Triton binding energy calculated from the $SU_6$ quark-model nucleon-nucleon interaction

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Abstract

Properties of the three-nucleon bound state are examined in the Faddeev formalism, in which the quark-model nucleon-nucleon interaction is explicitly incorporated to calculate the off-shell $T$-matrix. The most recent version, fss2, of the Kyoto-Niigata quark-model potential yields the ground-state energy $E(^{3}\text{H}) = -8.514$ MeV in the 34 channel calculation, when the $np$ interaction is used for the nucleon-nucleon interaction. The charge root mean square radii of the $^{3}\text{H}$ and $^{3}\text{He}$ are 1.72 fm and 1.90 fm, respectively, including the finite size correction of the nucleons. These values are the closest to the experiments among many results obtained by detailed Faddeev calculations employing modern realistic nucleon-nucleon interaction models.

21.45.+v, 21.30.-x, 13.75.Cs, 12.39.Jh
I. INTRODUCTION

All the present-day quark-model descriptions of nucleon-nucleon (NN) and hyperon-nucleon (YN) interactions incorporate important roles of the quark-gluon degrees of freedom in the short-range region and the meson-exchange processes dominating in the medium- and long-range parts of the interaction. For example, the Kyoto-Niigata quark-model potential employs a one-gluon exchange Fermi-Breit interaction and effective meson-exchange potentials (EMEP’s) acting between quarks, and has achieved accurate descriptions of the NN and YN interactions with a limited number of parameters. The early version, the model FSS, includes only the scalar (S) and pseudoscalar (PS) meson-exchange potentials as the EMEP’s. This model is superseded by the new model fss2, which has also introduced the vector (V) meson-exchange potentials and the momentum-dependent Bryan-Scott terms included in the S and V meson EMEP’s. Owing to the introduction of the V mesons, the model fss2 in the NN sector has attained the accuracy comparable to that of one-boson exchange potential (OBEP) models. For example, the $\chi^2$ values defined by

$$\chi^2 = \sum_{i=1}^{N} \left( \delta_{i}^{\text{cal}} - \delta_{i}^{\text{exp}} \right)^2 / N$$

for the $J \leq 2$ np phase-shift parameters in the energy range $T_{\text{lab}} = 25 - 300$ MeV are $\sqrt{\chi^2} = 0.59^\circ$ for fss2, which is compared with the values $1.10^\circ$, $1.40^\circ$ and $1.32^\circ$ for OBEP, Paris and Bonn potentials, respectively. The incorporation of the momentum-dependent Bryan-Scott term is favorable in extending our quark-model description of the NN scattering at the non-relativistic energies to the higher energies up to $T_{\text{lab}} = 800$ MeV, and also in describing reasonable asymptotic behavior of the nucleon s.p. potentials in the high-momentum region. The agreement of the higher partial waves up to $J=4$ with the phase shift analysis is also improved. In both models FSS and fss2, the existing data for the YN scattering are well reproduced and the essential feature of the $\Lambda N-\Sigma N$ coupling remains almost unchanged. Fixing the model parameters in the strangeness $S=0$ and $-1$ sectors, we proceed to explore interactions for any arbitrary combinations of octet baryons ($B_8$). The $B_8B_8$ interactions in $S = -2$, $-3$ and $-4$ sectors include the $\Lambda\Lambda$ and $\Xi N$ interactions, which are recently attracting much interest in the rapidly developing field of the hypernuclei and the strangeness nuclear matter. The interaction derived in these models may be used for realistic calculations in few-baryon systems, like the triton $^3\text{H}$, the hypertriton $^3\Lambda\text{H}$, and also in various types of baryonic matter. This project, however, involves a non-trivial problem of determining how to extract the effective two-baryon interaction from the microscopic quark-exchange kernel. The basic baryon-baryon interaction is formulated as a composite-particle interaction in the framework of the resonating-group method (RGM). If we rewrite the RGM equation in the form of a Schrödinger-type equation, the interaction term becomes non-local and energy dependent. Furthermore, the RGM equation sometimes involves redundant components, due to the effect of the antisymmetrization, which is related to the existence of the Pauli-forbidden states. In such a case, the full off-shell $T$-matrix is not well defined in the standard procedure, which usually assumes simple energy-independent local potentials. Since these features are related to the description of the short-range part in the quark model, it would be desirable if the quark-exchange kernel could be used directly in application to many-baryon systems.

In this paper, we will show some results of the Faddeev calculation which directly employs the quark-model NN interactions fss2 and FSS to derive the off-shell $T$-matrix. Following the notation in ref. 7,8, we write the RGM equation of the $(3q)-(3q)$ system in the form of
the Schrödinger-type equation
\[
\left[ \varepsilon - h_0 - V^{\text{RGM}}(\varepsilon) \right] \chi = 0 ,
\]
where \( \varepsilon \) is the total energy in the center-of-mass system, measured from the two-cluster threshold, \( \varepsilon = E - 2E_N \), \( h_0 \) is the kinetic-energy operator of the \( NN \) relative motion, and
\[
V^{\text{RGM}}(\varepsilon) = V_D + G + \varepsilon K ,
\]
is the RGM kernel composed of the direct potential \( V_D \), the sum of the exchange kinetic-energy and interaction kernels, \( G = G^K + G^V \), and the exchange normalization kernel \( K \). Since there is no Pauli forbidden state in the \( NN \) system, we can solve the Lippmann-Schwinger equation
\[
T(\omega, \varepsilon) = V^{\text{RGM}}(\varepsilon) + V^{\text{RGM}}(\varepsilon) g_0(\omega) T(\omega, \varepsilon) ,
\]
with \( g_0(\omega) = 1/(\omega - h_0 + i0) \), by assuming the \( 2N \) energy \( \varepsilon \) as a mere parameter. The Faddeev equation for the \( 3N \) bound state is given by the eigenvalue problem
\[
\lambda(E) \psi_\alpha = G_0^{(+)}(E) T^{(3)}_\alpha(E, \varepsilon_\alpha) (\psi_\beta + \psi_\gamma) ,
\]
with \( \lambda(E) = 1 \), where the two-body \( T \)-matrix in the three-body model space is given by
\[
T^{(3)}_\alpha(E, \varepsilon_\alpha) = T_\alpha(E - h_{0\alpha}, \varepsilon_\alpha) ,
\]
and \( G_0(E) = 1/(E - H_0 + i0) \) is the free Green’s function for the three-body kinetic-energy operator \( H_0 = h_{0\alpha} + h_{0\beta} \). The energy dependence of the two-cluster RGM kernel is self-consistently determined \[8,9\] through
\[
\varepsilon_\alpha = \langle \Psi | h_{0\alpha} + V^{\text{RGM}}(\varepsilon_\alpha) | \Psi \rangle = \frac{1}{3} E + \frac{1}{2} \langle \varphi_\alpha | H_0 | \Psi \rangle ,
\]
where \( \Psi = \varphi_\alpha + \varphi_\beta + \varphi_\gamma \) is the normalized total wave function for the \( 3N \) bound state. In practice, we start from some specific values of \( \varepsilon_\alpha \) and \( E \), and solve Eq. (1.4) to find a negative three-body energy \( E \) such that the eigenvalue \( \lambda(E) \) becomes 1. The normalized Faddeev component \( \varphi_\alpha \) yields a new value of \( \varepsilon_\alpha \) through Eq. (1.6). Since it is usually not equal to the starting value, we repeat the process by using the new value. This process of double iteration converges very fast if the starting values of \( \varepsilon_\alpha \) and \( E \) are appropriately chosen.

For the numerical calculation, we discretize the continuous momentum variables \( p \) and \( q \) for the Jacobi coordinate vectors, using the Gauss-Legendre \( n_1 \)- and \( n_2 \)-point quadrature formulas, respectively, for each of the three intervals of \( 0 - 1 \) \( \text{fm}^{-1} \), \( 1 - 3 \) \( \text{fm}^{-1} \) and \( 3 - 6 \) \( \text{fm}^{-1} \). The small contribution from the intermediate integral over \( p \) beyond \( p_0 = 6 \) \( \text{fm}^{-1} \) in the \( 2N \) \( T \)-matrix calculation is also taken into account by using the Gauss-Legendre \( n_3 \)-point

\[1\] As a system of identical three particles, \( \varepsilon_\alpha, \varepsilon_\beta \) and \( \varepsilon_\gamma \) are all equal and are expressed as \( \varepsilon \) in Table III.
quadrature formula through the mapping \( p = p_0 + \tan(\pi/4)(1 + x) \). The momentum region \( q = 6 \text{ fm}^{-1} - \infty \) is also discretized by the \( n_3 \) point formula just as in the \( p \) discretization case. We take \( n_1-n_2-n_3=10-10-5 \), for which well converged results are obtained at least for 2 and 5 channel calculations. The partial-wave decomposition of the \( 2N \) RGM kernel is carried out numerically using the Gauss-Legendre 20-point quadrature formula. The modified spline interpolation technique developed in [10] is employed for constructing the rearrangement matrix. For the diagonalization of the large non-symmetric matrix, the Arnoldi-Lanczos algorithm recently developed in the ARPACK subroutine package [11] is very useful.

Tables I and II list the deuteron properties and the \( NN \) effective range parameters predicted by fss2 and FSS, respectively. All the calculations in the present paper are carried out in the isospin basis. For a realistic calculation of the \( ^3\text{H} \) binding energy, it is essential to use the \( NN \) interaction that reproduces the correct \( D \)-state probability (\( P_D \)) of the deuteron and the effective range parameters of the \( ^1S_0 \) scattering. Since all the realistic \( NN \) interactions reproduce the \( NN \) phase shifts more or less correctly, the strength of the central attraction is counterbalanced with that of the tensor force. Namely, if the interaction has a weaker tensor force, then it should have a stronger central attraction. Generally speaking, the effect of the tensor force is reduced in the nuclear many-body systems, in comparison with the bare two-nucleon collision. This implies that the \( NN \) interaction with a weaker tensor force is favorable, in order to obtain sufficient binding energies of the nuclear many-body systems. The weak tensor force, however, causes various problems like a too small value for the deuteron quadrupole moment \( Q_d \) and some disagreement of the mixing parameter \( \varepsilon_1 \) of the \( ^3S_1 + ^3D_1 \) coupling. For example, the Reid soft core potential (RSC) [13] gives \( P_D = 6.5 \% \) and predicts too small \( ^3\text{H} \) binding energy, \( B_t = 7.35 \text{ MeV} \), compared with the experimental value \( B_t^{\text{exp}} = 8.48 \text{ MeV} \). A series of the Bonn potentials reproduce the \( NN \) phase shifts very accurately, but they have a tendency that the tensor force is generally rather weak. [3] The model C has the strongest tensor force \( P_D = 5.61 \% \), yielding \( B_t = 7.99 \text{ MeV} \). The value \( P_D \) becomes smaller for models B and A, and the value of \( B_t \) becomes larger, correspondingly. The following results are given in ref. [3]: model-B \( (P_D = 5.0 \%, \ B_t = 8.13 \text{ MeV}) \), model-A \( (P_D = 4.4 \%, \ B_t = 8.32 \text{ MeV}) \). These results are all obtained in the 34 channel calculations (including the \( 2N \) total angular momentum \( J \leq 4 \)), and by using the \( np \) interaction. In fact, the effects of the charge dependence and the charge asymmetry are important for the detailed discussion, and it is estimated to be about 190 keV in refs. [3] and [14]. The most recent Faddeev calculation employing the CD-Bonn potential [15] incorporates these effect, and predicts \( B_t = 8.014 \text{ MeV} \) [13] for \( P_D = 4.85 \% \). The present status of the \( ^3\text{H} \) binding energy calculation is summarized as that more than 0.5 MeV is missing if the two-nucleon force of any realistic \( NN \) interactions is only employed. [17]

On the other hand, our result of \( P_D \) in Table is about 5.5 % both in the fss2 and FSS cases. We think that this is a reasonable value, in spite of the fact that \( Q_d \) of fss2 is too small. This is because a careful evaluation of the meson-exchange current contributions to \( Q_d \), which could be as large as 0.01 fm\(^2\) [13,14], must be made. Our results of the effective

\(^2\)These \( n_3 \) points for \( p \) are not included for solving the Faddeev equation [14], since it causes a numerical inaccuracy for the interpolation.
range parameters in Tables are not as perfect as those of the Bonn B potential. It should be noted that the effects of the higher-order terms of the Coulomb interaction are not incorporated in these calculations. The deuteron binding energy and the scattering length \(a_s\) for the \(^1S_0\) state are fit in determining our quark-model parameters.

Table list the results of the Faddeev calculations by fss2 and FSS in various types of truncations of the model space. The 5 channel calculation with \(J \leq 1^+\) incorporates only the partial waves \(^3S_1 + ^3D_1\) and \(^3S_0\) for the \(2N\) \(T\)-matrix. Similarly, the 18 and 34 channel calculations incorporate the partial waves with \(J \leq 2\) and \(J \leq 4\), respectively. We find that the energy gain in the 5 channel to 34 channel calculation is about 330 - 360 keV, which is the same tendency for the realistic \(NN\) potentials with a strong tensor force, such as the RSC and Paris potentials. The convergence is not enough even in the 34 channel calculation, and we expect the further energy gain of the order of 10 - 20 keV. The model fss2 predicts \(B_t = 8.51\) MeV and seems to give too large binding energy, compared with experiment. In fact, it underbinds by 150 - 160 keV, if the effects of the charge dependence and the charge asymmetry of the \(NN\) interaction is taken into account. The scenario assuming the most favorable Bonn A potential is given in Table 11.1 of ref. [6], which tells us that the corrected value due to the charge dependence and the charge asymmetry of the two-body force is 8.13 MeV and the rest, 350 keV, is attributed to the combined contribution of the three-body force and the medium effect of the two-body force. Our result using the quark-model potentials indicates that one can reduce the net effect besides the two-nucleon force to less than half of the OBEP values, by keeping the deuteron \(D\)-state probability in a reasonable magnitude.

Note that the \(2N\) energy \(\varepsilon_0\) in Eq. (1.3) is directly related to the separation of the total energy \(E(\alpha)\) into the kinetic-energy contribution, \(\langle H_0 \rangle = 2(3\varepsilon_0 - E)\), and the potential-energy contribution, \(\langle V \rangle = 3(E - 2\varepsilon_0)\). In the 34 channel calculations, these are given by \(\langle H_0 \rangle = 43.95\) MeV, \(\langle V \rangle = -52.47\) MeV for fss2, and \(\langle H_0 \rangle = 41.83\) MeV, \(\langle V \rangle = -50.22\) MeV for FSS. If we compare these with the results of the CD-Bonn potential \(\langle H_0 \rangle = 37.42\) MeV, \(\langle V \rangle = -45.43\) MeV) and the AV-18 potential \(\langle H_0 \rangle = 46.73\) MeV, \(\langle V \rangle = -54.35\) MeV), we find that our quark-model potentials give a moderate amount of the kinetic-energy contribution just between the CD-Bonn and AV18 potentials.

Table also shows the calculated charge root mean square (rms) radii of \(^3H\) and \(^3\text{He}\) obtained by fss2 and FSS. The finite size corrections of the nucleons are made through

\[
\langle r^2 \rangle_{^3\text{H}} = R_C(^3\text{H})^2 + (0.8502)^2 - 2 \times (0.3563)^2,
\]

\[
\langle r^2 \rangle_{^3\text{He}} = R_C(^3\text{He})^2 + (0.8502)^2 - \frac{1}{2} \times (0.3563)^2,
\]

where \(R_C^2\) stands for the square of the charge rms radius for the point nucleons. Since our \(3N\) bound state wave functions are given in the momentum representation, we first calculate the charge form factors \(F_C(Q^2)\), according to the formulation given in ref. [24]. \(R_C^2\) is then extracted from the power series expansion of \(F_C(Q^2)\) with respect to \(Q^2\). We have employed 20 points, \(Q = 0.05 \times n\) fm\(^{-1}\) with \(n = 1 - 20\), for the extrapolation to \(Q = 0\). In the present calculation, the Coulomb force and the relativistic correction terms of the charge current operator are entirely neglected. The experimental values are rather difficult to determine, as discussed in ref. [24]. Here we compare our results with two empirical values.
We find that the agreement with the experiment is satisfactory both for fss2 and FSS.

The Faddeev calculations for $^3$H, using the quark-model $NN$ potentials, have been carried out by Takeuchi, Cheon and Redish [28], and recently by the Salamanca-Jülich group [29]. In the former calculation, the model QCM-A, by the Tokyo University group gives the $NN$ phase shifts with almost the same accuracy as our model FSS. The model QCM-A predicts $P_D = 5.58\%$ for the deuteron $D$-state probability and $B_t = 8.01 - 8.02$ MeV for the $^3$H binding energy in the 5 channel calculation. This is very similar to our results for the model FSS. On the other hand, the Salamanca-Jülich group predicts $B_t = 7.72$ MeV, in spite of the very small $D$-state probability $P_D = 4.85\%$. It is not clear to us how they treated the energy dependence of the RGM kernel at the process of the separable expansion for solving the Faddeev equation. They have to improve the fit of the $NN$ phase shifts for higher partial waves (especially, for the $P$ waves), in order to extend their calculation to more than 5 channels.

In summary, we have carried out the Faddeev calculation for the three-nucleon bound state, by explicitly incorporating the off-shell $T$-matrix derived from the RGM exchange kernel of the quark-model $NN$ interaction. The energy dependence of the two-cluster RGM kernel is self-consistently treated. [8,9] For the two models fss2 [3,4] and FSS [4,5], we have obtained $E(^3\text{H}) = -8.514$ MeV (fss2), and $-8.390$ MeV (FSS) in the 34 channel calculation using the $np$ interaction. The charge rms radii of the $^3\text{H}$ and $^3\text{He}$ are in fair agreement with experiment: $\sqrt{\langle r^2\rangle}^{^3\text{H}} = 1.72$ fm (fss2), 1.74 fm (FSS) and $\sqrt{\langle r^2\rangle}^{^3\text{He}} = 1.90$ fm (fss2), 1.92 fm (FSS). In these calculations, the Coulomb force and the relativistic correction terms are neglected. In view of the fact that the $NN$ phase shifts of FSS are not that excellent, the results of fss2 are more meaningful. These numbers are the closest to the experiments among many results obtained by Faddeev calculations employing modern realistic $NN$ interaction models. Since both models fss2 and FSS have a common feature in describing the short range correlation by the quark exchange kernel, it is important to clarify the mechanism in which the quark-model potentials give larger $^3\text{H}$ binding energy than the meson-exchange potentials. The off-shell behavior of the RGM $T$-matrix is closely connected to this alternative description of the short-range correlations. More detailed study on this point is now under way.

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TABLES

TABLE I. The deuteron properties by fss2 and FSS in the isospin basis. The results by the Bonn B potential [6] are also shown for comparison. A small difference in FSS from Table IV of [3] is due to the numerical inaccuracy in the previous calculation. The effect of the meson exchange current is not included in the calculated values of $Q_d$ and $\mu_d$.

|                | FSS  | fss2 | Bonn B | Expt.                     | Ref. |
|----------------|------|------|--------|---------------------------|------|
| $\epsilon_d$ (MeV) | 2.256| 2.225| 2.2246 | $2.224644 \pm 0.000046$    | [20] |
| $P_D$ (%)       | 5.86 | 5.49 | 4.99   |                           |      |
| $\eta = A_D/A_S$| 0.0267| 0.0253| 0.0264 | $0.0256 \pm 0.0004$       | [21] |
| rms (fm)        | 1.963| 1.960| 1.968  | $1.9635 \pm 0.0046$       | [20] |
| $Q_d$ (fm²)     | 0.283| 0.270| 0.278  | $0.2860 \pm 0.0015$       | [22] |
| $\mu_d$ ($\mu_N$)| 0.8464| 0.8485| 0.8514| $0.857406 \pm 0.000001$   | [23] |

TABLE II. The NN effective range parameters calculated by fss2 and FSS in the isospin basis. The results by the Bonn B potential [6] are also shown for comparison. The higher-order terms of the Coulomb force are not included. The experimental values are taken from [20].

|                | FSS  | fss2 | Bonn B | Expt.                     |
|----------------|------|------|--------|---------------------------|
| $a_s$ (fm)    | −23.64| −23.76| −23.75 | −23.748 ± 0.010           |
| $r_s$ (fm)    | 2.62 | 2.58 | 2.71   | 2.75 ± 0.05               |
| $a_s$ (fm)    | 5.41 | 5.399| 5.424  | 5.424 ± 0.004             |
| $r_s$ (fm)    | 1.76 | 1.730| 1.761  | 1.759 ± 0.005             |
TABLE III. The three-nucleon bound state properties predicted by the Faddeev calculation with fss2 and FSS. The \(np\) interaction is used in the isospin basis. The discretization points of \(p\) and \(q\) are specified by the values of \(n_1-n_2-n_3=10-10-5\) (see the text). The column “channels” implies the number of two-nucleon channels included, and \(n_{\text{max}} = n(3n_1)(3n_2 + n_3)\) for the \(n\) channel calculation is the dimension of the diagonalization for the Faddeev equation. \(E(\text{\(^3\)H})\) is the ground state energy, and \(\sqrt{\langle r^2 \rangle_{\text{\(^3\)H}}}\) and \(\sqrt{\langle r^2 \rangle_{\text{\(^3\)He}}}\) are the charge rms radii for \(\text{\(^3\)H}\) and \(\text{\(^3\)He}\), respectively, with the proton and neutron size corrections introduced by Eq. (1.7). The Coulomb force and the relativistic corrections are neglected. \(\varepsilon\) is the \(2N\) expectation value, Eq. (1.6), determined self-consistently.

| model | channels | \(n_{\text{max}}\) | \(\varepsilon(2N)\) (MeV) | \(E(\text{\(^3\)H})\) (MeV) | \(\sqrt{\langle r^2 \rangle_{\text{\(^3\)H}}}\) (fm) | \(\sqrt{\langle r^2 \rangle_{\text{\(^3\)He}}}\) (fm) |
|-------|----------|-----------------|-----------------|-----------------|-----------------|-----------------|
| fss2  | 2 ch     | 2,100           | 2.361           | −7.807          | 1.80            | 1.96            |
|       | 5 ch     | 5,250           | 4.341           | −8.189          | 1.75            | 1.92            |
|       | 10 ch    | 10,500          | 4.249           | −8.017          | 1.76            | 1.94            |
|       | 18 ch    | 18,900          | 4.460           | −8.439          | 1.72            | 1.90            |
|       | 34 ch    | 35,700          | 4.488           | −8.514          | 1.72            | 1.90            |
| FSS   | 2 ch     | 2,100           | 2.038           | −7.674          | 1.83            | 1.99            |
|       | 5 ch     | 5,250           | 3.999           | −8.034          | 1.78            | 1.95            |
|       | 10 ch    | 10,500          | 3.934           | −7.909          | 1.78            | 1.97            |
|       | 18 ch    | 18,900          | 4.160           | −8.342          | 1.74            | 1.93            |
|       | 34 ch    | 35,700          | 4.175           | −8.390          | 1.74            | 1.92            |