A description of $^{103-107}$Pd Isotopes in the Interacting Boson-Fermion Model

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Abstract: Energy levels and B(E2) transitions for palladium isotopes with proton number $Z=46$ and neutron numbers ($n$) between 57 and 67 have been calculated through the interacting boson-fermion model. The set of parameters used in this calculation is the best approximation that has been carried out so far. Good agreement was found from comparison between the calculated energy levels and the transition probabilities B(E2) with those of experimental.

Keywords: Interacting Boson-Fermion Model, Pd isotopes, Energy levels, Positive parity, B(E2).

1. Introduction

The interacting boson model (IBM) proposed by Arima and Iachello [1, 2] has been widely applied to the structure of low-lying states in even-even nuclei and has considerable success. In the IBM, the even-even nucleus is assumed to be a collection of interacting s and d bosons with angular momentum (L)= 0 and 2, respectively. This model is associated with an inherent group structure, which allows the introduction of limiting symmetries called U(5), SU(3) and O(6) [1, 3]. The interacting boson model represents a significant step forward in our understanding of nuclear structure. It offers a simple Hamiltonian, capable of describing collective nuclear properties across a wide range of nuclei, and is founded on rather general algebraic group theoretical techniques, which have also found recent application to problems in atomic, molecular, and high-energy physics [4]. The application of this model to deformed nuclei is currently a subject of considerable interest and controversy. The interacting boson model (IBM-1) [5] and its extension to the odd-A nuclei, the interacting boson-fermion model (IBFM-1) [6], have proved to be able to give a successful description of widely varying classes of nuclei situated away from closed shell configurations.

The even-even palladium isotopes Pd ($Z=46$) are one of the most important nuclei which characterized by shape changes between spherical and deformed. Many experimental and
theoretical studies on the structure of energy level and electromagnetic transition properties of the even-even rare-earth isotopes had been investigated [7-17]. The aim of the present work is to do a microscopic study of the even-even and even-odd Hg isotopes within the IBM and the IBFM to give a comprehensive view of these isotopes in rather simple way. The results of the IBFM multilevel calculations for $^{103-107}$Pd isotopes will present for energy levels and transitions probabilities and will compare with the corresponding the experimental data. Also, the IBM-1 will apply to calculate the low-energy levels according to arrangement of bands (gr-, γ- and β-) and the B(E2) value for even-even $^{103-107}$Pd isotopes. Then study of the wave function structure for Hg isotopes.

2. Theoretical

The Interacting Boson-Fermion Model (IBFM) its building blocks a set of N-bosons with $L = 0, 2$ and an odd nucleon (either a proton or a neutron), and $M$-fermions occupying single-particle orbits with angular moment $j_1, j_2, j_3, \ldots$. The components of the fermion angular moment are the m-dimensional space of the group $U(m)$ with $m = \sum_j (2j_i + 1)$. The fermions creation $a_i^\dagger$ and annihilation $a_i$ operators for the single-particle in addition to the boson creation $b_i^\dagger$ and annihilation $b_i$ operators for the collective degrees of freedom. The fermion operators satisfy anti-commutation relations [18]:

$$\{a_i, a_j^\dagger\} = \{a_i^\dagger, a_j\} = \{a_i, a_j\} = \{a_i^\dagger, a_j^\dagger\} = 0$$

The bilinear products of fermion creation $a_i^\dagger$ and annihilation $a_i$

The IBFM Hamiltonian has an interesting algebraic structure, that proposes the possible occurrence of dynamical symmetries in odd-A. In the single-j case, the value of m is $m = 2j+1$, in general, a chain of algebras is:

$$U(2j+1) \supset SU(2j+1) \supset SP(2j+1) \supset SU(2j) \supset O(2j)$$

In this study, the calculations of energy levels of $^{103-107}$Pd isotopes have been done using interacting boson-fermion model. Positive parity state energies, the reduce probabilities of $E2$ and transitions ($B(E2)$ values) are calculated and compared with the available experimental data.

In the IBFM odd-A nuclei are described in terms of a mixed system of interacting bosons and fermions, the concept of dynamical symmetries has to be generalized. Under the restriction, that both the boson and fermion states have good angular momentum, the respective group chains should contain the rotation group $O(3)$ for boson and $SU(2)$ for fermion as subgroup [18,19].

If one of subgroups of $U_B(6)$ is isomorphic to one of the subgroups of $U_F(m)$, the boson and fermion group chains can be combined into a common boson-fermion group chain. When the Hamiltonian is written in terms of Casimir invariants of the combined boson-fermion group chain, dynamical boson-fermion symmetry arises. The odd-A nuclei are described by the coupling of the odd fermionic quasi-particle to a collective boson core. The total Hamiltonian consists of three parts and is given by the following equation [18, 20]:

$$H = H_B + H_F + V_{BF}$$

which contains one-body terms only and given by

$$H_F = \sum_{\mu} \epsilon_j a_{j\mu}^\dagger a_{j\mu}$$

where $\epsilon_j$ are the quasi-particle energies and $a_{j\mu}^\dagger a_{j\mu}$ is the creation (annihilation) operator for the quasi-particle in the eigenstate $|jm\rangle$.

$V_{BF}$ is the boson-fermion interaction that describes the interaction between the odd quasi-nucleon and the even-even core nucleus:

$$V_{BF} = \sum_j A_j \left[ (d^\dagger \times \bar{d})^{(0)} \times (a_j^\dagger \times \bar{a}_j)^{(0)} \right]_0^{(0)} + \sum_{jj'} f_{jj'} [Q^{(2)} \times (a_j^\dagger \times \bar{a}_j)^{(2)}]_0^{(2)}$$

(6)
where $Q^{(2)}$ is the core boson quadrupole operator.

The parameters $A_j$, $I_{jj'}$ and $A'_{jj'}$ are defined by the following equations:

$$A_j = A_0 \sqrt{2j + 1}$$

$$I_{jj'} = \sqrt{5} I_0 (u_j u_{j'} - v_j v_{j'}) Q_{jj'}$$

$$A'_{jj'} = -\sqrt{5} A_0 \left[ (u_j v_{j'} + v_j u_{j'}) Q_{jj'} \beta_{jj'} + (u_j v_{j'} + v_j u_{j'}) Q_{jj'} \gamma_{jj'} \right]$$

where: $A_0$ is the monopole interaction, $I_0$ is the quadrupole interaction and $A_0$ is the exchange of a quasi-particle with one of the two fermions forming a boson, dynamical boson-fermion symmetry associated with the O(6) limit and the odd nucleon occupying single-particle orbits with spin $j = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$. In this case, the fermion space is decomposed into a pseudo- orbital part with $K = 0, 2$ and a pseudo-spin part with $s = \frac{1}{2}$.

### 3. Results and discussions

#### 3.1 Energy Level

In Pd nuclei have 46 protons and have 57-67 neutron numbers and one fermion according to framework of interaction boson-fermion model (IBFM). The calculations have been performed with IBFM-1 and hence, no distinction made between neutron and fermion. For the analysis of excitation energies in Pd isotopes it was tried to keep to minimum the number of free parameters in Hamiltonian. The explicit expression of Hamiltonian adopted in calculations is [18].

$$H = H_B + H_F + V_{BF}$$

Tables 1 and 2 shows the best values of the parameters which give the best fitting between theoretical and the experimental energy levels of the above isotopes. All parameters are given in MeV. The energy levels of the 1, 2 and 3-bands for even-odd Pd isotopes have calculated using IBFM with PHINT code [22].

**Table 1.** Adopted parameters which is used for IBFM calculations. All parameters are given in MeV.

| Parameters | $^{103}$Pd | $^{105}$Pd | $^{107}$Pd |
|------------|-------------|-------------|-------------|
| BFE        | 0.26        | 0.21        | 0.22        |
| BFQ        | 0.02        | 0.02        | 0.01        |
| BFM        | -0.27       | -0.65       | -0.22       |

**Table 2.** Adopted values for the parameters used for IBFM-1 calculations. The parameter $\epsilon_j$ is given in MeV.

| Parameters | $^{103}$Pd | $^{105}$Pd | $^{107}$Pd |
|------------|-------------|-------------|-------------|
| $\epsilon_j$ | 2.433 | 2.127 | 1.414 |
| $\nu_j^2$  | 0.268 | 0.084 | 0.774 |
Figure 1 shows that the calculated 1, 2 and 3-bands and the experimental data [23-26] for even-odd Pd isotopes. From this figure, the calculated energy levels are in good agreement with the experimental ones for all isotopes. Levels with "( )" in 1, 2 and 3-bands correspond to cases for which the spin and/or parity of the corresponding states are not well established experimentally.

Figure 1: (Color online) comparisons between the calculated IBFM-1 and the experimental data [23-26] for $^{103-107}$Pd Isotope

3.2 $B(E2)$ Transition
The calculation of $E2$ transition strengths will be discussed and compared with the available experimental data. Calculation of electromagnetic transitions gives a good test of the nuclear model wave functions. In general, the electromagnetic transition operators can be written as sum of two terms, the first of the boson part of the wave function and second only on the fermion part [18,19], in the case of a fermion occupying $j=1/2, 3/2$, and $5/2$ sublevel. The values of effective
charge ($e_B$) are calculated and presented in the table (3). The values of effective charge ($e_F$) were estimated from the selection rules $\Delta \sigma_1 = \Delta \sigma_2 = \Delta \sigma_3 = 0$ and $\Delta (\tau_1 + \tau_2) = \pm 1$ transitions, which are allowed for $e_B (\alpha_2) = e_F (f_2)$, and $\Delta \sigma_1 = \Delta \sigma_2 = \pm 1$, $\Delta (\tau_1, \tau_2) = e_B$, and $\Delta \sigma_1 = \Delta \sigma_2 = \Delta \sigma_3 \neq 0$, $\Delta (\tau_1, \tau_2) = e_F$. At $e_B (\alpha_2) = e_F$, the effective charge ($e_F$) can be reproduced from the experimental $B(E2; /g_{1836}/g_{3036} \to /g_{1836}/g_{3033})$ and can be written as [18].

$$B(E2; (N, 1, 0), (\tau_1, \tau_2), L_i, I_i \to (N + 1, 0, 0), (\tau_1, \tau_2)f, L_f, I_f) = (\alpha_2 - f_2)^2 \frac{2(N+1)(N+2)}{5(N+1)(N+2)}$$

The values of effective charge ($e_F$) are given in the table (3). The calculated $B(E2)$ values are presented in the table 4.

Table 3. Parameters (in eb) used to reproduce $B(E2)$ values for $^{103-107}$Pd isotopes.

| Isotopes | $e_B$ (eb) | $e_F$ (eb) |
|----------|------------|------------|
| $^{103}$Pd | 0.1004 | -0.1934 |
| $^{105}$Pd | 0.0935 | 0.0083 |
| $^{107}$Pd | 0.0926 | 0.0264 |

Table 4. The IBM-1 and experimental [23-26] values of $B(E2)$ (in e$^2$ b$^2$).

| $I_i$ $\to$ $I_f$ | $^{103}$Pd EXP. | $^{103}$Pd IBM-1 | $^{105}$Pd EXP. | $^{105}$Pd IBM-1 | $^{107}$Pd EXP. | $^{107}$Pd IBM-1 |
|------------------|----------------|----------------|----------------|----------------|----------------|----------------|
| $3/2^+ \to 5/2^+$ | 0.0329 | 0.0492 | 0.0028 | 0.0041 | -- | 0.0721 |
| $5/2^+ \to 5/2^+$ | 0.0086 | 0.0067 | 0.0044 | 0.0029 | -- | 0.0695 |
| $5/2^+ \to 5/2^+$ | 0.0229 | 0.0249 | -- | 0.0216 | -- | 0.0121 |
| $7/2^+ \to 3/2^+$ | 0.00005 | 0.0043 | -- | 0.0120 | -- | 0.0016 |
| $3/2^+ \to 5/2^+$ | -- | 0.0018 | -- | 0.0000 | -- | 0.0005 |
| $1/2^+ \to 3/2^+$ | -- | 0.0025 | <0.0279 | 0.0243 | -- | 0.0689 |
| $7/2^+ \to 5/2^+$ | -- | 0.0724 | 0.0441 | 0.0850 | -- | 0.1112 |
| $1/2^+ \to 5/2^+$ | -- | 0.0394 | 0.0102 | 0.0092 | 0.0017 | 0.0020 |
| $9/2^+ \to 7/2^+$ | -- | 0.0073 | -- | 0.0209 | -- | 0.0066 |
| $11/2^+ \to 9/2^+$ | -- | 0.0285 | -- | 0.0307 | -- | 0.0407 |
| $9/2^+ \to 5/2^+$ | -- | 0.0684 | 0.0588 | 0.0765 | -- | 0.0872 |
4. Conclusion
The low-lying positive parity states, 2 and 3-bands (energy levels), electric transition probabilities $B(E2)$ values have been calculated using Interacting Boson-Fermion Model (IBFM). These nuclei assumed to be as medium mass nuclei. All the results were compared with experimental data and acceptable agreement obtained. Electric quadruple transitions probability $B(E2)$ were calculated and compared with available experimental data.

Acknowledgements
We thank University of Kerbala, College of Science, Department of Physics for supporting this work.

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