DUALITIES, COMPOSITENESS AND SPACETIME
STRUCTURE OF 4D EXTREME STRINGY BLACK HOLES

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Abstract
We study the BPS black hole solutions of the (truncated) action for heterotic string theory compactified on a six-torus. The $O(3, Z)$ duality symmetry of the theory, together with the bound state interpretation of extreme black holes, is used to generate the whole spectrum of the solutions. The corresponding spacetime structures, written in terms of the string metric, are analyzed in detail. In particular, we show that only the elementary solutions present naked singularities. The bound states have either null singularities (electric solutions) or are regular (magnetic or dyonic solutions) with near-horizon geometries given by the product of two 2d spaces of constant curvature. The behavior of some of these solutions as supersymmetric attractors is discussed. We also show that our approach is very useful to understand some of the puzzling features of charged black hole solutions in string theory.

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1 Introduction

The recent developments in string theory have pointed out the crucial role played by non-perturbative structures such as BPS solitons, black holes, D-branes, etc. [1]. Though we are still far away from having full control of the non-perturbative regime of string theory, the investigation of these structures represents a way to check new ideas, for example the dualities between different string theories, and to try to address in a new setting old, still unsolved, problems of black hole physics.

From this perspective, the fact that for the first time the black hole entropy has been calculated by counting microstates is very encouraging [2]. Moreover, the new scenario represent an unifying framework, in which most of the known solutions of low-energy string theory find their natural place. This is achieved, for instance, by relating typically string structures, e.g. D-branes [3], with more conventional spacetime structures, e.g. black holes.

One important issue, which has been widely investigated, is the interpretation of the geometrical structures, appearing in the low-energy theory, in terms of the fundamental string dynamics [4, 5, 6]. Generically, one expects the elementary (solitonic) string excitations to be related with electric (magnetic or dyonic) charged extreme black hole solutions of the 4d low-energy string theory. This correspondence should be particularly instructive because extreme black holes are BPS saturated states, so that one is tempted to use supersymmetry as a cosmic censor [7].

The implementation of the previous idea was only partially successful. For example, whereas extremal dilatonic black holes with dilaton coupling \( a = 0 \) are regular or have only null singularities, the \( a = \sqrt{3} \) solution has a naked singularity. Supersymmetry alone fails as universal cosmic censor. In the context of \( N \geq 2 \) supergravity theories, the requirement that the solutions preserve some of the supersymmetries puts strong constraints on the form of the admissible spacetime structures. The fact that the Bertotti-Robinson geometry behaves as an attractor for a wide class of \( N = 2 \) supersymmetric black holes is an example of this highly constrained dynamics [8, 9, 10]. Moreover, it gives us a general principle to compute the black hole entropy as an extremum of the central charge [8].

Despite the relevance of these results, some of the open questions that have been with us since the early investigations on charged black holes in string theory, remain still unanswered. Why is the spacetime structure of the extreme \( a = 0 \) solution (the Reissner-Nordstrom solution of general relativity) so drastically different from that of the other supersymmetric, \( a = 1, \sqrt{3}, 1/\sqrt{3} \) solutions? Why does the former have non-vanishing, charge-dependent, entropy, whereas the others have zero entropy? Why does the near-horizon geometry of the former behave as an attractor whereas this is not the case for the others?

A possible starting point for trying to answer these questions is to assume that the different behavior of the solutions of the low-energy theory can be explained in terms of the fundamental string dynamics. A first step in this direction is the interpretation of the black hole solutions in terms of string states. Progress along this line has been achieved by considering the \( a = \sqrt{3} \) solution as elementary and the other supersymmetric solutions as bound states [11, 12]. This idea was further supported by the interpretation of the black holes as intersections of D-branes [3].

In this paper we investigate, using as guideline the idea of compositeness together with that of duality symmetry, the extreme black hole solutions of a truncated \( d = 4 \) low-energy
string action. The model is particularly interesting because it gives an unified description of $d = 4$ black hole solutions in string theory. We show that the model has an $O(3, Z)$ duality symmetry that is generated by the duality (or mirror) symmetries connecting different string theories (heterotic, $IIA$, $IIB$) and by $S - T - U$ dualities. We use the $O(3, Z)$ symmetry and the bound state interpretation of extremal black holes to generate the BPS spectrum of the theory. We try to explain the features of the solutions by considering the correspondence with string states. The geometrical structures of the solutions are given in terms of the string metric. We show that whereas the elementary solutions have naked singularities, the bound states correspond to spacetimes with null singularities, in the electric, case and to regular spacetimes for the magnetic or dyonic solutions.

The structure of the paper is as follows. In sect. 2 we present the model we are going to investigate and its $O(3, Z)$ duality symmetry. In sect. 3 we consider the BPS black hole solutions and we explain how $O(3, Z)$ can be used as a spectrum-generating symmetry. The elementary solutions and the rules for constructing bound states are presented in sect. 4. In sect. 5 we construct the bound states of elementary solutions. In sect. 6 we discuss the corresponding spacetime structures. In sect. 7 we investigate the behavior of the near-horizon solutions as supersymmetric attractors. We also use a minimization procedure to calculate the entropy. Finally, in sect. 8 we present our conclusions.

2 Truncated $d = 4$ string action

The starting point of our discussion is the truncated version of the bosonic action for heterotic string theory compactified on a six-torus [12, 13],

$$S_H = \frac{1}{16\pi G} \int d^4 x \sqrt{-g} \left\{ R - \frac{1}{2} \left[ (\partial \eta)^2 + (\partial \sigma)^2 + (\partial \rho)^2 \right] - \frac{e^{-\eta}}{4} \left[ e^{-\sigma+\rho} F_1^2 + e^{-\sigma-\rho} F_2^2 + e^{\sigma+\rho} F_3^2 + e^{\sigma-\rho} F_4^2 \right] \right\}. \quad (1)$$

In the bosonic action of heterotic string theory toroidally compactified to $d = 4$, we have set to zero the axion fields and all the $U(1)$ field strengths but four, two Kaluza-Klein fields $F_1, F_2$ and two winding modes $F_3, F_4$. The scalar fields are related to the standard definitions of the string coupling, Kähler form and complex structure of the torus, through the equations

$$e^{-\eta} = \text{Im} S, \quad e^{-\sigma} = \text{Im} T, \quad e^{-\rho} = \text{Im} U. \quad (2)$$

Owing to the $SL(2, Z)_S \times O(6, 22, Z)_{TU}$ duality symmetry of the heterotic string in $d = 4$, the truncated theory [4] should have as duality group $Z_2 \times O(2, Z)$, where $Z_2 \subset SL(2, Z)_S$ is the strong/weak coupling duality and $O(2, Z) \subset O(6, 22, Z)_{TU}$ is a symmetry of the action [4], not only of its equations of motion. However, one can easily verify that the equations of motion following from the action [4] are invariant under the bigger group $O(3, Z)$. This enhancement of the duality symmetry is a consequence of the string/string/string triality discussed in ref. [4], which relates one with the other the heterotic, type $IIA$ and type $IIB$ strings in four dimensions. From this point of view, the action [4] can be thought of as a truncated, $S - T - U$ symmetric, action that gives a unified description of the low-energy solutions of string theory.
In the next sections we will make a broad use of the $O(3,Z)$ duality symmetry of the model [14]. In particular, we will use it to generate the BPS spectrum and to discuss the features of the corresponding spacetime structures. For this reason, we will first present some general properties of this group, its realization in terms of the fields appearing in the action (3) and its relationship with the string/string/string triality of ref. [14].

The $O(3,Z)$ group has 48 elements and can be realized as rotations (with integer entries) of the space $(\eta, \sigma, \rho)$, transforming, additionally, the field strengths $F_i$ ($i = 1,..,4$). The transformations of the $O(2,Z)_{TU}$ subgroup are off-shell symmetries of the theory (they correspond to mirror symmetries of string theory), whereas the other transformations are on-shell symmetries. An useful way to describe the $O(3,Z)$ group is to generate it by using the three $S-T-U$ duality transformations $\tau_S, \tau_T, \tau_U$ together with the permutation group $P_3$ acting on the scalar fields $\eta, \sigma, \rho$:

$$\tau_S : \eta \rightarrow -\eta, \quad F_1 \rightarrow \tilde{F}_3, \quad F_3 \rightarrow \tilde{F}_1, \quad F_2 \rightarrow \tilde{F}_4, \quad F_4 \rightarrow \tilde{F}_2;$$

$$\tau_T : \sigma \rightarrow -\sigma, \quad F_1 \leftrightarrow F_4, \quad F_2 \leftrightarrow F_3;$$

$$\tau_U : \rho \rightarrow -\rho, \quad F_1 \leftrightarrow F_2, \quad F_3 \leftrightarrow F_4;$$

(3)

$$P_3 : \begin{align*}
\sigma &\leftrightarrow \rho, \quad F_2 \leftrightarrow F_4; \\
\eta &\leftrightarrow \sigma, \quad F_3 \rightarrow \tilde{F}_4, \quad F_4 \rightarrow \tilde{F}_3; \\
\eta &\rightarrow \sigma, \quad \sigma \rightarrow \rho, \quad \rho \rightarrow \eta, \quad F_4 \rightarrow F_2, \quad F_2 \rightarrow \tilde{F}_3, \quad F_3 \rightarrow \tilde{F}_4; \\
\eta &\leftrightarrow \rho, \quad F_2 \rightarrow \tilde{F}_3, \quad F_3 \rightarrow \tilde{F}_2; \\
\eta &\rightarrow \rho, \quad \sigma \rightarrow \eta, \quad \rho \rightarrow \sigma, \quad F_2 \rightarrow F_4, \quad F_3 \rightarrow F_2, \quad F_4 \rightarrow F_3; \\
\end{align*}$$

(4)

where $\tilde{F}_1 = e^{-\eta-\sigma-\rho}F_1$, $\tilde{F}_2 = e^{-\eta-\sigma+\rho}F_2$, $\tilde{F}_3 = e^{-\eta+\sigma+\rho}F_3$, $\tilde{F}_4 = e^{-\eta+\sigma-\rho}F_4$ (* denotes the Hodge dual).

The previous transformations generate the whole $O(3,Z)$ group. Each element of $O(3,Z)$ can be written as a product of transformations appearing in (3,4). Moreover, these group-generating transformations have a simple interpretation in terms of string duality transformations. $\tau_S, \tau_T, \tau_U$ are the well-known $S-T-U$ string dualities that act on the moduli and on the charges. $\tau_S$ interchanges electrically (magnetically) charged Kaluza-Klein (KK) states with magnetically (electrically) charged Winding (W) states. $\tau_T$ maps electrically (magnetically) charged KK states into electrically (magnetically) charged W state. $\tau_U$ maps electrically (magnetically) charged KK (W) states into electrically (magnetically) charged KK (W) states.

The transformations (4), represent string/string dualities (or mirror symmetries). The permutation group $P_3$ generates the string/string/string triality diagram of ref. [14]. The five transformations (4) are, respectively, the five $d = 4$ string/string low-energy dualities (or mirror symmetries): $H_{STU} \rightarrow H_{SUT}$, $H_{STU} \rightarrow (IIA)_{TU}$, $H_{STU} \rightarrow (IIA)_{UT}$, $H_{STU} \rightarrow (IIB)_{UTS}$, $H_{STU} \rightarrow (IIB)_{UTS}$, where $H, IIA, IIB$ are the three string types (heterotic, $IIA, IIB$).

Besides the duality symmetries, an other important feature of the model under consideration is the possibility to describe some of its solutions as the solutions of a single scalar,
single gauge field action [15]. In fact, it has been shown that, for particular values of the integration constants (the charges and the asymptotic values of the scalar fields) the solutions can be obtained from a single scalar, single gauge field, action with dilaton coupling parameter \( a = \sqrt{3}, 1, 1/\sqrt{3}, 0 \) [11]. This fact has been used to give a bound state interpretation of extremal black holes in string theory. We will come back to this point in sects. 4 and 5, when we will generate the BPS spectrum using the idea of compositeness.

Let us conclude this section with some remarks about the supersymmetries of the model we are considering. Generically, compactification of the heterotic string on a six-torus produces \( N = 4 \) supergravity in four dimensions. However, we can also embed the solutions of the action (1) into \( N = 8 \) supergravity or, equivalently, we can consider eq. (1) as a truncated (bosonic) action of \( N = 8 \) supergravity. In the following, we will consider this second option, which give rise to four central charges \( Z_i \).

3 BPS states and \( O(3, Z) \) as spectrum-generating symmetry

The solutions of the action (1) describing extreme black holes are by now well known [16, 8]. We will consider here only supersymmetric solutions, i.e. BPS states that saturate some of the Bogomol’nyi bounds of \( N = 8 \) supergravity. We want to analyze the action of the duality symmetries on the space of the solutions. In general, duality transformations change the constant values of the scalar fields at infinity, which will be denoted here as \( g = e^{\eta_0/2}, h = e^{\sigma_0/2}, l = e^{\rho_0/2} \). The form of the solution given in ref. [8] is the most suitable one for our purposes, because it makes the dependence of the solutions on the moduli \( g, h, l \) manifest,
in eqs. (5), (6). The generic BPS saturated state will be labeled by the $U(1)$-charge vectors $Q_i, P_i (i = 1..4)$, with entries given respectively by the electric and magnetic charges $q, p$, the moduli vector $G_\alpha = (g, h, l), \alpha = 1, 2, 3$, the scalar-charge vector $\Sigma_\alpha = (\Sigma_\eta, \Sigma_\sigma, \Sigma_\rho)$ and the ADM mass $M$. $\Sigma_\alpha$ is defined in terms of the asymptotic behavior of the scalar fields,

$$2\eta = 2\eta_0 + \frac{\Sigma_\eta}{r} + O\left(\frac{1}{r^2}\right), \quad 2\sigma = 2\sigma_0 + \frac{\Sigma_\sigma}{r} + O\left(\frac{1}{r^2}\right), \quad 2\rho = 2\rho_0 + \frac{\Sigma_\rho}{r} + O\left(\frac{1}{r^2}\right), \quad \text{(7)}$$

whereas $M$ is defined, as usual, in terms of the asymptotic behavior of the metric. The previous parameters are not independent. In the following, we will consider $\Sigma_\alpha$ and $M$ as a function of the independent parameters $Q_i, P_i$ and $G_\alpha$.

In the usual realization (see eqs. (3), (4)), the duality group $O(3,\mathbb{Z})$ acts on the moduli $G_\alpha$ and on the charges $Q_i, P_i, \Sigma_\alpha$, but it leaves the mass of the state unchanged, because the metric is left invariant. A spectrum-generating technic for BPS saturated states requires transformations at fixed values of the moduli. The use for this purpose of duality symmetries is therefore problematic [18]. In order to make $O(3,\mathbb{Z})$ a suitable spectrum-generating symmetry, we use a realization of the group such that each group transformation, realized as in eq (3), (4) is followed by the inverse transformation of the moduli. In this way $O(3,\mathbb{Z})$ acts at fixed values of the moduli, whereas the mass of the state is changed. Denoting with $G_\alpha \rightarrow G'_\alpha(G)$ the transformation of the moduli, we have the following realization of $O(3,\mathbb{Z})$:

$$G_\alpha \rightarrow G_\alpha, \quad Q_i \rightarrow Q'_i, \quad P_i \rightarrow P'_i,$$

$$\Sigma_\alpha \rightarrow \Sigma'_\alpha \quad M(Q, P, G) \rightarrow M(Q, P, G'^{-1}). \quad \text{(8)}$$

Note that the transformations $G_\alpha \rightarrow G'_\alpha(G)$, acting on the moduli at fixed $U(1)$-charges, represent itself a realization of the $O(3,\mathbb{Z})$ group. Their action on the spectrum is to move around in the moduli space solutions of given $U(1)$-charge. Eqs. (8) translate immediately in a particular realization of the group-generating transformations (3), (4). In the next sections we will use this realization, together with the bound state interpretation, to generate the spectrum of extremal black hole solutions of the theory and to discuss their physical properties.

4 Elementary solutions and rules for the construction of bound states

The extremal black holes (5) can be viewed as bound states of elementary constituents [11]. It has been shown that extreme dilatonic black holes with dilaton coupling $a = 1, 1/\sqrt{3}, 0$ arise as bound states, with zero binding energy, of respectively 2, 3 or 4, elementary, $a = \sqrt{3}$ black holes [11]. This idea found support from the fact that these black hole solutions can be interpreted as intersections of 10-dimensional D-branes, yielding after compactification to 4d the $a = 1, 1/\sqrt{3}, 0$ black holes [3]. Here, we will develop further this idea of compositeness, and we will use it, together with duality symmetry, to generate the spectrum of the BPS saturated states of the model (1). Let us first list and discuss the elementary solutions, which will be used to build the bound states.
We consider as elementary those states that have only one \(U(1)\)-charge different from zero. For fixed values of the moduli and of the \(U(1)\)-charge, there are 8 of such states, which we denote with \(S_i, \bar{S}_i\). They can be generated from

\[
S_1 : Q_i = (q, 0, 0, 0), \quad \Sigma_\alpha = qghl(-1, -1, -1),
\]

acting with the \(O(3, Z)\) group-generating transformations (9). We have

\[
\begin{align*}
S_2 & : Q_i = (0, q, 0, 0), \quad \Sigma_\alpha = \frac{g}{h}(-1, 1, 1), \\
S_3 & : Q_i = (0, 0, q, 0), \quad \Sigma_\alpha = \frac{g}{hl}(-1, 1, 1), \\
S_4 & : Q_i = (0, 0, 0, q), \quad \Sigma_\alpha = \frac{g}{hl}(1, 1, 1), \\
\bar{S}_1 & : P_i = (q, 0, 0, 0), \quad \Sigma_\alpha = \frac{1}{ghl}(1, 1, 1), \\
\bar{S}_2 & : P_i = (0, q, 0, 0), \quad \Sigma_\alpha = \frac{l}{gh}(1, 1, -1), \\
\bar{S}_3 & : P_i = (0, 0, q, 0), \quad \Sigma_\alpha = \frac{hl}{g}(1, -1, -1), \\
\bar{S}_4 & : P_i = (0, 0, 0, q), \quad \Sigma_\alpha = \frac{h}{gl}(1, -1, 1).
\end{align*}
\]

The state \(\bar{S}_i\) can be obtained from the state \(S_i\) by acting with the transformation \(\tau_S \tau_T \tau_U\). Two generic states verifying this relation will be called throughout this paper dual states.

The charges characterize completely the state because the duality transformations act at fixed values of the moduli and the mass of the single state is related to the scalar-charge through the simple relation

\[
M = \frac{1}{4} |\Sigma_\eta|.
\]

The states \(S_i\) are characterized by \(M \propto g\), are electrically charged, correspond to elementary string excitations \([4]\) and are expected to be dominant in the weak coupling regime of the theory. The states \(\bar{S}_i\) are characterized by \(M \propto 1/g\), are magnetically charged, correspond to solitonic string excitations \([4]\) and are expected to be dominant in the strong coupling regime of the theory. This characterization holds for the \(S\)-String. Consistently with the idea of triality we have besides the \(S\)-string also a \(T\)-string and a \(U\)-string, whose dilaton/axion fields are given respectively by \(T\) and \(U\) \([14]\). A state which is electric for the \(S\)-string can have an other characterization for the other two strings.

The information about the electric or solitonic character of the state is encoded in the multiplet structure (9),(10). Of particular relevance is the transformation law of the states under the permutation group \(P_3\) (this is the group that switches between the \(S - T - U\) string description) and the invariance group \((\subset O(3, Z))\) of the state. If a state is invariant under \(P_3\), then this state will have the same characterization in the three string descriptions. This is the case of the state \(S_1(\bar{S}_1)\), which is electric (magnetic). On the other hand, if the invariance group of the state is not \(P_3\), it will look differently for the \(S - T - U\) strings and the multiplet structure gives account of this fact. This is what happens to the the
other 6 elementary states in the spectrum. For example, the state $S_2$ corresponds to an electric excitation of the $S-$ and $T$-string but to a solitonic excitation of the U-string. In fact, $S_2$ is invariant under the second transformation in eq. (4) (in terms of string/string dualities this corresponds to $H_{STU} \rightarrow (IIA)_{TSU}$) but is mapped into the state $\bar{S}_3$ by the fourth transformation in eq. (4) (in terms of string/string dualities this corresponds to $H_{STU} \rightarrow (IIB)_{UTS}$).

By putting together elementary solutions (9),(10), one can build bound states. Starting point of this procedure are the results of ref. [11], where bound states of 2, 3 and 4 solutions have been constructed. In order to construct the whole spectrum, we need, however, some simple and general rules. When we form bound states of two or more elementary states of different $U(1)$-charges, we get a multiplicity of states that have the same non-vanishing components of the charge vectors $P$ and $Q$ and that differ one from the other just in a permutation of the entries of the these vectors. This is an unpleasant feature because there is no relevant physical information related to the permutation of the entries of the $U(1)$-charge vectors. Taking into account these permutations would only make our notation awkward. For this reason, we will consider as equivalent, states that can be transformed one into the other just by a permutation of the entries of the $U(1)$-charge vectors. We will use for this equivalence class of composite states (up to four elementary states) the notation, 

$$(S_i, S_j, \ldots). \quad (12)$$

States with a given number of constituents transform as different multiplets under $O(3,\mathbb{Z})$. We will denote these multiplets with $n = 1, 2, 3, 4$.

A crucial and simple feature of the composite solutions is that all the parameters characterizing the solutions are additive, i.e. we can express the charges $Q, P, \Sigma$ and mass $M$ of the bound state in terms of the charges $Q_I, P_I, \Sigma_I$ and masses $M_I$ of the elementary constituents as follows:

$$Q = \sum Q_I, \quad P = \sum P_I, \quad \Sigma = \sum S_I, \quad M = \sum M_I. \quad (13)$$

The additivity of the masses implies that all the bound states we can construct have zero binding energy. Note that the states belonging to a given equivalence class (12) in general are not degenerate. The charges and masses of the single states can be obtained from those of the state that represents the equivalence class, by permuting the entries of the $U(1)$-charge vectors.

Let us now define the invariance group $G_0$ of the state (12). $G_0$ is defined as the subgroup of $O(3,\mathbb{Z})$ that maps the equivalence class (12) in itself. In general $G_0$ is not the invariance group of the single state. This will be given by the product of $G_0$ with a subgroup of the permutation group that acts on the entries of the $U(1)$-charge vectors.

The previous rules enable one to write down explicitly the form of the solution corresponding to a given bound state of elementary solutions. However, not all possible combinations of elementary solutions are a priori allowed. The easiest way to write down the selections rules one needs to construct the spectrum, is to use $O(3,\mathbb{Z})$ duality arguments. States with $n = 2, 3, 4$ transform as different multiplets under the $O(3,\mathbb{Z})$ group and, owing to the duality symmetry of the spectrum, every state within a given multiplet can be generated, acting with a transformation of $O(3,\mathbb{Z})$, from the states: $(S_1, S_3)$ for $n = 2$, $(S_1, \bar{S}_2, S_3)$ for $n = 3$, $(S_1, S_2, S_3, \bar{S}_4)$ for $n = 4$. This statement represents a strong selection rule that rules out
from the BPS spectrum some of the states that, in principle, can be build by combining
\( n = 1 \) elementary states. In the next section we will list and study the allowed bound states.

5 Bound states

5.1 The \( n = 2 \) multiplet

By constructing all the possible combinations of two elementary states we obtain the \( n = 2 \)
bound states. Only 12 of these states are allowed. In the \( S \)-string description we have 2
electric states with \( M \propto g \),
\[
(S_1, S_3), (S_2, S_4); \tag{14}
\]
2 magnetic states with \( M \propto 1/g \) (the duals of the previous states), and 8 dyonic states,
\[
(S_1, \bar{S}_2), (\bar{S}_3, S_4), M \propto h; \tag{15}
\]
\[
(S_1, \bar{S}_4), (S_2, \bar{S}_3), M \propto l, \tag{16}
\]
together with the dual states characterized, respectively, by \( M \propto 1/h \) and \( M \propto 1/l \).

From the point of view of the \( T- \) and \( U \)-string the role of the states are correspondingly
reversed. For instance, the \( T \)-string sees the states (15) as electric, the dual states as
magnetic and the states (14), (16) together with their duals, as dyonic. It is interesting to
see how the existence of dyonic states is a consequence of the \( O(3, Z) \) duality symmetry of
the spectrum. The following diagram explains the action of the \( O(3, Z) \) group-generating
transformations on the \( n = 2 \) multiplet:

\[
(14) \leftrightarrow (15) \leftrightarrow (16)
\]
\[
\downarrow \tau_S \quad \uparrow \tau_T \quad \uparrow \tau_U \quad \downarrow \tau_S
\]

The states in the second row are the dual states of those appearing in the first row. The
horizontal arrows represent transformations of the permutation group \( P_3 \) whereas the vertical
arrows represent the \( S - T - U \) dualities. The \( n = 2 \) multiplet is generated by horizontal
symmetries that switch between different string descriptions (e.g. they interchange states
with \( M \propto g \) with states with \( M \propto h \)) together with vertical symmetries that map electric
into magnetic states (e.g. \( M \propto g \) into \( M \propto 1/g \) states).

Each state in the multiplet has an invariance group \( G_0 \) that is generated by the permutation
group \( P_2 \), which interchanges two moduli, and by the product of two dualities in (3),
which act on the same moduli. For example, the states in the first column of (17) have a
invariance group generated by the permutation of \( (T, U) \) and by \( \tau_T \tau_U \).

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invariance group generated by the permutation of \( (T, U) \) and by \( \tau_T \tau_U \).

5.2 The \( n = 3 \) multiplet

The \( n = 3 \) multiplet has only 8 states. In fact the \( O(3, Z) \) symmetry allows only for the
following bound states of 3 elementary solutions:
\[
(S_1, \bar{S}_2, S_3), (S_1, S_3, \bar{S}_4), (\bar{S}_1, S_2, S_4), (S_2, \bar{S}_3, S_4), \tag{18}
\]
together with the dual states. At first sight, it could seem that these states have dyonic character, because they are bound states of elementary solutions some of which carry magnetic, others electric, \( U(1) \)-charges. However, it turns out that we can assign to each state either an electric or magnetic character. Each \( n = 3 \) state can be put in correspondence with a \( n = 1 \) state, in the following way:

\[
S_1 \sim (S_2, S_3, S_4), \quad S_2 \sim (S_1, S_3, S_4), \quad S_3 \sim (S_1, S_2, S_4), \quad S_4 \sim (S_1, S_2, S_3). \tag{19}
\]

Given this correspondence, the \( n = 3 \) multiplet spans the same representation of the \( O(3, Z) \) group as the \( n = 1 \) multiplet. As a consequence, the algebraic features discussed for the \( n = 1 \) multiplet hold also for the \( n = 3 \) multiplet. In particular, in view of (19), the states (18) can be considered as electric whereas the dual states can be considered as magnetic (S-string description).

Though the underlying \( O(3, Z) \) algebraic structure of the \( n = 3 \) and \( n = 1 \) multiplets is the same, the solutions behave differently. As we shall see in the next section, they give rise to different spacetime structures and break \( N = 8 \) supersymmetry in two different ways.

5.3 The \( n = 4 \) multiplet

The \( n = 4 \) multiplet contains only two states, the state \( X = (S_1, \bar{S}_2, S_3, \bar{S}_4) \) and its dual \( \bar{X} = (\bar{S}_1, S_2, \bar{S}_3, S_4) \). These two states transform one into the other under the action of the \( S - T - U \) dualities (3) but are invariant under the permutation group (4). The state \( X \) and \( \bar{X} \) are, therefore, truly dyonic because all the three strings \((S,T,U)\) see them as bound states of two magnetic and two electric elementary states. This behavior is to be compared with that of the \( n = 2 \) dyonic states. The states \( X \) and \( \bar{X} \) are also invariant under the action of the group

\[
\mathcal{H} = \{\tau_S \tau_T, \tau_S \tau_U, \tau_T \tau_U\}. \tag{20}
\]

\( P_3 \) together with \( \mathcal{H} \) closes to form the full invariance group, \( \mathcal{G}_0 \), of the state, which is a group isomorph to \( SO(3, Z) \).

6 Spacetime structure and compositeness

In the previous sections we have been mainly concerned with the algebraic features of the BPS spectrum. We turn now to discuss the spacetime structures associated with the solutions. Again, the idea of compositeness will play a crucial role. In particular, we will see that there is a non-trivial interplay between the singularities, the topological features of the spacetime and the interpretation of the solutions as bound states.

Let us first write the solution (4) in terms of the string metric, the metric to which strings naturally couple. A consequence of the triality symmetry is the existence of three string metrics, \( g_s, g_t, g_u \), to which the \( S - T - U \) strings respectively couple. They are related to the canonical metric in eq. (4) \( g_C \), through the equations

\[
g_s = e^\phi g_C, \quad g_t = e^\sigma g_C, \quad g_u = e^\rho g_C. \tag{21}
\]
The use of the string metric(s) for the discussion of the spacetime structures is particularly useful because it makes the transformation of the metric under the \( O(3, Z) \) duality group explicit. In fact, whereas the duality transformations leave invariant \( g_C \) (usual realization of the \( O(3, Z) \) symmetry) or change only the moduli dependence of \( g_C \) (our realization of the \( O(3, Z) \) symmetry), they change drastically the string metrics.

The presence of the three string metrics (21) in the same multiplet is a consequence of the \( O(3, Z) \) duality symmetry, in particular of its \( P_3 \) subgroup. In fact, either a state is invariant under the action of \( P_3 \) and in this case the three string metrics (21) are the same, or a state is not \( P_3 \) invariant and in this case the multiplet must contain the three metrics (21). For this reason, it is enough to consider just one string metric (we choose here \( g_S \)), the \( O(3, Z) \) symmetry of the spectrum will take automatically into account the other two.

In terms of the string metric, the extremal black hole solutions have the same form as in eqs. (5), but with metric given by

\[
\begin{align*}
\mathcal{d}s^2 &= -\psi_1 \psi_3 \, dt^2 + (\chi_2 \chi_4)^{-1} \left( dr^2 + r^2 d\Omega^2 \right). 
\end{align*}
\] (22)

Choosing the appropriate values of the \( U(1) \)-charges, and applying \( O(3, Z) \) transformations we can generate the \( n = 1, 2, 3, 4 \) solutions discussed in the previous section. We begin the discussion with some features of the general solution.

We are particularly interested in the near-horizon geometry, that is in the solution that describes the spacetime near \( r = 0 \). In this region the metric (22) has the form

\[
\begin{align*}
\mathcal{d}s^2 &= -\frac{m_c^2}{c^2} \, dt^2 + \frac{b^2}{r^s} \left( dr^2 + r^2 d\Omega^2 \right). 
\end{align*}
\] (23)

Where \( m, s = 0, 1, 2 \) are, respectively, the number of electric and magnetic solutions participating to the bound state, \( c \) is a constant which depends only on the moduli and on the electric charges, whereas \( b \) depends only on the moduli and on the magnetic charges. Eq. (23) is an exact solution of the Weyl rescaled version of the model (1). This solution gives a general description of the near-horizon geometry of the extremal black hole. The information about the nature of the state is encoded in the parameters \( m, s, c, b \). To discuss the singularities of the spacetime (23), which are the same as those of the spacetime (22), it is useful to write down the scalar curvature. It turns out that, independently of the parameter \( m \), the spacetimes with \( s = 2 \) (the bound states of 2 magnetic elementary constituents) have constant, \( m \)-dependent, curvature

\[
\begin{align*}
R &= \left( 2 - \frac{m^2}{2} \right) \frac{1}{b^2}. 
\end{align*}
\] (24)

Moreover, for \( s = 2 \), the spacetime (23) is the product of two two-dimensional spaces of constant curvature:

\[
H^2 \times S^2 \tag{25}
\]

where \( H^2 \) is a two-dimensional spacetime and \( S^2 \) is the two-sphere. For \( s \neq 2 \) the scalar curvature is

\[
\begin{align*}
R &= \frac{1}{2} \left[ m^2 + 2m - ms - 4 + (2 - s)^2 \right] \frac{r^{s-2}}{b^2}. 
\end{align*}
\] (26)

In this case \( r = 0 \) is always a curvature singularity of the spacetime.
6.1 Elementary states

The states of the \( n = 1 \) multiplet are characterized by one non-vanishing value of the \( U(1) \)-charges. Therefore they give rise to two spacetime structures, which carry either electric or magnetic charge. These spacetime structures, with \( h = l = 1 \), are also solutions of the dilaton gravity model with dilaton coupling parameter \( a = \sqrt{3} \). Near the horizon the solutions are given by eq. (23) with, respectively, \( m = 1, s = 0 \) and \( m = 0, s = 1 \),

\[
\begin{align*}
 ds^2 &= g^2 \left[ -\frac{r}{q_1 gh l} dt^2 + (dr^2 + r^2 d\Omega^2) \right], \\
 ds^2 &= g^2 \left[ dt^2 + \frac{p_1}{gh l r} (dr^2 + r^2 d\Omega^2) \right].
\end{align*}
\]

The previous equations describe the \( S_1 \) and the \( \bar{S}_1 \) state, but with slightly modifications (the moduli and charge dependence) also the other electric \( (S_i) \) and magnetic \( (\bar{S}_i) \) states of the \( n = 1 \) multiplet. Both the electric and magnetic spacetimes have a curvature singularity at \( r = 0 \), which is null in the electric case and naked (timelike) in the magnetic case. As we shall see later in detail, the \( n = 1 \) states are the only states in the spectrum that produce geometrical structures with naked singularities. The elementary solution have a maximal number \( (N = 4) \) of unbroken supersymmetries. In fact, they saturate the four Bogomol'nyi bounds. This two facts are consistent with the idea of compositeness: the elementary states are truly singular and possess a maximal amount of supersymmetries whereas the bound states, having an internal structure, are non-singular and possess a lesser number of unbroken supersymmetries.

6.2 \( n = 2 \) bound states

\( n = 2 \) states can be build in three distinct ways: a) with two electric, b) with two magnetic and c) with one electric and one magnetic \( U(1) \)-charge. The solutions with equal values of the charges and appropriate values of the moduli, are also solutions of the \( a = 1 \) dilaton gravity model \([13]\). Near the horizon the solutions are, respectively,

\[
\begin{align*}
 ds^2 &= -\frac{r^2}{q_1 q_3} dt^2 + g^2 \left( dr^2 + r^2 d\Omega^2 \right), \\
 ds^2 &= -g^2 dt^2 + \frac{p_1 p_3}{r^2} \left( dr^2 + r^2 d\Omega^2 \right), \\
 ds^2 &= -\frac{g}{h l q_1} r dt^2 + \frac{p_2 g l}{h} \frac{1}{r} \left( dr^2 + r^2 d\Omega^2 \right).
\end{align*}
\]

The equations (29), (30), (31) represent the (string) metric associated, respectively, with the states \( (S_1, S_3), (\bar{S}_1, \bar{S}_3), (S_1, \bar{S}_2) \). The metric solutions associated with the other states in the \( n = 2 \) multiplet can be easily obtained by using \( O(3, Z) \) duality transformations. They differ from the expressions (29), (30), (31) just in the charge and moduli dependence. The electric (29) and the dyonic (31) spacetime are both singular, but the singularity at \( r = 0 \) is a null singularity. On the other hand the magnetic spacetime (30) is perfectly regular, has constant curvature \( R = 2/p_1 p_3 \) and has the topology (25) with \( H^2 \) given by two-dimensional
Minkowski space. These features have a nice interpretation. The electric solutions correspond to elementary string excitation whereas the magnetic solutions correspond to solitonic string excitations. The dyonic solutions have a special status. As we have already noted in sect. 5.1, their existence is related to the string triality symmetry and their character (electric, magnetic or dyonic) depends on the string picture \((S, T, U)\) one is using. They can not be regular because, as noted at the beginning of this section, a bound state requires two elementary magnetic solutions in order to be a regular spacetime. All the \(n = 2\) BPS states preserve \(N = 2\) supersymmetry because they saturate two Bogomol’nyi bounds.

6.3 \(n = 3\) bound states

We have seen in sect. 5.2 that the \(n = 3\) multiplet can be put in correspondence with the \(n = 1\) multiplet. For this reason, we have also in this case only two spacetimes structures, the electric one, build from two electric and one magnetic elementary states and the magnetic one, built from two magnetic and one electric elementary states. Both spacetime, with equal values of the \(U(1)\)-charges and \(g = h = l = 1\), are solutions of the \(a = 1/\sqrt{3}\) dilaton gravity model. Near the horizon these solutions become,

\[
ds^2 = -\frac{r^2}{q_1 q_3} dt^2 + \frac{p_2 g l}{h r} \left( dr^2 + r^2 d\Omega^2 \right), \tag{32}
\]

\[
ds^2 = -\frac{r}{q_1 h l} dt^2 + \frac{p_4 p_3}{r^2} \left( dr^2 + r^2 d\Omega^2 \right) \tag{33}
\]

Eqs. (32),(33) correspond respectively to the \((S_1, \bar{S}_2, S_3)\) and to the \((S_1, \bar{S}_2, \bar{S}_4)\) states. The other solutions of the \(n = 3\) multiplet can be easily generated using \(O(3, Z)\) duality transformations. They have the same structure as (32) and (33). As expected, the electric spacetime (32) has a null singularity at \(r = 0\) whereas the magnetic one is regular. The latter is a spacetime with constant curvature \(R = 3/(2p_2 p_4)\) and of topology \((27)\), with \(H^2\) given by two-dimensional anti-de Sitter spacetime. Four-dimensional spaces of this kind have already been studied in the context of four-dimensional string effective theories in ref. [20]. The \(n = 3\) solutions saturate only one Bogomol’nyi bound, they are BPS states with only one unbroken supersymmetry.

6.4 \(n = 4\) bound states

Associated with the \(n = 4\) multiplet there is only one spacetime structure, which appears also as solution of the \(a = 0\) dilaton gravity model. The uniqueness of the solution is related to the high degree of symmetry of the \(n = 4\) states. In fact the two states of the \(n = 4\) multiplet have truly dyonic character and can be constructed by putting together two electric and two magnetic elementary states. The near-horizon geometry is given by the well-known Bertotti-Robinson spacetime

\[
ds^2 = -\frac{r^2}{q_1 q_3} dt^2 + \frac{p_2 p_4}{r^2} \left( dr^2 + r^2 d\Omega^2 \right). \tag{34}
\]

The spacetime is again given by the product of two two-dimensional spaces of constant curvature \((24)\) \((H^2\) is also here a two-dimensional anti-de Sitter spacetime). Apart from
the uniqueness, this solution has other features that make it peculiar with respect to the other solitonic solutions discussed in this section. First of all, the solution has the same form when expressed in terms of the string or canonical metric. We have already noted in sect. 5.3 that the the $S - T - U$ string descriptions of the $n = 4$ states are the same. Now we learn that the string and canonical description of these states are also the same. Second, the scalar curvature of the spacetime (34) not only is constant but is actually zero; the positive curvature of the two-sphere $S^2$ compensate exactly the negative curvature of $H^2$. Third, the near-horizon geometry (34) is moduli independent and depends only on the charges [8, 9]. Last, whereas the solution (5) preserve only $N = 1$ supersymmetry, for the near-horizon geometry we have the doubling of unbroken supersymmetries to $N = 2$ [8, 19]. These features are a consequence of the symmetries of the state and can be used to explain why, near the horizon, the $n = 4$ solutions behave as supersymmetric attractors, whereas this is not the case for the $n \neq 4$ supersymmetric solutions. This will be discussed in detail in the next section.

7 Supersymmetric attractors

Ferrara and Kallosh have shown that in the context of $N = 2$ supergravity theories the near-horizon geometry of a wide class of extreme black holes behaves has a supersymmetric attractor [8, 9]. The near-horizon geometry is moduli independent and depends only on the charges. A procedure to determine the entropy of the extremal black hole was also given. It consists in taking the extrema of the ADM mass, which for BPS saturated states coincides with the largest eigenvalue of the central charge matrix, with respect to the moduli at fixed $U(1)$-charges. One can show that the area of the horizon and then the entropy of the black hole, is proportional to $M^2$ evaluated in its minimum.

A nice example of this kind of behavior is given by the $n = 4$ solution discussed in the previous section [8]. All the features of the solution fit very well in this schema. The near-horizon geometry (34) is independent of the moduli. The vanishing of the Weyl tensor (related to the vanishing of the scalar curvature) for the Bertotti-Robinson geometry is responsible for the doubling of supersymmetries near the horizon [8, 19]. By minimizing the ADM mass of the solution

$$M = \frac{1}{4} \left\{ q_1 g h l + q_3 \frac{g}{l} h + p_2 \frac{l}{g h} + p_4 \frac{h}{g l} \right\},$$

(35)

with respect to the moduli $g, h, l$ and calculating the value of $M$ at the minimum one finds a non-vanishing charge-dependent, value of the entropy.

Though the Ferrara-Kallosh mechanism works very well for the $n = 4$ states, it cannot be used (at least in its original form) for $n \neq 4$. In fact, one easily realizes that for the $n \neq 4$ states the near-horizon metric (both in the canonical and string description) depends on the moduli. Moreover, the ADM mass does not have local extrema in the moduli space. The existence of supersymmetric attractors and the validity of the previous minimization procedure does not seem to be features of generic $N = 2$ BPS saturated states, but they seem to be related with the Bertotti-Robinson form of the near-horizon geometry. In the following, using duality symmetry arguments, we will show that a minimization procedure
can also be used to calculate the entropy of the \( n \neq 4 \) states and we will explain why for these states the near-horizon solution cannot be considered as an attractor.

It is a general fact, known since early works on the subject, that dilatonic extremal black holes with dilaton coupling \( a \neq 0 \) have zero area of the horizon [21], i.e. zero entropy. This is not only true for the solutions of the model that admit a single scalar description, but also for the generic solutions with \( n \neq 4 \). Only the states of the \( n = 4 \) multiplet are characterized by a non-vanishing value of the entropy.

The vanishing value of the entropy in the \( n \neq 4 \) case can be also obtained by looking for the minima of the corresponding ADM masses in the moduli space at fixed \( U(1) \)-charges. Differently from the \( n = 4 \) case we now look for generic, i.e. absolute or local, extrema of \( M \).

Let us see how it does work in detail for the \( n = 1, 2, 3 \) multiplets.

For \( n = 1 \), the ADM mass of the solutions can be easily calculated using eqs. (9), (10), (11). The minima of \( M \) occur at \( g = 0 \) for the electric solutions and at \( g = \infty \) for the magnetic ones. They are characterized by finite, undetermined values of the other two moduli \( h, l \) and by \( (M)_{\text{min}} = 0 \). The permutation symmetry of the spectrum implies that a pattern of \( M = 0 \) minima can be obtained either with \( h = 0, \infty \) and \( g, l \) undetermined or with \( l = 0, \infty \) and \( g, h \) undetermined. For example, for the state \( S_2 \) the minimum of \( M \) can be reached for \( g = 0 \) or \( h = 0 \) or \( l = \infty \). The physical meaning beyond this, at first sight complicated, pattern is simple: given a string description, the electric states are dominant (i.e. have zero mass) in the weak coupling regime, whereas the magnetic ones are dominant in the strong coupling regime.

The 12 states of the \( n = 2 \) multiplet belong to three classes having respectively 1) \( M \propto g \) or \( M \propto 1/g \); 2) \( M \propto h \) or \( M \propto 1/h \); 3) \( M \propto l \) or \( M \propto 1/l \). For instance, the states \((S_1, S_3), (\bar{S}_1, \bar{S}_3)\) have respectively

\[
M = \frac{1}{4} g \left( q_1 hl + \frac{q_3}{hl} \right), \tag{36}
\]

\[
M = \frac{1}{4} g \left( p_3 hl + \frac{p_1}{hl} \right). \tag{37}
\]

The minima of \( M \) fix only one modulus and are given respectively by 1) \( g = 0 \) or \( g = \infty \); 2) \( h = 0 \) or \( h = \infty \); 3) \( l = 0 \) or \( l = \infty \). As in the \( n = 1 \) case, the minima occur at \( M = 0 \) and leave undetermined two moduli. However, the nature of the indetermination here is different. Whereas for \( n = 1 \) the minima can occur for vanishing (or divergent) values of more than one modulus, for \( n = 2 \) we have minima for vanishing (or divergent) values of a single modulus. This has a simple explanation. If a state of the \( n = 2 \) multiplet is electric (or magnetic) in a string description it will be dyonic in the other two strings descriptions. For instance the state \((S_1, S_3)\) corresponds to elementary electric states of the \( S \)-string but to dyonic states of the \( T \)- or \( U \)-string. Therefore it will be dominant in the weak coupling regime of the \( S \)-string but it will decouple in both the weak/strong coupling regimes of the \( T \)- and \( U \)-string.

For \( n = 3 \), we have a similar behavior as for \( n = 1 \). The points of the moduli space which are minima for the \( n = 3 \) states are also minima for the corresponding (see eq. (19)) \( n = 1 \) states. There is one important difference. For \( n = 3 \) the indetermination that we have for \( n = 1 \) is removed, so that now minima take place on points not on curves of the moduli.
space. For example, the state \((S_2, \bar{S}_3, S_4)\) has mass

\[
M = \frac{1}{4} \left\{ g \left( \frac{h q_2}{l} + \frac{l q_4}{h} \right) + \frac{h l p_3}{g} \right\}.
\]  

(38)
The previous function reaches the minimum at \(g = h = l = 0\). The mass of the corresponding \(n = 1\) state, \(S_1\), reaches the minimum at \(g = 0\), \(h\) and \(l\) undetermined, \(h = 0\), \(g\) and \(l\) undetermined, \(l = 0\), \(g\) and \(h\) undetermined.

Let us try to understand how the previous features are related to the duality symmetries of the spectrum. A crucial role is played by the invariance group \(G_0\). We have already noted in sect. 4 that the single state belonging to the equivalence class of states \((12)\) is invariant under a group \(G_0 \times L_0\), where \(L_0\) is a subgroup of the permutation group acting on the entries of the two \(U(1)\)-charge vectors \(P, Q\). It is clear that \(G_0 \times L_0\) leaves also invariant the ADM mass of the solution and the near-horizon metric. One can show that if this group contains as subgroup the three duality transformations \(\tau_S, \tau_T, \tau_U\), or a group generated by taking products of these transformations, then a) the minimum of \(M\) takes place at finite, non-vanishing, charge-dependent values of the moduli and of \(M\) itself; b) the near-horizon geometry associated with the solution behaves as an attractor (it is moduli-independent and depends only on the \(U(1)\)-charges). To prove the statement a) one has to take into account that, as a consequence of the invariance of \(M\), both the point of minimum in the moduli space and \((M)_{\text{min}}\) must be fixed points of the group \(G_0 \times L_0\). Because zero and infinity cannot be fixed points of the transformations \((3)\) and because the group \(L_0\) acts on the charges \(Q, P\), it follows immediately the statement a). Also the statement b) follows from similar reasoning. Invariance of the near-horizon geometry under \(G_0 \times L_0\) implies that the moduli and charge dependence of the metric must be in an \(G_0 \times L_0\)-invariant combination. Because the only invariant of this kind that can be constructed by combining moduli and charges is the ADM mass \(M\) (or a function of \(M\)) and because \(M\) appears as the \(1/r\) coefficient in the asymptotic expansion of the metric, it follows that only a charge dependence of the near-horizon metric is allowed, i.e. statement b).

Now we can easily understand why the \(n = 4\) states are characterized by a non-vanishing value of the entropy and why for these states the near-horizon geometry behaves as an attractor. This two features simply follow from the fact that the invariance group of the state contains the group \(H\) in eq. (21), which is generated by the dualities \(\mathfrak{R}\). On the other hand, we can also explain why the states of the \(n = 1, 2, 3\) multiplets have zero entropy and why in this case the near-horizon geometry does not behave as an attractor. The invariance group of the \(n = 1, 3\) states is \(P_3\) and that of the \(n = 2\) states is generated by \(P_2\) and by the product of only two of the duality transformations \(\mathfrak{R}\) (see section 5.1). Both groups do not contain all the three dualities in eq. \(\mathfrak{R}\).

It is interesting to note that the states of the \(n = 2\) multiplet points out an intermediate behavior between the states of the \(n = 1, 3\) and of the \(n = 4\) multiplets. Though the invariance group of the \(n = 2\) states does not contain all the three duality transformations of eq. \(\mathfrak{R}\), it is generated by two of them. As a consequence though the near-horizon geometry is not fully moduli independent, it depends only on one modulus. The independence of the near-horizon geometry from two moduli means that, owing to the symmetries of the state, part of the attractor behavior of the \(n = 4\) states still survives for \(n = 2\).
8 Conclusions

One important result of our investigation of $d = 4$ extreme black hole solutions of string theory is that duality symmetries reveal all their predictive power when used together with the bound state interpretation. This is at least true for the truncated model we have analyzed in this paper, but one expects it to hold for more general and realistic models.

The use of these two ideas not only has given us a powerful and simple tool to generate the spectrum of the BPS black holes of the theory, but also a key concept to interpret the features of the black holes in terms of string states. Though our arguments are mostly based on the low-energy, truncated, string action $\mathcal{I}$, the structure of the various multiplets reflects the underlying $O(3, \mathbb{Z})$ duality symmetry, whose origin is of string theoretical nature. In particular, using the string metric to describe the corresponding spacetime structures, we have found how the cosmic censor hypothesis can be implemented in the case when one considers black holes as bound states. Only the elementary black hole solutions present naked singularities. The bound states have either null singularities (electrically charged black holes), or are regular geometric structure (magnetic or truly dyonic black holes). Moreover in the latter case the near-horizon geometry is the product of two two-dimensional spaces of constant curvature.

Our approach has also helped us to understand some old (and new) puzzling features of charged black holes in string theory. String theory allows for two kinds of extreme charged black holes. We have extreme black holes with non-vanishing entropy and attractor behavior of the near-horizon geometry on the one side and extreme black holes with zero entropy and moduli-dependent near-horizon geometry on the other side. We have seen that this can be explained in terms of the duality symmetry of the spectrum and using the bound state interpretation.

Our investigation has been mainly based on the model $\mathcal{I}$ that describes the solutions of (truncated) heterotic string theory compactified on a six-torus or the, through string/string dualities, related solutions of type $IIA, IIB$, string theories. It would be very interesting to test the validity of our approach and of our results on the complete (not truncated) theory. In this case one has to do with duality groups much bigger than $O(3, \mathbb{Z})$ and with a huge number of $U(1)$ field strengths that may introduce some complications in the construction of the bound states. An other interesting way to check and to improve our results is to use the interpretation of 4d black holes as intersections of D-branes. We expect this approach to be particularly fruitful because D-brains are solitonic excitation of string theory, so that their use represent a natural framework to implement both the ideas of duality and compositeness.

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