EXCITED BARYONS AND CHIRAL SYMMETRY BREAKING OF QCD

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N* masses in the spin-1/2 and spin-3/2 sectors are computed using two non-perturbative methods: lattice QCD and QCD sum rules. States with both positive and negative parity are isolated via parity projection methods. The basic pattern of the mass splittings is consistent with experiments. The mass splitting within the same parity pair is directly linked to the chiral symmetry breaking QCD.

1. Introduction

There is increasing experimental information on the baryon spectrum from JLab and other accelerators (as tabulated in the particle data group), and the associated desire to understand it from first principles. The rich structure of the excited baryon spectrum provides a fertile ground for exploring how the internal degrees of freedom in the nucleon are excited and how QCD works in a wider context. One outstanding example is the parity splitting pattern in the low-lying N* spectrum. The splittings must be some manifestation of spontaneous chiral symmetry breaking of QCD because without it, QCD predicts parity doubling in the baryon spectrum.

2. Lattice QCD

Lattice QCD plays an important role in understanding the N* spectrum. One can systematically study the spectrum sector by sector, with the ability to dial the quark masses, and dissect the degrees of freedom most responsible. Given that state-of-the-art lattice QCD simulations have produced a ground-state spectrum that is very close to the observed values, it is important to extend the success beyond the ground states. There exist already a number of lattice studies of the N* spectrum, focusing mostly on the spin-1/2 sector. All established a clear splitting from the
ground state. Here, we focus on calculating the excited baryon states in the spin-3/2 sector. We consider the following interpolating field with the quantum numbers $I^J = \frac{1}{2} \left( \frac{3}{2}^+ \right)$, \[ \chi^N_{\mu} = \epsilon^{abc}(u_a^T C\gamma_5 \gamma_\mu d_b)\gamma_5 u_c. \] (1) The interpolating fields for other members of the octet can be found by appropriate substitutions of quark fields. Despite having an explicit parity by construction, these interpolating fields couple to both positive and negative parity states. A parity projection is needed to separate the two. In the large Euclidean time limit, the correlator with Dirichlet boundary condition in the time direction and zero spatial momentum becomes \[ G_{\mu\nu}(t) = \sum_{x} \langle 0| \chi^e_\mu(x) \bar{\chi}^e_\nu(0)|0\rangle \] \[ = f_{\mu\nu} \left[ \lambda^2 \frac{\gamma_4 + 1}{2} e^{-M_+ t} + \lambda^2 \frac{-\gamma_4 + 1}{2} e^{-M_- t} \right] \] (3) where $f_{\mu\nu}$ is a function common to both terms. The relative sign in front of $\gamma_4$ provides the solution: by taking the trace of $G_{\mu\nu}(t)$ with $(1 \pm \gamma_4)/4$, one can isolate $M_+$ and $M_-$, respectively.

It is well-known that a spin-3/2 interpolating field couples to both spin-3/2 and spin-1/2 states. A spin projection can be used to isolate the individual contributions in the correlation function $G_{\mu\nu}$. Numerical test of spin projection in the spin-3/2+ channel reveals two different exponentials in $G(t)$ from the spin-3/2+ and spin-1/2+ parts, with the spin-3/2+ state heavier than the spin-1/2+ one, which is consistent with experiment. One would mistake the dominant spin-1/2+ state for the spin-3/2+ state without spin projection.

Figure 1 presents preliminary results for mass ratios extracted from the correlation functions for the $\frac{3}{2}^+$ N* states to the nucleon ground state as a function of $(\pi/\rho)^2$. Mass ratios have minimal dependence on the uncertainties in determining the scale and the quark masses, so that a more reliable comparison with experiment can be made. These ratios appear headed in the right direction compared to experiment where available, but more study is needed to address the systematics. Figure 2 shows the similar plots for the $\frac{3}{2}^-$ N* states.

3. QCD sum rules

The QCD Sum Rule method is a time-honored method that has proven useful in revealing a connection between hadron phenomenology and the
non-perturbative nature of the QCD vacuum via only a few parameters (the vacuum condensates). It has been successfully applied to a variety of observables in hadron phenomenology, providing valuable insights from a unique, QCD-based perspective, and continues an active field. The method is analytical (no path integrals!), is physically transparent (one can trace
back to the QCD operators responsible), and has minimal model dependence. The accuracy of the approach is limited due to limitations inherent in the operator-product-expansion (OPE), but well understood.

One progress in this area is the use of Monte Carlo-based analysis to explore the predictive ability of the method for \( N^\ast \) properties. The idea is to probe the entire QCD parameter space and map the error distribution on the OPE side to the phenomenological side. It is found that some QCD sum rules are truly predictive for \( N^\ast \) masses, while others are marginal.

Another progress is that a parity separation similar to that in lattice QCD can be performed in the QCD sum rule approach, resulting in the so-called parity-projected sum rules which have the general structure

\[
A(M, w_+) + B(M, w_+) = \tilde{\lambda}_+^2 \exp\left(-\frac{m_+^2}{M^2}\right), \tag{4}
\]
\[
A(M, w_-) - B(M, w_-) = \tilde{\lambda}_-^2 \exp\left(-\frac{m_-^2}{M^2}\right), \tag{5}
\]

where \( M \) is the Borel mass parameter, \( (m_B, \tilde{\lambda}^2, w) \) are the phenomenological parameters (mass, coupling, threshold). The term \( B \) controls the mass splitting: if \( B = 0 \), then \( m_+ = m_- \). Term \( B \) involves only dimension-odd condensates, such as the quark condensate \( \langle \bar{q}q \rangle \) and the mixed condensate \( \langle \bar{q}g\sigma \cdot Gq \rangle \). So a direct link is established between the mass splitting of parity pairs and dynamical chiral symmetry breaking of QCD. Fig. 3 shows a numerical confirmation in the case of nucleon. As \( \langle \bar{q}q \rangle \) is decreased, both masses decrease, but with a different rate. \( N_{1^+}^\ast \) falls faster than \( N_{1^-}^\ast \). As a result, the mass splitting decreases. In the limit that chiral-symmetry is restored (\( \langle \bar{q}q \rangle = 0 \)), it is expected that \( N_{1^+}^\ast \) and \( N_{1^-}^\ast \) become degenerate.

In conclusion, we can compute the baryon spectrum in the spin-1/2 and spin-3/2 sectors for all particle channels using two methods: lattice QCD and QCD sum rules. Parity projection further reveals that the mass splitting within the same baryon pair is directly controlled by spontaneous chiral symmetry breaking of QCD.

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Figure 3. Mass splitting between $N_{1/2}^-$ and $N_{1/2}^+$ as a function of the quark condensate. The ratio is relative to the standard value of $a = -(2\pi)^2 \langle \bar{q}q \rangle = 0.52 \text{ GeV}^3$.

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