Adaptive robust tracking control of a class of nonlinear systems with input delay

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Abstract In this paper, the tracking control problem of a class of uncertain Euler–Lagrange systems subjected to time-varying input delay and bounded disturbances is addressed. To this front, a novel delay-dependent control law referred as adaptive robust outer loop control (AROLC) is proposed. Compared to the conventional predictor-based approaches, the proposed controller is capable of negotiating any input delay, within a stipulated range, without knowing the delay variation. The maximum allowable input delay is computed through Razumikhin-type stability analysis. AROLC also provides robustness against the disturbances due to the input delay, parametric variations and unmodelled dynamics through switching control law. The novel adaptive law allows the switching gain to modify itself online in accordance with the tracking error without any prerequisite of the uncertainties. The uncertain system, employing AROLC, is shown to be uniformly ultimately bounded. As a proof of concept, experimentation is carried out on a nonholonomic wheeled mobile robot with various time varying as well as fixed input delay, and better tracking accuracy of the proposed controller is noted compared to predictor-based methodology.

Keywords Adaptive robust control · Euler–Lagrange systems · Input delay · Razumikhin theorem · Wheeled mobile robot

1 Introduction

1.1 Background and motivation

Time delays could be inherent in the system or may be a result of some sources such as transportation and transmission lags, communication delays. A few applications where delays are evident include chemical and biological processes, teleoperated robotic systems, rolling mills, delays due to sensor response and input delay in mobile robots, engine cycle delays in internal combustion engine, control over network [1–3]. Left unattended, such delay reduces performance of the system and may create potential instability to the system. The survey articles [1–5] document the recent advances, challenges and controllers developed to tackle the unwanted delay for linear systems [1–4] and nonlinear systems [5]. The control strategies reported in the literature can broadly be classified in two categories, one for linear systems [6–26] and other for nonlinear systems [27–40].

Controllers developed for linear systems are first discussed. Control law that predicts the input delay to the system is one the most studied as well as used method and has a root to the Smith predictor-based approach [6]. However, performance of this method
depends on modelling accuracy of the system and can be applied to stable systems only. On the contrary, Artstein model reduction [7], finite spectrum assignment [8], can be applied to unstable and multivariable plants. The predictor-based approach transforms the delayed system into a delay-free system using the control input history. The prediction scheme of [8] was extended in [9] when state delay is also present along with the input delay. A discrete time predictor-based controller is reported in [10] for uncertain linear systems. A reduction method to design a delayed feedback based controller for uncertain linear systems with time-varying input delay is proposed by Yue and Han [11]. However, this approach requires exact knowledge of the input delay to compensate its effect. A robust controller for time-varying input delay is designed in [12] by combining Lyapunov–Krassovskii functional and a neutral transformation, assuming that bound of the first derivative of delay is available. A predictor-based state feedback controller is proposed in [13, 14] for stabilization of uncertain discrete time systems via LMI technique. In the predictor-based approach of [15], an equivalent transformation is used on the characteristic equation to convert the predictor feedback control into a stabilization problem of neutral time-delay system for constant time delay. Predictor feedback control law, based on small gain analysis, for linear time invariant systems with unknown time varying delay is designed in [16]. The problem of absolute stability of Lurie system with interval time varying and known bounded uncertainty is addressed in [17]. A stabilizing controller is reported in [18] for linear systems for state delay as well as neutral-type delay with predefined uncertainty bound. The concept of sampled data control is investigated in [19, 20] for input delay compensation of linear systems within known system uncertainty.

Adaptive control techniques are addressed in [21–23] for linear systems. An adaptive control law is reported by Bekiaris-Liberis and Kristic [21] for feedforward linear systems subjected to both state and input delays, where the unknown time delay is predicted adaptively using a projection function without considering plant uncertainties. The case of linearly parameterized uncertainty in plants with known bounds is considered in [22]. Adaptive backstepping method is used in [23] for robust stabilization problem of linear non-minimum phase systems subjected to unknown dynamics and input delay. Sliding mode control strategies are considered in [24–26]. However, Li et al. [24] only considered constant time delay and uncertainties in plant parameters were ignored. Integral sliding mode control is used by Zhang et al. [25] for uncertain linear systems under time-varying delay at the input, but it requires known bounds of the uncertainties as well as its first time derivative and can only tackle slowly varying input delay. On the other hand, Roh and Oh [26] used adaptive sliding mode control for linear input-delayed systems where bound on the uncertainties is estimated online. However, estimation of the uncertainties becomes a monotonically increasing function of states for nonzero values of states. This may cause very high switching gain and consequent chattering.

Nonlinear systems with input delay pose a greater challenge since linear boundedness of plant model cannot be incorporated in the stability proof [5]. Compared to linear systems, results for nonlinear systems with time delay at the input are limited. A delay-dependent Lyapunov–Razumikhin-based approach for controlling nonlinear systems is provided by Jankovic [27]. Delay-independent Razumikhin approach for nonlinear time-delay systems with triangular structure is proposed in [28]. The system was shown to be uniformly ultimately bounded (UUB), assuming that the delay disturbances are bounded. However, no control input was designed to negotiate the time-delayed uncertainties and the delay-independent solution is conservative. Smith predictor-based globally linearizing controller is used by Henson and Seborg [29] for known nonlinear system. A backstepping-based predictor approach is proposed by Kristic [30] for delay compensation of forward complete and strict-feedforward nonlinear systems utilizing ODE–PDE cascade transformation. Controllers for rigid and flexible link manipulators are developed in [31, 32] by linearizing the system. Here, the controllers developed for delay-free system are shown to be stable under some delay-dependent conditions. The controller reported in [33] aims for delay compensation in states and dead zone at the input. However, the above-mentioned control laws do not consider modelling uncertainties and require exact knowledge of the models which is difficult for nonlinear systems. This issue was addressed in [34] where a predictor-based approach is proposed for constant input delay based on Lyapunov–Krassovskii method. In this case, uncertainty in the inertia matrix was considered. The same approach was later extended for time-varying input delay in [35] and also for state delay [36]. A filtered tracking error-based control law is used in [37].
to tackle the effect of known constant input time delay and used fuzzy logic systems of first type to approximate the unknown disturbances with predefined uncertainty bound. A stabilizing controller for spacecraft with unknown but constant input delay is proposed in [38] with approximated uncertainty bound. However, in practice delay may not always be constant and defining prior uncertainty bounds for the uncertainties is not always feasible for nonlinear uncertain systems due to unmodelled dynamics and unknown disturbances. Adaptive control may play crucial role for estimating the controller gains in face of unknown uncertainties. Adaptive sliding mode-based control law is developed in [39] for complex network systems under time-varying network delays for perturbation compensation. Again, Jin et al. [40] reported an adaptive law for evaluation of coupling strength for synchronization of chaotic systems under network-induced delay. However, the adaptive laws reported in both [39] and [40] do not allow the gains to decrease even when error reduces. Application of such adaptive law creates an overestimation of switching gains and compromises the controller accuracy [41]. Adaptive laws were also proposed in [42,43] to estimate the gains, but these works did not consider time-delayed systems. Moreover, He et al. [42] require predefined bound on disturbances to design the control law. Also, according to the adaptive laws developed in [42,43], the gains decrease or increase depending on a time-varying threshold value which create overestimation–underestimation problem of the gain [41].

1.2 Contributions

In this paper, a delay-dependent control law, christened as adaptive robust outer loop control (AROLC), is proposed for tracking control of a class of Euler–Lagrange systems subjected to time-varying input delay with unknown variation and bounded uncertainty. Two major advantages of the proposed control scheme are stated below,

(i) The proposed control law does not require any explicit knowledge of the variation in input delay to design the control law unlike the ones reported in [34–37] and can negotiate any input delay within a maximum delay range. The maximum allowable input delay is obtained from the delay-dependent stability criterion of the closed-loop system employing the Razumihkin-type stability approach [44].

(ii) As opposed to the controllers developed in [34–38], the proposed adaptive robust law does not necessitate the predefined knowledge of uncertainty bound. The switching gain is determined based on an adaptive law which solely depends on a function of tracking error and does not make the switching gain a monotonically increasing function of error. Further, the evaluation of switching gain does not involve any threshold value and thus alleviates the overestimation–underestimation problem of switching gain. The proposed controller possesses the flexibility that a user can specify any error function based on which the switching gain would be modified while maintaining the same stability criterion. The closed-loop stability analysis of the system dynamics is carried out in the sense of UUB.

(iii) Experimental validation of the proposed AROLC law is also carried out on PIONEER-3 wheeled mobile robot (WMR), and the results are compared with the predictive control approaches reported in [34,35].

1.3 Notations

The following notations are used throughout the paper: any variable $\rho$ delayed by an amount $h$ as $\rho(t-h)$ would be denoted as $\rho_h$; $\lambda_{\text{min}}(\cdot)$ and $\|\cdot\|$ represent minimum eigen value and Euclidean norm of the argument, respectively; $I$ represents identity matrix.

1.4 Organization

The article is organized as follows: the detailed problem formulation is first carried out in Sect. 2. This is followed by the proposed adaptive robust control methodology and its in-depth analysis. Section 3 presents the experimental results of the proposed controller and its comparison with [34,35]. Section 4 concludes the entire work.

2 Controller design

2.1 Problem formulation and objective definition

In general, an Euler–Lagrange system possessing second-order dynamics and input delay can be written as
\[ M(q) \ddot{q} + N(q, \dot{q}) = \tau(t - h(t)), \]  

where \( q(t) \in \mathbb{R}^n \) is the system state, \( \tau(t) \in \mathbb{R}^n \) is the control input, \( M(q) \in \mathbb{R}^{n \times n} \) is the mass/inertia matrix and \( N(q, \dot{q}) \in \mathbb{R}^n \) denotes combination of other system dynamics terms based on system properties. In practice, it can be assumed that unmodelled dynamics and disturbances are subsumed by \( N \). \( h(t) \) is a time-varying input delay which may be a result of computation delay, communication delay between the controller and the actuator. Let, \( q^d(t) \) be the desired trajectory to be tracked and \( e_1(t) = q^d(t) - q(t) \) is the tracking error. Before introducing the proposed control law, a brief overview of the control structure of predictive control law, reported in [34,35], is provided below.

### 2.1.1 Predictive controller [34,35]

**Assumption 1** All the disturbances and their first time derivative are bounded by some known constant.

**Assumption 2** Input delay \( h(t) \) and \( \dot{h}(t) \) are bounded by a known constant and \( \dot{h}(t) < 1 \).

Let \( \varrho \) be a measurable filtered tracking error and defined as

\[
\varrho = \dot{e}_1 + \kappa e_1 - \vartheta e_z.
\]

\[
e_z = \int_{t-h(t)}^{t} \tau(\theta) \, d\theta
\]  

where \( \kappa \) is a positive constant and \( \vartheta \) is a positive definite matrix. Taking time derivative of (2) and multiplying it by \( M \) and then using (1) and (3), we get

\[
M \dot{\varrho} = M \ddot{q}^d + N - \dot{\varrho} \eta - \vartheta \epsilon - \dot{\vartheta} h_1 M \dot{e}_1,
\]

where \( \eta = \vartheta - M^{-1} \). The control input is selected as \( \tau = k_b \varrho \) to track the desired trajectory provided Assumptions 1 and 2 hold. Selection methods of \( k_b \) and \( \kappa \) are detailed in [34,35].

**Remark 1** In practice, proper bound estimation of unmodelled dynamics and unknown disturbances is almost impossible. Moreover, according to Assumption 2, knowledge of the bound of \( \dot{h} \) and \( h \) is required and the controller can only tackle slowly varying time delay [35]. However, this is difficult to attain since variation in time delay \( h \) can be arbitrary in practical scenario. Hence, it is not always possible to maintain Assumptions 1 and 2.

So, the aim of this paper is to design a control law which simultaneously fulfills the following two objectives:

**O1:** To maintain system stability of the input-delayed system (1), within a stipulated maximum time delay, while it follows a predefined desired path. This objective is to be met irrespective and without any knowledge of the nature of the variation of delay \( h(t) \).

**O2:** To provide robustness to system (1) against bounded but unknown uncertainties and disturbances without any prior knowledge of the bounds.

### 2.2 Adaptive robust outer loop controller

Towards achieving the outlined objectives, a novel controller, named adaptive robust outer loop controller (AROLC), is proposed. The structure of the proposed control law is selected to be

\[
\tau = \hat{M} u + \hat{N},
\]

where \( u \) is the auxiliary control input; \( \hat{M} \) and \( \hat{N} \) are the nominal values of \( M \) and \( N \), respectively. In practice, it is always possible to have a certain amount prior knowledge of some system parameters. For example, in case of a mobile robot, some prior knowledge of the system mass, robot’s width and length, diameter of the wheels, etc. are always available. The prior knowledge of those parameters is termed as nominal value of the corresponding parameters. Evaluation of system dynamics with those nominal parametric values provides \( \hat{M} \) and \( \hat{N} \). So, \( \hat{M} \) and \( \hat{N} \) constitute the nominal dynamics of a system. In general, \( M \) and \( N \) can be represented as the following:

\[
M = \hat{M} + \Delta M,
\]

\[
N = \hat{N} + \Delta N,
\]

where \( \Delta M \) and \( \Delta N \) denote the perturbations in \( M \) and \( N \), respectively. However, it is not possible to have a nominal structure of the unmodelled dynamics and external disturbances and these can be considered to be subsumed by \( \Delta N \).

The auxiliary control input \( u \) is defined in the following way,

\[
u = \hat{u} + \Delta u,
\]
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where $\hat{u}$ and $\Delta u$ are nominal and switching control input, respectively, and they are evaluated as

$$\hat{u} = \ddot{q}^d(t) + K_2 \dot{e}_1(t) + K_1 e_1(t),$$

$$\Delta u = \begin{cases} 
ac(e, t) \frac{s}{||s||} & \text{if } ||s|| \geq \epsilon, \\
ac(e, t) \frac{s}{\epsilon} & \text{if } ||s|| < \epsilon,
\end{cases}$$

where $K_1$ and $K_2$ are two positive definite matrices with appropriate dimensions, $s = B^T Pe$, $e = [e_1^T \dot{e}_1^T]^T$, $\dot{c}$ is the switching gain responsible to tackle the uncertainties, $\alpha > 0$ is a scalar adaptive gain and $\epsilon > 0$ represents a small scalar.

To accomplish the second objective, an adaptive control law is needed to be formulated for evaluation of $\dot{c}$. Two pioneering works were carried out in [39] and [40] where the controller gains are modified through an adaptive law. However, the adaptive laws proposed in [39] and [40] make the controller gains a nondecreasing function of error due to the application requirement. In fact, it is mentioned in [40] that such “large enough” gains (referred as coupling strength in [40]) are necessary to maintain certain conditions [read as (12)–(13) in [40]].

However, in case of switching law such as (10), high switching gain would cause overestimation problem and consequent reduction in controller accuracy [41]. Furthermore, it was verified in [45] that for robotic systems, such adaptive law causes excessive gain and reduces tracking accuracy. Considering such scenario, the following adaptive law is proposed in this paper:

$$\dot{c} = \begin{cases} 
\parallel s \parallel & \dot{c} > \gamma, s^T \dot{s} > 0, \\
-\parallel s \parallel & \dot{c} > \gamma, s^T \dot{s} \leq 0, \\
\gamma & \dot{c} \leq \gamma,
\end{cases}$$

where $\gamma > 0$ is a small scalar to keep $\dot{c}$ always positive. According to the adaptive law (11), $\dot{c}$ increases (resp. decreases) whenever the error trajectories move away from (resp. move closer to) $s = 0$.

**Remark 2** It can be noted from (11) that $\dot{c}$ increases or decreases according to tracking error incurred by the system. Hence, the proposed adaptive law does not make the switching gain $\dot{c}$ a monotonically increasing function of error like [39,40]. Moreover, the adaptive law (11) does not involve any threshold value like [42, 43] and alleviates the overestimation–underestimation problem.

Nevertheless, due to the input delay $h$, the error dynamics of (1) employing (5) and (8) is found to be

$$\dot{e}_1 = -K_2 \dot{e}_1h - K_1 e_1h + \sigma - \Delta u_h,$$

where $\sigma = (I - M^{-1}(q) \dot{M}(q_h))u_h + M^{-1}(q)(N(q, \dot{q}) - \dot{N}(q, \dot{q}_h)) + \ddot{q}^d(t) - \ddot{q}_h^d$ and denotes the overall uncertainty. Further, (12) can be formulated in state space as

$$\dot{e} = A_1 e + B_1 e_h + B(-\Delta u_h + \sigma),$$

where $A_1 = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 & 0 \\ -K_1 & -K_2 \end{bmatrix}$ and $\Delta u$ attempts to negotiate $\sigma$. Noting that, $e_h = e(t) - \int_0^h \dot{e}(t + \theta) d\theta$, where the derivative inside the integral is with respect to $\theta$; the error dynamics (13) is modified as

$$\dot{e}(t) = A e(t) - B_1 \int_{-h}^0 \dot{e}(t + \theta) d\theta$$

$$+ B(-\Delta u_h + \sigma),$$

where $A = A_1 + B_1$. The controller gains $K_1$ and $K_2$ are selected in a way such that $A$ is Hurwitz which is always possible by noting the structure of $A$. The stability analysis of the proposed controller is carried out in the sense of UUB and is detailed subsequently.

2.2.1 **Stability analysis of AROLC**

**Assumption 3** All the states, i.e. $q, \dot{q}$, are available.

**Assumption 4** The uncertainties are bounded as $||\sigma|| \leq c$, where $c$ is an unknown scalar quantity. Knowledge of $c$ is not required to compute the control law of AROLC. However, consideration of $c$ is necessary for stability analysis.

**Assumption 5** Let, $V(e)$ be a Lyapunov function candidate. Then, following the Razumikhin-type theorem [44], for some constant $r > 1$, let the following inequality holds:

$$V(e(\xi)) < rV(e(t)), \quad t - 2h \leq \xi \leq t.$$  

To study the stability aspect of the overall closed-loop system (1), the following Lyapunov function is considered:

$$V(e) = V_1(e) + V_2(e),$$

where $V_1(e) = \frac{1}{2}e^T Pe$, $V_2(e) = \frac{1}{2}(\dot{c} - c)^2$, $P > 0$ is
the solution of the Lyapunov equation $A^T P + PA = -Q$ for some $Q > 0$.

To simplify the stability analysis of the overall system, the time derivative of $V_1(e)$ is evaluated through Lemma 1, which further allows to attain the maximum allowable input delay. The result of Lemma 1 facilitates the analysis of the UUB stability notion of overall system as stated in Theorem 1.

**Lemma 1** (i) The time derivative of $V_1(e)$ can be simplified by using (5)–(10) as

$$
\dot{V}_1(e) \leq -\frac{1}{2} e^T [Q - hE] e \\
+ \Gamma + s^T (-\Delta u + \sigma) + s^T \Theta,
$$

(17)

where

$$
E = \beta PB_1 \left(A_1 P^{-1} A_1^T + B_1 P^{-1} B_1^T + P^{-1}\right) \\
\times B_1^T P + 2(r/\beta) P
$$

$$
\Theta = \Delta u - \Delta u_h
$$

(ii) The maximum allowable input delay for system (1) is obtained as

$$
h < \frac{\lambda_{\min}(Q)}{||E||},
$$

(18)

provided gains $K_1$, $K_2$ and scalar design parameter $r > 1$, $\beta > 0$ are selected in a manner which satisfies

$$
\lambda_{\min}(Q) > h||E|| \quad \forall h.
$$

(19)

**Proof** (i) Using (14), the time derivative of $V_1(e)$ yields

$$
\dot{V}_1(e) = -\frac{1}{2} e^T Q e - e^T PB_1 \int_{-h}^{0} \hat{e}(t + \theta) d\theta \\
+ s^T (-\Delta u_h + \sigma)
$$

(20)

Again, using (13), the second term of (20) is simplified as

$$
- e^T PB_1 \int_{-h}^{0} \hat{e}(t + \theta) d\theta \\
= - \int_{-h}^{0} e^T PB_1 \left[ A_1 e(t + \theta) + B_1 e(t - h + \theta) + B_2 \sigma_2(t + \theta) \right] d\theta,
$$

(21)

where $\sigma_2(t) = -\Delta u_h + \sigma(t)$. Applying (15) into (16), the following relation is obtained

$$
e^T(\xi) Pe(\xi) < re^T(t) Pe(t) + \varphi(\xi),
$$

(22)

where $\varphi(\xi) = r(\hat{c}(t) - c)^2 - (\hat{c}(\xi) - c)^2$. For any two nonzero vectors $\zeta_1$ and $\zeta_2$, there exists a constant $\beta > 0$ and matrix $D > 0$ such that the following inequality holds

$$
- 2 \zeta_1^T D^{-1} \zeta_1 + (1/\beta) \zeta_2^T D \zeta_2.
$$

(23)

Applying (23) into (21) and taking $D = P$, the following inequalities are obtained

$$
- 2 \int_{-h}^{0} e^T PB_1 A_1 \left\{ e(t + \theta) \right\} d\theta \\
\leq \int_{-h}^{0} \left\{ \beta e^T PB_1 A_1 P^{-1} A_1 B_1^T P e \\
+ r/\beta) Pe(t + \theta) \right\} d\theta
$$

(24)

$$
\leq h e^T \left\{ \beta PB_1 A_1 P^{-1} A_1 B_1^T P + (r/\beta) P \right\} e \\
+ \int_{-h}^{0} (1/\beta) \varphi(t + \theta) d\theta
$$

(25)

$$
- 2 \int_{-h}^{0} e^T PB_1 [B \sigma_2(t + \theta)] d\theta
$$

(26)

Assuming the uncertainties to be integrable within the delay (i.e. system is locally Lipschitz within the delay), there exists a scalar $\Gamma > 0$ such that the following inequality holds over the stipulated time delay:
The second, third and fourth terms of (28) along with (5) control input for the system (1), it can be derived from (28) in the following way.

(ii) For the stability of the overall system, the first term of (28) is required to be negative definite, i.e.

\[ Q - hE > 0 \Rightarrow \lambda_{\min}(Q) > h||E|| \quad \forall h \]

So, the controller gains \( K_1, K_2 \) and the scalar variables \( r, \beta \) are to be selected in a manner such that \( \lambda_{\min}(Q) > h||E|| \quad \forall h \). Hence, the maximum allowable delay can be found from (29) as

\[ h < \frac{\lambda_{\min}(Q)}{||E||} \quad (30) \]

The second, third and fourth terms of (28) along with \( \dot{V}_2(e) \) help to define various error bounds of the UUB conditions as derived through Theorem 1.

The stability analysis of system (1) employing AROLC is stated in Theorem 1 in the sense of UUB using a common Lyapunov function (16).

**Theorem 1** The system (1) employing AROLC with the control input (5)–(11) is UUB with the following error bounds,

\[ \sigma_i = \mu_i + \sqrt{\frac{2(\Gamma + i)}{\lambda_{\min}(\Psi)}} \mu_i^2 \quad i = 1, 2, 3 \quad \text{for} \quad ||s|| \geq \epsilon \]

\[ \sigma_i = \sqrt{\frac{4\alpha(\Gamma + i)c + \epsilon \mu_i^2}{2\alpha c\lambda_{\min}(\Psi)}} \quad i = 4, 5, 6 \quad \text{for} \quad ||s|| < \epsilon \]

where

\[ \Psi = Q - hE \]

\[ \mu_1 = \frac{||\Theta||||BT P||}{\lambda_{\min}(\Psi)} \]

\[ \mu_2 = \frac{(2c - (\alpha + 1)c + ||\Theta||||B^T P||)}{\lambda_{\min}(\Psi)} \]

\[ \mu_3 = \frac{(c - \alpha c + ||\Theta||)}{\lambda_{\min}(\Psi)} \]

\[ \mu_4 = c + ||\Theta||, \quad \mu_5 = 2c - \hat{c} + ||\Theta||, \]

\[ \mu_6 = c + ||\Theta|| \]

\[ i = \gamma^2 \quad \text{for} \quad \hat{c} \leq \gamma \quad \text{and} \quad i = \mu \quad \text{for} \quad \hat{c} > \gamma \]

**Proof** Exploring the various possible combinations of \( \Delta u \) and \( \hat{c} \) in (10) and (11), respectively, the following six different cases have been identified:

**Case (i):** \( \hat{c} > \gamma, \quad ||s|| \geq \epsilon \quad \text{and} \quad s^T \hat{c} > 0 \)

**Case (ii):** \( \hat{c} > \gamma, \quad ||s|| \geq \epsilon \quad \text{and} \quad s^T \hat{c} < 0 \)

**Case (iii):** \( \hat{c} \leq \gamma, \quad ||s|| \geq \epsilon \quad \text{and} \quad s^T \hat{c} \leq 0 \)

**Case (iv):** \( \hat{c} > \gamma, \quad ||s|| < \epsilon \quad \text{and} \quad s^T \hat{c} > 0 \)

**Case (v):** \( \hat{c} > \gamma, \quad ||s|| < \epsilon \quad \text{and} \quad s^T \hat{c} \leq 0 \)

**Case (vi):** \( \hat{c} \leq \gamma, \quad ||s|| < \epsilon \quad \text{and} \quad s^T \hat{c} \leq 0 \)

Utilizing Lemma 1, (8)–(11) and (16), each of the six cases can be shown to be UUB. The individual cases are analysed as follows:

**Case (i):** \( \hat{c} > \gamma, \quad ||s|| \geq \epsilon \quad \text{and} \quad s^T \hat{c} > 0 \)

Utilizing (11) and (17), the time derivative of (16) yields

\[ \dot{V}(e) \leq -\frac{1}{2}e^T \Psi e + \Gamma + s^T (\Delta u + \sigma) + s^T \Theta \\
+ (\hat{c} - c)||s|| \\
= -\frac{1}{2}e^T \Psi e + \Gamma + s^T (\alpha \hat{c} \frac{s}{||s||} + \sigma) + s^T \Theta \\
+ (\hat{c} - c)||s||. \quad (31) \]

Again, third term of (31) can be expanded as:

\[ s^T (\alpha \hat{c} \frac{s}{||s||} + \sigma) = -\alpha c s^T \frac{s}{||s||} + s^T \sigma \]

\[ \leq (-\alpha \hat{c} + c)||s|| \quad (32) \]

Using (31) and (32), we have

\[ \dot{V}(e) \leq -\frac{1}{2}\lambda_{\min}(\Psi)||e||^2 \\
- (\alpha - 1)\hat{c}||s|| + ||\Theta||||s|| + \Gamma \]

So, for a choice of \( \alpha > 1 \), \( \dot{V}(e) < 0 \) would be established if \( \lambda_{\min}(\Psi)||e||^2 > 2\Gamma + 2||\Theta||||s|| \). Again, we have \( ||s|| \leq ||B^T P||||e|| \). Thus, (1) would be UUB with the error bound,
Using (11), (17) and (36), the time derivative of (16) gives
\[
\dot{V}(e) \leq -\frac{1}{2}e^T \Psi e + \Gamma + s^T (\Delta u + \sigma) \\
+ s^T \Theta + (\hat{c} - c)||s|| \\
\leq -\frac{1}{2}\lambda_{\min}(\Psi)||e||^2 + \frac{s^T (\Delta u + \sigma)}{||s||} \\
+ ||\Theta|| ||s|| + (\hat{c} - c)||s||. 
\]

The combination of third, fourth and fifth terms of (37) take the maximum value of \((e\mu_5^2)/(2\alpha \hat{c})\) for \(||s|| = (e\mu_5)/2\alpha \hat{c}\). Thus, \(\dot{V}(e) < 0\) would be achieved if \(\lambda_{\min}(\Psi)||e||^2 > 2\Gamma + (e\mu_5^2)/(2\alpha \hat{c})\). So, the system is UUB and the error bound is calculated to be
\[
||e|| = \sqrt{\frac{4\alpha \hat{c} + e\mu_5^2}{2\alpha \hat{c}\lambda_{\min}(\Psi)}} = \sigma_5 
\]

Using (11), (17) and (36),
\[
\dot{V}(e) \leq -\frac{1}{2}e^T \Psi e + \Gamma + s^T (\Delta u + \sigma) + s^T \Theta \\
+ (\hat{c} - c)||s|| \\
\leq -\frac{1}{2}\lambda_{\min}(\Psi)||e||^2 + \Gamma + s^T \left(-\frac{\alpha \hat{c}^2}{\epsilon} + \frac{s}{||s||}\right) \\
+ ||\Theta|| ||s|| + (\hat{c} - c)||s||. 
\]

The combination of third, fourth and fifth terms of (39) take the maximum value of \((e\mu_5^2)/(2\alpha \hat{c})\) for \(||s|| = (e\mu_5)/2\alpha \hat{c}\). Thus, \(\dot{V}(e) < 0\) would be achieved if \(\lambda_{\min}(\Psi)||e||^2 > 2\Gamma + (e\mu_5^2)/(2\alpha \hat{c})\). So, the system is UUB and the error bound is calculated to be
\[
||e|| = \sqrt{\frac{4\alpha \hat{c} + e\mu_5^2}{2\alpha \hat{c}\lambda_{\min}(\Psi)}} = \sigma_5 
\]

Case (vi): \(\hat{c} \leq \gamma, \quad ||s|| < \epsilon\) and any \(s^T \dot{s} \leq 0\)

Using similar procedure, time derivative of (16) for Case (vi) yields
\[
\dot{V}(e) \leq -\frac{1}{2}e^T \Psi e + \Gamma + s^T (\Delta u + \sigma) + s^T \Theta + \gamma^2 \\
+ (\hat{c} - c)||s|| \\
\leq -\frac{1}{2}\lambda_{\min}(\Psi)||e||^2 + \Gamma + s^T \left(-\frac{\alpha \hat{c}^2}{\epsilon} + \frac{s}{||s||}\right) \\
+ ||\Theta|| ||s|| + (\hat{c} - c)||s|| + \gamma^2. 
\]

The combination of third, fourth and fourth terms of (41) take the maximum value of \((e\mu_5^2)/(4\alpha \hat{c})\) for \(||s|| = (e\mu_5)/(4\alpha \hat{c})\). Thus, \(\dot{V}(e) < 0\) would be achieved if \(\lambda_{\min}(\Psi)||e||^2 > 2\Gamma + (e\mu_5^2)/(4\alpha \hat{c})\). So, the system is UUB and the error bound is calculated to be
\[
||e|| = \sqrt{\frac{4\alpha \hat{c} + e\mu_5^2}{4\alpha \hat{c}\lambda_{\min}(\Psi)}} = \sigma_5 
\]
\( (\epsilon \mu_6) / 2\alpha \dot{c} \). Thus, \( \dot{V}(e) < 0 \) would be achieved if 
\( \lambda_{\text{min}}(\Psi)||e||^2 > 2\Gamma + (\epsilon \mu_5^2) / 2\alpha \dot{c} + 2\gamma^2 \). So, the 
system is UUB and the error bound is defined as

\[
||e|| = \sqrt{\frac{4\alpha \dot{c}(\Gamma + \gamma^2) + \epsilon \mu_5^2}{2\alpha \dot{c}\lambda_{\text{min}}(\Psi)}} = \sigma_6 \tag{42}
\]

Let \( \mathcal{S} \) denote the smallest level surface of \( V \) containing
the ball \( B_{\sigma_i}, i = 1, \ldots, 6 \) with radius \( \sigma_i \) centred at 
\( e = 0 \). For initial time \( t_0 \), if \( e(t_0) \in \mathcal{S} \) then the solution
remains in \( \mathcal{S} \). If \( e(t_0) \notin \mathcal{S} \), then \( V \) decreases as long as 
\( e(t) \notin \mathcal{S} \). The time required to reach \( \sigma_i \) is zero when 
\( e(t_0) \in \mathcal{S} \); otherwise, \( e(t_0) \notin \mathcal{S} \) the finite time \( t_{r_i} \) to
reach the error bounds \( \sigma_i \) can be computed as [46],

\[
t_{r_i} - t_0 \leq (V(||e(t_0)||) - V(\sigma_i))/c_0, \tag{43}
\]

where \( \dot{V}(t) \leq -c_0 \) for some \( c_0 > 0 \).

\( \square \)

Remark 3  The performance of AROLC can be charac-
terized by the various error bounds \( \sigma_i, i = 1, \ldots, 6 \)
under various conditions. The scalars \( \sigma_i \)'s are function
of \( \alpha \) and delay \( h \). It can be seen that as \( \alpha \) increases
and \( h \) drops, better tracking accuracy can be achieved.
However, too large \( \alpha \) may result in high control input.
Also, one may choose different values of \( \alpha \) for \( s^T \bar{s} \leq 0 \)
and \( s^T \bar{s} > 0 \). Again, it is to be noticed that instead of
\( s^T \bar{s} \) user can select any suitable error function while
keeping the same stability notion.

Comparison with the existing results

- It can be observed from (18) that the proposed sta-
bility approach is independent of the rate of change
in \( h \) and thus can negotiate any arbitrarily time-
varying delay within the stipulated maximum delay
bound. On the contrary, [35,36] can negotiate only
slowly varying time delay and also bound of \( \bar{h} \) is
required.
- Computation of switching gain \( \dot{c} \) of AROLC using
(11), unlike [34–38], does not require predefined
bound of the uncertainties and helps to attain the
tracking objective. Accomplishment of this objective
in turn reduces the tedious modelling effort of
complex nonlinear systems. To illustrate the fact
with an example, \( \hat{N} \) does not need to include fric-
tion, slip, skid, etc. for wheeled mobile robot and
these terms can be treated as uncertainties. Further,
through the adaptive law (11), \( \dot{c} \) avoids a monoton-
ically increasing nature (compared to [39,40])
and alleviates the overestimation–underestimation
problem (compared to [42,43]).

3 Application: nonholonomic WMR

Nonholonomic WMR provides a unique platform to
test the proposed control law since under practical cir-
stances; a WMR is always subjected to uncertainties
like friction, slip, skid, etc. These terms are difficult
to model, and in many cases they are not considered
while modelling. The dynamic equation of a WMR
after solving the Lagrange multiplier can be written
as follows [47],

\[
\tilde{M}(q)\ddot{q} + \tilde{V}(\dot{q}, \dot{q}) = \tilde{G}u, \tag{44}
\]

where

\[
\tilde{M} = \begin{bmatrix}
m & 0 & K \sin \phi & k_1 & k_2 \\
0 & m & -K \cos \phi & k_3 & k_4 \\
k_1 & k_3 & -k_5 & I_u & 0 \\
k_2 & k_4 & k_5 & 0 & I_u
\end{bmatrix},
\]

\[
\tilde{G} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0
\end{bmatrix},
\]

\[
\tilde{V} = \begin{bmatrix}
md\dot{\phi}^2 \cos \phi + m\ddot{r} \sin \phi (\dot{\theta}_2^2 - \dot{\theta}_1^2) / 2b \\
md\dot{\phi}^2 \cos \phi - m\ddot{r} \sin \phi (\dot{\theta}_1^2 - \dot{\theta}_2^2) / 2b \\
K\dot{r}^2 (\dot{\theta}_1^2 - \dot{\theta}_2^2) / 2b \\
-K\dot{r}\dot{\phi}^2 / 2 \\
-K\dot{r}\dot{\phi}^2 / 2
\end{bmatrix}
\]

\[
k_1 = \sin \phi (md\ddot{r} - K\dot{r})/b - m\ddot{r} \cos \phi / 2,
\]

\[
k_2 = \sin \phi (K\ddot{r} - md\ddot{r})/b - m\ddot{r} \cos \phi / 2,
\]

\[
k_3 = \cos \phi (K\ddot{r} - md\ddot{r})/b - m\ddot{r} \sin \phi / 2,
\]

\[
k_4 = \cos \phi (md\ddot{r} - K\dot{r})/b - m\ddot{r} \sin \phi / 2,
\]

\[
k_5 = \ddot{r}(I - Kd)/b,
\]

Here, \( q \in \mathbb{R}^5 = \{x_c, y_c, \phi, \theta_r, \theta_t\} \) is the
generalized coordinate vector of the system. The position of
the WMR can be specified by three generalized coordinates
\((x_c, y_c, \phi)\) where \((x_c, y_c)\) are the coordinates of the cen-
tre of mass of the system and \( \phi \) is the heading angle.
\((\theta_r, \theta_t)\) and \((u_r, u_t)\) are rotation and torque inputs of the
right and left wheels, respectively. $m$ and $\bar{I}$ represent the mass and inertia of the overall system. Definition of other system parameters is detailed in [47].

3.1 Experimental results and comparison

AROLC is employed in “PIONEER-3” WMR while the robot is directed to track the following circular path:

$$x_c^d = 1.25 \sin(0.35t) + 0.1,$$
$$y_c^d = 1.25 \cos(0.35t) + 1.35,$$
$$\phi^d = 0.25t, \quad \theta_r^d = 3t, \quad \theta_l^d = 2t.$$

The control architecture of the proposed AROLC is depicted in Fig. 1. Tracking performance of AROLC is compared with the predictive controller (PCON) reported in [35]. For a choice of $K_1 = K_2 = Q = 1, \beta = 1, r = 1.1$, the maximum allowable delay is found to be $h_m = 125$ ms. Other parameters to design AROLC are defined as $\alpha = 2, \epsilon = 0.1, \gamma = 0.001$. The following time-varying input delay was induced into the system for both the controllers:

- **S1**: $h(t) = 20 + 80 \abs{\sin(t)}$ ms.
- **S2**: $h(t) = 5 + 120 \abs{\sin(0.1t)}$ ms.

The input delay is envisaged by halting the programme for $h(t)$ amount of time in the VC++ programming environment between the control input computation and feeding it to the system. Again, to create a dynamic payload variation, a further 3.5 kg payload is added and kept for 5 s and then removed. This process is carried out for the entire duration of experimentation. A time gap 5 s is maintained between two successive instances of addition of the payload. However, the payload was added randomly at different places on the robotic platform every time to create dynamic variation in centre of mass and inertia.

The trajectory tracking performance between AROLC and PCON is depicted in Fig. 2 for the condition **S1**. The corresponding $x_c$ and $y_c$ position error comparison is illustrated through Figs. 3 and 4, respectively. All the error plots in this paper are in absolute value. In the situation **S2**, Figs. 5 and 6 depict the $x_c$ and $y_c$ position error comparison, respectively. Due to the robustness property against the unmodelled dynamics, AROLC provides better accuracy over PCON through its switching control logic. This can easily be comprehended from these error plots. To infer the performance of the individual controllers, absolute average
Adaptive robust tracking control of a class of nonlinear systems

Fig. 3 $x_c$ position error comparison for input delay S1

Fig. 4 $y_c$ position error comparison for input delay S1

Fig. 5 $x_c$ position error comparison for input delay S2

Fig. 6 $y_c$ position error comparison for input delay S2

e in $x_c$ (AE-$x_c$) and $y_c$ position (AE-$y_c$) is provided in Table 1 for S1 and S2. The percentage error is calculated with respect to the diameter of the circular path. The tabulated data further establish the superior performance of AROLC over PCON.

The performance of AROLC is also tested for the following two cases when the input delay is fixed and compared with the predictive controller reported in [34] (denoted here as PCONf):

- S3: $h(t) = 60$ ms.
- S4: $h(t) = 120$ ms.

The comparative performance of AROLC and PCONf is provided in Table 2 for S3 and S4. Superior performance of the proposed controller over PCONf is clearly evident from the tabulated data. However, tracking accuracy of AROLC degrades as $h$ increases and this is commensurate with the fact that the error bands for AROLC increases with high input delay. The total variation (TV), a measure of smoothness of input, is denoted as [48],

$$TV = \frac{1}{n} \sum_{i=1}^{n-1} |u_r(i + 1) - u_r(i)| + |u_l(i + 1) - u_l(i)|$$

where $n$ is the length of the samples in $u_r$ and $u_l$ accumulated during experimentation. High value of TV denotes excessive usage of control input [48]. TV for the three controllers under various cases is provided in Table 3. It can be noticed that fixed time delay resulted in more control input requirement than the time-varying delay for all the controllers. However, AROLC consumed the least control input for all the cases, which further augment its superior performance compared to PCON and PCONf. Some boxes in Table 3 are left blank since those particular controllers were not used for the corresponding conditions.
### Table 1 $x_c$ and $y_c$ position error (mm) comparison for S1 and S2

| Delay (ms) | Controller          | AE-$x_c$ | % AE-$x_c$ | AE-$y_c$ | % AE-$y_c$ |
|------------|---------------------|----------|------------|----------|------------|
| S1         | PCON [35]           | 58.30    | 2.33       | 53.09    | 2.12       |
|            | AROLC (proposed)    | 23.33    | 0.93       | 22.92    | 0.92       |
| S2         | PCON [35]           | 78.46    | 3.14       | 81.37    | 3.25       |
|            | AROLC (proposed)    | 36.42    | 1.46       | 39.66    | 1.59       |

### Table 2 $x_c$ and $y_c$ position error (mm) comparison for S3 and S4

| Delay (ms) | Controller          | AE-$x_c$ | % AE-$x_c$ | AE-$y_c$ | % AE-$y_c$ |
|------------|---------------------|----------|------------|----------|------------|
| S3         | PCONF [34]          | 99.63    | 3.99       | 85.79    | 3.43       |
|            | AROLC (proposed)    | 45.66    | 1.82       | 51.84    | 2.07       |
| S4         | PCONF [34]          | 136.91   | 5.48       | 102.19   | 4.79       |
|            | AROLC (proposed)    | 82.29    | 3.29       | 69.05    | 2.76       |

### Table 3 TV of controllers for S1, S2, S3 S4

| Controller           | S1   | S2   | S3   | S4   |
|----------------------|------|------|------|------|
| PCONF [34]           | –    | –    | 8.74 | 10.73|
| PCON [35]            | 5.45 | 6.39 | –    | –    |
| AROLC (proposed)     | 0.42 | 0.55 | 2.23 | 3.36 |

### Conclusion and future scope

In this paper, the tracking problem for a class of uncertain nonlinear systems in the presence of input delay is solved. The proposed delay-dependent control law is insensitive towards the variation in the delay and can negotiate any delay within the stipulated maximum delay. The maximum sustainable delay is determined by utilizing the Razumikhin approach. Furthermore, through its novel adaptive law, AROLC is able to provide robustness against the parametric and unmodelled uncertainties without any prior knowledge of their bounds. Experimental results, using a nonholonomic wheeled mobile robot, validates the superiority of the proposed controller compared to the predictor-based control approach.

However, the overall uncertainties of Euler-Lagrange systems have exclusive dependence on states. So, prior bound assumption is restrictive in nature and removal of such assumption paves the way for an exciting future work.

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