Three–dimensional mathematical model of a convective cloud with detailed account for the thermodynamic, microphysical and electric processes

V A Shapovalov, A V Shapovalov, M A Sherieva
High Mountain Geophysical Institute, Nalchik, Russia
atajuk@mail.ru

Abstract. A three–dimensional numerical model of a convective cloud developed by the authors, taking into account thermodynamic, microphysical and electrical processes, is presented. The model is characterized by the use of detailed microphysics with a large number of particle size gradations. There are taken into account: the accumulation of charges in the cloud, the electric field, the electric coagulation of cloud particles. The technique of formation of three–dimensional initial data at initialization of model is developed. On the basis of the developed model with a detailed description of the processes for the first time is studied the formation of macro- and micro structural and electrical parameters. The dynamics of changes in the characteristics of thunderstorm clouds at the stage of growth and maximum development are determined. Electrical characteristics of thunderstorm clouds at different times and their relationship with micro structural parameters are investigated. The spatial structure of voluminous electric charges in a cloud, three–dimensional distribution of electric field intensity are determined. A quantitative assessment of the effect of electric coagulation on the rate of precipitation in powerful clouds is obtained.

1. Introduction
In the physics of clouds, for objective reasons, electrical processes are least studied, although this direction is given much attention. Mathematical modeling allows to investigate many aspects of electrical processes in convective clouds, which due to danger, inaccessibility and other reasons could not be studied [1–15]. With the help of modeling it is important to investigate the interaction of microphysical and electrical processes, which plays a significant role in the formation of precipitation in powerful clouds [10–19].

This paper presents the developed three–dimensional unsteady numerical model of convective clouds with detailed consideration of thermodynamic, microphysical and electrical processes [1, 3]. The model differs from its analogues [10–15] by the fact that it uses detailed microphysics with several tens of gradations of the liquid and solid particles’ sizes, taking into account the accumulation of charges in the cloud, the electric coagulation of cloud particles, the potential and the electric field strength.

2. Model and research methods
The hydrothermodynamic block of the model consists of motion equations describing wet convection in the Boussinesq approximation. The equations take into account advective and turbulent transport, buoyancy forces, friction and baric gradients. The microphysical block of the model describes the
processes of nucleation, condensation, coagulation of droplets with droplets, sublimation, accretion, freezing of droplets, deposition of cloud particles in the gravity field, their transfer by air flows, as well as the interaction of cloud particles under the influence of the electric field of the cloud [1].

The problem statement of the mathematical model of convective cloud includes the following equations of thermodynamics, microphysics and electrostatics:

$$\frac{\partial u}{\partial t} + (\vec{V} \cdot \nabla) u = -\nabla \rho' + \Delta' u + lv, \quad (1)$$

$$\frac{\partial v}{\partial t} + (\vec{V} \cdot \nabla) v = -\nabla \rho' + \Delta' v - lu, \quad (2)$$

$$\frac{\partial w}{\partial t} + (\vec{V} \cdot \nabla) w = -\nabla \rho' + \Delta' w + \rho \left( \frac{\partial \theta}{\partial t} + 0.61s' - Q_s \right), \quad (3)$$

continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \quad (4)$$

equations of thermodynamics:

$$\frac{\partial \theta}{\partial t} + (\vec{V} \cdot \nabla) \theta = \frac{L_a}{c_p \rho} \frac{\partial M_a}{\partial \theta} + \frac{L_c}{c_p \rho} \frac{\partial M_c}{\partial \theta} + \frac{L_c}{c_p \rho} \frac{\partial M_c}{\partial \theta} + \Delta' \theta, \quad (5)$$

$$\frac{\partial S}{\partial t} + (\vec{V} \cdot \nabla) S = -\frac{\partial M_K}{\partial \theta} - \frac{\partial M_c}{\partial \theta} + \Delta' S, \quad (6)$$

equations for the distribution functions of droplets, crystals and fragments of freezing by mass:

$$\frac{\partial f_1}{\partial t} + u \frac{\partial f_1}{\partial x} + v \frac{\partial f_1}{\partial y} + (w-V_1) \frac{\partial f_1}{\partial z} = \left( \frac{\partial f_1}{\partial t} \right)_K + \left( \frac{\partial f_1}{\partial t} \right)_{AK} + \left( \frac{\partial f_1}{\partial t} \right)_3 + \Delta' f_1 + I_1, \quad (7)$$

$$\frac{\partial f_2}{\partial t} + u \frac{\partial f_2}{\partial x} + v \frac{\partial f_2}{\partial y} + (w-V_2) \frac{\partial f_2}{\partial z} = \left( \frac{\partial f_2}{\partial t} \right)_c + \left( \frac{\partial f_2}{\partial t} \right)_{AK} + \left( \frac{\partial f_2}{\partial t} \right)_3 + \Delta' f_2 + I_2 + I_{AB}, \quad (8)$$

$$\frac{\partial f_3}{\partial t} + u \frac{\partial f_3}{\partial x} + v \frac{\partial f_3}{\partial y} + (w-V_3) \frac{\partial f_3}{\partial z} = \left( \frac{\partial f_3}{\partial t} \right)_c + \left( \frac{\partial f_3}{\partial t} \right)_{AK} + \Delta' f_3, \quad (9)$$

equations for calculating the amount of electricity

$$\rho_e = a \int_0 f_m \rho_d - \lambda \rho_e - \gamma \sum \rho_i, \quad (10)$$

Poisson’s equation for electrostatic field potential

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = -\frac{\rho_e}{\varepsilon_0}, \quad (11)$$

The initial conditions for equations (1)–(11) are as follows:

$$u(\vec{r}, 0) = u_0(\vec{r}), \quad v(\vec{r}, 0) = v_0(\vec{r}), \quad w(\vec{r}, 0) = w_0(\vec{r}), \quad \theta(\vec{r}, 0) = \theta_0(\vec{r}), \quad s(\vec{r}, 0) = s_0(\vec{r}), \quad (12)$$

$$f_1(\vec{r}, m, 0) = f_2(\vec{r}, m, 0) = f_3(\vec{r}, m, 0) = 0, \quad \rho_e(\vec{r}, 0) = \rho_e(\vec{r}, 0) = 0. \quad (13)$$

Boundary condition:
The system of equations is used for the space–time domain

\[ 0 \leq x \leq L_x, \quad 0 \leq y \leq L_y, \quad 0 \leq z \leq L_z, \quad 0 \leq m < \infty, \quad t > 0. \]  

Symbols are used:

\[ \mathbf{V} = \{u, v, w\} \] – velocity vector, \( u(\mathbf{r}, t), v(\mathbf{r}, t), w(\mathbf{r}, t) \) – components of the velocity vector; \( l \) – parameter that takes into account inertial forces; \( \theta(\mathbf{r}) \) – potential temperature; \( \pi(\mathbf{r}) = \epsilon H(P(z)/1000) \) – dimensionless pressure; \( \bar{\theta} \) – average potential temperature; \( \bar{R} \) – gas constant; \( s(\mathbf{r}) \) – specific humidity of air; \( Q(\mathbf{r}) \) – total mixing ratio of liquid and solid phases in the cloud; \( \sigma(z) \) – parameter that takes into account the change of air density with height; \( P(z) \) and \( T(\mathbf{r}) \) – respectively, pressure and temperature; \( c_p \) – heat capacity of air at constant pressure; \( L_K, L_O, L_3 \) – respectively, the specific heat of condensation, sublimation and freezing; \( \pi_d(\mathbf{r}), \theta_d(\mathbf{r}), s_d(\mathbf{r}) \) – deviations of dimensionless pressure, potential temperature and specific humidity from their background values in the ambient atmosphere \( \pi_d(\mathbf{r}), \theta_d(\mathbf{r}), s_d(\mathbf{r}) \); \( \frac{\delta M_1}{\delta t}, \frac{\delta M_2}{\delta t} \) – changes in specific humidity due to the diffusion of steam on droplets and crystals; \( \frac{\delta M_3}{\delta t} \) – mass of droplet water freezing per unit time per unit volume of air; \( K(\mathbf{r}) \) – coefficient of turbulent diffusion; \( V_1(m), V_2(m) \) – steady–state rates of incidence of liquid and solid particles; \( \frac{\partial f_1}{\partial t}, \frac{\partial f_2}{\partial t}, \frac{\partial f_3}{\partial t} \) – changes in the distribution function of droplets due to microphysical condensation processes, coagulation of droplets, accretion of droplets and crystals, crushing and freezing, respectively; \( \frac{\partial f_1}{\partial t}, \frac{\partial f_2}{\partial t}, \frac{\partial f_3}{\partial t} \) – changes in the distribution function of crystals due to sublimation, accretion and freezing of droplets; \( \frac{\partial f_1}{\partial t}, \frac{\partial f_2}{\partial t}, \frac{\partial f_3}{\partial t} \) – changes in the distribution function of supercooled cloud droplets and their accretion with crystals; \( I_1 \) and \( I_2 \) – sources of droplets and crystals; \( I_{AB} \) – source of artificial crystals under active influence; \( \rho_{\ell}(\mathbf{r}, t) \) – total volume electric charge, \( \varepsilon_0 \) – dielectric constant of vacuum.

The model takes into account in detail the processes of electrification of cloud particles on the basis of the obtained patterns of thunderstorm activity in the clouds and the values of the charge separation coefficients associated with the freezing of water droplets, the growth of cereals and hailstones and the interaction of hailstones with ice crystals and supercooled drops.

Due to the microphysical processes of freezing drops and accretion there is an accumulation of negative charge on the ice particles in the cloud. At the same time, a positive charge is formed,
consisting of charges of individual particles – fragments of freezing droplets.

Microscopic fragments of freezing are carried by streams to the upper part of the cloud, where a predominantly positive volumetric charge is formed \( \rho_+ (\vec{r}, t) \). The area of concentration of negatively charged ice particles forms a zone of predominantly negative volumetric charge \( \rho_- (\vec{r}, t) \).

In the simulation at each time step, the volume charges in the cloud, the potential of the electrostatic field generated by these charges, as well as the horizontal \( E_x (\vec{r}) \), \( E_y (\vec{r}) \) and vertical components \( E_z (\vec{r}) \) of the electric field of the cloud are calculated.

The value of the total (positive and negative) volume charges \( \rho (\vec{r}) = \rho_+ (\vec{r}) - \rho_- (\vec{r}) \) is used to determine the potential \( U (\vec{r}) \) of the electrostatic field created by them. For this purpose, a three–dimensional Poisson equation (11) is solved at each time step. The values of the electric field intensity were taken into account in the work to calculate the coefficients of the electric coagulation of cloud particles. For this purpose, approximation formulas based on existing theoretical and experimental data for this parameter were used.

The radar reflectance of the cloud at wavelengths of 3, 5 and 10 cm is calculated for comparison with the observational data in the model.

The system of equations of the model (1) – (17) was solved by methods of splitting by physical processes and coordinate–wise splitting.

3. Result of calculation
Below are presented the results of studies of thermodynamic, micro structural and electrical parameters’ formation of convective clouds at different states of atmosphere.

The size of the spatial region in the calculations was set from 40 to 80 km in the horizontal and 16 km in the vertical. The grid spacing in the X, Y coordinates were 500–1000 m, in the Z coordinate was 250–500 m. The X Axis is directed eastward, Y – northward, Z – vertically. The cloud was initiated by setting a pulse near the earth's surface with overheating \( \Delta T=1–4 \, ^\circ\text{C} \), the shape and size of the pulse also varied in numerical experiments.

In performing numerical experiments, the data of upper–air sensing at Mineralnye Vody airport and, in a number of experiments – three–dimensional data on thermodynamic parameters and horizontal wind were used. There were selected days when in fact within the radius of the representative ballooned probe observed showers, thunderstorms, hail. Radar observations were used for comparison a simulation clouds with real clouds.

Formation and accumulation of electric charges in the cloud occurs as a result of freezing of droplets, accretion (interaction of droplets and crystals) and collisions of crystals. Due to different speeds of falling in the air of micro–splinters (charging mainly positively) and larger particles, cereals and hail (charging mainly negatively), there is a spatial separation of charges: a positive volumetric charge prevails in the pre–summit part of the cloud, below — a negative one. The density of positive charge reached a \( 2.8 \times 10^{-9} \, \text{C/m}^3 \) in 20 minutes, the negative is \( 1.5 \times 10^{-9} \, \text{C/m}^3 \), the electric potential was \( 1.4 \times 10^9 \, \text{V} \). Components \( E_x, E_y \) of the field intensity were approximately 1300 V/cm, \( E_z \approx 2000 \, \text{V/cm} \). By the results of simulations the spatial distribution of the total space charge in the cloud at different points in time is investigated (figure 1). The electrostatic field strength calculated at each moment of time in the nodes of the spatial grid was taken into account when calculating the values of the coagulation coefficients of droplets and crystals.

Numerical experiments were carried out taking into account the electric coagulation of cloud particles and without it. Comparison of the precipitation formation time in these two cases showed that due to electric coagulation, the growth time of precipitation particles in a powerful convective cloud is significantly reduced by 10–14 minutes (20–30 %).

The results of calculations based on a three–dimensional model with detailed consideration of hydrodynamic, thermodynamic, microphysical and electrical processes show that these processes in convective clouds mutually affect each other. This nonlinear interaction is very complex and plays an important role in the formation of the microstructure of clouds. Dynamic processes deform the fields of thermodynamic parameters in the cloud, which in turn determine the microphysical processes and the growth of precipitation particles. Electrical parameters affect the development of precipitation.
Figure 1. Contours of the space charge and potential in the vertical plane passing through the cloud, on a background of isosurfaces of radar reflectivity $Z_{\text{rad}} = 10 \text{ dBZ}$, and at time: 

- (a) $t=30 \text{ min}$, 
- (b) $t=33.5 \text{ min}$, 
- (c) $t=37 \text{ min}$, 
- (d) $t=40.5 \text{ min}$.

Contour of the space charge (closed contours) have values: $-5.0 \cdot 10^{-9}$, $-1.0 \cdot 10^{-9}$, $-1.0 \cdot 10^{-10}$, $1.0 \cdot 10^{-10}$, $1.0 \cdot 10^{-9}$, $5.0 \cdot 10^{-9} \text{ C/m}^3$ and the potential created by the charge levels (in order from the outer contour to the center) $-5.0 \cdot 10^8$, $-2.5 \cdot 10^8$, $-1.0 \cdot 10^8$, $-5.10^7$, $-2.5 \cdot 10^7$, $-1.0 \cdot 10^7$, $-5.0 \cdot 10^6$, $5.0 \cdot 10^6$, $1.0 \cdot 10^7$, $2.5 \cdot 10^7$, $5.0 \cdot 10^7$, $1.0 \cdot 10^8$, $2.5 \cdot 10^8$, $5.0 \cdot 10^8$ \text{ B}.

In numerical experiments, it was found that there is a positive feedback between the growth of ice particles’ mass and electric charge volume in the cloud.

The need to study the interaction of processes in convective clouds is associated with their important role in cloud – and sedimentation.

4. Summary

A three–dimensional unsteady numerical model of a convective cloud with a detailed account of thermo hydrodynamic, microphysical and electrical processes is developed. The process of accumulation of electric charge during the freezing of drops and accretion is formalized. The algorithms for calculating the system of equations of the model, the potential and intensity of the electrostatic field, electric coagulation are presented. The model allows to calculate thermodynamic, micro structural and electrical parameters of convective clouds at different stages of cloud development. When initializing the model, the three–dimensional initial distribution of thermodynamic parameter fields in the domain, constructed from the data of the global GFS model, was used.

The important aspects of the mechanism of electric charge and field’s formation in clouds taking into account the interaction of thermodynamic, microphysical and electrical processes are investigated.
The spatial distribution and quantitative values of the volume electric charges and the field strength in and around the cloud at successive points in time during evolution are determined. It is shown that there is a positive feedback between the growth of precipitation particles and the field strength in the cloud and the increase in the electrostatic field. According to the simulation results, the most intensive growth of liquid and solid precipitation occurs at the stage of maximum development of the convective cloud due to electric coagulation.

References
[1] Ashabokov B A, Shapovalov A V, Kuliev D D, Prodan K A and Shapovalov V A 2014 Numerical simulation of thermodynamic, microstructural, and electric characteristics of convective clouds at the growth and mature stages Radiophysics and Quantum Electronics 56 Issue 11 pp 811–817
[2] Adzhieva A K, Stasenko V N, Shapovalov A V and Shapovalov V A 2016 Atmospheric electric field strength and thunderstorms in the North Caucasus Russian Meteorology and Hydrology 41 pp 186–192
[3] Bychkov A A and Shapovalov V A 2017 Formation of Bulk Electric Charges and Fields during Development of Thunderstorm Clouds International Journal of Applied Engineering Research ISSN 0973–4562 12 pp 13142–13149
[4] Ashabokov B A, Fedchenko L M, Shapovalov A V and Shapovalov V A 2017 Physics of clouds and active influences on them Nalchik: publishing House “Printing yard” p 240
[5] Veremey N E, Dovgaluk Yu A, Zatevakhin M A, Ignatiev A A, Morozov V N and Pastushkov R S 2016 Description of the underlying unsteady three–dimensional numerical models of convective clouds. Proceedings of MGO 582 pp 45 – 91
[6] Dovgaluk Yu A, Veremey N E, Vladimirov S A, Drofa A S, Zatevakhin M A, Ignatiev A A, Morozov V N, Pastushkov R S, Sinkevich A A and Shapovalov A V 2016 The concept of development of a numerical unsteady three–dimensional model of evolution of a sedimentary convective cloud under natural conditions and under active influences Proceedings of MGO 582 pp 7 – 44
[7] Dovgaluk Yu A, Veremey N E, Vladimirov S A, Drofa A S, Zatevakhin M A, Ignatiev A A, Morozov V N, Pastushkov R S, Sinkevich A A, Stasenko V N, Stepanenko V D, Shapovalov A V, Schukin G G 2008 The concept of development of a three–dimensional model of a sedimentary convective cloud. I. model Structure and basic equations of hydrothermodynamic unit Proceedings of MGO 558 pp 102–142
[8] Dovgaluk Yu A, Veremey N E, Vladimirov S A, Drofa A S, Zatevakhin M A, Ignatiev A A, Morozov V N, Pastushkov R S, Sinkevich A A, Stasenko V N, Stepanenko V D, Shapovalov A V and Schukin G G 2010 The concept of development of a three–dimensional model of a sedimentary convective cloud. II. Microphysical block of the model Proceedings of MGO 562 pp 7–39
[9] Kogan E A, Mazin I P, Sergeev B N and Khvorostyanov V I 1984 Numerical simulation of clouds Moscow: Hydrometeoizdat p 184
[10] Berry E X and Reinhard R L 1974 An analysis of cloud drop growth by collection. Part I. Double distributions J. Atmos. Sci. 31 No. 7 pp 1825–1831
[11] Clark T 1979 Numerical Simulation with a Tree–Dimension Cloud Model: lateral Boundary Condition Experiments and Multiceller seven Storm Simulations. J. Atmos. Sci. 36 No. 11 pp 2191–2215
[12] Cotton W R, Stephens M A, Nehrkorn T, Tripoli G J 1982 The Colorado State University three–dimensional cloud model. Part II: An ice phase parameterization J. Rech. Atmos. 16 pp 295–320
[13] Farley R B 1987 Numerical Modeling of Hailstone Growth. Part III: Simulation of an Alberta Hailstorm Natural seed Cases. J. Claim. Appl. Met. 26 No. 7 pp 789–812
[14] Orville R D and Kopp F J 1977 Numerical simulation of the life history of a hailstorm J. Atm. Sci. 34 No. 10 pp 1596–1618
[15] Straka J M 2009 Cloud and precipitation microphysics. Principles and Parameterizations
Cambridge University Press p 392
[16] Rawlins F 1982 A numerical study of thunderstorm electrification using a three dimensional
model incorporating the ice phase Quarter. Jour. of the Royal Met. Society 108 pp 779–801
[17] Shapovalov V, Shapovalov A, Koloskov B, Kalov R and Stasenko V 2018 Numerical Study of
the Dynamic, Thermodynamic and Microstructural Parameters of Convective Clouds
Natural Science 10 pp 63–69
[18] Sinkevich A A, Dovgalyuk Y A, Veremei N E et al 2017 Investments of the development of
thunderstorm with hail. Part 3. Numerical simulation of cloud evolution Russian
Meteorology and Hydrology 42 pp 494–502
[19] Veremey N E, Dovgalyuk Y A, Zatevakhin M A et al 2014 Study of the Evolution of the
Electric Structure of a Convective Cloud Using the Data of a Numeric Nonstationary Three–
Dimensional Model. Radiophysics and Quantum El 56 Issue 11–12 pp 801–810