Canonical interpretation of the $D_{s0}(2590)^+$ resonance

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The $D_{s0}(2590)^+$ resonance observed by LHCb Collaboration is a strong candidate of the $D_s(2S)_{0}$ state according to its spin parity and strong decay mode. However, the measured mass seems relatively lower than the previous theoretical predictions, which interests the coupled channel interpretations in the literature. In this work, we adopt an alternate approach, taking into account the screening effects in the potential model, to describe the $D_{s0}(2590)^+$ resonance. The mass spectrum and strong decays of the excited charmed-strange mesons are investigated within the modified relativized quark model and $^3P_0$ model. The calculated mass and width of the $D_{s0}(2590)^+$ are consistent with the experimental observations, which indicate that it can be reasonably interpreted as the $D_s(2S_{0})$ state.

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and the mass gap between two $2S$ charmed states $D'_1(2600)$ and $D_0(2550)$ with the latest measurements of LHCb Collaboration is [31]

$$m[D'_1(2600)] - m[D_0(2550)] = 124 \text{ MeV}. \quad (2)$$

The approximately equal mass splittings of charmed and charmed-strange sectors strongly suggest that the $D_{s0}(2590)$ should be the partner of $D_0(2550)$ and can be assigned as the $D_s(2S_0)$ state as the LHCb Collaboration suggested.

Instead of the unquenched approaches with higher Fock states, the potential model including screening effects is an alternate approach to lower the mass spectrum, which has been extensively employed to study the properties of conventional mesons and achieved significant success. The advantage of the screening potential is that one can bring down the masses of excited states while avoiding involving higher Fock components. Hence, we expect that the potential model including screening effects may relieve the tension between measured mass and theoretical predictions under the assignment of $D_{s0}(2590)$ as $D_s(2S_0)$ state. Moreover, it is natural and necessary to explore the possible conventional descriptions for a newly observed particle before introducing more complicated and exotic configurations. In this work, we apply the Godfrey-Isgur’s relativized quark model including screening effects to revisit the mass spectrum of the charmed-strange mesons, and then adopt the obtained wave functions to study their strong decay behaviors in the $\Upsilon_0$ model. Our results show that the calculated mass and width of the $D_{s0}(2590)$ are consistent with the experimental observations, which suggest that it can be reasonably interpreted as the $D_s(2S_0)$ state.

This article is organized as follows. In Sec. II, we briefly introduce the relativized quark models and $\Upsilon_0$ models. The results and discussions of charmed-strange mesons are presented in Sec. III. Finally, a summary is given in the last section.

II. MODELS

A. The relativized quark model

In this subsection, we will give a brief introduction of the relativized quark model proposed by Godfrey and Isgur (GI model) [14]. This model has been extensively adopted to investigate the properties of conventional hadrons [6, 14, 32–38] and tetraquarks [39–49], and give a unified description of different flavor sectors. In particular, for the low-lying states, the relativized quark model plays an important role in studying their mass spectra and provides an effective criterion to distinguish conventional mesons from exotics.

For a two-body system, the relevant Hamiltonian can be written as

$$H = H_0 + V^{\text{voge}} + V^{\text{conf}}, \quad (3)$$

where

$$H_0 = \sqrt{p^2 + m_1^2} + \sqrt{p^2 + m_2^2}$$

is the relativistic kinetic energy, $V^{\text{voge}}$ is the one gluon exchange potential, and $V^{\text{conf}}$ corresponds to the confining potential. The induced spin-dependent interactions are also included in the $V^{\text{voge}}$ and $V^{\text{conf}}$.

More explicitly, the potentials $V^{\text{voge}}$ and $V^{\text{conf}}$ can be expressed as

$$V^{\text{voge}} = \beta_{12} G(r) \delta_{12}^{1/2} + \frac{2 S_1 \cdot S_2}{3 m_1 m_2} \nabla^2 G(r) \delta_{12}^{1/2},$$

$$+ \delta_{12}^{1/2} \frac{S_1 \cdot S_2}{m_1 m_2} \cdot \frac{L}{r} \frac{1}{\delta_{12}^{1/2}}$$

and

$$V^{\text{conf}} = \tilde{S}(r) - \delta_{12}^{1/2} \frac{S_1 \cdot S_2}{m_1 m_2} \cdot \frac{L}{r} \frac{1}{\delta_{12}^{1/2}}$$

Here, the $G(r)$ and $\tilde{S}(r)$ are the smeared potentials, and can be written as

$$G(r) = -\frac{3}{r} \sum_{k=1}^{\infty} \frac{4 \kappa_k}{3 r} \text{erf}(\tau_{12} r) \quad (7)$$

and

$$\tilde{S}(r) = b r \left[ \frac{e^{-\sigma_{12}^2 r^2}}{\sqrt{2 \sigma_{12}^2}} + \left( \frac{1}{2} + \frac{1}{2 \sigma_{12}^2} \right) \text{erf}(\sigma_{12} r) \right] + c \quad (8)$$

with

$$\frac{1}{\sigma_{12}^2} = \frac{1}{\sigma_{11}^2} + \frac{1}{\sigma_{12}^2}$$

and

$$\sigma_{12}^2 = \sigma_{10}^2 \left[ \frac{1}{2} + \frac{1}{2} \left( \frac{4 m_1 m_2}{(m_1 + m_2)^2} \right)^2 \right] + \sigma_{11}^2 \left( \frac{2 m_1 m_2}{m_1 + m_2} \right)^2 \quad (10)$$

The definition of $\delta_{11}$, $\delta_{12}$, $\delta_{22}$, and $\beta_{12}$ are

$$\delta_{ij} = \frac{m_i m_j}{(p^2 + m_i^2)^{1/2} (p^2 + m_j^2)^{1/2}} \quad (11)$$

and

$$\beta_{12} = 1 + \frac{p^2}{(p^2 + m_1^2)^{1/2} (p^2 + m_2^2)^{1/2}} \quad (12)$$

The $p$ is the magnitude of the relative momentum between the quark and antiquark. The $m_1$ and $m_2$ are masses of the quark and antiquark, respectively. The $\alpha$, $\gamma$, $b$, $c$, $\sigma_{10}$, $s$ and $\epsilon_i$ are the parameters introduced in the relativized quark model.
B. Screened potential

For high excited states, it is necessary to introduce the screening effects to the relativized model, because the linear confining potential will be screened and softened by the vacuum polarization effects at a large distance [50–52]. Also, the modified relativized model (MGI model) including screening effects turns out to be able to give a better description of the mass spectra for the radial and orbital excitations [43, 53–59].

To incorporate the screening effects in the relativized quark model, we should replace the confining potential \( S(r) \) with a screened potential. The \( S(r) \) actually arises from the linear confinement according to the smearing transformation. For an arbitrary potential \( f(r) \), the smeared ones \( \tilde{f}(r) \) can be expressed as

\[
\tilde{f}(r) = \int d^3r' \rho_{12}(r-r')f(r')
\]

with

\[
\rho_{12}(r-r') = \frac{\rho_{12}^0 e^{-r_{12}^2}}{r_{12}^2}.
\]

It can be noticed that the linear confining potential \( S(r) = br + c \) indeed leads to \( S(r) \) through the above smearing transformation. Here, the constant \( c \) always attaches to the confining potential for the same convention as Ref. [14], which can be fixed by the mass of the ground state.

In the literature, the following replacement is often employed to modify the linear confining potential in the quark model [51, 52],

\[
S(r) = br + c \rightarrow V^{scr}(r) = \frac{b(1 - e^{-\mu r})}{\mu} + c.
\]

If \( r \) is small enough, one has \( V^{scr}(r) = V(r) \). Therefore, this replacement will minimally affect the ground states, and reduce the excited states significantly. The parameter \( \mu \) is related to the strength of the screening effects, and one can roughly understand that the screening effects begin to work from \( r \sim 1/\mu \). With the smearing transformation, one have

\[
V^{scr}(r) = \frac{b}{\mu r^2} \left( e^{\frac{\mu r}{2}} \left( 1 + \int_0^{\frac{\mu r}{2}} e^{-x^2} dx - \frac{1}{2} \right) \right) + \frac{2r e^{-r/2}}{2r_{12}^2} + \frac{r - e^{-r/2}}{2r_{12}^2} \left( \frac{1}{\sqrt{\pi}} \int_0^{\frac{2r_{12}}{r}} e^{-x^2} dx - \frac{1}{2} \right) + c.
\]

Finally, by replacing the \( S(r) \) with \( V^{scr} \) in the original relativized quark model, we obtain the modified relativized quark model including the screening effects. The mass spectrum and wave functions of the mesons can be obtained by solving the relativized Hamiltonian, and the wave functions are used as inputs to investigate the subsequent strong decays for mesons.

C. The \( ^3P_0 \) model

In addition to the mass spectrum, the decay widths are crucial to identify the assignments for mesons. Here, we give a brief introduction of the \( ^3P_0 \) model which is widely used in studying two-body OZI-allowed strong decays of mesons [9, 38, 60–76]. In the \( ^3P_0 \) model, the strong decay of a meson takes place by producing a quark-antiquark pair with vacuum quantum number \( J^{PC} = 0^{++} \). The newly created quark-antiquark pair, together with the \( q\bar{q} \) in the initial meson, regroups into two outgoing mesons in all possible quark rearrangements. Some detailed reviews on the \( ^3P_0 \) model can be found in Refs. [62, 63, 68–70].

The transition operator \( T \) of the decay \( A \to BC \) in the \( ^3P_0 \) model is given by

\[
T = -3\gamma \sum_{m} \langle 1,m; 1,-m|0,0 \rangle \int d^3p_3 d^3p_4 \delta^3(p_3 + p_4)
\]

\[
y_m^3 \frac{1}{2} \left( \frac{P_3 - \vec{p}_4}{P_3 - \vec{p}_4} \right)^{\chi_3^{1/2}} m_3^{1/2} \omega_0^{1/2} b_3^m(p_3) d_4^m(p_4),
\]

where the \( \gamma \) is a dimensionless parameter denoting the production strength of the quark-antiquark pair \( q\bar{q} \) with quantum number \( J^{PC} = 0^{++} \). \( p_3 \) and \( p_4 \) are the momenta of the created quark \( \bar{q} \) and antiquark \( q \), respectively. \( m_3^{1/2} \), \( \omega_0^{1/2} \), and \( \chi_3^{1/2} \) are the spin, flavor, and color wave functions of \( q\bar{q} \) pair, respectively. The solid harmonic polynomial \( y_m^3(p) \equiv |p|^3 Y_m^3(\theta_p, \phi_p) \) reflects the momentum-space distribution of the \( q\bar{q} \) pair.

The \( S \) matrix of the process \( A \to BC \) is defined by

\[
(BC|S|A) = I - 2\pi i \delta(E_A - E_B - E_C)(BC|T|A),
\]

where \( |A \rangle (|B \rangle, |C \rangle) \) is the mock meson defined by [77]

\[
|A(\eta_A^{2S+1}J_A M_A, \Lambda_A)(p_A)\rangle \equiv \sqrt{2E_A} \sum_{M_A, M_A} \langle L_A M_L, S_A M_S |J_A M_L \rangle \times \int d^3p_3 \psi(n_{1A} M_{1A} n_{2A} M_{2A})(p_3) \chi_{L_A}^{1/2} \phi_{S_A}^{1/2} \omega_{J_A}^{1/2} \times \left| q_1 \right|^m \left( m_{1A} = m_1 + p_1 \right) \left| q_2 \right|^m \left( m_{2A} = m_2 + p_2 \right) - \Lambda_A \rangle.
\]

Here, \( m_1 \) and \( m_2 \) \((p_1 \) and \( p_2 \)) are the masses (momena) of the quark \( q_1 \) and the antiquark \( \bar{q}_2 \), respectively; \( p_A = p_1 + p_2 \), \( p_A = \frac{m_1 p_1 + m_2 p_2}{m_1 + m_2} \), \( \chi_{L_A}^{1/2} \), \( \phi_{S_A}^{1/2} \), \( \omega_{J_A}^{1/2} \), and \( \psi(n_{1A} M_{1A} n_{2A} M_{2A})(p_3) \) are the spin, flavor, color, and space wave functions of the meson \( A \) composed of \( q_1 \bar{q}_2 \) with total energy \( E_A \), respectively. \( n_A \) is the radial quantum number of the meson \( A \). \( S_A = s_{q_1} + s_{\bar{q}_2} \), \( J_A = L_A + S_A \), \( s_{q_1}(s_{\bar{q}_2}) \) is the spin of \( q_1(\bar{q}_2) \), and \( L_A \) is the relative orbital angular momentum between \( q_1 \) and \( \bar{q}_2 \).

The transition matrix element \( (BC|T|A) \) can be written as

\[
(BC|T|A) = \delta^3(p_A - p_B - p_C)M^{M_A M_S M_C}(p),
\]
where the helicity amplitude $M_{M_A M_B M_C}(p)$ is

$$M_{M_A M_B M_C}(p) = \sqrt{8E_A E_B E_C} \sum_{M_{A_4}} \sum_{M_{B_4}} \sum_{M_{C_4}} \langle L_A M_{A_4} S_A M_S | J_A M_L \rangle$$

$$\times \langle L_B M_{B_4} S_B M_S | J_B M_L \rangle \phi_{M_{C_4}} \langle J_C M_L | M_C \rangle$$

$$\times (1 + m 00) \chi_{M_{S_4} M_{C_4} M_{L_4}} \chi_{M_{S_4} M_{C_4} M_{L_4}}$$

$$\sum_{f_1} (-1)^{1 + S_4 + S_B + S_C} f_2 (-p, m_1, m_2, m_3) \right) \right) \right),$$

with $f_1 = \langle \phi_{M_{C_4}} | \phi_{M_{C_4}} \rangle$ and $f_2 = \langle \phi_{M_{C_4}} | \phi_{M_{C_4}} \rangle$, and

$$I(p, m_1, m_2, m_3) = \int d^3p |\psi_{m_1 M_A, S_A, M_S}|^2 - p_b + p \rangle$$

$$\times |\psi_{m_2 M_B, S_B, M_S}|^2 - p_b + p \rangle \chi_{M_{S_4} M_{C_4} M_{L_4}}.$$

Various $\gamma_0$ models exist in literature and typically differ in the choices of the pair-production vertex, the phase space conventions, and the meson wave functions employed. In this work, we restrict to the simplest vertex as introduced originally by Micu [79] which assumes a spatially constant pair creation strength $\gamma$ for the $u\bar{u}$ and $d\bar{d}$ pairs. For the $s\bar{s}$ pair, the creation strength is multiplied by a factor $m_s/m_t$. The wave functions can be obtained from the modified relativized quark model including the screening effects. With the relativistic phase space, the decay width $\Gamma(A \rightarrow BC)$ can be expressed in terms of the partial wave amplitude

$$\Gamma(A \rightarrow BC) = \frac{\pi|p|^4}{4M_A} \sum_{LS} |M^{LS}(p)|^2,$$

where $|p| = \sqrt{M_A^2 - (M_B + M_C)^2} |M_B^2 - (M_B + M_C)^2 | / 2M_A$, and $M_A$, $M_B$, and $M_C$ are the masses of the mesons $A$, $B$, and $C$, respectively.

### III. Calculation and Results

#### A. Mass spectrum

The relevant parameters used in the original relativized quark model are listed in Table I [14]. When the screening effects are included, and extra parameter $\mu$ is introduced, which reflects the strength of screening effects. In present work, we can get the parameter $\mu$ by reproducing the experimental data of low-lying states. As mentioned in the Introduction, seven states, $D_1$, $D_2^*$, $D_{13}(2536)$, $D_{12}^*(2573)$, $D_{14}^*(2700)$, $D_{15}^*(2860)$, and $D_{16}^*(2860)$, can be reasonably classified in the conventional charmed-strange mesons. Since the $D_{13}(2536)$ is a mixture of the $D_1(1^1 P_1)$ and $D_1(1^3 P_1)$ states, we do not include it when determining the parameter $\mu$. Also, the overall constant $c$ is readjusted by fixing the mass of $D_{13}(1^3 S_0)$ to 1968 MeV when the $\mu$ varies.

The mass spectrum of the charmed-strange meson with $\mu$ from 0.04 to 0.05 GeV is listed in Table II. For comparison, the experiment data and predictions of the original relativized quark model are also presented. It can be seen that the measured masses of the low-lying states can be well reproduced and the predicted spectrum in the screened potential is improved significantly. Moreover, we can estimate the corresponding $\chi^2$ and present them in Table II. Here the $\chi^2$ can be defined as

$$\chi^2 = \sum_i \left( \frac{A_{th}(i) - A_{exp}(i)}{\text{Error}(i)} \right)^2,$$

where $A_{th}(i)$, $A_{exp}(i)$, and Error$(i)$ are theoretical values, experimental values, and experimental errors, respectively. With the reasonable range of $\mu$, the $\chi^2$ of screened potential is significantly smaller than that of original relativized quark model.

It should be mentioned that the $\chi^2$ is not the only criterion of the performances for different predictions in quark models. From Eq. (25), if the experimental accuracies of several states are high enough, the model with the smallest $\chi^2$ may only reproduce these few states and fail to describe the whole mass spectrum. Phenomenologically, we also expect the absolute value $|A_{th}(i) - A_{exp}(i)|$ for each state is not too large, such that these states can be interpreted in the conventional $c\bar{s}$ picture. In the range of $\mu = 0.04 \sim 0.05$ GeV, the results meet the above requirements.

Hence, we prefer to choose $\mu = 0.045$ GeV to calculate the mass spectrum of charmed-strange mesons, and take the masses with $\mu = 0.04$ and 0.05 GeV as the theoretical uncertainties. With $\mu = 0.045$ GeV, one can obtain the constant $c$ equals to $-0.243$. The theoretical predictions together with experimental data are shown in Figure 1. It can be seen that the $D_{13}(2590)^+$ can be assigned as the $D_1(2^3 S_0)$ state according to its mass. Moreover, we compare the predictions of different models in Table III, and find that they give rather different predictions for the higher states. Also, the screening

![Table 1: Parameters in the Godfrey-Isgur's relativized quark model [14].](image)

| Parameter value | Parameter value | Parameter value | Parameter value |
|-----------------|-----------------|-----------------|-----------------|
| $m_0$ (GeV)     | 0.22            | $b$ (GeV$^2$)   | 0.18            |
| $m_1$ (GeV)     | 0.22            | $c$ (GeV)       | -0.253          |
| $m_2$ (GeV)     | 0.419 $\sigma_0$ (GeV) | 1.8 | $\epsilon_{2s}$ | -0.035 |
| $m_3$ (GeV)     | 1.628           | $s$             | 1.55            |
| $\epsilon_{2s}$ | +0.055          | $\epsilon_{2s}$ | +0.055          |
TABLE II: Comparison of the experimental data and theoretical results with different $\mu$. We take $\mu=0.04$, 0.045, 0.05 GeV to show the results with the modified relativized quark model with screened potential. We also list the $\chi^2$ values for different models.

| $^{n^2s^2}L_J$ | Experimental values | GI model | Modified GI model | $\mu = 0.04$ | $\mu = 0.045$ | $\mu = 0.05$ |
|----------------|---------------------|----------|------------------|-------------|-------------|-------------|
| $D^+$          | $^1S_0$             | 1968.34±0.07 | 1979 | 1968 | 1968 | 1968 |
| $D^*_{s1}$     | $^1S_1$             | 2112.2±0.4  | 2129 | 2114 | 2114 | 2113 |
| $D^*_{s1}(2573)$ | $^1P_2$             | 2569.1±0.8  | 2592 | 2559 | 2556 | 2555 |
| $D^*_{s1}(2700)$ | $^3S_1$             | 2708±4.0    | 2732 | 2681 | 2675 | 2670 |
| $D^*_{s1}(2860)$ | $^1D_1$             | 2859±12±24  | 2899 | 2839 | 2833 | 2827 |
| $D^*_{s1}(2860)$ | $^3D_1$             | 2860±2.6±6.5| 2917 | 2858 | 2852 | 2846 |
| $\chi^2$       |                     | 666        | 55.17 | 86.23 | 119.39 |   |

FIG. 1: Mass spectrum of the charmed-strange mesons in units of MeV. The black lines show the MGI model with $\mu = 0.045$ GeV and the shaded regions stand for the theoretical uncertainties with $\mu = 0.04 - 0.05$ GeV. The dark blue dot denote the experimental data [80] and the vertical lines represent the errors.

Effects become increasingly important as the masses go up. The information on highly excited states is crucial to distinguish these different models and test our screened potential.

### B. Strong decays

Besides the mass spectrum, the strong decay behaviors are essential to clarify the internal structure of a new resonance. In this work, the $^3P_0$ model is adopted to investigate the strong decays of the $D_{s0}(2590)$. While we calculate the mass spec-
TABLE III: Our predicted masses of charmed-strange mesons compared with the experimental data and other quark model predictions. The units are in MeV.

| State | \(J^P\) | Ours | NLZ [3] | EFG [4] | ZVR [5] | GM [6] | LNR [7] | DE [8] | LJM [9] | GI [14] | Exp [80] |
|-------|---------|------|---------|---------|---------|--------|--------|--------|--------|---------|---------|
| \(D_s(1^P_S_0)\) | 0+ | 1968 | 1969 | 1940 | 1979 | 1975 | 1965 | 1969 | 1979 | 1968.34±0.07 |
| \(D_s(1^P_S_1)\) | 1+ | 2114 | 2112 | 2111 | 2130 | 2129 | 2180 | 2113 | 2107 | 2129 | 2112±0.4 |
| \(D_s(2^P_S_0)\) | 0+ | 2620 | 2649 | 2668 | 2610 | 2673 | 2659 | 2700 | 2640 | 2673 | 2591±6±7 |
| \(D_s(2^P_S_1)\) | 1+ | 2675 | 2737 | 2731 | 2730 | 2722 | 2806 | 2714 | 2732 | 2708±4.4 |
| \(D_s(3^P_S_0)\) | 0+ | 3072 | 3126 | 3219 | 3090 | 3154 | 3044 | 3029 | - | - | - |
| \(D_s(3^P_S_1)\) | 1+ | 3036 | 3196 | 3242 | 3190 | 3193 | 3087 | 3345 | - | - | - |

TABLE IV: Decay widths of \(D_s(2^P_0)\), \(D_s(2^P_1)\), \(D_s(2^P_2)\), \(D_s(1^P_0)\), and \(D_s(1^P_1)\) with fitted \(\gamma = 9.32\) (in MeV).

| Mode | \(D_s(2^P_0)\) | \(D_s(2^P_1)\) | \(D_s(2^P_2)\) | \(D_s(1^P_0)\) | \(D_s(1^P_1)\) |
|-------|---------------|---------------|---------------|---------------|---------------|
| DK | 12.07 | 61.13 | 150.72 | 25.65 |
| DK’ | 1.27 | 116.86 | 76.36 | 17.27 |
| \(D_s^*K^0\) | 0.03 | 3.69 | 45.22 | 1.32 |
| \(D_s^*\eta\) | - | 1.61 | 10.76 | 0.52 |
| \(D_s^*\eta'\) | - | - | 3.72 | 0.16 |
| Total width | 13.37 | 182.48 | 286.80 | 44.92 |
| Experiment | 16.9±0.7 | 122±10 | 159±23±7 | 53±7±7 |

TABLE V: Decay widths of \(D_0(2590)\) as the \(D_s(2^1S_0)\) state with fitted \(\gamma = 9.32\) (in MeV).

| Mode | \(D_0(2590)\) |
|-------|---------------|
| \(D_0^*K^0\) | 35.52 |
| \(D_0^*\eta\) | 39.38 |
| Total width | 74.90 |
| Experiment | 89±16±12 |

...trum, the corresponding wave functions of mesons are also obtained. Then, only one parameter \(\gamma\) in the \(\bar{P}_0\) model needs to determine. We can assume that the charmed-strange mesons share the same \(\gamma\), and fit this parameter from the known states. Among the seven reasonably classified states, \(D_s\) only decays though weak processes, \(D_s^*\) has no OZI-allowed strong decay, and the strong decays of \(D_s(2536)\) depend on the mixing angle sensitively. Hence, we adopt the remaining resonances, \(D_s^*(2573), D_s^*(2700), D_s^*(2860),\) and \(D_s^*(2860),\) to fit the parameter \(\gamma\).

According to the fitting process, the \(\gamma = 9.32\) is obtained, and the strong decay behaviors of the \(D_s(2573), D_s(2700), D_s^*(2860),\) and \(D_s^*(2860),\) are listed in Table IV. It can be seen that the calculated widths of \(D_s^*(2573)\) and \(D_s^*(2700),\) are consistent with the experimental data within errors, and the theoretical width of \(D_s^*(2700)\) and \(D_s^*(2860)\) seems a little bit larger. These differences may arise from the theoretical uncertainties of the \(\bar{P}_0\) model or the possible complicated S - D mixing mechanism for the \(D_s^*(2700)\) and \(D_s^*(2860)\) states. Hence, with the \(\gamma = 9.32,\) the strong decay behaviors of these four states are fairly described. We employ this value to investigate the strong decays of \(D_0(2590).\)

The results of \(D_0(2590)\) as the \(D_s(2^1S_0)\) state are listed in Table V. The calculated width is about 75 MeV, which agrees well with the experimental data 89±16±12 MeV. Also, the dependence on the mass of initial state is shown in Fig.2. When the mass of initial \(D_s(2^1S_0)\) state varies from 2570 to 2610 MeV, the total width lies in the range of 51 to 98 MeV. Our results indicate that the \(D_0(2590)^*\) observed by LHC Collaboration can be interpreted as the conventional \(D_s(2^1S_0)\) state.
In this work, we investigate the mass spectrum of charmed-strange mesons with the modified relativized quark model including the screening effects. With reasonable strength of screening effects, the calculated mass spectrum can explain the $D_{s0}(2590)^+$ as well as other known charmed-strange mesons. The information on highly excited states is crucial to distinguish various predictions and test our results with screened potential.

Besides the mass spectrum, the strong decays of $D_{s0}(2590)^+$ as $D_s(2S)_0$ state are also investigated in the 3P model with the obtained relativistic wave functions. The calculated width is about 75 MeV, which agrees well with the experimental data $89 \pm 16 \pm 12$ MeV. Our results indicate that the $D_{s0}(2590)^+$ can be interpreted as the conventional $D_s(2S)_0$ state.

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