Spectroscopic Study of Strangeness=-3 Ω− Baryon

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Abstract: Ω− baryon with sss quarks has been scarcely observed in the experiments so far and has been studied through many theoretical studies only. Here, an attempt has been made to explore properties of Ω with hypercentral Constituent Quark Model (hCQM) with a linear confining term. The resonance mass spectra has been obtained for 1S-4S, 1P-4P, 1D-3D and 1F-2F respectively. The Regge trajectory has been investigated for the linear nature based on calculated data alongwith the magnetic moment. The present work has been compared with various approaches and known experimental findings.

Keywords: Mass spectra, Strange baryon, Regge trajectory, Magnetic moment

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1 Introduction

The discovery of Ω baryon dates back to 1964 and yet till today it is the least observed in the experiments worldwide. Ω− holds a place in decuplet family with isospin I=0 and strangeness S=-3 with sss quarks. The search for missing resonances is the prime aim of the hadron spectroscopy so as to understand the internal dynamics of the quarks inside the system ranging from light hadrons to heavy as well as exotic hadrons as depicted by few recent reviews[2, 3]. The motivation behind the current study is to exploit all resonance mass with possible spin-parity assignment. This is the extension of the previous study for non-strange [4, 5] and strange baryons with S=-1,-2 [6].

Ω baryon belongs purely to the decuplet representation in the similar manner as Δ, while Σ and Ξ can in principle be realized as a mixture of octet and decuplet states. In terms of multi-strangeness, Ω baryon is in a similar position like Ξ as both are not easily observed in experiments and not more information has been readily available since bubble chamber data. A recent study of strong decays in constituent quark model with relativistic corrections, it is highlighted that unlike other light baryons, Ω with only strange quarks may be a tool to reach the valence quarks in the baryons when other kaon cloud effects are somehow excluded [7]. Pervin et al. [8] has vividly described that multi-strange baryons are produced only as a part of final state and also with very small production cross-section, making the analysis more complicated to study them. Recent studies at Belle experiments have provided with some results as Ω(2012) through e+e− annihilations and into Ξ−K− as well as Ξ−K0 decay channels [9]. Earlier, BaBar collaboration attempted to study the spin of Ω−(1672) for J=3/2 through processes like Ω0 → Ω−K+ and Ξ0 → Ω−π+ [10]. All these observations pose a challenge towards the underlying mystery of multi strange baryons especially for S=-3. The upcoming experimental facility at FAIR, PANDA-GSI is expected to perform a dedicated study of hyperons especially at low energy regime [11] as well as a part of BESIII experiment shall be including the strange quark systems [12] and J-PARC facility [13].

The three star state Ω(2012) has been a puzzling one appearing in table 1 as the second known state. The discovery of Ω(2012) by Belle collaboration sparked a lot of theoretical work on the issue, with pictures inspired by quark models as well as molecular pictures based on the meson-baryon interaction. In various quark models, the masses of the first orbital excitations of states were deemed to Ω(2012). A recent study has proposed this state to be a molecular one. This state is slightly below Ξ(1530)K threshold so that the binding mechanism could be a coupled channel dynamics [15]. Its characteristic signature could be a three body channel ΞKπ. There are other studies which signify that present information is not enough to consider it as a molecular state [16] as well as some other disapproving the proposed state [14]. Also, a study has been revisited to check the compatibility of molecular picture of 2012 within the coupled channel unitary approach [18]. Xiao
et al. has studied the strong decays within chiral quark model to understand the structure of \( \Omega(2012) \) state [19]. There are several models to study the \( \Omega \) baryon properties through theoretical and phenomenological such as quark pair-creation [20], QCD Sum rule [21], Glozman-Riska model [22], algebraic model by Bijker [23] and large-\( N_c \) analysis [24, 25]. Recently A. Arifi et al. has studied the strong decays within chiral quark due to gluonic strings and the potential increases linearly at the same time. The quarks are pictured to be connected by gluonic strings and the potential increases linearly with the radius \( x \). The reduced masses with Jacobi coordinates are given by

\[
\rho = \frac{1}{\sqrt{2}}(r_1 - r_2) \quad (1a)
\]

\[
\lambda = \frac{(m_1 r_1 + m_2 r_2 - (m_1 + m_2) R_3)}{\sqrt{m_1^2 + m_2^2 + (m_1 + m_2)^2}} \quad (1b)
\]

The hyperradius \( x \) is a one-dimensional coordinate that encloses the effects of the three-body interaction at the same time. The quarks are pictured to be connected by gluonic strings and the potential increases linearly with the radius \( x \). The reduced masses with Jacobi coordinates \( \rho \) and \( \lambda \) given by

\[
m_\rho = \frac{2m_1 m_2}{m_1 + m_2}; \quad m_\lambda = \frac{2m_3(m_1^2 + m_2^2 + m_1 m_2)}{(m_1 + m_2)(m_1 + m_2 + m_3)} \quad (3)
\]

The kinetic energy operator in the center of mass frame is written as

\[
-h^2 \frac{m}{2m} (\Delta + \Delta) = \frac{h^2}{2m} \left( \frac{\partial^2}{\partial x^2} + \frac{5}{x} \frac{\partial}{\partial x} + \frac{L^2(\Omega)}{x^2} \right) \quad (4)
\]

Here, \( L^2(\Omega) = L^2(\Omega_\rho, \Omega_\lambda, \xi) \) is the quadratic Casimir operator for the six-dimensional rotational group whose eigenfunctions are hyperspherical harmonics satisfying

\[
L^2(\Omega_\rho, \Omega_\lambda, \xi) Y_{\gamma l \rho \gamma l \lambda}(\Omega_\rho, \Omega_\lambda, \xi) = -\gamma(\gamma + 4)Y_{\gamma l \rho \gamma l \lambda}(\Omega_\rho, \Omega_\lambda, \xi) \quad (5)
\]

The hyper-radial part of the wave-function as determined by hypercentral Schrodinger equation is

\[
\left[ \frac{d^2}{dx^2} + \frac{5}{x} \frac{d}{dx} + \frac{\gamma(\gamma + 4)}{x^2} \right] \psi(x) = -2m[E - V(x)]\psi(x) \quad (6)
\]

Here \( l(l + 1) \rightarrow \frac{15}{4} + \gamma(\gamma + 4) \). The hyperradial wavefunction \( \psi(x) \) is completely symmetric for exchange of the quark coordinates using the orthogonal basis [34].

The expansion of quark interaction term as

\[
\sum_{i<j} V(r_{ij}) = V(x) + \ldots \quad (7)
\]

The present study is based on the hypercentral Constituent Quark Model (hCQM), a non-relativistic approach [32, 33]. The baryons are composed of three quarks confined within and interacting by a potential which here is considered to be hypercentral. The hyperspherical coordinates are given by the angles \( \Omega_\rho, \Omega_\lambda \) along with hyperradius \( x \) and hyperangle \( \xi \) which are written in terms of Jacobi coordinates as in figure [1].

### Table 1: PDG \( \Omega \) baryon [1]

| State      | \( J^P \) | Status |
|------------|----------|--------|
| \( \Omega(1672) \) | \( \frac{3}{2}^+ \) | **** |
| \( \Omega(2012) \) | ?^- | *** |
| \( \Omega(2250) \) | | *** |
| \( \Omega(2380) \) | | ** |
| \( \Omega(2470) \) | | ** |

2 Theoretical Background

The present study is based on the hypercentral Constituent Quark Model (hCQM), a non-relativistic approach [32, 33]. The baryons are composed of three quarks confined within and interacting by a potential which here is considered to be hypercentral. The hyperspherical coordinates are given by the angles \( \Omega_\rho, \Omega_\lambda \) along with hyperradius \( x \) and hyperangle \( \xi \) which are written in terms of Jacobi coordinates as in figure [1].
The potential with first term itself gives the hyper-central approximation, which has three-body character as not a single pair of coordinates can be disentangled from the third one. The Hamiltonian of the system is written with potential term solely depending on hyper-radius \( x \) of three body systems.

\[
H = \frac{p_i^2}{2m} + V^0(x) + V_{SD}(x)
\]

(8)

where \( m = \frac{2m_qm_\lambda}{m_q+m_\lambda} \) is the reduced mass. The potential is solely hyperradius dependent. So, it consists of a Coulomb-like term and a linear term acting as confining part.

\[
V^0(x) = -\frac{\tau}{x} + \alpha x
\]

(9)

Here, \( \tau = \frac{2}{3} \alpha_s \) with \( \alpha_s \) being running coupling constant.

\[
\alpha_s = \frac{\alpha_s(\mu_0)}{1 - \frac{31-2n_f}{12\pi} \alpha_s(\mu_0) \ln \left( \frac{m_1+m_2+m_3}{\mu_0} \right)}
\]

(10)

Here, \( \alpha_s \) is 0.6 at \( \mu_0 = 1 \text{GeV} \) and \( n_f \) is the number of active quark flavors whose value here is 3. \( \alpha \) is string tension of the confinement part of the potential. Also, \( \alpha \) is state dependent and is obtained by fixing the value using experimental ground state mass of the baryon \([38, 39]\). The constituent quark mass is taken to be \( m_s = 0.500 \) GeV.

If considering the chiral quark model, the low-energy regime shall be well established as the spontaneously broken SU(3) chiral symmetry scale is different from that of confinement scale of QCD. For three body higher excited states, the relative position of positive and negative parity states can be fixed by the interplay of relativistic kinematics and pion exchange interaction, playing the role of one-gluon exchange potential. Thus, the higher terms in Goldstone exchange will allow us to incorporate the hyperfine, and spin-singlet and triplet splitting \([40, 12]\).

The \( V_{SD}(x) \) is added for incorporating spin-dependent contributions through \( V_{SS}(x) \), \( V_{S\lambda}(x) \) and \( V_T(x) \) as spin-spin, spin-orbit and tensor terms respectively. These interactions arise due to \( \frac{g_s^2}{\rho} \) effects in non-relativistic expansion and by the standard Breit-Fermi expansion as described by Voloshin \([13]\).

\[
V_{SD}(x) = V_{SS}(x)(S_x \cdot S_\lambda) + V_{S\lambda}(x)(\gamma \cdot S) + V_T(S^2 - \frac{3(S \cdot x)(S \cdot x)}{x^2})
\]

(11)

\[
V_{SS}(x) = \frac{1}{3 m_q m_\lambda} \nabla^2 V_V
\]

(12)

\[
V_{S\lambda}(x) = \frac{1}{2 m_q m_\lambda x} \left( 3 \frac{dV_V}{dx} - \frac{dV_\lambda}{dx} \right)
\]

(13)

\[
V_T(x) = \frac{1}{6 m_q m_\lambda} \left( 3 \frac{d^2 V_V}{dx^2} - \frac{1}{x} \frac{dV_V}{dx} \right)
\]

(14)

where \( V_V = \frac{\tau}{x} \) and \( V_\lambda = \alpha x \) are the vector and scalar part of potential. However, in place of spin-spin interaction presented by delta function, we have employed a smear function of the form, details of which can be found in previous works \([33, 44, 55]\). Also, \( S = S_x + S_\lambda \) where \( S_x \) and \( S_\lambda \) are the spin vector associated with the \( \rho \) and \( \lambda \) variables.

\[
V_{SS}(x) = -\frac{A}{6 m_q m_\lambda x x_0^2} \frac{e^{\frac{-x}{x_0}}}{x_0}
\]

(15)

Here, \( x_0 \) being the hyperfine parameter, with value \( x_0 = 1 \) and \( A \) is a state dependent parameter consisting of an arbitrary constant. The form of \( A \) is chosen as \( A = A_0/\sqrt{n+l+\frac{1}{2}} \), wherein the value of \( A_0 = 28 \) for determining the ground state value \((1S_{1/2}^1) 1672 \text{ MeV}\) as well as other radially excited states. Similarly, the other parameters are determined for obtaining the experimentally known ground state mass, i.e. 1672 MeV in the case of \( \Omega \). In addition to this, masses with first order correction as \( \frac{1}{m} V^1(x) \) are taken into account through

\[
V^1(x) = -C_F C_A \frac{\alpha_s^2}{4x^2}
\]

(16)
Table 2: Ground state model parameters

| $m_0$(GeV) | $\alpha_s$ | $\alpha$(GeV²) |
|------------|------------|----------------|
| 0.500      | 0.5109     | 0.0129         |

where $C_F = \frac{2}{3}$ and $C_A = 3$ are Casimir elements of fundamental and adjoint representation.

Numerical solutions of Schrodinger equation has been obtained through Mathematica notebook [43].

3 Results and Discussion for the Resonance Mass Spectra

In the present work, 1S-4S, 1P-4P, 1D-3D and 1F-2F states have been obtained for $S = \frac{1}{2}$ and $S = \frac{3}{2}$ spin configurations with all possible $J^P$ values in tables [3] to [6]. Also, $Mass_{cal1}$ and $Mass_{cal2}$ signify the resonance masses for without and with first order correction term. Tables [7] and [8] give a comparison of obtained results with various approaches for positive and negative parity states. The ground state $\Omega(1672)$ is nearly same for many the approaches with a variation of 20-30 MeV in few cases depending on the approach.

Faustov et al. [46] has employed a relativistic quark model approach considering quark-diquark system. The lower excited states are very much in accordance but for higher excitations exact comparison is not possible. In ref. [47] has studied the spectrum through hyperfine interactions due to two-gluon exchange. For the available states, the present results are very near within 50 MeV compared to theirs. Another non-relativistic constituent quark model approach has been utilized by [48]. Y. Oh [49] has investigated with $\Omega$ spectrum using Skyrme model. [50] and [51] have exploited quark model based on chromodynamics and some of the present states are also in accordance. E. Klempt [52] has reproduced the few known states through a new baryon mass formula. U. L"orng et al. has studied the whole light spectrum within relativistic covariant quark model based on Bethe-Salpeter equation [53]. The BGR collaboration [54] results are based on chirally improved (CI) quarks. For higher $J^P$ values, not many approaches are available for comparison.

One puzzling question remains with $\Omega(12012)$ state, however our results donot exactly reproduce but vary by 30 MeV. Also, this study is not able to precisely comment on the proposed molecular nature of this state. So, the future experimental results would serve as a key towards its understanding.

The results described in tables [3] to [6] have been summarized in the increasing order for each $J^P$ value including positive parity in table [7] and negative parity in table [8]. All the mentioned models appearing in the table for comparison are not sufficient to segregate each state based on $J^P$ value. For the ground state $\Omega(1672)$ with $J^P = \frac{3}{2}^+$ is near to [46], [47], [48] and [51]. The first state with $J^P = \frac{1}{2}^+$ is nearly comparable to the relativistic approach by Faustov et al. however, due to limited data obtained by various compared models, exact state-wise comparison is not possible. Thus, this study is expected to provide a possible range of masses for upcoming experiments which shall identify the existence of a particular state.

4 Regge Trajectories

Regge trajectories have been of importance in spectroscopic studies. The plot of total angular momentum $J$ and principal quantum number $n$ against the square of resonance mass $M^2$ are drawn to obtain the non-intersecting and linearly fitted lines. Figure 2 shows a linear behaviour with almost all the points following the trend for $n - M^2$. Figures 3 and 4 are plotted with few natural and unnatural parity states for available results. These plots point that the spin-parity assignment of a given state in present calculation could possibly be correct. The linear fitting parameters are mentioned in the
Table 3: Resonance masses of S-state 1S-4S for without and with first order correction to the potential (in MeV)

| State | \( J^p \) | \( \text{Mass}_{\text{cal}1} \) | \( \text{Mass}_{\text{cal}2} \) |
|-------|-----------|----------------|----------------|
| 1S    | \( \frac{3}{2}^+ \) | 2057           | 2068           |
| 2S    | \( \frac{3}{2}^+ \) | 2076           | 2086           |
| 3S    | \( \frac{3}{2}^+ \) | 2101           | 2111           |
| 4S    | \( \frac{3}{2}^+ \) | 2126           | 2136           |

Table 4: Resonance masses of P-state 1P-4P for without and with first order correction to the potential (in MeV)

| State | \( J^p \) | \( \text{Mass}_{\text{cal}1} \) | \( \text{Mass}_{\text{cal}2} \) |
|-------|-----------|----------------|----------------|
| 1^2P_{1/2} | \( \frac{1}{2}^- \) | 1987           | 1996           |
| 1^2P_{3/2} | \( \frac{3}{2}^- \) | 1978           | 1985           |
| 1^4P_{1/2} | \( \frac{1}{2}^- \) | 1902           | 1909           |
| 1^4P_{3/2} | \( \frac{1}{2}^- \) | 1893           | 1899           |

Table 5: Resonance masses of D-state 1D-3D for without and with first order correction to the potential (in MeV)

| State | \( J^p \) | \( \text{Mass}_{\text{cal}1} \) | \( \text{Mass}_{\text{cal}2} \) |
|-------|-----------|----------------|----------------|
| 1^4D_{3/2} | \( \frac{1}{2}^+ \) | 2269           | 2288           |
| 1^2D_{5/2} | \( \frac{3}{2}^+ \) | 2250           | 2267           |
| 1^4D_{1/2} | \( \frac{1}{2}^+ \) | 2291           | 2311           |
| 1^4D_{3/2} | \( \frac{1}{2}^+ \) | 2276           | 2295           |

Table 6: Resonance masses of F-state 1F-2F for without and with first order correction to the potential (in MeV)

| State | \( J^p \) | \( \text{Mass}_{\text{cal}1} \) | \( \text{Mass}_{\text{cal}2} \) |
|-------|-----------|----------------|----------------|
| 2^2F_{3/2} | \( \frac{3}{2}^- \) | 2671           | 2703           |
| 2^2F_{5/2} | \( \frac{5}{2}^- \) | 2646           | 2676           |
| 2^4F_{1/2} | \( \frac{1}{2}^+ \) | 2699           | 2733           |
| 2^4F_{3/2} | \( \frac{1}{2}^+ \) | 2681           | 2713           |

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Table 7: Comparison of present masses with other approaches based on $J^P$ value with positive parity described in the increasing order for all possible spin-parity assignment (in MeV)

| $J^P$ | Mass$_{cal}1$ | Mass$_{cal}2$ | 46 | 47 | 48 | 49 | 50 | 51 | 53 | 54 |
|-------|---------------|---------------|-----|-----|-----|-----|-----|-----|-----|-----|
| $\frac{1}{2}^+$ | 2291          | 2311          | 2301 | 2182 | 2232 | 2175 | 2140 | 2220 | 2190 | 2322 | 2350(63) |
|       | 2699          | 2733          | 2202 | 2191 | 2255 | 2210 | 2256 | 2481(51) |
|       | 3157          | 3201          | 2276 | 2332 | 2263 | 2245 | 2345 | 2265 | 2287 |
| $\frac{3}{2}^+$ | 1672          | 1672          | 1678 | 1673 | 1672 | 1656 | 1694 | 1635 | 1675 | 1642(17) |
|       | 2057          | 2068          | 2173 | 2078 | 2159 | 2170 | 2282 | 2165 | 2065 | 2177 | 2470(49) |
|       | 2269          | 2288          | 2304 | 2208 | 2188 | 2182 | 2280 | 2215 | 2236 |
|       | 2276          | 2295          | 2332 | 2263 | 2245 | 2345 | 2265 | 2287 |
|       | 2429          | 2449          | 2671 | 2703 | 2681 | 2713 | 2852 | 2885 | 3122 | 3166 |
|       | 3134          | 3178          |      |      |      |      |      |      |      |      |
| $\frac{5}{2}^+$ | 2250          | 2267          | 2401 | 2224 | 2303 | 2178 | 2280 | 2225 | 2253 |
|       | 2257          | 2275          | 2260 | 2252 | 2210 | 2345 | 2265 | 2312 |
|       | 2646          | 2676          | 2656 | 2686 | 3092 | 3135 | 3102 | 3146 |
|       | 3102          | 3146          |      |      |      |      |      |      |      |
| $\frac{7}{2}^+$ | 2233          | 2249          | 2369 | 2205 | 2321 | 2183 | 2295 | 2210 | 2292 |
|       | 2623          | 2652          | 3065 | 3107 |      |      |      |      |      |

Fig. 2: Regge trajectory $n \rightarrow M^2$ for S, P, D and F state masses and linearly fitted.
Table 8: Comparison of present masses with other approaches based on $J^P$ value with negative parity described in the increasing order for all possible spin-parity assignment (in MeV)

| $J^P$ | $Mass_{cal1}$ | $Mass_{cal2}$ | [46] | [47] | [48] | [5] | [6] | [7] | [8] | [9] | [10] |
|-------|---------------|---------------|------|------|------|-----|-----|-----|-----|-----|-----|
| $\frac{1}{2}^-$ | 1987 | 1996 | 1941 | 2015 | 1957 | 1923 | 1837 | 1950 | 2020 | 1992 | 1944(56) |
|       | 1983 | 2001 | 2463 | 2410 | 2456 | 2716(118) |
|       | 2345 | 2363 | 2580 | 2490 | 2498 |
|       | 2352 | 2370 | 2550 |
|       | 2758 | 2788 |
|       | 2767 | 2797 |
|       | 3218 | 3264 |
|       | 3229 | 3276 |
| $\frac{3}{2}^-$ | 1978 | 1985 | 2038 | 2012 | 1953 | 1978 | 2000 | 2020 | 1976 | 2049(32) |
|       | 1983 | 1991 | 2537 | 2604 | 2440 | 2446 | 2755(67) |
|       | 2332 | 2349 | 2636 | 2495 | 2507 |
|       | 2339 | 2356 |
|       | 2622 | 2653 |
|       | 2740 | 2770 |
|       | 2749 | 2779 |
|       | 3072 | 3115 |
|       | 3196 | 3240 |
|       | 3207 | 3252 |
| $\frac{5}{2}^-$ | 1970 | 1997 | 2653 | 2490 | 2528 |
|       | 2321 | 2338 |
|       | 2585 | 2614 |
|       | 2595 | 2625 |
|       | 2726 | 2755 |
|       | 3027 | 3069 |
|       | 3039 | 3081 |
|       | 3178 | 3221 |
| $\frac{7}{2}^-$ | 2562 | 2590 | 2599 | 2531 |
|       | 2998 | 3040 |
| $\frac{9}{2}^-$ | 2521 | 2548 | 2649 | 2606 |
|       | 2949 | 2999 |

Table 9: Regge slopes and intercepts for $(n,M^2)$

| Trajectory | $b$ | $b_0$ |
|------------|-----|------|
| S          | 1.76838 ± 0.12837 | 0.84426 ± 0.35156 |
| P          | 2.07004 ± 0.1839  | 1.52458 ± 0.50364 |
| D          | 2.20397 ± 0.17905 | 2.67895 ± 0.3868  |
| F          | 2.34116            | 4.01428           |

Table 10: Comparison of ground state magnetic moment (in $\mu_N$)

|                | Present | Exp  | [56] | [56] | [57] | [58] | [59] | [60] | [61] | [62] |
|----------------|---------|------|------|------|------|------|------|------|------|------|
|                | -1.68   | -2.02| -1.67| -1.90| -2.06| -1.95| -1.61| -2.08| -2.01| -1.84|
Fig. 3: Regge trajectory $J \rightarrow M^2$ for natural parity

![Graph showing the Regge trajectory for natural parity with two linear fits: $y = 1.8319x + 4.5176$ and $y = 1.367x + 3.209$.](image)

Fig. 4: Regge trajectory $J \rightarrow M^2$ for unnatural parity

![Graph showing the Regge trajectory for unnatural parity with two linear fits: $y = 1.4889x + 1.832$ and $y = 1.1785x + 0.9691$.](image)
It is noteworthy that the current findings could not comment on the debated state of Ω(2012) for molecular structure, however the mass varies within 30 MeV with $J^P = \frac{3}{2}^-$ which may be identified as a negative parity state of 1P family. As the $J^P$ value for any other state is not experimentally known, exact comparison still depends upon more future findings. However, the probable spin-parity assignment according to the obtained value can be given. Thus, Ω(2250) could be 1D2+ state; Ω(2380) be 2P1−, and Ω(2470) may be associated with 3S with $\frac{3}{2}^+$. The magnetic moment differs by 0.5μB from PDG and other results. The Regge trajectories show the linear nature giving the hint for spin-parity assignments could possibly be correct. However, the validation of any of the results depends on the future experimental facilities to exclusively study the strange baryon properties especially by PANDA at FAIR-GSI [11] and BESIII [12].

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