Cosmological Constraints from Primordial Black Holes

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Abstract

Primordial black holes may form in the early Universe, for example from the collapse of large amplitude density perturbations predicted in some inflationary models. Light black holes undergo Hawking evaporation, the energy injection from which is constrained both at the epoch of nucleosynthesis and at the present. The failure as yet to unambiguously detect primordial black holes places important constraints. In this article, we are particularly concerned with the dependence of these constraints on the model for the complete cosmological history, from the time of formation to the present. Black holes presently give the strongest constraint on the spectral index $n$ of density perturbations, though this constraint does require $n$ to be constant over a very wide range of scales.

Key words: Black holes, inflationary cosmology
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1 Introduction

Black holes are tenacious objects, and any which form in the very early Universe are able to survive until the present, unless their Hawking evaporation is important. The lifetime of an evaporating black hole is given by

$$\frac{\tau}{10^{17}\text{ sec}} \simeq \left(\frac{M}{10^{15}\text{ grams}}\right)^3.$$  (1)

From this we learn that a black hole of initial mass $M \sim 10^{15}$g will evaporate at the present epoch, while for significantly heavier black holes Hawking evaporation is negligible. Another mass worthy of consideration is $M \sim 10^9$g, which leads to evaporation around the time of nucleosynthesis, which is well enough understood to tolerate only modest interference from black hole evaporation by-products.
Several mechanisms have been proposed which might lead to black hole formation; the simplest is collapse from large-amplitude, short-wavelength density perturbations. They will form with approximately the horizon mass, which in a radiation-dominated era is given by

\[ M_{\text{HOR}} \simeq 10^{18} \text{ g} \left( \frac{10^7 \text{ GeV}}{T} \right)^2, \]  

where \( T \) is the ambient temperature. This tells us that any black holes for which evaporation is important must have formed during very early stages of the Universe’s evolution. In particular, formation corresponds to times much earlier than nucleosynthesis (energy scale of about 1 MeV), which is the earliest time that we have any secure knowledge concerning the evolution of the Universe. Any modelling of the evolution of the Universe before one second is speculative, and especially above the electro-weak symmetry breaking scale (about 100 GeV) many possibilities exist. Note also that although we believe we understand the relevant physics up to the electro-weak scale, the cosmology between that scale and nucleosynthesis could still be modified, say by some massive but long-lived particle. In this article we will consider the standard cosmology and two alternatives [1,2].

We define the mass fraction of black holes as

\[ \beta \equiv \frac{\rho_{\text{pbh}}}{\rho_{\text{tot}}}, \]  

and will use subscript ‘i’ to denote the initial values. In fact, we will normally prefer to use

\[ \alpha \equiv \frac{\rho_{\text{pbh}}}{\rho_{\text{tot}} - \rho_{\text{pbh}}} = \frac{\beta}{1 - \beta}, \]  

which is the ratio of the black hole energy density to the energy density of everything else. Black holes typically offer very strong constraints because after formation the black hole energy density redshifts away as non-relativistic matter (apart from the extra losses through evaporation). In the standard cosmology the Universe is radiation dominated at these times, and so the energy density in black holes grows, relative to the total, proportionally to the scale factor \( a \). As interesting black holes form so early, this factor can be extremely large, and so typically the initial black hole mass fraction is constrained to be very small.

The constraints on evaporating black holes are well known, and we summarize them in Table 1. This table shows the allowed mass fractions at the time of evaporation. An additional, optional, constraint can be imposed if one
| Constraint | Range | Reason |
|------------|-------|--------|
| $\alpha_{\text{evap}} < 0.04$ | $10^9 \text{ g} < M < 10^{13} \text{ g}$ | Entropy per baryon at nucleosynthesis [4] |
| $\alpha_{\text{evap}} < 10^{-26} \frac{M}{m_{\text{Pl}}}$ | $M \simeq 5 \times 10^{14} \text{ g}$ | $\gamma$ rays from current explosions [5] |
| $\alpha_{\text{evap}} < 6 \times 10^{-10} \left(\frac{M}{m_{\text{Pl}}}\right)^{1/2}$ | $10^9 \text{ g} < M < 10^{11} \text{ g}$ | n$\bar{n}$ production at nucleosynthesis [6] |
| $\alpha_{\text{evap}} < 5 \times 10^{-29} \left(\frac{M}{m_{\text{Pl}}}\right)^{3/2}$ | $10^{10} \text{ g} < M < 10^{11} \text{ g}$ | Deuterium destruction [7] |
| $\alpha_{\text{evap}} < 1 \times 10^{-59} \left(\frac{M}{m_{\text{Pl}}}\right)^{7/2}$ | $10^{11} \text{ g} < M < 10^{13} \text{ g}$ | Helium-4 spallation [8] |

imagines that black hole evaporation leaves a relic particle, as these relics must then not over-dominate the mass density of the present Universe [3]. For black holes massive enough to have negligible evaporation, the mass density constraint is the only important one (though in certain mass ranges there are also microlensing limits which are somewhat stronger).

We will study three different cosmological histories in this paper, all of which are currently observationally viable. The first, which we call the standard cosmology, is the minimal scenario. It begins at some early time with cosmological inflation, which is necessary in order to produce the density perturbations which will later collapse to form black holes. Inflation ends, through the pre-heating/reheating transition (which we will take to be short), giving way to a period of radiation domination. Radiation domination is essential when the Universe is one second old, in order for successful nucleosynthesis to proceed. Finally, radiation domination gives way to matter domination, at a redshift $z_{\text{eq}} = 24 000 \Omega_0 h^2$ where $\Omega_0$ and $h$ have their usual meanings, to give our present Universe.

The two modified scenarios replace part of the long radiation-dominated era between the end of inflation and nucleosynthesis. The first possibility is that there is a brief second period of inflation, known as thermal inflation [9]. Such a period is unable to generate significant new density perturbations, but may be desirable in helping to alleviate some relic abundance problems not solved by the usual period of high-energy inflation. The second possibility is a period of matter-domination brought on by a long-lived massive particle, whose eventual decays restore radiation domination before nucleosynthesis. For definiteness, we shall take the long-lived particles to be the moduli fields of superstring theory, though the results apply for any non-relativistic decaying particle.
Fig. 1. Here $\alpha_i$ is the initial fraction of black holes permitted to form. The dotted line assumes black hole evaporation leaves a Planck-mass relic, and is optional.

2 The standard cosmology

Once the cosmology is fixed, the limits on the mass fraction at evaporation shown in Table 1, along with the constraints from the present mass density, are readily converted into limits on the initial mass fraction [10,1]. The limits in different mass ranges are shown in Figure 1; typically no more than about $10^{-20}$ of the mass of the Universe can go into black holes at any given epoch. This limits the size of density perturbations on the relevant mass scale.

These limits apply down to the lightest black hole which is able to form, which is governed by the horizon size at the end of inflation.

The black hole constraint limits density perturbations on scales much shorter than conventional measures of the density perturbation spectrum using large-scale structure and the microwave background (though the scales corresponding to the black hole formation are similar to those on which gravitational waves may be probed by the LIGO, VIRGO and GEO interferometer projects [11]). However, an interesting application of the black hole constraints shown in Figure 1 can be made if one has a definite form for the power spectrum. The simplest possibility is a power-law spectrum, whose spectral index $n$ is
assumed to remain constant across all scales, the interesting case being where \( n > 1 \) — a so-called blue spectrum. The constancy of \( n \) is in fact predicted by some hybrid inflation models, which are the most natural way of obtaining a blue spectrum.

With \( n > 1 \) the shortest-scale perturbations dominate, and the black hole constraints were explored by Carr et al. [12], who found that \( n \) was limited to be less than around 1.4 to 1.5. We have redone their analysis and corrected two significant errors. First, they used an incorrect scaling of the horizon mass during the radiation era, which should read

\[
\sigma_{\text{hor}}(M) = \sigma_{\text{hor}}(M_{\text{eq}}) \left( \frac{M}{M_{\text{eq}}} \right)^{(1-n)/4}.
\]  

(5)

Secondly, their normalization of the spectrum to the COBE observations, to fix the long wavelength behaviour, was incorrect (too low) by a factor of around twenty. With these corrections, the constraint on \( n \) tightens considerably to become \( n \lesssim 1.25 \) [1].

Ignoring for the time being uncertainties in cosmological modelling, this should be regarded as a very hard limit; it is not useful to try and think of it as representing some confidence level. Because the density perturbations are assumed gaussian, the formation rate of black holes is extremely sensitive to the amplitude of perturbations on the scale under consideration. Hence a very modest change in \( n \) gives a huge change in the predicted black hole abundance, which makes a rapid transition between totally negligible and enormously excessive as \( n \) crosses the quoted limit. In fact, to obtain a black hole density near the present limits requires a considerable fine-tuning, as we saw in Figure 1 that only about \( 10^{-20} \) or so of the mass of the Universe must be channelled into black holes.

However, the previous paragraph did not take into account uncertainties in cosmological modelling, and that is what we will quantify in the remainder of this article. We will also note that a lifting of the gaussianity assumption, presently a controversial topic [13,14], makes little change.

3 With thermal inflation

Thermal inflation is a brief period of inflation occurring at an intermediate energy scale. We model it as occurring from \( T = 10^7 \) GeV down to the supersymmetry scale \( T = 10^3 \) GeV, then reheating back up to \( 10^7 \) GeV, which is the standard thermal inflation scenario. It drives \( \ln(10^7 \text{GeV}/10^3 \text{GeV}) \simeq 10 \)
e-foldings of inflation. As we have seen [Eq. (2), most of the interesting mass region contains black holes forming before $T = 10^7$ GeV, which implies that the constraints are affected by thermal inflation. Three effects are important:

- Dilution of black holes during thermal inflation.
- A change in the correspondence of scales: COBE scales leave the horizon closer to the end of inflation.
- A mass range which enters the horizon before thermal inflation, but leaves again during it. No new perturbations are generated on this scale during thermal inflation, so from the horizon mass formula we find a missing mass range between $10^{18}$ g and $10^{26}$ g in which black holes won’t form. Thermal inflation at higher energy could exclude masses below $10^{15}$ g.

The dilution effect is shown in Figure 2; typical constraints now lie around $10^{-10}$ rather than $10^{-20}$. Taking all the effects into account, the constraint on the spectral index weakens to $n \lesssim 1.3$ [1].

4 Cosmologies with moduli domination

A prolonged early period of matter domination is another possible modification to the standard cosmology [2]. For example, moduli fields may dominate,
and in certain parameter regimes can decay before nucleosynthesis. Various assumptions are possible; here we’ll assume moduli domination as soon as they start to oscillate (around $10^{11}$ GeV). Part of the interesting range of black hole masses forms during moduli domination rather than radiation domination. Figure 3 shows the constraints in this case, and with moduli domination the limit on $n$ again weakens to $n \lesssim 1.3$ [2].

5 Conclusions

Although black hole constraints are an established part of modern cosmology, they are sensitive to the entire cosmological evolution. In the standard cosmology, a power-law spectrum is constrained to $n < 1.25$, presently the strongest observational constraint on $n$ from any source. Alternative cosmological histories can weaken this to $n < 1.30$, and worst-case non-gaussianity [13] can weaken this by another 0.05 or so, though hybrid models giving constant $n$ give gaussian perturbations. Finally, we note that while the impact of the cosmological history on the density perturbation constraint is quite modest due to the exponential dependence of the formation rate, the change can be much more significant for other formation mechanisms, such as cosmic strings where the black hole formation rate is a power-law of the mass per unit length $G\mu$. 
After all, the permitted initial mass density of black holes does increase by many orders of magnitude in these alternative cosmological models.

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