A new method to search for a cosmic ray dipole anisotropy

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Abstract. We propose a new method to determine the dipole (and quadrupole) component of a distribution of cosmic ray arrival directions, which can be applied when there is partial sky coverage and/or inhomogeneous exposure. In its simplest version it requires that the exposure only depends on the declination, but it can be easily extended to the case of a small amplitude modulation in right ascension. The method essentially combines a \( \chi^2 \) minimization of the distribution in declination to obtain the multipolar components along the North–South axis and a harmonic Rayleigh analysis for the components involving the right ascension direction.

Keywords: ultra high energy cosmic rays, cosmic rays

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1. Introduction

The search for a dipolar anisotropy in the arrival directions of cosmic rays (CRs) is of crucial importance to better understand the CR origin, the distribution of their sources and the way they propagate through the magnetized galactic medium. Several analyses have looked for a first harmonic in the right ascension distribution of the arrival directions, and some positive findings have been reported. These include some giving few per thousand anisotropies at energies $\sim 10^{15}$ eV, others giving per cent level anisotropies at $\sim 10^{16}$ eV (which are however in contradiction with more recent data from Kascade, which obtains upper limits below these findings) [1], and also a 4% anisotropy at $10^{18}$ eV has been reported by the Agasa collaboration [2]. Due to the small size of the anisotropies observed it is clear that these measurements are extremely delicate since non-uniformities in the sky coverage, due to e.g. asymmetries of the experimental set-up or to weather effects not properly accounted for, could mimic the signal searched.

We will be focusing here on ground arrays, which run essentially steadily and hence achieve an almost uniform exposure as a function of right ascension. The computation of the exposure for fluorescence detectors is generally quite involved, since these detectors only run in clear moonless nights and hence have a significantly less uniform coverage of the sky. Moreover, the detection efficiency is different e.g. for showers pointing towards or away from the telescopes. As a result, the isotropic expectations are usually evaluated by shuffling methods based on the data themselves rather than on the detailed knowledge of the detector’s acceptance and running conditions. In spite of this, one could mention that some attempts have been made to recover large scale anisotropy patterns with HiRes data [3].

On the theoretical side, realistic models of CR diffusion in the Galaxy, including the drift component, which actually turns out to give the dominant contribution to the CR macroscopic currents for energies above that of the knee and up to the ankle [4], predict anisotropies steadily increasing with energy, with a typical size of $10^{-3}$ at $10^{16}$ eV and reaching the per cent level at $10^{18}$ eV. These predictions of course depend on the details of the regular and turbulent galactic magnetic field models adopted and on the...
CR composition assumed, as well as on the distribution of cosmic ray sources, which are believed to be galactic below the ankle.

The Auger experiment will gather the largest CR statistics at EeV energies and above, and can hence study accurately the large scale anisotropies in these energy regimes. This should allow us to test Agasa’s findings at EeV energies and also to study the ankle region, which is of particular interest because it is there that a transition from a galactic to an extragalactic component is believed to occur, and hence the behaviour of the anisotropies at these energies could provide important clues to understand this transition.

Besides the traditional 2D harmonic analysis [5], which determines the phase in right ascension (in addition to the Rayleigh amplitude), it is clearly desirable to reconstruct the CR dipole in 3D, i.e. getting its actual amplitude and direction in the sky (both in right ascension and declination). A procedure to achieve this in the case of an experiment with full sky coverage was put forward by Sommers some time ago [6], and was recently generalized by Aublin and Parizot [7] to the case of partial exposure relevant for present experiments. The basis of Sommers’ method is to take each event direction \( \hat{n}_i \), with an associated exposure \( \omega_i \), and obtain the dipole through

\[
\vec{D} = \frac{3}{N} \sum_i \hat{n}_i \omega_i, \tag{1}
\]

with \( N \equiv \sum_i \omega_i^{-1} \) (\( i = 1, \ldots, N \) labels the different events observed). This approach (and its generalization to partial sky coverage as well, hereafter referred to as the SAP method) has the virtue of being very easy to implement once the exposure is estimated, but has the following possible drawbacks.

- Since each event is weighted by the inverse of its associated exposure, different regions in the sky do not have their real statistical weight in the determination of the dipole, as they have all been artificially uniformized by dividing by the exposure. Moreover, events in low exposure regions have very large weights and can hence give rise to enhanced fluctuations in the results.
- The uncertainties in both the amplitude and the direction of the dipole are not obtained from the data but rather using Monte Carlo simulations with a known dipole anisotropy and comparing the input dipole parameters with the reconstructed values.
- The method has no means of diagnosing whether the departure from isotropy has a dipolar shape, and for instance an intrinsic quadrupole or even a localized excess in a region of the sky will lead to a non-vanishing dipole amplitude. In these situations the uncertainties estimated from the MC simulations will clearly have little or no meaning.

2. The \( \chi^2 + \) Rayleigh method

We will here consider an alternative method to determine a dipole signature which tries to overcome some of the difficulties just mentioned and apply it to simulated data sets.

Let \( \hat{d} \) be the dipole’s direction, and \( \gamma = \arccos(\hat{n} \cdot \hat{d}) \) the angle between the dipole and the event’s arrival direction \( \hat{n} \). A dipolar distribution should give rise to a CR flux
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(incident on the Earth) of the form
\[ \frac{d\Phi}{d\Omega} \propto (1 + a \cos \gamma), \]  
\[ \text{(2)} \]
where \( a \) is the dipole’s amplitude (i.e. \( \vec{D} = ad \)). When this flux is observed by an experiment with non-uniform exposure (integrated over time) \( \omega(\hat{n}) \), the expected event rates should behave as
\[ \frac{dN}{d\Omega} = N\omega(\hat{n})(1 + a \cos \gamma) \]
\[ \text{(3)} \]
with the normalization, which depends on \( a \) and \( \hat{d} \), fixed to reproduce the total number of events observed. Suppose that now we try to fit a dipole signal along a direction \( \hat{d}' \), along which the expected distribution would be
\[ dN/d\cos \gamma' = N\int_0^{2\pi} d\theta' \omega(\hat{n})(1 + a \cos \gamma'), \]
\[ \text{(4)} \]
where \( \theta' \) is the azimuthal angle around the axis \( \hat{d}' \), \( \cos \gamma' \equiv \hat{d}' \cdot \hat{n} = \cos \beta \cos \gamma' + \sin \beta \sin \gamma' \cos(\theta' - \theta'_d) \),
\[ \text{(5)} \]
where \( \cos \beta \equiv \hat{d} \cdot \hat{d}' \) and \( \theta'_d \) is the azimuthal angle, measured around \( \hat{d}' \), of the dipole vector.

It is then clear that if \( \omega \) were uniform in the sky, the term proportional to \( \cos(\theta' - \theta'_d) \) in the above integral would vanish, leading to the behaviour \( dN/d\cos \gamma' \propto (1 + a \cos \beta \cos \gamma') \), and hence the dipole’s amplitude inferred, barring statistical fluctuations, would be \( a \cos \beta \), which is just the dipole component along the \( \hat{d}' \) axis. One may then envisage that the dipole’s direction could be obtained as the one maximizing the reconstructed dipole’s amplitude, but however, for non-uniform exposures the \( \cos(\theta' - \theta'_d) \) term does not average to zero in general, so that this procedure could lead to a biased result.

There is however a particularly relevant case where this bias is absent, which is when one considers \( \hat{d}' = \hat{z} \), i.e. the NS equatorial axis, since for the case of uniform exposure in right ascension (with an arbitrary declination dependence), the \( \cos(\theta' - \theta'_d) \) term in the integral will indeed vanish\(^1\), hence leading to a behaviour \( dN/d\cos \gamma' \propto (1 + a_z \cos \gamma') \), with \( a_z = a \sin \delta \) being the amplitude of the \( z \) component of the dipole. This then suggests to use a \( \chi^2 \) method to fit a dipolar distribution along the \( \hat{z} \) direction to get an unbiased estimate of \( a_z \).

To fit a dipole signal along a generic direction \( \hat{d}' \), one can just take \( n_\gamma \) bins of \( \cos \gamma' \) (e.g. \( n_\gamma = 10 \) is a reasonable choice, and we checked that using a larger value does not improve significantly the results), and compute the number of events in each bin \( N_j \) and the corresponding expected values
\[ \bar{N}_j = \int_{\Delta\Omega_j} d\Omega \frac{dN}{d\Omega}, \]
\[ \text{(6)} \]
where \( \Delta\Omega_j \) is the solid angle for which \( \cos \gamma' \) falls in the \( j \)th bin, with \( dN/d\Omega \) given by equation (3). We can then write the \( \chi^2 \) function as
\[ \chi^2(a) = \sum_{j=1}^{n_\gamma} \frac{(N_j - \bar{N}_j)^2}{N_j}, \]
\[ \text{(7)} \]
\(^1\) The other situation in which the result is unbiased is of course when \( \hat{d}' = \hat{d} \), since in this case \( \beta = 0 \).
where the sum is clearly restricted to the bins\(^2\) with \(\tilde{N}_j > 0\). One can then determine the value \(a_m\) for which \(\chi^2(a)\) is minimized, and obtain \(\Delta a\) such that \(\chi^2(a_m + \Delta a) = \chi^2(a_m) + 1\).

We hence apply this method along the \(\hat{z}\) direction to obtain first \(a_z\). Concerning the dipole component orthogonal to the \(\hat{z}\) axis, the natural strategy is to apply Rayleigh’s method. In particular, it has been noticed in [7] that this method gives, at least in the case of partial sky coverage, a better determination of the dipole’s right ascension than the SAP method. One can use then Rayleigh’s method to get the best estimate of the dipole’s right ascension and first harmonic amplitude. This method\(^3\) consists simply of computing the quantities (here \(\alpha\) is the right ascension)

\[
A = \frac{2}{N} \sum_{i=1}^{N} \cos \alpha_i \quad \text{and} \quad B = \frac{2}{N} \sum_{i=1}^{N} \sin \alpha_i
\]

and then get the first harmonic amplitude \(r\) and phase \(\Psi\) through

\[
r = \sqrt{A^2 + B^2} \quad \text{and} \quad \Psi = \tan \frac{B}{A}\]

In addition, as shown in [7], there is a simple relation between \(r\) and the original dipole components \(a_z\) and \(a_\perp \equiv a \cos \delta\) in the case in which the exposure is independent of \(\alpha\), which is

\[
r = \left| \frac{c_3 a_\perp}{c_1 + c_2 a_z} \right|
\]

where

\[
\begin{align*}
c_1 &= \int_{\delta_{\min}}^{\delta_{\max}} \omega(\delta) \cos \delta \, d\delta \\
c_2 &= \int_{\delta_{\min}}^{\delta_{\max}} \omega(\delta) \cos \delta \sin \delta \, d\delta \\
c_3 &= \int_{\delta_{\min}}^{\delta_{\max}} \omega(\delta) \cos^2 \delta \, d\delta
\end{align*}
\]

One can then reconstruct completely the dipole’s direction and amplitude from the previous expressions. Moreover, once the dipole’s direction is obtained, it is useful to redetermine the dipole’s amplitude \(a\) using the \(\chi^2\) minimization method for that fixed direction, and in this way one also evaluates the \(\chi^2/dof\) for this fit, which provides a check of the dipolar shape of the data distribution.

\(^2\) In case one bin ends up with a very small number of events, it may just be convenient to choose a different number of bins for the fit.

\(^3\) The Rayleigh method is based on the assumption that the experimental exposure is uniform in right ascension. This may require eventually to select a subset of the data corresponding to sidereal days in which this assumption holds to a specified level of accuracy or, as discussed further below, to generalize the method so as to include the effects of right ascension non-uniformities, if these can be properly modelled.
3. Results

As an example, we show the results of applying this method to data sets of $3 \times 10^4$ simulated events with an intrinsic dipole of 5% amplitude ($a = 0.05$) pointing towards $(\delta, \alpha) = (-45^\circ, 0^\circ)$, assuming an experiment at the Auger location (latitude $-35.2^\circ$) with an ideal geometric exposure (uniform in right ascension and with a zenith angle dependence $dN/d\theta \propto \sin \theta \cos \theta$, assuming a maximum zenith angle for the events analysed of $60^\circ$).

Let us for instance describe in more detail the results of one particular simulated data set. For this simulation the reconstructed dipole has an amplitude $a = 0.059 \pm 0.013$ pointing towards $(\delta, \alpha) = (-52^\circ, -6^\circ)$, which is $\sim 10^\circ$ apart from the input direction of the simulations. Let us notice that the error $\Delta a$ depends essentially only on the dipole’s declination and the statistics at hand, scaling with the overall statistics as $N^{-1/2}$, as expected. We find indeed that, for the assumed detector’s location coincident with the Auger latitude and for maximum zenith angles of $60^\circ$, the empirical expression

$$\Delta a \simeq \sqrt{\frac{3}{N}(1 + 0.6 \sin^3 |\delta|)}$$

(12)

reproduces quite reasonably the results of many simulations performed with different input parameters.

Figure 1 shows the distribution of the events (solid line), the expectations of an isotropic sky (dashed line) and the best fit obtained through the method outlined (dotted line), as a function of $\cos \gamma \equiv \hat{d} \cdot \hat{n}$.

The $\chi^2$ value of the best fit is here 13.1, and notice that the number of degrees of freedom is just $n'_\gamma - 1$, where $n'_\gamma$ is the number of bins which are not empty (which can be less than $n_\gamma$ due to the partial sky coverage, and moreover in this last case it will depend on the dipole’s orientation). For this example $n'_\gamma = 10$, giving a value for $\chi^2/dof = 1.46$,
which is not unreasonable. A plot of the $\chi^2$ function for the best fit direction, as well as
the fitting function $\chi^2(a) = \chi^2(a_m) + ((a - a_m)/\Delta a)^2$, are displayed in figure 2.

Making this analysis for a set of 500 simulations like the one just described, we got
the amplitude, right ascension and declination values (average and dispersion) which are
displayed in table 1. It is apparent from this table that both methods give average values
quite close to the input ones, so that their biases are small, and the $\chi^2$ + Rayleigh method
leads to dispersions in the recovered values smaller by about 20% than SAP’s method.
This is probably related to the possible large enhancement of the effects of statistical
fluctuations in the low exposure regions near the boundaries of the observed sky, which
affect the Aublin–Parizot generalization of Sommers’ method for partial sky coverage.

In the last row of table 1 we display the values of the dipole amplitude obtained
as $\sqrt{a_z^2 + a_\perp^2}$, but for comparison the values of $a$ which result from the $\chi^2$
minimization once the direction of the dipole has been fixed using $a_z$, $r$ and $\Psi$, lead to an
average value $\langle a_m \rangle = 5.4 \pm 1.4\%$, and the $\chi^2$ values of the corresponding fits are characterized by
$\langle \chi^2/dof \rangle = 0.88 \pm 0.45$.

4 Clearly part of the discrepancy between the input and reconstructed values is just due to statistical fluctuations
in the simulated data, while another piece will be associated with the reconstruction method itself.

Table 1. Results from MC simulations (as described in the text) for the average
values and dispersion in the reconstructed dipole’s amplitude, declination and
right ascension. The input values used were $a_{\text{input}} = 5\%$, $\delta = -45^\circ$ and $\alpha = 0^\circ$.

|      | $\langle a \rangle$ (%) | $\sigma_a$ | $\langle \delta \rangle$ | $\sigma_\delta$ | $\langle \alpha \rangle$ | $\sigma_\alpha$
|------|------------------------|------------|--------------------------|----------------|------------------------|------|
| SAP  | 5.8                    | 1.7        | $-44.6^\circ$            | 19.1$^\circ$    | $-0.3^\circ$         | 21.6$^\circ$
| $\chi^2$ + Rayleigh | 5.4        | 1.3        | $-44.5^\circ$            | 16.1$^\circ$    | 0.2$^\circ$           | 17.6$^\circ$ |
Another advantage of the method proposed here is that it really exploits the dipolar shape of the signal searched. For instance, we performed 200 MC simulations with $3 \times 10^4$ events each with a distribution consisting of an isotropic background plus a quadrupole, taken for definiteness with symmetry of revolution around an axis $\hat{q}$, i.e. such that

$$\frac{d\Phi}{d\Omega} \propto \left(1 + \frac{Q}{2} [3(\hat{n} \cdot \hat{q})^2 - 1]\right).$$

(13)

We adopted a quadrupole amplitude $Q = 0.1$ because this value leads to an inferred dipole $a \simeq 0.05$, as in the examples discussed previously, and the orientation $\hat{q}$ adopted also points towards $(\delta, \alpha) = (-45^\circ, 0^\circ)$. Applying the method introduced above to reconstruct a dipole leads to $\langle a \rangle = 6.3 \pm 1.1\%$ (with $\langle a_m \rangle = 5.3 \pm 1.1\%$), but the key point is that the best fit results have associated values of $\langle \chi^2/dof \rangle = 3.5 \pm 1.3$, which are quite poor, and hence this configuration cannot be mistaken with a dipolar one. Applying the SAP method to these simulations gives the reconstructed dipole’s amplitudes with $\langle a \rangle = 4.2 \pm 1.5\%$ and the associated uncertainty in $a$ that would have been estimated for this case is just the same as given in table 1, i.e. $\Delta a \simeq 1.7\%$.

Since the dipole’s reconstruction accuracy scales with the number of events observed as $N^{-1/2}$, it is clear that to be sensitive to smaller dipole amplitudes would require us to consider much larger statistics. For instance, for a $5\sigma$ measurement of a $1\%$ dipole (i.e. for $\Delta a \simeq 0.002$) the number of events required would be $N \simeq 10^6$ (see equation (12)), and we verified that this is indeed the case by applying our method to a simulated data set of this size.

Let us mention that in order to achieve sensitivities to amplitudes at the $1\%$ level or below it is crucial to ensure the high quality of the data used, since for instance non-symmetric arrays or border effects could induce systematic uncertainties in the reconstructed anisotropies. These undesirable effects are usually suppressed by considering only events which are well contained inside the array (through an appropriate quality trigger requirement). Also, temperature or pressure effects could induce a modulation in solar time which can eventually partially leak to the sidereal time distribution, and hence mimic a large scale anisotropy. These effects are mostly relevant for event energies near the detector threshold, for which small weather induced trigger fluctuations could affect the rates of detected showers in a non-uniform way. These modulation effects tend to be averaged out however when the experiment runs for a long period of time, and the residual effects can eventually also be accounted for. This is indeed necessary when dipole amplitudes below the per cent level are being searched.

4. Incorporating right ascension modulation effects

A basic underlying assumption of the Rayleigh method is the uniformity of the exposure of the experiment with respect to right ascension. This however does not always hold, due e.g. to power or to communication failures, so that the strategy usually followed in order to apply this method is to first select a subset of the data corresponding to whole sidereal days in which the running conditions of the experiment under consideration were stable. This procedure can however significantly reduce the number of data available. For instance, the Kascade Collaboration [8] was left with only $\sim 20\%$ of their original data when this kind of selection was performed.
We will here introduce a procedure which allows us to take into account right ascension modulation effects, due to e.g. non-uniform running conditions of the experiment or to the weather effects mentioned above, making it possible hence to use the whole statistics to obtain the amplitude and phase of the right ascension modulation of the incident cosmic ray flux.

Suppose one has obtained \[ \bar{\omega}(\delta, \alpha) \]. This allows us then to introduce an averaged exposure depending only on declination through

\[
\bar{\omega}(\delta) = \frac{1}{2\pi} \int_{0}^{2\pi} d\alpha \omega(\delta, \alpha).
\]  

It is now possible to generalize Rayleigh’s method by introducing

\[
\hat{A} \equiv \frac{2}{\bar{N}} \sum_{i} \cos \alpha_{i} \hat{\omega}(\delta_{i}) / \omega(\delta_{i}, \alpha_{i}),
\]  

and

\[
\hat{B} \equiv \frac{2}{\bar{N}} \sum_{i} \sin \alpha_{i} \hat{\omega}(\delta_{i}) / \omega(\delta_{i}, \alpha_{i}),
\]  

where

\[
\bar{N} \equiv \sum_{i} \hat{\omega}(\delta_{i}) / \omega(\delta_{i}, \alpha_{i}).
\]

Notice that now the contribution of each event to \( \hat{A}, \hat{B} \) and \( \bar{N} \) is weighted by the factor \( \hat{\omega}(\delta_{i}) / \omega(\delta_{i}, \alpha_{i}) \), which takes into account the right ascension non-uniformities, but being of order unity does not overweight the contribution of events in regions of relatively low exposure.

By analogy with the standard approach, a modified Rayleigh amplitude and phase can now be introduced through

\[
\tilde{\Psi} = \tan^{-1} \frac{\tilde{B}}{\tilde{A}}, \quad \tilde{r} = \sqrt{\tilde{A}^2 + \tilde{B}^2}.
\]  

It is now possible to generalize equations (10) and (11) by identifying

\[
\hat{A} = \frac{2}{N} \int d\Omega \omega(\delta, \alpha) \frac{d\Phi}{d\Omega}(\delta, \alpha) \cos \alpha \frac{\hat{\omega}(\delta)}{\omega(\delta, \alpha)}
\]  

\[
= \frac{2}{N} \Phi_{0} \int d\alpha \cos \alpha \cos \delta \cos(\alpha - \alpha_{d})
\]  

\[
= \frac{2\pi}{N} \Phi_{0} \hat{c}_{3} a_{\perp} \cos \alpha_{d}
\]  

where we have assumed that the cosmic ray flux is dipolar, i.e. following \( \Phi = \Phi_{0}(1 + a\hat{d} \cdot \hat{n}) \), and \( a_{\parallel} = a_{z} \) while \( a_{\perp} \) is the amplitude of the dipole vector in the \( xy \) plane, with \( \alpha_{d} \) being the right ascension of the dipole vector.

Similarly, one can find that

\[
\hat{B} = \frac{2\pi}{N} \Phi_{0} \hat{c}_{3} a_{\perp} \sin \alpha_{d}
\]  

(19)
and
\[ \tilde{N} = 2\pi \Phi_0 (\bar{c}_1 + a_{\|} \bar{c}_2) \] (21)

with
\[ \begin{align*}
\bar{c}_1 &= \int d\delta \bar{\omega}(\delta) \cos \delta \\
\bar{c}_2 &= \int d\delta \bar{\omega}(\delta) \cos \delta \sin \delta \\
\bar{c}_3 &= \int d\delta \bar{\omega}(\delta) \cos^2 \delta.
\end{align*} \] (22)

One then finds that
\[ \tilde{r} = \left| \frac{\bar{c}_3 a_{\perp}}{\bar{c}_1 + \bar{c}_2 a_{\|}} \right| \] (23)

while the Rayleigh phase $\tilde{\Psi}$ turns out to be just the right ascension of the dipole $\alpha_d$.

5. Including the quadrupole

One of the main concerns in the reconstruction of a given multipole from data of a partially covered sky, for example the dipole reconstruction discussed at the beginning, is the possible mixing with the other multipoles. For example, as we have shown, a non-vanishing quadrupole can lead to a dipolar signal when only a region of the sky is observed with an inhomogeneous exposure. A first control on the true dipolar character of the large scale anisotropy is given by the value of the $\chi^2$/dof. A more careful analysis can be performed by reconstructing the next multipole, namely the quadrupole, and quantifying in this way their relative strength and the amount of leaking among them.

We propose here an extension of the $\chi^2 + $ Rayleigh method to reconstruct a dipolar + quadrupolar signal. This distribution would give rise to a CR flux
\[
\frac{d\Phi}{d\Omega} = \Phi_0 \left( 1 + a_z \sin \delta + a_x \cos \delta \cos \alpha + a_y \cos \delta \sin \alpha + \frac{Q_{zz}}{2} \sin^2 \delta \\
+ \frac{Q_{xx}}{2} \cos^2 \delta \cos^2 \alpha + \frac{Q_{yy}}{2} \cos^2 \delta \sin^2 \alpha + Q_{xy} \cos^2 \delta \sin \alpha \cos \alpha \\
+ Q_{xz} \cos \delta \sin \delta \cos \alpha + Q_{yz} \cos \delta \sin \delta \sin \alpha \right),
\] (24)

where we have taken the $\hat{z}$ axis along the north pole direction, and the quadrupole tensor is symmetric and traceless. The observed flux, assuming for simplicity that the exposure only depends$^5$ on $\delta$, is given by
\[
\frac{dN}{d\Omega} = N(\delta) \frac{d\Phi}{d\Omega},
\] (25)

where the normalization, which now depends on the amplitude and orientation of the dipole and quadrupole, is fixed to reproduce the total number of events.

$^5$ The extension for $\omega(\delta, \alpha)$ is straightforward along the lines proposed in the previous section.
The first step is again to fit the distribution of events in $\delta$ integrated over $\alpha$, taking advantage of the fact that the exposure is uniform in $\alpha$. Now this distribution will be a function of not only $a_z$, but also of $Q_{zz}$, namely

$$\frac{dN}{d\sin\delta} \propto \left(1 - \frac{Q_{zz}}{4} + a_z \sin\delta + \frac{3Q_{zz}}{4} \sin^2\delta\right).$$

(26)

The values of $a_z$ and $Q_{zz}$ are obtained from a $\chi^2$ minimization similar to that performed in equations (6), (7) (notice that $\cos \gamma' = \sin \delta$ for an axis along the $\hat{z}$ direction).

The next step is to use a generalized Rayleigh method including higher order harmonics in order to obtain the rest of the dipole and quadrupole components. This is performed by computing the quantities

$$A = \frac{2}{N} \sum_{i=1}^{N} \cos \alpha_i, \quad B = \frac{2}{N} \sum_{i=1}^{N} \sin \alpha_i,$$

$$C = \frac{2}{N} \sum_{i=1}^{N} \cos \alpha_i \sin \alpha_i, \quad D = \frac{2}{N} \sum_{i=1}^{N} \sin^2 \alpha_i,$$

$$E = \frac{2}{N} \sum_{i=1}^{N} \cos \alpha_i \cos \delta_i, \quad F = \frac{2}{N} \sum_{i=1}^{N} \sin \alpha_i \cos \delta_i.$$  

(27)

These quantities can be easily related to the multipole coefficients in equation (24) using $\sum_i f(\delta_i, \alpha_i) = \int d\Omega \omega(\delta) d\Phi / d\Omega f(\delta, \alpha)$. We thus obtain

$$A = \frac{2\pi \Phi_0}{N} \left(a_x \frac{\cos \delta}{\cos \delta \sin \delta} + Q_{xz} \frac{\cos \delta}{\cos \delta \sin \delta}\right)$$

$$B = \frac{2\pi \Phi_0}{N} \left(a_y \frac{\cos \delta}{\cos \delta \sin \delta} + Q_{yz} \frac{\cos \delta}{\cos \delta \sin \delta}\right)$$

$$C = \frac{\pi \Phi_0}{2N} Q_{xy} \cos^2 \delta$$

$$D = \frac{2\pi \Phi_0}{N} \left(1 + a_z \frac{\sin \delta}{\cos \delta \sin \delta} + Q_{zz} \left(\frac{1}{2} \frac{\sin^2 \delta}{\cos^2 \delta} - \frac{1}{8} \cos^2 \delta \right) + \frac{1}{4} Q_{yy} \cos^2 \delta\right)$$

$$E = \frac{2\pi \Phi_0}{N} \left(a_x \frac{\cos^2 \delta}{\cos \delta \sin \delta} + Q_{xz} \cos \delta \sin \delta\right)$$

$$F = \frac{2\pi \Phi_0}{N} \left(a_y \frac{\cos^2 \delta}{\cos \delta \sin \delta} + Q_{yz} \cos \delta \sin \delta\right).$$

(28)

Finally, the total number of events is related to $\Phi_0$, $a_z$ and $Q_{zz}$ through

$$N = 2\pi \Phi_0 \left(1 + a_z \frac{\sin \delta}{\cos \delta \sin \delta} + Q_{zz} \left(3 \frac{\sin^2 \delta}{4} - 1\right)\right).$$

In the previous expression we have used the notation

$$f(\delta) = \int_{\delta_{\text{min}}}^{\delta_{\text{max}}} d\delta \cos \delta \omega(\delta) f(\delta).$$
The remaining dipolar and quadrupolar coefficients are then obtained as

\[
\begin{align*}
    a_x &= \frac{K_1}{K_2} \left( A \cos^2 \delta \sin \delta - E \cos \delta \sin \delta \right) \\
    a_y &= \frac{K_1}{K_2} \left( B \cos^2 \delta \sin \delta - F \cos \delta \sin \delta \right) \\
    Q_{xx} &= \frac{K_1}{K_2} \left( E \cos \delta - A \cos^2 \delta \right) \\
    Q_{yz} &= \frac{K_1}{K_2} \left( F \cos \delta - B \cos^2 \delta \right) \\
    Q_{xy} &= \frac{4}{\cos^2 \delta} C K_1 \\
    Q_{yy} &= \frac{4}{\cos^2 \delta} \left( K_1 D - \mathbf{I} - a_z \sin \delta - \frac{Q_{zz}}{8} \left( 4 \sin^2 \delta - \cos^2 \delta \right) \right) \\
    Q_{xx} &= -Q_{yy} - Q_{zz},
\end{align*}
\]

with

\[
\begin{align*}
    K_1 &= \frac{N}{2 \pi \Phi_0} = \mathbf{I} + a_z \sin \delta \cos^2 \delta - \cos \delta \sin \delta \cos \delta, \\
    K_2 &= \cos \delta \sin \delta \cos \delta - \cos \delta \sin \delta \cos \delta.
\end{align*}
\]

As an example, we generated two datasets of \(4 \times 10^5\) events each, one with a dipole and the other with a quadrupole. For the first, we took a dipole amplitude \(a = 0.1\) pointing along \((\delta, \alpha) = (-45^\circ, 0^\circ)\). Applying the method introduced above, we found from the \(\chi^2\) minimization along the \(z\)-axis that \(a_z = -0.085 \pm 0.011\) and \(Q_{zz} = -0.018 \pm 0.017\). The generalized Rayleigh method then gave \(a_x = 0.075, a_y = -0.005, Q_{xx} = 0.007, Q_{yy} = 0.011, Q_{xy} = -0.004, Q_{zz} = 0.002\) and \(Q_{yz} = -0.004\), in perfect agreement with the input values, which correspond to \((a_x, a_y, a_z) = (0.0707, 0, -0.0707)\) and \(Q_{ij} = 0\). Notice that applying to this same dataset the \(\chi^2 + \text{Rayleigh}\) method introduced initially, i.e. without allowing for a quadrupole contribution, leads to \(a_z = -0.075 \pm 0.005, a_\perp = 0.072, a = 0.104 \pm 0.004\), and pointing only \(2^\circ\) away from the input direction.

Regarding the dataset with an input quadrupole, for which we took the expression given in equation (13), with \(Q = 0.1\) and \(\hat{q}\) also pointing towards \((\delta, \alpha) = (-45^\circ, 0^\circ)\), we got from the \(\chi^2\) minimization that \(a_z = -0.014 \pm 0.010\) and \(Q_{zz} = 0.045 \pm 0.016\). For the other components we obtained \(a_x = 0.010, a_y = -0.002, Q_{xx} = 0.055, Q_{yy} = -0.100, Q_{xy} = 0.000, Q_{xz} = -0.129\) and \(Q_{yz} = -0.011\), in perfect agreement with the input values, which for the quadrupole assumed correspond to \(Q_{zz} = Q/2, Q_{xx} = Q/2, Q_{yy} = -Q\) and \(Q_{xz} = -3Q/2\), with all other parameters vanishing.

6. Summary

In this work we have introduced a method to obtain large scale anisotropies in cosmic ray arrival directions in three dimensions. In its simplest version, the method allows us to

\footnote{In this case, since we have to determine two parameters, it is convenient to take a larger number of bins of \(\cos \gamma\), e.g. \(n_\gamma = 20\).}
obtain the dipole vector describing the anisotropy of the incident flux, assuming that the exposure is uniform in right ascension, as approximately holds for surface arrays. The $z$ component $a_z$ is obtained from a $\chi^2$ fit to the event distribution along the NS axis, and the perpendicular component is obtained with a 2D Rayleigh analysis in the orthogonal plane. We compared this method with the one recently introduced by Aublin and Parizot, showing that the dispersions in the values obtained are typically 20% smaller, and also the $\chi^2$ fit along the reconstructed dipole’s direction provides a measure of the agreement between the arrival direction distribution and a dipolar shape.

We then generalized the method to the case in which a known modulation induces non-uniformities of the exposure with respect to right ascension. Finally, we extended the method to also include a quadrupole anisotropy, and showed with simulated datasets that this allows us to recover the three components of the dipole and the six independent components of the quadrupole reliably.

The methods here introduced should be applicable to detectors like Auger, and should help determine the large scale patterns of the cosmic ray distribution, which can provide crucial information about the origin of the cosmic rays and the way they propagate through the Galaxy.

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