The existence of extra chiral generations with all fermions heavier than $M_Z$ is strongly disfavored by the precision electroweak data. The exclusion of one additional generation of heavy fermions in SUSY extension of Standard Model is less forbidden if chargino and neutralino have low degenerate masses with $\Delta m \simeq 1$ GeV. However the data are fitted nicely even by a few extra generations, if one allows neutral leptons to have masses close to 50 GeV. Such heavy neutrino can be searched in the reaction $e^+e^- \rightarrow N\bar{N}\gamma$ at LEP-200 with total final luminosity of $2600pb^{-1}$.

1 Introduction

The straightforward generalization of the Standard Model (SM) through inclusion of extra chiral generation(s) of heavy fermions, quarks ($q = U, D$) and leptons ($l = N, E$), is an example of New Physics at high energies which does not decouple at “low” ($\sim m_Z$) energies. New particles contribute to physical observables through self-energies of vector and axial currents. This gives corrections $\delta V_i$ to the functions $V_i(i = A, R, m)$ which determine the values of physical observables (axial coupling $g_A$, the ratio $R = g_V / g_A$, and the ratio $m_W / m_Z$).

We consider the case of several lepton and quarks $SU(2)_L$ doublets and their right-handed singlet companions: $(UD)_L, U_R, D_R, (NE)_L, N_R, E_R$. In what follows we will assume that the mixing among new generations and the three existing ones is small, hence new fermions contribute only to oblique corrections (vector boson self energies).

2 LEPTOP fit to experimental data

We compare theoretical predictions for the case of the presence of extra generations with experimental data, with the help of the code LEPTOP. These experimental data are the latest updates presented at this conference and they are well fitted by Standard Model. We perform the four parameter $(m_t, m_H, \alpha_s, \bar{\alpha})$ fit to 18 experimental observables.

The fitted parameters together with the values of the predicted observables and their pulls from the experimental data are given in the Table. Only the experimental value of the forward-backward asymmetry in the $Z$ decay into the pair of b-quarks $A_{FB}$ shows a hint for disagreement with Standard Model. We take $m_D = 130$ GeV – the lowest value allowed for the new quark mass from Tevatron search and take $m_U \gtrsim m_D$. As for the leptons from the extra generations, their masses are independent parameters. To simplify the analyses we start with $m_N = m_U$.

---

Footnotes:

*a* The mass of Z-boson in the fit was fixed to the latest experimental value $M_Z = 91.1875(21)$ GeV.

*b* During this conference the new results on the electron-positron annihilation into hadrons in the range $\sqrt{s} = 2 - 5$ GeV from BES were released. With $\alpha^{-1} = 128.945(60)$ recalculated using these new BES results, we get from LEPTOP fit slightly higher prediction for the higgs mass $m_H = 76^{+53}_{-32}$ GeV, $m_t = 174.1(4.5)$ GeV, $\alpha_s = 0.1182(27)$, $\bar{\alpha}^{-1} = 128.927(58)$ and $\chi^2/ndf = 21.1/14$. 
Table 1. LEPTOP fit of the precision observables.

| Observ. | Exper. data | LEPTOP fit | Pull |
|---------|-------------|-------------|------|
| $\Gamma_Z$ [GeV] | 2.4952(23) | 2.4964(16) | -0.5 |
| $\sigma_h$ [nb] | 41.541(37) | 41.479(15) | 1.7 |
| $R_t$ | 20.767(25) | 20.739(18) | 1.1 |
| $A_{FB}^t$ | 0.0171(10) | 0.0164(3) | 0.7 |
| $A_t$ | 0.1439(42) | 0.1480(13) | -1.0 |
| $A_e$ | 0.2165(7) | 0.2157(1) | 1.2 |
| $R_b$ | 0.1709(34) | 0.1723(1) | -0.4 |
| $A_{FB}^b$ | 0.0090(20) | 0.1038(9) | 0.8 |
| $A_{FB}^c$ | 0.0689(35) | 0.0742(7) | -0.5 |
| $s_{2l}^U$ | 0.2255(21) | 0.2231(2) | 0.7 |
| $s_{2l}^D$ | 0.2321(27) | 0.2314(12) | -0.5 |
| $m_W$ [GeV] | 80.434(37) | 80.397(23) | 1.0 |
| $s_{2W}$ (νN) | 0.2312(10) | 0.2314(2) | -0.5 |
| $m_H$ [GeV] | 174.3(5.1) | 174.0(4.2) | 1.0 |
| $\hat{\alpha}_s$ | 0.1183(27) | 0.1183(27) | 0.3 |
| $\alpha^{-1}$ | 128.88(9) | 128.85(9) | 0.3 |
| $\chi^2/n_{dof}$ | 21.4/14 | 21.4/14 | 21.4/14 |

$m_{E} = m_{D}$. Any value of higgs mass above 113.3 GeV is allowed in our fits, however $\chi^2$ appears to be minimal for $m_H = 113$ GeV.

In Figure 1 the excluded domains in coordinates $(N_g, \Delta m)$ are shown (here $\Delta m = (m^2_U - m^2_D)^{1/2}$). Minimum of $\chi^2$ corresponds to $N_g = 0.1$. We see that one extra generation corresponds to $2\sigma$ approximately.

We checked that similar bounds are valid for the general choice of heavy masses of leptons and quarks. In particular we found that for $m_N = m_D = 130$ GeV and $m_E = m_U$ one extra generation is excluded at 1.5 $\sigma$ level, while for $m_E = m_U = 130$ GeV and $m_N = m_D$ the limits are even stronger than in Fig. 1. So the extra generations are excluded by the electroweak precision data, if all extra fermions are heavy: $m \gtrsim m_Z$.

3 Extra generations in case of SUSY

When SUSY particles are heavy they decouple and the same standard model exclusion plots shown in Fig. 1 are valid. One possible exception is a contribution of the third generation squark doublet, enhanced by large stop-bottom splitting. In this way we get noticeable positive contributions to functions $V_i$, which may help to compensate negative contributions of degenerate extra generations. We analyze the simplest case of the absence of $\tilde{t}_L - \tilde{t}_R$ mixing in Fig. 2. In this figure the case of degenerate extra generations with common mass 130 GeV is considered (contributions of superpartners of new generations to $V_i$ are negligible since new up- and down-particles are degenerate). Exclusion plot is presented in coordinates $(N_g, m_{bottom})$. We see that with inclusion of SUSY new heavy generations are also disfavoured.

Situation changes in case of light chargino and neutralino. The latter are still not excluded - dedicated search at LEP II
still allows the existence of such particles with masses as low as 68 GeV (gaugino region with light sneutrino) or 77 GeV (higgsino case) if their mass difference is $\approx 1$ GeV. Analytical formulas for corrections to the functions $V_i$ from quasi degenerate chargino and neutralino were derived and analyzed in 11. Corrections are big and this allows one to get lower bounds on masses of chargino and neutralino: $m_\chi > 54$ GeV for the case of higgsino domination and $m_\chi > 61$ GeV for the case of wino domination at 95% CL.

Fig. 2 demonstrates how presence of chargino-neutralino pair (dominated by higgsino) with mass 80 GeV slightly relaxes the bounds shown on Fig. 1. We see that one extra generation of heavy fermions is allowed within 1.5$\sigma$ domain in case of the light chargino.

4 Heavy neutrino with $m_N < m_Z$

For particles with masses of the order of $m_Z/2$ oblique corrections drastically differ from what we have for masses $\gtrsim m_Z$. In particular, renormalization of $Z$-boson wave function produces large negative contribution to $V_A$. Quasi-stable neutral lepton $N$ should have the mass slightly above $m_Z/2$ to avoid increasing the invisible $Z$-width and it should have the mixing angle with three known generations smaller than $10^{-6}$ to avoid desintegration in the detector. We consider new heavy neutrino with Dirac mass and we suppose that the Majorana mass of $N_R$ is negligible. From the analysis of the initial set of precision data in papers 14, 15 it was found that the existence of additional light fermions with masses $\approx 50$ GeV is allowed. Now analyzing all precision data and using bounds from direct searches we conclude, that the only presently allowed light fermion is neutral lepton $N$.

As an example we take $m_U = 220$ GeV, $m_D = 200$ GeV, $m_E = 100$ GeV and draw exclusion plot in coordinates $(m_N, N_g)$, see Fig. 3. From this plot it is clear that for the case of fourth generation with $m_N \approx 50$ GeV...
description of the data is not worse than for the Standard Model and that even two new generations with $m_{N_1} \approx m_{N_2} \approx 50$ GeV are allowed within $1.5\sigma$.

5 Possibility for the direct search of the 50 GeV heavy neutrino

The direct search of the heavy neutrino is possible in $e^+e^-$-annihilation into a pair of heavy neutrinos with the emission of initial state bremsstrahlung photon

$$e^+e^- \rightarrow \gamma + \bar{N}N \quad (1)$$

The main background is the production of the pairs of conventional neutrinos with initial state bremsstrahlung photon

$$e^+e^- \rightarrow \gamma + \nu_i \bar{\nu_i} \quad (2)$$

where $i = e, \mu, \tau$. These background neutrinos are produced in decays of real and virtual $Z$. In case of $\nu_e \bar{\nu_e}$, two mechanisms contribute, through $s$-channel $Z$ boson and from $t$-channel exchange of $W$ boson. We calculated the signal and background distributions and rates using CompHEP computer code.

In Fig. 5 the distribution on “invisible” mass $M_{inv}$ (invariant mass of the neutrino pair) is represented for SM background and the $N\bar{N}$ signal for $\sqrt{s} = 200$ GeV and different values of $N$ masses, $M_N = 46 - 100$ GeV. Here we applied kinematical cuts on the photon polar angle and transverse momentum, $|\cos \theta_{\gamma}| < 0.95$ and $p_T^{\gamma} > 0.0375\sqrt{s}$, being the ALEPH selection criteria. The photon detection efficiency 74% is assumed. For highest significance of the $N\bar{N}$ signal, evaluated as $N_S/\sqrt{N_B}$, one should include whole interval on $M_{inv}$ allowed kinematically, so we applied $M_{inv} > 2m_N$ cut.

On Fig. 6 the signal significances are represented as a function of $m_N$. One can derive that only the analysis based on combined data from all four experiments both from 1997-1999 runs ($\sqrt{s} = 182 - 202$ GeV) and from the current run, in total $\sim 2600$ pb$^{-1}$, can exclude at 95% CL the interval of $N$ mass up to $\sim 50$ GeV.

Another possibility is to search for 50 GeV neutrino at the future TESLA $e^+ - e^-$ electron-positron linear collider. The in-
increase in energy leads to the decrease both of the signal and the background, but it is compensated by the proposed increase of luminosity of 300 fb$^{-1}$/year. Further advantage of the linear collider is the possibility to use polarized beams. This is important in suppressing the cross section of $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ as this reaction goes mainly through the $t$-channel exchange of the $W$ boson. However, even without exploiting the beam polarization the advantage of TESLA in the total number of events is extremely important. Thus, Standard Model is expected to give approximately 0.3 million single photon events for $M_{inv} > 100$ GeV while the number of 50 GeV neutrino pairs would be about 4000. Although the signal over background ratio is still small (2.3-0.5% for $m_N = 45 - 100$ GeV correspondingly) the significance of the signal is excellent, higher than 5 standard deviations for $m_N < 60$ GeV.

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1 Guidelines

1.1 Producing the Hard Copy

The hard copy may be produced using the instructions given in the file htmlstyle.1st, which are repeated in this section. You should have three files in total.

- readme.txt — the preliminary guide.
- ws-p10x7.cls — the style file that provides the higher level latex2e commands for the proceedings. Don’t change these parameters.
- ws-p10x7.tex — the main text.

You can delete our sample text and replace it with your own contribution to the volume, however we recommend keeping an initial version of the file for reference. Strip off any mail headers and then latex the tex file. The command for latexing is latex ws-p10x7, do this twice to sort out the cross-referencing.

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1.2 Using Other Word-Processing Packages

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files as guidelines; then please follow the style given here for headings, table and figure captions, and the footnote and citation marks. For this size of volume, the final page dimensions will be 10 by 7 inches, however you should submit the copy on standard A4 paper. The text area (which includes the page numbers) should be 8.25 by 5.6 inches and the separation between the columns is 0.20 in.
The text should be in 10 pt roman for the title, section heads and the body of the text, using capitals for the title and authors, bold face for the title and headings, and italics for the subheadings. The abstract, footnotes, figure and table captions should be in 8pt.

It’s also important to reproduce the spacing of the text and headings as shown here. Text should be slightly more than single-spaced; use a baselineskip (which is the average distance from the base of one line of text to the base of an adjacent line) of 13 pts and 10 pts for footnotes. All headings should be separated from the text preceding it by a baselineskip of about 26 pts and use a baselineskip of about 18 pts for the following text.

Paragraphs should have a first line indented by about 0.25 in (6mm), except where the paragraph is preceded by a heading, and the abstract should be indented on both sides by 0.25 in (6mm) from the main body of the text.

1.3 Headings and Text

Please preserve the style of the headings, text fonts and line spacing to provide a uniform style for the proceedings volume. In a two column format there are more difficulties when finding suitable line and page breaks. We recommend that you leave such problems until preparing the final draft and after you have checked the placing of the two-column wide tables etc. (See Sec. 1.7 below).

1.4 Equations

Equations should be confined to one column wherever possible, as in Eq. (2), and the eqnarray environment may be used to split equations into several lines, for example in Eq. (3), or to align several equations. An alternative method is given in Eq. (4) for long sets of equations where only one referencing equation number is wanted.

If it’s essential to have a two-column wide equation then use the method of Eq. (1) above. The surrounding environment is important here. In the text file \texttt{ws-p10xt.tex} make sure that you keep the declarations \texttt{\begin{table*}} and \texttt{\end{table*}} and only change the equation and its label within the inner equation environment.

For problems of placement of a wide equation, see Sec. 1.7 below. Please note, do not use square brackets in two-column

\begin{equation}
U = D(\delta_1, \delta_2, \delta_3, \delta_4) R_{12}(a, \delta_5) R_{13}(b, \delta_6) R_{14}(c, \delta_7) R_{23}(d, \delta_8) R_{24}(e, \delta_9) R_{34}(f, \delta_{10})
\end{equation}

Table 1. This is a Small Table.

| Title                           | $\epsilon'$ | $\lambda$ | $\gamma$ |
|---------------------------------|--------------|-----------|----------|
| 3.5687                          | 3.4567       | 3.8746    | 2.8934   |
| Trans Process for Decay         |              |           |          |
| 6.8977                          | 8.9087       | 2.8989    | 4.2928   |

Figure 1. Caption for the figure will come here.
wide figures, tables and equations. This is a bug due to the declaration \texttt{\textbackslash twocolumn[...]} which is hidden in the definition of \texttt{table*} and \texttt{figures}.

1.5 Tables
The tables are designed to have a uniform style throughout the proceedings volume. It doesn't matter how you choose to place the inner lines of the table, but we would prefer the border lines to be of the style shown in Tables 1 and 2. For either a single or a double column table, the top and bottom horizontal lines should be single (using \texttt{\textbackslash hline}), and there should be single vertical lines on the perimeter. (using \texttt{\textbackslash begin\{tabular\}\{}[\ldots]\textbackslash end\{tabular\}}) as in Tables 1 and 2. For the inner lines of the table, it looks better if they are kept to a minimum. We've chosen more complicated examples purely as an illustration of what is possible.

We recommend the use of single-column wide tables wherever possible. For the page wide tables, use the environment given in the example of Table 2. Do not change the latex commands from \texttt{\textbackslash begin\{table\}} to \texttt{\textbackslash begin\{tabular\}}, or from \texttt{\textbackslash end\{tabular\}} to \texttt{\textbackslash end\{table\}}, apart from inserting your own caption heading and table label. The caption heading for a table should be placed at the top of the table.

1.6 Figures
The same arguments apply as are given above for tables, i.e. it is preferable to have figures that fit into a column of the text. If this is not possible, then use the example of figure 2 and use the commands \texttt{\textbackslash begin\{figure\}} and \texttt{\textbackslash end\{figure\}}. The \texttt{\textbackslash figurebox\{}\texttt{\{}\texttt{\{}\texttt{\}}} command is defined with three arguments. First argument is for the figure height, second argument is for figure width, and the third will be for the actual figure/image file name, (i.e. eps/ps file name).

If you wish to ‘embed’ a \texttt{eps/ps} file name, then include the figure/image (\texttt{eps/ps}) file name in the third argument with an extension, (i.e. \texttt{figure.eps} or \texttt{figure.ps}, etc. While embedding the figure it is not necessary to fill the first and second arguments and if the third argument is empty, the system will read the first two arguments and generate the box based on the measurements provided.

If you like to reduce or enlarge (scaling) the \texttt{eps/ps} figure, please remove the \texttt{\%} mark from the declaration of the \texttt{\textbackslash epsfig} and provide the exact width of the figure. \texttt{LaTeX2e} will scale the figure based on the width provided and adjust the height accordingly. You can use any form of the units of measurement described in the \texttt{LaTeX} Book.

If instead you wish to use some other method, then it’s most important to leave the right amount of vertical space in the figure declaration to accommodate your figure (i.e. remove the lines and change the space in the example). Send the original hard copy of the figure on a separate page, clearly identifying where it should be placed on the final hard copy. The original figure you send should be correctly scaled as this ensures that the details will be visible in the final version. The figure caption should be placed below the figure.

1.7 Limitations on the Placement of Equations, Tables and Figures
In the final stages of preparing the document, try to declare the two-column wide figures, tables or equations at a point in \texttt{\textit{us-10x7.tex}} that is prior to the top of the column of hardcopy text where you would like the item to appear.\footnote{By “declaring” we refer to placing the chunk of material describing the table or whatever at a particular point in the text file.} Very large figures and tables should be placed on a page by themselves. One can use the instruction \texttt{\textbackslash begin\{figure\}\{}\texttt{\}{\} or \texttt{\textbackslash begin\{table\}\{}\texttt{\}{\} to position these, and they...}
Table 2. Experimental Data bearing on $\Gamma(K \to \pi\pi\gamma)$ for the $K_S^0$, $K_L^0$ and $K^-$ mesons.

| Meson | $\Gamma(\pi^+\pi^-)$ s$^{-1}$ | $\Gamma(\pi^+\pi^-\gamma)$ s$^{-1}$ | Additional Notes |
|-------|-----------------|-----------------|-----------------|
| $K_S^0$ | $0.769 \times 10^{10}$ | $5.46 \times 10^7$ | No DE observed, not even (IB)-E1 interference, despite large statistics, for $E_\gamma^* > 20$ MeV. |
| $K_L^0$ | $3.93 \times 10^4$ | $0.90 \times 10^3$ \hspace{1cm} (DE = $0.62 \times 10^3$) | DE prominent, exceeding IB over the range of measurement $20 < E_\gamma^* < 160$ MeV. |
| $K^-$ | $1.711 \times 10^7$ | $2.22 \times 10^4$ \hspace{1cm} (DE = $1.46 \times 10^3$) | No (IB)-E1 interference seen but data shows excess events relative to IB over $E_\gamma^* = 80$ to $100$ MeV |

will appear on a separate page devoted to figures and tables. Again, we would recommend making any necessary adjustments to the layout of the figures and tables only in the final draft. It is also simplest to sort out line and page breaks in the last stages.

1.8 Footnotes, the Bibliography, Appendices and Acknowledgments

Acknowledgments to funding bodies etc. may be placed in a separate section at the end of the text, before the Appendices. This should not be numbered so use \textit{Acknowledgements}.

It’s preferable to have no appendices in a brief article, but if more than one is necessary then simply copy the \textit{Appendix} heading and type in Appendix A, Appendix B etc. between the brackets.

Footnotes are denoted by a letter superscript in the text, and references are denoted by a number superscript. We have used \textit{bibtex} to produce the bibliography. Citations in the text use the labels defined in the bibtex declaration, for example, the first paper by Jarlskog\textsuperscript{4} is cited using the command \texttt{\cite{ja}}.

1.9 Final Manuscript

The final hard copy that you send must be absolutely clean and unfolded. It will be printed directly without any further editing. Use a printer that has a good resolution (300 dots per inch or higher). There should not be any corrections made on the printed pages, nor should adhesive tape cover any lettering. Photocopies are not acceptable.

The manuscript will not be reduced or enlarged when filmed so please ensure that indices and other small pieces of text are legible.

2 Sample Text

The following may be (and has been) described as ‘dangerously irrelevant’ physics. The Lorentz-invariant phase space integral for a general n-body decay from a particle
with momentum $P$ and mass $M$ is given by:

$$I((P-k_i)^2, m_i^2, M) = \frac{1}{(2\pi)^6} \int \frac{d^4k_i}{2\omega_i} (P-k_i).$$

(2)

The only experiment on $K^\pm \to \pi^\pm \pi^0 \gamma$ since 1976 is that of Bolotov et al.\textsuperscript{3} The photon spectrum observed certainly exceeds the IB spectrum for $E^*_\gamma \geq 70$ MeV. These authors report definite evidence for DE, however they "conclude that [IB-DE] interference is not observed" in their experiment but such interference is expected to be seen.

Although PDG has recorded the Serplukov experiment as $K^+$ decay, it is in fact a $K^-$ experiment. This is not a trivial difference. Assuming CPT invariance to be satisfied, CP violation is equivalent with T violation. The latter has to do with phases in the DE processes. Part of the calculated phase will arise from the final-state interactions, but there will also be a non-zero phase in the initial Lagrangian which may feed through to the phases in the final amplitudes. These Lagrangian phases will be different for the $K^+$ and $K^-$ processes, and can give rise to differences between spectra and rates for $K^+$ and $K^-$ mesons.

### 2.1 Parametrizations of the CKM Matrix

It is emphasised that there are two necessary conditions required for any acceptable parametrization of the quark mixing matrix. The first is that the matrix must be unitary, and the second is that it should contain a CP violating phase $\delta$. In Sec. 2.2 the connection between invariants (of form similar to $J$) and unitarity relations will be examined further for the more general $n \times n$ case. For the present, it’s sufficient to note that $J$ is equal to just twice the area of any one of these triangles. This does not mean that a non-zero $J$ follows from unitarity alone; if $J$ equalled zero, the unitarity constraint would still hold, but the triangle would collapse to a straight line and the measurement of $J$ would be of no use in determining the existence of CP violation if the quark mixing matrix was in fact represented by such a matrix. The reason is that such a matrix is not a faithful representation of the group, i.e. it does not cover all of the parameter space available.

$$T = \text{Im}[V_{11}V_{12}^*V_{21}^*V_{22}^*] + \text{Im}[V_{22}V_{23}^*V_{23}^*V_{22}^*] - \text{Im}[V_{22}V_{23}^*V_{22}^*V_{23}^*] - \text{Im}[V_{23}V_{23}^*V_{13}V_{12}^*].$$

(3)

There are only 162 quark mixing matrices using these parameters which are to first order in the phase variable $e^{i\delta}$ as is the case for the Jarlskog parametrizations, and for which $J$ is not identically zero.\textsuperscript{6} It should be emphasised that these are physically identical and form just one true parametrization.

### 2.2 Four and N-Generation Mixing Matrix

Murnaghan\textsuperscript{2} provided the most general representation of a $4 \times 4$ unitary matrix given in Eq. (1). We have calculated the possible combinations and have found that there are eight distinct parametrizations.

The unitary nature of the matrix imposes eight conditions on the connections between adjacent rows and columns, analogous to the six unitarity triangles for the three family case, but for four generations the unitarity condition forms a quadrilateral in the imaginary plane.

We have found only one set of invariants that are independent of their positions in the matrix, i.e. for which one can choose any element to be the ‘starting point’ element $V_{j,\alpha}$ in the definitions of $K, L$ and $M$ given below.

\textsuperscript{6}An example of a matrix which has elements containing the phase variable $e^{i\delta}$ to second order, i.e. elements with a phase variable $e^{2i\delta}$ is given at the end of this section.
Figure 2. Dynamics of two limit cycle oscillators. The repulsive case ($K < 0$) and attractive case ($K > 0$) are shown. The rate of phase separation, $f$, is plotted versus the phase separation between the oscillators.

(where the invariants are the sums or differences of the imaginary parts of four plaquettes). These are not however independent of the choice of parametrization but are invariant for each of the individual choices.

$$K = 3[V_{j,a}V_{j,a+1}V_{j+1,a}V_{j+1,a+1}]$$
$$+ 3[V_{k,a+2}V_{k,a+3}V_{k+1,a+2}V_{k+1,a+3}]$$
$$+ 3[V_{j+2,3}V_{j+2,3+1}V_{j+3,3}V_{j+3,3+1}]$$
$$+ 3[V_{k+2,3+2}V_{k+2,3+3}V_{k+3,3+2}]$$

$$L = 3[V_{j+1,a}V_{j+1,a+1}V_{j+2,a}V_{j+2,a+1}]$$
$$- 3[V_{j,a}V_{j,a+1}V_{j+1,a+2}V_{j+1,a+3}]$$
$$+ 3[V_{j+3,3}V_{j+3,3+1}V_{j+2,3+2}V_{j+2,3+3}]$$
$$- 3[V_{j+2,3}V_{j+2,3+1}V_{j+3,3+2}V_{j+3,3+3}]$$

$$M = 3[V_{j,a+1}V_{j,a}V_{j+1,a+1}V_{j+1,a}]$$
$$+ 3[V_{k,a+2}V_{k,a+3}V_{k+1,a+2}V_{k+1,a+3}]$$
$$+ 3[V_{j+2,3}V_{j+2,3+1}V_{j+3,3}V_{j+3,3+1}]$$
$$+ 3[V_{k+2,3+2}V_{k+2,3+3}V_{k+3,3+2}]$$

where $k = j$ or $j + 1$ and $\beta = a$ or $a + 1$, but if $k = j + 1$, then $\beta \neq a + 1$ and similarly, if $\beta = a + 1$ then $k \neq j + 1$.

Acknowledgments

This is where one places acknowledgments for funding bodies etc. Note that there are no section numbers for the Acknowledgments and Appendix. The style file will automatically generate the heading for references.

Appendix

We can insert an appendix here and place equations so that they are given numbers such as Eq. (5).

$$x = y. \quad (5)$$

References

1. C. Jarlskog in *CP Violation*, ed. C. Jarlskog (World Scientific, Singapore, 1988).
2. L. Maiani, *Phys. Lett.* B 62, 183 (1976).
3. J.D. Bjorken and I. Dunietz, *Phys. Rev. D* 36, 2109 (1987).
\[ \frac{N_s}{\sqrt{N_B}} \]

\( \sqrt{s} = 360 \text{ GeV} \)

\( \mathcal{L} = 300 \text{ fb}^{-1} \)
$m_H > 113$ GeV
$m_\ell = m_\nu = 130$ GeV

$\Delta m_4$ (GeV)

$N_{\text{generation}}$