An Extremal Optimization approach to parallel resonance constrained capacitor placement problem

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Abstract

Installation of capacitors in distribution networks is one of the most used procedure to compensate reactive power generated by loads and, consequently, to reduce technical losses. So, the problem consists in identifying the optimal placement and sizing of capacitors. This problem is known in the literature as optimal capacitor placement problem. Nevertheless, depending on the location and size of the capacitor, it may become a harmonic source, allowing capacitor to enter into resonance with the distribution network, causing several undesired side effects. In this work we propose a parsimonious method to deal with the capacitor placement problem that incorporates resonance constraints, ensuring that every allocated capacitor will not act as a harmonic source. This proposed algorithm is based upon a physical inspired metaheuristic known as Extremal Optimization. The results achieved showed that this proposal has reached significant gains when compared with other proposals that attempt repair, in a post-optimization stage, already obtained solutions which violate resonance constraints.

Keywords — Capacitor Placement, Combinatorial Optimization, Distribution System Planning, Extremal Optimization, Metaheuristics.

1 Introduction

One of the leader causes of technical losses in electric power distribution networks can be assigned to resistance in distribution and transmission lines, subject to active and reactive currents. One possible strategy to reduce technical losses is by means of capacitor allocation (Bunch et al., 1982).

Capacitors are used in power distribution networks with the intention of reducing losses related to reactive power, power-factor correction, power-flow control and improvement of network stability (Madeiro et al., 2011).

The optimal capacitor placement problem (CPP) is to define location, size and number of capacitors which should be installed in a network, aiming to minimize a function that takes into account the capacitors installation investment and the reduction in power loss.

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CPP is a non-linear and non-differentiable mixed integer optimization problem with a set of operating constraints (Yu et al., 2004). So, traditional optimization methods are not capable of solving large instance of this hard combinatorial problem. Therefore, techniques conceived to reach high-quality solutions in a reasonable execution time, such as heuristics and metaheuristics, emerge as promising approaches.

Some metaheuristics have been successfully applied to CPP, such as: genetic algorithms (Delfanti et al., 2000; Mendes et al., 2005), particle swarm optimization (Yu et al., 2004), tabu search (Huang et al., 1996), hybrid evolutionary algorithms (Mendes et al., 2005), and genetic fuzzy systems (Das, 2008).

However, one of the main drawbacks of population metaheuristics, like genetic algorithms, particle swarm optimization, and ant colony optimization is their high computational cost. Since these algorithms handle a population of candidate solutions, a large number of fitness evaluations (performance of each candidate solution) must be done, which may be expensive, mainly in real world problems (Jin, 2005).

In the CPP, for each fitness evaluation an execution of a power-flow estimation method is needed. Only after having power-flow estimates we are allowed to figure out the effective contribution of a given set of allocated capacitors. In real world power distribution networks, with thousands of nodes, this procedure becomes computationally expensive.

Another relevant aspect that is commonly relaxed in CPP, or just taken into account in a post-optimization procedure, is the fact that the capacitor may enter into resonance with the power distribution network, depending on some factors, such as the distance between the capacitor location and the network feeder, and the capacitor size. A possible consequence of resonance is an interruption of the power network, that may bring several implications to the population and to the electricity concessionaire. So, before installing capacitors we need to ensure that the resonance will not show up.

Aiming at mitigating this problem, we propose a parsimonious extremal optimization method which takes into account the resonance constraints. Extremal optimization algorithms have reached good results with relatively small computational cost, when compared with alternative population metaheuristics (Lu, 2009) and (Lu, 2007).

The remainder of the text is structured as follows: in the next section we discuss about resonance in distribution network. The mathematical modeling of the resonance constrained CPP is described in Section 3. Extremal Optimization is depicted in Section 4. Our proposal is outlined in Section 5. Experiments carried out are described in Section 6, and the results are analyzed in Section 7. Finally, concluding remarks and future works are pointed out in Section 8.

## 2 Resonance in power distribution networks

The size and location of capacitors are critical factors in a distribution system’s response to the harmonic sources (Gönen, 1986). Combination of capacitors and the system reactance cause parallel resonance frequencies for the circuit. The possibility of resonance between a capacitors and the rest of the system, at a harmonic frequency, can be determined by calculating equal order of harmonic $h$ at which resonance may take place, given by (Gönen, 1986)
\[ h = \sqrt{\frac{S_{cc}}{Q_c}}, \]  

where \( S_{cc} \) is the three-phase short-circuit power of system at the place of capacitor installation, in VA, and \( Q_c \) is the capacitor size, given in var.

Parallel resonance frequency, \( f_p \), can be expressed as

\[ f_p = f_1 \cdot h, \]  

where \( f_1 \) is the fundamental frequency (60 Hz in Brazil). Replacing Eq. 1 in Eq. 2, we have

\[ f_p = f_1 \cdot \sqrt{\frac{S_{cc}}{Q_c}}. \]  

The characteristics of the most common harmonic loads in power distribution networks include the third, fifth and seventh harmonics. Thus, the purpose is to avoid these odd harmonics. In this work, we consider a range of 10 Hz above and below the frequency \( f_p \), that is, the \( n \)-th harmonic is characterized by the interval, \([ (n \cdot f_1) - 10Hz, (n \cdot f_1) + 10Hz ]\).

Problems resulting from harmonics include (among others) (Mahmoud et al., 1983):

- Inductive interference with telecommunication systems;
- Capacitor failure from dielectric breakdown or reactive power overload;
- Excessive losses in - and heating of - induction and synchronous machines;
- Dielectric instability of insulated cables resulting from harmonic overvoltages on the system.

Effects of harmonics in capacitors include: (i) capacitor overheating, (ii) overvoltage in the capacitor, and (iii) losses in the capacitor.

Among the harmonics control techniques we can cite (Gönen, 1986): (1) strategically identify the location of the capacitors for installation, (2) select the capacitors size properly, and (3) remove capacitors that act as harmonic sources. Our proposal include the first two control techniques, avoiding the third one. As we shall see in Section 7, the third strategy is not always a good choice.

3 Mathematical modeling

Capacitor placement problem seeks to minimize the cost of power losses and the investment made in capacitors. We can note that both objectives are specified in monetary terms. In fact, the higher the investment in capacitors, the smaller the power loss and, consequently, the smaller the cost of power loss. So, the algorithm needs to find a balanced solution between these two criteria: cost of losses and investment made in capacitors. It is worth mentioning that although this problem could be seen from a multiobjective perspective, we performed an equal-weighted criteria approach.

The entire network loss can be taken from Eq. 4.
\[ f_p(P, Q, V) = \sum_{n \in N} \sum_{a \in A_n} r_a \left( \frac{P_{(n,a)}^2 + Q_{(n,a)}^2}{V_{(n,a)}^2} \right), \] (4)

where \( N \) is the number of graph nodes (since the distribution network is represented by a graph (Cavallucci and Lyra, 1997)), \( A_n \) correspond to the arc set which emanates from node \( n \), \( r_a \) is the resistance in the path \( a \), \( P_{(n,a)} \) and \( Q_{(n,a)} \) are active and reactive power, respectively, flowing through arc \( a \), for a given period of time (1 hour). For the sake of simplicity, we will assume that voltage values \( (V_{(n,a)}) \) are approximately 1 pu, for all network nodes. Now, reformulating Eq. 4 to include capacitors, the network losses are defined as

\[ f_p(P, Q, \bar{Q}) = \sum_{n \in N} \sum_{a \in A_n} r_a (P_{(n,a)}^2 + Q_{(n,a)}^2 - \bar{Q}_{(n,a)}^2). \] (5)

In this work, the cost of the entire network loss is calculated for a period of one year (8760 hours). From this, we are able to define the optimization problem to be solved, which can be described by Eq. 6.

\[
\text{minimize } \sum_{u_n} \left\{ 8.76 \cdot \text{Cost} \cdot f_p(P, Q, \bar{Q}) + \sum_{n \in N} f_c(u_n) \right\} \\
\text{subject to } \begin{align*}
    P_{n-1} &= P_{L_n} + \sum_{a \in A_n} P_a \\
    Q_{n-1} &= Q_{L_n} - Q_n + \sum_{a \in A_n} Q_a \\
    V_a &\approx 1 \text{ p.u.} \\
    \bar{Q} &\in \Omega \bar{Q} \\
    h_i &\text{ is even, } \forall i = 1, 2, ..., N
\end{align*} \] (6)

where \( \bar{Q}_n \) is the installed capacitor at the location \( n \), \( \text{Cost} \) is the energy price (in U$) per MWh, \( h_i \) is the harmonic on the \( i \)-th network bar, that will be better discussed in Section 5, and \( f_c(u) \) is amortised capacitor cost. In the problem formulation, Eq. 6, the 8.76 value concerns the number of hours in a year (8760) divided by 1000, since losses are measured in kWh and the energy price in U$/MWh.

The amortised capacitor cost is given by:

\[ f_c(u_n) = \begin{cases} 
    \frac{i \cdot g_c(u_n)}{1 - 1/1 + i k}, & \text{if there is a capacitor in } n \\
    0, & \text{otherwise}
\end{cases} \] (7)

The capacitors cost described in Eq. 6 corresponds to the total cost subtracted by annual gain. So, it is necessary to define an amortization constant \( k \) for the equipment and an interest annual rate \( i \) (Eq. 7). The period of recovery generally corresponds to the useful life of the equipment. In our work we have adopted used an annual rate equal to 0.12 (12%) and a period \( k \) equal to five years.
For the cost of the capacitors $g_c(u_n)$ it was used the following table of capacitor available for installation ($\Omega_Q$ set):

Table 1: Types of capacitor available for installation.

| Type ($u_n$) | Size (kvar) | Cost (U$) | Cost/kvar |
|--------------|-------------|-----------|-----------|
| 1            | 150         | 1498      | 10,00     |
| 2            | 300         | 1604      | 5,35      |
| 3            | 450         | 1620      | 3,60      |
| 4            | 600         | 1823      | 3,04      |
| 5            | 900         | 2550      | 2,83      |
| 6            | 1200        | 2955      | 2,46      |

The resonance constraint is incorporated into the problem by means of a frequency scanning technique, looking for odd frequencies, given that they are commonly found in distribution networks. This procedure is better explained in Section 5.

4 Extremal Optimization

Extremal optimization (EO) (Boettcher and Percus, 1999, 2000) is a general-purpose local search heuristic based upon recent progress in understanding far-from-equilibrium phenomena in terms of self-organized criticality (SOC) (Boettcher and Percus, 2002). The dynamic of EO was inspired by self-organized criticality, a concept introduced to describe emergent complexity in physical systems, where an optimized structure emerges naturally by simple selection against the extremelly bad. EO method, as well as Simulated Annealing (SA) (Kirkpatrick et al., 1983) and Genetic Algorithm (GA) (Goldberg, 1989), are inspired by observations of natural systems.

Unlike GA, which is a population algorithm, EO handles only one solution at a time and seeks to improve the quality of this solution through local perturbations. Originally, this algorithm was proposed to deal with combinatorial optimization problems, particularly problems which can be represented by a graph. In these applications, EO has been shown competitive with more elaborate general-purpose heuristics on testbeds of constrained optimization problems with up to $10^5$ variables, such as bipartitioning, coloring, and satisfiability (Boettcher and Percus, 2002).

In a graph representation of CPP, variables are nodes and the influence between variables are represented by the arcs. So, a node perturbation will affect directly its neighbors (parent and children in the case of a tree).

In evolutionary algorithms, a quality measure is assigned to each solution, called fitness. Differently from these approaches, EO assigns a fitness to each variable (although that is not essential (Boettcher and Percus, 2000)), $\lambda_i$, being the total cost of solution, $C(S)$, obtained as follows

$$C(S) = \sum_{i=1}^{n} \lambda_i.$$  \hspace{1cm} (8)

As aforementioned, perturbations are made in a selected variable, the one with the smallest fitness value (maximization problem). Due to its influence on the neighboring variables (neighbor graph
nodes), this perturbation will also reflect in these variables. The pseudocode of the basic EO algorithm is presented in Algorithm 1.

Algorithm 1: Framework of Extremal Optimization.

Result: $S_{\text{best}}$ and $C(S_{\text{best}})$.

\begin{algorithm}
\begin{algorithmic}[1]
\State \textbf{begin}
\State Define an initial solution $S$; and set $S_{\text{best}} = S$
\State \textbf{repeat}
\State \hspace{1em} Evaluate $\lambda_i$ for each variable $x_i$
\State \hspace{1em} Find $j$ satisfying $\lambda_j \leq \lambda_i$, for all $i$
\State \hspace{1em} Choose $S' \in N_S$ such that $x_j$ must change
\State \hspace{1em} $S \leftarrow S'$
\State \hspace{1em} \textbf{if} $C(S) < C(S_{\text{best}})$ \textbf{then}
\State \hspace{2em} $S_{\text{best}} \leftarrow S$
\State \textbf{until} stopping condition met
\State \textbf{end}
\end{algorithmic}
\end{algorithm}

An EO variation was proposed by (Boettcher and Percus, 2001), such that the selection of the variable with the worst fitness is not done deterministically, but follows a probability distribution, favoring the worst variables, but also providing a chance to the others. A power law probability distribution was employed to perform this task, given by

$$P(k) \propto k^{-\tau} \quad (1 \leq k \leq n),$$

where $\tau \in \mathbb{R}$ is a probability distribution parameter. This variation was called $\tau$-EO. For $\tau = 0$, the algorithm is a local-random search algorithm. On the other hand, for $\tau \to \infty$, this is a deterministic local search algorithm, where the variable with the worst fitness is always selected to be updated. For the traveling salesman problem, $\tau$ values between 1.6 and 2 achieved good results (Boettcher and Percus, 2002).

Another variation of EO, proposed by (Yu-Wang Chen and Chen, 2007), was adopted here. In this method, besides employing a probability distribution to select a variable to be updated, another power law distribution is applied to select, among a set of neighboring candidate solutions, which one will replace the current solution. This approach lead to a more informative EO version and, as a result, it is possible to find good solutions faster than the original version.

5 EO approach to resonance constrained CPP

In this section, we describe an extremal optimization algorithm to deal with resonance constrained CPP. The pseudocode of this proposal is shown in Algorithm 2.

The main reason to employ an extremal optimization algorithm, instead of any other metaheuristics, is due to the fact that EO was initially developed to large combinatorial optimization problems which can be represent by a graph, that is precisely the inherent nature of CPP. Another reason,
which will become clear later, is the reduction in computational resource when handling resonance constraints.

### Algorithm 2: Proposed algorithm.

**Result:** $S_{best}$ and $C(S_{best})$

```plaintext
begin
  Set the initial solution, $S$, as a null vector
  $S_{best} \leftarrow S$
  Calculate fitness, $\lambda_i$, of each node (variable)
  repeat
    Sort variables by increasing fitness
    Select a variable by following a power law pdf
    Generate neighbors from the selected variable
    Calculate $\lambda_i$ and total cost of all neighbors
    Sort neighbors by decreasing fitness
    Select a neighbor by following a power law pdf
    Update $S_{best}$ if needed
  until stopping criterion is not satisfied;
end
```

Solutions are represented by an integer-valued vectors belonging to the interval $[0,6]$, being 0 if there is no capacitor in that network location and 1 to 6 if it has one of those six types of capacitor shown in Table 1.

For the initial solution we assume that there is no capacitor installed, that is, the initial solution is an $n$-dimensional vector consisting of all zeros, where $n$ is the number of nodes in the tree, representing the distribution network.

For each node of the tree, the cost function is evaluated. The losses $f_p(\cdot)$ of a given node are accumulated up to this network point, which is calculated by a power-flow estimation algorithm. Figure 1 shows how the losses of a node is calculated. The idea is to isolate the subnetwork rooted by the interested node and calculate the losses for this subnetwork.

In this work, a simplified version of the power-flow estimation algorithm proposed by Baran and Wu (Baran and Wu, 1989) was used.

![Figure 1: Subnetwork used to compute the node loss.](image-url)
After the cost function be evaluated for each node, they are sorted by increasing fitness and one node is chosen, following a power law probability distribution (Eq. 9), to be perturbated. Perturbation mechanism is viewed as a local exploration of the current solution, generating a set of neighboring candidate solutions \( N_S \), which are slightly different from \( S \).

At this point the resonance control procedure is applied. From the selected node, its neighborhood is generated by Algorithm 3.

**Algorithm 3:** Generate neighbors.

**Input:** \( i, S \)

**Result:** \( N_S \).

1. \[ N_S \leftarrow \emptyset \]
2. \[ \text{if } S[i] \neq 0 \text{ then} \]
3. \[ S'[i] \leftarrow 0 \]
4. \[ N_S \leftarrow N_S \cup S' \]
5. \[ \text{if } S[i] \neq 0 \text{ then} \]
6. \[ S'[i] \leftarrow \min(S[i] + 1, 6) \]
7. \[ N_S \leftarrow N_S \cup S' \]
8. \[ \text{if } S[i] \neq 0 \text{ then} \]
9. \[ S'[i] \leftarrow \max(S[i] - 1, 0) \]
10. \[ N_S \leftarrow N_S \cup S' \]
11. \[ \text{if } S[i] = 0 \text{ then} \]
12. \[ S'[i] \leftarrow \text{rand.integer}(1, 6) \]
13. \[ N_S \leftarrow N_S \cup S' \]
14. \[ \text{if } S[i] \neq 0 \text{ then} \]
15. \[ S'[\text{parent}_i] \leftarrow S[i] \]
16. \[ S'[i] \leftarrow 0 \]
17. \[ N_S \leftarrow N_S \cup S' \]
18. \[ \text{if } S[i] \neq 0 \text{ then} \]
19. \[ \text{for } k \text{ in children do} \]
20. \[ S''[k] \leftarrow S[i] \]
21. \[ S'[i] \leftarrow 0 \]
22. \[ N_S \leftarrow N_S \cup S' \]

It can be seen that there is a minimum and maximum quantity of solutions in its neighborhood. When there is no capacitor allocated in that position, just one neighbor is generated (lines 14-17). Otherwise, we get the maximum number of neighbors, \( 4+k \), where \( k \) is the number of descendant nodes (children).

A solution is only inserted in the neighbor set if, and only if, the parallel resonance constraint is satisfied. That is a penalty function constraint-handling method, which is known as death-penalty approach (Mezura-Montes and Coello, 2011). Even though there are many methods to handle constraints in nature-inspired algorithms, death-penalty is a very simple one and, as we will see in the results section, it has reached good performance. Algorithm 4 shows the routine developed to
Algorithm 4: Check resonance constraint routine.

Input: $n$
Result: $satisfy$

1. begin
2. $f_p = f_1 \cdot \sqrt{S_{cc}(n)/Q(n)}$
3. $h = \text{round}(f_p/f_1)$
4. if $h$ is even then
5. \hspace{1em} $satisfy = \text{True}$
6. \hspace{1em} $satisfy = \text{False}$

A key point of our extremal optimization approach is that only one network node has to be verified at each generation, unlike a genetic algorithm after the application of a uniform crossover operator, for example, in which a high number of nodes must be checked. Thus, the analysis over resonance constraint is made in only one network point whatever the network size. This may save a substantial computational resource, mainly in real power distribution networks, composed of thousands of nodes. That is a significant advantage of extremal optimization approaches in relation to another metaheuristic.

Once the set of neighboring solutions has been determined, they are evaluated and sorted by decreasing total cost $C(\cdot)$ and, then, one of these neighbors is selected to replace the current solution $S$, following another power law distribution, defined as (Guo-Qiang Zeng and Mao, 2010)

$$P(k) \propto e^{-\mu k} \quad (1 \leq k \leq n),$$

where $\mu \in \mathbb{R}_+$ is a distribution parameter.

Update the best solution found so far, $S_{best}$, if necessary, and check the stopping criterion.

6 Experiments

Our algorithm was applied to a power distribution network initially described in Baran and Wu (Baran and Wu, 1989). This network is composed of 33 nodes and 34 arcs. Although of small size, that network was widely used in the literature to compare performance of algorithms designed to cope with power distribution system problems, like CPP and network reconfiguration (Madeiro et al., 2011; Jeon et al., 2002; Jeon and Kim, 2004). Baran and Wu (BW) network is illustrated in Figure 2.

The performance of the considered algorithms was analyzed in relation to the number of function evaluations (FEs), instead of number of generations. This is due to the fact that populational algorithms, like GA, execute a large amount of function evaluations per generation, depending on the population size, whereas single individual algorithms, such as EO, perform a reduced amount of function evaluations. Function evaluation phase is very expensive in this type of problem, since it is necessary to run a power-flow estimation procedure for each candidate solution.
6.1 Comparative analysis

To comparatively analyze the performance of our EO method, a memetic algorithm proposed by (Mendes et al., 2005), designed to deal with CPP, was implemented. Despite not taking the resonance constraint into account, the memetic algorithm has reached good results for real power distribution networks. To get a final feasible solution, some post-optimization strategies (described in what follows) are applied, if the final solution does not satisfies the resonance constraints.

This memetic algorithm is a genetic algorithm that uses a hierarchically tree-structured population, composed of 13 individuals and a local search procedure, applied to the best individual in the population (placed at the tree root). Uniform crossover and punctual mutation operators are employed.

Some of the commonly used strategies (in practice) to repair a final unfeasible solution for CPP is described below. These strategies was named STRTG1, STRTG2 and STRTG3, respectively.

- **STRTG1**: Removing capacitors that has entered into resonance with the distribution network;
- **STRTG2**: Shifting capacitors that has entered into resonance with the network to their respective parents;
- **STRTG3**: Shifting capacitors that has entered into resonance with the network to their respective children.

If the constraint is still not satisfied, these solutions are dropped from analysis. Otherwise, their gains will be compared with the one achieved by our algorithm.

It is worth mentioning that STRTG1 always will return a feasible solution, whereas in the remaining two it is not guaranteed.

Both algorithms and the power-flow estimation procedure were implemented in Python 2.7 using Numpy. Simulations were performed on a Intel Core tm 2 Quad Q6600 @ 2.40 Ghz and 2 GB RAM.
7 Results and Discussion

In our simulations the following parameter values were used for GA (the same values used by Mendes et al. (Mendes et al., 2005)): $rate_{cross} = 1.5$, $p_{mut} = 0.1$ and structured population with 13 individuals. In the case of EO, the values (defined by a grid search procedure) are $\tau = 2$ and $\mu = 0.5$. For both algorithms, the number of fitness evaluations was limited in 50,000 and the results were analyzed over 30 independent runs.

Table 2 shows the results in terms of mean and standard deviation of the amount of saved money when using the best solution reached by each algorithm, varying linearly the energy price, from 50 to 150 dollars per MWh. The absence of results for MA+STRTG3 is because this approach was not able to produce feasible solutions.

Table 2: Mean and standard deviation of the amount of saved money for each algorithm when varying the energy price.

| Energy Price (U$) | EO             | MA+STRTG1     | MA+STRTG2     | MA+STRTG3     |
|------------------|----------------|---------------|---------------|---------------|
| 50               | 22,573.80 (± 229.01) | 17,643.85 (± 9.87) | 19,902.41 (± 9.87) | -             |
| 60               | 27,591.14 (± 268.27) | 21,424.15 (± 7.2e-12) | 24,217.54 (± 1.4e-11) | -             |
| 70               | 32,547.35 (± 305.18) | 25,200.06 (± 13.82) | 28,528.27 (± 13.82) | -             |
| 80               | 37,419.25 (± 289.14) | 28,967.14 (± 19.74) | 32,830.17 (± 19.74) | -             |
| 90               | 42,354.99 (± 345.82) | 32,770.64 (± 4.74) | 37,168.50 (± 4.74) | -             |
| 100              | 47,536.34 (± 364.21) | 36,597.07 (± 6.37) | 41,529.75 (± 6.37) | -             |
| 110              | 52,332.52 (± 425.12) | 40,421.99 (± 5.90) | 45,889.49 (± 5.90) | -             |
| 120              | 57,468.45 (± 356.68) | 44,248.70 (± 6.19) | 50,251.03 (± 6.19) | -             |
| 130              | 62,285.08 (± 413.51) | 48,074.14 (± 6.85) | 54,611.30 (± 6.85) | -             |
| 140              | 67,365.68 (± 473.19) | 51,899.50 (± 7.50) | 58,971.48 (± 7.50) | -             |
| 150              | 72,301.80 (± 554.78) | 55,728.08 (± 6.94) | 63,334.88 (± 6.94) | -             |

It can be seen that our proposal has reached better feasible solutions than memetic algorithm with post-optimization repair procedure, for all problem configurations. The difference in performance between algorithms becomes higher when energy price increases.

A statistical comparison between these results was done by means of the $t$-test. For all energy prices, the proposed EO-based algorithm statistically outperformed the other ones with $p$-value around 1e-50, i.e., the equality hypothesis is utterly rejected.

The obtained results gives us some evidences that STRTG3 is not a good post-optimization repair strategy, whereas STRTG1 and STRTG2 have succeeded in getting feasible solutions in all cases. Among the strategies 1 and 2, shifting capacitors to its parents (STRTG2) seems to be the more promising one, once it has reached better solutions for all situations.

Figures (3a), (3b), and (3c) depicts the mean and standard deviation of the quantity of each capacitor types allocated by each algorithm, varying the energy price. Due to the fact that both, STRTG1 and STRTG2, are slight modification of the same (unfeasible) solution, their curves have a quite similar behavior.

As we can see in these plots, the number of capacitors installed by EO is higher, in general, than the number proposed by the three other approaches. In addition, its solutions are composed of a larger variety of capacitor types in relation to the solutions of the other approaches. It can might be the result of a better exploration of the search space.
From now we will focus on the behavior of the algorithms when capacitor prices are changed. It is expected that the number of allocated capacitors will reduce as the capacitor price increases. Figures (4a), (4b), and (4c) shows the mean and standard deviation of the number of capacitors allocated by each type. We vary the capacitor prices from -20 to 20 percent, in other words, we construct scenarios of reduction and increase in the capacitor prices.

In contrast to what was expected, the number of allocated capacitors did not change significantly as the capacitor prices increase. It shows signs of robustness of the algorithms, although more simulations, mainly considering real large-sized power distribution networks, are required.

The EO algorithm performs only one slight modification of the current solution per step. So, generally, more steps are necessary to reach high-quality solutions, as might be expected. However, the final number of fitness evaluations is still smaller than the one required by the population-based memetic algorithm, even adopting a structured population.

A solution returned by the EO algorithm is illustrated in Figure 5.
Figure 4: Quantity of each type of capacitors allocated by the algorithms varying the capacitors cost.

Figure 5: A solution obtained by the EO algorithm.
8 Conclusion

It is known that the proper installation of capacitors brings some benefits to power distribution networks, such as maintenance of network stability and reduction of loss due to reactive currents. However, this procedure could allocate capacitors which become harmonic sources in the network. In this work an Extremal Optimization based algorithm was proposed to tackle the optimal capacitor placement problem, besides including, in a parsimonious way, parallel resonance constraints. The results showed that this approach reaches better performance when compared with another one which does not take resonance constraint into account, and attempts to repair candidate solutions by means of some post-optimization procedure. Assuming that the optimal solution of the unconstrained problem was found, trying to make it feasible, using local information, does not guarantee its optimality in the constrained problem.

It should also be interesting in future works to study new ways to generate neighbors, maybe using some heuristics to create good solutions. Another investigation is the behavior of our methodology when dealing with real large-sized power distribution networks, in order to account for its scalability. As mentioned earlier, it is possible to analyze the balance between cost of losses and investment made in capacitors from a multiobjective perspective, this will also be the focus of future works.

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