Automatic calibration of 3D anisotropic yield criteria using a parallel evolutionary algorithm

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Abstract. Advanced 3D non-quadratic anisotropic yield criteria are usually required to describe highly anisotropic materials such as aluminium alloys. One issue related to the advanced anisotropic yield criteria is that they often require the identification of many parameters that are difficult to calibrate. An automatic and reliable technique for the determination of the coefficients of 3D anisotropic yield criteria is presented here. The error between predictions and experimental data is minimised by finding the yield criteria coefficients using a modification of the evolutionary algorithm described in (Pedroso et al. 2017 Applied Soft Computing 61 995-1012). The method is implemented for parallel computation to both speed up calculations and to make sure the results are consistent after several runs. The results show that the proposed method can produce good coefficients for the 3D anisotropic yield criteria.

1. Introduction

One of the important factors for simulating the sheet metal forming processes is the plastic anisotropy. The rolling process and thermomechanical treatment when manufacturing metal sheets cause non-uniformed orientations of the crystal-texture which leads to anisotropic behaviour. Hill [1] introduced one of the most well-known quadratic anisotropic yield criterion based on von Mises criterion [2]. However, several researchers have shown that the quadratic base criterion cannot accurately describe highly anisotropic materials such as aluminium alloys and titanium alloys [3]. This is because the second order homogeneous polynomial criterion is unable to model materials with large variation in both yield stresses and R-values [4].

In order to accurately characterise anisotropic materials, many advanced anisotropic plasticity criteria have been proposed in the past few decades [4-10]. In this work, we consider two fully 3D anisotropic yield criteria; the yield model of Cazacu and Barlat [6] (CB2001) which extends Drucker’s yield criterion to include orthotropic case by using the theory of representation, and the model of Barlat et al. [8] (Yld2004) which introduces anisotropy into a Hershey like yield criterion by considering two linear transformations of the deviatoric stress. Although both of these models can accurately characterise highly anisotropic materials, they each require the identification of 18 parameters that are difficult to
calibrate. In this paper, we will use a parallel evolutionary algorithm to automatically calibrate the material coefficients for CB2001 and Yld2004 models.

An evolutionary algorithm is a derivative-less optimisation technique which can handle many complex (non-convex) problems [11, 12]. However, owing to the stochastic nature of the evolutionary algorithm, the results are not necessarily converged. The current algorithm is implemented by using CPU parallelisation to enhance the algorithm's efficiency.

In Section 2, the 3D anisotropic plasticity yield criteria will be reviewed. Section 3 shows the objective error functions and identification procedure used in this paper. The procedures for the parallel evolutionary algorithm are explained in Section 4. Then, some examples with discussions of the results are present in Section 5, and finally, Section 6 gives some conclusive remarks.

2. 3D Anisotropic Plasticity Models

In this section, the expressions for the yield criteria of CB2001 and Yld2004 will be reviewed.

2.1. CB2001 anisotropic yield criterion

The CB2001 yield criterion [6] is an extension of Drucker yield criterion and is given by

\[
F_{CB} = J_2^2 - c_0 J_3^2 = 27 \left( \frac{\sigma_y}{\sigma} \right)^6,
\]

where \( \sigma_y \) is the uniaxial yield stress related to isotropic hardening, \( J_2 \) is the modified second invariant of the stress deviator given by

\[
J_2 = \frac{a_1}{6} (\sigma_x - \sigma_y)^2 + \frac{a_2}{6} (\sigma_y - \sigma_z)^2 + \frac{a_3}{6} (\sigma_z - \sigma_x)^2 + a_4 \sigma_{xy}^2 + a_5 \sigma_{yz}^2 + a_6 \sigma_{xz}^2,
\]

and \( J_3 \) is the modified third invariant of the stress deviator given by

\[
J_3 = \frac{\sigma_x^3}{27} (b_1 + b_2) + \frac{\sigma_y^3}{27} (b_3 + b_4) + \frac{\sigma_z^3}{27} \left[ 2(b_1 + b_4) - b_2 - b_3 \right] - \frac{\sigma_x^2}{9} (b_1 \sigma_y + b_2 \sigma_z)
\]

\[
- \frac{\sigma_y^2}{9} (b_3 \sigma_x + b_4 \sigma_z) - \frac{\sigma_z^2}{9} \left[ (b_1 - b_2 + b_3) \sigma_x + (b_1 - b_3 + b_4) \sigma_y \right]
\]

\[
+ \frac{2 \sigma_x \sigma_y \sigma_z}{9} (b_1 + b_4) - \frac{\sigma_x^2}{3} \left[ 2 b_9 \sigma_y - 2 b_9 \sigma_z - (2 b_9 - b_9) \sigma_x \right]
\]

\[
- \frac{\sigma_y^2}{3} \left[ 2 b_{10} \sigma_z - b_7 \sigma_y - (2 b_{10} - b_7) \sigma_x \right] - \frac{\sigma_z^2}{3} \left[ -b_6 \sigma_y - b_7 \sigma_z + (b_6 - b_7) \sigma_x \right]
\]

\[
+ 2 b_{11} \sigma_{xy} \sigma_{yz} \sigma_{xz}.
\]

Here \( \sigma_x \) is the stress in the rolling direction, \( \sigma_y \) is the stress in the transverse direction, \( \sigma_z \) is the stress in the normal direction, and \( \sigma_{xy}, \sigma_{yz}, \sigma_{xz} \) are the shear stresses. The coefficients \( a_1, a_2, ..., b_{11}, c_0 \) are the 18 parameters which characterise the six-dimensional yield surface. These parameters are chosen by fitting Eq. (1) to a given set of data, usually obtained experimentally (e.g. via uniaxial tension tests, bulge tests, and/or disk compression tests), or through simulation (e.g. polycrystal modelling). In order to ensure the resulting yield surface is convex, the condition \(-\frac{27}{8} < c_0 < \frac{18}{8}\) has to be satisfied [6]. However, we find this condition cannot always guarantee the convexity of the yield surface. Additional check of the convexity has to be applied.

2.2. Yld2004 anisotropic yield criterion

The Yld2004 yield criterion is based on two linear transformations of the stress deviator, and is given by

\[
F_{Yld} = |S_{1} - S'_{1}|^a + |S_{1} - S''_{1}|^a + |S_{1} - S''_{1}|^a + |S_{2} - S'_{1}|^a + |S_{2} - S'_{1}|^a + |S_{3} - S'_{1}|^a + |S_{3} - S'_{1}|^a + |S_{3} - S'_{1}|^a = 4\sigma_y^a.
\]
The exponent $a$ is dependent on the material crystal structure and is set to 6 for Body Centred Cubic (BCC) and 8 for Face Centred Cubic (FCC) materials [8]. The $S_i^T$ and $S_i^{TT}$ ($i = 1, 2, 3$) are the principal values of the deviatoric stress $s$ subject to two (in general different) linear transformations;

$$
\hat{S}^{(TT)} = \begin{bmatrix}
S_x(k) & S_y(k) & S_z(k) \\
S_x(k) & S_y(k) & S_z(k) \\
S_x(k) & S_y(k) & S_z(k)
\end{bmatrix}
\begin{bmatrix}
0 & -c_{12}^{(k)} & -c_{13}^{(k)} \\
-c_{21}^{(k)} & 0 & -c_{23}^{(k)} \\
-c_{31}^{(k)} & -c_{32}^{(k)} & 0
\end{bmatrix}
\begin{bmatrix}
S_x \\
S_y \\
S_z
\end{bmatrix},
$$

where the superscript $(k)$ stands for either $'$ or $''$. The $c_{ij}^{(k)}$ are the 18 parameters which define a class of six-dimensional yield surfaces. Analogous to the $a_1, a_2, ..., b_{11}, c_0$ of the CB2001 model, these parameters are typically found by fitting Eq. (4) to experimentally or numerically generated test data. We note that the Yld2004 yield criterion defines a convex yield surface [8].

3. Objective Error Functions

In this work, we identify the material coefficients of the CB2001 and Yld2004 models by minimising a new proposed error function designed for the evolutionary algorithm

$$
E = E_{\sigma} \cdot E_{r} + E_{\sigma} + E_{r},
$$

where

$$
E_{\sigma} = \sum_k w_k \left( \frac{\sigma_y^m - \sigma_{y}^{m}}{\max \left( \frac{\sigma_y^m}{\sigma_0^m} \right) - \min \left( \frac{\sigma_y^m}{\sigma_0^m} \right)} \right)^2, \text{ and } E_{r} = \sum_i w_i \left( \frac{r_i - r_{i}^0}{\max \left( \frac{r_i}{r_{i}^0} \right) - \min \left( \frac{r_i}{r_{i}^0} \right)} \right)^2.
$$

Here, $\sigma_k^m = [\sigma_y^m, \sigma_z^m, \sigma_{tyz}^m, \sigma_{txz}^m, \sigma_{sz}^m, \sigma_{szz}^m]$ represents the modelling flow stresses calculated by the yield criterion. $\sigma_y^m$ is the uniaxial tension tested at $0$ degree from the rolling direction (e.g. $\sigma_y^m$ is the tension flow stress in rolling direction) given by

$$
\frac{\sigma_y^m}{\sigma_y} = \frac{\sigma_y^m}{\sigma_y(\sigma_x, \sigma_y, \sigma_z, \sigma_y, \sigma_y, \sigma_z)} = \frac{\sigma_y^m}{\sigma_y(\sigma_y^m \cos^2 \theta \sigma_y^m \sin^2 \theta \sigma_y^m \cos \theta \sin \theta \sigma_y^m)};
$$

$\sigma_y^m$ is the in-plane balanced biaxial tension given by

$$
\frac{\sigma_y^m}{\sigma_y} = \frac{\sigma_y^m}{\sigma_y(\sigma_x, \sigma_y, \sigma_z, \sigma_y, \sigma_y, \sigma_z)} = \frac{\sigma_y^m}{\sigma_y(0,0,0,0,0,0)};
$$

$\sigma_{tyz}$ is the 45° tension stress in (y-z, transverse - normal direction) plane given by

$$
\frac{\sigma_{tyz}^m}{\sigma_y} = \frac{\sigma_{tyz}^m}{\sigma_y(\sigma_x, \sigma_y, \sigma_z, \sigma_y, \sigma_y, \sigma_z)} = \frac{\sigma_{tyz}^m}{\sigma_y(\frac{1}{2} \sigma_{tyz}^m \frac{1}{2} \sigma_{tyz}^m \sigma_{tyz}^m \sigma_{tyz}^m \sigma_{tyz}^m \sigma_{tyz}^m)};
$$

$\sigma_{txz}^m$ is the 45° tension stress in (x-z, rolling - normal direction) plane given by

$$
\frac{\sigma_{txz}^m}{\sigma_y} = \frac{\sigma_{txz}^m}{\sigma_y(\sigma_x, \sigma_y, \sigma_z, \sigma_y, \sigma_y, \sigma_z)} = \frac{\sigma_{txz}^m}{\sigma_y(\frac{1}{2} \sigma_{txz}^m \frac{1}{2} \sigma_{txz}^m \sigma_{txz}^m \sigma_{txz}^m \sigma_{txz}^m \sigma_{txz}^m)};
$$

$\sigma_{syz}$ is the simple shear stress in (y-z, transverse - normal direction) plane given by

$$
\frac{\sigma_{syz}^m}{\sigma_y} = \frac{\sigma_{syz}^m}{\sigma_y(\sigma_x, \sigma_y, \sigma_z, \sigma_y, \sigma_y, \sigma_z)} = \frac{\sigma_{syz}^m}{\sigma_y(0,0,0,0,0,0)};
$$

and $\sigma_{sz}^m$ is the simple shear stress in (x-z, rolling - normal direction) plane given by

$$
\frac{\sigma_{sz}^m}{\sigma_y} = \frac{\sigma_{sz}^m}{\sigma_y(\sigma_x, \sigma_y, \sigma_z, \sigma_y, \sigma_y, \sigma_z)} = \frac{\sigma_{sz}^m}{\sigma_y(0,0,0,0,0,0)};
$$

and
\[
\frac{\sigma_{m}^{m}}{\sigma_{y}} = \frac{\sigma_{m}^{m}}{\sigma_{y}(\sigma_{x}, \sigma_{y}, \sigma_{z}, \sigma_{xy}, \sigma_{xz}, \sigma_{yz})} = \frac{\sigma_{m}^{m}}{\sigma_{y}(0,0,0,0,\sigma_{xz}^{2})}.
\]

Similarly, \(r_{b}^{m} = [r_{b}^{m} \cdot r_{b}^{m}]\) stands for the modelling R-value given by the plastic yield criterion, where R-value at \(\theta\)-degree from the rolling direction is [8]

\[
r_{b}^{m} = -1 - \left(\sigma_{y}^{-1} \frac{\partial \sigma_{y}}{\partial \sigma_{z}}(\cos^{2} \theta, \sin^{2} \theta, 0, \cos \theta \sin \theta, 0, 0)\right)^{-1},
\]

and the R-value given by the balanced biaxial tension is

\[
r_{b}^{m} = -1 - \left(\sigma_{y}^{-1} \frac{\partial \sigma_{y}}{\partial \sigma_{z}}(0,0,1,0,0,0)\right)^{-1}.
\]

The superscript \(e\) stands for the corresponding experimental or polycrystal model data. The \(w_{k}\) and \(w_{l}\) are the weighting parameters. In this study we set \(w_{0,45,90} = 150, w_{15,30,60,75} = 80, w_{k} = 100\) and all other weights to \(w = 20\). We note that values may not be optimal and further study of these parameters is required.

4. Parallel Evolutionary Algorithm

The evolutionary algorithm used in this study is heavily based on the method proposed in [12], but with some minor changes. The major steps of the current algorithm are:

1. A set of \(N_{sol}\) trial solutions are randomly generated for each of \(N_{pros}\) groups.
2. Within each group the trial solutions are evaluated in parallel according to Algorithms 1-8 given in [12]. The best trial solutions [according to the cost function Eq. (6) and identification of the Pareto front of the two objectives given in Eq. (7)] are then selected and passed to the next iteration. All groups perform \(N_{evol}\) iterations.
3. After Step 2, the best \(N_{pros}/2\) solutions [according to the objective Eq. (6)] are selected from the current, as well as past, iteration loops.
4. \(N_{pros}/2\) (unique) groups are then randomly selected. For each of these groups, a unique solution from the best \(N_{pros}/2\) solutions is chosen to replace an existing (randomly selected) within-group solution.
5. Step 1-4 are repeated until the minimum \(E\) is less than a target threshold. The total number of evolutionary iterations \(N_{iter}\) (loops through Steps 1-4), as well as the (globally) best solution (i.e. the best material coefficients) are then returned.

In this study, we set \(N_{sol} = 20, N_{pros} \geq 4\) and \(N_{evol} = 1000\). These setting where chosen based on the observed convergence of the solution and may not be optimal. Additionally, the solution was constrained to have all material parameters (for both yield models) lie in the interval [0.5, 2] by resetting values outside this interval to the closest boundary value in Step 2. This choice was also empirically based and further study is required to ascertain the validity of this assumption.

5. Results and Discussion

In this section, we investigate the performance of the established parallel evolutionary algorithm by automatically calibrating the material coefficients of two aluminium alloys sheets: AA6111-T4 and AA2090-T3. We use the experimental data from [7, 8], which is reproduced in Tables 1 and 2. Note that we use an exponent of \(a=8\) for the YLD2004 model.

| \(\sigma_{x}^{2}\) | \(\sigma_{y}^{2}\) | \(\sigma_{z}^{2}\) | \(\sigma_{xy}^{2}\) | \(\sigma_{xz}^{2}\) | \(\sigma_{yz}^{2}\) | \(r_{1}^{2}\) | \(r_{2}^{2}\) | \(r_{3}^{2}\) |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 1.000          | 1.000          | 0.990          | 0.970          | 0.960          | 0.960          | 0.949          | 1.010          | 1.080          | 1.050          |
| \(\sigma_{x}^{2}\) | \(\sigma_{y}^{2}\) | \(\sigma_{z}^{2}\) | \(\sigma_{xy}^{2}\) | \(\sigma_{xz}^{2}\) | \(\sigma_{yz}^{2}\) | \(r_{1}^{2}\) | \(r_{2}^{2}\) | \(r_{3}^{2}\) |
| 0.677          | 0.657          | 1.000          | 1.000          | 0.829          | 0.685          | 0.685          | 0.707          | 0.707          | 1.409          |
The results in Figures 1 and 2 show the developed evolutionary algorithm can give better material coefficients for the Yld2004 criterion compared to the reference data given in [8]. We find good solutions can be obtained in less than 10 evolutionary iterations. The coefficients determined for aluminium AA6111-T4 are listed in Table 3. We note that for both CB2001 and Yld2004 models, we found similar quality fits with different material parameters [according to criterion Eq. (6)]. This is likely due to that not all parameters are independent [13] or this may be due to insufficient experimental data.

### Table 3. AA6111-T4’s anisotropic coefficients given by evolution algorithm.

| $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $b_1$ | $b_2$ | $b_3$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0.786 | 0.859 | 0.522 | 0.723 | 0.592 | 0.560 | 1.065 | 1.987 | 1.887 |
| $b_4$ | $b_5$ | $b_6$ | $b_7$ | $b_8$ | $b_9$ | $b_{10}$ | $b_{11}$ | $c_0$ |
| -0.258 | 1.129 | 0.244 | 2.000 | -0.441 | 0.896 | 1.228 | 1.677 | -0.494 |

### Table 2. Experimental data for AA2090-T3 referenced from [7, 8].

| $\sigma_y$ | $\sigma_z$ | $\sigma_x$ | $\sigma_{yx}$ | $\sigma_{yz}$ | $\sigma_{zx}$ | $\sigma_{xy}$ | $\sigma_{xz}$ | $\sigma_{yz}$ |
|------------|------------|------------|---------------|---------------|---------------|---------------|---------------|---------------|
| 1.000      | 0.960      | 0.831      | 0.805         | 0.787         | 0.910         | 1.035         | 0.900         | 0.890         |
| $\sigma_{yx}$ | $\sigma_{yz}$ | $\sigma_{zx}$ | $\sigma_{xy}$ | $\sigma_{xz}$ | $\sigma_{yz}$ | $\sigma_{xy}$ | $\sigma_{xz}$ | $\sigma_{yz}$ |
| 0.230      | 1.429      | 3.533      | 7.524         | 5.000         | 2.381         | 3.286         | 3.190         |

### Figure 1. Experimental and calculated normalised flow stress and R-value for AA6111-T4 with CB2001. The value of the objective function is $E = 5.87$.

### Figure 2. Experimental and calculated normalised flow stress and R-value for AA6111-T4 with Yld2004. The value of the objective function is $E = 0.32$ (present) and $E = 4.54$ [8].

### Figure 3. Experimental and calculated normalised flow stress and R-value for AA2090-T3 with CB2001. The value of the objective function is $E = 23.33$ (present) and $E = 336.2$ [6].
The results in Figures 3 and 4 show the developed evolutionary algorithm can give better material coefficients for the CB2001 and Yld2004 criteria compared to the reference material coefficients given in [6, 8]. We find good solutions after running less than 10 evolutionary iterations. The material coefficients found for aluminium AA2090-T3 are listed in Table 4. Similar to the results for AA6111-T4, we found multiple sets of material coefficients for both models which yielded similar fitting errors.

**Table 4.** AA2090-T3’s anisotropic coefficients given by evolution algorithm.

| CB2001 |       |       |       |       |       |       |       |       |
|--------|-------|-------|-------|-------|-------|-------|-------|-------|
| a₁     | 0.674 | 0.526 | 0.936 | 0.729 | 1.615 | 0.324 | 0.949 |
| a₂     | 0.508 |       |       |       |       |       |       |
| a₃     |       |       |       |       |       |       |       |
| a₄     |       |       |       |       |       |       |       |
| a₅     |       |       |       |       |       |       |       |
| a₆     |       |       |       |       |       |       |       |
| b₁     |       |       |       |       |       |       |       |
| b₂     |       |       |       |       |       |       |       |
| b₃     |       |       |       |       |       |       |       |
| b₄     |       |       |       |       |       |       |       |

| Yld2004 (a=8) |       |       |       |       |       |       |       |       |
|---------------|-------|-------|-------|-------|-------|-------|-------|-------|
| c₁₁          | 0.639 | 0.496 | 0.667 | 1.066 | 0.958 | -0.432 | 0.892 | 0.904 |
| c₁₂          | -0.209 | 1.125 | 0.622 | 1.052 | 0.433 | 0.990 | 1.101 | 1.143 |
| c₁₃          | 0.121 | 0.151 | 0.432 | 0.990 | 1.101 | 1.143 | 1.047 | 1.047 |
| c₂₂          | 0.639 | 0.496 | 0.667 | 1.066 | 0.958 | -0.432 | 0.892 | 0.904 |
| c₂₃          | 0.121 | 0.151 | 0.432 | 0.990 | 1.101 | 1.143 | 1.047 | 1.047 |
| c₃₃          | 0.639 | 0.496 | 0.667 | 1.066 | 0.958 | -0.432 | 0.892 | 0.904 |

6. Conclusion Remarks

This research shows that the developed parallel evolutionary algorithm can automatically produce significantly better material coefficients compared to the reference material parameters given by [6, 8]. As a final comment, we note that this algorithm can be used for any yield criterion without modification.

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