Type II Theories Compactified on Calabi-Yau Threefolds in the Presence of Background Fluxes

Jan Louis and Andrei Micu

Fachbereich Physik, Martin-Luther-Universität Halle-Wittenberg, Friedemann-Bach-Platz 6, D-06099 Halle, Germany

ABSTRACT

Compactifications of type II theories on Calabi-Yau threefolds including electric and magnetic background fluxes are discussed. We derive the bosonic part of the four-dimensional low energy effective action and show that it is a non-canonical $N = 2$ supergravity which includes a massive two-form. The symplectic invariance of the theory is maintained as long as the flux parameters transform as a symplectic vector and a massive two-form which couples to both electric and magnetic field strengths is present. The mirror symmetry between type IIA and type IIB compactified on mirror manifolds is shown to hold for R-R fluxes at the level of the effective action. We also compactify type IIA in the presence of NS three-form flux but the mirror symmetry in this case remains unclear.

February 2002

1Work supported by: DFG – The German Science Foundation, GIF – the German–Israeli Foundation for Scientific Research, European RTN Program HPRN-CT-2000-00148 and the DAAD – the German Academic Exchange Service.

2email: j.louis@physik.uni-halle.de, micu@physik.uni-halle.de
1 Introduction

Calabi-Yau compactifications of heterotic and type II theories have been studied intensively in the past since they lead to consistent string theories below the critical dimension \(d = 10\). In particular compactifications on Calabi-Yau threefolds \(Y_3\) result in four flat Minkowskian space-time dimensions \((d = 4)\) and a low number of unbroken supersymmetries. Their effective theories are supergravities coupled to a set of vector- and matter multiplets with \(N = 1\) supersymmetry in the case of the heterotic string and \(N = 2\) supersymmetry for type II strings. The low energy effective theories all share the feature that they contain a (large) number of gauge neutral moduli multiplets which are flat directions of the effective potential and thus parameterize the vacuum degeneracy of the theory.

Generalization of Calabi-Yau compactifications are possible if one allows a \(p\)-form field strength \(F_p\) to take a non-trivial background value \(e_I\) along appropriate cycles \(\gamma_I\) in the compact Calabi-Yau manifold, i.e. \(\int_{\gamma_I} F_p = e_I\). Depending on the choice of these background fluxes the metric is deformed and the direct product of a four-dimensional Minkowskian space times a Calabi-Yau threefold is replaced by a warped product \([1, 2]\). This generically introduces a potential for the moduli and turns an ordinary supergravity into a gauged or massive supergravity. The consistency of such generalized compactifications was discussed in \([3–6]\) while various other aspects have been studied previously in refs. \([1, 2, 7–19]\).

The background fluxes \(e_I\) are quantized in units of the string scale \([7, 9]\) and thus do not represent a continuous deformations of the Calabi-Yau compactification. However, in the low energy supergravity they do appear as continuous parameters and hence can be discussed as continuous deformations of the well known low energy effective theories derived for vanishing flux background \([9]\). In the gauged supergravity the flux parameters play the role of masses and gauge charges.

The purpose of this paper is to perform a Kaluza-Klein reduction of the ten-dimensional type II supergravities on compact Calabi-Yau threefolds with all possible background fluxes turned on. We derive the bosonic part of the resulting low energy effective action and show - whenever possible - its consistency with gauged supergravity.\(^3\) We find that if magnetic charges are turned on a two-form becomes massive and the resulting \(N = 2\) supergravity is not easily related to the known \(N = 2\) supergravities \([20–22]\). We show that the symplectic invariance of the ungauged \(N = 2\) supergravity continues to hold as long as the background flux parameters transform as a symplectic vector. This leads to a symplectically invariant potential which for compactifications of type IIB theories was previously derived in refs. \([11–13]\). However, the issue of the symplectic invariance was not resolved and here we show that the presence of a massive two-form is crucial for the symplectic invariance of the theory. We believe that this is a more general feature of gauged supergravity and that the existing \(N = 2\) supergravities have to be amended by including the possibility of massive two-forms. Only in this more general framework a symplectically invariant theory can arise.

A second aspect of our paper concerns the mirror symmetry between type IIA compactified on \(Y_3\) and type IIB compactified on the mirror manifold \(\tilde{Y}_3\). This duality holds

\(^3\)For the heterotic string we performed a similar analysis in ref. \([18]\).
for vanishing background fluxes but its validity in the presence of fluxes is unclear. We show that for the background fluxes of Ramond-Ramond (R-R) \( p \)-forms mirror symmetry holds at the level of the effective theories while for background fluxes in the Neveu-Schwarz (NS) sector it is not easily established \([12, 16, 19, 23]\). A similar problem arises for the non-perturbative dualties relating type IIA compactified on \( K3 \) to heterotic on \( T^4 \) \([24]\) and type IIA compactified on \( Y_3 \) to heterotic on \( K3 \times T^2 \) \([16, 18]\).

This paper is organized as follows. In section 2 we focus on type IIA supergravity compactified on Calabi-Yau threefolds \( Y_3 \) in the presence of background fluxes. It turns out that in order to establish the mirror symmetry to type IIB compactifications it is necessary to start from the \( N = 2 \) massive version of the ten-dimensional type IIA supergravity where the NS two-form \( B_2 \) is massive \([27]\). We first briefly recall this theory including its symmetry properties. In section 2.1 we then perform the compactification on \( Y_3 \) turning on the \( 2h_{1,1} \) possible background values of the R-R two-form field strength \( \hat{F}_2 \) and the R-R four-form field strength \( \hat{F}_4 \). (\( h_{1,1} \) is the Hodge number of the cohomology group \( H^{1,1}(Y_3) \).) Furthermore, the four-dimensional low energy effective theory includes a three-form (with a four-form field strength) which in \( d = 4 \) is Poincaré dual to a constant. This constant is an additional parameter of the theory so that together with the ten-dimensional mass parameter of \( B_2 \) the theory depends on \( 2h_{1,1} + 2 \) ‘fluxes’. The resulting low energy effective action is not a standard \( N = 2 \) gauged supergravity in that also in \( d = 4 \) the two-form \( B_2 \) is generically massive. The mass terms depend on the magnetic flux parameters and vanish for purely electric fluxes. As far as we know this situation has not been discussed previously in the supergravity literature.

In section 2.2 we discuss the gauge invariance of this massive theory and furthermore show that as long as the flux parameters are appropriately transformed, the equations of motions are invariant under a generalized electric-magnetic duality which is part of an \( Sp(2h_{1,1} + 2) \) transformation. To establish this symmetry it is crucial to start from the massive type IIA theory in \( d = 10 \) and to also include the constant dual of the three-form. A similar observation was made in \([24]\) where \( K3 \) compactifications of type IIA in \( d = 6 \) were studied. The perturbative \( SO(4,20) \) T-duality which is present in the absence of background fluxes can only be maintained if one starts from the massive ten-dimensional IIA theory and furthermore transforms the flux parameter in the vector representation of \( SO(4,20) \). Nevertheless the appearance of the symplectic invariance in the \( d = 4 \) theory is somewhat unexpected since it is usually lost in gauged supergravities.

In section 2.3 we discuss the relation with the standard gauged \( N = 2 \) supergravities. By fixing the symplectic invariance one can go to a particular gauge where all magnetic charges vanish and \( B_2 \) remains massless. We show that in this gauge the theory is a special case of the known \( N = 2 \) gauged supergravities. However, in an arbitrary gauge \( B_2 \) is massive and the corresponding supergravity couplings are not independently known. Nevertheless, in \( d = 4 \) a massive two-form is dual to a massive one-form \([28-30]\) and therefore one might suspect that in the dual formulation consistency with the standard gauged supergravity is achieved. In section 2.3 (and appendix E.3) we explicitly perform the duality transformation and show that also in the dual basis the gauged supergravity is non-canonical.

In section 2.4 we turn our attention to non-trivial fluxes of the NS three-form \( \hat{H}_3 \)

\[\text{4An interesting suggestion for its cure has been put forward in refs.}\ [25, 26]\]
which have not been discussed previously in the literature. We derive the low energy effective theory and establish the consistency with $N=2$ gauged supergravity. In this case the potential depends non-trivially on the scalar in the hypermultiplets whereas the dependence on the scalars in the vector multiplets is only via an overall volume factor. This is exactly opposite to the case of R-R fluxes which induce a potential for the vector-scalars leaving the hyper-scalars (except the dilaton) undetermined. Furthermore, only the graviphoton participates in the gauging but no charged states with respect to the other gauge fields appear.

In section 3 we briefly discuss type IIB compactifications on $Y_3$ with non-trivial R-R three-form flux $\hat{F}_3$. This case has been considered previously in refs. [11–14, 17] and therefore we keep our presentation short. However, we do establish the fact that also in this case $B_2$ becomes massive for non-vanishing magnetic charges. With this result we are able to establish the mirror symmetry to type IIA compactified on the mirror threefold in the presence of R-R background fluxes at the level of the low energy effective theories.

Section 4 contains our conclusions and some of the more technical aspects of this paper are relegated to five appendices. Appendix A summarizes our notation. Appendix B briefly recalls $N=2$ supergravity in $d=4$. Appendix C assembles the necessary facts about the moduli space of Calabi-Yau threefolds. Appendix D redoes the compactification of massless type IIA on $Y_3$ with special emphasis on the role of the R-R three-form $C_3$ and its Poincaré dual constant. Already in this simpler case the constant is an additional parameter of the effective theory and turns the ordinary type IIA supergravity into a gauged supergravity. Finally appendix E discusses the Poincaré dualities of massless and massive two-forms and of three-forms in $d=4$.

2 Compactification of massive type IIA supergravity on Calabi-Yau threefolds with background fluxes

Let us start by compactifying the ten-dimensional massive type IIA theory [27] on a Calabi-Yau threefold in the presence of background fluxes. The compactification of ordinary massless type IIA supergravity on Calabi-Yau threefolds was performed in ref. [31], which we briefly recall in appendix D. The massless modes of the ten-dimensional IIA theory comprise in the NS-NS sector the metric, a two form $\hat{B}_2$ and the dilaton $\hat{\phi}$ while the RR-sector contains a vector field $\hat{A}_1$ and a three-form $\hat{C}_3$. In the massive version $\hat{B}_2$ is massive and a cosmological constant is present. The action reads [27]

\[
S = \int \left[ e^{-2\hat{\phi}} \left( \frac{1}{2} \hat{R} \ast 1 + 2d\hat{\phi} \wedge \ast d\hat{\phi} - \frac{1}{4} \hat{H}_3 \wedge \ast \hat{H}_3 \right) 
- \frac{1}{2} \left( \hat{F}_2 \wedge \ast \hat{F}_2 + \hat{F}_4 \wedge \ast \hat{F}_4 \right) + \mathcal{L}_{top} - \frac{1}{2} m^2 \ast 1 \right],
\]

where the field strengths are defined as

\[
\hat{F}_2 = d\hat{A}_1 + m\hat{B}_2, \quad \hat{F}_4 = d\hat{C}_3 - \hat{B}_2 \wedge d\hat{A}_1 - \frac{m}{2}(\hat{B}_2)^2, \quad \hat{H}_3 = d\hat{B}_2,
\]

This differs from [27] by a redefinition of the mass parameter $m \rightarrow -m$. 

\[5\]
and the topological terms read

\[ \mathcal{L}_{\text{top}} = -\frac{1}{2} \left[ \hat{B}_2 \wedge d\hat{C}_3 \wedge d\hat{C}_3 - (\hat{B}_2)^2 \wedge d\hat{C}_3 \wedge d\hat{A}_1 + \frac{1}{3} (\hat{B}_2)^3 \wedge d\hat{A}_1 \wedge d\hat{A}_1 \\
- \frac{m}{3} (\hat{B}_2)^3 \wedge d\hat{C}_3 + \frac{m}{4} (\hat{B}_2)^4 \wedge d\hat{A}_1 + \frac{m^2}{20} (\hat{B}_2)^5 \right]. \]  

(2.3)

Throughout the paper we use differential form notation (summarized in appendix A) and denote differential forms in \( d = 10 \) by a hat `\(^\wedge\)`. Furthermore we abbreviate \( \hat{B}_2 \wedge \hat{B}_2 = (\hat{B}_2)^2 \) etc. The massive IIA supergravity has an unbroken ten-dimensional \( N = 2 \) supersymmetry and in the limit \( m \to 0 \) the action for ordinary type IIA theory (recorded in (D.1)) is recovered. The action (2.1) is invariant under the following three Abelian gauge transformations (with parameters \( \Theta, \Sigma_2, \Lambda_1 \))

\[
\delta \hat{A}_1 = d\Theta, \quad \delta \hat{C}_3 = d\Sigma_2, \\
\delta \hat{B}_2 = d\Lambda_1, \quad \delta \hat{C}_3 = \hat{\Lambda}_1 \wedge d\hat{A}_1, \quad \delta \hat{A}_1 = -m\hat{\Lambda}_1. 
\]  

(2.4)

As already mentioned, in the limit \( m \to 0 \) we recover the standard type IIA supergravity and hence the degrees of freedom described by the two theories is the same. However, in the massive theory these degrees of freedom are redistributed in that \( \hat{B}_2 \) describes a massive two-form (which carries \( (\hat{\alpha})^2 = 36 \) degrees of freedom) while \( \hat{A}_1 \) carries no degree of freedom since it can be gauged away using (2.4). This is the analog of the unitary gauge in the standard Higgs mechanism where \( A_1 \) plays the role of the Goldstone boson which is eaten by \( \hat{B}_2 \).

Compactification of massive IIA supergravity on a Calabi-Yau threefold \( Y_3 \) results in an effective theory in \( d = 4 \) with \( N = 2 \) supersymmetry and proceeds as in the massless type IIA case (cf. appendix D). The ten-dimensional metric gives rise to \( h_{1,1} + 2h_{1,2} \) scalar fields related to \( h_{1,1} \) Kähler deformations \( v^i, i = 1, \ldots, h_{1,1} \) and the \( h_{1,2} \) (complex) deformations \( z^a, a = 1, \ldots, h_{1,2} \) of the complex structure. The two-form \( \hat{B}_2 \) decomposes into a two form \( B_2 \) in \( d = 4 \) and \( h_{1,1} \) scalar fields \( b^i \) according to

\[
\hat{B}_2 = B_2 + b^i \omega_i, \quad i = 1, \ldots, h_{1,1},
\]  

(2.5)

where \( \omega_i \) are harmonic \((1,1)\)-forms which form a basis of \( H^{1,1}(Y_3) \). The \( b^i \) combine with the \( v^i \) to form complex fields \( t^i = b^i + iv^i \). The three-form \( \hat{C}_3 \) decomposes into a four-dimensional three form \( C_3 \), \( h_{1,1} \) vector fields \( A^i \) and \( 2h_{1,2} + 2 \) (real) scalar fields \( \xi^A, \tilde{\xi}_A \)

\[
\hat{C}_3 = C_3 + A^i \wedge \omega_i + \xi^A \alpha_A + \tilde{\xi}_A \beta_A, \quad A = 0, \ldots, h_{1,2},
\]  

(2.6)

where \((\alpha_A, \beta_A)\) are harmonic three-forms which form a real basis of \( H^3(Y_3) \) (cf. appendix C.2). Finally the 1-form in \( d = 10 \) only results in a four-dimensional 1-form i.e. \( A_1 = A^0 \). Together these (bosonic) fields assemble into the \( N = 2 \) gravity multiplet \((g_{\mu\nu}, A^0)\), \( h_{1,1} \) vector multiplets \((A^i, t^i)\), \( h_{1,2} \) hypermultiplets \((z^a, \xi^a, \tilde{\xi}_a)\) and one tensor multiplet \((B_2, \phi, \xi^0, \tilde{\xi}_0)\). In \( d = 4 \) a two-form is dual to a scalar and hence the tensor multiplet can be dualized to an additional (universal) hypermultiplet. A four-dimensional three form \( C_3 \) is dual to a constant and thus the compactified massive type IIA theory already

\(^6\)A summary of the Calabi-Yau geometry is assembled in appendix C.
contains two parameters, the mass $m$ of the ten-dimensional action and the constant $e_0$ which is dual to $C_3$. Both of these constants are related to the cosmological constant and as we will see shortly they turn the ordinary $N=2$ supergravity into a gauged supergravity.

2.1 Turning on R-R fluxes

Let us now turn to the compactification of massive type IIA supergravity in the presence of background fluxes for the RR field strengths $\hat{F}_2$ and $\hat{F}_4$. This will add $2h_{1,1}$ parameters $(e_i, m^i)$ into the action. We assume that the fluxes are turned on perturbatively so that the light $d=4$ spectrum is not modified. More specifically we start from the standard reduction Ansatz (cf. appendix D)

$$\hat{A}_1 = A^0, \quad \hat{B}_2 = B_2 + b^i \omega_i, \quad \hat{C}_3 = C_3 + A^i \wedge \omega_i + \xi^A \alpha_A + \xi_A \beta^A,$$

(2.7)

and modify the R-R-field strength according to

$$\hat{C}_3 \rightarrow d\hat{C}_3 + e_i \tilde{\omega}^i, \quad d\hat{A}_1 \rightarrow d\hat{A}_1 - m^i \omega_i.$$

(2.8)

The flux parameters $(e_i, m^i)$ are constants and $\tilde{\omega}^i$ are harmonic $(2,2)$-forms which form a basis of $H^{2,2}(Y_3)$ and which are dual to the $(1,1)$-forms $\omega_i$, i.e. they obey the normalization of eq. (C.8). Notice that we can consistently do this modification at the level of the action since in (2.1) the fields $\hat{A}_1$ and $\hat{C}_3$ appear only through their Abelian field strengths $d\hat{A}_1$ and $d\hat{C}_3$. In terms of the field strengths defined in (2.2) turning on fluxes according to (2.8) amounts to

$$\hat{H}_3 = dB_2 + dB^i \omega_i, \quad \hat{F}_2 = dA^0 + mB_2 - (m^i - mb^i) \omega_i, \quad \hat{F}_4 = dC_3 - B_2 \wedge dA^0 - \frac{m}{2}(B_2)^2 + (dA^i - dA^0 b^i + m^i B_2 - mB_2 b^i) \wedge \omega_i$$

$$+ (d\xi^A \alpha_A + d\xi_A \beta^A) + (b^i m^j - \frac{1}{2} mb^i b^j) K_{ijk} \tilde{\omega}^k + e_i \tilde{\omega}^i,$$

(2.9)

where $K_{ijk} = \int_{Y_3} \omega_i \wedge \omega_j \wedge \omega_k$ and the last equation used (C.9).

To derive the four-dimensional effective action we insert (2.9) into (2.1). Before giving the final result let us discuss the ‘new’ terms which arise due to the presence of the parameters $(e_0, e_i, m, m^i)$ and are absent in the standard IIA theory. The kinetic term of $\hat{A}_1$ gives a contribution to the potential

$$V_1 = 2K(m^i - mb^i)(m^j - mb^j)g_{ij},$$

(2.10)

where $g_{ij}(v) = \frac{1}{K} \int_{Y_3} \omega_i \wedge^* \omega_j$ is the metric on the space of Kähler deformations and $K$ defined in (C.3) denotes the volume of $Y_3$. In addition the following interaction and mass

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7In ref. [31] the case $e_0 = 0$ was considered. In appendix D we derive the action of massless type IIA compactified on $Y_3$ for arbitrary $e_0$. The quantization condition on $e_0$ has been discussed in refs. [32].

8The minus sign in the last relation was chosen to make the symplectic invariance explicit later.
terms for $B_2$ arise

$$\delta \mathcal{L}_{\text{int}} = -m K B_2 \wedge^* dA^0 - \frac{m^2 K}{2} B_2 \wedge^* B_2.$$  \hfill (2.11)

The kinetic term of $\hat{C}_3$ also contributes to the potential

$$V_3 = \frac{1}{8K} (e_i + b^k m^i K_{ikl} - \frac{1}{2} m b^k b^l K_{ikl}) (e_j + b^m m^n K_{jmn} - \frac{1}{2} m b^m b^n K_{jmn}) g^{ij},$$  \hfill (2.12)

where $g^{ij} = 4K \int \omega^i \wedge^* \omega^j$ is the inverse metric obeying $g_{ij} g^{jk} = \delta^i_k$. In addition the following interaction terms arise

$$\delta \mathcal{L}_{\text{int}} = -4K (m^i - mb^i) B_2 \wedge^* (dA^i - dA^0 b^i) g_{ij} \quad \text{ (2.13)}$$

$$-2K (m^i - mb^i) (m^j - mb^j) g_{ij} B_2 \wedge^* B_2$$

Finally the topological terms $[2,3]$ are modified according to

$$\delta \mathcal{L}_{\text{top}} = -B_2 \wedge (dA^i e_i + b^j dA^i m^k K_{ijk} - b^i e_i dA^0 - b^i b^j m^k K_{ijk} dA^0) \quad \text{ (2.14)}$$

$$-\frac{1}{2} (2b^i e_i + b^j b^k m^k K_{ijk} - \frac{m}{3} b^i b^j b^k K_{ijk}) dC_3 + \frac{m}{2} B_2 \wedge (dA^i - dA^0 b^i) b^j b^k K_{ijk}$$

$$-\frac{1}{2} (m^i e_i - mb^i e_i + b^j m^k K_{ijk} - \frac{3m}{2} b^i b^j m^k K_{ijk} + \frac{m^2}{2} b^i b^j b^k K_{ijk}) (B_2)^2.$$

To arrange the above expressions in the form of a the standard gauged $N = 2$ supergravity we are going to proceed as in appendix [D]. First we dualize the three-form $C_3$ to a constant. Collecting all couplings of $C_3$ we obtain

$$\mathcal{L}_{C_3} = \frac{K}{2} (dC_3 - B_2 \wedge dA^0 - \frac{m}{2} (B_2)^2) \wedge^* (dC_3 - B_2 \wedge dA^0 - \frac{m}{2} (B_2)^2)$$

$$-(b^i e_i + \frac{1}{2} b^j b^k m^k K_{ijk} - \frac{m}{6} b^i b^j b^k K_{ijk}) dC_3.$$  \hfill (2.15)

The dualization of a three-form in $d = 4$ is summarized in appendix [E2]. Applying the formulae of this appendix yields the dual action where the three-form $C_3$ is traded for a constant $e_0$

$$\mathcal{L}_{C_3} \rightarrow \mathcal{L}_{e_0} = -\frac{1}{2K} (e_0 + e_i b^i + \frac{1}{2} b^j b^k m^k K_{ijk} - \frac{m}{6} b^i b^j b^k K_{ijk})^2 \wedge^* 1$$

$$-(e_0 + e_i b^i + \frac{1}{2} b^j b^k m^k K_{ijk} - \frac{m}{6} b^i b^j b^k K_{ijk}) (B_2 \wedge dA^0 + \frac{m}{2} (B_2)^2).$$  \hfill (2.16)

Let us stress that the appearance of the parameter $e_0$ obtained by dualizing $C_3$ does not depend on the fact that we have turned on other fluxes. $\mathcal{L}_{e_0}$ does not vanish in the limit $m = m^i = e_j = 0$ and thus is also present in the compactification of massless type IIA supergravity without any fluxes turned on. In appendix [D] we show that $e_0$ becomes the charge of the scalar $a$ which is dual to $B_2$ and a potential consistent with the standard $N = 2$ gauged supergravity is induced.
Defining the four-dimensional dilaton via $e^{-2\phi} = e^{-2\tilde{\phi}} K$, using the formulae (D.3), (D.6) together with (2.10) – (2.14) and (2.16) the low energy effective action takes the form

$$S = \int e^{-2\phi}\left(\frac{1}{2}R^I + 2d\phi \wedge d\phi - \frac{1}{4}H_3 \wedge H_3 - g_{ab}dz^a \wedge dz^b - g_{ij}dt^i \wedge dt^j\right)$$

$$+ \frac{1}{2} \left(\text{Im} \, M^{-1}\right)^{AB} \left[d\tilde{\xi}_A + M_{AC}d\xi^C\right] \wedge \left[d\tilde{\xi}_B + \tilde{M}_{BD}d\xi^D\right]$$

$$+ \frac{1}{2} H_3 \wedge (\tilde{\xi}_A d\xi^A - \xi^A d\tilde{\xi}_A) + \frac{1}{2} \text{Im} \, N_{IJ} F^I \wedge * F^J + \frac{1}{2} \text{Re} \, N_{IJ} F^I \wedge F^J$$

$$- B_2 \wedge J_2 - \frac{1}{2} M^2 B_2 \wedge * B_2 - \frac{1}{2} M_T^2 B_2 \wedge B_2 - V,$$

(2.17)

where $I = 0, \ldots, h_{1,1}$ and $N_{IJ}, M_{AB}$ are standard supergravity couplings defined in (C.4) and (C.23). The ‘new’ couplings $J_2, M^2, M_T^2$ which depend on the fluxes only appear as couplings to $B_2$ and are found to be

$$J_2 = (e_I F^I - m^i G_i),$$

$$M^2 = -m^I \text{Im} \, N_{IJ} m^J,$$

$$M_T^2 = -m^I \text{Re} \, N_{IJ} m^J + m^i e_i,$$

(2.18)

where we denoted $m$ by $m^0$ and introduced the vectors $m^I = (m^0, m^i)$, $e_I = (e_0, e_i)$. Furthermore, we introduced the magnetic dual of $F^I \equiv dA^I$ by

$$G_I \equiv \text{Im} \, N_{IJ} * F^J + \text{Re} \, N_{IJ} F^J.$$  

(2.19)

Finally the string frame potential in (2.17) is found to be

$$V = -\frac{1}{2} (e_I - \tilde{N}_{IK} m^K) (\text{Im} \, N)^{-1IJ} (e_J - N_{IJ} m^L),$$

(2.20)

where $(\text{Im} \, N)^{-1}$ is given in (C.11).

The action (2.17) together with the definitions (2.18) is our first non-trivial result. It gives the low energy effective action for massive type IIA supergravity compactified on a Calabi-Yau threefold in the presence of R-R-background flux. As we see the $2h_{1,1}$ flux parameters $(e_i, m^i)$ naturally combine with the mass parameter $m^0$ of the ten-dimensional massive IIA theory and the dual $e_0$ of the four-dimensional three-form $C_3$ to form the vectors $(e_I, m^I)$. As we are going to show next these vectors enjoy an action of a symplectic group $Sp(2h_{1,1} + 2)$. Furthermore the flux parameters introduce a potential $V$, Green-Schwarz type couplings $B_2 \wedge J_2$, a regular and a topological mass term $M, M_T$ for $B_2$.

### 2.2 Gauge and symplectic invariance

Let us first focus on the gauge invariance of the action (2.17). First of all (2.17) is manifestly invariant under the standard one-form gauge transformation $\delta A^I = d\Theta^I$. However,

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9 Strictly speaking also the Kähler moduli $t^i$ have to be redefined by a dilaton dependent factor [3]. In order not to overload the notation we use the same symbol $t^i$ also for the redefined moduli.

10 Note that $M^2$ is positive since in our conventions $\text{Im} \, N_{IJ}$ is negative definite.
the gauge invariance related to the two-form $B_2$ is less obvious. After compactification
the ten-dimensional gauge transformations (2.4) of the two-form $B_2$ become
\[
\delta B_2 = d\Lambda_1 , \quad \delta C_3 = \Lambda_1 \wedge dA^0 , \quad \delta A^I = -m^I\Lambda_1 . \tag{2.21}
\]
As in $d = 10$, the three-form $C_3$ transforms under this gauge transformations. However,
in the dualization of $C_3$ the gauge invariant combination $dC_3 - dA^0 \wedge B_2 - \frac{m}{2}B_2 \wedge B_2$
appeared and is dual to the (gauge invariant) constant $e_0$. The gauge invariance in the
dual action is most easily seen by rewriting the action (2.17) as
\[
S = \int e^{-2\phi} \left( \frac{1}{2} R^* 1 + 2d\phi \wedge \ast d\phi - \frac{1}{4} H_3 \wedge \ast H_3 - g_{a\dot{a}}dz^a \wedge \ast dz^\dot{a} - g_{ij}dt^i \wedge \ast d\bar{t}^j \right)
+ \frac{1}{2} \left( \text{Im } \mathcal{M}^{-1} \right)^{AB} \left[ d\tilde{\xi}_A + \mathcal{M}_{AC} d\xi^C \right] \wedge \ast \left[ d\tilde{\xi}_B + \tilde{\mathcal{M}}_{BD} d\xi^D \right]
+ \frac{1}{2} H_3 \wedge (\tilde{\xi}_A d\xi^A - \xi_A d\tilde{\xi}_A) + \frac{1}{2} \text{Im } \mathcal{N}_{IJ} \tilde{F}^I \wedge \ast \tilde{F}^J + \frac{1}{2} \text{Re } \mathcal{N}_{IJ} \tilde{F}^I \wedge \bar{\tilde{F}}^J
- \frac{1}{2} B_2 \wedge (\tilde{F}^I + dA^I)e_I - V , \tag{2.22}
\]
where we defined
\[
\tilde{F}^I \equiv dA^I + m^I B_2 . \tag{2.23}
\]
Under the transformations
\[
\delta B_2 = d\Lambda_1 , \quad \delta A^I = -m^I\Lambda_1 , \tag{2.24}
\]
$\tilde{F}^I$ is invariant and it can be easily checked that the action (2.22) is also not modified.

Let us discuss the symplectic invariance of the theory described by (2.22). The un-
gauged $N = 2$ supergravity is invariant under generalized electric-magnetic duality trans-
formations which are part of a symplectic $Sp(2h_{1,1} + 2)$ invariance [20,22]. However, this
is not a symmetry of the action, but it leaves the equations of motion and Bianchi iden-
tities invariant. In gauged supergravity this invariance is generically broken as charged
states appear and the action is no longer expressed in terms of only the field strength
$F^I$. However, for the case at hand a symplectic invariance can be maintained as long as
the fluxes $(m^I, e_I)$ are also transformed. To see this consider the Bianchi identities and
the equations of motions derived from the action (2.22)
\[
d dA^I = d\tilde{F}^I - m^I dB_2 = 0 ,
\]
\[
\frac{\partial L}{\partial A^I} = d\tilde{G}_I - e_I dB_2 = 0 , \tag{2.25}
\]
\[
\frac{\partial L}{\partial B_2} = \frac{1}{2} d(e^{-2\phi} dB_2) + m^I \tilde{G}_I - e_I \tilde{F}^I = 0 ,
\]
where
\[
\tilde{G}_I \equiv \text{Re } \mathcal{N}_{IJ} \tilde{F}^J + \text{Im } \mathcal{N}_{IJ} \ast \bar{\tilde{F}}^J . \tag{2.26}
\]
These equations are invariant under the symplectic transformations given in (B.10) with $(m^I, e^I)$ and $(\tilde{F}^I, \tilde{G}_I)$ transforming as symplectic vectors

\[
\begin{pmatrix} m^I \\ e^I \end{pmatrix} \rightarrow \begin{pmatrix} U & Z \\ W & V \end{pmatrix} \begin{pmatrix} m^I \\ e^I \end{pmatrix}, \quad \begin{pmatrix} \tilde{F}^I \\ \tilde{G}_I \end{pmatrix} \rightarrow \begin{pmatrix} U & Z \\ W & V \end{pmatrix} \begin{pmatrix} \tilde{F}^I \\ \tilde{G}_I \end{pmatrix},
\]

(2.27)

where $U, V, W, Z$ obey (B.11). Similarly one checks the invariance of $V$ given in (2.20).

The symplectic invariance of the equations (2.25) is our second non-trivial result. In particular it shows that the symplectic invariance of the potential as observed in [12, 13] has its deeper origin in the symplectic invariance of (2.25). However, the supergravity which displays this invariance is not the canonical one but instead features a massive two-form $B_2$ with very specific couplings to the gauge fields. It would be interesting to investigate this situation in more detail from a purely supergravity point of view without any reference to a flux background of string theory.

### 2.3 Relation with gauged supergravity

Let us now investigate the relation between the action derived in (2.17) or (2.22) and the standard gauged $N = 2$ supergravity as summarized in appendix B. The new ingredients in the action (2.17) are the mass terms for $B_2$. Let us first observe that they all vanish for $m^I = 0$. Since we have established the symplectic invariance of the theory we can always do a symplectic transformation on the vector $(m^I, e^I)$ and go to a basis where all $m^I$ vanish\footnote{We should stress again that from a pure supergravity point of view the fluxes $m^I$, $e^I$ are just continuous parameters and so there always exist an $Sp(2h_{1,1} + 2, \mathbb{R})$ transformation such that the rotated magnetic fluxes vanish. In a quantum theory however, the fluxes become quantized and the $Sp(2h_{1,1} + 2, \mathbb{R})$ invariance is generically broken to $Sp(2h_{1,1} + 2, \mathbb{Z})$. In this case, it is impossible to set the magnetic charges to zero by an $Sp(2h_{1,1} + 2, \mathbb{Z})$ rotation.}

\[
\begin{pmatrix} m^I \\ e^I \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ e'_I \end{pmatrix}.
\]

(2.28)

In this basis the ‘new’ couplings considerably simplify and from (2.18) and (2.20) one immediately obtains

\[
M = M_T = 0, \quad J_2 = e'_I F^I, \quad V = -\frac{1}{2} e'_I (\text{Im} \mathcal{N'})^{-1IJ} e'_J,
\]

(2.29)

where the prime indicates the rotated basis. The drawback of this basis is that also the gauge couplings $\mathcal{N}$ of the action (2.17) change according to (B.13) and the relation to the prepotential as given in (B.7) is more complicated. So we have the choice to work either with the standard gauge couplings and a set of complicated interactions of $B_2$ or to transform to a new basis where the gauge couplings are more complicated but $B_2$ remains massless. In this latter basis the consistency with gauged supergravity is easily established so let us first discuss this case.
For \( m^I = 0 \) \( B_2 \) is massless and thus can be dualized to a scalar \( a \) as in appendix E.1. After a Weyl rescaling \( g_{\mu\nu} \to e^{2\phi} g_{\mu\nu} \) the dual action reads

\[
S = \int \left[ \frac{1}{2} R^* 1 + \frac{1}{2} \text{Im} N'_{IJ} F'^I \wedge^* F'^J + \frac{1}{2} \text{Re} N'_{IJ} F'^I \wedge F'^J - g_{ij} dt^i \wedge^* d\bar{t}^j - h_{uv} Dq^u \wedge^* Dq^v - V_E \right],
\]

(2.30)

where

\[
h_{uv} Dq^u \wedge^* Dq^v = d\phi \wedge^* d\phi + e^{2\phi} \left[ Da + (\xi_A d\xi^A - \xi^A d\tilde{\xi}_A) \right] \wedge^* \left[ Da + (\bar{\xi}_A d\xi^A - \xi^A d\tilde{\xi}_A) \right] - \frac{e^{2\phi}}{2} \left( \text{Im} M^{-1} \right)^{AB} \left[ d\tilde{\xi}_A + M_{AC} d\xi^C \right] \wedge^* \left[ d\xi_B + \bar{M}_{BD} d\xi^D \right],
\]

(2.31)

and

\[
Da = da + 2e_I^' A^I.
\]

(2.32)

The covariant derivative of \( a \) arises from the Green-Schwarz type interaction \( B_2 \wedge J_2 \) in (2.17) and as a consequence \( a \) couples like a Goldstone boson and is charged under an Abelian gauge symmetry with gauge charges \( e_I' \). \( V_E \) represents the potential in the Einstein frame and is given by

\[
V_E = -\frac{e^{4\phi}}{2} e_I^' (\text{Im} N')^{-1IJ} e_J^'.
\]

(2.33)

In ref. [33] it was shown that \( h_{uv} \) of (2.31) is a quaternionic metric in accord with the constraints of \( N = 2 \) supergravity that the scalars in the hypermultiplets span a quaternionic manifold. In order to establish the further consistency with gauged \( N = 2 \) supergravity we need to show that the potential (2.33) is consistent with the general form of the potential (B.18) of gauged supergravity. Let us first note that only one scalar \( a \) in the hypermultiplets carries gauge charge while the scalars \( t^i \) in the vector multiplets remain neutral. In terms of the Killing vectors defined in (B.14) and (B.17) eq. (2.32) implies

\[
k_{I}^u = -2e_I^' \delta_{ua}, \quad k_{I}^i = 0.
\]

(2.34)

Inserted into (B.18) using (2.31) one arrives at

\[
V_E = -\frac{1}{2} \left[ (\text{Im} N')^{-1} \right]^{IJ} P_I^x P_J^x + 4e^K X^I \bar{X}^J (e^{4\phi} e_I^' e_J^' - P_I^x P_J^x).
\]

(2.35)

We are left with the computation of the Killing prepotentials \( P_I^x \) defined in (B.10). Following ref. [11] one first observes that for the constant (field independent) Killing vectors as in (2.34) eqs. (B.16) are solved by

\[
P_I^x = k_I^u \omega^x_u, \quad x = 1, 2, 3,
\]

(2.36)
where \( \omega^x_u \) is the \( SU(2) \) connection on the quaternionic manifold. For the case at hand \( \omega^x_u \) has been computed in [33] and here we only need their result \( \omega^x_u = \frac{i}{2} e^{2\phi} \delta^{3x} \). Inserted into (2.36) using (2.34) we obtain

\[
P_i^1 = P_i^2 = 0, \quad P_i^3 = e^{2\phi} e_i'
\]

which implies \( P_i^x P_j^y = h_{uv} k_i^u k_j^v \). Thus the last term in (2.33) vanishes while the first one reproduces the potential (2.33). This establishes the consistency with \( N=2 \) gauged supergravity.

Let us return to the discussion of the action in the unrotated basis where both \( e_I \) and \( m^I \) are non-zero. In this case \( B_2 \) is massive and the relation with the standard gauged supergravity is not obvious and, as far as we know, has not been discussed in the literature. However, one can use the fact that a massive two-form in \( d=4 \) is Poincaré dual to a massive vector [28–30]. This generic duality is briefly summarized in appendix E.3. In the following we perform the duality transformation and display the dual action in terms of only vector fields.

Starting from the action (2.17) it is straightforward to apply the results in appendix E.3. Denoting by \( A^H \) the dual of the massive \( B_2 \) the resulting action reads

\[
S = \int e^{-2\phi} \left( \frac{1}{2} R^I 1 + 2 d\phi \wedge^* d\phi - g_{ab} d\zeta^a \wedge^* d\zeta^b - g_{ij} dt^i \wedge^* d\bar{t}^j \right) + \frac{1}{2} (\text{Im } M^{-1})^{AB} \left[ d\bar{\xi}_A + M_{AC} d\xi^C \right] \wedge^* \left[ d\bar{\xi}_B + \bar{M}_{BD} d\xi^D \right] - V
\]

\[
+ \frac{1}{2} \text{Im } N_{IJ} F^I \wedge^* F^J + \frac{1}{2} \text{Re } N_{IJ} F^I \wedge F^J - e^{2\phi} A^H \wedge^* A^H
\]

\[
- \frac{1}{2} \frac{M_2^2}{M^4 + M_T^4} (F^H - J'_2)^* (F^H - J'_2) + \frac{1}{2} \frac{M_T^2}{M^4 + M_T^4} (F^H - J'_2) \wedge (F^H - J'_2)
\]

(2.38)

where

\[
F^H = dA^H, \quad J'_2 = J_2 + \frac{1}{2} (\bar{\xi}_A d\xi^A - \xi^A d\bar{\xi}_A),
\]

(2.39)

and the quantities \( M, M_T \) and \( J_2 \) are defined in (2.18). The above action contains an explicit mass term for the vector field \( A^H \) which can equivalently be written as the covariant derivative of a Goldstone boson

\[
e^{2\phi} A^H \wedge^* A^H = \frac{1}{4} e^{2\phi} D_a \wedge^* D_a
\]

(2.40)

where

\[
D_a = da + 2A^H
\]

(2.41)

(\( A'^H \) denotes the gauge transformed vector potential.) Inserting (2.40) into (2.38) and absorbing \( \frac{1}{2} (\xi_A d\xi^A - \xi^A d\xi_A) \) into a further redefinition of \( A^H \) results in

\[
S = \int \frac{1}{2} R^I 1 - g_{ij} dt^i \wedge^* d\bar{t}^j - h_{uv} Dq^u \wedge^* Dq^v + \frac{1}{2} \text{Im } \hat{N}_{ij} F^i \wedge^* F^j + \frac{1}{2} \text{Re } \hat{N}_{ij} F^i \wedge F^j - V_E
\]

(2.42)
where also a Weyl rescaling $g_{\mu\nu} \rightarrow e^{2\phi}g_{\mu\nu}$ has been performed and we introduced the index $\hat{I} = (I, H)$. $V_E$ is the Weyl rescaled potential related to $V$ of (2.20) by $V_E = e^{4\phi}V$. $h_{uv}Dq^u \wedge^* Dq^v$ is again the standard quaternionic metric given in (2.31) with the only difference that (2.32) is replaced by (2.41). Moreover, the ‘new’ $(h_{1,1} + 2) \times (h_{1,1} + 2)$ dimensional gauge coupling matrix $\hat{N}_{\hat{I}\hat{J}}$ is given by

$$\hat{N}_{\hat{I}\hat{J}} = N_{IJ} - i\mu(e^I - N_{IK}m^K)(e^J - N_{JL}m^L), \quad \hat{N}_{HH} = -i\mu,$$

(2.43)

One easily shows that $\hat{N}_{\hat{I}\hat{I}}m^J = e_I$ and hence $\text{Im} \hat{N}_{\hat{I}\hat{J}}$ has a null vector while $\text{Re} \hat{N}_{\hat{I}\hat{J}}$ has one constant eigenvalue. This implies that one (linear combination) of the vector fields only has a topological coupling.

The dualization of $B_2$ resulted in an additional massive vector $A^H$ and we chose to write the mass term as the coupling of a Goldstone boson $a$. The number of physical degrees of freedom is of course unchanged since the action (2.38)/(2.42) is still invariant under the gauge transformations (2.24) which after dualization become

$$\delta A^I = -m^I\Lambda_1, \quad \delta A^H = 0.$$

(2.44)

$A^H$ being the Poincaré dual of $H_3$ is invariant under (2.24) but one of the other $h_{1,1} + 1$ vector fields in (2.42) can be gauged away by (2.44). In this ‘unitary gauge’ the symplectic invariance is lost. Thus the theory can be formulated in terms of only vector fields but symplectic invariance demands the presence of an additional auxiliary vector field with only topological couplings. In any physical gauge the symplectic invariance is broken.

To conclude this section let us discuss another aspect of the dualization of the massive $B$-field. Eq. (2.23) can be solved for $\tilde{F}^I$ and $\tilde{G}_I$ in terms of electric and magnetic potentials $A^I$ and $\tilde{A}_I$

$$\tilde{F}^I = m^IB_2 + dA^I, \quad \tilde{G}_I = e_IB_2 + d\tilde{A}_I.$$

(2.45)

Now the equation of motion for $B_2$ becomes

$$\frac{1}{2}d(e^{-2\phi}\ast dB_2) + m^I d\tilde{A}_I - e_IdA^I = 0.$$

(2.46)

This suggests that we can introduce a scalar field $a$ (the dual of $B_2$) which obeys

$$e^{-2\phi}dB_2 = Da \equiv da - 2m^I d\tilde{A}_I + 2e_IdA^I.$$

(2.47)

This definition has the feature that it maintains explicitly the symplectic invariance closely related to the proposal of [11, 13]. However, in (2.43) $B_2$ and not $dB_2$ appears and thus it is not possible to give an action in terms of the dual scalar $a$ with electric and magnetic couplings. Nevertheless, one can compute the electric and magnetic Killing prepotentials corresponding to the gauging (2.47) as suggested in [11, 13]. They are very similar to the ones found only for the electrically charged particles (2.37).

$$P_I^3 = e^{2\phi}e_I, \quad \tilde{P}_I^{13} = e^{2\phi}m^I, \quad P_I^1 = P_I^2 = \tilde{P}_I^{11} = \tilde{P}_I^{12} = 0.$$

(2.48)

We thank B. de Wit and S. Vandoren for discussions on this point.
Using the formula for the potential suggested in \[11\]

\[
V_E = 4e^K X^I \tilde{X}^J h_u (k^u - \tilde{k}^u N_{KI})(k^v - \tilde{k}^v N_{KI}) \tag{2.49}
\]

\[\left[ \frac{1}{2} (\text{Im} \mathcal{N})^{-1} J + 4e^K X^I \tilde{X}^J \right] (P^x_i - \tilde{P}^{Kx} N_{KI})(P^x_j - \tilde{P}^{Kx} N_{KI}) \]

which is the symplectic invariant extension of \[ (B.18) \], one immediately recovers the potential obtained in \[ (2.20) \].

### 2.4 NS fluxes

So far we concentrated on non-trivial flux related to the modification of the R-R field strength \( \hat{C}_3 \) and \( \hat{A}_1 \). In this section we discuss the modifications which appear in the type IIA compactification due to the presence of NS fluxes. For the consistency of the procedure, we require that the fields which acquire a background value appear in the action only via the corresponding field strengths. This is easily achieved for the R-R fields \( \hat{A}_1 \) and \( \hat{C}_3 \) as can be seen in the form of the actions \[ (2.1) \]. Clearly, for massive type IIA theory, the NS-NS field \( \hat{B}_2 \) cannot appear only via its field strength so we only have a chance to turn on NS fluxes if we start from the massless type IIA theory given in \[ (D.1) \]. However we need to perform the field redefinition \( \hat{C}_3 \rightarrow \hat{C}_3 + \hat{A}_1 \wedge \hat{B}_2 \) in order to have the action only depend on the field strength \( \hat{H}_3 \) but not \( \hat{B}_2 \). This turns \[ (D.1) \] into

\[
S = \int e^{-2\phi} \left( \frac{1}{2} \tilde{R}^{*1} + 2d\phi \wedge *d\phi - \frac{1}{4} \hat{H}_3 \wedge *\hat{H}_3 \right) - \frac{1}{2} \left( \hat{F}_2 \wedge *F_2 + \hat{F}_4 \wedge *F_4 \right) + \frac{1}{2} \hat{H}_3 \wedge \hat{C}_3 \wedge d\hat{C}_3, \tag{2.50}
\]

where \( \hat{F}_4 = d\hat{C}_3 - \hat{A}_1 \wedge \hat{H}_3 \) and \( \hat{H}_3 = d\hat{B}_2 \). Note that this field redefinition changes the form of the gauge transformations \[ (D.3) \] which now become

\[
\delta \hat{B}_2 = d\hat{\Lambda}_1, \quad \delta \hat{C}_3 = d\hat{\Sigma}_2, \\quad \delta \hat{A}_1 = d\hat{\Theta}, \quad \delta \hat{C}_3 = \hat{\Theta} \wedge d\hat{B}_2. \quad (2.51)
\]

The compactification of this action proceeds as in appendix \[ D \] with the following modification of the reduction Ansatz \[ (D.4) \]

\[
\hat{H}_3 = H_3 + db^i \omega_i + p^A \alpha_A + q_A \beta^A, \tag{2.52}
\]

where \( H_3 = dB_2 \) and \( (p^A, q_A) \) are the \( 2h_{1,2} + 2 \) possible flux parameters of \( \hat{H}_3 \). Using \[ (2.52) \] \( \hat{F}_4 \) is reduced according to

\[
\hat{F}_4 = dC_3 - A^0 \wedge H_3 + (dA^i - A^0 dB^i) \wedge \omega_i + D\xi^A \wedge \alpha_A + D\tilde{\xi}_A \wedge \beta^A, \tag{2.53}
\]

where

\[
D\xi^A = d\xi^A - p^A A^0, \quad D\tilde{\xi}_A = d\tilde{\xi}_A - q_A A^0. \tag{2.54}
\]
Inserting (2.52) and (2.53) into the action (2.50) results in terms very similar to (D.13). The differences are that the kinetic term for \( C_3 \) now contains the covariant derivatives \( D\xi^A, D\tilde{\xi}_A \) instead of the ordinary ones \( d\xi^A, d\tilde{\xi}_A \). Furthermore, the kinetic term for \( \tilde{B}_2 \) (D.13) now induces a potential

\[
V = -\frac{1}{4} \frac{e^{-2\phi}}{K} (q_A + M_{AC} p^C)(\text{Im } \mathcal{M})^{-1AB} (q_B + \tilde{M}_{BD} p^D), \tag{2.55}
\]

where \( \mathcal{M} \) is defined in (C.23). The last modification due to (2.52) appears from the topological term in (2.50) which reads

\[
\delta \mathcal{L}_{\text{top}} = (p^A \tilde{\xi}_A - q_A \xi^A) dC_3. \tag{2.56}
\]

As before we have to dualize the three-form \( C_3 \) to a constant. Collecting the terms including \( C_3 \) we find

\[
\mathcal{L}_{C_3} = -\frac{K}{2} (dC_3 - A^0 \wedge H_3) \wedge^* (dC_3 - A^0 \wedge H_3) + (p^A \tilde{\xi}_A - q_A \xi^A) dC_3. \tag{2.57}
\]

By using the formulas of appendix E.1 we obtain the following action for the dual scalar

\[
\mathcal{L}_{C_3} \to \mathcal{L}_e = -\frac{1}{2K} (p^A \tilde{\xi}_A - q_A \xi^A + e)^2 \, \mathbf{1} + (p^A \tilde{\xi}_A - q_A \xi^A + e) A^0 \wedge H_3, \tag{2.58}
\]

where \( e \) is an arbitrary constant, the dual of \( C_3 \). The first term in the above expression contributes together with (2.53) to the scalar potential which reads

\[
V = -\frac{e^{-2\phi}}{4K} (q + p \mathcal{M})(\text{Im } \mathcal{M})^{-1}(q + p \tilde{\mathcal{M}}) + \frac{1}{2K} (p^A \tilde{\xi}_A - q_A \xi^A + e)^2. \tag{2.59}
\]

The final thing to do in order to have the compactified action in the standard form of a gauged supergravity is to dualize \( B_2 \) to a scalar. Collecting the terms including \( B_2 \) we find

\[
\mathcal{L}_{B_2} = -\frac{e^{-2\phi}}{4} H_3 \wedge^* H_3 + \frac{1}{2} H_3 \wedge \left[ \bar{\xi}_A d\xi^A - \xi^A d\bar{\xi}_A - 2(p^A \tilde{\xi}_A - q_A \xi^A + e)A^0 \right] \tag{2.60}
\]

\[
= -\frac{e^{-2\phi}}{4} H_3 \wedge^* H_3 + \frac{1}{2} H_3 \wedge \left[ \bar{\xi}_A D\xi^A - \xi^A D\bar{\xi}_A - (p^A \tilde{\xi}_A - q_A \xi^A + 2e)A^0 \right].
\]

Using appendix E.1, we obtain the following action for the dual scalar \( a \)

\[
\mathcal{L}_{B_2} \to \mathcal{L}_a = -\frac{e^{2\phi}}{4} \left[ Da + (\bar{\xi}_A D\xi^A - \xi^A D\bar{\xi}_A) \right] \wedge^* \left[ Da + (\bar{\xi}_A D\xi^A - \xi^A D\bar{\xi}_A) \right], \tag{2.61}
\]

where

\[
Da = da - (p^A \tilde{\xi}_A - q_A \xi^A)A^0 - 2e A^0. \tag{2.62}
\]

The final form of the action action is obtained after going to the Einstein frame and is similar to the one found in the massless case (D.13)

\[
S = \int \left[ \frac{1}{2} R^* \mathbf{1} - g_{ij} dt^i \wedge^* dt^j - h_{uv} Dq^u \wedge^* Dq^v + \frac{1}{2} \text{Im } \mathcal{N}_{IJ} F^I \wedge^* F^J + \frac{1}{2} \text{Re } \mathcal{N}_{IJ} F^I \wedge F^J - V_E \right], \tag{2.63}
\]

\[
\int \left[ \frac{1}{2} R^* \mathbf{1} - g_{ij} dt^i \wedge^* dt^j - h_{uv} Dq^u \wedge^* Dq^v + \frac{1}{2} \text{Im } \mathcal{N}_{IJ} F^I \wedge^* F^J + \frac{1}{2} \text{Re } \mathcal{N}_{IJ} F^I \wedge F^J - V_E \right], \tag{2.63}
\]
where now

\[ h_{uv}Dq^u \wedge ^* Dq^v = d\phi \wedge ^* d\phi + g_{ab}dz^a \wedge ^* dz^b \tag{2.64} \]

\[ + \frac{e^{4\phi}}{4} \left[ Da + (\xi_A D\xi^A - \xi^A D\xi_A) \right] \wedge ^* \left[ Da + (\bar{\xi}_A D\xi^A - \xi^A D\bar{\xi}_A) \right] \]

\[ - \frac{e^{2\phi}}{2} (\text{Im } \mathcal{M}^{-1})^{AB} \left[ D\xi_A + \mathcal{M}_{AC} D\xi^C \right] \wedge ^* \left[ D\bar{\xi}_B + \bar{\mathcal{M}}_{BD} D\xi^D \right], \]

with the covariant derivatives \( D\xi^A, D\bar{\xi}_A \) and \( Da \) given in (2.54) and (2.62). The Einstein frame potential will only differ from (2.59) by a factor of \( e^{4\phi} \)

\[ V_E = -\frac{e^{2\phi}}{4\mathcal{K}} (q + p\mathcal{M})(\text{Im } \mathcal{M})^{-1} (q + p\bar{\mathcal{M}}) + \frac{e^{4\phi}}{2\mathcal{K}} (p^A \bar{\xi}_A - q_A \xi^A + e)^2 . \tag{2.65} \]

The crucial difference to the previous case of R-R fluxes is that now the potential depends on the scalars in the hypermultiplets but not on the scalars in the vector multiplets.

As before the fluxes turn ordinary supergravity into a gauged supergravity where a certain isometry of the scalar manifold has been gauged. The corresponding gauge invariance is just the compactified version of the 10 dimensional gauge invariance (2.51). Inserted into the compactification Ansatz (D.4) modified according to (2.52), the four-dimensional gauge invariance reads

\[ \delta A^0 = d\Theta , \quad \delta dA^i = \Theta \wedge db^i , \quad \delta C_3 = \Theta \wedge H_3 , \]

\[ \delta \xi^A = p^A \Theta , \quad \delta \bar{\xi}_A = q_A \Theta . \tag{2.66} \]

The covariant derivatives defined in (2.54) and the action (2.64) is invariant under this transformations provided \( a \) transforms according to

\[ a \rightarrow a + \left[ 2\epsilon + (p^A \bar{\xi}_A - q_A \xi^A) \right] \Theta . \tag{2.67} \]

As before let us establish the consistency of the theory with the standard gauged supergravity. Compared to the massless type IIA theory the effect of the non-trivial \((p^A, q_A)\) NS-fluxes is the replacement of ordinary derivatives by the covariant derivatives (2.54), (2.62) and the appearance of the potential (2.59). Otherwise the structure of the theory is unchanged. To show agreement with gauged supergravity we need to demonstrate that with the gaugings (2.54) and (2.62) the potential (B.18) reduces to (2.63). The key point here is to notice that for the case at hand (B.18) considerably simplifies in the sense that the term which contains the Killing prepotentials vanishes. To see this we first note that since only one vector field \( A^0 \) participates in the gaugings (2.54) and (2.62), the only non-trivial components of \( P^I_I \) will be the ones for which \( I = 0 \). Using (C.3), (C.11) and the fact that \( X^0 = 1 \) one immediately sees that

\[ \frac{1}{2} (\text{Im } \mathcal{N})^{-100} + 4e^K X^0 \bar{X}^0 = 0 . \tag{2.68} \]

Inserting (2.68) into (B.18) we arrive at

\[ V_E = 4e^K h_{uv} h^{u \bar{v}} , \tag{2.69} \]
where we already used $k_i = 0$. The Killing vectors $k_0^a$ can be read off from the covariant derivatives (2.54) and (2.62) to be

$$
k_0^\xi = p^B, \quad k_0^\tilde{\xi} = q_B, \quad k_0^a = 2e + (p^A \tilde{\xi}_A - q_A \xi_A),$$

(2.70)

Using the metric components of the charged scalars from (2.64), the evaluation of (2.69) precisely results in the potential (2.59) and thus establishes the consistency with gauged supergravity.

As we said before the difference for NS-fluxes is that the potential depends on the scalars in hypermultiplets $(\phi, z^\alpha, \xi^A, \tilde{\xi}^A)$ while the vector multiplet scalars $t^i$ remain undetermined. Furthermore, the only gauge field which appears in the covariant derivatives is the graviphoton $A^0$ while all other $A^i$ do not participate in the gauging.

3 Type IIB compactified on Calabi-Yau threefolds with background fluxes and mirror symmetry

So far we concentrated on type IIA theories compactified on Calabi-Yau threefolds $Y_3$ and derived the four-dimensional effective theory when non-trivial background fluxes are turned on. Without fluxes these theories are equivalent to type IIB compactified on the mirror threefold $\tilde{Y}_3$. At the level of the low energy effective action the precise map between these compactifications was derived in refs. [33-35]. However, when background flux is turned on, the validity of perturbative and non-perturbative dualities become obscure and is not fully understood at present [16, 19, 23-26, 36]. Before we discuss this issue in more detail let us focus on type IIB compactification with background fluxes. Without fluxes the effective theory is given in [34, 35] while turning on three-form flux was considered in refs. [11] [14, 17]. Here we do not redo the computation in detail but focus on the situation where both electric and magnetic charges $(e_A, m_A)$ are present. For this case the potential has been computed in [11] but the complete couplings of the two forms were not derived. (Ref. [17] only considered the case $m^A = 0$.) The purpose of this section is to give the complete bosonic Lagrangian including the couplings and mass terms of the NS two-form $B_2$ and to discuss the mirror symmetry to type IIA compactification with fluxes as derived in the previous section. For simplicity we only turn on the background fluxes of the R-R two-form $C_2$.

Let us start by recalling the structure of the ten-dimensional type IIB supergravity. The NS-NS sector features the graviton, a two-form $\hat{B}_2$ and the dilaton $\hat{\phi}$, while the R-R sector contains a second scalar $l$, a second two form $\hat{C}_2$ and a four form $\hat{A}_4$ with a self-dual field strength. The bosonic part of the low energy effective action in ten dimensions reads [37, 38]

$$
S_{IIB} = \int e^{-2\hat{\phi}} \left( \frac{1}{2} \hat{\phi}^* \hat{\phi} + 2 d\hat{\phi} \wedge^* d\hat{\phi} - \frac{1}{4} \hat{H}_3 \wedge^* \hat{H}_3 \right) - \frac{1}{2} \left( dl \wedge^* dl + \hat{F}_3 \wedge^* \hat{F}_3 + \frac{1}{2} \hat{F}_5 \wedge^* \hat{F}_5 \right) - \frac{1}{2} \hat{A}_4 \wedge \hat{H}_3 \wedge dl \hat{C}_2, \quad (3.1)
$$

\[13\] In this action the self-duality condition on the 5-form field strength has not been imposed. A covariant action including the self-dual 4-form has been constructed in [38]. The field equations for type IIB supergravity were originally derived in refs. [39].
where the field strengths are defined as

\[ \hat{H}_5 = dB_2, \quad \hat{F}_3 = dC_2 - l dB_2, \quad \hat{F}_5 = d\hat{A}_4 + \hat{B}_2 \wedge d\hat{C}_2. \] (3.2)

When compactified on a Calabi-Yau threefold the resulting low energy spectrum contains as in type IIA \( h_{1,1} \) Kähler deformations \( v^i, i = 1, \ldots, h_{1,1} \) of the Calabi-Yau metric and \( h_{1,2} \) complex deformations \( z^a, a = 1, \ldots, h_{1,2} \) of the complex structure. The doublet of two-forms \( \hat{B}_2, \hat{C}_2 \) and the four-form \( \hat{A}_4 \) decompose according to

\[ \hat{B}_2 = B_2 + b^i \omega_i, \quad \hat{C}_2 = C_2 + c^i \omega_i, \]

\[ d\hat{A}_4 = dD^i \wedge \omega_i + F^A \alpha_A - G_B \beta^B + d\rho_i \tilde{\omega}^i, \] (3.3)

where as before \( (\alpha_A, \beta^B) \) span a real \((2h_{2,1} + 2)\)-dimensional basis of \( H^3(Y_3) \), \( \omega_i \) is a \( h_{1,1} \)-dimensional basis of \( H^{1,1}(Y_3) \) and \( \tilde{\omega}^i \) is the dual basis on \( H^{2,1}(Y_3) \). The normalizations of these basis are given in appendix C. The self-duality condition \( *F_5 = F_5 \) implies that the \( G_B \) are the dual magnetic field strengths of \( F^A \) while \( \rho_i \) are the duals of the tensors \( D^i \). Together these fields combine into a gravitational multiplet with bosonic components \((g_{\mu\nu}, A^\alpha)\), a double-tensor multiplet \((B_2, C_2, \phi, l)\), \( h_{1,1} \) tensor multiplets \((D^i, v^i, b^i, c^i)\) and \( h_{1,2} \) vector multiplets \((A^a, z^a)\)\(^{14}\). Dualizing the two-forms to scalars turns the tensor and double tensor multiplets into hypermultiplets each containing four real scalars. In this dual basis the low energy spectrum features \( h_{1,1} + 1 \) hypermultiplets and \( h_{1,2} \) vector multiplets apart from the gravitational multiplet.

Next we turn on background fluxes for the R-R three-form by modifying \( d\hat{C}_2 \)

\[ d\hat{C}_2 \rightarrow d\hat{C}_2 + m^A \alpha_A - e_A \beta^A, \] (3.4)

where \( e^A_1, m^A_1 \) are the constant background fluxes. The field strength \( \hat{F}_3 \) turns into

\[ \hat{F}_3 = F_3 - lH_3 + (dc^i - ld\tilde{b}^i) \omega_i + m^A \alpha_A - e_A \beta^A, \] (3.5)

where \( F_3 = dC_2 \) and \( H_3 = dB_2 \) and the only modification due to \((3.4)\) coming from the term \( F_3 \wedge *F_3 \) is a potential

\[ V = -\frac{1}{2} (e_A - m^C \mathcal{M}_{CA})(\text{Im} \mathcal{M})^{-1AB} (e_B - \mathcal{M}_{BD} m^D). \] (3.6)

The matrix \( \mathcal{M} \) is defined in \((C.22)\) and to obtain the above expression we used \((C.20)\) and \((C.22)\). Furthermore, \( d\hat{C}_2 \) appears in the definition of \( \hat{F}_5 \) \((3.2)\) which is modified according to

\[ \hat{F}_5 = (dD^i + b^i dC_2 + B_2 \wedge dc^i) \wedge \omega_i + \hat{F}^A \alpha_A - \hat{G}_A \beta^A + (d\rho_i + b^i d\tilde{c}^k \kappa_{ijk}) \wedge \tilde{\omega}^i, \] (3.7)

where we defined

\[ \hat{F}^A \equiv F^A + m^A B_2, \quad \hat{G}_A \equiv G_A + e_A B_2. \] (3.8)

\(^{14}\)Commonly one uses the convention \( F'_5 = dA'_4 - \frac{1}{2} \hat{C}_2 \wedge d\hat{B}_2 + \frac{1}{2} \hat{B}_2 \wedge d\hat{C}_2 \) which is obtained from \( F_5 \) by the redefinition \( A_4 = A'_4 - \frac{1}{2} B_2 \wedge C_2 \). Note that this redefinition modifies the topological term only by a total derivative while the self-duality constraint \( *F_5 = F_5 \) remains unchanged.

\(^{15}\)We consider \( A^A \) to be electric potentials, i.e. \( F^A = dA^A \)
and used (C.9) to express the product of harmonic two forms as a harmonic four form. Finally, the topological term produces a Green-Schwarz type interaction

$$\delta \mathcal{L}_{\text{top}} = -\frac{1}{2} (F^A e_A - G_A m^A) \wedge B_2.$$  \hfill (3.9)

In the standard compactification of the type IIB theory the four dimensional action is obtained after imposing the self-duality condition for $\tilde{F}_5$. This however, cannot be done directly since imposing $\tilde{F}_5 = *\tilde{F}_5$ in (3.1) the kinetic term for $\tilde{A}_4$ vanishes identically and the same happens with the kinetic terms of the fields which come from the reduction of $\tilde{A}_4$. The correct 4 dimensional action is obtained by adding appropriate Lagrange multipliers in order to impose the self-duality condition. At the level of the reduced fields this condition can be obtained from (3.7) using the expressions for the Hodge duals of the harmonic forms on a Calabi-Yau threefold (C.9), (C.20), (C.22) and reads

$$\tilde{G}_A = \text{Im} \mathcal{M}_{AB} * \tilde{F}_B + \text{Re} \mathcal{M}_{AB} \tilde{F}_B.$$  \hfill (3.10)

Similarly one obtains

$$d\rho_i + K_{ijk} b^j d\rho_k = 4 \mathcal{K}_{ij} (dD^j + b^j dC_2 + B_2 \wedge d\rho^i).$$  \hfill (3.11)

Strictly speaking this relation should involve field strengths as in (3.10) rather than only exterior derivatives of some potentials. However, since the fluxes play no role in this relation we do not need to be more precise here and instead rely to the standard Calabi-Yau compactification of type IIB theory. On the other hand, (3.10) does depend on the flux parameters through $\tilde{F}$ and $\tilde{G}$ defined in (3.8) and reduces to the usual formula (B.8) which relates the electric and magnetic field strengths only for vanishing fluxes. To obtain the Lagrangian for the vector fields we proceed as in the massless case [17]. First, treating $F^A$ and $G_A$ as independent fields, inserting (3.7) into (3.1) and taking into account (3.9) we obtain

$$\mathcal{L}(F, G) = \frac{1}{4} (\text{Im} \mathcal{M})^{-1AB} (\tilde{G}_A - \tilde{F}_C \mathcal{M}_{AC}) \wedge (* \tilde{G}_B - * \tilde{F}_D \mathcal{M}_{BD})$$

$$- \frac{1}{2} (F^A e_A - G_A m^A) \wedge B_2.$$  \hfill (3.12)

Furthermore, to impose (3.10) as the equation of motion for $G$ we have to add to (3.12) the term $\frac{1}{2} F^A \wedge G_A$. Eliminating $G_A$ using (3.10) one is left with the following Lagrangian for $F^A$

$$\mathcal{L}(F) = \frac{1}{2} F^A \wedge G_A - \frac{1}{2} (F^A e_A - G_A m^A) \wedge B_2$$

$$= \frac{1}{2} \text{Im} \mathcal{M}_{AB} \tilde{F}^A \wedge * \tilde{F}_B + \frac{1}{2} \text{Re} \mathcal{M}_{AB} \tilde{F}^A \tilde{F}_B - \frac{1}{2} (F^A e_A + \tilde{F}^A e_A) \wedge B_2.$$  \hfill (3.13)

The scalar sector is not modified by the introduction of fluxes in (3.4) and so we can
use the results in the literature \cite{33,35} to obtain the following 4 dimensional action

\[ S = \int e^{-2\phi} \left( \frac{1}{2} R^{\ast} + 2d\phi \wedge \ast d\phi - \frac{1}{4} H_3 \wedge \ast H_3 - g_{ab}dz^a \wedge \ast d\phi^b - g_{ij}dt^i \wedge \ast dt^j \right) \]

\[ + \frac{1}{2} \left( \text{Im} N^{-1} \right)^{IJ} \left[ d\tilde{\xi}_I + N_{IK}d\xi^K \right] \wedge \left[ d\tilde{\xi}_J + N_{JL}d\xi^L \right]. \]

\[ + \frac{1}{2} H_3 \wedge (\tilde{\xi}_A d\xi^A - \xi_A d\tilde{\xi}_A) + \frac{1}{2} \text{Im} M_{AB} \tilde{F}^A \wedge \ast \tilde{F}^B + \frac{1}{2} \text{Re} M_{AB} \tilde{F}^A \wedge \tilde{F}^B \]

\[ - \frac{1}{2} B_2 \wedge (\tilde{F}^A + dA^A)e_A - V, \]

where \( V \) is given in \((3.14)\) and the scalars \( \xi^I \) and \( \tilde{\xi}_I \) are functions of \( l, c^i, \rho_i \) and the dual of \( C_2 \) specified by the mirror map \cite{35}.

Mirror symmetry at the level of the low energy effective actions is now obvious since the \((2.22)\) and \((3.14)\) have the same form. The only difference is the range of indices and the fact that the matrices \( N \) and \( M \) are interchanged which just expresses the mirror symmetry \( h_{1,1} \leftrightarrow h_{1,2} \). Moreover the potentials \((3.6)\) and \((2.20)\) are the same and the flux parameters \((m^A, e_A)\) in type IIB case are the mirror partners of \((m^I, e_I)\) in type IIA. Due to the appearance of \( \tilde{F}^A \) in \((3.14)\) \( B_2 \) is also massive in IIB – a fact which has not been stressed previously. Finally, the two extra parameters in type IIA, the one coming from the mass \( m \) of the massive type IIA supergravity and the other coming from the dualization of the 4 dimensional \( C_3 \) to a constant \( e_0 \), are crucial in order to make mirror symmetry work when R-R fluxes are turned on.

A similar analysis can be performed for the NS three-form. Apart from the factor of the dilaton one obtains the same result as computed in refs. \cite{11,12,17}. However, in this case there is no obvious mirror dual set of fluxes in type IIA. This has led to the proposition \cite{23} that in type IIA the holomorphic three-form \( \Omega \) ceases to be holomorphic and effectively an NS two- and four-form is induced. It would be interesting to make this proposal more explicit at the level of the effective action. Similarly there is no obvious mirror dual in type IIB of the NS-fluxes \((p^A, q_B)\) discussed in section 2.4.

4 Conclusions

In this paper we studied the compactification of type II theories on Calabi-Yau threefolds in the presence of background fluxes. First we concentrated on the massive type IIA theory and considered non-trivial background values for the R-R fields \( \tilde{F}_2, \tilde{F}_4 \). These fluxes are in one to one correspondence with harmonic \((1,1)\)- and \((2,2)\)-forms on the Calabi-Yau manifold and therefore induce \( 2h_{1,1} \) flux parameters into the effective action. Furthermore, dualizing the three-form \( C_3 \) in the \( d = 4 \) effective action to a constant provides an additional parameter. Together with the \((ten-dimensional)\) mass of massive type IIA theory one finds a total of \( 2h_{1,1} + 2 \) flux parameters which naturally combine into symplectic vector of \( Sp(2h_{1,1} + 2) \). Furthermore, we showed that in fact the entire theory is invariant under \( Sp(2h_{1,1} + 2) \) rotations due to the presence of a massive two-form \( B_2 \) which couples to both electric and magnetic field strengths. In our analysis we just followed the compactification of type IIA supergravity but there is obviously a
It would be interesting to construct the most general gauged supergravity including massive two-forms and investigate the conditions for the symplectic $Sp(2h_{1,1} + 2)$ invariance of the theory entirely within supergravity and without any particular reference to flux backgrounds of string theory.

For vanishing magnetic fluxes $m^I = 0$ the two-form $B_2$ is massless and can be dualized to a scalar $a$. This dual basis is a standard gauged $N = 2$ supergravity with a potential (2.33) depending on the scalars in the vector multiplets. Only $a$ is charged but with respect to linear combination of all $h_{1,1} + 1$ gauge fields. The form of the potential coincides with the potential found in [18] for the case of the heterotic string compactified on $T^2 \times K3$ when a set of very specific fluxes along $K3$ are turned on. In particular, in the heterotic case more than one scalar is generically charged and thus the fluxes have to be chosen to point in the direction of $a$ in order to find agreement with the type IIA case [16].

The perturbative duality of type IIA compactified on $Y_3$ to type IIB compactified on the mirror threefold $\tilde{Y}_3$ was established at the level of the effective Lagrangian for R-R fluxes turned on in both theories. For non-vanishing $m^I$ also the type IIB effective theory features a massive $B_2$. Furthermore, the validity of the duality crucially depends on including the ten-dimensional mass parameter $m^0$ and the dual constant $e_0$ of three-form $C_3$.

The situation for NS-fluxes remains murky. We derived the effective theory of type IIA in the presence of NS three-form flux $H_3$. In this case a standard gauged $N = 2$ supergravity is found but contrary to R-R flux backgrounds the potential depends non-trivially on the scalars in the hypermultiplet. Furthermore, the hyper-scalars are charged only with respect to the graviphoton but not with respect to any of the other $h_{1,1}$ gauge fields.

In the type IIB compactification we found no obvious set of NS mirror fluxes and leave this puzzle for further studies. As suggested in [23] it might be related to the fact that the Calabi-Yau geometry is deformed and the precise nature of the deformation has to be taken into account in more detail.

We also did not address the issue of holomorphic superpotentials in relation with spontaneous $N = 2 \to N = 1$ supersymmetry breaking [12] which we leave to a separate publication [40].

Appendix

A Notations and conventions

In this appendix we assemble the conventions used throughout the paper.

- The space-time metric has signature $(-, +, +, \ldots)$. 

Appendix
• The components of a differential $p$-form are defined as follows
\[ A_p = \frac{1}{p!} A_{\mu_1 \ldots \mu_p} dx^{\mu_1} \wedge \ldots \wedge dx^{\mu_p}. \] (A.1)

• A hat on a $p$-form, e.g. $\hat{A}_p$ denotes differential forms in $d = 10$. $p$-forms without the hat are four-dimensional quantities.

• The Hodge operation $^*$ is defined in such a way that
\[ dA_p \wedge ^* dA_p = \frac{\sqrt{-g}}{p!} (dA)_{\mu_1 \ldots \mu_{p+1}} (dA)^{\mu_1 \ldots \mu_{p+1}} d^4x \] (A.2)

reproduces the correct kinetic term for a $p$-form in $d$ space-time dimensions. In particular we denote $^*1 = \sqrt{-g} d^4x$.

• After compactification the Hodge operation splits into a Hodge-star on the four-dimensional space and another one acting on the internal Calabi-Yau space. For example, in the expansion of a $p$ form one encounters terms like $\hat{A}_p = \ldots + A_{p-k} \omega_k + \ldots$, where $\omega_k$ is some harmonic $k$ form on the internal space. The Hodge dual is given by
\[ ^*\hat{A}_p = \ldots + (-1)^{k(p-k)} A_{p-k}^* \omega_k + \ldots, \] (A.3)

where the first $^*$ on the RHS acts only in space-time while the second acts only in the internal space. The $(-1)^{k(p-k)}$ assures that the kinetic term of $\hat{A}_p$ produces
\[ \int_{Y_3} \hat{A}_p \wedge ^* \hat{A}_p = \ldots + A_{p-k} \wedge ^* A_{p-k} \int_{Y_3} \omega_k \wedge ^* \omega_k + \ldots. \] (A.4)

• The indices $i, j, k, \ldots$ label harmonic (1, 1) and (2, 2) forms on the Calabi-Yau threefold and run from 1 to $h_{1,1}$; the indices $I, J, \ldots$ label the vector fields in type IIA compactifications and include the zero $I = 0, 1, \ldots, h_{1,1}$. The indices $a, b \ldots$ run from 1 to $h_{1,2}$ and label (1, 2)-forms on $Y_3$. The indices $A, B \ldots$ include the zero and label all three-forms including the (3, 0)-form, i.e. $A = 0, 1, \ldots, h_{1,2}$. $A, B, \ldots$ also label vector fields in type IIB compactifications.

B $N = 2$ supergravity in $d = 4$

The purpose of this appendix is to give a short review about $N = 2$ supergravity in $d = 4$ [20–22, 41]. A generic spectrum contains the gravitational multiplet, $n_V$ vector multiplets and $n_H$ hypermultiplets. The vector multiplets contain $n_V$ complex scalars $t^i, i = 1, \ldots, n_V$ while the hypermultiplets contain $4n_H$ real scalars $q^a, u = 1, \ldots, 4n_H$ [16]. Due to supersymmetry the scalar manifold factorizes
\[ \mathcal{M} = \mathcal{M}_V \otimes \mathcal{M}_H, \] (B.1)

16There also exist $N = 2$ tensor multiplets which contain a two-form $B_2$ and three real scalars as bosonic components. Upon dualizing $B_2$ to a scalar the tensor multiplet can be treated as an additional hypermultiplet. With this in mind we do not discuss tensor multiplets in this appendix.
where the component $\mathcal{M}_V$ is a special Kähler manifold spanned by the scalars $t^i$ while $\mathcal{M}_H$ is a quaternionic manifold spanned by the scalars $q^u$.

A special Kähler manifold is a Kähler manifold whose geometry obeys an additional constraint [20]. This constraint states that the Kähler potential $K$ is not an arbitrary real function but determined in terms of a holomorphic prepotential $\mathcal{F}$ according to

$$K = -\ln \left( iX^I(t)\mathcal{F}_I(X) - iX^I(t)\bar{\mathcal{F}}_I(\bar{X}) \right), \quad (B.2)$$

where $X^I, I = 0, \ldots, n_V$ are $(n_V + 1)$ holomorphic functions of the $t^i$. $\mathcal{F}_I$ abbreviates the derivative, i.e. $\mathcal{F}_I \equiv \frac{\partial \mathcal{F}(X)}{\partial X^I}$ and $\mathcal{F}(X)$ is a homogeneous function of $X^I$ of degree 2, i.e. $X^I \mathcal{F}_I = 2 \mathcal{F}$.

The $4n_H$ scalars $q^u, u = 1, \ldots, 4n_H$ in the hypermultiplets are coordinates on a quaternionic manifold [21]. This implies the existence of three almost complex structures $(J^w)^x_v, x = 1, 2, 3$ which satisfy the quaternionic algebra

$$J^y J^z = -\delta^{xy} + i\epsilon^{xyz} J^z. \quad (B.3)$$

Associated with the complex structures is a triplet of Kähler forms

$$K^x_v = h_{uw} (J^x)^w_v, \quad (B.4)$$

where $h_{uw}$ is the quaternionic metric. The holonomy group of a quaternionic manifold is $Sp(2) \times Sp(2n_H)$ and $K^x$ is identified with the field strength of the $Sp(2) \sim SU(2)$ connection $\omega^x$, i.e.

$$K^x = d\omega^x + \frac{1}{2} \epsilon^{xyz} \omega^y \wedge \omega^z. \quad (B.5)$$

The bosonic part of the (ungauged) $N = 2$ action is given by

$$S = \int \left[ \frac{1}{2} R^* \mathbf{1} - g_{ij} dt^i \wedge d\bar{t}^j - h_{uw} dq^u \wedge dq^v + \frac{1}{2} \text{Im} N_{IJ} F^I \wedge \ast F^J + \frac{1}{2} \text{Re} N_{IJ} F^I \wedge F^J \right], \quad (B.6)$$

where $g_{ij} = \partial_i \partial_j K$, $F^I = dA^I$ ($F^0$ denotes the field strength of the graviphoton) and the gauge coupling functions are given by

$$N_{IJ} = \bar{\mathcal{F}}_{IJ} + 2i \frac{\text{Im} \mathcal{F}_{IK} \text{Im} \mathcal{F}_{JL} X^K X^L}{\text{Im} \mathcal{F}_{KL} X^K X^L}. \quad (B.7)$$

The equations of motion of the action [B.4] are invariant under generalized electric-magnetic duality transformations. From (B.6) one derives the equations of motions

$$\frac{\partial \mathcal{L}}{\partial A^I} = dG_I = 0, \quad G_I = \text{Re} N_{IJ} F^J + \text{Im} N_{IJ} \ast F^J, \quad (B.8)$$

while the Bianchi identities read

$$dF^I = 0. \quad (B.9)$$
These equations are invariant under the generalized duality rotations:

\[ F^I \rightarrow U^I_J F^J + Z^{IJ} G_J, \]
\[ G_I \rightarrow V_I^J G_J + W_{IJ} F^J, \]  

(B.10)

where \( U, V, W \) and \( Z \) are constant, real, \((n_V + 1) \times (n_V + 1)\) matrices which obey

\[ U^T V - W^T Z = V^T U - Z^T W = 1, \]
\[ U^T W = W^T U, \quad Z^T V = V^T Z. \]  

(B.11)

Together they form the \((2n_V + 2) \times (2n_V + 2)\) symplectic matrix

\[ O = \begin{pmatrix} U & Z \\ W & V \end{pmatrix}, \quad O \in Sp(2n_V + 2). \]  

(B.12)

\((F^I, G_I)\) form a \((2n_V + 2)\) symplectic vector; \((X^I, F_I)\) enjoy the same transformations properties and also transforms as a symplectic vector under (B.10). Clearly, the Kähler potential (B.2) is invariant under this symplectic transformation. Finally, the matrix \( \mathcal{N} \) transforms according to

\[ \mathcal{N} \rightarrow (V \mathcal{N} + W) (U + Z \mathcal{N})^{-1}. \]  

(B.13)

Let us now turn to gauged \( N = 2 \) supergravity [22]. One can gauge the isometries on the scalar manifold \( M \). Such isometries are generated by the Killing vectors \( k^u_I(q) \), \( k^i_I(t) \)

\[ \delta q^u = L^I k^u_I(q), \quad \delta t^i = L^I k^i_I(t). \]  

(B.14)

\( k^u_I(q) \), \( k^i_I(t) \) satisfy the Killing equations which in \( N = 2 \) supergravity can be solved in terms of four Killing prepotentials \((P_I, P_I^x)\). The Killing vectors on \( M_V \) are holomorphic and obey

\[ k^i_I(t) = g^{ij} \partial_j P_I, \]  

(B.15)

while the Killing vectors on \( M_H \) are determined by a triplet of Killing prepotentials \( P_I^x(q) \) via

\[ k^u_I K^x_{uv} = -D_u P^x_I \equiv - (\partial_u P^x_I + \epsilon^{xyz} \omega^u_{v} P^x_I). \]  

(B.16)

Gauging the isometries (B.14) requires the replacement of ordinary derivatives by covariant derivatives in the action

\[ \partial_\mu q^u \rightarrow D_\mu q^u = \partial_\mu q^u - k^u_I A^I_\mu, \quad \partial_\mu t^i \rightarrow D_\mu t^i = \partial_\mu t^i - k^i_I A^I_\mu. \]  

(B.17)

Furthermore the potential

\[ V_E = e^K [X^I \bar{X}^J (g_{ij} k^i_I k^j_J + 4 h_{uv} k^u_I k^v_J) + g^{ij} D_i X^I D_j \bar{X}^J P_I^x P_J^x - 3 X^I \bar{X}^J P_I^x P_J^x] \]
\[ = e^K X^I \bar{X}^J (g_{ij} k^i_I k^j_J + 4 h_{uv} k^u_I k^v_J) - \left[ \frac{1}{2} (\text{Im} \mathcal{N})^{-11} + 4 e^K X^I \bar{X}^J \right] P_I^x P_J^x, \]  

(B.18)

\( ^{17} \) This is often stated in terms of the self-dual and anti-self-dual part of the field strength \( F^{\pm J} \) and the dual quantities \( G^{+ J}_I \equiv \mathcal{N} J^+_I F^{+ J}, \quad G^{- J}_I \equiv \mathcal{N} J^{- J} F^{- J}. \)
has to be added to the action in order to preserve supersymmetry. The bosonic part of
the action of gauged $N = 2$ supergravity is then given by

$$ S = \int \frac{1}{2} R^* \mathbf{1} - g_{ij} D^i \wedge^* D^j - h_{uv} D q^u \wedge^* D q^v + \frac{1}{2} \text{Im} \mathcal{N}_{IJ} F^I \wedge^* F^J + \frac{1}{2} \text{Re} \mathcal{N}_{IJ} F^I \wedge F^J - V_E . $$

(B.19)

The symplectic invariance of the ungauged theory is generically broken since the action
now explicitly depends on the gauge potentials $A^I$ through the covariant derivatives
$D^i, D q^u$.

C The moduli space of Calabi-Yau threefolds

The moduli space of Calabi-Yau threefolds $Y_3$ splits into the space of Kähler deforma-
tions of the Calabi-Yau metric and deformations of the complex structure. The Kähler
deformations are harmonic $(1,1)$-forms and thus are elements of $H^{1,1}(Y_3)$. The complex
structure deformations are harmonic $(1,2)$-forms and thus are elements of $H^{1,2}(Y_3)$. In
string theory a two-form $B_2$ always appears together with the (space-time) metric in the
NS-NS sector. Its compactification on a Calabi-Yau threefold produce $h_{1,1}$ additional
scalars which together with the Kähler class deformations form the the “complexified Kähler cone” $\mathcal{M}_{1,1}$. The moduli space $\mathcal{M}$ of Calabi-Yau manifolds is be a direct product
of $\mathcal{M}_{1,1}$ and the space $\mathcal{M}_{1,2}$ spanned by the complex structure deformations

$$ \mathcal{M} = \mathcal{M}_{1,1} \times \mathcal{M}_{1,2} . $$

(C.1)

$\mathcal{M}_{1,1}$ and $\mathcal{M}_{1,2}$ are both special Kähler manifolds and $\mathcal{M}_{1,1}$ describes the vector multiplet
sector in type IIA theories while $\mathcal{M}_{1,2}$ characterizes the vector multiplet sector in type
IIB case. In this appendix we briefly summarize these geometries following refs. [41–44].

C.1 The complexified Kähler cone

The Kähler class deformations together with the zero modes of $B_2$ are harmonic $(1,1)$-
forms on $Y_3$. Hence both the Kähler form $J$ and $B_2$ can be expanded in a basis $\omega_i$ of
$H^{1,1}(Y_3)$

$$ B_2 + iJ = (b^j + iv^j)\omega_j \equiv \theta^j \omega_j , \quad j = 1, \ldots, h_{1,1} . $$

(C.2)

It is useful to define the following quantities:

$$ \mathcal{K} = \frac{1}{6} \int_{Y_3} J \wedge J \wedge J , \quad \mathcal{K}_i = \int_{Y_3} \omega_i \wedge J \wedge J , $$

$$ \mathcal{K}_{ij} = \int_{Y_3} \omega^i \wedge \omega^j \wedge J , \quad \mathcal{K}_{ijk} = \int_{Y_3} \omega^i \wedge \omega^j \wedge \omega^k . $$

(C.3)

Note that we have introduced the factor $\frac{1}{6}$ in the definition of $\mathcal{K}$ so that it is precisely the
volume of $Y_3$. The metric on the complexified Kähler cone $\mathcal{M}_{1,1}$ is Kähler, i.e. $g_{ij} = \partial_i \partial_j \mathcal{K}$
and given by \[ 31, 42 \]

\[
g_{ij} = \frac{1}{4K} \int_{Y_3} \omega_i \wedge^* \omega_j = -\frac{1}{4} \left( \frac{K_{ij}}{K} - \frac{1}{4} \frac{K_i K_j}{K^2} \right) = -\partial_i \partial_j \left( -\ln 8 \mathcal{K} \right) = \partial_i \partial_j \left( -\ln 8 \mathcal{K} \right).
\]

Furthermore, the Kähler potential \( K \) is determined in terms of a holomorphic prepotential \( \mathcal{F} \) via

\[
e^{-K} = 8 \mathcal{K} = i \left( \bar{X}^I \mathcal{F}_I - X^I \bar{\mathcal{F}}_I \right), \quad \mathcal{F}_I \equiv \partial_I \mathcal{F}, \quad I = 0, \ldots, h_{1,1},
\]

where

\[
\mathcal{F} = -\frac{1}{3!} \frac{K_{ijk} X^i X^j X^k}{X^0}.
\]

The (complex) Kähler class deformations \( t^i \) are the so-called special coordinates related to the \( X^I \) via \( X^I = (1, t^i) \). \( (X^I, \mathcal{F}_I) \) transforms as a symplectic vector under (B.12) and \( K \) is a symplectic invariant.

The scalar manifold \( M_{1,1} \) describes the moduli space of the vector multiplets in the low energy effective action of type IIA supergravity compactified on a Calabi-Yau threefold. Therefore the matrix \( \mathcal{N} \) defined in (B.7) plays the role of generalized gauge couplings. Inserting (C.6) into (B.7) it is straightforward to derive

\[
\begin{align*}
\text{Re} \mathcal{N}_0 &= -\frac{1}{3} K_{ijk} b^i b^j b^k, & \text{Im} \mathcal{N}_0 &= -K + \left( K_{ij} - \frac{1}{4} \frac{K_i K_j}{K} \right) b^i b^j, \\
\text{Re} \mathcal{N}_i &= \frac{1}{2} K_{ijk} b^i b^j b^k, & \text{Im} \mathcal{N}_i &= \left( K_{ij} - \frac{1}{4} \frac{K_i K_j}{K} \right) b^j, \\
\text{Re} \mathcal{N}_{ij} &= -K_{ijk} b^k, & \text{Im} \mathcal{N}_{ij} &= \left( K_{ij} - \frac{1}{4} \frac{K_i K_j}{K} \right).
\end{align*}
\]

In the main text we encounter the inverse matrices \( g^{ij} \) and \( (\text{Im} \mathcal{N})^{-1} \). These can be expressed in terms of harmonic (2, 2) forms. On a Calabi-Yau threefold \( H^{2,2}(Y_3) \) is dual to \( H^{1,1}(Y_3) \) and it is useful to introduce the dual basis \( \tilde{\omega}^i \) normalized by

\[
\int_{Y_3} \omega_i \wedge \tilde{\omega}^j = \delta^i_j.
\]

With this normalization the following relations hold

\[
g^{ij} = 4K \int_{Y_3} \tilde{\omega}^i \wedge^* \tilde{\omega}^j, \quad * \omega_i = 4K g_{ij} \tilde{\omega}^j, \quad * \tilde{\omega}^i = \frac{1}{4K} g^{ij} \omega_j, \quad \omega_i \wedge \omega_j \sim K_{ijk} \tilde{\omega}^k.
\]

where the symbol \( \sim \) denotes the fact that the quantities are in the same cohomology class. Introducing \( K^{ij} \) via

\[
K^{ij} K_{jk} = \delta^i_k,
\]

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one derives

\[
(\text{Im} \mathcal{N})^{-1} = \begin{pmatrix}
-\frac{1}{K} & -\frac{b_i}{K} \\
-\frac{b_i}{K} & K^{ij} - \frac{v^i v^j}{2K}
\end{pmatrix}.
\] (C.11)

Using (C.10) one can also write an explicit formula for the inverse metric \(g^{ij}\)

\[
g^{ij} = -4K \left( K^{ij} - \frac{v^i v^j}{2K} \right). \tag{C.12}
\]

C.2 The special geometry of \(H^3\)

The complex structure deformations of a threefold parameterize \(H^{1,2}(Y_3)\). However, it turns out to be convenient to discuss the entire \(H^3(Y_3) = H^{3,0} \oplus H^{2,1} \oplus H^{1,2} \oplus H^{0,3}\) including \(H^{3,0}\) and \(H^{0,3}\) which have only the holomorphic \((3,0)\)-form \(\Omega\) and the complex conjugate \((0,3)\)-form \(\bar{\Omega}\) as elements. One commonly chooses a real basis \((\alpha^A, \beta^B)\) on \(H^3\) which obeys

\[
\int_{Y_3} \alpha^A \wedge \beta^B = \delta^B_A = -\int_{Y_3} \beta^B \wedge \alpha_A, \quad A, B = 0, \ldots, h_{1,2},
\]

\[
\int_{Y_3} \alpha^A \wedge \alpha_B = \int_{Y_3} \beta^A \wedge \beta^B = 0. \tag{C.13}
\]

Note that these relations are invariant under symplectic rotations

\[
\begin{pmatrix}
\beta \\
\alpha
\end{pmatrix} \to \begin{pmatrix}
U & Z \\
W & V
\end{pmatrix} \begin{pmatrix}
\beta \\
\alpha
\end{pmatrix}, \tag{C.14}
\]

where the matrices \(U, V, W, Z\) satisfy (B.11). The unique holomorphic \((3,0)\) \(\Omega\) can be expanded in terms of this basis according to

\[
\Omega = Z^A \alpha_A - G_A \beta^A, \tag{C.15}
\]

where \(Z^A, G_A\) are the periods of \(\Omega\) defined as

\[
Z^A = \int_{Y_3} \Omega \wedge \alpha_A, \quad G_A = \int_{Y_3} \Omega \wedge \alpha_A. \tag{C.16}
\]

\(\Omega\) is invariant under (C.14) and hence \((Z^A, G_A)\) transforms as a symplectic vector. The \(G_A\) are functions of \(Z^A\) and determined in terms of a homogenous function of degree two \(\mathcal{G}(Z)\) as

\[
G_A = \frac{\partial \mathcal{G}}{\partial Z_A} \equiv \partial_A \mathcal{G}. \tag{C.17}
\]

Furthermore, \(\Omega\) is homogenous of degree one in \(Z\), i.e. \(\Omega = Z^A \partial_A \Omega\) with

\[
\partial_A \Omega = \alpha_A - \mathcal{G}_{AB} \beta^B. \tag{C.18}
\]
The deformations of the complex structure \( z^a, a = 1, \ldots, h_{1,2} \) which reside in \( H^{1,2}(Y_3) \) are related to the coordinates \( Z^A \) via \( z^a = Z^a / Z^0 \) or in other words one can choose \( Z^A = (1, z^a) \). The metric \( g_{ab} \) on the space of complex structure deformations \( \mathcal{M}_{1,2} \) is Kähler \( g_{\bar{a} \bar{b}} = \partial_{\bar{a}} \partial_{\bar{b}} K \) with the Kähler potential \( K \) given by

\[
K = - \ln i \int_{Y_3} \Omega \wedge \bar{\Omega} = - \ln i \left( \bar{Z}^A \mathcal{G}_A - Z^A \bar{\mathcal{G}}_A \right). 
\]

(C.19)

As we see \( K \) is determined in terms of the holomorphic prepotential \( \mathcal{G}(Z) \) and hence \( \mathcal{M}_{1,2} \) is a special Kähler manifold. Note that \( K \) is symplectically invariant.

Finally, let us discuss the action of the Hodge * on the basis (C.13). \( *\alpha_A \) and \( *\beta_B \) are both three-forms again so that they can be expanded in terms of \( \alpha \) and \( \beta \) according to

\[
*\alpha_A = A^B \alpha_B + B^{AB} \beta^B, \quad *\beta_A = C^{AB} \alpha_B + D^A_B \beta^B. 
\]

(C.20)

Using (C.13) one derives

\[
B^{AB} = \int_{Y_3} \alpha_A \wedge^* \alpha_B = \int_{Y_3} \alpha_B \wedge^* \alpha_A = B_{BA}, 
\]

\[
C^{AB} = - \int_{Y_3} \beta^A \wedge^* \beta^B = - \int_{Y_3} \beta^B \wedge^* \beta^A = C^{BA}, 
\]

\[
A^B = - \int_{Y_3} \beta^B \wedge^* \alpha_A = - \int_{Y_3} \alpha_A \wedge^* \beta^B = -D^B_A. 
\]

(C.21)

Furthermore, the matrices \( A, B, C \) can be determined in terms of a matrix \( \mathcal{M} \)

\[
A = (\text{Re} \mathcal{M}) (\text{Im} \mathcal{M})^{-1}, \\
B = - (\text{Im} \mathcal{M}) - (\text{Re} \mathcal{M}) (\text{Im} \mathcal{M})^{-1} (\text{Re} \mathcal{M}), \\
C = (\text{Im} \mathcal{M})^{-1}, 
\]

(C.22)

where

\[
\mathcal{M}_{AB} = \bar{\mathcal{G}}_{AB} + 2i \frac{\text{Im} \mathcal{G}_{AC} Z^C (\text{Im} \mathcal{G}_{BD}) Z^D}{Z^C (\text{Im} \mathcal{G})_{CD} Z^D}. 
\]

(C.23)

\( \mathcal{M}_{AB} \) determines the gauge couplings in type IIB compactifications on \( Y_3 \).

D Massless type IIA supergravity compactified on Calabi-Yau threefolds without fluxes

In this section we recall the compactification of massless type IIA supergravity on a Calabi-Yau threefold first performed in ref. [31]. The purpose of this appendix is twofold. One the one hand we need to redo the computation in order to fix the notation and convention for the case studied in section 2 where fluxes are turned on. On the other hand, the detailed dualization of the three-form \( C_3 \) to our knowledge has not been presented previously. This is of importance for our analysis in section 2 as the three-form \( C_3 \) turns
out to be dual to a constant $e_0$ which nicely combines with other flux parameters to build
symplectic invariant combinations.\textsuperscript{18} Furthermore, as we will show $e_0$ is the charge of the
scalar $a$ which is dual to $B_2$ and a potential consistent with the standard $N = 2$ gauged
supergravity is induced.

Let us start from the ten-dimensional action of massless type IIA supergravity. It
features the graviton, a two form $\hat{B}_2$ and the dilaton $\hat{\phi}$ in the NS-NS sector, a vector field
$\hat{A}_1$ and a three-form $\hat{C}_3$ in the R-R-sector and reads

$$S = \int e^{-2\hat{\phi}} \left( \frac{1}{2} \hat{R}^1 + 2d\hat{\phi} \wedge *d\hat{\phi} - \frac{1}{2} \hat{H}_3 \wedge *\hat{H}_3 \right) - \frac{1}{2} (\hat{F}_2 \wedge *\hat{F}_2 + \hat{F}_4 \wedge *\hat{F}_4) + \mathcal{L}_{\text{top}} ,$$

where

$$\hat{F}_4 = d\hat{C}_3 - d\hat{A}_1 \wedge \hat{B}_2 , \quad \hat{F}_2 = d\hat{A}_1 , \quad \hat{H}_3 = d\hat{B}_2 ,$$

$$\mathcal{L}_{\text{top}} = -\frac{1}{2} (\hat{B}_2 \wedge d\hat{C}_3 \wedge d\hat{C}_3 - (\hat{B}_2)^2 \wedge d\hat{A}_1 \wedge d\hat{A}_1 + \frac{1}{3} (\hat{B}_2)^3 \wedge d\hat{A}_1 \wedge d\hat{A}_1) .$$

We have chosen the conventions such that (D.1) is the $m \to 0$ limit of the action (2.1).\textsuperscript{19}
In these conventions (D.1) is invariant under the following three Abelian gauge transformations

$$\delta \hat{A}_1 = d\hat{\Theta} , \quad \delta \hat{C}_3 = d\hat{\Sigma}_2 ,$$

$$\delta \hat{B}_2 = d\hat{\Lambda}_1 , \quad \delta \hat{C}_3 = \hat{A}_1 \wedge d\hat{\Lambda}_1 .$$

In order to compactify the action (D.1) on a Calabi-Yau threefold $Y_3$ we expand
the ten-dimensional fields in harmonic forms on $Y_3$. As already reviewed in section 2
the Kähler class deformations of the metric are in one to one correspondence with the
harmonic (1,1)-forms while the complex structure deformations are in one to one corre-

cpondence with the harmonic (1,2)-forms on $Y_3$. Furthermore, the forms $\hat{A}_1, \hat{B}_2, \hat{C}_3$ are expanded according to

$$\hat{A}_1 = A^0 ,$$

$$\hat{B}_2 = B_2 + b^i \omega_i ,$$

$$\hat{C}_3 = C_3 + A^i \wedge \omega_i + \xi^A \alpha_A + \xi_B \beta^B ,$$

where as reviewed in appendix C, $\omega_i, i = 1, \ldots, h_{1,1}$ are harmonic (1,1) forms on $Y_3$ and
$(\alpha_A, \beta^A), A = 0, \ldots, h_{1,2}$ is a real basis on $H^3(Y_3)$. $A^0$ is the graviphoton and together
with the graviton $g_{\mu \nu}$ describe the bosonic components of the gravitational multiplet.

The other $h_{1,1}$ vector fields $A^i$ combine with the $v^a = b^i + iv^j$ into $h_{1,1}$ vector-multiplets.

The $h_{1,2}$ complex structure deformations $z^a$ together with $\xi^a, \xi_a$ form $h_{1,2}$ hypermultiplets
and $B_2, \phi, \xi^0, \xi_0$ form a tensor multiplet. In $d = 4$ the two-form $B_2$ can be dualized to a scalar and hence the tensor multiplet can be turned into an additional (universal)
hypermultiplet. $C_3$ carries no degrees of freedom in $d = 4$ but is dual to a constant.

\textsuperscript{18}In ref. [31] the case $e_0 = 0$ was considered.

\textsuperscript{19}There is an ambiguity in the definition of $\hat{C}_3$ in that $\hat{C}_3 \to \hat{C}_3 + \hat{A}_1 \wedge \hat{B}_2$ changes the form of the action and the form of the gauge transformations. It is this second formulation that we use in section 2.4 where we turn on NS three-form flux.
Using (C.3), (C.22) and (D.4) the various terms in the Lagrangian integrated over the Calabi-Yau space become

\[-\frac{1}{4} \int_{Y_3} \dot{H}_3 \wedge^* \dot{H}_3 = -\frac{\mathcal{K}}{4} dB_2 \wedge^* dB_2 - \mathcal{K} g_{ij} db^i \wedge^* db^j ,\]

\[-\frac{1}{2} \int_{Y_3} \dot{F}_2 \wedge^* \dot{F}_2 = -\frac{\mathcal{K}}{2} dA^0 \wedge^* dA^0 , \quad (D.5)\]

\[-\frac{1}{2} \int_{Y_3} \dot{F}_4 \wedge^* \dot{F}_4 = -\frac{\mathcal{K}}{2} (dC_3 - dA^0 \wedge B_2) \wedge^* (dC_3 - dA^0 \wedge B_2) \]

\[-2\mathcal{K} g_{ij} (dA^i - dA^0 b^i) \wedge^* (dA^j - dA^0 b^j) \]

\[+ \frac{1}{2} (\text{Im } \mathcal{M}^{-1})^{AB} \left[ d\tilde{\xi}_A + \mathcal{M}_{AC} d\xi^C \right] \wedge^* \left[ d\tilde{\xi}_B + \mathcal{M}_{BD} d\xi^D \right] ,\]

where \( g_{ij} \) and \( \mathcal{K} \) were defined in (C.3), (C.4). Finally, for the topological terms we find

\[\int_{Y_3} L_{\text{top}} = -\frac{1}{2} \left[ B_2 \wedge d(\tilde{\xi}_A d\xi^A - \xi^A d\tilde{\xi}_A) + b^i dA^i \wedge dA^k \mathcal{K}_{ijk} \right. \]

\[\left. - b^i b^j dA^k \wedge dA^0 \mathcal{K}_{ijk} + \frac{1}{3} b^i b^j b^k dA^0 \wedge dA^0 \mathcal{K}_{ijk} \right] . \quad (D.6)\]

Defining the four-dimensional dilaton \( \phi \) via \( e^{-2\phi} = e^{-2\tilde{\phi}} \mathcal{K} \) the action in \( d = 4 \) becomes

\[S = \int e^{-2\phi} \left( \frac{1}{2} R^1 + 2d\phi \wedge^* d\phi - \frac{1}{4} H_3 \wedge^* H_3 - g_{ij} dt^i \wedge^* dt^j - g_{ab} dz^a \wedge^* d\bar{z}^b \right) \]

\[-\frac{1}{2} \int \left[ \mathcal{K} F_2 \wedge^* F_2 + 4\mathcal{K} g_{ij} (dA^i - dA^0 b^i) \wedge^* (dA^j - dA^0 b^j) \right] \]

\[+ \frac{1}{2} (\text{Im } \mathcal{M}^{-1})^{AB} \left[ d\tilde{\xi}_A + \mathcal{M}_{AC} d\xi^C \right] \wedge^* \left[ d\tilde{\xi}_B + \mathcal{M}_{BD} d\xi^D \right] \]

\[+ \frac{1}{2} \int H_3 \wedge (\tilde{\xi}_A d\xi^A - \xi^A d\tilde{\xi}_A) \]

\[+ \frac{1}{2} \int \left[ b^i dA^i \wedge dA^k - b^i b^j dA^k \wedge dA^0 + \frac{1}{3} b^i b^j b^k dA^0 \wedge dA^0 \right] \mathcal{K}_{ijk} \]

\[+ \frac{1}{2} \int \mathcal{K} (dC_3 - dA^0 \wedge B_2) \wedge^* (dC_3 - dA^0 \wedge B_2) . \quad (D.7)\]

The next step is the dualization of \( C_3 \) following appendix E.2. Using the results in this appendix we find that the dual of the Lagrangian

\[\mathcal{L}_{C_3} = -\frac{\mathcal{K}}{2} (dC_3 - dA^0 \wedge B_2) \wedge^* (dC_3 - dA^0 \wedge B_2) \quad (D.8)\]

\[20\text{Strictly speaking also the Kähler moduli } t^i \text{ have to be redefined by a dilaton dependent factor } \mathcal{K}_i. \]

In order not to overload the notation we use the same \( t^i \) also for the redefined moduli.
is given by
\[ \mathcal{L}_{e_0} = -\frac{1}{2\mathcal{K}}e_0^2 * 1 - e_0 dA^0 \wedge B_2, \] (D.9)
with \( e_0 \) being an arbitrary constant parameter. Replacing (D.8) by (D.9) in the action (D.7) and collecting the terms involving \( H_3 \) we obtain (after partial integration)
\[ \mathcal{L}_{H_3} = \left[ -\frac{1}{4}e^{-2\phi}H_3 \wedge ^* H_3 + \frac{1}{2}H_3 \wedge \left( \xi_A d\xi^A - \xi^A d\xi_A \right) + e_0 H_3 \wedge A^0 \right]. \] (D.10)
We see that \( e_0 \) induces a four-dimensional Green-Schwarz term \( H_3 \wedge A^0 \) into action. The standard form of the type IIA action is obtained by also dualizing \( B_2 \) to an axionic scalar \( a \). For details we refer the reader to appendix E.1 while here we only record the final result
\[ \mathcal{L}_{H_3} \rightarrow \mathcal{L}_a = -\frac{e^{2\phi}}{4} \left[ Da + (\xi_A d\xi^A - \xi^A d\xi_A) \right] \wedge ^* \left[ Da + (\xi_A d\xi^A - \xi^A d\xi_A) \right], \] (D.11)
where
\[ Da = da + 2e_0 A^0. \] (D.12)
As anticipated the axionic scalar \( a \) is charged under a local Peccei-Quinn gauge symmetry with \( e_0 \) being the gauge charge. Thus even ordinary Calabi-Yau compactifications of type IIA give a one-parameter family of four-dimensional effective theories which are generically gauged rather than ordinary supergravities. For \( e_0 = 0 \) one recovers the standard type IIA supergravity of ref. [31].

The final task of this appendix is to rewrite the action in the form of standard gauged supergravity [22]. To do this we use the dual action (D.11) instead of (D.10) and perform the Weyl rescaling \( g_{\mu\nu} \rightarrow e^{2\phi} g_{\mu\nu} \) in order to go to the Einstein frame. Furthermore the scalars \((\phi, a, z^a, \xi^A, \tilde{\xi}_A)\) which together form \( h_{1,2} + 1 \) hypermultiplets are denoted collectively by \( q^a \). In these variables the action (D.7) reads
\[ S = \int \frac{1}{2} R^* 1 - g_{ij} dt^i \wedge ^* d\bar{t}^j - h_{uv} Dq^u \wedge ^* Dq^v + \frac{1}{2} \text{Im} \mathcal{N}_{IJ} F^I \wedge ^* F^J + \frac{1}{2} \text{Re} \mathcal{N}_{IJ} F^I \wedge F^J - V_E. \] (D.13)
where
\[ h_{uv} Dq^u \wedge ^* Dq^v = d\phi \wedge ^* d\phi + g_{ab} dz^a \wedge ^* dz^b \]
\[ + \frac{e^{4\phi}}{4} \left[ Da + (\xi_A d\xi^A - \xi^A d\xi_A) \right] \wedge ^* \left[ Da + (\xi_A d\xi^A - \xi^A d\xi_A) \right] \]
\[ - \frac{e^{2\phi}}{2} \left( \text{Im} \mathcal{M}^{-1} \right)^{AB} \left[ d\tilde{\xi}_A + \mathcal{M}_{AC} d\xi^C \right] \wedge ^* \left[ d\tilde{\xi}_B + \mathcal{M}_{BD} d\xi^D \right], \]
and \( \mathcal{M}_{AB} \) was defined in (C.23). In ref. [33] it was shown in that \( h_{uv} \) is a quaternionic metric in accord with the constraints of \( \mathcal{N} = 2 \) supergravity that the scalars in the hypermultiplets span a quaternionic manifold.
Using (C.4) it is straightforward to show that the gauge couplings in (D.13) are given by (C.7). Finally the potential $V_E$ in (D.13) is given by

$$V_E = \frac{e^{4\phi}}{2K} e_0^2 1. \quad (D.15)$$

This potential coincides with (2.33) for $e_i = 0$ and thus the consistency with gauged supergravity can be shown analogously to section 2.3.

E Poincaré dualities

For an arbitrary $p$-form in $d$ dimensions one always has the choice to describe the action in terms of a Poincaré dual form. The nature of the dual form differs in the massless and massive case. A massless $p$-form in $d$ dimensions describes $\binom{d-2}{p}$ physical degrees of freedom while a massive $p$-form in $d$ dimensions contains $\binom{d-1}{p}$ degrees of freedom. The difference can be easily understood from a generalized Higgs mechanism where a $p$-form ‘eats’ a massless $p-1$-form and thus the number of degrees of freedom change by $\binom{d-2}{p-1}$. Therefore a massless $p$-form in $d$ dimensions is dual to a $(d-p-2)$-form while a massive $p$-form is dual to a $(d-p-1)$-form. A massless $(d-1)$-form is special in that it is dual to a constant. In $d = 4$ this implies that a massless three-form is dual to a constant, a massless two-form is dual to a scalar while a massive 2-form is dual to a vector (a 1-form). Let us discuss these cases in turn.

E.1 Dualization of a massless $B_2$

Let us first consider the dualization of a massless two-form $B_2$ with field strength $H_3 = dB_2$ to a scalar $a$. We start from the generic action

$$S_{H_3} = -\int \left[ \frac{g}{4} H_3 \wedge^* H_3 - \frac{1}{2} H_3 \wedge J_1 \right], \quad (E.1)$$

where $g$ is an arbitrary function of the scalars while $J_1$ is a generic 1-form depending on the scalars and possibly some gauge field $A_1$. The dualization can be carried out by introducing a scalar field $a$ as a Lagrange multiplier and adding the term $H_3 \wedge da$ to $S_{H_3}$. Treating $H_3$ as an independent three-form (not being $dB_2$) the equation of motion for $a$ implies $H_3 = dB_2$ while the equation of motion for $H_3$ reads $^* H_3 = \frac{1}{g}(da + J_1)$. Inserted back into the action (E.1) one obtains the dual action

$$S_a = -\int \frac{1}{4g} (da + J_1) \wedge^* (da + J_1). \quad (E.2)$$

There is an another way of treating the dualizations which turns out to be useful in understanding the dualization of a three-form in four dimensions. Consider the equation of motion for $B_2$

$$d(g^* H_3 - J_1) = 0, \quad (E.3)$$

\[\text{We thank F. Quevedo for educating us on this subject.}\]
which can be derived from (E.1). It is solved by

\[ g^* H_3 - J_1 = da , \]  

(E.4)

with \( a \) being some arbitrary scalar field. The equation of motion for this field is dictated by the Bianchi identity of \( H_3 \)

\[ 0 = dH_3 = d \left[ \frac{1}{g} *(da + J_1) \right] , \]  

(E.5)

which in turn can be obtained from the action (E.2). This implies that the two ways described for the dualization of \( B_2 \) are equivalent.

### E.2 Dualization of the three-form

Next we consider the dualization of a three-form in 4 dimensions. We start from a generic action for a three-form \( C_3 \) possibly coupled to two-forms, 1-forms and scalars

\[ S_{C_3} = - \int \left[ \frac{g}{4} (dC_3 - J_4) \wedge * (dC_3 - J_4) + \frac{h}{2} dC_3 \right] , \]  

(E.6)

where \( g, h \) denote two arbitrary scalar functions and \( J_4 \) is a 4-form which can depend on the two-forms, 1-forms and scalars present in the spectrum.

For the field strength of a three-form in 4 dimensions there is no proper Bianchi identity since no 5-forms exist. That is why the second way of dualizing forms presented in the previous section, by exchanging the equation of motion with the Bianchi identity, can not work in this case. The only consistent way to proceed is to add a Lagrange multiplier to the action (E.6)

\[ S_{C_3} = - \int \left[ \frac{g}{4} (dC_3 - J_4) \wedge * (dC_3 - J_4) + \frac{h}{2} dC_3 + \frac{e_0}{2} dC_3 \right] , \]  

(E.7)

where \( e_0 \) is a constant. The equation of motion for \( dC_3 \) imply

\[ \frac{g}{2} \ast (dC_3 - J_4) = \frac{h + e_0}{2} . \]  

(E.8)

Inserted back into the action (E.8) and using \( **dC_3 = -dC_3 \) one obtains

\[ S_{e_0} = - \int \left[ \frac{1}{4g} (h + e_0)^2 \ast 1 + \frac{1}{2}(h + e_0) J_4 \right] . \]  

(E.9)

As we see a potential for the scalar fields is induced and \( e_0 \) play the role of a cosmological constant.

### E.3 Dualization of a massive two-form

Finally, let us discuss the dualization of the a massive two-form \( B_2 \). We start from a generic action

\[ S_{B_2} = - \int [gH_3 \wedge * H_3 + M^2 B_2 \wedge * B_2 + M_T^2 B_2 \wedge B_2 + B_2 \wedge J_2 ] , \]  

(E.10)
where \( g, M, M_T \) can be field dependent couplings and \( J_2 \) is a two-form which can depend on the gauge potential \( A_1 \) and/or some scalar fields. (\( J_2 \) does not depend on \( B_2 \).) We can treat \( B_2 \) and \( H_3 \) as independent fields and ensure \( H_3 = dB_2 \) by the equations of motion. This is achieved in the action

\[
S'_{B_2} = - \int \left[ -gH_3 \wedge^* H_3 + 2gH_3 \wedge dB_2 + M^2B_2 \wedge^* B_2 + M_T^2B_2 \wedge B_2 + B_2 \wedge J_2 \right],
\]

(E.11)

which indeed has \( H_3 = dB_2 \) as the equation of motion for \( H_3 \). So by inserting \( H_3 = dB_2 \) into (E.11) we obtain (E.10). On the other hand one can eliminate \( B_2 \) through its equation of motion and obtain an action expressed only in terms of \( H_3 \). The equation of motion for \( B_2 \) from (E.11) is

\[
2M^2B_2 + 2M_T^2B_2 + J_2 - 2d^*(gH_3) = 0,
\]

(E.12)

which is solved by

\[
B_2 = \frac{1}{M^4 + M_T^4} \left[ M^2d^*(gH_3) + M_T^2d^*(gH_3) - \frac{M^2}{2} J_2 - \frac{M_T^2}{2} J_2 \right]
\]

or

\[
B_2 = \frac{1}{M^4 + M_T^4} \left[ M^2d^*(gH_3) - M_T^2d^*(gH_3) + \frac{M^2}{2} J_2 - \frac{M_T^2}{2} J_2 \right].
\]

(E.13)

Inserted back into the action (E.11) results in

\[
S''_{B_2} = \int \left[ gH_3 \wedge^* H_3 - \frac{M^2}{M^4 + M_T^4} \left( d^*(gH_3) - \frac{1}{2} J_2 \right) \wedge^* \left( d^*(gH_3) - \frac{1}{2} J_2 \right) \right. \\
\left. + \frac{M_T^2}{M^4 + M_T^4} \left( d^*(gH_3) - \frac{1}{2} J_2 \right) \wedge \left( d^*(gH_3) - \frac{1}{2} J_2 \right) \right].
\]

(E.14)

We can now replace \( H_3 \) by its Poincaré dual one-form \( A^H = g^*H_3 \) and the dual action for the massive field \( A^H \) is

\[
S_{A^H} = - \int \left[ \frac{1}{g} A^H \wedge^* A^H + \frac{M^2}{M^4 + M_T^4} \left( dA^H - \frac{1}{2} J_2 \right) \wedge^* \left( dA^H - \frac{1}{2} J_2 \right) \\
- \frac{M_T^2}{M^4 + M_T^4} \left( dA^H - \frac{1}{2} J_2 \right) \wedge \left( dA^H - \frac{1}{2} J_2 \right) \right].
\]

(E.15)

As promised this is the action for a massive one-form \( A^H \).

**Acknowledgments**

This work is supported by DFG – The German Science Foundation, GIF – the German–Israeli Foundation for Scientific Research, the European RTN Program HPRN-CT-2000-00148 and the DAAD – the German Academic Exchange Service.

We have greatly benefited from conversations with L. Andrianopoli, J.-P. Derendinger, B. de Wit, R. Grimm, B. Gunara, S. Gurrieri, D. Lust, P. Mayr, T. Mohaupt, F. Quevedo, H. Singh, A. Strominger, C. Vafa, S. Vandoren, A. Van Proeyen, and M. Zagermann.
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