A High-Frequency/Power Ratio Wilkinson Power Divider Based on Identical/Non-Identical Multi-T-Sections With Short-Circuited Stubs

ABHISHEK SAHU1 (Member, IEEE), KHAIR A. AL SHAMAILEH2 (Member, IEEE), PETER H. AAEN3 (Senior Member, IEEE), SAID A. ABUSHAMLEH4 (Member, IEEE), AND VIJAY K. DEVABHAKTUNI2 (Senior Member, IEEE)

1Wireless Division, Anaren Inc., Syracuse, NY 13057, USA
2Electrical and Computer Engineering Department, Purdue University Northwest, Hammond, IN 46323, USA
3Department of Electrical Engineering, Colorado School of Mines, Golden, CO 80401, USA
4Department of Electrical Engineering, Indiana Tech University, Fort Wayne, IN 46803, USA

This article was recommended by Associate Editor B. D. Sahoo.

CORRESPONDING AUTHOR: K. A. AL SHAMAILEH (e-mail: kalshama@pnw.edu)

ABSTRACT In this article, a systematic procedure is presented for the design of a dual-band unequal-split Wilkinson power divider (WPD) with high frequency and power division ratios. The design methodology is based on replacing the impractical high-impedance transmission line in the conventional divider with cascaded T-section structures with short-circuited stubs. The proposed methodology is simple and adopts distributed elements without reactive components to achieve a large power division ratio. The use of short-circuited stubs allows for circuit miniaturization in comparison to counterpart open-stub techniques. Furthermore, a high frequency ratio is obtained through optimizing the electrical and physical parameters of the T-sections based on a transmission lines theory framework as demonstrated in a simulated and measured prototype. To validate the proposed procedure, two WPD prototypes operating at 1/2.7 GHz and 1/4.4 GHz with a 1:10 power division ratio are simulated and measured. The theoretical response, supported by electromagnetic simulations and measurements, justifies the design concept.

INDEX TERMS Dual-band, microstrip lines, T-section, unequal-split, Wilkinson power divider (WPD).

I. INTRODUCTION

MERGING multi-standard wireless communication systems necessitate microwave components with dual- or multi-band operation [1]–[3]. The Wilkinson power divider (WPD) is ubiquitous in many front-end modules, such as antenna feed networks, I/Q vector modulators, and power amplifiers due to its planar structure, high isolation and matched conditions at all ports. Numerous designs of dual-band WPDs were reported in literature [4]–[35]; many of which employed lumped reactive components [4]–[6], composite-right/left-hand (CRLH) materials [7], transmission lines theory [8]–[10], output ports shifting [11], port impedance matching [13], [14], and coupled-lines [15]–[21]. However, none of these designs addressed unequal power division, which is a key element in applications including phased-arrays and beamforming networks. In recent years, attempts were made to design dual-band unequal-split WPDs using multi-section transmission lines [22]–[26], π- and T-configurations [27], [28], coupled lines [29]–[31], and RLC lumped components [32], [33]; all of which showed an excellent performance. Nevertheless, the limited frequency ratio and parasitic effects at high frequencies continue to be major challenges; not to mention that higher unequal power divisions impose impedances that cannot be realized with the methods reported in [22]–[33].

Very recently, we proposed a new approach for the design of a dual-band unequal-split WPD with arbitrary frequency/power division ratios to overcome some of the
above-mentioned limitations [34]. We showed that replacing the high impedance branch in the conventional design with cascaded-T-sections of open-stubs allows for a high power division at the operating frequencies. The work presented herein expands that demonstrated in [34] and introduces the following new contributions:

1) The utilization of short-circuited stubs facilitates a miniaturized physical area, which in turn results in reduced losses (e.g., radiation losses) as compared to open-stubs.

2) The optimization-driven framework which is based on transmission lines theory optimizes the electrical and physical parameters of the T-sections for higher frequency ratios.

In what follows, in Section II-A, the design equations of dual-band identical multi-T-sections with short-circuited-stubs are derived. Next, an optimization-driven framework is elaborated in Section II-B to develop non-identical multi-T-sections with higher frequency ratios. Steps for designing a dual-band unequal-split WPD are discussed in Section II-C. The resulting analytical response is given in Section III. Electromagnetic (EM) simulations and measurements are given in Section IV, followed by conclusions in Section V.

II. THEORY AND DESIGN EQUATIONS

In this section, the design procedure of the proposed high frequency and power division ratios dual-band WPD is elaborated. The major difference between the proposed methodology and the conventional divider design is in the use of identical multi-T-sections that facilitate a dual-band operation, and at the same time, eliminate the need for impractical high impedance transmission lines in high power division scenarios. Moreover, based on transmission lines theory, an optimization routine for non-identical structures is established to increase frequency ratio. In Sections II-A–II-C, the design equations, optimization process, and divider implementation are presented in greater detail.

A. HIGH-IMPEDANCE CHARACTERIZATION USING MULTI-T-SECTIONS

Fig. 1 shows a transmission line with electrical length \( \theta_h \) and high characteristic impedance \( Z_h \), subdivided into segments each with a length of \( \theta_h < \theta_h \) and the proposed dual-band cascaded T-section network. Each T-section consists of two identical transmission lines \( Z_u \) of length \( \theta_u \) and one short-circuited stub \( Z_s \) of length \( \theta_s \). The mathematical formulation of these T-sections begins with matching the ABCD parameters of a single T-section with those in the conventional high-impedance transmission line as follows:

\[
\begin{bmatrix}
\cos \theta_h & jZ_h \sin \theta_h \\
\bar{Z}_h^{-1} \sin \theta_h & \cos \theta_h
\end{bmatrix}
\begin{bmatrix}
\cos \theta_u & jZ_u \sin \theta_u \\
\bar{Z}_u^{-1} \sin \theta_u & \cos \theta_u
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
-jZ_s^{-1} \cot \theta_s & 1
\end{bmatrix}
\begin{bmatrix}
\cos \theta_h & jZ_h \sin \theta_h \\
\bar{Z}_h^{-1} \sin \theta_h & \cos \theta_h
\end{bmatrix}
\begin{bmatrix}
\cos \theta_u & jZ_u \sin \theta_u \\
\bar{Z}_u^{-1} \sin \theta_u & \cos \theta_u
\end{bmatrix}
= 
\begin{bmatrix}
\cos \theta_h & jZ_h \sin \theta_h \\
\bar{Z}_h^{-1} \sin \theta_h & \cos \theta_h
\end{bmatrix}
\begin{bmatrix}
\cos \theta_u & jZ_u \sin \theta_u \\
\bar{Z}_u^{-1} \sin \theta_u & \cos \theta_u
\end{bmatrix}
\begin{bmatrix}
\sin \bar{\theta}_h & 0 \\
0 & \bar{Z}_h^{-1} \cot \bar{\theta}_h
\end{bmatrix}
(1)
\]

By utilizing trigonometric identities on the parameters \( A \) and \( B \) at both sides, the following expressions are obtained:

\[
cot \theta_s = \frac{2Z_h}{Z_u} \left( \frac{\cos \bar{\theta}_h - \cos 2\theta_u}{\sin 2\theta_u} \right) \quad (2a)
\]

\[
Z_h \sin \bar{\theta}_h = 2Z_u \sin \theta_u \cos \theta_u + \frac{Z_u^2}{Z_s} \sin^2 \theta_u \cot \theta_s. \quad (2b)
\]

An expression of the T-section impedance, \( Z_u \), is obtained by substituting (2a) in (2b) as follows:

\[
Z_u = Z_h \left( \frac{\cot \theta_u \sin \bar{\theta}_h}{1 + \cos \bar{\theta}_h} \right). \quad (3)
\]

Finally, an expression of the stub impedance, \( Z_s \), is derived by substituting (3) in (2a):

\[
Z_s = \frac{1}{2} \left( \frac{Z_u \cot \theta_u \sin 2\theta_u}{\cos \bar{\theta}_h - \cos 2\theta_u} \right). \quad (4)
\]

It is noteworthy to point out that losses, which result in complex \( A \) \( B \) parameters, have a negligible impact on the T-section characteristics, especially when low-loss substrates are used. Therefore, a lossless transmission line model is used. According to (3) and (4), the proposed T-section maintains a dual-band operation under the following conditions:

\[
Z_u \left[ 1 + \cos \theta_u \right] \tan \theta_u = \pm Z_h \quad (5a)
\]

\[
\sin \bar{\theta}_h \theta_u = \mathbf{m} \pi \quad (5b)
\]

\[
\theta_u = \mathbf{m} \pi \quad (5c)
\]

where \( \theta_u(f_1, f_2) \) and \( \theta_u(f_1, f_2) \) are the transmission line and short-circuited stub lengths, respectively, at the operating frequencies \( f_1 \) and \( f_2 (f_2 > f_1) \); whereas \( m \) and \( n \) are positive integers. The frequency ratio, \( R \), is defined as
T-sections can be non-identical as Fig. 3 suggests. The cas-
certains discussed in the later sections.

design a dual-band identical multi-T-section with

\[ R = \theta_{u(f_2)}/\theta_{u(f_1)} = f_2/f_1. \]

Thus, a design equation with reference to \( f_1 \) can be derived from (5b) as:

\[ \theta_{u(f_1)} = n\pi/(R \pm 1). \]  

(6)

In this article, \( n = 1 \) and the positive sign in (6) are
considered to realize a compact power divider design. In
addition, \( \theta_{s} = \theta_{u} \) is set to obtain a physically realizable \( Z_{t} \);
whereas \( \theta_{s} = 2\theta_{h} \) in the case of multi-T-sections with open
stubs [34]. Hence, circuit miniaturization is achieved. Fig. 2
shows \( Z_{u} \) and \( Z_{s} \) with different values of \( \theta_{h} \) as a function of \( R \). Here, a 0.813-mm-thick Rogers RO4003C substrate with a
relative permittivity of 3.55 is considered, which allows for
a maximum microstrip impedance of \( Z_{max} = 140\,\Omega \). The
impedance \( Z_{h} \) is chosen as 295 \( \Omega \) for a reason that will be
discussed in the later sections.

B. NON-IDENTICAL MULTI-T-SECTION OPTIMIZATION

As shown in Figs. 1 and 2, a high impedance of electrical
length \( \theta_{h} = 90^\circ \) can be replaced with a dual-band identical
single T-section with practical impedances \( (Z_{u,\,s} \leq 140\,\Omega) \).
Furthermore, a higher \( R \) is obtained by subdividing \( Z_{h} \) into
segments each with a length of \( \tilde{\theta}_{h} < \theta_{h} \). However, for
\( \tilde{\theta}_{h} = 45^\circ \), the maximum value of \( R \) is 3.2. In other words,
to design a dual-band identical multi-T-section with \( R > 3.2 \),
\( \tilde{\theta}_{h} = 30^\circ \) must be considered (\( R_{max} = 4.1 \)). To increase \( R \) for a
given \( \tilde{\theta}_{h} \), the transmission line parameter matrices of the cascaded
T-sections can be non-identical as Fig. 3 suggests. The cas-
caded \( ABCD \) parameters of the T-sections, \([AB; CD]_{T}\), can be
obtained by multiplying the \( ABCD \) matrices of all segments
as follows:

\[
[AB; CD]_{T} = [AB; CD]_{11} \times [AB; CD]_{s1} \times [AB; CD]_{12}\times [AB; CD]_{21} \times [AB; CD]_{s2} \times [AB; CD]_{22}
\]  

(7)

where \([AB; CD]_{ij}\) is the \( ABCD \) matrix of \( \phi^{th} \) impedance in the
\( i^{th} \) section \((i,j) = 1, 2\) and \([AB; CD]_{ij}\) is the \( ABCD \) matrix
of the short-circuited stub in the \( i^{th} \) section. The \( ABCD \)
parameters are computed at \( f_{1,2} \) and are given as:

\[
A_{ij} = D_{ij} = \cos(\theta_{uij})
\]  

(8a)

\[
B_{ij} = jZ_{uij}\sin(\theta_{uij}) \quad C_{ij} = jZ_{uij}^{-1}\sin(\theta_{uij})
\]  

(8b)

\[
\theta_{uij} = \frac{2\pi}{c} f_{1,2} \sqrt{\varepsilon_{eff}} d_{uij} \quad D_{ij} = D_{sij} = 1
\]  

(8c)

\[
A_{sl} = D_{sl} = 0 \quad C_{sl} = -jZ_{sl}^{-1}\cot(\theta_{sl})
\]  

(9a)

\[
\theta_{sl} = \frac{2\pi}{c} f_{1,2} \sqrt{\varepsilon_{eff}} d_{sl}
\]  

(9b)

\[
\theta_{sl} = \frac{2\pi}{c} f_{1,2} \sqrt{\varepsilon_{eff}} d_{sl}
\]  

(9c)

where \( c \) is the speed of light. The effective dielectric constant,
\( \varepsilon_{eff} \), of each section is found using the microstrip line
formulas in [35]. In (8)-(9), \( Z_{uij} \) and \( Z_{sl} \) are the impedances
of the transmission lines and short stubs, respectively, and
\( \theta_{uij}, \, d_{uij} \) and \( \theta_{sl}, \, d_{sl} \) are the electrical and the physical
lengths of the transmission lines and short stubs, respectively, for
the cascaded non-identical T-sections. The input impedance, \( Z_{in} \),
of the cascaded T-sections terminated by a load impedance,
\( Z_{L} \), is expressed in terms of the total \( ABCD \) matrix in (7)
as follows [35]:

\[
Z_{in} = A_{T} Z_{L} + B_{T}\left(\frac{C_{T} Z_{L}}{A_{T} Z_{L} + B_{T}}\right)
\]  

(10)
Once \( Z_{in} \) is determined, the reflection coefficient, \( \Gamma, \) at \( f_{1,2} \) can be calculated as follows:

\[
\Gamma = \frac{Z_{in} - Z_s}{Z_{in} + Z_s} \tag{11}
\]

where \( Z_s \) is the source impedance. Then, an error function at each frequency is defined as:

\[
E = |\Gamma(f)|^2. \tag{12}
\]

Subsequently, the error vector resulting from applying (12) to both frequencies \( f_{1,2} \) is used to formulate and minimize the following objective function:

\[
\text{Objective} = |\Gamma(f_1)|^2 + |\Gamma(f_2)|^2. \tag{13}
\]

The parameters vector to be optimized in (13) consists of 12 elements: six physical lengths \( [d_{u(i,i=1,2)}, d_{s(i=1,2)}] \) and six impedance values \( [Z_{u(i,i=1,2)}, Z_{s(i=1,2)}] \). The optimized vector must be within reasonable fabrication tolerances and meet matching conditions. Therefore, the following physical constraints are set:

\[
1\text{mm} \leq d_{u(i,i=1,2)}, d_{s(i=1,2)} \leq 30\text{mm} \tag{14a}
\]

\[
20\Omega \leq Z_{u(i,i=1,2)}, Z_{s(i=1,2)} \leq 140\Omega. \tag{14b}
\]

The constraints presented in (14) confine the transmission lines within minimum and maximum lengths such that miniaturization is maintained; and ensure impedance values within milling tolerance. The sequential quadratic programming algorithm is used to minimize (13) subject to (14) due to its performance in constrained optimization problems. Once this procedure is completed, corresponding widths and lengths of the transmission lines and short-circuited stubs are found from the impedances and electrical lengths based on the well-known formulas reported in [35]. Algorithm 1 presents a pseudo-code of the design steps for a non-identical multi-T-section with a high frequency ratio.

### C. DESIGN OF UNEQUAL SPLIT WPDS WITH HIGH R

Figs. 4 and 5 show the schematics of two dual-band unequal-split divider examples with high frequency ratios. The schematics include the conventional branches of a WPD and the counterpart T-section replacement. The impedances of the conventional branches, \( Z_i(i=1..4) \) are calculated as \( Z_1 = Z_0\sqrt{k(2+1)k} = 295\Omega, Z_2 = Z_0\sqrt{k} = 88.91\Omega, Z_3 = Z_0\sqrt{1/k(1+1/k^2)} = 29.50\Omega \) and \( Z_4 = Z_0\sqrt{T/k} = 28.10\Omega. \)

Two 1: \( k^2 \) WPDS, where \( k^2 = 10 \), are designed to verify the design procedure. The first WPD is designed to operate at 1 and 2.7 GHz (\( \theta_u = \theta_s = 48.64^\circ \)) taking into account the substrate mentioned earlier. Here, the high-impedance branches \( Z_1 \) and \( Z_2 \) in the conventional WPD are subdivided into two 45° segments and replaced with identical T-sections; whereas the low-impedance branches, \( Z_{3,4} \), are replaced...
Algorithm 1 Dual-Band High Frequency Ratio WPD Design

Given: \([\varepsilon_r, h]\) - Substrate Parameters; 
\([d_{i,j}=1,2, d_{si}=1,2]\) - Physical Lengths; 
\([Z_{i,j}=1,2, Z_{si}=1,2]\) - Impedance Elements; 
\([f_1, f_2]\) - Target Frequencies; 
\([Z_{S}, Z_{L}]\) - Source and load impedances; 
\([Z_{min}, Z_{max}]\) - Impedance Constraints; 
\([d_{min}, d_{max}]\) - Length Constraints;

1: Procedure: Dualband_High_R_WPD()
2: Loop: for each frequency do
3: for each section do
4: \([AB; CD]=ABCD\_Matrix()\);
   //initial unknown parameters assumed
5: end for
6: \([AB; CD]_T = Overall\_ABCD\_Matrix()\);
   //\([AB; CD]_T\) denotes the total ABCD matrix
   //of two cascaded T-sections (\(\theta_h=45^\circ\))
7: \([Z_{in}] = Total\_Input\_Impedance\), 
   \((Z_{L}, [AB; CD]_T)\);
8: \([\Gamma] = Reflection\_Coefficient\), 
   \((Z_{in}, Z_o)\);
9: \([E] = Set\_Error\_Value\), 
   \((\Gamma, [f_1, f_2])\);
10: end for
11: \((d_{i,j}=1,2, d_{si}=1,2), [Z_{i,j}=1,2, Z_{si}=1,2], Objective\)
   =Minimize_Sum_Of_Errors(\([E]\))
12: Repeat Loop until optimal \((d_{i,j}=1,2, d_{si}=1,2), [Z_{i,j}=1,2, Z_{si}=1,2]\); // or predefined iterations
13: end Procedure

| TABLE 1. Impedances and lengths of the conventional 1:10 WPD and proposed dual-band design with \(R = 2.7\). |
|---|---|---|---|---|---|---|
| \(Z_i\) | Conv. Imp. (\(\Omega\)) | \(\theta_i(\circ)\) | \(Z_{ui}, Z_{si}(\Omega)\) | \(\bar{\theta}_h(\circ)\) | \(W_{ui,si}(\text{mm})\) |
| \(Z_1\) | 295.0 | 90 | 107.5, 56.28 | 45 | 0.37, 1.49 |
| \(Z_2\) | 88.91 | 90 | 32.41, 16.96 | 45 | 3.43, 7.79 |
| \(Z_3\) | 29.50 | 90 | 25.86, 89.22 | 90 | 4.60, 0.60 |
| \(Z_4\) | 28.10 | 90 | 24.75, 85.05 | 90 | 4.89, 0.67 |

| TABLE 2. Impedances and lengths of the identical T-sections for the proposed 1:10 WPD and dual-band Design with \(R = 4.4\). |
|---|---|---|---|---|
| \(Z_i\) | Conv. Imp. (\(\Omega\)) | \(\theta_i(\circ)\) | \(Z_{ui}, Z_{si}(\Omega)\) | \(\bar{\theta}_h(\circ)\) |
| \(Z_2\) | 88.91 | 90 | 55.99, 125.7 | 45 | 1.51, 0.24 |
| \(Z_3\) | 29.50 | 90 | 18.58, 41.70 | 45 | 6.98, 2.40 |
| \(Z_4\) | 28.10 | 90 | 17.71, 39.75 | 45 | 7.39, 2.58 |

with their equivalent dual-band single T-section structure. As the maximum frequency ratio with identical T-sections considering \(\bar{\theta}_h = 45^\circ\) is 3.2, the optimization routine is not necessary. The resulting impedances and electrical lengths are given in Table 1. The isolation resistor \(R_is\) is given as \(Z_0(k^2 + 1)/k = 174\Omega\) [35].

The second WPD example is designed to operate at 1 and 4.4 GHz. The impedances \(Z_{i(c=1,\ldots,4)}\) are subdivided into two 45° segments. According to the value of \(R\), \(Z_{i(c=2,\ldots,4)}\) are replaced with identical T-sections; whereas \(Z_1\) is replaced...
with non-identical T-sections as $R$ for $Z_1$ exceeds the maximum value with identical T-sections (i.e., $R_{\text{max}} = 3.2$). The impedances and electrical lengths of the identical T-sections for this case are given in Table 2; whereas the optimized non-identical T-section parameters are reported in Table 3. As shown in Table 3 all optimized parameters are within the constraints described in (14a) and (14b). It is paramount to point out that there is no unique solution for the optimized parameters, and each optimization results in different sets of impedances and lengths. However, the optimal analytical response adjoined with a compact size is considered.

The flowchart in Fig. 6 summarizes the design steps of a dual-band unequal-split WPD with high frequency ratio.

Meanwhile, it is imperative to discuss the key factors that limit the performance of the proposed WPDs. To this end, the effect of power division ratio, $k^2$ on maximum attainable frequency ratio, $R_{\text{max}}$ is studied. Based on Figs. 4 and 5, $Z_1$ has the maximum impedance. By utilizing (3)-(4), one can conclude that for a given $\bar{\theta}_h$, $R_{\text{max}}$ for $Z_1$ is the minimum for all impedance branches, $Z_{c(=1,\ldots,4)}$. Hence, $Z_1$ governs $R_{\text{max}}$ for multi-T-section configurations. Therefore, a study of power split versus frequency ratio for $Z_1$ would suffice. For any given $k^2$, $Z_1$ can be expressed as: $Z_1 = Z_0 \sqrt{(k^2 + 1)k}$, where $Z_0$ is the characteristic impedance. Next, by adopting (3)-(4) considering $Z_1 = Z_0$, the values of $Z_{\text{branch}}$ are obtained, which in turn provide $R_{\text{max}}$ for the corresponding $k^2$. Fig. 7 demonstrates $R_{\text{max}}$ for different values of $k^2$. It is observed that $R_{\text{max}}$ reduces with the increase of $k^2$ for any given $\bar{\theta}_h < \theta_h$. The lower/upper limits on $R_{\text{max}}$ for a given $k^2$ can also be derived from the plot.

III. ANALYTICAL RESULTS

In this section, the analytical results for the two WPD examples are presented. Matching transformers are included to match the output ports to the 50 Ω standard connectors. The analytical $S$-parameters of the divider with $R = 2.7$ are shown in Figs. 8(a) and 8(b); while those for $R = 4.4$ are shown in Figs. 8(c) and 8(d). As shown in Figs. 8(a) and 8(b), the designed $R = 2.7$ WPD provides excellent input/output port matching parameters $S_{11}, S_{22}, S_{33}$, which are better than $-30$ dB at the two design frequencies. The isolation parameter, $S_{32}$, is better than $-50$ dB; whereas the transmission parameters $S_{21}$ and $S_{31}$ are $-10.4$ dB and $-0.4$ dB, respectively, at the design frequencies. Furthermore, as shown in Figs. 8(c) and 8(d), the designed $R = 4.4$ divider provides ports matching and isolation better than $-30$ dB with transmission parameters of $-10.5$ dB and $-0.6$ dB at design frequencies. Hence, the proposed framework demonstrates an excellent dual-band performance of unequal split WPDs with high frequency ratio.

Next, a sensitivity analysis is performed to evaluate the manufacturing tolerances in the proposed design. Worst-case tolerances (i.e., ±10%) in the physical parameters for WPD1 (e.g., widths and lengths of the transmission lines and short-circuited stubs) are studied. Figs. 9 and 10 show the effect of the tolerances in widths ($W_{\text{ui}(i=1,\ldots,4)}$, $W_{\text{si}(i=1,\ldots,4)}$),
FIGURE 9. Effect of worst-case tolerances (i.e., ±10%) in widths on the S-parameters. (a) Input port matching $S_{11}$. (b) Output ports matching $S_{22}$ and $S_{33}$. (c) Transmission $S_{21}$ and $S_{31}$. (d) Isolation $S_{32}$. Solid, dashed and dotted lines represent the S-parameters for nominal widths, nominal width $-10\%$ and nominal width $+10\%$, respectively.

Table 1) and lengths ($d_{ui(i=1...4)}$, $d_{si(i=1...4)}$, Table 1) on the S-parameters, respectively. As illustrated in Fig. 9, the proposed design handles tolerances encountered in widths. However, such tolerances in length introduce a shift in the operating frequencies, as shown in Fig. 10. Therefore, the proposed design is more sensitive for the variation in the lengths of the transmission lines and short-circuited stubs in the multi-T-sections.

The proposed methodology differs from other previous efforts in the following aspects: 1) The resulting design is planar and is built on a single-layer. 2) Circuit miniaturization is achieved by employing short stubs with $\theta_s = \theta_u$. On the other hand, $\theta_s = 2\theta_u$ in the case where open stubs are utilized [34]. 3) Unlike [33], a distributed structure is adopted without reactive components. As such, characteristic distortions are avoided at high frequencies [27]. 4) The optimization-driven framework for the non-identical T-sections facilitates higher frequency ratios. Based on what was presented so far, it is not possible to achieve $R > 4$ without utilizing reactive components [33]. Our technique, however, achieves high $R$ with only distributed transmission lines.

IV. SIMULATIONS AND MEASUREMENTS

In this section, EM simulations and measurements are presented and discussed for the aforementioned dual-band 1:10 WPD examples. The prototypes are fabricated with a standard milling machine. The full-wave simulator ANSYS HFSS is used to simulate the two designs. Measurements are performed with a Rhode & Schwartz ZNB20 network analyzer. An off-wafer calibration was performed prior to the measurements to eliminate hardware error. To this end, a 3.5-mm short-open-load-through kit was employed to shift the measurement reference plane to the end of the probe tips.

Fig. 11 shows photographs of the fabricated prototypes; whereas simulations and experimental results are shown in Figs. 12 and 13. Figs. 12(a–b) show that the input/output ports matching for the WPD example with $R = 2.7$ are below $-30$ dB at the design frequencies. Fig. 12(c) shows that the simulated and measured values of $S_{21}$ and $S_{31}$ at $f_1 = 1$ GHz and $f_2 = 2.7$ GHz are $-10.8$ dB and $-0.85$ dB, respectively ($-10.41$ dB and $-0.41$ dB are the theoretical values for a 1:10 power division). Finally, Fig. 12(d) shows that the isolation between the output ports is better than $-30$ dB at the design frequencies.

Fig. 13 shows the simulated and measured S-parameters for the WPD example with $R = 4.4$. Input/output ports matching below $-30$ dB are achieved at $f_1 = 1$ GHz and $f_2 = 4.4$ GHz; whereas the isolation between output ports is better than $-20$ dB. Finally, the transmission parameters, $S_{21}$ and $S_{31}$, are found to be $-11.5$ dB and $-1.83$ dB, respectively, at the design frequencies. It is noteworthy to point out that the simulations and measurements demonstrated
FIGURE 10. Effect of worst-case tolerances (i.e., ±10%) in physical lengths on the $S$-parameters parameters. (a) Input port matching $S_{11}$. (b) Output ports matching $S_{22}$ and $S_{33}$. (c) Transmission $S_{21}$ and $S_{31}$. (d) Isolation $S_{32}$. Solid, dashed, and dotted lines represent the $S$-parameters for nominal lengths, nominal length −10% and nominal length +10%, respectively.

FIGURE 11. (a) Fabricated prototype of power divider I with $R = 2.7$. (b) Fabricated prototype of power divider II with $R = 4.4$.

in Figs. 8, 12, and 13 represent a dual-frequency (i.e., selective) operation. In other words, the desired electrical performance (e.g., matching, transmission) is met at the design frequencies. Future investigations can be devoted towards achieving a similar response over a broadband frequency range. In both WPD examples, the agreement between simulations and measurements are acceptable at the design frequencies. The measurements for WPD2 are slightly different from simulations, which can be attributed to fabrication errors. Fig. 14 depicts the measured phase difference between the output ports. As can be seen, the measured phase difference around the design frequencies in both examples is in the range of ±10 degrees.

A comparison between the proposed dual-band WPDs with other state-of-the-art designs is given in Table 4. The adopted technique, frequency ratio, power division ratio, $S$-parameters and circuitry area are set as benchmarks. It is observed that the proposed method facilitates higher power and frequency division ratios, $k^2$ and $R$, respectively, as compared to the state-of-the-art techniques. While it is possible to achieve a frequency ratio $> 4$ as reported in [33], the use of reactive components imposes distortions and parasitic effects.
at higher frequencies. The proposed method also facilitates compact designs in contrast to [27], [33], [34] due to adopting short-circuited stubs. Therefore, the proposed method entails the design and realization of dual-frequency power dividers with high power/frequency ratios and intertwines miniaturized circuitry, ease in fabrication and inherent good performance.

V. CONCLUSION
A design methodology of dual-band WPDs with high frequency and power split ratios are proposed. First, a theoretical investigation to derive analytical formulas of cascaded identical T-sections with short-circuited stubs to replace impractical transmission lines in the conventional power divider is performed. Then, with the use of transmission line modeling, an optimization framework is proposed to increase the frequency ratio with non-identical multi-T-sections. To demonstrate the proposed method, two 1:10 WPDs with frequency ratios of 2.7 and 4.4 are designed, simulated, and measured taking into account rigorous mathematical analysis, optimization, and full-wave simulations. The two prototypes are planar and do not require reactive components. Furthermore, the use of short-circuited stubs together with an optimization framework facilitates
higher power/frequency ratios, while occupying minimal physical area. Future efforts can be devoted toward designing multi frequency/power ratios with wide/controllable fractional bandwidths and exploring different optimization techniques to maximize the frequency ratio for a given power ratio in non-identical multi-T-section configurations.
ACKNOWLEDGMENT

The authors acknowledge the start-up funding received by Dr. Al Shamaileh from Dean, Dr. Holford, of the College of Engineering and Sciences, Purdue University Northwest. The authors also acknowledge the EECS Department, University of Toledo, for providing graduate assistantship to Dr. Sahu. The authors acknowledge the donation of substrates from the Rogers Corporation.

REFERENCES

[1] U. T. Ahmed and A. M. Abbosh, “Modified Wilkinson power divider using coupled microstrip lines and shunt open-ended stubs,” IET Elect. Lett., vol. 51, no. 11, pp. 838–839, May 2015.
[2] R. Gómez-García, R. Loeches-Sánchez, D. Psychogiou, and D. Peroulis, “Single/multi-band Wilkinson-type power dividers with embedded transversal fltering sections and application to channelized filters,” IEEE Trans. Circuits Syst. I, Reg. Papers, vol. 62, no. 6, pp. 1518–1527, Jun. 2015.
[3] R. Gómez-García, F. M. Ghannouchi, N. B. Carvalho, and H. C. Luong, “Advanced circuits and systems for CR/SDR applications,” IEEE I. Sel. Emerg. Topics Circuits Syst., vol. 3, no. 4, pp. 485–488, Dec. 2013.
[4] L. Wu, Z. Sun, H. Yilmaz, and M. Berroth, “A dual-frequency Wilkinson power divider,” IEEE Trans. Microw. Theory Tech., vol. 54, no. 1, pp. 278–284, Jan. 2006.
[5] X. Wang, I. Sakagami, Z. Ma, A. Mase, M. Yoshikawa, and M. Ichimura, “Miniaturized dual-band Wilkinson power divider with self-compensation structure,” IEEE Trans. Compon. Packag. Manuf. Technol., vol. 5, no. 3, pp. 389–397, Mar. 2015.
[6] N. Gao, G. Wu, and Q. Tang, “Design of a novel compact dual-band Wilkinson power divider with wide frequency ratio,” IEEE Microw. Wireless Compon. Lett., vol. 24, no. 2, pp. 81–83, Feb. 2014.
[7] A. Genc and R. Baktur, “Dual- and triple-band Wilkinson power dividers based on composite right- and left-handed transmission lines,” IEEE Trans. Compon. Packag. Manuf. Technol., vol. 1, no. 3, pp. 327–334, Mar. 2011.
[8] K. K. M. Cheng and F. L. Wong, “A new Wilkinson power divider design for dual band application,” IEEE Microw. Wireless Compon. Lett., vol. 17, no. 9, pp. 664–666, Sep. 2007.
[9] K. K. M. Chang and C. Law, “A novel approach to the design and implementation of dual-band power divider,” IEEE Trans. Microw. Theory Tech., vol. 56, no. 2, pp. 487–492, Feb. 2008.
[10] M.-J. Park and B. Lee, “A dual-band Wilkinson power divider,” IEEE Microw. Wireless Compon. Lett., vol. 18, no. 2, pp. 85–87, Feb. 2008.
[11] Y. Shin, B. Lee, and M.-J. Park, “Dual-band Wilkinson power divider with shifted output ports,” IEEE Microw. Wireless Compon. Lett., vol. 18, no. 7, pp. 443–445, Jul. 2008.
[12] G. Wu, L. Yang, Y. Zhou, and Q. Xu, “Wilkinson power divider design for dual-band applications,” IET Electron. Lett., vol. 50, no. 14, pp. 1003–1005, Jul. 2014.
[13] M. A. Maktoomi and M. S. Hashimi, “A performance enhanced port extended dual-band Wilkinson power divider,” IEEE Access, vol. 5, pp. 11832–11840, 2017.
[14] M.-J. Park and B. Lee, “Wilkinson power divider with extended ports for dual-band operation,” IET Electron. Lett., vol. 44, no. 15, pp. 916–917, Jul. 2008.
[15] M.-J. Park, “Two-section cascaded coupled line Wilkinson power divider for dual-band applications,” IEEE Microw. Wireless Compon. Lett., vol. 19, no. 4, pp. 188–190, Apr. 2009.
[16] Y. Wu, Y. Liu, and Q. Xue, “An analytical approach for a novel coupled-line dual-band Wilkinson power divider,” IEEE Trans. Microw. Theory Tech., vol. 59, no. 2, pp. 286–294, Feb. 2011.
[17] X. Tang and K. Moutaamah, “Compact dual-band power divider with single allpass coupled lines sections,” IET Electron. Lett., vol. 46, no. 10, pp. 688–689, May 2010.
[18] M.-J. Park, “Dual-band Wilkinson divider with coupled output port extensions,” IEEE Trans. Microw. Theory Tech., vol. 57, no. 9, pp. 2232–2237, Sep. 2009.

[19] W.-H. Chen, Y.-C. Liu, X. Li, Z.-H. Feng, and F. M. Ghannouchi, “Design of reduced-size unequal power divider for dual-band operation with coupled lines,” IET Electron. Lett., vol. 47, no. 1, pp. 59–60, Jan. 2011.
[20] Y.-C. Liu, W.-H. Chen, X. Li, and Z.-H. Feng, “Compact design of dualband power divider with coupled-line shunt elements,” IET Electron. Lett., vol. 47, no. 4, pp. 262–263, Feb. 2011.
[21] Y. Liu, W. Chen, X. Li, and Z. Feng, “Design of compact dual-band power dividers with frequency-dependent division ratios based on multisection coupled line,” IEEE Trans. Compon. Packag. Manuf. Technol., vol. 3, no. 3, pp. 467–475, Mar. 2013.
[22] I. Sakagami, X. Wang, K. Takahashi, and S. Okamura, “Generalized two-way two-section dual-band Wilkinson power divider with two absorption resistors and its miniaturization,” IEEE Trans. Microw. Theory Tech., vol. 59, no. 11, pp. 2833–2847, Nov. 2011.
[23] C. Feng, G. Zhao, X.-F. Liu, and F.-S. Zhang, “A novel frequency unequal Wilkinson power divider,” Microw. Opt. Technol. Lett., vol. 50, no. 6, pp. 1695–1699, Jun. 2008.
[24] M. Honari, L. Mirzavand, R. Mirzavand, A. Abdipour, and P. Mousavi, “Theoretical design of broadband multisection Wilkinson power dividers with arbitrary power split ratio,” IEEE Trans. Compon. Packag. Manuf. Technol., vol. 6, no. 4, pp. 605–612, Apr. 2016.
[25] S.-Y. Yin, J.-L. Li, and S.-S. Gao, “Compact dual-band five way Wilkinson power divider,” IET Electron. Lett., vol. 53, no. 13, pp. 866–888, Jun. 2017.
[26] A. Qrooit, N. Dib, and A. Gheethan, “Design methodology of multi-frequency un-equal split Wilkinson power dividers using transmission line transformers,” Prog. Electromagn. Res., vol. 22, pp. 1–21, Jan. 2010.
[27] Y. Wu, Y. Liu, Y. Zhang, J. Gao, and H. Zhou, “A dual band unequal Wilkinson power divider without reactive components,” IEEE Trans. Microw. Theory Tech., vol. 57, no. 1, pp. 216–222, Jan. 2009.
[28] T. Kim, B. Lee, and M.-J. Park, “Dual-band unequal Wilkinson power divider with reduced length,” Microw. Opt. Technol. Lett., vol. 52, no. 5, pp. 1187–1190, May 2010.
[29] X. Li et al., “Design of unequal Wilkinson power divider for dual-band operation with isolation stubs,” IET Electron. Lett., vol. 45, no. 24, pp. 1245–1247, Nov. 2009.
[30] Y.-H. Pang and Z.-H. Le, “Dual-band bandpass Wilkinson power divider of controllable bandwidths,” IET Electron. Lett., vol. 52, no. 7, pp. 537–539, Apr. 2016.
[31] B. Li, X. Wu, N. Yang, and W. Wu, “Dual-band equal/unequal Wilkinson power dividers based on coupled-line section with short-circuited STUB,” Prog. Electromagn. Res., vol. 111, pp. 163–178, Jan. 2011.
[32] S.-H. Ahn, J. W. Lee, C. Cho, and T. K. Lee, “A dual-band unequal Wilkinson power divider with arbitrary frequency ratios,” IEEE Microw. Wireless Compon. Lett., vol. 19, no. 12, pp. 783–785, Dec. 2009.
[33] X. Wang, I. Sakagami, K. Takahashi, and S. Okamura, “A general- ized dual-band Wilkinson power divider with parallel L, C, and R components,” IEEE Trans. Microw. Theory Tech., vol. 60, no. 4, pp. 952–964, Apr. 2012.
[34] K. Al Shamaileh, N. Dib, and S. Abushamleh, “A dual-band 1:10 Wilkinson power divider based on multi-T-section characterization of high-impedance transmission lines,” IEEE Microw. Wireless Compon. Lett., vol. 27, no. 10, pp. 897–899, Oct. 2017.
[35] D. M. Pozar, Microwave Engineering, 3rd ed. New York, NY, USA: Wiley, 2005.

ABHISHEK SAHU (Member, IEEE) was born in Bhubaneswar, India, in 1990. He received the B.Tech. degree in electronics and communication engineering from the Biju Patnaik University of Technology, Rourkela, India, in 2012, and the Ph.D. degree in engineering from the University of Toledo, USA, in 2018. He joined the Wireless Division, Anaren Inc., as an RF Engineer in 2018. His current research interests include applied electromagnetics, mm-wave components design, and microwave measurement techniques. He was a recipient of the Best Student Paper Award at the Applied Electromagnetic Conference, India, in 2011, and the ARFTG Microwave Symposium, Atlanta, GA, USA, in 2015.
KHAIR A. AL SHAMAILEH (Member, IEEE) received the B.Sc. degree in communications and electronics engineering and the M.Sc. degree in wireless communications engineering from the Jordan University of Science and Technology in 2009 and 2011, respectively, and the Ph.D. degree in engineering from Toledo University, USA, in 2015. He joined the ECE Department, Purdue University Northwest as an Assistant Professor in 2016. His research interests include microwave modeling, multi- and broad-band RF circuits design, wireless security, sensor networks, localization algorithms, and applied optimization techniques to engineering problems.

PETER H. AAEN (Senior Member, IEEE) received the B.A.Sc. degree in engineering science and the M.A.Sc. degree in electrical engineering from the University of Toronto, Toronto, ON, Canada, in 1995 and 1997, respectively, and the Ph.D. degree in electrical engineering from Arizona State University, Tempe, AZ, USA, in 2005. He was the Manager of RF Division, RF Modeling and Measurement Technology Team, Freescale Semiconductor, Inc., a company which he joined in 1997, then the Semiconductor Product Sector, Motorola, Inc. In 2013, he joined the Faculty of Engineering and Physical Sciences, University of Surrey, Guildford, U.K., where he was a Reader of Microwave Semiconductor Device Modeling. He was also the Director of the Nonlinear Microwave Measurement and Modeling Laboratory, a joint University of Surrey/National Physical Laboratory, and the Director of National Physical Laboratory, Teddington, U.K. In 2019, he joined the Indiana Tech University as an Assistant Professor of Electrical Engineering. His research areas include design and analysis of microstrip antennas for mobile radio systems, electromagnetic band gap structures and their applications in antenna engineering, planar soft surfaces for mutual coupling reduction between patch antennas, microwave filters, and metamaterials.

SAID A. ABUSHAMILEH (Member, IEEE) received the B.Sc. degree in electrical and computer engineering from Hashemite University, Zarqa, Jordan, in 2007, the M.Sc. degree in wireless communications from Lund University, Lund, Sweden, in 2009, and the Ph.D. degree in antenna engineering and electromagnetics from the University of Arkansas, Little Rock, AR, USA, in 2015. From 2015 to 2016, he was a Postdoctoral Research Associate with the Advanced Radar Research Center, University of Oklahoma, Norman, OK, USA. In 2016, he joined the Department of Physics—Engineering Foundation Program, University of Nebraska, Kearney, NE, USA, where he was an Assistant Professor of Electrical Engineering. In 2019, he joined the Indiana Tech University as an Assistant Professor of Electrical Engineering Department. His research areas include design and analysis of microstrip antennas for mobile radio systems, electromagnetic band gap structures and their applications in antenna engineering, planar soft surfaces for mutual coupling reduction between patch antennas, microwave filters, and metamaterials.

VIJAY K. DEVABHAKTUNI (Senior Member, IEEE) received the B.Eng. degree in electrical and electronics engineering, the M.Sc. degree in physics from the Birla Institute of Technology and Science, Pilani, India, in 1996, and the Ph.D. degree in electronics from Carleton University, Ottawa, Canada, in 2003. He held the competitive Natural Sciences and Engineering Research Council of Canada (NSERC) Postdoctoral Fellowship and spent the tenure researching with Dr. J. W. Haslett with the University of Calgary, Calgary, Canada, from 2003 to 2004. In 2005, he taught with Penn State Behrend. From 2005 to 2008, he held the Canada Research Chair of Computer-Aided High-Frequency Modeling and Design with Concordia University, Montreal, Canada. In 2008, he joined the Department of Electrical Engineering and Computer Science, University of Toledo, Toledo, as an Associate Professor, and was promoted to a Professor in 2013. In 2018, he joined Purdue University Northwest, Hammond, as a Chair of the Department of Electrical and Computer Engineering. He secured external funding close to $5M in his research areas (sponsoring agencies include AFOSR, AFRL, CFI, NASA, NIST, NSERC, NSF, ONR, and industry partners). He has authored 250 peer-reviewed papers. His interests are applied electromagnetics, biomedical applications of wireless sensor networks, computer-aided design, device modeling, image processing, infrastructure monitoring, neural networks, RF/microwave design, unmanned aerial vehicles, and virtual reality. In Canada and USA, he graduated 75 theses students at M.S. and Ph.D. levels and won student nominated teaching excellence awards. He served as an Associate Editor for the International Journal of RF and Microwave Computer-Aided Engineering under the Editor-in-Chief Dr. I. Bahl. He is a Professional Engineer of the Association of Professional Engineers and Geoscientists of Alberta.