Solitonic Gluons

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We describe a physical mechanism for creating multi-soliton bound states where solitons are glued together by attraction between the beams they guide, solitonic gluons. We verify the concept of the solitonic gluons experimentally, observing a suppression of the repulsion between dark solitons due to an attractive force between bright guided beams.

It has been established for many models of nonlinear physics that solitons behave as effective particles when interacting with external fields or other solitons [1]. In the case of bright solitons described by the nonlinear Schrödinger (NLS) equation, the coherent interaction of solitons depends on the relative phase between them, so that identical solitons with opposite phases repel each other, whereas in-phase solitons attract each other [2]. Interaction of dark solitons is unconditionally repulsive, in all types of models described by the generalized NLS equation [3].

If a nonlinear system supports propagation of two, or more, waves of different frequencies or polarization, vector solitons consisting of more than one components can be formed. Indeed, when two vector solitons are closely separated, they may form a bound state if the sum of all forces acting between different soliton components is zero. As an example, let us consider interaction of two vector solitons consisting of dark and bright components in a defocusing optical medium. Two dark solitons always repel each other, and as a result, they can not form a bound state [3]. However, if we introduce out-of-phase bright components guided by each of the dark solitons, their attractive interaction creates a proper balance of forces, which results in a stationary two-soliton bound state (see Fig. 1).

The mechanism leading to the formation of bound states of the compound solitons described above, makes it somewhat tempting to draw an analogy between the soliton interaction and the fundamental theory of quarks and gluons – quantum chromodynamics. As is well known [4], gluons mediate the strong force and are responsible for binding the quarks into protons and neutrons, for example. Unless forming a bound state with quarks, gluons do not exist as separate particles. Taking these properties of gluons into consideration, we believe that it would be plausible to call bright components guided by dark solitons in a defocusing medium ‘solitonic gluons’. On the one hand, they do not ‘live’ in a defocusing medium by themselves, i.e. they diffract when separated from the dark solitons; and on the other hand, they ‘glue’ the dark solitons together in a bound state which otherwise can not be formed.

![FIG. 1. Schematic presentation of the force balance for the interacting dark–bright compound solitons. Shown are the intensities of dark and bright components in the fields $u$ and $w$, respectively; $\phi$ is a relative phase between the bright guided beams.](image)

To demonstrate this general concept on a particular example and verify it later experimentally, we consider two incoherently interacting linearly polarized beams in a photorefractive medium. The corresponding physical model is described by the equations for the normalized envelopes $U$ and $W$ [3]:

$$
\begin{align*}
\frac{i}{\rho} \frac{\partial U}{\partial z} + \frac{1}{2} \frac{\partial^2 U}{\partial x^2} + \frac{\beta (1 + \rho) U}{1 + |U|^2 + |W|^2} &= 0, \\
\frac{i}{\lambda} \frac{\partial W}{\partial z} + \frac{1}{2} \frac{\partial^2 W}{\partial x^2} + \frac{\beta (1 + \rho) W}{1 + |U|^2 + |W|^2} &= 0.
\end{align*}
$$

Here $\rho = I_\infty / I_d$, $\beta = (k_0 x_0)^2 n_{r33}^4 E_0 / 2$, where $I_\infty$ stands for the total power density in the limit $x \to \infty$, $I_d$ is the so-called dark irradiance, $k_0$ is the propagation constant, $x_0$ is the spatial width of the beam, and $n_{r33}^4 E_0$ is a correction to the refractive index due to the external field applied to a crystal along the $x$-axis [5].

Solitonic solutions of Eqs. (1) can be sought for in the form $U = \sqrt{5} u \exp[\sigma (1 - s) z]$ and $W = \sqrt{5} w \exp[\sigma (1 - s) \lambda z]$, where $s < 1$ and $\lambda < 1$ are two dimensionless parameters and $\sigma \equiv \text{sgn} \beta$ defines the type of nonlinearity, namely defocusing ($\sigma = +1$) or focusing ($\sigma = -1$). Amplitudes $u$ and $w$ satisfy the normalized equations:

$$
\begin{align*}
\frac{i}{\rho} \frac{\partial u}{\partial z} + \frac{1}{2} \frac{\partial^2 u}{\partial x^2} - \frac{\sigma (|u|^2 + |w|^2) u}{1 + s(|u|^2 + |w|^2)} + u &= 0, \\
\frac{i}{\lambda} \frac{\partial w}{\partial z} + \frac{1}{2} \frac{\partial^2 w}{\partial x^2} - \frac{\sigma (|w|^2 + |u|^2) w}{1 + s(|u|^2 + |w|^2)} + \lambda w &= 0,
\end{align*}
$$

where the spatial coordinates $z$ and $x$ are measured in the units of $s/|\beta|(1 + \rho)$ and $(s/|\beta|(1 + \rho))^{1/2}$, respectively.
In Eq. (2), the parameter $s$ characterizes nonlinearity saturation.

First, assuming $\sigma = +1$, we look for stationary, $z$-
-independent solutions of the system (2) in the form
of dark-bright solitons, where $u(x)$ has nonvanishing
asymptotics but $w(x)$ vanishes for $|x| \to \infty$. Then, a
compound dark-bright solitary wave $(u, w)$, emerges from
the one-component dark solution $(u, 0)$ at a certain bi-
furcation point $\lambda = \lambda_0(s)$. Near the bifurcation point,
we can treat the dark component as an effective wave-
guide which guides the spatially localized mode $w$. Such
compound solitons form a continuous family in $\lambda$, for a
fixed $s$. The existence of this type of vector dark-bright
solitons in the photorefractive media has been earlier es-

tablished theoretically [3] and verified experimentally [3].
The families of two-component dark-bright solitons have
been numerically found in Ref. [3].

Interaction between two weakly overlapping dark-
bright solitons can be described by the effective interac-
tion energy as a function of the soliton separation
$x_0$, $V_{\text{eff}}(x_0) = V_d(x_0) + a^2 V_b(x_0) \cos \phi$, where the first
and second terms stand for the interaction between dark
and bright soliton components, respectively, with $\phi$
being a relative phase between the bright components of
the equal amplitude $a$. In the case of small $s$, the effec-
tive interaction energies $V_d(x_0)$ and $V_b(x_0)$ can be found
analytically [3], $V_d(x_0) \approx 2 \exp(-4x_0)$ and $V_b(x_0) \approx
4x_0 \exp(-2x_0)$. Thus, interaction between two dark-
bright solitons is crucially affected by the forces acting
between the bright components guided by the closely sep-


arated dark solitons. If the bright components are out of
phase ($\phi = \pi$), an attractive force acting between them
can balance (or even overbalance) the repulsion between
the dark solitons, thus creating a bound state. This ob-
ervation has been confirmed for Eqs. (2) by numerically
identifying the families of two-soliton bound states [3].

To demonstrate the concept of force balance, we have
investigated numerically the interaction of a pair of dark
solitons generated by a localized input in the shape of a
box-like notch on a cw background. Figures 2(a) and
2(b) show the results of our simulations for two distinct
cases. First, the box-like input is launched without
丰富多彩 component. It generates a pair of scalar
(one-component) dark solitons which move away from
each other as they propagate [see Fig. 2(a)] due to their
mutual repulsion [3]. However, the propagation dyna-

mics is drastically modified by introducing an additional
bright beam. Figure 2(b) shows the results for the input
field as in Fig. 2(a) but with the added $w$-component in
the form of the first (one-node) mode of the symmetric
box-shaped waveguide. Such an amplitude profile of the
bright component has been chosen to mimic a solitonic
gluon binding two closely separated dark solitons. As is
seen from Fig. 2(b), the attraction between out-of-phase
bright guided beams compensates for the repulsive force
between dark solitons, so that two dark-bright solitons
propagate almost in parallel.

FIG. 2. Generation of a pair of dark solitons from a lo-
calized box-like input: (a) no bright component introduced,
(b) in the presence of the bright component. Bottom row:
corresponding input beam profiles.

FIG. 3. Bound states of bright solitons in the model (2)
at $\sigma = -1$. (a),(b) Examples of compound solitons with
the first and second guided modes, respectively. (c),(d) Inter-
action of two bright solitons from (a), without and with the
gluon component, respectively.

The concept of solitonic gluons can be readily extended
to describe multi-soliton bound states composed of var-
ious types of bright vector solitons [3]. Bright solitons
of Eqs. (2) are possible for a self-focusing nonlinear-
ity, $\sigma = -1$. Continuous in $\lambda$, families of such solu-

tions originate at the points of bifurcation of the scalar
($(u$-component only) bright soliton families. Similar to
the case of dark-bright solitons, the $w$– field near the
bifurcation points can be described as a linear mode of
an effective waveguide induced by the $u$–soliton, the lat-

ter can support more than one mode, depending on the
value of $s$.

A rich variety of the soliton structures, consisting of
a fundamental bright soliton and a $n$–node mode of the
soliton-induced waveguide, results in complex shapes of
solitonic gluons of the focusing nonlinearity. Interaction
between the bright components depends on their relative
phase, and therefore the bound states of bright vector
solitons should be constructed from out-of-phase (mutu-
ally repelling) and in-phase (mutually attracting) com-
ponents. Some vector solitary waves of this type have
already been described [3]. In Figs. 3(a, b) we present
two examples of such soliton bound states, found numeri-
cally for Eqs. (2), where the repulsion of the out-of-phase
fundamental (no nodes) solitons of the $u$– component is
balanced by an attractive force acting between two types
of solitonic gluons, formed by the in-phase first or second modes of effective soliton-induced waveguides.

Numerical simulations of the beam propagation, with the input of the type presented in Fig. 3(a), revealed a drastic change in the soliton interaction as shown in Figs. 3(c,d) [cf. Fig. 2(a,b)]. The binding force of the solitonic gluons makes the otherwise mutually repelling out-of-phase solitons propagate in parallel [Fig. 3(d)].

To verify the concept of solitonic gluons experimentally, we have studied the interaction of dark solitons in a photorefractive medium, using the experimental setup for generating dark-bright soliton pairs earlier reported in Refs. [6,10]. A cw argon-ion laser beam (488 nm) is collimated and split by a polarizing beam splitter. The ordinary polarized beam is used as uniform background illumination to mimic the dark irradiance, and the extraordinary polarized beam is split into two soliton-forming beams. One of these two beams is used to generate a dark notch on reflection, by illuminating a wire. The dark notch is then imaged onto the input face of a SBN:61 crystal with the background beam covering the entire input face. One half of the other beam (the bright component) goes through a tilted thin glass that introduces a $\pi$ phase jump at its centre, thereby creating a field profile resembling the first mode of a box-like waveguide, i.e. a bright beam with a node [Fig. 4(a)]. We make the dark and bright input beams mutually incoherent by having their optical path difference greatly exceed the coherence length of the laser.

![Figure 4](https://example.com/fig4.png)

**FIG. 4.** Photographs showing the effect of the solitonic gluons via coupling of a bright (lower row) and dark (upper row) incoherently interacting beams: (a) an input at $z = 0$ mm, and three outputs at $z = 11.7$ mm for (b) normal diffraction of coupled beams, (c) stationary two-soliton bound state, (d) decoupled beam propagation.

Figures 4(a-d) show photographs of the bright and dark beams taken on the input [Fig. 4(a)] and output [Figs. 4(b,c,d)] faces of a 11.7-mm-long SBN crystal. Without a dc field, both beams merely diffract in a similar fashion, as shown in Fig. 4(b). When we launch only a dark-notch-bearing beam in the presence of an appropriate dc field in the $x$ direction ($|c|$ axis), it generates a Y-junction created by a pair of repelling dark solitons, as is seen in Fig. 4(d) (upper row) and earlier reported in Ref. [11]. On the other hand, the uncoupled bright beam diffracts even more strongly due to effectively defocusing nonlinearity, Fig. 4(d) (lower row).

Repulsion of a pair of dark solitons created by a wire is suppressed dramatically when we simultaneously launch a bright beam in the form of a second waveguide mode, thus creating two bright beams with opposite phases. As can be seen from Figs. 4(c) and 4(d), the output distance between the dark solitons is subsequently reduced from 53$\mu$m to 43$\mu$m. In addition, we have verified that when the repulsion between the dark solitons is enhanced when the bright beams are in-phase.

In conclusion, we have suggested and verified experimentally a concept of solitonic gluons featuring a balance of attractive and repulsive forces acting between different components of compound solitons as a physical mechanism responsible for the formation of bound states of vector solitons. This novel type of interaction of compound solitons is a rather fundamental phenomenon which should also be observed for many other multi-component nonlinear models, e.g. incoherent solitons.

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