A mid-infrared magneto-optical trap of metastable strontium for an optical lattice clock

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We report on the realization of a magneto-optical trap (MOT) for metastable strontium operating on the 2.92 µm transition between the energy levels 5s5p 3P2 and 5s4d 3D3. The strontium atoms are initially captured in a MOT operating on the 461 nm transition between the energy levels 5s2 1S0 and 5s5p 3P1, prior to being transferred into the metastable MOT and cooled to a final temperature of 6 µK. Challenges arising from aligning the mid-infrared and 461 nm light are mitigated by employing the same pyramid reflector to realize both MOTs. Finally, the 2.92 µm transition is used to realize a full cooling sequence for an optical lattice clock, in which cold samples of 87Sr are loaded into a magic-wavelength optical lattice and initialized in a spin-polarized state to allow high-precision spectroscopy of the 5s2 1S0 to 5s5p 3P0 clock transition.

I. INTRODUCTION

Laser cooling of alkaline earth and alkaline-earth-like atoms is a key tool for realizing our most precise atomic clocks [1,2], with further applications in atom interferometry [3,4], quantum computing and quantum simulation [5,6]. The ability to cool alkaline earth atoms quickly to a few µK is especially important for high performance atomic clocks, as the cooling time usually limits the frequency stability of the clock via the Dick effect [7,8]. To address this need, efficient methods of laser cooling alkaline earth metals have already been developed in the pioneering work of several groups over the past few decades [4,6,10–12]. In this report, we add to this body of work by demonstrating a metastable magneto-optical trap (MOT) for strontium based on the transition between the 5s5p 3P2 and 5s4d 3D3 states separated by 2.92 µm (see Figure 1), a cycling transition similar to that used previously to create metastable MOTs for calcium [13] and magnesium [14]. We show that the metastable MOT for strontium reaches a final temperature of 6 µK, and that the same transition at 2.92 µm can be used to prepare spin-polarized samples of cold 87Sr atoms in an optical lattice for realizing a high-precision optical clock.

The motivation to pursue a metastable MOT for strontium is that it offers advantages over the typically used ‘red’ MOT based on the 5s2 1S0 → 5s5p 3P1 transition at 689 nm. The red MOT capitalizes on a narrow transition linewidth of 7.5 kHz to reach temperatures as low as 250 nK [12]—close to the limit set by a single 689 nm photon recoil—but the low scatter rate also limits the maximum radiation force to 15 g, making the red MOT relatively shallow and slow to reach equilibrium. The red MOT is complicated further for 87Sr because the lack of electronic angular momentum in the 5s2 1S0 ground state severely reduces the MOT confinement force unless an additional ‘stirring’ laser is introduced [11]. In contrast, the broader transition linewidth of 2π×57 kHz [17] in the metastable MOT leads to a higher Doppler limit of 1.3 µK, but relaxes the requirements for laser frequency stabilization, while the electronic angular momentum of the metastable strontium results in an efficient magneto-optical trapping force for 87Sr even without the need of an extra stirring laser. Additionally, the long wavelength of the trapping light offers an intriguing possibility to investigate long-range dipolar interactions [15] and sets a recoil limited temperature of 12 nK which could potentially be accessible via sub-Doppler cooling. A final advantage of the metastable MOT is that it does not strongly perturb the intercombination transitions 1S0 → 3P0 or 1S0 → 3P1. This opens the door toward realizing a continuous source of cold atoms for an atom laser [19], zero-dead-time clock [20], superradiant laser [21,22] or atom interferometer [5].

II. LASER SYSTEM AND OPTICAL SETUP

The 2.92 µm light needed to operate the metastable MOT is synthesized via difference frequency generation (DFG) from light at 813 nm and 636 nm, and is delivered to the atoms using the optical setup in Figure 2. The two optical beams are combined on a dichroic mirror and then coupled into a waveguide embedded in a crystal of periodically-poled lithium niobate [23]. As the two optical beams propagate through the waveguide, they interact non-linearly to generate a mid-infrared beam at 2.92 µm. At the output, all three wavelengths of light are collimated on an off-axis parabolic mirror and then separated from each other using a dispersive CaF2 prism cut close to the Brewster angle. The powers in the transmitted beams at 813 nm and 636 nm are actively stabilized on separate photodiodes, allowing feedback to acousto-optic modulators in the optical beam paths for high-precision dynamic control of the 2.92 µm power. The 2.92 µm beam is then expanded using a 1:10
parabolic mirror telescope to a final collimated beam waist $w_0 = 26 \text{ mm}$ incident on the pyramid MOT. The telescope also provides an opportunity to combine the 2.92 $\mu$m beam with the 461 nm MOT beam using a knife-edge mirror placed at the focus. With 125 mW at 813 nm and with up to 25 mW at 636 nm exiting from the waveguide, the DFG module generates up to 700 $\mu$W of power at 2.92 $\mu$m. Given that the transition saturation intensity is $I_{\text{sat}} = 0.5 \text{ W cm}^{-1}$, a peak intensity up to 140 $I_{\text{sat}}$ can therefore be generated in the center of the MOT beam.

The 636 nm light comes from an extended-cavity diode laser (ECDL) with 120 mW single-mode output power. The 813 nm light is generated using a Ti:Sapphire laser, which also serves as the source of the magic-wavelength optical lattice light used to trap the atoms during spectroscopy of the clock transition—see section III. The frequency stability of both lasers is ensured using Pound-Drever-Hall locks [24] to a shared Fabry-Pérot cavity [25]. The tunable cavity is further locked to an ultra-stable, low drift clock laser [26] to transfer long-term frequency stability to the 2.92 $\mu$m light. The frequency mismatch between the cavity modes and the $^{3}\text{P}_2 \rightarrow ^{3}\text{D}_3$ resonance is bridged using an ‘electronic sideband’ configuration, implemented using waveguide EOMs placed between each seed laser and the cavity [27]. As the 813 nm laser must be operated precisely at the magic wavelength to ensure accuracy of the optical clock [1], the 2.92 $\mu$m cooling light is set to the desired frequency by tuning the offset of the 636 nm laser from the cavity.

During preparation of the manuscript, we became aware of similar work using DFG of 1062 nm and 779 nm light to generate 2.92 $\mu$m light which was used to perform an absolute frequency measurement of the same $^{3}\text{P}_2 \rightarrow ^{3}\text{D}_3$ transition [28].

III. THE METASTABLE MOT SEQUENCE

The Sr atomic beam is initially slowed using a permanent-magnet Zeeman slower [29]. Then, all stages of cooling, trapping and clock spectroscopy are implemented inside the pyramid MOT structure depicted in figure 1—comprised of six radial silver-coated prism mirrors arranged in a hexagon, plus a CaF$_2$ prism positioned at the apex. Details regarding the prism mirror assembly, together with data illustrating the performance of the first stage ‘blue’ MOT, have been presented in a separate work [30]. For the metastable MOT described in this report, the pyramidal retro-reflector has one notable advantage: it greatly simplifies optical alignment and polarization control. In contrast, setting up out-of-vacuum optics for a conventional six beam MOT would be more challenging due to the high cost of detectors and optics in the mid-infrared.

The energy level diagram for strontium is shown in figure 1. In the first cooling stage, atoms exiting the Zeeman slower are gathered into a ‘blue’ MOT operating at 461 nm on the $5s^2 \, ^1S_0 \rightarrow 5s5p \, ^1P_1$ transition [31]. The transition is not perfectly closed as approximately one in every 50,000 photons scattered causes the atom to be shelved into the metastable 5s5p $^3P_2$ state. To recycle atoms in the $^3P_2$ state back into the MOT, a 497 nm repump beam is used. The repump excites the metastable atoms to $5s5d \, ^3D_2$, which then decays to the ground state via $5s5p \, ^3P_1$. The 497 nm source is based on a wavemeter-stabilized ECDL operating at 994 nm which is coupled into a waveguide second-harmonic generation module [23]. To ensure all five hyperfine sub-levels of the $^3P_2$ state are repumped efficiently, the laser is frequency modulated using a resonant electro-optic modu-
MOT we initially set a relatively large laser detuning and maximum of $0.1 \, \text{MHz}$. From $0 \, \text{MHz}$ to $0.1 \, \text{MHz}$, the mean detuning is ramped linearly from $0 \, \text{MHz}$ to $2 \, \text{MHz}$ while the field gradient ramps from $7 \, \text{mT cm}^{-1}$ to $3 \, \text{mT cm}^{-1}$. At the end of the broadband stage, the atom cloud has a full width at half maximum of $0.7 \, \text{mm}$. To increase the density and reduce the temperature of the cloud further, we implement a final stage of narrowband MOT cooling during which the frequency modulation is switched off. In the narrowband MOT we initially set a relatively large laser detuning and intensity, $-159 \, \text{kHz} \times (1.7 \times \Gamma)$ and $0.7 \, I_{\text{sat}}$ respectively, to minimize losses between the broadband and narrowband MOT. Following this, throughout the $70 \, \text{ms}$ duration of the narrowband MOT the magnetic field gradient is ramped linearly from $0.3 \, \text{mT cm}^{-1}$ to $0.15 \, \text{mT cm}^{-1}$, the intensity is ramped to $0.1 \, I_{\text{sat}}$, and the detuning is ramped to a final value in the range $-200 \, \text{kHz}$ to $-20 \, \text{kHz}$ depending on the desired final size of the cloud (see Figure 5).

IV. CHARACTERISATION OF THE METASTABLE MOT

The temperature of the metastable MOT is characterized using a time-of-flight sequence, in which a variable delay is introduced between release of the MOT and the imaging pulse. The Gaussian spatial width of the cloud $\sigma(t)$ as a function of expansion time is related to atomic temperature $T$ using the following expression [32].
As

\[
\sigma(t) = \sqrt{\sigma_0^2 + k_b T t^2/m},
\]

(1)

An example time-of-flight measurement is depicted in Figure 4 for final MOT detuning and intensity of \(-68\) kHz and 0.1 \(I_{\text{sat}}\) respectively. The images are gathered by the following protocol: after completing the sequence described in section III the narrowband MOT is held for an additional 70 ms in steady state to allow the position and temperature of the atoms to reach equilibrium before the cloud is released and allowed to propagate for a variable time-of-flight between 0 ms and 14 ms. The atoms are then pumped to the ground state using a 1 ms pulse of 497 nm light, and finally imaged using fluorescence from a pulse of resonant 461 nm light lasting 50 \(\mu\)s. A 2D Gaussian is then fit to each fluorescence image to extract the cloud size.

Repeating this procedure for various final laser detunings \(\Delta\), the measured MOT temperature, averaged over the horizontal and vertical directions, is plotted against laser frequency in Figure 5. According to Doppler cooling theory, the temperature should depend on detuning as

\[
T(\Delta) = \Delta T_{\text{min}} \frac{1 + 4(\Delta/\Gamma_E)^2}{4|\Delta|/\Gamma_E}.
\]

(2)

where \(\Gamma_E = \Gamma + s_{\text{tot}}\) is the power-broadened transition linewidth and \(T_{\text{min}} = \hbar \Gamma_E/2k_B\) is the generalized Doppler cooling limit including the effect of total saturation \(s_{\text{tot}} \approx 64k_B/I_{\text{sat}}\) due to all MOT beams. The overall scaling term \(\alpha\) should be unity in the ideal Doppler cooling model, which assumes a two-level atom and no additional heating processes beyond spontaneous emission, but a scaling is included here to match the treatment in [13].

Leaving \(\alpha\), \(s_{\text{tot}}\), and the overall detuning as free parameters for the fit curve in Figure 5, we measure a minimum temperature of \(5.9(6)\ \mu\)K, corresponding to \(\alpha = 2.0(1)\). Furthermore, from the fit we estimate total saturation \(s_{\text{tot}} = 3.4(3)\), which is factor of two above the measured value based on the beam intensity and trap geometry. The discrepancy in the measured saturation parameter and observation that the temperature did not reduce with intensity, indicates there may be underlying heating effects which we have not fully accounted for.

![Figure 4](image1.png)

**FIG. 4.** Time-of-flight measurement used to track the expansion rate of the atoms released from the triplet MOT. The expansion rate gives an estimated average temperature of \(5.9\ \mu\)K.

![Figure 5](image2.png)

**FIG. 5.** Average temperature of the MOT as measured from time-of-flight expansion. Error bars are estimated from the shot-to-shot noise in the data; they do not include any possible systematic effects. Inset are pictures of the cloud at various detunings. For large detuning, the MOT increases in size and falls under the force of gravity, resting on a shell of maximum force where the Zeeman shift is equal to laser detuning. Similar observations have been made in red MOTs of strontium [12], in both cases indicating that the atom temperature is low enough for the gravitational potential to play a substantial role in the MOT dynamics.

**V. STATE PREPARATION FOR AN OPTICAL LATTICE CLOCK**

In this section we outline the additional steps that are needed to use the metastable MOT as the basis for an optical lattice clock—specifically loading the atoms in a magic-wavelength lattice [1] at 813 nm and optically pumping the sample into a specific Zeeman sub-level for spectroscopy.

To load atoms into the optical lattice, the lattice light is kept on throughout the final narrowband metastable MOT stage described in section III so that a significant fraction of the atoms stays trapped after the MOT light is switched off. Using a lattice waist of 100 \(\mu\)m and an axial trapping frequency of 62 kHz for ground-state atoms, a transfer efficiency of 10% into the lattice is observed. It is critical that atoms are cooled to the low \(\mu\)K regime and held in a relatively shallow lattice, so as to constrain...
their motion while mitigating higher-order shifts from the lattice light \[34, 35\].

Before the system can be operated as an optical lattice clock, the atoms in the lattice must be pumped into one of the clock states, \(5s^2 \ ^1S_0\) or \(5s5p\ \ ^3P_0\). Furthermore, to eliminate the 1st-order Zeeman shift of the clock frequency we must average interleaved measurements using atoms prepared in different Zeeman sublevels with opposite linear magnetic field sensitivities. Typically this is achieved using spin polarized samples, alternating each sequence between the two \(M_F = \pm 9/2\) stretched states. In order to prepare the atoms into the \(5s^2 \ ^1S_0\) \(M_F = 9/2\), we operate a molasses for 15 ms on the 2.9 \(\mu\)m transition in a bias field of 64 \(\mu\)T, with the laser frequency detuned by 100 kHz from the red side of the \(^3P_2\) \(M_F = +13/2 \rightarrow ^3D_3\) \(M_F = +15/2\) (i.e. detuned by \(-1\) MHz from the zero-field atomic resonance). The molasses has little effect on the lattice-trapped atom number or temperature, but ensures that \(75\%\) of the atoms end up in the \(^1S_0\) \(M_F = +9/2\) state after a 10 ms stage of repumping with 497 nm and 679 nm light. Unfortunately, the molasses cannot directly be used to spin polarize the sample in the \(M_F = -9/2\) Zeeman level as this would require a positive frequency detuning which would heat the sample. However, atoms prepared in \(^1S_0\) \(M_F = +9/2\) can be ‘flipped’ to \(M_F = -9/2\) by non-adiabatically switching the magnetic field into the opposite direction in 100 \(\mu\)s. Finally, we adiabatically rotate the magnetic field back to the original direction so that the clock spectroscopy can be carried out in the same field on both the \(M_F = \pm 9/2\) states. Using a bias field of 64 \(\mu\)T in magnitude, we observe that a 40 ms rotation time is sufficiently long to have a negligible impact on spin polarization fraction.

At the end of all these cooling stages, the spin-polarization efficiency is \(75\%\) and the atomic temperature is a large fraction of the trap depth. This has the undesirable effects of limiting the contrast of the clock transition and exacerbating systematic shifts from line-pulling, collisions, and higher-order lattice-atom interactions. To address these problems, a simple state selection protocol is implemented. First, the lattice depth is linearly ramped down to around 5 \(\mu\)K in 20 ms, held at that depth for 20 ms, and then ramped back up to the operating depth of 13 \(\mu\)K in 20 ms. This ‘spilling’ protocol results in an atom temperature of 3 \(\mu\)K as measured via clock spectroscopy of the axial motional sidebands \[36\], but the reduction in temperature comes at the expense of losing approximately half of the atoms. The 20 ms ramp time is optimized experimentally, as the minimum time below which hotter samples are measured after the spilling stage. Next, the spin-polarized Zeeman state is selected by driving a resonant 30 ms Rabi \(\pi\) pulse on the \(5s^2 \ ^1S_0\) \(M_F = \pm 9/2\) to \(5s5p\ \ ^3P_0\) \(M_F' = \pm 9/2\) clock transition in a bias field of 64 \(\mu\)T, followed by a 5 ms flash of 461 nm cooling light which clears out any atoms remaining in the ground state. This ‘clearance pulse’ leaves more than 99\% of the atoms in the single internal state \(5s5p\ \ ^3P_0\)

\(M_F' = \pm 9/2\), ready for interrogation of the clock transition. We calibrate the final number of spin-polarized atoms using cavity-enhanced non-destructive detection \[37\], and compare against the fluorescence images of the spin-polarised sample to extract fluorescence signal per atom. With this method we measure a typical final atom number in the lattice of \(2 \times 10^4\) collected from a blue MOT containing \(4 \times 10^6\) atoms.

VI. CONCLUSION AND OUTLOOK

We have realized a mid-infrared MOT for metastable strontium, observing a temperature of 6 \(\mu\)K in the low-intensity limit. We use the mid-infrared transition to prepare \(^{87}\)Sr atoms in a spin polarized state in a magic-wavelength optical lattice for high-precision spectroscopy of the \(5s^2 \ ^1S_0\) to \(5s5p\ \ ^3P_0\) clock transition. The metastable MOT results in comparable loading time and atom number as alternative cooling methods for optical lattice clocks \[38, 39\], and achieves a lower temperature than metastable MOTs previously realized in calcium \[15\] and magnesium \[16\] by an order of magnitude.

Further optimization of the metastable MOT sequence, aimed at increasing atom number or reducing the cooling time, could potentially improve the performance of the optical lattice clock by reducing the Dick-effect frequency instability \[9\]. For example, better transfer into the metastable MOT might be achieved if more optical power were available at 2.92 \(\mu\)m: the double MOT could then be operated at a higher intensity and larger laser frequency modulation depth, thereby increasing the capture velocity and volume of the metastable MOT. For the broadband metastable MOT stage, replacing the sinusoidal modulation with a sawtooth-wave adiabatic-passage (SWAP) cooling protocol could increase phase space density more quickly \[40\]. Finally, improved transfer efficiency into the optical lattice could be facilitated by the addition of a 497 nm ‘drain’ laser overlapped with the lattice to selectively repump atoms into the ground state as they enter the capture volume—this method has already been successfully implemented as a means to load a dipole trap from a metastable calcium MOT \[41\].

Future work could pursue sub-Doppler cooling on the 2.92 \(\mu\)m transition, and discover how far metastable atoms can be cooled towards the recoil limit of 12 nK. Further, one could explore schemes to cool atoms on a continuous basis as proposed in \[42\]. Such a scheme would be compatible with various quantum sensors reliant on the narrow \(^1S_0 \rightarrow \ ^3P_0\) or \(^1S_0 \rightarrow \ ^3P_1\) transitions given the fact that these transitions are not strongly perturbed by the 2.92 \(\mu\)m light.

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