PRODUCTION OF MILISECOND DIPS IN SCO X-1 COUNT RATES BY DEAD TIME EFFECTS

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ABSTRACT

Chang and coworkers reported millisecond duration dips in the X-ray intensity of Sco X-1 and attributed them to occultations of the source by small trans-Neptunian objects (TNOs). We have found multiple lines of evidence that these dips are not astronomical in origin, but rather the result of high-energy charged particle events in the RXTE PCA detectors. Our analysis of the RXTE data indicates that at most 10% of the observed dips in Sco X-1 could be due to occultations by TNOs, and, furthermore, we find no positive or supporting evidence for any of them being due to TNOs. We therefore believe that it is a mistake to conclude that any TNOs have been detected via occultation of Sco X-1.

Subject headings: Kuiper Belt — solar system: general — X-rays: general — X-rays: individual (Sco X-1)

1 INTRODUCTION

Chang et al. (2006) found statistically significant 1–2 ms duration dips in the count rate during X-ray observations of the bright X-ray source Sco X-1 carried out with the Proportional Counter Array (PCA) on the Rossi X-Ray Timing Explorer (RXTE) and attributed them to occultations of the source by small objects orbiting the Sun beyond the orbit of Neptune, i.e., trans-Neptunian objects (TNOs). In all, Chang et al. (2006) found some 58 dips in approximately 322 ks of Sco X-1 observations. Given that the RXTE spacecraft moves through the diffraction-widened shadows of any TNOs at a velocity of ~30 km s$^{-1}$, dips of ~2 ms duration should correspond to a TNO size of ~$60$ m. If the identification of these dips with occultations by TNOs is correct, the dips would provide extremely valuable information on the number and distribution of solar system objects of ~$20$–100 m in size. We have found evidence that these dips are produced by electronic dead time as a result of high-energy charged particle events in the RXTE PCA detectors. Preliminary reports of our results were given by Jones et al. (2006, 2007); herein we give a more detailed and complete report.

2. AVERAGE PROPERTIES OF DIPS

IN THE SCO X-1 COUNT RATES

Subsequent to the report by Chang et al. (2006) we searched for dips of the type they describe in the archival data obtained in ~880 ks of RXTE PCA observations of Sco X-1. These observations were performed starting in early 1996 soon after the launch of the spacecraft. For the search we only used observations that provided count rates of events with a time resolution of ~0.25 ms. The total count rate depends on the strength of Sco X-1 at the time of observation as well as on the number of Proportional Counter Units (PCUs) that were operating and the location of the source within the field of view; it ranges from below 40,000 counts s$^{-1}$ to nearly 200,000 counts s$^{-1}$ (see Fig. 1).

Coincidences within a ~10 $\mu$s window among two or more of the measurement chains in a Proportional Counter Unit (PCU) are normally used to identify charged particle events (see Jahoda et al. 2006 and references therein for technical information on the PCA). However, the intensity of Sco X-1 is so high that there is a substantial count rate due to the detection of two X-ray photons in two distinct regions of the detector serviced by different measurement chains within the 10 $\mu$s window. For most of the Sco X-1 observations, the rates of such so-called two-level discriminator ("2-LLD") events were telemetered with submillisecond time resolution. In those cases where the 2-LLD event data are available, we add 2 counts for each 2-LLD event to the counts of single LLD events.

In our search, we looked for instances when the number of counts in a time "window" that is an integral multiple of 0.25 ms (2$^{-12}$ s to be precise) was less than that reasonably expected to occur given the mean count rate. The search was performed for each of seven window intervals, i.e., 0.75, 1, 1.5, 2, 2.5, 3, and 4 ms. The expected number of counts in each window was determined from the running average count rate in a time interval centered on the time window of 128 ms for the 0.75 and 1 ms windows, 192 ms for the 1.5 ms window, or 256 ms for the 2, 2.5, 3, and 4 ms windows.

Given N, the number of counts actually measured in the window, and $N_{\text{exp}}$, the expected number of counts in that window based on the running average, we computed $P(n \leq N|N_{\text{exp}})$, the probability that $N$ or fewer counts would be detected based on the simplistic assumption that the counts obey a Gaussian distribution with mean $N_{\text{exp}}$ and standard deviation $(N_{\text{exp}})^{1/2}$. We define the detection of a dip as any instance in which we found $P(n \leq N|N_{\text{exp}}) < 10^{-10}$. In the search, the same dip could be found multiple times, e.g., in contiguous intervals when it was longer than the window interval or for different window intervals. We generated a list of 203 dips in which these duplications had been eliminated. Of these, 198 occur in data in which 2-LLD events had been taken into account. All but three of the 58 dips of Chang et al. (2006) and all but 14 of the 107 dips identified by Chang et al. (2007) were identified in our search. Eight of the latter 14 dips were not found by us because we did not search five orbits of data that Chang et al. (2007) searched, and because six of the 14 did not meet our significance threshold (perhaps because we included 2-LLD events). We detected more dips than Chang et al. (2007) because we included 2-LLD events, because our detection threshold was slightly less strict in terms of required significance, and possibly because of other minor differences in the searches.

The frequency of occurrence of dips was found to be a function of count rate, and, as one should expect, tended to be higher...
at the higher count rates. Given our detection criteria, we found no dips at count rates below 43,000 counts s\(^{-1}\) and only three at count rates below 55,000 counts s\(^{-1}\). We searched for dips in data which included approximately 570 ks of observations in which the count rates were above 55,000 counts s\(^{-1}\) (Fig. 1). For each of the seven window durations, we computed \(N\) and \(N_{\text{exp}}\) for window intervals that started every 0.25 ms. Thus the number of independent trials must be less than \(7 \times 570,000/0.00025 \approx 1.6 \times 10^{10}\). If the actual probability of finding a dip due to a statistical fluctuation in each trial is \(10^{-10}\), then at most a few of the 203 dips could be the result of statistical fluctuations.

In every case, the actual probability of getting \(N\) or fewer counts differs from our computed value for at least two reasons. First, the Gaussian distribution we utilized overestimates the probabilities of small numbers of counts by large factors when compared with a Poisson distribution of the same mean. Second, the intensity of Sco X-1 is time variable, and there is some chance that the mean intensity at the time of a dip was lower than that corresponding to \(N_{\text{exp}}\). If source variability were important, our detection procedure would have led to numerous spurious detections, particularly for the longer window durations. However, only four diplike events were most significantly detected when using either the 3 or 4 ms window durations. We further checked the effects of source variability by, first, integrating properly normalized power density spectra of Sco X-1 count rate data over the frequency ranges of 5–300, 4–250, 4–200, 4–170, and 4–125 Hz, which are relevant to the 1.5, 2, 2.5, 3, and 4 ms window-duration searches, respectively. The resulting rms fractional amplitudes in the given frequency bands are typically ~2.5% and are almost always less than ~4.5%. Then, for each of the five window-duration searches, the relevant variability estimate was used together with the average count rates on an \((RXTE)\) orbit-by-orbit basis to estimate the effects on the dip detection probabilities. These estimated effects are consistent with the paucity of dips detected most significantly using the 3 and 4 ms window durations and indicate that statistical fluctuations in the presence of source variability did not produce more than ~5 spurious detections.

In order to obtain estimates of dip widths and depths, we used the IDL procedure \texttt{gaussfit} to fit each dip profile with a function \(f(t)\) representing a constant count rate plus a Gaussian shaped dip:

\[
f(t) = A - Be^{-(t-t_0)^2/2\sigma^2},
\]

where \(A\), \(B\), \(t_0\), and \(\sigma\) were the parameters to be determined. Examples of dip profiles and the fitted functions are shown in Figure 2. The values of the fitted widths as parameterized by the values of \(\sigma\), and of the fitted minimum normalized count rates, i.e., the values of \(1 - B/A\), of the ~200 dips are shown in Figure 3. The key features of this figure are the relatively narrow range of widths and absence of long dips. Nearly all of the events have \(0.4 \leq \sigma \leq 0.8\) ms, and there is only one dip with \(\sigma > 1.1\) ms. These aspects of the plot are discussed below (see § 4).

If the counting rate dips are the result of occultations, then we would expect that diffraction effects would produce small count rate increases on either side of the dips. The sizes of these sidelobes and indeed the other details of the dip profiles depend on a number of factors including the sizes, shapes, distances, and velocities of the occulting bodies, the impact parameters characterizing the occultation events, and the velocities of the \textit{RXTE} spacecraft at the times the dips were observed.

To quantify our expectations of the average diffraction sidelo size and other typical properties of dips produced by occultations by TNOs, we performed a Monte Carlo computer simulation of an ensemble of occultation events in which the occulting bodies were assumed to be opaque spheres, to have radii \(s\) that follow a distribution \(dn/ds \sim s^{-4}\), and to follow prograde circular orbits at a distance of 40 AU from the Sun. Since the effects of diffraction are wavelength dependent, we took the spectrum of detected X-rays to be that of a typical pulse height spectrum of Sco X-1 measured with the PCA. The simulation was carried out for each of four cases in which the relative velocity between the spacecraft and the shadows of
the occulting bodies was a fixed value, viz., 15, 25, 30, or 35 km s$^{-1}$.

Simulated dip profiles were computed for a large number of occultations with object size and occultation impact parameter chosen at random according to the appropriate distributions. The profiles were normalized to unity at times far from the occultation centers and were then inserted into the real PCA count rate data by multiplying the actual count rates by the dip profiles. The dip center times were chosen at random among the 880 ks of observations that were used for the present analysis. These data were then searched for dips using the search algorithm described above.

We fit the detected model dips with the function given by equation (1). The light curves of the PCA data containing the detected simulated dips were superposed after aligning the dip centroids and rescaling the timescale of each light curve in order to normalize all of the fitted widths to the value $\sigma = 0.85$ ms. Interpolation of the rescaled bin times to 0.25 ms time bins was required to superpose the rescaled light curves, and, as a consequence, adjacent bins in the superposition are not completely statistically independent. The results are shown in Figure 4.

The average profile of the real detected dips was similarly constructed by superposing the PCA light curves of 202 of the 203 detected dips after alignment and stretching; the one dip with width $\sigma \sim 2.3$ ms was excluded. The results are shown in Figure 5. To check this, we also superposed only those 109 dips with fitted widths in a narrow range, i.e., with $0.55 \text{ ms} < \sigma < 0.75 \text{ ms}$. The light curves containing those 109 were aligned and summed, but no stretching was done. The results are shown in Figure 6. A superposition of the profiles of (only) the 2-LLD events that occurred around the times of 198 dips looked similar to the profiles shown in Figures 5 and 6.

Per the results shown in Figure 4, we expected to see side-lobes with intensities as high as $\sim 8\%$ above the mean count rate determined substantially away from the superposed dips. The superposed light curves do not exhibit diffraction side-lobes as high as those evident in the average profile of the simulated dips, despite having statistics sufficient to reduce fluctuations to $\sim 1.6\% (1\sigma)$ of the mean count rate. While the differences are not sufficiently significant to be conclusive, they strongly suggest there may be a problem with the occultation hypothesis. We elaborate on this in the discussion section below.

Three additional potential problems with the occultation interpretation are manifest from the dip profiles. First, the summed dip profile is distinctly asymmetric in shape as Chang et al. (2006) suggested for many of the individual dips. Second, the distribution of dip widths is narrower than what one would expect from occultations by bodies with a power-law size distribution of.
index ~4, i.e., there are fewer than expected statistically significant dips with Gaussian FWHM widths greater than ~1 ms. Third, when diffraction effects are taken into consideration, one would expect to see a correlation between the fitted widths and the fitted minimum count rates, such that longer dips tend to be deeper on average. No such correlation is seen in Figure 3.

3. SEARCH FOR AN ALTERNATE EXPLANATION

These findings prompted us to further explore alternative explanations for the dips. Only one hypothesis appeared to be worthwhile to pursue, i.e., that the dips are caused by electronic dead time in response to some type of charged particle shower in the spacecraft. Unfortunately, no information with millisecond time resolution was available on the non-X-ray background during the Sco X-1 observations. Counts of good events, very large events (VLEs), propane-layer events, and a catch-all category of other types of events (hereafter SM1-other events) that includes events (VLEs), propane-layer events, and a catch-all category of other types of events (hereafter SM1-other events) that includes multiple LLD events are available at ~1 ms time resolution from the spacecraft. In order for the detector to be shut down for an extended period (>1 ms), an extraordinary amount of charge must be deposited on most of the six main measurement chains; it is unclear, at present, whether this can happen in response to a single charged particle.

Figure 7 shows counts of three different types of events from Standard Mode 1 in ~1 ms time bins superposed around the times of 201 dips. In each panel, the centers of the dips have been placed in the bin at time = 0 s. The top panel shows the rates of good events, i.e., those identified as being due to charged particles, in the main xenon layers of all operating PCUs, and clearly shows the superposed dips; two-LLD events are not included in these rates. The counting rate drops by only ~0.7% because of the dilution of a ~2 ms dip within a 128 ms bin. The counting rate in propane-layer-only events (not shown) drops by a similar fraction. In contrast, the middle panel shows the enhancement of the counting rate of SM1-other events in the vicinity of the dips. The peak is highly significant (~38 σ). The bottom panel corresponds to the VLE event rate superposed around the dip times. This peak is also statistically very significant (~7 σ). The increase in the VLE rate is nearly so large as to be consistent with the detection of ~1 VLE per PCU per dip. We discuss this further below.

To make sure that, e.g., the enhancement in the VLE rate is not dominated by a large number of events associated with a small number of dips, we repeated the superposition of the Standard Mode 1 data but did not include those profiles in which the excess number of SM1-other events was more than 6 σ (see the following section). The resulting plot was similar to that shown in Figure 7.

The enhancements in the SM1-other event and VLE rates around the times of the dips indicate that there is an increase in the rate of detection of non-X-ray events. We speculate that these non–X-ray events interrupt normal event processing for 1-2 ms
in most of the PCUs roughly once per hour due to the collection of very large amounts of charge. Such an energetic event may be the consequence of a particle shower produced by the collision of a high-energy cosmic ray with a nucleus in the RXTE spacecraft. In any case, further clarification of the causes of the observed dips would be of interest.

4. DISCUSSION

The observed dips have widths and depths that are approximately what one might expect to be produced by occultations by TNOs, even though much wider dips would be detectable in principle (given appropriate depths). Thus we are obligated to seriously consider the hypothesis that some or all of the observed dips are the product of TNO occultations. However, close examination of the RXTE PCA data reveals six signatures that independently indicate that few and possibly none of the observed dips are due to occultations by TNOs. The signatures are (1) the numbers of SM1-other events during the dips; (2) the numbers of VLE events during the dips; (3) the absence of the expected diffraction sidelobes; (4) the temporal asymmetry of the dips; (5) the almost total lack of dips longer than \( \sim 1 \) ms; and (6) the lack of correlation between dip duration and depth. We discuss each of these in turn.

1. SM1-other events.—Figure 7 shows that there is, on average for 201 dips, a large excess of SM1-other events at or near the times of the dips. On average, individual dips should show an excess of SM1-other events at the 2.7 \( \sigma \) level. In Figure 8 we show a histogram of the number of SM1-other events in the \( \frac{1}{8} \) s time bins corresponding to the dips, expressed in standard deviations above the mean. The mean value obtained from the histogram is \( \sim 2.7 \) \( \sigma \), as expected.

If one makes the reasonable assumption that the numbers of SM1-other events should not be affected by true occultations (other than by negligible increases due to reductions in the electronic dead time), then one may estimate the maximum fraction, \( f_{occ} \), of the observed dips that represent genuine occultation events that is consistent with this distribution of SM1-other events. We constructed the following simple function with which to fit the histogram, and thereby constrain \( f_{occ} \):

\[
\text{probability} = \frac{1}{\sqrt{2\pi}c} e^{-c^2/2} + \frac{1}{\sqrt{2\pi}c_{\sigma}^2} (1 - f_{occ}) e^{-(c - c_{\sigma})^2/2c_{\sigma}^2},
\]

(2)

where \( C \) is the number of excess SM1-other counts (in units of standard deviations of the counts per bin in each PCA light curve), and \( C_{\sigma} \) is the mean of \( C \) for those dips which are not the results of occultation events and which we take to be \( \approx 2.7/(1 - f_{occ}) \). The distribution of the numbers of excess SM1-other counts is wider than what would be expected from a Poisson distribution with a mean equal to the slightly increased (on average) number of SM1-other events per bin; the width of this component of the fitting function is adjusted by means of the parameter \( \sigma_{cr} \). Fits of the function to the histogram in Figure 8 were carried out with \( \chi^2 \) fits using both Gaussian and Cash (1979) statistics. If we neglect the tail of the distribution at high numbers of excess SM1-other events, i.e., at >9 \( \sigma \), we obtain formally acceptable fits with values of \( f_{occ} \) in the range 0.0–0.12 and values of \( \sigma_{cr} \) in the range 1.85–2.35 (based on Gaussian statistics; the limits represent the formal joint 95\% confidence range). Using Cash statistics, we obtain formally acceptable fits with values of \( f_{occ} \) in the range 0.0–0.11 and values of \( \sigma_{cr} \) in the range 1.65–2.15 (95\% confidence). These results indicate that fewer than 11\% of the 203 dips might be the product of TNO occultations.

As an independent check to see if the dips with low numbers of excess SM1-other counts might be unusual, we superposed (1) the profiles of the 13 dips for which the excess number of SM1-other events is less than zero, and (2) the profiles of the 38 dips for which the excess number of SM1-other events is less than 1 standard deviation. The superposed profiles of these sets of dips are not markedly different from the superposed profiles shown in Figures 5 and 6.

2. VLE events.—Figure 7 also shows that there is an excess of VLE events around the times of the dips. The difference between the background rate and that in the \( \frac{1}{8} \) s bin containing the dips is very close to 4 (actually 3.9 \pm 0.5) extra VLE events per dip. The peak in Figure 7 is significant at the 7 \( \sigma \) level. If there is precisely one VLE per operating PCU for each non-TNO dip, then we would expect on average an excess of 4.67 VLEs per non-TNO dip. If only the non-TNO dips contribute to the excess VLE events then there is an upper limit to \( f_{occ} \) that is consistent with the observations. If we further allow that the statistical mean excess number of VLE events per dip may have been as small as 2.9, then a simple calculation gives the limit \( f_{occ} \leq 0.38 \) (95\% confidence). This limit is weaker than for the SM1-other events, and, furthermore, is compromised by the possibility that more than one VLE event could be produced in a operating PCU in a single cosmic-ray induced dip.

3. Lack of diffraction sidelobes.—In Figure 4 we showed an average profile of simulated dips that had been inserted into actual PCA data. It should be compared to averages of the actual measured dip profiles in Figures 5 and 6. The average model dip profile shows a clear bump of \( \sim 8\% \) amplitude on either side of the dip due to diffraction, whereas the averages of the actual profiles show no significant evidence for diffraction sidelobes. Thus, we conclude that the fraction \( f_{occ} \) of legitimate TNO occultations...
can be no larger than ~30%, otherwise diffraction sidelobes likely would have been detected. Again, while this is a clear strike against the dips being due to TNOs, the limiting statistically significant constraint that can be set due to the lack of diffraction sidelobes is not as significant as for the SM1-other events.

4. Asymmetry.—A comparison of the simulated with the actual dip profiles (as in item 3) above clearly shows a marked asymmetry for the real dip events. This is physically implausible if the dips are the product of occultation events and therefore testifies against a TNO origin for most of the dip events. We estimate that the statistical significance of the asymmetry is ~6 σ. Unfortunately, there is no direct way to use this information to constrain the fraction of legitimate TNO occultations. The problem is that we do not know, a priori, how large the asymmetry is, on average, for non-TNO dips. Therefore, we cannot tell how “diluted” the non-TNO events are by potentially real ones. Nonetheless, this marked asymmetry is another solid indication that few of the dips are the product of TNO occultations.

5. Lack of dips longer than ~1 ms.—From Figure 3 we can see that all of the dips, except for a single event, have rms widths σ < 1.1 ms. In § 2 we described a computer simulation of the production, detection, and analysis of dips caused by TNO occultations. For a relative speed between the RXTE satellite and the shadows of the putative TNOs we find that the fraction of recovered simulated dips with σ > 1.1 ms is ~27%. For vrel = 35 km s⁻¹, 9% of the dips have σ > 1.1 ms. We estimate that the average relative velocity between RXTE and the shadows of any TNOs was not higher than vrel ~ 30 km s⁻¹. For this speed, 16% of the dips are characterized by σ > 1.1 ms. Therefore, if all of the dips are the result of TNO occultations the number of longer duration dips should be ~30, whereas the observed number is actually 1. On the other hand, if only 15% of the dips are due to TNO occultations, we would expect only ~5 dips with σ > 1.1 ms. This expected number is marginally statistically consistent, i.e., at ~5% confidence, with the detection of one dip with σ > 1.1 ms. Therefore, we conclude that the lack of longer dips allows an upper limit of 15% to be set on the fraction, focc, of potentially real TNO occultations.

6. Lack of correlation between width and depth.—If the dips were due to TNO occultations of Sco X-1, we would expect a strong correlation between the widths of the dips and their depths. This results from the fact that diffraction produces shallow occultations for the smaller size occultors, while it produces deeper more geometric-shadowing-like occultations for the larger occultors. As can be seen from the distribution of dip widths versus depths in Figure 3, there is no such correlation, with almost all of the dips confined to a narrow range of widths (between 0.4 and 0.8 ms) and depths that range all the way from 45% to nearly 100%. Thus, the fact that the dips we detect include a significant number, i.e., ~20%, that are both narrow (σ < 0.7 ms) and deep (minimum normalized count rate below 0.2) whereas only ~2% of the ‘detected’ simulated dips (for vrel ~ 30 km s⁻¹) are this narrow and deep, indicates that ~10% of the dips might be due to TNO occultations. Given the effects of statistical fluctuations the observed number of narrow deep dips and the fact that the simulation is based on somewhat uncertain parameters, it is more reasonable to use these numbers to set an upper limit of ~20%.

Summarizing the results from approaches (1) through (6) above, we find limits on the fraction of valid TNOs to be focc < 11%, <38%, <30%, <Q%, <15%, <20%, respectively, where “Q” denotes that a formal limit could not be set, but the approach provides an important independent indication that the dips are, for the most part, not the result of TNO occultations.

We believe that the combined upper limit on focc due to the joint application of all six approaches is simply the minimum value achieved by the most sensitive of these, i.e., the constraints cannot be combined. The reason, in short, is that the effects we explore serve only to statistically limit the number of events which could be due to TNOs rather than to identify specific qualifying events. Therefore, our final limit is simply focc ≤ 10%.

One might argue, as did Chang et al. (2007), that since ~10% of the observed dips cannot be formally eliminated as due to TNOs, they serve as viable potential candidates for TNO detections. However, we argue that if 90% of the dips can be securely eliminated as TNO occultations, and there are six different and independent indicators that point in the direction of a common cause due to cosmic ray interactions in the detector, then it is most plausible that all of the dips have this common origin.

While our results cast serious doubt on whether any true occultation events have been detected, one cannot yet conclude with a high degree of confidence that no such events have been detected. Further investigations of the dip phenomenon and its possible causes would be of interest. We are working to obtain a new measurement of, or upper limit on, the rate of occurrence of occultations of Sco X-1 by analyzing the data that are being obtained in a new series of RXTE observations of Sco X-1 with high-time-resolution information on VLE events.

5. IMPLICATIONS FOR THE POPULATION OF TNOs

Given that we detected 203 dips in data that covered 570 ks of observations with count rates ≥ 55,000 counts s⁻¹, our upper limit on focc corresponds to an upper limit of ≤ 20 occultations in the 570 ks of observations. If we adopt a model of the TNO characteristics, this upper limit of detected TNO occultations can be used to establish an upper limit on the abundance of small TNOs. For this purpose we use the assumptions and results of our model TNO simulations described above in § 2.

A TNO of radius s appears to sweep out a solid angle per unit time

$$\frac{d\Omega}{dt} = \frac{2(s + \delta)v_{rel}}{D^2},$$

where s + δ is the maximum impact parameter for which this body may produce a detectable dip, vrel is the apparent transverse velocity of the TNO, and D is the distance from Earth to the TNO. We find from our simulations of occultations that, on account of diffraction effects, δ ~ 7 m for the model TNOs in the relevant size range. We use vrel ~ 30 km s⁻¹ and D ~ 40 AU. The solid angle swept out in time Δt is

$$\Omega_{sw} = \frac{d\Omega}{dt} \Delta t = \frac{2(s + \delta)v_{rel}}{D^2} \Delta t.$$ (4)

For an ensemble of TNOs of various sizes, the average solid angle swept out per TNO of radius s > s_{min} is then

$$\overline{\Omega}_{sw} = \frac{1}{N(s > s_{min})} \int_{s_{min}}^{\infty} \Omega_{sw}(s) \frac{dN}{ds} ds = \frac{2v_{rel} \Delta t}{D^2} (3s_{min}/2 + \delta),$$ (5)

where we have assumed that the differential size distribution of TNOs is given by dN/ds ∝ s⁻¹ for s ≥ s_{min}. In our simulations we find s_{min} ~ 15 m. Using this and the above values, one obtains

$$\overline{\Omega}_{sw} \sim 2.8 \times 10^{-14} \text{ sr} \sim 9 \times 10^{-11} \text{ deg}^2.$$ (6)
Given that as many as 20 TNOs may have been detected, the upper limit on the areal density of TNOs is then

$$N(s \gtrsim 15 \text{ m}) \leq \frac{20}{9 \times 10^{-11} \text{ deg}^{-2}} \sim 2 \times 10^{11} \text{ deg}^{-2}. \quad (7)$$

This upper limit is compared to previous measurements of the size distribution of TNOs at larger radii in Figure 9, which summarizes the results of surveys of TNOs smaller than 1000 km.

We have repeated the above analysis for plausibly smaller and larger values of $v_{\text{rel}}$, namely, 25 and 35 km s$^{-1}$. The value of $v_{\text{rel}}$ affects $s_{\text{min}}$ and $\Omega^s_{\text{sw}}$. These different values of $v_{\text{rel}}$ imply TNO population upper limits, at the fixed radius of 15 m, that differ from the limit given above by about ±20%. These differences are relatively small compared to the likely factor of ~2 uncertainty in the upper limit.

The smallest TNOs which have been securely detected were found in the *Hubble Space Telescope* ACS survey reported by Bernstein et al. (2004) in which 3 TNOs of radius $s \approx 10$ km were found in 0.019 deg$^2$ of sky. We also show the population estimates from ground-based surveys that are summarized in Table 2 of Bernstein et al. (2004).

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