Optimal Training for Wireless Energy Transfer

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Abstract—Wireless energy transfer (WET) is potentially a promising solution to provide convenient and reliable energy supplies for energy-constrained networks, and has drawn growing interests recently. To overcome the significant propagation loss over distance, employing multi-antennas at the energy transmitter (ET) to more efficiently direct wireless energy to desired energy receivers (ERs), termed energy beamforming, is an essential technique for WET. However, the achievable gain of energy beamforming crucially depends on the available channel state information (CSI) at the ET, which needs to be acquired practically. In this paper, we study the optimal design of one efficient channel-acquisition method for a point-to-point multiple-input multiple-output (MIMO) WET system, by exploiting the channel reciprocity based on which the ET estimates the CSI via dedicated reverse-link training from the ER. Considering the limited energy availability at the ER, the training strategy should be carefully designed so that the channel can be estimated with sufficient accuracy, and yet without consuming excessive energy at the ER. To this end, we propose to maximize the net energy at the ER, which is the total energy harvested offset by that used for channel training. The optimal training design, including the number of receive antennas to be trained, as well as the training time and power allocated, is derived. Our result shows that training helps only when either the channel coherence time, or the number of antennas at the ET, or the effective signal-to-noise ratio (ESNR), is sufficiently large; otherwise, no training should be applied and isotropic energy transmission is optimal.

I. INTRODUCTION

For energy-constrained wireless networks such as sensor networks, harvesting energy from the environment is a convenient and sustainable solution to energy replenishment [1]. Apart from the traditional energy sources such as solar and wind, which may not be always available, the ambient radio-frequency (RF) power is a viable new source for energy scavenging, which can be made more reliable if dedicated energy transmitter (ET) is used [2]. To compensate for the rapid RF power attenuation over distance, multi-antenna beamforming can be applied at the ET to direct wireless energy more flexibly and efficiently to a set of energy receivers (ERs), termed energy beamforming [3]. In this regard, massive multiple-input multiple-output (MIMO) technique [4] is particularly suitable for RF-based wireless energy transfer (WET), where a very large number of antennas are equipped at the ET to achieve enormous beamforming gains; as a result, the end-to-end energy transfer efficiency can be greatly enhanced.

In practice, the benefit of energy beamforming in WET crucially depends on the available channel state information (CSI) at the ET, which needs to be acquired in practice at the cost of additional resources. Similar to wireless communication systems, one straightforward approach to obtain CSI at the ET is by sending pilot signals from the ET to the ER [5], based on which the ER estimates the channel and sends the estimation back to the ET via a feedback channel. However, since the training time required scales with the number of antennas at the ET, denoted by $M$, such channel-acquisition method is not suitable when $M$ is large [6]. Furthermore, as pointed out by [7], estimating the channel at the ER requires complex baseband signal processing, which may not be available at the ER due to its practical hardware limitation. A novel channel-learning algorithm for WET by taking into account the practical energy harvesting circuitry at the ER has thus been proposed in [7], which simplifies the processing at the ER and requires only one-bit feedback from the ER per feedback interval. Nevertheless, the number of feedback intervals required for the channel-learning scheme in [7] still increases quickly with $M$, which may be prohibitive for large $M$. Furthermore, it is worth noting that for both aforementioned schemes, feedback signals need to be sent from the ER to the ET using part of the harvested energy. Considering limited energy availability at the ER, the energy consumed for sending the feedback signals should be taken into account in the channel learning and feedback designs.

In this paper, we consider a point-to-point MIMO WET system in which channel reciprocity holds by assuming that the forward-link energy transmission from the ET to the ER and the reverse-link communication from the ER to the ET take place in a time-division duplexing (TDD) manner. Notice that channel reciprocity is a key enabling factor for the implementation of massive MIMO systems to reduce the channel-acquisition overhead [8]. Under channel reciprocity, it is a well-known technique in wireless communication that the CSI at the transmitter can be obtained via reverse-link channel training, where a fraction of the channel coherence time is assigned to the receiver for sending pilot signals to the transmitter to estimate the channel. However, applying this method to WET systems needs a more careful design of the training strategy, due to...
the following new trade-offs: too little training leads to coarsely estimated channel at the ET and hence reduced energy beamforming gain; whereas too much training consumes excessive energy harvested by the ER, and also leaves less time for energy transmission given a finite channel coherence time. To optimally resolve the above trade-offs, we propose to maximize the net energy at the ER, which is the total energy harvested offset by that used for channel training. The optimal training design, including the number of ER antennas to be trained, as well as the training time and power allocated, is derived. Simulation results are also provided to validate our study.

II. SYSTEM MODEL

We consider a point-to-point MIMO WET system as shown in Fig. 1, where the ET with \( M \) antennas is employed to deliver wireless energy to the ER, which is equipped with \( N \) antennas. We assume a quasi-static flat fading channel model, where the channel matrix \( \mathbf{H} \in \mathbb{C}^{N \times M} \) from the ET to the ER remains constant within each coherent block of \( T \) symbol durations, and may vary independently from one block to another. We further assume Rayleigh fading in this paper, which is a suitable model for the rich scattering environment in practice; while other channel models will be considered in our future work. The entries in \( \mathbf{H} \) are thus assumed to be independent and identically distributed (i.i.d.) zero-mean circularly symmetric complex Gaussian (CSCG) random variables each with the same variance of \( \beta \), i.e., \( \mathbf{H} \) is distributed according to

\[
\text{vec}(\mathbf{H}) \sim \mathcal{CN}(0, \beta \mathbf{I}_{MN}),
\]

(1)

where \( \text{vec}(\mathbf{A}) \) denotes the vector obtained by stacking the columns in \( \mathbf{A} \). Note that \( \beta \) models the large-scale fading, which includes the effects of both distance-dependent path loss and shadowing [8].

Within one coherent block of \( T \) symbols, the input-output relation for WET can be written as

\[
\mathbf{y}[t] = \mathbf{H}\mathbf{x}[t] + \mathbf{n}[t], \quad t = 1, \ldots, T,
\]

(2)

where \( \mathbf{y}[t] \in \mathbb{C}^{N \times 1} \) is the received signal vector at the ER in the \( t \)th channel use; \( \mathbf{n}[t] \sim \mathcal{CN}(0, \sigma_n^2 \mathbf{I}_N) \) denotes the additive Gaussian noise vector; and \( \mathbf{x}[t] \in \mathbb{C}^{M \times 1} \) represents the energy-bearing signal transmitted by the ET, which can be assumed to be generated from arbitrary distributions with zero mean and covariance matrix \( \mathbf{S} \), i.e., \( \mathbf{S} = \mathbb{E}[\mathbf{x}[t]\mathbf{x}[t]^H] \). Denote by \( P_f \) the transmit power constraint at the ET. We thus have \( \text{tr}(\mathbf{S}) \leq P_f \).

By ignoring the noise power which is practically too small for energy harvesting, the total energy harvested by the ER within the coherent block of \( T \) symbols can be expressed as [3]

\[
Q = \sum_{t=1}^{T} \eta \mathbb{E}[\|\mathbf{y}[t]\|^2] \approx \eta T \text{tr}(\mathbf{H}^H \mathbf{S} \mathbf{H}),
\]

(3)

where \( 0 < \eta \leq 1 \) denotes the energy harvesting efficiency at the ER. It has been shown in [3] that if perfect CSI, i.e., \( \mathbf{H} \), is available at the ET, the optimal transmit covariance matrix to maximize the harvested energy \( Q \) given in (3) is

\[
\mathbf{S}^* = P_f \mathbf{v}_{\text{max}}(\mathbf{H}^H \mathbf{H}) \mathbf{v}_{\text{max}}^H(\mathbf{H}^H \mathbf{H}),
\]

(4)

which achieves the maximum harvested energy

\[
Q_{\text{max}} = \eta T P_f \lambda_{\text{max}}(\mathbf{H}^H \mathbf{H}),
\]

(5)

with \( \lambda_{\text{max}}(\mathbf{A}) \) and \( \mathbf{v}_{\text{max}}(\mathbf{A}) \) denoting the dominant eigenvalue and the corresponding eigenvector of \( \mathbf{A} \), respectively. Note that \( Q_{\text{max}} \) in (5) is a random variable dependent on the channel realization \( \mathbf{H} \). With the distribution of \( \mathbf{H} \) given in (1), the maximum average energy harvested can be obtained as

\[
Q_{\text{max}} = \mathbb{E}_\mathbf{H}[Q_{\text{max}}] = \eta T P_f \beta \mathbb{E}_\mathbf{H}[(\mathbf{H}^H \mathbf{H})_{\text{max}}] \triangleq \eta T P_f \beta \Lambda(M,N),
\]

(6)

where \( \mathbf{H} \triangleq \mathbf{H}/\sqrt{\beta} \) is the normalized channel matrix whose entries are i.i.d. zero-mean CSCG random variables with unit variance. The probability density function (pdf) of the maximum eigenvalue \( \lambda_{\text{max}}(\mathbf{H}^H \mathbf{H}) \) has been extensively studied in the literature (see e.g. [9]), based on which the expectation \( \mathbb{E}[(\lambda_{\text{max}}(\mathbf{H}^H \mathbf{H}))] \) can be computed for different values of \( M \) and \( N \). Note that \( \mathbb{E}[(\lambda_{\text{max}}(\mathbf{H}^H \mathbf{H}))] \) depends only on the dimension of \( \mathbf{H} \) and hence is denoted as \( \Lambda(M,N) \) in (6). In the special cases of \( N = 1 \) or \( M = 1 \), it can be easily obtained that \( \Lambda(M,1) = M \) and \( \Lambda(1,N) = N \). For general \( M \) and \( N \), no closed-form expression for \( \Lambda(M,N) \) is available, whereas its numerical values can be easily computed, e.g., based on the algorithm proposed in [10].

In the practical case with imperfect CSI at the ET, the maximum average harvested energy given in (6) only serves a performance upper bound. In this paper, by exploiting the channel reciprocity between the forward (from the ET to the ER) and reverse (from the ER to the ET) links, we propose a two-phase protocol for the MIMO WET system for channel training and energy transmission, respectively. The first phase corresponds to the first \( \tau \leq T \) symbol durations in each coherent block, where pilot symbols are sent by the ER to the ET using the energy harvested in previous blocks for channel training. Based on the received pilot signals, the ET obtains an estimate of the MIMO channel. In the second phase
of the remaining $T - \tau$ symbol durations, based on the estimated channel, the ET transmits the energy-bearing signal with optimized transmit covariance matrix. These two phases are elaborated in detail in the next section.

III. TWO-PHASE PROTOCOL

A. Reverse-Link Channel Training

Denote by $\tau \leq T$ the number of symbol durations used for channel training in the first phase. To obtain an estimate for the complete channel matrix $H$, we need at least as many measurements at the ET as unknowns, which implies that $M\tau \geq MN$ or $\tau \geq N$ [11]. Nevertheless, training for all ER antennas can be strictly suboptimal in general, as can be seen by considering the special case with $T = N$, in which we have $\tau = T$ and hence no time is left for energy transmission in the second phase. Therefore, to obtain the optimal training scheme, we need to consider a more general strategy where only a sub-block of the channel matrix $H$ is to be estimated at the ET. Without loss of generality, $H$ is partitioned as

$$H = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix},$$

where $H_1 \in \mathbb{C}^{N_1 \times M}$ denotes the channel matrix corresponding to the first $N_1$ ER antennas that will be estimated at the ET, and $H_2 \in \mathbb{C}^{N_2 \times M}$ represents the remaining block of $H$ that will not be estimated. We obviously have $0 \leq N_1 \leq N$ and $N_2 = N - N_1$. Therefore, during the channel training phase, pilot symbols are only sent from the first $N_1$ ER antennas, which yields

$$Y = \sqrt{\frac{P_r}{N_1}} \Phi H_1 + Z,$$

where $Y \in \mathbb{C}^{\tau \times M}$ contains the received signals at the ET during the $\tau$ symbol durations used for training, with $N_1 \leq \tau \leq T$; $P_r$ denotes the training power used by the ER; $\Phi \in \mathbb{C}^{\tau \times N_1}$ denotes the orthogonal pilot signals sent by the ER with $\Phi^H \Phi = \tau I_{N_1}$; and $Z \in \mathbb{C}^{\tau \times M}$ represents the training noise received at the ET with i.i.d. zero-mean CSCG entries each with variance $\sigma_z^2$. The energy consumed at the ET due to channel training is thus given by

$$E_u = \left( \frac{P_r}{N_1} \right)^2_1 \Phi^H \Phi = P_r \tau.$$  

Since the entries in $H_1$ are assumed to be i.i.d. Gaussian, the minimum mean-square error (MMSE) estimate $\hat{H}_1$ of $H_1$ is the same as the maximum likelihood (ML) estimate, which is given by [12]

$$\text{vec}(H_1) = R_{H_1Y} R_Y^{-1} \text{vec}(Y),$$

where

$$R_{H_1Y} \triangleq \mathbb{E} \left[ \text{vec}(H_1) \text{vec}^H(Y) \right] = \frac{P_r}{N_1} \beta I_M \otimes \Phi^H,$$

$$R_Y \triangleq \mathbb{E} \left[ \text{vec}(Y) \text{vec}^H(Y) \right] = \frac{P_r \beta}{N_1} I_M \otimes \Phi \Phi^H + \sigma_z^2 I_{M\tau},$$

with $\otimes$ denoting the Kronecker product. Let $\hat{H}_1$ denote the channel estimation error, i.e., $\hat{H}_1 = H_1 - \tilde{H}_1$. Based on the well-known orthogonal property of the MMSE estimation for Gaussian random variables [12], $\hat{H}_1$ and $\tilde{H}_1$ are independent. The covariance matrices of $\hat{H}_1$ and $\tilde{H}_1$ can be obtained as

$$R_{\hat{H}_1} \triangleq \mathbb{E} \left[ \text{vec}(\hat{H}_1) \text{vec}^H(\hat{H}_1) \right] = \sigma_{\hat{H}_1}^2 I_{MN_1},$$

$$R_{\tilde{H}_1} \triangleq \mathbb{E} \left[ \text{vec}(\tilde{H}_1) \text{vec}^H(\tilde{H}_1) \right] = \sigma_{\tilde{H}_1}^2 I_{MN_1},$$

where

$$\sigma_{\hat{H}_1}^2 = \frac{\beta^2 P_r \tau}{P_r \tau \beta + \sigma_z^2 N_1}, \quad \sigma_{\tilde{H}_1}^2 = \frac{\beta \sigma_z^2 N_1}{P_r \tau \beta + \sigma_z^2 N_1}.$$  

B. Forward-Link Energy Transmission

After the training-based channel estimation, dedicated energy-bearing signals are transmitted by the ET based on the estimated channel during the remaining $T - \tau$ symbol durations. Similar to (3), the total harvested energy within one coherent block is given by

$$Q = \eta(T - \tau) \text{tr}(H^H S).$$

Since the ET only has the knowledge of imperfectly and partially estimated channel matrix $\hat{H}_1$, the optimal transmit covariance matrix $S^*$ given in (4) cannot be implemented. In this case, given $\hat{H}_1$, $S$ is optimized at the ET to maximize the conditional harvested energy $\hat{Q} = \mathbb{E}_{H_1} [Q | \hat{H}_1]$. With the channel partition in (7) and the identity $H_1 = \hat{H}_1 + \tilde{H}_1$, we have

$$\hat{Q} = \eta(T - \tau) \mathbb{E}_{H_1,H_2} \left[ \text{tr} \left( (H_1 H_1^H + H_2 H_2^H) S \right) | \hat{H}_1 \right]$$

$$= \eta(T - \tau) \mathbb{E}_{H_1,H_2} \left[ \text{tr} \left( (H_1 H_1^H + H_2 H_2^H) \hat{H}_1 + \hat{H}_1^H \hat{H}_1 + \hat{H}_1 H_2 H_2^H \hat{H}_1 \right) | \hat{H}_1 \right]$$

$$= \eta(T - \tau) \text{tr} \left( (H_1 \hat{H}_1^H + (\sigma_{\hat{H}_1}^2 N_1 + \beta N_2) I) S \right),$$

where (14) follows from (1) and (11), and also the independence between $\tilde{H}_1$ and $\tilde{H}_1$. With the transmit power constraint $P_f$ at the ET, similar to (4), the harvested energy $\hat{Q}$ in (14) is then maximized by

$$S = P_f v_E v_E^H,$$

where

$$v_E = \nu_{\text{max}} \left( \hat{H}_1^H \hat{H}_1 + (\sigma_{\hat{H}_1}^2 N_1 + \beta N_2) I \right).$$

The resulting maximum value of $\hat{Q}$ is then given by

$$\hat{Q}^* = \eta(T - \tau) P_f \nu_{\text{max}} \left( \hat{H}_1^H \hat{H}_1 + (\sigma_{\hat{H}_1}^2 N_1 + \beta N_2) I \right),$$

where $\nu_{\text{max}}$ is the maximum eigenvalue of the matrix $\hat{H}_1^H \hat{H}_1 + (\sigma_{\hat{H}_1}^2 N_1 + \beta N_2) I$. The total harvested energy within one coherent block is then

$$Q = \eta(T - \tau) \text{tr}(H^H S).$$
Therefore, the average harvested energy, defined as \( \tilde{Q} = \mathbb{E}_{\mathbf{H}_1}[\tilde{Q}] \), is given by
\[
\tilde{Q} = \eta(T-\tau) P_f \left( \mathbb{E}_{\mathbf{H}_1} [\lambda_{\max}(\mathbf{H}_1^H \mathbf{H}_1)] + \sigma_{H_1}^2 N_1 + \beta N_2 \right)
\]
\[
= \eta (T-\tau) P_f \left( \frac{2}{1} \mathbb{E} \left[ \lambda_{\max}(\mathbf{H}_1^H \mathbf{H}_1) + \sigma_{H_1}^2 N_1 + \beta N_2 \right] \right),
\]
where \( \mathbf{H}_1 \triangleq \mathbf{H}_1/\sigma_{H_1} \) is the normalized channel matrix that has i.i.d. CSCG entries with zero mean and unit variance. As discussed in Section II, the expectation \( \mathbb{E} [\lambda_{\max}(\mathbf{H}_1^H \mathbf{H}_1)] \) depends only on the dimension of \( \mathbf{H}_1 \) and thus is denoted as \( \Lambda(M, N_1) \). By substituting (12) into (17), we finally have
\[
\tilde{Q} = \eta (T-\tau) P_f \beta \left( \frac{P \tau \beta \Lambda(M, N_1) + \sigma^2_d N_1^2 + \tau \Lambda}{P \tau \beta + \sigma^2_d N_1^2} + N_2 \right).
\]

It is observed from (18) that the average harvested energy is given by a summation of two terms. The first term, which monotonically increases with the training energy \( P \tau \) and the number of ET antennas \( M \), is attributed to the first \( N_1 \) antennas of the ER whose channel matrix \( \mathbf{H}_1 \) is estimated at the ET. The second term is attributed to the \( N_2 \) non-trained ER antennas, which is independent of the number of ET antennas \( M \) since no beamforming gain can be achieved for the energy transmission over the channel \( \mathbf{H}_2 \).

The net average harvested energy, i.e., the average harvested energy offset by the energy consumed for sending pilot signals at the ER, is then given by
\[
\tilde{Q}_{\text{net}}(N_1, \tau, P_r) = \tilde{Q} - P_r \tau,
\]
where \( \tilde{Q} \) is given in (18). Note that \( \tilde{Q}_{\text{net}} \) is a function of the number of ER antennas \( N_1 \) to be trained, the number of symbol durations \( \tau \) used for training, as well as the training power \( P_r \). The problem of finding the optimal training design so that \( \tilde{Q}_{\text{net}} \) is maximized can then be formulated as
\[
\text{(P1)}: \max_{N_1, \tau, P_r} \tilde{Q}_{\text{net}}(N_1, \tau, P_r)
\]
subject to
\[
0 \leq N_1 \leq N, \quad N_1 \leq \tau \leq T, \quad P_r \geq 0.
\]

In the next section, we derive the optimal solution to (P1), based on which some insights for the effect of channel training on WET performance are obtained.

IV. OPTIMAL CHANNEL TRAINING

Lemma 1: The optimal solution \((N_1^*, \tau^*, P_r^*)\) to problem (P1) satisfies \( \tau^* = N_1^* \).

Proof: Lemma 1 can be shown by considering the two cases with \( N_1^* > 0 \) and \( N_1^* = 0 \) as follows.

Case 1: \( N_1^* > 0 \). In this case, we prove Lemma 1 by contradiction. Suppose, on the contrary, that the optimal solution to (P1) satisfies \( \tau^* > N_1^* \). Then it can be verified based on (19) that the following inequality holds:
\[
\tilde{Q}_{\text{net}} \left( N_1^*, \tau^*, P_r^* \right) > \tilde{Q}_{\text{net}}(N_1^*, \tau^*, P_r^*).
\]

In other words, there exists another tuple \((N_1', \tau', P_r')\) with \( N_1' = \tau' = N_1^* \) and \( P_r' = P_r^* \) satisfying the constraints of (P1) while strictly increasing the objective value \( \tilde{Q}_{\text{net}} \). This thus contradicts the assumption that \((N_1^*, \tau^*, P_r^*)\) is optimal. Therefore, \( \tau^* = N_1^* \) must hold.

Case 2: \( N_1^* = 0 \). In this case, it trivially follows that \( \tau^* = 0 \) and \( P_r^* = 0 \).

This thus completes the proof of Lemma 1.

With Lemma 1, (P1) can be recast as
\[
\text{(P2)}: \max_{N_1, P_r} \tilde{Q}_{\text{net}}(N_1, P_r)
\]
subject to
\[
0 \leq N_1 \leq N, \quad P_r \geq 0.
\]

Where \( \tilde{Q}_{\text{net}}(N_1, P_r) \) in (P2) is obtained by substituting \( \tau = N_1 \) into (19) and is explicitly given in (26) at the top of the next page.

To find the optimal solution to (P2), we first obtain the optimal training power with \( N_1 \) fixed by solving:
\[
\text{(P3)}: \max_{P_r \geq 0} \tilde{Q}_{\text{net}}(N_1, P_r).
\]

Denote the optimal solution and the optimal value of (P3) as \( P_r^*(N_1) \) and \( \tilde{Q}_{\text{net}}^*(N_1) \), respectively. When \( N_1 = 0 \), it follows trivially that \( P_r^*(0) = 0 \) and the corresponding net average harvested energy is
\[
\tilde{Q}_{\text{net}}^*(0) = \eta T P_f \beta N.
\]

For \( 1 \leq N_1 \leq N \), it can be verified that \( \tilde{Q}_{\text{net}}(N_1, P_r) \) given in (26) is a concave function with respect to \( P_r \); hence (P3) is a convex optimization problem, whose solution can be found as
\[
P_r(N_1) = \sqrt{\eta T P_f \beta} \left[ \frac{(\Lambda(M, N_1) - 1)}{N_1} \right]^+ \left[ \frac{1}{\sqrt{T}} \right].
\]

where \( [x]^+ \triangleq \max\{x, 0\} \), and \( \Gamma \triangleq \eta P_f \beta^2 \) is referred to as the two-way effective signal-to-noise ratio (ESNR). Note that the term \( \beta^2 \) in \( \Gamma \) does not capture the effect of two-way signal attenuation due to both the reverse-link training and the forward-link energy transmission. The optimal value of (P3) for \( 1 \leq N_1 \leq N \) can then be expressed as (30) given at the top of the next page, where the set \( N \) is defined as
\[
N \triangleq \left\{ 1 \leq N_1 \leq N : \left( \frac{\Lambda(M, N_1)}{N_1} - 1 \right) > \frac{\Gamma}{P_f} \right\}.
\]

Finding the optimal solution to (P2) now reduces to determining the optimal number of ER antennas \( N_1^* \) to be trained, which can be easily obtained by comparing \( \tilde{Q}_{\text{net}}^*(N_1) \) for the \( N + 1 \) possible values of \( N_1 \in \{0, 1, \cdots, N\} \). In fact, as evident from (28) and (30),
and the corresponding optimal value \( \bar{Q}_{\text{net}}(N_1) \) in (30) for \( N_1 \in N \) reduces to

\[
\bar{Q}_{\text{net}}(N_1) = \left\{ \begin{array}{ll}
(T - N_1)\eta P_f \beta + \eta P_f N_1 \left( \sqrt{(T - N_1) \left( \frac{\Lambda(M, N_1)}{N_1} - 1 \right)} - \frac{1}{\sqrt{T}} \right)^2, & \text{if } N_1 \in N \\
(T - N_1)\eta P_f \beta N, & \text{otherwise},
\end{array} \right.
\]

\[
\bar{Q}_{\text{net}}^*(N_1) = \left\{ \begin{array}{ll}
1, & \text{if } TM - T - M > \frac{1}{T} + \frac{2}{\sqrt{T}}, \\
0, & \text{otherwise},
\end{array} \right.
\]

which can be readily determined given the closed-form expressions (28) and (30).

It follows from (32) that \( N_1^* > 0 \) only when \( N \) is non-empty. Thus, it can be inferred from (31) that for the MIMO WET system, channel training helps only if at least one of the following conditions is true: (i) the channel coherence time \( T \) is large; (ii) the ratio \( \frac{\Lambda(M, N_1)}{N_1} \) is large, i.e., the number of ET antennas \( M \) is large; and (iii) the ESNR \( \Gamma \) is high. If none of the above conditions is satisfied, the benefit of channel training and hence the energy beamforming based on the estimated channel cannot compensate the time and the energy used for sending the pilot symbols by the ER, and thus no training should be applied; instead, it is optimal to assign all the \( T \) symbol durations to transmit the energy signals isotropically, as can be seen from (16) with \( N_1 = 0 \) and hence \( \bar{H}_1 = 0 \).

In the special cases of MISO (\( N = 1 \)) and massive MIMO (\( M \gg N \)), the optimal solution in (32), and hence that to problem (P1) can be obtained in closed forms, as we show next.

A. MISO Channel

For the MISO setup with \( N = 1 \), we have \( \Lambda(M, 1) = M \) and the set \( N \) in (31) reduces to

\[
N = \left\{ \begin{array}{ll}
1, & \text{if } (T - 1)(M - 1) > \frac{1}{T}, \\
0, & \text{otherwise},
\end{array} \right.
\]

Hence, the optimal solution in (32) can be found as

\[
N_1^* = \left\{ \begin{array}{ll}
1, & \text{if } TM - T - M > \frac{1}{T} + \frac{2}{\sqrt{T}}, \\
0, & \text{otherwise},
\end{array} \right.
\]

and the corresponding optimal value \( \bar{Q}_{\text{net}}^*(N_1) \) of (P1) is

\[
\bar{Q}_{\text{net}}^*(N_1) = \left\{ \begin{array}{ll}
(T - 1)\eta P_f \beta + \eta P_f N_1 \left( \sqrt{(T - N_1) \left( \frac{\Lambda(M, N_1)}{N_1} - 1 \right)} - \frac{1}{\sqrt{T}} \right)^2, & \text{if } N_1 \in N \\
(T - N_1)\eta P_f \beta N, & \text{otherwise}.
\end{array} \right.
\]

B. Massive MIMO

In the massive MIMO regime with \( M \gg N \), we have \( \bar{H}_1 \bar{H}_1^H \rightarrow M \bar{I}_{N_1} \), where \( \bar{H}_1 \in \mathbb{C}^{N_1 \times M} \) is defined in (17). It thus follows that

\[
\Lambda(M, N_1) = \mathbb{E} \left[ \lambda_{\text{max}}(\bar{H}_1^H \bar{H}_1) \right] = M, \quad \forall 1 \leq N_1 \leq N.
\]

Furthermore, we have

\[
(T - N_1) \left( \frac{\Lambda(M, N_1)}{N_1} - 1 \right) \geq \frac{1}{T}, \quad \forall N_1 < T.
\]

Therefore, \( \bar{Q}_{\text{net}}^*(N_1) \) in (30) for \( N_1 \in N \) reduces to

\[
\bar{Q}_{\text{net}}^*(N_1) = \eta P_f \beta (T - N_1)(M + N - N_1).
\]

It can be verified that \( \bar{Q}_{\text{net}}^*(N_1) \) in (36) strictly decreases with \( N_1 \) for large \( M \). We thus have

\[
\bar{Q}_{\text{net}}^*(1) \geq \bar{Q}_{\text{net}}^*(N_1), \quad \forall N_1 \in N.
\]

Furthermore, with (28) and (36), it can be obtained that \( \bar{Q}_{\text{net}}(1) > \bar{Q}_{\text{net}}^*(0) \) holds. Therefore, the optimal solution to (32) is \( N_1^* = 1 \), i.e., for the WET system with \( M \gg N \), it is optimal to train one ER antenna only.

V. NUMERICAL RESULTS

In this section, numerical examples are provided to corroborate our study. We assume that the ET has the maximum transmit power of 30 dBm, i.e., \( P_f = 1 \) Watt. The average signal attenuation from the ET to the ER is assumed to be 40dB, i.e., \( \beta = 10^{-4} \). Furthermore, the training noise power is assumed to be \( \sigma_f^2 = -50 \) dBm and the energy harvesting efficiency is \( \eta = 0.5 \).

In Fig. 2, by varying the number of trained ER antennas \( N_1 \), the net average harvested power, i.e., \( \bar{Q}_{\text{net}}^*(N_1) \) with \( \bar{Q}_{\text{net}}^*(N_1) \) given by (28) for \( N_1 = 0 \) or by (30) for \( 1 \leq N_1 \leq N \), is plotted for different channel coherence time \( T \) for a MIMO WET system with \( M = 5 \) and \( N = 10 \). For moderate block lengths of \( T = 25 \) and \( T = 50 \), Fig. 2 clearly shows the trade-offs in selecting the number of ER antennas to be trained. It is observed that the optimal number of trained ER antennas equals...
2 for $T = 25$ and increases to 5 for $T = 50$. As the block length $T$ increases to 100 symbol durations, the net average harvested power monotonically increases with $N_1$, and hence it is optimal to train all of the $N = 10$ available ER antennas.

In Fig. 3, the net average harvested power of the proposed training-based scheme, i.e., $Q^{\text{net}}_T$ with $Q^{\text{net}}_T$ denoting the optimal value of (P1), is plotted against the block length $T$ for a MIMO WET system with $M = N = 5$. The performances of two benchmark schemes, namely the ideal energy beamforming with perfect CSI and the isotropic transmission (i.e., $S = \frac{P}{M} I_M$) with no CSI, are also plotted. It is observed that the training-based WET significantly outperforms isotropic transmission, and its performance improves with the increasing of $T$. This is expected since for larger block length, it is more affordable to have longer training, and hence the channel can be more accurately estimated at the ET and the resulting energy beamforming is more effective. It is also observed that with sufficiently large $T$, the training-based WET approaches the performance upper bound with perfect CSI.

Last, for the MISO setup with $N = 1$, Fig. 4 shows the optimal training and no-training regions based on (34) for different pairs of $M$ and $T$ with $\Gamma = -10$ dB or $\Gamma = 10$ dB. Fig. 4 affirms our conclusion in Section IV that training helps for WET when either $T$, $M$, or $\Gamma$ is sufficiently large. In fact, as observed from Fig. 4, for large $\Gamma = 10$ dB, training should be applied for almost all $(M, T)$ pairs.

VI. CONCLUSION

Under the point-to-point and Rayleigh fading setups, this paper studies the optimal design of channel training for MIMO WET systems. Assuming channel reciprocity, the forward link channel is efficiently estimated at the ET based on the training signals sent by the ER in the reverse link. We derive the optimal training strategy to maximize the net average energy harvested at the ER by taking into account the energy consumed for channel training. In future work, we will extend the results in this paper to more general setups with more than one ERs as well as other practical fading channel models.

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