Development of Structure and Characteristics Calculation Method for $\Gamma$-shape Rope Vibration Insulator

Yury K. Ponomarev
Samara National Research University, Russia, 443086, Samara, Moskovskoe shosse, 34.
E-mail: ponomarev-ssau@yandex.ru

Abstract: The paper gives an overview of the design of rope vibration insulators with elastic elements of the center line in the form of two rectilinear and one curved section. In the Russian-language scientific literature this type of rope vibration insulators received a stable name "$\Gamma$-shaped vibration insulators" by analogy with the shape of the letter "gamma-$\Gamma$" of the Greek alphabet and a similar letter of the Cyrillic alphabet. Despite the wide using of vibration insulators designed on this shape, its mathematical calculation model has not yet been developed. In this connection, in this article, for the first time on the basis of the "Method of Forces" and the "Mohr Method", an analytical technique has been developed for calculating the characteristics of a vibration insulator in the directions of three mutually perpendicular axes. In addition, the article proposes a new structure of a vibration insulator consisting of several tiers of elements of this type, based on a new patented technology for manufacturing quasi-continuous woven rings, proposed by the author of this article in co-authorship with several employees of the Samara National Research University. Simple formulas are obtained for calculating the load characteristics in three mutually perpendicular directions. This makes it possible to calculate the corresponding stiffness and natural frequencies of mechanical vibration protection systems. It is established that the stiffness of the vibration insulator in the direction of the Z axis is greater than the stiffness in the X and Y axis directions, however, if a vibration insulator with equal, or close to equal characteristics, along three axes has to be designed according to the technical specification, this can be done by selecting the parameters included in the equations given in article for load characteristics.

1. Introduction
One of first designers of $\Gamma$-shape rope vibration insulators is Carlo Camossi [1]. A simple structure and manufacturing technology of vibration insulator presented on Figure 1 was proposed by him. This vibration insulator has three metal or plastic lathes with holes; sections of rope are passed through these holes. The lathes in the places of rope are pressed in some points to hold this rope. The middle lath is deformed at 90-degree angle after vibration insulator assembling. Two extreme lathes have holes for connection with protected object and a basement which is under a load of vibration.
L.S. Tomilin developed a spatial rope vibration insulator [2] as a group of Г-shape or П-shape elements uniformly distributed on a circle and fixed in two round supports (figure 2).

Structure of Γ-shape vibration insulator as a group of ribbons with elliptic cross-section is presented on figure 3. The ribbons are winded by wire under stress to obtain large press load and damping properties [3].

In [4] a structure of vibration insulator as comb of Γ-shape elements with central guiding element made of deformed profile is proposed by V.S. Il’insky. Empirical equations for calculation of stiff characteristics of this vibration insulator are presented in [4] too.

As it is known for author of a present paper, there is no published analytic research of stiff characteristics for Γ-shape vibration insulators. The only research is [5]. Calculation of load characteristics of vibration insulator for three perpendicular directions obtained by finite element method in this paper. Results are presented in special criterial coordinates. It allows recalculation of these load characteristics in dimensional parameters if size of cross-section, length of straight parts and elastic modulus of rope material are known.
The aim of the present paper is development of workable structure of rope vibration insulator with Γ-shape elastic elements and development of analytic model of deformation for this vibration insulator. The developed structure is presented on Figure 4.

A volume model is presented on figure 5. The shape of elastic elements is obtained by thermal fixation [6] in a special device.

The problem of development of mathematic model of load characteristic of vibration insulator for three perpendicular directions was solved by methods of strength of materials [5]. The used admissions are:

1. Deformations of wires of elastic elements are little;
2. Spiral shape of wires in ropes elements didn’t take into account;
3. Deformation of wire material is in accordance to Hook law;
4. Layers of rope elements have no interaction during vibration insulator deformation.

Because elastic elements of vibration insulator are fixed in supports, but one support has linear displacement relatively another one, elastic elements are non-determined statically. This problem was
solved by force method [7]. The main system of Γ-shape element was loaded for it consequently by linear loads \( P_x, P_y, P_z \), unit moments and unit forces in accordance with method in [7]. For solution of static indeterminate the canonical equations of “force method” were obtained. “Superfluous” variables and equivalent bending moments were obtained from these equations, and load characteristics of element and vibration insulator were obtained for \( x, y, z \) axis directions as dependencies on geometrical and physical characteristics of vibration insulator:

\[
P_x(x, l, R) = \frac{12NEJ_x(4l + \pi R)x}{(24Rl^3 - 48R^3l + 3\pi^2 R^4 - 24Rl^3 + 10l^4 + 48R^2l^2 + 4\pi Rl^3 + 24\pi R^3l)};
\]

(1)

\[
P_y(y, l, R) = P_x(x, l, R);
\]

(2)

\[
P_z(z, l, R) = \frac{144EJ_z(\mu + 2)(4l + \pi R)zN}{(1728Rl^3 - 1152R^3l - 768R^4\mu + 384l^4\mu + 72\pi^2 R^4 - 576R^4 + 624l^4 + 2304R^2l^2 - 256R^4\mu^2 + 27\pi^2 R^4\mu^2 + 192\pi Rl^3 + 864\pi R^3l + 1152R^3l\mu - 1920R^3\mu + 288\pi R^2l^2 + 90\pi^2 R^4\mu + 1536R^2l^2\mu - 768R^3l\mu^2 + 96\pi Rl^3\mu + 936\pi R^3\mu + 144\pi R^2l^2\mu + 252\pi R^3l\mu^2)}.
\]

(3)

**Figure 6.** Load characteristics of vibration insulator for \( OX, OY \) and \( OZ \) axis direction: \( l = 25 \text{ mm}; R = 5 \text{ mm}; \) rope diameter \( D = 5 \text{ mm}; \) number of wire in section \( n = 49; N = 8; E = 2 \times 10^5 \text{ N/mm}^2 \)

Here \( E \) is elastic modulus for material of wire of rope, \( \mu = 0.3 \) is Poisson ratio, \( l, R \) are length of straight part and radius of axial line for rope elastic element (Figure 4), \( J_z = \pi(d^4/64) \) is axial inertia moment for cross-section of rope, \( z \) is deformation of vibration insulator under static load, \( N \) is number of Γ-shape elements in vibration insulator.
Load characteristics of vibration insulator for three perpendicular directions for specific geometrical and physical parameters are presented on Figure 6 as an example. As it is seen from figure 6, stiff characteristics of vibration insulator for vertical and two horizontal directions are different. Beside it, stiffness of vibration insulator as first differential from equations (1–3) of respective displacement (static deformation) depends on geometrical parameters $l$, $R$. It allows obtaining of required stiffness for any mechanical system.

Acknowledgment
An experimental specimen of developed vibration insulator is tested on an aerospace unit. This work was supported by the Ministry of Education and Science of the Russian Federation in the framework of the implementation of the Program «Research and development on priority directions of scientific-technological complex of Russia for 2014–2020».

References
[1] United States Patent 3 360 225. Shock and Vibration Mounts of the Cable Support Type. Carlo Carnossi, 1967.
[2] USSR Patent 241156, MPC F16F. Vibration insulator/ L.S. Tomilin (USSR). No 1169642/25-28. Applied 04.07.67. Published 01.04.69. Bulletin No 13.1. [in Russian]
[3] USSR Patent 2179664. MPC F16F 7/08. Antipov V.A., Kalakutsky V.I., Ponomarev Yu.K. et al. Published in Bulletin No 5 (2002 year). [in Russian]
[4] Il’insky V.S. Protection of radio-electronic and precise equipment against dynamic load //M.: «Radio i svyaz», 1982, 296 pp. [in Russian]
[5] Ponomarev Yu.K., Kalakutsky V.I. Multi-layer all-metal vibration insulators with regular structure of elastic elements. Samara: Samara science center of Russian Academy of science, 2003. – 168 pp. [in Russian]
[6] Lakhtin Yu.M., Rakhshtadt A.G. Thermal Treatment in Machine Manufacturing. M.: 1980. – 775 pp. [in Russian]
[7] Makarov E.G. Strength of materials on a base of Mathcad. «BHV - Peterburg», SPb:, 2004, - 512 pp. [in Russian]