Translation correlations in anisotropically scattering media

Benjamin Judkewitz1,2*, Roarke Horstmeyer2†, Ivo M. Vellekoop3, Ioannis N. Papadopoulos1 and Changhuei Yang2

Controlling light propagation across scattering media by wavefront shaping holds great promise for a wide range of communications and imaging applications. But, finding the right shape for the wavefront is a challenge when the mapping between input and output scattered wavefronts (that is, the transmission matrix) is not known. Correlations in transmission matrices, especially the so-called memory effect, have been exploited to address this limitation. However, the traditional memory effect applies to thin scattering layers at a distance from the target, which precludes its use within thick scattering media, such as fog and biological tissue. Here, we theoretically predict and experimentally verify new transmission matrix correlations within thick anisotropically scattering media, with important implications for biomedical imaging and adaptive optics.

Focusing light through strongly scattering media is an important goal in optical imaging and communication. Long considered impossible, recent advances in the field of wavefront shaping1,2 changed this view by demonstrating that diffuse light can be focused through inhomogeneous media—as long as the correct input wavefront is used. With direct optical access to the target plane, the correct wavefront can be obtained by iterative optimization1, phase conjugation1, or by measuring the transmission matrix13,14. In many imaging scenarios, however, there is no direct access to the target plane. In those cases, nonlinear20, fluorescent21, kinematic22, acousto-optic10–12 and photo-acoustic13–15 guide stars can be used as reference beacons. However, these techniques provide wavefront information for only one target location at a time. Although transmission matrices can be sampled quickly with a photo-acoustic approach23, this method requires absorbing samples. As a result, many samples’ transmission matrices can be sampled only sparsely. Correlations within a transmission matrix can compensate for sparse sampling and could enable high-speed imaging. One of the most widely known transmission matrix correlations is the so-called ‘memory effect’17,18, which describes the following phenomenon: when an input wavefront reaching a diffusing sample is tilted within a certain angular range, the output wavefront is equally tilted, resulting in the translation of the far-field speckle pattern at a distance behind the sample (see Fig. 1).

The translation distance within which this effect holds (that is, the field of view (FOV)) is inversely proportional to diffuser thickness $L$ and directly proportional to the distance $s$ of the diffuser from the screen. It can be approximated by the equation $\text{FOV} \approx sL/\pi L$ (refs 19–22).

The memory effect has found numerous applications for point scanning21,24, direct image transfer25 and for computational image recovery21,22,23,26. Yet, in all of these applications, the target plane was at a distance from a thin diffuser with free space in between ($s > 0$) — which has limited use for imaging inside thick scattering media. As such samples are neither thin nor at a distance from the target area of interest, the correlations predicted by the ‘traditional’ memory effect should be minimal27. Here, we set out to examine whether there are other correlations that apply to such samples. We show that significant transmission correlations can exist in thick scattering media at zero distance, as long as scattering is directional.

Traditional memory effect

Although the memory effect has been extensively derived from first principles14, these derivations relied on assuming perfectly diffuse scattering—which does not apply to many biological samples. Here we will approach the problem without relying on diffusion, but instead using transmission matrices. Specifically, we are interested in the matrix $T \equiv T(x_a, x_b)$, which defines the relationship between the spatial input and output optical modes of a scattering slab (that is, from input plane $A$ to output plane $B$). For simplicity of graphical representation, we assume propagation of one-dimensional (1D) wavefronts in a 2D geometry, but all conclusions will be generalizable to 2D wavefronts in a 3D geometry. Owing to its discrete nature, the transmission matrix is especially amenable to experimental observation. We will first use our framework to analyse the traditional memory effect, and then calculate the speckle correlations in thick anisotropic media.

When assuming highly randomizing transmission, but incomplete measurement of input and output channels, the transmission matrix is often modelled as a random matrix with complex Gaussian elements. However, the transmission matrix will have an additional macroscopic structure, which will be particularly prominent for thin scattering slabs: a point source on the input plane of the slab would spread to a diffuse spot at the output plane (whose diameter would be of the order of $L$ for slabs with thickness $L$ larger than one transport mean free path—as predicted by the diffusion approximation). As a result, even though individual transmission matrix elements may not be known, the average amplitudes of the transmission matrix elements decrease with distance from the diagonal. The
The traditional memory effect. a. The traditional memory effect as described for light propagating through thin diffusing slabs. Tilting the input wavefront (plane A) reaching the slab tilts the scattered wavefront at the output (plane B), which shifts the far-field intensity speckle pattern projected on a screen (plane C). b. When the input wavefront is shaped to converge at a spot on the screen, tilting the input wavefront scans the spot laterally, which can be used for imaging by point scanning. The FOV of this approach is approximated by the equation $\text{FOV} \approx \frac{s}{\lambda L}$.

The correlation theorem may then help re-express the spatial intensity as a non-negative envelope along its near-diagonals. The cross-correlation theorem may then help re-express the spatial intensity as a non-negative envelope along its near-diagonals. The cross-correlation theorem may then help re-express the spatial intensity as a non-negative envelope along its near-diagonals. The cross-correlation theorem may then help re-express the spatial intensity as a non-negative envelope along its near-diagonals. The cross-correlation theorem may then help re-express the spatial intensity as a non-negative envelope along its near-diagonals. The cross-correlation theorem may then help re-express the spatial intensity as a non-negative envelope along its near-diagonals. The cross-correlation theorem may then help re-express the spatial intensity as a non-negative envelope along its near-diagonals. The cross-correlation theorem may then help re-express the spatial intensity as a non-negative envelope along its near-diagonals. The cross-correlation theorem may then help re-express the spatial intensity as a non-negative envelope along its near-diagonals.

In other words, one input plane wave (one mode in k-space) will result in a limited angular span of output waves.

As a result of such preserved directionality, anisotropically scattering media will have a macroscopic structure in the $k_x$ matrix (rather than $T_x$). Large amplitudes in $T_x$ will primarily concentrate near its main diagonal (see Fig. 2b). By analogy to our prior reasoning for the traditional memory effect, if $T_x$ is a band matrix, the entries within each diagonal of its associated $T_x$ will be correlated with one another. Now, a spatial shift of the optical field at the input plane will cause a spatial shift of the field at the output plane (this is in contrast to the traditional memory effect, in which a tilt at the input plane causes a tilt at the output plane).

The range of correlations in $T_x$ will depend on the width of the diagonally bordered envelope in $T_x$. Equivalent to equation (2), we may define a k-space intensity propagator using an ensemble average: $P_x(k_x, k_y) \equiv \langle |T_x(k_x, k_y)|^2 \rangle$. Here, the average intensity spread of each input plane wave into a finite output wavevector cone now specifies the envelope shape along the columns of $P_x$. Following equation (3), we may show that the band structure of $P_x$ creates correlations in space:

$$\mathcal{F}^{x \rightarrow k} P_x = \mathcal{C}_x(\Delta x, \Delta x) \delta_{\Delta k_x, \Delta k_y} \mathcal{F}^{k \rightarrow \Delta k} P_x(\Delta k)$$

where $\Delta x = x_b - x_a$ and $\delta$ is a Kronecker delta. Equation (3) is the well-known memory effect\(^{28}\). As can be seen directly from equation (2), the angular range over which the memory effect is significant is inversely proportional to the width of the intensity propagator, $P_x$. When considering intensity transmission, equation (3) corresponds to the lowest-order $C^{(1)}$ term\(^{16}\). Unlike such prior work, equation (2) also describes the general case of tilt/shift correlations for a geometry that is not invariant under lateral translation.

These considerations reconfirm our expectation that the traditional memory effect may be minimal in thick biological media. First, the average spread of intensity from the input to output surface will increase with sample thickness, $L$ (ref. 29). A wider $P_x(\Delta x)$ will subsequently reduce the range of tilt/shift correlations between the input and output planes. Second, the plane of interest is not at a distance from the sample, which means that the tilt at the output plane would not translate into a useful spatial shift at the target plane.

### Correlations in anisotropic media

We therefore investigated whether there might be other types of transmission matrix correlation in thick samples, such as biological media. We started by recognizing that in many samples scattering is anisotropic and occurs primarily in the forward direction. Scattering is particularly anisotropic in biological media, where the anisotropy parameter $g$ (the average cosine of the scattering angle) typically ranges from 0.9 to 0.98 (refs 30,31). This means that after a limited number of scattering events, the directionality of an input beam will be preserved to some extent as it reaches the output plane.

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### Scattering in biological media

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### Measurement of sample thickness

In equation (5), $P_x$ includes the effects of sample anisotropy. Its width scales inversely proportional with $g$ but will increase with sample thickness $L$ (see Supplementary Section C). If $P_x$ depends only on difference coordinates, the correlation function can be...
predicted by a simple experiment, namely by illuminating the sample with a plane wave $E_a(x_0) = 1$ and measuring the output wavefront, $E_b(x) = T_{bb} E_a(x_0)$. The spectrum of $E_b(x)$ indicates the average spread of wavevectors exiting the output surface:

$$P_x(\Delta k) \equiv |\mathcal{F}^{-1}_{x-b}[E_b(x_0)]|^2$$  \hspace{1cm} (6)

The Fourier transform of equation (6) subsequently yields the correlation between the electric field outputs from two spatially shifted inputs. In effect, the shape of the shift/shift correlation function equals the autocorrelation of $E_b(x)$. A more detailed derivation of equations (1)–(6) can be found in the Supplementary Information.

**Experimental validation**

To validate our predictions experimentally, we created four scattering samples with well-defined scattering properties (3-µm-diameter silica beads dispersed in agarose gel, $g = 0.978$ as calculated by Mie theory and scattering length $1/\mu_s = 175$ µm at 632 nm, slab thickness $L$ in micrometres: 140, 280, 560 and 1,120, or 1, 2, 4 and 8 spacers of 140-µm thickness). We then performed four different experiments with this scattering sample set, as detailed below.

First, we illuminated each sample with a random input wave and recorded the output field, $E_{in}$, using off-axis digital holography. We translated each sample laterally ($\Delta x$ ranging from $-10$ µm to 10 µm in 0.2 µm steps) and measured the absolute correlation $C(\Delta x)$ between the resulting output fields, $E_{out}$, and $E_{in}$: $C(\Delta x) = \operatorname{corr}(E_{in}, E_{out})$ (Fig. 3a,c). Second, to compare these results with our predictions in equation (6), we illuminated each of the samples with a plane wave and calculated the autocorrelation of the output speckle patterns (Fig. 3b,d). Third, our theory predicts that the correlation function for one sample thickness $L_0$ can be used to estimate the correlation function for any other thickness (see Supplementary Equation C3); for example, the correlation function for a slab of thickness $2L_0$ is simply the correlation function for a slab of thickness $L_0$, squared. We therefore used the speckle autocorrelation measured for the thinnest slab (blue curve in Fig. 3d) and calculated the remaining correlation functions using Supplementary Equation C4 (Fig. 3e).

Fourth, we used Mie theory to obtain a single-scattering phase function for our bead samples, and calculated the theoretical correlation function using Supplementary Equation C1 (Fig. 3f). We computed this last set of curves using only the refractive indices of the media, the bead diameter and slab optical thickness (that is, without using experimental data), as detailed in Supplementary Section C. Figure 3f combines all four strategies for determining the correlation function into one plot. It illustrates that the experimentally measured correlation function is in agreement with all three of our derived predictions.

The shift/shift correlations apply to any input field, including fields that are shaped to converge to a sharp focus. To demonstrate the use of these correlations for scanning a point focus across a biological sample, we first used optical phase conjugation to focus light (780 nm diode laser) through 500-µm- thick slices of chicken muscle tissue, employing off-axis holography for wavefront measurement and a spatial light modulator (SLM) for wavefront shaping (Fig. 4a). We projected a point source at one surface of the tissue slice (surface A) and detected the scattered wavefront propagating from this point through the tissue (exiting at surface B) to the SLM plane. In the next step, we displayed the phase conjugate of the detected wavefront, which travelled back through the tissue and formed a focus on surface A.

To validate the predicted shift/shift correlations, we then shifted the phase-conjugated wavefront laterally at surface B, testing whether the focus at surface A would be preserved and whether it moved. As expected, we noticed that motion of the shaped wavefront resulted in concurrent movement of the focus (Fig. 4b,c), while the focus intensity decreased with distance from the original position, following a bell-shaped curve (Fig. 4d,e). For the 500 µm slice the full-width at half-maximum (FWHM) was 5 µm, and the full-width at tenth-maximum (FWTM) was 10 µm. In the case of the 1,000 µm slice, the FWHM was 3 µm and the FWTM was 6 µm.
Published scattering parameters for chicken tissue vary, and owing to experimental limitations, the single-scattering phase function has not been determined. However, equation (6) provides a practical way to predict the shift/shift correlation function from the experimentally accessible speckle autocorrelation function. We therefore illuminated the samples with a plane wavefront and examined whether the shape of the spatial autocorrelation of the resulting speckle pattern followed the profile of the shift/shift correlations \(C(x)\), as derived in equation (6). Indeed, Fig. 4d,e shows that both profiles are in good experimental agreement.

**Discussion**

The traditional (tilt/tilt) memory effect has recently enabled the development of several modalities to image scattering 'walls'\(^{26-28}\). Intriguing as these methods are, they suffer from two limitations: the sample should be thin, and the object should be placed at a distance behind the sample.

Here, we demonstrated a complementary type of memory effect that suffers from neither limitation: the correlations are present even inside thick scattering media, as long as scattering is anisotropic and the mapping between input and output wavefronts preserves any level of directionality. This is the case up to a depth of about one transport mean free path.

We showed that the shift/shift memory effect is the Fourier complement of the traditional (tilt/tilt) memory effect, and that the extent of correlations can be directly determined from the spatial speckle autocorrelation function during plane-wave illumination.

Our theory is general in the sense that it applies to any linear propagation, which can be described by an input–output matrix. If this matrix is banded in the spatial frequency domain, there will be shift/shift correlations. This means that there will be an effect whenever the directionality of the input waves is maintained to some degree at a chosen output plane, where the ‘output’ plane could be inside the sample, on the opposite surface, or beyond the opposite surface in free space.

As compared with correlations measured at the surface, the extent of correlations within biological media may be affected by diffuse back-scattering from deeper layers. To estimate the extent of this effect, we can decompose the angular intensity propagator \(P_\text{b}\) into the forward-scattered component (equivalent to the one measured at the output surface of a slab with thickness \(t\)) and the-backscattered or diffuse component. In the case of highly anisotropic media such as biological tissue, we can expect that the forward-propagating component will dominate in the quasi-ballistic regime. For example, if the ratio of back-scattered versus forward-scattered power in the semi-infinite medium at depth \(t\) was 20%, we would...
still expect 80% of the field correlations present at the surface of an equivalent slab with thickness $t$.

Thus, our results pave the way for extending memory-effect-based imaging methods to also work inside biological tissue. On the basis of our measurements, we expect such methods to achieve diffraction-limited resolution at a depth of 1 mm inside muscle tissue, albeit at a limited FOV of $<10 \mu m$, initially.

We foresee several possibilities to further increase the FOV of our method, for example by tiling neighboring FOVs using multiple corrections or by interpolating a sparsely sampled transmission matrix, which can be under-sampled by two orders of magnitude. Furthermore, our results suggest that the extent of correlations will be largest for photons that have undergone few scattering events and little angular deviation—also called snake-photons. Hence, selective measurement and correction of snake-photons (for example, by temporal gating, coherence gating or spatial filtering) may considerably increase the extent of correlations and the imaging FOV.

Finally, we note that tilt/tilt and shift/shift correlations are not mutually exclusive. For example, recent work establishes that light within biological tissue exhibits much stronger tilt/tilt correlations than in low-$g$ media. This is because light spreads less across both angle and space in anisotropically scattering media, making both $P_l$ and $P_s$ highly non-uniform. Even though using strict tilt/tilt correlations for scanning or imaging still requires a finite distance between the scattering sample and target (and may thus not be applicable for imaging biological media), we anticipate that a potential combination of both memory effects into a joint model could further extend the translation correlations described here. Furthermore, we anticipate that there may be additional correlations present in biological media. Future work measuring complete transmission matrices in the adaptive optics and the complex wavefront shaping regime will shed light on spectral, temporal and spatial correlations. They may ultimately be used in combination with the shift/shift correlations reported here.

We note that the described shift/shift correlations are consistent with the set-up geometry of adaptive optics microscopy (Fig. 5), where wavefronts are corrected in the conjugate Fourier plane of the microscope objective. Tilting the incoming wavefront in the Fourier plane (for example, in a laser scanning microscope) leads to a shift of the wavefront reaching the sample and a resulting shift of the focus. In other words, adaptive optics microscopy implicitly already takes advantage of shift/shift correlations, albeit in the ballistic regime—as such it can be interpreted as a special case of the general shift/shift correlations derived here.

With further study of spatial, spectral and temporal transmission matrix correlations, these advances may lead to a unified understanding of adaptive optics and complex wavefront shaping and extend their use in thick biological tissues, enabling versatile imaging and photostimulation in a wide range of biologically relevant media.

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Author contributions
B.J. and R.H. conceived and developed the idea with essential help from I.M.V. B.J. and I.N.P. performed experiments. B.J., R.H. and I.M.V. wrote the manuscript. R.H. and I.M.V. wrote the mathematical supplement with help from B.J. B.J. and C.Y. supervised the project.

Additional information
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Competing financial interests
The authors declare no competing financial interests.