Constraining $f(R)$ gravity with PLANCK data on galaxy cluster profiles

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ABSTRACT

Models of $f(R)$ gravity that introduce corrections to the Newtonian potential in the weak field limit are tested at the scale of galaxy clusters. These models can explain the dynamics of spiral and elliptical galaxies without resorting to dark matter. We compute the pressure profiles of 579 galaxy clusters assuming that the gas is in hydrostatic equilibrium within the potential well of the modified gravitational field. The predicted profiles are compared with the average profile obtained by stacking the data of our cluster sample in the Planck foreground clean map SMICA. We find that the resulting profiles of these systems fit the data without requiring a dominant dark matter component, with model parameters similar to those required to explain the dynamics of galaxies. Our results do not rule out that clusters are dynamically dominated by Dark Matter but support the idea that Extended Theories of Gravity could provide an explanation to the dynamics of self-gravitating systems and to the present period of accelerated expansion, alternative to the concordance cosmological model.

1 INTRODUCTION

Measurements based on Supernovae Type Ia (SNeIa) have indicated that the Universe has entered a period of accelerated expansion [Riess et al. 2004; Astier et al. 2006; Clocchiati 2006]. Data on Cosmic Microwave Background (CMB) temperature anisotropies measured by the Wilkinson Microwave Anisotropy Probe (WMAP) [Hinshaw et al. 2013] and the Planck satellite [Planck Results XVI 2013, Planck Results XXI 2013], on Baryon Acoustic Oscillations (BAO) [Blake et al. 2011] and other observables, together with SNeIa data, favor the concordance $\Lambda$CDM model. In this model, the energy component that drives the current period of accelerated expansion is a cosmological constant $\Lambda$. The associated energy density is $\Omega_{\Lambda} \approx 0.7$, in units of the critical density. The second most important component is Dark Matter (DM), a matter component required to explain the formation of galaxies and the emergence of Large Scale Structure, with $\Omega_{DM} \approx 0.26$. More general models assume that the acceleration is due to an evolving form of Dark Energy (DE) characterized by an equation of state parameter $\omega \leq -1/3$. For these models, cosmological observations indicate that $w = -1.13^{+0.13}_{-0.10}$ [Planck Results XVI 2013] fully compatible with a cosmological constant ($w = -1$).

As an alternative, models involving extensions of General Relativity (GR) have also been widely considered (for comprehensive reviews see [Carroll et al. 2004, Sotiriou & Faraoni 2010, De Felice & Tsujikawa 2013, Nojiri & Odintsov 2011, Nojiri & Odintsov 2011] and Capozziello & De Laurentis 2011). In this approach, the Hilbert-Einstein action changes from being linear in the Ricci curvature scalar, $R$, to a more general function. The simplest extensions are $f(R)$ models, where the Lagrangian is a function (possibly analytic) of the Ricci scalar. In these models, the higher order gravity terms introduced in the action are responsible for the present period of accelerated expansion. In some Extended Theories of Gravity (ETG), the Newtonian limit is also modified and models have been constructed where the dynamics of galaxies can be explained without requiring a DM component. For instance, analytic $f(R)$ models give rise to Yukawa-like correction to the gravitational potential [Capozziello & De Laurentis 2011, 2012] that do not require DM to explain the flat rotation curves of spiral galaxies [Cardone & Capozziello 2011], or the velocity dispersion of ellipticals [Napolitano et al. 2012]. The constraints derived from planetary dynamics are weak since the Yukawa correction is negligible at those scales [Capozziello & Troisi 2003, Allemandi et al. 2003, Berry & Gair 2011]. ETG also modify the hydrostatic equilibrium of stars: Capozziello et al. (2011), Finelli et al. (2013) have compared the Lane-Endem solution of polytropic gases in both $f(R)$ and general relativity and found them to be compatible while Capozziello et al. (2012) analyzed Jeans instabilities in self-gravitating systems and studied star formation in $f(R)$ gravity.

Since the exact functional form of the Lagrangian is unknown, theoretical considerations need to be complemented with observations. Thus, it is important to test potential
models using all available data. At present, clusters of galaxies are the largest virialized objects in the Universe and offer the opportunity to test these alternative theories of gravity on scales larger than galaxy scales. Using the mass profiles of clusters of galaxies, Capozziello, De Filippis & Salzano (2009) showed that ETG provide a fit to the distribution of baryonic matter (stars+gas) derived from X-ray observations in 12 clusters without requiring DM. Nevertheless, in conventional cosmological models, the non-linear evolution and virialization of self-gravitating objects is studied using numerical simulations. $f(R)$ models have a much larger number of degrees of freedom and the study of galaxy and cluster formation requires more complex simulations, specific for each particular Lagrangian. A first attempt to constrain ETG using cluster abundances in numerical simulations has been carried out by Ferraro, Schimd & Hu (2011) and Schimd, Vikhlinin & Hu (2009). Other numerical constraints on $f(R)$ models can be found in Song, Hu & Sawicki (2007), Sawicki & Hu (2007), Hu & Sawicki (2007) and Lima & Lidelle (2013). More promising is the study of temperature fluctuations on the CMB. Galaxy clusters are reservoirs of hot gas that induces anisotropies by means of the Sunyaev-Zeldovich (SZ) effect (Sunyaev & Zeldovich 1972, 1980). Pressure profiles of galaxies can be computed in ETG assuming that the gas is in hydrostatic equilibrium within the potential well of clusters. This is in agreement with the results of numerical simulations based on the concordance cosmology that showed that gas is in hydrostatic equilibrium in the intermediate regions of clusters, while in the cluster cores, the physics of baryons is more complex and in the outer regions it is dominated by non-equilibrium processes (Kravtsov & Borgani 2012). Recently, hydro-numerical simulations are being carried out to study the properties of galaxy clusters and groups in ETG. Arnold, Puchwein & Springel (2013) showed that the intra-cluster medium temperature increases in $f(R)$ gravity in low mass halos but the difference disappears in massive objects. Based on these results we will assume that the physics of the gas will be weakly dependent on the underline theory of gravity.

The SZ anisotropies generated by individual clusters and by the unresolved cluster population have been measured by the Atacama Cosmology Telescope (ACT) (Hand et al. 2011; Hasselfield et al. 2013; Menanteau et al. 2013; Sehgal et al. 2011), the South Pole Telescope (SPT) (Benson et al. 2013; Staniszewski et al. 2009; Williamson et al. 2011; Vanderlinde et al. 2010) and the Planck satellite (Planck Intermediate Results V 2013; Planck Intermediate Results X & XX 2013; Planck Results XXI 2013). Gas profiles based on the Navarro-Frenk-White (hereafter NFW, Navarro et al. (1997)) profile, derived from numerical simulations, have been found to be in agreement with TSZ (Atrio-Barandela et al. 2005) and X-ray observations (Arnaud et al. 2011). Nevertheless, the contribution of the unresolved cluster population in WMAP 7yr data has been found to be smaller than expected based on theoretical and numerical modeling of clusters (Komatsu et al. 2011). For the Coma cluster, the analysis of Planck data (Planck Intermediate Results V 2013; Planck Intermediate Results X 2013) finds a normalization of $\sim 10 - 15\%$ lower compared with the parameters derived from XMM observations. These discrepancies can be related to the existence of complex structures and sub-structures in clusters of galaxies as well as to the limitations of the theoretical modeling [Fusco-Femiano et al. 2013], that is the approach we are going to consider here.

In this article, we will compare the pressure profiles of clusters of galaxies in $f(R)$ models with Planck data. To construct the pressure profiles, we will assume that the gas is in hydrostatic equilibrium within the potential well generated by the cluster. At this level, our assumption can not be applied to models not in equilibrium like the Bullet cluster (Clowe et al. 2006). We will restrict our analysis to $f(R)$ models of gravity that introduce Yukawa corrections to the Newtonian potential in order to test if the dynamics of clusters of galaxies can be also described without a dominant dark matter component. The paper is organized as follows: in Sec. 2, we consider the weak field limit of $f(R)$ gravity deriving the gravitational potential for self-gravitating objects; in Sec. 3, we present the pressure profiles based on the NFW profile and X-ray data most commonly used and we compute the pressure profile for $f(R)$ models; in Sec. 4 we describe the data used in our analysis; in Sec. 5, we discuss our results and, finally, in Sec. 6 we present our main conclusions.

## 2 YUKAWA CORRECTIONS TO THE NEWTONIAN POTENTIAL IN $f(R)$-GRAVITY

In $f(R)$ ETG, field equations are derived from the action

$$A = \frac{c^4}{16\pi G} \int d^4x \sqrt{-g} f(R) + L_m, \quad (1)$$

yielding

$$f'(R) R_{\mu\nu} - \frac{f(R)}{2} g_{\mu\nu} - f'(R)_{,\mu\nu} + g_{\mu\nu} \Box f(R) = 8\pi G T_{\mu\nu},$$

where $f'(R) = df(R)/dR$ is the first derivative with respect to the Ricci scalar, $\Box f(R) = \sigma^\alpha_{\alpha} f(R)$ is the d’Alembertian with covariant derivatives, $T_{\mu\nu} = -2(1-g)^{-1/2} \delta(\sqrt{-g} L_m)/\sqrt{-g}$ is the matter energy-momentum tensor, $T$ its trace, $g$ the determinant of the metric tensor $g_{\mu\nu}$. Greek indices run from 0 to 3.

We search for spherically symmetric solutions of the form

$$ds^2 = g_{tt} c^2 dt^2 - g_{rr} dr^2 - r^2 d\Omega^2,$$

where $d\Omega$ is the solid angle. Let us restrict our study to those $f(R)$-Lagrangians that can be expanded in Taylor series around a fixed point $R_0$

$$f(R) = \sum_n \frac{f_n(R_0)}{n!} (R - R_0)^n \simeq f_0 + f'_0 R + \frac{f''_0}{2} R^2 + \ldots.$$  (3)

The fixed point represents the Ricci-scalar in GR for the same mass distribution. In this case $f_0$ is a cosmological constant and $f'_0 = 1$. Then, the field Eqs. (2) can be solved at different orders in terms of the Taylor expansion. In the Newtonian limit the first correction is of order $c^2$. The metric
tended to (2) a new characteristic scale length. The paradigm can be ex-
gravity is fourth-order and contains a larger number of de-
restored at Solar System scales so 
servation laws are fulfilled like in GR. Third, GR is totally 
identities hold for 
only asymptotically. This is not a problem since Bianchi 
resulting in some important implications for the theory. First 
Newtonian limit (see Quandt & Schmidt (1991) for details), 
be seen as alternative to Dark Matter. The Yukawa correc-
analytic one (KSZ) due to the motion of the cluster as a whole.
• The X-ray emitting region of clusters of galax-
ysis on the CMB of two different type by means of the
the electron within the cluster potential well and kinemat-
the electron temperature and the slope 
need to be deter-
To compute the TSZ anisotropy we need to specify the 
pressure profile $n_e T_e$ of clusters. Using X-ray data and 
umerical simulations, several cluster profiles have appeared in 
the literature:
• The X-ray emitting region of clusters of galax-
s is well fit by the isothermal $\beta$-model profiles
in the model, the 

$$f(R) = \frac{GM(r)}{r} = \frac{\Phi_N(r)}{(1 + \delta)} \left( 1 + \delta e^{-\frac{x}{a}} \right),$$

$$\Phi_N(r) = \frac{GM(r)}{r}, \quad \Phi_{grav}(r) = \frac{\Phi_N(r)}{(1 + \delta)} \left( 1 + \delta e^{-\frac{x}{a}} \right),$$

Analytic $f(R)$ models that modify the Newtonian limit can 
be seen as alternative to Dark Matter. The Yukawa correction 
to the gravitational field allows us to study the dynamics 
of galaxies without requiring dark matter. The parameters 
($\delta, L$) are related to the coefficients in the Taylor 
expansion as: $f_0 = 1 + \delta$ and $L = [f_0/(6f_0)]^{1/2}$, where 
$\delta$ represents the deviation from GR at zero or-
and $L$ the scale length of the self-gravitating object 
(Capozziello & De Laurentis 2011, 2012). In the limit $\delta = 0$, 
we recover the Newtonian limit of GR, irrespective of the 
scalar parameter $L$. In ETG, $L$ depends on the scale of the 
system considered; it assumes different values for the various 
self-gravitating systems like galaxies or cluster of galax-
ies while its effects are totally negligible at Solar System 

The physical meaning of the characteristic length 
$L$ deserves further discussion. As pointed out in 
Capozziello & De Laurentis (2011), $L$ can be seen as an extra 
gravitational radius similar to the Schwarzschild radius. 

When CMB photons cross the potential wells of clusters 
of galaxies, they are scattered off by the electrons of the Intra-Cluster medium, inducing secondary temperature anisotropies on the CMB of two different type by means of the 

$$\Delta T / T_0 = g(\nu) \frac{\kappa_B \sigma_T}{m_e c^2} \int n_e T_e dl,$$

where $T_e$ is the electron temperature, $n_e$ the electron density 
and the integration is carried out along the line of sight $l$. In 
Eq. (9) $\kappa_B$ is the Boltzmann constant, $m_e c^2$ the electron 
anihilation temperature, $\nu$ the speed of light, $\nu$ the frequency 
of observation, $\sigma_T$ Thomson cross section and $T_0$ the mean 
temperature of the CMB. Finally, $g(\nu) = x \coth(x/2) - 4$ 
is the frequency dependence of the TSZ effect, with $x = \nu / KT_0$.

3 CLUSTER PRESSURE PROFILES IN F(R) GRAVITY

| Model  | $c_{500}$ | $\alpha_a$ | $\beta_a$ | $\gamma_a$ | $P_0$ |
|--------|----------|------------|-----------|------------|-------|
| Arnaud | 1.177    | 1.051      | 5.4905    | 0.3081     | 8.403/3/2 |
| Planck | 1.81     | 1.33       | 4.13      | 0.31       | 6.41  |
| Sayers | 1.18     | 0.86       | 3.67      | 0.67       | 4.29  |

Table 1. Parameters of the Generalized NFW, $\beta$ and $f(R)$ models represented from Arnaud et al. (2011) and Sayers et al. (2013), the $\beta$-model data corresponds to the Coma cluster and the $f(R)$ profile data is the best fit model to Planck data (see Sec. 5).

More recently, Arnaud et al. (2011) proposed a phenomeno-
logical parametrization of the electron pressure profile based on generalized Navarro-Frenk-White (GNFW) profiles derived from the numerical simulations of \cite{Nagai+2007}. This profile has the following functional form

$$p(z) = \frac{P_0}{(c_{500} r)^\gamma [1 + (c_{500} r)^\alpha]^\beta / \alpha},$$

In this expression $z$ is the radial distance in units of $r_{500}$, the radius where the average density is 500 times the critical density, and $c_{500}$ is the concentration parameter at $r_{500}$. Different groups have fit the model parameters [$c_{500}, \alpha, \beta, \gamma, P_0$] to X-ray or CMB data; their best fit values are given in Table 1.

GNFW models fit the DM distribution in numerical simulations that use newtonian gravity and therefore can not be used to describe the dynamics in the ETG we are considering. Instead, baryons reside in the potential well of the Coma cluster. We assume that it follows a polytropic equation of state

$$P(r) \propto \rho^\gamma(r),$$

Eqs. (11) and (12) together with mass conservation

$$\frac{dM(r)}{dr} = 4\pi \rho(r),$$

and the cluster gravitational potential given by eq. 8 form a close system of equations that can be solved numerically to obtain the pressure profiles of any given cluster as a function of two gravitational parameter ($\delta, L$) and the polytropic index $\gamma$. For illustration, in Fig. 1 we plot the different profiles integrated along the line of sight with the parameters given in Table 1. We particularize the models for the Coma cluster. For convenience, all distances are written in units of $r_{500}$ and the angular scale is $\theta_{500} = r_{500}/d_A$ where $d_A$ is the angular diameter distance of Coma. Dashed, solid and dash-dotted lines correspond to GNFW profiles with the Arnaud et al. (2010), Planck Intermediate Results V (2013) and Sayers et al. (2013) parameters, respectively. The long-dashed line corresponds to the $\beta$ model and the red solid line to the $f(R)$ model.

4 DATA.

To constrain the ETG model described in Sec. 2, we will use the pressure profiles of clusters of galaxies given in Sec. 3. To that purpose we shall use Planck data and a proprietary cluster catalog.

4.1 The Cluster Catalog

Our cluster catalogue contains 579 clusters selected from ROSAT All Sky-Survey (RASS). Those clusters are outside the minimal Planck mask that removes a $\sim 20\%$ of the sky in the Plane of the Galaxy. Clusters are drawn from the three flux limited cluster samples: the extended Brightest Cluster Sample (eBCS, \cite{Ebeling+1998, Ebeling+2000}), the ROSAT-ESO Flux Limited X-ray catalog (REFLEX, \cite{Bohringer+2004}), and the Clusters in the Zone of Avoidance (CIZA, \cite{Ebeling+2002, Kocevski+2007}). For each cluster, the catalog lists position, flux, and luminosity measured directly from RASS data and spectroscopically measured redshifts. The X-ray electron temperature is derived from the $L_X - T_X$ relation of \cite{White+1997}. For each cluster, the spatial profile of the X-ray emitting gas is fit to a $\beta$-model convolved with the RASS pointspread function to the RASS data. Due to the poor sampling of the surface brightness profile for all but the most nearby clusters, $\beta$ is fixed to the canonical value of $\beta = 2/3$. For each cluster, the catalog lists position, flux, and luminosity measured directly from RASS data and spectroscopically measured redshifts. The X-ray electron temperature is derived from the $L_X - T_X$ relation of \cite{White+1997}. For each cluster, the spatial profile of the X-ray emitting gas is fit to a $\beta$-model convolved with the RASS pointspread function to the RASS data. Due to the poor sampling of the surface brightness profile for all but the most nearby clusters, $\beta$ is fixed to the canonical value of $\beta = 2/3$. For each cluster, the catalog lists position, flux, and luminosity measured directly from RASS data and spectroscopically measured redshifts. The X-ray electron temperature is derived from the $L_X - T_X$ relation of \cite{White+1997}. For each cluster, the spatial profile of the X-ray emitting gas is fit to a $\beta$-model convolved with the RASS pointspread function to the RASS data. Due to the poor sampling of the surface brightness profile for all but the most nearby clusters, $\beta$ is fixed to the canonical value of $\beta = 2/3$.

4.2 Cosmic Microwave Background data.

The release of WMAP 3yr data \cite{Bennet+2013} at the end of 2012 was followed by the first data release of the Planck satellite in April 2013. Nine maps spanning a frequency range from 32 to 845GHz have been made publicly available by the Planck Collaboration\footnote{http://irsa.ipac.caltech.edu/Missions/planck.html}. While the WMAP team provided foreground clean maps of all Differencing Assemblies (DA), the Planck Collaboration did not validate foreground clean maps at all frequencies. Instead, they used component separation methods to construct a map of CMB

![Figure 1.](image-url)
Constraining $f(R)$ gravity with PLANCK data on galaxy cluster profiles

To each data point we associate an error bar obtained by evaluating 1,000 times the average profiles at 579 random positions in the SMICA and NILC maps. To avoid overlapping real and simulated clusters, we excise a disc of 80′ around each cluster in our sample. The results on both maps are also indistinguishable. For comparison, we analyzed the W-band of WMAP 9yr data. The results were very similar to those of Planck except for larger error bars. As remarked in Planck Results XII [2013], at high latitudes, outside the Galactic Plane, the amplitude of the foregrounds residuals present on the SMICA map is a few μK, smaller than those on the NILC map. Therefore, since NILC or WMAP do not provide extra information and since they are more affected by noise or foregrounds than SMICA, we will restrict our analysis to the latter data.

4.3 The average SZ profile

To compare cluster profiles with observations, we measure the angle subtended by every cluster in units of $\theta_{500}$. For each cluster, the radial scale $r_{500}$ can be derived using the following scaling relation [Boehringer et al. 2007]:

$$r_{500} = \frac{0.753 h^{-1}\text{Mpc}}{h(z)} \times \left(\frac{L_{X}}{10^{44}h^{-2}\text{erg s}^{-1}}\right)^{0.228}.$$  

The radius $r_{500}$ will allow us to test if the characteristic scale of our ETG, $L$, depends on the cluster properties or not. We checked that our results did not depend on the uncertainties of eq. (14) and we will not consider them any further. Similarly, we did not consider other scaling relations based on different data [Piffaretti et al. 2011, Planck Early Results XI 2013]. Eqs. (4), (11), (12) and (13) allow us to compute the pressure profile of all clusters in the data as a function of three parameters: ($\delta, L, \gamma$). These profiles are integrated along the line of sight to be compared with those measured in the SMICA map. As indicated in Sec. 2, $L$ characterizes the dependence of $f(R)$ gravity on the size of the gravitating system. We consider two parameterizations of $L$ to test if the theory depends on the properties of the clusters: (A) $L = \zeta r_{500}$ is different for each cluster but depends homogeneously on $r_{500}$ for the whole sample and (B) where $L$ is the same for all clusters. In Fig. 3 we plot the pressure profile integrated along of line of sight, convolved with a gaussian beam of 5′ resolution, for different model parameters. Our models only predict the profile but not the central anisotropy. For this reason, we normalize all our theoretical profiles to unity. The data is equally normalized by dividing all the averages by the mean temperature on a disc of $0.1\theta_{500}$.

Error bars are computed in the same manner, renormalizing the disc and rings at random positions on the sky by the mean on the central disc of $0.1\theta_{500}$. In Figs. 4a–c L is different for each cluster (Model A) and in Figs. 4d–f $L$ is the same for all clusters (Model B). To avoid overcrowding the plots, we fixed $\gamma = 1.2$. In each panel we show the variation of the pressure profile with $L$. Notice that in Model B, when $L \geq 20\text{Mpc}$, the variations on the profile are small. This is logical since $L$ is the scale length of the Yukawa correction, that becomes negligible for large values of $L$. For illustrative purposes we overplot the SMICA data shown in Fig. 2 normalized to unity, with their corresponding error bars.

![Figure 2](image-url)
5 RESULTS AND DISCUSSION

To determine the model parameters that best fit the SMICA data we generate pressure profiles for different values of the parameters \((\delta, L, \gamma)\), integrated along the line of sight and convolved with a Gaussian beam with the same resolution of the SMICA map to compare them with the data. On physical grounds, we fix our parameter space to be \(\delta = [-0.99, 1.0]\) since if \(\delta < -1\) the potential is repulsive and diverges at \(\delta = -1\). In the parametrization \(L = \zeta r_{500}\) we take \(\zeta = [0.1, 4]\). When \(L\) is the same for all clusters, we fix the interval to be \(L = [0.1, 20]\) Mpc, from the scale of cluster core radius to the typical mean cluster separation scale. Finally, the polytropic equation of state parameter is set to vary within the range \(\gamma = [1.0, 1.6]\), that corresponds to an isothermal and adiabatic monoatomic gas, respectively. We take 30 equally spaced steps in all intervals.

In Figs. 4 and 5 we compute the confidence contours for the different model parameters of Model A and Model B, respectively. The likelihood function \(\log L = -\chi^2/2\) is computed as

\[
\chi^2(p) = \sum_{i,j=0}^{N}(y(p, x_i) - d(x_i))C_{ij}^{-1}(y(p, x_j) - d(x_j))
\]

where \(N = 7\) is the number of data points. The mean profile \(y(p, x_i)\) of all the clusters in our sample depends on three parameters: \(p = (\delta, L, \gamma)\). In eq. (15), \(d(x_i)\) is the SMICA average profile and \(C_{ij}\) is the correlation function between bins. To compute the correlation function we choose 579 random positions outside the locations of known clusters and compute the average temperature anisotropy on discs and rings of size \(\theta_{500}\), different for each of the random clusters. The process is repeated 1,000 times and \(C_{ij}\) is the average correlation between bins of any given cluster, averaged over all clusters and all simulations.

The value of the model A and B parameters that maximize the likelihood are given in Table 2. In Fig. 4 we plot the 68% and 95% confidence contours for pairs of parameters of Model A. Fig. 5 shows the same contours for the Model B. Since the models are very similar to each other, the likelihood function is flat close to the maximum. The 1σ contours are cut by our physical boundaries on \(\delta\) and \(\gamma\). Consequently, 2D contours of the marginalized likelihoods of pairs of parameters of these Figs are not closed, and only lower or upper limits to the parameters can be derived from their marginalize 1D likelihoods. At the 68% and 95% confidence levels those limits are \(\delta < -0.46, -0.10, \zeta < 2.5, 3.7\) and \(\gamma > 1.35, 1.12\) for the Model A parameters and \(\delta < -0.43, -0.08, L < 12, 19\) Mpc and \(\gamma > 1.45, 1.2\) for the Model B. In general, model parameters are weakly
Constraining $f(R)$ gravity with PLANCK data on galaxy cluster profiles

7

constrained. In particular, the polytropic index constraints dominated by the physical boundary on this parameter. The characteristic scale length $L$ is similar in both models, whether it scales with $r_{500}$ or is identical for all clusters. In retrospect, this explains why the results of model A did not depend on the uncertainties in the scaling relation of $r_{500}$, given in eq. (13). But even if the parameters are weakly constrained, let us remark that in both models, A and B, the value $\delta = 0$ is excluded at more than a 95% confidence level. Since $\delta \approx 0$ corresponds to the standard Newtonian potential without DM then the data does rule out that baryons alone are the dominant matter component in clusters.

The open contours in Figs. 4 and 5 reflect the physical limitations of our model. We can not extend our parameter space beyond $\delta = -1$. The limitation stems from the use of first order perturbations with respect to a background model. The contours show that at the $1\sigma$ level $L$ is compatible with zero. Physically, at $L \simeq 0$ the gravitational field corresponds to the Newtonian potential generated by a mass $M' = M/(1+\delta)$. As $\delta + 1 \simeq 0$ then $M' \gg M$ and the gravitational field is that of a system that contains a large fraction of DM distributed like the baryonic gas. Briefly, while our results show that cluster TSZ profiles in ETG are compatible with the data, they do not rule out that clusters could contain a significant fraction of DM. In summary, in order to fit the TSZ data, clusters are either dominated by DM or the Newtonian potential includes a Yukawa correction.

Comparison of Figs. 1 and 2 also shows that the data does not have enough statistical power to discriminate between Models A and B. Importantly, the results are consistent with those obtained by Sanders (1984) and Napolitano et al. (2012) using spiral and elliptical galaxies, respectively. In model A we find the same correlation between the parameter values $L$ and $\delta$ that in the case of galaxies: to accommodate the data, larger values of $L$ require lower values of $\delta$, while the behavior is the opposite in Model B. This different scaling suggests that Model A is in better agreement with the dynamics of galaxies than Model B. Also, conceptually is the preferable model since $L$ scales with the size of the self-gravitating system. The agreement of the central values of $\delta$ and $L$ with those of galaxies, that correspond to a different linear scale, is very reassuring; the dynamics of galaxies and clusters can be equally described by ETG, without requiring DM. In other words, DM and alternative gravity models are equivalent descriptions that could be discriminated only by some signature at fundamental scales, i.e. the discovery of new particles non-interacting at electromagnetic level, or the clear evidence of some new gravitational mode not related to GR (Capozziello & De Laurentis (2011), Bogdanos et al. (2011)).

For comparison, we also compute the likelihood of each of the models given in Table 1 and their $\chi^2$ per degree of freedom are given in Table 2. For the $\beta$ model we generate the profile of each cluster using the data of our catalog. The $\beta$ model does not produce a good fit to the data, in agreement with our previous results using WMAP data (Atrio-Barandela et al. 2008), since this model only fits the X-ray emitting regions of the inner parts of clusters. Comparing the three GNFW parameters, the Arnaud et al. (2010) parameters, derived using the X-ray data of 33 clusters, performs better that either the Planck Intermediate Results VI (2013) or the Sayers et al. (2013) parameters, that were obtained from TSZ observations. These discrepancies are not relevant since we did not explore the parameter space to find the best fit values of GNFW models to the SMICA data. Nevertheless, the fact that our $f(R)$ profiles fit significantly better than any other model is a clear indication that our assumption of a polytropic gas in hydrostatic equilibrium in the cluster potential well is supported by the data.

6 CONCLUSIONS

We have constructed cluster pressure profiles based on the Yukawa-like correction to the Newtonian potential obtained in the weak field approximation of $f(R)$ gravity. These models do not require large fractions of DM and they have been shown to describe well the dynamics of spiral and elliptical galaxies. By fitting the pressure profiles measured in the foreground clean SMICA map released by the Planck Collaboration, we have found that clusters can also be accurately described in these models. We have used a proprietary catalog of 579 clusters, and have determined the parameter space that best fits data. Our results are predicated on the baryonic gas being in hydrostatic equilibrium in the potential wells of clusters. This hypothesis can only be tested using hydrodynamical simulations and if the gas turn out not to be in equilibrium, our conclusion will be severely weaken.

Models based on $f(R)$-gravity that do not require DM halos appear as a viable alternative to generalized NFW models. Due to foreground contamination, we cannot use single frequency maps. For instance, the 217GHz channel could be used to remove the intrinsic CMB component and the signal at other frequencies could be fit to the profile of each individual clusters. Lacking frequency information increases our error bars and makes our final contours wider than what they would be otherwise. Then, the constraints from pressure profiles could be further improve by using frequency information, by carrying out the analysis in foreground clean maps, using the 217GHz map to remove the cosmological CMB signal and fitting the profile of each individual cluster to the data. The conclusion of this and similar studies Cardone & Capozziello (2011), Napolitano et al. (2012) is that large amounts of DM are not required to describe self-gravitating systems, if we relax the hypothesis that gravity is strictly scale independent above the scale of Solar System.

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Table 2. $\chi^2$ per degree of freedom ($\chi^2_{\text{dof}}$) for the $\beta$-model, GNFW models with parameters given in Table 1 and for the two $f(R)$-parametrization considered in this work.

![Figure 4. Confidence contours for pairs of parameters of Model A. Contours are at the 68% and 95% confidence level.](image)
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