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INTEGRAL CALCULUS OF ONE-DIMENSIONAL FUNCTIONS

Personal tasks and Samples

Interactive textbook is developed for students who study on technical specialities
INTEGRAL CALCULUS OF ONE-DIMENSIONAL FUNCTIONS

Personal tasks and Samples

Electronic network tutorial edition

The interactive textbook is developed for English-speaking students whose study on Mathematical Calculus is based on classic programs of Ukrainian higher educational institutions. Its structure is as follows: the first part consists of 30 personal tasks with problems in one-dimensional integration (indefinite and definite integrals, geometric applications); the second part proposes the algorithmic solutions of the sample task with graphics built in Wolfram Mathematica 11.1, and 20 video-lessons guided by authors.

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Introduction

The changes in higher education during the last decade have increased the amount of individual work of students. As result, there appears the necessity of clear-developed guidelines and resources for learning (on-line learning) of fundamental sciences.

This textbook is created for English-speaking students whose study in Mathematical Calculus is based on modern programs of Ukrainian higher educational institutions. It includes personal tasks and samples on “Integral Calculus of One-Dimensional Functions”, one of the classic parts of Calculus course of the first year degree on technical specialities. All problems are the new ones; they were generated/tested by applying Wolfram Mathematica technologies. Guided by authors, 20 video-lessons are available for on-line learning on our educational YouTube channel.

Let us be more specific. The textbook contains 30 personal tasks and a solved sample task with guidelines (both written and interactive versions). The mathematical skeleton of each personal task consists of 4 parts:

- methods of the indefinite integration;
- evaluation of definite integrals;
- geometric applications of definite integrals, which are related to finding the metric characteristics of the curves, regions and solids of revolution;
- integration of improper integrals.

Authors illustrate on-line how to solve 20 problems from the sample task (an access from the textbook employs links and QR-codes).

In more detail, each personal task consists of 40 problems divided on 12 categories referring to fixed techniques ([2, Chapter 6], [3, Chapters 7-8], [4, Chapters 8-10], [5, Chapter 4] and [6, Part 3]):

1. to be solved, the proposed 4 problems use the reduction to the table of integration combined with general properties of indefinite integrals, and the method of substitution under the differential (see, [2, p. 267-273], [3, p. 193-202] and [6, Chapter 13]);

2. to be solved, the proposed 4 problems use the integration of fractions with quadratic functions (see, [2, p. 286-288] and [3, p. 214-216]);

3. to be solved, the proposed 6 problems use the integration by parts (3 common classes) or suitable substitutions (see, [2, p. 272-276] and [3, p. 200-203]);

4. to be solved, the proposed 3 problems use the integration techniques for polynomial fractions (see, [2, p. 276-281] and [3, p. 203-211]);
5. to be solved, the proposed 3 problems use the integration of trigonometric functions: products and rational functions of sines and cosines with equal arguments, products of mentioned functions with different arguments (see, [2, p.281-286], [3, p. 212-214] and [6, Chapter 15]);

6. to be solved, the proposed 2 problems use the integration techniques for fractions with radicals by applying the trigonometric and hyperbolic substitutions (see, [2, p. 286-289], [3, p. 214-219] and [6, Chapters 15, 17]);

7. to be solved, the proposed 4 problems use the integration techniques for definite Riemann integrals with applying the Newton-Leibnitz formula. Here, the general evaluation, integration by parts and applying of suitable substitutions are covered (see, [2, p. 290-300] and [3, p. 221-233]);

8. to be solved, the proposed 3 problems require the building of correct regions in Cartesian, Parametric and Polar coordinates, applying of suitable formulas and valid evaluation of definite integrals (see, [2, p. 306-311], [3, p. 237-242] and [6, p. 233-257]);

9. to be solved, the proposed 3 problems require the building of correct arc segments in Cartesian, Parametric and Polar coordinates, applying of suitable formulas and valid evaluation of definite integrals (see, [2, p. 311-314], [3, p. 242-245] and [6, p. 257-261]);

10. to be solved, the proposed 3 problems require the building of correct generatrices in Cartesian, Parametric and Polar coordinates, analysis of surfaces of revolution, applying of suitable formulas and valid evaluation of definite integrals (see, [2, p. 314-316], [3, p. 247-248] and [6, p. 267]);

11. to be solved, the proposed 3 problems require the building of correct rotating regions in Cartesian and Polar coordinates, analysis of solids of revolution, applying of suitable formulas and valid evaluation of definite integrals (see, [2, p. 317-319], [3, p. 245-247] and [6, p. 265-267]);

12. to be solved, the proposed 2 problems use the integration techniques for improper Riemann integrals of the 1st and 2nd kinds (see, [2, p. 300-305] and [3, p. 233-237]).

The following classic list of the curves are employed in the textbook:

- in Cartesian coordinates: ellipses, exponentials, circles, catenaries, logarithmic functions, squared and cubic parabolas (power functions), straight lines and general curves;
• in Parametric coordinates: astroids, ellipses, circles, cycloids, involutes of classic curves and spirals;

• in Polar coordinates: Archimedean and logarithmic spirals, cardioids, lemniscates (8-shaped and $\infty$-shaped) and petaled roses.

If a parametric range is omitted in the problem, it is supposed to be the whole region of the curve’s well-definiteness.

In order to visualize the shapes of the curves, regions, surfaces and solids, we strongly recommend students to use the following links:

• https://www.mathcurve.com;

• http://mathworld.wolfram.com;

• http://old.nationalcurvebank.org/volrev/volrev.htm

The suitable formulas for evaluation of metric characteristics are available as well.

The sample task consists on algorithmic solutions of the proposed 40 problems. Here, we point out the applied techniques for indefinite and definite integrals (categories 1–7), develop step-by-step algorithms with 2D/3D-simulations via Wolfram Mathematica 11.1 for geometric problems (categories 8-11) and solve the problems with improper integrals (category 12). Based on problems from the sample task, we created 20 illustrative video-lessons on techniques of integration. These lessons were prepared with KPI TV© Team and nowadays are available on our educational YouTube channel:

• https://www.youtube.com/channel/UCHrLMGhP7cM664BCF_e32Qg

The list of problems covered in video-lessons is attached below (see, pages 70-73); the fast access is possible to achieve within links and QR-codes.

This interactive textbook finishes with references on the best books in our framework complemented by useful external links.

To remove possible misunderstandings, the next page exhibits the differences between some notations of Post-Soviet countries and the foreign ones.
On notions used in the textbook

Following by mathematical traditions stated in Ukraine (Post-Soviet countries), we have used the following notions for some trigonometric, hyperbolic functions and their inverses:

- $\text{tg} \ x$ instead of $\tan x$, $\text{tg} \ x = \frac{\sin x}{\cos x}$;
- $\text{ctg} \ x$ instead of $\cot x$, $\text{ctg} \ x = \frac{\cos x}{\sin x}$;
- $\arctg \ x$ instead of $\arctan x$;
- $\arccctg \ x$ instead of $\arccot x$, $\arccctg \ x = \frac{\pi}{2} - \arctg \ x$;
- $\text{sh} \ x$ instead of $\sinh x$, $\text{sh} \ x = \frac{e^x - e^{-x}}{2}$;
- $\text{ch} \ x$ instead of $\cosh x$, $\text{ch} \ x = \frac{e^x + e^{-x}}{2}$;
- $\text{th} \ x$ instead of $\tanh x$, $\text{th} \ x = \frac{\text{sh} \ x}{\text{ch} \ x}$;
- $\text{cth} \ x$ instead of $\coth x$, $\text{cth} \ x = \frac{\text{ch} \ x}{\text{sh} \ x}$;
- $\text{arcsh} \ x$ instead of $\arcsinh x$, $\text{arcsh} \ x = \ln \left( x + \sqrt{x^2 + 1} \right)$;
- $\text{arcch} \ x$ instead of $\arccosh x$, $\text{arcch} \ x = \ln \left( x + \sqrt{x^2 - 1} \right), |x| \geq 1$;
- $\text{arcth} \ x$ instead of $\arctanh x$, $\text{arcth} \ x = \frac{1}{2} \ln \left( \frac{1 + x}{1 - x} \right), |x| < 1$;
- $\text{arccth} \ x$ instead of $\text{arccoth} \ x$, $\text{arccth} \ x = \frac{1}{2} \ln \left( \frac{x + 1}{x - 1} \right), |x| > 1$;

For more information about hyperbolic functions, inverse hyperbolic functions and their transforms see [2, p. 285-289] and [6, Chapter 17].
### Personal task 1

1. Integrate using the table and substitution under differential:
   a) \( \int x(3x^2 - 2)^3 \, dx \);  
   b) \( \int \tan^2 x \, dx \);  
   c) \( \int \sin x e^2 \cos x - 5 \, dx \);  
   d) \( \int \cos^2(1 - x) \, dx \).

2. Integrate the quadratic fractions:
   a) \( \int \frac{2 - 3x}{x^2 - 4} \, dx \);  
   b) \( \int \frac{dx}{\sqrt{20 + 24x - 9x^2}} \);  
   c) \( \int \frac{(x + 4) \, dx}{x^2 + 6x + 5} \);  
   d) \( \int \frac{(2x - 5) \, dx}{\sqrt{x^2 - 2x - 15}} \).

3. Integrate by parts or using the suitable substitutions:
   a) \( \int \frac{dx}{e^x + 1} \);  
   b) \( \int \frac{dx}{x \sqrt{x^2 - 1}} \);  
   c) \( \int \ln(x - 3) \, dx \);  
   d) \( \int (x^2 + 5) \cos 2x \, dx \);
   e) \( \int \arcsin 2x \, dx \);  
   f) \( \int \frac{\ln(\cos x)}{\cos^2 x} \, dx \).

4. Integrate the polynomial fractions:
   a) \( \int \frac{3x^2 + 14x + 19}{(x^2 + 4x + 3)(x + 5)} \, dx \);  
   b) \( \int \frac{x^3 + 1}{x^3 - 2x^2 + x} \, dx \);  
   c) \( \int \frac{10x + 6}{(x^2 + 2x + 5)(x - 1)} \, dx \).

5. Integrate trigonometric expressions:
   a) \( \int \sin 3x \cos x \, dx \);  
   b) \( \int \cos^4 3x \sin^2 3x \, dx \);
   c) \( \int \frac{\sqrt{9 - x^2}}{x^4} \, dx \).

6. Integrate the fractions with radicals:
   a) \( \int \frac{\sqrt{9 - x^2}}{x^4} \, dx \);  
   b) \( \int \frac{(1 - \sqrt{x + 1}) \, dx}{(1 + \sqrt{x + 1}) \sqrt{x + 1}} \).

7. Solve the definite integrals:
   a) \( \frac{e+1}{2} \int_0^x \ln(x - 1) \, dx \);  
   b) \( \int_0^{\pi/4} \sqrt[3]{\tan^2 x} \, dx \);  
   c) \( \int_0^3 \sqrt{x + 1} \, dx \);  
   d) \( \int_0^4 \sqrt{\frac{3}{3 + \sqrt[3]{(x - 2)^2}}} \, dx \).

8. Find the area of the figure bounded by the curves:
   a) \( y = 2x^2 - 8x + 6 \), \( y = x^2 - 3x \);  
   b) \( \begin{cases} x = 4 \cos^3 t, \\ y = 4 \sin^3 t \end{cases} \);  
   c) \( \rho = 6 \cos 3\phi \).

9. Find the arc-length of the curve:
   a) \( y = \ln x \), \( \sqrt{3} \leq x \leq \sqrt{8} \);  
   b) \( \begin{cases} x = 2(t - \sin t), \\ y = 2(1 - \cos t) \end{cases} \);  
   c) \( \rho = e^{-\phi} \), \( 0 \leq \phi \leq \pi \).

10. Find the area of the surface formed by rotating the curves around the \( l \)-axis:
    a) \( y = \frac{1}{3}x^3 \), \( -1 \leq x \leq 1 \), \( l = OX \);  
    b) \( \begin{cases} x = 1 + 2 \cos t, \\ y = 3 + 2 \sin t \end{cases} \);  
    c) \( \rho = 2\sqrt{\sin^2(\phi - \frac{\pi}{4})} \), \( l = \rho \).

11. Find the volume of the body formed by rotating the curves around the \( l \)-axis:
    a) \( y = x^2 - 2x + 1 \), \( y = x + 1 \), \( l = OX \);  
    b) \( x = -y^2 + 5y - 6 \), \( x = 0 \), \( l = OY \);
    c) \( \rho = 2(1 + \cos \phi) \), \( l = \rho \).

12. Solve the improper integrals:
    a) \( \int_0^\infty \frac{x}{x^4 + 16} \, dx \);  
    b) \( \int_0^{1/2} \frac{dx}{\sqrt{2 - 4x}} \).
Personal task 2

1. Integrate using the table and substitution under differential:
   a) \( \int \frac{(3 - \sqrt[3]{x})^2}{\sqrt[3]{x}} \, dx \); b) \( \int \frac{\cos 2x}{\cos^2 x \sin^2 x} \, dx \);
   c) \( \int x^3 \cdot 5x^2 - 2 \, dx \); d) \( \int \sin^2(1 - 2x) \, dx \).

2. Integrate the quadratic fractions:
   a) \( \int \frac{4x + 5}{x^2 - 1} \, dx \); b) \( \int \frac{dx}{\sqrt{5 - 8x - 4x^2}} \);
   c) \( \int \frac{(2x - 5) \, dx}{x^2 + 6x + 13} \); d) \( \int \frac{(4x + 1) \, dx}{\sqrt{x^2 + 4x - 12}} \).

3. Integrate by parts or using the suitable substitutions:
   a) \( \int \frac{(4x + 3) \, dx}{(x - 2)^3} \); b) \( \int \frac{dx}{x \sqrt{12x^2 + 4x + 1}} \);
   c) \( \int (1 - 5x + x^2) \sin 3x \, dx \);
   d) \( \int x^3 \ln x \, dx \); e) \( \int \arctg x \, dx \);
   f) \( \int \cos(\ln x) \, dx \).

4. Integrate the polynomial fractions:
   a) \( \int \frac{15x^2 + 15x - 54}{(x^2 + x - 2)(x - 2)} \, dx \);
   b) \( \int \frac{x^3 - 2x^2 - 2x + 1}{x^3 - x^2} \, dx \);
   c) \( \int \frac{2x^2 + 4x - 26}{(x^2 - 4x + 8)(x - 1)} \, dx \).

5. Integrate trigonometric expressions:
   a) \( \int \sin 3x \sin 9x \, dx \); b) \( \int \cos^3 x \sqrt{\sin^4 x} \, dx \);
   c) \( \int \frac{dx}{4 + 5 \sin^2 x - 3 \cos^2 x} \).

6. Integrate the functions with radicals:
   a) \( \int x^4 \sqrt{4 - x^2} \, dx \); b) \( \int \frac{x + 4}{x - 4 (x + 4)^2} \, dx \).

7. Solve the definite integrals:
   a) \( \int_0^1 (x - 1)^2 e^x \, dx \); b) \( \int_0^{2\pi} \cos^3 \left( \frac{x}{4} \right) \sin^3 \left( \frac{x}{4} \right) \, dx \);
   c) \( \int_0^{\pi/2} \frac{dx}{\sqrt{x^2 + 1}} \); d) \( \int_0^\infty \frac{dx}{x^6 + 1} \).

8. Find the area of the figure bounded by the curves:
   a) \( y = (x - 2)^2, \ y = 4x - 8 \);
   b) \( \begin{cases} x = 3 \cos t, \\ y = 4 \sin t; \ y = 2 \ (y \geq 2) \end{cases} \);
   c) \( \rho = 2 \sin 2\phi \).

9. Find the arc-length of the curve:
   a) \( y = \sqrt{1 - x^2}, \ -\frac{\sqrt{3}}{2} \leq x \leq 1 \);
   b) \( \begin{cases} x = 4e^t \cos t, \\ y = 4e^t \sin t; \ 0 \leq t \leq 2\pi \end{cases} \);
   c) \( \rho = 2(1 - \cos \phi) \).

10. Find the area of the surface formed by rotating the curves around the l-axis:
    a) \( y = e^x, \ 0 \leq x \leq \frac{1}{2} \ln 8, \ l = OX \);
    b) \( \begin{cases} x = 2(t - \sin t), \ 0 \leq t \leq 2\pi, \\ y = 2(1 - \cos t); \ l = OX \end{cases} \);
    c) \( \rho = 3 \sqrt{\cos 2\phi}, \ l = OP \).

11. Find the volume of the body formed by rotating the curves around the l-axis:
    a) \( y = x^2 + 2x + 1, \ y = 1 - x, \ l = OX \);
    b) \( x = -y^2 + 2y, \ x = 4y - 2y^2, \ l = OY \);
    c) \( \rho = 4 \sin \phi, \ l = OP \).

12. Solve the improper integrals:
    a) \( \int_1^\infty \frac{16x}{16x^4 - 1} \, dx \); b) \( \int_{\pi/4}^{\pi/2} \frac{\sin x}{\cos^3 x} \, dx \).
Personal task 3

1. Integrate using the table and substitution under differential:
   a) \( \int \sqrt{x(1 - 2x^4)} \, dx \); b) \( \int \text{ctg}^2 x \, dx \);
   c) \( \int \frac{ \ln^3 (2x - 5) }{2x - 5} \, dx \); d) \( \int \cos^2 (x + 3) \, dx \).

2. Integrate the quadratic fractions:
   a) \( \int \frac{6x - 5}{x^2 - 9} \, dx \); b) \( \int \frac{dx}{\sqrt{-55 + 16x - x^2}} \);
   c) \( \int \frac{(3x + 4) \, dx}{x^2 + 2x + 5} \); d) \( \int \frac{(2x + 3) \, dx}{\sqrt{x^2 + 2x - 15}} \).

3. Integrate by parts or using the suitable substitutions:
   a) \( \int \frac{e^x}{\sqrt{e^{2x} + 1}} \, dx \); b) \( \int \frac{\sqrt{4 + x^2}}{x} \, dx \);
   c) \( \int 8x^2 \sin 2x \, dx \); d) \( \int \frac{3x^2 + 6x - 3}{e^{3x}} \, dx \);
   e) \( \int \arccos 4x \, dx \); f) \( \int \frac{x}{\cos^2 x} \, dx \).

4. Integrate the polynomial fractions:
   a) \( \int \frac{4x^2 - 10x}{(x^2 - 4x + 3)(x - 2)} \, dx \);
   b) \( \int \frac{2x^3 + 3x^2 - x + 12}{x^3 + 4x^2} \, dx \);
   c) \( \int \frac{-3x^2 + 24x - 63}{(x^2 - 4x + 13)(x + 1)} \, dx \).

5. Integrate trigonometric expressions:
   a) \( \int \cos 7x \cos 3x \, dx \); b) \( \int \frac{\sin^3 x}{\cos^7 x} \, dx \);
   c) \( \int \frac{\cos x \, dx}{(\sin x + 1)^2 - \cos^2 x} \).

6. Integrate the fractions with radicals:
   a) \( \int \frac{dx}{\sqrt{x^2 - 1}} \); b) \( \int \frac{3/(x + 1)^2 + \sqrt[3]{x + 1}}{\sqrt{x + 1} + \sqrt[3]{(x + 1)^2}} \, dx \).

7. Solve the definite integrals:
   a) \( \int_0^{2e-1} \ln^2 (x + 1) \, dx \); b) \( \int_0^\pi \frac{\sin^8 x \, dx}{x + 1} \);
   c) \( \int_0^\sqrt{3} \frac{x^2}{1 + x^2} \, dx \); d) \( \int_0^5 \frac{dx}{2x + \sqrt[3]{3x + 1}} \).

8. Find the area of the figure bounded by the curves:
   a) \( x + y = 7, \ xy = 6 \);
   b) \( \begin{cases} x = 2 \cos^3 t, & y = 4 \sin^3 t, & y \geq 0 \\ y = 4(1 - \cos t), & 0 \leq t \leq \pi \\ \rho = \sqrt{\sin 2\varphi} \end{cases} \).

9. Find the arc-length of the curve:
   a) \( y = \frac{1}{3} x^3, -\frac{1}{2} \leq x \leq \frac{1}{2} \);
   b) \( \begin{cases} x = 4(t - \sin t), & y = 4(1 - \cos t), & 0 \leq t \leq \pi \\ \rho = e^{\frac{3\varphi}{\pi}}, & 0 \leq \varphi \leq 2\pi \end{cases} \).

10. Find the area of the surface formed by rotating the curves around the l-axis:
    a) \( y = \frac{1}{4} \text{ch} 4x, \ 0 \leq x \leq \ln 8, \ l = OX \);
    b) \( \begin{cases} x = 2 + \cos t, & l = OY \\ y = \sin t \end{cases} \).

11. Find the volume of the body formed by rotating the curves around the l-axis:
    a) \( y = x^2 - 2x + 5, \ y = x + 5, \ l = OX \);
    b) \( y = \ln x, \ x = 0, \ y = 0, \ y = \ln 4, \ l = OY \);
    c) \( \rho = 3(1 - \cos \phi), \ l = \text{op} \).

12. Solve the improper integrals:
    a) \( \int_0^\infty \frac{x^6}{\sqrt[5]{(x^7 + 1)^3}} \, dx \);
    b) \( \int_0^{\pi/2} \frac{\cos^2 x \, dx}{\sin^4 x} \).
Personal task 4

1. Integrate using the table and substitution under differential:
   a) \( \int \sqrt{x(1-x^2)} \cdot 3dx \);  
   b) \( \int \frac{(\sin 2x)dx}{(\cos x - 1)^2} \);  
   c) \( \int x^3e^{2x^4-1} \cdot dx \);  
   d) \( \int \text{th}^2 x \cdot dx \).

2. Integrate the quadratic fractions:
   a) \( \int \frac{-2x - 5}{x^2 - 4} \cdot dx \);  
   b) \( \int \frac{dx}{\sqrt{12 + 12x - 9x^2}} \);  
   c) \( \int \frac{(2x + 3)dx}{x^2 - 6x + 10} \);  
   d) \( \int \frac{(4x - 5)dx}{\sqrt{x^2 + 2x - 8}} \).

3. Integrate by parts or using the suitable substitutions:
   a) \( \int \frac{2x + 5}{(x + 3)^7} \cdot dx \);  
   b) \( \int \frac{dx}{(x + 1)^2 \sqrt{-2x - x^2}} \);  
   c) \( \int x^2(\cos x + \sin x) \cdot dx \);  
   d) \( \int \ln(x + 1) \cdot \frac{dx}{x + 1} \);  
   e) \( \int \sin(\ln x) \cdot dx \);  
   f) \( \int \arctg2x \cdot dx \).

4. Integrate the polynomial fractions:
   a) \( \int \frac{3x^2 + 42}{(x^2 + 5x + 4)(x - 2)} \cdot dx \);  
   b) \( \int \frac{4x^3 - 2x^2 - 9}{x^3 + 2x^2 + x} \cdot dx \);  
   c) \( \int \frac{5x^2 + 22x + 27}{(x^2 + 4x + 5)(x + 2)} \cdot dx \).

5. Integrate trigonometric expressions:
   a) \( \int \cos 4x \sin 7x \cdot dx \);  
   b) \( \int \frac{\cos^5 x}{\sin^7 x} \cdot dx \);  
   c) \( \int \frac{dx}{5 - 4 \sin x + 2 \cos x} \).

6. Integrate the fractions with radicals:
   a) \( \int \frac{\sqrt{9 - x^2}}{x^2} \cdot dx \);  
   b) \( \int \frac{\sqrt{x} + \sqrt{x}}{1 + \sqrt{x}} \cdot dx \).

7. Solve the definite integrals:
   a) \( \int_1^0 \frac{e^{-\frac{1}{x^2}}}{x^2} \cdot dx \);  
   b) \( \int_{\frac{\pi}{8}}^0 \cos^4 2x \sin^4 2x \cdot dx \);  
   c) \( \int_0^{\frac{1}{\ln 5}} \frac{12x + 5}{x^2 - 4} \cdot dx \);  
   d) \( \int_{-2}^2 x^3 \sqrt{4 - x^2} \cdot dx \).

8. Find the area of the figure bounded by the curves:
   a) \( y = (x + 3)^2; \ y = 2x + 6; \)
   b) \( \begin{cases} x = 4 \cos t; \\ y = 4 \sin t; \end{cases} \ x = 2 \ (x \geq 2); \)
   c) \( \rho = \sin 3\phi \).

9. Find the arc-length of the curve:
   a) \( y = \ln(\sin x), \ \frac{\pi}{3} \leq x \leq \frac{\pi}{2}; \)
   b) \( \begin{cases} x = 2 \cos t - \cos 2t; \\ y = 2 \sin t - \sin 2t; \end{cases} \ 0 \leq t \leq 2\pi; \)
   c) \( \rho = \phi, \ 0 \leq \phi \leq \pi \).

10. Find the area of the surface formed by rotating the curves around the l-axis:
    a) \( y = \sqrt{2x + 6}, \ 0 \leq x \leq 3, \ l = OX; \)
    b) \( \begin{cases} x = \cos^3 t; \\ y = \sin^3 t; \end{cases} \ l = OY; \)
    c) \( \rho = 3\cos 2\phi, \ l = \rho \).

11. Find the volume of the body formed by rotating the curves around the l-axis:
    a) \( y = x^2 - 4x + 4, \ y = x, \ l = OX; \)
    b) \( x = 9 - (y - 2)^2, \ x = 0, \ l = OY; \)
    c) \( \rho = 6 \sin \phi, \ l = \rho \).

12. Solve the improper integrals:
    a) \( \int_{-2}^2 \frac{4x - 2}{4x^2 - 4x - 3} \cdot dx \);  
    b) \( \int_0^1 \frac{dx}{\sqrt{3 - 2x - x^2}} \).
Personal task 5

1. Integrate using the table and substitution under differential:
   a) \( \int \sqrt{x}(3 + 2x\sqrt{x})^2 dx \);
   b) \( \int \frac{dx}{\cos 2x + \sin^2 x} \);
   c) \( \int \frac{x^3 dx}{\sqrt{x^4 + 9}} \);
   d) \( \int (1 - \sin \frac{x}{2})^2 dx \).

2. Integrate the quadratic fractions:
   a) \( \int \frac{-4x + 7}{x^2 - 16} dx \);
   b) \( \int \frac{dx}{\sqrt{15 + 6x - 9x^2}} \);
   c) \( \int \frac{(2x - 5)dx}{x^2 - 8x + 20} \);
   d) \( \int \frac{(6x - 3)dx}{\sqrt{x^2 + 4x - 12}} \).

3. Integrate by parts or using the suitable substitutions:
   a) \( \int \frac{dx}{\sqrt{9 + e^x}} \);
   b) \( \int \frac{dx}{x^2 \sqrt{x^2 + 4}} \);
   c) \( \int \ln^2 xdx \);
   d) \( \int (1 - 6x^2) \cos 3xdx \);
   e) \( \int e^{x+3} dx \);
   f) \( \int x \cdot \arctg x dx \).

4. Integrate the polynomial fractions:
   a) \( \int \frac{-5x^2 - 4x + 9}{(x^2 + 4x + 3)(x + 2)} dx \);
   b) \( \int \frac{x^3 + 3x^2 + 3x + 4}{x^3 + 4x^2 + 4x} dx \);
   c) \( \int \frac{5x^2 - 24x + 24}{(x^2 - 6x + 10)(x - 1)} dx \).

5. Integrate trigonometric expressions:
   a) \( \int \sin 2x \sin 4xdx \);
   b) \( \int \frac{\cos^2 x}{\sin^8 x} dx \);
   c) \( \int \frac{\cos x dx}{6 - \sin^2 x + 3 \cos^2 x} \).

6. Integrate the functions with radicals:
   a) \( \int x^2 \sqrt{1 - x^2} dx \);
   b) \( \int \sqrt{\frac{x + 1}{x - 1}(x - 1)^3} dx \).

7. Solve the definite integrals:
   a) \( \int_0^2 (2 - x)^2 e^{2x} dx \);
   b) \( \int_{-\pi/4}^{\pi/4} \sin^4 x \cos^6 x dx \);
   c) \( \int_{-1/2}^1 \frac{dx}{\sqrt{8 + 2x - x^2}} \); d) \( \int_0^7 \sqrt{(x + 1)^2} \frac{dx}{4 + \sqrt{(x + 1)^2}} \).

8. Find the area of the figure bounded by the curves:
   a) \( x + y = -3, \ xy = 2 \);
   b) \( \left\{ \begin{array}{l} x = 5 \cos t, \ \ y = 2 \sin t; \ y = 0 \ (y \geq 0) \end{array} \right. \);
   c) \( \rho = 4(1 + \sin \phi) \).

9. Find the arc-length of the curve:
   a) \( y = 3 + \chi x, \ 0 \leq x \leq \ln 2 \);
   b) \( \left\{ \begin{array}{l} x = 4 \cos^3 t, \ \ y = 4 \sin^3 t; \ 0 \leq t \leq \frac{\pi}{2} \end{array} \right. \);
   c) \( \rho = \sqrt{2} e^{\phi}, \ 0 \leq \phi \leq \frac{\pi}{2} \).

10. Find the area of the surface formed by rotating the curves around the \( l \)-axis:
    a) \( y = \frac{1}{2} x^2, \ 0 \leq x \leq 2\sqrt{6}, \ l = OY \);
    b) \( \left\{ \begin{array}{l} x = 3(t - \sin t), \ t \leq 2\pi, \ y = 3(1 - \cos t); \ l = OX \end{array} \right. \);
    c) \( \rho = 8 \sin \phi, \ l = \rho \).

11. Find the volume of the body formed by rotating the curves around the \( l \)-axis:
    a) \( y = x^2 - 2x + 2, \ y = x + 2, \ l = OX \);
    b) \( x = y^2 + 4, \ x = 0, \ |y| = 2, \ l = OY \);
    c) \( \rho = 2 \sin 2\phi, \ l = \rho \).

12. Solve the improper integrals:
    a) \( \int_{1/3}^{\infty} \frac{4x^2}{9x^6 + 1} dx \);
    b) \( \int_1^\infty \frac{dx}{x \sqrt{1 - \ln^2 x}} \).
1. Integrate using the table and substitution under differential:
   a)\( \int x(6\sqrt{x} - \sqrt{x})^2 \, dx \)
   b)\( \int \frac{\sin 2x \, dx}{(2 - \sin x)^2 - 4} \)
   c)\( \int \ln^7(3x + 1) \, dx \)
   d)\( \cos^2(2x - 5) \, dx \).

2. Integrate the quadratic fractions:
   a)\( \int \frac{12x + 7}{x^2 - 4} \, dx \)
   b)\( \int \frac{dx}{\sqrt{4x^2 - 20x + 24}} \)
   c)\( \int \frac{(-3x + 1) \, dx}{x^2 + 6x + 13} \)
   d)\( \int \frac{-4x + 3}{\sqrt{-2x - x^2}} \, dx \).

3. Integrate by parts or using the suitable substitutions:
   a)\( \int \frac{e^x \, dx}{\sqrt{e^{8x} + 16}} \)
   b)\( \int \frac{dx}{x^2\sqrt{3 - 4x + x^2}} \)
   c)\( \int (3 - 4x) \sin 4x \, dx \)
   d)\( \int \frac{5x^2 - 1}{e^{5x}} \, dx \)
   e)\( \int (\arcsin x)^2 \, dx \)
   f)\( \int \frac{\ln x}{x^3} \, dx \).

4. Integrate the polynomial fractions:
   a)\( \int \frac{x^2 + 14}{(x^2 - 5x + 4)(x - 2)} \, dx \)
   b)\( \int \frac{3x^3 + 7x^2 + 9x + 9}{x^3 + 3x^2} \, dx \)
   c)\( \int \frac{9x^2 - 24x + 46}{(x^2 - 6x + 10)(x + 2)} \, dx \).

5. Integrate trigonometric fractions:
   a)\( \int \sin 7x \cos 2x \, dx \)
   b)\( \int \frac{\sin^2 2x}{\cos^4 2x} \, dx \)
   c)\( \int \frac{5 \sin x \, dx}{14 - 12 \cos x - 9 \sin^2 x} \).

6. Integrate the fractions with radicals:
   a)\( \int \frac{\sqrt{x^2 - 4}}{x^5} \, dx \)
   b)\( \int \frac{x + 2}{x - 2(x + 2)^2} \, dx \).

7. Solve the definite integrals:
   a)\( \int \frac{e^{+2\ln^5(x - 2)}}{x - 2} \, dx \)
   b)\( \int \frac{\pi/2}{\cos^3 x \sin^8 x} \, dx \)
   c)\( \int \frac{\pi/2}{3} \, dx \)
   d)\( \int \frac{\sqrt{2x + 4}}{4 + \sqrt{2x + 4}} \, dx \).

8. Find the area of the figure bounded by the curves:
   a)\( x + 2y = 5, \quad xy = 2 \)
   b)\( \begin{cases} x = 3 \cos^3 t, \\ y = 3 \sin^3 t \end{cases} \quad x = 0, \quad (x \geq 0) \)
   c)\( \rho = 2 \cos 4\phi \).

9. Find the arc-length of the curve:
   a)\( y = \ln(x + \sqrt{x^2 - 4}), \quad 2 \leq x \leq 2\sqrt{5} \)
   b)\( \begin{cases} x = 2(t - \sin t), \\ y = 2(1 - \cos t) \end{cases} \quad 0 \leq t \leq 2\pi \)
   c)\( \rho = 6 \cos \phi - 6 \sin \phi \).

10. Find the area of the surface formed by rotating the curves around the l-axis:
    a)\( y = 1 + \frac{1}{3} \cosh 3x, \quad -1 \leq x \leq 1, \quad l = OX \)
    b)\( \begin{cases} x = 1 + 2 \cos t, \\ y = 2 \sin t \end{cases} \quad (x \geq 0), \quad l = OY \)
    c)\( \rho = 1 + \cos \phi, \quad l = O\rho \).

11. Find the volume of the body formed by rotating the curves around the l-axis:
    a)\( y = x^2 - 2x + 3, \quad y = x + 3, \quad l = OX \)
    b)\( y = \sqrt{1 - x^2}, \quad y = x, \quad x = 0, \quad l = OY \)
    c)\( \rho = 4\phi, \quad 0 \leq \phi \leq \pi, \quad l = O\rho \).

12. Solve the improper integrals:
    a)\( \int_0^\infty \frac{x^2}{x^2 + 1} \, dx \)
    b)\( \int_0^{\pi/2} e^{-\tan x} \, dx \).
1. Integrate using the table and substitution under differential:
   a) \( \int \frac{(x - 1)^2}{\sqrt{x}} \, dx \);  
   b) \( \int \frac{\tan^3 x}{\cos^2 x} \, dx \);  
   c) \( \int x\sqrt{x^2 - 4} \, dx \);  
   d) \( \int 2\sin^2 \left( \frac{x}{2} \right) \, dx \).  
2. Integrate the quadratic fractions:
   a) \( \int \frac{2x - 7}{x^2 - 9} \, dx \);  
   b) \( \int \frac{dx}{\sqrt{15 - 6x - 9x^2}} \);  
   c) \( \int \frac{(-2x + 9) \, dx}{x^2 + 4x + 20} \);  
   d) \( \int \frac{(6x + 3) \, dx}{\sqrt{x^2 - 2x - 8}} \).  
3. Integrate by parts or using the suitable substitutions:
   a) \( \int \frac{dx}{x\sqrt{x + 4}} \);  
   b) \( \int \frac{\sqrt{1 + \ln x}}{x \ln x} \, dx \);  
   c) \( \int (2 - 3x^2) \sin 2x \, dx \);  
   d) \( \int x^4 \, dx \);  
   e) \( \int \arctan \sqrt{x} \, dx \);  
   f) \( \int \ln(x^2 + 1) \, dx \).  
4. Integrate the polynomial fractions:
   a) \( \int \frac{3x^2 + 5x - 6}{(x^2 + 4x + 3)(x + 2)} \, dx \);  
   b) \( \int \frac{2x^3 + 11x^2 + 17x + 9}{x^3 + 5x^2 + 8x + 4} \, dx \);  
   c) \( \int \frac{x^2 + x - 2}{x^3 + 2x^2 + 2x} \, dx \).  
5. Integrate trigonometric expressions:
   a) \( \int \cos x \sin x \cos 3x \, dx \);  
   b) \( \int \cos^5 x \sqrt{\sin x} \, dx \);  
   c) \( \int \frac{dx}{5 - 3 \cos x} \).  
6. Integrate the functions with radicals:
   a) \( \int \sqrt{1 - 4x - x^2} \, dx \);  
   b) \( \int \frac{dx}{x(\sqrt{x} + \sqrt[3]{x^2})} \).  
7. Solve the definite integrals:
   a) \( \int \frac{\ln^2 x}{(x + 3)^2} e^x \, dx \);  
   b) \( \int_0^{\frac{\pi}{2}} \cos^2 \left( \frac{x}{2} \right) \sin \left( \frac{x}{2} \right) \, dx \);  
   c) \( \int_1^0 \frac{4x^3}{x^8 + 1} \, dx \);  
   d) \( \int_0^\infty \frac{1}{x^2 + 1} \, dx \);  
   e) \( \int_0^\infty \sqrt{1 - e^2 x} \, dx \).  
8. Find the area of the figure bounded by the curves:
   a) \( y = (x + 1)^2, \ y = 6x + 6 \);  
   b) \( \begin{cases} x = 12 \cos t; \\ y = 5 \sin t; \end{cases} \ x = 6 \ (x \geq 6) \);  
   c) \( \rho = 2\cos 2\phi \).  
9. Find the arc-length of the curve:
   a) \( y = \frac{1}{2} \ln(\cos 2x), \ 0 \leq x \leq \frac{\pi}{12} \);  
   b) \( \begin{cases} x = 4 \cos t; \\ y = 2 \ (y \geq 2) \end{cases} \);  
   c) \( \rho = 6 \sin^2 \left( \frac{\phi}{2} \right) \).  
10. Find the area of the surface formed by rotating the curves around the \( l \)-axis:
    a) \( y = \sqrt{3}x + 7, \ 0 \leq x \leq 2\sqrt{3} \); \( l = OX \);  
    b) \( \begin{cases} x = 3 \cos^3 t; \\ y = 3 \sin^3 t; \end{cases} \ l = OY \);  
    c) \( \rho = 2e^{-\phi}, \ 0 \leq \phi \leq \pi \); \( l = op \).  
11. Find the volume of the body formed by rotating the curves around the \( l \)-axis:
    a) \( y = (x - 2)^2, \ y = x + 4 \); \( l = OX \);  
    b) \( y = 9 - x^2, \ y = 9 - 3x^2, \ y = 0 \); \( l = OY \);  
    c) \( \rho = 4 \sin 2\phi \); \( l = op \).  
12. Solve the improper integrals:
    a) \( \int_1^\infty \frac{dx}{x^2(x + 1)} \);  
    b) \( \int_0^{2/\pi} \sin \left( \frac{1}{x} \right) \frac{dx}{x^2} \).
1. Integrate using the table and substitution under differential:
   a) \( \int \frac{x^2 - 2x^3 e^x + 4}{x^3} \, dx \);
   b) \( \int \frac{4 + \cos^2 x}{1 + \cos 2x} \, dx \);
   c) \( \int \frac{\ln^2 (3 - 2x)}{3 - 2x} \, dx \);
   d) \( \int (1 - \sin 3x)^2 \, dx \).

2. Integrate the quadratic fractions:
   a) \( \int \frac{-4x + 3}{x^2 - 16} \, dx \);
   b) \( \int \frac{1}{\sqrt{4x^2 - 12x + 8}} \, dx \);
   c) \( \int \frac{(4x + 1) \, dx}{x^2 - 2x + 17} \);
   d) \( \int \frac{(2x + 3) \, dx}{\sqrt{15 - 2x - x^2}} \).

3. Integrate by parts or using the suitable substitutions:
   a) \( \int \frac{\ln x}{x \sqrt{1 - \ln^2 x}} \, dx \);
   b) \( \int \frac{x^2 + 1}{x} \, dx \);
   c) \( \int (3 - 9x) \sin 3x \, dx \);
   d) \( \int (1 - 8x^2) e^{4x} \, dx \);
   e) \( \int \frac{\arctg x}{x^2} \, dx \);
   f) \( \int \frac{x}{\sin^2 x} \, dx \).

4. Integrate the polynomial fractions:
   a) \( \int \frac{4x^2 - 8x - 20}{(x^2 + 6x + 5)(x - 1)} \, dx \);
   b) \( \int \frac{x^3 + 3x^2 + 4x - 1}{x^3 + 5x^2 + 8x + 4} \, dx \);
   c) \( \int \frac{-7x^2 - 13x - 10}{(x^2 + 4x + 5)(x - 1)} \, dx \).

5. Integrate trigonometric expressions:
   a) \( \int \sin 7x \sin 4x \, dx \);
   b) \( \int \cos^2 x \sin^4 x \, dx \);
   c) \( \int \frac{2 - \cos x}{2 + \cos x} \, dx \).

6. Integrate the fractions with radicals:
   a) \( \int \frac{dx}{x \sqrt{1 + 4x - 5x^2}} \);
   b) \( \int \frac{xdx}{(\sqrt{x} + 2\sqrt{x})^2} \).

7. Solve the definite integrals:
   a) \( \int_0^1 (x + 1) \ln (x + 1) \, dx \);
   b) \( \int_0^1 \cos^4 4x \sin^3 4x \, dx \);
   c) \( \int_{-\pi/4}^{\pi/4} \frac{x - 1}{3 + x} \, dx \);
   d) \( \int_0^\infty \frac{e^{tg x}}{\cos^2 x} \, dx \).

8. Find the area of the figure bounded by the curves:
   a) \( y = (x - 1)^3 \), \( y = 4(x - 1) \);
   b) \( \begin{cases} 
   x = 3 \cos^3 t, & y = 0 \ (y \geq 0); \\
   y = \sin^3 t, & 0 \leq y \leq 2 \pi; \\
   \rho = 2 \sin 3 \phi. 
   \end{cases} \)

9. Find the arc-length of the curve:
   a) \( y = 4 - \frac{1}{2} \chi 2x \), \( 0 \leq x \leq \ln 4 \);
   b) \( \begin{cases} 
   x = 2(t - \sin t), & y = 2(1 - \cos t); \\
   \rho = \sqrt{8} e^{-3\phi}, & 0 \leq \phi \leq \pi. 
   \end{cases} \)

10. Find the area of the surface formed by rotating the curves around the \( l \)-axis:
    a) \( y = 2x^2 \), \( 0 \leq x \leq \sqrt{3} \);
    b) \( \begin{cases} 
   x = 1 + 3 \cos t, & l = OY; \\
   y = 3 \sin t; & l = OX; \\
   \rho = 4 \sqrt{\cos 2\phi}, & l = op. 
   \end{cases} \)

11. Find the volume of the body formed by rotating the curves around the \( l \)-axis:
    a) \( y = -x^2 + 4x \), \( y = 0 \);
    b) \( y = x^2 \), \( y = 5x^2 - 16 \);
    c) \( \rho = 2(1 - \cos \phi), \ l = op. \)

12. Solve the improper integrals:
    a) \( \int_{-1}^{\infty} \arctg^5 x \, dx \);
    b) \( \int_{-1}^{2/3} \frac{2x + 2}{\sqrt{x^2 + 2x - 3}} \, dx \).
Personal task 9

1. Integrate using the table and substitution under differential:
   a) $\int x^2(1 - 3\sqrt{x})^3 dx$;
   b) $\int \frac{\tan^7 x}{1 - \cos^2 x} dx$;
   c) $\int \frac{\arcsin^3 x + x}{\sqrt{1 - x^2}} dx$;
   d) $\int (\sin x + \cos x)^2 dx$.

2. Integrate the quadratic fractions:
   a) $\int \frac{-2x + 4}{x^2 - 25} dx$;
   b) $\int \frac{dx}{\sqrt{3 - 4x - 4x^2}}$;
   c) $\int \frac{(-2x - 7)dx}{x^2 + 6x + 13}$;
   d) $\int \frac{(6x + 8)dx}{\sqrt{x^2 + 4x - 5}}$;

3. Integrate by parts or using the suitable substitutions:
   a) $\int \frac{4x - 7}{(x - 1)^5} dx$;
   b) $\int \frac{dx}{(x - 1)\sqrt{2x - x^2}}$;
   c) $\int x(\sin x - 2 \cos x)dx$;
   d) $\int \frac{x^2 - 4x + 1}{e^{2x}} dx$;
   e) $\int \frac{\ln(x - 2)}{(x - 2)^2} dx$;
   f) $\int (\arccos x)^2 dx$.

4. Integrate the polynomial fractions:
   a) $\int \frac{7x^2 + 14x - 85}{(x^2 - 6x + 5)(x + 3)} dx$;
   b) $\int \frac{3x^3 + 15x^2 + 24x + 10}{x^3 + 5x^2 + 7x + 3} dx$;
   c) $\int \frac{4x^2 - 9x + 30}{(x^2 - 4x + 20)(x + 2)} dx$.

5. Integrate trigonometric expressions:
   a) $\int \cos x \sin 8x dx$;
   b) $\int \frac{\cos^3 x}{\sin^6 x} dx$;
   c) $\int \frac{2dx}{3 + \sin x + 5\cos x}$.

6. Integrate the fractions with radicals:
   a) $\int \frac{\sqrt{1 - x^2}}{x^4} dx$;
   b) $\int \frac{\sqrt{1 - x} dx}{1 + x}$.

7. Solve the definite integrals:
   a) $\int_{2/\pi}^{\pi/2} \cos \left(\frac{1}{x} x^2\right) dx$;
   b) $\int_{6/\pi}^{\pi/2} x \cos x \sin x dx$;
   c) $\int_0^{\sqrt{3}/2} 2x - 2 dx$;
   d) $\int_0^1 x^2\sqrt{1 - x^2} dx$.

8. Find the area of the figure bounded by the curves:
   a) $y = -e^2 + 5x + 1$, $y = \frac{5}{x}$;
   b) $\left\{\begin{array}{l} x = 1 + 2 \cos t, \\ y = 2 \sin t \end{array}\right.$ $y = 1 \ (y \geq 1)$;
   c) $\rho = \cos 2\phi$.

9. Find the arc-length of the curve:
   a) $y = 3 \ln(9 - x^2)$, $0 \leq x \leq 2$;
   b) $\left\{\begin{array}{l} x = 3e^t \cos t, \\ y = 3e^t \sin t \end{array}\right.$ $0 \leq t \leq \pi$;
   c) $\rho = 4 \cos^2 \left(\frac{\phi}{2}\right)$.

10. Find the area of the surface formed by rotating the curves around the l-axis:
    a) $y = 3 - \frac{1}{2} \sin 2x$, $-1 \leq x \leq 1$, $l = OX$;
    b) $\left\{\begin{array}{l} x = 2 \cos^3 t, \\ y = 2 \sin^3 t \end{array}\right.$ $l = OY$;
    c) $\rho = 6 \sin \phi$, $l = op$.

11. Find the volume of the body formed by rotating the curves around the l-axis:
    a) $y = x^2 + 6x + 9$, $y = -4x$, $l = OX$;
    b) $y = x^2 - 8$, $y = 7x$, $x = 0 \ (x \geq 0)$, $l = OY$;
    c) $\rho = 2\phi$, $0 \leq \phi \leq \pi$, $l = op$.

12. Solve the improper integrals:
    a) $\int_{0}^{\infty} \frac{x^2}{e^{x^2}} dx$;
    b) $\int_{0}^{1} \frac{1 + x - x^2}{\sqrt{1 - x^2}} dx$.
1. Integrate using the table and substitution under differential:
   a) \( \int x \sqrt{x(1 - 3x)}^2 dx \);
   b) \( \int \frac{\cos 2x}{\cos^2 x} dx \);
   c) \( \int \frac{1 - 2x}{\sqrt{1 - x^2}} dx \);
   d) \( \int (1 + \cos \frac{x}{2})^2 dx \).

2. Integrate the quadratic fractions:
   a) \( \int -6x + 5 \frac{dx}{x^2 - 9} \);
   b) \( \int \frac{dx}{\sqrt{4x^2 - 4x - 5}} \);
   c) \( \int \frac{(-4x + 2)dx}{x^2 - 2x + 17} \);
   d) \( \int \frac{-6x + 7}{\sqrt{6x^2 - x^2}} dx \).

3. Integrate by parts or using the suitable substitutions:
   a) \( \int \frac{1 - e^x}{1 + e^x} dx \);
   b) \( \int \frac{dx}{x^{\sqrt{x^2 - 9}}} \);
   c) \( \int (2 - 8x^2) \cos 2x dx \);
   d) \( \int (x + 1)^4 \ln(x + 1) dx \);
   e) \( \int \sin \sqrt{xdx} \);
   f) \( \int \frac{x^2 \arctg x}{1 + x^2} dx \).

4. Integrate the polynomial fractions:
   a) \( \int \frac{4x^2 + 12x + 6}{x^2 + 3x + 2}(x + 3) dx \);
   b) \( \int \frac{4x^3 - 7x^2 - 2x + 1}{x^2 - x + 1} dx \);
   c) \( \int \frac{-5x^2 - 6x + 15}{x^2 + 4x + 5}(x - 2) dx \).

5. Integrate trigonometric expressions:
   a) \( \int \cos x \sin x \cos 3x dx \);
   b) \( \int \sqrt{\cos^5 x \sin^5 x} dx \);
   c) \( \int \frac{dx}{\sin x + \cos x} \).

6. Integrate the fractions with radicals:
   a) \( \int \frac{x^4}{\sqrt{(1 - x^2)}^3} dx \);
   b) \( \int \frac{x dx}{(2 + x) \sqrt{1 + x}} \).

7. Solve the definite integrals:
   a) \( \int_{-1}^{\ln 4} (x + 1)e^{2x^2} dx \);
   b) \( \int_{0}^{\ln 1} \sin^{2} x \cos^{4} x dx \);
   c) \( \int_{-1}^{\frac{\pi}{3}} \frac{2x - 4}{\sqrt{3} - 2x - x^2} dx \);
   d) \( \int_{0}^{\frac{3}{\sqrt{x + 1}} + 1} dx \).

8. Find the area of the figure bounded by the curves:
   a) \( 2x + y = 5, \ xy = 2 \);
   b) \( \begin{cases} x = 3 \cos t, \ y = 6 \sin t; & x = 0 (x \geq 0) \end{cases} \);
   c) \( \rho = 2(1 + \sin \phi) \).

9. Find the arc-length of the curve:
   a) \( y = 5 - \chi x, \quad 0 \leq x \leq \ln 3 \);
   b) \( \begin{cases} x = 2 \cos^3 t, \ y = 2 \sin^3 t; & 0 \leq t \leq \pi \end{cases} \);
   c) \( \rho = 5e^\phi, \quad 0 \leq \phi \leq \pi \).

10. Find the area of the surface formed by rotating the curves around the l-axis:
    a) \( y = 2x^2, \quad 0 \leq x \leq \sqrt{6}, \ l = OY \);
    b) \( \begin{cases} x = t - \sin t, \ y = 1 - \cos t; & 0 \leq t \leq 2\pi, \ l = OX \end{cases} \);
    c) \( \rho = 3 \sin 2(\phi - \frac{\pi}{4}), \ l = o\rho \).

11. Find the volume of the body formed by rotating the curves around the l-axis:
    a) \( y = x^2 - 4x, \ y = 0, \ l = OX \);
    b) \( y = 9 - x^2, \ y = 8x, \ x = 0 (x \geq 0), \ l = OY \);
    c) \( \rho = 2 \cos \phi, \ l = o\rho \).

12. Solve the improper integrals:
    a) \( \int_{0}^{\infty} \frac{2x - 3}{x^2 + 4} dx \);
    b) \( \int_{1}^{e} \frac{\ln x - 1}{x \sqrt{1 - \ln^2 x}} dx \).
Personal task 11

1. Integrate using the table and substitution under differential:
   a) \( \int \frac{(-x^2 + 3)^3}{x^4} \, dx \);  
   b) \( \int (\tan x + \cot x)^2 \, dx \);
   c) \( \int xe^{-3x^4 + 1} \, dx \);
   d) \( \int \sin^2(x + 2) \, dx \).

2. Integrate the quadratic fractions:
   a) \( \int \frac{1 - 6x}{x^2 - 9} \, dx \);
   b) \( \int \frac{\sqrt{4 - x^2}}{x} \, dx \);
   c) \( \int 2x^2 \cos 4x \, dx \);
   d) \( \int \arcsin 3x \, dx \);
   e) \( \int e^{\frac{3}{2}x} \, dx \);
   f) \( \int \ln(x + \sqrt{x^2 + 1}) \, dx \).

3. Integrate by parts or using the suitable substitutions:
   a) \( \int \frac{4^x - 2^x}{\sqrt{4^x + 1}} \, dx \);
   b) \( \int \frac{\sqrt{4 - x^2}}{x} \, dx \);
   c) \( \int 2x^2 \cos 4x \, dx \);
   d) \( \int \arcsin 3x \, dx \);
   e) \( \int e^{\frac{3}{2}x} \, dx \);
   f) \( \int \ln(x + \sqrt{x^2 + 1}) \, dx \).

4. Integrate the polynomial fractions:
   a) \( \int \frac{3x^2 + 7x - 4}{(x^2 + 3x + 2)(x - 1)} \, dx \);
   b) \( \int \frac{x^4 + 2x^3 - 4x^2 - 10x - 12}{x^3 - 4x} \, dx \);
   c) \( \int \frac{4x^2 + 14x + 16}{(x^2 + 2x + 2)(x + 2)} \, dx \).

5. Integrate trigonometric expressions:
   a) \( \int \sin 5x \cos 2x \, dx \);
   b) \( \int \cos^4 2x \sin^2 2x \, dx \);
   c) \( \int \frac{dx}{3 - 2 \sin x + \cos x} \).

6. Integrate the fractions with radicals:
   a) \( \int \frac{\sqrt{x^2 - 9}}{x^3} \, dx \);
   b) \( \int \frac{dx}{(\sqrt{x + 3} - 1)\sqrt{x + 3}} \).

7. Solve the definite integrals:
   a) \( \int_1^e (2x - 1) \ln x \, dx \);
   b) \( \int_0^{\pi/8} \frac{\sqrt{\tan^3 2x}}{\cos^2 2x} \, dx \);
   c) \( \int_0^2 x^2 \sqrt{1 + x^3} \, dx \);
   d) \( \int_0^{26} \frac{\sqrt{x + 1}}{5 + \sqrt{x + 1}} \, dx \).

8. Find the area of the figure bounded by the curves:
   a) \( y = -x^2 + 4x + 1, \ y = -x + 5 \);
   b) \( \left\{ \begin{array}{l} x = 2 \cos^3 t, \ y = 2 \sin^3 t; \\
                              x = 0 \ (x \geq 0) \end{array} \right. \);
   c) \( \rho = 3 \cos 2\phi \).

9. Find the arc-length of the curve:
   a) \( y = 4 \ln x, \ 3 \leq x \leq \sqrt{15} \);
   b) \( \left\{ \begin{array}{l} x = 3(t - \sin t), \\
                              y = 3(1 - \cos t); \\
                              0 \leq t \leq 2\pi \end{array} \right. \);
   c) \( \rho = 3e^{-3\phi}, \ 0 \leq \phi \leq \pi \).

10. Find the area of the surface formed by rotating the curves around the l-axis:
    a) \( y = x^3, \ -2 \leq x \leq 2, \ l = OX \);
    b) \( \left\{ \begin{array}{l} x = 2 + \sin t, \\
                              y = 2 + \cos t; \\
                              l = OY \end{array} \right. \);
    c) \( \rho = 6 \sin^2 \left( \frac{\phi}{2} \right), \ l = \rho \).

11. Find the volume of the body formed by rotating the curves around the l-axis:
    a) \( y = x^2 + 4x + 4, \ y = 3x + 6, \ l = OX \);
    b) \( x = -y^2 + 4y, \ x = 0, \ l = OY \);
    c) \( \rho = 2 \sin \phi, \ l = \rho \).

12. Solve the improper integrals:
    a) \( \int_{-\infty}^{0} \frac{x^2}{x^6 + 4} \, dx \);
    b) \( \int_0^{1/2} \frac{dx}{\sqrt{(1 - 2x)^2}} \).
1. Integrate using the table and substitution under differential:
   a) \( \int \frac{3x^2 2^x - 4^x}{2x} dx \);  
   b) \( \int (5 - ctg^2 x) dx \);  
   c) \( \int x^2 2^{2x^3 - 1} dx \);  
   d) \( \int \cos^2 (3x + 1) dx \);  
2. Integrate the quadratic fractions:
   a) \( \int \frac{4x - 7}{x^2 - 4} dx \);  
   b) \( \int \frac{dx}{\sqrt{6x - 9x^2}} \);  
   c) \( \int \frac{(2x - 5)dx}{x^2 - 4x + 13} \);  
   d) \( \int \frac{(2x + 3)dx}{\sqrt{x^2 + 2x - 3}} \);  
3. Integrate by parts or using the suitable substitutions:
   a) \( \int x^2 (x + 2)^{10} dx \);  
   b) \( \int \frac{2dx}{x^2 \sqrt{2x^2 + 2x + 1}} \);  
   c) \( \int (4 - 12x^2) \sin 4x dx \);  
   d) \( \int (x - 1)^3 \ln (x - 1) dx \);  
   e) \( \int e^{\sqrt{2x - 4}} dx \);  
   f) \( \int \frac{x^2 \arccotg x}{x^2 + 1} dx \).  
4. Integrate the polynomial fractions:
   a) \( \int \frac{5x^2 - 11x - 6}{(x^2 - 3x + 2)(x + 2)} dx \);  
   b) \( \int \frac{-2x^3 - 12x^2 - 5x + 70}{(x + 2)(x^2 + 3x - 10)} dx \);  
   c) \( \int \frac{3x^2 + 6x + 10}{x^3 + 2x^2 + 2x} dx \);  
5. Integrate trigonometric expressions:
   a) \( \int \sin 2x \sin 5x dx \);  
   b) \( \int \cos^5 x \sqrt{x} \sin x dx \);  
   c) \( \int \frac{-2dx}{9 - 7 \cos x} \);  
6. Integrate the fractions with radicals:
   a) \( \int \frac{\sqrt{1 - 9x^2}}{x^4} dx \);  
   b) \( \int \sqrt{x + 1} \frac{dx}{x - 2 (x + 1)^2} \);  
7. Solve the definite integrals:
   a) \( \int_0^{\frac{3\pi}{2}} x^2 e^{x^2} + 2 dx \);  
   b) \( \int_{-2}^{0} \cos^4 \left(\frac{x}{3}\right) \sin^2 \left(\frac{x}{3}\right) dx \);  
   c) \( \int_0^{1/\sqrt{2}} 2 \arcsin x + x \frac{dx}{\sqrt{1 - x^2}} \);  
   d) \( \int_0^{\sqrt{x} + 1} \ln (x + 1) dx \);  
8. Find the area of the figure bounded by the curves:
   a) \( y = (x + 3)^2 \);  
   b) \( y = 4x + 12 \);  
   c) \( \rho = 2 \sin 2\phi \);  
9. Find the arc-length of the curve:
   a) \( y = 2 \ln (4 - x^2) \);  
   b) \( y = (3 - \cos t) \);  
   c) \( \rho = 4(1 - \cos \phi) \);  
10. Find the area of the surface formed by rotating the curves around the l-axis:
    a) \( y = \sqrt{1 - \frac{x^2}{9}} \);  
    b) \( y = \cos^3 t \);  
    c) \( \rho = \sqrt{2} \sin \phi \);  
11. Find the volume of the body formed by rotating the curves around the l-axis:
    a) \( y = -x^2 + 4x + 1 \);  
    b) \( x = -y^2 + 7y \);  
    c) \( \rho = 6\phi \);  
12. Solve the improper integrals:
    a) \( \int_{-1}^{\infty} \frac{4x - 1}{x^2 + 9} dx \);  
    b) \( \int_0^{\arccos x} \frac{dx}{\sqrt{1 - x^2}} \).
1. Integrate using the table and substitution under differential:
   a) $\int \frac{(2-x \sqrt{x})^3 dx}{x^2}$; b) $\int (\tan x - \cot x)^2 dx$;
   c) $\int \ln^4(2x + 3) dx$; d) $\int \cos^2(3-2x) dx$.

2. Integrate the quadratic fractions:
   a) $\int \frac{5x + 3}{x^2 - 16} dx$; b) $\int \frac{dx}{\sqrt{12 + 12x - 9x^2}}$;
   c) $\int \frac{(4x + 6) dx}{x^2 - 4x + 8}$; d) $\int \frac{(2x - 7) dx}{\sqrt{x^2 - 4x - 12}}$.

3. Integrate by parts or using the suitable substitutions:
   a) $\int dx \sqrt{1 - e^{2x}}$; b) $\int \frac{\sqrt{4 + x^2}}{x} dx$;
   c) $\int 12x \sin 3x dx$; d) $\int \frac{x^2 + 4x}{e^{2x}} dx$;
   e) $\int \arccos 2x dx$; f) $\int \frac{2x + 3}{\cos^2 x} dx$.

4. Integrate the polynomial fractions:
   a) $\int \frac{8x^2 - 31x + 25}{(x^2 - 5x + 6)(x - 1)} dx$;
   b) $\int \frac{2x^3 - 22x^2 + 52x + 49}{x^3 - 14x^2 + 49} dx$;
   c) $\int \frac{9x^2 - 10x + 34}{(x^2 - 2x + 10)(x - 2)} dx$.

5. Integrate trigonometric expressions:
   a) $\int \cos 9x \cos 2x dx$; b) $\int \sqrt{\frac{\cos x}{\sin^9 x}} dx$;
   c) $\int \frac{6 \sin x dx}{13 \cos^2 x + 4 \sin^2 x}$.

6. Integrate the functions with radicals:
   a) $\int x^2 \sqrt{9 - x^2} dx$; b) $\int \frac{(\sqrt{x} + 2 - 1) dx}{(x + 2)(1 + \sqrt{x} + 2)}$.

7. Solve the definite integrals:
   a) $\int_0^{e^{-1}} \sqrt{x + 1} dx$; b) $\int_0^\pi \cos^8 x \sin^3 x dx$;
   c) $\int_0^{\sqrt{3}} 2 \arctan x dx - x$; d) $\int_1^5 \frac{dx}{x + \sqrt{2x - 1}}$.

8. Find the area of the figure bounded by the curves:
   a) $x + y = 6, \ xy = 5$;
   b) $\left\{ \begin{array}{l} x = 3 \cos^3 t, \\ y = 3 \sin^3 t \end{array} \right\}$ \ for \ $0 \leq x \leq 0$ (\ $x \geq 0$);
   c) $\rho = 2 \sqrt{\cos 2\phi}$.

9. Find the arc-length of the curve:
   a) $y = 4 - e^x, \ 0 \leq x \leq \sqrt{15}$;
   b) $\left\{ \begin{array}{l} x = 2(t - \sin t), \\ y = 2(1 - \cos t) \end{array} \right\}$ \ for \ $0 \leq t \leq 2\pi$;
   c) $\rho = 2 \cos \phi + 2 \sin \phi$.

10. Find the area of the surface formed by rotating the curves around the $l$-axis:
    a) $y = 3x + 6, \ -2 \leq x \leq 4, \ l = OX$;
    b) $\left\{ \begin{array}{l} x = 2 + 4 \cos t, \\ y = 4 \sin t \end{array} \right\}$ \ for \ $x \geq 0$ \ $l = OY$;
    c) $\rho = 3e^{2\phi}, \ 0 \leq \phi \leq \pi, \ l = op$.

11. Find the volume of the body formed by rotating the curves around the $l$-axis:
    a) $y = -x^2 + 4x + 3, \ y = -x + 7, \ l = OX$;
    b) $y = \sqrt{4 - x^2}, \ y = x, \ x = 0, \ l = OY$;
    c) $\rho = 4(1 - \cos \phi), \ l = op$.

12. Solve the improper integrals:
    a) $\int_0^\infty xe^{-x} dx$; b) $\int_0^{\pi/2} \frac{\sin 2x dx}{\sqrt{1 - \cos^4 x}}$.
1. Integrate using the table and substitution under differential:
   a) \( \int \sqrt{x}(1 + \sqrt{x})^2 \, dx \); b) \( \int \frac{\sin^3 x}{1 + \cos x} \, dx \);
   c) \( \int x^3 + 3 \, dx \); d) \( \int \frac{dx}{\sin x + \cos x} \);

2. Integrate the quadratic fractions:
   a) \( \int -\frac{4x - 7}{x^2 - 9} \, dx \); b) \( \int \frac{8dx}{\sqrt{8 + 6x - 9x^2}} \);
   c) \( \int -\frac{2x - 12}{x^2 + 4x + 8} \, dx \); d) \( \int \frac{4x \, dx}{\sqrt{x^2 - 2x + 15}} \);

3. Integrate by parts or using the suitable substitutions:
   a) \( \int \frac{2x - 3}{(x - 2)^5} \, dx \); b) \( \int \frac{dx}{x\sqrt{5x^2 - 4x - 1}} \);
   c) \( \int 4(x \cos 2x - \sin 2x)^2 \, dx \);
   d) \( \int \ln(x - 1) \, dx \); e) \( \int \arctg x \, dx \);
   f) \( \int e^\frac{3x}{x-4} \, dx \);

4. Integrate the polynomial fractions:
   a) \( \int \frac{8x^2 + 7x - 30}{(x^2 - 3x + 2)(x + 2)} \, dx \);
   b) \( \int \frac{x^3 - 2x^2 - 14x + 21}{(x^3 - 5x^2 + 3x + 9) \, dx} \);
   c) \( \int \frac{9x + 64}{(x^2 + 6x + 10)(x - 4)} \, dx \);

5. Integrate trigonometric expressions:
   a) \( \int \cos 2x \sin 7x \, dx \); b) \( \int \frac{\cos^3 x}{\sin^9 x} \, dx \);
   c) \( \int \frac{dx}{5 + \sin x + 4 \cos x} \);

6. Integrate the fractions with radicals:
   a) \( \int \frac{x^2}{\sqrt{1 - x^2}} \, dx \); b) \( \int \frac{\sqrt{x}}{\sqrt{x - 3x^2}} \, dx \);

7. Solve the definite integrals:
   a) \( \int_{\pi/2}^{\pi/4} e^{\sin^2 x} \sin 2x \, dx \); b) \( \int_{0}^{\pi/4} \cos^4 2x \sin^3 2x \, dx \);
   c) \( \int_{0}^{\sqrt{3}} -12x + 9 \, dx \); d) \( \int_{0}^{\ln 2/3} 3e^{4x} \, dx \).

8. Find the area of the figure bounded by the curves:
   a) \( y = (x - 4)^2, \ y = 3x - 12 \);
   b) \( \begin{cases} x = 2 + 4 \cos t, \\ y = 2 \sin t \end{cases}; \ y = 1 (y \geq 1) \);
   c) \( \rho = 3 \sin 3\phi \).

9. Find the arc-length of the curve:
   a) \( y = 2 + \frac{1}{2} \ln(\sin 2x), \ \frac{\pi}{6} \leq x \leq \frac{\pi}{4} \);
   b) \( \begin{cases} x = 2 \cos t - \cos 2t, \\ y = 2 \sin t - \sin 2t \end{cases}; \ 0 \leq t \leq 2\pi \);
   c) \( \rho = 4\phi, \ 0 \leq \phi \leq \pi \).

10. Find the area of the surface formed by rotating the curves around the l-axis:
    a) \( y = \frac{1}{3} \mathrm{ch} 3x, \ 0 \leq x \leq 1 \);
    b) \( \begin{cases} x = 2(t - \sin t), \\ y = 2(1 - \cos t) \end{cases}; \ l = OX \);
    c) \( \rho = 4\sqrt{\cos 2\phi} \), \( l = op \).

11. Find the volume of the body formed by rotating the curves around the l-axis:
    a) \( y = -x^2 + 8x, \ y = -x \);
    b) \( x = 6 - y^2, \ x = 2 \);
    c) \( \rho = 2\cos \phi, \ l = op \).

12. Solve the improper integrals:
    a) \( \int_{0}^{\infty} \frac{4x - 5}{4x^2 - 4x + 2} \, dx \); b) \( \int_{0}^{2 \arcsin \left(\frac{x}{2}\right)} 2 \arcsin \left(\frac{x}{2}\right) - 2x \, dx \).
Personal task 15

1. Integrate using the table and substitution under differential:
   a) \( \int \frac{(3 - 2x)^3}{x^2} \, dx \);   b) \( \int \frac{1}{\sin x} \, dx \);
   c) \( \int \frac{x^4}{\sqrt{x^5 + 9}} \, dx \);   d) \( \int \frac{6\arcsin^5 x}{\sqrt{1 - x^2}} \, dx \).

2. Integrate the quadratic fractions:
   a) \( \int \frac{8x + 7}{x^2 - 4} \, dx \);   b) \( \int \frac{dx}{\sqrt{9x^2 - 4x - 12}} \);
   c) \( \int \frac{(-2x + 11)}{x^2 - 8x + 17} \, dx \);   d) \( \int \frac{6x + 3}{\sqrt{5 - 4x - x^2}} \, dx \).

3. Integrate by parts or using the suitable substitutions:
   a) \( \int \frac{dx}{\sqrt{e^x - 4}} \);   b) \( \int \frac{-4dx}{x^2\sqrt{5x^2 + 4x - 1}} \);
   c) \( \int 12x^2 \sin 3xdx \);   d) \( \int \cos^2 \frac{x}{e^2} \, dx \);
   e) \( \int \ln(4 - x^2) \, dx \);   f) \( \int \arctg \sqrt{x} \, dx \).

4. Integrate the polynomial fractions:
   a) \( \int \frac{6x^2 - 28x + 28}{(x^2 - 3x + 2)(x - 3)} \, dx \);
   b) \( \int \frac{2x^3 - 14x^2 + 30x - 22}{(x^2 - 4x + 3)(x - 1)} \, dx \);
   c) \( \int \frac{x^2 + 8x}{(x^2 + 4x + 8)(x + 2)} \, dx \).

5. Integrate trigonometric expressions:
   a) \( \int \cos x \sin x \cos 4xdx \);
   b) \( \int \sin^2 2x \cos^4 2xdx \);
   c) \( \int \frac{12 \cos x}{4 \sin^2 x - 5 \cos^2 x + 9} \, dx \).

6. Integrate the fractions with radicals:
   a) \( \int \frac{\sqrt{1 - x^2}}{x^4} \, dx \);   b) \( \int \frac{dx}{\sqrt{x + 3 + 3\sqrt{x + 3}}}.\)

7. Solve the definite integrals:
   a) \( \int_0^1 (x - 1)^2 e^{-x} \, dx \);   b) \( \int_0^{\pi/4} \sin^6 x \, dx \);
   c) \( \int_1^{5/2} (-2x + 5) \, dx \);   d) \( \int_0^{3/4} \sqrt[3]{(x - 2)^2} \, dx \).

8. Find the area of the figure bounded by the curves:
   a) \( x + 2y = 7, \ xy = 3 \);   b) \( \begin{cases}  x = 2 \cos t, \\ y = 2 \sin t \end{cases}; \ y = 1 \ (y \geq 1) \);
   c) \( \rho = 4(1 - \sin \phi) \).

9. Find the arc-length of the curve:
   a) \( y = 4 - \frac{1}{2} \text{ch} 2x, \ 0 \leq x \leq \frac{1}{2} \ln 3 \);
   b) \( \begin{cases} x = 2 \cos^3 t, \\ y = 2 \sin^3 t \end{cases}; \ 0 \leq t \leq \pi \);
   c) \( \rho = 5\sqrt{2}e^{-2\phi}, \ 0 \leq \phi \leq \frac{\pi}{2} \).

10. Find the area of the surface formed by rotating the curves around the l-axis:
    a) \( y = \frac{1}{8}x^2, \ 0 \leq x \leq 3 \); \ l = OY;
    b) \( \begin{cases} x = 2 + \cos t, \\ y = 3 + \sin t \end{cases}; \ l = OX; \)
    c) \( \rho = 6 \cos \phi, \ l = op \).

11. Find the volume of the body formed by rotating the curves around the l-axis:
    a) \( y = x^2 - 4x + 5, \ y = x + 5 \); \ l = OX;
    b) \( x = 4 - y^2, \ x = 8 - 2y^2 \); \ l = OY;
    c) \( \rho = 4 \sin 2\phi, \ l = op \).

12. Solve the improper integrals:
    a) \( \int_1^\infty \frac{2x^2 + 1}{x^4 + x^2} \, dx \);   b) \( \int_1^\infty \frac{dx}{x\sqrt{1 - \ln x}} \).
1. Integrate using the table and substitution under differential:
   a) \( \int \frac{4x^3e^x + 7e^{2x}}{e^x} \, dx \);  
   b) \( \int \frac{\sin 2xdx}{(\cos x + 3)^2 - 9} \);
   c) \( \int \frac{\ln^3(3 - 2x)}{3 - 2x} \, dx \);  
   d) \( \int \frac{\sin^2 \sqrt{x} \, dx}{\sqrt{x}} \).

2. Integrate the quadratic fractions:
   a) \( \int \frac{12x - 3}{x^2 - 9} \, dx \);  
   b) \( \int \frac{-10dx}{\sqrt{4x^2 - 4x - 24}} \);
   c) \( \int \frac{(12 - 6x)dx}{x^2 - 6x + 18} \);  
   d) \( \int \frac{7 - 4x}{\sqrt{8 - 2x - x^2}} \, dx \).

3. Integrate by parts or using the suitable substitutions:
   a) \( \int \sqrt{e^{2x} + 4} \, dx \);  
   b) \( \int \frac{6dx}{x \sqrt{9x^2 - 1}} \);
   c) \( \int 4x(\sin 2x + \cos 2x)^2dx \);
   d) \( \int (2x^2 - 6)e^{2x} \, dx \);  
   e) \( \int (x^2 + 4) \ln x \, dx \);
   f) \( \int \arcsin x \frac{dx}{\sqrt{x + 1}} \).

4. Integrate the polynomial fractions:
   a) \( \int \frac{-3x^2 - 4x - 11}{(x^2 - 5x + 4)(x + 1)} \, dx \);
   b) \( \int \frac{2x^3 + 10x^2 + 12x - 4}{x^3 + 4x^2 + 4x} \, dx \);  
   c) \( \int \frac{6x^2 + 4x + 36}{(x^2 + 6x + 18)(x - 2)} \, dx \).

5. Integrate trigonometric fractions:
   a) \( \int \sin 5x \sin 3x \, dx \);  
   b) \( \int \cos^2 x \sin^4 x \, dx \);
   c) \( \int \frac{4dx}{8 + \sin x + 7\cos x} \).

6. Integrate the fractions with radicals:
   a) \( \int \frac{x^5dx}{\sqrt{4 - x^2}} \);  
   b) \( \int \frac{3 \left( \frac{x + 1}{x - 2} \right)^2 \, dx}{(x + 1)^2} \).

7. Solve the definite integrals:
   a) \( \int_{\frac{\pi}{2}}^{2\pi+2} \frac{dx}{\sin(x - 2)} \);  
   b) \( \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{6\cos^2 x \, dx}{\sin^4 x} \);
   c) \( \int_{-1}^{\frac{\sqrt{31}}{2}} 3x^2 \sqrt{x^3 + 1} \, dx \);  
   d) \( \int_{-1}^{\sqrt{2}x} \frac{dx}{5 + \sqrt{2x - 4}} \).

8. Find the area of the figure bounded by the curves:
   a) \( y = 2x^2 - 13x + 16, \ x + y = 6; \)  
   b) \( \begin{cases} x = 2\cos^3 t, \\ y = 2\sin^3 t; \end{cases} \ y = 0(y \geq 0); \)
   c) \( \rho = 2\cos 2\phi \).

9. Find the arc-length of the curve:
   a) \( y = \ln(x + \sqrt{x^2 - 9}), \ 3 \leq x \leq 3\sqrt{5}; \)
   b) \( \begin{cases} x = 4(t - \sin t), \\ y = 4(1 - \cos t); \end{cases} \ 0 \leq t \leq 2\pi; \)
   c) \( \rho = 4 \sin \phi + 4 \cos \phi \).

10. Find the area of the surface formed by rotating the curves around the l-axis:
    a) \( x^2 + \frac{y^2}{4} = 1, \ l = OY; \)
    b) \( \begin{cases} x = e^t \cos t, \\ y = e^t \sin t; \end{cases} \ 0 \leq t \leq \pi, \ l = OX; \)
    c) \( \rho = 6 \sin \phi, \ l = \rho \).

11. Find the volume of the body formed by rotating the curves around the l-axis:
    a) \( y = -x^2 + 5x + 1, \ y = x + 1, \ l = OX; \)
    b) \( y = \sqrt{9 - x^2}, \ y = x, \ x = 0, \ l = OY; \)
    c) \( \rho = 5(1 - \cos \phi), \ l = \rho \).

12. Solve the improper integrals:
    a) \( \int_{-1}^{\infty} \frac{\arctg \left( \frac{x}{3} \right)}{x^2 + 9} \, dx \);  
    b) \( \int_{0}^{\frac{1}{2}} \frac{x + 1}{\sqrt{2x - x^2}} \, dx \).
1. Integrate using the table and substitution under differential:
   a) \( \int x(\sqrt{x} - 1)^3 dx \); b) \( \int \frac{dx}{\sin x + 2\cos x} \);
   c) \( \int x^2\sqrt{x^3 - 4} dx \); d) \( \int \frac{\tan^5 x}{\cos^4 x} dx \).

2. Integrate the quadratic fractions:
   a) \( \int \frac{-2x + 6}{x^2 - 9} dx \); b) \( \int \frac{dx}{\sqrt{15 - 4x - 4x^2}} \);
   c) \( \int \frac{4x - 7}{x^2 + 6x + 10} dx \); d) \( \int \frac{(4x - 5)dx}{\sqrt{x^2 + 2x - 3}} \).

3. Integrate by parts or using the suitable substitutions:
   a) \( \int \frac{3x + 7}{\sqrt{x} + 4} dx \);
   b) \( \int \frac{\ln x - 2}{x(4 + \ln^2 x)} dx \);
   c) \( \int 6x^2 \sin 2xdx \);
   d) \( \int x^2(2x - 8x) dx \);
   e) \( \int \ln(x - \sqrt{x^2 - 4}) dx \);
   f) \( \int x \frac{\sin x}{\cos^5 x} dx \).

4. Integrate the polynomial fractions:
   a) \( \int \frac{13x + 19}{(x^2 + x - 6)(x + 1)} dx \);
   b) \( \int \frac{2x^3 + 3x^2 - 28x - 22}{x^3 + x^2 - 8x - 12} dx \);
   c) \( \int \frac{8x^2 + 11x + 11}{(x^2 - 2x + 10)(x + 3)} dx \).

5. Integrate trigonometric expressions:
   a) \( \int \sin 2x \cos 5xdx \);
   b) \( \int \sqrt{\frac{\sin x}{\cos^9 x}} dx \);
   c) \( \int \frac{dx}{5 \sin x - 12 \cos x} \).

6. Integrate the functions with radicals:
   a) \( \int \frac{\sqrt{9 + x^2}}{x^4} dx \);
   b) \( \int \frac{dx}{\sqrt{x - 2\sqrt{3}} x} \).

7. Solve the definite integrals:
   a) \( \int (2x + 2)e^{2x} dx \);
   b) \( \int \sin^4\left(\frac{x}{2}\right) dx \);
   c) \( \int \frac{1/\sqrt{2} \arcsin^2 x - x}{\sqrt{1 - x^2}} dx \);
   d) \( \int \sqrt{\frac{3}{\pi}} \cos \sqrt{\frac{x}{3}} dx \).

8. Find the area of the figure bounded by the curves:
   a) \( y = (x + 2)^2, \ y = 4x + 8 \);
   b) \( \begin{cases} x = 2\cos t, \\ y = 5\sin t \end{cases} \quad x = 1 \ (x \geq 1) \);
   c) \( \rho = 4\sin 3\phi \).

9. Find the arc-length of the curve:
   a) \( y = \frac{1}{4} \ln(\cos 4x), \quad -\frac{\pi}{24} \leq x \leq \frac{\pi}{24} \);
   b) \( \begin{cases} x = 4e^{-t} \cos t, \\ y = 4e^{-t} \sin t \end{cases} \quad 0 \leq t \leq 2\pi \);
   c) \( \rho = 6\sin^2 \left(\frac{\phi}{2}\right) \).

10. Find the area of the surface formed by rotating the curves around the l-axis:
   a) \( y = 1 + \frac{1}{4} \text{ch}4x, \quad -1 \leq x \leq 1, \ l = OX \);
   b) \( \begin{cases} x = 3(t - \sin t), \\ y = 3(1 - \cos t) \end{cases} \quad 0 \leq t \leq 2\pi, \ l = OX \);
   c) \( \rho = 2\sqrt{\cos 2\phi}, \ l = o\rho \).

11. Find the volume of the body formed by rotating the curves around the l-axis:
   a) \( y = -x^2 + 5x + 1, \ xy = 5, \ l = OX \);
   b) \( y = 4 - x^2, \ y = 8 - 2x^2, \ l = OY \);
   c) \( \rho = 6\phi, \ 0 \leq \phi \leq \pi, \ l = o\rho \).

12. Solve the improper integrals:
   a) \( \int_1^\infty \frac{dx}{x^2(x^2 + 1)} \);
   b) \( \int_0^1 \left(\frac{1}{5}\right)^x dx \).
1. Integrate using the table and substitution under differential:
   a) \( \int \frac{3x^4 - x^2 3x}{x^2} \, dx \); b) \( \int \frac{4 - 3 \cos^2 x}{1 - \cos 2x} \, dx \);
   c) \( \int \frac{\ln^5(1 - 3x)}{1 - 3x} \, dx \); d) \( \int (\cosh x + \sinh x)^2 \, dx \).

2. Integrate the quadratic fractions:
   a) \( \int \frac{-4x - 8}{x^2 - 16} \, dx \); b) \( \int \frac{dx}{\sqrt{9x^2 - 6x - 8}} \);
   c) \( \int \frac{(-4x - 11)\, dx}{x^2 + 4x + 13} \); d) \( \int \frac{(2x + 10)\, dx}{\sqrt{15 + 2x - x^2}} \).

3. Integrate by parts or using the suitable substitutions:
   a) \( \int \frac{e^{2x} - 1 + e^x}{\sqrt{1 - e^{2x}}} \, dx \); b) \( \int \frac{\sqrt{x^2 + 2}}{x} \, dx \);
   c) \( \int (2x^2 - 6x + 8) \cos 2x \, dx \);
   d) \( \int \frac{3 - 6x^2}{e^{3x}} \, dx \); e) \( \int \frac{\arccos \sqrt{x}}{\sqrt{x}} \, dx \);
   f) \( \int 6(x^2 + 3) \ln(x^2 + 9) \, dx \).

4. Integrate the polynomial fractions:
   a) \( \int \frac{9x^2 - 31x + 20}{(x^2 - 5x + 6)(x + 1)} \, dx \);
   b) \( \int \frac{2x^3 + x^2 + 5x + 2}{x^3 - x^2 - x + 1} \, dx \);
   c) \( \int \frac{7x^2 - 36x + 13}{x^3 - 6x^2 + 13x} \, dx \).

5. Integrate trigonometric expressions:
   a) \( \int \sin 5x \sin 3x \, dx \); b) \( \int \frac{\cos^3 x}{\sin^9 x} \, dx \);
   c) \( \int \frac{dx}{5 + 3 \sin x} \).

6. Integrate the fractions with radicals:
   a) \( \int \frac{4(x + 2)^{-2} \, dx}{\sqrt{8 + 4x + x^2}} \);
   b) \( \int \frac{(\sqrt{x} + 1) \, dx}{\sqrt{x^3 + 4\sqrt{x}}} \).

7. Solve the definite integrals:
   a) \( \int_0^{e-1} \ln(x + 1) \, dx \); b) \( \int_0^\pi \cos^4 x \sin^4 x \, dx \);
   c) \( \int_1^9 \frac{x^2 - 16}{\sqrt{x} + 2} \, dx \); d) \( \int_0^{\pi/4} (e^{-\tan x} + \sin x) \, dx \).

8. Find the area of the figure bounded by the curves:
   a) \( y = (x + 1)^3, \ y = 5(x + 1) \);
   b) \( \begin{cases} x = 3 \cos^3 t, & y = 4 \sin^3 t; \\ y = 2 (y \geq 2) \end{cases} \);
   c) \( \rho = 4 \sqrt{\sin 2\phi} \).

9. Find the arc-length of the curve:
   a) \( y = 2 - \frac{1}{3} \cosh 3x, \ 0 \leq x \leq \frac{1}{3} \ln 3 \);
   b) \( \begin{cases} x = 3(t - \sin t), & y = 3(1 - \cos t); \\ 0 \leq t \leq 2\pi \end{cases} \);
   c) \( \rho = \sqrt{2} e^{-2\phi}, \ 0 \leq \phi \leq \pi \).

10. Find the area of the surface formed by rotating the curves around the l-axis:
    a) \( y = x^3, \ -3 \leq x \leq 3, \ l = OX \);
    b) \( \begin{cases} x = 3 + \cos t, & l = OY; \\ y = 2 + \sin t \end{cases} \);
    c) \( \rho = 6(1 + \cos \phi), \ l = \rho l \).

11. Find the volume of the body formed by rotating the curves around the l-axis:
    a) \( y = -x^2 + 6x, \ y = -2x^2 + 12x, \ l = OX \);
    b) \( y = x^2, \ y = 3x^2 - 18, \ l = OY \);
    c) \( \rho = 8 \cos \phi, \ l = \rho l \).

12. Solve the improper integrals:
    a) \( \int_0^\infty x e^{-x^2} \, dx \); b) \( \int_0^1 \frac{4x - 2}{\sqrt{2x - x^2}} \, dx \).
1. Integrate using the table and substitution under differential:
   a) \( \int \frac{(1 - 3\sqrt{x})^3}{\sqrt{x}} \, dx \);
   b) \( \int \frac{18\operatorname{ctg}^5x}{1 - \cos^2 x} \, dx \);
   c) \( \int \frac{x^3 - x}{\sqrt{1 - x^2}} \, dx \);
   d) \( \int (\cos x + \frac{3}{\cos x})^2 \, dx \).

2. Integrate the quadratic fractions:
   a) \( \int \frac{-2x + 6}{x^2 - 9} \, dx \);
   b) \( \int \frac{dx}{\sqrt{3 - 8x - 16x^2}} \);
   c) \( \int \frac{(4 - 2x)dx}{x^2 + 4x + 20} \);
   d) \( \int \frac{(6x - 9)dx}{\sqrt{x^2 - 4x + 5}} \).

3. Integrate by parts or using the suitable substitutions:
   a) \( \int \frac{2 - 5x}{(x + 2)^7} \, dx \);
   b) \( \int \frac{(2 + 4\ln x)dx}{x \sqrt{\ln^2 x + 4}} \);
   c) \( \int 16x \sin 4x \, dx \);
   d) \( \int \frac{x^2 - 4x}{e^{2x}} \, dx \);
   e) \( \int \ln(\sqrt{x} + 1) \, dx \);
   f) \( \int \frac{2x + 4}{\sin^2 x} \, dx \).

4. Integrate the polynomial fractions:
   a) \( \int \frac{7x^2 + 24x - 15}{x^3 + 2x^2 - 3x} \, dx \);
   b) \( \int \frac{4x^3 + 17x^2 + 27x + 16}{(x^2 + 3x + 2)(x + 1)} \, dx \);
   c) \( \int \frac{x^2 + 14x - 15}{(x^2 - 2x + 5)(x + 2)} \, dx \).

5. Integrate trigonometric expressions:
   a) \( \int \cos x \sin 9x \, dx \);
   b) \( \int \frac{\cos^2 x}{\sin^6 x} \, dx \);
   c) \( \int \frac{4dx}{4 + 4\sin x + 5\cos x} \).

6. Integrate the fractions with radicals:
   a) \( \int \frac{dx}{x^3\sqrt{1 - x^2}} \);
   b) \( \int \sqrt{\frac{x + 2}{x - 1} (x + 2)^2} \, dx \).

7. Solve the definite integrals:
   a) \( \int_{\pi/4}^{\pi/2} (1 - \sin \sqrt{x}) \, dx \);
   b) \( \int_{\pi/9}^{\pi/2} x \cos^2 x \, dx \);
   c) \( \int_{0}^{\sqrt{3}} \frac{2x + 4}{x^2 + 1} \, dx \);
   d) \( \int_{0}^{3} x^5 \sqrt{9 - x^2}^3 \, dx \).

8. Find the area of the figure bounded by the curves:
   a) \( y = -x^2 + 4x + 1 \), \( xy = 4 \);
   b) \( \begin{cases} x = 2 \cos t, \\ y = 2 + 4 \sin t \end{cases} \), \( x = 1 \) \( (x \geq 1) \);
   c) \( \rho = 3 \cos 3\phi \).

9. Find the arc-length of the curve:
   a) \( y = 2 - e^{2x}, \ \frac{1}{4} \ln 2 \leq x \leq \frac{1}{4} \ln 6 \);
   b) \( \begin{cases} x = 3(2 \cos t - \cos 2t), \\ y = 3(2 \sin t - \sin 2t) \end{cases} \), \( 0 \leq t \leq 2\pi \);
   c) \( \rho = 2 \cos \phi + 2 \sin \phi \).

10. Find the area of the surface formed by rotating the curves around the l-axis:
    a) \( y = 3x + 9 \), \( -3 \leq x \leq 3 \), \( l = OX \);
    b) \( \begin{cases} x = 4 \cos^3 t, \\ y = 4 \sin^3 t \end{cases} \), \( l = OY \);
    c) \( \rho = \sqrt{\sin 2(\phi - \frac{\pi}{4})} \), \( l = op \).

11. Find the volume of the body formed by rotating the curves around the l-axis:
    a) \( y = (x - 2)^2 \), \( y = x + 4 \), \( l = OX \);
    b) \( y = x^2 - 8 \), \( y = 2x \), \( x = 0 \) \( (x \geq 0) \), \( l = OY \);
    c) \( \rho = \phi \), \( 0 \leq \phi \leq \pi \), \( l = op \).

12. Solve the improper integrals:
    a) \( \int_{-1/2}^{\infty} \arctg^3 \left( \frac{x}{2} \right) \, dx \);
    b) \( \int_{1}^{2} \frac{2(x + 1) \, dx}{\sqrt{4x - x^2 - 3}} \).
Personal task 20

1. Integrate using the table and substitution under differential:
   a) $\int \sqrt{x(2-x\sqrt{x})^2} \, dx$; b) $\int \frac{1 + \cos 2x}{1 - \cos 2x} \, dx$;
   c) $\int \frac{-8x}{\sqrt{1 - 4x^2}} \, dx$; d) $\int (1 - 3\sin \frac{x}{2})^2 \, dx$.

2. Integrate the quadratic fractions:
   a) $\int \frac{7 - 6x}{x^2 - 1} \, dx$; b) $\int \frac{dx}{\sqrt{12 + 4x - 4x^2}}$;
   c) $\int \frac{-4x \, dx}{x^2 - 2x + 5}$; d) $\int \frac{(14 - 6x) \, dx}{\sqrt{x^2 - 8x + 25}}$.

3. Integrate by parts or using the suitable substitutions:
   a) $\int \frac{(1 - e^x)^2}{1 + e^x} \, dx$; b) $\int \frac{dx}{(x - 3)^2 \sqrt{6x - x^2}}$;
   c) $\int (4 - 8x^2) \cos 4x \, dx$; d) $\int \ln^2(x + 2) \, dx$; e) $\int 2\sqrt{x^2 + 2} \, dx$;
   f) $\int \arccos(x^2 - 1) \, dx$.

4. Integrate the polynomial fractions:
   a) $\int \frac{4x^3 - 8x - 36}{x^3 + x^2 - 9x - 9} \, dx$;
   b) $\int \frac{2x^3 - 9x^2 + x + 22}{x^3 - 2x^2 - 4x - 8} \, dx$;
   c) $\int \frac{4x^2 + 10x + 26}{(x^2 + 6x + 13)(x - 1)} \, dx$.

5. Integrate trigonometric expressions:
   a) $\int \cos 2x \cos 5x \, dx$; b) $\int \frac{3\cos^4 x \sin^3 x \, dx}{12x}$;
   c) $\int \frac{4 \cos^2 x - 4 \sin x \cos x - 8 \sin^2 x}{x^2} \, dx$.

6. Integrate the fractions with radicals:
   a) $\int \frac{x^2 \, dx}{\sqrt{(4 - x^2)^3}}$; b) $\int \frac{dx}{(x + 5) \sqrt{9 + x}}$.

7. Solve the definite integrals:
   a) $\int_1^{\ln^2+1} (x - 1)e^{x - 1} \, dx$; b) $\int_0^\pi \sin^4 x \cos^2 x \, dx$;
   c) $\int_1^{\sqrt{\frac{5}{2}}} (2x + 7) \, dx$; d) $\int_{-1}^{\frac{\sqrt{8}}{2}} \frac{\sqrt{x + 1 - 2}}{x + 2} \, dx$.

8. Find the area of the figure bounded by the curves:
   a) $2x + y = 7$, $xy = 3$; b) $\begin{cases} x = 4 \cos t, \\ y = 2 + 4 \sin t \end{cases}$, $x = 2 \ (x \geq 2)$;
   c) $\rho = 4(1 - \sin \phi)$.

9. Find the arc-length of the curve:
   a) $y = 4 \ln(16 - x^2)$, $-2 \leq x \leq 2$; b) $\begin{cases} x = 3 \cos^3 t, \\ y = 3 \sin^3 t \end{cases}$, $0 \leq t \leq \frac{\pi}{4}$;
   c) $\rho = \phi$, $0 \leq \phi \leq 2\pi$.

10. Find the area of the surface formed by rotating the curves around the l-axis:
    a) $y = 4\cosh \left(\frac{x}{4}\right)$, $0 \leq x \leq 4$, $l = OX$;
    b) $\begin{cases} x = 2(t - \sin t), \\ y = 2(1 - \cos t) \end{cases}$, $0 \leq t \leq 2\pi$;
    c) $\rho = 8 \cos \phi$, $l = \rho \phi$.

11. Find the volume of the body formed by rotating the curves around the l-axis:
    a) $y = x^2 - 6x$, $y = 2x^2 - 12x$, $l = OX$;
    b) $y = 9 - x^2$, $x = 8$, $l = OY$;
    c) $\rho = 4 \sin 2\phi$, $l = \rho \phi$.

12. Solve the improper integrals:
    a) $\int_0^\infty \frac{-2x + 5}{x^2 + 1} \, dx$; b) $\int_0^{\pi/2} \frac{3 \cos x}{\sqrt{1 - \sin x}} \, dx$. 

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Personal task 21

1. Integrate using the table and substitution under differential:
   a) $\int \frac{(1 - 3x^2)^3}{x^5} dx$; b) $\int \frac{2 - \sin x}{\sin^2 x} dx$;
   c) $\int x^3 e^{2x^4 + 5} dx$; d) $\int \frac{(\sin 2x - 6 \sin x) dx}{\cos^2 x + 9}$.

2. Integrate the quadratic fractions:
   a) $\int \frac{4 - 8x}{x^2 - 4} dx$; b) $\int \frac{dx}{\sqrt{8 + 6x - 9x^2}}$;
   c) $\int \frac{(6x + 7) dx}{x^2 + 4x + 5}$; d) $\int \frac{(2x - 6) dx}{\sqrt{x^2 - 2x - 3}}$.

3. Integrate by parts or using the suitable substitutions:
   a) $\int \sqrt{e^{2x} + 16} dx$; b) $\int \frac{dx}{x^2 \sqrt{1 - 4x^2}}$;
   c) $\int (x^3 - 6) \sin x dx$; d) $\int x \ln(x + 1) dx$;
   e) $\int \arcsin \sqrt{x} dx$; f) $\int \cos (\frac{1}{2} \ln x) dx$.

4. Integrate the polynomial fractions:
   a) $\int \frac{x^2 + 5x - 20}{(x^2 - x - 2)(x - 3)} dx$;
   b) $\int \frac{-2x^3 - 10x^2 + x + 20}{x^3 + 3x^2 - 4} dx$;
   c) $\int \frac{-3x^2 - 4x + 44}{(x^2 - 8x + 20)(x + 2)} dx$.

5. Integrate trigonometric expressions:
   a) $\int \sin 5x \cos 2x dx$; b) $\int \cos^4 2x \sin^2 2x dx$;
   c) $\int \frac{-6 \sin x dx}{10 - \sin^2 x + 3 \cos^2 x}$.

6. Integrate the fractions with radicals:
   a) $\int \frac{\sqrt{x^2 + 9}}{x^4} dx$; b) $\int \frac{(4 - \sqrt{x}) dx}{(2 + \sqrt{x}) \sqrt{x}}$.

7. Solve the definite integrals:
   a) $\ln 2 + 1 \int_1^{\pi/4} \frac{\sin^3 x}{\cos^7 x} dx$; b) $\int_0^{\sqrt{10}} \frac{\sin x}{\sqrt{x^3 - 1}} dx$;
   c) $\int_0^{\sqrt{y + 1}} \frac{3 \sqrt{x + 1} dx}{4 + \sqrt{(x + 1)^2}}$.

8. Find the area of the figure bounded by the curves:
   a) $y = 2x^2 - 9x + 10$, $y = -x^2 + 3x + 1$; b) $\left\{ \begin{array}{l} x = 2 \cos^3 t, \\ y = 4 \sin^3 t; \end{array} \right.$ $x = 1$ $(x \geq 1)$;
   c) $\rho = 2 \cos 4\phi$.

9. Find the arc-length of the curve:
   a) $y = 2 + \ln x$, $1 \leq x \leq \sqrt{15}$; b) $\left\{ \begin{array}{l} x = 3(t - \sin t), \\ y = 3(1 - \cos t); \end{array} \right.$ $0 \leq t \leq 2\pi$;
   c) $\rho = \sqrt{2} e^{-2\phi}$, $0 \leq \phi \leq \pi$.

10. Find the area of the surface formed by rotating the curves around the $l$-axis:
    a) $y = \sqrt{\frac{x}{6}} (x - 12)$, $0 \leq x \leq 12$, $l = OX$;
    b) $\left\{ \begin{array}{l} x = 1 + 2 \cos t, \\ y = 2 \sin t; \end{array} \right.$ $(x \geq 0)$ $l = OY$;
    c) $\rho = \sqrt{\cos 2\phi}$, $l = op$.

11. Find the volume of the body formed by rotating the curves around the $l$-axis:
    a) $y = x^2 - 6x + 10$, $y = x + 10$, $l = OX$;
    b) $x = -y^2 + 6y - 8$, $x = 0$, $l = OY$;
    c) $\rho = 4(1 + \cos \phi)$, $l = op$.

12. Solve the improper integrals:
    a) $\int_0^\infty \frac{x + \arctg x}{x^2 + 1} dx$; b) $\int_0^{1/3} \frac{-x + 3}{\sqrt{1 - 9x^2}} dx$. 
1. Integrate using the table and substitution under differential:
   a) \( \int \frac{(2 + \sqrt{x})^3}{x} \, dx \);  
   b) \( \int \frac{dx}{\text{sh} x - \text{ch} x} \);  
   c) \( \int x^5 3x^3 - 2 \, dx \);  
   d) \( \int \frac{dx}{\sqrt{1 - \cos^4 x}} \).

2. Integrate the quadratic fractions:
   a) \( \int \frac{6x - 5}{x^2 - 1} \, dx \);  
   b) \( \int \frac{dx}{\sqrt{5 + 8x - 4x^2}} \);  
   c) \( \int \frac{(4x - 2) \, dx}{x^2 - 4x + 13} \);  
   d) \( \int \frac{(-4x - 10) \, dx}{\sqrt{x^2 + 6x + 5}} \).

3. Integrate by parts or using the suitable substitutions:
   a) \( \int \frac{(3x - 7) \, dx}{(x + 1)^5} \);  
   b) \( \int \frac{dx}{(x - 3)^2\sqrt{6x - x^2}} \);  
   c) \( \int 6x^2 \sin 2x \, dx \);  
   d) \( \int e^x (\cos 4x + 2) \, dx \);  
   e) \( \int x^6 \ln x \, dx \);  
   f) \( \int \frac{\arctg \sqrt{x}}{\sqrt{x}} \, dx \).

4. Integrate the polynomial fractions:
   a) \( \int \frac{x^2 + 20x + 24}{(x^2 - 3x - 4)(x + 2)} \, dx \);  
   b) \( \int \frac{-3x^3 + 3x^2 + 4x - 1}{x^3 - 2x^2 + x} \, dx \);  
   c) \( \int \frac{4x^2 - 28x - 4}{(x^2 + 2x + 2)(x - 4)} \, dx \).

5. Integrate trigonometric expressions:
   a) \( \int \sin x \sin 7x \, dx \);  
   b) \( \int \frac{\cos^3 x}{\sin^9 x} \, dx \);  
   c) \( \int \frac{dx}{5 - 2 \sin x + 4 \cos x} \).

6. Integrate the fractions with radicals:
   a) \( \int \frac{x^3 \, dx}{\sqrt{(9 - x^2)^3}} \);  
   b) \( \int \frac{x + 1}{x - 2} \frac{dx}{(x + 1)^2} \).

7. Solve the definite integrals:
   a) \( \int_0^{\log_2 3} x^2 2^x \, dx \);  
   b) \( \int_0^{2\pi} \cos^4 \left(\frac{x}{4}\right) \sin^2 \left(\frac{x}{4}\right) \, dx \);  
   c) \( \int_0^{\frac{\sqrt{2} - 1}{2}} -12x^3 \, dx \);  
   d) \( \int_1^e \frac{\ln x \, dx}{x \sqrt{1 + \ln x}} \).

8. Find the area of the figure bounded by the curves:
   a) \( y = (x + 2)^2, \ y = 4x + 8 \);  
   b) \( \begin{cases} x = 4 \cos t, & y = 3 \sin t, \ x = 2 \ (x \geq 2) \end{cases} \);  
   c) \( \rho = 3 \cos 2\phi \).

9. Find the arc-length of the curve:
   a) \( y = \sqrt{4 - x^2}, \ \sqrt{2} \leq x \leq 2 \);  
   b) \( \begin{cases} x = 3e^{-t} \cos t, & y = 3e^{-t} \sin t, \ 0 \leq t \leq 2\pi \end{cases} \);  
   c) \( \rho = 6(1 + \cos \phi) \).

10. Find the area of the surface formed by rotating the curves around the l-axis:
    a) \( y = \sqrt{4x + 12}, \ -3 \leq x \leq 1, \ l = OX \);  
    b) \( \begin{cases} x = \cos^3 t, & y = \sin^3 t, \ l = OY \end{cases} \);  
    c) \( \rho = 2 \sin \phi, \ l = op \).

11. Find the volume of the body formed by rotating the curves around the l-axis:
    a) \( y = x^2 + 4x + 4, \ y = 4 - x, \ l = OX \);  
    b) \( x = -y^2 + 3y, \ x = -2y^2 + 6y, \ l = OY \);  
    c) \( \rho = \sin 2\phi, \ l = op \).

12. Solve the improper integrals:
    a) \( \int_1^\infty \frac{-18x}{4x^4 - 9} \, dx \);  
    b) \( \int_{-\ln 2}^{\ln 2} \frac{e^x}{\sqrt{1 - e^{2x}}} \, dx \).
Personal task 23

1. Integrate using the table and substitution under differential:
   a) \( \int \frac{(3 - 2x^4)^2}{x^2} \, dx \);    b) \( \int (3 + \tan^2 x) \, dx \);
   c) \( \int \ln^7 (5 - 3x) \, dx \);    d) \( \int \cos^2 (2x + 5) \, dx \).

2. Integrate the quadratic fractions:
   a) \( \int \frac{6x + 5}{x^2 - 9} \, dx \);    b) \( \int \frac{dx}{\sqrt{5 + 4x - 4x^2}} \);
   c) \( \int \frac{6dx}{x^2 + 2x + 10} \);    d) \( \int \frac{(4x + 4) \, dx}{\sqrt{7 - 6x - x^2}} \).

3. Integrate by parts or using the suitable substitutions:
   a) \( \int \frac{\sqrt{2x + 3} \, dx}{4 + \sqrt{2x + 3}} \);    b) \( \int \frac{x \, dx}{x^2 \sqrt{9 + x^2}} \);
   c) \( \int \frac{x^2 - 2x \, dx}{e^{2x}} \);    d) \( \int (8 - 4x) \sin 4xdx \);
   e) \( \int 2x \frac{\cos x \, dx}{\sin^3 x} \);    f) \( \int \arccos (1 - x^2) \, dx \).

4. Integrate the polynomial fractions:
   a) \( \int \frac{2x^2 + 10x - 10}{(x^2 - 7x + 10)(x + 1)} \, dx \);
   b) \( \int \frac{-2x^3 + 2x^2 + 15x - 16}{x^3 - 4x^2 + 4x} \, dx \);
   c) \( \int \frac{4x^2 + 19x - 29}{(x^2 - 2x + 10)(x + 3)} \, dx \).

5. Integrate trigonometric expressions:
   a) \( \int \cos 2x \cos 6xdx \);    b) \( \int \cos^4 x \sin^5 x \, dx \);
   c) \( \int \frac{-6dx}{9 \sin^2 x - 6 \sin x \cos x - 3 \cos^2 x} \).

6. Integrate the fractions with radicals:
   a) \( \int \frac{dx}{x^3 \sqrt{x^2 - 4}} \);    b) \( \int \frac{dx}{\sqrt{x} + 4 \sqrt[3]{x^2}} \).

7. Solve the definite integrals:
   a) \( \frac{e^{x+1}}{2} \);    b) \( \frac{\pi/2 \cos^2 x}{\sin^4 x} \);
   c) \( \frac{3\sqrt{3} \, x^2 \, dx}{9 + x^2} \);    d) \( \frac{4 \, 6dx}{x + \sqrt{3x + 4}} \).

8. Find the area of the figure bounded by the curves:
   a) \( x + y = 9, \ xy = 8 \);
   b) \( \begin{cases} x = 2 \cos^3 t, \\ y = 2 \sin^3 t; \quad x = 1 (x \geq 1) \end{cases} \);
   c) \( \rho = 4 \sin 2\phi \).

9. Find the arc-length of the curve:
   a) \( y = \ln(x + \sqrt{x^2 - 9}), \ 3 \leq x \leq 3\sqrt{6} \);
   b) \( \begin{cases} x = 2(t - \sin t), \\ y = 2(1 - \cos t); \quad 0 \leq t \leq 2\pi \end{cases} \);
   c) \( \rho = 3\sqrt{2}e^{-2\phi}, \ 0 \leq \phi \leq \pi \).

10. Find the area of the surface formed by rotating the curves around the l-axis:
    a) \( y = 2 \arcsinh \left(\frac{x}{2}\right), \ -2 \leq x \leq 2, \ l = OX \);
    b) \( \begin{cases} x = 3 + \cos t, \\ y = 1 + \sin t; \quad l = OY \end{cases} \);
    c) \( \rho = 4 \sqrt{\cos 2\phi}, \ l = O\rho \).

11. Find the volume of the body formed by rotating the curves around the l-axis:
    a) \( y = x^2 - 2x + 10, \ y = x + 10, \ l = OX \);
    b) \( y = \ln x, \ x = 0, \ y = 0, \ y = \ln 5, \ l = OY \);
    c) \( \rho = 2(1 - \cos \phi), \ l = O\rho \).

12. Solve the improper integrals:
    a) \( \int_0^\infty \frac{2xdx}{\sqrt{(x^2 + 1)^2}} \);    b) \( \int_0^{\pi/4} \frac{\sin \sqrt{x} - 1}{\sqrt{x}} \, dx \).
Personal task 24

1. Integrate using the table and substitution under differential:
   a) $\int x^2(4 - 3x^2)^3 \, dx$;  
   b) $\int \frac{\sin 2x \, dx}{4 - (\sin x - 2)^2}$; 
   c) $\int x^5 3^x e^{-2 \, dx}$;  
   d) $\int \text{th}^4 x \, dx$.

2. Integrate the quadratic fractions:
   a) $\int \frac{-4x - 8}{x^2 - 16} \, dx$;  
   b) $\int \frac{\sqrt{12x - 9x^2}}{x - 1} \, dx$; 
   c) $\int (2x + 6) \, dx$;  
   d) $\int (3x - 9) \, dx$;  
   e) $\int \frac{\sqrt{x^2 + 4x - 5}}{x^2 - 2x + 5} \, dx$.

3. Integrate by parts of using the suitable substitutions:
   a) $\int \frac{(e^x + 1)^2}{\sqrt{e^{2x + 1}}} \, dx$; 
   b) $\int \frac{dx}{(x - 1) \sqrt{2x - x^2}}$; 
   c) $\int (4 - 2x^2) \cos 2x \, 2x \, dx$; 
   d) $\int (2 - 3x) 3^x \, dx$;  
   e) $\int \frac{4 \ln x}{x^{\sqrt{x}}} \, dx$; 
   f) $\int (5x^4 - 1) \arctg x \, dx$.

4. Integrate the polynomial fractions:
   a) $\int \frac{-x^2 + 3x - 6}{(x^2 - 3x + 2)(x - 3)} \, dx$; 
   b) $\int \frac{3x^3 - 4x^2 - 8x - 10}{x^3 - 3x - 2} \, dx$;  
   c) $\int \frac{4x^2 - 10x - 40}{x^3 + 4x^2 + 20x} \, dx$.

5. Integrate trigonometric expressions:
   a) $\int \cos 3x \sin 9x \, dx$;  
   b) $\int \frac{\cos^3 2x}{\sin^7 2x} \, dx$; 
   c) $\int \frac{dx}{9 - 3 \sin x + 5 \cos x}$.

6. Integrate the fractions with radicals:
   a) $\int \frac{\sqrt{9 - x^2}}{x^2} \, dx$;  
   b) $\int \frac{\sqrt{x} + \sqrt{x}}{1 + \sqrt{x}} \, dx$.

7. Solve the definite integrals:
   a) $\int_0^2 (4x - 8)e^{-2x} \, dx$;  
   b) $\int_0^\pi/6 \cos^2 3x \sin^5 3x \, dx$; 
   c) $\int_{\sqrt{2}}^2 \frac{2x - 6}{x^2 - 9} \, dx$;  
   d) $\int_1^x x^2 \sqrt{1 - x^2} \, dx$.

8. Find the area of the figure bounded by the curves:
   a) $y = (x - 4)^2$, $y = 3x - 12$; 
   b) $\begin{cases} x = t - \sin t, & y = 1 - \cos t; \\ y = 0, & 0 \leq t \leq 2\pi; \end{cases}$
   c) $\rho = 3 \cos 3\phi$.

9. Find the arc-length of the curve:
   a) $y = \frac{1}{3} + \frac{1}{3} \ln(\sin 3x)$, $\frac{\pi}{3} \leq x \leq \frac{2\pi}{3}$; 
   b) $\begin{cases} x = 2(2 \cos t - \cos 2t), & 0 \leq t \leq 2\pi; \\ y = 2(2 \sin t - \sin 2t); \end{cases}$
   c) $\rho = 4\phi$, $0 \leq \phi \leq \pi$.

10. Find the area of the surface formed by rotating the curves around the l-axis:
    a) $x^2 + \frac{y^2}{16} = 1$, $l = OY$; 
    b) $\begin{cases} x = e^t \cos t, & 0 \leq t \leq \pi; \\ y = e^t \sin t; \end{cases}$ $l = OX$; 
    c) $\rho = 4\sqrt{\sin 2\left(\phi - \frac{\pi}{4}\right)}$, $l = \rho$.

11. Find the volume of the body formed by rotating the curves around the l-axis:
    a) $y = -x^2 + 6x + 1$, $xy = 6$, $l = OX$; 
    b) $x = 4y - y^2$, $x = 8y - 2y^2$, $l = OY$; 
    c) $\rho = 4 \sin \phi$, $l = \rho$.

12. Solve the improper integrals:
    a) $\int_{-1}^{\infty} -8x - 10 \, dx$;  
    b) $\int_{1/\sqrt{2}}^{x} \frac{dx}{\sqrt{1 - x^2}}$. 

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Personal task 25

1. Integrate using the table and substitution under differential:
   a) \( \int \frac{3x^2e^x - e^{2x}}{e^x} \, dx \); b) \( \int \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \, dx \);
   c) \( \int \frac{x^4 \, dx}{\sqrt{x^5 + 9}} \); d) \( \int \frac{\sin 2x - 2\sin x}{\cos^2 x - 4} \, dx \).

2. Integrate the quadratic fractions:
   a) \( \int \frac{4x - 7}{x^2 - 1} \, dx \); b) \( \int \frac{dx}{\sqrt{15 - 6x - 9x^2}} \);
   c) \( \int \frac{(2 - 8x - 12x^2)}{x^2 - 8x + 12} \, dx \); d) \( \int \frac{(4x - 2) \, dx}{\sqrt{x^2 - 4x - 5}} \).

3. Integrate by parts or using the suitable substitutions:
   a) \( \int \frac{\sqrt{x} + 2}{x - 9} \, dx \); b) \( \int \frac{dx}{\sqrt{x} + 4} \);
   c) \( \int (2 - 8x - 12x^2) \cos 2xdx \); d) \( \int \sin 3x \, e^{4x} \, dx \);
   e) \( \int \frac{\ln(x - 2)}{(x - 2)^3} \, dx \);
   f) \( \int \frac{\arcsin x}{\sqrt{4x + 4}} \, dx \).

4. Integrate the polynomial fractions:
   a) \( \int \frac{10x + 14}{(x^2 + 3x + 2)(x - 1)} \, dx \);
   b) \( \int \frac{2x^3 - 2x^2 - 22x + 4}{x^3 + 2x^2 - 4x - 8} \, dx \);
   c) \( \int \frac{4x^2 - 24x - 54}{(x^2 + 6x + 18)(x - 3)} \, dx \).

5. Integrate trigonometric expressions:
   a) \( \int 6 \sin x \sin 4xdx \); b) \( \int \cos^4 3x \sin^2 3xdx \);
   c) \( \int \frac{9 + 5 \sin^2 x + 6 \cos^2 x}{9 + 5 \sin^2 x + 6 \cos^2 x} \, dx \).

6. Integrate the functions with radicals:
   a) \( \int \sqrt{\frac{x^3 + 4}{x^2}} \, dx \); b) \( \int \sqrt{\frac{x + 4}{x - 1}(x + 4)^2} \, dx \).

7. Solve the definite integrals:
   a) \( \int_0^1 (x - 1)^2 e^{3x} \, dx \); b) \( \int_0^{\pi/4} \sin^2 x \, dx \);
   c) \( \int_{-2}^0 2xdx \); d) \( \int_2^4 \frac{x + 3}{\sqrt{x - 1}} \, dx \).

8. Find the area of the figure bounded by the curves:
   a) \( y = 2x^2 - 9x + 10, \quad y = -x^2 + 3x + 1 \);
   b) \( \begin{cases} x = 6 \cos t, & x = 0 \ (x \geq 0); \\ y = 2 \sin t, & x = 0 \ (x \geq 0) \end{cases} \);
   c) \( \rho = 2(1 + \sin \phi) \).

9. Find the arc-length of the curve:
   a) \( y = 2 - \frac{1}{2} \chi 2x, \quad -\ln 2 \leq x \leq \ln 2 \);
   b) \( \begin{cases} x = \cos^3 t, & 0 \leq t \leq \pi; \\ y = \sin^3 t, & 0 \leq t \leq \pi \end{cases} \);
   c) \( \rho = \sqrt{2} e^{-\phi}, \quad 0 \leq \phi \leq 2\pi \).

10. Find the area of the surface formed by rotating the curves around the l-axis:
    a) \( y = x^2, \quad 0 \leq x \leq 2\sqrt{5}, \quad l = OY \);
    b) \( \begin{cases} x = 2(t - \sin t), & 0 \leq t \leq 2\pi, \\ y = 2(1 - \cos t), & l = OX \end{cases} \);
    c) \( \rho = 8 \cos \phi, \quad l = op \).

11. Find the volume of the body formed by rotating the curves around the l-axis:
    a) \( y = (x - 1)^3, \quad y = 16x - 16, \quad (x \geq 1), \quad l = OX \);
    b) \( x = 8 + y^2, \quad x = 16 - y^2, \quad l = OY \);
    c) \( \rho = \sin 2\phi, \quad l = op \).

12. Solve the improper integrals:
    a) \( \int_{-\sqrt{2}}^\infty \frac{4x^3 - 16x}{x^4 + 4} \, dx \); b) \( \int_1^e \frac{\sin(ln x) \, dx}{x} \).
1. Integrate using the table and substitution under differential:
   a) \( \int \frac{(x-2)^3}{x\sqrt{x}} \, dx \)
   b) \( \int (4 - 3\text{ctg}^2 x) \, dx \)
   c) \( \int \frac{x^3}{e^{3x^2-9}} \, dx \)
   d) \( \int 8\sin^2 \frac{3x}{\sqrt{x^2}} \, dx \)

2. Integrate the quadratic fractions:
   a) \( \int \frac{2x - 16}{x^2 - 4} \, dx \)
   b) \( \int \frac{dx}{\sqrt{15 + 4x - 4x^2}} \)
   c) \( \int \frac{(-6x + 14)dx}{x^2 - 6x + 13} \)
   d) \( \int \frac{4x + 6}{\sqrt{x^2 + 6x}} \, dx \)

3. Integrate by parts or using the suitable substitutions:
   a) \( \int \frac{(2\ln x + 8)dx}{x\sqrt{4 - \ln^2 x}} \)
   b) \( \int \frac{dx}{(x - 2)^2\sqrt{4x - x^2}} \)
   c) \( \int 16x^2 \sin 4x \, dx \)
   d) \( \int \frac{4x^2 - 8x + 12}{2x} \, dx \)
   e) \( \int (x + 3)^3 \ln x \, dx \)
   f) \( \int (\text{arccos } 2x)^2 \, dx \)

4. Integrate the polynomial fractions:
   a) \( \int \frac{2x^2 - 2x + 12}{x^3 - x^2 - 4x + 4} \, dx \)
   b) \( \int \frac{3x^3 - 10x^2 - 22x + 6}{x^3 - 2x^2 - 7x - 4} \, dx \)
   c) \( \int \frac{8x^2 - 36x}{(x^2 - 4x + 8)(x - 4)} \, dx \)

5. Integrate trigonometric fractions:
   a) \( \int \sin 7x \cos 4x \, dx \)
   b) \( \int \cos^2 3x \sin^4 3x \, dx \)
   c) \( \int \frac{-18dx}{9 \sin^2 x + 6 \sin x \cos x - 8 \cos^2 x} \)

6. Integrate the fractions with radicals:
   a) \( \int \frac{\sqrt{x^2 - 1}}{x^4} \, dx \)
   b) \( \int \frac{\sqrt{x} + 2}{\sqrt{x + \sqrt{x^2}}} \, dx \)

7. Solve the definite integrals:
   a) \( \int_{-1}^{2} -2\ln^3(x + 2) \, dx \)
   b) \( \int_{\pi/4}^{\pi/2} \cos^3 x \, dx \)
   c) \( \int_{0}^{1} x^2 + 2x \, dx \)
   d) \( \int_{0}^{3} 2\sqrt{x-3} \, dx \)

8. Find the area of the figure bounded by the curves:
   a) \( 2x + y = 7, \ xy = 3 \)
   b) \( \begin{cases} x = \cos^3 t, \ y = 2\sin^3 t; & x = 0 \ (x \geq 0) \\ \phi = 3\sin 4\phi \end{cases} \)
   c) \( \rho = 2\cos \phi + 2\sin \phi \)

9. Find the arc-length of the curve:
   a) \( y = \ln(x + \sqrt{x^2 - 9}), \ 3 \leq x \leq 6 \)
   b) \( \begin{cases} x = 4(t - \sin t), \ y = 4(1 - \cos t); & 0 \leq t \leq 2\pi \\ \phi = 2\cos \phi + 2\sin \phi \end{cases} \)

10. Find the area of the surface formed by rotating the curves around the \( l \)-axis:
    a) \( y = 2x + 6, \ 0 \leq x \leq 2, \ l = OX \)
    b) \( \begin{cases} x = 3 + 2\cos t, \ y = 1 + 2\sin t; & l = OY \\ \phi = \sqrt{\cos 2\phi}, \ l = op \end{cases} \)

11. Find the volume of the body formed by rotating the curves around the \( l \)-axis:
    a) \( y = x^2 + 4x + 6, \ y = x + 6, \ l = OX \)
    b) \( y = \sqrt{4 - x^2}, \ y = x, \ y = 0, \ l = OY \)
    c) \( \rho = 8\sin^2 \left(\frac{\phi}{2}\right), \ l = op \)

12. Solve the improper integrals:
    a) \( \int_{-\infty}^{0} e^{x} \cos x \, dx \)
    b) \( \int_{1}^{4} \frac{dx}{x\sqrt{x - 1}} \)
1. Integrate using the table and substitution under differential:
   a) \( \int \frac{(2x - 1)^3}{\sqrt[3]{x}} \, dx \);  
   b) \( \int \frac{\cos 2x - 1}{\cos 2x + 1} \, dx \);  
   c) \( \int 5x \sqrt{x^2 - 9} \, dx \);  
   d) \( \int \left(1 - 3\tan x\right)^2 \, dx \).

2. Integrate the quadratic fractions:
   a) \( \int \frac{4x + 12}{x^2 - 9} \, dx \);  
   b) \( \int \frac{dx}{\sqrt{8 - 6x - 9x^2}} \);  
   c) \( \int \frac{(-4x + 16)\, dx}{x^2 - 4x + 20} \);  
   d) \( \int \frac{12dx}{\sqrt{x^2 + 4x - 12}} \).

3. Integrate by parts or using the suitable substitutions:
   a) \( \int \frac{8dx}{\sqrt{e^{4x} - 16}} \);  
   b) \( \int \frac{\sqrt{9 + x^2}}{x} \, dx \);  
   c) \( \int 6x^2(\sin x + \cos x) \, dx \);  
   d) \( \int x^3 \, dx \);  
   e) \( \int \arctg \sqrt{x} \cdot \frac{dx}{\sqrt{x}} \);  
   f) \( \int \ln(x^2 - 9) \, dx \).

4. Integrate the polynomial fractions:
   a) \( \int \frac{4x^2 - 4x + 6}{(x^2 - 4x + 3)(x + 2)} \, dx \);  
   b) \( \int \frac{-2x^3 - 6x^2 - 11x - 8}{x^3 + 4x^2 + 4x} \, dx \);  
   c) \( \int \frac{x^2 + 26x - 15}{(x^2 - 2x + 5)(x + 5)} \, dx \).

5. Integrate trigonometric expressions:
   a) \( \int \sin x \cos 5x \, dx \);  
   b) \( \int \cos^3 x \sqrt{\sin^2 x} \, dx \);  
   c) \( \int \frac{60dx}{13 + 12\sin x} \).

6. Integrate the fractions with radicals:
   a) \( \int \frac{\sqrt{x^2 - 4}}{x^3} \, dx \);  
   b) \( \int \frac{\sqrt{x - 3}}{x + 1(x - 3)^2} \, dx \).

7. Solve the definite integrals:
   a) \( \int_0^4 (x - 4) e^x \, dx \);  
   b) \( \int_0^{2\pi} \cos^2(\frac{x}{4}) \sin^4(\frac{x}{4}) \, dx \);  
   c) \( \int_0^\sqrt{2} (\frac{4x^3 + 8x}{\sqrt{16 - x^4}}) \, dx \);  
   d) \( \int_1^{e^2-1} \ln x \, dx \).

8. Find the area of the figure bounded by the curves:
   a) \( y = (x - 1)^2, \ y = 6x - 6 \);  
   b) \( \begin{cases} x = 6 \cos t, & y = 5 \sin t; \ x = 3 \ (x \geq 3); \\ y = 3 \cos^3 t, & x = 0 \ (x \geq 0); \\ \rho = 8 \cos^2 \left(\frac{\phi}{2}\right) \end{cases} \).

9. Find the arc-length of the curve:
   a) \( y = \frac{1}{3} \ln(\cos 3x), \ -\frac{\pi}{18} \leq x \leq \frac{\pi}{18} \);  
   b) \( \begin{cases} x = 3 \cos^3 t, & y = 3 \sin^3 t; \ x = 0 \ (x \geq 0); \\ \rho = 8 \cos^2 \left(\frac{\phi}{2}\right) \end{cases} \).

10. Find the area of the surface formed by rotating the curves around the l-axis:
    a) \( x^2 + y^2 = 4, \ l = OY \);  
    b) \( \begin{cases} x = t - \sin t, & 0 \leq t \leq \pi, \ l = OX; \\ y = 1 - \cos t; \end{cases} \rho = 6 \sin \phi, \ l = o\rho \).

11. Find the volume of the body formed by rotating the curves around the l-axis:
    a) \( y = -x^2 + 4x + 1, \ xy = 4, \ l = OX \);  
    b) \( y = 4 - x^2, \ y = 12 - 3x^2, \ l = OY \);  
    c) \( \rho = 3e^{-\phi}, 0 \leq \phi \leq \pi, \ l = o\rho \).

12. Solve the improper integrals:
    a) \( \int_1^\infty \frac{dx}{x + \sqrt[3]{x}} \);  
    b) \( \int_0^1 \frac{x^2}{\sqrt{1-x^2}} \, dx \).
Personal task 28

1. Integrate using the table and substitution under differential:
   a) \( \int \frac{x^3 - 3x^2}{x^2} dx \);   b) \( \int (\tan x - \cot x)^2 dx \);
   c) \( \int \frac{3\arcsin^2 x - x}{\sqrt{1 - x^2}} dx \); d) \( \int \frac{\cos x + \sin x}{\sin x - \cos x} dx \).

2. Integrate the quadratic fractions:
   a) \( \int \frac{8x - 3}{x^2 - 9} dx \); b) \( \int \frac{dx}{\sqrt{8 - 4x - 4x^2}} \);
   c) \( \int \frac{(-4x + 5)dx}{x^2 - 2x + 2} \); d) \( \int \frac{(-4x - 4)dx}{\sqrt{x^2 - 4x - 12}} \).

3. Integrate by parts or using the suitable substitutions:
   a) \( \int \frac{\ln^2 x dx}{x^3 \sqrt{4 + \ln x}} \); b) \( \int \frac{\sqrt{2x - 5} dx}{1 + \sqrt{2x - 5}} \);
   c) \( \int 12x^2 \cos^2 2x dx \); d) \( \int e^x \sin 3x dx \);
   e) \( \int \frac{x \arccos x}{\sqrt{1 - x^2}} dx \); f) \( \int (2x + 1)^2 \sin xdx \).

4. Integrate the polynomial fractions:
   a) \( \int \frac{-5x^2 + 10x - 8}{x^3 - x^2 - 4x + 4} dx \); b) \( \int \frac{dx}{(x^2 + 4x - 5)(x - 1)} \);
   c) \( \int \frac{5x^2 - 35}{x^2 + 4x + 13}(x + 1) dx \).

5. Integrate trigonometric expressions:
   a) \( \int \sin 8x \sin 4xdx \); b) \( \int \sqrt[3]{\frac{\sin^2 x}{\cos^8 x}} dx \);
   c) \( \int \frac{8 \sin x dx}{7 + 9 \sin^2 x + 13 \cos^2 x} \).

6. Integrate the fractions with radicals:
   a) \( \int \frac{x^2 dx}{\sqrt{(9 - x^2)^3}} \); b) \( \int \frac{dx}{\sqrt{x + 1 + 4\sqrt{x + 1}}} \).

7. Solve the definite integrals:
   a) \( \int_{e}^{e^2} \ln^2 x dx \); b) \( \int_{0}^{\pi} \cos^4 2x \sin^4 2x dx \);
   c) \( \int_{0}^{4} \frac{\sqrt{x} dx}{x + 4} \); d) \( \int_{0}^{2} x^3 \sqrt{x^2 - 1} dx \).

8. Find the area of the figure bounded by the curves:
   a) \( y = (x + 2)^3, \ y = 9(x + 2) \);
   b) \( \left\{ \begin{array}{l} x = 4 \cos^3 t, \\
                                  y = 2 \sin^3 t; \\
                   \end{array} \right. \quad y = 1 \ (y \geq 1) \);
   c) \( \rho = 6(1 - \sin \phi) \).

9. Find the arc-length of the curve:
   a) \( y = 2 - e^x, \ 0 \leq x \leq \frac{1}{2} \ln 15 \);
   b) \( \left\{ \begin{array}{l} x = 3(t - \sin t), \\
                                  y = 3(1 - \cos t); \\
                   \end{array} \right. \quad 0 \leq t \leq 2\pi \);
   c) \( \rho = \sqrt{2e^{-2\phi}}, \ 0 \leq \phi \leq 4\pi \).

10. Find the area of the surface formed by rotating the curves around the l-axis:
   a) \( 9y^2 = 4x^3, \ 0 \leq x \leq 1, \ l = OY \);
   b) \( \left\{ \begin{array}{l} x = 2 + \cos t, \\
                                  y = 1 + \sin t; \\
                   \end{array} \right. \quad l = OX \);
   c) \( \rho = 3\sqrt{\cos 2\phi}, \ l = o\rho \).

11. Find the volume of the body formed by rotating the curves around the l-axis:
   a) \( y = -x^2 + 5x + 1, \ y = 6 - x, \ l = OX \);
   b) \( y = 1 - x^2, \ y = 3 - 3x^2, \ l = OY \);
   c) \( \rho = 2 \sin 2\phi, \ l = o\rho \).

12. Solve the improper integrals:
   a) \( \int_{1}^{\infty} \arctg \frac{3x}{1 + x^2} dx \); b) \( \int_{0}^{\pi/4} \frac{\cos^2 \sqrt{x} dx}{\sqrt{x}} \).
Personal task 29

1. Integrate using the table and substitution under differential:
   a) \( \int \frac{(2 + 3\sqrt{x})^3}{x^2} \, dx \); b) \( \int \tan^4 x \, dx \);
   c) \( \int \frac{x^3}{\sqrt{1 - x^4}} \, dx \); d) \( \int (\text{th} x + \text{cth} x)^2 \, dx \).

2. Integrate the quadratic fractions:
   a) \( \int \frac{2x - 6}{x^2 - 25} \, dx \); b) \( \int \frac{dx}{\sqrt{8 + 4x - 4x^2}} \);
   c) \( \int \frac{(4x - 9)dx}{x^2 - 6x + 18} \); d) \( \int \frac{(6x - 6)dx}{\sqrt{5 - 4x + x^2}} \).

3. Integrate by parts or using the suitable substitutions:
   a) \( \int \frac{5x + 6}{(x + 1)^7} \, dx \); b) \( \int \frac{dx}{(x - 2)^2 \sqrt{4x - x^2}} \);
   c) \( \int (4x^2 - 8x) \sin x \cos x \, dx \);
   d) \( \int e^{4x} \sin 3x \, dx \); e) \( \int \frac{\ln x}{x^3} \, dx \);
   f) \( \int (3x^2 + 1) \arctan x \, dx \).

4. Integrate the polynomial fractions:
   a) \( \int \frac{-x^2 + 15x - 20}{(x^2 - 5x + 6)(x + 1)} \, dx \); b) \( \int \frac{2x^3 - 3x^2 - 17x - 26}{x^3 + x^2 - 8x - 12} \, dx \); c) \( \int \frac{14x + 80}{(x^2 - 2x + 10)(x + 4)} \, dx \).

5. Integrate trigonometric expressions:
   a) \( \int \cos 2x \sin 8x \, dx \); b) \( \int \frac{\cos^2 x}{\sin^6 x} \, dx \);
   c) \( \int \frac{4dx}{\sqrt{1 - 2 \sin x + 6 \cos x}} \).

6. Integrate the fractions with radicals:
   a) \( \int \frac{\sqrt{4 + x^2}}{x^4} \, dx \); b) \( \int \frac{\sqrt{x + 3}}{x - 2 (x + 3)^2} \, dx \).

7. Solve the definite integrals:
   \[ \int_{1/\ln 2}^{1/\ln 4} \sqrt{e} \cdot \frac{dx}{x^2} ; \quad \int_0^\pi \cos^4 x \sin^2 x \, dx ; \]
   \[ \int_0^{\sqrt{3}/2} \frac{2x + 4}{x^2 + 1} \, dx ; \quad \int_0^1 x^3 \sqrt{1 - x^2} \, dx . \]

8. Find the arc-length of the curve:
   a) \( y = 2x^2 - 11x + 13, \quad y = -x^2 + 4x + 1 \);
   b) \( \begin{cases} x = 4 \cos t, \quad y = 2 + 4 \sin t; \\ x = 2 (x \geq 2) \end{cases} \);
   c) \( \rho = 3 \sin 2\phi \).

9. Find the area of the figure bounded by the curves:
   a) \( y = 4 \ln (16 - x^2), \quad -2 \leq x \leq 2 \);
   b) \( \begin{cases} x = 2e^{-t} \cos t, \\ y = 2e^{-t} \sin t; \quad 0 \leq t \leq 2\pi \end{cases} \);
   c) \( \rho = 4 \cos^2 \left( \frac{\phi}{2} \right) \).

10. Find the area of the surface formed by rotating the curves around the l-axis:
    a) \( y = \frac{1}{5} \tan 5x, \quad 0 \leq x \leq 1, \quad l = OX \);
    b) \( \begin{cases} x = 3 \cos^3 t, \\ y = 3 \sin^3 t; \quad l = OY \end{cases} \);
    c) \( \rho = 2 \cos \phi, \quad l = \rho \).

11. Find the volume of the body formed by rotating the curves around the l-axis:
    a) \( y = x^2 - 4x + 4, \quad y = 6x - 12, \quad l = OX \);
    b) \( y = x^2 - 6, \quad y = 5x, \quad x = 0 (x \geq 0), \quad l = OY \);
    c) \( \rho = 6\phi, \quad 0 \leq \phi \leq \pi, \quad l = \rho \).

12. Solve the improper integrals:
    a) \( \int_0^\infty x^3 e^{-x^2} \, dx \); b) \( \int_0^1 \frac{x^3 - 4x}{\sqrt{1 - x^4}} \, dx \).
1. Integrate using the table and substitution under differential:
   a) \( \int \sqrt{x}(1 + 2x\sqrt{x})^2 \, dx; \)
   b) \( \int \frac{\cos 2x + 1}{\sin^2 x} \, dx; \)
   c) \( \int \frac{3\arccos^2 x - 2x}{\sqrt{1 - x^2}} \, dx; \)
   d) \( \int \left( \text{sh}(x + 1) - 2 \right)^2 \, dx. \)

2. Integrate the quadratic fractions:
   a) \( \int \frac{-4x + 6}{x^2 - 9} \, dx; \)
   b) \( \int \frac{dx}{\sqrt{9x^2 - 6x - 8}}; \)
   c) \( \int \frac{(-4x + 4) \, dx}{x^2 + 2x + 17}; \)
   d) \( \int \frac{-6x + 6}{\sqrt{6x - x^2}} \, dx. \)

3. Integrate by parts or using the suitable substitutions:
   a) \( \int \frac{\sqrt{x} - 3 \, dx}{x + 4\sqrt{x} - 3}; \)
   b) \( \int \frac{dx}{x^2 + 4 - x^2}; \)
   c) \( \int 9x^2 \cos 3x \, dx; \)
   d) \( \int x^3 \ln(x - 1) \, dx; \)
   e) \( \int e^{\sqrt{x} + 4} \, dx; \)
   f) \( \int x \cdot \arcsin(x^2) \, dx. \)

4. Integrate the polynomial fractions:
   a) \( \int \frac{3x^2 + 5x - 4}{(x^2 + 3x + 2)(x + 3)} \, dx; \)
   b) \( \int \frac{-x^3 + 4x^2 + 6x - 28}{(x^2 - 6x + 8)(x - 2)} \, dx; \)
   c) \( \int \frac{5x^2 - 16x - 45}{(x^2 + 2x + 5)(x - 4)} \, dx. \)

5. Integrate trigonometric expressions:
   a) \( \int \sin 5x \cos 3x \, dx; \)
   b) \( \int \sin^2 x \cos^4 x \, dx; \)
   c) \( \int \frac{12 \, dx}{4 \sin^2 x - 12 \sin x \cos x + 5 \cos^2 x}. \)

6. Integrate the fractions with radicals:
   a) \( \int \frac{x^3}{\sqrt{x^2 + 4}} \, dx; \)
   b) \( \int \frac{(\sqrt{x} + 1) \, dx}{\sqrt{x^2 + 2\sqrt{x} - 3}}. \)

7. Solve the definite integrals:
   a) \( \int_0^1 \arctg x \, dx; \)
   b) \( \int_0^{\pi/4} \frac{\sin x}{\cos^5 x} \, dx; \)
   c) \( \int_1^2 \frac{4x + 2}{\sqrt{4x - x^2}} \, dx; \)
   d) \( \int_{-2}^2 \frac{x^2 \sqrt{4 - x^2}}{\sqrt{x} \, dx}. \)

8. Find the area of the figure bounded by the curves:
   a) \( y = (x + 2)^3, \quad y = 4x + 8; \)
   b) \( \begin{cases} x = 4 \cos t, & x = 2 (x \geq 2); \\
y = 6 \sin t; & \end{cases} \)
   c) \( \rho = 3 \cos 2\phi. \)

9. Find the arc-length of the curve:
   a) \( y = 1 - \frac{1}{3} \text{ch} 3x, \quad 0 \leq x \leq \frac{1}{3} \ln 2; \)
   b) \( \begin{cases} x = 4 \cos^3 t, & 0 \leq t \leq 2\pi; \\
y = 4 \sin^3 t; & \end{cases} \)
   c) \( \rho = 6 \cos \phi + 6 \sin \phi. \)

10. Find the area of the surface formed by rotating the curves around the l-axis:
    a) \( y = \sqrt{2x + 8}, \quad -4 \leq x \leq 4, \quad l = OX; \)
    b) \( \begin{cases} x = 3(t - \sin t), & 0 \leq t \leq 2\pi, \\
y = 3(1 - \cos t); & \end{cases} \)
    c) \( \rho = 2(1 + \cos \phi), \quad l = op. \)

11. Find the volume of the body formed by rotating the curves around the l-axis:
    a) \( y = x^2 - 4x, \quad y = 2x^2 - 8x, \quad l = OX; \)
    b) \( x = 9 - y^2, \quad x + 3y = 9, \quad l = OY; \)
    c) \( \rho = e^{-\phi}, \quad 0 \leq \phi \leq \pi, \quad l = op. \)

12. Solve the improper integrals:
    a) \( \int_0^\infty \frac{2x + 5}{x^2 + 1} \, dx; \)
    b) \( \int_1^\infty \frac{e \, dx}{x \sqrt{1 - \ln x}}. \)
Sample task

1. Integrate using the table and substitution under differential:
   a) \( \int x^3(1 - x^2)^2dx \);  
   b) \( \int (3 + \tan^2 x)dx \); 
   c) \( \int x^4e^{2x^5-1}dx \);  
   d) \( \int \sin^2(1 + 3x)dx \).

2. Integrate the quadratic fractions:
   a) \( \int \frac{-4x + 5}{x^2 - 4}dx \);  
   b) \( \int \frac{dx}{\sqrt{6x - 9x^2}} \);  
   c) \( \int \frac{(2x - 7)dx}{x^2 + 6x + 10} \);  
   d) \( \int \frac{(8x - 5)dx}{\sqrt{x^2 + 4x - 5}} \).

3. Integrate by parts or using the suitable substitutions:
   a) \( \int \frac{3x + 2}{\sqrt{x + 4}}dx \);  
   b) \( \int \frac{dx}{x\sqrt{3x^2 - 4x + 1}} \);  
   c) \( \int \frac{(4x^2 - 3)dx}{e^{2x}} \);  
   d) \( \int x^2 \ln xdx \);  
   e) \( \int \sin \sqrt{x + 1}dx \);  
   f) \( \int x \frac{\sin x}{\cos^3 x}dx \).

4. Integrate the polynomial fractions:
   a) \( \int \frac{-3x^2 + 2x + 13}{x^3 + 2x^2 - x - 2}dx \);  
   b) \( \int \frac{3x^3 - 32x + 56}{x^3 - 2x^2 - 4x + 8}dx \);  
   c) \( \int \frac{(x^2 + 4x + 5)(x - 1)dx}{x^2 - 2x - 9} \).

5. Integrate trigonometric functions:
   a) \( \int \sin 10x \sin 3xdx \);  
   b) \( \int \frac{\sin x}{\cos^{13} x}dx \);  
   c) \( \int \frac{dx}{4 \cos x + 3 \sin x + 6} \).

6. Integrate the fractions with radicals:
   a) \( \int \frac{dx}{x^2\sqrt{4 - x^2}} \);  
   b) \( \int \frac{x + 1}{x - 1(x - 1)^3}dx \).

7. Solve the definite integrals:
   a) \( \int_0^1 \ln(1 + x^2)dx \);  
   b) \( \int_0^{\pi/2} \sin^3 x\sqrt{\cos x}dx \);  
   c) \( \int_1^{\sqrt{5/2}} 4x^3dx \);  
   d) \( \int_0^1 \frac{dx}{\sqrt{x + \sqrt{x}}} \).

8. Find the area of the figure bounded by the curves:
   a) \( y = 2x^2 - 10x + 6, \ y = x^2 - 3x \);  
   b) \( \begin{cases} \quad x = 4 \cos^3 t, \\ y = 4 \sin^3 t, \quad y = \frac{1}{2} (y \geq \frac{1}{2}) \end{cases} \);  
   c) \( \rho = 4 \sin 3\varphi \).

9. Find the arc length of the curves:
   a) \( y = \frac{1}{3} \ln(\cos 3x), \ 0 \leq x \leq \frac{\pi}{18} \);  
   b) \( \begin{cases} \quad x = 4(t - \sin t), \\ y = 4(1 - \cos t) \end{cases}, \ 0 \leq t \leq 2\pi \);  
   c) \( \rho = \frac{10}{\sqrt{101}}e^{\frac{r}{10}}, \ 0 \leq \varphi \leq 2\pi \).

10. Find the area of the surface formed by rotating the curves around the l-axis:
   a) \( y = \frac{1}{2} \text{ch} 2x, \ -1 \leq x \leq 1, \ l = OX \);  
   b) \( \begin{cases} \quad x = 3 + \cos t, \\ y = 2 + \sin t \end{cases}, \ l = OY \);  
   c) \( \rho = \sqrt{\cos 2\varphi}, \ 0 \leq \varphi \leq \pi \ l = \rho \).

11. Find the volume of the body formed by rotating the curves around the l-axis:
   a) \( y = x^2 + 2x + 5, \ y = 5 - x, \ l = OX \);  
   b) \( x = 5 + 4y - y^2, \ x = 5, \ l = OY \);  
   c) \( \rho = 6(1 + \cos \varphi), \ l = \rho \).

12. Solve the improper integrals:
   a) \( \int_{-2}^{\infty} \frac{2x - 1}{x^2 + 4}dx \);  
   b) \( \int_0^{\pi/2} e^{-\tan x} \frac{dx}{\cos^2 x} \).
1.(a) \[ I = \int x^3(1-x^2)^2 \, dx = \left| \text{Applying the Newton binom to the function under the integral} \right| = \]

\[ = \int x^3(1 - 2x^2 + x^4) \, dx = \int (x^3 - 2x^5 + x^7) \, dx = \frac{1}{4}x^4 - \frac{2}{6}x^6 + \frac{1}{8}x^8 + C = \]

\[ = -\frac{1}{4}x^4 - \frac{1}{3}x^6 + \frac{1}{8}x^8 + C. \]

Result: \[ I = \frac{1}{4}x^4 - \frac{1}{3}x^6 + \frac{1}{8}x^8 + C, \, C \in \mathbb{R}. \]

* Solution of Problem 1(a) guided by Ricard Riba is available on-line: https://youtu.be/x48CikKf9c

1.(b) \[ I = \int (3 + \tan^2 x) \, dx = \int (2 + (1 + \tan^2 x)) \, dx = \left| \text{Applying the trigonometric identity} \right| = \]

\[ = \int \left( 2 + \frac{1}{\cos^2 x} \right) \, dx = 2x + \int \frac{dx}{\cos^2 x} = 2x + \tan x + C. \]

Result: \[ I = 2x + \tan x + C, \, C \in \mathbb{R}. \]

* Solution of Problem 1(b) guided by Irina Blazhievska is available on-line: https://youtu.be/1MqmZbQ-3qM

1.(c) \[ I = \int x^4e^{2x^5-1} \, dx = \left| \text{Checking the exact differential of the power} \right| = \]

\[ \frac{d(2x^5 - 1)}{10} = 10x^4 \, dx; \quad x^4 \, dx = \frac{1}{10}d(2x^5 - 1) \]

\[ = \frac{1}{10} \int e^{2x^5-1}d(2x^5 - 1) = \frac{1}{10}e^{2x^5-1} + C. \]

Result: \[ I = \frac{1}{10}e^{2x^5-1} + C, \, C \in \mathbb{R}. \]

* Solution of Problem 1(c) guided by Irina Blazhievska is available on-line: https://youtu.be/kSn2UvdXWVs
1. (d) \[ \dot{I} = \int \sin^2(1 + 3x) \, dx = \begin{align*} & \text{Applying the reduction power formula} \\ & \sin^2(1 + 3x) = \frac{1}{2} (1 - \cos(2 + 6x)) \\ & = \frac{1}{2} \int (1 - \cos(2 + 6x)) \, dx = \frac{1}{2} \left( \int dx - \int \cos(2 + 6x) \, dx \right) \\ &= \text{Substitution under the differential} \\ & dx = \frac{1}{6} d(2 + 6x) \\ & = \frac{1}{2} \left( x - \frac{1}{6} \int \cos(2 + 6x) \, d(2 + 6x) \right) + C = \frac{1}{2} x - \frac{1}{12} \sin(2 + 6x) + C. \end{align*} \]

Result: \[ \dot{I} = \frac{1}{2} x - \frac{1}{12} \sin(2 + 6x) + C, \ C \in \mathbb{R}. \]

2. (a) \[ \dot{I} = \int \frac{-4x + 5}{x^2 - 4} \, dx = \begin{align*} & \text{Checking the differential of the denominator} \\ & d(x^2 - 4) = 2x \, dx \\ & \text{Decomposing the numerator} \\ & -4x + 5 = -2(2x) + 5 \\ & = -2 \int \frac{2x \, dx}{x^2 - 4} + 5 \int \frac{dx}{x^2 - 4} = -2 \int \frac{d(x^2 - 4)}{x^2 - 4} + 5 \int \frac{dx}{x^2 - 2^2} \\ & = -2 \ln |x^2 - 4| + \frac{5}{4} \ln \left| \frac{x - 2}{x + 2} \right| + C = -\frac{3}{4} \ln |x - 2| - \frac{13}{4} \ln |x + 2| + C. \end{align*} \]

Result: \[ \dot{I} = -\frac{3}{4} \ln |x - 2| - \frac{13}{4} \ln |x + 2| + C, \ C \in \mathbb{R}. \]

2. (b) \[ \dot{I} = \int \frac{dx}{\sqrt{6x - 9x^2}} = \begin{align*} & \text{Completing the square inside the radical} \\ & 6x - 9x^2 = 1 - (3x - 1)^2 \\ & = \int \frac{dx}{\sqrt{1 - (3x - 1)^2}} = \frac{1}{3} \int \frac{d(3x - 1)}{\sqrt{1 - (3x - 1)^2}} = \frac{1}{3} \arcsin(3x - 1) + C. \end{align*} \]

Result: \[ \dot{I} = \frac{1}{3} \arcsin(3x - 1) + C, \ C \in \mathbb{R}. \]

* Solution of Problem 2(b) guided by Irina Blazhievksa is available on-line: [https://youtu.be/KQuEzkh6AqQ](https://youtu.be/KQuEzkh6AqQ)
2.(c) \[ \dot{I} = \int \frac{-2x - 7}{x^2 + 6x + 10} \, dx = \]

Checking the differential of the denominator:
\[ d(x^2 + 6x + 10) = (2x + 6) \, dx; \]
Decomposing the numerator
\[ -2x - 7 = -(2x + 6) - 1 \]

\[ = - \int \frac{2x + 6}{x^2 + 6x + 10} \, dx - \int \frac{dx}{x^2 + 6x + 10} \]

Completing the square:
\[ x^2 + 6x + 10 = (x + 3)^2 + 1 \]

\[ = - \ln(x^2 + 6x + 10) - \arctg(x + 3) + C. \]

Result: \[ \dot{I} = - \ln(x^2 + 6x + 10) - \arctg(x + 3) + C, \ C \in \mathbb{R}. \]

2.(d) \[ \dot{I} = \int \frac{8x - 5}{\sqrt{x^2 + 4x - 5}} \, dx = \]

Checking the differential of the quadratic function:
\[ d(x^2 + 4x - 5) = (2x + 4) \, dx; \]
Decomposing the numerator
\[ (8x - 5) = 4(2x + 4) - 21 \]

\[ = 4 \int \frac{2x + 4}{\sqrt{x^2 + 4x - 5}} \, dx - 21 \int \frac{dx}{\sqrt{x^2 + 4x - 5}} \]

Completing the square:
\[ x^2 + 4x - 5 = (x + 2)^2 - 3^2 \]

\[ = 4 \int \frac{d(x^2 + 4x - 5)}{\sqrt{x^2 + 4x - 5}} - 21 \int \frac{d(x + 2)}{\sqrt{(x + 2)^2 - 3^2}} \]

\[ = 8\sqrt{x^2 + 4x - 5} - 21 \ln \left|x + 2 + \sqrt{x^2 + 4x - 5}\right| + C. \]

Result: \[ \dot{I} = 8\sqrt{x^2 + 4x - 5} - 21 \ln \left|x + 2 + \sqrt{x^2 + 4x - 5}\right| + C, \ C \in \mathbb{R}. \]

3.(a) \[ \dot{I} = \int \frac{3x + 2}{\sqrt{x + 4}} \, dx = \]

Performing the change of variable:
\[ t = \sqrt{x + 4}; \]
\[ x = t^2 - 4, \ dx = 2tdt \]

\[ = \int \frac{3(t^2 - 4) + 2}{t} \cdot 2tdt = 2 \int (3t^2 - 10) \, dt = 2(t^3 - 10t) + C = \]
Undoing the change of variable: 
\[ t = (x + 4)^{1/2} \]

Result: \( \dot{I} = 2(x + 4)^{3/2} - 20(x + 4)^{1/2} + C, \ C \in \mathbb{R}. \)

* Solution of Problem 3(a) guided by Irina Blazhievska is available on-line: [https://youtu.be/ZOFSo2oDVzQ](https://youtu.be/ZOFSo2oDVzQ)

\[ \dot{I} = \int \frac{dx}{x\sqrt{3x^2 - 4x + 1}} = \left| \begin{array}{c} \text{Performing the change of variable: } t = \frac{1}{x}; \\ x = \frac{1}{t}, \ dx = -\frac{1}{t^2}dt; \\ \sqrt{3x^2 - 4x + 1} = \frac{\sqrt{t^2 - 4t + 3}}{t} \end{array} \right| = \left| \begin{array}{c} \text{Undoing the change of variable: } \quad t = \frac{1}{x}; \\ \sqrt{t^2 - 4t + 3} = \frac{\sqrt{3x^2 - 4x + 1}}{x} \end{array} \right| \\
\] 

\[ \dot{I} = -\int \frac{-\frac{1}{t^2}dt}{\frac{1}{t} \cdot \sqrt{\frac{t^2 - 4t + 3}{t}}} = -\int \frac{dt}{\sqrt{t^2 - 4t + 3}} = -\int \frac{d(t - 2)}{\sqrt{(t - 2)^2 - 1}} = \]

\[ = -\ln \left| t - 2 + \sqrt{t^2 - 4t + 3} \right| + C = \left| \begin{array}{c} \text{Using integration by parts: } \\ \int udv = uv - \int vdu, \\ u = 4x^2 - 3; \ \ dv = e^{-2x}dx; \\ du = 8xdx; \ \ v = -\frac{1}{2}e^{-2x} \end{array} \right| = \left| \begin{array}{c} \text{Using integration by parts: } \\ \int udv = uv - \int vdu, \\ u = 4x^2 - 3; \ \ dv = e^{-2x}dx; \\ du = 8xdx; \ \ v = -\frac{1}{2}e^{-2x} \end{array} \right| \\
\] 

\[ \dot{I} = -\ln \left| \frac{1}{x} - 2 + \frac{\sqrt{3x^2 - 4x + 1}}{x} \right| + C, \ C \in \mathbb{R}. \)

\[ \dot{I} = \int \frac{4x^2 - 3}{e^{2x}}dx = \int (4x^2 - 3)e^{-2x}dx = \left| \begin{array}{c} \text{Using integration by parts: } \\ \int udv = uv - \int vdu, \\ u = 4x^2 - 3; \ \ dv = e^{-2x}dx; \\ du = 8xdx; \ \ v = -\frac{1}{2}e^{-2x} \end{array} \right| = \left| \begin{array}{c} \text{Using integration by parts: } \\ \int udv = uv - \int vdu, \\ u = 4x^2 - 3; \ \ dv = e^{-2x}dx; \\ du = 8xdx; \ \ v = -\frac{1}{2}e^{-2x} \end{array} \right| \\
\] 

\[ = -\frac{1}{2}e^{-2x}(4x^2 - 3) - \int \left( -\frac{1}{2} \right) 8xe^{-2x}dx = \]

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\[
\begin{align*}
-\frac{1}{2} e^{-2x}(4x^2 - 3) + 4 \int xe^{-2x} dx &= \left| \begin{array}{c}
\text{Using integration by parts:} \\
\int uv' = uv - \int v'u
\end{array} \right| = \\
&= -\frac{1}{2} e^{-2x}(4x^2 - 3) + 4 \left( -\frac{1}{2} xe^{-2x} - \int \left( -\frac{1}{2} \right) e^{-2x} dx \right) = \\
&= -\frac{1}{2} e^{-2x}(4x^2 - 3) - 2xe^{-2x} - e^{-2x} + C = \left( -2x^2 - 2x + \frac{1}{2} \right) e^{-2x} + C.
\end{align*}
\]

Result: \( \dot{I} = (-2x^2 - 2x + \frac{1}{2}) e^{-2x} + C, \ C \in \mathbb{R}. \)

3.(d) \( \dot{I} = \int x^2 \ln x \, dx \) = \( \left| \begin{array}{c}
\text{Using integration by parts:} \\
\int uv' = uv - \int v'u
\end{array} \right| = \frac{1}{3} x^3 \ln x - \int \frac{1}{3} x^3 \, dx \) = \( \frac{1}{3} x^3 \ln x - \frac{1}{3} \int x^2 \, dx = \frac{1}{3} x^3 \ln x - \frac{1}{3} \cdot \frac{1}{3} x^3 + C = \frac{1}{9} x^3 (3 \ln x - 1) + C. \)

Result: \( \dot{I} = \frac{1}{9} x^3 (3 \ln x - 1) + C, \ C \in \mathbb{R}. \)

* Solution of Problem 3(d) guided by Ricard Riba is available on-line: https://youtu.be/hDYt-m7ZCgM

3.(e) \( \dot{I} = \int \sin \sqrt{x+1} \, dx \) = \( \left| \begin{array}{c}
\text{Performing the change of variable:} \ t = \sqrt{x+1}; \\
x = t^2 - 1, \ dx = 2t \, dt
\end{array} \right| = \)

\( = \int 2t \sin t \, dt = \left| \begin{array}{c}
\text{Using integration by parts:} \\
\int uv' = uv - \int v'u
\end{array} \right| = -2t \cos t - \int (-\cos t)2 \, dt = \)

\( = -2t \cos t + 2 \sin t + C = \left| \begin{array}{c}
\text{Undoing the change of variable:} \ t = \sqrt{x+1}
\end{array} \right| = \)

\( = -2\sqrt{x+1} \cos \sqrt{x+1} + 2 \sin \sqrt{x+1} + C. \)

Result: \( \dot{I} = -2\sqrt{x+1} \cos \sqrt{x+1} + 2 \sin \sqrt{x+1} + C, \ C \in \mathbb{R}. \)
3. (f) \[ I = \int x \frac{\sin x}{\cos^3 x} \, dx = \left| \begin{array}{l}
\text{Using integration by parts: } \int udv = uv - \int vdu \\
u = x; \quad v = \int \frac{\sin x}{\cos^3 x} \, dx = -\int \frac{d(\cos x)}{\cos^3 x} = \frac{1}{2 \cos^2 x}; \\
\frac{du}{dx} = \frac{dx}{x}; \quad \frac{dv}{dx} = \frac{\sin x}{\cos^3 x} \, dx
\end{array} \right| =
\frac{x}{2 \cos^2 x} - \frac{1}{2} \int \frac{dx}{\cos^2 x} = \frac{x}{2 \cos^2 x} - \frac{1}{2} \tan x + C = \frac{1}{2} \left( \frac{x}{\cos^2 x} - \tan x \right) + C.
\]

Result: \[ I = \frac{1}{2} \left( \frac{x}{\cos^2 x} - \tan x \right) + C, \quad C \in \mathbb{R}. \]

4. (a) \[ I = \int \frac{-3x^2 + 2x + 13}{x^3 + 2x^2 - x - 2} \, dx. \]

Algorithm:

**Step 1.** The function under the integral is a suitable polynomial fraction:

\[ f(x) = \frac{Q_2(x)}{P_3(x)}. \]

**Step 2.** Finding the roots of the denominator \( P_3(x) = x^3 + 2x^2 - x - 2 \). Possible integer roots may be \( \pm 1; \pm 2 \).

\[ P_3(1) = 1^3 + 2 \cdot 1^2 - 1 - 2 = 0 \quad \Rightarrow \quad x_1 = 1 \quad \text{is a root of } P_3(x) \]

\[ \Rightarrow \quad P_3(x) \quad \text{is divisible by } (x - 1). \]

The decomposition \( P_3(x) = (x - 1)(x^2 + 3x + 2) = (x - 1)(x + 2)(x + 1) \), implies that all roots are simple and real-valued.

**Step 3.** Applying the method of unknown coefficients to the suitable fraction:

\[ f(x) = \frac{-3x^2 + 2x + 13}{(x - 1)(x + 2)(x + 1)} = \frac{A}{x - 1} + \frac{B}{x + 1} + \frac{C}{x + 2} =
\]

\[ = \frac{A(x + 2)(x + 1) + B(x - 1)(x + 2) + C(x - 1)(x + 1)}{(x - 1)(x + 2)(x + 1)}. \]

Numerator’s equality:

\[ -3x^2 + 2x + 13 = A(x + 2)(x + 1) + B(x - 1)(x + 2) + C(x - 1)(x + 1). \]
Applying equalities by zeros:

\[ x = 1 \Rightarrow -3 + 2 + 13 = A(1 + 1)(1 + 2); \quad 12 = 6A \Rightarrow A = 2 \]
\[ x = -1 \Rightarrow -3 - 2 + 13 = B(-1 - 1)(-1 + 2); \quad 8 = -2B \Rightarrow B = -4 \]
\[ x = -2 \Rightarrow -3 \cdot 4 - 4 + 13 = C(-2 - 1)(-2 + 1); \quad -3 = 3C \Rightarrow C = -1. \]

The resulting decomposition is as follows:

\[ f(x) = \frac{-3x^2 + 2x + 13}{(x-1)(x+2)(x+1)} = \frac{2}{x-1} - \frac{4}{x+1} - \frac{1}{x+2}. \]

**Step 4.** Substitution of the fully decomposed fraction under the integral:

\[
\int \frac{-3x^2 + 2x + 13}{x^3 + 2x^2 - x - 2} \, dx = \int \left( \frac{2}{x-1} - \frac{4}{x+1} - \frac{1}{x+2} \right) \, dx = 2 \ln |x - 1| - 4 \ln |x + 1| - \ln |x + 2| + C.
\]

**Result:** \( \dot{I} = 2 \ln |x - 1| - 4 \ln |x + 1| - \ln |x + 2| + C, C \in \mathbb{R}. \)

4.(b)

\[
\dot{I} = \int \frac{3x^3 - 32x + 56}{x^3 - 2x^2 - 4x + 8} \, dx
\]

**Algorithm:**

**Step 1.** The function under the integral is a non-suitable polynomial fraction:

\[ f(x) = \frac{Q_3(x)}{P_3(x)}. \]

Separation of integer part leads us to the following sum of 0-degree polynomial and a suitable polynomial fraction:

\[ f(x) = 3 + \frac{6x^2 - 20x + 32}{x^3 - 2x^2 - 4x + 8}. \]

**Step 2.** Finding the roots of the denominator \( P_3(x) = x^3 - 2x^2 - 4x + 8. \) Possible integer roots may be \( \pm 1, \pm 2, \pm 4, \pm 8. \)

\[ P_3(2) = 2^3 - 2 \cdot 2^2 - 4 \cdot 2 + 8 = 0 \Rightarrow x_1 = 8 \text{ is a root of } P_3(x) \]
\[ \Rightarrow P_3(x) \text{ is divisible by } (x - 2). \]

The decomposition \( P_3(x) = (x - 2)(x^2 - 4) = (x - 2)^2(x + 2), \) implies that \( P_3 \)
has two real-valued roots, a double root \( x = 2 \) and a simple root \( x = -2. \)
**Step 3.** Applying the method of unknown coefficients to the suitable fraction:

\[
\frac{6x^2 - 20x + 32}{x^3 - 2x^2 - 4x + 8} = \frac{6x^2 - 20x + 32}{(x - 2)^2(x + 2)} = A \frac{x - 2}{x - 2} + B \frac{1}{(x - 2)^2} + C \frac{1}{x + 2} = \\
= \frac{A(x - 2)(x + 2) + B(x + 2) + C(x - 2)^2}{(x - 2)^2(x + 2)}.
\]

Numerator’s equality:

\[
6x^2 - 20x + 32 = A(x - 2)(x + 2) + B(x + 2) + C(x - 2)^2 = \\
x^2(A + C) + x(B - 4C) + (-4A + 2B + 4C).
\]

Mix of equalities by zeros with equality of coefficients of some monomial:

\[
x = 2 \Rightarrow 6 \cdot 2^2 - 20 \cdot 2 + 32 = B(2 + 2); \quad 16 = 4B \Rightarrow B = 4
\]
\[
x = -2 \Rightarrow 6 \cdot (-2)^2 + 32 = C(-2 - 2)^2; \quad 96 = 16C \Rightarrow C = 6
\]
\[
x^2: \quad 6 = A + C; \quad A = 6 - C = 6 - 6 = 0 \Rightarrow A = 0.
\]

The resulting decomposition is as follows:

\[
\frac{6x^2 - 20x + 32}{x^3 - 2x^2 - 4x + 8} = \frac{6x^2 - 20x + 32}{(x - 2)^2(x + 2)} = \frac{4}{(x - 2)^2} + \frac{6}{x + 2}.
\]

**Step 4.** Substitution of the fully decomposed fraction under the integral:

\[
\int \frac{3x^3 - 32x + 56}{x^3 - 2x^2 - 4x + 8} dx = \int \left( 3 + \frac{4}{(x - 2)^2} + \frac{6}{x + 2} \right) dx = \\
= 3x - \frac{4}{x - 2} + 6 \ln |x + 2| + C.
\]

**Result:** \[I = 3x - \frac{4}{x - 2} + 6 \ln |x + 2| + C, \quad C \in \mathbb{R}.
\]
Algorithm:

**Step 1.** The function under the integral is a suitable polynomial fraction:

\[ f(x) = \frac{Q_2(x)}{P_3(x)}. \]

**Step 2.** Finding the roots of the denominator \( P_3(x) = (x^2 + 4x + 5)(x-1) \).

The polynomial \( x^2 + 4x + 5 \) is indecomposable quadratic function since its discriminant is negative. The decomposition of \( P_3(x) \) implies that the unique real-valued root of \( P_3(x) \) is \( x_1 = 1 \).

**Step 3.** Applying the method of unknown coefficients to the suitable fraction:

\[
\begin{align*}
  f(x) &= \frac{x^2 - 2x - 9}{(x^2 + 4x + 5)(x-1)} = \frac{Ax + B}{x^2 + 4x + 5} + \frac{C}{x-1} = \\
  &= \frac{(Ax + B)(x - 1) + C(x^2 + 4x + 5)}{(x^2 + 4x + 5)(x - 1)}. \\
\end{align*}
\]

Numerator's equality:

\[
\begin{align*}
  x^2 - 2x - 9 &= (Ax + B)(x - 1) + C(x^2 + 4x + 5) = \\
  &= x^2(A + C) + x(-A + B + 4C) + (-B + 5C). \\
\end{align*}
\]

Mix of equalities by zeros with equality of coefficients of some monomial:

- \( x = 1 \Rightarrow 1 - 2 - 9 = C(1 + 4 + 5); \quad -10 = 10C \Rightarrow C = -1 \)
- \( x^2 : 1 = A + C; \quad A = 1 - C = 1 - (-1) = 2 \Rightarrow A = 2 \)
- \( x^0 : -9 = -B + 5C; \quad B = 9 + 5C = 9 + 5 \cdot (-1) = 4 \Rightarrow B = 4. \)

The resulting decomposition is as follows:

\[
\begin{align*}
  f(x) &= \frac{x^2 - 2x - 9}{(x^2 + 4x + 5)(x-1)} = \frac{2x + 4}{x^2 + 4x + 5} - \frac{1}{x - 1}. \\
\end{align*}
\]

**Step 4.** Substitution of the fully decomposed fraction under the integral:

\[
\begin{align*}
  \int \frac{x^2 - 2x - 9}{(x^2 + 4x + 5)(x-1)} \, dx &= \int \left( \frac{2x + 4}{x^2 + 4x + 5} - \frac{1}{x - 1} \right) \, dx = \\
  &= \int \frac{d(x^2 + 4x + 5)}{x^2 + 4x + 5} - \int \frac{dx}{x - 1} = \ln(x^2 + 4x + 5) - \ln |x - 1| + C. \\
\end{align*}
\]

**Result:** \( I = \ln(x^2 + 4x + 5) - \ln |x - 1| + C, C \in \mathbb{R}. \)
5.(a) \[ \dot{I} = \int \sin 10x \sin 3x \, dx = \left| \begin{array}{c}
Applying the decomposition of a product of sinus functions: \\
\sin \alpha \sin \beta = \frac{1}{2} \left( \cos(\alpha - \beta) - \cos(\alpha + \beta) \right)
\end{array} \right| =
\]

\[ = \frac{1}{2} \int \left( \cos(10x - 3x) - \cos(10x + 3x) \right) \, dx = \frac{1}{2} \int \left( \cos 7x - \cos 13x \right) \, dx =
\]

\[ = \frac{1}{2} \cdot \frac{\sin 7x}{7} - \frac{1}{2} \cdot \frac{\sin 13x}{13} + C = \frac{1}{14} \sin 7x - \frac{1}{26} \sin 13x + C.
\]

Result: \[ \dot{I} = \frac{1}{14} \sin 7x - \frac{1}{26} \sin 13x + C, \, C \in \mathbb{R}. \]

---

5.(b) \[ \dot{I} = \int 3 \sqrt{\frac{\sin x}{\cos^3 x}} \, dx = \left| \begin{array}{c}
Applying the integration of a product \sin^m x \cos^n x \text{ with } m = 0, \, n = -4, \text{ decompose:} \\
\cos^{-13/3} x = \cos^{-1/3} x \cos^{-4} x; \\
\frac{1}{\cos^2 x} = 1 + \tan^2 x; \frac{dx}{\cos^2 x} = d(\tan x)
\end{array} \right| =
\]

\[ = \int 3 \sqrt{\frac{\sin x}{\cos x \cos^4 x}} \, dx = \int \tan^{1/3} x \frac{1}{\cos^2 x \cos^2 x} \, dx = \int \tan^{1/3} x (1 + \tan^2 x) \, d(\tan x) =
\]

\[ = \int \left( \tan^{1/3} x + \tan^{7/3} x \right) \, d(\tan x) = \frac{3}{4} \tan^{4/3} x + \frac{3}{10} \tan^{10/3} x + C.
\]

Result: \[ \dot{I} = \frac{3}{4} \tan^{4/3} x + \frac{3}{10} \tan^{10/3} x + C, \, C \in \mathbb{R}. \]

---

5.(c) \[ \dot{I} = \int \frac{dx}{4 \cos x + 3 \sin x + 6} = \left| \begin{array}{c}
R(\sin x, \cos x) = \frac{1}{4 \cos x + 3 \sin x + 6} \text{ has a general form.} \\
Applying the universal substitution: } t = \tan \frac{x}{2};
\end{array} \right| =
\]

\[ = \int \frac{2dt}{1 + t^2}; \sin x = \frac{2t}{1 + t^2}, \cos x = \frac{1 - t^2}{1 + t^2}
\]

\[ = \int \frac{2dt}{4(1 - t^2) + 6t + 6(1 + t^2)} = \int \frac{dt}{t^2 + 3t + 5} =
\]

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Completing the square: \( t^2 + 3t + 5 = \) 
\[ = (t + \frac{3}{2})^2 - \left(\frac{3}{2}\right)^2 + 5 = \left( t + \frac{3}{2} \right)^2 + \left( \frac{\sqrt{11}}{2} \right)^2 \] 
\[ = \int \frac{d \left( t + \frac{3}{2} \right)}{\left( t + \frac{3}{2} \right)^2 + \left( \frac{\sqrt{11}}{2} \right)^2} = \] 
\[ = \frac{1}{\sqrt{11}} \arctg \left( \frac{t + \frac{3}{2}}{\frac{\sqrt{11}}{2}} \right) + C \]

Result: \( \dot{I} = \frac{2}{\sqrt{11}} \arctg \left( \frac{2 \tan \frac{x}{2} + 3}{\sqrt{11}} \right) + C, \ C \in \mathbb{R} \).

* Solution of Problem 5(c) guided by Irina Blazhievska is available on-line: https://youtu.be/loxi3dwTmho

6.(a) \( \dot{I} = \int \frac{dx}{x^2 \sqrt{4 - x^2}} = \) 
Performing the change of variable: \( x = 2 \sin t; \ dx = 2 \cos t \ dt; \) 
\[ \sqrt{4 - x^2} = 2 \cos t \] 
\[ = \int \frac{2 \cos t \ dt}{(2 \sin t)^2 2 \cos t} = \] 
\[ = \frac{1}{4} \int \frac{dt}{\sin^2 t} = - \frac{1}{4} \ctg t + C \] 
Undoing the change of variable: 
\[ \ctg t = \frac{2 \cos t}{2 \sin t} = \frac{\sqrt{4 - x^2}}{x} \] 
\[ = - \frac{1}{4} \sqrt{\frac{4 - x^2}{x}} = - \frac{1}{4} \sqrt{\frac{4}{x^2}} - 1 + C. \]

Result: \( \dot{I} = - \frac{1}{4} \sqrt{\frac{4}{x^2}} - 1 + C, \ C \in \mathbb{R} \).

* Solution of Problem 6(a) guided by Ricard Riba is available on-line: https://youtu.be/ba4kyucyLVk
• Alternative solution 1

\[ \dot{i} = \int \frac{dx}{x^2 \sqrt{4 - x^2}} = \int \frac{dx}{x^3 \sqrt{\frac{4}{x^2} - 1}} = \left| \begin{array}{c}
\text{Substitution under the differential} \\
d\left(\frac{4}{x^2} - 1\right) = -8 \frac{dx}{x^3} \\
\frac{dx}{x^3} = -\frac{1}{8} d\left(\frac{4}{x^2} - 1\right)
\end{array} \right| = \left| \begin{array}{c}
\left(\frac{4}{x^2} - 1\right)^{-1/2} d\left(\frac{4}{x^2} - 1\right) = -\frac{2}{8} \left(\frac{4}{x^2} - 1\right)^{1/2} + C = -\frac{1}{4} \sqrt{\frac{4}{x^2} - 1} + C.
\end{array} \right| \]

• Alternative solution 2

\[ \dot{i} = \int \frac{dx}{x^2 \sqrt{4 - x^2}} = \left| \begin{array}{c}
\text{Performing the change of variable: } t = \frac{1}{x} \\
x = \frac{1}{t}, \; dx = -\frac{1}{t^2} dt
\end{array} \right| = \int \frac{-\frac{1}{t^2} dt}{\frac{1}{t^2} \sqrt{4 - \left(\frac{1}{t}\right)^2}} = \int \frac{-1}{\sqrt{4t^2 - 1}} dt = \left| \begin{array}{c}
d\left(4t^2 - 1\right) = 8tdt; \\
dt = \frac{1}{8} d\left(4t^2 - 1\right)
\end{array} \right| = -\frac{1}{8} \int \frac{d\left(4t^2 - 1\right)}{\sqrt{4t^2 - 1}} = \left| \begin{array}{c}
\sqrt{4t^2 - 1} + C = \left| \begin{array}{c}
\text{Undoing the change of variable: } t = \frac{1}{x} \\
x = \frac{1}{t}, \; \sqrt{4 - x^2} = \sqrt{4 - 4\frac{1}{t^2}} = \frac{2}{\text{ch} t}
\end{array} \right| = -\frac{1}{4} \sqrt{\frac{4}{x^2} - 1} + C.
\end{array} \right| \]

• Alternative solution 3

\[ \dot{i} = \int \frac{dx}{x^2 \sqrt{4 - x^2}} = \left| \begin{array}{c}
\text{Performing the change of variable:} \\
x = 2 \text{th } t, \; dx = \frac{2dt}{\text{ch}^2 t}; \\
\sqrt{4 - x^2} = \sqrt{4 - 4\text{th}^2 t} = \frac{2}{\text{ch} t}
\end{array} \right| = \int \frac{2dt}{(2\text{th } t)^2 \frac{2}{\text{ch} t}} = \left| \begin{array}{c}
\text{Undoing the change of variable:} \\
\text{sh} t = 2 \text{th } t \frac{\text{ch } t}{2} = \frac{x}{\sqrt{4 - x^2}}
\end{array} \right| = \left| \begin{array}{c}
\frac{1}{4} \int \frac{\text{ch } t dt}{\text{sh}^2 t} = \frac{1}{4} \int \frac{d(\text{sh } t)}{\text{sh}^2 t} = -\frac{1}{4} \frac{1}{\text{sh } t} + C = \left| \begin{array}{c}
\frac{\sqrt{4 - x^2}}{x} + C = -\frac{1}{4} \sqrt{\frac{4}{x^2} - 1} + C.
\end{array} \right|
\end{array} \right| \]
6. (b) \( I = \int \sqrt[3]{\frac{x+1}{x-1}} \frac{dx}{(x-1)^3} = \) 

\[
\begin{aligned}
\text{Performing the change of variable:} \\
t &= \frac{3}{x+1}; \quad \frac{x+1}{x-1} = t^3; \\
x &= \frac{t^3 + 1}{t^3 - 1} = 1 + \frac{2}{t^3 - 1}; \quad \frac{2}{t^3 - 1}; \\
dx &= \frac{-6t^2}{(t^3 - 1)^2} dt \\
= \int t \cdot \frac{-6t^2}{(t^3 - 1)^2} \left( \frac{t^3 - 1}{2} \right)^3 dt = \frac{-6}{8} \int t^3(t^3 - 1) dt = -\frac{3}{4} \int (t^6 - t^3) dt = \\
= -\frac{3}{4} \left( \frac{t^7}{7} - \frac{t^4}{4} \right) + C = -\frac{3}{28} t^7 + \frac{3}{16} t^4 + C = \\
= \left| \begin{align*}
\text{Undoing the change of variable:} \\
t &= \left( \frac{x+1}{x-1} \right)^{1/3} \\
&= -\frac{3}{28} \left( \frac{x+1}{x-1} \right)^{7/3} + \frac{3}{16} \left( \frac{x+1}{x-1} \right)^{4/3} + C, \ C \in \mathbb{R}.
\end{align*} \right|
\end{aligned}
\]

Result: \( I = -\frac{3}{28} \left( \frac{x+1}{x-1} \right)^{7/3} + \frac{3}{16} \left( \frac{x+1}{x-1} \right)^{4/3} + C, \ C \in \mathbb{R}. \)

7. (a) \( I = \int_{0}^{1} \ln(1 + x^2) dx = \) 

\[
\begin{aligned}
\text{Using integration by parts:} \int_{a}^{b} u dv = uv \bigg|_{a}^{b} - \int_{a}^{b} v du, \\
u = \ln(1 + x^2); \quad dv = dx; \\
du = \frac{2x \, dx}{1 + x^2}; \quad v = x.
\end{aligned}
\]

\[
\begin{aligned}
x \ln(1 + x^2) \bigg|_{0}^{1} - \int_{0}^{1} \frac{2x^2}{1 + x^2} \, dx &= \ln 2 - 2 \int_{0}^{1} \frac{x^2 + 1 - 1}{1 + x^2} \, dx = \\
= \ln 2 - 2 \int_{0}^{1} \left( 1 - \frac{1}{1 + x^2} \right) \, dx = \ln 2 - 2 \left[ x - \arctg x \right]_{0}^{1} = \\
= \ln 2 + (-2 + 2 \arctg 1) - (0 + 2 \arctg 0) = \ln 2 + \frac{\pi}{2} - 2.
\end{aligned}
\]

Result: \( I = \ln 2 + \frac{\pi}{2} - 2. \)

* Solution of Problem 7(a) guided by Irina Blazhiievská
is available on-line: https://youtu.be/jfbI2G23U2M
7.(b) \[ I = \int_0^{\pi/2} \sin^3 x \sqrt{\cos x} \, dx = \frac{\pi}{2} \int_0^{\pi/2} \sin^3 x \cos x \, dx \]

Applying the integration of a product \( \sin^m x \cos^n x \) with \( m = 3 \), \( n = 0 \) decompose:

\[ \sin^3 x = \sin^2 x \sin x; \quad \sin^2 x = 1 - \cos^2 x; \quad \sin x \, dx = -d(\cos x) \]

\[ = - \int_0^{\pi/2} (1 - \cos^2 x) \cos^{1/4} x \, d(\cos x) = \int_0^{\pi/2} (\cos^{9/4} x - \cos^{1/4} x) \, d(\cos x) = \]

\[ = \left[ \frac{4}{13} \cos^{13/4} x - \frac{4}{5} \cos^{5/4} x \right]_0^{\pi/2} = 0 - \left[ \frac{4}{13} - \frac{4}{5} \right] = \frac{32}{65}. \]

Result: \( I = \frac{32}{65} \).

* Solution of Problem 7(b) guided by Irina Blazhievska is available on-line: [https://youtu.be/s4VH2LvXh7M](https://youtu.be/s4VH2LvXh7M)

7.(c) \[ I = \int_1^{\sqrt{2}/2} \frac{4x^3 \, dx}{\sqrt{4 - x^8}} = \int_1^{\sqrt{2}/2} \frac{4x^3 \, dx}{\sqrt{2^2 - (x^4)^2}} = \frac{\sqrt{2}}{2} \int_1^{\sqrt{2}/2} \frac{d(x^4)}{\sqrt{2^2 - (x^4)^2}} = \text{arcsin} \frac{x^4}{2} \left|_1^{\sqrt{2}/2} \right. = \text{arcsin} \frac{\sqrt{2}}{2} - \text{arcsin} \frac{1}{2} = \frac{\pi}{4} - \frac{\pi}{6} = \frac{\pi}{12}. \]

Result: \( I = \frac{\pi}{12} \).

7.(d) \[ I = \int_1^{16} \frac{1 + \sqrt{x}}{\sqrt{x} + \sqrt{x}} \, dx = \left| \begin{array}{c}
\text{Performing the change of variable: } t = \sqrt{x} \\
x = t^4, \ dx = 4t^3 \, dt; \\
x = 1 \rightarrow t = 1, \\
x = 16 \rightarrow t = 2 \\
\begin{array}{c|c|c}
x & 1 & 16 \\
t & 1 & 2
\end{array}
\end{array} \right| =
\]
\[ = 4 \int_1^2 \frac{t^4 + t^2}{1 + t} \, dt = 4 \int_1^2 \frac{t^4 + t^2}{1 + t} \, dt = \left| \begin{array}{c}
\text{Separating the fraction’s integer part} \\
\frac{t^2 + t^4}{1 + t} = t^3 - t^2 + 2t - 2 + \frac{2}{1 + t}
\end{array} \right| =
\]
\[ = 4 \int_1^2 \left( t^3 - t^2 + 2t - 2 + \frac{2}{1 + t} \right) \, dt = \left[ \frac{t^4}{4} - \frac{4}{3} t^3 + 4t^2 - 8t + 8 \ln |1 + t| \right]_1^2 =
\]
\[
\left(16 - \frac{32}{3} + 16 - 16 + 8 \ln 3\right) - \left(1 - \frac{4}{3} + 4 - 8 + 8 \ln 2\right) = \frac{29}{3} + 8 \ln \frac{3}{2}.
\]

Result: \( \dot{I} = \frac{29}{3} + 8 \ln \frac{3}{2} \).

8.(a) Find the area of the figure bounded by the curves:

\[
y = 2x^2 - 10x + 6; \quad y = x^2 - 3x.
\]

The written version of solution is proposed below. Algorithm:

**Step 1.** Building the picture.

\( y = 2x^2 - 10x + 6 \) is a \( \cup \)-shaped parabola with vertex \( \left( \frac{5}{2}, -\frac{13}{2} \right) \) and points of intersection with OX: \( x_1 = \frac{5 - \sqrt{13}}{2} \approx 0, 7 \) and \( x_2 = \frac{5 + \sqrt{13}}{2} \approx 4, 3 \).

\( y = x^2 - 3x \) is a \( \cup \)-shaped parabola with vertex \( \left( \frac{3}{2}, -\frac{9}{4} \right) \) and points of intersection with OX: \( x_1 = 0 \) and \( x_2 = 3 \).

**Step 2.** Finding the points of intersection between the curves.

\[
\begin{align*}
2x^2 - 10x + 6 &= x^2 - 3x; \\
x^2 - 7x + 6 &= 0; \\
(x - 6)(x - 1) &= 0;
\end{align*}
\]

Abscises of points: \( x_1 = 1, x_2 = 6 \).
Step 3. Analytical description of the region in Cartesian coordinates:
\[ \Omega = \left\{ 1 \leq x \leq 6; 2x^2 - 10x + 6 \leq y \leq x^2 - 3x \right\}. \]

Step 4. Applying the suitable formula to find the area.

\[
S(\Omega) = \int_{a}^{b} (y_2(x) - y_1(x)) \, dx = \int_{1}^{6} (x^2 - 3x - (2x^2 - 10x + 6)) \, dx = \int_{1}^{6} (7x - x^2 - 6) \, dx = \left[ \frac{7}{2}x^2 - \frac{1}{3}x^3 - 6x \right]_{1}^{6} = \frac{7}{2}(36 - 1) - \frac{1}{3}(216 - 1) - 6(6 - 1) = 5 \left( \frac{147 - 86 - 36}{6} \right) = \frac{125}{6}.
\]

Result: \( S(\Omega) = \frac{125}{6} \) (square units).

8. (b) Find the area of the figure bounded by the curves:

\[
x(t) = 4 \cos^3 t, \quad y(t) = 4 \sin^3 t, \quad y = \frac{1}{2} \left( y \geq \frac{1}{2} \right).
\]

Algorithm:

Step 1. Building the picture.

\( x(t) = 4 \cos^3 t, \ y(t) = 4 \sin^3 t \) is a parametric representation of an astroid inscribed inside a circle of radius 4.

\( y = \frac{1}{2} \) is the straight line passing through \( (0, \frac{1}{2}) \) and parallel to OX.

Step 2. Finding the points of intersection between the curves.

\[
\begin{align*}
\begin{cases}
y = 4 \sin^3 t \\
y = \frac{1}{2}
\end{cases} \quad \Rightarrow \quad \sin t = \frac{1}{2}
\end{align*}
\]

\( \Rightarrow \) Parameters of points:

\[ t_1 = \frac{\pi}{6}, \ t_2 = \frac{5\pi}{6}. \]
**Step 3.** Analytical description of the region in Parametric coordinates.

The OY-symmetry of region $\mathcal{D}$ implies that its area is:

$$ S(\mathcal{D}) = 2S(\mathcal{D}^+), $$

where $\mathcal{D}^+$ is the subregion contained in the first quadrant,

$$ \mathcal{D}^+ = \left\{ 0 \leq x \leq 4 \cos^3 t; \frac{1}{2} \leq y \leq 4 \sin^3 t, \frac{\pi}{6} \leq t \leq \frac{\pi}{2} \right\}. $$

**Step 4.** Applying the suitable formula to find the area.

$$ S(\mathcal{D}) = 2S(\mathcal{D}^+) = 2 \int_{\alpha}^{\beta} x(t)y'(t)dt = \left| x(t) = 4 \cos^3 t, \quad y'(t) = (4 \sin^3 t)' = 12 \sin^2 t \cos t \right| = $$

$$ = 2 \cdot 4 \cdot 12 \int_{\pi/6}^{\pi/2} \cos^4 t \sin^2 t dt = 96 \int_{\pi/6}^{\pi/2} \frac{1 + \cos 2t}{2} \cdot \frac{\sin 2t}{4} dt = $$

$$ = 12 \int_{\pi/6}^{\pi/2} \left( \frac{1 - \cos 4t}{2} + \cos 2t \sin^2 2t \right) dt = 12 \left[ \frac{t}{2} - \frac{1}{8} \sin 4t + \frac{1}{6} \sin^3 2t \right] \Bigg|_{\pi/6}^{\pi/2} = $$

$$ = 12 \left[ \frac{1}{2} \left( \frac{\pi}{2} - \frac{\pi}{6} \right) - \frac{1}{8} \left( \sin 2\pi - \sin \frac{4\pi}{6} \right) + \frac{1}{6} \left( \sin^3 \pi - \sin^3 \frac{\pi}{3} \right) \right] = $$

$$ = 12 \left[ \frac{\pi}{6} + \frac{\sqrt{3}}{16} - \frac{1}{6} \left( \frac{\sqrt{3}}{2} \right)^3 \right] = 12 \frac{\pi}{6} = 2\pi. $$

**Result:** $S(\mathcal{D}) = 2\pi$ (square units).

8.(c) Find the area of the figure bounded by the curve:

$$ \rho = 4 \sin 3\phi. $$

**Algorithm:**

**Step 1.** Building the picture.

$\rho = 4 \sin 3\phi$ is a polar representation of a 3-petaled rose inscribed inside a circle of radius 4. Since this curve is constructed from a sin-function and negative radius is not allowed, it has OY-symmetry and it is well-defined when $\frac{2\pi}{3}k \leq \phi \leq \frac{\pi}{3} + \frac{2\pi}{3}k$, $k \in \{0, 1, 2\}$.  

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The fact that this curve has period $\frac{2\pi}{3}$ implies that the rose’s k-petal is created by rotating 0-petal in counterclockwise direction around the pole an angle $\frac{2\pi}{3}k$, $k \in \{1, 2\}$. Next we show the building of the 0-petal. For this, we set $\phi \in [0, \frac{\pi}{3}]$ and consider the additional table.

| $\phi$ | 0 | $\frac{\pi}{18}$ | $\frac{\pi}{12}$ | $\frac{\pi}{9}$ | $\frac{\pi}{6}$ | $\frac{2\pi}{9}$ | $\frac{\pi}{4}$ | $\frac{5\pi}{18}$ | $\frac{\pi}{3}$ |
|--------|---|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $\rho = 4 \sin 3\phi$ | 0  | 2               | $2\sqrt{2}$    | $2\sqrt{3}$    | 4               | $2\sqrt{3}$    | $2\sqrt{2}$    | 2               | 0               |

**Step 2.** Analytical description of the region in Polar coordinates.

Since the rose is formed by rotation of 0-petal around the pole, all petals have the same metric characteristics. Then the 3 petals have the same area. The multiplication of the 0-petal’s area by 3 give us the area of the whole region $\mathcal{D}$:

$$S(\mathcal{D}) = 3S(\mathcal{D}_0),$$

where $\mathcal{D}_0$ is the 0-petal’s region,

$$\mathcal{D}_0 = \left\{0 \leq \phi \leq \frac{\pi}{3}; \ 0 \leq \rho \leq 4 \sin 3\phi\right\}$$

**Step 3.** Applying the suitable formula to find the area.

$$S(\mathcal{D}) = 3S(\mathcal{D}_0) = \frac{3}{2} \int_{\frac{\pi}{3}}^{\beta} \rho^2(\phi)d\phi = \frac{3}{2} \int_0^{\frac{\pi}{3}} (4 \sin 3\phi)^2 d\phi = 24 \int_0^{\frac{\pi}{3}} \sin^2 3\phi d\phi =$$

$$= 12 \int_0^{\frac{\pi}{3}} (1 - \cos 6\phi) d\phi = 12 \left[ \phi - \frac{1}{6} \sin 6\phi \right]_0^{\frac{\pi}{3}} = 12 \left( \frac{\pi}{3} - \frac{1}{6} \sin 2\pi \right) = 4\pi.$$

**Result:** $S(\mathcal{D}) = 4\pi$ (square units).

9.(a) Find the arc length of the curve

$$y = \frac{1}{3} \ln(\cos 3x), \ 0 \leq x \leq \frac{\pi}{18}.$$
Algorithm:

**Step 1.** Building the picture.

\[ y = \frac{1}{3} \ln(\cos 3x) \]

is a curve constructed from a cos-function. Since the logarithm is only defined for positive values, this curve is only defined on regions \([-\frac{\pi}{6} + \frac{2\pi k}{3}, \frac{\pi}{6} + \frac{2\pi k}{3}]\), \(k \in \mathbb{Z}\).

On each region it has a \(\cup\)-like form with zeros-maxima at \(x = \frac{2\pi}{3} k\), and vertical asymptotes:

\[ x = \pm \frac{\pi}{6} + \frac{2\pi k}{3}, k \in \mathbb{Z}. \]

**Step 2.** Analytical description of the arc segment in Cartesian coordinates:

\[ \Gamma = \left\{ y = \frac{1}{3} \ln(\cos 3x); \ 0 \leq x \leq \frac{\pi}{18} \right\} \]

Note that \([0, \frac{\pi}{18}]\) belongs to the region of curve’s well-definiteness.

**Step 3.** Applying of the suitable formula to find the arc length.

\[
\begin{align*}
\ell(\Gamma) &= \int_{a}^{b} \sqrt{1 + (y'_{x})^{2}} \, dx = \left| y = \frac{1}{3} \ln(\cos 3x); \ y'_{x} = \frac{1}{3} \cdot \frac{-3 \sin 3x}{\cos 3x} = -\tan 3x; \quad 1 + (y'_{x})^{2} = 1 + (\tan 3x)^{2} = \frac{1}{\cos^{2} 3x} \right| \\
&= \int_{0}^{\pi/18} \sqrt{1 + \tan^{2} 3x} \, dx = \int_{0}^{\pi/18} \frac{1}{\cos 3x} \, dx = \int_{0}^{\pi/18} \frac{1}{\cos 3x} \, dx = \\
&= \int_{0}^{\pi/18} \cos 3x \, dx = -\frac{1}{3} \int_{0}^{\pi/18} \frac{d(\sin 3x)}{\sin^{2} 3x - 1} \, dx = -\frac{1}{3} \frac{1}{2} \ln \left| \frac{\sin 3x - 1}{\sin 3x + 1} \right|_{0}^{\pi/18} = \\
&= -\frac{1}{6} \ln \left(3 \cdot \frac{\pi}{18} \right) - 1 + \frac{1}{6} \ln \left(0 - 1 \right) = -\frac{1}{6} \ln \left(\frac{1}{2} + 1 \right) = \frac{1}{6} \ln 3.
\end{align*}
\]

**Result:** \(\ell(\Gamma) = \frac{1}{6} \ln 3 \) (units).
9.(b) Find the arc length of the curve

\[ x(t) = 4(t - \sin t), \quad y(t) = 4(1 - \cos t), \quad t \in [0, 2\pi]. \]

Algorithm:

**Step 1.** Building the picture.

This curve is generated by the trajectory of a marked point in a circle of radius 4 rolling on the "positive" side of the base. The parameter region \( 0 \leq t \leq 2\pi \) creates one full wave of the curve, which intersects OX at points \( x = 0 \) and \( x = 8\pi \), has maxima at \( (4\pi, 8) \) and an axis of symmetry at \( x = 4\pi \).

**Step 2.** Analytical description of the arc segment in Parametric coordinates:

\[ \Gamma = \{ x = 4(t - \sin t), \quad y = 4(1 - \cos t); \quad 0 \leq t \leq 2\pi \}. \]

**Step 3.** Applying the suitable formula to find the arc length.

\[
\begin{align*}
    l(\Gamma) &= \int_{\alpha}^{\beta} \sqrt{(x'_t)^2 + (y'_t)^2} \, dt = \\
    &= 4 \int_{0}^{2\pi} \sqrt{(1 - \cos t)^2 + \sin^2 t} \, dt \\
    &= 4 \int_{0}^{2\pi} \sqrt{2(1 - \cos t)} \, dt \\
    &= 8 \int_{0}^{2\pi} \sin \frac{t}{2} \, dt = \\
    &= 8 \left[ -16 \cos \frac{t}{2} \right]_{0}^{2\pi} = -16(\cos \pi - \cos 0) = 32.
\end{align*}
\]

**Result:** \( l(\Gamma) = 32 \) (units).
9.(c) Find the arc length of the curve

\[ \rho = \frac{10}{\sqrt{101}} e^{\phi/10}, \quad 0 \leq \phi \leq 2\pi. \]

* Solution of Problem 9(c) guided by Irina Blazhievska is available on-line: [https://youtu.be/AJbHsr1eOes](https://youtu.be/AJbHsr1eOes)

The written version of solution is proposed below. Algorithm:

**Step 1.** Building the picture.

\[ \rho = \frac{10}{\sqrt{101}} e^{\phi/10} \] is a polar representation of a logarithmic spiral. It has 2 parameters; the initial radius \( \frac{10}{\sqrt{101}} \) and the rate of spiral’s increasing \( k = \frac{1}{10} \) \( (k = \frac{\rho'_{\phi}}{\rho}) \).

Since \( k > 0 \), the spiral rotates around the pole in counterclockwise direction with increasing polar radius.

The angle domain \( 0 \leq \phi \leq 2\pi \) generates one full turn with continuous increase of radius from \( \frac{10}{\sqrt{101}} \) to \( \frac{10}{\sqrt{101}} e^{\pi/10} \).

**Step 2.** Analytical description of the arc segment in Polar coordinates:

\[
\Gamma = \left\{ \rho = \frac{10}{\sqrt{101}} e^{\phi/10}; \quad 0 \leq \phi \leq 2\pi \right\}.
\]

**Step 3.** Applying the suitable formula to find the arc length.

\[
l(\Gamma) = \int_{\alpha}^{\beta} \sqrt{\rho^2 + (\rho'_\phi)^2} d\phi = \left| \rho = \frac{10}{\sqrt{101}} e^{\phi/10}; \rho'_\phi = \frac{10}{\sqrt{101}} e^{\phi/10}, \frac{1}{10} e^{\phi/10} = \sqrt{\frac{10}{101}} e^{\phi/10} \right| =
\]

\[
= \int_{0}^{2\pi} \sqrt{\left(e^{\phi/10}\right)^2} d\phi = \int_{0}^{2\pi} e^{\phi/10} d\phi = 10 e^{\phi/10} \bigg|_{0}^{2\pi} = 10(e^{2\pi/10} - e^{0}) = 10(e^{\pi/5} - 1).
\]

**Result:** \( l(\Gamma) = 10(e^{\pi/5} - 1) \) (units).
10.(a) Find the surface area generated by rotating the curve
\[ y = \frac{1}{2} \cosh 2x, \ -1 \leq x \leq 1, \] around the axis \( l = OX \).

Algorithm:

**Step 1.** Building the picture.

The surface of revolution of the catenary curve around OX is a *catenoid*. Note that the catenoid has a minimal surface area.

**Step 2.** Analytical description of the rotating arc segment (the generatrix) in Cartesian coordinates:

\[ \Gamma = \left\{ y = \frac{1}{2} \cosh 2x; \ -1 \leq x \leq 1 \right\} \]

**Step 3.** Applying the suitable formula to find the surface area.

\[
S_{OX} = 2\pi \int_{a}^{b} y(x)\sqrt{1 + (y'_x)^2} \, dx = \left. \begin{array}{l}
y = \frac{1}{2} \cosh 2x; \ y'_x = \frac{1}{2} \cdot 2 \sinh 2x = \sinh 2x \\
1 + (y'_x)^2 = 1 + (\sinh 2x)^2 = \cosh 2x
\end{array} \right| =
\]

\[
= 2\pi \int_{-1}^{1} \frac{1}{2} \cosh 2x \sqrt{1 + (\sinh 2x)^2} \, dx = \pi \int_{-1}^{1} \cosh 2x \, dx = \pi \int_{-1}^{1} \frac{1 + \cosh 4x}{2} \, dx =
\]

\[
= \frac{\pi}{2} \left[ x + \frac{1}{4} \sinh 4x \right]_{-1}^{1} = \frac{\pi}{2} \left( 2 + \frac{1}{2} \sinh 4 \right) = \pi \left( 1 + \frac{1}{4} \sinh 4 \right).
\]

**Result:** \( S_{OX} = \pi \left( 1 + \frac{\pi}{4} \sinh 4 \right) \) (square units).
10.(b) Find the surface area generated by rotating the curve

\[ x = 3 + \cos t, \ y = 2 + \sin t, \] around the axis \( l = OY. \)

* Solution of Problem 10(b) guided by Irina Blazhievska is available on-line: https://youtu.be/nXzXyHIw0w8

The written version of solution is proposed below. Algorithm:

**Step 1.** Building the picture.

\[
x(t) = 3 + \cos t \\
y(t) = 2 + \sin t
\]
is a parametric representation of a circle of radius 1 with center in \((3, 2)\): \((x - 3)^2 + (y - 2)^2 = 1^2\). The empty intersection of the curve with OY implies that there are no restrictions for parameter’s domain: \(0 \leq t \leq 2\pi\). The surface of revolution with this rotating curve is a ring-torus.

**Step 2.** Analytical description of the rotating arc segment (the generatrix) in Parametric coordinates:

\[
\Gamma = \left\{ \begin{array}{l}
x = 3 + \cos t, \\
y = 2 + \sin t; \\
0 \leq t \leq 2\pi
\end{array} \right\}
\]

**Step 3.** Applying the suitable formula to find the surface area.

\[
S_{OY} = 2\pi \int_{\alpha}^{\beta} x(t) \sqrt{(x'(t))^2 + (y'(t))^2} \, dt = \left| \begin{array}{l}
x(t) = 3 + \cos t; \ y(t) = 2 + \sin t; \\
x'(t) = (3 + \cos t)' = -\sin t; \\
y'(t) = (2 + \sin t)' = \cos t; \\
(x'(t))^2 + (y'(t))^2 = (-\sin t)^2 + (\cos t)^2 = 1
\end{array} \right| = \]

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\[=2\pi \int_{0}^{2\pi} (3 + \cos t) \sqrt{(-\sin t)^2 + (\cos t)^2} \, dt = 2\pi \int_{0}^{2\pi} (3 + \cos t) \, dt =
\]
\[=2\pi \left[ 3t + \sin t \right]_{0}^{2\pi} = 2\pi \left[ 3(2\pi - 0) + (\sin 2\pi - \sin 0) \right] = 12\pi^2. \]

**Result:** \( S_{OY} = 12\pi^2 \) (square units).

10.(c) Find the area surface area generated by rotating the curve \( \rho = \sqrt{\cos 2\phi} \) around the axis \( l = o\rho \)

**Algorithm:**

**Step 1.** Building the picture.

\( \rho = \sqrt{\cos 2\phi} \) is a Polar representation of an \( \infty \)-shaped lemniscate inscribed inside a circle of radius 1. This curve is known as *Bernoulli’s lemniscate*. Since this curve is constructed from a cos-function and negative radius is not allowed, it has OX-symmetry and it is well-defined on \( \phi \in [\frac{-\pi}{4}, \frac{\pi}{4}] \cup [\frac{3\pi}{4}, \frac{5\pi}{4}] \).

OX-symmetry of the curve implies the restriction on polar angle domain for rotating part: \( \phi \in [0, \frac{\pi}{4}] \cup \left[ \frac{3\pi}{4}, \pi \right] \).

The surface of revolution generated by the lemniscate is "Hourglass"-shaped and is attached on the right.

**Step 2.** Analytical description of the rotating arc segment (the generatrix) in Polar coordinates.

The mirror symmetry of the surface with respect to YZ-plane implies that both Hourglass’ sides have the same metric characteristics.
The multiplication of right-side surface area $S_{op}^+$ by 2 gives us the surface area of the whole solid of revolution: $S_{op} = 2S_{op}^+$, where the generatrix of $S_{op}^+$ is located in the first quadrant:

$$\Gamma^+ = \left\{ \rho = \sqrt{\cos 2\phi}; \ 0 \leq \phi \leq \frac{\pi}{4} \right\}.$$ 

**Step 3.** Applying the suitable formula to find the surface area.

$$S_{op} = 2S_{op}^+ = 2 \cdot 2\pi \int_{\alpha}^{\beta} \rho(\phi) \sin \phi \sqrt{(\rho)^2 + (\rho')^2} \ d\phi = \left. \begin{array}{c} \rho = \sqrt{\cos 2\phi}; \\ \rho' = \frac{\sin 2\phi}{\sqrt{\cos 2\phi}}; \\ (\rho)^2 + (\rho')^2 = \frac{1}{\cos 2\phi} \end{array} \right|_{\rho = \sqrt{\cos 2\phi}; (\rho')^2 = \frac{1}{\cos 2\phi}} =$$

$$= 4\pi \int_{0}^{\pi/4} \sqrt{\cos 2\phi} \sin \phi \sqrt{\frac{1}{\cos 2\phi}} \ d\phi = 4\pi \int_{0}^{\pi/4} \sin \phi d\phi = 4\pi \left[ \sin \phi \right]_{0}^{\pi/4} =$$

$$= -4\pi \left( \cos \left( \frac{\pi}{4} \right) - \cos 0 \right) = -4\pi \left( \frac{\sqrt{2}}{2} - 1 \right) = 2\pi (2 - \sqrt{2}).$$

**Result:** $S_{op} = 2\pi (2 - \sqrt{2})$ (square units).

11.(a) Find the volume of the body formed by rotating region between the curves

$$y = x^2 + 2x + 5, \ y = 5 - x,$$ around the axis $l = OX$.

**Algorithm:**

**Step 1.** Building the picture.

![Rotating region](image)

$y = x^2 + 2x + 5$ is a $\cup$-shaped parabola with vertex $(-1, 3)$; it has empty intersection with $OX$, but intersects $OY$ at $(0, 5)$.

$y = 5 - x$ is a straight line passing through the points $(5, 0), (0, 5)$. 

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Step 2. Finding the points of intersection between the curves.

\[
\begin{align*}
\begin{cases} 
  y = x^2 + 2x + 5 \\
  y = 5 - x 
\end{cases} \Rightarrow 
  x^2 + 2x + 5 = 5 - x;
\end{align*}
\]

Abscises of points: \( x_1 = -3, \ x_2 = 0 \).

Step 3. Analytical description of the rotating region in Cartesian coordinates:

\( D = \{ -3 \leq x \leq 0; \ x^2 + 2x + 5 \leq y \leq 5 - x \} \)

The body generated by revolution of the region \( D \) around \( OX \) is attached below, on the right.

Step 4. Applying the suitable formula to find the volume of the body.

\[
V_{OX} = V_2 - V_1 = \pi \int_{a}^{b} (y_2^2(x) - y_1^2(x)) \, dx =
\]

\[
= \pi \int_{-3}^{0} (5 - x)^2 - (x^2 + 2x + 5)^2 \, dx =
\]

\[
= \pi \int_{-3}^{0} (-x^4 - 4x^3 - 13x^2 - 30x) \, dx =
\]

\[
= \pi \left[ -\frac{x^5}{5} - x^4 - 13 \frac{x^3}{3} - 30 \frac{x^2}{2} \right]_{-3}^{0} =
\]

\[
= \pi \left[ \frac{-243}{5} + 99 \right] = \frac{252}{5} \pi = 50.4\pi.
\]

Result: \( V_{OX} = 50.4\pi \) (cubic units).

11.(b) Find the volume of the body formed by rotating the region between the curves

\( x = 5 + 4y - y^2, \ x = 5, \) around the axis \( l = OY. \)

Algorithm:

Step 1. Building the picture.
\[ x = 5 + 4y - y^2 = 9 - (y - 2)^2 \] is a \( \triangleright \)-shaped horizontal parabola with vertex \((9, 2)\) and points of intersection with \(OY\): \( y_1 = -1 \) and \( y_2 = 5 \).

\( x = 5 \) is a straight line passing through \((5, 0)\) and parallel to \(OY\).

**Step 2.** Finding the points of intersection between the curves.

\[
\begin{cases}
x = 5 + 4y - y^2 \\
x = 5
\end{cases} \Rightarrow
\]

\[ 5 + 4y - y^2 = 5; -y^2 + 4y = 0; \]
\[ y(y - 4)=0 \]

Ordinates of points: \( y_1 = 0, \ y_2 = 4 \).

**Step 3.** Analytical description of the rotating region in Cartesian coordinates:

\[
\mathcal{D} = \left\{ 0 \leq y \leq 4; \ 5 \leq x \leq 5 + 4y - y^2 \right\}.
\]

The body generated by the revolution of the region \( \mathcal{D} \) around \( OY \) is attached on the left.

**Step 4.** Applying the suitable formula to find the volume of the body.

\[
V_{OY} = V_2 - V_1 = \pi \int_c^d \left( x_2^2(y) - x_1^2(y) \right) dy = \pi \int_0^4 \left( (5 + 4y - y^2)^2 - 5^2 \right) dy =
\]

\[
= \pi \int_0^4 (y^4 - 8y^3 + 6y^2 + 40y) dy = \pi \left[ \frac{1}{5}y^5 - 2y^4 + 2y^3 + 20y^2 \right]_0^4 =
\]

\[
= \pi \left[ \frac{1}{5}4^5 - 2 \cdot 4^4 + 2 \cdot 4^3 + 20 \cdot 4^2 \right] = \pi \cdot 4^3 \left[ \frac{16}{5} - 1 \right] = \frac{704}{5} \pi = 140, 8\pi.
\]

**Result:** \( V_{OY} = 140, 8\pi \) (cubic units).
11.(c) Find the volume of the body formed by rotating the curve

$$\rho = 6(1 + \cos \phi), \text{ around the axis } l = o\rho.$$  

* Solution of Problem 11(c) guided by Irina Blazhievska is available on-line: https://youtu.be/tr2pXX5GJJE

The written version of solution is proposed below. Algorithm:

**Step 1.** Building the picture.

$\rho = 6(1 + \cos \phi)$ is a polar representation of a cardioid with a parameter $a = 6$. This curve has a heart’s shape with a "stalk" at the pole and $o\rho$-symmetry, with angle domain $0 \leq \phi \leq 2\pi$. Since it is constructed from a cos-function, it has OX-symmetry, which implies the restriction on polar angle domain for rotating part: $\phi \in [0, \pi]$.

**Step 2.** Analytical description of the rotating region in Polar coordinates:

$$\mathcal{D} = \{0 \leq \phi \leq \pi, \ 0 \leq \rho \leq 6(1 + \cos \phi)\}$$

The body generated by revolution of the region $\mathcal{D}$ around the $o\rho$-axis has a shape of an apple with the stalk at the origin; it is attached on the right.
Step 3. Applying the suitable formula to find the volume of the body.

\[ V_{\rho} = \frac{2\pi}{3} \int_{\alpha}^{\beta} \rho^3(\phi) \sin \phi \, d\phi = \frac{2\pi}{3} \int_{0}^{\pi} (6(1 + \cos \phi))^3 \sin \phi \, d\phi = \]
\[ = -\frac{2\pi}{3} \cdot 6^3 \int_{0}^{\pi} (1 + \cos \phi)^3 \, d(1 + \cos \phi) = -6^2 \pi \frac{1}{4} [(1 + \cos \phi)^4]_{0}^{\pi} = \]
\[ = -36\pi [(1 + \cos \pi)^4 - (1 + \cos 0)^4] = -36\pi [0 - 2^4] = 476\pi. \]

Result:  \( V_{\rho} = 476\pi \) (cubic units).

12.(a) \( \dot{I} = \int_{-2}^{\infty} \frac{2x - 1}{x^2 + 4} \, dx = \)
\[ f(x) = \frac{2x - 1}{x^2 + 4} \text{ is a bounded continuous function for all } x \in [-2, +\infty), \]
\[ \Rightarrow \text{Improper integral of 1st kind} \]
\[ = \lim_{N \to \infty} \int_{-2}^{N} \frac{2x - 1}{x^2 + 4} \, dx = \lim_{N \to \infty} \int_{-2}^{N} \left( \frac{2x}{x^2 + 4} - \frac{1}{x^2 + 2^2} \right) \, dx = \]
\[ = \lim_{N \to \infty} \left[ \int_{-2}^{N} \frac{d(x^2 + 4)}{x^2 + 4} - \int_{-2}^{N} \frac{dx}{x^2 + 2^2} \right] = \lim_{N \to \infty} \left[ \ln(x^2 + 4) - \frac{1}{2} \arctg \frac{x}{2} \right]_{-2}^{N} = \]
\[ = \lim_{N \to \infty} \left[ \ln(N^2 + 4) - \frac{1}{2} \arctg \left( \frac{N}{2} \right) - \ln((-2)^2 + 4) + \frac{1}{2} \arctg \left( \frac{-2}{2} \right) \right] = \]
\[ = \lim_{N \to \infty} \left[ \ln(N^2 + 4) - \frac{1}{2} \arctg \left( \frac{N}{2} \right) - \ln 8 - \frac{\pi}{8} \right] = +\infty - \frac{3\pi}{8} - \ln 8 = +\infty. \]

Result: The integral is divergent: \( \dot{I} = +\infty. \)

* Solution of Problem 12(a) guided by Irina Blazhievksa is available on-line: https://youtu.be/r1GZaS5Hooo
12. (b) \( \hat{I} = \int_{0}^{\pi/2} e^{-\tan x} \frac{dx}{\cos^2 x} = \int_{0}^{\pi/2-\varepsilon} e^{-\tan x} \frac{dx}{\cos^2 x} = \lim_{\varepsilon \to 0^+} \frac{\pi/2 - \varepsilon}{\varepsilon} \int_{0}^{\pi/2-\varepsilon} e^{-\tan x} d(tg x) = -\lim_{\varepsilon \to 0^+} e^{-\tan x} \bigg|_{0}^{\pi/2-\varepsilon} = -\lim_{\varepsilon \to 0^+} \left[ e^{-\tan(\pi/2-\varepsilon)} - e^{\tan 0} \right] = 1 - \lim_{\varepsilon \to 0^+} e^{-\ctg \varepsilon} = 1 - 0 = 1. \)

Result: The integral is convergent: \( \hat{I} = 1. \)

* Solution of Problem 12(b) guided by Irina Blazhievksa is available on-line: [https://youtu.be/lbBT5mV_pc4](https://youtu.be/lbBT5mV_pc4)
### Indefinite Integration

| Task Description                                                                 | Guide                          | Video Link                      |
|----------------------------------------------------------------------------------|--------------------------------|---------------------------------|
| Reduction to the table of integrals:                                             | Ricard Riba Garcia             | [https://youtu.be/x48CikKlF9c](https://youtu.be/x48CikKlF9c) |
| \[ \int x^3(1 - x^2)^2 \, dx \]                                                   |                                |                                 |
| Reduction to the table of integrals:                                             | Irina Blazhievska              | [https://youtu.be/1MqmZbQ-3qM](https://youtu.be/1MqmZbQ-3qM) |
| \[ \int (3 + \tan^2 x) \, dx \]                                                 |                                |                                 |
| Substitution under the differential:                                             | Irina Blazhievska              | [https://youtu.be/kSn2UvdXWVs](https://youtu.be/kSn2UvdXWVs) |
| \[ \int x^4 e^{2x^5 - 1} \, dx \]                                               |                                |                                 |
| Integration of fractions with quadratic functions:                               | Irina Blazhievska              | [https://youtu.be/KQuEzkh6AqQ](https://youtu.be/KQuEzkh6AqQ) |
| \[ \int \frac{dx}{\sqrt{6x - 9x^2}} \]                                          |                                |                                 |
| Integration of fractions with quadratic functions:                               | Ricard Riba Garcia             | [https://youtu.be/RXJcMkNz8Zg](https://youtu.be/RXJcMkNz8Zg) |
| \[ \int \frac{(-2x - 7) \, dx}{x^2 + 6x + 10} \]                                |                                |                                 |
| Indefinite Integration                                                                 |
|---------------------------------------------------------------------------------------|
| **Change of variable:**                                                               |
| \[ \int \frac{(3x + 2)dx}{\sqrt{x + 4}} \]                                        |
| Guide: Irina Blazhievska                                                              |
| [https://youtu.be/ZOFSo2oDVzQ](https://youtu.be/ZOFSo2oDVzQ)                           |
| **Integration by parts:**                                                              |
| \[ \int x^2 \ln x \, dx \]                                                            |
| Guide: Ricard Riba Garcia                                                              |
| [https://youtu.be/hDYt-m7ZCgM](https://youtu.be/hDYt-m7ZCgM)                           |
| **Integration of polynomial fractions:**                                               |
| \[ \int \frac{(3x^3 - 32x + 56)\, dx}{x^3 - 2x^2 - 4x + 8} \]                      |
| Guide: Irina Blazhievska                                                              |
| [https://youtu.be/X_300OO1i8A](https://youtu.be/X_300OO1i8A)                          |
| **Integration of trigonometric functions (products):**                                 |
| \[ \int \sin 10x \sin 3x \, dx \]                                                    |
| Guide: Irina Blazhievska                                                              |
| [https://youtu.be/-Zsc5t-YIOk](https://youtu.be/-Zsc5t-YIOk)                          |
| **Integration of trigonometric rational function (fractions):**                        |
| \[ \int \frac{dx}{4 \cos x + 3 \sin x + 6} \]                                      |
| Guide: Irina Blazhievska                                                              |
| [https://youtu.be/loxi3dwTmho](https://youtu.be/loxi3dwTmho)                         |
| **Integration of fractions with radicals:**                                            |
| \[ \int \frac{dx}{x^2 \sqrt{4 - x^2}} \]                                            |
| Guide: Ricard Riba Garcia                                                              |
| [https://youtu.be/ba4kyucyLVk](https://youtu.be/ba4kyucyLVk)                         |
## Definite Integration

### Integration by parts:

\[ \int_{0}^{1} \ln(1 + x^2) \, dx \]

Guide: Irina Blazhievska

https://youtu.be/jfbI2G23U2M

### Substitution under the differential:

\[ \int_{0}^{\pi/2} \sin^3 x \sqrt{\cos x} \, dx \]

Guide: Irina Blazhievska

https://youtu.be/s4VH2LvXh7M

### Change of variable:

\[ \int_{1}^{16} \left( \frac{1 + \sqrt{x}}{\sqrt{x} + \sqrt{x}} \right) \, dx \]

Guide: Irina Blazhievska

https://youtu.be/gCX8MgQrd7A

## Geometric Applications of Definite Integrals

- **Cartesian coordinates:**
  Area of the figure bounded by the curves:
  \[ y = 2x^2 - 10x + 6, \ y = x^2 - 3x \]
  Guide: Irina Blazhievska
  
  https://youtu.be/w4wx7Dd547w

- **Polar coordinates:**
  Arc length of the curve:
  \[ \rho(\phi) = \frac{10}{\sqrt{101}} e^{\phi}, \ \phi \in [0, 2\pi] \]
  Guide: Irina Blazhievska
  
  https://youtu.be/AJbHsr1eOes
| Geometric Applications of Definite Integrals |
|---------------------------------------------|
| **• Parametric coordinates:** Surface area generated by rotating the curve around OY-axis: |
| \[ x = 3 + \cos t, \quad y = 2 + \sin t \] |
| Guide: Irina Blazhievska |
| [https://youtu.be/nXzXyHlw0w8](https://youtu.be/nXzXyHlw0w8) |

| **• Cartesian coordinates:** Volume of the body generated by rotating around OX-axis the region bounded by the curve: |
| \[ \rho(\phi) = 6(1 + \cos \phi) \] |
| Guide: Irina Blazhievska |
| [https://youtu.be/tr2pXX5GJJE](https://youtu.be/tr2pXX5GJJE) |

| Improper Integrals |
|--------------------|
| **• Improper integral of 1st kind:** |
| \[ \int_{-2}^{\infty} \frac{(2x - 1)dx}{x^2 + 4} \] |
| Guide: Irina Blazhievska |
| [https://youtu.be/r1GZaS5Hooo](https://youtu.be/r1GZaS5Hooo) |

| **• Improper integral of 2nd kind:** |
| \[ \int_{0}^{\pi/2} e^{-\tan x} \frac{dx}{\cos^2 x} \] |
| Guide: Irina Blazhievska |
| [https://youtu.be/1bBT5mV_pc4](https://youtu.be/1bBT5mV_pc4) |
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- https://math.stackexchange.com
- http://old.nationalcurvebank.org/volrev/volrev.htm