Flexible fiber transport by a fluid flow in fractures with smooth and rough walls.

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Abstract. The transport of flexible fibers by a flowing fluid has been studied experimentally in transparent model fractures. Both finite length segments (20 mm \( \leq l \leq 150 \) mm) and continuous fibers penetrating freely into the model were used; their motion is monitored by means of a digital camera and of an image thresholding technique and is induced by the flow of water or of a polymer solution at a mean velocity \( U \) (50 \( \leq U \leq 400 \) mm.s\(^{-1}\)). In a model with plane smooth parallel walls, the influence of the friction with the walls is small: fiber segments reach quickly a constant velocity as their distance \( x_m \) to the inlet increases, the velocity of continuous fibers increases slower with distance before reaching a constant value. The second model fracture has two complementary rough self-affine walls with a relative lateral shift; it displays, in addition, a gradient of the aperture in the direction transverse to the mean flow. For this model, the transport of the fibers by flowing water is only possible in the region of largest aperture and is of a “stop and go” type at low velocities. If water is replaced by the shear thinning polymer solution, the fibers move faster and continuously in high aperture regions; fiber transport also becomes possible in narrower regions.

1. Introduction
Understanding the dynamics of fiber transport by flowing fluids is relevant to many fields of physics, biology and engineering. Examples include paper pulp [1], fiber-reinforced composites [2], the dynamics and rheology of biological polymers [3] and the motility of microscopic organisms [4, 5]. Recently, in-situ measurements on natural water flows by means of long optical fibers have been suggested [6]. Yet, relatively little work has been devoted to flow channels with rough walls such as fractures of natural rocks or of materials of major industrial interest in civil, environmental and petroleum engineering. In that case, the interactions of the fibers with the walls are particularly strong and may stop their motion and/or lead to a blockage of the flow channels. In addition to these applications, such processes raise important fundamental questions on the motion of flexible solid bodies in complex flow fields, particularly regarding the coupling between their mechanical deformations and the flow.

The present paper reports laboratory experiments performed on a model of a single fracture and investigating the transport of a single fiber by a fluid flow. Two experimental fracture
models have been considered: the first one has two plane parallel flat walls and the roughness of the walls of the second one reproduces that of natural fractures. In this second model, the roughness and the relative position of the walls creates long range correlations in the aperture field; also, the interaction of the fiber with the walls becomes very strong and may block the motion of the fiber. Several factors may influence this blockage: it would seem that, to ease the motion in the fracture, the fiber would need to be very thin and flexible in order to pass through local constrictions and to remain in the central part of the gap where the velocity is highest. However, additional factors may prevent the fiber from following the streamlines and lead to its trapping. One needs in particular to take into account tension forces (i.e. the mechanical cohesion of the fiber along its length), elastic forces (non-zero stiffness of the fibers), and, also, the rheology of the fluids which influence the hydrodynamic forces on the fibers. The objective of the experiments reported here is to analyze the transport (and/or the trapping) of a fiber in a smooth and a rough model fracture and their dependence on these different parameters.

2. Experimental set-up and procedure

The fiber used in this work is a commercial Polyester fine thread of radius varying from 110 to 170µm and of the type used for needlework. The thread is made of two filaments twisted together. The density of the fiber is \( \rho = 1.8 \pm 0.1 \text{g.cm}^{-3} \) and its bending modulus is of the order of \( 10^3 \text{Pa} \) [7].

Both finite length and continuous fibers have been used. In the first case, fiber segments of length \( 20 \text{mm} \leq l \leq 150 \text{mm} \) are cut out of longer samples. The continuous fibers are longer than the fracture and are dragged into the fracture from its upper side by hydrodynamic forces and by their own weight during the experiments.

The model fractures are obtained by milling with a computer controlled machine two plexiglas plates which are then clamped together with, between them, a gap saturated by the flowing fluid. A detailed description of the procedure is given in ref. [8]. Two fracture models were machined. The first one, referred to in the following as the smooth walled fracture, has two plane walls at a fixed distance \( a(x, y) = a_0 = 0.65 \text{mm} \). The second model has rough self-affine walls and is referred as the rough fracture in the rest of the paper. There is a small angle between the mean planes of these rough walls: as a result the mean distance between them is constant in the direction \( x \) of the mean flow \( \vec{U} \) but varies linearly in the transverse direction \( y \). The minimum aperture (averaged along \( x \)) is \( a_x(y = 0) = 0.8 \text{mm} \) while the maximum one is reached on the opposite side with \( a_x(y = w) = 1.1 \text{mm} \) where \( w = 90 \text{mm} \) is the width of the cell. The average of the aperture over the full model surface is \( \pi_{x,y} = 0.91 \text{mm} \). This wedge-like shape of the fracture reproduces aperture gradients often encountered near the edge of natural fractures.

The model rough fracture surfaces used in this work have statistical properties similar to those of real rocks, i.e. they are self-affine with and the amplitude of the roughness is of the order of \( 25 \text{mm} \) [9, 10]. The two walls have complementary geometries and can be brought in contact over their full area. In the model, both a normal relative displacement equal to \( a_x(y) \) and a lateral shear displacement \( \delta = 0.65 \text{mm} \) normal to \( \vec{U} \) (i.e. \( \parallel y \)) have been introduced with respect to the contact position. The shear displacement introduces a random disorder of the aperture field with a standard deviation \( \sigma_a = 0.127 \text{mm} \).

The models are held vertically and saturated with a fluid; a vertical downward flow is established by sucking this fluid at the bottom side through a leak-tight injector part distributing evenly the flow by means of a gear pump. In order to establish a continuous permanent flow, the pumped fluid is reinjected into a rectangular bath attached onto the upper side of the model and connected to the fracture gap through a 5mm wide slot. The mean flow velocity is \( 50 \leq U \leq 400 \text{mm.s}^{-1} \). For water, this corresponds to a range of Reynolds numbers \( Re = U\pi\rho/\mu: 40 \leq Re \leq 320 \) for which substantial inertial effects are expected [11, 12], particularly for rough fractures. The fibers are injected into the model through the free surface of the bath: in order
to ease their insertion, the top part of the fracture gap has a Y-shaped section with an aperture increasing upward and acting like a funnel for the fiber.

The back of the models is illuminated by a light panel and a digital camera allows one to record images of the moving fiber at a rate of 30 frames per second. Each image has $1024 \times 768$ pixels; it corresponds to a field of view of vertical length 150 mm with its upper boundary located at 110 mm below the upper side of the model and of width corresponding to that of the fracture. Finally, the location of the fiber is determined for each picture by a binary thresholding technique. Quantitatively, the velocity of the fibers is characterized by the time variation of the distance $x_m$ of their center of mass (for segments) or $x_t$ of their tip (for continuous fibers) to the inlet of the model. Unless otherwise mentioned, $dx_m/dt$ or $dx_t/dt$ are referred to as the fiber velocity $V_f$.

The Newtonian fluid used in the experiments is high purity water (Millipore - Milli-Q grade) with a density close to $\rho = 1 \text{ g cm}^{-3}$ and a dynamic viscosity of $10^{-3} \text{ Pa.s}$. The shear thinning fluid is a solution of concentration $C_p = 1000 \text{ ppm}$ of high molecular weight Scleroglucan (Sanofi Bioindustries) in water. The variation of its effective viscosity $\mu$ with the shear rate $\dot{\gamma}$ (see Figure 2 of ref. [13]) is well adjusted by the Carreau function:

$$\mu = \frac{1}{(1 + (\dot{\gamma}/\dot{\gamma}_0)^2)^{\frac{n-1}{2}}}(\mu_0 - \mu_\infty) + \mu_\infty.$$  

(1)

In Eq. (1), the parameter $\dot{\gamma}_0$ corresponds to the transition between a “Newtonian plateau” domain at low shear rates ($\dot{\gamma} < \dot{\gamma}_0$) in which $\mu = \mu_0$ and a domain in which $\mu$ decreases with $\dot{\gamma}$ following the power law $\mu \propto \dot{\gamma}^{(n-1)}$ ($\dot{\gamma} > \dot{\gamma}_0$). For the present solution, rheological measurements give $n = 0.26$, $\dot{\gamma}_0 = 26 \pm 4 \times 10^{-3} \text{ s}^{-1}$ and $\mu_\infty$ is taken equal to the viscosity of the solvent, i.e. water with $\mu_\infty = 10^{-3} \text{ Pa.s}$. The corresponding transition value of $\dot{\gamma}$ may be estimated by

\[ \begin{align*}
& (a) \quad \text{Figure 1. Experimental pictures of the motion of fibers in two model fractures at successive times } t_1 < t_2 < t_3. \quad (a): 50 \text{ mm long fiber segment moving in the fracture with flat parallel walls.} \\
& (b) \quad \text{(b) Transient pinning event at a point } P \text{ for a continuous fiber in the fracture with rough walls.}
\end{align*} \]
approximating the rheological curve at high shear rates by a truncated power law, leading to \( \dot{\gamma}_\infty \sim 2300 \text{s}^{-1} \) [14].

The maximum value of the shear rate is reached at the walls and is equal to \( \dot{\gamma}_w = 6U/a \) for a Poiseuille Newtonian flow (the actual value will be higher because of the shear thinning properties of the fluid at intermediate shear rates). In the present experiments for which \( 50 \leq U \leq 400 \text{mm.s}^{-1} \), this leads to the rough estimation \( 300 \leq \dot{\gamma}_w \leq 2400 \text{s}^{-1} \): therefore, at the walls, \( \dot{\gamma}_w \) is much larger than \( \dot{\gamma}_o \) and may become of the order of \( \dot{\gamma}_\infty \). Since, in the center of the pipe, \( \dot{\gamma} \) always cancel out, we conclude that, in the upper range of velocities \( U \) of the present experiments, both limiting effective viscosities \( \mu_\infty \) and \( \mu_o \) are reached respectively at the walls and in the center of the pipe with a “shear thinning” region in between.

3. Experimental results

3.1. Fiber transport by water in the fracture with smooth walls

A first series of experiment was realized in the fracture with flat walls, considered as a reference case. In a first step, the influence of the length and the flow velocity on the transport of fiber segments has been studied. Fibers with four different lengths \( 20 \leq l \leq 150 \text{mm} \) have been used and 20 measurements of the fiber velocity \( V_f \) were performed for each value of \( l \) at velocities \( 50 \leq U \leq 400 \text{mm.s}^{-1} \). Only experiments in which the angle of the fibers with respect to \( \vec{U} \) is less than \( 10^\circ \) are retained for the analysis. In all these cases, the fibers have a constant velocity inside the field of view and do not get deformed during their motion (see Fig. 1a). Figure 2 displays the variation of the normalized fiber velocity \( V_f/U \) as a function of \( U \) for different values of the length \( l \). The dispersion of these experimental values may reflect a variability of the initial injection and, also, the natural curvature of the fiber. For a given length \( l \), the distribution of the values of \( V_f/U \) becomes narrower at high velocities; also, there is no clear trend for the variation of \( V_f/U \) with \( U \), particularly if one takes into account the broad range of values of the \( U \) (almost a decade). At most, \( V_f/U \) increases slightly with \( U \) for the longer fibers and decreases for the shorter ones: this latter trend is also visible for \( l = 50 \text{mm} \) in Fig. 3 (open symbols).
At the highest velocities, \( V_f/U \) is of the order of 1 (0.9 ≤ \( V_f/U \) ≤ 1.2): this implies that the relative velocity of the fluid and the fiber is small and that the fiber remains in the center part of the gap.

Overall, \( V_f/U \) also decreases for longer fibers, particularly at low velocities (nearly a factor of 2 in this case): long fibers will more often than shorter ones be curved or, even, display meanders. They are then more likely to be in the lower velocity zones close to the walls over, at least, a part of their length. Friction with the walls may also increase and recirculation inside meanders may dissipate energy.

![Figure 3](image URL)

**Figure 3.** Full symbols: variation of the normalized velocities \( V_f/U \) of the tip of the continuous fibers as a function of \( x_t \): (■) \( U = 110\text{ mm.s}^{-1} \), (●) \( U = 190\text{ mm.s}^{-1} \), (▲) \( U = 290\text{ mm.s}^{-1} \). Open symbols: variation of the normalized velocity \( V_f/U \) of the center of mass of fiber segments (\( l = 50\text{ mm} \)) as a function of \( x_m \): (□) \( U = 110\text{ mm.s}^{-1} \), (○) \( U = 190\text{ mm.s}^{-1} \), (△) \( U = 290\text{ mm.s}^{-1} \).

A second series of experiments has been performed using a continuous long fiber sucked into the model by hydrodynamic forces at the inlet side with no external applied force. The variations of the corresponding values of \( V_f/U = (1/U)dx_t/dt \) with the distance \( x_t \) are compared in Figure 3 to the corresponding values of \( V_f/U \) for a 50mm long segment plotted as a function of \( x_m \). While the velocity of the segments is globally independent of distance \( x_m \) (open symbols), the normalized velocity of the continuous fiber clearly increases with the distance \( x_t \) from the inlet (full symbols) before leveling off for \( v_t \sim 250\text{ mm} \). The variations are identical for the two highest mean velocities. This variation may be interpreted as reflecting the larger local aperture both in the upper part of the model (of height equal to 52 mm) which has an Y-shaped section and in the upper bath. This higher aperture reduces the local velocity and, therefore, the drag force on the fiber: the velocity \( dx_t/dt \) corresponds then to an average of the mean velocities over the full distance from the free surface down to the tip. This explains why the normalized velocity \( V_f/U \) of the tip of the continuous fiber is at most 0.7, i.e. significantly lower than that of short segments. This effect should be independent on the velocity provided the fiber retains the same shape and remains at the same distance from the walls in the gap.
Figure 4. Variation of the normalized fiber velocity $V_f/U$ in a water flow as a function of the distance $x_m$ (fiber segments) or $x_t$ (continuous fibers) from the inlet of the rough model. Continuous fibers: (●) $U = 165\,\text{mm.s}^{-1}$, (▲) $U = 220\,\text{mm.s}^{-1}$. Fiber segments ($l = 50\,\text{mm}$): (◦) $U = 165\,\text{mm.s}^{-1}$, (△) $U = 220\,\text{mm.s}^{-1}$.

3.2. Fiber transport by water in the rough fracture

As mentioned above, the rough model has a wedge-like section transverse to the flow and we observed that the fiber could penetrate deeply into the fracture only on the most open third of the width. Figure 4 displays the normalized velocity of continuous fiber and of a fiber segment ($l = 50\,\text{mm}$) as a function of distance along their path for two different mean velocities $U$; the initial location of the fiber at the inlet is the same for all 4 experiments. Compared to the smooth flat fracture, a striking feature is the very large fluctuations of $V_f/U$ with the distance. Also, the mean velocity is significantly larger for the segment of fiber than for the continuous one. Moreover, the mean velocity of the continuous fiber does not increase steadily with distance as it does for the smooth fracture. Several pinning events were observed for both types of fibers, particularly at lower mean velocities $U$: they remained stopped during some time before getting released with a sharp jump of the velocity (see Fig. 1b). For different $U$ values, a given fiber slows down at the same locations but, at the higher $U$’s, it does not stop before accelerating again.

Figure 5 displays the variations of the geometry of a continuous fiber during a pinning event. While the tip of the fiber remains motionless, it displays first a global curvature and then two meanders with their curvatures in opposite directions. The local curvatures increase with time which results in an accumulation of bending elastic energy. When this energy becomes large enough, depinning occurs and the fiber pursues its progression in the model fracture.

3.3. Fiber transport by a polymer solution in the rough fracture

The influence of the fluid rheology on these results has been studied by performing experiments with the same continuous fibers as above but in a flow of the polymer solution with the rheological properties discussed in Sec. 2. Since its effective viscosity will be larger than that of water, except near the walls, one may expect that the hydrodynamic forces will be significantly enhanced.
Figure 5. Time variation of the shape of a fiber during a pinning event in a water flow with $U = 230 \text{mm.s}^{-1}$. Time elapsed after pinning: $t = 0 \text{s}$ (dots), $t = 0.33 \text{s}$ (dash-dots), $t = 0.66 \text{s}$ (dashes), $t = 1 \text{s}$ (continuous line).

Figure 6. Variation of the normalized fiber velocity $V_f/U$ of the tip of a continuous fiber in a water-polymer flow as a function of the distance $x_t$ of the tip from the inlet of the rough model: (●) $U = 165 \text{mm.s}^{-1}$, (▲) $U = 220 \text{mm.s}^{-1}$.

At a similar velocity (Fig. 6), the ratio $V_f/U$ is larger for the shear thinning solution than for water, except at short distances. The amplitude of the velocity fluctuations is also smaller but the minima of the velocity are still mostly located at the same distances $x_t$ as the pinning sites observed for water flows at low velocities $U$. The motion of the fiber is therefore still influenced by the local constrictions encountered along its path, although much more weakly.
Also, the global increase of the fiber velocity of these continuous fibers with the distance $x_t$ which had been discussed in the case of a water flow and of smooth walls is again observed here: this suggests that using the polymer solution instead of water reduces significantly the relative influence of the friction forces between the fibers and the rough walls.

4. Conclusions

In the present work, the transport of fibers in two model transparent fractures with (a) smooth parallel walls and (b) rough walls has been compared. The transport of both finite length fibers and continuous ones has displayed very different characteristics and a strong dependence on the rheology of the fluids.

For the model with smooth walls, the influence of the interaction of the fibers with the walls is small, even for a low viscosity fluid like water. The hydrodynamic forces applied to the fibers are transmitted all along their length and the fibers remain in the vicinity of the central part of the gap of the fracture during their motion (Their velocity is then of the order of the fluid velocity). For the rough model, the friction of the fibers with the walls is much larger and reduces strongly their mobility, particularly for a water flow: however, in this case, the transport of the fibers along the fracture still remains possible as long as the aperture is about twice the fiber diameter or larger. Also the propagation of the hydrodynamic forces along the fiber is more difficult so that the influence of the length of the fibers on their velocity is reduced, particularly for continuous fibers. An important result of the present work is the enhancement of the mobility of the fibers when water is replaced by a water-polymer solution with a high viscosity at low shear rates. Further studies will allow to evaluate the feasibility of fiber transport through narrow constrictions when such fluids are used.

An important feature of fiber transport in the rough fracture, particularly at low velocities and in regions of narrow aperture, is the occurrence of pinning resulting in an intermittent, stop and go-like, progression of the tip of the fiber. In the pinning events, the deformability of the fibers is a key factor: for relatively rigid fibers and moderate pinning forces, the fiber becomes increasingly deformed by the hydrodynamic forces until the total force on the fiber is large enough to induce depinning. New studies will be focused on the dynamics of the deformation of the fiber during the pinning stage.

Finally, it will be necessary to extend these results to the transport of fibers in natural fracture networks. A first issue is the influence of the orientation of a channelization of the fracture with respect to the direction of the motion of the fiber (and of the mean flow). For that purpose, experiments with channels perpendicular to the mean flow will, in particular, be needed [15]. Another important step towards the application of these results to fracture networks will be the transport of fibers at an intersection between fractures.

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