Strange Quark Matter with $\beta$-equilibrium condition

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Abstract. Present study explores the properties of strange quark matter (SQM) or strange quark star (SQS) within the Polyakov extended chiral $SU(3)$ quark mean-field (PCQMF) model. Using $\beta$-equilibrium condition in the PCQMF model, the analysis of pressure density, and equation of state (EoS) of SQM for different values of vector coupling constant is carried out. Three different conditions of Proto-Quark Star (PQS) along the star evolution ($S/n_B = 1, Y_l = 0.4; S/n_B = 2, Y_l = 0; S/n_B = 0, Y_l = 0$) are considered to perform the theoretical simulation. Providing a significant vector coupling constant, the change in pressure with baryon density is found to be more as compared to zero vector interaction. Further, pressure density shows monotonically and smoothly increasing behavior with an increase in the energy density. The study thus carried out, anticipated to give a better insight in understanding the properties of matter inside the core of supermassive stars in the universe.

1. Introduction

The study of phase structure of hot and dense matter created in relativistic heavy-ion collisions and dense compressed baryonic matter which may be a part of compact star, shows significant attention since last few decades. Many experimental facilities are dedicated to investigate the properties of strongly interacting matter such as the Large Hadron Collider (LHC) at CERN, Relativistic Heavy Ion Collider (RHIC) at BNL, Facility for Anti-Proton and Ion Research (FAIR), and Nuclotron-based Ion Collider fAcility (NICA). The baryon density for inner core of compact stars are expected to be around $6\rho_0$ ($\rho_0 \approx 0.15$ fm$^{-3}$, is normal nuclear density), and this may lead to the various possibilities such as: existence of hyperons [1], quark matter core [2], color superconducting phase [3] and meson condensations [4]. Bodmer [5] and Witten [6] suggested a conjecture that the matter with an equal number of $u$, $d$, and $s$ quarks also termed as strange quark matter (SQM) may be the ground state of nuclear matter. Many efforts are devoted to study the conversion from neutron star (NS) to quark star (QS) which may consist of SQM, lepton like electron and muon due to $\beta$-equilibrium and charge neutrality [6, 7]. The EoS of SQM is an important input to understand the structure of QSs or hybrid stars. Recently observed massive compact stars, e.g. PSR J1614-2230 [8] and PSR J0348+0432 [9], have put strong constraints on the EoS of strongly interacting matter. Different phenomenological models based on quark degree of freedom have been proposed to explore the EoS of SQM such as MIT bag model [10], PQMC model [11], NJL model [12], PNJL model [13], PLSM model [14], chiral $SU(3)$ quark mean-field (CQMF) model [15] and Polyakov extended CQMF (PCQMF) model [17]. The primary objective of present work is to study the influence of vector interaction on pressure and EoS of SQM using Polyakov chiral $SU(3)$ quark mean-field model with various condition during the evolution of Proto-quark star (PQS) and QS.
2. Methodology

The Chiral $SU(3)$ Quark Mean Field (CQMF) Model \cite{17} respects the non-linear realization of chiral symmetry \cite{15,16}, spontaneous symmetry breaking, broken scale invariance \cite{21,22}, and trace anomaly properties \cite{22}. Polyakov loop potential incorporated in CQMF model to explain the properties of deconfinement in phase transition. The effective Lagrangian density and thermodynamic potential density for SQM in Polyakov chiral $SU(3)$ quark mean field model can be elucidated as

\[ \mathcal{L}_{PCQMF} = \mathcal{L}_{q0} + \mathcal{L}_{qm} + \mathcal{L}_{\Sigma} + \mathcal{L}_{V} + \mathcal{L}_{SB} + \mathcal{L}_{\Delta m} + \mathcal{L}_{h} - U(\Phi, \bar{\Phi}, T), \]  

(1)

and

\[ \Omega = -\gamma_i k_B T \sum_{q,l} \int_0^{\infty} \frac{d^3k}{(2\pi)^3} \left[ \ln \left( 1 + e^{-3(E_i^*(k) - \nu^*_i)/k_B T} + 3\Phi e^{-(E^*_i(k) - \nu^*_i)/k_B T} \right) 
+ 3\Phi e^{-2(E^*_i(k) - \nu^*_i)/k_B T} \right] + \ln \left( 1 + e^{-3(E_i^*(k) + \nu^*_i)/k_B T} + 3\Phi e^{-(E^*_i(k) + \nu^*_i)/k_B T} \right) 
+ 3\Phi e^{-2(E^*_i(k) + \nu^*_i)/k_B T} \right] - \mathcal{L}_{\Sigma} - \mathcal{L}_{VV} - \mathcal{L}_{SB} - \mathcal{L}_{\text{vac}} + \mathcal{U}(\Phi, \bar{\Phi}, T), \]  

(2)

respectively. In above, summation runs over constituent quarks ($q=u, d$ and $s$) and leptons ($l=e, \mu, \nu_e$ and $\nu_\mu$). Moreover, the value of spin degeneracy factor, $\gamma_i$ is 6 for quarks while 2 for leptons and $E_i^*(k) = \sqrt{m_i^2 + k^2}$ is the effective single particle energy of quarks. The vacuum energy term, $\mathcal{L}_{\text{vac}}$ is subtracted to attain zero vacuum energy. Additionally, the effective chemical potential $\nu_i^*$ and effective constituent mass of quarks are inscribed as per relation

\[ \nu_i^* = \mu_i - g_i^q \omega - g_i^q \phi - g_i^q \rho, \]  

(3)

and

\[ m_i^* = -g_i^q \sigma - g_i^q \zeta - g_i^q \delta + m_{j0}. \]  

(4)

In the above equations, $g_i^q$, $g_i^q$, $g_i^q$, $g_i^q$, $g_i^q$ and $g_i^q$ are the coupling strength of quarks with vector and scalar meson fields.

For Polyakov potential, best known logarithmic form \cite{13,23,24} is considered in current work, that fulfills the $Z(N_C)$ symmetry of pure gauge Lagrangian and is written by

\[ \frac{U(\Phi, \bar{\Phi}, T)}{T^4} = -\frac{a(T)}{2} \Phi \Phi + b(T) \ln \left[ 1 - 6\Phi \Phi + 4(\Phi^3 + \Phi^3) - 3(\Phi \Phi)^2 \right]. \]  

(5)

In order to calculate the scalar fields $\sigma$, $\zeta$ and $\delta$, the dilaton field $\chi$, the vector fields $\omega$, $\rho$ and $\Phi$ and the Polyakov field $\Phi$ and its conjugate $\bar{\Phi}$, we have minimized $\Omega$ with respect to these fields, i.e.,

\[ \frac{\partial \Omega}{\partial \sigma} = \frac{\partial \Omega}{\partial \zeta} = \frac{\partial \Omega}{\partial \delta} = \frac{\partial \Omega}{\partial \omega} = \frac{\partial \Omega}{\partial \rho} = \frac{\partial \Omega}{\partial \phi} = \frac{\partial \Omega}{\partial \Phi} = \frac{\partial \Omega}{\partial \bar{\Phi}} = 0. \]  

(6)

The weak $\beta$ equilibrium condition for SQM can be expressed as \cite{12}

\[ \mu_d = \mu_s = \mu_u + \mu_e - \mu_{\nu_e}, \]  

(7)

\[ \mu_\mu = \mu_e \quad \text{and} \quad \mu_{\nu_\mu} = \mu_{\nu_e}. \]  

(8)

Additionally, the condition for electric charge neutrality condition can be expressed as

\[ \frac{2}{3} \rho_u = \frac{1}{3} \rho_d + \frac{1}{3} \rho_s + \rho_e + \rho_\mu. \]  

(9)
By using thermodynamical potential density, \( \Omega \), one can define the pressure, \( p = -\Omega \), and the energy density, \( \epsilon = \Omega + \sum_i \nu_i \rho_i + TS \). In the following study, two conditions are taken into consideration. In the first situation, neutrinos are assumed to be inside of a star and the later one comes into picture just after the escape of neutrinos. We explore the following: (I) \( s = 1, \ Y_l = 0.4; \) (II) \( s = 2, \ Y_\nu = 0; \) (III) \( s = 0, \ Y_\nu = 0 \), where \( s \) is the entropy per baryon for star matter, \( Y_l \) and \( Y_\nu \) stand for the lepton fraction and neutrino fraction, respectively.

3. Results and Discussion

In this section, the effect of vector interaction, \( g_v \) on the pressure density, \( P \) and equation of state (EoS) of strange quark matter by imposing \( \beta \)-equilibrium and charge neutrality condition is presented. Various parameters necessary for the calculation can be found in [17].

Figure 1. (Color online) The pressure density as a function of baryonic density, \( \rho_B \) (in units of nuclear saturation density \( \rho_0 \)) for different conditions of Proto-quark star for \( g_v = 0 \) and 10.92.

Figure II depicts the pressure density against the total baryonic density for different conditions of star evolution ( \( s = 1, \ Y_l = 0.4; \ s = 2, \ Y_\nu = 0; \ s = 0, \ Y_\nu = 0 \)). At the initial stage of PQS, when \( s = 1 \) and \( Y_l = 0.4 \), the pressure density is increasing monotonically from 0 to 200 MeV/fm\(^3\) with an increase in \( \rho_B \). After 10-20 seconds, neutrinos start diffusing and heat the quark matter by increasing the entropy density, \( s = 2 \) [12]. The absence of neutrinos causes a decrease in pressure at higher \( \rho_B \). After this stage, the star initiates cooling by radiating neutrino pairs, and hence forming a cold quark star. For cold quark stars, \( P \) is smaller when compared with the previous condition. To incorporate the effect of vector interaction we consider \( g_v = 10.92 \) and observe that the pressure density increases gradually with baryon density but the value is larger than \( g_v = 0 \). The range of \( P \) is almost double for finite \( g_v \) at \( s = 1 \) and \( Y_l = 0.4 \).

The EoS of PQS, for \( g_v = 0 \) and 10.92, is presented in figure II. At \( g_v = 0 \), the EoS for finite lepton fraction is observed to be stiff as compared to zero lepton fraction for all values of entropy per baryon, because a higher value of lepton fraction contributes to a large electron fraction which conceals the other negative charge particles to maintain the electric charge neutrality condition. The effect of lepton fraction on EoS is also explained in the NJL model and MIT bag model with \( \beta \)-equilibrium and charge neutrality condition [22]. The behavior of EoS is the same for
Figure 2. (Color online) The pressure density as a function of energy density for proto-quark star for $g_v = 0$ and 10.92.

NJL model but different in MIT model because the concept of lepton fraction does not apply to the MIT bag model due to the absence of chiral restoration. If we include the effect of vector interaction, the EoS for $Y_l = 0.4$ and $s = 1$ becomes softer than for $Y_e = 0$. At $g_v = 10.92$, the value of pressure density is more at fixed energy density for with and without trapped neutrinos matter.

4. Summary
In above study, we considered three conditions of star evolution to study the strange quark matter in Polyakov chiral $SU(3)$ quark mean-field model. We have studied the effects of vector interaction and lepton fraction on pressure density and EoS and it is observed that the stiffness of EoS is decreased as proto-quark star evolves to the cold quark star at $g_v = 0$. In the future, with the help of present EoS, we will calculate the mass-radius relation and tidal-deformability of strange quark stars.

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