Dispersion relations of mesons in symmetric nuclear matter

L. Mornas

Universidad de Oviedo, Departamento de Física, E-33007 Oviedo, Spain

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Abstract

We calculate dispersion relations and propagators for the $\sigma$, $\omega$, $\pi$, $\rho$, $\delta$, $\eta$ and $a_1$ mesons in relativistic, dense, hot, symmetric nuclear matter. In addition to the usual mixing of the $\sigma - \omega$ system, we obtain mixing of the $\delta$ with the longitudinal $\rho$ mode and of the transverse $\rho$ with the transverse $a_1$ mode. Finally, the component of the $a_1$ polarization along the transferred momentum modifies the in-medium pion propagator in a way similar to the Migdal contact interaction, but with the opposite sign. The spurious pion condensate as well as the additional contribution from the $a_1$ meson are removed by a contact term. We compare two ways of implementing contact term subtraction.

1 Introduction

This work investigates some points concerning the dispersion relations and effective masses of mesons in dense, hot, symmetric nuclear matter. The dispersion relations will be calculated in the random phase approximation (RPA), by performing a linear response analysis [1]–[5] around the homogeneous Hartree ground state. This paper reproduces previous results and extends them in two aspects.

Equations are presented for the six mesons of the Bonn model [6], i.e. $\sigma$, $\omega$, $\pi$, $\rho$, $\delta$ and $\eta$. In addition, the contribution of NN loops to the dispersion relation of the $a_1$ is investigated, since this meson can be of interest from various points of view. One is the construction of chiral Lagrangians, since the $a_1$ is the chiral partner of the $\rho$ meson [7] – [9]. Another is the issue of the dilepton production in relativistic heavy ion collisions. As a matter of fact, the $a_1$ [10] couples to the $\rho$ meson and may modify its spectral properties. The $a_1$ may also be used to describe part of the nucleon-nucleon interaction which is mediated by correlated $\pi$-$\rho$ exchange in the S-wave channel [12, 18]. The inclusion of correlated $\pi$-$\rho$ exchange is known to improve the Bonn potential [13]; however one should keep in mind that the structure of correlated $\pi$-$\rho$ exchange is more complex than what can be described by exchange of particles with a sharp mass [18]. Finally, the $a_1$ meson has sometimes been quoted [20, 21] to provide a means to improve the behaviour of the differential nucleon-nucleon cross section for backwards scattering angle. This is related to the behavior of the tensor part of the NN potential near $\vec{r} = 0$.

As a first part of this work, a discussion of the $a_1$ meson dispersion relation and mixing effects with the $\rho$ and $\pi$ mesons is presented in section §2. It will be seen that the transverse mode of the $a_1$ mixes with that of the $\rho$. Since the axial current is not conserved, we will have a part of the $a_1$ polarization proportional to $q^\mu q^\nu$, where $q^\mu$ is the transferred quadrimomentum. This part mixes

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with the pion and renormalizes its polarization in a way which is formally similar to that produced
by a contact interaction of the Landau-Migdal type. However, it has the opposite sign, so that the
\( a_1 \) would enhance the unobserved pion zero sound mode. Since experimental evidence shows that
pion condensation does not occur at saturation density in symmetric matter, this spurious mode
should be removed. The standard procedure to implement this is to introduce a contact interaction.

We are thus lead in a second part to make some considerations on the relativistic generalization
of the contact term. We will compare in section \( \S 3 \) the results obtained using the Ansatz of
Horowitz et al. [22, 23], who modify the pion propagator in an ad-hoc way, to a more standard
implementation simply consisting of adding one or more contact pieces to the Lagrangian density.
It will be seen that both procedures may lead to different results, when the former is not used
with some caution. In particular, this may explain the discrepancy recently obtained in calculating
the RPA corrections to neutrino-nucleon scattering in proto neutron star matter [24, 25, 26]. We
advocate for the latter procedure.

Finally, numerical results are presented in section \( \S 4 \). In symmetric matter, the dispersion
relation factors in several subsystems. First there is the \( \sigma-\omega \) sector, with a transverse \( \omega \) mode
and a longitudinal mixed \( \sigma-\omega \) mode. Since this mode has already been studied extensively in the
litterature, no numerical results are shown here. Then we have a \( \delta-\rho-\pi a_1 \) sector, which is composed
of three parts: a longitudinal mixed \( \delta-\rho \) mode, a transverse mixed \( \rho-a_1 \) mode and a mixed \( \pi-a_1 \)
mode. Dispersion relations are calculated for these modes and the behaviour of the effective masses
of the mesons is investigated. We mention the problem of vacuum fluctuations and of the choice
of a renormalization procedure. As already noticed in the case of the \( \rho \) meson [3], different results
are obtained depending of the choice of the renormalization conditions. At the time of writing, we
must regard this as an unresolved issue.

Among the possible applications of the present results, one can quote the calculation of screened
nucleon-nucleon potentials [2, 27] or cross sections [28, 29] and their application to nuclear dynamics
[30], dilepton production in relativistic heavy ion collisions [14], the calculation of the electronic re-
sponse function [13] or of the compressional modes of nuclear [12, 31, 32]. Another field of application
concerns the physics of supernovae explosions and protoneutron star cooling. In conditions of high
density and temperature reached in the early phases of neutron star formation, it has recently been
argued that RPA corrections to the neutrino-nucleon scattering cross section could be responsible
for a sizeable modification of the neutrino mean free path [24, 25, 26]. The results obtained in this
work may be applied in the limiting case of pure neutron matter, by appropriately modifying the
degeneracy factor. In astrophysical applications nevertheless, one has to deal with an asymmetric
environment. The dispersion relation of neutral mesons in asymmetric nuclear matter is presented
in a companion paper [35]. The case of propagation of charged mesons in asymmetric matter is
currently under consideration.

## 2 Meson exchange model

### 2.1 Discussion about the mesons included in this work

The \( \sigma \), \( \omega \), \( \rho \) and \( \pi \) mesons are standard mesons which are included in the description of the NN interaction
in hadronic models, whose prototype is the Quantum Hadrodynamics (QHD) developped
by Walecka and coworkers [36]. A vast amount of literature has been devoted to study their dispersion relations \([1] - [6], [37] - [44]\). This section presents a discussion on the relevance of extending
the approach to study more mesons.

The \( \delta \) and \( \eta \) mesons belong to the set of mesons exchanged in the Bonn potential [1]. However,
they have been less studied than the previous ones. One reason is that the authors of [1] found that
they only bring a small adjustment to the form of the NN potential. In mean field studies of nuclear
matter, the $\delta$ is usually not considered on the argument that the exchange of $\rho$ mesons is enough to reproduce the asymmetry energy at the mean field level (see however \[45, 47\]). In asymmetric matter, on the other hand, the delta meson could be important, since it carries isospin. Interest has been shown for the $\delta$ meson in the context of Dirac-Brueckner calculations of the equation of state in asymmetric matter. When developing equivalent mean field theories with density dependent couplings, in order to reproduce the results of the full many-body calculation, de Jong and Lenske \[46\] or Shen et al. \[47\] have found that it was necessary to introduce the $\delta$ meson with a significant coupling strength, e.g. $g^2_{\delta}(m_{\text{sat}})//(4\pi) = 4.61$ at saturation density.

The $\delta$ meson mixes with the $\rho$ meson, just as does the $\sigma$ with the $\omega$ in the isoscalar sector. It is well known that $\sigma$-$\omega$ mixing is very strong (see e.e \[2, 43\]), so that one may ask whether the same occurs in the case of $\delta$-$\rho$ mixing. In relation with the debated case of the interpretation of dilepton measurements in heavy ion collisions, the effect of $\delta$-$\rho$ mixing has been studied by Teoredescu et al. \[44\].

The $a_1$ is very massive ($m_a=1260$ MeV), and is for this reason usually not included in meson exchange models, which take 1 GeV as a reasonable cutoff energy scale. There are however theoretical reasons to consider this meson. As mentioned in the introduction, there is a need of keeping this meson from symmetry arguments since the $a_1$ is the chiral partner of the $\rho$.  \[7 - 15\].

For the purpose of reproducing the dilepton measurements of CERES and HELIOS, an enormous amount of work has been spent on calculating the $\rho$ spectral function, as this meson yields the dominant contribution to dilepton production in the vector dominance model. The controversy arised, as to whether the dilepton enhancement at lower invariant mass was due to a reduction of the $\rho$ meson mass in the medium and a signal of chiral restoration in the Brown-Rho scaling picture \[48\] (B/R scenario), or could it be explained by purely hadronic models as an increased width and shifted strength due $\rho\pi\pi$ coupling \[19\] (R/W scenario). It was suggested \[10, 50\] that one may expect a substantial contribution to the width from $\rho\pi a_1$ loops.

An other ground to study the $a_1$ meson is to determine whether it could simulate short range corrections in the pion exchange potential \[21, 22\]. As a matter of fact, Fourier transforming the expression obtained for the pion exchange contribution to the transition matrix yields a $\delta(r)$ piece in the NN potential in coordinate space. The standard procedure is to introduce the so-called Landau-Migdal parameter $g'$ as a contact interaction to remove this piece. This term is necessary to achieve a good description of the proton-neutron cross section at large scattering angles $\theta \simeq 180^\circ$ \[51, 22\]. The parameter $g'$ then also enters in the expression of the pion polarization and further has the nice property of removing spurious zero-sound modes from the pion dispersion relation, which would be responsible for the onset of pion condensation at unrealistically low densities \[33\].

In a relativistic formalism, it is less clear how to implement this term in a simple fashion. For example, Horowitz et al. \[22, 23\] have suggested an Ansatz making a replacement in the pion propagator. An other possibility would be to add to the Lagrangian a contact term with pseudovector coupling. This is the method used by Schäfer et al. \[54\], who add to their Lagrangian a term $g_a(\bar{\psi}\gamma_5\gamma^\mu\tau\psi)(\bar{\psi}\gamma_5\gamma^\mu\tau\psi)$ in order to fit the NN cross section at large scattering angles. Equivalently Engel et al. \[21\] suggest to add to the Lagrangian a piece $g_A\bar{\psi}\gamma_5\gamma^\mu\tau a_A$ and take the limit of infinitely heavy $a^\mu$ field. In section 3, we compare the effects of the Ans"atze of Horowitz et al. or of Schäfer et al on the one hand, and of true $a_1$ exchange on the other hand, on the dispersion relation of the $\pi$ and $\rho$ mesons. It will be seen that the mixing of the $a_1$ meson with the pion acts as a Landau-Migdal term with the “wrong” sign.

2.2 Outline of derivation

We will take the following Lagrangian density

$$\mathcal{L} = \mathcal{L}_{NN} + \mathcal{L}_{N\Phi} + \mathcal{L}_{\Phi\Phi} + \mathcal{L}_{\Phi_1\Phi_2} + \mathcal{L}_{\text{CT}}$$

(1)
\[ L_{NN} = \bar{\psi} \left[ \frac{i}{2} \gamma \cdot \slashed{D} - m \right] \psi \quad ; \quad L_{N\Phi} = \bar{\psi} \Phi(x) \psi \]  

with \( \Phi(x) = g_\sigma \sigma + g_\rho \vec{\rho} \cdot \vec{\sigma} - 2 m_\rho \gamma^\mu \omega_{\rho \mu} - \frac{f_\pi}{m_\pi} \gamma^\mu \gamma_5 \partial_{\mu} \vec{\pi} \cdot \vec{\tau} - ig_\eta \gamma_5 \eta \)  

\[ L_{\Phi\Phi} = \frac{1}{2} \partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} \partial^\mu \vec{\delta} \partial_\mu \vec{\delta} - \frac{1}{2} m_\rho^2 \vec{\omega}_\rho \cdot \vec{\sigma} + \frac{1}{2} \partial^\mu \vec{\pi} \partial_\mu \vec{\pi} - \frac{1}{2} m_\pi^2 \vec{\pi} \cdot \vec{\pi} + \frac{1}{2} \partial^\mu \vec{\eta} \partial_\mu \vec{\eta} - \frac{1}{2} m_\eta^2 \vec{\eta} \cdot \vec{\eta} \]

\[ L_{\Phi_1\Phi_2} \geq - \frac{1}{3} \beta m \sigma^3 - \frac{1}{4} c \sigma^4 - \frac{1}{2} g_{\sigma\pi} m_\sigma \sigma \pi^2 + g_{\rho\pi} \vec{\rho} \cdot (\partial \vec{\pi} \times \vec{\pi}) + \frac{1}{4} g_{\rho\pi} (\vec{\rho} \times \vec{\pi}) \cdot (\vec{\rho} \times \vec{\pi}) \]

The first two lines are the Lagrangian for the nucleon and its interaction with the meson fields. We chose the pseudovector coupling for the pion since its phenomenology is better, namely it permits to reproduce the pion-nucleon scattering. The pseudoscalar coupling will also be considered in section 3.4.

The following two lines contain the kinetic terms for the mesons. The last two lines contain some meson-meson interactions which are important from the point of view of phenomenology. In particular, we may have \( \sigma \pi \pi \), \( \rho \pi \pi \) and \( \rho a_1 \) couplings, which contribute to the spectral width of the \( \sigma \), \( \rho \) and \( a_1 \) mesons. The sigma self couplings \( b m \sigma^3 + c \sigma^4 / 4 \) improve the description of the effective nucleon mass and incompressibility modulus.

One could consider more complicated forms of the Lagrangian, which would be obtained by enforcing chiral symmetry \([\text{see } 11]\). Accordingly, more meson-meson coupling terms would appear in \( L_{\Phi_1\Phi_2} \). Since our purpose here was mainly to investigate NN loops, we will not consider such terms in this work. In fact, as will be seen in the following, the derivation method based on linear response analysis, which we follow here, discards meson-meson correlations at an early stage of the calculation, so that terms involving meson couplings only appear at the mean field level. Since we deal in this paper with symmetric matter, most of these terms actually disappear, since they would involve expectation values of the pion, rho or \( a_1 \) fields, all of which vanish in this approximation. Only terms involving the \( \sigma \) field will survive. Meson loops should of course appear in a more realistic treatment.

The term \( L_{CT} \) represents a counterterm Lagrangian in order to handle the vacuum divergences; its form is given in Appendix A.3.

The dispersion relations are derived from a linear response analysis in the Wigner operator formalism. By applying a perturbation to the relativistic Hartree equilibrium, one obtains homogeneous equations for the perturbed meson fields \( \phi_1(q) \) in the form \( D(q) \phi_1(q) = 0 \), where \( \phi_1(q) \) is a vector formed by the components of the perturbations to the various fields \( \sigma, \omega, \pi, \rho, \delta, \eta \) and \( a_1 \), and \( D(q) \) is a matrix which contains the dispersion relations. The condition that these homogeneous equations admit non trivial solutions is that the determinant of \( D(q) \) vanishes. As \( D(q) \) is in general non diagonal, the determinant will only partially factorize. A standard example is the case of the \( \sigma-\omega \) subsystem, where the \( \omega \) transverse mode factorizes out, while the longitudinal mode of the \( \omega \) mixes with the \( \sigma \) mode \([\text{see } 2]\). It was shown in this work that the results obtained coincide with the one-loop approximation from Green's function formalism.

We define the Wigner operator

\[ F_{\alpha\beta}(x, p) = \int \frac{d^4 R}{(2\pi)^4} e^{-ip \cdot R} \bar{\psi}_\beta(x + \frac{R}{2}) \otimes \psi_\alpha(x - \frac{R}{2}) \]  

(5)
It obeys the kinetic equation (as well as a conjugate equation)

\[
\left[ i\gamma^\mu \partial_\mu + (\gamma^\nu p - m) \right] F(x, p) = -\int \frac{d^4 R}{(2\pi)^4} d^4 \xi \ e^{-i(p-\xi) \cdot R} \Phi(x - \frac{R}{2}) F(x, \xi)
\]

((6))

with \(\Phi(x)\) defined as in Eq. (3). At this level, \(F(x, p)\) as well as the fields \(\sigma(x), \omega(x), \pi(x), \delta(x) ...\) etc contained in \(\Phi\) are operators. In order to obtain the RPA approximation, two basic assumptions are made:

— Correlations are neglected. When taking the statistical average of Eq. (8), it amounts to replacing in the right hand side the average of the product \(<\Phi F>\) by the product of averages \(<\Phi><F>\). It is at this point that we neglect the contribution of meson loops to the dispersion relations. They could be restored by releasing this assumption, or be reintroduced by hand at the end of the calculation. We will not consider them in this work. We will omit the \(<>\) denoting statistical averages henceforward in order to simplify the notations.

— It is assumed that there exists an uniform, unpolarized equilibrium given by the relativistic Hartree approximation, and that one may perform an expansion around this equilibrium (remember the notations \(F, \sigma \ldots\) represent from now on the statistical averages \(<F>, <\sigma> ...\))

\[
\begin{align*}
F^\mu(x, p) &= F_H(p) + F_1(x, p) , \quad \sigma(x) = \sigma_H + \sigma_1(x), \quad \delta(x) = \delta_H + \delta_1(x) \\
\omega^\mu(x) &= \omega_H^\mu + \omega_1^\mu(x) , \quad \rho^\mu(x) = \rho_H^\mu + \rho_1^\mu(x) \\
\bar{\pi}(x) &= \bar{\pi}_1(x), \quad \eta(x) = \eta_1(x) , \quad \bar{\sigma}^\mu(x) = \bar{\sigma}_1^\mu(x)
\end{align*}
\]

((7))

In eqs. (8) the pion, eta and \(a_1\) contributions vanish due to parity arguments in unpolarized matter. Moreover, in symmetric matter the chemical potentials of the proton and neutron coincide, so that \(\rho_H\) and \(\delta_H\) will vanish as well, since they are proportional to the difference between the proton and neutron density, and between the proton and neutron effective masses respectively. (We do not take into account the tiny difference due to the electromagnetic interaction.)

With these approximations, we obtain after linearizing and Fourier transforming Eq. (3) the first order perturbation to the nucleon Wigner function

\[
F_1(q, p) = G \left( p - \frac{q}{2} \right) \Phi(q) F_H \left( p + \frac{q}{2} \right) \right) + F_H \left( p - \frac{q}{2} \right) \Phi(q) G \left( p + \frac{q}{2} \right)
\]

((8))

with

\[
\begin{align*}
\Phi(q) &= -g_\sigma \sigma_1(q) + g_\omega \gamma^\mu \omega_1^\mu(q) - g_\rho \rho_1(q) \cdot \bar{\tau} + \frac{i f_\pi}{m_\pi} \gamma_5 \gamma^\mu q_\mu \bar{\pi}_1(q) \cdot \bar{\tau}
+ \frac{f_\rho}{2m} \sigma_{\mu\nu} q_\mu \rho_1(q) \cdot \bar{\tau} + g_a \gamma_5 \gamma^\mu \bar{\sigma}_1^\mu(q) \cdot \bar{\tau}
\end{align*}
\]

((9))

\[
G(p) = \frac{\gamma P + M}{P^2 - M^2 \pm i\epsilon}
\]

((10))

\[
F_H(p) = S(p) \varphi(p)
\]

((11))

with

\[
\varphi(p) = \frac{1}{(2\pi)^3} \delta(P^2 - M^2) \left[ \theta(p_0) m(p) + \theta(-p_0) \bar{m}(p) - \theta(-p_0) \right]
\]

In symmetric nuclear matter, \(d = 2\) is the isospin degeneracy. \(n(p)\) and \(\bar{n}(p)\) are the Fermi-Dirac distribution functions for the (quasi)-particles and antiparticles respectively. \(M = m - g_\sigma \sigma_H\) is the effective mass and \(P^\mu = p^\mu - g_\omega \omega_H^\mu\) is the effective momentum. Finally, we insert this solution in the linearized equations of the mesons. For example, we have for the \(a_1\) meson

\[
\left[ q^\mu q^\nu + (q^2 + m_{a_1}^2) q_{\mu\nu} \right] \bar{a}_1(q) = -g_{a\rho\pi} \bar{\pi}_1(q) \times \bar{\rho}_H + g_a \int d^4 p Tr[\gamma_5 \gamma^\mu \bar{\tau} F_1(q, p)]
\]

((12))
Since we assume in this work that we are in symmetric matter, the Hartree component of the $\rho$ field vanishes. Inserting (8) in this expression, we can recast it in the form

$$\left[-q^\mu q^\nu + (q^2 - m_a^2)g^{\mu\nu}\right] \bar{a}_{1\nu}(q) = \Pi^\mu_{a\pi}(q) \bar{\pi}_1(q) + \Pi^\mu_{a\rho}(q) \bar{\rho}_1(q) + \Pi^\mu_{a\sigma}(q) \bar{a}_1(q)$$

where, for example

$$\Pi^\mu_{a\rho} = \int d^4p \text{Tr} \left[ g_a\gamma_5\gamma^\mu\pi_i S(p - \frac{q}{2}) \left( g_\rho\gamma^\nu + \frac{f_\rho}{2m_\rho} q^\nu q_\lambda \right) \tau_j S(p + \frac{q}{2}) \right] \left\{ \varphi(p + \frac{q}{2}) - \varphi(p - \frac{q}{2}) \right\} \left\{ \frac{\varphi(p + \frac{q}{2}) - \varphi(p - \frac{q}{2})}{2p.q} - i\epsilon \right\}$$

Terms such as $\Pi_{a\omega}$ vanish in symmetric matter since the trace over isospin will involve the difference of proton and neutron distribution functions. The dispersion relations may be summarized in matrixial form $D(q)\phi(q) = 0$. In symmetric matter the dispersion relation decouples in three blocks: $(\sigma, \omega)$, $(\eta)$ and $(\rho, \delta, a_1, \pi)$:

$$D(q) = \begin{pmatrix}
D_{\sigma\sigma} & D_{\sigma\omega}^\nu & 0 & 0 & 0 & 0 \\
D_{\omega\sigma} & D_{\omega\omega}^\nu & 0 & 0 & 0 & 0 \\
0 & 0 & D_{\eta\eta} & 0 & 0 & 0 \\
0 & 0 & 0 & D_{\delta\delta} & D_{\delta\rho} & D_{\delta\omega} \\
0 & 0 & 0 & D_{\rho\rho} & D_{\rho\rho}^\nu & 0 \\
0 & 0 & 0 & 0 & D_{a\sigma} & D_{a\rho} \\
0 & 0 & 0 & 0 & 0 & -D_{a\pi}^{\mu\nu} & D_{a\pi}^{\mu\nu}
\end{pmatrix} \quad \phi(q) = \begin{pmatrix}
\sigma_1 \\
\omega_1 \nu \\
\eta_1 \\
\delta_1 \\
\rho_1 \nu \\
a_1 \nu \\
\pi_1
\end{pmatrix}$$

with

$$D_{\sigma\sigma} = q^2 - m_\sigma^2 + \Pi_{\sigma\sigma} \quad ; \quad D_{\sigma\omega}^\nu = q^\mu q^\nu + (m_\omega^2 - q^2)g^{\mu\nu} + \Pi_{\omega\omega}^{\mu\nu} \quad ; \quad D_{\omega\sigma} = \Pi_{\omega\sigma}^{\mu\nu}$$

$$D_{\eta\eta} = q^2 - m_\eta^2 + \Pi_{\eta\eta} \quad ; \quad D_{\delta\delta} = q^2 - m_\delta^2 + \Pi_{\delta\delta}$$

In the preceding equations, the meson self couplings were not taken into account. If they are present, they appear as mean field modifications. The dispersion relations in this case are given by making the following replacements for the $\sigma$ and $\pi$ masses

$$m_\sigma^2 \to M_\sigma^2 = m_\sigma^2 + 2bm_\sigma\sigma_H + 3c\sigma_H^2 \quad , \quad m_\pi^2 \to M_\pi^2 = m_\pi^2 + g_{\sigma\pi} m_\sigma \sigma_H$$

We will decompose the polarizations on the usual orthogonal set of tensors and vectors as follows

$$\Pi_{\omega\omega}^{\mu\nu} = -\Pi_{\omega L} L^{\mu\nu} - \Pi_{\omega T} T^{\mu\nu} \quad ; \quad \Pi_{\omega\sigma}^{\mu\nu} = \Pi_{\sigma\omega}^{\mu\nu}$$

$$\Pi_{\rho\rho}^{\mu\nu} = -\Pi_{\rho L} L^{\mu\nu} - \Pi_{\rho T} T^{\mu\nu} \quad ; \quad \Pi_{\delta\rho}^{\mu\nu} = \Pi_{\rho\delta}^{\mu\nu}$$

$$\Pi_{a\sigma}^{\mu\nu} = -\Pi_{a L} L^{\mu\nu} - \Pi_{a T} T^{\mu\nu} - \Pi_{a Q} Q^{\mu\nu} \quad ; \quad \Pi_{a\rho}^{\mu\nu} = i \Pi_{a\rho}^{\mu\nu} q_\alpha \eta_\beta$$

with

$$\eta^\mu = u^\mu - \frac{q.\mu}{q^2} q^\mu$$

$$L^{\mu\nu} = \frac{\eta^\mu \eta^\nu}{\eta^2} \quad ; \quad T^{\mu\nu} = g^{\mu\nu} - \frac{\eta^\mu \eta^\nu}{\eta^2} - \frac{q^\mu q^\nu}{q^2}$$

$$Q^{\mu\nu} = \frac{q^\mu q^\nu}{q^2} \quad ; \quad E^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta} q_\alpha \eta_\beta$$
The explicit expression of the polarizations can be found in the Appendix.

We obtain the dispersion relations by equating the determinant of the matrix $D$ to zero

\[
\text{Det}(D) = m_\rho^2 m_\omega^2 \left[ (q^2 - m_\omega^2 + \Pi_{\omega L})(q^2 - m_\sigma^2 + \Pi_\sigma) + \eta^2 \Pi_{\sigma \omega} \right] \times \left[ q^2 - m_\rho^2 + \Pi_{\rho T} \right]^2 \times
\]

\[
\times \left[ q^2 - m_\sigma^2 + \Pi_{\sigma L} \right] \times \left( m_\rho^2 - \Pi_{\rho Q} \right) \times \left[ q^2 - m_\pi^2 + \Pi_\pi - \frac{q^2 \Pi_{\alpha \pi}^2}{m_\rho^2 - \Pi_{\rho Q}} \right] \times
\]

\[
\times \left[ (q^2 - m_\rho^2 + \Pi_{\rho T})(q^2 - m_\rho^2 + \Pi_{\omega T}) + \eta^2 q^2 \Pi_{\rho \omega}^2 \right]^2 \times
\]

\[
\times \left[ (q^2 - m_\rho^2 + \Pi_{\rho L})(q^2 - m_\delta^2 + \Pi_\delta) + \eta^2 \Pi_{\sigma \rho}^2 \right] \times \left[ q^2 - m_\eta^2 + \Pi_\eta \right] = 0 \quad (18)
\]

### 2.3 Propagators for the $\sigma$-$\omega$-$\pi$-$\rho$-$\delta$-$\eta$-$a_1$ model

The propagators $G$ are obtained by inverting the dispersion matrix $D$.

The $\sigma$-$\omega$ sector has already been studied thoroughly (see e.g. [2, 3, 38, 40, 41, 43]). We obtain as usual

\[
G_\sigma = \frac{1}{q^2 - m_\sigma^2 + \Pi_\sigma} \quad (19)
\]

\[
G_\omega^{\mu \nu} = -G_{\omega T} T^{\mu \nu} - G_{\omega L} L^{\mu \nu} - G_{\omega Q} Q^{\mu \nu} \quad (20)
\]

with

\[
G_{\omega L} = \frac{q^2 - m_\sigma^2 + \Pi_\sigma}{(q^2 - m_\omega^2 + \Pi_{\omega L})(q^2 - m_\sigma^2 + \Pi_\sigma) + \eta^2 \Pi_{\sigma \omega}^2} \quad (21)
\]

\[
G_{\omega T} = \frac{1}{q^2 - m_\omega^2 + \Pi_{\omega T}} \quad (22)
\]

\[
G_{\omega Q} = \frac{-1}{m_\omega^2} \quad (23)
\]

\[
G_{\sigma \omega}^{\mu} = \frac{-\Pi_{\sigma \omega} \eta^\mu}{(q^2 - m_\omega^2 + \Pi_{\omega L})(q^2 - m_\delta^2 + \Pi_\delta) + \eta^2 \Pi_{\sigma \rho}^2} \quad (24)
\]

The $\sigma$ meson mixes with the longitudinal part of the $\omega$ meson. The mixing effect is known to be strong. A zero sound mode may appear in the longitudinal mode, with a strength which depends on the values of the cutoff parameters in the nucleon form factors [3, 43].

The $\eta$ meson decouples from the other ones in symmetric matter, therefore the calculation of its propagator is trivial. We have:

\[
G_\eta = \frac{1}{q^2 - m_\eta^2 + \Pi_\eta} \quad (25)
\]

If we had not taken into account the $a_1$ meson, the $\rho$-$\delta$ system would decouple and reproduce the pattern of the $\sigma$-$\omega$ in the isovector sector. Now the $\rho$ also couples to the $a_1$ through the polarization $\Pi_{\rho \rho}$. Nevertheless, the calculation shows that the dispersion relation of the $(\rho$-$\delta$-$a_1$-$\pi)$ subsystem factorizes in three terms: a mode where the $\delta$ meson mixes with the longitudinal part of the $\rho$, a mode where the transverse contribution from the $a_1$ mixes with the transverse part of the $\rho$, and a mode where the pion dispersion relation is modified by a term coming from the part of the $a_1$ meson which is parallel to $q^a q^b$. We have

\[
G_{\delta} = \frac{q^2 - m_\rho^2 + \Pi_{\rho L}}{(q^2 - m_\rho^2 + \Pi_{\rho L})(q^2 - m_\delta^2 + \Pi_\delta) + \eta^2 \Pi_{\rho \delta}^2} \quad (26)
\]

\[
G_{\rho \rho}^{\mu \nu} = -G_{\rho T} T^{\mu \nu} - G_{\rho L} L^{\mu \nu} - G_{\rho Q} Q^{\mu \nu} \quad (27)
\]

with

\[
G_{\rho L} = \frac{q^2 - m_\delta^2 + \Pi_\delta}{(q^2 - m_\rho^2 + \Pi_{\rho L})(q^2 - m_\delta^2 + \Pi_\delta) + \eta^2 \Pi_{\rho \delta}^2} \quad (28)
\]
exactly the Landau-Migdal form in the relativistic case with the replacement

\[ G_{\rho T} = \frac{(q^2 - m_\rho^2 + \Pi_{\rho T})(q^2 - m_\rho^2 + \Pi_{\rho T}) + \eta^2 q^2 \Pi_{\rho a}^2}{(q^2 - m_\rho^2 + \Pi_{\rho L})(q^2 - m_\rho^2 + \Pi_{\rho L}) + \eta^2 \Pi_{\rho a}^2} \]  

(29)

\[ G_{\rho Q} = -\frac{1}{m_\rho} \]  

(30)

\[ G_{a T}^{\mu} = \frac{\Pi_{a T} \eta^\mu}{(q^2 - m_\rho^2 + \Pi_{a T})(q^2 - m_\rho^2 + \Pi_{a T}) + \eta^2 \Pi_{\rho a}^2} \]  

(31)

\[ G_{a L}^{\mu} = \frac{-1}{q^2 - m_a^2 + \Pi_{a L}} \]  

(32)

\[ G_{a Q}^{\mu} = \frac{1}{(m_a^2 - \Pi_{a Q})(q^2 - m_\rho^2 + \Pi_{a T}) + \eta^2 q^2 \Pi_{\rho a}^2} \]  

(33)

\[ G_{a T} = \frac{(q^2 - m_\rho^2 + \Pi_{a T})}{(q^2 - m_\rho^2 + \Pi_{a T})(q^2 - m_a^2 + \Pi_{a T}) + \eta^2 q^2 \Pi_{\rho a}^2} \]  

(34)

\[ G_{a Q} = \frac{-1}{(m_a^2 - \Pi_{a Q})(q^2 - m_\rho^2 + \Pi_{a T}) + \eta^2 q^2 \Pi_{\rho a}^2} \]  

(35)

\[ G_{a T} = \frac{1}{q^2 - m_a^2 + \Pi_{a T}} ; \quad \Pi_{a T} = \Pi_{\rho a} - \frac{q^2 \Pi_{\rho a}^2}{m_a^2 - \Pi_{a Q}} \]  

(36)

\[ G_{a \pi}^{\mu} = -i G_{a T} q^\mu \]  

(37)

\[ G_{a \rho}^{\mu} = \frac{-i \Pi_{a \rho} \epsilon^{\mu \nu \rho \lambda} q_\nu \eta_\lambda}{(q^2 - m_\rho^2 + \Pi_{a T})(q^2 - m_a^2 + \Pi_{a T}) + \eta^2 q^2 \Pi_{\rho a}^2} \]  

(38)

The mixing with the \( a_1 \) meson acts as a modification to the pion polarization. Using the relations existing between \( \Pi_{\rho a} \), \( \Pi_{a T} \) and \( \Pi_{a Q} \) (see Appendix A), it can be recast into the form

\[ \Pi_{\rho a} = \frac{q^2 \Pi_{\rho a}}{q^2 + \frac{g_a^2}{m_a^2} \Pi_{a T}} \]  

(39)

It therefore comes out in a form similar as would a Landau-Migdal interaction. We would obtain exactly the Landau-Migdal form in the relativistic case with the replacement

\[ \frac{g_a^2}{m_a^2} \to -q^2 \frac{f_{\pi}}{m_{\pi}} \]  

(40)

We must note however the sign in the previous equation. One would need a fictitious \( a_1 \) field with an imaginary coupling so that equation (40) may be fulfilled. Contrary to some expectations raised in the literature [20], the \( a_1 \) meson cannot be used to improve the short range behaviour of the pion-nucleon interaction; instead it contributes an additional term which must be compensated by e.g. a residual contact interaction.

Also when calculating the potential generated by the exchange of \( \pi \) and \( a_1 \) mesons in vacuum, one obtains after taking the semiclassical limit \( k/m \ll 1 \) and also \( k/m_a \ll 1 \)

\[ V_{\pi a}^{\pi a} = -\left( \frac{f_{\pi}}{m_\pi} \right)^2 \frac{1}{3} \left[ \bar{\sigma}_1 \bar{\sigma}_2 - \frac{m_{\pi}^2}{k^2 + m_{\pi}^2} \bar{\sigma}_1 \bar{\sigma}_2 + \frac{S_{12}}{k^2 + m_{\pi}^2} + \frac{g_a^2}{m_a^2} \right][ -\bar{\sigma}_1 \bar{\sigma}_2 + \mathcal{O}(k^2)] \]  

(41)

and the term responsible for the \( \delta(\vec{r}) \) singularity \( \bar{\sigma}_1 \bar{\sigma}_2 \) after Fourier transforming to position space would be removed by the same choice \( \frac{g_a^2}{m_a^2} \to -(1/3)(f_{\pi}^2/m_{\pi}^2). \)
3 Short range behavior

As is well known, and also will be seen in section 4.4, the dispersion relations derived in the preceding paragraph would lead to an excessive softening of the pion mode and to pion condensation, which is not observed experimentally. This is due to the fact that we have not yet taken into account the effect of short range corrections at this level of approximation. A related shortcoming of the pion exchange model is that it gives a vanishing differential cross section at scattering angle \( \theta = 180^\circ \) in the neutron-proton exchange reaction, in glaring contradiction with the experiment. Both features are due to the fact that the contribution to the NN potential arising from pion exchange \( V_\pi(r) \) contain a piece \( \delta(\vec{r}) \) singular at the origin. This is an artefact of the model due to the assumption that the particles are pointlike. Such singular pieces in fact appear for all kinds of meson exchange.

Whereas it is enough for the \( \sigma, \omega \) terms entering the definition of the central potential to smooth this divergence by folding it with a form factor of the type

\[
g_\alpha^2 \to g_\alpha^2 \left( \frac{\Lambda_\alpha^2 - m_\alpha^2}{\Lambda_\alpha^2 - q^2} \right) \quad \alpha \in \{\sigma, \omega, \pi, \rho, \ldots\},
\]

it is necessary in the case of the pion to remove the \( \delta(\vec{r}) \) function in order to arrive at a correct description of the NN cross cross section and spin transfer observables [51].

Short range corrections of the desired type would arise in a many body calculation from higher order correlations at the \( \pi NN \) vertex [56]. In a simpler approach, the usual practice is to Fourier transform the potential to coordinate space and there remove the delta function by hand. This is equivalent to adding to the potential the Migdal contact term \( g' \sigma_1 \sigma_2 \tau_1 \tau_2 \). The delta subtraction procedure for the pion alone amounts to taking \( g' = 0.333 \). A further contribution comes from the rho. Phenomenology favors higher values \( g' = 0.5 - 0.9 \). A recent analysis [57] extracted \( g' = 0.6 \) from data on the Gamow Teller resonance. This procedure works well in the non-relativistic limit [58] but has the disadvantage of not being covariant.

In the following we examine two procedures which have been suggested in the literature to introduce the Landau-Migdal short range correction in a covariant way.

3.1 Propagator replacement Ansatz

Horowitz et al [22, 23] suggest to replace the vertex and propagator of the pion as follows:

\[
\begin{align*}
\Gamma_\pi &= \frac{\gamma_5 \gamma^\mu q_\mu}{q^2 - m_\pi^2} \\
G_\pi^0 &= \frac{\gamma_5 \gamma^\mu q_\mu}{q^2 - m_\pi^2} \rightarrow \left\{ \begin{array}{l}
\Gamma_\pi^0 = \frac{q^\mu q^\nu}{q^2 - m_\pi^2} - g' g^{\mu\nu} \\
G_\pi^{0,\mu\nu} = \frac{q^\mu q^\nu}{q^2 - m_\pi^2} + \frac{\Pi^{\mu\nu}_{\pi\rho}}{g'[q^2 - g'(q^2 - m_\pi^2)]} - g' g^{\mu\nu} + \Pi^{\mu\nu}_{\pi\pi}
\end{array} \right. (42)
\end{align*}
\]

In the medium, the pi and rho propagators obey the coupled Dyson equation \( G = G^0 + G^0 \Pi G \) or equivalently, \( [G]^{-1} = [G_0]^{-1} - \Pi \), or explicitly

\[
[G]^{-1} = \left( \frac{q^\mu q^\nu + (m_\rho^2 - q^2) g^{\mu\nu} + \Pi^{\mu\nu}_{\pi\rho}}{\Pi^{\mu\nu}_{\pi\pi}} - g' \frac{q^\mu q^\nu}{g'[q^2 - g'(q^2 - m_\pi^2)]} - g' g^{\mu\nu} + \Pi^{\mu\nu}_{\pi\pi} \right) (43)
\]

The “pion” polarization which appears in this expression is given by

\[
\Pi^{\mu\nu}_{\pi\pi} = -\left( \frac{f_\pi}{m_\pi} \right)^2 \int d^4p Tr \left[ \gamma_5 \gamma^\mu S(p - \frac{q}{2}) \gamma_5 \gamma^\nu S(p + \frac{q}{2}) \right] \left\{ \frac{\varphi(p - \frac{q}{2}) - \varphi(p + \frac{q}{2})}{2p.q - i\epsilon} \right\} (44)
\]

\[
= -\Pi_{\pi T} T^{\mu\nu} - \Pi_{\pi L} L^{\mu\nu} - \Pi_{\pi Q} Q^{\mu\nu} (45)
\]

and the usual pseudovector pion polarization is recovered by contracting with \( q_\mu q_\nu \)

\[
\Pi^{PV} = -\Pi^{\mu\nu}_{\pi\pi} q_\mu q_\nu = -q^2 \Pi_{\pi Q} (46)
\]
This “pion” mixes with the ρ meson, due to the polarization

\[
\Pi_{\pi\rho}^{\mu\nu} = i \int d^4p \text{Tr} \left[ \left( \frac{f_\pi}{m_\pi} \gamma_5 \gamma^\mu \right) S(p - \frac{q}{2}) \left( g_\rho \gamma^\nu + \frac{f_\rho}{2m} \sigma^{\nu\lambda} q_\lambda \right) S(p + \frac{q}{2}) \right] \left\{ \frac{\varphi(p - \frac{q}{2}) - \varphi(p + \frac{q}{2})}{2p \cdot q - i\epsilon} \right\}
\]

\[
= \Pi_{\pi\rho} \epsilon^{\mu\nu\rho\lambda} q_\rho \eta_\lambda
\]

(47)

The full dispersion relation in the π-ρ sector is obtained by taking the determinant of \((43)\). We have

\[
\text{Det}(G^{-1}) = m_\rho^2 \left[ \left( \frac{1}{g'} + \Pi_{\pi\rho}T \right)(q^2 - m_\rho^2 + \Pi_{\pi\rho}L) - \eta^2 q^2 \Pi_{\pi\rho}^2 \right]^2 \times \left[ q^2 - m_\rho^2 + \Pi_{\pi\rho}L \right] \times \left[ q^2 - m_\rho^2 - \frac{q^2 \Pi_{\piQ}}{1 + g' \Pi_{\piQ}} \right] \times \frac{(1 + g' \Pi_{\piQ})}{g'(q^2 - g'(m_\rho^2 - q^2))}
\]

(48)

By inversion of \((43)\), we obtain the propagators

\[
G^{(H)\mu\nu}_\rho = -G^{(H)}_{\rho\rho} L^{\mu\nu} - G^{(H)}_{\rho\rho T} T^{\mu\nu} - G^{(H)}_{\rho\rho Q} Q^{\mu\nu}
\]

\[
G^{(H)}_{\rho\rho} = \frac{1}{q^2 - m_\rho^2 + \Pi_{\rho\rho L}}
\]

\[
G^{(H)}_{\rho\rho T} = \frac{1}{q^2 - m_\rho^2 + \Pi_{\rho\rho T} - \eta^2 q^2 \Pi_{\rho\rho T}^2}
\]

\[
G^{(H)}_{\rho\rho Q} = \frac{-1}{m_\rho^2}
\]

\[
G^{(H)\mu}_\rho = G^{(H)\mu\nu}_\rho \epsilon^{\nu\rho\lambda\lambda}
\]

\[
G^{(H)\rho\rho} = \frac{-\Pi_{\rho\rho}}{(q^2 - m_\rho^2 + \Pi_{\rho\rho T})(\frac{1}{g'} + \Pi_{\rho\rho T}) - \eta^2 q^2 \Pi_{\rho\rho T}^2}
\]

\[
G^{(H)\rho\rho T} = \frac{(q^2 - m_\rho^2 + \Pi_{\rho\rho T})(\frac{1}{g'} + \Pi_{\rho\rho T}) - \eta^2 q^2 \Pi_{\rho\rho T}^2}{(m_\rho^2 - q^2)(1 + g' \Pi_{\rho\rho Q}) + q^2 \Pi_{\rho\rho Q}}
\]

\[
G^{(H)\rho\rho Q} = \frac{g'}{(m_\rho^2 - q^2)(1 + g' \Pi_{\rho\rho Q}) + q^2 \Pi_{\rho\rho Q}}
\]

(52)

(53)

(54)

(55)

Using the relations between the polarizations \(\Pi_{\alpha\rho}, \Pi_{\pi\rho}, \Pi_{\alpha L}, \Pi_{\pi L}\), it can be checked that one can go from the equations for the \(\rho-a_1-\pi\) system to those resulting from the Ansatz of Horowitz by taking the limit \(q^2/m_\alpha^2 \to 0\) and replacing \((g_\alpha/m_\alpha)^2(m_\pi/f_\pi)^2\) by \(-g'\), so that we see again that the mixing with the \(a_1\) meson acts as a Landau-Migdal term with the opposite sign.

### 3.2 Contact term

The procedure consisting of adding a contact term to the Lagrangian is straightforward

\[
\mathcal{L} \ni \overline{\psi} \left[ \frac{i}{2} \gamma^\mu \partial_\mu \mp \frac{f_\pi}{m_\pi} \gamma_5 \gamma^\mu \partial_\mu \mp \bar{\psi} \right] \psi - g_A (\overline{\psi} \gamma_5 \gamma^\mu \partial_\mu \bar{\psi})(\overline{\psi} \gamma_5 \gamma^\mu \bar{\psi})
\]

(56)
The new term does not directly modify the field equations for the pion meson. Rather, it contributes an additional pseudovector term to the evolution equation for the Wigner function. Following the derivation method used above, after neglecting all correlations, we obtain at first order

\[
\left[ \gamma^\mu (p_\mu - \frac{q_\mu}{2}) - m \right] F_L(x, p) \cong \left[ i \frac{f_\pi}{m_\pi} \gamma_5 \gamma^\mu q_\mu \vec{n}_1(q) + 2g_A \gamma_5 \gamma^\mu \vec{F}_1(q) \right] F_H(p + \frac{q}{2})
\]

with

\[
\vec{F}_1(q) = \int d^4k \, \text{Tr} \left[ \gamma_5 \gamma^\mu \vec{F}_1(q, k) \right]
\]

Taking the trace of Eq. (8), where \( \Phi(q) \) should now be replaced by \( \Phi(q) + 2g_A \gamma_5 \gamma_\mu \vec{F}_1(q) \), with \( \gamma_5 \gamma_\mu \) and integrating, yields a self consistent equation for \( P_\nu q_\nu \)

\[
P_\nu q_\nu = i \left( \frac{f_\pi}{m_\pi} \right) \frac{q^2 \Pi_{AA}^Q}{1 - 2g_A \Pi_{AA}^Q} \vec{n}_1
\]

The polarization

\[
P_{\mu\nu}^{AA} = \int d^4p \, \text{Tr} \left[ \gamma_5 \gamma^\mu \tau G(p - \frac{q}{2}) \gamma_5 \gamma^\nu \tau F_H(p + \frac{q}{2}) + (G \leftrightarrow F_H) \right]
\]

\[
= -\Pi_{AA}^Q Q^{\mu\nu} - \Pi_{AA}^T \tau^{\mu\nu} - \Pi_{AA}^L L^{\mu\nu}
\]

has already been met before. The equation is easily solved as

\[
P_\nu q_\nu = i \left( \frac{f_\pi}{m_\pi} \right) \frac{q^2 \Pi_{AA}^Q}{1 - 2g_A \Pi_{AA}^Q} \vec{n}_1
\]

Replacing this solution in the first order term of the field equation for the pion, we obtain the dispersion relation of the pion modified by a contact term

\[
q^2 - m_\pi^2 = \frac{(f_\pi/m_\pi)^2 q^2 \Pi_{AA}^Q}{1 - 2g_A \Pi_{AA}^Q} = 0
\]

With \(- (f_\pi/m_\pi)^2 q^2 \Pi_{AA}^Q = \Pi_{\rho \nu} \), and defining \( 2g_A (m_\pi/f_\pi)^2 = g' \), we recover the standard expression for the pion pseudovector polarization modified by the Landau-Migdal \( g' \) parameter.

\[
\bar{\Pi}_\pi = - \left( \frac{f_\pi}{m_\pi} \right)^2 \frac{q^2 \Pi_{AA}^Q}{1 - 2g_A \Pi_{AA}^Q} \frac{\Pi_{\rho \nu}}{q^2 - g' \Pi_{\rho \nu}}
\]

Apart from redefining the pion polarization, the \( g_A \) contact term also mixes with the transverse part of the \( \rho \) meson. The result is given in Eq. (73). Let us nevertheless study before a more general choice of the contact interaction. The rho meson exchange potential also gives rise to a delta function in the coordinate representation of the potential (see eq. (114)). This delta can be eliminated independently from the one appearing in the pion potential if we add another contact term for the \( \rho \) meson

\[
\mathcal{L} \supset \bar{\psi} \left[ \frac{i}{2} \gamma^\mu \partial_\mu - m - g_\rho \gamma^\mu \vec{p}_\mu \cdot \vec{\tau} \right] - \frac{1}{4} R_{\mu\nu} R^{\mu\nu} + \frac{1}{2} m_\rho^2 p_\mu p^\mu - g_R (\bar{\psi} \gamma^\mu \vec{\tau} \psi) (\bar{\psi} \gamma^\mu \vec{\tau} \psi)
\]
When solving the equation for the Wigner function, there is an additional contribution to the isovector density. At first order of the linearization procedure, we have

\[
\begin{align*}
  \left[ \gamma^\mu (p_\mu - q_\mu) - m - \gamma^\nu (g_\rho \rho H_\nu + 2g_R I_{H_\mu}) \right] F_1(x, p) \\
  = \left[ g_\rho \gamma^\mu \bar{\tau} \rho_1 \mu(q) + 2g_R \gamma^\mu \bar{\tau} \rho L_1 \mu(q) \right] F_H(p + \frac{q}{2}) + \ldots
\end{align*}
\]

with

\[
\tilde{R}_H^\mu(q) = \int d^4k \, \text{Tr} \left[ \gamma^\mu \bar{\tau} F_1(q, k) \right]
\]

In symmetric matter, the mean value of the rho Hartree field and the contribution to the isospin current \( I_H^\mu = \int d^4p \, Tr[\gamma^\mu \bar{\tau} F_H] \) vanish. \( \tilde{R}_H^\mu(q) \) obeys the self-consistent equation

\[
\tilde{R}_H^\mu(q) = -g_\rho \Pi_{VV}^{\mu\nu} \bar{\rho}_1 \nu - 2g_R \Pi_{VV}^{\mu\nu} \tilde{R}_1 \nu
\]

where \( \Pi_{VV}^{\mu\nu} \) is the vector contribution to the \( \rho \) polarization \( \Pi_{\rho V}^{\mu\nu} = g_\rho^2 \Pi_{VV}^{\mu\nu} + 2g_\rho f_\rho \Pi_{TT}^{\mu\nu} + f_\rho^2 \Pi_{TT}^{\mu\nu} \). It is solved by projecting this relation on \( T^{\mu\nu} \), \( L^{\mu\nu} \) and \( Q^{\mu\nu} \). In the general case that pion, rho and \( a_1 \) fields are present, and we moreover introduce both the contact terms \( g_A(\bar{\psi} \gamma_5 \gamma_\mu \tau \psi)^2 \) and \( g_R(\bar{\psi} \gamma_\mu \tau \psi)^2 \), the various components obey coupled equations. As before, the longitudinal modes decouple, whereas the transverse mode of the \( a_1 \) is mixed with the \( \rho \) and the pion mode is modified by the part of the \( a_1 \) polarization which is parallel to \( q^\mu q^\nu \). We arrive at the dispersion relations

\[
\begin{align*}
  \left[ q^2 - m_\pi^2 - \left( \frac{f_\pi}{m_\pi} \right)^2 \right] \frac{\Pi_{A3}^Q q^2}{1 - (2g_A + (g_\rho^2/m_\rho^2)) \Pi_{A3}^Q} \bar{\rho}_1(q) &= 0 \quad (68) \\
  (q^2 - m_\rho^2 + \bar{\Pi}_{\rho T}) T^{\mu\nu} \bar{\rho}_1 \nu + \bar{\Pi}_{aT} T^{\mu\nu} \bar{a}_1 \nu &= 0 \quad (69) \\
  (q^2 - m_a^2 + \bar{\Pi}_{aT}) T^{\mu\nu} \rho_1 \nu + \bar{\Pi}_{aT} T^{\mu\nu} \bar{a}_1 \nu &= 0 \quad (70) \\
  (q^2 - m_\rho^2 + \bar{\Pi}_{\rho L}) L^{\mu\nu} \bar{\rho}_1 \nu &= 0 \quad (71) \\
  (q^2 - m_a^2 + \bar{\Pi}_{aL}) L^{\mu\nu} \bar{a}_1 \nu &= 0 \quad (72)
\end{align*}
\]

The explicit form of the polarizations corrected by the contact terms \( g_A, g_R \) is given in Appendix B. When \( g_R=0 \), the polarizations of the \( \rho \) reduce to

\[
\begin{align*}
  \bar{\Pi}_{\rho T} &= \Pi_{\rho T} + \frac{2g_A(\Pi_{AR})^2}{1 - 2g_A \Pi_{A3}^T} \\
  \bar{\Pi}_{\rho L} &= \Pi_{\rho L}
\end{align*}
\]

(73)

The equations (63, 73) are in fact the same as obtained from Horowitz propagator replacement Ansatz (see the third factor in eq. (18) and eq. (50)). One should notice however that the later Ansatz, apart from producing the desired result for the pi and transverse rho propagators, also introduces additional spurious terms in the dispersion relation (48) arising from the unphysical part of the new “pion” propagator orthogonal to \( q^\mu q^\nu \) like \( e.g. \, 1 + g' \Pi_{\pi T} \) and \( 1 + g' \Pi_{\pi L} \).

4 Dispersion relations: numerical results

In this section we study the dispersion relations which result from the models described in the previous sections. We will concentrate on the \( \delta-\rho-\pi-a_1 \) sector, since the \( \sigma-\omega \) branches have been studied extensively elsewhere.


4.1 Model parameters

When not explicitly stated otherwise, the parameters are chosen to be those of the Bonn potential [6]. They are summarized in the following table.

| meson | $m_\alpha$ [MeV] | $\frac{g^2_\alpha}{4\pi}$ | $\Lambda_\alpha$ |
|-------|------------------|-----------------|----------------|
| $\sigma$ | 550. | 8.2797 | 2000. |
| $\omega$ | 782.6 | 20 | 1500. |
| $\pi$ | 138. | 14.6 | 1300. |
| $\rho$ | 769. | 0.81 | 2000. |
| $\delta$ | 983. | 1.1075 | 2000. |
| $\eta$ | 548.8 | 5 | 1500. |

The coupling of the pion is taken in this Bonn model to be $g^2_\pi/(4\pi) = 14.6$. One usually prefers now a somewhat lower value $g^2_\pi/(4\pi) \approx 13.67$ so that $f_\pi = m_\pi g_\pi/(2m) \approx 0.965$. An other issue is the value of the cutoff, which is taken to be 1300 MeV in the Bonn model, whereas most calculations and experimental determinations of this parameter would give $\Lambda_\pi \approx 800$ MeV.

Data for the coupling of the $a_1$ meson to the nucleon is scarce. A simple realization of the chiral symmetry is $g_\alpha = g_\rho$. From early chiral models by Weinberg and Wess & Zumino [7] one obtains the relation $g_\alpha = m_\alpha (f_\pi/m_\pi)$. With $m_\alpha=1260$ MeV and $f_\pi \approx 0.965$ as determined from NN scattering data, this yields $g_\alpha \approx 8.79$. More recent implementations of the chiral model [11, 15] point out the difficulty in adjusting all known data on the width and mass of the $a_1$ and quote values ranging from 3.8 to 18 [15]. Another value used in the literature is $g_\alpha = 6.44$ [17]. Reference [59] also provides us with a rather low value for the cutoff $\Lambda_{aNN} = 809$ MeV. Here we will work with $g_\alpha = 8.8$ and a higher value of the cutoff $\Lambda_{aNN} = 2000$ MeV.

4.2 Longitudinal $\delta$-$\rho$ mode

The dispersion relations were calculated with the parameters of Machleidt’s Bonn potential [6] using three alternative renormalization schemes. As discussed e.g. in [3], where the dispersion relation of the rho was studied without mixing, the polarizations contain a diverging contribution from vacuum fluctuations which has to be renormalized. One may define several subtraction schemes, which, unfortunately, lead to very different behaviors of the mesons effective masses. In [3] two classes of renormalization procedures were identified

- a first class which renormalizes divergences of the form $M^2/\epsilon$ by subtracting a general counterterm $A_0 + A_1 \sigma + A_2 \sigma^2$, and adjust the constants so that the polarization $\Pi$ and its derivatives ($\partial \Pi/\partial \sigma$), ($\partial^2 \Pi/\partial \sigma^2$) vanish at some point. In this way, one tries to minimize the effect of introducing new couplings with the sigma field in the counterterm Lagrangian, which are not present in the original physically motivated Lagrangian. In references [2, 3], the renormalization point is chosen to be the mass shell $q^2 = m_\alpha^2$ for the polarization as well as its derivatives. In [38] Kurasawa and Suzuki subtract the polarization at $q^2 = m_\alpha^2$ but the derivatives at $q^2 = 0$. Using this procedure, the $\omega$ meson mass decreases with density, however the $\rho$ mass increases due to the tensor coupling $f_\rho$.

- a second class which preserves the structure of contributions $M^2/\epsilon = (m - g_\rho \sigma)^2/\epsilon$ by subtracting a counterterm in the form $A(m - g_\rho \sigma)^2$. $A$ is determined by setting $A = 0$ at some point (at $q^2 = m_\alpha^2$). Since we have only one parameter $A$, it is not possible to minimize the new couplings to the $\sigma$ field introduced in the counterterm Lagrangian. Using this procedure, the $\rho$ meson mass decreases with density.
The reader is referred to [5] and Appendix A.2 for further details. Once a renormalization procedure is chosen, it should be used for all mesons.

The first renormalization scheme (scheme A) used in this paper belongs to the first class for all mesons. For the ρ meson, it is the “scheme 2” described in the Appendix C of [3]. For the δ meson, it is obtained by replacing everywhere $m_\sigma$ and $g_\sigma$ by $m_\delta$ and $g_\delta$ in the expression for the vacuum polarization of the σ given in Ref. [3].

The second renormalization scheme (scheme B) belongs to the second class for all mesons. For the ρ meson, it is the “scheme 3” described in the Appendix C of [3].

A third renormalization scheme (scheme C) is used for comparison with calculations published in the literature [14]. For the ρ meson, it uses as in [14] the procedure of Shiomi and Hatsuda [12], which according to the classification given above, belongs to the second class. Moreover, the polarization obtained in this paper subtract the vacuum polarization at all $k$, so that it vanishes identically in the vacuum. In contrast, the vacuum polarization of schemes A and B does not vanish away from the mass shell. For the δ meson, it uses as in [14] the procedure of Kurasawa and Suzuki (KS) (thus belonging to the first class defined above) and then subtract the vacuum polarization at all $k$, $\Pi_{\text{vac}}(M, k) = \Pi_{\text{KS}}(M, k) - \Pi_{\text{KS}}(m, k)$.

Several parameter sets were tried besides those given in the table of [11]. In all cases, we find normal mode branches for the δ and longitudinal ρ (see Fig. 1). For the ρ, there are moreover two heavy meson branches. Contrary to the very similar σ-ω system, no zero sound mode was found.

Effective masses can be defined as usual as the solution(s) $m^*_i = \omega$ of the dispersion relation $D(\omega, \vec{k}) = 0$ at $\vec{k} = 0$. With the renormalization scheme A, the ρ meson mass increases with density and the δ meson mass also slightly increases (see Fig. 2). With renormalization scheme B, the ρ meson mass first decreases, reaches a minimum at $\sim 1.8 \rho_0$ and then slowly increases, whereas the delta meson mass increases reaches a maximum at $\sim 0.8 \rho_0$ and then decreases. The ρ and δ masses are nearly constant with temperature (see Fig. 3) with both $m^*_\rho$ and $m^*_\delta$ slightly decreasing. This figure was plotted using renormalization scheme A. In scheme C we reproduced the results of [14], with both the rho and delta masses decreasing as a function of density.

This illustrates again the difficulties met with the standard renormalization techniques, which preclude a reliable prediction of the behavior of the effective meson masses in the medium. One cannot simply drop the contribution of the vacuum term by performing a normal ordering, since the contribution left out depends on the density through the effective mass of the nucleon. Moreover, pathologies appear in the dispersion relation (kinks, no clean normal modes) if one attempts to do so [3, 5, 39]. One could in principle apply a subtraction procedure at the one-loop order, however the various schemes used in the literature [2, 38, 12] lead to widely different results. This is related to the fact that we are dealing with an effective theory which should enforce the scalings and symmetries of the underlying more fundamental theory, whereas the approximation made to the full many-body theory (here, RPA) blurs these concepts. There is some hope that one could solve the problem by applying “naturalness” and symmetry arguments [60, 61]. At the time of writing however, we must consider this a still unsolved problem.

One sometimes defines a mixing angle [12] by

$$\theta_{\delta p L} = \frac{1}{2} \arctan \left[ \frac{2 \sqrt{\eta^2} |\Pi_{\rho L}|}{m_\delta^2 - m_\rho^2 - \Pi_{\rho L} + \Pi_{\delta}} \right]$$

which is obtained from the diagonalization of the mass matrix for the mixed dispersion relation of two mesons A and B in the timelike region $q^2 > 0, \eta^2 < 0$:

$$\mathcal{M} = \begin{bmatrix} m_A^2 - \Pi_A & -\sqrt{\eta^2} \Pi_{AB} \\ -\sqrt{\eta^2} \Pi_{BA} & m_B^2 - \Pi_B \end{bmatrix}$$

$$\det [q^2 I - \mathcal{M}] = (q^2 - m_A^2 + \Pi_A)(q^2 - m_B^2 + \Pi_B) + \eta^2 \Pi_{AB}^2$$

14
The mixing angle is calculated at the solutions of the dispersion relation, there is therefore one for each branch. The mixing angle is represented for renormalization schemes A and B in Fig. 4 as a function of density at vanishing temperature for a momentum $k = 300$ MeV. Schemes B and C yield values of $\theta_{\delta \rho L}$ of the order of a few degrees. The higher values found in scheme A results in fact no so much from the strength of the mixing, but rather from the fact that the difference of the effective masses of the $\delta$ and $\rho$ happens to be smaller in this scheme.

The behavior with $k$ is non monotonous. It is represented in Fig. 5 in renormalization scheme B at $T = 0$ and $n_B = 3 n_{\text{sat}}$. We also calculated the effect of finite temperature. The mixing angle decreases with $T$ in all renormalization schemes. This might reduce the effectiveness of $\delta$-$\rho$ mixing as a mechanism invoked by the authors of [44] for dilepton production.

### 4.3 Transverse $a_1$-$\rho$ mode and longitudinal $a_1$

Here we investigate the dispersion relation of the $a_1$ meson, and in what measure does the mixing with the $a_1$ meson modify the transverse $\rho$ dispersion relation.

The dispersion relation for the longitudinal $a_1$ mode is represented on Fig. 6 with renormalization schemes A and B. Only normal branches appear.

The $a_1$-$\rho$ mixing vanishes at $\vec{k} = 0$. It therefore does not affect the effective masses, and $m_{a_1}^*$ coincides as calculated from the transverse or longitudinal dispersion relations. The $a_1$ effective mass is plotted on Fig. 7. With renormalization scheme A, it decreases slightly with increasing density. When using renormalization scheme B, a non monotonous behavior is obtained for the $a_1$ mass, which first presents a steep increase at low density, and then decreases again, in contradiction with the behavior expected from QCD sum rules [63, 64, 65]. In the case of the $a_1$ meson therefore, the renormalization scheme A seems in better agreement with the results expected from other QCD-based models, whereas we saw that for the $\rho$ meson, scheme B would have seemed preferable.

A third renormalization scheme was also tried (let us call it scheme C), using the same procedure as for the $\delta$ in [44], (that is, subtracting the vacuum at all $k$ from the expression obtained from the procedure of Kurasawa and Suzuki, see previous section and Appendix A.2). In this scheme, the effective mass of the $a_1$ decreases. The behavior of $m_{a_1}^*$ as a function of temperature was also investigated; for all renormalization schemes it is almost constant as a function of temperature.

At finite $k$, the $a_1$-$\rho$ mixing sets on. It does not appreciably affect the position of the normal branches. Nevertheless, it amplifies somewhat the (spurious) zero sound branch which may appear in the transverse part of the dispersion relation at high density. This mode corresponds to the $\rho$ meson and is due to the high value of the tensor coupling $f_\rho/g_\rho = 6.1$ of the Bonn potential. It is weaker and appears at a higher density with the vector dominance value $f_\rho/g_\rho = 3.7$ and disappears completely if $f_\rho = 0$. Note however that a high value of $f_\rho$ is also supported by QCD sum rule calculations [16]. This mode is present in both schemes a high enough density, but is stronger in scheme A. Such a mode would lead to divergences, or at least an enhancement of Friedel oscillations of the $\rho$ contribution to the screened nucleon-nucleon potential [8]. It could be eliminated if a strong cutoff is applied to the $\rho$ meson, of the order of 1200 - 1300 MeV. An other possibility is to use the contact term introduced in section §3.2, which we will need anyway for the pion. It was checked that the spurious zero sound branch appearing in the $\rho$ dispersion relation is removed at all densities by the contact term used in next section.

The strength of the $a_1$-$\rho$ mixing can be estimated by calculating the mixing angle, as explained in the previous section. In Fig. 8 it is shown at $T = 0$ and $k = 300$ MeV.

### 4.4 Pion mode

If the dispersion relation is calculated with pure pseudovector coupling, and neither mixing with the $a_1$ meson nor Ansätze for short range corrections (à la Horowitza or Landau-Migdal contact term)
are taken into account, a zero sound mode is found already at saturation density in renormalization scheme A. The short range corrections must be added since such a mode is not observed. Before we pass to examine this point, let us first make a few more observations.

Contrary to the non relativistic case, the zero sound mode is found to disappear again at higher density. Such a behavior was already noticed by Dawson and Piekarewicz [67]. The zero-sound problem is in part due to the high value of the cutoff $\Lambda$ of the Bonn potential model, whereas several studies favor a lower value $\Lambda = 800$ MeV. If we choose $g_\sigma^2/(4\pi)$ and $\Lambda = 800$ MeV, there is only a tiny zero sound branch around $k = 250$ MeV, between $n_B = 0.72 n_{sat}$ and $n_B = 1.2 n_{sat}$.

In renormalization scheme B, no zero sound mode is found when pure pseudovector coupling of the pion is used without mixing with the $a_1$. However this renormalization scheme appears less favorable when we consider the behavior of the effective pion mass. Whereas the pion mass stays approximately constant with scheme A, it strongly decreases with density when scheme B is used.

In both cases, it is possible to adjust the effective mass at saturation density as measured in recent experiments [68] by adding a small admixture of pseudoscalar coupling \( g \sigma \pi \). With this value, the pion mass can be adjusted to the value \( m_\pi (n_{sat}) = 1.1 \pm 0.03 \), with \( \alpha = 0.0735 \) in renormalization scheme A and \( \alpha = 0.118 \) in renormalization scheme B.

As was to be expected, the zero sound branches are amplified when the mixing with the $a_1$ meson is taken into account, since it modifies the pion dispersion relation formally as would a Landau-Migdal contact term. Then it is present in renormalization scheme B as well. The zero sound branch can be eliminated as before by adding a contact term. The relevant formula when a PS admixture and a $\sigma \pi^2$ term are also present is

\[
q^2 - m_\pi^2 = g_{\sigma \pi} m_\sigma \sigma + \bar{\Pi}_\pi = 0
\]

\[
\bar{\Pi}_\pi = \left[ \left( \frac{f_\pi}{m_\pi} \right)^2 \Pi_{PV} + g_\pi^2 \Pi_{PS} + 2 g_{\sigma \pi} f_\pi \frac{m_\pi}{m_\pi} \Pi_{mix} \right] - \frac{\left( g_\pi^2 \Pi_{PS} q_\mu + f_\pi \Pi_{PV} q_\mu \right)^2}{q^2 + \left( g_{\sigma \pi}^2 m_\sigma^2 - 2 g_A \Pi_{PV} \right)}
\]

\[
\Pi_{mix} = \frac{1}{2M} \Pi_{PV} , \quad \Pi_{PS} = \left( \frac{1}{2M} \right)^2 \Pi_{PV} - 8 \int d^4 p \varphi(p)
\]

\[
\Pi_{\sigma \pi} q_\mu = \Pi_{PV} , \quad \Pi_{\sigma \pi} q_\mu = \frac{1}{2M} \Pi_{PV}
\]

(For clarity we draw the coupling constants out of the definition of the polarizations in this equation.) Mixing with the $a_1$ can somewhat increase the effective pion mass. This effect is less important after including the pseudoscalar admixture.

The Horowitz Ansatz was also studied. The part of the dispersion relation which corresponds to the pion mode has the required form of a pseudovector coupling with a short range correction of the Landau-Migdal contact type. This removes the zero sound branch which appeared there for a pure pseudovector coupling. However, this Ansatz does also affect the rho meson mode through mixing with the fictitious "pion" introduced there. It is seen in dispersion relation Eq. [43] that there are also factors $1 + g \Pi_{\sigma T}$, $1 + g \Pi_{\sigma L}$. It was found that these terms give spurious branches in the spacelike region whatever renormalization scheme was used. The most favorable case was that...
of scheme A, which removes the branch due to $1 + g\Pi_{\pi L}$ entirely and part of that due $1 + g\Pi_{\pi T}$ at low density. These branches are an artefact of the unphysical components of the auxiliary pseudovectorial “pion” and should be removed carefully by hand at the end of the calculation. We therefore prefer using the contact Lagrangian of section §3.2, since this is free from such problems from the onset, moreover it permits easily to allow for a PS admixture, whereas Horowitz Ansatz can only be implemented for a pure pseudovector coupling.

5 Conclusions

This work gathers and extends results on the dispersion relations of mesons in relativistic nuclear matter at high density and finite temperature, as obtained in the RPA approximation to the quantum hadrodynamics. Besides the “standard” mesons $\sigma, \omega, \pi, \rho$, results for additional mesons $\delta, a_1$ are given. Both mix with the $\rho$ meson, the $\delta$ with the longitudinal mode and the $a_1$ with the transversal mode. They therefore may represent an additional source of modification of the dilepton production at finite density and temperature. Also, new interest arises in the $\delta$ meson in the context of the description of asymmetric matter in density dependent mean field theories [46].

Various ways of introducing short range corrections in order to eliminate unobserved zero sound modes at saturation density were examined. It is seen that the $a_1$ meson mixes with the pion so that it acts exactly as would a Landau-Migdal contact term, but with the ”wrong” sign. The Ansatz by Horowitz comes with the right sign, however it introduces spurious branches in the transverse channel which badly affect the dispersion relation in the spacelike region. A simple standard contact interaction in the Lagrangian does the job best.

It was shown that the results are affected by the way the renormalization is performed in order to regulate the high momenta divergences. Without any renormalization, there appear kink structures in the effective meson masses as a function of density, no matter what a strong cutoff is applied. The expected branches are recovered when applying a renormalization procedure. Among the possible subtraction procedures, two schemes A and B were introduced and their predictions compared. Such important results as the behavior of the effective meson masses or the presence of zero sound modes differ widely whether one or the other scheme is used. In order to be consistent, a same scheme should be used for all mesons. Schemes of the A class have been used in previous literature to study $\sigma, \omega$ and $\pi$ mesons. Scheme A yields results more in agreement with other models for the behavior of the $\pi, \sigma$ and $a_1$ masses, but would predict an increasing $\rho$ meson mass. On the other hand, schemes of the B class which give a decreasing rho meson mass, as favored by theoretical models and experimental data, would predict strongly increasing $\sigma$ and $a_1$ masses, and a rapidly dropping pion mass. This last result stands in strong disagreement with the expected behavior of a Goldstone boson and the recent experimental determination of [68].

A renormalization procedure which would respect the scalings and symmetries [60] of the underlying more fundamental theory of which the effective Lagrangian means to be a low energy approximation is clearly needed.

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Appendix A: Polarizations

There are contributions from the meson self interaction terms and from the particle hole insertions
The (retarded) particle hole insertions are given by

\[ \Pi^{AB}(p) = \int d^4p \text{Tr} \left[ \Gamma^A S(p - \frac{k}{2}) \Gamma^B S(p + \frac{k}{2}) \right] \left\{ \frac{\varphi(p + \frac{q}{2}) - \varphi(p - \frac{q}{2})}{2p.q - i \epsilon \text{sign}(p_0)} \right\} \]  

(77)

where

\[ S(p) = \gamma.P + M, \]
\[ \varphi(p) = \frac{d}{(2\pi)^3} \delta(P^2 - M^2) [\theta(p_0)n(p) + \theta(-p_0)\bar{n}(p) - \theta(-p_0)], \]

\( d = 2 \) is the isospin degeneracy, and the \( \Gamma^A \) are the vertices, respectively for the \( \sigma, \omega, \pi, \rho, a_1 \) couplings to the nucleon

\[ \Gamma^A \in \left( g_\sigma, g_\omega \gamma^\mu, ig_\pi \gamma_5, i(f_\pi/m_\pi)\gamma^\mu g_\mu, g_\rho \gamma^\mu + \frac{f_\rho}{2M} g_\mu q_\nu, g_{a_1} \gamma^\mu \gamma^\nu \right) \]  

(78)

As an example, we have

\[ \text{Tr}\left[\{g_1 \gamma_5 \gamma^\mu\} G_N(p - \frac{k}{2})\{g_1 \gamma_5 \gamma^\nu\} G_N(p + \frac{k}{2})\right] = \]
\[ 4g_1^2 \{(-p^2 - M^2 + \frac{k^2}{4})g^{\mu\nu} \frac{1}{2}k^\mu k^\nu + 2p^\mu p^\nu\} \]

(79)

\[ \text{Tr}\left[\{\rho^\mu\} - \frac{f_\rho}{2m} \gamma^a k_\alpha \right] G_N(p - \frac{k}{2})\{g_1 \gamma_5 \gamma^\nu\} G_N(p + \frac{k}{2}) = \]
\[ \text{Tr}\left[\{g_1 \gamma_5 \gamma^\mu\} G_N(p - \frac{k}{2})\{g_\rho \gamma^\nu + \frac{f_\rho}{2m} \gamma^a k_\alpha\} G_N(p + \frac{k}{2})\right] = \]
\[ 4ig_1 (g_\rho + f_\rho) e^{\mu\nu\alpha\beta} k_\alpha p_\beta \]  

(80)

In symmetric nuclear matter, the polarization matrix have the components

\[
\begin{pmatrix}
\Pi_{\pi\pi}^\nu & \Pi_{\pi\omega}^\nu & 0 & 0 & 0 & 0 \\
\Pi_{\pi\omega}^\nu & \Pi_{\pi\eta}^\nu & 0 & 0 & 0 & 0 \\
0 & 0 & \Pi_{\pi\eta}^\nu & 0 & 0 & 0 \\
0 & 0 & 0 & \Pi_{\pi\sigma}^\nu & \Pi_{\pi\rho}^\nu & \Pi_{\pi\delta}^\nu \\
0 & 0 & 0 & \Pi_{\pi\rho}^\nu & \Pi_{\pi\delta}^\nu & \Pi_{\pi\delta}^\nu \\
0 & 0 & 0 & 0 & \Pi_{a_1\rho}^\nu & \Pi_{a_1\sigma}^\nu \\
0 & 0 & 0 & 0 & 0 & -\Pi_{a_1\pi}^\nu & \Pi_{a_1\pi}^\nu
\end{pmatrix}
\]  

(81)

In asymmetric nuclear matter the mixing polarizations \( \Pi_{\omega\rho}^\nu, \Pi_{\sigma\rho}, \Pi_{\sigma\delta}, \Pi_{\delta\omega}, \Pi_{\omega a_1}^\nu \) ... would not vanish. Asymmetric matter is discussed in a companion paper [35].

The polarizations involving the \( a_1 \) meson are related to others by the following equations

\[ \Pi_{\pi\pi}^{\mu\nu} = \frac{g_1^2}{g_\omega} \Pi_{\pi\omega}^{\mu\nu} + \frac{g_\omega^2}{(f_\pi/m_\pi)^2} \frac{g_1^2}{q^2} \Pi_{\pi\nu}^{\mu\nu} \Rightarrow \Pi_{\pi Q} = -\frac{g_1^2}{(f_\pi/m_\pi)^2} \frac{1}{q^2} \Pi_{\pi\pi}^{\mu\nu} \]  

\[ \Pi_{\pi\pi}^{\mu\nu} = \frac{g_1 a}{(f_\pi/m_\pi)^2} \frac{g_1^2}{q^2} \Pi_{\pi\pi}^{\mu\nu} \]  

(82)

These relations are used to simplify the expression of the pion propagator appearing in Eq. (26)

\[ \Pi_{\pi} = \Pi_{\pi\pi} - q^2 \frac{\Pi_{\pi\pi}^2}{m_\pi^2 - \Pi_{\pi Q}} = \frac{q^2 \Pi_{\pi\pi}}{q^2 + \frac{g_1^2 a}{m_\pi^2} \frac{m_\pi^2}{f_\pi^2} \Pi_{\pi\pi}} \]  

(83)
A.1 - Polarizations, real matter part

The matter part of the polarizations is given by

\[
\Pi^{(\text{mat})}_{\sigma} = -d \frac{g_\sigma^2}{2\pi^2} \left[ 2X_0 + \left( \frac{q^4}{4} - M^2q^2 \right) T_0^\sigma \right]
\]

(84)

\[
\Pi^{(\text{mat})}_{\omega} = -d \frac{g_\sigma g_\omega}{2\pi^2} Mq^2 \left[ T_0^1 - \frac{\omega}{k} T_1^0 \right] \eta^\mu
\]

(85)

\[
\Pi^{(\text{mat})}_{\omega\nu} = -\Pi_{\omega LL}^\nu\mu - \Pi_{\omega T}^\mu\nu ;
\]

(86)

\[
\Pi^{(\text{mat})}_{\omega L} = -d \frac{g_\omega^2}{2\pi^2} \left[ T_0^1 - T_2^0 \right]
\]

(87)

\[
\Pi^{(\text{mat})}_{\omega T} = -d \frac{g_\omega^2}{4\pi^2} \left[ M^2q^2T_0^0 + (\omega^2 + k^2) \left( T_0^1 + T_2^0 \right) - 4\omega k T_1^1 \right]
\]

(88)

\[
\Pi^{(\text{mat})}_{\rho} = -d \frac{g_\rho^2}{2\pi^2} \left[ 2X_0 + \left( \frac{q^4}{4} - M^2q^2 \right) T_0^0 \right]
\]

(89)

\[
\Pi^{(\text{mat})}_{\rho\nu} = -\Pi_{\rho LL}^\nu\mu - \Pi_{\rho T}^\mu\nu ;
\]

(90)

\[
\Pi^{(\text{mat})}_{\rho L} = -d \frac{g_\rho^2}{4\pi^2} \left[ T_0^2 - T_2^0 \right] - d \left( \frac{f_\rho}{2m} \right) g_\rho \frac{M q^4}{2\pi^2} T_0^0
\]

(91)

\[
\Pi^{(\text{mat})}_{\rho T} = -d \frac{g_\rho^2}{4\pi^2} \left[ M^2q^2T_0^0 + (\omega^2 + k^2) \left( T_0^1 + T_2^0 \right) - 4\omega k T_1^1 \right]
\]

(92)

\[
\Pi^{(\text{mat})}_{\Pi_{PV}} = -d \frac{q^4M^2}{2\pi^2} \left( \frac{f_\pi}{m_\pi} \right)^2 T_0^0
\]

(93)

\[
\Pi^{(\text{mat})}_{\Pi_{Q_1}} = -\Pi_{\Pi_{L}}^L L^\nu\mu - \Pi_{\Pi_{T}}^T T^\mu\nu - \Pi_{\Pi_{Q}}^Q Q^\mu\nu ;
\]

\[
\Pi^{(\text{mat})}_{\Pi_{L}} = -d \frac{g_\omega^2}{2\pi^2} \left[ M^2T_0^2 - T_2^0 + T_2^0 \right]
\]

(94)

\[
\Pi^{(\text{mat})}_{\Pi_{T}} = -d \frac{g_\omega^2}{4\pi^2} \left[ M^2q^2T_0^0 + (\omega^2 + k^2) \left( T_0^1 + T_2^0 \right) + 4\omega k T_1^1 \right]
\]

(95)

\[
\Pi^{(\text{mat})}_{\Pi_{Q}} = -d \frac{g_\rho^2}{2\pi^2} q^2 M^2 T_0^0
\]

(96)

\[
\Pi^{(\text{mat})}_{\Pi_{\nu\rho}} = \Pi_{\Pi_{\nu\rho}} = -d \frac{q^4}{4\pi^2} \left( g_\rho + 2M \frac{f_\rho}{2m} \right) e^{\mu\alpha\beta} q_\alpha \eta_\beta q^2 \left[ T_0^1 - \frac{\omega}{k} T_1^0 \right]
\]

The integrals \( T_m^n \) which appear in these expressions are given by

\[
T_m^n(\omega, k) = \int_0^\infty dp \, p^2 \left( \sqrt{p^2 + M^2} \right)^{n-1} I_m(p, \omega, k) \left[ \eta(p) + (-1)^{m+n} \bar{n}(p) \right]
\]

(97)

\[
I_m(p, \omega, k) = \int_{-1}^{+1} du \frac{\left( p^2 + M^2 \right)^m \left( p u + q^2/2 \right) \left( p u + q^2/2 \right) \left( p u - q^2/2 \right) \left( p u - q^2/2 \right)}{(w^2 + M^2 - pku - q^2/2)(w^2 + M^2 - pku + q^2/2)}
\]
The vacuum part of the polarizations comes from the $-\theta(-p^\mu)$ term in the definition of the relativistic Hartree approximation to the Wigner function. It is divergent but can be renormalized by dimensional regularization and subtraction of appropriate counterterms. The standard procedure is exposed in [2, 17]. In the case of the $\rho$ and $\pi$ mesons, the derivative coupling makes the model nonrenormalizable in the usual sense. In practice however, a renormalization scheme can be defined at a given order of perturbation / loop expansion. One has the choice between several renormalization schemes, which were divided in [5] into two subclasses: the “increasing rho mass” and the “decreasing rho mass” classes, in reference to the much debated issue of the interpretation of dilepton production in heavy ion collisions and the Brown-Rho scaling conjecture. To the former class belong earlier procedures by Chin or Kurasawa and Suzuki [37, 38]. In the latter class we find more recent procedures used by Shiomi and Hatsuda [42, 44]. Both classes have advantages and drawbacks. In any case, a same scheme should be used for all mesons.

To the Lagrangian (1) we will have to add the following counterterm Lagrangian in order to perform the dimensional regularization and renormalization of diverging vacuum terms

$$\mathcal{L}_{\text{CT}} = Z_\sigma(\partial_\mu \sigma)^2 + A_\sigma \sigma^2 + B_\sigma \sigma^3 + C_\sigma \sigma^4 + Z_\omega F^{\mu\nu} F_{\mu\nu} +$$

$$+ (Z_\rho + Y_\rho \sigma + X_\rho \sigma^2) R^{\mu\nu} + W_\rho(\partial^\rho R_{\mu\lambda})(\partial_\nu R^{\mu\lambda})$$

$$+ (A_\pi + B_\pi \sigma + C_\pi \sigma^2) \pi^2$$

$$+ Z_\alpha A^{\mu\nu} A_{\mu\nu} + \left(A_a + B_a \sigma + C_a \sigma^2\right) a^\mu a_\mu + \left(F + G_\sigma \sigma + H_\sigma \sigma^2\right) (a^\mu \partial_\mu \pi)$$

(98)

**Renormalization scheme A**

The first renormalization scheme is obtained by imposing that the polarization vanish in vacuum ($M = m$) on the mass shell $q^2 = m^2_\alpha$, for each meson $\alpha = \sigma, \omega, \delta, \rho, \pi, a_1, \eta$. We also require that the derivative with respect to $q^2$ vanishes on the mass shell for the mesons $\rho$ and $a_1$. For all mesons except the $\omega$, divergences of the type $\frac{M^2}{\epsilon^2}$, $\epsilon \to 0$ appear, which require for their cancellation counterterms like $A_\sigma \sigma^2 + B_\sigma \sigma^3 + C_\sigma \sigma^4$. The additional constants are fixed by imposing that the derivatives with respect to the $\sigma$ field vanish.

$$\frac{\partial \Pi}{\partial \sigma}\bigg|_{\text{shell}} = 0, \quad \frac{\partial^2 \Pi}{\partial \sigma^2}\bigg|_{\text{shell}} = 0$$

(99)

The expressions given here are that of reference [4] for the $\sigma$ and $\omega$. We have:

$$\Pi^{(\text{vac}A)}_{\sigma\sigma} = \frac{d}{2} \frac{g^2}{2\pi^2} \left[ 6M^2 \log M - q^2(\log M + \theta) + 4M^2 \theta - (4 - m^2_\sigma) \theta_\sigma ight.$$

$$+ (q^2 - m^2_\sigma)(\theta_\sigma - (4 - m^2_\sigma) \theta_\sigma)$$

$$+ (1 - M) \left( 6m^2_\sigma + (4 - m^2_\sigma) \theta_{\sigma m} + 8 \theta_\sigma \right)$$

$$- \frac{1}{2}(1 - M)^2 \left( 18 + m^2_\sigma + (4 - m^2_\sigma) \theta_{\sigma mm} + 8 \theta_\sigma + 16 \theta_{\sigma m} \right) \right]$$

(100)

where we express all quantities in units of the nucleon mass (“m=1”) and with the definitions

$$\theta = \theta(q^2, M^2) = y \int_0^\infty \frac{dx}{(x^2 + y)\sqrt{x^2 + 1}}$$

with $y = 1 - \frac{q^2}{4M^2}$.
\[
\theta_\sigma = \theta(m_\sigma^2, m^2) \quad , \quad \theta_{\sigma m} = \left. \frac{\partial \theta}{\partial M} \right|_{q^2 = m_\sigma^2, M = m} \quad , \quad \theta_{\sigma m m} = \left. \frac{\partial^2 \theta}{\partial M^2} \right|_{q^2 = m_\sigma^2, M = m}
\]

The expression for \(\Pi^{(\text{vac})}_\partial\) is identical to that for \(\Pi^{(\text{vac}A)}_\partial\) when replacing \(g_\sigma, m_\sigma, \theta_\sigma, \ldots\) by \(g_\delta, m_\delta, \theta_\delta, \ldots\). For the \(\omega\) meson the renormalized vacuum polarization is given by

\[
\Pi^{\mu\nu(\text{vac}A)}_{\omega\omega} = \frac{d}{2} \frac{g_\omega^2}{3\pi^2} \left[ \log M + \theta + \frac{2M^2}{q^2} (\theta - 1) - \frac{1}{m_\omega^2} \left( (2 + m_\omega^2) \theta_\omega - 2 \right) \right] \left\{ q^2 g^{\mu\nu} - q^\mu q^\nu \right\}
\]

For the \(\rho\) we take the “scheme 2” expression of reference [3].

\[
\Pi^{\mu\nu(\text{vac}A)}_{\rho\rho} = \frac{d}{2} \left\{ \frac{g_\rho^2}{3\pi^2} \left[ \log M + \theta + \frac{2M^2}{q^2} (\theta - 1) - \frac{1}{m_\rho^2} \left( (2 + m_\rho^2) \theta_\rho - 2 \right) \right] \\
+ \frac{2}{\pi^2} \left( \frac{f_\pi}{2m_\rho} \right)^2 \left[ M (\log M + \theta) - \theta_\rho - (1 - M) (\theta_\rho + 1 + \theta_{\rho m}) \right] \\
+ \left( \frac{f_\pi}{2m_\rho} \right)^2 \frac{1}{6\pi^2} \left[ 6M^2 \log M + 8M^2\theta + q^2 (\log M + \theta) - (8 + m_\rho^2) \theta_\rho \right.
\]
\[
\left. + (1 - M) \left( 6 + m_\rho^2 + 16 \theta_\rho + (8 + m_\rho^2) \theta_{\rho m} \right) \\
- \frac{1}{2} (1 - M)^2 \left( 18 - m_\rho^2 + 16 \theta_\rho + 32 \theta_{\rho m} + (8 + m_\rho^2) \theta_{\rho mm} \right) \\
- \left( q^2 - m_\rho^2 \right) \left( 8 + m_\rho^2 \theta_\rho + \theta_\rho \right) \right\] \left\{ q^2 g^{\mu\nu} - q^\mu q^\nu \right\}
\]

For the pion with pseudovector coupling, we obtain

\[
\Pi^{(\text{vac}A)}_{\pi\pi} = - \frac{d}{2} \frac{2}{\pi^2} \left( \frac{f_\pi}{m_\pi} \right)^2 q^2 \left[ M^2 (\log M + \theta) - \theta_\pi + (1 - M) (1 + 2\theta_\pi + \theta_{\pi m}) \right] \\
\frac{1}{2} (1 - M)^2 \left( 3 + 2\theta_\pi + 4\theta_{\pi m} + \theta_{\pi mm} \right)
\]

\[
\Pi^{(\text{vac}A)}_{a\pi} = \frac{i}{2} \frac{q^\mu}{q^2} \frac{g_a}{(f_\pi/m_\pi)} \Pi^{(\text{vac}A)}(\pi^\mu A^\nu)
\]

Since the polarization of the \(a_1\) meson can be written as \(\Pi_a^{\mu\nu} = (g_a/g_\omega)^2 \Pi_\omega^{\mu\nu} + g_a^2 (m_\pi/f_\pi)^2 \Pi_\pi^{\mu\nu} q^{\mu\nu}/q^2\), it is tempting to apply the renormalization procedure with this decomposition. In this case, we will have \(\Pi_a^{\mu\nu(\text{vac})} = \Pi_1 (q^2 g^{\mu\nu} - q^\mu q^\nu) + \Pi_2 g^{\mu\nu}\) where the expression of the \(\Pi_1\) contribution is obtained from that of \(\Pi_1^{(\text{vac})}\) by replacing \(g_\omega, m_\omega, \theta_\omega\) by \(g_a, m_a, \theta_a\), and that of \(\Pi_2\) is proportional to \(\Pi^{(\text{vac})}_a\) when replacing \(f_\pi/m_\pi, m_\pi, \theta_\pi\) by \(g_a, m_a, \theta_a\). However, the \(\Pi_2\) term is not orthogonal to \(q^\mu q^\nu\), whereas the polarization of the \(a_1\) enters in the dispersion relations with the decomposition \(\Pi_a T^{\mu\nu} + \Pi_a L L^{\mu\nu} + \Pi_a Q q^{\mu} q^{\nu}/q^2\). It is therefore preferable to apply the renormalization procedure to \(\Pi_a^{\mu\nu(\text{vac})} = \Pi_a^{(\text{vac})}(q^\mu q^\nu/q^2) + \Pi_a^{(\text{vac})} q^{\mu} q^{\nu}/q^2\). The \(\Pi_a^{(\text{vac})}\) then happens to have the same structure as the polarization of the \(\sigma\) meson. We obtain

\[
\Pi^{\mu\nu(\text{vac}A)}_{aa} = \frac{d}{2} \frac{g_a^2}{3\pi^2} \left[ 6M^2 \log M - q^2 (\log M + \theta) + 4M^2\theta - (4 - m_a^2) \theta_a \right.
\]
\[
\left. + (q^2 - m_a^2) \left( \theta_a - (4 - m_a^2) \theta_{aa} \right) \\
+ (1 - M) \left( 6 - m_a^2 + (4 - m_a^2) \theta_{am} + 8 \theta_a \right) \right]
$$\frac{1}{2}(1 - M)^2 \left(18 + m_a^2 + (4 - m_a^2)\theta_{amm} + 8\theta_a + 16\theta_{am}\right) \left\{ g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right\}$$

$$\frac{d}{2} 2g^2 \left[ M^2 (\log M + \theta) - \theta_a + (1 - M) \left(1 + 2\theta_a + \theta_{am}\right) \right]$$

$$\frac{1}{2}(1 - M)^2 (3 + 2\theta_a + 4\theta_{am} + \theta_{amm}) \left\{ g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right\}$$

(106)

The vacuum contribution to the mixed polarizations $\Pi_{\sigma\omega}^\mu$, $\Pi_{\sigma\rho}^\mu$ and $\Pi_{\alpha\rho}^{\mu\nu}$ vanishes.

**Renormalization scheme B**

In this second renormalization scheme, we notice that there appears in the polarizations diverging contributions of the type $M/\epsilon$, $M^2/\epsilon$, $\epsilon \to 0$. Instead of cancelling these divergences with counterterms of the form $a + b\sigma + c\sigma^2$, the original structure is preserved by subtracting only counterterms in the combination $a(m - g_\sigma \sigma)$ or $a(m - g_\sigma \sigma)^2$. The unknown constant $a$ is determined by imposing $\Pi_{\text{shell}} = 0$. No conditions are imposed on the derivatives.

Originally this scheme was motivated by the fact that it yields a decreasing effective $\rho$ mass \[\text{(3)}\].

When applying it to the other mesons, we obtain

$$\Pi_{\sigma}^{(\text{vac B})} = \frac{d}{2} \left\{ \frac{g_\rho^2}{3\pi^2} \left[ \ln M + \theta + \frac{2M^2}{q^2} (\theta - 1) - \frac{1}{m_\rho^2} \left(2 + m_\rho^2\right) \theta_\rho - 2 \right] \right.$$  

$$+ \left(\frac{f_\rho}{2m}\right)^2 \frac{1}{6\pi^2} \left[ M^2 \ln M + 8M^2\theta - 8 + m_\rho^2\right) \theta_\rho$$

$$+ (q^2 - m_\rho^2) \left(8 + m_\rho^2\right) \theta_{\rho q} + \theta_\rho + (M^2 - 1) \left(m_\rho^2 (8 + m_\rho^2) \theta_{\rho q} - 8\theta_\rho \right)$$

$$+ \frac{2}{\pi^2} \left(\frac{f_\rho}{2m}\right) g_\rho \left[M (\ln M + \theta - \theta_\rho)\right] \left\{ q^2 g^{\mu\nu} - q^\mu q^\nu \right\}$$

(107)

The polarization of the $\delta$ meson is identical when relacing $g_\sigma$, $m_\sigma$, $\theta_\sigma$ ... by $g_\delta$, $m_\delta$, $\theta_\delta$ ...

Since no divergences proportional to $M$ or $M^2$ occur in the polarization of the $\omega$ meson, its expression is left unchanged by this renormalization scheme and its expression still given by Eq. (102). We recall the expression obtained for the $\rho$ meson in scheme B ($\equiv$ scheme 3 of \[\text{(3)}\])

$$\Pi_{\rho}^{(\text{vac B})} = \frac{d}{2} \left\{ \frac{g_\rho^2}{3\pi^2} \left[ \ln M + \theta + \frac{2M^2}{q^2} (\theta - 1) - \frac{1}{m_\rho^2} \left(2 + m_\rho^2\right) \theta_\rho - 2 \right] \right.$$  

$$+ \left(\frac{f_\rho}{2m}\right)^2 \frac{1}{6\pi^2} \left[ M^2 \ln M + 8M^2\theta - 8 + m_\rho^2\right) \theta_\rho$$

$$+ (q^2 - m_\rho^2) \left(8 + m_\rho^2\right) \theta_{\rho q} + \theta_\rho + (M^2 - 1) \left(m_\rho^2 (8 + m_\rho^2) \theta_{\rho q} - 8\theta_\rho \right)$$

$$+ \frac{2}{\pi^2} \left(\frac{f_\rho}{2m}\right) g_\rho \left[M (\ln M + \theta - \theta_\rho)\right] \left\{ q^2 g^{\mu\nu} - q^\mu q^\nu \right\}$$

(108)

For the pion with pseudovectorial coupling, we obtain

$$\Pi_{\pi \rho\nu}^{(\text{vac B})} = -\frac{d}{2} \frac{g_\pi^2}{\pi^2} \left(\frac{f_\pi}{m_\pi}\right)^2 q^2 M^2 \left[\ln M + \theta - \theta_\pi\right]$$

(109)

Finally, the polarization of the $a_1$ meson is given in scheme B by

$$\Pi_{a a}^{(\text{vac B})} = -\Pi_{\alpha LT}^{(\text{vac B})} \left( g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) - \Pi_{\alpha Q}^{(\text{vac B})} \frac{q^\mu q^\nu}{q^2}$$

$$\Pi_{a LT}^{(\text{vac B})} = \frac{d}{2} \frac{g_\sigma^2}{3\pi^2} \left[ 6M^2 \ln M + 4M^2\theta - q^2 (\log M + \theta) - (4 - m_\sigma^2)\theta_a + (M^2 - 1) \left((4 - m_\sigma^2)m_\sigma^2 \theta_{aq} - 4\theta_a\right) + (q^2 - m_\sigma^2) \left(\theta_a - (4 - m_\sigma^2)\theta_{aq}\right) \right]$$

(110)

$$\Pi_{\alpha Q}^{(\text{vac B})} = 2 \frac{d}{2} \frac{g_\sigma^2}{\pi^2} M^2 \left[\ln M + \theta - \theta_\sigma\right]$$

(111)
All imaginary parts can be expressed in terms of three integrals $E_1$, $E_2$, $E_3$. At finite temperature, for the calculation of the retarded polarizations, we have

- for spacelike momentum:

\[
E_n = \int dy \left[ \left( y + \frac{\omega}{2} \right)^{n-1} \theta(y - y_L) \{ n(y) - n(y + \omega) \} + (-1)^n \left( y - \frac{\omega}{2} \right)^{n-1} \theta(y - y_U) \{ \pi(y) - \pi(y - \omega) \} \right]
\]

(112)

- for timelike momentum with $q^2 < 4M^2$, the imaginary parts vanish: $E_n = 0$.

- for timelike momentum with $q^2 > 4M^2$:

\[
E_n = (-1)^{n-1} \int_M^\infty dy \left( y - \frac{\omega}{2} \right)^{n-1} \left[ \theta(y - y_L) - \theta(y - y_U) \right] \{ n(y - \omega) + \pi(y - 1) \}
\]

(113)

with

\[
n(y) = \left[ e^{\beta(y-\mu)} + 1 \right]^{-1}, \quad \pi(y) = \left[ e^{\beta(y+\mu)} + 1 \right]^{-1}
\]

\[
y_L = \left| \frac{k \sqrt{\Delta - \omega}}{2} \right|, \quad y_U = \left| \frac{k \sqrt{\Delta + \omega}}{2} \right|, \quad \Delta = 1 - 4 \frac{M^2}{q^2}
\]

In the following equations $d$ is the degeneracy parameter ($d = 2$ for symmetric nuclear matter).

\[
\Im \Pi_\sigma = -d \frac{g^2_\sigma}{2\pi k} \left( M^2 - \frac{q^2}{4} \right) E_1
\]

\[
\Im \Pi_\omega L = d \frac{g^2_\omega}{2\pi k} \left[ \frac{q^2}{4} E_1 - \frac{q^2}{k^2} E_3 \right]
\]

\[
\Im \Pi_\omega T = d \frac{g^2_\omega}{4\pi k} \left( M^2 + \frac{q^2}{4} \right) E_1 + \frac{q^2}{k^2} E_3
\]

\[
\Im \Pi_\sigma \omega = -d \frac{g_\sigma g_\omega}{2\pi k^3} q^2 M E_2
\]

\[
\Im \Pi_\rho L = \frac{d}{2\pi k} \left\{ g_\rho \left[ \frac{q^2}{4} E_1 - \frac{q^2}{k^2} E_3 \right] + g_\rho \frac{f_\rho}{2m} q^2 M E_1 + \left( \frac{f_\rho}{2m} \right)^2 \left[ M^2 q^2 E_1 + \frac{q^4}{k^2} E_3 \right] \right\}
\]

\[
\Im \Pi_\rho T = \frac{d}{4\pi k} \left\{ g_\rho \left[ \left( M^2 + \frac{q^2}{4} \right) E_1 + \frac{q^2}{k^2} E_3 \right] + 2g_\rho \frac{f_\rho}{2m} q^2 M E_1 \right. \]

\[
\left. + \left( \frac{f_\rho}{2m} \right)^2 \left[ \left( M^2 q^2 + \frac{q^4}{4} \right) E_1 - \frac{q^4}{k^2} E_3 \right] \right\}
\]

\[
\Im \Pi_\pi^\nu = -d \frac{M^2 q^2}{2\pi k} \left( \frac{f_\pi}{m_\pi} \right)^2 E_1
\]

\[
\Im \Pi_\pi^S = -d \frac{g^2_\pi q^2}{8\pi k} E_1
\]

\[
\Im \Pi_\alpha L = -d \frac{g^2_\alpha}{2\pi k} \left[ \left( M^2 - \frac{q^2}{4} \right) E_1 + \frac{q^2}{k^2} E_3 \right]
\]
\[ \text{Im } \Pi_{aT} = -d \frac{g_2^2}{4\pi k} \left[ \left( M^2 - \frac{q^2}{4} \right) E_1 - \frac{q^2}{k^2 E_3} \right] \]
\[ \text{Im } \Pi_{aQ} = -d \frac{g_1^2}{2\pi k} M^2 E_1 \]
\[ \text{Im } \Pi_{a\pi} = d \frac{g_a}{2\pi k} \left( \frac{f_\pi}{m_\pi} \right) \left( \frac{\alpha}{M} + 1 - \alpha \right) M^2 E_1 \]
\[ \text{Im } \Pi_{a\rho} = d \frac{g_a (g_\rho + 2M(f_\rho/2m))}{4\pi k^3} q^2 E_2 \]
\[ \text{Im } \Pi_{\eta}^{PS} = -d \frac{g_\eta q^2}{8\pi k} E_1 \]

**Appendix B: Polarizations modified by contact terms**

In meson exchange models of the NN interaction, the potential is obtained by calculating the transition amplitude \( \mathcal{M} = \sum_{a,b=\sigma,\omega,\pi,\rho,\delta,\lambda,\eta} (U_3 \Gamma_a U_1) G^{ab} (U_3 \Gamma_b U_1) \), with \( \Gamma^a \) and \( G^{ab} \) being the relevant vertices and meson propagators. After taking the semiclassical limit and multiplying by the minimal relativity factors, one finally performs a Fourier transformation to obtain the potential in coordinate space. Besides the desired Yukawa-like terms with ranges characterized by the inverses of the meson masses, one also obtains a singular contribution at the origin \( \delta(\vec{r}) \) due to the assumption that all particles are pointlike.

In a \( \sigma-\omega-\pi-\rho \) model, the singular contribution is given by

\[ V(r) \equiv -\left[ \frac{g_2^2}{4m^2} + \frac{g_2^2}{2m^2} + \frac{g_1^2}{2m^2} \frac{(g_\rho + f_\rho)^2}{2m^2} \frac{\vec{r}_1 \cdot \vec{r}_2}{2m^2} \right] \delta(\vec{r}) \]
\[ -\left[ \frac{g_2^2}{2m^2} + \frac{g_2^2}{4m^2} \frac{\vec{r}_1 \cdot \vec{r}_2}{2m^2} + \frac{(g_\rho + f_\rho)^2}{2m^2} \frac{\vec{r}_1 \cdot \vec{r}_2}{2m^2} \right] \delta(\vec{r}) \frac{1}{3} \vec{r}_1 \cdot \vec{r}_2 \]  

(114)

It is not always enough to smooth this singularity by convoluting the result with form factors, especially in the case of the pion. It is then desirable to remove the singular piece by subtracting one or more contact terms. If this method is chosen, the dispersion relations of the mesons will be modified as follow.

**\( \pi-a_1-\rho \) sector**

When a contact interaction is introduced as

\[ \mathcal{L} \ni -g_A (\vec{\psi}\gamma_5 \gamma_\mu \vec{\tau} \vec{\psi}) (\vec{\psi}\gamma_5 \gamma^\mu \vec{\tau} \vec{\psi}) - g_R (\vec{\psi}\gamma_\mu \vec{\tau} \vec{\psi}) (\vec{\psi}\gamma^\mu \vec{\tau} \vec{\psi}) \]

the dispersion relations of the \( \pi-a_1-\rho \) sector are given by (72). The polarizations which appear in these expressions are given explicitly in this Appendix.

\[ \Pi_{\rho T} = \left( g_\rho + 2g_R \frac{f_\rho}{2m} \right) R_{\rho T} + \left( 2g_A \frac{f_\rho}{2m} \Pi_{\rho T} \right) P_{\rho T} + \frac{f_\rho}{2m} \left( g_\rho \Pi_{\rho T} + \frac{f_\rho}{2m} \Pi_{\rho T} \right) \]  

(116)

\[ \Pi_{a L} = \frac{g_2^2}{1 - 2g_A \Pi_{\rho T}^L} \]  

(117)

\[ \Pi_{a T} = \frac{f_\rho}{2m} g_2 \Pi_{\rho T}^L + \left( g_\rho + 2g_R \frac{f_\rho}{2m} \right) R_{a T} + \left( 2g_A \frac{f_\rho}{2m} \Pi_{\rho T} \right) P_{a T} \]  

(118)
\[ T^{\mu \nu} P_{1 \nu} = P_{\rho T} T^{\mu \nu} \rho_{1 \nu} + P_{a T} T^{\mu \nu} a_{1 \nu} \]  
\[ T^{\mu \nu} R_{1 \nu} = R_{\rho T} T^{\mu \nu} \rho_{1 \nu} + R_{a T} T^{\mu \nu} a_{1 \nu} \]  
\[ P_{\rho T} = \left( 1 - 2g_R \Phi_{1 \nu}^{R T} \right) \left( g_R \Phi_{1 \nu}^{R T} + \frac{f_R}{2m} \Phi_{1 \nu}^{R T} \right) + 2g_R \Phi_{1 \nu}^{R T} \left( g_R \Phi_{1 \nu}^{R T} + \frac{f_R}{2m} \Phi_{1 \nu}^{R T} \right) / D \]  
\[ P_{a T} = \left( 1 - 2g_A \Phi_{1 \nu}^{A T} \right) g_A \Phi_{1 \nu}^{A T} + 2g_R g_A \Phi_{1 \nu}^{A T} / D \]  
\[ R_{\rho T} = \left( 1 - 2g_A \Phi_{1 \nu}^{A T} \right) \left( g_R \Phi_{1 \nu}^{R T} + \frac{f_R}{2m} \Phi_{1 \nu}^{R T} \right) + 2g_A \Phi_{1 \nu}^{R T} \left( g_R \Phi_{1 \nu}^{R T} + \frac{f_R}{2m} \Phi_{1 \nu}^{R T} \right) / D \]  
\[ R_{a T} = \left( 1 - 2g_A \Phi_{1 \nu}^{A T} \right) g_A \Phi_{1 \nu}^{A T} + 2g_A \Phi_{1 \nu}^{A T} / D \]  
\[ D = \left( 1 - 2g_A \Phi_{1 \nu}^{A T} \right) \left( 1 - 2g_R \Phi_{1 \nu}^{R T} \right) - 4g_A g_R \Phi_{1 \nu}^{A T} \Phi_{1 \nu}^{R T} \]  

with the definitions \[ \Pi_{\mu \rho}^{\mu \nu} = g^2 g_{\mu \nu} + 2g_{\rho} \left( \frac{f_R}{2m} \right) \Phi_{\mu \nu} + \left( \frac{f_R}{2m} \right)^2 \Phi_{\mu \nu}, \]  
\[ \Pi_{\mu \nu}^{\mu \nu} = g^2 \Phi_{\mu \nu}^{\mu \nu} \]  
\[ \Pi_{\mu \rho}^{\mu \nu} = g_{\rho} g_{\mu \nu} \Phi_{\mu \nu}^{\mu \nu} \]  
\[ \sigma 1-\omega \text{ sector} \]

Contact terms of the form \( \delta(\vec{r}) \) also appear in the contributions of the \( \sigma \) and \( \omega \) mesons exchange to the NN potential. Although the problem is less acute than in the case of the spin dependent part of the interaction mediated by the pion, one could wish for consistency to remove these \( \delta(\vec{r}) \) contributions by the introduction of further contact terms \[ \mathcal{L} \ni - g_V (\overline{\psi} \gamma_\mu \psi)(\overline{\psi} \gamma^\mu \psi) - g_S (\overline{\psi} \psi)(\overline{\psi} \psi) \]

These terms modify the dispersion relations as follows \[ \begin{align*}
[-q^2 + m^2_\sigma] \sigma_1 &= g^2 g_{\sigma} \Phi_{\sigma} \sigma_1 + g_{\sigma} g_{\omega} \Phi_{\omega} \omega_1 \mu \\
[-q^2 + m^2_\omega] \omega_1 \nu &= g_{\theta} g_{\omega} \Phi_{\omega} \omega_1 \nu + g_{\omega} \left( -\Phi_{\omega} T^{\mu \nu} \Phi_{\omega} L^{\mu \nu} \right) \omega_1 \nu \end{align*} \]  
\[ \phantom{\begin{align*}} \text{with} \quad \Phi_{\sigma} = \frac{(1 - 2 g_V \Phi_{\omega} L) \Phi_{\sigma} - 2 g_V \Phi_{\omega} \Phi_{\omega} \eta \sigma}{(1 - 2 g_V \Phi_{\omega} L) (1 + 2 g_S \Phi_{\sigma}) - 4 g_S g_V \Phi_{\sigma} \Phi_{\omega} \eta^2} \]  
\[ \Phi_{\omega} T = \frac{(1 - 2 g_V \Phi_{\omega} L) \Phi_{\omega} T}{(1 - 2 g_V \Phi_{\omega} L) (1 + 2 g_S \Phi_{\sigma}) - 4 g_S g_V \Phi_{\sigma} \Phi_{\omega} \eta^2} \]  
\[ \Phi_{\omega} L = \frac{(1 + 2 g_S \Phi_{\sigma}) \Phi_{\omega} L + 2 g_S \Phi_{\omega} \Phi_{\omega} \eta^2}{(1 - 2 g_V \Phi_{\omega} L) (1 + 2 g_S \Phi_{\sigma}) - 4 g_S g_V \Phi_{\sigma} \Phi_{\omega} \eta^2} \]

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Fig. 1 Dispersion relation of the $\delta$-$\rho$ longitudinal mode, as calculated with renormalization scheme A (left panel) or B (right panel), at vanishing temperature and three values of the density: 0.5, 1.5 and 3 times the saturation density.
Fig. 2 Effective masses of the $\delta$ and $\rho$ mesons as a function of density, as calculated with renormalization scheme A (left panel) or B (middle panel) or C (right panel), at vanishing temperature.
**Fig. 3** Effective masses of the $\delta$ and $\rho$ as a function of temperature at three times saturation density in renormalization scheme A.

**Fig. 4** Mixing angle between the $\delta$ and $\rho$ in the longitudinal mode at $T = 0$ and $k = 300$ Mev, represented as a function of density, and calculated with renormalization scheme A (left panel) or B (right panel).
Fig. 5 Mixing angle between the $\delta$ and $\rho$ in the longitudinal mode represented as a function of momentum (left panel) at $n_B = 3\, n_{sat}$ and $T = 0$, and as a function of temperature (right panel) at $n_B = 3\, n_{sat}$ and $k = 300$ MeV. Both figures were obtained with renormalization scheme B.

Fig. 6 Dispersion relation of the $a_1$ longitudinal mode, as calculated with renormalization scheme A (left panel) or B (right panel). The curves are labelled by the value of the density in saturation units $n_B/n_{sat}$. 
Fig. 7 Effective mass of the $a_1$ meson, as calculated with renormalization schemes A, B or C, plotted as a function of density at $T = 0$ (left panel), and as a function of temperature $T$ at $n_B = 3 \, n_{\text{sat}}$ (right panel).

Fig. 8 Mixing angle in the $a_1$-$\rho$ transverse mode at $k=300$ MeV and $T = 0$, as a function of density. On the left panel with renormalization scheme A; on the right, with scheme B.
Fig. 9 Dispersion relation for the pion with pure pseudovector coupling, with and without mixing with the \(a_1\). The curves are labelled by the value of the density in saturation units \(n_B/n_{\text{sat}}\) and were obtained with renormalization scheme A.
The experimental determination of the effective pion mass at $n_B = n_{sat}$ is indicated on the figures.