Gravitino Dark Matter
with Weak-Scale Right-Handed Sneutrino

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Abstract

We consider cosmological implications of supersymmetric models with right-handed (s)neutrinos where the neutrino masses are purely Dirac-type. We pay particular attention to the case where gravitino is the lightest superparticle while one of the right-handed sneutrinos is next-to-the-lightest superparticle. We study constraints from big-bang nucleosynthesis and show that the constraints could be relaxed compared to the case without right-handed sneutrinos. As a result, the gravitino-dark-matter scenario becomes viable with relatively large value of the gravitino mass. We also discuss constraints from the structure formation; in our model, the free-streaming length of the gravitino dark matter may be as long as $O(1\text{ Mpc})$, which is comparable to the present observational upper bound on the scale of free-streaming.
1 Introduction

Existence of dark matter in our universe, which is strongly supported by a lot of recent cosmological observations [1, 2, 3], requires physics beyond the standard model. This is because there is no viable candidate for dark matter in the particle content of the standard model. Many possibilities of dark matter have been discussed in various frameworks of particle physics models so far [4]. Importantly, properties of the dark matter particle depend strongly on the particle physics model we consider.

In this article, we adopt supersymmetry (SUSY) as new physics beyond the standard model. In such a case, probably the most popular candidate for dark matter is thermally produced lightest neutralino which is usually assumed to be the lightest superparticle (LSP); in some part of the parameter space of the minimal supersymmetric standard model (MSSM), the relic density of the lightest neutralino well agrees with the present mass density of dark matter observed. However, as we discuss in the following, the lightest neutralino is not the only possibility of dark matter in supersymmetric models.

If we try to build a supersymmetric model which accommodates with all theoretical and experimental requirements, we expect that there exist new exotic particles which are not superpartners of the standard model particles. For example, if local SUSY is realized in nature, gravitino \( \psi_\mu \) which is the superpartner of the graviton should exist. In addition, superpartners of right-handed neutrinos, which are strongly motivated to explain neutrino masses indicated by the neutrino oscillation experiments, may also exist. In particular, if the neutrino masses are Dirac-type, superpartners of the right-handed neutrinos are expected to be as light as other MSSM superparticles in the framework of gravity-mediated SUSY breaking. Importantly, one of those new particles may be the lightest superparticle and hence may be dark matter. In addition, existence of these exotic superparticles may significantly change the phenomenology of dark matter in supersymmetric models.

In this paper, we consider the supersymmetric model in which the neutrino masses are Dirac-type and discuss cosmological implications of such a scenario. The possibility of the right-handed sneutrino LSP has already been discussed in Ref. [5]; it has been pointed out that, if one of the right-handed sneutrinos is the LSP, the present relic density of the right-handed sneutrino \( \tilde{\nu}_R \) may be as large as the dark matter density and hence the scenario of \( \tilde{\nu}_R \) dark matter can be realized. Here, we consider another case where the gravitino is the LSP and one of the right-handed sneutri-
nos is the next-to-the-lightest superparticle (NLSP). If the gravitino is the LSP, it may be a viable candidate for dark matter also in the case without the right-handed sneutrinos \[6, 7, 8\]. In such a case, however, stringent constraints on the scenario are obtained from the study of the gravitino production at the time of the reheating after inflation and also from the study of the big-bang nucleosynthesis (BBN) reactions. With the right-handed sneutrino NLSP, we reconsider cosmological constraints on the gravitino LSP scenario. We pay particular attention to the BBN constraints and also to the constraints from the structure formation of the universe. We will see that the BBN constraints could be significantly relaxed if there exists the right-handed sneutrino NLSP. In addition, we will also see that the free-streaming length of the gravitino dark matter may be a few Mpc, which is as long as the present sensitivity to the free-streaming length from observations. Thus, detailed understanding about the mechanism of structure formation has important impact on our scenario.

The organization of this paper is as follows. In the next section, we introduce the model based on which our analysis will be performed. In Section 3, we discuss the BBN constraints on the gravitino LSP scenario with right-handed sneutrinos. We will see that the constraints could be significantly relaxed compared to the case without right-handed sneutrinos. Constraints from the structure formation will be discussed in Section 4. Application of our discussion to the scenario where all the gravitino dark matter is produced by the decay of other superparticle will be discussed in Section 5. Section 6 is devoted to conclusions and discussion.

### 2 Model Framework

In this section, we summarize the model. As we mentioned, we consider the case where gravitino is the LSP while right-handed sneutrino is the NLSP. We assume that neutrino masses are purely Dirac-type, and the superpotential of the model is written as

\[
W = W_{\text{MSSM}} + y_\nu \hat{L} \hat{H}_u \hat{\nu}_R^c,
\]

where \(W_{\text{MSSM}}\) is the superpotential of the MSSM, \(\hat{L} = (\hat{\nu}_L, \hat{e}_L)\) and \(\hat{H}_u = (\hat{H}_u^+, \hat{H}_u^0)\) are left-handed lepton doublet and up-type Higgs doublet, respectively. In this article, “hat” is used for superfields, while “tilde” is for superpartners. Generation indices are omitted for simplicity. In this model, neutrinos acquire their masses only through Yukawa interactions as \(m_\nu = y_\nu \langle H_u^0 \rangle = y_\nu v \sin \beta\), where \(v \simeq 174\).
GeV is the vacuum expectation value (VEV) of the standard model Higgs field and \( \tan \beta = \langle H_u^0 \rangle / \langle H_d^0 \rangle \). Thus, the neutrino Yukawa coupling is determined by the neutrino mass as

\[
y_{\nu} \sin \beta = 3.0 \times 10^{-13} \times \left( \frac{m_{\nu}^2}{2.8 \times 10^{-3} \text{ eV}^2} \right)^{1/2}.
\]

(2)

Mass squared differences among neutrinos have already been determined accurately by neutrino oscillation experiments \([9, 10]\), and are given by

\[
[\Delta m_{\nu}^2]_{\text{atom}} \simeq 2.8 \times 10^{-3} \text{ eV}^2, \quad [\Delta m_{\nu}^2]_{\text{solar}} \simeq 7.9 \times 10^{-5} \text{ eV}^2.
\]

(3)

In this article, we assume that the spectrum of neutrino masses is hierarchical, hence the largest neutrino Yukawa coupling is of the order of \(10^{-13}\). We use \( y_{\nu} = 3.0 \times 10^{-13} \) for our numerical analysis.

With right-handed (s)neutrinos, it is also necessary to introduce soft SUSY breaking terms related to right-handed sneutrinos: right-handed sneutrino mass terms and tri-linear coupling terms called \( A_{\nu} \)-terms. Soft SUSY breaking terms relevant to our analysis are

\[
\mathcal{L}_{\text{SOFT}} = -M_L^2 \bar{L}^T L - M_{\tilde{\nu}_R}^2 \tilde{\nu}_R^c \tilde{\nu}_R + \left( A_{\nu} L H_u \tilde{\nu}_R^c + h.c. \right),
\]

(4)

where all breaking parameters, \( M_L, M_{\tilde{\nu}_R} \) and \( A_{\nu} \), are defined at the electroweak (EW) scale. We parametrize \( A_{\nu} \) by using the dimensionless constant \( a_{\nu} \) as

\[
A_{\nu} = a_{\nu} y_{\nu} M_L.
\]

(5)

We adopt gravity-mediated SUSY breaking scenario and, in such a case, \( a_{\nu} \) is expected to be \( O(1) \). Though the \( A_{\nu} \)-term induces the left-right mixing in the sneutrino mass matrix, the mixing is safely neglected in the calculation of mass eigenvalues due to the smallness of neutrino Yukawa coupling constants. Thus, the masses of sneutrinos are simply given by

\[
m_{\tilde{\nu}_L}^2 = M_L^2 + \frac{1}{2} \cos(2\beta) m_Z^2, \quad m_{\tilde{\nu}_R}^2 = M_{\tilde{\nu}_R}^2,
\]

(6)

where \( m_Z \approx 91 \text{ GeV} \) is the Z boson mass. In the following discussion, we assume that all the right-handed sneutrinos are degenerate in mass for simplicity.

In this article, we consider the gravitino LSP scenario with right-handed sneutrino NLSP. In such a case, the next-to-next-the-LSP (NNLSP) plays an important role in the thermal history of the universe. However, there are many possibilities
of the NNLSP, depending on the detail of SUSY breaking scenario. In our study, we concentrate on the case that the NNLSP is the lightest neutralino whose composition is almost Bino. This situation is easily obtained if we consider the so-called constrained-MSSM type scenario [11]. It is not difficult to extend our discussion to the scenario with other NNLSP candidates. If Bino is the NNLSP, only the right-handed sneutrino which is the superpartner of the heaviest neutrino plays important roles in the Bino decay and other right-handed sneutrinos are hardly produced in this decay. Thus, $m_{\tilde{\nu}_R}$ given in Eq. (6) should be understood as the mass of the superpartner of the heaviest neutrino.

3 Constraints from BBN

It is well known that models with the gravitino LSP usually receive stringent constraints from BBN. In these models, the lightest superparticle in the MSSM sector (which we call MSSM-LSP) has a long lifetime, sometimes much longer than one second due to Planck-scale suppressed interactions. Then, MSSM-LSP produced in the early universe decays into gravitino by emitting standard-model particles after BBN starts, which may spoil the success of BBN. In this section, we show that, when the right-handed sneutrino is the NLSP, the thermal history of the universe could be significantly altered, resulting in weaker constraints from BBN.

Now, we consider the thermal history of the universe. In the early universe when the temperature is higher than the masses of MSSM particles, all the standard model particles and their superpartners are in thermal equilibrium. Gravitino and right-handed sneutrinos are, however, never thermalized due to the weakness of their interactions. When the temperature becomes as low as $m_B/20$ ($m_B$: mass of Bino-like neutralino), Bino-like neutralino $\tilde{B}$ decouples from the thermal bath. Then, Bino-like neutralino decays into right-handed sneutrino as well as into gravitino. After that, right-handed sneutrino decays into gravitino emitting right-handed neutrino. As can be easily understood, the decay of right-handed sneutrino is harmless for the BBN scenario, because only right-handed neutrino is emitted. On the other hand, the decay of Bino-like neutralino into gravitino affects BBN.

Constraints from BBN including hadronic decay modes are intensively studied in Refs. [12, 13]; according to the studies, the BBN constraints give the upper bound on $Y_X E_{\text{vis}}$ as a function of $\tau_X$, where $X$ stands for a long-lived but unstable particle, $Y_X \equiv [n_X/s]_{\tau < \tau_X}$ (with $n_X$ and $s$ being the number density of $X$ and the entropy
density of the universe, respectively), $E_{\text{vis}}$ is the mean energy of visible particles emitted in the $X$ decay, and $\tau_X$ is the lifetime of $X$. We use the upper bound on $Y_X E_{\text{vis}}$ obtained in Ref. [12]. In our analysis, we adopt the line of $Y_p$(IT) in that work as the constraint from the $p \leftrightarrow n$ conversion.

3.1 Decay of Bino-like neutralino

First, we will take a closer look at the decay of the MSSM-LSP, which is Bino-like neutralino. Main decay modes are the following two-body decays shown in Fig 1: $\tilde{B} \rightarrow \tilde{\nu}_R \tilde{\nu}_L$, $\tilde{B} \rightarrow \psi \mu \gamma$, and $\tilde{B} \rightarrow \psi \mu Z$. Decay widths of these processes are given by

\begin{equation}
\Gamma_{\tilde{B} \rightarrow \tilde{\nu}_R \tilde{\nu}_L} = \frac{g_Y^2}{64\pi} m_{\tilde{B}} \left[ \frac{A_{\nu}}{m_{\tilde{\nu}_L}^2 - m_{\tilde{\nu}_R}^2} \right]^2 \left[ 1 - \frac{m_{\tilde{\nu}_R}^2}{m_{\tilde{B}}^2} \right],
\end{equation}

\begin{equation}
\Gamma_{\tilde{B} \rightarrow \psi \mu \gamma} = \frac{\cos^2 \theta_W}{48\pi M_*^2} m_{\tilde{B}}^5 \left[ 1 - \frac{m_{3/2}^2}{m_{\tilde{B}}^2} \right]^3 \left[ 1 + 3 \frac{m_{3/2}^2}{m_{\tilde{B}}^2} \right],
\end{equation}

\begin{equation}
\Gamma_{\tilde{B} \rightarrow \psi \mu Z} = \frac{\sin^2 \theta_W}{48\pi M_*^2} m_{\tilde{B}}^5 F \left[ \left( 1 - \frac{m_{3/2}^2}{m_{\tilde{B}}^2} \right)^2 \left( 1 + 3 \frac{m_{3/2}^2}{m_{\tilde{B}}^2} \right) - \frac{m_Z^2}{m_{\tilde{B}}^2} G \right],
\end{equation}

where $m_{3/2}$ is the gravitino mass and functions $F$ and $G$ are defined as

\begin{equation}
F(m_{\tilde{B}}, m_{3/2}, m_Z) = \left[ 1 - \left( \frac{m_{3/2}^2 + m_Z^2}{m_{\tilde{B}}^2} \right) \right]^{1/2} \left[ 1 - \left( \frac{m_{3/2}^2 - m_Z^2}{m_{\tilde{B}}^2} \right) \right]^{1/2},
\end{equation}

\begin{equation}
G(m_{\tilde{B}}, m_{3/2}, m_Z) = 3 + \frac{m_{3/2}^3}{m_{\tilde{B}}^3} \left( -12 + \frac{m_{3/2}^2}{m_{\tilde{B}}^2} \right) + \frac{m_{3/2}^4}{m_{\tilde{B}}^4} - \frac{m_Z^2}{m_{\tilde{B}}^2} \left( 3 - \frac{m_{3/2}^2}{m_{\tilde{B}}^2} \right).\end{equation}

Here, $g_Y$ is the $U(1)_Y$ gauge coupling constant, $M_* \simeq 2.4 \times 10^{18}$ is the reduced Planck mass, and $\theta_W$ is the Weinberg angle. The lifetime of Bino-like neutralino is given by
Figure 2: Contour plot of the branching ratio \( \text{Br}(\tilde{B} \rightarrow \psi \mu \gamma) \) (left figure) and the lifetime of \( \tilde{B} \) (right figure) on the \((m_{3/2}, m_{\tilde{B}})\) plane: we take \( m_{\tilde{\nu}_R} = 100 \text{ GeV}, a_\nu = 1, \) and \( m_{\tilde{\nu}_L} = 1.5m_{\tilde{B}} \) in both figures.

\[
\tau_{\tilde{B}}^{-1} = \Gamma_{\tilde{B}} \simeq 2\Gamma_{\tilde{B}\rightarrow\tilde{\nu}_R\bar{\nu}_L} + \Gamma_{\tilde{B}\rightarrow\psi\mu\gamma} + \Gamma_{\tilde{B}\rightarrow\psi\mu\gamma},
\]

where the factor 2 in front of \( \Gamma_{\tilde{B}\rightarrow\tilde{\nu}_R\bar{\nu}_L} \) comes from the contribution of the CP conjugate final state.

The branching ratio of the mode \( \tilde{B} \rightarrow \psi\mu\gamma \) is shown in Fig.2 (left figure) on the \((m_{3/2}, m_{\tilde{B}})\) plane, where we take \( m_{\tilde{\nu}_R} = 100 \text{ GeV}, a_\nu = 1, \) and \( m_{\tilde{\nu}_L} = 1.5m_{\tilde{B}} \).

Importantly, the decay mode \( \tilde{B} \rightarrow \tilde{\nu}_R\bar{\nu}_L \) competes with the mode \( \tilde{B} \rightarrow \psi\mu\gamma \) or it even dominates the total decay rate when the gravitino mass is larger than 0.1 GeV\(^1\). The lifetime of \( \tilde{B} \) is \(10^{2-3}\) seconds on most of the parameter region shown in Fig.2 (right figure).

Without right-handed sneutrinos, \( \tilde{B} \rightarrow \psi\mu\gamma/Z \) is the main decay mode, and significant amount of visible particles (including hadrons) are produced. As a result, the gravitino mass is strictly constrained as \( m_{3/2} \lesssim 0.1 \text{ GeV} \) for \( \tau_{\tilde{B}} \lesssim 1 \) second from BBN\(^1\). In our scenario with the right-handed sneutrino NLSP, however, less visible particles are emitted, though the Bino-like neutralino is long-lived\(^2\). Therefore, constraints from BBN is expected to be relaxed.

Next, we consider three- or four-body decay modes of \( \tilde{B} \). Although branching ratios of these processes are much smaller than 1, they have impacts on the BBN

\(^1\)We have checked that \( \Gamma_{\tilde{B}\rightarrow\psi\mu\gamma} \) is about one order of magnitude smaller than \( \Gamma_{\tilde{B}\rightarrow\psi\mu\gamma} \) on the parameter region of Fig.2

\(^2\)Left-handed neutrinos injected by the decay might possibly change the abundance of \(^4\)He\(^\text{15}\). However, we have checked that the BBN constraints on the neutrino injection are much weaker than those on hadron injection from three- or four-body decay.
scenario. In particular, the hadronic decay modes give more severe constraints on the model than radiative ones via hadro-dissociation and $p \leftrightarrow n$ conversion processes.

In our model, three- or four-body decay modes of the Bino-like neutralino are $\tilde{B} \rightarrow \psi \mu q\bar{q}$, $\tilde{B} \rightarrow \tilde{\nu}_R \tilde{\nu}_L q\bar{q}$, and $\tilde{B} \rightarrow \tilde{\nu}_R \tilde{\nu}_L q\bar{q}$ shown in Fig.3, where $q$ and $\bar{q}$ denote quark and anti-quark, respectively. Since the hadronic branching ratio is comparable to radiative one in our scenario, the hadronic processes give the most severe constraints. Thus, we concentrate on hadro-dissociation and $p \leftrightarrow n$ conversion processes in order to derive BBN constraints. We calculate $B_{\text{had}} Y_{\tilde{B}} E_{\text{vis}}$ as a function of $\tau_{\tilde{B}}$, where $B_{\text{had}}$ is the hadronic branching ratio. In order to constrain the model quantitatively, we use the upper bound on $Y_X E_{\text{vis}}$ obtained in Ref.\cite{12}.

The product of the hadronic branching ratio and the visible energy, which is the mean energy of emitted hadrons from the three- and four-body decays, is given by

$$B_{\text{had}} E_{\text{vis}} = \frac{1}{\Gamma_{\tilde{B}}} \left[ \sum_q \left\{ 2\Gamma_{\tilde{B} \rightarrow \tilde{\nu}_R \psi \mu q\bar{q}} \langle E_{\text{vis}}^{(\tilde{\nu}_R \psi \mu q\bar{q})} \rangle + 2\Gamma_{\tilde{B} \rightarrow \tilde{\nu}_R \tilde{\nu}_L q\bar{q}} \langle E_{\text{vis}}^{(\tilde{\nu}_R \tilde{\nu}_L q\bar{q})} \rangle \right\} + \Gamma_{\tilde{B} \rightarrow \psi \mu Z} B_{\text{had}}^Z \langle E_{\text{vis}}^{(Z)} \rangle \right],$$

(9)

where the factor 2 in first and second terms are from contributions of CP conjugate

\footnote{In some parameter regions where the $Z$ boson in $\tilde{B} \rightarrow \psi \mu q\bar{q}$ is off-shell, the photo-dissociation effect caused by $\tilde{B} \rightarrow \psi \mu \gamma$ might be comparable to the hadro-dissociation effect. In such a parameter region, however, $\tilde{B} \rightarrow \tilde{\nu}_R \tilde{\nu}_L$ dominates in total decay as seen in Fig.2 Therefore, the photodissociation effect is negligible, which is consistent with previous work [14].}
final states, $B_{\text{had}}^{Z} \simeq 0.7$ is the hadronic branching ratio of $Z$ boson, $\langle E_{\text{vis}}^{(\cdots)} \rangle$ is the averaged energy of hadrons emitted in each decay process, and $E_{\text{vis}}^{(Z)}$ is the energy of the $Z$ boson,

$$E_{\text{vis}}^{(Z)} = \left[m_{Z}^{2} + \frac{m_{B}^{2}}{4} \left(1 - 2 \frac{m_{3/2}^{2} + m_{\tilde{B}}^{2}}{m_{B}^{2}} + \frac{(m_{3/2}^{2} - m_{\tilde{B}}^{2})^{2}}{m_{B}^{4}}\right)\right]^{1/2}. \quad (10)$$

### 3.2 Constraints

In order to evaluate $B_{\text{had}}Y_{\tilde{B}}E_{\text{vis}}$, we have to determine the primordial abundance of Bino-like neutralino. The abundance depends highly on parameters in the MSSM sector such as masses of other superparticles. We use the following formula for the (would-be) density parameter of $\tilde{B}$ [14]:

$$\Omega_{\tilde{B}} h^{2} = C_{\text{model}} \times 0.1 \left[\frac{m_{\tilde{B}}}{100 \text{ GeV}}\right]^{2}, \quad (11)$$

where the additional parameter $C_{\text{model}}$ is introduced to take the model dependence into account: $C_{\text{model}} \sim 1$ for the neutralino in the bulk region, $C_{\text{model}} \sim 0.1$ for the co-annihilation or funnel region, and $C_{\text{model}} \sim 10$ for the pure Bino case without co-annihilation. Then, the yield of the Bino-like neutralino is given by

$$Y_{\tilde{B}} = C_{\text{model}} \times 3.6 \times 10^{-12} \frac{m_{\tilde{B}}}{100 \text{ GeV}}. \quad (12)$$

Our numerical results are shown in Fig.4, where the BBN constraints are depicted on the $(m_{3/2}, m_{\tilde{B}})$ plane. We take $C_{\text{model}} = 10, 1, 0.1$ in the left, middle, and right figures, respectively. Other parameters are the same as those used in Fig.2.

Shaded regions are ruled out by BBN. As shown in these figures, the constraints are drastically relaxed compared to those in models without right-handed sneutrinos [14].

As shown in the figures, new allowed region appears; for example, for $C_{\text{model}} = 1, 0.1 \text{ GeV} \lesssim m_{3/2} \lesssim 40 \text{ GeV}$. In that region, $m_{\tilde{B}}$ is bounded from above due to the BBN constraints from four-body decays, $\tilde{B} \rightarrow \tilde{\nu}_{R} e_{L}^{+} q q'$ and $\tilde{B} \rightarrow \tilde{\nu}_{R} \nu_{L} q q$. (Notice that the hadronic branching ratio $B_{\text{had}}$ and mean energy $E_{\text{vis}}$ are enhanced when $m_{\tilde{B}}$ is large.) On the contrary, in the $0.01 \text{ GeV} \lesssim m_{3/2} \lesssim 0.1 \text{ GeV}$ region, Bino-like neutralino decays mainly into the gravitino through the $\tilde{B} \rightarrow \psi_{\mu} \gamma$ process with the lifetime $\tau_{\tilde{B}} \lesssim 1 \text{ second}$. Since the decay occurs before BBN starts, it does not affect the BBN scenario. This situation also holds in the usual gravitino LSP scenario without right-handed sneutrinos, and the same allowed region can be seen in Ref. [14].
Figure 4: Constraints from the BBN on the \((m_{3/2}, m_{\tilde{B}})\) plane: Parameters are chosen to be \(m_{\tilde{\nu}_R} = 100\) GeV, \(a_\nu = 1\), and \(m_{\tilde{\nu}_L} = 1.5m_{\tilde{B}}\). We set \(C_{\text{model}} = 10\), 1, and 0.1 in the left, middle, and right figures, respectively. The middle-shaded regions are ruled out by BBN, while dark and light-shaded regions are excluded by the WMAP measurement and the structure formation of the universe, respectively.

In the case of \(C_{\text{model}} = 10\)\,(0.1), the constraints from BBN is more (less) stringent than the \(C_{\text{model}} = 1\) case. As a result, the upper bound on \(m_{\tilde{B}}\) becomes smaller (larger). Results also depend on left-right mixing angle \(\theta_{\tilde{\nu}_L-\tilde{\nu}_R} \equiv |A_{\nu}/(m_{\tilde{\nu}_L}^2 - m_{\tilde{\nu}_R}^2)|\) in the sneutrino mass matrix. As one can see in Eq.(7), branching ratio of the mode \(\tilde{B} \rightarrow \psi_\mu\gamma/Z\) is more suppressed for larger \(\theta_{\tilde{\nu}_L-\tilde{\nu}_R}\), which leads to less stringent constraints from BBN. On the contrary, when \(a_\nu\) in Eq.(5) is much smaller than 1, BBN constraints are severe and our constraints become close to those for the case without right-handed sneutrinos. We also find that the region \(m_{\tilde{B}} \simeq 100\) GeV and \(0.1\) GeV \(\lesssim m_{3/2} \lesssim 1\) GeV is excluded in the left and middle figures. In these regions, Bino is almost degenerate with right-handed sneutrino in mass. As a result, the process \(\tilde{B} \rightarrow \tilde{\nu}_R\tilde{\nu}_L\) is kinematically suppressed and branching ratio of the process \(\tilde{B} \rightarrow \psi_\mu Z\) is enhanced.

In addition to the BBN constraints, we also depict other cosmological bounds in Fig.4 the gravitino abundance originating in \(\tilde{B}\) must not exceed the value observed in the WMAP, \(\Omega_{\text{DM}} h^2 \simeq 0.105\). Gravitino abundance from the decay is given by \(\Omega_{3/2}^{\text{dec}} = (m_{3/2}/m_{\tilde{B}}) \Omega_{\tilde{B}}\). Using Eq.(11), the cosmological constraint on the \((m_{3/2}, m_{\tilde{B}})\) plane is obtained as \(m_{\tilde{B}} m_{3/2} < 10^4\) GeV/\(C_{\text{model}}\), which is shown as a dark-shaded region in Fig.4. This constraint gives the upper bound on \(m_{3/2}\). Another constraint, \(\Omega_{3/2}^{\text{dec}} < 0.4\Omega_{\text{DM}}\), is also depicted as a light-shaded region, which comes from the structure formation of the universe, which is discussed in the next section.
4 Constraints from Structure Formation

As shown in the previous section, larger value of $m_{3/2}$ is allowed compared to the case without right-handed sneutrinos. In the newly allowed parameter region, the MSSM-LSP decays mainly into right-handed sneutrino $\tilde{\nu}_R$, and $\tilde{\nu}_R$ decays into the gravitino. Since the gravitino is produced with large velocity dissipation at the late universe, it behaves as a warm dark matter, and as a result, may affect the structure formation of the universe. In this section, we consider the constraints from the structure formation.

4.1 Decay of right-handed sneutrino

Once the gravitino is produced from the decay of $\tilde{\nu}_R$, it is expected to freely stream in the universe and smooth out the (small scale) primordial density fluctuation. Since the lifetime of Bino is much shorter than that of right-handed sneutrino in our scenario, the free-streaming length of the gravitino is estimated as

$$\lambda_{FS} = \int_{t_{\nu R}}^{t_{EQ}} \frac{dt}{a(t)} = \frac{2t_{EQ}u(t_{EQ})}{a(t_{EQ})} \left( \ln \left[ \frac{1}{u(t_{EQ})} + \sqrt{1 + \frac{1}{u^2(t_{EQ})}} \right] - \ln \left[ \frac{1}{u(\tau_{\tilde{\nu} R})} + \sqrt{1 + \frac{1}{u^2(\tau_{\tilde{\nu} R})}} \right] \right), (13)$$

where $v(t)$ is the velocity, $u(t) = p(t)/m_{3/2}$ with $p(t)$ being the momentum of gravitino, and $a(t)$ is the cosmic scale factor. Here, the time of the matter-radiation equality and the lifetime of $\tilde{\nu}_R$ are denoted by $t_{EQ}$ and $\tau_{\tilde{\nu} R}$, respectively.

Right-handed sneutrino decays into the gravitino through the two-body decay process $\tilde{\nu}_R \rightarrow \psi_\mu \nu_R$ shown in Fig.5. Notice that three- and four-body decay processes such as $\tilde{\nu}_R \rightarrow \psi_\mu \nu_L Z$, $\tilde{\nu}_R \rightarrow \psi_\mu e^- W^+$, $\tilde{\nu}_R \rightarrow \psi_\mu \nu_L q\bar{q}$, and $\tilde{\nu}_R \rightarrow \psi_\mu e^- q\bar{q}'$.

Figure 5: Diagram of the decay of $\tilde{\nu}_R$: $\tilde{\nu}_R \rightarrow \psi_\mu \nu_R$. 
are negligible, because these are strongly suppressed by the small neutrino mass compared to the two-body decay process. Thus, the decay width of $\tilde{\nu}_R$ is given by

$$\Gamma_{\tilde{\nu}_R \rightarrow \psi \mu} = \tau_{\tilde{\nu}_R}^{-1} = \frac{1}{48\pi M^2} \frac{m^5_{\tilde{\nu}_R}}{m_{3/2}^2} \left[ 1 - \frac{m_{3/2}^2}{m_{\tilde{\nu}_R}^2} \right]^4.$$  \hspace{1cm} (14)

The lifetime of right-handed sneutrino turns out to be $10^2$-$10^8$ seconds with $m_{\tilde{\nu}_R} = 100$ GeV for $0.1$ GeV $\lesssim m_{3/2} \lesssim 100$ GeV.

When $m_{3/2}$ is small enough compared to $m_{\tilde{\nu}_R}$, $t_{eq} \gg \tau_{\tilde{\nu}_R}$ is satisfied and free-streaming length is approximately proportional to $u(\tau_{\tilde{\nu}_R})^{1/2}$. Since $u(\tau_{\tilde{\nu}_R})^{-1}$ and $\tau_{\tilde{\nu}_R}^{1/2}$ are both proportional to $m_{3/2}$, $\lambda_{FS}$ becomes independent of $m_{3/2}$. With the use of the lifetime obtained in the above equation, the free-streaming length turns out to be $\lambda_{FS} \approx 6$ Mpc when $m_{\tilde{\nu}_R} = 100$ GeV unless $m_{3/2}$ is very close to $m_{\tilde{\nu}_R}$. This fact indicates that the component of the dark matter (i.e., gravitino) from sneutrino decay acts as a warm dark matter (WDM).

In addition to the sneutrino decay, gravitinos are also produced by the thermal scattering at the reheating epoch after inflation. The abundance of the gravitino from the scattering process is determined by the reheating temperature and the gravitino mass \cite{6}. Since the gravitino from the scattering is non-relativistic at the time of the structure formation, it acts as a cold dark matter (CDM). Thus, we have to consider the constraints from the structure formation of the universe on the WDM + CDM scenario.

### 4.2 Constraints

Constraints from the structure formation on the WDM + CDM scenario are studied in recent works \cite{17, 18}. According to these studies, it turns out that the matter power spectrum has a step-like decrease around the free-streaming scale of the WDM component, $k \sim 2\pi/\lambda_{FS}$. This is because only the power spectrum of the WDM component dumps at $2\pi/\lambda_{FS}$. The power spectrum is estimated from the observations of the cosmic microwave background \cite{1, 3}, the red shift surveys of galaxies \cite{2}, and so on. The WDM + CDM scenario is viable if the step-like decrease is within the uncertainty of the observed power spectrum. In this article, we adopt the following constraint: the power spectrum with the step-like decrease should be consistent with observational data \cite{1} at 95% confidence level.

In our model, the energy density of dark matter is composed of two components, $\rho_{DM} = \rho_{3/2}^{dec} + \rho_{3/2}^{th}$, where $\rho_{3/2}^{dec}$ and $\rho_{3/2}^{th}$ are the energy densities of gravitino produced
by the decay and by the thermal scattering processes, respectively. Introducing the fraction of WDM component $f$, we rewrite $\rho_{\text{DM}}$ as

$$\rho_{\text{DM}} = \rho_{\text{dec}}^{3/2} + \rho_{\text{th}}^{3/2} = f\rho_{\text{pureWDM}} + (1 - f)\rho_{\text{pureCDM}},$$

(15)

where $\rho_{\text{pureWDM}}$ and $\rho_{\text{pureCDM}}$ are the energy densities of pure WDM and CDM scenario, respectively, where $\Omega_{\text{pureCDM}}h^2 = \Omega_{\text{pureWDM}}h^2 \simeq 0.1$. We consider the adiabatic density fluctuation, then the power spectrum for the scale $k^{-1}$ is written as

$$P_{\text{DM}}(k) = \left[ fP_{\text{pureWDM}}^{1/2}(k) + (1 - f)P_{\text{pureCDM}}^{1/2}(k) \right]^2.$$  

(16)

In order to evaluate the magnitude of the step-like decrease, it is convenient to define the ratio of the CDM component in the total power spectrum at $k \gtrsim 2\pi/\lambda_{\text{FS}}$:

$$r \equiv \frac{P_{\text{DM}}(k \gtrsim 2\pi/\lambda_{\text{FS}})}{P_{\text{pureCDM}}(k \gtrsim 2\pi/\lambda_{\text{FS}})} = (1 - f)^2.$$  

(17)

The lower bound on $r$ is obtained from the ratio of the lower and upper bounds on the observed power spectrum. In our scenario, $\lambda_{\text{FS}} \simeq 6$ Mpc and hence $k \simeq 1$ Mpc$^{-1}$. For such a wavelength, we obtain $r > 0.35$ at the 95% confidence level [1], which leads to $f < 0.4$. In terms of the density parameter, it indicates $\Omega_{3/2}^{\text{dec}} < 0.4\Omega_{\text{DM}}$, which gives the upper bound on the gravitino mass as $m_{3/2} < 40$ GeV (4 GeV) for $C_{\text{model}} = 1$ (10) (light-shaded regions in Fig.4).

The constraint does not depend highly on the detail of the observational data. In fact, in other recent observations, it is claimed that the observational error on the power spectrum is about 15% [2, 17], leading to the constraint as $f \lesssim 0.2$, which is of the same order of magnitude as the result above.

5 Gravitino Dark Matter from Decay

In our model, it is also possible to realize the scenario in which all the SuperWIMP (in our case, gravitino) dark matter is produced by the decay of other superparticle [7, 19]. In such a scenario, the SuperWIMP dark matter originates in MSSM-LSP$^4$. It is well known that the relic abundance of the MSSM-LSP explains the observed dark matter abundance if the MSSM-LSP is stable. Therefore, even if the MSSM-LSP decays into the SuperWIMP at the late universe, the dark matter abundance

$^4$Thus, it is implicitly assumed that the reheating temperature is low enough in order to suppress the gravitino production from the thermal scattering at the reheating epoch.
Figure 6: BBN constrains to the gravitino dark matter scenario on the $(m_\tilde{B}, m_\tilde{\nu}_R)$ plane, where $m_{\tilde{\nu}_L} = 1.5m_\tilde{B}$. The region $m_{\tilde{\nu}_R} > m_\tilde{B}$ is irrelevant because we consider the case that $\tilde{\nu}_R$ is the NLSP. In the left figure, we set $C_{\text{model}} = 1$, while $C_{\text{model}} = 0.1$ in the right figure. The constraints are shown as light-, middle-, and dark-shaded regions, for $a_\nu = 1$, 3, and 5, respectively.

is still explained as far as the mass of the SuperWIMP is of the same order of that of the MSSM-LSP because of

$$\Omega_{\text{SuperWIMP}} = \frac{m_{\text{SuperWIMP}}}{m_{\text{MSSM-LSP}}} \Omega_{\text{MSSM-LSP}}.$$  (18)

When the SuperWIMP is gravitino, the scenario receives stringent constraints from BBN [14], because not only gravitino but also visible particles are emitted in the decay of MSSM-LSP. In order to avoid the BBN constraints, one may assume that the lifetime of the MSSM-LSP is much shorter than one second or the MSSM-LSP is highly degenerate with the gravitino in mass. However, such possibilities often lead to fine-tunings of parameters in the model. On the other hand, in our model, the MSSM-LSP (i.e., Bino-like neutralino) decays mainly into the right-handed sneutrino NLSP, then right-handed sneutrino decays into gravitino. The lifetime of $\tilde{B}$ is shorter than that in model without right-handed sneutrinos, and the amount of visible particles emitted in the decay is suppressed. Furthermore, the decay of $\tilde{\nu}_R$ into gravitino does not produce visible particles. Thus, the last decay is harmless for BBN, though the lifetime of sneutrino is much longer than one second. As a result, the constraints on the scenario are relaxed.

In Fig[6] we show the BBN constraints to the scenario on the $(m_\tilde{B}, m_\tilde{\nu}_R)$ plane, where we take $m_{\tilde{\nu}_L} = 1.5m_\tilde{B}$. In the left (right) figure, we set $C_{\text{model}} = 1 \ (0.1)$. Notice
that the gravitino mass is determined by Eqs. (11) and (18) with \( \Omega_{\text{SuperWIMP}} \simeq 0.1. \) The excluded regions from BBN are the light-, middle-, and dark-shaded regions for \( a_\nu = 1, 3, \) and 5, respectively. In these figures, contours of the mass and the free-streaming length of the gravitino are also depicted.

Since the gravitino is produced in the decay process, it may act as a warm dark matter. As a result, the scenario receives the constraint from the structure formation of the universe. From observation, the free-streaming length should be shorter than about 1 Mpc; with such a bound, we obtain the lower bound on right-handed sneutrino mass as \( m_{\tilde{\nu}_R} \gtrsim 400 \text{ GeV}. \)

As shown in Fig. 6, when \( C_{\text{model}} = 1, \) only the small region, \( m_{\tilde{B}} \sim 600 \text{ GeV} \) and \( m_{\tilde{\nu}_R} \sim 400 \text{ GeV}, \) is consistent with both of the constraints from BBN and the structure formation of the universe. The upper bound on \( m_{\tilde{B}} \) comes from the BBN constraints on three- and four-body decays as discussed in previous sections. On the other hand, the upper bound on \( m_{\tilde{\nu}_R} \) is due to those from the decay of \( \tilde{B} \) into the gravitino. When \( C_{\text{model}} = 0.1, \) the BBN constraints are relaxed, because the gravitino mass is larger than that in the \( C_{\text{model}} = 1 \) case and the decay rate into gravitino is suppressed. The constraints from the structure formation are also milder. As a result, a wide range of the parameter space is consistent with the constraints, which indicates that the scenario with gravitino dark matter from the MSSM-LSP can be naturally realized in our model if the annihilation cross section of the MSSM-LSP is large enough.

Recently, it is pointed out that there are various discrepancies between numerical simulations for the structure formation based on the CDM scenario and the observations of substructures in galaxies \cite{20, 21, 22}. Interestingly, a warm dark matter whose free-streaming length is slightly less than 1 Mpc may solve the discrepancies \cite{19, 21, 25}. The free-streaming length as long as \( \lambda_{\text{FS}} \sim 1.0-0.4 \text{ Mpc} \) is suggested as a solution to the missing satellites problem \cite{20}, which can be easily realized in our scenario. For cusp problem \cite{21}, however, parameter region which solves the problem receives noticeable change by analysis methods. For example, in \cite{19}, it is claimed that the cusp and missing-satellites problem can be solved simultaneously when \( \lambda_{\text{FS}} \) is in appropriate range even if the lifetime of the decaying particle is as short as \( \sim 10^5 \text{ sec}; \) then the cusp problem can be also solved in our scenario. On the other hand, in \cite{24}, it is claimed that a simultaneous solution to both problems are

\footnote{There are also discussions that the observations using the gravitational lensing is quite consistent with not the WDM scenario but the CDM one \cite{23}.}
hardly obtained unless the lifetime of the decaying particle is longer than $\sim 10^{10}$ sec; such a long lifetime cannot be realized in our scenario. The detailed analysis of the small-scale structure problems are out of the scope of this article and we leave them as future studies.

6 Conclusions and Discussion

In this paper, we have studied the cosmological implications of the gravitino LSP scenario with the right-handed sneutrino NLSP in the framework where neutrino masses are purely Dirac-type. In the case that MSSM-LSP is Bino-like neutralino, it mainly decays into the right-handed sneutrino with the lifetime $\tau_{\tilde{B}} \sim 10^2$-$10^3$ seconds in the wide range of the parameter region. Though the MSSM-LSP is long-lived, no visible particles are produced in the leading process, thus constraints from BBN can be relaxed compared to the case without right-handed sneutrinos. With the quantitative analysis of the BBN constraints, we have found the new allowed region, $0.1$ GeV $\lesssim m_{3/2} \lesssim 40$ GeV, when $m_{\tilde{\nu}_R} = 100$ GeV. In this region, the BBN constraints give the upper bound on the Bino mass as $m_{\tilde{B}} \lesssim 200$-$400$ GeV, which mainly comes from hadronic four-body decays. On the other hand, the upper bound on the gravitino mass is given by the constraints from the structure formation of the universe. In our scenario, some part of the gravitino is produced by the decay of right-handed sneutrino at the late universe. As a result, the gravitino freely streams in the universe and acts as a WDM. The gravitino is also produced from thermal scattering processes, which acts as a CDM. Taking the CDM contribution into account, we have considered the constraints on the WDM + CDM scenario from the observations of (small scale) structure formation, and finally found the upper bound on the gravitino mass.

So far, we have concentrated on the case with Bino MSSM-LSP. Another well-motivated candidate for the MSSM-LSP is the lighter stau $\tilde{\tau}_1$. With $\tilde{\tau}_1$-NNLSP, the main decay mode of $\tilde{\tau}_1$ is $\tilde{\tau}_1 \to \tilde{\nu}_R W^\pm$. We have also analyzed this case and derived upper bound on $m_{\tilde{\tau}_1}$; we found $m_{\tilde{\tau}_1} \lesssim 400$ GeV for $m_{\tilde{\nu}_R} = 100$ GeV and $1$ GeV $\lesssim m_{3/2} < 100$ GeV. Here we take $U_{1L}^2 = 0.1$, where $U_{1L}$ is left-handed stau component in lighter stau. In addition, the lifetime of lighter stau is $1$-$10^2$ seconds if this process is kinematically forbidden, $\tilde{\tau}_1$ mainly decays as $\tilde{\tau}_1 \to \psi \mu \tau$ and the right-handed sneutrino plays no significant role. Constraints on such a scenario are already analyzed in [14, 26], where the upper bound on the gravitino mass is given as $m_{3/2} \lesssim 10$ GeV.
in the allowed region. Thus, the lighter stau may be seen as a long-lived charged track in future colliders.

We have also discussed the possibility to realize the scenario where gravitino dark matter is produced from the decay of other superparticle. In the scenario, it is postulated that the gravitino dark matter originating in the Bino-like neutralino accounts for the total dark matter abundance. Considering the mass parameter region $m_{\tilde{B}} = 400 \text{ GeV-1 TeV}$, we have found that the free-streaming length is $\lambda_{FS} = 1-0.3 \text{ Mpc}$, which allows to solve the small scale structure problems of galaxies. As in the case above, BBN constraints give the upper bound on the Bino mass as $m_{\tilde{B}} \lesssim 600 \text{ GeV-1 TeV}$, which corresponds to $m_{3/2} \lesssim 80-250 \text{ GeV}$. As a result, all superparticle masses are within 100 GeV-1 TeV, thus the gravitino dark matter scenario in our framework seems to be natural.

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