Formation of double-layers and super-solitons in a six-component cometary dusty plasma

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Abstract. The formation and main features of the fully nonlinear structure in a six-component dusty plasma system have been investigated using Sagdeev potential approach. By means of pseudo-potential technique, the fluid equations describing the plasma system were diminished into a single nonlinear differential equation, called energy equation modified by presenting the parameters of addition plasma components. For certain values of the configurationally plasma parameters, our findings show that in addition to the solitary waves, double-layers and super-solitons exist and propagate in this plasma system. The main features of these waves have been studied by investigating the associated phase-portrait trajectories and potential curves as well and are found to be very sensitive to the variation of these parameters. The obtained results are mainly relevant to the cometary plasma and it may also provide better and helpful understanding of the nonlinear nature of space, astrophysical and cometary plasmas.

1 Introduction

The study of the fully nonlinear structures in a dusty plasma plays a significant role in astrophysical environments and space plasma such as cometary tails, comets and planetary rings. In the upper atmosphere and the lower magnetosphere, the study of nonlinear waves such as solitons, double-layers and super-solitons has also become one of the most important and interesting topics. Also, the study of nonlinear waves has a significant role in the fusion and high-power laser devices [1–7].

The double-layers are considered as an ending line for the solitons to be existed. But just when nearly approaching the double-layers formation domain, a new phenomenon appears that is “super-soliton.” One may identify the existence of super-soliton, while applying Sagdeev pseudo-potential technique [8], by determining three local maxima in the pseudo-potential curve. The concept of this new interesting plasma feature (localized super-soliton) was reported by Dubinov and Kolotkov [9–12], for the first time, in multispecies plasma systems. Then, Verheest et al. [13–17] discussed the existence of ion-acoustic super-solitons in various multispecies systems of plasma. The clear point about these models, which supports the existence of super-solitons, is that it should contain, at least, three components in order to form this amazing feature. Of course, a more complicated Sagdeev potential structure may appear, as the number of components of plasma system increases. Hence, a non-trivial phase-portrait topology may show the existence of such waves. It is a bit tricky to differentiate the true existence of the super-soliton from the regular soliton. However, one may safely confirm the existence of the super-soliton by obtaining the signature associated with the electric-field (wiggles) like shape. It is also important to mention that super-solitons are not a KdV-type class of small amplitude waves. In other words, the reductive perturbation method that based on small amplitude limitation is no longer valid to investigate the existence of super-solitons. El-Wakil et al. [18] investigated super-soliton waves, via Sagdeev potential technique, in four-component plasma system with non-extensive electrons and ions. The formation of nonlinear structure is found to be sensitive to the configurational plasma parameters such as non-extensive value, Mach number and the ratio of dust grains temperature.

In many astrophysical environments in the universe, the dust particles exist and play a central role in formation of nonlinear structures in dusty space plasmas [19–21]. Also, it was noticed that the famous Maxwellian distribution, which is highly effective and informative distribution, is no longer valid to accurately describe and represent the plasma particles behavior, where the dust particles do not exist in a stationary state. Thus, the plasma particles experience a clearly noticeable deviation from the well-known familiar Maxwellian distribution. The main reason behind this type of deviation is caused by the energetic particles present in the comet tail. Thus, Vasyliunas [22] considered kappa distribution function in stead-of the Maxwellian one. The plasma of a comet is made up of a pair of Oxygen ions (ion-pair), light ion (Hydrogen ion)
and electrons with distinct temperatures. Thus, for a reasonable and an accurate modeling, a plasma system should at least contain five components. Michael et al. [23] investigated a five-component of cometary plasma system, using the reductive perturbation method and ion-acoustic shocks were found to be existed. Most of dusty plasma in both space and laboratories are found to contain negative dust particles [24]. However, positive dust particles can exist in some regions in space or laboratories [25–27]. The existence of localized solitary wave structures in six-component dusty cometary plasma have discussed and investigated [28]. It was found that the localized rarefactive and compressive solitary waves may (co-) exist in this system of plasma within the reasonably discussed Mach number intervals. In addition to the solitary waves, multicomponent plasma model may support the existence of another expected localized nonlinear waves, called double-layer. In some limiting cases more complicated nonlinear structures (super-soliton) can be observed when the solitary waves occur beyond the existence range of double-layer. On the other side, in some limiting cases another type of nonlinear waves called periodic and super-periodic waves arise. The occurrence of such types of waves has been extensively studied by several authors [28–30].

Therefore, we aim here to extend the analysis of our previous work [31] to see whether the plasma model under consideration admits new types of nonlinear localized waves, such as double-layer and super-soliton. Our plasma model consists of Oxygen ion pair, negative dust particles, kappa distributed ions of Hydrogen, hot solar electrons and cold electrons of the comet tail [31]. The layout of this paper is organized as, the fluid equations describing our system in section II, then we drive an energy equation and introducing the pseudo-potential in section III, in section IV we discuss the analysis of the double-layers and super-solitons then finally conclusions can be found in section V, where we summarize the discussed ideas and results.

2 Basic equations

In the present investigation, we consider a six-component cometary dusty plasma system consisting of cold Oxygen ion pair, cold negative dust particles, and kappa distributed hot light ions of Hydrogen, hot solar electrons and electrons of the comet tail. In equilibrium, the neutrality condition

\[ Z_1 n_{10} - Z_2 n_{20} + Z_d n_{d0} + n_{ce0} + n_{se0} - n_{H0} = 0, \]

should be fulfilled. Here, \( Z_i \) is the charge number of inertial species (\( j = 1, 2, d \) with \( j = 1 \) denotes cold negative Oxygen ions, \( j = 2 \) used for cold positive ions of Oxygen and \( j = d \) refers to cold negative dust particles) and \( n_{i0} \) denotes the equilibrium density for inertial and inertialless species (\( k = j, i \) and \( i = ce, se, H \) where, \( i = ce \) points to comet tail electrons, \( i = se \) shows solar hot electrons and \( i = H \) represents protons (Hydrogen ions)). In this work, heavy ions pair of Oxygen and the negative dust particles are considered to be inertial species. The governing normalized fluid equations [31] describing the plasma model are

\[
\begin{align*}
\partial n_1/\partial t + \partial (n_1 u_1)/\partial x &= 0, \\
\partial u_1/\partial t + u_1 \partial u_1/\partial x &= (\alpha_1/\beta_1) \partial \varphi/\partial x, \\
\partial n_2/\partial t + \partial (n_2 u_2)/\partial x &= 0, \\
\partial u_2/\partial t + u_2 \partial u_2/\partial x &= -(\alpha_2/\beta_2) \partial \varphi/\partial x, \\
\partial n_d/\partial t + \partial (n_d u_d)/\partial x &= 0, \\
\partial u_d/\partial t + u_d \partial u_d/\partial x &= \partial \varphi/\partial x,
\end{align*}
\]

where \( n_j \) and \( u_j \) refer to normalized number density \( (n_j = N_j/N_{i0}) \), and velocity of inertial \( j \) species, \( j=1 \) for negative Oxygen ions, \( 2 \) for positive Oxygen ions and \( d \) for negative dust grains. Here, \( \varphi \) is the normalized electrostatic potential. \( \alpha_j = m_j/m_d, \beta_j = Z_d/Z_j \) are the dust mass ratio and dust charge ratio, respectively. To make this system self-consistence, the following Poisson’s equation may be introduced as

\[
\partial^2 \varphi/\partial x^2 = \mu_1 n_1 - \mu_2 n_2 + n_d + \mu_{ce} n_{ce} + \mu_{se} n_{se} - \mu_H n_H, \tag{5}
\]

where the density number ratios are

\[
\begin{align*}
\mu_1 &= (Z_1 n_{10})/(Z_d n_{d0}), & \mu_2 &= (Z_2 n_{20})/(Z_d n_{d0}), \\
\mu_{ce} &= n_{ce0}/(Z_d n_{d0}), & \mu_{se} &= n_{se0}/(Z_d n_{d0}), \\
\mu_H &= n_{H0}/(Z_d n_{d0}), \tag{6b}
\end{align*}
\]

and \( n_i \) are the normalized number densities of inertialless species, which are normalized with respect to their equilibrium values \( N_{i0} \) i.e., \( n_i = N_i/N_{i0} \) and assumed to be taken in the “kappa distribution” forms as:

\[
n_i = \{1 \mp \varphi/|\sigma_i(\kappa_i - 3/2)|\}^{-(\kappa_i + 1/2)}, \tag{7}
\]

where \( i=ce \) for cold comet electrons, \( se \) for hot solar electrons and \( H \) for Hydrogen ions, where the negative and positive signs stand for electron and ion, respectively. Here \( \kappa_i \) point to spectral index (kappa parameter) for inertialless species \( i \), and \( \sigma_i=T_i/T_d \) is the ratio between the temperature of the inertialless species \( i \) and the temperature of dust species \( T_d \). The velocities of inertial species in Eqs. (2)–(4) are normalized by the acoustic speed \( C_s \) i.e. \( u_j = V_j/C_s, C_s = \omega_p/\lambda_{Deff} \) with effective Debye length \( \lambda_{Deff} = 1/(\sum_i \lambda_{Di}^2)^{1/2} \), where \( \lambda_{Di} = [4\pi e^2 N_{i0}/(k_B T_i)][(2\kappa_i - 1)/(2\kappa_i - 3)]^{-1/2} \),
and effective plasma frequency $\omega_{p\text{eff}} = (\sum \omega_p^2)^{1/2}$, where $\omega_{p_j} = (4\pi e^2 Z_j^2 N_{j0}/m_j)^{1/2}$. $\varphi = Z_d e \phi / m_d C_d^2 \equiv e\phi / k_B T_d$ is the normalized electrostatic potential, $m_d$ is the mass of dust particles and $k_B$ is the well-known Boltzmann’s constant. $x$ is the normalized space coordinate using the effective Debye length $\lambda_{D,\text{eff}}$ while $t$ is the normalized time coordinate using the inverse of effective plasma frequency as $\omega_{p\text{eff}}^{-1}$.

Here, we deal with the normalized set of Eqs. (2)–(4) to investigate the formation and existence range of the fully nonlinear structures in a six-component dusty plasma system. In order to achieve this, it is reasonable to employ Sagdeev pseudo-potential technique [8, 28].

### 3 Sagdeev pseudo-potential analysis

By introducing the travelling wave transformation $\zeta = x - M t$, where $M$ is the Mach number (the velocity of nonlinear waves normalized with respect to the acoustic speed $C_d$), the system of Eqs. (2)–(5) are reduced to a set of nonlinear ordinary differential equations (ODEs). Solving this set of ODEs with the boundary conditions $n_j \rightarrow 1, u_j \rightarrow 0, \varphi \rightarrow 0$ as $\zeta \rightarrow \pm \infty$, one gets after some mathematical manipulations, the energy equation

$$
(d\varphi/d\zeta)^2 / 2 + V(\varphi) = E,
$$

(8)

where $V(\varphi)$ is the Sagdeev pseudo-potential,

$$
V(\varphi) = M^2 - M \sqrt{M^2 + 2\varphi} + \mu_1(\beta_1/\alpha_1)[M^2 - M \sqrt{M^2 + 2(\alpha_1/\beta_1)\varphi}] + \mu_2(\beta_2/\alpha_2)[M^2 - M \sqrt{M^2 - 2(\alpha_2/\beta_2)\varphi}] + \mu_{se}\sigma_{se} - \mu_{se}\sigma_{se}\{1 - \varphi/|\sigma_{se}(\kappa_{se} - 3/2)|\}^{-\kappa_{se}+3/2} + \mu_{ce}\sigma_{ce} - \mu_{ce}\sigma_{ce}\{1 - \varphi/|\sigma_{ce}(\kappa_{ce} - 3/2)|\}^{-\kappa_{ce}+3/2} + \mu_{H}\sigma_H - \mu_{H}\sigma_H\{1 + \varphi/|\sigma_H(\kappa_H - 3/2)|\}^{-\kappa_H+3/2}
$$

(9)

Equation (8) can be considered as the energy-balance equation of an oscillating pseudo-particle with mass of unity and velocity $d\varphi/d\zeta$ at a position $\zeta$ moves under the effect of pseudo-potential $V(\varphi)$ with total energy $E$.

With the knowledge of the explicit form of Sagdeev pseudo-potential $V(\varphi, M)$, one may examine the existence range of the electrostatic solitary waves, under the conditions:

i. $V(\varphi, M) = dV/d\varphi = 0$ and $d^2V/d\varphi^2 < 0$ at $\varphi = 0$, that mean $V(\varphi, M)$ is maximum.

ii. There exists a non-trivial root $\varphi_m$ at a maximum value of $\varphi$, i.e; $V(\varphi_m, M) = 0$.

iii. Furthermore, $V(\varphi, M) < 0$ in the intervals $(\varphi_m, 0]$ and $[0, \varphi_m)$, and $dV/d\varphi|_{\varphi=\varphi_m} > 0$.

The condition (i) yields the minimum value of the Mach number ($M_{\text{min}}$) for the existence of fully nonlinear solitary waves,

$$
M_{\text{min}} = [1 + \mu_1\alpha_1/\beta_1 + \mu_2\alpha_2/\beta_2]/
\left[\mu_{se}(2\kappa_{se} - 1) + \mu_{ce}(2\kappa_{ce} - 1) + \mu_{H}(2\kappa_{H} - 1)\right]^{1/2}
$$

(10)

Also, the same condition, $d^2V/d\varphi^2 < 0$ at $\varphi = 0$, leads to the normalized neutrality condition of the plasma system as $\mu_1 - \mu_2 + \mu_{se} + \mu_{ce} - \mu_H + 1 = 0$.

On the other side, double-layers can be also predicted under the following conditions:

$$
V(\varphi_m = 0, dV/d\varphi|_{\varphi=\varphi_m} = 0, d^2V/d\varphi^2|_{\varphi=\varphi_m} < 0.
$$

Now, by employing the general procedure of Sagdeev pseudo-potential technique, the system of fluid Eqs. (2)–(5) is reduced to a single nonlinear ordinary differential Eq. (8). Thus, finding the solutions of the system of fluid Eqs. (2)–(4) is equivalent to solve the energy Eq. (8). By applying the boundary conditions $\varphi \rightarrow 0$ and $d\varphi/d\zeta \rightarrow 0$ as $\zeta \rightarrow \pm \infty$, the solution can be directly obtained by integrating the following equation

$$
\zeta = \int_{\varphi_0}^{\varphi} d\varphi' / \sqrt{-V(\varphi')}
$$

(11)

Through this integration, one may determine the explicit form of localized electrostatic pulse $\varphi(\zeta)$ and the associated electrostatic field $E(\zeta)$ (via $E(\zeta) = -d\varphi/d\zeta$). Unfortunately, due to the clear complexity involved in the mathematical structure of $V(\varphi)$, the integration process remains impossible. Therefore, it would be better to deal with such equation numerically.

### 4 Double-layer and supersoliton analysis

To be more realistic, the observed values of configurational plasma parameters should be related to those of comet Halley plasma system [19–21, 23, 32–34]. This means that $Z_1 = Z_2 = 1, Z_d = 10^3, m_1 \approx m_2 \approx 2.6561 \times 10^{-23} \text{gm}, m_d \approx 2.5 \times 10^{-19} \text{gm}, N_{10} \approx 0.05 \text{cm}^{-3}, N_{20} \approx 0.5 \text{cm}^{-3}, N_{H_2} \approx 4.95 \text{cm}^{-3}, N_{se} \approx 4.95 \text{cm}^{-3}, N_{ce} \approx 0.45 \text{cm}^{-3}, T_H \approx 8 \times 10^K, T_{se} \approx 2 \times 10^K, T_{ce} \approx 2 \times 10^4 \text{K}$. Based on these values we have $\alpha_1 = \alpha_2 = \alpha$ and $\beta_1 = \beta_2 = \beta = 10^3$ while the densities ratios are $\mu_1 \approx 0.12195, \mu_2 \approx 1.2195, \mu_H \approx 12.073, \mu_{se} \approx 12.073, \mu_{ce} \approx 0.0976$. We have to emphasize that the neutrality condition should be always satisfied along the analysis.

In order to investigate the behavior of the fully nonlinear structures involved in such plasma model, we consult the bifurcation analysis in which the Sagdeev pseudo-potential function $V(\varphi)$, and the associated phase-portrait $(\varphi, d\varphi/d\zeta)$ should be first sketched versus the electrostatic potential $\varphi$ for different values of
some relevant plasma parameters. The advantage of using the bifurcation analysis is threefold: (i) to classify the allowed different types of nonlinear waves propagating in the plasma, (ii) to investigate the impact of some configurational plasma parameters on the formation of the nonlinear waves, (iii) to specify the existence range in which the nonlinear waves can propagate in the plasma under consideration. However, our findings can graphically be summarized as in Figs. 1, 2, 3, 4.

Generally, by looking upon these figures enable us to distinguish four types of localized electrostatic solitary pulses namely positive amplitude (compressive) solitary waves, double-layers, super-solitons and negative amplitude (rarefactive) solitary waves. To be more specific, the variation of Sagdeev pseudo-potential and phase-portrait topologies are depicted (Fig. 1a and b) for different values of mass ratio $\alpha (\alpha = 1420, 1408.5853, 1405, 1393)$, where the other parameters follow $M = 1.398, \sigma_{ce} = 1.112, \sigma_{se} = 250, \sigma_{H} = 50, \kappa_{ce} \approx 2.167, \kappa_{se} \approx 1.551, \kappa_{H} \approx 2.8118$. This figure shows obviously that the amplitude (width) of solitary wave decreases (increases) with increasing the mass ratio $\alpha$. The profile of both positive and negative electrostatic solitary waves $\varphi(\zeta)$ and electric field $E(\zeta)$ ($E = -d\varphi/d\zeta$) are graphically shown, as in Fig. 1c and d. Decreasing the value of the mass ratio $\alpha (\alpha = 1408.5853)$, the corresponding curves (dotted curves) in Fig. 1a and b support the coexistence of both positive double-layer and rarefactive solitary waves. This implies that positive double-layer and rarefactive solitary waves coexist
and propagate in plasma as $\alpha$ decreases and the amplitude get larger. Further slightly decreasing in the mass ratio $\alpha$ than that of double-layer (at $\alpha = 1405$), the dashed curves in the bifurcation diagrams in Fig. 1a and b refer to super-soliton on the positive side of $\varphi$ and rarefactive mode on negative side of $\varphi$ propagating simultaneously. The profile of the Sagdeev pseudo-potential and its phase-portrait at $\alpha = 1405$ follow clearly the behavior of the super-solitons [11], where there exist three local extrema between $\varphi = 0$ and $\varphi_m$ ($V(\varphi_m) = 0$). The variation of electrostatic pulses $\varphi(\zeta)$ and the associated electric field $E(\zeta)$ versus the variable $\zeta$ illustrates obviously the difference between a super-soliton and an ordinary soliton. The presence of subsidiary local three extrema makes the super-soliton profile is relatively distorted compared to the ordinary soliton. Accordingly, the electric field curve in Fig. 1d has two peaks, where the first peak is clearly higher than the second one which generates the electric field signature for existence of super-soliton. This implies that at mass ratio $\alpha = 1405$, the supersoliton coexists with rarefactive pulse and propagates in this plasma configuration. For lower value of mass ratio $\alpha$, the bifurcation curves indicate that super-soliton is no longer exist at $\alpha = 1393$(dot-dashed curves) and we are only left with rarefactive solitary waves. Now it is evident to conclude that changing the values of mass ratio $\alpha$ plays a significant role for the formation of different types of nonlinear structures propagating in the plasma model.

Next, we aim to determine to what extent are the Mach number effects of the waves necessary for the formation of the nonlinear structures in the plasma model. To illustrate the effects of the Mach number, we first depict the bifurcation diagrams (pseudo-potential $V(\varphi)$

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**Fig. 2** Plot of (a) Sagdeev potential, (b) Phase-portrait, (c) Electrostatic potential and (d) Electrostatic Field at different values of $M$
Fig. 3 Plot of a Sagdeev potential, b Phase-portrait, c Electrostatic potential and d Electrostatic Field at different values of \( \kappa_{se} \) and phase-portrait \( (\varphi, d\varphi/d\zeta) \) for different values of the Mach number \( M (M = 1.390, 1.395368, 1.399, 1.4045) \), as shown in Fig. 2a and b. At \( M = 1.390 \) (solid curves), one can see that two opposite polarity ordinary solitary waves (compressive and rarefactive) can coexist and propagate in the plasma configuration. The corresponding profiles of the localized electrostatic potential solitary wave \( \varphi(\zeta) \) and the associated electric field \( E(\zeta) \) are clearly shown in Fig. 2c and d, where the amplitude (width) of solitary wave increases (decreases) with increasing the Mach number. This means that solitary waves with larger amplitude move faster than smaller amplitude waves. For a higher value of the Mach numbers \( M \) (viz. \( M = 1.395368 \)), the dotted curves in Fig. 2a and b indicate the existence of positive amplitude double-layer that coexists with negative amplitude solitary wave, where there exist only one local extremum between \( \varphi = 0 \) and \( \varphi = \varphi_m \). The corresponding profile of the electrostatic modes \( \varphi(\zeta) \) and the electric field \( E(\zeta) \) at \( M = 1.395368 \) in Fig. 2c and d are precisely the same as for a double-layer and rarefactive solitary waves. This implies that the upper Mach number limit for positive solitary wave can be found once positive double-layer appears. Moving to higher value of the Mach number \( M \) than that the value of double-layer, (viz. \( M = 1.399 \)), there is a distinct structure (dashed curves), where three local extrema appear in the Sagdeev potential and phase-portrait, as shown in Fig. 2a and b. This could support the possibility of occurrence of super-solitons at the Mach numbers.
Fig. 4 Plot of a Sagdeev potential, b Phase-portrait, c Electrostatic potential and d Electrostatic Field at different values of $\sigma$.

$M = 1.399$. The occurrence of super-solitons can be terminated once two of the extrema could coalesce and then we go back to ordinary solitons, accordingly. It is noting that the existence of double-layer provides us the minimum value of Mach number range for the existence of super-soliton. The main features and properties of positive supersoliton $\varphi(\zeta)$ and the corresponding electric field $E(\zeta)$ can be obviously observed in Fig. 2c and d. This means that, at $M = 1.399$ the plasma configurations support the coexistence of both positive super-solitons and rarefactive solitary waves, where its amplitude increases as the Mach number $M$ increases. Further increasing in the value of the Mach number, say, $M = 1.4045$, the existence of positive double-layers or positive supersolitons is no longer possible and the remaining type is the rarefactive solitary wave. At this point, one may obviously see that the variation of the Mach number $M$ plays a significant role for the formation and existence range of nonlinear structures propagating in the plasma model.

The impact of solar electron kappa parameter $\kappa_{se}$ on the formation of the admissible nonlinear structures can be also illustrated by plotting the bifurcation diagrams, as in Fig. 3a and b. Here, we fix the value of Mach number at $M = 1.398$ and mass ratio at $\alpha = 1403$, the Sagdeev potential $V(\varphi)$ and phase-portrait ($\varphi, d\varphi/d\zeta$) are depicted for different values of $\kappa_{se}$ as shown in Fig. 3a and b. In these figures, one can see two opposite polarities of solitary waves coexist at $\kappa_{se} = 1.556$ (solid curves), whereas moving toward the lower value of $\kappa_{se}$ ($\kappa_{se} = 1.552075$), plasma configurations support the coexistence of positive double-layer with negative solitary pulses, where its amplitude (width) increases (decreases) as $\kappa_{se}$ decreases. For lower
values of $\kappa_{sc}$, say, $\kappa_{sc} = 1.548$, the super-solitons coexist with negative amplitude solitary waves, whereas, in case of $\kappa_{sc} = 1.539$ (dot-dashed curves) the supersoliton is no longer exist and we are only left with the negative potential pulse propagating in our plasma system. From above, one can see that the existence range and the formation of nonlinear structures are found to be very sensitive to the magnitude of solar electron kappa parameter $\kappa_{sc}$.

Additionally, the influences of other parameters such as the kappa parameters for both comet electrons $\kappa_{ce}$ and Hydrogen ions $\kappa_{H}$ and temperature ratios for the three inertialess components $\sigma_{ce}$, $\sigma_{se}$ and $\sigma_{H}$ can be also examined in the same manner. For example, the effects of the temperature ratio of comet electrons $\sigma_{ce}$ can be graphically shown, as in Figs. (4). Plotting $V(\varphi)$ and $(\varphi, d\varphi/d\zeta)$ for different values of $\sigma_{ce}$ ($\sigma_{ce}=1.125, 1.117559, 1.112, 1.103$), one can clearly observe that positive and negative amplitude solitary waves coexist at $\sigma_{ce} = 1.125$ (solid curve). For lower values of $\sigma_{ce}$ ($\sigma_{ce} = 1.117559$) positive double-layer may be generated and coexist with negative amplitude solitary pulse, while at $\sigma_{ce} = 1.112$ there exist a supersoliton propagating with the negative pulse, even simultaneously. Moving toward the lower values of $\sigma_{ce}$ ($\sigma_{ce} = 1.103$), supersoliton suddenly disappeared and only left with negative solitary waves in this plasma configuration.

From above analysis, one may lead to the conclusion that the formations and the existence range of the fully nonlinear structures (soliton, double-layer and supersoliton) are found to be very sensitive to the variation of the configurational plasma parameters.

Finally, it is mentioned that another type of nonlinear waves called nonlinear periodic and super-periodic waves can exist by investigating the solid curves in the Sagdeev potential diagrams, Figs. 1a, 2a, 3a, 4a. As an example, the solid curve in the potential diagram, Fig. 1a, has obviously three fixed points. The corresponding trajectories of the phase-portrait show clearly unstable saddle point at $(0,0)$ and two stable centers at $(\varphi_1,0)$ and $(\varphi_2,0)$, where $\varphi_1$ and $\varphi_2$ are the real roots of $dV/d\varphi = 0$. Due to the complexity involving in solving the equation $dV/d\varphi = 0$ for $\varphi$, it is reasonable to find out $\varphi_1$ and $\varphi_2$ graphically, as in Fig. 5. It can be seen that the periodic orbits around the two centers with different values of the total energy (E) refer to a family of periodic solutions. On the other side, the trajectory that is going from the saddle point and returning to it (homoclinic orbit) refers to the coexistence of compressive ($\varphi > 0$) and rarefactive ($\varphi < 0$) solitary waves. Moreover, if E exceeds the value of the homoclinic orbit, another feature of nonlinear structure arises, where the existence and propagation of nonlinear super-periodic waves can also be observed in this plasma configuration, as shown in Fig. 5. It is noted that in small amplitude approximation the behavior and property of the nonlinear periodic and super-periodic waves propagating in different plasma systems are extensively studied by means of the bifurcation technique [28–30]. In case of arbitrary amplitude, the existence and propagation of nonlinear periodic and super-periodic solutions in our plasma system is in progress considering the impact of some relevant plasma parameters on the features of such waves.

5 Conclusion

In this article, the existence and propagation of fully nonlinear structures in a six-component plasma have been investigated by means of Sagdeev pseudo-potential technique and bifurcation analysis. Based on this analysis, one can distinguish four types of localized nonlinear waves (compressive solitons, rarefactive solitons, double-layers and super-solitons) depending mainly on the values of the configurational plasma parameters. The properties of the nonlinear waves propagating in the desired plasma are shown graphically. While plotting the figures the numerical values considered are $Z_1 = Z_2 = Z = 1$, $Z_d = 10^3, m_1 \approx m_2 = m \approx 2.6561 \times 10^{-23} gm, m_d \approx 2.5 \times 10^{-19} gm, N_{10} \approx 0.05 cm^{-3}, N_{20} \approx 0.5 cm^{-3}, N_{H^0} \approx 4.95 cm^{-3}, N_{sc} \approx 4.95 cm^{-3}, N_{ce} \approx 0.45 cm^{-3}, T_H \approx 8 \times 10^4 K, T_{sc} \approx 2 \times 10^5 K$ and $T_{ce} \approx 2 \times 10^4 K$, which are found in space plasma such as Halley comet tail and interplanetary plasma. The occurrence and the existence range of the nonlinear waves are found to be very sensitive to the magnitude of the mass ratio $\alpha$, Mach number $M$, kappa parameters $\kappa_i$ and the temperature ratios $\sigma_i$. For instance, at certain value of the Mach number $(M = 1.390)$ the coexistence of both negative and positive localized electrostatic modes can be observed. Moving to the higher value of the Mach number $(M = 1.395368)$ the positive amplitude double-layer coexist with negative amplitude solitary wave, while positive amplitude super-soliton can be generated and propagated with negative localized solitary wave, simultaneously. The supersoliton can be identified by the signature wiggle shape of electrostatic field $E(\zeta)$. This of course makes the profile of the electrostatic potential $\varphi(\zeta)$ of the super-soliton is sharper than that of the regular soliton, accordingly. Increasing the Mach number $M$ (at $M = 1.4045$), the propagation of the super-soliton is no longer exist and we are only left with negative amplitude solitary waves. However, the nonlinear structures get stronger as $M$ increases. Obviously, the Mach number effects start plying central role in the formation of double-layer and supersoliton when $M$ is equal or greater than $M = 1.395368$. On the other side, our nonlinear analysis, the mass ratio $\alpha$, kappa parameters $\kappa_i$, and the temperature ratios $\sigma_i$ have clearly opposite effects on the formation of the fully nonlinear structures.

The bipolar electric field signature associated with the supersoliton makes our findings very important and significant because one may easily visualize and distinguish the satellite observations of nonlinear electrostatic localized waves in space plasmas. Since the results in this work are based on real values related to that of
comet Halley, the obtained results may support a better understanding of the cometary plasma environments and astrophysical plasma field in general.

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**Conflict of interest** The authors declare that they have no conflict of interest.

**Data Availability Statement** This manuscript has associated data in a data repository. [Authors’ comment: All data generated or analyzed during this study are included in this published article.]

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**Fig. 5** Plot of a Sagdeev potential, b Phase-portrait at different values of total energy $E$
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