Unsteady Hydromagnetic Flow and Radiation Heat Transfer Past a Stretching Surface Embedded in a Porous Medium with Dissipation Effects

S. Anne Susan ¹, S.P. Anjali Devi, S. Nagarani
¹Sri Ramakrishna Institute of Technology, Coimbatore-641 010, Tamil Nadu, INDIA
E-mail: annesusan.sh@srit.org

Abstract. This paper presents a mathematical analysis on the effects of viscous dissipation over unsteady MHD flow and radiation heat transfer past a stretching surface subjected to suction embedded in a porous medium. The governing nonlinear partial differential equations of the problem are transformed into nonlinear ordinary differential equations adopting suitable similarity transformations. The numerical solution is obtained by employing Nachtsheim-Swigert shooting iteration method together with Fourth order Runge-kutta integration method. Numerical results for the dimensionless velocity and temperature distribution as well as for the local skin friction coefficient and nondimensional rate of heat transfer are obtained and displayed graphically for pertinent physical parameters.

1. Introduction

Research in the dynamics of fluid flow due to the stretching of a boundary along with radiative heat transfer has been contemplated by researchers worldwide because of its importance in the polymer industry. The dynamic initiative in this area was done by Sakiadis [1] and who investigated the boundary layer flow on a continuously stretching surface with an uninterrupted speed. Crane [2] further studied the flow of a Newtonian fluid over a linearly stretching surface and in view of these, various aspects of the flow and heat transfer problems for stretching surfaces moving in the infinite fluid medium have been investigated.

Thermal radiation effects play a significant role in controlling heat transfer processes in polymer industry and many works have been reported on flow and heat transfer over a stretching surface in the presence of radiation. However, only a few investigations are available on unsteady flow over a stretching surface along with Porous media which arise frequently in numerous industrial and transpiration processes. Unsteady flow past a stretching sheet has been discussed by Pop and Na [3]. Their work involved only the momentum equation, whereas Elbashbeshy et al. [4] solved both momentum and energy equations. The magnetohydrodynamic flow past a plate by the presence of radiation was analyzed by Raptis and Massalas [5]. Further, Sharidan et al. [6] analyzed unsteady flow and heat transfer over a stretching sheet in a viscous and incompressible fluid which is at rest. Radiation effect on the flow and heat transfer over an unsteady stretching sheet was analyzed by Mohamed Abd El-Aziz [7]. His work forms an extension to that of [4] by including the effect of thermal radiation in their analysis.

Soundalgekar [8] studied the viscous dissipation effect on unsteady free convection flow past an infinite, vertical porous plate with constant suction. The effects of variable viscosity on hydromagnetic flow past a continuously moving porous boundary was analysed by Seddeek [9]. Seddeek [10] also studied the effect of radiation and variable viscosity on an MHD free convection flow past a semi-infinite flat plate within an aligned magnetic field in the case of unsteady flow. Mukhopadhayay [11] presented solutions for unsteady boundary layer flow and heat transfer over a stretching surface with variable fluid viscosity and thermal diffusivity in presence of wall suction. Israel et al. [12] investigated on the influence of viscous dissipation and radiation on unsteady MHD free-convection flow past an infinite heated vertical plate in a porous medium with time-dependent suction.
The Combined effect of conducting and viscous dissipation on magneto hydrodynamic free convection flow along a vertical flat plate was discussed by Al-Mamun et al. [13]. Rafael Cortell [14] analyzed the effects of viscous dissipation and radiation on the thermal boundary layer over a nonlinearly stretching sheet. Viscous dissipation effects on nonlinear MHD flow in a porous medium over a stretching porous surface have been studied by Anjali Devi and Ganga [15].

In view of all these, the present work deals with unsteady MHD flow with radiation and dissipation effects over a stretching surface subjected to suction embedded in a porous medium.

2. Formulation of the problem
A two-dimensional, unsteady hydromagnetic boundary layer flow with effects of viscous dissipation and radiation of an incompressible, viscous, electrically conducting and radiating fluid over a stretching surface subjected to suction embedded in a porous medium is considered. The \( x \)-axis is taken along the stretching sheet and is in the direction of the flow of the fluid. The \( y \)-axis is taken perpendicular to the fluid flow. The velocity of the stretching sheet is given as

\[
U_v(x, t) = \frac{bx}{1-\alpha t}
\]

where both \( b \) and \( \alpha \) are constants where the constant \( \alpha \) has the dimension reciprocal to time. \( b \) is the initial stretching rate and \( b / (1 - \alpha t) \) is the effective stretching rate which varies with time. The properties of the material of the sheet may change with respect to time in case of polymer extrusion. However, in this case, the surface undergoes A variable magnetic field \( \mathbf{B} \) is applied along the \( y \)-axis and is given by,

\[
\mathbf{B}(x, t) = B_0 x \left(1 - \alpha t\right)^{\frac{m-1}{2}} \hat{y}
\]

where \( B_0 \) is the magnetic field strength, \( m \) is the positive integer with \( m=1 \) and \( \alpha \) is a constant. The temperature of the stretching sheet is given in the following form

\[
T_v(x, t) = T_\infty + T_0 \left[ \frac{bx^2}{2\nu^2} \left(1 - \alpha t\right)^\frac{3}{2} \right]
\]

where \( T_\infty \) is temperature of the fluid in the mainstream. \( T_0 \) is the reference temperature and \( \nu \) is the kinematic viscosity of the ambient fluid. The expressions for (1), (2) and (3) are valid only for time \( t < \alpha^{-1} \) unless \( \alpha = 0 \).

The Governing equations for such a boundary layer problem are given by

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\nu \partial^2 u}{\partial y^2} - \frac{\sigma B^2 u}{\rho} - \frac{v}{k_p}
\]

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_0 \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y} + \frac{v}{C_p} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\sigma B^2 u^2}{\rho C_p}
\]

where 
- \( u \) - velocity component along the \( x \)-axis
- \( v \) - velocity component along the \( y \)-axis
- \( \nu \) - kinematic coefficient of viscosity
- \( \sigma \) - fluid electrical conductivity
- \( B \) - magnetic field strength
- \( \rho \) - density of the fluid
- \( k_p \) - permeability of the medium
- \( T \) - fluid temperature
- \( \alpha_0 = (K / \rho C_p) \) - thermal diffusivity with \( K \) as the fluid thermal conductivity and \( C_p \) is the heat capacity at constant pressure and \( q_r \) is the radiative heat flux.
The fluid is considered to be gray, absorbing, emitting but non-scattering medium and the Rosseland approximation is used to describe the radiative heat flux which is given by

\[ q_r = -\frac{4\sigma^* T^4}{3k_1} \frac{\partial T}{\partial y} \]  

where \( \sigma^* \) is the Stefan-Boltzman constant and \( k_1 \) is the mean absorption coefficient.

\( T^4 \) is expressed as a linear function of temperature by assuming the temperature difference within the fluid to be very small and by expanding using Taylor series. Hence, it is obtained as,

\[ T^4 \approx 4T^4_T - 3T^4_T \] (neglecting higher-order terms) 

In view of the equations (7) and (8), equation (6) becomes

\[ \frac{\partial T}{\partial t} + \frac{u}{a} \frac{\partial T}{\partial x} + \frac{v}{a} \frac{\partial T}{\partial y} = \alpha_0 \frac{\partial^2 T}{\partial y^2} + \frac{v}{a} \left( \frac{\partial u}{\partial y} \right)^2 + \sigma \frac{\partial^2 u^2}{\rho C_p} \left( \frac{\partial y}{\partial y} \right)^2 \]  

The associated boundary conditions are given by

\[ u = U_w(x, t), \quad v = v_w(t), \quad T = T_w(x, t) \]  

at \( y = 0 \)

\[ u \to 0, \quad T \to T_\infty \] as \( y \to \infty \) 

where \( v_w = -\frac{v_0}{\sqrt{1 - \alpha t}} \) is the suction velocity \( (v_0 > 0) \) of the fluid. This expression for \( v_w(t) \) is valid only for time \( t < \alpha^{-1} \) unless \( \alpha = 0 \).

3. Method of solution

The following similarity variable \( \eta \) and the dimensionless variables \( f \) and \( \theta \) are introduced

\[ \eta = \left( \frac{b}{v(1 - \alpha t)} \right) \frac{1}{\sqrt{1 - \alpha t}} y \]  

\[ \psi = \left( \frac{b}{v(1 - \alpha t)} \right) \frac{1}{\sqrt{1 - \alpha t}} \]  

\[ \frac{1}{\sqrt{1 - \alpha t}} \]  

\[ T = T_w + T_\infty \left[ \frac{b^2}{2v} \right] (1 - \alpha t)^{-\frac{1}{2}} \cdot \theta(\eta) \]  

The mass conservation equation (4) is ascertained by the \( \psi \) which is the physical stream function. Here \( \theta(\eta) = \frac{T - T_w}{(T_\infty - T_w)} \) is the non-dimensional temperature.

The components of velocity are obtained as

\[ u = \frac{\partial \psi}{\partial y} = U_w(\eta, \theta), \quad v = -\frac{\partial \psi}{\partial x} = -(vb)^{1/2} (1 - \alpha t)^{-1/2} f(\eta) \]  

The mathematical problem defined in equations (4), (5) and (9) are then transformed into a set of nonlinear ordinary differential equations and their associated boundary conditions are

\[ f'' - \frac{\alpha}{b} \left[ \frac{\eta}{2} f'' + f' \right] - (f')^2 + f'' - Mf'' - \frac{1}{D} f' = 0 \]  

\[ (3R + 4) \theta'' + 3\Pr R \left[ \left( f - \frac{A}{2} \right) \theta' + \left( Ec f^2 \right) \theta' + 2f - \frac{A}{2} \right] = 0 \]  

where \( A = \frac{\alpha}{b} \) is the unsteadiness parameter, \( D = \frac{v}{\beta k} \) is the permeability parameter

\( M = \frac{\sigma B^2}{\rho b} \) is the magnetic parameter, \( R = \frac{K k_1}{4\sigma T_\infty^4} \) is the radiation parameter, \( \Pr = \frac{v}{\alpha} \) is the Prandtl number, \( \Ec = \frac{U_w}{C_p(T_\infty - T_w)} \) is the Eckert number, \( S = \frac{\sqrt{v_0}}{\sqrt{vb}} \) is the Suction parameter.

The boundary conditions corresponding to (15) and (16) are given by
f′′(η) = 1, f(η) = S, θ(η) = 1 at η = 0
f′′(η) → 0, θ(η) → 0 as η → ∞ (17)

4. Local Skin Friction and Local Nusselt Number
The local skin friction is also called as the skin friction drag and the non-dimensional rate of heat transfer or the local Nusselt number are other physical quantities of interest in this problem.

The skin friction drag is caused by the viscosity of the fluid as it moves on a surface and the local skin friction coefficient is given by

\[ C_f = \frac{2\mu (\partial u/\partial y)_{y=0}}{\rho U_\infty^2} = 2\Re_{\infty}^{1/2} f'(0) \] (18)

Convective heat transfer relationships are expressed in terms of Nusselt number as a function of Reynolds’s number and the local Nusselt number is given by

\[ Nu_x = \frac{x}{T_y} (\partial T/\partial y)_{y=0} = -\frac{1}{2} (1-\alpha^1) \Re_{\infty}^{1/2} 0'(0) \] (19)

5. Numerical solution of the problem
The nonlinear boundary value problem which is established by the equations (15) and (16) are solved numerically using Nachtsheim-Swigert iteration scheme along with Runge–Kutta iteration method. Suitable values for f′′(0) and θ′(0) are chosen and the criteria for the convergence of the problem depends upon the values that are guessed for the initial conditions in the shooting technique. The solution of the initial value problem is determined using Runge-Kutta fourth order method.

6. Results and discussion
The numerical solutions of the problem concerned with the effects of MHD, radiation and viscous dissipation on the unsteady flow over a stretching surface embedded in a porous medium are obtained for various values of the physical parameters involved in the problem. In the absence of porous medium and dissipation effects, the result of the present work is identical to that of Mohammed Abd El-Aziz (2009). In order to illustrate the numerical results pertaining to dimensionless longitudinal velocity and temperature, Figs. 1 to 13 are presented.

Fig. 1 shows the effect of the Unsteadiness parameter A over the dimensionless velocity. For increasing values of A, the velocity gets decelerated. This shows that the effect of the unsteadiness parameter is to reduce the velocity of the fluid flow. Fig. 2 depicts the dimensionless velocity for various values of the Permeability parameter D. There is an insignificant decrease in the velocity for an increase in D. The effect of the Magnetic interaction parameter M over the velocity is portrayed through Fig 3. The applied magnetic field produces a drag in the form of Lorentz force thereby decreasing the magnitude of the velocity and hence the velocity decreases due to the effect of magnetic interaction parameter. The effect of the suction parameter S over the dimensionless velocity is portrayed through Fig. 4. It shows that the velocity gets decelerates as the value of S increases.

Figures Fig. 5 to Fig. 10. depicts the influence of all the parameters involved in the dimensionless temperature. It is evident from Fig. 5 that the unsteadiness parameter A has a decreasing effect over the temperature and for various values of Eckert number the dimensionless temperature decreases which is shown in Fig.6. The influence of magnetic field is to reduce both the temperature of the fluid and the thermal boundary layer thickness. These effects are evident through Fig.7 which is due to the increasing frictional drag due to the Lorentz force which is responsible for increasing the thermal boundary layer thickness.

The nondimensional temperature for different values of Prandtl number is shown in Fig.8. It is seen that the effect of Prandtl number is to decrease the temperature with an accompanying decrease in the thermal boundary layer thickness. This is because a higher Prandtl number fluid has a relatively low thermal conductivity which reduces conduction, and thereby reduces the thermal boundary layer thickness Fig. 9 illustrates the effect of radiation parameter over dimensionless temperature. It is evident from this
figure that the temperature decreases for an increase in radiation parameter which is a clear vindication to the physical fact that the effect of radiation is to enhance the temperature. Variation in dimensionless temperature due to the variation in suction parameter is portrayed in Fig. 6. As the suction parameter increases, the temperature is reduced with an accompanying decrease in the thermal boundary layer thickness.

Effects of the permeability parameter $D$, magnetic interaction parameter $M$ and the suction parameter $S$ over the local skin friction coefficient are presented in Fig. 11, Fig. 12 and Fig. 13 respectively. All these parameters have a decreasing effect over the skin friction coefficient. Figs. 14 and 15 shows the nondimensional rate of heat transfer decreases for different values of Eckert number and magnetic interaction parameter respectively. Fig. 16 shows that the Prandtl number has an increasing effect over the nondimensional rate of heat transfer. The nondimensional rate of heat transfer for various values of radiation parameter and suction parameter is illustrated through Fig 17 and 18. It indicates that the nondimensional rate of heat transfer increases due to the effect of radiation and suction parameter.

7. Conclusion
The unsteady hydromagnetic nonlinear boundary layer flow over a stretching surface subjected to suction and embedded in a porous medium with the effects of radiative heat transfer and viscous dissipation is analyzed and the results are predicted through numerical computations. In general, the flow field and temperature distribution are affected by the thermophysical parameters involved in the problem. In the absence of porous medium and the dissipation effects the results are identical to Mohammed Abd El-Aziz (2009). From the numerical results thus obtained, the following conclusions are arrived.

- The dimensionless longitudinal velocity is decreased due to the effect of unsteadiness parameter, magnetic field, permeability parameter and suction parameter. It is interesting to note that the momentum boundary layer thickness is reduced due to the effect of unsteadiness parameter, local magnetic interaction parameter and suction parameter.
- The influence of unsteadiness parameter, Suction parameter, Eckert number, Prandtl number and the Radiation parameter is to decrease the dimensionless temperature as well as the thermal boundary layer thickness whereas the interaction of magnetic field enhances the temperature.
- There is a decreasing Effect of the permeability parameter $D$, magnetic interaction parameter $M$ and the suction parameter $S$ over the skin friction coefficient.
- The non-dimensional rate of heat transfer increases due to the effect of Suction parameter, Radiation parameter and Prandtl number. However, the magnetic interaction parameter and Eckert number have a decreasing effect over the non-dimensional rate of heat transfer.

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Fig. 7 Effect of $M$ over the dimensionless temperature profiles

Fig. 8 Dimensionless temperature profiles for various $\Pr$

Fig. 9 Dimensionless temperature profiles for different $R$

Fig. 10 Dimensionless temperature distribution for various $S$

Fig. 11 Variation of $D$ over $f''(0)$

Fig. 12 Variation of $M$ over $f''(0)$
\[ f''(0) \]

- \( A = 1.2, Ec = 0.01, D = 0.2, Pr = 0.71, R = 5.0, M = 3.0 \)
- \( S = 0.5, 1.0, 1.5, 2.0 \)

\[ \theta'(0) \]

- \( A = 1.2, D = 0.2, M = 3.0, Pr = 0.71, R = 5.0, S = 1.0 \)
- \( Ec = 0.001, 0.01, 0.05, 0.1 \)

Fig. 13 S variation over \( f''(0) \)

Fig. 14 Variation of Ec over \( -\theta'(0) \)

Fig. 15 M variation over \( -\theta'(0) \)

Fig. 16 Variation of Pr over \( -\theta'(0) \)

Fig. 17 Effect of R over \( -\theta'(0) \)

Fig. 18 S variation over \( -\theta'(0) \)