Optimal sensor placement for stochastic sources in machine tools

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Accuracy requirements for machine tools demand an accurate positioning of the tool center point (TCP). At the same time the tools are comprised of many components with complicated geometries. Each component is subject to a variety of thermal loads, amongst them are the thermal couplings, the thermal loss near chip of and the cooling with fluids. The latter two are examples for unknown sources in space, where at most the average of the thermal power in space is known in advance.

We are interested in reducing the effect of the uncertainty in the exact heat source location in space on the accurate displacement of the tool center point (TCP). Furthermore we assume that only temperature sensors are available. After discretization in space, the evolution of the temperature field is described by the simplified state space system

\begin{align}
 MT\dot{t} &= AT + Bu + u_s u_d(t) \quad & (1.1a) \\
y_t(t) &= C_T T(t) \quad & (1.1b) \\
y_u(t) &= C_u T(t) \quad & (1.1c)
\end{align}

where the coefficient matrices $M$ and $A$ represent the discrete mass and the discretized Laplacian, together with parts of the robin-type boundary conditions, respectively. In addition, the ambient temperatures and further deterministic sources are part of the inputs $u$.

The stochastic distribution of the sources in space are described by the stochastic vector $u_s$. We assume a normally distributed vector $u_s$ with zero mean. The covariance of $u_s$ after the measurements is given by Equation (1.2d). The outputs $y_T$ and $y_u$ represent the temperature measurements and the displacement of the TCP, respectively.

The uncertainty of the TCP displacement is described by the covariance of the displacement. In our model, this corresponds to the covariance $C_u$ of the outputs $y_u$, for given measurements $y_T$. Therefore we aim to minimize the size of the covariance ellipsoids. As a measure for the size we use the logarithm of the volume at final time $t_f$. The optimization variables are the weights of $n$ possible sensor locations. From all locations, we seek to select the best locations for $m$ sensors.

Thus the associated integer minimization problem reads

\begin{align}
 \min_{w \in \{0,1\}^n} & \quad \log \det C_u \quad & (1.2a) \\
 1_m \cdot w &= m \quad & (1.2b)
\end{align}

with the displacements covariance

\begin{equation}
 C_u = S_u(t_f) \Gamma_u(w) S_u(t_f)^T \quad (1.2c)
\end{equation}

and the posterior covariance

\begin{equation}
 \Gamma_u(w)^{-1} = \Gamma_{\text{prior}} + \sigma_T^{-2} \sum_k S_T(t_k)^T \text{diag}(w) S_T(t_k) . \quad (1.2d)
\end{equation}

In these equations the matrices $S_u(t_f)$ and $S_T(t_k)$ represent the sensitivities of the displacement at the final time and the temperatures at all measurement times with respect to the uncertain sources, respectively. As a prior model $\Gamma_{\text{prior}}$ we use a

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Laplacian-like prior, whose parameters are derived from the physical parameters. Due to the linearity of the outputs $y_T(t)$ and $y_u(t)$ with respect to the sources $u$, the sensitivities do not depend on the source strengths and, in particular, they do not depend on the measured values. The latter independency renders the optimization independent of the measurements. Therefore the optimal placement can be computed in advance.

2 Results and conclusions

The optimization problem in Equation (1.2) is a binary optimization problem. Thus the optimization variables have to be relaxed appropriately.

We compare the simplicial decomposition (SD) method from [1] to the proximal extrapolated gradient method (PGMA) method from [2]. Both methods were used to compute the optimal sensor weights. Afterwards we round the largest weights to one and the others to zero, meaning no sensor at that position. Figure 2.1 depicts the objective with respect to the number of iterations to obtain the optimal solutions and the objective after rounding with a dot in the same color. Despite the much higher iteration number, the placement obtained by PGMA is slightly better than the SD method, even after rounding. Furthermore we observe a smaller deviation of the objective before and after rounding to the binary solution. For the comparison of the iteration numbers, one has to take the number of iterations for the restricted master problem in the SD method into account. These were 15 and 19 iterations for PGMA and the multiplicative algorithm from Torsney [3], respectively. Hence the total number of PGMA iterations using SD is higher than using PGMA directly.

In the Figure 2.2 we also show the selected sensors as colored spheres. Gray spheres correspond to sensor positions not selected. The other sphere colors correspond to the method in Figure 2.1. Interestingly, the PGMA method also adds sensors farther away from the TCP, which is located at the center.

The numerical results show a successful optimal sensor placement on a thermo-mechanical model. The numerical effort strongly depends on the ratio of number of possible locations to the number of available sensors. Furthermore, the solutions of the relaxed optimization problems are not necessarily near an optimal binary solution. Hence a binary approach, like branch and bound, might be more efficient.

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