Gain-induced topological response via tailored long-range interactions

Yuzhou G. N. Liu, Pawel S. Jung, Midya Parto, Demetrios N. Christodoulides and Mercedeh Khajavikhan

The ability to tailor the hopping interactions between the constituent elements of a physical system could enable the observation of unusual phenomena that are otherwise inaccessible in standard settings. In this regard, a number of recent theoretical studies have indicated that an asymmetry in either the short- or long-range complex exchange constants can lead to counterintuitive effects, for example, the possibility of a Kramer’s degeneracy, even in the absence of spin 1/2 or the breakdown of the bulk-boundary correspondence. Here we show how such tailored asymmetric interactions can be realized in photonic integrated platforms by exploiting non-Hermitian concepts, enabling a class of topological behaviours induced by optical gain. As a demonstration, we implement the Haldane model, a canonical lattice that relies on asymmetric long-range hopping to exhibit quantum Hall behaviour without a net external magnetic flux. The topological response observed in this lattice is a result of gain and vanishes in a passive but otherwise identical structure. Our findings not only enable the realization of a wide class of non-trivial phenomena associated with tailored interactions, but also open up avenues to study the role of gain and nonlinearity in topological systems in the presence of quantum noise.

The interplay between long- and short-range interactions is known to play a pivotal role in many and diverse areas of physical sciences. Such interactions naturally manifest themselves in Rydberg atoms and ionic systems and are known to drastically modify the metal–insulator transitions in Bose–Hubbard models. In the realm of topological physics, the possibility of introducing asymmetries in the complex exchange constants of a system has been actively pursued along several theoretical fronts in an effort to unravel novel effects and behaviours. In this respect, a host of intriguing topological phenomena have been predicted, ranging from the quantum anomalous Hall and non-Hermitian skin effects, to Anderson transitions in the Hatano–Nelson model and the emergence of Gaussian symplectic ensembles in the absence of spin.

An archetypical lattice in which a topological phase can appear because of non-symmetric long-range exchange interactions was proposed by Haldane in 1988. This proposition trail-blazed a path towards the discovery of topological insulator materials based on processes such as spin–orbit interactions. Yet, despite a series of ground-breaking advancements, to this day, the original Haldane lattice has remained an elusive crystal within the context of solid-state physics and is yet to be synthesized in the laboratory. What makes the implementation of this system and other related topological models challenging is the difficulty of realizing the aforementioned non-symmetric long-range exchange mechanisms in condensed matter. It is clearly of interest to explore alternative platforms where this type of interactions can be supported.

Photonic systems have so far provided a versatile testbed to study a variety of topological effects through artificial gauge fields and synthetic dimensions. However, most interactions in current photonic topological lattices tend to be of the nearest-neighbour (NN) type and symmetric. Ideally, it will be beneficial if one had the freedom to establish any arbitrary interconnectivity, irrespective of whether it is symmetric/asymmetric or short/long-range. In this work, we show how optical gain and non-Hermiticity can be utilized to provide an advantage to tailor the hopping long-range exchange between resonant elements in a photonic arrangement. We demonstrate this approach by implementing a Haldane lattice operating in its topological Chern regime. As we will show, the topological features of this lattice are manifested solely because of non-Hermiticity and nonlinearity. In this respect, we showcase the potential of active photonic systems in designing a class of topological lattices that are uniquely enabled by optical gain.

The Haldane structure features a honeycomb lattice, composed of two species of atom (Fig. 1a). Besides NN interactions among detuned neighbouring atoms (which break inversion symmetry), an asymmetric exchange is established between the next-nearest neighbouring (NNN) elements that can be expressed through antisymmetric complex hopping terms of the form \( \epsilon \mathrm{e}^{\imath \phi} \). The Hamiltonian associated with this lattice is expressed as follows:

\[
\hat{H} = M \sum_{p \in A} \hat{a}_p^\dagger \hat{a}_p - M \sum_{p \in B} \hat{a}_p^\dagger \hat{a}_p + t_1 \sum_{\text{NN}} \hat{a}_p^\dagger \hat{a}_q + t_2 \sum_{\text{NNN}} \epsilon \mathrm{e}^{\imath \phi} \hat{a}_p^\dagger \hat{a}_q
\]

where \( \hat{a}_p^\dagger \) and \( \hat{a}_p \) are the bosonic creation and annihilation operators at sites \( p \) and \( q \), respectively. \( M \) represents a relative detuning (energy difference) associated with the two sublattice sites \( A \) and \( B \), \( t_1 \) is the coupling strength between neighbouring elements, and \( t_2 \) stands for the NNN hopping coefficient, with \( \phi_\alpha = -\phi_\beta \) being the phase associated with the NNN exchange. Both \( t_1 \) and \( t_2 \) are taken here to be real and positive. A characteristic feature of the Haldane two-band model is a phase transition between trivial and topological states, depending on the complex tunnelling amplitude among NNN sites. Here, by varying the values of \( M \), \( t_1 \), and \( t_2 \), the Haldane lattice can transition from a topologically trivial to a non-trivial phase. When terminated with a proper arrangement, for example with a zigzag edge, this topological structure can support novel effects and behaviours.
a unidirectional and scatter-free edge current. Figure 1b–d displays the topological and trivial regimes arising at different values of the parameters $M$, $t_2$ and $\phi$ (when $t_1 = 1$). As is evident from this figure, when $t_2 = 0$, the array is in a topologically trivial phase with a Chern number $C = 0$. On the other hand, by tuning the NNN hopping amplitude to $M/t_2 = 1.25$ and $\phi = \pi/2$, this system will behave in the Chern insulating regime ($C = +1$).

In our study, we realized a topological Haldane lattice by using an active photonic structure as shown in Fig. 2. Its unit cell, depicted in the inset, is composed of two sets of ring-type cavities, A and B (colour-coded by blue and red). These two species of resonators have slightly different perimeters, thus supporting eigenfrequencies that are somewhat detuned. A fan-shaped construct is incorporated into each active resonator that generates an exceptional point and enforces a unidirectional flow of light due to the interplay of gain, spontaneous emission and gain-induced nonlinearity with the geometrical features of the cavity (Supplementary Sections 1 and 2).
The phase $\phi$ is determined by the length of link $L$. In the experiments, only one of the three resonators is pumped. The direction of the power flow (white arrows) towards the unpumped resonators changes as the phase $\phi$ varies from $-\pi$ to $\pi$ (indicated in the panels) for sublattices A (f-h) and B (j-l).

As previously indicated, the most crucial feature of this lattice is the complex antisymmetric long-range NNN hopping between elements of the same species ($A \leftrightarrow A$ and $B \leftrightarrow B$). Typically, in passive optical arrangements, the Haldane model requires time-reversal symmetry breaking, which is difficult to achieve without the use of magneto-optic materials that rely on external magnetic fields. Here we propose an innovative approach to address this issue. We couple two unidirectional active cavities (of the same type) through a combination of directional couplers and waveguides involved. On the other hand, for light travelling from resonator 2 to 1, it has to follow the ‘cross’ ports path, resulting in an additional $\pi$ phase shift (that is, the overall phase is now $\pi + \phi$).

In either direction, the magnitude of the complex NNN hopping coefficient has the same value $t_\perp$. By properly choosing the length of the link ($L$) so $\phi = \pi$, the hopping coefficients assume the values $i\delta$ and $-i\delta$, when considering the paths $1 \rightarrow 2$ and $2 \rightarrow 1$, respectively. Because, in this case the resulting $\phi_{\text{tot}} = \pi/2$, the equivalent Haldane system is placed in the Chern insulating regime ($C = +1$) as depicted in Fig. 1b. This complex antisymmetric coupling between NNN elements of the same species is what actually enables the Haldane model to acquire a topological phase. Finally, to provide coupling between all the NNN resonators in the hexagonal unit cell, it is imperative to allow the waveguide sections to intersect each other. For high-contrast waveguides (InGaAsP core-SiO$_2$ claddings, as used here), electromagnetic simulations confirm that the crosstalk between intersecting channels is negligible provided that the crossing angle is $\sim 90^\circ$ (refs. 31") (Supplementary Section 5). More details about the design are provided in the Methods and Extended Data Fig. 3.

In the present work, the Haldane lattice was fabricated on a wafer with InGaAsP multiple quantum wells. Details of the fabrication process are provided in the Methods and Extended Data Fig. 3. The lattice is terminated with zigzag edges, naturally resulting in an equilateral triangular structure. It should be noted that other edges, such as armchairs, can also be realized, resulting in ribbon-like lattices (Supplementary Section 6). Overall, this system contains 166 microresonators (or 66 unit cells). In addition, we incorporated three gratings at the corners of the structure to monitor the light exiting the lattice at these points. Scanning electron microscopy and optical microscope images of our photonic Haldane lattice are shown in Fig. 2b–d. The fabricated samples were characterized in a photoluminescence measurement station, with an optical mask placed in the path of the pump beam to generate the desired pump profile on the sample. To promote the topological edge mode, the outer perimeter of this Haldane lattice was optically pumped with a pulsed laser. The emission profile was then captured by an InGaAs infrared camera, while the lasing spectrum was collected with a monochromator accompanied by an InGaAs detector array. The sample was placed in a cryostat with an electric heater to fine-tune the output gratings. When the desired phase ($\phi = \pi$ or $\phi = 2\pi$) is achieved, the spectrum indicates that there are two lasing modes. The schematic design of the three-element systems used for sublattice A (e) and sublattice B (i).

**Fig. 3** | Experimental results for two-element and three-element subunits. a, Schematic design of the two-element system. Two unidirectional microring resonators with S-bends are coupled by a link. The phase $\phi$ is determined by the length of the link. To facilitate interferometric measurements, the field in each ring is sampled and sent to a 3 dB coupler, where each output arm incorporated with a grating. b–d, Experimental results of the intensity distributions for $\phi = \pi/2$ (b) or $\phi = \pi$ or $2\pi$ (c) or $\phi = 3\pi/2$ (d). Insets show the spectra at the output gratings. When the desired phase ($\phi = \pi$ or $\phi = 2\pi$) is achieved, the spectrum indicates that there are two lasing modes. e, Schematic design of the three-element systems used for sublattices A (e) and sublattice B (i). The phase $\phi$ is determined by the length of link $L$. In the experiments, only one of the three resonators is pumped. f–h and j–l. The direction of the power flow (white arrows) towards the unpumped resonators changes as the phase $\phi$ varies from $-\pi$ to $\pi$ (indicated in the panels) for sublattices A (f–h) and B (j–l).
In our experiments, we first characterized the complex antisymmetric hopping behaviour in a constituent subunit composed of two identical unidirectional resonators, as shown in Fig. 3a. The two grating parts are incorporated to probe the emission from the structure at a wavelength of $\lambda \approx 1,550$ nm. This coupled system supports two eigenmodes $[1 \pm i]^{\dagger}$ with corresponding eigenvalues of $\pm te^{i\phi}$ (Supplementary Section 4). It should be noted that, if the two involved resonators are not enforced to be unidirectional, an additional coupling will form between the counter-propagating modes in these two resonators, which can prevent the system from displaying a topological response (Supplementary Section 8). When this structure is pumped, depending on the value of $\phi$, various emission profiles and spectra are observed. If, for example, $\phi = \pi/2$ or $3\pi/2$, the two eigenvalues become purely imaginary (with opposite signs), hence one of the two modes experiences a substantially higher gain. Consequently, the emission spectra of the corresponding lasers become single-moded, and the output emissions from the two resonators interfere constructively, leading to a power build-up in one of the two gratings (Fig. 3b,d). On the other hand, when $\phi = 0$ or $\pi$, the two eigenvalues are real and therefore one should observe the simultaneous lasing of two modes with a frequency splitting of $2t$. As a result, the power is equally distributed between the two grating parts (Fig. 3c). As indicated earlier, this is the phase condition required for NNN hopping (generally $\phi = n\pi$, where $n$ is an integer). Similarly, we examined the behaviour of the three-element subunit cell (composed of identical triangular resonators). Here, when one of the three elements is pumped, the energy is expected to flow in one of the two counter-propagating directions around the subunit cell (Fig. 3f–l). As shown in Fig. 3j–l, for phases $\phi$ that are centred at odd products of $\pi$, the mode circulates in the clockwise direction, while for $\phi$ values positioned around even multiple integers of $\pi$, it selects the counterclockwise path. In this regard, when integrating these subunits in the greater Haldane lattice, the direction of the flow of light can be flipped by either altering the chirality of all the individual resonators involved (by flipping the fan-shaped structures) or by changing the value of $n$ by an odd integer increment.

In characterizing the topological Haldane lattice, we look for its most prominent signature, that is, a unidirectional flow of light around the perimeter when the array is terminated with zigzag edges. Here, a metallic mask is used to selectively pump the periphery, while an additional attenuating filter is used to cover one of the sides of the lattice from the pump beam (details about the metallic mask fabrication are provided in the Methods). To observe the
unidirectional flow of light around the structure, one edge of the lattice has been kept almost unpumped, while the other two edges are fully pumped. Under these conditions, we expect to observe a power difference between the ports located at the two ends of the unpumped edge because of the unidirectional flow of light around the perimeter (Supplementary Section 9). By monitoring and comparing the intensity of light exiting the grating couplers (incorporated at the corners of the structure), the direction of light transport around the edges of the array is determined. Figure 4 shows the related experimental results. In particular, a comparison between the light intensity levels as they exit the gratings at the two ends of the unpumped side reveals that in the topological Haldane lattice, light circulates in one of the two counter-propagating directions (in this case, counterclockwise), as expected from the Haldane model. This chiral behaviour is consistently observed as we rotate the unpumped edge (Fig. 4f,g). These results are in full agreement with our theoretical predictions and simulations (Supplementary Section 9). It should be noted that, because the output intensity at the grating couplers is a combination of stimulated and spontaneous emissions, the power ratios reported in the caption of Fig. 4e–g assume finite values. We intentionally chose a structure in which the direction of energy flow around the edge differs from that of the power circulation in the individual resonators involved. The corresponding bandgap of the Haldane topological lattice is calculated to be ~200 GHz (Supplementary Section 10). To compare with the trivial case, we fabricated a non-topological lattice by placing the link farther apart from the resonators, thus reducing the strength of NNN hopping. The reported unidirectional power flow is only observed in topological lattices (where $C=1$), whereas in a trivial Haldane lattice, light shows no preferential direction when circulating around the array; that is, the intensity emitted by the two gratings is almost identical (Fig. 4i–k). Finally, for this structure to support a chiral edge mode, the directionality of the complex conjugate NNN hoppings for both sets of resonators ($A \leftrightarrow A$ and $B \leftrightarrow B$) must be the same. Given that this condition has to be satisfied while $\theta = n\pi$ is reached, it limits the number of longitudinal modes that can lase in a topological edge state. In this regard, above threshold, the topological Haldane lattice is expected to operate as a single mode laser. This behaviour is observed in our experiments, as is apparent from the emission spectrum provided in Fig. 4h. By contrast, the trivial lattice, having no constraint of this kind, tends to display a multimode lasing behaviour (Fig. 4i).

In conclusion, we have shown how tailored short- and long-range interactions can be realized in active optical platforms using some of the unique properties offered by gain materials. By adopting this approach, we have demonstrated a topological Haldane lattice operating in its Chern regime. Our work may inspire further developments in the field of topological physics by introducing higher degrees of tailored interconnectivity in Haldane-like networks that could display previously unattainable topological phases.

Online content
Any additional references, Nature Research reporting summaries, source data, extended data, supplementary information, acknowledgements, peer review information; details of author contributions and competing interests; and statements of data and code availability are available at https://doi.org/10.1038/s41567-021-01185-4.

Received: 18 June 2020; Accepted: 22 January 2021; Published online: 25 February 2021

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 Modal fields in the two identical rings can be effectively described through a set of directional couplers. The coupling strengths between the rings and bus waveguides respectively. The angular frequency $\omega$ is determined by the resonance condition for each resonator in the absence of coupling. Using either temporal or spatial coupled mode analysis, one can readily show that the coupling from resonator 1 to 2 is given by $k(2\pi)/L$, while that from 2 to 1 is expressed by $-k(2\pi)/L$. Here, r and t represent the through and cross-coupling terms for the directional coupler. The coupling strengths between the rings and bus waveguides are all set to be $k$ and the effective propagation constant is $k = 2\pi/\lambda$. Finally, the overall length of the delayed coupling part is $L = L_1 + 2L_2$, where $L_1$ and $L_2$ are the lengths of the waveguide sections between the two directional couplers and from each directional coupler to the adjacent ring, respectively. In the case of $r = t = 0$, the two coupling coefficients become $k_+ = -i\kappa/2$ and $k_- = i\kappa/2$. Consequently, this system exhibits the asymmetric imaginary (yet Hermitian) hopping coefficients that are required to realize the Haldane Hamiltonian.

Electromagnetic and wafer design. The dimensions of the resonators, the fan-shaped couplers and the links are shown in Extended Data Fig. 2a–c. To realize the required detuning between the two species of resonators, the perimeters of the two rings are designed to be slightly different ($L_1 = 62.525\mu m$, $L_2 = 62.534\mu m$). The gain material used in this study consists of partially embedded In$_{0.53}$Ga$_{0.47}$As quantum wells of $0.10 \text{nm}$ thick/In$_{0.53}$Ga$_{0.47}$As quantum wells of $0.20 \text{nm}$ thick, with an overall height of $500 \text{ nm}$ grown on type-I InP substrate. The quantum wells are covered by a $10-nm$-thick InP overlay for protection. The optical waveguiding sections are designed to have a width of $500 \text{ nm}$ and a thickness of $200 \text{ nm}$, and are partially embedded in a silicon dioxide ($\text{SiO}_2$) cladding, to support the TE$_0$ mode at a wavelength of $1,064 \text{ nm}$. The TE$_0$ mode (Extended Data Fig. 2d) has an effective index of $n_eff = 2.272$ and a group index of $n_g = 4$.

Fabrication. The fabrication steps that followed to realize the proposed lattices are shown in Extended Data Fig. 3. An XR-1541 hydrogen silsesquioxane (HSQ) solution in methyl isobutyl ketone was used as a negative electron-beam resist and was spun onto the wafer (thickness of $50\text{ nm}$) and soft-baked at a temperature of $180^\circ \text{C}$ (Extended Data Fig. 3a). The rings were patterned by electron-beam lithography (Extended Data Fig. 3b), then the wafer was immersed in tetramethylammonium hydroxide for $120 \text{ s}$ to develop the patterns, then rinsed and served as a mask for the subsequent reactive-ion-etching (RIE) processes. To perform dry etching, a mixture of $\text{H}_2\text{C}_2\text{H}_2\text{Ar}$ gases was used at a ratio of $10:1:7:1$ s.c.c.m. Both radiofrequency and inductively coupled plasma (ICP) powers were set at $150 \text{ W}$ and the chamber pressure was held at $35 \text{ mTorr}$ (Extended Data Fig. 3c). The wafer was cleaned with oxygen plasma to remove the organic contaminations and polymers that form during the dry etching process (O$_2$, $50 \text{ sccm}$ flow; ICP power, $150 \text{ W}$; chamber pressure, $50 \text{ mTorr}$). The sample was then submerged in buffered oxide etch for $10 \text{ s}$ to remove the HSQ mask (Extended Data Fig. 3d). After this, a $2 \mu m$ layer of SiO$_2$ was deposited onto the wafer using plasma-enhanced chemical vapour deposition (Extended Data Fig. 3e). We used SU-8 3010 photoresist to bond the wafer to a glass substrate for mechanical support (Extended Data Fig. 3f). After spinning the photoresist, the sample was placed on the glass, with the pattern side facing down, and exposed for $30 \text{s}$ on both sides. Finally, the remaining InP substrate was entirely removed by wet etching in hydrochloric acid for $100 \text{ min}$ (Extended Data Fig. 3g).

In the experiments, a metallic mask was placed in the pump branch to create the desired pump profile on the sample (see next section). This mask was fabricated by depositing metals on a glass slide. A pattern was first transferred onto the glass slide using photolithography (negative photoresist, NR7-3000; developer, RD6), then a layer of gold with a thickness of $10 \text{ nm}$ was deposited (for adhesion purposes) using electron-beam evaporation, followed by a layer of titanium with a thickness of $100 \text{ nm}$ to ensure that the transmittance of the pump beam ($\lambda = 1,064 \text{ nm}$) remained below $1\%$. Finally, the metal was removed from undesired places by dissolving its underlying resist in acetone.

Characterization setup. A micro-photoluminescence ($\mu$-PL) set-up, depicted in Extended Data Fig. 4, was used to characterize the structures. The sample was optically pumped by a pulsed laser (duration, $15 \text{ ns}$; repetition rate, $290 \text{ kHz}$) operating at a wavelength of $1,064 \text{ nm}$ (SPI fibre laser). A beam-shaping system with additional metal mask and knife edges was designed to realize the desired pump size/shape. A $\times 10$ microscope objective (NA, 0.26) was used to project the pump beam onto the structure and also to collect the photoluminescence emission from the sample. For temperature tuning, the sample was inserted into a cryostat (Janis ST-500). The surface of the sample was imaged by two cascaded $4-f$ imaging systems into an infrared camera (Xenics). A broadband amplified spontaneous emission device was used to illuminate the sample to properly position the pump beam with respect to the pattern. A notch filter was placed in the path of emission to attenuate the pump beam. The output spectra were obtained by a monochromator equipped with an attached linear array InGaAs detector. A power meter was inserted at the focus of the emitted beam to measure the output power.

Data availability. Data that support the plots within this paper and other findings of this study are available from the corresponding author upon reasonable request. Source data are provided with this paper.

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Acknowledgements

This work was supported by DARPA (D18AP00058), the Office of Naval Research (N00014-20-1-2522, N00014-20-1-2789, N00014-16-1-2640, N00014-18-1-2347 and N00014-19-1-2052), the Army Research Office (W911NF-17-1-0481), the Air Force Office of Scientific Research (FA9550-14-1-0037 and FA9550-20-1-0322), the National Science Foundation (CBET 1805200, ECCS 2000538, ECCS 2011171), the US-Israel Binational Science Foundation (BSF; 2016381) and the Polish Ministry of Science and Higher Education (Mobility Plus, 1654/MOB/V/2017). Y.G.N.L. thanks the following individuals for their support: A. U. Hassan in design and analysis, F. O. Wu in analysis, W. E. Hayenga in characterization and fabrication, O. Hemmatyar in analysis and graphics, and M. P. Hokmabadi in design and fabrication. The fan-shaped couplers are named after F. O. Wu, D.N.C. and M.K. acknowledge the technical discussions with M. Segov.

Author contributions

Y.G.N.L., D.N.C. and M.K. conceived the idea. Y.G.N.L. designed the structures and experiments. Y.G.N.L. fabricated and characterized the lattices and sublattices. Y.G.N.L., P.S. and M.P. performed the simulations. Y.G.N.L., P.S., M.P., D.N.C. and M.K. developed the theoretical analysis. All authors contributed to preparing the manuscript.

Competing interests

The authors declare no competing interests.

Additional information

Extended data is available for this paper at https://doi.org/10.1038/s41567-021-01185-4. Supplementary information The online version contains supplementary material available at https://doi.org/10.1038/s41567-021-01185-4. Correspondence and requests for materials should be addressed to D.N.C. or M.K.

Peer review information Nature Physics thanks Shuang Zhang and the other anonymous, reviewer(s) for their contribution to the peer review of this work. Reprints and permissions information is available at www.nature.com/reprints.
Extended Data Fig. 1 | Schematic of a two-element system with unidirectional microring resonators and a link structure. The directional couplers provide a π phase difference between the coupling terms $\kappa_{1 \rightarrow 2}$ and $\kappa_{2 \rightarrow 1}$. The coupling phase $\kappa L$ can be changed by varying the length $L = 2L_c + L_m$. 
Extended Data Fig. 2  | Schematics and dimensions of the waveguides, resonators, fan-shape constructs and the link. a, The triangular microring resonator. b, The fan-shaped structure to be incorporated in the triangular resonators. c, The perimeters of two adjacent resonators are denoted by $L_A$ and $L_B$. d, Transverse electric field distribution in a single waveguide. The black arrows indicate the electric field vector.
Extended Data Fig. 3 | Schematic of the fabrication procedure of microring lasers. a, HSQ e-beam resist is spun onto the wafer. b, The wafer is patterned by e-beam lithography. c, A dry etching process to define the rings. d, The sample is immersed in BOE to remove the masking HSQ. e, A 2 μm layer of SiO$_2$ is deposited via PECVD. f, The wafer is flipped upside-down and bonded to a glass substrate by SU-8 photoresist to provide mechanical support. g, Lastly, the InP substrate is wet etched by HCl.
**Extended Data Fig. 4 | Schematic of the μ-PL characterization set-up.** The microrings are pumped by a pulsed laser (15 ns pulse width, 290 kHz repetition rate). The pump beam is focused onto the sample with a 10x objective, this objective in turns also collects the emission from the samples. Light is then either directed to a linear array detector for spectral measurements or to an IR camera for intensity profile observation.