THE DECAY WIDTHS OF THE \( Z_{cs}(3985/4000) \) BASED ON RIGOROUS QUARK-HADRON DUALITY

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Abstract

In this work, we explore the hadronic coupling constants \( G_{ZJ/\psi K}, G_{Zc_{cs}K^*}, G_{Zc_{cs}D_s} \) of the exotic states \( Z_{cs}(3985/4000) \) both in the pictures of the tetraquark states and molecular states with the tentative assignments \( J^{PC} = 1^{+} \) based on the rigorous quark-hadron duality. Then we obtain the total widths \( \Gamma_{Z_{cs}}^T = 15.31 \pm 0.06 \) MeV and \( \Gamma_{Z_{cs}}^T = 83.51 \pm 21.09 \) MeV, which are consistent with the experimental data 13.8\(^{+8.1}_{-5.2}\) ± 4.9 MeV from the BESIII collaboration and 131 ± 15 ± 26 MeV from the LHCb collaboration, respectively, and support assigning the \( Z_{cs}(3985) \) and \( Z_{cs}(4000) \) to be the hidden-charm tetraquark state and molecular state with the \( J^{PC} = 1^{+} \), respectively.

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Key words: Tetraquark state, QCD sum rules

1 Introduction

In 2013, the BESIII and Belle collaborations explored the process \( e^+e^- \rightarrow \pi^+\pi^- J/\psi \), and observed a structure \( Z_c^+(3900) \) in the \( \pi^\pm J/\psi \) mass spectrum \cite{1,2}. Also in 2013, the BESIII collaboration investigated the process \( e^+e^- \rightarrow \pi D\bar{D}^* \), and observed a structure \( Z_{cs}^+(3885) \) in the \( (D\bar{D}^*)^\pm \) mass spectrum \cite{3}. The \( Z_c(3900/3885) \) have the spin-parity \( J^P = 1^+ \) \cite{3,4}.

In 2020, the BESIII collaboration observed a structure \( Z_{cs}^- \) in the \( K^+ \) recoil-mass spectrum in the processes of the \( e^+e^- \rightarrow K^+(D_-^+ D^{0^+} + D^{*+} D^0) \) \cite{5}. The Breit-Wigner mass and width are 3985.2\(^{+2.1}_{-2.0}\) ± 1.7 MeV and 13.8\(^{+8.1}_{-5.2}\) ± 4.9 MeV, respectively \cite{5}.

In 2021, the LHCb collaboration observed two new exotic states \( Z_{cs}^+(4000) \) and \( Z_{cs}^+(4220) \) in the \( J/\psi K^+ \) mass spectrum in the process \( B^+ \rightarrow J/\psi \phi K^+ \) \cite{6}. The most significant state, \( Z_{cs}^+(4000) \), has the Breit-Wigner mass and width 4003 ± 6\(^{+4}_{-3}\) MeV and 131 ± 15 ± 26 MeV, respectively, and the spin-parity \( J^P = 1^+ \) \cite{6}.

The \( Z_c(3900/3885) \) and \( Z_{cs}(3985/4000) \) have analogous decay modes,

\[
\begin{align*}
Z_c^+ (3900) & \rightarrow J/\psi \pi^\pm, \\
Z_{cs}^+ (4000) & \rightarrow J/\psi K^+, \\
Z_{cs}^+ (3885) & \rightarrow (D\bar{D}^*)^\pm, \\
Z_{cs}^- (3985) & \rightarrow D_s^- D^{0^+}, D^{*+} D^0,
\end{align*}
\]

which lead to the possible outcome that they should have analogous quark structures. Although the Particle Data Group takes it for granted that the \( Z_c(3900) \) and \( Z_{cs}(3885) \) are the same particle according to the analogous masses and widths \cite{7}; however, it is difficult to explain the ratio \cite{3},

\[
R_{exp} = \frac{\Gamma(Z_c(3885) \rightarrow D\bar{D}^*)}{\Gamma(Z_c(3900) \rightarrow J/\psi \pi)} = 6.2 \pm 1.1 \pm 2.7
\]

in a satisfactory way. On the other hand, the widths of the \( Z_{cs}^-(3985) \) and \( Z_{cs}^+(4000) \) are inconsistent with each other. The \( Z_c(3900/3885) \) and \( Z_{cs}(3985/4000) \) may be four distinguished particles indeed.

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The exotic states $Z_{cs}(3985/4000)$ lie above the $D^-_s D^{*0}$ and $D^{*0} D^0$ thresholds 3975.2 MeV and 3977.0 MeV, respectively, if they are molecular states indeed, we should introduce the coupled-channel effects to account for the mass gaps and decay widths [5, 6, 7, 11, 12], or just re-scattering effects [13, 14]. It is more natural to reproduce their masses in the diquark-antidiquark type tetraquark pictures than in the color-singlet-color-singlet type tetraquark pictures, if we choose the theoretical framework of potential quark models [15, 16].

In the QCD sum rules, we usually choose the color antitriplet-triplet type and singlet-singlet type tetraquark pictures than in the color-singlet-color-singlet type tetraquark pictures, if we choose the theoretical framework of potential quark models [15, 16].

In Ref. [26], we assign the $Z_{cs}^+ (3900)$ to be the diquark-antidiquark type tetraquark state with the quantum numbers $J^{PC} = 1^{-+}$, calculate the hadronic coupling constants $G_{Z_{cs}/\psi K}$, $G_{Z_{cs}/\eta K}$, $G_{Z_{cs}/D}$, with the three-point QCD sum rules based on the rigorous duality for the first time, then obtain the partial decay widths to diagnose the nature of the $Z_{cs}^+ (3900)$. Thereafter, the rigorous duality has been successfully applied to study the strong decays of the exotic states $X(4140)$, $X(4660)$, $Z_c(4000)$ and $P_c(4312)$ [27, 28, 29, 30, 31, 32].

In this work, we extend our previous work on the masses of the hidden-charm tetraquark states $Z_{cs}(3985/4000)$ [17, 18, 19, 20, 21] and molecular states $[17, 21, 22, 23, 24, 25]$. There may exist two $Z_{cs}$ states, an antitriplet-triplet type tetraquark state and singlet-singlet type molecular state, or only one $Z_{cs}$ state, which has both the antitriplet-triplet type and singlet-singlet type components. We have to explore the two-body strong decays to diagnose their nature.

In Ref. [26], we assign the $Z_{cs}^+ (3900)$ to be the diquark-antidiquark type tetraquark state with the quantum numbers $J^{PC} = 1^{-+}$, calculate the hadronic coupling constants $G_{Z_{cs}/\psi K}$, $G_{Z_{cs}/\eta K}$, $G_{Z_{cs}/D}$, with the three-point QCD sum rules based on the rigorous duality for the first time, then obtain the partial decay widths to diagnose the nature of the $Z_{cs}^+ (3900)$. Thereafter, the rigorous duality has been successfully applied to study the strong decays of the exotic states $X(4140)$, $X(4660)$, $Z_c(4000)$ and $P_c(4312)$ [27, 28, 29, 30, 31, 32].

In this work, we extend our previous work on the masses of the hidden-charm tetraquark states and molecular states with the strangeness [19, 25], to explore the two-body strong decays of the $Z_{cs}(3985/4000)$ with the possible assignments $J^{PC} = 1^{-+}$, so as to diagnose their nature.

The article is arranged as follows: we acquire the QCD sum rules for the hadronic coupling constants $G_{Z_{cs}/\psi K}$, $G_{Z_{cs}/\eta K}$, $G_{Z_{cs}/D}$, in section 2; in Sect.3, we present the numerical results and discussions; and Sect.4 is reserved for our conclusion.

## 2 The hadronic coupling constants of the $Z_{cs}(3985/4000)$

We investigate the two-body strong decays $Z_{cs}^+ (3985/4000) \rightarrow J/\psi K^-, \eta K^+, D^{*0} D^-_s, D^0 D^-_s$ using the three-point correlation functions $\Pi_{\mu\nu}^1 (p, q)$, $\Pi_{\mu\nu}^2 (p, q)$ and $\Pi_{\mu\nu}^3 (p, q)$, respectively,

\[
\begin{align*}
\Pi_{\mu\nu}^1 (p, q) &= i^2 \int d^4 x d^4 y e^{i p x} e^{i q y} \langle 0 | T \left\{ J_{\mu J/\psi} (x) J_{\mu K^-} (y) J_{\mu 0} (0) \right\} | 0 \rangle, \\
\Pi_{\mu\nu}^2 (p, q) &= i^2 \int d^4 x d^4 y e^{i p x} e^{i q y} \langle 0 | T \left\{ J_{\mu \eta} (x) J_{\mu K^+} (y) J_{\mu 0} (0) \right\} | 0 \rangle, \\
\Pi_{\mu\nu}^3 (p, q) &= i^2 \int d^4 x d^4 y e^{i p x} e^{i q y} \langle 0 | T \left\{ J_{\mu D^*} (x) J_{\mu D^-} (y) J_{\mu 0} (0) \right\} | 0 \rangle,
\end{align*}
\]

where the currents

\[
\begin{align*}
J_{\mu J/\psi} (x) &= \bar{c}(x) \gamma_\mu c(x), \\
J_{\mu K^+} (y) &= \bar{u}(y) \gamma_5 s(y), \\
J_{\mu \eta} (x) &= \bar{c}(x) i \gamma_5 c(x), \\
J_{\mu K^-} (y) &= \bar{u}(y) \gamma_\mu s(y), \\
J_{\mu D^*} (x) &= \bar{u}(x) \gamma_\mu c(x), \\
J_{\mu D^-} (y) &= \bar{c}(y) i \gamma_5 s(y), \\
J_{\mu 0} (0) &= J_{\mu 0}^T (0), J_{\mu 0}^{M1} (0),
\end{align*}
\]
interpolate the mesons $J/\psi, K, \eta_c, K^*, D^*, D_s, Z_{cs}^T$ and $Z_{cs}^M$, respectively, the superscripts $T$ and $M$ denote the tetraquark-type and molecule-type four-quark currents/tetraquarks, respectively [19] [25].

We insert a complete set of intermediate hadronic states with the same quantum numbers as the currents into the three-point correlation functions, and isolate the ground state contributions,

\[
J_{\nu}^{\text{T}1}(0) = \frac{\varepsilon_{ijk}i\mathbf{mn}}{\sqrt{2}} \left\{ c^{T_i(0)}C \gamma_\nu u^{m(0)}c^{T_j(0)}C \gamma_\nu u^{n(0)} - c^{T_i(0)}C \gamma_\nu u^{n(0)}c^{T_j(0)}C \gamma_\nu u^{m(0)} \right\},
\]
\[
J_{\nu}^{\text{M}1}(0) = \frac{1}{\sqrt{2}} \left\{ \bar{s}(0)\gamma_\nu c(0)\bar{c}(0)i\gamma_\nu u(0) + \bar{s}(0)i\gamma_\nu c(0)\bar{c}(0)\gamma_\nu u(0) \right\},
\]

(6)

interpolate the mesons $J/\psi, K, \eta_c, K^*, D^*, D_s, Z_{cs}^T$ and $Z_{cs}^M$, respectively, the superscripts $T$ and $M$ denote the tetraquark-type and molecule-type four-quark currents/tetraquarks, respectively [19] [25].

We insert a complete set of intermediate hadronic states with the same quantum numbers as the currents into the three-point correlation functions, and isolate the ground state contributions,

\[
\Pi_{\mu\nu}^{\text{T}1}(p, q) = \left\{ \frac{2}{m_u + m_s} \left( \begin{array}{c} f_K M_K f_{J/\psi} M_{J/\psi} \lambda_{Z_{J/\psi}} G_T^{Z_{J/\psi}} \lambda_{Z_{J/\psi}} G_T^{Z_{J/\psi}} \\ M_{J/\psi} - p^2 - (M_{J/\psi} - p^2)(M_{J/\psi} - q^2) \end{array} \right) \right\} g_{\mu\nu} + \cdots
\]

(7)

\[
\Pi_{\mu\nu}^{\text{M}1}(p, q) = \left\{ \frac{2}{m_u + m_s} \left( \begin{array}{c} f_K M_K f_{J/\psi} M_{J/\psi} \lambda_{Z_{J/\psi}} G_T^{Z_{J/\psi}} \lambda_{Z_{J/\psi}} G_T^{Z_{J/\psi}} \\ M_{J/\psi} - p^2 - (M_{J/\psi} - p^2)(M_{J/\psi} - q^2) \end{array} \right) \right\} g_{\mu\nu} + \cdots
\]

(8)

\[
\Pi_{\mu\nu}^{\text{T}2}(p, q) = \left\{ \frac{2}{2m_c} \left( \begin{array}{c} f_K M_K f_K M_K \lambda_{Z_{J/\psi}} G_T^{Z_{J/\psi}} \lambda_{Z_{J/\psi}} G_T^{Z_{J/\psi}} \\ M_{K^*} - p^2 - (M_{K^*} - p^2)(M_{K^*} - q^2) \end{array} \right) \right\} g_{\mu\nu} + \cdots
\]

(9)

\[
\Pi_{\mu\nu}^{\text{M}2}(p, q) = \left\{ \frac{2}{2m_c} \left( \begin{array}{c} f_K M_K f_K M_K \lambda_{Z_{J/\psi}} G_T^{Z_{J/\psi}} \lambda_{Z_{J/\psi}} G_T^{Z_{J/\psi}} \\ M_{K^*} - p^2 - (M_{K^*} - p^2)(M_{K^*} - q^2) \end{array} \right) \right\} g_{\mu\nu} + \cdots
\]

(10)

\[
\Pi_{\mu\nu}^{\text{T}3}(p, q) = \left\{ \frac{2}{2m_c} \left( \begin{array}{c} f_K M_K f_K M_K \lambda_{Z_{J/\psi}} G_T^{Z_{J/\psi}} \lambda_{Z_{J/\psi}} G_T^{Z_{J/\psi}} \\ M_{K^*} - p^2 - (M_{K^*} - p^2)(M_{K^*} - q^2) \end{array} \right) \right\} g_{\mu\nu} + \cdots
\]

(11)

\[
\Pi_{\mu\nu}^{\text{M}3}(p, q) = \left\{ \frac{2}{2m_c} \left( \begin{array}{c} f_K M_K f_K M_K \lambda_{Z_{J/\psi}} G_T^{Z_{J/\psi}} \lambda_{Z_{J/\psi}} G_T^{Z_{J/\psi}} \\ M_{K^*} - p^2 - (M_{K^*} - p^2)(M_{K^*} - q^2) \end{array} \right) \right\} g_{\mu\nu} + \cdots
\]

(12)
where \( p' = p + q \), the \( f_{J/\psi}, f_K, f_{\eta_c}, f_{K^*}, f_{D^*}, f_{D_s}, \lambda_Z^2 \) and \( \lambda_Z^M \) are the decay constants of the mesons \( J/\psi, K, \eta_c, K^*, D^*, D_s, Z_{cs}^T \) and \( Z_{cs}^M \), respectively,

\[
\begin{align*}
\langle 0 | J_{\mu}^{J/\psi}(0) | J/\psi(p) \rangle &= f_{J/\psi} M_{J/\psi} \xi_\mu, \\
\langle 0 | J_{\mu}^{K^*}(0) | K^*(q) \rangle &= f_{K^*} M_{K^*} \xi_\mu, \\
\langle 0 | J_{\mu}^{D^*}(0) | D^*(p) \rangle &= f_{D^*} M_{D^*} \xi_\mu, \\
\langle 0 | J_{\mu}^{K}(0) | K(q) \rangle &= \frac{f_K M_K^2}{m_u + m_s}, \\
\langle 0 | J_{\mu}^{D^*}(0) | D_s(q) \rangle &= \frac{f_{D_s} M_{D_s}^2}{m_c + m_s}, \\
\langle 0 | J_{\nu}^{J/\psi}(0) | Z_{cs}^M (p') \rangle &= \lambda_Z^{T/M} \xi_\nu,
\end{align*}
\]

and the \( \xi, \varepsilon, \zeta \) and \( \zeta \) are polarization vectors of the \( J/\psi, K^*, D^* \) and \( Z_{cs}^{T/M} \), respectively, the hadronic coupling constants \( G_{ZJ/\psi K}, G_{ZJ/\psi K^*}, G_{Z\eta_c K^*}, G_{Z\eta_c K}, G_{ZD^* D_s}, G_{ZD^* D_s} \), are defined by

\[
\begin{align*}
\langle J/\psi(p) K(q) | Z_{cs}^T(p') \rangle &= -\xi^*(p) \cdot \zeta(p') G_{ZJ/\psi K}^T, \\
\langle \eta_c(p) K^*(q) | Z_{cs}^T(p') \rangle &= -\varepsilon^*(q) \cdot \zeta(p') G_{Z\eta_c K^*}^T, \\
\langle D^*(p) D_s(q) | Z_{cs}^T(p') \rangle &= -\zeta^*(p) \cdot \zeta(p') G_{ZD^* D_s}^T, \\
\langle J/\psi(p) K(q) | Z_{cs}^M(p') \rangle &= i\xi^*(p) \cdot \zeta(p') G_{ZJ/\psi K}^M, \\
\langle \eta_c(p) K^*(q) | Z_{cs}^M(p') \rangle &= i\varepsilon^*(q) \cdot \zeta(p') G_{Z\eta_c K^*}^M, \\
\langle D^*(p) D_s(q) | Z_{cs}^M(p') \rangle &= i\zeta^*(p) \cdot \zeta(p') G_{ZD^* D_s}^M.
\end{align*}
\]

The hadronic spectral densities \( \rho_{Z_{JS}}(p^2, p^2, u) \), \( \rho_{Z_{LS}}(p^2, s, q^2) \), \( \rho_{Z_{JS}}(s', p^2, q^2) \), \( \rho_{Z_{S}}(s', p^2, q^2) \) in the component \( \Pi_{L}^2(p^2, p^2, q^2) \) and \( \Pi_{L}^2(p^2, s, q^2) \) stand for the transitions between the ground states and continuum states (including the first radial excited states). In fact, such hadronic spectral densities also exist in the big \( \{ \} \) in the components \( \Pi_{L}^2(p^2, p^2, q^2), \Pi_{L}^1(p^2, p^2, q^2) \), \( \Pi_{L}^3(p^2, p^2, q^2) \), \( \Pi_{L}^4(p^2, p^2, q^2) \), \( \Pi_{L}^5(p^2, p^2, q^2) \), we neglect them for simplicity. In this work, we choose the tensor structure \( g_{\mu\nu} \) to explore the hadronic coupling constants \( G_{ZJ/\psi K}^T, G_{Z\eta_c K^*}^T, G_{Z\eta_c K}^T, G_{ZD^* D_s}^T, G_{ZD^* D_s}^M \), and neglect other tensor structures for simplicity.

We accomplish the operator product expansion up to the vacuum condensates of dimension 5 and neglect the tiny gluon condensate contributions, just like in our previous works\[25, 27, 28, 29, 30]\], then obtain the QCD spectral densities \( \rho_{QCD}(p^2, s, u) \) through double dispersion relation, and write the correlation functions at the QCD side in the form,

\[
\Pi_{QCD}(p^2, p^2, q^2) = \int_{\Delta_2^2}^\infty ds \int_{\Delta_2^2}^\infty du \rho_{QCD}(p^2, s, u) \frac{\rho_{QCD}(p^2, s, u)}{(s - p^2)(u - q^2)},
\]

where the \( \Delta_2^2 \) and \( \Delta_2^2 \) are the thresholds. While at the hadron side, we obtain the hadron spectral densities \( \rho_H(s', s, u) \) through triple dispersion relation, and write the correlation functions in the form,

\[
\Pi_H(p^2, p^2, q^2) = \int_{\Delta_2^2}^\infty ds' \int_{\Delta_2^2}^\infty ds \int_{\Delta_2^2}^\infty du \frac{\rho_{QCD}(s', s, u)}{(s - p^2)(s - p^2)(u - q^2)},
\]

according to Eqs.\[11-12\], where the \( \Delta_2^2 \) are the thresholds. We match the hadron side with the QCD side of the correlation functions below the continuum thresholds according to the dispersion.
relation at the QCD side to warrant rigorous quark-hadron duality,

$$\int_{\Delta s_0}^{s_0} ds \int_{\Delta u_0}^{u_0} du \frac{\rho_{\text{QCD}}(p'^2, s, u)}{(s - p'^2)(u - q'^2)} = \int_{\Delta s_2}^{s_0} ds \int_{\Delta u_2}^{u_0} du \left[ \int_{\Delta s}^{s_0} ds' \frac{\rho_H(s', s, u)}{(s' - p'^2)(u - q'^2)} \right]$$

(17)

where the $s_0$ and $u_0$ are the continuum thresholds, we accomplish the integral over $ds'$ firstly, and introduce some unknown parameters to parameterize the contributions involving the higher resonances and continuum states in the $s'$ channel [26, 27]. we take account of the higher resonances and continuum states rather than neglecting them. For example, according to Eq.(7), we write down the correlation function at the hadron side explicitly,

$$\int_{\Delta s_2}^{s_0} ds \int_{\Delta u_2}^{u_0} du \frac{\rho_{\text{QCD}}(p'^2, s, u)}{(s - p'^2)(u - q'^2)} = \frac{A_{ZJ/\psi K}}{(M_Z^2 - p'^2)(M_{J/\psi}^2 - p'^2)(M_K^2 - q'^2)} + \frac{C_{J/\psi K}^{T^*}}{(M_{J/\psi}^2 - p'^2)(M_K^2 - q'^2)}$$

(18)

where

$$A_{ZJ/\psi K} = \frac{f_K M_K^2 f_{J/\psi} M_{J/\psi}^2 \lambda_{2Z} G_{ZJ/\psi K}^{T^*}}{m_u + m_s},$$

$$C_{J/\psi K}^{T^*} = \int_{s_0^2}^{s_0^2} ds' \frac{\rho_{ZJ/\psi K}(s', p'^2, q'^2)'}{s' - p'^2}$$

(19)

where the unknown parameter $C_{J/\psi K}^{T^*}$ parameterizes the contributions involving the higher resonances and continuum states with the same quantum numbers as the $Z_{cs}$ (in the $s'$ channel), the unknown parameters $C_{J/\psi K}^M$, $C_{n, K}^T$, $C_{n, K}^T$, $C_{D, D_s}^T$, and $C_{D, D_s}^M$ in Eqs.(21)-(25) are implied in the same way.

We accomplish the integral over the variable $ds'$ firstly, and do not need the continuum threshold parameters $s_0'$, and the duality is rigorous in the sense that we do not set $s_0 = s_0' = s_0^2$ approximately. If we set $s_0 = s_0^0 = s_0^0$ in the present case, then $\sqrt{s_0^0} > M_Z > \psi' > \eta' > M_{D_s} > M_{D_s}$, the contaminations from the excited states are out of control. In fact, without accomplishing the integral over the variable $ds'$ firstly, the representations at the QCD side and at the hadron side cannot match with each other, up to now, no one can approve that they match with each other without merging the $s$ and $s'$ channels $s_0 = s_0^0 = s_0^0$ at the hadron side by hand, to be serious, such approximations $s_0 = s_0^0$ are wrong, as the $s$ and $s'$ channels are quite different.

In the $s$ and $u$ channels, we deal with the conventional mesons, and choose the standard vector and pseudoscalar currents to interpolate them, direct calculations indicate that we can reproduce their masses satisfactorily with the two-point QCD sum rules below the continuum thresholds $s_0$ and $u_0$, respectively, the quark-hadron duality is reliable.

We set $p'^2 = p^2$ and $p'^2 = 4p^2$ in the correlation functions $\Pi_{T/M}^{1/2}(p'^2, p^2, q'^2)$ and $\Pi_{T/M}^3(p'^2, p^2, q'^2)$, respectively, and accomplish the double Borel transform in regard to the variables $P'^2 = -p'^2$ and
\( Q^2 = -q^2 \) respectively, then set the Borel parameters \( T_2^q = T_2^0 = T^2 \) to get the six QCD sum rules,

\[
\frac{f_K M_K^2 f_{J/\psi} M_{J/\psi} \lambda_{Z}^T G_{Z,J/\psi,K}}{m_u + m_s} \left[ \frac{1}{M_Z^2 - M_{J/\psi}^2} \right] \exp \left( -\frac{M_{J/\psi}^2}{T^2} \right) - \exp \left( -\frac{M_{J/\psi}^2}{T^2} \right) \exp \left( -\frac{M_{K}^2}{T^2} \right) + C_{J/\psi,K}^T \exp \left( -\frac{M_{J/\psi}^2 + M_{K}^2}{T^2} \right) = \frac{1}{64\sqrt{2}\pi^4} \int_{4m_c^2}^{u_K^0} ds \int_0^{u_K^0} du \sqrt{1 - \frac{4m_c^2}{s}} u (2s + 4m_c^2 - 3m_s) \exp \left( -\frac{s}{T^2} \right)
\]

\[
\exp \left( -\frac{s + u}{T^2} \right) - m_s \left[ \frac{[\bar{q}q] + \langle ss \rangle}{24\sqrt{2}\pi^2} \right] \int_{4m_c^2}^{u_K^0} ds \sqrt{1 - \frac{4m_c^2}{s}} (s + 2m_c^2) \exp \left( -\frac{s}{T^2} \right) \exp \left( -\frac{M_{J/\psi}^2}{T^2} \right) - \exp \left( -\frac{M_{J/\psi}^2}{T^2} \right) \exp \left( -\frac{M_{K}^2}{T^2} \right) + C_{J/\psi,K}^M \exp \left( -\frac{M_{J/\psi}^2 + M_{K}^2}{T^2} \right) = \frac{1}{128\sqrt{2}\pi^4} \int_{4m_c^2}^{u_K^0} ds \int_0^{u_K^0} du \sqrt{1 - \frac{4m_c^2}{s}} u (2s + 4m_c^2 - 3m_s) \exp \left( -\frac{s}{T^2} \right)
\]

\[
\exp \left( -\frac{s + u}{T^2} \right) - m_s \left[ \frac{[\bar{q}q] + \langle ss \rangle}{48\sqrt{2}\pi^2} \right] \int_{4m_c^2}^{u_K^0} ds \sqrt{1 - \frac{4m_c^2}{s}} (s + 2m_c^2) \exp \left( -\frac{s}{T^2} \right) \exp \left( -\frac{M_{J/\psi}^2}{T^2} \right) - \exp \left( -\frac{M_{J/\psi}^2}{T^2} \right) \exp \left( -\frac{M_{K}^2}{T^2} \right) + C_{J/\psi,K}^M \exp \left( -\frac{M_{J/\psi}^2 + M_{K}^2}{T^2} \right) = \frac{1}{96\sqrt{2}\pi^2 T^2} \int_{4m_c^2}^{u_K^0} ds \sqrt{1 - \frac{4m_c^2}{s}} s \exp \left( -\frac{s}{T^2} \right) + \frac{m_s (\bar{q}q) G_s}{8\sqrt{2}\pi^2} \int_{4m_c^2}^{u_K^0} ds \sqrt{1 - \frac{4m_c^2}{s}} s \exp \left( -\frac{s}{T^2} \right)
\]

\[
\exp \left( -\frac{s + u}{T^2} \right) + m_s \left[ \frac{[\bar{q}q] + \langle ss \rangle}{72\sqrt{2}\pi^2} \right] \int_{4m_c^2}^{u_K^0} ds \sqrt{1 - \frac{4m_c^2}{s}} s \exp \left( -\frac{s}{T^2} \right) - m_s \left[ \frac{[\bar{q}q] + \langle ss \rangle}{24\sqrt{2}\pi^2} \right] \int_{4m_c^2}^{u_K^0} ds \sqrt{1 - \frac{4m_c^2}{s}} s \exp \left( -\frac{s}{T^2} \right),
\]
\[
\frac{f_{\epsilon M_5^2 f_{K^*} M_{K^*} \lambda_z^M G_2^M \epsilon_{K^*}}}{2m_c} \left[ \frac{1}{M_Z^2 - M_{\tilde{Z}}^2} \right] \left[ \exp\left( -\frac{M_{\tilde{Z}}^2}{T^2} \right) - \exp\left( -\frac{M_{\tilde{Z}}^2}{T^2} \right) \right] \exp\left( -\frac{M_{K^*}}{T^2} \right) \\
+ C_{\epsilon_{K^*}}^M \exp\left( -\frac{M_{\tilde{Z}}^2 + M_{\tilde{K}^*}^2}{T^2} \right) = \frac{1}{128\sqrt{2\pi^4}} \int_0^{s_{m_c}} ds \int_0^u d\epsilon \left[ 1 - \frac{4m_c^2}{s} u (2s - 3m_c m_c) \right] \exp\left( -\frac{s_{m_c}}{T^2} \right) \\
\exp\left( -\frac{s + u}{T^2} \right) - \frac{m_s\langle \tilde{q}g_s\sigma G s \rangle}{192\sqrt{2\pi^2} T^2} \int_0^{s_{m_c}} ds \sqrt{1 - \frac{4m_c^2}{s}} s \exp\left( -\frac{s}{T^2} \right) \\
- \frac{m_s\langle \tilde{q}g_s\sigma G q \rangle}{72\sqrt{2\pi^2}} \int_0^{s_{m_c}} ds \frac{1 + 2m_c^2}{\sqrt{s} (s - 4m_c^2)} \exp\left( -\frac{s}{T^2} \right) \\
+ m_c \left[ \langle \tilde{q}g_s\sigma G q \rangle + \langle \tilde{q}g_s\sigma G s \rangle \right] \int_0^{s_{m_c}} ds \sqrt{1 - \frac{4m_c^2}{s}} s \exp\left( -\frac{s}{T^2} \right),
\] (23)

\[
\frac{f_{D^*} M_5^2 f_{D^*} M_{D^*} \lambda_z^M G_{2D^*} \epsilon_{D^*}}{4(m_c + m_s)} \left[ \frac{1}{M_Z^2 - M_{\tilde{Z}}^2} \right] \left[ \exp\left( -\frac{M_{\tilde{Z}}^2}{T^2} \right) - \exp\left( -\frac{M_{\tilde{Z}}^2}{T^2} \right) \right] \exp\left( -\frac{M_{D^*}^2}{T^2} \right) \\
+ C_{\epsilon_{D^*}}^T \exp\left( -\frac{M_{D^*}^2 + M_{D^*}^2}{T^2} \right) = -\frac{m_s\langle \tilde{q}g_s\sigma G s \rangle}{96\sqrt{2\pi^2}} \int_0^{s_{m_c}} ds \left( \frac{9}{2} - \frac{10m_c^2}{s} + \frac{3m_s^4}{2s^2} \right) \exp\left( -\frac{s + m_c^2}{T^2} \right) \\
+ \frac{m_s\langle \tilde{q}g_s\sigma G q \rangle}{96\sqrt{2\pi^2}} \int_0^{s_{m_c}} ds \left( \frac{3}{2} - \frac{4m_c^2}{u} - \frac{3m_s^4}{2u^2} \right) \exp\left( -\frac{u + m_c^2}{T^2} \right) \\
+ \frac{m_c\langle \tilde{q}g_s\sigma G q \rangle}{96\sqrt{2\pi^2}} \int_0^{s_{m_c}} ds \left( \frac{1}{u - m_c^2} \right) \left( u - 9m_c^2 - \frac{5m_s^4}{u} - \frac{3m_s^6}{u^2} \right) \exp\left( -\frac{u + m_c^2}{T^2} \right),
\] (24)
There exist end-divergences at the endpoints $s = 4m_c^2$ and $u = m_c^2$; we regularize the endpoint divergences with the simple replacements $\frac{1}{\sqrt{s-4m_c^2}} \to \frac{1}{\sqrt{s-4m_c^2+4m^2}}$ and $\frac{1}{u-m_c^2} \to \frac{1}{u-m_c^2+4m^2}$ by adding a small squared s-quark mass $4m_s^2$. Furthermore, just like in previous works \cite{26, 27, 28, 29, 30}, empirical, we take a short digression to illustrate the end-point divergences, we often encounter the typical integral, 

$$I_{21} = \int d^4k_1 \frac{1}{(k_1^2 - m_c^2)^2 ((p-k_1)^2 - m_c^2)},$$

and calculate it by using the Cutkosky's rules,

$$I_{21} = \frac{\partial}{\partial A} \int d^4k_1 \frac{1}{k_1^2 - A (p-k_1)^2 - m_c^2} \big|_{A \to m_c^2}$$

$$= \frac{\partial}{\partial A} \frac{(-2\pi i)^2}{2\pi i} \int_{(\sqrt{s+m_c^2})^2}^{\infty} \frac{dt}{t-p^2} \int d^4k_1k^4k_2\delta^4(k_1+k_2-p)\delta(k_1^2 - A)\delta(k_2^2 - m_c^2)$$

$$= \frac{\partial}{\partial A} \frac{(-2\pi i)^2}{2\pi i} \int_{(\sqrt{s+m_c^2})^2}^{\infty} \frac{dt}{t-p^2} \frac{\pi \sqrt{\lambda(t, A, m_c^2)}}{t} \big|_{A \to m_c^2}$$

$$= \frac{(-2\pi i)^2}{2\pi i} \int_{4m_c^2}^{\infty} \frac{dt}{t-p^2} \frac{\pi}{2 \sqrt{t(t-4m_c^2)}},$$

divergence at the end-point $t = 4m_c^2$ appears. Such terms are composed with the small s-quark mass $m_s$ and play a minor important role, we regularize the endpoint divergences by adding a small squared s-quark mass $4m_s^2$ empirically, just like in previous works \cite{26, 27, 28, 29, 30}. If we
vary the $4m_c^2$ by adding a small uncertainty $4m_c^2 \pm \delta$, and we can obtain almost invariant numerical results.

Furthermore, we smear the dependencies of the parameters $C_{J/\psi K}^T$, $C_{J/\psi K}^M$, $C_{\eta, K^*}^T$, $C_{\eta, K^*}^M$, $C_{D^*}^{T,D}$, and $C_{D^*}^{M,D}$ on the Lorentz invariants $p^2$, $p'^2$, $q^2$, and take them as free parameters, and search for the best values to delete the contaminations from the high resonances and continuum states to acquire stable QCD sum rules with variations of the Borel parameters $T^2$.

3 Numerical results and discussions

At the QCD side, we take the standard values $(\bar{q}q) = -(0.24 \pm 0.01 \text{ GeV})^3$, $(\bar{s}s) = (0.8 \pm 0.1)(\bar{q}q)$, $(\bar{q}g, \sigma Gq) = m_0^2(\bar{q}q)$, $(\bar{s}g, \sigma Gs) = m_0^2(\bar{s}s)$, $m_0^2 = (0.8 \pm 0.1) \text{ GeV}^2$ at the energy scale $\mu = 1 \text{ GeV}$ [43, 44, 45], and take the $MS$ quark masses $m_c(m_c) = (1.275 \pm 0.025) \text{ GeV}$ and $m_s(\mu) = (0.995 \pm 0.005) \text{ GeV}$ from the Particle Data Group [7]. We set $m_u = m_d = 0$ and take account of the energy-scale dependence of the input parameters,

\[
(\bar{q}q)(\mu) = (\bar{q}q)(1 \text{ GeV}) \left[ \frac{\alpha_s(1 \text{ GeV})}{\alpha_s(\mu)} \right]^{\frac{12}{32 T^2}},
\]

\[
(\bar{s}s)(\mu) = (\bar{s}s)(1 \text{ GeV}) \left[ \frac{\alpha_s(1 \text{ GeV})}{\alpha_s(\mu)} \right]^{\frac{12}{32 T^2}},
\]

\[
(\bar{q}g, \sigma Gq)(\mu) = (\bar{q}g, \sigma Gq)(1 \text{ GeV}) \left[ \frac{\alpha_s(1 \text{ GeV})}{\alpha_s(\mu)} \right]^{\frac{12}{32 T^2}},
\]

\[
(\bar{s}g, \sigma Gs)(\mu) = (\bar{s}g, \sigma Gs)(1 \text{ GeV}) \left[ \frac{\alpha_s(1 \text{ GeV})}{\alpha_s(\mu)} \right]^{\frac{12}{32 T^2}},
\]

\[
m_c(\mu) = m_c(m_c) \left[ \frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right]^{\frac{12}{32 T^2}},
\]

\[
m_s(\mu) = m_s(2 \text{ GeV}) \left[ \frac{\alpha_s(\mu)}{\alpha_s(2 \text{ GeV})} \right]^{\frac{12}{32 T^2}},
\]

\[
\alpha_s(\mu) = \frac{1}{b_0} \left[ 1 - b_1 \log t + b_2 \frac{(\log^2 t - \log t - 1)}{b_0^2} \right],
\]

from the renormalization group equation, $t = \frac{\mu^2}{\Lambda_{QCD}^2}$, $b_0 = 0$, $b_1 = \frac{12}{15 \frac{\pi^2}{24} + 0.07}$, $b_2 = \frac{2857 - 12 \frac{\pi^2}{24} + 0.07}{12 \frac{\pi^2}{24} + 0.07}$, $\Lambda_{QCD} = 210 \text{ MeV}$, $292 \text{ MeV}$ and $332 \text{ MeV}$ for the flavors $n_f = 5, 4$ and $3$, respectively [7, 37], and evolve all the input parameters to the typical energy scale $\mu = m_c(m_c) \approx 1.3 \text{ GeV}$ to extract hadronic coupling constants [19, 25, 38].

At the hadron side, we take the parameters as $M_K = 0.4937 \text{ GeV}$, $M_{K^*} = 0.8917 \text{ GeV}$, $M_{J/\psi} = 3.8969 \text{ GeV}$, $M_{\eta_c} = 2.9834 \text{ GeV}$, $M_{D^*} = 2.007 \text{ GeV}$ and $D_s = 1.969 \text{ GeV}$ [7], $f_K = 0.156 \text{ GeV}$ [27], $f_{K^*} = 0.220 \text{ GeV}$, $\sqrt{u_{K^*}} = 1.0 \text{ GeV}$, $\sqrt{u_{K^*}} = 1.3 \text{ GeV}$ [39], $f_{D_s} = 0.240 \text{ GeV}$, $\sqrt{u_{D_s}} = 2.6 \text{ GeV}$, $f_{D^*} = 263 \text{ MeV}$, $\sqrt{u_{D^*}} = 2.5 \text{ GeV}$ [40], $f_{J/\psi} = 0.418 \text{ GeV}$, $f_{\eta_c} = 0.387 \text{ GeV}$ [41], $\sqrt{u_{J/\psi}} = 3.6 \text{ GeV}$, $\sqrt{u_{J/\psi}} = 3.5 \text{ GeV}$ [7], $M_Z = 3.99 \text{ GeV}$, $\lambda_Z = 2.85 \times 10^{-2} \text{ GeV}^5$ [19], $\lambda_Z = 1.96 \times 10^{-2} \text{ GeV}^5$ [25], $\frac{f_{J/\psi} M_{J/\psi}^2}{m_{J/\psi} + m_{\psi}} = (\bar{q}q)(\bar{s}s)$ from the Gell-Mann-Oakes-Renner relation, $\delta_K = 0.50$ [42].

In calculations, we fit the unknown parameters to be $C_{J/\psi K}^T = 0.00045 + 0.00038 \times T^2 \text{ GeV}^8$, $C_{J/\psi K}^T = -0.00184 - 0.00068 \times T^2 \text{ GeV}^8$, $C_{J/\psi K}^M = 0.0004 + 0.00017 \times T^2 \text{ GeV}^8$, $C_{J/\psi K}^T = 0.00094 + 0.00033 \times T^2 \text{ GeV}^8$, and $C_{J/\psi K}^M = -0.023 - 0.00405 \times T^2 \text{ GeV}^8$ to acquire the flat Borel platforms $T_{max}^2 - T_{min}^2 = 1 \text{ GeV}^2$, where the max and min represent the
maximum and minimum values, respectively, see Fig.1 for example. In the picture of tetraquark states, the Borel parameters are $T^2_{J/ψK} = (2.7 - 3.7)$ GeV$^2$, $T^2_{ηcK*} = (2.5 - 3.5)$ GeV$^2$ and $T^2_{D^*D_s} = (3.1 - 4.1)$ GeV$^2$ for the hadronic coupling constants $G_{Z_J/ψK}^T$, $G_{Z_{ηcK*}}^T$, and $G_{Z_{D^*D_s}}^T$, respectively, while in the picture of molecular states, the Borel parameters $T^0_{J/ψK} = (2.7 - 3.7)$ GeV$^2$, $T^0_{ηcK*} = (2.3 - 3.3)$ GeV$^2$ and $T^0_{D^*D_s} = (2.3 - 3.3)$ GeV$^2$ for the hadronic coupling constants $G_{M_{Z_J/ψK}}^M$, $G_{M_{Z_{ηcK*}}}^M$, and $G_{M_{D^*D_s}}^M$, respectively, where we add the subscripts $J/ψK$, $ηcK*$ and $D^*D_s$ to denote the corresponding channels. The uncertainties originate from the uncertainties of the input parameters at the QCD side, in general, can be absorbed into the decay constants and coupling constants together, for example, the $f_J/ψ$, $f_K$, $λ^2$ and $G_{Z_J/ψK}$ in the QCD sum rules in Eq.(20).

If we use the symbol $ξ$ to stand for the input parameters at the QCD side, the uncertainties $ξ → ξ + δξ$ lead to the uncertainties $f_{J/ψf_K}λ^2Z_{Z_J/ψK}$ → $f_{J/ψf_K}λ^2Z_{Z_J/ψK}$ + $δf_{J/ψf_K}λ^2Z_{Z_J/ψK}$, $C_{J/ψK}^T → C_{J/ψK}^T + δC_{J/ψK}^T$, where

$$δ f_{J/ψf_K}λ^2Z_{Z_J/ψK} = f_{J/ψf_K}λ^2Z_{Z_J/ψK} \left( \frac{δf_{J/ψ}}{f_{J/ψ}} + \frac{δf_{K}}{f_{K}} + \frac{δλ^2Z_{Z_J/ψK}}{λ^2Z_{Z_J/ψK}} \right), \quad (29)$$

we can set $\frac{δf_{J/ψ}}{f_{J/ψ}} = \frac{δf_{K}}{f_{K}} = \frac{δλ^2Z_{Z_J/ψK}}{λ^2Z_{Z_J/ψK}}$ approximately, then

$$δ f_{J/ψf_K}λ^2Z_{Z_J/ψK} = f_{J/ψf_K}λ^2Z_{Z_J/ψK} \frac{4δG_{Z_J/ψK}^T}{G_{Z_J/ψK}^T}, \quad (30)$$

to avoid overestimating the uncertainty of the hadronic coupling constant. In calculations, we estimate the uncertainties according to Eq.(30) analogously and neglect the uncertainties of the unknown parameters $C_{J/ψK}^T$, $C_{ηcK*}^T$, $C_{D^*D_s}^T$, $C_{J/ψK}^M$, $C_{ηcK*}^M$, and $C_{D^*D_s}^M$, except in the case of the $δm_c(m_c)$ for the $C_{D^*D_s}^M$, where we have to take account of the uncertainty to acquire flat Borel platform.

Now let us obtain the values of the hadronic coupling constants routinely,

$$G_{Z_J/ψK}^T = 1.79 ± 0.10 \text{ GeV},$$
$$|G_{Z_{ηcK*}}^T| = 2.97 ± 0.22 \text{ GeV},$$
$$|G_{Z_{D^*D_s}}^T| = 0.71 ± 0.04 \text{ GeV},$$
$$G_{Z_J/ψK}^M = 1.23 ± 0.07 \text{ GeV},$$
$$G_{Z_{ηcK*}}^M = 2.20 ± 0.16 \text{ GeV},$$
$$|G_{Z_{D^*D_s}}^M| = 9.60 ± 1.27 \text{ GeV}. \quad (31)$$

Then we choose the masses $M_{K^−} = 0.4937 \text{ GeV}$, $M_{K^{+}} = 0.8917 \text{ GeV}$, $M_{J/ψ} = 3.0969 \text{ GeV}$, $M_{η_c} = 2.9834 \text{ GeV}$, $M_{D^{*−}} = 2.0069 \text{ GeV}$, $M_{D^{*+}} = 1.9690 \text{ GeV}$, $M_{D^{0}} = 1.8648 \text{ GeV}$, $M_{D^*} = 2.1122 \text{ GeV}$ [7], $M_{Z_c} = 3.99 \text{ GeV}$ [19, 25], and obtain the partial decay widths,

$$\Gamma(Z_{cs}^T → J/ψK^{−}) = 5.36 ± 0.60 \text{ MeV},$$
$$\Gamma(Z_{cs}^T → ηcK^{*−}) = 9.54 ± 1.41 \text{ MeV},$$
$$\Gamma(Z_{cs}^T → D^{*−}D^{−}_s) = 0.21 ± 0.02 \text{ MeV},$$
$$\Gamma(Z_{cs}^T → D^{0}D^{−}_s) = 0.20 ± 0.02 \text{ MeV}, \quad (32)$$

$$\Gamma(Z_{cs}^M → J/ψK^{−}) = 2.53 ± 0.29 \text{ MeV},$$
$$\Gamma(Z_{cs}^M → ηcK^{*−}) = 5.23 ± 0.76 \text{ MeV},$$
$$\Gamma(Z_{cs}^M → D^{*−}D^{−}_s) = 38.69 ± 10.24 \text{ MeV},$$
$$\Gamma(Z_{cs}^M → D^{0}D^{−}_s) = 37.06 ± 9.81 \text{ MeV}. \quad (33)$$
Figure 1: The hadronic coupling constants $G^T_{ZJ/\psi K}$ (I) and $G^M_{ZJ/\psi K}$ (II) with variations of the Borel parameter $T^2$, where the regions between the two vertical lines are the Borel windows.

and the total widths,

\[ \Gamma^T_{Zcs} = 15.31 \pm 2.06 \text{ MeV}, \]
\[ \Gamma^M_{Zcs} = 83.51 \pm 21.09 \text{ MeV}, \]

(34)

which are consistent with the experimental data $13.8^{+8.1}_{-5.2} \pm 4.9$ MeV from the BESIII collaboration [5] and $131 \pm 15 \pm 26$ MeV from the LHCb collaboration [6], respectively. There are two $Z_{cs}$ states with analogous masses, just like in the case of the $Z_c^\pm$ states, there exist a tetraquark candidate $Z_c(3900)$ and a molecule candidate $Z_c(3885)$ [25].

The physical $Z_{cs}$ states maybe have both the diquark-antidiquark type and color-singlet-color-singlet type tetraquark Fock components, it is better to choose the mixing currents or more physical currents to interpolate them,

\[ J_\nu(x) = J^T_\nu(x) \cos \theta + J^M_\nu(x) \sin \theta, \]

(35)

with the mixing angle $\theta$. As both the currents $J^T_\nu(x)$ and $J^M_\nu(x)$ can lead to a mass about $3.99 \pm 0.09$ GeV in the case of choosing the same constraints [19, 25]. The additional parameter $\theta$ cannot affect the predicted mass remarkably, but can affect the predicted width remarkably. We can estimate the hadronic coupling constants $G$ with the simple replacement,

\[ G \to G^T \cos \theta + G^M \sin \theta, \]

(36)

and compare the predicted total widths and branching fractions to the precise experimental data in the future to estimate the mixing angle $\theta$, the branching fractions have not been measured yet. The significance of the $Z^-_{cs}(3985)$ is about $5.3 \sigma$ [5], while the significance of the $Z^+_{cs}(4000)$ is about $15 \sigma$ [6], such large significances only indicate that there maybe exist those resonance structures indeed, more experimental data are still needed even to distinguish (and confirm) the $Z_{cs}(3985)$ and $Z_{cs}(4000)$ unambiguously. In the present work, at least, we can obtain the conclusion tentatively that larger color-singlet-color-singlet component in the $Z_{cs}$ state leads to larger decay width, the $Z_{cs}(3985)$ maybe have large diquark-antidiquark type Fock component, while the $Z_{cs}(4000)$ maybe have large color-singlet-color-singlet type Fock component.

4 Conclusion

In this work, we explore the hadronic coupling constants $G_{ZJ/\psi K}$, $G_{Z\eta_c K^*}$, $G_{ZD^+\bar{D}^0}$, of the exotic states $Z_{cs}(3985/4000)$ both in the pictures of the tetraquark states and molecular states with the
tentative assignments \( J^{PC} = 1^{+-} \) based on the rigorous quark-hadron duality. We write down the three-point correlation functions, and accomplish the operator product expansion up to the vacuum condensates of dimension-5, and neglect the tiny gluon condensate contributions, just like in our previous works, and obtain the spectral densities through dispersion relation, then we acquire the rigorous quark-hadron duality below the continuum thresholds \( s_0 \) and \( u_0 \) by accomplishing the integral over the variable \( ds' \) firstly, and obtain six QCD sum rules for the hadronic coupling constants. We investigate the two-body strong decays of the axialvector tetraquark state and molecular state, respectively, and obtain the total widths \( \Gamma_{Z_{cs}}^{\pm} = 15.31 \pm 2.06 \text{ MeV} \) and \( \Gamma_{Z_{cs}}^{0} = 83.51 \pm 21.09 \text{ MeV} \), which are consistent with the experimental data \( 13.8^{+8.1}_{-5.2} \pm 4.9 \text{ MeV} \) from the BESIII collaboration and \( 131 \pm 15 \pm 26 \text{ MeV} \) from the LHCb collaboration, respectively. The present calculations support assigning the \( Z_{cs}(3985) \) and \( Z_{cs}(4000) \) to be the hidden-charm tetraquark state and molecular state with the \( J^{PC} = 1^{+-} \), respectively. Or at least, the \( Z_{cs}(3985) \) maybe have large diquark-antidiquark type Fock component, while the \( Z_{cs}(4000) \) maybe have large color-singlet-color-singlet type Fock component.

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