General Gauge Field Theory And Its Application

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Abstract

A gauge field model, which simultaneously has strict local gauge symmetry and contains massive general gauge bosons, is discussed in this paper. The model has $SU(N)$ gauge symmetry. In order to introduce the mass term of gauge fields directly without violating the gauge symmetry of the theory, two sets of gauge fields will be introduced into the theory. After some transformations, one set of gauge fields obtain masses and another set of gauge fields keep massless. In the limit $\alpha \to 0$ or $\alpha \to \infty$, the gauge field model discussed in this paper will return to Yang-Mills gauge field model. Finally, some applications of this model are discussed.

1 Introduction

Yang and Mills founded non-Abel gauge field theory in 1954[1]. Since then, gauge field theory has been extensively applied to elementary particle theories. Now, it is generally believed that four kinds of fundamental interactions, i.e. strong interactions, electromagnetic interactions, weak interactions and gravitation, are all gauge interactions. From theoretical point of view, the requirement of gauge invariant determines the forms of interactions. But for Yang-Mills gauge theory, if lagrangian has strict local gauge symmetry, the masses of gauge fields must be zero. On the other hand, physicists found that the masses of intermediate bosons are very large in the forties[2]. After introducing spontaneously symmetry breaking and Higgs mechanism, Glashow[3], Weinberg[4] and Salam[5] founded the well-known unified electroweak standard model. The standard model is consonant well with experiments and intermediate bosons $W^\pm$ and $Z^0$ have already been found by experiments. But Higgs particle has not been found by experiments until now. We know that Higgs particle is necessitated by the standard model. Whether Higgs particle exists in nature? If there were no Higgs particle, how should we construct the unified electroweak model?
A possible way to solve this problem is to use general gauge field theory[6]. The general gauge field theory not only has strict local gauge symmetry, but also contains massive general gauge bosons. Using general gauge field theory, we could construct an electroweak model which contains no Higgs particle[7].

2 The lagrangian of the model

Suppose that $N$ fermion fields $\psi_l(x) \ (l = 1, 2, \ldots, N)$ form a multiplet of matter fields. The state of matter fields is denote as:

$$\psi(x) = \left( \begin{array}{c} \psi_1(x) \\ \psi_2(x) \\ \vdots \\ \psi_N(x) \end{array} \right) \quad (2.1)$$

The system has $SU(N)$ symmetry. The representative matrices of generators of $SU(N)$ group are denoted by $T_i \ (i = 1, 2, \ldots, N^2 - 1)$. They satisfy:

$$[T_i, T_j] = i f_{ijk} T_k \quad (2.2)$$
$$Tr(T_i T_j) = \delta_{ij} K. \quad (2.3)$$

A general element of the $SU(N)$ group can be expressed as:

$$U = e^{-i \alpha^i T_i} \quad (2.4)$$

with $\alpha^i$ the real group parameters. $U$ is a unitary $N \times N$ matrix.

We need two kinds of gauge fields $A_\mu(x)$ and $B_\mu(x)$. They can be expressed as linear combinations of generators:

$$A_\mu(x) = A^i_\mu(x) T_i \quad (2.5a)$$
$$B_\mu(x) = B^i_\mu(x) T_i. \quad (2.5b)$$

where $A^i_\mu(x)$ and $B^i_\mu(x)$ are component fields. In the present model, there are two gauge fields corresponds to one gauge symmetry. Corresponds to two kinds of gauge fields, there are two kinds of gauge covariant derivatives

$$D_\mu = \partial_\mu - ig A_\mu \quad (2.6a)$$
$$D_\mu = \partial_\mu + i \alpha g B_\mu. \quad (2.6b)$$

The strengths of gauge fields are:

$$A_{\mu \nu} = \frac{1}{-ig} [D_\mu, D_\nu]$$
$$= \partial_\mu A_\nu - \partial_\nu A_\mu - ig [A_\mu, A_\nu] \quad (2.7a)$$
\[ B_{\mu\nu} = \frac{1}{i\alpha g} [D_{b\mu}, D_{b\nu}] \]
\[ = \partial_\mu B_{\nu} - \partial_\nu B_{\mu} + i\alpha g [B_{\mu}, B_{\nu}]. \]  
\[(2.7b)\]

Similarly, \( A_{\mu\nu} \) and \( B_{\mu\nu} \) can also be expressed as linear combinations of generators:

\[ A_{\mu\nu} = A^i_{\mu\nu} T_i \]  
\[ (2.8a) \]
\[ B_{\mu\nu} = B^i_{\mu\nu} T_i, \]  
\[ (2.8b) \]

where

\[ A^i_{\mu\nu} = \partial_\mu A^i_{\nu} - \partial_\nu A^i_{\mu} + g f^{ijk} A^j_{\mu} A^k_{\nu} \]  
\[ (2.9a) \]
\[ B^i_{\mu\nu} = \partial_\mu B^i_{\nu} - \partial_\nu B^i_{\mu} - \alpha g f^{ijk} B^j_{\mu} B^k_{\nu}. \]  
\[ (2.9b) \]

The lagrangian density of the model is:

\[ \mathcal{L} = -\bar{\psi} (\gamma^\mu D_\mu + m) \psi \]
\[ -\frac{1}{4\kappa} Tr (A^{\mu\nu} A_{\mu\nu}) - \frac{1}{4\kappa} Tr (B^{\mu\nu} B_{\mu\nu}) \]
\[ -\frac{\mu^2}{2\kappa (1 + \alpha^2)} Tr [(A^\mu + \alpha B^\mu) (A^\mu + \alpha B^\mu)] \]  
\[ (2.10) \]

where \( \alpha \) is a constant.

Local gauge transformations are

\[ \psi \rightarrow \psi' = U \psi, \]  
\[ (2.11) \]
\[ A_\mu \rightarrow U A_\mu U^\dagger - \frac{1}{ig} U \partial_\mu U^\dagger \]  
\[ (2.12) \]
\[ B_\mu \rightarrow U B_\mu U^\dagger + \frac{1}{i\alpha g} U \partial_\mu U^\dagger \]  
\[ (2.13) \]

then,

\[ A_{\mu\nu} \rightarrow U A_{\mu\nu} U^\dagger \]  
\[ (2.14) \]
\[ B_{\mu\nu} \rightarrow U B_{\mu\nu} U^\dagger \]  
\[ (2.15) \]
\[ (A_\mu + \alpha B_\mu) \rightarrow U (A_\mu + \alpha B_\mu) U^\dagger \]  
\[ (2.16) \]

It is easy to prove that the lagrangian is invariant under the above local gauge transformations. Therefore the model has strict local gauge symmetry. In the above lagrangian, we have introduced mass terms of gauge fields without violating the gauge symmetry of the model.
3 Masses of general gauge fields

If we select $A_\mu$ and $B_\mu$ as basis, the mass matrix of gauge fields is:

$$M = \frac{1}{1 + \alpha^2} \begin{pmatrix} \mu^2 & \alpha \mu^2 \\ \alpha \mu^2 & \alpha^2 \mu^2 \end{pmatrix}$$  \hspace{1cm} (3.1)

The masses of gauge fields are given by eigenvalues of mass matrix. Mass matrix has two eigenvalues:

$$m_1^2 = \mu^2, \quad m_2^2 = 0.$$  \hspace{1cm} (3.2)

The wave functions of physical particles are given by eigenvectors of mass matrix. Two eigenvectors of mass matrix are:

$$\begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}, \begin{pmatrix} -\sin \theta \\ \cos \theta \end{pmatrix}$$  \hspace{1cm} (3.3)

where

$$\cos \theta = \frac{1}{\sqrt{1 + \alpha^2}}, \quad \sin \theta = \frac{\alpha}{\sqrt{1 + \alpha^2}}.$$  \hspace{1cm} (3.4)

The wave functions of physical particles are defined as:

$$C_\mu = \cos \theta A_\mu + \sin \theta B_\mu$$  \hspace{1cm} (3.5a)

$$F_\mu = -\sin \theta A_\mu + \cos \theta B_\mu.$$  \hspace{1cm} (3.5b)

The above transformations are pure field transformations. They can be regarded as redefinition of gauge fields. They do not affect the symmetry of the lagrangian. So, the local gauge symmetry of the lagrangian is still strictly preserved after the above transformations.

After the above transformations, the lagrangian density changes into

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1,$$  \hspace{1cm} (3.6)

$$\mathcal{L}_0 = -\bar{\psi}(\gamma^\mu \partial_\mu + m)\psi - \frac{1}{4} C^{ij\mu} \bar{C}_0^{ij\mu} - \frac{1}{4} F^{ij\mu} \bar{F}_0^{ij\mu} - \frac{\mu^2}{2} C^{ij\mu} \bar{C}_0^{ij\mu}.$$  \hspace{1cm} (3.7)

$$\mathcal{L}_1 = \frac{ig}{2} \bar{\psi} C_\mu \psi - \frac{1}{4} \alpha^2 g f^{ijk} C_0^{ij\mu} C_\mu C_\nu C_\sigma + \frac{1}{2} g f^{ijk} C_0^{ij\mu} F_\mu F_\nu + \frac{1}{4} g f^{ijk} C_0^{ij\mu} F_\mu F_\nu - \frac{1}{4} \alpha^2 g f^{ijk} F_0^{ij\mu} C_\mu C_\nu.$$  \hspace{1cm} (3.8)
From free lagrangian $\mathcal{L}_0$, we could see that the mass of gauge field $C_\mu$ is $\mu$ and the mass of gauge field $F_\mu$ is zero.

\[ m_c = \mu, \quad m_F = 0. \quad (3.9) \]

Local gauge transformations of $C_\mu$ and $F_\mu$ respectively are:

\[ C_\mu \rightarrow UC_\mu U^\dagger \quad (3.10) \]

\[ F_\mu \rightarrow UF_\mu U^\dagger + \frac{1}{ig\sin\theta}U\partial_\mu U^\dagger \quad (3.11) \]

The form of local gauge transformation of $C_\mu$ is homogeneous. But obviously, $C_\mu$ is not an ordinary vector field or an ordinary matter field, because $C_\mu$ is a linear combination of the standard gauge fields $A_\mu$ and $B_\mu$ and transmits gauge interactions between matter fields. So, we call $C_\mu$ general gauge field for the moment.

## 4 Yang-Mills Limits

In a proper limit, general gauge field theory can return to Yang-Mills gauge field theory. There are two kinds of Yang-Mills limits.

The first kind of Yang-Mills limit corresponds to very small parameter $\alpha$. Let:

\[ \alpha \rightarrow 0, \quad (4.1) \]

then

\[ \cos\theta = 1, \quad \sin\theta = 0. \quad (4.2) \]

\[ C_\mu = A_\mu, \quad F_\mu = B_\mu. \quad (4.3) \]

In this case, the lagrangian density becomes

\[ \mathcal{L} = -\bar{\psi}[\gamma^\mu(\partial_\mu - igC^i_\mu T^i) + m]\psi \]

\[ -\frac{1}{4}C^{i\mu\nu}C^i_{\mu\nu} - \frac{1}{4}F^{i\mu\nu}F^i_{\mu\nu} - \frac{\mu^2}{2}C^i_\mu C^i_\mu. \quad (4.4) \]

We could see that, only massive gauge field directly interacts with matter field. This limit corresponds to the case that gauge interactions are mainly transmitted by massive gauge field. But if $\alpha$ strictly vanishes, the lagrangian does not have gauge symmetry and the theory is not renormalizable.

The second kind of Yang-Mills limit corresponds to very large parameter $\alpha$. Let

\[ \alpha \rightarrow \infty, \quad (4.5) \]
then
\[ \cos \theta = 0, \quad \sin \theta = 1. \] (4.6)
\[ C_\mu = B_\mu, \quad F_\mu = -A_\mu. \] (4.7)

Then, the lagrangian density becomes
\[
\mathcal{L} = -\bar{\psi} i \gamma^\mu \left( \partial_\mu + ig F_\mu^i T^i \right) + m \bar{\psi} \psi - \frac{i}{4} F_{\mu\nu}^i F_i^{\mu\nu} - \frac{1}{4} C_{\mu\nu}^i C_i^{\mu\nu} - \frac{\mu^2}{2} C_\mu C^\mu. \] (4.8)

In this case, only massless gauge field directly interacts with matter fields. This limit corresponds to the case that gauge interactions are mainly transmitted by massless gauge field.

In the particles’ interaction model, the parameter \( \alpha \) should be finite,
\[ 0 < \alpha < \infty. \] (4.9)
In this case, both massive gauge field and massless gauge field directly interact with matter fields, and gauge interactions are transmitted by both of them.

5 Is the theory renormalizable

We know that the propagator of a massive vector field usually has the following form:
\[
\Delta F_{\mu\nu} = \frac{-i}{k^2 + \mu^2 - i\varepsilon} \left( g_{\mu\nu} + \frac{k_\mu k_\nu}{\mu^2} \right). \] (5.1)

So, when we let
\[ k \longrightarrow \infty \] (5.2)
then,
\[ \Delta F_{\mu\nu} \longrightarrow \text{const.} \] (5.3)

According to the power counting law, a massive vector field model is not renormalizable in most case. Though general gauge field theory contains massive vector fields, it is renormalizable. The key reason is that the lagrangian has local gauge symmetry[8].

The general gauge field theory has maximum local \( SU(N) \) gauge symmetry. When we quantize the general gauge field theory in the path integral formulation, we must select gauge conditions first[9]. In order to make gauge transformation degree of freedom completely fixed, we must select two gauge conditions simultaneously: one is for massive gauge field \( C_\mu \) and another is for massless gauge field \( F_\mu \). For example, if we select temporal gauge condition for massless gauge field \( F_\mu \):
\[ F_4 = 0, \] (5.4)
there still exists remainder gauge transformation degree of freedom, because temporal gauge condition is unchanged under the following local gauge transformation:

\[ F_\mu \rightarrow UF_\mu U^\dagger + \frac{1}{ig\sin\theta} U\partial_\mu U^\dagger \]  (5.5)

where

\[ \partial_t U = 0, \quad U = U(x). \]  (5.6)

In order to make this remainder gauge transformation degree of freedom completely fixed, we had better select another gauge condition for gauge field \( C_\mu \). For example, we could select Lorentz gauge condition for gauge field \( C_\mu \):

\[ \partial^\mu C_\mu = 0. \]  (5.7)

If we select two gauge conditions simultaneously, when we quantize the theory in path integral formulation, there will be two gauge fixing terms in the effective lagrangian. The effective lagrangian could be written as:

\[ \mathcal{L}_{\text{eff}} = \mathcal{L} - \frac{1}{2\alpha_1} f_1^a f_1^{a} - \frac{1}{2\alpha_2} f_2^a f_2^{a} + \bar{\eta}_1 M f_1 \eta_1 + \bar{\eta}_2 M f_2 \eta_2 \]  (5.8)

where

\[ f_1^a = f_1^a(F_\mu), \quad f_2^a = f_2^a(C_\mu) \]  (5.9)

We could select

\[ f_2^a = \partial^\mu C_\mu^a. \]  (5.10)

In this case, the propagator of massive gauge field \( C_\mu \) is

\[ \Delta_{F_{\mu\nu}}^{ab}(k) = \frac{-i\delta^{ab}}{k^2 + \mu^2 - i\varepsilon} \left( g_{\mu\nu} - (1 - \frac{1}{\alpha_2}) \frac{k_\mu k_\nu}{k^2 - \mu^2/\alpha_2} \right). \]  (5.11)

If we let \( k \) approach infinity, then

\[ \Delta_{F_{\mu\nu}}^{ab}(k) \sim \frac{1}{k^2}. \]  (5.12)

According to the power counting law, general gauge field theory is a kind of renormalizable theory.

The local \( SU(N) \) gauge symmetry will also give a Ward-Takahashi identity which will eventually make the theory renormalizable. So, in the renormalization of the general gauge field theory, local gauge symmetry plays the following two important roles: 1) to make the propagators of massive gauge bosons have the renormalizable form; 2) to give a Ward-Takahashi identity which plays a key role in the renormalization of the general gauge field theory.
6 Electroweak model without Higgs particle

As an example, let’s discuss electroweak interactions of leptons. For the sake of convenience, let $e$ represent leptons $e, \mu$ or $\tau$, and $\nu$ represent the corresponding neutrinos $\nu_e, \nu_\mu$ or $\nu_\tau$. According to the standard model, $e$ and $\nu$ form left-hand doublet $\psi_L$ which has $SU(2)_L$ symmetry and right-hand singlet $e_R$. Neutrinos have no right-hand singlets. The definitions of these states are:

$$\psi_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L, \quad y = -1$$

$e_R$, $y = -2$, \hspace{1cm} (6.1)

$y$ is the quantum number of weak hypercharge $Y$.

Four gauge fields are needed in the new electroweak theory. They are two non-Abel gauge fields $F_{1\mu}$ and $F_{2\mu}$ corresponding to the $SU(2)_L$ symmetry and two Abel gauge fields $B_{1\mu}$ and $B_{2\mu}$ corresponding to the $U(1)_Y$ symmetry. The strengths of four gauge fields are respectively defined as:

$$F_{1\mu\nu} = \partial_\mu F_{1\nu} - \partial_\nu F_{1\mu} - ig[F_{1\mu}, F_{1\nu}], \hspace{1cm} (6.3a)$$

$$F_{2\mu\nu} = \partial_\mu F_{2\nu} - \partial_\nu F_{2\mu} + ig\tan\alpha[F_{2\mu}, F_{2\nu}], \hspace{1cm} (6.3b)$$

$$B_{m\mu\nu} = \partial_\mu B_{m\nu} - \partial_\nu B_{m\mu}, \quad (m = 1, 2). \hspace{1cm} (6.3c)$$

In order to introduce symmetry breaking and masses of all fields, a vacuum potential is needed. It has mass dimension. It has no kinematic energy term in the lagrangian. So, it has no dynamical degree of freedom. The coupling between vacuum potential and matter fields can be regarded as a kind of interactions between vacuum and matter fields.

The lagrangian density of the model is:

$$\mathcal{L} = \mathcal{L}_l + \mathcal{L}_g + \mathcal{L}_{v-l}, \hspace{1cm} (6.4)$$

where

$$\mathcal{L}_l = -\overline{\psi}_L \gamma^\mu (\partial_\mu + \frac{i}{2}gB_{1\mu} - igF_{1\mu})\psi_L - \overline{\tau}_R \gamma^\mu (\partial_\mu + igB_{1\mu})e_R$$

$$\mathcal{L}_g = -\frac{1}{4}F_{1\mu\nu}F_{1}^{\mu\nu} - \frac{1}{4}F_{2\mu\nu}F_{2}^{\mu\nu} - \frac{1}{4}B_{1\mu\nu}B_{1\mu\nu} - \frac{1}{4}B_{2\mu\nu}B_{2\mu\nu}$$

$$- v^\dagger [\cos\theta_W (\cos\alpha F_{1}^{\mu} + \sin\alpha F_{2}^{\mu}) - \sin\theta_W (\cos\alpha B_{1}^{\mu} + \sin\alpha B_{2}^{\mu})]$$

$$\cdot [\cos\theta_W (\cos\alpha F_{1}^{\mu} + \sin\alpha F_{2}^{\mu}) - \sin\theta_W (\cos\alpha B_{1}^{\mu} + \sin\alpha B_{2}^{\mu})] v$$

$$\mathcal{L}_{v-l} = -f (\overline{\tau}_R v^\dagger \psi_L + \overline{\psi}_L ve_R), \hspace{1cm} (6.6)$$

(6.7)
where $f$ is a dimensionless parameter, $\alpha$ is a constant, $g$, $g'$ are coupling constants and $\theta_W$ are Weinberg angle. $v$ is the vacuum potential.

In the original lagrangian density, $v$ has gauge transformation degree of freedom. But in our real physical world, the state of vacuum can not be varied freely and the properties of vacuum are rather stable, it has no gauge transformation degree of freedom. In the local inertial coordinate system, vacuum is invariant under space-time translation. So $v$ is well-distributed, it is a constant. Suppose that $v$ has the following value

$$v = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix},$$  
(6.8)

where $v_1$ and $v_2$ satisfy the following relation:

$$v_1^2 + v_2^2 = \mu^2 / 2,$$  
(6.9)

Make a global $SU(2)_L$ gauge transformation so as to make $v$ change into the following form

$$v = \begin{pmatrix} 0 \\ \mu / \sqrt{2} \end{pmatrix},$$  
(6.10)

When $v$ takes fixed value, the symmetry of the lagrangian is broken simultaneously.

Gauge fields $F_{1\mu}$, $F_{2\mu}$, $B_{1\mu}$ and $B_{2\mu}$ are not eigenvectors of mass matrix. In order to obtain eigenvectors of mass matrix, we will make the following two sets of transformations of fields. The first set of transformations are:

$$W_\mu = \cos \alpha F_{1\mu} + \sin \alpha F_{2\mu}$$  
(6.11a)

$$W_{2\mu} = -\sin \alpha F_{1\mu} + \cos \alpha F_{2\mu}$$  
(6.11b)

$$C_{1\mu} = \cos \alpha B_{1\mu} + \sin \alpha B_{2\mu}$$  
(6.11c)

$$C_{2\mu} = -\sin \alpha B_{1\mu} + \cos \alpha B_{2\mu}.$$  
(6.11d)

The second set of transformations are:

$$Z_\mu = \sin \theta_W C_{1\mu} - \cos \theta_W W_{3\mu}$$  
(6.12a)

$$A_\mu = \cos \theta_W C_{1\mu} + \sin \theta_W W_{3\mu}$$  
(6.12b)

$$Z_{2\mu} = \sin \theta_W C_{2\mu} - \cos \theta_W W_{3\mu}$$  
(6.12c)

$$A_{2\mu} = \cos \theta_W C_{2\mu} + \sin \theta_W W_{3\mu}.$$  
(6.12d)
After all these transformations, the lagrangian densities of the model change into:

\[ L_l + L_v = -\bar{\psi} (\gamma^\mu \partial_\mu + \frac{1}{\sqrt{2}} f_\mu) e - \bar{\nu} L \gamma^\mu \partial_\mu \nu_L \]

\[ + \frac{i}{2} \sqrt{g^2 + g'^2} \sin2\theta_W j^e_{\mu} \]

\[ - \sqrt{g^2 + g'^2} j^\nu L (\cos^2 \theta_W - \sin^2 \theta_W) \]

\[ + \frac{\sqrt{2}}{2} i g \partial_L \gamma^\mu (\cos^2 \theta_W - \sin^2 \theta_W) \]

\[ + \frac{\sqrt{2}}{2} i g \partial_L \gamma^\mu \nu_L (\cos^2 \theta_W - \sin^2 \theta_W) \]

\[ (6.13) \]

\[ L_g = -\frac{1}{2} W^+_{\mu\nu} W^-_{\nu\mu} - \frac{1}{4} Z^\mu \nu Z_{\mu\nu} - \frac{1}{4} A^\mu \nu A_{\mu\nu} \]

\[ + \frac{1}{2} W^+_{\mu\nu} W^-_{\nu\mu} - \frac{1}{4} Z^\mu \nu Z_{\mu\nu} - \frac{1}{4} A^\mu \nu A_{\mu\nu} \]

\[ (6.14) \]

where \( L_g \) only contains interaction terms of gauge fields. Definitions of field strengths are:

\[ W^\pm_{\mu\nu} = \frac{1}{\sqrt{2}} (W^1_{\mu\nu} = i W^2_{\mu\nu}) \] (6.15)

\[ W^\pm_{\mu\nu} = \partial^\mu W^\pm_{\nu\mu} - \partial^\nu W^\pm_{\mu\nu} \] (6.16)

\[ Z_{\mu\nu} = \partial^\mu Z_{\nu\mu} - \partial^\nu Z_{\mu\nu} \] (6.17)

\[ A_{\mu\nu} = \partial^\mu A_{\nu\mu} - \partial^\nu A_{\mu\nu} \] (6.18)

The currents in the above lagrangian are defined as:

\[ j^e_{\mu} = -\bar{\psi} \gamma^\mu e \]

\[ (6.19) \]

\[ j^Z = j^3_\mu - \sin^2 \theta_W j^e_{\mu} = i \bar{\psi} L \gamma_\mu \frac{\tau^3}{2} \psi_L - \sin^2 \theta_W \bar{\psi} L \gamma_\mu e. \]

\[ (6.20) \]

From the above lagrangian, we could see that the mass of fermion \( e \) is \( \frac{1}{\sqrt{2}} f_\mu \), the mass of neutrino is zero, the masses of charged intermediate gauge bosons \( W^\pm \) are \( \mu \cos \theta_W \), the mass of neutral intermediate gauge boson \( Z \) is \( \mu = \frac{m_W}{\cos \theta_W} \) and all other gauge fields are massless. That is

\[ m_e = \frac{1}{\sqrt{2}} f_\mu \quad m_\nu = 0 \]

\[ (6.21) \]

\[ m_W = \mu \cos \theta_W \quad m_Z = \mu = \frac{m_W}{\cos \theta_W} \]

\[ (6.22) \]

\[ m_A = m_{A2} = m_{W2} = m_{Z2} = 0 \]

\[ (6.23) \]

It is easy to see that, in this model, the expressions of the masses of fermions and intermediate gauge bosons are the same as those in the standard model.
In a proper limit, the present model will approximately return to the standard model. Suppose that parameter $\alpha$ is much smaller than 1,

$$\alpha \ll 1,$$

(6.24)

then, in the leading term approximation,

$$\cos \alpha \approx 1, \quad \sin \alpha \approx 0.$$  

(6.25)

In this case, the lagrangian density for fermions becomes:

$$\mathcal{L}_l + \mathcal{L}_{\nu - l} = -\bar{e}(\gamma^\mu \partial_\mu + \frac{1}{\sqrt{2}} f_\mu)e - \bar{\nu}_L \gamma^\mu \partial_\mu \nu_L$$

$$+ e j^\text{em}_\mu A^\mu - \sqrt{\frac{g^2 + g'^2}{2} Z^\mu}$$

$$+ \sqrt{\frac{g}{2}} i g \bar{\nu}_L \gamma^\mu e_L W^\mu_L + \sqrt{\frac{g}{2}} i g \bar{e}_L \gamma^\mu \nu_L W^-_L$$

(6.26)

where

$$e = \frac{gg'}{\sqrt{g^2 + g'^2}}.$$  

(6.27)

We see $\mathcal{L}_l + \mathcal{L}_{\nu - l}$ is the same as the corresponding lagrangian density in the standard model. In this approximation, except for the terms concern Higgs particle, the lagrangian of the model discussed in this paper is almost the same as that of the standard model: they have the same mass relation of intermediate gauge bosons, the same charged currents and neutral current, the same electromagnetic current, the same coupling constant of electromagnetic interactions, the same effective coupling constant of weak interactions \cdots etc.. On the other hand, we must see that there are two fundamental differences between the new electroweak model and the standard model: 1) there is no Higgs particle in the new electroweak model, so there are no interaction terms between Higgs particle and leptons, quarks or gauge bosons; 2) compare with the standard model, we have introduced two sets of gauge fields in the new electroweak model. These new gauge bosons are all massless.

From the above lagrangian, we could see that there is no Higgs particle exist in the present electroweak theory. Except for Higgs particle and interactions between Higgs particle and leptons, the new electroweak model keeps almost all other dynamical properties of the standard model. Because the theoretical predictions of the standard model coincide well with experimental results, we could anticipate that the parameter $\alpha$ will be very small. Though vacuum potential $v$ is very like Higgs field, they have essential differences. The most important difference is that Higgs field has kinematical energy terms but vacuum potential has no kinematical energy terms. In the standard model, masses of all fields, include quark fields, lepton fields and gauge fields, are generated from their interactions with Higgs field. In the present model, we could think that masses of all fields are generated from their interactions with vacuum. If the parameter $\alpha$ is small, the cross section caused by the interchange of massless gauge bosons will be extremely small. So, there will exists no contradictions between high energy experiments and the new electroweak model.
7 Comments

In general gauge field theory, if the local gauge symmetry of the lagrangian is strictly preserved, the mass of general gauge field can be non-zero. The interaction properties between matter fields and gauge fields of the general gauge field theory are the same as those of Yang-Mills gauge field theory. In a proper limit, the general gauge field theory can return to Yang-Mills gauge field theory. It keeps Yang-Mills gauge field theory as one of its special case.

Because the lagrangian has strict local gauge symmetry, general gauge field theory is renormalizable[8].

If we apply general gauge field theory to electroweak interactions, we could construct an electroweak model in which Higgs mechanism is avoided. So, Higgs particle is not a necessary part of an acceptable electroweak model[7].

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