Mathematical imaginations and constructing algebraic concept

I Dwijayanti*, A A Nugroho

Department of Mathematics Education, Universitas PGRI Semarang. 24 East Sidodadi Street, Dr. Cipto-Semarang, Central Java 50125, Indonesia

*Corresponding author: idadyana@gmail.com

Abstract. Imagination has a valuable role in the process of education for humans. The purpose of the study to determine the role of mathematical imagination to construct an understanding of mathematical concepts. The purpose of this qualitative exploratory study was to analyze students’ gestures in understanding algebraic expression. It involved 32 in Junior high school students in Semarang city. Time triangulation was utilized to assure data validity. The results showed that students employed: 1) imagination involved to understand the concept of variables, coefficients or constants is perception visualization that presents the possibility of actions representing a container, representing sum of objects in a container and representing representing sum of objects; 2) imagination in other processes is perception visualization that presents the possibility of action giving examples of algebraic expression, giving examples rather than algebraic expression, and identifying equations. This study yields an important description of misconceptions that are detected through students' imagination.

1. Introduction

Concept understanding is student's ability to find and explain, translate, interpret, and conclude a mathematical concept based on the formation of his own knowledge [1]. Students self-knowledge construction involves knowledge that comes from the concepts that have been owned or that come from life experiences that have been experienced. Knowledge and experience are visualized by students during the process of knowledge formation, so that mental activity occurs in shaping the visualization. Visualization of perception that presents the possibilities of action, presents the readiness to take possible action is known as mathematical imagination [2].

Many researchers have researched the beneficial of imagination in understanding mathematical concepts; including Wilke utilize the use of stories to stimulate students' imagination in numerical learning. Wilke focuses on the benefits of story problems in the formation of imagination by students. Nemirovsky investigated the development of perceptoumotor integration and its role in mathematical thinking and learning [3]. In particular, the component of this project research aims to investigate various ways in which verbal and physical activities through a tool that has been prepared to stimulate the imagination of visitors in an exhibition to express and shape their understanding of ratios and proportions. In particular, the component of this project research aims to investigate various ways in which verbal and physical activities through a tool that has been prepared to stimulate the imagination of visitors in an exhibition to express and shape their understanding of ratios and proportions. There is also research on the mathematical imagination that has been done focuses on mathematical problem solving [4]. In addition, research also looks at the influence of gender on the appearance of imagination. This study was inspired by research conducted by Nemirovsky that explored the workings of a tool in shaping the imagination [3]. In this study, the research focuses on the form of imagination involved in understanding concepts. In addition, this research also reveals how imagination plays a role in
constructing concepts so that the formed understanding is meaningful so that it could misunderstanding concepts. The problems raised in this study are: "What is the role of students' mathematical imagination in constructing the mathematics concept?". So the purpose of this research is to determine the role of imagination to construct of algebraic concepts, in order to minimize misunderstanding mathematical concepts.

2. Methods
This study involved 32 seventh-grade students in Semarang city, Indonesia. Data on students’ understanding and students’ imaginations were respectively obtained through test and interview as follows: (1) administering test 1 to examine students’ understanding of the algebraic concepts, (2) interviewing the students to explore their understanding on test 1, (3) analysing the results of test 1 and interview, (5) administering test 2, (6) repeating procedure 2 and 3 based on test 2.

We carefully developed test instrument and interview guideline to respectively examine students’ conceptual understanding on algebra and students’ imagination [5]. Test the validity and reliability of the instrument has been done through expert justment and field trials and produced a conclusion that the instrument can assess the imagination of students with fixed results for each different data collection [6, 7].

Data analysis followed three stages: data condensation, data display, and drawing and verifying conclusion [8]. In the preparation of data analysis, data on students’ imaginations to get detailed students’ imaginations when explaining the concepts of variables, coefficients and constants. This process was repeated in the analysis of the results of the second interview. At the stage of data condensation, we grouped students’ imaginations based on the indicators of conceptual understanding. Unrelated gestures were ignored, for example the students scratched their heads or explanations cancelled by the students. After the data was grouped, it was presented in the form of narrative dialogue. At last stage, the results of students’ gestures in the first and second interviews were then compared to see whether their imaginations are consistent when explaining each concept (variables, coefficients and constants). If the results were the same, then the conclusion was drawn.

3. Results and Discussion

3.1 Results
Understanding the concept of algebra subject in the class, the algebraic material is described based on cognitive processes of understanding. The algebraic understanding of the test results obtained data about the value of students conceptual understanding as shown in Table 1.

| Value Range | Total students |
|-------------|----------------|
| 91 – 100    | 7              |
| 81 – 90     | 13             |
| 71 – 80     | 5              |
| 61 – 70     | 7              |

Imagination involved to understand the concept of variables, coefficients or constants is perception visualization that presents the possibility of actions representing a container, representing sum of objects in a container and representing representing sum of objects. Imagination in other processes is perception visualization that presents the possibility of action giving examples of algebraic expression, giving examples rather than algebraic expression, and identifying equations. The descriptions of students’ imaginations are presented to explain their understanding of algebraic expression. Students’ imaginations on algebraic conceptual understanding would be seen in Table 2.
Students’ imaginations on Algebraic Conceptual Understanding

| Steps       | Profile Of Students Algebraic Conceptual Understanding |
|------------|-------------------------------------------------------|
| Interpreting | Interpreting information about two container of candy and five candies up to $2x + 5$ |
| Exemplifying | Exemplifying $2x + 1$ as an algebraic expression with the thought of 2 dolls and 1 toy car |
| Classifying  | Classifying $3p + 2$ as an algebraic expression, because there are variables $p$ and constants are $2$ and $3$; and $2a + 4 = 8$ as not algebraic because they are equations |
| Summarizing  | Attribute of algebraic expression is have constants and variables. |
| Inferring    | Definition of algebraic expression as numbers described |
| Comparing    | Comparing the differences between algebraic and non-algebraic expression. An algebraic expression have constants, variables but no symbols "=" |
| Explaining   | Explaining the algebraic expression changes to another algebraic expression by examining variables, constants and coefficients as elements of the algebraic expression. |

Table 2 shows the quality of students algebraic conceptual understanding. Misconceptions about the concept of variables seen in the exemplifying stage, where students imagine the variable $x$ as a puppet and the variable $y$ as a car. Based on Algebraic Expression book shows that the variable is a member of the universe of conversation whose value is unknown.

3.2 Discussion

Explanation of the mathematical imagination comes from constructivist theory. In the interpretation process, show that students do imagination about algebraic concepts through objects that they recognize in everyday life. This imagination arises because of the use of the context of everyday life in the test items. The selection of content questions that involve everyday events helps facilitate students to understand the context of the conversation they want to appear without limiting the imagination that will be raised [9].

But in the exemplifying stage, students look wrong to understand the concept of algebra. students imagine $2x + 1$ as 2 dolls and 1 car. This shows that students' understanding of the concepts of variables and constants is meaningless. Construction representation of the environment in a visualization carried out by the subject in his mind will form a network of concepts called schemes. These schemes are used to form new schemes that have the same information structure through the assimilation process. Connecting concepts that have been possessed in a visualization concept is used by students. This justifies the theory that understanding concepts is a key factor for the development of subject mathematics knowledge. On the other side, these results are in accordance with the opinion about an unrevealed error rooted in the subject's mind will be the biggest threat to the formation of subject knowledge [10]. The results of this study support several results of previous research which states that engaging the imagination will make a concept clearer so that it is easier to understand by the subject [11]. In addition, the results of this study support the results of research on the involvement of old schemes in constructing new schemes. The absence of gestures which are manifestations of sensorimotor information is the cause of errors in understanding the concept of students [12]. These results supported the theory which states that the initial sensorimotor information possessed by a child is an important aspect to encourage cognitive development [13]. In addition, the results of the study support the research on how embodied cognition is derived from experiences owned by children which contribute as a cognitive foundation in the process of conceptual development [14].

Students' imagination during the process of understanding mathematical concepts can also be explained through Schlegel's mental workplace theory. Stimulation obtained through statements presented at the interpretation stage is manipulated by students into a ladder form. On the other hand, students cannot manipulate the statements presented in the process of interpretation in which students are unable to present an appropriate imagination at the exemplifying stage. This happens because the prefrontal cortex part of the process of manipulation of representation is not done perfectly by students. In the inferring stage, students do not present either the imagination that comes from experience and the imagination that comes from previous concepts so that when students define algebraic forms it produces illogical definitions. This logic is consistent with the theory that if the activities that occur in
the frontal, parietal, and occipital cortex occur continuously, it will affect one's logical reasoning [15]. This is in line with the theory that visual image manipulation (imagination) that occurs in the frontal and parietal area networks is involved in many high-level cognitive abilities in humans [16, 17].

4. Conclusion

The conclusion of this study is that the construction of the mathematical imagination as follow: imagination involved to understand the concept of variables, coefficients or constants is perception visualization that presents the possibility of actions representing a container, representing sum of objects in a container and representing representing sum of objects; 2) imagination in other processes is perception visualization that presents the possibility of action giving examples of algebraic expression, giving examples rather than algebraic expression, and identifying equations. This study yields an important description of misconceptions that are detected through students' imagination.

Acknowledgments

Civitas academics of Universitas PGRI Semarang and SMP Negeri 2 Semarang as the research site.

References

[1] Duffin J M & Simpson A P 2000 J. Math. Behav. 18 415
[2] Pellerone M, Passanisi A, and Bellomo M 2015 Psychol. Res. Behav. Manag. 8 239
[3] Nemirovsky R, Kelton M L, and Rhodehamel B 2013 J. Res. Math. Educ. 44 372
[4] Pirrone C 2015 Mediterr. J. Soc. Sci. 6 152.
[5] Dwijayanti I, I K Budayasa and T Y E Siswono 2018 J. Phys.: Conf, Ser. 983 012132
[6] Azwar S 2014 Psikologi inteligensi (Yogyakarta: Pustaka Pelajar)
[7] Dwijayanti I, I K Budayasa and T Y E Siswono 2019: J. Tadris Mat. 12 133
[8] Miles M B, Huberman A M, and Saldana J 2014 Qualitative Data Analysis, A Methods Sourcebook, Edition 3 (USA: Sage Publications)
[9] Wilke J 2006 Educ. Perspect. 9 15
[10] Legutko M 2008 Handbook for Mathematics Teaching: Teacher Experiment. A Tool for Research, 141-152
[11] Nemirovsky R, Kelton M L, and Rhodehamel B 2012 Gesture 12 130
[12] Laakso A 2011 Cogn. Brain Behav 15 409
[13] Glenberg A M & Gallese V 2012 Cortex 48 905
[14] Gabbard C 2013 Early Child Dev. Care 183 643
[15] Green AE, Kraemer DJM, Fugelsang JA, Gray JR, Dunbar KN 2012 J Exp Psychol Learn Mem Cogn 38 264
[16] Salazar RF, Dotson NM, Bressler SL, Gray CM 2012 Science 338 1097
[17] Kazak S, Wegerif R, and Fujita T 2015 Educ. Stud. Math. 90 105 = 17