I. INTRODUCTION

For the past two decades, modern network theory has set a new standard for understanding several features exhibited by diverse types of complex systems [1–5], such as the cell [6], the World Wide Web [7], the transportation network [8, 9], the fracture networks [10, 11], the earthquake network [12–15], etc. The ability to quantify the complexity in the system is possibly the most remarkable achievement of the network theory and now, this quantitative characterization is recognized as the first step towards understanding the underlying mechanism that leads to such complex structure. Many complex dynamical phenomena, including the rich get richer [3], six degrees of separation [16], public opinion formation, and epidemic spreading [17] can be comprehensively understood using the framework of network theory.

In the network theory, a system is represented by a set of “nodes” and “links” connecting different pairs of nodes based on some interaction between them. The topology of a large number of real-world networks ranging from biological, chemical to social, have been studied extensively and it is well known that they are far from being fully random. Rather, they are found to exhibit some common features, such as they are simultaneously scale-free and small-world [5]. While scale-free indicates highly heterogeneous network characterized by the power-law decay of the degree distribution function $p(k) \sim k^{-\gamma}$ with $k$ being the degree (i.e., the number of links attached to a node) of the nodes, the small-world nature describes that any arbitrary pair of nodes can be reachable within a few number of steps through the links of intermediate nodes despite the large size $N$ of the network. Specifically, for a small-world network the average shortest path length between the nodes grows as $\ln N$.

Inspired by the exceptional success of the network theory in recent years, the analysis of time series from the perspective of complex network has received considerable attention due to the standing requirement of understanding the non-trivial dynamical processes behind time series data [18–22]. If a time series is mapped into a complex network, one may expect that such a network reflects some inherent properties of the original time series. Thus, one can utilize the recent graph-theoretical tools to extract novel properties hidden in the time series.

Among several other methods [21, 22], the visibility graph [20] has become popular due to its simplicity and wide range of applicability. This method has demonstrated its potential in extracting several characteristic features of the time series such as the periodicity, fractality, self-similarity, chaoticity, and more [20, 23]. A merit of the visibility graph method is its ability to capture some correlations in non-stationary time series without introducing any other procedures such as detrending. In particular, it has been shown that the visibility graph corresponding to the time series generated from a fractional Brownian motion (fBm) is scale-free. Moreover, the exponent $\gamma$ for the degree distribution is dependent on the Hurst exponent ($H$) of the fBm as [24]:

$$\gamma = 3 - 2H.$$  

(1)

Since the fBm generates $f^{-\beta}$ power spectrum with $\beta = 1 + 2H$, the exponent $\gamma$ of the visibility graph should correspond to $\beta$ as

$$\gamma = 4 - \beta.$$  

(2)

Subsequently, the method has been applied to extract the fBm-like nature of time series in several contexts such as finance [23], health science [26, 27], and geophysics [28, 29].

In this paper, we study the nature of correlation in earthquake time series by means of visibility graph.
particular, we focus on the two important quantities: the magnitude and the inter-event time (IET) between two consecutive earthquakes. To gain more general insights on earthquake time series, we investigate three different categories of earthquakes: regular earthquakes, earthquake swarms, and tectonic tremors. (These three categories are outlined in the next section.) Based on the analysis of the visibility graph, we argue against systematic correlation in the magnitude time series of earthquakes, whereas an evidence for correlation in the IETs is apparent. We also show that the time series of three different types of earthquakes can be distinguished in the topology of the associated visibility graph.

The paper is organized as follows. We start by describing the visibility graph algorithm and the characteristics of the three categories of earthquakes including the specifications of the studied seismogenic zones in Sec. II. The existence of memory in the time series of magnitudes and IETs have been investigated in Secs. III and IV respectively. We discuss the topology of the visibility graph for both magnitude and inter-event time series in Sec. V. Finally, we summarize in Sec. VI.

II. CONSTRUCTION OF VISIBILITY GRAPH FROM SEISMIC SEQUENCES

Given the time sequence of the occurrence of seismic events, the visibility graph is constructed by considering each event as a node and linking the nodes based on mutual visibility of the corresponding data heights. The data recorded at time \( t_k \) is represented as the height \( h_k \) of the \( k \)-th node. Specifically, any arbitrary pair of data values \( (t_i, h_i) \) and \( (t_j, h_j) \) \( (t_i < t_j) \) are visible to each other if the straight line joining the two data points does not intersect any intermediate data heights, as illustrated in Fig. I. One can easily note that if there exists visibility, the slope \( s_{ij} \) of the line between the nodes \( i \) and \( j \) must be the maximum of the slopes \( s_{ik} \) for all \( i < k < j \). Therefore, a link is placed between two nodes \( i \) and \( j \) in the visibility graph if and only if for all \( t_i < t_k < t_j \) the following criteria is satisfied:

\[
h_k < h_i + (h_j - h_i) \frac{t_k - t_i}{t_j - t_i}.
\]

Clearly, every node is visible at least from its left and right nearest neighbors and thus one obtains a completely connected network.

The “divide & conquer” algorithm [30] has been used to efficiently transform a time series into its corresponding visibility graph. This algorithm takes advantage of the fact that the node with the maximum height divides the time series into two segments in the sense that the nodes situated at one side of the maximum are not visible from the another side. Therefore, it is not required to check the visibility between the two sides of each separated segments. In each step, the visibility of the node with the maximum height to the other nodes at its right and left sides is determined. Each new segment is then treated independently and the same procedure is repeated until every segment contains one single node. The CPU time taken by the algorithm scales with the size \( N \) of a time series as \( N \log N \).

A. Characteristics of the seismic sequences

The fundamental difference among the three types of earthquakes studied here lies in their generation mechanisms and the time scale associated with the released energy.

A time series of regular earthquakes includes mainshock-aftershock sequences and the background activity. While the latter is a Poissonian process, the former is generally clustered in space and time. Aftershocks are triggered usually by the static stress change associated with the mainshock, as well as some other post-seismic relaxation processes such as afterslip or fluid flow. Major fraction of the total energy is released almost instantaneously at the time of the mainshock and slowly decreases in time. It is observed that the magnitude-frequency distribution \( P(M) \) obeys an exponential distribution, namely, the Gutenberg-Richter (GR) law [31]:

\[
P(M) \propto 10^{-bM},\ \text{with} \ b \ \text{taking a value around 1 in the active fault zones}\ [32]. \text{On the other hand, the temporal decay of the frequency of aftershocks is described by the Omori-Utsu law}\ [33, 34].
\]

The same phenomenology is not observed for the other two categories of earthquakes. In contrast to mainshock-aftershock sequence, a seismic swarm is defined as a cluster of earthquakes with similar magnitudes, which usually occur in a volcanic or geothermal tectonic setting. The intrusion of fluids can reduce the resistance of faults and redistribute the stress in such a manner that the energy is released gradually and almost equally among
TABLE I. The summary of the catalog data analyzed for investigating the correlations between the earthquake events.

| Earthquake type | Region       | $\theta_{\text{min}}$ | $\phi_{\text{min}}$ | $\theta_{\text{max}}$ | $\phi_{\text{max}}$ | Period                        | $M_c$  | $N_t$  |
|-----------------|--------------|------------------------|----------------------|------------------------|------------------------|-------------------------------|--------|--------|
| Regular         | Tohoku       | 34.00                  | 135.00               | 42.00                  | 145.0                  | 01/01/2000 – 30/11/2019      | 2.0    | 147021 |
|                 | Kumamoto     | 32.40                  | 130.40               | 33.40                  | 131.6                  | 01/01/2000 – 30/11/2019      | 1.0    | 44486  |
|                 | Southern California | 30.00              | -124.00              | 39.00                  | -111.0                 | 01/01/1990 – 08/12/2019      | 1.5    | 222491 |
| Swarm           | Hakone       | 35.15                  | 138.90               | 35.35                  | 139.1                  | 06/04/1995 – 03/10/2015      | 0.1    | 16279  |
|                 | Izu          | 34.60                  | 138.95               | 35.15                  | 139.5                  | 01/01/1995 – 30/11/2019      | 0.0    | 38657  |
| Tremor          | Shikoku      | 33.66                  | 131.61               | 34.28                  | 134.5                  | 01/04/2004 – 01/09/2016      | 77701  |
|                 | Cascadia     | 37.50                  | -118.20              | 51.00                  | -128.7                 | 09/01/2005 – 30/12/2014      | 30084  |

The largest shocks. The Omori-Utsu law does not generally hold for swarms.

Tectonic tremors represent weak and repetitive seismic signals emitted from a plate boundary in a subduction zone. To the current belief, fluids generated by slab dehydration may be a cause of tremors. Similar to swarm earthquakes, the tectonic tremor activity is characterized by hypocentre migration but on a different spatial and temporal scale: tremors migrate up to several hundreds kilometers, whereas swarms are more local. The statistical laws are largely unknown for tremors.

B. Description of the seismic catalog

In a seismic catalog, an event is described by the location of the hypocenter, the time of occurrence, and the magnitude (M). Since the seismic activity varies from region to region, we aim to study separately the statistical properties of the event sequences in different regions for a given type of earthquake. We select representative regions from Japan and California, as the two areas are well-known for intense seismic activity and dense monitoring networks. We used catalog data provided by the Japanese Meteorological Agency [35], the Hot Spring Research Institute [36], the Southern California Earthquake Center [37], the World Tremor Database [38] and Slow Earthquake Database [39].

A selected region is described by the minimum and the maximum of the latitude ($\theta$) and longitude ($\phi$) coordinates, i.e., the values of ($\theta_{\text{min}}, \phi_{\text{min}}$) and ($\theta_{\text{max}}, \phi_{\text{max}}$). We consider only the crustal events within the depth of 50 km. For the regular and the swarm earthquakes, we also indicate the magnitude of completeness $M_c$, i.e., the minimum magnitude above which all the events are recorded in the catalog. We determined these values using the Zmap software tool [42]. For tremors, we consider all detected events recorded in the two previously mentioned database [43, 44]. The total number of events in a catalog is denoted by $N_t$. The detailed specifications of these catalogs data are given in Table I.

For the regular earthquake time series we analyzed 3 active seismic regions located in different tectonic settings, respectively subduction, compression and active faulting. The region named Tohoku corresponds to an offshore area of the Japan Trench subduction zone where the 2011 earthquake of moment magnitude $M_w$9.0 and its aftershocks were recorded. Time series before and after the $M_w$9.0 event are referred here as Tohoku1 and Tohoku2, respectively. The Southern California region is located in a complex compressional tectonic setting dominated by the southern part of the San Andreas Fault system, but also includes earthquakes generated by the slow uplifting of the Sierra Nevada Mountain range, as well as volcanic and geothermal related activity. The Kumamoto region mostly includes the recent seismic activity generated by the 2016 $M_w$7.0 Kumamoto earthquake around the active Futagawa-Hinagu fault and the surrounding active volcanic region of Aso-Yufuin-Beppu. Thus, most earthquakes in the Kumamoto catalog are aftershocks. In the Hakone volcanic region, significant swarm activity was detected since 2001 [45]. Although many different swarm episodes were recorded, they don’t exhibit any specific temporal pattern. An increase in the seismicity level was observed in 2015 due to a volcanic eruption [46]. The Izu volcanic region is characterized by magma-intrusion episodes which generate frequent swarm activity [47]. Concerning the tremor activity, we selected two areas where the largest number of detected events is available, such as Cascadia in North America and Shikoku around the Nankai Trough in Japan.

III. CORRELATION BETWEEN THE EARTHQUAKE MAGNITUDES

To investigate whether the magnitude of earthquakes has any correlations, we study the degree distribution of the visibility graph constructed from the magnitude time series. First we check if the degree distribution is power law. Typically, a power law distribution is characterized by a long tail that develops with the network size $N$ in such a manner that the average maximum nodal degree $k_{\text{max}}(N)$ grows as $k_{\text{max}}(N) \propto N^\alpha$. This signifies the existence of power-law degree distribution for the infinitely large network, $N \rightarrow \infty$. In order to do this analysis, the original time series is divided into several segments such that each segment contains exactly $N$
number of events.

We first describe our results for regular earthquakes in the Tohoku region. Since the period of Tohoku 2 is exceptionally active after the occurrence of the magnitude 9.0 earthquake, we have analyzed the data for Tohoku 1 and Tohoku 2 separately. In Figs. 2(a) and (b), the degree distribution of the visibility graph is shown on a double logarithmic scale for four values of $N$ starting from $2^{10}$ to $2^{13}$, at each step $N$ being increased by a factor of 2. For all the four values of $N$ in both the cases (Tohoku 1 and Tohoku 2), the curves have certain amount of curvature and the tails of the degree distributions do not elongate significantly as $N$ increases. To see this dependence more clearly, we have plotted the average maximum nodal degree $\langle k_{\text{max}}(N) \rangle$ against $N$ on a semilog scale in Fig. 2(c). Clearly, this implies that $\langle k_{\text{max}}(N) \rangle \sim \ln N$, demonstrating that the degree distribution is not a power law: namely, the absence of fBm-like structure in the magnitude time series.

Specifically, the degree distribution appears to follow a stretched exponential function:

$$p(k) = A \exp[-(k/k_0)^\tau].$$  \hspace{1cm} (4)
In Figs. 3(a) and (b), we have plotted the degree distribution $p(k)$ of the visibility graph on a log-log scale for the whole time series of Tohoku_1 and Tohoku_2 containing 55824 and 91197 events, respectively. The logarithmically binned data for both the series fits quite well with the above functional form in the range of $k$ between 6 to approximately 100. This is shown more explicitly in Fig. 3(c), where $p(k)$ is replotted against $k^\tau$ on a semilog scale. The curves are straight in the intermediate region, indicating that the distribution follows an exponentially decaying function of $k^\tau$.

To confirm the ubiquity of the stretched exponential nature of the degree distribution, we analyze the other six earthquake catalogs. Figure 4 shows the degree distribution presented similarly to those in Fig. 3(c) for Southern California (regular), Hakone (swarms), and Shikoku (tremors). Apparently, the degree distribution is fitted with the stretched-exponential function irrespective of the region or the earthquake type.

We further remark some important points regarding the robustness of the degree distribution. First, it does not significantly change even when the cutoff magnitude $M_c$ is set to be lower than the completeness magnitude: Namely, the result is unchanged irrespective of the presence of some undetected smaller events. This is because the tail of the degree distribution is solely controlled by the larger magnitude events that generally have higher visibility. Second, we confirm that the degree distribution is unaltered even if the time series is with respect to the event index instead of the real occurrence time. (Namely, $h(t)$ is replaced by $\{h_i\}$, where $i = 1, 2, \cdots$.)

The absence of power law in the degree distribution of the visibility graph indicates the absence of fBm-like structure in the time series. Hereafter we further argue the absence of any correlations in the magnitude time series. To this end, we first analyze the visibility graph corresponding to the shuffled time series. Namely, by randomly choosing a pair of events, their respective magnitudes are swapped. This process is repeated by $N_t$ times. This procedure preserves the probability density function of the magnitudes, but destroys any correlations between them. By averaging $10^5$ shuffled sequences, the degree distribution $p(k)$ is shown in Fig. 5. As in the case of original sequence, in a wide range of $k$ the data of $p(k)$ fits the stretched exponential distribution, validating the absence of any correlations in the original sequence.

Next we investigate a random time series for which the height values $\{h_i\}$ are drawn randomly and independently from an exponential distribution $p(h) \sim e^{-\lambda h}$ between $[2, 9]$. Note that $\lambda$ is proportional to the b-value in the GR law as 2.303. In Fig. 6 (main panel), we have shown the degree distribution $p(k)$ for various values of $\lambda$. Unlike the exponential decay observed for the uniformly distributed heights (20), here each curve is seen to follow the stretched exponential form in Eq. (4). Similar to the original earthquake data, $k_{\text{max}}(N)$ is also observed to grow logarithmically with $N$ [Fig. 6 (inset)]. Other network based statistical measures are also in close correspondence with the original data (mentioned in Tab. 

![FIG. 4. Plot of the degree distribution $p(k)$ against $k^\tau$ on a semilog scale for the time series of Southern California (black), Hakone (red) and Shikoku (blue). The $\tau$ values are 0.364, 0.364, and 0.280, respectively. The plot indicates exponential decay of all the curves. For visual clarity, a linear shift is given to the black curve $[p(k) = p(k)/2]$.]

![FIG. 5. Log-log plot of the degree distribution for the shuffled sequences of the data of (a) Tohoku_1 and (b) Tohoku_2. The data points for both the time series fit (solid curves) with the stretched exponential form given in Eq. (4). The parameter values are $\lambda = 52.23$ and 42.70, $1/k_0 = 37.30$ and 31.72, and $\tau = 0.340$ and 0.345, respectively. The corresponding $p(k)$ vs $k^\tau$ plot on a semilog scale has been shown in the inset.]

...
ble II of Sec. V. All the findings above lead us to conclude that there exists no systematic memory in the original earthquake magnitude series. Namely, the magnitudes follow the statistics of the independent and identically distributed random variables for all the categories of earthquakes investigated here.

To illustrate the effect of systematic memory, we study another synthetic time series belonging to Markov process. Specifically, the time series is generated by simulating the Brownian motion of a particle in one dimension subjected to a linear potential $U(x) = c|x|$. Starting from $x = 0$ at time $t = 0$, the position of the particle is updated in steps of $dt = 10^{-6}$ according to the following discretized version of the Langevin equation:

$$x(t + dt) = \begin{cases} x(t) - cd t + \sqrt{dt} \xi & \text{for } x \geq 0, \\ x(t) + cd t + \sqrt{dt} \xi & \text{for } x < 0, \end{cases}$$

(5)

where $\xi$ is a Gaussian white noise with zero mean and unit variance. At any time instant $t$, the position $x(t)$ of the particle is recognized as the height value $h$ in the time series. Therefore, at long times the height distribution $p(h)$ follows the Boltzmann-Gibbs distribution, i.e., exponential distribution, as shown in Fig. 7(a). In contrast to the time series mentioned above, the degree distribution of the visibility graph is observed to follow a power-law. In Fig. 7(b), we have shown the degree distribution plot on a double logarithmic scale for four different system sizes $N$. Additionally, the dependence of $\langle k_{\text{max}}(N) \rangle$ on $N$ has been exhibited in the inset of Fig. 7(b). As expected, $\langle k_{\text{max}}(N) \rangle$ grows as a power-law with $N$: $\langle k_{\text{max}}(N) \rangle \sim N^{0.486(5)}$. This signifies that any systematic single step memory in the time series leads to a scale-free network and hence, it strengthens our conclusion that the earthquake magnitudes are statistically uncorrelated.

![Fig. 6](image1.png)

**FIG. 6.** Main panel: Plot of the degree distribution $p(k)$ against $k$ with $\tau = 0.36$ for the visibility graph associated with a random time series of $N = 2^{20}$ exponentially distributed data values on a semilog scale for $\lambda = 1$ (black), 2 (red), and 3 (blue). Inset: Plot of the average maximum nodal degree $\langle k_{\text{max}}(N) \rangle$ against $N$ on a lin-log scale for $\lambda = 2$.

![Fig. 7](image2.png)

**FIG. 7.** (a) Plot of the height distribution $p(h)$ of the time series generated from the Brownian motion of a particle confined in a linear potential $U(x) = c|x|$ on a semilog scale for $c = 1$ (black), 2 (red), and 3 (green). The slopes of the curves are found to be 1.98(3), 3.95(3), and 5.98(3), respectively. (b) Main panel: Log-log plot of the degree distribution $p(k)$ of the visibility graph corresponding to the time series of $c = 1$ for $N = 2^{16}$ (black), $2^{18}$ (red), $2^{20}$ (green), and $2^{22}$ (blue). The dotted line is the guide to the eye with slope 2.01. Inset: The variation of the average maximum nodal degree $\langle k_{\text{max}}(N) \rangle$ against $N$ on a log-log scale. The results are based on the averages of at least $10^3$ independent trajectories.

### IV. CORRELATION BETWEEN THE INTER-EVENT TIMES

Next we focus on the interevent time (IET) series of earthquakes. Here the IET series is obtained from earthquake catalogs by calculating the intervals between two consecutive events and plotting them against the event index. In Fig. 8(a), we have plotted the cumulative degree distribution $P(k)$, i.e., the probability of finding a node with degree at least $k$ in the visibility graph, for the IET series of Tohoku$_1$ and Tohoku$_2$ on a double logarithmic scale. For both cases, the degree distribution is found to be heavy-tailed distribution and the tail can approximately be fitted with a power law. From the slopes we have estimated the exponent: $\gamma = 2.34(5)$ for the Tohoku$_1$ and $\gamma = 2.60(8)$ for the Tohoku$_2$. We also observed that the average maximum nodal degree varies as a power law: $\langle k_{\text{max}}(N) \rangle \sim N^{\alpha}$, where $\alpha = 0.77(3)$.
and 0.53(4) for the Tohoku₁ and Tohoku₂, respectively (not shown here). This behavior supports the power law nature of the degree distribution. Thus, the visibility graphs constructed from the IET series exhibit typical signatures of a scale-free network, indicating the existence of fBm-like correlations in the time series.

To validate the presence of correlation in a contrasting manner, we have analyzed the shuffled sequences of the IET data. Here we found that the degree distribution $p(k)$ can be fitted with the stretched exponential function given in Eq. (4). In Fig. 8(b), we have plotted $p(k)$ with $k^\tau$ using a semilog scale for the shuffled sequences of IET data for Tohoku₁ and Tohoku₂. The straight line here confirms the stretched exponential form of the degree distribution. In addition, we have found that $\langle k_{\text{max}}(N) \rangle \sim \ln N$ (not shown). Evidently, the shuffled data produces the properties of a random time series and therefore confirms the existence of correlation in the original time series.

Similar set of analyses are carried out for regular earthquakes in different regions, as well as for swarms and tremors. The results are shown in Fig. 9. In Figs. 9(a), (b) and (c), the cumulative degree distribution is plotted for regular earthquakes, swarms, and tremors. For every case, a heavy-tailed distribution has been observed. While for regular earthquakes and tremors a power law regime extending more than one decade is quite apparent, the data for swarms shows more complex behavior. However, an approximate power law variation can fit the data in the intermediate region. The power-law exponent $\gamma$ is estimated as 1.73(8), 2.64(5), 1.81(9), 1.79(9), 2.51(5) and 2.13(5) for Kumamoto (regular), Southern California (regular), Hakone (swarm), Izu (swarm), Cascadia (tremor), and Shikoku (tremor), respectively. In addition, the power law dependence of the average largest degree $\langle k_{\text{max}}(N) \rangle$ with $N$ has been observed for every set.
of data (not shown), supporting the power law nature of the degree distribution.

The tail part of the degree distribution is characterized by the exponent $\gamma$, which seems to depend on the seismic activity of the specific region: i) Earthquake swarms (Izu and Hakone) have a common value, $\gamma \simeq 1.8$. ii) Regular earthquakes may also have a common value, $\gamma \simeq 2.6$ (Tohoku and Southern California), while it is somewhat smaller (2.3) before the Tohoku $M_w 9.0$ earthquake (Tohoku1). iii) Kumamoto is exceptional with $\gamma \simeq 1.7$. This value is rather close to swarms, although the data mainly consist of aftershocks of 2016 Kumamoto earthquake. There may be two reasons for this discrepancy. First, the data is not a usual mainshock-aftershocks sequence, but rather a foreshocks-mainshock-aftershocks sequence. Alternatively, we may interpret it as two mainshocks ($M_w 6.2$ and 7.0) that occurred within only thirty hours. In any case, it is rather anomalous seismic activity. The second potential reason is an active volcano (Mt. Aso) located in the proximity of the main fault. The $M_w 7.0$ mainshock triggered many earthquakes in the volcanic area, including an $M_w 5.9$ event and its own aftershocks. Thus, the overall seismic activity is influenced by the nearby volcanic field and this may explain the resemblance to swarms.

If we suppose the relation between the fractional Brownian motion and the power-law degree distribution, i.e., Eq. (2), the exponent for the power spectrum $\beta$ can be determined. For example, swarms have $\beta \simeq 2.2$ and $H \simeq 0.6$. They are close to those for standard Brownian motion ($\beta = 2$ and $H = 0.5$) but yet slightly larger, corresponding to superdiffusion. Regular earthquakes ($\gamma \simeq 2.6$) have $\beta = 1.4$ and $H = 0.2$, corresponding to subdiffusion. Extraction of these exponents from actual seismic data is difficult using other standard methods such as autocorrelation functions due to the strong nonstationary nature of the seismic record. In this sense, these exponents might not be considered as that for fBm itself, but should represent some counterpart in seismic activities.

Shuffling the sequence of data we found that for every case, the visibility graph resembles the properties of a random time series and therefore confirm the fact that the original IET series corresponding to all three types of earthquakes possess fBm-like correlations. This result does not contradict the previous studies on regular earthquakes obtained using some different methods [48, 49]. Here we have confirmed the correlation using complex network based approach, and more importantly, found correlations in tremors and swarms.

V. DETAILED STRUCTURE OF VISIBILITY GRAPH

The detailed characterization of the topology of the network has served to identify several non-trivial features exhibited by diverse types of real-world systems including the basic principles that played role in the network formation [1, 3, 5]. In order to extract more properties hidden in the seismic records, the following graph-theoretical quantities have been analyzed.

Since our visibility graph is connected and undirected, there always exists at least one path between any arbitrary pair of nodes $i$ and $j$ through the links of intermediate nodes. The path with the minimal links traversed is called the shortest path length $d_{ij}$, and the average shortest path length is defined as,

$$l = \frac{1}{N(N-1)} \sum_{i,j \neq j} d_{ij}.$$  \hspace{1cm} (6)

In Figs. (a) and (b), we show the variation of $l(N)$ with $N$ on a semilog scale for both the magnitude and IET series, respectively. The best fit of the data by a straight line indicates its logarithmic scaling and hence, the network is small-world. Although the data for IET series of Shikoku has some curvature, the linear behavior...
is quite apparent for large values of $N$. For IET series of swarms, $l(N)$ grows more slowly than $\ln N$.

Another important quantity associated with the network is the clustering coefficient which measures the three point correlation among the neighbors. Specifically, the clustering coefficient $C_i$ of node $i$ measures the probability that the two neighbors of $i$ are connected. If there exists $E_i$ links among the $k_i$ neighbors of node $i$ then, $C_i = 2E_i/k_i(k_i-1)$. In the case of $k_i < 2$, $C_i = 0$. The global clustering coefficient is expressed as,

$$C = \langle C_i \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{2E_i}{k_i(k_i-1)}.$$  \hspace{1cm} (7)

By varying $N$ from $2^9$ to $2^{16}$ we have observed that $C$ is almost independent of $N$ (values differ only at 4-th decimal place) for both magnitude and IET time series of different types of earthquakes. Further, the clustering coefficient $\langle C(k) \rangle$ for the nodes with degree $k$ has been found to decay as $\langle C(k) \rangle \sim k^{-\nu}$ with $\nu \approx 1$, as shown in Fig. 11. This is the universal feature of a hierarchical network observed in many real-world networks [50]. The clustering coefficient $C$ assumes its highest value for the IET series of tremors.

We have also calculated the Pearson correlation coefficient $r$ to investigate whether a high degree node tends to be linked with a high degree node (assortative mixing, $r > 0$) or a low degree node (disassortative mixing, $r < 0$). We have calculated $r$ using the following formula [51],

$$r = \frac{L^{-1} \sum_i k_{i1}k_{i2} - [L^{-1} \sum_i \frac{1}{2}(k_{i1} + k_{i2})]^2}{L^{-1} \sum_i \frac{1}{2}(k_{i1}^2 + k_{i2}^2) - [L^{-1} \sum_i \frac{1}{2}(k_{i1} + k_{i2})]^2},$$  \hspace{1cm} (8)

where, $k_{i1}$ and $k_{i2}$ are the degrees of nodes at the ends of link $i$ with $i = 1, 2, \cdots , L$. We found that for all earthquake types, the magnitude series shows assortative nature (last column of Table II). In contrast, in case of IET series we obtain a value of $r \approx 0$ for the regular earthquakes and for swarms and tremors $r < 0$ (last column of Table III). Moreover, the graph associated with the IET series of swarms has been found to be more disassortative than that of tremors. This means that for swarms the high degree nodes show more preference towards linking with the low degree nodes. This indicates that the smaller heights are abundant in both the time series, however, there are a few very large heights (i.e., long quiescence periods) in the swarms which are even larger than the largest height in the tremor series. Therefore, swarms are more intermittent than tremors.

For a detailed comparison of the characteristic differences among the three different types of earthquakes, the above quantities have been calculated for a fixed value of $N = 2^{12}$ and the obtained values are listed in Table III and Table IV for the magnitude and the IET series, respec-

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**TABLE II.** Average values of the maximum degree $k_{\text{max}}$, average degree $\langle k \rangle$, clustering coefficient $C$, shortest path length $l$, and Pearson correlation coefficient for the visibility graph of the magnitude time series with $N = 2^{12}$. The synthetic catalog corresponds to the exponentially distributed heights with $\lambda = 2.303$ (i.e., $b = 1$).

| Region         | $k_{\text{max}}$ | $\langle k \rangle$ | $C$ | $l$ | $r$ |
|----------------|------------------|---------------------|-----|-----|-----|
| Tohoku1        | 101              | 6.76                | 0.770 | 5.49 | 0.118 |
| Tohoku2        | 82               | 6.36                | 0.764 | 5.66 | 0.167 |
| Kumamoto       | 86               | 6.61                | 0.769 | 5.64 | 0.128 |
| Southern California | 94           | 6.58                | 0.765 | 5.64 | 0.133 |
| Hakone         | 108              | 6.92                | 0.766 | 5.28 | 0.118 |
| Izu            | 110              | 6.69                | 0.762 | 5.80 | 0.125 |
| Cascadia       | 109              | 6.88                | 0.751 | 5.84 | 0.158 |
| Shikoku        | 129              | 7.05                | 0.759 | 5.43 | 0.092 |
| Synthetic Catalog | 82          | 6.64                | 0.780 | 5.67 | 0.122 |

**TABLE III.** Average values of the maximum degree $k_{\text{max}}$, average degree $\langle k \rangle$, clustering coefficient $C$, shortest path length $l$, and Pearson correlation coefficient for the visibility graph of the inter-event time series with $N = 2^{12}$. The data for swarms and tremors show disassortative degree mixing.

| Region         | $k_{\text{max}}$ | $\langle k \rangle$ | $C$ | $l$ | $r$ |
|----------------|------------------|---------------------|-----|-----|-----|
| Tohoku1        | 435              | 8.52                | 0.785 | 4.99 | -0.008 |
| Tohoku2        | 148              | 7.01                | 0.782 | 5.54 | 0.097 |
| Kumamoto       | 477              | 8.71                | 0.780 | 5.23 | 0.021 |
| Southern California | 188         | 7.20                | 0.784 | 5.32 | 0.071 |
| Hakone         | 1750             | 17.06               | 0.790 | 3.24 | -0.211 |
| Izu            | 1714             | 15.99               | 0.796 | 3.55 | -0.223 |
| Cascadia       | 701              | 11.89               | 0.816 | 3.98 | -0.107 |
| Shikoku        | 1185             | 13.78               | 0.828 | 3.45 | -0.162 |
tively. Clearly, they can be distinguished by the values of different graph-theoretical quantities obtained from their individual IET series.

VI. DISCUSSION AND CONCLUSION

Characterizing correlations between the earthquake magnitudes is a subject of great importance as it has relevance to the forecast of major earthquakes. However, to this date, the issue on the existence of correlations has not been settled [48, 52, 54]. For instance, it was reported that regular earthquakes occurring close in space and time are correlated in their magnitudes [52]. A counterargument was given in Ref. [53] that these were pseudo correlations due to the magnitude incompleteness and the modified Omori law. To shed a new light to this long-standing problem, we have made use of the complex network theory and analyzed the visibility graph to extract any correlations in magnitude time series.

Although the previous studies involve regular earthquakes only, here we extend the analysis to the other types of earthquakes [55] to consider this problem in a more general perspective. We have analyzed several seismic time series in seven seismogenic zones by constructing networks using the visibility graph algorithm. The degree distribution appears to be fitted with a stretched exponential function, Eq. (1), for all the types of earthquakes analyzed here. Furthermore, we found that those constructed from shuffled catalog (Fig. 5) or synthetic data drawn randomly from the GR law (Fig. 6) are also fitted with the stretched exponential function. This means that the visibility graph constructed from uncorrelated magnitude time series are characterized by the degree distribution with stretched exponential form. Contrarily, another synthetic magnitude time series with one-step memory leads to a power law degree distribution. All these findings indicate the statistical independence of the magnitudes of natural seismic events, since they follow the statistics of the uncorrelated events. However, we cannot rule out some irregular short-range correlations because such correlations may not be detected via the visibility graph.

The memoryless nature of earthquake magnitudes is a basic assumption in the epidemic-type aftershock sequences (ETAS) model, which is the most successful statistical model for earthquake time series [50]. Although the model has been already accepted commonly, the results given here may be a further evidence for the assumption in the model.

The degree distributions of stretched exponential form appear to contradict some previous studies [57, 58], in which the power law tails are concluded for the magnitudes of regular earthquakes. In view of Eq. (1), this may imply a fBm-like correlation in the magnitude time series. Interestingly, however, they also analyzed randomly shuffled sequences of magnitudes and did not find any significant difference in the degree distributions. This rather contradicts the existence of a correlation. Additionally, the degree distribution obtained in Ref. [57] spans approximately one decade only, and the tails are noisy. Thus one needs to be careful to draw a conclusion based on these data alone. In Ref. [58], the tails of the degree distributions are less noisy, but they appear to fall off from the power law at the tails. Thus, their degree distributions might be fitted with a stretched exponential function. However, the degree distribution produced from Mexican catalog appears to develop a tail that is still different from stretched exponential. We noticed that the magnitude data in the Mexican catalog do not always obey the GR law, and this may be the reason for the deviation from the stretched exponential function. However, the Mexican data require more careful and dedicated analyses to draw any decisive conclusions on magnitude correlation.

We extend the visibility-graph analysis for the inter-event time (IET) series. Contrary to the magnitude time series, we find an evidence of fBm-like correlations between the inter-event times. The network associated with the IET series has a scale-free nature with the exponents $\gamma$, which depends on the essential characteristics of seismic activity. In the context of the $f^{-\beta}$ noise, the exponent $\gamma$ is directly related to $\beta$. These exponents may work as a generalized and unified quantification of the intermittent nature of seismic time series. For instance, we find that the IET series for swarms are similar to superdiffusive Brownian motion, whereas those for regular earthquakes correspond subdiffusion. However, the interpretation of super- or subdiffusive nature in the IET series is yet unclear from the mechanical point of view, and should be pursued in the subsequent studies.

One can also consider more elaborated methods for the graph construction. For instance, the visibility graph constructed here is the undirected and unweighted. Taking into consideration the time directionality and assigning weights to the links based on the inter-event distances, the resulting graph theoretical properties would be interesting to study in the future. Additionally, since the spatial information of the seismic events has been disregarded here, the extension of the visibility graph method to space-time may be an promising attempt.

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