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Bin Zuo, Jianping Li, Cheng Sun & Xin Zhou

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A new statistical method for detecting trend turning

Bin Zuo 1 · Jianping Li 2,3 · Cheng Sun 2 · Xin Zhou 4

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Abstract

When long time series are analyzed, two nearby periods may show significantly different trends, which is known as trend turning. Trend turning is common in climate time series and crucial when climate change is investigated. However, the available detection methods for climate trend turnings are relatively few, especially for the methods which have the ability of detecting multiple trend turnings. In this article, we propose a new methodology named as the running slope difference (RSD) \( t \) test to detect multiple trend turnings. This method employs a \( t \)-distributed statistic of slope difference to test the sub-series trend difference of the time series, thereby identifying the turning points. We compare the RSD \( t \) test method with some other existing trend turning detection methods in an idealized time series case and several climate time series cases. The results indicate that the RSD \( t \) test method is an effective tool for detecting climate trend turnings.

Keywords Climate trend turning · Turning point estimation · Statistical detection method · Time series analysis · Global warming

1 Introduction

Global mean surface air temperature (GMT) rose roughly 0.85 °C from 1880 to 2012 (IPCC 2013), attributing mainly to an increase in atmospheric greenhouse gases (GHGs). The earth’s climate is a multiple-factor complex system; although GHGs have been monotonically increasing, GMT may result in little or no warming over some decadal timescale periods (Chen et al. 2015; Li et al. 2013, 2018; England et al. 2014; Trenberth and Fasullo 2013). Temperature change from warming to little or no warming (or vice versa) is a typical example of climate trend turning (also known as trend change or structural change of the trend). Trend turning is a commonly observed phenomenon in many climate parameters, such as precipitation (Alexander et al. 2006; Groisman and Easterling 1994), tropical cyclone frequency (Cheung et al. 2015), haze days (Zhao et al. 2016), and the index of some climate patterns like NAO (Hurrell 1995; Li and Wang 2003). Understanding trend turning is important because it can influence the interpretation of climate variation as well as guiding future research directions.

Periods of time exhibiting GHGs increases without simultaneous GMT warming are known as warming hiatus (or warming slowdown). From the 1940s to the early 1970s, there was a three-decade-long hiatus period (England et al. 2014; Karl et al. 2000; Li et al. 2013; Xing et al. 2017) with the CO₂ concentration rapidly increasing since the 1950s (IPCC 2013). And recently, a relatively slow rate of GMT warming since the early 2000s to 2014 was considered as another hiatus event by many climate scientists (Chen and Luo 2017; Easterling and Wehner 2009; England et al. 2014; Feng et al. 2017; Huang et al. 2017; Kosaka and Xie 2013; Li et al. 2013; Trenberth and Fasullo 2013; Yu and Lin 2018; Yu et al. 2017; Li et al. 2018). A recent work by Meehl et al. (2014) shows that only a few of IPCC climate models correctly projected the global warming slowdown in the early 2000s (10 out of 262 uninitialized CMIP5 simulations actually projected the observed warming trend), and this result indicates that we are still lacking knowledge about decadal timescale variability of global warming and more efforts should be made to improve future climate projection.
In the previous study, significant research attention has been focused on long-term linear trends of climate. Many well-known trend test methodologies such as Student’s t test, Mann-Kendall trend test (Mann 1945; Kendall 1975), Theil-Sen slope estimator (Theil 1950; Sen 1968), and innovative trend analysis (Sen 2012) have been widely used in climate research. However, the available detection methods for climate trend turning are relatively few. And many of these available methods are restricted to only a single turning point (Chu and White 1992; Andrews 1993; Ploberger and Kramer 1996; Perron and Zhu 2005; Toms and Lesperance 2003).

Previous studies have pointed out that the ability of detecting multiple changes is important for climate detection methods (Jiang et al. 2002; Xiao and Li 2007). Currently, there are mainly two types of methods for detecting multiple trend turnings. One is the optimal piecewise linear regression (OPLR; Liu et al. 2010; Tomé and Miranda 2004; Yao et al. 2017), and the other one is running trend test (Maher et al. 2014; Meehl et al. 2011; Thanasis et al. 2011). However, both existing methods usually fail to stress the statistical significance test of the sub-series trend difference during their detections. The OPLR method finds a piecewise regression solution which minimizes the residual sum under the condition that two nearby sub-series trends must have opposite sign (Tomé and Miranda 2004). But opposite sign of two trends cannot be equal to significant difference of two trends. Using opposite sign as the condition for identifying the turning points (also mentioned as trend breakpoints or trend change points) will reject trend turnings in which both sub-series have the same sign, and may increase the risk of false alarm. The running trend test methods detect trend turnings making use of existing trend slope significant test on running windows along the time series (Thanasis et al. 2011). Like the OPLR method, this method may reject trend turnings in which both sub-series have the same sign as well.

Based on the above facts, we have studied the characteristics of climate trend turning. The most basic characteristic of trend turning is the sub-series trends difference between both sides of the turning-point, so we propose a new trend turning detection methodology by testing the sub-series trend difference. This new method uses a statistical t test of slope differences to identify the turning points and we name it as running slope difference (RSD) t test. The RSD t test has the capacity to detect multiple times of trend turnings and provides a test of significance for these trend turnings.

The remainder of this article is as follows. In Section 2, we discuss some major characteristics of the trend turning. The RSD t test detection method is given in Section 3. And next in Section 4, we compare the RSD t test method with some existing methods in an idealized time series case and several climate time series cases. At last, in Section 5, we draw our conclusions.

### 2 Characteristics of trend turning

Turning type is a major characteristic of trend turning. According to the different combination of trends that occur before and after turning, trend turning can be divided into three types: TR (trend reversal), TN (trend vs. no trend), and TD (trends are different only in degree, and are in same sign). All of trend turning types and their sub-types are listed in Table 1. TR represents trend reversal between statistically significant positive and negative trends; therefore, it includes two sub-types. TN represents trend turning from no trend to significant trend (or vice versa); it includes four sub-types. TD

| Basic types of trend turning | TR | TN | TD |
|-----------------------------|----|----|----|
| trend turning between significant positive trend and significant negative trend | (+, −) | trend turning from no trend to significant trend (or vice versa) | (+, +) |
| Sub-types of trend turning | trend turning from significant positive trend to significant negative trend | (+, 0) | trend turning from relatively slow increasing trend to relatively rapid increasing trend |
| | trend turning from significant negative trend to significant positive trend | (−, 0) | trend turning from relatively rapid increasing trend to relatively slow increasing trend |

(+), (−), and (0) are the symbols for significant positive trend, significant negative trend, and no trend. For TD type, (+ +) represents a relatively large positive trend which is significantly different from the nearby (+) trend, and (− −) represents a relatively small negative trend which is significantly different from the nearby (−) trend.

| Basic types of trend turning | TR | TN | TD |
|-----------------------------|----|----|----|
| trend turning between significant positive trend and significant negative trend | (+, −) | trend turning from no trend to significant trend (or vice versa) | (+, +) |
| Sub-types of trend turning | trend turning from significant positive trend to significant negative trend | (+, 0) | trend turning from relatively slow increasing trend to relatively rapid increasing trend |
| | trend turning from significant negative trend to significant positive trend | (−, 0) | trend turning from relatively rapid increasing trend to relatively slow increasing trend |
| | trend turning from no trend to significant positive trend | (0, +) | trend turning from relatively slow decreasing trend to relatively rapid decreasing trend |
| | trend turning from significant negative trend | (0, −) | trend turning from relatively rapid decreasing trend to relatively slow decreasing trend |

| Basic types of trend turning | TR | TN | TD |
|-----------------------------|----|----|----|
| trend turning between significant positive trend and significant negative trend | (+, −) | trend turning from no trend to significant trend (or vice versa) | (+, +) |
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| | trend turning from significant negative trend to significant positive trend | (−, 0) | trend turning from relatively rapid increasing trend to relatively slow increasing trend |
| | trend turning from no trend to significant positive trend | (0, +) | trend turning from relatively slow decreasing trend to relatively rapid decreasing trend |
| | trend turning from significant negative trend | (0, −) | trend turning from relatively rapid decreasing trend to relatively slow decreasing trend |
represents trend turning between two same-sign trends (either significant positive or negative); it includes four sub-types. Existing detection methods usually ignored the TD type of trend turning. However, the TD type of trend turning is also important in climate research and we will further discuss it in Section 4.

Analyzing all turning types, the common characteristic of trend turning is the sub-series trend difference between both sides of the turning point. With a previous trend transition into a different trend after the turning point, the slopes before and after turning will exhibit large differences. Therefore, the statistical significance of sub-series slope difference can be used as an indicator of trend turning. The RSD test is designed based on this fact, and it has the ability of detecting all three types of trend turning.

For trend turning detection, the main scientific problem is to identify the turning points of the sample time series. When turning points are determined, the original time series can be subdivided into several trend phases (linear segments). Then, the existing trend significant test can be used to further analyze the trend of each phase. In this study, we focus on identification of the turning points and three basic features of each trend phase: (1) the duration of each phase, (2) the least square trend value of each phase, and (3) the least square regression of each phase.

3 The running slope difference t test detection method

3.1 Statistical test of slope difference

The statistical test of slope difference is performed with a t-distribution statistic. Let \( Y : \{y_i = \beta_1 + \alpha_1 + \varepsilon_i \ | \ 1 \leq i \leq n\} \) and \( Z : \{z_j = \beta_2 + \alpha_2 + \varepsilon_j \ | \ 1 \leq j \leq m\} \) be the two sample series; the length of \( Y \) is \( n \) and \( Z \) is \( m \). \( \beta_1 \) and \( \beta_2 \) are the slopes of sample series \( Y \) and \( Z \), \( \alpha_1 \) and \( \alpha_2 \) are the intercepts, and \( \varepsilon_i \) and \( \varepsilon_j \) are the error terms. The null hypothesis here is \( \beta_1 = \beta_2 \). Assume that \( \varepsilon_i \) and \( \varepsilon_j \) are normally distributed independent random variables with zero mean and variance \( \sigma^2 \). Let \( \bar{Y} : \{\bar{y}_i \ | \ 1 \leq i \leq n\} \) and \( \bar{Z} : \{\bar{z}_j \ | \ 1 \leq j \leq m\} \) be the least-squares linear regressions of \( Y \) and \( Z \) and \( \bar{\beta}_1 \) and \( \bar{\beta}_2 \) are the least-squares estimators of slopes \( \beta_1 \) and \( \beta_2 \). The general form of the slope difference t-distribution statistic \( t_{slope} \) between \( Y \) and \( Z \) is:

\[
t_{slope} = \frac{\bar{\beta}_1 - \bar{\beta}_2}{S_{\beta_1, \beta_2}}
\]

where

\[
S_{\beta_1, \beta_2}^2 = \frac{1}{C} \frac{1}{n + m - 4} \left( \sum_{i=1}^{n} (y_i - \bar{y}_i)^2 + \sum_{j=1}^{m} (z_j - \bar{z}_j)^2 \right)
\]

is the variance of the regression errors and \( C = \frac{NM}{N + M} \), \( N = \sum_{i=1}^{n} (i - \frac{n+1}{2})^2 \), and \( M = \sum_{j=1}^{m} (j - \frac{m+1}{2})^2 \) are known constants.

Since \( \varepsilon_i \) and \( \varepsilon_j \) are assumed to be normally distributed independent random variables with zero mean, the distribution of both regression errors (residuals \( y_i - \bar{y}_i \) and \( z_j - \bar{z}_j \)) follows the normal distribution and both are independent for each other; then, the sampling distribution of \( X_Y = \frac{1}{\sigma} \sum_{i=1}^{n} (y_i - \bar{y}_i)^2 \sim \chi^2(n-2) \) and \( X_Z = \frac{1}{\sigma} \sum_{j=1}^{m} (z_j - \bar{z}_j)^2 \sim \chi^2(m-2) \) is \( \chi^2 \) distribution (see, e.g., von Storch and Zwiers 1999). As the least-squares estimators are unbiased, the expectation and variance of the slope difference \( \bar{\beta}_1 - \bar{\beta}_2 \) are \( E(\bar{\beta}_1 - \bar{\beta}_2) = \bar{\beta}_1 - \bar{\beta}_2 \) and \( \text{Var}(\bar{\beta}_1 - \bar{\beta}_2) = \frac{1}{\sigma^2} \), respectively; thereby, \( Z = \sqrt{C} \times (\frac{\bar{\beta}_1 - \bar{\beta}_2}{\sigma}) \sim N(0, 1) \) is normally distributed (see, e.g., von Storch and Zwiers 1999). From \( X_{\bar{Y}}, X_{\bar{Z}}, \) and \( Z \), it can be inferred that \( t_{slope} = \frac{X_{\bar{Y}} - X_{\bar{Z}}}{\sqrt{(X_{\bar{Y}} + X_{\bar{Z}})/n + m - 4}} \) is the t-distribution and the degrees of freedom are \( f = n + m - 4 \). The null hypothesis of no slope difference between \( Y \) and \( Z \) is rejected at the significance level \( \alpha \) if \( |t_{slope}| \geq t_{n+m-4, 1 - \alpha/2} \).

For climate time series, the regression errors here may not be independent of each other; this means degrees of freedom estimated by the above formula \( n + m - 4 \) may be more than the effective ones (Santer et al. 2000; Xiao and Li 2007). In practice, the degrees of freedom above are replaced by the effective degrees of freedom \( N_{eff} \) in consideration of the autocorrelation effect (Bretherton et al. 1999; Ebisuzaki 1997; Santer et al. 2000). \( N_{eff} \) is evaluated following the work of Bartlett (1946) as:

\[
\frac{1}{N_{eff}} \approx \frac{1}{N} + \frac{2}{N} \sum_{j=1}^{N-1} \frac{(N-j)}{N} \rho_j,
\]

where \( \rho_j \) is the autocorrelation coefficient with a lagged scale \( j \) and \( N = n + m \).

3.2 Detection process

Climate variability is frequently characterized by the multimodal structures (Ji et al. 2014; Wang et al. 2009). And trend turning of a sample time series usually for different timescales may also be different (Fig. 2, in Tomé and Miranda 2004). Therefore, users of the RSD test method should determine the trend turning timescale \( T \) according to their specific study issue before the trend turning detection. The parameter \( T \) is the basic timescale which is used to analyze the trend of sub-series. After the parameter \( T \) is determined, only the sub-series:......
series greater than or equal to timescale $T$ can be detected (the same as in the parameter $T$ in the OPLR method by Tomé and Miranda 2004).

To locate the turning points using $t_{\text{slope}}$, the RSD $t$ test adopts a running test technique which first finds all the potential turning points of possible trend turnings (candidates of the turning points). Let $L: \{l_i \mid 1 \leq i \leq k\}$ be the sample time series. Set $\tau$ to be the running window, parameter $\tau$ is determined based on the timescale $T$. We suggest that the value of $\tau$ should be a little smaller than $T$ considering that some time series may have a length $T$ trend phase (usually we set $\tau = T - 1$ or $\tau = T - 2$, and $\tau$ should not be less than $3/4T$). For each point $l_i (a \in [\tau + 1, k - \tau])$, the difference between the slope of sub-time series $L_{a1}: \{l_i | a - \tau \leq i \leq a - 1\}$ and the slope of sub-time series $L_{a2}: \{l_i | a + 1 \leq i \leq a + \tau\}$ is tested using $t_{\text{slope}}$. If the slope of $L_{a1}$ is significantly different from the slope of $L_{a2}$, it means that a potential turning may occur at point $l_i$. All the points which have passed the running slope difference test will form several continuous intervals, with each interval corresponding to a possible trend turning. The RSD $t$ test picks the point which has the maximum absolute slope difference value in each interval as the potential turning point.

Not all these potential turning points are true turning points, some of which might be false turnings caused by the running test. Thus, a check process is designed to eliminate these false turnings in the RSD $t$ test. Assume that we got $k$ potential turning points, they subdivided the sample time series $L$ into $k + 1$ trend phases. For each potential turning point, the two nearby trend phases pair will be checked again using $t_{\text{slope}}$ to ensure that they exhibit significant trend differences. Usually, the length of each trend phase is longer than the running window $\tau$; thereby, some potential turning points may not pass the test in the check process since the test samples have changed (the two nearby trend phases pair usually contains more points than $L_{a1}$ and $L_{a2}$). Potential turning points which fail to pass the check are the false turnings. Removing all the false turnings, the rest points are turning points of sample time series $L$.

In this article, we will compare the trend turning detection results of the RSD $t$ test with two existing methods which are commonly used for detecting climate trend turnings. One method is the optimal piecewise linear regression (OPLR) method from Tomé and Miranda (2004), and the other one is a running trend test (R-MK) based on the MK trend test (Mann 1945; Kendall 1975). The statistic of MK trend test used here is a modified version from Yue and Wang (2004), which can be applied in both independent and autocorrelated data.

4 Application

4.1 Idealized time series case

To further illustrate the performance of the RSD $t$ test method and compare it with existing methods, we built an idealized sample case of three time series. The first time series $I_1$ (Fig. 1a, thick line) is a series with six times of different type trend turnings, the length of each trend phase being 60–90 points. The information of turning points of $I_1$ is listed in Table 2 (trend turning true value, rows 2–4). The second time series $I_2$ (Fig. 1d, thick line) is a series with no trend turning but has the same overall slope value as $I_1$. The third time series $I_3$ (Fig. 1g, thick line) is a series with no trend turning and zero overall slope value. We superpose these series with randomly generated noise and detect their trend turnings using the RSD $t$ test method, as well as the OPLR and R-MK methods.

In the idealized case, we blended two types of randomly generated noise: the independent noise and the autocorrelated noise. The independent noise $\varepsilon_t$ is a random, independent, and normally distributed series with zero mean (the standard deviation of the noise is set as 15). The autocorrelated noise $x_t$ is generated by a first-order autocorrelation process (AR (1); Box et al. 1976)

$$x_{t+1} = \Phi x_t + \varepsilon_t,$$

where $\varepsilon_t$ is the independent noise; $\Phi$ is the lag-one autocorrelation coefficient. For the autocorrelated noise $x_t$, the autocorrelation coefficient is set as $\Phi = 0.6$. According to the work of Bartlett (1935), statistical tests which are based on hypothesis of independent sample and have no consideration of autocorrelation will have a very low accuracy when the autocorrelation coefficient come to 0.6 or more.

To compare the results of different trend turning detection methods, the timescale $T$ of three methods are all set as 50 points. Other detection parameters for each method are as follows: for the RSD $t$ test method, running window is set as $\tau = T - 2$ and statistical test confidence level is set at 99%; for the OPLR method, the maximum number of trend turnings is set as 8; for the R-MK method, the threshold of significant trend is set at 95%.

Figure 1 is the trend turning detection of idealized time series with independent noise using the RSD $t$ test method. For series $I_1$, six times of different types of trend turnings have all been identified (Fig. 1c and the 6th row in Table 2). All the turning points correspond to the local maximum or minimum of the slope difference curve (Fig. 1b). These detection results are also stable and insensitive to the small changes of detection parameters $T$ and $\tau$. Table 3 shows a comparative experiment which uses different pairs of parameters $T$ and $\tau$ (with only a small difference), all the detections having resulted similar results. For series $I_2$ and $I_3$, detection results show there
is no trend turning just as we have designed (Fig. 1f, i). Although the running test process may bring several false turnings (Fig. 1e, h), these false turnings can be removed through the elimination process of false trend turning. Therefore, the final results of the RSD $t$ test method are correct, with no false turnings.

Figure 2 is the trend turning detection results of idealized time series with independent noise using the OPLR method.

Table 2  Trend turning detection results of idealized time series $I_{1}$ by different methods

| Trend turning true value (without noise) |
|-----------------------------------------|
| Turning points   | 80 | 140 | 230 | 300 | 365 | 440 |
| Type            | TR | TN | TN | TD | TR | TD |
| Sub-type        | (+, 0) | (+, +) | (−, +) | (+, +) | (−−) | (+, +) |

| Trend turning estimation (independent noise) |
|---------------------------------------------|
| RSD $t$ test | 81 | 137 | 213 | 298 | 364 | 440 |
| OPLR      | 83 | 135 | 187 (II) | 244 | (I) | 384 | (I) |
| R-MK      | 71 | 146 | 221 | (I) | 367 | (I) |

| Trend turning estimation (autocorrelated noise) |
|-----------------------------------------------|
| RSD $t$ test | 88 | 140 | 209 | 299 | 384 | 442 |
| OPLR      | 82 | 136 | 190 (II) | 246 | (I) | 401 | (I) |
| R-MK      | 69 | 145 | 280 | (I) | 382 | 440 |

Trend turning type symbols (+), (−), (0), (+ +), and (− −) as in Table 1. (I) and (II) represent false rejection and false alarm, respectively.
Trend turning estimation (autocorrelated noise) line) of idealized time series $I_1$ with independent noise (thin line). Dash points at 300 and 440 in $I_1$. Furthermore, this method has the ability of identifying both the TR and TN types of trend turnings; however, it can hardly identify all the TD type of trend turnings as in the independent noise case, and the results between two cases are quite similar (rows 11–13 in Table 2). The RSD $t$ test method has the ability of identifying all three types of trend turnings (Fig. 4c and Table 2), while the OPLR and R-MK methods can hardly identify all the TD type of trend turnings as in the independent noise case (the OPLR method has missed two TD-type turning points at 300 and 440 in $I_1$, the R-MK method having missed one at 300). The OPLR method still has a relatively high chance of triggering false alarm when the slope of sample time series is near 0 in the autocorrelated noise case (Fig. 5a and Table 3). The RSD $t$ test method, OPLR, and R-MK methods, the effect of autocorrelation is considered by using the effective degrees of freedom. And in the R-MK method, the statistic of the MK trend test is replaced by a modified version (the rationale of this modified version is also using the effective sample size) from Yue and Wang (2004). Figures 4, 5, and 6 are the trend turning detection results of idealized time series with autocorrelated noise using the RSD $t$ test, OPLR, and R-MK methods. The detection results show that all three methods can be used to identify the trend turnings of the autocorrelated noise case as in the independent noise case, and the results between two cases are quite similar (rows 11–13 in Table 2). The RSD $t$ test method has the ability of identifying all three types of trend turnings (Fig. 4c and Table 2), while the OPLR and R-MK methods can hardly identify all the TD type of trend turnings as in the independent noise case (the OPLR method has missed two TD-type turning points at 300 and 440 in $I_1$, the R-MK method having missed one at 300). The OPLR method still has a relatively high chance of triggering false alarm when the slope of sample time series is near 0 in the autocorrelated noise case (Fig. 5a and c, one false alarm at point 190 in $I_1$ and six false alarms in $I_3$).

As shown in Fig. 2a and the 7th row in Table 2, the OPLR method has the ability of identifying both the TR and TN types of trend turnings; however, it can hardly identify the TD type of trend turnings, missing all two TD-type turning points at 300 and 440 in $I_1$. Furthermore, this method has a relatively high chance of triggering false alarm when the slope of sample time series is near 0 (Fig. 2a and c, one false alarm at point 187 in $I_1$ and six false alarms in $I_3$). These estimation errors mainly resulted from the lack of the statistical test of the sub-series trend difference. Figure 3 is the trend turning detection results of idealized time series with independent noise using the R-MK method. Like the OPLR method, the R-MK method also has the ability of identifying both the TR and TN types of trend turnings; however, it can hardly identify all the TD type of trend turnings as well (Fig. 3b and the 8th row in Table 2, R-MK has missed all two TD-type turning points at 300 and 440 in $I_1$).

Assumption of independent observations could result in erroneous conclusions when the sample time series are autocorrelated. The statistic test of $t_{\text{slope}}$ used in the RSD $t$ test method and the statistic trend test of MK used in the R-MK method both have such assumption. In the RSD $t$ test method,

![Table 3](#)

| Trend turning estimation (independent noise) |
|--------------------------------------------|
| $T = 50, \tau = T - 1$                      |
| 81 137 216 293 365 441                     |
| $T = 50, \tau = T - 2$                      |
| 81 137 213 298 364 440                     |
| $T = 52, \tau = T - 1$                      |
| 85 140 220 295 363 440                     |
| $T = 52, \tau = T - 2$                      |
| 85 163 220 295 364 440                     |
| $T = 48, \tau = T - 1$                      |
| 80 137 213 297 361 440                     |
| $T = 48, \tau = T - 2$                      |
| 81 138 213 296 371 438                     |

4.2 Changes in the growth rate of global atmospheric CO$_2$

In this section, we are testing the multidecadal trend turning of global atmospheric CO$_2$ concentration about half a century. The atmospheric concentration of CO$_2$ has increased since 1750 due to human activities and exceeded the pre-industrial levels by about 40% (IPCC 2013). Emissions of CO$_2$ alone have caused a radiative forcing of 1.68 W m$^{-2}$, which has a very high level of confidence to be the primary driver of global warming (IPCC 2013). Figure 7a shows the global atmospheric CO$_2$ concentration from 1750 to 2005; this data is obtained from the work of Meinshausen et al. (2011). CO$_2$ concentration has been monotonically increasing, with an obviously increasing growth rate.

To detect the multidecadal trend turning of CO$_2$ concentration using the RSD $t$ test, OPLR, and R-MK methods, the
timescale $T$ is set as 50 years. Other detection parameters are set the same as in the idealized time series case.

Figure 7 is the trend turning detection results of CO$_2$ concentration using the RSD $t$ test method. Two TD-type trend turnings are identified respectively around the years of 1840 and 1954. These turning points divide the original time series into three phases: 1750–1839, 1841–1953, and 1955–2005. The growth rates of CO$_2$ in the latter two phases are 4.5 and
24.6 times greater than that in the first phase, respectively. We note that the two turning points around the years of 1840 and 1954 correspond to the initial stages of the second industrial revolution and information revolution, and these results may suggest that technological advancements contribute strongly to anthropogenic CO2 emissions.

Figure 8 is the trend turning detection results of CO2 concentration using the OPLR method. The OPLR method also finds two turning points (around the years of 1843 and 1894); however, the regression error of the OPLR method is obviously greater than that of the RSD t test method (standard deviation of regression error is 8.0:1.4). Furthermore, the estimated trend value (− 0.01 × 10^2 ppm per decade) of the second phase (1844–1894) using the OPLR method has an opposite sign compared with its least square slope (2.72 × 10^2 ppm per decade) or Theil-Sen slope (2.72 × 10^2 ppm per decade). Figure 9 is the trend turning detection results of CO2 concentration using the R-MK method. The R-MK method shows that there are no trend turnings in CO2 concentration time series.

Comparing these results of different detection methods, the RSD t test method is much more appropriate for identifying TD types of trend turnings. TD type of trend turning is common in the anthropogenic factors of climate such as atmospheric CO2 concentration; therefore, analysis using the RSD t test method may contribute to the understanding of how these factors changed in different time periods.

4.3 Trend turning in global temperature

In this section, we will use the RSD t test method to detect the decadal trend turnings of GMT time series. Three datasets which are common for studying global warming are used to detect the decadal trend turnings of GMT time series. These three datasets are HadCRUT4 dataset (Fig. 10a; Morice et al. 2012) which is developed by the Climatic Research Unit (University of East Anglia) in conjunction with the Hadley Centre (UK Met Office), GISTEMP dataset (Fig. 10d; Hansen et al. 2010) which is developed by the Goddard Institute for Space Studies (NASA), and NOAA Global Temp (NOAA-GT, used to known as MLOST) dataset (Fig. 10g; Smith et al. 2008; Vose et al. 2012) which is developed by the National Climatic Data Center (NOAA).

To detect the decadal trend turning of GMT using the RSD t test, OPLR, and R-MK methods, the timescale T is set as 15 years following the work of Karl et al. (2000). The test sample size in this case is much smaller than that in the idealized time series case; thereby, the running window for the

![Fig. 5](image-url) The same as in Fig. 2, but for trend turning detection of idealized time series with autocorrelated noise

![Fig. 6](image-url) The same as in Fig. 3, but for trend turning detection of idealized time series with autocorrelated noise
Fig. 7  Trend turning detection of the global CO₂ concentration time series using RSD t test method (detection parameter $T = 50$ years, $\tau = T - 2$, confidence level at 99%). a CO₂ concentration time series. b The same as in Fig. 1b, but for the detection process of CO₂ concentration time series. c The trend turning estimations (colored thick line) of CO₂ concentration time series (thin line). Dash vertical lines denote the turning points (1840 and 1954). “Tr” and “D” represent the least square trend value and duration of each trend phase, respectively. Note that the least square trend value unit here is $10^2$ ppm per decade and the duration unit is year.

Fig. 8  Trend turning detection of the global CO₂ concentration time series using OPLR method (detection parameter $T = 50$). The trend turning estimations (colored thick line) of CO₂ concentration time series (thin line). Dash vertical lines denote the turning points (1843 and 1894). “Tr” and “D” the same as in Fig. 7.

Fig. 9  Trend turning detection of the global CO₂ concentration time series using R-MK method (detection parameter $T = 50$, confidence level at 95%). a The running MK test statistic $Z$ (black curve) of CO₂ concentration time series. Colored horizontal lines denote the confidence level threshold. b The trend turning estimations of CO₂ concentration time series (thin line). Dash vertical lines denote the turning points (no trend turning).

A new statistical method for detecting trend turning

RSD t test method is set as $\tau = T - 1$ and statistical test confidence level is set at 95%. Other detection parameters are set the same as in the idealized time series case.

Figures 10, 11, and 12 are the trend turning detection results of GMT time series using the RSD t test, OPLR, and R-MK methods. The detection results of different methods and datasets during the period of 1880–1998 are generally consistent with each other, that is, all the results show that there are three times of trend turnings around the years of 1910, 1940, and 1970 during this period. In phases 1910–1940 and 1970–1998, GMT increases rapidly with trend values about 0.10–0.13 and 0.17–0.18 °C per decade, respectively. But in phases 1880–1910 and 1940–1970, although during these
periods the CO$_2$ concentration is still growing (Fig. 7). GMT trend shows a cooling or a warming hiatus about $-0.06\sim-0.05$ and $-0.03\sim-0.02$ °C per decade, respectively. These results are consistent with the IPCC assessment report (2013) and many other previous researches based on both eye inspect and statistical detection (Easterling and Wehner 2009; England et al. 2014; Karl et al. 2000; Li et al. 2013; Rahmstorf et al. 2017; Trenberth and Fasullo 2013).

For the first 15 years of the twenty-first century, despite the fact that GMT warming during 2000–2014 is indeed lower than that during 1970–1998, this change significant enough to be considered as the “global warming hiatus/slowdown” is still under controversy. The detection results of different methods and datasets are inconsistent during this period. Detection results of the RSD $t$ test method show that in HadCRUT4 dataset, there is a significant trend turning around 1910, 1942, 1973, and 1999. “Tr” and “D” the same as in Fig. 7, but the least square trend value unit here is °C per decade. a, b, and c The same as in a, b, and c, but for GISTEMP and NOAA-GT datasets (both turning points at 1910, 1942, and 1973).

For the first 15 years of the twenty-first century, despite the fact that GMT warming during 2000–2014 is indeed lower than that during 1970–1998, this change significant enough to be considered as the “global warming hiatus/slowdown” is still under controversy. The detection results of different methods and datasets are inconsistent during this period. Detection results of the RSD $t$ test method show that in HadCRUT4 dataset, there is a significant trend turning around 1910, 1942, 1973, and 1999. “Tr” and “D” the same as in Fig. 7, but the least square trend value unit here is °C per decade. a, b, and c The same as in a, b, and c, but for GISTEMP and NOAA-GT datasets (both turning points at 1910, 1942, and 1973).
the year of 1998 known as TD type, and phase 2000–2014 has a lower but still warming trend compared with phase 1970–1998. In GISTEMP and NOAA-GT datasets, there is no significant trend turning around the year of 1998, but they still have a weak negative trend changing signal (Fig. 10e, h). In the detection results of the R-MK method, there are also signals of negative trend change in the running MK statistic around the year of 1998 (Fig. 12a, c, e).

The insignificant negative trend change of GMT around the year of 1998 may arise from the spatial difference of climate change. In some regions, there is no warming hiatus/slowdown around the year of 1998. For example, in the European region (40–60° N, 0–40° E), warming rates for 1970–1998 and 2000–2014 periods are about 0.16–0.22 and 0.22–0.27 °C per decade, respectively. But in other regions, there is robust warming hiatus/slowdown or even cooling signals around the year of 1998. For example, in the southeast tropical Pacific region (10–30° S, 100–150° W), warming rates for 1970–1998 and 2000–2014 periods are about 0.20–0.21 and −0.26 to −0.18 °C per decade, respectively. To further understand the climate change around the year of 1998, a systematic study of the trend turnings in both space and time is needed. We believe that the RSD $t$ test method will make contribution to this kind of climate research subjects.

5 Conclusions

We propose a new methodology, the RSD $t$ test, to relatively objectively detect multiple trend turnings of time series. This method uses a slope difference statistic $t_{\text{slope}}$ to test the sub-series trend difference, thereby identifying the turning points. The RSD $t$ test has the capacity to detect all three types (TR, TN, and TD) of trend turning which are discussed above.

In this article, we compare the RSD $t$ test method with some existing methods in an idealized time series case and several climate time series cases. The trend turning detection results of these cases show that the RSD $t$ test method has a superior performance when detecting multiple trend turnings of climate time series. And as we envisioned, the RSD $t$ test method is much more appropriate for identifying TD type of trend turning compared with existing methods.

The new test statistic $t_{\text{slope}}$ is a $t$-distribution slope difference statistic. This statistic $t_{\text{slope}}$ can be used for both equal and unequal test samples thereby suitable for detecting trend turning. By replacing the degrees of freedom from the assumption of independent observations to the effective degrees of freedom, the RSD $t$ test method may still be effective when detecting the trend turning in autocorrelated time series. This feature is important because autocorrelation is common in the climate data.

At last, it needs to be pointed out that the research on climate trend turning is still few in the literature. And currently, there has been no general standard for judging different trend turning detection methods. Therefore, the reliability and applicability of the RSD $t$ test also need to be further verified and developed through more climate series analysis in climate change research.
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