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On semilinear equations with free boundary conditions on stratified Lie groups. (English)
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Summary: In this paper we establish existence of a solution to a semilinear equation with free boundary conditions on stratified Lie groups. In the process, a monotonicity condition is proved, which is quintessential in establishing the regularity of the solution.

MSC:
35J61 Semilinear elliptic equations
35R03 PDEs on Heisenberg groups, Lie groups, Carnot groups, etc.
35R35 Free boundary problems for PDEs

Keywords:
Dirichlet free boundary value problem; stratified Lie group; sub-Laplacian

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