Brane Localized Curvature for Warped Gravitons

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Abstract

We study the effects of including brane localized curvature terms in the Randall-Sundrum (RS) model of the hierarchy. This leads to the existence of brane localized kinetic terms for the graviton. Such terms can be induced by brane and bulk quantum effects as well as Higgs-curvature mixing on the brane. We derive the modified spectrum of Kaluza-Klein (KK) gravitons and their couplings to 4-dimensional fields in the presence of these terms. We find that the masses and couplings of the KK gravitons have considerable dependence on the size of the brane localized terms; the weak-scale phenomenology of the model is consequently modified. In particular, the weak-scale spin-2 graviton resonances which generically appear in the RS model may be significantly lighter than previously assumed. However, they may avoid detection as their widths may be too narrow to be observable at colliders. In the contact interaction limit, for a certain range of parameters, the experimental reach for the scale of the theory is independent of the size of the boundary terms.

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1 Introduction

The Randall-Sundrum (RS) model provides a natural geometric picture for the hierarchy between the weak scale and the apparent gravitational scale, $M_{pl}$. In this model, two 3-branes truncate a 5-dimensional curved spacetime. In the original RS construction, the Standard Model (SM) fields reside on the SM (or TeV) brane where a scale, which is exponentially smaller than the fundamental 5-d scale, is induced by the geometry. Gravity is localized on the second brane, known as the Planck brane. This exponential hierarchy is controlled by the distance $r_c$ between the two branes and can generate the weak scale on the SM brane. The physics of the graviton Kaluza-Klein (KK) modes is governed by the same scale. It has been shown that $r_c$ can be stabilized by bulk scalar fields, resulting in the appearance of a single weak scale radion field.

The most generic and distinctive signature of this model is the presence of TeV-scale spin-2 graviton KK resonances at colliders. It is important to examine whether modifications arising in a more realistic RS scenario could affect the KK graviton phenomenology. One such modification is the introduction of brane curvature terms for the graviton. These terms respect all 4-d symmetries, and are expected to be present in a 4-d effective theory, leading to Brane Localized Kinetic Terms (BLKT’s) for the graviton. It has been argued that brane quantum effects can generate BLKT’s, and that these terms are required as brane counter terms for bulk quantum effects. In addition, since the Higgs field is assumed to reside on the SM brane, a Higgs-curvature mixing term can be included in the brane action. Such a term is allowed by all 4-d symmetries and leads to Higgs-radion mixing with important phenomenological consequences. We observe that the same Higgs-curvature mixing term can also induce a BLKT for the graviton on the SM brane. Thus, there are many good theoretical reasons for assuming the presence of such terms for gravitons.

The effects of BLKT’s on graviton physics have been studied for flat spacetimes and result in novel features. Boundary terms have also been shown to significantly modify the KK phenomenology of bulk gauge fields in flat and warped geometries. For the case of an alternative RS model with only one brane and an uncompactified 5th dimension, the gravitational and cosmological effects of these terms have been analyzed. However, the case of the compactified RS model, which is relevant for weak scale physics, in principle involves graviton boundary terms on both branes and has yet to be examined. We study this case here and investigate the effects on the phenomenology of the weak scale KK gravitons.
in the presence of localized curvature terms.

We derive the modified graviton KK spectrum and couplings and show that a significant dependence on the size of such terms emerges. We illustrate that this leads to considerable changes in the low energy 4-d phenomenology of the RS model. These changes could have important implications for future searches and the potential discovery of model signatures. In addition, we derive bounds on the magnitude of the localized curvature terms on the Planck and TeV branes, by requiring the absence of graviton and radion ghost states.

The outline of this paper is as follows. In the next section, we derive the necessary formalism. Our numerical analysis is presented in section 3 and section 4 contains some concluding remarks.

2 Formalism

Here, we present our formalism and notation. We work in the RS background, as given by the following metric
\[ ds^2 = e^{-2\sigma} \eta_{\mu\nu} dx^\mu dx^\nu + r_c^2 d\phi^2, \tag{1} \]
where \( \sigma = k r_c |\phi| \), \( k \) is the curvature scale of order \( M_{Pl} \), \( r_c \) is the radius of compactification, and \( \phi \in [-\pi, \pi] \). The extra dimension is here compactified on an \( S^1/Z_2 \) orbifold. The gravitational action \( S_G \), augmented by brane localized curvature terms on the Planck and TeV branes, is assumed to be given by

\[ S_G = \frac{M_5^3}{4} \int d^4x \int r_c d\phi \sqrt{-G} \left\{ R^{(5)} + [g_0 \delta(\phi) + g_\pi \delta(\phi - \pi)] R^{(4)} + \ldots \right\}, \tag{2} \]
where \( M_5 \) is the 5-d reduced Planck mass, \( G \equiv \det(G_{\mu\nu}) \) and \( g_0, \pi \) are numerical coefficients which are treated as phenomenological inputs in our calculations; one may expect \( g_0, \pi \sim 1 \) based on naturalness arguments. Here, \( R^{(5)} \) denotes the 5-d gravitational curvature and \( R^{(4)} \) is the 4-d curvature which is constructed only out of 4-d Minkowski metric components and derivatives. The terms proportional to \( g_0, \pi \) represent the only modifications to the original RS proposal [1] that we study in this work. Note that we have assumed that the form of the brane terms are exactly the same as the bulk term except that they are calculated with the relevant 4-d metrics. In principle, one can imagine brane terms which are quadratic or even
higher order in $R$, but these terms are small and would violate the philosophy of the original model where classical general relativity is applicable. As we will see below, brane terms of the type we consider here only alter the detailed structure while retaining the general nature of the usual RS solutions.

We will consider metric perturbations of the form

$$G_{\mu\nu} = e^{-2\sigma}(\eta_{\mu\nu} + \kappa_5 h_{\mu\nu}), \quad (3)$$

with $\kappa_5 = 2M_5^{-3/2}$; $h_{\mu\nu}$ is the 5-d graviton field. The transverse-traceless gauge will be implemented in our calculations; this corresponds to taking $\partial^\mu h_{\mu\nu} = h^\mu_\mu = 0$. We use a KK expansion for $h_{\mu\nu}$ of the form

$$h_{\mu\nu}(x, \phi) = \sum_n h^{(n)}_{\mu\nu}(x) \chi^{(n)}(\phi) \sqrt{r_c}, \quad (4)$$

where $h^{(n)}_{\mu\nu}(x)$ are the KK modes and $\chi^{(n)}(\phi)$ are the wavefunctions along the extra dimension. With this expansion, Eqs. (2) and (3) imply the following equation of motion

$$\frac{d}{d\phi} \left( e^{-4\sigma} \frac{d}{d\phi} \chi^{(n)}(\phi) \right) + \left[ 1 + g_0 \delta(\phi) + g_\pi \delta(\phi - \pi) \right] e^{-2\sigma} r_c^2 m_n^2 \chi^{(n)}(\phi) = 0, \quad (5)$$

where $m_n$ is the mass of the $n^{th}$ KK mode. Away from the boundaries at $\phi = 0, \pi$, for $n \geq 1$, we have [3]

$$\chi^{(n)}(\phi) = \frac{e^{2\sigma}}{N_n} \zeta_2(z_n), \quad (6)$$

where $N_n$ are normalization constants, $\zeta_q(z_n) \equiv J_q(z_n) + \alpha_n Y_q(z_n)$, with $z_n(\phi) \equiv (m_n/k) e^{\sigma}$, and $\alpha_n$ are numerical coefficients to be fixed below; $J_q$ and $Y_q$ are Bessel functions of order $q$.

Let $\varepsilon_n \equiv z_n(0)$ and $x_n \equiv z_n(\pi)$. Integrating Eq. (5) around $\phi = 0, \pi$ yields

$$\zeta_1(\varepsilon_n) + \gamma_0 \varepsilon_n \zeta_2(\varepsilon_n) = 0 \quad (7)$$

and

$$\zeta_1(x_n) - \gamma_\pi x_n \zeta_2(x_n) = 0, \quad (8)$$
respectively, with \( \gamma_{0,\pi} \equiv g_{0,\pi} kr_c/2 \). Addressing the hierarchy problem requires \(kr_c \sim 10\), hence, we expect \(|\gamma_{0,\pi}| \lesssim 10\). Eq.(7) yields
\[
\alpha_n = -\frac{J_1(\varepsilon_n) + \gamma_0 \varepsilon_n J_2(\varepsilon_n)}{Y_1(\varepsilon_n) + \gamma_0 \varepsilon_n Y_2(\varepsilon_n)}.
\]
(9)

Here, we note that for \( \gamma_0 \neq -1/2 \), \( \alpha_n \sim \varepsilon_n^2 \), where \( \varepsilon_n \sim 10^{-15} \) for the lowest lying KK modes. The roots of Eq.(8) give the masses \( m_n = x_n ke^{-kr_c \pi} \). Note that with \( \alpha_n \) sufficiently small we can safely neglect the \( Y_q \) components of the eigenvalue equation. This implies that the masses of the KK graviton tower are \( \gamma_0 \) independent unlike what happens in the corresponding case with gauge bosons in the bulk. It can be shown that for \( \gamma_{\pi} \gg 1 \), an additional root \( x^* \ll 1 \) is obtained, where
\[
x^* \approx 2 (\gamma_{\pi} + 1/2)^{-1/2}.
\]
(10)
This implies that there is a light mode in the spectrum for \( \gamma_{\pi} \gg 1 \). We will show that this signals an instability in this background.

Next, we will fix the normalization \( N_n \) of the KK modes by requiring
\[
\int d\phi e^{-2\sigma} [1 + g_0 \delta(\phi) + g_{\pi} \delta(\phi - \pi)] \chi^{(n)} \chi^{(n)*} = 1.
\]
(11)

For \( n = 0 \), this equation yields the zero mode wavefunction
\[
\chi^{(0)} = \left( \frac{kr_c}{1 + 2 \gamma_0} \right)^{1/2},
\]
(12)
which demonstrates that we must have \( \gamma_0 > -1/2 \) in order for the zero mode to be a physical state. For \( n \geq 1 \), Eq.(11) gives
\[
N_n^2 = \frac{e^{2kr_c \pi} \zeta_2^0(x_n)}{kr_c} (1 + \gamma_{\pi}^2 x_n^2 - 2 \gamma_{\pi}),
\]
(13)
where terms of order \( e^{-2kr_c \pi} \) have been ignored. In the limit that we can neglect the \( \alpha_n \), this form implies that the graviton wavefunctions are \( \gamma_0 \) independent. Note that this normalization factor remains positive for all values of \( \gamma_{\pi} \) so that this parameter is not constrained by these considerations.

†Later we will show that \( \gamma_0 = -1/2 \) is not a physically allowed value.
We can now obtain the couplings of the KK gravitons to the energy momentum tensor \( T_{\mu\nu} \) of the fields residing on the TeV brane at \( \phi = \pi \). To do this, first we note that the action (2) yields

\[
\overline{M}_{Pl}^2 = \frac{M_5^3}{k} (1 + 2\gamma_0), \tag{14}
\]

where \( \overline{M}_{Pl} \) is the 4-d reduced Planck mass and terms of order \( e^{-2kr_c\pi} \) have been ignored. The Lagrangian for the interactions of \( h_{\mu\nu} \) and the 4-d fields on the TeV brane is given by

\[
\mathcal{L} = \frac{1}{M_5^{3/2}} h_{\mu\nu}(x, \pi) T^{\mu\nu}(x). \tag{15}
\]

Eqs. (3), (12), (13), and (14) then yield

\[
\mathcal{L} = \frac{1}{\overline{M}_{Pl}} h_{\mu\nu}^{(0)}(x) T^{\mu\nu}(x) + \frac{1}{\Lambda_\pi} T^{\mu\nu}(x) \sum_{n=1}^{\infty} \lambda_n h_{\mu\nu}^{(n)}(x), \tag{16}
\]

with \( \Lambda_\pi \equiv \overline{M}_{Pl} e^{-kr_c\pi} \) as usual and

\[
\lambda_n \equiv \left( \frac{1 + 2\gamma_0}{1 + \gamma_\pi x_n^2 - 2\gamma_\pi} \right)^{1/2}. \tag{17}
\]

As expected, the zero mode coupling is \( \overline{M}_{Pl}^{-1} \) and for \( \gamma_0, \gamma_\pi \to 0 \), the original RS model coupling for the non-zero mode KK states, \( \Lambda_\pi^{-1} \) [3], is retrieved. Choosing \( \gamma_\pi \neq 0 \) renders the KK couplings mode dependent, through \( \lambda_n \). We see that for \( \gamma_\pi x_n \gg 1 \) these couplings have suppressions \( \sim (\gamma_\pi x_n)^{-1} \).

Before continuing, we consider if there are any further constraints which can be imposed upon the parameters \( \gamma_0, \gamma_\pi \). Though here we are concerned with the effect of brane terms on the masses and couplings of the KK gravitons, the RS metric yields another excitation, the radion, whose properties may also be modified by such terms. Using the results of Csaki et al. [8], for example, the effects of the brane terms on the radion [14] can be determined since they are similar in form to those which induce Higgs-radion mixing. A short analysis shows that the brane terms lead to a multiplicative correction to the usual radion kinetic term, in the absence of the Higgs field, by a factor of \( 1 - \gamma_\pi \); the corresponding \( \gamma_0 \)-dependent contribution is found to be suppressed by powers of the exponential warp factor and we will ignore it in the following discussion. To avoid a radion ghost this implies that \( \gamma_\pi \leq 1 \),
suggesting that negative or small positive values of $\gamma_\pi$ are allowed. This multiplicative factor can then be absorbed by a suitable field rescaling so that the radion field becomes canonically normalized. However, once the Higgs-radion mixing term is included, this bound becomes a bit more complex since mixing and brane term contributions can compensate for one another. If the brane localized curvature for gravitons arises from solely the same source as Higgs-radion mixing [8, 14] then we find that similar bounds on $\gamma_\pi$ can also be obtained. Of course, such brane terms can arise from many other sources so that a much wider range of values of $\gamma_\pi$ are certainly possible. Naturalness suggests that $\gamma_\pi \geq -10$ as discussed above and we will assume this soft bound in our discussion below.

3 Analysis

The first step in our analysis is to examine how the mass spectrum of the graviton KK modes is influenced by the presence of the brane kinetic terms parameterized via $\gamma_{0,\pi}$. As was noted above, both the root equation and the wave functions for the graviton KK modes in the extra dimension are found to be independent of $\gamma_0$ when powers of the warp factor, $e^{-\pi k r_c}$, are neglected; this implies that the $x_n$ for the range of interest to us are essentially $\gamma_0$ independent. Recall that the masses of the KK states are now given by $m_n = x_n(\gamma_\pi)ke^{-kr_c\pi}$. Fig. 1 displays these roots as functions $\gamma_\pi$. (For purposes of comparison we note that the first root in the usual RS model without brane terms is given by $x_1 \simeq 3.83$.) For all KK modes, other than the case $n = 1$, the $x_n$ are essentially constant away from the transition region near $\gamma_\pi = 0$. As we see from the root equation above, once $|\gamma_\pi|$ gets large the $x_n$ are essentially just the roots of the equation $J_2(x_n) = 0$ which accounts for their mostly $\gamma_\pi$ independent behaviour. The root $x_1$ behaves somewhat differently and scales as $\sim 2/\sqrt{\gamma_\pi}$ for large positive $\gamma_\pi$ as seen from Eq.(10) above. Note that this causes the ratio of the first two roots $x_2/x_1$ to grow substantially in the large and positive $\gamma_\pi$ region.

Once the $x_n$ are known the effective couplings of the KK modes to SM matter on the TeV brane can be immediately calculated and are shown in Fig. 2 for the case $\gamma_0 = 0$; for other values of $\gamma_0$ these results are scaled by a factor of $\sqrt{1 + 2\gamma_0}$. Except for the lowest KK mode when $\gamma_\pi > 0$, the couplings of all KK modes are observed to fall off drastically as the

\[^{3}\text{In the notation of Csaki et al. [8], we find that the induced value of } \gamma_\pi \text{ is given by } \gamma_\pi = \xi(1 + 2\gamma_0)(v/\Lambda_\pi)^2, \text{ where } v \text{ is the SM Higgs vev and } \xi \text{ is the radion-Higgs mixing parameter. Putting in some reasonable numbers, we see that the induced } \gamma_\pi \text{ is of order unity or less in magnitude.}\]
Figure 1: The first six roots $x_n$ as functions of $\gamma_\pi$. These results are independent of $\gamma_0$ up to terms suppressed by warp factors. As discussed in the text, the region $\gamma_\pi < 1$ is required by the absence of radion ghosts taking the results of Ref. [8].
magnitude of $\gamma_{\pi}$ increases, demonstrating the $1/|n\gamma_{\pi}|$ behavior, as expected. Clearly, the reduced couplings of these graviton KK modes will have an important impact on the ability to detect these states either directly or indirectly at colliders. This will be true in particular in the case of a positive $\gamma_{\pi}$, where as we saw above the second KK mode is significantly heavier than the first mode and has a much weaker coupling to the SM brane fields. However, we note that the absence of a radion ghost would exclude this extreme situation if we apply the analysis as given in Ref. [8].

Figure 2: The couplings of the first five KK graviton tower modes as functions of $\gamma_{\pi}$ assuming $\gamma_0 = 0$. The top curve corresponds to the lightest KK state with the more massive tower modes corresponding to the subsequently lower curves. The same restrictions on $\gamma_{\pi}$ apply as in the previous figure.

Perhaps the most important experimental signature for the RS model is the appearance of spin-2 KK resonances at colliders. As shown above, when $\gamma_{\pi}$ is non-zero the masses and couplings of these KK states are altered from the usual expectations for which the phenomenology has by now been well explored. Figs. 3 and 4 show some of the possible modifications in the presence of the boundary terms to the production cross section and KK spectrum for the cases $k/M_{Pl} = 1$ and 0.1, respectively, for the process $e^+e^- \rightarrow \mu^+\mu^-$ at a linear collider assuming the mass of the first KK resonance is 600 GeV. For the case of $k/M_{Pl} = 1$ in the usual RS model, which corresponds to $\gamma_{\pi} = 0$ and is represented by the dot-
ted curve in Fig. 3, the KK states are sufficiently strongly coupled to smear out the individual resonance structures and produce only a large shoulder in the cross section. For non-zero $\gamma_\pi$, there is a significant change in these expectations due to the reduced values of the couplings and the additional mass spectrum modifications. This is seen explicitly in these two figures. Note that the widths of these graviton resonances scale as $\Gamma_n/m_n \sim (x_n \lambda_n k/M_{Pl})^2$ where $\lambda_n$ is the relative coupling strength for the $n$-th KK mode calculated in Eq.(17) and shown in Fig. 2. For positive values of $\gamma_\pi$ the lowest KK mode is well separated in mass from the next more massive state. For $\gamma_\pi < 0$ the strong constructive interference between the resonances rapidly vanishes and individual peaks become observable. As $\gamma_\pi$ decreases further this results in a series of very narrow, spike-like peaks once values of $\sim -10$ are reached.

Figure 3: Cross section for $e^+e^- \rightarrow \mu^+\mu^-$ at a linear collider assuming $m_1 = 600$ GeV and $k/M_{Pl} = 1$. The dotted curve is the standard RS result while from top to bottom on the right-hand side the curves correspond to $\gamma_\pi = 1(-1,-2,-10)$ represented by cyan(red, magenta, blue). In all cases $\gamma_0 = 0$ is assumed.

When $k/M_{Pl} = 0.1$ the effects of the brane terms are in some sense much more severe as can be seen from Fig. 4 due to the further reduction in the resonance widths by a factor of 100. The usual resonances, the first few of which are reasonably narrow in the standard RS scheme in this case, are converted into narrow spikes some if not all of which will yield event rates which are quite small compared to the continuum background in any given mass bin.
For $\gamma_{\pi}$ positive and of order unity the unique feature here is the large mass gap between the first and second resonances. For $\gamma_{\pi}$ negative the resonance structures are also narrow but with a smaller mass gap. With this or even smaller values of $k/M_{Pl}$, it may be difficult to observe anything, even the first KK graviton excitation, over much of the range of $\gamma_{\pi}$ unless use is made of radiative returns or one is lucky choosing the collider center of mass energy in performing a scan. It is clear from our discussion that it is important to see at least two of these resonance peaks to uniquely identify the model and extract out the relevant parameters.

Figure 4: Same as the previous figure but now for $k/M_{Pl} = 0.1$. The rapidly rising curve is the usual RS scenario prediction. The resonances for the case $\gamma_{\pi} = -10$ have become essentially a spike here due to the resolution in our plotting routine.

The shifts in the mass spectrum and couplings of the KK resonances induced by the brane kinetic terms are so large that even for values of $|\gamma_{\pi}| = 0.2$, there are substantial modifications to the shape of the production cross section. This is seen explicitly in Fig. 5.

If the graviton KK modes are kinematically accessible they will most likely be first observed at a hadron collider. As discussed in our earlier work, the lack of observation of KK graviton resonances in, e.g., the Drell-Yan channel at the Tevatron already places a constraint on the RS model parameter space[3]. This analysis can be modified in order to examine the
Figure 5: Same as the previous figure with $k/M_{Pl} = 0.1$ but now for $\gamma_\pi = -0.2, 0,$ and 0.2 corresponding to the red, blue and green curves, respectively, which are represented by the middle, top and bottom curves on the right-hand side.

influence of brane kinetic terms for the graviton; the results are presented in Figs. 6, 7 and 8 keeping in mind that values of $k/M_{Pl} < \sim 0.1$ are theoretically preferred. These figures show the search reach for the first graviton KK resonance in Drell-Yan production as a function of $\gamma_\pi$ for different values of $k/M_{Pl}$.

The search reach from Run I of the Tevatron, which constitutes the present lower bound on the mass of the first KK graviton excitation, is shown in Fig. 6. Here we see that, as expected, the search reach degrades quite rapidly as we move away from the canonical value of $\gamma_\pi = 0$. Note that for negative values of $\gamma_\pi$ it is still quite possible for the mass of this lightest state to be less than 200 GeV (provided this is not excluded by LEP searches for narrow resonances). For negative values of $\gamma_\pi$ the corresponding bound on $\Lambda_\pi$ can be approximately obtained from the $m_1$ constraint by dividing by $\sim 5k/M_{Pl}$.

Fig. 7 shows that future running of the Tevatron during Run II can be useful in covering some of the large $|\gamma_\pi|$, small KK mass region. The higher energy and integrated luminosity will result in a reasonable increase in the overall parameter space coverage compared to Run I but, as can be seen from the figure, will still allow for the existence of very
Figure 6: Search reach for the first graviton KK resonance employing the Drell-Yan channel at Tevatron Run I as a function of $\gamma_\pi$ assuming $\gamma_0 = 0$. From bottom to top on the RHS of the plot, the curves correspond to $k/M_{Pl} = 0.01, 0.025, 0.05, 0.075, 0.10, 0.125$ and $0.15$, respectively. The unshaded region is that allowed by naturalness considerations and the requirement of a ghost-free radion sector.
light KK gravitons.

Fig. 8 shows the potential search reach for the first KK graviton resonance at the LHC. Even in this case one sees that there is a significant degradation in the reach away from $\gamma_\pi = 0$. Unlike in the standard RS scenario without brane terms we see that the LHC can no longer cover all of the interesting parameter space for this model due to the reduced coupling of the first KK resonance. For example we see from the figure that for large negative $\gamma_\pi$, a first KK graviton resonance with a mass of 600 GeV and $k/M_{Pl} = 0.01$ may still miss detection - even at the LHC.

It is possible that the indirect effects of graviton KK tower exchange in particle pair production may be observed. This may be more important than direct graviton searches in some regions of parameter space. These indirect effects are similar to those of contact interactions, except that the graviton exchange operator is dimension-8. Graviton virtual effects have been computed for the case of large extra dimensions in Ref. [15], where it was noted that the angular distributions of the final state particles may reveal the spin-2 nature of the indirect graviton exchange. Here, the 4-fermion matrix element is obtained from that
for the large extra dimensions scenario with the replacement

\[ \frac{\lambda}{M_H^4} \rightarrow \frac{M_{Pl}^2}{8k^2\Lambda^4} \sum_n \frac{\lambda_n(\gamma_{\pi}, \gamma_0)^2}{x_n(\gamma_{\pi})^2}, \]

in the limit of \( m_n^2 \gg s \). In this model the sum rapidly converges. The coupling weighted sum over the KK tower exchanges differs from the conventional RS model by the ratio

\[ \mathcal{R} = \frac{\sum_n \lambda_n(\gamma_{\pi}, \gamma_0)^2}{\sum_n \frac{\lambda_n(0)^2}{x_n(0)^2}}, \]

which is displayed in Fig. 9. We see from the figure that for positive values of \( \gamma_{\pi} \) the effects of the suppressed couplings and the modified roots exactly compensate each other. The standard RS predictions are thus maintained in this region. The contact interaction bounds on \( \Lambda_{\pi} \) obtained from the lepton pair invariant mass spectrum and forward-backward asymmetry in Drell-Yan production at the Tevatron and LHC are presented in Fig. 10 as a function of \( \gamma_{\pi} \). The results from unpolarized (and polarized in the case of the Linear Collider) angular distributions, summing over \( e, \mu, \tau, c, b \) (and \( t \) if kinematically accessible)
Figure 9: Sum over the graviton KK towers weighted by the appropriate coupling strength in comparison to that obtained in the standard RS model without brane kinetic terms as a function of $\gamma_\pi$ assuming $\gamma_0 = 0$. 
final states, as well as $\tau$ polarization distributions are shown in Fig. 11 for LEP II and the Linear Collider. From these figures we see that the constraints from 4-fermion interactions are rather weak for negative values of $\gamma_\pi$, even for the higher energy colliders. The searches from both direct graviton production at hadron colliders and the indirect exchange of graviton KK states in fermion pair production are thus difficult in this parameter space region and low mass graviton states may escape detection. It is possible that searches for narrow s-channel resonances in $e^+e^-$ collisions or direct graviton production in $e^+e^- \to \gamma G$ may extend the discovery reach in this region.

Figure 10: Contact interaction constraints on the scale $\Lambda_\pi$ from the run II Tevatron with an integrated luminosity of $2 \text{ fb}^{-1}$ (blue lower set of curves) and the LHC (red higher set of curves) with an integrated luminosity of $100 \text{ fb}^{-1}$. The dashed (solid, dotted) curves correspond to $k/M_{Pl} = 0.01\,(0.1, 1)$, respectively. The shading is as in the previous figures.

4 Conclusions

The most generic and distinctive signature of the RS model of the hierarchy is the appearance of weak scale, spin-2 KK resonances of the 5-d graviton. The discovery of such resonances at a collider will provide strong evidence for the RS picture. It is therefore important to
Figure 11: Same as the previous figure but now for LEPII (red lower set of curves), a 500 GeV Linear Collider (green, middle set), or a 1 TeV Linear Collider (black, higher set). The LC integrated luminosity is assumed to be 500 $fb^{-1}$.
Understand how the spectrum and the couplings of these states could be affected by well-motivated theoretical modifications of the original proposal. The addition of brane localized curvature terms is such a modification. These terms yield contributions to the graviton BLKT’s and can be generated from brane and bulk quantum effects. They can also arise at tree level due to Higgs-curvature mixing.

In this paper, we studied the consequences of adding brane curvature terms in the RS model. We derived the modified spectrum and couplings of the graviton KK states to the fields on the SM brane. The presence of localized curvature on the SM brane leads to important changes in the weak scale phenomenology. Requiring that the effective theory is free of KK and radion ghosts places lower and upper bounds on the coefficients $\gamma_0$ and $\gamma_{\pi}$ of these terms on the Planck and SM branes, respectively.

We showed that, within the natural and allowed values of these coefficients, the variation in the shape and production cross sections of the resonances is significant. In particular, the KK resonances become very narrow for some regions of the parameter space and may be too narrow to be observed at colliders. We found that very light KK gravitons, of order 200–600 GeV may escape detection at the Tevatron and LHC. In the contact interaction limit, where the KK masses are too large for direct production, we observed that the collider reach for the SM scale $\Lambda_{\pi} \sim \text{TeV}$ varies considerably with $\gamma_{\pi} < 0$. However, for $\gamma_{\pi} > 0$, this reach remains constant. The cause of this behavior is the interplay of the modified masses and couplings of the KK states. In brief, we found that the modifications resulting from brane localized curvature terms in the RS model have interesting features and that they are important for future phenomenological studies and experimental searches.

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