High Energy Hadron-Hadron Scattering

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ABSTRACT

The elastic hadron-hadron scattering at high energies is one of the most fundamental subjects of all particle physics problems and yet is least understood in spite of many advances in quantum chromodynamics (QCD) at the conceptual level. We review here the recent theoretical and experimental status of the subject as well as the rigorous results of the high energy hadron-hadron scattering. Surprisingly enough, the high-energy models for the elastic and diffractive scattering abstracted from the Regge-Pomeron field theory is still phenomenologically successful to explain the high-energy scattering data though no rigorous derivation out of the QCD is yet available.

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2Permanent Address
I Introduction

With the recent report from the UA4/2 group [1] on new determination of $\rho$ for the forward $p\bar{p}$ scattering amplitude at CERN $Sp\bar{p}S$ energy $\sqrt{s} = 541$ GeV which replaced the earlier high $\rho$ value [2] and with the new measurements of the total cross-section $\sigma_T$ by the CDF group [3] and the E710 group [4] at Fermilab Tevatron energy $\sqrt{s} = 1.8$ TeV, it seems appropriate and necessary to review the status of the high energy hadron-hadron scattering and put our current understanding of the high-energy strong interactions into perspective. Indeed these are basically the complete information that we will have on the high-energy elastic scattering until the LHC is built and becomes operative.

Though we have in principle the exact theory of the strong interactions, QCD, which can give good descriptions of the hadron interactions at short distances, i.e. the high-energy deep-inelastic reactions, the interactions at large distances, namely; near forward scattering can not reliably be calculated in the framework of QCD. However the two domains are intimately related to each other and the description of the high-energy elastic and diffractive scattering based on the microscopic gauge theory of colored quarks and gluons, QCD, presents a major challenge. Indeed it is a central problem of the modern theory of strong interactions to establish a smooth connection and a relationship between the hard Pomeron obtained from perturbative QCD for the hard processes and the soft Pomeron emerged from phenomenological models for the high-energy near forward hadron-hadron scattering[5]. The discovery of the semihard processes in QCD for the high-energy inclusive interactions may eventually lead to matching of the two regions but the gap between the phenomenological models for the high-energy hadron scattering and the microscopic gauge theory of the strong interaction is so wide for the moment that it is difficult to imagine any relationship between them.

Another view is that the strong interactions at large distances can at least be approximately calculated by lattice QCD. But the continuum limit of the realistic lattice QCD, exhibiting the color confining and unitary hadron interactions and asymptotically free interactions at short distances, seems to be as remote as the asymptotic energy region, asymptopia, of the hadron interactions.

The concept of the Pomeron was originally introduced to understand the high-energy behavior of total cross sections which is controlled by a right-most Regge pole with vacuum quantum number in the analytic complex an-
gular momentum plane. With the advent of the analytic S-matrix consistent with the requirement of the s-channel unitarity, the dual resonance model was invented, which can be identified as the tree-level amplitudes in the string theory, thus raising a hope to find the theory of the strong interactions in the string theory approach for the colored quarks and gluons that may lead to the Reggeon calculus based on the microscopic structure of QCD. But the search for the realistic string theory with the desired QCD properties in both infrared and ultraviolet regions has not yet been successful. It is however clear that in any of these developments the Pomeron plays the central role in describing the high-energy analytic S-matrix consistent with the unitarity requirement.

In the Reggeon calculus that was invented before the gauge theory of QCD, it is well-known that one must include the multi-Pomeron exchanges to consistently describe the high energy analytic S-matrix because of the unitarity. This means that the single bare Pomeron has to be renormalized by rescattering of two Pomerons via multi-Pomeron exchanges. It turns out that this procedure can be compared to the eikonal formalism which guarantees the manifest s-channel unitarity and converts the bare Pomeron power increase of total cross sections to a logarithmic increase of the renormalized Pomeron consistent with the rigorous unitarity limit. Constructing a microscopic theory of colored quarks and gluons that can give the bare Pomeron intercept slightly larger than one is then a major task of the Reggeon calculus. Indeed it is well-known that the ladder diagram for scattering of two reggeized gluons in the t-channel in the leading-log approximation can generate a fixed branch-point singularity in the angular momentum plane, i.e., the Lipatov Pomeron [6], at $j = 1 + \epsilon$ with $\epsilon = \frac{g^2}{\pi^2} 2 \ln 2$ so that $\sigma_T \sim s^\epsilon$ which violates the Froissart bound. When the perturbative leading-log QCD is extended to the exchange of N-gluons, it produces a sequence of branch points on an interval $j = 1 + \epsilon_N$ where $\epsilon_N \leq \frac{1}{2} N(N - 1)\epsilon$. Thus it seems very likely that such a dense sequence of singularities gives such a complicated Pomeron structure that it may not have a chance to reconcile the s-channel unitarity. In addition, the calculations leading to $\epsilon$ and $\epsilon_N$ produce amplitudes [7] which have zero transverse momentum singularities (but not divergences) due to the exchange of massless gluons thus conflicting with the color confinement and absence of zero mass particles in the strong interactions. The discovery of the semihard processes, i.e., the production of quark- and gluon-jets with relatively large transverse momenta but with a small fraction of the initial
energy may help to get around some of the difficulties for the leading-log approximation of perturbative QCD [8] but it appears one needs additional phenomenological inputs such as the structure functions of the partons in the initial hadrons and the fragmentation functions of the quarks and gluons into the secondary hadrons. In addition, a smooth matching of the semihard Pomeron to the soft Pomeron responsible for large cross sections of the elastic and diffractive scattering has not yet been explicitly demonstrated for the realistic color interactions.

There is another somewhat pessimistic view that matching the low $p_T$ physics and the ultraviolet QCD of parton models and perturbation theory would require a new technology [9], whereby proposing a program to extract the low $p_T$ Pomeron from QCD which is based on an analysis of the infrared divergences (as the gauge symmetry is restored) of a reggeon diagram description of spontaneously broken QCD. Needless to say, the description of the large cross section physics at high energies in terms of the microscopic structure of QCD is far from the reality and one must still rely on the phenomenological models that satisfy the general principles required for the analytic S-matrix and utilize the general QCD concepts only.

In building phenomenological models for the high-energy scattering, asymptotic analytic amplitude analysis based on the general principles of unitarity, analyticity and crossing-symmetry has proven to be useful even though the currently accessible energy region is not in asymptopia. In particular, asymptotic amplitude analysis led to the quasilocal analyticity relations expressed in terms of derivative dispersion relations (DDR) with respect to $\ln s$ variable at energies sufficiently higher than the typical resonance domain for both even- and odd-signatured amplitudes [10]. Phenomenological models can then be constructed as the solution of DDR that satisfy the unitarity bound, and have proven to be successful in understanding the high-energy scattering data, though not asymptotic, as well as predicting the high-energy behavior of the hadron-hadron scattering amplitude [11]. The total cross sections for $pp$ and $p\bar{p}$ scattering do indeed smoothly rise in the ISR energies, suggesting a priori justification of the use of the quasilocal analyticity relation at the current high energies. Also there have been comprehensive eikonal amplitude analyses [12] based on QFD expectations and motivations that mimic QCD. Though different in derivations, they both can give the same asymptotic term responsible for the characteristic rise of the total cross section at high energies: the asymptotically increasing term in the analytic amplitude.
analysis is due to a leading $j-$plane singularity near the forward direction constrained by and consistent with rigorous asymptotic theorems, while such a term can also be constructed out of an eikonal that is motivated by general field theory expectations. The asymptotic term in the amplitude has a natural interpretation as coming from the leading $j-$plane singularity, i.e., that has the intercept 1 in the forward direction. Within the general framework of QCD, the asymptotic term in the crossing-even amplitude $F_+(s,t)$ can be thought of as the renormalized or physical Pomeron that is achieved by unitarization of the bare Pomeron of color-singlet and C-even two-gluon exchanges through rescattering of two gluons to include multigluon exchanges. If the leading bare $j-$plane singularity due to the C-odd and color-singlet three-gluon exchange has the intercept $\alpha_o(0) = 1 + \epsilon_o > 1$, this will imply a nonvanishing odd-signatured amplitude $F_-(s, t = 0)$ as $s \to \infty$. This C-odd leading $j-$plane singularity is the C-odd counterpart of the Pomeron and became to be known as the Odderon.

One can easily introduce both the Odderon and Pomeron in the asymptotic amplitude analysis as solutions of the DDR [10] that maximally saturate the rigorous bounds of the amplitudes for both particle-particle and antiparticle-particle scattering so that both $F_\pm$ can grow as fast as maximally possible at asymptotic energies. The maximal Odderon model seems to have no support from the near forward scattering experiments by now, although it can arise naturally as a reggeized three-gluon exchange in the leading-log approximation of perturbative QCD. It is however not clear if its intercept is indeed larger than 1. In addition, even if the bare Odderon intercept is assumed to be bigger than one, it is surprisingly difficult to generate large odd-signatured amplitude within the eikonal formalism as it is incapable of giving the maximal Odderon behavior [13]. Though the Odderon is irrelevant for the forward scattering, one can not rule out its role in the non-forward region, in particular if there is a non-zero difference between the differential cross sections of $pp$ and $\bar{p}p$. It has been suggested [14] that the three-gluon exchange in the form of a Regge cut might be important in order to understand the energy dependence of the diffractive minimum and secondary maximum region of the differential cross sections through the interference with the Pomeron and Pomeron-Pomeron cut. The new data for $\rho$ from UA4/2 has washed out any need to include the Odderon contribution to the high-energy forward elastic scattering.

There are a number of rigorous results for the analytic S-matrix at high
energies that are based on some very general principles such as unitarity, analyticity and crossing-symmetry. Strictly speaking, these results are valid in asymptotic energy limit, i.e., the asymptotic theorems. Nevertheless, they are useful to guide the phenomenological models for the high-energy hadron-hadron scattering, in either the asymptotic analytic amplitude analysis or the eikonal types mimicking general QCD properties, and predicting the asymptotic form of the rising cross sections, which has direct implications on the structure of the physical Pomeron. We will see, however, from the asymptotic amplitude analysis that the currently available high-energy data do not necessarily prefer \( \ln^2 s \) rise over \( \ln s \) behavior of the total cross sections in terms of the statistics, despite many common beliefs. In fact, the asymptotic theorems have often been misinterpreted and even misused in some cases. For these reasons we will summarize some of these theorems in Section III as soon as we establish the notations, definitions, and concepts of the hadron-hadron scattering at high energies in Section II, in which general features of the high-energy scattering data will also be reviewed. We then will review in Section IV the status of the large cross section processes, namely, the high-energy near forward scattering based on the asymptotic amplitude analysis and give predictions of the total cross section at the LHC energy. We shall be mostly discussing the case of the two equal mass particle scattering, namely \( pp \) and \( p\bar{p} \) scattering amplitudes, for which there are data available up to the ISR energies. In addition we have high-energy data for \( p\bar{p} \) at the CERN \( Sp\bar{p} \) S and Fermilab Tevatron energies, which provide a raison d’être for reviewing the features and status of the high-energy hadron-hadron scattering. Even though the elastic scattering is one of the simplest subjects, we will see that it is one of the least understood problem from the basic theory of QCD and also still poses problems for experiments to clarify.

II Scattering Amplitudes, Cross Sections and Experimental Data

Following the notations of Ref. [11], we use the normalization convention for the scattering amplitude such that the imaginary part of the forward elastic scattering amplitude \( F(s,t = 0) \) is related to the total cross section \( \sigma_T \) by
the optical theorem as
\[ \sigma_T = \frac{1}{s} \text{Im} F(s, t = 0) \]  
for both \( pp \) and \( p\bar{p} \) scattering. The crossing-even and -odd amplitudes \( F_\pm \) are given by
\[ F_\pm = \frac{1}{2} (F_{pp} \pm F_{\bar{p}p}) \] 
in terms of the \( pp \) and \( \bar{p}p \) scattering amplitudes, so that
\[ F_{pp} = F_+ + F_- \quad F_{\bar{p}p} = F_+ - F_- \] 
In addition to the total cross section, experiments measure the ratio of the real part to the imaginary part of the forward elastic amplitude,
\[ \rho_i(s) = \frac{\text{Re} F_i(s, t = 0)}{\text{Im} F_i(s, t = 0)} \quad (i = pp, p\bar{p}) \] 
The differential cross sections for \( pp \) and \( p\bar{p} \) scattering get the Coulomb contribution [15] in addition to the hadronic amplitudes,
\[ \frac{d\sigma}{dt} = \frac{1}{16\pi s^2 (hc)^2} |F_C + F|^2, \] 
where
\[ F_C = \frac{8\pi s\alpha (hc)^2}{|t|} G^2(t) \exp^{-i\alpha\phi(t)}, \] 
\[ \phi(t) = \ln\left(\frac{0.08}{|t|}\right) - 0.577, \] 
\( G(t) \) being the electromagnetic form factor of the proton which is usually parametrized as a dipole form, \( G(t) = (1 + \frac{|t|}{0.71})^{-2} \). Here \( t \) is in units of \( \text{GeV}^2 \). For small \( t \) within the diffractive region, experimentalists use a simple optical model form with a single exponential term for the hadronic amplitudes,
\[ F(s, t) = s \sigma_T(s) (\rho(s) + i) \exp^{-\frac{1}{2} B|t|} \] 
where \( B \) is the nuclear slope parameter, though it depends on \( s \) and \( t \) strictly speaking. In principle \( \sigma_T, \rho, \) and \( B \) are the three quantities that can be determined from the experimental data of differential cross sections at a given
energy. This is the case in the E710 experiment. In addition, $\sigma_T$ can be obtained from the simultaneous measurements of the elastic differential cross section at small $t$ and the total inelastic rates, i.e., the luminosity independent method [16], which was used in the ISR, UA4, and CDF experiments. Basically the luminosity independent method is the only way to determine the Tevatron luminosity and the total cross section at Fermilab [3]. This method is based on the observation that the total cross section is given by the sum of the total elastic and inelastic rates,

$$\sigma_T = \frac{1}{L} (R_e + R_i),$$

while the differential cross section is related to the differential rate with respect to $t$,

$$\frac{d\sigma}{dt} = \frac{1}{L} \frac{dR_e}{dt}.$$  \hspace{1cm} (10)

Because of (5), we then get

$$(1 + \rho^2)\sigma_T = 16\pi(\hbar c)^2 \left[ \frac{dR_e}{dt} \right]_{t=0} / (R_e + R_i).$$  \hspace{1cm} (11)

Using this method the CDF collaboration recently obtained $(1 + \rho^2) \sigma_T(s) = 62.64 \pm 0.95 \text{ mb}$ and $81.83 \pm 2.29 \text{ mb}$ at $\sqrt{s} = 54.6 \text{ GeV}$ and $1800 \text{ GeV}$ respectively which agrees with the UA4 value reported earlier $63.3 \pm 1.5 \text{ mb}$ at the CERN $SpS$ energy $\sqrt{s} = 546 \text{ GeV}$ but disagrees somewhat with the E710 value $73.6 \pm 3.3 \text{ mb}$ based on the three parameter fits to the $t$-distribution of the differential rates at the Fermilab Tevatron energy. The new UA4/2 $\rho$-value is extracted from the $t$-distribution of the differential rate with the UA4 value of $(1 + \rho^2)\sigma_T$ as input. The differential cross section exhibit a sharp Coulomb peak near $|t| = 0$ and an interference structure around $|t| = 10^{-3} \text{ GeV}^2$ followed by a diffractive pattern of the hadronic reactions which has a characteristic $e^{-B|t|}$ decrease to a minimum followed by a shoulder. In order to extract the hadronic scattering parameters, the Coulomb contribution has to be carefully accounted for by studying the interference region and beyond. The minimum $|t|$ achieved by both the new E710 and UA4/2 experiments is $0.75 \times 10^{-3} \text{ GeV}^2$ well inside the Coulomb-hadron interference region.

Fortunately the Coulomb contributions decrease rather rapidly as $|t|$ increases, e.g., it is only about 1% at $|t| = 0.025 \text{ GeV}^2$ the minimum value in
the CDF experiment. The region of maximum interference is expected from where \( F_C \sim F \), i.e., \( |t| \sim 8\pi\alpha(hc)^2/\sigma_T \), which gives \( |t| \sim 1.1 \times 10^{-3} \text{ GeV}^2 \) for \( \sigma_T = 60 \text{ mb} \), a reasonable value of the total cross section around \( \sqrt{s} = 540 \text{ GeV} \). We give in Table 1 basically the complete information of the high-energy forward scattering parameters that are available now.

| \( \sqrt{s}(\text{GeV}) \) | \( (1+\rho^2)\sigma_T(\text{mb}) \) | \( \sigma_T \) | \( \rho \) | \( B (\text{GeV}^{-2}) \) |
|---|---|---|---|---|
| UA4/2 | 541 | 63.3 \( \pm \) 1.5 * | 62.2 \( \pm \) 1.5 | 0.135 \( \pm \) 0.015 | 15.52 \( \pm \) 0.07 |
| CDF | 546 | 62.64 \( \pm \) 0.95 | 61.26 \( \pm \) 0.93 | 0.15 * | 15.2 \( \pm \) 0.6 |
| CDF | 1800 | 81.83 \( \pm \) 2.29 | 80.03 \( \pm \) 2.24 | 0.15 * | 17.0 \( \pm \) 0.25 |
| E710 | 1800 | 73.6 \( \pm \) 3.3 | 72.2 \( \pm \) 2.7 | 0.134 \( \pm \) 0.069 | 16.72 \( \pm \) 0.44 |

Table 1. High-energy \( \bar{p}p \) scattering parameters from most recent experiments. The assumed values are marked with ASTERISKS(*). The E710 results are from the three parameter fits.

The experimental information about the scattering parameters for both \( pp \) and \( \bar{p}p \) scatterings are available only up to the ISR energies and at high energies only the \( \bar{p}p \) scattering data are known as given in Table 1. The total cross sections started rising with the energy already at ISR and the \( \bar{p}p \) cross section continues to increase up to the Tevatron energy. However the difference between the CDF and E710 cross sections is posing a difficulty in predicting a unique asymptotic behavior of the cross sections as they both can be continued smoothly from the ISR data and the phenomenological models based on \( \chi^2 \) fit should favor neither of them as emphasized in [17] (See Figs. 1 and 2 in Ref.[17]). The absence of \( pp \) cross section data at higher energies is another reason why the asymptotic theorems are useful in model building.

The \( t \)-dependence of the differential cross sections away from the forward direction shows the diffraction pattern, i.e., a sharp diffraction peak followed by a diffractive minimum and secondary shoulder. The nuclear slope parameter \( B \) is strictly speaking given by

\[
B = \frac{d}{dt} \left[ \ln \left( \frac{d\sigma}{dt} \right) \right]_{t=0}
\]  
(12)
and refers to the slope of the diffraction peak. Experimentally $B$ shows a slow rise in energy which makes the diffraction peak to shrink as one can expect from a simple Regge behavior. A distinctive feature of $B$ at the ISR and also at the $Sp\bar{p}S$, though less pronounced, is the break in the slope around $|t| = 0.1$ $GeV^2$, which however seems to have almost disappeared at the Tevatron energy[18] (See Figs. 4, 5 and 6 in Ref.[18]). At $\sqrt{s} = 53$ $GeV$, the ISR data[19] show that the $pp$ and $\bar{p}p$ have almost identical diffraction structure with a diffractive minimum around $|t| = 1.3$ $GeV^2$ and secondary maximum at larger $|t|$-values except that there appears to be an extra shoulder filling in the $\bar{p}p$ minimum. The $\bar{p}p$ data at the $Sp\bar{p}S$[20] shows almost no diffractive minimum but a broad shoulder which could be due to an experimental resolution. The identity of the $pp$ and $\bar{p}p$ differential cross sections within the diffractive minimum is expected from the asymptotic theorems as we will see in the next Section but the question of difference around the diffractive minimum at high energies can not be answered from experiments for the indefinite future. Apparent difference at the diffractive minimum between $pp$ and $\bar{p}p$ at ISR and the rise of the secondary peak(shoulder) with energy for $\bar{p}p$ differential cross section prompted some to suggest the need of three-gluon exchange or Odderon beyond the Pomeron and two-Pomeron cut (See Figs. 8 and 9 in Ref.[18]) as we mentioned before.

One of the most striking experimental features of the total cross sections for $pp$ and $\bar{p}p$ is that they have rather smooth behavior at energies above a few $GeV$ up to the known Tevatron energy. This may suggest that the hadron-hadron scattering at high energies can be described by a simpler analyticity representation. Indeed a suggestion[10] was made in 1974 to employ certain quasi-local DDR to describe the high energy scattering data instead of the full dispersion relation which must be used at low energies to take into account the rich structure of total cross sections.

One can derive DDR from the Sommerfeld-Watson-Regge(SWR) representation

$$F_{\pm}(s, t) = -\frac{1}{2i} \int_C dj \frac{1 \pm e^{-i\pi j}}{\sin \pi j} s^j F(j, t)$$

with the assumption that the asymptotic behavior is controlled by the rightmost singularity of $F(j, t)$ in the complex $j$-plane which in particular is located near $j = 1$ in forward scattering. The SWR representation LEADS to the quasi-local analytic relation in which the real and imaginary parts of $F_{\pm}$ are related to each other by certain derivatives with respect to $\ln s$. The
leading terms of DDR at high energies are

\[ \text{Re} \left( F_+(s, t) / s \right) \simeq \frac{\pi}{2} \frac{\partial}{\partial \ln s} \text{Im} \left( F_+(s, t) / s \right) \] (14)

\[ \text{Im} \left( F_-(s, t) / s \right) \simeq -\frac{\pi}{2} \frac{\partial}{\partial \ln s} \text{Re} \left( F_-(s, t) / s \right) \] (15)

which reduce to the relationships between the total cross section \( \sigma_\pm(s) \) and real parts \( \rho_\pm(s) \) in the forward direction.

It is easy to see from the SWR relation (13) that a simple Regge pole in the complex \( j \)-plane at \( j = \alpha_k(t) \) gives

\[ F_+(k)(s, t) = C_+^{(k)} \left[ i - \cot \left( \frac{\pi}{2} \alpha_k(t) \right) \right] s^{\alpha_k(t)} \] (16)

\[ F_-(k)(s, t) = -C_-^{(k)} \left[ i + \tan \left( \frac{\pi}{2} \alpha_k(t) \right) \right] s^{\alpha_k(t)} \] (17)

The Regge pole \( \alpha_k(t) \) is called exchange-degenerate if it contributes to both even- and odd-signatured amplitudes with \( C_+^{(k)} = C_-^{(k)} \). One can easily see that \( F_\pm^{(k)}(s, t) \) are the solutions of the DDR (14) and (15) when \( \alpha_k(t) \) is near 1. DDR possess the correct analyticity property as given by the SWR representation.

The concept of the Pomeron was introduced originally as a simple Regge pole at \( j = \alpha_p(t) \) that has the quantum numbers of the vacuum and thus contributes to the even-signatured amplitude only so that both \( F_{pp} \) and \( F_{pp} \) get the same contribution \( F_+^{(P)} \sim s^{\alpha_p(t)} \) at high energies. As the cross sections can not increase faster than \( \ln^2 s \) from the rigorous results as we will see, \( \alpha_p(0) \) has to be very close to 1. The slow and smoothly rising behavior of the total cross sections can then be approximated by a small power increase \( s^{\epsilon_p} \), leading to an empirical trajectory for the Pomeron

\[ \alpha(t) = 1 + \epsilon_p + \alpha' t = 1.08 + \alpha' t \] (18)

The slope of the Pomeron trajectory \( \alpha' \) can be determined from the slow increasing behavior of the nuclear slope parameter \( B \) within the diffraction peak because the simple Pomeron picture gives the explicit energy dependence of the slope parameter

\[ B = B_o + 2 \alpha' \ln s \] (19)
and makes the diffraction peak to shrink as the energy increases. Experimentally $\alpha' = 0.2 GeV^{-2}$ as it has been known for about two decades [21](See also Fig. 2 in Ref. [18]). The simple Regge pole picture of the Pomeron has a few other striking predictions. For example, the $\rho$-value for $pp$ as well as $\overline{p}p$ both should be given by $\frac{\pi}{2} \epsilon_p \simeq 0.12$ at high energies and the elastic cross section $\sigma_e$ must increase faster than the total cross-section $\sigma_T$ because

$$\sigma_e = \sigma_T^3 / 16\pi B$$

(20)

For the optical model type parametrization (8). Obviously this effect can not continue forever.

There are in fact some authors [22] who do not want to be constrained by the asymptotic bounds and theorems at present energies and continue to prefer a simple bare Pomeron picture. The need to unitarize the bare Pomeron behavior at high energies is however generally recognized, because the asymptotic bound is based on general principles involving more than just the unitarity and therefore less constraining. Thus one can violate the unitarity bound well below the rigorous asymptotic bound, which in fact is the case for the picture of the bare Pomeron [23]. Physically speaking, one may say the small but positive $\epsilon_p$ is a reflection of heavy flavour production above a certain energy scale. Thus $\epsilon_p$ should be scale-dependent through new flavour production. One should then unitarize the bare Pomeron behavior by eikonalization. If the bare Pomeron is due to the reggeized color-singlet two-gluon exchanges, one may say it should be renormalized through rescattering of two gluons via multi-gluon exchanges. This procedure will convert the bare Pomeron behavior to a more tamed $\ln s$ ($\gamma \leq 2$) behavior. The solutions we get from DDR corresponding to a singularity at $j = 1$ in the forward direction correspond directly to the renormalized and physical Pomeron.

The solution of DDR that gives the asymptotic behavior of the total cross section to be $\ln^2 s$ is

$$F_+^{(P2)}(s, 0) = i s [A_+ + B_+ (\ln \frac{s}{s_+} - i\frac{\pi}{2})^2]$$

(21)

which is sometimes called the unitarized or physical Pomeron term.

On the other hand, the odd-signatured counterpart of the Pomeron can also be constructed from DDR by assuming that the difference of the $pp$ and $\overline{p}p$ cross sections $\Delta \sigma_T = \sigma_{\overline{p}p} - \sigma_{pp}$ increases like $\ln s$ while the total cross sections increase like $\ln^2 s$ [10, 24],
This odd-signatured Pomeron-like object is called the maximal Odderon. One can also construct the Pomeron amplitude that gives the total cross section increasing with energy as \( \ln s \),

\[
F_+(P_1)(s, 0) = i s \left[ A_+ + B_+ \left( \ln \frac{s}{s^+} - i\frac{\pi}{2} \right) \right]
\]

(23)

In this case, \( \Delta \sigma_T \) can be a non-zero constant asymptotically,

\[
F_-^{(O_1)}(s, 0) = s \left[ A_- + B_- \left( \ln \frac{s}{s^-} - i\frac{\pi}{2} \right) \right]
\]

(24)

Experimentally \( \Delta \sigma_T \sim s^{\alpha(0)-1} \) with \( \alpha(0) \simeq 0.5 \) through the ISR energies. It was shown from the eikonal formalism[13] that the maximal Odderon behavior is difficult to generate while either behavior of the two Pomeron amplitudes are easy and natural to derive.

In the eikonal formalism, the near-forward scattering amplitude for \( pp \) and \( \bar{p}p \) scattering is written as

\[
F_k(s, t) = 4\pi i s \int_0^\infty b \, db \, J_0(b\sqrt{-t}) \{1 - e^{-\Omega_k(s, b)}\}, \quad (k = pp, \bar{p}p)
\]

(25)

in the impact parameter \( b \) space. Here \( \Omega_k(s, b) \) are the eikonals. One can then decompose the eikonals into \( \Omega_{\pm} \) in analogous form as (2) so that

\[
F_+(s, 0) = 8\pi i s \int_0^\infty b \, db \{1 - e^{-\Omega_+(s, b)}\} \cosh \Omega_-(s, b)
\]

(26)

\[
F_-(s, 0) = 8\pi i s \int_0^\infty b \, db \, e^{-\Omega_+(s, b)} \sinh \Omega_-(s, b)
\]

(27)

There are several classes of the eikonals that can lead to \( \ln^2 s \) behavior for \( \sigma_T \) but in no cases \( \Delta \sigma_T \sim \ln s \) behavior is possible to achieve.

Finally if \( \sigma_T(s) \sim s^{\alpha_p(0)-1} \) asymptotically, the Pomeron should also couple to the diffractive process \( p + p \rightarrow p + X \) with \( X \) having an invariant mass \( M \) and diffractively produced, i.e., into the forward direction with the same quantum number as \( p \). Then through the unitarity one will get the contributions from the diffractive process in the intermediate states through the
optical theorem. Such a process will produce the so-called triple – Pomeron coupling contribution to the total cross section when both \( s \) and \( M^2 \) are large compared to a characteristic scale beyond which the simple Regge picture makes sense. This process will obviously present a self-consistent check on the Pomeron dominance at high energies. The CDF group\[25]\ tested the standard triple-Pomeron Regge formula for single diffraction dissociation

\[
s \frac{d^2\sigma_{SD}}{dt\,dM^2} = G(t) \left( \frac{s}{s_0} \right)^{\alpha_p(0)-1} \left( \frac{s}{M^2} \right)^{2\alpha_p(t)-\alpha_p(0)}
\]

at the Tevatron energies \( \sqrt{s} = 546 \, GeV \) and 1800 \( GeV \) for the regions \( M^2/s < 0.2 \) and \( 0 \leq -t \leq 0.4GeV^2 \). They found in particular the need to include the screening corrections in the triple-Pomeron Regge model because the single diffraction total cross section shows a flat \( s \)-dependence rather than \( s^{2\alpha_p} \) as expected from the model.

The Pomeron exchange diagram in the diffraction scattering has an obvious similarity with the one in the deep-inelastic lepton scattering where the exchanged object is instead a virtual photon. Because of the similarity between the Pomeron and photon coupling to the quarks, there is a quantitative relation between the two processes and the deep-inelastic lepton scattering may be used to calculate the diffraction dissociation. The recent experimental efforts at HERA are in fact to learn about, amongst others, the quark and gluon densities in the Pomeron, i.e., the partonic structure of the Pomeron based on the similarity with the diffraction dissociation processes \[26\].

### III Asymptotic Theorems

As we mentioned before, experiments show that the nuclear slope parameter \( B \) increases slowly with energy like \( \ln s \) as expected from the simple Regge Pomeron picture so that the elastic total cross section increases faster than the total cross section at the current energies. For example, we have \( \sigma_e/\sigma_T \approx 0.18 \) at ISR energies \( \sqrt{s} = 30 \) to 60 \( GeV \); 0.215±0.005 (UA4)\[16\] and 0.210±0.002 (CDF)\[3\] at \( \sqrt{s} = 546 \, GeV \); 0.23 ± 0.012 (E710)\[27\] and 0.248 ± 0.005 (CDF)\[3\] at \( \sqrt{s} = 1800 \, GeV \).

Obviously the elastic total cross section can not continue to grow faster than the total cross section indefinitely so that from (20) the nuclear slope
$B$ should grow at least as fast as the total cross section at asymptotic energies within the framework of the optical model parametrization. In fact the rigorous statement is that the absorptive slope $B_A$ defined by the absorptive part of the amplitude

$$B_A(s, t) = 2 \frac{d}{dt} (\ln A(s, t))$$

satisfies the inequality

$$B_A(s, t = 0) > \frac{\sigma_T^2}{18\pi\sigma_e^A} > \frac{\sigma_T}{18\pi}$$

a result which has been with us for three decades [28]. As for the full nuclear slope defined by (12), there can be complications because of the real part contributions and the best one can say[29] is that $B(s, t) < c(t)\ln s$ for $t < 0$ under certain extra assumptions of the infinite sequence of uniformly continuous functions. While it is then clear that we are far from the asymptotic energy, the bound on the nuclear slope $B$ is an example indicating how general the rigorous statements are.

Nevertheless asymptotic theorems are very powerful and useful to guide what could be expected at high energies. The most famous case of the rigorous statements is the Froissart bound $\sigma_T < C \ln^2 s/s_+$ with $C \leq \pi/m^2 \simeq 60\,mb$ [30]. Besides the unitarity condition for the partial waves, one needs to assume certain polynomial boundedness on the analytic scattering amplitude itself or its absorptive part to derive the Froissart bound. As mentioned before, the Froissart bound is rather general and is not violated numerically for a reasonable choice of the scale parameter $s_+$ around a few GeV$^2$ by the known experiments at current energies. It is interesting to know if nature chooses to follow $\ln^2 s$ behavior for the total cross section at the asymptotic energy. The discrepancy between the CDF and E710 $\sigma_T$ at the Tevatron energy makes it difficult to draw an unambiguous conclusion in the asymptotic amplitude analysis as we will see in the next section.

There are several general statements that can follow when the Froissart bound is saturated. One of them is the lower bound of the absorptive slope $B_A(s, t = 0)$ as discussed above. Another one is the Auberson, Kinoshita and Martin (AKM) theorem[31] which says if $\sigma_t \propto \ln^2 s$, $F_{\pm}(s, t)/F_{\pm}(s, 0)$ tends to a non-trivial entire function of order $\frac{1}{2}$ of the variable $\tau = t \ln^2 s$ in the region $|t| < R/\ln^2 s$. In this case, the diffraction peak shrinks like $1/\ln^2 s$. 

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With the increasing cross sections, the question of similarity between the particle-particle and antiparticle-particle scattering cross sections, i.e., the original Pomeranchuk theorem, $\sigma_{T}^{pp} - \sigma_{T}^{p\bar{p}} \rightarrow 0$ as $s \rightarrow \infty$, has to be replaced by the statement that the ratio $\sigma_{T}^{p\bar{p}}/\sigma_{T}^{pp}$ tends to unity as $s \rightarrow \infty$. In fact, more rigorous examinations revealed that the Pomeranchuk theorem could be proven only when $\sigma_{T}^{pp}$ or $\sigma_{T}^{p\bar{p}}$ goes to infinity as $s \rightarrow \infty$[32]. In this case, the difference $\Delta \sigma_T$ between $\sigma_{T}^{pp}$ and $\sigma_{T}^{p\bar{p}}$ does not necessarily tend to zero and the best one can say[33] is that $|\Delta \sigma_T| \leq C \ln s/s_0$ so that the maximal or other finite Odderon contributions are not contradicting with the rigorous statements at the asymptotic energy.

As for the amplitudes, if one allows all possible asymptotic behaviours for the even-signatured part consistent with the Froissart bound, i.e.,

$$F_+(s, t = 0) \sim i B_+ s \left[ \ln(s \, e^{-i\pi/2}) \right]^{\beta_+}$$

with $\beta_+ \leq 2$, the odd-signatured part can take any of the following form asymptotically [34],

$$F_-(s, t = 0) \sim B_- s \left[ \ln(s \, e^{-i\pi/2}) \right]^{\beta_-}$$

with $\beta_- \leq \beta_+/2+1$ and $\beta_- < \beta_+ + 1$. In other words, there is a large domain of $(\beta_+, \beta_-)$ that is allowed by the general principles of analyticity, unitarity and positivity of the absorptive part of $F_{pp}$ and $F_{p\bar{p}}$. The maximal Odderon behavior corresponds to the point $\beta_+ = \beta_- = 2$.

We note that experimentally $\sigma_- = \frac{1}{2} (\sigma_T^{pp} - \sigma_T^{p\bar{p}})$ is negative up to the ISR energies but when the total cross sections increase with energy there is no guarantee that $\sigma_-$ will continue to stay negative at higher energies in principle. However $\sigma_-$ is consistent with $s^{-1/2}$ behavior from the known experiments. If the odd-signature amplitude is negligible at high energies, one has $\rho_{pp} = \rho_{p\bar{p}}$ which will tend to 0 as $s \rightarrow \infty$. But with $F_-$ contributions, one can have all sorts of possibilities for $\rho$. The rigorous statement of $\rho$ [35] based on general principles is that $\rho \leq \frac{\pi}{m_\pi} \ln(s/s_0) \sqrt{\sigma_T}$.

The rigorous statement on the ratio of the differential cross sections for $pp$ and $p\bar{p}$ at high energies is that inside the diffraction peak, the ratio $(d\sigma/dt)_{pp}/(d\sigma/dt)_{p\bar{p}}$ tends to some limiting values that contain unity[36]. The proof of this statement involves several qualified assumptions such as those about the phase and imaginary parts of the amplitudes. But experimentally the ratio of the particle and antiparticle differential cross sections seems
to approach to each other, i.e., the ISR data \([19]\) at \(\sqrt{s} = 52.8 \, GeV\) show remarkable equality except for the dip region as we mentioned above.

IV  Asymptotic Amplitude Analysis

It is clear that the various asymptotic analytic representations for the Pomeron and Odderon discussed in Section II are all consistent with the rigorous asymptotic statements. The asymptotic amplitude analysis is then to construct the \(pp\) and \(\overline{pp}\) amplitudes from (3) where \(F_+\) is made of various different combinations of the Pomeron \(P_1\) or \(P_2\) and Regge pole terms and \(F_-\) given by Regge terms alone or plus the Odderon term;

\[
F_+ = F_+^{(P_i)}(s, 0) + \sum_k F_+^{(k)}(s, 0), \quad (i = 1 \text{ or } 2) \quad \text{(33)}
\]

\[
F_- = F_-^{(0)}(s, 0) + \sum_k F_-^{(k)}(s, 0), \quad \text{or } \quad = \sum_k F_-^{(k)}(s, 0) \quad \text{(34)}
\]

which are then tested against the existing experimental data \([11,17,37]\).

It is crucial then to select as complete a set of data as possible without leaving out any experimental group of data that are published and not retracted, nor reducing errors to artificially enhance the weight to account for the paucity of higher energy results. Such a compilation of data, including statistical merging of data points at a given energy, already exists \([38]\).

Comprehensive \(\chi^2\)-fits and analysis have been carried out for two data sets differing only by the lowest value of \(\sqrt{s}\) allowed, i.e., 9.7 \(GeV\) and 5 \(GeV\). There are 171 experimental points in the larger set of data for \(\sqrt{s} \geq 5 \, GeV\) and 111 experimental points for \(\sqrt{s} \geq 9.7 \, GeV\). They are distributed as follows: 97 values of \(\sigma_T\) (40 for \(\overline{pp}\) and 57 for \(pp\)), 65 values of \(\rho\) (12 for \(\overline{pp}\) and 53 for \(pp\)), and 9 values of \(\Delta\sigma_T\) in the larger set, while 58 values of \(\sigma_T\) (22 for \(\overline{pp}\) and 36 for \(pp\)) and 53 values of \(\rho\) (12 for \(\overline{pp}\) and 41 for \(pp\)) in the smaller set. The details of the analysis can be found in \([17]\). Because the majority of precise data is at lower energies ( \(\sqrt{s} < 63 \, GeV\) ), it is expected that somewhat detailed Regge terms beyond just one exchange degenerate Regge pole term should be needed. In addition because there is no new anomaly in higher energy experimental points, one can expect several forms of the combinations in \(F_+\) and \(F_-\) to do more or less equally well in terms of \(\chi^2\). In fact this was found to be the case for any model that contains more
than just one exchange-degenerate Regge term. In particular, Model (B) in Ref. 17 in which $F_+ = F_{+}^{(P_2)} + F_{+}^{R_+} + F_{+}^R$ and $F_- = F_{-}^{R_+} + F_{-}^R$ where $F_{\pm}^{R_\pm}$ and $F_{\pm}^R$ denote the exchange-degenerate and non-degenerate Regge terms respectively, fits the data slightly better than Model (D) of Ref. 17 in which $F_{+}^{(P_2)}$ is replaced by $F_{+}^{(P_1)}$ for $\sqrt{s} > 5 GeV$ while the situation is reversed for the data set with $\sqrt{s} > 9.7 GeV$. But the $\chi^2$-difference per d.o.f. in either case is only no larger than 0.04, thus making both models equally acceptable in terms of statistics. Note that in the first model, $\sigma_+ \sim \ln^2 s$ as $s \to \infty$ while in the second model, $\sigma_+ \sim \ln s$ as $s \to \infty$, thus making it difficult to favor either of the asymptotic forms of $\sigma_T$. In either models however $F_- \to 0$ as $s \to \infty$ and in particular there is no need of the Odderon term in the asymptotic amplitude analysis. In fact the maximal Odderon model fared better for the larger data set with $\sqrt{s} \geq 5 GeV$ than the data set with $\sqrt{s} \geq 9.7 GeV$, implying that the Odderon term is effectively improving the low-energy fit and not relevant for the high-energy fit.

If the new CDF $\sigma_T$ is to withstand further experimental scrutiny, it may find a natural explanation in terms of a threshold slightly above the $Sp\bar{p}S$ energy. Detailed fit with Model (G) in Ref.17 which is the modification of Model (D) in Ref.17 by a threshold term [39] gives $\sigma_T = 77.1 mb$ at $\sqrt{s} = 1800 GeV$ somewhat smaller than the CDF $\sigma_T$ while the standard model (B) or (D) gives $\sigma_T = 74 \sim 76 mb$ at the Tevatron energy which is consistent with the E710 value. Since the UA4/2 data does not seem to require more than one exponential conforming to the standard expectation of $\rho$, the effective strength of the threshold component, if it exists, should be rather weak leaving at most a few mb level of new physics at the Tevatron energy. Finally Model (B) predicts $\sigma_T = 109 mb$ at the LHC while Model (D) gives $\sigma_T = 99 \sim 104 mb$ compared to Model (G) which predicts $\sigma_T = 101 \sim 105 mb$.

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References

1. UA4/2 Collab., C. Augier et al., Phys. Lett. B316, 448(1993).
2. UA4 Collab., D. Bernard et al., Phys. Lett. B198, 538(1987).
3. CDF Collab., F. Abe et al., Fermilab-Pub-93/234-E; See also P. Giromini, in Proc. Vth Blois Workshop (Providence, RI, 1993).
4. E710 Collab., S. Sadr, in Proc. Vth Blois Workshop; N. Amos et al., Phys. Lett. B243, 158(1990); Phys. Rev. Lett. 68, 2433(1992).
5. E. M. Levin and M. G. Ryskin, Phys. Rep. 189, No.6, 267(1990); E. Levin, Fermilab-Pub-93/062-T.
6. L. Lipatov, in Perturbative QCD, ed. A. H. Mueller (World Scientific, Singapore, 1989).
7. G. J. Daniell and D. A. Ross, Phys. Lett. B224, 166(1989); P. Gauron, L. Lipatov and B. Nicolescu, Phys. Lett. B260, 407(1991).
8. L. V. Gribov, E. M. Levin and M. G. Ryskin, Phys. Rep. 100, 1(1983); See also E. M. Levin (Ref. 5).
9. A. R. White, Mod. Phys. A6, 1859(1991); also ANL-HEP-preprint.
10. K. Kang and B. Nicolescu, Phys. Rev. D11, 2461(1974).
11. K. Kang, Nucl. Phys. B(Proc. Suppl.) 12, 64(1990).
12. H. Cheng, J. K. Walker and T. T. Wu, Phys. Lett. 44B, 97(1973); C. Bourrely, J. Soffer and T. T. Wu, Phys. Rev. D19, 3249(1979); M. M. Block, in : Proc. Vth Blois Workshop where further related references can be found.
13. J. Finkelstein, H. M. Fried, K. Kang and C-I Tan, Phys. Lett. B232, 257(1989).
14. A. Donnachie and P. V. Landshoff, Phys. Lett. 122B, 345(1983); Nucl. Phys. B231, 189(1983); B244, 322(1984).
15. G. B. West and D. R. Yennie, Phys. Rev. 172, 1413(1968); N. H. Buttimore, E. Gostman and E. Leader, Phys. Rev. D18, 694(1978); D35, 407(1987); R. Cahn, Z. Phys. C15, 253(1982).
16. U. Amaldi et al., Nucl. Phys. B145, 367(1978); UA4 Collab., M. Bozzo et al., Phys. Lett. 147B, 392(1984).
17. K. Kang, P. Valin and A. R. White, in : Proc. Hadron ’93 (Como, Italy 1993) and Brown-HET-924.
18. M. M. Block, K. Kang and A. R. White, Mod. Phys. A7,
444(1992), which reviewed the status of high energy elastic
scattering as of October 1991 before the UA4/2 and new CDF
results.

19. A. Breakstone et al., Phys. Rev. Lett. 54, 2180(1985);
   S. Erhan et al., Phys. Lett. B152, 131(1985).
20. UA4 Collab., M. Bozzo et al., Phys. Lett. 155B, 197(1985).
21. P. D. B. Collins, F. D. Gault and A. Martin, Nucl. Phys.
   B85, 141(1977).
22. A Donnachie and P. V. Landshoff, Nucl. Phys. B267, 690(1986).
   See also P. V. Landshoff, in : Proc. First Blois Workshop
   (Château de Blois, France 1985); Proc. 3rd Blois Workshop
   (Evanston, IL, 1989).
23. U. Maor, in : Proc. Vth Blois Workshop.
24. L. Lukaszuk and B. Nicolescu, Nuovo Cim. Lett. 8, 405(1973).
25. CDF Collab., S. Belforte et al., CDF/ANAL/CDF/CDFR/2050.
   See also P. Giromini in : Proc. Vth Blois Workshop.
26. G. Ingleman and P. Schline, Phys. Lett. B152, 256(1985);
   A. Donnachie and P. V. Landshoff, Nucl. Phys. B244 (Ref. 14)
27. N. Amos et al., Phys. Lett. B243 (Ref. 4)
28. S. W. Mac Dowell and A. Martin, Phys. Rev. 135, 960(1964)
29. A. Martin, in : Proc. First Blois Workshop in which other
   asymptotic bounds are also critically reviewed.
   See also P. Valin, Phys. Rep. 203(4), 233(1991).
30. M. Froissart, Phys. Rev. 123, 1053(1961);
   A. Martin, Nuovo Cimento 42, 930(1966);
   L. Lukaszuk and A. Martin, Nuovo Cimento 47A, 265(1967).
31. G. Auberson, T. Kinoshita and A. Martin, Phys. Rev.
   D3, 3185(1971).
32. T. Kinoshita, in : Perspectives in Modern Physics, ed. R. Marshak
   (John Wiley and Sons, 1966); R. J. Eden, Phys. Rev. Lett. 16,
   39(1966); G. Grunberg and T. N. Truong, Phys. Rev. Lett. 31,
   63(1973).
33. S. M. Roy and V. Singh, Phys. Lett. 32B, 50(1970);
   R. J. Eden, Rev. Mod. Phys. 43, 15(1971).
34. H. Cornille, Nuovo Cimento 70A, 165(1970).
35. N. N. Khuri and T. Kinoshita, Phys. Rev. B140, 706(1965).
36. H. Cornille and A. Martin, Nucl. Phys. B48, 104(1972).
37. K. Kang, P. Valin and A. R. White, in: Proc. Vth Blois Workshop and Brown-HET-916; M. M. Block et al., in: Proc. Multiparticle '93 Conference (Aspen, Co, 1993).

38. S. Hadjitheodoridis, Ph. D. thesis, Brown University (May 1989) which is updated with high energy data by K. Kang, P. Valin and A. R. White (Refs. 17 and 37).

39. K. Kang and S. Hadjitheodoridis, Phys. Lett. B208, 135(1988).