The need for an early anti-neutrino run of NOνA

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The moderately large value of θ13, measured recently by reactor experiments, is very welcome news for upcoming accelerator experiments. In particular, the NOνA experiment, with 3 years of ν run followed by an equal ν run, will be able to determine the mass hierarchy if one of the following two favorable combinations is true: normal hierarchy with −180° ≤ δCP ≤ 0 or inverted hierarchy with 0 ≤ δCP ≤ 180°. In this letter, we study the hierarchy reach of the first 3 years of NOνA data. Since sin2 θ13 is measured to be non-maximal, θ23 can be either in the lower or higher octant. The true octant of θ23 has a deep impact on the hierarchy reach of early NOνA data. With the present uncertainty of 10% in sin2 θ13, equal 1.5 year ν and ν run have better hierarchy determination capability compared to a pure 3 year ν run. Daya Bay expects to reduce the uncertainty in sin2 θ13 to 5%. Such a reduction improves the hierarchy reach of a 3 year ν run for two of the four octant-hierarchy combinations, but still fails to give any sensitivity for the other two. However, equal 1.5 year ν and ν runs have reasonable hierarchy sensitivity for all four combinations.

PACS numbers: 14.60.Pq, 14.60.Lm, 13.15.+g

Introduction: Neutrino oscillations are the first and, so far, the only evidence we have for physics beyond standard model. During the past year and a half, a lot of excitement has been created after the reactor neutrino experiments measured the last neutrino mixing angle θ13 to be non-zero [1–3]. In fact, its measured value is moderately large and is just below the upper limit established earlier. Daya Bay experiment gives the most precise value of sin2 2θ13 = 0.089 ± 0.01 [1]. By the end of its run, the uncertainty is expected to be reduced from the present 10% to 5% [4]. Another important recent discovery is the precision measurement of sin2 θ23 by MINOS, which found it to be non-maximal [5]. This raises the problem of determining the true octant of θ23.

Neutrino oscillations depend on two mass-squared differences, ∆21 = m2 2 − m1 2 and ∆31 = m2 3 − m1 2, three mixing angles and a CP violating phase δCP. Here m1, m2 and m3 are the masses of three mass eigenstates. The present oscillation data determine all oscillation parameters reasonably well except δCP, on which there is no information [6–8]. The observed energy dependence of the solar neutrino survival probability requires ∆21 to be positive. But, present data allow ∆31 to be either positive or negative. Given that |∆31| ⪆ ∆21, we have two possibilities: m3 ⪰ m2 > m1, called normal hierarchy (NH) or m2 > m1 ⪰ m3, called inverted hierarchy (IH). Determination of the neutrino mass hierarchy, the octant of θ23 and the search for CP violation in neutrino sector are the important physics goals of current and future oscillation experiments.

A number of models are proposed to explain the observed pattern of neutrino masses and mixing. Among these, the models predicting NH are qualitatively different from those predicting IH. Therefore, the determination of the neutrino mass hierarchy will enable us to distinguish between different types of models. A large number of these models predict θ13 to be zero and θ23 to be maximal. A precise measurement of the deviations from these predictions will enable us to discern the pattern of symmetry breaking in the models. Ever since the possibility of generating baryon asymmetry via leptogenesis was raised [9], the search leptonic CP violation has acquired great significance.

A simple way to achieve the above three goals is to measure the probabilities for νµ → νe oscillation (Pµe) and νµ → νe oscillation (Pµµ). The leading term in both these probabilities is proportional to sin2 θ13 sin2 θ23. Therefore, the moderately large value of θ13 makes it possible for the current experiments to address the problems of both hierarchy and the octant of θ23. In particular, the NOνA experiment can determine the mass hierarchy [10, 11] and octant of θ23 [12, 13] for about half the possible values of δCP.

The leading term in Pµe and in Pµµ [14] are driven by the matter modified ∆31, which depends on hierarchy. The second term, suppressed by the small parameter α = ∆21/∆31, depends on δCP. For NH (IH), the first term in Pµe becomes larger (smaller). For Pµµ, the situation is reverse. The change in the first term can be canceled by changing the value of δCP in second term, leading to hierarchy-δCP degeneracy [15, 16]. This is illustrated in fig. 1, where Pµe and Pµµ for NOνA are plotted. We see that, Pµe and Pµµ for NH and δCP in the upper half plane (UHP) (0 ≤ δCP ≤ 180°) are very close to or degenerate with those of IH and δCP in the lower half plane (LHP) (−180° ≤ δCP ≤ 0). For these unfavorable combinations, NOνA has no hierarchy sensitivity [11]. Addition of T2K data gives rise to a small sensitivity [16, 17]. On the other hand, for NH and δCP in LHP the values of Pµe (Pµµ) are reasonably greater (lower) than the values of Pµe (Pµµ) for IH and any value of δCP. Similarly, for IH and δCP in UHP the values of Pµe (Pµµ) are reasonably lower (greater) than the values of Pµe (Pµµ) for NH and any value of δCP. Hence, for these combinations NOνA is
In fig. 2 we have shown the hierarchy determination potential of NOvA assuming a 10% uncertainty in $\sin^2 2\theta_{13}$ and maximal $\theta_{23}$. In the left (right) panel, the true hierarchy is taken to be NH (IH).

FIG. 1: $P_{\mu e}$ (top panel) and $P_{\bar{\mu}e}$ (bottom panel) vs. energy for NOvA. Variation of $\delta_{CP}$ in the range $[-180^\circ, 180^\circ]$ leads to the blue (red) bands for NH (IH). The curves for $\delta_{CP} = \pm 90^\circ$ are emphasized in each case. The plots are drawn for maximal $\theta_{23}$ and other neutrino parameters given in the text.

FIG. 2: Hierarchy sensitivity assuming 10% uncertainty in $\sin^2 2\theta_{13}$ and maximal $\theta_{23}$. In the left (right) panel, the true hierarchy is taken to be NH (IH).

capable of determining the hierarchy at a confidence level (C.L.) of 2$\sigma$ or better, with 3 years each of $\nu$ and $\bar{\nu}$ runs.

In this letter, we study the possible hierarchy reach of the first three years of NOvA data. We consider two options: (a) a 3 year $\nu$ run (3$\nu$) and (b) equal $\nu$ and $\bar{\nu}$ runs of 1.5 years each (1.5$\nu$ + 1.5$\bar{\nu}$). We find that the combined $\nu$ and $\bar{\nu}$ runs have a superior hierarchy determination capability compared to the pure neutrino run.

**Simulations:** NOvA experiment [18] consists of a 14 kiloton totally active scintillator detector (TASD), placed 810 km away from Fermilab, situated at a 0.8° off-axis location from the NuMI beam. The $\nu$ flux peaks sharply at 2 GeV, close to the oscillation maximum energy of 1.4 GeV. It is scheduled to have equal $\nu$ and $\bar{\nu}$ runs of 3 years each, with a NuMI beam power of 700 kW, corresponding to $6 \times 10^{20}$ protons on target per year. In our simulations, we have used the re-tuned signal acceptance and background rejection factors taken from [17, 19]. In the numerical simulations, we took the solar oscillation parameters to be $\sin^2 \theta_{12} = 0.30$ and $\Delta_{21} = 7.5 \times 10^{-5}$ eV$^2$, which have been kept fixed. The other parameters used are $\sin^2 \theta_{13} = 0.089$ and $\Delta m^2_{32} = \pm 2.4 \times 10^{-3}$ eV$^2$, where the positive (negative) sign is for NH (IH). $\Delta_{31}$ is derived from $\Delta m^2_{32}$ from the expression given in [20]. In the case of $\theta_{23}$, we considered the cases of both maximal and non-maximal mixing. For maximal mixing (MM), $\sin^2 \theta_{23} = 0.5$. For non-maximal mixing, we have used the two degenerate best-fit values of the global fits: 0.41 for $\theta_{23}$ in the lower octant (LO) and 0.59 for $\theta_{23}$ in the higher octant (HO) [8]. The number of electron neutrino appearance events $N_e$ and the electron anti-neutrino appearance events $N_{\bar{e}}$ are first computed for an assumed true hierarchy. The same quantities are calculated again for the wrong hierarchy and the $\Delta \chi^2$ is computed between the event numbers for the true and the wrong hierarchies. The event number simulations and the $\Delta \chi^2$ calculation are done by using the software GLoBES [21, 22]. The minimum $\Delta \chi^2$ is computed by doing a marginalization over the neutrino parameters. We took $\sigma(\Delta m^2_{32}) = 3\%$ [23] and $\sigma(\sin^2 2\theta_{13}) = 10\%$ in the preliminary calculations and 5% in later calculations. For both these parameters, the marginalization was done over 2$\sigma$ range. The marginalization range for $\sin^2 \theta_{23}$ is its 3$\sigma$ allowed range: [0.35, 0.65] and that of $\delta_{CP}$ is the full range $[-180^\circ, 180^\circ]$.

**Effect of precision of $\sin^2 2\theta_{13}$** In fig. 2 we have shown the hierarchy determination potential of NOvA assuming a 10% uncertainty in $\sin^2 2\theta_{13}$. The plots show $\Delta \chi^2$ vs. $\delta_{CP}$ (true) for $\theta_{23} = 45^\circ$, for both 3$\nu$ and 1.5$\nu$ + 1.5$\bar{\nu}$ runs. The left panel is for (NH and LHP) and the right panel is for (IH and UHP). We see from these plots that a 2$\sigma$ hierarchy determination is possible for about 50% of the favorable half plane for 1.5$\nu$ + 1.5$\bar{\nu}$ runs, whereas a 3$\nu$ run can determine hierarchy for only a smaller range. In particular, if (IH and UHP) is true, a 2$\sigma$ hierarchy determination is not possible for any $\delta_{CP}$. The lower sensitivity of 3$\nu$ run is due to the marginalization over $\theta_{13}$. The 1.5$\nu$ + 1.5$\bar{\nu}$ run is less sensitive to this marginalization. This occurs because of the following reason. Let us assume (NH and LHP) is true. Because $\delta_{CP}$ values are in favorable half plane, for a given $\theta_{13}$, $P_{\mu e}(\text{NH})$ is greater than $P_{\mu e}(\text{IH})$ for any $\delta_{CP}$, implying a clean hierarchy separation. But, when we marginalize over $\theta_{13}$, it is possible to have $P_{\mu e}(\text{IH}, \theta_{13}')$ close to $P_{\mu e}(\text{NH}, \theta_{13})$, where $\theta_{13}' > \theta_{13}$. Hence, the $\Delta \chi^2$ for 3$\nu$ run will be moderate. However, for $\bar{\nu}$ case, $P_{\bar{\mu}e}(\text{IH}, \theta_{13}')$ will be farther away from $P_{\bar{\mu}e}(\text{NH}, \theta_{13})$. Thus, if the value of $\theta_{13}'$ is chosen to minimize $|N_e(\text{NH}, \theta_{13}') - N_e(\text{IH}, \theta_{13}')|$, then $|N_e(\text{NH}, \theta_{13}) - N_e(\text{IH}, \theta_{13}')|$ becomes larger. Varying $\theta_{13}'$ away from the true value of $\theta_{13}$ can make $\Delta \chi^2$ very small but it will also make $\Delta \chi^2$ very large, leading to the total $\Delta \chi^2 = \Delta \chi^2_{\nu} + \Delta \chi^2_{\bar{\nu}}$ also being very large. Be-
cause of this, for $1.5\nu + 1.5\bar{\nu}$ run, the minimum of $\Delta \chi^2$ occurs when $\theta_{13}$ very close to $\theta_{13}$ even when the marginalization range of $\theta_{13}$ is large. Therefore $1.5\nu + 1.5\bar{\nu}$ run is less sensitive to marginalization in $\theta_{13}$ and gives a better hierarchy reach compared to $3\nu$ run, if the uncertainty in $\sin^2 2\theta_{13}$ is large. If the uncertainty in $\sin^2 2\theta_{13}$ is reduced to 5%, the hierarchy reach for $3\nu$ does improve and becomes equal to that of $1.5\nu + 1.5\bar{\nu}$ run.

**Non-maximal $\theta_{23}$**: We now assume that $\sigma(\sin^2 2\theta_{13}) = 5\%$ and take $\theta_{23}$ to be non-maximal. Once again we limit ourselves to the favorable hierarchy-$\delta_{CP}$ combinations, (NH and LHP) and (IH and UHP). But, because of the octant degeneracy of $\theta_{23}$, we must consider four possible combinations of octant and hierarchy: LO-NH, HO-NH, LO-IH and HO-IH.

In fig. 3, we show the hierarchy capability assuming (NH and LHP). The left (right) panel corresponds to $\theta_{23}$ in LO (HO). In fig. 4, we do the same for (IH and UHP), where again the left and the right panels have the same interpretation. From these figures, we see that for HO-NH and LO-IH, $3\nu$ run does have a better hierarchy reach compared to $1.5\nu + 1.5\bar{\nu}$ run and is capable of giving a better than $2\sigma$ hierarchy discrimination for more than half of the favorable half-plane. But, for the other two possibilities, LO-NH and HO-IH, $3\nu$ run has no hierarchy sensitivity whereas $1.5\nu + 1.5\bar{\nu}$ run has reasonable hierarchy sensitivity. Addition of 5 year $\nu$ data from T2K leads only to a small improvement.

The very small values of $\Delta \chi^2$, for the $3\nu$ run, occur due to the marginalization over $\sin^2 \theta_{23}$ and $\delta_{CP}$. As mentioned before, the dominant term in $P_{\mu\bar{\nu}}$ is proportional to $\sin^2 2\theta_{13} \sin^2 \theta_{23}$. Matter effects in NH make this term larger and choosing HO makes it even larger. Hence, for $\delta_{CP}$ in LHP, $P_{\mu\bar{\nu}}(\text{HO-NH})$ is significantly higher than $P_{\mu\bar{\nu}}(\text{IH})$ for any values of neutrino parameters. Because of the double increase in the probability, the statistics for HO-NH will be quite large. Hence, this combination has 2$\sigma$ hierarchy discrimination for $87\%$ (68\%) of the favorable half-plane for $3\nu$ ($1.5\nu + 1.5\bar{\nu}$) run. Matter effects in IH make the leading term in $P_{\mu\bar{\nu}}$ smaller and choosing LO makes it even smaller. So, for $\delta_{CP}$ in UHP, $P_{\mu\bar{\nu}}(\text{LO-IH})$ is significantly smaller than $P_{\mu\bar{\nu}}(\text{NH})$ for any values of neutrino parameters. This double decrease in probability, leads to the lowest statistics for LO-IH. Here, $3\nu$ ($1.5\nu + 1.5\bar{\nu}$) run can determine hierarchy at $2\sigma$ for $35\%$ (20\%) of favorable half-plane. However, it must be emphasized that, in these two cases HO-NH and LO-IH, the hierarchy reach of $1.5\nu + 1.5\bar{\nu}$ is only slightly worse than that of $3\nu$.

But, for the combination of LO-NH, the choice of NH increases $P_{\mu\bar{\nu}}$ whereas the choice of LO lowers it. Similarly, for the combination HO-IH, the choice of IH lowers $P_{\mu\bar{\nu}}$ and the choice of HO increases it. The marginalization over $\theta_{23}$ and $\delta_{CP}$ leads to a wrong hierarchy probability being very close to that of the true hierarchy. Thus, it is possible to have $P_{\mu\bar{\nu}}(\text{LO-NH}, \delta_{CP})$ mimic $P_{\mu\bar{\nu}}(\text{HO-IH}, \delta_{CP})$, where $\delta_{CP}$ and $\delta_{CP}$ may or may not be equal. But, in the case of $\nu$, both the choices LO and NH lead to a reduction in the probability and both the choices HO and IH increase the probability. Whenever it is possible to have $P_{\mu\bar{\nu}}(\text{LO-NH}) \approx P_{\mu\bar{\nu}}(\text{HO-IH})$, the corresponding values of $P_{\mu\bar{\nu}}$ will be far apart. This is illustrated in fig. 5 for two cases, where the two left panels have $\delta_{CP} = \delta_{CP}$ and the two right panels have $\delta_{CP} \neq \delta_{CP}$. The large separation in $P_{\mu\bar{\nu}}$ leads to a far better hierarchy discrimination for $1.5\nu + 1.5\bar{\nu}$ run compared to $3\nu$ run. All the results discussed above are neatly summarized in the table I. In all cases, the $1.5\nu + 1.5\bar{\nu}$ data is insensitive to the uncertainty in $\sin^2 2\theta_{13}$. Except for the no-sensitivity combinations, LO-NH and HO-IH, the $3\nu$ data shows noticeable improvement when the uncertainty is reduced to 5\% but none with further reduction to 2\%.
FIG. 5: Illustration of degenerate $P_{\mu e}$ and non-degenerate $P_{\mu e}$ for the following two cases. Left: (LO-NH, $\delta_{CP} = -45^\circ$) and (HO-III, $\delta_{CP} = -45^\circ$). Right: (LO-NH, $\delta_{CP} = -90^\circ$) and (HO-III, $\delta_{CP} = -45^\circ$).

TABLE I: Hierarchy discrimination reach of NO$\nu$A data for 1.5$\nu$ + 1.5$\bar{\nu}$ and 3$\nu$ runs. The upper (lower) half is for NH-LHP (IH-UHP) true. In each case, the $\Delta \chi^2$ values are shown for $\theta_{23}$ being in LO, maximal and in HO and for three values of $\delta_{CP}$, covering half of the favorable half plane. $\Delta \chi^2$ values for the other half are nearly symmetric about $\delta_{CP} = \pm 90^\circ$, as can be seen from figs. 3 and 4. The three lines in each small box correspond to 10%, 5% and 2% precision in $\sin^2 2\theta_{13}$ respectively.

Conclusions: NO$\nu$A experiment plans to have a 3 year $\nu$ run followed by a 3 year $\bar{\nu}$ run. The hierarchy reach of the 3$\nu$ run depends on the uncertainty in $\sin^2 2\theta_{13}$. Even when this uncertainty is reduced to 5%, the 3$\nu$ run fails to give any hierarchy discrimination, if the true combinations are LO-NH or HO-III. Equal $\nu$ and $\bar{\nu}$ runs of 1.5 years each have good hierarchy discrimination for all octant-hierarchy combinations, which is independent of the uncertainty in $\sin^2 2\theta_{13}$. Therefore, it is imperative for NO$\nu$A to plan an early $\bar{\nu}$ run to get a first hint of hierarchy.

Acknowledgement: U. R. thanks Council for Scientific and Industrial Research (CSIR), Government of India, for financial support. We thank Sanjib Agarwalla for discussions on implementing retuned NO$\nu$A efficiencies in GLoBES.