Random matrix ensemble with random two-body interactions in presence of a mean-field for spin one boson systems

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Abstract

For \( m \) number of bosons, carrying spin (\( S=1 \)) degree of freedom, in \( \Omega \) number of single particle orbitals, each triply degenerate, we introduce and analyze embedded Gaussian orthogonal ensemble of random matrices generated by random two-body interactions that are spin (\( S \)) scalar [BEGOE(2)-S1]. The embedding algebra is \( U(3) \supset G \supset G1 \otimes SO(3) \) with \( SO(3) \) generating spin \( S \). A method for constructing the ensembles in fixed-(\( m, S \)) space has been developed. Numerical calculations show that the form of the fixed-(\( m, S \)) density of states is close to Gaussian and level fluctuations follow GOE. Propagation formulas for the fixed-(\( m, S \)) space energy centroids and spectral variances are derived for a general one plus two-body Hamiltonian preserving spin. In addition to these, we also introduce two different pairing symmetry algebras in the space defined by BEGOE(2)-S1 and the structure of ground states is studied for each paring symmetry.

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I. INTRODUCTION

Embedded Gaussian orthogonal ensembles of one- plus two-body interactions for finite isolated interacting spin-less many boson systems [denoted by BEGOE(1+2)] were introduced and studied in detail in the last decade in [1–5]. Going beyond spin-less boson systems, very recently embedded Gaussian orthogonal ensemble of random matrices for two spices boson systems with $F$-spin degree of freedom for Hamiltonians that conserve the total $F$-spin of the $m$-boson systems [called BEGOE(1+2)-$F$] is introduced and its spectral properties are analyzed in detail in [6]; the $F$-spin for the bosons is similar to the $F$-spin in the proton-neutron interacting boson model (pnIBM) of atomic nuclei [7]. Another interesting extension of BEGOE is to a system of bosons carrying spin one ($S = 1$) degree of freedom. With random two-body interactions preserving many boson spin $S$ then generates the ensemble called hereafter BEGOE(2)-$S1$. In the presence of a mean-field the corresponding ensemble is BEGOE(1+2)-$S1$. The purpose of the present paper is to introduce this ensemble and report results of the first analysis, both numerical and analytical, of this ensemble. BEGOE(1+2)-$S1$ ensemble will be useful for spinor BEC discussed in [8, 9] and in the analysis of IBM-3 model of atomic nuclei (here spin $S$ is isospin $T$ of the bosons in IBM-3) [10, 11]. Moreover, there is a considerable interest in analyzing a variety of embedded ensembles as they can be used as generic models for many-body chaos [12, 13] and hence useful to analyze BEGOE(1+2)-$S1$ ensemble. Now we will give a preview.

In Section II, introduced is the embedded ensemble BEGOE(1+2)-$S1$ [also BEGOE(2)-$S1$] for a system of $m$ bosons in $\Omega$ number of sp orbitals that are triply degenerate with total $S$-spin being a good symmetry. A method for the numerical construction of this ensemble in fixed-$(m, S)$ space is described. In Section III, embedding algebra, $U(\Omega) \otimes [SU(3) \supset SO(3)]$ for BEGOE(1+2)-$S1$ is described. Section IV contains some numerical results for the ensemble averaged eigenvalue density, nearest neighbor spacing distribution (NNSD), width of the fluctuations in energy centroids and spectral variances. In addition, propagation formula for fixed-$(m, S)$ energy centroids for general one- plus two-body Hamiltonians that preserve $S$ and a method to propagate the spectral variances are given. In Section V, two types of pairing in BEGOE(1+2)-$S1$ space are introduced and some numerical results for ground state structure vis-a-vis the two different pairing interactions are presented. Finally, Section VI gives conclusions and future outlook.
Let us consider a system of \( m \) \((m > 2)\) bosons with spin 1 \((S = 1)\) degree of freedom and occupying \( \Omega \) number of sp levels. For convenience, in the remaining part of this section, we use the notation \( s \) for the spin quantum number of a single boson, \( s \) for the spin carried by a two boson system and for \( m > 2 \) boson systems \( S \) for the spin. Therefore, \( s = 1; s = 0, 1 \) and 2; \( S = m, m - 1, \ldots, 0 \). Similarly, the \( \hat{S}_z \) (‘hat’ denoting operator) eigenvalue is denoted by \( m_s, m_s \) and \( M_S \) respectively. Now on, the space generated by the sp levels \( i = 1, 2, \ldots, \Omega \) is referred as orbital space. Then the sp states of a boson are denoted by \( |i; s = 1, m_s\rangle \) with \( i = 1, 2, \ldots, \Omega \) and \( m_s = +1, 0 \) and \(-1\). With \( \Omega \) number of orbital degrees of freedom and three spin \((m_s)\) degrees of freedom, total number of sp states is \( N = 3\Omega \). Going further, two boson (normalized) states that are symmetric in the total orbital \( \times \) spin space are denoted by \( |(ij); s, m_s\rangle \) with \( s = 1 \times 1 = 0, 1 \) and 2; however, for \( i = j \) only \( s = 0, 2 \) are allowed.

For one plus two-body Hamiltonians preserving \( m \)-particle spin \( S \), the one-body Hamiltonian \( h(1) \) is defined by the sp energies \( \epsilon_i; i = 1, 2, \ldots, \Omega \), with average spacing \( \Delta \),

\[
\hat{h}(1) = \sum_{i=1}^{\Omega} \epsilon_i \hat{n}_i
\]

where \( \hat{n}_i = \sum_{m_s} \hat{n}_{i;m_s} \) counts number of bosons in the \( i \)-th orbit. Similarly the two-body Hamiltonian \( V(2) \) is defined by the two-body matrix elements \( V_{ijkl}^{s}(2) = \langle (kl)s, m_s | \hat{V}(2) | (ij)s, m_s \rangle \) with the two-particle spin \( s \) taking values 0, 1 and 2. These matrix elements are independent of the \( m_s \) quantum number. The \( V(2) \) matrix in two-particle space will be a direct sum three matrices generated by the three \( \hat{V}_s(2) \) operators respectively. Now the BEGOE(1+2)-S1 Hamiltonian is

\[
\left\{ \hat{H}(1 + 2) \right\} = \hat{h}(1) + \lambda_0 \left\{ \hat{V}^{s=0}(2) \right\} + \lambda_1 \left\{ \hat{V}^{s=1}(2) \right\} + \lambda_2 \left\{ \hat{V}^{s=2}(2) \right\}
\]

with three parameters \((\lambda_0, \lambda_1, \lambda_2)\). Now, BEGOE(2)-S1 ensemble for a given \((m, S)\) system is generated by defining the three parts of \( \hat{V}(2) \) in two-particle space to be independent GOE(1)s [i.e., matrix elements are independent Gaussian variables with zero center and variance unity for off-diagonal matrix elements and 2 for diagonal matrix elements] and then propagating each member of the \( \left\{ \hat{H}(1 + 2) \right\} \) to the \( m \)-particle space with a given spin \( S \) by using the geometry (direct product structure) of the \( m \)-particle space. A method for carrying out the propagation is discussed ahead. With \( \hat{h}(1) \) given by Eq. (1), the sp levels
will be triply degenerate with average spacing $\Delta$. Without loss of generality we put $\Delta = 1$ so that the $\lambda$s in Eq. (2) will be in units of $\Delta$.

For generating a many-particle basis, firstly, the sp states are arranged such that the first $\Omega$ number of sp states have $m_s = 1$, next $\Omega$ number of sp states have $m_s = 0$ and the remaining $\Omega$ sp states have $m_s = -1$. Now, the many-particle states for $m$ bosons can be obtained by distributing $m_1$ bosons in the $m_s = 1$ sp states, $m_2$ bosons in the $m_s = 0$ sp states and similarly, $m_3$ bosons in the $m_s = -1$ sp states with $m = m_1 + m_2 + m_3$. Thus, $M_S = (m_1 - m_3)$. Let us denote each distribution of $m_1$ bosons in $m_s = 1$ sp states by $m_1$, $m_2$ bosons in $m_s = 0$ sp states by $m_2$ and similarly, $m_3$ for $m_3$ bosons in $m_s = -1$ sp states. Configurations defined by $(m_1, m_2, m_3)$ will form a basis for constructing $H$ matrix in $m$-boson space. Action of the Hamiltonian operator defined by Eq. (2) on $(m_1, m_2, m_3)$ basis states with fixed-$(m, M_S = 0)$ generates the ensemble in $(m, M_S)$ space. It is important to note that the construction of the $m$-particle $H$ matrix in fixed-$(m, M_S = 0)$ space reduces to the problem of BEGOE(1+2) for spinless boson systems and hence Eq. (4) of [1] will apply. For this, we need to convert the $H$ operator into $M_S$ representation. Two boson states in $M_S$ representation can be written as $|i, m_s; j, m'_s\rangle; m_s = m_s + m'_s$. The the two particle matrix elements are

$$V'_{i,m'_s;k,m''_s;l,m'''_s}(2) = \langle i, m'_s; j, m''_s | \hat{V}(2) | k, m'_s; l, m'''_s \rangle.$$  

It is easy to apply angular momentum algebra and derive formulas for these in terms of $V_{ijkl}^s(2)$. The final formulas are,

$$V'_{i,1;j,1;k,1;l,1}(2) = V_{ijkl}^{s=2}(2),$$

$$V'_{i,1;j,0;k,1;l,0}(2) = \frac{\sqrt{(1 + \delta_{ij})(1 + \delta_{kl})}}{2} \left[ V_{ijkl}^{s=1}(2) + V_{ijkl}^{s=2}(2) \right],$$

$$V'_{i,1;j,-1;k,1;l,-1}(2) = \frac{\sqrt{(1 + \delta_{ij})(1 + \delta_{kl})}}{6} \left[ 2 V_{ijkl}^{s=0}(2) + 3 V_{ijkl}^{s=1}(2) + V_{ijkl}^{s=2}(2) \right],$$

$$V'_{i,0;j,0;k,0;l,0}(2) = \left[ \frac{1}{3} V_{ijkl}^{s=0}(2) + \frac{2}{3} V_{ijkl}^{s=2}(2) \right],$$

$$V'_{i,1;j,-1;k,0;l,0}(2) = \frac{\sqrt{(1 + \delta_{ij})}}{3} \left[ V_{ijkl}^{s=2}(2) - V_{ijkl}^{s=0}(2) \right].$$

All other $V'$ matrix elements follow by symmetries. The fact that the sp energies $\epsilon$ are
independent of \(m_s\), Eq. (3) above and Eq. (4) of [1] will allow one to construct the \(H\)-matrix in \((m_1, m_2, m_3)\) basis for a given value of \(m\) and \(M_S = 0\). Then, \(\hat{S}^2\) operator is used for projecting states with good \(S\), i.e. to covert the \(H\)-matrix into direct sum of matrices with block matrices for each allowed \(S\) value. Matrix elements of \(\hat{S}^2\) in \(s = 0, 1\) and \(2\) space are \(-4\), \(-2\) and \(2\) respectively. This procedure has been implemented and computer programmes are developed. Some numerical results obtained using these programmes will be discussed in Section III. Let us add that the BEGOE(1+2)-S1 ensemble is defined by five parameters \((\Omega, m, \lambda_0, \lambda_1, \lambda_2)\) with \(\lambda_s\) in units of \(\Delta\).

### III. \(U(\Omega) \otimes [SU(3) \supset SO(3)]\) EMBEDDING ALGEBRA

Embedding algebra for BEGOE(1+2)-S1 is not unique and following the earlier results for the IBM-3 model of atomic nuclei [10, 11], it is possible to identify two algebras. They are: (i) \(U(3\Omega) \supset U(\Omega) \otimes [U(3) \supset SO(3)]\); (ii) \(U(3\Omega) \supset SO(3\Omega) \supset SO(\Omega) \otimes SO(3)\). Here we will consider (i) and later in Section V we will consider briefly (ii).

Firstly, the spectrum generating algebra \(U(3\Omega)\) is generated by the \((3\Omega)^2\) number of operators \(u^k_q(i, j)\) where

\[
u^k_q(i, j) = \left( \begin{array}{c} b_{i; s=1}^\dagger \tilde{b}_{j; s=1} \\ \tilde{b}_{i; s=1} \end{array} \right)^k_q ; k = 0, 1, 2 \text{ and } i, j = 1, 2, \ldots, \Omega .
\]

Note that \(u^k\) are given in angular momentum coupled representation with \(k = s \times s = 0, 1, 2\). Also, \(b^\dagger\) are one boson creation operators, \(b\) are one boson annihilation operators and \(\tilde{b}_{i; 1, m_a} = (-1)^{1+m_s} b_{i; 1, -m_a}\). The quadratic Casimir invariant of \(U(3\Omega)\) is

\[
\hat{C}_2(U(3\Omega)) = \sum_{i,j,k} u^k(i, j) \cdot u^k(j, i).
\]

Note that \(T^k \cdot U^k = (-1)^k \sqrt{(2k + 1)} (T^k U^k)^0\). In terms of the number operator \(\hat{n}\),

\[
\hat{n} = \sum_{i, m_a} b_{i; 1, m_a}^\dagger b_{i; 1, m_a},
\]

we have

\[
\hat{C}_2(U(3\Omega)) = \hat{n}(\hat{n} + 3\Omega - 1).
\]

All \(m\)-boson states transform as the symmetric irrep \(\{m\}\) w.r.t. \(U(3\Omega)\) algebra and

\[
\left\langle \hat{C}_2(U(3\Omega)) \right\rangle^{(m)} = m(m + 3\Omega - 1).
\]
Using the results given in [14] it is easy to write the generators of the algebras $U(\Omega)$ and $SU(3)$ in $U(3\Omega) \supset U(\Omega) \otimes SU(3)$. The $U(\Omega)$ generators are $g(i, j)$ where,

$$g(i, j) = \sqrt{3} \left( b_{i,s=1}^\dagger b_{j,s=1}^\dagger \right)^0 ; i, j = 1, 2, \ldots, \Omega \quad (9)$$

and they are $\Omega^2$ in number. Similarly, $SU(3)$ algebra is generated by the eight operators $h_k^{1,2}$ where,

$$h_k^k = \sum_i \left( b_{i,s=1}^\dagger b_{i,s=1} \right)_q^k ; k = 1, 2 \quad (10)$$

It is useful to mention that $(h^0, h_1^1, h_2^2)$ generate $U(3)$ algebra and $U(3) \supset SU(3)$. The quadratic Casimir invariants of $U(\Omega)$ and $SU(3)$ algebras are,

$$\hat{C}_2(U(\Omega)) = \sum_{i,j} g(i, j) \cdot g(j, i) ,$$

$$\hat{C}_2(SU(3)) = \frac{3}{2} \sum_{k=1,2} h^k \cdot h^k \quad (11)$$

The irreps of $U(\Omega)$ can be represented by Young tableaux $\{f\} = \{f_1, f_2, \ldots, f_\Omega\}$, $\sum_i f_i = m$. However, as we are dealing with boson systems (i.e. the only allowed $U(3\Omega)$ irrep being $\{m\}$), the irreps of $U(\Omega)$ and $U(3)$ should be represented by the same $\{f\}$. Therefore, $\{f\}$ will be maximum of three rows. The $U(\Omega)$ and $SU(3)$ equivalence gives a relationship between their quadratic Casimir invariants,

$$\hat{C}_2(U(\Omega)) = \hat{C}_2(U(3)) + (\Omega - 3) \hat{n} ,$$

$$\hat{C}_2(U(3)) = \sum_{k=0,1,2} h^k \cdot h^k = \frac{2}{3} \hat{C}_2(SU(3)) + \frac{\hat{n}^2}{3} \quad (12)$$

These relations are easy to prove using Eqs. (9)- (11). Given the $U(\Omega)$ irrep $\{f_1 f_2 f_3\}$, the corresponding $SU(3)$ irrep in Elliott’s notation [15] is given by $(\lambda\mu)$ where $\lambda = f_1 - f_2$ and $\mu = f_2 - f_3$. Thus,

$$\{m\}_{U(3\Omega)} \rightarrow \left[ \{f_1 f_2 f_3\}_{U(\Omega)} \right] [ (\lambda\mu)_{SU(3)} ] ;$$

$$f_1 + f_2 + f_3 = m, \quad f_1 \geq f_2 \geq f_3 \geq 0 \quad ,$$

$$\lambda = f_1 - f_2, \quad \mu = f_2 - f_3 \quad .$$

(13)

Using Eq. (13) it is easy to write, for a given $m$, all the allowed $SU(3)$ and equivalently $U(\Omega)$ irreps. Eigenvalues of $\hat{C}_2(SU(3))$ are given by

$$\left\langle \hat{C}_2(SU(3)) \right\rangle^{(\lambda\mu)} = C_2(\lambda\mu) = [ \lambda^2 + \mu^2 + \lambda\mu + 3(\lambda + \mu) ] \quad .$$

(14)
Let us add that the $SU(3)$ algebra also has a cubic invariant $C_2(SU(3))$ and its matrix elements are [16],
\[
\left\langle \hat{C}_3(SU(3)) \right\rangle^{(\lambda \mu)} = C_3(\lambda \mu) = \frac{2}{9} (\lambda - \mu)(2\lambda + \mu + 3)(\lambda + 2\mu + 3) .
\]

The $SO(3)$ subalgebra of $SU(3)$ generates spin $S$. The spin generators are
\[
S^i_q = \sqrt{2} h^i_q , \quad \hat{S}^2 = C_2(SO(3)) = S^1 \cdot S^1 , \quad \left\langle \hat{S}^2 \right\rangle^S = S(S + 1) .
\]

Given a $(\lambda \mu)$, the allowed $S$ values follow from Elliott’s rules [15] and this introduces a ‘$K$’ quantum number,
\[
K = \min(\lambda, \mu), \min(\lambda, \mu) - 2, \ldots, 0 \text{ or } 1 ,
\]
\[
S = \max(\lambda, \mu), \max(\lambda, \mu) - 2, \ldots, 0 \text{ or } 1 \text{ for } K = 0 ,
\]
\[
= K, K + 1, K + 2, \ldots, K + \max(\lambda, \mu) \text{ for } K \neq 0 .
\]

Eq. (17) gives $d_{(\lambda \mu)}(S)$, the number of times a given $S$ appears in a $(\lambda \mu)$ irrep. Similarly the number of substates that belong to a $U(\Omega)$ irrep $\{f_1 f_2 f_3\}$ are given by $d_\Omega(f_1 f_2 f_3)$ where [17],
\[
d_\Omega(f_1 f_2 f_3) = \begin{vmatrix} d_\Omega(f_1) & d_\Omega(f_1 + 1) & d_\Omega(f_1 + 2) \\ d_\Omega(f_2 - 1) & d_\Omega(f_2) & d_\Omega(f_2 + 1) \\ d_\Omega(f_3 - 2) & d_\Omega(f_3 - 1) & d_\Omega(f_3) \end{vmatrix} .
\]

Here, $d_\Omega(\{g\}) = (\Omega + g - 1)$ and $d_\Omega(\{g\}) = 0$ for $g < 0$. Note that the determinant in Eq. (18) involves only symmetric $U(\Omega)$ irreps. Using the $U(3\Omega) \supset U(\Omega) \otimes [U(3) \supset SO(3)]$ algebra, $m$ bosons states can be written as $|m; \{f_1 f_2 f_3\} \alpha; (\lambda \mu) K S M_S\rangle$; The number of $\alpha$ values is $d_\Omega(f_1 f_2 f_3)$, $K$ values follow from Eq. (17) and $-S \leq M_S \leq S$. Note that $m$ and $(\lambda \mu)$ give a unique $\{f_1 f_2 f_3\}$. Therefore $H$-matrix dimension in fixed-$(m, S)$ space is given by
\[
d(m, S) = \sum_{\{f_1 f_2 f_3\} \in m} d_\Omega(f_1 f_2 f_3) d_{(\lambda \mu)}(S) ,
\]
and they will satisfy the sum rule $\sum_S (2S + 1)d(m, S) = \binom{3\Omega + m - 1}{m}$. Also, the dimension $D(m, M_S = 0)$ of the $H$-matrix in the basis discussed earlier is $D(m, M_S = 0) = \sum_{S \leq m} d(m, S)$. For example, for $(\Omega = 4, m = 8)$, the dimensions $d(m, S)$ for $S = 0 - 8$ are 714, 1260, 2100, 1855, 1841, 1144, 840, 315 and 165 respectively. Similarly, for $(\Omega = 6, m = 10)$, the dimensions for $S = 0 - 10$ are 51309, 123585, 183771, 189630, 178290, 133497, 94347, 51645, 27027, 9009 and 3003 respectively. Because of these very large dimensions, numerical analysis of BEGOE(1+2)-S1 ensemble is quite difficult.
IV. RESULTS FOR SPECTRAL PROPERTIES: PROPAGATION OF ENERGY CENTROIDS AND SPECTRAL VARIANCES

A. Eigenvalue density and NNSD: numerical results

Using the method described in Section II, in some examples BEGOE(2)-S1 ensemble has been constructed and analyzed are eigenvalue density and spectral fluctuations. Figure 1 presents the results for the ensemble-averaged fixed-\((m, S)\) eigenvalue density \(\rho_{m, S}(E)\) for the BEGOE(2)-S1 ensemble defined by \(h(1) = 0\) in Eq. (2). We have considered a 100-member BEGOE(2)-S1 ensemble with \(m = 8\) and \(\Omega = 4\). The strengths of the two-body interaction in the \(s = 0\), \(s = 1\) and \(s = 2\) channels are chosen to be equal i.e. \(\lambda_0 = \lambda_1 = \lambda_2\). In the construction of the ensemble averaged eigenvalue densities, the spectra of each member of the ensemble are first zero centered and scaled to unit width. The eigenvalues are then denoted by \(\hat{E}\). Given the fixed-\((m, S)\) eigenvalue centroids \(E_c(m, S)\) and spectral widths \(\sigma(m, S)\), \(\hat{E} = [E - E_c(m, S)]/\sigma(m, S)\). Then the histograms for the density are generated by combining the eigenvalues \(\hat{E}\) from all the members of the ensemble. In the figure, histograms are constructed with a bin size equal to 0.2. Results are shown in Fig. 1 for \(S = 0, 4\) and \(8\) values. It is clearly seen that the eigenvalue densities are close to Gaussian also the agreements with Edgeworth (ED) corrected Gaussians are excellent.

The nearest neighbor spacing distribution (NNSD), which gives information about level repulsion, is of GOE type for spin-less BEGOE(2) [1] and BEGOE(2)-F [6]. In Fig. 2 NNSD results are shown for BEGOE(2)-S1 with \(m = 8\) and \(\Omega = 4\) for selected spin values. The NNSDs are obtained by unfolding each spectrum in the ensemble, using the method described in [1], with the smooth density as a corrected Gaussian with corrections involving up to 6th order moments of the density function. In the calculations, 80% of the eigenvalues (dropping 10% from both ends of the spectrum) from each member are employed. It is clearly seen from the figures that the NNSDs are close to the GOE (Wigner) form.

Previously it was shown that BEGOE(1+2) for spinless boson systems [1, 3] and BEGOE(1+2)-F for two species boson systems [6] generate Gaussian eigenvalue densities in the dense limit and fluctuations follow GOE in absence of the mean-field. Therefore, combining these with the results in Figs. 1,2, we can conclude that for finite isolated interacting boson systems the eigenvalue density will be generically of Gaussian form and
FIG. 1. Ensemble averaged eigenvalue density $\rho^{m,S}(\hat{E})$ vs normalized energy, $\hat{E} = \frac{E - E_c(m,S)}{\sigma}$, for a 100 member BEGOE(2)-S1 ensemble with $\Omega = 4$, $m = 8$ and spin $S=0$, 4 and 8. The red curves give Gaussian representation while the green curves are Edgeworth corrected Gaussians (ED). The ensemble averaged values of excess ($\gamma_2$) parameters are as shown in figure. Note that Skewness $\gamma_1 \sim 0$ in all cases. In the plots, the state densities, for a given spin $S$, are normalized to dimension $d(m,S)$. Note that the total dimensionality of $H$-matrix here is $\sum_S d(m,S) = 10234$.

fluctuations, in absence of the mean-field, follow GOE. As discussed in [3, 6], with mean-field, the interaction strength has to be larger than a critical value for the fluctuations to change from Poisson like to GOE.

B. Propagation of energy centroids and spectral variances

As the eigenvalue density is close to Gaussian, it is useful to derive formulas for energy centroids and spectral variances in terms of sp energies $\epsilon_i$ and the two-particle $V(2)$ matrix elements $V_{ijkl}$. They will also allow us to study, numerically, fluctuations in energy centroids and spectral variances. Simple propagation equation for the fixed-$(m,S)$ energy centroids $\langle H \rangle_{m,S}$ in terms of the scalars $\hat{n}$ and $S^2$ operators [their eigenvalues are $m$ and $S(S+1)$] is not possible. This is easily seen from the fact that upto 2 bosons, we have 5 states ($m = 0, S = 0; m = 1, S = 1; m = 2, S = 0, 1, 2$) but only 4 scalar operators $(1, \hat{n}, \hat{n}^2, \hat{S}^2)$. For the missing operator we can use $\hat{C}_2(SU(3)$ but then only fixed-$(m, (\lambda \mu)S)$ averages will
FIG. 2. Ensemble averaged Nearest Neighbor Spacing Distribution (NNSD) histogram for a 100 member BEGOE(2)-S1 with $m = 8$ and $\Omega = 4$. Results are shown for the spin values $S=0$, 4 and 8. Here $x$ is in the units of local mean spacing. Results are compared with Poisson and GOE (Wigner) forms.

propagate [18]. The propagation equation is,

$$
\langle \hat{H}(1 + 2)^{m,(\lambda \mu),S} \rangle = \langle \hat{h}(1) + \hat{V}(2)^{m,(\lambda \mu),S} \rangle = m \langle \hat{h}(1) \rangle^{1,(10),1}
$$

$$
+ \left[ \frac{m}{6} + \frac{m^2}{18} + \frac{C_2(\lambda \mu)}{9} - \frac{S(S + 1)}{6} \right] \langle \hat{V}(2) \rangle^{2,(20),0}
$$

$$
+ \left[ -\frac{5m}{6} + \frac{5m^2}{18} + \frac{C_2(\lambda \mu)}{18} + \frac{S(S + 1)}{6} \right] \langle \hat{V}(2) \rangle^{2,(20),2}
$$

$$
+ \left[ \frac{m}{2} + \frac{m^2}{6} - \frac{C_2(\lambda \mu)}{6} \right] \langle \hat{V}(2) \rangle^{2,(01),1} \right] .
$$

(20)

Now summing over all $(\lambda \mu)$ irreps that contain a given $S$ will give $\langle \hat{H}(1 + 2)^{m,S} \rangle$. This is used to verify the codes we have developed for constructing BEGOE(1+2)-S1 members. Propagation equation for spectral variances $\langle [\hat{H}(1 + 2)]^2 \rangle^{m,S}$ is more complicated. Just as with energy centroids, it is possible to propagate the variances $\langle [\hat{H}(1 + 2)]^2 \rangle^{m,(\lambda \mu),S}$. Towards this, first it should be noted that upto $m = 4$, there are 19 states as shown in Table 1. Therefore, for propagation we need 19 $SO(3)$ scalars that are of maximum body
TABLE I. \{f\}, (\lambda \mu) and S labels for \(m \leq 4\) bosons and the averages of \(\hat{X}_3\) and \(\hat{X}_4\) operators

| \(m\) | \{\(f\)\} | (\(\lambda \mu\)) | \(S\) | \langle \hat{X}_3 \rangle | \langle \hat{X}_4 \rangle |
|---|---|---|---|---|---|
| 0 | \{0\} | (00) | 0 | 0 | 0 |
| 1 | \{1\} | (10) | 1 | 5 | −25 |
| 2 | \{2\} | (20) | 0 | 0 | 0 |
| | | 2 | 21 | −147 |
| | \{11\} | (01) | 1 | −5 | −25 |
| 3 | \{3\} | (30) | 1 | 9 | −81 |
| | | 3 | 54 | −486 |
| | \{21\} | (11) | 1 | 0 | −135 |
| | | 2 | 0 | −81 |
| | \{111\} | (00) | 0 | 0 | 0 |
| 4 | \{4\} | (40) | 0 | 0 | 0 |
| | | 2 | 33 | −363 |
| | | 4 | 110 | −1210 |
| | \{31\} | (21) | 1 | −7 | −121 |
| | | 2 | 21 | −459 |
| | | 3 | 18 | −246 |
| | \{22\} | (02) | 0 | 0 | 0 |
| | | 2 | −21 | −147 |
| | \{211\} | (10) | 1 | 5 | −25 |

rank 4. For this the invariants \(\hat{n}, \hat{S}^2, \hat{C}_2(SU(3))\) and \(\hat{C}_3(SU(3))\) will not suffice as they will give only 15 scalar operators. The missing three operators can be constructed using the \(SU(3) \supset SO(3)\) integrity basis operators \(\hat{X}_3\) and \(\hat{X}_4\) that are 3– and 4–body in nature; see
Formulas for the averages $X_i((\lambda \mu), S) = \langle \hat{X}_i \rangle^{(\lambda \mu), S}$ are given by Eqs. (8)-(10) of [16] and they involve $SU(3) \supset SO(3)$ reduced Wigner coefficients. Using the programmes for these, given in [19], averages for $\hat{X}_3$ and $\hat{X}_4$ in the 19 states with $m \leq 4$ are calculated and the results are given in Table 1. Eqs. (14) and (15) respectively will give $C_2(\lambda \mu)$ and $C_3(\lambda \mu)$. Propagation equation for spectral variances over fixed-$(\lambda \mu), S$ space can be written as,

$$\langle \hat{H}^2 \rangle^{m,(\lambda \mu),S} = \sum_{i=1}^{19} a_i C_i ;$$

where

- $C_1 = 1$,
- $C_2 = m$,
- $C_3 = m^2$,
- $C_4 = m^3$,
- $C_5 = m^4$,
- $C_6 = C_2(\lambda \mu)$,
- $C_7 = m C_2(\lambda \mu)$,
- $C_8 = m^2 C_2(\lambda \mu)$,
- $C_9 = S(S+1)$,
- $C_{10} = m S(S+1)$,
- $C_{11} = m^2 S(S+1)$,
- $C_{12} = S(S+1) C_2(\lambda \mu)$,
- $C_{13} = [S(S+1)]^2$,
- $C_{14} = [C_2(\lambda \mu)]^2$,
- $C_{15} = C_3(\lambda \mu)$,
- $C_{16} = m C_3(\lambda \mu)$,
- $C_{17} = X_3[(\lambda \mu), S]$,
- $C_{18} = m X_3[(\lambda \mu), S]$,
- $C_{19} = X_4[(\lambda \mu), S]$.

As we know $\langle C_i \rangle^{m,(\lambda \mu),S}$ for $m \leq 4$, we can use Using $\langle \hat{H}^2 \rangle^{m,(\lambda \mu),S}$ for $m \leq 4$ as inputs (they can be calculated by explicit construction of the Hamiltonian matrices using the method discussed in Section II) one can solve Eq. (22) to obtain the $a_i$’s. Then, Eq. (22) can be used to calculate $\langle \hat{H}^2 \rangle^{m,(\lambda \mu),S}$ for any $m$, $(\lambda \mu)$ and $S$. However we still need to evaluate numerically $X_3[(\lambda \mu), S]$ and $X_4[(\lambda \mu), S]$. Their values are shown for $m = 6$ and $8$ examples in Table 2. Spectral variances $\langle \hat{H}^2 \rangle^{m,S}$ over fixed-$S$ space can be obtained easily using $\langle \hat{H}^2 \rangle^{m,(\lambda \mu),S}$. Let us add that there are methods [20], though much more cumbersome, that will give directly $\langle \hat{H}^2 \rangle^{m,S}$. One such method is to use $(m_1, m_2, m_3)$ configurations introduced in Section II and evaluate traces over these spaces. Here, trace propagation is simple for both $H$ and $H^2$ averages and also $(m_1, m_2, m_3)$ configuration have a definite $M_S$ value. Now, a subtraction procedure using $\langle H^p \rangle^{(m_1, m_2, m_3)}$, $p = 1, 2$ will give fixed $(m, S)$ energy centroids and spectral variances. This procedure is being implemented and results of this will be reported elsewhere.

Calculation of energy centroids and spectral variances for each member of the ensemble will allow us to examine the covariances in these quantities. For example, normalized
covariances in energy centroids is defined by

\[
\Sigma_{11}(m, S : m', S') = \frac{\langle H \rangle^{m,S} \langle H \rangle^{m',S'} - \left\{ \langle H \rangle^{m,S} \right\} \left\{ \langle H \rangle^{m',S'} \right\}}{\sqrt{\left\{ \langle H^2 \rangle^{m,S} \right\} \left\{ \langle H^2 \rangle^{m',S'} \right\}}} \tag{23}
\]

For \((m, S) = (m', S')\) they will give information about fluctuations and in particular about level motion in the ensemble [1]. For \((m, S) \neq (m', S')\), the covariances (cross correlations) are non-zero for BEGOE while they will be zero for independent GOE representation for the \(m\) boson Hamiltonian matrices with different \(m\) or \(S\) (with fixed \(\Omega\)). We have computed self-correlations \([\Sigma_{11}(m, S : m, S)]^{1/2}\) as a function of spin \(S\) for 100 member BEGOE(2)-S1 with \((m = 8, \Omega = 4)\) and the results are shown in Fig. 3. It is seen that the centroid fluctuations are large as \([\Sigma_{11}]^{1/2} \sim 28\%\). However, the variation with spin \(S\) is weak. We have also calculated in some examples the variation of the average of spectral variances with \(S\) and the width of the fluctuations of the spectral widths over the ensemble. Results are shown in Fig. 4 for a \((m = 8, \Omega = 4)\) system. It is clearly seen from the figure that the variances are almost constant for lower spins and increases for \(S\) close to the maximum value of \(S\); a similar result is known for fermion systems [21]. Also, as seen from the figure, the width of the fluctuations in spectral widths is much smaller unlike the width of the fluctuations in energy centroids. Let us add that near constancy of widths is a feature of many-body chaos [22, 23].

V. PAIRING ALGEBRAS AND GROUND STATE STRUCTURE

In the BEGOE(1+2)-S1 space, it is possible to identify two different pairing algebras (each defining a particular type of pairing) and they follow from the results in [11, 14, 24]. One of them corresponds to the \(SO(\Omega)\) algebra in \(U(3\Omega) \supset [U(\Omega) \supset SO(\Omega)] \otimes [U(3) \supset SO(3)]\) and we refer to this as \(SO(\Omega) \sim SU(3)\) pairing. The other corresponds to the \(SO(3\Omega)\) in \(U(3\Omega) \supset SO(3\Omega) \supset SO(\Omega) \otimes SO(3)\). Note hat both the algebras have \(SO(3)\) subalgebra that generates the spin \(S\). Here below we will give some details of these pairing algebras. Inclusion of pairing Hamiltonians in BEGOE(1+2)-S1 \(H\) will alter the structure of ground states and this will be discussed in Section V C.
FIG. 3. $[\Sigma_{11}(m, S : m, S)]^{1/2}$ giving width of the fluctuations in energy centroids scaled to the spectrum width, as a function of spin $S$ for BEGOE(2)-S1 with $(m = 8, \Omega = 4)$.

A. $SO(\Omega) - SU(3)$ pairing

Following the results given in [11, 14, 24] it is easy to identify the $\Omega(\Omega - 1)/2$ number of generators $U(i, j), i < j$ of $SO(\Omega)$ in $U(3\Omega) \supset [U(\Omega) \supset SO(\Omega)] \otimes [U(3) \supset SO(3)]$,

$$U(i, j) = \sqrt{\alpha(i, j)} \left[ g(i, j) + \alpha(i, j) g(j, i) \right], \quad i < j;$$

$$|\alpha(i, j)|^2 = 1, \quad \alpha(i, j) = \alpha(j, i), \quad \alpha(i, j)\alpha(j, k) = -\alpha(i, k).$$

Note that $g(i, j)$ are defined in Eq. (9). The quadratic Casimir invariant of $SO(\Omega)$ is,

$$\hat{C}_2(SO(\Omega)) = \sum_{i < j} U(i, j) \cdot U(j, i).$$

Applying Eq. (24) now gives,

$$\hat{C}_2(SO(\Omega)) = \sum_{i < j} \alpha(i, j) \left[ g(i, j) \cdot g(i, j) + g(j, i) \cdot g(j, i) + 2\alpha(i, j) g(i, j)g(j, i) \right]$$

$$= \sum_{i \neq j} g(i, j) \cdot g(j, i) + \sum_{i \neq j} \alpha(i, j) g(i, j) \cdot g(i, j)$$

$$= \hat{C}_2(U(\Omega)) - \sum_{i,j} \beta_i\beta_j g(i, j) \cdot g(i, j);$$

$$\beta_i\beta_j = -\alpha(i, j), \text{ for } i \neq j, \quad |\beta_i|^2 = 1.$$
Here we have introduced $\beta_i$’s and the $\alpha(i, j)$ are defined in Eq. (24). Now defining the pairing operator $P_q^k$, $k = 0, 2$ as

$$P_q^k = \sum_i \beta_i \left( b_{i;1}^\dagger b_{i;1}^\dagger \right)_q^k; \quad k = 0, 2$$  \hspace{1cm} (27)$$

it is easy to see that,

$$H_P = \sum_{k=0,2/q} P_q^k (P_q^k)\dagger = \hat{C}_2(U(\Omega)) - \hat{C}_2(SO(\Omega)) - \hat{n}$$

$$= \frac{2}{3} \hat{C}_2(SU(3)) - \hat{C}_2(SO(\Omega)) - (\Omega - 4)\hat{n} + \frac{\hat{n}^2}{3}. \hspace{1cm} (28)$$

In the final form above we have used Eqs. (12). Thus the pairing Hamiltonian in the $U(3\Omega) \supset [U(\Omega) \supset SO(\Omega)] \otimes [U(3) \supset SO(3)]$ algebra is a sum of $k = 0$ and 2 pairs and it is simply related to the $SO(\Omega)$ and $SU(3)$ algebras. It is possible enumerate the irreps of $SO(\Omega)$ given a $U(\Omega)$ or equivalently $SU(3)$ irrep (for a given $m$); see [24] and references therein. In terms of these irrep labels and $SU(3)$ labels $(\lambda \mu)$, eigenvalues of $H_P$ will follow from Eq. (28). This and the complimentary non-compact $sp(6)$ pairing algebra generated by $P_q^k$, $(P_q^k)\dagger, h_q^1, h_q^2$ and $\hat{n}$ will be discussed elsewhere. For a recent review on complimentary
TABLE II. $(\lambda \mu), S, \hat{X}_3$ and $\hat{X}_4$ values for $m = 6$ and 8.

| $m$ | $(\lambda \mu)$ | $S$ | $\hat{X}_3$ | $\hat{X}_4$ | $m$ | $(\lambda \mu)$ | $S$ | $\hat{X}_3$ | $\hat{X}_4$ |
|-----|----------------|-----|------------|-------------|-----|----------------|-----|------------|-------------|
| 6   | (60)          | 0   | 0          | 0           | 8   | (61)          | 6   | 375        | −7965       |
|     | (60)          | 2   | 45         | −675        |     | (61)          | 7   | 340        | −5116       |
|     | (60)          | 4   | 150        | −2250       |     | (42)          | 0   | 0          | 0           |
|     | (60)          | 6   | 315        | −4725       |     | (42)          | 2   | 0          | −1131       |
|     | (41)          | 1   | −9         | −297        |     | (42)          | 3   | 0          | −2046       |
|     | (41)          | 2   | 27         | −891        |     | (42)          | 4   | 91         | −2566       |
|     | (41)          | 3   | 36         | −702        |     | (42)          | 5   | 117        | −3567       |
|     | (41)          | 4   | 132        | −2466       |     | (42)          | 6   | 105        | −1701       |
|     | (41)          | 5   | 117        | −1431       |     | (50)          | 1   | 13         | −169        |
|     | (30)          | 1   | 9          | −81         |     | (50)          | 3   | 78         | −1014       |
|     | (30)          | 3   | 54         | −486        |     | (50)          | 5   | 195        | −2535       |
|     | (03)          | 1   | −9         | −81         |     | (23)          | 1   | −11        | −193        |
|     | (03)          | 3   | −54        | −486        |     | (23)          | 2   | 33         | −1107       |
|     | (11)          | 1   | 0          | −135        |     | (23)          | 3   | −33        | −1314       |
|     | (11)          | 2   | 0          | −81         |     | (23)          | 4   | −44        | −1954       |
|     | (00)          | 0   | 0          | 0           |     | (23)          | 5   | −39        | −879        |
|     | (22)          | 0   | 0          | 0           |     | (31)          | 1   | −2         | −319        |
|     | (22)          | 2   | 0          | −603        |     | (31)          | 2   | 6          | −297        |
|     | (22)          | 3   | 0          | −990        |     | (31)          | 3   | 63         | −1164       |
|     | (22)          | 4   | 0          | −450        |     | (31)          | 4   | 55         | −640        |
| 8   | (80)          | 0   | 0          | 0           |     | (04)          | 0   | 0          | 0           |
|     | (80)          | 2   | 57         | −1083       |     | (04)          | 2   | −33        | −363        |
|     | (80)          | 4   | 190        | −3610       |     | (04)          | 4   | −110       | −1210       |
|     | (80)          | 6   | 399        | −7581       |     | (12)          | 1   | 7          | −121        |
|     | (80)          | 8   | 684        | −12996      |     | (12)          | 2   | −21        | −459        |
|     | (61)          | 1   | −11        | −553        |     | (12)          | 3   | −18        | −246        |
|     | (61)          | 2   | 33         | −1467       |     | (20)          | 0   | 0          | 0           |
|     | (61)          | 3   | 54         | −1398       |     | (20)          | 2   | 21         | −147        |
|     | (61)          | 4   | 166        | −3994       |     | (01)          | 1   | −5         | −25         |
|     | (61)          | 5   | 171        | −2919       |     |

Algebras see [25]. Finally, the two-particle matrix elements of $H_P$ are $V_{ii'jj'}^{s=0} = 1$, $V_{ii'jj'}^{s=2} = 1$ and all other matrix elements are zero.

Before going further, it is useful to mention that the Majorana operator ($\hat{M}$) that changes the space labels $(i, j)$ in a two-particle states without changing the spin labels $m_s$ related in
a simple manner to $\hat{C}_2(U(3))$. Denoting the spin labels by $\alpha, \beta, \ldots$, we have

$$\hat{M} = \sum_{i,j,\alpha,\beta} b_{j,\alpha}^\dagger b_{i,\beta}^\dagger \left( b_{i,\alpha}^\dagger b_{j,\beta}^\dagger \right)^\dagger = \hat{C}_2(U(3)) - 3\hat{n}.$$  \hspace{1cm} (29)

**B. $SO(3\Omega)$ pairing**

Second pairing algebra follows from the recognition that $U(3\Omega)$ admits $SO(3\Omega)$ subalgebra and as we will see ahead, the pairing here is generated by $k = 0$ pairs $b_i^\dagger \cdot b_i^\dagger$ alone. Following the results in [14] the generators of $SO(3\Omega)$ are easy to identify and they are,

$$u_k^{q=1}(i,i) \ ; \ i = 1, 2, \ldots, \Omega ,$$

$$V_k^{q}(i,j) = \sqrt{(-1)^k \alpha(i,j)} \left[ u_q^k(i,j) + \alpha(i,j) (-1)^k u_q^k(j,i) \right] , \ i < j ;$$

$$|\alpha(i,j)|^2 = 1, \ \alpha(i,j) = \alpha(j,i), \ \alpha(i,j)\alpha(j,k) = -\alpha(i,k).$$

The operators $u_q^k$ are defined by Eq. (4). Carrying out angular momentum algebra the following relation between the quadratic Casimir invariants $\hat{C}_2(SO(3\Omega))$ and $\hat{C}_2(U(3\Omega))$, of $SO(\Omega)$ and $U(3\Omega)$, can be established using Eqs. (30) and (5),

$$\hat{C}_2(SO(3\Omega)) = 2 \sum_i u_i^1(i,i) \cdot u_i^1(i,i) + \sum_{i < j} V_k(i,j) \cdot V_k(i,j)$$

$$= \hat{C}_2(U(3\Omega)) - \sum_{i,k} (-1)^k u_k(i,i) \cdot u_k(i,i) + \sum_{i\neq j,k} (-1)^k \alpha(i,j) u_k(i,j) \cdot u_k(i,j).$$

Introducing the pairing operator $P_+$,

$$P_+ = \sum_i \gamma_i P_+(i) = \frac{1}{2} \sum_i \gamma_i b_{i,1}^\dagger \cdot b_{i,1}^\dagger , \ \ P_- = (P_+)^\dagger$$

we can prove the following relationship between $\hat{C}_2(SO(3\Omega))$ and the pairing Hamiltonian $H_P = 4P_+P_-,$

$$4H_P = 4P_+P_- = -\hat{n} + \hat{C}_2(U(3\Omega)) - \hat{C}_2(SO(3\Omega))$$

$$= \hat{n}(\hat{n} + 3\Omega - 2) - \hat{C}_2(SO(3\Omega)) ;$$

$$\gamma_i\gamma_j = -\alpha(i,j), \ \text{for} \ i \neq j, \ \ |\gamma_i|^2 = 1 .$$

The $\beta \leftrightarrow \alpha$ relation is needed for the correspondence between $H_P$ and $\hat{C}_2(SO(3\Omega))$. Important point now being that the three operators $P_+$, $P_-$ and $P_0 = (\Omega + \hat{n})/2$ will form a $SU(1,1)$ algebra complimentary to $SO(3\Omega)$. Thus the $SO(3\Omega)$ pairing is much simpler. With $U(3\Omega)$ irreps being $\{m\}$, the $SO(3\Omega)$ irreps are labeled by the seniority quantum number $\omega$ where,

$$\omega = m, m - 2, \ldots, 0 \ or \ 1 .$$

(33)
and $H_P$ eigenvalues are

$$\langle H_P \rangle^{m,\omega} = \frac{1}{4} (m - \omega)(m + \omega + 3\Omega - 2) . \quad (35)$$

The two particle matrix elements of $H_P$ are simply $V_{iijj}^{s=0} = 1$ and all other matrix elements are zero.

C. Ground state structure

With two different pairings in the BEGOE(1+2)-S1 space, analysis of properties of spin one boson systems with the following extended Hamiltonian $H_{ext}$ will be interesting and useful,

$$\left\{ \hat{H}_{ext} \right\} = \hat{h}(1) + \lambda_0 \left\{ \hat{V}^{s=0}(2) \right\} + \lambda_1 \left\{ \hat{V}^{s=1}(2) \right\} + \lambda_2 \left\{ \hat{V}^{s=2}(2) \right\} + \lambda_{p1} H_P + \lambda_{p2} H_P + \lambda_S \hat{S}^2 . \quad (36)$$

As an example, shown in Fig. 5 are the results for the probability that the ground state spin is $S = S_{max} = m$. It is seen that even with strong random interaction the probability is not 100% and with increasing $\lambda_S$, the probability rapidly comes down to zero. The situation here is different from the result seen for BEGOE(1+2)-F where strong enough interaction generates states with maximum spin with 100% probability. With $SO(3\Omega)$ pairing the drop in probability is more rapid compared to the situation with $SO(\Omega) - SU(3)$ pairing. In addition, we have also calculated the expectation values of the two pairing Hamiltonians and $C_2(SU(3))$ and the results are shown in Fig. 6. We have considered BEGOE(1+2) Hamiltonian defined by Eq. (2) with $\lambda_0 = \lambda_1 = \lambda_2 = \lambda = 0.2$, i.e. in the region of chaos generated by random two-body interactions in the presence of a mean-field. It is seen that the expectation values are largest near the ground states and then decrease as we move towards the center of the spectrum. Due to finiteness of the model space, the curves are essentially symmetric around the center. The calculated results are in good agreement with the prediction [6] that for boson systems (just as it was well verified for fermion systems), expectation values will be ratio of Gaussians; see Section VI and Eq. (43) in [6]. Results in the figure show that with repulsive pairing, ground states will be dominated by low seniority structure. Also, with random interactions, there is no clear distinction between the two different pairing structures. Variation with the $\lambda$ parameter for larger systems ($\Omega$ and $m$ large) may show the difference but these calculations are not attempted as the matrix dimensions will be very large. This exercise will be attempted in future.
FIG. 5. (a) Probability for ground state to have maximum spin as a function of $\lambda_S$ for a 250 member $(m = 6, \Omega = 4)$ BEGOE(1+2)-$S1$ system with $H$ defined by Eq. (36). In all the calculations $\lambda_0 = \lambda_1 = \lambda_2 = \lambda$ and the results are shown for $\lambda = 0.2$ and 0.5. (a) results with $SO(\Omega) - SU(3)$ pairing parameter $\lambda_{p1} = 0.3$ and $SO(3\Omega)$ pairing parameter $\lambda_{p2} = 0$ in Eq. (36). (b) Same as (a) but with $SO(\Omega) - SU(3)$ pairing parameter $\lambda_{p1} = 0$ and $SO(3\Omega)$ pairing parameter $\lambda_{p2} = 0.3$ in Eq. (36).

VI. CONCLUSIONS AND FUTURE OUTLOOK

Introduced in this paper is the embedded Gaussian orthogonal ensemble of random matrices generated by random two-body interactions in presence of a mean-field for spin one boson systems. Presented are some first analytical and numerical results for this ensemble. Due to large fixed-$(m, M_S = 0)$ matrix dimensions, only restricted numerical calculations (with dimensions less than 10000) could be carried out at present. Some results for spectral properties including the form of eigenvalue density close to Gaussian, NNSD following GOE for sufficiently strong interaction strength and also for lowest two moments of the two point function are presented. It is possible to deal with much larger space if we use direct construction of $H$ matrix in a good $S$ basis. This is being attempted and using this in future a more detailed study with much larger size examples will be reported. Preliminary aspects of one of the embedding algebras $SU(\Omega) \otimes SU(3)$ and also two pairing algebras in the space defining BEGOE(1+2)-$S1$ are discussed in the paper. More detailed study of the effects of
FIG. 6. Expectation values of the two pairing Hamiltonians and $C_2(SU(3))$ vs $\hat{E}$ for a 250 member BEGOE(1+2) systems with $H$ defined by Eq. (2) and $(\Omega = 4, m = 6)$. Results are shown for spins $S = 0$ and $S = 4$. (a) expectation values of $H_P$, (b) expectation values of $H_P$ and (c) expectation value of $C_2(SU(3))$. Ensemble averaged results are shown by histograms while continuous curves are ratio of Gaussians given by EGOE theory [6]. See text for further details.

random interactions in presence of the two pairing interactions will be discussed elsewhere. Extension of BEGOE(2)-S1 to BEGUE(2)-S1 and to the more restricted BEGUE(2)-SU(3) with $H$ preserving $SU(3)$ symmetry for spin one boson systems are possible; see [26] for preliminary results for BEGUE(2)-SU(3). Finally, applications of BEGOE(1+2)-S1 ensemble to spin one BEC should be possible in future.
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