TWO TRACE INEQUALITIES FOR OPERATOR FUNCTIONS

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Abstract. In this paper we show that for a non-negative operator monotone function \( f \) on \([0, \infty)\) such that \( f(0) = 0 \) and for any positive semidefinite matrices \( A \) and \( B \),

\[
\text{Tr}((A - B)(f(A) - f(B))) \leq \text{Tr}(|A - B|f(|A - B|)).
\]

When the function \( f \) is operator convex on \([0, \infty)\), the inequality is reversed.

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