Detection probability of a low-mass planet for triple lens events: implication of properties of binary-lens superposition

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ABSTRACT
In view of the assumption that any planetary system is likely to be composed of more than one planet, and that a multiple planet system with a large-mass planet has a greater chance of detailed follow-up observations, the multiple planet system may be an efficient way to search for sub-Jovian planets. We study the central region of the magnification pattern for the triple lens system composed of a star, a Jovian mass planet and a low-mass planet to answer the question of if the low-mass planet can be detected in high-magnification events. We compare the magnification pattern of the triple lens system with that of a best-fitted binary system composed of a star and a Jovian mass planet, and check the probability of detecting the low-mass secondary planet whose signature will be superposed on that of the primary Jovian mass planet. Detection probabilities of the low-mass planet in the triple lens system are quite similar to the probability of detecting such a low-mass planet in a binary system with a star and only a low-mass planet, which shows that the signature of a low-mass planet can be effectively detected even when it is concurrent with the signature of the more massive planet, implying that the binary superposition approximation works over a relatively broad range of planet mass ratio and separations, and the inaccuracies thereof do not significantly affect the detection probability of the lower-mass secondary planet. Since the signature of the Jovian mass planet will be larger and lasting longer, thereby warranting more intensive follow-up observations, the actual detection rate of the low-mass planet in a triple system with a Jovian mass can be significantly higher than that in a binary system with a low-mass planet only. We conclude that it may be worthwhile to develop an efficient algorithm to search for ‘super-Earth’ planets in the paradigm of the triple lens model for high-magnification microlensing events.

Key words: gravitational lensing: micro – planets and satellites: general.

1 INTRODUCTION
To detect and characterize extrasolar planets various techniques have been employed so far, which include the radial velocity technique (Mayor & Queloz 1995), the transit method (Charbonneau et al. 2007), direct imaging (Chauvin et al. 2004), pulsar timing analysis (Wolszczan & Frail 1992) and microlensing (Bond et al. 2004; Udalski et al. 2005; Beaulieu et al. 2006; Gould et al. 2006; Bennett et al. 2008; Gaudi et al. 2008; Dong et al. 2009; Janczak et al. 2010; Sumi et al. 2010). Compared to other techniques, the microlensing method has the important advantage of being applicable to planets to which other methods are generally insensitive; the microlensing technique is sensitive to detecting low-mass planets and cool planets, or even free-floating planets (Bennett & Rhie 2002; Han et al. 2004, 2005). This sensitivity is very important for testing the core accretion theory of planet formation, which predicts that the dominant planets in any planetary system should form in the vicinity of the ‘snow line’, which is located at a few au from the host star (Laughlin, Bodenheimer & Adams 2004; Ida & Lin 2005; Kennedy, Kenyon & Bromley 2006).

When a microlensing event occurs in a planetary system, the planetary signal appears as a short-duration perturbation to the standard light curve induced by the lens star (Mao & Paczynski 1991; Gould & Loeb 1992; Bennett & Rhie 1996). The planetary lensing signal induced by a planet with the mass of Jupiter lasts for a duration of ~1 d and that by a planet with the mass of the Earth lasts ~1.5 h. Therefore, the discovery of a terrestrial planet would only be possible by high-cadence anomaly monitoring. In fact, the detection of a significant number of terrestrial extrasolar planets requires

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well-coordinated efforts involving a network (e.g. MOA-II; Sumi et al. 2010; OGLE-IV: Udalski et al. 2005). MOA-II has reported two low-mass planets with their survey, MOA-2007-BLG-192Lb (Bennett et al. 2008) and OGLE-2007-BLG-368Lb (Sumi et al. 2010), and is preparing one, MOA-2009-BLG-266Lb (in preparation). The planetary perturbation occurs when the source star crosses the caustic or passes close to it. Caustic-crossing events cause conspicuous double-peaks over the smooth light curve induced by a lensing star. However, the perturbations due to caustic-crossing occur without any prior warning so that the current microlensing follow-up observations are focused on high-magnification events for the sake of practicality. For high-magnification events, the source trajectories always pass close to the perturbation region around the central caustic induced by the planet and thus the timing can be predicted fairly accurately (Griest & Safizadeh 1998; Han & Kim 2001; Bond et al. 2002; Rattenbury et al. 2002; Yoo et al. 2004). Griest & Safizadeh (1998) have shown that planets with masses as low as $10M_{\oplus}$ could be detected with significant probability in events with magnification of $\sim 50$ by monitoring the peaks of the events with a photometric precision of $\sim 1$ per cent. Rhie et al. (2000) have first showed that high-magnification events are sensitive to low-mass (Earth-mass) planets (for the lower limit of the planet mass most recently reported, see Yee et al. 2009).

Since the discovery of the first extrasolar planet, orbiting a solar-type star (Mayor & Queloz 1995), the Extrasolar Planet Encyclopedia lists 429 entries, including 45 multiple planetary systems, as of 2010 February 9. The detectable mass of exoplanets is becoming smaller and below the $10M_{\oplus}$ regime, with the discoveries of Gliese 876d with a mass of $\sim 7.5M_{\oplus}$ (Rivera et al. 2005); three planets around HD 40307 with masses of $\sim 4.2, \sim 6.9$ and $\sim 9.2M_{\oplus}$ (Mayor et al. 2009a); and Gliese 581e with a mass of $\sim 1.9M_{\oplus}$ (Mayor et al. 2009b) obtained using the radial-velocity technique, as well as OGLE 2005-BLG-390Lb detected by microlensing at a mass of $\sim 5.5M_{\oplus}$ (Beaulieu et al. 2006; Bennett et al. 2008).

Many of the discovered 'super-Earth' planets have been revealed through the close re-examination of planetary signals that have already proved the existence of their big brother. Considering that a planetary system is likely to be composed of more than one planet, this kind of strategy to find low-mass planets in the multiple planet systems may become an efficient way to search for terrestrial planets in the sense that it is easier to detect subtle signals when one knows what to look for. The magnification pattern due to the triple-lens systems is known to be well approximated by the superposition of the magnifications due to the planetary caustics (Han et al. 2001), or central caustics (Han 2005), of individual planets. Therefore, we may expect that the detection probability of low-mass planets in a triple system with a Jovian mass planet plus a low-mass planet should be quite similar to the detection probability in a binary system with only one low-mass planet. However, this is known only for a limited range of planet mass ratio and separations (Han 2005), and also the effect of the deviation from the superposition approximation on the detection probability is not known. Moreover, it is not clear if the smaller deviation in the magnification due to a low-mass planet, when superposed on the larger deviation due to a more massive Jovian mass planet, can be effectively detected in high-magnification events when the 'adjusted' binary lens model is fitted to incorporate the additional deviation from the low-mass planet, possibly removing the signature of the low-mass planet. So in this paper, we study the triple lens system (a lens star, a Jovian mass planet and a low-mass planet) for a broader range of planetary masses and separations than in previous studies, motivated by the fact that all planets in the lensing zone will substantially affect the central caustics and thus the existence of the multiple planets can be inferred by analyzing additionally deformed anomalies in the light curve of high-magnification microlensing events (Gaudi, Naber & Sackett 1998; Gaudi et al. 2008). We further calculate the probability of the detection of a low-mass planet in the high-magnification events of a triple lens system, and compare that to that of a binary lens system (a lens star and a low-mass planet). Since the dependence of the size of the central caustic on the planet/star mass ratio $q$ is linear (Griest & Safizadeh 1998; Chung et al. 2005; Han 2006), detecting signals for Earth-mass planets with $q \sim 10^{-3}$ from the planet-search strategy of monitoring the high-magnification event of the binary lens system is observationally challenging (e.g. Bennett & Rhie 1996, 2002; Gaudi & Sackett 2000). Therefore, it is crucial and timely to address the question of if the detection probability of a low-mass planet for the high-magnification events of the triple lens system is comparable to that of the binary lens system. Answers to this question may well have implications on interpretations of the light curve of high-magnification microlensing events. Indeed, recently there have been efforts to re-analyze the observational data along this line. For example, Kubas et al. (2008) have attempted to estimate the detection probability for the secondary planet by re-analyzing the observational data of OGLE 2005-BLG-390. Their assumptions may be justified for their purpose in that the impact parameter of OGLE 2005-BLG-390 is large, i.e. $u_0 = 0.359$. On the contrary, in this paper we specifically concentrate on the high-magnification microlensing events ($u_0 \lesssim 0.01$).

In Section 2, we begin with a brief description of the multiple lens systems. In Section 3, we construct the fractional deviation of the magnification map induced by the secondary planet. In Section 4, we present the detection probability and compare the triple lens case with the binary lens case. In Section 5, we conclude with a summary of our results.

2 MULTIPLE LENS SYSTEMS

When a source star is gravitationally lensed by a point-mass lens, the source is split into two images with the total magnification given as the simple analytic form of

$$A_0 = \frac{u^2 + 2}{u\sqrt{u^2 + 4}},$$

where $u$ is the lens-source separation normalized by the angular Einstein ring radius (Paczyński 1986). The angular Einstein ring radius is related to the physical parameters of the lens by

$$\theta_E = \sqrt{\frac{4GM}{c^2}} \left( \frac{1}{D_{ol}} - \frac{1}{D_{ol}} \right)^{1/2},$$

where $M$ is the mass of the lens and $D_{ol}$ and $D_{os}$ represent the distances from the observer to the lens and source, respectively.

If the lens star accommodates a planetary companion, the latter may further perturb the image and thereby change the magnification. When a lensing event is caused by the multiple lens system, locations of the individual image are obtained by solving the lens equation expressed in complex notations by

$$\zeta = \zeta + \sum_{j=0}^{N-1} \frac{q_j}{\zeta_j - \zeta},$$

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where \( q_j \) and \( z_j \) represent the mass fractions of individual lenses (\( j = 0 \) for the central lens star, \( j = 1 \) for the primary planet and \( j = 2 \) for the secondary planet and so on) such that \( \sum_j q_j = 1 \) and the positions of the lenses, respectively \( \zeta = \xi + i\eta \) and \( z = x + iy \), are the positions of the source and images, and \( \bar{z} \) denotes the complex conjugate of \( z \) (Witt 1990). Note that all these lengths are normalized by the combined Einstein ring radius, which is equivalent to the Einstein ring radius of the single lens with a mass equal to the total mass of the system. The total magnification is the sum of magnifications of the individual images, \( A = \sum_i A_i \). The magnification of each image evaluated at the position of each image, \( A_i \), is given by

\[
A_i = \frac{1}{|\det J|},
\]

where

\[
\det J = 1 - \frac{\partial \xi}{\partial \bar{z}} \frac{\partial \bar{\zeta}}{\partial \bar{z}}.
\]

The fundamental difference in the geometry of the multiple lens system from that of the single point-mass lens system is the formation of caustics. The caustic refers to the source position on which the magnification of a point source event becomes infinity, i.e. \( \det J = 0 \). The set of caustics forms closed curves. For example, for the planetary lensing system composed of a single star and a single planet, there exist two sets of disconnected caustics: one ‘central’ caustic is located close to the lens star, while the other ‘planetary’ caustic(s) is (are) located away from the lens star. The number of the planetary caustics is one or two depending on whether the planet lies outside or inside the Einstein ring. When the planetary companion is close to the Einstein ring, the planetary and central caustics merge into a single ‘resonant caustic’. The central caustic plays a crucial role in current microlensing planet searches, as stated earlier, in that the central caustic occurring due to the planet has a high probability of perturbing the light curve of high-magnification events, because of which it is detectable as the planetary signal. For the triple lens case, the lens equation becomes a rather complicated

\[\text{Figure 1. Geometry of triple lens system composed of a lens star, a primary planet and a secondary planet. In the upper panel, the coordinate } (\xi, \eta) \text{ are centred at the position of the lens star, and } \theta \text{ is the angle between position vectors of two planets } (q_1, q_2). \text{ All lengths are normalized by the radius of the Einstein ring. The grey-scale represents the magnification and brighter tone represents higher magnification. The lower panel shows the zoom of the boxed region of the high magnification within } a_0 \leq 0.01. \]
10th-order polynomial equation that has 4, 6, 8 or 10 solutions, corresponding to physical images depending on the configuration of the lens system and the location of the source (Rhie 2002). In the triple lens system, patterns of the planetary caustics may well be described by the superposition of those of the single-planet systems (Han et al. 2001; Han 2005). They are barely affected by each other unless the projected positions of the planets are close. On the contrary, in the central region of the magnification map the anomaly pattern induced by one planet can be significantly affected by the existence of another planet.

3 DEVIATIONS DUE TO SECONDARY PLANET

Deviation maps are presented in terms of the lens position of the secondary planet both for the angle between the position vectors of the two planets, $\theta$, and for the separations, $s_2$ (see Fig. 1).

In Fig. 2, we show the fractional deviation of the magnification map obtained by fitting the binary lens model to the magnification map induced by triple lens systems, which is defined by

$$\epsilon \equiv \frac{A_{\text{tri}} - A_{\text{bin}}}{A_{\text{bin}}},$$

where $A_{\text{tri}}$ and $A_{\text{bin}}$ represent the magnification map generated with the primary plus secondary planets and that obtained by fitting the binary lens model in which the secondary planet is absent, respectively. The inverse ray-shooting technique is used to obtain the magnification map (Kayser, Refsdal & Stabell 1986; Schneider & Weiss 1986; Wambsganss 1997). Each map of the fractional deviation is calculated as a function of the source position $(\xi, \eta)$. The coordinates are set so that the lens star is at the centre. We concentrate only on regions of $|\xi| \leq 0.01$ and $|\eta| \leq 0.01$ for which high-magnification events are relevant. In generating the magnification map induced by triple lens systems, the primary planet is fixed

![Figure 2](https://academic.oup.com/mnras/article-abstract/412/1/503/985955)
to be located on the \( \xi \) axis, \((x_1, y_1, z_1) = (-1.3, 0, 0)\). We set the mass ratio of the primary planet to the lens star \( q_1 = 3 \times 10^{-3} \) and that of the secondary planet \( q_2 = 1 \times 10^{-3} \), corresponding to Jupiter-mass and Saturn-mass planets orbiting a \( 0.3 \text{-} \text{M}_\odot \) star, respectively. Unless otherwise stated, in all computations throughout this paper we adopt the typical values of distances for Galactic bulge events: \( D_{\odot} = 6 \text{ kpc} \), \( D_{\odot} = 8 \text{ kpc} \). We also assume the source radius \( \theta_* \) to be \( \theta_*/\theta_E = 0.001 \), which corresponds to a typical main-sequence star. Note that in the fitting procedure parameters of the binary lens model composed of a lens star and the primary planet are constrained to find their best-fitting values. To minimize the difference between the \( \varepsilon \) we generated using the triple lens and one we modelled by the binary lens model, the minimization process has been performed by the least-squares method by varying the mass ratio of the primary planet, \( q_1 \), and its position, \( s_1 \). Contours are drawn at the levels of \( \varepsilon = \pm 1, \pm 5, \pm 10 \) and \( \pm 20 \) per cent, and the regions of negative \( \varepsilon \) are shown in blue and positive \( \varepsilon \) in red. The colour changes into light shade as the \( |\varepsilon| \) level decreases.

As one may expect, distorted regions due to the secondary planet are confined to the central caustic of the primary planet due to the non-linear interference between perturbations produced by the two planets. On the other hand, it should be noted that when \( \theta = 0^\circ \) there may exist a degenerate case where two-planet geometries will give rise to exactly the same magnification map as obtained with a single planet of a slightly larger mass, as previously noted by Gaudi et al. (1998). This can be seen in our particular example, in the case of \( s_2 = 1.26 \) since the primary planet is located at \((-1.3, 0.0)\). We also note that the degeneracy between \( s_2 \) and \( s_2^{-1} \) can be seen due to the additional planet. Regions deformed by the secondary planet revolve as the angle between the two planets varies, and become broader as \( s_2 \) approaches unity. To explore the effect of \( q_2 \), we show the deviation maps for several \( s_2 \) and \( q_2 \) in Fig. 3. We set the mass ratio of the primary planet to the lens star \( q_1 = 3 \times 10^{-3} \) and the angle between the position vectors of the two planets \( \theta = 30^\circ \). As in Fig. 2, contours are drawn at the levels of \( \varepsilon = \pm 1, \pm 5, \pm 10 \) and \( \pm 20 \) per cent, and the regions of negative \( \varepsilon \) are shown in blue and positive \( \varepsilon \) in red. From the maps, one finds that the lower limit of the mass of the secondary planet that may cause sufficient deformations in the magnification map is between \( 1 \times 10^{-5} \lesssim q_2 \lesssim 1 \times 10^{-4} \), in which \( q_2 = 1 \times 10^{-4} \) corresponds to a 10-\text{-}M_\oplus \) planet orbiting a \( 0.3 \text{-} \text{M}_\odot \) star. In other words, an Earth-mass planet can only be possibly detectable in a narrow region of the parameter space unless...

**Figure 3.** Similar maps as Fig. 2, except that we set the angle between the position vectors of two planets \( \theta = 30^\circ \) and vary the mass of the secondary planet. As in Fig. 2, contours are drawn at the levels of \( \varepsilon = \pm 1, \pm 5, \pm 10 \) and \( \pm 20 \) per cent, and the regions of negative \( \varepsilon \) are shown in blue and positive \( \varepsilon \) in red.
the photometric accuracy allows one to examine the observational data in the levels of $|\varepsilon| \sim 1\%$.

4 COMPARISON OF DETECTION PROBABILITY

To ‘discover’ a planet, one may demand that $\Delta \chi^2$ between the light curve calculated by the planetary lens model and the observed data should be smaller than a carefully chosen critical value. Or, as another way to disclose a planet, the $\Delta \chi^2$-based criterion can be translated into the so-called Gould & Loeb criterion, assuming a pre-determined photometric accuracy and non-white noise (Gould & Loeb 1992). Gould & Loeb criterion considers deviations as a planetary signal when a few observational points deviated consecutively from the single-lens light curve. In the current exercise, we are going to follow the Gould & Loeb criterion. Thus, we will count a signal of planetary detection as successful if at least one point in the deviation map has the deviation amplitude larger than a certain threshold. We have defined the detection probability of the low-mass planet as follows: First, for given $q_2$, $s_2$ and $\theta$, we calculate the area in which the value of $|\varepsilon|$ in the deviation map, e.g. shown in Figs 2 and 3, is greater than 5 per cent. Bearing in mind that monitoring high-magnification events should be an efficient strategy of detecting ‘super-Earth’ planets in view of practical purposes, we only consider this quantity in the range of $|u_0| \leq 0.01$ as commonly employed. Having done that, we average the obtained area over the position angle in the range of $0^\circ \leq \theta \leq 180^\circ$. Then, we normalize the averaged area with the area of $|u_0| \leq 0.01$. In this way, we obtain the detection probability as a function of $q_2$ and $s_2$. Note that we repeat the same calculation for the binary lens with a planet that has the same mass as the secondary planet in the triple lens system, except that we construct fractional deviation maps given as

$$\varepsilon \equiv \frac{A_{\text{bin}} - A_{\text{single}}}{A_{\text{single}}} \quad (7)$$

where $A_{\text{bin}}$ and $A_{\text{single}}$ represent the magnification map generated with the binary lens system having a planet and that obtained by fitting of the single lens model, respectively.

In Fig. 4, we compare the probability of detecting the low-mass planet in triple systems with that in binary systems, provided that the detection threshold is $|\varepsilon| > 5\%$. The detection probability of the secondary low-mass planet in the triple lens system is presented by grey-scale such that a darker shade represents a higher probability as shown in the scale bar. We note that for a given $q_2$ the probability of detecting the secondary planet becomes higher as $s_2$ approaches unity, as expected in plots such as Fig. 2. We find that in the triple lens system the detection probabilities of the low-mass planet are $\sim 50$, $\sim 10$ and $\sim 1\%$ for $q_2 = 10^{-3}$, $10^{-4}$ and $10^{-5}$, respectively, if we only consider the secondary planet residing in the lensing zone, $0.6 \lesssim s_2 \lesssim 1.6$. For comparison, we also present as dotted contours the probability of detecting the low-mass planet in binary systems. The detection probabilities for low-mass planets have been calculated (Gould & Loeb 1992; Bennett & Rhie 1996; Gaudi et al. 1998; Griest & Safizadeh 1998; Rhie et al. 2000; Bennett & Rhie 2002; Rattenbury et al. 2002; Kubas et al. 2008). Interestingly enough, for a given $q_2$ and $s_2$ the detection probability

Figure 4. Probabilities of detecting the low-mass planet in binary systems and in triple systems. The detection probability of the secondary low-mass planet in the triple lens system is represented by grey-scale, and is drawn such that the darker shade represents a higher probability as indicated in the grey index. For comparison, we also present as dotted contours the probability of detecting the same low-mass planet if it is in a binary system. Probabilities are calculated such that the value of $|\varepsilon|$ in the deviation map is greater than 5 per cent, considering only $|u_0| \leq 0.01$ events.
Detection probability of a low-mass planet

Figure 5. Similar maps as Fig. 4, except that probabilities are calculated such that the value of $|\varepsilon|$ in the deviation map is greater than 1 per cent, considering only $|u_0| \leq 0.01$ cases.

of the low-mass planet in the triple lens system is very similar to that in the binary lens system. It makes sense to recall that the magnification pattern due to the triple-lens systems is well approximated by the superposition of the magnifications due to the individual planets (Han et al. 2001; Han 2005). The fundamental reason why the detection probability of low-mass planets in a triple-lens system with a Jovian mass planet could be similar to the detection probability in a binary-lens system is that in most cases the binary superposition works well even in the central region. If the interference between caustics due to two planets destroys its original shapes then there is no reason that we end up with residuals implying a second planet. Having said so, another necessary condition for the detection probability of low-mass planets in a triple-lens system with a Jovian mass planet to be similar to the detection probability in a binary-lens system is that the minimization should find a solution for the Jovian planet accurately enough.

In Fig. 5, we present the detection probability for the lower threshold of $|\varepsilon| > 1$ per cent, provided that photometric uncertainties of $\geq 1$ per cent are achievable in the future. It is natural to find that the probability in the traditional lensing zone becomes higher. That is, we find that in the triple lens system the detection probabilities of the low-mass planet are $\sim 80$, $\sim 40$ and $\sim 5$ per cent for $q_3 = 10^{-3}$, $10^{-4}$ and $10^{-5}$, respectively, if we only consider the secondary planet residing in the lensing zone. It is also found that for a high-mass planet of $q_3 \gtrsim 10^{-4}$ corresponding to a 10-M$_\odot$ planet orbiting a 0.3-M$_\odot$ star, there exist reasonably high probabilities in the broader range than the lensing zone, i.e. $s_2 \lesssim 0.6$ or $s_2 \gtrsim 1.6$. The detection probability of the low-mass secondary planet in the triple lens system is also comparable to that in the binary lens system, except when $q_2 \lesssim 3 \times 10^{-6}$ corresponding to a 0.3-M$_\odot$ planet orbiting a 0.3-M$_\odot$ star. In that parameter space the detection probability in the binary lens system is somewhat higher than that in the triple lens system.

5 CONCLUSION

We have computed the detection probability of the low-mass planet specifically for the high-magnification events of the triple lens system, motivated by the fact that the central caustic is distorted by any companion to the lens star. Having done that, we have compared the detection probability of the low-mass secondary planet for high-magnification events of the triple lens system with that of the same low-mass planet but in the binary lens system. It should be stressed that the detection probability of the low-mass planet for high-magnification events of the triple lens system given in this paper is a kind of relative probability in the sense that our criteria for the discovery of a planet is actually closer to one of signals for the planet. The detection of signals for a low-mass planet may not be sufficient for the discovery of such a planet. Having compared results of two cases with the same criteria, however, probabilities given here still shifts the emphasis to the light curves of the triple lens system.

Our main findings are as follows.

(1) In the triple lens case where the secondary planet resides in the lensing zone, the detection probabilities of the low-mass planet for the high-magnification events are $\sim 50$, $\sim 10$ and $\sim 1$ per cent for $q_3 = 10^{-3}$, $10^{-4}$ and $10^{-5}$, respectively, when the detection criterion is $|\varepsilon| > 5$ per cent where $\varepsilon$ is the deviation in magnification.

(2) When the detection criterion is $|\varepsilon| > 1$ per cent, those probabilities increase to $\sim 80$, $\sim 40$ and $\sim 5$ per cent for $q_3 = 10^{-3}$, $10^{-4}$.
and $10^{-5}$, respectively. For high-mass planets of $q_2 \gtrsim 10^{-4}$, there exist reasonably high probabilities outside the usual lensing zone, $s_2 \lesssim 0.6$ or $s_2 \gtrsim 1.6$.

(3) For a given $q_2$ and $s_2$, the detection probability of the low-mass planet in the triple lens system is comparable to that in the binary lens system. Therefore, it is quite necessary to develop an efficient algorithm search for 'super-Earth' planets in the paradigm of the triple lens model as well as of the binary lens model.

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REFERENCES

Beaulieu J.-P. et al., 2006, Nat, 439, 437
Bennett D. P., Rhie S. H., 1996, ApJ, 472, 660
Bennett D. P., Rhie S. H., 2002, ApJ, 574, 985
Bennett D. P. et al., 2008, ApJ, 684, 663
Bond I. A. et al., 2002, MNRAS, 333, 71
Bond I. A. et al., 2004, ApJ, 606, L155
Charbonneau D., Brown T. M., Burrows A., Laughlin G., 2007, Protostars and Planets V, 701
Chauvin G., Lagrange A.-M., Dumas C., Zuckerman B., Mouillet D., Song I., Beuzit J.-L., Lowrance P., 2004, A&A, 425, L29
Chung S.-J. et al., 2005, ApJ, 630, 535
Dong S. et al., 2009, ApJ, 698, 1826
Gaudi B. S., Sackett P. D., 2000, ApJ, 528, 56
Gaudi B. S., Naber R. M., Sackett P. D., 1998, ApJ, 502, L33
Gaudi B. S. et al., 2008, Sci, 319, 927
Gould A., Loeb A., 1992, ApJ, 396, 104
Gould A. et al., 2006, ApJ, 644, L37
Griest K., Safizadeh N., 1998, ApJ, 500, 37
Han C., 2005, ApJ, 629, 1102
Han C., 2006, ApJ, 638, 1080
Han C., Kim Y.-G., 2001, ApJ, 546, 975
Han C., Chang H.-Y., An J. H., Chang K., 2001, MNRAS, 328, 986
Han C., Chung S.-J., Kim D., Park B.-G., Ryu Y.-H., Kang S., Lee D., 2004, ApJ, 604, 372
Han C., Gaudi B. S., An J. H., Gould A., 2005, ApJ, 618, 962
Iida S., Lin D. N. C., 2005, ApJ, 626, 1045
Janczak J. et al., 2010, ApJ, 711, 731
Kayser R., Refsdal S., Stabell R., 1986, A&A, 166, 36
Kennedy G. M., Kenyon S. J., Bromley B. C., 2006, ApJ, 650, L139
Kubas D. et al., 2008, A&A, 483, 317
Laughlin G., Bodenheimer P., Adams F. C., 2004, ApJ, 612, L73
Mao S., Paczynski B., 1991, ApJ, 374, L37
Mayor M., Queloz D., 1995, Nat, 378, 333
Mayor M. et al., 2009a, A&A, 493, 639
Mayor M. et al., 2009b, A&A, 507, 487
Paczynski B., 1986, ApJ, 304, 1
Rattenbury N. J., Bond I. A., Skuljan J., Yock P. C. M., 2002, MNRAS, 335, 159
Rhie S. H., 2002, preprint (arXiv:astro-ph/0202294)
Rhie S. H. et al., 2000, ApJ, 533, 378
Rivera E. J. et al., 2005, ApJ, 634, 625
Schneider P., Weiss A., 1986, A&A, 164, 237
Sumi T. et al., 2010, ApJ, 710, 1641
Udalski A. et al., 2005, ApJ, 628, L109
Wambsganss J., 1997, MNRAS, 284, 172
Witt H. J., 1990, A&A, 236, 311
Wolszczan A., Frail D. A., 1992, Nat, 355, 145
Yee J. C. et al., 2009, ApJ, 703, 2082
Yoo J. et al., 2004, ApJ, 616, 1204

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