RG and BV-formalism

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Abstract

In present paper a quantization scheme proposed recently by Morris [arXiv:1806.02206[hep-th]] is analyzed. This method is based on idea to combine the renormalization group with the BV-formalism in an unique quantization procedure. It is shown that the BV-formalism and the new method should be considered as independent approaches to quantization of gauge systems both provided by global supersymmetry.

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1 Introduction

At present the BRST symmetry [1, 2] is considered as a fundamental principle of Modern Quantum Field Theory allowing suitable quantum description of a given dynamical system [3, 4]. This principle is underlying the powerful quantization methods known in covariant formalism as the Batalin-Vilkovisky (BV) method [5, 6] and in canonical formulation as the Batalin-Fradkin-Vilkovisky (BFV) approach [7, 8] (for recent developments of these methods see [9, 10, 11, 12, 13]). Application of these methods to a given dynamical system guarantees gauge-independence of physical results thanks to the BRST symmetry.

The Gribov-Zwanziger theory [14, 15, 16] and the functional renormalization group approach [17, 18] belong to a class of non-perturbative formulations of quantum theory of gauge fields with violation of the BRST symmetry. In its turn the breakdown of the BRST symmetry in both these cases leads to gauge dependence of effective action even on-shell [19, 20, 21, 22, 23] making physical interpretation of the results impossible.

Recently it has been proposed to combine methods of the BV-formalism with the exact renormalization group (ERG)[24]. To do this regularized versions of the antibracket and Delta-operator have been introduced in a way dictated by main concepts of the renormalization group with preserving (almost) all basic properties known from the BV-formalism. In contrast with standard formulation of the functional renormalization group approach [17, 18] the regulators should be introduced in a way that preserving gauge invariance of regularized action [24, 25, 26].

In present paper we analyze basic assumptions of new quantization approach [24] in general gauge theories. As a result we conclude that the BV-formalism and the ERG method [24] should be considered as independent quantization schemes. It means in particular that one needs to study basic properties of the new quantization procedure and among them a gauge fixing procedure, a gauge dependence problem, existence of global supersymmetry and so on. As in the case of BV-formalism the ERG can be provided with a global supersymmetry (regularized BRST symmetry).

The paper is organized as follows. In section 2 a short presentation of the BV-formalism is given. Basic ingredients and relations of the ERG method are considered in section 3. In section 4 anticanonical transformations in the ERG method and relations between the BV-formalism and the ERG method are studied. In section 5 the gauge fixing procedure for general gauge theories within the ERG method is introduced. In section 6 the existence of global supersymmetry (regularized BRST symmetry) in the ERG method for the general gauge and the Yang-Mills type theories is proved. Finally, our conclusions and remarks are presented in section 7.

In the paper the DeWitt’s condensed notations are used [27]. We employ the notation $\varepsilon(A)$ for the Grassmann parity of any quantity $A$. The right and left functional derivatives with respect to fields and antifields are marked by special symbols ” $\leftarrow$ ” and ” $\rightarrow$ ” respectively.
2 BV-formalism in short

The BV-formalism presents a powerful method of covariant quantization of general gauge theories [5, 6]. It is based on a number of fundamental assumptions about the properties of the systems in question. It is assumed that a given system of fields $A^i$, $\varepsilon(A^i) = \varepsilon_i$, is described by an initial classical action $S_0[A]$ being invariant under gauge transformations, $\delta \varepsilon A^i = R^i_\alpha(A)\xi^\alpha$, where $R^i_\alpha(A)$, $\varepsilon(R^i_\alpha) = \varepsilon_i + \varepsilon_\alpha$, are generators of gauge transformations and $\xi^\alpha = \xi^\alpha(x)$, $\varepsilon(\xi^\alpha) = \varepsilon_\alpha$, are arbitrary functions. In general algebra of gauge generators may be (ir)reducible and (or) open and structure coefficients may depend on fields. Taking into account the structure of gauge algebra one defines a minimal antisymplectic space parameterized by fields $\Phi^A_{min}$, $\varepsilon(\Phi^A_{min}) = \varepsilon_A$ and antifields $\Phi^*_{min}$, $\varepsilon(\Phi^*_{min}) = \varepsilon_A + 1$. For irreducible gauge algebra the set $\Phi^A_{min} = \{A^i, C^\alpha\}$ includes initial fields $A^i$ and ghost fields $C^\alpha$, $\varepsilon(C^\alpha) = \varepsilon_\alpha + 1$. In turn the set of corresponding antifields takes the form $\Phi^*_{min} = \{A^*_i, C^*_\alpha\}$. For reducible theories the set $\Phi^A_{min}$ looks more complicate and includes a pyramid of ghost for ghost fields and a pyramid of auxiliary fields but here we are not going to details. In the minimal antisymplectic space a solution, $S_{min} = S_{min}[\Phi^A_{min}\Phi^*_{min}]$, to the classical master equation, $(S_{min}, S_{min}) = 0$, $S_{min}|_{\Phi^*_{min}=0} = S_0[A]$, is constructed in the Taylor expansion with respect to ghost fields. Then full antisymplectic space of fields $\Phi = \{\Phi^A\}$ and antifields $\Phi^* = \{\Phi^*_A\}$ is introduced. For irreducible gauge algebra the explicit content of these sets are $\Phi^A = \{A^i, C^\alpha, \tilde{C}^\alpha, B^\alpha\}$, $\Phi^*_A = \{A^*_i, C^*_\alpha, \tilde{C}^*_\alpha, B^*_\alpha\}$ where $\tilde{C}^\alpha$ and $B^\alpha$ are antighost and auxiliary (Nakanishi-Lautrup) fields correspondingly. In full antisymplectic space the action $S = S[\Phi, \Phi^*]$ constructed by the rule $S = S_{min} + \tilde{C}^*_\alpha B^\alpha$ satisfies the quantum master equation $(1/2)(S, S) = i\hbar\Delta S$. This action is the initial object of the BV-formalism in construction of quantum description of a given gauge system. Making use a special type of anticanonical transformation with the help of Fermion functional $\Psi = \Psi[\Phi]$ the gauge-fixing functional $S_\Psi = S_\Psi[\Phi, \Phi^*]$ is introduced. The action $S_\Psi$ satisfies the quantum master equation as well, $(1/2)(S_\Psi, S_\Psi) = i\hbar\Delta S_\Psi$. With the help of $S_\Psi$ the generating functional of Green functions, $Z[J, \Phi^*]$, in the form of functional integral over fields $\Phi$ is defined. Vacuum functional, $Z[\Phi^*] = Z[J = 0, \Phi^*]$ obeys very important property of independence on specific choice of gauge-fixing functional $\Psi$ as a consequence that $S_\Psi$ satisfies the quantum master equation. In turn it means the gauge independence of physical quantities constructed in the BV-formalism due to the equivalence theorem [25].

3 Morris’s construction

In the paper [24] a generalization of the antibracket and the Delta-operator of the BV-formalism has been proposed. For any two functionals $F = F[\Phi, \Phi^*]$ and $G = G[\Phi, \Phi^*]$ the new antibracket is given by the rule

$$
(F, G)_\Lambda = \int dx F\left(\frac{\xi}{\delta \Phi^A(x)} K_\Lambda(x) \frac{\delta}{\delta \Phi^*_A(x)} - \frac{\xi}{\delta \Phi^*_A(x)} K_\Lambda(x) \frac{\delta}{\delta \Phi^A(x)}\right) G,
$$

(3.1)
or in the DeWitt’s condensed notation

\[(F, G)_\Lambda = F(\overleftarrow{\partial}_{\Phi^A} K_A \overrightarrow{\partial}_{\Phi^*_A} - \overleftarrow{\partial}_{\Phi^*_A} K_A \overrightarrow{\partial}_{\Phi^A}) G.\]  

(3.2)

Here \(\Phi = \{\Phi^A\}, \varepsilon(\Phi^A) = \varepsilon_A\) and \(\Phi^* = \{\Phi^*_A\}, \varepsilon(\Phi^*_A) = \varepsilon_A + 1\) are sets of fields and antifields correspondingly and a regulator operator \(K_A\) is introduced,

\[K_A(x) = K(\Box/\Lambda^2), \quad \Box = \partial_\mu \partial^\mu, \quad \varepsilon(K_A) = 0,\]  

(3.3)

with the following properties: \(K_A(0) = 1\) and \(K_A(x) \to 0\) for \(x \to \infty\).

Taking into account that the integration by parts reads

\[\int dx f(x) K_A(x) g(x) = \int dx g(x) K_A(x) f(x)(-1)^{\varepsilon(f)\varepsilon(g)} = \int dx (K_A(x) f(x)) g(x),\]  

(3.4)

one can check that the standard properties of the antibracket in the BV-formalism \([5, 6]\) hold for the new antibracket (3.1) as well:

1. Grassmann parity

\[\varepsilon((F, G)_\Lambda) = \varepsilon(F) + \varepsilon(G) + 1 = \varepsilon((G, F)_\Lambda)\]  

(3.5)

2. Generalized antisymmetry

\[(F, G)_\Lambda = -(G, F)_\Lambda(-1)^{\varepsilon(F)\varepsilon(G) + 1},\]  

(3.6)

3. Leibniz rule

\[(F, GH)_\Lambda = (F, G)_\Lambda H + (F, H)_\Lambda G(-1)^{\varepsilon(G)\varepsilon(H)},\]  

(3.7)

4. Generalized Jacobi identity

\[((F, G)_\Lambda, H)_\Lambda(-1)^{\varepsilon(F)\varepsilon(H) + 1} + \text{cycle}(F, G, H) \equiv 0.\]  

(3.8)

The generalized Delta-operator has the form\(^2\)

\[\Delta_\Lambda = \int dx (-1)^{\varepsilon_A} \overleftarrow{\delta} K_A(x) \overrightarrow{\delta} = (-1)^{\varepsilon_A} \overleftarrow{\partial}_{\Phi^A} K_A \overrightarrow{\partial}_{\Phi^*_A}, \quad \varepsilon(\Delta_\Lambda) = 1,\]  

(3.9)

and obeys the nilpotency property

\[\Delta^2_\Lambda = 0.\]  

(3.10)

\(^2\)As in the BV-formalism the operator \(\Delta_\Lambda\) is ill-defined due to local nature of differential operation entering in (3.9) and leading to the \(\delta(0)\) problem. This problem can be solved by using for example the dimensional regularization when \(\delta(0) = 0\).
Action of the generalized Delta-operator on the antibracket (3.1) takes the standard form in the BV-formalism

\[ \Delta_\Lambda(F, G) = (\Delta_\Lambda F, G)_\Lambda - (F, \Delta_\Lambda G)_\Lambda(-1)^{\varepsilon(F)}. \]  

(3.11)

The same statement is valid for action of the generalized Delta-operator on product of two functionals. The result reads

\[ \Delta_\Lambda (F \cdot G) = (\Delta_\Lambda F) \cdot G + F \cdot (\Delta_\Lambda G)(-1)^{\varepsilon(F)} + (F, G)_\Lambda(-1)^{\varepsilon(F)}. \]  

(3.12)

In deriving (3.11) and (3.12) the integration by parts (3.4) was intensively used.

For a given gauge theory with initial classical action \( S_0[A] \) having properties described in Sec. 2 it is assumed that there exists a possibility to construct a regularized action \( S_0\Lambda[A] \) which will be invariant under (regularized) gauge transformations. It is exactly the case of paper [26] where regularization of kinematic part of full action is presented in explicit form and existence of regularized part of interaction assumed. In paper [24] the regularization of interaction terms of full action in the case of non-Abelian gauge theories is given perturbatively. Then it is expected that all the consequences of the BV formalism can be applied. This would be so if the new formalism were equivalent to the BV formalism. We are going to prove non-equivalence of the BV-formalism and the new quantization method.

4 Anticanonical transformations

Anticanonical transformation in the BV-formalism preserves structure of any relation involving the antibracket and the Delta-operator. Relations listed in Section 3 confirmed that.

But there exist an essential difference between these two approaches. It is related with canonical relations in the BV-formalism,

\[ (\Phi^A, \Phi^*_B) = \delta^A_B, \]  

(4.1)

and relations with the regularized antibracket,

\[ (\Phi^A, \Phi^*_B)_\Lambda = K_\Lambda \delta^A_B. \]  

(4.2)

The relations (4.1) can be rewritten in the form

\[ (\Phi^A, K^{-1}_\Lambda \Phi^*_B)_\Lambda = (K^{-1}_\Lambda \Phi^A, \Phi^*_B)_\Lambda = \delta^A_B. \]  

(4.3)

When \( K_\Lambda \neq 1 \) there is no anticanonical transformation reproducing the relations

\[ \Phi'^A = \Phi^A, \quad \Phi'^*_A = K_\Lambda \Phi^*_A \quad \text{or} \quad \Phi'^A = K^{-1}_\Lambda \Phi^A, \quad \Phi'^*_A = \Phi^*_A. \]  

(4.4)

Indeed, let \( F = F[\Phi, \Phi^*], \varepsilon(F) = 1 \) be generator of anticanonical transformation,

\[ \Phi'^A = \frac{\delta}{\delta \Phi^*_A} F[\Phi, \Phi^*], \quad \Phi'^*_A = F[\Phi, \Phi^*] \frac{\delta}{\delta \Phi^*_A}. \]  

(4.5)
Then from (4.4) we have, in particular,

$$\delta_{\Phi^*} F[\Phi, \Phi^*] = \Phi^A$$  \hspace{1cm} (4.6)

and therefore

$$F[\Phi, \Phi^*] = \Phi^*_A \Phi^A + X[\Phi], \quad \varepsilon(\Psi) = 1,$$  \hspace{1cm} (4.7)

with some odd functional $X = X[\Phi]$. The second relation in (4.4) allows us to specify the functional $X[\Phi]$,

$$(K_A - 1) \Phi^*_A = X[\Phi] \delta_{\Phi^A},$$  \hspace{1cm} (4.8)

with the results $X[\Phi] = \text{const}$, $K_A - 1 = 1$. Because anticanonical transformations (automorphism of CME) [29] and anticanonical master-transformations (automorphism of QME) [30, 31, 9] rule all scheme of the BV-formalism we conclude that the ERG method is independent quantization procedure for general gauge theories.

The regularized antibracket is not invariant under anticanonical transformations accepted in the BV-formalism, where anticanonical transformations play very important role in solving all the principal problems concerning the gauge fixing procedure, the gauge invariant renormalization, the gauge dependence problem and so on. As an independent approach the ERG method based on using the regularized quantum master equation formulated in terms of regularized antibracket and regularized Delta-operator requires first of all to study the invariance properties of the regularized antibracket. Now we have to consider the relation

$$(\Phi^A(x), \Phi^*_B(y)) = \delta^A_B K_A(x) \delta(x - y)$$  \hspace{1cm} (4.9)

as the basic one in the ERG method. It is clear that the identical transformation leaving the regularized antibracket invariant is described by the functional $X_0[\Phi, \Phi^*]$,

$$X_0[\Phi, \Phi^*] = \int dx \Phi^*_A(x) K_A(x) \Phi^A(x).$$  \hspace{1cm} (4.10)

We consider now transformations of variables which infinitesimally differ of the identical ones and describe by the functional $F[\Phi, \Phi^*]_\Lambda = X_0[\Phi, \Phi^*] + \varepsilon[\Phi, \Phi^*]$, so that

$$\Phi^A(x) = \delta_{\Phi^*} F_A[\Phi, \Phi^*] = K_A(x) \Phi^A(x) + \delta_{\Phi^*_A} \varepsilon[\Phi, \Phi^*],$$

$$\Phi^*_A(x) = F_A[\Phi, \Phi^*] \delta_{\Phi^A} = \Phi^*_A(x) K_A(x) + \varepsilon[\Phi, \Phi^*] \delta_{\Phi^A}.$$  \hspace{1cm} (4.11)

In the first order in $\varepsilon[\Phi, \Phi^*]$ we obtain

$$(\Phi^A(x), \Phi^*_B(y))_\Lambda = \delta^A_B K_A(x) \delta(x - y) +$$

$$+ \delta_{\Phi^*_A} \varepsilon[\Phi, \Phi^*] \delta_{\Phi^B} - \Phi^*_A(x) K_A(x) \delta_{\Phi^*_A} \varepsilon[\Phi, \Phi^*] \delta_{\Phi^B} K_A^{-1}(y).$$  \hspace{1cm} (4.13)
In general, the regularized antibracket is not invariant under the transformations (4.11), (4.12). Nevertheless, there exist the two cases of transformations (4.11), (4.12) preserving the form of regularized antibracket. They correspond to the following choice of functional $\varepsilon[\Phi, \Phi^*]$,

$$\varepsilon[\Phi, \Phi^*] = \Psi[\Phi] \quad \text{or} \quad \varepsilon[\Phi, \Phi^*] = X[\Phi^*]$$

for arbitrary odd functionals $\Psi[\Phi], X[\Phi^*]$. We can refer to these cases as reduced anticanonical transformations in the ERG method.

### 5 Gauge fixing procedure

Now we are in position to describe the gauge fixing procedure in the new quantization approach in general case of any initial system of gauge fields $A^i$ with an action $S_0[A]$ which is invariant under the gauge transformations $\delta A^i = R^i_\alpha(A)\xi^\alpha$. Main idea of the EFG method is to save the gauge invariance of regularized action $S_{0\Lambda}[A]$ corresponding to $S_0[A]$,

$$S_{0\Lambda}[A] \overset{\Lambda \to 0}{\to} R^i_\Lambda(A)\xi^\alpha = 0.$$  

(5.1)

The regularized action $S_{0\Lambda}[A]$ and regularized gauge generators $R^i_\Lambda(A)$ satisfy the properties

$$\lim_{\Lambda \to 0} S_{0\Lambda}[A] = S_0[A], \quad \lim_{\Lambda \to 0} R^i_\Lambda(A) = R^i_\alpha(A).$$

(5.2)

Requirement of gauge invariance of the regularized action (5.1) differs from usually accepted regularization of kinematic part of full action in the standard FRG approach [17, 18] which leads to breakdown of gauge symmetry and causes the gauge dependence of effective average action even on-shell [21, 22].

Now let $S_\Lambda = S_\Lambda[\Phi, \Phi^*]$ be an action satisfying the regularized quantum master equation

$$\frac{1}{2}(S_\Lambda, S_\Lambda)_\Lambda = i\hbar \Delta_\Lambda S_\Lambda \leftrightarrow \Delta_\Lambda \exp \left\{ \frac{i}{\hbar} S_\Lambda \right\} = 0,$$

(5.3)

and the boundary condition

$$S_\Lambda|_{\Phi^* = \hbar = 0} = S_{0\Lambda}[A].$$

(5.4)

For any odd functional $\Psi = \Psi[\Phi]$ we derive the relation

$$[\Delta_\Lambda, \Psi] = -\int dx \overleftarrow{K_\Lambda(x)} \overrightarrow{\partial_\Phi} \Psi[\Phi] \partial_\Phi K_\Lambda(x),$$

(5.5)

which allows to state that

$$\exp \left\{ -[\Delta_\Lambda, \Psi] \right\} W[\Phi, \Phi^*] = W[\Phi, \Phi^*] + \Psi[\Phi] \overleftarrow{\partial_\Phi} K_\Lambda,$$

(5.6)

where $W[\Phi, \Psi^*]$ is arbitrary functional. In particular, we can construct the action $S_\Lambda\Psi = S_{\Lambda\Psi}[\Phi, \Phi^*]$ as

$$S_{\Lambda\Psi}[\Phi, \Phi^*] = \exp \left\{ -[\Delta_\Lambda, \Psi] \right\} S_\Lambda[\Phi, \Phi^*] = S_\Lambda[\Phi, \Phi^*] + \Psi[\Phi] \overleftarrow{\partial_\Phi} K_\Lambda,$$

(5.7)
where $S_A[\Phi, \Phi^*]$ is a solution to equations (5.3), (5.4). The functional (5.7) satisfies the quantum master equation (5.3) as well. Indeed, let us compute the commutator of regularized Delta-operator $\Delta_A$ with $[\Delta_A, \Psi]$. The result reads

$$[\Delta_A, [\Delta_A, \Psi]] = -\int dxdy(-1)^{\epsilon_A(\epsilon_B+1)}(\Psi[\Phi]\overrightarrow{\partial}_{\Phi^B(y)}\overrightarrow{\partial}_{\Phi^A(x)})K_A(y)\overrightarrow{\partial}_{\Phi^B(y)}K_A(x)\overrightarrow{\partial}_{\Phi^A(x)}. \quad (5.8)$$

Due to the symmetry properties of integrand in (5.8) we conclude that

$$[\Delta_A, [\Delta_A, \Psi]] = 0. \quad (5.9)$$

From Eqs. (5.3), (5.7), (5.9) it follows

$$0 = \exp\{ -[\Delta_A, \Psi]\} \Delta_A \exp\{ -\frac{i}{\hbar}S_A \} = \Delta_A \exp\{ -[\Delta_A, \Psi]\} \exp\{ -\frac{i}{\hbar}S_A \} = \Delta_A \exp\{ -\frac{i}{\hbar}S_A \Psi \} = 0. \quad (5.10)$$

We consider the relation (5.7) as gauge fixing procedure in the ERG method. In the limit $\Lambda \rightarrow 0$ the procedure described coincides with the gauge fixing procedure in the BV-formalism where $\Psi[\Phi]$ is the gauge fixing functional.

6 Global supersymmetry

Now we are going to prove that there exists a possibility to provide the ERG method with a global supersymmetry similarly to the BV-formalism. Starting point is the generating functional of Green function

$$Z_A[J, \Phi^*] = \int D\Phi \exp\left\{ \frac{i}{\hbar} \left( S_A[\Phi, \Phi^*] + J_A \Phi^A \right) \right\}, \quad (6.1)$$

where $J_A \ (\varepsilon(J_A) = \varepsilon(\Phi^A) = \varepsilon_A)$ are external sources to fields $\Phi^A$. To discuss the existence of global supersymmetry within the ERG method we consider the vacuum functional $Z_A = Z_A[J = 0, \Phi^* = 0]$,

$$Z_A = \int d\Phi \exp\left\{ \frac{i}{\hbar} S_A[\Phi, \Psi[\Phi] \overrightarrow{\partial}_\Phi K_A] \right\}, \quad (6.2)$$

It is convenient to present (6.2) in the form

$$Z_A = \int d\Phi d\Phi^* d\lambda \exp\left\{ \frac{i}{\hbar} \left( S_A[\Phi, \Phi^*] + (\Phi^*_A - \Psi[\Phi] \overrightarrow{\partial}_\Phi K_A)\lambda^A \right) \right\}, \quad (6.3)$$

with the help of auxiliary fields $\lambda^A, \ \varepsilon(\lambda^A) = \varepsilon_A + 1$.

Then the integrand in (6.3) is invariant under the following global supertransformations:

$$\delta_A \Phi^A = K_A \lambda^A \mu, \quad \delta_A \Phi^*_A = \mu K_A (S_A[\Phi, \Phi^*] \overrightarrow{\partial}_\Phi), \quad \delta_A \lambda^A = 0, \quad (6.4)$$
where \( \mu \) is a constant anticommuting parameter. In deriving this result we take into account the following facts: a) the Jacobian is equal to

\[
J = \exp \{ \Delta \Lambda S_\Lambda \},
\]

(6.5)
b) the functional \( S_\Lambda \) satisfies the regularized quantum master equation (5.3), c) the equality holds

\[
\int dx dy (-1)^{\epsilon A(\epsilon B + 1)} (\Psi[\Phi] \stackrel{\rightarrow}{\partial}_{\Phi B}(y) \stackrel{\leftarrow}{\partial}_{\Phi A}(x)) K_\Lambda(y) \lambda^B(y) K_\Lambda(x) \lambda^A(x) = 0
\]

(6.6)
due to the symmetry properties in the integrand (6.6).

The transformations (6.4) represent the global supersymmetry for the ERG approach in the space of variables \( \Phi, \Phi^* \), \( \lambda \). They may be named as regularized BRST transformations. Notice that in general the regularized BRST symmetry is not symmetry of some action in full agreement with situation in the BV-formalism. There is another property of transformations (6.4) similarly to the BRST transformations in the BV-formalism, namely they do not depend on choice of gauge fixing condition. It is very important to realize that the existence of this symmetry is the consequence of the fact that the bosonic functional \( S_\Lambda \) satisfies the regularized quantum master equation (5.3).

In the case when the regularized action \( S_0[A] \) belongs to Yang-Mills type of gauge theories, the action \( S_\Lambda \Psi[\Phi, \Phi^*] \) can be constructed in explicit form. To this end we assume gauge invariance of the action \( S_0[A] \),

\[
S_{0A,i}[A] R_{\Lambda \alpha}^i (A) = 0,
\]

(6.7)

where the gauge generators \( R_{\Lambda \alpha}^i (A) \) form a linear independent set in the index \( \alpha \) and satisfy the gauge algebra

\[
R_{\Lambda \alpha,j}^i (A) R_{\Lambda \beta,j}^i (A) - (-1)^{\epsilon_\alpha \epsilon_\beta} R_{\Lambda \beta,j}^i (A) R_{\Lambda \alpha,j}^i (A) = -R_{\Lambda \gamma}^i (A) F_{\alpha \beta}^\gamma.
\]

(6.8)

Here \( F_{\alpha \beta}^\gamma \) are structure coefficients which do not depend on fields \( A^i \) and the notation \( G^i = G \stackrel{\rightarrow}{\partial}_{A^i} \) is used. Now the action \( S_\Lambda \Psi[\Phi, \Phi^*] \) is constructed by the Faddeev-Popov rules [32] and has the form

\[
S_\Lambda \Psi[\Phi, \Phi^*] = S_0[A] + \Psi[\Phi] \stackrel{\rightarrow}{\partial}_{\Phi A} R_\Lambda^A(\Phi) + \Phi^* R_\Lambda^A(\Phi),
\]

(6.9)

where \( \Phi^A = (A^i, B^\alpha, C^\alpha, \bar{C}^\alpha) \) are fields appearing in the Faddeev-Popov method, and

\[
R_\Lambda^A(\Phi) = (R_{\Lambda \alpha}^i (A) C^\alpha, 0, -(1/2) (-1)^{\epsilon_\beta} F_{\alpha \beta}^\gamma C^\gamma C^\beta, (-1)^{\epsilon_\alpha} B^\alpha)
\]

(6.10)

are generators of the regularized BRST transformations

\[
\delta_{BA} \Phi^A = R_\Lambda^A(\Phi) \mu, \quad \mu = \text{const}, \quad \epsilon(\mu) = 1.
\]

(6.11)

Due to the nilpotency of transformations (6.11),

\[
R_\Lambda^A(\Phi) \stackrel{\rightarrow}{\partial}_{\Phi B} R_\Lambda^B(\Phi) = 0,
\]

(6.12)

the action (6.9) is invariant under the regularized BRST transformations (6.11).
7 Discussion

In the paper we have analyzed basic assumptions of new approach for quantization of gauge systems to combine attractive features of the BV-formalism with main idea of the renormalization group [24]. We have confirmed basic algebraic properties and the Jacobi identity for regularized antibracket and Delta-operator introduced in [24] which are similarly to corresponding relations in the BV-formalism. Nevertheless the BV-formalism [5, 6] and the new method [24] should be considered as independent approaches to quantization of gauge systems because there does not exists an anticanonical transformations allowing to connect the canonical relations between fields and antifields (4.1) in the BV-formalism and corresponding relations in Morris’s approach (4.2). We have found transformations of variables of antisymplectic space leading to the invariance of the regularized antibracket. These transformations have been called as the reduced anticanonical transformations.

Notice that the cornerstone of the new quantization approach is construction of regularized action for an initial gauge theory. It is not trivial problem because it is required for the regularized action to be gauge invariant. In this point there is essential difference between the standard FRG approach [17, 18] and the new ERG method [26, 24, 25]. Regularization of kinematic part of full classical action accepted in the FRG violates the gauge invariance that causes the gauge dependence problem of effective average action even on-shell [21, 22]. The regularization of initial gauge invariant action in the ERG method needs in special efforts [26, 24, 25]. We want to illustrate problems arising in this way using pure Yang-Mills theory with action

\[ S_0[A] = -\frac{1}{4} \int dx F^a_{\mu\nu}(x) F^a_{\mu\nu}(x) = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}, \]  

\[ F^a_{\mu\nu}(x) = \partial_\mu A^a_\nu(x) - \partial_\nu A^a_\mu(x) + f^{abc} A^b_\mu(x) A^c_\nu(x), \]

where \( f^{abc} \) are structure coefficients of the \( SU(N) \) Lie group satisfying the Jacobi identity,

\[ f^{abc} f^{cde} + f^{ace} f^{bde} + f^{ade} f^{ceb} \equiv 0. \]  

Let us try to introduce the regularization in the form preserving the geometric description in terms of field strength \( F^a_{\mu\nu} \). It can be done for example in the form,

\[ S_0[A] = \left[ -\frac{1}{4} \int dx F^a_{\mu\nu}(x) K^{-1}_A(x) F^a_{\mu\nu}(x) = -\frac{1}{4} F^a_{\mu\nu} K^{-1}_A F^a_{\mu\nu}. \]  

The action (7.4) is invariant under the following gauge transformation

\[ \delta_{\xi A} F^a_{\mu\nu} = f^{abc} K_A F^b_{\mu\nu} \xi^c, \quad \delta_{\xi A} S_0[A] = 0. \]

From (7.2) it follows the presentation of this variation in terms of field variations

\[ \delta_{\xi A} F^a_{\mu\nu} = \partial_\mu \delta_{\xi A} A^a_\nu - \partial_\nu \delta_{\xi A} A^a_\mu + f^{abc} [\delta_{\xi A} A^b_\mu A^c_\nu + A^b_\mu \delta_{\xi A} A^c_\nu]. \]
In the Yang-Mills theory ($K_\Lambda = 1$) the gauge transformations of the field strength $F^a_{\mu\nu}$, $\delta_\xi F^a_{\mu\nu} = f^{abc} F^b_{\mu\nu} \xi^c$, can be rewritten in terms of gauge transformations of fields $A^a_\mu$, $\delta_\xi A^a_\mu = D^{ab}_\mu(A) \xi^b$. Let us try to present the variation (7.5) in the form (7.6). The result reads

$$\delta_\xi \Lambda F^a_{\mu\nu} = \partial_\mu (K_\Lambda D^a_\nu(A) \xi^b) - \partial_\nu (K_\Lambda D^a_\mu(A) \xi^b) + f^{abc} [K_\Lambda, A^b_\mu] D^c_\nu(A) \xi^d + f^{abc} [K_\Lambda, A^b_\nu] D^d_\mu(A) \xi^c. \quad (7.7)$$

For Abelian Lie group $f^{abc} = 0$ one can formulate the gauge invariance of the regularized initial action in terms of gauge transformations of fields $A_\mu$ as $\delta_\xi A_\mu = K_\Lambda \partial_\mu \xi$ and after that to apply the BV-formalism to construct suitable quantum description of renormalization group respecting BRST symmetry. In particular the BRST transformations in the sector of fields $A_\mu$ are described by the relations $\delta_{BA} A_\mu = K_\Lambda \partial_\mu C$.

In general from (7.7) it follows that gauge invariance of the action (7.3) cannot be expressed in terms of gauge transformations of fields $A^a_\mu$. In particular the gauge transformations of fields $A^a_\mu$,

$$\delta_\xi A^a_\mu = K_\Lambda D^a_\nu(A) \xi^b, \quad (7.8)$$
do not present symmetry transformations of the regularized action (7.3). It means that the regularized version of initial classical action proposed here does not give a possibility to apply the BV-formalism. From point of view of the new ERG method this feature forbids to use the regularization procedure proposed here.

We have proved that the ERG method can be provided with reduced anticanonical transformations preserving the regularized antibracket and with global supersymmetry (regularized BRST symmetry) if assumptions listed in Secs. 5, 6 are fulfilled. Main of them are the regularized initial gauge-invariant action (5.1), the existence of solutions to quantum master equation (5.3) with boundary condition (5.4) and the gauge fixing procedure proposed (5.7).

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