The contribution to the decay $b \to s\gamma$ in the Minimal Supersymmetric Standard Model with radiative electroweak breaking is presented. The impact of a detection of this inclusive decay at the SM level is discussed. Results for other inclusive $b \to s$ transitions are also mentioned.

The data collected in the last few years at CESR and DORIS II have sensibly reduced the upper bounds on some rare B processes. In particular, the CLEO vertex detector allows to put a limit on the inclusive decay $b \to s\gamma$, thus circumventing the still sizeable theoretical uncertainty in the evaluation of the matrix element $\langle K^* | \bar{s}_L i \sigma^{\mu \nu} b_R | B \rangle$. The present limit on $BR(b \to s\gamma)$ of $8.4 \times 10^{-4}$, only about a factor of two away from the Standard Model (SM) prediction for $m_t \simeq 130\text{ GeV}$, makes therefore the detection of this process quite imminent.

It comes then natural to ask oneself where the past claims of sensitivity of rare B processes to new physics stand today. The first point to be inquired is whether these claims have survived the steady increase in the experimental lower bounds for the mass of particles predicted by models beyond the standard one. In case they have, then, the other question to be addressed is what type of restriction on the allowed parameter-space of these models can be enforced by a detection of $b \to s\gamma$ at the SM level. The aim of this talk is to answer these two questions in the case of Supersymmetry (SUSY). In this framework, a possible enhancement with respect to the SM predictions had been claimed in the past for the inclusive radiative decays $b \to s\gamma$ and $b \to sg$, while no sizeable deviation from the SM level had been considered possible for other flavour changing processes as $b \to sq\bar{q}$, $b \to s\ell^+\ell^-$, $b \to s\nu\bar{\nu}$, and for the oscillations $B_d^0 - \bar{B}_d^0$, $B_s^0 - \bar{B}_s^0$.

The reasoning behind these claims can be quickly explained. Before doing so, I shall list here the effective operators relevant for the previous $b \to s$ transitions and briefly recall the impact of QCD corrections on the relative Wilson coefficients. The two-quarks effective operators contributing to the radiative decays $b \to s\gamma$, $b \to sg$ are:

$$
\mathcal{O}_{LR}^{ph} \equiv (\bar{s}_L i \sigma^{\mu \nu} b_R) m_b q_{\nu} \epsilon_{\mu} \quad \mathcal{O}_{LR}^{gl} \equiv (\bar{s}_L i \sigma^{\mu \nu} T^a b_R) m_b q_{\nu} \epsilon_{\mu}^a
$$

(1)

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where $\sigma^{\mu\nu} \equiv (i/2)[\gamma^\mu, \gamma^\nu]$; $\epsilon_\mu$ and $\epsilon^a_\mu$ are photon and gluon fields, respectively; $T^a$ is a generator of $SU(3)$; $q_\nu$ is the momentum of the emitted gauge boson and finally the subindices $L,R$ indicate the left- or right-handedness of each fermion. Similarly, the four-quarks effective operators contributing to the remaining set of processes mentioned before can be divided in the groups:

\begin{align}
\mathcal{O}_{(LL)V} &\equiv (\bar{s}_L\gamma^\mu T b_L) \left( \bar{f} \gamma_\mu T f \right) \\
\mathcal{O}_{(LR)V} &\equiv (\bar{s}_L i\sigma^{\mu\nu} T b_R) m_b \frac{q_\nu}{q^2} \left( \bar{f} \gamma_\mu T f \right) \\
\mathcal{O}_{(LL)(LL)} &\equiv (\bar{s}_L\gamma^\mu T b_L) \left( \bar{f}_L \gamma_\mu T f_L \right) \\
\mathcal{O}_{(LL)(RR)} &\equiv (\bar{s}_L\gamma^\mu T b_L) \left( \bar{f}_R \gamma_\mu T f_R \right)
\end{align}

where $f$ is a generic fermion $q, \ell, \nu$ etc.; the matrix $T$ (which can be the identity matrix $1$ or a generator $T^a$) accounts for a possible colour structure of the two currents in each operator, and the subindex $V$ is used to denote a vector-current. Subleading operators proportional to the $s$-quark mass are neglected and all the coupling constants are also omitted, for simplicity. Penguin diagrams with exchange of massless bosons, i.e. photons and gluons ($q_\nu$ is the momentum of the exchanged boson), contribute to the operators $\mathcal{O}_{(LL)V}$ and $\mathcal{O}_{(LR)V}$, while $Z$-mediated penguins contribute to $\mathcal{O}_{(LL)V}$ and $\mathcal{O}_{(LL)(LL)}$. Box diagrams account for $\mathcal{O}_{(LL)(LL)}$ and $\mathcal{O}_{(LL)(RR)}$. The last operator, as well as $\mathcal{O}_{(LL)(LL)}$ with $T = T^a$, not present in the SM, arise in the supersymmetric case from box diagrams with virtual Majorana fermions (gluinos $\tilde{g}$) running in the loop.

The special role played by the operator $\mathcal{O}_{(LR)^{pl,gl}}$ and $\mathcal{O}_{(LR)V}$ within the SM, was quickly realized as soon as interest started gathering around these flavour changing processes. It was found that, due to mixings with the operators $(\bar{c}_L\gamma^\mu b_L)$ $(\bar{s}_L\gamma_\mu c_L)$, the Wilson coefficients relative to $\mathcal{O}_{(LR)^{pl,gl}}$ and $\mathcal{O}_{(LR)V}$ undergo a substantial strong correction during the evolution from $M_W$ to $m_b$ (for a discussion of these issues see [2] and references therein). These coefficients, with a typical implementation of the GIM mechanism of power-type, $\sim m_f^2/M_W^2$, roughly speaking, acquire additive logarithmic terms $\sim \ln(m_f^2/m_b^2)$. The effect is a significant change of their value on one side, and a reduction of their sensitivity to the top quark mass, $m_t$, on the other. Indeed, an enhancement bigger than 100% can be found for the decay $b \to s \gamma$, while destructive interferences lead to a suppression of $b \to s g$.

The situation is more complicated for $b \to s \ell^+ \ell^-$. The strong enhancement relative to $\mathcal{O}_{(LR)V}$ makes the contribution of this operator to $\Gamma(b \to s \ell^+ \ell^-)$ roughly of the same size as the contributions due to $\mathcal{O}_{(LL)V}$ and $\mathcal{O}_{(LL)(LL)}$. The QCD correction to the photon-mediated penguin contributing to $\mathcal{O}_{(LL)V}$, already exhibiting a logarithmic GIM suppression, do not have the same impact they had in the case of $\mathcal{O}_{(LR)V}$. Finally, $Z$-mediated penguins and box diagrams, with a power-type implementation of the GIM mechanism, do not obtain strong corrections. The net result is a modest enhancement of the inclusive decay $b \to s \ell^+ \ell^-$, never exceeding 20%. No correction is present in $b \to s \nu \bar{\nu}$ and related decay modes as $B_s \to \tau^+ \tau^-$. Similar to $b \to s \ell^+ \ell^-$ is the case of the transition $b \to s \tilde{g} \tilde{g}$. Although potentially interesting since it can give together with $b \to s g g$ (and $b \to s g$) non-trivial rates to hadronic decays of the $B$ meson, as $B \to K_S \phi$, I shall neglect
it in the following discussion. A re-analysis of the supersymmetric contribution to the non-charmed hadronic decay $b \to sq\bar{q}$, in fact, has not been performed; it exists only for $b \to sg$. Finally, suppression factors around 15% are obtained for the mixings $B^0 - \bar{B}^0$.

The reason for dwelling so long upon the type of modifications that QCD corrections bring to each operator, is that one can draw interesting analogies with the corrections due to the exchange of supersymmetric particles in the internal loops. Again, $O^{\text{ph}, \text{gl}}_{\text{LR}}$ and $O^{(\text{LR})V}_{\text{LR}}$ are, in principle, the likely operators to induce a sizeable enhancement of supersymmetric signals over the SM ones. The explanation is quite simple. For long time, the effective tree-level coupling $\bar{g} - q - \bar{q}'$, with quarks and squarks, $q, \bar{q}'$, of different flavours, was considered the key ingredient to induce flavour change in quark transitions. In this case, in fact, the presence of the strong coupling $\alpha_S$ versus the weak one, typical of the SM, produces a net gain for SUSY if all the other ingredients entering in the loop calculation, contribute numerically in the same way in the two frameworks. Even assuming that the generational splitting among squarks is of the same size as the one among the quarks running in the SM loop, the higher average squark mass gives a stronger squark degeneracy and, numerically, a supersymmetric GIM suppression at least as effective as the power-type suppression. Therefore, the correct strategy is to consider only processes involving operators which already have a strong GIM suppression in the SM. Since Z-mediated penguins and box diagrams can be excluded as unlikely possibilities, see [4], the only viable operators one is left with are $O^{\text{ph}, \text{gl}}_{\text{LR}}$ and $O^{(\text{LR})V}_{\text{LR}}$. The expectations are then shaped on the results obtained in the case of the strong corrections: while a possible enhancement of the supersymmetric contribution of $O^{(\text{LR})V}_{\text{LR}}$ to $b \to s\ell^+\ell^-$ would be diluted by the contributions coming from different operators, the radiative processes $b \to s\gamma, b \to sg$ can give, in principle, a clear signature of SUSY above the SM. These are the conclusions reached in [3] for squarks and gluinos masses taken at the weak scale and when the abovementioned splitting among squarks of different generations is assumed.

However, it is clear that the increase of the experimental lower bounds of supersymmetric particles does not allow to single out the effective vertex $\bar{g} - d - \bar{d}'$ as the dominant source of flavour change and that all the possible sources contributing to $b - s$ transitions have to be considered. They are distinguishable according to the virtual particles running in the loop mediating the $b - s$ transition: 1) $W^-$-boson and $u$-quarks; 2) charged Higgs $H^-$ and $u$-quarks; 3) charginos $\tilde{\chi}^-$ and $u$-squarks; 4) neutralinos $\tilde{\chi}^0$ and $d$-squarks; 5) gluinos $\tilde{g}$ and $d$-squarks.

The calculation of all these contributions and their possible interference effects can be reasonably performed within the minimal supersymmetric standard model with radiative breaking of the electroweak sector. Apart from $m_t$ and $\tan \beta$ (the ratio of vacuum expectation values giving rise to the $u$- and $d$-quark masses), only two new parameters are needed to fully specify this model. We choose them to be $m$ and $M$, the common masses which all scalars and gauge fermions acquire respectively at the Planck scale, after the soft breaking of supersymmetry. Once the value of these parameters is fixed, the correct boundary conditions at a grand unified scale are kept in account, and the low-energy input values for $\alpha, \alpha_S, \sin^2 \theta_W$ and $m_b$ are considered, all mass parameters and couplings present in the model can be calculated. They are obtained by integrating the relative renormalization group equations and by requiring that the scalar potential acquires at
the electroweak scale the minimum needed for the correct breaking of $SU(2)_L \times U(1)_Y$. The details of the procedure followed for this analysis can be found in [4, 5].

It turns out that the constraint of radiative breaking of $SU(2)_L \times U(1)_Y$ is rather stringent. Interesting relations among the supersymmetric masses are obtained, depending on the particular values of $m_t$, $\tan \beta$, $m$ and $M$. One feature, though, is quite general and has strong consequences on the size of the various contributions to the $b - s$ transitions discussed here: the lightest $d$-squark, $\tilde{d}_1$, is in general heavier than the lightest $u$-squark, $\tilde{u}_1$. Moreover, the implementation of a modest lower bound on $m_{\tilde{u}_1}$ ($25 - 30$ GeV) can push $m_{\tilde{d}_1}$ towards much higher values. This fact, together with the requirement that $m_{\tilde{g}}$ is above $100 - 150$ GeV, strongly suppresses the size of the gluino contribution to $b \rightarrow s\gamma$. The neutralino contribution, further penalized by the smallness of its coupling, is then completely negligible. The two biggest supersymmetric contributions come from the exchange of $H^-$ and $u$-quarks and of $\tilde{\chi}^-$ and $u$-squarks. The elements playing an important role are obviously the presence of the top-Yukawa-coupling in the first case and the contribution of the lightest squark $\tilde{u}_1$ in the second one. However, both contributions are below the SM prediction: at most $60\%$ of the contribution coming from the $W^-$ and $u$-quarks exchange can be obtained by the exchange of $H^-$ and $u$-squarks for $m_t = 130$ GeV and $\tan \beta = 2$. These contributions add constructively to give a band of supersymmetric results almost completely above the SM prediction, as shown in Fig. 1 for $\tan \beta = 2, 8$ and $m_t = 130$ GeV. The width of this band is due to the dependence on the remaining supersymmetric parameter $M$. QCD corrections are also implemented in these results (for details see [1]). An enhancement of a factor $2 - 3$, is obtained in the case of $b \rightarrow s\gamma$.

As expected, $b \rightarrow s\nu\bar{\nu}$ and $B_s \rightarrow \tau\bar{\tau}$ do not show any deviation from the SM prediction and the same is true for the oscillations $B^0 - \bar{B}^0$ [8]. Interesting is instead the result obtained for $b \rightarrow s\ell^+\ell^-$. The shape of the supersymmetric band of values for $BR(b \rightarrow s\ell^+\ell^-)$, similar to the one for $BR(b \rightarrow s\gamma)$, is a clear indication that the enhancement observed in this case is due to the operator $O_{(LR)V}$. This enhancement, though, would not be visible if the QCD corrections to this operators had not increased its contribution to $BR(b \rightarrow s\ell^+\ell^-)$ and if destructive interferences among the various components contributing to the remaining operators had not occurred. Although the enhancement factor over the SM prediction is similar to the one obtained for $b \rightarrow s\gamma$, this decay is much less interesting at the moment since it is still quite far from experimental detection.

**Fig. 1** Branching Ratios of various loop induced $b - s$ transitions in the Minimal Supersymmetric Standard Model for $m_t = 130$ GeV and $\tan \beta = 2, 8$, as a function of the soft breaking scalar mass $m$. The horizontal lines correspond to the SM predictions ($f_{B_s} = 150$ MeV). From ref. [4].
As for $b \to s\gamma$, the enhancement obtained in the framework of Minimal Supersymmetry with radiatively induced breaking of $SU(2)_L \times U(1)_Y$ is smaller than the values claimed in the past. However, even this reduced enhancement makes $b \to s\gamma$ sensitive to region in the supersymmetric parameter space not yet excluded by collider searches. As can be seen in Fig. 2, the experimental detection of $BR(b \to s\gamma)$ at 1.5 times the SM prediction for $m_t = 130$ GeV and $\tan \beta$ can induce restrictions of the parameter space competitive with the ones due to negative results in searches of supersymmetric particles at LEP I and LEP II.

Fig. 2 Comparison between the regions of the plane $m_{\tilde{g}} - \mu$ excluded at LEP I (area enclosed by the thick solid line), the projected limits obtainable at LEP II (area below the dashed line) and the regions excluded by: i) the requirement of radiative electroweak breaking (A); ii) a bound on $BR(b \to s\gamma)$ 50% above the SM prediction (A+B); iii) 15% above the SM prediction (A+B+C). From ref. [6]

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