Dark Halo and Disk Galaxy Scaling Laws

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Abstract. I highlight recent progress in our understanding of the origin of disk galaxy scaling laws in a hierarchically clustering universe. Numerical simulations of galaxy formation in Cold Dark Matter (CDM) dominated universes indicate that the slope and scatter of the I-band Tully-Fisher (TF) relation are well reproduced in this model, although not, as proposed in recent work, because of the cosmological equivalence between halo mass and circular velocity, but rather as a result of the dynamical response of the halo to the assembly of the luminous component of the galaxy. The zero-point of the TF relation is determined mainly by the stellar mass-to-light ratio ($\Upsilon_I$) as well as by the concentration ($c$) of the dark halo. For $c \sim 10$, as is typical of halos formed in the “concordance” ΛCDM model, we find that this requires $\Upsilon_I \sim 1.5$, in reasonable agreement with the mass-to-light ratios expected of stellar populations with colors similar to those of TF galaxies. This conclusion supersedes that of Navarro & Steinmetz (2000a,b), who claimed the ΛCDM halos were too concentrated to be consistent with the observed TF relation. The disagreement can be traced to an incorrect normalization of the power spectrum used in that work. Our new results show that simulated disk galaxies in the ΛCDM scenario are not clearly inconsistent with the observed I-band Tully-Fisher relation. On the other hand, their angular momenta is much lower than observed. Accounting simultaneously for the spin, size and luminosity of disk galaxies remains a challenge for hierarchical models of galaxy formation.

1. Introduction

The structural parameters of dark matter halos are tightly related through simple scaling laws that reflect the cosmological context of their formation. These regularities are likely the result of the approximately scale-free process of assembly of collisionless dark matter into collapsed, virialized halos. One example is the relation between halo mass and size; a direct result of the finite age of the universe (see, e.g., Eke, Navarro & Frenk 1998 and references within). A second example concerns the angular momentum of dark halos, which is also linked to mass and size through simple scaling arguments (Peebles 1969, White...
Finally, similarities are also apparent in the internal structure of dark halos (Navarro, Frenk & White 1996, 1997, hereafter NFW). It has long been thought that the scaling properties of dark halos relate directly to analogous correlations between structural parameters of disk galaxies, a question that we have addressed in detail over the past few years using increasingly sophisticated N-body/gasdynamical simulations (Navarro & Steinmetz 1997, Steinmetz & Navarro 1999, Navarro & Steinmetz 2000a,b, hereafter NS00a,b). This work has shown that the velocity scaling of luminosity and angular momentum in spiral galaxies arise naturally in hierarchical galaxy formation models such as CDM. Large discrepancies, however, were observed in the zero-point of both correlations: at fixed rotation speed, simulated disks were found to be too small and too faint compared with their observational counterparts.

The failure of simulations to match the angular momentum of disk galaxies was ascribed to the assembly of the galaxy through a sequence of mergers, where the bulk of the angular momentum of the gas is transferred to the halo, as first suggested by Navarro & Benz (1991). Matching the spin of observed spirals appears to demand a large injection of energy (presumably from supernovae or AGNs) that prevents gas at early times from cooling and condensing into protogalaxies, shifting the bulk of star formation to later times and alleviating the angular momentum losses associated with major mergers.

The trouble with the zero-point of the Tully-Fisher relation may be traced to the “concentration” of dark halos, which determine the contribution of dark matter to the circular speed of galaxy disks: the higher the halo concentration the faster a disk of given mass must rotate to achieve centrifugal equilibrium. Thus the higher the concentration the lower the stellar mass-to-light ratio needed for galaxies to remain within the observed Tully-Fisher relation. As described by NS00, this property can be used to rule out of the “standard” CDM model ($\Omega = 1$, $h = 0.5$, $\sigma_8 = 0.6$, hereafter sCDM): halos formed in this scenario are so concentrated that the mass-to-light ratio required is unacceptably small.

NS00 also argued that a similar problem afflicts the currently popular “concordance” $\Lambda$CDM model ($\Omega_0 \sim 0.3$, $\Lambda \sim 0.7$, $h \sim 0.7$, $\sigma_8 \sim 1$), a result that added to an uncomfortably long list of concerns regarding the success of CDM on the scale of individual galaxies, such as the survival of a large number of halos within halos (at odds with the few satellites observed around the Milky Way; the “substructure” problem, Klypin et al 1999, Moore et al 1999) as well as evidence for constant density dark matter “cores” in some low surface brightness dwarfs (at odds with the steeply divergent dark matter density profiles expected in CDM universes, see, e.g., NFW).

Taken together, the evidence appears to warrant a radical revision of one or more of the premises of the CDM paradigm, and there has been no shortage of suggestions: self-interacting dark matter (Spergel & Steinhardt 2000), warm dark matter (Dalcanton & Hogan 2000, Bode et al 2000), fluid dark matter (Peebles 2000), etc., all aim to provide a model that behaves like CDM on large scales but with reduced substructure and concentration on the scale of individual galactic halos. Although the introduction of these alternative dark matter models has generated great interest, it is important to note that the presumed CDM “failures” that motivate them are not beyond doubt. For example, as noted by van den Bosch et al (2000) and van den Bosch & Swaters (2000), the...
evidence for constant density “cores” is sometimes weak and, at best, confined to a handful of galaxies. At the same time, arguments against the presence of a large number of “substructure” halos in the vicinity of the Milky Way (as CDM predicts) are indirect and so far inconclusive (see, e.g., White 2000).

I revisit below the argument of NS00 against ΛCDM based on the large concentration of dark halos formed in this scenario. As it turns out, the power spectrum of the ΛCDM simulations reported by NS00 was incorrectly normalized: the amplitude of mass fluctuations on $8h^{-1}$ Mpc scales was effectively $\sigma_8 \approx 1.6$ rather than the quoted 1.14. Our new simulations, reported fully in Eke, Navarro & Steinmetz (2000), show that the concentration of halos formed in the “concordance” ΛCDM model are not obviously inconsistent with constraints posed by dynamical observations of the Milky Way and by the zero-point of the I-band Tully-Fisher relation. I begin this contribution by reviewing briefly the theoretical motivation for halo scaling laws and their relation to disk galaxies, and then concentrate on our new results for the Tully-Fisher relation in the ΛCDM scenario. I am grateful to my collaborators, Vincent Eke and Matthias Steinmetz, for allowing me to discuss these results in advance of publication.

2. Scaling Laws

2.1. Mass, Radius, and Circular Velocity

The “size” of dark halos is usually associated with the distance from the center at which mass shells are infalling for the first time. This “virial” radius (a misnomer, since there is really nothing “virial” about it) sets a firm upper limit to the baryonic mass of the galaxy inside each halo: baryons beyond this radius have not had time yet to accrete onto the central galaxy. Virial radii, $r_\Delta$, are defined, at $z = 0$, by the region that contains a mean inner density contrast (relative to critical), of order $\Delta \sim 178\sqrt{\Omega_0}$. In terms of the circular velocity at the virial radius, $V_\Delta$, halo masses are given by,

$$M_\Delta (V_\Delta) = 1.9 \times 10^{12} \left( \frac{\Delta}{200} \right)^{1/2} \left( \frac{V_\Delta}{200 \text{ km s}^{-1}} \right)^3 h^{-1} M_\odot, \quad (1)$$

This power-law dependence on velocity is similar to that of the I-band Tully-Fisher relation of late-type spirals,

$$L_I \approx 2.0 \times 10^{10} \left( \frac{V_{\text{rot}}}{200 \text{ km s}^{-1}} \right)^3 h^{-2} L_\odot, \quad (2)$$

a coincidence that suggests a direct cosmological origin for this scaling law. Introducing the parameters $M_{\text{disk}}$ and $\Upsilon_I = M_{\text{disk}}/L_I$ to represent the mass of the disk and the disk mass-to-light ratio in solar units, respectively, eqs. 1 and 2 can be combined to yield $M_{\text{disk}}$ as a fraction of the total mass,

$$f_{\text{mdisk}} = \frac{M_{\text{disk}}}{M_\Delta} = 8.5 \times 10^{-3} h^{-1} \left( \frac{\Delta}{200} \right)^{1/2} \Upsilon_I \left( \frac{V_{\text{rot}}}{V_\Delta} \right)^3, \quad (3)$$

or, in terms of the total baryonic mass within $r_\Delta$ (assuming $\Omega_b = 0.0125 h^{-2}$),

$$f_{\text{bdisk}} = \frac{M_{\text{disk}}}{(\Omega_b/\Omega_0)M_\Delta} \approx 0.85 \Omega_0 h \Upsilon_I \left( \frac{\Delta}{200} \right)^{1/2} \left( \frac{V_{\text{rot}}}{V_\Delta} \right)^3. \quad (4)$$
Figure 1.  (a) **Left panel:** The disk mass fraction versus the ratio between disk rotation speed and halo circular velocity. The thick dashed and solid lines correspond to the constraint imposed on these two quantities by the Tully-Fisher relation (eq. 3) in the ΛCDM and sCDM scenarios, respectively. Dotted lines correspond to the relation expected for galaxies assembled in NFW halos of constant “concentration” parameter, as labeled. Constant disk mass-to-light ratios and Δ = 200 are assumed throughout; Υ_I = 2 in the upper panel and Υ_I = 1 in the lower one, respectively.  (b) **Right panel:** Specific angular momentum as a function of circular velocity. Dots are data on TF galaxies compiled from the literature. Symbols correspond to sCDM and ΛCDM models. Two models are shown for sCDM, corresponding to different choices of the feedback parameters $\epsilon_v$ and $c_*$ (see NS00 for references and details).

Therefore, the slope and zero-point of the Tully-Fisher relation imply, for a given cosmogony, a delicate balance between $M_{\text{disk}}$, Υ_I, and the ratio $V_{\text{rot}}/V_\Delta$.

The simplest way to satisfy eqs. 3 and 4 is that argued by Mo, Mao & White (1998), who suggest approximately constant values of all these parameters for all galaxies: $f_{\text{mdsk}} \sim 5 \times 10^{-2}$, Υ $\sim 1.7h$, and $V_{\text{rot}} \sim 1.5V_\Delta$. Although plausible, this assumption is at odds with numerical experiments, which show that these parameters vary widely from halo to halo (NS00). This suggests another possibility: that $f_{\text{mdsk}}$, Υ_I and $V_{\text{rot}}/V_\Delta$ are not constant from halo to halo but correlated in the manner prescribed by equation 3. Such correlation may actually arise as a result of the dynamical response of the dark halo to the assembly of the galaxy. This is illustrated in Figure 1a, where we show, for two choices of Υ_I, the relation between $V_{\text{rot}}/V_\Delta$ and $f_{\text{mdsk}}$ computed under the assumption that the structure of the halo can be approximated by an NFW profile and that it responds “adiabatically” to the assembly of the disk (Mo et al 1998). The thick solid and dashed lines correspond to the constraint enunciated in eq. 3 for two different cosmological models; sCDM and ΛCDM. The rightmost
point in each of the thick lines corresponds to the maximum disk mass fraction allowed by the baryonic content of the halo. Dotted lines show the results of applying the adiabatic contraction approximation to the halo for different values of the NFW “concentration” parameter, \( c \). Figure 1a shows that \( \Upsilon_I \) and the cosmological parameters determine in practice the range of halo concentrations consistent with the zero-point of the TF relation. For example, sCDM halos must have \( c \lesssim 5 \) if \( \Upsilon_I \approx 1 \). This effectively rules out the sCDM scenario, since N-body simulations show that sCDM halos have typically much higher concentrations, \( c \sim 15-20 \) (NFW). Higher concentrations are acceptable for ΛCDM, mainly as a result of the different value of the Hubble constant assumed in that model, which makes all galaxies dimmer at a given rotation speed. Concentrations as high as \( c \sim 10-12 \) are acceptable if \( \Upsilon_I \approx 1 \).

Another important point illustrated in Figure 1a is that the structure and dynamical response of the halo to the assembly of the disk may be responsible for the small scatter in the Tully-Fisher relation. For illustration, consider two halos of the same mass, and therefore approximately similar concentration, where the fraction of baryons collected into the central galaxy, \( f_{\text{mdsk}} \), differs substantially. Provided that \( f_{\text{mdsk}} > 0.02 \), where the “adiabatic contraction” dotted curves are approximately parallel to the observational constraint delineated by the thick lines, these two galaxies will lie approximately along the same Tully-Fisher relation: galaxies scatter along the Tully-Fisher relation due to the halo response. Even if the concentration of the two halos were to differ greatly, its effect on the scatter of the Tully-Fisher relation would be relatively minor: at fixed \( f_{\text{mdsk}} \), \( V_{\text{rot}}/V_{\Delta} \) changes by only about 20% when \( c \) changes by a factor of two.

### 2.2. Circular Velocity and Angular Momentum

Another similarity between the properties of dark halos and galaxy disks concerns their angular momentum. N-body simulations show that, in terms of the dimensionless rotation parameter, \( \lambda = J/E^{1/2}/GM_{\Delta}^{5/2} \) (\( J \) and \( E \) are the total angular momentum and binding energy of the halo, respectively), the distribution of halo angular momenta is approximately independent of mass, redshift, and cosmological parameters, and peaks at around \( \lambda \sim 0.05 \) (Cole & Lacey 1996 and references therein). The binding energy depends on the internal structure of the halos but the structural similarity between dark halos established by NFW implies that \( E \) is to good approximation roughly proportional to \( M_{\Delta}V_{\Delta}^2 \), with a very weak dependence on concentration (see Mo et al 1998 for further details). The specific angular momentum of the halo then may be written as,

\[
j_{\Delta} \approx 2 \frac{\lambda}{\Delta^{1/2}} \frac{V_{\Delta}^2}{H_0} = 2.8 \times 10^3 \left( \frac{\Delta}{200} \right)^{-1/2} \left( \frac{V_{\Delta}}{200 \text{ km s}^{-1}} \right)^2 \text{ km s}^{-1} h^{-1} \text{ kpc},
\]

where we have used the most probable value of \( \lambda = 0.05 \) in the second equality. The simple velocity-squared scaling of this relation is identical to that illustrated in Figure 1b between the specific angular momentum of disks and their rotation

\[1 \text{c} = r_{\Delta}/r_s, \text{ where } r_s \text{ is the scale radius of the NFW density profile, } \rho(r) \propto (r/r_s)^{-1}(1+r/r_s)^{-2} \]
speed (solid line in Figure 1b),

\[ j_{\text{disk}} \approx 1.3 \times 10^3 \left( \frac{V_{\text{rot}}}{200 \text{ km s}^{-1}} \right)^2 \text{ km s}^{-1} \text{h}^{-1} \text{kpc} \]  

(6)

suggestive, as in the case of the Tully-Fisher relation, of a cosmological origin for this scaling law.

Combining eqs. 5 and 6, we can express the ratio between disk and halo specific angular momenta as,

\[ f_j = \frac{j_{\text{disk}}}{j_\Delta} \approx 0.45 \left( \frac{\Delta}{200} \right)^{1/2} \left( \frac{V_{\text{rot}}}{V_\Delta} \right)^2. \]  

(7)

If the rotation speeds of galaxy disks are approximately the same as the circular velocity of their surrounding halos, then disks must have retained about one-half of the available angular momentum during their assembly.

The velocity ratio may be eliminated using eq. 4 to obtain a relation between the fraction of baryons assembled into the disk and the angular momentum ratio,

\[ f_j \approx 0.5 \left( \frac{\Delta}{200} \right)^{1/6} \left( \frac{f_{b\text{disk}}}{\Omega_0 h \Upsilon_I} \right)^{2/3}. \]  

(8)

This combined constraint posed by the Tully-Fisher and the angular momentum-velocity relation is shown in Figure 2a for two different cosmological models. As in Figure 1a, thick solid lines correspond to the “standard” cold dark matter model, sCDM, and thick dashed lines to the ΛCDM model. Each curve is labeled by the value adopted for the disk mass-to-light ratio, \( \Upsilon_I \). The precise values of \( f_{b\text{disk}} \) and \( f_j \) along each curve are determined by the ratio \( V_{\text{rot}}/V_\Delta \), and are shown by starred symbols for the case \( V_{\text{rot}} = V_\Delta \) and \( \Upsilon_I = 1 \).

One important point illustrated by Figure 2a is that disk galaxies formed in a low-density universe, such as ΛCDM, need only accrete a small fraction of the total baryonic mass to match the zero-point of the Tully-Fisher relation, but must draw a comparably much larger fraction of the available angular momentum to be consistent with the spins of spiral galaxies. For example, if \( V_{\text{rot}} = V_\Delta \) and \( \Upsilon_I = 1 \), disk masses amount to only about 30% of the total baryonic mass of a ΛCDM halo but contain about 60% of the available angular momentum. This is intriguing and, at face value, counterintuitive. Angular momentum is typically concentrated in the outer regions of the system (see, e.g., Figure 9 in Navarro & Steinmetz 1997, NS97), presumably the ones least likely to cool and be accreted into the disk, so it is puzzling that galaxies manage to tap a large fraction of the available angular momentum whilst collecting a small fraction of the total mass. The simulations in NS97, which include the presence of a strong photo-ionizing UV background, illustrate exactly this dilemma; the UV background suppresses the cooling of late-infalling, low-density, high-angular momentum gas and reduces the angular momentum of cold gaseous disks assembled at the center of dark matter halos.

The situation is less severe in high-density universes such as sCDM; we see from Figure 2a that disks are required to collect similar fractions of mass and of angular momentum in order to match simultaneously the Tully-Fisher
and the spin-velocity relations. As a result, any difficulty matching the angular momentum of disk galaxies in sCDM will become only worse in a low-density ΛCDM universe.

Note as well that the problem becomes more severe the lower the mass-to-light ratio of the disk. Indeed, from the point of view of this constraint, it would be desirable for disks formed in the ΛCDM scenario to have \( \Upsilon_I > 2 \); in this case \( f_j \sim f_{\text{bdsk}} \) would be consistent with the constraint posed by observed scaling laws. However, this is the opposite of what is required to reconcile highly-concentrated halos with the zero-point of the Tully-Fisher relation. This conundrum illustrates the fact that accounting simultaneously for the mass and angular momentum of disk galaxies represents a serious challenge to hierarchical models of galaxy formation.

3. Numerical Scaling Laws

3.1. The Tully-Fisher relation

The symbols with horizontal error bars in Figure 2b show the numerical Tully-Fisher relation obtained in our N-body/gasdynamical simulations. These simulations include the main physical ingredients considered relevant to the formation of galaxies: self-gravity, hydrodynamical shocks, radiative cooling, photoheating, star formation and feedback. Solid (open) circles denote the luminosities and rotation speeds of galaxy models formed in the ΛCDM (sCDM) scenario. Error bars span the range in luminosities corresponding to assuming either a Salpeter or a Scalo stellar initial mass function. As is clear from Figure 2b, the slope of the numerical TF relation in both cosmologies is consistent with the observed values, and the scatter is much smaller than observed (only 0.12 mag in the case of ΛCDM).

A detailed analysis confirms that this is because of the response of the halo to the assembly of the disk, as discussed in §2 and in NS00. The main difference with the latter work concerns the ΛCDM results: the zero-point of this relation is offset only by 0.5 mag, compared with the 1.5 mag reported by NS00. The reason for the discrepancy can be traced to the fact that NS00 used an outdated transfer function for the ΛCDM transfer function, which led to a significantly higher effective normalization than intended; instead of \( \sigma_8 = 1.14 \), NS00’s simulations had effectively \( \sigma_8 = 1.6 \). Halos in a \( \sigma_8 = 1.6 \) universe collapse much earlier and have concentrations about twice as high as in the “concordance” \( \sigma_8 \approx 1 \) ΛCDM model (for details see Eke, Navarro & Steinmetz 2000), leading to a much larger zero-point offset than seen in Figure 2b.

The remaining 0.5 mag difference between simulation and observation is perhaps not too worrying, given that the simulated galaxies have colors that are slightly too red compared with their TF counterparts. Indeed, the average \( B-R \) color in the simulations is \( \sim 1.0 \), compared with the \( \sim 0.8 \) average in the sample of Courteau (1997). This suggests that star formation in the simulations proceeds too quickly and too early; any modification to the feedback algorithm that remedies this will also tend to make the stellar population mix in the simulated galaxies brighter. If this correction can bring \( \Upsilon_I \) down by \( \sim 50\% \) then the remaining 0.5 mag offset between simulations and observations should be possible to bridge. To summarize, it appears that, if \( \Upsilon \approx 1.5 \), then galaxies formed
Figure 2.  (a) **Left panel:** The fraction of the baryons assembled into a disk galaxy ($f_{bdsk}$) versus the ratio between the specific angular momenta of the disk and its surrounding halo ($f_j$). Thick solid and dashed lines correspond to the constraints imposed by the Tully-Fisher relation and by the relation between rotation speed and angular momentum (eq. 8). The solid (dashed) thick line corresponds to the sCDM ($\Lambda$CDM) scenario, shown for different values of $Y_I$, as labeled. Symbols correspond to simulated galaxy models as per the labels in Figure 1b. (b) **Right panel:** The I-band Tully-Fisher relation compared with the results of the numerical simulations. Dots correspond to the observational samples of Mathewson, Ford & Buchhorn, (1992), Giovanelli et al (1997), and Han & Mould (1992). Error bars in the simulated magnitudes correspond to adopting a Salpeter or a Scalo IMF.

in $\Lambda$CDM halos have properties that are consistent with the slope, scatter, and zero-point of the I-band Tully-Fisher relation.

### 3.2. The angular momentum problem

Despite its success at accounting for the TF relation, simulated galaxies fail to match the observed spin of spiral galaxies in both sCDM and $\Lambda$CDM scenarios. As in our previous work, we associate these problems with the assembly of galaxies through merging. The magnitude of the problem is shown in Figure 1b, where it is clear that the angular momentum of simulated disks is well below observed values, as a result of the loss of the bulk of their angular momentum (see symbols in Figure 2a). This serious difficulty may reflect limitations in our implementation of feedback processes or may indicate a fundamental flaw in our current hierarchical picture of galaxy assembly. Nevertheless, it makes clear that accounting simultaneously for the luminosity, velocity, and angular momentum of spiral galaxies remains a challenging problem for the popular $\Lambda$CDM cosmogony.
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