Critical and excess current through an open quantum dot: Temperature and magnetic field dependence

H. Ingerslev Jørgensen,† K. Grove-Rasmussen,‡ K. Flensberg,† and P. E. Lindeløf†

†Nano-Science Center, Niels Bohr Institute, University of Copenhagen, Universitetsparken 5, DK-2100 Copenhagen Ø, Denmark

We present measurements of temperature and magnetic field dependence of the critical current and excess current in a carbon nanotube Josephson quantum dot junction. The junction is fabricated in a controlled environment which allows for extraction of the full critical current. The measurements are performed in the open quantum dot regime, and fitted to theory with good qualitative agreement. We also show how to extract level spacing, level broadening, and charging energy of an open quantum dot from a bias spectroscopy plot.

PACS numbers: 74.45.+c, 73.23.Ad, 73.63.Fg, 74.50.+r

Nanoscale Josephson quantum dot junctions are intriguing devices showing several interesting physical phenomena. Supercurrent, Andreev reflections, quasiparticle transport, and excess current have all been studied in junctions where a nanotube or nanowire constitute the quantum dot [1, 2, 3, 4, 5, 6]. Furthermore, the interplay between these Josephson junction related phenomena and correlations as the Kondo effect [8, 9, 10, 11, 12] and the 0-π transition for more weakly coupled junctions has been explored [13, 14, 15].

In this paper, we present experimental results in the strongly coupled regime for a Josephson quantum dot junction realized in a carbon-nanotube. Inspired by Ref. [16], we utilize a designed external circuit in order to control the phase fluctuations which enables us to infer the true magnitude of the critical current, $I_C$, from the measurable critical current/switching current, $I_m$, by a fitting procedure [13]. $I_m$ can significantly differ from $I_C$ as demonstrated previously for nanotube-based Josephson junctions [3, 4, 13]. Here we analyze the magnetic field dependence and temperature dependence of both the critical current and excess current.

The devices are fabricated on a degenerately doped silicon wafer with a 0.5 μm layer of SiO$_2$. Carbon nanotubes are grown from islands of catalyst material and contacted by small electrodes of superconducting trilayers of 5nm Ti, 60nm Al and 5nm Ti. The superconducting electrodes are kept small to reduce junction capacitance. Each superconducting electrode is contacted by two normal metal leads to bonding pads which enables four probe measurements. The measurements are performed in a $^3$He-$^4$He dilution fridge with a base electron temperature of 75 mK. Inside the dashed square in Fig. 1(a) we show a schematic circuit diagram of the on-chip components of the full Josephson junction. The fabrication is similar to Ref. [13]. The superconductor-nanotube-superconductor junction is represented by a Josephson element (cross), a junction resistor $R_J$, and a junction capacitor $C_J$. The Josephson element has a current-phase relation, which we in the fitting procedure (see below) assume to be $I(\phi) = I_C \sin(\phi)$, with $\phi$ being the phase difference between the two superconducting and $I_C$ the critical current. However, the sinusoidal form of this relation is not in general true and this may cause some inaccuracy in the determination of the critical current. In Ref. [13] we show that the difference between the extracted $I_C$ using either $\sin(\phi)$ or the correct functional form of the current-phase relation is in fact small and moreover largest near...

![FIG. 1: (color online) (a) Inside the dashed square: Schematic circuit of on-chip components of the Josephson junction. Outside dashed square: Four probe voltage bias setup for measuring junction voltage $V_J$ vs. current $I$. The full Josephson junction consists of both the superconductor-nanotube-superconductor junction, represented by a Josephson element $I_C$ in parallel with a junction capacitor $C_J$ and junction resistor $R_J$, and on-chip resistors $R$ and capacitances $C$. (b) Bias spectroscopy plot of differential conductance versus source-drain, and gate voltage. (c) Schematic of a Fabry-Perot diamond. (d-f) Schematic transport diagrams at zero-bias resonance (d), positive-bias resonance (e), and negative-bias resonance (f).]
the 0-π transition relevant only for closed dots. We therefore expect the simpler relation also to be a reasonable approximation in the case of open dots (which allows us to use the Ivanchenko-Zil’berman relation in Eq. [6]). At sub-gap bias voltages $R_J$ accounts for current due to multiple Andreev reflections and at higher bias voltages it accounts for quasi-particle transport. The capacitance between the superconducting electrodes ($C_J \sim 5 \text{fF}$), and between bonding pads ($C \sim 1 \text{pF}$) is estimated as a parallel plate capacitance through the back gate. We have fabricated long thin metal leads with a measured resistance of $R \sim 1 \text{kΩ}$ between the bonding pads and the superconducting electrodes. The separation between these two rides is $\sim J_{\text{sd}}$ and capacitances in gate ($C_g$) and bias ($C_{\text{sd}}$) (see Fig. 3) can be set up the following two equations:

$$\frac{e}{C_{\text{sd}}} W_g = \Gamma + N' U_C,$$  \hspace{1cm} (5)

$$\frac{e}{C_{\text{sd}}} W_{\text{sd}} \approx 2 \Gamma,$$  \hspace{1cm} (6)

where the second equation is a good approximation when the asymmetry of the capacitive or tunnel coupling is not too large (see appendix). $N'$ is the number of electrons added to the dot between $V_{\text{gate}} = \pm W_g/2$ from resonance. We estimate $N'$ by integrating a Lorentzian density of state for each energy level on the dot:

$$N' = \int_{-W_g/2}^{W_g/2} \sum_j \frac{4}{\pi} \frac{1}{(e + j \Delta V_g)^2 + (\frac{1}{2} W_g)^2} \text{de}$$  \hspace{1cm} (7)

where the sum should include an appropriate number of energy levels. If only one energy level is included ($j = 0$) $N' \sim 2$, but for increasing number energy levels included $N'$ saturates at a higher number (since the tails of the other levels contribute). For the device analyzed in paper it saturates at $N' \sim 2.5$.

By solving the equations above we can find expressions for the following parameters:

$$\Delta E = e \Delta V_{\text{sd}},$$

$$U_C = \frac{W_C \Delta E - e \Delta V_g W_{\text{sd}} \frac{1}{2}}{e C_{\text{sd}}} \sim 0.5 \text{meV},$$

$$C_g = \frac{\Delta E}{(e C_{\text{sd}} + 4) \Delta V_g} \sim 4.3 \text{aF},$$

$$C_s = \frac{e C_J \Delta V_{\text{sd}} - 4 - 4 \Delta E}{e C_{\text{sd}} \Delta V_g} \sim 152 \text{aF},$$

$$C_d = \frac{C_s - C_J - C_g}{U_C} \sim 158 \text{aF},$$

$C = C_s + C_d + C_g \sim 315 \text{aF},$ 

$\Gamma = \frac{W_{\text{sd}}}{2} \epsilon \sim 4.3 \text{meV},$ 

$\Gamma_s = \frac{\Gamma_{\alpha+1}}{(\alpha + 1)^{\alpha+1}} \sim 1 \text{meV},$ 

$\Gamma_d = \frac{\Gamma_{\alpha+1}}{(\alpha + 1)^{\alpha+1}} \sim 3.3 \text{meV},$

We have in the right hand column estimated the parameters for the device analyzed in this paper [22]. Note that $\Delta E > U_C > \Delta_0$, where $\Delta_0 = 0.11 \text{meV}$ is the superconducting energy gap (see below).

We now return to the measurements shown in Fig. 3(b), where two parallel conductance ridges are observed at low bias due to the density of states in the superconducting electrodes. The separation between these two ridges is $\Delta_0/e$, yielding $\Delta_0 \sim 0.11 \text{meV}$. In the following we focus on measurements performed on and off zero-bias resonance at the two indicated positions in Fig. 3(b). Current versus junction voltage ($IV_J$ curves) off resonance for large scale voltages is shown in Fig. 2(a), where the black curve is with superconducting electrodes and the red curve is with a small magnetic field ($150 \text{mT}$) to suppress the superconductivity. At high bias $V_{\text{sd}} > 2\Delta_0/e$ transport is governed by quasiparticle transport and one Andreev reflection processes yielding an excess current, while at sub-gap bias $V_{\text{sd}} < 2\Delta_0/e$ transport are governed by Andreev reflections and supercurrent [4]. A
switching and hysteresis are observed whenever the full state, and red curve with a small magnetic field (150 mT) applied to suppress the superconductivity. (b) Close-up of the supercurrent branch from (a), measured with a voltage bias setup (circles) and a current bias setup (triangles). (c-d) Dependence of the diffusive supercurrent branch on temperature and magnetic field (d).

close-up at very low bias voltages, shown in Fig. 2(b), reveals a pronounced supercurrent branch with finite resistance, a so-called diffusive supercurrent branch (c). The black circles are measured with a voltage bias setup as shown in Fig. 1(a), while the green triangles are measured with a current bias setup (sweeping from negative to positive current). For voltage bias measurements we have observed no hysteresis or switching in the IVJ curves at any gate voltages. But for current bias measurements switching and hysteresis are observed whenever the full IVJ-curve has local minima and maxima, as observed in Fig. 2(b). Such local minima and maxima will for current bias measurements lead to switching in voltage and result in a hysteretic IVJ-curve. To resolve the full IVJ-curve we have therefore used voltage bias measurements in this paper.

In Fig. 2(c) and (d) we show the temperature and magnetic field dependence of the diffusive supercurrent branch, which we will analyze in the following. The zero bias slope of the diffusive supercurrent branch in Fig. 2 yields a resistance of the order kilo ohm. For a Josephson quantum dot junction with only two channels as for a nanotube the Josephson energy $E_J = \hbar I_C/2e$ can be comparable to the temperature of the cryostat. Thermal fluctuations will therefore lead to fluctuations in the phase difference across the junction, and consequently give a supercurrent branch with finite resistance. In order to dampen these phase fluctuations and thereby increase the size of the supercurrent branch, we have designed the environment of the superconductor-nanotube-superconductor junction as described in Ref. [13]. The quality factor for the junction is $Q < 0.5$, i.e., strongly damped. The full IVJ-curve for a damped Josephson junction including the external components (without $R_J$) was calculated by Ivanenko and Zil’berman[18] and used with great success by Steinbach et. al. [10]. Since this device has considerable current contribution from multiple Andreev reflections at sub-gap bias voltage we have to a rough approximation included a constant resistor $R_J$. The full IVJ-curve can then be calculated as

$$I(V_{sd}) = I_C J_m \left( \frac{\text{Im}(E_J/k_BT)}{\text{Im}(E_J/k_BT)} \right) + \frac{V_J}{R_J}$$

where $I_n(x)$ is the modified Bessel function of complex order, and $\eta = (hV_{sd})/(2eRk_BT)$. To plot $I(V_{sd})$ versus $V_J$ instead of $V_{sd}$ we can use that $V_J = V_{sd} - RI(V_{sd})$. There are two fitting parameters in this theory, the temperature dependent critical current $I_C(T)$ and $R_J$. In Fig. 3 we show three I versus $V_J$ curves measured at the same gate voltage for increasing temperatures. From left to right: 75 mK, 150 mK, and 300 mK. The black circles are the measurement and the solid red curve is theoretical fit with Eq. (9). The three fits are made with $R_J = 7.7k\Omega$, and the temperature at which it is measured, the only free fitting parameter is $I_C(T)$ yielding 4.8, 4.8, and 4.6 nA respectively. Eq. (9) fits the measured IVJ curves very well for all temperatures with $I_C$ as the only fitting parameter. Above ~300 mK smaller and smaller critical currents are needed to make a good fit. Critical currents versus temperature found by these fits are plotted in Fig. 4(a). At temperatures lower than ~300 mK the current critical is saturated at ~5 nA, while at higher temperatures it decreases more rapid than a BCS-gap dependence. In Fig. 4(a) we also plot the excess current versus temperature, measured at $V_{sd} = 4\Delta_0/e$. We compare the measurement with theory for a superconducting quantum point contact $I_{pk}[19,20,21]$. We use Eq. 1 and 2 in Ref. [14] with $\Delta = \Delta(T)$ having a BCS temperature dependence to fit the measured temperature dependence of the critical and excess current, solid red and blue curve.

FIG. 2: (color online) Current versus junction voltage on (a-b) and off (c-d) resonance at positions indicated in Fig. 1(b). (a) Black curve is with the electrodes in the superconducting state, and red curve with a small magnetic field (150 mT) applied to suppress the superconductivity. (b) Close-up of the supercurrent branch from (a), measured with a voltage bias setup (circles) and a current bias setup (triangles). (c-d) Dependence of the diffusive supercurrent branch on temperature (c) and magnetic field (d).

FIG. 3: (color online) Current versus junction voltage on resonance at position indicated in Fig. 1(b) for three different temperatures. From left to right: 75 mK, 150 mK, and 300 mK. Four probe voltage bias measurement (circles), and fit (solid red line) using Eq. (9) with $R_J = 7.7k\Omega$, $R = 1k\Omega$, the temperature at which the curve is measured, and $I_C = 4.8\text{nA}$, 4.8 nA, and 4.6 nA from left to right.

FIG. 4: (color online) Current versus junction voltage on resonance at position indicated in Fig. 1(b) for three different temperatures. From left to right: 75 mK, 150 mK, and 300 mK. The black circles are the measurement and the solid red curve is theoretical fit with Eq. (9). The three fits are made with $R_J = 7.7k\Omega$, and the temperature at which it is measured, the only free fitting parameter is $I_C(T)$ yielding 4.8, 4.8, and 4.6 nA respectively. Eq. (9) fits the measured IVJ curves very well for all temperatures with $I_C$ as the only fitting parameter. Above ~300 mK smaller and smaller critical currents are needed to make a good fit. Critical currents versus temperature found by these fits are plotted in Fig. 4(a). At temperatures lower than ~300 mK the current critical is saturated at ~5 nA, while at higher temperatures it decreases more rapid than a BCS-gap dependence. In Fig. 4(a) we also plot the excess current versus temperature, measured at $V_{sd} = 4\Delta_0/e$. We compare the measurement with theory for a superconducting quantum point contact $I_{pk}[19,20,21]$. We use Eq. 1 and 2 in Ref. [14] with $\Delta = \Delta(T)$ having a BCS temperature dependence to fit the measured temperature dependence of the critical and excess current, solid red and blue curve.
FIG. 4: (color online) (a) Temperature dependence of the measured critical current (squares) and excess current (diamonds) on resonance. (b) Magnetic field dependence of the measured critical current on and off resonance (squares and circles), and excess current on resonance (diamonds). The normal state zero-bias conductance is $2.6e^2/h$ on resonance and $1.4e^2/h$ off resonance. The solid lines in both (a) and (b) are the predicted curves for a superconducting quantum point contact multiplied by a constant factor of 0.25 for the critical current and 0.7 for the excess current. Insert shows the magnetic field dependence of the sub gap structure of a similar device in the Coulomb blockade regime.

in Fig.4(a). The magnitude of the measured critical and excess current is 0.25 and 0.7 lower than the theory predicts, while their qualitative dependence on temperature fits well with theory.

In Fig.4(b) we plot the magnetic field dependence of the critical current on and off resonance (see arrows in Fig.4(b)), and excess current on resonance. The critical currents are found by the same method as above by fitting Eq. (9) to each measured $IV_J$ curve in Fig.2(d). We compare the measurement to the same theory as above, but with $\Delta = \Delta(B) = (1 - B/B_C)\Delta_0$, where $B_C \sim 90\text{mT}$ is the critical field. We use a linear dependence because, as shown in the insert of Fig.4(b), the sub-gap structure has approximately a linear dependence on magnetic field. The theory seems to fit qualitatively well to the measurement. But the magnitude of the measured critical and excess current is, as above for the temperature dependence, 0.25 and 0.7 lower than theory.

The dot is in the open regime with a charging energy of $U_C \sim 0.5\text{meV}$ as discussed in the beginning of the paper, which is several times larger than the superconducting energy gap ($\Delta_0 \sim 0.11\text{meV}$). We speculate that the discrepancy of the factor 0.25 between the measured critical current and theory could be due to the charging energy being larger than the gap thus suppressing the Cooper pair transport. The 0.7 discrepancy for the excess current has been seen before [4], but we have no good explanation for that.

FIG. 5: (color online) (a) Differential conductance versus bias and gate voltage using Eq. (17) and (18). (b) and (c) Differential conductance versus gate voltage at $V_{sd} = 0\text{mV}$ (b), and versus bias voltage at resonance (c). Red squares are experimental data (measured with $B = 150\text{mT}$), Solid black line is a Lorentzian fit to the measurement yielding $W_g = 0.4\text{V}$ and $W_{sd} = 8.5\text{mV}$, and blue circles are numerical theory extracted from (a).
MEAN FIELD DESCRIPTION OF FABRY-PEROT RESONANCES IN A NANOTUBE QUANTUM DOT.

The electronic states in the nanotube can be described by

\[ H_{\text{cnt}} = \sum_{m\eta\sigma} \Delta E n_{m\eta\sigma} + \frac{1}{2} U_C \tilde{N}^2 - e V_{\text{eff}} \tilde{N} \]  

(10)

\[ V_{\text{eff}} = \sum_{\beta=g,s,d} \frac{V_{\beta} C_{\beta}}{C} \]  

(11)

where

\[ \tilde{N} = \sum_{m\eta\sigma} n_{m\eta\sigma}, \quad U_C = \frac{e^2}{C} \]  

(12)

and \( \Delta E \) is the level spacing. The quantum numbers \( m, \sigma, \eta \) describe the orbital, spin and pseudospin degrees of freedom, respectively. The subscripts \( g, s, \) and \( d \) refer to gate, source and drain. In the experiment we apply asymmetric bias, i.e., \( V_s = V_d \) and \( V_d = 0 \). In the mean-field approximation (which is valid when \( \Gamma \gg U_C \)), the Hamiltonian is

\[ H_{\text{cnt}} \approx \sum_{m\eta\sigma} \Delta E n_{m\eta\sigma} + U_C \langle \tilde{N} \rangle - e V_{\text{eff}} \tilde{N}, \]  

(13)

where the total occupation \( \langle \tilde{N} \rangle \) should be determined self-consistently

\[ \langle \tilde{N} \rangle = \sum_{m\eta\sigma} \langle n_{m\eta\sigma} \rangle, \]  

(14)

with

\[ \langle n_{m\eta\sigma} \rangle = \sum_{\alpha=s,d} \frac{\Gamma_{\alpha}}{\Gamma} \int \frac{d\omega}{2\pi} n_F(\omega + e V_{\alpha}) A_{m\eta\sigma}(\omega). \]  

(15)

Assuming all levels to be simple Lorentzians with equal widths, the spectral functions are

\[ A_{m\eta\sigma}(\omega) = \frac{\Gamma}{(\omega - m \Delta E - U_C \langle \tilde{N} \rangle + e V_{\text{eff}})^2 + (\Gamma/2)^2}. \]  

(16)

Inserting this into the integral, summing over quantum numbers, and setting \( T = 0 \), then gives the self-consistency equation

\[ \langle \tilde{N} \rangle = \sum_{m} \sum_{\alpha=s,d} 4 \frac{\Gamma_{\alpha}}{\Gamma} \left[ \frac{1}{2} - \frac{1}{\pi} \tan^{-1} \left( \frac{m \Delta E + e V_{\alpha} + U_C \langle \tilde{N} \rangle - e V_{\text{eff}}}{\Gamma/2} \right) \right]. \]  

(17)

This equation can be solved numerically. Once we know the total occupation for given gate, source and drain voltages, the current is given by

\[ I = \frac{e}{h} \frac{4 \Gamma_s \Gamma_d}{\pi \Gamma} \sum_{m\eta\sigma} \int \frac{d\omega}{2\pi} \left[ n_F(\omega + e V_s) - n_F(\omega + e V_d) \right] A_{m\eta\sigma}(\omega) \]

\[ = \frac{4e}{h} \frac{4 \Gamma_s \Gamma_d}{\pi \Gamma} \sum_{m} \left[ \tan^{-1} \left( \frac{m \Delta E + e V_d + U_C \langle \tilde{N} \rangle - e V_{\text{eff}}}{\Gamma/2} \right) \right] - \tan^{-1} \left( \frac{m \Delta E + e V_s + U_C \langle \tilde{N} \rangle - e V_{\text{eff}}}{\Gamma/2} \right). \]  

(18)

In Fig. 5(a) we plot the differential conductance versus bias and gate voltage using Eq. (17) and (18) with the parameters found in Eq. (8). We compare the theory with experimental data measured with \( B = 150 \) mT in Fig. 5(b) and (c). In (b) we make a gate trace at zero bias and in (b) we make a bias trace at the resonance indicated in Fig. 1(a).
[5] A. Y. Kasumov, R. Deblock, M. Kociak, B. Reulet, H. Bouchiat, I. I. Khodos, Y. B. Gorbatov, V. T. Volkov, C. Journet, and M. Burghard, Science 284, 1508 (1999).

[6] E. Pallecchi, M. Gaaß, D. A. Ryndyk, and C. Strunk, Applied Physics Letters 93, 072501 (2008), 0804.0168.

[7] Y. Zhang, G. Liu, and C. N. Lau, Nano Res. 1, 145 (2008).

[8] A. Eichler, R. Deblock, M. Weiss, C. Karrasch, V. Meden, C. Schönenberger, and H. Bouchiat, ArXiv e-prints (2008), 0810.1671.

[9] M. R. Buitelaar, W. Belzig, T. Nussbaumer, B. Babić, C. Bruder, and C. Schönenberger, Physical Review Letters 91, 057005 (2003).

[10] M. R. Buitelaar, T. Nussbaumer, and C. Schönenberger, Physical Review Letters 89, 256801 (2002).

[11] K. Grove-Rasmussen, H. Ingerslev Jørgensen, and P. E. Lindelof, New Journal of Physics 9, 124 (2007).

[12] T. Sand-Jespersen, J. Paaske, B. M. Andersen, K. Grove-Rasmussen, H. I. Jørgensen, M. Angesen, C. B. Sørensen, P. E. Lindelof, K. Flensberg, and J. Nygård, Physical Review Letters 99, 126603 (2007).

[13] H. I. Jørgensen, T. Novotný, K. Grove-Rasmussen, K. Flensberg, and P. E. Lindelof, Nanoletters 7, 2441 (2007).

[14] J. A. van Dam, Y. V. Nazarov, E. P. A. M. Bakker, S. D. Franceschi, and L. P. Kouwenhoven, Nature 442, 667 (2006).

[15] J.-P. Cleuziou, W. Wernsdorfer, V. Bouchiat, T. On-darcu hu, and M. Monthioux, Nature Nanotechnology 1, 53 (2006).

[16] A. Steinbach, P. Joyez, A. Cottet, D. Esteve, M. H. Devoret, M. E. Huber, and J. M. Martinis, Phys. Rev. Lett. 87, 137003 (2001).

[17] W. Liang, M. Bockarth, D. Bozovic, J. H. Hafner, M. Tin-kham, and H. Park, Nature 411, 665 (2001).

[18] Y. M. Ivanchenko and L. A. Zil’berman, Sov. Phys. JETP 28, 1272 (1969).

[19] A. Martín-Rodero, A. Levy Yeyati, and J. C. Cuevas, Superlattices and Microstructures 25, 925 (1999).

[20] V. S. Shumeiko, E. N. Bratus, and G. Wendin, Low Temp. Phys. 23, 181 (1997).

[21] J. C. Cuevas, A. Martín-Rodero, and A. L. Yeyati, Phys. Rev. B 54, 7366 (1996).

[22] $\Delta V_g = 0.8 \text{ V}, \Delta V_{g1} = 0.45 \text{ V}, \Delta V_{g2} = 0.35 \text{ V}, \Delta V_{sd} = 9 \text{ mV}, W_g = 0.4 \text{ V}, W_{sd} = 8.5 \text{ mV}$