Non-abelian $q\bar{q}$ contributions to small-$x$

anomalous dimensions

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Abstract

By using $k$-factorization, we derive resummation formulas for the non-abelian $q\bar{q}$ contributions to both heavy flavour production by gluon fusion, and to the next-to-leading BFKL kernel. By combining this result with previous ones by Fadin et al. on the virtual terms, we also compute in closed form the complete $q\bar{q}$ contribution to the gluon anomalous dimension in the $Q_0$-scheme. We find that $q\bar{q}$ resummation effects are important for heavy flavour production, but are instead small in the anomalous dimension eigenvalues, because of a cancellation between abelian and non abelian contributions.

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1 Introduction

Hard processes in the small-x regime $s \gg Q^2 \gg \Lambda^2$ play an increasingly important role at present colliders [1] and are characterized by the fact that the effective QCD coupling constant $\alpha_s(Q^2) \log(1/x)$ is sizeable and the corresponding perturbative contributions need to be resummed.

Such resummations are performed, at leading log level, on the basis of the BFKL equation [2] for the anomalous dimensions, and of the $k$-factorization formulae [3-6] for the coefficient functions. Particular attention has been devoted in the literature to heavy quark production processes [3, 6] and to the (abelian) light quark contributions to the $qg$ and $qq$ entries of the anomalous dimension matrix [5]. Furthermore, an impressive program of evaluation of next-to-leading (NL) kernels of the BFKL equation is under way [7-10].

In this note, we focus our attention on the (non-abelian) $q\bar{q}$ contribution to the $k$-factorization program, which is relevant in two ways: firstly, as a hard subprocess in Drell-Yan heavy-flavour production and secondly, in the massless quark limit, as an important NL contribution to the gluon anomalous dimension. We shall then derive QCD resummation formulas in both cases and in particular we shall obtain in closed form all NL terms of the form $\alpha_s N_f (\alpha_s C_A \log(1/x))^n$, contributing to the gluon anomalous dimension in the so-called $Q_0$-scheme [11, 12].

The evaluation of such contributions is particularly important for structure functions. Since the gluon couples to the proton only through $q\bar{q}$ states, the NL anomalous dimension $\gamma_{qg}$ (determined essentially by the abelian $q\bar{q}$ kernel) contributes to scaling violations at the same level as $\gamma_{gg}$. It is then important to perform a complete calculation of NL $q\bar{q}$ contributions (in particular, the non-abelian one), in order to check the pattern described above, and to reduce the factorization scheme dependence of the theoretical estimates of scaling violations at HERA [13, 14].
2 $Q\bar{Q}$ Hadroproduction

Let us start considering the cross-section for the hadroproduction of a (heavy) quark-antiquark pair, which at high energies is dominated by the gluon fusion process in Fig. 1. This contribution may be expressed in the $k$-factorized form [3]:

$$M^2\sigma = \int \frac{d^2k_1}{z_1} \frac{d^2k_2}{z_2} \hat{\sigma}(k_1, k_2, M^2, z_1 z_2 s) F^{(1)}(z_1, k_1) F^{(2)}(z_2, k_2),$$

where $F$ denotes the unintegrated gluon density in the hadron and $\hat{\sigma}$ the high-energy projection of the (Regge) gluon fusion process $g(k_1)g(k_2) \to Q\bar{Q}$.

Since the evolution of the structure functions is simpler in the space of $z$ and $k$ moments it is useful to express the high energy factorization formula (2.1) in terms of the double-Mellin transformed structure functions

$$F^{(i)}(\gamma) = \int_0^1 dz z^{\omega-1} \int_0^\infty d^2k \left( \frac{k^2}{Q_0^2} \right)^\gamma F^{(i)}(z, k)$$

and hard cross-section coefficient function $^1$

$$\left( \frac{\alpha_s}{\pi} \right) H(\gamma_1, \gamma_2) = \int_0^\infty \frac{d^2k_1}{k_1^2} \frac{d^2k_2}{k_2^2} \left( \frac{k_1^2}{M^2} \right)^{\gamma_1} \left( \frac{k_2^2}{M^2} \right)^{\gamma_2} \int_0^\infty ds \left( \frac{M^2}{s} \right)^\omega \hat{\sigma}(k_1, k_2, M^2, s),$$

so that it takes the form:

$$M^2\sigma \left( \frac{M^2}{s} \right) = \left( \frac{\alpha_s}{\pi} \right) \int_{\frac{1}{2} - i\infty}^{\frac{1}{2} + i\infty} i^{s-2} \frac{d\gamma_1 d\gamma_2 d\omega}{(2\pi)^3} \left( \frac{s}{M^2} \right)^\omega \left( \frac{M^2}{Q_0^2} \right)^{\gamma_1 + \gamma_2} F^{(1)}(\gamma_1) F^{(2)}(\gamma_2) H(\gamma_1, \gamma_2).$$

For large enough $M^2$, the $\gamma$ integrals will be dominated by the BFKL anomalous dimension, $\gamma_1 \simeq \gamma_2 \simeq \gamma_L(\bar{\alpha}_s/\omega)$. This means that the high-energy resummation effects on the cross section we are considering are embodied in the $\gamma_1, \gamma_2$-dependence of the hard coefficient function $H(\gamma_1, \gamma_2)$, which is precisely what we wish to compute.

$^1$This function is related by the change of normalization $h = \frac{4\pi}{\alpha_s} \gamma_1 \gamma_2 H$ to the $h$-functions of Refs [3, 5, 6].
The squared matrix element for the process under consideration was computed in Ref. [3] (see also Ref. [8]), and contains two terms, with colour factors $C_F$ and $C_A$, that we shall call respectively the abelian and the non abelian contributions.

The abelian contribution $H^a$ is known [3] and provides the quark anomalous dimension [5]. Let us then concentrate on the non abelian one $H^{na}$.

First, we rewrite the squared matrix element of Ref. [3] in terms of the Sudakov parametrization for the exchanged gluons' momenta

$$k_1^\mu \simeq z_1 p_1^\mu + k_1^\mu, \quad k_2^\mu \simeq z_2 p_2^\mu + k_2^\mu,$$  (2.5)

and for the momentum transfer

$$\Delta^\mu = z_1 x_1 p_1^\mu - z_2 x_2 p_2^\mu + \Delta^\mu,$$  (2.6)

where $p_1$, $p_2$ denote the (light-like) momenta of the incoming hadrons.

By using explicit expressions for the invariants

$$\hat{s} = (k_1 + k_2)^2, \quad \hat{t} = \Delta^2, \quad \hat{u} = (k_1 - k_2 - \Delta)^2, \quad \nu = \hat{s} + (k_1 + k_2)^2,$$  (2.7)

the non abelian squared matrix element of Ref. [3] can be rewritten in the form

$$A^{na} = \pi^2 \alpha_s \left[ -\frac{1}{(M^2 - \hat{t})(M^2 - \hat{u})} + \frac{1}{\hat{s}} \left( \frac{1}{M^2 - \hat{u}} - \frac{1}{M^2 - \hat{t}} \right) (1 - x_1 - x_2) + \frac{2}{\nu \hat{s}} + \frac{2}{k_1^2 k_2^2} \left( \frac{1}{2} - \frac{(1 - x_1)(1 - x_2)\nu}{M^2 - \hat{t}} + \frac{\nu}{2}(1 - x_1 - x_2) - \frac{k_1^2(1 - x_2) + k_2^2(1 - x_1) - k_1 \cdot k_2 + \Delta \cdot (k_2 - k_2)}{\hat{s}} \right) \times \left( \frac{1}{2} - \frac{x_1 x_2 \nu}{M^2 - \hat{u}} - \frac{\nu}{2}(1 - x_1 - x_2) + \frac{k_1^2(1 - x_2) + k_2^2(1 - x_1) - k_1 \cdot k_2 + \Delta \cdot (k_2 - k_2)}{\hat{s}} \right) \right]$$  (2.8)
By then integrating over the phase space and performing the moments of Eq. (2.3) we obtain the non abelian coefficient function $H^{na}_{\omega}(\gamma_1, \gamma_2)$ as follows

$$\frac{\alpha_s}{\pi} H^{na}_{\omega}(\gamma_1, \gamma_2) = \frac{1}{8 \pi^4} \int \frac{d\nu}{\nu^2} \frac{d^2k_1}{\pi k_1^2} \frac{d^2k_2}{\pi k_2^2} \frac{d^2\Delta}{\pi} dx_1 dx_2 \delta(x_2(1-x_1)\nu - (k_1 - \Delta)^2 - M^2) \times$$

$$\times \delta(x_1(1-x_2)\nu - (k_2 + \Delta)^2 - M^2) \left( \frac{k_1^2}{M^2} \right)^{\gamma_1} \left( \frac{k_2^2}{M^2} \right)^{\gamma_2} A^{na}(k_1, k_2, \Delta, x_1, x_2, \nu, M^2) \quad (2.9)$$

This expression can be evaluated analytically (at least in the limit $\omega = 0$, relevant at high energies) with a careful choice of the order of the integrations. We find it convenient to start eliminating the variable $\nu$ by integration of one mass-shell delta function, and then computing the $\Delta$-integral with the aid of a Feynman parametrization of denominators. The remaining integrals, first over the transverse momenta $k_1$, $k_2$ and then over the Feynman parameters and longitudinal momentum fractions, can be evaluated in terms of Gamma and Beta functions, and give, after some algebra, the result

$$H^{na}_{\omega}(\gamma_1, \gamma_2) =$$

$$= \frac{C_A \alpha_s}{2 \pi} \left[ \Gamma(1-\gamma_1-\gamma_2)\Gamma(\gamma_1)\Gamma(\gamma_2) \cdot \frac{B(1-\gamma_1,1-\gamma_1)B(1-\gamma_2,1-\gamma_2)}{4} + \right.$$

$$+ \frac{B(1-\gamma_1,2-\gamma_1)B(1-\gamma_2,2-\gamma_2)}{(3-2\gamma_1)(3-2\gamma_2)} (1 + (1-\gamma_1)(1-\gamma_2)) + \right.$$  

$$+ B(\gamma_1,1-\gamma_1)B(\gamma_2,1-\gamma_2) \left( \frac{\Gamma(2-\gamma_1-\gamma_2)}{\Gamma(4-2\gamma_1-2\gamma_2)} - \frac{\Gamma(3-\gamma_1-\gamma_2)}{\Gamma(6-2\gamma_1-2\gamma_2)} \right)$$

$$+ \left. 2B(\gamma_1,1-\gamma_1-\gamma_2)B(\gamma_2,1-\gamma_1-\gamma_2)B(3-\gamma_1-\gamma_2,3-\gamma_1-\gamma_2) \right] \frac{1}{(1-\gamma_1-\gamma_2)}. \quad (2.10)$$

The details of the calculation are reported elsewhere [13].

Let us note the triple pole singularity at $\gamma_1 + \gamma_2 = 1$ of the last term of Eq. (2.10), which is related to the collinear-singular behaviour of the partonic cross section in the massless limit. In fact, if $M^2 \ll (k_1 + k_2)^2 \equiv q^2 \ll k_1^2 \simeq k_2^2$, the hard cross section approaches the singular limit

$$\hat{\sigma}_{\omega=0}(k_1, k_2, M^2) = \hat{\alpha}_s \frac{\alpha_s}{\pi} \int_0^1 dx_1 \frac{x_1(1-x_1)}{q^2} \log \left( 1 + \frac{x_1(1-x_1)q^2}{M^2} \right) \simeq \hat{\alpha}_s \left[ \frac{\log \frac{q^2}{M^2} - \frac{5}{3}}{2} \right] \frac{1}{q^2}$$

$$\left( \hat{\alpha}_s = \frac{\alpha_s C_A}{\pi} \right), \quad (2.11)$$
which, after taking the $k^2$-moments, yields precisely a triple pole. The factor in front of the logarithm is related to the $g \to q\bar{q}$ GLAP splitting function \cite{3,6}.

Such singular behaviour was already discussed in Ref. \cite{3} and yields an interesting enhancement of the cross section when $\gamma_1$ and $\gamma_2$ approach the BFKL saturating value $\gamma = 1/2$. Here we further notice that, after combination with virtual terms in a BFKL kernel, the behaviour (2.11) is responsible for the $q\bar{q}$ contribution to the running coupling constant, as we shall see in the following.

Indeed, the same cross section $\hat{\sigma}$ occurs, in the massless limit, as a light quark contribution to the BFKL kernel, given by the inverse of Eq. (2.3), i.e., by

$$\frac{\pi}{\alpha_s M^2} \delta_{\omega=0} \left( k_1, k_2, M^2 \right) = \frac{1}{M^2} \int_{\frac{1}{2}+i\infty}^{\frac{1}{2}-i\infty} d\gamma_1 d\gamma_2 \frac{H_0(\gamma_1, \gamma_2)}{M^2} \left( \frac{k_1^2}{M^2} \right)^{-\gamma_1} \left( \frac{k_2^2}{M^2} \right)^{-\gamma_2}. \quad (2.12)$$

After subtraction of the singular contribution of Eq. (2.11), the eigenvalue of the finite part of the kernel is directly obtained by the massless limit of Eq. (2.12), which singles out the residue at the (single) pole at $\gamma_1 + \gamma_2 = 1$, as follows

$$H_{\text{real}}^{\text{na}}(\gamma) = \lim_{\gamma_2 \to 1-\gamma} \left( H_{\text{na}}^{\text{na}}(\gamma, \gamma_2) - H_{\text{s}}^{\text{na}}(\gamma, \gamma_2) \right) (1 - \gamma - \gamma_2)$$

$$= \left( \frac{\alpha_s}{\pi} \right) C_A \left[ \Gamma^2(\gamma) \Gamma^2(1-\gamma) \left( \frac{1}{3} - (2 + 3\gamma(1-\gamma)) \frac{\Gamma(2-\gamma)\Gamma(1+\gamma)}{\Gamma(4-2\gamma)\Gamma(2+2\gamma)} \right) \right], \quad (2.13)$$

for each quark flavour.

Note that the various terms of Eq. (2.13) show spurious double poles at $\gamma = 0$ and $\gamma = 1$, which would signal the presence of a collinear singularity in the $t$-channel. They cancel however in the sum, as they should because the real emission non-abelian diagrams do not contribute to the $t$-channel singularity.

## 3 Next-to-leading gluon anomalous dimension

Let us now come to the important issue of NL contributions to the flavour singlet anomalous dimension matrix $\gamma_{ab}(\alpha_s)$ $(a, b = q, g)$. As is known, the gluon entries start at leading level

$$\gamma_{gg} = \gamma_L \left( \frac{\alpha_s}{\omega} \right) + \alpha_s \gamma_{NL} \left( \frac{\alpha_s}{\omega} \right) + \ldots, \quad \gamma_{gq} = \frac{C_F}{C_A} \gamma_L (1 + O(\alpha_s)), \quad (3.1)$$
while the quark entries \( \gamma_{gg} \) start at NL level and, in the \( Q_0 \)-scheme \( \frac{1}{3\pi} \) are given by

\[
\begin{align*}
\gamma_{gg} &= N_f \gamma^2_L H_2(\gamma_L), \\
\gamma_{qq} &= \frac{C_F}{C_A} (\gamma_{gg} - \frac{N_f \alpha_s}{3\pi}).
\end{align*}
\]

(3.2)

Here \( H_2 \) is the \( k \)-factorization vertex measured by the \( F_2 \) structure function and, similarly to Eq. (2.13), is given by the abelian \( q\bar{q} \) contribution as \( \gamma_L \)

\[
H_2(\gamma) = C_F^{-1} \lim_{\gamma_2 \rightarrow 1-\gamma} H^a_{o}(\gamma, \gamma_2) (1 - \gamma - \gamma_2) = \\
\frac{\alpha_s \Gamma^2(\gamma) \Gamma^2(1-\gamma) \Gamma(2-\gamma) \Gamma(1+\gamma) \Gamma(4-2\gamma) \Gamma(2+2\gamma)}{\pi \Gamma(2+2\gamma) \Gamma(1+\gamma) \Gamma(4-2\gamma)} (2 + 3\gamma(1-\gamma)).
\]

(3.3)

The NL terms of \( \gamma_{gg} \) occur in the larger of the anomalous dimension eigenvalues

\[
\gamma_+ \simeq \gamma_{gg} + \frac{C_F}{C_A} \gamma_{gg}, \quad \gamma_- \simeq -\frac{C_F N_f \alpha_s}{3\pi C_A}
\]

(3.4)

and require a full discussion of the BFKL kernel

\[
K_{\omega}(\alpha_s, k_1, k_2) = \frac{\bar{\alpha}_s(k_1^2)}{\omega} (K_0(k_1, k_2) + \alpha_s K_1(k_1, k_2)).
\]

(3.5)

Here \( K_0 \) is the leading kernel, whose \( \gamma \)-dependent eigenvalue \( \chi_0(\gamma) \)

\[
\chi_0(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma)
\]

(3.6)

determines the anomalous dimension \( \gamma_L(\bar{\alpha}_s/\omega) \)

\[
1 = \frac{\bar{\alpha}_s}{\omega} \chi_0(\gamma_L), \quad \gamma_L = \frac{\bar{\alpha}_s}{\omega} + 2\zeta(3) \left( \frac{\bar{\alpha}_s}{\omega} \right)^2 + ... .
\]

(3.7)

Furthermore, both \( K_0 \) and the NL kernel \( K_1 \) are scale-invariant, because the non-invariant NL part has been factored out in Eq. (3.5) inside the running coupling constant at the scale \( k_1^2 \)

\[
\alpha_s(k_1^2) = \alpha_s(\mu^2) - b_0 \alpha_s^2(\mu^2) \log \frac{k_1^2}{\mu^2} + ..., \quad \mu = O(Q).
\]

(3.8)
We shall compute the NL contributions (which are factorization scheme dependent \[11, 18\]), in the so-called \(Q_0\)-scheme of Ref. \[12\], further discussed in a forthcoming paper \[15\]. In this scheme we define the quark sea density as in the physical DIS scheme \[19\], i.e. by the structure function \(F_2\), similarly to the parton model.

Instead, the gluon density is defined by the procedure of \(k\)-factorization itself, which provides a gauge invariant off shell continuation, so that the initial gluon virtuality can be fixed at \(Q_0\). Therefore, gluon states carry a high-energy polarization which in general does not match the collinear definition at higher orders. For this reason, the expression of \(\gamma_{gg}\) in Eq. (3.2) does not carry the R-factors of the \(MS\) type schemes \[11, 5\], and a leading similarity transformation \[12\] is needed to relate the former scheme to the latter.

The key advantage of the \(Q_0\)-scheme is that the anomalous dimension \(\gamma_+\) of Eq. (3.4) is computed from the eigenvalue of the scale invariant kernel \(K_0 + \alpha_s K_1\), directly in four dimensions, as follows:

\[
1 = \frac{\bar{\alpha}_s}{\omega} (\chi_0(\gamma_+) + \alpha_s \chi_1(\gamma_+)) , \quad \bar{\alpha}_s = \bar{\alpha}_s(\mu^2) .
\]

(3.9)

By expanding and using Eq. (3.7), we obtain the NL contribution to \(\gamma_+\)

\[
\gamma_+^{NL} \left( \frac{\bar{\alpha}_s}{\omega} \right) = -\alpha_s \frac{\chi_1(\gamma_L(\bar{\alpha}_s/\omega))}{\chi_0(\gamma_L(\bar{\alpha}_s/\omega))} = \gamma_{gg}^{NL} + \frac{C_F}{C_A} \gamma_{qg} ,
\]

(3.10)

which contains, in particular, the \(q\bar{q}\) contribution.

In order to compute \(\chi_1^{na}\), the non-abelian part of \(\chi_1^{q\bar{q}}\), we need to combine the singular real emission contribution in Eq. (2.11) with the virtual corrections, and then to add the resulting finite term to the nonsingular real emission part in Eq. (2.13).

The virtual \(q\bar{q}\) contribution to the one gluon emission kernel was computed in Ref. \[9\] and, after azimuthal averaging, takes the form

\[
\frac{\alpha_s N_f}{6\pi} \left( \frac{1}{q^2} \log \frac{M^2}{\mu^2} - \frac{1}{k_1^2 - k_2^2} \log \frac{k_1^2}{k_2^2} \right) , \quad (q = k_1 + k_2)
\]

(3.11)
in which $\mu^2 = O(Q^2)$ is the factorization scale, and $M$ is the (small) quark mass. By combining Eq. (3.11) with the singular $q\bar{q}$ emission part in Eq. (2.11) the $M^2$-dependence cancels out and we get
\[
\frac{\alpha_s N_f}{6\pi} \left[ \left( \log \frac{k_1^2}{\mu^2} + \log \frac{q^2}{k_1^2} - \frac{5}{3} \right) \left( \frac{1}{q^2} - \frac{1}{k_1^2 - k_2^2} \log \frac{k_1^2}{k_2^2} \right) \right]. \tag{3.12}
\]

A similar calculation can be performed \cite{11} directly at $M = 0$ and $D = 4 + 2\varepsilon$ dimensions, to yield the $\varepsilon$-dependent kernel
\[
\frac{\alpha_s N_f}{6\pi} \left[ \frac{1}{\varepsilon} \frac{\Gamma(1 + \varepsilon)}{\Gamma(1 + 2\varepsilon)} \left( \frac{q^2}{\mu^2} \right)^\varepsilon - \frac{1}{\varepsilon} \frac{1}{q^2} - \frac{1}{k_1^2 - k_2^2} \log \frac{k_1^2}{k_2^2} \right]. \tag{3.13}
\]

In order to regularize the $q^2 = 0$ singularity in Eq. (3.12) and (3.13), one should combine the above results with the purely virtual term, computed in Ref. \cite{10}. This can be done in dimensional regularization by means of Eq. (3.13), and the result \cite{11} can be read off directly in four dimensions as follows: the logarithmic term at scale $k_1$ in Eq. (3.12) is absorbed in the running coupling factor of Eq. (3.5) and the remaining (regularized) collinear part becomes
\[
\alpha_s K_1^{(v)} + K_1^{na,s} = \frac{\alpha_s N_f}{6\pi} \left[ \left( \log \frac{q^2}{k_1^2} - \frac{5}{3} \right) \left( \frac{1}{q^2} \right) \right] \bigg|_R - \frac{1}{k_1^2 - k_2^2} \log \frac{k_1^2}{k_2^2} \bigg] \tag{3.14}
\]
where we have introduced the notation
\[
f(k_1, q) \bigg|_R = f(k_1, q) - \delta^2(q) \int_{\lambda^2}^{k_1^2} d^2 q f(k_1, q) \tag{3.15}
\]
and $q^2 > \lambda^2$ is understood in all $q$-integrations. It is now straightforward to diagonalize the kernel (3.14) and to combine it with the finite real emission part in Eq. (2.13) to get the complete non-abelian eigenvalue expressed in terms of $\chi_0(\gamma)$ of Eq. (3.6) and of $\psi(\gamma)$
\[
\alpha_s \chi_1^{(na)}(\gamma) = \frac{N_f \alpha_s}{6\pi} \left[ \frac{1}{2} (\chi_0' + \chi_0^2) - \frac{5}{3} \chi_0 - (\psi'(\gamma) + \psi'(1 - \gamma)) + \left( \frac{\pi}{\sin \pi \gamma} \right)^2 \left( 1 - 3 \cos(\pi \gamma) \frac{1 + \frac{3}{2} \gamma (1 - \gamma)}{(3 - 2\gamma)(1 - 2\gamma)(1 + 2\gamma)} \right) \right], \tag{3.16}
\]
together with its abelian counterpart of Eq. (3.3):

\[
\alpha_s \chi_1^{(a)} = \frac{C_F \alpha_s N_F}{C_A} \left( \frac{\pi}{\sin \pi \gamma} \right)^2 \cos(\pi \gamma) \frac{1 + \frac{3}{2} \gamma (1 - \gamma)}{(3 - 2 \gamma)(1 - 2 \gamma)(1 + 2 \gamma)}. \tag{3.17}
\]

Note that both eigenvalues are symmetrical for \( \gamma \to 1 - \gamma \), apart from the first term in square brackets of Eq. (3.16) \( \sim \chi_0' \) which comes from the fact that we have factorized the running coupling effects in the upper mass \( k_1^2 \).

In order to extract from Eqs. (3.16) and (3.17) the anomalous dimension eigenvalue \( \gamma_+ \) we have to use Eq. (3.10), i.e., we have to divide it by \( \chi_0' \). Since \( \chi_0' (\gamma) \simeq -1/\gamma^2 + O(\gamma) \) for small \( \gamma \), the one-loop and two-loop results in the DIS scheme are read off from the small \( \gamma \) behaviour of Eqs. (3.16), (3.17), that is

\[
\alpha_s \chi_1^{(a)} (\gamma) \simeq \frac{C_F \alpha_s N_f}{C_A} - \frac{13}{6 \gamma} + O(1), \quad \alpha_s \chi_1^{(na)} (\gamma) \simeq \frac{\alpha_s N_f}{6\pi} \left( -\frac{1}{\gamma^2} - \frac{23}{6\gamma} + O(1) \right). \tag{3.18}
\]

In fact, by using Eq. (3.10) we check directly the known low-order expressions in the DIS scheme

\[
\gamma_{qg} = \frac{N_f \alpha_s}{3\pi} \left( 1 + \frac{13}{6} \frac{\alpha_s}{\omega} + \ldots \right), \quad \gamma_{gg}^{NL} = -\frac{N_f \alpha_s}{6\pi} \left( 1 + \frac{23}{6} \frac{\alpha_s}{\omega} + \ldots \right). \tag{3.19}
\]

Furthermore, Eqs. (3.16), (3.17) provide an all-order resummation in the \( Q_0 \)-scheme. Since \( \gamma_{qg} \) is known from Eq. (3.2), and \(-\chi_0' \gamma^2 = 1 + O(\gamma^3)\), we find that \( \chi_1^a \) is not perfectly cancelled by \((C_F/C_A)\gamma_{qg}\), and therefore \( \gamma_{gg}^{NL} \) takes also contributions of type \( \frac{N_f C_F \alpha_s}{\pi C_A} \left( \frac{\alpha_s}{\omega} \right)^n \) from three loops on. Finally, the terms \( \frac{N_f \alpha_s}{\pi} \left( \frac{\alpha_s}{\omega} \right)^n \) are resummed by \( \chi_1^{na} \), which is typically of negative sign, as it appears already at low order in Eq. (3.19).

The total \( q\bar{q} \) eigenvalue \( \chi_1^{q\bar{q}} (\gamma) = \chi_1^a (\gamma) + \chi_1^{na} (\gamma) \) is plotted against \( \gamma \) in Fig. 2. It is apparent that a cancellation between abelian and non abelian terms is at work, and that resummation effects are small, in their sum, even for \( \gamma \) values which approach the saturation limit \( \gamma = 1/2 \). Only for \( \gamma \to 1 \) is the abelian term dominant, because of its stronger \( \gamma = 1 \) singularity.

This remark is made more significant by the observation that the scheme dependence of \( \gamma_+ \) is a simple one. For instance, the transformation from the DIS-\( Q_0 \) scheme to the
DIS-\overline{MS} scheme, involves a scale change by a factor \( R(\gamma_L(\alpha_s(t))) \) \cite{11, 12}, under which \( \gamma_+ \) changes by running coupling effects only, i.e.,

\[
\tilde{\gamma}_+ = \gamma_+ - \frac{\dot{R}}{R}.
\] (3.20)

We tentatively conclude that \( q\bar{q} \) resummation effects in scaling violations are small in a combination proportional to \( F_q + (C_A/C_F)F_g \) which isolates the \( \gamma_+ \) eigenvalue, and is accessible experimentally through proper combinations of \( F_2 \) and \( F_L \) \cite{13, 18}, when the latter will be measured. It is important to check this conclusion against the purely gluonic NL terms, which are still under investigation \cite{7, 8}.

A final comment is needed for the running coupling effects (to which the \( q\bar{q} \) channel contributes) which remain an important NL feature of the BFKL equation. We have decided to factorize the \( \alpha_s \) dependence at the upper scale \( k_1^2 \) \cite{17} in Eq. (3.5). It can be shown \cite{13} that, in the regime \( k_1^2 \gg Q_0^2 \gg \Lambda^2 \), with \( \bar{\alpha}_s(Q_0^2)/\omega < 1 \), this dependence is equivalent to a renormalization-group evolution due to the anomalous dimension \( \gamma_+ \) of Eq. (3.9), apart from a calculable coefficient factor, common to quark the gluon in the \( Q_0 \)-scheme, given by the expression

\[
\left[ \gamma_L \left( \frac{\bar{\alpha}_s}{\omega} \right) \sqrt{-\chi_0(\gamma_L)} \right]^{-1}.
\] (3.21)

Since the latter differs from 1 only from 3 loops on, the choice we have made is good at perturbative level. The factor (3.21) can be reinterpreted in terms of an additional contribution to the gluon anomalous dimension, along the lines of Eq. (3.20).

To sum up, in this paper we have resummed in closed form the non abelian \( q\bar{q} \) contributions to both heavy flavour production and to the small-\( x \) anomalous dimensions. While non abelian resummation effects are quite important in the former case, they nearly cancel with the abelian ones in the latter, due to virtual contributions in the anomalous dimension eigenvalue.

The most significant NL effects (apart from the running coupling) seem to remain the rather large scaling violations due to \( \gamma_{qg} \) \cite{13}, which somehow represent a leakage from
the gluon density to the quark sea, which unfortunately depends on how the gluon is defined, even in the class of DIS-schemes. It is therefore important to measure a quantity independent of $F_2$, for instance $F_L$, in order to disentangle the gluon density from the quark sea and/or to look for combinations with smaller scaling violations, as is suggested by the result on $\gamma_+$ of the present work.

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Fig. 1: Kinematics of the $Q\bar{Q}$ hadroproduction. Wavy lines denote (Regge) gluon exchanges.
Fig. 2: $q\bar{q}$ contributions to the BFKL eigenvalue as function of the anomalous dimension variable $\gamma$. 