Investigation on Maximum Electromagnetic Torque of Permanent-Magnet Synchronous Machines

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ABSTRACT Many application fields need motors that could output large torque in a short span of time such as missile and aircraft, but larger torque needs larger volume that could influence the system performance. Therefore, promoting the motor’s maximum torque with small volume is the new challenge for motors. So, this paper presents a method for analyzing maximum electromechanical torque (with rate speed) of PMSM without the consideration of thermal effect produced by copper loss. This paper also discusses the factors that influence maximum torque of PMSM under the condition of \( i_d = 0 \) control mode. A function between motor torque and the inner parameters is first obtained under the constraint of electrical and magnetic saturation. For verifying the accuracy of the function and its results, the characteristics of the series motors with different inner parameters that are obtained in the function are testified by finite element method (FEM) software. Finally, a prototype motor is selected to be manufactured to verify the analysis result. Based on the torque experiments result, it could verify the accuracy of the function that describes the rule of PMSM maximum torque. Based on FEM software and prototype test verifying, the method could supply the theoretical support for PMSM maximum torque design.

INDEX TERMS Permanent magnet machines, finite element analysis, numerical analysis, torque.

I. INTRODUCTION

In the field of missile and aerospace, the motor is not required to have a long lifetime, and is only used for a short period. At the same time, the fields that require motors should have high power density and high torque density for high dynamic characteristic [1], [13]. Therefore, such application fields require an electrical actuator output maximum torque under the condition of rated speed, so they could reach the highest dynamic response ability. The traditional motor theory adopts a large volume and mass to ensure large torque, however, large volume brings large inertia, and high torque density and high dynamic response level cannot be reached by Permanent-Magnet Synchronous Machines (PMSM). Thus, the motor with small volume and large torque output characteristic is required urgently.

With the development of magnetic material, high power density and high torque density motors will be an increasing trend for PMSM [3], [4]. Most motor researchers seek a method to improve the motor’s power density and torque density [1], [4], [5]. The motor’s maximum torque characteristic in low speed is studied by many researchers [6]–[8], and the PMSM motor’s maximum torque characteristic under a rated speed is researched less [9]. Hence, under the conditions of a rated constant speed, the PMSM maximum torque characteristic lacks direct theoretical formulation against the motor’s inner parameters.

The factors that influence the PMSM maximum torque characteristics could be divided into two kinds. First is that the electromagnetic torque of a motor is fundamentally determined by its magnetic and electric loads. The magnetic load could be equivalent to the air-gap flux density, which influences the stator and rotor iron material’s saturation point, and the electric load could be equivalent to the armature ampereturn number, which is limited by the outer voltage and control model.

The other kind is the temperature of motor components, and the motor important component is the armature winding.
When motors run in long-time duty, the temperature of motor components is the main restrictions for safety [10]–[12]. The temperature of motor components should not increase when motors run in short-time duty because the thermal conduction needs more time.

Consequently, this paper analyzes the factors that influence PMSM maximum torque with short-time duty. And the short time is 10 ms.

On the other hand, with the development of electronic technology, the vector controller would be a better choice for PMSM [13]. Under the condition of the vector controller, the output torque characteristics of PMSM are different from the PMSM without controllers because its output torque characteristics are influenced by the controller [14]. With the controller, the relationship between the input voltage and the inner parameters is not clear. Based on the motor back EMF, and direct and quadrature axis reactance are the functions of coil turn.

The vector diagram of PMSM based on the voltage function is shown in Fig. 2.

![FIGURE 2. PMSM running vector diagram.](image)

Therefore, the power of PMSM is shown as (2)

\[
P_{em} = \frac{mU[E_0(X_d \sin \theta - R_1 \cos \theta) + R_1 U + 0.5U(X_d - X_q) \sin 2\theta]}{R_1^2 + X_dX_q}
\]

(2)

It can be seen from (2) that the output power is limited by the direct and quadrature axis reactance under the condition of rate speed.

When neglecting stator resistance, the power function can be shown as follows:

\[
P_{em} = \frac{mE_0U}{X_d} \sin \theta + \frac{mU^2}{2} \left( \frac{1}{X_q} - \frac{1}{X_d} \right) \sin 2\theta
\]

(3)

It can be seen that the output power is in inverse proportion to the direct axis reactance under the condition of neglected resistance. However, the relationship between output power and the inner parameters is not clear. Based on the motor theory, the function between EMF and coil turn is a linear function, and the function between direct and quadrature axis reactance and coil turn is a quadratic function. In (3), the motor back EMF, and direct and quadrature axis reactance are the functions of coil turn.

When the motor runs with maximum load, the stator resistance is a non-negligible parameter because the resistance voltage-loss would be large. So, (2) and (3) cannot describe the relationship between the maximum power and the motor’s inner parameters.

A. DRIVE CONTROL MODE

From Fig. 1, maximum power of PMSM is influenced not only by the motor’s inner parameters, but also by the control method. With the development of motor vector control technology, the motor’s running characteristics are limited by the control algorithm and method. The vector control method is adopted to control the current vector phase and amplitude. It would control the motor output characteristics when the motor’s excitation flux and armature reactance are ensured. The current vector can be divided into two current
scalars, $i_d$, $i_q$, which control the motor characteristics so the electromagnetic torque could be described as shown in (4).

$$T_{em} = p \left[ \psi_f i_q + (L_d - L_q)i_d i_q \right] \tag{4}$$

Based on the application, the traditional control method could include the $i_d = 0$ control method, the maximum torque/current control method, the $\cos \phi = 1$ control method, and the weaken magnetic field control method. The $i_d = 0$ control method is adopted by most PMSMs, as it can control the direct current constant of zero, so that there is only quadrature current output for the power and torque. In this paper, the function between the ultimate power and inner parameters is analyzed with the $i_d = 0$ control mode for sampling model.

**B. ULTIMATE POWER ANALYSIS OF PMSM**

Under the condition of $i_d = 0$ mode, the vector diagram of PMSM is shown in Fig. 3.

![PMSM running vector diagram with id = 0 control mode.](image)

In equations (2) and (3), the variables are also the functions of motor inner parameters, hence, this output power cannot describe the relationship between the motor property and the inner parameters. Therefore, this paper presents a novel analysis method to describe the impact factors that are limited for the considerations of maximum power running.

When the PMSM is running in the maximum power condition, its electrical and structural parameters are limited by (5) and (6):

$$U^2 = (E_0^2 + I \cdot R)^2 + (I \cdot X_q)^2 \tag{5}$$

$$(t_s + b_s)t_s \cdot B_0 + B_{Feom} = B_{th} \tag{6}$$

Equation (5) is the limited condition of electrical saturation, which is obtained through the motor vector diagram. It describes the maximum input current ability with control mode $i_d = 0$. The equation (6) is the limited condition of magnetic saturation, which describes that the flux density of tooth cannot go over the material magnetic saturation point.

When the PMSM output torque reaches maximum, there are two restrictive conditions: one is the iron magnetic saturation, which is composed of air-gap magnetic field and stator armature current; the other is the controller’s electrical saturation, particularly, the controller output current achieves saturation point in condition of special voltage and back electromotive force. Therefore, when the motor output reaches maximum torque, the magnetic circuit and electrical circuit should achieve saturation point simultaneously. The reason is that when stator iron magnetic field achieves saturation point, the motor controller could output more current into the armature winding, but the current cannot transform the electrical energy to the motor’s torque because of stator iron saturation. Furthermore, when the motor reaches electrical saturation, because of the motor voltage-current characteristic limitations, the motor controller cannot output current into armature winding, but the stator iron magnetic flux density does not achieve saturation point at the same time, then it would lead to the waste of motor volume and material. Therefore, when PMSM output maximum torque, the armature current, which motor controller output into motor, should achieve maximum value, and the stator iron magnetic flux density should achieve saturation point at the same time.

Subsequently, through the magnetic saturation condition (6) and electrical saturation condition (5), the function between the maximum armature winding current and motor inner parameters could be resolved.

For analyzing the relationship of electromagnetic parameters, these are the assumptions:

1) The motor runs under the saturation point of ferromagnetic material, and its magnetic pressure drop is kept constant but on the saturation point of ferromagnetic material, the motor’s performance reaches the ultimate condition.

2) Under the condition of ultimate torque, the impacts of thermal capacity and thermal transformation are not considered.

3) Without the consideration of couple between the direct axis and quadrature axis, there is a proportional relation between the direct axis, quadrature axis reactance and square of coil turns.

4) Without the consideration of thermal effect produced by copper loss, the method is analyzed for ultimate torque in unit volume.

Therefore, the flux density of tooth that produced by armature winding can be obtained as shown in (7). The detailed derivation is shown in Appendix.

$$B_{Feom} = \frac{3\sqrt{2}}{\pi} \frac{\mu_0 N k_{NM} I}{2(\delta + h_{IM})p} \tag{7}$$

$$k_E = 4.44 \cdot f \cdot S \tag{8}$$

$$k_f = \frac{3\sqrt{2}}{\pi} \frac{\mu_0 k_{NM}}{2(\delta + h_{IM})p} \tag{9}$$

$$k_q = 2\pi \cdot f \cdot \delta_q \tag{10}$$

$$k_R = \frac{4pL_d}{a\pi d^2} \tag{11}$$

$$k_{th} = \frac{L_s}{t_s + b_s} \tag{12}$$

Therefore, some of the motor’s parameters are rewritten as the following:

$$\begin{align*}
E_0 &= k_E \cdot N \cdot B_0 \\
B_{Feom} &= k_f \cdot N \cdot I \\
X_q &= k_q \cdot N^2 \\
R &= k_R \cdot N
\end{align*} \tag{13}$$
Using (6), (9) - (13), it can be obtained that:

\[
B_0 = t_s \left( B_{th} - \frac{3\sqrt{2}}{2} \frac{\mu_0 k_{NM}}{q} N \cdot I \right)
\]

Then, applying all the coefficients and (14) into (5), the function of the current is obtained as shown in (15).

\[
\begin{cases}
I^2 \cdot a + I \cdot b + c = 0 \\
a = (k_E k_F k_{th} \cdot N - k_R)^2 + N^2 k_q^2 \\
b = 2k_E k_F k_{th} B_{th}(1 - k_E k_{th} \cdot N) \\
c = k_E^2 k_{th}^2 B_{th}^2 - U^2/N^2
\end{cases}
\]

From (15), the condition that the equation has a solution is shown as follows:

\[U^2 > k_E^2 k_{th}^2 B_{th}^2 \cdot N^2\] (16)

The physical meaning of (16) is that the back EMF produced by the air flux density should not exceed the input voltage. When the motor runs in drive mode, the input voltage always exceeds the back EMF, so (16) always has a solution, and the current solution should only be a positive value.

So, the motor’s maximum current function is obtained as:

\[
I = \frac{k_E k_{th} B_{th}}{a} \sqrt{d^2 - a[1 - e]}
\]

\[
d = k_F (1 - k_E k_{th} \cdot N) \\
e = U^2/(k_E^2 k_{th}^2 B_{th}^2 \cdot N^2)
\]

Applying (17) into (7), it can be obtained that:

\[B_{Feom} = \frac{k_E k_{th} B_{th} \cdot N}{a} \sqrt{d^2 - a(1 - e)}\] (18)

where, the value of coil turns \(N\) should be an integer and limited by the mechanical condition as shown in (19)

\[N \leq \frac{4b_s h_s f_N}{N_2 \pi d^2}\] (19)

Therefore, PMSM’s maximum input power could be calculated by (20) as shown:

\[P_{in} = mUI \cos \theta = mEI + mI^2 R\] (20)

where, \(mEI\) is the electromagnetic power, and \(mI^2 R\) is the winding copper loss.

Thus, the equation (20) could be adopted to calculate the motor maximum power when the winding phase current reaches maximum value.

### III. INFLUENCE RULE BETWEEN PMSM ULTIMATE POWER AND COIL TURNS

To verify the theory mentioned above, a six-pole, nine-slot PMSM was analyzed. This motor’s parameters are shown in Table 1.

From the data above, the parameters in (16) and (17) can be calculated using Matlab software. In these equations, the variable is the coil turn \(N\), and the variable \(N\) is restrained by (19). Based on the (18), the slot width \(b_s\) is adopted as the global variable, then the coil turn could be calculated by (19). The calculation program flow chart is shown in Fig. 4.

The relationship between the output power of PMSM and the number of coil turns is shown in Fig. 5.
could reach 216 kW, and at this point, the copper loss reaches maximum.

However, this condition cannot be reached because under the condition of low turns, the air flux density produced by rotor permanent magnets cannot reach the required value. It will lead that the stator tooth is not saturated and has low torque output. The relation between the coil turn, the armature current, and air-gap flux density is shown in Fig. 6 and Fig. 7. At the same time, the winding copper loss is also large, as shown in Fig. 5. With one coil turn, the copper loss reaches 2.67 MW, with input power of 2.89 MW and output power of 216 kW.

From Fig. 6 and Fig. 7, it can be seen that under the condition of a low number of turns, the air flux density is large because the parameters $k_{th}$ is large, which allows higher air flux density to enter stator tooth without magnetic saturation. For traditional PMSM, the remnant flux density $B_r$ of the permanent magnets located in the rotor is limited, and thus cannot reach the required air flux density when the number of turns is low. From Fig. 5, when the number of turn is one, the air flux density after an equivalent square wave is 1.3 T.

Therefore, when the coil turn is one, the motor could output large torque, but this maximum torque is restricted to air-gap flux density and stator winding temperature, the design is not adopted in engineering application.

With the increasing of turns, under the condition of stator magnetic saturation, the air flux density produced by the rotor appears to rise, and then reduce. Because the maximum value of air flux density produced by rotor is 1.0T, and the waveform of air flux density is sinuous waveform, the square waveform amplitude of the air flux density is converted to 0.7 T. So, the number of coil turns is selected as 14 as seen from Fig. 3, therefore, the input current is 150 A.

The motor’s parameters with six-poles and nine-slots are shown in Table 2.

TABLE 2. Structure parameters of PMSM with six poles and nine slots.

| Item                                      | Value       |
|-------------------------------------------|-------------|
| Coil turn number                          | 14          |
| Diameter of copper wire                   | 0.49 mm     |
| Number of conductors in group             | 18          |
| Tooth width                               | 11 mm       |
| Maximum current                           | 150 A       |
| Maximum power                             | 15000 W     |
| Back EMF (eff)                            | 33.35 V     |
| Maximum current density                   | 44.21 A/mm² |
| Working time with maximum torque          | 10 ms       |
| Maximum copper loss                       | 3.2 kW      |
| Motor Mass $^*$                           | 4.47 kg     |
| Motor torque density                      | 8.02 Nm/kg  |
| Maximum torque                            | 35.81 Nm    |

$^*$ motor mass should contained stator iron, copper wire, insulating materials, shaft, magnets, motor frame, end covers, position sensor and aviation plug.

IV. FEM ANALYSIS AND TEST OF PMSM ULTIMATE POWER

A. FEM ANALYSIS OF PROTOTYPE MOTOR

For verifying the method analyzed the maximum power, the prototype motor with 14 turns is simulated.

The motor model is simulated by Cedrat Flux based on the data mentioned above, and the model is shown in Fig. 8.

Based on FEM analysis, the simulation result without load is shown in Fig. 8. It can be seen from Fig. 9 that the back EMF amplitude is 50 V, the effective value is 35.3 V, and the air flux density amplitude value without load is 1.1 T.

EMF amplitude is 50 V, the effective value is 35.3 V, and the air flux density amplitude value without load is 1.1 T.

The maximum performance was simulated in Fig. 11. It can be seen that when the output torque is 35.39 Nm, the phase back EMF effective value is 55 V. At the same time, the tooth flux density is 1.85 T, and the yoke flux density is 1.78T. The tooth and yoke flux densities show that they reach the
saturation point of ferromagnetic material, and the motor’s output performance reaches its maximum.

Under the condition when current density is 50 A/mm², and the tooth and yoke flux densities reach saturation point, the back EMF value cannot reach 65 V. Accordingly, the current is increased to 150A, and the electromagnetic torque increasing tendency begins to slow down. Fig. 13 shows the current–back EMF curve and Fig. 14 shows the current-torque curve.

From Fig. 14, it can be seen that the motor’s ultimate torque is 35.39 Nm when the current is 150 A. At this moment, the electrical saturation limited condition cannot be reached, but the magnetic saturation limited condition can be reached. In these conditions, the difference between the maximum torque calculated by FEM and by analysis is low.

Therefore, the simulation calculated by FEM is not the same as the result calculated by the numerical analysis (35.81 Nm). The difference between FEM and the numerical analysis is approximately 2%, but the special difference of saturation torque is low. Because the parameters of the function are derived from the experimental function, the difference is acceptable.

**B. FEM ANALYSIS OF SERIES MOTORS**

Form the above analysis, the motor with 14 turns is analyzed by FEM software. The simulation result proves that the maximum torque, which is obtained through the analytic method, corresponds with FEM simulation result. Then, for verifying the analytic method comprehensively, the series motor with different turns is also analyzed.
Because the value of coil turn $N$ should be an integer, when the slot width adjusts continuously, the coil turn may maintain the same value. So, the different value of coil turn is selected for solving in FEM, the one is selected for FEM solving in same value of coil turn.

The series motors’ characteristic is shown in Table 3.

**TABLE 3.** Series motor’s characteristic with six poles and nine slots.

| Coil TRUN | Maximum Power | Maximum Current |
|-----------|---------------|-----------------|
| 1         | 216.14 kW     | 16499 A         |
| 2         | 170.32 kW     | 6114 A          |
| 4         | 100.96 kW     | 1784 A          |
| 6         | 63.75 kW      | 790 A           |
| 8         | 41.42 kW      | 441 A           |
| 10        | 28.91 kW      | 277 A           |
| 12        | 20.71 kW      | 195 A           |
| 16        | 10.55 kW      | 123 A           |

So, the eight FEM models are established with different coils turn and slot width, the current-power curve is shown in Fig. 15.

From Fig.15, when the armature current is lower than 1 kA, the maximum power results calculated by FEM and numerical method are closed. However, when the armature current is higher than 1 kA, the maximum power calculated by FEM is higher than the result calculated by numerical method because the stator iron saturation condition is not serious based on the FEM calculation result, and the stator iron flux distortion is not considered by numerical method, then the tooth flux density calculated by numerical method is higher.

So, the simulation result calculated by FEM software is closed to the result calculated by analysis method, and this result supports that the method could calculate the PMSM’s maximum power and this method should be adopted for analyzing the PMSM’s maximum power rule.

**V. TEST OF PROTOTYPE MOTOR**

For testifying the simulation result, a prototype motor, which parameters are from Table 2, is manufactured to test its maximum torque, and the prototype motor is shown in Fig. 17. The best test method is adopted HD812 hysteretic brake dynamometer, and adopted an assistant equivalent static test method. The equivalent static test method diagram is shown in Fig. 16. This test requires the following equipment: servo motor and its drive, 30AG torque master, LEM current sensor, TEK2024 oscilloscope, and 24V lead-acid battery. Those equipment and test scene are shown in Fig. 17.

The static test is adopted when the armature current is higher than 80A, and when the armature current is lower than 80A the prototype motor is tested by dynamic test. The dynamic test scene is shown in Fig. 18.

The torque experimental results are obtained through the static and dynamic tests, as shown in Fig. 19. It can be seen from the test result that when the armature current density is 5 A/mm² (this is the rated condition, armature current is 12.7 A, and the rated speed is 4000rpm), the output torque is 3.1 Nm, which is loaded by the motor’s $T-I$ curve. When the armature current is increased to 100 A (the current density is 40 A/mm²), the output torque is 23.1 Nm compared with the simulation result of 23 Nm.
When the armature current is 150 A, the output torque is 35.395 Nm. And when the armature current continues to be increased, the output torque does not increase because the motor’s electrical saturation and flux saturation limited the torque. This ultimate torque of 35.395 Nm is close to the result calculated by (14) which could be seen in Table 2.

VI. CONCLUSION
This paper presents a novel analysis method to analyze the rule between PMSM maximum power/torque and its inner parameters under the condition of id = 0 control mode. The output torque property rule is described by a function that has variables per pole phase winding turn, tooth width, and slot width. Using the function, the output torque quantization rule with per-pole per-phase winding turn can be achieved. Therefore, this function can be adopted to design the maximum PMSM property. Based on simulation results and experimental validation, four main conclusions can be drawn as follows:

a) the maximum torque is only limited by the temperature of stator. For short time working duty, the maximum torque could output less than 10ms, and for special field, the motor is a throwaway product, the motor’s maximum torque could be designed in a more reasonable way as presented in this paper.

b) the value of maximum torque should concern the stator iron saturation and the electrical saturation. Those factors influence respectively the magnetic field and armature current. Only the stator saturation and electrical saturation achieved at the same time, the maximum torque should be designed reasonable.

c) the output torque quantization rule with per-pole per-phase winding turn can be achieved. Form this rule, we could choose the major parameters for designing the maximum torque.

d) A PMSM prototype is manufactured and tested for proved the function correctness, and the simulation result is closer to the experiment result.

APPENDIX
When the motor is running in maximum torque, based on the motor vector diagram with id = 0 control mode, the electrical saturation function and magnetic saturation function are shown as fellow:

\[ U^2 = (E_0^2 + I \cdot R)^2 + (I \cdot X_q)^2 \]  
\[ \frac{t_s + b_s}{t_s} B_0 + B_{Fom} = B_{th} \]

For describing the \( B_{Fom} \), the stator armature winding magnetic field could be calculated by the simplified magnetic circuit as shown in Fig. 20.

From Fig. 1, air-gap flux density produced by armature winding can be obtained as:

\[ B_{Feom} = \frac{3\sqrt{2}}{\pi} \frac{\mu_0 Nk_N M I}{2(\delta + h_M)p} \]

Base on the motor theory, the EMF is obtained as follows:

\[ E_0 = 4.44 \cdot f \cdot N \cdot \phi \]

where, \( E_0 \) is the EMF without load, \( \phi \) is per pole magnetic flux, and it can be described as:

\[ \phi = B_0S \]

The quadrature axis reactance is a variable which is related to the magnetic conductivity parameter \( \lambda_q \), and it can be described as:

\[ X_q = 2\pi \cdot f \cdot \lambda_q \cdot N^2 \]

When the motor structure parameter, winding parallel branch number, and multiple winding number stay the same, the motor phase resistor is directly proportional to the coil turn number, and it can be described as:

\[ R = \frac{4\rho L_4}{a\pi d^2} \cdot N \]

So, with those assumed conditions in this paper, the five coefficients which could adopt (23)-(27) as special function with variables such coil turn \( N \), armature current \( I \), and air-gap flux density without load \( B_0 \), are defined as follows:

\[ k_E = 4.44 \cdot f \cdot S \]
\[ k_F = \frac{3\sqrt{2}}{\pi} \frac{\mu_0 k_N}{2(\delta + h_M)p} \]
\[ k_q = 2\pi \cdot f \cdot \lambda_q \]
\[ k_R = \frac{4\rho L_4}{a\pi d^2} \]
\[ k_{th} = \frac{t_s}{t_s + b_s} \]
Therefore, the equations (23-27) could be adopted as follows:

\[
\begin{align*}
E_0 &= k_F \cdot N \cdot B_0 \\
B_{Feom} &= k_F \cdot N \cdot I \\
X_q &= k_q \cdot N^2 \\
R &= k_R \cdot N 
\end{align*}
\]  
(33)

Those functions could be put into (22), it can be rewritten as:

\[
B_0 = \frac{ts}{ts + b_s} [B_{ih} - \frac{3\sqrt{2}}{\pi} \frac{\mu_0 k_{NM}}{2(\delta + h_M) p} N \cdot I] = k_{ih}(B_{ih} - k_F \cdot N \cdot I)
\]  
(34)

Then, the equation (1) is rewritten as:

\[
U^2 = [(k_E N B_{ih})^2 + I \cdot k_R \cdot N^2 + (I \cdot k_q N)^2
\]  
(35)

Then, the function \( B_0 \) is integrated into (21) and is shown in (36):

\[
U^2 = [(k_E k_i h N(B_{ih} - k_F \cdot N \cdot I))^2 + I \cdot k_R \cdot N^2 + (I \cdot k_q N)^2
\]  
(36)

After solving (36), the quadric equation in armature current \( I \) can be obtained through:

\[
\begin{align*}
t^2 \cdot a + I \cdot b + c &= 0 \\
a &= (k_E k_i k_F \cdot N - k_R)^2 + N^2 k_q^2 \\
b &= 2k_E k_i k_{ih} B_{ih}(1 - k_E k_{ih} \cdot N) \\
c &= k_E^2 k_i h^2 B_{ih}^2 - \frac{U^2}{N^2}
\end{align*}
\]  
(37)

After solving quadratic equation, the efficient solution is shown as:

\[
\begin{align*}
l &= \frac{k_E k_i h B_{ih}}{a} \sqrt{d^2 - a[1 - e]} \\
d &= k_F(1 - k_E k_{ih} \cdot N) \\
e &= \frac{k_E^2 k_i h^2 B_{ih}^2}{U^2} \cdot N^2
\end{align*}
\]  
(38)

The solved armature current \( I \) is integrated into (14), the stator iron flux density is derived as:

\[
B_{Feom} = \frac{k_E k_i h B_{ih} \cdot N}{a} \sqrt{d^2 - a(1 - e)}
\]  
(39)

So, with the \( id = 0 \) control mode, the motor input power could be described as:

\[
P = mU I \cos \varphi
\]  
(40)

Based on the motor vector diagram, the \( U \cos \varphi \) in (20) should be described as:

\[
U \cos \varphi = E_0 + IR
\]  
(41)

Then, the input power \( P \) should be adopted as:

\[
P = m(E + IR)I = mEI + mL^2R
\]  
(42)

where, \( mEI \) is the electrical power that stator side transform to rotor side, and \( mL^2R \) is stator copper loss.

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C. Jiwei et al.: Investigation on Maximum Electromagnetic Torque of PMSMs

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