Universality of miscible–immiscible phase separation dynamics in two-component Bose–Einstein condensates

Xunda Jiang1,2, Shuyuan Wu1,2, Qinzhou Ye1,2,3 and Chaohong Lee1,2,3

1 Laboratory of Quantum Engineering and Quantum Metrology, School of Physics and Astronomy, Sun Yat-Sen University (Zhuhai Campus), Zhuhai 519082, People’s Republic of China
2 Key Laboratory of Optoelectronic Materials and Technologies, Sun Yat-Sen University (Guangzhou Campus), Guangzhou 510275, People’s Republic of China
3 Guangdong Provincial Key Laboratory of Quantum Engineering and Quantum Materials, School of Physics and Telecommunication Engineering, South China Normal University, Guangzhou 510006, People’s Republic of China

E-mail: lichaoh2@mail.sysu.edu.cn

Keywords: universal dynamics, quantum phase separation, Bose–Einstein condensate, critical exponent

Abstract

We investigate the non-equilibrium dynamics across the miscible–immiscible phase separation in a binary mixture of Bose–Einstein condensates. The excitation spectra reveal that the Landau critical velocity vanishes at the critical point, where the superfluidity spontaneously breaks down. We analytically extract the dynamical critical exponent \( z = 2 \) and static correlation length critical exponent \( \nu = 1/2 \) from the Landau critical velocity. Moreover, by simulating the real-time dynamics across the critical point, we find the average domain number and the average bifurcation delay show universal scaling laws with respect to the quench time. We then numerically extract the static correlation length critical exponent \( \nu = 1/2 \) and the dynamical critical exponent \( z = 2 \) according to Kibble–Zurek mechanism. The scaling exponents \((\nu = 1/2, z = 2)\) in the phase separation driven by quenching the atom–atom interaction are different from the ones \((\nu = 1/2, z = 1)\) in the phase separation driven by quenching the Rabi coupling strength \((2009 \text{ Phys. Rev. Lett. 102} 070401; 2011 \text{ Phys. Rev. Lett. 107} 230402)\). Our study explores the connections between the spontaneous superfluidity breakdown and the spontaneous defect formation in the phase separation dynamics.

1. Introduction

The non-equilibrium dynamics of quantum phase transitions have attracted great interest in many branches of physics, including cosmology, particle physics and condensed matter physics [1–3]. When a system is driven across a phase transition and enters a symmetry broken phase, one of the most nontrivial results is the creation of topological defects, such as domains [4–13], vortices [14–16] and solitons [17–19]. The possibility to engineer a quantum phase transition and recover its universality from topological defects is of great significance in non-equilibrium physics [20]. In recent years, due to their high controllability and robust quantum coherence, atomic Bose–Einstein condensates (BECs) become an excellent candidate for exploring non-equilibrium dynamics across phase transitions [12, 20–37].

In recent years, multi-component BECs have been widely investigated in both experiments [38–43] and theories [5–11, 16, 44–50]. Up to now, great efforts have been made to create multi-component BECs with different atomic species [38, 39], isotopes [40] or spin states [41–43]. Multi-component BECs exhibit rich physics not accessible in a single-component BEC, including phase separation with symmetry breaking [5, 44–49], Josephson oscillation [31] and domain walls [6–11]. Remarkably, the phase separation in multi-component BECs has been observed in several experiments [38–41, 43]. Through controlling the intra- and inter-component interaction via Feshbach resonance [52–54], multi-component BECs offer an ideal test bed to study the non-equilibrium physics of phase separation. However, there is few work on non-equilibrium dynamics in multi-component BECs, in particular, the dynamics of phase transition is still unclear.
In this paper, we investigate the non-equilibrium dynamics of phase separation in a binary mixture of atomic BECs. When the system is driven across the critical point at a finite rate, the critical dynamics across a miscible–immiscible phase transition is studied. Through calculating the Bogoliubov excitation spectrum, we find that the Landau critical velocity vanishes and the superfluidity breaks down at the critical point, and we analytically extract the dynamical critical exponent and static correlation length critical exponent from the Landau critical velocity. To show how the non-equilibrium dynamics appears, we numerically simulate the real-time dynamics in quench process, in which the intra-component interaction strength is linearly swept through the critical point. From the non-equilibrium dynamics far from the critical point, we numerically extract two universal scalings for the average domain number and the average bifurcation delay with respect to the quench rate, and we find the critical exponents derived from the numerical results consist well with the analytical exponents. These scaling exponents in the phase separation induced by tuning the atom–atom interaction are different from the ones [5–8], in which the phase separation is induced by tuning the Rabi coupling strength.

The paper is organized as follows. In section 2, we describe the model and discuss its ground states. In section 3, we implement the Bogoliubov–de Gennes (BdG) analysis and obtain the Landau critical velocity. In section 4, we analytically extract the Kibble–Zurek (KZ) scalings and numerically simulate the real-time dynamics of the phase transition and extract the universal scalings. Finally, we give a brief summary and discussion in section 5.

2. Model

We consider a mixture of two weakly interacting BECs in a ring trap. When the transverse frequency $\omega_{\perp}$ is sufficiently strong, we can integrate out the transverse degrees of freedom and obtain the one-dimension Hamiltonian $\hat{H} = \hat{H}_0 + \hat{H}_I$. Given the radius $r$ and the angle $\theta$, introducing $x = r\theta$, we have the single-body part

$$\hat{H}_0 = \int dx \sum_{j=1,2} \hat{\psi}_j^\dagger(x) \left[ -\frac{\hbar^2}{2m_j} \frac{\partial^2}{\partial x^2} + V(x) \right] \hat{\psi}_j(x),$$

and the two-body part

$$\hat{H}_I = \int dx \left\{ \sum_{j=1,2} \left[ \frac{g_{jj}}{2} \hat{\psi}_j^\dagger(x) \hat{\psi}_j^\dagger(x) \hat{\psi}_j(x) \hat{\psi}_j(x) \right] \right\} + \int dx \{ g_{12} \hat{\psi}_1^\dagger(x) \hat{\psi}_2^\dagger(x) \hat{\psi}_2(x) \hat{\psi}_1(x) \}. \tag{2}$$

Here, $g_{jj} = 4\pi \hbar^2 a_{jj}/m_j > 0$ and $g_{12} = 2\pi \hbar^2 a_{12}(m_1 + m_2)/(m_1 m_2) > 0$ characterize the intra- and inter-component interactions, with $m_j$ being the mass of a atom in component $j$, $a_{jj}$ and $a_{12}$ respectively denoting the intra-component and inter-component s-wave scattering lengths. In experiments, $g_{jj}$ and $g_{12}$ can be adjusted by using Feshbach resonance [52–54]. For simplicity, we only consider the homogeneous case, namely the external trapping potential $V(x) = 0$.

In the mean-field (MF) theory, the system obeys the Gross–Pitaevskii equations (GPEs)

$$i\hbar \frac{\partial \psi_j}{\partial t} = \left[ -\frac{\hbar^2}{2m_j} \frac{\partial^2}{\partial x^2} + g_{jj} |\psi_j|^2 + g_{12} |\psi_{1-j}|^2 \right] \psi_j. \tag{3}$$

The nature of the ground states is determined by the competition between the intra-component and inter-component interactions [44–46]. If the intra-component interaction dominates, i.e. $g_{12} > g_{12}^c$, the energy is minimized when the two components occupy all available volume. In such a miscible phase, the two BEC wave-functions coexist at all positions. However, if the inter-component interaction dominates, i.e. $g_{12} < g_{12}^c$, the energy of the system is minimized when the two BECs are separated in space. In such an immiscible phase, the two BEC wave-functions occupy different spatial regions. For the ground state in the immiscible phase, it is not easy to obtain the exact solution analytically. However, by utilizing the imaginary time propagation method, we numerically solve equation (3) with a split-step method [55] with the Wick rotation $t = -it$ and obtain the ground state under the period boundary condition. In figure 1, we show a typical ground state in the immiscible phase. The two BEC wave-functions indeed occupy different space. We observe an increase of the interface with the intra-component interaction strength $g_{jj}$. If the intra-component interaction strength $g_{jj}$ is stronger than a threshold $g_{jj}^c$ (or the inter-component interaction $g_{12}$ is smaller than a threshold $g_{12}^c$), the interface will cover the whole space, which is referred as miscible–immiscible phase transition.
To understand the critical dynamics near the phase transition, we introduce a dimensionless distance
\[ \epsilon(t) = \frac{|g_{22}(t) - g_{22}^c|}{g_{22}^c}, \]
where \( g_{22}^c = \frac{g_{12}^2}{g_{11}} \) is the critical point, and
\[ g_{22}(t) = g_{22}^c \left(1 - \frac{t}{\tau_Q}\right) \]
is a linearly quenched parameter with \( \tau_Q \) being the quench time. Below, we quench the intra-component interaction strength \( g_{22} \) with different quench time \( \tau_Q \) over several orders of magnitude. Then, from our numerical simulation, we extract the relation between the number of topological defects, the bifurcation delay and the quench time \( \tau_Q \).

3. Bogoliubov excitation and spontaneous superfluidity breakdown near the critical point

In this section, we analyse the Bogoliubov excitations near the critical point of phase separation and investigate the spontaneous breakdown of superfluidity. According to the Landau criterion, if the superfluid velocity is smaller than the Landau critical velocity, elementary excitations are prohibited due to the conservation of energy and momentum. However, around the critical point, the Landau critical velocity vanishes and so that elementary excitations appear spontaneously.

We implement a Bogoliubov analysis to obtain the excitation modes over the ground states. In the miscible phase, the nonlinear Schrödinger equations have an obvious homogenous solution
\[ NL_{j,j} = f_j^0, \]
with \( f_j^0 \) the ground state wave-function of the \( j \)th component and \( L \) the length of the system. The chemical potentials are given as
\[ \mu_1 = g_{11} \rho_1 + g_{12} \rho_2 \]
and
\[ \mu_2 = g_{22} \rho_2 + g_{12} \rho_1, \]
and \( g_{11} = g_{12} \) in our system. To derive the Bogoliubov excitation spectrum, we consider the perturbed ground state
\[ \psi_j(x, t) = [\phi_j + \delta\phi_j(x, t)] e^{-i\omega_j t / \hbar}. \]

Inserting the state (6) into the equation (3) and keeping the first-order terms, in units of \( \hbar = m = \omega_\perp = 1 \), the linearized equations for the perturbations are given as,
\[ i \frac{\partial \phi_1}{\partial t} = \left( -\frac{1}{2} \frac{\partial^2}{\partial x^2} + 2g_{11}|\rho_1| + g_{12}|\rho_2| - \mu_1 \right) \delta\phi_1 + g_{11} \phi_1^* \phi_1^* \delta\phi_2 + g_{12} \phi_1^* \phi_2^* + g_{12} \phi_1^* \phi_2^* \delta\phi_2, \]
\[ i \frac{\partial \phi_2}{\partial t} = \left( -\frac{1}{2} \frac{\partial^2}{\partial x^2} + 2g_{22}|\rho_2| + g_{12}|\rho_1| - \mu_2 \right) \delta\phi_2 + g_{22} \phi_2^* \phi_2^* \delta\phi_1 + g_{12} \phi_2^* \phi_1^* + g_{12} \phi_2^* \phi_1^* \delta\phi_1. \]
The perturbations \( \delta\phi_{1,2} \) can be written as
\[ \begin{pmatrix} \delta\phi_1 \\ \delta\phi_2 \end{pmatrix} = \begin{pmatrix} u_{1,q} \\ u_{2,q} \end{pmatrix} e^{iqx - i\omega t} + \begin{pmatrix} v_{1,q}^* \\ v_{2,q}^* \end{pmatrix} e^{iqx + i\omega t}, \]
in which \( q \) is the excitation quasimomentum, \( \omega \) is the excitation frequency, \( u_{j,q} \) and \( v_{j,q} \) \((j = 1, 2)\) are the Bogoliubov amplitudes. Inserting equation (9) into the linearized equation equation (7),(8) and comparing the coefficients for the terms of \( e^{iqx - i\omega t} \) and \( e^{iqx + i\omega t} \), one obtains the BdG equations.
In the vicinity of the critical point, we can simply expand the equation \( \psi(z) \) to linear order in \( z \), \[ \psi(z) \approx \psi(0) + \xi \partial_z \psi(0) + \ldots, \] where \( \xi \) is the correlation length, and extract another universal critical exponent 

\[ \eta_\xi = \lim_{\eta \to 0} \frac{\xi(\eta)}{\eta} = \frac{\rho}{4}(g_{11} + g_{22} \pm \sqrt{(g_{11} - g_{22})^2 + 4g_{12}^2}). \]

When \( g_{12} < 0 \), \( \eta_\xi \) is a negative value, \( \omega_- \) becomes imaginary in the long-wavelength limit. This means that the phase transition corresponds to the dynamical instability of Bogoliubov excitations. In \([5, 6] \), the KZ mechanism originates from the gradually vanishing of the Higgs modes when the system approaches to the critical point, that is, the gapless Higgs modes at the critical point. However, in our work, the long-wavelength excitations are always gapless, and the KZ mechanism originates from the spontaneous superfluidity breakdown induced by the dynamical instability.

On the other hand, at the critical point, \( g_{22} = g_{22}^- = g_{11}/g_{12} \), the Landau critical velocity becomes \n
\[ v_L = \lim_{\eta \to 0} \frac{\omega_- (\eta)}{\eta} = \sqrt{\eta} = 0. \]

The vanishing critical velocity at the critical point results in the spontaneous superfluidity breakdown and spontaneous elementary excitations.

4. KZ scalings

In this section, we extract two universal scalings from the Bogoliubov excitation in the miscible phase. As the softening of the phonon mode near the critical point, we can extract the dynamical critical exponent \( z \) from the Landau critical velocity. Furthermore, we can make use of the Landau critical velocity to define correlation length, and extract another universal critical exponent \( \nu \). At the same time, we perform numerical simulations of non-equilibrium dynamics in the GPE (3). Starting from the ground state in the miscible phase, according to equation (5), we linearly sweep the intra-component interaction strength \( g_{12} \) to drive the system across the critical point \( g_{22}^- \). When the evolution time \( t \) increases from \( t < 0 \) to \( t > 0 \), the system goes from the miscible phase to the immiscible phase. Through analysing the domains and bifurcation delay, we numerically obtain two universal critical exponents, which are consist well with the analytical exponents.

The dynamical critical exponent \( z \) can be derived from the low-energy long-wave excitations. At the critical point, the low-energy excitation in the long-wavelength limit behave as

\[ \lim_{\eta \to 0} \frac{\omega_- (\eta)}{\eta} = \frac{q^2}{2}. \]

Since \( \omega_- (q) \propto |q|^p \) as \( q \to 0 \) \([1, 20, 56] \), we have the dynamical critical exponent \( z = 2 \).

The static correlation length critical exponent \( \nu \) can be derived from the divergence of the correlation length \( \xi \) near the critical point. The Landau critical velocity \( v_L \) provide a definition of the correlation length \( \xi \) according to \( \xi = \frac{\hbar}{(mv_L)} \) \([57] \). Therefore, the Landau critical velocity \( v_L \) should have a power-law scaling behavior around the critical point as

\[ v_L \propto \xi^{-1} \propto |\varepsilon|^\nu. \]

In the vicinity of the critical point, we can simply expand \( \eta_- \) to the leading order of \( \varepsilon \), that is

\[ \eta_- = \frac{\rho g_{22}^-}{4} (2 + |\varepsilon| - \sqrt{|\varepsilon|^2 + 4}) \approx \frac{\rho g_{22}^-}{4} |\varepsilon|. \]

As \( v_L = \sqrt{\eta_-} \), thus we have the static correlation length critical exponent \( \nu = 1/2 \). In figure 2, we show our numerical results of \( v_L \) for different \( |\varepsilon| \) near the critical point in a log-log coordinate. Through linear fitting, we find a power-law \( v_L \propto \varepsilon ^\nu \) with the static correlation length critical exponent \( \nu = 0.4996 \pm 0.0001 \), which agrees well with the analytical one \( \nu = 1/2 \).
In addition, one can also derive the critical exponent from the sound velocity \[ c \propto \sqrt{q} \] \[ \text{(18)} \] In the long-wavelength limit \((q \to 0)\), the sound velocity reads \[ c(q=0) = \frac{\partial \omega}{\partial q} \Bigg|_{q=0} = \sqrt{\eta_\pm}. \] We can expand \( c \) to the leading order of \( \epsilon \), that is, \( c_\pm \propto |\epsilon|^{1/2} \). If \( g_{22}^2 > g_4 b_2 \), \( c_\pm \) becomes negative and so that the system becomes unstable. Therefore, the fastest growing modes with the wave number \( q_f \) are given by minimizing \( \omega_q \) with respect to \( q \), and these modes grow initially at a rate \( E_q = |c_-|^{1/2} \). We expect that these modes will set the length scale of the pattern that is formed, so that the condensates will separate into domain structure, and the length scale \( \xi \) of domain is given as \[ \xi = \frac{2\pi}{q_f} = \frac{2\pi}{|c_-|^{1/2}}. \] At the same time, we can estimate the time scale \( \tau \) for the formation process of domains \[ \tau = 1/E_q = 1/|c_-| \propto |\epsilon|^{-1}. \] Combining the equations (19) and (20), one can analytically obtain the two critical exponents \((v = 1/2, z = 2)\).

In our simulations, we choose quench times \( \tau_Q \) over four orders of magnitude and perform 100 runs of simulation for each \( \tau_Q \). Since the quantum fluctuations that trigger the phase transition are ignored in the MF approximation \[ \text{(50)} \], we introduce random noises to the initial state and so that the dynamics of spontaneous topological defects can be studied by the MF theory. As the interaction strength \( g_2 \) is quenched according to equation (5), unlike the static bifurcation, in which the bifurcation exactly occurs at the critical point, the dynamical bifurcation takes place after the system crossing the critical point.

The bifurcation delay, \( b_d = |g_{22}^* - g_{22}^0| \propto |\hat{\epsilon}| \), is obtained by analyzing spatial fluctuation of the local spin polarization \[ \Delta J_L = \left| \frac{1}{\sqrt{L^2}} \int J_L^2(x) dx - \left| \frac{1}{L} \int J_L(x) dx \right|^2 \right|, \] where \( n_j(x) = |\psi_j(x)|^2 \) and \( J_L = [n_1(x) - n_2(x)]/[n_1(x) + n_2(x)] \). Before the bifurcation, there is no spatial fluctuation, i.e. \( \Delta J_L = 0 \). The dynamical bifurcation occurs at \( g_{22}^* \) where \( \Delta J_L \) reaches a small nonzero value \( \delta \), which is chosen as 0.05 in our calculations. Actually, based upon our calculations, similar conclusions can be obtained for other small \( \delta \) between 0.05 and 0.2.

In figure 3, we show the dependence of the bifurcation delay on the quench time. Clearly, there is a significant delay between the growth of local spin fluctuation and the static phase transition, see the inset of figure 3. For a smaller quench time, the system has a larger bifurcation delay \( b_d \). In our numerical results, it is clearly illustrate that the average bifurcation delay \( \bar{b}_d \) follows a power-law scaling with respect to the quench time \( \tau_Q \), \[ \bar{b}_d \propto \tau_Q^{-d_2} \] with the scaling exponent \( d_2 = 0.4937 \). According to the KZ mechanism, we have \[ \hat{\epsilon} \propto \tau_Q^{-z} \] \[ \propto \tau_Q^{-0.4937}. \] In order to extract the critical exponents, we also analyse the universal scaling of domain number \( N_d \) versus the quench time \( \tau_Q \). Typical examples of domain formation dynamics for different quench times are illustrated.
One can find that the average domain size increases with the quench time. In each run, we count the number of domains \( N_d \) by identifying the number of zero crossings of \( J_z \) at the end of evolution, when the domain shape becomes stable. The number of domains for different \( \tau_Q \) are shown in figure 4. We observe that the average domain number follows a power-law scaling \( N_d \propto \tau_Q^{-d_1} \) with the scaling exponent \( d_1 = 0.2510 \).

According to the KZ mechanism, we have

\[
N_d \propto \tau_Q^{-v} = \tau_Q^{-0.5084}.
\]

Combining equations (22) and (23), we obtain the static correlation length critical exponent \( v = 0.5084 \) and the dynamical critical exponent \( z = 2.0172 \). These exponents well consist with the ones \( (v = 1/2, z = 2) \) obtained from the Landau critical velocity and the correlation length. These critical exponents are in the same universal class as in the previous work \([11, 13, 16, 17, 34]\). However, in a binary BEC with Rabi coupling \([5–8]\) across the phase separation driven by quenching the Rabi coupling strength, the exponents are given as \( (v = 1/2, z = 1) \). This means that, although the phase separation can be induced by quenching either the Rabi coupling or the atom–atom interaction, they belong to different universal classes.

5. Summary and discussion

In summary, we have investigated the non-equilibrium dynamics across a phase separation transition in a mixture of two-component BECs. Through analyzing the Bogoliubov excitation spectrum, we find that the
Landau critical velocity vanishes at the critical point, which results in the spontaneous superfluidity breakdown and creation of elementary excitations, and we extract the critical exponents from the Landau critical velocity and the correlation length near the phase transition. When the system is driven across the critical point with a finite quench rate, the domains are spontaneously created due to the vanishing of the spontaneous superfluidity breakdown. On the other hand, by numerically simulating the GPEs, we find that domains appear after the system crossing the critical point \( g^{c}_{22} \), and we count the domains number at the end of evolution. Through quenching the system with various \( \tau_{Q} \), we find two universal scalings of the average bifurcation delay and average domain number versus the quench rate. We then numerically extract two critical exponents \( (\nu = 1/2, z = 2) \) from two universal scalings.

Although all this work and the previous ones [5–8] study miscible–immiscible transitions, there are significant differences between their KZ scalings. Firstly, the miscible–immiscible transitions are induced by different sources. In the previous works [5–8], the miscible–immiscible transitions are controlled by the Rabi coupling. On the other hand, in our system, the miscible–immiscible transition is controlled by the interaction coefficient. Secondly, the pictures for understanding the KZ mechanism are different. In the previous works [5–8], the KZ mechanism is understood via the gapless excitations. However, in our manuscript, the KZ mechanism is understood via the divergence of correlation length derived from the Landau critical velocity. So far, we analytically extract the dynamical critical exponent from the Landau critical velocity and determine the static correlation length critical exponent from the correlation length derived from the Landau critical velocity. Thirdly, and most importantly, the KZ scaling exponents are different. In our manuscript, the scaling exponents are given as \( (\nu = 1/2, z = 2) \). However, in the previous works [5–8], the scaling exponents are given as \( (\nu = 1/2, z = 1) \). This means that the miscible–immiscible transition driven by quenching the atom–atom interaction and the miscible–immiscible transition driven by quenching the Rabi coupling belong to different universal classes. In the previous works [5–8], the KZ mechanism originates from the gradually vanishing of the Higgs modes when the system approaches to the critical point, that is, the gapless Higgs modes at the critical point. However, in our work, the long-wavelength excitations are always gapless, and the KZ mechanism originates from the spontaneous superfluidity breakdown induced by the dynamical instability.

Based upon current available techniques for atomic BECs, it is possible to probe the above universal scalings. The mixture of two-component BECs can be prepared with different atomic species \([38, 39]\), isotopes \([40]\) or spin states \([41–43]\). The phase separation has been observed in several experiments \([40, 43]\) and the high tunability of the interaction strength makes the quenching across the phase separation transition possible. Furthermore, the bifurcation delay and the size of domains can be extracted by measuring the local spin polarization via the time-of-flight. Considering an ensemble of \(^{87}\text{Rb} \) atoms in a ring trap with the total number of atoms \( N = 2 \times 10^6 \) and the transverse trapping frequency \( \omega_{\perp} = 2\pi \times 1 \text{ kHz} \), \( L = 96 \) in our simulation corresponds to 32.6 um, and \( g_{11} = 0.5 \) corresponds to the scattering length \( a_{11} = 13.54 \text{ nm} \).

Acknowledgments

This work was supported by the National Natural Science Foundation of China (Grants No. 11874434, No. 11574405).
ORCID iDs
Chaohong Lee © https://orcid.org/0000-0001-9883-5900

References
[1] Sachdev S 2011 Quantum Phase Transitions 2nd edn (Cambridge: Cambridge University Press)
[2] Morikawa M 1995 Cosmological inflation as a quantum phase transition Prog. Theor. Phys. 93 685
[3] Kibble T W B 1980 Some implications of a cosmological phase transition Phys. Rep. 67 183
[4] Kibble T W B 1976 Topology of cosmic domains and strings J. Phys. A: Math. Gen. 9 1387
[5] Lee C 2009 Universality and anomalous mean-field breakdown of symmetry-breaking transitions in a coupled two-component bose–einstein condensate Phys. Rev. Lett. 102 070401
[6] Sabbatini J, Zurek W H and Davis M J 2011 Phase separation and pattern formation in a binary Bose–Einstein condensate Phys. Rev. Lett. 107 230402
[7] Sabbatini J, Zurek W H and Davis M J 2012 Causality and defect formation in the dynamics of an engineered quantum phase transition in a coupled binary Bose–Einstein condensate New. J. Phys. 14 095030
[8] Xu J, Wu S-Y, Qin X-Z, Huang J-H, Ke Y-G, Zhong H-H and Lee C 2016 Kibble–Zurek dynamics in an array of coupled binary Bose condensates Europhys. Lett. 113 5003
[9] Świłosz T, Witkowska E, Dziarmaga J and Matuszewski M 2013 Double universality of a quantum phase transition in spinor condensates: modification of the Kibble–Zurek mechanism by a conservation law Phys. Rev. Lett. 110 045303
[10] Hofmann J, Natu S S and Sarma S D 2014 Coarsening dynamics of binary Bose condensates Phys. Rev. Lett. 113 095702
[11] Wu S-Y, Ke Y-G, Huang J-H and Lee C 2017 Kibble–Zurek scalings of continuous magnetic phase transitions in spin-1 spin-orbit-coupled Bose–Einstein condensates Phys. Rev. A 95 063606
[12] Navon N, Gaunt A L, Smith R P and Hadzibabic Z 2015 Critical dynamics of spontaneous symmetry breaking in a homogeneous Bose gas Science 347 167
[13] Ye Q-Z, Wu S-Y, Jiang X-D and Lee C 2018 Universal dynamics of zero-momentum to plane-wave transition in spin-orbit-coupled Bose–Einstein condensates J. Stat. Mech. 053110
[14] Weiler C N, Neely T W, Scherer D R, Bradley A S, Davis M J and Anderson B P 2008 Spontaneous vortices in the formation of Bose–Einstein condensates Nature 455 948
[15] Su S W, Gou S C, Bradley A, Fialko O and Brand J 2013 Kibble–Zurek scaling and its breakdown for spontaneous generation of Josephson vortices in Bose–Einstein condensates Phys. Rev. Lett. 110 215302
[16] Wu S-Y, Qin X-Z, Xu J and Lee C 2016 Universal spatiotemporal dynamics of spontaneous superfluidity breakdown in the presence of synthetic Gauge fields Phys. Rev. A 94 043606
[17] Damski B and Zurek W H 2010 Soliton creation during a Bose–Einstein condensation Phys. Rev. Lett. 104 160404
[18] Witkowka E, Deuar P, Gaidai M and Rzatewski K 2011 Solitons as the early stage of quasicondensate formation during evaporative cooling Phys. Rev. Lett. 106 135301
[19] Zurek W H 2009 Causality in condensates: gray solitons as relics of BEC formation Phys. Rev. Lett. 102 105702
[20] Polkovnikov A, Sengupta K, Silva A and Vengalattore M 2011 Nonequilibrium dynamics of closed interacting quantum systems Rev. Mod. Phys. 83 863
[21] Sadler J E, Higbie J M, Leslie S R, Vengalattore M and Stamper-Kurn D M 2006 Spontaneous symmetry breaking in a quenched ferromagnetic spinor Bose–Einstein condensate Nature 443 312
[22] Meldgin C, Ray U, Russ P, Chen D, Ceperley D M and DeMarco B 2016 Probing the Bose glass-superfluid transition using quantum quenches of disorder Nat. Phys. 12 646
[23] Baumann K, Motto R, Brennecke F and Esslinger T 2011 Exploring symmetry breaking at the Dicke quantum phase transition Phys. Rev. Lett. 107 140402
[24] Bloch I, Dalibard J and Zwerger W 2008 Many-body physics with ultracold gases Rev. Mod. Phys. 80 885
[25] Dziarmaga J 2010 Dynamics of a quantum phase transition and relaxation to a steady state Adv. Phys. 59 1063
[26] Lamporesi G, Donadello S, Serafini S, Dalfovo F and Ferrari G 2013 Spontaneous creation of Kibble–Zurek solitons in a Bose–Einstein condensate Nat. Phys. 9 656
[27] Anquez M, Robbins B A, Bharath H M, Boguslawski M, Hoang T M and Chapman M S 2016 Quantum Kibble–Zurek mechanism in a spin-1 Bose–Einstein condensate Phys. Rev. Lett. 116 155301
[28] Barnett R, Polkovnikov A and Vengalattore M 2011 Prethermalization in quenched spinor condensates Phys. Rev. A 84 023606
[29] Klings J, Keler H, Wolke M, Mathey L and Hemmerich A 2015 Dynamical phase transition in the open Dicke model Proc. Natl. Acad. Sci. 112 3290
[30] Nicklas E, Karl M, Höfer M, Johnson A, Muessel W, Strobel H and Tomkovic J 2015 Observation of scaling in the dynamics of a strongly quenched quantum gas Phys. Rev. Lett. 115 245301
[31] Feng L, Clark I W, Gaj A and Chin C 2018 Coherent inflationary dynamics for Bose–Einstein condensates across a quantum critical point Nat. Phys. 14 269
[32] Pelissetto A and Vicari E 2017 Dynamic off-equilibrium transition in systems slowly driven across thermal first-order phase transitions Phys. Rev. Lett. 118 030602
[33] Coulamly I B, Sagui A and Sarandy M S 2017 Dynamic off-equilibrium transition in systems slowly driven across thermal first-order phase transitions Phys. Rev. E 95 023602
[34] Clark I W, Feng L and Chin C 2016 Universal space–time scaling symmetry in the dynamics of bosons across a quantum phase transition Science 354 650
[35] Mistakidis S I, Katstsouris C, Kevrekidis P G and Schmelcher P 2018 Correlation effects in the quench-induced phase separation dynamics of a two-species ultracold quantum gas New. J. Phys. 20 043052
[36] Utesov O, Baglay M J and Andreev S V 2018 Effective interactions in a quantum Bose–Bose mixture Phys. Rev. A 97 053617
[37] Francuz A, Dziarmaga J, Gardos B and Zurek W H 2016 Space and time renormalization in phase transition dynamics Phys. Rev. B 93 075134
[38] McCarron D J, Cho H W, Jenkin D L, Köppinger M P and Cornish S L 2011 Dual-species Bose–Einstein condensate of 87Rb and 133Cs Phys. Rev. A 84 011603
[39] Modugno G, Modugnocchi M, Riboli F, Roati G and Inguscio M 2002 Two atomic species superfluid Phys. Rev. Lett. 89 19
[40] Papp SB, Pino JM and Wieman CE 2008 Tunable miscibility in a dual-species Bose–Einstein condensate Phys. Rev. Lett. 101 040402
[41] Lin Y-J, Jiménez-Garcia K and Spielman IB 2011 Spin-orbit coupled Bose–Einstein condensates Nature 471 83
[42] Hall DS, Matthews MR, Ensher JR, Wieman CE and Cornell EA 1998 Dynamics of component separation in a binary mixture of Bose–Einstein condensates Phys. Rev. Lett. 81 1539
[43] Tojo S, Taguchi Y, Masuyama Y, Hayashi T, Saito H and Hirano T 2010 Controlling phase separation of binary Bose–Einstein condensates via mixed-spin-channel feshbach resonance Phys. Rev. A 82 033609
[44] Timmermans E 1998 Phase separation of Bose–Einstein condensates Phys. Rev. Lett. 81 5718
[45] Ao P and Chui ST 1998 Binary Bose–Einstein condensate mixtures in weakly and strongly segregated phases Phys. Rev. A 58 4836
[46] Trippenbach M, Góral K, Rzazewski K, Malomed B and Band YB 2000 Structure of binary Bose–Einstein condensates J. Phys. B 33 4017
[47] Takeuchi H and Kasamatsu K 2013 Nambu-Goldstone modes in segregated Bose–Einstein condensates Phys. Rev. A 88 043612
[48] Alexandrov AS and Kabanov VV 2002 Excitations and phase segregation in a two-component Bose–Einstein condensate with an arbitrary interaction J. Phys.: Condens. Matter 14 L327
[49] Esry BD 1998 Impact of spontaneous spatial symmetry breaking on the critical atom number for two-component Bose–Einstein condensates Phys. Rev. A 58 R3599
[50] Saito H, Kawaguchi Y and Ueda M 2007 Kibble-Zurek mechanism in a quenched ferromagnetic Bose–Einstein condensate Phys. Rev. A 76 043613
[51] Williams J, Walser R, Cooper J, Cornell E and Holland M 1999 Nonlinear Josephson-type oscillations of a driven two-component Bose–Einstein condensate Phys. Rev. A 59 R31
[52] Courteille P, Freeland RS and Heinzen DJ 1998 Observation of a feshbach resonance in cold atom scattering Phys. Rev. Lett. 81 69
[53] Cornish SL, Clausen NR, Roberts JL, Cornell EA and Wieman CE 2000 Stable 85Rb Bose–Einstein condensates with widely tunable interactions Phys. Rev. Lett. 85 1795
[54] Inouye S, Andrews MR, Stenger J, Miesner HJ, Stamper-Kurn DM and Ketterle W 1998 Observation of feshbach resonances in a Bose–Einstein condensate Nature 392 151
[55] Javanainen J and Ruostekoski J 2006 Symbolic calculation in development of algorithms: split-step methods for the Gross-Pitaevskii equation J. Phys. A: Math. Gen. 39 L179
[56] Robinson M 2011 Symmetry and the Standard Model (New York: Springer)
[57] Giorgini S, Pitaevskii LP and Stringari S 2008 Theory of ultracold atomic Fermi gases Rev. Mod. Phys. 80 1215