Quantum field theory of interacting plasmon–photon–phonon system

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Abstract
This work is devoted to the construction of the quantum field theory of the interacting system of plasmons, photons and phonons on the basis of general fundamental principles of electrodynamics and quantum field theory of many-body systems. Since a plasmon is a quasiparticle appearing as a resonance in the collective oscillation of the interacting electron gas in solids, the starting point is the total action functional of the interacting system comprising electron gas, electromagnetic field and phonon fields. By means of the powerful functional integral technique, this original total action is transformed into that of the system of the quantum fields describing plasmons, transverse photons, acoustic as well as optic longitudinal and transverse phonons. The collective oscillations of the electron gas is characterized by a real scalar field $\phi(x)$ called the collective oscillation field. This field is split into the static background field $\phi_0(x)$ and the fluctuation field $\zeta(x)$. The longitudinal phonon fields $Q^l(x), Q^o(x)$ are also split into the background fields $Q^l_0(x), Q^o_0(x)$ and dynamical fields $q^l(x), q^o(x)$ while the transverse phonon fields $Q^T(x), Q^o(x)$ themselves are dynamical fields $q^T(x), q^o(x)$ without background fields. After the canonical quantization procedure, the background fields $q^l_0(x), Q^T_0(x), Q^o_0(x)$ remain the classical fields, while the fluctuation fields $\zeta(x)$ and dynamical phonon fields $q^l(x), q^o(x)$ become quantum fields. In quantum theory, a plasmon is the quantum of Hermitian scalar field $\sigma(x)$ called the plasmon field, longitudinal phonons as complex spinless quasiparticles are the quanta of the effective longitudinal phonon Hermitian scalar fields $\theta^l(x), \theta^o(x)$, while transverse phonons are the quanta of the original Hermitian transverse phonon vector fields $q^T(x), q^o(x)$. By means of the functional integral technique the original action functional of the interacting system comprising electron gas, electromagnetic field and phonon fields is transformed into the total action functional of the resultant system comprising plasmon scalar quantum field $\sigma(x)$, longitudinal phonon effective scalar quantum fields $\theta^l(x), \theta^o(x)$ and transverse phonon vector quantum fields $q^T(x), q^o(x)$.

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1. Introduction

Since the early works on the collective motion of charged particles in plasma, including the interacting electron gas in solids, it was shown that there exists a resonance of the collective oscillations at some frequency called the plasma frequency. This resonance phenomenon was interpreted as the appearance of an elementary excitation—a complex quasiparticle called a plasmon—and the plasma frequency was also called plasmon frequency (the references on early works on plasmons can be found in the literature [1–3]). In the physical processes with the presence of plasmon the plasmon–photon interaction plays the main role. Moreover, in the electron gas of solids there always exists the electron–phonon interaction.
leading to the effective plasmon–phonon interaction. Therefore the
knowledge on the mutual interaction of plasmon, photon and phonons is necessary for both theoretical and experimental studies on the physical processes and phenomena involving plasmon. The present work is devoted to the elaboration of the quantum field theory of the plasmon–photon–phonon interacting system by applying the functional integral technique [4–7]. The assumptions comprise only the fundamental principles of electrodynamics and quantum theory of many-body system.

For the application of mathematical tools of functional integral technique, the physical content of the theory of phonons in solids must be presented in the languages of the quantum field theory. This will be done in section 2. Here there is a distinction between longitudinal and transverse phonons. While the transverse phonons are described by the transverse phonon vector fields as other transverse vector fields in the theory of the elementary particles, for simplifying the presentation of the formulae related to longitudinal phonons we propose to describe them by some effective scalar fields similar to the quantum fields of spinless particles. Moreover, because the interaction of longitudinal phonons with electron is much stronger than that of transverse ones, in the study of physical phenomena and processes with the dominant competition of longitudinal phonons we can neglect the contribution of transverse phonons. Thus the transverse phonon fields will be retained only in the particular cases when they play the essential role.

Section 3 is devoted to the establishment of the expression of total action functional of the interacting plasmon–photon–phonon system. It contains all three types of fields: (i) collective oscillation field \( q(x) \); (ii) transverse electromagnetic vector field \( A(x) \) and (iii) all phonon fields, both acoustic and optic phonon fields \( Q_{\mu}(x) \), \( a_{\mu}(x) \), index \( \mu \) labeling the phonon branches. By grouping suitable terms from the formula of total action functional of the whole system it is possible to derive expressions of action functional of different subsystems of related fields. The fundamental subsystem is the collective oscillation field \( q(x) \). A short review of the results of previous works related to this field in the harmonic approximation is presented.

The construction of quantum fields of interacting plasmon–phonon system is the content of section 4. In the harmonic approximation with respect to the collective oscillation field as well as to the fields of both acoustic and optical phonons, the action functional of the subsystem comprising interacting collective oscillation field \( q(x) \) as well as both acoustic and optic phonon fields \( Q_{\mu}(x) \) and \( a_{\mu}(x) \) is derived. Each of longitudinal phonon fields \( Q_{\mu}(x) \) and \( a_{\mu}(x) \) is split into two parts, background field \( Q_{0\mu}(x) \) or \( a_{0\mu}(x) \) and dynamical field \( q_{\mu}(x) \) or \( a_{\mu}(x) \), while transverse phonon fields \( Q_{\mu}(x) \), \( a_{\mu}(x) \) themselves are dynamical ones \( q_{\mu}(x) \) and \( a_{\mu}(x) \). The dynamical phonon fields generate the physical phonons playing the role of dynamical quasiparticles in physical phenomena and processes.

The construction of the quantum fields of the whole interacting plasmon–photon–phonon system is the content of the section 5. The expression of total action functional of this whole system, described by background fields \( q_{0\mu}(x) \), \( Q_{0\mu}(x) \) and \( a_{0\mu}(x) \), fluctuation field \( q(x), A(x) \), electromagnetic field \( A(x) \) and dynamical phonon fields \( q_{\mu}(x), a_{\mu}(x) \) and \( q_{\mu}(x), a_{\mu}(x) \), is derived in the harmonic approximation with respect to each of three types of fields: (i) fluctuation field, (ii) electromagnetic field and (iii) all dynamical phonon fields. The characterizing features of different subsystems of the whole system are briefly investigated. From the obtained expression of total action functional of the whole system it is possible to derive the expressions of the action functional of different interaction vertices. The conclusion and discussions are presented in section 6.

2. Phonon quantum fields

For using in the study of the interaction of phonons with other quasiparticles in solids by means of the functional integral technique let us construct the quantum fields of phonons. There exist many types of phonons with various characteristics in different materials [8]. In the present work we limit to the frequently investigated solids: elastic media [3] and crystalline lattices [3, 9]. The quantum fields of acoustic and optic phonons will be constructed separately. For simplifying formulae we use the notations proposed in our previous works [4–6] and the unit system with \( \hbar = c = 1 \).

Consider first the acoustic phonons. In both above-mentioned types of solids there exist one longitudinal and two transverse acoustic phonon branches. Denote \( Q_{\mu}(x) \) their quantum fields, where \( \mu = 1, 2 \) for transverse phonons and \( \mu = 3 \) for longitudinal one. For a definite \( \mu \)th branch between angular frequency \( \omega \) and wave vector \( k \) at small values of \( k = |k| \) there exists a linear relation

\[
\omega = v_{\mu}k.
\]

We assume that this formula is the dispersion law of the acoustic phonon in general. It looks like that of a massless relativistic particle, except for the scaling of spatial coordinates

\[
x \rightarrow x^* = \frac{x}{v_{\mu}}, \quad k \rightarrow k^* = v_{\mu}k.
\]

On the basis of the analogy with the free field of relativistic massless particles we have following Lagrange function and action functional of the acoustic phonon in \( \mu \)th branch

\[
\mathcal{L}_{0\mu}(Q_{\mu}, \frac{\partial Q_{\mu}}{\partial t}, \frac{\partial a_{\mu}}{\partial x_i}) = \frac{1}{2} \left[ \left( \frac{\partial Q_{\mu}}{\partial t} \right)^2 - \omega_{\mu}^2 \sum_{i=1}^{3} \left( \frac{\partial a_{\mu}}{\partial x_i} \right)^2 \right]
\]

and

\[
I_{0\mu}(Q_{\mu}) = \int dx \mathcal{L}_{0\mu}(Q_{\mu}, \frac{\partial Q_{\mu}}{\partial t}, \frac{\partial a_{\mu}}{\partial x_i}).
\]

Now we consider the optic phonons. In a crystalline lattice with \( s \) non-equivalent ions per a primitive cell, \( s \neq 1 \),...
beside three acoustic phonon branches there exist 3(s-1) branches of optic phonons with non-vanishing limiting angular frequency \( \Omega_m \) at \( k = 0 \). Denote \( \mathbf{Q}^{\mu}(x) \) the optic phonon field in the \( \mu \)th branch. Since \( k \)-dependent terms in the dispersion law of optic phonon are very small in comparison with the constant terms \( \Omega_m \), let us neglect them. Then the optic phonon field \( \mathbf{Q}^{\mu}(x) \) has following Lagrange function and action functional

\[
\mathcal{L}^\mu_0 \left( Q^{\mu}, \frac{\partial Q^{\mu}}{\partial t}, \frac{\partial Q^{\mu}}{\partial x_i} \right) = \frac{1}{2} \left( \left( \frac{\partial Q^{\mu}}{\partial t} \right)^2 - \Omega^2_\mu (Q^{\mu})^2 \right)
\]

(3)

and

\[
I_0^\mu (Q^{\mu}) = \int dx \mathcal{L}^\mu_0 \left( Q^{\mu}, \frac{\partial Q^{\mu}}{\partial t}, \frac{\partial Q^{\mu}}{\partial x_i} \right).
\]

(4)

In the special case of isotropic crystals with \( s = 2 \) non-equivalent ions per a primitive cell, there exist one longitudinal and two degenerate transverse optic phonon branches with limiting angular frequencies \( \Omega_l \) and \( \Omega_t \) at \( k = 0 \). Between \( \Omega_l \) and \( \Omega_t \) there exists following relation

\[
\Omega_l > \Omega_t
\]

(5)

and

\[
\frac{\Omega_l^2 - \Omega_t^2}{\Omega_t^2} = \frac{\epsilon_i - \epsilon_\infty}{\epsilon_\infty},
\]

(6)

where \( \epsilon_0 \) is the static dielectric constant of the medium and \( \epsilon_\infty \) is the square of the refractive index of the medium at optical frequencies.

In solids there always exists the electron-phonon interaction. In most cases the interaction of longitudinal acoustic or optic phonons with electron is much stronger than that of transverse acoustic or optic phonon, respectively. In these cases the longitudinal phonons play a much more important role than the corresponding transverse phonons do, so that the interaction between longitudinal phonons and electron has been intensively studied during a long time. It was shown that for various solids the Hamiltonians of the interaction between electron and longitudinal acoustic and optic phonons have following expression [3, 9]

\[
H_{int}^{al} = g_a \int dx \psi(x) \psi(x) \nabla Q^{o.l}(x)
\]

(7)

and

\[
H_{int}^{ol} = g_o \int dx \bar{\psi}(x) \psi(x) \nabla Q^{o.l}(x),
\]

(8)

where \( \psi(x) \) is the electron field operator, \( \bar{\psi}(x) \) is its Hermitian conjugate. The coupling constants \( g_a \) and \( g_o \) depend on the crystalline and electronic structures of solids.

Meanwhile, the interaction between electron and transverse phonons was much less known. Let us consider the simple case of the lattice with 2 non-equivalent ions per a primitive cell, \( s = 2 \). Then besides the two degenerate acoustic transverse phonon branches with wave function \( Q^{al}(x) \) there exist also only two degenerate optic phonon branches with wave function \( Q^{ol}(x) \). Since the physical origin of the appearance of phonons is the oscillation of ions in solids and the coupling of phonons with electron is caused by the photon exchange between ion and electron, it is natural to believe that the Hamiltonian of the interaction between transverse phonons with electron have the expressions similar to the electron-phonon interaction Hamiltonian in the transverse gauge.

Therefore we assume following expressions of the transverse phonon–electron interaction Hamiltonians:

\[
H_{int}^{al} = -i g_a \int dx \left[ \bar{\psi}(x) \left( \frac{\partial}{\partial x} - \frac{\partial}{\partial x} \right) \psi(x) \right] Q^{al}(x)
\]

(9)

for acoustic transverse phonon and

\[
H_{int}^{ol} = -i g_o \int dx \left[ \bar{\psi}(x) \left( \frac{\partial}{\partial x} - \frac{\partial}{\partial x} \right) \psi(x) \right] Q^{ol}(x)
\]

(10)

for optic transverse phonon.

The interaction of ions in the lattice with the electromagnetic wave, in principle, can also generate the direct coupling of electron with transverse acoustic and optic phonons. In the transverse gauge the effective interaction Hamiltonians have the expressions

\[
H_{int}^{al} = g_a' \int dx \mathbf{Q}^{al}(x) \mathbf{A}(x)
\]

(11)

for acoustic phonon and

\[
H_{int}^{ol} = g_o' \int dx \mathbf{Q}^{ol}(x) \mathbf{A}(x)
\]

(12)

for optic phonon.

3. Total functional integral

As the extension of total functional integral of the interacting plasmon–photon system studied in the previous work [14] we have following total functional integral of the plasmon–phonon–photon system

\[
Z_{tot} = \int [D\psi] [D\bar{\psi}] \int [D\mathbf{A}] \delta \left( \frac{\partial \mathbf{A}}{\partial x} \right) \int [D\mathbf{Q}^{al}] \times \int [D\mathbf{Q}^{ol}] \exp \left\{ i\int_{\text{int}} \left[ \bar{\psi}(x) \psi(x) + \mathbf{Q}^{al}(x) \mathbf{A}(x) + \mathbf{Q}^{ol}(x) \mathbf{A}(x) \right] \right\},
\]

(13)

where \( I_{tot} \left[ \bar{\psi}, \psi; \mathbf{A}; Q^{al}, Q^{ol} \right] \) is the total action functional of this system:

\[
I_{tot} \left[ \bar{\psi}, \psi; \mathbf{A}; Q^{al}, Q^{ol} \right] = I_{00}^{al}[\mathbf{A}] + I_{00}^{ol}[\mathbf{A}] + I_{int}^{al}[\bar{\psi}, \psi; \mathbf{A}; Q^{al}] + I_{int}^{ol}[\bar{\psi}, \psi; \mathbf{A}; Q^{ol}]
\]

(14)

\[
I_{0}^{al}[\mathbf{A}] \) is the action functional of the transverse free electromagnetic field in the transverse gauge

\[
I_{0}^{al}[\mathbf{A}] = \frac{1}{2} \int dx \left( \epsilon_0 \left( \frac{\partial \mathbf{A}(x)}{\partial x} \right)^2 - \left( \frac{\partial}{\partial x} \wedge \mathbf{A}(x) \right)^2 \right),
\]

(15)
\(e_0\) being the static dielectric constant of the medium, \(I_0^{a} [Q^{\mu}]\) and \(I_0^{o} [Q^{\mu}]\) are the action functional of two systems of all acoustic phonon fields and all optic phonon fields, respectively

\[
I_0^{a} [Q^{\mu}] = \sum_{\mu=1}^{3} I_0^{a\mu} [Q^{\mu}],
\]

\[
I_0^{o} [Q^{\mu}] = \sum_{\mu=1}^{3} I_0^{o\mu} [Q^{\mu}],
\]

where \(I_0^{a\mu} [Q^{\mu}]\) and \(I_0^{o\mu} [Q^{\mu}]\) being determined by formulae (1)–(4), \(I^I [\psi, \bar{\psi}]\) is the action functional of two systems of all electrons mutually interacting through the Coulomb repulsion. \(I^I [\psi, \bar{\psi}]\) consists of two parts

\[
I^I [\psi, \bar{\psi}] = I_0^I [\psi, \bar{\psi}] + I_{\text{int}}^{\psi} [\psi, \bar{\psi}],
\]

where \(I_0^I [\psi, \bar{\psi}]\) is the action functional of free electron moving in the electrostatic field of ions in the crystalline lattice

\[
I_0^I [\psi, \bar{\psi}] = \int dx \bar{\psi}(x) \left\{ i \frac{\partial}{\partial x_0} - H \left( -i \frac{\partial}{\partial x}, x \right) \right\} \psi(x),
\]

\[
H \left( -i \frac{\partial}{\partial x}, x \right) \text{ is the quantum mechanical Hamiltonian of single electron}
\]

\[
H \left( -i \frac{\partial}{\partial x}, x \right) = \frac{m}{2} \left( -i \frac{\partial}{\partial x} \right)^2 + U(x),
\]

\(m\) is the effective mass of electron, \(I_{\text{int}}^{\psi} [\psi, \bar{\psi}]\) is the action functional of electron-electron Coulomb interaction

\[
I_{\text{int}}^{\psi} [\psi, \bar{\psi}] = \frac{1}{2} \int dx \int dy \bar{\psi}(x) \psi(y) u(x - y) \times \bar{\psi}(y) \psi(y),
\]

\[
u(x - y) = \frac{\delta(x_0 - y_0)}{u(x - y)},
\]

\[
u(x - y) = \frac{e^2}{\epsilon_0 |x - y|},
\]

\(-e\) is the electron charge. It is straightforward to show that

\[
I_{\text{int}}^{\mu} [\psi, \bar{\psi}; A] = ie^2 \int dx \bar{\psi}(x) \left( \frac{\partial}{\partial x} - \bar{\psi} \right) \psi(x) \frac{\partial}{\partial x} A(x) + \frac{e^2}{2m} \int dx \bar{\psi}(x) \psi(x) A(x)^2.
\]

According to formulae (7)–(10) for the electron–phonon interaction Hamiltonians we have

\[
I_{\text{int}}^{\mu} [\psi, \bar{\psi}; Q^{\mu}] = -\frac{g_0}{\mu} \int dx \bar{\psi}(x) \psi(x) \bar{Q}^{\mu}(x)
\]

\[
+ i\hbar \int dx \left( \bar{\psi}(x) \left( \frac{\partial}{\partial x} - \bar{\psi} \right) \psi(x) \right) Q^{\mu}(x),
\]

(24)

\[
I_{\text{int}}^{\mu} [\psi, \bar{\psi}; Q^{\mu}] = -g_0 \int dx \bar{\psi}(x) \psi(x) \bar{Q}^{\mu}(x)
\]

\[
+ i\hbar \int dx \left( \bar{\psi}(x) \left( \frac{\partial}{\partial x} - \bar{\psi} \right) \psi(x) \right) Q^{\mu}(x),
\]

(25)

From expression (11) and (12) of the Hamiltonians describing the coupling of transverse phonons with photon it follows that

\[
I_{\text{int}}^{\mu} [A; Q^{\mu}] = -\frac{g_0}{\mu} \int dx \bar{Q}^{\mu}(x) A(x)
\]

and

\[
I_{\text{int}}^{\mu} [A; Q^{\mu}] = -\frac{g_0}{\mu} \int dx \bar{Q}^{\mu}(x) A(x).
\]

The Coulomb interaction functional (20) is bilinear with respect to the electron density \(\bar{\psi}(x) \psi(x)\). This expression can be linearized by means of the Hubbard-Stratonovich transformation

\[
\exp \left\{ -\frac{i}{2} \int dx \int dy \bar{\psi}(x) \psi(y) u(x - y) \bar{\psi}(y) \psi(y) \right\}
\]

\[
= \frac{1}{Z^0} \int [D\psi] \exp \left\{ -\frac{i}{2} \int dx \int dy \varphi(x) u(x - y) \varphi(y) \right\}
\]

\[
\times \exp \left\{ -i \int dx \int dy \varphi(x) u(x - y) \varphi(y) \right\},
\]

(28)

where

\[
Z^0 = \int [D\psi] \exp \left\{ -\frac{i}{2} \int dx \int dy \varphi(x) u(x - y) \varphi(y) \right\},
\]

(29)

as this was proposed in references [4, 5]. The bosonic real integration variable \(\varphi(x)\) describing collective oscillations of electron gas was called the collective oscillation field. Using formulae (14), (17) and (28), we rewrite the total functional integral (13) in the new form

\[
Z_{\text{tot}} = \frac{Z_0^0}{Z^0} \int [D\varphi] \exp \left\{ -\frac{i}{2} \int dx \int dy \varphi(x) u(x - y) \varphi(y) \right\}
\]

\[
\times \left[ \int [DA] \delta \left( \frac{\partial A}{\partial x} \right) \exp \left\{ iI_0^a [A] \right\} \int [DQ^{\mu}] \right.
\]

\[
\times \exp \left\{ iI_0^o [Q^{\mu}] \right\} \int [DQ^{\mu}] \exp \left\{ iI_0^o [Q^{\mu}] \right\}
\]

\[
\times \exp \left\{ iI_{\text{int}}^{\psi} [\psi, \bar{\psi}] \right\} \exp \left\{ iI_{\text{int}}^{\mu} [\psi, \bar{\psi}; Q^{\mu}] \right\}
\]

\[
\times F \left[ \varphi; A; Q^{\mu}, Q^{\mu} \right],
\]

(30)
and

\[
F[\phi; \omega; Q^{\text{eff}}, Q^{\text{tot}}] = \frac{1}{\Lambda^2} \int [D\phi] [D\tilde{\phi}] \exp \left\{ i \int_0^\infty \left[ i \delta_{\text{eff}}[\phi, \phi] + i \int d\delta \left[ i \delta_{\text{int}}[\phi, \phi] \right] \right] \right\}
\]

\[
\times \exp \left\{ -i \int dx \int d\varphi \varphi(x) u(x-y) \varphi(y) \right\}
\]

(32)

Expanding four last exponential functions in rhs of relation (32) into power series, neglecting the very small terms proportional to 1 m\(^{-2}\) and performing the functional integration over the Grassmann variables, after lengthy but standard calculations we obtain following expression of the functional (32)

\[
F[\phi; \omega; Q^{\text{eff}}, Q^{\text{tot}}] = \exp \left\{ i W[\phi; A; Q^{\text{eff}}, Q^{\text{tot}}] \right\},
\]

(33)

where \(W[\phi; A; Q^{\text{eff}}, Q^{\text{tot}}]\) is a functional power series of \(\phi(x), A(x), Q^{\text{eff}}(x), Q^{\text{tot}}(x)\) as the functional variables:

\[
W[\phi; A; Q^{\text{eff}}, Q^{\text{tot}}] = \sum_{m=0}^{\infty} \sum_{n=p=0}^{\infty} \sum_{q=0}^{\infty} W^{(m,n,p,q)}[\phi; A; Q^{\text{eff}}, Q^{\text{tot}}],
\]

(34)

the term \(W^{(m,n,p,q)}[\phi; A; Q^{\text{eff}}, Q^{\text{tot}}]\) being a homogeneous functional polynomial of \(n, n, p, q\) orders with respect to the functions \(\phi(x), A(x), Q^{\text{eff}}(x), Q^{\text{tot}}(x)\), respectively. Substituting expression (33) of functional \(F[\phi; A; Q^{\text{eff}}, Q^{\text{tot}}]\) into rhs of formula (30), we transform the total functional \(I[\phi]\) into power series, neglecting the very small terms

\[
I[\phi] = \int dx^2 dw \left[ \frac{1}{2} \int d\varphi \varphi(x) u(x-y) \varphi(y) + \int d\delta \int d\delta \left[ i \delta_{\text{int}}[\phi, \phi] \right] \right] \phi(x)
\]

(37)

Since the functional \(W[\phi; A; Q^{\text{eff}}, Q^{\text{tot}}]\) is a series of the system of four interacting fields \(\phi(x), A(x), Q^{\text{eff}}(x), Q^{\text{tot}}(x)\) has following expression

\[
I[\phi] = \int dx^2 dw \left[ \frac{1}{2} \int d\varphi \varphi(x) u(x-y) \varphi(y) + \int d\delta \int d\delta \left[ i \delta_{\text{int}}[\phi, \phi] \right] \right] \phi(x)
\]

(36)

By grouping suitable terms from the expression in rhs of formula (37), we can derive the expression of total action functional of any subsystem of above-mentioned interacting system of four-fields \(\phi(x), A(x), Q^{\text{eff}}(x), Q^{\text{tot}}(x)\).

The first subsystem is the collective oscillation field \(\phi(x)\). In references [4, 5] it was shown that this field is split into two parts

\[
\phi(x) = \phi_0(x) + \zeta(x),
\]

(38)

where \(\phi_0(x)\) is the static background field, \(\phi_0(x) = \phi_0(x, t) = \phi_0(x)\) corresponding to the extreme value of the action functional \(I_0[\phi]\) of this field in the harmonic approximation

\[
I_0[\phi] = \int dx \int d\varphi \varphi(x) u(x-y) \varphi(y) + \int d\delta \int d\delta \left[ i \delta_{\text{int}}[\phi, \phi] \right] \phi(x)
\]

(39)

\[
\text{and } \zeta(x) \text{ is the field of small fluctuations around background field } \phi_0(x). \text{ We call } \zeta(x) \text{ the fluctuation field. In terms of } \phi_0(x) \text{ and } \zeta(x) \text{ the action functional } I_0[\phi] \text{ has the expression}
\]

\[
I_0[\phi_0 + \zeta] = I_0[\phi_0] + I_{\text{eff}}[\zeta],
\]

(40)

where

\[
I_{\text{eff}}[\zeta] = \int dx \int d\varphi \varphi(x) K(x-y) \zeta(y),
\]

(41)

\[
K(x-y) = u(x-y) + \int dx' \int dx'' \Pi(x'-y') u(x'-y' - y),
\]

(42)

\[
\Pi(x-y) = -i G(x-y) G(y-x),
\]

(43)

\(G(x-y)\) is the two-point Green function of free electron. Denote \(\zeta[k, \omega]\) and \(K[k, \omega]\) the Fourier transforms of the field \(\zeta(x)\) and the kernel \(K(x-y)\). It was shown that in the case of a homogeneous electron gas

\[
\hat{K}(k, \omega) = \frac{4\pi e^2}{\varepsilon_0 k^2 \omega^2} (\omega^2 - \omega_0^2 - \gamma^2 k^2),
\]

(44)

where \(\omega_0\) is the plasma frequency of the electron gas

\[
\omega_0^2 = \frac{4\pi e^2 n_0}{\varepsilon_0 m},
\]

(45)

\(n_0\) is the electron density and

\[
\gamma^2 = \frac{3}{5} \frac{p_F^2}{m^2},
\]

(46)

\(p_F\) is the electron momentum at the Fermi surface. In terms of the Fourier transforms \(\zeta[k, \omega]\) and \(K[k, \omega]\) formula (73) becomes

\[
I_{\text{eff}}[\zeta] = \frac{1}{(2\pi)^3} \int dk \int d\omega \int d\omega \frac{1}{2} \hat{K}(k, \omega)^* \hat{\zeta}(k, \omega)
\]

× \(K(k, \omega) \zeta(k, \omega).
\]

(47)

Setting

\[
\delta(k, \omega) = \sqrt{\frac{4\pi e^2}{\varepsilon_0 \omega^2} \zeta(k, \omega)}
\]

(48)
and introducing the scalar field \( \sigma(x, t) \) with the Fourier transforms \( \hat{\sigma}(\mathbf{k}, \omega) \), we rewrite \( I_{0}[\zeta] \) in the new form

\[
I_{0}[\zeta] = I_{0}^{\mu}[\sigma] = \int dx \int dt \left\{ \frac{1}{2} \left[ \left( \frac{d\sigma(x, t)}{dt} \right)^{2} - \gamma^{2} \left[ \nabla \sigma(x, t) \right]^{2} - \omega_{0}^{2} \sigma(x, t) \right] \right\}.
\]

(49)

Functional (49) is the action functional of free plasmon field. It has the form similar to the action functional of the Klein–Gordon real scalar field in relativistic quantum field theory [10–13], except for a scaling factor \( \gamma \) at the spatial coordinates. After the canonical quantization procedure, real scalar field \( \sigma(x, t) \) becomes a Hermitian quantum field, whose quantum is plasmon: the quantum plasmon field. The expression of \( \sigma(x, t) \) in terms of the destruction and creation operators of plasmon was known.

Thus the quantum plasmon field based on the study of collective oscillation field \( \phi \) as the fundamental subsystem has been constructed. Another important subsystem is that of phonon fields developed in the preceding section 2. The quantum fields of interacting plasmon–phonon subsystem were also constructed in reference [14]. The quantum field theory of plasmon–phonon subsystem is the subject of the next section.

4. Quantum fields of interacting plasmon–phonon system

Now we study in detail the interacting plasmon–phonon system. In order to avoid lengthy expression we limit to the harmonic approximation with respect to two types of fields: (i) the collective oscillation field and (ii) both acoustic and optic phonon fields. Moreover, since the interaction of longitudinal phonons with electron is much stronger than that of transverse phonons, we can neglect the contribution of transverse phonons in the phenomena and processes in which there exists the competition of longitudinal phonons, and retain the electron–transverse phonon interaction only when the transverse phonons play the essential role. In particular, the contribution of transverse phonons must be taken into account when we consider the phenomena and processes with the participation of the transverse electromagnetic field, as this will be performed in the next section.

First we note that the electron–phonon interaction leads to the interaction of phonons with the collective oscillation field. The action functional of the interaction between the fields \( \phi \), \( Q^{aw}(\chi) \), \( Q^{ow}(\chi) \) has the expression of the form

\[
I_{\text{int}}[\phi; Q^{aw}, Q^{ow}] = \sum_{m=1}^{\infty} \sum_{p=1}^{\infty} I_{(m,p,a)}^{(a)}[\phi; Q^{aw}, Q^{ow}]
+ \sum_{m=1}^{\infty} \sum_{p=1}^{\infty} I_{(m,p,q)}^{(a)}[\phi; Q^{aw}, Q^{ow}]
+ \sum_{m=1}^{\infty} \sum_{p=1}^{\infty} I_{(m,p,q)}^{(o)}[\phi; Q^{aw}, Q^{ow}],
\]

(50)

where

\[
I_{(m,p,a)}^{(a)}[\phi; Q^{aw}, Q^{ow}] = W_{(m,p,a)}^{(a)}[\phi; A; Q^{aw}, Q^{ow}].
\]

(51)

Because the plasmon is the quasiparticle generated by the fluctuation \( \zeta(x) \) around the background field \( \phi_{0}(x) \), the state \( \phi_{0}(x) \) must be considered as the physical vacuum of the plasmon field. Therefore the term \( I_{\text{int}}[\phi_{0}; Q^{aw}, Q^{ow}] \) must be included into the total action functional of the subsystem comprising only phonon fields \( Q^{aw} \) and \( Q^{ow} \):

\[
I_{\text{int}}^{(p)}[Q^{aw}, Q^{ow}] = I_{0}^{(p)}[Q^{aw}] + I_{0}^{(p)}[Q^{ow}] + I_{\text{int}}[\phi_{0}; Q^{aw}, Q^{ow}].
\]

(52)

In order to avoid the lengthy and complicated formulae and as the simple example, let us limit to the first order approximation \( (m=1) \) with respect to the field \( \phi_{0}(x) \) in the expression (50) of \( I_{\text{int}}[\phi_{0}; Q^{aw}, Q^{ow}] \). Then in the harmonic approximation with respect to the phonon fields \( \rho+q \leq 2 \) we have following action functional of the subsystem of phonon fields \( Q^{aw} \) and \( Q^{ow} \) interacting with the background field \( \phi_{0}(x) \) of the collective oscillations of the interacting electron gas

\[
I_{\text{int}}^{(p)}[Q^{aw}, Q^{ow}] = \sum_{l} \int dx \frac{1}{2} \left[ (\dot{Q}^{aw}(x))^{2} - \phi_{0}^{2}(x) \right] + \int dx \frac{1}{2} \left[ (\dot{Q}^{ow}(x))^{2} - \phi_{0}^{2}(x) \right] + \int dx \int dy \left[ g_{a}(\dot{Q}^{aw}(x)) + g_{o}(\dot{Q}^{ow}(x)) \right] \times u(x - y) \Pi(x - y) \left[ g_{a}(\dot{Q}^{aw}(x)) + g_{o}(\dot{Q}^{ow}(x)) \right] + \int dx \int dy \left[ g_{a}(\dot{Q}^{aw}(x)) + g_{o}(\dot{Q}^{ow}(x)) \right] \times \Pi(x - y) \left[ g_{a}(\dot{Q}^{aw}(x)) + g_{o}(\dot{Q}^{ow}(x)) \right] + \int dx \int dy \left[ g_{a}(\dot{Q}^{aw}(x)) + g_{o}(\dot{Q}^{ow}(x)) \right] \times \Pi(x - y) \left[ g_{a}(\dot{Q}^{aw}(x)) + g_{o}(\dot{Q}^{ow}(x)) \right]
+ \frac{1}{2} \int dx \int dy \left[ g_{a}(\dot{Q}^{aw}(x)) + g_{o}(\dot{Q}^{ow}(x)) \right] \Pi(x - y) \left[ g_{a}(\dot{Q}^{aw}(x)) + g_{o}(\dot{Q}^{ow}(x)) \right],
\]

(53)

where

\[
\Pi(x - y) = -[G(x - y)G(y - z)G(z - x) + G(x - z)G(z - y)G(y - x)].
\]

(54)

It consists of two parts

\[
I_{\text{int}}^{(p)}[Q^{aw}, Q^{ow}] = I_{0}^{(p)}[Q^{aw}] + I_{0}^{(p)}[Q^{ow}] + I_{\text{int}}[\phi_{0}; Q^{aw}, Q^{ow}].
\]

(55)
where the first part
\[
I_{\delta}[\mathbf{Q}^{al}, \mathbf{Q}^{ql}] = \int dx \frac{1}{2} \left[ \left( \mathbf{Q}^{al}(x) \right)^2 - u_i^2 \sum_{i=1}^{3} (\partial_i \mathbf{Q}^{al}(x))^2 \right] + \int dx \frac{1}{2} \left[ \left( \mathbf{Q}^{ql}(x) \right)^2 - \Omega_i^2 (\partial_i \mathbf{Q}^{ql}(x))^2 \right]
\]
(56)

is the action functional of the free transverse phonons, and
\[
I_{\delta}[\mathbf{Q}^{ol}, \mathbf{Q}^{fl}] = \int dx \frac{1}{2} \left[ \left( \mathbf{Q}^{ol}(x) \right)^2 - u_i^2 \sum_{i=1}^{3} (\partial_i \mathbf{Q}^{ol}(x))^2 \right] + \int dx \frac{1}{2} \left[ \left( \mathbf{Q}^{fl}(x) \right)^2 - \Omega_i^2 (\partial_i \mathbf{Q}^{fl}(x))^2 \right] - \delta \int dx \left[ g_u \left( \nabla \mathbf{Q}^{fl}(x) \right) + g_u \left( \nabla \mathbf{Q}^{ol}(x) \right) \right]
\]

\[
+ \int dx \int dx' \int dy \left( \phi_{ol}(x', y) \Pi(x - y) \right) + \int dx \int dx' \int dy \left( \phi_{fl}(y') \Pi(x - y) \right)
\]
\[
+ \frac{1}{2} \int dx \int dx' \int dy \left[ g_u \left( \nabla \mathbf{Q}^{ol}(x) \right) + g_u \left( \nabla \mathbf{Q}^{fl}(x) \right) \right] + \frac{1}{2} \int dx \int dx' \int dy \left( \phi_{ol}(x', y) \Pi(x - y) \right)
\]
\[
+ \Pi(x - y, x - z) \left[ g_u \left( \nabla \mathbf{Q}^{ol}(y) \right) + g_u \left( \nabla \mathbf{Q}^{fl}(y) \right) \right]
\]
\[
+ \frac{1}{2} \int dx \int dx' \int dy \left[ g_u \left( \nabla \mathbf{Q}^{ol}(x) \right) + g_u \left( \nabla \mathbf{Q}^{fl}(x) \right) \right]
\]
(57)

is the total action functional of the acoustic as well as optic longitudinal phonons interacting with the background field \( \phi_{ol}(x) \) of collective oscillation field \( \phi(x) \). The interaction action functional in this expression leads to the mixing between acoustic and optic phonons.

From the extreme action principle
\[
\frac{\delta I_{\delta}[\mathbf{Q}^{al}, \mathbf{Q}^{ol}]}{\delta \mathbf{Q}^{al}(x)} = \frac{\delta I_{\delta}[\mathbf{Q}^{ol}]}{\delta \mathbf{Q}^{ol}(x)} = 0
\]
(58)

it follows the system of differential-integral equations for the background phonon fields \( \mathbf{Q}^{al}(x) \) and \( \mathbf{Q}^{ol}(x) \) corresponding to the extreme value of the action functional (57):
\[
\frac{\partial^2 \mathbf{Q}^{al}(x)}{\partial t^2} - u_i^2 \sum_{i=1}^{3} (\partial_i \mathbf{Q}^{al}(x))^2 = g_u \nabla n(x)
\]
\[
- g_u \int dy \int dy' \nabla \Pi(x - y) u(y - y') \phi_{ol}(y')
\]
\[
- g_u^2 \int dy \nabla I \Pi(x - y) \left( \nabla \mathbf{Q}^{ol}(y) \right)
\]
\[
- g_u g_u \int dy \nabla \Pi(x - y) \left( \nabla \mathbf{Q}^{ol}(y) \right) - g_u^2 \int dy \int dz \nabla \Pi(x - y, z - \zeta) \left( \nabla \mathbf{Q}^{ol}(y) \right) u(z - \zeta) \phi_{ol}(\zeta)
\]
\[
- g_u g_u \int dy \int dz \int dz' \nabla \Pi(x - y, z - \zeta) \left( \nabla \mathbf{Q}^{ol}(y) \right) u(z - \zeta) \phi_{ol}(\zeta),
\]
(59)
In order to exhibit the property of these fields to be longitudinal let us use following modified Fourier expansion

\[
\mathbf{q}^{pl, ol}(x, t) = \mathbf{q}^{pl, ol}(x, t) = -i \frac{1}{(2\pi)^2} \int dk \frac{k}{|k|} \tilde{\theta}^{\alpha, \sigma}(k)
\]

so that

\[
\nabla \mathbf{q}^{pl, ol}(x) = \frac{1}{(2\pi)^2} \int dk e^{ikx} |k| \tilde{\theta}^{\alpha, \sigma}(k).
\]

In terms of the modified Fourier transforms $\tilde{\theta}^{\alpha, \sigma}(k)$ the action functional (63) becomes

\[
\mathcal{I}_{\text{eff}}^{\alpha} \left[ \mathbf{q}^{\alpha}, \mathbf{q}^{\alpha} \right] = \frac{1}{(2\pi)^2} \int dk \frac{1}{2} \left\{ \tilde{\theta}^{\alpha}(k)^* \left( k_0^2 - u_0^2 k^2 \right) \tilde{\theta}^{\alpha}(k) + \tilde{\mathbf{P}} \left( \tilde{\theta}^{\alpha}(k) + g_0 \tilde{\theta}^{\alpha}(k) \right) \right\}
\]

\[
\times |k|^2 \tilde{\mathbf{F}}(k) \left[ g_0 \tilde{\theta}^{\alpha}(k) + g_0 \tilde{\theta}^{\alpha}(k) \right] + \frac{1}{(2\pi)^2} \int dk \frac{1}{2} \int dl \frac{1}{2} \tilde{p}_0(l) \tilde{u}(l) \tilde{\theta}^{\alpha}(k - l) + g_0 \tilde{\theta}^{\alpha}(k - l) \right\}
\]

\[
\times |k - l| \tilde{\mathbf{F}}(k - l, k) \left[ g_0 \tilde{\theta}^{\alpha}(k) + g_0 \tilde{\theta}^{\alpha}(k) \right].
\]

Thus the total action functional of longitudinal phonon fields in the harmonic approximation consists of two parts

\[
\mathcal{I}_{\text{eff}}^{\alpha} \left[ \mathbf{q}^{\alpha}, \mathbf{q}^{\alpha} \right] = \mathcal{I}^{\text{nl}}_{\alpha} \left[ \mathbf{q}^{\alpha}, \mathbf{q}^{\alpha} \right] + \mathcal{I}^{\text{int}}_{\alpha} \left[ \mathbf{q}^{\alpha}, \mathbf{q}^{\alpha} \right],
\]

where

\[
\mathcal{I}^{\text{nl}}_{\alpha} \left[ \mathbf{q}^{\alpha}, \mathbf{q}^{\alpha} \right] = \int dx \int dy \left[ g_0 \mathbf{q}^{\alpha}(x) + g_0 \mathbf{q}^{\alpha}(y) \right] \times \mathbf{F}(x - y) \left[ g_0 \mathbf{q}^{\alpha}(y) + g_0 \mathbf{q}^{\alpha}(y) \right]
\]

is the interacting functional describing the elastic scattering of phonons in the effective potential field $\mathbf{F}(x - y)$ as well as the mixing between longitudinal acoustic and optic phonons.

Note that the effective potential $\mathbf{F}(x - y)$ is both non-local and non-instantaneous.

Since plasmons are generated by the fluctuation $\zeta(x)$ of the collective oscillation field $\mathbf{q}(x)$ around its static background $\mathbf{q}_0(x)$, in order to study the plasmon–phonon interaction first we derive the expression of the action functional $\mathcal{I}_{\text{pl}}^{\beta}[\zeta; \mathbf{Q}^{\alpha}, \mathbf{Q}^{\alpha}]$ of the interaction between the fluctuation field and phonon fields. We have

\[
\mathcal{I}_{\text{pl}}^{\beta} \left[ \zeta; \mathbf{Q}^{\alpha}, \mathbf{Q}^{\alpha} \right] = \mathcal{I}_{\text{pl}}^{\text{nl}} \left[ \zeta; \mathbf{Q}^{\alpha}, \mathbf{Q}^{\alpha} \right] + \mathcal{I}_{\text{pl}}^{\text{int}} \left[ \zeta; \mathbf{Q}^{\alpha}, \mathbf{Q}^{\alpha} \right],
\]

where $\mathcal{I}_{\text{pl}}^{\text{nl}}[\zeta; \mathbf{Q}^{\alpha}, \mathbf{Q}^{\alpha}]$ is determined by formula (50) and (51). Because the terms containing transverse phonon fields are very small, we discard them and retain only the terms containing longitudinal fields. According to formula (61) each of them consists of two parts: the background longitudinal field $\mathbf{Q}^{pl} \alpha(x)$ or $\mathbf{Q}^{pl} \alpha(x)$ and the dynamical longitudinal phonon field $\mathbf{q}^{pl}(x)$ or $\mathbf{q}^{pl}(x)$. Consider again the case of homogeneous electron gas. Then in the harmonic approximation with respect to both types of fields (fluctuation field and phonon fields) the action functional can be represented in the
general form as follows:

\[
I_{\text{int}}^{(a)} \left[ \zeta, Q^{(a)} \right] = \int dx \int dy \, \zeta(x) \times \sum_{i=1}^{3} \left[ F_{i}^{(a)}(x, y) Q^{(a)}(y) + F_{i}^{(a)}(y, x) Q^{(a)}(x) \right] + \int dx_{1} \int dx_{2} \int dy \, \zeta(x_{1}) \zeta(x_{2}) \times \sum_{i=1}^{3} \left[ F_{i}^{(a)}(x_{1}, x_{2}, y) Q^{(a)}(y) + F_{i}^{(a)}(x_{1}, y, x_{2}) Q^{(a)}(x_{2}) \right] + \int dx \int dy_{1} \int dy_{2} \, \zeta(x) \times \sum_{i=1}^{3} \sum_{j=1}^{3} \left[ F_{i}^{(a)}(x, y_{1}, y_{2}) Q^{(a)}(y_{1}, y_{2}) + F_{j}^{(a)}(x, y_{1}, y_{2}) Q^{(a)}(y_{2}, y_{1}) \right]
\]

(77)

The Fourier transform \( \tilde{\zeta}(k, \omega) \) of the fluctuation field \( \zeta(x) \) is expressed in terms of the Fourier transform \( \tilde{\sigma}(k, \omega) \) of the fluctuation field \( \sigma(x) \) according to formula (48), i.e.

\[
\tilde{\zeta}(k, \omega) = \sqrt{\frac{\varepsilon_{0} \omega^{2} k^{2}}{4 \pi c^{2}}} \tilde{\sigma}(k, \omega).
\]

(78)

Therefore between \( \zeta(x) \) and \( \sigma(x) \) there exists following linear functional relation

\[
\zeta(x) = \int dy \, T(x - y) \sigma(y),
\]

(79)

where

\[
T(x - y) = \frac{1}{(2\pi)^{2}} \int dk \int d\omega \, e^{i[k(x-y)-\omega(y_{0}-y_{0})]} \times \sqrt{\frac{\varepsilon_{0} \omega^{2} k^{2}}{4 \pi c^{2}}}.
\]

(80)

Similarly, according to formula (64), between the dynamical longitudinal phonon fields \( q^{(\sigma)}(x) \) and the scalar fields \( \theta^{(\sigma)}(x) \) there exists following linear functional relation

\[
q^{(\sigma)}(x) = \int dy \, R(x - y) \theta^{(\sigma)}(y),
\]

(81)

where

\[
R(x - y) = -\frac{i}{(2\pi)^{2}} \int dk \int d\omega \, e^{i[k(x-y)-\omega(x_{0}-y_{0})]} \frac{k}{|k|}.
\]

(82)

It is straightforward to derive the expression of the action functional of the interaction of plasmon with longitudinal phonons from formulae (61), (77), (79) and (81).

5. Quantum fields of interacting plasmon–photon–phonon system

On the basis of the results obtained in preceding sections we consider now the whole system of interacting plasmon, photon and phonons. The total action functional of this system has the expression (37). The collective oscillation scalar field \( q_{0}(x) \) and the fluctuation field \( \zeta(x) \). Each longitudinal phonon field \( Q^{(a)}(x) \), \( Q^{(\sigma)}(x) \) is also split into two parts: the static background field \( q_{0}(x) \) and the dynamical ones without the background fields: \( Q^{(a)}(x) = q^{(a)}(x) \) and \( Q^{(\sigma)}(x) = q^{(\sigma)}(x) \). Meanwhile the transverse acoustic and optic phonon are the quanta of the original transverse vector fields \( q^{(a)}(x) \) and \( q^{(\sigma)}(x) \). The corresponding quasiparticles are called effective longitudinal phonon fields. In order to shorten lengthy formulae, they are introduced and used instead of the original longitudinal vector fields \( q^{(a)}(x) \) and \( q^{(\sigma)}(x) \). Above-mentioned quantized fields have following expansions in terms of the destruction and creation operators of the corresponding quasiparticles:

\[
\sigma(x, t) = \frac{1}{(2\pi)^{3}} \int dk \int d\omega \, \frac{1}{\sqrt{2\omega_{0}^{(\sigma)}}} \left[ c_{k} e^{i[k_{x}-\omega_{0}(k)t]} + c_{k}^{*} e^{-i[k_{x}-\omega_{0}(k)t]} \right],
\]

(83)

where \( c_{k} \) and \( c_{k}^{*} \) are the destruction and creation operators of the plasmon with momentum \( k \) and \( \omega_{0}(k) \) is its energy,

\[
\theta^{(\sigma)}(x, t) = \frac{1}{(2\pi)^{3}} \int dk \int d\omega \, \frac{1}{\sqrt{2\omega_{0}^{(\sigma)}}} \left[ a_{k} e^{i[k_{x}-\omega_{0}(k)t]} + a_{k}^{*} e^{-i[k_{x}-\omega_{0}(k)t]} \right],
\]

(84)

and

\[
\theta^{(a)}(x, t) = \frac{1}{(2\pi)^{3}} \int dk \int d\omega \, \frac{1}{\sqrt{2\omega_{0}^{(a)}}} \left[ b_{k} e^{i[k_{x}-\omega_{0}(k)t]} + b_{k}^{*} e^{-i[k_{x}-\omega_{0}(k)t]} \right],
\]

(85)

where \( a_{k} \) or \( b_{k} \) and \( a_{k}^{*} \) or \( b_{k}^{*} \) are the destruction and creation operators, respectively, of the longitudinal acoustic or optic phonon with momentum \( k \) and energy.
\( a_\omega(k) \approx v_\text{f} k \) or \( a_\omega(k) \approx \Omega_i, \)

\[
q_{\text{fl}}(x, t) = \frac{1}{(2\pi)^3} \int \frac{dk}{2\omega_i(k)} \sum_{i=1}^{2} \xi_i^{(\omega)}(k) \times \left[ \frac{a_i^+(k)e^{i(kx-v_\text{f}k)t} + a_i^-(k)e^{-i(kx-v_\text{f}k)t}}{\xi_i^{(\omega)}(k)} \right] 
\]

and

\[
q_{\text{at}}(x, t) = \frac{1}{(2\pi)^3} \int \frac{dk}{2\omega_i(k)} \sum_{i=1}^{2} \xi_i^{(\omega)}(k) \times \left[ \frac{b_i^+(k)e^{i(kx-v_\text{f}k)t} + b_i^-(k)e^{-i(kx-v_\text{f}k)t}}{\xi_i^{(\omega)}(k)} \right],
\]

where \( a_i^+(k) \) or \( b_i^+(k) \) and \( a_i^-(k) \) or \( b_i^-(k) \) are the destruction and creation operators, respectively, of the transverse acoustic or optic phonon with momentum \( k \), energy \( \omega_i(k) \approx v_\text{f} k \) or \( a_\omega(k) \approx \Omega_i \), and polarization vector \( \xi_i^{(\omega)}(k) \), \( s \) being the index indicating the polarization state of the transverse phonons with momentum \( k \)

\[
k \xi_i^{(\omega)}(k) = 0. \quad (88)
\]

The total action functional of the system of interacting plasmon, photon and phonons has the expression of the form

\[
I_{\text{tot}} \left[ \phi_0 + \zeta ; A ; Q_{\text{fl}}^{\text{ph}} + Q_{\text{at}}^{\text{ph}} + Q_{\text{at}}^{\text{ph}} \right] = \frac{1}{2} \int_0^\infty \int dx \int dy \left[ \phi_0(x) + \zeta \right] u(x - y) \times \left[ \phi_0(y) + \zeta(y) \right] + I_0^\text{fl} \left[ A ; Q_{\text{fl}}^{\text{ph}} + Q_{\text{at}}^{\text{ph}} \right] + I_0^\text{at} \left[ A ; Q_{\text{at}}^{\text{ph}} \right] + I_0^\text{at} \left[ A ; Q_{\text{at}}^{\text{ph}} \right] + I_0^\text{at} \left[ A ; Q_{\text{at}}^{\text{ph}} \right] + I_0^\text{at} \left[ A ; Q_{\text{at}}^{\text{ph}} \right] + Q_{\text{fl}}^{\text{ph}} + Q_{\text{at}}^{\text{ph}} + Q_{\text{at}}^{\text{ph}} \right]
\]

with \( Q_0^{\text{fl}}(x) = Q_0^{\text{at}}(x) = 0 \). After the quantization procedure the fluctuation field \( \zeta(x) \), the fluctuating longitudinal phonon fields \( \phi_{\text{fl}}(x) \) and \( \phi_{\text{at}}(x) \), the transverse phonon fields \( \phi_{\text{at}}(x) \) and \( \phi_{\text{at}}(x) \) become the field operators, while the background fields \( \phi_0(x), Q_0^{\text{fl}}(x) \) and \( Q_0^{\text{at}}(x) \) remain to be classical fields. Quantum fluctuation field is expressed in terms of the plasmon quantum field \( \sigma(x) \) through the linear functional relation (79), longitudinal phonon fields are expressed in terms of the longitudinal phonon effective scalar fields \( \theta^\text{fl}(x), \theta^\text{at}(x) \) by means of the linear functional relation (81). The quantum of \( \sigma(x) \) is plasmon, the quanta of \( \theta^\text{fl}(x) \) and \( \theta^\text{at}(x) \) are longitudinal phonons, and the quanta of \( \phi_{\text{at}}(x) \) are transverse phonons. Thus in the whole system of interacting plasmon, photon and phonons there are three types of dynamical fields:

(1) Fluctuation field \( \zeta(x) \),
(2) Transverse electromagnetic field \( A(x) \),
(3) Longitudinal as well as transverse acoustic and optic phonon fields \( \phi_{\text{fl}}(x), \phi_{\text{at}}(x) \) and \( \phi_{\text{at}}(x), \phi_{\text{at}}(x) \).

The total action functional of the whole system can be represented in the form

\[
I_{\text{tot}} \left[ \zeta ; A ; Q^{\text{fl}}, Q^{\text{at}}, Q^{\text{at}} \right] = I_0 \left[ \zeta ; A ; Q^{\text{fl}}, Q^{\text{at}}, Q^{\text{at}} \right] + I_{\text{int}} \left[ \zeta ; A ; Q^{\text{fl}}, Q^{\text{at}}, Q^{\text{at}} \right],
\]

where the first term in rhs of equation (90) is the action function of free fields and second term is the interaction action functional. Action functional of free fields is the sum of action functional of all six free dynamical fields

\[
I_0 \left[ \zeta ; A ; Q^{\text{fl}}, Q^{\text{at}}, Q^{\text{at}}, Q^{\text{at}} \right] = I_{0\text{fl}} \left[ Q^{\text{fl}}, Q_0^{\text{fl}} \right] + I_{0\text{at}} \left[ Q^{\text{at}}, Q^{\text{at}}_0 \right] + I_{0\text{at}} \left[ Q^{\text{at}}, Q^{\text{at}}_0 \right] + I_{0\text{at}} \left[ Q^{\text{at}}, Q^{\text{at}}_0 \right] + I_{0\text{at}} \left[ Q^{\text{at}}, Q^{\text{at}}_0 \right] + I_{0\text{at}} \left[ Q^{\text{at}}, Q^{\text{at}}_0 \right].
\]

All six terms in rhs of equation (91) were given in preceding sections. The expression of interaction action functional has the general form

\[
I_{\text{int}} \left[ \zeta ; A ; Q^{\text{fl}}, Q^{\text{at}}, Q^{\text{at}}, Q^{\text{at}} \right] = \sum_{i=0}^\infty \sum_{j=0}^\infty \sum_{k=0}^\infty \sum_{l=0}^\infty \sum_{m=0}^\infty \sum_{n=0}^\infty \sum_{p=0}^\infty \sum_{q=0}^\infty \sum_{l_1=0}^\infty \sum_{l_2=0}^\infty \sum_{l_3=0}^\infty \sum_{l_4=0}^\infty \left( I^{(i,j,k,l_1,l_2)} \right) \left[ \zeta ; A ; Q^{\text{fl}}, Q^{\text{at}}, Q^{\text{at}}, Q^{\text{at}} \right] \]

The term \( I^{(i,j,k,l_1,l_2)}(\zeta; A; Q^{\text{fl}}, Q^{\text{at}}, Q^{\text{at}}, Q^{\text{at}}) \) is a homogeneous functional polynomial of \( \zeta \), \( A \), \( Q^{\text{fl}} \), \( Q^{\text{at}} \) and \( Q^{\text{at}} \), \( Q^{\text{at}} \) and \( Q^{\text{at}} \), \( Q^{\text{at}} \) and \( Q^{\text{at}} \), respectively. According to formulae (79) and (81), the fluctuation field \( \zeta(x) \) is a linear functional of plasmon field \( \sigma(x) \), and longitudinal phonon fields \( Q^{\text{fl}}(x) \) are the linear functional of two effective scalar phonons as two spinless quasiparticles. Substituting rhs of formulae (79) and (81) into the expression (92), we obtain the interaction action functional in the form of a functional of the scalar fields \( \sigma(x), \theta^\text{fl}(x), \theta^\text{at}(x) \) and \( \theta^\text{at}(x) \) of plasmon and longitudinal phonons, and transverse vector fields \( \phi_{\text{fl}}(x) \) and \( \phi_{\text{at}}(x) \) of transverse phonons.

In the diagrammatic representation of the finally derived expression of the interaction action functional, each term is represented by a corresponding vertex with definite external lines. The number of external lines of each type indicate the number of corresponding particles or quasiparticles participating in the represented interaction fact.

6. Conclusion and discussions

In the present work, by means of the powerful functional integral technique, the quantum fields of the interacting system of plasmons, photons and phonons in electron gas of solids were constructed. The starting assumptions are the fundamental principles of electrodynamics and quantum
theory of many-body systems. The general form of the formula of total action functional was established. The whole system is described by a set of six fields: the scalar plasmon field $\sigma(x)$, transverse electromagnetic field $A(x)$, the effective scalar fields $\theta^a(x)$ and $\theta^0(x)$ of longitudinal phonons as spinless quasiparticles, the transverse phonon vector fields $q^{at}(x)$ and $q^{ot}(x)$.

$$\nabla q^{at}(x) = \nabla q^{ot}(x) = 0.$$  

The total interaction action functional of the whole system is a series in which each term represents a definite interaction fact.

The derived interaction action functional has following particular feature: all terms in its expression represent the non-local and non-instantaneous interaction between the involved quantum fields. The Lagrangian and Hamiltonian similar to those in traditional quantum field theory do not exist. Although each term in the expression of the interaction action functional is the matrix element of a physical process in the first order approximation, the matrix elements of physical processes in higher order approximations cannot be calculated by means of the traditional perturbation theory. Therefore it is necessary to elaborate the new method for calculating the matrix elements of physical processes in higher orders from the formula of total action functional derived in the present work. Thus a lot of theoretical works should be performed in order to fulfill the construction of a complete quantum theory of physical phenomena and processes with the participation of plasmon.

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