Scale-corrected minimal skew simplex sampling UKF for BLDCM sensorless control

Zhugang Ding, Guoliang Wei*, Xueming Ding and Haidong Lv

Shanghai Key Lab of Modern Optical System, Department of Control Science and Engineering, University of Shanghai for Science and Technology, Shanghai 200093, People’s Republic of China

(Received 19 December 2014; accepted 22 February 2015)

In this paper, a scale-corrected minimal skew simplex sampling unscented Kalman filter (UKF) algorithm for the permanent magnet (PM) brushless DC motors (BLDCM) sensorless control has been studied to cancel the position sensor by the use of a systematical and analytical approach. Compared with the general UKF, the sampling method with the least Sigma points called minimal skew simplex sampling is adopted to reduce amount of computation and increase the estimation precision. Moreover, the scale-corrected strategy is introduced into the minimal skew simplex sampling UKF to overcome the nonlocal effects. On the other hand, for more easily calculating the value of back-EMF, the shape function of counter electromotive force is approximated by a series of sine and cosine functions based on the law of Fourier series. The purpose of the problem addressed is, by the method of scale-corrected minimal skew simplex sampling UKF, to properly estimate the rotor speed and position without installing encoders. Finally, the effectiveness of the proposed sensorless control technique is verified by simulation in MATLAB/Simulink.

Keywords: unscented Kalman filter; sensorless control; scale-corrected minimal skew simplex sampling; BLDCM

1. Introduction

With the dramatic improvement of power electronics, inverter control technologies and permanent magnet (PM) materials, brushless DC motor (BLDCM) has undergone great developments in communication equipments, industrial automation systems, space industries and medical devices, which is also ascribed to its advantages, such as high efficiency, low power consumption, low maintenance and compact structure. In the actual BLDCM control system, it requires that the currents in the windings of BLDCM must be synchronized to the instantaneous position of the rotor, therefore, resolvers or encoders may be used to measure rotor position. However, the applications of position sensors inevitably increase the cost of system and decrease the reliability. In addition, position sensors are quite sensitive to the environments, and they even cannot work in the condition of high temperature and high humidity.

Hence, it would be a great deal of sense to obtain the rotor position signal indirectly with the easily available information of stator terminal voltage, stator current, counter electromotive force (EMF) and motor parameters instead of installing resolvers or encoders. Over the past decades, the elimination of rotor position or velocity sensors has been the focus of intensive research, which mainly includes the back-EMF method, freewheel diode method, inductance testing method, rotor flux method, state observer method and some other special methods (Acarnley & Watson, 2006). Among them, the back-EMF method (Shao, Nolan, Teissier, & Swanson, 2003) is the most technically matured, the simplest and most popular for detecting the rotor position in the trapezoidal brushless DC motor. However, in practice, there still exist some obstacles and challenges to detect the zero point information; therefore, some other indirect methods have been put forward, such as the terminal voltage method (Kim, Lee, & Kwon, 2006; Lai & Lin, 2011), third harmonic method (Moreira, 1996) and back-EMF integration method (Jahns, Becerra, & Ehsani, 1991). The freewheeling diode method was first proposed in 1991 by Ogawara and Akagi (1991), which detected current flowing through a freewheeling diode in the silent phase to determine the rotor position. The freewheeling diode method is essentially based on the principle of the back-EMF zero-crossing, so it also has a position error of commutation points in the transient state. In Kulkarni and Ehsani (1992), Rodriguez and Emadi (2007) and Jang, Sul, Ha, Ide, and Sawamura (2003), the inductance-based method has been proposed for the motor with severe saliency such as switched reluctance motors and interior PM motors, which utilized the relationship between the motor winding inductance and the rotor position to calculate rotor position. The rotor flux method has been investigated in Tatematsu, Hamada, Uchida, Wakao, and Onuki (2000) and Kim and

*Corresponding author. Email: guoliang.wei1973@gmail.com

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Ehsani (2004), and by virtue of measured voltages and currents, the flux linkage is estimated. This method, however, requires a large amount of computation and is sensitive to the variation of motor parameters. State observer methods (Zhu, Kaddouri, Dessaint, & Akhrif, 2001) contain the sliding-mode observer, extended Kalman filter (EKF) method (Bolognani, Oboe, & Zigliotto, 1999; Terzic & Jadric, 2001) and model reference adaptive system observer. Most of the state observer methods take advantage of the known information, such as the stator current and terminal voltage, to estimate the rotor position in real time.

In the past few decades, the filtering problems have been extensively investigated (Dong, Wang, & Gao, 2013; Hu, Wang, Shen, & Gao, 2013; Liang, Sun, & Liu, 2014; Shen, Wang, Ding, & Shu, 2013). Accordingly, the filter theory has been successfully applied in many branches of practical domains, such as computer vision, communications, navigation and tracking systems. It is well known that the traditional Kalman filter (KF) serves as an optimal filter in the least mean square sense for linear systems with the assumption that the system model is exactly known. In the case that the system model is nonlinear and/or uncertain, there has been an increasing research effort to improve KF with hope to enhance their capabilities of handling nonlinearities. The EKF has been shown to be an effective way for tackling the nonlinear system estimation problems. In fact, EKF has recently gained particular research attention with promising application potentials, see, e.g. Hu, Wang, Gao, and Stergioulas (2012), Kallapur, Petersen, and Anavatti (2009) and Kluge, Reif, and Brokate (2010). Unfortunately, the EKF also have three well-known drawbacks: (1) linearisation can produce highly unstable filter performance, if the timestep intervals are not sufficiently small. (2) The derivations of the Jacobian matrices are non-trivial in most applications and often lead to significant implementation difficulties. (3) Sufficiently small timestep intervals usually imply high computational overhead as the number of calculations demanded for the generation of the Jacobian and the predictions of state estimate and covariance are large. Therefore, in Julier and Uhlmann (2004), put forward a new filter, unscented Kalman filter (UKF), which approximated the Gaussian distribution associated with each state variable rather than approximated nonlinear function transformation. The core idea of UKF lies in unscented transformation (UT) which propagates the mean and covariance through a nonlinear function based on a set of chosen sample points, known as sigma points, and preserves the nonlinear nature of the system. Considering development up to now, the UT sampling strategies include: basic UT, general UT, simplex UT, spherical UT and high-order UT. In the above several sampling methods, the numbers of the sigma points of the basic and general UT are $2n$ and $2n + 1$ (Julier, Uhlmann, & Durrant-Whyte, 2000), however, in order to reduce computing cost, the minimal skew simplex UT (Julier, 2003; Julier & Uhlmann, 2002) with the least sigma points is applied into UKF in this paper. Meanwhile, the scale-corrected method (Julier, 2002) is introduced into minimal skew simplex UT to overcome the nonlocal effects caused by the increase in states or to decrease the error of high-order item, which allows any set of sigma points to be scaled by an arbitrary scaling factor. While a few practical applications of the UKF have been studied by scholars, mainly in radar tracking, signal processing and robotics. However, up to now, there is still little research involved in the scale-corrected minimal skew simplex sampling UKF for the BLDCM sensorless control.

Motivated by the above discussions, in this paper, the scale-corrected minimal skew simplex sampling UKF sensorless control technology is designed for the BLDCM drive. The main contributions are as follows: (1) UKF approach is designed to estimate the position sensors for the BLDCM sensorless control; (2) the minimal skew simplex UT is used to replace the basic UT for reducing the computing cost; (3) to overcome the nonlocal effects, the scale-corrected method of UT is introduced into the minimal skew simplex sampling UKF; and (4) the back-EMF is represented by a shape function combining with Fourier series without judging the angle range.

### 2. BLDCM model description

A typical block diagram of BLDCM drive system is shown in Figure 1 consisting primarily of PM motor ontology, detectors and power switch device (inverter). In this paper, we assume that the magnetic flux of the motor is not saturated, the motor three-phase stator windings are a star connection, the influence of the magnetic hysteresis is negligible and the back-EMF of BLDCM is trapezoidal. Then, we have the following PM BLDCM model in the three-phase stationary frame.

$$
\begin{bmatrix}
U_{ag} \\
U_{bg} \\
U_{cg}
\end{bmatrix}
= 
\begin{bmatrix}
R & 0 & 0 \\
0 & R & 0 \\
0 & 0 & R
\end{bmatrix}
\begin{bmatrix}
i_a \\
i_b \\
i_c
\end{bmatrix}
+ 
\begin{bmatrix}
L_d - M & 0 & 0 \\
0 & L_d - M & 0 \\
0 & 0 & L_d - M
\end{bmatrix}
\frac{d}{dt}
\begin{bmatrix}
i_a \\
i_b \\
i_c
\end{bmatrix}
+ 
\begin{bmatrix}
e_a \\
e_b \\
e_c
\end{bmatrix}
+ 
\begin{bmatrix}
U_n \\
U_n \\
U_n
\end{bmatrix},
$$

(1)

where $U_{ag}, U_{bg}$ and $U_{cg}$ are the terminal voltages of three-phase windings, $i_a, i_b$ and $i_c$ are the stator three-phase currents, $e_a, e_b$ and $e_c$ are the three-phase back-EMFs, $U_n$ is the neutral-point voltage, $R$ is the phase resistance, $L_d$ is the self-inductance of three-phase windings and $M$ is the mutual inductance of three-phase windings.
For the symmetrically distributed star connected three-phase windings, the three-phase currents obey
\[ i_a + i_b + i_c = 0. \] (2)

Then, from Equations (1) and (2), \( U_n \) can be represented as
\[ U_n = \left( U_{ag} + U_{bg} + U_{cg} \right) - \left( e_a + e_b + e_c \right) \] (3)

Substituting Equation (3) into the BLDCM model (1), we can have
\[
\frac{d}{dt} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \frac{-R}{L_s} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \frac{1}{3(L_s - M)} \begin{bmatrix} U_{ab} - U_{ca} \\ U_{bc} - U_{ab} \\ U_{ca} - U_{bc} \end{bmatrix} + \frac{1}{3(L_s - M)} \begin{bmatrix} e_b + e_c - 2e_a \\ e_a + e_c - 2e_b \\ e_a + e_b - 2e_c \end{bmatrix}, \] (4)

where \( U_{ab}, U_{bc} \) and \( U_{ca} \) are line-to-line voltages, and \( U_{ab} = U_{ag} - U_{bg}, U_{bc} = U_{bg} - U_{cg}, U_{ca} = U_{cg} - U_{ag}. \)

The electromagnetic torque is calculated as follows:
\[ T_e = \frac{e_a i_a + e_b i_b + e_c i_c}{\omega_r}, \] (5)

where \( \omega_r \) is the mechanical angular velocity of motor.

From Equation (5), we conclude that, to produce a steady electromagnetic torque, the sum of \( e_a i_a, e_b i_b \) and \( e_c i_c \) has to be constant as far as a certain speed is concerned. We assume that the air gap magnetic field distribution of BLDCM is an ideal trapezoidal wave, then the distribution of the magnetic induction intensity and the back-EMF are consistent. Therefore, to ensure a constant torque, the armature current with ideal square waveform should be in phase with back-EMF waveform, as shown in Figure 2.

Nevertheless, the back-EMF cannot be directly measured in the running process of the motor. To solve this problem, considering the relationship of the rotor position, speed and back-EMF, the back-EMF can be expressed in the following form:
\[ e = \omega_r K_e B_g(\theta), \] (6)
where \( K_e \) is the back-EMF coefficient which is a constant, \( B_g(\theta) \) is a shape function that depends on the rotor position. As shown in Figure 3, \( B_g(\theta) \) can be written as the following piecewise function:
\[
B_g(\theta) = \begin{cases} 
\frac{6\theta}{\pi}, & 0 \leq \theta \leq \frac{\pi}{6}, \\
1, & \frac{\pi}{6} \leq \theta \leq \frac{5\pi}{6}, \\
\frac{6(\theta - \pi)}{\pi}, & \frac{5\pi}{6} \leq \theta \leq \frac{7\pi}{6}, \\
-1, & \frac{7\pi}{6} \leq \theta \leq \frac{11\pi}{6}, \\
\frac{6(\theta - 2\pi)}{\pi}, & \frac{11\pi}{6} \leq \theta \leq 2\pi.
\end{cases} \] (7)

Remark 1 For convenience, combining with the principle of Fourier series, the piecewise function \( B_g(\theta) \) which
back-EMFs can be expressed as the difference of 120 electrical degrees. Then, Equation (7) can be replaced by the approximate mathematical expression (8),

\[ B_g(\theta) = 1.215 \sin \theta + 0.270 \sin 3\theta + 0.049 \sin 5\theta - 0.025 \sin 7\theta - 0.030 \sin 9\theta \cdots . \]  

(8)

Three phases A, B, and C are similar only with the difference of 120 electrical degrees. Then, three-phase back-EMFs can be expressed as

\[
\begin{bmatrix}
  e_a \\
  e_b \\
  e_c
\end{bmatrix} = w_e K_e \begin{bmatrix}
  B_g(\theta - \frac{2\pi}{3}) \\
  B_g(\theta - \frac{4\pi}{3}) \\
  B_g(\theta)
\end{bmatrix}.
\]

(9)

From Equations (9), (4) and (5) can be rewritten as

\[
\frac{d}{dt} \begin{bmatrix}
  i_a \\
  i_b \\
  i_c
\end{bmatrix} = -R \begin{bmatrix}
  i_a \\
  i_b \\
  i_c
\end{bmatrix} + \frac{1}{3(L_a - M)} \begin{bmatrix}
  U_{ab} - U_{ca} \\
  U_{bc} - U_{ab} \\
  U_{ca} - U_{bc}
\end{bmatrix}
\]

\[
+ \frac{w_e K_e}{3(L_a - M)} \begin{bmatrix}
  B_g(\theta - \frac{2\pi}{3}) + B_g(\theta - \frac{4\pi}{3}) - 2B_g(\theta) \\
  B_g(\theta) + B_g(\theta - \frac{4\pi}{3}) - 2B_g(\theta - \frac{2\pi}{3}) \\
  B_g(\theta) + B_g(\theta - \frac{2\pi}{3}) - 2B_g(\theta - \frac{4\pi}{3})
\end{bmatrix}
\]

\[
\times \begin{bmatrix}
  B_g(\theta - \frac{2\pi}{3}) + B_g(\theta - \frac{4\pi}{3}) - 2B_g(\theta) \\
  B_g(\theta) + B_g(\theta - \frac{4\pi}{3}) - 2B_g(\theta - \frac{2\pi}{3}) \\
  B_g(\theta) + B_g(\theta - \frac{2\pi}{3}) - 2B_g(\theta - \frac{4\pi}{3})
\end{bmatrix}
\]

(10)

and

\[ T_e = p K_e \left[ B_g(\theta)i_a + B_g(\theta - \frac{2\pi}{3})i_b + B_g(\theta - \frac{4\pi}{3})i_c \right], \]

(11)

where \( p = w_e/w_r \), which is the number of pole pairs.

Based on the mechanical equation of BLDCM

\[
J \frac{dw_e}{dt} = T_e - T_L - B w_e
\]

and Equation (11), we can obtain

\[
\frac{dw_e}{dt} = \frac{p T_e - p T_L - B w_e}{J}
\]

\[
p^2 K_e \left[ B_g(\theta)i_a + B_g(\theta - \frac{2\pi}{3})i_b + B_g(\theta - \frac{4\pi}{3})i_c \right] - p T_L - B w_e, \]

(12)

where \( J \) is the rotor equivalent inertia, \( B \) is the viscous friction coefficient and \( T_L \) is the load torque.

3. Scale-corrected minimal skew simplex sampling

**UKF**

UKF provides an alternative estimation methodology for nonlinear applications which does not rely upon any linearization procedure. The principle of UKF is based on performing the state estimation by approximating the probability distribution instead of approximating the nonlinearity itself. To approximate the probability distribution, a set of sigma points is chosen and propagated through the nonlinearity, then, the state mean and covariance could be obtained by these sigma points. The detailed process of this algorithm is introduced in the following.

Consider the following discrete-time nonlinear system and measurement model:

\[
\begin{align*}
x_{k+1} &= f_k(x_k, u_k) + \omega_k \\
z_{k+1} &= h_k(x_{k+1}) + v_{k+1},
\end{align*}
\]

(13)

where \( x_k \in \mathbb{R}^{n \times 1} \) is the state vector, \( z_k \in \mathbb{R}^{m \times 1} \) is the measurement vector, \( f_k(x_k, u_k) \) is a known nonlinear state transition vector, \( h_k(x_{k+1}) \) is a known nonlinear measurement transition vector. The \( \omega_k \) is system noise and \( v_{k+1} \) is measurement noise. They are both additive white Gaussian noise with zero mean, and both obey the following statistical properties:

\[
\begin{align*}
E(\omega_k) &= 0, & \text{Cov}(\omega_k, \omega_j) &= Q_{\delta_{kj}}, \\
E(v_k) &= 0, & \text{Cov}(v_k, v_j) &= R_{\delta_{kj}}, \\
& \text{Cov}(\omega_k, v_j) &= 0,
\end{align*}
\]

where \( \delta_{kj} \) is the kronecker – \( \delta \) function.

3.1. Minimal skew simplex UT

The minimal skew simplex sigma points are chosen by the following algorithm:

(1) Choose the weight \( 0 \leq W_0 < 1 \).
3.3. Whole process of algorithm

We assume that the initial values of state variable \( x_0 \) and the noise \( \omega_k, v_k \) are mutually independent. The initial state statistical property is assumed as follows:

\[
\begin{align*}
\hat{x}_0 &= \mathbb{E}(x_0), \\
P_0 &= \text{Var}(x_0) = \mathbb{E}[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T],
\end{align*}
\]

(1) **Time update**

Project the sigma points in time using the following nonlinear transformation:

\[
\hat{x}_{i,k|k-1} = f_{x,k-1}(\xi_{i,k-1}, u_{k-1}), \quad i = 0, 1, 2, \ldots, j + 1.
\]

Calculate the predicted states mean

\[
\hat{x}_{k|k-1} = \sum_{i=0}^{j+1} W^m_i \hat{x}_{i,k|k-1}
\]
and the predicted error covariance

\[ P_{\kappa|k-1} = \sum_{i=0}^{j+1} W_i (\hat{\chi}_{i,k|k-1} - \hat{x}_{k|k-1}) (\hat{\chi}_{i,k|k-1} - \hat{x}_{k|k-1})^T + Q_{k-1}. \]

(2) Measurement update:

Solution of the classical form of KF is adapted in UKF, so measurement update algorithm is performed in the similar way to that in classical KF and also requires output covariance \( P_{z_k} \) and cross-covariance matrix \( P_{z_kx_k} \).

Recalculate the sigma points

\[ \hat{x}_{i,k|k-1} = \hat{x}_{k|k-1} + \alpha \sqrt{P_{k|k-1}} \xi_i. \]

Project the sigma points through the following observation function:

\[ \hat{y}_{i,k|k-1} = h_k(\xi_{i,k|k-1}), \quad i = 0, 1, 2, \ldots, j + 1. \]

Calculate the predicted measurements mean

\[ \hat{z}_{k|k-1} = \sum_{i=0}^{j+1} W_i \hat{y}_{i,k|k-1}. \]

The predicted covariance matrix of observation

\[ P_{z_k} = \sum_{i=0}^{j+1} W_i (\hat{y}_{i,k|k-1} - \hat{z}_{k|k-1}) (\hat{y}_{i,k|k-1} - \hat{z}_{k|k-1})^T + R_k. \]

The cross-covariance matrix

\[ P_{z_kx_k} = \sum_{i=0}^{j+1} W_i (\hat{y}_{i,k|k-1} - \hat{z}_{k|k-1}) (\hat{\chi}_{i,k|k-1} - \hat{x}_{k|k-1})^T. \]

Correct the predicted states and covariance matrix

\[
\begin{align*}
K_k &= P_{z_kx_k} P_{x_k}^{-1}, \\
\hat{x}_k &= \hat{x}_{k|k-1} + K_k (\hat{z}_k - \hat{z}_{k|k-1}), \\
P_k &= P_{k|k-1} - K_k P_{z_k} K_k^T.
\end{align*}
\]

4. BLDCM discrete-time model

To implement the above algorithm on a simulation computer, the system dynamic equation should be transformed into a discrete-time state equation. Assume that the controller has a short sampling time, hence, the rotor electrical angular velocity can be regarded as a constant within a sampling period,

\[ \theta(k + 1) = T_s \omega_{\kappa}(k) + \theta(k), \]

where \( T_s \) is the sampling time.

The model (10) and (12) must be discretized for the convenience to implement the algorithm on the computer.

\[
\begin{align*}
\begin{bmatrix}
i_a(k + 1) \\
i_b(k + 1) \\
i_c(k + 1)
\end{bmatrix} &= \begin{bmatrix}
1 - \frac{T_s R}{L - M} & & \\
& 1 - \frac{T_s R}{L - M} & \\
& & 1 - \frac{T_s R}{L - M}
\end{bmatrix}\begin{bmatrix}
i_a(k) \\
i_b(k) \\
i_c(k)
\end{bmatrix} \\
&+ \frac{T_s}{3(L - M)} [u_{ab}(k) - u_{ca}(k)] \\
&+ \frac{T_s}{3(L - M)} [B_g(\theta_k - 2\pi/3) + B_g(\theta_k - 4\pi/3) + 2B_g(\theta_k)] \\
&\quad - 2B_g(\theta_k - 2\pi/3) - \omega_{\kappa}(k),
\end{align*}
\]

Furthermore, Equation (15) can be rewritten in the form of the UKF algorithm

\[
\begin{align*}
x_{k+1} = F_x(x_k) x_k + G_x u_k + \omega_x, \\
z_{k+1} = H x_{k+1} + v_{k+1},
\end{align*}
\]
algorithm and the traditional algorithm, the general UT UKF and the scale-corrected minimal skew simplex UT UKF have been operated in the same simulation model. The simulation model is given in Figure 4. The block in yellow is the embedded simulation program (M-file) with scale-corrected minimal skew simplex UT, and the red is the embedded simulation program (M-file) with general UT.

In this simulation model, the parameters of BLDCM are as follows:

| Parameter | Value |
|-----------|-------|
| Phase resistance | 0.62 Ω |
| Stator inductance of three-phase windings | 10⁻³ mH |
| Back-EMF coefficient | 0.066 |
| Number of pole pairs | 4 |
| Rotor equivalent inertia | 3.62 × 10⁻⁴ kg · m² |
| Viscous friction coefficient | 9.444 × 10⁻⁵ N · m · s |

By iterative experimentations and previous studies, good transient response and steady-state performance can be obtained by selecting the appropriate system noise covariance matrix \( Q_\kappa \) and the measurement noise covariance matrix \( R_\kappa \),

\[
Q_\kappa = \text{diag}[0.1 \ 0.1 \ 1 \times 10^{-6} \ 1 \times 10^{-6} \ 0],
\]

\[
R_\kappa = \text{diag}[0.1 \ 0.1 \ 1 \times 10^{-3}],
\]

and \( \alpha = 0.009, \beta = 2 \).
Figure 5.  Speed estimated performance under abrupt load variation (scale-corrected minimal skew simplex UT).

Figure 6.  Position estimated performance under abrupt load variation (scale-corrected minimal skew simplex UT).

Figure 7.  Position error under abrupt load variation (scale-corrected minimal skew simplex UT).
Figure 8. Speed estimated performance when reference speed varies (scale-corrected minimal skew simplex UT).

Figure 9. Position estimated performance when reference speed varies (scale-corrected minimal skew simplex UT).

Figure 10. Position error when reference speed varies (scale-corrected minimal skew simplex UT).
Table 1. general UT.

| Times | Elapsed time(s) |
|-------|-----------------|
| 1     | 0.000278        |
| 2     | 0.000271        |
| 3     | 0.000292        |
| 4     | 0.000284        |
| 5     | 0.000269        |
| 6     | 0.000340        |
| 7     | 0.000308        |
| 8     | 0.000308        |
| 9     | 0.000277        |
| 10    | 0.000295        |
| ...   | ...             |
| n     |                 |
| Total (n = 10,000) | 3.431525 |

State estimation starts with turning the motor system on, and initial values of state estimation are chosen as

\[
\hat{x}_0 = E(x_0) = [0 0 0 0 0]^T, \\
P_0 = \text{Var}(x_0) = E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T] = \text{diag}[0.01 0.01 0.01 0.01 0.01].
\]

Then, the effectiveness and practicability of the speed estimated method for BLDCM are proved by two different groups of simulation experiments with the changes of reference speed and load torque. Figures 5–7 show the estimations of the motor speed, position when the load torque changes from 0 to 5 N·m at \( t = 0.2 \) s under the condition that the reference speed is always 2000 r/min. Figures 8–10 show the estimations of the motor speed, position when the reference speed changes from 1500 to 2000 r/min at \( t = 0.2 \) s under the condition that the load torque \( T_L \) is always 2 N·m.

As shown in Figures 5 and 8, the speed observer designed in this paper can accurately estimate the motor real speed and track the speed quickly when the reference speed changes. When the outside load torque changes, the motor speed can be fed into a stable state in a very short time. Figures 6, 7, 9 and 10 verify that this method has a high accuracy of estimation of rotor position.

About the computation time of the two different UTs (the two blocks in colour), we give Tables 1 and 2 and the single running time and the total running time are also listed in them. From the tables, it is obviously that the computation time of the scale-corrected minimal skew simplex UT is less than the general UT.

Table 2. scale-corrected minimal skew simplex UT.

| Times | Elapsed time(s) |
|-------|-----------------|
| 1     | 0.000227        |
| 2     | 0.000234        |
| 3     | 0.000233        |
| 4     | 0.000227        |
| 5     | 0.000218        |
| 6     | 0.000240        |
| 7     | 0.000232        |
| 8     | 0.000225        |
| 9     | 0.000226        |
| 10    | 0.000229        |
| ...   | ...             |
| n     |                 |
| Total (n = 10,000) | 2.510272 |

6. Conclusions
The sensorless control technique can effectively enhance the reliability and cut down the hardware cost. Therefore, this paper is concerned with the problem of BLDCM sensorless control by the use of scale-corrected minimal skew simplex sampling UKF. In this algorithm, we use the minimal skew simplex UT which has the least number of sigma points to capture the mean and covariance so as to save computational time. In addition, to overcome the drawback of nonlocal effects caused by the increase in state dimensions, the scaled UT is introduced into this algorithm. On the flip side, we use a back-EMF shape function based on Fourier series instead of the back-EMF function described by a piecewise linear function in order to streamline program codes. In the end, simulation results of the UKF for BLDCM driven with noise illustrate that the proposed method is effective and practical.

Recently, under pressure from high-energy prices, immediate actions should be taken to strengthen the study of motor control methods for BLDC motor to save energy. In addition, in the most recent sensorless drive methods, rotor position estimation precision depends on motor parameters and measured quantities. However, the ambient environment and motor ageing will greatly affect the accuracies of parameters and measurements that lead to the poor control performance. Therefore, adapting the artificial intelligence control which can compensate parameter variations is necessary. Also, benefiting from the rapid development of the microprocessor technology, it is possible to accomplish the complicated control algorithms such as neural network, model predictive control (Fu, Aghezzaf, and Keyser, 2014) and fuzzy logic with the fast DSP chips.

Disclosure statement
No potential conflict of interest was reported by the authors.

Funding
This work was supported in part by the National Natural Science Foundation of China under Grant (61374039, 61203143), Shanghai Pujiang Program under Grant 13PJ1406300, Shanghai Natural Science Foundation of China under Grant 13ZR1428500,
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