Balanced SPADE detection for distance metrology

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We implement Hermite-Gaussian (HG) spatial-mode demultiplexing (SPADE), boosted by homodyne detection, for high-precision measurements of the transverse distance between two point-sources. Routinely used for telecommunication, SPADE has been adopted to measure distances in the transverse plane, demonstrating extraordinary performances [1–5]. These measurements are typically limited by cross-talk [6]. Here we improve on these performances by exploiting, through balanced detection, the simultaneous information contained in the lowest HG modes. In a first set of measurements, we simulate a real acquisition setup with large cross-talk between the channels. We show that balanced detection allows us to suppress the effect of cross-talks, yielding a resolution much below the Rayleigh limit, down to one hundredth of the FWHM. A reproducibility test confirms the reliability of this setup. In a second set of measurements, we repeat the experiment with an improved setup optimised for minimal cross-talks. In this improved scenario we demonstrate precision down to few thousands of the FWHM, with the power to resolve two sources within 0.0055 FWHM. To the best of our knowledge this is an improvement of one order of magnitude compared with the best measurement of this type [2].

Over the time, several criteria have been developed to define, in a more or less quantitative way, the resolving power of an optical imaging system. Among the most popular ones there is the Rayleigh criterion [7], which states, in a heuristic way, that the wavelength sets the minimum resolvable transverse separation between two point sources A and B. According to the Rayleigh criterion, two point-sources can be resolved when the first diffraction minimum of the image of A coincides with the maximum of the image of B. Indeed, this criterion is strictly related to the point-spread function (PSF) of the optical system. In practice, the knowledge of the PSF alone is not sufficient to determine the resolution power, as it also depends on signal-to-noise ratio (SNR). A number of techniques have been developed to circumvent the Rayleigh limit [8]. These includes switching the emission on and off [9], near field probing [10], or exploiting optical non-linearities [11], just to name a few. Most of this techniques rely on source engineering, which is not an option for astronomical observations.

Recently, Tsang et al. [1] proposed a technique to achieve far-field super-resolution of a pair of natural, incoherent point-sources. This is enabled by linear optics and photon detection in the photon-counting regime, and exploits the additional information contained in the phase and in the spatial correlations of the optical field. Such information is ignored in direct imaging but can be extracted through a coherent processing of the field before detection, using interferometric techniques as SPAtial mode DE-multiplexing (SPADE) [12–15] or Super-Localization via Image-inVERsion interferometry (SLIVER) [16–18]. These and other interferometric techniques [3, 19, 20] have been demonstrated for super-resolution imaging and high-precision distance measurements [21, 22], especially for the problem of estimating the transverse separation between two point-sources [2–5], both in the photon-counting regime [22] and for bright sources [24].
In a very fine experiment [5], Boucher et al. demonstrated separation measurements with high precision. In their work, the sensitivity is quantified by the ratio \( r = d_m/w_0 \), where \( d_m \) is the minimum measurable separation, and \( w_0 \) is the beam waist. They reported \( r \simeq 5 \cdot 10^{-2} \) using a Multi-Plane Light Conversion system as a demultiplexer [26, 27]. Their observed sensitivity is limited by the cross-talk \( x_{0n} \) between HG\(_{00}\) and generic HG\(_{nm}\) channels quantified as

\[
x_{0n} = 10 \log_{10} \left( \frac{P_{nm}}{P_{00}} \right),
\]

where \( P_{nm} \) represents the output power on the HG\(_{nm}\) channel (fiber-coupled in our case) when a HG\(_{00}\) is injected with a power equal to \( P_{00} \) (free-space coupled). Here we exploit balanced detection to suppress the effects of cross-talks. To test the system, we intentionally lower the matching of coupling optics. This induces large cross-talks between the channels (see table I), which in turn simulate in-field experimental conditions (e.g., cells for a microscope, binary stars for a telescope). Balanced detection yields noise cancellation and leads to an effective zero-background measurement. With this technique we improve the ratio \( d_m/w_0 \) by a factor \( \simeq 4 \): this is a very promising result for application of SPADE in real observation campaigns.

The experimental setup is shown in figure 1. We combine two telecom fiber lasers (1.55 \( \mu \)m) on a non-polarizing beam splitter (NPBS) to mimic the point-like sources. Each fiber laser is coupled with a collimator and partially mode-matched using two lenses. Two polarizers are used to change the beams power in controlled way. The beams are combined on the NPBS through a pair of steering mirrors that are also used to couple each beam in the input (free-space) port of the demultiplexer. The second mirror of each beam is mounted on a translation stage to move the beams, within transverse plane, with micrometric resolution. The demultiplexer, PROTEUS-C from Cailabs, allows us to perform intensity measurements on six Hermite-Gaussian modes. It accepts radiation from the free-space input port, and decomposes it in the lowest-order modes (HG\(_{00}\), HG\(_{01}\), HG\(_{10}\), HG\(_{11}\), HG\(_{20}\), HG\(_{02}\)). The latter are coupled with six single-mode fibers following conversion into the HG\(_{00}\) mode. Finally, the intensities of modes HG\(_{01}\) and HG\(_{00}\) are combined for balanced detection using commercial balanced detectors that produce a signal \( S_1 \) proportional to \( \alpha_1 \)HG\(_{00}\), where \( \alpha_1 \) is an attenuation factor selected to set \( S_1 \) to zero when the beams displacement is zero (i.e., the beams overlap completely) and the overall source has a circular symmetry. The same procedure is repeated for the modes HG\(_{10}\) and HG\(_{00}\), yielding the signal \( S_2 \) and the optimised value of the factor \( \alpha_2 \).

To simulate a situation where we do not know whether there is a single source or there are two sources, we align the system on the centroid of the two sources by maximizing the HG\(_{00}\) output. Then, we acquire the signals \( S_1 \) and \( S_2 \) for different separations of the two simulated sources between 0 and 200 \( \mu \)m (beams FWHM \( \simeq 360 \mu \)m). The beams positions are controlled using a pair of translation stages, shifted in opposite directions by the same amount in order to keep the centroid aligned with demultiplexer optics. We note that using only one translation stage, and keeping the other source aligned with the demultiplexer, one would obtain a better resolution. However, in a practical scenario one can only hope to align the demultiplexer with the centroid, since the positions of the sources are unknown.

Figure 2 shows the measured signals \( S_1 \) and \( S_2 \) (blue points) for several values of the separation \( d \) between the simulated sources. Using \( S_1 \) and \( S_2 \) as calibration curves, it is possible to

| \( x_{00} \) | \( x_{10} \) | \( x_{02} \) | \( x_{20} \) | \( x_{11} \) |
|-------|-------|-------|-------|-------|
| -8.7  | -7.7  | -5.9  | -6.2  | -12.1 |

TABLE I: Cross-talk values from HG\(_{00}\) mode into the HG\(_{nm}\) modes. The values are enhanced on purpose to simulate a real observation with reduced control and misalignment.
FIG. 1: **Experimental setup.** Two telecom fiber lasers (1.55 \( \mu \text{m} \)) exit from collimators (C) and are mode-matched using a simple lenses (L) system. The intensities are tuned by changing the relative orientation of a pair of polarizers (P). The beams are combined on the NPBS through a pair of steering (M) mirrors that are also used to couple each beam in the input free space port of the demultiplexer. The second mirror of each beam is mounted on a translation stage to move the beams, within transverse plane, with micrometric resolution. The demultiplexer, PROTEUS-C from Cailabs, allows to perform intensity measurements on six Hermite-Gaussian mode. The HG\textsubscript{01}/HG\textsubscript{00} modes and HG\textsubscript{10}/HG\textsubscript{00} modes are detected by balanced detection.

FIG. 2: **Measured signals.** Measured signals \( S_1 \) and \( S_2 \) (upper plots, blue points) at different values of separation \( d \) between simulated sources. Using the \( S_1 \) and \( S_2 \) as calibration curves, it is possible to estimate the beam separation by simply measuring \( S_1 \) or \( S_2 \) and by finding the corresponding beam separation \( d \). In the lower graphs, the red points represent the uncertainties \( \delta d \) associated to the estimation of the beam separation \( (d) \). Using the upper graphs as calibration curves, \( \delta d \simeq \frac{\partial d_i}{\partial S_i} \delta S_i \).
measure the beam separation by simply measuring $S_1$ or $S_2$ and by finding the corresponding beam separation $d$. In the lower graphs of figure 2, the red points represent the uncertainties of the beam separation estimates using these calibration curves. We estimate the uncertainties (red points) $\delta d$ through the following procedure: (1) Inversion of the data (blue points) represented in the upper graphs $S_i(d) \rightarrow d(S_i)$; (2) Calculation of the numerical derivative $\partial d(S_i)/\partial S_i$; (3) Multiplication by the uncertainty $\delta S_i$ (vertical error bars of the blue points) obtaining: $\delta d \approx \delta S_i \partial d(S_i)/\partial S_i$. We obtain a very low uncertainty, which allows us to resolve the two beams within one hundredth of the FWHM even in presence of the large cross-talk simulating a real observation. The $S_1$ and $S_2$ signals show different responses (and thus different sensitivities) on $d$. This is due to the fact that HG$_{01}$ forms an angle lower than $45^\circ$ with the direction of $d$, whereas the angle of HG$_{10}$ is larger than $45^\circ$. In principle, by rotating the image in the transverse plane, it would be possible to find the direction of max sensitivity of, e.g., $S_1$ (which in turn corresponds to min sensitivity of $S_2$). This further optimisation could be implemented by placing a Dove prism on a rotation stage carefully aligned with optical axis. Finally, we remark that this technique is not an absolute distance measurement as it is based on a calibration curve.

After demonstrating a high resolving power in high cross-talk condition (to emulate a real observation), we test the system for robustness and reproducibility. We repeated the measurement for the more sensitive signal $S_1$ two days after the first measurement, obtaining $S_1r$. We found excellent reproducibility as shown in figure 3 also thanks to finely controlled environmental conditions. In fact, the setup is placed on a 460 mm optical table with active vibration isolation and self-leveling in a humidity and temperature controlled laboratory. Figure 3 shows the difference between signals measured in two different days $\Delta = S_1(d) - S_1(r,d)$. All the differences are within $\delta S$, ensuring a high level of reproducibility. The balanced detection we used can be especially advantageous in passive observation as it is independent on unpredictable source fluctuations and in real measurement campaigns where the cross-talk among the channels in not negligible.

After demonstrating the benefit of balanced detection for real observations, we prepared an improved setup to increase the performance even further by lowering the cross-talk and using more advanced translation stages. We matched the laser waist with the demultiplexer waist as much as possible, obtaining the cross-talk values as in table II. We replaced the previous translation stages with a higher resolution translators (250 nm each and 500 nm for symmetric separation) but a more limited travel. We acquired the signal on oscilloscope for HG$_{01}$ and HG$_{10}$ modes at different values of the source separation with this improved setup. The results are shown in figure 4. The upper graphs show the intensities of HG$_{01}$ and HG$_{10}$ modes for different values of source separations. The lower graphs show the uncertainty calculated using the same procedure as figure 2. Using this improved setup we obtain an uncertainty of a few thousands of the FWHM, and we are able to resolve two sources within a distance of 0.0055 FWHM.

In conclusion, we have demonstrated in a proof-of-principle experiment, a very simple yet robust and reliable scheme for high-precision distance metrology in transverse plane in the presence of strong cross-talk between channels, exploitable in real observations where the imperfections cannot be fully controlled. The measurement relies on spatial demultiplexing combined with balanced homodyne detection and we obtained a minimum measurable separation

| x_{01} | x_{10} | x_{02} | x_{20} | x_{11} |
|-------|-------|-------|-------|-------|
| -20.0 | -22.6 | -15.9 | -18.5 | -34.1 |

TABLE II: Cross-talk values from HG$_{00}$ mode into the HG$_{nm}$ modes in good alignment conditions.
between sources lower than one hundredth of FWHM. We test the reproducibility with good results proving the reliability of the setup for real observations. Finally, using an improved setup with lower cross-talk and high resolution translation stages, we repeated the measurements by recording the intensities of $H_{G01}$ and $H_{G10}$ modes obtaining a resolution of 0.0055 FWHM. To the best of our knowledge this is an improvement of about one order of magnitude compared with best measurement of this type \cite{5}. The technique is not an absolute technique since it is based on a calibration curve but can be used both in real observations or for development of inertial sensors in the two dimensions of the transverse plane at the same time.

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\[ \text{Repeatability.} \] Difference $\Delta$ between signals $S_1$ and $S_r$ measured in two different days. All $\Delta$ values are included in $\pm \delta S$ ensuring an excellent measurement reproducibility.

\[ \text{FIG. 3: Repeatability.} \] Difference $\Delta$ between signals $S_1$ and $S_r$ measured in two different days. All $\Delta$ values are included in $\pm \delta S$ ensuring an excellent measurement reproducibility.

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FIG. 4: High resolution measurements. Measured intensities HG$_{01}$ and HG$_{10}$ (upper graphs) at different values of the separation $d$ between simulated sources. Using HG$_{01}$ and HG$_{10}$ intensities as calibration curves, it is possible to estimate the beam separation by simply measuring HG$_{01}$ and HG$_{10}$ and by finding the corresponding beam separation value. In the lower graphs, the red points represent the associated uncertainties $\delta d$, obtained using the upper graphs as calibration curves, $\delta d \approx \frac{\partial d}{\partial HG_{01}} \delta HG_{01}$ and $\delta d \approx \frac{\partial d}{\partial HG_{10}} \delta HG_{10}$.

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