A Matched-Filter-Based Coherent Integration Method for Passive Bistatic Radar Using Frequency-Agile Radar as the Illuminator

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Abstract. Frequency agile radar (FAR) has been widely used due to its good ability in anti-jamming and low probability of interception, which will benefit passive bistatic radar (PBR) using FAR as the illuminator. However, the traditional moving target detection (MTD) method cannot be utilized by FAR due to the random phase fluctuation of its echo signals. To solve this problem, we propose a modified MTD method of PBR for target detection using FAR as the illuminator. This method first removes the initial carrier frequency of echo signals. Then a bank of matched filters are constructed according to the echo signals with the initial carrier frequency removed. After being matched filtered, the phase fluctuation caused by frequency agility is eliminated. Consequently, coherent integration can be achieved by MTD processing under certain criteria. Furthermore, the factors that affect the performance of the proposed method are analyzed. Finally, numerical simulations are performed to verify the effectiveness and validity of the proposed method in target detection.

1. Introduction
Owing to better detection capabilities, less costs and lower probability of being detected compared with active radars, passive bistatic radar (PBR) has attracted increasing attentions and become the subject of intensive investigation among nations and related organizations. Generally, most of illuminators focus on FM/TV broadcasting, cellular signals, GPS signals, etc., due to the excellent coverage and high transmitter power[1-4]. However, PBR using dedicated pulse radar system as the illuminator such as frequency-agile radar (FAR) is rare. Different from traditional radar with a constant carrier frequency, FAR transmits a sequence of pulses in which each pulse is arbitrarily assigned frequency covering a bandwidth. The frequency agility of FAR provides the advantages of enhancing the ability in anti-jamming, reducing the possibility of interception and suppressing the sea clutter[5-6]. Yet at the same time, the random phase fluctuation introduced by frequency agility will destroy the consistency between pulse-to-pulse radar echo and cause difficulties when it comes to coherent processing of target detection.

As for the target detection, the early studies are mainly focused on incoherent integration. For example, the Hough transform (HT) and Radon transform (RT) based methods proposed by Carlson et al. [7-10] are the typical methods. However, the incoherent method has integration loss compared with the coherent method. Consequently, some researchers proposed the target detection method based on coherent integration. Moving target detection (MTD) and the Radon Fourier transform (RFT) are two
typical coherent integration methods, but they require a constant carrier frequency of echo signals and thus cannot be directly employed by PBR with FAR as the illuminator [11-14]. Recently, some coherent integration methods of FAR for target detection have been proposed [15,16]. These methods can solve the problem of the phase fluctuation due to frequency agility by compensation. Nevertheless, the compensation is based on joint searching along range and velocity direction, which will consume lots of computational burden without the prior knowledge of the motion parameters. Furthermore, C. Chen et al. [17, 18] proposed to avoid the compensation of the phase fluctuation by transmitting signals with parameters deliberately set. However, this kind of method requires special design of radar parameters and hardware, which will increase the cost. D. Zhao et al. [19] introduced a new Doppler processor based on nonuniform fast Fourier transform (NUFFT) into the coherent processing of frequency agility waveform. However, this method suffers from relatively high grating lobes and needs to design the mismatch weights to suppress grating lobes, which will complicate the signal processing.

In this paper, we model the target echo, outline the difficulties associated with the processing of frequency agility and then propose a matched-filter-based coherent integration method of PBR using FAR as the illuminator for target detection. This method can eliminate the phase fluctuation caused by frequency agility and achieve coherent integration without the prior knowledge of the motion parameters. In the numerical simulations, the proposed method is compared with MTD and frequency agile coherent Radon transform (FA-CRT) [15]. Simulation results verify the effectiveness of the proposed method and demonstrate its detection probability in the presence of noise.

2. Signal Model

In this work, a PBR system using the fully coherent FAR as the illuminator is considered. Irrespective of the height of the point target, Figure 1. shows the geometry relationship of the PBR system, where $R_{0T}, R_{TR}$ and $L$ denote the transmitter-to-target range, the target-to-receiver range and the baseline range between the transmitter and receiver, respectively.

![Figure 1. Geometry relationship of PBR.](image)

The transmitted signal of FAR is a sequence of $N$ pulses and the frequencies of the pulses are modulated by a random sequence $b_n$, which can be formulated as

$$x(t) = \sum_{n=0}^{N-1} u(t-nT_p) \exp[-j2\pi f_n(t-nT_p)]$$  \hspace{1cm} (1)$$

where $u(t) = \text{rect}(\frac{t}{T_p}) \exp(j\pi \frac{B}{T_p} t^2)$ is the baseband waveform of the chirp, $T_p$ is the pulse duration, $B$ denotes the bandwidth of the chirp, and $T_r$ stands for the pulse repetition interval (PRI). Furthermore, $f_n = f_0 + b_n f_s$ represents the transmitted frequency of the $n^{th}$ pulse, which is randomly spaced among the working band. Meanwhile, $f_0$ and $f_s$ represent the initial carrier frequency and the minimum frequency step between two transmitted frequencies, respectively.

According to the stop and hop approximation, the received echo signal backscattered by the point target with a constant velocity of $v$ can be expressed as
\[ r(t) = \sum_{n=0}^{N-1} \sigma_u(t-nT_r - R_b(n)/c) \exp\left( -j2\pi f_c(t-nT_r - R_b(n)/c) \right) \]  

(2)

where \( R_b(n) \) is the bistatic range, \( c \) means the speed of the radio and \( \sigma \) denotes the echo amplitude. According to Figure 1, the bistatic range can be written as

\[ R_b(n) = \sqrt{R_{T0}^2 + (vnT_r)^2 - 2R_{T0}vnT_r \cos(\delta - \beta/2)} + \sqrt{R_{R0}^2 + (vnT_r)^2 - 2R_{R0}vnT_r \cos(\delta + \beta/2)} - L \]  

(3)

where \( \beta \) denotes the bistatic angle and \( \delta \) means the angle between the moving direction and the angular bisector of \( \beta \).

Apply the Taylor expansion to \( R_b(n) \) and omit the third and higher order terms, the bistatic range can be simplified as

\[ R_b(n) = r_0 - v_0 nT_r + \frac{1}{2} a_0(nT_r)^2 \]  

(4)

where \( r_0 = R_{T0} + R_{R0} - L \) is the initial bistatic range, \( v_0 = 2v\cos(\delta)\cos(\beta/2) \) is the initial bistatic velocity and \( a_0 = \frac{1}{2} \left( \sin^2(\delta - \beta/2)/R_{T0} + \sin^2(\delta + \beta/2)/R_{R0} \right) v^2 \) denotes the initial target acceleration. Based on (4), the bistatic Doppler frequency can be given by

\[ f_d = \frac{1}{\lambda} \frac{dR_b}{dnT_r} = \frac{-v_0 + a_0 nT_r}{\lambda} \]  

(5)

In addition, the bistatic range resolution and Doppler resolution can be expressed as

\[ \rho_r = \frac{c}{2B \cos(\beta/2)} \]

\[ \rho_d = \frac{1}{NT_r} \]  

(6)

(7)

If there is no Doppler expansion or range curvature caused by the acceleration term within the coherent processing interval (CPI), namely[20]

\[ NT_r < \min\left( \frac{1}{\sqrt{2a_0}}, \frac{\sqrt{2\rho_r}}{\sqrt{\rho_d}} \right) = \frac{\sqrt{2}}{a_0} \]  

(8)

thus (4) can be approximated by

\[ R_b(n) = r_0 - v_0 nT_r \]  

(9)

For the traditional MTD processing, the echo signal is down converted to baseband, thus (2) can be rewritten as

\[ r'(t) = \sum_{n=0}^{N-1} \sigma_u(t-nT_r - R_b(n)/c) \exp(j2\pi f_c R_b(n)/c) \]  

(10)

Equation (10) reveals that the second term, \( \exp(j2\pi f_c R_b(n)/c) \), is due to the interaction of changing frequency with target range. For a conventional pulse-Doppler radar with a constant carrier frequency, we can derive the Doppler frequency and achieve coherent integration by MTD. However, the phase fluctuation of the second term induced by frequency agility destroys the consistency of echo phase and results in the energy dispersion and bistatic Doppler frequency shift of the target. Hence, the conventional MTD method cannot be employed directly by PBR using FAR as the illuminator.

3. Proposed Method

In this paper, a matched-filter-based coherent integration method of PBR using FAR as the illuminator is proposed. In order to eliminate the phase fluctuation due to frequency agility, we first remove the
initial frequency $f_0$ instead of the agile frequency $f_n$ by down conversion and then (2) can be rewritten as

$$\hat{r}(t) = \sum_{n=0}^{N-1} \sigma_n u\left(t-nT_r - \frac{R_n(n)}{c}\right) \exp[-j2\pi b_n f_n (t-nT_r - \frac{R_n(n)}{c})] \exp(j2\pi f_0 \frac{R_n(n)}{c}) \cdot$$

(11)

Notice from (11), the Doppler frequency of the moving target is preserved in the last term, $\exp(j2\pi f_0 R_n(n)/c)$, given by the product of a constant frequency $f_0$ with time delay $R_n(n)/c$, which is essential and remained for MTD processing. Meanwhile, the rest terms can be viewed as a chirp modulated by the carrier frequency of $b_n f_\lambda$. After removing the initial frequency $f_0$, a set of filters matched to modulated chirps can be constructed as

$$h_n(t) = u(t) \exp(j2\pi b_n f_\lambda t)$$

(12)

In essence, the matched filter is the correlator of the transmitted waveform, namely the chirp signal modulated by the frequency $b_n f_\lambda$. Therefore, the echo signals are matched filtered by varying the filter and correlating against (11), which can be formulated as

$$s(t) = \sum_{n=0}^{N-1} A_n \sin[\pi B (t-nT_r - \frac{R_n(n)}{c})] \cdot \exp[-j2\pi b_n f_\lambda (t-nT_r - \frac{R_n(n)}{c})] \exp(j2\pi f_0 \frac{R_n(n)}{c})$$

(13)

Furthermore, the output of (13) can be given by

$$s(t) = \sum_{n=0}^{N-1} A_n \sin[\pi B (t-nT_r - \frac{R_n(n)}{c})] \cdot \exp[-j2\pi b_n f_\lambda (t-nT_r - \frac{R_n(n)}{c})] \exp(j2\pi f_0 \frac{R_n(n)}{c})$$

(14)

where $A_n$ is the amplitude of echo after being matched filtered. Substituting (9) into (14) yields

$$s(t) = \sum_{n=0}^{N-1} A_n \sin[\pi B (t-nT_r - \frac{R_n(n)}{c})] \cdot \exp[-j2\pi b_n f_\lambda (t-nT_r - \frac{R_n(n)}{c})] \exp(j2\pi f_0 \frac{R_n(n)}{c})$$

(15)

Generally, in a CPI, the across range unit (ARU) effect is usually neglected when the ARU range migration is less than the bistatic range resolution $\rho_r$, namely

$$v_r N T < \frac{c}{2B \cos(\beta/2)}$$

(16)

In this condition, (15) can be condensed as

$$s(t) = A_r \exp(j2\pi f_0 \tau_0) \sum_{n=0}^{N-1} \sin[\pi B (t-nT_r - \tau_0)] \cdot \exp[-j2\pi b_n f_\lambda (t-nT_r - \tau_0 + \frac{v_r n T} {c})] \exp(-j2\pi f_0 \frac{v_r n T} {c})$$

(17)

where $\tau_0 = \frac{\tau_0}{c}$. When the signal of (17) is sampled at $t = nT_r + \tau_0$, the output can be expressed as

$$E(n) = A_r \exp(j2\pi f_0 \tau_0) \sum_{n=0}^{N-1} \exp(-j2\pi b_n f_\lambda \frac{v_r n T} {c}) \cdot \exp(-j2\pi f_0 \frac{v_r n T} {c})$$

(18)

Irrespective of the constant terms, (18) demonstrates that the phase of $E(n)$ consists of two parts, the phase error term $\exp(-j2\pi b_n f_\lambda v_r n T / c)$ and the Doppler frequency term $\exp(-j2\pi f_0 v_r n T / c)$. The phase error term will disperse the mainlobe energy of the output Doppler profile into neighbouring bins and introduce Doppler frequency shift after fast Fourier transform (FFT) [21]. But its effect will be negligible within a CPI if the range migration is short enough, namely [20]

$$v_r N T < \frac{c}{4b_n f_\lambda} = \frac{c}{4Mf_\lambda}$$

(19)

where $M$ means the maximum value of $b_n$. In this condition, (18) can be approximated by

$$E(n) = A_r \exp(j2\pi f_0 \tau_0) \exp\left(-j2\pi f_0 \frac{v_r n T} {c}\right)$$

(20)
Notice from (20) that the signal is well suited to MTD processing and then we can easily achieve coherent integration by feeding (20) to FFT. According to (8), (16) and (19), the maximum velocity of this proposed method can be given by

$$v_{\text{max}} = \frac{c}{4NMf_T}$$

(21)

However, for the velocity larger than $v_{\text{max}}$, compensation needs to be added to avoid peak degradation after FFT, which can be formulated as

$$C(n) = \exp\left( \frac{j4\pi b_n f_0 k v_{\text{max}} n T_p}{c} \right) \quad |k| = \pm 1, \pm 2, \ldots \pm \text{round}\left( \frac{4NMf_0}{f_0} \right)$$

(22)

In fact, ARU compensation is also included, which is beyond the scope of the present manuscript. In summary, the procedure of the matched-filter-based coherent integration method for PBR using FAR as the illuminator can be given as follows.

Step 1) Remove the initial frequency $f_0$ of the echoes by down conversion.
Step 2) Construct the matched filters for the echoes with the initial frequency $f_0$ removed and then perform the matched filtering to eliminate the phase fluctuation.
Step 3) Make a velocity compensation for the matched filtered signals to reduce the peak degradation.
Step 4) Apply FFT to the compensated signal in Step 3 to achieve the coherent integration.

4. Simulation and Analysis

In this section, the influence of the phase error term is analyzed first and then the proposed method is evaluated and compared with the existing methods, MTD and FA-CRT. The parameters used in this section are given in Table 1.

| Table 1. Parameters set for the simulation |
|-------------------------------------------|
| Parameter          | Value  |
| Initial carrier frequency $f_0$           | 1 GHz   |
| Frequency step $f_\Delta$                 | 1 MHz   |
| Sampling rate $f_s$                       | 40 MHz  |
| Pulse duration $T_p$                      | 10 us   |
| PRI $T_r$                                 | 1/1500 s|
| Bandwidth $B$                             | 1 MHz   |
| Baseline $L$                              | 100 km  |
| Pulse number $N$                          | 128     |

4.1. Analysis of phase error term

To analyze the influence of the phase error term on the output of the proposed method, we simulated with different values of the initial bistatic $v_0$. The random sequence $b_n$ hops within (1,64) and the outputs of the phase error term after FFT are shown in Figure 2.

**Figure 2.** Outputs of the phase error term with different velocities.

**Figure 3.** Outputs of the phase error term with different $b_n$. 
Figure 2 displays that the peaks have decayed and broadened compared with the stationary target. The peak almost decays half when \( v_0 = 2v_{\text{max}} \) and vanishes when \( v_0 = 4v_{\text{max}} \). In addition, we note that the peak has shifted as well, which will result in the Doppler frequency shift. Furthermore, different random sequences \( h_n \) are also investigated to test the validity of the conclusion above and the outputs are plotted together in Figure 3.

Note from Figure 3. that the x-axis means the initial bistatic velocity ranging from \(-15v_{\text{max}}\) to \(15v_{\text{max}}\), and the y-axis represents the output peak of the phase error term after FFT. Figure 3 demonstrates that the peaks with different \( h_n \) almost decay half when their initial bistatic velocity reaches \( v_0 = 2v_{\text{max}} \) and vanishes when \( v_0 = 4v_{\text{max}} \) free of random sequence approximately. Additionally, it also suggests that the compensation velocity in (22) is well appropriate and acceptable with little loss.

4.2. Output of the proposed method

Consider the scenario shown in Figure 1, we start by assuming that there is a point target \( T(20\text{km},50\text{km}) \) with a constant velocity value of 100m/s and the coordinates of the transmitter and receiver are \((-50\text{km},0)\) and \((50\text{km},0)\), respectively. With the given radar parameters in Table 1, the output of the matched-filter-based coherent integration method with \( \delta = \pi/4 \), as well as its range response slice and velocity response slice, are shown in Figure 4 (a)-(c), respectively.

It is obvious that the echo energy is coherently integrated in Figure 4 (a), proving that the theoretical result agrees well with the simulation result. Figure 4(b) is the bistatic velocity response slice and we know the noise pedestal is introduced by the phase error term. It is clear from the bistatic range response slice, shown in Figure 4(c), that there are some sidelobes in company with the peak, due to range error. When the signal of (17) is sampled at \( t = nT_r + \tau_0 - \Delta R \) with a range error \( \Delta R \), it will introduce a range error term, namely

\[
\Delta P(n) = \sin c(\pi B\Delta R) \cdot \exp\left(\frac{j2\pi n f_r \Delta R}{c}\right)
\]  

Equation (21) implies that the sampling points will be coherently integrated to produce the sidelobes when \( f_r \Delta R \) is dividable by \( c \).

\[\text{(a) 2-D output of the proposed method} \quad \text{(b) Range response slice} \quad \text{(c) Velocity response slice}\]

Figure 4. Output of the proposed method.

\[\text{(a) Integration result of MTD} \quad \text{(b) Integration result of FA-CRT} \quad \text{(c) Integration result of the proposed method}\]

Figure 5. Integration results of different methods.
4.3. Comparison of methods
In this simulation, we compare the performance of the proposed method with MTD and FA-CRT. With the given parameters in Table 1, and the SNR of echo signals after pulse compression is -15dB. Figure 5(a), shows the result of MTD and it suggests that MTD cannot be used directly for coherent integration due to frequency agility. The results of the proposed method and FA-CRT are displayed in Figure 5(b) and (c), respectively. It is obvious that both methods have succeeded in coherent integration and generated a peak in the noise background, but the proposed method surpasses FA-CRT in computational burden.

5. Conclusion
In this paper, a matched-filter-based coherent integration method of PBR using FAR as the illuminator is proposed for target detection. This method can overcome the shortcomings of MTD and eliminate the phase fluctuation due to frequency agility. Compared with the 2-D searching method, this proposed method can achieve coherent integration without the prior knowledge of the motion parameters and thus, the computational burden is effectively reduced. In addition, the factors affecting the performance of the proposed method are analyzed. Finally, simulation results validate the effectiveness of the proposed method and indicate its good detection probability in the presence of noise. However, the inherent sidelobe of this method will introduce a reduction in low SNRs, and the next step is to suppress it.

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