Black-hole ejecta by frame-dragging along the axis of rotation

Maurice H.P.M. van Putten
LIGO Laboratory, MIT 17-161, 175 Albany Street, Cambridge, MA 02139
(Dated: March 20, 2022)

An energy $E = \omega J$ is derived for gravitational spin-orbit coupling by frame-dragging $\omega$, acting on angular momentum $J$. This interaction defines a no-boundary mechanism for linear acceleration of magnetized matter along the axis of rotation of black holes. We explain GRB030329/SN2003dh by centered nucleation of a high-mass black hole with rapid rotation, producing baryon-poor ejecta by gravitational spin-orbit coupling. The duration of the burst is attributed to spin-down in the emission of gravitational-waves by a surrounding non-axisymmetric torus.

PACS numbers: Valid PACS appear here

GRB030329/SN2002dh showed that Type Ib/c supernovae are the parent population of cosmological gamma-ray bursts. Type II and Type Ib/c supernovae are believed to represent core-collapse of massive stars [1, 2]. These events probably take place in binaries, such as in the Type II/Ib event SN1993J [3]. This binary association suggests a hierarchy, in which hydrogen-rich, envelope retaining Type II and Type Ib are associated with increasingly compact binaries [2-4]. By tidal coupling in the latter, the primary star rotates at the orbital period. Consequently, core-collapse of the primary with an evolved core [5] produces a rapidly rotating black hole [6, 7, 15].

Here, we identify the potential energy induced by spin-orbit interactions. This is a completely new result, which expresses a powerful interaction of frame-dragging along the axis of rotation of a black hole. We begin with a new four-covariant derivation of curvature-spin coupling. This result radically differs from the common view, that black-hole energetic processes are confined to frame-dragging in the ergosphere.

The world-line $x^a$ of a particle moving in a periodic orbit about the orbital center describes helical motion about the time-axis in spacetime. Fig. 1 shows the closed curve $\gamma$ of a single orbit of period $T$ as measured in a local restframe, consisting of an open curve plus closing line-segment

$$\gamma : x^b(t) \quad (0 < t < T), \quad \gamma'' : t \quad (0 < t < T).$$

The surface enclosed by $\gamma$ may be taken to be sum of the curved spiral surface $S$ and a closing wedge $W$

$$S^{ab} = \int_{\gamma'} x^{[a}v^{b]} ds, \quad W^{ab} = T u^a_{\nu} u^b_{\mu} = T x^{[a} u^{b]},$$

where $' = d/ds$ and $v^b = dx^b/ds$ denotes the unit tangent to the particle world-line. This introduces the separation vector and four-velocity

$$[x_b(t)] = x^b(t + T) - x^b(t), \quad u^b = \frac{x^b(t)}{T}$$

of the particle between two consecutive orbits.

For parallel transport of a vector $\xi^b$ along $\gamma$ we have, according to the definition of the Riemann tensor [23],

$$\frac{\Delta \xi}{T} = \frac{1}{2T} R_{abc} v^c \xi^d + \frac{1}{2} R_{abcd} w^b u^c \xi^d.$$

Localizing to orbits of small radius $x^a$, the surface $S^{ab}$ is orthogonal to $u^b$. Consider, therefore,

$$\dot{S}^{ab} = \frac{1}{T} \int_{\gamma'} x^{[a} v^{b]} ds = \epsilon^{ab}_{cd} S^{c} u^{d}$$

where $' = d/d\tau$ expressed in terms of the specific angular momentum $s^b$: a spatial vector orthogonal to $\dot{S}^{ab}$ whose
variation satisfies magnitude equals the rate of change of surface area. The variation satisfies
\begin{equation}
\xi_c = \frac{1}{2} \epsilon_{abcdef} R^{ef}_{\quad cd} s^a u^b \xi^d + \frac{1}{2} R_{abcd} u^{ab} \xi^d.
\end{equation}

In case of a point symmetric mass-distribution about the orbital center, such as two particles attached to the end-points of a rod, a continuous mass-distribution in a solid ring or charged particles in a magnetic field, we can integrate over the mass-distribution. Since \( u^{ab} \) is 2\( \pi \)-periodic in the angular position of the wedge, only the term coupled to \( s^b \) survives. Taking \( \xi^b = u^b \), the acceleration of the orbital center satisfies
\begin{equation}
\dot{u}_c = \frac{1}{2} \epsilon_{abcdef} R^{ef}_{\quad cd} s^a u^b u^d.
\end{equation}
This is curvature coupling to specific angular momentum. The particle trajectory becomes completely specified by Fermi-Walker transport of \( s^b \).

The Kerr metric describes curvature induced by spin. This is the converse of above. Spinning bodies hereby couple to spinning bodies by analogy to magnetic momentum-magnetic moment interactions, although there is a difference in sign. To study this in the Kerr metric, we study spin-orbit interactions along the axis of rotation. Based on dimensional analysis, the gravitational potential for spin aligned interactions should satisfy
\begin{equation}
E = \omega J,
\end{equation}
where \( \omega \) refers to the frame-dragging angular velocity produced by the massive body and \( J \) is the angular momentum of the spinning object.

The non-zero components of the Riemann tensor of the Kerr metric have been expressed in Boyer-Lindquist coordinates relative to tetrad 1-forms \( e_0 = \alpha dt, \ e_1 = \frac{\Sigma}{\rho} (d\phi - \omega dt) \sin \theta, \ e_2 = \frac{\Sigma}{\Delta} dr, \) and \( e_3 = \rho d\theta \) by Chandrasekhar. Here, we use the expressions
\begin{equation}
\Sigma^2 = (r^2 + a^2)^2 - a^2 \Delta \sin \theta, \quad \rho^2 = r^2 + a^2 \cos^2 \theta
\end{equation}
and \( \Delta = r^2 + a^2 - 2Mr \) for a black hole of mass \( M \) and specific angular momentum \( a \). In particular, one of the Riemann tensor component satisfies
\begin{equation}
R_{1302} = AD,
\end{equation}
where \( A = aM \rho^{-6}(3r^2 - a^2 \cos^2 \theta) \) and \( D = \Sigma^{-2}[2(r^2 + a^2)^2 + a^2 \Delta \sin^2 \theta] \). On-axis \( (\theta = 0) \), we have
\begin{equation}
2A = -\partial_t \omega = \frac{2aM}{\rho^6}(3r^2 - a^2), \quad D = 2.
\end{equation}
According to, this component of the Riemann tensor creates the radial force
\begin{equation}
F_2 = J R_{3120} = JAD = -\partial_\phi \omega J.
\end{equation}
The assertion follows from
\begin{equation}
E = \int_r^\infty F_2 ds = \omega J.
\end{equation}
The result also follows from a completely independent derivation, by considering the difference in total energy between two particles in counter rotating orbits about the axis of rotation of the black hole. Let \( u^b \) denote the velocity four-vector and \( u^a/u^t = \Omega \) the angular velocities of either one of these, \(-1 = u^a u_c = [g_{tt} + g_{\phi\phi}(\Omega - 2\omega)](u^t)^2 \). This normalization condition has the two roots
\begin{equation}
\Omega_\pm = \omega \pm \sqrt{\omega^2 - (g_{tt} + (u^t)^{-2})/g_{\phi\phi}}.
\end{equation}
We insist that these two particles have angular momenta of opposite sign and equal magnitude, \( J_\pm = g_{\phi\phi} u^t(\Omega_\pm - \omega) \),
\begin{equation}
J_\pm = g_{\phi\phi} u^t \sqrt{\omega^2 - (g_{tt} + (u^t)^{-2})/g_{\phi\phi}} = \pm J.
\end{equation}
This shows that \( u^t \) is the same for each particle. The total energy of the particles is given by \( E_\pm = (u^t)^{-1} + \Omega_\pm J_\pm \), and hence one-half their difference
\begin{equation}
E = \frac{1}{2}(E_+ - E_-) = \omega J.
\end{equation}
The above shows that curvature-angular momentum coupling is universal: it applies whether the angular momentum is mechanical, electromagnetic or quantum mechanical in origin.

A magnetized blob of perfectly conducting fluid is characterized by rigid rotation with angular velocity $\Omega_h$. In the frame of zero angular momentum observers, the local charge-density is given by the Golreich-Julian charge density $22$. In their lowest energy state characterized by vanishing canonical angular momentum, their angular momentum satisfies $J = e A_\phi$, where $e$ denotes the unit of electric charge and $A_\phi$ the $\phi$-component of the electromagnetic vector potential. Consider a magnetized blob about the axis of rotation of the black hole shown in Fig. 2. The number density $N(s)$ of particles per unit distance $s$ along the axis of rotation – the number density per unit scale-height for a given magnetic flux – satisfies

$$N(s) = \frac{\rho}{e} = (\Omega_b - \omega) A_\phi.$$  \hfill (18)

A pair of blobs in both directions along the spin-axis of scale height $h$ hereby receives an energy

$$E_{\text{blob}} = \omega J N h = \omega (\Omega_b - \omega) A_\phi^2 h.$$  \hfill (19)

Expressed in dimensionful form, we have

$$E_{\text{blob}} = (1 \times 10^{57} \text{ erg}) B_{15} h_M^3 H,$$  \hfill (20)

where $h_M = h/M$ denotes the linear dimension of the blob, $B_{15} = B/10^{15}$. Here, we use the normalized function $H = 4\dot{\omega} \left( \frac{\Omega_b}{\Omega_h} - \dot{\omega} \right)$, where $\dot{\omega} = \omega/\Omega_h$ and $\Omega_b = \Omega_b/\Omega_h$.

We propose that blobs are ejected along the axis of rotation both to infinity ($H > 0$) and into the black hole ($H < 0$). This happens, for example, by break-up of a blob of perfectly conducting fluid with vanishing total angular momentum into two peaces, by tidal interaction of gravitational spin-orbit coupling given that $\omega$ is strong near the black hole and weak at larger distances.

A discrete, intermittent process produces "pancakes," which produce gamma-rays by shocks when they collide (see 31 for a review). This forms an alternative to continuous outflows in the form of baryon-poor jets (see 32 for a recent account), wherein shocks appear either by steepening due to temporal fluctuations in the interaction with the environment.

We explain GRB-supernova GRB030329/SN2003dh in terms of core-collapse supernova by centered nucleation of a rapidly rotating black hole in a massive star, followed by the ejection of baryon-poor blobs and radiative spin-down of the black hole against gravitational radiation catalyzed by a surrounding non-axisymmetric torus. We attribute the supernova to irradiation of the remnant stellar envelope from within, by high-energy photons produced in the dissipation of a subdominant torus wind.

Acknowledgment. The author thanks A. Levinson, E. Schuryak and R.P. Kerr for constructive comments. This research is supported by the LIGO Observatories, constructed by Caltech and MIT with funding from NSF under cooperative agreement PHY 9210038. The LIGO Laboratory operates under cooperative agreement PHY-0107417. This paper has been assigned LIGO document number LIGO-P0400XX-00-R.

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