CMB B-mode auto-bispectrum produced by primordial gravitational waves

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Gravitational waves from inflation induce polarization patterns in the cosmic microwave background (CMB). It is known that there are only two types of non-Gaussianities of the gravitational waves in the most general covariant scalar field theory having second-order field equations, namely, generalized G-inflation. One originates from the inherent non-Gaussianity in general relativity, and the other from a derivative coupling between the Einstein tensor and the scalar field. We calculate polarization bispectra induced by these non-Gaussianities by transforming them into separable forms by virtue of the Laplace transformation. It is shown that future experiments can constrain the new one but cannot detect the general relativistic one.

Subject Index E03, E63, E80

1. Introduction

Inflation in the early Universe [1–5] produces both primordial density perturbations [6–9] and gravitational waves [10,11] out of quantum fluctuations. (For a review of inflation, see, e.g., Ref. [12].) The simplest model of slow-roll inflation predicts that they are almost Gaussian and that the primordial density perturbations have only small non-Gaussianities of the order of the slow-roll parameters [13,14]. Deviations from the simplest model may result in non-Gaussianities large enough to be detected through temperature and E-mode polarization bispectra of the cosmic microwave background (CMB), and CMB experiments [15,16] have targeted such large non-Gaussianities to verify the model of inflation. The non-Gaussianities are classified into several types, which have distinct wavenumber dependence mostly for the sake of simplicity of calculation (e.g., Refs. [14,17–23]). However, up to now even the most precise CMB observation has not yet detected any statistically significant deviation from Gaussianity [16].

On the other hand, primordial non-Gaussianities of the gravitational waves differ from those of the curvature perturbations. They are known to be classified into only two types [24] in the framework of the generalized G-inflation [25], which was originally the most general single-field inflation model based on the Horndeski theory [26,27] with second-order field equations. One is inherent in the general relativity and results in primordial non-Gaussianities of the order of unity. The other originates from a certain kind of a non-minimal derivative coupling of the scalar field with the Einstein tensor. Detection of the former would provide evidence that the standard quantum field...
theory calculation in curved spacetime works well, whereas that of the latter would be a smoking gun of the modification of gravity.

Since primordial density perturbations cannot give rise to B-mode polarization [28–31], its primary auto-bispectrum just reflects the non-Gaussianities of gravitational waves. In this sense, the B-mode auto-bispectrum is suitable for investigating the non-Gaussianities of tensor perturbations and is worthy of consideration. So far only the power spectrum of B-mode polarization has been detected, which is induced not by primordial gravitational waves but by lensing of E-modes and Galactic dust emission [32–36]. Thus there has been little study concerning the B-mode auto-bispectrum with a few exceptions of models with parity violation [37,38].

The rest of the paper is organized as follows. We first state our bispectrum notation in Sect. 2. In Sect. 3 we show the non-Gaussianity of tensor perturbations in the generalized G-inflation with the Lagrangian of the Horndeski theory. In Sect. 4 we calculate the bispectrum under linear evolution. Finally we show the results in Sect. 5 and conclude in Sect. 6.

2. CMB bispectrum and reduced bispectrum

We decompose a CMB anisotropy map \( A(\hat{n}) \) into a set of coefficients of spherical harmonics:

\[
A(\hat{n}) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\hat{n}).
\]  

(1)

The CMB bispectrum is the three-point correlation of the \( a_{\ell m} \):

\[
B_{\ell_1 \ell_2 \ell_3}^{m_1 m_2 m_3} = \langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} \rangle.
\]

(2)

If the Universe is statistically isotropic, then the bispectrum can be angle averaged without any loss of information, i.e.,

\[
B_{\ell_1 \ell_2 \ell_3} = \sum_{m_1 m_2 m_3} \left( \begin{array}{ccc} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{array} \right) B_{\ell_1 \ell_2 \ell_3}^{m_1 m_2 m_3},
\]

(3)

where \( \left( \begin{array}{ccc} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{array} \right) \) is the Wigner-3j symbol. We also define the reduced bispectrum [20] as

\[
b_{\ell_1 \ell_2 \ell_3} = G_{\ell_1 \ell_2 \ell_3}^{-1} B_{\ell_1 \ell_2 \ell_3},
\]

(4)

with

\[
G_{\ell_1 \ell_2 \ell_3} = T_{\ell_1 \ell_2 \ell_3}^{0 0 0},
\]

(5)

where we define

\[
T_{\ell_1 \ell_2 \ell_3}^{s_1 s_2 s_3} = \sqrt{(2\ell_1 + 1)(2\ell_2 + 1)(2\ell_3 + 1) \frac{(\ell_1 \ell_2 \ell_3)}{4\pi}}.
\]

(6)

However, \( G_{\ell_1 \ell_2 \ell_3} \) vanishes when \( \ell_1 + \ell_2 + \ell_3 = \text{odd} \), and in such a case we redefine \( G_{\ell_1 \ell_2 \ell_3} \) by using identities of Wigner-3j symbols as [39]

\[
G_{\ell_1 \ell_2 \ell_3} = \frac{2\sqrt{\ell_2(\ell_2 + 1)\ell_3(\ell_3 + 1)}}{[\ell_1(\ell_1 + 1) - \ell_2(\ell_2 + 1) - \ell_3(\ell_3 + 1)]} T_{\ell_1 \ell_2 \ell_3}^{0 - 1 1}.
\]

(7)
3. Primordial non-Gaussianities in the generalized G-inflation

The generalized G-inflation [25] is the most general single-field inflation based on the Horndeski theory [26], which is the most general single scalar–tensor theory with second-order field equations. The Lagrangian has four arbitrary functions of a scalar field $\phi$ and $X \equiv -\partial_\mu \phi \partial^\mu \phi / 2$:

$$\mathcal{L} = \sum_{i=2}^{5} \mathcal{L}_i,$$

$$\mathcal{L}_2 = K(\phi, X),$$

$$\mathcal{L}_3 = -G_3(\phi, X) \Box \phi,$$

$$\mathcal{L}_4 = G_4(\phi, X) R + G_{4X} \{ (\Box \phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2 \},$$

$$\mathcal{L}_5 = G_5(\phi, X) G_{\mu\nu} \nabla^\mu \nabla^\nu \phi - \frac{1}{6} G_{5X} \{ (\Box \phi)^3 - 3(\Box \phi)(\nabla_\mu \nabla_\nu \phi)^2 + 2(\nabla_\mu \nabla_\nu \phi)^3 \},$$

where $R$ is the Ricci scalar and $G_{\mu\nu}$ is the Einstein tensor, $G_{iX} = \partial G_i / \partial X$, $(\nabla_\mu \nabla_\nu \phi)^2 = \nabla_\mu \nabla_\nu \phi \nabla^\mu \nabla^\nu \phi$, and $(\nabla_\mu \nabla_\nu \phi)^3 = \nabla_\mu \nabla_\nu \phi \nabla^\nu \nabla^\lambda \phi \nabla_\lambda \nabla^\mu \phi$. We use the unitary gauge in which $\phi = \phi(t)$ and write the metric as

$$g_{00} = -1, \quad g_{0i} = 0, \quad g_{ij} = a^2(t) (e^h)_{ij},$$

where

$$(e^h)_{ij} = \delta_{ij} + h_{ij} + \frac{1}{2} h_{ik} h_{kj} + \frac{1}{6} h_{ik} h_{kl} h_{lj} + \cdots ,$$

to focus on the tensor perturbations, which are transverse and traceless. Then the quadratic and cubic actions for $h_{ij}$ are obtained as below:

$$S^{(2)} = \frac{1}{8} \int dt \, d^3 x \, a^3 \left[ G_T \dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2} (\partial_k h_{ij})^2 \right],$$

$$S^{(3)} = \int dt \, d^3 x \, a^3 \left[ \mathcal{F}_T \left( h_{ik} h_{jl} - \frac{1}{2} h_{ij} h_{kl} \right) \partial_k \partial_l h_{ij} + X \phi G_{5X} \frac{\dot{h}_{ij} \dot{h}_{jk} \dot{h}_{ki}}{12} \right],$$

where

$$\mathcal{F}_T = 2[G_4 - X(\dot{\phi} G_{5X} + G_{5\phi})],$$

$$G_T = 2[G_4 - 2X G_{4X} - X(H \dot{\phi} G_{5X} - G_{5\phi})].$$

From the quadratic action, the primordial power spectrum of gravitational waves, $\xi^{(s)}(k) \equiv h_{jk}(k)e^{i \omega(k)}(k)$, in which $(s)$ stands for the helicity, is given by

$$\langle \xi^{(s)}(k) \xi^{* (s')}(k') \rangle = (2\pi)^3 \delta^{(3)}(k - k') \delta_{ss'} \frac{\pi^2}{k^3} P_h(k),$$

$$P_h(k) = \frac{2H^2 G_T^{1/2}}{\mathcal{F}_T^{3/2}},$$

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where $e^{(s)}_{jk} = e^{(s)}_{jk}(k_i)$ is a transverse and traceless polarization tensor and it obeys the normalization condition $e^{(s)}_{jk}(k) e^{* (s')}_{jk} = \delta_{ss'}$. From the cubic action, we get the bispectrum of primordial gravitational waves with $P_h^2$ [24]:

$$
\langle \xi^{(s_1)}(k_1) \xi^{(s_2)}(k_2) \xi^{(s_3)}(k_3) \rangle = (2\pi)^7 \delta^{(3)}(k_1 + k_2 + k_3) P_h^2 \frac{S_{(GR)}^{S_{(S2S3)}} + S_{(5X)}^{S_{(S2S3)}}}{k_1^2 k_2^2 k_3^2},
$$

where $k_r = k_1 + k_2 + k_3$, and $f_{SX}$ is given by

$$
f_{SX} = \frac{-2 HX \phi G_{SX}}{G_T}. \tag{18}
$$

4. Calculation of the bispectrum

Under linear evolution of the perturbations, the $a_{lm}$ are given as

$$
da^{(s)}_{lm} = 4\pi (-i)^l \int \frac{k^2 dk}{(2\pi)^3} T^{(s)}_l(k) \int d\Omega_k - 2 f_{SXM} \xi^{(s)}(k), \tag{19}
$$

where the superscript $(s)$ stands for the helicity of cosmological perturbations that induce the CMB anisotropies [29,40]. Then the bispectrum is represented as the sum of all the helicity contributions:

$$
B_{\ell_1 \ell_2 \ell_3} = \sum_{S_{(S2S3)}} B_{\ell_1 \ell_2 \ell_3}^{S_{(S2S3)}}, \tag{20}
$$

$$
B_{\ell_1 \ell_2 \ell_3}^{S_{(S2S3)}} = \sum_{m_1 m_2 m_3} {\ell_1 \ell_2 \ell_3 \choose m_1 m_2 m_3} \times \prod_{j=1}^{3} \left[ 4\pi (-i)^l \int \frac{k_j^2 dk}{(2\pi)^3} T^{(s_j)}_l(k_j) \int d\Omega_{k_j} - 2 f_{SXM} \xi^{(s_j)}(k_j) \right] \langle \xi^{(s_1)}(k_1) \xi^{(s_2)}(k_2) \xi^{(s_3)}(k_3) \rangle. \tag{21}
$$

Since the Horndeski theory does not violate the parity symmetry, the bispectrum vanishes by summing over the helicities when the total multipole moment is even or odd, depending on the parity of the types of CMB fluctuations. For example, the auto-bispectrum of B-mode polarization is

$$
B_{\ell_1 \ell_2 \ell_3} = 0 \text{ for } \ell_1 + \ell_2 + \ell_3 = \text{even}, \tag{22}
$$

and the auto-bispectrum of E-mode polarization is

$$
B_{\ell_1 \ell_2 \ell_3} = 0 \text{ for } \ell_1 + \ell_2 + \ell_3 = \text{odd}. \tag{23}
$$
We assume that the spectrum of primordial gravitational waves is scale invariant, and that their
functions of \( k \) of primordial gravitational waves with the Wigner-9 symbol (for derivation of the expression, see, e.g., Ref. [41]):

\[
B^{(s_1,s_2,s_3)}_{\ell_1\ell_2\ell_3} = \sum_{L_1 L_2 L_3} T^{s_1} \tilde{T}^{s_2} \tilde{T}^{s_3} \begin{vmatrix}
\ell_1 & \ell_2 & \ell_3 \\
\ell_1' & \ell_2' & \ell_3' \\
L_1 & L_2 & L_3
\end{vmatrix} \times \int x^2 dx \prod_{j=1}^{3} \left[ \frac{2}{\pi} i^{\ell_j - \ell_j'} \int dk_j \tilde{T}^{(s_j)}_{\ell_j'} (k_j) j_{\ell_j'} (k_j x) \right] \times (2\pi)^4 P_{k} \mathcal{K} (k_1, k_2, k_3),
\]

(24)

where \( \mathcal{K} = \mathcal{K}_{(GR)} + \mathcal{K}_{(5X)} \) represents the wavenumber dependence of the non-Gaussianities with

\[
\mathcal{K}_{(GR)} = \frac{(4\pi)^{3/2}}{80} \left( k_1 \frac{\sum_{i\neq j} k_i^2 k_j + 4 k_1 k_2 k_3}{k_i^2} \right) \left[ \frac{k_3}{k_1 k_2} \delta_{\ell_1'} ^2 \delta_{\ell_2'} ^2 \left( \frac{5\delta_{\ell_3'} ^4}{\sqrt{21}} - \frac{\delta_{\ell_3'} ^4}{\sqrt{7}} \right) + 2 \text{circulations} \right],
\]

(25)

\[
\mathcal{K}_{(5X)} = \frac{(4\pi)^{3/2}}{80} f_{5X} \sqrt{\frac{7}{3}} k_1^2 k_2 k_3 \delta_{\ell_1'} ^2 \delta_{\ell_2'} ^2 \delta_{\ell_3'} ^2.
\]

(26)

Due to the complicated form of \( \mathcal{K} (k_1, k_2, k_3) \), \( k_j \) integration in Eq. (24) cannot be performed separately. By using the Laplace transformation,

\[
p^{-\nu - 1} = [\Gamma (\nu + 1)]^{-1} \int_0^\infty r^\nu e^{-rt} dt \quad (\text{Re}[\nu] > -1),
\]

(27)

however, \( k_j ^{-\nu - 1} \) becomes a product of the functions of the \( k_j \), to make each term in Eq. (24) a product of the functions of \( k_1, k_2, \) and \( k_3 \). Although a new integration variable must be introduced here, the exponential factor decays so rapidly that the computation cost can be reduced as in separable templates of non-Gaussianities [42]. We emphasize that the Laplace transformation enables us to get the exact primary bispectra of B-mode polarization up to the nonlinearity parameter \( f_{5X} \), in contrast with the temperature bispectrum from primordial scalar perturbations that have been studied based on a few templates that are not necessarily derived from specific models of inflation [43,44].

We calculated the evolution of linear perturbations with the Boltzmann code CLASS [45] with the cosmological parameters estimated by the latest observation [46] to obtain the transfer functions. We assume that the spectrum of primordial gravitational waves is scale invariant, and that their amplitude is parametrized by the tensor-to-scalar ratio \( r \), which is defined by \( P_R (k) = r P_R (k_s) \) at \( k_s = 0.05 \text{Mpc}^{-1} \).

5. Results

Once the B-mode auto-bispectrum produced by primordial gravitational non-Gaussianity has been calculated, the maximum possible signal-to-noise ratio where observational errors are dominated by the cosmic variance is given by \( (S/N) = 1/\sqrt{F^{-1}} \). As the non-Gaussianity of temperature anisotropy is known to be small and it is natural to expect the same is the case for B-mode polarization, we may...
Fig. 1. Signal-to-noise ratio (SNR) when B-mode polarization is observed up to the maximum of the multipole moment $\ell_{\text{max}}$. The solid lines are the case of general relativity and the dotted lines represent the contribution of a new interaction with $f_{5X} = 1$. The upper lines for the respective modes are for a tensor-to-scalar ratio $r = 0.1$, and the lower lines are for $r = 0.01$. Without delensing the signal-to-noise ratios are suppressed by lensing-induced B-mode polarization, as shown by the thick lines. If lensing-induced B modes are completely removed, we can improve the signal-to-noise ratios shown by the thin lines.

use the Fisher information $F$ defined by

$$F = \sum_{\ell_1, \ell_2, \ell_3 \leq \ell_{\text{max}}} \frac{B_{\ell_1 \ell_2 \ell_3}^2}{6 C_{\ell_1} C_{\ell_2} C_{\ell_3}}. \quad (28)$$

Since the power spectrum $C_{\ell_j}$ is proportional to the tensor-to-scalar ratio $r$ and the bispectrum $B_{\ell_1 \ell_2 \ell_3}$ to $r^2$, the signal-to-noise ratio has the dependence $(S/N) \propto \sqrt{r}$.

First, we supposed that $f_{5X} = 0$ and found that the signal is too small to be detected. We show the signal-to-noise ratios in the zero-noise ideal experiment as thin solid lines in Fig. 1. Next we assume the case in which $f_{5X}$ is large enough for its non-Gaussianity to dominate. We computed the signal-to-noise ratios with the noise of lensing-induced B modes and show the result as thick lines in Fig. 1. They indicate that without delensing it is enough to observe the B-mode polarization up to $\ell_{\text{max}} = 200$. In that case, we are able to detect the bispectrum when the tensor-to-scalar ratio $r = 0.1$ or 0.01, only if the coefficient $f_{5X}$ is as large as $10^7$ or $10^8$, respectively.

We have calculated the shape correlator between the bispectra induced by general relativity and nonzero $G_{5X}$ and obtained a value $-0.35$, which is smaller than 0.51, that between the temperature bispectra of the equilateral model and the local model [47]. Hence we can say that these two modes are distinct from each other.

We have also calculated the CMB temperature bispectrum induced by this mode and found that even if $f_{5X}$ takes such a large value, the signal-to-noise ratio of the resultant temperature bispectrum is too small to be detected, so that it would not contradict any current observational data.

We show the shapes of the primordial non-Gaussianities and the reduced B-mode bispectrum (see Figs. 2 and 3). Their shapes are similar in both general relativity and the new class of gravity in which $G_{5X}$ does not vanish.

The B-mode bispectrum $B_{\ell_1 \ell_2 \ell_3}$ is zero along the lines (see Fig. 4) where at least two of the $\ell_j$ are equal, since the Wigner-3$j$ symbol $\begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$ is antisymmetric to a replacement of columns when
Fig. 2. The shape function $|S_{G^5 X}^{(GR,S)}|$ of non-Gaussianity of primordial gravitational waves from (a) general relativity and (b) nonzero $G_{5X}$. The vertical axis is normalized to unity for $k_2/k_1 = k_3/k_1 = 1$.

Fig. 3. The B-mode bispectrum $B_{\ell_1 \ell_2 \ell_3}$ induced by (a) general relativity and (b) nonzero $G_{5X}$ where $\ell_1 + \ell_2 + \ell_3 = 33$. Here we divide $B_{\ell_1 \ell_2 \ell_3}$ by (a) $iP_{h}^{2}$ and (b) $iP_{h}^{2}f_{5X}$, respectively.

$\ell_1 + \ell_2 + \ell_3 = \text{odd}$. This is because the B-mode bispectrum has odd parity. The bispectrum decreases toward the lines and so the signal-to-noise ratio is smaller than that of the E-mode auto-bispectrum, which is not suppressed because it has even parity.

6. Conclusion

In the present paper, we have calculated the B-mode auto-bispectrum induced by tensor perturbations produced during inflation in the early Universe based on the most general single-field inflation model [24]. In this theory it is known that the primordial bispectrum of tensor perturbations consists of two distinct terms, one present in general relativity, and the other proportional to $f_{5X}$. We have found that the former contribution, whose nonlinearity parameter is always unity, cannot be detected by the CMB, whereas the new term may be detectable only if complete delensing is achieved in the case in which $f_{5X} \gtrsim 10^5$. 

Fig. 4. An example of points where at least two of the $\ell_j$ are equal in the multipole moment space $(\ell_1, \ell_2, \ell_3)$ where $\ell_1 + \ell_2 + \ell_3 = \text{const.}$ Where the point is red, the B-mode auto-bispectrum must vanish.

As a future prospect, when 21 cm maps of the atomic hydrogen distribution in the dark ages become available, the curl mode of the gravitational lensing effect [48] may be the key to detection of the non-Gaussianity of primordial gravitational waves, because 21 cm fluctuations can probe up to $\ell \sim 10^7$ in principle and there are many statistically independent fluctuation modes.

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