Loop Calculations with Anomalous Gauge Boson Couplings

David London\textsuperscript{a} and C.P. Burgess\textsuperscript{b}

\textsuperscript{a} Laboratoire de Physique Nucléaire, Université de Montréal
C.P. 6128, Montréal, Québec, CANADA, H3C 3J7.

\textsuperscript{b} Physics Department, McGill University
3600 University St., Montréal, Québec, CANADA, H3A 2T8.

Abstract

Analyses which use loop calculations to put constraints on anomalous trilinear gauge boson couplings (TGC’s) often give bounds which are much too stringent. The reason has nothing to do with gauge invariance, in contrast to the recent claims of de Rújula et. al., since the lagrangians used in these calculations are gauge invariant, with the $SU(2)_L \times U(1)_Y$ symmetry nonlinearly realized. We trace the true cause of the problem to the improper interpretation of cutoffs in the calculation. The point is that the cutoff dependence of a loop integral does not necessarily reflect the true dependence on the heavy physics scale $M$. We illustrate that, if done carefully, one finds that the true constraints on anomalous TGC’s are much weaker.

\textsuperscript{†} Invited talk presented by David London at the Beyond the Standard Model III conference, Ottawa, Canada, June 1992
One of the most important tasks of LEP200 will be to directly measure the trilinear gauge boson couplings (TGC’s), thereby testing the gauge nature of the $W$- and $Z$-bosons. Of course it is hoped that these measurements will reveal the presence of new physics, either through deviations of TGC’s from their standard model predictions, or through the appearance of anomalous gauge boson vertices. Much work has been done in determining the precision with which TGC’s can be measured, both at LEP200 and at other facilities.

If one wishes to look at the physics of TGC’s, it is convenient to use an effective lagrangian. In this approach the effects of any unknown new heavy physics are parametrized through the nonrenormalizable interactions, such as anomalous TGC’s, induced among the light particles. In order to construct the low-energy effective lagrangian, one needs to specify only two things – the low-energy particle content and the symmetries of the lagrangian. Typically one has three possibilities:

1. Linearly Realized $SU(2)_L \times U(1)_Y$: In this formulation the low-energy particle content includes the standard model Higgs doublet.

2. Nonlinearly Realized $SU(2)_L \times U(1)_Y$: This framework is also known under the general rubric of “chiral lagrangians”. Here the unknown symmetry breaking sector at low-energy consists only of the three Nambu-Goldstone bosons which are eaten to produce the massive $W^\pm$ and $Z^0$ particles.

3. Only $U(1)_{em}$ Gauge Invariance: The only bosonic fields included here are the massive $W^\pm$ and $Z^0$ and the massless photon. The lagrangian is constrained to respect only Lorentz invariance and electromagnetic gauge invariance.

Most analyses in the past have used this third formulation in performing their calculations (although we will return to this point later). The TGC sector then has the following form for on-shell gauge bosons [1]:

$$
\mathcal{L}_{WWV} \sim ig_1^V (W^\dagger_{\mu \nu} W^{\mu \nu} - W^{\mu \nu} W^\dagger_{\mu \nu} V_{\nu}) + i\kappa_V W^\dagger_{\mu \nu} W^{\mu \nu} V_{\nu} \\
+ i\frac{\lambda_V}{M^2} W^\dagger_{\lambda \mu} W^{\mu \nu} V^{\nu \lambda} - g_4^Z W^\dagger_{\mu \nu} W_{\nu} (\partial_{\mu} Z^{\nu} + \partial^{\mu} Z^{\nu}) + \cdots
$$

where we have included only a subset of all possible TGC’s. Here, $V^\mu$ represents either the photon or the $Z$, $W^\mu$ is the $W^-$ field, and $W_{\mu \nu} = \partial_{\mu} W_{\nu} - \partial_{\nu} W_{\mu}$ (and similarly for $V_{\mu \nu}$).

Since we will have to wait several years before these TGC’s are measured directly, it is reasonable to ask if it is possible to put any constraints on anomalous TGC’s now, using current data. Clearly the effects of these new vertices will only appear at the loop level, and many authors have calculated the contributions of anomalous TGC’s to such loop-induced processes as the $W$- and $Z$-masses, $\Delta \rho$, $(g-2)_\mu$, and others [2].
A typical such calculation might go as follows. Consider the contribution of the above $g_4^Z$ term (which is called the anapole coupling) to the $\rho$-parameter. A convenient way of representing the effects of new physics on $\Delta \rho = \rho - 1$ is given in terms of the transverse piece of the gauge boson vacuum polarizations, $\pi^{\mu \nu}_T (q) = g^{\mu \nu} \pi(q^2) + ...$ [3]:

$$\frac{\delta \pi_{WW} (0)}{M_W^2} - \frac{\delta \pi_{ZZ} (0)}{M_Z^2} = \alpha (M_Z) T. \tag{2}$$

Present constraints on $\Delta \rho$ imply that $T$ satisfies roughly $|T| < 0.8$ [4]. Since the anapole is CP violating, there is a nonzero effect only when the anapole coupling appears at both vertices of the vacuum polarization diagrams, as in Fig. 1. Furthermore, since this coupling is nonrenormalizable, these diagrams will diverge and must be regularized in some way. The most common regularization method is to use a cutoff, $\Lambda$. Calculating the diagrams of Fig. 1 in this way, and keeping only the highest divergence in each case, one obtains

$$\delta \pi_{WW} (q^2) = - \frac{(g_4^Z)^2}{6\pi^2} \frac{\Lambda^6}{M_W^2 M_Z^2},$$

$$\delta \pi_{ZZ} (q^2) = - \frac{(g_4^Z)^2}{144\pi^2} \frac{\Lambda^4}{M_W^2} q^2. \tag{3}$$

If we assume, as is done in the literature, that $\Lambda$ actually represents the scale of new physics, say 1 TeV, then we can use Eq. (2) and the bound on $T$ to put an extremely stringent constraint on $g_4^Z$:

$$g_4^Z < 3.5 \times 10^{-4} \left( \frac{1 \text{ TeV}}{\Lambda} \right)^3. \tag{4}$$

Fig. 1: Contribution of anapole anomalous TGC (blob) to gauge boson propagators.

Recently de Rújula and coworkers [5] have made the claim that this type of analysis is wrong, that it yields constraints which are considerable overestimates. The cause of this, they say, is that the lagrangian in Eq. (1) is not gauge invariant under $SU(2)_L \times U(1)_Y$. 3
This is a red herring. In fact, the lagrangian in Eq. (1) is equivalent, term by term, to a chiral lagrangian in which \( SU(2)_L \times U(1)_Y \) is present, but nonlinearly realized. In other words, although we listed above three possibilities for a low-energy effective lagrangian, in fact two of them (the formulations with nonlinearly realized \( SU(2)_L \times U(1)_Y \) and with only \( U(1)_{em} \) gauge invariance) are equivalent. We will not prove this, but rather refer the reader to Ref. 6 for the details.

It is nevertheless true that results such as Eq. (4) overestimate considerably the effects of anomalous TGC’s in loops. The real reason has nothing to do with gauge invariance – rather it is the improper use of cutoffs in the calculations of such loop diagrams [7].

Suppose we knew the full theory at scale \( M \gg m \), where \( m \) represents the mass scale of some light particle, such as the \( W \) or \( Z \). And suppose we now calculate the contribution to a light particle mass, \( \delta \mu^2 \), as a function of these two mass scales. The answer will in general have the following form:

\[
\delta \mu^2(m, M) = aM^2 + bm^2 + c\frac{m^4}{M^2} + \cdots
\]  

in which the dots represent terms that are suppressed by more than two powers of \( m/M \). Notice that there are no terms of the form \( M^4/m^2 \). These are forbidden because only logarithmic infrared divergences are possible at zero temperature in four dimensions [8]. Note also that the dimensionless coefficients may depend logarithmically on the large mass ratio \( M/m \).

Now suppose that we split the calculation into a “high-energy” piece and a “low-energy” piece by choosing a cutoff \( \Lambda \) which satisfies \( M \gg \Lambda \gg m \). The contributions from the two pieces might have the following form:

\[
\delta \mu_{he}^2(m, \Lambda, M) = a'M^2 + b'\Lambda^2 + c'\frac{\Lambda^4}{m^2} + \cdots
\]

\[
\delta \mu_{le}^2(m, \Lambda, M) = b''\Lambda^2 + c''\frac{\Lambda^4}{m^2} + \cdots
\]  

Since the calculation with the cutoff just represents a reorganization of the full calculation, we must have

\[
\delta \mu^2(m, M) = \delta \mu_{he}^2(m, \Lambda, M) + \delta \mu_{le}^2(m, \Lambda, M),
\]  

and since the full calculation (Eq. (5)) is independent of \( \Lambda \), it follows that

\[
a = a' , \quad b' = -b'' , \quad c' = -c''.
\]
In other words, all quadratic and higher dependence on $\Lambda$ found in the low-energy calculation is simply cancelled by counterterms coming from the high-energy piece of the calculation! In this way one sees that there is no physical significance to terms containing the cutoff $\Lambda$. Therefore, due to the faulty interpretation of the significance of the cutoff, results such as that of Eq. (4) clearly overestimate the bounds which can be placed on anomalous TGC’s from loop calculations. We note in passing that the only term which depends strongly on the heavy mass scale, $a M^2$ in Eq. (5), cannot be calculated solely within the low-energy theory – the coefficient $b''$ of Eq. (6) is completely unrelated to the coefficient of $M^2$ in the full theory.

The point is that cutoffs do not, in general, accurately track the heavy mass contributions to low-energy processes. The one exception is in the case of a logarithmic divergence. Suppose there were a term in the full calculation of the form

$$\delta \mu^2 \sim d \, m^2 \log \left( \frac{M^2}{m^2} \right).$$

(9)

In this case the high-energy and low-energy contributions would have the form

$$\delta \mu^2_{\text{he}} \sim d' \, m^2 \log \left( \frac{M^2}{\Lambda^2} \right),$$

$$\delta \mu^2_{\text{le}} \sim d'' \, m^2 \log \left( \frac{\Lambda^2}{m^2} \right),$$

(10)

so that the cancellation of the $\Lambda$ dependence requires $d = d' = d''$. This is the only case in which the heavy mass dependence is accurately tracked by the cutoff.

Let us now return to our original example of the anapole contribution to $\Delta \rho$. Given that the naive use of a cutoff doesn’t yield accurate constraints, how does one get bounds which are physically significant? The easiest method is not to use cutoffs at all, but to use dimensional regularization and the decoupling subtraction renormalization scheme. Here, the divergent contributions to the $W$- and $Z$-propagators are

$$\delta \pi_{WW} \left( q^2 \right) \mid_{q^2=0} = - \frac{\left(g_Z^2\right)^2}{4\pi^2} \frac{3M_W^2}{2} \left[ 1 + \frac{M_Z^2}{M_W^2} - \frac{M_H^4}{M_W^4} \right] \left[ 1 + \frac{M_Z^2}{M_W^2} - \frac{M_H^4}{M_W^4} \right] \ln \left( \frac{\mu^2}{\mu'^2} \right).$$

(11)

where $\epsilon = n - 4$ in $n$ spacetime dimensions. These divergences renormalize the bare $T$ parameter, which is also present in the effective lagrangian:

$$\alpha T(\mu^2) = \alpha T \left( \mu'^2 \right) - \frac{3}{8\pi^2} \left(g_Z^2 \left( \mu'^2 \right) \right)^2 \left[ 1 + \frac{M_Z^2}{M_W^2} - \frac{M_H^4}{M_W^4} \right] \ln \left( \frac{\mu^2}{\mu'^2} \right).$$

(12)
This shows how the two operators $T$ and $g_4^Z$ mix when the lagrangian is renormalized and evolved down from scale $\mu'$ to scale $\mu$ (in the absence of mass thresholds). Note that, although there may be a strong quadratic dependence on the heavy mass scale $M$, this is completely contained in the incalculable initial condition $T(M^2)$.

The important point to realize here is that, even in an effective lagrangian, bare parameters must be renormalized. This seems to have been overlooked in most previous analyses. Since the effective lagrangian is nonrenormalizable, in general renormalization will introduce an infinite number of counterterms. However, this creates no problems, since the effective lagrangian already contains an infinite number of terms.

We can now use Eq. (12) to put a bound on $g_4^Z$. Taking $\mu' \approx 1 \text{ TeV}$, $\mu = M_W$, and assuming no accidental cancellations between the two terms, one finds

$$g_4^Z < 0.24.$$  \hspace{1cm} (13)

This is 3 orders of magnitude weaker than the bound found using cutoffs (Eq. (4))!

In summary, bounds on anomalous trilinear gauge boson couplings are often significantly overestimated. This is also true for predictions of large loop-induced effects due to new effective operators. These conclusions are unrelated to issues of gauge invariance. In fact, any lagrangian which obeys Lorentz invariance and electromagnetic gauge invariance is automatically $SU(2)_L \times U(1)_Y$ gauge invariant, with the $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$ symmetry breaking pattern nonlinearly realized. The real source of the problem is the incorrect use of cutoffs in estimating the effect of the heavy mass scale in the loops. In general, the cutoff dependence does not correctly reflect the true dependence on the heavy mass scale, $M$. (The only exception is in the case of a logarithmic divergence.) An easier way to do the analysis is to use dimensional regularization, supplemented by the decoupling subtraction renormalization scheme. This shows explicitly how operators mix as the effective lagrangian is renormalized. In addition, one sees that the logarithmic dependence on the heavy mass is simply due to the renormalization group evolution of the lagrangian down from the scale $M$ to low energies.

Acknowledgments

This work was supported in part by the Natural Sciences and Engineering Research Council of Canada, and by FCAR, Québec.
1. References

[1] K.J.F. Gaemers and G.J. Gounaris, *Zeit. Phys.* C1 (1979) 259; K. Hagiwara, R.D. Peccei, D. Zeppenfeld and K. Hikasa, *Nucl. Phys.* B282 (1987) 253.

[2] For references to the literature, see Refs. 5, 7.

[3] M.E. Peskin and T. Takeuchi, *Phys. Rev. Lett.* 65 (1990) 964; W.J. Marciano and J.L. Rosner, *Phys. Rev. Lett.* 65 (1990) 2963; D.C. Kennedy and P. Langacker, *Phys. Rev. Lett.* 65 (1990) 2967; *Phys. Rev.* D44 (1991) 1591.

[4] P. Langacker, U. Penn preprint UPR-0492T (1991).

[5] A. de Rújula, M.B. Gavela, P. Hernandez and E. Massó, CERN preprint CERN-Th.6272/91, 1991.

[6] M.S. Chanowitz, M. Golden and H. Georgi, *Phys. Rev.* D36 (1987) 1490; C.P. Burgess and David London, preprint McGill-92/04, UdeM-LPN-TH-83, 1992.

[7] C.P. Burgess and David London, preprint McGill-92/05, UdeM-LPN-TH-84, 1992.

[8] S. Weinberg, *Phys. Rev.* 140 (1965) B516; in *Asymptotic Realms Of Physics* (Cambridge, 1981).