Correlation length scaling laws in drift-Alfvén edge turbulence computations

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Abstract

The effect of changes in plasma parameters, that are characteristic near or at an L–H transition in fusion edge plasmas, on fluctuation correlation lengths are analysed by means of drift-Alfvén turbulence computations. Scalings by density gradient length, collisionality, plasma beta and by an imposed shear flow are considered. It is found that strongly sheared flows lead to the appearance of long-range correlations in electrostatic potential fluctuations parallel and perpendicular to the magnetic field.

(Some figures may appear in colour only in the online journal)

1. Introduction

The interplay between long-range correlations of turbulent fluctuations, radial electric fields and edge bifurcations in fusion plasmas has received recent interest in the form of various experimental studies [1–8]. This interest is motivated by a missing mechanism behind the formation of edge transport barriers at the transition from L- to H-mode plasma states. The central link between the appearance of radial electric field $E_r$ and associated sheared $E \times B$ flows to suppression of small-scale turbulence, the reduction in turbulent transport, and a steepening of the pedestal profile, is generally accepted [8]. However, the causal chain of mechanisms behind this transport barrier formation is as yet unclear.

It has been speculated that turbulence generated zonal flows could be able to trigger the mean shear flow bifurcation. Long-range correlations in turbulent fluctuations have been associated with enhanced zonal flow activity. In L-mode experiments with imposed shear flow an increase in correlation length of the fluctuating electrostatic potential [2] and density [4] has been found along and across magnetic field lines.

The influence of single possible players behind the formation of long-range correlations cannot always easily be determined by experiments, but may be straightforwardly studied with numerical simulation. In this work, local drift-Alfvén flux-tube turbulence computations are applied to analyse correlation statistics for various L-mode parameters in scalings that are characteristic for the approach to the H-mode. In particular, scaling effects by the background density gradient length, the collisionality, the plasma beta and imposed $E \times B$ shear flows on correlation statistics are studied. It is found that only strong imposed shear flows are able to generate significant long-range correlations in these simulations.

The work is organized as follows: in section 2 the numerical model and reference parameters are described. In section 3 the evaluation of correlation functions from fluctuating simulated quantities is reviewed. In section 4 the individual scaling relations are analysed, followed by the conclusions in section 5.

2. Numerical model: drift-Alfvén turbulence

Electromagnetic drift wave turbulence can be regarded as a paradigmatic model for turbulent transport and zonal flow structure formation in tokamak edge plasmas [9]. The use of a cold-ion four-field fluid model is appropriate if the ion temperature gradient (ITG) is sufficiently low such that the
The plasma edge of tokamaks is characterised by a value of all frequency determines the resistive coupling between species through ITG turbulence, and the collisionality is sufficiently high to allow for a fluid approach. Both conditions are well met in typical L-mode tokamak edge plasmas.

Then the drift-Alfvén turbulence dynamics is controlled mainly by three dimensionless parameters in the model: a plasma beta parameter \( \hat{\beta} = \hat{\varepsilon} \beta \), a mass ratio \( \hat{\mu} = \hat{\varepsilon} (m_i/m_e) \), and a collisionality parameter \( \hat{C} = 0.5 \hat{\varepsilon} (m_i/m_e) (\nu_c L) / c_s \).

The factor \( \hat{\varepsilon} = (q R / L) \) gives the ratio between parallel length scale \( L \parallel = q R \) and perpendicular length scale \( L \perp \), where \( q \) is the safety factor and \( R \) is the major torus radius. The plasma edge of tokamaks is characterised by a value of all parameters \( \hat{\beta}, \hat{C} \) and \( \hat{\mu} \) in the order of unity.

The electron and ion masses are \( m_e \) and \( m_i \), respectively. The collisionality parameter determines the resistive coupling between species through the parallel current and depends on the electron collision frequency \( \nu_c \), scaled by \( L \perp / \nu_c \), where \( \nu_c = \sqrt{\hat{\varepsilon} T_e / m_i} \) is the sound speed. The collisionality parameter is related to the classical thermal resistivity by \( \eta_T = (m_e v_c) / (m_n e^2) \) through the collision frequency, and to the experimentally often used ‘nu star’ parameter by \( \nu^* = \hat{C} (0.5 \hat{\mu}^{1/2} r^{3/2}) \) where \( \hat{\mu} \) is the inverse aspect ratio. For \( \epsilon = 0.3 \) the above base parameters correspond to \( \nu^* = 6 \).

The four-field drift-Alfvén model is solved numerically using the local flux-tube code ATTEMPT [10]. The model describes the evolution of fluctuations of the electrostatic potential \( \phi \), particle density \( n \), vector potential \( A \parallel \) and parallel ion velocity \( u \):

\[
\frac{dn}{dt} = \frac{1}{e} \nabla J - n \nabla u - n \nabla (\phi + \frac{T_e}{e} \nabla (n \phi)), \tag{1}
\]

\[
nm \frac{dw}{dt} = \nabla J + T_e \nabla (n \phi), \tag{2}
\]

\[
\frac{dA}{dt} + \frac{m_e}{e^2 n} \frac{dJ}{dt} + T_e \nabla (n \phi - \eta_T J), \tag{3}
\]

\[
nm \frac{du}{dt} = -T_e \nabla n. \tag{4}
\]

This set of equations is coupled to the solution of Poisson’s and Ampere’s equations for the vorticity \( w \) and the vector potential \( A \parallel \):

\[
w = \nabla^2 \phi \quad \text{and} \quad \mu_0 J = -\nabla^2 A. \tag{5}
\]

Operator abbreviations have been introduced as follows:

\[
d \frac{d}{dt} = \frac{\partial}{\partial t} + v_E \cdot \nabla \quad \text{with} \quad v_E \cdot \nabla = \frac{B}{B^2} \cdot (\nabla \phi \times \nabla), \tag{6}
\]

\[
\nabla A = \nabla \cdot \left( \frac{B \times \nabla}{B^2} \right), \tag{7}
\]

\[
\nabla J = \frac{B}{B} \cdot \nabla - \frac{B}{B^2} \cdot (\nabla A \times \nabla). \tag{8}
\]

\[
\nabla^2 = \nabla^2 - \nabla \cdot \left( \frac{B \times \nabla}{B^2} \right). \tag{9}
\]

A static toroidal equilibrium background magnetic field \( B \) is assumed. The model describes nonlinear electromagnetic \( E \times B \) drift motions of electrons and ions with charges \( q = \pm e \). The ion and electron particle densities are equal, \( n_e = n_i = n \), obeying quasi-neutrality.

A local approximation is applied, where the density gradient is linear and constant in time with \( L^{-1} = |\nabla \ln n| \) with axisymmetric background density \( n_0 \) and the density \( n = n_0 + n \) split into a static and a fluctuating part. In the following the tilde on the fluctuating density will be avoided for better readability. A partially field-aligned flux-tube coordinate system \((x, y, z)\) is introduced and the standard drift normalization is applied, which are described in detail in the appendix of [10], where the coordinates \((x, y, z)\) are denoted by \((\chi, \eta, \sigma)\). Parallel derivatives (in the \( z \) direction) are normalized with respect to the parallel connection length \( L \parallel = q R_0 \), perpendicular derivatives with \( L \perp \), and time scales with \((c_s / L \perp)\). The density gradient length then enters via \( \lambda_n = L \perp / L_n = |\partial \eta / \partial \ln n| \). The normalized set of equations for the fluctuations is

\[
\frac{dn}{dt} = -[\phi, n] - \lambda_n \hat{\mu} \hat{\beta} \hat{C} \hat{\Phi} \hat{K} (n - \phi) + \nabla \phi (J - u), \tag{10}
\]

\[
\frac{dw}{dt} = -[\phi, w] + \nabla \phi (J + \hat{K} (n)), \tag{11}
\]

\[
\hat{\beta} \frac{dA}{dt} + \hat{\mu} \frac{dJ}{dt} = -[\phi, J] + \nabla (n - \phi) - \hat{C} J, \tag{12}
\]

\[
\hat{\epsilon} \frac{du}{dt} = -[\phi, u] - \nabla n. \tag{13}
\]

In a large aspect ratio toroidal circular flux-tube geometry, the Poisson bracket is \( [f, g] = \partial_x f \partial_g - \partial_x g \partial_f \), the curvature operator is \( K \phi = \omega_B [\cos(z), \partial, f, \sin(z), \partial, g] \), the parallel derivative \( \nabla \phi = \partial_x f - \beta A \parallel \) and the Laplacian becomes \( \nabla^2 = \partial_x^2 + \partial_y^2 \). The numerical methods using a higher-order Adams–Bashforth/Arakawa scheme are detailed in [10].

This work is a computational study on inherent scalings within the local drift-Alfvén turbulence model, and in this sense purely theoretical. It is not the intention to do any modelling for some specific fusion experiment or a direct comparison with measurements. The dimensionless model parameters are generically in the order of unit in the plasma edge (closed flux surface pedestal) region of present tokamaks and stellarators. By varying the parameters independently we can thus scan a wide and typical range of experimental parameters across devices.

The dimensionless model base parameters are here chosen as \( \hat{\beta} = 1.0, \hat{C} = 1.0 \) and \( \hat{\mu} = 4.7 \). In order to complete a consistent base parameter set we here refer to ‘typical’ mid-pedestal edge parameters of smaller tokamaks like, for example, TEXTOR. In physical units our parameter set corresponds to a local edge electron temperature \( T_e = 52 \text{ eV} \), plasma density \( n_0 = 5.6 \times 10^{18} \text{ m}^{-3} \), magnetic field strength \( B_0 = 1.0 \text{T} \), a background density gradient reference scale \( L \perp = 0.053 \text{ m} \), and a parallel connection scale \( L \parallel = q R = 4.7 \text{ m} \). This relates a scale ratio of about \( \hat{\varepsilon} = (L \parallel / L \perp)^2 = 7. In order to complete a consistent base parameter set we here refer to ‘typical’ mid-pedestal edge parameters of smaller tokamaks like, for example, TEXTOR. In physical units our parameter set corresponds to a local edge electron temperature \( T_e = 52 \text{ eV} \), plasma density \( n_0 = 5.6 \times 10^{18} \text{ m}^{-3} \), magnetic field strength \( B_0 = 1.0 \text{T} \), a background density gradient reference scale \( L \perp = 0.053 \text{ m} \), and a parallel connection scale \( L \parallel = q R = 4.7 \text{ m} \). This relates a scale ratio of about \( \hat{\varepsilon} = (L \parallel / L \perp)^2 =
directions, and by evaluation at every time step a time series of the self-length analysis. At several ‘probe’ position, and are subjected to a correlation times. The AC function is evaluated in the interval of a fluctuation signal analysis are reviewed. The auto-correlation (AC) function in the following section correlation functions used in the following analysis are displayed.

A windowed AC analysis on the computed time series is performed by shifting a slice of the data \( f \) of size \( \Delta T \) by \( \delta_T \) for every step. Here we use \( \delta_T = 0.3 L_\perp/c_s \), which is an optimal value well smaller than typical turbulent decorrelation times but again large enough to give smooth PDFs, and \( \Delta T = 60 L_\perp/c_s \), which was found to be much larger than any resulting correlation times. The AC function is evaluated in the interval \([t_i, t_i + \Delta T]\) with \( t_0 = 0, t_1 = \delta_T, t_1 = i \delta_T:\n
\gamma_{\text{auto}}(\tau, t_i) = \frac{1}{\Delta T} \frac{1}{f(t)^2} \sum_{t = t_i}^{t + \tau} f(t + \tau) f(t). \quad (15)\n
By evaluation at every time step a time series of the self-correlation time \( \tau_{AC}(t_i) \) defined by \( \gamma_{\text{auto}}(\tau_{AC}(t_i), t_i) \approx 0.5 \) is obtained. The statistical properties of \( \tau_{AC}(t_i) \) are displayed using probability density functions (PDFs):

\[ P(\tau_{AC}) = P(l_{b-1} \prec \tau_{AC} < l_b = l_{b-1} + dl_b) = \frac{1}{N} \sum_{l_{b-1} \prec \tau_{AC} < l_b} \delta(t - \tau_{AC}), \quad (16)\n
where \( N \) is the length of the fluctuation time series \( f(t), t_b \) is the position of a bin centre, with \( dt_b = (\max \tau_{AC} - \min \tau_{AC})/N_b \) the width of a bin and \( N_b \) the number of bins used. \( P \) gives the probability of finding the auto-correlation time \( \tau_{AC} \) in the fluctuation time series.

Spatial correlation lengths are analysed by means of the cross-correlation function of two time series \( f(t) \) and \( g(t) \) are fluctuation time series at two spatial positions:

\[ \gamma_{gf}(\tau) = \frac{1}{T} \frac{1}{\sigma(f) \sigma(g)} \sum_{t=0}^{T} (f(t + \tau) - \bar{f})(g(t) - \bar{g}) \quad (17)\n
with

\[ \bar{f} = \frac{1}{T} \sum_{t=0}^{T} f(t) \quad \text{and} \quad \sigma(f) = \sqrt{\frac{1}{T} \sum_{t=0}^{T} (f(t) - \bar{f})^2} \quad (18)\n
To obtain a measure for the spatial coherence of fluctuations the cross-correlation coefficient \( CC(f, g) = \gamma_{gf}(0) \) is evaluated as the correlation function \( \gamma_{gf}(\tau) \) in the limit \( \tau = 0 \). The spatial correlation function \( L_{CC} \) is calculated as the cross-correlation between a fluctuation signal \( f \) taken at a probe at position \( l_0 \): \( f(l_0, t) \) and at a spatially shifted position \( l_j \): \( f(l_j, t) \), with the distance between the probes \( \delta l := |l_j - l_0| \):

\[ L_{CC}(\delta l) = CC(f(l_0), f(l_0 + \delta l)) \quad (19)\n
A statistical description is used, where the data \( f(t, l) \) are cut into pieces of length \( \Delta T \), as for the auto-correlation time above. The correlation length \( \lambda_i(t_i) \) is evaluated for a time window \( [i\delta_T : (i+1)\delta_T] \). A time series of half width times \( \lambda_i(t_i) \) results with \( t_0 = 0, t_1 = \delta_T, t_i = i \delta_T:\n
\[ L_{CC}(\delta l) = CC(f(t_i, l_0), f(t_i, l_0 + \delta l)) \quad (20)\n
The correlation length \( \lambda_i(t_i) \) is defined as the half width of the correlation function at \( L_{CC}(\lambda_i(t_i), t_i) = 0.5 \). \( \lambda_i(t_i) \) is binned into a histogram and normalized to one, giving a probability density function

\[ P(\lambda_i) = P(l_{b-1} \prec \lambda_i < l_b = l_{b-1} + dl_b) = \frac{1}{N} \sum_{l_{b-1} \prec \lambda_i < l_b} \delta(t - \tau_{AC}) \quad (21)\n
where \( l_{b-1}, l_b \) are bin centres, \( dl_b \) is the width of the bins.

4. Scalings of correlation lengths

When the L–H transition is approached from an L-mode state, several parameters of the edge pedestal can change in theory that characteristically influence the turbulence and transport [14]. Here we model this approach by independent variations of the parameters while remaining well within the L-mode (i.e. in the cold ion) limit.

In the transition to the H-mode the pedestal density and temperature rise. The collisionality parameter \( \tilde{\gamma} \sim 1/\nu_e \) is reduced as the edge temperature grows, the plasma beta parameter \( \beta \) increases, and the density gradient length \( L_n \) becomes smaller.
Figure 1. Global averages of the fluctuation kinetic energy $\langle E \times B \rangle = \langle v^2 E \times B \rangle$ (thin solid line), zonal flow energy $\langle (\partial_x \varphi)^2 \rangle$ (bold solid line), radial transport $\langle I_x \rangle = \langle v_{E \times B} \rangle$ (dashed–dotted line), density free energy $\langle |n| \rangle$ (bold dashed–dotted line) and average zonal flow shear $\langle \omega \rangle = \langle \partial^2_x \varphi_0 \rangle$ (bold dashed line). Scalings with varying (a) inverse density gradient length $1/L_n$, (b) collisionality parameter $\tilde{C}$, (c) normalized plasma beta $\tilde{\beta}$, (d) imposed flow shear $\Omega_0 = (\partial v_{ZF}/\partial x)$. Temporal standard deviations of the fluctuating global quantities are shown as error bars.

4.1. Density gradient length scaling

First, the steepening of the edge density gradient is modelled by varying the density gradient length $L_n$ while all other parameters remain constant at their nominal L-mode levels. A reference simulation is performed with initial gradient length $L_n,0 = 0.035$ m, and four simulations with steepened gradient lengths $L_n/L_{n,0} = (1.25, 1.5, 1.75, 2.0, 2)$.

In figure 1(a) global averages (over the whole computational domain except radial boundary dissipation regions $dx = 8\Delta x$) of energetic quantities are shown. The global mean is in addition averaged over time during the saturated turbulent phase of the simulations, and the temporal standard deviations of the fluctuating global quantities are shown as error bars. The values are normalized with respect to the reference simulation $L_n = L_{n,0}$.

The zonal flow strength $v_{ZF}^2$ is doubled when the gradient is steepened corresponding to half the reference gradient length. The zonal flow strength $v_{ZF}^2 = (\partial^2 \phi_0)^2$, with $\phi_0(x) = (\phi)^{\gamma_e}$, is doubled when the gradient is steepened corresponding to half the reference gradient length. The zonal flow shear $\langle \omega \rangle = (\partial^2_x \varphi_0)$ is also doubled when the gradient is steepened.

Around the L–H transition a mean $E \times B$ flow shear layer would develop within the edge pedestal region. As turbulence codes to date are unable to self-consistently account for realistic H-mode shear flow development [15–17], we model this effect by imposing a background vorticity on the turbulence.

The influence of these respective parameter scalings, which model the approach to an H-mode state, on fluctuation correlation statistics is analysed in the following. The reference ‘probe’ position, at which the time series are recorded, is located in the centre of the computational domain, corresponding to mid-pedestal radius $(L/2)$ at the torus outboard midplane position. Further analyses have been performed for a number of radial ‘probe’ positions $(L/4, 3L/8, 5L/8, 3L/4)$, which showed very similar results concerning scaling relations compared with the radial reference position. Therefore only results for this reference position are presented.
flow shear \( \omega_0 = \langle \partial_t \phi_0 \rangle \) increases by an order of magnitude, similar to the average free energy density \( \langle |n| \rangle \). The radial density transport, defined as \( \Gamma_n = n v_{E,B} \), increases by nearly two orders of magnitude. The enhanced gradient drive is thus found to increase all turbulent activities.

The PDFs of the fluctuation auto-correlation \( P(\tau_{AC}) \) and for the correlation lengths \( P(\lambda_{x,y,z}) \) are computed for electrostatic potential perturbations \( \phi \) and density perturbations \( n \). The positions of the maxima of the PDFs for both \( \phi \) and \( n \) are determined for each value of the density gradient length \( L_n/L_\perp \). The uncertainty in determination of the maxima is estimated by the sensitivity of the PDFs to the free correlation parameters like bin width (but not actually calculated by some statistical procedure).

In figure 2 the maxima \( t(\max, AC) \) and \( L_{x,y,z} \), respectively, of (a) auto-correlation PDFs \( P(\tau_{AC}) \), (b) perpendicular correlation lengths \( P(\lambda_{x,y}) \) and (c) parallel correlation lengths \( P(\lambda_z) \), are drawn as a function of \( L_n/L_\perp \) for both \( \phi \) and \( n \).

The temporal correlations (a) are found to change only insignificantly with varying density gradient length within the estimated uncertainties.

The spatial scales are consistent with typical \( E \times B \) turbulent vortex structures of the order of a few \( \rho_s \) in the perpendicular direction (b) and elongated in the order of \( L_\parallel \) in the parallel directions (c).

Note that the gradient length is reduced during an approach to H-mode: larger values of \( L_n/L_\perp \) correspond to less steep profiles and less turbulent drive. This apparently does not affect perpendicular correlation lengths for density fluctuations (red/grey bold lines in (b)), which are roughly constant within the methodological uncertainty. The radial lengths \( L_r \) are depicted with a solid line, and the poloidal lengths \( L_y \) with a dashed line.

The perpendicular correlation lengths for potential fluctuations (black lines in (b)) are slightly reduced by around one quarter when the density gradient length is halved from \( L_n/L_\perp = 1.0 \) to 0.5. This reflects the influence of stronger zonal flow shear \( \langle \partial_t \phi \rangle \) on the \( E \times B \) vortices (compare figure 1(a)).

The parallel correlation lengths (c) are increased (by around 15\%) for both density and potential fluctuations when reducing the gradient length from 1.0 to 0.5.

4.2. Collisionality parameter scaling

Next, the effect of a reduced collisionality parameter on edge turbulence correlations is analysed. The collisionality \( \hat{C} \propto 1/T_e^2 \) scales inversely with the 3rd power of the electron temperature \( T_e \). In the following the parameters used for this simulation series are summarized:

\[
\hat{C} = 1.0 : T_e = 51.8 \text{ eV} ; n_0 = 5.56 \times 10^{19} \text{ m}^{-3} ; L_\perp/c_s = 0.80 \mu\text{s} ; \rho_s = 1.00 \text{ mm} \\
\hat{C} = 2.5 : T_e = 38.2 \text{ eV} ; n_0 = 7.60 \times 10^{19} \text{ m}^{-3} ; L_\perp/c_s = 0.93 \mu\text{s} ; \rho_s = 8.93 \text{ mm} \\
\hat{C} = 5.0 : T_e = 30.3 \text{ eV} ; n_0 = 9.51 \times 10^{19} \text{ m}^{-3} ; L_\perp/c_s = 1.00 \mu\text{s} ; \rho_s = 7.95 \text{ mm} \\
\hat{C} = 7.5 : T_e = 26.5 \text{ eV} ; n_0 = 10.9 \times 10^{19} \text{ m}^{-3} ; L_\perp/c_s = 1.12 \mu\text{s} ; \rho_s = 7.43 \text{ mm} \\
\hat{C} = 10 : T_e = 24.0 \text{ eV} ; n_0 = 12.0 \times 10^{19} \text{ m}^{-3} ; L_\perp/c_s = 1.18 \mu\text{s} ; \rho_s = 7.10 \text{ mm}
\]

A collisionality parameter \( \hat{C} = 1 \) corresponds to a roughly doubled temperature compared with \( \hat{C} = 10 \). All time scales are as usual normalized to the drift time scale \( L_\perp/c_s \). In this series of simulations, however, \( L_\perp/c_s \) is varied in addition to \( \hat{C} \).

In figure 1(b) the energetics scaling with \( \hat{C} \) is shown. Except for the zonal flow energy \( \langle \omega_E^2 \rangle \) all global energy averages increase linearly with this collisionality parameter. For the radial density transport \( \Gamma_n \) not only the mean value, but the fluctuation width (drawn as vertical deviation bars around the mean) increases.

In figure 3 the maxima \( t(\max, AC) \) and \( L_{x,y,z} \), respectively, of (a) auto-correlation PDFs \( P(\tau_{AC}) \), (b) perpendicular correlation lengths \( P(\lambda_{x,y}) \) and (c) parallel correlation lengths \( P(\lambda_z) \), are drawn as a function of \( \hat{C} \) for both \( \phi \) and \( n \).
The temporal correlations (a) are found to change only insignificantly also with varying collisionality parameter within the estimated uncertainties.

$L_1$ of $\phi$ and $n$ and $L_2$ of $\phi$ vary insignificantly with $\hat{C}$. Only the poloidal correlation $L_3$ of $n$ shows some reduction by around one third when the collisionality parameter is reduced from $\hat{C} = 10$ to 1. The parallel correlation lengths are slightly increased by around 10% for both density and potential fluctuations when reducing the parameter $\hat{C}$ from 10 to 1.

4.3. Plasma beta scaling

In the H-mode the plasma pressure in the pedestal is elevated. For constant magnetic field strength, the plasma beta rises. This motivates the following simulation series, where the magnetic beta is increased, while keeping the collisionality constant. The variation of the electron temperature $T_e$, the particle density $n$, the time and space scales for the various simulation runs are listed in the following:

\[
\begin{align*}
\hat{\beta} = 1.0 & : \quad T_e = 52 \text{ eV}; \quad n_0 = 5.56 \times 10^{18} \text{ m}^{-3}; \quad L_1/c_s = 0.80 \mu s; \quad \rho_s = 1.00 \text{ mm} \\
\hat{\beta} = 2.5 & : \quad T_e = 70 \text{ eV}; \quad n_0 = 10.2 \times 10^{18} \text{ m}^{-3}; \quad L_1/c_s = 0.69 \mu s; \quad \rho_s = 1.21 \text{ mm} \\
\hat{\beta} = 5.0 & : \quad T_e = 88 \text{ eV}; \quad n_0 = 16.2 \times 10^{18} \text{ m}^{-3}; \quad L_1/c_s = 0.61 \mu s; \quad \rho_s = 1.36 \text{ mm} \\
\hat{\beta} = 7.5 & : \quad T_e = 101 \text{ eV}; \quad n_0 = 21.3 \times 10^{18} \text{ m}^{-3}; \quad L_1/c_s = 0.57 \mu s; \quad \rho_s = 1.45 \text{ mm} \\
\hat{\beta} = 10 & : \quad T_e = 111 \text{ eV}; \quad n_0 = 25.8 \times 10^{18} \text{ m}^{-3}; \quad L_1/c_s = 0.55 \mu s; \quad \rho_s = 1.52 \text{ mm}
\end{align*}
\]

The global energetics are shown in figure 1(c). A reduction of zonal flows (drawn as solid line) to about a third for the $\hat{\beta} = 10$ compared with the reference parameter case ($\hat{\beta} = 1$), is caused by the enhanced Maxwell stress [18]. The zonal flow shear, the density free energy, the $E \times B$ energy as well as the radial $E \times B$ energy transport increase with beta.

In figure 4 the maxima ($t_{\text{max}}$, AC) and $L_{x,y,z}$, respectively) of (a) auto-correlation PDFs $P(t_{\text{AC}})$, (b) perpendicular correlation lengths $P(\lambda_z)$ and (c) parallel correlation lengths $P(\lambda_x)$, are drawn as a function of $\hat{\beta}$ for both $\phi$ and $n$.

The temporal correlations (a) are again found to change only insignificantly with varying collisionality parameter within the estimated uncertainties, except for an initial drop between $\hat{\beta} = 1$ and 2.

The perpendicular correlations in (b) for the potential fluctuations (black thin lines) are strongly enhanced with rising $\hat{\beta}$, about a factor of two for the radial direction (solid line) and a factor three in poloidal direction (dashed line) when beta is increased from 1 to 10. This reflects that on the one hand the turbulent drive (and thus fluctuation amplitudes) are enhanced with beta, and on the other hand the zonal flows are depleted by increasing Maxwell stress. Both factors lead to an increasing size of potential ($E \times B$ vortex) structures in the perpendicular plane.

The scale of density correlations (red/grey bold lines) increasingly differs from the potential correlations due to the higher non-adiabaticity (via magnetic flutter) of the electrons, which decouples density and potential fluctuations.

Along magnetic field lines $L_z$ drawn in (c) shows a clear decrease in correlation lengths of both density and potential, dropping to around on half for $\phi$ and one third for $n$ when beta is increased from 1 to 10.

4.4. Imposed shear flow scaling

Further, a simulation series has been performed with the aim of analysing the impact of an imposed sheared $E \times B$ mean flow (in addition to the self-consistently obtained zonal flows) on correlations. An external electrostatic zonal potential field $\Phi_0(x)$ is applied through the nonlinear advection operators as $[\phi, f] \rightarrow [\phi + \Phi_0, f]$:

\[
\Phi_0(x) = (1/2)\Omega_0 (x + L_x/2)^2,
\]

\[
v_0(x) = \Omega_0 (x + L_x/2),
\]

\[
\partial_x v_0(x) = \Omega_0.
\]
The application of $\Phi_0$ results in a radially increasing $E \times B$ mean drift flow with velocity $v_0$ and constant flow shear $\Omega_0 = (0, 250, 500, 1000, 2000)$. In figure 1(d) the turbulent transport and fluctuation amplitudes show a reduction for all levels of an imposed flow shear. The zonal flow amplitude increases strongly for moderate $\Omega_0 = 250$–500 due to enhanced zonal vorticity coupling $v_0^2 \sim \langle \Omega \rangle \langle R_e \rangle$ of the Reynolds stress drive. For larger $\Omega_0$ the zonal flows appear strongly reduced, when the Reynolds stress $R_e = \partial_x \phi \partial_y \phi$ is lowered by the quenched fluctuation amplitudes.

The temporal auto-correlation PDF for the shear flow experiment in figure 5(a) shows for $\Omega_0 = 250$–500 a shift of the maximum to higher time scales (around 3–4 $L_s/c_s$) for both density and potential fluctuations. This reflects the initial effect on zonal flow amplitude. For larger values of $\Omega_0$ the correlation time scales are reduced as expected.

The perpendicular correlation lengths show an increase by around 50% for potential perturbations $\phi$ when the mean flow strength is increased from $\Omega_0 = 500$–2000, whereas the density correlation lengths remain largely unaffected within the given uncertainties. The parallel correlation lengths for both $n$ and $\phi$ are enhanced by around one quarter for the same change in background flow strength.

The general trend of these results is in accordance with experimental measurements (e.g. [2, 3]), although of course a quantitative comparison between our somewhat artificial computational setup and actual experiments is not feasible. This was clearly not the intention of our present study, which should rather examine trends in this particular theoretical model for edge turbulence, and thus help us to identify basic physics mechanisms within the model.

5. Conclusions

Correlation lengths of density and electrostatic potential fluctuations for conditions relevant to an L-mode tokamak edge plasma near the L–H transition have been analysed by numerical simulation of drift-Alfvén turbulence. Five parameter scalings have been independently performed: density gradient length $L_n$, the collisionality parameter $\hat{C}$, the plasma beta $\hat{\beta}$, and by imposing a flow shear $\Omega_0$. 
A reduction of $L_n$ (corresponding to a pedestal density profile steepening) results in reduced perpendicular correlations lengths for $\phi$, and enhanced parallel correlation lengths (in the order of 15–25% across the varied range). Reducing the collisionality parameter $\hat{C}$ (corresponding to a rise in pedestal electron temperature) leads to a slight reduction of poloidal density correlations but leaves the other perpendicular correlations largely unaffected. The parallel correlation is found to be slightly enhanced (by around 10% over the varied range). These trends for $L_n$ and $\hat{C}$ are contrary to the effects of the following variable parameters $\hat{\beta}$ and $/Omega_0$.

Increasing the plasma beta $\hat{\beta}$ has different effects on density and potential correlations. Perpendicular correlation lengths for $\phi$ increase strongly (around a factor of 2–3 for the investigated range), whereas they remain nearly unchanged for $n$. Parallel correlation lengths are strongly reduced (by one third to one half) for both $n$ and $\phi$. Externally imposing a flow shear $/Omega_0$ was found to significantly enhance (by around 50% over the investigated range) the poloidal and parallel correlations lengths of $\phi$ for strong shearing rates. Density correlation lengths are increased by around 20% in all directions for strong shear flows.

It can be concluded from these parameter scaling simulations that experimentally observed long-range correlations near or at the transition to H-mode states are likely caused by the straining effect of strongly sheared flows on the turbulence. The parameters $L_n$ and $\hat{C}$, which mainly act as turbulent drive mechanisms, showed slight scaling trends contrary to experimental expectations. The parameters $\hat{\beta}$ and $/Omega_0$ had the strongest impact on perpendicular correlation lengths with trends in accordance to observations. The enhanced plasma beta leads to a reduction of zonal flow amplitude and thus allows larger perpendicular turbulence structures. The parallel correlation lengths are, however, strongly reduced with increasing beta, which is in contradiction to the experiment.

The only mechanism which had the experimentally expected effect in all directions (i.e. an increase in correlations lengths in both perpendicular and parallel directions) was an externally imposed mean shear flow. We can therefore conclude (within the limitations of our present model) that the experimentally observed increase in correlation lengths during the transition from L- to H-mode is likely to be mediated by the growing mean flow shear, however, it is generated.

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