Self-interacting Dark Matter with Scalar Dilepton Mediator

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The cold dark matter (CDM) candidate with weakly interacting massive particles can successfully explain the observed dark matter relic density in cosmic scale and the large-scale structure of the Universe. However, a number of observations at the satellite galaxy scale seem to be inconsistent with CDM simulation. This is known as the small-scale problem of CDM. In recent years, it has been demonstrated that self-interacting dark matter (SIDM) with a light mediator offers a reasonable explanation for the small-scale problem. We adopt a simple model with SIDM and focus on the effects of Sommerfeld enhancement. In this model, the dark matter candidate is a leptonic scalar particle with a light mediator. We have found favored regions of the parameter space with proper masses and coupling strength generating a relic density that is consistent with the observed CDM relic density. Furthermore, this model satisfies the constraints of recent direct searches and indirect detection for dark matter as well as the effective number of neutrinos and the observed small-scale structure of the Universe. In addition, this model with the favored parameters can resolve the discrepancies between astrophysical observations and N-body simulations.
1. Introduction

The evidence of dark matter is usually inferred from its gravitational interactions. However, weakly interacting massive particles (WIMPs) provide intuitive candidates as cold dark matter (CDM). Stable invisible WIMPs with proper mass and coupling strength can lead to a matter density that is consistent with the observed DM relic density in cosmic scale structure of the Universe [2]. In addition, CDM can account for the consistency of large scale structure ($\gtrsim 1$ Mpc) in the Universe between the astrophysical observations [3] and $N$-body simulations [4].

There are discrepancies between CDM simulations and astrophysical observations on the small scale structure of the Universe. The first one is the cusp-core problem (CCP) [5, 6]. The observed mass distributions are more flat in the central region of dwarf galaxies without a steep cusp predicted from CDM simulations. The second one is the missing satellite problem (MSP) [7]. The observed number of dwarf satellite galaxies in the Milky Way is much less than that predicted from CDM simulations. In recent year there is another problem originally from the MSP, which is called the too-big-to-fail (TBTF) [8, 9]. Most massive sub-halos generated from the CDM simulation are too massive in the Milky Way halo with circular velocity larger than 30 km/s, whereas the observed maximum circular velocities of dwarf spheroidals are less than 25 km/s. All these small scale problems can be resolved if the CDM particles are self-interacting with a light mediator to give a large self-interacting cross section (SICS) [10].

In 2018, a simple and elegant model with a self-interacting leptonic scalar dark matter ($\chi$) and a light mediator ($\zeta$) was proposed [13] to provide a CDM candidate and to solve the small scale problems of the Universe. To satisfy cosmological requirements, the mediator ($\zeta$) is chosen to have special Yukawa couplings such that it would not decay into electrons and photons [13]. We have adopted this model with a focus on Sommerfeld enhancement to determine the CDM relic density more precisely. In addition, we find the allowed parameter space that satisfies all constraints of (a) recent direct searches (b) indirect detection experiments, (c) the observed relic density, (d) effective number of neutrinos, and (e) the astrophysical observation of small-scale structure of Universe.

2. Leptonic Scalar Dark Matter Model

The leptonic scalar dark matter (LSDM) model [13] is a simple extension of the Standard Model (SM) with conservation of a $U(1)_L$ lepton number. There exist a singlet scalar ($\chi$) chosen to be the DM candidate with $L = 1$, one light singlet scalar ($\zeta$) as a mediator with $L = 2$, and three right-handed neutrinos: $\nu_{Ri}$ ($i = 1, 2, 3$).

The general scalar potential consisting of $\chi$, $\zeta$ and the SM Higgs doublet is given by [13]

$$V = \mu_0^2 \Phi^\dagger \Phi + \mu_1^2 \chi^\dagger \chi + \mu_2^2 \zeta^\dagger \zeta + (\mu_{12} \chi^\dagger \zeta + H.c.) + \frac{1}{2} \lambda_0 (\Phi^\dagger \Phi)^2 + \frac{1}{2} \lambda_1 (\chi^\dagger \chi)^2 + \frac{1}{2} \lambda_2 (\zeta^\dagger \zeta)^2 + \lambda_{01} (\Phi^\dagger \Phi) (\chi^\dagger \chi) + \lambda_{02} (\Phi^\dagger \Phi) (\zeta^\dagger \zeta) + \lambda_{12} (\chi^\dagger \chi) (\zeta^\dagger \zeta).$$  \hspace{1cm} (1)

The scalar masses have the following relations

$$m_H^2 = \lambda_0 v^2 \approx (125 \text{ GeV})^2, \quad m_\chi^2 = \mu_1^2 + \frac{1}{2} \lambda_{01} v^2, \quad m_\zeta^2 = \mu_2^2 + \frac{1}{2} \lambda_{02} v^2, \hspace{1cm} (2)$$

and the Higgs vacuum expectation value is $v = 246$ GeV.
For simplicity, let us consider a CP-conserving scalar potential with eight free real parameters:

\[ m_\chi, m_\zeta, \mu_{12}, \lambda_1, \lambda_2, \lambda_{01}, \lambda_{02}, \text{ and } \lambda_{12}. \]  

The values of \( \mu_0 \) and \( \lambda_0 \) are fixed by the minimization condition of the scalar potential and the measured Higgs mass. The \( \mu_{12}\zeta\chi^2 \) term serves as the source to enhance the self-interaction of \( \chi\chi^* \to \chi\chi^* \) through the exchange of \( \zeta \).

3. Direct Search for Leptonic Scalar Dark Matter

In this section, we focus on the search for nuclear recoils generated by the WIMP-nucleon scattering. We evaluate \( \chi \)-nucleon elastic scattering cross section for the leptonic scalar dark matter (\( \chi \)). In addition, we place limits on the relevant parameters \( \lambda_{01} \) and \( m_\chi \) with XENON1T results.

In the LSDM model with a scalar dark matter (\( \chi \)) and a light mediator (\( \zeta \)), there are eight free parameters as shown in Eq. (3). In our analysis, the scan is performed with the log-prior distributions for the input parameters as shown in the below: (i) \( m_H/2 \leq m_\chi \leq 1 \) TeV, such that \( \chi\chi \to \zeta H \) can occur, (ii) \( 0.2 \text{ GeV} \leq m_\zeta \leq 1.2 \) GeV, (iii) \( 1 \text{ GeV} \leq \mu_{12} \leq 1 \) TeV, and (iv) \( 10^{-6} \leq \lambda \leq O(1) \sim \sqrt{4\pi} \) for \( \lambda = \lambda_1, \lambda_2, \lambda_{01}, \text{ or } \lambda_{12} \).

Note that \( \lambda_{02} \) is chosen to be \( 10^{-6} \leq \lambda_{02} \leq 10^{-2} \). It is constrained by the SM Higgs invisible decay width (\( H \to \zeta\zeta^* \)), i.e.

\[ \Gamma(H \to \zeta\zeta^*) = \frac{\lambda_{02}^2 v^2}{16\pi m_H}. \]

Assuming that the invisible width is less than 10% of the Higgs width \( \Gamma_H \sim 4.12 \) MeV, we obtain the maximal value of \( \lambda_{02} \sim 6.5 \times 10^{-3} \). In addition, \( m_\zeta \) must be greater than 0.2 GeV to satisfy the cosmological constraint of effective number of neutrinos.

The scaled SI and SD cross sections (\( \sigma_{SI}^{\chi P} \) and \( \sigma_{SD}^{\chi P,n} \)) depend on two parameters: (i) the mass of leptonic scalar dark matter (\( m_\chi \)), and (ii) the coupling \( \lambda_{01} \). We apply the constraint from XENON1T experiment [15] with the upper limits of SI WIMP-nucleon scattering cross section. Fig. (1a) shows the spin independent cross section \( \sigma_{SI}^{\chi P} \) versus \( m_\chi \) with random sampling for DM (\( \chi \)) scattered off the nuclei \( ^{129,131}\text{Xe} \). In addition, a scatter plot for the same samples projected to the the plane of \( (m_\chi, \lambda_{01}) \) with the corresponding \( \lambda_{01} \) is presented in Fig. (1b). There are three groups of samples: (a) all samples with red “x” above the the upper limits of XENON1T experiment [15] are ruled out, (b) those with green “o” between the upper limits of XENON1T experiment and the curve of neutrino background [16–18] are allowed and could be detectable in future detectors, and (c) the samples with blue “.” below the curve of neutrino background, and they are allowed as well. However, it is challenging to distinguish the DM events from neutrino events.

4. Relic Density and Indirect Search

The Planck collaboration has measured cosmological parameters with very high precision [2]. The updated cold dark matter relic density is

\[ \Omega_{CDM} h^2 = 0.120 \pm 0.001. \]
Let us take a conservative approach and choose the relic density at $3\sigma$ allowed range as

$$\Omega_\chi h^2 \leq 0.123.$$  \hfill (6)

This assumption also includes the central value $\Omega_\chi h^2 \approx 0.12$.

### 4.1 Indirect Search for Leptonic Scalar Dark Matter

In the halo of the Milky Way and nearby galaxies, WIMP DM annihilation might generate high energy gamma-rays and appear in detectors such as Fermi-LAT [19], H.E.S.S. [20], HAWC [21], MAGIC [22], or VERITAS [23]. In addition, WIMP dark matter would lose energy when they pass through massive stars such as the Sun. They become gravitationally trapped and accumulate. WIMP annihilation could be sources of high energy neutrinos and might be detected by ANTARES and IceCube [24].

We evaluate the leptonic scalar DM annihilation cross section $\langle \sigma_{\text{ann}} v \rangle$ in different channels, and investigate the discovery potential as well as determine favored parameters with the Fermi-LAT and the H.E.S.S. data. In the LSDM model, the dark matter particle ($\chi$) can annihilate into a pair of SM particles such as $W^+W^-$, $Z^0Z^0$, $HH$, fermion pairs $f\bar{f}$, or $\zeta\zeta^*$ through s-channel exchange of SM Higgs boson $H$. In addition, The leptonic scalar DM can also annihilate into a pair of $\zeta\zeta^*$ or $HH$ through 4-point interactions and t-channel exchange of $\chi$, or $\nu_R\nu_R$ through s-channel exchange of $\zeta$, or a pair of $\zeta H$.

### 4.2 Sommerfeld Enhancement Effect

When the DM particles froze out in the early universe, they became non-relativistic and the non-perturbative Sommerfeld enhancement effect becomes important [25–29]. In fact, Sommerfeld effect contains an infinite series of the ladder diagrams.
We find that pairs of DM particles $\chi\chi$ can form a bound state and the wave function $\psi(\vec{r})$ satisfies the following Schrödinger equation:

$$\frac{-1}{2\mu} \nabla^2 \psi(\vec{r}) + V(\vec{r}) \psi(\vec{r}) = E \psi(\vec{r}) = \frac{1}{2} \mu v^2 \psi(\vec{r}).$$  \tag{7}$$

where $\mu = m_\chi/2$ is the reduce mass of the bound state $(\chi\chi)$, and $E = |\vec{p}|^2/2\mu \equiv \mu v^2/2$ is the total kinetic energy with the relative velocity $v = v_{\text{lab}}$. $V(r)$ is a Yukawa potential

$$V(r) = -\alpha_X \frac{e^{-mxr}}{r},$$  \tag{8}$$

where $\alpha_X = g^2_X/4\pi$ and $m_X$ is the mass of mediator.

It is well known that there is no analytic solution with a Yukawa potential in Eq. (7), but the Hulthén potential maintains the same short and long distance behavior of the Yukawa potential and has an analytic solution for s-wave function. Hence it is a good approximation to employ the Hulthén potential to obtain $|\psi|_{\text{H}}(r = 0)$ with a Yukawa potential [30].

4.3 Numerical Results for Relic Density and Indirect Search

For indirect searches, we compare our theoretical results with data from the Fermi-LAT [19] and the H.E.S.S. [20]. Fermi-LAT provides upper limits on $\langle \sigma_{\text{ann}} v \rangle$ for DM annihilating into $W^+W^-$ and the SM fermion pairs: $b\bar{b}, \mu\mu, \tau^+\tau^-, \mu^+\mu^-$ at 95% confidence level with WIMPs masses between 2 GeV to 10 TeV, while H.E.S.S. gives the upper limits on $\langle \sigma_{\text{ann}} v \rangle$ for DM annihilating into $W^+W^-$ and the SM fermion pairs: $t\bar{t}, b\bar{b}, \tau^+\tau^-, \mu^+\mu^-$ with masses from 160 GeV to 70 TeV.

Fig. 2 presents $\langle \sigma_{\text{ann}} v \rangle$ for lepton scalar DM $(\chi\chi)$ annihilating into $W^+W^-, t\bar{t}, \tau^+\tau^-, \zeta H$ and $\zeta\zeta^*$. The samples above the upper limits of Fermi-LAT and H.E.S.S are ruled out. The plots on the left-handed do not include the Sommerfeld effect but the Sommerfeld effect are considered in the plots on the right-handed side. In each plot on the left-handed side, the same color scheme as presented in Fig. 1 is used. In the panels of the right column, we show the Sommerfeld effect for the data survived from XENON1T limits, namely the samples taken from those green “o” and blue “x” in the left panels.

Without considering the Sommerfeld effect, we see that DM can only be detected with $m_\chi \gtrsim 1$ TeV via $W^+W^-$ or the Higgs resonance annihilation via $b\bar{b}$ and $\tau^+\tau^-$ channel. Clearly, the cross sections can be enhanced by the Sommerfeld effect. It is interesting that the cross section of the channel $\zeta\zeta^*$ is overall enhanced. We see that DM annihilating to a pair of $\zeta\zeta^*$ is dominant while this channel is not detectable because $\zeta$ eventually decays to $\nu_R$.

Fig. 3 shows the leptonic scalar DM thermal relic density $\Omega_\chi h^2$ as a function of $m_\chi$: (a) without Sommerfeld enhancement, and (b) with Sommerfeld enhancement effects. The horizontal line denotes the observed relic density [2]: $\Omega_{\text{obs}} h^2 = 0.120$. Since the relic density is roughly proportional to the inverse of the total $\langle \sigma_{\text{ann}} v \rangle$, the same parameters will lead to a smaller relic density with Sommerfeld enhancement. Thus, there are more regions of the parameter space satisfying the relic density requirement $\Omega_\chi h^2 < 0.123$.

5. Conclusions

We have adopted a special model that has a leptonic scalar dark matter (LSDM) $(\chi)$ with lepton number $L_\chi = 1$ and a light scalar mediator $(\zeta)$ with $L_\zeta = 2$ and three flavors of neutrino $\nu_R$. In the
Figure 2: Thermally averaged annihilation cross section times velocity for the lepton scalar dark matter (χ) ⟨σ_{ann}v⟩ in different channels. [Left] Without the Sommerfeld effect: red “×”, green “□” and blue “○” denote the scenario excluded by XENON1T, testable soon by future underground detectors, and below the neutrino floor, respectively. [Right] the Sommerfeld effect for those samples surviving from XENON1T limit, namely green “●” and blue “○” in the left panels.

LSDM model, a light mediator (ζ) makes the LSDM (χ) self interacting cross section reasonable large. Furthermore, we evaluate the the Sommerfeld effects and find significant enhancement for the SICS. That makes the LSDM model suitable to explain the small scale structure of the Universe.
We apply selection requirements for small scale structure, cold dark matter relic density (Planck), direct searches (XENON1T), and indirect detection (Fermi-LAT and H.E.S.S.), as well as cosmological constraints on right-handed neutrinos. Large regions of the parameter space in the LSDM model are found to be consistent with astrophysical and cosmological observations and collider Higgs properties. A summary is in the following for the favored ranges of parameters: (i) $0.2 \text{ GeV} \leq m_\chi \leq 0.814 \text{ GeV}$ (BBN, CMB), (ii) $276 \text{ GeV} \leq m_\nu \leq 1176 \text{ GeV}$ (implied by $m_\chi$), (iii) $75 \text{ GeV} \leq \mu_{12} \leq 634 \text{ GeV}$, and (iv) $\lambda_{02} \leq 10^{-2}$ (Higgs invisible width). In addition, the upper bound of $\lambda_{01}$, $\lambda_{02}$ and $\lambda_{12}$ are 0.27, 0.01 and 0.51, respectively.

It is interesting that almost all regions of parameter space satisfying astrophysical and cosmological observations lead to a cold dark matter relic density with the most restrictive $3\sigma$ requirement [2]. That is $0.117 \leq \Omega_\chi h^2 \leq 0.123$. A more realistic requirement should be $\Omega_\chi h^2 \leq 0.123$. That will enlarge the favored parameter space and accommodate more types of dark matter particles.

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