INTRODUCTION AND PRELIMINARIES

Fixed point theory is a beautiful mixture of Analysis, Topology and Geometry. Fixed points Theory has been playing a vital role in the study of nonlinear phenomena. In 1922, Banach[1] proved a fixed point theorem, which ensures the existence and uniqueness of a fixed point under appropriate conditions. This result of Banach is known as Banach fixed point theorem and ensures the existence and uniqueness of a fixed point under appropriate conditions. This result of Banach is known as Banach fixed point theorem or contraction mapping principle. In 1987, Guo and Lakshmikantham [2] initiated the notion of coupled fixed point and they established some coupled fixed point results. Afterward, the coupled fixed point theorems in partially ordered metric spaces has been developed by Bhaakar and Lakshmikantham [3]. Subsequently, many authors established coupled fixed point theorems in different spaces (see [4]-[13]).

This work is motivated by the recent work on extension of Banach contraction principle on Bipolar metric spaces, which has been done by Mutlu and Gu’rdal [14]. Also, they investigated some fixed point and coupled fixed point results on this spaces (see [14],[15]).

The aim of this paper is to initiate the study of a common coupled fixed point results for two covariant mappings in Bipolar metric spaces. We have also illustrated the validity of the hypotheses of our results are achieved. Also, we give an illustration which presents the applicability of the achieved results. Moreover, we give an application of nonlinear integral equations as well as Homotopy theory by using fixed point theorems.

2010 AMS Classification:54H25, 47H10, 54E50

Keywords and phrases: Bipolar metric space, ω-compatible mappings, coupled fixed point, common fixed point.

Definition 1.1 ([14]) Let (A, B, d) be a bipolar metric space, \( A \times B \rightarrow [0, \infty) \) is a mapping satisfying the following properties:

\[
\begin{align*}
(B_1) & \quad d(a,b) = 0 \text{ then } a = b \text{ for all } a, b \in A \times B, \\
(B_2) & \quad d(a,b) = 0 \text{ for all } a, b \in A \times B, \\
(B_3) & \quad d(a,b) = d(b,a) \text{ for all } a, b \in A \times B, \\
(B_4) & \quad \text{If } d(a_1,b_1) \leq d(a_1,b_2) + d(a_2,b_1) \text{ for all } a_1, b_1, a_2, b_2 \in A \times B, \\
& \quad \text{Then the mapping } d \text{ is called a bipolar-metric on the pair } (A, B) \\
& \quad \text{and the triple } (A, B, d) \text{ is called a bipolar-metric space.}
\end{align*}
\]

Definition 1.2 ([14]) Assume \((A_1, B_1)\) and \((A_2, B_2)\) as two pairs of sets.

The function \( F : A_1 \cup B_1 \rightarrow A_2 \cup B_2 \) is said to be a covariant map, if \( F(A_1) \subseteq A_2 \) and \( F(B_1) \subseteq B_2 \) and denote this as \( F : (A_1, B_1) \equiv (A_2, B_2) \).

The mapping \( F : A_1 \cup B_1 \rightarrow A_2 \cup B_2 \) is said to be a contravariant map, if \( F(A_1) \subseteq B_2 \) and \( F(B_1) \subseteq A_2 \) and this as \( F : (A_1, B_1) \equiv (A_2, B_2) \).

In particular, if \( d_1 \) and \( d_2 \) are bipolar metrics in \((A_1, B_1)\) and \((A_2, B_2)\) respectively. Then in some times we use the notations \( F : (A_1, B_1, d_1) \equiv (A_2, B_2, d_2) \) and \( F : (A_1, B_1, d_1) \equiv (A_2, B_2, d_2) \).

Definition 1.3. ([11]) Assume \((A, B, d)\) be a bipolar metric space. A point \( v \in A \cup B \) is termed as a left point if \( v \in A \), a right point if \( v \in B \) and a central point if both.

Similarly, a sequence \( (a_n) \) on the set \( A \) and a sequence \( (b_n) \) on the set \( B \) are called a left and right sequence respectively. In a bipolar metric space, sequence is the simple term for a left or right sequence.

A sequence \( (v_n) \) as considered convergent to a point \( v \), if and only if \( v_n \) is a left sequence, \( v \) is a right point and \( \lim_{n \to \infty} d(v_n, v) = 0 \) or \( (v_n) \) is a right sequence, \( v \) is a left point and \( \lim_{n \to \infty} d(v, v_n) = 0 \).

A sequence \((a_n, b_n) \) on \((A, B, d)\) is sequence on the set \( A \times B \). If the sequence \((a_n) \) and \((b_n) \) are convergent, then the sequence \((a_n, b_n) \) is said to be convergent. \((a_n, b_n) \) is Cauchy sequence, if \( \lim_{n \to \infty} (a_n, b_n) = 0 \).

A bipolar metric space is called complete, if every Cauchy sequence is convergent, hence biconvergent.

Definition 1.4 ([15]) Let \((A, B, d)\) be a bipolar metric space, \( F : (A_2, B_2) \rightarrow (A, B) \) be a covariant mapping if \( F(a, b) = a \) and
F(b, a) = b for (a, b) ∈ A2 ∪ B2 then (a, b) is called a coupled fixed point of F.

**MAIN RESULTS**

In this section, we give some common fixed point theorems for two covariant mappings satisfying various contractive conditions in bipolar metric spaces.

**Definition 2.1** Let (A, B) be a bipolar metric space, F : (A2, B2) ⇒ (A, B) and f : (A, B) ⇒ (A, B) be two covariant mappings. An element (a, b) is said to be a coupled coincident point of F and f. If F (a, b) = fa and f (b) = fb, then (a, b) is called a coupled fixed point of F and f.

**Definition 2.2** Let (A, B, D) be a bipolar metric space, F : (A2, B2) ⇒ (A, B) and f : (A, B) ⇒ (A, B) be two covariant mappings. An element (a, b) is said to be a common coupled fixed point of F and f if F (a, b) = fa and f (b, a) = fb.

**Definition 2.3** Let (A, B, D) be a bipolar metric space, F : (A2, B2) ⇒ (A, B) and f : (A, B) ⇒ (A, B) be two covariant mappings. F and f are called ω-compatible if f (F (a, b)) = F (fa, fb) and f (f (b, a)) = F (fb, fa) whenever F (a, b) = fa and f (b, a) = fb.

**Theorem 2.4** Let (A, B, D) be a bipolar metric spaces, suppose that F : (A2, B2) ⇒ (A, B) and f : (A, B) ⇒ (A, B) be a covariant mappings satisfying:

\[
\begin{align*}
\text{(2.4.1)} & \quad d(F(a, b), F(p, q)) \leq \theta \max\{d(fa, fp), d(fb, fq)\} \\
\text{(2.4.2)} & \quad F(A2) \subseteq f(A2) \cap B2, \\
\text{(2.4.3)} & \quad \text{Either } F, f \text{ is } \omega \text{-compatible.} \\
\text{(2.4.4)} & \quad f(A \cup B) \text{ is complete.}
\end{align*}
\]

Then the mappings F : A2 ∪ B2 → A ∪ B and f : A ∪ B → A ∪ B have a unique common fixed point of the form (u, v).

**Proof.** Let a0, b0 ∈ A and p0, q0 ∈ B and from (2.4.2), we construct the bisquence \{(a2n), (b2n), (o2n), (v2n)\} in (A, B) as

\[
\begin{align*}
F(a2n, b2n) &= fa2n+1 = o2n \\
F(p2n, q2n) &= fp2n+1 = v2n \\
F(b2n, a2n) &= fb2n+1 = v2n \\
F(q2n, p2n) &= fq2n+1 = v2n
\end{align*}
\]

for n = 0, 1, 2, ....

Now from (2.4.1), we have

\[
\begin{align*}
d(o2n, v2n + 1) &= d(F(a2n,b2n), F(p2n + 1, q2n + 1)) \\
&\leq \theta \max\{d(fa2n, fp2n + 1), d(fb2n,fq2n + 1)\} \\
&\leq \theta \max\{d(o2n - 1, v2n - 1, o2n, v2n)\} \\
&\leq \theta \max\{d(o2n - 1, v2n - 1, o2n, v2n)\}
\end{align*}
\]

and

\[
\begin{align*}
d(o2n, v2n + 1) &= d(F(b2n,a2n), F(q2n + 1, p2n + 1)) \\
&\leq \theta \max\{d(fb2n, fq2n + 1), d(fa2n,fp2n + 1)\} \\
&\leq \theta \max\{d(o2n - 1, v2n - 1, o2n, v2n)\}
\end{align*}
\]

Combining (2.4.5) and (2.4.6), we get that

\[
\begin{align*}
\text{max}\{d(o2n, v2n + 1), d(o2n, v2n + 1)\} \\
&\leq \theta \max\{d(o2n - 1, v2n), d(o2n - 1, v2n)\} \\
&\leq \theta \max\{d(o2n - 1, v2n), d(o2n - 1, v2n)\}
\end{align*}
\]

Thus \[d(o2n, v2n + 1) \leq \theta \max\{d(o2n - 1, v2n), d(o2n - 1, v2n)\}\]

On the other hand

\[
\begin{align*}
d(o2n + 1, v2n) &= d(F(a2n + 1, b2n + 1), F(p2n,q2n)) \\
&\leq \theta \max\{d(fa2n + 1, fp2n), d(fb2n + 1, fq2n)\} \\
&\leq \theta \max\{d(o2n, v2n - 1), d(o2n, v2n - 1)\}
\end{align*}
\]

Combining (2.4.8) and (2.4.9), we get that

\[
\begin{align*}
\text{max}\{d(o2n + 1, v2n), d(o2n + 1, v2n)\} \\
&\leq \theta \max\{d(o2n - 1, v2n - 1), d(o2n - 1, v2n - 1)\}
\end{align*}
\]

Moreover,

\[
\begin{align*}
d(o2n, v2n) &= d(F(a2n, b2n), F(p2n,q2n)) \\
&\leq \theta \max\{d(fa2n, fp2n), d(fb2n, fq2n)\} \\
&\leq \theta \max\{d(o2n - 1, v2n - 1, o2n, v2n - 1)\}
\end{align*}
\]

Combining (2.4.11) and (2.4.12), we get

\[
\begin{align*}
\text{max}\{d(o2n, v2n), d(o2n, v2n)\} \\
&\leq \theta \max\{d(o2n - 1, v2n - 1), d(o2n - 1, v2n - 1)\}
\end{align*}
\]

Thus \[d(o2n, v2n) \leq \theta \max\{d(o2n - 1, v2n - 1), d(o2n - 1, v2n - 1)\}\]

Thus, d(o2n, v2n) ≤ θ max\{d(o2n - 1, v2n - 1), d(o2n - 1, v2n - 1)\}.
Using the property (B4),
\[
d([\omega_2, \varphi_2]) \leq \sum d([\omega_2, \varphi_2, 1]) + d([\omega_2, \varphi_2, 1, \varphi_2])
\]
and
\[
d([\omega_2, \varphi_2]) \leq \sum d([\omega_2, \varphi_2, 1]) + d([\omega_2, \varphi_2, 1, \varphi_2])
\]
for each \( n, m \in \mathbb{N} \) with \( n < m \).

Therefore, \( d([\omega_2, \varphi_2]) \) converges in \( A \) and \( B \) with
\[
limit_{n \to \infty} [\omega_{2n}, \varphi_{2n}] = \lim_{n \to \infty} ([\omega_{2}, \varphi_{2}]) = 0
\]
Since \( f(A \cup B) \) is a complete subspace of \( (A, B, d) \),
\( (\omega_{2n+1}, \varphi_{2n+1}) \in f(A \cup B) \) converges in the complete bipolar metric space \( (A, B, d) \).
Therefore, there exist \( u, v \in f(A) \) and \( w, z \in f(B) \) with
\[
limit_{n \to \infty} [\omega_{2n+1}, \varphi_{2n+1}] = \lim_{n \to \infty} \omega_{2n} + 1 = z
\]
Since \( f(A \cup B) \) is a complete subspace of \( (A, B, d) \), there exist \( u, v \in f(A) \) and \( w, z \in f(B) \) such that
\[
\lim_{n \to \infty} \omega_{2n+1} = w, \lim_{n \to \infty} \omega_{2n} + 1 = z
\]
From (2.4.11) and (B4), we have
\[
d(f(I, m), w) \leq \max\{d(F(I, m), \omega_{2n+1}), d(F(I, m), \varphi_{2n+1})\}
\]
and
\[
d(f(I, m), w) \leq \max\{d(F(I, m), \omega_{2n+1}), d(F(I, m), \varphi_{2n+1})\}
\]
for all \( n \in \mathbb{N} \).

Therefore,
\[
d(F(I, m), w) = 0 \implies F(I, m) = w \in f \text{r}
\]
Similarly, we can prove that \( F(I, m) = w \in f \text{r} \).
Since \( f(I, m, f) \) are \( \omega \)-compatible mappings, we have
\[
F(u, v) = f(u, v) = f(z, w) = f(z, w)
\]
We prove that \( u = v, v = v \) and \( f(u, v) = f(z, w) = f(z, w) \).

Now
\[
d(fu, \varphi_2) = d(f(u, v), f(v, u)) \leq \max\{d(fu, \varphi_2), f(v, u)\}
\]
and
\[
d(fv, \varphi_2) = d(f(v, u), f(u, v)) \leq \max\{d(fv, \varphi_2), f(u, v)\}
\]
for all \( n \in \mathbb{N} \).

Therefore,
\[
d(fu, \varphi_2) = d(f(u, v), f(v, u)) \leq \max\{d(fu, \varphi_2), f(v, u)\}
\]
and
\[
d(fv, \varphi_2) = d(f(v, u), f(u, v)) \leq \max\{d(fv, \varphi_2), f(u, v)\}
\]
for all \( n \in \mathbb{N} \).

Similarly, if \( u = v \in E2 \) and \( v = v \in E2 \), then we have \( u = u \) and \( v = v \).
Then \( u = v \in A2 \cap B2 \) is unique coupled fixed point of covariant mappings \( F \) and \( f \).

Now we prove the uniqueness, we begin by taking \( u^*, v^* \) in \( A2 \cap B2 \) to be another fixed point of \( f \) and \( f \).
If \( u^*, v^* \) in \( A2 \), then we have
\[
d(u, u) = d(f(u, v), F(u, v)) \leq \max\{d(f(u, v), f(v, u))\}
\]
and
\[
d(v, v) = d(f(v, u), F(v, u)) \leq \max\{d(f(v, u), f(u, v))\}
\]
for all \( n \in \mathbb{N} \).

Therefore, \( d(fu, f(v)) \) and \( d(fv, f(u)) \) are \( \omega \)-compatible mappings, we have
\[
F(u, v) = f(u, v) = f(z, w) = f(z, w)
\]
and
\[
F(v, u) = f(v, u) = f(z, w) = f(z, w)
\]
Finally we will show that $u = v$.

$$d(u, v) = d(F(u, v), F(v, u)) \leq \theta \max \{d(fu, vu), dv, vu\}$$

(2.4.21)

and

$$d(u, v) = d(F(u, v), F(v, u)) \leq \theta \max \{d(fu, vu), d(vu, fu)\}$$

(2.4.22)

Combining (2.4.21) and (2.4.22), we get

$$\max \{d(u, v), d(v, u)\} \leq \theta \max \{d(fu, vu), d(fb, fb)\}$$

for all $a, b \in A$ and $p, q \in B$ with $\theta \in (0, 1)$.

$$(2.9.1) \quad d(F(a, p), F(q, b)) \leq \theta \max \{d(a, q), d(b, p)\}$$

(2.9.2)

$$(a, b) \cup (B \times A) \subseteq f(A \cup B).$$

(2.9.3) Either $f$ is $\omega$-compatible.

(2.9.4) $f(A \cup B)$ is complete.

Then the mappings $F : (A \times B, d) \rightarrow A \cup B$ and $F : A \cup B \rightarrow A \cup B$ have a unique common fixed point of the form $(u, u)$.

**Example 2.5** Let $U \times V$ and $L \times M$ be the set of all $m \times m$ upper and lower triangular matrices over $R$. Define

$$d : U \times V \rightarrow [0, \infty)$$

as

$$d(P, Q) = \sum_{i,j=1}^{m}[p_{ij} - q_{ij}]$$

for all $P = (p_{ij})_{m \times m} \in U \times V$ and $Q = (q_{ij})_{m \times m} \in L \times M$.

Then obviously $(U \times V)$, $(L \times M) \subset (D, d)$ is bipolar metric space.

**Theorem 3.1** Let us consider the integral equation $\alpha(x) = \int_{E_1}^{E_2} (a(x, t) + K_1(x, t)) d\theta + F(x)$. Here, $H = f(0, \delta(0)) + g(0, \sigma(0))$ Where $\{E_1, E_2\} \subset E_1 \cup E_2$ is a Lebesgue measurable set.

Suppose,

i. $K_1 : E_1 \rightarrow E_2 \rightarrow [0, \infty)$, $K_2 : E_1 \rightarrow E_2 \rightarrow (-\infty, 0]$ and $f \in \text{L} \in \text{E} \cup \text{E} \cup \text{E}$ are integrable.

ii. There exists $i, j \in (0, 1)$ such that $0 \leq \sigma(0) - \tau(0) \leq i(\sigma(0) - \tau(0)) \leq g(0, v) - g(0, v) \leq 0$ for $v \in E_1 \cup E_2$ and $i, j \in (0, 1)$.

iii. $\int_{E_1}^{E_2} (a(x, t) + K_1(x, t)) d\theta + F(x)$

then the equation has a unique solution in $\text{L} \cap \text{E} \cap \text{E}$.
\begin{align*}
\lim_{\substack{\alpha \to \beta \\
\gamma \to \xi}} V(K_\alpha + g(\theta, \alpha) - \theta U + \gamma v) &= \sum_{E^1 \cup E^2} K^2(v, \theta) \left( f(\theta, \alpha(v)) + g(\theta, \alpha(v)) \right) d\theta + F(v), \forall e_{E^1} \\
\lim_{\substack{\alpha \to \beta \\
\gamma \to \xi}} V(K_\alpha + g(\theta, \alpha) - \theta U + \gamma v) &= \sum_{E^1 \cup E^2} K^2(v, \theta) \left( f(\theta, \alpha(v)) + g(\theta, \alpha(v)) \right) d\theta - \\
\lim_{\substack{\alpha \to \beta \\
\gamma \to \xi}} V(K_\alpha + g(\theta, \alpha) - \theta U + \gamma v) &= \sum_{E^1 \cup E^2} K^2(v, \theta) \left( f(\theta, \alpha(v)) + g(\theta, \alpha(v)) \right) d\theta
\end{align*}

Now we have, $d(S(\alpha, \beta), S(\xi, \gamma)) = ||S(\alpha, \beta) - S(\xi, \gamma)||_\infty$.

Let us first evaluate the following expression:

\[ |S(\alpha, \beta) - S(\xi, \gamma)(v)| \]

\begin{align*}
&= \left| \int_{E^1 \cup E^2} K^1(v, \theta) f(\theta, \alpha(v)) d\theta \right| \\
&\leq \theta \max \{d(u_0, x_1), d(v_0, y_1)\}
\end{align*}

Therefore, (un, xn) and (vn, yn) are Cauchy bisquence in (U, V) with (κ, ζ) → (κ, ζ) as $n \to \infty$. We will show that (κ, ζ) ∈ X2 ∩ Y.

Since (κ0, ζ0) ∈ (X, Y) for $n = 0, 1, 2, 3, \ldots$, there exists bisequences $\{u_n\}_{n=0}^{\infty}$ and $\{v_n\}_{n=0}^{\infty}$ such that $\|u_{n+1} - u_n\| \to 0$ as $n \to \infty$. We will show that $\{u_n\}_{n=0}^{\infty}$ is a Cauchy sequence in (U, V).

\begin{align*}
\|u_{n+1} - u_n\| &\leq \theta \max \{d(u_0, x_1), d(v_0, y_1)\} \\
&\leq \theta^2 \max \{d(u_{n-2}, x_{n-1}), d(v_{n-2}, y_{n-1})\}
\end{align*}

\begin{align*}
\|u_{n+1} - u_n\| &\leq \theta^3 \max \{d(u_{n-3}, x_{n-2}), d(v_{n-3}, y_{n-2})\}
\end{align*}

\begin{align*}
\|u_{n+1} - u_n\| &\leq \theta^n \max \{d(u_0, x_1), d(v_0, y_1)\}
\end{align*}

Hence, applying Corollary 2.1 we get the desired result.

\section*{APPLICATION TO HOMOTOPY}

\textbf{Theorem 4.1} Let (A, B, d) be complete bipolar metric space, (U, V) be an open subset of (A, B) and (\bar{U}, \bar{V}) be closed subset of (A, B) such that (U, V) ⊂ (\bar{U}, \bar{V}) ⊂ \{0, 1\} → U ⊂ B be an operator with following conditions satisfying:

\begin{align*}
&\text{for each } u, v \in U \cup V \\
&\text{for all } u, v \in \bar{U}, x, y \in \bar{V} \text{ and } x \in [0, 1], \theta \in [0, 1]
\end{align*}

\text{for every } u, v \in \bar{U} \text{ and } x, y \in \bar{V} \text{ and } x, \xi \in [0, 1], \text{ and} \\xi \in [0, 1].

Then $H(\cdot, \cdot)$ has a fixed point $\Rightarrow H(\cdot, \cdot)$ has a fixed point.

\textbf{Proof.} Let the set

\[ X = \{ x \in [0, 1] : u = H(u, v, k), v = H(v, u, k) \text{ for some } (u, v) \in U \cup V \}. \]

\textbf{Consider,}

\[ d(u_0, x_1) = d(H(u_0, v_0, \kappa), H(x_1, y_1, \zeta)), \]

\[ \leq \theta \max \{d(u_0, x_1), d(v_0, y_1)\} \]

\text{And}

\[ d(v_0, y_1) = d(H(v_0, u_0, \kappa), H(v_0, y_1, \zeta)) \]

\text{max} \{d(u_0, x_1), d(v_0, y_1)\}

\text{Thus}

\[ d(u_0, x_1) \leq \theta \max \{d(u_0, x_1), d(v_0, y_1)\} \]

\[ d(v_0, y_1) \leq \theta \max \{d(u_0, x_1), d(v_0, y_1)\} \]

\text{Similarly, we can prove}

\[ d(u_1, x_0) \leq \theta \max \{d(u_0, x_0), d(v_0, y_0)\} \]

\[ d(v_1, y_0) \leq \theta \max \{d(u_0, x_0), d(v_0, y_0)\} \]

\text{Thus}

\[ d(u_1, x_0) \leq \theta \max \{d(u_0, x_0), d(v_0, y_0)\} \]

\[ d(v_1, y_0) \leq \theta \max \{d(u_0, x_0), d(v_0, y_0)\} \]

\text{for each } n, m \in N, n < m \text{ using the property (B4) and (4.6)}, \text{ we have}

\[ d(u_n, x_m) + d(v_n, y_m) \leq \theta \max \{d(u_0, x_1), d(v_0, y_1)\} + d(u_n, x_m) + d(v_n, y_m) \]

\text{Similarly, we can show}

\[ d(u_n, x_m) \leq \theta \max \{d(u_0, x_1), d(v_0, y_1)\} \]

\text{Limit}

\[ d(u_n, x_m) \to 0 \text{ as } n, m \to \infty. \]

\text{Therefore, (u_n, x_n) and (v_n, y_n) are Cauchy bisequence in (U, V).}

\text{By completeness, there exist } \xi, \zeta \in \bar{U} \text{ and } \delta, \eta \in \bar{V} \text{ with}

\[ \lim_{n \to \infty} u_n = \delta, \text{ and } \lim_{n \to \infty} v_n = \eta \]

\text{Then}

\[ \lim_{n \to \infty} u_n = \delta, \text{ and } \lim_{n \to \infty} v_n = \eta \text{ since } H(\cdot, \cdot) \text{ is a fixed point}. \]

\text{Now consider}

\begin{align*}
\text{Proof.} \text{ Let the set } X = \{ x \in [0, 1] : u = H(u, v, k), v = H(v, u, k) \text{ for some } (u, v) \in U \cup V \}. \\
\end{align*}
\[ d(H(\xi, v, \kappa), \delta) \leq d(H(\xi, v, \kappa), x_{n+1}) + d(u_{n+1}, x_{n+1}) + d(u_{n+1}, \delta) \]
\[ \leq d(H(\xi, v, \kappa), H(x_n, y_n, \zeta)) + d(H(u_{n+1}, v_{n+1}, \kappa), H(x_{n+1}, y_{n+1}, \zeta)) \]
\[ + d(H(u_{n+1}, v_{n+1}, \kappa), H(x_{n+1}, y_{n+1}, \zeta)) \]
\[ \leq \theta \max \{d(u_0, x_0), d(v_0, y_0)\} + M|\kappa_n - \zeta_n| + d(u_{n+1}, x_{n+1}) + d(v_{n+1}, y_{n+1}) \]
\[ \leq \theta \max \{d(u_0, x_0), d(v_0, y_0)\} + M|\kappa_n - \zeta_n| + d(u_{n+1}, x_{n+1}) + d(v_{n+1}, y_{n+1}) \]
\[ \leq \theta \max \{d(u_0, x_0), d(v_0, y_0)\} + M|\kappa_n - \zeta_n| + d(u_{n+1}, x_{n+1}) + d(v_{n+1}, y_{n+1}) \]
\[ \leq \theta \max \{d(u_0, x_0), d(v_0, y_0)\} + M|\kappa_n - \zeta_n| + \frac{r}{2} \]
\[ \vdash H(\delta, \xi) = \delta \]
\[ \text{Thus} \ H(\delta, \xi) = \delta \]

The authors contribute equally and significantly in writing this article. All authors read and approved the final manuscript.
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