Numerical modeling results of vibroseismic monitoring of volcanic structures with different shape of the magma chamber

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Abstract. The paper considers the possibility of active monitoring of volcanic structures based on the simulation of vibroseismic transmission using the dynamical theory of elasticity. A solution method, a parallel algorithm, and a program system based on a finite-difference approach together with CFS-PML method of absorbing boundaries are developed. Different variations of a magmatic stratovolcano approximate model are considered, comparative numerical calculations are carried out. The calculated wave field inside the volcanic structure has a compound structure depending on the geometry of studied objects and their rheological characteristics. Based on the studies, it may be concluded that monitoring of the considered type of volcanoes using precision vibroseismic sources and seismic observation systems is possible. A variant of vibroseismic monitoring of Elbrus volcano using the observation system in the tunnel of the Baksan Neutrino Observatory is proposed.

1. Introduction
Eruption of magmatic volcanoes is a dangerous natural disaster. Predicting such phenomena is a difficult task. Currently, they are mainly forecasted using precursors or using earthquakes or industrial explosions for transmission of chamber zones. However, it is not always possible to predict the time and place of a catastrophic eruption with their help, since unstable sources of seismic waves are used, which does not allow monitoring the status of chamber zones and output channels. Unfortunately, there is currently no reliable description of the magma chambers geometry and the processes occurring in them. Several mathematical models of varying complexity have been proposed that describe the process of a volcanic eruption which has already begun or continues for some time. From works [1–2] it follows that the main processes leading to eruptions are associated with the boiling of melts and the occurrence of cracks (dykes) in the roof of the magma chamber. If the pressure at the tip of the dyke overcomes the rock’s tensile strength, then an eruption occurs. Probably, magma pre-accumulates in the expanding chamber and advances in dykes. A dike breaking through to the surface forms an eruption channel. Perhaps this explains the appearance of two peaks at Elbrus. Based on this mechanism, the authors propose creating a monitoring system that allows one to investigate the geometric dimensions of the surface magma chamber and magma output channels. The main element of such system is precision vibroseismic sources developed at the SB RAS, which have high source...
stability for monitoring magmatic volcanoes. In this paper, we consider the behavior of the wave field for two geometrical models of the source — elliptical and conical.

2. The physical model of stratovolcano Elbrus

Previously, the authors of this work proposed a geophysical model of Elbrus magmatic chambers in the form of two ellipsoids with cylindrical supply channels included in a layered medium [3–5]. The model of central type volcanoes (Fedotov et al., 1991) and the data presented in the study [6–7] for Elbrus have taken as the base of such a model. Also data given in [8–11] are used.

In this paper, we consider another model of the upper magma chamber, which is proposed in [12]. According to the authors, the lower boundary of the crustal volcanic chamber is approximately 8–15 km below sea level. At depths of the order of 5 km, the width of the chamber reaches 8 km and gradually decreases as it moves toward the surface. A rapid chamber decrease begins with a depth of about 2 km (where its width doesn’t exceed 5 km), and at a depth of 1 km its characteristic dimensions do not exceed 2×2.5 km. It is assumed that the chamber has shape of a cone with vertex upwards. The data for a horizontally layered medium are used the same as in the work [4], but limited to a depth of 22 km (see table 1). In the magma chamber we take \( \rho = 2.1 \text{ g/cm}^3 \), \( V_p = 2.2 \text{ km/s} \), the diameter of the upper channel is 130 m, and the diameter of the channel connecting the upper and parent magma chambers 260 m. At the same time, we assume that in an equilibrium state the upper channel is plugged and merges with the layers that include it.

**Table 1.** Layered medium parameters for the geophysical model of Elbrus volcano.

| Depth interval (km) | \( V_p \) (km/s) | \( V_s \) (km/s) | \( \rho \) (g/cm\(^3\)) |
|--------------------|-----------------|-----------------|--------------------------|
| Layer +II          | 2.85            | 1.65            | 2.4                      |
| Layer +I           | 3.1             | 1.79            | 2.66                     |
| Layer I            | 3.2             | 1.82            | 2.7                      |
| Layer II           | 5.9             | 3.42            | 2.85                     |
| Layer III          | 6.22            | 3.59            | 2.62                     |
| Layer IV           | 5.82            | 3.37            | 2.7                      |
| Layer V            | 5.97            | 3.45            | 2.75                     |

Based on these data, a geophysical model of the medium with the upper magma chamber of the Elbrus stratovolcano has compiled (see figure 1). The lower acute angles of the cone wrong represent the shape of the chamber from the point of view of the molten rock physics. Therefore they are smoothed out by inscribed circles.

The following monitoring option is proposed for the upper magma chamber of the volcano. To register signals from the vibrator, the tunnel of the Baksan Neutrino Observatory of INR RAS (BNO) is used. The entrance of the tunnel coordinates are 43°16.338’N 42°40.878’E. The height above sea level is 1740 m. The distance to the top of Elbrus is 21.9 km. Azimuth of the tunnel is 150°37’. The tunnel is located in the village of Neutrino under the mountain Mount Andyrchi and has a length of more than four kilometers [7]. Thus, we can place in the tunnel a seismic sensors line with a length of more than 4 km to receive vibroseismic signals from a source located on the opposite side of the volcano. Note that this arrangement of sensors minimizes industrial interference.

A vibroseismic source is placed on the volcano opposite side at a distance from the observation system of about 30–40 km in order to transmission the deep structure of the medium to 15–20 km. A CV-40 portable vibrator developed at the SB RAS can be used as a source. It develops vibrational forces of 40–50 tons and has an operating range of 5–15 Hz [13]. Since the monitoring system in this case is fixed, for detailed monitoring of dynamic changes in the chamber and output channels, it will be necessary to set a profile along which the vibrator and fixed sensing points on this profile will move. By conducting regular soundings with a given periodicity, we get the opportunity to study the behavior of the magma chamber and output channels. Based on the results of numerical experiments,
it will be possible to determine the optimal number of sensing points on the vibrator placement profile. In figure 1, the possible location of the vibrator and detectors in the BNO tunnel is indicated.

![Figure 1](image)

**Figure 1.** An approximate geophysical model of Elbrus volcano structure with conical upper magma chamber and vibroseismic monitoring scheme.

3. Mathematical simulation of vibroseismic transmission of volcanic structures

Simulation of the seismic waves propagation in inhomogeneous elastic media is carried out on the base of a numerical solution of system (1) of elasticity equations written down in terms of the displacement velocity vector $\mathbf{u} = (U, V, W)^T$ and the stress tensor $\mathbf{\sigma} = (\sigma_{xx}, \sigma_{xy}, \sigma_{xz}, \sigma_{yy}, \sigma_{yz}, \sigma_{zz})^T$:

$$
\rho \frac{\partial \mathbf{u}}{\partial t} = [A] \mathbf{\sigma} + \mathbf{F}(t, x, y, z), \quad \frac{\partial \mathbf{\sigma}}{\partial t} = [B] \mathbf{u},
$$

(1)

$$
A = \begin{bmatrix}
\frac{\partial}{\partial x} & 0 & 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} & 0 \\
0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial x} & 0 & \frac{\partial}{\partial z} \\
0 & 0 & \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} & \frac{\partial}{\partial y}
\end{bmatrix}, \quad B = \begin{bmatrix}
(\lambda + 2\mu) \frac{\partial}{\partial x} & \lambda \frac{\partial}{\partial y} & \lambda \frac{\partial}{\partial z} \\
\lambda \frac{\partial}{\partial x} & (\lambda + 2\mu) \frac{\partial}{\partial y} & \lambda \frac{\partial}{\partial z} \\
\lambda \frac{\partial}{\partial x} & \lambda \frac{\partial}{\partial y} & (\lambda + 2\mu) \frac{\partial}{\partial z} \\
\mu \frac{\partial}{\partial y} & \mu \frac{\partial}{\partial x} & 0 \\
\mu \frac{\partial}{\partial z} & 0 & \mu \frac{\partial}{\partial x} \\
0 & \mu \frac{\partial}{\partial z} & \mu \frac{\partial}{\partial y}
\end{bmatrix},
$$
where $t$ is time, $\rho(x,y,z)$ is density, and $\lambda(x,y,z), \mu(x,y,z)$ are the Lame coefficients. The simulated domain is a parallelepiped with an isotropic inhomogeneous elastic medium with complex geometry.

The initial and boundary conditions at the free surface are

$$
\sigma \mid_{t=0} = 0, \quad \bar{u} \mid_{t=0} = 0, \quad \sigma_{\nu} \mid_{z=0} = 0, \quad \sigma_{\nu} \mid_{z=-d} = 0.
$$

To numerically solve the problem (1) – (2) we apply well-proven finite difference schemes on a staggered grid [14]. The calculation of its difference coefficients uses integral conservation laws. The schemes of the 2nd order of approximation in time and the 2nd and 4th order of approximation in space are used.

To dismiss false reflections from boundaries of the simulated domain we use absorbing boundary conditions CFS-PML [15]. The parallelepiped boundaries, except the free surface, are surrounded by absorbing layers in which components with damping parameters are added to the initial finite difference equations. At points outside this zone calculations are carried out according to the primary scheme.

Based on these numerical methods, a complex of parallel algorithms and programs for numerically solving problem (1) – (2) has been developed, adapted to the architecture of computing clusters equipped with modern multi-core processors and general purpose graphics accelerators using MPI and CUDA technologies [4–5]. It includes a set of software modules for 2D and 3D modeling:

— arrays builder with coefficients of medium typical of magma volcanoes;

— parallel finite difference solvers with absorbing layers;

— writer of output field snapshots and theoretical seismograms for a given observation system.

The developed complex has high scalability of about 80% and effectively uses the GPU architecture [16]. The calculation of a full-scale 3D model at the nodes of the hybrid cluster of the Siberian Supercomputing Center ICM&MG SB RAS lasts several hours, and the calculation time of the 2D model is only about half an hour on one GPU. This allows one to quickly select the kinematic characteristics of the studied medium.

4. The results of numerical simulations

To study the influence of the upper magma chamber shape on the wave field nature and the information content of the signal reaching the observation system in the BNO tunnel, 2D modeling of the seismic waves propagation inside the volcanic structure was performed. 2D modeling requires less computational resources, and it is quite informative for the initial analysis of the wave pattern, the model refinement and reduction of number of computationally expensive 3D calculations.

The presence of appropriate roads suggests the possibility of placing a vibrator in the area of the Bitik-Tybyu valley at a distance of about 9 km from the peak of Elbrus and 30 km from the observation system in the BNO tunnel. At the same time, the potential vibrator location is almost in line with the top of Elbrus and the BNO tunnel. Thus, to conduct the numerical experiments, we considered a 2D section passing through this line.

Let us note that the 2D modeling does not take into account the terrain, which, due to its static and linearity of wave field, will not have a large impact on monitoring the wave field changes associated with changes in the internal structure of the volcano during a possible eruption.

To conduct numerical experiments, we consider the models of the geophysical structure of Elbrus volcano presented in section 2 with the elliptical and conical structures of the upper magma chamber included in the six-layer medium with the parameters of layers $+1–V$ from table 1. The size of the computational domain is 36 km along the horizontal axis $X$ and 24 km along vertical axis $Z$. The observation system is a line of 9 receivers with 500 m step located respectively in the BNO tunnel. The excitation system is a point source of pressure center type with the frequency of 8 Hz lying near the free surface on the upper left corner of the computational domain and 9 km from axis of rotation of the magma chamber.
The calculations have carried out on the node with the NVIDIA K40 graphics accelerator, which is part of the SSCC hybrid cluster. The size of the computational grid is 7200 × 4800 nodes. One calculation for 56000 time steps takes on average about 25 minutes.

To present the results of numerical modeling, theoretical seismograms and snapshots of the wave field are presented. The visualization is created using the Aspis software developed by the Sibneftegeofizika company.

Figure 2. Snapshots of the wave field $u$ component at different time points. A is ellipsoidal chamber; B is cone-shaped chamber. The green line indicates the location of the detectors line.
Figure 2 presents the wave field snapshots at different points in time for a model with an elliptical chamber (A) and for a model with a cone-shaped chamber (B), the lines indicate the boundaries of layers and inclusions. In both cases, the plugged upper channel is considered. It can be seen from the snapshots that the wave field has a compound structure with a significant influence of the chamber shape on the shape of the waves passing through it. In this case, the most contrasting boundary between the layers is the boundary between I and II layers, which also significantly affects the overall picture. In case B, the camera generates multiple reflections inward. It leads to “stuck” of field in it, and the waves reflected from the bottom of the camera and going beyond its boundary (B2) come to the surface without reaching the detector line (B3).

To analyze the information on inclusions arriving at the detectors, we exclude the layers influence on the wave field. To do this, we subtract the calculation results for a layered medium without inclusions from the calculation field for cases A and B. This will, in particular, exclude powerful Rayleigh waves near the surface. Results of this manipulation are shown in figure 3 (wave field snapshots) and 4 (theoretical seismograms).

![Wave field snapshots](image1)

**Figure 3.** The difference between snapshots of wave field $u$ component for a layered medium with and without magmatic inclusions at different time points. A is ellipsoidal chamber; B is cone-shaped chamber. The green line indicates the location of the detectors line.

In figure 3, in snapshots A1 and B1, the zone of wave exit from the magma chamber is marked, and in snapshots A2 and B2, the arrival of the same waves in the region of the observation system is noted. On seismograms you can also observe the arrival of these waves at appropriate times. We note that in the elliptical camera case (A), we observe the arrival of waves reflected from the bottom of the ellipse at seismograms, and in the cone-shaped camera case (B), a large number of waves arrive at the
detectors reflected from the upper diffraction points of the camera and propagating inside layers + I and I. Thus, comparing the wave field snapshots and theoretical seismograms, we can analyze the kinematic characteristics of the wave field due to magmatic inclusions.

![Figure 4. The difference between the theoretical seismograms of \( u \) component for a layered medium with and without magmatic inclusions from the line of receivers corresponding to the BNO tunnel. A – the ellipsoidal chamber, B – the cone-shaped chamber. The vertical axis is plotted in seconds.](image)

The presented experiments show that numerical simulation can provide significant information for conducting real experiments and interpreting their results. The wave field dependence on the geometrical configuration of the magma chamber is obvious, that can be used to detect the eruption preparation by the vibroseismic monitoring method. However, it should be noted that for a more complete concept of the object geometry, 3D modeling is necessary. If it is supposed to use an observation system located only in the BNO tunnel, the issue of orientation of the vibrator relative to the aperture and the possibility of installing the vibrator in an optimal place from the point of view of delivery becomes relevant. The calculations, in the case of a model with a cone-shaped camera, showed that the geometry of the camera can be such that for some arrangement of the vibrator, the information from the boundaries of the camera that gets to the detectors will be insignificant. Therefore, a change in the vibrator position and the study of the possibility of placing additional detectors will be required. The complexity of the wave field may require various methods of processing seismic information, which can be tested on the results of numerical simulation.

5. Conclusion
The paper considers two geophysical models of the upper magma chamber structure and a scheme of the vibroseismic monitoring of Elbrus volcano with reference to the terrain. A complex of parallel algorithms and programs for numerically simulating the seismic waves propagation during vibroseismic monitoring of magmatic volcanoes has been developed. Numerical experiments and a
comparative analysis of the features of the calculated wave field have carried out using the developed set of programs for two models of the upper magma chamber of Elbrus volcano structure.

The obtained results show a significant difference in the observed wave field pattern for different shapes of the upper magma chamber (which allows monitoring processes in the chamber zone), as well as the possible applicability of the proposed monitoring scheme using the BNO tunnel in both cases. The work involves the study continuation of 2D and 3D modeling results for the proposed internal structure of Elbrus volcano medium.

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