Parton Energy Loss at Twist-Six in Deeply Inelastic e-A Scattering

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Within the framework of the generalized factorization in pQCD, we investigate the multiple parton scattering and induced parton energy loss at twist-6 in deeply inelastic e-A scattering with the helicity amplitude approximation. It is shown that twist-6 processes will give rise to additional nuclear size dependence of the parton energy loss due to LPM interference effect while its contribution is power suppressed.

Jet quenching, or parton energy loss induced by multiple scattering in high-energy nuclear collisions has been proposed as a very sensitive probe of the hot and dense medium \([1, 2]\), which recently has given a compelling theoretical explanation for many exciting experimental phenomena observed at RHIC, such as the suppression of high \(p_T\) single hadron production, and the disappearance of away-side two hadron correlation \([3]\). To investigate the radiative energy loss, many theoretical methods have been developed so far \([4, 5, 6, 7, 8, 9, 10, 11]\). Recently, based on the generalization of high twist \([17, 18]\), we will derive the semi-inclusive hadronic tensor at twist-6 with the approach of helicity amplitude approximation (HAA) \([10]\) and the corresponding parton energy loss. Also we will show the nuclear size dependence of parton energy loss at twist-6 due to the Landau-Pomeranchuk-Migdal (LPM) interference effect \([12]\).

In this letter, we will extend the twist expansion approach to study the parton multiple scattering at twist-6 for e-A DIS in nuclei. In the framework of generalized factorization of high twist \([17, 18]\), we will derive the semi-inclusive hadronic tensor at twist-6 with the approach of helicity amplitude approximation (HAA) \([10]\) and the corresponding parton energy loss. Also we will show the nuclear size \(R_A\) dependence of parton energy loss at twist-6 due to the Landau-Pomeranchuk-Migdal (LPM) interference effect \([12]\).

We consider the semi-inclusive process in e-A DIS, \(e(L_1) + A(p) \rightarrow e(L_2) + h(\ell_h) + X\). The differential cross section for this process has a form as

\[
\frac{d\sigma}{dL_2d\ell_h} = \frac{\alpha_{EM}^2}{2\pi s} \frac{1}{Q^4} \frac{2\pi s}{Q^4} L_{\mu \nu} E_{\ell_h} \frac{dW^{\mu \nu}}{d^3\ell_h},
\]

where \(p = \left[p^+, 0, 0, 1\right]\) is the momentum per nucleon in the nucleus with the atomic number \(A\), \(q = L_2 - L_1 = [-Q^2/2q^-, q^-, 0, 1]\) is the momentum transfer, the Bjorken variable is defined as \(x_B = Q^2/2p^+q^-\), \(s = (p + L_1)^2\) and \(\alpha_{EM}\) is the electromagnetic (EM) coupling constant. The leptonic tensor is \(L_{\mu \nu} = 1/2 \text{Tr}(\gamma^\mu L \gamma^\nu)\), \((L_2 \gamma^\nu)\) while the semi-inclusive hadronic tensor is defined as

\[
\frac{dW^{\mu \nu}}{d^3\ell_h} = \frac{1}{2} \sum_X (A|J_\mu(0)\rangle\langle h,X|J_\nu(0)\rangle A) \times 2\pi \delta^4(q + p - p_X - \ell_h),
\]

where \(\sum_X\) runs over all possible final states and \(J_\mu = \sum q_\psi^\dagger q_\psi^\mu \psi_\mu\) is the hadronic EM current.

In the parton model with collinear factorization approximation, the leading-twist contribution at the lowest order of single scattering can be factorized as,

\[
\frac{dW^{S}_{\mu \nu}}{dz_h} = \sum_q \int dx f_q^A(x, \mu_f^2) \times H_{\mu \nu}^{(0)}(x, p, q) D_{q\rightarrow h}(z_h, \mu^2)
\]

where the momentum fraction carried by the hadron is defined as \(z_h = \ell_h^+ / q^-\). \(H_{\mu \nu}^{(0)}\) is the hard partonic tensor, \(\mu_f^2\) and \(\mu^2\) are the factorization scales for the initial quark distributions \(f_q^A(x, \mu_f^2)\) in a nucleus and the fragmentation functions \(D_{q\rightarrow h}(z_h, \mu^2)\), respectively. Including all leading log radiative corrections, the renormalized quark fragmentation function \(D_{q\rightarrow h}(z_h, \mu^2)\) satisfies the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations \([20]\).

In the nuclei the propagating quark in DIS will experience additional scatterings with other partons from the nucleus. The rescatterings may induce additional gluon radiation which will cause the energy loss of the leading quark. Such phenomena are associated with the
high twist effect. Previous results \[6, 10, 11, 12, 14, 15\] have discussed the double scattering which gives twist-4 contribution. Here we would like to go beyond double scattering and extend the study to investigate the twist-6 (triple scattering) processes as illustrated in Fig. 1 and their contribution to the parton energy loss.

According to the generalized factorization theorem of high twist processes \[16, 17, 18\], the leading twist-6 (triple scattering) contribution with nuclear medium effect to the semi-inclusive hadronic tensor can be formulated as

\[
\frac{dW^T_{\mu\nu}}{dz_h} = \sum_q \int \frac{dz}{z} D_{q\to h}(z_h/z) \\
\times \frac{1}{2\pi} dy_1 dy_2 dy_3 dy_4 T^A_{qgq}(y^-, y_1^-, y_2^-, y_3^-, y_4^-) \\
\times (\frac{1}{2} g^{\alpha\beta}(-\frac{1}{2} g^{\alpha\beta}) \left[ \frac{1}{4!} \frac{\partial^4}{\partial k_{T\alpha} \partial k_{T\beta}} \delta (k_{T\gamma} - k_{T\gamma}') \right]_{k_T=0},
\]

where \(k_T\) is the initial gluon’s intrinsic transverse momentum and \(T^A_{qgq}\) is related to the parton correlation function of one quark and two gluons in a nucleus,

\[
T^A_{qgq}(y^-, y_1^-, y_2^-, y_3^-, y_4^-) = \frac{1}{2} \langle A|\tilde{q}(0)\gamma^+ F_+^{\mu}(y_1^-) F_+^{\nu}(y_2^-) F_+^{\sigma}(y_3^-) \psi_q(y^-)|A\rangle.
\]

Because of the collinear approximation, the tensor structure of the triple scattering is generally the same as in the leading order single scattering, \(\overline{W}^T_{\mu\nu}\) in Eq. (4) can be expressed as

\[
\overline{W}^T_{\mu\nu}(y^-, y_1^-, y_2^-, y_3^-, y_4^-, k_T, p, q, z) = \int dx H^O_{\mu\nu}(x, p, q)
\]

At twist-4 \(6, 10, 11, 12, 14\) there are 23 cut-diagrams, while at twist-6 there are 201 cut-diagrams. To simplify our calculations, we adopt the helicity amplitude approximation \(6, 10\) with the soft gluon radiation \(z_g = (1 - z) \to 0\), where \(z_g\) is the momentum fraction carried by the radiated gluon and \(z\) is the momentum fraction carried by the leading quark. With helicity amplitude approximation, it is similar to the twist-4 processes that the leading order contribution comes from 49 central-cut diagrams for the twist-6 processes.

For the contributions of the diagrams in Fig. 1, the results can be expressed as

\[
\overline{W}^T_{(a)} \propto \frac{1}{\ell_T^2} (1 - e^{ixLP^+y_1^-}) \\
\times e^{i(xB + x_A)p^+ y_1^- + ixLP_+^+(y_2^- - y_3^- + y_4^-)} \\
\times (1 - e^{ixLP_+^-(y^- - y_1^-)),
\]

\[
\overline{W}^T_{(b)} \propto \frac{\ell_T \cdot (\ell_T - k_T^p)}{\ell_T^2 (\ell_T - k_T^p)^2} (1 - e^{ixLP_+^+(y_4^- - y_1^-))} \\
\times \left( e^{i(xB + x_A)p^+ y_4^- + ixLP_+^+(y_2^- + y_3^- + y_4^-)} \\
\times \left( e^{ixLP_+^-(y^- - y_1^-)} - e^{ixLP_+^-(y^- - y_1^-))/(1 - z) \right).\n\]

It is similar to the results of Eqs. (11) and (12), one can check that the contributions from 40 diagrams among the 49 central-cut diagrams have the form

\[
\overline{W}^T \propto \frac{\ell_T \cdot (\ell_T - \eta k_T^\perp)}{\ell_T^2 (\ell_T - \eta k_T^\perp)^2} e^{ixP^+Y^-}.
\]

where we introduced a symbol \(\eta = 0, 1\), \(X\) and \(Y^-\) represent the longitudinal momentum fraction and the spatial coordinates, respectively. In Eq. (11), the form of \(\overline{W}^T_{(a)}\) corresponds to the case for \(\eta = 0\) and \(\overline{W}^T_{(b)}\) in Eq. (12) is the case for \(\eta = 1\). Other 38 diagrams have the similar results. According to the above factorization formula, we can prove that the fourth derivative of the above expression with respect to \(k_T\) vanishes at \(k_T=0\) when we keep only the leading terms as \(\ell_T \to 0\). Therefore, the contributions from these 40 central-cut diagrams to the hadronic tensor will vanish. As a result, only 9 central-cut diagrams as shown in Fig. 2 give non-zero contribution to the hadronic tensor with helicity amplitude approximation.

For the central-cut diagrams in Fig. 2 the contributions have the following form

\[
\overline{T}^i_{\hat{x}_i} = \int \frac{d\ell_T^2}{(\ell_T - k_T^p)^2} \frac{\alpha_s}{2\pi} \frac{1 + z^2}{1 - z} 16\pi^2 \alpha_s^2 \\
\times e^{i(xB + x_A)p^+ y_1^- + ixLP_+^+(y_2^- - y_3^- + y_4^-)} \\
\times \theta(y_1^- - y_2^-) \theta(y_1^- - y_3^-) \overline{T}^i_{\hat{x}_i}.
\]

For each diagram in Fig. 2 \(T_i\) can be expressed as

\[
\overline{T}_i = (e^{-i(x_D^p - x_D)p^+ (y_4^- - y_2^-)} - ixLP_+^+ y_2^-)
\]
The variables $x_L$, $x_D^0$ and $x_D$ in the above expressions
are defined as
\[ x_L = \frac{\ell_T^2}{2p^+q^-z(1-z)}, \]
\[ x_D^0 = \frac{k_T^2}{2p^+q^-}, \]
\[ x_D = \frac{k_T^2 - 2\overline{K}_T \cdot \ell_T}{2p^+q^-z}, \]
where \( \ell_T \) is the transverse momentum of the radiated gluon, and \( z = \ell_T^-/q^- \) is the momentum fraction carried by the final quark. Four terms of each expression in Eqs. (21-22) represent the contributions from different processes and their interferences. In the collinear limit as \( \ell_T \to 0 \) and \( k_T \to 0 \), there will be a clear cancelation of these contributions due to the LPM effect. We will see later that the LPM effect will give an additional nuclear size dependence to the parton energy loss.

Adding up all contributions together, we can derive the semi-inclusive hadronic tensor for triple scattering processes at twist-6,
\[
\frac{W_{\mu\nu}^T}{dz_h} = \sum_q \int dx H_{\mu\nu}^{(0)}(x,p,q) \frac{\alpha_s}{2\pi} \int \frac{dz}{z} D_{q\to h}(z_h/z) 
\times \frac{2}{3} \int d\ell_T^2 16\pi^2 \alpha_s^2 \int_1^1 \frac{dz}{z} T_{ggg}(x,x_L) 
\times \delta(z-1) \Delta T_{ggg}(x,\ell_T^2)),
\]
where the "\( + \)" functions have a form
\[
\int_0^1 dz \frac{F(z)}{1-z} = \int_0^1 dz \frac{F(z) - F(1)}{1-z},
\]
\( F(z) \) being any function which is sufficiently smooth at \( z = 1 \). Here we also include the virtual corrections in Eq. (28), which is obtained by using the unitarity requirement and ensures the final result to be infrared safe.

The parton correlation function \( T_{ggg}(x,x_L) \) in Eq. (22) is expressed as
\[
T_{ggg}(x,x_L) = \int dy^- d\overline{y}^- dy^- d\overline{y}^- dy^- e^{i(x+xl)p^+y^-} \times \theta(y^- - y^-) \theta(y^+ - y^-) \theta(y^+ - y^-) 
\times \langle I(x_L) T_{ggg}(y^- y^- y^- y^-), \rangle,
\]
where function \( I(x_L) \) is defined as
\[
I(x_L) = \sum_{i=1,2} \sum_{j=3,4} C_{ij} \left(1 - e^{-ix_L p^+ y^-}\right) \times \left(1 - e^{-ix_L p^+ y^-}\right).
\]

Here \( C_{ij} \) are associated with the color factors of each cut-diagram, and we have \( C_{14} = \frac{55}{96} \), \( C_{24} = C_{13} = -\frac{3}{64} \), and \( C_{23} = \frac{1}{12} \). In principle, the parton correlation function \( T_{ggg}(x,x_L) \) is not calculable and can only be measured in the experiments. However, under some assumptions, one can relate this parton correlation function to parton distributions and estimate its value using the LPM effect (15, 16, 18). The definition of \( \Delta T_{ggg}(x,\ell_T^2) \) in Eq. (23) is
\[
\Delta T_{ggg}(x,\ell_T^2) = \int_0^1 dz \frac{1}{1-z} \left[ 2\Delta T_{ggg}(x,x_L) \right]_{z=1} 
\times (1+z^2) T_{ggg}(x,x_L).
\]

Considering the high-twist contributions up to twist-6, we can express the hadronic tensor as
\[
dW_{\mu\nu} = \sum_q \int dx \overline{f}_q(x,\mu_T^2) H_{\mu\nu}^{(0)}(x,p,q) 
\times \overline{D}_{q\to h}(z_h,\mu_T^2),
\]
where the quark distribution function \( \overline{f}_q(x,\mu_T^2) \) include also the twist-6 contributions. Up to twist-6 the modified quark fragmentation function \( \overline{D}_{q\to h}(z_h,\mu_T^2) \) have the form
\[
\overline{D}_{q\to h}(z_h,\mu_T^2) = D_{q\to h}(z_h,\mu_T^2) + D^{(twist-4)}_{q\to h}(z_h,\mu_T^2) 
+ D^{(twist-6)}_{q\to h}(z_h,\mu_T^2),
\]
\[
\Delta \gamma_{q\to gg}^{twist-6}(z_h,\mu_T^2) = \Delta \gamma_{q\to gg}^{twist-6}(z_h,\mu_T^2) + \Delta \gamma_{q\to gg}^{twist-6}(z_h,\mu_T^2)
\times \left[ \frac{1+z^2}{1-z} T_{ggg}(x,x_L) + \delta(1-z) \Delta T_{ggg}(x,\ell_T^2) \right],
\]
where quark fragmentation function \( D_{q\to h}^{(twist-4)}(z_h,\mu_T^2) \) together with corresponding modified splitting functions \( \Delta \gamma_{q\to gg}^{twist-4}(z_h,\mu_T^2) \) and \( \Delta \gamma_{q\to gg}^{twist-6}(z_h,\mu_T^2) \) at twist-4 have been derived in Ref. 17, 18, the third term \( \Delta \gamma_{q\to gg}^{twist-6}(z_h,\mu_T^2) \) in Eq. (29) is the additional corrections to quark fragmentation function at twist-6. In Eq. (29) the modified splitting functions at twist-6 are
\[
\Delta \gamma_{q\to gg}^{twist-6}(z_h,\mu_T^2) = \Delta \gamma_{q\to gg}^{twist-6}(z_h,\mu_T^2) + \Delta \gamma_{q\to gg}^{twist-6}(z_h,\mu_T^2)
\times \left[ \frac{1+z^2}{1-z} T_{ggg}(x,x_L) + \delta(1-z) \Delta T_{ggg}(x,\ell_T^2) \right],
\]
Here, we replace \( 1/\ell_T^2 \) in Eq. (23) with \( 1/\ell_T^2(\ell_T^2 + (k_T^2)^2) \) in order to restore the collinear structure of the scattering amplitude (19). In Eq. (23) for the hadronic tensor at twist-6, there is an additional factor \( 1/\ell_T^2 \) as compared to the case at twist-4 (16, 17, 18). For large \( \ell_T^2 \), LPM effect becomes unimportant and one can neglect the interference contribution. In these processes, large final transverse momentum \( \ell_T^2 \sim Q^2 \) will lead to a \( 1/Q^2 \) suppression as compared to the case at twist-4.
an intermediate value, we cannot ignore LPM effect. One can check in Eq. (26), if $t_T \to 0$, there will be a clear cancelation by the interferences. The LPM interference effect restricts the radiated gluon to have a minimum transverse momentum $t_T^2 \sim Q^2/A^{1/3} \pi$. Therefore, the additional factor $1/t_T^2$ in Eq. (23) as compared to the case at twist-4 has a suppression of $R_A/Q^2$ for intermediate $Q^2$ at twist-6 due to the LPM interference effect, here $R_A = 1.12A^{1/3}$ is the radius of the nucleus.

As shown in Ref. [10, 12], the fractional energy loss of the leading quark can be given by the fractional energy carried away by the induced gluon, the fractional energy loss of the leading quark at twist-4 can be expressed as

$$\langle \Delta z_g \rangle_{\text{twist-4}} = \int_0^{t_T^2} \frac{d\ell_T^2}{\ell_T^2} \int_0^1 dz \frac{\alpha_s}{2\pi} z \times \Delta z_q^{(t_T^2 - 4)}(z, x_B, x_L, \ell_T^2)$$

where $f_q^A(x_B)$ is the quark distribution function up to the twist-4. For the twist-4 processes, the parton correlation function $T_{qq}^A(x, x_L)$ contributes a $R_A$ dependence [11, 12]. After taking into account $1/t_T^2 \sim R_A/Q^2$ resulting from the LPM effect, the nuclear correction to the fragmentation function and the parton energy loss due to double parton scattering (twist-4) processes will then be in the order of $\alpha_s^2 R_A^2 / t_T^2 \sim \alpha_s^2 R_A^2 / Q^2$. As shown in Ref. [11, 12], the order of the fractional energy loss for leading quark has a form

$$\langle \Delta z_g \rangle_{\text{twist-4}} \sim \alpha_s^2 \frac{x_B}{x_A^2 Q^2}.$$

where $x_A = 1/m_N R_A$ with $m_N$ is the nucleon’s mass.

At twist-6 it is the same as in the case of the twist-4 the fractional energy loss of the leading quark is given by the fractional energy carried away by the induced gluon, then the fractional energy loss of the leading quark has a form

$$\langle \Delta z_g \rangle_{\text{twist-6}} = \int_0^{t_T^2} \frac{d\ell_T^2}{\ell_T^2} \int_0^1 dz \frac{\alpha_s}{2\pi} z \times \Delta z_q^{(t_T^2 - 6)}(z, x_B, x_L, \ell_T^2)$$

where quark distribution function $f_q^A(x_B)$ contains the contribution of the twist-6 processes. The parton correlation function $T_{qq}^{A}(x, x_L)$ at twist-6 will contribute $R_A^2$ dependence because $T_{qq}^{A}(x, x_L)$ has an additional $R_A$ dependence due to the extra gluon pair as compared to the case at twist-4. Here we adopt also the assumption

\[ \langle \Delta z_g \rangle_{\text{twist-6}} \sim \alpha_s^3 \frac{x_B^2}{x_A^2 Q^4}. \]

One can see $\langle \Delta z_g \rangle_{\text{twist-6}}$ has $\alpha_s R_A^2 / Q^2$ suppression as compared to $\langle \Delta z_g \rangle_{\text{twist-4}}$ for fixed $x_B$.

The twist-6 calculation indicates that the twist expansion by taking into account the multiple scattering in nuclear medium is actually the expansion with the parameter $\alpha_s R_A^2 / Q^2$ due to LPM effect. Whereas the LPM effect can be neglected, the expanding parameter will be $\alpha_s R_A / Q^2$, so that we can see the LPM effect plays an important role to give an additional nuclear size dependence for the parton energy loss. Furthermore, we can see, as $Q^2$ is large and $R_A$ is not very large, the parameter $\alpha_s R_A^2 / Q^2$ gives a small value and it is reasonable to consider only the double scattering processes because all high twist contribution will be suppressed by the power of $\alpha_s R_A^2 / Q^2$. However, for the intermediate $Q^2$ and large $R_A$ the high twist processes will give considerable contribution and the resummation of all high twist contribution will be needed. This resummation would be similar to resuming all power corrections to the nuclear structure functions in e-A DIS [21], except that in our case the calculations may be complicated by the additional induced gluon radiation. We expect the resummation of all high twist contributions to parton energy loss may adopt a similar reaction operator formalism as derived by GLV to compute the radiative energy loss to all orders in opacity in a hot QCD medium [13], and this work is in progress.

In summary, we have calculated the semi-inclusive hadronic tensor and the corresponding parton energy loss at twist-6 in e-A DIS with helicity amplitude approximation by utilizing the twist expansion approach. The energy loss induced by gluon radiation at triple scattering (twist-6) processes has a $\alpha_s R_A^2 / Q^2$ suppression due to LPM interference effect as compared to the contribution of double scattering (twist-4) processes. The expanding parameter of high twist-expansion is discussed and we find the resummation of all high twist processes should be needed for intermediate $Q^2$ and heavy nucleus with large nucleus radius.

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