Designing of Timber Bolt Connection Subjected To Double Unequal Shears

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Abstract. The paper deals with load-carrying capacity of bolted connections subjected to unequal double shear with thin plates as outer members and inner timber member. This type of connection is usually widespread and in building support structures made of wood is commonly used. This may occur for example in skeletal structures which contain structural elements based on wood, but also for smaller wooden buildings. Specifically, this type of connection can be found in ceiling structures in the joint joists and beams. If one joist greater margin than the second, bringing the load on the side of the joists of a larger span greater loads than on the side with a smaller span joist. Structure engineer, who is designing such a connection, must use for the design of the connection design procedures and formulas from which he or she calculates the design resistance in order to carry out further assessment of the reliability of the connection in the ultimate limit state. The load-carrying capacity of this connections type can be calculated at present according to Johansen’s equations, which are also contained in present European standard for the design timber structures -Eurocode 5. These Johansen’s equations assume that the loads which act on the outer plates are equal. For this reason, the structure engineer is often forced to use formulas intended for the timber bolt connection subjected to double equal shear and he or she must find ways how to use them although the formulas are not suitable. This paper deals with the case, when the loads acting on the outer plates are unequal.

1. Introduction
One of the most important issues of timber structures design is the design of connections between their members. In the past timber structure connections were made as scribed wooden joints without certain steel parts. Nowadays, but the steel parts are widely used in connections of timber structures, like are bolts, dowels, nails, plates or different steel welded elements. For the design of connections in timber structures is nowadays possible to use the equations which were derived by Johansen [1]. These Johansen’s equations are also used in [4] and can be also found for example in [2] or [3], where the main principles of their derivation are also presented. According to Johansen’s equations are calculated the characteristic load–carrying capacity of connections and they described the behavior of connections in the ultimate limit state.

2. Investigated connection
This paper deals with the connection type, which is presented in Figure 1.
Figure 1. Bolted connection subjected to double shear with thin plates as outer members and inner timber member

In Figure 1 $F_{d1}$ and $F_{d2}$ are design loads acting on the steel plates. If we assume, that $F_{d1} = F_{d2} = F_d$ and also assume that the outer steel plates are thin ($t_p \leq 0,5d$), we can calculate the characteristic load–carrying capacity per shear plane of the one bolt according to the following Johansen’s equation.

$$R_k = \min \left\{ 1,15 \cdot \sqrt{\frac{0,5 \cdot f_{h,k} \cdot t_2 \cdot d}{2 \cdot M_{y,Rk} \cdot f_{h,k} \cdot d}} \right\}$$

where:
- $f_{h,k}$ is the characteristic embedment strength of the timber member.
- $t_2$ is the thickness of the timber member,
- $d$ is the bolt diameter,
- $M_{y,Rk}$ is the characteristic bolt yield moment.

The top member of the equation (1) corresponds with the failure mode, when the $M_{y,Rk}$ of the bolt is not exceeded and it stays straight. The failure begins only by reaching of the characteristic embedment strength in the timber member. The bottom member of the equation (1) corresponds with the failure mode, when the $M_{y,Rk}$ of the bolt is exceeded and the plastic hinge occurs in it.
In EN 1995-1-1 the bottom member of the equation (1) is increased about the contribution of the rope effect. This rope effect is determined as \(\frac{R_{ax,Rk}}{4}\), but it can’t be higher than 25% of the bottom member in the equation (1). The \(R_{ax,Rk}\) is the characteristic axial load-carrying capacity of the bolt. So the equation (1) for \(R_k\) with the contribution of the rope effect is:

\[
R_k = \min \left\{ 0.5 \cdot f_{h,k} \cdot t_2 \cdot d , 1.15 \cdot \sqrt{2 \cdot M_{y,Rk} \cdot f_{h,k} \cdot d} + \frac{R_{ax,Rk}}{4} \right\}
\]  

(2)

The design load–carrying capacity per shear plane of the one bolt \(R_d\) is then calculated according to EN 1995-1-1 as follows:

\[
R_d = k_{mod} \cdot \frac{R_k}{\gamma_M}
\]

(3)

where:

- \(k_{mod}\) is a modification factor taking into account the effect of the duration of load and the moisture content,
- \(\gamma_M\) is partial factor for a material property.

The reliability condition for the connection is:

\[
\frac{F_{Ed}}{R_d} \leq 1
\]

(4)

where \(F_{Ed}\) is the design load of the connection.

The previous equations describe only the situation, when the \(F_{d1} = F_{d2}\). But in practice there are many cases, when the outer steel plates are loaded by different loads \(F_{d1} \neq F_{d2}\). For the following reflections is assumed that \(F_{d1} > F_{d2}\).

One way how to deal with this problem is that we calculate separately two cases “A” and “B”. For case “A” we calculate the design load–carrying capacity for the connection in single shear with one steel plate, which is loaded by \(F_{d1}\). The design load–carrying capacity for this case \(R_{dA}\) is calculated according to EN 1995-1-1 as follows:

\[
R_{kA} = \min \left\{ 0.4 \cdot f_{h,k} \cdot t_2 \cdot d , 1.15 \cdot \sqrt{2 \cdot M_{y,Rk} \cdot f_{h,k} \cdot d} + \frac{R_{ax,Rk}}{4} \right\}
\]

(5)

\[
R_{dA} = k_{mod} \cdot \frac{R_{kA}}{\gamma_M}
\]

(6)

Then we calculate case “B” where we assume that the connection is in double shear and has both outer steel plates loaded by \(F_{d1}\). The design load–carrying capacity for this case \(R_{dB}\) we can calculate from equations (2) and (3).

The resultant design load–carrying capacity of the connection \(R_d\) we will find as the minimum from \(R_{dA}\) and \(R_{dB}\), so:
It is evident that the previous method does not describe the real behaviour of the connection in the ultimate limit state. It is evident on the safe side and is not economical. In the following part of the paper a new method is suggested, which enables to calculate the design load-carrying capacity of the connection presented in Figure 1, when $F_{d1} \neq F_{d2}$ and $F_{d1} \geq F_{d2}$.

3. Results and discussions

For the determination of the design load-carrying capacity we have to consider different failure modes. The failure modes are shown in Figure 2. The pictures a) and b) show the connection in the ultimate limit state. The capacities indicated from the pictures are characteristic.

**Failure mode a):** This failure mode is caused by creating of the plastic hinge in the bolt. If the loads of the outer steel plates are unequal, it can be assumed that the plastic hinge will occur on the side with the higher load. It is the load $R_{k1}$. Over the other side of the bolt the plastic hinge will not begin. Under this assumption we can use the bottom member of equation (2), than

$$R_{k1} = 1.15 \cdot \sqrt{2 \cdot M_{y,Rk} \cdot f_{h,k} \cdot d + \frac{F_{ax,Rk}}{4}}$$

(8)

The corresponding design load-carrying capacity:

$$R_{d1} = k_{mod} \cdot \frac{R_{k1}}{\gamma_M}$$

(9)

The reliability of the connection for this failure mode results from the condition

$$\frac{F_{d1}}{R_{d1}} \leq 1$$

(10)
**Failure mode b):** For the determination of the characteristic load–carrying capacity the Figure 3 will be used.

![Figure 3](image)

**Figure 3.** Scheme for the determination of the characteristic load–carrying capacity

Moment equilibrium condition to the point B can be written:

\[-f_{h,2,k} \cdot d \cdot a^2 + f_{h,2,k} \cdot d \cdot \frac{b^2}{2} - R_{k2} \cdot t_2 = 0\]  \hspace{1cm} (11)

where:

- \( f_{h,2,k} \) is the characteristic embedment strength in the timber member,
- \( t_2 \) is the thickness of the middle timber member,
- \( a, b \) are dimensions shown in the figure,
- \( d \) is the bolt diameter,
- \( M_{y,b} \) is the characteristic bolt yield moment,
- \( R_{k1} \) and \( R_{k2} \) are the characteristic load–carrying capacities of the outer steel plates.

In accordance with the Figure 3 the dimension

\[ a = \frac{t_2 - b}{2} \]  \hspace{1cm} (12)

After substitution \( a \) and related modifications, we will get the following quadratic equation:
\[ b^2 + 2 \cdot t_2 \cdot b - \left( t_2^2 + \frac{4 \cdot R_{k2} \cdot t_2}{f_{h,2,k} \cdot d} \right) = 0 \]  

(13)

From the equation (13) we can get the dimension \( b \):

\[
    b = \sqrt{2 \cdot t_2^2 + \frac{2 \cdot R_{k2} \cdot t_2}{f_{h,2,k} \cdot d}} - t_2
\]

(14)

In accordance with Figure 3 the force equilibrium condition must be fulfilled:

\[
    R_{k1} + R_{k2} = f_{h,2,k} \cdot d \cdot b
\]

(15)

By substitution and related modification, we get the formula for \( R_{k1} \):

\[
    R_{k1} = f_{h,2,k} \cdot d \cdot \left( \sqrt{2 \cdot t_2^2 + \frac{2 \cdot R_{k2} \cdot t_2}{f_{h,2,k} \cdot d}} - t_2 \right) - R_{k2}
\]

(16)

In the formula (16) the load-carrying capacities are characteristic. For static calculations and checks, we need to work with the design load-carrying capacities. The relation between the design load-carrying capacities and the characteristic load-carrying capacities are given by following formulas:

\[
    R_{d1} = k_{mod} \cdot \frac{R_{k1}}{\gamma_M} ; \quad R_{d2} = k_{mod} \cdot \frac{R_{k2}}{\gamma_M}
\]

(17)

**Figure 4.** Dependence \( R_{d1} \) on the \( R_{d2} \).
It can be transformed formula (17):

\[ R_{k1} = \frac{\gamma_M}{k_{mod}} \cdot R_{d1} ; \quad R_{k2} = \frac{\gamma_M}{k_{mod}} \cdot R_{d2} \quad (18) \]

The formula (18) we can substitute into the formula (16) and after some mathematical equation editing we get the formula for the design load-carrying capacity \( R_{d1} \):

\[ R_{d1} = \frac{k_{mod}}{\gamma_M} \cdot f_{h,2,k} \cdot d \cdot \left( \sqrt{2} \cdot \left[ \frac{2 \cdot \gamma_M \cdot R_{d2} \cdot t_2}{k_{mod} \cdot f_{h,2,k} \cdot d} - t_2 \right] - R_{d2} \right) \quad (19) \]

The dependence of the \( R_{d1} \) on the \( R_{d2} \) is shown in Figure 4. The graph is calculated for the following parameters:

\( f_{h,2,k} = 16,46 \) MPa, \( t_2 = 140 \) mm, \( d = 16 \) mm, \( k_{mod} = 0,8 \), \( \gamma_M = 1,3 \).

Let’s assume the situation when \( R_{d2} = 0 \) kN. This case corresponds to the connection in single shear with one outer steel plate. If we substitute \( R_{d2} = 0 \) kN into the equation (19), we get:

\[ R_{d1} = \frac{k_{mod}}{\gamma_M} \cdot f_{h,2,k} \cdot d \cdot \left( \sqrt{2} - 1 \right) \cong \frac{k_{mod}}{\gamma_M} \cdot 0,4 \cdot f_{h,2,k} \cdot d \cdot t_2 \quad (20) \]

The formula (20) is the same like the top member in equation (5), which describes a connection in single shear with one outer steel plate. If we again assume \( f_{h,2,k} = 16,46 \) MPa, \( t_2 = 140 \) mm, \( d = 16 \) mm, \( k_{mod} = 0,8 \), \( \gamma_M = 1,3 \) we get:

\[ R_{d1} = \frac{0,8}{1,3} \cdot 16,46 \cdot 16 \cdot 140 \cdot (\sqrt{2} - 1) = 9,40 \text{ kN} \quad (21) \]

The graph in the Figure 4 ends in point, where \( R_{d1} = R_{d2} \). This point corresponds to a connection in double shear with two outer steel plates loaded by equal loads. This can be proved when we substitute the values into the top member of equation (2) and calculate it:

\[ \frac{k_{mod}}{\gamma_M} \cdot 0,5 \cdot f_{h,2,k} \cdot d \cdot t_2 = \frac{0,8}{1,3} \cdot 0,5 \cdot 16,46 \cdot 16 \cdot 140 = 11,35 \text{ kN} \quad (22) \]

There is seen, that the value results from the equation (22) is the same as the value results from the formula (19), if we substitute \( R_{d2} = 11,35 \) kN into it.

4. Conclusions
In the paper the formulas which can be used for calculation of load-carrying capacity of timber structure bolt connections subjected to double unequal shears were derived. There was also derived, that the connections subjected to double equal shears or to the single shears are only special cases of the ones subjected to double unequal shears.

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