Abstract. We reconstruct shear maps and angular power spectra from simulated weakly lensed total intensity ($TT$) and polarised ($EB$) maps of the Cosmic Microwave Background (CMB) anisotropies, obtained using Born approximated ray-tracing through the N-body simulated Cold Dark Matter (CDM) structures in the Millennium Simulations (MS). We compare the recovered signal with the ΛCDM prediction, on the whole interval of angular scales which is allowed by the finite box size, extending from the degree scale to the arcminute, by applying a quadratic estimator in the flat sky limit; we consider PRISM-like instrumental specification for future generation CMB satellites, corresponding to arcminute angular resolution of $3.2\arcmin$ and sensitivity of $2.43 \mu K\arcmin$. The noise contribution in the simulations closely follows the estimator prediction, becoming dominated by limits in the angular resolution for the $EB$ signal, at $\ell \simeq 1500$. The recovered signal shows no visible departure from predictions of the weak lensing power within uncertainties, when considering $TT$ and $EB$ data singularly. In particular, the reconstruction precision reaches the level of a few percent in bins with $\Delta \ell \simeq 100$ in the angular multiple interval $1000 \lesssim \ell \lesssim 2000$ for $T$, and about 10% for $EB$. Within the adopted specifications, polarisation data do represent a significant contribution to the lensing shear, which appear to faithfully trace the underlying N-body structure down to the smallest angular scales achievable with the present setup, validating at the same time the latter with respect to semi-analytical predictions from ΛCDM cosmology at the level of CMB lensing statistics. This work demonstrates the feasibility of CMB lensing studies based on large scale simulations of cosmological structure formation in the context of the current and future high resolution and sensitivity CMB experiment.

Keywords: CMBR polarisation, weak gravitational lensing, cosmological simulations

ArXiv ePrint: 1311.7112
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1 Introduction

The anisotropies in the Cosmic Microwave Background (CMB) are one of the most important observables of modern cosmology. Their study substantially contributed to the establishment of a reference cosmological model consistent with a flat Friedmann Robertson Walker metric and made of three main components, baryons and leptons representing about 4% of the total energy density, Cold Dark Matter (CDM, about 26%) which constitutes the large part of the gravitational potential around cosmological structures, and about 70% of a Dark Energy (DE) component, similar or coincident with a Cosmological Constant (Λ, CC), responsible for a late time phase of accelerated expansion. The early Universe phenomenology is consistent with the inflationary picture, with a Gaussian and almost scale invariant initial perturbation spectrum dominated by scalar modes. Modern observations allow to measure these quantities to percent precision or better (see [1] and references therein).

More and more efforts are being undertaken for measuring second order effects, i.e. physical phenomena which occur after the last scattering surface, the epoch at which CMB photons decouple from the rest of the system and the first order anisotropies are imprinted. In order to constrain the dark cosmological components, and the DE in particular, the observation and characterization of the weak lensing of the CMB induced by forming structures along the line of sight of photons at the epoch in which the DE overcomes the CDM component [2] is gathering more and more attention. CMB lensing, in fact, allows us to break the degeneracies present in the measurements of cosmological parameters through CMB observations only [3] as well as providing more constraining power on the same parameters [4, 5]. Moreover, as lensing is closely related to the underlying gravitational theory, it can be used to test the possibility that the late-time accelerated expansion is not given by a DE component, but rather by a modified theory of gravity [6]. As CMB lensing is a second order effect in cosmological perturbations, sourced by forming structures onto primary anisotropies, it acts on the total intensity (T) anisotropies through a smearing of the acoustic peaks on sub-degree angular scales, as well as the transfer of power to the arcminute scale [7]; the same effect is induced on the scalar-type mode of CMB polarisation, the $E$-mode, while a fraction of power in the latter leaks to the curl component, the $B$-mode, causing a characteristic peak centered on the scales of a few arcminutes. Furthermore, since primordial gravitational waves contribute to the $B$ polarization modes on the degree scale, the latter effect in particular from gravitational lensing must be correctly taken into account when information about the ratio between tensor and scalar modes in cosmology ($r$) is to be obtained (see [8] and references therein).

This CMB lensing signal, first detected correlating the data from the Wilkinson Microwave Anisotropy Probe (WMAP) with matter tracers [9], has been then measured through CMB observations alone by the ACT collaboration [10], the South Pole Telescope (SPT) [11] and with spectacular confidence in the first release of the PLANCK data [12]; future CMB observations from the PLANCK second data release and other sub-orbital experiments are expected to provide more and more precise measurements of this signal; recently, the cross-correlation of the $B$-modes as predicted by the cross-correlation between the SPT measurement of the $E$-mode polarisation with Herschel data and the actual polarisation $B$ measurement from SPT itself yielded a detection of the lensing $B$-modes to high confidence [13].

In this scenario, our capability of understanding the lensing signal to extreme accuracy is most important and a necessary condition for that is to be able to model it appropriately and to the accuracy needed by modern cosmological observations. In the recent past, ef-
forts were made in order to simulate lensed CMB maps in the context of modern N-body simulations, which, once validated, have the potential and crucial capability of enabling us to estimate the constraining power which we will have from CMB lensing in particular on the underlying cosmological model [see 14, and references therein] and most importantly in view of cross-correlating CMB lensing measurements with those of the Large Scale Structures (LSS) which are responsible for the lensing itself, culminating with the launch of the Euclid satellite in about one decade [15]. In this paper we progress on this line, by extracting and characterizing, for the first time, the lensing signal in simulated CMB temperature and polarization maps using ray tracing through N-body simulations. We exploit a flat sky lensing extraction pipeline developed and exploited in [16] onto the CMB lensed maps which were constructed by performing ray tracing in the Born approximation using the Millennium Simulations (MS) in [17, 18].

The paper is organized as follows. In Section 2 we introduce and discuss the details of the N-body simulations used to reconstruct the CMB maps, also specifying the methods used to produce these maps. In Section 3 we briefly review the theoretical background for lensing extraction methods and we detail the formalism and procedure followed. Section 4 contains the application of our extraction pipeline on the CMB maps reconstructed from the N-body simulations, assuming observational errors compatible with current and upcoming CMB surveys. We discuss our results in the concluding Section 5.

2 From N-Body simulations to CMB maps

Weak lensing of the CMB deflects photons coming from an original direction \( \hat{n}' \) on the last scattering surface to a direction \( \hat{n} \) on the observed sky, and the lensed CMB field is given by \( \tilde{X}(\hat{n}) = X(\hat{n}') \) in terms of the unlensed field \( X = T, Q, U \). The vector \( \hat{n}' \) is obtained from \( \hat{n} \) by moving its end on the surface of a unit sphere by a distance \( |\nabla \hat{n} \phi(\hat{n})| \) along a geodesic in the direction of \( \nabla \hat{n} \phi(\hat{n}) \), where \( \nabla \hat{n} \) is the angular derivative in the direction transverse to the line-of-sight pointing along \( \hat{n} \equiv (\vartheta, \varphi) \) [19–22]. Here the field \( \phi \) is the so-called “lensing potential”, and \( |\nabla \hat{n} \phi(\hat{n})| \) is assumed to be constant between \( \hat{n} \) and \( \hat{n}' \). Therefore the lensed temperature and polarization fields are given by

\[
\tilde{T}(\hat{n}) = T \left[ \hat{n} + \hat{n} \nabla \phi(\hat{n}) \right],
\]

\[
(\tilde{Q} + i\tilde{U})(\hat{n}) = (Q \pm iU) \left[ \hat{n} + \hat{n} \nabla \phi(\hat{n}) \right].
\]

In what follows we will consider only the small angle scattering limit, i.e. the case where the change in the comoving separation of CMB light rays, owing to the deflection caused by gravitational lensing from matter inhomogeneities, is small compared to the comoving separation between the undeflected rays. In this case it is sufficient to calculate all the relevant integrated quantities, i.e. the lensing potential and its angular gradient, the so-called deflection angle, along the undeflected rays. This small angle scattering limit corresponds to the Born approximation.

Under this approximation, adopting conformal time and comoving coordinates [23], the integral for the projected lensing potential due to scalar perturbations in the absence of anisotropic stress reads

\[
\phi(\hat{n}) = -2 \int_{0}^{D_{r}} dD \frac{D_{r} - D}{DD_{r}} \psi(D\hat{n}, D),
\]
where $D$ and $D_*$ are, respectively, the comoving angular diameter distances to the lens and to the CMB last scattering surface, and $\psi$ is the physical peculiar gravitational potential generated by density perturbations [19, 24–26]. Let us notice that $\phi$ is connected to the convergence field $\kappa$ via $\hat{\nabla}^2 \phi = -2\kappa$.

If the gravitational potential $\psi$ is Gaussian, the lensing potential is also Gaussian. However, the lensed CMB is non-Gaussian, as it is a second order cosmological effect produced by matter perturbations onto CMB anisotropies, yielding a finite correlation between different scales and thus non-Gaussianity. This is expected to be most important on small scales, due to the non-linearity already present in the underlying properties of lenses.

In the present work we analyse the full sky $T$, $Q$, $U$ maps lensed by the matter distribution of the MS and generated by [18] via a modification of the publicly available LENSPix code\(^1\) (LP), described in [21]. In its original version this code lenses the primary CMB intensity and polarization fields using a Gaussian realisation, in the spherical harmonic domain, of the lensing potential power spectrum as extracted from the publicly available Code for Anisotropies in the Microwave Background (CAMB\(^2\)). The modification made by the authors consists in forcing LP to deflect the CMB photons using the fully non-linear and non-Gaussian lensing potential realization obtained from MS exploiting the procedure briefly summarized below, and presented in [18]; we refer the reader to this paper for further details.

The MS is a high resolution $N$-body simulation for a $\Lambda$CDM cosmology consistent with the WMAP 1 year results [27], carried out by the Virgo Consortium [28]. It uses about 10 billion collisionless particles with mass $8.6 \times 10^8 h^{-1} M_\odot$, in a cubic region $500 h^{-1} $Mpc$^3$ on a side which evolves from redshift $z^* = 127$ to the present, with periodic boundary conditions. The map-making procedure developed in [17] is based on ray-tracing of the CMB photons in the Born approximation through the three-dimensional field of the MS peculiar gravitational potential. In order to produce mock lensing potential maps that cover the past light-cone over the full sky, the MS peculiar gravitational potential grids are stacked around the observer located at $z = 0$, and the total volume around the observer up to $z^*$ is divided into spherical shells, each of thickness $500 h^{-1} $Mpc: all the MS boxes falling into the same shell are translated and rotated with the same random vectors generating a homogeneous coordinate transformation throughout the shell, while randomization changes from shell to shell. The peculiar gravitational potential at each point along a ray in direction $\hat{n}$ is interpolated from the pre-computed MS potential grids which possess a spatial resolution of about $195 h^{-1} $kpc.

Being repeated on scales larger than the box size, the resulting weak lensing distortion lacks large scale power, which manifests itself in the lensing potential power spectrum as an evident loss of large scale power with respect to semi-analytic expectations, most noticeable at multipoles smaller than $l \simeq 400$. This has been cured in [17] by augmenting large scale power (LS-adding) directly in the angular domain, a procedure which we exploit here as well, since large scale modes in the lensing potential field are transferred to small scales in the CMB field, causing, e.g., the increasing of the so-called temperature damping tail with respect to the unlensed field. This mode coupling effect, which produces the characteristic non-Gaussianity of the CMB lensed field, is indeed exploited for the reconstruction of the underlying matter deflecting field. Nonetheless, in this work, we are mostly interested in studying the lensing reconstruction of the MS matter field, which corresponds to scales $l > 400$, and therefore, while still using all sky CMB lensed maps as input, we will exploit the flat sky lensing

\(^1\)http://cosmologist.info/lenspix/
\(^2\)http://camb.info/
extraction pipeline for the reconstructed lensing potential output, as described in the next Section.

For the construction of the all sky lensed CMB input maps, in [18] the LS-adding technique has been implemented directly into the LP code where the spherical harmonics domain has been split into two multipole ranges: $0 \leq l \leq 400$, where the MS fails in reproducing the correct lensing potential power due to the limited box-size of the simulation, and $l > 400$, where instead the power spectrum is reproduced correctly. On the latter interval of multipoles, the corresponding ensemble $\phi_{lm}^{MS}$ of lensing potential spherical harmonic coefficients produced by the MS lens distribution has been extracted. The LP code has been modified to read and use these MS harmonic coefficients on the corresponding range of multipoles. On the interval $0 \leq l \leq 400$, instead, LP generates its own ensemble of spherical harmonic coefficients $\phi_{lm}^{LP}$, which are a realisation of a Gaussian random field characterised by the CAMB semi-analytic non-linear lensing potential power spectrum inserted as input in the LP parameter file. Since on multipoles $0 \leq l \leq 400$ the effects of non-Gaussianity from the non-linear scales are negligible and the $\phi_{lm}$ are independent, every time that we run the MS-modified-LP, we generate a joined ensemble $\tilde{\phi}_{lm}$, where $\tilde{\phi}_{lm} = \phi_{lm}^{LP}$ for $0 \leq l \leq 400$ and $\tilde{\phi}_{lm} = \phi_{lm}^{MS}$ for $l > 400$. This technique reproduces correctly the non-linear and non-Gaussian effects of the MS non-linear dark matter distribution on multipoles $l > 400$, including at the same time the contribution from the large scales at $l \leq 400$, where the lensing potential follows mostly the linear trend.

To generate the lensed $T$, $Q$, $U$ fields from the MS-modified-LP code, we adopt the method described in [21], using a high value of the multipole $l_{\text{max}}$ to maximize the accuracy. This allows running the simulation several times without excessive consumption of CPU time and memory. We work under the assumption that tensor modes are absent in the early Universe, so that the produced $B$-mode polarization is due only to the power transfer from the primary scalar $E$-modes into the lensing induced $B$-modes. We choose $l_{\text{max}} = 6143$ and a HEALPix$^3$ pixelisation parameter $N_{\text{side}} = 2048$, which corresponds to an angular resolution of $\sim 1.72'$ [29], with $12N_{\text{side}}^2$ pixels in total.

3 CMB Lensing Extraction

As mentioned in the previous Section, we will work in the so-called “flat-sky approximation” for the reconstruction of the lensing potential. In this limit the lensing potential can be written as [20]:

$$\phi(\hat{n}) = \int \frac{d^2 L}{(2\pi)^2} \phi(L) e^{i L \cdot \hat{n}}$$  \hspace{1cm} (3.1)

where the polar and azimuthal angles have been replaced by the displacement $L$. The corrections due to lensing in the Fourier moments of temperature and polarization fields can be

http://healpix.jpl.nasa.gov/
expressed, at the linear order in $\phi$, as [19]

$$
\delta \tilde{T}(l) = \int \frac{d^2 l'}{(2\pi)^2} T(l') W(l', L), 
$$

(3.2)

$$
\delta \tilde{E}(l) = \int \frac{d^2 l'}{(2\pi)^2} \left[ E(l') \cos 2\varphi_{1l} - B(l') \sin 2\varphi_{1l} \right] W(l', L), 
$$

$$
\delta \tilde{B}(l) = \int \frac{d^2 l'}{(2\pi)^2} \left[ B(l') \cos 2\varphi_{1l} + E(l') \sin 2\varphi_{1l} \right] W(l', L), 
$$

where the azimuthal angle difference is $\varphi_{1l} \equiv \varphi_l - \varphi_{1l}$, $L = 1 - l'$, and

$$
W(l, L) = [-\mathbf{l} \cdot \mathbf{l}'] \phi(L). 
$$

(3.3)

These equations show that lensing couples the gradient of the primordial CMB modes $l'$ to that of the observed modes $l$. Furthermore, even if primordial $B$-modes are vanishing, $B(l') = 0$, lensing generates $B$-mode anisotropies in the observed map given the leakage from the $E$ and $T$-modes.

We will consider noise in the CMB maps assumed homogeneous and white, characterized by a Gaussian beam. The power spectrum of the detector noise is [30]

$$
C_l^X = \sigma_{\text{pix}}^2 \Omega_{\text{pix}}, 
$$

(3.4)

where $\sigma_{\text{pix}}$ is the r.m.s. noise per pixel and $\Omega_{\text{pix}}$ is the solid angle subtended by each pixel. The observed CMB temperature and polarization fields, $X \in [T, E, B]$, and their power spectra, $\tilde{C}_l^X$, are

$$
\tilde{X}_1^\text{obs} = \tilde{X}_1 e^{-\frac{1}{2} l^2 \sigma_b^2} + N_1^X, 
$$

$$
\tilde{C}_l^X \text{obs} = \tilde{C}_l^X e^{-l^2 \sigma_b^2} + C_l^X, 
$$

(3.5)

where $N_1^X$ is the Fourier mode of the detector noise, and $\sigma_b$ is related to the Full Width at Half Maximum (FWHM) of the telescope beam, $\theta_{\text{FWHM}}$, as $\sigma_{\text{FWHM}} = \theta_{\text{FWHM}} \Delta \ln 2$.

We exploit the quadratic estimator formalism [31–33], built in the context of the convergence estimators [34, 35], in order to extract the lensing information from the simulated CMB maps used in the analysis. The estimator is uniquely determined by requiring each component to be unbiased over an ensemble average of the CMB temperature and polarization fields $X$ and $Y$ ($\langle \tilde{\kappa}^{XY}(\hat{n}) \rangle = \kappa(\hat{n})$) and the variance of the estimator to be minimal,

$$
\langle \tilde{\kappa}_1^{XY} \tilde{\kappa}_1^{XY} \rangle = (2\pi)^2 \delta^D(\mathbf{1} - \mathbf{1}) (C_l^{\kappa \kappa} + N_l^{\kappa \kappa, XY}), 
$$

(3.6)

where the $N_l^{\kappa \kappa, XY}$ term represents the noise contribution which is also predicted by the estimator, as we see below. In real space the convergence estimators are constructed on the basis of appropriate filters of the observed fields, weighted in the harmonic domain by their power spectra, which are given by [35]

$$
G_{XY} (\hat{n}) = \int \frac{d^2 \mathbf{l}}{(2\pi)^2} \mathbf{i} \tilde{X}_1^\text{obs} \frac{C_l^{XY} \tilde{C}_l^{xY} \text{obs}}{C_l^{XY} \text{obs}} \left\{ e^{2i\varphi_l} e^{-\frac{1}{2} l^2 \sigma_b^2 + i \mathbf{l} \cdot \hat{n}} \right\} 
$$

(3.7)

$$
W_Y (\hat{n}) = \int \frac{d^2 \mathbf{l}}{(2\pi)^2} \mathbf{i} \tilde{X}_1^\text{obs} \frac{\tilde{C}_l^{xY} \text{obs}}{C_l^{XY} \text{obs}} \left\{ e^{2i\varphi_l} e^{i \mathbf{l} \cdot \hat{n}} \right\} e^{-\frac{1}{2} l^2 \sigma_b^2 + i \mathbf{l} \cdot \hat{n}}
$$

(3.8)
Figure 1. Noise spectrum for different experimental specifications. For graphical purposes we show the convergence power spectrum which is connected to the potential through $C_\kappa = \ell^2 C_{\phi \phi}$. In this figure, the prediction for the convergence spectrum using as input cosmology the estimated cosmological parameters coming from Planck 1 year observations (red solid line), the noise for the minimum variance quadratic estimator for a Planck-like experiment (green dashed line) and the the noise for the same estimator for a PRISM-like experiment (blue dashed line) are plotted.

where $\phi_1$ is the azimuthal angle of the wave vector $\ell$; the two phase factors in braces are applied when $Y = E, B$ respectively, and are unity when $Y = T$. Also, $C^{XY}_\ell = C^{XE}_\ell$ for $Y = B$. In the construction of these fields the map beam deconvolution is incorporated, hence the beam factors $e^{-\ell^2 \sigma_b^2}$ appearing on both fields.

Given the two filtered fields in Eq. (3.7) and Eq. (3.8), the convergence estimators are then given by

$$\hat{\kappa}^{XY}_\ell = -\frac{A^{XY}_\ell}{2} i \ell \cdot \int d^2 \hat{n} \Re \{ G^{XY}(\hat{n}) W^*_Y(\hat{n}) \} e^{-i \ell \cdot \hat{n}}. \quad (3.9)$$

The normalization coefficients, $A^{XY}_\ell$, are related to the noise power spectrum, $N^{\kappa \kappa,XY}_\ell$, of the estimators $\hat{\kappa}^{XY}(\hat{n})$ by $N^{\kappa \kappa,XY}_\ell = \ell^2 A^{XY}_\ell / 4$, and can be expressed as

$$\frac{1}{A^{XY}_\ell} = \frac{1}{\ell^2} \int \frac{d^2 l_1}{(2\pi)^2} (1 \cdot l_1) \frac{C^{XY}_{\ell_1} f^{XY}_{\ell_1,\ell_2} C^{X,obs}_{\ell_1} C^{Y,obs}_{\ell_2}}{C^X_{\ell_1,obs} C^Y_{\ell_2,obs}} \times \left\{ \cos 2 \Delta \varphi \right\} e^{-l_1^2 \sigma_b^2} e^{-l_2^2 \sigma_b^2}, \quad (3.10)$$
with \( l = l_1 + l_2 \), \( \Delta \varphi = \varphi_{l_1} - \varphi_{l_2} \), and \( \langle X_{l_1} Y_{l_2} \rangle = f^{XY}_{l_1 l_2} \phi_l \), where [31]

\[
\begin{align*}
  f^{TT}_{l_1 l_2} &= (1 \cdot l_1) C^T_{l_1} + (1 \cdot l_2) C^T_{l_2}, \\
  f^{TE}_{l_1 l_2} &= (1 \cdot l_1) C^E_{l_1} \cos 2\Delta \varphi + (1 \cdot l_2) C^E_{l_2}, \\
  f^{TB}_{l_1 l_2} &= (1 \cdot l_1) C^E_{l_1} \sin 2\Delta \varphi, \\
  f^{EE}_{l_1 l_2} &= [(1 \cdot l_1) C^E_{l_1} + (1 \cdot l_2) C^E_{l_2}] \cos 2\Delta \varphi, \\
  f^{EB}_{l_1 l_2} &= (1 \cdot l_1) C^E_{l_1} \sin 2\Delta \varphi. 
\end{align*}
\]

On the basis of the relations above, we stress that a careful estimation of the noise contribution to lensing depends on how accurately the observed spectra are known, as well as how much the exponential representation of the high \( l \) cutoff due to instrumental beam in (3.10) is indeed faithful. Our code for estimating the convergence using the quadratic estimator formalism is a direct implementation of the above equations, Eq. (3.4)–(3.11), and was exploited in [16]. In that work, the CMB lensing signal was directly simulated on flat sky. In the present one, we need to project a curved sky onto a flat patch, in order to proceed with the analysis. We exploit a gnomonic projection scheme validating it in the next Section.

| Experiment | FWHM (arcmin) | \( \sigma_{\text{pix}} \) (\( \mu \text{K} \cdot \text{arcmin} \)) |
|------------|---------------|----------------------------------|
| PLANCK     | 7.18'         | 43.1                             |
| PRISM      | 3.2'          | 2.43                             |

Table 1. PLANCK and PRISM performance specifications. Beam FWHM is given in arcminutes, and the sensitivity for \( T \) per pixel in \( \mu \text{K} \cdot \text{arcmin} \). The channels used are 143 GHz for PLANCK and 160 GHz for PRISM. The polarization sensitivity for both \( E \) and \( B \)–modes is \( \sqrt{2} \Delta T / T \).

In Fig. 1 we show the forecasted noise spectra for the minimum variance estimator in a PLANCK-like case [36] and in a PRISM-like case [37] (for the adopted specifications see Tab. 1), computed for the PLANCK 1 year cosmology [38].

By comparing the amplitude of the noise contribution between the PLANCK and PRISM cases, we can conclude that the precision of the PLANCK experiment, despite being extremely powerful on the already delivered temperature spectrum, does not allow for a detection with high signal to noise ratio at the large scales targeted in this work, both due to the beam amplitude and to the sensitivity, whereas in the case of a future survey with the PRISM specifications the quality of the measurement will improve significantly, permitting to obtain a highly precise reconstruction also at very small angles. For this reason, in this work we will adopt the PRISM specifications to address the contribution coming from non-linearities.

4 The recovered lensing signal

In this Section we discuss the results of our lensing extraction, showing maps of recovered shear lensing signal, and quantitative comparisons of its power spectrum against the \( \Lambda \text{CDM} \) predictions in the interval of angular scales which is made accessible by the present simulation setup. First, let us do a few considerations on the noise spectra in the angular region of interest. It is known that the noise spectra of all the possible combinations \( TT, TE, TB, EE, EB \) for the convergence spectrum are relatively flat on large scales, just having a difference in amplitude, but not exhibiting a particular dependence on \( l \) (see [39]). As already
explained in the previous Sections, we are interested in lensing reconstruction ranging from the arcminute to the degree scale, where the noise spectrum is comparable or lower than the signal to extract only for the TT and EB cases. Thus, we focus our analysis on these two observables as they are most significant for the experimental configurations we will examine.

We apply the flat sky lensing estimator procedure described in the previous Section by adopting a $15^\circ$ patch side. The lensing extraction pipeline proceeds as follows. From the all sky lensed maps, we extract 296 squared patches, with centres distributed following [40]. The shear angular power spectra from each single patch are then stacked for producing the final result. The noise contribution as predicted by the lensing estimator is subtracted. In order to validate our simulation setup, we perform a test run using a simulated LP map by adopting the PRISM specifications and a WMAP 1 year fiducial set of cosmological parameters [27]. The resulting convergence spectra as output by the lensing extraction pipeline and obtained by subtracting the noise contribution are shown in Fig. 2 and exhibit a complete agreement with the theoretical prediction both for the TT and the EB case. The zoomed regions
in the $800 \leq \ell \leq 2000$ show numerical instabilities which are showing up at the highest multipoles. The figure also anticipates some of the features which will be highlighted for the cases of the run on the N-body CMB lensed maps, precisely in the shape and amplitude of the noise contributions, for the $TT$ and $EB$ cases. The $TT$ case appears to be noise dominated on all angular scales, while the effects of the limited angular resolution are visible at the largest scales in the $EB$ case, in the shape of the noise contribution, reflected by the error bar increase in the recovered signal at $\ell \gtrsim 1500$. The $\Lambda$CDM predicted power is recovered very accurately on all scales, reflecting the precision in the evaluation of the noise contribution. Finally, with the adopted specifications, the polarised data do represent a significant contribution to the recovery of the signal, with comparable precision up to $\ell \simeq 1500$. It is also interesting to look at the reconstruction precision, reaching a few percent in bins with $\Delta \ell \simeq 100$ in the angular multiple interval $1000 \lesssim \ell \lesssim 2000$ for $T$, and about 10% for $EB$.

We now turn to the study of results on the N-body lensed CMB maps. The angular power spectra from the shear maps stacking are shown in Fig. 3, where the two panels corresponding to the result of the $TT$ (left) and $EB$ (right) estimators, respectively, show the reconstructed lensing potential evaluated by stacking the lensing spectra extracted in each of the regions considered. As expected, the noise contributions for the two cases are the same as the LP case in Fig. 2. The solid lines corresponding to the the spectra after subtraction of the noise contribution show no visible departure from predictions of the weak lensing power as predicted by the $\Lambda$CDM cosmology, within uncertainties, for both cases, in particular on the angular scales which are less affected by Cosmic Variance, e.g. corresponding to $\ell \simeq 1000$ and beyond. The consistency between the two cases keeps validity up to the extreme angular resolution, as it is clear by comparing the zoomed areas in this and 2 cases, indicating that the behaviour at the largest scales is actually a numerical feature to be attributed to the estimator rather than to the simulated CMB lensing maps. It is to be noted that the results in Fig. 3 are obtained from the MS-modified-LP as defined in Sec. 2, whereas the panels shown in 2 have been obtained with the standard unmodified LP version. This result, validating the whole scheme of the simulation pipeline, constructed using N-body structures out of theoretical power predicted semi-analytically, ray traced and then inspected at the level of the CMB lensing extraction precision, has the immediate consequence that the biases from inaccuracies across the pipeline are well below the high precision performance of next generation CMB experiments for lensing extraction. The outline procedure should then allow to characterize departures from $\Lambda$CDM predictions within the redshift interval which is contributing significantly to the lensing power, within the assumed instrumental accuracy. It should be noted that this is true in particular in the small scale part, where the corrections from mildly non-linear matter evolution, described through the HALOFIT package into CAMB contribute and are faithfully reconstructed.

Before concluding we perform a last visual study of our results, showing how the lensing signal is consistent in different renderings. In Fig. 4 the four panels show the modulus of the input and reconstructed deflection angle compared with the difference between the lensed and unlensed CMB maps for $T$ and the polarisation amplitude $P = \sqrt{Q^2 + U^2}$. A first immediate evidence is the marked non-Gaussianity of the lensing field, e.g. in the $T$ difference; the structures there represent the line of sight integral of MS DM lenses acting on the background CMB field. The same holds for the polarisation field difference, with a clear correlation with the $T$ field, as expected, as well as a finer structure in the lensing
contribution. The bottom panels show the input and reconstructed noisy lensing potential field, again featuring an evident correlation between input and output, despite of the noise pattern, which is also well evident. A similar analysis, on the whole sky, was performed by [18], without applying a full lensing extraction pipeline as we do in the present work.

5 Concluding remarks

We presented here the first extraction of lensing shear and quantitative comparison with semi-analytical expectations of Cosmic Microwave Background (CMB) lensing simulations.
obtained through ray-tracing across N-body structure formation. We consider the lensed total intensity and polarisation CMB maps obtained by displacing the background field with Born approximated deflection angles evaluated from the Millennium Run simulation, stacked to fill up the whole Hubble volume. We test our pipeline by making use of simulated realisations of CMB lensing fields where the polarisation angle is assumed to have a Gaussian statistics and a power spectrum as given by semi-analytic predictions. We adopt the specification of future high resolution and sensitivity CMB satellites, corresponding to arcminute and µK-arcminute angular resolution and sensitivity, respectively. The geometry of the simulation setup, corresponding to a N-body box size of 500$h^{-1}$ Mpc and a pixelisation with 1.7′ pixel size, gives us access to angular scales covering the arcminute and reaching about a degree in the sky. For that we use a flat sky approximated version of the lensing extraction pipeline based on a quadratic estimator.

We inspect separately the extracted lensing pattern from total intensity and polarisation. We discuss the lensing contribution as predicted by the lensing estimator in the two cases, finding the signal to be completely signal dominated for total intensity, while the effect of limited angular resolution is evident in the polarisation noise contribution at the small scale edge of the relevant interval.

By applying the extraction pipeline, we find that the reconstructed weak lensing shear power spectra are featureless as in the case of the simulated maps, following the theoretically predicted power within the assumed uncertainties, separately for the total intensity and polarisation based estimator. Within the assumed instrumental specifications, we find that the polarisation field has comparable relevance in constraining the lensing signal.

The performed analysis is relevant in the context of the current and planned CMB and LSS large observational campaigns. In this context, galaxy-galaxy and CMB lensing are predicted to be most important observables for constraining the dark cosmological components, and the control and reliability of the corresponding simulated signal possesses a crucial importance in the forecasting phase, as well as for the interpretation of the data. For this reason, it is important in particular for CMB lensing to gather the different pieces of the simulations in a single pipeline and to study the results. This work represents a first significant step in this direction, demonstrating not only that the inaccuracies of the simulated cosmological structure, ray tracing scheme and lensing extraction provide no significant disturbance to the lensing recovery on the entire interval of angular scale considered, but also that this procedure can be upgraded by adopting more sophisticated simulations, both in terms of general architecture of the N-body and/or ray tracing procedure, as well as underlying cosmologies. These aspects are indeed the subject of our future works in this direction.
Acknowledgements

This work was supported by the INFN PD51 initiative. CB, MM and CA acknowledge support by the Italian Space Agency through the ASI contracts Euclid-IC (I/031/10/0). YF is supported by ERC Grant 277742 Pascal and acknowledges funding from the research council of Norway. CC acknowledges the INAF Fellowships Programme 2010. Part of this work was conducted at the Institute of Theoretical Astrophysics, University of Oslo. YF and CA acknowledge support from this institution. Also, CA thanks the Institute of Cosmology and Gravitation and the Perimeter Institute for Theoretical Physics for hospitality during this work.
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