The magnetar emission in the IR band: the role of magnetospheric currents

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Abstract There is a general consensus about the fact that the magnetar scenario provides a convincing explanation for several of the observed properties of the Anomalous X-ray Pulsars and the Soft Gamma Repeaters. However, the origin of the emission observed at low energies is still an open issue. We present a quantitative model for the emission in the optical/infrared band produced by curvature radiation from magnetospheric charges, and compare results with current magnetars observations.

1 Introduction

The Anomalous X-ray Pulsars and the Soft Gamma Repeaters (AXPs and SGRs) are peculiar X-ray pulsators which exhibit erratic phases of bursting/outbursting activity (e.g. Mereghetti 2008). There is now a wide consensus that these sources indeed host an ultra-magnetized NS, or magnetar (Duncan & Thompson 1992), with a magnetic field $B \approx 10^{14}$–$10^{15}$ G, well in excess of the quantum critical field $B_Q \approx 4.4 \times 10^{13}$ G.

The magnetar scenario provides a quite coherent interpretation of the observational properties of SGRs/AXPs, in particular of both their bursting and persistent emission at X/γ-ray energies (Wood & Thompson 2006). In particular, it has been suggested that the magnetosphere of a magnetar is twisted ($\nabla \times B \neq 0$), in which case the density of magnetospheric currents is substantial and the observed soft X-ray spectrum ($< 10$ keV) can be explained in terms of resonant cyclotron scattering (RCS) of thermal surface photons onto mildly relativistic particles flowing along
the closed field lines (see Zane et al. 2009 for a recent application, and references therein).

Observations at IR/optical wavelengths led to the discovery of faint (K ∼ 19–21), and in some cases variable IR counterparts to several AXPs/SGRs (see again Mereghetti 2008). In two sources optical pulsations at the X-ray period were found (Kern & Martin 2002, Dhillon et al. 2009). However, the origin of the IR/optical emission is not clear as yet. The proposed scenarios involve emission/reprocessing by a fossil disk (see e.g. Perna et al. 2000), or non-thermal emission from the inner magnetosphere (Eichler et al. 2002, Beloborodov & Thompson 2007, hereafter BT07). Although this latter option appears promising, no detailed computations have been presented as yet. In this paper we reassess this issue, and provide a quantitative estimate of the IR emission expected from curvature radiation.

2 Pair production in the inner magnetosphere

In the inner magnetosphere, the magnetic field is so strong that the conversion of high-energy photons produced by RCS into pairs may give rise to a quasi-equilibrium configuration in which the $e^\pm$ Lorentz factor is locked at about the threshold value required to ignite the pair cascade ($50 < \gamma < 1000$, BT07). In order to estimate the spatial extension of this region, we proceed as follows. We assume that primary photons have a typical energy $\varepsilon \sim 1$ keV, and the magnetic field scales as $B \sim B_p(R/R_{NS})^{-3}$, where $B_p \sim 10B_q$ is the polar field and $R_{NS}$ is the star radius.

We assume that the magnetosphere is force free. Charges, flowing along $\mathbf{B}$, are accelerated by the parallel electric field which is provided by the decay of the twist, $\frac{\partial(B^2/8\pi)}{\partial t} = -E_{\parallel}j$. A thermal photon scatters where its energy in the particle frame, $\gamma(1 - \beta \cos \theta)\varepsilon$, equals the local cyclotron energy, $\varepsilon_B = m_e c^2 (B/B_Q)$ (here $\beta = v/c \sim 1$ and $\theta$ is the angle between the incident photon and the magnetic field), i.e. when the particle Lorentz factor is $\gamma = \gamma_{res} \sim (m_e c^2/\varepsilon)(B/B_Q)$. Because the potential drop at the endpoints of a flux tube (of length $L$) is enormous, $e\Phi_0 > 10^8 m_e c^2$ (see below), the electron Lorentz factor, which is $\sim 1$ at the star surface, monotonically increases up to a value which is much higher than $\gamma_{res,max} \sim$
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500(1 keV/ε)(B_p/B_Q) ≈ 5000. This ensures that the resonant condition (γ_{res} = γ) is met after the electron travelled a typical distance λ_{acc,res} = γ(dx/d(γβ)) ≈ γ_{res}L/(εΦ_0/m_eε^2) ≪ L. On the other hand, this is not a sufficient condition for RCS to occur: even if they can be accelerated up to γ_{res}, electrons are able to scatter only if λ < λ_{acc,res}, where λ ∼ (ε/m_eε^2)/(n_phσ_{eff}) ≈ 10^{-3}(T/1 keV)^{-2}(R/R_{NS})^2 cm is the particle mean free path for resonant scattering (here n_ph is the density of thermal photons and σ_{eff} = 3πσ_f/4α_e; e.g. Dermer 1990). This translates into:

\[ \frac{εΦ_0}{m_eε^2} < 5 \times 10^{11} \left( \frac{B}{B_Q} \right) \left( \frac{R}{R_{NS}} \right)^{-2} \left( \frac{T}{1\,\text{keV}} \right)^{2} \left( \frac{L}{10^6\,\text{cm}} \right). \]  

(1)

We approximate the left hand quantity with the potential drop which produces the conduction current in a 1D, relativistic double layer (where j ∝ Φ_0; Carlqvist 1982):

\[ \frac{εΦ_0}{m_eε^2} \approx 2 \times 10^{9} \frac{m_+}{m_e} \left( \frac{B}{B_Q} \right)^{1/2} \left( \frac{R}{R_{NS}} \right)^{-1/2} \left( \frac{L}{10^6\,\text{cm}} \right). \]  

(2)

where m_+ is the mass of the positive charge. If the magnetosphere is populated by e^-/e^+ pairs then m_+ = m_e, and for a polar magnetic field ∼ 10B_Q the inequality given by Eq. (1) is satisfied up to B/B_Q > 0.05, or R/R_{NS} < 6.

The upscattered photon produced in the region B/B_Q > 0.05 has an energy ε' ∼ γ_{res}'ε/(1 + γ_{res}'ε/m_eε^2) and may convert into a e^± pair in the strong magnetic field provided that the threshold condition ε' > 2m_eε^2/|sin θ'| is met, where θ' is the angle of the scattered photon with the magnetic field. For ε ∼ 1 keV, pair creation requires γ_{res} > 30 (i.e. B > 0.05B_Q). Since electrons are relativistic, the scattered photon initially moves parallel to B and a pair can be produced only after a large enough pitch angle has built up. Moreover, pair production is efficient only if its characteristic length-scale is less than the dimension of the circuit, i.e. α_± > 1/R_{NS} where α_± is the absorption coefficient for ordinary and extraordinary photons (see Baring 2007). Accordingly, the limiting value of magnetic field above which the process is effective decreases with increasing photon energy, and it is B ∼ 0.05B_Q for pairs created near threshold.

In summary, in the region B > 0.05B_Q a quasi-equilibrium configuration is reached with a pair multiplicity ∼ L/λ_{acc,res} of a few. Screening of the electric field limits the potential drop to eΦ_0/m_eε^2 ≈ γ_{res} ∼ 500(B/B_Q) and the maximum e^± Lorentz factor is γ_{res}. Charges undergo only few scatterings with thermal photons, but they loose most of their kinetic energy in each collision. A steady situation is maintained against such Compton losses because e^-/e^+ are re-accelerated by the electric field before they can scatter again. RCS may occur also at larger radii provided that the charges Lorentz factor is limited to ∼ γ_{res}, which decreases as B ∼ 1/R^3. The pairs energy depends on the electrostatic potential (in turn fixed by conduction current), and also on the efficiency of the radiative (Compton) drag. If pairs with γ ∼ γ_{res} are injected in the external region (B/B_Q < 0.05), the circuit is likely to behave much differently from a double layer, allowing the current to be conducted with only a small potential drop (see also Beloborodov, this volume).
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3 IR/Optical emission

In order to estimate the amount of curvature emission from the inner magnetosphere, as seen by a distant observer, we use a simple geometrical model. We approximate the twisted field with a dipole, \( B = B_p (R/R_{NS})^{-3}(\cos \theta, \sin \theta/2, 0) \), while the current is still taken to be \( j = (c/4\pi)|\nabla \times B|_{\text{twist}} \approx B(R/R_{NS})^{-1} \). The particle density is then \( n_\pm = B(R, \theta) (R/R_{NS})^{-1}/(4\pi e) \), for a pair velocity \( \approx c \). With reference to a generic flux tube labelled by \( \theta_0 (0 \leq \theta_0 \leq \pi/2) \), curvature photons will reach the observer if they are emitted in the volume \( \Delta V = XR_c \sin \theta_{\text{obs}} \left| dZ/d\theta_0 \right| \Delta \theta_0/\gamma^2 \), around the point \( P \) of cartesian co-ordinates \((X, Z)\) where \( B \) is parallel to the LOS (Fig. 1). Here \( \theta_{\text{obs}} \) is the viewing angle, i.e. the angle between the line-of-sight (LOS) and the magnetic axis. In general there are two emitting regions along each flux tube one above the other below the magnetic equator, because of electrons and positrons, and for any \( \theta_0 \) there are two azimuthally symmetric flux tubes.

The (polarization averaged) CR spectral power per particle can be expressed as
\[
p(\varepsilon) = (\dot{E}/\varepsilon) \left[ (\varepsilon/\varepsilon_c) \int_{\varepsilon_c}^{\varepsilon} K_{5/3}(x)dx \right] / \left[ \int_{c_0}^{\varepsilon} dx \int_{x_0}^{x} K_{5/3}(x')dx' \right],
\]
where \( K_{5/3}(x) \) is the modified Bessel function, \( \varepsilon_c = (3/2)\hbar c \gamma^3/R_c \), \( \dot{E} = (2/3)e^2 c^2 \gamma^4/R_c^2 \), and \( R_c \) is the curvature radius of the field line. The monochromatic luminosity emitted by the volume \( \Delta V \) is \( \Delta L = p(\varepsilon)n_\pm \Delta V \), and, since radiation is collimated in the solid angle \( \Delta \Omega = \pi/\gamma^2 \), the observer at infinity measures a luminosity \( 4\pi \Delta L/\Delta \Omega \). The total received power is then obtained summing the individual contributions over all the flux tubes, assuming the pair Lorentz factor is locked at \( \gamma_{\text{res}} \).

Fig. 2 (left panel) shows the computed CR spectrum for \( B_p = 4.3 \times 10^{14} \) and different viewing angles \( \theta_{\text{obs}} \), together with the optical/IR luminosities observed for a sample of AXPs. It should be noted that all these sources typically show variability.

Fig. 2 Model spectra for different values of \( \theta_{\text{obs}} \) and \( B_p = 4.3 \times 10^{14} \) G. The XMM-Newton X-ray spectrum of 1RXS J1708-4009 is from Rea et al. (2008; red solid line). The AXPs IR/optical data are from Duncan & van Kerkwijk 2005 (4U 0142+614 and 1E 1048-5937) and Mignani et al. 2007 (XTE J1810-197 and 1E 2259+586). The adopted distances for de-reddening are 5 kpc (4U 0142+614, 1RXS J1708-4009), 3 kpc (1E 1048-5937, 1E 2259+586), 4 kpc (XTE J1810-197), 8.5 kpc (1E 1841-045). Left: Spectra computed for incoherent curvature emission. Right: Same as in the left panel, but accounting for particle bunching.
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in all energy bands. The data shown in Fig. 2 were not selected according to any particular criterion and are just meant to be representative of the IR emission of magnetar candidates. As it appears from Fig. 2 model spectra severely underpredicts optical/IR data. Basically, this is due to the fact that the particle density is too low for incoherent CR emission to produce the required power. The point has been already noted, on the basis of a qualitative analysis, by BT07 who concluded that particle bunching would be likely present, providing the required amplification of CR emission.

Many models have been suggested to explain the origin of the interactions which push particles together and can lead to the formation of bunches of charged particles localized in phase-space, but the most promising explanation seems to be connected with plasma instabilities, like the two-stream (electron-positron/electron-ion) instability. If \( N \) particles are contained in a bunch of spatial scale \( l_B \), they radiate like a single particle of charge \( Q = Ne \). This collective emission produces an amplification of the radiated power by a factor \( N \) with respect to the standard, non-coherent emissivity by \( N \) individual charges. The details of the emission depend on the shape of the bunch. In the simplest, one-dimensional case the power per unit frequency emitted by a bunch is

\[
p_B(v) \sim N^2 p(v) \sin^2(\pi v / v_1)/(\pi v / v_1)^2 = N^2 \mathcal{P}(v) \quad (\text{Sagion 1975}).
\]

The main source of uncertainties lies in the size of the bunch, and here we assume that \( l_B \) is associated to the local plasma frequency,

\[
l_B \sim c/\nu_{pe} = \pi m_e c^2/(\nu_e^2 n_e) \quad (\text{e.g. Lesch 1998}).
\]

Since a single bunch emits a spectral power

\[
\langle n_e l_B^3 \rangle^2 \mathcal{F}(v)
\]

and there are \( N_B \sim \Delta V/l_B^3 \) bunches in each emitting volume, the total spectral power radiated within \( \Delta V \) is

\[
\Delta L_B = N_B \langle n_e l_B^3 \rangle^2 \mathcal{F}(v) = \langle n_e l_B^3 \rangle \Delta L.
\]

We stress that it has to be \( N = n_e l_B^3 > 1 \) for the process to be effective, meaning that only emission at frequencies \( \nu < \nu_l < \nu_{co} = n_e^{1/3} c \) is efficiently amplified. Where \( N < 1 \) emission is from individual particles. Finally, because of the strong absorption below the plasma frequency, a low-energy cut-off is present around \( \nu_{pe} \).

We have recalculated the emergent spectrum including particle bunching (Fig. 2 right panel), finding that coherent curvature radiation can indeed produce sufficient IR/optical emission to account for the observed one. Given the limitations of our model, we are not in a position to attempt any fit of the data. What we aim to, instead, is verifying that magnetospheric CR emission is able of reproduce the gross characteristics of the observed low-energy emission, its energetics in particular. The predicted spectrum vary with the viewing angle, and the luminosity is higher when the star is viewed nearly pole-on. The curvature spectrum is far below the observed one in the soft X-rays, indicating that a different mechanism is necessary to account for the X-ray emission below \( \sim 10 \) keV. In the twisted magnetosphere scenario this is RCS onto mildly relativistic pairs populating the external magnetosphere.

4 Discussion and Conclusion

Despite the magnetar scenario is now regarded as the most likely interpretation for the observed properties of SGRs and AXPs, and the many investigations aimed at
producing detailed models for the spectral and timing properties of magnetars, a number of key issues are still unresolved. In particular, the origin of the low-energy emission (in the IR/optical band) of a magnetar is still under debate. In this paper we gave a quantitative estimate of the IR/optical emission, under the assumption that it arises from curvature radiation from pairs in the inner magnetosphere. A comparison with observations of AXPs shows that the predictions of the model are in general agreement with observations, provided that curvature radiation is not coherent and a bunching mechanism is at work at long wavelengths.

Our model is based on a number of simplifying assumptions. For instance, the computation is based on a globally twisted dipolar magnetosphere while in some magnetars, the transient AXPs in particular, the twist might affect only a limited bundle of field lines close to a magnetic pole (Beloborodov 2009). If the inner part of the magnetosphere is (or becomes) untwisted, no currents are present that can produce the IR/optical curvature radiation. Also, we used a simple double layer model (eq. 1) for the linear accelerator, which may not be valid in presence of intense pair creation (BT07). Furthermore, and more important, the details of the charges motion in the external region of a twisted magnetosphere, which are essential ingredients in providing a model for the high energy emission in the soft and hard X-ray band ($\sim 0.5$–$200$ keV), are still unclear. In order to reach firmer conclusions about the entire multi-wavelength spectrum a more detailed study of the magnetosphere is required, and will be matter of future work (see also Beloborodov, this volume).

Acknowledgements LN and RT are partially supported by the INAF/ASI grant AAE-I/088/06/0. The authors thank the organizers of the Sant Cugat forum for the hospitality and support.

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