Cascading Constraints from Neutrino-emitting Blazars: The Case of TXS 0506+056

Anita Reimer1, Markus Böttcher2, and Sara Buson3,4,5

1 Institute for Astro- & Particle Physics, University of Innsbruck, A-6020 Innsbruck, Austria; Anita.Reimer@uibk.ac.at
2 Centre for Space Research, North-West University, Potchefstroom, 2531, South Africa
3 NASA Postdoctoral Program Fellow, Universities Space Research Association, USA; Markus.Bottcher@nwu.ac.za, buson@astro.uni-wuerzburg.de

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Abstract

We present a procedure to generally constrain the environments of neutrino-producing sites in photomeson production models of jetted active galactic nuclei (AGNs) where any origin of the dominant target photon field can be accommodated. For this purpose we reconstruct the minimum target photon spectrum required to produce the (observed) neutrino spectrum, and derive the distributions of all corresponding secondary particles. These initiate electromagnetic cascades with an efficiency that is linked to the neutrino production rate. The derived photon spectra represent the minimum radiation emerging from the source that is strictly associated with the photohadronically produced neutrinos. Using the 2014/15 neutrino spectrum observed by IceCube from TXS 0506+056, we conduct a comprehensive study of these cascade spectra and compare them to the simultaneous multiwavelength emission. For this set of observations, photopion production from a cospatially produced (comoving) photon target can be ruled out as well as a setup where synchrotron- or synchrotron–Compton-supported cascades on a stationary (AGN rest frame) target photon field operate in this source. However, a scenario where Compton-driven cascades develop in the stationary soft X-ray photon target, which photohadronically produced the observed neutrinos, appears feasible with required proton kinetic jet powers near the Eddington limit. The source is then found to produce neutrinos inefficiently, and emits GeV photons significantly below the observed Fermi-Large Area Telescope flux. Hence, the neutrinos and the bulk of the gamma-rays observed in 2014/15 from TXS 0506+056 cannot have been initiated by the same process.

Key words: galaxies: active – galaxies: jets – gamma rays: galaxies – neutrinos – radiation mechanisms: non-thermal – relativistic processes

1. Introduction

Active galactic nuclei (AGNs) have long been considered to be among the source populations responsible for the ultra-high-energy cosmic rays observed at Earth. Indeed, members of this source population fulfill the Hillas criterion (Hillas 1984) as well as energetics requirements to accelerate charged particles to such extreme energies, in principle. These relativistic hadrons interact in the highly radiative environment of AGNs, if above the threshold for photomeson and/or Bethe–Heitler pair production, or interact in dense material associated with the AGN via inelastic nucleon–nucleon interactions, to produce secondary particles, among them electron–positron pairs, neutrinos, and high-energy photons. Hence, for just as long, γ-ray-loud AGNs have been predicted as sources of neutrinos (Mannheim & Biermann 1989; Stecker et al. 1991; Mannheim et al. 1992, 2001; Protheroe & Szabo 1992; Mannheim 1993; Szabo & Protheroe 1994; Mastichiadis 1995; Bednarek & Protheroe 1997, 1999; Protheroe 1999; Atoyan & Dermer 2001, 2003; Mücke & Protheroe 2001; Mücke et al. 2003; Protheroe et al. 2003; Reimer et al. 2004; Dermer et al. 2009, 2012, 2014; Dimitrakoudis et al. 2012; Böttcher et al. 2013; Halzen 2013; Murase et al. 2014, 2018; Gao et al. 2017).

Until a few years ago, multimessenger astrophysics was merely a prediction by (some) cosmic-ray theorists. It became reality with the first detections of high-energy neutrinos of astrophysical origin (Aartsen et al. 2013). Observationally, no individual sources could be associated with these neutrino events, which were found to be isotropically distributed over the sky (Aartsen et al. 2014). Most interestingly, the total diffuse flux of these neutrinos follows a differential spectrum not harder than $dN/dE \propto E^{-2}$ above a few tens of TeV and ranging to almost 10 PeV (Aartsen et al. 2015). The detection of these neutrinos initiated an avalanche of hadronic emission models for various possible high-energy source populations, among them blazars, a sub-class of jetted AGNs with the line of sight close to the jet axis. Blazars are the most numerous sources in the extragalactic GeV–TeV γ-ray sky (e.g., Reimer & Böttcher 2013; Acero et al. 2015; Ackermann et al. 2016). The fact that in hadronic interactions, roughly equal powers of high-energy photons + pairs and neutrinos are produced (e.g., Mannheim & Schlickeiser 1994; Mücke et al. 2000b) makes these γ-ray blazars plausible sources of the detected neutrinos (e.g., Dimitrakoudis et al. 2014; Krauss et al. 2014; Cerutti et al. 2015; Padovani et al. 2015; Petropoulou et al. 2015; Kadler et al. 2016). One-zone blazar emission models, where neutrinos are produced in photohadronic interactions, typically predict neutrino spectra peaking at or beyond PeV energies, with extremely hard spectral shapes below (e.g., Mücke et al. 2003; Dermer et al. 2012) when applied to GeV γ-ray sources.

Recently, neutrino astrophysics experienced a further boost when the BL Lac object TXS 0506+056 was found within the 50% containment region of the single IceCube event IceCube-170922A (Aartsen et al. 2018a), and simultaneously in an elevated flux state in the Fermi-Large Area Telescope (LAT) energy range (Aartsen et al. 2018a). Furthermore, an extended neutrino excess (designated the “neutrino flare” in the following) consisting of $\sim 13$ neutrino events detected in a 110±24 days period in 2014 September–2015 March was found from the same source (Aartsen et al. 2018b). This has motivated the suggestion that TXS 0506+056 is the source
of these detected neutrinos (Aartsen et al. 2018a), implicitly assuming a causal connection between the neutrino and γ-ray emission. Such a link is justified if at least part of these ≥GeV photons have been initiated by the same process responsible for the neutrino flux, or have at least been produced in the same emission region of the source. It is, however, questionable whether a ≥GeV γ-ray-emitting source can simultaneously be an efficient producer of neutrinos in the 10–100 TeV energy range, in the framework of a photodisintegration emission model (e.g., Begelman et al. 1990). The latter requires a photons production optical depth $\tau_\gamma > 1$ in a field of target photons at UV-to-X-ray energies (see Section 3.1), while the former requires GeV photons to escape the source, hence the $\gamma\gamma$ pair production optical depth $\tau_{\gamma\gamma} \ll 1$ for target photons at X-ray energies. Hence, the same target radiation field necessary for neutrino production is also a source of opacity for ≥GeV photons. Observe that a ratio of $\tau_p/\tau_\gamma > 1$ is required in order for the radiation to escape, while $\tau_p/\tau_\gamma \approx \sigma_p/\sigma_\gamma \ll 1$ is guaranteed by the nature of the processes.

In this work we present a procedure that provides, in the framework of photodisintegrationally produced neutrinos in jetted AGNs, constraints on their associated broadband photon spectral energy distribution (SED) in a way that is independent of the origin of the dominant target photon field. It is expected to guide searches for neutrino source associations with electromagnetic counterparts for cases of highly radiative sources where neutrinos are produced predominantly photodisintegrationally. We present this procedure by applying it to the 2014/15 neutrino flare of TXS 0506+056. The resulting constraints are then discussed to infer limits on the required source power and multiband (MWL) predictions of the photon SED. The comparison to MWL data allows us in turn to constrain models.

In the following, we first (Section 2) present the quasi-simultaneous multimessenger data acquired for the time period of the 2014/15 neutrino flare, to be placed into a generic theoretical setup where neutrinos are produced photodisintegrationally in a relativistic jet. Section 3 describes the simulations of electromagnetic cascades initiated by photodisintegration processes and the constraints that can be deduced from them. Section 4 then discusses implications for the nature of the target photon field for photodisintegration processes responsible for the neutrino emission, where we will show that the only plausible scenario of photopion production in the jet of TXS 0506+056 requires a stationary UV–X-ray target photon field external to the jet that leads to Compton-supported, photopion-induced cascades. We find that in this setup the neutrino-production optical depth is surprisingly low during the neutrino flare unlike the case of efficient neutrino production, $\tau_\gamma > 1$. This in turn allows photon escape of any origin from the neutrino production site only below $\sim 10^{-5} E_{\nu,\text{obs}}$ (with $E_{\nu,\text{obs}}$ the observed neutrino energy). The γ-ray flux strictly associated with the neutrino flare is shown to lie significantly below the GeV flux detected contemporaneous with the IceCube neutrino flare, implying no common production origin between the observed IceCube neutrinos and LAT γ-rays. In Section 5 we provide a summary of our results and conclusions. Throughout the paper, primed quantities refer to the comoving jet frame of the emission region. Physical quantities are parameterized as $Q = 10^Q Q$ in c.g.s units.

2. Quasi-simultaneous Multimessenger Data of the Jet

2.1. Neutrino Data

The neutrino flux from the 2014/15 neutrino flare of TXS 0506+056 was best described by a spectrum between 32 TeV and 3.6 PeV of the form $\Phi_\nu(E) = \Phi_0 E^{-\Gamma/2}$ with $\Phi_0 = 2.2 \times 10^{-13}$ cm$^{-2}$ s$^{-1}$ TeV$^{-1}$ and $E_{\nu,\text{iso}} = E_\nu/(100 \text{ TeV})$ (Aartsen et al. 2018b). For a redshift of TXS 0506+056 as measured by Ajello et al. (2014) and Paiano et al. (2018), $z = 0.3365$, which corresponds to a luminosity distance of $d_\text{L} \approx 1.8 \text{ Gpc} \approx 5.5 \times 10^{27}$ cm, the IceCube neutrino flux corresponds to an isotropic-equivalent luminosity in muon neutrinos of $L_{\nu,\text{iso}} \approx 5.8 \times 10^{46}$ erg s$^{-1}$.

We assume that the neutrinos are produced by relativistic hadrons in the high-energy emission region (sometimes referred to as the “blob”) of (jet-frame) size $R_j$, moving relativistically along the jet with constant Lorentz factor $\Gamma \equiv 10 \Gamma_{10}$ and viewing angle $\theta_{\text{obs}}$, resulting in relativistic Doppler boosting by the Doppler factor $D = (\Gamma [1 - \beta \cos \theta_{\text{obs}}])^{-1} \equiv 10 \, D_1$ with $\beta_c$ the bulk velocity. In that case, the total neutrino luminosity (comprising all neutrino flavors and assuming complete flavor mixing) produced in the comoving frame of the emission region is reduced to $L'_{\nu} \approx 1.7 \times 10^{43} D_1^{-1}$ erg s$^{-1}$.

2.2. Photon Data

To study the electromagnetic behavior of TXS 0506+056 during the neutrino flare we collect data in the optical, X-ray, and γ-ray bands. Optical V-band and g-band data are obtained from the All-Sky Automated Survey for Supernovae (ASAS-SN; Shappee et al. 2014; Kochanek et al. 2017). Data are publicly available from the ASAS-SN website and available between 2014 February 2 and 2018 September 25. At X-ray energies, we use data from the Burst Alert Telescope (BAT), on board the Neil Gehrels Swift Observatory, to derive a constraint at keV energies for a time interval simultaneous with the neutrino flare. Fermi-LAT has been monitoring the whole sky for almost 10 years, including the relevant period of the neutrino flare as described in Section 2.2.2.

2.2.1. Swift-BAT Data Analysis

The BAT survey data were retrieved from the HEASARC public archive and processed using the BATIMAGER code (Segreto et al. 2010), dedicated to the processing of coded mask instrument data, which performs processing and cleaning of the data, source detection, and production of background subtracted images and other scientific products. TXS 0506+056 is not detected in the whole set of survey data spanning 2004 December to 2017 November. Using the 20–85 keV sensitivity map, for the time period spanning 2014 September 16 to 2017 February 28 we have derived an upper limit of $9.12 \times 10^{-12}$ erg cm$^{-2}$ s$^{-1}$ to its flux, at a 5σ level.

2.2.2. Fermi-LAT Data

In this work we consider 10 years of Fermi-LAT data, collected between 2008 August 4 and 2018 September 29 (MJD 54682–58390). Events are selected in a $10^6 \times 10^6$ square centered on the position of TXS 0506+056, from 100 MeV up to 300 GeV. To minimize the contamination from TPC...
γ-rays produced in the Earth’s atmosphere, we apply a zenith angle cut of $\theta < 90^\circ$. Time periods during which the LAT detected bright solar flares and γ-ray bursts were excluded. We perform a binned likelihood analysis with the package fermipy\(^9\) (v0.17.3), based on the standard LAT ScienceTools\(^10\) (v11-07-00) and the P8R2_SOURCE_V6 instrument response functions. The model of the region accounts for all sources included in the Fermi-LAT preliminary eight year source list,\(^11\) and contained in a $15^\circ$ region from the position of TXS 0506+056, as well as the isotropic and Galactic diffuse γ-ray emission models (iso_P8R2_SOURCE V6 v06.txt and gll_iem_v06.fits). The spectral parameters of the bright γ-ray object located $1^\circ$2 from TXS 0506+056 and associated with the blazar PKS 0502+049, are left free to vary in the fit. As in Garrappa et al. (2018), the residual map of the region does not show evidence for significant structures, indicating that our best-fit model adequately describes the region.

To investigate the γ-ray data of TXS 0506+056 during the neutrino flare, we adopt as definition for the time interval the “box window” identified by Aartsen et al. (2018b), i.e., 158 days (MJD 56937 to MJD 57096, that is 2014 October 7–2015 March 15). For this period, the γ-ray SED of TXS 0506+056 is well modeled with a power-law spectral shape. A maximum likelihood fit yields the best-fit ($>100\text{MeV}$) flux value of $(3.3 \pm 0.9) \times 10^{-8} \text{cm}^{-2} \text{s}^{-1}$, with a photon spectral index of $1.9 \pm 0.1$, and the source is detected at a significance of more than $12\sigma$. This gives an isotropic equivalent γ-ray luminosity in the LAT-energy range during the neutrino flare of $1.9 \times 10^{38} \text{erg s}^{-1}$, or $1.9 \times 10^{41}D^{-4}_{100} \text{erg s}^{-1}$ in the comoving frame of the jet.

Downturns or upturns in the MeV–GeV spectrum of TXS 0506+056 could be interesting to confirm/rule out features generated by the cascade scenarios. A previous work reported a hint for a hardening of the TXS 0506+056 spectrum in the $\gamma > 2 \text{GeV}$ LAT band (Padovani et al. 2018) during the 2014/15 “neutrino flare” interval. Issues arising from source confusion at the lower LAT energies prevented the same authors from conducting a full investigation of the blazar spectrum in the $>100\text{MeV}$ range. In our analysis we are able to overcome these issues by accounting for the nearby known bright γ-ray blazar PKS 0502+049 in our model for the region of interest, and allowing its spectral parameters to vary in the fit. During the neutrino flare, we find that the source flux is consistent with a quiescent state and the spectral shape with the average one in the $>100\text{MeV}$ range (see also Garrappa et al. 2018). Including this full broadband information provided by the LAT allows us to rule out a large variety of cascade scenarios, as explained in more detail in Section 3.2.

Figure 1 shows the γ-ray and optical light curves. The Fermi-LAT light curve is computed with an eight week binning, integrating energies above 300 MeV. The time bin edges are chosen to conveniently overlap with the neutrino flare, highlighted by the green shaded area in 1. Notably, the detection of the high-energy neutrino IC170922A (red line) coincides with the major γ-ray outburst experienced by TXS 0506+056 in the 10 years of γ-ray monitoring, reaching a peak flux of $(1.20 \pm 0.07) \times 10^{-7} \text{cm}^{-2} \text{s}^{-1}$. The more recent 2018 data highlight the fading trend of the flare, while the flux remains still at higher-than-average values. The behavior in the optical band overall matches quite well the major γ-ray variations, displaying a lower-than-average optical flux during times coincident with the neutrino flare. The brightest prolonged variations are coincident with the major γ-ray flare and IC170922A.

3. Theoretical Setup

This section is devoted to deriving the minimal requirements on the target photon field and the relativistic proton population to explain the observed neutrino (flare) flux and spectrum in a way that is as independent of model assumptions as is possible. We will then discuss implications for the environment where the neutrino production occurs. The focus lies on the region where the neutrino production takes place. Our proposed procedure as illustrated by the flow chart in Figure 2 does not

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9 \url{https://fermipy.readthedocs.io}

10 \url{http://fermi.gsfc.nasa.gov/ssc/data/analysis/}

11 \url{FLS4Y preliminary catalog: https://fermi.gsfc.nasa.gov/ssc/data/access/lat/48y/}.
require a full multimessenger modeling, but nevertheless will be able to constrain the family of emission models by comparing with MWL data.

In our minimal setup we consider all neutrinos to be produced photohadronically in the jet of TXS 0506+056. The calculations of all particle–photon and photon–photon interactions are carried out in the jet frame, where we assume, for simplicity, the target radiation field to be isotropic. For external target radiation fields the angular distribution of the target photons in the comoving jet frame may become anisotropic and will lead to an increase or lowering of the particle–photon and photon–photon collision rate by an amount that depends essentially on the relative location of the emission region with respect to the source of the target photons (e.g., Protheroe et al. 1992; Dermer & Schlickeiser 1993; Roustazadeh & Böttcher 2010; Dermer et al. 2012; Sitarek & Bednarek 2012). We expect only minor changes of the corresponding photon emission for this case as discussed below (Section 3.2). A quantitative comprehensive study of general anisotropy effects for the present setup is in progress and will be presented in a forthcoming work (A. Reimer et al. 2019, in preparation).

The resulting yields are then transformed into the observer frame. The secondary particles from particle–photon interactions initiate pair cascades in the emission region which we follow to calculate the emerging photon SED. The pair cascading is considered to be linear, i.e., the pair compactness is modest or low: \( L_i^e \sigma_T / (R' m_p c^2) < 10 \), where \( L_i^e \) is the injected particle power, \( \sigma_T \) the Thomson cross section, and \( R' \) the size of the emission region (Svensson 1987), and the calculations consider the steady-state case.

### 3.1. Proton–Photon Interactions and Neutrino Production

Neutrinos are produced through hadronic interactions of relativistic protons of (jet-frame) energy \( E_p^i = \gamma_p^i m_p c^2 \) with target photons of (jet-frame) energy \( \epsilon' \) when the center-of-momentum frame energy \( \sqrt{s} \) of the interaction is above threshold, i.e., \( \sqrt{s} > \sqrt{s_{thr}} = (m_p + m_p) c^2 \approx 1.08 \text{GeV} \). The nucleon energy required to produce neutrinos at hundreds of TeV energies, \( E_{\nu} \approx 100 E_{14} \text{TeV} \) is \( E_{\nu} \approx 200 E_{14}/(D_{50,05}) \text{TeV} \) (i.e., \( \gamma_p^i = E_p^i / m_p c^2 \approx 2 \times 10^2 E_{14}/(D_{50,05}) \)) where \( \xi = 0.05 \) is the average target photon energy per injected nucleon energy in photohadronic interactions (Mücke et al. 2000a).

Protons of this energy are very inefficient emitting synchrotron radiation in the magnetic fields typically assumed in hadronic blazar models \( (B \sim 10–100 \text{G}) \) with observed synchrotron cooling timescales of \( t_{\text{obsy}} \approx 25 D_{1}^{-1} B_{1}^{-2} (\gamma/(6 \times 10^6))^{-1} \text{yr} \), where \( B_1 \equiv B/(10 \text{G}) \). The Larmor radius of protons with such energy is \( r_L \approx 2 \times 10^{2} (\gamma/(6 \times 10^6))B_{1}^{-1} \text{cm} \), indicating that they are expected to be well confined within the emission region and can plausibly be accelerated by standard mechanisms, such as diffusive shock acceleration.

With the threshold condition \( \epsilon' E_p^i \geq 0.14 \text{GeV}^2 \), one finds the minimum energy of the target photons required to produce neutrinos at tens of TeV energies, \( \epsilon' \geq 0.7 D_{1} \xi_{50,05}/E_{14} \text{keV} \). For pion (and neutrino) production at the \( \Delta^+ \) resonance, where \( s = D_{\Delta}^2 = (1232 \text{MeV})^2 \), target photons of \( \epsilon' \geq 1.6D_{50,05}/E_{14} \text{keV} \) are required. In order to produce neutrinos in a broad spectral range we consider the injection of relativistic protons with a power-law distribution \( dN_p^i/dE_p^i \propto E_p^{\alpha_p} \) in the range \( E_p^i = [m_p c^2, E_{p,\text{max}}] \) in a target photon field of photon energy density \( u_\nu \), and use the SOPHIA Monte Carlo code (Mücke et al. 2000b)

to calculate all resulting yields. We parameterize the target field using a simple power law for its differential photon spectrum \( n' \propto \epsilon'^{-\alpha_t} \) in the energy range \( \epsilon' = [\epsilon'_{\text{min}}, \epsilon'_{\text{max}}] \) and reconstruct (using the SOPHIA code) the minimal target photon field, irrespective of its origin, that is required to explain the observed neutrino spectrum upon injection of a power-law proton distribution.\(^\text{12}\)

Figure 3 shows the resulting muon neutrino (dotted and dashed-triple-dotted black lines) and electron neutrino (dashed and dashed-dotted black lines) spectra at production for a “blob” that is moving with \( D = 10 \). The spectra are produced assuming that protons with a power-law spectrum with index \( \alpha_p = 2 \) and energies up to \( E_{p,\text{max}} = 30 \text{ PeV} \) interact with a distribution of target photons with spectral index \( \alpha_t = 1 \), between energies \( \epsilon'_{\text{min}} = 10 \text{ keV} \) and \( \epsilon'_{\text{max}} = 60 \text{ keV} \). Neutrino-decay neutrinos would appear at \( \sim 10^{-2} \) smaller energies, and are not considered here. We adopted somewhat higher target photon energies than naively estimated so that the simulated neutrino spectrum matches the best-fit spectrum for the 2014/15 neutrino flare reported by the IceCube collaboration. Looking at the SED of TXS 0506+056, the required energy of the target photon field coincides with the rising part of the second, high-energy hump, where the electromagnetic spectrum follows roughly \( dN/dE \propto E^{-\delta} \). This motivates the choice of \( \alpha_v = 1 \). The observational uncertainties of the neutrino spectrum are well captured by varying the proton spectral index, in the range \( \alpha_p = 2.0 \pm 0.1 \), along with a corresponding proton spectrum normalization variation by \( \pm 10\% \) (see Figure 4). The calculation of the total neutrino spectrum (black solid lines in Figures 3 and 4) takes into account neutrino oscillations, and is compared to the IceCube observations (violet bow tie) of the neutrino flare. Once the

\(^{12}\) The minimal target photon field is the narrowest possible soft photon spectrum with a shape and normalization that yields the observed neutrino spectrum upon injection of a power-law distribution of protons interacting photohadronically with these soft photons. The required normalization depends strongly on the proton distribution, unlike the photon spectral shape for sufficiently narrow target field spectra.
target photon spectral shape and Doppler factor are fixed, the neutrino flux normalization depends further on the injected proton energy density $u_p$ (mildly coupled to $\alpha_p$) and, through the photomeson production opacity $\tau_p$, on the target photon energy density $u_\gamma$ and the size $R'$ of the emission region (see Figure 2). The combination of these parameters is chosen to fit the observed neutrino flux (and hence is determined by observables), while each parameter individually is not required to be fixed. In the case of target photon fields external to the jet, particle–photon collisions may become anisotropic in the comoving jet frame, which in turn affects the collision rates. The observed neutrino flux level would then be matched by adjusting the injected proton energy density correspondingly.

Secondary pairs are produced along with the neutrinos, and are depicted as the blue dashed-line bump at high energies ($\gtrsim 10$ TeV) in Figure 3, while the produced $\gamma$-ray spectrum from meson decay is shown by the solid blue line. The ratio of the secondary pair and photon number density to that of the neutrinos remains constant, even in the case of scatterings involving target photon populations that are anisotropic in the jet frame. Electromagnetic proton–photon interactions (Bethe–Heitler pair production) are guaranteed at sites where photohadronic interactions occur, and are therefore taken into account here (see Figure 2).

We simulate Bethe–Heitler pair production in the same target photon field using the Monte Carlo code of Protheroe & Johnson (1996), and using the same proton injection distribution and flux normalization. The resulting pair yields upon production are shown as the blue dashed line lower-energy (10 MeV–30 GeV) hump in Figure 3 for a Doppler factor $D = 10$ as an example, and its spread due to the observational uncertainties of the neutrino spectrum is depicted in Figure 4. Note that the ratio of the pair-to-pion production rates in the same target photon field (see Figure 5) is fixed by the nature of the interaction.

Figure 5 depicts interaction rates for various processes. It illustrates the dominance of losses due to Bethe–Heitler pair production for low proton energies while, above the threshold for meson production, photohadronic interactions dominate. While the minimal energy range of the target photon field is fixed to some extent by the energy range of the observed neutrino spectrum (modulo the Doppler factor in jetted neutrino sources) its shape is less constrained. Indeed, by choosing softer (e.g., $\alpha_t = 2$–3) or harder (e.g., $\alpha_t = 0$–0.5) power-law target photon fields we were able to find equally good representations of the observed neutrino spectrum, with only very small adjustments of the remaining parameters (e.g., $\alpha_p$, etc.). We attribute this behavior to the narrowness of the observed neutrino spectrum. Changing the target photon spectral index in the considered narrow energy range is expected to cause only minor changes in the associated pair cascade spectrum (see Section 3.2), well within the uncertainties implied by the observed neutrino spectrum. The situation changes somewhat when extending sufficiently peaked target photon spectra to a broader energy range. For example, by extending the $\alpha_t = 1$ target photon spectrum down to $\epsilon_{\gamma,\min} = 1$ eV, we were also able to find a representation of the observed neutrino spectrum for $\alpha_p = 2.2$ and $E_{p,\max}' = 20$ PeV. While the corresponding impact on the synchrotron-supported cascades turns out minor (except for a broadening of the absorption troughs), the Compton-supported cascades will become broader (see Section 3.2.1).

Finally, varying the Doppler factor shifts the energy range of the minimal target photon field and injected proton spectrum correspondingly, while the required flux normalization to reach the observed neutrino flux level can be adjusted by changing the injected proton energy density accordingly. We scanned from $D = 1$ (suitable for misaligned jetted AGNs) up to $D = 50$ (suggested by minute-timescale $\gamma$-ray flux variations found in some blazars; e.g., Begelman et al. 2008) to find satisfactory representations of the observed neutrino spectrum using $\epsilon_{\gamma,\min}' = 1 \ldots 170$ keV, $\epsilon_{\gamma,\max}' = 4 \ldots 1000$ keV, and $E_{p,\max}' = 3 \ldots 300$ PeV.

To summarize, Figure 3 shows the distribution of all secondary pairs and $\gamma$-rays that are inevitably produced along with the (observed) neutrinos. By varying the proton spectral index $\alpha_p = 2.0^{+0.2}_{-0.1}$ we propagate the observational uncertainties of the neutrino spectrum to the corresponding pair and $\gamma$-ray distributions. These pairs and $\gamma$-rays typically initiate
electromagnetic cascades (see Figure 2) which re-distribute their power to energies where the produced photons can eventually escape the source. The $\gamma\gamma$-pair production optical depth and its energy dependence determine the spectral escape probability of the photons, and is therefore central for calculating the emerging photon spectrum associated with the neutrino spectrum.

3.2. Pair Cascading

The high-energy pairs and $\gamma$-rays that were produced along with the neutrino flux in the minimal target radiation field are injected into a pair-cascading code. In a radiative and magnetized environment the electromagnetic cascade develops rapidly in the target photon field through photon–photon pair production, and is either driven by mainly inverse Compton scattering in the same radiation field ("Compton-supported cascades") if $u'_\gamma \gg u'_B$ (with $u'_B$ the magnetic field energy density), by synchrotron radiation ("synchrotron-supported cascades") if $u'_\gamma \ll u'_B$, or by both ("synchrotron–Compton cascades") in cases where the magnetic field energy density turns out to be comparable to the target photon energy density (see Figure 2). Once the cascade photons reach down to energies too low for pair production, the cascade development ceases, and the remaining pairs lose their energy via inverse Compton and/or synchrotron emission. The emerging photon spectrum is governed by both the energy-dependent photon–photon pair production optical depth and the dominating radiative dissipation process.

The pair-cascading code employs the matrix multiplication method described by Protheroe & Stanev (1993) and Protheroe & Johnson (1996). Monte Carlo programs for photon–photon pair production and inverse Compton scattering are used to calculate the mean interaction rates (see Figure 5) and the secondary particle and photon yields due to the interaction with the target radiation field. Note that inverse Compton scattering during the cascading proceeds in the Klein–Nishina regime, and transitions to the Thomson regime only below the threshold for pair production. The calculation of the synchrotron yields follows Protheroe (1990); see also Pacholczyk (1970). The yields are then used to build up transfer matrices which describe the change in the electron and photon spectra after propagating a given time step $d\tau$, which we chose as the dynamical timescale of the problem. The escape probability for photons is calculated using the formula given in Osterbrock & Ferland (2006) for a spherical region, while charged particles are assumed not to escape. For steady-state spectra we continue the transfer process until convergence is reached. In each time step energy conservation is verified.

In the following, all the photon models are generated with a normalization that yields the observed neutrino flux.

3.2.1. Compton-supported Cascades

If inverse Compton scattering supports the cascade ($u'_\gamma \gg u'_B$), the emerging photon spectra are fully determined once the opacity due to photon–photon pair production is fixed, because $\tau_{\gamma\gamma} \propto \tau_{\text{IC}}$ (with $\tau_{\text{IC}}$ the inverse Compton optical depth). This energy-dependent opacity in turn is related to the optical depth of photon–photon interactions, which itself is linked to the neutrino spectral flux (see Section 3.1). Compton-supported cascading on the minimal target photon field (as determined in Section 3.1)\(^{13}\) therefore provides the corresponding minimal cascade flux for $u'_\gamma \gg u'_B$. In the following we choose to vary the maximum of the energy-dependent photon–photon opacity, $\tau_{\gamma\gamma,\text{max}}$ (which here serves as a proxy for $(R' \cdot u'_\gamma)$) to calculate the corresponding cascade spectra. Figures 6–9 show our results for Doppler factors $D = 1, 10$, and 50 and $\log(\tau_{\gamma\gamma,\text{max}}) = -6, -5, \ldots, 0, 1, 2$, covering the optically thin and thick cases, and compares them to the MWL.

\(^{13}\) The range of the minimal target photon field used here is $\epsilon' = 1 \ldots 4$ keV for $D = 1, \epsilon' = 10 \ldots 60$ keV for $D = 10$, and $\epsilon' = 0.17 \ldots 1$ MeV for $D = 50$.  

Figure 6. Compton-supported cascade spectra arriving at Earth without (thin lines) and including absorption in the extragalactic background light (thick lines) using the model of Franceschini & Rodighiero (2017) for $\log(\tau_{\gamma\gamma,\text{max}}) = -4$ (short dashed line), $-3$ (dashed-dotted line), $-2$ (dashed-triple-dotted line), $-1$ (long dashed line) as indicated, and $D = 10$. The shaded areas represent the spread of the cascade spectra, for the case $\tau_{\gamma\gamma,\text{max}} = 10^{-4}$ as an example, propagated from the uncertainties of the observed neutrino spectrum. The red data points (ASAS-SN, SWIFT-BAT, Fermi-LAT) depict the quasi-simultaneous observations while the gray data points represent archival data (Aartsen et al. 2018a). Very high-energy data (all MAGIC data from Ansoldi et al. 2018, all VERITAS data from Abeysekara et al. 2018, HAWC archival data from Aartsen et al. 2018a) are not simultaneous to the 2014/15 neutrino flare and are included in gray.

Figure 7. Compton-supported cascade spectra arriving at Earth without (thin lines) and including absorption in the extragalactic background light (thick lines) for $\log(\tau_{\gamma\gamma,\text{max}}) = 0, 1, 2$ as indicated, and $D = 10$. The shaded areas represent the spread of the cascade spectra, for the case $\tau_{\gamma\gamma,\text{max}} = 1$ as an example, propagated from the uncertainties of the observed neutrino spectrum. For the $\tau_{\gamma\gamma,\text{max}} = 1$ case we have also added the corresponding proton synchrotron radiation component for two sub-equipartition ($u'_p < u'_\gamma$) field strengths: 0.5 G (solid line), 3 G (dashed line). The data points are the same as shown in Figure 6.
is much larger than the indicated, and the MWL indicated, and D = 1. The data points are the same as shown in Figure 6.

The Compton optical depth \( \tau_{\gamma\gamma} \) is directly linked to the electromagnetic fields that are anisotropic in the keV range. This is because the photon energy associated with the lowest-energy radiating electron is lower for \( \tau_{\gamma\gamma} \) and the emerging cascade spectra presented here are proxied by EBL-corrected data. Therefore, in these cases, the flux level of the MWL (in particular, Fermi-LAT data). Therefore, in these cases, the photon flux has to be produced by emission processes occurring in the source that were not initiated by proton–photon interactions. When increasing the opacity further, \( \tau_{\gamma\gamma} \) deep absorption troughs in the GeV range appear. Any photons in the GeV range produced co-spatially to the neutrino flux will inevitably suffer from internal absorption and will subsequently be cascaded to lower energies, independent of their production process. In this case of an efficient neutrino producer, the observed GeV flux has to be produced either in a region within the jet of TXS 0506+056 that has no dense radiation fields at keV energies, or in a different source altogether. As a consequence, there is no causal connection between the observed neutrino flux and observed GeV flux. These findings are similar for all values of the Doppler factor inspected (see Figures 8 and 9).

As pointed out in Section 3.1 broader target radiation fields may also lead to photohadronically produced neutrino spectra that fit the observations (e.g., extending the target field \( n_f \propto \epsilon^{-1} \) to lower energies, e.g., \( \epsilon_{\text{min}} = 1 \) eV). Inspecting the associated Compton-supported cascade spectra shows broader SEDs than for the case of narrow target photon fields (see Figure 10). This is because the photon energy associated with the lowest-energy radiating electron is lower for \( \epsilon_{\text{min}} = 1 \) eV (broad target field) than for \( \epsilon_{\text{min}} = 10 \) keV (narrow target field). In addition, the overall cascade flux increases notably below the GeV regime when increasing the energy range of the target photon field. We note that in such cases X-ray data may be able to place stringent constraints on models.

The Compton optical depth \( \tau_{\gamma\gamma} \) may be altered when scatterings sample photon fields that are anisotropic in the comoving frame. Since \( \tau_{\gamma\gamma} \) is directly linked to \( \tau_{\gamma\gamma} \) (see Figure 5) and the emerging cascade spectra presented here are proxied by \( \tau_{\gamma\gamma} \), no significant changes of the Compton cascade spectra for a given \( \tau_{\gamma\gamma} \) are expected in such cases.

### 3.2.2. Synchrotron-supported Cascades

If the magnetic energy density \( u_B \) is much larger than the target photon field energy density \( u_f \), the electromagnetic cascades are supported by synchrotron radiation. In blazar jet environments the magnetic field strength \( B \) (assumed constant here) is typically \( \ll B_\gamma = 2m_e^2c^3/\epsilon e = 4.4 \times 10^{13} \) G, the critical magnetic field strength for radiating pairs, which is therefore also our assumption here. As an example we show in Figures 11–13 the case where \( u_B = 10 u_f \) with the above specified target photon spectrum, again corrected for absorption.
Figure 11. Synchrotron-supported cascade spectra arriving at Earth without (thin lines) and including absorption in the extragalactic background light (thick lines) for \( n_B = 10 n_{\text{target}} \) and \( \log(\tau_{\gamma\gamma,\max}) = -6 \) (solid line), -5 (dotted line), -4 (short dashed line), -3 (dashed-dotted line), -2 (dashed-triple-dotted line), -1 (long dashed line), 0, 1, 2 (solid lines) as indicated, and \( D = 10 \). The shaded areas represent the spread of the cascade spectra, for the cases \( \tau_{\gamma\gamma,\max} = 10^{-6}, 1, 100 \) as examples, propagated from the uncertainties of the observed neutrino spectrum. The data points are the same as shown in Figure 6.

Figure 12. Same as Figure 11 except \( D = 1 \).

Figure 13. Same as Figure 11 except \( D = 50 \).

Figure 14. Synchrotron-supported cascade spectra, including the proton synchrotron radiation components, arriving at Earth without (thin lines) and including absorption in the extragalactic background light (thick lines) for \( n_B = 10 n_{\text{target}} \) (solid line), \( n_B = 10^2 n_{\text{target}} \) (dotted line), \( n_B = 10^3 n_{\text{target}} \) (dashed line), \( \tau_{\gamma\gamma,\max} = 10^{-6} \), and \( D = 10 \). The data points are the same as shown in Figure 6.

in the EBL. Synchrotron-self absorption is not taken into account. The uncertainties in the IceCube flux are propagated again to the corresponding cascade spectra, and depicted in Figures 11–13 as shaded areas for the cases \( \tau_{\gamma\gamma,\max} = 10^{-6}, 1, 100 \). In the optically thin case the standard synchrotron cooled spectra for a two-component (meson decay and the lower energy Bethe–Heitler pairs) pair population is recovered, with increasing cooling for increasing field strength (see Figure 14). Note that for \( B \ll B_{\text{cr}} \) the energy loss rate is \( \propto \gamma_e^2 \) for all energies whereas for Compton-loss-dominated pair cascades the interaction probability changes from \( \propto \gamma_e \) in the Klein–Nishina regime to \( \propto \gamma_e^2 \) in the Thomson regime. This is reflected in the resulting shape of the cascade spectra. Because the secondary pair distribution resulting from proton–photon interactions is fixed by the observed neutrino spectrum the corresponding synchrotron-supported cascade emission in the optically thin case is expected to be unaffected by a possible anisotropy of the collisions. In the optically thick case, \( \tau_{\gamma\gamma,\max} \gg 1 \), the number of pairs produced from synchrotron photons increases dramatically in the cascade. The amount of reprocessing is given by \( \tau_{\gamma\gamma} \), and the resulting cascade flux for a given \( \tau_{\gamma\gamma,\max} \) is therefore independent of whether it is produced by isotropic or anisotropic scatterings, as long as the contribution to the scattering rate per unit scattering angle is energy-independent. Also, the produced pairs are more energetic in environments with high magnetic fields than in sites of low magnetization (see Figure 15). As for the optically thick Compton-supported cascades, we note deep absorption troughs at GeV energies leading to the same implications for the association between the neutrino and GeV source, i.e., the lack of a causal connection between the observed neutrino and GeV flux. Note that the hump-like feature at high energies (\( \sim 1 \text{ GeV}–1 \text{ TeV} \)) in the optically thick cases disappears by sufficiently broadening the target photon spectrum to lower energies. As compared to the Compton-supported case, the synchrotron-supported cascades extend down to much lower energies: whereas the inverse Compton photon spectrum extends down to \( \propto \gamma_{\min}^2 (B/B_{\text{cr}}) m_e c^2 \) with typically
When compared to the quasi-simultaneous observed SED we note that deep optical-to-X-ray observations are able to provide sensitive constraints, in particular on the efficiency of neutrino production. For the present case we find that the BAT upper limit constrains the neutrino-producing environment to $\tau_{\gamma\gamma,\text{max}} < 100$ for $D < 10$, whereas the Fermi-LAT spectrum excludes $\tau_{\gamma\gamma,\text{max}} < 10$.

### 3.2.3. Synchrotron–Compton Cascades

Generally in environments where $u_B \sim u'$, both synchrotron and Compton losses have to be considered when following electromagnetic cascades. In the high-energy regime, down to the threshold for pair production, the synchrotron–Compton-supported (see Figures 16–18) and synchrotron-supported cascade SEDs are very similar. This is because Compton-scattering proceeds in the Klein–Nishina regime here at a rate much smaller than the synchrotron loss rate, and the resulting spectra are determined by synchrotron radiation and pair production. At lower energies, photon production takes place through synchrotron and inverse Compton scattering on an equal footing, which is reflected in the resulting cascade spectra. With increasing opacity the number of (cascade) pairs rises. At MeV energies the resulting SED is dominated by inverse Compton photons from the primary Bethe–Heitler pairs and cascade pairs, and synchrotron photons from the $\pi^0$-decay pairs, while the SED at lower energies is due to synchrotron radiation from the Bethe–Heitler and cascade pairs.

By comparing also here with the quasi-simultaneous observed SED we find again the BAT upper limit to place constraints on the neutrino production efficiency $\tau_{\nu}$: Only $\tau_{\gamma\gamma,\text{max}} \sim \text{a few tens}$ for small Doppler factors $D < 10$, $\tau_{\gamma\gamma,\text{max}} \sim 10$ if $D = 10$, and $\tau_{\gamma\gamma,\text{max}} \sim \text{a few hundreds}$ for large $D \gg 10$ do not violate the minimal cascade fluxes computed, similar to the synchrotron-supported cascades.
3.3. On Efficient Neutrino Production

Since the cross section maximum for photon–photon pair production (near its threshold) compares to the maximum cross section value for photomeson production (at the $\Delta_{123}-$resonance near threshold) as $\sigma_{\gamma\gamma,\text{max}} \approx 300 \sigma_{\gamma\gamma,\text{max}}$, we expect in typical AGN environments the maximum optical depth of the respective interactions in the same target photon field to approximately scale correspondingly. In particular, for the case of an efficient neutrino producer, i.e., $\tau_{\gamma\gamma} > 1$, one yields $\tau_{\gamma\gamma} \gg 1$, and deep absorption troughs appear in the SED at energies $E_{\gamma}$ where $\gamma\gamma$-pair production sets in (see also discussions in, e.g., Begelman et al. 1990; Dermer et al. 2007; Murase et al. 2016):

$$E_{\gamma,\text{thr}} = \frac{s_{\gamma\gamma,\text{thr}}}{2e'(1 - \cos \theta)} = \frac{4m_e^2 e^4}{2e'(1 - \cos \theta)}$$

(1)

where $s_{\gamma\gamma,\text{thr}}$ is the center-of-momentum frame threshold energy of the interaction, $\epsilon'$ the target photon energy and $\theta$ the interaction angle. Considering that neutrinos are produced in the same target photon field via photomeson production dominantly in the $\Delta$-resonance region, the energy of the target photons is related to the observed neutrino energy $E_{\nu,\text{obs}}$ through

$$\epsilon' \approx \frac{s_{\Delta} - m_p^2 e^2}{2E_{\nu,\text{obs}}(1 - \cos \theta)} = \frac{(s_{\Delta} - m_p^2 e^2)}{2E_{\nu,\text{obs}}(1 - \cos \theta)} \xi$$

(2)

with $s_{\Delta} \approx (1.232 \text{GeV})^2$ and assuming $\beta_p = 1$. Note that the interaction angle for proton–photon and photon–photon interactions is in this case expected to be similar for highly relativistic protons. By combining Equations (1) and (2) one can then relate the (observer frame) photon energy $E_{\gamma,\text{obs}}$ to the detected neutrino energy through

$$E_{\gamma,\text{obs}} \lesssim \frac{4m_e^2 e^4 E_{\nu,\text{obs}}}{(s_{\Delta} - m_p^2 e^2)} \xi \approx 3.3 \times 10^{-5} E_{\nu,\text{obs}} \xi^{-1}$$

(3)

If neutrinos are very efficiently produced photohadronically, the associated cascade photons therefore can only escape the source at energies $> E_{\gamma,\text{obs}}$. For a neutrino flare spectrum in the 30 TeV–3 PeV range, the corresponding cascade spectrum is hence expected at $< 1 \text{ GeV}$. This applies also to $> \text{GeV}$ photons from alternative radiation processes. As a consequence, one does not expect a causal connection between the GeV flux detected by the LAT from TXS 0506+056 and the observed “flare” IceCube neutrinos for an efficient neutrino producer, irrespective of the dominant dissipation process.

4. Implications on the Jet Power and Target Photon Fields

We now derive constraints on the jet power using the observed neutrino flux in combination with limits for their production efficiency, derived via the minimal cascading flux (see Section 3.2). This can then be used, in a subsequent step, to deduce limits on the origin of the target photon field. Here, we will distinguish two possible scenarios, which can be thought of as extreme, limiting cases: (a) a target photon field that is comoving with the emission region (such as the electron–synchrotron emission), or (b) a stationary target photon field in the AGN rest frame. In case (a), the target photon energy $\epsilon'$ corresponds to an observed photon energy of $\epsilon_{\text{obs}} > 16 D_{14}^{2} 0.05 / E_{14} \text{ keV}$ (i.e., hard X-rays), while in case (b), the external (stationary) target photon field is Doppler boosted into the blook frame, so that $\epsilon_{\text{obs}} > 0.16 0.05 / E_{14} \text{ keV}$ (i.e., UV to soft X-rays).

4.1. Target Photon Energy Density and Proton Kinetic Power

As in Section 3 we consider a proton spectrum of the form $N_p(\gamma_{p}) = N_{p,\gamma}^{-\gamma_{p}}$ with $\alpha_p = 2$. The comoving neutrino luminosity (see Section 2.1) can be related to the proton energy loss rate due to photoproduction, given by Kelner & Aharonian (2008):

$$\gamma_{p,\gamma} \approx -c \langle \sigma p_{\gamma} \rangle n_{ph}(\epsilon'_{p}) \epsilon'_{p} \gamma_{p}$$

(4)

where $\epsilon'_{p} = \epsilon'(m_e^2 c^2)$ and $\langle \sigma p_{\gamma} \rangle \approx 10^{-28} \text{ cm}^2$ is the inelasticity-weighted $p_{\gamma}$ interaction cross section. The factor $n_{ph}(\epsilon'_{p}) \epsilon'_{p}$ provides a proxy for the comoving energy density of the target photon field, $u_{\gamma} \approx m_e c^2 n_{\gamma}(\epsilon'_{p}) \epsilon'_{p}$. Considering that the energy lost by protons in $p_{\gamma}$ interactions is shared approximately equally between photons and neutrinos, the 30 TeV–3 PeV neutrino luminosity is given by

$$L_{\nu} \approx \frac{1}{2} N_0 m_{p} c^2 \int_{\gamma_{\text{obs}}}^{\gamma_{\text{thr}}} \gamma_{p,\gamma}^{-\gamma_{p}} d\gamma_{p}$$

$$\approx 1.3 \times 10^{-14} N_0 u_{\gamma} \text{ cm}^3 \text{ s}^{-1}$$

(5)

for $\alpha_p = 2$. The limits in the integral in Equation (5) are given by $\gamma_1 = 6.4 \times 10^8 (D_{14} 0.05)^{-1}$ and $\gamma_2 = 6.4 \times 10^9 (D_{14} 0.05)^{-1}$, and $\alpha_p = 2$. Setting the neutrino luminosity from Equation (5) equal to the comoving neutrino luminosity inferred from IceCube observations, yields

$$N_0 u_{\gamma} \approx 1.3 \times 10^{57} D_{14}^{-4} \text{ erg cm}^{-3}$$

(6)

We now have two independent constraints on the target photon field. First is the direct observational constraint that the observed photon flux corresponding to the target photon field cannot exceed the observed flux in the relevant energy range. This provides a direct upper limit on $u_{\gamma}$, and will be used in Sections 4.2 and 4.3.

Second is the limit on the $\gamma\gamma$ opacity provided by the target photon field. In Section 3.2, we had found that, in order not to violate constraints from the observed optical, X-ray, and/or Fermi-LAT flux, the system needs to be either in a regime where cascades are Compton dominated (as the minimal value of the Compton cascade emission turned out to lie always below the observed SED, irrespective of the value of $\tau_{\gamma\gamma,\text{max}}$, or synchrotron (or synchrotron–Compton) dominated for $\tau_{\gamma\gamma,\text{max}} \sim$ a few tens (hundreds) for small $D \lesssim 10$ (large $D \gg 10$) Doppler factor. Compton-supported cascades will occur if

$$u_{\gamma,\text{Comp}} \gg u_{\gamma} \approx 4 B_{1}^{2} \text{ erg cm}^{-3}$$

(7)

in the comoving frame with $B = B_{1} 10$ G the magnetic field strength.

Using a simple $\delta$-function approximation for the $\gamma\gamma$ pair production cross section, we can estimate $u_{\gamma,\text{Comp}} \sim \gamma_{p} u_{\gamma} R_{16}^2 / (3 m_e^2 c^2) \sim 2.7 \times 10^{-3} u_{\gamma} R_{16}$, where $u_{\gamma} = u_{\gamma} / (\text{erg cm}^{-3})$ parameterizes the target photon energy density, and $R_{16} = R / (10^{16} \text{ cm})$ is the size of the emission region. Thus, the condition $\tau_{\gamma\gamma,\text{max}} \sim$ a few tens (hundreds) for $D \lesssim 10$ ($D \gg 10$)
translates into a limit of

\[ u'_{\text{syn}} \leq 10^4 R_{16}^{-1} \text{erg cm}^{-3} \text{ for } D \leq 10, \]
\[ u'_{\text{syn}} \leq 10^3 R_{16}^{-1} \text{erg cm}^{-3} \text{ for } D \gg 10. \]  (8)

Combining with \( u'_{\text{syn}} = u_0' \approx 4 B_2^2 \text{erg cm}^{-3} \) for synchrotron (synchrotron–Compton) cascades one gets

\[ R_{16} B_2^2 \geq 2.5 \times 10^3 (2.5 \times 10^4) \text{ for } D \leq 10 (D \gg 10). \]  (9)

This indicates the need for large field strengths and/or emission regions in the case of synchrotron (synchrotron–Compton) cascades operating in the source.

A limit on the proton kinetic power can then be derived by combining Equation (6) with (8) in the case where synchrotron (synchrotron–Compton) cascades determine the dissipation process. The comoving relativistic proton energy density, assuming that the proton spectrum extends as an unbroken power law with index \( \alpha_p = 2 \) from \( \gamma_{\min} = 1 \) to \( \gamma_2 \gg \gamma_{\min} \), is calculated by

\[ u'_p = m_p c^2 V' \int_{\gamma_1}^{\gamma_2} \frac{d\gamma}{\gamma} \gamma_1^{-\alpha_p} \approx 3.6 \times 10^{-52} N_0 R_{16}^3 (15.7 - \eta) \text{ erg cm}^{-3} \]  (10)

with \( \eta = \ln(D_{1.5,0.05}) \) and \( V' \) the comoving volume of the emission region. Since typically \( \eta \approx 0 \) with only small deviations expected, we will neglect the \( \eta \)-term in the following. The relativistic proton kinetic power \( L_p = 2n R_{16}^2 c^5 \gamma'_p \) carried by the jet in highly magnetized environments where pair synchrotron losses cannot be neglected can then be evaluated as

\[ L_p \gg 1.4(14) \times 10^{47} (\Gamma_1/D^2_1)^2 \text{ erg s}^{-1} \text{ for } D \gg 10 (D \leq 10), \]  (11)

with \( \Gamma \) the bulk Lorentz factor \( \Gamma_1 \equiv 10^{\Gamma_1} \), and a magnetic field power (using Equation (9)) of

\[ L_B \gg 0.2(2) \times 10^{50} \Gamma_1^{-2} R_{16}^{-1} \text{ erg s}^{-1} \text{ for } D \leq 10 (D \gg 10). \]  (12)

Hence, the total jet power turns out to be in this case at least of order \( \sim 10^{59-60} \text{erg s}^{-1} \) (or higher), independent of the origin of the target photon field.

We now use the observed SED in the energy range of the target photons (here \( \epsilon'_\min = 1...170 \text{keV}, \epsilon'_\max = 4...1000 \text{keV} \) for \( D = 1...50 \)) to derive a direct limit on \( u'_p \) which will then be combined with the previous constraints.

### 4.2. Case A: Comoving Target Photon Field

If the target photon field is comoving with the emission region, the required target photon energy corresponds to hard X-rays at an observed energy of \( \epsilon'_\text{obs} \gg 16 D_1^2 \epsilon_{0.05}/E_{44} \text{ keV} \). The observed X-ray flux from TXS 0506+056 is of the order of \( \epsilon'_X \sim 10^{-12} \text{erg cm}^{-2} \text{ s}^{-1} \), implied from archival data. The flux received directly from the target photon field, present in an assumed spherical emission region of size \( R'_1 \equiv 10^{16} R_{16} \text{ cm} \), is \( F'_\text{obs} = \frac{\pi}{4} c u'_p D^4 \) and must be equal to or less than the observed value. This constrains the comoving target photon energy density to be

\[ u'_p \lesssim 9 \times 10^{-4} R_{16}^{-2} D_1^{-4} \text{ erg cm}^{-3} \]  (13)

and in turn the proton-spectrum normalization factor in Equation (6):

\[ N_0 \geq 1.5 \times 10^{60} R_{16}^2. \]  (14)

The corresponding total kinetic power carried by the jet can then be evaluated as

\[ L_p \sim 1.55 \times 10^{55} \Gamma_1^3 R_{16} \text{ erg s}^{-1}. \]  (15)

which appears unreasonably high to be powered by an AGN accretion flow, as it exceeds the Eddington luminosity of even the most massive known supermassive black holes \( M_{\text{bh}} \lesssim 10^{10} M_\odot \) by several orders of magnitude.

The photon energy density limit from Equation (13) can also be used to calculate the proton cooling timescale due to photoproduction:

\[ t_{\text{p}}^{\text{obs}} = \frac{m_p c^2}{D \left( \epsilon_{0.05} f \right) u'_p} \geq 3 \times 10^{13} R_{16}^0 D_1^3 \text{ s}, \]  (16)

which can also be used to calculate the proton cooling timescale due to photoproduction:

\[ t_{\text{py}}^{\text{obs}} \sim 9 \times 10^8 B_1^{-2} D_1^{-1} \left( \frac{\gamma_p}{6 \times 10^6} \right) \text{ s}, \]  (17)

even for protons of moderate Lorentz factors of \( \gamma_p \sim 6 \times 10^6 \). In fact, this allows us to calculate the relative radiative power output in proton synchrotron- versus photopion-induced emissions:

\[ \frac{L_{\text{py}}}{L_{\text{p}}} \sim \frac{t_{\text{p}}^{\text{obs}}}{t_{\text{py}}^{\text{obs}}} \sim 4 \times 10^4 R_{16}^2 D_1^2 B_1^2 \left( \frac{\gamma_p}{6 \times 10^6} \right)^2 \]  (18)

with proton-synchrotron emission peaking at

\[ \nu_{\text{p}}^{\text{obs}} \sim 9 \times 10^{16} B_1 \left( \frac{\gamma_p}{6 \times 10^6} \right)^2 D_1 \text{ Hz}. \]  (19)

i.e., in the extreme UV/soft X-ray band. This implies that, in this scenario, the protons producing the 0.03-3 PeV neutrino flux are expected to produce proton synchrotron emission in the X-ray regime at a \( \nu F_\nu \) flux value larger than the corresponding \( \nu F_\nu \) neutrino flux value by a factor given by Equation (18). Reducing this to the limit based on the observed X-ray flux \( (L_{\text{py}}/L_{\text{p}} \gtrsim 0.1) \) would require an unusually low magnetic field value \( (B \lesssim 1 \text{G}) \) and/or a very low Doppler factor \( D \sim 1 \).

This is demonstrated in Figures 6 and 13 where we have added the proton synchrotron radiation on top of the minimal cascade component for various model parameters.

Based on these results, we may confidently rule out a scenario in which the IceCube neutrino flare flux from TXS 0506+056 is photohadrondically produced in a target
4.3. Case B: Stationary Target Photon Field in the AGN Frame

In the case of a target photon field that is stationary in the AGN rest frame, the target photon energy (in the AGN frame) should be $\epsilon_{\text{obs}} \geq 0.16 D_{15} S_{0.05}/E_{14}$ keV, and the AGN-frame radiation energy density is boosted into the comoving frame as $u' = u' \Gamma^2 u$. In the external photon field case, the spatial extent of the photon field may be scaled in units of a typical broad-line region size, $R_i \equiv 10^{17} R_{i,17} \text{ cm (AGN frame)}$, and the corresponding flux received is $F^\text{UV}_\text{comoving} = \frac{S^\prime}{4 \pi} \epsilon u'_i \Gamma^{-2}$.

For the Compton-dominated cascade case we now insert the energy density from Equation (7) to derive

$$F^\text{obs}_\text{UV} \approx 4.3 \times 10^{-15} (B R_{i,17}/\Gamma)^2 \text{ erg cm}^{-2} \text{ s}^{-1}$$

which is below the observed SED from archival data for not too large field strengths. Inspecting now the case for a highly magnetized environment where synchrotron losses become non-negligible we derive a UV–soft X-ray flux of

$$F^\text{obs}_\text{UV} \approx (10^{-9} \ldots 10^{-8}) R_{i,17}^2 (R_{i,17}/\Gamma)^2 \text{ erg cm}^{-2} \text{ s}^{-1}$$

for $D \leq 10$ ($D \gtrsim 10^3$) by making use of Equation (8). This flux level seems several orders of magnitude higher than the observed SED from archival data implies, and hence gives strong arguments to rule out an environment of the neutrino emission region where synchrotron (synchrotron–Compton) cascades operate.

From the observed SED a conservative upper limit on the $\nu F_\nu$ flux at UV–soft X-rays may be placed at $\nu F_\nu \lesssim 10^{-11} \text{ erg cm}^{-2} \text{ s}^{-1}$. The comoving target photon energy density can then be constrained as

$$u'_i \lesssim 94 \Gamma^{-2} R_{i,17}^2 \text{ erg cm}^{-3}$$

giving a corresponding pair production optical depth of

$$\tau_{\gamma\gamma, \text{max}} \lesssim 0.25 R_{16} \Gamma^2 R_{i,17}^2$$

and a photodisproportion optical depth of

$$\tau_{p\gamma, \text{max}} \lesssim 8 \times 10^{-4} R_{16} \Gamma^2 R_{i,17}^2$$

above the thresholds of the interactions. For moderate magnetic fields, $B \lesssim 10^4$, this may lead to Compton-supported cascades, thus not violating limits on the multiwavelength cascade flux. Equation (6) then constrains the proton-spectrum normalization to

$$N_0 \gtrsim 1.4 \times 10^{55} D_{15}^{-4} \Gamma^{-2} R_{i,17}^2$$

yielding a required proton kinetic jet power of

$$L_p \gtrsim 1.5 \times 10^{50} D_{15}^{-4} R_{i,17}^2 R_{16}^{-1} \text{ erg s}^{-1}$$

Due to the strong inverse dependence on the Doppler factor, for $D$ significantly exceeding 10 (and/or a larger target photon field radius or smaller emission region), this may be within the range plausibly powered by accretion onto a supermassive black hole of mass $\sim 3 \times 10^8 M_\odot$ (Padovani et al. 2019). The relativistic proton power from Equation (6) may be compared to the power carried along the jet in magnetic fields,

$$L_R = 3.8 \times 10^{45} R_{16}^2 \Gamma^2 \text{ erg s}^{-1}$$

Thus, for $B \lesssim 10^9$, the jet would have to be strongly dominated by the kinetic power of the relativistic protons.

For the target photon field of Equation (22), the proton photopion cooling timescale is, analogous to Equation (16),

$$t^\text{obs}_p \approx 2.9 \times 10^7 \tau^\text{max}_{\text{p}} \text{ yr}$$

Also in this case, proton–synchrotron radiation, at X-ray frequencies according to Equation (19), cannot be neglected, and the ratio of proton–synchrotron- to photopion-induced radiative (and neutrino) output will be

$$\frac{L^\text{psy}}{L_p} \approx 0.4 \Gamma^{-2} D_1 R_{i,17}^2 B_1^2 \left( \frac{\gamma_p}{6 \times 10^6} \right)$$

thus predicting a X-ray proton–synchrotron emission component with a flux of $\nu F_\nu(X) \lesssim 10^{-10} \Gamma^{-2} D_1 R_{i,17}^2 B_1^2 (\gamma_p/(6 \times 10^6)) \text{ erg cm}^{-2} \text{ s}^{-1}$. For a magnetic field of $B \sim 10^{-4}$, this does not seem to violate observational limits (see also Figure 6).

We thus conclude that a scenario appears feasible in which an external soft X-ray photon field provides the targets for photopion production generating the IceCube neutrinos of TXS 0506+056 in an environment where dominantly Compton scattering supports pair cascading. Equations (24) and (23) imply in this case still very inefficient neutrino production, and a corresponding pair cascade flux that lies significantly below the $\gamma$-ray flux level observed by the LAT during the neutrino flare (see Figures 6–10). Further high-energy radiation processes are therefore needed to explain the observed $\gamma$-ray SED, which is unlikely proton-initiated, during the neutrino flare.

5. Discussion and Conclusions

In this work we present a procedure to derive constraints on the broadband photon SED for a given (observed) neutrino spectrum and in the framework of photodisproportionionally produced neutrinos from relativistic protons accelerated in jetted AGNs. This procedure can accommodate any origin for the dominant target photon field. For this purpose we first reconstruct, in the comoving jet frame, the minimum target photon spectrum required to produce the neutrino spectrum, and calculate all corresponding further secondary particles produced through these interactions (photomeson and Bethe–Heitler pair production). A rather narrow energy range at hard X-rays (in the comoving jet frame) is sufficient to explain the 2014/15 IceCube neutrino flare spectrum from the direction of TXS 0506+056 by photomeson production of relativistic protons from an injection spectrum with index $\alpha_p = 2.0 \pm 0.1$, extending to PeV–hundreds of PeV energies. For typical spatial scales of the emission region and magnetic field strengths these protons are well contained in this region.

The produced high-energy photons and electron–positron pairs initiate electromagnetic cascades in the neutrino emission region with an efficiency that is directly linked to the neutrino production rate. Both the energy-dependent pair production optical depth and the radiation process (inverse Compton scattering, synchrotron radiation) that feed the cascades determine the resulting shape of the photon spectrum escaping the source. The so-derived photon spectrum, for a given
Doppler factor, can be considered as the minimum radiation emerging from the source that is strictly associated with the photopionically produced neutrinos. By comparing our simulated cascade spectra to broadband SEDs from observations carried out quasi-simultaneously to the neutrino detection, one can then derive constraints on the environment of the neutrino-producing site. A summary of the tested setups and resulting constraints is provided in Table 1.

A comprehensive study of these (steady-state) minimum cascade SEDs, when operating in a narrow X-ray target photon field, reveals rather low flux levels at GeV energies if inverse Compton radiation feeds the cascade, even for an optically thin radiative environment. In the case of highly magnetized environments where pair synchrotron losses cannot be neglected, the synchrotron- and synchrotron–Compton-supported cascades extend across a much wider energy range than the Compton-supported cascades, down to radio/optical frequencies. This allows deep optical to X-ray measurements to set limits on the efficiency of neutrino production in this region.

The efficient photopion production of \( \sim \)PeV neutrinos requires a high target photon density, leading to \( \gamma \gamma \) absorption depths \( \tau_{\gamma \gamma} \gg 1 \) for \( \sim \)GeV photons. We therefore conclude that efficient photopion production of IceCube neutrinos in blazars does not predict a causal connection between quasi-contemporaneous GeV \( \gamma \)-ray and neutrino emission here.

We then combined the derived limits on the \( \gamma \gamma \) opacity from the cascade SEDs and comoving neutrino luminosity of TXS 0506+056 with direct observational constraints from its MWL SED in the expected energy range of the required minimum target photon field. Considering first the extreme case of a co-spatially produced (comoving) target photon field for photopion production of the neutrino flare of TXS 0506+056, we deduced extreme values for the required proton kinetic power and a too high proton synchrotron component that would violate the observed photon SED at X-ray energies. This makes such a scenario implausible. Also regarding the case of a stationary (in the AGN frame) target photon field as the other extreme case results in too high expected flux values in the soft X-ray regime as compared to the observed SED if synchrotron or synchrotron–Compton cascades operate in the neutrino emission region.

The only viable scenario appears to be a stationary soft X-ray photon field providing the targets for photopion production of the observed neutrinos with the resulting cascades being fed by inverse Compton radiation only. The required proton kinetic powers appear in a plausible range for a jet powered by accretion onto a supermassive black hole, and the expected proton synchrotron component does not exceed the observed X-ray flux for not too large field strengths. In this environment, however, neutrino production is very inefficient \( \langle \tau_{\gamma\gamma} \rangle \sim 10^{-3} \) for typical blazar model parameters) during the neutrino flare and the associated pair cascade GeV flux too low to explain the LAT observations. The origin of the LAT spectrum observed during the neutrino flare period therefore cannot be associated with the neutrino-producing mechanism, and the GeV flux has to be emitted from zones in the jet other than the neutrino zone. Any link between the neutrino and GeV flux is then determined by a corresponding connection between the \( \gamma \)-ray and neutrino-emitting zones.

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### Table 1

**Summary of Tested Setups and Results**

| Origin of Target Photon Field | Further Setup Constraints | Results |
|-------------------------------|--------------------------|---------|
| All target photon fields      | \( u'_i > u''_i \)      | no constraints |
| (cascading constraints)       | \( u'_i \leq u''_i \)    | \( \tau_{\gamma\gamma,\max} \sim (a few) \) 10 for \( D \leq 10 \) |
|                               |                          | \( \tau_{\gamma\gamma,\max} \sim (a few) \) \( 10^{1-2} \) for \( D \geq 10 \) |
| Comoving target photon field  |                          | ruled out: too high \( L_\gamma \) (Equation (15)); p syn overshoots X-ray archival flux (Equations (18), (19)) |
| Stationary target photon field| \( u'_i \leq u''_i \)    | ruled out: target photon flux\(^2 \) overshoots UV/soft X-ray archival flux (Equation (21)) |
|                               | \( u'_i \gg u''_i \)    | viable: target photon flux below UV/soft X-ray archival flux (Equation (20)); \( L_\gamma \) (Equation (26)) acceptable for sufficiently large \( D \) |

**Note.**

* Includes cascading constraints.
Erratum: “Cascading Constraints from Neutrino-emitting Blazars: The Case of TXS 0506+056” (2019, ApJ, 881, 46)

Anita Reimer1, Markus Böttcher2, and Sara Buson3,4,5,6

1 Institute for Astro- & Particle Physics, University of Innsbruck, A-6020 Innsbruck, Austria; Anita.Reimer@uibk.ac.at
2 Centre for Space Research, North-West University, Potchefstroom, 2531, South Africa; Markus.Bottcher@nwu.ac.za
3 Universities Space Research Association, USA; buson@astro.uni-wuerzburg.de

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We wish to correct the definition of the comoving target photon energy density in Section 4 of the published article which should read

\[ u_t' \approx m_e c^2 n_{\text{ph}}'(\epsilon_t') \epsilon_t'^2 \]

with \( \epsilon_t' = \epsilon'/(m_e c^2) \) the dimensionless comoving photon energy of the target field with density \( n_{\text{ph}}'(\epsilon_t') \) (differential in energy; see Section 3.1 of the published article). As a consequence Equation (5) becomes

\[ L_p' \approx 1.3 \times 10^{-14} N_0 u_t' \epsilon_t'^{-1} \text{ cm}^3 \text{ s}^{-1}, \]

and Equation (6) changes to

\[ N_0 u_t' \approx 1.3 \times 10^{37} D_1^{-4} \epsilon_t' \text{ erg cm}^{-3} \]

with \( D \) the Doppler factor \( D \equiv 10 D_1 \). The relativistic proton kinetic power as evaluated in Equation (11) then gives

\[ L_p \gg 1.4(14) \times 10^{47} (\Gamma_1/D_1)^3 \epsilon_t' \text{ erg s}^{-1} \text{ for } D \gg 10 (D_1 \ll 10), \]

with \( \Gamma \) the bulk Lorentz factor \( \Gamma \equiv 10 \Gamma_1 \). Inserting the required target photon energy \( \epsilon_t' \approx 3.1 \times 10^{-3} D_1 \xi_{0.05} E_{14}^{-1} \) (see Section 3.1) yields

\[ L_p \gg 4.4(44) \times 10^{44} \Gamma_1^2 D_1^{-3} \xi_{0.05} E_{14}^{-1} \text{ erg s}^{-1}. \]

If the target photon field is comoving with the emission region (Section 4.2) the proton spectrum normalization factor (Equation (14)) results in \( N_0 \gtrsim 1.5 \times 10^{60} \Gamma_1^{2.5} \epsilon_t' \approx 4.7 \times 10^{57} \Gamma_1^{2.5} D_1 \xi_{0.05} E_{14}^{-1} \) and the corresponding kinetic jet proton power (Equation (15)) yields values as high as

\[ L_p \gtrsim 4.9 \times 10^{52} \Gamma_1^2 R_{1.16} D_1 \xi_{0.05} E_{14}^{-1} \text{ erg s}^{-1}, \]

in this case. This significantly exceeds the Eddington luminosity for even the most supermassive black holes known in the universe by several orders of magnitude. Our original conclusions for this case therefore remain unchanged. In Section 4.2 we also wish to correct a typo in Equation (17) of the published article, which should correctly read

\[ \tau_{\text{obs}}^\text{psyn} \approx 9 \times 10^8 B_{1}^{-2} D_1^{-1} \left( \frac{\gamma}{6 \times 10^6} \right)^{-1} \text{ s}. \]

For the case of a target photon field that is stationary in the active galactic nucleus rest frame (Section 4.3) the proton spectrum normalization (Equation (25)) is correctly constrained to

\[ N_0 \gtrsim 1.4 \times 10^{55} D_1^{-4} \Gamma_1^{-2} R_{1.17} \epsilon_t' \]

yielding a proton kinetic jet power (Equation (26)) of

\[ L_p \gtrsim 4.7 \times 10^{47} D_1^{-3} R_{1.17} \xi_{0.05} E_{14}^{-1} \text{ erg s}^{-1}, \]

which is well within the range plausibly powered by accretion onto a supermassive black hole of mass \( 3 \times 10^8 M_\odot \) as has been suggested for TXS 0506 + 056. This even relaxes the energetics requirement by a factor of \( \sim 300 \) with respect to our original result, thereby supporting the conclusions outlined in the published article.

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1 NASA Postdoctoral Program Fellow.
2 Centre for Space Research, North-West University, Potchefstroom, 2531, South Africa; Markus.Bottcher@nwu.ac.za
3 NASA Postdoctoral Program Fellow.
4 Based at NASA Goddard Space Flight Center, Greenbelt, MD 20771, USA.
5 Universities Space Research Association, USA; buson@astro.uni-wuerzburg.de
6 Now at University of Würzburg, D-97074 Würzburg, Germany.

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