Error analysis and kinematic calibration of a new 3-PRS parallel mechanism

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Abstract. Taking a new 3-PRS parallel mechanism as the research object, error analysis and kinematics calibration are carried out. Combining with inverse kinematics, the parameter error of the mechanism is analyzed comprehensively. A calibration method of inverse kinematics based on structural parameters and genetic algorithm is proposed, and the error calibration model of 3-PRS parallel mechanism is deduced. The calculation by MATLAB shows that the reverse motion error calculated by nominal machine tool structural parameters is larger than that obtained by calibrated structural parameters. The position of the three cylindrical and spherical hinges after calibration is closer to the nominal position. It shows that the error calibration can correct the machine tool structure and guide the subsequent installation and debugging of the machine tool.

1. Introduction

From the point of view of mechanism, parallel machine tool has the advantage of high accuracy, but in practical application, due to the existence of machining tolerance and assembly error, there are always deviations between the actual and nominal values of the kinematic parameters of parallel machine tool, which limits the actual accuracy of parallel machine tool [1]. Therefore, it is necessary and important to calibrate the kinematics parameters of the actual parallel machine tool system.

At present, the calibration methods of parallel machine tools can be roughly divided into external calibration method and self-calibration method [2]. The calibration of kinematics parameters of mechanism is actually an estimation of kinematics parameters, so it is also one of the tasks of system control [3], which has been extensively studied. Li et al. [4] introduced the kinematics calibration method of parallel machine tool from the aspects of error modeling, pose measurement, error identification and error compensation, Zhang et al. [5] uses Theory and Cooperative Coevolutionary Network to calibrate the Optimal Kinematic Calibration of Parallel Manipulators, Li [6] proposed a method for optimizing the measurement configuration of residual proportions and the algorithms for optimal measurement configuration selection and robust source error identification, Wang et al. [7] proposed a parameter calibration method for 3-DOF parallel robot based on self-learning method, Fang et al. [8] proposed a kinematic calibration method for serial and parallel robots based on quantum particle swarm optimization algorithm, Zhang et al. [9] realized error compensation and
kinematics calibration by identifying error model parameters and spatial interpolation based on inverse distance ratio method, Ding et al. [10] established a calibration model based on inverse kinematics solution and solved the actual value of structural parameters by Gauss-Newton nonlinear least squares method, Li et al. [11] established kinematic calibration model based on geometric error transfer model, Liu et al. [12] proposed an index for evaluating measurement trajectory based on the description method of non-orthogonal degree of vector, Zhao et al. [13] proposed a new generalized method for parallel robot static error modeling based on the principles of parallel robot influence coefficient and virtual displacement, Mei et al. [14] established a mapping model between the complete home position errors and the errors of end-effector based on the closed-loop vector equations of a single limb and proposed a fast calibration approach based on the rotary angle errors of end-effector. In this paper, the inverse kinematics calibration method based on structural parameters and genetic algorithm is used to calibrate the kinematics of 3-PRS parallel machine tool. It does not depend on the continuity of the objective function. It only needs to evaluate the fitness of the objective function without the knowledge of search space and other auxiliary information. This method does not need to select the initial iteration value and complex mathematical derivation. It is universal and easy to implement.

2. Mechanism model and inverse kinematics analysis

2.1. Summary of 3-PRS parallel mechanism

The schematic diagram of the 3-PRS parallel mechanism is shown in Figure 1. The 3-PRS parallel mechanism comprises cutter, fixed platform and moving platform, three horizontal guide rails with 120° distribution $A_iB_i$ ($i = 1, 2, 3$), three vertical slides $P_iD_i$ ($i = 1, 2, 3$) and three identical branched chains $P_iS_i$ ($i = 1, 2, 3$). The cutter is mounted vertically in the center of the moving platform, and each branched chain contains a connecting rod, a prismatic pair (P), a Revolute (R) and a spherical pair (S). The fixed coordinate system $O – XYZ$ is located on the static platform. Axis $OX$ at the point $A_1$, and center $O$ is the midpoint of the $A_1A_2$. The dynamic coordinate system $O^T – xyz$ is located on the moving platform. Center $O^T$ is located at the tip (or shaft end), and axis $O^T x$ at the point $S_1$. Axis $O^T z$ along the axis of the spindle.

![Figure 1. The schematic diagram of the 3-PRS parallel mechanism.](image-url)
$R$ is circumradius of $A_iA_2A_3$, which is located on the inside of the horizontal guide rail. $r$ is circumradius of $S_iS_2S_3$, which is located on the moving platform. $H_i(i=1,2,3)$ is the height of $PD_i(i=1,2,3)$. $\theta_i(i=1,2,3)$ is the included angle between $P_iS_i$ and $P_iD_i$. $h_i$ is the distance between the tip and the center of the moving platform. $R_i(i=1,2,3)$ is the distance between the slider $D_i(i=1,2,3)$ and the center of the pedestal $O'$. 

### 2.2. Inverse kinematics analysis of 3-PRS parallel mechanism

The inverse kinematics analysis of the 3-PRS parallel mechanism is that the position and attitude parameters of the end effector are known, and the position parameters of the vertical sliding block are solved. In the fixed coordinate system $O−XYZ$, the vector displacement of $D_i$ and $P_i$ are as follows:

$$
\begin{align*}
\vec{D}_1 &= [R_1 + \frac{1}{2}R,0,0]^T \\
\vec{D}_2 &= [\frac{1}{2}(R - R_2), \frac{\sqrt{3}}{2}R_2,0]^T \\
\vec{D}_3 &= [\frac{1}{2}(R - R_3), -\frac{\sqrt{3}}{2}R_3,0]^T \\
\vec{P}_1 &= [R_1 + \frac{1}{2}R, 0, H_1]^T \\
\vec{P}_2 &= [\frac{1}{2}(R - R_2), \frac{\sqrt{3}}{2}R_2, H_2]^T \\
\vec{P}_3 &= [\frac{1}{2}(R - R_3), -\frac{\sqrt{3}}{2}R_3, H_3]^T
\end{align*}
$$

(1)

In dynamic coordinate system $O^T−xyz$, the vector displacement of $S_i$ is as follows:

$$
\begin{align*}
\vec{s}_1 &= [r,0,0]^T \\
\vec{s}_2 &= [-\frac{1}{2}r, \frac{\sqrt{3}}{2}r, h]^T \\
\vec{s}_3 &= [-\frac{1}{2}r, -\frac{\sqrt{3}}{2}r, h]^T
\end{align*}
$$

(3)

In dynamic coordinate system $O^T−xyz$, the vector displacement of $O^T$ is as follows:

$$
\vec{O}^T = [x_T, y_T, z_T]^T
$$

(4)

The transition matrix $[T]$ of moving coordinate system with respect to fixed coordinate system is as follows:

$$
[T] = \begin{bmatrix}
k_1 & m_1 & n_1 \\
k_2 & m_2 & n_2 \\
k_3 & m_3 & n_3
\end{bmatrix}
$$

(5)

where, $\alpha, \beta, \gamma$ are the rotation angles which around the X, Y and Z axes in the moving coordinate system. The displacement of spherical hinge $S_i$ can be expressed as follows in the fixed coordinate system $O−XYZ$:

$$
[S_i]_{xyz} = [T][S_i]_{xyz} + O^T, i = 1,2,3
$$

(6)

Equations (3), (4) and (5) into Equation (6), the displacement of spherical hinge $S_i$ in the fixed coordinate system $O−XYZ$ is rewritten:
\[ \vec{S}_i = \begin{bmatrix} \vec{r}_k + h\vec{n} + x_T \\ \vec{r}_2 + h\vec{n} + y_T \\ \vec{r}_3 + h\vec{n} + z_T \end{bmatrix} \] (7)

\[ \vec{S}_i^o = \begin{bmatrix} \vec{r}_k - \sqrt{3}/2 \vec{r}_I + h\vec{n} + x_T \\ \vec{r}_2 - \sqrt{3}/2 \vec{r}_I + h\vec{n} + y_T \\ \vec{r}_3 - \sqrt{3}/2 \vec{r}_I + h\vec{n} + z_T \end{bmatrix} \] (8)

\[ \vec{S}_j = \begin{bmatrix} \vec{r}_k - \sqrt{3}/2 \vec{r}_I + h\vec{n} + x_T \\ \vec{r}_2 - \sqrt{3}/2 \vec{r}_I + h\vec{n} + y_T \\ \vec{r}_3 - \sqrt{3}/2 \vec{r}_I + h\vec{n} + z_T \end{bmatrix} \] (9)

Each connecting rod can move only in the corresponding space plane because of the rotation constraint, and the expression for the constraint space plane is as follows:

\[ \begin{align*}
\Omega_1 : Y &= 0 \\
\Omega_2 : Y &= -\sqrt{3}(X - \frac{1}{2}R) \\
\Omega_3 : Y &= \sqrt{3}(X - \frac{1}{2}R)
\end{align*} \] (10)

Equations (7), (8), (9) into the constraint equation, as follows can be obtained.

\[ \begin{align*}
rk_2 + h\vec{n} + y_T &= 0 \\
-\frac{1}{2}rk_2 + \sqrt{3}/2 rm_2 + h\vec{n} + y_T &= -\sqrt{3}(-\frac{1}{2}rk_1 + rm_1 + h\vec{n} + x - \frac{1}{2}R_z) \\
-\frac{1}{2}rk_2 - \sqrt{3}/2 rm_2 + h\vec{n} + y_T &= \sqrt{3}(-\frac{1}{2}rk_1 + rm_1 + h\vec{n} + x - \frac{1}{2}R_z)
\end{align*} \] (11)

Simplify Equation (10), it would become:

\[ \begin{align*}
\gamma &= f(\alpha, \beta) = -\arctan \left( \frac{\sqrt{3}R_z - \sqrt{3}R_3 + 6\alpha \sin \alpha \sin \beta}{6\beta (\cos \alpha + \cos \beta)} \right) \\
x_T &= \frac{1}{2} (\cos \beta \cos \gamma \sin \alpha \sin \beta \sin \gamma - \cos \alpha \cos \gamma) - h \sin \phi \frac{R_z - \frac{R_3}{4}}{4} \\
y_T &= h \sin \alpha \cos \beta - \cos \alpha \cos \gamma \\
z_T &= \frac{R_3}{2}
\end{align*} \] (12)

Equation (11) indicates that the parameter \( \gamma \) depends on \( \alpha \) and \( \beta \), and \( x_T, y_T \) are functions of \( \alpha, \beta, r, h \) and \( R(i=1,2,3) \). Given \( z_T, \alpha \) and \( \beta \), the position of the spherical hinge \( S_i \) can be obtained by Equations (7), (8), (9) in the fixed coordinate system \( O-XYZ \). The length of the connecting rod is fixed, and the position component of the movable pair on the Z axis can be determined by the following type:

\[ H_i = \sqrt{[l_i - (P_{ix} - S_i x)]^2 - (P_{iy} - S_i y)^2 + S_i z} i = 1,2,3 \] (13)

3. Establishment and calibration of error model

According to the inverse kinematics analysis, the moving components of each branch of the parallel machine tool can be represented as the vector chain shown in Figure 2. The vector form of the link between the cylindrical hinge and the spherical hinge can be expressed as:

\[ L_i = [T] S_i + O_T - P_i, (i = 1,2,3) \] (14)
This equation is the basic equation of kinematics of parallel machine tool. $P_i$ is the vector of the central motion position of the cylindrical hinge in the formula.

Assuming that the elastic deformation and clearance of the mechanism are not taken into account, the errors of each hinge point are caused by the errors of position installation and rod length. In order to make the theoretical value of slide displacement close to or equal to the actual value, the nominal structural parameter $P_i, S_i, l_i (i = 1, 2, 3)$ should be revised, and their corresponding structural parameters are to be $r, h, R_i, R_2, R_3, l_1, l_2, l_3$.

3.1. Structural fitness function
In the process of kinematic calibration, the following fitness function can be constructed:

$$F = \min \sum_{j=1}^{n} \sum_{i=1}^{3} E_{i,j}^2, i = 1, 2, 3$$

(14)

Where, $E_{i,j} = H_{i,j}^c - H_{i,j}^b = H_{i,j}^c - \left( \sqrt{h_i^2 - (P_{i,j,x} - S_{i,y})^2} - (P_{i,j,y} - S_{i,\zeta})^2 + S_{i,\xi} \right), i$ is the three branch numbers of the parallel machine tool, $j$ is the number of times measured for the attitude of the spindle platform, $n$ is the number of different positions, $H_{i,j}^c$ is the actual value (the actual displacement of the measured feed slider), $H_{i,j}^b$ is the calibration value.

So, the fitness function can be expressed as:

$$F = \min \sum_{j=1}^{n} \sum_{i=1}^{3} E_{i,j}^2 = \min \sum_{j=1}^{n} \left( E_{i,1,j}^2 + E_{i,2,j}^2 + E_{i,3,j}^2 \right), i = 1, 2, 3, j = 1, ..., n$$

(15)

The fitness function is the minimum value function optimized by genetic algorithm. The smaller the fitness function, the higher the fitness of the corresponding individual. On the contrary, the lower the fitness function.

3.2. Strategies and steps of kinematics calibration
The process of inverse kinematics calibration of machine tools using genetic algorithm is as follows:

(1) Parameter assignment. Population size $M = 200$, termination optimization algebra $T = 200$, number of variables $v = 8$, crossover probability $P_c = 0.95$, mutation probability $P_m = 0.05$.

(2) Set variables. According to the kinematics analysis of the machine tool, eight variables are set to be the radius of the outer circle of the moving platform $r$, the distance between the tool tip and the center of the platform $h$, the radius of the outer circle of the fixed platform $R_i (i = 1, 2, 3)$, the length of the connecting rod $l_i (i = 1, 2, 3)$, and the range of values of eight variables from [149 349 349 349 819 819 819] to [151 151 351 351 351 821 821 821].
(3) The calculation of fitness value. Within the range of variables obtained in step (2), a random initial population is generated and then calculated in Equation (15).

(4) Selection of operations. A regenerated individual is selected according to the fitness value. The individuals with higher fitness value are preferentially selected and those with lower fitness value are excluded.

(5) Cross operation. The crossover probability \( P_c = 0.95 \). A fixed crossover method is used to generate new individuals.

(6) Mutation operation. The mutation probability \( P_m = 0.05 \), a fixed mutation method is used to generate new individuals.

(7) The fitness values of all individuals in the local optimal solution are obtained by the above steps, and the optimal individuals are preserved.

(8) Set the termination of evolutionary algebra \( T = 100 \), when the evolutionary algebra is less than 100, then jump to step (4), otherwise output the best individual and the best solution, and end the operation.

3.3. Data processing and results
Nominal structural parameters in machine tool controllers are shown in Table 1, i.e., \( r = 150\, \text{mm} \), \( h = 20\, \text{mm} \), \( R_1 = 350\, \text{mm} \), \( R_2 = 350\, \text{mm} \), \( R_3 = 350\, \text{mm} \), \( l_1 = 820\, \text{mm} \), \( l_2 = 820\, \text{mm} \), \( l_3 = 820\, \text{mm} \).

| number | \( s_x \) | \( s_y \) | \( s_z \) | \( P_c \) | \( P_m \) | \( l_i \) |
|--------|----------|----------|----------|----------|----------|----------|
| 1      | 325.00   | 0.00     | -130     | 525.00   | 0.00     | 820.00   |
| 2      | 100.00   | 129.904  | -130     | 0.00     | 303.109  | 820.00   |
| 3      | 100.00   | -129.904 | -130     | 0.00     | -303.109 | 820.00   |

Table 1. Nominal parameters of parallel machine tool structure (unit: mm).

| data number | Moving Platform Direction(\( \alpha \), \( \beta \), \( \gamma \)) | Location of moving platform (\( x_T \), \( y_T \), \( z_T \)) | Location of sliding seat (\( H_i^1 \), \( H_i^2 \), \( H_i^3 \)) |
|-------------|---------------------------------|-------------------|-------------------|
| 1           | 1.0096, -0.8891, -2.5759        | 210.7558, -29.7855, -164.9740 | 446.6967, 465.6980, 699.1149 |
| 2           | 1.0197, -1.0368, -2.9878        | 201.7233, -87.9964, -145.3661 | 475.6233, 379.1517, 753.8037 |
| 3           | 0.8230, -0.9864, -3.7620        | 172.8483, -125.8431, -156.2109 | 546.5950, 347.9791, 756.6492 |
| 4           | 0.8927, -0.8863, -4.2369        | 150.0954, -115.2423, -146.2131 | 624.2652, 395.0579, 708.0127 |
| 5           | 0.8579, -0.6530, -4.8230        | 154.0516, -77.8823, -159.3254 | 711.5471, 491.6801, 619.5769 |
| 6           | 0.7535, -0.7133, -5.1449        | 219.4207, 137.8137, -165.6462 | 571.7235, 749.6493, 436.4422 |
| 7           | 0.7536, -0.6477, -5.4662        | 167.9936, -26.4509, -163.7833 | 749.6940, 599.0657, 567.0780 |
| 8           | -0.9138, -1.0444, -6.5459       | 171.1122, -83.3429, -156.3960 | 727.2474, 465.3826, 692.7828 |
| 9           | 1.0340, 0.8529, -5.9486         | 186.2497, -105.5877, -168.7248 | 631.4234, 662.8321, 534.9616 |
| 10          | -0.7990, -0.9132, -5.3464       | 221.2757, -143.4413, -168.7516 | 596.8728, 395.5739, 766.3084 |
| 11          | -0.9990, -0.8774, -5.0297       | 238.7600, -118.1360, -165.3164 | 530.2253, 440.5661, 734.0920 |
| 12          | -0.9703, -0.9758, -4.5727       | 242.3548, -78.9121, -145.0624 | 482.7788, 502.6812, 669.2598 |
| 13          | -0.9662, -1.0095, -4.0457       | 234.6553, -11.1632, -137.9863 | 452.8080, 620.3581, 551.0329 |
| 14          | -1.0329, -0.8911, -3.4471       | 197.3096, 61.6392, -156.5555 | 456.9459, 737.3390, 411.7630 |
| 15          | -0.8904, -0.9394, -2.8631       | 181.0056, 107.2611, -146.8561 | 504.8242, 758.8146, 356.5914 |
| 16          | -0.9177, -0.8106, -2.3303       | 153.9830, 114.5957, -161.7873 | 580.3521, 726.0617, 372.4234 |
| 17          | -1.0440, -0.7976, -1.9704       | 140.8599, 93.5018, -146.3689 | 647.9224, 765.5908, 426.6989 |
| 18          | -1.0116, -0.6991, -1.4176       | 150.1273, 53.1810, -163.9769 | 723.2654, 596.7166, 517.9724 |
| 19          | -0.8486, -0.7059, -0.9506       | 162.6250, 26.8278, -155.2768 | 748.5891, 567.0485, 584.0438 |
| 20          | -0.9886, -0.5310, -0.5820       | 187.2655, -22.0628, -168.0192 | 747.3927, 503.5236, 670.7678 |

In the workspace of a parallel machine tool, 20 terminal actuator positions are randomly selected, and the corresponding input parameters are shown in Table 2. In order to fully excite the kinematics parameters of parallel machine tools and facilitate the actual calibration, the randomly selected
position of the end effector should be all over the workspace. The parameters of the actual parallel machine tool are calibrated by genetic algorithm. The results of calibration by MATLAB are shown in Table 3. The errors of calibrated structural parameters relative to nominal parameters are shown in Table 4. The optimum fitness values and variable values are shown in Figure 3.

Table 3. Calibration results of machine tool structural parameters (unit: mm).

| number | $s_x$  | $s_y$  | $s_z$  | $p_x$  | $p_y$  | $p_z$  |
|--------|--------|--------|--------|--------|--------|--------|
| 1      | 324.928| 0.00   | -130.112| 524.920| 0.00   | 820.026|
| 2      | 100.018| 129.864| -130.112| 0.00   | 303.101| 820.050|
| 3      | 100.074| -129.864| -130.112| 0.00   | -303.196| 820.065|

Table 4. Errors of machine tool structural parameters (unit: mm).

| number | $\Delta s_x$ | $\Delta s_y$ | $\Delta s_z$ | $\Delta p_x$ | $\Delta p_y$ | $\Delta p_z$ |
|--------|-------------|-------------|-------------|-------------|-------------|-------------|
| 1      | 0.072       | 0.00        | 0.112       | 0.080       | 0.000       | -0.026      |
| 2      | -0.018      | 0.040       | 0.112       | 0.000       | 0.008       | -0.050      |
| 3      | -0.074      | -0.040      | 0.112       | 0.000       | 0.087       | -0.065      |

As can be seen from Table 4, there are some corresponding relationships among the three branch errors, which is consistent with the hypothesis that the position of three cylindrical or spherical hinges is an ideal equilateral triangle. The position error of cylindrical hinge is larger than that of connecting rod length error and spherical hinge error, which is related to the installation position error of slide, the accumulation error of cylindrical hinge and sliding seat.

As can be seen from Figure 3, the best fitness value is 0.00960358, and the average fitness value is 0.630292. The values of eight parameter variables are 149.954, 19.888, 349.946, 349.990, 350.101, 820.016, 820.050, 820.065. By substituting these six parameters into the inverse kinematics equation, the average displacement of the slider can be calculated, $S_{1,mean} = 597.8396mm$, $S_{2,mean} = 554.0385mm$, $S_{3,mean} = 591.4688mm$.

Figure 3. The diagram of optimal fitness variation and variable value.

3.4. Verification of calibration results

By substituting nominal structural parameters into inverse kinematics equation, slide displacement $S_{i,j}^m$ can be obtained, and then $E_{i,j1} = S_{i,j}^c - S_{i,j}^m$ can be calculated. By substituting the calibrated structural parameters into the inverse kinematics equation, slide displacement $S_{i,j}^b$ can be obtained, and then $E_{i,j2} = S_{i,j}^c - S_{i,j}^b$ can be calculated. Finally, the errors of $E_{i,j1}$ and $E_{i,j2}$ are calculated. The results are shown in Table 5, including average, maximum and standard deviation.
Table 5. Statistical comparison of inverse kinematics errors.

|              | $E_1$ (mm) | $E_2$ (mm) |
|--------------|------------|------------|
| $E_{\min}$  | 0.0056     | 0.0025     |
| $E_{\max}$  | 0.0050     | 0.0022     |
| $S_{E_1}$   | 0.00039    | 0.00045    |

From Table 5, it can be seen that the reverse kinematics error obtained by nominal structural parameters is larger than that obtained by calibrated structural parameters.

4. Conclusions

In this paper, an inverse kinematics calibration method based on structural parameters and genetic algorithm is proposed, and the error calibration model of parallel machine tool is derived. The calculation shows that the reverse motion error calculated by nominal machine tool structural parameters is larger than that obtained by calibrated structural parameters. The position of the three cylindrical and spherical hinges after calibration is closer to the nominal position. It shows that the error calibration can correct the machine tool structure and guide the subsequent installation and debugging of the machine tool.

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