Effects of MHD and slip on heat transfer boundary layer flow over a moving plate based on specific entropy generation

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1. Introduction

Mechanism of heat transport on a roving solid surface plays a significant role in extrusion of plastic sheets, crystal growth, polymer industry, paper production, liquid film and in condensation processes. Sakiadis [1] stood as a pioneer who investigated the behaviour of the boundary layer flow with constant velocity over a solid moving plate; then Erickson et al. [2] extended the said problem for heat and mass transport in moving surface with suction or blowing. Tsou et al. [3] testified together theoretical and experimental for the continuous moving flat plate. Abdelhafiz [4] examined heat transfer on the continuous flat surface in a parallel free stream and talked about the special cases of Blasius and Sakiadis in the said problem. Afzal et al. [5] studied the combined velocity in the form \( U = U_w + U_\infty \) along with the single boundary condition instead of taking separately as reported by Abdelhafiz [4]. Rashidi et al. [6] have used similarity variables to examine the mixed convective flow with MHD, slip and thermal convective boundary condition along with a moving vertical flat surface. Some related problems with diverse physical conditions on moving boundary can be found in Ref. [7].

Moreover, the fundamental doctrine of Navier–Stokes principle that is no slip boundary condition has been investigated by several researchers under certain circumstances. There are some situations where no-slip condition is not suitable and is swapped by a partial slip condition, as boundary slip has numerous uses like the refining of inside cavities besides artificial heart valves. Martin and Boyd [8] discussed the partial slip effect on laminar boundary layer flow and heat transfer over a flat surface. Sheikholeslami et al. [9] analysed MHD flow effect in thermal diffusion heat-generation flow past an oscillating vertical plate through porous medium. Pal and Talukdar [10] explored unsteady MHD boundary under the effect of radiation with momentum slip. Recently, Bhattacharyya et al. [11,12] presented the impact of uniform magnetic field embedded with porous medium on boundary layer flow with slip effect over flat plate. The influence of momentum slip boundary condition on Newtonian fluid flow owing to stretched surface is described by Andersson [13] and Wang [14].

Furthermore, magneto-hydrodynamic (MHD) flow and heat transfer towards moving or fixed flat surface have become a vibrant problem in the current field and huge applications in the field of engineering problems, for instance, petroleum engineering, plasma studies, geothermal energy abstractions, aerodynamics and much more. Particularly, to regulate the behaviour of boundary layer, numerous artificial approaches have been established. The impact of induced magnetic field on free convection heat transport on isothermal plate is deliberated by Sparrow and Cess [15]. Gupta [16] discussed electrically conducting vertical flow with moveable wall to describe heat flux under the impact of magnetic field. Riley [17], Watanabe and Pop [18] elucidated MHD boundary layer flow past a moving flat surface with the existence of hall impact. Damseh et al. [19] inspected the influence of radiation and uniform transverse magnetic field on forced convective and transfer of energy with suction or injection from a porous surface.
In addition, an innovative variation in thermodynamics perceives by many investigators nowadays. Idea of entropy generation is also one of the utmost visible forms of this variation. It plays a crucial role to recognize the diverse singularities in different areas, particularly in the process of energy conversion, which typically tends to an irreversible intensification in entropy; therefore, reduction of entropy generation consequence is well-organized in energy systems. Bejan [20] gave a technique called minimization of entropy generation to extend and improve the disorder or incompetence generated during a process. Aspect of entropy generation, MHD and heat can be found in the notable studies listed in Refs. [20–37].

Inspired by the previous researchers, the determination of this effort is to observe theatrically the entropy generation analysis on boundary layer flow past a flat moving surface under the combined impact of induced magnetic field and momentum slip condition. The influence of interesting physical parameters is done through graphs to characterize the flow and heat transfer performance thoroughly. The transformed resulting system is solved numerically with Bvp4c and shooting algorithm via Matlab. This paper proceeds thru the subsequent results and discussion is given. Key observations are summarized in the last section.

2. Problem design and solution procedure

An incompressible flow past a flat moving plate which moves in the same and opposite directions is considered as shown in Figure 1. Assume that the wall temperate is \( T_w > T_\infty \), and \( u_\infty \) represents uniform and constant free stream velocity.

![Figure 1](image_url)  
**Figure 1.** Geometry of flat moving plates when (a) \( \lambda < 1 \) and (b) \( \lambda > 1 \).

The equation of continuity, momentum and energy transport in the form of Cartesian coordinates are: [38]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,  
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - \frac{\partial^2 u}{\partial y^2} - \frac{\sigma \mu^2}{\rho} u = 0,  
\]

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} - \frac{\sigma^2 T}{\partial y^2} = 0.  
\]

Along with boundary relations are:

\[
v = 0, \quad u = u_w + A \frac{\partial u}{\partial y}, \quad T = T_w \text{ at } y = 0,  
\]

\[
u = u_\infty, \quad T = T_\infty \quad \text{as } y \to \infty.  
\]

A stream function \( \psi(x, y) \) with similarity transformation

\[
\psi = \sqrt{2\nu xu_\infty f(\eta)}, \quad \eta = \sqrt{\frac{u_\infty}{2\nu x}},  
\]

\[
\theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad u = \frac{\partial \psi}{\partial y}, v = \frac{\partial \psi}{\partial x}.  
\]

By utilizing the above similarity transformation, we get the following dimensionless form that satisfies the continuity relation:

\[
f''f - 2Mf' + f''c = 0,  
\]

\[
\theta'' + \rho \beta / f = 0.  
\]

The corresponding boundary equations

\[
f(0) = 0,  
\]

\[
f'(0) = \alpha f'(0) + \lambda = 0 \quad \text{and} \quad \theta(0) = 1,  
\]

\[
f' \to 1 \text{ as } \eta \to \infty,  
\]

\[
\theta \to 0 \text{ as } \eta \to \infty.  
\]

in which \( \lambda = u_w/u_\infty \) represents velocity ratio of plate, \( \alpha = A \sqrt{2\nu x} \) is slip parameter, \( M = \frac{x \sigma \rho^2 \beta}{\rho u_\infty} \) is MHD parameter and \( Pr = \nu/\beta \) is the Prandtl number.

Most important measures of practical attention are friction factor and heat transfer rate.

\[
C_f = \frac{\tau_w}{\rho u_\infty^2}, \quad Nu = \frac{x q_w}{k (T_W - T_\infty)}.  
\]

Here \( \tau_w \) is the wall shear stress while \( q_w \) represents the heat flux at wall.
\[ y = 0 \text{ and } q_w + k \left( \frac{\partial T}{\partial y} \right) = 0, \text{ at } y = 0. \] (11)

Using the expression given in (5) into Equations ((10) and (11)), we get

\[ \sqrt{2} Re C_f = f''(0), \quad \left( \frac{Re}{2} \right)^{\frac{1}{2}} Nu = -\theta'(0), \] (12)

where \( Re \) denotes local Reynolds number, \( f''(0) \) and \( -\theta'(0) \) represent reduced skin friction and Nusselt number.

### 2.1. Entropy generation

The entropy generation equation [39] stated as:

\[ S''_{gen} = \left[ \frac{k}{T^2} (\nabla T)^2 \right] + \left[ \frac{\mu}{\rho T} \phi \right] + \frac{\sigma P_0^2}{T} u^2. \] (13)

Putting Equation (5) into Equation (13), one can drive the following entropy expression for a moving flat surface

\[ S = \frac{\theta^2}{2(\theta + \theta_\infty)^2 E_p R_e} \left( 1 + \frac{u^2}{2 R_e} \right) + \frac{1}{2} \left( \frac{f''}{(\theta + \theta_\infty) R_e} + Mf' \right). \] (14)

So, non-dimensional form of entropy generation is

\[ S = \frac{\nu^2 \Delta T}{u_\infty \mu} S''_{gen}. \] (15)

In Equation (14), first term \( S_h \) is due to transfer of heat and second term \( S_f \) is owing to fluid friction. Another irreversibility parameter called Bejan number is stated as:

\[ Be = \frac{S_h}{S_h + S_f}. \] (16)

It is quite clear that the value of \( Be \) varies from zero to one; when \( Be = 1 \), irreversibility will be dominated for the case of heat transfer. When \( Be = 0 \), the irreversibility overcome in the case of fluid friction and \( Be = 0.5 \) is the situation when entropy generation and fluid friction are equal due to heat transfer.

### 3. Results and discussion

The nonlinear differential Equations (6) and (7) along with relevant boundary relations (8) and (9) are tackled numerically with the help of Bvp4c Matlab package, by transforming it into an initial value problem. The computed numerical results are achieved for numerous values of non-dimensional constraints involved in the equations, like magnetic parameter \( M \), and velocity slip parameter \( \alpha \) on moving plate are demonstrated through pictorial form and explained physically as follows:

#### 3.1. Velocity profile

Figure 2 shows that the velocity profile of the fluid can be affected by the plate’s velocity because MHD boundary-layer profiles are different for \( \lambda > 1 \) and \( \lambda < 1 \). It is found that if \( \lambda < 1 \) then fluid velocity takes an increasing trend with moving toward the wall, however, for \( \lambda > 1 \), fluid velocity approaches free stream. For \( \lambda > 1 \) or \( \lambda < 1 \) the MHD parameter is augmented Lorentz drive related to MHD field that influences boundary layer more slender. Figure 3 reflects that the velocity profile of fluid is badly affected by the plate’s velocity. Obviously slip boundary layer profiles are different for \( \lambda > 1 \) and \( \lambda < 1 \). In this case, if \( \lambda > 1 \) then \( \alpha \) enhances boundary layer thickness and as a result velocity reduces. By this, one can conclude that the velocity slip parameter \( \alpha \) compares to augment in both temperature and focus profile. This is because slip velocity presence on extending surface. Correspondingly when \( \lambda < 1 \) the slip parameter \( \alpha \) increases energy boundary layer thickness and henceforth augments the velocity. By this we implied that the velocity slip parameter \( \alpha \) compares to increase in both temperature and
focus profile. It is because of the presence of slip velocity on the extending surface.

3.2. Velocity gradient profile

Figure 4 elaborates the velocity gradient profile of the fluid, which can be affected by the plate’s velocity gradient. It is obvious because MHD boundary-layer profiles differ for $\lambda > 1$ and $\lambda < 1$. It is seen that the magnetic lines of force travel past the plate towards wall velocity and as a consequence, the fluid caught a push from magnetic field, which was initially slow down because of thick force. Subsequently, for $\lambda > 1$ or $\lambda < 1$, velocity of the fluid declines as the magnetic parameter $M$ increments. Henceforth, $\lambda > 1$ or $\lambda < 1$, the velocity gradient of the fluid reduces for the increasing values of magnetic parameter $M$. Figure 5 exhibits that the slip parameter $\alpha$ built the thickness of force boundary layer and henceforth diminishes the velocity gradient. On the extending surface, slip parameter $\alpha$ augments the thickness of force boundary layer and thus results in an increase in the velocity gradient for $\lambda > 1$. By this we implied that the velocity gradient slip parameter $\alpha$ compares to augment in both temperature and focus profile.

3.3. Temperature profile

Figure 6 portrays the effects of magnetic parameter for numerous values of $\lambda$ on temperature distribution. For $\lambda > 1$ and $\lambda < 1$, increasing trend of temperature profile by moving outward wall is observed. In case when $\lambda > 1$ or $\lambda < 1$, the magnetic parameter $M$ augments temperature distribution that is thermal boundary layer thickness augments by augmenting the magnetic parameter $M$. It is because of extra heating in the stream structure due to viscous dissipation.

Figure 7 offers the variation the slip boundary-layer for various values of $\lambda$ for temperature profile. It is seen that the slip boundary layer profiles are different for $\lambda > 1$ and $\lambda < 1$. In both cases, temperature profile approaches to free stream. For $\lambda > 1$ the thermal boundary-layer thickness increases by increasing the
values of slip parameter whereas decrease rate of heat transfer is perceived at the surface for $\lambda < 1$. The associated graph uncovers that the wall temperature $\theta(\eta)$ and thermal boundary-layer thickness diminishes.

### 3.4. Temperature gradient profile

Figure 8 illustrates the variation of MHD boundary layer parameter for temperature gradients against different values of $\lambda$. From figure, a noticeable change is perceived in boundary-layer for $\lambda > 1$ and $\lambda < 1$. For all values of $\lambda$, temperature gradient takes an increasing trend when moving outward from the wall. The same behaviour is noted for magnetic parameter $M$. The thermal boundary-layer thickness increases by increasing the values of magnetic parameter $M$. The viscous dispersal, as a heat generation inside the fluid, augments the mass fluid, temperature gradient. Here it might be credited to the extra heating in the stream system because of thick dispersal. Figure 9 elucidates the variation of temperature gradients against slip boundary layer designed a number of $\lambda$. The slip boundary layer profiles is established different for $\lambda > 1$ and $\lambda < 1$. In case $\lambda > 1$, temperature gradient takes an increasing trend with moving outward from the wall. The same behaviour is noted. However, for $\lambda > 1$, thermal boundary-layer thickness augment for slip parameter $\alpha$. This is the cause of reduction in rate of heat which moves at the surface. Similarly, the slip parameter $\alpha$ increases when $\lambda < 1$. The graph reveals that the wall temperature gradient $\theta'(\eta)$ and thermal boundary layer thickness diminishes.

### 3.5. Effects of Bejan number

Figure 10 points out the effects of Bejan number for different values of $\lambda$. It is depicted that the Bejan number increases by increasing values of magnetic parameter $M$. Bejan number approaching to free stream is also detected. Figure 11 explains the role of Bejan number for different values of $\lambda$. First we see that temperature approaches to free stream. In both cases, slip $\alpha$ increases when Bejan number declines.
3.6. Effects of entropy generation

Figure 12 describes the formation of entropy on MHD boundary-layer for various values of \( \lambda \). It is obvious that MHD boundary layer profiles are different for \( \lambda > 1 \) and \( \lambda < 1 \). Entropy approaches free stream for \( \lambda > 1 \) and \( \lambda < 1 \). It is perceived that when MHD parameter increases, then as a result generated entropy decreases and finally approaches zero for increasing values of magnetic parameter \( M \) for \( \lambda > 1 \) and \( \lambda < 1 \). A resistance of the particles of fluid which cause heat to be created decreases in the fluid is noticed. Figure 13 expresses the formation of entropy in the slip boundary-layer for various values of \( \lambda \). For \( \lambda > 1 \) and \( \lambda < 1 \) entropy generation approaches to free stream, whereas slip boundary layer profiles are different. Also, the slip parameter \( \alpha \) augments the measure of formation entropy diminishes lastly stop to zero.

4. Conclusion

This paper is concerned with the combined influence of MHD and slip over a flat moving plate with entropy analysis for numerous physical parameters. The nonlinear equations are solved numerically by utilizing Bvp4c Matlab package. Impact of magnetic parameter, slip parameter and velocity ration parameter are examined. Key observations are summarized as follows:

- Bejan number decreases for \( \lambda > 1 \) and increases for \( \lambda < 1 \) with the variation of slip parameter.
- Velocity gradient flattening for both cases as \( \eta \to 1 \) and approaches to zero against \( M \).
- Entropy generation reduces for \( \lambda > 1 \) and increases for \( \lambda < 1 \) against slip parameter when \( \eta \to 1 \).
- Temperature upturns by rising values of MHD parameter.

Disclosure statement

No potential conflict of interest was reported by the authors.

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