Vector Supersymmetry and Finite Quantum Correction of Chern-Simons Theory in the Light-Cone Gauge

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Abstract

Vector supersymmetry is shown to exist also in light-cone gauge Chern-Simons theory. Using a gauge invariant regularization scheme, we demonstrate explicitly that the finite quantum correction to the coupling constant of Chern-Simons theory is intimately associated with the breaking of vector supersymmetry at the regularization level. The advantage of investigating such a quantum phenomenon in the light-cone gauge is emphasized and the BRST and vector supersymmetry invariance of quantum effective action is discussed.

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1. Introduction

Gauge fixing is an indispensable procedure in quantizing a gauge field theory. Different gauge choices define distinct hypersurfaces which intersect the gauge orbits in the configuration space of the gauge field and lead to different gauge-fixed effective actions. The gauge-fixed effective action breaks the local gauge symmetry but preserves a rigid BRST symmetry. The latter is known to play a fundamental role, since it ensures not only the renormalizability and unitarity of the underlying gauge theory, but also guarantees its gauge independence. In addition to this rigid BRST symmetry, there exists a BRST-like vector supersymmetry which was discovered in three-dimensional Chern-Simons theory and which can arise with certain gauge choices, such as the Landau gauge [1]. Although this vector supersymmetry survives only in a particular gauge, it was nevertheless beneficial in analyzing the dynamical structure of Chern-Simons theory, for instance in proving its perturbative finiteness [2,3]. Moreover, it was argued that this vector supersymmetry may not only play

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a role in constructing physical observables and, therefore, possesses actual physical relevance \[4\], but might also be the symmetry origin for the infrared safety of topologically massive Yang-Mills theory \[5\]. For these very reasons it would be desirable to clarify the physical status of vector supersymmetry.

The dynamical effects of vector supersymmetry in the Landau gauge have been investigated by various authors \[2,3,6\]. They showed that Landau vector supersymmetry imposes subtle constraints on quantum Chern-Simons gauge theory \[2,3\], preventing the theory from receiving any quantum corrections and thereby keeping the quantum theory identical to its classical counterpart. So much for the theoretical end of things. But what happens to vector supersymmetry in an actual calculation, i.e. in a perturbative calculation, which is known to require an intermediate regularization procedure in order to handle the UV divergence? The question is nontrivial since we know that BRST symmetry and Landau vector supersymmetry cannot survive simultaneously at the regularization level. In short, there exists no regularization procedure preserving both the BRST symmetry and Landau vector supersymmetry. Moreover, if a regularization scheme preserves BRST symmetry, but breaks Landau vector supersymmetry, the coupling constant of Chern-Simons theory will receive a finite quantum correction \[1-3\]. By contrast, if the Landau vector supersymmetry is respected and the BRST symmetry violated, there is no quantum correction and the coupling constant keeps its classical value \[10,11\]. Accordingly, since BRST symmetry is the most fundamental symmetry a gauge theory can possess, it seems reasonable to work with a regularization scheme that will preserve that symmetry \[6\].

A few years ago researchers discovered — again in the framework of Chern-Simons theory — that vector supersymmetry manifests itself in noncovariant gauges as well, specifically in gauges of the axial type \[12,13\]. A detailed practical calculation was performed in the light-cone gauge by the authors of Ref. \[14\]. Employing a BRST-invariant regularization, consisting of higher covariant-derivative regularization and dimensional regularization, these authors demonstrated the appearance of a finite quantum correction to the coupling constant. This result motivated us to check whether the finite quantum correction is connected with the breaking of vector supersymmetry at the regularization level.

2. Symmetries in light-cone Chern-Simons theory

The classical Euclidean action of $SU(N)$ Chern-Simons theory reads

$$S = -\frac{i k}{4\pi} \int d^3x \epsilon_{\mu\nu\rho} \left( \frac{1}{2} A^a_{\mu} \partial_{\rho} A^a_{\nu} + \frac{1}{3!} f^{abc} A^a_{\mu} A^b_{\rho} A^c_{\nu} \right), \quad \mu, \nu, \rho = 1, 2, 3,$$

where $f^{abc}$ are the structure constants of the gauge group $SU(N)$ and $k$ is the bare statistical parameter of Chern-Simons theory. It is convenient to write $|k| = 4\pi/g^2$ and to rescale the gauge field $A^a_{\mu}$ as $A \rightarrow gA$. Hence, in the homogeneous light-cone gauge $n \cdot A^a = 0$, $n^2 = 0$, the gauge-fixed Chern-Simons action becomes

$$S = -i \text{sign}(k) \int d^3x \epsilon_{\mu\nu\rho} \left( \frac{1}{2} A^a_{\mu} \partial_{\rho} A^a_{\nu} + \frac{1}{3!} g f^{abc} A^a_{\mu} A^b_{\rho} A^c_{\nu} \right)$$

$$+ \int d^3x \left( B^a n^\mu A^a_{\mu} + \partial^a n^\mu D^a_{\mu} n^\nu \partial^a A^c_{\nu} \right),$$

(2)
where $D^a_{\mu} = \partial^a_{\mu} + g f^{abc} A^c_{\mu}$ is the covariant derivative. As in four-dimensional Yang-Mills, the theory possesses the usual BRST symmetry, namely,

$$s A^a_\mu = D^a_{\mu} c^b, \quad sc^a = -\frac{1}{2} g f^{abc} c^b c^c, \quad sc^a = B^a, \quad s B^a = 0.$$  \hfill (3)$$

In addition, Chern-Simons theory exhibits, just as in the Landau gauge, the vector supersymmetry

$$v_\nu A^a_\mu = i \text{sign}(k) \epsilon_{\nu\rho\alpha} n^\alpha c^a, \quad v_\nu c^a = 0, \quad v_\nu \overline{c}^a = A^a_\nu, \quad v_\nu B^a = -\partial_\nu c^a,$$  \hfill (4)$$

and the anti-vector supersymmetry

$$\overline{v}_\nu A^a_\mu = i \text{sign}(k) \epsilon_{\nu\rho\alpha} n^\alpha \overline{c}^a, \quad \overline{v}_\nu c^a = 0, \quad \overline{v}_\nu \overline{c}^a = -A^a_\nu, \quad \overline{v}_\nu B^a = -\partial_\nu \overline{c}^a,$$  \hfill (5)$$

leading to the following supersymmetric variations,

$$v_\nu S = \int d^3 x \left[ \partial_\rho \left( n_\rho c^a A^a_\nu \right) - \partial_\nu \left( n_\rho c^a A^a_\rho \right) \right] = 0; \quad \overline{v}_\nu S = \int d^3 x \left[ \partial_\rho \left( n_\rho \overline{c}^a A^a_\nu \right) - \partial_\nu \left( n_\rho \overline{c}^a A^a_\rho \right) \right] = 0.$$  \hfill (6)$$

### 3. Breaking of vector supersymmetry in BRST-invariant regularization

Let us first analyze how the supersymmetries in Eqs. (4) and (5) are broken in a BRST-invariant regularization method. As we know, dimensional regularization is generally regarded as a reliable and convenient regularization scheme, preserving gauge invariance in a natural way. Of course, it is also well known that the presence of $\gamma_5$ or, equivalently, of the epsilon tensor, requires a special treatment, such as the consistent dimensional regularization scheme proposed by ‘t Hooft and Veltman [15,16]. Unfortunately, as shown in Ref. [9], a consistent dimensional regularization continuation does not exist for pure Chern-Simons gauge theory, since the three-dimensional antisymmetric tensor $\epsilon_{\mu\nu\rho}$ prevents inversion of the kinetic term in $d$-dimensional space, even after gauge-fixing. To obtain a regularized Chern-Simons theory, consistent with gauge symmetry, one must adopt a kind of hybrid regularization — a combination of higher covariant-derivative regularization and consistent dimensional regularization. The simplest higher covariant expression is the three-dimensional Yang-Mills term $S_{YM}$,

$$S_{YM} = \frac{1}{4m} \int d^3 x F^a_{\mu\nu} F^a_{\mu\nu},$$  \hfill (7)$$

$m$ being the regulator mass. The $S_{YM}$ term should be added to the Chern-Simons action before performing the dimensional continuation according to the ‘t Hooft-Veltman prescription [13,14,15]. The regularized theory has now two regulators: the dimensional parameter $\epsilon = 3 - d$ and the mass regulator $m$, the order of taking the limits being first $d \to 3$, then $m \to \infty$.

Although the BRST symmetry (3) is now preserved at the regularization level, the introduction of the Yang-Mills term $S_{YM}$ violates the supersymmetries (4) and (5) explicitly. A straightforward calculation gives
The broken vector supersymmetry manifests itself through the second term on the R.H.S. of Eq. (12), and the anti-ghost field \( \overline{c}^a \). In momentum space, namely, the field \( \overline{c}^a \) is related to the composite operator \( v_\nu S_{YM} \), and is the 1PI Green function

\[
\mathcal{G}_{\mu\nu}(p) = i \text{sign}(k) \epsilon_{\mu\nu\rho} n_\rho S(p) + G_{\mu\nu}(p) \Omega_{\rho\nu}(p) S(p),
\]

where \( G_{\mu\nu}(p) \) and \( S(p) \) denote the propagators for the gauge field and ghost field in momentum space, respectively; \( \Omega_{\mu\nu}(p) \) is related to the composite operator \( v_\nu S_{YM} \) and is the 1PI part of the Green function \( \langle A_{\mu}^a(x) \overline{c}^b(y) v_\nu S_{YM} \rangle \) in momentum space, namely,
\[ \left\langle A^\mu_\nu(x) e^\beta(y) v_{\nu} S_{\text{YM}} \right\rangle = \int d^3 u d^3 v G_{\mu\nu}(x-u) \Omega_{\rho\nu}(u-v) S(v-y) = \int \frac{d^3p}{(2\pi)^3} G_{\mu\nu}(p) \Omega_{\rho\nu}(p) S(p) e^{ip(x-y)}. \]  

(14)

4. Finite quantum correction as breaking effect of vector supersymmetry

In the following, we shall check identity (13) up to the one-loop level in order to convince ourselves that the finite quantum correction is indeed associated with the breaking of vector supersymmetry. To this effect, we first expand the various quantities in (13) to one-loop order,

\[
G_{\mu\nu}(p) = G_{(0)}^{(0)}(p) + G_{\mu\lambda}(p) \Pi_{\lambda\nu}(p) G_{(0)}^{(0)}(p) + \mathcal{O}(g^4);
\]

\[
S(p) = S^{(0)}(p) + S^{(0)}(p) \Sigma(p) S^{(0)}(p) + \mathcal{O}(g^4);
\]

\[
\Omega_{\mu\nu}(p) = \Omega_{(0)}^{(0)}(p) + \Omega_{(1)}^{(0)}(p) + \mathcal{O}(g^4),
\]

where \(G_{(0)}^{(0)}(p)\) and \(S^{(0)}(p)\) are the tree-level propagators of the Chern-Simons gauge field and ghost field, respectively. In Euclidean space-time [14],

\[
G_{(0)}^{(0)}(p) = \frac{m}{p^2(p^2 + m^2)} \left[ -m \text{sign}(k) \epsilon_{\mu\nu\rho} p_\rho + \frac{m}{n \cdot p} \text{sign}(k) (p_\mu \epsilon_{\nu\lambda\rho} p_\lambda n_\rho - p_\nu \epsilon_{\mu\lambda\rho} p_\lambda n_\rho) \right.
\]

\[
+ p^2 \delta_{\mu\nu} - \frac{p^2}{n \cdot p} (p_\mu n_\nu + p_\nu n_\mu) \right],
\]

\[
= \frac{m}{(p^2 + m^2)(n \cdot p)} \left[ -m \text{sign}(k) \epsilon_{\mu\nu\rho} n_\rho + n \cdot p \delta_{\mu\nu} - (p_\mu n_\nu + p_\nu n_\mu) \right], \quad (16a)
\]

\[
S^{(0)}(p) = \frac{i}{n \cdot p}. \quad (16b)
\]

Here, we have employed the identity [14]

\[
\frac{1}{n \cdot p} \epsilon_{\mu\nu\rho} n_\rho = \frac{1}{p^2} \left[ \epsilon_{\mu\nu\rho} p_\rho - \frac{1}{n \cdot p} (p_\mu p_\rho \epsilon_{\nu\lambda\rho} - p_\nu p_\rho \epsilon_{\mu\lambda\rho}) \right]. \quad (17)
\]

The above propagators are seen to contain the spurious light-cone gauge singularity \((n \cdot p)^{-1}\), which can be treated by the following prescription [17–19] in Minkowski space:

\[
\frac{1}{p \cdot n} = \lim_{\epsilon \to 0} \frac{p \cdot n^*}{(p \cdot n)(p \cdot n^*) + i \epsilon}, \quad \epsilon > 0,
\]

\[
n_\mu = (n_0, n), \quad n^*_\mu = (n_0, -n), \quad n^2 = n^{*2} = 0. \quad (18)
\]

In order to verify identity (13), we require not only the one-loop vacuum polarization tensor \(\Pi_{\mu\nu}(p)\) and the ghost field self-energy \(\Sigma(p)\), but also the 3-point and 4-point vertices of topologically massive Yang-Mills theory (Figs. 1a and 1b), namely,

\[
V_{\mu\nu\rho}^{abc}(p, q, r) = ig f^{abc} \left\{ \text{sign}(k) \epsilon_{\mu\nu\rho} + \frac{1}{m} [(p - q)_\rho \delta_{\mu\nu} + (q - r)_\mu \delta_{\nu\rho} + (r - p)_\nu \delta_{\mu\rho}] \right\}
\]
\[ \equiv ig f^{abc} \left[ \text{sign}(k)\epsilon_{\mu\nu\rho} + \frac{1}{m} \tilde{V}_{\mu\nu\rho}(p, q, r) \right], \quad p + q + r = 0; \] (19a)

\[ V^{abcd}_{\mu\nu\lambda\rho}(p, q, r, s) = -\frac{g^2}{m} \left[ f^{ead} f^{fcd} (\delta_{\mu\lambda}\delta_{\nu\rho} - \delta_{\nu\lambda}\delta_{\mu\rho}) + f^{ebd} f^{fed} (\delta_{\lambda\rho}\delta_{\mu\nu} - \delta_{\nu\rho}\delta_{\lambda\mu}) \right. \]
\[ + \left. f^{ebe} f^{edc} (\delta_{\mu\lambda}\delta_{\nu\rho} - \delta_{\nu\mu}\delta_{\lambda\rho}) \right], \quad p + q + r + s = 0, \] (19b)

along with the ghost-ghost-gauge vertex (Fig. 1c),

\[ V^{abc}_{\mu} = -gf^{abc}n_{\mu}. \] (19c)

\[ \Omega^{(0)}_{\mu\nu}(p) \] and \[ \Omega^{(1)}_{\mu\nu}(p) \] may be evaluated with the help of the Feynman rules for the composite operators (cf. Eq. (8), see Fig. 2):

\[ v^{(0)}_{\mu}(p, q) = \frac{i}{m} \text{sign}(k)\epsilon_{\nu\alpha\beta}n_{\beta}\delta^{ab} \left( p^2\delta_{\alpha\mu} - p_\alpha p_\mu \right), \quad p + q = 0; \] (20)

\[ v^{(1)}_{\mu}(r, p, q) = -\frac{1}{m} \text{sign}(k)\epsilon_{\nu\alpha\beta}n_{\beta}gf^{abc} \left[ \delta_{\mu\rho} (p_\alpha - q_\alpha) + \delta_{\rho\alpha} (q_\mu - r_\mu) \right. \]
\[ + \left. \delta_{\alpha\mu} (r_\rho - p_\rho) \right] = -\frac{1}{m} \text{sign}(k)\epsilon_{\nu\alpha\beta}n_{\beta}gf^{abc} \tilde{V}_{\alpha\mu\rho}(r, p, q), \quad p + q + r = 0; \] (21)

\[ v^{(2)}_{\mu}(s, p, q, r) = -\frac{i}{m} \text{sign}(k)\epsilon_{\nu\alpha\beta}n_{\beta}g^2 \left[ f^{eda} f^{ebc} (\delta_{\mu\rho}\delta_{\lambda\alpha} - \delta_{\nu\alpha}\delta_{\lambda\rho}) \right. \]
\[ + \left. f^{eac} f^{edb} (\delta_{\mu\rho}\delta_{\lambda\alpha} - \delta_{\nu\alpha}\delta_{\lambda\rho}) + f^{edc} f^{eab} (\delta_{\lambda\mu}\delta_{\rho\alpha} - \delta_{\nu\rho}\delta_{\lambda\mu}) \right] \]
\[ = i \text{sign}(k)\epsilon_{\nu\alpha\beta}n_{\beta}V^{dabc}_{\alpha\mu\rho\lambda}(s, p, q, r), \quad p + q + r + s = 0. \] (22)

Inserting the expansions (15) into the identity (13), we obtain both the tree-level relation

\[ G^{(0)}_{\mu\nu}(p) = i \text{sign}(k)\epsilon_{\mu\rho\nu\rho}n_{\rho}S^{(0)}(p) + G^{(0)}_{\mu\rho}(p)\Omega^{(0)}_{\rho\nu}(p)S^{(0)}(p), \] (23)

and the one-loop relation,

\[ G^{(0)}_{\mu\alpha}(p)\Pi_{\lambda\rho}(p)G^{(0)}_{\rho\nu}(p) = i \text{sign}(k)\epsilon_{\mu\rho\nu\rho}n_{\rho}S^{(0)}(p)\Sigma(p)S^{(0)}(p) \]
\[ + G^{(0)}_{\mu\rho}(p)\Omega^{(0)}_{\rho\nu}(p)S^{(0)}(p)\Sigma(p)S^{(0)}(p) \]
\[ + G^{(0)}_{\mu\rho}(p)\Omega^{(0)}_{\rho\nu}(p)S^{(0)}(p) \]
\[ + G^{(0)}_{\mu}(p)\Pi_{\sigma\lambda}(p)G^{(0)}_{\lambda\rho}(p)\Omega^{(0)}_{\rho\nu}(p)S^{(0)}(p). \] (24)

The tree-level relation (23) is satisfied trivially, as can be seen by applying the Feynman rules in Eqs. (16) and (19).

Our real interest lies with the one-loop relation (24) to see whether or not the shift in the Chern-Simons parameter \( k \) does indeed arise from the breaking of the vector supersymmetry (4). In order to verify (24), we shall proceed as follows. We shall begin by calculating the one-loop quantities \( \Sigma(p) \) and \( \Omega^{(1)}_{\mu\nu}(p) \), comparing our result with the value of the vacuum polarization tensor \( \Pi^{\nu\mu} \) obtained in Ref. [14], and then ascertain whether or not relation (24) is consistent with the aforementioned finite quantum correction.

We now turn to the evaluation of \( \Sigma(p) \) and \( \Omega^{(1)}_{\mu\nu}(p) \). Although the dimensionally continued gauge propagator is fairly complicated due to the presence of the \( \epsilon_{\mu\nu\rho} \) tensor, a detailed analysis [14] shows that the gauge propagator may actually be “simplified” by decomposing
it into a $d$-dimensional part and an evanescent part. The advantage of this separation is that the evanescent portion exhibits an improved UV behaviour and consequently vanishes in the limit as $d \to 3$. Accordingly, it is perfectly safe to work with the propagator (16a). We also recall that in the light-cone gauge,

$$n_\mu G^{(0)}_{\mu \nu}(p) = 0,$$

so that the ghost field self-energy (Fig. 3), given by

$$\Sigma(p)\delta^{ab} = \int \frac{d^d k}{(2\pi)^d} \left( -gf^{acd}n_\mu \right) G^{(0)}_{\mu \nu}(k) \left( -gf^{bde}n_\nu \right) S^{(0)}(k + p),$$

actually vanishes:

$$\Sigma(p) = 0.$$

Moreover, the last term on the R.H.S. of Eq. (24) also vanishes in the limit $m \to \infty$, since $\Omega^{(0)}_{\mu \nu}(p)$ is proportional to $1/m$, while $\Pi_{\mu \nu}(p)$ is finite in the large $m$ limit. Hence, the one-loop identity (24) reduces to

$$G^{(0)}_{\mu \lambda}(p)\Pi_{\lambda \rho}(p)G^{(0)}_{\rho \nu}(p) = G^{(0)}_{\mu \rho}(p)\Pi^{(1)}_{\rho \nu}(p)S^{(0)}(p).$$

(28)

It remains to evaluate $\Omega^{(1)}_{\mu \nu}(p)$ and see whether its value is consistent with the vacuum polarization obtained in Ref. [14], namely,

$$\Pi_{\mu \nu}(p) = \frac{1}{8\pi} \text{sign}(k)c_v g^2 \left[ 4\epsilon_{\mu \rho \nu} p_\rho - \frac{3p \cdot n^*}{n \cdot n^*} \epsilon_{\mu \nu \rho} n_\rho 
\right.
+ \frac{3p \cdot n^*}{(n \cdot n^*)(p \cdot n)} (n_\mu \epsilon_{\nu \lambda \rho} - n_\nu \epsilon_{\mu \lambda \rho}) n_\lambda p_\rho \right].$$

(29)

The first term in Eq. (29) contributes to the local quantum effective action, while the third term proportional to $p \cdot n^*$ gives a nonlocal contribution which may be absorbed into the local effective action by a finite, non-multiplicative wave function renormalization [14].

As shown in Fig. 4, $\Omega^{(1)}_{\mu \nu}(p)$ gives rise to five one-loop diagrams, only two of which are non-zero (Figs. 4a and 4b). The contribution from Fig. 4a reads

$$\Omega^{(1)ab}_{(a)\mu \nu}(p) = \frac{1}{2} \int \frac{d^d k}{(2\pi)^d} \bar{\nu}_{\rho \lambda \mu}(-p, k, -k, p)G^{(0)}_{\rho \lambda}(k)\delta^{cd}\bigg|_{d \to 3}$$

$$= \frac{i}{2} \text{sign}(k)g^2 c_v \delta^{ab} \epsilon_{\mu \nu \rho \lambda} \frac{1}{(2\pi)^d n \cdot k(k^2 + m^2)} \left[ (\epsilon_{\mu \rho \lambda} \delta_{\mu \lambda} n_\beta + \epsilon_{\nu \lambda \beta} \delta_{\nu \beta} n_\lambda - 2\delta_{\mu \rho} \epsilon_{\nu \beta \lambda} n_\beta (msign(k)\epsilon_{\rho \lambda \alpha} n_\alpha + n \cdot k \delta_{\rho \lambda} - k_\lambda n_\rho - k_\mu n_\lambda) \right]_{d \to 3}$$

$$= i \text{sign}(k)g^2 c_v \delta^{ab} \epsilon_{\nu \lambda \beta} n_\beta n_\mu \int \frac{d^d k}{(2\pi)^d n \cdot k(k^2 + m^2)} k_\lambda \bigg|_{d \to 3}$$

$$= \frac{i}{4\pi} g^2 c_v \delta^{ab} m \text{sign}(k)\epsilon_{\nu \lambda \beta} n_\beta n_\mu n_\lambda \frac{n_\mu n_\lambda}{n \cdot n^*},$$

(30)

where we have used prescription (18), together with the formula (see Appendix of Ref. [14]),
Adding Eqs. (30) and (35), we get
\[ \lim_{d \to 3} \int \frac{d^dk}{(2\pi)^d} \frac{k_\lambda}{n \cdot k (k^2 + m^2)} = \frac{1}{4\pi} \frac{mn^*_\gamma}{n \cdot n^*}. \] (31)
c_V is the quadratic Casimir operator in the adjoint representation given by \( f^{a cd} f^{b ed} = c_V \delta^{ab} \). Evaluation of \( \Omega^{(1)ab}_{(b)\mu\nu}(p) \) in Fig. 4b is somewhat lengthier. We find that
\[
\Omega^{(1)ab}_{(b)\mu\nu}(p) = \frac{1}{2} \int \frac{d^dk}{(2\pi)^d} \frac{ig f^{a de} V_{\mu\lambda\rho}(p, -k - p, k) G^{(0)}_{\lambda\alpha}(k) V^{bdc}_{\nu\alpha\sigma}(p, -k, k + p)}{G^{(0)}_{\sigma\rho}(k + p)}
= \frac{i}{2} \frac{c_V g^2}{(2\pi)^d} \int \frac{d^dk}{(2\pi)^d} \frac{V_{\mu\lambda\rho}(p, -k - p, k) G^{(0)}_{\lambda\alpha}(k) G^{(0)}_{\sigma\rho}(k + p) - 1}{m} \epsilon_{\nu\beta\gamma} n_{\gamma} \tilde{V}_{\alpha\sigma}
= \frac{i}{2} \frac{c_V g^2}{(2\pi)^d} \int \frac{d^dk}{(2\pi)^d} \frac{V_{\mu\lambda\rho}(p, -k - p, k) G^{(0)}_{\lambda\alpha}(k) G^{(0)}_{\sigma\rho}(k + p) \epsilon_{\nu\beta\gamma} n_{\gamma} [-V_{\alpha\sigma} + \text{sign}(k) \epsilon_{\beta\alpha\sigma}]}{V^{(0)}_{\nu\sigma\rho}(k + p)}
\times \epsilon_{\nu\beta\gamma} n_{\gamma} \epsilon_{\beta\alpha\sigma} \right] \}. \] (32)
The second term in Eq. (32) vanishes identically, since \( n_\mu G_{\mu\nu} = 0 \), i.e.
\[
\int \frac{d^dk}{(2\pi)^d} \frac{V_{\mu\lambda\rho}(p, -k - p, k) G^{(0)}_{\lambda\alpha}(k) G^{(0)}_{\sigma\rho}(k + p) \epsilon_{\nu\beta\gamma} n_{\gamma} \epsilon_{\beta\alpha\sigma}}{V^{(0)}_{\nu\sigma\rho}(k + p)}
= \int \frac{d^dk}{(2\pi)^d} \frac{V_{\mu\lambda\rho}(p, -k - p, k) G^{(0)}_{\lambda\alpha}(k) G^{(0)}_{\sigma\rho}(k + p) \left( \delta_{\nu\sigma} n_{\alpha} - \delta_{\nu\alpha} n_{\sigma} \right)}{V^{(0)}_{\nu\sigma\rho}(k + p)} = 0. \] (33)
The next step is to insert the value for \( \Pi^{(b)\mu\beta}(p) \), which represents the vacuum polarization tensor from a gluon loop with two three-gauge vertices and two gauge propagators. In the limits as \( d \to 3 \) and \( m \to \infty \), \( \Pi^{(b)\mu\beta}(p) \) becomes \[ \Pi^{(b)\mu\beta}(p) = \frac{i}{8\pi} c_V g^2 \left[ 4 \text{sign}(k) \epsilon_{\mu\beta\gamma} p_{\gamma} - \frac{p \cdot n^*}{n \cdot n^*} \text{sign}(k) \epsilon_{\mu\beta\gamma} n_{\gamma} + 3 \frac{p \cdot n^*}{(n \cdot n^*)(p \cdot n)} \text{sign}(k) (n_\mu \epsilon_{\beta\gamma\delta} - n_\beta \epsilon_{\mu\gamma\delta}) n_{\gamma} p_{\delta} + 2m \frac{n_\mu n^*_\beta}{n \cdot n^*} \right], \] (34)
so that
\[
\Omega^{(1)ab}_{(b)\mu\nu}(p) = \frac{1}{2\pi} g^2 c_V \delta^{ab} \left[ (n \cdot p_{\delta\mu\nu} - n_\mu p_{\nu}) - \frac{i}{2} m \text{sign}(k) \epsilon_{\nu\beta\gamma} n_{\gamma} \frac{n_\mu n^*_\beta}{n \cdot n^*} \right]. \] (35)
Adding Eqs. (30) and (34), we get
\[
\Omega^{(1)ab}_{\mu\nu} = \frac{1}{2\pi} g^2 c_V \delta^{ab} \left( n \cdot p \delta_{\mu\nu} - n_\mu p_{\nu} \right), \] (36)
where \( \Omega^{(1)ab}_{\mu\nu} \), unlike \( \Pi^{(b)\mu\beta}(p) \) in Eq. (29), is obviously independent of \( n_\mu^* \). Inserting \( \Omega^{(1)ab}_{\mu\nu} \) and \( \Pi^{(b)\mu\beta}(p) \) into the relation (28) and taking the limit \( m \to \infty \), we find that (28) is indeed satisfied.
In this context, the following two points seem worth emphasizing. First, looking at Eqs. (24) or (28), we notice again how inextricably the polarization tensor $\Pi^{\mu\nu}$ is linked with the composite operator $\Omega^{(1)}_{\mu\nu}$. Appearance of a non-zero $\Omega^{\mu\nu}$ in the original identities (12) and (13) is clearly a signal of vector supersymmetry breaking. This observation brings us to our second point, namely the impact of the chosen regularization scheme on the corresponding symmetries. There are basically two possibilities: we can (a) either adopt a regularization scheme which preserves BRST invariance, but violates vector supersymmetry, or (b) employ a scheme that preserves vector supersymmetry at the expense of BRST symmetry.

The only meaningful choice, in our opinion, is to work with a regularization procedure that respects BRST symmetry. The latter unquestionably outranks vector supersymmetry, a topological symmetry, in both overall effectiveness and field-theoretic importance. Accordingly, the observed shift in the Chern-Simons statistical parameter $k$ may be attributed directly to a BRST-preserving regularization scheme, in other words, to the breaking of vector supersymmetry at the regularization level. A similar conclusion holds for the anti-vector supersymmetry (5) and regardless whether the gauge is noncovariant – as in the present paper – or covariant [6,8].

Having established the fact that there is no regularization in the light-cone gauge that preserves both BRST and vector supersymmetry, the question remaining is whether or not perhaps the renormalized effective action could be made both BRST and vector supersymmetry invariant. To answer this question we are guided by the arguments given in [1] for the Landau gauge, where it was demonstrated that the quantum effective action could indeed be expressed in an explicit BRST and vector supersymmetry invariant form by an appropriate definition of the fields. To appreciate this line of reasoning, we recall what happens in the simple renormalization scheme for a finite theory, defined by

$$\Phi_R = Z_\Phi^{-1/2} \Phi, \quad \Phi = (A^a, B^a, c^a, \bar{c}^a),$$

with

$$Z_A = Z_B^{-1} = Z_c = Z_{\bar{c}} = 1.$$  \hspace{1cm} (38)

Here, the one-loop quantum effective action is given by [14]

$$\Gamma_R = \left( 1 + \frac{C_V g^2}{4\pi} \right) S_{CS}[A_R] + \int d^3x \left( B_R^a n_\mu A_R^a_{\mu} + \bar{c}_R^a n_\mu D_R^{ab} c_R^b \right),$$

where the subscript $R$ denotes the renormalized quantity, $D_R^{ab} = \partial_\mu \delta^{ab} + g f^{abc} A_R^c$, and $Z_\Phi$ is the wave function renormalization constant of the $\Phi$ field. In this particular renormalization scheme, where the bare quantities equal the renormalized ones, the quantum effective action (39) can only be BRST invariant. However, if we choose the alternative renormalization scheme defined by

$$\Phi^- = \bar{Z}_\Phi^{-1/2} \Phi,$$

with

$$\bar{Z}_A = \bar{Z}_B^{-1} = \bar{Z}_c = \bar{Z}_{\bar{c}}^{-1} = 1 - \frac{g^2 C_V}{4\pi},$$

(41)
the one-loop quantum effective action will become
\[
\Gamma_R = S_{\text{CS}}[A_R] + \int d^3 x \left( B^\alpha_R n_\mu A^\alpha_{R \mu} + \bar{c}_R^\mu n_\mu D^\mu c_R^b \right) - \frac{1}{8\pi g^2 C_V} \int d^3 x \left( \frac{1}{3!} \text{sign}(k) g f^{abc} \epsilon_{\mu\nu\rho} A_\mu^{a R} A_\nu^{b R} A_\rho^{c R} - g f^{abc} c_\mu^{a R} A_\mu^{b R} c_\rho^{c R} \right). \tag{42}
\]

It can be shown that the effective action \( \Gamma_R \) satisfies the Ward identities corresponding to both BRST symmetry and vector (anti-vector) supersymmetry.

5. Conclusion

In this article, we have examined the origin of finite quantum corrections in perturbative Chern-Simons theory in the noncovariant light-cone gauge, \( n \cdot A^a(x) = 0, \ n_\mu = (n_0, n) \), \( n^2 = 0 \). Our analysis consisted of five basic steps:

- Variation of the Chern-Simons effective action \( S \), namely \( v_\nu S \), where \( v_\nu \) is the supersymmetric vector operator defined in Eq. (4).
- Application of a gauge-invariant regularization scheme which defines Chern-Simons theory as the large mass limit of topologically massive Yang-Mills theory. The chosen regularization scheme consists of dimensional regularization and the higher covariant-derivative term \( S_{\text{YM}} \) in Eq. (7). This hybrid regularization preserves BRST symmetry, but violates the vector supersymmetry in Eq. (4).
- Use of the \( n^*_\mu \)-prescription in handling the spurious singularities of \( (n \cdot q)^{-1} \) in the gauge and ghost propagators, where \( n^*_\mu = (n_0, -n) \).
- Derivation of the Ward identity (12) in coordinate space or, equivalently, Eq. (13) in momentum space.
- Discussion on the BRST and vector supersymmetry invariance of the quantum effective action in the light-cone gauge in the spirit of Ref. [6].

Our results may be summarized as follows:

1. The Ward identity in Eq. (13) was shown to be satisfied both at the tree level, Eq. (23), and at the one-loop level, Eq. (28).

2. We have demonstrated that the composite operator \( \Omega_{\mu\nu}(p) \), which reflects vector symmetry breaking, is inextricably linked with the vacuum polarization tensor \( \Pi_{\mu\nu} \) in Eq. (23). We note that the latter is UV finite and transverse, but contains one nonlocal, gauge-dependent term.

3. The finite shift in the Chern-Simons statistical parameter \( k \), i.e. the finite quantum correction of the coupling constant, was shown to arise specifically from the breaking of vector supersymmetry.
4. The quantum effective action can be defined as being both BRST invariant and light-cone vector supersymmetric invariant with an appropriate choice of renormalization scheme.

Conclusions similar to the above also hold for other BRST-invariant regularization procedures, such as the hybrid scheme consisting of higher covariant-derivative regularization plus Pauli-Villars regularization \[8\]. Nevertheless, it must be emphasized that the light-cone gauge is particularly well suited in pinpointing the origin of quantum corrections. The reason can be found by looking at the two terms on the right-hand side of the identity \([13]\), both containing the ghost propagator \(S(p)\). Since \(S(p)\) receives no quantum corrections in the light-cone gauge, the first term proportional to \(\epsilon_{\mu\nu\rho}n_\rho S(p)\) maintains its classical value. Consequently, the observed quantum corrections must necessarily originate from the second term, specially from the composite operator \(\Omega_{\rho\nu}(p)\). No such conclusion is possible in the Landau gauge, since quantum corrections are now also generated in the ghost propagator \(S(p)\).

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FIGURES

FIG. 1. Vertices of topologically massive Yang-Mills theory: the wavy line represents the gauge field and the dashed line represents the ghost field.

FIG. 2. Composite vertices relevant to vector supersymmetry-breaking terms

FIG. 3. Self-energy of ghost field

FIG. 4. Feynman diagrams contributing to $\Omega^{(1)}_{\mu\nu}$. Diagrams (c), (d) and (e) vanish.