The Gravity Probe B Experiment and Early Results

John W. Conklin for the Gravity Probe B Collaboration

Stanford University
E-mail: johnwc@stanford.edu

Abstract. The NASA Gravity Probe B orbiting gyroscope test of General Relativity, launched from Vandenberg Air Force Base on 20 April, 2004 tests two consequences of Einstein’s theory: 1) the predicted 6.6 arcs/yr geodetic effect due to the motion of the gyroscope through the curved space-time around the Earth; 2) the predicted 0.039 arcs/yr frame-dragging effect due to the rotating Earth. The mission required the development of many technologies that did not exist when experiment was conceived in 1960. Cryogenic gyroscopes with drift-rates 7 orders of magnitude better than the best inertial navigation gyroscopes, a < 1 marcs star tracking telescope, and other essential technologies were developed as a result of an intensive collaboration between Stanford physicists and engineers, NASA and industry. Gravity Probe B collected science data from August 27, 2004 through September 29, 2005. Analysis of the data began during the mission and is on-going. This paper describes the main features and challenges of the experiment and presents the preliminary results to date.

1. Introduction

Gravity Probe B (GP-B) is a space-borne physics experiment designed to test two predictions of General Relativity. The concept of the experiment, depicted in Figure 1, is to place precision gyroscopes in a low Earth polar orbit and measure their long term precession relative to a guide star that represents an inertial reference frame. Using precision gyroscopes to test Einstein’s general theory of relativity (GR) was independently proposed by Pugh [1] and Schiff [2; 3] in 1960. Both pointed out that according to the general theory of relativity, the angular momentum axis of a gyroscope in orbit about the Earth will precess about a direction normal to the orbital plane due to the gravitational interaction of the spinning gyroscope with its orbital motion, and simultaneously about the direction of the Earth’s rotation axis due to the interaction of the spinning gyroscope with the angular momentum of the Earth, as shown in Figure 1. The first effect is known as the geodetic effect, and the second is known as the frame-dragging or Lense-Thirring effect. The instantaneous precession rate in Einstein’s general relativity for a spherical massive body is given by

$$\vec{\Omega} = \frac{3GM}{2c^2R^3} \vec{R} \times \vec{v} + \frac{GI}{c^2R^3} \left[ \frac{3}{R^2} \left( \vec{\omega} \cdot \vec{R} \right) - \vec{\omega} \right]$$

(1)

1 M. Adams, W. J. Bencze, R. W. Brumley, S. Buchman, B. Clarke, D. B. DeBra, M. Dolphin, C. W. F. Everitt, D. N. Hipkins, T. Holmes, J. H. Goebel, D. Gill, G. Green, M. Heifetz, G. M. Keiser, J. Kolodziejczak, J. Li, J. Lipa, J. M. Lockhart, J. Mester, B. Muhlfelder, Y. Oshima, B. Parkinson, M. Salomon, P. Shestople, A. S. Silbergleit, V. Solomonik, K. Stahl, M. A. Taber, J. P. Turneaure, S. Wang and P. Worden
Figure 1. Gravity Probe B experiment concept showing gyroscope precession due to geodetic and frame-dragging effects predicted by general relativity.

Here $G$ is the gravitational constant, and $M$, $I$ and $\vec{\omega}$ are the mass, moment of inertia, and the angular velocity of the massive body. The vectors $\vec{R}$ and $\vec{v}$ are the position and velocity of the gyroscope relative to the center of mass of the body. In a 640 km polar orbit, the gyroscope precession due to the orbital motion about the Earth is 6.6 arcs/yr (32 $\mu$rad/yr), while the orbital average precession rate due to the Earth’s angular momentum is 0.039 arcs/yr (0.20 $\mu$rad/yr). In addition to the geodetic effect due to the orbital motion about the Earth, there is also a contribution due to the orbital motion about the Sun, where the predicted magnitude is 0.019 arcs/yr (92 nrad/yr), perpendicular to the plane of the ecliptic. The magnitude of the solar frame-dragging effect is below the anticipated accuracy of the experiment [4].

During the 40+ years since GP-B was first proposed, there have been considerable advances in testing GR and also in understanding the significance of different tests [5], though it is remarkable how restricted the testing has been. Among the principal advances have been the Shapiro time delay effect [6–8], greatly improved measurements of light deflection [9], and above all the strong evidence for gravitational wave damping in binary pulsars [10; 11]. Another intriguing observation related to GP-B’s geodetic measurement has been the determination to 2% of the de Sitter (i.e. geodetic) effect on the motion of the Earth-Moon system around the sun [12]. Expressed in terms of the Parametrized Post-Newtonian (PPN) formalism [13], light deflection and the time delay effect depend on $1 + \gamma$, where $\gamma$ is the curvature parameter, while the geodetic effect is proportional to $1 + 2\gamma$. Within this framework, therefore, the three effects are related, with the first two determining $\gamma$ by photons and the geodetic effect determining it by massive bodies.

The concept of frame-dragging was first forwarded by J. Lense and H. Thirring in 1918 by considering possible GR effects on orbit planes of the moons of Jupiter and other planets [14]. They and subsequent authors recognized that the measurement would be extremely difficult owing to the presence of much larger Newtonian effects. One very bold attempt to perform such
a measurement has been through a combination of laser ranging measurements to the LAGEOS and LAGEOS II satellites around the Earth [15]. The claimed accuracy is 10%. This requires separating the Einstein effect from Newtonian precessions that are $1.5 \times 10^7$ times larger and also elaborate treatment of Earth models. The results remain controversial [16].

Discussion of the frame-dragging effect is often expressed in terms of an analogy with electromagnetism, where a rotating massive body generates a gravitomagnetic field analogous to the electromagnetic field around a rotating charged body (see e.g. Ref. [17]). The analogy potentially extends the experimental question into new areas. For example, K. Nordtvedt has pointed out that theories fitting the analogy require a gravitomagnetic term in the Earth-Moon motion observed through Lunar laser ranging (LLR) [18]. However, corrections to Nordtvedt’s original calculation, e.g. by S. Robertson [19], show that the amplitude of the term is at or below the current accuracy of LLR.

Gravity Probe B, like Gravity Probe A [20], is intellectually distinct from most previous tests of GR, which consist of astrophysical observations, in that GP-B is an experiment under the control of physicists.

2. The Gravity Probe B Instrument

The Gravity Probe B science instrument consists primarily of four science gyroscopes and a star tracking telescope, both rigidly mounted to a fused-quartz block which acts a metrology reference for the experiment. The spin axes of the gyroscopes and the position of the guide star are both measured with respect to a spacecraft reference frame defined by the quartz block. The instrument is mounted in a low-temperature probe, which serves as a vacuum chamber. This probe is inserted into a 2300 L superfluid liquid helium dewar, which maintains the science instrument at $\sim 2$ K. Operating at low temperatures increases mechanical stability, significantly lowers gas pressure and hence gyroscope damping torques, and allows for a London moment gyroscope readout described in Section 2.1.1. The liquid helium dewar is also critical for the attenuation of external magnetic fields by $< 2 \times 10^{-12}$ and the residual magnetic field at the location of the gyroscopes to < 0.1 nT (1 $\mu$G). The liquid helium dewar is mounted in the frame of the spacecraft which incorporates typical spacecraft systems, such as an Electric Power System, an Attitude and Translation Control system, a Global Positioning System and a communications system. The entire instrument rolls about the line of site to the guide star with a period of 77.5 s. Rolling the spacecraft shifts the inertial pointing signal from low frequency to the roll frequency, averages spacecraft body-fixed classical disturbance torques towards zero, and reduces the effect of body-fixed pointing biases.

2.1. Gyroscope

Each of the Gravity Probe B gyroscopes shown in Figure 2 (a) consist of a spherical rotor and a housing. The rotors are 38.2 mm diameter, 63 g, fused-quartz spheres made round by a unique lapping and polishing process developed at Stanford University. The spheres are coated with a 1.3 $\mu$m (50 $\mu$m) thick layer of niobium that is uniform to better than 0.3% [21]. This thin layer of niobium provides an electrically conducting surface for the electrostatic suspension system as well as a superconducting coating for the London moment readout. All fight quality rotors have an asphericity of less than 25 nm (1 $\mu$m), a mass unbalance less than 25 nm (1 $\mu$m), and a fractional difference in their moments of inertia better than $3 \times 10^6$.

The gyroscope housing, shown in Figure 2 (a), is a fused-quartz cylinder with an interior spherical cavity that has a radius 31 $\mu$m larger than that of the coated rotor. Three pairs of mutually orthogonal electrodes are delineated by circular cuts in the interior of the housing. Within this area, a 2.5 $\mu$m (100 $\mu$m) thick 7-layer Ti-Cu-. . .-Cu-Ti electrode is sputter deposited onto the surface [22; 23]. Each of the gyroscopes is spun up with a helium gas spin-up system, where the gas flowed through a spin-up channel cut into the housing [24]. A fiber optic cable
Figure 2. (a) Photograph of a Gravity Probe B gyroscope and (b) star-tracking telescope delivers ultraviolet light to the gyroscope from a mercury lamp to control rotor charge. A 2 mm diameter electrode concentric with the termination of the UV cable can be biased +3, 0, or -3 V to control the direction of the flow of photoelectrons allowing the rotor to charge or discharge.

2.1.1. Gyroscope Readout

The London magnetic moment generated by the spinning, superconducting rotor is used for the gyroscope readout [25; 26]. A spinning spherical superconductor spontaneously develops a magnetic dipole moment aligned with and proportional to its angular velocity vector. For a gyroscope spinning at 80 Hz the magnitude of an equivalent uniform field in the center of the sphere is 5.7 nT (57 µG). Changes in the orientation of the gyroscope spin axis are determined by measuring changes in the magnetic flux through a 4-turn niobium thin-film pickup loop, sputter deposited and lithographically patterned on the parting plane of the gyroscope housing [27]. A superconducting cable connects the pickup loop to a DC SQUID (Superconducting Quantum Interference Device). The gyroscope spin axis is nearly aligned with the plane of the pickup loop, so that changes in the magnetic flux through the loop are proportional to the angle between the gyroscope spin axis and the pickup loop plane.

A calibration signal derived from a voltage reference stable to several parts in $10^5$ over one year is injected into the feedback loop and measured by the readout system in order to determine the zero-frequency component of the gyroscope readout scale factor. In addition, the long-term stability of the voltage reference is compared to the flux quantum in the SQUID.

2.1.2. Electrostatic Suspension System

The electrostatic suspension system measures and controls the position of the rotor relative to each pair of electrodes along a given axis [28]. A capacitance bridge operating at 34 kHz for each electrode axis measures the rotor position relative to the electrodes. A 40 mV peak-to-peak excitation at 34 kHz is applied to each capacitance bridge, while the phase of the excitation on the capacitance bridge for the three electrode axes is shifted by 120° to maintain the rotor at a virtual ground. The demodulated potential difference between each electrode pair is a direct measure of the position of the rotor relative to the electrodes. The noise in each capacitance bridge is less than 0.1 nm/√Hz.

The electrostatic suspension system is designed to operate over a wide dynamic range of applied acceleration and force for operation during gyroscope spin-up, science data collection and possible micrometeorite impulses. In addition the suspension system is highly reliable since the loss of control on any one of the gyroscopes while it is spinning at high speed would very likely destroy that gyroscope and possibly the science instrument assembly.
2.2. Telescope
A cryogenic telescope, shown in Figure 2 (b), is used to measure the apparent position of the guide star IM Pegasi [29–31]. The folded Cassegrain telescope is constructed from fused-quartz, and has an aperture of 144 mm, a focal length of 3.8 m, and field of view of 320 $\mu$rad (66 arcs). The light from the guide star passes through four windows in the cryogenic probe, then is reflected from the primary, secondary, and tertiary mirrors and enters an image divider assembly mounted on the front plate of the telescope. Within the image divider assembly, a beam splitter divides the light in half, and each half is focused on the vertex of a roof prism. If the telescope is pointed directly at the star then light from each roof prism is divided equally from each side of its vertex. The light beams from each half of the roof prism are sent through another image divider and measured by redundant pairs of silicon photodiodes. With flight data calibration, the telescope pointing angle can be derived from the readout by a simple cubic model with an error less than 0.1 marcs within the pointing range of 2 $\mu$rad (400 marcs). The combined pointing noise from all four detector pairs is 2.3 nrad (0.48 marcs) per orbit.

2.3. Spacecraft
Unlike most spacecraft, the GP-B spacecraft and payload are intricately connected to achieve drag-free control performance. The drag-free control system reduces inertially-fixed accelerations transverse to the satellite roll axis to less than $2 \times 10^{-11}$ m/s$^2$ in the frequency band from less than 0.01 mHz to 10 mHz [32; 33]. Boil-off gas from the superfluid liquid helium is exhausted through a porous plug [34] and used to drive 16 proportional thrusters to control the attitude and translation of the spacecraft. The GP-B spacecraft is shown in Figure 3.

Power for the spacecraft is provided by four fixed gallium arsenide solar arrays, while batteries provide power when in eclipse. A sun shield is attached to the warm end of the low-temperature probe to prevent the light from the sun from entering the telescope. A passive thermal shield, mounted close to the top of the low temperature probe, encloses the most sensitive electronics boxes, and critical circuit boards within these electronics boxes are temperature controlled. The center of mass for the spacecraft lay between the telescope and the closest gyroscope. Five mass control mechanisms move 10 or 20 kg masses along thread screws to align the principal axes of the spacecraft with the telescope axis and provide a fine adjustment for the radial mass unbalance of the spacecraft. A pair of modified Trimble TANS Vector III GPS receivers [35] are used to determine the position and velocity of the spacecraft, and a laser retroreflector is attached to the aft end of the spacecraft. Commercial star trackers and rate gyroscopes are used for roll and coarse attitude control. Two omnidirectional antennas, one mounted near the tip of the sunshade and the other mounted on the aft end of the spacecraft provide communications with NASA ground stations or through NASA’s Tracking and Data Relay Satellite System (TDRSS). Additional information on the flight hardware may be found in Refs. [36; 37].

3. Flight Operations and On-orbit Performance
The Gravity Probe B spacecraft was launched on 20 April 2004, at 09:57:24 PDST from Vandenberg Air Force Base in southern California. A Delta II 7920-10 launch vehicle placed GP-B into a 640 km altitude circular polar orbit.

Telemetry is recorded to a solid-state recorder and telemetered to the ground stations at a rate of 32 kbit/s. This telemetry is sent over the NASA close I/O network to the Mission Operations Center at Stanford University, decommutated, and stored in a database. The telemetry data rate through the TDRSS network is 1-2 kbit/s lasting ~ 20 minutes per contact. The Mission Operations Center at Stanford operated 24 hours a day from launch to helium depletion, which ended the science mission on 29 September 2005. The mission is divided into three phases.
3.1. Initial On-orbit Commissioning

The IOC phase consisted of the initial check out of the various subsystems and extensive operations with the spacecraft’s attitude and translation control system. It included acquisition of the guide star, adjustments to the attitude control system to allow rapid acquisition of the guide star following its occultation by the Earth every orbit, and tests of the various operating modes of the drag-free control system. The electrostatic suspension system levitated the gyroscopes and measured the rotor potential. Then, the ultraviolet charge control system discharged the gyroscopes. The residual trapped magnetic flux in each of the rotors was measured, and a planned magnetic flux reduction procedure, which consisted of heating the gyroscopes above the superconducting transition temperature and slowly cooling them again was used to reduce the trapped magnetic flux in the gyroscope rotors.

As each of the gyroscopes was spun up to their full speed, there was a decrease in the spin speed of the other three gyroscopes because of the increased helium gas pressure within the low-temperature probe. Once the four gyroscopes were spinning at their final speeds of 79.4, 61.8, 82.1, and 64.9 Hz, a low-temperature bakeout heated the gyroscopes, the interior of the vacuum probe, and the sintered titanium cryopump from 2 K to > 6 K for 8 hours. After this operation, the measured spin down time constants for the four gyroscopes ranged from 0.29 to 1.4 $\mu$Hz/hr, indicating that the pressure was less than $2.0 \times 10^{-9}$ Pa ($1.5 \times 10^{-11}$ Torr). However, differences in the spin-down time constants of the four gyroscopes and the lack of observable variation in the spin-down time constant with temperature indicated that the residual gas pressure was not the dominant spin-down torque acting on the gyroscopes. Later tests confirmed a pressure of

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**Figure 3.** The Gravity Probe B spacecraft
approximately $10^{-12}$ Pa ($10^{-14}$ Torr).

Finally, the spin axis of each of the four gyroscopes was aligned so that the spin axis and the satellite roll axis over the course of the year was less than 50 $\mu$rad (10 arcs). To align the gyroscopes, torques were applied with the electrostatic suspension system taking advantage of the equatorial bulge of approximately 25 nm of the rotors due to the combined effects of the centrifugal forces and the static rotor shape.

The IOC phase, initially planned for 60 days, was only completed after 123 days with three gyroscopes spinning at full speed and aligned on 28 August 2004 and the completion of the final alignment of gyroscope 4 on 13 September, 2004. The increased length of the initialization phase was due to the limited communication with the spacecraft, the caution in all on-orbit operations, and the longer than anticipated time required to adjust the control systems of the payload and spacecraft.

### 3.2. Science Data Collection

Science data collection for gyroscopes 1, 2, and 3 began on August 28, 2004, and on September 13, 2004 for gyroscope 4. The intent is to disturb the gyroscopes as little as possible in order to collect adequate data to determine the orientations of the gyroscopes.

It quickly became evident that the polhode period of each of the four gyroscopes was changing with time. The polhode period is determined from the modulation in the amplitudes of each harmonic of the spin frequency. The changing polhode period indicates that the maximum moment of inertia of the rotor is slowly becoming aligned with its angular velocity vector. Although the changing polhode has no impact on the London moment signal, it slowly changes the $\sim 1\%$ contribution of the trapped magnetic flux to the total gyroscope readout signal.

During science data collection, standard operations were interrupted nine times because of spacecraft anomalies. These anomalies to the normal operation of the spacecraft were most frequently due to reboots of the spacecraft’s flight computer probably caused by multiple single bit errors or double bit errors (two consecutive bit flips) attributed to charged particle radiation. Another example of an anomaly which began on January 20, 2005, was caused by increased solar proton flux with energies greater that 10 MeV which increased from less than 0.1 to greater than 100 particles/(cm$^2$ s sr) for several hours. The flux of protons and secondary particles rendered the telescope signal unusable, causing the spacecraft to wonder $\sim 3$ mrad from the guide star. The data from each of these anomalies is carefully evaluated to determine the extent to which additional parameters have to be included in the data analysis to account for any offset in the orientation of the gyroscope spin axis during these periods.

### 3.3. Post-science Calibration Phase

During the post-science calibration phase, operating conditions of the gyroscopes and spacecraft were deliberately changed to place limits on potential systematic errors. Early in the calibration phase low risk operations were performed which included changing the bias of the SQUID readout system, changing the frequency and amplitude of the calibration signal injected into the readout system, and turning off or changing the set point of the temperature control on critical circuit boards and the SQUID itself. Then operations expected to significantly increase the classical torques acting on the gyroscopes were performed. These included increasing the average voltages on the electrostatic suspension system, deliberately modulating the increased preload voltages at the spacecraft roll frequency, changing the position of the gyroscope within the housing, and modulating the position of the gyroscope at the spacecraft roll frequency. Finally calibrations involving the entire spacecraft were performed. These operations included accelerating the spacecraft up to the $10^{-7}$ g and changing the orientation of the spacecraft roll axis to place limits on gyroscope torques proportional to the misalignment between the spacecraft roll axis and the gyroscope spin axis. In these operations, the spacecraft’s attitude
control system was commanded to move the satellite roll axis to a direction of another star, acquire the star and remain in that orientation for up to one day, and then return to the guide star, IM Pegasi. A total of 17 different maneuvers were made with the majority of maneuvers to stars or virtual stars (chosen orientations where no star is visible) within 1° of the guide star. During some of these maneuvers, the operating conditions of the suspension system was changed and a spacecraft inertial acceleration was applied. In the final days of the calibration phase, the spacecraft roll rate was decreased to search for additional potential systematic errors that depend on the spacecraft roll rate.

One clear indication of additional classical torques acting on the gyroscopes was found during the calibration phase. Observations of the gyroscope orientation before and after maneuvering the spacecraft away from the guide star indicates that there are torques acting on the gyroscopes that are larger than expected. The analysis of the data from these maneuvers shows that for each of the gyroscopes there is a torque acting which is proportional to the misalignment between the gyroscope spin axis and the spacecraft roll axis. In addition, the direction of the torque causes the gyroscope spin axis to precess about the direction of the spacecraft roll axis. The relation between the drift rate and the misalignment angle is linear to < 10% for misalignments less than 1° but becomes very nonlinear for larger misalignment angles.

Further investigations indicate that the magnitude of the classical torque varies over time and from gyro to gyro. But typically, the classical torque during the science data collection phase causes a drift rate in the gyroscopes that is only ~ 5% of the expected drift rate due to the geodetic effect predicted by GR. In Section 4.4, discussing the data analysis methods, we show that the classical torque induced drift rate is clearly separable from a uniform drift caused by relativity.

4. Data Analysis
More than 1 terabyte of telemetry data was collected from the Gravity Probe B spacecraft during the ~ 1.5 years of operation. The majority of these data are from the science instrument assembly which determines a continuous time history of the orientation of the gyroscopes relative to the guide star. During the science data acquisition phase, preliminary estimates of the gyroscope spin axis orientation time histories were made in order to plan for the post-science calibration phase. After the depletion of the liquid helium on the spacecraft, a more detailed analysis of the data started.

4.1. Determination of Gyroscope Spin Axis Orientation
The SQUID and telescope readout electronics respectively provide signals which determine the orientations of spin axes of the gyroscopes and the apparent orientation of the guide star with respect to the quartz block. When the guide star is not occulted by the Earth the attitude control system uses the output of the science telescope to continuously point the telescope at the apparent position of the guide star. Once the guide star is acquired, the spacecraft’s attitude control system typically maintains the telescope axis within ±1 µrad rms (200 marcs) of the apparent position of the guide star. In addition, because of the careful initial alignment and low drift rate of the gyroscopes, the spin axes of each of the four gyroscopes remains within 150 µrad (30 arcs) of the satellite roll axis over the course of the mission.

The gyroscope and telescope signals are combined to determine the orientation of the gyroscope spin axis relative to the apparent direction to the guide star. To determine the relative scale factor between the gyroscope and telescope readout systems, a sinusoidal dither signal is injected into the pitch and yaw axes of spacecraft’s attitude control system. Measuring the amplitudes of the gyroscope and telescope readout signals at the dither frequency then provides the scale factor relating the telescope readout to each of the gyroscope readouts.
The combined gyroscope and telescope signal is modulated at both the satellite roll frequency \( 1/(77.5 \text{ s}) \) and the orbital frequency \( 1/(97.5 \text{ min}) \). Roll star trackers on the spacecraft’s attitude reference platform measure the phase of the roll to an accuracy < 50 µrad (10 arcs). The magnitude and phase of the roll frequency signal determines the orientation of the gyroscope spin axis relative to the apparent direction to the guide star. The modulation at the orbital frequency is caused by the change in the apparent direction to the guide star due to the aberration of starlight, which provides a known calibration signal for the complete readout system. The aberration of starlight is due to changes in the transverse velocity of the telescope relative to the direction to the guide star. The exact aberration signal including both classical and special relativistic effects is calculated using the known spacecraft orbit [38]. The component of aberration, which is modulated at the orbital frequency is referred to as the orbital aberration. It has a magnitude of approximately 25 µrad (5 arcs) and varies sinusoidally in the plane of the orbit at the orbital frequency. Since the position and velocity of the spacecraft are known to a fractional accuracy of better than \( 10^{-3} \), the orbital aberration signal is known to this accuracy. The known orbital aberration signal is used to find the scale factor of the gyroscope readout \( C_g \) and orientation of the pickup loop relative to the star trackers \( \delta \phi \).

By properly accounting for the total aberration of starlight, which includes the changing velocity of the spacecraft, the gravitational deflection of the light from the guide star by the Sun, and the parallax due to the Earth’s annual motion about the Sun, the orientation of the gyroscope in an inertial frame defined by the true position of the guide star is determined. In addition to the orbital aberration, there is a component referred to as the annual aberration. This signal has a magnitude of 100 µrad (20 arcs), and its direction slowly varies in an elliptical path. The annual aberration is determined from the position and velocity of the Earth relative to the barycenter of the solar system as given by the JPL Earth ephemeris.

It is now useful to define the inertial coordinate system used for the data analysis. The unit vector \( \hat{e}_S \) defines the true direction to the guide star. The West-East direction \( \hat{e}_{WE} \) is defined as the cross product of \( \hat{e}_S \) with the Earth’s rotation axis on 1 January 2000. A third orthonormal basis vector in the North-South direction is defined by the cross product \( \hat{e}_{NS} = \hat{e}_{WE} \times \hat{e}_S \). The component of this spin axis along the unit vector \( \hat{e}_{NS} = S_{NS} \) and the component of the spin axis along the unit vector \( \hat{e}_{WE} = S_{WE} \).

The measurement equation for the roll frequency components of the combined gyroscope and telescope signal \( z(t) \) is

\[
z(t) = C_g \left[ (S_{NS}(t) - A_{NS}(t)) \cos(\phi_r + \delta \phi) + (S_{WE}(t) - A_{WE}(t)) \sin(\phi_r - \delta \phi) \right]
\]

Here, \( C_g \) is the scale factor of the combined gyroscope and telescope signal, \( \phi_r(t) \) is the roll phase as measured by the star trackers on the attitude reference platform, \( \delta \phi \) is a fixed phase difference between the normal to the pickup loop and the star trackers, and \( A_{NS}(t) \) and \( A_{WE}(t) \) are the known combined effects of the aberration of starlight, deflection of starlight, and parallax.

4.2. Time Varying Gyroscope Scale Factor

The scale factor \( C_g \) for each gyroscope readout contains a small contribution due to trapped magnetic flux. Since the trapped flux is fixed in the body-fixed frame of the rotor, its contribution is modulated at the harmonics of the gyroscope polhode frequency. The magnitude of this modulation is gyroscope and time dependent and ranges from 0.4% to 3% of the total \( C_g \).

Over short intervals of up to several days, the amplitude of this modulation may be treated as constant, and the magnitude and phase of the modulation may be determined by fitting the data to a nonlinear model which includes the harmonics of the measured polhode frequency. This method of determining the gyroscope readout scale factor is called the Low Frequency (LF) determination of \( C_g \).
Because of the changing polhode in each gyroscope, the magnitude and phase of the modulation at each of the harmonics of the polhode frequency slowly changes. In addition, there is a small but slowly varying contribution to the zero-frequency component of the gyroscope scale factor due to the trapped magnetic flux. As a result, the LF determination of \( C_g \) has a limiting accuracy of \( \sim 3 \times 10^{-4} \) over intervals of 5 days due to the SQUID noise. Over longer intervals, it may be necessary to take advantage of the information provided by the high frequency channel of the SQUID readout system.

The signals at harmonics of the gyroscope spin speed contain detailed information on the distribution of the trapped magnetic flux on the surface of each gyroscope rotor. An alternate method of determining the gyroscope scale factor variations known as \textit{Trapped Flux Mapping} uses the measured harmonics of the spin speed to fit a non-linear model for the three Euler angles defining the time history of the body-fixed orientation of the rotor with respect to the SQUID pick-up loop. Using the estimated rotor orientation, a linear least squares fit for the coefficients in a spherical harmonic expansion of the trapped magnetic potential is performed. With both the orientation and trapped magnetic potential, the variations in \( C_g \) can be predicted. The two methods for determining the gyroscope readout scale factor agree to \( \sim 1 \) part in 10\(^3\) of the total scale factor, boosting confidence in the overall estimation of \( C_g \).

4.3. \textit{Classical Misalignment Torques}

A general model that assumes arbitrary patch potential distributions on both the surface of the rotor and the housing predicts the general behavior of the observed classical torques acting on the gyroscopes. The patch effect is usually described in terms of a surface dipole layer [39] above the underlying conducting surface, which produces an electrostatic potential frozen on the surface, independent of external applied fields. The electric field is not necessarily perpendicular to the surface, and it is possible to produce torques on a perfectly spherical rotor. Similar effects have been discussed for planar surfaces in Ref. [40]. For small angular misalignments between the spacecraft roll axis and the gyroscope spin axis, the analytical model predicts that the magnitude of the disturbance drift rate is proportional to the misalignment and its direction is perpendicular to the misalignment. Even though this model clearly explains the observed effects, some of the details of the interaction between the potentials on the housing and the rotor are not known.

The analytical model also explains an additional observation. When a harmonic of the rotor’s polhode frequency coincides with the satellite roll frequency, offsets in the gyroscope spin axis as large as 0.1 arcs are observed over intervals of \( \sim 1 \) day. As the harmonic of the slowly changing polhode frequency drifts through the roll frequency, there is a net torque which has a nonzero average value in the inertial frame.

4.4. \textit{“Algebraic” and “Geometric” Data Processing}

The primary focus of the data analysis is to separate a uniform drift rate (relativity) from the drift rate induced by classical misalignment torques. We associate a uniform drift rate in the gyroscopes with general relativity, but in fact, a uniform drift rate could be caused by other theories that may agree or disagree with GR. The GP-B data analysis methods must produce the best estimate of the uniform drift rate in the presence of the classical misalignment torque with no prior assumptions regarding the uniform drift rate magnitude or direction. This is done by taking advantage of the observed magnitude and orientation of the classical torque-induced drift rate relative to the magnitude and orientation of the misalignment between the gyroscope spin axis and the spacecraft roll axis. At any given time, the gyroscope drift rate may be divided into two components: one component in a direction parallel to the misalignment between the gyroscope spin axis and the spacecraft roll axis (the radial component) and another component perpendicular (the azimuthal component). The azimuthal component of the drift
Figure 4. (a) Separation of the uniform precession and classical torque induced drift. (b) Geometric Method of data processing: radial drift rate versus misalignment phase and best fit. Rate has contributions from the misalignment torque and the uniform drift rate, but the radial component is entirely due to the uniform drift rate, as depicted in Figure 4 (a). As the direction of the misalignment changes, primarily due to the annual aberration, a uniform drift rate will vary sinusoidally as a function of the misalignment phase (the direction of the misalignment in an inertial reference frame). The amplitude of the sine wave is equal to the magnitude of the uniform drift rate, while the phase of the sinusoidal variation determines the direction of the uniform rate. A plot of the radial component of the drift rate versus the misalignment phase is shown in Figure 4 (b). Data is not used at those times when a harmonic of the polhode frequency is close to the roll frequency. This method of processing the gyroscope orientation data to estimate the uniform drift rate is the simplest form of a Geometric Method of data processing.

Although the simple Geometric Method of analyzing flight data shows that the uniform drift rate may be clearly separated from the effects of the misalignment torque, other methods of analyzing the data will very likely be more precise. This form of the Geometric Method requires that the data are divided into segments several days long to determine the drift rate for each segment. Dividing the data into short segments decreases the potential accuracy of the drift rate estimates from \( \sqrt{2/L} t^{-3/2} \) where \( t \) is the total measurement time (\( \sim 1 \) year) to \( \sqrt{2/L} T^{-1} t^{-1/2} \) where \( T \) is the batch length (\( \sim 5 \) days).

An alternate approach to the data analysis is the Algebraic Method, which simultaneously estimates the uniform and torque-induced drifts, in order to overcome the limitation due to the short data segments. Instead of using the geometry to eliminate the misalignment torque from the analysis, the Algebraic Method models the misalignment torque based upon its known behavior. Another important difference between the two approaches is that the Algebraic Method is an orientation-based approach whereas the Geometric Method is a rate-based approach. The Algebraic Method uses the following relations to model the uniform drift rate.
rate and misalignment torque.

\[ \vec{R} = \frac{d\vec{S}}{dt} = \vec{\tau} + \vec{R}_{\text{torque}} \]  

(3)

Here, \( \vec{R} \) is the total drift rate (derivative of the orientation \( \vec{S} \)), \( \vec{\tau} \) is the uniform drift rate (relativity) and \( \vec{R}_{\text{torque}} \) is the drift rate caused by the misalignment torque, which takes the form,

\[ \vec{R}_{\text{torque}} = (\hat{\tau} \times \hat{S}) k(t) \]  

(4)

Equation (4) states that the misalignment torque is proportional and perpendicular to the angle between the spacecraft roll axis \( \hat{\tau} \) and the gyroscope spin axis \( \hat{S} \). The constant of proportionality \( k(t) \) is allowed to vary with time. Assuming \( k(t - t_0) \ll 1 \), the solution to (3) is,

\[
S_{NS}(t) = S_{NS}(t_0) + \left[ r_{NS} - k S_{WE}(t_0) \right] (t - t_0) + k \int_{t_0}^{t} \tau_{WE}(t') \, dt' \\
S_{WE}(t) = S_{WE}(t_0) + \left[ r_{WE} + k S_{NS}(t_0) \right] (t - t_0) - k \int_{t_0}^{t} \tau_{NS}(t') \, dt' 
\]  

(5)

Equation (5) forms the backbone of the Algebraic Method. The gyroscope orientation time histories \( S_{NS}(t) \) and \( S_{WE}(t) \) are computed from the combined gyroscope and telescope readout using (2). The time history of the orientation of the spacecraft roll axis \( \tau(t) \) can also be computed from the telescope and gyroscope signals. The Algebraic Method estimates the uniform drift rate \( \vec{\tau} \) along with the time varying torque coefficient \( k(t) \) using either a piece-wise constant or a Fourier harmonic expansion for \( k \) over short segments of data. When a harmonic of the polhode frequency is close to the roll frequency, a linear ramp is used to model the gyroscope orientation with parameters accounting for the amplitude and direction of the ramp estimated for each of these events.

5. Preliminary Results

As of July 2007 the Geometric Method gives a uniform drift rate of \(-6638 \pm 97 \) marcs/yr in the plane of the orbit (North-South direction). This result may be compared with the geodetic drift rate predicted by General Relativity of \(6571 \pm 1 \) marcs/yr. The GR prediction includes the relativistic geodetic effect of the sun and guide star proper motion. The experiment error is dominated by systematic errors as indicated by the disagreement between the measured drift rates of the four gyroscopes. At this time, the Geometric Method estimate for the drift rate in the West-East direction is consistent with frame-dragging predicted by GR, but additional work is needed before a clear comparison between the measurements and the predicted effect may be made.

Preliminary uniform drift rate estimates and statistical errors for representative Algebraic Method runs in December 2007 are shown in Figure 5. Estimates from two data analysis runs are shown. The first (blue) uses data from all four gyroscopes over a period of 45 days spanning 1 January 2005 to 15 February 2005, and the second (magenta) uses data from gyroscopes 1, 3 and 4 over a period of 85 days spanning 12 December 2004 to 4 March 2005. The solid error ellipses are derived from the measured SQUID and telescope noises, while the dashed error ellipses are scaled based on the magnitude of the post-fit residuals which account for some, but not all, of the unmodeled systematic errors. The results from the 85-day analysis is \(-6632 \pm 43 \) marcs/yr in the North-South direction and \(-82 \pm 13 \) marcs/yr in the West-East direction using the SQUID and telescope noises. These estimates are consistent with the GR prediction of \(-6571 \pm 1 \) marcs/yr and \(-75 \pm 1 \) marcs/yr in the North-South and West-East directions respectively.

Although these results are quite promising, especially in light of the reduction in the statistical error when more data is included, there is evidence that some systematics effects that are not yet
accounted for or completely understood may be several times larger than the statistical errors shown in Figure 5. Evidence for these systematic errors come from variations in the uniform drift rate estimated when different combinations of gyroscopes and data segments are used. The main focus of the data analysis team in the coming months is to understand and model these systematic effects in order to produce a robust and consistent estimate of the uniform drift rate for the four gyroscopes over the course of the mission. Once this work is completed, the final results from Gravity Probe B can be properly compared with the predictions of general relativity.

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