Index Cohesive Force Analysis Reveals That the US Market Became Prone to Systemic Collapses Since 2002

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Abstract

Background: The 2007–2009 financial crisis, and its fallout, has strongly emphasized the need to define new ways and measures to study and assess the stock market dynamics.

Methodology/Principal Findings: The S&P500 dynamics during 4/1999–4/2010 is investigated in terms of the index cohesive force (ICF - the balance between the stock correlations and the partial correlations after subtraction of the index contribution), and the Eigenvalue entropy of the stock correlation matrices. We found a rapid market transition at the end of 2001 from a flexible state of low ICF into a stiff (nonflexible) state of high ICF that is prone to market systemic collapses. The stiff state is also marked by strong effect of the market index on the stock-stock correlations as well as bursts of high stock correlations reminiscent of epileptic brain activity.

Conclusions/Significance: The market dynamical states, stability and transition between economic states was studies using new quantitative measures. Doing so shed new light on the origin and nature of the current crisis. The new approach is likely to be applicable to other classes of complex systems from gene networks to the human brain.

Introduction

The current financial crisis began with the collapse of the subprime bubble at the end of 2007 [1,2], and then spread to the global financial markets and economies worldwide. In the past, and in the aftermath of the crisis, much work has been devoted to the study and characterization of financial bubbles [1,3,4,5,6,7,8,9]. In a recent study, Sornette et al. [1] have presented a general framework in which they propose that the fundamental cause of the crisis was in fact an accumulation of several bubbles in the markets, and the interplay between these bubbles.

The formation of bubbles in the markets is followed by a strong herding phenomenon amongst traders [9], and the burst of these bubbles is accompanied by strong synchrony in the markets reminiscent of epileptic seizures. For example, Lillo et al. [10,11] have investigated the dynamics of markets following crashes. Such synchrony in the markets can be used as a predictive measure for the formation of bubbles, and more importantly, for the burst of such bubbles. As such, it is crucial to develop new quantitative measures to fully capture, characterize and understand the market dynamical states, stability and transition between economic states. Currently, in this regard, much work is focused on the analysis of zero lagged [12] or higher-order lagged correlations [13], a detrending approach to the study of cross correlations [14,15,16], and other measures to study co-movement and synchronization in stock markets [17,18].

Here, we use a new, physics motivated, analysis framework to investigate the dynamics of markets, during the past decade. We show that the fragility of the market could be detected as early as the beginning of 2002, when the market dynamics went through a rapid change that was marked by a jump in the index cohesive force (ICF), and a decline in the correlation Eigenvalue entropy. This transition in the market dynamical state created a significant change in the structure of the market, due to an abnormal dominance of the market index on the stock correlations. The outcome was a rapid transition into a stiff market state that lacked a sufficient degree of freedom and internal flexibility of response to extreme changes. Hence, the index dominance rendered the market prone so systemic collapses as in the case of the sub-prime crisis.

We investigated the time dynamics of the S&P500 index, and 418 of its constituting stocks (not all 500 stocks were traded for the entire time period), during the last decade – from April 1999 to April 2010 (see also Text S1, for full description of the dataset). The investigations were carried out in terms of the index cohesive force (ICF) - the balance between the raw stock correlations that include the index effect and the residual stock correlations (or partial correlations) after subtraction of the index effect [19,20]. The ICF provides a means to identify structural changes in the market, which significantly alter the stability of these markets. For
additional assessments of the results we also inspected the time evolvement of the correlation entropy - the Eigenvalue entropy [19,21,22] of the matrices of stock correlations, during the last decade.

Methods

Raw Stock Correlations

The similarity between stock price changes is commonly calculated by the Pearson’s correlation coefficient [20]. The raw stock correlations [20,23] are calculated for time series of the log of the daily return, given by:

\[ r_i(t) = \log[P_i(t)] - \log[P_i(t-1)] \]  

(1)

Where \( P_i(t) \) is the daily adjusted closing price of stock \( i \) at day \( t \). The raw stock correlations are calculated using the Pearson’s correlation coefficient \( C_{ij} \) between every pair of stocks \( i \) and \( j \), where

\[ C_{ij} = \frac{\langle (r_i - \langle r_i \rangle)(r_j - \langle r_j \rangle) \rangle}{\sigma_i \sigma_j} \]  

(2)

\( \langle \rangle \) denotes average, and \( \sigma \) are the standard deviations (STD).

Residual Correlations

Recently, we have made use of partial correlations to calculate the residual correlation between stocks, after removing the affect of the index [20]. Partial correlation is a powerful tool to investigate how the correlation between two stocks depend on the correlation of each of the stocks with a third mediating stock or with the index as is considered here. The residual, or partial, correlation \( \rho_{ij|m} \) between stocks \( i \) and \( j \), using the Index (\( m \)) as the mediating variable is defined by [19,20,24]

\[ \rho_{ij|m} = \frac{C_{ij} - C_{im}C_{jm}}{\sqrt{(1-C^2_{im})(1-C^2_{jm})}} \]  

(3)

Note that according to this definition, \( \rho_{ij|m} \), can be viewed as the residual correlation between stocks \( i \) and \( j \), after subtraction of the contribution of the correlation between each of the stocks with the Index.

The index cohesive force

Recently, we have shown that the market index has a cohesive effect on the dynamics of the stock correlations [20]. This refers to the observed affect the index has on stock correlations, where we have found that larger changes of the index result in higher stock correlations, and as such more cohesive [20]. Here we expand this analysis and introduce a quantitative measure of the index cohesive force. We define \( ICF(\tau) \) - the index cohesive force calculated over a time window \( \tau \), as a measure of the balance between the raw and residual correlations given by,

\[ ICF(\tau) = \frac{\langle C_{ij} \rangle_{\tau}}{\langle \rho_{ij|m} \rangle_{\tau}} \]  

(4)

where \( \tau \) the time window, during which the average correlation and average residual correlation are calculated, denoted by \( \langle \rangle_{\tau} \). The size of the time window is selected following the considerations presented further below and in the Text S2 (see also Figure S7).

Eigenvalue entropy

To further asses the market stiffness, we computed the eigenvalue (spectral) entropy of the raw correlation matrices. Qualitatively, the entropy of a system refers to the changes in the status quo of the system, and is used as a measure for the order and information content of the system. The spectral entropy [19,21,22,25,26], \( SE \), is defined as

\[ SE = -\frac{1}{\log(N)} \sum_{i=1}^{N} \Omega_i \log[\Omega_i] \]  

(5)

where \( \Omega_i \) - the normalized eigenvalues \( \lambda_i \) of the diagonalized matrix (correlation matrix) - are defined as

\[ \Omega_i = \frac{\lambda_i^2}{\sum_{i=1}^{N} \lambda_i^2} \]  

(6)

Note that the \( 1/\log(N) \) normalization was incorporated to ensure that \( SE = 1 \) for the maximum entropy limit of flat spectra (equal eigenvalues). We associate the market stiffness with one minus the SE [19,21,22,25,26].

Results

The average raw correlation between stocks has been investigated in the past [27,28,29,30], with the focus being on large time windows (200 to 500 days) to reduce the statistical variations. Here we selected a shorter, 22 trading days (corresponding to one work month), time window. We validated that while these short time windows retained limited variations (as shown by the results), they are successful in capturing short time events in the market dynamics. Such short time localized events are averaged out and cannot be deciphered when long time windows are used. In particular, we will show that using these short time windows enabled us to reveal changes in the index cohesive force that are very rapid and of high magnitude (see also Text S2 and Figure S7).

Time dynamics of the raw and residual correlations and market stiffness

We begin our investigation by studying the dynamics of the stocks’ raw correlations (Figure 1B) and residual correlations (Figure 1C), in comparison to the dynamics of the S&P500 index (Figure 1A). Such analysis reveals a transition in the market, taking place at the end of 2001. Following the transition, the market entered into a state dominated by the index as is reflected by the very small residual correlations in the new dynamical state. This state is characterized by an abnormal dominance of the market index, and a state in which the effect other processes such as the influence of different economic sectors is drastically reduced. We propose, in light of the recent global financial events, that the outcome is that the strong index influence rendered the market into a stuff state that is less adaptable to financial changes and therefore is more prone to crises. In other words, being a complex system [20,31], when the average interactions between the market stocks becomes very large, the market becomes inflexible and more sensitive to external changes and thus more prone to crises (see Text S1 and Figures S1, S2, S3, S4, S5 for validation tests of the results,
and Text S3 and Figures S8, S9, S10 for error estimation of the measures used to quantify the dynamics of correlations).

Market seizure-like behavior

The anomalous dominance of the index and the market dangerous stiffness of this market state since the end of 2001, is manifested by the emergence of market seizure-like behavior - bursts of very high stock raw correlations that usually coincide with local minima in the index (Figure 1B). Performing our analysis using longer time windows resulted in qualitatively similar results, in which the transition in the market was still captured, while the localized bursts of correlation were no longer present.

Dynamics of the index cohesive force

In Figure 2 we present the time evolution of the ICF, versus the average stocks-index correlations. In the left panel we use the same coloring scheme as in Figure 1A. The results well depict the significant difference between the two market states. In the right panel of Figure 2, we highlight the time period of 2010, using a color scheme from light yellow at the beginning of the year to black at the end of April. Using this color code, we observe that during early 2010 the market dynamics moved back towards the stable state, but this trend was reversed at the end of March (see also Text S1 and Figure S1).

To further assess the current state of the market, we calculated the ICF for the entire year of 2010. In Figure 3A we present the time evolution of the ICF for 2010. We divide the entire year into 5 periods, based on the changes in the ICF. As was observed in Figure 2B, we find a drop in the ICF at the beginning of 2010 (blue circle), followed by a dramatic jump in the ICF (green circle). In addition to the strong peak in the ICF observed for April 2010, we observe additional somewhat weaker peaks, in June and August of 2010. Finally, as presented in Figure 2B, we compare the ICF to the average stock-index correlation, for the entire year of 2010 (Figure 3B, color coded according to Figure 3A). We note that in general, the year of 2010 was dominated by high values of the ICF, which remains high at the end of the year. Furthermore, comparing Figure 3B to Figure 2A, we observe that the market is still in the abnormal stiff state so it continues to be prone to systemic collapses.
Reflections on the widely used systemic risk parameter

In finance, the capital asset pricing model (CAPM) [32,33] is used to determine a theoretically appropriate required rate of return of an asset, if that asset is to be added to an already well-diversified portfolio, given that asset’s non-diversifiable risk. The model takes into account the asset’s sensitivity to non-diversifiable risk (also known as systematic risk or market risk), often represented by the systemic risk parameter beta ($\beta$) in the financial industry, as well as the expected return of the market and the expected return of a theoretical risk-free asset. The correlation $C_{(i,m)}$ between the return of the given stock $i$ and the daily return of the market index $r_m(t)$, is similar to $\beta_i$ - the systematic risk parameter of this stock which is defined within the security characteristic line (SCL) theory [32,34,35]. More specifically, using these parameters, the return of the asset on the return of the index is given by,

$$r_i(t) = \alpha_i + \beta_i \cdot r_m(t) + \varepsilon_i(t)$$

(7)

Where $\varepsilon_i(t)$ is a random variable and the regression parameters $\alpha_i$ and $\beta_i$ are given by:

$$\beta_i = \frac{\text{cov}(r_i,r_m)}{\text{var}(r_m)} = C(i,m) \cdot \frac{\sigma_i}{\sigma_m}$$

(8)

According to these definitions, the residual correlation $\rho(i,j|m)$ can be viewed as the correlation between the residuals $\varepsilon_i(t)$, after removing the dependency of the given stock on the index. In Figure 4 we show that the average of the systematic risk $\langle \beta_i \rangle$ over all stocks (blue curve) differs from the average of the stock-index correlations $\langle C(i,m) \rangle$ (red curve). As in the case of the average correlation, we observe a jump in $\langle \beta_i \rangle$ at the beginning of 2002. However, we did not decipher a trend reverse in the value of $\langle \beta_i \rangle$ as we found for the ICF during the first months of 2010. We note that in a market which behaves as described by the Capital Asset Pricing Model (CAPM) [36], the $\langle \beta_i \rangle$ of the market should equal 1. In such market, as a result of its definition, the ICF should diverge. Hence, our results might indicate that the market dynamics do not follow the CAPM.

Furthermore, we present in Figure 5 a comparison of the ICF to the $\langle \beta_i \rangle$ as a function of time, color-coded according to Figure 1A. It is evident that the two parameters are very different, especially following the transition at the end of 2001.

Dynamics of Eigenvalue entropy

In Figure 6 we show the evolvement of the spectral entropy during the last decade. We note a sharp fall in the correlation

$$z_i = \langle r_i \rangle - \beta_i \cdot \langle r_m \rangle$$

(9)
entropy at the end of 2001, followed by strong entropy fluctuations. A second significant entropy fall is detected at September 2008 when the index dynamics switched from a negative trend to a positive trend. In Figure 7 we present the values of the entropy versus the average stock-index correlation, color-coded according to Figure 1A. This representation provides additional support that the market underwent a rapid transition between two very different dynamical states.

In a previous paper [19], we have studied how the entropy (information) content of a stock correlation matrix changes, when the market mode is removed. Either analyzing the partial correlation matrix, or looking at the eigenvalue spectrum without the principal eigenvalue can achieve this. Preliminary results (not shown) reveal that removing the principal eigenvalue dramatically influences the spectral entropy; while this is consistent with the rationale, the results are still inconclusive in this regard, and further research is necessary.

**Manifestation of the transition at the end of 2001**

The dramatic differences between the flexible and stiff (inflexible) market states are best manifested in the 3-dimensional scatter plot presented in Figure 8A. The axes of this 3D space are the average Stocks-Index correlations, the average raw correlations, and the average residual correlations. The color code makes transparent the fact that the market dynamical state was not determined by the Index trend (positive or negative): The stiff state started in the midst of a decline in the Index and continued unchanged as the Index trend changed several times. To demonstrate this change, we show in Figure 8B a scatter plot in a different 3D space – the axes are the spectral entropy SE, the average beta coefficient, \( \langle \beta_i \rangle \), and the average residual correlations. Clearly the two scatter plots capture the same phenomenon. We also note that repeating the analysis while using the financial sector Index instead of the S&P500 Index yielded similar results.

**Discussion**

In summary, we presented new approaches to quantify the dynamics of the stock market, using the correlation entropy and the index cohesive force (ICF). The ICF parameter provides a new quantitative measure to investigate different financial states of the market, and the transitions between these states.

Using this approach we discovered a rapid transition in the market dynamical state at the end of 2001. This transition is manifested by a jump in the stock correlations, and a sharp fall in the stock residual correlations. After the transition the market entered into a high ICF stiff state. In this state the index predominantly affects the market dynamics while it shades the effect of other degrees of freedom that can contribute to the market flexibility. Thus, we suggest that during this state the market is highly prone to systematic collapses, even due to relatively small external perturbations, leaving it incapable of coping with crises. This interpretation is consistent with the fact that following the burst of the subprime bubble and the fall of Lehman Brothers [37,38], the market collapsed. It is also reasonable to assume that this rapid transition at the end of 2001 might have been a consequence of the “dot-com” bubble crisis, combined with the traumatic events which took place in the
U.S. at the beginning of the decade and the outcome of the rapid interest cuts [39] and other financial policies employed to overcome the fallout effect of those. One such important financial policy was the implementation of the Decimal Pricing system in the American stock markets. The process of implementation was finalized in the NYSE at January 2001, and in the NASDAQ at April 2001. However, the observed transition in the market uncovered by the ICF took place at December 2001; thus, this change in tick size is one more contributing factor to the transition in the market.

The time period studied here covers the two largest crises that took place in the past decade – the 2000–2001 “.com” crisis, and the 2007–2009 credit crunch crises. During the “.com” period, internet and technological companies were hit hard by the crisis, while other sectors were less affected. This was a local crisis, and the bubble-crash was unevenly distributed among these sectors. This means that the residual correlations during this period should be unusually high, as indeed we found. The credit crunch crisis was a systemic (global) one, which spilled over from the financial sector into all other sectors. As such, the entire market dynamics exhibited high synchrony, as is reflected by the high values of the ICF measure introduced here. As we have shown, during the first part of 2010 there seemed to be a recovery in the markets, which was accompanied by a drop in the values of the ICF. However, a jump in the ICF, and indeed a renewed dangerous process in the ICF, and indeed a renewed dangerous process in the market is still in the abnormal state, and still strongly prone to systematic collapse.

Comparing between the ICF and the risk parameter $\beta$, we found that the ICF provides better representation of the state of the market: While $\beta$ represents the coupling of a given stock to the index, the ICF represents the full state of the system, and can be considered as a system level measure of the state of the market. Probably for this reason, while the ICF revealed that during the first three months of 2010 the market was on its way to recovery, and then the trend was drastically changed at the end of March back into still state, this phenomenon is not revealed by the $\beta$ parameter. Finally, the ICF parameter presented here can be further generalized, such as by a normalization of the volatility, or standard deviation of correlations; we propose this normalized ICF parameter as an Herding factor, which allows a quantification of herding in financial markets. A brief example of this Herd factor is presented in Text S1 (see Figure S6), and we plan on presenting a thorough investigation of this issue in the future.

In conclusion, we propose the ICF as a new system-level parameter, which provides an efficient measure to describe and quantify the market dynamical state, and which can be used as a tool to monitor the stability of stock markets. The stability of the markets is crucial for the world’s economies, thus this tool can be very important to governments and regulation agencies worldwide.

**Supporting Information**

**Figure S1** Comparison of the ICF to the average stock-index correlation, for the period of 2010. The ICF and average correlation were computed for the 500 S&P500 stocks (left) and the 418 S&P500 stocks used for the entire analysis. We use a color code to present the chronological time progression, from dark blue for the beginning of 2010, to dark red, for April 2010. Comparing the two panels, we note that there is a high qualitative similarity between the two.

**Figure S2** Calculation of the ICF for a sub-set of 300 stocks. To validate the results of the ICF for the full dataset, we randomly chose 300 stocks, calculate the average stock, stock-index, and partial correlation, and the ICF. We perform this selection 4 times. The values of the ICF is presented for each of the 4 iterations, using a different color.

**Figure S3** A three-dimensional scatter plot of the market dynamical evolution of stocks belonging to the S&P500 index in the past decade, as presented in Figure 8. We first calculate the average value of the raw, stock-index and partial correlations, over the 4 iterations of random selection of the 300 stock sub-set. The color code used is the same as in Figure 8.

**Figure S4** Eigenvalue entropy versus the average stock-index correlation, as function of time, color coded according to Figure 1A. This is presented for the 300 stock subset, as in Figure S1, S2, S3. We first calculate the average value of the entropy and the stock-index correlation over all 4 iterations.

**Figure S5** Comparison of the ICF calculated using the S&P500 index (red curve) and the ICF calculated using a synthetic index.
The synthetic index was calculated using only the stocks included in the dataset, as a weighted average of these stocks, using their original weights from the S&P500 index. While the ICF calculated using the synthetic index is noisier, the two are qualitatively very similar, with a correlation of 0.65, which is probably strongly affected by the fact that the ICF(synthetic) is much noisier in the pre-2002 period.

**Figure S6** Comparison of the H factor to the ICF, color coded for time according to the code presented in Figure 1A.

**Figure S7** ICF analysis of the S&P500 dataset, using a sliding window of 50, 100, 200, 300, 400, and 500 days. The transition, observed using the 22-day window, is qualitatively observed for all other window sizes, around the same period.

**Figure S8** The average stock correlation (left) and average stock partial correlation (right), as presented in Figure 1A and B respectively, with the addition of error lines. The error lines were estimated using the standard deviation for each parameter separately, marked by a dotted red line.

**Figure S9** The average Beta coefficient, as presented in Figure 4, with the addition of error, estimated using the standard deviation, marked with a dotted red line.

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