numerical results for the velocity, temperature, local skin friction coefficients and rate of heat transfer associated with the different values of related parameters are presented graphically in Fig. 2-9, respectively.

Fig. 2: (a) Variation of velocity and (b) variation of temperature against \( \eta \) for varying of \( M \) with \( R = 0.001, J = 0.01 \) and \( Q = 0.01 \).

The effects of different values of magnetic parameter \( M \) on the velocity profiles and temperature distribution are illustrated in Fig. 2(a) and Fig. 2(b), respectively. The applied magnetic field produces Lorentz force due to the interaction
with the flowing fluid particles. This force opposes the motion of the fluid, as a result the velocity of the fluid decreases with the increasing of $M$ observed in Fig.2 (a). It reveals that the shape of the velocity profile increases near the interface and then begins to decrease thereafter. The temperature increases within the boundary layer for the increasing values of magnetic field parameter due to interaction of fluid flow and magnetic field which is illustrated in Fig.2 (b).

Figure 3 illustrates the skin friction coefficient and the rate of heat transfer for the variation of magnetic parameter $M$ while $R = 0.001$, $J = 0.01$ and $Q = 0.01$. Since the velocity of the fluid decreases for increasing values of magnetic parameter as mentioned earlier, accordingly, the skin friction on the plate decreases as observed in Fig.3 (a). Moreover, the temperature within the boundary layer increases (Fig. 2(b)) for the increasing $M$, which result, the heat transfer rate from the plate to fluid decreases as shown in Fig. 3 (b).

Figure 4 (a) and Fig. 4 (b) depict the velocity and temperature distributions, respectively for some selected values of the radiation parameter $R$ together with a certain value of $M$, $J$ and $Q$. Fluid absorbed heat while radiation imitates from the heated plate. As a result the motion and the temperature of the fluid increase within the flow region. It is also observed that the velocity profiles shift upward and the position of peak velocity moves outward from the plate for the increasing $R$ (4(a)).

The skin friction coefficient and the rate of heat transfer against $x$ for various values of radiation parameter $R$ are illustrated in Fig.5 (a) and Fig.5 (b), respectively. The increased values of the radiation parameter accelerate the fluid motion (Fig 4 (a)) within the boundary layer. As a result, the friction between the surface of the plate and the fluid increases as observed in Fig.5 (a). The skin friction increase significantly for a particular value of radiation parameter $R$ with the increasing value of $x$. Increasing temperature in the flow region decreases temperature difference between the outer surface of the plate and the flow field. Consequently, the rate of heat transfer decreases for increasing value of radiation parameter as shown in Fig.5(b).
The above figures (6(a) and 6(b)) represent the effect of Joule heating parameter $J$ on velocity and temperature field while the controlling parameters are $M = 0.50$, $R = 0.001$ and $Q = 0.01$. As Joule heating parameter produce temperature in the conductor, therefore the velocity and temperature of the fluid increase associated with the increasing values of $J$.

Figure 7(a) and Fig. 7(b) illustrate the local skin friction coefficient and heat transfer rate for different values of $J$ with increasing $x$ and fixed value of controlling parameters. Since the velocity of the fluid increase with the increasing of $J$ that has been shows in Fig. 6(a), accordingly, the corresponding skin friction coefficient increases. The increased temperature due to increasing of Joule heating parameter increases the interfacial temperature, for that reason the heat transfer rate from the plate to fluid decreases which is shown in Fig. 7(b).

The velocity of the fluid increases by the increasing value of $Q$, because of the effects of $Q$ generates heat in the flow region as a result, the temperature of the fluid increase within the thermal boundary layer. These phenomenon are presented in the Fig. 8 (a) and 8 (b), respectively. On the other hand, heat generation parameter effects positively (increase) in the skin friction and negatively (decrease) in the heat transfer rate which are shown in the Fig.9 (a) and 9(b), respectively. These effects actually occur due to the increased rate of velocity by $Q$, that leads to increase of skin friction along the surface of the vertical flat plate and the presence of heat generator creates a hot fluid layer adjacent to the surface due to the heat generation mechanism, accordingly, the heat transfer rate decrease from the surface to the fluid.
4. Conclusion

In this analysis the effect of radiation and heat generation on magnetohydrodynamic (MHD) natural convection flow along a vertical flat plate in presence of Joule heating has been analyzed for some selected values of pertinent parameters including magnetic parameter, radiation parameter, Joule heating parameter and heat generation parameter. From the present investigation, it may be concluded that the velocity of the fluid and the skin friction at the interface decrease with the increasing magnetic parameter while they increase with the increasing radiation parameter, Joule heating parameter and heat generation parameter. The temperature of the fluid increases with the increasing magnetic parameter, radiation parameter, Joule heating parameter and heat generation parameter but opposite results arise for heat transfer rate.

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