Isovector deformation and its link to the neutron shell closure

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Abstract. DWBA analysis of the inelastic $^{30-40}\text{S}(p,p')$ and $^{18-22}\text{O}(p,p')$ scattering data measured in the inverse kinematics has been performed to determine the isoscalar ($\delta_0$) and isovector ($\delta_1$) deformation lengths of the $2^+_1$ excitations in the Sulfur and Oxygen isotopes using a compact folding approach. A systematic $N$-dependence of $\delta_0$ and $\delta_1$ has been established which shows a link between $\delta_1$ and the neutron-shell closure. Strong isovector deformations were found in several cases, e.g., the $2^+_1$ state in $^{20}\text{O}$ where $\delta_1$ is nearly three times larger than $\delta_0$. These results confirm the relation $\delta_1 > \delta_0$ anticipated from the core polarization by the valence neutrons in the open-shell (neutron rich) nuclei. The effect of neutron shell closure at $N = 14$ or 16 has been discussed based on the folding model analysis of the inelastic $^{22}\text{O}+p$ scattering data at 46.6 MeV/u measured recently at GANIL.

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The neutron and proton contributions to the structure of the lowest $2^+$ excited states are known to be quite different in the neutron-rich nuclei due, in particular, to a strong polarization of the core by valence neutrons. In the distorted-wave Born approximation (DWBA) for the inelastic hadron scattering, the different neutron and proton contributions to the nuclear excitation are explicitly determined by the isospin dependence of the inelastic form factor (FF).

In general, the isospin-dependent nucleon optical potential (OP) can be written in terms of the isoscalar (IS) and isovector (IV) components as

$$U(R) = U_0(R) \pm \varepsilon U_1(R), \quad \varepsilon = (N - Z)/A,$$  \hspace{1cm} (1)

where the $+$ sign pertains to incident neutron and - sign to incident proton. While the strength of the Lane potential $U_1$ has been studied since a long time in the $(p,p)$ and $(n,n)$ elastic scattering and $(p,n)$ reaction studies, very few attempts were made to study the isospin dependence of the inelastic FF. Within a collective-model prescription, the inelastic FF for the nucleon-nucleus scattering is obtained by “deforming” the OP with scaling factors known as the nuclear deformation lengths

$$F(R) = \frac{\delta}{dR} = \delta_0 \frac{dU_0(R)}{dR} \pm \varepsilon \delta_1 \frac{dU_1(R)}{dR}. \hspace{1cm} (2)$$

The explicit knowledge of the isoscalar ($\delta_0$) and, especially, isovector ($\delta_1$) deformation lengths would give us vital information on the structure of the nuclear excitation under study. There are only two types of experiment that might allow one to determine the IV deformation length $\delta_1$ based on the prescription (2):

i) $(p,n)$ reaction leading to the excited isobar analog state. It was shown, however, that the two-step mechanism usually dominates this process and the calculated DWBA cross sections are not sensitive to $\delta_1$.

ii) Another way is to extract $\delta_0$ and $\delta_1$ from the $(p,p')$ and $(n,n')$ inelastic scattering measured at the same incident energy and exciting the same target state. Such double measurements are presently not feasible with the beams of unstable nuclei.

We have recently suggested a compact folding method to determine $\delta_0(1)$ based on the DWBA analysis of the $(p,p')$ data only. In this approach, instead of deforming the OP, we build up the proton and neutron transition densities of a $2^+$-pole excitation ($\lambda \geq 2$) by using the Bohr-Mottelson prescription separately for protons and neutrons

$$\rho^\tau_s(r) = -\delta, \frac{dp^\tau_s(r)}{dr}, \quad \text{with} \quad \tau = p, n. \hspace{1cm} (3)$$

Here $\rho^\tau_s(r)$ are the proton and neutron ground-state (g.s.) densities and $\delta_\tau$ are the corresponding deformation lengths. Given the explicit proton and neutron transition densities, one can obtain from the folding model the inelastic proton-nucleus FF in terms of the IS and IV parts as

$$F(R) = F_0(R) - \varepsilon F_1(R), \hspace{1cm} (4)$$

where $F_0(R)$ and $F_1(R)$ are determined from the sum ($\rho^p_s + \rho^n_s$) and difference ($\rho^p_s - \rho^n_s$) of the neutron and proton transition densities, respectively. One can see that $F_1(R)$ is just the prototype of the Lane potential in the inelastic nucleon scattering. It is natural to represent...
the IS and IV parts of the nuclear transition density as

$$
\rho^{(1)}_{\lambda}(r) = \rho^{n}_{\lambda}(r) \pm \rho^{p}_{\lambda}(r).
$$

On the other hand, $\rho^{(1)}_{\lambda}(r)$ can be obtained using the same Bohr-Mottelson method, by deforming the IS and IV parts of the g.s. density

$$
\rho^{(1)}_{\lambda}(r) = -\delta_{(1)} \frac{d[\rho^{n}_{\lambda}(r) \pm \rho^{p}_{\lambda}(r)]}{dr}.
$$

It is straightforward to derive in this compact approach a consistent one-to-one correspondence between $\delta_{n}$ and $\delta_{p}$, which can be used to determine $\delta_{(1)}$ values from $\delta_{p(n)}$ values given by the DWBA analysis. If one assumes that the excitation is purely isoscalar and the neutron and proton densities have the same radial shape, scaled by the ratio $N/Z$, then $\delta_{n} = \delta_{p} = \delta_{0} = \delta_{1}$. Therefore, any significant difference between $\delta_{0}$ and $\delta_{1}$ would directly indicate a different isospin distribution in the structure of the nuclear excitation under study.

In the present work, we have studied the elastic and inelastic $^{30,32,36}S+p$ scattering data at 53 MeV/u [8] and $^{34,36,38,40}S+p$ data at energies of 28 to 39 MeV/u. The IS and IV contributions of the inelastic FF were considered explicitly to find out a systematic behavior of $\delta_{1}$ along the Sulfur isotopic chain, passing by the magic number $N=20$. Then, the folding + DWBA analysis of the elastic and inelastic $^{18,20}O+p$ data at 43 MeV/u [13] and $^{22}O+p$ data at 46.6 MeV/u [14] has been done to find out the $N$-dependence of $\delta_{1}$ in the Oxygen case.

To have the accurate “distorted” waves for the DWBA calculation of inelastic scattering, the optical model (OM) analysis of the elastic data was done using the real folded potential [7] obtained with the density- and isospin-dependent CDM3Y6 interaction and nuclear g.s. densities given by the Hartree-Fock-Bogoliubov (HFB) calculation [15]. The imaginary part of the OP was taken in the Woods-Saxon (WS) form from the global systematics CH89 [17]. All the considered elastic data were well reproduced (see Figs. 2 and 4) with the depth of the WS imaginary potential slightly adjusted by the OM fit (keeping the radius and diffuseness unchanged). The experimental reduced electric transition rate $B(E2 \uparrow)$ was used in each case to fix $\delta_{p}$ value in the expression (3) for $\rho^{p}_{\lambda}(r)$ which is further used in the folding calculation [3]. As the only parameter, $\delta_{n}$ was adjusted iteratively in the folding + DWBA calculation to fit the measured inelastic cross section. Since the CDM3Y6 interaction is real, the imaginary nuclear FF was obtained by deforming the imaginary part of the OP with $\delta_{0}$ and $\delta_{1}$ values at each iteration step of the folding + DWBA fit to the inelastic data. In each case, the final set of deformation lengths $\delta_{0}$ and $\delta_{1}$ was fixed only after the best-fit $\delta_{0}$ has been obtained.

An earlier folding + DWBA analysis [7] of the same inelastic $^{30-40}S+p$ scattering data, using inelastic FF given by the microscopic transition densities obtained in the Quasiparticle Random Phase Approximation (QRPA) [8], has shown that the neutron and proton contributions to the $2^{+}_{1}$ excitation in $^{30,32,34}S$ follow approximately the isoscalar rule which implies $\delta_{0} \approx \delta_{1}$. The present folding model analysis using the collective-model transition densities [3] has shown about the same results (see $\delta_{0}$ and $\delta_{1}$ values extracted for $^{30,32,34}S$ in Fig. 1). With the neutron shell becomes closed at $N=20$, a significant “damping” of the neutron transition strength occurs and suppresses strongly the IV deformation length $\delta_{1}$ of the $2^{+}_{1}$ state of $^{36}S$. In fact, $\delta_{1}$ is reaching its minimum as the neutron number $N$ approaches the magic number 20. The $N$-dependence of the IV deformation length $\delta_{1}$ is well correlated with the $N$-dependence of the reduced transition rate $B(E2 \uparrow)$ which also reaches its minimum at $N=20$ (see Fig. 1). The shell closure effect is so strong in this case that the proton and neutron QRPA transition densities needed to be scaled down by a factor of 0.63 and 0.88, respectively, for a correct description of the measured $2^{+}$ cross section in our earlier DWBA analysis [7] of the inelastic $^{30}S+p$ scattering. The contribution by the two valence neutrons in $^{38}S$ to the $2^{+}_{1}$ excitation is quite strong and $\delta_{n}$ turned out to be larger than $\delta_{p}$ by around 30% (see Fig. 2). This difference between the proton and neutron transition strengths results on the IV deformation length $\delta_{1}$ larger than $\delta_{0}$ by about 64% (see Fig. 1). Note that such a strong core polarization by the two valence neutrons in the $^{38}S$ case could not be fully accounted for by the QRPA calculation and the QRPA neutron transition density has been scaled by a factor of 1.25 for a good agreement of calculated DWBA cross section with
of the IV deformation length of 2\textsuperscript{0}\textsubscript{O} scattering data has shown an interesting N-dependence of \( \delta_1 \). Except for the double-magic \((N=Z)\) \textsuperscript{16}\textsuperscript{O}, where one has exact relation \( \delta_0 = \delta_1 \), the results obtained for \textsuperscript{18}\textsuperscript{O} already indicate a rather strong IV deformation for the 2\textsuperscript{+} state of this nucleus (see Fig. 3). We recall that the IS and IV deformation parameters of the 2\textsuperscript{+} state of \textsuperscript{18}\textsuperscript{O} have been determined long ago by Grabmayr et al. in a simultaneous DWBA analysis of the \((p,p')\) and \((n,n')\) inelastic scattering data at 24 MeV using the collective form factor. It is easy to deduce from the results of Ref. the corresponding deformation lengths \( \delta_0 \approx 1.1 \) and \( \delta_1 \approx 2.6 \pm 1.3 \) fm for the 2\textsuperscript{+} state of \textsuperscript{18}\textsuperscript{O}. These values agree reasonably with the results of our folding + DWBA analysis (see Fig. 3) of the inelastic \textsuperscript{18}\textsuperscript{O}+\(p\) scattering data at 43 MeV/u. Note that the \( \delta_1 \) value extracted from our analysis is a much smaller uncertainty compared to that deduced from the results of Ref. The error bars for \( \delta_{0(1)} \) plotted in Figs. were accumulated from the experimental uncertainties of the measured \( B(E2) \) values and \((p,p')\) cross section.

As already mentioned above, a double measurement of \((p,p')\) and \((n,n')\) inelastic scattering is not possible with unstable Oxygen isotopes, and our folding method is the only alternative way to determine \( \delta_1 \). The present analysis (with a more consistent treatment of the IV part of the imaginary FF) has confirmed again the large IV deformation length of the 2\textsuperscript{+} state of \textsuperscript{20}\textsuperscript{O} found earlier in Ref. With the IV deformation about three times the IS deformation (see Fig. 3), the contribution by the Lane form factor \( F_1 \) to the 2\textsuperscript{+} excitation of \textsuperscript{20}\textsuperscript{O} amounts up to 40-50\% of the total inelastic cross section. If we consider \textsuperscript{20}\textsuperscript{O} as consisting of the \textsuperscript{16}\textsuperscript{O} (or \textsuperscript{18}\textsuperscript{O}) core and four (or two) valence neutrons, then a large value of IV deformation length \( \delta_1 \) indicates a strong core polarization by the valence neutrons in the 2\textsuperscript{+} state of \textsuperscript{20}\textsuperscript{O}.

In such a “core + valence neutrons” picture, it is natural to expect that the 2\textsuperscript{+} state of \textsuperscript{22}\textsuperscript{O} should be more collective and have a larger IV deformation length due to the contribution of two more valence neutrons. However, the inelastic \textsuperscript{22}\textsuperscript{O}+\(p\) scattering data at 46.6 MeV/u measured recently at GANIL show clearly the opposite effect, with the \((p,p')\) cross section about 3 to 4 times smaller than that measured for the 2\textsuperscript{+} state of \textsuperscript{20}\textsuperscript{O} at 43 MeV over a wide angular range. The folding + DWBA analysis of these data using the QRPA transition densities for the 2\textsuperscript{+} state of \textsuperscript{22}\textsuperscript{O} has pointed to a much weaker neutron transition strength compared to that of the 2\textsuperscript{+} state of...
20. O. Given a significantly higher excitation energy of this state (1.5 MeV higher than that of the 2+_1 state of 20O), the newly measured inelastic 22O+p scattering data were suggested [14] as an important evidence for the neutron shell closure at N = 14 or 16.

24O are highly desirable for a definitive conclusion on a new magic number N = 16 in the neutron rich nuclei.

In summary, we have shown that the behavior of the dynamic isovector deformation of the 2+_1 states in neutron rich nuclei is closely correlated with the evolution of the valence neutron shell. This interesting result emphasizes again the importance of (p, p') reactions measured with unstable nuclei in the inverse kinematics.

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