On the Approach to Optimization of Restoration Works at Railway Facilities

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Abstract. The article considers an approach to solving the problem of finding the optimal option for the work teams to carry out a complex of restoration works at railway facilities destroyed as a result of an emergency. An approach to solving this problem is proposed, based on the methods of graph theory, fuzzy set theory, and mathematical programming. The proposed approach is illustrated by a meaningful example.

1. Introduction

Let's give a verbal statement of the specified problem. Let's assume that several railway facilities were destroyed as a result of an emergency. It is necessary to plan the use of existing work commands in order to quickly restore these objects\textsuperscript{[1,2]}.

When restoring railway objects, it is necessary to use successively different working commands, i.e., stage-by-stage (step-by-step) restoration of all destroyed railway objects by working teams should be performed\textsuperscript{[3,4]}.

Each object must be restored, and any working team can work on no more than one object, performing no more than one stage on it from its beginning to its end. At the same time, it is required that the reliability of the restoration of railway facilities is maximized, which implies that each stage will be completed within the planned time for its implementation.

2. Problem statement

To solve the formulated problem, methods of graph theory, fuzzy set theory, and methods for solving the maximum flow problem are used.

To indicate possible sequences of implementation of the stages of reconstruction of railway objects, it is proposed to use a directed connected graph $G = (V, E)$, where $V$ is the set of vertices of the graph under consideration, and $E$ is the set of its arcs. Let $V = \{v_1, v_2, \ldots, v_{m+n}, t\}$, where $n$ is the number of objects to restore, and $m$ is the number of working commands\textsuperscript{[5,6]}.

Vertices of the graph with numbers 1,2,...,n correspond to objects, vertices with numbers from $n+1$ to $n+m$ correspond to working commands. In the future, work teams and railway objects will be identified with the corresponding vertices. The arc with the beginning at the vertex $u \in V$ and the end at the vertex $w \in V$ will be denoted. The $(u, t)$ arc means that the object where the last stage of restoration work was carried out by the $u$ work team has been restored. The presence on the graph of the arc $(u, w)$ in $w \neq t$ means that the object, the restoration work which was conducted by a team...
u can continue restoration work team working w. Each arc \((u, w) \in E\), with \(w \neq t\) mapped to a number \(c_{uw}\) - the weight (the degree of our confidence in that object, the restoration work which was conducted by the working team, \(u\) will hold, given the planned rehabilitation work, the working team \(w\), \(c_{ut} = 1\) for any arc \((u, t) \in E\).

3. Relevance

The article proposes an algorithm for solving the problem under consideration, which is a generalization of the algorithm proposed in [1] and uses methods for solving the maximum flow problem.

An example of graph \(G\) is shown in figure 1. Here, vertices numbered 1 and 2 correspond to destroyed railway objects, and the other vertices, except for vertex \(t\), correspond to working teams[7,8].

![Figure 1. The original graph \(G\).](image)

We will consider the described graph \(G\) as a transport network with communication capacities (arcs) equal to one, on which the sources are the vertices corresponding to railway objects, and the drain is the vertex \(t\). Recall that the Ford-Fulkerson method requires a single source, and we have their number equal to the number of objects. Therefore, we introduce an additional vertex \(s\) – the source, and all other vertices, except for the vertex \(t\), will be considered intermediate. Connect the vertex \(s\) with arcs \((s, u)\) with any vertex \(u\) corresponding to the railway object, the capacity of such an arc will be considered equal to one, assuming \(c_{su} = 1\) for any arc \((s, u), u \in V\).

To solve this problem, we will use a well-known technique in the literature, used, for example, by the authors in [2]. To do this, we will build a graph \(G^*\) from graph \(G\), for which each vertex of the transport network represented by graph \(G\) corresponding to the working command will be considered as a pair of vertices, one of them (the first) receives the flow, and the other second leaves this vertex. These vertexes are connected by a single communication. It is assumed that its throughput is equal to one. On the resulting transport network, the maximum flow is searched[9,10].

Let, for example, at railway facilities, work teams carry out restoration work in accordance with the scheme set by the graph in figure 1. Then the transport network constructed in accordance with the modification of the vertices corresponding to the working commands, taking into account the input of the source \(s\), will be defined by the graph in figure 2.
Graph theory and fuzzy set theory are used to solve the problem [3-9]. We present some concepts of fuzzy set theory and fuzzy logic.

4. Theoretical part
The concept of a fuzzy set is an attempt to mathematically formalize fuzzy information to build mathematical models. It is assumed that the elements that make up a given set, which have a common property, can have this property in different degrees and, therefore, belong to this set with different degrees. When saying that an element belongs to a given set, it is necessary to indicate to what extent it satisfies the properties of this set.

The following concepts and results of fuzzy set theory and fuzzy logic are borrowed from [11-12]. Fuzzy sets are defined on universal sets, which are ordinary sets. For example, if we are talking about the volume of destruction as a result of an emergency, then as a universal one we can take the set of all possible values of these volumes.

$$\hat{A}$$ fuzzy set on a universal set $U$ is a set of pairs $\left(\mu_{\hat{A}}(u), u\right)$, where $\mu_{\hat{A}}(u)$ is the membership function (degree of membership, reliability), i.e. the degree of membership of an element $u \in U$ to the fuzzy set $\hat{A}$. The degree of membership is a number from the segment $[0; 1]$, the higher it is, the more the element corresponds to the properties of a fuzzy set, the more likely it is an element of this fuzzy set.

We define, following Zadeh's work, a set-theoretic intersection operation for fuzzy sets.

The intersection of the fuzzy sets $\hat{A}$ and $\hat{B}$ given on the universal set $U$ is the fuzzy set $\hat{C}$ with the membership function $\mu_{\hat{C}}(u)$, where

$$\mu_{\hat{C}}(u) = \min\{\mu_{\hat{A}}(u), \mu_{\hat{B}}(u)\} \text{ for all } u \in U.$$  \hspace{1cm} (1)

We will need logical operations with fuzzy statements [10, 14].

In fuzzy logic, statements are considered that can be true or false to some extent, such statements are called fuzzy. The degree of truth of a fuzzy statement takes values from the closed interval $[0; 1]$. A fuzzy statement with a degree of truth of zero is perceived as "false", with a degree of truth of 1-as "true".

Logical operations are introduced on fuzzy statements, in particular conjunction operations.

The degree of truth of a fuzzy statement $\tilde{A}$ will be denoted by $\mu_{\tilde{A}}$. 

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Figure 2. Transformed network, graph $G^*$. 

Graph theory and fuzzy set theory are used to solve the problem [3-9]. We present some concepts of fuzzy set theory and fuzzy logic.
Let be given fuzzy statements A and B. The fuzzy logical operation And (conjunction) by analogy with the set-theoretic intersection operation (1) is performed by the rule:

\[
\mu_{A \cap B} = \min \{\mu_A, \mu_B\}
\]  

(2)

Let's return to the problem mentioned at the beginning of the article. So, consider the graph \(G^{*}\). Find the maximum flow on the corresponding transport network from the source to the drain.

An arc whose flow is greater than zero (and therefore equal to one) is called a loaded arc, and the path from the source to the drain, all arcs of which are loaded arcs, is called a loaded path. In relation to the problem under consideration, the path from the source to the drain corresponds to a possible sequence of stages, as a result of which the railway object is restored.

Note 1. Consider a graph \(G\) has a path from the top of the corresponding rail object to the vertex \(t\). Let this path pass through vertex \(v_1, \ldots, v_k, \ldots, v_r\). In a graph \(G^{*}\) there is a single path from the source to the drain, contain all the vertices of the considered path, and not containing any other vertex is a path passing through the vertices of \(s, v'_1, v'_1, \ldots, v'_k, v'_k, \ldots, v'_r, v'_r, t\). Consider this path in a graph \(G^{*}\) corresponds to the graph \(G\) considered in the way and vice versa. Each path from the vertex corresponding to the railway object to the vertex \(t\) on the graph \(G\) corresponds to a certain path from the source to the drain on \(G^{*}\) and vice versa.

Note 2. Loaded paths on graph \(G^{*}\) do not intersect. Indeed, due to the conditions imposed on the flow, the flow to each vertex must equal the flow from it. On the graph \(G^{*}\), exactly one arc enters or exits each vertex, not counting the source and drain. Therefore, none of the specified vertices can lie on two loaded paths.

Note 3. The number of loaded paths is equal to the maximum flow value.

To solve the considered problem is rather on a graph \(G\) find loaded \(n\) disjoint paths whose total weight max (under a total weight of the system of tracks is proposed, following (2) to understand a minimum of the weights of the arcs of these paths). Found the way, and it will set the optimal set sequence of stages in the restoration of railway facilities[13,17].

To find the specified set of paths, an algorithm consisting of a sequence of steps is proposed.

At the initial (zero) stage, the graph \(G_1^{*}\) is constructed according to the conditions of the problem.

At the stage with the number \(\tau (\tau=1,2,3,\ldots)\) for the graph \(G_\tau^{*}\), the maximum flow is found, using, for example, the Ford-Fulkerson method [15, 16]. Let the number of loaded paths from source to drain be \(M_\tau\). If \(M_\tau < n\), then the problem has no solution. Let \(M_\tau \geq n\). We consider the corresponding set of paths on the graph \(G\) and determine its weight, let it be equal to \(g_\tau\). We remove from the graph \(G_\tau^{*}\) arcs whose weight does not exceed \(g_\tau\). If an isolated vertex appears, then remove it as well. We get the graph \(G_{\tau+1}^{*}\). Let's move on to the next stage.

The calculations are carried out until we get the maximum flow, the value of which is less than \(n\).

We consider the loaded paths obtained at the penultimate stage. The paths corresponding to them on the graph \(G\) are the solution of the problem under consideration.

Example. Let's consider the problem formulated at the beginning of the article, assuming that there are two railway objects destroyed as a result of an emergency, 10 working teams and possible sequences of implementation of the stages of restoration of railway objects are presented using graph \(G\) (fig. 1).
5. Results of experimental research

The reliability of the fact that the working team will perform its operation efficiently (in a given time, with a given quality, etc.), taking into account the specified possible sequences of implementation of the stages of restoration of railway facilities, is indicated in table 1. These dependences will be called the dependences of the corresponding arcs\([18,20]\). The reliability of the remaining arcs on the graph \(G^*\) (arcs \((u,u')\), \(u = 3, 4, \ldots, 10\) and arcs going from the source and to the drain are equal to one).

Note that the arc \((u,v)\), at \(u,v \geq 3\), on graph \(G\) corresponds to the arc \((u',v)\) on graph \(G^*\).

| № of the initial vertex | 3' | 3' | 4' | 4' | 5' | 5' | 6' | 6' | 6' | 7' |
|-------------------------|----|----|----|----|----|----|----|----|----|----|
| № of the end vertex     | 4  | 6  | 5  | 7  | 7  | 8  | 7  | 9  | 10 | 4  |
| Reliability             | 0.9| 0.7| 0.9| 0.4| 0.5| 0.5| 0.7| 0.5| 0.8| 0.8|

Table 1. The values of the reliabilities of arcs.

| № of the initial vertex | 3' | 3' | 4' | 4' | 5' | 5' | 6' | 6' | 6' | 7' |
|-------------------------|----|----|----|----|----|----|----|----|----|----|
| № of the end vertex     | 8  | 10 | 10 |    |    |    |    |    |    |    |
| Reliability             | 0.9| 0.6| 0.6|    |    |    |    |    |    |    |

Step 0. Build a graph \(G\), see figure 2.

Step 1. Find the maximum flow on \(G^*_1\). Its value is \(M_1 = 2 = n\), loaded paths, for example, can be paths \(s-1-3-3'-6-6'-9-9'-t\), \(s-2-5-5'-8-8'-t\). The reliability of this set of paths is \(g_1 = 0.5\). We remove arcs from graph \(G^*_1\) with a reliability of no more than 0.5, we get table 2 and graph \(G^*_2\) (fig. 3). Note that in the following tables, the removed arcs are highlighted in bold [21,22].

| № of the initial vertex | 3' | 3' | 4' | 4' | 5' | 5' | 6' | 6' | 6' | 7' |
|-------------------------|----|----|----|----|----|----|----|----|----|----|
| № of the end vertex     | 4  | 6  | 5  | 7  | 7  | 8  | 7  | 9  | 10 | 4  |
| Reliability             | 0.9| 0.7| 0.9| 0.4| 0.8| 0.5| 0.7| 0.5| 0.8| 0.8|

Table 2. The values of the reliabilities of arcs for a graph \(G^*_2\)

| № of the initial vertex | 3' | 3' | 4' | 4' | 5' | 5' | 6' | 6' | 6' | 7' |
|-------------------------|----|----|----|----|----|----|----|----|----|----|
| № of the end vertex     | 8  | 10 | 10 |    |    |    |    |    |    |    |
| Reliability             | 0.9| 0.6| 0.6|    |    |    |    |    |    |    |
Step 2. Find a maximum flow for $G_2^*$. Its value is $M_2 = 2 = n$. Loaded paths, for example, can be paths $s-1-3-3'-6-6'-10-10'-t$, $s-2-5-5'-7-7'-8-8'-t$. The reliability of this set of paths is $g_2 = 0.7$.

We remove arcs from graph $G_2^*$ with a reliability of no more than 0.7, we get table 3 and graph $G_3^*$ (fig. 4).

**Table 3.** The values of the reliabilities of arcs for a graph $G_3^*$

| № of the initial vertex | 3' | 3' | 4' | 4' | 5' | 5' | 6' | 6' | 6' | 7' |
|-------------------------|----|----|----|----|----|----|----|----|----|----|
| № of the end vertex     | 4  | 6  | 5  | 7  | 7  | 8  | 7  | 9  | 10 | 4  |
| Reliability             | 0.9| 0.7| 0.9| 0.4| 0.8| 0.5| 0.7| 0.5| 0.8| 0.8|
| № of the initial vertex | 7' | 7' | 9' |    |    |    |    |    |    |    |
| № of the end vertex     | 8  | 10 | 10 |    |    |    |    |    |    |    |
| Reliability             | 0.9| 0.6| 0.6|    |    |    |    |    |    |    |

Step 3. The value of the maximum flow on the graph $G_3^*$ is equal to $M_3 = 1 < 2 = n$. 

![Figure 3. Graph $G_2^*$](image)
This is the end of the algorithm.

We return to step 2. The paths loaded on graph \( G_2^* \) s-1-3-3'-6-6'-10-10'-t and s-2-5-5'-7-7'-8-8'-t on graph \( G \) correspond to paths 1-3-6-10-t and 2-5-7-8-t.

As a result of the calculations to the recovery of the first train of the subject to attract the third, sixth and tenth laborers, to restore the second to fifth, seventh and eighth teams, while their use is required in the specified order[23]. The reliability of this option is maximum and is equal to 0.7.

6. Conclusion
1. Thus, the optimal set of stage sequences for the restoration of railway objects is found.
2. What is new in the proposed approach is that the "assignment problem" is considered on an arbitrary directed graph.
3. Of undoubted practical interest is the construction of effective algorithms for solving similar problems, but with other efficiency criteria, for example, total costs in financial terms or in team hours, an interesting indicator is the time spent on performing all the work.
4. The task can be easily modified for the case when working teams can be consistently involved in more than one stage of restoration of railway facilities.

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