Model updating based on particle swarm optimization

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Abstract. This paper is a bridge model correction based on particle swarm optimization. The main method is the combination of response surface method (RSM) and particle swarm optimization (PSO), and it is used to verify the model of four span truss bridge. The results show that the method is suitable for model updating of bridge structures.

1. Introduction
Structural finite element model updating is to identify structural physical parameters through structural test information. The main method is the combination of response surface method (RSM) and particle swarm optimization (PSO), and by the combined method of response surface method and particle swarm optimization method, we conducted model correction of a four span truss model.

2. Response surface method
RSM is an approximation optimization method. The basic idea of RSM is to use an explicit function (response surface function, RSF) to approximate the relationship between variables and responses, which is usually implicit and difficult to interpret. Regression analysis of the experimental samples based on least square method (LSM) is used to obtain RSF. As a meta-model, RSM provides an efficient way to calculate response value under different variables. It has been applied in many engineering fields, such as aerodynamics, structural reliability analysis, electronic package optimization, etc.

The main steps of RSM include design of experiment (DOE), RSF regression, and RSF verification.

3. Particle swarm optimization
PSO, a population-based stochastic optimization technique, is one of the evolutionary optimization algorithms. It was developed based on the swarm behavior of birds, fish and bees when they are searching for available food. A single bird (or other animal) is called a member or particle in PSO algorithm. The particles fly around in multidimensional search space in pursuit of most fertile food location, or, mathematically, to find the optimal location that minimizes specific objective function. The positions of individual particles are adjusted according to their own best position and also to other particles' best position, i.e., the local best and global best. Eventually, it is expected over a number of generations the whole swarm will move to the global optimal location.

In PSO, each particle i has a position vector $x_i(t)$ and a velocity vector $v_i(t)$ at generation t in multidimensional space. With the positions of individual particles, the algorithm calculates the fitness of each particle according to objective function, and then adjusts the position to optimal location. The position $x_i(t)$ is updated by

$$
x_i(t+1) = x_i(t) + v_i(t) \Delta t
$$

(2)
where \( t \) is the generation number, and \( \Delta t \) is taken equal to 1. The velocity \( v_i \) is updated by

\[
v_i(t + 1) = \omega v_i(t) + c_1 \phi_1 (p_{bi}(t) - x_i(t)) + c_2 \phi_2 (p_{gi}(t) - x_i(t))
\]

where \( \omega \) is inertia weight, \( p_{bi}(t) \) is the local best position found so far by particle \( i \), \( p_{gi}(t) \) is the global best position found by the swarm. \( c_1 \) and \( c_2 \) are the positive constants called cognition and social coefficients, and \( \phi_1 \) and \( \phi_2 \) are the random number uniformly distributed between 0~1.

4. Numerical simulation of a simply supported truss

Fig.1. The simulated steel truss under study

As shown in Figure 1, the truss model is established, and the total length of the truss model is 2m, using a circular section with an area of 0.00232 square meters. The finite element model of truss is composed of 21 elements and 12 nodes. The initial values of the Young’s modulus and density of the truss are 2.06x10^5 MPa, 7850 kg/m³, respectively. In the middle of the truss, 1000N concentrated force is applied.

4.1. Sensitivity analysis of truss model

Through the sensitivity analysis of the parameters of the truss model, the parameters which are sensitive to the response of the truss model are obtained.

Four sensitive parameters were selected and artificially given the damage value. The density of elements 1 (\( \rho_1 \)), elements 5 (\( \rho_5 \)), elements 9 (\( \rho_9 \)), elements 20 (\( \rho_{20} \)) declined 20%, 15%, 30%, 10% to the values of 9420 kg/m³, 9027.5 kg/m³, 10205 kg/m³, respectively. Thus, the undamaged model represents the initial model, while the damaged model stands for the “true” model. The purpose of FE model updating is to update the initial model, making its responses more close to the true model. The proposed RSM and PSO method is then applied to achieve this purpose, i.e., to find out the change in Young’s modulus and density of the damaged elements.

| Items | Initial model (Undamaged) | True model (Damaged) | Difference (%) |
|-------|---------------------------|----------------------|----------------|
| f1 (Hz) | 35.887 | 35.579 | 0.86% |
| f2 (Hz) | 56.348 | 55.940 | 0.73% |
| f3 (Hz) | 78.870 | 78.187 | 0.87% |
| f4 (Hz) | 79.937 | 79.603 | 0.42% |
| f5 (Hz) | 126.384 | 125.128 | 1.00% |
| f6 (Hz) | 132.490 | 130.722 | 1.35% |
| f7 (Hz) | 154.734 | 153.518 | 0.79% |
| f8 (Hz) | 206.898 | 204.491 | 1.18% |
| f9 (Hz) | 226.610 | 225.384 | 0.54% |
Table 1 shows the first six frequencies ($f_1$~$f_6$) true model, respectively.

5. Experimental design

Before updating the initial model, the updating parameters need to be determined. In this section, it is assumed that the updating parameters are exactly the damaged elements, i.e., the density of elements 1 ($\rho_1$), elements 5 ($\rho_5$), elements 9 ($\rho_9$), elements 20 ($\rho_{20}$). Then The Latin hypercube sampling is used to test the 4 parameter. As shown in Fig.2, total of 40 Latin hypercube sampling are used, they can be generated by the Matlab. In this paper, we select two of the parameters, namely, $\rho_1$ and $\rho_5$, and draw the scatter plot of sample points.

![Fig.2. Latin sample distribution scatter plot](image)

The expressions of “$Xs$” are used to represent factors to indicate that they are independent variables. The baseline values of the six factors are 7850kg/m$^3$, 7850kg/m$^3$, 7850kg/m$^3$, and 7850kg/m$^3$.

![Fig.3. Response surface diagram of Latin Hypercube Design fitting](image)

The frequencies and displacement at mid-span are extracted by numerical modal analysis and static tests based on ANSYS. Based on the parameters and responses, the coefficients of the RSF can be computed using LSM. As shown in Fig.3, the response surface between the response value and the parameter is plotted. The calculated full quadratic form of the RSF can be expressed as:

$$[f_1 \ f_2 \ f_3 \ f_4 \ f_5 \ f_6 \ f_7 \ f_8 \ f_9] = X \beta$$ \hspace{1cm} (4)

where $f_1$~$f_9$ are the first six natural frequencies; a concentrated load, 1000N.

$$X = \begin{pmatrix}
    1 & x_1 & x_3 & x_4 & x_1^2 & x_2^2 & x_3^2 & x_4^2
\end{pmatrix}$$ \hspace{1cm} (5)

$x_1$~$x_4$ represent the six factors, i.e., respectively. $\beta$ is the coefficient matrix expressed as:
Table 2. The R² and RMSE values of the fitted full quadratic RSF model

| Criteria | f1  | f2  | f3  | f4  | f5  | f6  | f7  | f8  | f9  |
|----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| R²       | 1.00| 1.00| 1.00| 1.00| 1.00| 1.00| 1.00| 1.00| 1.00|
| RMSE     | 1.07E-04 | 3.25E-04 | 3.09E-04 | 1.96E-04 | 3.10E-04 | 1.24E-04 | 1.46E-04 | 6.65E-04 | 1.56E-04 |

After the explicit function between responses and updating parameters is determined by RSF, the accuracy test of the RSF model is performed. As can be seen in Table 2, the R² values of five factors are all closer to 1 and RMSE are closer to 0, which means the RSF model is well fitted from the trials in DOE and can be further used in model updating.

An objective function is then built up using the response residuals between true model and RSF model. For the true model, the frequencies and displacement values are shown in Table 1. For the RSF model, the values of frequencies and displacement can be directly calculated using updating parameters based on Eq.(2). Thus, the objective function for model updating can be defined as

\[ F = \left( \sum_{i=1}^{n} \left( \frac{f_i' - f_i^r}{f_i'} \right)^2 \right)^{1/2} \]  

(6)

where \( F \) is the objective function value, \( f_i' \) are the frequencies and displacement, and the superscript “t” and “r” stand for true model and RSF model, respectively. In the numerical study, the weight factors are all set to 1, as both frequency and displacement can be accurately calculated and they have equal importance.

PSO algorithm is then applied to update the four parameters to find the minimum value of the objective function. The lower and upper thresholds for the six parameters are set as [7850kg/m³, 7850kg/m³, 7850kg/m³, 7850kg/m³] and [11775kg/m³, 11775kg/m³, 11775kg/m³, 11775kg/m³], respectively. A population of 200 particles is used in PSO, and the maximum iterations are set to 1000. The inertial weight \( \gamma \) is selected as 0.729, the cognition and social coefficients are set to 2 and 2. The updated parameters and their true values are shown in Fig.4.
Fig. 4. Updating with 4 parameters: the comparison of updating parameters between true values and updated values by using full quadratic RSF

6. Data analysis
As can be seen from the figure, the errors for the six parameters are all under 4%, the largest error is 3.75% for the density of element 20 (ρ20). Considering the comparatively small absolute true values of the three parameters, the accuracy of the updated values is quite satisfactory.

7. Conclusion
It should be noted that in this numerical study, the damage locations were assumed to be known. While in real world application, the exact location of damages is unknown, and the selection of updating parameters mainly rely on sensitive analysis and the experiences of analyst, as will be shown later in the application section.

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