Neutrinoless quadruple beta decay

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Abstract – We point out that lepton number violation is possible even if neutrinos are Dirac particles. We illustrate this by constructing a simple model that allows for lepton number violation by four units only. As a consequence, neutrinoless double beta decay is forbidden, but neutrinoless quadruple beta decay is possible: (A, Z) → (A, Z + 4) + 4e−. We identify three candidate isotopes for this decay, the most promising one being 150Nd due to its high Q_{0ν4β}-value of 2 MeV. Analogous processes, such as neutrinoless quadruple electron capture, are also possible. The expected lifetimes are extremely long, and experimental searches are challenging.

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Introduction. – Of all the open questions concerning neutrinos — mass scale and hierarchy, possible CP violation, origin of the mixing pattern — the conceptually most interesting has to be its very nature: is the neutrino its own antiparticle, and hence a Majorana fermion, or do neutrino and antineutrino differ, making the neutrino a Dirac particle like all the other fermions of the Standard Model (SM)? The key observation here would be neutrinoless double beta decay (0ν2β) [1], (A, Z) → (A, Z + 2) + 2e−, because the observation of this ΔL = 2 process unambiguously confirms the Majorana nature of neutrinos [2]. Other ΔL = 2 signatures, ranging from low-energy processes like neutrinoless double electron capture to collider processes, have also been proposed and tested (a collection of references can be found in refs. [1,3]), but all experiments came up empty so far.

The necessary lepton number violation (LNV) by two units, ΔL = 2, can be realized directly with a tree level Majorana mass term, or indirectly via diagrams containing two vertices with ΔL = 1, one example being R-parity-violating supersymmetry [4]. However, it is most often overlooked that LNV and Majorana neutrinos are not necessarily connected. For instance, there are non-perturbative processes in the SM that violate the lepton (and baryon) number by three units [5], ΔL = ΔB = 3, which obviously do not lead to Majorana neutrinos, and are in fact perfectly compatible with Dirac neutrinos.

In this letter we will entertain the possibility that LNV

occurs only by four units, and that ΔL = 2 processes are forbidden; neutrinos are then Dirac particles. We will realize those lepton-number-violating Dirac neutrinos in a simple model based on a spontaneously broken U(1)_{B-L}. As an interesting consequence, neutrinoless quadruple beta decay (0ν4β),

(A, Z) → (A, Z + 4) + 4e−, (1)

is allowed. This novel nuclear decay process plays for our framework the role that neutrinoless double beta decay plays for Majorana neutrinos: it will be the dominant possible LNV process for Dirac neutrinos, which is surely of great conceptual interest even if the decay rates that we estimate are tiny1.

We will first construct a simple toy model that forbids Majorana neutrinos but allows for LNV by four units. Then we will search for interesting isotopes that can undergo 0ν4β and estimate the expected lifetimes. Interestingly, all three isotopes that we identify as potential 0ν4β-emitters (96Zr, 136Xe, and 150Nd) are familiar from searches for neutrinoless double beta decay. The most interesting candidate is 150Nd, with a Q_{0ν4β}-value of 2.079 MeV. We also identify four candidates for neutrinoless quadruple electron capture and related processes (124Xe, 130Ba, 148Gd, and 154Dy). More detailed studies

1One might think that 0ν3β should be the next probable neutrinoless beta decay after 0ν2β. This process would however violate Lorentz symmetry, similarly to neutrinoless single beta decay n → p + e−.
regarding model-building aspects, collider phenomenology and cosmological aspects will be presented elsewhere.

**Simple model for \( \Delta(B - L) = 4 \).** We introduce three right-handed neutrinos \( \nu_R \) (RHNs) to the SM, which results in Dirac masses for the neutrinos after spontaneous electroweak symmetry breaking. A striking feature of the chiral fermion content of the SM + \( \nu_R \) is the existence of a new, accidental, anomaly-free symmetry \( U(1)_{B - L} \), which can therefore be consistently gauged in addition to the SM gauge group. Breaking \( B - L \) by a scalar \( \phi \) with charge \( |B - L| = 4 \) can then lead to a remaining discrete symmetry group \( \mathbb{Z}_4^L \) in the lepton sector, which protects the Dirac structure of neutrinos and still allows for LNV processes. Quartic LNV operators for Dirac neutrinos were also mentioned in a study of anomaly-free discrete \( R \)-symmetries in ref. [6].

For a simple realization of this idea, we work with a gauged \( U(1)_{B - L} \) symmetry, three RHNs \( \nu_R \approx -1 \), one scalar \( \phi \approx 4 \), and one scalar \( \chi \approx -2 \), all of which are SM singlets. The Lagrangian takes the form

\[
\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{kinetic}}(\nu_R, \phi, \chi) + \mathcal{L}_{Z'} - V(H, \phi, \chi)
\]

\[
+ \left( y_{\alpha\beta} L^a H \nu_{R,\alpha} + \kappa_{\alpha\beta} \chi \overline{\nu}_{R,\alpha} \nu_{R,\beta} + \text{h.c.} \right), \tag{2}
\]

\( H \) being the SM Higgs doublet. The phenomenology of the accompanying \( Z' \) boson, described in \( \mathcal{L}_{Z'} \), is not important here. Working in the diagonal charged-lepton basis, the neutrinos obtain the Dirac mass matrix \( M_{\alpha\beta} \equiv \langle H \rangle y_{\alpha\beta} \) upon electroweak symmetry breaking. A unitary transformation can be used to diagonalize this mass matrix via \( U^\dagger M V = \text{diag}(m_1, m_2, m_3) \), where \( U \) is the lepton mixing matrix relevant for electroweak charged-current interactions. Contrary to other models with Dirac neutrinos, the right-handed transformation \( V \) does not drop out, but can be absorbed by the complex symmetric Yukawa coupling matrix \( \kappa_{\alpha\beta} = \kappa_{\beta\alpha} \), which is non-diagonal in general.

The scalar potential of our model is of the simple form

\[
V(H, \phi, \chi) = \sum_{\chi = H, \phi, \chi} \left( \mu_\chi^2 |X|^2 + \lambda_\chi |X|^4 \right)
\]

\[
+ \lambda_H |H|^2 |\phi|^2 + \lambda_H \lambda_\chi |H|^2 |\chi|^2 + \lambda_\chi |\chi|^2 |\phi|^2
\]

\[
- (\mu \phi^2 + \text{h.c.}) \tag{3}
\]

Here, the coefficients \( \mu_\chi, \lambda_\chi \) and \( \lambda_H \) have mass dimension one and zero, respectively. Assuming \( \mu_H^2, \mu_\chi^2 < 0 < \mu_\chi^2 \) and appropriate signs and magnitudes of the \( \lambda_i \), we can biunitarily construct a potential that is bounded from below and breaks \( SU(2)_L \times U(1)_Y \times U(1)_{B - L} \) to \( U(1)_{\text{EM}} \times \mathbb{Z}_4^L \). In order to forbid Majorana neutrinos, it is imperative that \( \chi \) does not acquire a vacuum expectation value; without the last line in eq. (3), the necessary condition for this would be

\[
m_\chi^2 \equiv \mu_\chi^2 + \lambda_H |H|^2 + \lambda_\chi |\phi|^2 > 0, \tag{4}
\]

but the \( \mu \)-term modifies this condition. To see how, let us first note that we can choose \( \mu \) and \( \langle \phi \rangle \) real and positive written in full by us, using phase and \( B - L \) gauge transformations. The \( \mu \)-term will then induce a mass splitting between the properly normalized real (pseudo)scalar fields Re \( (\chi) \) and Im \( (\chi) \),

\[
m_{\text{Re}(\chi)}^2 = m_\chi^2 - 2 \mu (\phi), \quad m_{\text{Im}(\chi)}^2 = m_\chi^2 + 2 \mu (\phi), \tag{5}
\]

so the condition \( (\chi) = 0 \) becomes equivalent to \( m_{\text{Re}(\chi)}^2 > 0 \), which can be easily satisfied.

Neutrinos are hence Dirac particles, but we also obtain effective \( \Delta L = 4 \) four-neutrino operators by integrating out \( \chi \) at energies \( E \ll m_{\text{Re}(\chi)}, m_{\text{Im}(\chi)} \):

\[
\Delta L = 4 \rightarrow \frac{1}{2} \left( m_{\text{Im}(\chi)}^2 - m_{\text{Re}(\chi)}^2 \right) \left( \kappa_{\alpha\beta} \overline{\nu}_{R,\alpha} \nu_{R,\beta} \right)^2 + \text{h.c.,} \tag{6}
\]

see fig. 1 for the relevant Feynman diagrams. For simplicity, we will assume physics at the TeV scale as the source of our four-neutrino operators throughout this paper; a discussion of more constrained light mediators, as well as of other and more complicated models that generate effective four-neutrino operators with left-handed neutrinos, will be presented elsewhere. We note that our particular example uses a gauged \( B - L \) framework; in general, however, the observation and the model-building possibilities that might lead to lepton-number–violating Dirac neutrinos are much broader.

**Candidates for \( 0\nu 4\beta \).** – Our model from the last section gave us the effective dimension-six \( \Delta L = 4 \) operator \( \langle \nu_R \nu_R \rangle^2 \), which can lead to an interesting signature in beta decay measurements: four nucleons undergo beta decay, emitting four neutrinos; these four meet at the effective \( \Delta L = 4 \) vertex and remain virtual. We only see four electrons going out, so at parton level we have \( 4d \rightarrow 4u + 4e^- \), and on hadron level \( 4n \rightarrow 4p + 4e^- \) (fig. 2).

Obviously this neutrinoless quadruple beta decay (0\( \nu 4\beta \)) is highly unlikely —more so than 0\( \nu 2\beta \), as it is of fourth order—but one can still perform the exercise of identifying candidate isotopes for the decay and estimating the lifetime; constraining the lifetime experimentally is of course also possible. Besides 0\( \nu 4\beta \), one can imagine analogous processes such as neutrinoless double electron capture (0\( \nu 4\text{EC} \)), neutrinoless quadruple positron decay (0\( \nu 4\beta^+ \)), neutrinoless double electron capture double positron decay (0\( \nu 2\text{EC}2\beta^+ \)), etc. We will find potential candidates for 0\( \nu 4\beta \), 0\( \nu 2\text{EC}2\beta^+ \), 0\( \nu 3\text{EC}\beta^+ \), and 0\( \nu 4\text{EC} \).

We will now identify those candidate isotopes for \( \Delta L = 4 \) processes. We need to find isotopes which are
more stable after the flip \((A, Z) \rightarrow (A, Z \pm 4)\). Normal beta decay has to be forbidden in order to handle backgrounds and make the mother nucleus sufficiently stable. Using nuclear data charts [7], we found seven possible candidates: three for \(0\nu\beta\beta\), four for neutrinoless quadruple electron capture and related decays. They are listed in table 1, together with their \(Q\)-values, competing decay channels, and natural abundance. It should be obvious that not all \(0\nu\beta\beta\) candidates \((A, Z)\) make good \(0\nu\beta\beta\) candidates, as \((A, Z + 4)\) can have a larger mass than \((A, Z)\); it is less obvious that there exist no \(0\nu\beta\beta\) candidates with beta-unstable daughter nuclei. Using the semi-empirical Bethe-Weizsäcker mass formula, one can however show that

\[
\frac{M[A(Z - 2)] - M[A(Z + 2)]}{M[A(Z - 1)] - M[A(Z + 1)]} = 2, \tag{7}
\]

where \(M[AZ]\) denotes the mass of the neutral atom \(AZ\) in its ground state. Applied to our problem, this means that the mass splitting of the odd-odd states in fig. 3 (shown in red) is expected to be smaller than the mass splitting of the two \(\Delta Z = 4\) nuclei (which is just the \(Q\)-value, see below), which implies that beta-stable \(0\nu\beta\beta\) candidates will decay into beta-stable nuclei (this simple argument is confirmed with data charts [7]).

The \(Q\)-values in table 1 can be readily calculated in analogy to \(0\nu\beta\beta\). In general, the total kinetic energy of the emitted electrons/positrons in a \(0\nu\beta\beta\) decay,

\[
AZ \rightarrow A(Z \pm n) + ne^\mp, \tag{8}
\]

is given by the \(Q\)-value, and can be calculated via

\[
Q_{0\nu\beta\beta^-} = M[AZ] - M[A(Z + n)], \tag{9}
\]

\[
Q_{0\nu\beta\beta^+} = M[AZ] - M[A(Z - n)] - 2nm_e. \tag{10}
\]

The term \(-2nm_e\) in \(Q_{0\nu\beta\beta^+}\) already makes \(0\nu\beta\beta^+\) very rare, but neutrinoless quadruple positron decay \(0\nu\beta\beta^+\) impossible. Electron capture with the emission of up to two positrons is however permitted, as the \(Q\)-value for the EC process

\[
AZ + k e^- \rightarrow A(Z - n) + (n - k)e^+ \tag{11}
\]

is given by \(Q_{0\nu\beta\beta^+} = Q_{0\nu\beta\beta^-} + 2km_e\), allowing all for neutrinoless quadruple electron capture \(0\nu\beta\beta\) in four isotopes (table 1).

Having identified all \(\Delta L = 4\) candidates, we discuss their experimental prospects and challenges in more
detail: Let us first take a look at the most promising element for 0ν4β: 150Nd. The following decay channels are possible (see also fig. 3):

- $^{150}_{60}$Nd $\rightarrow$ $^{150}_{62}$Sm via 2ν2β, i.e., via the forbidden intermediate odd-odd state $^{150}_{61}$Pm. Two neutrinos and two electrons are emitted; the electrons hence have a continuous energy spectrum and total energy $E_{\nu,1} + E_{\nu,2} < 3.371$ MeV. This decay has been observed with a half-life of $7 \times 10^{18}$ y.

- $^{150}_{60}$Nd $\rightarrow$ $^{150}_{64}$Gd via 0ν4β. Four electrons with continuous energy spectrum and summed energy $Q_{0\nu4\beta} = 2.079$ MeV are emitted. In this special case, the daughter nucleus is α-unstable with half-life $t_{1/2}^{α}(^{150}_{64}$Gd $\rightarrow$ $^{146}_{62}$Sm) $\simeq 2 \times 10^8$ y.

A sketch of the summed electron energy spectrum is shown in fig. 4. $Q_{0\nu4\beta}$ will always sit somewhere in the middle of the continuous spectrum, so one would have to identify the four electrons in order to remove the 2ν2β background. This still leaves other backgrounds to be considered, e.g., the scattering of the two 2ν2β electrons off of atomic electrons, which can effectively lead to four emitted electrons (and two neutrinos). Since $Q_{0\nu4\beta} < Q_{2\nu2\beta}$, the sum of the electron energies will be continuously distributed and can overlap the discrete $Q_{0\nu4\beta}$ peak. A dedicated discussion of this and other possible backgrounds goes far beyond the scope of this letter.

As an alternative to direct searches, one could even omit an energy measurement and just look at the transmutation $^{150}_{60}$Nd $\rightarrow$ $^{150}_{62}$Gd using, e.g., chemical methods; as the background for $^{150}_{60}$Nd $\rightarrow$ $^{150}_{62}$Gd is basically non-existent —the SM-allowed 4ν4β is killed by the Q-dependence of the eight-particle phase space $G_{4\nu4\beta} \sim Q^{24}$ (compared to the four-particle phase space $G_{2\nu2\beta} \sim Q^{11}$), and 2ν2β would be seen long before we ever see the double 2ν2β that mimics 0ν4β. Hence, this transmutation suffices to test 0ν4β. In case of $^{150}_{60}$Nd, the instability of the daughter nucleus $^{150}_{62}$Gd can even be advantageous, as the resulting alpha particle provides an additional handle to look for the decay3. The necessary macroscopic number of daughter elements will of course result in weak limits compared to dedicated 0ν4β searches in 0ν2β experiments. However, for elements not under consideration in 0ν2β experiments, this could be a viable and inexpensive way to test 0ν4β.

There is also the possibility of decay into an excited state, $^{150}_{60}$Nd $\rightarrow$ $^{150}_{64}$Gd* via 0ν4β. The excited final state will reduce the effective Q-value —by 0.638 MeV (1.207 MeV) for the lowest 2+ (0+) state—and produce detectable photons.

2We note that if neutrinos are Majorana particles, the decay $^{150}_{60}$Nd $\rightarrow$ $^{150}_{62}$Sm via 0ν2β is possible. Two mono-energetic electrons would be emitted with total energy $Q_{0\nu2\beta} = 3.371$ MeV.

The alpha decay is however too slow to be used in coincidence with 0ν4β.

All the above holds similarly for $^{96}_{46}$Zr and $^{136}_{54}$Xe as well. Both have much smaller Q-values—which theoretically reduces the rate—but a-stable daughter nuclei. The non-solid structure of xenon makes it in principle easier to check for the transmutation into cerium; furthermore, the EXO [8] 0ν2β experiment is currently running and could check for 0ν4β, should their detector be sensitive at these energies and not flooded by backgrounds. $^{96}_{46}$Zr is a better candidate due to its higher Q-value, but there are no dedicated $^{96}_{46}$Zr experiments planned. Still, the NEMO Collaboration could set limits on $^{96}_{46}$Zr $^{0.629}_{4}$Ru by reanalyzing their data from ref. [9]. $^{150}_{64}$Nd is by far the best candidate, due to the high $Q_{0\nu4\beta}$-value. Coincidentally, it also has a high $Q_{0\nu2\beta}$-value, which makes it a popular isotope to test for 0ν2β, with some existing and planned experiments [1]. Once again, NEMO might already be able to constrain $^{150}_{60}$Nd $^{2.079}_{\nu3\beta79}$ $^{150}_{64}$Gd with their data [10].

The 0ν4EC channels in table 1 lead to a similar transmutation behavior as discussed above for 0ν4β, and can be checked in the same way. Note that the energy gain $Q_{0\nu4EC}$ will here be carried away by photons instead of electrons; the captured electrons will be taken out of the K and L shells, resulting in a cascade of X-ray photons. The Q-values of $^{148}_{72}$Gd and $^{154}_{70}$Dy are high enough to also undergo 0ν3ECβ+; $^{154}$Dy is the only isotope capable of 0ν2EC2β+. This can give rise to distinguishable signatures due to the additional 511 keV photons from electron-positron annihilation. The comparatively fast α-decay of $^{148}_{72}$Gd and $^{154}_{70}$Dy—and the fact that they have to be synthesized from scratch—make them however very challenging probes for $\Delta L = 4$, despite their large Q-values. $^{124}$Xe might then be the best element to test for 0ν4EC; unfortunately, the enriched xenon used by EXO contains almost no $^{124}$Xe, so 0ν4EC is currently hard to test (dark-matter experiments using xenon can in principle be used, as they contain $^{124}$Xe). Resonant enhancement of the 0ν4EC rates, as discussed for the 0ν2EC mode [11], might boost the signal.

Apparently, $\Delta L = 4$ signals are in general easier to test via the 0ν4β channels, with both $^{96}_{46}$Zr and $^{150}_{64}$Nd as more favorable isotopes when it comes to Q-values and natural abundance.

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Fig. 4: Sum of kinetic electron energies in the beta decays 0ν2β, 2ν2β, and 0ν4β. Relative contribution not to scale.
Rates for $0\nu 4\beta$. – Let us estimate some rates. Similarly to $0\nu 2\beta$, the half-life of $0\nu 4\beta$ can approximately be factorized as

$$
\left[ \tau_{1/2}^{0\nu 4\beta} \right]^{-1} = G_{0\nu 4\beta} |\mathcal{M}_{0\nu 4\beta}|^2,
$$

(12)

where $G_{0\nu 4\beta}$ denotes the phase space and $\mathcal{M}_{0\nu 4\beta}$ the nuclear transition matrix element (including the particle physics parameters) facilitating the process. Using an effective $\Delta L = 4$ vertex ($p_L^\nu p_L^\nu / \Lambda^2$) gives $G_{0\nu 4\beta} \propto G_F^2 / G^4_L$, just by counting propagators. For the virtual neutrino momentum $p_\nu$ we will use the inverse distance between the decaying nucleons, $p_\nu \sim |q| \sim 1\,\text{fm}^{-1} \simeq 100\,\text{MeV}$. The phase-space factor for the four final particles is the same as the one in $2\nu 2\beta$ (proportional to $Q^{11}$ for $Q \gg m_e$ [12]), which also tells us that each of the four electrons will be distributed just like the electrons in $2\nu 2\beta$, with a different $Q$-value, of course. Purely on dimensional grounds we can then estimate the dependence of the half-life on our parameters as

$$
\left[ \tau_{1/2}^{0\nu 4\beta} \right]^{-1} \propto Q^{11} \left( \frac{G_F^2}{q^2 \Lambda^2} \right)^2 q^{18},
$$

(13)

where the last factor is included to obtain the correct mass dimension. The above estimate is only valid for large $Q$-values, as it assumes massless electrons; the low $Q_{0\nu 4\beta}$ of most elements in table 1 render (some of) the four electrons non-relativistic and make necessary a more accurate calculation of the phase space. To partially cancel the uncertainties, we can approximate that the phase space for $0\nu 4\beta$ and $2\nu 2\beta$ is overall similar and consider the ratio (for $^{150}\text{Nd}$ and $|q| \simeq 100\,\text{MeV}$)

$$
\left( \frac{\tau_{1/2}^{0\nu 4\beta}}{\tau_{1/2}^{2\nu 2\beta}} \right) = \left( \frac{Q_{0\nu 4\beta}}{Q_{0\nu 2\beta}} \right)^{11} \left( \frac{\Lambda^4}{q^{12} G^2_F} \right) \simeq 10^{46} \left( \frac{\Lambda}{\text{TeV}} \right)^4.
$$

(14)

This is of course a rough estimate, and a better calculation, dropping the implicitly used closure approximation, including effects of the nuclear Coulomb field etc., will certainly change this rate. To this effect we stress a difference between $0\nu 2\beta$ and $0\nu 4\beta$: while the former decay proceeds via a kinematically forbidden intermediate state, the latter also features an energetically preferred intermediate state $X$, only to rush past it on the mass parabola (see fig. 3). Since excited states of $X$ can still have a mass lower than our initial nucleus, the summation over all these states is important and cannot be approximated away as easily as the excited states of an already forbidden intermediate state.

Finally, in our simple model from above, we generate the $\Delta L = 4$ operator with RHNs, $(\mathbf{\tau}_R \mathbf{\tau}_L)^2$, so each of the neutrinos in fig. 2 requires a mass flip in order to couple to the $W$ bosons. The particle physics amplitude is therefore further suppressed by a factor $(m_\nu / q)^4 \simeq 10^{-37}$, making this process all the more unlikely. These mass flips can be avoided in left-right–symmetric extensions of our model, at the price of replacing the four $W$ bosons in fig. 2 with their heavier $W_R$ counterparts.

Even with all our approximations leading to the above estimates, one can safely conclude that the half-life for neutrinoless quadruple beta decay is very large, at least if physics at the TeV scale is behind it in any way. This may be a too conservative approach, because four-neutrino interactions do not suffer from such stringent constraints as other four-fermion interactions [13]. The effective LNV operator $(\mathbf{\tau}_L \mathbf{\tau}_L)^2 / \Lambda$ discussed here has not been constrained so far, and the contribution to the well-measured invisible $Z$-width via $Z \rightarrow 4\nu$ only gives $\Lambda > 1/\mathcal{O}(10)\sqrt{GF} \sim 20\,\text{GeV}$. This, of course, only holds if the mediator is heavy enough to be integrated out in the first place. Light mediators can significantly increase the rate; the lifetime will be minimal if the exchanged particles have masses of the order of $|q| \simeq 100\,\text{MeV}$. For neutrinoless double beta decay the gain factor for the half-life is about $10^{15}$ [14], and we can expect something similar here. Given that we have four-neutrino propagators, the rate might be enhanced by a sizable factor, and therefore experimental searches for $0\nu 4\beta$ should be pursued.

While the expected rates for $0\nu 4\beta$ in our proof-of-principle model are observably small, more elaborate models —invoking resonances— might overcome this obstacle. Most importantly, the experimental and nuclear-physical aspects of $0\nu 4\beta$ are completely independent of the underlying mechanism, and can therefore be readily investigated.

Conclusion. – Contrary to popular belief, Majorana neutrinos are not a prerequisite for lepton number violation, and we have given a simple counterexample of lepton-number–violating Dirac neutrinos in this work. This gives rise to previously undiscussed $\Delta L = 4$ processes, the most striking of which would be neutrinoless quadruple beta decay, which can in principle be observable in three nuclei. The most promising isotope is $^{150}\text{Nd}$ due to its high $Q_{0\nu 4\beta}$-value and natural abundance (see table 1), and existing experiments could already be used to test $0\nu 4\beta$.

Let us stress that the decay should be constrained experimentally, as our theoretical estimates for TeV-scale-physics–induced $0\nu 4\beta$ might be too conservative. Not only is it a novel possible decay channel on the nuclear physics side, but it contains very interesting conceptual information about the fate of the classically conserved lepton number symmetry.

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