A Chebyshev multidomain adaptive mesh method for reaction-diffusion equations.

Summary: Reaction-diffusion equations can present solutions in the form of traveling waves with stiff wave fronts. Such solutions evolve in different spatial and temporal scales, and it is desired to construct numerical methods that can adopt spatial refinement at locations where the solution becomes stiff. In this work, we develop a high-order adaptive mesh method based on Chebyshev spectral methods with a multidomain approach for the traveling wave solutions of reaction-diffusion equations. The proposed method uses the non-conforming and non-overlapping spectral multidomain method with the temporal adaptation of the computational mesh. The spectral multidomain methods have been used for solving PDEs including the reaction-diffusion equations. However, the non-conformal, non-overlapping and adaptive mesh spectral methods have not been used in the reaction-diffusion community. The proposed multidomain spectral method solves the given reaction-diffusion equations in each subdomain locally first and the boundary and interface conditions are enforced in a global manner. In this way, the method can be parallelizable and is efficient for large reaction-diffusion systems. We show that the proposed method is stable and accurate for solving reaction-diffusion equations with stiff traveling wave solutions. We provide both one- and two-dimensional numerical results that show the efficacy of the proposed method. The application of the adaptive multidomain spectral methods to the reaction-diffusion equations, that yield stiff traveling wave solutions, is new and needs further investigation.

MSC:

65Mxx Numerical methods for partial differential equations, initial value and time-dependent initial-boundary value problems
92Cxx Physiological, cellular and medical topics
35Kxx Parabolic equations and parabolic systems

Keywords:
Chebyshev multidomain spectral method; reaction-diffusion equations; adaptive mesh method

Full Text: DOI arXiv

References:

[1] Alhumaizi, K.; Aris, R., Surveying a Dynamical System: a Study of the Gray-Scott Reaction in a Two Phase Reactor (1995), Longman: Longman Harlow - Zbl 0854.92024
[2] Artlurs, C. J.; Bishop, M. J.; Kay, D., Efficient simulation of cardiac electrical propagation using high order finite elements, J. Comput. Phys., 231, 3946-3962 (2012) - Zbl 1237.92005
[3] Baltensperger, R.; Trummer, M. R., Spectral differencing with a twist, SIAM J. Sci. Comput., 24, 1465-1487 (2006) - Zbl 1034.65016
[4] Barillot, E.; Boissonade, J., Asymptotic pseudospectral method for reaction-diffusion systems, J. Phys. Chem., 97, 1566-1570 (1993)
[5] Barkley, D., A model for fast computer simulation of excitable media, Physica D, 49, 61-70 (1991)
[6] Brauer, F.; Castillo-Chavez, C., Mathematical Models in Population Biology and Epidemiology, Texts in Applied Mathematics (2000), Springer: Springer New York - Zbl 0967.92015
[7] Bueno-Orovio, A.; Pérez-García, V. M.; Fenton, F. H., Spectral methods for partial differential equations in irregular domains: the spectral smoothed boundary method, SIAM J. Sci. Comput., 28, 886-900 (2006) - Zbl 1114.65119
[8] Bueno-Orovio, A.; Kay, D.; Burrage, K., Fourier spectral methods for fractional-in-space reaction-diffusion equations, BIT Numer. Math., 54, 937-954 (2014) - Zbl 1306.65265
[9] Chamakuri, N., Parallel and space-time adaptivity for the numerical simulation of cardiac action potentials, Appl. Math. Comput., 353, 406-417 (2019) - Zbl 1428.92006
[10] Cherry, E. M.; Greenside, H. S.; Henriquez, C. S., A space-time adaptive method for simulating complex cardiac dynamics, Phys. Rev. Lett., 84, 1343-1346 (2000)
Fenton, F. H.; Cherry, E. M.; Models of cardiac cell, Scholarpedia, 3, 1868 (2008)

Fenton, F. H.; Cherry, E. M.; Hastings, H. M.; Evans, S. J., Multiple mechanisms of spiral wave breakup in a model of cardiac electrical activity, Chaos, 12, 852-892 (2002)

Gray, P.; Scott, S. K., Autocatalytic reactions in the isothermal, continuous stirred tank reactor. Oscillations and instabilities in the system \( (A + 2 B \to 3 B; B \to C) \), Chem. Eng. Sci., 39, 1087-1097 (1984)

Hoermann, J. M.; Bertoglio, C.; Kronbichler, M.; Pfäller, M. R.; Chabiniok, R.; Wall, W. A., An adaptive hybridizable discontinuous Galerkin approach for cardiac electrophysiology, Int. J. Numer. Methods Biomed. Eng., 34, Article e2959 pp. (2018), (1-18)

Hu, G.; Qiao, Z.; Tang, T., Moving finite element simulations for reaction-diffusion systems, Adv. Appl. Math. Mech., 4, 365-381 (2012)

Huang, W.; Ren, Y.; Russell, R. D., Moving mesh partial differential equations (MMPDES) based on the equidistribution principle, SIAM J. Numer. Anal., 31, 709-730 (1994) · Zbl 0806.65092

Jones, W. B.; O’Brien, J. J., Pseudo-spectral methods and linear instabilities in reaction-diffusion fronts, Chaos, 6, 219-228 (1996)

Keener, J. P.; Sneyd, J., Mathematical Physiology, Interdisciplinary Applied Mathematics (1998), Springer: Springer New York · Zbl 0913.92009

Krause, D.; Döcker, T.; Pute, M.; Krause, R., Towards a large-scale scalable adaptive heart model using shallow tree meshes, J. Comput. Phys., 298, 79-94 (2015) · Zbl 1349.76940

Kuramoto, Y., Chemical Oscillations, Waves and Turbulence (1984), Dover: Dover New York · Zbl 0558.76051

Lee, K.; McCormick, W. D.; Pearson, J. E.; Swinney, H. L., Experimental observation of self-replication spots in a reaction-diffusion system, Nature, 1994, 215-218 (1994)

Liu, H.; Yan, J., The direct discontinuous Galerkin (DDG) methods for diffusion problems, SIAM J. Numer. Anal., 47, 675-698 (2009) · Zbl 1189.65227

Manukian, V., On travelling waves of the Gray-Scott model, Dyn. Syst., 2-27 (2015)

Muratov, C. B.; Osipov, V. V., Spike autosolitons and pattern formation scenarios in the two-dimensional Gray-Scott model, Eur. Phys. J. B, 22, 213-221 (2001)

Murray, J., Mathematical Biology I and II, Interdisciplinary Applied Mathematics (2002), Springer: Springer New York

Omos, D.; Shizgal, B., A spectral method of solution of Fisher’s equation, J. Comput. Appl. Math., 193, 219-242 (2006) · Zbl 1092.65088

Omos, D.; Shizgal, B. D., Pseudospectral method of solution of the Fitzhugh-Nagumo equation, Math. Comput. Simul., 79, 2258-2278 (2009) · Zbl 1166.65382

Owolabi, K. M.; Patidar, K. C., Higher-order time-stepping methods for time-dependent reaction-diffusion equations arising in biology, Appl. Math. Comput., 240, 30-50 (2014) · Zbl 1334.65136

Owolabi, K. M.; Patidar, K. C., Numerical solution of singular patterns in one-dimensional Gray-Scott-like models, Int. J. Nonlinear Sci. Numer. Simul., 15, 437-462 (2014) · Zbl 1401.65101

Pearson, J. E., Complex patterns in a simple system, Science, 216, 189-192 (1993)

Petrov, V.; Scott, S.; Shoval, K., Excitability, wave reflection and wave splitting in a cubic autocatalysis reaction diffusion system, Philos. Trans. R. Soc. A, 347, 631-642 (1994) · Zbl 0987.35047

Qu, Z.; Garfinkel, A., An advanced algorithm for solving partial differential equation in cardiac conduction, IEEE Trans. Biomed. Eng., 46, 1166-1168 (1999)

Rodríguez-Padilla, J.; Olmos-Liceaga, D., Numerical solutions of equations of cardiac wave propagation based on Chebyshev multidomain pseudospectral methods, Math. Comput. Simul., 151, 29-53 (2018) · Zbl 07316242

Rodríguez-Padilla, J.; Olmos-Liceaga, D., Chebyshev multidomain pseudospectral method to solve cardiac wave equations with rotational anisotropy, Int. J. Model. Simul. Sci. Comput., 9, Article 1850025 pp. (2018), (1-25)

Shankar, V.; Wright, G. B.; Kirby, R. M.; Fogelson, A. L., A radial basis function (RBF)-finite difference (FD) method for diffusion and reaction-diffusion equations on surfaces, J. Sci. Comput., 63, 745-768 (2015) · Zbl 1319.65079

Shen, J.; Tang, T.; Wang, L.-L., Spectral Methods. Algorithms, Analysis and Applications, Springer Series in Computational Mathematics, vol. 41, 109 (2011), Springer in Computational Mathematics, 41, 109 (2011)

Trangenstein, J.; Kim, C., Operator splitting and adaptive mesh refinement for the Luo-Rudy I model, J. Comput. Phys., 196, 645-679 (2004) · Zbl 1056.92014

Tyson, J. J., What everyone should know about the Belousov-Zhabotinsky reaction, (Levin, S., Frontiers in Mathematical Biology, Frontiers in Mathematical Biology, Lecture Notes in Biomathematics (1994), Springer: Springer Berlin, Heidelberg), 569-587 · Zbl 0925.92068

Tyson, J. J.; Keener, J. P., Singular perturbation theory of traveling waves in excitable media, Physica D, 32, 327-361 (1988) · Zbl 0656.76018
Yanagida, E., Stability of fast travelling pulse solutions of the FitzHugh-Nagumo equations, J. Math. Biol., 22, 81-104 (1985) · Zbl 0566.92009

Zhang, J.; Lin, S.; Wang, J., Stability and convergence analysis of Fourier pseudo-spectral method for FitzHugh-Nagumo model, Appl. Numer. Math., 157, 563-578 (2020) · Zbl 1447.65104

Zhang, R.; Yu, X.; Zhu, J.; Loula, A. F.D., Direct discontinuous Galerkin method for nonlinear reaction-diffusion systems in pattern formation, Appl. Math. Model., 38, 1612-1621 (2014) · Zbl 1427.65272

Zhang, Y.; Cohen, J.; Davidson, A. A.; Owens, J. D., A hybrid method for solving tridiagonal systems on the GPU, (Hwu, Wen-mei W., GPU Computing Gems (2012), Elsevier), 117-132

Zhu, J.; Zhang, Y.-T.; Newman, S. A.; Alber, M., Application of discontinuous Galerkin methods for reaction-diffusion systems in developmental biology, J. Sci. Comput., 40, 391-418 (2009) · Zbl 1203.65194

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.