Parameter estimation of the homodyned K distribution based on neural networks and trainable fractional-order moments

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Abstract—Homodyned K (HK) distribution has been widely used to describe the scattering phenomena arising in various research fields, such as ultrasound imaging or optics. In this work, we propose a machine learning based approach to the estimation of the HK distribution parameters. We develop neural networks that can estimate the HK distribution parameters based on the signal-to-noise ratio, skewness and kurtosis calculated using fractional-order moments. Compared to the previous approaches, we consider the orders of the moments as trainable variables that can be optimized along with the network weights using the back-propagation algorithm. Networks are trained based on samples generated from the HK distribution. Obtained results demonstrate that the proposed method can be used to accurately estimate the HK distribution parameters.

Index Terms—homodyned K distribution, neural networks, parameter estimation, quantitative ultrasound.

I. INTRODUCTION

Homodyned K (HK) distribution has been widely used to describe the scattering phenomena arising in various research fields. In ultrasound (US) imaging, the HK distribution has been utilized to model the backscattered echo amplitude and quantitatively assess tissue structure [1]. For example, the HK distribution was applied for ultrasound based temperature monitoring and tissue characterization [2], [3], [4], [5].

Various methods have been developed for the estimation of the HK distribution parameters. Hruska and Oelze proposed a level-set estimation technique based on the signal-to-noise ratio, skewness and kurtosis parameters calculated using fractional-order moments [6]. Destrempes et al. proposed an iterative estimation technique based on the first moment of the intensity and two log-moments, namely the X- and U-statistics [7]. Building on the previous works, Zhou et al. utilized an artificial neural network (ANN) to estimate the parameters of the HK distribution [8]. Authors utilized the signal-to-noise ratio, skewness, kurtosis, X- and U-statistics as the input to the feed-forward neural network.

In this work, we propose a machine learning based technique for the estimation of the HK distribution parameters. Similar to Zhou et al., we train our neural network based on the SNR, skewness and kurtosis statistics [8]. However, in our case the orders of the moments used for the calculations are not fixed. Hruska and Oelze presented that the choice of the moments is important for the accurate estimation of the HK distribution parameters [6]. To improve the estimation, we treat the orders of the moments as trainable variables that can be optimized along with the network weights using the back-propagation algorithm.

II. METHODS

A. Homodyned K distribution

The probability density function of the HK distribution can be expressed in the following way:

\[ p(A) = A \int_0^\infty h J_0(\sigma h^3) J_0(Ah) \left(1 + \frac{h^2 \sigma^2}{2u} \right)^{-u} dh, \]  

(1)

where \( A \) stands for the amplitude, \( J_0 \) is the zero-th order Bessel function of the first kind and variable \( h \) is used for the integration. Parameters \( \sigma^2 \) and \( \mu^2 \) stand for the coherent and diffusive signal power. HK distribution has two parameters used for the quantitative assessment of the scattering phenomena in US. The first parameter, \( \mu \), is the scatterer clustering parameter reflecting the number of the scatterers in the resolution cell. The second quantitative parameter of the HK distribution is expressed as the ratio \( k = \frac{\mu}{\sigma} \) and is related to the spatial periodicity of the scatterer distribution.

B. The RSK estimator

Hruska and Oelze proposed the level-set method for the estimation of the HK distribution parameters based on the signal-to-noise ratio (R), skewness (S) and kurtosis (K) of the amplitude, denoted as the RSK estimator [6]. These three can be calculated with the following equations:

\[ R(v) = \left( \frac{E[A^v]}{(E[A^{2v}] - E^2[A^v])^{1/2}} \right), \]  

(2)

\[ S(v) = \frac{E[A^3v] - 3E[A^v]E[A^{2v}] + 2E^3[A^v]}{(E[A^{2v}] - E^2[A^v])^{3/2}}, \]  

(3)
determined for the HK distribution with specific
parameters. Additionally, each dense layer was equipped with
the batch-normalization layer and sigmoid activation function.

Following the work of Zhou et al., the network was trained to
output the \( u \) and \( k \) parameters for ranges of \( \log_{10}(u) \in [-1, 2] \)
and \( k \in [0, 2] \), additionally taking into account the scaling
required for the sigmoid activation function \([8]\). The mean
absolute percentage error (MAPE) and the Adam optimizer
with the learning rate of 0.001 were used for the training.

Network was trained based on samples generated from the
HK distribution for \( \log_{10}(u) \in [-1, 2] \) and \( k \in [0, 2] \). Batch
size was set to 16. For each batch, the number of the amplitude
samples was drawn at random from the interval [500, 2000].

TensorFlow was utilized for the calculations \([9]\).

D. Evaluation

We followed the same approach to the evaluation as in the
previous studies \([6], [7]\). Amplitudes were sampled from the
HK distribution with \( u \) and \( k \) parameters in the domains of \( u \in \{1, 2, ..., 9, 10\} \)
and \( k \in \{0.1, 0.2, ..., 0.9, 1.0\} \), respectively. The mean intensity of the HK distribution was constant. For
each pair of the parameters, we simulated 1000 sets, each with
the number of samples equal to 1000. Based on the estimates
\( \hat{u} \) and \( \hat{k} \) determined for each set, we calculated the relative
bias \( \sqrt{\text{Var}[\hat{u}]/u} \) and \( \sqrt{\text{Var}[\hat{k}]/k} \) as well as the normalized
standard deviations \( \sqrt{\text{Var}[\hat{u}]/u} \) and \( \sqrt{\text{Var}[\hat{k}]/k} \). Moreover, the
relative root mean squared errors (RMSEs) \( \sqrt{\text{Var}[\hat{u}]/u} \)
and \( \sqrt{\text{Var}[\hat{k}]/k} \) were computed. The proposed approach
was evaluated in two settings. First, we assessed the estimation
performance of an ensemble including 100 networks trained
with different initial weights. In this case, the outputs of the
networks were averaged to obtain the final prediction. Second,
the better performing network in respect to the average RMSEs
was selected and evaluated separately. The initial values of the
\( u \) parameters were set to 0.5, which corresponded to the mid-
point of the range used by Hruska and Oelze in the case of
the grid-search range of \([0, 1]\) \([6]\). Additionally, the proposed
approach was compared with the RSK and XU estimators as
well as with a network trained with the \( v \) value equal to 2
as in Zhou et al. \([8]\). Evaluations were performed in Matlab
(MathWorks, USA).

\[ K(v) = \frac{E[A^{4v}] - 4E[A^{v}]E[A^{3v}] + 6E[A^{2v}]E^2[A^v] - 3E^4[A^v]}{(E[A^{2v}] - E^2[A^v])^2}, \]

where \( v \) is a positive number used to adjust the orders of
the amplitude moments. In the case of the level-set approach,
the \( R, S \) and \( K \) calculated based on the amplitude samples
are compared with the theoretical values of these parameters
determined for the HK distribution with specific \( u \) and \( k \)
parameters. Hruska and Oelze presented that the choice of the
\( v \) parameter in eq. 2-4 has a large impact on the performance
of the level-set method \([6]\). Authors reported that the better
estimation performance of the level-set method could be
achieved based on six level curves corresponding to \( R, S \) and
\( K \) calculated for two values of the \( v \) parameter, namely 0.72
and 0.88. These values of the \( v \) parameter were selected using
the grid-search algorithm for \( v \) ranging from 0.02 to 1, with
an increment of 0.02.

C. Neural network based estimation

Zhou et al. developed a neural network to estimate the
parameters of the HK distribution based on the \( R, S, K, X \)-
and \( U \)-statistics \([8]\). \( R, S \) and \( K \) were calculated for a
fixed value of the \( v \) parameter equal to 2. Similar to Zhou
et al., we utilize a feed-forward neural network to determine
the \( u \) and \( k \) parameters of the HK distribution. However,
in our case we treat the \( v \) parameter as a trainable variable
that can be separately optimized for \( R, S \) and \( K \) in eq. 2-4.
During the training of the network, we utilize the back-
propagation algorithm to adjust the values of the \( v \) parameters.
Compared to the grid-search algorithm used by Hruska and
Oelze to select \( v \), in our work the order of the moments are
determined automatically \([6]\). Scheme of the proposed method
is presented in Fig. 1. The input of the network consisted of
three units corresponding to the \( R, S \) and \( K \) parameters.
Next, three hidden dense layers were utilized with the number
of units set to 6, 12 and 6, respectively. The output of the
network consisted of 2 units designed to calculate the \( u \) and \( k \)
parameters. Additionally, each dense layer was equipped with

![Trainable Model](image_url)

**Fig. 1.** Scheme presenting the proposed approach to the estimation of the HK distribution parameters. In our work, the order of the moments, related to the
\( v \) value, used to calculate SNR \( R \), skewness \( S \) and kurtosis \( K \) are trainable in the same way as the weights of the neural network.
Fig. 2. Relative biases, normalized standard deviations (SD) and rooted mean squared errors obtained for the HK distribution parameters estimator based on the ensemble of neural networks.
Fig. 3. Histograms of the $v$ values obtained in the training procedure for the SNR $R$, skewness $S$ and kurtosis $K$ for networks constituting the ensemble.

III. RESULTS AND DISCUSSION

Table I presents the performance of the network ensemble. Here, we can observe that the method based on networks with trainable orders of moments achieved better performance than the network with the fixed value of $v$ equal to 2. Similar results are presented in Table II for the single networks. Moreover, both Tables present that the network based approaches outperformed the RSK and XU estimators for the majority of metrics. Error plots calculated for the ensemble are presented in Fig. 2.

Fig. 3 shows the histograms of the $v$ values obtained for the $R$, $S$ and $K$ parameters in the case of the ensemble. The networks constituting the ensemble utilized lower $v$ values in majority, which may partially explain the lower performance of the network with the fixed value of $v$ equal to 2.

IV. CONCLUSION

In this work, we developed and evaluated a neural network for the estimation of the HK distribution parameters. Results demonstrated that the proposed method can be used to accurately estimate the parameters.

CONFLICTS OF INTEREST

The authors do not have any conflicts of interest to disclose.

ACKNOWLEDGEMENT

This work has not received funding.

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