CosMIn: The Solution to the Cosmological Constant Problem

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Abstract

The current acceleration of the universe can be modelled in terms of a cosmological constant $\Lambda$. We show that the extremely small value of $\Lambda \approx 3.4 \times 10^{-122}$, the holy grail of theoretical physics, can be understood in terms of a new, dimensionless, conserved number CosMIn, which counts the number of modes crossing the Hubble radius during the three phases of evolution of the universe. Theoretical considerations suggest that $N \approx 4\pi$. This single postulate leads us to the correct, observed numerical value of the cosmological constant! This approach also provides a unified picture of cosmic evolution relating the early inflationary phase to the late accelerating phase.

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Introduction: Our description of the cosmos is very tantalizing! It has three distinct phases of evolution, bearing no apparent relation to each other: An early inflationary phase, driven possibly by a scalar field, a late-time accelerated phase, dominated by dark energy, and a transient phase in between, dominated by radiation and matter.

The first and the last phases are approximately de Sitter, with Hubble radii $H_{\text{inf}}^{-1}$ and $H_{\Lambda}^{-1}$, characterized by two dimensionless ratios $\beta^{-1} \equiv H_{\text{inf}} L_P$ and $\Lambda L_P^2$, where $L_P \equiv (G\hbar/c^3)^{1/2}$ is the Planck length. If inflation took place at GUTs scale ($\sim 10^{15}$ GeV), then $\beta \approx 3.8 \times 10^7$, while observations\footnote{Essay selected for Honorable Mention in the Gravity Research Foundation Essay Contest 2013.} suggest that $\Lambda L_P^2 \approx 3.4 \times 10^{-122} \approx 3 \times e^{-281}$. It is expected that physics at, say, the GUTs scale will (eventually) determine $\beta$. But no fundamental principle has been suggested to explain the extremely small value of $\Lambda L_P^2$, which is related (directly or indirectly) to the cosmological constant problem. Understanding this issue \footnote{Essay selected for Honorable Mention in the Gravity Research Foundation Essay Contest 2013.} from first principles is considered very important in theoretical physics today.

In this essay, we will describe an approach which tackles this problem (for more details, see \footnote{Essay selected for Honorable Mention in the Gravity Research Foundation Essay Contest 2013.}) and also provides a unified picture of cosmic evolution. We show that $\ln(\Lambda L_P^2)$ is related to a dimensionless number (‘Cosmic Mode Index’,
or CosMIn, \( N_c \) that counts the number of modes within the Hubble volume that cross the Hubble radius between the end of inflation and the beginning of late-time acceleration. CosMIn is a characteristic number for our universe and it is possible to argue [9] that the natural value for \( N_c \) is about \( 4\pi \); i.e., \( N_c = 4\pi \mu \) with \( \mu \sim 1 \). This single postulate allows us to determine the numerical value of \( \Lambda L_p^2 \): We obtain \( \Lambda L_p^2 = C\beta^{-2}\exp(-24\pi^2\mu) \), where \( C \) depends on \( n_{\gamma}/n_m \), the ratio between the number densities of photons and matter. This leads to the correct observed value of the cosmological constant for a GUTs scale inflation and the range of \( C \) permitted by cosmological observations.

**CosMIn and the cosmological constant:** A proper length scale \( \lambda_{prop}(a) \equiv a/k \) (labelled by a co-moving wave number, \( k \)) crosses the Hubble radius whenever the equation \( \lambda_{prop}(a) = H^{-1}(a) \), i.e., \( k = aH(a) \) is satisfied. For a generic mode (see Fig.1 line marked \( ABC \)), this equation has three solutions: \( a = a_A \) (during the inflationary phase; at \( A \)), \( a = a_B \) (during the radiation/matter dominated phase; at \( B \)), \( a = a_C \) (during the late-time accelerating phase; at \( C \)). But modes with \( k < k_- \) exit during the inflationary phase and never re-enter. Similarly, modes with \( k > k_+ \) remain inside the Hubble radius and only exit during the late-time acceleration phase.

The modes with comoving wavenumbers in the range \((k, k + dk)\) where \( k = aH(a) \) and \( dk = |d(aH)/da|da \) cross the Hubble radius during the interval \((a, a + da)\). The number of modes in a comoving Hubble volume \( V_{com} = (4\pi H^{-3}/3a^3) \)
with wave numbers in the interval \((k, k + dk)\) is 
\[ dN = V_{\text{com}} d^3 k / (2\pi)^3. \]
Hence, the number of modes that cross the Hubble radius in the interval \((a_1 < a < a_2)\) is given by
\[ N(a_1, a_2) = \int_{a_1}^{a_2} \frac{V_{\text{com}} k^2}{2\pi^2} \frac{dk}{da} da = \frac{2}{3\pi} \int_{a_1}^{a_2} \frac{d(Ha)}{Ha} = \frac{2}{3\pi} \ln \left( \frac{H_2 a_2}{H_1 a_1} \right), \tag{1} \]
where we have used \(V_{\text{com}} = 4\pi/3H^3a^3\) and \(k = Ha\).

All the modes which exit the Hubble radius during \(a_A < a < a_X\) enter the Hubble radius during \(a_X < a < a_B\) (and again exit during \(a_Y < a < a_Q\)). So the number of modes which do this during \(PX, XY\) or \(YQ\) is a characteristic, ‘conserved’ number (“CosMIn”) in a universe having three distinct phases. The epochs \(P\) and \(Q\), limiting the otherwise semi-eternal de Sitter phases, now have a special significance \([6, 7, 8]\). Modes which exit the Hubble radius before \(a = a_P\) never re-enter. On the other hand, the epoch \(a = a_Q\) denotes (approximately) the time when the CMBR temperature falls below the de Sitter temperature \([6, 7, 8]\). The special role of \(PXQY\) makes the value of CosMIn significant. As shown in Fig.1 these modes in \(PXQY\) (with \(k_- < k < k_+\)) cross the Planck length during \(a_- < a < a_+\). Based on holographic considerations, it is possible to argue \([9]\) that Planck scale physics imposes the condition \(N_c = N(a_-, a_+) \approx 4\pi\) at this stage. So, by computing CosMIn for the universe, and equating to \(4\pi\), we can determine \(\Delta L_P^2\).

As a quick check on the paradigm \(N_c \approx 4\pi\), let us approximate the intermediate phase of the universe as purely radiation dominated \((H(a) \propto a^{-2})\) and assume Planck scale inflation \((\beta = 1)\), thereby eliminating all free parameters. The above procedure now \([9]\) gives:
\[ \Delta L_P^2 = \frac{3}{4} \exp(-24\pi^2 \mu); \quad \mu \equiv \frac{N_c}{4\pi}. \tag{2} \]
Thus, \(\Delta L_P^2\) is directly related to CosMIn and, in this simple model, there are no other adjustable parameters. Eq. (2) leads to the observed value \(\Delta L_P^2 = 3.4 \times 10^{-122}\) when \(\mu = 1.18\), showing we are clearly on the right track!

The presence of matter and the fact that the inflationary scale may not be the Planck scale in our universe \((\beta \neq 1)\), surprisingly, make the postulate of \(N_c = 4\pi\) work better in the real universe and reproduce the observed value of the cosmological constant. In this case, it is simpler to express \(\Lambda\) in terms of \(N_c\), \(\beta\) and a variable \(\sigma\) defined through \(\sigma^4 \equiv (\Omega_R^4/\Omega_m^4)[1 - \Omega_m - \Omega_R]\). Even though the values for \(\Omega_R\) and \(\Omega_m\) depend on the epoch \(t = t_e\) at which they are measured, the value of \(\sigma\) is the same at all epochs. (It is an example of an epoch-invariant parameter and, of course, the value of \(\Delta L_P^2\) can only depend on such epoch-invariant parameters \([5]\)). Determining \(N_c\) in terms of \(\Delta L_P^2\), \(\beta\) and \(\sigma\), and expressing \(\Delta L_P^2\) in terms of the other parameters, we get:
\[ \Delta L_P^2 = \beta^{-2} C(\sigma) \exp[-24\pi^2 \mu]; \quad \mu \equiv \frac{N_c}{4\pi}. \tag{3} \]
where \( C(\sigma) = 12(\sigma r)^{4}(3r + 4)^{-2} \) and \( r \) satisfies the quartic equation \( \sigma^{4}r^{4} = (1/2)r + 1 \). Given the numerical value of \( \sigma \), the inflation scale determined by \( \beta \), and our postulate \( \mu = 1 \), we can calculate the value of \( \Lambda L_{p}^{2} \) from Eq. (3).

The result in Eq. (3) is summarized in Fig. 2. The thick black curve is obtained from Eq. (3) if we take \( \mu = 1 \) and \( \beta = 3.83 \times 10^{7} \) (corresponding to the inflationary energy scale of \( V_{\text{inf}}^{1/4} = 1.16 \times 10^{15} \text{ GeV} \)) and leads to the observed (mean) value of \( \Lambda L_{p}^{2} = 3.39 \times 10^{-122} \) (horizontal unbroken, blue line). Observational constraints [1, 2] lead to \( \sigma = 0.003^{+0.004}_{-0.001} \) (three vertical, red lines) and \( \Lambda L_{p}^{2} = (3.03 - 3.77) \times 10^{-122} \) (horizontal, broken blue lines). This cosmologically allowed range in \( \sigma \) and \( \Lambda L_{p}^{2} \) is bracketed by the two broken black curves obtained by varying \( \beta \) in the range \( (2.64 - 7.29) \times 10^{7} \) (i.e., \( V_{\text{inf}}^{1/4} = (0.84 - 1.4) \times 10^{15} \text{ GeV} \)). So, for an acceptable range of energy scales of inflation, and for the range of \( \sigma \) allowed by cosmological observations, our postulate \( N_{c} = 4\pi \) gives the correct value for the cosmological constant.

Since our results only depend on the combination \( \beta^{-2} \exp(-24\pi^{2}\mu) \), the same set of curves arise in a Planck scale inflationary model (\( \beta = 1 \)) with \( \mu \) in the range \( (1.144 - 1.153) \). There are three conceptually attractive features about Planck scale inflation with \( \beta = 1 \). First, it eliminates one free parameter, \( \beta \), and gives a direct relation between the two scales \( \Lambda \) and \( L_{p}^{2} \) which occur in the Einstein-Hilbert action. (The dependence of the result on \( \sigma \) is weak and can be thought of as a matter of detail, like, for example, the fine structure correction to spectral lines beyond Bohr’s model). Second, we can think of the intermediate
phase as a mere transient connecting two de Sitter phases (the chicken is just
the egg’s way of making another egg!), both of which are semi-eternal. Since
the de Sitter universe is time-translationally invariant, it is a natural candidate
to describe the geometry of the universe dominated by a single length scale —
$L_P$ in the initial phase and $\Lambda^{-1/2}$ in the final phase. The quantum instability
of the de Sitter phase at the Planck scale can lead to cosmogenesis and the
transient radiation/matter dominated phase, which gives way, eventually, to
the late-time acceleration phase. Finally, the argument for $N_c \approx 4\pi$ is quite
natural with Planck scale inflation. The transition at $X$, entrenched in Planck
scale physics in such a model, can easily account for deviations of $\mu$ from unity.

**An integrated view of cosmology:** The standard approach to cosmology
treats the evolution of the universe in a fragmentary manner, with Planck scale
physics, the inflationary era, the matter sector properties and the late-time ac-
celeration each introducing their own parameters — like $L_P^2$, $E_{\text{inf}}$, $(n_m/n_\gamma)$, $\Lambda$ — all independently specified, bearing no relation with each other. Even if
GUTs scale physics (eventually) determines $E_{\text{inf}}$ and $(n_m/n_\gamma)$, there is still no
link between these parameters, $L_P$ and $\Lambda$.

In striking contrast, our paradigm, the postulate $N_c = 4\pi$ acts as the con-
necting thread leading to a unified, holistic approach to cosmic evolution. In
fact, when $\sigma \ll 1$, one can write Eq. (3) (with $\mu = 1$) as:

$$L_P^4 \rho_\Lambda = K \left( \frac{M_P}{m} \right)^2 \frac{n_\gamma}{n_m} \beta^{-3} \exp(-36\pi^2)$$  \hspace{1cm} (4)

where $K \equiv \left( \frac{\pi^{11/2}}{\zeta(3)^2} \right) (1/6^{3/2}) \approx 0.055$, $\rho_m \equiv mn_m$ with $m$ being the
(mean) mass of the particle contributing to matter density, and $M_P$ being the
Planck mass. In a consistent quantum theory of gravity, we expect inflation
(which determines $\beta$) and genesis of matter (which determines $m$ and $n_m/n_\gamma$)
to be related to Planck scale physics such that our fundamental relation in
Eq. (4) holds.

Solving the cosmological constant problem by actually determining its nu-
merical value has not been attempted before. This approach is similar in spirit
to the Bohr model of the hydrogen atom, which used the postulate $J = nh$
to explain the hydrogen spectrum. Here, our postulate $N_c = 4\pi$, captures the
essence and explains the value of $\Lambda L_P^2$. This is simpler and more elegant than
many other ad-hoc assumptions made in the literature \[3, 4\] to solve the cos-
mological constant problem. More importantly, we do know that this postulate
is correct! The value of $\text{CosMIn}$ can be determined directly from the observed
value of $\Lambda$ as well as other cosmological parameters. We would then find that
it is indeed very close to $4\pi$.

**Why does it work?** Recent work \[9\] has shown that cosmic evolution can be thought of as a quest for *holographic equilibrium*. One can associate with the
*proper* Hubble volume $V_{\text{prop}} \equiv 4\pi/3H^3$, the numbers,

$$N_{\text{surf}} \equiv 4\pi H^{-2} L_P^2; \hspace{1cm} N_{\text{bulk}} \equiv -\epsilon E/(1/2)k_B T$$  \hspace{1cm} (5)

which count the surface and bulk degrees of freedom, where $E = (\rho + 3p)V_{\text{prop}}$
is the Komar energy, $T = H/2\pi$ is the analogue of the horizon temperature and
\( \epsilon = \pm 1 \) is chosen to keep \( N_{\text{bulk}} \) positive. Clearly, \(|E| = (1/2)N_{\text{bulk}} k_B T \) denotes equipartition of energy. Holographic equipartition is the demand that \( N_{\text{sur}} = N_{\text{bulk}} \), which holds in a de Sitter universe with \( p = -\rho, \epsilon = 1, H^2 = (8\pi/3)\rho \).

When the universe is not pure de Sitter, we expect the holographic discrepancy between \( N_{\text{sur}} \) and \( N_{\text{bulk}} \) to drive the expansion of the universe, which suggests \[ \] the law:
\[
\frac{dV_{\text{prop}}}{dt} = L_p^2(N_{\text{sur}} - \epsilon N_{\text{bulk}}) \tag{6}
\]

Incredibly, this leads to the standard Friedmann equation for cosmic expansion, but now obtained without using the field equations of general relativity! The right hand side is (nearly) zero in the initial and final phases (with \( V_{\text{prop}} \approx \) constant) and cosmic expansion in the transient phase can be interpreted as a quest towards holographic equipartition. It is then natural to associate a number \( N_{\text{sur}} = 4\pi L_p^2 / L_p^2 = 4\pi \) with the modes, when they cross the Planck scale.

Clearly, such a quantum gravitational imprint has far reaching consequences, culminating in the solution to the cosmological constant problem itself.

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