Comment on “New Experimental Limit on the Electric Dipole Moment of the Neutron”

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Abstract

A new limit for the neutron electric dipole moment has been recently reported. This new limit is obtained by combining the result from a previous experiment with the result from a more recent experiment that has much worse statistical accuracy. We show that the old result has a systematic error possibly four times greater than the new limit, and under the circumstances, averaging of the old and new results is statistically invalid. The conclusion is that it would be more appropriate to quote two independent but mutually supportive limits as obtained from each experiment separately.

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Recently, a new experimental limit for the neutron permanent electric dipole moment (EDM) has been reported [1] with a substantial increase in sensitivity as compared to the previous result [2]. The increase in sensitivity results from combining the old experimental result with new data that has considerably worse statistical accuracy. As we discuss below, the averaging of the old and new results is not justified in this situation and one would better quote two independent results that are mutually consistent with and supportive of each other. It makes no sense to average the results because there are unexplained systematic errors in the old data that exceed the new 95% confidence limit given in [1] by a factor of at least two.

The new quoted 90% confidence limit of $|d_n| < 6.3 \times 10^{-26} \text{ cm}$ was obtained by combining the result of the new (1999) experiment

$$d_n = 1.9 \pm 5.4 \times 10^{-26} \text{ cm}; \quad \chi^2/\nu = 0.4$$

(see Fig. 1, where $\nu = 9$) with the previous (1990) result,

$$-3 \pm 5 \times 10^{-26} \text{ cm} \quad (1)$$

for which the error is dominated by systematic uncertainty in the data. The 95% confidence limit for the 1990 data is given as

$$|d_n| < 12 \times 10^{-26} \text{ cm}. \quad (2)$$

Without attempting to correct for systematic effects, the average of the 1990 data shown in Fig. 1 is

$$d_n = -1.9 \pm 2.2 \times 10^{-26} \text{ cm}; \quad \chi^2/\nu = 3.2 \quad (3)$$

and $\nu = 14$. Unfortunately, there is no clear way to use the discrete Rb magnetometers employed in the 1990 version of the experiment to correct for the magnetic systematic effect that is evidenced through the large $\chi^2/\nu$ and bimodal character of the data; simply subtracting the average of the Rb magnetometer signals gives the result

$$d_n = -(3.4 \pm 2.6) \times 10^{-26} \text{ cm}, \quad \chi^2/\nu = 2.2; \quad (4)$$

where the reduction in $\chi^2$ is due to the increase in the effective standard deviation due to the magnetometer noise, not a decrease in the scatter of the data, as is stated in [2].

The discussion associated with Eq. (5) of [1] (also labelled here as Eq. (5)),

$$d_n = -3.4 \pm 3.9 \text{ stat} \pm 3.1 \text{ syst} \times 10^{-26} \text{ cm} \quad (5)$$

does not accurately describe the source of errors in the final quoted result. In particular, the statistical error in Eq. (5) is obtained by multiplying the uncertainty in Eq. (4) by the square root of reduced $\chi^2$, $2.6 \times \sqrt{2.2} = 3.9$. Therefore, the statistical error given in Eq. (5) has a substantial systematic contribution. Because there is no physical basis for subtracting the average Rb magnetometer signal from the neutron EDM result (as we will describe), if instead we take the intrinsic neutron EDM statistical error as that given in Eq. (3), Eq. (5) implies a systematic error of $\sqrt{3.9^2 - 2.2^2} = 3.2 \times 10^{-26} \text{ cm}$; this should be
added in quadrature to the systematic error given in Eq. (5) yielding an overall systematic uncertainty of at least

\[ 4.8 \times 10^{-26} \text{ cm} \]  

and represents a guess at how well the average systematic fluctuations averaged to zero for the entire 1990 data set. This systematic error, when combined with the intrinsic statistical error, yields the combined uncertainty given in Eq. (1).

In fact, the final systematic uncertainty was chosen so that the 95\% confidence limit would encompass the obvious excess scatter in the 1990 data, reflected by the large \( \chi^2/\nu = 3.2 \). The final 95\% confidence limit was chosen to reflect this deviation in the data because it was accepted that there is no reason to assume the random systematic fluctuations average to zero; Eq. (1) above might very well be optimistic. These fluctuations are correlated with disassembly and reassembly of the apparatus, and therefore are possibly the result of subtle changes in, for example, high-voltage leakage current paths within the magnetic shields.

We would like to take a different approach to combining the 1990 and 1999 data groups; this approach gives a more accurate estimate of a possible systematic error in the final combined number. As shown in Fig. 1, the 1990 data is better described by a bimodal distribution. Curve A is a \( 1/\sigma^2 \) bimodal weighted fit to two Gaussian probability distributions for the 1990 data alone. Curve B shows the bimodal probability distribution for the 1990 and 1999 combined data sets, while C is the combined data set when the standard errors of the 1990 data are multiplied by \( \sqrt{3.2} \). It is evident that the distribution for the combined data set, for both B and C, is still strongly bimodal and the use of standard statistical techniques as in [1] is not justified.

This is demonstrated in Fig. 2 which gives the 68.3\% and 95\% confidence regions (\( \Delta \chi^2 = 2.3 \) and \( \Delta \chi^2 = 6.17 \), respectively, for two degrees of freedom [3]) for the two means determined in the fit to the bimodal distribution (a total of six parameters in the fit). Of particular interest is Fig. 2 A; \( \chi^2_{19} \) for the optimum fit to two Gaussian distributions (each Gaussian has three fit parameters so there are \( 25 - 6 = 19 \) degrees of freedom) is 8.3 as compared to \( \chi^2_{22} = 48 \) for a fit to a single Gaussian distribution. The reduced \( \chi^2 \) for the bimodal fit is \( \chi^2_{19}/19 = 0.44 \). However, the statistical significance is best illustrated by use of the \( F \)-test [4]. Adding the second Gaussian distribution is equivalent to adding three fit parameters, and

\[ F = \frac{[\chi^2_{22} - \chi^2_{19}]/3}{\chi^2_{19}/19} = \frac{(48 - 8.3)/3}{(0.44 \rightarrow 1)} = 13.2 \]

(the division by the new reduced \( \chi^2/\nu \) is valid only when it is greater than 1). The statistical confidence of the new bimodal description of the combined data can be calculated (see Appendix C-5 of [4]); the probability of such a large \( F \) being due to statistical fluctuations is about 3\%, i.e. the statistical confidence is 97\%. There is no justification for artificially increasing the errors of the 1990 data by \( \sqrt{3.2} \) because the final combined data set without doing so as described by the bimodal distribution has a reduced \( \chi^2/\nu = 0.44 \). We again point out that even if the 1990 errors are increased by \( \sqrt{3.2} \) the full data set is still bimodal, as shown in Fig. 1 C and Fig. 2 B.

The bimodal means are of opposite sign, but of remarkably close magnitudes:
and represent the magnitude of the random systematic in the data. In Eq. (1), there is an implied faith that the systematic errors in the data average to zero, and therefore Eq. (1) must be treated with a certain caution. The Particle Data Group does indeed recommend scaling of the data by $\sqrt{\chi^2}$ as done in [1]. However they justify this by the remark that this approach has the property that if there are two values with comparable errors separated by much more than their stated errors the error on the mean is increased so that it is approximately half the interval between the two discrepant values. Applying this criterion to the present case and taking the bimodal means for the 1990 data as two discrepant values shows a systematic error of

$$\frac{d_{n1} - d_{n2}}{2} = 12 \times 10^{-26} \text{ ecm}$$

which is rather larger than the weighted error when scaled by $\sqrt{3.2}$, i.e. a factor of two greater than Eq. (6). This implies a 95% confidence limit of about $20 \times 10^{-26} \text{ ecm}$, about a factor of two greater than Eq. (2). In addition, both $\chi^2$ plots in Fig. 2 indicate a net 95% confidence limit of about $20 \times 10^{-26} \text{ ecm}$. Under the circumstances, the 95% confidence limit given in [2], Eq. (2) above, requires additional justification.

Furthermore, other analysis techniques support this magnitude of random systematic for the 1990 data. For example, when only data where the magnetometer signals were statistically zero was included in the average, a greater than two-sigma hint of a non-zero value the neutron EDM was obtained [6],

$$d_n = -(17 \pm 6) \times 10^{-26} \text{ ecm}.$$  \hspace{1cm} (10)

without correcting for the magnetometers. There is no statistical or physical basis for subtracting the average magnetometer signal, as it has been proven by use of correlation techniques [4,8] that even when the average magnetometer signal is zero, the individual magnetometers are still correlated with the electric field, and with the neutron EDM signal. This implies that the magnitude, and possibly sign, of the correlated field measured by the Rb magnetometers is different among the three magnetometers and the neutron storage cell.

A better correction for a possible systematic effect was obtained by determining the correlation coefficient between each magnetometer and the neutron EDM signal. Subtracting the correlated Rb magnetometer signal made the neutron data internally more consistent; in this analysis, before subtracting the correlated signal, $d_n = -(2.8 \pm 3.0) \times 10^{-26} \text{ ecm}$, $\chi^2 = 3.8$, and after subtraction, $d_n = -(5.0 \pm 3.8) \times 10^{-26} \text{ ecm}$, $\chi^2 = 1.4$. The decrease in $\chi^2$ in this case results from a reduction of scatter rather than an increase in the effective per point standard deviation due to magnetometer noise.

Agreement between the various analyses was used to estimate the residual random systematic in the EDM signal that did not average away, and in particular, was chosen so that the 95% confidence limit of the combined statistical and systematic error encompassed the range of systematic fluctuation, as mentioned above. Although the correlation technique is compelling, it is not proof that the systematic contribution has been fully accounted for. The implication is that the average field seen by the magnetometers does not necessarily
represent the field seen by the neutrons, and was the motivation for construction of the $^{199}$Hg co-magnetometer [3].

To our knowledge, the correlation techniques were not applied to the new data, and therefore the 1999 data cannot be used to infer anything about the source of the random systematic in the 1990 data. In particular, we disagree with the statements associated with Eq. (5) of [1]; the sources of the scatter in the previous data are still unknown, and experimental evidence suggests they are associated with the magnetic fluctuations generated with the application of high voltage; this is the systematic of principal concern. Although the statements in [1] regarding magnetic fluctuations are technically correct, the “other, unknown, systematics” suggested in [1], if they even exist, are likely irrelevant to any discussion. Again, the important point is whether the magnetic fluctuations that contaminated the data are associated with the high voltage or with other noise sources.

The new data sheds no light on the previous result, particularly when one considers the vast array of changes with the new configuration of the apparatus. For example, the much larger ultracold neutron storage vessel in the new experiment means that a discrete magnetometer placed within the shield would be relatively farther from the geometrical center of the neutron storage region, so the relative effect of a leakage current magnetic field could be quite different between the previous and present configurations of the apparatus; although Rb magnetometers were included in the new version of the apparatus, no presentation or discussion of experimental data from them is given in [1]. Of more critical concern, in [1] the apparent $^{199}$Hg EDM in the co-magnetometer data is not given, and is crucial toward understanding possible systematic effects. The graph given in Fig. 3 of [1] might seem compelling, but tells us nothing in regard to subtle effects with application of high voltage; the fluctuations shown in Fig. 3 of [1] are likely primarily due to external influences. In regard to the old data, it was accepted that external magnetic influences on the neutron EDM are well-represented by the average Rb magnetometer signal, while local fields, particularly those associated with the high voltage supply or leakage current, are not.

Under the circumstances, the result for the 1990 data, Eq. (5), should be treated with a certain caution. The final uncertainty in this result was chosen so that the 95% confidence limit would encompass the magnitude of the random systematic, $\approx 12 \times 10^{-26}$ ecm. Nothing associated with the new data would allow one to assume that the rather large random systematic averaged to zero in the previous data. It therefore is imprudent to average the old and new results. Hopefully, in the not-too-distant future, a co-magnetometer neutron EDM experiment will be possible with ultracold neutron density equal to or much greater than that obtained in connection with the 1990 version of the experiment. Only then will a truly new and improved limit for the neutron EDM, without concerns of systematic contamination of up to $12 \times 10^{-26}$ ecm or greater, be possible.

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FIGURES

FIG. 1. Previous and new neutron EDM data averaged over subsequent several-week reactor operation cycles. The horizontal axis is arbitrary and labels the chronological order of the data subsets. Particularly for the 1990 data, the apparatus was dis- and re-assembled between reactor cycles. A: Fit to a bimodal distribution for the 1990 data. B: Fit to a bimodal distribution for the combined 1990 and 1999 data. C: Fit to a bimodal distribution with the 1990 data errors multiplied by $\sqrt{3.2}$.

FIG. 2. 68.3% and 95% confidence regions for the two means in the bimodal fits. A: Combined 1990 and 1999 data. B: Combined data, 1990 data error multiplied by $\sqrt{3.2}$.
A \chi^2_{\text{min}} = 8.3; 19 \text{ d.o.f.}

B \chi^2_{\text{min}} = 4.0; 19 \text{ d.o.f.}
neutron edm \((10^{-26} \text{ e cm})\)

1990: \((-1.9 \pm 2.2) \quad \chi^2/\nu=3.2\)

1999: \((1.7 \pm 5.4) \quad \chi^2/\nu=0.4\)

all data: \((-1.3 \pm 2.1) \quad \chi^2/\nu=2.0\)