Effect of Humidity on Concrete Creep Predictions

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Abstract. An accurate estimation of creep strains is mandatory to maintain the stability and integrity of concrete structures. Different parameters and conditions affect concrete creep predictions more precisely the relative humidity. To study its effect, a large database coming from international laboratories and research centers was applied to compare the experimental measurements to the theoretical values predicted by Eurocode 2 model. An inaccurate estimation was detected; therefore, an amelioration of the Eurocode 2 model was elaborated according to the relative humidity parameter. These calculations allow to predict accurately concrete creep and to avoid prejudicial consequences in structures due to creep phenomenon.

1. Introduction

The concrete is a metastable material that “lives in time”. It has a significant time-dependent behaviour. During its lifetime, concrete is subjected to deformations manifested under sustained external load known as creep strains. These strains have an important effect on the stability and integrity of concrete structures. Thus, an accurate estimation of creep strains is required [1, 2]. Different parameters affect concrete creep more precisely the relative humidity which is the scope of this paper. To study its effect, a large worldwide database of creep tests coming from international laboratories and research centers [3] was applied to compare the experimental measurements to the theoretical values predicted by Eurocode 2 (EC2) model using the CEB statistical methods. An inaccurate estimation of the Eurocode 2 creep compliance according to the relative humidity categories is noted.

To overcome this difference, a calibration is required by implementing corrective coefficients to the Eurocode 2 equations. These correction coefficients are quantified by applying the Approximate Bayesian Computation (ABC) method. Using these correction coefficients will help to predict accurately the creep deformations in the design stage and hence, the long-term deflection.

2. Eurocode 2 creep model evaluation

2.1. Eurocode 2 creep model

According to Eurocode 2 model [4], the compliance function can be predicted using the following equation:
\[ J(t, t_0) = \frac{1}{E_{cm}(t_0)} + \frac{\phi(t, t_0)}{E_{cm28}} \]  

(1)

Where; \( J(t, t_0) \) is the compliance function, \( E_{cm28} \) and \( E_{cm}(t_0) \) are the modulus of elasticity of concrete at 28 days after casting and at the loading age \( t_0 \) respectively, and \( \phi(t, t_0) \) is the creep coefficient.

2.2. Creep database

Bažant and Panula were the first researchers that have collected a large worldwide creep and shrinkage database at the Northwestern University (NU) [5]. This database established from American and European institutions consisted of approximately 400 creep tests and 300 shrinkage tests and was included in the papers presenting the BP Model [5]. During the international ACI Fall 1979 Convention, a joint ACI-RILEM committee was organized to extend the Northwestern University database. The pursuit of this work led to the RILEM-ACI 209 database in 1992. After that, in 2008 and 2010, Bažant and Li [6] and Kim [7] have expanded the database at the Northwestern University (NU). The latest database was assembled at NU during 2010-2013 [3]. Information was extracted from many reports, conference proceedings and journal articles. This database is constituted of approximately 1433 creep tests and 1827 shrinkage tests. The tests in the database are performed using different concrete mix composition and placed in various environmental conditions.

2.3. Evaluation methods

The creep compliance \( J(t, t_0) \) which is the time-dependent strain per unit stress is calculated according to the Eurocode 2 model for each test in the database. Then the predicted values are compared to the experimental measurements using the CEB mean deviation (\( M_{CEB} \)), the CEB coefficient of variation (\( V_{CEB} \)), the CEB mean square error (\( F_{CEB} \)), and the BP coefficient of variation (\( \varpi_{BP} \)). In fact, the CEB statistical indicators were suggested by Muller and Hilsdorf in 1990, while the BP coefficient of variation was developed by Bažant and Panula in 1978 [8].

2.3.1. The CEB mean deviation (\( M_{CEB} \)). The \( M_{CEB} \) method calculates an average gap and indicates if a model overestimates or underestimates systematically experimental measurements [8]. It may be calculated using the following formulas:

\[ M_{CEB} = \frac{\sum_{i=1}^{N} M_i}{N} = \frac{\sum_{i=1}^{N} \left( \frac{1}{n} \sum_{j=1}^{n} (\text{Cal}X_{ij} - \text{Obs}X_{ij}) \right)}{N} \]  

(2)

Where; \( \text{Cal}X_{ij} \) refers to the predicted creep compliance at time \( j \) of experiment \( i \), \( \text{Obs}X_{ij} \) refers to the experimental creep compliance at time \( j \) of experiment \( i \), \( n \) is the total number of measurements at time \( j \) of experiment \( i \), \( N \) is the total number of experiments, and \( M_i \) is the mean deviation of experiment \( i \). When \( M_{CEB} \) coefficient is near 1, the results of the predicted compliance are close to the experimental values and the model predicts accurately experimental measurements. If the \( M_{CEB} \) coefficient exceeds 1, this means that the calculated values are bigger than experimental values and the model overestimates experimental measurements. If the \( M_{CEB} \) coefficient is less than 1, then the model underestimates experimental measurements.

2.3.2. The CEB coefficient of variation (\( V_{CEB} \)). The \( V_{CEB} \) method calculates an average coefficient of variation to evaluate a model relatively to the experimental database. It can be calculated using:

\[ Y_i = \frac{\sum_{j=1}^{n} Y_{ij}}{n} \]  

(3)

\[ S_i = \left[ \frac{1}{n-1} \sum_{j=1}^{n} (Y_{ij} - Y_i)^2 \right]^{1/2} \]  

(4)
Small values of $V_{CEB}$ show that the predicted compliance is almost equal to the experimental measurements.

2.3.3. The CEB mean square error ($F_{CEB}$). The $F_{CEB}$ method calculates the mean square error of the predicted values using the difference between the calculated and observed values relative to the observed value. It may be calculated according to the following formulas:

\[ f_j = \frac{(\text{Cal}X_{ij} - \text{Obs}X_{ij})}{(\text{Obs}X_{ij})} \times 100 \]  

\[ F_i = \sqrt{\frac{1}{n-1} \sum_{j=1}^{n} f_j^2} \]  

\[ F_{CEB} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} F_i^2} \]  

Like the coefficient of variation criterion, small values of $F_{CEB}$ show that the predicted compliance is close to the experimental measurements.

2.3.4. The BP coefficient of variation ($\sigma_{BP}$). This coefficient developed by Bažant and Panula in 1978 is calculated by assigning weight to each point in the data set based on the decade in which it falls and on the number of data points in that particular decade [8]. Data points in each logarithmic decade are considered as one group. The overall coefficient of variation for all data sets is the root mean square of the data set values. The coefficient is given by:

\[ \sigma_{ij} = \frac{n}{n_d n_k} \]  

\[ O_i = \frac{1}{n_w} \sum_{j=1}^{n} \sigma_{ij} \times \text{Obs}X_{ij} \]  

\[ \sigma_i = \frac{1}{O_i} \sqrt{\frac{1}{n-1} \sum_{j=1}^{n} \sigma_{ij}(\text{Cal}X_{ij} - \text{Obs}X_{ij})^2} \]  

\[ \sigma_{BP} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \sigma_i^2} \]  

Where; $n_d$ refers to the number of decades on the logarithmic scale spanned by measured data in data set i, $n_k$ refers to the number of data points in the k-th decade, $n_w$ refers to the sum of the weights of all data points in a data set, $\sigma_{ij}$ refers to the weight assigned to the j-th data point in data set number i, $\sigma_i$ is the coefficient of variation for data set number i, and $O_i$ is the average of the weighted measured value of the creep compliance for the i-th data point in data set number j.

Small values of $\sigma_{BP}$ show that the predicted compliance is close to experimental measurements.

2.4. Results of the Eurocode 2 Evaluation

Since Eurocode 2 model calculates the creep differently according to the concrete strength $f_{cm}$ less than or exceeding 35 MPa and since the relative humidity RH has an important effect on concrete strains [9], Table 1 indicates the evaluation method results for each category as shown below.
For a concrete with a compressive strength less than 35 MPa, \( \frac{\phi(t, t_0)}{E_{cm28}} \) overestimates harmonically the real creep values regardless of the relative humidity. As for compressive strength exceeding 35 MPa, \( \frac{\phi(t, t_0)}{E_{cm28}} \) estimates accurately the experimental creep results for a relative humidity less than 60% and overestimates them for RH exceeding 60%.

3. Approximate Bayesian computation (ABC) method

To overcome this inaccurate estimation, correction coefficients are implemented in Eurocode 2 formula as shown in the below equations:

For \( f_{cm} \leq 35 \text{ MPa} \):

\[
\frac{\phi(t, t_0)}{E_{cm28}} = O \times \left[ \frac{16.8x10^0.3}{22000 \times (0.1 + \phi_0^{1.7})} \times \left( 1 + \frac{10}{\sqrt{h_0}} \times \frac{(1-t_0)^{0.3}}{(1-t)^{0.3}} \times \frac{RH(1-t_0)^{0.3}}{10x\sqrt{h_0} \times (1-t_0)^{0.3}} \right) \right] \times E_{cm}^{0.8 \times xP} \tag{13}
\]

For \( f_{cm} > 35 \text{ MPa} \):

\[
\frac{\phi(t, t_0)}{E_{cm28}} = Q \times \left[ \frac{35^{0.2} \times 16.8 \times (1-t_0)^{0.3}}{22000 \times (0.1 + \phi_0^{1.7})} \times \left( 1 + \frac{(100 - RH)^{3.5} \times 35^{0.7}}{(10x\sqrt{h_0}) \times xP^{0.7}} \right) \times \left[ 1.8 \frac{(1-t_0)^{0.3}}{1+(0.012 \times RH)^{0.8}} + 1.0 + 2.5 \times \frac{35^{0.5} xP^{0.5}}{E_{cm}^{0.8}} \times (1-t_0)^{0.7} \times \right] \right] \times E_{cm}^{1-R} \tag{14}
\]

Where O, P, Q and R, are correction coefficients related to each category of \( f_{cm} \).

To quantify these correction coefficients, the Bayesian calibration is applied more specifically the Approximate Bayesian Computation (ABC) method.

The Bayesian calibration provides an automated process for calibrating models by multiplying the expert knowledge known as a priori distribution by the likelihood coming from the database [10-12]. Therefore, an a posteriori distribution will be defined, which is an update of the knowledge already known by using the latest database. Different methods of Bayesian calibration may be used. In this study, the Approximate Bayesian Computation (ABC) rejection algorithm is applied [13]. The Approximate Bayesian Computation algorithms can be seen as calibration techniques for implicit stochastic models, inferring parameter values in light of computer model, data, prior beliefs about the parameter values and any measurement or model errors [14]. ABC algorithms, sometimes called likelihood-free algorithms, enable inference using only simulations generated from the model, and don’t require any evaluation of the likelihood. The most basic form of the ABC algorithm is based on the rejection algorithm. The basic approximate rejection algorithm can be interpreted as performing exact inference in the presence of uniform model or measurement error.

The Approximate Bayesian Computation rejection algorithm is based on generating a random vector for each correction coefficient following an a priori distribution [13]. For each random value of the correction coefficient and for all tests in database, the compliance is calculated according to the
proposed new equations. These calculations are performed at different time of measurements as provided in the database. Once the updated compliance is calculated, it will be compared to experimental measurement. If the difference between the updated and experimental compliances is less than the threshold, the random variable is accepted; if not, it is rejected. In this study, the rejection parameter is equal to the ratio between predicted and experimental creep. The mean value of the rejection rate is equal to one with a tolerance margin since the predicted values cannot be exactly equal to the experimental measurements. Once this procedure is applied, a set of correction coefficient values is obtained which follows a known posterior distribution or an empirical posterior distribution.

4. Results
Table 2 summarizes the correction coefficients values that have resulted from the application of the Approximate Bayesian Computation method to the creep coefficient ratio \( \frac{\dot{\varepsilon}(t,t_0)}{E_{cm28}} \) for both categories of \( f_{cm} \) (less than 35 MPa and exceeding 35 MPa) and for both subcategories of RH (less than 60% and exceeding 60%).

| \( \frac{\dot{\varepsilon}(t,t_0)}{E_{cm28}} \) | Evaluation Methods | RH ≤ 60% | RH > 60% |
|---------------------------------|------------------|----------|----------|
| \( f_{cm} \leq 35 \text{ MPa} \) | O = 2.5; P = 1.4 | O = 2.44; P = 1.45 |
| \( f_{cm} > 35 \text{ MPa} \)  | Q = 1; R = 1     | Q = 2.2; R = 1.22 |

It can be noticed that for \( f_{cm} \leq 35 \text{ MPa} \), the correction coefficients are similar for both categories of the relative humidity (RH). As for \( f_{cm} > 35 \text{ MPa} \), and RH < 60%, the correction coefficients are equal to 1 which confirms that no correction is required for this category. Contrary for RH > 60%, the correction coefficients are equal to 2.2 and 1.22 respectively.

To evaluate the accuracy of these correction coefficients, the evaluation methods described in paragraph 2.3 are applied to compare the updated Eurocode 2 formulas with correction coefficients to the experimental measurements.

Table 3. Eurocode 2 evaluation results before and after the Approximate Bayesian Computation (ABC) method correction.

| \( \frac{\dot{\varepsilon}(t,t_0)}{E_{cm28}} \) | Evaluation Methods | \( M_{\text{CEB}} \) (expected value 1) | \( V_{\text{CEB}} \) (expected value 0) | \( F_{\text{CEB}} \) (expected value 0) | \( \varphi_{\text{BP}} \) (expected value 0%) |
|---------------------------------|------------------|-------------------|-----------------|-----------------|-----------------|
| \( f_{cm} \leq 35 \text{ MPa} \) | Before correction | After correction | Before correction | After correction | Before correction | After correction |
| RH ≤ 60% | 1.52 | 1.3 | 59.4 | 43.6 | 128.3 | 104.6 | 63.60% | 47.40% |
| 1.72 | 1.22 | 148.5 | 98.5 | 447.4 | 302.7 | 155.40% | 102.80% |
| RH > 60% | 1.03 | 1.03 | 52.5 | 52.5 | 147.8 | 147.8 | 53% | 53% |
| \( f_{cm} > 35 \text{ MPa} \) | Before correction | After correction | Before correction | After correction | Before correction | After correction |
| RH ≤ 60% | 1.24 | 1.14 | 150.6 | 150 | 213 | 205 | 161.40% | 143.70% |
| 1.24 | 1.14 | 150.6 | 150 | 213 | 205 | 161.40% | 143.70% |

Table 3 summarizes the results of \( M_{\text{CEB}} \), \( V_{\text{CEB}} \), \( F_{\text{CEB}} \) and \( \varphi_{\text{BP}} \) before and after applying the correction to the Eurocode 2 model for each category of the concrete strength and relative humidity using the Approximate Bayesian Computation (ABC) method. It can be noticed that all the above parameters have decreased towards their target values after correction.
5. Conclusions
Creep strains may affect the stability and integrity of concrete structures if they are not estimated accurately at design stage. To study this phenomenon, a comparison between the compliance predicted by Eurocode 2 and the experimental measurements is performed according to the categories of concrete strength $f_{cm}$ and relative humidity RH using the CEB and $\sigma_{eq}$ evaluation methods. This study shows that Eurocode 2 model overestimates creep in most cases and a correction is required to improve the estimation of concrete creep. Therefore, the Approximate Bayesian Computation (ABC) method is applied to quantify the correction coefficients. The evaluation of the updated model shows an improvement in the accuracy of creep predictions. The Approximate Bayesian Computation rejection algorithm has proven to be an effective solution for the improvement of the creep prediction according to Eurocode 2.

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