Magneto-optical double zero-index media and their electromagnetic properties in the bulk

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Abstract

Double-zero-index media (DZIM) with zero permittivity and permeability are one important class in zero-refractive index photonics. Here, we extended the concept of DZIM and proposed a more general type, i.e., the magneto-optical DZIM (MODZIM), of which the permittivity and the determinant of the Hermitian permeability tensor are simultaneously zero. By formulating the Maxwell’s equations in the basis of complex-valued axes and using some mathematical principles, we studied the electromagnetic (EM) properties in the bulk of the MODZIM with different boundaries and impurities. Inside the MODZIM which is infinite along in the out-of-plane direction, it is shown that the scalar (out-of-plane) field is not uniform in general, in contrast to traditional DZIM where the scalar field is always uniform in the bulk. Nevertheless, for a normal incidence, the uniform scalar field inside the MODZIM can be achieved by optimizing the boundary conditions and doping some types of impurities, such as resonant round cylinders and arbitrary shaped media with a zero permeability. As long as the scalar field is uniform, the propagation of the EM wave inside the MODZIM can be analyzed with closed-form expressions. Our work will extend the study of zero-refractive-index photonics and provide deeper understanding of wave dynamics in the bulk of MODZIM.

1. Introduction

Photonic zero-index media (ZIM) with one or more constitutive parameters being zero exhibit many unique features that enable the exploration of unprecedented light–matter interactions [1–10] and intriguing optical phenomena [11–19]. Compared with the single-ZIM where only the relative permittivity or relative permeability approaches zero, the double-zero-index media (DZIM) [20–24] whose relative permittivity and permeability are simultaneously zero possess the advantages of impedance-matching with air, non-zero group velocity and lower optical loss [25]. Moreover, the DZIM can be implemented using all dielectric media in the optical regime to realize the on-chip applications [26, 27]. On the other hand, the concept of ZIM has also been extended to the electro-optical (magneto-optical) media whose relative permittivity (permeability) is an asymmetric tensor rather than a simple scalar. Studies were mainly focused on the single ZIM where the relative permeability is unit and the determinant of the relative permittivity tensor is zero [28–30]. Recently, we have demonstrated that the metamaterial with simultaneous zero effective permittivity and zero-determinant effective permeability tensor can be realized by using a magneto-optical photonic crystal that carries an unpaired Dirac cone at the Brillouin zone center [31]. Due to the breaking of time-reversal symmetry since \( \mu^* \neq \mu \), the magneto-optical materials are widely used in topological photonics [32–34]. Owing to this, conventional studies are mainly focused on the boundary effect of the magneto-optical ZIM (MOZIM) [28–30, 35], in contrast to the extensive investigations of bulk state properties of the traditional isotropic ZIM. Recently, several works have also been focused on enhancing the light–matter interactions by the MOZIMs [36, 37].
For traditional isotropic ZIM which are infinite in the out-of-plane direction, the decoupling of electricity and magnetism makes the out-of-plane scalar field uniform inside the ZIM. The uniformity greatly simplifies the theoretical analysis and enables some fascinating applications, such as the geometry-free resonant cavities [38, 39], photonic doping [40, 41] and so on. However, the scalar field inside the MOZIM is not necessarily homogeneous. The propagation of electromagnetic (EM) wave in the bulk of the MOZIM and the preconditions of uniform scalar field inside the MOZIM are still need to be explored.

In this work, first, we proposed the concept of magneto-optical double-zero-index media (MODZIM) of which the relative permittivity and determinant of relative Hermitian permeability tensor are simultaneously zero. The MODZIM can be regarded as an extension of the traditional DZIM but possess some peculiarity that are different from the traditional DZIM. Then, using the Maxwell’s equations in the basis of the complex-valued axes which are eigenvectors of the permeability tensor, we studied the propagation and distribution of EM waves in the bulk of the MODZIM. The MODZIM with different boundaries and impurities were studied. It is shown that the out-of-plane scalar field inside the MODZIM is inhomogeneous generally, which is quite different from the traditional ZIM. However, by optimizing the boundaries of the MODZIM, uniform scalar field can be achieved and therefore the propagation of the EM waves can be analytically analyzed. Also, the scalar field is uniform when the impurities are resonant cylinders and geometry symmetry-free \( \mu \)-zero media (the relative permeability is zero). By doping cylinders into the \( \mu \)-zero material impurities, the reflection and transmission of EM waves can be flexibly tuned according to an analytical formula. Although this work is focused on the MODZIM, the theory can be directly applied to the electro-optical DZIM according to the duality.

### 2. The concept and basic properties of MODZIM

Since the magneto-optical effect is absent for the \( H_z \) polarization, here we only consider about the \( E_z \) polarization where the electric field is constant along the out-of-plane (z) direction. Thus, the electric field is a scalar field while the magnetic field is a vector field. The general in-plane (xy-plane) relative permeability tensor of the magneto-optical medium in the Cartesian coordinate is given by

\[
\begin{align*}
\hat{\mu} &= \mu_x \hat{x} \hat{x} + \mu_y \hat{y} \hat{y} - i \mu_z (\hat{x} \hat{y} - \hat{y} \hat{x}), \\
\end{align*}
\]

where \( \mu_x > 0, \mu_y > 0, \mu_z \neq 0 \) are purely real numbers, making sure that \( \hat{\mu} \) is Hermitian. And the out-of-plane relative permittivity is \( \varepsilon \) which is also purely real. As such, the magneto-optical medium we consider here is lossless. As the relative permeability tensor is Hermitian, it can also be rewritten as

\[
\begin{align*}
\hat{\mu} &= \lambda_1 \nabla Z_1^* \otimes \nabla Z_1 + \lambda_2 \nabla Z_2^* \otimes \nabla Z_2, \\
\end{align*}
\]

where \( \otimes \) denotes the tensor product operator, \( \lambda_1, \lambda_2 \) are eigenvalues of \( \hat{\mu} \), and \( \nabla Z_1^*, \nabla Z_2^* \) are the corresponding normalized eigenvectors with \( Z_i = \alpha_i x + i \beta_i y \) being complex coordinates. \( \nabla Z_i \) must be along the complex-valued axis, otherwise \( \mu_z \) goes to zero according to equation (2), in contradiction to that the medium is magneto-optical. Due to the Hermiticity of \( \hat{\mu} \), the eigenvalues \( \lambda_1, \lambda_2 \) are purely real, and the eigenvectors satisfy the following relations

\[
\nabla Z_i \cdot \nabla Z_j^* = \delta_{ij}, \hat{z} \times (\nabla Z_i) = \nabla Z_j^*,
\]

where \( \hat{z} \) represents the unit normal vector along the z direction. The eigenvalues \( \lambda_1, \lambda_2 \) characterize the relative permeabilities along the two complex-valued principle axes \( \nabla Z_1, \nabla Z_2 \) of the magneto-optical medium.

The refractive indices along the \( x \) and \( y \) directions of the magneto-optical medium are [31]

\[
\begin{align*}
n_x &= \sqrt{\frac{\lambda_1 \lambda_2}{\mu_x} \varepsilon}, \\
n_y &= \sqrt{\frac{\lambda_1 \lambda_2}{\mu_y} \varepsilon}. \\
\end{align*}
\]

According to equation (4), the refractive indices are zeros when either \( \lambda_1 \) or \( \lambda_2 \) is zero. Therefore, we refer the magneto-optical medium whose permeability tensor has a zero determinant (\( \det | \hat{\mu} | = \lambda_1 \lambda_2 \)) to the MOZIM. In reference [28], the authors have pointed out that the zero determinant is corresponding to the transition point for the medium from opaque to transparency.
Figure 1. A Gaussian beam incident to (a) a DZIM slab and (b) a MODZIM slab. The DZIM and MODZIM are embedded in air. The real parts of the electric field are shown. Periodic boundary conditions are applied along the $y$ direction. The frequency and beam waist of the Gaussian beam are 165 MHz and 4 m, respectively. The constitutive parameters of the MODZIM and MOZIM are approaching 0, while for the MODZIM, $\lambda_1 = 0$, $\lambda_2 = 2$, $Z_1 = (-ix + y)/\sqrt{2}$, $Z_2 = (ix + y)/\sqrt{2}$.

Since a zero eigenvalue induces that the permeability tensor is constructed by only one eigenvector, the MOZIM is also called the complex axis nihility medium in some references [29, 30]. According to equation (1), when $\lambda_1 = 0$, the other eigenvalue and the two normalized eigenvectors of the permeability tensor are expressed as

$$\lambda_2 = \mu_x + \mu_y,$$

$$\nabla Z_1^* = \sqrt{\frac{\mu_x}{\mu_x + \mu_y}} \left( \frac{i\mu_x}{\mu_x} \hat{e}_x + \hat{e}_y \right), \quad \nabla Z_2^* = \sqrt{\frac{\mu_y}{\mu_x + \mu_y}} \left( \frac{-i\mu_y}{\mu_y} \hat{e}_x + \hat{e}_y \right).$$

(5)

If the permittivity $\varepsilon$ approaches zero simultaneously, the impedance goes from infinity to a finite number and can be matched with air [31]. We refer this magneto-optical medium with a zero permittivity and a zero-determinant permeability tensor to the MODZIM, as an extension of the traditional DZIM. Compared with the MOZIM, the MODZIM has been rarely studied, especially about the bulk transport of EM waves. Moreover, the traditional DZIM can be regarded as a special case of the MODZIM, corresponding to that both eigenvalues $\lambda_1, \lambda_2$ go to zeros. Therefore, the MODZIM can be regarded as the general type of DZIM.

In figure 1(b), we showed a Gaussian beam normally incident into a MODZIM. As a comparison, the incidence of the same Gaussian beam into a traditional DZIM is shown in figure 1(a). The EM wave can travel through the MODZIM and possesses the identical phase on the outgoing surface, indicating that the medium has a finite impedance and a zero refractive index. However, in stark contrast to that the out-of-plane electric field is uniform everywhere inside the DZIM, see figure 1(a), the electric field inside the MODZIM is inhomogeneous, see figure 1(b). In the following, we will show that the inhomogeneity of the out-of-plane electric field inside MODZIM is quite general if we do not carefully optimize the boundary conditions.

For the sake of mathematical simplicity and without affecting the physics, hereafter, we assume $\varepsilon_0 = \mu_0 = c = 1$, where $\varepsilon_0, \mu_0$ and $c$ are the permittivity, permeability and light speed of vacuum, respectively. In the basis of $(\nabla Z_1^*, \nabla Z_2^*)$, the in-plane magnetic field inside the magneto-optical medium can be expressed as

$$\mathbf{H} = H_1(\nabla Z_1^*) + H_2(\nabla Z_2^*),$$

(6)

where $H_1, H_2$ are the magnetic field components along the two complex-valued axes, and the nabla operator is given by

$$\nabla = (\nabla Z_1^*) \frac{\partial}{\partial Z_1} + (\nabla Z_2^*) \frac{\partial}{\partial Z_2} = (\nabla Z_1^*) \frac{\partial}{\partial Z_1} + (\nabla Z_2^*) \frac{\partial}{\partial Z_2}.$$  

(7)
inside the MODZIM could be analyzed. We first consider that the system preserves continuous translational

Making use of equations (8)–(10) and knowing about the boundary conditions, the properties of EM waves

3. MODZIM with different boundaries

Figure 2. A $E_z$ polarized plane wave normally incident to (a) a MODZIM slab and (b) a MOZIM slab. The real parts of the electric field are shown. The unit of the electric field is $E_0$ which is the amplitude of the incident field. Periodic boundary conditions are applied along the $y$ direction. The relative permittivities of the MODZIM and MOZIM are $\varepsilon \to 0$ and $\varepsilon = 5$, respectively, while for the relative Hermitian permeability tensors, $\lambda_1 = 0$, $\lambda_2 = 2$, $Z_1 = (-ix + y)/\sqrt{2}$, $Z_2 = (ix + y)/\sqrt{2}$.

Substituting equations (6) and (7) into the Maxwell’s equations and using the relations equation (3), for a time-harmonic wave ($e^{-i\omega t}$) we obtain [29]

\[
\nabla \cdot \mathbf{B} = \lambda_1 \frac{\partial H_1}{\partial Z_1} + \lambda_2 \frac{\partial H_2}{\partial Z_2} = 0,
\]

\[
\nabla \times \mathbf{E} = (\nabla Z_1) \frac{\partial E_z}{\partial Z_2} - (\nabla Z_2) \frac{\partial E_z}{\partial Z_1} = i\omega \mu^+ \cdot \mathbf{H} = i\omega \lambda_1 H_1 \left(\nabla Z_1^*\right) + i\omega \lambda_2 H_2 \left(\nabla Z_2^*\right),
\]

\[
\nabla \times \mathbf{H} \cdot \hat{z} = \frac{\partial H_2}{\partial Z_1} - \frac{\partial H_1}{\partial Z_2} = -i\omega\varepsilon E_z.
\]

For MOZIM, without loss of generality, we assume $\lambda_1 = 0, \lambda_2 \neq 0$. According to the first and second of equation (8), we easily obtain that

\[
\frac{\partial H_2}{\partial Z_2} = 0, \quad \frac{\partial E_z}{\partial Z_2} = 0, \quad \frac{\partial E_z}{\partial Z_1} = -i\omega \lambda_2 H_2.
\]

Therefore, $E_z$ and $H_2$ are independent of $Z_2$, or in other words, they are analytic functions of the single complex variable $Z_1$. Since $Z_2$ is a linear combination of $Z_1$ and $Z_1^*$, and $Z_2$ is independent of $Z_1^*$, we obtain $\frac{\partial H_1}{\partial Z_1} = \frac{\partial H_1}{\partial Z_1^*} = \frac{\partial H_2}{\partial Z_2} = 0$, leading to that $E_z$ and $H_2$ are also independent of $Z_1^*$ or they are also analytic functions of the single complex variable $Z_1^*$. As a consequence, $\partial H_2/\partial Z_1^* = 0$ yielding that the third of equation (8) becomes

\[
\frac{\partial H_1}{\partial Z_1^*} = i\omega\varepsilon E_z.
\]

Inside the MODZIM where only $\lambda_1$ is zero, from the third of equation (9), the out-of-plane electric field will be inhomogeneous once $H_2$ does not vanish inside the whole MODZIM. For the Gaussian beam incident, according to the continuity of the normal nonzero magnetic induction field across the left boundary $\hat{x} \cdot \mathbf{B} = \hat{x} \cdot (\mu^+ \cdot \mathbf{H}) = \hat{x} \cdot \nabla Z_1^* H_2$, $H_2$ must be nonzero near the boundary, making the electric field inhomogeneous. As long as $\varepsilon$ approaches zero, equation (10) yields $\partial H_1/\partial Z_1^* = 0$. As a result, the magnetic field component $H_1$ is an analytic function of the single complex variable $Z_2$ inside the MODZIM.

However, the out-of-plane electric field is generally constant with the coordinates ($\partial E_z/\partial Z_1 = \partial E_z/\partial Z_2 = 0$) inside the traditional isotropic ZIM when the other eigenvalue $\lambda_2$ is also zero, according to the third of equation (9).

3. MODZIM with different boundaries

Making use of equations (8)–(10) and knowing about the boundary conditions, the properties of EM waves inside the MODZIM could be analyzed. We first consider that the system preserves continuous translational
symmetry along one real axis, for example the $y$ direction. Then the invariance of the electric field $E_y$ along the $y$ direction yields

$$\frac{\partial E_y}{\partial y} = \frac{\partial E_y}{\partial Z_1} \frac{dZ_1}{dy} + \frac{\partial E_y}{\partial Z_2} \frac{dZ_2}{dy} = \frac{\partial E_y}{\partial Z_1} \frac{dZ_1}{dy} = 0. \quad (11)$$

Since $Z_1, Z_2$ are complex combinations of $x$ and $y$ which guarantees $dZ_1/dy \neq 0$ and $dZ_2/dy \neq 0$, equation (11) is valid only when $\partial E_y/\partial Z_1 = 0$. Combining with the second of equation (9), the electric field is independent of the position once the system preserves continuous translational symmetry along any direction in the real space.

Moreover, since $\varepsilon \rightarrow 0$, using equation (10), and the invariance of $H_1$ along the $y$ direction which reads

$$\frac{\partial H_1}{\partial y} = \frac{\partial H_1}{\partial Z_1} \frac{dZ_1}{dy} + \frac{\partial H_1}{\partial Z_2} \frac{dZ_2}{dy} = \frac{\partial H_1}{\partial Z_1} \frac{dZ_1}{dy} = 0, \quad (12)$$

the magnetic field are also uniform inside the MODZIM. In conjunction with the boundary conditions that the electric field is continuous across the boundary, the MODZIM acts as a ‘single point’ electromagnetically [7], leading to the total transmission of the EM waves, which is verified by the numerical result as shown in figure 2(a).

However, the transmission will be reduced for the MOZIM where $\varepsilon \neq 0$, as shown in figure 2(b). Although the electric field is still uniform as $H_2$ vanishes everywhere, because $\partial H_1/\partial Z_1 \neq 0$ for $\varepsilon \neq 0$ according to equation (10), $\partial H_1/\partial Z_2$ is not zero either according to the second of equation (12). Therefore, the magnetic field inside the MOZIM is not uniform even if continuous translational symmetry is preserved. This nonuniformity of magnetic field leads to the reflection of EM waves.

The second and third of equation (9) reveal that the scalar field is uniform when $H_2$ vanishes inside the whole medium and vice versa. Suppose that the MODZIM has a perfect magnetic conductor (PMC) boundary. Since the PMC boundary forces the magnetic field to be perpendicular to it, the magnetic field must be polarized along a real axis at the boundary, in contradiction to $H_2 = 0$ which requires the magnetic field polarized along the complex-valued axis $\nabla Z_1^*$ according to equation (6). Therefore, the out-of-plane electric field cannot be uniform when the MODZIM has PMC boundaries, whether the PMC boundaries are external or internal, see figures 3(a) and (b).

If the MODZIM possesses the perfect electric conductor (PEC) boundary, according to the uniqueness theorem that the analytic function of a single complex variable vanishes everywhere when it is zero for a region bigger than a point [42], the electric field vanishes in the whole MODZIM, as shown in figures 3(c) and (d). Therefore, the $E_z$ polarized EM wave cannot be coupled into the MODZIM when it has a PEC boundary.

4. MODZIM with impurities

As shown in figure 4(a), embedding an impurity into the MODZIM to break continuous translational symmetry along the $y$ direction, the magnetic field inside the MODZIM becomes inhomogeneous.
Figure 4. A $E_z$ polarized plane wave normally incident to a MODZIM slab with dielectric cylinders embedded inside from left. Periodic boundary conditions are applied along the $y$ direction. For the relative permeability tensors, $\lambda_1 = 0$, $\lambda_2 = 2$, $Z_1 = (ix + y)/\sqrt{2}$, $Z_2 = (ix + y)/\sqrt{2}$. The frequency of the plane wave is 165 MHz. The electric field is in unit of $E_0$.

(a) The relative permittivity and radius of the cylinder are $\varepsilon_s = 5$, $r_0 = 0.7$ m. (b) $J_1(\kappa r_0) = 0$ is satisfied. (c) One cylinder satisfies $J_1(\kappa r_0) = 0$ and one cylinder does not. (b) The electric field along the white dashed line marked in (g). The relative permittivities of the cylinders are (b) and (c) $\varepsilon_s = 5$, (d) $\varepsilon_s = 4.115$, (e) $\varepsilon_s = 8.654$, (f) $\varepsilon_s = 5.115$, (g) $\varepsilon_s = 5$, $\varepsilon_s = 5$. The radii of the cylinders are (b) $r_0 = 0.495$ m, (c) $r_0 = 0.907$ m, (d) and (e) $r_0 = 1.0$ m, (f) $r_0 = 0.907$ m, $r_0 = 1.0$ m, (g) $r_0 = 0.907$ m, $r_0 = 1.2$ m.

Therefore, the electric field is usually inhomogeneous inside the MODZIM when continuous translational symmetry is broken. This inhomogeneity will bring about difficulty when we want to predict the transmission of such a system or achieve some geometry symmetry-free novel phenomena, i.e., the photonic doping [40].

However, the electric field can still be uniform for at least two types of impurities. As discussed previously, the sufficient and necessary condition to achieve the uniform electric field is that $H_2$ vanishes everywhere inside the MODZIM. According to the maximum modulus principle [42], the local maximum modulus of an analytic function of a single complex variable is always on the boundary of the definition domain. Therefore, if we let $H_2 = 0$ on all boundaries of the MODZIM, then $H_2 = 0$ everywhere.

At the MODZIM boundary, combining with equations (2) and (6), the continuity of the normal component of the magnetic induction field reads

$$\hat{n} \cdot (\hat{n} \cdot \mathbf{H}) = (\hat{n} \cdot \nabla Z_2^\ast) \lambda_2 H_2 = \mu_0 H_{on},$$

where $\hat{n}$ denotes the unit normal vector of the boundary in the real space, $\mu_0$ is the relative permeability on the other side of the boundary and $H_{on}$ is magnetic field along the normal direction on the other side of the boundary. Note that $\hat{n} \cdot \nabla Z_2^\ast \neq 0$ because $\nabla Z_2^\ast$ is always along the complex-valued axis. Either $\mu_0 = 0$ or $H_{on} = 0$ will make $H_2 = 0$ at the boundary. For the configuration shown in figure 4, $H_{on} = 0$ at the ingoing (on the left) boundary is guaranteed by the normal incidence, and $H_{on} = 0$ at the outgoing (on the right) boundary is ensured because the EM wave is always normally refracted. For the interface between the MODZIM and the impurity, we can either choose $\mu$-zero impurity or using resonance to realize $H_{on} = 0$ at the interface.
4.1. Dielectric cylinder embedded inside the MODZIM

We consider the impurity as a dielectric cylinder with a radius \( r_0 \), the relative permittivity \( \varepsilon_r \). To apply the Mie theory, the MODZIM is considered to be isotropic on the xy plane (\( \mu_x = \mu_y = \mu_r \)), i.e., \( \hat{\mu} = \mu_r(\hat{x} \hat{x} + \hat{y} \hat{y}) - \mu_k(\hat{x} \hat{y} - \hat{y} \hat{x}) \). According to the Mie theory [43–45], the EM fields inside the cylinder are expressed as

\[
E_{\text{ins}} = \sum_n c_n N_n^{(1)}(k, r), \quad H_{\text{ins}} = \frac{k}{\omega} \sum_n s_n M_n^{(1)}(k, r),
\]

where \( c_n \) is the expansion coefficient, \( k = \sqrt{\varepsilon \omega / \varepsilon} \) is the wavenumber inside the cylinder, \( \mathbf{r} \) is the cylindrical coordinate with the origin at the center of the cylinder, and the vector cylindrical wave function (VCWFs) are expressed as [44, 45]

\[
N_n^{(j)} = \nabla \times z_n(\rho) e^{i\rho},
\]

\[
M_n^{(j)} = \frac{1}{k} \nabla \times N_n^{(j)} = \left( \frac{\text{im}}{\rho} z_n(\rho) \hat{r} - \frac{\text{d} z_n(\rho)}{\text{d}\rho} \hat{\varphi} \right) e^{i\rho},
\]

\[
L_n^{(j)} = \hat{z} \times M_n^{(j)} = \left( \frac{\text{d} z_n(\rho)}{\text{d}\rho} \hat{r} + \frac{\text{im}}{\rho} z_n(\rho) \hat{\varphi} \right) e^{i\rho},
\]

with \( \rho = kr, z_n = J_n(\rho) \) for \( j = 1 \) and \( z_n = H_n^{(1)}(\rho) \) for \( j = 3 \), \( J_n \) being the Bessel function of the first kind, \( H_n^{(1)} \) being the Hankel function of the first kind, and \( \hat{r}, \hat{\varphi} \) representing the unit vectors along the radial and azimuthal directions. In the background magneto-optical medium, the incident and scattered fields in terms of the VCWFs are [45]

\[
E_{\text{inc}} = \sum_n q_n N_n^{(1)}(k', r),
\]

\[
H_{\text{inc}} = \frac{\omega \varepsilon}{i} \sum_n \left[ \frac{1}{k'_{\text{b}}} q_n M_n^{(1)}(k', r) - \frac{i}{k'_{\text{b}}} \frac{\mu_k}{\mu_r} s_n L_n^{(1)}(k', r) \right],
\]

and

\[
E_{\text{sca}} = \sum_n b_n N_n^{(3)}(k', r),
\]

\[
H_{\text{sca}} = \frac{\omega \varepsilon}{i} \sum_n \left[ \frac{1}{k'_{\text{b}}} b_n M_n^{(3)}(k', r) - \frac{1}{k'_{\text{b}}} \frac{\mu_k}{\mu_r} b_n L_n^{(3)}(k', r) \right],
\]

where \( k'_{\text{b}} = \sqrt{\varepsilon (\mu_k^2 - \mu_r^2)/\mu_r \omega / \varepsilon} = m' \varepsilon \omega / \varepsilon \) denotes the wavenumber in the MODZIM. Using the boundary conditions that the tangential EM fields should be continuous and combining equations (14)–(17), we obtain

\[
c_n J_0(x_0) = q_n J_0(x_0) + b_n J_n^{(1)}(x_0),
\]

\[
\frac{k}{\omega} c_n J_n'(x_0) = \frac{\omega \varepsilon}{i k'_{\text{b}}} \left[ q_n J_n'(x_0) + \frac{i \mu_k}{\mu_r} q_n J_0(x_0) + b_n J_n^{(1)}(x_0) + \frac{i \mu_k}{\mu_r} b_n J_0(x_0) \right],
\]

where \( x_0 = k'_{\text{b}} x, x = kr_0 \) for brevity. Solving equation (18) yields

\[
\frac{\varepsilon}{\mu_k} J_n(x_0) = \frac{H_n^{(1)}(x_0) J_0'(x_0) - J_n^{(1)}(x_0) J_0(x_0)}{q_n},
\]

\[
\frac{\varepsilon}{\mu_k} J_n'(x_0) = \frac{H_n^{(1)}(x_0) J_0(x_0) - J_n^{(1)}(x_0) J_0'(x_0)}{q_n}.
\]

In the limit \( \mu_k \to \mu_r \), using the Taylor series, \( c_n / q_n \) for all orders vanish except the following two

\[
\frac{c_{-1}}{q_{-1}} = -\frac{2}{\eta'_{\text{b}} \sqrt{\varepsilon} J_1(x_0)}, \quad \frac{\varepsilon}{q_0} = \frac{1}{J_0(x_0)},
\]

where \( \eta'_{\text{b}} = m' \varepsilon / \varepsilon \) is the impedance of the background medium. For the MODZIM, \( x_0 \to 0 \) due to \( k'_{\text{b}} \to 0 \) and \( |\mu_k| \to |\mu_r| \). Since the background is the zero-index medium where the wavelength is extremely large, in the long-wavelength limit, the second term of equation (14) is dominated by the zeroth, \(-1\)st and 1st order. In the limit \( |\mu_k| \to |\mu_r| \), the following two equations hold.

\[
\frac{c_{-1}}{q_{-1}} = -\frac{2}{\eta'_{\text{b}} \sqrt{\varepsilon} J_1(x_0)}, \quad \frac{\varepsilon}{q_0} = \frac{1}{J_0(x_0)}.
\]
Therefore, the EM fields inside the cylinder are dominated by −1st and 0th order terms in equation (14). Easily, we know that for \( \mu \) the electric field inside the MODZIM with impurities is not uniform generally. The electric field keeps whole volume.

According to equation (21), \( H_2 = 0 \) at interface is satisfied when \( J_1(kr_0) = 0 \). While the condition is satisfied, the electric field is homogeneous on the interface between the MODZIM and \( \phi \)-independent. According to the Mie theory, it means that only the 0th order term of the EM fields inside the MODZIM will be kept when \( J_1(kr_0) = 0 \) is satisfied, namely \( c_{-1} \) vanishes for \( \mu \) ≠ \( \mu_r \).

In figures 4(b)–(f), we showed several typical cases that \( J_1(kr_0) = 0 \) is satisfied. The dielectric cylinders in figures 4(b) and (c) have the same relative permittivity but different radii, while the cylinders in figures 4(d) and (e) possess the same radius but different relative permittivities. In figure 4(f), we also embed two types of cylinders into the MODZIM. It is seen that the electric field is uniform inside the MODZIM for all the cases. As long as the electric field is uniform inside the MODZIM, we can easily obtain that the transmission is unit, see appendix A. When there is an additional cylinder that \( J_1(kr_0) = 0 \) is not fulfilled, the electric field is no longer homogeneous and the total transmission will be ruined, see figures 4(g) and (h).

Although both the PMC and the dielectric impurity satisfying \( J_1(kr_0) = 0 \) impel zero tangential magnetic induction fields at their boundaries, the key difference is that the normal magnetic induction fields are always nonzero on the PMC boundary but vanishing on the boundary of the dielectric impurity. As such, the electric field inside the MODZIM is non-uniform once there is any PMC boundary but could be uniform when the embedded dielectric impurity satisfies the condition \( J_1(kr_0) = 0 \).

4.2. \( \mu \)-zero impurity

When the impurity is \( \mu \)-zero material, namely \( \mu_0 = 0 \), \( H_2 = 0 \) at the interface is fulfilled irrespective of the geometry of the impurity and the MODZIM can be anisotropic on the \( xy \) plane (\( \mu_x \neq \mu_y \)), according to equation (13). In figure 5(a), we showed that the electric field is uniform in the MODZIM when the \( \mu \)-zero impurity of an arbitrary geometry is embedded. In contrast, when the permeability of the impurity is not zero, the air for example, the electric field inside the MODZIM becomes inhomogeneous, see figure 5(b).

Transmission coefficient for the MODZIM with \( \mu \)-zero impurity of an arbitrary shape can be easily obtained as (see appendix B)

\[
T = \frac{2h}{2h - i\omega \varepsilon_m A_m / c},
\]

where \( h \) is the width of the MODZIM, \( \varepsilon_m \) and \( A_m \) are the relative permittivity and volume of the impurity, respectively. Based on equation (22), the transmission can be tuned by adjusting \( \varepsilon_m \) and \( A_m \). For multiple \( \mu \)-zero impurities, the transmission coefficient can be easily deduced as

\[
T = \frac{2h}{2h - ik_0 \sum_n \varepsilon_m^{(i)} A_m^{(i)}},
\]

where \( k_0 = \omega / c \), \( \varepsilon_m^{(i)}, A_m^{(i)} \) denote the relative permittivity and volume of the \( i \)th \( \mu \)-zero impurity.

Because the electric field is uniform inside the MODZIM whatever the location and geometry of the \( \mu \)-zero impurity are, the \( \mu \)-zero impurity can serve as a kind of dopant for the MOZIM to tailor the EM properties of MODZIM. Based on the idea of photonic doping [40], the effective constitutive parameters of the doped MODZIM are given by

\[
\varepsilon_{\text{eff}} = \frac{\sum_i \varepsilon_m^{(i)} A_m^{(i)}}{S}, \quad \mu_{\text{eff}} = 0
\]

where \( \varepsilon_m^{(i)}, A_m^{(i)} \) denote the relative permittivity and volume of the \( i \)th \( \mu \)-zero impurity, and \( S \) represents the whole volume.

As the out-of-plane field in traditional ZIM is always uniform irrespective of the locations and geometries of the impurities, any kinds of impurities can be doped to ZIM [40]. However, we showed that the electric field inside the MODZIM with impurities is not uniform generally. The electric field keeps uniform only when the impurities are carefully selected. Therefore, different from traditional ZIM, the photonic doping for MODZIM works only for particular types of impurities.
Figure 5. A $E_z$ polarized plane wave normally incident to a MODZIM slab with (a) a $\mu$-zero impurity and (b) an air impurity from left. The impurities have an arbitrary shape and relative permittivity $\varepsilon_m = 5$. Periodic boundary conditions are applied along the $y$ direction. For the MODZIM, $\lambda_1 = 0$, $\lambda_2 = 3$, $Z_1 = (-ix + \sqrt{2}y)/\sqrt{3}$, $Z_2 = (\sqrt{2}ix + y)/\sqrt{3}$. The electric field is in unit of $E_0$.

Figure 6. $\mu$-zero impurities doped by cylinders. Periodic boundary conditions are applied along the $y$ direction. The relative permittivity of the $\mu$-zero impurities is $\varepsilon_m = 5$. For the MODZIM, $\lambda_1 = 0$, $\lambda_2 = 2$, $Z_1 = (-ix + y)/\sqrt{2}$, $Z_2 = (ix + y)/\sqrt{2}$. The cylinders have the identical radii $r_0 = 2.5$ m. (a) The relative permittivity of the dielectric cylinder is $\varepsilon_s = 12.5$. (b) A PMC cylinder is doped. (c) A PEC cylinder is doped. (d) The is a very thin air gap between the PEC cylinder and the $\mu$-zero impurity. (e) The total reflection. The doped dielectric cylinder has a relative permittivity $\varepsilon_s = 1.86$, fulfilling $J_0(kr_0) = 0$. (f) The total transmission when equation (26) is fulfilled. The relative permittivity of the dielectric cylinder is $\varepsilon_s = 3.02$, and the $\mu$-zero impurity has a green square shape with the side length 6.04 m. The plane wave is normally incident from left with the frequency 165 MHz. The unit of the electric field is $E_0$.

However, the photonic doping for MODZIM processes some peculiar advantages compared with traditional photonic doping, such as responding to certain spin polarization and retaining the intriguing boundary effects.

5. Total transmission and reflection

From equation (22), we see that the total transmission cannot be achieved unless there is no $\mu$-zero impurity ($A_m \to 0$) or $\varepsilon_m \to 0$ (namely, the impurity is DZIM). However, the situation will be different if we dope cylinders into the $\mu$-zero impurity. In figure 6, we showed the results for four types of dopants, a dielectric cylinder (a), a PMC cylinder (b), a PEC cylinder (c) and a PEC cylinder with an air gap (d). As we can see, the electric field inside the MODZIM is still uniform, which is expected because $H_2 = 0$ on all boundaries of the MODZIM still holds. When the dopant is a PEC, the uniformity and continuity of the electric field force the electric field inside the MODZIM to be zero. Thus, no EM field can be coupled into the MODZIM. The PEC cylinder with an air gap is just equivalent to the PMC cylinder, see figures 6(b) and (d).

For the dopant being a dielectric cylinder, the transmission coefficient is derived as (see appendix B)

$$T = \frac{2h}{2h - i\varepsilon_m A_m^2 - 2i\sqrt{\varepsilon_m} \pi r_0 \frac{J_1(kr_0)}{J_0(kr_0)}}.$$  (25)
Based on equation (25), the transmission can be tuned not only by adjusting the parameters of the \( \mu \)-zero impurity, but also by well selecting the parameters of the dielectric cylinder. According to equation (25), the total transmission takes place when

\[
\varepsilon_m A_m \frac{\omega}{c} + 2\sqrt{\varepsilon_m} \varepsilon_0 f_0 = 0.
\]

(26)

On the other hand, when \( f_0 = 0 \), the denominator of equation (25) diverges, leading to the total reflection \( (T \to 0) \), whatever the permittivity and volume of the \( \mu \)-zero impurity are.

In figure 6(e), the doped the cylinder which satisfies \( f_0 = 0 \). We can see that the electric field vanishes inside the ZIM region everywhere and on the right-hand side of the MODZIM, indicating that the total reflection occurs and no EM field is coupled to the MODZIM. On the other hand, when the parameters of the doped cylinder are well chosen to validate equation (26), the normally incident plane wave can travel through the MODZIM slab without any reflection, see figure 6(f) where the magnitude of the transmitted wave is equivalent to that of the incident wave. In figure 6(f), the \( \mu \)-zero impurity is chosen as square in order to facilitate the calculation of its volume \( A_m \). In fact, according to the idea of photonic doping [40], the \( \mu \)-zero impurity doped with a dielectric cylinder fulfilling \( f_0 = 0 \) is equivalent to a PEC material, while the \( \mu \)-zero impurity doped with a dielectric cylinder fulfilling equation (25) is equivalent to a DZIM. However, the mechanisms of the \( \mu \)-zero impurity doped with a dielectric cylinder fulfilling \( f_0 = 0 \) and the PEC material for EM field shielding are essentially different. The EM fields cannot be coupled to the former because it corresponds a BIC mode with an infinite quality factor [39], while the EM fields are forbidden in the latter due to the infinite conductivity.

6. Loss effect and a possible real structural design

In this section, we will briefly discuss the loss effect and a possible real structural design of the MODZIM. Loss is inevitable in practice. In our work, all the results and conclusions are based on the assumption that the system is Hermitian, namely the loss is absent or very weak. This is on one hand reasonable in the microwave regime, and on the other hand simplifies the analytical derivations. If the non-Hermitian effects are significant caused by a large loss, the mathematical method we used no longer works and the conclusions may be ruined. Nevertheless, if the loss is not too large, the conclusions are still applicable. As shown in figure 7, we considered that the relative permittivity of MODZIM has an imaginary part 0.001 (the loss tangent is 10), the electric field inside the MODZIM is still uniform when the dielectric impurity satisfies \( f_0 = 0 \), and the transmission is still very close to 1.

A feasible realization of MODZIM is using the magneto-optical photonic crystal exhibiting an unpaired Dirac cone at the Brillouin zone center [31]. As shown in figure 8(a), an unpaired Dirac cone formed by the second and third bands emerges when the external magnetic field is well tuned. The inclusions of the photonic crystal are made of ferrite material with an elliptical geometry. The relative permittivity of the inclusion is 12.5 and the relative permeability tensor components of the ferrite material with external magnetic field \( H \) applied along the z direction are given by [46]

\[
\mu_r = 1 + \frac{\omega_m \omega}{\omega_0^2 - \omega^2}, \quad \mu_k = \frac{\omega_k \omega}{\omega_1^2 - \omega^2},
\]

(27)

where \( \omega_m = g \mu_0 M_s, \omega_0 = g \mu_0 H, \) with \( g = 1.76 \times 10^7 \text{ C kg}^{-1} \) being the gyromagnetic ratio and \( M_s = \frac{1780}{4\pi} \text{ G} \) being the saturation magnetization. The Dirac point forms when the external field is \( H = 50 \text{ G} \), which is small and easily achieved in experiments. The Dirac point frequency is 16.6 GHz. As shown in figure 8(b), at the frequency of the Dirac point, when a plane wave normally incidences into the magneto-optical photonic crystal, the total transmission takes place and the phases on the incident and outgoing boundaries are the same, indicating that the magneto-optical photonic crystal is a MODZIM.

Finally, we also considered about the incidence of a Gaussian beam. Although the beam axis is perpendicular to the incident boundary, the normal magnetic induction fields off the beam center are nonzero because of the oblique incident angle. Thus, the electric field cannot be uniform inside the MODZIM, as shown in figure 1(b). However, we can coat a thin layer, such as a metasurface [22] and DZIM, on the incident boundary, to make any aplanatic beam normally incident to the MODZIM. As shown in figure 9, a Gaussian beam is incident from left to a MODZIM with a thin DZIM layer coated. For both the homogeneous MODZIM and the MODZIM with a dielectric impurity satisfying the condition, the electric fields inside them are uniform.
Figure 7. A $E_z$ polarized plane wave normally incident to a lossy MODZIM slab with a dielectric cylinder satisfy $J_1(\kappa r_0) = 0$ inside from left. Periodic boundary conditions are applied along the $y$ direction. For the relative permeability tensors, $\lambda_1 = 0$, $\lambda_2 = 2$, $Z_1 = (ix + y)/\sqrt{2}$, $Z_2 = (ix + y)/\sqrt{2}$. The frequency of the plane wave is 165 MHz. The electric field is in unit of $E_0$. The relative permittivity and radius of the cylinder are $\varepsilon_s = 5$, $r_0 = 0.907$ m. (a) The relative permittivity of MODZIM is $\varepsilon = 10^{-4} + i \cdot 10^{-3}$ with a remarkable loss tangential 10. (b) Shows the absolute value of the electric field at the white line in (a) which indicates almost total transmission with a small loss.

Figure 8. (a) An unpaired Dirac cone at the Brillouin zone center of a magneto-optical photonic crystal. The unit cell is shown in the inset. The lattice constant is $a = 1$ cm, the semi-major and semi-minor axes of the inclusion are $r_a = 0.17a$, $r_b = 1.205r_a$. The relative permittivity is $\varepsilon_r = 12.5$, and the permeability tensor components are given by equation (27) with external magnetic field $H = 50$ G. (b) A $E_z$ polarized plane wave normally incident to the magneto-optical photonic crystal at the Dirac point frequency. Periodic conditions are applied along the $y$ direction.
Figure 9. A Gaussian beam incident to a MODZIM slab with a thin DZIM layer coated on the left surface. The real parts of the electric field are shown. Periodic boundary conditions are applied along the $y$ direction. The frequency and beam waist of the Gaussian beam are 165 MHz and 4 m, respectively. The constitutive parameters of the DZIM and MODZIM are approaching scalar 0, while for the permeability MODZIM, $\lambda_1=0$, $\lambda_2=2$, $Z_1=(-ix+y)/\sqrt{2}$, $Z_2=(ix+y)/\sqrt{2}$. (b) The embedded cylinder satisfies $J_1(kr_0)=0$ with the relative permittivity and radius are $\varepsilon_s=5$, $r_0=0.907$ m.

7. Conclusions

In summary, we have proposed the concept of MODZIM which possess a zero permittivity and a Hermitian permeability tensor of zero determinant and will reduce to the traditional DZIM when the Hermitian permeability tensor becomes zero diagonal, and investigated the EM properties of the MODZIM by formulating the Maxwell’s equations in the basis of the two complex-valued eigenvectors of the Hermitian permeability tensor. We have shown that the out-of-plane electric field and the magnetic field components along the two complex-valued axes are all analytic functions of a single complex variable. As such, the electric field inside the MODZIM is usually not uniform. Particularly, the electric field is definitely inhomogeneous when the MODZIM possess any PMC boundary. For the MODZIM have any PEC boundary, the $E_z$ polarized EM wave cannot be coupled into the MODZIM. However, using the principle of maximum modulus for analytic functions, we can make the out-of-plane electric field inside the MODZIM uniform by optimizing the boundary conditions, such as imposing the continuous translational symmetry and interfacing the MODZIM with $\mu$-zero media. Moreover, the electric field can be uniform even when some particular types of impurities are embedded in the MODZIM slab, including the resonant round cylinders and $\mu$-zero media of an arbitrary shape. As long as the electric field inside the MODZIM is homogeneous, we can analytically study the scattering and transporting of EM waves, just like what we have done for the traditional DZIM. Our work will extend the study of zero-refractive-index photonics and pave the way for the applications of the MODZIM in EM wave manipulations.

In this work, we focused the study on the two-dimensional systems which are infinite along the out-of-plane direction. It was demonstrated that the wave dynamics in three-dimensional ZIM are very different from those in two-dimensions [47, 48]. This difference will be more significant for the MODZIM since the MODZIM is anisotropic intrinsically and the boundary effects are more striking, such as the one-way boundary transport. It will be interesting to explore the wave dynamics in three-dimensional MODZIM in the future. In order to simply the theoretical analysis, we assumed the slab geometry of the MODZIM in this work. However, the basic conclusions such the electric field is generally non-uniform inside the MODZIM and reaches uniform only when the normal magnetic induction fields on all boundaries of the MODZIM are zero, are also valid for other geometries. For geometries with finite sizes, besides the normal incidence, additional efforts are needed to ensure the uniformity of the electric field inside the MODZIM.
Figure A1. Sketch for deriving the transmission of a plane wave normally incident on the MODZIM with impurities. (a) The impurity is a dielectric cylinder. (b) The impurity is a geometry-free \( \mu \)-zero material with a cylinder embedded inside. The red arrows denote the propagation direction of the incident wave, the black arrows denote the directions of the outer and inner boundaries of the MODZIM, while the cyan arrow denotes the direction of the outer boundary of the cylinder. Periodic boundaries conditions are applied along the \( y \) direction.

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Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

Appendix A. Transmission for the MODZIM with the dielectric round impurity fulfilling \( J_1(kr_0) = 1 \)

In this appendix, we will derive the transmission for the system shown in figure A1(a). Periodic boundary condition is applied along the \( y \) direction. Assume the reflection and transmission coefficients are \( R \) and \( T \), respectively, then the electric field at the ingoing and outgoing boundaries are \((1 + R)E_0\) and \(TE_0\), where \(E_0\) is the amplitude of the incident electric field. Using that the electric field is homogeneous inside the MODZIM, we obtain

\[ 1 + R = T. \tag{A1} \]

Since the plane wave in the air propagates along the positive-\( x \) direction, the magnetic fields along the positive-\( y \) direction for the incident, reflected and transmitted waves are \(-E_0\), \(RE_0\) and \(-TE_0\), respectively. According to the Ampere–Maxwell law and using the Stokes theorem, inside the MODZIM

\[ \oint_S \nabla \times \mathbf{H} \cdot \hat{z} \, dS = \int_{I_o} \mathbf{H} \cdot d\mathbf{l}_o + \int_{I_i} \mathbf{H} \cdot d\mathbf{l}_i = -i\omega \int_S \varepsilon E_z \, dS, \tag{A2} \]

where \( S \) is the volume of the MODZIM, \( I_o \) and \( I_i \) are the outer and inner boundaries of the MODZIM as shown in figure A1. Because \( \varepsilon \) approaches zero, the right-hand side of equation (A2) vanishes. Therefore,

\[ h(1 - R - T)E_0 + \int_{I_i} \mathbf{H} \cdot d\mathbf{l}_i = -2hRE_0 + \int_{I_i} \mathbf{H} \cdot d\mathbf{l}_i = 0, \tag{A3} \]

where \( h \) is the width of the MODZIM, see figure A1(a). In section 4.1, we have proved that the magnetic field inside the cylinder is only contributed by the 0th order term in equation (14). Since \( J_1(kr_0) = 0 \), the
arbitrary shape

μ

reflection and transmission coefficients are

E

Here, we will derive the transmission for the system shown in figure A1(b), where the impurity is an azimuthal magnetic field on the interface l. The last equality of equation (A5) is due to the boundary condition. If there is no cylinder embedded, the second integral on left-hand side of equation (A5) vanishes, substituting equation (A5) into equation (A3) we obtain

which yields R = 0. As a result, the total transmission takes place when the dielectric cylinder satisfies J_1(kr_0) = 0.

Appendix B. Transmission for the MODZIM with the μ-zero impurity

Here, we will derive the transmission for the system shown in figure A1(b), where the impurity is an arbitrary shape μ-zero medium with or without cylinders embedded inside. Similarly, we assume the reflection and transmission coefficients are R and T, respectively, and the amplitude of the incident field is E_0. Then equations (A1) and (A3) are still valid. Therefore, the reflection coefficient is obtained by calculating \int_l \mathbf{H} \cdot d\mathbf{l}. Utilizing the Stokes theorem and the Maxwell's equation, we obtain

where \lambda_m is the volume of the μ-zero medium, \varepsilon_m is the permittivity of the μ-zero medium, \lambda_i is the boundary of the embedded cylinder. Thus, equation (A3) reduces to

Substituting equation (A11) into equation (A3), the transmission coefficient is calculated as

\text{Then reflection and transmission coefficients are}

\begin{align}
R &= \frac{i\omega \varepsilon_m A_m / \epsilon}{2h - i\omega \varepsilon_m A_m / \epsilon},
T &= 1 + R = \frac{2h}{2h - i\omega \varepsilon_m A_m / \epsilon}.
\end{align}

If there is a dielectric cylinder inside the μ-zero medium, we should calculate \int_i \mathbf{H} \cdot d\mathbf{l}^i. Because the electric field is uniform inside the μ-zero medium, as shown in section 4.1, the EM fields inside the cylinder must be contributed only by the zeroth order term based on the Mie theory,

\begin{align}
E_z(r) &= c_0 J_0(kr),
H(r) &= \frac{k}{i\omega} c_0 j_0'(kr) \hat{\varphi},
\end{align}

where c_0 is the coefficient, k = \sqrt{\varepsilon_\omega / \epsilon} is the wavenumber inside the cylinder. On the interface between the cylinder and the μ-zero medium, the electric field is continuous, yielding

\begin{align}
E_z(kr_0) &= E_z = TE_0,
\end{align}

where r_0 is the radius of the cylinder. Combining equations (A8) and (A9) to obtain H(r) and substituting it into the second integral of equation (A5), we obtain

\begin{align}
\int \mathbf{H} \cdot d\mathbf{l}^i &= \frac{k}{i\omega} c_0 j_0'(kr_0) 2\pi r_0 = \frac{T i k}{\omega} 2\pi r_0 j_0'(kr_0) E_0.
\end{align}

Inserting equation (A10) into equation (A5), we obtain

\begin{align}
\int \mathbf{H} \cdot d\mathbf{l} = \frac{i\omega}{\epsilon} \varepsilon_m TE_0 A_m - \frac{T i k}{\omega} 2\pi r_0 j_0'(kr_0) E_0.
\end{align}

Substituting equation (A11) into equation (A3), the transmission coefficient is calculated as

\begin{align}
T &= \frac{2h}{2h - i\varepsilon_m A_m \frac{J_0'}{J_0(kr_0)}}.
\end{align}
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