Scaling Properties of Pion Production

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Abstract

The scaling of pion multiplicities with system size in proton-nucleus and nucleus-nucleus reactions at 200 $A$ GeV is investigated. A nuclear geometry calculation of incoherent multiple scattering including effects of deceleration of the participating particles is presented. Pion multiplicities at high transverse momenta agree relatively well with an overall scaling with the number of binary collisions. The deviations from this gross scaling for p+A and S+Au reactions at 200 AGeV are compared to model predictions and the implications are discussed. Within this picture no universal scaling law for high $p_T$ particle production is obtained. In addition to the conceptual problems of such an incoherent model, it is shown that the implied kinematic relation between a possible $p_T$-broadening and deceleration leads to contradictions. The scaling of pion production at intermediate $p_T$ with system size does not indicate a multiple scattering enhancement.

1 Introduction

Hadron production at high transverse momenta in very high energy collisions is thought to be described by perturbative QCD. The transverse momentum and energy dependence of the cross section is however not consistent with simple qq scattering processes. Higher order contributions and scaling violations have to be included. At somewhat lower energies ($\sqrt{s} \leq 20$ GeV) it is still unclear whether meson degrees of freedom contribute to high $p_T$ scattering via the constituent interchange mechanism (for reviews see e.g. [1,2]).

The interest in high $p_T$ production has been renewed by the observation of the so-called Cronin-effect in p+A collisions at Fermilab [3]. Here it was seen that the target mass dependence of particle production when parameterized as $A^\alpha$ reveals a power larger than one for $p_T > 2$ GeV/c, an effect, which was later interpreted as the result of multiple scattering of the incident partons [4,5].

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In nucleus-nucleus collisions there is still a strong enhancement of particle production at high $p_T$. It has been proposed to parameterize the nuclear dependence as \cite{6}:

$$E \frac{d^3\sigma}{dp^3} (AB) = (A \cdot B)^{\alpha(p_T)} E \frac{d^3\sigma}{dp^3} (pp).$$ \hspace{1cm} (1)

For minimum bias nucleus-nucleus collisions this parameterization yields exponents at high $p_T$ which are comparable to those in p+A, however, for heavy nuclei the statistical significance was so far limited. Recently high precision neutral pion data from 200 AGeV S+Au collisions have become available \cite{7}, which corroborate the above result. It is also emphasized in \cite{7} that this enhancement must not necessarily be an isolated feature at high $p_T$, but that differences of the shape of the momentum spectra over the whole $p_T$ range are important.

Still the similarity of the exponents may be accidental, as a lot of information from the heavy ion reaction is washed out due to the impact parameter averaging. In a naïve picture, the total cross section in nucleus-nucleus collisions should scale like $(R_A + R_B)^2 \propto (A^{1/3} + B^{1/3})^2$, while the yield of produced particles for a given nuclear collision should depend mainly on the apparent thicknesses like $A^{1/3} \cdot B^{1/3}$. For proton-nucleus this reduces to a simple dependence, because $A \ll B$. This is not generally true for a heavy-ion collisions ($A \neq B$), with an exception of the special case where every possible binary collision contributes the same amount to the total cross section without further modification. Then the cross section must be $AB\sigma_{pp}$. In case of an anomalous nuclear enhancement as observed in \cite{3} such a simple relation does not hold.

I will try to establish a way of investigating scaling properties which can be applied to centrality selected heavy-ion data. Special attention will be given to high $p_T$ production. There it is reasonable to assume a scaling with the number of binary collisions as a baseline estimate. Modifications due to initial state multiple scattering should scale with the thickness of the colliding nuclei. The rms-thickness properly averaged over all possible nucleon-nucleon collisions will be used here. If the nuclear enhancement is caused by initial state multiple scattering, one expects the particle yield per binary collision to be a continuous and monotonous function of this thickness parameter.

For the intermediate $p_T$ range there have been attempts \cite{8} to describe the change in slope of the spectra for heavier systems as a $p_T$-broadening due to initial state multiple scattering. There only the shapes of the spectra have been compared, no attempt was made to predict the absolute scale of particle production. As such a model must also account for changes in the multiplicity of particles, it is of interest to simultaneously compare the concepts of multiple scattering also to data at lower $p_T$. 

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In this paper I use a simple simulation of incoherent multiple scattering in the initial state to predict possible scaling laws for particle production. One has to note, however, that the assumption of incoherence and the decomposition of the reaction into subsequent binary collisions is very naive and most likely not fulfilled.

I will calculate a certain transverse rapidity kick from the effective thickness with the help of a hydrodynamically inspired formula. This recipe may be questionable, but is used merely to study the change in shape of the spectra – which it should allow – and does not imply any physical significance of the hydrodynamic origin of the formula. In addition, a possible energy degradation is calculated, and the altered $\sqrt{s}$ used in the following collisions. This relies heavily on the incoherence of the collisions and is in this respect very similar to a naive cascade model. It is actually not expected to be applicable in high energy reactions.

Also, the transverse rapidity kick is only relevant for the produced particles. The number of collisions the nucleons suffer is calculated by assuming that they continue to travel on a straight line. It is further assumed that both the rapidity kick and the longitudinal rapidity loss are independent of the $\sqrt{s}$ of the collision in question. This will obviously not be true once $\sqrt{s}$ gets very small. This will be neglected because the concept of energy degradation has its problems, and we will only make limited use of it in this paper. For similar reasons the effects of varying “source rapidity” depending on the amount of multiple scattering will be neglected.

The predictions will be compared to pion multiplicities in proton-nucleus collisions at 200 GeV measured at a pseudorapidity $\eta = 3.26$ [3] and S+Au data for different impact parameters at 200 A GeV in the rapidity range $2.1 \leq y \leq 2.9$ [7]. Such comparisons might help in understanding the limitations of incoherent (“random-walk”) multiple scattering for the description of particle production in nucleus-nucleus collisions.

2 The Model

The model calculations use the following recipe: For a given reaction system nucleons are randomly distributed in the nuclei according to Woods-Saxon distributions:

$$\rho(r) \propto \frac{1}{1 + \exp\left[\frac{(r - r_0)}{0.54}\right]}$$
using $r_0 = 1.16A^{1/3} - 1.35A^{-1/3}$. It is taken care that the hard cores of the nucleons ($R_{hc} = 0.6\text{ fm}$) do not overlap. The two nuclei collide at a given impact parameter. A projectile nucleon is supposed to interact with a target nucleon, if their minimum distance on straight line trajectories is less than their interaction distance ($d_{int} = 1.025\text{ fm}$).\footnote{Mostly nucleon-nucleon interactions are presumed, but it should not really matter whether the nucleon or the parton picture determines the underlying dynamics. One of the assumptions of the model is, however, that all individual binary collisions in the nuclei are incoherent.} The calculations are repeated for a large number of random configurations of nucleons within the nuclei.

From this model the following variables are calculated:

- The average number of binary collisions $N_{\text{coll}}$, which may happen in a nuclear reaction. This is the main variable controlling particle multiplicities at high momenta in a model of independent production.
- The reduction in particle multiplicity due to the deceleration of participant nucleons. It is assumed that a particle suffers a constant rapidity shift $\delta y$ per collision. The number of collisions a nucleon may undergo before a given collision which is responsible for the production of the particle of interest is calculated. I will call this the number of prescatterings $N_{\text{pre}}$. It determines the total rapidity loss and thereby the center-of-mass energy of the collision in question $\sqrt{s_1}$, which will in general be lower than the $\sqrt{s_0}$ of the incident nucleons. From this I can deduce an attenuation factor $X$ in different ways:

  1. For high $p_T$ production it is assumed that the multiplicities scale according to $(1 - x_T)^9$ \cite{9} with $x_T \equiv 2p_T/\sqrt{s}$, so one gets:

     $$X_{\text{high}} = \frac{(1 - 2p_T/\sqrt{s_1})^9}{(1 - 2p_T/\sqrt{s_0})^9}. \quad (2)$$

  2. For low $p_T$ production a scaling with $\ln(s)$ is assumed \cite{10}:

     $$X_{\text{low}} = \frac{0.88 + 0.44\ln(s_1) + 0.118\ln^2(s_1)}{0.88 + 0.44\ln(s_0) + 0.118\ln^2(s_0)}. \quad (3)$$

  3. The most radical assumption is that of complete absorption. A nucleon produces particles only in the very first collision:

     $$X_{\text{abs}} = \begin{cases} 1, & N_{\text{pre}} = 0, \\ 0, & N_{\text{pre}} > 0. \end{cases} \quad (4)$$

This corresponds to the limit of maximum momentum transfer in cases 1) and 2).
These factors are averaged over all possible binary collisions and attenuation factors \( x_i = \langle X_i \rangle \) are obtained.

- The effective number of prescatterings \( N_{\text{eff}} \). For this, the squareroot of the number of prescatterings in each individual binary collision is calculated. This is then averaged over all possible binary collisions. It should control the effects of initial state multiple scattering. The value of \( N_{\text{eff}} \) for a specific reaction depends on the way the averaging over the different binary collisions is done. For each binary collision the corresponding attenuation factor \( X_i \) has to be taken into account as a weight, because it reduces the multiplicity. This introduces an implicit dependence on the longitudinal rapidity shift \( \delta y \) – a larger deceleration will result in a smaller effective number of prescatterings.

A certain transverse rapidity shift \( \delta \rho \) is attributed to every prescattering – this is the shift apparent in the final particle production, so the rapidity shift a single nucleon gets is actually \( 2 \delta \rho \). Since \( N_{\text{eff}} \) already includes the effects of random relative orientation of these prescatterings by adding the contributions quadratically, the average transverse rapidity shift in a given nuclear reaction is just \( \rho(N_{\text{eff}}) = \delta \rho \cdot N_{\text{eff}} \). The effect of the additional transverse velocity is then calculated with the help of a formula frequently used to describe hydrodynamical transverse flow [11,12]:

\[
\frac{1}{p_T} \cdot \frac{dN}{dp_T} = C \cdot m_T \cdot I_0 \left( \frac{p_T \sinh \left[ \rho(N_{\text{eff}}) \right]}{T} \right) K_1 \left( \frac{m_T \cosh \left[ \rho(N_{\text{eff}}) \right]}{T} \right).
\]

With \( N_i \) being the number of pre-scatterings for a given binary collision, the two parameters explained above are different averages \( N_{\text{pre}} = \langle N_i \rangle_i \) and \( N_{\text{eff}} = \langle \sqrt{N_i} \rangle_i \). They both have a well defined role within the model studied here, however, they also provide in general (esp. in the case \( \delta y = 0 \)) measures of the effective thickness of the two colliding nuclei. It is therefore illustrative to study particle production as a function of these variables.

Figure 1 shows sample results of this calculation for \( p+A \) and \( S+Au \) collisions at 200AGeV. The attenuation factor (left side of figure 1) must obviously be \( x = 1 \) for \( p+p \) collisions (i.e. \( N_{\text{coll}} = 1 \)). It decreases for larger system size. There is however no unique dependence on \( N_{\text{coll}} \), the behaviour is slightly different for \( S+Au \) compared to \( p+A \). One also notices immediately that the effects are much stronger for high \( p_T \) than for low \( p_T \) – this is a direct consequence of the stronger \( \sqrt{s} \) dependence of the high \( p_T \) production.

The effective number of prescatterings (right side of figure 1) is also by definition \( N_{\text{eff}} = 0 \) for \( p+p \) collisions and increases with system size. There is a similar discontinuity when going from \( p+A \) to \( S+Au \) collisions. The largest

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One should note, however, that while \( N_{\text{eff}} \) contains an influence from energy degradation there is no such effect included in \( N_{\text{pre}} \).
values for $N_{\text{eff}}$ are obtained without any deceleration effect ($\delta y = 0$), a finite rapidity loss per collision of $\delta y = 0.5$ reduces these numbers slightly for low $p_T$ and drastically for high $p_T$.

## 3 Analysis and Discussion

The pion cross sections for $p_T = 0.77$ GeV/c and $p_T = 3.08$ GeV/c from [3] have been normalized with the nuclear inelastic cross section for the corresponding nuclei [13] to obtain multiplicities/event. Neutral pion equivalent multiplicities have been calculated by averaging the results for negative and positive pions.

The neutral pion cross sections for S+Au can be very well described by a fit with a power-law like formula [7,14]:

$$E \frac{d^3\sigma}{dp^3} = C' \left( \frac{p_0}{p_T + p_0} \right)^n$$

From this parameterization the pion multiplicities have been calculated at the same $p_T$ values as above. The systematic uncertainty of the pion yield given in [7] will be used as error.

The reaction cross sections from [7] for the different centrality classes selected by the transverse energy $E_T$ are used to calculate impact parameter estimates assuming a monotonic increase of $E_T$ with $b$. This may lead to relatively large uncertainties for the most peripheral reactions which are affected most strongly by the experimental trigger bias. While the results presented below include an estimate of the systematic error of the model predictions from variations of the model assumptions, this may not completely account for the trigger bias in the most peripheral trigger class.

Scaled pion multiplicities are then calculated for different input parameters of the model:

$$R_\pi = \frac{1}{N_{\text{coll}}} \cdot E \frac{d^3N}{dp^3} (p_T) \cdot \frac{1}{X_i(\delta y)}.$$  

Figure 2 displays the scaled multiplicities at $p_T = 3.08$ GeV/c for p+A and S+Au as a function of the effective number of prescaterrings $N_{\text{eff}}$. The upper left part shows the data without any correction for deceleration effects. One immediately realizes that the multiplicity per collision rises with $N_{\text{eff}}$ (i.e. with the target mass) in p+A collisions (open circles) – this is just a different representation of the anomalous nuclear enhancement.
The p+A data have been fitted with equation 5 according to the following procedure: The slope parameter $T$ was tuned to describe p+p at high $p_T$ – this yields $T = 240$ MeV. With $T$ fixed the other parameters were then fitted to the data for the heavier targets in two different ways: a) keeping the normalization fixed to describe p+p and b) leaving the normalization as a free parameter. Both curves are shown in figure 2. The fit (a) does not correctly describe all the p+A data, while fit (b) accounts well for the heavier targets but misses the p+p point. The same analysis can be done assuming a finite amount of deceleration of the participant nucleons, the qualitative picture of the dependence being unaltered. The fit results are summarized in table 1. It can be seen that the necessary transverse rapidity shift $\delta \rho$ increases with larger (longitudinal) rapidity loss $\delta y$ of the participants. One should note that for $\delta y = 1$ a transverse rapidity shift of $\delta \rho \approx 7$ would be necessary to account for the rise in the data. This is completely unphysical.

In this context it is of interest to compare the parameters necessary to account for the enhancement with the kinematics of scattering processes. Every gain in net transverse momentum $p'_T$ is related to a loss in momentum $p'_L$ available for particle production. For elastic scattering one can easily deduce from momentum conservation a relation between the transverse rapidity shift $\delta \rho_{\text{elast}}$ a nucleon obtains in a collision and the longitudinal rapidity loss $\delta y_{\text{elast}}$ it suffers:

$$\left[\tanh(y_0)\right]^2 = \left[\tanh(y_0 - \delta y_{\text{elast}})\right]^2 + \left[\tanh(\delta \rho_{\text{elast}})\right]^2. \quad (8)$$

This relation is displayed in figure 3 as a solid line. For any inelastic process, the transverse rapidity can only be smaller than this limit for a given longitudinal rapidity loss. This allowed region is indicated in figure 3 as a grey area. As a good approximation this relation can also be used as a limit for the transverse shift obtained in a multiple scattering picture. The symbols show the parameter values extracted from the fits of equation 5 to the data. It is obvious that these values are not consistent with scattering kinematics. The maximum transverse rapidity shift allowed for a given $\delta y$ would not suffice to explain the enhancement observed in p+A collisions.

This observation can most easily be interpreted as a failure of the underlying picture of incoherent scatterings. This is actually expected because of the short time scales involved in hard scattering processes. In a coherent model there is no possibility of an energy degradation between two “individual” collisions like it is implemented in the model used here, and thus the kinematical considerations do not apply in the same way. The results obtained may be

\footnote{The estimate would be different for cases, where the collision axis of the binary collision is strongly tilted relative to the overall collision axis - this will not be considered here.}
regarded as a warning against too naïve use of such simplified models. I will therefore not discuss these cases further.

The S+Au data have also been included in figure 2. For all cases the data agree with the extrapolation from p+A for intermediate centralities, however, the increase with \( N_{\text{eff}} \) is much stronger than predicted by the extrapolation. The difference in system size between p+A and S+Au data may be relevant: While the parameter \( N_{\text{eff}} \) is comparable e.g. for p+W and peripheral S+Au, the number of binary collisions in peripheral S+Au is larger by a factor of two.

Ignoring the data points for the peripheral S+Au collisions the heavy ion data would be consistent with the heavier target p+A data. In this case, however, the p+p data cannot be described. There is no universal quantitative description of both p+A and S+Au data within this multiple scattering picture, although it seems to be able to predict the scaling behaviour for both groups of data qualitatively.

Still, as calculations using a “random-walk” picture have been advocated for the description of nucleus-nucleus collisions [8], I will compare the present model to particle production at lower transverse momentum \( (p_T = 0.77 \text{ GeV/c}) \). Figure 4 shows the scaled multiplicities for this case. Without any deceleration effects (upper left) the multiplicity per collision decreases for increasing target size in p+A reactions. Like for the high \( p_T \) case this reflects the value of the “Cronin”-exponent \( \alpha \) – here one finds \( \alpha < 1 \), which indicates that the nuclei are not at all transparent for these softer processes.

Introducing a correction for the attenuation with intermediate values of \( \delta y = 0.25 - 0.5 \) does not really compensate for the decrease. If one assumes complete “absorption”, so that only the first binary collision contributes to particle production, the scaled multiplicities show a small increase\(^5\). The particle yields at low \( p_T \) in p+A can obviously be understood without any strong enhancement via initial state multiple scattering. They are very close to the expectation from production only at the surface of the nuclei. The multiplicities in S+Au collisions show an additional increase compared to the expectation from p+A.

\(^5\) A rapidity loss \( \delta y \geq 2 \) is necessary to compensate for the decrease. This is for practical purposes very similar to the complete absorption case.
4 Conclusions

Pion production at high $p_T$ scales approximately with the number of binary collisions. The enhancement relative to this scaling in p+A and S+Au has been studied as a function of an effective thickness parameter. Results indicate that it will be difficult to describe the enhancement in both data sets with a unique function of the thickness.

The description of the nuclear enhancement at high $p_T$ in a picture of incoherent scatterings (“random-walk”) in the present calculation would require parameters which contradict momentum conservation in individual scatterings. This confirms that the $p_T$-broadening at high $p_T$ from initial state multiple scattering should not be described incoherently. Recent perturbative QCD calculations including the effects of intrinsic $p_T$ and initial state multiple scattering [15] were able to reproduce data for neutral pion production from central S+S reactions [7] as well as preliminary results for central Pb+Pb collisions [16,17]. It would be of interest to see, whether a QCD calculation with coherent multiple scattering effects would be able to describe also the scaling of particle multiplicity with centrality in heavy ion collisions.

Particle production at intermediate $p_T$ in p+A is consistent with production only at the nuclear surface. In this scenario there is little room for an enhancement by initial state multiple scattering. It must therefore be considered very unlikely that e.g. the increase in inverse slope of the momentum spectra with system size also at intermediate $p_T$ can be attributed to this mechanism in the way discussed in [8].

A simple explanation of increasing slopes with larger system size might be an admixture of the two components, where the soft component dominates at low $p_T$ with increasing importance of the hard component with increasing $p_T$. The ratio hard/soft would roughly scale as volume/surface and would thus increase the high $p_T$ part of the spectra more rapidly with increasing system size. This could explain the behaviour in p+A collisions at intermediate $p_T$. The particle production in S+Au collisions might call for additional thermal or hydrodynamical production not present in p+A, while for the explanation of the high $p_T$ production more refined calculations of hard scattering appear to be necessary.

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References

[1] J.F. Owens, Rev. Mod. Phys. 59 No. 2 (1987) 465–503.
[2] W.M. Geist et al., Phys. Rep. 197 (1990) 263–374.
[3] D. Antreasyan et al., Phys. Rev. D 19 (1979) 764.
[4] A. Krzywicki et al., Phys. Lett. B 85 (1979) 407.
[5] M. Lev and B. Petersson, Z. Phys. C 21 (1983) 155–161.
[6] H.R. Schmidt and J. Schukraft, J. Phys. G 19 (1993) 1705–1796.
[7] R. Albrecht et al., Eur. Phys. J. C 5 (1998) 255.
[8] A. Leonidov, M. Nardi, and H. Satz, Hadron spectra from nuclear collisions, Nucl. Phys. A 610 (1996) 124c–131c, in: Quark Matter ’96, Proceedings of the Twelfth International Conference on Ultra-Relativistic Nucleus-Nucleus Collisions.
[9] D. Antreasyan et al, Phys. Rev. Lett. 38 (1977) 112.
[10] W. Thomé et al., Nucl. Phys. B 129 (1977) 365.
[11] J. Sollfrank, P. Koch, and U. Heinz, Z. Phys. C 52 (1991) 593.
[12] E. Schnedermann, J. Sollfrank, and U. Heinz, Phys. Rev. C 48 (1993) 2462.
[13] M. Aguilar-Benitez et al., Phys. Rev. D 50 (1994) 1173–1826, Particle Data Group.
[14] D. Stüken, 1999. PhD thesis, Universität Münster, in preparation.
[15] X.-N. Wang, Phys. Rev. Lett. 81 (1998) 2655.
[16] WA98 Collaboration, M. Aggarwal et al., Nucl. Phys. A 610 (1996) 200c.
[17] WA98 Collaboration, T. Peitzmann, et al., Quark Matter ’97, Proceedings of the 13th International Conference on Ultrarelativistic Nucleus-Nucleus Collisions, Tsukuba, Japan.
Table 1
Transverse rapidity shift per collision $\delta \rho$ from a fit of equation 5 to data at $p_T = 3.08 \text{GeV}/c$ for p+p and p+A reactions at 200 GeV [3].

| $\delta y$ | $\delta \rho$ (p+p and p+A) | $\delta \rho$ (only p+A) |
|-----------|-----------------------------|-------------------------|
| 0         | 0.087 ± 0.006               | 0.061 ± 0.015           |
| 0.25      | 0.357 ± 0.011               | 0.348 ± 0.029           |
| 0.5       | 0.882 ± 0.022               | 0.903 ± 0.073           |
| 1         | 6.8 ± 0.2                   | 7.1 ± 0.5               |

Fig. 1. Parameter values obtained in the model calculation for p+A and centrality selected S+Au collisions. Left: attenuation factors $x_i$ for low- and high-$p_T$ production assuming $\delta y = 0.5$, right: effective number of precollisions $N_{eff}$ calculated without deceleration ($\delta y = 0$) and for low- and high-$p_T$ production assuming $\delta y = 0.5$. The filled symbols show results relevant for high $p_T$, the open symbols for low $p_T$. 
Fig. 2. Normalized particle multiplicities at $p_T = 3.08$ GeV/$c$ for p+A and S+Au collisions as a function of the effective number of prescatterings $N_{eff}$. The lines show fits of the multiple scattering model (equation 5) to the p+A data including (a) and excluding (b) the p+p data point.
Fig. 3. Transverse rapidity $\delta \rho$ as a function of the longitudinal rapidity loss $\delta y$ for nucleon-nucleon scattering processes at $\sqrt{s} = 19.4$ GeV. The solid line shows the relation for elastic scattering (equation 8), the symbols show the fit parameters of the multiple scattering model given in table 1, where the black squares make use of the p+p data point and the grey circles are obtained without it.

Fig. 4. Normalized particle multiplicities at $p_T = 0.77$ GeV/$c$ for p+A and S+Au collisions as a function of the effective number of prescatterings $N_{eff}$ assuming no rapidity loss (left) and complete absorption (right). The lines show exponential fits to guide the eye. In the representation of the data in the right plot the same values as for $\delta y = 0$ have been used for $N_{eff}$, because the true values would be $N_{eff} \equiv 0$. 