Two $\Lambda(1405)$ states in a chiral unitary approach with a fully-calculated loop function

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Abstract

The Bethe-Salpeter equation is solved in the framework of unitary coupled-channel approximation by using the pseudoscalar meson-baryon octet interaction. The loop function of the intermediate meson and baryon is deduced accurately in a fully dimensional regularization scheme, where the off-shell correction is supplemented. Two $\Lambda(1405)$ states are generated dynamically in the strangeness $S = -1$ and isospin $I = 0$ sector, and their masses, decay widths and couplings to the meson and the baryon are similar to those values obtained in the on-shell factorization. However, the scattering amplitudes at these two poles become weaker than the cases in the on-shell factorization.

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I. INTRODUCTION

There are some different views about the structure of \( \Lambda(1405) \), and its structure has been challenging the standard view of baryons made of three quarks for decades. Some theorists think that the \( \Lambda(1405) \) could be a kind of molecular state arising from the interaction of the \( \pi \Sigma \) and \( \bar{K}N \) channels [1–3]. Furthermore, the number of poles is also a puzzle for the \( \Lambda(1405) \) resonance. Some people predict one pole corresponds to the \( \Lambda(1405) \) resonance [4–6], however, the calculation results in the unitary coupled-channel approximation show there are two poles in the 1400MeV region [7–11]. It should be pointed out that only one pole is observed around 1400MeV in the experiments [12–15]. It seems the results in Ref. [4–6] are more consistent to the experimental data. However, in Ref. [16], the authors think there should be two \( \Lambda(1405) \) resonances, and the single peak observed in the experiment should be a consequence from the overlap of the two-pole contributions.

The mass and width of the \( \Lambda(1405) \) are calculated in the unitary coupled-channel approximation by solving the Bethe-Salpeter equation [7]. Two poles of the amplitude are detected in the complex energy plane, and these two poles were explained as combinations of a single state and an octet state when the \( SU(3) \) symmetry breaks up [8]. However, a loop function of the meson and the baryon in the on-shell factorization is used when the Bethe-Salpeter equation is solved, and some important elements might be eliminated in this approximation, which would result in the uncertainty of the calculation. In this article, we calculate the loop function accurately in a fully dimensional regularization scheme, and then solve the Bethe-Salpeter equation in the unitary coupled-channel approximation.

This manuscript is organized as follows. In Section II, the framework of unitary coupled-channel approximation is discussed, especially, the accurate formula of the loop function of the pseudoscalar meson and the baryon octet is obtained in the fully dimensional regularization. In Sections III and IV, the cases with isospin \( I = 0 \) and \( I = 1 \) are calculated, respectively. Some discussions and a conclusion are given in Section V.

II. FRAMEWORK

We mainly focus on the low-energy pseudoscalar meson-baryon octet scattering processes in the isospin space with strangeness \( S = -1 \). The lowest order meson-baryon chiral La-
The Lagrangian is given as

\[ L = \langle B i \gamma^\mu \frac{1}{4 f^2} [(\Phi \partial_\mu \Phi - \partial_\mu \Phi \Phi)] B - B(\Phi \partial_\mu \Phi - \partial_\mu \Phi \Phi) \rangle, \]

(1)

where the symbol \( \langle ... \rangle \) denotes the trace of matrices in the SU(3) space. The matrices of the pseudoscalar meson and the baryon octet are given as follows

\[ \Phi = \begin{pmatrix} \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}} \eta \end{pmatrix}, \]

(2)

and

\[ B = \begin{pmatrix} \frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}} \Lambda \end{pmatrix}. \]

(3)

Ten coupled channels are considered in the case of pseudoscalar meson-baryon octet scattering processes at the low energy region, which are \( K^- p, \bar{K}^0 n, \pi^0 \Lambda, \pi^0 \Sigma^0, \pi^+ \Sigma^-, \pi^- \Sigma^+, \eta \Lambda, \eta \Sigma^0, K^+ \Xi^- \) and \( K^0 \Xi^0 \).

The potential of the baryon octet and the pseudoscalar meson can be obtained from the lowest order meson-baryon chiral Lagrangian in Eq. (1). In this article, we adopt the form in Ref. [8]

\[ V_{ij} = -C_{ij} \frac{1}{4 f^2} (2\sqrt{s} - M_i - M_j) \left( \frac{M_i + E}{2M_i} \right)^{\frac{1}{2}} \left( \frac{M_j + E'}{2M_j} \right)^{\frac{1}{2}}, \]

(4)

where the coefficient \( C_{ij} \) is shown in Table I and the decay constant \( f = 1.123 f_\pi \) with the pion decay constant \( f_\pi = 92.4 \text{MeV} \). In Eq. (4), \( M_i \) and \( M_j \) denote the initial and final baryon masses, respectively, while \( E \) and \( E' \) the initial and final baryon energies in the center of mass system, respectively.

The scattering amplitude can be constructed by solving the Bethe-Salpeter equation in the unitary coupled-channel approximation

\[ T = [1 - VG]^{-1}V. \]

(5)

The symbol \( G \) in the equation (5) denotes the loop function of a pseudoscalar meson and a baryon, which can be written as

\[ G_t = i \int \frac{d^4 \mathbf{q}}{(2\pi)^4} \frac{\mathbf{q} + M_i}{q^2 - M_i^2 + i\epsilon} \frac{1}{(P - q)^2 - M_j^2 + i\epsilon} \]

(6)
\[
\begin{array}{cccccccc}
K^p & \bar{K}^0 & n & \pi^0 & \lambda & \eta & \Sigma^0 & \pi^+ & \Sigma^- & K^+ & \Xi^- & K^0 & \Xi^0 \\
K^p & 2 & 1 & \frac{\sqrt{3}}{2} & \frac{1}{2} & \frac{3}{2} & \frac{\sqrt{3}}{2} & 0 & 1 & 0 & 0 & 0 & 0 \\
\bar{K}^0 & n & 2 & -\frac{\sqrt{3}}{2} & \frac{1}{2} & \frac{3}{2} & -\frac{\sqrt{3}}{2} & 1 & 0 & 0 & 0 & 0 & 0 \\
\pi^0 & \lambda & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} & 0 & 0 & 0 & 0 \\
\pi^0 & \Sigma^0 & 0 & 0 & 0 & 0 & 2 & 2 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\
\eta & \lambda & 0 & 0 & 0 & 0 & 0 & \frac{3}{2} & \frac{3}{2} & 0 & 0 & 0 & 0 \\
\eta & \Sigma^0 & 0 & 0 & 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
\pi^+ & \Sigma^- & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\pi^- & \Sigma^+ & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
K^+ & \Xi^- & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
K^0 & \Xi^0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

TABLE I: The coefficient \( C_{ij} \) in the pseudoscalar meson-baryon octet interaction in the \( S = -1 \) channel, \( C_{ij} = C_{ji} \).

with \( P \) the total momentum of the system, \( m_l \) the meson mass, and \( M_l \) the baryon mass, respectively.

In Ref. [17], a dimensional regularization form of the loop function \( G \) is deduced in the on-shell factorization approximation, which is denoted as

\[
G_i(s) = \frac{2M_l}{16\pi^2} \left\{ a_l(\mu) + \ln \frac{M_l^2}{\mu^2} + \frac{m_l^2 - M_l^2 + s}{2s} \ln \frac{m_l^2}{M_l^2} + \right. \\
+ \left. \frac{\bar{q}_l}{\sqrt{s}} \left[ \ln(s - (M_l^2 - m_l^2) + 2\bar{q}_l\sqrt{s}) + \ln(s + (M_l^2 - m_l^2) + 2\bar{q}_l\sqrt{s}) \\
- \ln(-s + (M_l^2 - m_l^2) + 2\bar{q}_l\sqrt{s}) - \ln(-s - (M_l^2 - m_l^2) + 2\bar{q}_l\sqrt{s}) \right] \right\} (7)
\]

with \( \mu = 630\text{MeV} \) the scale of dimensional regularization and the symbol \( a_l(\mu) \) the subtraction constant.

According to Ref. [11], the values of \( a_l(\mu) \) are give as follows

\[
a_{KN} = -1.84, \quad a_{\pi\Sigma} = -2.00, \quad a_{\pi\Lambda} = -1.83, \quad a_{\eta\Lambda} = -2.25, \quad a_{\eta\Sigma} = -2.38, \quad a_{K\Xi} = -2.67.
\]

In Eq. (7), \( \bar{q}_l \) denotes the three-momentum of the vector meson or the octet baryon in
the center of mass system and is given by
\[ \bar{q}_l = \frac{\lambda^{1/2}(s, m_l^2, M_l^2)}{2\sqrt{s}} = \frac{\sqrt{s - (M_l + m_l)^2} \sqrt{s - (M_l - m_l)^2}}{2\sqrt{s}}, \] (9)
where \( \lambda \) is the triangular function and \( M_l \) and \( m_l \) are the masses of octet baryons and vector mesons, respectively.

Actually, the loop function in Eq. (6) can be calculated accurately in the dimensional regularization without the on-shell approximation taken into account. Thus the loop function takes the form of
\[ G_l = \frac{\gamma_\mu P^\mu}{32P^2\pi^2} \left[ (a(\mu) + 1)(m_l^2 - M_l^2) + (m_l^2\ln\frac{m_l^2}{\mu^2} - M_l^2\ln\frac{M_l^2}{\mu^2}) \right] + \left( \frac{\gamma_\mu P^\mu[P^2 + M_l^2 - m_l^2]}{4P^2M_l} + \frac{1}{2} \right) G'_l. \] (10)
Since the total three-momentum \( \vec{P} = 0 \) in the center of mass system, only the \( \gamma_0 P^0 \) parts remain in Eq. (10). If the anti-baryon is not considered in the calculation, \( \gamma_0 P^0 \) can be treated as the total energy of the system \( P^0 = \sqrt{s} \) in Eq. (10). Therefore, the loop function in Eq. (10) is simplified as
\[ G_l = \frac{\sqrt{s}}{32\pi^2s} \left[ (a(\mu) + 1)(m_l^2 - M_l^2) + (m_l^2\ln\frac{m_l^2}{\mu^2} - M_l^2\ln\frac{M_l^2}{\mu^2}) \right] + \left( \frac{s + M_l^2 - m_l^2}{4M_l\sqrt{s}} + \frac{1}{2} \right) G'_l. \] (11)

The real part and imaginary part of the loop function of a \( \pi \) meson and a \( \Sigma \) baryon as functions of the total energy with \( \text{Im} \sqrt{s} = 0 \) in the center of mass system are depicted in Fig. 1. The solid lines denote the case in Eq. (11), while the dash lines correspond to the case in Eq. (7), which is deduced in the on-shell factorization approximation. It can be seen that the absolute value of the real part of the loop function is larger than that of the loop function in the on-shell factorization approximation. However, the imaginary parts of them are identical.

Assuming the amplitudes to behave as
\[ T_{ij} = \frac{g_ig_j}{z - z_R}, \] (12)
with \( z_R \) the position of the pole in the complex \( \sqrt{s} \) plane, and \( i \) and \( j \) being the initial and final channels, respectively, we can obtain the size of the couplings \( g_i \) by evaluating the residues of the diagonal elements \( T_{ii} \). When the strongest coupled channel is determined, the couplings to the other channels, \( g_j \), can be evaluated with the residues of the non-diagonal elements \( T_{ij} \) and the largest coupling \( g_i \) by using Eq. (12).
III. THE ISOSPIN $I = 0$ SECTOR

We shall discuss the scattering amplitude in the isospin states, and thus we must use the average mass for the $\pi(\pi^+, \pi^0, \pi^-)$, $K(K^+, K^0)$, $\bar{K}(\bar{K}^0, K^-)$, $N(p, n)$, $\Sigma(\Sigma^+, \Sigma^0, \Sigma^-)$ and $\Xi(\Xi^0, \Xi^-)$ particles. There are four coupled states with isospin $I = 0$ and strangeness $S = -1$, which are $\bar{K}N, \pi\Sigma, \eta\Lambda$ and $K\Xi$.

The phase conventions $|\pi^+\rangle = - |1, 1\rangle$, $|K^-\rangle = - |\frac{1}{2}, -\frac{1}{2}\rangle$, $|\Sigma^+\rangle = - |1, 1\rangle$ and $|\Xi^-\rangle = - |\frac{1}{2}, -\frac{1}{2}\rangle$ are used for the isospin states, which are consistent with the structure of the $\Phi$ and $B$ matrices, and then the isospin state with $I = 0$ can be written as

$$|\bar{K}N, I = 0\rangle = \frac{1}{\sqrt{2}}(\bar{K}^0 n + K^- p),$$

$$|\pi\Sigma, I = 0\rangle = - \frac{1}{\sqrt{3}}(\pi^+ \Sigma^- + \pi^0 \Sigma^0 + \pi^- \Sigma^+),$$

$$|K\Xi, I = 0\rangle = - \frac{1}{\sqrt{2}}(K^0 \Xi^0 + K^+ \Xi^-).$$

(13)

The correspond coefficients $C_{ij}$ for the isospin states with $I = 0$ are listed in Table II. With these coefficients, the amplitudes $T$ with isospin $I = 0$ can be obtained by solving the Bethe-Salpeter equation in Eq. (5).

|       | $\bar{K}N$ | $\pi\Sigma$ | $\eta\Lambda$ | $K\Xi$ |
|-------|-----------|-------------|---------------|--------|
| $\bar{K}N$ | 3         | $-\frac{\sqrt{3}}{2}$ | $\frac{3}{\sqrt{2}}$ | 0      |
| $\pi\Sigma$ | 4         | 0           | $\frac{\sqrt{3}}{2}$ |        |
| $\eta\Lambda$ |          | 0           | $-\frac{3}{\sqrt{2}}$ |        |
| $K\Xi$ |            |             | 3              |        |

TABLE II: The coefficients $C_{ij}$ for the isospin states with $I = 0$, $C_{ij} = C_{ji}$.

The squared amplitudes $|T|^2$ in the $\pi\Sigma$ channel with isospin $I = 0$ in the complex $\sqrt{s}$ plane are shown in Fig. 2. Some poles might be generated dynamically when the Bethe-Salpeter equation is solved in the unitary coupled-channel approximation. In Fig. 2, the poles generated dynamically with the accurate loop function in Eq. (11) are labeled with NEW, while the label OLD denotes the poles generated with the original loop function in the on-shell factorization in Eq. (7). It can be seen that there are three poles generated dynamically in the isospin $I = 0$ sector, which locate at $1384 + i53$MeV, $1424 + i22$MeV and...
1675 + i18MeV on the complex $\sqrt{s}$ plane, respectively. The two poles around 1400MeV are both above the $\pi\Sigma$ threshold, and it implies that these two poles might correspond to the $\Lambda(1405)$ resonance in the data of Particle Data Group (PDG) [18]. The couplings of them to different baryons and pseudoscalar mesons are listed in Table III where the label *New* denotes the results with the accurate loop function in Eq. (11), while the label *OLD* means the results calculated with the loop function in the on-shell factorization, as in Eq. (7). Although the two poles in the 1400MeV region are close to each other, their couplings are different. The resonance at 1384 + i53MeV couples strongly to the $\pi\Sigma$ channel, however, the resonance at 1424 + i22MeV couples strongly to the $\bar{K}N$ channel. It implies that these two poles might correspond to two different resonances, or the single and octet states of the $\Lambda(1405)$ resonance. At this point, the result calculated with the accurate loop function in Eq. (11) is consistent with the original conclusion in Ref. [8]. The pole at the position of $\sqrt{s} = 1675 + i18MeV$ can be regarded as a candidate of the $\Lambda(1670)1/2^-$ resonance, which couples strongly to the $K\Xi$ channel.

Although the positions and couplings of these poles are similar to those calculated with the loop function in the on-shell factorization approximation, respectively, as shown in Fig. 2 and Table III. The amplitudes take different values from those calculated in the original scheme. In Fig. 2, The peaks at the poles 1384 + i53MeV and 1424 + i22MeV are lower than their counterparts in the on-shell factorization approximation, while the peak at the pole 1675 + i18MeV is higher than the original pole at 1680 + i20MeV calculated with the loop function in the on-shell factorization approximation.

Since the absolute value of the real part of the accurate loop function are larger than that in the on-shell factorization approximation, as depicted in Fig. 1, it is apparent that the amplitudes are different from those original ones. However, the singularity of the accurate loop function near the threshold is the same as that of the original loop function in the on-shell factorization approximation, so it is easy to understand that the new scheme gives the similar pole positions and couplings to the original ones.

**IV. THE ISOSPIN $I = 1$ SECTOR**

In the isospin $I = 1$ sector, we have five coupled states, $\bar{K}N$, $\pi\Sigma$, $\pi\Lambda$, $\eta\Sigma$ and $K\Xi$. 

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The isospin state with $I = 1$ can be written as

$$|\bar{K}N, I = 1\rangle = \frac{1}{\sqrt{2}}(\bar{K}^0 n - \bar{K}^- p),$$

$$|\pi\Sigma, I = 1\rangle = \frac{1}{\sqrt{2}}(\pi^- \Sigma^+ - \pi^+ \Sigma^-),$$

$$|K\Xi, I = 1\rangle = \frac{1}{\sqrt{2}}(K^0 \Xi^0 - K^+ \Xi^-).$$

The coefficients $C_{ij}$ for the isospin states with $I = 1$ can be constructed by using Eq. (14) and Table II which are given in Table IV.

There are two peaks of squared amplitudes $|T|^2$ detected at 1434 + i169MeV and 1570 + i234MeV in the complex $\sqrt{s}$ plane, respectively. as seen in Table V. However, the peak at 1434 + i169MeV is too low and it couples weakly to the baryon and the pseudoscalar meson. Thus it can be neglected when the $SU(3)$ symmetry is broken completely.
TABLE IV: The coefficients $C_{ij}$ for the isospin states with $I = 1$, $C_{ij} = C_{ji}$.

|     | $KN$ | $\pi\Sigma$ | $\pi\Lambda$ | $\eta\Sigma$ | $K\Xi$ |
|-----|------|--------------|--------------|--------------|--------|
| $KN$ | 1    | -1           | $-\sqrt{\frac{3}{2}}$ | $-\sqrt{\frac{3}{2}}$ | 0      |
| $\pi\Sigma$ | 2    | 0            | 0            | 1            |        |
| $\pi\Lambda$ | 0    | 0            | $-\sqrt{\frac{3}{2}}$ |        |        |
| $\eta\Sigma$ | 0    | $-\sqrt{\frac{3}{2}}$ |            |        |        |
| $K\Xi$ | 1    |              |              |              |        |

The resonance at $1570 + i234\text{MeV}$ is similar to the $\Sigma(1580)3/2^-$ state in the PDG data. Nevertheless, in the S-wave approximation, the total angular momentum of the resonance at $1570 + i234\text{MeV}$ is $J = 1/2$, and its parity is negative, furthermore, the decay width is too large, so it can not be $\Sigma(1580)3/2^-$ apparently. The resonance at $1570 + i234\text{MeV}$ is more possible to correspond to the $\Sigma(1620)1/2^-$ state in the PDG data although its mass is lower about $50\text{MeV}$ than the latter. From Table IV we can know that this resonance couples to the $K\Xi$ channel strongly.

The squared amplitude $|T|^2$ in the $\pi\Sigma$ channel with strangeness $S = -1$ and isospin $I = 1$ as a function of the total energy $\sqrt{s}$ in the center of mass system is depicted in Fig. 3, where the label OLD denotes the results calculated with the loop function in the on-shell factorization, and the label NEW means those of the accurate loop function in the fully dimensional regularization in Eq. (11). The pole positions are different in these two schemes. In the on-shell factorization approximation, the pole locates at $1579 + i264\text{MeV}$, it indicates that both the mass and the decay width of the resonance are larger than the values in the accurate dimensional regularization. Meanwhile, the squared amplitude at the pole calculated in the on-shell factorization is larger than its corresponding value calculated with the accurate dimensional regularization.

V. DISCUSSION AND CONCLUSION

In this article, the accurate formula of the loop function in the Bethe-Salpeter equation is deduced in the fully dimensional regularization scheme. Comparing with the loop function in the on-shell factorization approximation, the relativistic high-order correction and off-shell
The interaction between the pseudoscalar meson and the baryon octet is studied in the strangeness $S = -1$ sector, and some resonances are generated dynamically in the unitary coupled-channel approximation by solving the Bethe-Salpeter equation with the accurate loop function of the pseudoscalar meson and the baryon in the fully dimensional regularization. The masses and decay widths of these resonances are similar to the corresponding values obtained with the loop function in the on-shell factorization approximation in Ref. [8], moreover, the couplings to the meson and the baryon almost take the same values as those in Ref. [8]. Especially, two $\Lambda(1405)$ resonances are produced when the accurate loop function of the pseudoscalar meson and the baryon in the fully dimensional regularization is adopted in the calculation, and one couples strongly to the $\pi\Sigma$ channel, while the other mainly couples to the $\bar{K}N$ channel. At this point, the calculation results are consistent to those in Ref. [8]. However, the amplitudes in the energy region where the resonances generated are different from the values calculated with the loop function in the on-shell factorization approximation, and it might have an influence on the calculation results on some photo-production or electro-production processes of hadrons.
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Appendix: A fully-calculated loop function in the dimensional regularization scheme

In the dimensional regularization scheme, the loop function in the Bethe-Salpeter equation can be deduced as

\[ G_t(s) = i \int \frac{d^4q}{(2\pi)^4} \frac{q^i + M_l}{q^2 - M_l^2 + i\epsilon (P - q)^2 - m_l^2 + i\epsilon} \]

\[ = -PA(s) + \frac{1}{2}G'(s), \tag{15} \]

where

\[ G'(s) = i \int \frac{d^4q}{(2\pi)^4} \frac{2M_l}{q^2 - M_l^2 + i\epsilon (P - q)^2 - m_l^2 + i\epsilon} \]

\[ = \frac{2M_l}{16\pi^2} \left\{ a_l(\mu) + \ln \frac{M_l^2}{\mu^2} + \frac{m_l^2 - M_l^2 + s}{2s} \ln \frac{m_l^2}{M_l^2} + \frac{\bar{q}_l}{\sqrt{s}} \left[ \ln(s - (M_l^2 - m_l^2)) + 2\bar{q}_l \sqrt{s} \right] + \ln(s + (M_l^2 - m_l^2) + 2\bar{q}_l \sqrt{s}) \right. \]

\[ \left. - \ln(-s + (M_l^2 - m_l^2) + 2\bar{q}_l \sqrt{s}) - \ln(-s - (M_l^2 - m_l^2) + 2\bar{q}_l \sqrt{s}) \right\} \tag{16} \]

and

\[ P^\mu A(s) = -i \int \frac{d^4q}{(2\pi)^4} \frac{q^\mu}{q^2 - M_l^2 + i\epsilon (P - q)^2 - m_l^2 + i\epsilon}. \tag{17} \]

Since \(-2P \cdot q = [(P - q)^2 - m_l^2] - [q^2 - M_l^2] + m_l^2 - M_l^2 - P^2\),

\[ -2P^2 A(s) = -i \int \frac{d^4q}{(2\pi)^4} \frac{-2P \cdot q}{q^2 - M_l^2 + i\epsilon (P - q)^2 - m_l^2 + i\epsilon} \]

\[ = -i \int \frac{d^4q}{(2\pi)^4} \frac{[(P - q)^2 - m_l^2] - [q^2 - M_l^2] + m_l^2 - M_l^2 - P^2}{[(P - q)^2 - m_l^2 + i\epsilon][(P - q)^2 - m_l^2 + i\epsilon]} \]

\[ \left. = -i \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - M_l^2 + i\epsilon} - i \int \frac{d^4q}{(2\pi)^4} \frac{-1}{(P - q)^2 - m_l^2 + i\epsilon} \right. \]

\[ \left. - i \int \frac{d^4q}{(2\pi)^4} \frac{m_l^2 - M_l^2 - P^2}{[q^2 - M_l^2 + i\epsilon][(P - q)^2 - m_l^2 + i\epsilon]} \right\} \tag{18} \]

\[ = - \frac{M_l^2}{16\pi^2} \left( 1 + a_l + \ln \left( \frac{M_l^2}{\mu} \right) \right) + \frac{m_l^2}{16\pi^2} \left( 1 + a_l + \ln \left( \frac{m_l^2}{\mu} \right) \right) - \frac{(P^2 + M_l^2 - m_l^2)}{2M_l} (-G'(s)). \]
Thus the formula of $A(s)$ can be written as

$$A(s) = \frac{1}{-2P^2} \left\{ -\frac{M^2}{16\pi^2} \left( 1 + a_l + \ln\left( \frac{M^2}{\mu} \right) \right) + \frac{m_i^2}{16\pi^2} \left( 1 + a_l + \ln\left( \frac{m_i^2}{\mu} \right) \right) - \frac{(P^2 + M_i^2 - m_i^2)}{2M_i} \left( -G'(s) \right) \right\},$$

with $P^2 = s$.

By replacing the $G'(s)$ and $A(s)$ in Eq. (15) with Eqs. (16) and (19), respectively, the formula of the loop function in Eq. (10) is obtained.
Figure Captions

**Fig. 1** Loop functions in the $\pi\Sigma$ channel with $\text{Im}\sqrt{s} = 0$.

**Fig. 2** Comparison of poles in the strangeness $S = -1$ and isospin $I = 0$ sector. *NEW* denotes the case calculated from the accurate loop function in Eq. (11), while *OLD* stands for the case of the loop function in the on-shell factorization approximation in Eq. (7).

**Fig. 3** Same as Fig. 2 but for the strangeness $S = -1$ and isospin $I = 1$ sector.
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