Non-linear control over a continuous current to continuous current power converter using exact linearization

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Abstract. Taking as a starting point the exact feedback linearization to transform a non-linear system into a linear one through coordinate transformation and state-space feedback based on its non-linear model, we have presented in this document the modelling, analysis and non-linear control of a boost-type power converter, with the objective of comparing the results with a proportional, integral and derivative controller. This technique was then applied on a power converter from continuous current to continuous current and the dynamics were analyzed zero to verify the validity of the controller, which was simulated in the Simulink-Matlab software. Finally, the simulation results showed that the system with the exact linearization controller is not only free of excess, but also improves the stability in the output voltage.

1. Introduction
In physics as in mathematics and its applications, linearization refers to the process of finding the linear approximation to a function at a given point; in particular, in dynamical systems, linearization is a method to study the local stability of an equilibrium point of a system of non-linear differential equations. This procedure is used in fields such as physics, biology, economics, and engineering. Our interest is focused on the theory of feedback linearization whose foundation corresponds to various mathematical concepts of differential geometry such as the gradient of a scalar function, the Jacobian of a vector function and the directional derivative of a scalar field in the direction of a vector field, also known as Lie derivative and Frobenius' theorem. In that sense, we will deal with feedback linearization, which is a design method for non-linear control systems that has experienced great popularity and acceptance in recent years.

The main idea is to transform, partially or totally, the non-linear dynamics of the system to be controlled into linear dynamics. The fact of obtaining a linear resulting dynamic allows applying linear techniques to obtain a desired closed-loop system. On the other hand, the main drawback of the method is that it requires having an exact model of the system [1-4].

In this work, the non-linear model of the continuous current to continuous current power converter was developed. Then, exact feedback linearization was applied to transform the system into a linear one using coordinate transformation and state space feedback based on its non-linear model. In addition, zero dynamics was analyzed to verify the validity of the controller. Finally, the simulation results show that the system with the exact linearization controller is not only free of excess, but also improves the stability in the output voltage.
2. Content
Next, the mathematical formalism is presented in order to define some concepts. Suppose that \( h(x) \in \mathbb{R}^n \) is a scalar function, its gradient is a vector of dimensions \((1 \times n)\) defined by Equation (1).

\[
\nabla h = \frac{\partial h}{\partial x} = \begin{bmatrix} \frac{\partial h}{\partial x_1} & \ldots & \frac{\partial h}{\partial x_n} \end{bmatrix}.
\]

If \( f(x) \) is a vector function of the \((n\)-dimensional) state, also called the vector field, the Jacobian is given by an array of dimensions \((n \times n)\) \([5]\), Equation (2).

\[
\nabla f = \frac{\partial f}{\partial x} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \ldots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \ldots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}.
\]

Having said the above, we will proceed to define what the exact linearization consists of considering a non-linear system described by the following Equation (3).

\[
\dot{x} = f(x) + g(x)u, \quad y = h(x),
\]

and it is posed what conditions are needed for there to be feedback of states \( u = \alpha(x) + \beta(x)v \) this refers to the "Jacobian" linearization that converts the system into a linear one \([6-10]\). The Boost type power converter is a voltage booster circuit, which uses the characteristics of the inductor and the capacitor as energy storage elements to raise the current from the power source and use it to inject it into the capacitor, thus producing higher voltage levels in the load than the source.

The switch in Figure 1 consists of two elements: a fast-switching element such as a bipolar junction transistor, a metal oxide semiconductor field-effect transistor (MOSFET) or the most commonly used insulated gate bipolar transistor and the other, a diode with a much shorter recovery time than the control signal period; The function of the latter is to prevent the discharge current of the capacitor from returning since it is desired that when the source is disconnected from the capacitor and the load resistance to store energy in the coil, the current is supplied to the load by the discharge of the condenser; when the transistor is in conduction (switch in 1), the inductance stores energy and then supplies it simultaneously to the load and to the capacitor at another voltage level in the intervals in which the transistor is cut (switch in 2).

![Figure 1. CC-CC type Boost convert circuit.](image)

3. Application
Consider the power converter circuit. Boost-type continuous current to continuous current (CC-CC) shown in Figure 1 which is described by the system of Equation (4). In the circuit we can see that there is a commutation moment which generates some circuit conditions before and after said moment, said
moment makes it difficult to propose a model that describes its information through translational methods such as the Laplace transform since it ignores the conditions initials.

\[
\begin{align*}
\dot{x}_1 &= -w_0 x_2 + u w_0 x_2 + b, \\
\dot{x}_2 &= w_0 \dot{x}_1 - w_1 x_2 - u w_0 x_1, \\
y &= x_2,
\end{align*}
\]

(4)

where \(x_1 = 1/\sqrt{L}, x_2 = V/\sqrt{C}\), represent the standard variables of the input current to the coil L of the converter and the output voltage of the capacitor C, respectively.

\[
w = E/L > 0
\]

is the normalized numerical value of the external constant voltage source E. The counters \(w_0 = 1/\sqrt{LC}\), and \(w_1 = 1/RC\) are named respectively, natural oscillation frequency of the input LC circuit and time constant of the RC circuit of output, the variable or denotes the position function of the switch or switch, which acts with the control variable taking values in the discrete set \(\{0, 1\}\). This signal synthesizes by transistors and diodes. Table 1 shows the parameters of the DC-CC type Boost converts [11,12].

| Table 1. Converter parameters. |
|--------------------------------|
| Element | Description  | Value  |
|---------|--------------|--------|
| R       | Load resistance | 50 Ω   |
| C       | Condenser     | 25 μF  |
| L       | Inductance    | 400 μH |
| R       | Inductance resistance | 0.1 Ω |
| E       | Supply voltage | 100 V  |

To linearize the system, first find the equilibrium points to determine the necessary linear model, the system of Equation (5) and Equation (6) has the following.

\[
\begin{align*}
ul + \frac{V}{R} &= 0, \\
E - IR - uV &= 0.
\end{align*}
\]

(5)

(6)

Due to the simplicity of the equations, the solution for the equilibrium points is not difficult to obtain considering a duty cycle of \(u = D = 0.5\). The solutions are found to be: \(V = 198.412V\) and \(I = 7.936A\). With the previous equilibrium points we now proceed to find a linear model of the form Equation (7).

\[
\Delta \dot{x} = \frac{\partial x}{\partial x} \Delta x + \frac{\partial x}{\partial u} \Delta u,
\]

(7)

where you have to Equation (8).

\[
\frac{\partial x}{\partial x} = \begin{bmatrix}
\frac{\partial x_1}{\partial x_1} & \frac{\partial x_1}{\partial x_2} \\
\frac{\partial x_2}{\partial x_1} & \frac{\partial x_2}{\partial x_2}
\end{bmatrix} = \begin{bmatrix}
\frac{-r}{L} & \frac{-u}{L} \\
\frac{u}{L} & \frac{-1}{LC}
\end{bmatrix}
\]

and \(\frac{\partial x}{\partial u} = \begin{bmatrix}
\frac{\partial x_1}{\partial u} & \frac{\partial x_2}{\partial u}
\end{bmatrix} = \begin{bmatrix}
\frac{-V}{L} \\
\frac{1}{1/C}
\end{bmatrix}.
\]

(8)

By replacing the equilibrium points the following linearized system is obtained Equation (9) and Equation (10).

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} = \begin{bmatrix}
-250 & -1250 \\
20000 & -800
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + \begin{bmatrix}
-5 \times 10^5 \\
3.2 \times 10^5
\end{bmatrix},
\]

(9)
\[
\begin{bmatrix}
  x_1 \\
  x_2
\end{bmatrix}
= 
\begin{bmatrix}
  1 & 0 \\
  0 & 1
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  x_2
\end{bmatrix}
+ 
\begin{bmatrix}
  0 \\
  0
\end{bmatrix}.
\]

(10)

4. Results

In this section we will compare the output voltage in a non-linear DC-DC converter without the control action (the Boost converter) versus the output voltage when a controller for feedback of state variables and another using a PID intervenes in the system. The Figure 2 shows the output voltage of the non-linear DC-DC converter without the control action. We can see that the voltage has peaks which are unwanted for a real system since these could damage the end-use equipment, so it is necessary to be able to mitigate this effect.

![Figure 2. Boost converter output voltage without control action.](image)

Once the system was linearized by the exact linearization method described in the previous section, two controllers were designed, one by feedback of state variables and the other using a PID. Figures 3 present the response of the system. Figure 3(a) shows the output voltage of the controller by feedback of state variables as observed by this controller means that the output voltage does not present values above the reference that for this case are 200 V, which makes it ideal for use equipment. In the end without damage.

![Figures 3. The response of the system (a) output voltage, (b) error dynamics and, (c) control signal feedback status variables.](image)
The problem seen in this driver is mainly the time it takes to reach the reference; despite this, as can be seen in Figure 3(b) where the dynamics of the error are shown and in Figure 3(c) which shows the control signal, the system is much more efficient than the previous one.

Figure 4 shows the response of the system to a PID controller. Figure 4(a) shows the output voltage of the converter, which reaches the output voltage in a shorter time, but this speed is paid by making the system oscillate which is not ideal in this type of system, Figure 4(b) shows the dynamics of the error and Figure 4(c) the control signal.

Figure 4. System response to a PID controlled (a) output voltage, (b) error dynamics and (c) control signal.

The control strategy is highly effective since it brings the control signal to a reference which guarantees the least possible damage to the elements connected to the circuit, in addition the error is tolerable for this type of systems and in time quickly brought to zero and finally the state variables are subordinated to the control system.

5. Conclusions
As shown in the Figure 3 and Figure 4, the response of the most optimal system is that of feedback of state variables since it is in spite of the fact that it takes a few more seconds to reach the reference does not present oscillations as if the PID, in addition if they were adjusted parameters to the PID so that it does not present these oscillations the time to reach the reference would be even greater than the controller by feedback of state variables. For the aforementioned, an ideal controller for this type of systems where the control signal is expected not to exceed the reference is ideal. The PID controller is faster than the state feedback control. However, the latter reduces sensitivity to variation in the Load resistance.

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