Study on Multi-scale Time-frequency Analysis of Cutting Force Based on EMD Method

Ying Wu
College of Mechanical Engineering, Shenyang ligong University, Shenyang, Liaoning, 110159, China
* wying420@126.com

Abstract. A method for establishing the multi-scale cutting force spectrum under complex random load history is presented according to the non-stationary random characteristics of the cutting force-time history. The IMF components of different time scales are obtained by EMD decomposition of complex random cutting force-time history data. The probability density distribution of cutting force in certain frequency band is obtained as the load spectrum on this time scale. This method avoids the disadvantage of the conventional method that the load or stress is regarded as a constant or variable amplitude load under one frequency to establish the load spectrum. The method for establishing the multi-scale load spectrum can provide a basis for fatigue life calculation and fatigue reliability analysis of mechanical components.

1. Introduction
The external excitation of CNC machine tool mainly comes from cutting force. The cutting force and cutting speed of CNC machine tools are random [1]. The cutting process is a complex non-stationary random process [2, 3]. In terms of the spectrum analysis of the cutting force, several load components with different frequency and amplitude can be decomposed. In the relevant literature [4, 5], the multi-axis non identical frequency analysis and dual-frequency load analysis methods are proposed, without intensive study on the establishment of load spectrum on the different frequency band of the multi-frequency load-time-history. In fact, the impact of load of different frequency bands on fatigue life and fatigue reliability is different. Therefore, it is necessary to deeply study the establishment method of load spectrum on the different frequency band of the multi-frequency load-time-history.

EMD method is a method proposed by Norden e. Huang in 1998 to analyze nonlinear and non-stable signals [6, 7]. EMD can adaptively decompose non-stationary signals into several independent IMF, which can express the local characteristics of signals [8].

IMF components at different time scales were obtained by EMD decomposition of cutting force-time process data. Hilbert transform was performed on each IMF component to obtain the Hilbert marginal spectrum of each IMF component. And the load band of each component can be determined. Data statistics analysis of the load cycle in IMF signals of each frequency band were conducted. The probability density distribution of the load on the frequency band can be obtained as the load spectrum on the time scale. This method avoids the disadvantages of the conventional method in which the load or stress is regarded as constant or variable load with only one frequency. The multi-scale load spectrum established by this method can provide more accurate basis for the fatigue life calculation and fatigue reliability analysis of mechanical parts.
2. EDM Time-Frequency Analysis Theory

EDM method realized the decomposition of IMF function through the screening process. And multiple IMF components \( C_n(t) \) and residual term \( r \) from high frequency to low frequency can be obtained \([9, 10]\). The screening process ends until a given termination condition is satisfied. Finally, the original signal can be represented as formula (1).

\[
x(t) = \sum_{i=1}^{k} C_i(t) + r
\]

Where each IMF component \( C_1(t), C_2(t), \ldots, C_k(t) \) represents the components of the original signal from high frequency to low frequency.

The Cauchy convergence criterion (as shown in (2)) is used to constrain the screening process.

\[
SD = \sum_{t=0}^{T} \left[ \frac{|h_{1(k-1)}(t) - h_{1k}(t)|^2}{h_{1(k-1)}^2(t)} \right]
\]

Where SD is the screen threshold value, generally in 0.2-0.3; \( T \) is the time of the signal sequence; \( h_{1k}(t) \) is the obtained data for the \( k \)-th screening; \( h_{1(k-1)}(t) \) is the obtained data for the \((k-1)\)-th screening.

The Hilbert-Huang spectrum is defined as shown in equation (3).

\[
H(\omega, t) = \text{Re} \sum_{i=1}^{n} a_i(t) \exp(i \int \omega_i(t) dt)
\]

Where \( a_i(t) \) is the amplitude, \( \omega_i(t) \) is the instantaneous frequency. The marginal spectrum can be defined as shown in equation (4).

\[
h(\omega) = \int_{0}^{T} H(\omega, t) dt
\]

Where \( h(\omega) \) is the Hilbert marginal spectrum, \( T \) is the sampling time. Hilbert marginal spectrum can accurately describe the amplitudes of each frequency component and has a high resolution.

3. Establishment of Multi-Scale Cutting Force Spectrum

Multi-frequency load analysis for the measured load-time history or the simulated load-time history is performed. Firstly, the stationarity and ergodicity of the signals are tested. Secondly, EMD decomposition is performed on load-time history data to obtain IMF components at different time scales. The Hilbert marginal spectrum of each IMF component is obtained by Hilbert transformation. Then the appropriate load cycle counting method is selected for the IMF signals of each frequency band, and the data statistics are carried out respectively. The probability density distribution of cutting force in this frequency band is obtained as the load spectrum on this time scale. The technical flow chart of this method is shown in figure 1.

4. Establishment of Cutting Force Spectrum in a Milling Process

Milling force experiment was tested on the TC500 vertical drilling and tapping center. Test instruments include Kistler9257B three-dimensional dynamometer and 5017B multichannel charge amplifier. In the process of experiment, the three-dimensional dynamometer was installed on the worktable, and the workpiece was fixed on the dynamometer as shown in figure 2, and the cutting force of the machining center can be measured. The experimental parameters are shown in table 1.

Most of the cutting conditions of this T500 are milling. In this paper, the multi-scale load spectrums of cutting force are established by taking the cutter as the end milling cutter.

| Table 1. Machining parameters of milling force test on the TC500. |
|-----------------|-----------------|
| Parameter values | Parameter values |
| spindle speed n (r/min) | 1200 |
cutting depth $\alpha_p$ (mm) 0.2

Feeding speed $F$ (mm/min) 300

Figure 1. Technical flowchart of multi-scale load spectrum establishment based on EMD theory.

Figure 2. Milling force test experiment on the TC500.
(1) The load-time history of the main cutting force $F_c$. The main cutting force-time history measured from the experiment is shown in figure 3.

![Figure 3. The main cutting force-time history diagram.](image)

(2) Test the stationarity and ergodicity of measured $F_c$ signal. The test of stationarity is carried out by the method of round test. Here number of rounds is $r = 13$ and all are in the $(8,18)$ region. So stationary hypothesis is accepted at significant level $\alpha = 0.05$.

(3) The main cutting force is decomposed by emd method, and the IMF components of different time scales are obtained.

(4) Hilbert marginal spectrum of each IMF component is shown in figure 4. And the load band of each component can be determined.

![Figure 4. Hilbert marginal spectrum of each IMF component.](image)
As can be seen from figure 4, the eighth order frequency of IMF decomposition is very low. In this paper, only the first 7 IMF components with relatively high amplitude and frequency are considered to establish the load spectrum.

(5) The load cycle and cyclic counting method are determined for the IMF signals in each frequency band. Each rotation of the spindle of the TC500 is used as a load cycle. The number of spindles rotated is the corresponding number of load cycles, as shown in formula (5)

$$N = \frac{L}{F}$$  \hspace{1cm} (5)

Where is the cutting length, mm; F is the feed speed, mm/r.

(6) Probability density function of load amplitude in each IMF component is fitted. Statistical analysis, parameter estimation and hypothesis testing were performed for each IMF component. The distribution histogram of the load amplitude of each IMF component is obtained, as shown in figure 5. For the complex histogram, the mixed Gaussian distribution or mixed Weibull distribution can be used to fit the histogram.

$\chi^2$ test is used to test the load amplitude distribution obtained by fitting. According to the test, the probability density function of the amplitude of each IMF component conforms to different parameter distributions respectively, and all of them follow the original hypothesis. The probability density function of the load amplitude of each IMF component is shown in the formula (6) ~ (12).
5. Conclusions

In this paper, a multi-scale load spectrum establishment method based on EMD time-frequency analysis theory is proposed. Through EMD decomposition of the complex random load-time history data, the probability density distribution of loads on each frequency band is obtained as the load spectrum on this time scale. The multi-scale load spectrum of cutting force of a milling process on a
TC500 machine tool was established, and the feasibility of the method was verified. The establishing method of multi-scale load spectrum can provide a basis for fatigue life calculation and fatigue reliability analysis of mechanical components.

Acknowledgments
The authors gratefully acknowledge the support of the Natural Science Foundation of Liaoning Province, China (No. 2019-ZD-0263) and High-level Talent Research Foundation of Shenyang Ligong University (No.1050002000604).

References
[1] Li Bin, Liu Xiaolong, Liu Hongqi, et al. 2013. Estimation of dynamic milling force by feeding motor currents. Journal Huazhong University of Science and Technology (Natural Science Edition) (Jun. 2013), 1-6.
[2] A.M.M. Sharif Ullah, Khalifa H. Harib. 2010. Simulation of cutting force using nonstationary Gaussian process. J Intell Manuf (Feb. 2010), 681-691.
[3] Dong Gao, Zhirong Liao, Zekun Lv, et al. 2015. Multi-scale statistical signal rocessing of cutting force in cutting tool condition monitoring. Int J Adv Manuf Technol (Oct. 2015), 1843-1853.
[4] Huang Yuhua, Wang Dejun, Mao Guiting, et al. 1990. A Study on The Effect of Fatigue Character Under Dual-frequency Loading. Mechanical Strength (Jun. 1990), 19-21.
[5] Zhai Lifang. 2011. Research on Fatigue Life Prediction Method under Multiaxial Out-of-frequency Loading. Northeastern University.
[6] Huang N E, Shen Z, Long S R. 1998. The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis. Proc . R . Soc . Lond A (Mar. 1998), 903-995.
[7] Huang N E, Shen Z, Long S R. 1999. A new view of nonlinear water waves: the hilbert spectrum. Annu. Rev. FluidMech (Jan. 1999), 417-457.
[8] Sun Weiwei, Huang Min, Gao Yan. 2017. CNC Tools Wearing Fault Diagnosis Based on EMD-HMM. J Intell Manuf 45 (Jul. 2017), 178-181.
[9] Guan Shan, Wang Longshan, Nie peng. 2011. Identification method of tool wear based on empirical mode decomposition and least squares support vector machine. Journal of Beijing University of Aeronautics and Astronautics (Feb. 2011), 144-148.
[10] Zhang Xiaoliang. 2003. Application of EMD to Diagnosis of Machine Fault. Ocean University of China.