REPLICAS SYMMETRY BREAKING IN AN AXIAL MODEL OF QUADRUPOLAR GLASS

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We perform the replica symmetry breaking (RSB) in the vicinity of the point of instability of the replica symmetric solution in the model of axial quadrupolar glass. It is shown that the solution with the first stage RSB is stable against the second stage RSB. Although there is no reflection symmetry the 1RSB solution bifurcates continuously from the RS one. These facts mean that our model can not be associated with one of two classes usually considered in spin glass theory.
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1. In recent years the interest to non-Ising spin glasses without reflection symmetry is renewed. The theory of these models is often associated with the theory of real structural glasses. At present time there is no satisfactory microscopic model of the liquid–glass transition, although a great number of real and computer experimental data exist, as well as some phenomenological theories. At the same time the investigations of the spin glasses, orientational glasses and other models with frozen disorder are actively carried out. They generated lots of ideas and methods that are used for investigation of other systems, for example neural networks, proteins, rubbers and real glasses.

Usually two main links between the spin glass theory and the theory of real glasses are marked. First, the spin glass theory of some class of models is used as a possible scenarium for the liquid–glass transition in many-particle systems (see, for example, [1]–[3]). Second, the methods of the spin glass theory are used to construct a model for glass-forming systems of real particles [4]–[6].

In fact, there is a third link: the approaches are developed where the transition to the multipole glass state appears as a part of the liquid–glass transition [7, 8]. It is worth to notice that the physical meaning of the order parameters is different in different approaches. The problem considered in this paper can be useful in connection with the first and the third aspects of links.

As the recent works have shown, experimental characteristics of relaxation processes are described well enough by the mode coupling theory [9]. The similar equations can be obtained also as a result of the dynamic investigation of some spin glasses. This similarity is most pronounced (the first observation of this type is due to the authors of [10]) in the class of mean-field spin glasses without reflection symmetry. In this case the static transition at the temperature $T_c$ (the replica symmetry breaking – RSB) is discontinuous and the one-step RSB (1RSB) solution occurs to be stable, while the full Parisi scheme fails. The absence of the reflection symmetry means that the internal symmetry group does not contain the element transforming dynamical variable $Q$ to $-Q$. This results in cubic terms in Ginzburg–Landau functional for non-random case [11] and in the special form of RSB free energy functional for random interactions [12]. The temperature of dynamical transition $T_d$ in these models is higher than $T_c$. As a result of the dynamical transition the system
is trapped in the state that is less energetically favorable than the RSB state and stays there for a long time. The examples of such models are the $p$-spin model, orientational glasses, random Potts model and etc. They can be considered to be a prototype of real glasses. Some of these models, especially the $p$-spin spherical model, have already been investigated in detail in the middle of the 90ties (see the review [1]). The others are under investigation today [13]–[15]. It is important that the set of the specific features either appears as a whole or is absent as in Sherrington-Kirkpatrick model with Parisi solution. The set contains the absence of the reflection symmetry, the order parameter discontinuity, the stability of 1RSB solution and the fact that $T_d > T_c$. So traditionally the idea of two classes of models is used [1, 2, 16].

However, there are some indications that this classification is not quite correct. For example, the situation is not clear for the three state Potts model ([17]–[19]). The 1RSB solution has no discontinuity also in the case of three spin spherical model for the certain values of the external field [20].

In this paper we consider the quadrupolar glass model with infinite-range random interaction [21, 22] without reflection symmetry. We show that in contrast to the usual classification the 1RSB solution is continuous but stable. May be, this is due to the influence of the quadrupole operator algebra, that causes internal fields in the system with the effect similar to the external field effect in [20]. (For more details about internal fields in quadrupolar glasses see [23] and [24].)

2. So, we investigate the system of $N$ axial quadrupoles that are in the sites $i,j$ of the regular lattice with the Hamiltonian [21, 22]

$$H = -\frac{1}{2} \sum_{i \neq j} J_{ij} Q_i Q_j,$$

where $Q = 3J_z^2 - 2$, $J = 1$, $J_z = 1, 0, -1$, and the values of the coupling constants $J_{ij}$ are distributed following the Gaussian law:

$$P(J_{ij}) = (\sqrt{2\pi}J)^{-1} \exp \left[ -(J_{ij} - J_0)^2 / 2J^2 \right],$$

and $J_0 = \tilde{J}_0/N$, $J = \tilde{J}/N^{1/2}$.

Using the standard procedure of the replica method, we get the expression for the free energy, corresponding to the Hamiltonian [1] [21, 22]:

$$\frac{(F)_J}{NkT} = -\lim_{n \to 0} \frac{1}{n \operatorname{max}} \left\{ \frac{\tilde{J}^2n}{(kT)^2} - \frac{1}{2} \sum_{(\alpha\beta)} (q^{\alpha\beta})^2 - \frac{1}{2} \sum_\alpha (x^{\alpha})^2 + \right.$$ 

$$+ \ln \operatorname{Tr} \exp \left[ \frac{\tilde{J}}{kT} \sum_{(\alpha\beta)} q^{\alpha\beta} Q^{\alpha} Q^{\beta} + \left( \frac{\tilde{J}_0}{kT} + \frac{\tilde{J}^2}{2(kT)^2} \right) \frac{1}{\sqrt{2}} \sum_\alpha Q^{\alpha} x^{\alpha} - \right.$$ 

$$- \left( \frac{\tilde{J}}{kT} \right)^2 \sum_\alpha Q^{\alpha} \right\}.$$
Here \((\alpha\beta)\) means the sum over the couples of replicas, \(n\) – the number of replicas, and \(x^\alpha \sim \langle (Q^\alpha T) \rangle_J\), \(q^{\alpha\beta} \sim \langle (Q^\alpha Q^\beta) T \rangle_J\). Further, we suppose for simplicity that \(J_0 = 0\).

The case of symmetric replicas was investigated in details in [21, 22] and the solutions of the equations for the order parameters were obtained. It was shown that they grow continuously with the decrease of the temperature. No phase transition (in the traditional sense) was found. This fact agrees with the experiment [25] for \(N_2 – Ar\) mixture. There was also obtained the behaviour of the specific heat with the wide maximum and linear dependence on the temperature for low temperatures, that corresponded to the experiment of that time, which had not reached ultralow temperatures.

Later this model was investigated in detail in many papers (see, for example [23]–[30] and references therein), however the replica symmetry breaking was not carried out. However the investigation of the stability of the replica symmetric solution using Almeida-Thouless method [31] showed the instability against RSB [24, 30].

In this work we carry out the replica symmetry breaking for the model (1) in the vicinity of the instability point of the RS solution. We find the solution in this vicinity, that corresponds to 1RSB and we show, that it is stable for further replica symmetry breaking. It is important that despite of absence of reflection symmetry, 1RSB solution bifurcates continuously that contradicts the usual classification.

To carry out 1RSB we divide \(n\) replicas into \(n/m\) groups with \(m\) replicas in each group. We take \(q^{\alpha\beta}\) equal to \(q_1\) if \(\alpha\) and \(\beta\) belong to one group and \(q^{\alpha\beta}\) equal to \(q_0\) in the opposite case.

Now:

\[
\sum_{\alpha \neq \beta} (q^{\alpha\beta})^2 = q_1^2 n(m - 1) + q_0^2 n(n - m),
\]

\[
\sum_{\alpha \neq \beta} (q^{\alpha\beta}) Q^\alpha Q^\beta = q_0 \left( \sum_1^n Q^\alpha \right)^2 + (q_1 - q_0) \left[ \left( \sum_1^m Q^\alpha \right)^2 + \ldots + \left( \sum_{n-m}^n Q^\alpha \right)^2 \right] - 2nq_1 + q_1 \sum_1^n Q^\alpha.
\]

Substituting (3) and (4) into (2), using the expression for linearization of the exponent:

\[
\exp(\lambda a^2) = \frac{1}{\sqrt{2\pi}} \int dx \exp\left[-\frac{x^2}{2} + \sqrt{2\lambda} ax\right]
\]

and using new variables \(q_1 \rightarrow t(p + v), q_0 \rightarrow tp, x \rightarrow \sqrt{\frac{t}{2}} x\), where \(t = \frac{J}{kT}\), we obtain for the free energy:

\[
\frac{F}{NkT} = -t^2 + \frac{t^2 x^2}{4} + \frac{t^2}{4} \left(-mp^2 + (p + v)^2(m - 1) + 4(p + v)\right) - \]


\[-\frac{1}{m} \int dy^G \ln \int dz^G \Psi^m (\theta), \quad (6)\]

where
\[da^G = \frac{1}{\sqrt{2\pi}} dae^{-a^2/2},\]
\[\Psi = 2e^\theta + e^{-2\theta},\]
\[\theta = ty\sqrt{p} + tz\sqrt{v} + \frac{t^2}{2} (p + v - 2 + x),\]
and \(x, p, v, m\) satisfy the equations that express the extremum condition of the functional \((6)\):
\[x = \int dy^G \left[ \frac{dz^G \Psi^{m-1} \Psi'}{\int dz^G \Psi^m} \right], \quad (7)\]
\[p + v = \int dy^G \left[ \frac{dz^G \Psi^{m-2} (\Psi')^2}{\int dz^G \Psi^m} \right], \quad (8)\]
\[p = \int dy^G \left[ \frac{dz^G \Psi^{m-1} \Psi'}{\int dz^G \Psi^m} \right]^2, \quad (9)\]
\[-\frac{t^2}{4} m \left( (p + v)^2 - p^2 \right) = \frac{1}{m} \int dy^G \ln \int dz^G \Psi^m -
\int dy^G \left[ \frac{dz^G \Psi^{m-1} \Psi'}{\int dz^G \Psi^m} \right] \ln \Psi. \quad (10)\]

From the expression for free energy \((6)\) we get the entropy in form:
\[\frac{S}{Nk} = -t^2 - \frac{3}{4} t^2 x^2 - \frac{3}{4} t^2 \left[ -mp^2 + (p + v)^2 (m - 1) \right] - t^2 (p + v) + 2t^2 x +
\frac{1}{m} \int dy^G \ln \int dz^G \Psi^m (\theta). \quad (10)\]

3. Expanding \((7)–(10)\) in \(t\) at \(t \to 0\), it is easy to show, that there is only one solution for the system of new equations at high temperatures and it is the same as for RS equations \([21, 22]\). One can obtain the RS equations from \((6)–(10)\) if \(v = 0:\)
\[x' = 2 \int dy^G \left[ \frac{e^\theta - e^{-2\theta}}{2e^\theta + e^{-2\theta}} \right], \quad (11)\]
\[p' = 4 \int dy^G \left[ \frac{e^\theta - e^{-2\theta}}{2e^\theta + e^{-2\theta}} \right]^2, \quad (12)\]
\[\theta' = ty\sqrt{p'} + \frac{t^2}{2} (p' + x' - 2).\]

The solutions \(x = x', p = p', v = 0\) are the solutions of the system of equations \((11)–(14)\) for any \(m\) at all temperatures. Now let us investigate the behaviour of
1RSB system near the possible point of instability. To do this we expand the free energy (6) in series near the arbitrary point \( t_c \):

\[
F = F_0 + \frac{1}{2} x_1^2 (F_{xx} + \tau F_{xxt}) + \frac{1}{2} p_1^2 (F_{pp} + \tau F_{ppt}) + \\
+ x_1 p_1 (F_{xp} + \tau F_{xpt}) + \frac{1}{2} v^2 (F_{vv} + \tau F_{vvt}) + \\
+ x_1 v (F_{vx} + \tau F_{vxt}) + p_1 v (F_{vp} + \tau F_{vpt}) + \\
+ \frac{1}{6} x_1^3 F_{xxx} + \frac{1}{3} p_1^3 F_{ppp} + \frac{1}{6} v^3 F_{vvv} + \\
+ \frac{1}{2} x_1^2 (p_1 F_{xxp} + v F_{xxv}) + + \frac{1}{2} x_1^2 (p_1 F_{xxp} + v F_{xxv}) + \\
+ \frac{1}{2} p_1^2 (x_1 F_{ppx} + v F_{ppv}) + + \frac{1}{2} v^2 (x_1 F_{vux} + p_1 F_{vvp}) + \\
+ x_1 p_1 v F_{xpv}, \tag{13}
\]

where \( F_0 \) – free energy of the RS solution, \( \tau = t - t_c \), and \( x_1, p_1, v \) are deviations from RS solutions \( (p = p' + p_1, x = x' + x_1) \). Let us look for small solutions of the equations for the extremum condition of the functional (13), following the general bifurcation theory (see, for example [32]). Introducing new variable \( A = F/(Nkt) \) and using the expressions for \( A_{ab} \) from Appendix 1, we write the extremum condition in variables \( x, p, v \) in the form:

\[
x_1 A_{xx} + p_1 A_{px} - v(m - 1) A_{px} = B_x, \tag{14}
\]

\[
x_1 A_{px} + p_1 A_{pp} - v(m - 1) A_{pp} = B_p, \tag{15}
\]

\[
x_1 A_{px} + p_1 A_{pp} + v[A_{pp} - 2mD] = B_v, \tag{16}
\]

where \( D \) is defined by the expression

\[
A_{vv} = -(m - 1)[A_{pp} - 2mD],
\]

\( B_x, B_p, B_v \) are bilinear in \( x_1, p_1, v, \tau \). The equations (14 - 16) have always the trivial solution that corresponds to RS solution. And at the singular point which is determined from the condition that the determinant of the linear homogeneous system is equal to zero:

\[
\det \begin{pmatrix}
A_{xx} & A_{xp} & A_{xv} \\
A_{px} & A_{pp} & A_{pv} \\
A_{vx} & A_{vp} & A_{vv}
\end{pmatrix} = 0 \tag{17}
\]

the solution ceases to be unique. The condition (17) can be rewritten in the form:

\[
m (m - 1) (A_{pp} - 2D) (A_{xx} A_{pp} - A_{px}^2) = 0.
\]

Here the coefficients are for RS solution. Using [21, 22], it is easy to see that

\[
D' = A_{xx} A_{pp} - A_{px}^2
\]

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nowhere turns to zero (see also [30]), and the condition
\[ A_{pp} - 2D = 0 \] (18)
defines the temperature of the possible bifurcation:
\[ t = t_c = \frac{1}{1.367} \].

At this point RS solution is
\[ x'(t_c) = -0.581; \quad p'(t_c) = 1.449. \] (19)

If the condition (18) is fulfilled, then the fourth equation of the system – the extremum condition in \( m \) – has the form:
\[ x_1A_{px} + p_1A_{pp} - v(m - 1)A_{pp} = B_m, \] (20)
so in fact it plays a role of the compatibly condition.

4. To find small solutions of the system (14 – 16, 20) in the vicinity of \( t_c \), we have to (see [32]) solve the system of two equations with the determinant not equal to zero \( D' \neq 0 \) for variables \( x_1 \) and \( p_1 \), and to define \( v \) and \( m \) from the bifurcation equations. From the system
\[ x_1A_{xx} + p_1A_{px} = v(m - 1)A_{px} + B_x, \] (21)
\[ x_1A_{px} + p_1A_{pp} = v(m - 1)A_{pp} + B_p \] (22)
we obtain
\[ x_1 = \frac{B_pA_{px} - B_xA_{pp}}{D'}, \] (23)
\[ p_1 = v(m - 1) + \frac{B_xA_{px} - B_pA_{xx}}{D'}, \] (24)
so in the main order
\[ x_1 = 0; \quad p_1 = v(m - 1). \] (25)

Substituting the obtained solutions into the initial equations (14 - 16, 20) (in the l.h.s. we need to substitute the full solutions (23) and (24), while for r.h.s. (25) is enough), we obtain the bifurcation equations, that in our case lead simply to
\[ B_p = B_v; \quad B_m = B_v \] (26)
or
\[ \tau G_1 = -v\frac{t^6}{16}(G_2 + mG_3); \] (27)
\[ \tau \frac{2m - 1}{m} G_1 = -v\frac{t^6}{48}[-G_2 + 2m(G_2 - G_3) + 3m^2G_3]. \] (28)
The expressions for the coefficients $G_i$ and their numerical values for RS solution are given in the Appendix 1. Solving (27) – (28), we finally obtain:

$$m = \frac{G_2}{2G_2 + G_3} = 0.427,$$

$$v = \tilde{v}\tau; \quad \tilde{v} = -\frac{16}{t_0^6} \frac{G_1}{G_2 + mG_3} = 3, 13$$

(29)

(30)

So, we managed to find the small 1RSB solution of the system (7–10) near $T_c$. The 1RSB solution $x = x', p = p' + \tilde{v}\tau(m - 1), v = \tilde{v}\tau$ at $m = 0.427$ bifurcates continuously from the RS solution.

5. Now let us investigate the stability of 1RSB against the further replica symmetry breaking (2RSB). Now we divide each former group of replicas with $m$ elements into $m/m_1$ groups with $m_1$ elements. Parameter $q_{\alpha\beta}$ is assigned $p_3$, if replicas $\alpha$ and $\beta$ belong to the same smallest group (the number of $p_3$ is $\frac{n(m-1)}{2}$), $p_2$ – if $\alpha$ and $\beta$ belong to the same ”middle” group, but to the different smallest groups (they are $\frac{n(m-m_1)}{2}$ there), and $p_0$ – if $\alpha$ and $\beta$ belong to the different groups of $m$ replicas (their number is $\frac{n(n-m)}{2}$). As usual (see, for example, [33]), we deal with fluctuations inside one group of $m$ replicas and intergroup fluctuations, connected with the corresponding overlap of replicas $q_{\alpha\beta}$. The term in the free energy, corresponding to the intergroup replicon mode, which defines the stability of 1RSB solution in our case, has the form:

$$\Delta F_1 = \frac{1}{2}v_1^2(m - 1)(m - m_1)(m_1 - 1)(P_0 - 2Q_4 + R_5),$$

(31)

and

$$P_0 = 2 - 2t^2[4 - 4x + p + v - (p + v)^2],$$

$$Q_4 = -2t^2[2(p + v) - (p + v)^2 - t_3],$$

$$R_5 = -2t^2[r_4 - (p + v)^2],$$

where $v_1 w_1$ (see Appendix 2) are defined by the expressions

$$p_3 = w_1 - (m - m_1)v_1,$$

$$p_2 = w_1 + (m_1 - 1)v_1.$$

The eigenvalue is

$$\Lambda' = P_0 - 2Q_4 + R_5 = 2 - 2t^2[4 - 4x - 3(p + v) + 2t_3 + r_4],$$

where

$$t_3 = \int dy G \int dz G \Psi^{m-3}(\Psi')^3 \frac{1}{\int dz^G \Psi^m},$$

(32)
\[ r_4 = \int dy^G \int dz^G \frac{\Psi^{m-4}(\Psi')^4}{\int dz^G \Psi^m}. \] (33)

The other terms in the free energy are given in the Appendix 2 (38). With the help of RS and 1RSB solutions we can obtain the value of \( \Lambda' \) near \( t_c \). Taking into account that \( \Lambda'(t_c) = 0 \) and expanding \( t_3 \) and \( r_4 \) in series in variables \( p, v \) and \( t \), using the expressions

\[
\frac{\partial t_3}{\partial p} = 3t^2 \langle \alpha W(W + \alpha^2) \rangle, \quad (34)
\]

\[
\frac{\partial r_4}{\partial v} = \frac{\partial v_4}{\partial p} - 4mt^2 \langle \alpha^4 W \rangle, \quad (37)
\]

we obtain for the solution (23, 29, 30):

\[ \Lambda' = 6t^2 m \tau \tilde{v}(1 - 2t^2 G_4). \]

Here we take \( G_4 \) for RS solution:

\[ G_4 = \langle W(\alpha^4 + 2\alpha^3 - \alpha^2 - 2\alpha) \rangle = 0, 27, \]

so finally

\[ \Lambda' = 2, 9\tau, \]

that means the stability of 1RSB solution in this vicinity at \( T < T_c \) and \( m_1 < m < 1 \).

6. In conclusion, we have investigated the behaviour of the model (1) near the instability point \( t_c \) of its RS solution. We have shown that in the vicinity of \( t_c \) there is a more favorable solution which corresponds to the one-step replica symmetry breaking. We have also shown, that the new solution is stable relative to further replica symmetry breaking, i.e. the full Parisi scheme does not work in this case, at least in its classical variant. It is worth to notice that in spite of absence of reflection symmetry, 1RSB solution bifurcates continuously from the RS solution, that contradicts the idea of division in two classes.

However, one can await that at further decrease of the temperature, our 1RSB solution will become unstable. For complete investigation of this case it is necessary to solve the system of equations (6–10) for all temperatures and use the obtained solution for the definition of the sign of \( \Lambda_1 \) everywhere. May be, the low-temperature form of the 1RSB solution will explain the behaviour of experimental data [34] at ultralow temperatures \( T \approx 0, 05 K \), where the deviations from the RS solution are noticed, i.e. the linear dependence of the specific heat on \( T \) changes to the quadratic one.
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1 Appendix 1

Let us take into account that \( v = 0 \) for the RS solution. So \( \Psi \) does not depend on \( z \), and the internal integrals in (6 - 9) can be taken directly. Let us denote:

\[
\Psi' \bigg|_{t=t_c, x=x_0, p=p_0, v=0} = \alpha,
\]

\[
W = \alpha^2 + \alpha - 2,
\]

\[
\langle ... \rangle = \int dy G ...
\]

In these notations

\[
A_{xx} = 1 + \frac{t^2}{2} \langle W \rangle,
\]

\[
A_{px} = -\frac{t^2}{2} \langle W \alpha \rangle,
\]

\[
A_{pp} = -1 + \frac{t^2}{2} \langle W (W + 2\alpha^2) \rangle,
\]

\[
A_{ex} = -(m - 1) A_{px},
\]

\[
A_{pe} = -(m - 1) A_{pp},
\]

\[
A_{vv} = -(m - 1) [A_{pp} - 2mD],
\]

\[
D = t^2 \langle W \alpha^2 \rangle,
\]

\[
A_{pp} - 2D = -1 + t^2 \langle W^2 \rangle,
\]

At the bifurcation point \( t_c = 1,449 \), \( x' = -0,581 \), \( p' = 1,449 \) and we have for the RS solution

\[
\langle W \rangle = -1,132,
\]

\[
\langle W \alpha \rangle = 0,6035,
\]

\[
\langle W \alpha^2 \rangle = -0,997,
\]

\[
\langle W \alpha^3 \rangle = 1,232,
\]

\[
\langle W \alpha^4 \rangle = -1,981.
\]

The coefficients \( B_x, B_p, B_v \) contain the third and \( B_m \) - the fourth derivatives of \( A \) at the point \( t_c \), and their numerical values are obtained from the RS solution. With their help we get:
\[ G_1 = -t + t^5(W[(x'(t_c) - 2)(2\alpha^3 + 3\alpha^2 - 3\alpha - 2) + +p'(t_c)(-10\alpha^4 + 18\alpha^3 + 15\alpha^2 + 19\alpha - 6)]) = -0.4, \]
\[ G_2 = 4(W[4\alpha^4 + 8\alpha^3 - 3\alpha^2 - 7\alpha - 2]) = 11, \]
\[ G_3 = 4(W[-10\alpha^4 - 20\alpha^3 + 12\alpha^2 + 22\alpha + 10]) = 4. \]

2 Appendix 2

\[ 4\Delta F_2 = p_0^2 m[-2P_1 + 4mQ_1 - 4(m - 1)Q_3 - 6m^2R_1 + 8m(m - 1)R_3 - 2(m - 1)^2R_7] + +w_t^2(m - 1)[2P_0 + 4Q_4(m - 2) + R_5(m - 2)(m - 3) - R_6 m(m - 1)] + +p_0 w_1 4m(m - 1)[-2Q_2 + mR_2 - (m - 2)R_4]. \] (38)

Here the coefficients \( P, Q, R \) are for 1RSB solution and are as follows:

\[ P_1 = 2 - 2t^2[4 - 4x + p - p^2], \]
\[ Q_1 = -2t^2[2p - p^2 - t_{111}], \]
\[ Q_2 = -2t^2[2p - p(p + v) - t_{21}], \]
\[ Q_3 = -2t^2[2p(p + v) - p^2 - t_{21}], \]
\[ R_1 = -2t^2[r_{1111} - p^2], \]
\[ R_2 = -2t^2[r_{211} - p(p + v)], \]
\[ R_3 = -2t^2[r_{211} - p^2], \]
\[ R_4 = -2t^2[r_{31} - p(p + v)], \]
\[ R_6 = -2t^2[r_{22} - (p + v)^2], \]
\[ R_7 = -2t^2[r_{22} - p^2], \]

where

\[ t_{111} = \int dy^G \left[ \frac{\int dz^G \Psi^{m-1} \Psi'}{\int dz^G \Psi^m} \right]^3, \] (39)
\[ t_{21} = \int dy^G \frac{\int dz^G \Psi^{m-1} \Psi' \int dz^G \Psi^{m-2} (\Psi')^2}{\left[ \int dz^G \Psi^m \right]^2}, \] (40)
\[ r_{1111} = \int dy^G \left[ \frac{\int dz^G \Psi^{m-1} \Psi'}{\int dz^G \Psi^m} \right]^4, \] (41)
\[ r_{31} = \int dy^G \frac{\int dz^G \Psi^{m-1} \Psi' \int dz^G \Psi^{m-3} (\Psi')^3}{\left[ \int dz^G \Psi^m \right]^2}, \] (42)
\[ r_{22} = \int dy^G \left[ \frac{\int dz^G \Psi^{m-2} (\Psi')^2}{\int dz^G \Psi^m} \right]^2. \] (43)
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