CP violating asymmetries in single top quark production at the Tevatron $p\bar{p}$ collider

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Abstract

Analytic expressions for the angular distributions of the $b$-quarks associated with single $t$-quark production in $pp \rightarrow W^* \rightarrow t\bar{b} \rightarrow b\bar{b}W$ and of the leptons from the subsequent decay $W \rightarrow lv$ are obtained in the laboratory system. CP violation in the $t$-production vertex is assumed. Different angular and total cross section CP violating asymmetries are considered. Relations testing CP violation in the $t$-decay vertex are also given. A numerical analysis is performed in the MSSM with a CP violating phase of the trilinear coupling $A_{t}$. The asymmetries are typically of the order $10^{-3} - 10^{-4}$.

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1 Introduction

The experiments at the Tevatron $p\bar{p}$ collider offer the possibility of thoroughly studying
the properties of the top quark. In particular, in top physics the Standard Model (SM) gives
negligible CP violating effects [1] due to the GIM mechanism. Looking for CP violation in top
quark production and/or decays is therefore one of the best ways to probe New Physics. In
extensions of the SM as, for instance, in two Higgs doublet models or in supersymmetric (SUSY)
models, CP violating phases appear rather naturally. They can then cause CP violating effects
in processes with top quarks at one–loop level.

In this paper we study CP violating asymmetries in $t\bar{b}$ and $\bar{t}b$ production in $p\bar{p}$ collisions,
induced by CP violating form factors in the $tbW$ vertex. More precisely, we consider the two
CP conjugate processes

\[ p\bar{p} \rightarrow W^{+} \rightarrow t\bar{b} \quad \text{and} \quad p\bar{p} \rightarrow W^{-} \rightarrow b\bar{t} \]  

followed by the decays $t \rightarrow bW^{+} \rightarrow bl^{+}\nu_{l}$, $\bar{t} \rightarrow \bar{b}W^{-} \rightarrow \bar{b}l^{-}\bar{\nu}_{l}$. The possibility of testing CP
violation in $p\bar{p} \rightarrow t\bar{b} \rightarrow W^{+}b\bar{b}$ was already considered in [2, 3] and in more details in a review
on this subject in [4]. In this article, we present new additional CP violating asymmetries. In
particular, we define two types of asymmetries: (i) with $b$ quarks and (ii) with leptons in the
final states. For the asymmetries with $b$ quarks we assume that the $b$’s from production can
be distinguished from those from $t$ decays. This is justified by the requirement that the whole
event be reconstructed in order to distinguish the events of single $t$ production in (1) from
the background events [5]. We thus define separate asymmetries for $b$ quarks from production,
from decay, and from both production and decay. The asymmetries with leptons are in general
easier to observe. However, the cross section is smaller by the branching ratio $W \rightarrow l\nu$, which
is roughly $1/3$ if all leptons $l = e, \mu, \tau$ are counted.

For both $b$ quarks and leptons, we consider asymmetries in the total number of $b$ and $\bar{b}$ ($l^{+}$ and
$l^{-}$) and asymmetries in their angular distributions (forward–backward asymmetries). When
considering the angular distributions of the decay products we assume CP violation only in the
production vertex of the $t$ quark. As shown in [3] CP violation in $t$-decay is suppressed by the
polarization of the $t$-quark. Moreover, CP violation in the decay would mean that in addition to CP violating phases there are also new decay modes of the top quark, which—according to the present experimental limits—does not seem to be the case. However, we will also discuss the contribution of CP violation in the $t$ decay to the total cross section asymmetries defined in Sections 4.2 and 5.

The paper is organized as follows: In Section 2 we briefly give our notation. In Section 3 we define the CP violating asymmetries for $b$ quarks from production, namely total cross section asymmetries and forward–backward asymmetries. In Section 4 we derive the formulae for the asymmetries for $b$ quarks and leptons from the $t$ decay. We again consider total cross section asymmetries and forward–backward asymmetries. The forward–backward asymmetry for $b$ quarks from both production and decay is given in Section 5. In Section 6 we give a numerical analysis and discussion of the resulting asymmetries in the framework of the Minimal Supersymmetric Standard Model (MSSM) with complex parameters. The necessary formulae for the form factors, mass matrices, and couplings in the MSSM are given in the Appendices.

2 Notation

The quark subprocesses which we consider for single $t$ and $\bar{t}$ production are

$$u + \bar{d} \rightarrow W \rightarrow t + \bar{b}, \quad (2)$$

$$d + \bar{u} \rightarrow W \rightarrow b + \bar{t}. \quad (3)$$

The matrix elements including CP violation read ($m_b = 0$):

$$M^{\bar{b}}_{t} = -\frac{g^2}{2} \bar{u}(-\bar{p}_d) \gamma_\alpha P_L u(p_a) \frac{i}{\hat{s} - m_W^2} \bar{u}(p_t) \Gamma^\alpha u(-p_b), \quad (4)$$

$$M^{\bar{d}}_{b} = -\frac{g^2}{2} \bar{u}(-\bar{p}_u) \gamma_\alpha P_L u(p_d) \frac{i}{\hat{s} - m_W^2} \bar{u}(p_b) \bar{\Gamma}^\alpha u(-p_{\bar{t}}) \quad (5)$$

with

$$\Gamma^\alpha = \gamma^\alpha P_L (1 + i f_L^{CP}) + \frac{P^\alpha}{m_t} P_L i g_R^{CP}, \quad (6)$$

$$\bar{\Gamma}^\alpha = \gamma^\alpha P_L (1 - i f_L^{CP}) - \frac{P^\alpha}{m_t} P_L i g_R^{CP}. \quad (7)$$
\( P^\alpha = p^\alpha_\ell - p^\alpha_\bar{\ell}, \bar{P}^\alpha = p^\alpha_\bar{\ell} - p^\alpha_\ell \) and \( P_L = (1 - \gamma_5)/2 \). \( f_{L}^{CP} \) and \( g_{R}^{CP} \) are the CP violating form factors of the \( tbW \) vertex. They are complex functions. The asymmetries considered in this paper measure the absorptive parts of these form factors — \( \Im m f_{L}^{CP} \) and \( \Im m g_{R}^{CP} \). The real part of \( g_{R}^{CP} \), \( \Re e g_{R}^{CP} \), can be measured through triple product correlations. As there is no CP violation at tree level, \( \Re e f_{L}^{CP} \) has no physical meaning and cannot enter measurable quantities.

### 3 \( b \) quarks from production

In the centre–of–mass system (CMS) of \( u \) and \( \bar{d} \) or \( \bar{u} \) and \( d \), with the \( z \) axis pointing along the momentum \( p_u \) or \( p_d \), the angular distributions of the \( b \) and \( \bar{b} \) quarks in the subprocesses (3) and (\( \bar{3} \)) are:

\[
\begin{align*}
\hat{\sigma}_{1}^{\bar{b}} &= C \left\{ a_{0}^{b} + a_{1}^{b} \cos \theta_{b}^{*} + a_{2}^{b} \cos^{2} \theta_{b}^{*} \right\} d \cos \theta_{b}^{*}, \\
\hat{\sigma}_{1}^{\bar{b}} &= C \left\{ a_{0}^{\bar{b}} - a_{1}^{\bar{b}} \cos \theta_{b}^{*} + a_{2}^{\bar{b}} \cos^{2} \theta_{b}^{*} \right\} d \cos \theta_{b}^{*}.
\end{align*}
\]

Here

\[
\begin{align*}
\tilde{a}_{i}^{\bar{b}} &= a_{i}^{SM} \pm a_{i}^{CP}, \\
\tilde{a}_{0}^{SM} &= -(m_{t}^{2} + \hat{s})/2, \\
\tilde{a}_{1}^{SM} &= -\hat{s}, \\
\tilde{a}_{2}^{SM} &= (m_{t}^{2} - \hat{s})/2, \\
\tilde{a}_{0}^{CP} &= 2 a_{0}^{SM} \Im m f_{L}^{CP}(\hat{s}) + (\hat{s} - m_{t}^{2}) \Im m g_{R}^{CP}(\hat{s}), \\
\tilde{a}_{1}^{CP} &= 2 a_{1}^{SM} \Im m f_{L}^{CP}(\hat{s}), \\
\tilde{a}_{2}^{CP} &= 2 a_{2}^{SM} \left( \Im m f_{L}^{CP}(\hat{s}) + \Im m g_{R}^{CP}(\hat{s}) \right),
\end{align*}
\]

and

\[
C = \frac{-\pi a_{w}^{2}(\hat{s} - m_{t}^{2})^{2}}{16[\hat{s}(\hat{s} - m_{W}^{2})]^{2}}.
\]

\( \hat{s} \) stands for

\[
\hat{s} = \begin{cases} 
\hat{s}_{\pm} = (p_u + p_d)^2 & \text{for } \hat{\sigma}_{\bar{b}}^{\bar{b}} \\
\hat{s}_{\pm} = (p_u + p_d)^2 & \text{for } \hat{\sigma}_{\bar{b}}^{\bar{b}}
\end{cases}
\]
The polar angle $\cos \theta$ in the laboratory frame — the CMS of $p$ and $\bar{p}$ — is related to the angle $\cos \theta^*$ in the CMS of the initial quarks by:

$$\cos \theta^* = \frac{\cos \theta - v}{1 - v \cos \theta},$$

where $v$ is the velocity of the laboratory system in the CMS of the quarks, $v = (x_1 - x_2)/(x_1 + x_2)$. Here $\cos \theta$ stands for $\cos \theta_b$ or $\cos \bar{\theta}_b$, and $x_1$ and $x_2$ are the fractions of the longitudinal momenta of the quarks. The angular distribution of the $b$ and $\bar{b}$ quarks in the laboratory system thus is

$$d\hat{\sigma}_{tb}^b d\cos \theta_b = d\hat{\sigma}_{tb}^\bar{b} d\cos \bar{\theta}_b = \frac{C(1 - v^2)}{(1 - v \cos \theta_b)^4} \left[ a^b_0(1 - v \cos \theta_b)^2 + a^b_1(\cos \theta_b - v)(1 - v \cos \theta_b) + a^b_2(\cos \theta_b - v)^2 \right],$$

and

$$d\hat{\sigma}_{tb}^\bar{b} d\cos \bar{\theta}_b = \frac{C(1 - v^2)}{(1 - v \cos \theta_b)^4} \left[ a^\bar{b}_0(1 - v \cos \theta_b)^2 - a^\bar{b}_1(\cos \theta_b - v)(1 - v \cos \theta_b) + a^\bar{b}_2(\cos \theta_b - v)^2 \right].$$

Taking into account the two possibilities $x_1 = x_u, x_2 = x_d$ and $x_1 = x_d, x_2 = x_u$, and using the parton distribution functions $f_u$ and $f_d$ of the proton, we get in the laboratory system:

$$\frac{d\sigma_{tb}^b}{d\cos \theta_b} = \frac{1}{2} \int \frac{d\sigma_{tb}^b(\hat{s}, v)}{d\cos \theta_b} \left[ f_u(x_u)f_d(x_d) + f_u(x_d)f_d(x_u) \right] dx_u dx_d,$$

and

$$\frac{d\sigma_{tb}^\bar{b}}{d\cos \theta_b} = \frac{1}{2} \int \frac{d\sigma_{tb}^\bar{b}(\hat{s}, v)}{d\cos \theta_b} \left[ f_u(x_u)f_d(x_d) + f_u(x_d)f_d(x_u) \right] dx_u dx_d$$

with

$$\hat{s} = x_u x_d s \quad \text{and} \quad v = (x_u - x_d)/(x_u + x_d).$$

### 3.1 Total cross section asymmetries for $b$ from production

For the total cross sections of (2) and (3) we obtain:

$$\frac{d\sigma_{tb}^b}{d\cos \theta_b} = \frac{1}{2} \int \frac{d\sigma_{tb}^b(\hat{s}, v)}{d\cos \theta_b} \left[ f_u(x_u)f_d(x_d) + f_u(x_d)f_d(x_u) \right] dx_u dx_d,$$

and

$$\frac{d\sigma_{tb}^\bar{b}}{d\cos \theta_b} = \frac{1}{2} \int \frac{d\sigma_{tb}^\bar{b}(\hat{s}, v)}{d\cos \theta_b} \left[ f_u(x_u)f_d(x_d) + f_u(x_d)f_d(x_u) \right] dx_u dx_d$$

with

$$\frac{d\sigma^{b\bar{b}}}{d\cos \theta_b} = 2C \left( a^b_0 + a^\bar{b}_2 / 3 \right)$$

$$= \left( \hat{\sigma}_1 \right)^{SM}(\hat{s}) \left\{ 1 \pm 2 \Im m f_L^{CP} \pm \frac{m^2_{\ell} - \hat{s}}{m^2_{\ell} + 2\hat{s}} 2 \Im m g_R^{CP} \right\}$$

(25)
where
\[(\hat{\sigma}_1)^{SM}(\hat{s}) = \frac{\pi \alpha_w^2 (\hat{s} - m_t^2)^2 (m_t^2 + 2\hat{s})}{24[\hat{s}(\hat{s} - m_W^2)]^2}, \tag{26}\]
and \(\alpha_w = g^2/4\pi\). In (23) the upper sign stands for \(\hat{\sigma}^b\) and the lower one for \(\hat{\sigma}^{\bar{b}}\). The CP violating asymmetry for the total number of \(b\) and \(\bar{b}\) quarks from the production processes (4) is thus given by
\[R_1^{tot}(b) = \frac{(\sigma_1)^b - (\sigma_1)^{\bar{b}}}{(\sigma_1)^b + (\sigma_1)^{\bar{b}}}. \tag{27}\]

From (22), (23) and (25) we obtain:
\[R_1^{tot}(b) = \frac{\int C \left(a_0^{CP} + a_2^{CP}/3\right) f_u(x_u) f_d(x_d) dx_u dx_d}{\int C \left(a_0^{SM} + a_2^{SM}/3\right) f_u(x_u) f_d(x_d) dx_u dx_d} = \frac{2 \int \hat{\sigma}^{SM}(\hat{s}) \left[3m f_u^{CP}(\hat{s}) + \frac{m_t^2 - \hat{s}}{m_t^2 + 2\hat{s}} 3m g_R^{CP}(\hat{s})\right] f_u(x_u) f_d(x_d) dx_u dx_d}{\int \hat{\sigma}^{SM}(\hat{s}) f_u(x_u) f_d(x_d) dx_u dx_d}. \tag{28}\]

This asymmetry was already obtained and discussed in [3]. We recall it here for completeness and also include it in our numerical analysis.

### 3.2 Forward–backward asymmetries for \(b\) from production

We next define two CP violating forward–backward asymmetries for the production process:
\[A_1^{FB}(b) = \frac{(\sigma_1)^b_F - (\sigma_1)^{\bar{b}}_F}{(\sigma_1)^b_F + (\sigma_1)^{\bar{b}}_F} = \frac{\int [(\hat{\sigma}_1)^b_F - (\hat{\sigma}_1)^{\bar{b}}_F] [f_u(x_u) f_d(x_d) + f_u(x_d) f_d(x_u)] dx_u dx_d}{\int [(\hat{\sigma}_1)^b_F + (\hat{\sigma}_1)^{\bar{b}}_F] [f_u(x_u) f_d(x_d) + f_u(x_d) f_d(x_u)] dx_u dx_d}, \tag{29}\]
and
\[A_2^{FB}(b) = \frac{(\sigma_1)^b_B - (\sigma_1)^{\bar{b}}_B}{(\sigma_1)^b_B + (\sigma_1)^{\bar{b}}_B} = \frac{\int [(\hat{\sigma}_1)^b_B - (\hat{\sigma}_1)^{\bar{b}}_B] [f_u(x_u) f_d(x_d) + f_u(x_d) f_d(x_u)] dx_u dx_d}{\int [(\hat{\sigma}_1)^b_B + (\hat{\sigma}_1)^{\bar{b}}_B] [f_u(x_u) f_d(x_d) + f_u(x_d) f_d(x_u)] dx_u dx_d}. \tag{30}\]

Here \((\sigma_1)^b_{F(B)}\) are the number of \(b\) quarks from production in the forward (backward) direction. Note that in single \(t\) production, contrary to \(t\bar{t}\) pair production, we have \(|A_1^{FB}| \neq |A_2^{FB}|\). This
is due to the fact that CP violation leads to a difference in the total cross sections of $b$ and $\bar{b}$ production. In terms of form factors we obtain:

$$A_{1}^{FB}(b) = \frac{\int C \left( a_0^{CP} + (1 - v^2) a_1^{CP} / 2 + a_2^{CP} / 3 \right) f_u(x_u) f_d(x_d) \, dx_u \, dx_d}{\int C \left( a_0^{SM} + (1 - v^2) a_1^{SM} / 2 + a_2^{SM} / 3 \right) f_u(x_u) f_d(x_d) \, dx_u \, dx_d} = \frac{2 \int h(\hat{s}) [(2m_t^2 + \hat{s}(7 - 3v^2)) \Re m f_L^{CP} + 2(m_t^2 - \hat{s}) \Im m g_R^{CP}] f_u(x_u) f_d(x_d) \, dx_u \, dx_d}{\int h(\hat{s}) [2m_t^2 + \hat{s}(7 - 3v^2)] f_u(x_u) f_d(x_d) \, dx_u \, dx_d},$$

(31)

$$A_{2}^{FB}(b) = \frac{\int C \left( a_0^{CP} - (1 - v^2) a_1^{CP} / 2 + a_2^{CP} / 3 \right) f_u(x_u) f_d(x_d) \, dx_u \, dx_d}{\int C \left( a_0^{SM} - (1 - v^2) a_1^{SM} / 2 + a_2^{SM} / 3 \right) f_u(x_u) f_d(x_d) \, dx_u \, dx_d} = \frac{2 \int h(\hat{s}) [(2m_t^2 + \hat{s}(3v^2 + 1)) \Re m f_L^{CP} + 2(m_t^2 - \hat{s}) \Im m g_R^{CP}] f_u(x_u) f_d(x_d) \, dx_u \, dx_d}{\int h(\hat{s}) [2m_t^2 + \hat{s}(3v^2 + 1)] f_u(x_u) f_d(x_d) \, dx_u \, dx_d},$$

(32)

with

$$h(\hat{s}) = \left[ \frac{m_t^2 - \hat{s}}{\hat{s}(\hat{s} - m_W^2)} \right]^2. \quad (33)$$

4 Secondary $b$ quarks and leptons

Let us now turn to the $b$ quarks and leptons which originate from the $t$ decays:

$$t \to bW, \quad t \to bl^+\nu. \quad (34)$$

Following [3, 3] in the narrow width approximation ($\Gamma_t \ll m_t$) we obtain in the CMS of $u\bar{d}$:

$$d\hat{\sigma}_2^x = \left( \frac{d\hat{\sigma}_1^t}{d \cos \theta_t^*} \right) d \cos \theta_t^* \frac{d \Gamma(t \to x...)}{\Gamma_{tot}} \frac{E_t^*}{m_t} = \left( \frac{d\hat{\sigma}_1^t}{d \cos \theta_t^*} \right) d \cos \theta_t^* \frac{d \Gamma(t \to x...)}{\Gamma_{tot}} Br(t \to xX),$$

(35)

and analogously for the decay of $\bar{t}$. In (35), $d\hat{\sigma}_1^t / d \cos \theta_t^*$ is the distribution of the $t$ quarks in $U$:

$$\frac{d\hat{\sigma}_1^t}{d \cos \theta_t^*} = C \left\{ a_0^t + a_1^t \cos \theta_t^* + a_2^t \cos \theta_t^{*2} \right\}, \quad (36)$$
and $E_t^*$ is the energy of the decaying $t$. $\frac{d\Gamma}{\Gamma}(t \to x + ...)$ is the angular distribution of the secondary particle $x$ ($x = b, l^+$) in this frame with $t$ polarized, normalized to the partial decay width $\Gamma(t \to x)$. The branching ratio $Br(t \to x X)$ stands for $Br(t \to bW^+)$ or $Br(t \to b l^+ \nu_l)$. If $\beta_t^*$ is the velocity of $t$ in the CMS of $u\bar{d}$ we have [9]:

$$\left( \frac{d\Gamma}{\Gamma} \right)(t \to x + ...) = \frac{d\Omega_x^*}{4\pi} \frac{m_t^2}{E_t^*} \left( 1 - \beta_t^* \cos \theta_{tx}^* \right)^2 \left\{ 1 + \frac{\alpha_x}{(p_t p_x)} \left( \xi p_x \right) \right\},$$

(37)

where $\alpha_x$ determines the sensitivity of the particle $x = b, l$ to the polarization of the $t$ quark:

$$\alpha_b = \frac{m_t^2 - 2 m_W^2}{m_t^2 + 2 m_W^2},$$

(38)

$$\alpha_l = -1.$$  

(39)

$\xi$ is the polarization four–vector of $t$, which is determined by the production process (2), and $\cos \theta_{tx}^*$ is the angle between the momenta $p_t$ and $p_x$:

$$\cos \theta_{tx}^* = \sin \theta_t^* \sin \phi_x^* + \cos \theta_t^* \cos \phi_x^*.$$  

(40)

The treatment of the $t$ polarization four–vector and the general formula for the differential cross section, in the CMS of $(u\bar{d})$, in terms of the SM- and CP-violating components of $\xi$, needed to derive the angular distribution $d\hat{\sigma}_x^2/d\cos \theta_x$ are given in Appendix A.

### 4.1 Angular distribution of the decay products

Integrating (38) over $d\cos \theta_t^*$ and $d\phi_x^*$ we obtain the angular $\cos \theta_x^*$ distribution of the decay products $x = b, l^+$ in the CMS of $u\bar{d}$:

$$\frac{d\hat{\sigma}_x^2}{d\cos \theta_x^*} = B \left[ b_0 + b_1 \cos \theta_x^* + b_2 \cos^2 \theta_x^* \right]$$

(41)

where

$$b_i = b_i^{SM} + b_i^{CP},$$

(42)

$$b_i^{SM} = c_i^{SM} + \alpha_x a_i^{SM},$$

(43)

$$b_i^{CP} = c_i^{CP} + \alpha_x a_i^{CP}.$$  

(44)
For the CP conserving part we obtain:

\[
c_0^{SM} = \frac{1}{2m_t^2} \left[ 2\hat{s}m_t^2(m_t^2 + \hat{s}) \ln \frac{m_t^2}{\hat{s}} - (m_t^6 + 3\hat{s}m_t^4 - 5\hat{s}^2m_t^2 + \hat{s}^3) \right], \quad (45)
\]

\[
c_1^{SM} = \frac{\hat{s}}{m_t^2} \left[ -2\hat{s}m_t^2 \ln \frac{m_t^2}{\hat{s}} + m_t^4 - \hat{s}^2 \right], \quad (46)
\]

\[
c_2^{SM} = \frac{1}{2m_t^2} \left[ -6m_t^2\hat{s}(m_t^2 + \hat{s}) \ln \frac{m_t^2}{\hat{s}} + (m_t^2 - \hat{s})(m_t^4 + 10\hat{s}m_t^2 + \hat{s}^2) \right], \quad (47)
\]

\[
d_0^{SM} = -\hat{s} \left[ (m_t^2 + \hat{s}) \ln \frac{m_t^2}{\hat{s}} - 2(m_t^2 - \hat{s}) \right], \quad (48)
\]

\[
d_1^{SM} = 2\hat{s} \left[ \hat{s} \ln \frac{m_t^2}{\hat{s}} - (m_t^2 - \hat{s}) \right], \quad (49)
\]

\[
d_2^{SM} = -3d_0^{SM}. \quad (50)
\]

For the CP violating part we have:

\[
c_0^{CP} = -2c_0^{SM} \Im f_L^{CP} + m_t^2 + \hat{s} \left[ -2\hat{s} m_t^2 \ln \frac{m_t^2}{\hat{s}} + m_t^4 - \hat{s}^2 \right] \Im m g_R^{CP}, \quad (51)
\]

\[
c_1^{CP} = -2c_1^{SM} \Im f_L^{CP} \quad (52)
\]

\[
c_2^{CP} = -2c_2^{SM} \Im f_L^{CP} \quad (53)
\]

\[
d_0^{CP} = -2d_0^{SM} \Im f_L^{CP} \quad (54)
\]

\[
d_1^{CP} = -2d_1^{SM} \Im f_L^{CP} + \frac{2\hat{s}}{m_t^2} \left[ -2\hat{s} m_t^2 \ln \frac{m_t^2}{\hat{s}} + m_t^4 - \hat{s}^2 \right] \Im m g_R^{CP}, \quad (55)
\]

\[
d_2^{CP} = -3d_0^{CP} \quad (56)
\]

The coefficient $B$ is:

\[
B = \frac{-\pi \alpha_w^2 m_t^2 \text{Br}(t \to XX)}{8|\hat{s} - m_W^2|^2}. \quad (57)
\]

Relations (54) and (56) ensure that the polarization of the $t$ does not contribute to the total number of $b$ quarks from $t$ decay. The angular distribution in the laboratory system is obtained from (41) by the Lorentz boost (19):

\[
\frac{d^2\sigma}{d\cos \theta_x} = \frac{1}{2} \int \frac{d\hat{\sigma}^2}{d\cos \theta_x} \left[ f_u(x_u) f_d(x_d) + f_u(x_u) f_d(x_u) \right] dx_u dx_d. \quad (58)
\]
with
\[ \frac{d\hat{\sigma}_2^x}{d \cos \theta_x} = \frac{\mathcal{B}(1 - v^2)}{(1 - v \cos \theta_x)^4} \left[ b_0(1 - v \cos \theta_x)^2 
\right. \\
\left. + b_1(\cos \theta_x - v)(1 - v \cos \theta_x) + b_2(\cos \theta_x - v)^2 \right], \]
(59)
and \( \hat{s} \) and \( v \) defined in (24). The angular distribution of the decay products from \( \bar{t} \) decay is obtained from (59) by CP conjugation.

### 4.2 Total cross section asymmetries for the decay products

Analogously to (28) we define a total cross section asymmetry \( R_{2\text{tot}}^x \) for the secondary particles \( x \):
\[ R_{2\text{tot}}^x = \frac{\sigma_x^x - \sigma_{\bar{x}}^x}{\sigma_x^x + \sigma_{\bar{x}}^x}, \quad x = b, l^+ . \]
(60)
This asymmetry was first suggested for \( b \) quarks in [3], but without giving an explicit analytic expression. Using our result (58) and (59) we obtain:
\[ R_{2\text{tot}}^x = \frac{\int \mathcal{B}(b_0^{CP} + b_2^{CP}/3) f_u(x_u) f_d(x_d) dx_u dx_d}{\int \mathcal{B}(b_0^S + b_2^S/3) f_u(x_u) f_d(x_d) dx_u dx_d} \]
\[ = \frac{\int \mathcal{B}(c_0^{CP} + c_2^{CP}/3) f_u(x_u) f_d(x_d) dx_u dx_d}{\int \mathcal{B}(c_0^S + c_2^S/3) f_u(x_u) f_d(x_d) dx_u dx_d} . \]
(61)
As there is no dependence on the \( t \) polarization, (61) implies that the total cross section asymmetries for secondary \( b \) quarks and leptons are equal in magnitude:
\[ R_{2\text{tot}}^b = R_{2\text{tot}}^{l^+} . \]
(62)
This is a consequence of CP invariance in the decay \( W \to l\nu \). Eq. (61) also implies that
\[ R_{2\text{tot}}^x = -R_{1\text{tot}}^x, \quad x = b, l^+ . \]
(63)
However, (63) is not valid in general. As already pointed out in [3], \( R_{2\text{tot}}^x \) receives contributions from both the production and decay vertices. In the general case, (63) reads:
\[ R_{2\text{tot}}^x = -R_{1\text{tot}}^x + \delta_{1CP}^x, \quad x = b, l^+ . \]
(64)
where \( \delta_{1CP}^x \) is due to CP violation in the \( t \)-decay [10]
\[ \delta_{1CP}^x = \frac{\Gamma(t \to bW^+) - \Gamma(\bar{t} \to \bar{b}W^-)}{\Gamma(t \to bW^+) + \Gamma(\bar{t} \to \bar{b}W^-)} . \]
(65)
Testing (64) would be a model independent test of CP violation in the \( t \)-decay vertex.
4.3 Forward–backward asymmetries for the secondary products

For the secondary products \( x \) we can again define two forward–backward asymmetries in the laboratory frame:

\[
R_{1}^{FB}(x) = \frac{(\sigma_{\uparrow}^{F}_x - \sigma_{\downarrow}^{F}_x)}{(\sigma_{\uparrow}^{F} + \sigma_{\downarrow}^{F})} = \frac{\int [(\hat{\sigma}_{\uparrow}^{F}_x - \hat{\sigma}_{\downarrow}^{F}_x)] f_u(x_u) f_d(x_d) \, dx_u \, dx_d}{\int [(\hat{\sigma}_{\uparrow}^{F} + \hat{\sigma}_{\downarrow}^{F})] f_u(x_u) f_d(x_d) \, dx_u \, dx_d},
\]

\[
R_{2}^{FB}(x) = \frac{(\sigma_{\downarrow}^{B}_x - \sigma_{\uparrow}^{B}_x)}{(\sigma_{\downarrow}^{B} + \sigma_{\uparrow}^{B})} = \frac{\int [(\hat{\sigma}_{\downarrow}^{B}_x - \hat{\sigma}_{\uparrow}^{B}_x)] f_u(x_u) f_d(x_d) \, dx_u \, dx_d}{\int [(\hat{\sigma}_{\downarrow}^{B} + \hat{\sigma}_{\uparrow}^{B})] f_u(x_u) f_d(x_d) \, dx_u \, dx_d}.
\]

From (71) and (72) we obtain:

\[
R_{1}^{FB}(x) = \frac{\int B \left(c_0^{CP} + c_2^{CP}/3 - (v^2 - 1)b_1^{CP}/2 \right) f_u(x_u) f_d(x_d) \, dx_u \, dx_d}{\int B (c_0^{SM} + c_2^{SM}/3 - (v^2 - 1)b_1^{SM}/2) f_u(x_u) f_d(x_d) \, dx_u \, dx_d}
= -2 \frac{\int D(\hat{s}) \{k_1(\hat{s}) \Im m f_L + l_1(\hat{s}) \Im m g_R \} f_u(x_u) f_d(x_d) \, dx_u \, dx_d}{\int D(\hat{s}) k_1(\hat{s}) f_u(x_u) f_d(x_d) \, dx_u \, dx_d}
\]

with

\[
k_1(\hat{s}) = (m_t^2 - \hat{s}) \left[ 2m_t^4 - \hat{s}^2(7 - 3v^2) - \hat{s}m_t^2((1 - 3v^2) - 6\alpha_x(1 - v^2)) \right] + 6(1 - \alpha_x)(1 - v^2) m_t^2 \hat{s}^2 \ln \frac{m_t^2}{\hat{s}},
\]

\[
l_1(\hat{s}) = (m_t^2 - \hat{s}) \left[ 2(m_t^2 - \hat{s})^2 + 3\alpha_x(1 - v^2)(m_t^2 + \hat{s}) \right] - 6\alpha_x(1 - v^2) m_t^2 \hat{s}^2 \ln \frac{m_t^2}{\hat{s}},
\]

and

\[
R_{2}^{FB}(x) = \frac{\int B \left(c_0^{CP} + c_2^{CP}/3 + (v^2 - 1)b_1^{CP}/2 \right) f_u(x_u) f_d(x_d) \, dx_u \, dx_d}{\int B (c_0^{SM} + c_2^{SM}/3 + (v^2 - 1)b_1^{SM}/2) f_u(x_u) f_d(x_d) \, dx_u \, dx_d}
= -2 \frac{\int D(\hat{s}) \{k_2(\hat{s}) \Im m f_L + l_2(\hat{s}) \Im m g_R \} f_u(x_u) f_d(x_d) \, dx_u \, dx_d}{\int D(\hat{s}) k_2(\hat{s}) f_u(x_u) f_d(x_d) \, dx_u \, dx_d}
\]

with

\[
k_2(\hat{s}) = (m_t^2 - \hat{s}) \left[ 2m_t^4 - \hat{s}^2(1 + 3v^2) + \hat{s}m_t^2((5 - 3v^2) - 6\alpha_x(1 - v^2)) \right] - 6(1 - \alpha_x)(1 - v^2) m_t^2 \hat{s}^2 \ln \frac{m_t^2}{\hat{s}},
\]

\[
l_2(\hat{s}) = (m_t^2 - \hat{s}) \left[ 2(m_t^2 - \hat{s})^2 - 3\alpha_x(1 - v^2)(m_t^2 + \hat{s}) \right] + 6\alpha_x(1 - v^2) m_t^2 \hat{s}^2 \ln \frac{m_t^2}{\hat{s}}.
\]
The factor $D(\hat{s})$ is

\[ D(\hat{s}) = [\hat{s}(\hat{s} - m_W^2)]^{-2}. \]  

(74)

Note that $R_{1,2}^{FB}$ are polarization asymmetries, i.e. they measure different combinations of the CP violating contributions to the $t$ quark polarization. This can also be seen from the explicit expressions for $k_{1,2}$ and $l_{1,2}$. As a consequence, the forward–backward asymmetries for the $b$ quarks are different from those for the leptons. Note, moreover, that the contribution from $t$ polarization enters only through the term $b_1$ which is linear in $\cos \theta_x$, which ensures that polarization does not contribute to the total cross section.

5 $b$ quark forward–backward asymmetry for the sum of the cross sections

An asymmetry that seems most convenient what concerns statistics is the forward–backward asymmetry for the total number of $b$ and $\bar{b}$ quarks from both production and decay:

\[ A_{FB}^{FB}(b) = \frac{\sigma_F^{b} - \sigma_{\bar{b}}^{b}}{\sigma_F^{b} + \sigma_{\bar{b}}^{b}} \]  

(75)

where

\[ \sigma_F^{b\bar{b}} = (\sigma_1)_F^{b\bar{b}} + (\sigma_2)_F^{b\bar{b}}, \quad \sigma_{\bar{b}b} = (\sigma_1)_{\bar{b}}^{b} + (\sigma_2)_{\bar{b}}^{b}. \]  

(76)

We have:

\[ A_{FB}^{FB}(b) = \frac{\int [C a_1^{CP} + B b_1^{CP}] (1 - v^2) f_u(x_u) f_d(x_d) dx_u dx_d}{\int [4 C (a_0^{SM} + a_2^{SM}/3) + (1 - v^2) (C a_1^{SM} + B b_1^{SM})] f_u(x_u) f_d(x_d) dx_u dx_d}, \]  

(77)

where we have used the fact that in the SM the total numbers of $b$ quarks from production and decay are the same:

\[ (C a_0^{SM} + B b_0^{SM}) + (C a_2^{SM} + B b_2^{SM})/3 = 2C(a_0^{SM} + a_2^{SM}/3). \]  

(78)

Explicitly, in terms of the CP violating form factors, we obtain:

\[ A_{FB}^{FB}(b) = \frac{6 \int D(\hat{s}) [k(\hat{s}) \Im f_F^{CP} + l(\hat{s}) \Im g_R^{CP}] f_u(x_u) f_d(x_d) dx_u dx_d}{\int D(\hat{s}) m(\hat{s}) f_u(x_u) f_d(x_d) dx_u dx_d}. \]  

(79)
with

\[
  k(\hat{s}) = 2\hat{s} \left[ (\hat{s} - m_t^2)(\hat{s} + (2 - 3\alpha_b)m_t^2) + 3(1 - \alpha_b)m_t^2 \hat{s} \ln \frac{m_t^2}{\hat{s}} \right], \tag{80}
\]

\[
  l(\hat{s}) = 3\alpha_b \hat{s} \left[ m_t^4 - \hat{s}^2 - 2m_t^2 \hat{s} \ln \frac{m_t^2}{\hat{s}} \right], \tag{81}
\]

\[
  m(\hat{s}) = -2 \left[ (m_t^2 - \hat{s}) \left(2m_t^4 + m_t^2 \hat{s}(2 + 3\alpha_b(1 - v^2)) - \hat{s}^2(7 - 3v^2)\right) + 3m_t^2 \hat{s}^2 (1 - \alpha_b) (1 - v^2) \ln \frac{m_t^2}{\hat{s}} \right]. \tag{82}
\]

Assuming CP violation only in the $t$-production vertex, the total cross sections $\sigma^b$ and $\sigma^{\bar{b}}$ for the $b$ and $\bar{b}$ quarks from both production and decay are equal, i.e. the total cross section asymmetry $R_{\text{tot}}^b$ is zero:

\[
  R_{\text{tot}}^b = \frac{\sigma^b - \sigma^{\bar{b}}}{\sigma^b + \sigma^{\bar{b}}} = 0. \tag{83}
\]

However, when CP violation in $t$-decay is also considered, we have:

\[
  R_{\text{tot}}^b = \frac{\delta_{\text{CP}}}{2}. \tag{84}
\]

In the general case, we thus have three different total cross section asymmetries and they measure three different quantities. The total cross section asymmetry of the $b$-quarks from production, $R_{1\text{tot}}^b$, measures CP violation in the production vertex, the total cross section asymmetry of the $t$-decay products, $R_{2\text{tot}}^x$, $x = b,l$ measures CP violation both in the production and decay vertices, the total cross section asymmetry of the $b$-quarks from both production and decay, $R_{\text{tot}}^b$, measures CP violation in the $t$-decay vertex.

Neglecting CP violation in $t$-decay, we are left with only one forward–backward asymmetry $A_{\text{FB}}$ when all $b$ and $\bar{b}$ quarks from single $t$-quark production are counted.

6 Numerical results and discussion within the MSSM

In this Section we present numerical results for the discussed asymmetries in the framework of the Minimal Supersymmetric Standard Model (MSSM) with CP violating phases. In the
MSSM one has the gaugino mass parameters $M_1 = M'$, $M_2 = M$, $M_3 = m_{\tilde{g}}$ corresponding to the groups U(1), SU(2), SU(3), where $m_{\tilde{g}}$ is the gluino mass. We assume the GUT relations:

$$m_{\tilde{g}} = (\alpha_s/\alpha_2) M \sim 3M, \quad M' = (5/3) \tan^2 \theta_W M \sim \frac{1}{2} M$$

with a common phase of $M_i$ which can be rotated away by an $R$ rotation. The only complex parameters relevant for our discussion thus are: the Higgsino mass parameter $\mu = |\mu| e^{i\phi_\mu}$ and the SUSY breaking trilinear couplings of the stops and sbottoms, $A_t = |A_t| e^{i\phi_t}$ and $A_b = |A_b| e^{i\phi_b}$. As we work in the limit of $m_b = 0$ the phase $\phi_b$ does not play any role. The phase of $\mu$ is strongly constrained by the upper bounds of the electric dipole moments of the electron and neutron. $\phi_\mu$ must be very small except the SUSY masses are large ($> 1$ TeV) [11] or there are strong cancellations between the different contributions [12]. We therefore take $\phi_\mu = 0$. After this the only relevant phase we are left with for the considered CP violating asymmetries is the phase $\phi_t$.

In the numerical analysis we assume CP violation only in the production vertex and not in the decay of the $t$ quark: CP violation in the decay would mean that in addition to CP violating phases there are also new decay modes of the top quark. According to the present experimental limits there is only a small window left for the decay $t \rightarrow \tilde{\chi}_1^0 + \tilde{t}_1$. A detailed analysis of CP violation in $t$-decay was performed in [3].

At one–loop level, the reactions $u\bar{b} \rightarrow t\bar{b}$, $d\bar{u} \rightarrow b\bar{t}$ receive radiative corrections from triangle and box graphs with charginos $\chi^+_i$, neutralinos $\chi^0_j$, squarks $\tilde{q}_i$ ($i = 1, 2; j = 1...4$), and gluinos $\tilde{g}$ in the loops. The analytic expressions for the form factors due to these diagrams have been worked out in [3]. Following [3], in the limit $\phi_\mu \rightarrow 0$, $m_{u,d} \rightarrow 0$, only the graphs with $(\chi^+ \chi^0 t)$ and $(\tilde{t} \tilde{b} \tilde{g})$ loops (see Fig. 1) contribute to CP violation and the contribution from the box diagrams is negligible. We thus base our numerical analysis on the two contributions – from the $(\chi^+ \chi^0 t)$ and $(\tilde{t} \tilde{b} \tilde{g})$ loops. Our formulae for the form factors agree with those in [3] and are given in Appendix B.

We now want to analyze the influence of a possibly large phase of $A_t$ on the CP violating asymmetries defined in the previous Sections. For this purpose we choose three scenarios of gaugino–higgsino mixing: (i) a gaugino scenario with $M = 116$ GeV, $\mu = 400$ GeV, (ii)
a Higgsino scenario with $M = 400$ GeV, $\mu = 116$ GeV, and (iii) a ‘mixed’ scenario with $M = \mu = 168$ GeV. Further, we take $\tan \beta = 4$, $m_{\tilde{Q}} = 300$ GeV, $m_{\tilde{U}} = 270$ GeV, and $m_{\tilde{D}} = 330$ GeV. The SM parameters are: $m_t = 174$ GeV, $m_W = 80.03$ GeV, $\sin^2 \theta_W = 0.23$ $\alpha(m_Z) = 1/129$, and $\alpha_s(m_Z) = 0.12$ (we neglect the bottom mass, $m_b = 0$). For the structure functions of the proton we use the CTEQ5 parton distribution function cteq5m1 [13]. We leave out the theoretical uncertainties associated with the QCD corrections and parton distribution functions discussed in [14] since these uncertainties, being CP-even, cannot mimic the CP-violating asymmetry discussed here.

Figures 2–4 show the resulting CP violating asymmetries for the three scenarii as a function of $\phi_t$. $|A_t|$ is chosen such that $m_{\tilde{t}_1} \simeq 96$ GeV. Moreover, we have $m_{\tilde{\chi}_1^+} \simeq 104$ GeV and $m_{\tilde{\chi}_1^0} \sim 50–100$ GeV. The asymmetries are typically of the order of $10^{-4}$. Note, however, that the masses in our scenarii are just at the borderline of the experimentally allowed values [15]. If the mass spectrum becomes heavier the CP violating asymmetries quickly decrease. The asymmetries also decrease with increasing $\tan \beta$.

The asymmetries $A_{2FB}(b)$ and $R_{2FB}(b)$ seem to be the most promising ones — they reach up to $\sim 0.1\%$. Our results are an order of magnitude smaller then the estimates obtained in [3]. A general result for the two types of forward-backward asymmetries is that

$$A_{2FB} \geq R_{1}^{tot} \geq A_{1FB}, \quad R_{2FB} \geq R_{1FB}, \quad (86)$$

and $A_{2FB}$ ($R_{2FB}$) in some cases can be $\sim 2–3$ times bigger than $A_{1FB}$ ($R_{1FB}$). Another general feature is that the forward-backward asymmetries for the decay products $R_{1(2)}^{FB}$ are almost equal for $b$-quarks and leptons:

$$R_{1(2)}^{FB} (b) \simeq R_{1(2)}^{FB} (l). \quad (87)$$

The forward-backward asymmetry $A^{FB}$ for the sum of $b$ quarks from both production and decay turns out to be extremely small $\lesssim 0.02\%$. 

14
7 Conclusions

We have studied the process of single $t$-quark production in $p\bar{p}$ collisions. Assuming CP violation in the $t$-production vertex we have defined different angular and total-cross-section asymmetries as measurable quantities. General analytic expressions for these asymmetries in terms of the corresponding CP violating form factors are obtained. Relations sensitive to CP violation in the $t$-decay vertex are defined. We want to emphasize that the formulae are valid independently of the origin of CP violation.

We have performed a numerical analysis within the MSSM with complex phases. In accordance with the constraints from the measurements of the electric dipole moments of the electron and the neutron we have only kept the influence of the phase of $A_t$. The effects turn out to be rather small. The discussed asymmetries are of the order $10^{-3} - 10^{-4}$ and they decrease as the mass spectrum and $\tan\beta$ increase.

The cross section for $p\bar{p} \to W^+ \to t\bar{b}$ is $\sim 340$ fb. This means $\sim 10^4$ events for RUN II of the Tevatron with an integrated luminosity of $30$ fb$^{-1}$ (15 fb$^{-1}$ per experiment). To measure an asymmetry of the order of $10^{-4}$, one would need about $10^8$ events. This is far beyond the observability of such an asymmetry at the Tevatron. Nevertheless, it would be worthwhile to look for CP violation in top physics as it would imply not only physics beyond the Standard Model, but also beyond the Minimal Supersymmetric Standard Model.

8 Acknowledgements

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A Differential cross section for the $t$ decay

A.1 The $t$ polarization four–vector

Let us write the amplitude of the process (2) in the form:

$$M = \bar{u}(p_t) M_1 u(-p_b)$$

(88)

where

$$M_1 = \frac{g^2}{2} \bar{u}(-p_d) \gamma_\alpha \frac{1 - \gamma_5}{2} u(p_u) \frac{-i}{\hat{s} - m_W^2} \Gamma^\alpha .$$

(89)

The polarization four–vector $\xi_\alpha$ of the $t$ quarks is determined by

$$\xi_\alpha = \left( g_{\alpha\beta} - \frac{p_\alpha p_\beta}{m_t^2} \right) \frac{Tr \left\{ M_1 \Lambda(-p_b) \bar{M}_1 \Lambda(p_t) \gamma^\beta \gamma^5 \right\} }{Tr \left\{ M_1 \Lambda(-p_b) \bar{M}_1 \Lambda(p_t) \right\} } , \quad \Lambda(p_t) = p_t + m_t .$$

(90)

In the most general form the covariant decomposition of $\xi^\alpha$ reads

$$\xi^\alpha = Q_1^\alpha \mathcal{P}_1^t + Q_2^\alpha \mathcal{P}_2^t + \varepsilon(\alpha, p_u, p_d, p_t) \mathcal{D}_t .$$

(91)

Here

$$Q_1^\alpha = p_u^\alpha - \frac{(p_u p_t)}{m_t^2} p_t^\alpha , \quad Q_2^\alpha = p_d^\alpha - \frac{(p_d p_t)}{m_t^2} p_t^\alpha$$

(92)

are two four–vectors in the production plane orthogonal to $p_t$, and $\varepsilon(\alpha, p_u, p_d, p_t)$ is a four–vector orthogonal to $p_t$ and to the production plane. Using the notation

$$\mathcal{N}(c_t) = a_0^b + a_1^b c_t + a_2^b c_t^2 , \quad c_t \equiv \cos \theta_t ,$$

(93)

and the usual Mandelstam variables

$$\hat{s} = (p_u + p_d)^2 , \quad \hat{t} = (p_u - p_b)^2 = (p_d - p_t)^2 , \quad \hat{s} + \hat{t} + \hat{u} = m_t^2$$

(94)

we obtain in the CMS of $u\bar{d}$:

$$\mathcal{P}_1^t = \frac{-1}{(m_t^2 - \hat{s}) \mathcal{N}(c_t)} \frac{3m_\tilde{g}_R^{CP}}{m_t} \left\{ (m_t^2 - \hat{t}) \left[ 4(\hat{s} - m_t^2) - 2(m_t^2 - \hat{u}) - t \right] 
- \hat{u}(m_t^2 - \hat{u}) + m_t^2(4\hat{u} + 3\hat{s}) - \hat{s}^2 \right\} ,$$

(95)
\[
\mathcal{P}_2' = \frac{1}{(m_t^2 - \hat{s}) \mathcal{N}(c_t)} \left\{ -4 m_t \hat{t} (1 - 2 \Im m f_{CP}^L) \right.
\]
\[
- \frac{\Im m g_{CP}^R}{m_t} \left[ (m_t^2 - \hat{t}) [-2(m_t^2 - u) - t] - \hat{u}(m_t^2 - \hat{u}) - m_t^2 (4\hat{t} - 3\hat{s}) - \hat{s}^2 \right] \right\}, \quad (96)
\]
\[
\mathcal{D}' = \frac{1}{(m_t^2 - \hat{s}) \mathcal{N}(c_t)} \frac{4 \Re e g_{CP}^R}{m_t} (\hat{s} - m_t^2). \quad (97)
\]

### A.2 General formula for the differential cross section

From (95), (97) and (99) we obtain the general formula for the differential cross section for the production of the $t$ quarks and their subsequent decay in the CMS of $u\bar{d}$:

\[
d\sigma_2^x = \frac{C}{4\pi} \frac{m_t^2 Br(t \rightarrow xX)}{E_t^2 (1 - \beta_t \cos \theta_{tx})^2} \left\{ \mathcal{N}^{SM} + \mathcal{N}^{CP} \right. \]
\[
+ \alpha_x \frac{m_t \sqrt{\hat{s}}}{2 E_t (1 - \beta_t \cos \theta_{tx})} \left[ (\mathcal{P}_+^{SM} + \mathcal{P}_+^{CP}) \left( 1 - \frac{E_t^2}{m_t^2} (1 - \beta_t \cos \theta_{tx}) \right) \right. \]
\[
- (\mathcal{P}_-^{SM} + \mathcal{P}_-^{CP}) \left( \cos \theta_x - \beta_t \cos \theta_{tx} \frac{E_t^2}{m_t^2} (1 - \beta_t \cos \theta_{tx}) \right) \]
\[
\left. \left. - \mathcal{D} \frac{\hat{s}}{2} |\vec{p}_t| (\vec{n}_u \vec{n}_t \vec{n}_x) \right] \right\} \ d \cos \theta_t \ d \cos \theta_x \ d\phi_x, \quad (98)
\]

where we have used the notation

\[
\mathcal{P}_+^{SM} = \mathcal{P}_1' + \mathcal{P}_2', \quad \mathcal{P}_+^{CP} = \mathcal{P}_+^{SM} + \mathcal{P}_+^{CP}, \quad \mathcal{D} = \mathcal{D}', \quad (99)
\]

and

\[
\mathcal{N}(c_t) = \mathcal{N}^{SM}(c_t) + \mathcal{N}^{CP}(c_t), \quad (100)
\]
\[
E_t = \frac{\hat{s} + m_t^2}{2\sqrt{\hat{s}}}, \quad |\vec{p}_t| = \frac{\hat{s} - m_t^2}{2\sqrt{\hat{s}}}, \quad \beta_t = \frac{\hat{s} - m_t^2}{\hat{s} + m_t^2}, \quad (101)
\]

$(\vec{n}_u \vec{n}_t \vec{n}_x)$ denotes the triple product

\[
(\vec{n}_u \vec{n}_t \vec{n}_x) = \vec{n}_u \times \vec{n}_t \cdot \vec{n}_x \quad (102)
\]

with $\vec{n}_u$ the unit vector pointing in the direction of $\vec{p}_u$, etc.
B The CP violating form factors in the MSSM

B.1 Chargino–neutralino–stop loop

The CP violating form factors from the $\tilde{t}_n \tilde{\chi}_j^+ \tilde{\chi}_k^0$ ($n, j = 1, 2; k = 1...4$) loop are:

\[
f^L_{CP}(\tilde{\chi}^+) = \frac{\alpha_w}{4\pi} \left\{ \mathcal{O}_1 \left[ 2 C_{00} - m_t^2 C_{12} + s (C_{22} + C_{12}) \right] + \mathcal{O}_2 m_t \tilde{m}_k^0 C_2 - \mathcal{O}_3 m_t \tilde{m}_j^+ (C_0 + C_2) - \mathcal{O}_4 \tilde{m}_j^+ \tilde{m}_k^0 C_0 \right\}
\]

(103)

and

\[
g^R_{CP}(\tilde{\chi}^+) = \frac{\alpha_w}{4\pi} m_t \left\{ \mathcal{O}_1 m_t C_{12} - \mathcal{O}_2 \tilde{m}_k^0 C_2 + \mathcal{O}_3 \tilde{m}_j^+ (C_0 + C_1 + C_2) \right\}.
\]

(104)

Here we use the notation $\tilde{m}_j^+ = m_{\tilde{\chi}_j^+}$ and $\tilde{m}_k^0 = m_{\tilde{\chi}_k^0}$. The $C_X$ are the standard three–point functions [16] for which we follow the convention of [17]. In this case,

\[
C_X \equiv C_X(m_b^2, m_t^2, \hat{s}, m_{\tilde{\chi}_j^+}^2, m_{\tilde{\chi}_k^0}^2), \quad X \in \{0, 1, 2, 00, 11, 12, 22\}.
\]

(105)

See Appendix B.3 for the explicit definition of the $C_X$. The couplings $\mathcal{O}_i \equiv \mathcal{O}_{i^njk}$ are

\[
\mathcal{O}_1^{njk} = -\sqrt{2} \Im (l_{nj}^i O_{kj}^L a_{nk}^i),
\]

(106)

\[
\mathcal{O}_2^{njk} = -\sqrt{2} \Im (l_{nj}^i O_{kj}^L b_{nk}^i),
\]

(107)

\[
\mathcal{O}_3^{njk} = -\sqrt{2} \Im (l_{nj}^i O_{kj}^R b_{nk}^i),
\]

(108)

\[
\mathcal{O}_4^{njk} = -\sqrt{2} \Im (l_{nj}^i O_{kj}^R a_{nk}^i),
\]

(109)

with $l_{nj}^i$, $O_{kj}^{L,R}$, $a_{nk}^i$, and $b_{nk}^i$ given in Appendix C. Notice that in (103) and (104) one has to sum over $n, j, k$.

B.2 Gluino–stop–sbottom loop

The $\tilde{g} \tilde{t}_n \tilde{b}_m$ loop gives:

\[
f^L_{CP}(\tilde{g}) = 0,
\]

(110)

\[
g^R_{CP}(\tilde{g}) = -\frac{2}{3} \frac{\alpha_s}{\pi} m_t m_{\tilde{g}} \mathcal{O}_5 C_1,
\]

(111)
where
\[ C_1 \equiv C_1(m_b^2, m_t^2, \hat{s}, m_{\tilde{b}_m}^2, m_{\tilde{g}}^2, m_{\tilde{t}_n}^2) \] (112)
and
\[ O_5 \equiv O_5^{mn} = |\Gamma_{Lm}^\tilde{b}|^2 \Im \left( \Gamma_{Ln}^\tilde{t} \Gamma_{Rn}^\tilde{t} \right), \] (113)

With the explicit expression for the stop mixing matrix we have, see Appendix C,
\[ |\Gamma_{Lm}^\tilde{b}|^2 = \{ \cos^2 \theta_{\tilde{b}}, \sin^2 \theta_{\tilde{b}} \}, \quad m = 1, 2, \] (114)
\[ \Im \left( \Gamma_{Ln}^\tilde{t} \Gamma_{Rn}^\tilde{t} \right) = \frac{1}{2} \sin \varphi_{\tilde{t}} \sin 2 \theta_{\tilde{t}} \{ 1, -1 \}, \quad n = 1, 2. \] (115)

Again, in (111) a summation over \( m, n \) is assumed.

### B.3 Three point functions

Here we give the definition of the Passarino–Veltman three point functions [16] used above in the convention of [17]. For the general denominators we use the notation
\[ D^0 = q^2 - m_0^2 \quad \text{and} \quad D^j = (q + p_j)^2 - m_j^2. \] (116)

Then the loop integrals in \( D = 4 - \epsilon \) dimensions are as follows:
\[ C_0 = \frac{1}{i\pi^2} \int d^D q \frac{1}{D^0 D^1 D^2}, \] (117)
\[ C_\mu = \frac{1}{i\pi^2} \int d^D q \frac{q_\mu}{D^0 D^1 D^2} = p_{1\mu} C_1 + p_{2\mu} C_2, \] (118)
\[ C_{\mu\nu} = \frac{1}{i\pi^2} \int d^D q \frac{q_\mu q_\nu}{D^0 D^1 D^2} \]
\[ = g_{\mu\nu} C_{00} + p_{1\mu} p_{1\nu} C_{11} + (p_{1\mu} p_{2\nu} + p_{2\mu} p_{1\nu}) C_{12} + p_{2\mu} p_{2\nu} C_{22}. \] (119)

where the \( C \)'s have \( (p_1^2, (p_1 - p_2)^2, p_2^2, m_0^2, m_1^2, m_2^2) \) as their arguments.

### C Masses, Mixing Matrices, and Couplings

The neutralino mass matrix in the basis of
\[ \Psi_j^0 = \left( -i\lambda', -i\lambda^3, \psi_{H_1}^0, \psi_{H_2}^0 \right) \] (120)
\[
\mathcal{M}^N = \begin{pmatrix}
M' & 0 & -m_Z \sin \theta_W \cos \beta & m_Z \sin \theta_W \sin \beta \\
0 & M & m_Z \cos \theta_W \cos \beta & -m_Z \cos \theta_W \sin \beta \\
-m_Z \sin \theta_W \cos \beta & m_Z \cos \theta_W \cos \beta & 0 & -\mu \\
m_Z \sin \theta_W \sin \beta & -m_Z \cos \theta_W \sin \beta & -\mu & 0
\end{pmatrix}
\]

Here \( \tan \beta = v_2/v_1 \). This matrix is diagonalized by the unitary neutralino mixing matrix \( N \):

\[
N^* \mathcal{M}^N N^\dagger = \mathcal{M}_D^N
\]

where \( \mathcal{M}_D^N \) is a diagonal matrix with non-negative elements — \( \tilde{m}_1^0, \tilde{m}_2^0, \tilde{m}_3^0, \tilde{m}_4^0 \) — the masses of the physical neutralino states.

The chargino mass matrix is:

\[
\mathcal{M}^C = \begin{pmatrix}
M & \sqrt{2}m_W \sin \beta \\
\sqrt{2}m_W \cos \beta & \mu
\end{pmatrix}
\]

It is diagonalized by the two unitary matrices \( U \) and \( V \):

\[
U^* \mathcal{M}^C V^\dagger = \mathcal{M}_D^C
\]

where \( \mathcal{M}_D^C \) is a diagonal matrix with non-negative entries — \( \tilde{m}_1^+, \tilde{m}_2^+ \) — the masses of the physical chargino states.

The mass matrix of the stops in the basis \((\tilde{t}_L, \tilde{t}_R)\) is

\[
\mathcal{M}_t^2 = \begin{pmatrix}
m_Z^2 + m_Z^2 \cos 2\beta (\frac{1}{2} - \frac{2}{3} \sin^2 \theta_W) + m_t^2 & (A_t^* - \mu \cot \beta) m_t \\
(A_t - \mu \cot \beta) m_t & m_Z^2 + \frac{2}{3} m_Z^2 \cos 2\beta \sin^2 \theta_W + m_t^2
\end{pmatrix}
\]

\( \mathcal{M}_t^2 \) is diagonalized by the rotation matrix \( \Gamma \) such that \( \Gamma^\dagger \mathcal{M}_t^2 \Gamma = \text{diag}(m_{t_1}^2, m_{t_2}^2) \) and \( (\tilde{t}_L^\dagger) = \Gamma (\tilde{t}_L) \). We have:

\[
\Gamma = \begin{pmatrix}
\Gamma_{L1}^\dagger & \Gamma_{L2}^\dagger \\
\Gamma_{R1}^\dagger & \Gamma_{R2}^\dagger
\end{pmatrix} = \begin{pmatrix}
e^{-\frac{i}{2} \varphi_t} \cos \theta_t & -e^{-\frac{i}{2} \varphi_t} \sin \theta_t \\
e^{\frac{i}{2} \varphi_t} \sin \theta_t & e^{\frac{i}{2} \varphi_t} \cos \theta_t
\end{pmatrix}
\]
The interaction Lagrangian which we need is:

\[ \mathcal{L}_{b\bar{t}_n\tilde{\chi}^j} = g\bar{b}(k_{nj}^i P_L + \bar{t}_{nj}^i P_R) (\tilde{\chi}_j^+ \tilde{\chi}_n) + \text{h.c.} \]  \hfill (125)

\[ \mathcal{L}_{i\bar{t}_n\tilde{\chi}_k^0} = g\bar{t}(b_{nk}^i P_L + a_{nk}^i P_R) \tilde{\chi}_k^+ \tilde{\chi}_n + \text{h.c.} \]  \hfill (126)

\[ \mathcal{L}_{\tilde{\chi}_j^+ \tilde{\chi}_k^0 W} = g\chi_0^+ \gamma^\alpha (O_{kj}^L P_L + O_{kj}^R P_R) \tilde{\chi}_j^+ W_{\alpha}^- + \text{h.c.} \]  \hfill (127)

The chargino–stop–bottom couplings are:

\[ k_{nj}^i = +h_b U_{j2}^\Gamma \tilde{\Gamma}_L, \]  \hfill (128)

\[ \bar{t}_{nj}^i = -V_{j1}^\Gamma \Gamma_L + h_t V_{j2}^\Gamma \Gamma_R, \]  \hfill (129)

with

\[ h_t = \frac{m_t}{\sqrt{2} m_W \sin \beta}, \quad h_b = \frac{m_b}{\sqrt{2} m_W \cos \beta}. \]  \hfill (130)

The neutralino–stop–top couplings are:

\[ a_{nk}^i = f_{Lk}^i \Gamma_L, \]  \hfill (131)

\[ b_{nk}^i = h_{Lk}^i \Gamma_L + f_{Rk}^i \Gamma_R, \]  \hfill (132)

with

\[ f_{Lk}^i = -\frac{1}{\sqrt{2}} (N_k^2 + \frac{1}{3} \tan \theta_W N_k), \]  \hfill (133)

\[ f_{Rk}^i = \frac{2\sqrt{2}}{3} \tan \theta_W N_k^*, \]  \hfill (134)

\[ h_{Lk}^i = -h_t N_k^*, \]  \hfill (135)

in the basis \( \Psi^0_j = (-i\lambda', -i\lambda^3, \psi_{H1}^0, \psi_{H2}^0). \) The chargino–neutralino–W couplings are given by:

\[ O_{kj}^L = -\frac{1}{\sqrt{2}} N_{k4} V_{j2}^* + N_{k2} V_{j1}^*, \]  \hfill (136)

\[ O_{kj}^R = \frac{1}{\sqrt{2}} N_{k4}^* U_{j2} + N_{k2}^* U_{j1}. \]  \hfill (137)
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Figure 1: One-loop Feynman diagrams for $u\bar{d} \rightarrow t\bar{b}$.
Figure 2: CP violating asymmetries (in units of $10^{-4}$) for the gaugino scenario: $M = 116$ GeV, $\mu = 400$ GeV, $\tan \beta = 4$, $m_{\tilde{Q}} = 300$ GeV, $m_{\tilde{U}} = 270$ GeV, $m_{\tilde{D}} = 330$ GeV, $m_{\tilde{\chi}^+} \simeq 104$ GeV, and $m_{\tilde{t}_1} \simeq 96$ GeV.
Figure 3: CP violating asymmetries (in units of $10^{-4}$) for the higgsino scenario: $M = 400$ GeV, $\mu = 116$ GeV, $\tan \beta = 4$, $m_{\tilde{Q}} = 300$ GeV, $m_{\tilde{U}} = 270$ GeV, $m_{\tilde{D}} = 330$ GeV, $m_{\tilde{\chi}^+} \simeq 104$ GeV, and $m_{\tilde{t}_1} \simeq 96$ GeV.
Figure 4: CP violating asymmetries (in units of $10^{-4}$) for the ‘mixed’ scenario: $M = \mu = 168$ GeV, $\tan \beta = 4$, $m_Q = 300$ GeV, $m_{\tilde{U}} = 270$ GeV, $m_{\tilde{D}} = 330$ GeV, $m_{\tilde{\chi}^+} \simeq 104$ GeV, and $m_{\tilde{t}_1} \simeq 96$ GeV.