A Chaotic Quadratic Oscillator with Only Squared Terms: Multistability, Impulsive Control, and Circuit Design

Dhinakaran Veeman 1, Ahmad Alanezi 2, Hayder Natiq 3, Sajad Jafari 4,5 and Ahmed A. Abd El-Latif 6,*

Abstract: Here, a chaotic quadratic oscillator with only squared terms is proposed, which shows various dynamics. The oscillator has eight equilibrium points, and none of them is stable. Various bifurcation diagrams of the oscillator are investigated, and its Lyapunov exponents (LEs) are discussed. The multistability of the oscillator is discussed by plotting bifurcation diagrams with various initiation methods. The basin of attraction of the oscillator is discussed in two planes. Impulsive control is applied to the oscillator to control its chaotic dynamics. Additionally, the circuit is implemented to reveal its feasibility.

Keywords: quadratic oscillator; bifurcation; multistability; impulsive control; chaotic circuit

1. Introduction

Chaotic flows have attracted lots of attention recently [1,2]. Many systems with various features have been proposed to study the chaotic dynamics [3,4]. Some of the proposed oscillators are discussed from the viewpoint of their quadratic or cubic terms [5,6]. Some other studies have focused on the equilibrium points [7]. Oscillators with no equilibria [8], with stable equilibria [9], with curves of equilibria [10], and with a peanut-shaped equilibrium curve [11] are some examples. A hyperjerk oscillator has been investigated in [12]. A multi-dimensional chaotic system was discussed in [13]. In [14], a multi-scroll chaotic circuit was analyzed. Various dynamics of the Sprott B system were studied in [15]. The dynamics of coupled neurons were investigated in [16]. Chaotic dynamics have many applications, such as encryption [17–19]. In [20,21], a discrete chaotic dynamic was used in image encryption. A chaotic encryption method and its application in the internet of things were studied in [22]. A plain-text-related image encryption method using Chen oscillator was proposed in [23].

Multistability is an exciting feature of dynamical systems [24–26]. Multistable oscillators have various applications [27]. A multistable oscillator with various attractors was discussed in [28]. In [29,30], the multistability of the series hybrid electric vehicle was investigated. Extreme multistability is a particular case of multistability [31]. In [32], the extreme multistability of a fractional-order oscillator was studied. The multistability of a Chua system was discussed in [33]. Memristive chaotic oscillators are very interesting [34].
A memristive neural system was studied in [35]. A memristive Chua system was discussed in [36]. In [37], a memristive oscillator with a unique attractor was investigated. A memristive oscillator with a fractional-order difference was studied in [38]. In [39], the multistability of a five-value memristive oscillator was investigated.

Control and synchronization of dynamical systems have been a hot topic [40–42]. Many algorithms have been proposed to control chaotic oscillators [43]. Control of the Chen oscillator was investigated in [44]. A fuzzy-based controller was studied in [45]. In [46], control of a piecewise linear oscillator was discussed. Adaptive control of a chaotic oscillator was discussed in [47]. Delayed feedback control of chaotic oscillators was investigated in [48]. Impulsive control is a valuable method for this purpose [49]. Event-triggered impulsive control was studied in [50–52]. Synchronization of the Chen oscillator was discussed in [53]. Impulsive control for synchronization of a chaotic network was investigated in [54].

Chaotic circuits show the feasibility of these dynamics [55,56]. Implementing circuits for chaotic oscillators has been an exciting topic [57]. A memristive circuit was studied in [1]. Various dynamics of the system were discussed. A jerk circuit with the arcsinh function was proposed in [58], and its behaviors were investigated. The application of a chaotic circuit on image encryption was discussed in [59]. The circuit design of a 5D hyperchaotic oscillator was studied in [60], and its multistability was discussed. In [61], the circuit design of a 3D system was studied.

Here, a quadratic oscillator with only squared terms is presented. The comparison of the proposed system with some previous important literature is presented in Table 1 to highlight the distribution of this paper. The proposed system is a simple quadratic system with only squared terms, and some important properties of the system are investigated. The chaotic behavior of the oscillator is discussed in Section 2. Additionally, its equilibrium points and their stabilities are investigated. In Section 3, various oscillator dynamics are discussed by changing its three parameters. 1D and 2D bifurcation diagrams are used to investigate the dynamics of the oscillator. Lyapunov exponents (LEs) help to investigate the type of dynamics in various parameters. Then by plotting bifurcation diagrams with various initial conditions, the multistability of the oscillator is investigated. The basin of attraction for the oscillator shows the variation of dynamics by changing the initial values of its three variables. In Section 4, the impulsive control of the oscillator is designed, and its results are discussed. Then, the circuit of the oscillator is designed in Section 5. In Section 6, the conclusion of the paper is discussed.

### Table 1. Comparison of the proposed system with the previous ones.

| Ref. | Dimension | Type of Terms                  | Number of Terms | Number of Equilibrium Points | Multistability | Circuit |
|------|-----------|--------------------------------|-----------------|-----------------------------|----------------|---------|
| [62] | 4         | Cubic                          | 9               | Infinite                    | ✓              | ✓       |
| [63] | 3         | Cubic & tanh(.)                | 7               | 5                           | ✓              | ✓       |
| [64] | 4         | Quadratic                      | 9               | 0                           | ✓              | ✓       |
| [16] | 3         | Linear & Tanh                  | 10              | 3–7                         | ✓              | ✓       |
| This work | 3       | Quadratic with only squared terms | 9               | 8                           | ✓              | ✓       |

### 2. The Proposed Oscillator

A quadratic oscillator is presented as:

\[
\begin{align*}
\dot{x} &= -y^2 + z^2 + C_2x^2 + C_1 \\
\dot{y} &= -z^2 + x^2 + 2 \\
\dot{z} &= z^2 + C_3x^2
\end{align*}
\]
To propose the system, a parametric quadratic system with only squared terms and constants in each variable is designed. Then a computer search is applied to compute the value of parameters and initial values for chaotic solutions. The oscillator only has squared terms and not a multiplication of two variables. It is important to investigate if this system is symmetric or has offset boosting properties. Offset boosting and symmetry experiments are significant features of chaotic systems [65,66]. We examine the existence of offset boosting and symmetry by adding a constant excitation force to all the right-hand sides of equations one by one. However, there was no offset boosting. The system shows chaotic dynamics in $C_1 = -0.6$, $C_2 = -0.7$, $C_3 = -2$ and initial conditions $(0, 0, 0)$. Figure 1 presents the time series of chaotic dynamics in part (a), the 3D chaotic attractor in part (b), and its three 2D projections in $X - Y$, $Y - Z$, and $X - Z$ planes with gray color.

![Figure 1](image)

Figure 1. Chaotic dynamics of the presented oscillator in $C_1 = -0.6$, $C_2 = -0.7$, $C_3 = -2$ and initial conditions $(0, 0, 0)$; (a) time series; (b) phase space.

The equilibrium points of the system are calculated by setting zeros on the right-hand side of Equation (1). The system has eight equilibrium points, as shown in Table 1. To investigate the stability of equilibrium points, the characteristic equation and eigenvalues should be computed for each of them. The corresponding eigenvalues of the equilibrium points are presented in Table 2. All of the equilibrium points have at least one positive real part of the eigenvalue, so they are unstable.

| # | Equilibrium | Eigenvalues |
|---|-------------|-------------|
| $E_1$ | $(\sqrt{2}, \sqrt{2}, 2)$ | $\lambda_1 = -1.2036$, $\lambda_{2,3} = 1.6118 \pm 4.8979i$ |
| $E_2$ | $(-\sqrt{2}, \sqrt{2}, 2)$ | $\lambda_1 = 8.9257$, $\lambda_{2,3} = -1.4729 \pm 1.1899i$ |
| $E_3$ | $(\sqrt{2}, -\sqrt{2}, 2)$ | $\lambda_1 = 3.1356$, $\lambda_{2,3} = -0.5577 \pm 3.1455i$ |
| $E_4$ | $(-\sqrt{2}, -\sqrt{2}, 2)$ | $\lambda_1 = 6.2323$, $\lambda_2 = -2.3957$, $\lambda_3 = 2.1432$ |
| $E_5$ | $(\sqrt{2}, \sqrt{2}, -2)$ | $\lambda_1 = -6.2323$, $\lambda_2 = 2.3957$, $\lambda_3 = -2.1432$ |
| $E_6$ | $(-\sqrt{2}, \sqrt{2}, -2)$ | $\lambda_1 = -3.1356$, $\lambda_{2,3} = 0.5577 \pm 3.1455i$ |
| $E_7$ | $(\sqrt{2}, -\sqrt{2}, -2)$ | $\lambda_1 = -8.9257$, $\lambda_{2,3} = 1.4729 \pm 1.1899i$ |
| $E_8$ | $(-\sqrt{2}, -\sqrt{2}, -2)$ | $\lambda_1 = 1.2036$, $\lambda_{2,3} = -1.6118 \pm 4.8979i$ |

3. Dynamical Properties

The oscillator has three crucial parameters that significantly affect its dynamics. The first studied parameter is $C_1$. Figure 2 presents the bifurcation diagram of Oscillator (1) by
varying $C_1$. The other parameters are kept constant as $C_2 = -0.7, C_3 = -2$. The maximum values of three variables of the oscillator with the forward continuation method are plotted in parts (a−c). The oscillator shows a period-doubling route to chaos. Part (d) of the Figure 2d shows the oscillator’s LEs by changing $C_1$. A positive LE can prove the existence of chaos. Additionally, one LE approaches zero by approaching bifurcation points.

Figure 2. Bifurcation diagram by varying $C_1$ with forwarding continuation method; The first initial conditions are $(0, 0, 0)$; (a) peak values of $x$ variable; (b) peak values of $y$ variable; (c) peak values of $z$ variable; (d) LEs.

The bifurcation diagram is discussed by changing $C_2$ in Figure 3. The diagram is plotted using a forward continuation method in constant parameters $C_1 = -0.6, C_3 = -2$. A period-doubling route to chaos can be seen by changing the parameter. LEs of the oscillator confirm the chaotic dynamics in small $C_2$.

To better investigate various dynamics of the oscillator, the 2D bifurcation diagrams are discussed by changing parameters $C_1$ and $C_2$ in Figure 4. A classic bifurcation diagram presents the dynamics by changing one parameter. The 2D bifurcation diagram is helpful since it shows the variations by changing two parameters. The bifurcation diagram by changing $C_1$ is plotted for nine values of parameter $C_2$. The diagram helps to investigate various dynamics by changing these two parameters. The results show that in the studied interval of $C_1$, increasing $C_2$ causes a decrease in the complexity of dynamics.
Figure 3. Bifurcation diagram by varying $C_2$ with forwarding continuation method; The first initial conditions are $\langle 0,0,0 \rangle$; (a) peak values of $x$ variable; (b) peak values of $y$ variable; (c) peak values of $z$ variable; (d) LEs.

Figure 4. Bifurcation diagram of the oscillator by changing parameters $C_1$ and $C_2$; the diagrams are plotted by constant initial values at origin; (a) maximum values of $x$; (b) maximum values of $y$; (c) maximum values of $z$. 
The multistability of the oscillator can be revealed by plotting a bifurcation diagram with different initial conditions. In Figure 5, various bifurcations are plotted using different colors in parameters $C_1 = -0.6$ and $C_2 = -0.7$. The magenta color shows a bifurcation diagram in $C_3 \in [-2, -1.953]$. It is plotted by the forward continuation method and the first initial conditions at the origin. The blue one is the forward continuation bifurcation diagram with origin as the first initial conditions. The green color is the forward bifurcation in $C_3 \in [-1.9836, -1.983]$ and the first initial conditions as $(0, 0, 0)$. Comparison of the magenta color diagram with blue and green ones reveals the coexisting attractors in various intervals of $C_3$.

Figure 5. Forward bifurcation diagram with origin as the first initial conditions; the blue diagram in $C_3 \in [-1.974, -1.965]$, the magenta diagram in the interval $C_3 \in [-2, -1.953]$, and the green diagram in $C_3 \in [-1.9836, -1.983]$; (a) peak values of $x$ variable; (b) peak values of $y$ variable; (c) peak values of $z$ variable.

The basin of attraction of the oscillator in $C_1 = -0.6, C_2 = -0.7, C_3 = -1.9722$ is discussed to investigate the initial conditions that result in chaotic and periodic dynamics as presented in Figure 5. In Figure 6, the basin of attractions is plotted in two planes as $z_0 = 0$ and $z_0 = 1$. The pink color shows chaotic regions, and the white one presents the periodic regions. The gray color depicts unbounded regions. In each plane, the dynamics in the intervals $x_0 \in [-3, 7], y_0 \in [-4, 4]$ are computed for constant $z_0$. So the variations of dynamics by changing $x_0$ and $y_0$ can be seen in each plane. The effect of $z_0$ can be seen by comparing the two planes. Four sets of initial conditions are selected from the two planes to show the coexisting chaotic and periodic attractors (Figure 7).
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Figure 6. Basin of attraction in \(C_1 = -0.6, C_2 = -0.7, C_3 = -1.9722\); pink color shows the chaotic region, white shows the periodic region, and gray color presents unbounded regions; two planes are computed for two \(z_0\).

Figure 7. Attractors of the oscillator in \(C_1 = -0.6, C_2 = -0.7, C_3 = -1.9722\) and (a) \((x_0, y_0, z_0) = (-0.05, -1.4, 0)\); (b) \((x_0, y_0, z_0) = (-0.05, -1.44, 0)\); (c) \((x_0, y_0, z_0) = (-0.05, -2.14, 1)\); (d) \((x_0, y_0, z_0) = (-0.05, -2.12, 1)\); the 2D projections of the attractors are plotted in gray color.
4. Impulsive Control

In this section, impulsive control [67,68] is applied to stabilize the proposed oscillator. As was discussed in Table 2, the system does not have an equilibrium point in origin. So in the first step, the change of variables \( x_{\text{new}} = x_{\text{old}} - \sqrt{2}, y_{\text{new}} = y_{\text{old}} - \sqrt{2}, z_{\text{new}} = z_{\text{old}} - 2 \) is used to move the equilibrium point \( (x^*, y^*, z^*) = \left(\sqrt{2}, \sqrt{2}, 2\right) \) to the origin. So the transformed oscillator is as follows:

\[
\begin{align*}
\dot{x} &= -(y + y^*)^2 + (z + z^*)^2 - 0.7(x + x^*)^2 - 0.6 \\
\dot{y} &= -(z + z^*)^2 + (x + x^*)^2 + 2 \\
\dot{z} &= -(z + z^*)^2 - 2(x + x^*)^2
\end{align*}
\]  

(2)

where \( x_{\text{new}}, y_{\text{new}}, z_{\text{new}} \) are called \( x, y, z \) in Equation (2). Then Oscillator (2) can be rewritten as:

\[
P = A \times P + \phi(P)
\]

(3)

where \( P \) is the vector of variables \( [x, y, z]^T \), \( A \times P \) is the linear term of Equation (2), and \( \phi(P) \) is the nonlinear term. From Equation (2), we have:

\[
A = \begin{bmatrix}
-1.4 \times x^* & -2 \times y^* & 2 \times z^* \\
2 \times x^* & 0 & -2 \times z^* \\
-4 \times x^* & 0 & 2 \times z^*
\end{bmatrix}, \quad \phi(P) = \begin{bmatrix}
-y^2 + z^2 - 0.7x^2 \\
-z^2 + x^2 \\
z^2 - 2x^2
\end{bmatrix}
\]

(4)

Now the controlled oscillator can be written as:

\[
\begin{align*}
\dot{P} &= g(t, P) = A \times P + \phi(P) \quad t \neq \tau_i \\
&= (t^+) - P(t^-) = B \times P \quad t = \tau_i, i = 1, 2, \ldots
\end{align*}
\]

(5)

So \( B, \tau_i \) should be calculated for this control method.

**Definition 1.** If \( VV : R_+ \times R^n \rightarrow R_+ \), then \( VV \) belong to the class \( VVV_0 \), if:

1. \( VV \) is continuous in \( (\tau_{i-1}, \tau_i] \times R^n \) and for each \( P \in R^n, i = 1, 2, \ldots \), \( (t, Y) \rightarrow (\tau_i^+, P) \)
   \( \lim VV(t, Y) = VV(\tau_i^+, P) \) exists;

2. \( VV \) is locally Lipschitzian in \( P \).

**Definition 2.** For \( (t, P) \in (\tau_{i-1}, \tau_i] \times R^n \), we have,

\[
D^+VV(t, P) \triangleq h \rightarrow 0 + \lim \sup_{\eta \rightarrow 0} \frac{1}{h} |VV(t + h, P + hg(t, P) - VV(t, P)|
\]

**Definition 3.** Comparison system: \( VV \in VVV_0 \) and consider:

\[
D^+VV(t, X) \leq a(t, VV(t, P)), t \neq \tau_i; \quad \text{and} \quad VV(t, P + UU(t, P)) \leq \Psi_i(VV(t, P)), t = \tau_i,
\]

where \( a : R_+ \times R_+ \rightarrow R \) is continuous and \( \Psi_i : R_+ \rightarrow R_+ \) is non-decreasing. Then the following system is the comparison system:

\[
\begin{align*}
\omega \omega &= a(t, \omega), \; t \neq \tau_i \\
\omega \omega(\tau_i^+) &= \Psi_i(\omega \omega(\tau_i)) \\
\omega \omega(\tau_i^0) &= \omega \omega 0 \geq 0
\end{align*}
\]

(6)

**Theorem 1.** The following conditions are considered:

1. \( VV : R^n \times R^n \rightarrow R_+ \), \( VV \in VVV_0 \), \( KK(t)D^+VV(t, P) + D^+KK(t)VV(t, P) \leq a(t, KK(t)VV(t, P)), t \triangleq li, \) when \( a \) is continuous in \( (\tau_{i-1}, \tau_i] \times R^n \) for each \( P \in R^n \), \( i = 1, 2, \ldots \), \( (t, y) \rightarrow (\tau_i^+, P) \) \( \lim \omega(a(t, y)) = a(\tau_i^+, P) \) exists. \( KK(t) \geq mm > 0, \) t →
\( \tau_i^+ \lim_{t \to t} KK(t) = KK(\tau_i), t \to \tau_i^+ \lim_{t \to t} KK(t) \) exists, \( i = 1, 2, \ldots, D^+ KK(t) = h \to 0+ \lim\sup \left( \frac{1}{t} \right) KK(t) = h \lim \sup (1) \); 

\[ KK(\tau_i + 0)VV(\tau_i + 0, P + UU(kk, P)) \leq \Psi_i(VK(T)VV(\tau_i, P)), i = 1, 2, \ldots; \]

\[ VV(t, 0) = 0 \text{ and } a(||P||) \leq VV(t, P) \text{ on } R_+ \times R^n, \text{ when } a(\cdot) \in R \text{ (continuous strictly increasing function class } a : R_+ \to R_+ \text{ so that } a(0) = 0) \text{ are satisfied.} \]

The global asymptotic stability for the solution \( \omega \omega = 0 \) of the comparison system implies global asymptotic stability of impulsive system trivial solution.

**Theorem 2.** Consider \( a(t, \omega \omega) = \lambda(t) \omega \omega, \Psi_i(\omega \omega) = d_i \omega \omega, d_i \geq 0 \text{ for all } i \geq 1 \). Consequently, the system origin is global asymptotically stable if Theorem 1 conditions and the following conditions are kept:

1. \( \lambda(t) \) is non-decreasing, \( t \to \tau_i^+ \lim \lambda(t) = \lambda(\tau_i), t \to \tau_i^+ \lim \lambda(t) = \lambda(\tau_i^+) \) exists, for all \( i = 1, 2, \ldots; \)

2. \( \sup \{ d_i \exp(\lambda(\tau_i + 1) - \lambda(\tau_i^+) ) \} = \varepsilon_0 < \infty; \)

3. There is a \( r > 1 \) such that \( \lambda(\tau_i + 1) + \lambda(\tau_i^+) + \lambda(\tau_i^+) \) is held for all \( d_i \geq d_i > 0, i = 1, 2, \ldots, \) or there is a \( r > 1 \) such that \( \lambda(\tau_i + 1) + \lambda(\tau_i^+) \) is for all \( i; \)

4. \( VV(t, 0) = 0, \) and we have a \( \cdot \) in class \( N \) such that \( a(||P||) \leq VV(t, P). \)

**Theorem 3.** The origin is an asymptotically stable equilibrium for the proposed oscillator if there is a \( \xi > 1 \) and a differentiable at \( t \neq \tau_i \) and non-increasing function \( KK(t) \) which satisfies:

\[
\begin{align*}
-KK(t) & \leq q + r \leq \frac{1}{(1+\varepsilon)\Delta_2} \ln \left( \frac{KK(\tau_i^+) KK(\tau_i^+) \lambda(\tau_i^+) \lambda(\tau_i^+)}{KK(\tau_i^+) KK(\tau_i^+) \lambda(\tau_i^+) \lambda(\tau_i^+)} \right) \\
-KK(t) & \leq q + r \leq \frac{1}{\max(\Delta_1, \Delta_2)} \ln \left( \frac{KK(\tau_i^+) \lambda(\tau_i^+) \lambda(\tau_i^+)}{KK(\tau_i^+) \lambda(\tau_i^+) \lambda(\tau_i^+)} \right) \\
r & = \begin{cases} 
0, & \text{if } K = I \\
2M \sqrt{\frac{2}{\lambda_1}}, & \text{if } K \neq I 
\end{cases}
\end{align*}
\]

where \( q \) is defined as the largest eigenvalue of \( (A + K^{-1}A^T K), K \) is a positive definite matrix, and \( \lambda_1 > 0 \) and \( \lambda_2 > 0 \) are the smallest and the largest eigenvalues of \( K \), respectively. \( \rho(V) \) is the spectral radius of \( V \) and \( d = r^2(I + B) \). \( M \) is considered as \( |x_i| < M, |y_i| < M, |z_i| < M \). \( KK(t) \) is as in Theorem 1, \( \tau_i^+ \): \( i = 1, 2, \ldots \) should satisfy:

\[
\Delta_1 = \sup_{1 \leq i < \infty} (T_{2i+1} - T_{2i}) < \infty \\
\Delta_2 = \sup_{1 \leq i < \infty} (T_{2i} - T_{2i-1}) < \infty
\]

For a constant \( \varepsilon \), we have:

\[ T_{2i+1} - T_{2i} \leq \varepsilon(T_{2i} - T_{2i-1}), \forall i \in 1, 2, \ldots, \infty \]

The theorem’s proof can be seen in [67].

**Remark 1.** Theorem 3 estimates the upper bound \( \Delta_{\text{max}} \) and \( \Delta_{\text{max}} \) of impulsive intervals.

\[
\Delta_1 = \frac{1}{(1+\varepsilon)(q+2\max(1))} \ln \left( \frac{KK(\tau_i^+) \lambda(\tau_i^+) \lambda(\tau_i^+)}{KK(\tau_i^+) \lambda(\tau_i^+) \lambda(\tau_i^+)} \right) \\
\Delta_2 = \varepsilon \Delta_1
\]
For controlling the proposed oscillator, the matrix $B$ is considered as:

$$
B = \begin{bmatrix}
-1.1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & -1
\end{bmatrix}
$$

(11)

Here, $q$ is defined as the largest eigenvalue of $(A + K^{-1}A^TK)$, where $K$ is a positive definite matrix. By considering $K = I$, $q$ is calculated as the maximum eigenvalue of $(A + A^T)$. For the Oscillator (2), $q = 9.8272$.

Another parameter in this control method is $d$ which is defined as:

$$
d = \rho^2(I + B)
$$

(12)

where $\rho(V)$ is the spectral radius of $V$. So, we have $d = (-1.1 + 1)^2 = 0.01$. Then the intervals of applying the controller are computed as:

$$
T_{2j+1} - T_{2j} = T_{2j} - T_{2j-1} = \Delta < (\Delta_1 = \Delta_2)
$$

$$
\Delta_1 = \Delta_2 = \frac{-\ln(\xi d)}{q}
$$

(13)

where $\xi$ is considered 1.1; we have $\Delta_1 = 0.4589$. So $\Delta$ is considered 0.45. Figure 8 presents the results of applying the discussed controller. Part (a) of the figure shows the controlled system (2), while in part (b), the original time series of the oscillator for various variables are plotted in the same time interval. So the controller makes all variables approach zero.

![Figure 8](image-url)

**Figure 8.** Time series of (a) controlled system (b) original system for various variables by initial conditions $(-\sqrt{2}, -\sqrt{2}, -2)$.

5. Circuit Design

Here the circuit of the oscillator is investigated in $C_1 = -0.6$, $C_2 = -0.7, C_3 = -2$, as its schematic is shown in Figure 9. The circuit is implemented with OrCAD-Pspice. The values of resistors are considered as $Res_1 = Res_2 = Res_5 = Res_6 = Res_8 = 70 \text{k}\Omega$, $Res_3 = 100 \text{k}\Omega$, $Res_4 = 17500 \text{k}\Omega$, $Res_7 = 5250 \text{k}\Omega$, $Res_9 = 35 \text{k}\Omega$, $Res_{10} = Res_{11} = Res_{12} = Res_{13} = Res_{14} = Res_{15} = 100 \text{k}\Omega$. The capacitors are selected as $Cap_1 = Cap_2 = Cap_3 = 10 \text{nF}$. Here, AD633 was used as a multiplier, and OPA404 was used as the operational amplifier. The positive voltage source is set to 15 V. The initial values of voltage in capacitors are considered as $(0, 0, 0)$. Figure 10 presents the results of the designed circuit for the Oscillator (1). Part (a) of the figure shows the time series of the chaotic circuit, while the other parts show the projection of its dynamics in three different planes. The results are wholly matched with the dynamics of Figure 1. In other words, the chaotic oscillator...
was completely implemented without any issue, and its feasibility was realized. So the
oscillator with only squared terms is physically realizable.

Figure 9. Schematic of the chaotic circuit.

Figure 10. (a) time series of the chaotic circuit; the chaotic attractor in (b) $X - Y$ plane; (c) $X - Z$ plane; (d) $Y - Z$ plane.
6. Conclusions

A novel quadratic chaotic oscillator was proposed here. The attractors of the oscillator were studied. Investigating the oscillator has shown the existence of eight equilibrium points, and none of them are stable; 1D and 2D bifurcation diagrams were studied to investigate the various dynamics of the oscillator. The results have shown the rich dynamics of the oscillator. LEs have revealed the types of dynamics. Studying bifurcation diagrams of the system by different initial values has shown coexisting attractors in different parameter regions. The basin of attractions was discussed in two planes. In addition, some of the multistable attractors were shown. Impulsive control was applied to the oscillator to force the chaotic dynamics approaches to the origin. The results have shown the high potential of this controller. Then the circuit of the oscillator was designed, which presents the feasibility of the chaotic dynamics. The complex dynamics of the oscillator make it a proper choice for random number generators and encryption applications.

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References

1. Li, C.; Yang, Y.; Du, J.; Chen, Z. A simple chaotic circuit with magnetic flux-controlled memristor. *Eur. Phys. J. Spec. Top.* 2021, 230, 1723–1736. [CrossRef]

2. Kamal, F.; Elsonbaty, A.; Elsaid, A. A novel fractional nonautonomous chaotic circuit model and its application to image encryption. *Chaos Solitons Fractals* 2021, 144, 110686. [CrossRef]

3. Ma, C.; Mou, J.; Xiong, L.; Banerjee, S.; Liu, T.; Han, X. Dynamical analysis of a new chaotic system: Asymmetric multistability, offset boosting control and circuit realization. *Nonlinear Dyn.* 2021, 103, 2867–2880. [CrossRef]

4. Bonny, T. Chaotic or hyper-chaotic oscillator? Numerical solution, circuit design, MATLAB HDL-coder implementation, VHDL code, security analysis, and FPGA realization. *Circuits Syst. Signal Process.* 2021, 40, 1061–1088.

5. Mathale, D.; Goufo, E.F.D.; Khumalo, M. Coexistence of multi-scroll chaotic attractors for a three-dimensional quadratic autonomous fractional system with non-local and non-singular kernel. *Alex. Eng. J.* 2021, 60, 3521–3538. [CrossRef]

6. Mammeri, M. A 2-D Discrete Cubic Chaotic Mapping with Symmetry: Una cartografía cónica cúbica discreta con simetría. *South Fla. J. Dev.* 2021, 2, 5111–5121. [CrossRef]

7. Chen, M.; Wang, Z.; Nazarimehr, F.; Jafari, S. A novel memristive chaotic system without any equilibrium point. *Integration* 2021, 79, 133–142. [CrossRef]

8. Wei, Z. Dynamical behaviors of a chaotic system with no equilibria. *Phys. Lett. A* 2011, 376, 102–108. [CrossRef]

9. Wang, X.; Chen, G. A chaotic system with only one stable equilibrium. *Commun. Nonlinear Sci. Numer. Simul.* 2012, 17, 1264–1272. [CrossRef]

10. Sambas, A.; Vaidyanathan, S.; Bonny, T.; Zhang, S.; Hidayat, Y.; Gundara, G.; Mamat, M. Mathematical Model and FPGA Realization of a Multi-Stable Chaotic Dynamical System with a Closed Butterfly-Like Curve of Equilibrium Points. *Appl. Sci.* 2021, 11, 788. [CrossRef]

11. Sambas, A.; Vaidyanathan, S.; Tlelo-Cuautle, E.; Abd-El-Atty, B.; El-Latif, A.A.A.; Guillén-Fernández, O.; Hidayat, Y.; Gundara, G. A 3-D multi-stable system with a peanut-shaped equilibrium curve: Circuit design, FPGA realization, and an application to image encryption. *IEEE Access* 2020, 8, 137116–137132. [CrossRef]

12. Rajagopal, K.; Duraisamy, P.; Tadesse, G.; Volos, C.; Nazarimehr, F.; Hussain, I. A fractional-order ship power system: Chaos and its dynamical properties. *Int. J. Nonlinear Sci. Numer. Simul.* 2021. [CrossRef]
41. Zhou, Z.-X.; Grebogi, C.; Ren, H.-P. Parameter impulse control of chaos in crystal growth process. *J. Cryst. Growth* **2021**, 563, 126079. [CrossRef]

42. Mahmoud, E.E.; Higazy, M.; Althagafi, O.A. A novel strategy for complete and phase robust synchronizations of chaotic nonlinear systems. *Symmetry* **2020**, 12, 1765. [CrossRef]

43. Boccaletti, S.; Grebogi, C.; Lai, Y.-C.; Mancini, H.; Maza, D. The control of chaos: Theory and applications. *Phys. Rep.* **2000**, 329, 103–197. [CrossRef]

44. Yassen, M. Chaos control of Chen chaotic dynamical system. *Chaos Solitons Fractals* **2003**, 15, 271–283. [CrossRef]

45. Zhou, S.-S.; Jahanshahi, H.; Din, Q.; Bekiros, S.; Alcaraz, R.; Alessafi, M.O.; Alsaaadi, F.E.; Chu, Y.-M. Discrete-time macroeconomic system: Bifurcation analysis and synchronization using fuzzy-based activation feedback control. *Chaos Solitons Fractals* **2021**, 142, 110378. [CrossRef]

46. Fu, S.; Liu, Y.; Ma, H.; Du, Y. Control chaos to different stable states for a piecewise linear circuit system by a simple linear control. *Chaos Solitons Fractals* **2020**, 130, 109431. [CrossRef]

47. Pham, V.-T.; Vaidyanathan, S.; Volos, C.; Jafari, S.; Alsaaadi, F.E.; Alsaaadi, F.E. Chaos in a simple snap system with only one nonlinearity, its adaptive control and real circuit design. *Arch. Control. Sci.* **2019**, 29, 73–96.

48. Pyragas, K. Delayed feedback control of chaos. *Philos. Trans. R. Soc. A Math. Phys. Eng. Sci.* **2006**, 364, 2309–2334. [CrossRef]

49. Wu, S.; Li, X.; Ding, Y. Saturated impulsive control for synchronization of coupled delayed neural networks. *Neural Netw.* **2021**, 141, 261–269. [CrossRef]

50. Peng, D.; Li, X.; Rakkiyappan, R.; Ding, Y. Stabilization of stochastic delayed systems: Event-triggered impulsive control. *Appl. Math. Comput.* **2021**, 401, 126054. [CrossRef]

51. Wang, M.; Wu, S.; Li, X. Event-triggered delayed impulsive control for nonlinear systems with applications. *J. Franklin. Inst.* **2021**, 358, 4277–4291. [CrossRef]

52. Li, X.; Li, P. Input-to-state stability of nonlinear systems: Event-triggered impulsive control. *IEEE Trans. Autom. Control* **2021**, 1–7. [CrossRef]

53. Zhang, X.; Li, X.; Han, X. Design of hybrid controller for synchronization control of Chen chaotic system. *J. Nonlinear Sci. Appl.* **2017**, 10, 3320–3327. [CrossRef]

54. Li, X.; Rakkiyappan, R. Impulsive controller design for exponential synchronization of chaotic neural networks with mixed delays. *Commun. Nonlinear Sci. Numer. Simul.* **2013**, 18, 1515–1523. [CrossRef]

55. Leutcho, G.D.; Kengne, J.; Kengne, L.K.; Akgul, A.; Pham, V.-T.; Jafari, S. A novel chaotic hyperjerk circuit with bubbles of bifurcation: Mixed-mode bursting oscillations, multistability, and circuit realization. *Phys. Scr.* **2020**, 95, 075216. [CrossRef]

56. Yu, F.; Shen, H.; Liu, L.; Zhang, Z.; Huang, Y.; He, B.; Cai, S.; Song, Y.; Yin, B.; Du, S. CCII and FPGA realization: A multistable modified fourth-order autonomous Chua’s chaotic system with coexisting multiple attractors. *Complexity* **2020**, 2020, 5212601. [CrossRef]

57. Li, C.; Wei, Z.; Zhang, W. Periodic solutions and circuit design of chaos in a unified stretch-twist-fold flow. *Eur. Phys. J. Spec. Top.* **2021**, 230, 1971–1978. [CrossRef]

58. Leutcho, G.D.; Kengne, J.; Kengne, L.K.; Chedjou, J.C.; Nosirov, K. A simple anti-parallel diodes based chaotic jerk circuit with arcsinh function: Theoretical analysis and experimental verification. *Analogue Integr. Circuits Signal Process.* **2021**, 108, 597–623. [CrossRef]

59. Hu, H.; Cao, Y.; Xu, J.; Ma, C.; Yan, H. An image compression and encryption algorithm based on the fractional-order simplest chaotic circuit. *IEEE Access* **2021**, 9, 22141–22155. [CrossRef]

60. Wan, Q.; Zhou, Z.; Ji, W.; Wang, C.; Yu, F. Dynamic analysis and circuit realization of a novel no-equilibrium 5D memristive hyperchaotic system with hidden extreme multistability. *Complexity* **2020**, 2020, 7106861. [CrossRef]

61. Lai, Q.; Kuate, P.D.K.; Liu, F.; Iu, H.H.-C. An extremely simple chaotic system with infinitely many coexisting attractors. *IEEE Trans. Circuits Syst. II Express Briefs* **2019**, 67, 1129–1133. [CrossRef]

62. Lai, Q.; Wan, Z.; Kengne, L.K.; Kuate, P.D.K.; Chen, C. Two-memristor-based chaotic system with infinite coexisting attractors. *IEEE Trans. Circuits Syst. II Express Briefs* **2020**, 68, 2197–2201. [CrossRef]

63. Yu, F.; Shen, H.; Zhang, Z.; Huang, Y.; Cai, S.; Du, S. A new multi-scroll Chua’s circuit with composite hyperbolic tangent-cubic nonlinearity: Complex dynamics, Hardware implementation and Image encryption application. *Integration* **2021**, 81, 71–83. [CrossRef]

64. Lai, Q.; Wan, Z.; Kuate, P.D.K. Modelling and circuit realisation of a new no-equilibrium chaotic system with hidden attractor and coexisting attractors. *Electron. Lett.* **2020**, 56, 1044–1046. [CrossRef]

65. Li, C.; Lei, T.; Wang, X.; Chen, G. Dynamics editing based on offset boosting. *Chaos Interdiscip. J. Nonlinear Sci.* **2020**, 30, 063124. [CrossRef]

66. Li, C.; Wang, X.; Chen, G. Diagnosing multistability by offset boosting. *Nonlinear Dyn.* **2017**, 90, 1335–1341. [CrossRef]

67. Sun, J.; Zhang, Y. Impulsive control of Lorenz systems. In Proceedings of the Fifth World Congress on Intelligent Control and Automation (IEEE Cat. No. 04EX788), Hangzhou, China, 15–19 June 2004.

68. Zhang, Y.; Sun, J. Controlling chaotic Lu systems using impulsive control. *Phys. Lett. A* **2005**, 342, 256–262. [CrossRef]