The $D_s$-meson leading-twist distribution amplitude within the QCD sum rules and its application to the $B_s \rightarrow D_s$ transition form factor

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We make a detailed study on the $D_s$ meson leading-twist LCDA $\phi_{2,D_s}$ by using the QCD sum rules within the framework of the background field theory. To improve the precision, its moments $\langle \xi^n\rangle_{2,D_s}$ are calculated up to dimension-six condensates. At the scale $\mu = 2\text{GeV}$, we obtain: $\langle \xi^n\rangle_{2,D_s} = -0.261^{+0.020}_{-0.020}$, $\langle \xi^2\rangle_{2,D_s} = 0.184^{+0.012}_{-0.012}$, $\langle \xi^3\rangle_{2,D_s} = -0.111^{+0.007}_{-0.012}$ and $\langle \xi^4\rangle_{2,D_s} = 0.075^{+0.005}_{-0.005}$. Using those moments, the $\phi_{2,D_s}$ is then constructed by using the light-cone harmonic oscillator model. As an application, we calculate the transition form factor $f_{B_s^0 \rightarrow D_s^+}(q^2)$ within the light-cone sum rules (LCSR) approach by using a right-handed chiral current, in which the terms involving $\phi_{2,D_s}$ dominates the LCSR. It is noted that the extrapolated $f_{B_s^0 \rightarrow D_s^+}(q^2)$ agrees with the Lattice QCD prediction. After extrapolating the transition form factor to the physically allowable $q^2$-region, we calculate the branching ratio and the CKM matrix element, which give $\mathcal{B}(B_s^0 \rightarrow D_s^+\nu\bar{\nu}) = (2.03^{+0.35}_{-0.49}) \times 10^{-2}$ and $|V_{cb}| = (40.00^{+4.98}_{-4.08}) \times 10^{-3}$.

I. INTRODUCTION

Since the first measurement of the ratio $\mathcal{R}(D^*(s))$ of the branching fractions $\mathcal{B}(B \rightarrow D^+(s)\tau\nu\bar{\tau})$ and $\mathcal{B}(B \rightarrow D^+(s)\ell\nu\ell)$, where $\ell$ stands for the light lepton $e$ or $\mu$, had been reported by the BaBar Collaboration, the $B \rightarrow D^{(*)}$ semileptonic decays have attracted great attentions due to large differences between the experimental measurements [1–4] and the standard model (SM) predictions [5–14]. Such difference has been considered as an evidence of new physics. Comparing with the $B^{0,+}$ decays, because its background contamination from the partial reconstruction decay could be less serious, the $B_s \rightarrow D_s\ell\nu\ell$ decay is experimentally attractive. A natural question is whether there is also evidence of new physics in the semileptonic decay $B_s \rightarrow D_s\ell\nu\ell$. This decay could also be an important channel for determining the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $|V_{cb}|$.

The LHCb collaboration reported the measurement of $|V_{cb}|$ by using $B^0 \rightarrow D_s^-\mu^+\nu$ and $B^+ \rightarrow D_s^-\mu^+\nu$ decays [15], in which the data of the proton-proton collision at the center-of-mass energies of 7 and 8 TeV with the integrated luminosity about 3 fb$^{-1}$ had been used in the analysis. By using the Caprini-Lellouch-Neubert (CLN) and the Boyd-Grinstein-Lebed (BGL) parameterization [16–19] for $B_s \rightarrow D_s$ transition form factor (TFF), the determined $|V_{cb}|$ are $(41.4^{+0.6}_{-0.2}\pm0.9\pm1.2) \times 10^{-3}$ and $(42.3^{+0.8}_{-0.8}\pm0.9\pm1.2) \times 10^{-3}$, respectively. The LHCb collaboration also measured the ratio of the branching fractions $\mathcal{B}(B_s^0 \rightarrow D_s^-\mu^+\nu) \mathcal{R}(B_s^0 = D_s^-\mu^+\nu)$, i.e. $\mathcal{R} = 1.09 \pm 0.05 \pm 0.06 \pm 0.05$, which then gives $\mathcal{B}(B_s^0 \rightarrow D_s^-\mu^+\nu) = (2.49 \pm 0.12 \pm 0.14 \pm 0.16) \times 10^{-2}$.

The accuracy of theoretical predictions on the branching fraction $\mathcal{B}(B_s \rightarrow D_s \ell \nu \ell)$ depends heavily on the TFF $f_{B_s^0 \rightarrow D_s^+}(q^2)$. It has been calculated within several approaches, such as the quark models [20–22], the QCD light cone sum rules (LCSR) [23, 24], and the lattice QCD (LQCD) [25–27]. Similar to the $B \rightarrow \pi$ TFF's [28], the LQCD prediction is reliable in large $q^2$-region, the QCD factorization prediction or the quark model prediction is reliable in large recoil region $q^2 \sim 0$, and the LCSR is reliable in low and intermediate $q^2$-regions. Predictions under various methods are complementary to each other. Because the LCSR prediction is applicable in a wider region and could be adapted for all $q^2$-region via proper extrapolations, and in this paper, we will adopt the LCSR approach to calculate $f_{B_s^0 \rightarrow D_s^+}(q^2)$.

Generally, contributions from the light-cone distribution amplitude (LCDA) suffers from the power counting rules basing on the twists, i.e. the high-twist LCDAs are usually powered suppressed to the lower twist ones in large $Q^2$-region. The high-twist LCDAs may have sizable contributions to the LCSR, and how to "design" a proper correlator is a tricky problem for the LCSR approach. By choosing a proper correlator, one can not only study the properties of the hadrons but also simplify the theoretical uncertainties effectively. As the usual treatment, the correlator is constructed by using the currents with definite quantum numbers, such as those with definite $J^P$, where $J$ is the total angular momentum and $P$ is the parity of the bound state. Such a construction of the correlator is not the only choice suggested in the literature, e.g. the chiral correlator with a chiral current in between the matrix element has also been suggested to suppress the hazy contributions from the uncertain LCDAs [29–34]. In the paper, we adopt a chiral correlator to do the LCSR cal-

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calculation, and we shall find that the leading-twist LCDA $\phi_{2,D_s}$ provides dominant contributions. Therefore, if an accurate $\phi_{2,D_s}$ has been achieved, we shall obtain an accurate prediction on $f_{B_s^+ \to D_s^+}(q^2)$.

Till now, there are few calculations on the $D_s$-meson leading-twist LCDA $\phi_{2,D_s}$: recently, it has been studied by using the light-front quark model [35]. We shall first construct a light-cone harmonic oscillator model for $\phi_{2,D_s}$ based on the well-known BHL-description [36–38] as we have done for $\pi$, $\rho$, $D$ and heavy meson LCDA [39–46]. Then its input parameters shall be fixed by using reasonable constraints such as the probability of finding the leading Fock-state in $D_s$-meson Fock-state expansion, the normalization condition, and the calculated LCDA moments $\langle \xi^n \rangle_{2,D_s}$ or the Gegenbauer moments $a^{D_s}_n$. All those moments shall be computed by using the QCD sum rules [47] within the framework of background field theory (BFT) [48] up to dimension-six operators.

The remaining parts are organized as follows. The LCSR for $B_s^0 \to D_s^0$ TFF, the QCD sum rules of the moments of $\phi_{2,D_s}$ and the light-cone harmonic oscillator model for $\phi_{2,D_s}$ are given in Sec.II. Numerical results and discussions are presented in Sec.III. Section IV is reserved for a summary. The useful functions for calculating the $\phi_{2,D_s}$ moments are listed in the Appendix.

## II. CALCULATION TECHNOLOGY

### A. The LCSR for $B_s^0 \to D_s^0$ TFF

The $B_s^0 \to D_s^0$ TFF $f_{B_s^0 \to D_s^0}(q^2)$ and $\tilde{f}_{B_s^0 \to D_s^0}(q^2)$ are usually defined as:

$$\langle D_s(p)[\bar{c}\gamma_\mu b]\rangle \rightarrow f_{B_s^0 \to D_s^0}(q^2) + \tilde{f}_{B_s^0 \to D_s^0}(q^2),$$

(1)

where $p$ is the momentum of $D_s$-meson, $q$ is the momentum transfer. In this paper, we focus on the semileptonic decay $B_s^0 \to D_s^0 \ell\bar{\nu}_\ell$ with $\ell = (e, \mu)$. The masses of light-leptons are negligible, and then due to chiral suppression, only $f_{B_s^0 \to D_s^0}(q^2)$ is relevant for our present analysis.

To derive the LCSR of $f_{B_s^0 \to D_s^0}(q^2)$, we adopt the following chiral correlation function (correlator):

$$\Pi_\mu(p,q) = \int d^4xe^{ipx} \langle 0| \bar{D}_s(p)| \bar{c}\gamma_\mu(x)b(x)\rangle \langle 0|0 \rangle$$

(2)

It is done by using the b-quark propagator

$$\langle 0|Tb(x)\bar{b}(0)|0 \rangle = \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \frac{k^\mu + m_b}{k^2 - m_b^2} + \cdots.$$  

(5)

By matching the hadronic representation (4) and the OPE of the correlator (2) with the help of the dispersion relation, the LCSR of $f_{B_s^0 \to D_s^0}(q^2)$ can be obtained

$$f_{B_s^0 \to D_s^0}(q^2) = \frac{e^{m_{B_s}^2/M^2}}{m_{B_s}^2f_{B_s}} \left[ F_0(q^2, M^2, s_{B_s}^B) \right] + \frac{\alpha_sC_F}{4\pi} F_1(q^2, M^2, s_{B_s}^B).$$

(6)

where $C_F = 4/3$, $M$ is the Borel parameter. Here the Borel transformation has been adopted to suppress continuum contributions. The leading-order (LO) contribution of $f_{B_s^0 \to D_s^0}(q^2)$ takes the form:

$$F_0(q^2, M^2, s_{B_s}^B) = \frac{m_{B_s}^2f_{B_s}}{m_{B_s}^2f_{B_s}} e^{m_{B_s}^2/M^2} \int_\Delta \frac{du}{u} \phi_{2,D_s}(u)$$

$$\times \exp \left[ -\frac{m_{B_s}^2 - \tilde{u}(q^2 - um_{D_s}^2)}{uM^2} \right].$$

(7)
with the $D_s$-meson decay constant $f_{D_s}$ and

$$\Delta = \frac{1}{2m_{D_s}^2} \left[ \sqrt{(s_0^{B_s} - q^2 - m_{D_s}^2)^2 + 4m_{D_s}^2(m_s^2 - q^2)} - (s_0^{B_s} - q^2 - m_{D_s}^2) \right].$$

The NLO contribution of $f_{D_s}^{B_s \to D_s}(q^2)$ reads

$$F_1(q^2, M^2, s_0^{B_s}) = \frac{f_{D_s}}{\pi} \int_{s_0^{B_s}}^{\infty} ds e^{-s/M^2} \int_0^1 du \text{Im} T_1(q^2, s, u) \phi_{2; D_s}(u).$$

The imaginary part of the next-to-leading order amplitude $T_1$ can be read from Ref.[49]. Due to the present choice of the chiral correlator (2), contributions from the twist-3 $D_s$-meson LCDA exactly vanish in the LCSR. Thus the terms from omitted gluonic field in $b$-quark propagator (5) and hence contributions from even higher-twist terms are negligibly small and can be safely neglected. Our remaining task is then to achieve a precise $\phi_{2; D_s}$.

### B. Sum rules for the moments of the $D_s$-meson leading-twist LCDA $\phi_{2; D_s}$

The $D_s$-meson leading-twist LCDA $\phi_{2; D_s}$ is defined as

$$\langle 0| \bar{c}(z) \gamma_5 s(z) | D_s(q) \rangle = i(z \cdot q) f_{D_s} \int_0^1 dx e^{i(2x - 1)(z \cdot q)} \phi_{2; D_s}(x),$$

where $f_{D_s}$ is the $D_s$ meson decay constant. The moments of $\phi_{2; D_s}(x)$ can be derived by expanding the left-hand-side of Eq.(9) around $z = 0$ and the exponent in the right-hand-side of Eq.(9) as a power series, e.g.

$$\langle 0| \bar{c}(0) \gamma_5 (z \cdot D)^n s(0) | D_s(q) \rangle = i f_{D_s} (z \cdot q)^{n+1} \langle \xi^n \rangle_{2; D_s},$$

where the $n_{th}$-moment is defined as

$$\langle \xi^n \rangle_{2; D_s} = \int_0^1 dx (2x - 1)^n \phi_{2; D_s}(x).$$

The 0th-moment satisfies the normalization condition

$$\langle \xi^0 \rangle_{2; D_s} = 1.$$  (12)

The sum rules of those moments can be derived by using the following correlator

$$\Pi^{(n,0)}_{2; D_s}(z, q) = i \int d^4 x e^{i q \cdot x} \langle 0| T \{ J_n(x) J_0^\dagger(0) \} | 0 \rangle = (z \cdot q)^{n+2} I^{(n,0)}_{2; D_s}(q^2),$$

where $n = (0, 1, 2, ...)$, and the currents

$$J_n(x) = \bar{c}(x) \gamma_5 (iz \cdot D)^n s(x),$$

$$J_0^\dagger(0) = \bar{s}(0) \gamma_5 c(0).$$

By applying the OPE for the correlator (13) in deep Euclidean region based on the BFT [48], we obtain

$$\Pi^{(n,0)}_{2; D_s}(z, q) = i \int d^4 x e^{i q \cdot x}$$

$$\times \left\{ - \text{Tr}(0| S_{c\bar{c}}^{D_s}(0, x) \bar{c}(0) \gamma_5 (iz \cdot D)^n s(x) \gamma_5 | 0 \rangle \right\}$$

$$+ \text{Tr}(0| S_{c\bar{c}}^{D_s}(0, x) \bar{c}(0) \gamma_5 (iz \cdot \bar{D})^n \bar{s}(0) s(x) \gamma_5 | 0 \rangle \right\}$$

$$+ \cdots,$$

where $S_{c\bar{c}}^{D_s}(0, x)$ and $S_{c\bar{c}}^{D_s}(0, x)$ are $c$ and $s$ quark propagators in the BFT, $(iz \cdot \bar{D})^n$ stands for the vertex operators, and “...” indicates the even higher-order terms.

There are totally 40 Feynman diagrams for the present considered accuracy, e.g. up to dimension-6 operators, the first and second terms in Eq.(16) contain 35 and 5 Feynman diagrams, respectively. Typical Feynman diagrams are shown in Figure. 1 and Figure. 2, other diagrams can be obtained by permutation. In those two figures, the left big dot and the right big dot stand for the vertex operators $\not\! D_s(z \cdot \bar{D})^n$ and $\not\! D_s$ in the currents $J_n(x)$ and $J_0^\dagger(0)$, respectively; the cross symbol indicates the gluonic background field. There are also cases in which the cross symbol stands for the $s$-quark background field. In deriving the QCD sum rules for the moments, we need to know the propagators and vertex operators under the BFT up to dimension-six operators, and tedious expressions of them can be found in Ref.[39]. Here different from the case of the $D$-meson, the mass effect in the denominator of $s$-quark propagator cannot be ignored. However, considering that the $s$-quark mass is not large, we expand the $s$-quark propagator as a power series over $m_s$ and keep only the first power of $m_s$. In this way, we can use the corresponding calculation technology described in detail in Ref. [42] to do the calculation.

Following the standard procedures of QCD sum rules [50, 51], we obtain the sum rules for the moments of $D_s$-meson leading-twist LCDA, i.e.

$$\langle \xi^n \rangle_{2; D_s} f_{D_s} \frac{\bar{f}_{D_s}}{M^2} e^{-m_{D_s}^2/M^2}$$

$$= \frac{1}{\pi} \frac{1}{M^2} \int_{m_{D_s}}^{s_0^{B_s}} ds e^{-\frac{s}{m_{D_s}^2}} \text{Im} I_{\text{pert}}(s) + \hat{B}_{M^2} I_{(\bar{s}s)}(-q^2)$$

$$+ \hat{B}_{M^2} I_{(\bar{G}s)}(-q^2) + \hat{B}_{M^2} I_{(\bar{G}s)s)}(-q^2) + \hat{B}_{M^2} I_{(\bar{s}s)}s(-q^2),$$

The analytical expressions of the perturbative and non-perturbative terms are
\[ \text{Im} I_{\text{pert}}(s) = \frac{3}{8\pi^2 M^2(n+1)(n+3)} \left\{ \frac{1}{v} \left( 1 + \sqrt{1 - \frac{4m_2^2v^2}{s}} \right) - 1 \right\}^{n+1} \left\{ 1 - \frac{n+1}{2v} \left( 1 + \sqrt{1 - \frac{4m_2^2v^2}{s}} \right) \right\} \left( 1 - \frac{\sqrt{1 - \frac{4m_2^2v^2}{s}}}{s} \right)^{-1} \left( 1 - \frac{\sqrt{1 - \frac{4m_2^2v^2}{s}}}{s} \right)^{-2} \right\} \]  

\[ \mathcal{B}_{M^2} I_{1; D_s}^{(G^3)}(-q^2) = \frac{(\alpha_s G^2)}{12\pi M^4} \left[ 2(n-1) \mathcal{H}(n-2, 1, 3, 2) + \mathcal{H}(n, 0, 2, 2) - 2m_e^2 \mathcal{H}(n, 1, 1, 3) \right], \]  

\[ \mathcal{B}_{M^2} I_{1; G^3}(-q^2) = \frac{(g_s f G^3)}{120\pi^2} \left[ -10 (n-1) n (n+1) \mathcal{H}(n-2, 1, 4, 3) - 30m_e^2 n(n-1) \mathcal{H}(n-2, 1, 4, 4) - 15m_e^2 \mathcal{H}(n, 1, 1, 4) - 5m_e^2 \mathcal{H}(n, 0, 2, 4) + 5nm_e^2 \mathcal{H}(n-1, 1, 2, 4) + 36m_e^2 \mathcal{H}(n, 1, 1, 5) \right] \]
with $v = s/(s - m_c^2 + m_{\perp}^2)$. Here the functions $\mathcal{F}_{1,2}(n, a, t_{\text{min}}, t_{\text{max}}), \mathcal{G}_{1,2}(n, a), \mathcal{H}(n, a, b, c)$ and Borel transformations which are collected in the Appendix IV.

C. The light-cone harmonic oscillator model for the $D_s$-meson leading-twist LCDA $\varphi_{2;D_s}$

Based on the BHL-description [36-38], similar to the case of $D$-meson leading-twist LCDA [42], we construct a light-cone harmonic oscillator model of the $D_s$-meson leading-twist wavefunction $\Psi_{2;D_s}(x, k_{\perp})$ as

$$\Psi_{2;D_s}(x, k_{\perp}) = \chi_{2;D_s}(x, k_{\perp}) \Psi_{2;D_s}^R(x, k_{\perp}),$$

(24)

where $k_{\perp}$ is the transverse momentum, $\chi_{2;D_s}(x, k_{\perp})$ is the spin-space wavefunction and $\Psi_{2;D_s}^R(x, k_{\perp})$ indicates the spatial wavefunction. The spin-space wavefunction $\chi_{2;D_s}(x, k_{\perp})$ reads [52]

$$\chi_{2;D_s}(x, k_{\perp}) = \frac{\hat{m}_c x + \hat{m}_s (1-x)}{\sqrt{k_{\perp}^2 + [\hat{m}_c x + \hat{m}_s (1-x)]^2}},$$

(25)

where $\hat{m}_c$ and $\hat{m}_s$ are constituent quark masses of $D_s$, and we adopt $\hat{m}_c = 1.5\text{GeV}$ and $\hat{m}_s = 0.5\text{GeV}$. The spatial wavefunction takes the form

$$\Psi_{2;D_s}^R(x, k_{\perp}) = A_{D_s} \varphi_{2;D_s}(x) \times \exp \left[ -\frac{1}{\beta_{D_s}} \left( \frac{k_{\perp}^2 + \hat{m}_c^2}{1-x} + \frac{k_{\perp}^2 + \hat{m}_s^2}{1-x} \right) \right],$$

(26)

where $A_{D_s}$ is the normalization constant, $\beta_{D_s}$ is the harmonious parameter that dominates the wavefunction’s transverse distribution, and function $\varphi_{2;D_s}(x)$ dominates the wavefunction’s longitudinal distribution. $\varphi_{2;D_s}(x)$ can be taken as the first few terms of the Gegenbauer series, here we take

$$\varphi_{2;D_s}(x) = 1 + \sum_{n=1}^4 B^D_n C_n^3(2x - 1).$$

(27)

By using the relationship between the $D_s$-meson leading-twist wavefunction, one can get its LCDA at the scale $\mu_0$,

$$\phi_{2;D_s}(x, \mu_0) = \frac{2\sqrt{6}}{f_{D_s}} \int_{|k_{\perp}| \leq \mu_0^2} \frac{d^2k_{\perp}}{16\pi^3} \Psi_{2;D_s}(x, k_{\perp}),$$

(28)

which, after integrating over the transverse momentum $k_{\perp}$, becomes

$$\phi_{2;D_s}(x, \mu_0) = \frac{6A_{D_s} \beta_{D_s}}{\pi^2 f_{D_s}} x(1-x) \varphi_{2;D_s}(x) \times \exp \left[ -\frac{\hat{m}_c^2 x + \hat{m}_s^2 (1-x)}{8\beta_{D_s}^2 x (1-x)} \right] \times \left[ 1 - \exp \left[ -\frac{\mu_0^2}{8\beta_{D_s}^2 x (1-x)} \right] \right],$$

(29)

where $\mu_0 \sim \Lambda_{\text{QCD}}$ is the factorization scale. Because $\hat{m}_c \gg \Lambda_{\text{QCD}}$, the spin-space wavefunction $\chi_{D_s} \to 1$. The above model (24, 29) is for $D_s^-$-meson. The leading-twist wavefunction and the LCDA for $D_s^+$-meson can be obtained by replacing $x$ with $(1-x)$ in Eqs.(24, 29).

The model parameters $A_{D_s}, B^D_n$ and $\beta_{D_s}$ are scale dependent, their values at an initial scale $\mu_0$ can be determined by reasonable constraints, and their values at any other scale $\mu$ can be derived via the evolution equation [53]. More explicitly, we shall adopt the following constraints to fix the parameters:
The normalization condition,
\[
\int_0^1 dx \phi_{2,D_s}(x, \mu_0) = 1.
\] (30)

The probability of finding the leading Fock-state \(|\bar{c}s\rangle\) in \(D_s\)-meson Fock-state expansion,
\[
P_{D_s} = \frac{A_D^2 \beta_D^2}{4\pi^2} x(1-x) \varphi_D^2(x) \times \exp \left[ -\frac{m_s^2 x + m_c^2 (1-x)}{4\beta_D^2 x} (1-x) \right].
\] (31)

We will take \(P_{D_s} \approx 0.8\) [54] in subsequent calculations.

The Gegenbauer moments of \(\phi_{2,D_s}(x, \mu_0)\) can be derived via the following formula,
\[
a_n^{D_s}(\mu_0) = \frac{\int_0^1 dx \phi_{2,D_s}(x, \mu_0) C_n^{3/2}(2x-1)}{\int_0^1 dx 6x(1-x)[C_n^{3/2}(2x-1)]^2}.
\] (32)

and the \(\phi_{2,D_s}(x, \mu_0)\) moments are defined as
\[
\langle \xi^n \rangle_{2,D_s} = \int_0^1 dx (2x-1)^n \phi_{2,D_s}(x, \mu_0).
\] (33)

The values of the moments \(\langle \xi^n \rangle_{2,D_s}\) and the Gegenbauer moments \(a_n^{D_s}\) at the scale 2 GeV will be given in next subsection.

### III. NUMERICAL ANALYSIS

#### A. Input parameters

To do the numerical analysis on the moments of \(D_s\)-meson leading-twist LCDA, we take the \(D_s\)-meson mass \(m_{D_s} = 1.968 \pm 0.0007\) GeV, the \(c\)-quark current-quark mass \(m_c(m_c) = 1.275 \pm 0.02\) GeV, the \(s\)-quark mass \(m_s(2\text{ GeV}) = 0.093^{+0.03}_{-0.005}\) GeV and the decay constant of \(D_s\)-meson \(f_{D_s} = 0.256 \pm 0.0042\)MeV [55]. For the gluon vacuum condensates, we take \((\alpha_s G^2) = 0.038 \pm 0.011\) GeV\(^4\) and \((g_s^3 f G^3) = 0.045\) GeV\(^6\) [56]. For the remaining vacuum condensates, we adopt \((\bar{s}s) = \kappa \langle \bar{q}q \rangle\), \((g_s \bar{s}s T G s) = \kappa (g_s \bar{q}q T G q)\), and \((g_s \bar{s}s)^2 = \kappa^2 (g_s \bar{q}q)^2\), where \(\kappa = 0.74 \pm 0.03\) [57], \(\langle \bar{q}q \rangle = (-2.417^{+0.227}_{-0.114}) \times 10^{-2}\) GeV\(^2\), \(\langle g_s \bar{s}s T G q \rangle = (-1.934^{+0.188}_{-0.103}) \times 10^{-2}\) GeV\(^5\) and \(\langle g_s \bar{q}q \rangle^2 = (2.082^{+0.743}_{-0.697}) \times 10^{-3}\) GeV\(^6\) at \(\mu = 2\) GeV [50]. The scale evolution equations of those inputs are [50, 58, 59]

\[
\begin{align*}
\bar{m}_c(\mu) &= \bar{m}_c(m_c) \left[ \frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right]^{4/\beta_0}, \\
\bar{m}_s(\mu) &= \bar{m}_s(2\text{ GeV}) \left[ \frac{\alpha_s(\mu)}{\alpha_s(2\text{ GeV})} \right]^{4/\beta_0}, \\
\langle \bar{q}q \rangle(\mu) &= \langle \bar{q}q \rangle(2\text{ GeV}) \left[ \frac{\alpha_s(\mu)}{\alpha_s(2\text{ GeV})} \right]^{-4/\beta_0},
\end{align*}
\]

\[
\langle g_s \bar{q}q T G q \rangle(\mu) = \langle g_s \bar{q}q T G q \rangle(2\text{ GeV}) \left[ \frac{\alpha_s(\mu)}{\alpha_s(2\text{ GeV})} \right]^{-2/(3\beta_0)}, \\
\langle g_s \bar{q}q \rangle^2(\mu) = \langle g_s \bar{q}q \rangle^2 \left[ \frac{\alpha_s(\mu)}{\alpha_s(2\text{ GeV})} \right]^{-4/\beta_0}, \\
\langle \alpha_s G^2(\mu) = \langle \alpha_s G^2(2\text{ GeV}) \right]^{-4/\beta_0}, \\
\langle g_s^3 f G^3 \rangle(\mu) = \langle g_s^3 f G^3 \rangle(2\text{ GeV}).
\] (34)

where \(\beta_0 = 11 - 2n_f/3\) with \(n_f\) being the active quark flavors. In the following numerical calculation of the moments \(\langle \xi^n \rangle_{2,D_s}\), the scale \(\mu\) will be set as the Borel parameter as usual, i.e., \(\mu = M\). For the continuous threshold \(s_0^{D_s}\), it is usually taken as the squared mass of the \(D_s\)-meson’s first exciting state, and we take \(s_0^{D_s} \simeq 6.5\) GeV\(^2\).

#### B. The moments \(\langle \xi^n \rangle_{2,D_s}\) from QCD sum rules

| \(n\) | \(M^2\) | \(\langle \xi^n \rangle_{2,D_s}\) |
|-----|-------|----------------|
| 1   | [1.517, 5.840] | [−0.304, −0.263] |
| 2   | [2.165, 4.164] | [0.168, 0.193] |
| 3   | [2.162, 7.185] | [−0.107, −0.104] |
| 4   | [1.928, 5.524] | [0.069, 0.077] |

FIG. 3: The \(D_s\)-meson leading-twist LCDA moments \(\langle \xi^n \rangle_{2,D_s}\) at the scale \(\mu = M\) with \(n = (1, \cdots, 4)\) versus the Borel parameter \(M^2\), where all input parameters are set to be their central values.

To get the numerical value of moments \(\langle \xi^n \rangle_{2,D_s}\) of \(\phi_{2,D_s}(x, \mu)\), one need to fix the Borel window \(M^2\) which is introduced to depress the contributions from both the continuum states and the highest dimensional condensates. Usually, the continuum contribution and the dimension-six condensate contribution are taken to be
less than 30% and 10% respectively, while the value of \(\langle \xi^n \rangle_{2;D_s} \) is required to be as stable as possible in the allowed Borel window. In this paper, the continuum state contribution for \(\langle \xi^n \rangle_{2;D_s} |_{\mu} \) with \( n = (1, 2, 3, 4) \) is required to be less than 20%, 25%, 10%, 30%, respectively, and each of the dimension-six condensates contributions is no more than 1%. The determined Borel windows and the corresponding \( D_s \)-meson leading-twist LCDA moments \( \langle \xi^n \rangle_{2;D_s} \) at the scale \( \mu = 2 \) GeV with \( n = (1, \cdots, 4) \) are presented in Table I, where all input parameters are taken to be their central values. We present the \( D_s \)-meson leading-twist LCDA moments \( \langle \xi^n \rangle_{2;D_s} \) with \( n = (1, \cdots, 4) \) at \( \mu = 2 \) GeV versus \( M^2 \) in Fig. 3. To be consistent with Table I, those moments are stable over the allowable Borel windows.

If setting \( \mu = 2 \) GeV, by taking all uncertainty sources into consideration, we obtain

\[
\begin{align*}
\langle \xi^1 \rangle_{2;D_s} |_{\mu = 2 \text{GeV}} &= -0.261^{+0.020}_{-0.020}, \\
\langle \xi^2 \rangle_{2;D_s} |_{\mu = 2 \text{GeV}} &= +0.184^{+0.012}_{-0.012}, \\
\langle \xi^3 \rangle_{2;D_s} |_{\mu = 2 \text{GeV}} &= -0.111^{+0.007}_{-0.012}, \\
\langle \xi^4 \rangle_{2;D_s} |_{\mu = 2 \text{GeV}} &= +0.075^{+0.005}_{-0.005},
\end{align*}
\]

where the errors are squared averages of all the mentioned error sources.

C. Determination of the model parameters of \( \phi_{2;D_s} \)

According to the constraints of \( D_s \)-meson leading-twist LCDA \( \phi_{2;D_s}(x, \mu) \), i.e., Eqns. (30-32), we need to know the Gegenbauer moments \( a_n^{D_s}(\mu) \) to fix the parameters \( A_{D_s}, B_n^{D_s} \) and \( \beta_{D_s} \). The Gegenbauer moments \( a_n^{D_s}(\mu) \) using their relations to the LCDA moments \( \langle \xi^n \rangle_{2;D_s} |_{\mu} \) [40], we obtain

\[
\begin{align*}
a_1^{D_s}(2 \text{ GeV}) &= -0.436^{+0.033}_{-0.033}, \\
a_2^{D_s}(2 \text{ GeV}) &= -0.047^{+0.035}_{-0.035}, \\
a_3^{D_s}(2 \text{ GeV}) &= +0.004^{+0.010}_{-0.020}, \\
a_4^{D_s}(2 \text{ GeV}) &= -0.004^{+0.025}_{-0.026}.
\end{align*}
\]

We present all the determined input parameters at the scale \( \mu = 2 \) GeV in Table II. The accuracy of \( \phi_{2;D_s}(x, \mu) \) is dominated by the magnitudes of the Gegenbauer moments \( a_n^{D_s}(\mu) \). As we have pointed out in Ref. [42, 43], the Gegenbauer moments \( a_n^{D_s}(\mu) \) are correlated to each other and can not be changed independently within their own error regions. Then Table II associates the uncertainty of \( \phi_{2;D_s}(x, \mu) \) with the error of Gegenbauer moments \( a_n^{D_s}(\mu) \), which facilitates our further discussion on the impact of \( \phi_{2;D_s}(x, \mu) \) as an input parameter to the \( B_s \to D_s \) decay.

Figure. 4 shows the \( D_s \)-meson leading-twist LCDA \( \phi_{2;D_s}(x, \mu) \) with typical values of the input parameters exhibited in Table II. The solid, the dash-dotted and the dashed lines are for the parameters exhibited in second, third and forth lines of Table II. Our model of \( \phi_{2;D_s}(x, \mu) \) prefers a broader behavior in low \( x \)-region. It has a peak around \( x \approx 0.35 \). Figure. 5 shows the \( D_s \)-meson leading-twist LCDA \( \phi_{2;D_s}(x, \mu) \) at different scales, where the solid, the dashed, the dotted and the dash-dotted lines are for \( \mu = 2, 3, 10, 100 \) GeV, respectively. It shows that with the increament of \( \mu \), \( \phi_{2;D_s}(x, \mu) \) becomes broader and broader and becomes more symmetric, e.g. the peak moves closer to \( x = 0.5 \). When \( \mu \to \infty \), \( \phi_{2;D_s}(x, \mu) \) tends to the known asymptotic form, i.e. \( \phi_{2;D_s}(x, \mu \to \infty) = 6x(1-x) \).
TABLE II: Typical $D_s$-meson leading-twist LCDA model parameters at scale $\mu = 2$ GeV. The first line stand for the central value, the secon/third lines mean the upper/lower limit for the LCDA.

| Parameter | Lower | Middle | Upper |
|-----------|-------|--------|-------|
| $a^{D_s}_{\mu}$ | $-0.436_{-0.047}^{+0.033}$ | $0.004_{-0.026}^{+0.010}$ | $2.760_{-0.313}^{+0.185}$ |
| $b^{D_s}_{\mu}$ | $0.004_{-0.026}^{+0.010}$ | $0.004_{-0.026}^{+0.010}$ | $0.083_{-0.008}^{+0.008}$ |
| $\beta_{D_s}^{\mu}$ | $4.521_{-4.567}^{+4.567}$ | $4.484_{-4.567}^{+4.567}$ | $4.521_{-4.567}^{+4.567}$ |

D. Numerical results of $B_s \to D_s$ TFF and its applications

Our inputs for the $B_s \to D_s$ TFF $f_{B_s \to D_s}^T(q^2)$ are [55]

\[ m_B = 5.36688 \pm 0.00017 \text{ GeV}, \]
\[ \bar{m}_b = 4.18_{-0.03}^{+0.04} \text{ GeV}, \]
\[ f_{B_s} = 266 \pm 19 \text{ MeV}. \]

![Graph showing the TFF $f_{B_s \to D_s}^T(q^2)$ for some typical $q^2$ values versus the Borel parameter $M^2$.](image)

TABLE III: The parameters $a$ and $b$ for the TFF extrapolation. The lowest, middle and the highest TFFs are adopted for such a determination.

| Parameter | $a$ | $b$ |
|-----------|-----|-----|
| $f_{B_s \to D_s}^T(0)$ | 0.639 | 1.350 |
| $f_{B_s \to D_s}^T(0)$ | 0.583 | 1.345 |
| $f_{B_s \to D_s}^T(0)$ | 0.714 | 1.320 |

There are still two parameters to be fixed, the continuum threshold $s_0^{B_s}$ and the Borel window $M^2$. We set $s_0^{B_s} = 38 \pm 1 \text{ GeV}^2$ and $M^2 = (20 - 30) \text{ GeV}^2$ with the scale $\mu \simeq 3 \text{ GeV}$ which is close to $\sqrt{m_{B_s}^2 - m_b^2}$. Such a choice makes the TFF $f_{B_s \to D_s}^T(q^2)$ be stable within the allowable Borel window as can be seen from Fig. 6. In large recoil point $q^2 = 0$, we obtain

\[ f_{B_s \to D_s}^T(0) = 0.639_{-0.009}^{+0.056} f_{B_s} + 0.005 f_{D_s} + 0.014 f_{D_s}^{\phi}. \]

and in zero recoil region $q^2 = q_{\text{max}}^2$, we obtain

\[ f_{B_s \to D_s}^T(q_{\text{max}}^2) = 1.189 \pm 0.125, \]

where all the uncertainties have been added up in quadrature, and the errors from $\phi_{2,D_s}(x, \mu)$ and $f_{B_s}$ dominate the uncertainties. It agrees with the lattice QCD predictions within errors, $f_{B_s \to D_s}^T(0) = 0.656^{(31)}_{(26)}$ and $f_{B_s \to D_s}^T(0) = 0.666^{(12)}_{(27)}$.

Fig. 6 also shows that for larger $q^2$-values, the TFF will show sizable dependence on $M^2$, which agrees with the convention that the LCSR approach cannot be applied for very large $q^2$-value. We adopt the TFF $f_{B_s \to D_s}^T(q^2)$ within the region of $[0, 9 \text{ GeV}^2]$ as a basis to extrapolate it to all physical $q^2$-value. For the purpose, we adopt the double-pole-extrapolation method [60] to do the extrapolation, i.e.

\[ f_{B_s \to D_s}^T(q^2) = \frac{f_{B_s \to D_s}^T(0)}{1 - a(q^2/m_{B_s}^2) + b(q^2/m_{B_s}^2)^2}. \]

We put the fitted parameters in Table III.

![Graph showing the extrapolated LCSR prediction for the TFF $f_{B_s \to D_s}^T(q^2)$, where the lighter shaded band shows its uncertainty. The Lattice QCD prediction and its extrapolated results given in year 2017 [26] have also been presented as a comparison, the thicker shaded band shows its uncertainty.](image)
the lighter shaded band shows its theoretical uncertainty, in which the uncertainties from all the mentioned error sources, such as \( \phi_{2,D_s}(x, \mu) \), \( s_0^{B_s} \), \( f_{B_s} \), \( f_{D_s} \), \( m_b \) and etc., have been added up in quadrature. As a comparison, the Lattice QCD predictions for large \( q^2 \)-points and its extrapolation to all \( q^2 \)-region have also been presented and the thicker shaded band represents the errors [26]. Our results agree well with the Lattice QCD predictions, especially the arising trends over the changes of \( q^2 \).

### TABLE IV: A comparison of \(|V_{cb}|\) under various approaches and the experimental measured values.

| References                  | \(|V_{cb}| \times 10^{-3}\) |
|-----------------------------|-----------------------------|
| This work                   | 40.003 ± 1.075              |
| LHCb(CLN) [15]              | 41.4 ± 6.0 ± 0.9 ± 1.2      |
| LHCb(BGL) [15]              | 42.3 ± 6.0 ± 0.9 ± 1.2      |
| HPQCD [12]                  | 39.6 ± 1.7 ± 0.2            |
| PDG [55]                    | 41.0 ± 1.4                  |
| BaBar [63]                  | 38.36 ± 0.9                |
| BELLE(CLN+LQCD) [64]        | 38.4 ± 0.2 ± 0.6 ± 0.6      |
| BELLE(BGL+LQCD) [64]        | 38.3 ± 0.3 ± 0.7 ± 0.6      |
| LQCD [65]                   | 41.3 ± 2.2                 |

As applications, we adopt the LCSR prediction for the TFF to make a prediction on the CKM matrix element \(|V_{cb}|\) and the branching ratio \(B(B_s \to D_s \ell \bar{\nu}_\ell)\).

The TFF at the zero recoil point, \( f_{B_s \to D_s}^{B_s \to D_s}(q^2_{\text{max}}) \), is often quoted as

\[
G(1) = \frac{2\sqrt{m_B m_{D_s}}}{m_B + m_{D_s}} \times f_{B_s \to D_s}^{B_s \to D_s}(q^2_{\text{max}}).
\]

Using the averaged value given by the BaBar collaboration via the measurements on the semi-leptonic decay \( B \to D \ell \bar{\nu}_\ell \) [61, 62], \( \eta_{\text{ew}} G(1) |V_{cb}| = (42.65 \pm 1.53) \times 10^{-3} \), one obtain \(|V_{cb}| = (40.003 \pm 1.075) \times 10^{-3}\). In Table IV, we present a comparison of \(|V_{cb}|\) with the LHCb measured values under CLN and BGL approaches [15], the HPQCD prediction [12], the PDG averaged value [55], the BaBar measured value [63], the BELLE measured values under CLN+LQCD and BGL+LQCD approaches [64] and the Lattice QCD prediction [65].

We adopt the extrapolated TFF \( f_{B_s \to D_s}^{B_s \to D_s}(q^2) \) to calculate the branching ratio \(B(B_s \to D_s \ell \bar{\nu}_\ell)\), which can be derived by using the following formula

\[
B(B_s \to D_s \ell \bar{\nu}_\ell) = \tau_{B_s} \int_0^{(m_{B_s} - m_{D_s})^2} dq^2 \frac{d\Gamma(B_s \to D_s \ell \bar{\nu}_\ell)}{dq^2},
\]

where the differential decay width is

\[
\frac{d\Gamma(B_s \to D_s \ell \bar{\nu}_\ell)}{dq^2} = \frac{G_F^2 |V_{cb}|^2}{192\pi^3 m_{B_s}^5} \lambda^{3/2}(q^2) |f_{B_s \to D_s}^{B_s \to D_s}(q^2)|^2,
\]

where \( G_F = 1.1663787(6) \times 10^{-5}\) GeV\(^{-2}\), and the phase-space factor \( \lambda(q^2) = (m_{B_s}^2 + m_{D_s}^2 - q^2)^2 - 4m_{B_s}^2 m_{D_s}^2 \). We present the differential decay width \(1/|V_{cb}|^2 \times d\Gamma/dq^2\) with \( \ell = (e, \mu) \). As a comparison, we also present the Lattice QCD predictions in large \( q^2 \) points [26].

### IV. SUMMARY

In this work, we have made a detailed study on the \( D_s \)-meson leading-twist LCDA \( \phi_{2,D_s} \). Its moments have been calculated by using the QCD sum rules within the framework of BPT, and its first four moments have been given in Eqs.(35, 36, 37, 38), which then result in the Gegenbauer moments \( a_1^D(2\text{GeV}) = \ldots \), \( a_2^D(2\text{GeV}) = \ldots \), \( a_3^D(2\text{GeV}) = \ldots \), \( a_4^D(2\text{GeV}) = \ldots \) and etc., as applications, we adopt the LCSR prediction for the TFF to make a prediction on the CKM matrix element \(|V_{cb}|\) and the branching ratio \(B(B_s \to D_s \ell \bar{\nu}_\ell)\), whose behavior is constrained by the normalization condition, the probability of finding the leading Fock-state \(|\bar{c}s\rangle\) in \( D_s \)-meson Fock-state expansion, and the known Gegenbauer moments. As the key input for studying the high-energy processes involving \( D_s \)-meson, our suggested \( \phi_{2,D_s} \) shall be of great importance.

Using the present model of \( \phi_{2,D_s} \), we calculate the \( B_s \to D_s \) TFF \( f_{B_s \to D_s}^{B_s \to D_s}(q^2) \) within the QCD LCSR approach by adopting a chiral current correlator, in which the leading-twist terms dominant over the LCSR. At the large recoil region, we obtain \( f_{B_s \to D_s}^{B_s \to D_s}(0) = 0.639^{+0.046}_{-0.055} \). By using the extrapolated TFF with the double-pole-extrapolation method, we obtain \( B(B_s \to D_s \ell \bar{\nu}_\ell) = \ldots \) and the CKM element \(|V_{cb}| = (40.00 \pm 4.92) \times 10^{-3}\), which is consistent with the various measurements within reasonable errors.
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Appendix: Useful functions for calculating the moments of $\phi_{2,D_s}$

The functions $\mathcal{F}_{1,2}(n, a, b, l_{\text{min}}, l_{\text{max}})$, $\mathcal{G}_{1,2}(n, a)$, $\mathcal{H}(n, a, b, c)$ used in the sum rules Eqs. (18)-(23) are:

\[
\mathcal{F}_1(n, a, b, l_{\text{min}}, l_{\text{max}}) = \sum_{k=0}^{n} \frac{(-1)^k n! \Gamma(k+a)}{k!(n-k)!} \sum_{l=0}^{l_{\text{max}}} \frac{\Gamma(l+b) \Gamma(n-1-k+l)}{\Gamma(n-1+l+a)} \left( \frac{m_c^2}{M^2} \right)^{l-i} \times \frac{1}{l!(l-i)!(l-1-i+b)!} \left( \frac{m_c^2}{M^2} \right)^{l-i},
\]

\[
\mathcal{F}_2(n, a, b, l_{\text{min}}, l_{\text{max}}) = \sum_{k=0}^{n} \frac{(-1)^k n! \Gamma(k+a)}{k!(n-k)!} \sum_{l=0}^{l_{\text{max}}} \frac{\Gamma(l+b) \Gamma(n-k+l)}{\Gamma(n+l+a)} \times \frac{1}{l!(l-i)!(l-1-i+b)!} \left( \frac{m_c^2}{M^2} \right)^{l-i},
\]

\[
\mathcal{G}_1(n, a) = \sum_{k=0}^{n-2} \frac{(-1)^k n! \Gamma(k+a) \Gamma(n-1-k)}{k!(n-k)! \Gamma(n-1+a)},
\]

\[
\mathcal{G}_2(n, a) = \sum_{k=0}^{n-1} \frac{(-1)^k n! \Gamma(k+a) \Gamma(n-k)}{k!(n-k)! \Gamma(n+a)},
\]

\[
\mathcal{H}(n, a, b, c) = \int_0^1 dx (2x-1)^n x^a (1-x)^b \exp \left[ -\frac{m_c^2}{M^2(1-x)} \right] = \frac{1}{(c-1)! (M^2)^c} \int_0^1 dx (2x-1)^n x^a (1-x)^b \exp \left[ -\frac{m_c^2}{M^2(1-x)} \right].
\]

And the Borel transformation formulas are:

\[
\hat{B}_{M^2} \frac{1}{(-q^2 + m_c^2)k} \ln \frac{-q^2 + m_c^2}{\mu^2} = \frac{1}{(k-1)! M^{2k}} \frac{1}{\mu^2} e^{-m_c^2/M^2} \left[ \ln \frac{M^2}{\mu^2} + \psi(k) \right] \quad (k \geq 1),
\]

\[
\hat{B}_{M^2}(-q^2 + m_c^2)k \ln \frac{-q^2 + m_c^2}{\mu^2} = (-1)^{k+1} k! M^{2k} e^{-m_c^2/M^2} \quad (k \geq 0),
\]

\[
\hat{B}_{M^2} \frac{(-q^2)^l}{(-q^2 + m_c^2)^{l+\tau}} = \begin{cases} \sum_{i=0}^{l-1} \frac{l!}{l!(l-i)!(l-1-i)!} \left( \frac{m_c^2}{M^2} \right)^{l-i} e^{-m_c^2/M^2}, & \tau = 0, l > 0; \\ \sum_{i=0}^{l-1} \frac{l!}{l!(l-i)!(l+\tau-i-1)!} \left( -\frac{m_c^2}{M^2} \right)^{l-i} \frac{1}{M^{2\tau}} e^{-m_c^2/M^2}, & \tau > 0, l \geq 0. \end{cases}
\]
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