On Stable Throughput of Cognitive Radio Networks With Cooperating Secondary Users

Kedar Kulkarni, Student Member, IEEE, and Adrish Banerjee, Senior Member, IEEE

Abstract—In this paper, we study cooperative cognitive radio networks consisting of a primary user and multiple secondary users. Secondary users transmit only when primary user is sensed as silent and may interfere with primary transmission due to imperfect sensing. When primary activity is sensed correctly, secondary users cooperate with primary user by assisting retransmission of failed packets of primary user. We analyze packet throughput of primary and secondary users for three variations of proposed cooperation method. Signal flow graph (SFG) based approach is employed to obtain closed form expressions of packet throughput. The analysis is done for two cases; individual sensing and cooperative sensing. Further, we characterize optimal transmission probability of secondary users that maximizes individual secondary packet throughput keeping all queues in the system stable. Results present a comparison of throughput performance of proposed cooperation methods under different scenarios and show their benefits for both primary as well as secondary user throughput.

Index Terms—Cognitive radio, cooperative relaying, queue stability, signal flow graph, stable throughput

I. INTRODUCTION

Studies have shown that currently allocated wireless spectrum is highly underutilized in temporal and spectral domain [1]. Cognitive radio (CR) is considered as a potential technology for efficient use of allocated spectrum [2]. In cognitive radio systems, unlicensed cognitive users, also called as secondary users (SUs) sense the spectrum for activity of licensed users or primary users (PUs). Depending on the sensing information, SUs make a decision to access the spectrum for their own communication. Commonly used spectrum access models are interweave mode and joint interweave-underlay mode. In interweave mode, SUs access the spectrum only when PUs are sensed to be silent. Due to errors in sensing, SU transmission may interfere with PU transmission. In joint interweave-underlay mode, SUs transmit even when PUs are sensed to be present. While opportunistically accessing the spectrum allocated to PU, SUs must ensure that quality of service (QoS) constraint of PUs is satisfied. Cognitive radio networks have been extensively studied in literature under information-theoretic framework with various objectives and constraints [3]–[5].

In practice, data transmission is of bursty nature. Data arrives at a transmitting source in form of packets of bits. In transmission, a whole packet is lost if not decoded correctly at the receiver. Study of such systems gives insights in network layer aspects like packet throughput. Generally at a source, packets generated in upper layers are stored in queues before transmission. In case of poor source-destination link or high interference, packet transmission fails. If queue length grows in unbounded manner, delivery of new packets cannot be guaranteed. Such a system is said to be unstable. Thus, it is important to study throughput of a stable system (called as stable throughput). In [6], authors studied stable throughput tradeoff between only-interweave and joint interweave-underlay mode for perfect sensing case. In [7], authors analyzed effect of energy availability at each user on stable throughput region of the system assuming perfect sensing. In [9], authors considered sensing errors and characterized optimal transmit power of SU that keeps PU queue stable under interweave mode. In [6]–[9], users transmit without any cooperation between them. SUs may act as relays for PU transmission and benefit due to cooperation as shown in [10]–[12] under information theoretic framework. In queue based systems, cooperation from SUs increases packet throughput of PU. As a result PU packet queue is emptied more often, providing more silent slots for SU to transmit. Shafie et al. analyzed stable throughput of a three-node network with one node acting as relay of finite queue size [13]. In [14], authors analyzed stable throughput for a primary multi-access system where SU receives packets from two PUs and relays them using superposition coding technique when PU slot is idle. Fodor et al. studied tradeoff between packet delay and energy consumption in a cooperative cognitive network [15]. A common cooperation method is cooperative relaying [16]. In cooperative relaying, SU receives unsuccessfully transmitted PU packets and relays them to PU destination on next transmission opportunity. An additional relay queue is needed at SU source for this purpose. In [17], authors considered a finite capacity relay queue and proposed packet admission control at relay queue to maximize SU packet throughput. Ashour et al. proposed admission control as well as randomized service at relay queue and analyzed stable throughput and packet delays [18]. Effect of energy availability on cooperation and stable throughput was studied in [19] for battery powered nodes.

A. Main results

In this paper, we propose a cooperation method where PU and SUs that have received unsuccessful PU packets form a virtual multiple-input single-output (MISO) system and retransmit the packet using distributed orthogonal space-time block code (D-OSTBC). Depending on which SUs assist PU transmission, we present three variations of the cooperation method and analyze stable throughput of the system. We take into account the effect of imperfect sensing. Only those SUs that sense presence of PU correctly, can receive PU packet. Misdetecting SUs may interfere with the packet transmission. Such cooperation model results in multiple tradeoffs as large number of SUs provide higher
cooperation to PU but can potentially cause more interference to assisted transmission due to imperfect sensing. Also, with increasing number of SUs, inter-SU interference increases and negatively affects individual SU packet throughput. Specifically, our contribution in this paper is as follows.

- We propose and model the basic cooperation protocol where multiple SUs that have received unsuccessful PU packet, retransmit the packet along with PU using D-OSTBC. We then propose three versions of the protocol, namely all relay cooperation (ARC), recurrent best relay cooperation (R-BRC) and non-recurrent best relay cooperation (NR-BRC). We analyze stable throughput of PU and SU for given protocols using signal flow graph (SFG) theory.
- The stable throughput analysis is done for two cases; individual sensing (IS) and cooperative sensing (CS). In individual sensing each SU senses spectrum and takes decision to transmit independently. In cooperative sensing, SUs share their sensing decisions (hard or soft decisions) and take a collective decision on availability of spectrum.
- We then characterize optimal transmission probability of SU that maximizes individual SU packet throughput while keeping PU queue stable.
- Finally, we present numerical results to study various trade-offs arising with different number of SUs, varying SU transmit power and sensing type (individual or cooperative).

B. Related work

In [16]-[19], stable throughput with cooperative relaying was studied under joint interweave-underlay mode for a simple two-user model. Fanous et al. extended the model for the case of multiple SUs where SUs receive unsuccessful PU packet and relay it using D-OSTBC in the next silent slot [20]. Due to joint interweave-underlay mode in [16], there is a chance that a PU packet being relayed by SU may collide with new packet transmitted by PU. Model in [20] solved this issue by mandating that SUs relay PU packets only in silent slots. However, it assumed perfect sensing. In contrast, we consider effect of imperfect sensing in this paper. Further, in cooperative relaying, a packet is removed from PU queue if it is successfully received at PU destination or at any of the SU sources. If a PU packet is received by a SU, responsibility of delivering the packet lies solely at SU. Such cooperation is ineffective if link between SU sources and PU destination is weak. Cooperation method proposed in this paper resolves this issue as a packet is removed from PU queue only when it is received successfully at PU destination. Thus, packet departure rate of PU queue equals goodput of PU, unlike cooperation method in [20].

Canzian et al. [21] studied throughput and average time for packet delivery in a non-cognitive network for two cooperation scenarios—forced cooperation where best relay retransmits a failed packet and voluntary cooperation where a user may act as relay to get higher access probability in return. In our paper, due to presence of PU, relaying capability of SUs is affected by sensing errors and interference by other users. In [21], authors modelled packet transmission process of automatic repeat request (ARQ) mechanism as Markov chain assuming that at most one retransmission per packet is allowed. We model transmission process of proposed protocols in a similar way but the signal flow graph approach employed for throughput analysis puts no restriction on number of retransmissions.

C. Organization

In Section II, we present the system model. In Section III, proposed cooperation method and its variations are explained. Also packet throughput expressions for given methods are derived. In Section IV and V, packet throughput of PU and SU is analyzed for individual sensing case and cooperative sensing case respectively. In Section VI, we present numerical results to demonstrate performance of proposed methods. Finally, we conclude in Section VII.

II. SYSTEM MODEL

As shown in Fig. 1, the system consists of a PU transmitting from source P to destination D. A secondary network of L SUs uses the same frequency band as that of PU to transmit packets from source $S_j$ to respective destination $R_j$, $j \in \{1, 2, \ldots, L\}$. All users store packets in queues before transmission. Each SU source has one queue for its own packets and one relay queue to store unsuccessful packets of PU. All queues have infinite storing capacity [15], [20]. This is a good approximation for practical systems with large queue sizes as packet loss probability due to buffer overflow is low. Similar to [20], we assume that all SUs have backlogged packet queues, that is each SU always has packets to transmit. Time is slotted such that duration of a slot equals time required to transmit one packet. Packet arrival at PU queue is a stationary Bernoulli process with average rate $\lambda_P \in [0, 1]$ packets per slot. Average packet departure rates of PU and SUs are denoted by $\mu_P$ and $\mu_{S_j}$, $j \in \{1, 2, \ldots, L\}$ respectively.

A. Stable throughput

A queue is said to be stable if the queue length does not increase in unbounded manner. Unlike an unstable queue where packets may get queued up indefinitely, packet delivery is guaranteed in a stable queue. For a system to be stable, all queues in the system should be stable. Packet throughput achieved in such
a system is called stable throughput. Thus, for the system of Fig. 1 to be stable, we need PU queue as well as relay queues at all SUs to be stable. In proposed cooperation protocol, relay queues at SUs are always stable due to the fact that relay queue length cannot exceed 1 as explained later in Section III. The PU queue evolves with time as

\[ Q_p^{t+1} = [Q_p^t - Y_p^t]^+ + X_p^t, \]

where \( Q_p^t \) denotes length of PU queue at the beginning of time slot \( t \), \( Y_p^t \) is number of packet departures in time slot \( t \) and \( X_p^t \) denotes number of arrivals in time slot \( t \). The operation \( [x]^+ \) denotes \( \max(0, x) \). PU queue is stable if for every \( x \in \mathbb{N}_0 \),

\[ \lim_{t \to \infty} P [Q_p^t < x] = F(x) \] with \( \lim_{x \to \infty} F(x) = 1 \). We use Loynes’ criteria for queue stability which states that, for jointly stationary packet arrival and departure processes, the queue is stationary packet arrival and departure processes, the queue is stable if average packet arrival rate is less than average packet departure rate [22]. Packet departure depends on channel fading, inference by SUs as well as cooperation offered by SUs, and is independent of packet arrivals. As PU packet departure rate is given by \( \mu_p = E[Y_p^t] \), condition for PU queue stability is

\[ \lambda_p < \mu_p. \]

Thus, to keep system stable, SUs should control the interference caused to PU and ensure PU queue stability.

### B. Physical layer model

All channels are independent Rayleigh block fading, that is, channel coefficients remain constant in one slot and change independently from slot to slot. Channel coefficient of link between source \( s \in \{P, S_1, S_2, \ldots, S_L\} \) and destination \( d \in \{D, R_1, R_2, \ldots, R_L\} \) is \( h_{sd} \sim \mathcal{CN}(0, \sigma_{sd}^2) \). We can classify the communication links in 6 classes; PU source to PU destination, PU source to SU destinations, SU source to SU destinations, SU sources to SU destination and SU source to other SU sources. Fig. 1 shows direct links between a source and its intended destination by continuous lines, interference links by dashed lines and cooperation links by dash-and-dotted lines. As done in [20], for mathematical tractability of our analysis, we consider a symmetric case where distribution parameters of all channels in one class are same. This is possible if all SU source nodes are in close vicinity and all SU destinations are in close vicinity. The proposed framework for analysis of stable throughput remains unchanged for a general asymmetric case. For the symmetrical case, channel properties are as follows.

- **PU source to PU destination** \( h_{PD} \sim \mathcal{CN}(0, \sigma_{PD}^2) \).
- **PU source to i-th SU destination** \( h_{PR_i} \sim \mathcal{CN}(0, \sigma_{PR_i}^2) \) for \( i \in \{1, 2, \ldots, L\} \).
- **PU source to j-th SU source** \( h_{PS_j} \sim \mathcal{CN}(0, \sigma_{PS_j}^2) \) for \( i \in \{1, \ldots, L\} \).
- **PU source to j-th SU destination** \( h_{SR_j} \sim \mathcal{CN}(0, \sigma_{SR_j}^2) \) for \( i \in \{1, 2, \ldots, L\} \).
- **SU source to SU destination** \( h_{SD} \sim \mathcal{CN}(0, \sigma_{SD}^2) \) for \( i \in \{1, 2, \ldots, L\} \).
- **SU source to j-th SU source** \( h_{SS_j} \sim \mathcal{CN}(0, \sigma_{SS_j}^2) \) for \( i \in \{1, 2, \ldots, L\}, \quad j \in \{1, 2, \ldots, L\} \).
- Noise is additive white Gaussian (AWGN) \( n \sim \mathcal{CN}(0, \sigma_n^2) \).

### C. Spectrum sensing and access

PU and SUs follow time slotted synchronous communication [23]. The slot is divided in two parts: first of duration \( T_s \) allocated to tasks like spectrum sensing and exchange of control information, second of duration \( T_t \) used for transmission of packets. In the first part, PU sends pilot signals to PU destination while SUs employ pilot based spectrum sensing to detect presence of PU transmission [24]. Due to channel fading, two types of sensing errors may occur, namely misdetection and false alarm. Misdetection happens when PU is active but a SU senses it as idle. False alarm occurs when PU is sensed as active when in fact it is silent. We denote probabilities of correct detection and false alarm by \( p_d \) and \( p_f \) respectively. Due to independence of fading channels between PU source and SU source nodes, misdetection and false alarm events are independent for all SU pairs. Furthermore, due to the assumption of symmetry, values of \( p_d \) and \( p_f \) are same for all SU pairs. In Section V, we also consider the case of cooperative sensing where SUs share their sensing data (hard sensing data or soft sensing data) and a collective decision is taken on the availability of spectrum [25–27]. Probabilities of correct detection and false alarm in this case are denoted as \( p_d^c \) and \( p_f^c \) respectively. In case of cooperative sensing, if PU is sensed as active, all SUs keep silent.

In the transmission duration, PU transmits using a feedback mechanism where destination node \( D \) sends an acknowledgement (ACK) to source \( P \) when a packet is correctly received. If packet transmission is unsuccessful, a negative acknowledgement (NACK) for that packet is sent. We make following assumptions regarding the system model as done in [16].

- The feedback channel is an error-free broadcast channel. Thus, SUs can overhear ACKs and NACKs sent by PU destination \( D \).
- Feedback is available immediately after packet transmission.
- SUs are able to receive packets transmitted by PU. This is possible if the SU sources lie in vicinity of PU source transmitting with an omni-directional antenna.
- SUs can either receive or transmit at a time but cannot do both actions simultaneously. This holds true in most practical cases where nodes are equipped with a single transceiver pair. Thus, a SU is able to receive PU packets only when it is silent.

If PU is sensed as silent, in transmission duration, each SU transmits independently with probability \( q \). If PU is sensed as present, a SU keeps silent (interweave mode) and cooperates using methods described in Section III. Transmission of SU’s own packets may occur in two cases; one when PU is active and SU misdetects, other when PU is silent and there is no false alarm. Usually the target detection probability is high, hence second case dominates achievable throughput [28]. Also probability of successful packet transmission of SU is less in case of interference from PU due to generally high PU transmit power. Thus, we restrict analysis of SU packet throughput to the second case which gives a lower bound on SU packet throughput performance. The bound is tight when PU transmit power is very high and link between PU source to SU destination is strong. The lower bound and actual performance coincide in case of perfect sensing, as PU and SUs do not interfere.
D. Probability of successful packet transmission

PU transmits with power $P_P$ and each SU transmits with power $P_S$. PU and SU packets have fixed length of $B$ bits. A packet is delivered successfully to the intended receiver in a slot if instantaneous channel capacity on the source-destination link is greater than $\frac{B}{Ts}$ bits/s. Instantaneous channel capacity can be given as $R = W \log_2 (1 + \gamma)$ bits/s where $\gamma$ is received signal to interference plus noise ratio (SINR) and $W$ is channel bandwidth. Then the probability of successful packet transmission is $\Pr \left[ \gamma > \frac{B}{Ts} \right]$. In rest of the paper, we use the notation $\beta = 2^{\frac{B}{Ts}} - 1$.

III. COOPERATION METHODS AND SIGNAL FLOW GRAPH REPRESENTATION

If a SU correctly detects PU, it can receive packet transmitted by PU. This enables SUs to cooperate with PU as follows. If the PU packet transmission is unsuccessful, a NACK is sent by PU destination which can be heard by all SUs. Upon receiving NACK, a SU puts the PU packet in its relay queue, provided the relay queue is non-empty. As PU does not give priority to PU packets, that is SUs always transmit packet ready to be transmitted. Each source can act as an antenna in a MISO channel and transmit the packet using orthogonal space-time block coding (OSTBC) scheme [20]. In OSTBC, the packet is encoded in blocks of bits which are distributed among different antennas and across time. For this purpose, all transmitting sources need channel state information (CSI) of other transmitting sources. Also each transmitting source must know which antenna it mimics in the virtual MISO, in order to transmit appropriate parts of the packet according to the corresponding space-time matrix. This can be achieved by coordination between the sources on a low bandwidth control channel or by prior indexing. In the next slot, PU as well as all the cooperating SUs retransmit the PU packet using distributed-OSTBC. Diversity results in higher SINR at PU receiver and probability of successful packet transmission increases. If an ACK is received after the retransmission, the packet is removed from PU queue and from relay queues of cooperating SUs. If a NACK is received, SUs and PU keep retransmitting the same packet until it is received successfully at PU destination. SUs which don’t have the PU packet continue to sense the spectrum and may interfere with assisted retransmission in case of misdetection. It should be noted that priority is given to PU packets, that is SUs always transmit from relay queue if the relay queue is non-empty. As PU does not transmit a new packet until the retransmitted packet is correctly received, there is at most 1 packet in any relay queue.

A case may arise where assisted retransmission is unsuccessful and some of the listening SUs receive the PU packet from assisted retransmission. Thus, more SUs can cooperate with PU in the next slot. However, due to multiple queue interactions between relay queues, keeping track of how many SUs receive packets from assisted retransmissions is complicated. Thus, we restrict our analysis to the case where same group of assisting SUs participates in retransmission if current retransmission is unsuccessful – that is, other SUs don’t receive packets from assisted retransmission. This gives us a lower bound on PU packet throughput performance.

A. Methods of cooperation

We analyze stable throughput for three variations of the proposed cooperation method.

1) All Relay Cooperation (ARC) - All SUs that receive unsuccessful PU packet transmit in retransmission phase.

2) Recurrent Best Relay Cooperation (R-BRC) - Out of all assisting SUs, only the SU that has highest instantaneous channel gain on SU source to PU destination link participates in retransmission. We refer to such a SU as “best” SU relay. Other assisting SUs remain silent and retain the packet. If retransmission is unsuccessful, best SU relay selection is repeated and new best SU assists PU transmission. Note that the same SU may be chosen as best SU relay again depending on the instantaneous channel gains. Best SU relay selection can be done in duration $T_s$ using time-out timers inversely proportional to channel gains as done in [30].

3) Non-recurrent Best Relay Cooperation (NR-BRC) - In this method, unlike R-BRC, other SUs discard PU packet after best SU relay selection is performed. If retransmission is unsuccessful, the same SU assists irrespective of whether it has the best SU source to PU destination channel. Other SUs continue sensing and accessing the spectrum with probability $q$.

B. Signal flow graph representation

In commonly used automatic repeat request (ARQ) protocols, source node attempts retransmission of an unsuccessful packet until ACK for the packet is received. Packet transmission process of ARQ protocols can be represented by signal flow graphs (SFG). SFG representation and subsequent graph reduction provides an efficient way to analyze packet throughput [31]–[33]. To calculate PU packet throughput, we first represent the cooperation methods by signal flow graphs.

1) ARC/R-BRC: Fig. 2 shows SFG of ARC and R-BRC methods for transmission of $M$ PU packets. We use $z$ as one slot length operator. PU packets originate from input $P$. State $D_j, j \in \{1, 2, \ldots, M\}$ represents the state where ACK for
jth packet is received and transmission of new packet begins. Transmission of M packets is over at output \( D_M \). Any path leading from \( P \) to \( D_M \) corresponds to successful transmission of M packets. State \( A_n \) represents the assist state where assistance from \( n \) SUs is possible for retransmission of a packet.

We first discuss transmission of a single packet from input \( P \) to output \( D_1 \). A packet is successfully transmitted without assistance from SU with probability \( s_{na} \), indicated by link \( P - D_1 \). With probability \( s_{ps,n} \), PU transmission is unsuccessful and the packet is received by \( n \) SUs as indicated by the link \( P - A_n \). As there are \( L \) SUs in the system, there can be at most \( L \) assist states. Self loop at \( P \) shows that PU transmission is unsuccessful and no SU is able to receive the unsuccessful packet. This happens with probability \( \bar{s}_{na} \). Thus, we have

\[
s_{na} + \bar{s}_{na} + \sum_{n=1}^{L} s_{ps,n} = 1. \tag{1}
\]

When \( n \) SUs assist, a packet retransmission is successful with probability \( s_{a,n} \) as shown by \( A_n - D_1 \) link. Probability \( s_{a,n} \) depends on whether all assisting SUs transmit (in ARC) or only the best SU transmits (in R-BRC). Self loop at \( A_n \) indicates that assisted transmission is unsuccessful. This happens with probability \( \bar{s}_{a,n} \). As we assume that non-assisting SUs do not receive packets from assisted retransmissions, there is no link between two assist states \( A_n - A_m, n \neq m \). Then we have

\[
s_{a,n} + \bar{s}_{a,n} = 1 \text{ for } n \in \{1, 2, \ldots, L\}. \tag{2}
\]

The links between \( P - D_1 \) are repeated for transmission of \( M \) packets.

**Claim 1.** PU packet throughput for cooperation protocol in Fig. 2 is given by

\[
\mu_P = (1 - s_{na}) \left[ 1 + \frac{\sum_{n=1}^{L} s_{ps,n} s_{a,n}}{s_{a,n}} \right]^{-1}. \tag{3}
\]

**Proof:** We first determine transfer function of the graph in Fig. 2. For single packet transmission from \( P \) to \( D_1 \), there are \((L + 1)\) parallel forward paths as listed below.

- **Forward path \( F_1 \):** \( P - A_1 - D_1 \)
- **Forward path \( F_2 \):** \( P - A_2 - D_1 \)
- **\vdots**
- **Forward path \( F_L \):** \( P - A_L - D_1 \)
- **Forward path \( F_{L+1} \):** \( P - D_1 \)

Each path \( F_n, n = 1, 2, \ldots, L \) has path gain \( G_{F_n} = s_{ps,n} s_{a,n} \bar{s}_{a,n} \) and it touches a loop at \( A_n \) with loop gain \( L_{F_n} = \bar{s}_{a,n} \bar{s}_{a,n} \). Path \( F_{L+1} \) has path gain \( G_{F_{L+1}} = s_{na,1} \). By Mason’s gain formula [4], we can write transfer function of each forward path as

\[
H_{F_n}(z) = \frac{s_{ps,n} s_{a,n} \bar{s}_{a,n} z^2}{1 - \bar{s}_{a,n} z} \quad \text{for } n = 1, 2, \ldots, L, \tag{4}
\]

\[
H_{F_{L+1}}(z) = s_{na,1}. \tag{5}
\]

We can replace \((L + 1)\) parallel branches connecting two nodes in the same direction by a single branch with path gain equal to sum of path gains of the parallel branches. After merging the parallel branches using (4) and (5), we get a single forward path from \( P \) to \( D_1 \) with gain

\[
G_F = s_{na,1} + \sum_{n=1}^{L} s_{ps,n} s_{a,n} \bar{s}_{a,n} z^2. \tag{6}
\]

The path touches a self-loop at \( P \) having loop gain \( L_F = \bar{s}_{a,n} \). Then using Mason’s gain formula, transfer function for transmission of single packet from \( P \) to \( D_1 \) is given by

\[
H(z) = \frac{s_{na,z} + \sum_{n=1}^{L} s_{ps,n} s_{a,n} \bar{s}_{a,n} z^2}{1 - \bar{s}_{a,n} z}. \tag{7}
\]

For transmission of \( M \) packets, there are \( M \) such branches in series. Thus, overall transfer function from input \( P \) to output \( D_M \) is

\[
H_M(z) = \left[ \frac{s_{na,z} + \sum_{n=1}^{L} s_{ps,n} s_{a,n} \bar{s}_{a,n} z^2}{1 - \bar{s}_{a,n} z} \right]^M. \tag{8}
\]

Average number of slots required to transmit \( M \) packets is given by

\[
\mu_P = \lim_{M \to \infty} \frac{M}{\left. \frac{dH_M(z)}{dz} \right|_{z=1}}. \tag{9}
\]

After solving (7) using (1) and (2), we get

\[
\mu_P = \frac{1 - s_{na}}{1 + \sum_{n=1}^{L} s_{ps,n} s_{a,n}}. \tag{10}
\]

2) **NR-BRC:** Fig. 5 shows SFG of NR-BRC method. As explained in the case of ARC/R-BRC, (1) and (2) hold true for NR-BRC. But unlike R-BRC, best relay selection is not performed again if assisted transmission is unsuccessful. Thus, if retransmission is unsuccessful, process does not return to the same assist state. Instead it goes to a “fresh attempt” state denoted by \( F \) where the same SU assists PU retransmission irrespective of whether it is the best SU or not. Packet transmission from fresh
attempt state is successful with probability \( s_f \) as shown by \( F - D_1 \) link. With probability \( \bar{s}_f \), the retransmission is unsuccessful and process remains in the same state. Thus, we have

\[
s_f + \bar{s}_f = 1. \tag{9}
\]

**Claim 2.** PU packet throughput for cooperation protocol in Fig. 3 is given by

\[
\mu_P = (1 - \bar{s}_{na}) \left[ 1 + \sum_{n=1}^{L} s_{ps,n} + \sum_{n=1}^{L} \frac{s_{ps,n} \bar{s}_{a,n}}{s_f} \right]^{-1}. \tag{10}
\]

**Proof:** We first list all forward paths for single packet transmission from \( P \) to \( D_1 \).

- Forward path \( F_1 : P - A_1 - D_1 \)
- Forward path \( F_2 : P - A_2 - D_1 \)
- \[\vdots\]
- Forward path \( F_{L+1} : P - A_1 - F - D_1 \)
- Forward path \( F_{L+2} : P - A_2 - F - D_1 \)
- \[\vdots\]
- Forward path \( F_{2L} : P - A_L - F - D_1 \)
- Forward path \( F_{2L+1} : P - D_1 \)

There are two self-loops; one at input \( P \) with loop gain \( L_1 = \bar{s}_{na} z \) and one at state \( F \) with loop gain \( L_2 = \bar{s}_f z \). As both loops are non-touching, graph determinant \( \Delta \) is given as

\[
\Delta = 1 - L_1 - L_2 + L_1 L_2 = 1 - \bar{s}_{na} z - \bar{s}_f z + \bar{s}_{na} \bar{s}_f z^2. \tag{11}
\]

Using (11), we get path gains and co-factors associated with each forward path as

\[
G_{F_n} = \begin{cases} 
  s_{ps,n} s_{a,n} z^2 & \text{for } n = 1, 2, \ldots, L \\
  s_{ps,n-1} s_{a,n-1} L \bar{s}_f z^3 & \text{for } n = L + 1, L + 2, \ldots, 2L, \\
  \bar{s}_{na} z & \text{for } n = 2L + 1
\end{cases}
\]

\[
\Delta_{F_n} = \begin{cases} 
  1 - \bar{s}_f z & \text{for } n = 1, 2, \ldots, L \\
  1 & \text{for } n = L + 1, L + 2, \ldots, 2L. \\
  1 - \bar{s}_f z & \text{for } n = 2L + 1
\end{cases}
\]

Using Mason’s gain formula, transfer function of \( P - D_1 \) link is given by

\[
H(z) = \frac{\sum_{n=1}^{2L+1} G_{F_n} \Delta_{F_n}}{\Delta}
\]

For transmission of \( M \) packets, we have \( H_M(z) = [H(z)]^M \).

Using same approach followed in ARC/R-BRC case, we get

\[
\mu_P = \frac{1 - \bar{s}_{na}}{1 + \sum_{n=1}^{L} s_{ps,n} + \sum_{n=1}^{L} \frac{s_{ps,n} \bar{s}_{a,n}}{s_f}}. \tag{12}
\]

In the next section, we derive values of probabilities used in (3) and (10). This allows us to calculate PU and SU packet throughput.

**IV. THROUGHPUT ANALYSIS - INDIVIDUAL SENSING (IS)**

We define the notation used to denote probabilities of successful packet transmission under various cases in Table 1. Exact closed form expressions for \( \mathcal{U}_a(n, m) \) and \( \mathcal{U}_b(n, m) \) are derived in Appendix A and Appendix B. Expressions of \( \mathcal{W}(m) \) and \( \mathcal{V}(m) \) can be derived as special cases of \( \mathcal{U}_a(n, m) \) and have been given in [20, Eq.(11), (17)]. Using results in (3) and (10), we now analyze packet throughput of PU and SU for the case where each SU senses individually and transmits based on own sensing decision.

**A. Primary user throughput**

1) ARC: When PU is present, a non-assisting SU interferes with PU in case of misdetection, that is with probability \( (1 - p_d) q \). When \( m \) SUs transmit and no SU assists, PU transmission is successful with probability \( \mathcal{U}_a(n, 0, m) \). Then probability of successful transmission of PU packet without any assistance is

\[
s_{na} = \sum_{m=0}^{L} \left( \frac{L}{m} \right) [(1 - p_d) q]^m [1 - (1 - p_d) q]^{L-m} \mathcal{U}_a(n, 0, m) .
\]

Substituting value of \( \mathcal{U}_a(n, 0, m) \) from (A.8), we get

\[
s_{na} = \exp \left( -\frac{\sigma_0^2 \beta}{P_o \sigma_p^2} \right) \left[ 1 - \frac{B_p (1 - q_d) q^L}{1 + B_p} \right]^{L} . \tag{13}
\]

where \( B_p = \frac{\sigma_0^2 \beta}{P_o \sigma_p^2} \).

SUs that correctly detect PU remain silent and try to receive PU packet. Misdetecting SUs in the vicinity may cause interference at the receiving SU sources. Let \( \mathcal{O}_{PD} \) be the event that PU transmission on \( P - D \) link is unsuccessful. Also let \( \mathcal{O}_{PS}(n) \) be the event that \( n \) SUs receive PU packet. Then overall probability that direct transmission of PU is unsuccessful but \( n \) SUs are able to receive PU packet is given by

\[
s_{ps,n} = \sum_{l=0}^{L} \Pr \{ l \text{ out of } L \text{ SUs detect PU} \}
\times \sum_{m=0}^{L-l} \Pr \{ m \text{ SUs out of } (L - l) \text{ interfere} \}
\times \Pr \{ \mathcal{O}_{PD} \cap \mathcal{O}_{PS}(n) | m \text{ SUs interfere} \} .
\]

Given that \( l \) SUs try to receive PU packet, probability that \( n \leq l \) SUs actually receive the packet under interference from \( m \) SUs is \( \left( \frac{l}{m} \right) \mathcal{W}(m)^n [1 - \mathcal{W}(m)]^{l-n} \). Noting that events \( \mathcal{O}_{PD} \) and \( \mathcal{O}_{PS}(n) \) are independent, we can write

\[
s_{ps,n} = \sum_{l=0}^{L} \left( \frac{L}{l} \right) p_d^l (1 - p_d)^{L-l} \sum_{m=0}^{L-l} \left( \frac{L-l}{m} \right) \times q^m (1 - q)^{L-m} [1 - \mathcal{U}_a(n, 0, m)]
\times \left( \frac{l}{m} \right) \mathcal{W}(m)^n [1 - \mathcal{W}(m)]^{l-n} . \tag{14}
\]

All SUs that receive PU packet assist with the packet transmission. Remaining SUs that don’t have the PU packet, continue sensing spectrum and may interfere in transmission duration
in case of misdetection. Thus, probability of successful PU transmission when \( n \) SUs assist is

\[
s_{a,n} = \sum_{m=0}^{L-n} \binom{L-n}{m} \left[ (1 - p_d) q \right]^m \times \left[ 1 - (1 - p_d) q \right]^{L-n-m} U_a(n,m).
\] (15)

Using values of \( s_{na} \), \( s_{ps,n} \), and \( s_{a,n} \), we calculate \( \bar{s}_{na} \) and \( \bar{s}_{a,n} \) from (1) and (2). PU packet throughput for ARC method \( \mu_{P,ARC} \) is obtained by substituting values of \( \bar{s}_{na} \), \( s_{ps,n} \), and \( s_{a,n} \) in (3).

2) \textit{R-BRC:} R-BRC differs from ARC only in the retransmission phase. Thus, probability of successful transmission without assistance \( s_{na} \) and probability of \( n \) SUs receiving PU packet \( s_{ps,n} \) remains same as derived in (13) and (14) for ARC. In R-BRC, only the best SU relay transmits in retransmission phase. Then similar to (15), we can write \( s_{a,n} \) as

\[
s_{a,n} = \sum_{m=0}^{L-n} \binom{L-n}{m} \left[ (1 - p_d) q \right]^m \times \left[ 1 - (1 - p_d) q \right]^{L-n-m} U_b(n,m).
\] (16)

Using (16), PU throughput in R-BRC \( \mu_{P,R-BRC} \) is calculated from (3).

3) \textit{NR-BRC:} In NR-BRC, unlike previous two methods, assisting SUs discard PU packet after best SU relay selection. If assisted retransmission is unsuccessful, in fresh attempt state, the same SU assists while all other SUs continue sensing spectrum and may interfere. Here values of \( s_{na} \), \( s_{ps,n} \), and \( s_{a,n} \) remain unchanged from the case of R-BRC. Overall probability that PU transmission is successful when one SU assists in fresh attempt state is given by

\[
s_{f} = \sum_{m=0}^{L-1} \binom{L-1}{m} \left[ (1 - p_d) q \right]^m \times \left[ 1 - (1 - p_d) q \right]^{L-1-m} U_b(1,m).
\] (17)

Using (17), we get PU throughput in NR-BRC \( \mu_{P,NR-BRC} \) from (10).

4) \textit{No cooperation (NC):} As a baseline for performance comparison, we consider a system with no cooperation between SUs and PU. In this case, SUs do not receive packets from PU. Thus, there is no assistance in transmission of unsuccessful PU packets. In this case, probability of successful packet transmission is same as \( s_{na} \) from (13) and is given by

\[
\mu_{P,NC} = \exp \left( \frac{-\sigma^2}{P_{S} \sigma_{PS}^2} \right) \left[ 1 - \frac{B_P (1 - p_d) q}{1 + B_P} \right]^L.
\] (18)

| Notation | Probability of successful transmission |
|----------|--------------------------------------|
| \( U_a(n, m) \) | PU source to PU destination when \( n \) SUs assist and \( m \) SUs interfere |
| \( U_b(n, m) \) | PU source to PU destination when best SU out of \( n \) SUs assists and \( m \) SUs interfere |
| \( V(m) \) | SU source to SU destination when PU doesn’t interfere and \( m \) SUs interfere |
| \( W(m) \) | PU source to SU source when \( m \) SUs interfere |

**Algorithm 1** Calculating PU and SU packet throughput in IS case

1. Calculate values of branch gains \( s_{na}, s_{ps,n}, s_{a,n} \) and \( s_f \) using Eq. (13), (14), (15) for ARC, Eq. (13), (14), (16) for R-BRC and Eq. (13), (14), (16), (17) for NR-BRC.
2. Calculate PU throughput for ARC/R-BRC using (5) and for NR-BRC using (10).
3. Calculate SU throughput using (19).

It can be seen that PU packet departure process depends on three events— direct transmission of PU packet without assistance, transmission from PU to SUs and assisted transmission to PU destination. These events in turn depend on channel fading processes and events of misdetection which are stationary. Consequently, PU packet departure process is stationary.

**B. Lower bound on secondary user throughput**

For given packet arrival rate \( \lambda_P \) and packet throughput \( \mu_P \), PU queue is stable if \( \lambda_P < \mu_P \). A stable queue is non-empty with probability \( \lambda_P / \mu_P \) \cite{35}. Thus, PU is silent with probability \( (1 - \lambda_P / \mu_P) \). SU packet throughput achieved in absence of interference from PU is of interest as explained in Section II-D.

When PU is inactive, each SU transmits with probability \( q \) when there is no false alarm. Then packet throughput of a single SU is

\[
\mu_S = \left( 1 - \frac{\lambda_P}{\mu_P} \right) \sum_{m=0}^{L-1} \binom{L-1}{m} \left( 1 - p_f q \right)^m \times \left[ 1 - (1 - p_f) q \right]^{L-1-m} V(m).
\]

Using expression for \( V(m) \) as derived in \cite{20 eq.(17)} and solving, we get

\[
\mu_S = \left( 1 - \frac{\lambda_P}{\mu_P} \right) q (1 - p_f) \exp \left( \frac{-\sigma^2}{P_{S} \sigma_{PS}^2} \right) \left[ 1 - \frac{q (1 - p_f) \beta}{1 + \beta} \right]^{L-1}.
\] (19)

We summarize the process of calculating PU and SU packet throughput in IS case in Algorithm 1.

To keep the system stable, transmission probability \( q \) should be chosen optimally such that SU packet throughput in \( \mu_{S} \) is maximized while ensuring PU queue stability. Due to various non-linearities involved, finding closed form expression of optimal transmission probability \( q^* \) is complicated. However \( q^* \) can be found numerically as explained later in Section VI-B.
special case where \( p_d = 1 \), \( \mu_P \) is independent of \( q \). Thus, by differentiating \( 19 \) and equating to zero, we get
\[
q^* = \min \left[ \frac{1 + \beta}{(1 - p_f) \beta L}, 1 \right].
\] (20)

V. THROUGHPUT ANALYSIS - COOPERATIVE SENSING (CS)

In cooperative sensing case, SUs share their sensing data with one of the SUs that acts as fusion center. In this case, there is no interference in assisted transmission as cooperating SUs can direct non-cooperating SUs to stay silent. With this property of no interference in assisted transmission as cooperating SUs can ensuing analysis as the analysis only uses detection probability sensing (hard data fusion or soft data fusion) does not change the CS case, we analyze PU and SU throughput using results and false alarm probability

\[ p_d^* \] and false alarm probability \( p_f^* \). Values of \( p_d^* \) and \( p_f^* \) may change depending on the underlying CS technique

A. Primary user throughput

1) ARC: If SUs sense PU presence correctly, there is no interference to PU transmission. In case of misdetection, each SU transmits and interferes with probability \( q \). Thus, probability of successful PU packet transmission without assistance is
\[
s_{na} = p_d^* U_a(0, 0) + (1 - p_d^*) \sum_{m=0}^{L} \frac{L}{m} q^m (1 - q)^{L-m} U_a(0, m). \] (21)

Substituting value of \( U_a(0, m) \) from (A.8), we get
\[
s_{na} = \exp \left( -\frac{\sigma_N^2 \beta}{P_P \sigma_D^2} \right) \left[ p_d^* + (1 - p_d^*) \left( 1 - \frac{B_P q}{1 + B_P} \right)^L \right], \] (22)

where \( B_P = \frac{\beta P_S \sigma_D^2}{P_P \sigma_D^2} \).

If PU is correctly detected, all SUs try to receive PU packet. Thus, probability that \( n \) SUs receive PU packet when direct transmission of PU is unsuccessful is
\[
s_{ps,n} = p_d^* (1 - U_a(0, 0)) \frac{L}{n} \left[ \frac{N}{W(0)} \right]^n \left[ 1 - \frac{N}{W(0)} \right]^{L-n}. \] (23)

As there is no interference in cooperation phase, probability of successful PU packet transmission when \( n \) SUs assist is
\[
s_{a,n} = U_a(n, 0). \] (24)

Using (1), (22), (23) and (24), we get PU throughput of ARC from (3).

2) R-BRC: In this case, values of \( s_{na}, s_{ps,n} \) remain same as in the case of ARC. When \( n \) SUs have received unsuccessful PU packet, only the best SU cooperates without any interference. Thus, we have
\[
s_{a,n} = U_b(n, 0). \] (25)

PU throughput for R-BRC can be found using (3).

3) NR-BRC: In this case, values of \( s_{na}, s_{ps,n} \) and \( s_{a,n} \) remain same as in the case of R-BRC. As there is no interference in cooperation phase, probability of successful transmission in fresh attempt state is
\[
s_f = U_b(1, 0). \] (26)

PU throughput for NR-BRC can be found using (10).

4) No cooperation (NC): In this case, PU packet throughput is same as \( s_{na} \) from (22) and is given by
\[
\mu_{P, NC} = \exp \left( -\frac{\sigma_N^2 \beta}{P_P \sigma_D^2} \right) \left[ p_d^* + (1 - p_d^*) \left( 1 - \frac{B_P q}{1 + B_P} \right)^L \right]. \] (27)

Similar to IS case, PU packet departure process in CS case is a stationary process as it is a function of stationary events.

B. Lower bound on secondary user throughput

When PU is inactive, all SUs transmit with probability \( q \) if there is no false alarm. Then SU throughput is given by
\[
\mu_S = \left( 1 - \frac{\lambda_P}{\mu_P} \right) \left( 1 - p_f^* \right) \sum_{m=0}^{L-1} \frac{L}{m} (1 - 1)^{L-m} q^m (1 - q)^{L-1-m} V(m). \] (28)

After simplifying (28), we get
\[
\mu_S = \left( 1 - \frac{\lambda_P}{\mu_P} \right) q (1 - p_f^*) \exp \left( -\frac{\sigma_N^2 \beta}{P_P \sigma_D^2} \right) \left[ 1 - \frac{\beta q}{1 + \beta} \right]^{L-1}. \] (29)

For special case where \( p_f^* = 1 \), optimal \( q \) that maximizes \( \mu_S \) is independent of false alarm probability \( p_f^* \) and is given by
\[
q^* = \min \left[ \frac{1 + \beta}{\beta L}, 1 \right].
\]

As process of calculating PU and SU packet throughput in CS case is similar to IS case as given in Algorithm [1], we omit the algorithm for CS case for brevity.

VI. NUMERICAL RESULTS AND DISCUSSION

We now present numerical results to study performance of proposed cooperation methods. Parameter values used are as follows. Transmit powers of PU and SU are \( P_P = P_S = 0.1 W \). Noise power is \( \sigma_N^2 = 0.1 W \). Average channel gains are \( \sigma_D^2 = -13 dB \), \( \sigma_{BR} = 0 dB \), \( \sigma_{PS} = -10 dB \), \( \sigma_{SR} = -10 dB \) and \( \sigma_{SD} = -10 dB \), unless stated otherwise. Value of SINR threshold for successful packet transmission is \( \beta = 0.1 \). Detection probability is \( p_f = 0.8 \) and false alarm probability is \( p_f = 0.1 \) unless mentioned otherwise. To demonstrate performance of cooperative sensing (CS) case, we use majority rule for hard decision combining [24]. In majority rule, probability of detection or false alarm is given by
\[
p_f^* = \sum_{n=\left[ \frac{L}{2} \right]}^{L} \left( \frac{L}{n} \right) p_f^n (1 - p_f)^{L-n}, j \in \{ d, f \}.
\]

A. PU throughput

1) Effect of cooperation: Fig. 3 plots PU throughput versus number of SUs for different cooperation methods. It shows that as number of SUs increases, more SUs are available for cooperation, resulting in increased PU packet throughput. ARC performs the best compared to R-BRC and NR-BRC. This is because all SUs participate in retransmission in case of ARC. NR-BRC performs worse than R-BRC. This is because, in NR-BRC the same SU participates in subsequent retransmissions of the same packet, even if it is not the best SU relay. However, all three methods result in higher PU throughput than no cooperation (NC) case.
2) Effect of sensing: Fig. 4 also shows that PU throughput in perfect sensing case is better than imperfect sensing case. This is because, there is no interference to PU transmission or assisted retransmissions in perfect sensing case. When SUs employ cooperative sensing, probability of detection is higher than that in IS case. Thus, the performance of ARC CS is close to the perfect sensing case.

3) Effect of SU transmit power: When cooperating SUs transmit with higher power, received SINR at PU destination increases, resulting in higher PU throughput. But interference caused to PU transmission and assisted retransmissions also increases as misdetecting SUs transmit with high power. This tradeoff is shown in Fig. 5. For IS case, PU throughput initially increases with increasing $P_S$. As $P_S$ increases further, effect of increased interference dominates effect of cooperation and $\mu_P$ decreases. In CS case, effect of interference dominates effect of cooperation only at very high values of $P_S$. When channel gains between SU sources to PU destination are high, $\mu_P$ increases due to better cooperation. But the tradeoff point is reached sooner and rate of decrease in $\mu_P$ is higher.

B. SU throughput

1) Optimal SU transmission probability: In case of imperfect sensing, PU packet throughput $\mu_P$ is a function of $q$. For packet arrival rate $\lambda_P$, system becomes unstable if $\mu_P(q) < \lambda_P$. To find optimal $q$ that maximizes SU packet throughput and ensures PU queue stability, we define SU packet throughput by an auxiliary function as

$$\hat{\mu}_S = \begin{cases} \mu_S & \text{for } \mu_P(q) \geq \lambda_P \\ 0 & \text{for } \mu_P(q) < \lambda_P \end{cases}$$

where $\mu_S$ is the lower bound on SU throughput derived in (19) for IS case and in (29) for CS case. It can be seen that $q$ that maximizes $\hat{\mu}_S$ also maximizes $\mu_S$ and ensures queue stability of PU. Fig. 5 plots $\hat{\mu}_S$ versus transmission probability $q$ for $\lambda_P = 0.1$ and $L = 15$. With increasing $q$, SU transmits more often and achieves higher packet throughput. But higher value of $q$ results in decreased PU throughput which lowers probability of PU being silent. Thus, $\hat{\mu}_S$ falls when $q$ increases further. This tradeoff is seen for cooperation as well as non-cooperation cases. When $\mu_P(q)$ falls below $\lambda_P$, system becomes unstable and $\hat{\mu}_S$ becomes zero. Using this tradeoff, optimal $q$ that maximizes $\hat{\mu}_S$ is found by numerical search.

2) Effect of cooperation: Fig. 7 plots lower bound on SU packet throughput derived in (19) and (29) versus number of SUs $L$ for IS and CS case. With increase in number of SUs, PU throughput increases, resulting in more silent slots. This results in improvement in SU throughput initially. However as $L$ increases further, inter-SU interference becomes dominant and $\mu_S$ decreases. Similar to the PU throughput performance ARC performs better than R-BRC and NR-BRC. NR-BRC performs worse than R-BRC.

3) Effect of sensing: In case of imperfect sensing, PU throughput is less than that in perfect sensing case. This results in lower probability of PU queue being empty. Also, with non-zero $p_f$, results in decreased PU throughput which lowers probability of PU being silent. Thus, $\hat{\mu}_S$ falls when $q$ increases further. This tradeoff is seen for cooperation as well as non-cooperation cases. When $\mu_P(q)$ falls below $\lambda_P$, system becomes unstable and $\hat{\mu}_S$ becomes zero. Using this tradeoff, optimal $q$ that maximizes $\hat{\mu}_S$ is found by numerical search.
SUs sense some silent slots as being active and do not transmit. Thus, SU throughput for imperfect sensing case is less as shown in Fig. 7. In cooperative sensing, $p_f^*$ is high and $p_d^*$ is significantly low. Thus, SU throughput in CS case is close to the perfect sensing case.

C. Stable throughput region

All tuples of PU packet arrival rate $\lambda_P$ and SU packet throughput $\mu_S$ that keep the system stable make up the stable throughput region. Fig. 8 plots SU packet throughput $\mu_S$ versus PU packet arrival rate $\lambda_P$ for IS case. With large number of SUs, PU throughput increases. Thus, PU can support higher packet arrival rate $\lambda_P$ while keeping the system stable. However, as inter-SU interference increases with increase in number of SUs, maximum achievable $\mu_S$ (at $\lambda_P = 0$) decreases. In contrast, for NC case, maximum SU throughput $\mu_S$ as well as maximum supported $\lambda_P$ decrease with increasing $L$.

D. Average PU packet delay: Comparison with [18]

In Fig. 9 we compare average delay performance of ARC with cooperative relaying (CR) protocol in [18] under perfect sensing case for $L = 1$. Average PU packet delay in CR has been derived in [18] Eqs. (14), (16), (17)]. Similarly, using Little’s law and Pollaczek-Khinchine formula [35], average delay experienced by PU packets in proposed cooperation method can be written as

$$D_P = \frac{1 - \lambda_P}{\mu_P - \lambda_P}.$$

In CR, some PU packets get queued up in two queues—PU queue and relay queue at SU—before reaching PU destination. Also SU relays PU packets only when PU queue is empty. This results in greater delay as compared to ARC. When SU source to PU destination link is weak, improvement in delay performance due to ARC is significant.

VII. Conclusion

In this paper, we proposed a cooperation method where unsuccessful PU packets are retransmitted by PU as well as cooperating SUs. Depending on how SUs are chosen for cooperation, three variations of the cooperation method were presented. We analyzed packet throughput of PU and SU in these methods by representing them as signal flow graph and using graph reduction tools. Individual sensing as well as cooperative sensing cases were considered. The cooperation methods result in significant packet throughput gains over systems with no cooperation. These performance gains are achieved by using minimum resources for relay queue at each SU. It was observed that higher number of SUs offer more cooperation to PU and result in higher PU packet throughput. But individual SU throughput decreases due to increased inter-SU interference. Optimal transmission probability to maximize SU throughput and keep PU queue stable can be found numerically.
APPENDIX A

DERIVATION OF SUCCESS PROBABILITY \( U_a(n, m) \)

In ARCh, all SUs that have received unsuccessful PU packet assist in PU transmission using D-OSTBC. When \( n > 0 \) SUs assist and \( m > 0 \) SUs interfere, we have

\[
U_a(n, m) = \Pr \left[ \frac{P_P |h_P|^2 + \sum_{j=1}^{n} P_S |h_S,j|^2}{\sigma_N^2 + \sum_{j=1}^{m} P_S |h_S,j|^2} > \beta \right]. \tag{A.1}
\]

We write (A.1) in the form

\[
U_a(n, m) = \Pr \left[ W > c + X - Y \right], \tag{A.2}
\]

where

- \( X = \beta \sum_{j=1}^{m} P_S |h_S,j|^2 \) is a Gamma random variable with probability density function (PDF) \( f_X(x) = \frac{\beta^m}{\Gamma(m)} x^{m-1} e^{-\beta x} \) with shape parameter \( m \) and rate parameter \( \beta_1 = \frac{1}{\beta_P P_S \sigma_D^2} \) where \( \Gamma(\cdot) \) is Gamma function defined as \( \Gamma(a) = \int_0^\infty x^{a-1} e^{-x} dx \). \( \text{[36]} \)
- \( Y = \sum_{j=1}^{n} P_S |h_S,j|^2 \) is a Gamma random variable with PDF \( f_Y(y) = \frac{\beta_2^y}{\Gamma(y)} y^{y-1} e^{-\beta_2 y} \) with shape parameter \( n \) and rate parameter \( \beta_2 = \frac{1}{P_S \sigma_D^2} \).
- \( W = P_P |h_P|^2 \) is an exponential random variable with PDF \( f_W(w) = \beta_3 e^{-\beta_3 w} \) with rate parameter \( \beta_3 = \frac{1}{P_P \sigma_P^2} \).
- \( c = \beta \sigma_N^2 \) is a constant.

To calculate probability in (A.2), we first find distribution of difference of Gamma distributed independent random variables \( X \) and \( Y \). Let \( Z = X - Y \). As \( X, Y \in [0, \infty) \), \( Z = X - Y \) takes on values in \( (-\infty, \infty) \). We have

\[
f_{X+Y}(z) = \int_{-\infty}^{\infty} f_X(x) f_Y(z-x) \, dx. \tag{A.3}
\]

Using (A.3) and noting that \( f_{-Y}(y) = f_Y(-y) \), we get

\[
f_{X-Y}(z) = \int_{-\infty}^{\infty} f_X(x) f_Y(x-z) \, dx
\]

\[
= \int_{-\infty}^{\infty} f_X(x) f_Y(x-z) \, dx. \tag{A.4}
\]

We can further write (A.4) as

\[
f_Z(z) = \begin{cases} \int_{-\infty}^{z} f_X(x) f_Y(x-z) \, dx & \text{for } z < 0 \\ \int_{-\infty}^{\infty} f_X(y+z) f_Y(y) \, dy & \text{for } z \geq 0 \end{cases}. \tag{A.5}
\]

Substituting expressions of PDFs of \( X \) and \( Y \) in (A.5) and simplifying we get PDF of \( Z \) as given in (A.9) on next page.

In the original problem of (A.2), \( W \) is exponentially distributed, hence \( W \in [0, \infty) \). Thus, we can write

\[
\Pr \left[ W > c + Z \right] = \begin{cases} 1 & \text{for } Z < -c \\ e^{-\beta_3(z+c)} & \text{for } Z \geq -c \end{cases}.
\]

Thus, overall probability of successful transmission in (A.1) is given as

\[
U_a(n, m) = \int_{-\infty}^{-c} f_Z(z) \, dz + \int_{-c}^{0} e^{-\beta_3(z+c)} f_Z(z) \, dz
\]

\[
+ \int_{0}^{\infty} e^{-\beta_3(z+c)} f_Z(z) \, dz. \tag{A.6}
\]

where \( I_A, I_B \) and \( I_C \) are found using (A.9). Let \( \Gamma(a, x) \) be the upper incomplete Gamma function defined as \( \Gamma(a, x) = \int_{x}^{\infty} t^{a-1} e^{-t} \, dt \). Also let \( \gamma(a, x) \) be the lower incomplete Gamma function defined as \( \gamma(a, x) = \int_{0}^{x} t^{a-1} e^{-t} \, dt \). Then the expressions for \( I_A, I_B, I_C \) are as given in (A.10), (A.11) and (A.12) on next page.

When no SU interferes \( (m = 0) \) and \( n > 0 \) SUs assist, using similar approach, we get probability of successful transmission as

\[
U_a(n, 0) = \begin{cases} \frac{\Gamma(n, \beta_2 c)}{\Gamma(n)} e^{-\beta_3 c} \frac{\beta_2^n}{\beta_1^n} \frac{\gamma(n, \beta_2 - \beta_3 c)}{\beta_1^n} & \text{for } \beta_2 \neq \beta_3 \\ \frac{\Gamma(n, \beta_2 c)}{\Gamma(n)} e^{-\beta_3 c} \frac{\beta_2^n}{\beta_1^n} & \text{for } \beta_2 = \beta_3 \end{cases}. \tag{A.7}
\]

When no SU assists \( (n = 0) \) but \( m > 0 \) SUs interfere, probability of successful transmission is same as that derived in [20 Eq.(11)] and is given by

\[
U_a(0, m) = e^{-\beta_3 c} \left[ 1 + \frac{\beta_3}{\beta_1} \right]^{-m}
\]

\[
= \exp \left[ \frac{-\sigma_2^2}{P_P \sigma_D^2} \right] \left[ 1 + \frac{\beta P_P \sigma_2^2}{P_P \sigma_D^2} \right]^{-m}. \tag{A.8}
\]

APPENDIX B

DERIVATION OF SUCCESS PROBABILITY \( U_b(n, m) \)

In R-BRC and NR-BRC, out of all SUs that have received PU packet, the SU having best SU source to PU destination channel is selected for cooperation. Only the best SU and PU transmit in retransmission phase using D-OSTBC. Non-assisting SUs may interfere in case of misdetection. Then probability of successful transmission of PU packet when \( n > 0 \) SUs participate in best relay selection and \( m > 0 \) SUs interfere is

\[
U_b(n, m) = \Pr \left[ \frac{P_P |h_P|^2 + \sum_{j=1}^{n} P_S |h_S,j|^2}{\sigma_N^2 + \sum_{j=1}^{m} P_S |h_S,j|^2} > \beta \right]. \tag{B.3}
\]

where \( |h_S|^2 \) is \( \text{max} \left\{ |h_S,1|^2, |h_S,2|^2, \ldots, |h_S,n|^2 \right\} \). Note that in (B.3), \( \{S_1, S_2, \ldots, S_n\} \) is a set of arbitrary SU sources that have received PU packet and the subscripts represent no particular order. We write (B.3) as

\[
U_b(n, m) = \Pr \left[ W > c + Z \right], \tag{B.4}
\]

where

- \( W = \max \{X_1, X_2, \ldots, X_n\} \) where \( X_i, i = 1, 2, \ldots, n \) is exponential random variable with PDF \( f_X(x) = \beta e^{-\beta x} \) with rate parameter \( \beta_1 = \frac{1}{P_S \sigma_D^2} \).
- \( Z = \beta \sum_{j=1}^{m} P_S |h_S,j|^2 - P_P |h_P|^2 \) is difference of two Gamma random variables, with PDF \( f_Z(z; m, \beta_1, 1, \beta_2) \) as given in (A.9) where \( \beta_1 = \frac{1}{P_S \sigma_D^2} \) and \( \beta_2 = \frac{1}{P_P \sigma_P^2} \).
- \( c = \beta \sigma_N^2 \) is a constant.
Then overall probability \( (B.1) \) and \( (B.2) \) on next page.

When no SU interferes \( (m = 0) \), using similar approach, we get

\[
\overline{U}_0(n, 0) = \sum_{j=0}^{n} \binom{n}{j} (-1)^j e^{-j \beta_3 c} C_j, \quad \text{for } z < 0
\]  

(B.7)

\[
C_j = \frac{1 - e^{-\beta_3 c}}{c} \quad \text{for } \beta_2 \neq j \beta_3.
\]

(B.8)

Due to independence of \( X_i \)'s, we get cumulative distribution function (CDF) of \( W \) as

\[
\Pr[W \leq w] = \Pr[\max\{X_1, X_2, \ldots, X_n\} \leq w]
\]

\[
= \prod_{i=1}^{n} \Pr[X_i \leq w]
\]

\[
= (1 - e^{-\beta_3 w})^n. \quad \text{(B.5)}
\]

Let \( \overline{U}_0(n, m) = 1 - \overline{U}_0(n, m) = \Pr[W \leq c + Z] \). As \( W \) takes values in \([0, \infty)\), we have

\[
\Pr[W \leq c + Z] = \begin{cases} 
0 & \text{for } Z < -c \\
(1 - e^{-\beta_3 c} e^{-\beta_3 z})^n & \text{for } Z \geq -c.
\end{cases}
\]

Then overall probability \( \overline{U}_0(n, m) \) is

\[
\overline{U}_0(n, m) = \int_{-c}^{\infty} \left(1 - e^{-\beta_3 c} e^{-\beta_3 z}\right)^n f_Z(z) \, dz.
\]

Using binomial expansion, we get

\[
\overline{U}_0(n, m) = \sum_{j=0}^{n} \binom{n}{j} (-1)^j e^{-j \beta_3 c}
\]

\[
\times \left[ \int_{-c}^{0} e^{-j \beta_3 z} f_Z(z) \, dz + \int_{0}^{\infty} e^{-j \beta_3 z} f_Z(z) \, dz \right].
\]

(B.6)

Using \( f_Z(z) \) from (A.9), we can write \( I_{B1} \) and \( I_{B2} \) as given in (B.1) and (B.2) on next page.

Using (B.4), we get \( \overline{U}_0(n, m) = 1 - \overline{U}_0(n, m) \).

REFERENCES

[1] M. A. McHenry, “NSF spectrum occupancy measurements project summary,” Shared Spectrum Company, 2005.

[2] S. Haykin, “Cognitive radio: brain-empowered wireless communications,” IEEE J. Sel. Areas Commun., vol. 23, no. 2, pp. 201–220, Feb 2005.

[3] A. Goldsmith, S. Jafar, I. Maric, and S. Srinivasa, “Breaking spectrum gridlock with cognitive radios: An information theoretic perspective,” Proc. IEEE, vol. 97, no. 5, pp. 949–914, May 2009.

[4] N. Devroye, P. Mitran, and V. Tarokh, “Achievable rates in cognitive radio channels,” IEEE Trans. Inf. Theory, vol. 52, no. 5, pp. 1813–1827, May 2006.

[5] A. Jovicic and P. Viswanathan, “Cognitive radio: An information-theoretic perspective,” IEEE Trans. Inf. Theory, vol. 55, no. 9, pp. 3945–3958, Sept 2009.

[6] K. Kalkarni and A. Banerjee, “Adaptive transmission strategies to maximize packet throughput of cognitive radio under primary user queue stability constraint,” in Proc. Int. Conf. Signal Process. and Comm. (SPCOM 2014), July 2014, pp. 1–6.

[7] J. Jeon and A. Ephremides, “The stability region of random multiple access under stochastic energy harvesting,” in Proc. IEEE Int. Symp. Inf. Theory (ISIT 2011), St. Petersburg, Russia, July 2011, pp. 1796–1800.

[8] R. Duan, M. Elmusrati, and R. Virrankoski, “Stable transmission for a cognitive-shared channel with rechargeable transmitters,” in Proc. IEEE Int. Conf. Comm. (ICC 2012), June 2012, pp. 4632–4636.

[9] O. Simeone, Y. Bar-Ness, and U. Spagnolini, “Stable throughput of cognitive radios with and without relaying capability,” IEEE Trans. Commun., vol. 55, no. 12, pp. 2351–2360, 2007.

[10] Q. Zhang, J. Jia, and J. Zhang, “Cooperative relay to improve diversity in cognitive radio networks,” IEEE Commun. Mag., vol. 47, no. 2, pp. 111–117, February 2009.

[11] Y. Zou, J. Zhu, B. Zheng, and Y.-D. Yao, “An adaptive cooperation diversity scheme with best-relay selection in cognitive radio networks,” IEEE Trans. Signal Process., vol. 58, no. 10, pp. 5438–5445, Oct 2010.

[12] J. Lee, H. Wang, I. Andrews, and D. Hong, “Outage probability of cognitive relay networks with interference constraints,” IEEE Trans. Wireless Commun., vol. 10, no. 2, pp. 390–395, February 2011.
[13] A. El Shafie, M. Khafagy, and A. Sultan, “Optimization of a relay-assisted link with buffer state information at the source,” IEEE Commun. Lett., vol. 18, no. 12, pp. 2149–2152, Dec 2014.

[14] I. Krikidis, N. Devroye, and J. Thompson, “Stability analysis for cognitive radio with multi-access primary transmission,” IEEE Trans. Wireless Commun., vol. 9, no. 1, pp. 72–77, 2010.

[15] L. Wang and V. Fodor, “Dynamic cooperative secondary access in hierarchical spectrum sharing networks,” IEEE Trans. Wireless Commun., vol. 13, no. 11, pp. 6068–6080, Nov 2014.

[16] S. Kompella, G. Nguyen, C. Kam, J. Wieselthier, and A. Ephremides, “Cooperation in cognitive underlay networks: Stable throughput tradeoffs,” IEEE/ACM Trans. Netw., vol. PP, no. 99, pp. 1756–1768, 2013.

[17] A. Elmahdy, A. El-Keyi, T. Elbatt, and K. Seddik, “On the stable throughput of cooperative cognitive radio networks with finite relaying buffer,” in Proc. IEEE 25th Int. Symp. Personal, Indoor, and Mobile Radio Commun. (PIMRC 2014), Washington DC, USA, Sept 2014, pp. 942–946.

[18] M. Ashour, A. Elsherif, T. Elbatt, and A. Mohamed, “Cognitive radio networks with probabilistic relaying: Stable throughput and delay tradeoffs,” IEEE Trans. Commun., vol. PP, no. 99, pp. 4002–4014, 2015.

[19] K. Kulkarni and A. Banerjee, “Stable throughput tradeoffs in cognitive radio networks with cooperating rechargeable nodes,” in Proc. IEEE Wireless Commun. Netw. Conf. (WCNC 2015), March 2015, pp. 1–6.

[20] A. Fanous and A. Ephremides, “Stable throughput in a cognitive wireless network,” IEEE J. Sel. Areas Commun., vol. 31, no. 3, pp. 523–533, 2013.

[21] L. Canzian, L. Badia, and M. Zorzi, “Promoting cooperation in wireless relay networks through stackelberg dynamic scheduling,” IEEE Trans. Commun., vol. 61, no. 2, pp. 700–711, Feb 2013.

[22] R. Loynes, “The stability of a queue with non-independent inter-arrival and service times,” in Proc. Cambridge Philos. Soc., vol. 58, no. 3. Cambridge Univ Press, 1962, pp. 497–520.

[23] Q. Zhao, L. Tong, A. Swami, and Y. Chen, “Decentralized cognitive MAC for opportunistic spectrum access in ad hoc networks: A POMDP framework,” IEEE J. Sel. Areas Commun., vol. 25, no. 3, pp. 589–600, April 2007.

[24] C. L. Wang and H. W. Chen, “A new signal structure for active sensing in cognitive radio systems,” IEEE Trans. Commun., vol. 62, no. 3, pp. 822–835, March 2014.

[25] E. Peh, Y.-C. Liang, Y. L. Guan, and Y. Zeng, “Optimization of cooperative sensing in cognitive radio networks: A sensing-throughput tradeoff view,” IEEE Trans. Veh. Technol., vol. 58, no. 9, pp. 5294–5299, Nov 2009.

[26] A. Ghasemi and E. Sousa, “Collaborative spectrum sensing for opportunistic access in fading environments,” in Proc. IEEE Int. Symp. New Frontiers in Dynamic Spectrum Access Netw. (DySPAN 2005), Nov 2005, pp. 131–136.

[27] I. F. Akyildiz, B. F. Lo, and R. Balakrishnan, “Cooperative spectrum sensing in cognitive radio networks: A survey,” Phys. Commun., vol. 4, no. 1, pp. 40–62, Mar. 2011. [Online]. Available: http://dx.doi.org/10.1016/j.phycom.2010.12.003

[28] Y.-C. Liang, Y. Zeng, E. Peh, and A. T. Hoang, “Sensing-throughput tradeoff for cognitive radio networks,” IEEE Trans. Wireless Commun., vol. 7, no. 4, pp. 1326–1337, April 2008.

[29] V. Tarokh, H. Jafarkhani, and A. Calderbank, “Space-time block codes from orthogonal designs,” IEEE Trans. Inf. Theory, vol. 45, no. 5, pp. 1456–1467, Jul 1999.

[30] Y. H. Ezzeldin, A. Sultan, and M. Youssef, “Best relay selection for underlay cognitive radio systems with collision probability minimization,” in Proc. Int. Conf. Comput. Netw. Commun. (ICNC 2014), Feb 2014, pp. 945–949.

[31] D.-L. Lu and J.-F. Chang, “Analysis of ARQ protocols via signal flow graphs,” IEEE Trans. Commun., vol. 37, no. 3, pp. 245–251, Mar 1989.

[32] ———, “Performance of ARQ protocols in nonindependent channel errors,” IEEE Trans. Commun., vol. 41, no. 5, pp. 721–730, May 1993.

[33] K. Ausavapattanakun and A. Nosratinia, “Analysis of selective-repeat ARQ via matrix signal-flow graphs,” IEEE Trans. Commun., vol. 55, no. 1, pp. 198–204, Jan 2007.

[34] S. Mason, “Feedback theory-some properties of signal flow graphs,” Proc. IRE, vol. 41, no. 9, pp. 1144–1156, Sept 1953.

[35] L. Kleinrock, Queueing Systems, Vol. 1: Theory. Wiley-Interscience, 1975.

[36] A. Papoulis and S. Pillai, Probability, random variables, and stochastic processes, ser. McGraw-Hill electrical and electronic engineering series. McGraw-Hill, 2002.