Possibility of parametrization of atmospheric muon angular flux using underwater data

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Abstract. We present the formula for angular distribution of integral flux of conventional ($\pi, K$) muons deep under water taking into account the sphericity of the atmosphere and fluctuations of muon energy losses. The accuracy of this formula for various sea level muon spectra is discussed. The possibility of reconstructing two parameters of sea level spectrum by fitting measured underwater angular intensity is shown for Baikal Neutrino Telescope NT–36 experimental data.

1 Introduction

The knowledge of expected angular distribution of integral flux of atmospheric muons deep underwater is of interest not only for cosmic ray physics but also for the estimation of the possible background for neutrino detection and at last for a test of the correctness of underwater telescope data interpretation using the natural flux of atmospheric muons as calibration source. The last item frequently implies the estimation with an appropriate accuracy (e.g., better than 5% for a given sea level spectrum) of the underwater integral muon flux for various sets of depths, cutoff energies and angular bins especially for telescopes of big spacial dimensions.

Up to now the presentation of the results of calculations of muon propagation through thick layers of water both for parent muon sea level spectra (especially for angular dependence taking into account the sphericity of atmosphere) and for underwater angular flux has not been quite convenient when applied to concrete underwater arrays. In addition, a part of numerical results is available only in data tables (often insufficient for accurate interpolation) and figures. The possibility of direct implementation of Monte Carlo methods depends on the availability of corresponding codes and usually assumes rather long computations and accurate choice of the grid for simulation parameters to avoid big systematic errors. Therefore, there remains the necessity of analytical expressions for underwater muon integral flux. In addition, the possibility of reconstructing the parameters of a sea level spectrum by fitting measured underwater flux in the case of their direct relation looks rather attractive.

In this paper we present rather simple method allowing one to analytically calculate the angular distribution of integral muon flux deep under water for cutoff energies ($1–10^4$) GeV and slant depths of ($1–16$) km for conventional ($\pi, K$) sea level atmospheric muon spectra fitted by means of five parameters. The fluctuations of muon energy losses are taken into account.

The possibility of reconstructing two parameters of sea level spectrum by fitting measured underwater angular intensity is shown for Baikal Neutrino Telescope NT–36 experimental data.

2 Basic formulas

According to the approach of work (Klimushin et al., 2001) the analytical expression for calculations of underwater angular integral flux above cutoff energy $E_f$ for a slant depth $R = h/cos \theta$ seen at vertical depth $h$ at zenith angle $\theta$ and allowing for the fluctuations of energy loss is based on the relation

$$F_{\theta \theta}(\geq E_f, R, \theta) = \frac{F_{\theta \theta}(\geq E_f, R, \theta)}{C_f(\geq E_f, R, \theta)}$$

where correction factor $C_f$ is expressed, by definition, by the ratio of theoretical integral flux calculated in the continuous loss approximation to that calculated by exact Monte Carlo, and $F_{\theta \theta}(\geq E_f, R, \theta)$ is the angular flux based on continuous energy losses.

In principle, the correction factor $C_f$ can be calculated using known codes for muon propagation through matter. In this work we apply for this aim the MUM code described in work (Sokalski et al., 2001).

The values of correction factors calculated for the same slant depth $R$ at vertical direction and at zenith angle $\theta$ differ weakly. It is illustrated in Fig. 1, where one can see that
The dependencies of correction factor on $\theta$ for any geometrical shape of surface. Right hand side of (2) depends on $\theta$ because, generally, $R = R(\theta)$. So, in the particular case of a flat surface the angular dependence of the correction factor appears, in our approximation, only through the relation $R = h/\cos \theta$ (where $h$ is a vertical depth).

The accuracy of formula (2) for $E_f=1\text{--}100\text{GeV}$ is better than $\pm 2\%$ for slant depths $R$ as large as 22 km and is not worse than $\pm 3\%$ for $E_f=1\text{ TeV}$ up to $R=17\text{ km}$ and for $E_f=10\text{ TeV}$ up to $R=15\text{ km}$. Fig. 1 shows that for $E_f < 100\text{ GeV}$ the total energy loss may be treated as quasi-continuous (at level of $C_f > 0.9$) only for slant depths $R < 2.5\text{ km}$ but for $E_f=10\text{ TeV}$ the fluctuations should be taken into account at level of 15\% already for slant depth as small as $R=1\text{ km}$.

The dependence of correction factor $C_f$ on different sea-level vertical spectra is illustrated by Fig. 2. The correction factors calculated for $E_f=10\text{ GeV}$ using sea level spectrum (6) with spectral index $\gamma$ of 2.5 and 3.0 (instead of 2.72) differ more than on a factor of 2 starting from slant depth of $R=12\text{ km}$. Nevertheless, the values of $C_f$ calculated using sea level spectra having $\gamma=2.65\text{--}2.78$ are already within $\pm 5\%$ corridor. For $E_f=1\text{ TeV}$ this corridor is larger on 2\%. This fact results in the possibility to extrapolate the parametrization (2) based on sea level spectrum having $\gamma=2.72$ to other spectra at least up to slant depths of (12–13) km without introduction of additional spectral corrections.

The angular flux $F_{\alpha,\beta}(\geq E_f, R, \theta)$ based on effective linear continuous energy losses $\alpha + \beta E$ having 2 slopes, is calculated by the following rule:

$$F_{\alpha,\beta}(\geq E_f, R, \theta) = \begin{cases} F_{\alpha,\beta}(\geq E_f, R, \theta; \alpha_1, \beta_1) & \text{for } R \leq R_{12}, \\ F_{\alpha,\beta}(\geq E_{12}, (R - R_{12}), \theta; \alpha_2, \beta_2) & \text{for } R > R_{12}. \end{cases}$$  

Here $E_{12}$ is the energy in the point of slope change from $(\alpha_1, \beta_1)$ to $(\alpha_2, \beta_2)$ and $R_{12}$ is the muon path from the energy $E_{12}$ till $E_f$ which is given by

$$R_{12} = \frac{1}{\beta_1} \ln \left( \frac{\alpha_1 + E_{12} \beta_1}{\alpha_1 + E_f \beta_1} \right).$$

The formula for integral muon angular flux in the assumption of linear continuous energy losses is as follows:

$$F_{\alpha,\beta}(\geq E_f, R, \theta; \alpha, \beta) = \frac{e^{-\beta R \gamma}}{\gamma}.$$
The distributions for cut-off energy $E_f=10$ GeV are given. Solid curves correspond to numerical computations using sea level spectrum defined by Eq. (6) with varying spectral index $\gamma$. Open circles correspond to numerical computations using VZK sea level spectrum (Volkova et al., 1979), closed circles – Gaisser’s sea level spectrum (Gaisser, 1990), squares – MACRO (Ambrosio et al., 1995) sea level spectrum. All distributions are shown for the following parameters of a sea level spectrum gives the possibility of their direct best fit using the experimental underwater distribution.

Flux value in (4) is expressed in units of $(\text{cm}^{-2}\text{s}^{-1}\text{sr}^{-1})$ and all energies are in (GeV), slant depth $R$ in units of $(\text{g cm}^{-2})$, loss terms $\alpha$ and $\beta$ in units of $(10^{-3}\text{GeV cm}^2\text{g}^{-1})$ and $(10^{-6}\text{cm}^2\text{g}^{-1})$, correspondingly. For the description of effective linear continuous energy losses we use the following values of parameters when substituting in (3): $(\alpha_1=2.67, \beta_1=3.40)$ and $(\alpha_2=-6.5, \beta_2=3.66)$ with $E_{12}=35.3$ TeV.

To examine the angular behaviour of a flux given by the formula (1) by means of the comparison with numerical calculations we used the following parameters of the sea level muon spectrum:

$$
D_{0x} = 0.175, \quad D_{0\kappa} = 6.475 \times 10^{-3}, \quad E_{0x}^c(0^\circ) = 103 \text{ GeV}, \quad E_{0\kappa}^c(0^\circ) = 810 \text{ GeV}, \quad \gamma = 2.72.
$$

These values have been chosen according to splines computed in this work via the data tables kindly given us by authors of Ref. (Misaki et al., 1999). When checking the values of fit spectrum for $\cos \theta=(0.05–1.0)$ we realized that the standard description of effective cosine (with geometry of spherical atmosphere and with definite value of effective height of muon generation) is not enough and one should introduce an additional correction $S(\theta)$ leading to (10–20) % increase of effective cosine value for $\cos \theta < 0.1$. The reason of an appearing of this correction is that the concept of an effective generation height is approximate one. It fails at large zenith angles where the real geometrical size of the generation region becomes very large.

The resulting fit of angular sea level spectrum in units of $(\text{cm}^{-2}\text{s}^{-1}\text{sr}^{-1}\text{GeV}^{-1})$ is given by

$$
D(E_0, \theta) = 0.175 E_0^{-2.72} \times \left( \frac{1}{1 + \frac{E_0 \cos \theta^{**}}{103}} + \frac{0.037}{1 + \frac{E_0 \cos \theta^{**}}{810}} \right),
$$

with modified effective cosine expressed by

$$
\cos \theta^{**} = S(\theta) \cos \theta^*,
$$

where $\cos \theta^*$ is derived from spherical atmosphere geometry and is given by the polynomial fit:

$$
\cos \theta^* = \sum_{i=0}^{4} c_i \cos^i \theta,
$$

we use the following parametrization:

$$
D(E_0, \theta) = E_0^{-\gamma} \sum_{i=\pi, K} \frac{D_{0i}}{1 + E_0/E_0^c(i\theta)}, \quad (5)
$$

where $\gamma$ is a spectral index and $E_{0\pi, K}^c (\theta)$ have approximate sense of critical energies of pions and kaons for given zenith angle and $E_{0\pi, K}^c (0^\circ)$ are those for vertical direction. The corresponding angular distribution should be introduced using an analytical description of effective cosine $\cos \theta^*$ taking into account the sphericity of atmosphere. It should be noted that the description of underwater angular flux with the 5 parameters of a sea level spectrum gives the possibility of their direct best fit using the experimental underwater distribution.

$\times \sum_{i=\pi, K} D_{0i} E_0^{\pi, K}(\theta)(E_f + y_i)\gamma(1 - z_i)^{1-\gamma} S(z_i, \gamma)$, (4)

where subscript $i$ stands over both pion ($\pi$) and kaon ($K$) terms and

$$
y_i = \frac{\alpha}{\beta} (1 - e^{-\beta R}) + E_0^{\pi, K}(\theta) e^{-\beta R},$$

$$
z_i = \frac{E_0^{\pi, K}(\theta) e^{-\beta R}}{E_f + y_i}, \quad E_0^{\pi, K}(\theta) = \frac{E_0^{\pi, K}(0^\circ)}{\cos \theta^*},$$

$$
S(z, \gamma) = 1 + \sum_{n=1}^{\infty} n! z^n \left( \prod_{j=1}^{n} (\gamma + j) \right)^{-1} = 1 + \frac{z}{\gamma + 1} + \frac{2z^2}{(\gamma + 1)(\gamma + 2)} + \frac{6z^3}{(\gamma + 1)(\gamma + 2)(\gamma + 3)} + \ldots .
$$

When using expression (4) for slant depths $R > R_{12}$ one must substitute $R \to (R - R_{12})$ and $E_f \to E_{12}$ and use the values $(\alpha_2, \beta_2)$ for a loss description. For slant depths $R \leq R_{12}$ the use of (4) remains unchangeable and the loss values are expressed by $(\alpha_1, \beta_1)$. This algorithm may be extended to computations with any number of slopes of the energy losses.

The 5 parameters $(D_{0\pi}, D_{0\kappa}, E_{0\pi}^c (0^\circ), E_{0\kappa}^c (0^\circ), \gamma)$ are those of the differential sea level muon spectrum, for which

Fig. 2. Correction factor $C_f$ as a function of spectral index $\gamma$ of sea level spectrum for various depths in water for vertical direction. The distributions for cut-off energy $E_f=10$ GeV are given. Solid curves correspond to numerical computations using sea level spectrum defined by Eq. (6) with varying spectral index $\gamma$. Open circles correspond to numerical computations using VZK sea level spectrum (Volkova et al., 1979), closed circles – Gaisser’s sea level spectrum (Gaisser, 1990), squares – MACRO (Ambrosio et al., 1995) sea level spectrum. All distributions are shown for the following values of vertical depth in pure water: 1.15 km (a), 3 km (b), 5 km (c), 7 km (d), 9 km (e), 11 km (f), 13 km (g), 15 km (h), 17 km (i), and 21 km (j), from top to bottom.
The examination of (4) showed rather quick convergence of series \( S(z, \gamma) \) with increase of \( R \) and \( E_f \). Therefore, for the accuracy of \( F_{E_f} \) computation better than 0.1 % it is quite enough to take only four first terms of this series (up to \( z^4 \)) for all values \( R > 1 \) km and \( E_f \) in \((1–10^4) \) GeV. Even using the two terms leads to the accuracy of 1.3 % for \((R=1.15 \) km, \( E_f=1 \) GeV) and < 0.5 % for \((R > 2.5 \) km, \( E_f > 1 \) GeV).

Fig. 4 shows the comparison of underwater angular integral fluxes allowing for loss fluctuations at different basic depths \( h \) (of location of existing and planned telescopes) calculated both numerically using MUM code (Sokalski et al., 2001) for parent sea level spectrum and analytically (1) for the spectrum given by (6).

We realized that the error given by formula (1) for all mentioned sea level spectra is within the corridor of \( \pm(4–6) \) % for all cutoff energies \( E_f=(1–10^3) \) GeV and slant depths \( R=(1–16) \) km (corresponding angle is expressed by \( \cos \theta = h/R \) for a given vertical depth \( h \)). This is proved for \( h \) in a range \((1–3) \) km. For bigger cutoffs of \( E_f=(1–10) \) TeV the corridor of errors is \( \pm(5–7) \) % for \( R=(1–13) \) km. Note that for the sea level spectrum (6), just used for \( C_f \) parametrization, the errors are smaller than 2 %.

The accuracy of the parametrization, used for the correction factor as a function of \( E_f \) and slant depth \( R \) is rather high and is about \( \pm 5 \) % for all angles and kinds of the sea level spectrum (assuming that the spectral index \( \gamma \) is approximately within \((2.65–2.78) \)) (Fig. 5). It results in the possibility to use it for an estimating numerically from various sea level spectra the value of an angular integral flux allowing for fluctuations of losses without direct Monte Carlo simulations.

Note that the expression (1) may be directly used for an ice after the substitution \( R \rightarrow R/\rho \), with \( \rho \) being the ice density, and, with an additional error of \( \sim 2 \) %, for sea water. In spite
Table 2. Coefficients \(c_i\) of the fitting formula (8) for effective cosine with the maximum relative errors.

| \(\cos \theta\) | \(c_0\) | \(c_3\) | \(c_2\) | \(c_3\) | \(c_4\) | Max. err. [%] |
|-----------------|--------|--------|--------|--------|--------|----------------|
| 0±0.002         | 0.11137| 0      | 0      | 0      | 0      | 0.004          |
| 0.002±0.2       | 0.11148| -0.03427| 5.2053 | -14.197| 16.138 | 0.3            |
| 0.2±0.8         | 0.06714| 0.71578| 0.42377| -0.19634| -0.021145| 0.7          |

The validity of this analytical expression with an accuracy of \(\pm(5–7)\)% for \(E_f=(10^3–10^4)\) GeV and slant depths of \((1–12)\) km gives also the possibility of estimation the angular underwater differential spectrum (by means of numerical differentiation) with error smaller than \(\pm(6–8)\)% for energies of \((30–5\times10^3)\) GeV.

![Fig. 5. The accuracy of formula (1) as function of cutoff energy \(E_f\) for different vertical sea level spectra. Four pictures are shown for various vertical depths \(h\): 2 km (a), 5 km (b), 10 km (c), and 15 km (d), correspondingly. Thick solid curves correspond to comparison with numerical computations using sea level muon spectrum based on data tables from (Misaki et al., 1999) and MUM code of muon propagation. Thin solid curves – sea level spectrum defined by Eq. (5) by fitting with MINUIT least square method the corresponding underwater angular intensity expected at vertical depth \(h=1.15\) km and expressed by formula 
\[ E_{\theta}^{\text{w}}(0\degree) = 103 \text{ GeV}, \quad E_{\theta}^{\text{w}}(0\degree) = 810 \text{ GeV}. \]

The results of reconstructing of two free parameters \((D_{0\pi}, \gamma)\) of sea level spectrum are as follows.

(i) For a range of zenith angles within \(\cos \theta=(0.17–0.99)\) we have obtained formally \((D_{0\pi} = 0.26, \gamma = 2.79)\). It is illustrated by Fig. 6. In spite of this result coincides with MACRO (Ambrosio et al., 1995) and LVD (Aglietta et al., 1999) best fits, its confidence level (CL) is close to 0. The artificial increase of errors in 3 times due to additional systematic errors leads to \((D_{0\pi} = 0.17, \gamma = 2.73)\) with CL=87%.

(ii) For vertical directions with \(\cos \theta=(0.61–0.99)\) the reconstructed sea level spectrum is extremely steep with \((D_{0\pi} = 1.0, \gamma = 3.0)\) and CL=0.5% but the increase of errors in 2 times results in \((D_{0\pi} = 0.19, \gamma = 2.74)\) with CL=40%.

(iii) For horizontal directions with \(\cos \theta=(0.13–0.61)\) the reconstructed sea level spectrum is flat, as \((D_{0\pi} = 0.1, \gamma = 2.65)\) with CL=70%, and for \(\cos \theta=(0.17–0.61)\) as \((D_{0\pi} = 0.12, \gamma = 2.68)\) with CL=40%. The result of this best fit is shown in Fig. 7.
Fig. 6. Zenith angle distribution of the muon intensity at vertical depth of 1.15 km. Experimental points - NT-36 data (Belolaptikov et al., 1997). Solid curve results from the best fit of 2 parameters \( (D_0 = 0.26, \gamma = 2.79) \) of sea level spectrum. Dashed curve results from analytical expression (1) using the sea level spectrum (6) and is consistent with experimental data also with CL=0. Only statistical errors have been taken into account.

It should be pointed out that the implementation of Gaisser’s set of 3 parameters \( (D_0, E_{\theta}^{cr}(0^\circ), E_{\theta}^{cr}(0^\circ)) \) (Gaisser, 1990) gives almost the same results of reconstructing of \( (D_0, \gamma) \), as well as when using the recalculated depth-intensity curve. The fact that sea level spectrum changes the slope from vertical directions to horizontal ones may be explained probably by unproper taking into account the muon bundles when unfolding the measured intensity.

5 Conclusions

The analytical expression presented in this work allows to estimate for fluctuating losses the integral flux of atmospheric muons in pure water expected for different zenith angles, \( \cos \theta = (0.05–1.0) \), at various vertical depths at least of \( h = (1–3) \) km for different parametrizations of the sea level muon spectra. The errors of this expression are estimated to be smaller than \( \pm (4–6) \% \) for cutoff energies ranged in \( E_f = (1–10^3) \) GeV and slant depths in \( h/\cos \theta = (1–16) \) km. The main advantage of the presented formula consists in the possibility of the direct best fit of at least 2 parameters of parent sea level spectrum using angular distribution of underwater integral flux measured experimentally at a given vertical depth.

The fitted sea level spectrum for NT-36 data is too steep for vertical directions \( (\gamma = 3.0) \) and flat for horizontal ones \( (\gamma = 2.65–2.68) \). It leads to the necessity of proper introducing of systematic errors mainly resulted from muon bundles.

The artificial increase of statistical errors in 2–3 times results in sea level spectra closer to \( \) (Klimushin et al., 2001) and \( \) (Gaisser, 1990).

The proposed method may be adapted to estimations in rock after corresponding description of the correction factor and continuous effective losses.

References

Klimushin, S. I., et al., Phys. Rev. D64, 014016, 2001, [hep-ph/0012032].
Sokalski, I. A., et al., Phys. Rev. D64, 074015, 2001, [hep-ph/0010322].
Misaki, A., et al., in Proceedings of the 26th ICRC, Salt Lake City, Utah, 1999, edited by D. Kieda, M. Salamon, and B. Dingus. Vol. 2, p. 139.

Volkova, L. V., et al., Yad. Fiz. 29, 1252, 1979 [Sov. J. Nucl. Phys. 29, 645, 1979].
Gaisser, T. K., Cosmic Rays and Particle Physics (Cambridge University Press, Cambridge), 1990.
LVD Collaboration, Aglietta, M., et al., Phys. Rev. D60, 112001, 1999.
MACRO Collaboration, Ambrosio, M., et al., Phys. Rev. D52, 3793, 1995.
Baikal Collaboration, Belolaptikov, I. A., et al., Astropart. Phys. 7, 263, 1997.