MAXIMAL OPERATORS OF $T$ MEANS WITH RESPECT TO WALSH–KACZMARZ SYSTEM

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Abstract. In this paper we prove and discuss some new $(H_p, L_p, \infty)$ type inequalities of the maximal operators of $T$ means with monotone coefficients with respect to Walsh-Kaczmarz system. It is also proved that these results are the best possible in a special sense. As applications, both some well-known and new results are pointed out. In particular, we apply these results to prove a.e. convergence of such $T$ means.

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