Enhancement of 3D Mass and Heat Transfer within a Porous Ceramic Exchanger by Flow-Induced Vibration

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ABSTRACT: This work investigates heat transfer enhancement for a porous ceramic heat exchanger. The effect of flow-induced vibration of exchanging air flow through porous tube banks has been tested. A numerical model able to assess the vibration effect on heat and mass transfer inside a porous ceramic exchanger has been carefully developed. A three-dimensional unstructured control volume finite element method (CVFEM) is developed to simulate the transport phenomena that arise during convective exchange. In this respect, several numerical tests have been conducted. The time evolution of temperature, liquid saturation, and pressure of the porous domain are analyzed and compared for two cases: with and without vibration. It is found that the vibration highly enhances the heat and mass transfer inside the ceramic exchanger. As a result, the gain of exchanging time to reach the thermal equilibrium between the hot air and the porous domain was 75% for the case of air vibration under sawtooth type at a frequency of $f_v = 5$ Hz and $V_{max} = 10$ m/s compared to nonvibrating exchange.

1. INTRODUCTION

Heat transfer advancements in heat exchanger devices such as heat exchangers have received a lot of attention. One of the promising techniques is the use of porous media that can highly improve heat and mass transport in energy systems. Therefore, porous media have been utilized for a variety of industrial purposes, but there is still room to investigate new uses for porous media, particularly in heat exchangers. In this context, several researchers have studied the heat and mass transfer inside an exchanger. Sun et al. have explained the theoretical and the experimental study on heat and mass transfer of a porous ceramic tube. Also, Tuz and Lebed have studied the heat and mass transfer during adiabatic fluid boiling in channels of contact exchangers. Very recently, the heat and mass transfer of an evaporative cooler with elliptic tube heat exchangers were experimentally studied by Chien et al. Wei et al. have developed an experimental and numerical study on heat and mass transfer characteristics in a direct-contact total heat exchanger for flue gas heat recovery.

In the last years, the intensification of heat and mass transfer in porous media has attracted considerable research. This interest has been motivated by many industrial applications (food, wood, paper, ceramic).

In this context, considerable investigations have used more efficient technologies (the ultrasonic and the acoustic techniques). Recently, the development of the vibration method in porous media has been an interesting subject in a great number of fields and has been found to enhance the heat and mass transfer process. Moreover, Rzig et al. have developed a numerical model to simulate the vibration effects on heat and mass transfer during the drying process. Very recently, Ben Khedher et al. have studied a numerical comparison of triangular and sinusoidal external vibration effects on the 3D porous drying process.

The usage of standard heat exchanger materials must be reconsidered if concentrated solar power heat exchangers are to be effective. Since it is unknown if the materials can withstand the high temperatures required for good molten salt–CO$_2$ heat exchange. Ceramics and ceramic composites are showing considerable promise in terms of effectively transferring heat in such a harsh environment. Some ceramic materials have features that make them ideal for heat exchangers. However, the mechanism that arises during employing porous ceramic heat exchangers is the coupled heat and mass transfer. Referring to the theory of Luikov and later of Whitaker, a numerical mathematical model for transport in porous media has been developed with dynamic content variables (temperature, saturation, pressure). The finite difference method has been widely used in the simulation of transport in porous media.
Moreover, the analytical method\(^5\) and the computational fluid dynamics method\(^19\) were studied for the comprehensive parameter study. Recently, the control volume finite element method (CVFEM) has been developed.\(^{19–21}\) The broad literature review shows that the unstructured mesh control volume methods have been used to simulate the physics of transport (heat and mass) in porous media.\(^{22}\)

The aim of the present research is to study a numerical model that can explain the vibration effects on heat and mass transfer inside a ceramic exchanger. In this context, a 3D numerical model is employed to describe the transport phenomena from two cases: classical and vibrating heat and mass exchange.

Several numerical results are presented and explained.

2. MATHEMATICAL MODELING

The problem is an industrial problem treating the heat and mass transfer inside a ceramic exchanger. The geometrical configuration of the heat exchanger and the relative dimensions are shown in Figure 1.

![Ceramic exchanger dimensions](https://doi.org/10.1021/acsomega.2c00907)

The considered exchanger is assumed as a porous medium composed of:
- Inert and rigid solid phase.
- Liquid phase.
- Gaseous phase.

2.1. Governing Equations. Referring to the Whitaker theory,\(^2\) a mathematical model of heat and mass transfer is created for the unsaturated porous media.

To have a closed set of governing macroscopic equations, the following assumptions were made:
- The porous medium is homogeneous and isotropic.
- The solid, liquid, and gaseous phases are in local thermodynamic equilibrium.
- The viscous dissipation and compression work are negligible.
- In the thermodynamic sense, the gaseous phase is ideal.
- The dispersion and tortuosity terms are used as diffusion terms.
- The radiative heat transfer is negligible.

Based on these assumptions and two other refs 10 and 11, the macroscopic equations governing heat and mass transfer in porous media are as follows:

### 2.1.1. Generalized Darcy’s Law

Using Darcy’s law, the average velocities of the liquid phase (\(\bar{v}_l\)) and the gaseous phase (\(\bar{v}_g\)) are formulated as given below

#### 2.1.1.1. Liquid Phase

\[
\bar{v}_l = -\frac{K_{Kl}}{\mu_l} [\nabla (P_g^s - P) - P_{1l}^s] \tag{1}
\]

where \(P_{1l}^s\) is the capillary pressure.

#### 2.1.1.2. Gaseous Phase (without Applying the Gravitational Effect)

\[
\bar{v}_g = -\frac{KK}{\mu_g} [\nabla P_g^s] \tag{2}
\]

#### 2.1.2. Mass Conservation

##### 2.1.2.1. Liquid Phase

The mass conservation equation of the liquid phase is (liquid density is constant) given as follows:

\[
\frac{\partial \bar{v}_l}{\partial t} + \nabla (\bar{v}_l \bar{V}_l) = -\frac{m_i}{\rho_l} \tag{3}
\]

where \(m_i\) is the mass rate of evaporation and \(\bar{V}_l\) is the volume fraction of the liquid phase.

##### 2.1.2.2. Gaseous Phase

The mass conservation equation of the gaseous phase is

\[
\frac{\partial \bar{v}_g}{\partial t} + \nabla (\bar{v}_g \bar{V}_g) = m_v \tag{5}
\]

\[
\bar{p}_g^s \bar{V}_v = \bar{p}_g^s \bar{V}_g - \bar{p}_g^s D_{eff} \nabla \left( \frac{\bar{v}_g}{\bar{p}_g^s} \right) \tag{6}
\]

where \(D_{eff}\) is the coefficient of the effective diffusion of vapor in a porous medium (this coefficient takes into account the resistance to the diffusion due to the effects of constriction and tortuosity).

##### 2.1.3. Energy Conservation

\[
\frac{\partial}{\partial t} (\rho C_p \bar{T}) + \nabla \left( \bar{v}_l P_l C_p \bar{T} + \sum_{k=1}^{n} \bar{p}_k^s C_p \bar{V}_k \right) = \nabla (\lambda_{eff} \nabla \bar{T}) - \Delta H_{vap} m_v \tag{7}
\]

where \(\Delta H_{vap}\) is the latent heat of vaporization at temperature \(T\) (K), \(\lambda_{eff}\) is the effective thermal conductivity of a porous medium, and \(\rho C_p\) is the heat capacity of the porous medium given by

\[
\bar{rho} C_p = \bar{rho}_{Cs} C_{ps} + \bar{rho}_{Cl} C_{pl} + \bar{rho}_{Cv} C_{pv} \tag{8}
\]

where \(\bar{rho}_{Cs}, \bar{rho}_{Cl}, \bar{rho}_{Cv}\) are the mass heat capacities of the solid, liquid, vapor, and air.

### 2.1.4. Thermodynamic Relations

The vapor partial pressure is
$P_v = P_{veq}(T, S)$

where $S$ is the liquid saturation explained by

$$S = \frac{\rho_l}{\rho_l + \rho_g}$$  \hspace{1cm} (9)

The gaseous phase (ideal mixture of perfect gases)

$$\bar{P}_i = \frac{\rho_i}{M_i}RT_i; \ i = a, v$$

$$\bar{P}_g = \bar{P}_i + \bar{P}_v = \bar{P}_i + \bar{P}_v$$  \hspace{1cm} (10)

The vapor pressure is defined by

$$\frac{P_v}{P_{vs}} = \exp \left(-\frac{2 \cdot \sigma \cdot M_v}{r \cdot \rho_l \cdot R \cdot T}\right)$$  \hspace{1cm} (11)

2.2. Boundary Conditions. Initially, the temperature, the liquid saturation, and the gaseous pressure are uniform in the ceramic exchanger (Figure 2). The liquid saturation inside the porous domain initially equals 50%. Similarly, the temperature of the porous domain initially equals 20 °C.

On the exchanging faces (only red holes, as depicted in Figure 2)

$$[\lambda_{eff} \frac{\partial (T)}{\partial X_i} + \Delta H_{cap} \rho_l \langle V_i \rangle n_i] = h_l(\langle T \rangle - T_{\infty})$$  \hspace{1cm} (12)

The mass flow (evaporation and evacuation of water)

$$[\rho_l(V_i) + \rho_g \langle V_g \rangle n_i = h_m(C_{vs} - C_{v_{\infty}})$$  \hspace{1cm} (13)

The gaseous pressure on exchanging face is defined as the atmospheric pressure

$$[\bar{P}_g] = P_{atm}$$  \hspace{1cm} (14)

The other faces (right, left, up, bottom, and yellow holes) are adiabatic and impermeable faces.

$$[\lambda_{eff} \frac{\partial (T)}{\partial X_i} + \Delta H_{cap} \rho_l \langle V_i \rangle n_i] = 0$$  \hspace{1cm} (15)

$$[\rho_l(V_i) + \rho_g \langle V_g \rangle n_i = 0$$  \hspace{1cm} (16)

$$\left[\frac{\partial \bar{P}_g}{\partial X_i}\right] = 0$$  \hspace{1cm} (17)

The convective heat and mass transfer coefficients are explained in Table 1.

| Table 1. Convective Coefficients$^a$ |
|--------------------------------------|
|                        | circular holes$^{23}$ | planar faces$^{24}$ |
| $h_l$      | $\frac{\lambda \times 0.023 \times R^4 \times \rho_l^{4/5}}{D}$ | $\frac{\lambda \times 0.023 \times R^4 \times \rho_l^{4/5}}{L}$ |
| $h_m$      | $\frac{D_{AB} \times 0.023 \times R^4 \times \rho_l^{4/5}}{D}$ | $\frac{D_{AB} \times 0.023 \times R^4 \times \rho_l^{4/5}}{L}$ |
| Validity   | $10^4 < Re < 1.0 \times 10^5$ | $Re < 5 \times 10^5$ |
| $0.6 \leq Pr \leq 160$ | $Pr \geq 0.6$ |
| $0.6 \leq Sc \leq 160$ | $Sc \geq 0.6$ |
| $Re$       | $\frac{\rho_l \nu}{\rho_v}$ | $\frac{\rho_l \nu}{\rho_v}$ |
| $Pr = \frac{\rho_l \nu}{\rho_v}$ | $Sc = \frac{\rho_l \nu}{\rho_v}$ |

$^a$Where $h_l$ is the convective heat transfer coefficient (W/m² ºC), $h_m$ is the convective mass transfer coefficient (m/s), $D_{AB}$ is the vapor diffusion in air defined by $D_{AB} = D_{vap,air} = 0.26 \times 10^{-4}$ (m²/s), $D = 0.01$ m is the diameter of the holes, and $L_C = 0.02$ m is the characteristic length of the exchanger.

3. SOLUTION, GRID TEST, AND VALIDATION

The equation system has been solved with the control volume finite element method (CVFEM).25,26 This method ensures flux conservation. Also, the control volume is composed of triangular elements that improve the grid flexibility.

For the mesh generation, we use the free mesh generator Gmsh.27 The exchanger domain is divided into six-node prisms. Also, the centroids of the triangular elements (prism’s base) are joined to the midpoints of the corresponding sides (Figure 3).

Then, we developed a Fortran program. The coupled nonlinear equations are solved using the iterative BICG-STAB solver. The convergence criteria for continuity, momentum, and energy equations were set at $10^{-6}$.

3.1. Grid Dependency. Knowing the importance of the grid size in the accuracy of the numerical results, grid independency tests were carefully carried out. For this purpose, for four grid different sizes (Table 2), the corresponding liquid saturation and the temperature of the porous medium are compared under the same boundary conditions, as shown in Figures 4 and 5.
Three other time-step sizes were also tested, with the results shown in Figure 6. The mesh set \( (N = 37,850) \) and the time step \( (t = 0.5 \, \text{s}) \) were chosen to carry out all of the simulations in this research to retain an acceptable degree of accuracy while minimizing computing costs as much as possible.

### 3.2. Validation

To verify the accuracy and correctness of the results, validation of the results by former research is a vital step. For the experimental validation, the coupled heat and mass transfer model is validated by a convective drying experiment elaborated by Chemkhi.\(^{28}\)

The used conditions are listed in Table 3.

#### Table 3. First Validation Case Conditions

| Parameter       | Value 1 | Value 2 |
|-----------------|---------|---------|
| \( T_{\text{amb}} \) (°C) | 40      | 50      |
| \( T_{\text{ini}} \) (°C) | 25      |         |
| initial water content (kg/kg b-s) | 0.1995  |         |
| \( C_{\text{vamb}} \) | 0.01    |         |
| \( P_{\text{amb}} \) (atm) | 1       |         |
| relative humidity (%) | 40      |         |
| air velocity (m/s) | 2       |         |

In Figure 7, we present the time evolution of water content. Comparing both experimental data to the current model findings, it can be seen that there is an overall good agreement.

The research of Hussain et al.\(^{29}\) is the second case of validation. The authors investigated experimentally the heat and mass transfer through porous tubular ceramic membranes (porous aluminum oxide). Combined heat and mass transfer are integrated into this analysis, aiming to give insight into how mass transfer is influenced by temperature distribution and heat transfer through porous membranes. In this study, experiments have been carried out with tubular ceramic membranes, with a length of \( L = 250 \, \text{mm} \), an inner radius of \( r_{m,i} = 10.5 \, \text{mm} \), and an outer radius of \( r_{m,o} = 16 \, \text{mm} \).

In these experiments, the inlet flow velocity of the gas in the annulus ranged \( u_{\text{in}} = 0.44 \, \text{m/s} \) and the imposed heat flux corresponds to the heat flow rate of \( Q = 50 \, \text{W} \).

With the excellent agreement shown in Figure 8, it can be concluded that the model built to simulate mass and heat transfer inside porous exchangers is sufficiently accurate in its predictions and reliable enough to be implemented in this study.

### 4. RESULTS AND DISCUSSION

The numerical results obtained from the compilation of numerical codes of heat and mass transfer inside the ceramic porous exchanger are explained in this section.

The numerical model is a comparison of transfer inside the ceramic exchanger between two modes of transfer (hot air velocity)

#### 4.1. Classical Transfer (without Vibration)
4.2. Vibrating Transfer of Sawtooth Type.

\[ V_a = 10 \text{(m/s)} \]  \hspace{1cm} (18)

\[ V_a = a \times t + b \text{(m/s)} \]

\[ a = \frac{3 \times A_v}{T_v}, \quad b = 0; \quad 0 \leq t \leq \frac{T_v}{3} \]

with:

\[ a = \frac{-6 \times A_v}{T_v}, b = 3 \times A_v, \quad \frac{T_v}{3} \leq t \leq 2 \times \frac{T_v}{3} \]

\[ a = \frac{3 \times A_v}{T_v}, b = -3 \times A_v, \quad \frac{2 \times T_v}{3} \leq t \leq T_v \]  \hspace{1cm} (19)

where \( A_v (A_v = 10 \text{ m/s}) \) is the maximum amplitude of vibrating velocity, \( T_v (T_v = 0.2 s) \) is the period of vibrating velocity, \( f_v (f_v = 5 \text{ Hz}) \) is the frequency of vibrating velocity, \( t \) is the time, and \( a \) is the acceleration.

For the vibrating transfer, the periodic evolution of hot air velocity is presented in Figure 9.

To simulate the three-dimensional heat and mass transfer inside the ceramic exchanger, we use the free mesh generator Gmsh to achieve 3D meshing (Figure 10).

The operating conditions used in this numerical model are defined in Table 4.

Figures 11–13 show the evolution of temperature, liquid saturation, and gaseous pressure for two modes of transfer (classical and vibrating modes).

Based on Figure 11, we note that the temperature evolution of the vibrating mode is more rapid than the classical mode of transfer inside the ceramic exchanger. Therefore, the vibrating transfer is more intense, and it is clearly noticeable during decreasing rate phase that is characterized by evaporation inside the environment and diffusion to the surface.

In Figure 12, we observe that the vibrating transfer has a rapid decrease in liquid saturation.

From Figure 13, we can notice that the vibrating heat transfer is more intense and rapid compared to the classical process, which took a long time. Since a longer time period of gaseous pressure is noticed for the case of vibration.

Figure 14 presents a comparison of water content evolution (classical and vibrating process). As transfer progresses, the water content decreases in time until it approaches the zero value, and this decrease is more rapid for the vibrating mode.

To better understand the three-dimensional heat and mass transfer inside the vibrating process of the ceramic exchanger, we have shown in Figures 15–17 the spatiotemporal evolutions of...
temperature, liquid saturation, and pressure inside the porous exchanger for three times of transfer:

- 30 min
- 1 h
- 2 h

Referring to these results, we can note that:

- The temperature gradient supplies the energy necessary for evaporation. Referring to the migration of liquid at the exchanging faces, the vibrating process forces the temperature profile to be more rapid and intense. Hence, the heat exchange is enhanced.
- The presence of vibrating flow favors the migration of liquid at the exchanging faces with high quantity and to be extracted faster. Due to the vibration, the liquid fraction of water existing in the porous heat exchanger is rapidly lowered.
- The gaseous pressure resulted from the cross-diffusion effect of vapor and air, and it is clear that the vibration encourages the pressure profile to be more intense, especially near the exchanging faces. The gaseous pressure
plays an important role to extract embedded water in the porous structure of the heat exchanger.

At this level, to exhibit clearly the 3D vibration effect on heat and mass transfer inside the ceramic exchanger, it is interesting to choose the sectional view to present the evolution of temperature, liquid saturation, and pressure during the exchange process (Figure 18).

After 2 h of transfer inside the ceramic exchanger, the heat and mass transfer process are well established. Consequently, the vibrating flow of hot air obligates the transfer process to be faster than the classical mode. Moreover, given the fact of the vibration, it is clear that the vibration favors the exchange inside the exchanger and forces the heat and mass transfer phenomena to be more rapid and intense. Also, the temperature profile is more intense, and the pressure evolution is intensively observed inside the ceramic exchanger. This is due to the vibration, which encourages liquid evaporation to occur inside the porous ceramic. In addition, the liquid saturation decreases gradually from the adiabatic faces and the center toward the exchanging faces, and this decrease is more rapid for the vibrating mode.

5. CONCLUSIONS

A 3D numerical model of heat and mass transfer inside a porous ceramic exchanger for two modes of transfer (classical and flow-induced vibration) has been established. This numerical model highlights the importance of vibration phenomena in transfer inside the porous heat exchanger. The mesh is generated by the

Figure 16. Distribution of temperature, liquid saturation, and pressure after 1 h of the vibrating process.

Figure 17. Distribution of temperature, liquid saturation, and pressure after 2 h of the vibrating process.
free mesh generator Gmsh, and a numerical code has been developed for three-dimensional simulation of the vibrating transfer in the ceramic exchanger.

The numerical model successfully depicts the effect of the flow-induced vibration of hot air on the heat and mass transfer rate within a porous ceramic exchanger. As a result, the heat exchange becomes more rapid and intense with vibration compared to passive and classical heat exchange. Moreover, the gain of exchanging time to reach the thermal equilibrium between the hot air and the porous domain was 75% for the case of air vibration under sawtooth type at a frequency of $f_v = 5$ Hz and $V_{\text{max}} = 10$ m/s compared to nonvibrating exchange. The findings of this study can be useful for the design of innovative heat exchangers that are highly improved by the vibration effect.

Compared to previous works, this study presents a detailed contribution to the 3D mass and heat transfer using the porous ceramic heat exchanger in the presence of induced vibration flow.

**APPENDIX**

The physical characteristics of a ceramic exchanger and air used in the simulation (Table A1) are as follows:

### Table A1. Ceramic Porous Exchanger

| material properties | values used for the computations |
|---------------------|----------------------------------|
| porosity            | 0.36                             |
| density of solid matrix | $2.6 \times 10^3$ kg/m$^3$ |
| specific heat capacity of solid matrix | $8.79 \times 10^3$ J/kg K |
| specific heat capacity of liquid | $4.22 \times 10^3$ J/kg K |
| specific heat capacity of vapor | $1.86 \times 10^3$ J/kg K |
| liquid dynamic viscosity | $6.5 \times 10^{-4}$ kg/ms |
| vapor dynamic viscosity | $1.86 \times 10^{-5}$ kg/ms |
| intrinsic liquid permeability | $2.5 \times 10^{-14}$ m$^2$ |
| relative liquid permeability | $K_l = X_\ast$ with $X_\ast = X_l - X_{\text{ir}}$ and $X_{\text{ir}} = 0.3$ |
| relative gaseous permeability | $K_{rg} = 1 + (2X_\ast - 3)X_\ast^2$ |
| capillary pressure | $P_c = (\varepsilon/K)^{1/2}\sigma(T)$ with $J(s) = 0.364[1 - \exp(-40(1 - s))] + 0.221(1 - s) + \frac{0.05}{0.07}$ and $\sigma(T) = (121.2 - 0.167T) \times 10^{-3}$ |
| vapor pressure | $P_v = \langle \sigma(T) \rangle$ with $\log_{10}(r) = 2.16 \times 10^2 + 43.85 = 253.5 \times 10^2 + 794.54 \times 10^2 - 1337.1 \times 10^2 + 1111 \times 10^2 - 352.5 \times 10^2$ |
| thermal conductivity | $\lambda_{\text{eff}} = (\lambda_g \ast (\varepsilon - \varepsilon_l) + \lambda_l \ast (1 - \varepsilon_l))^{1/4}$ (W/mK) with $a = 0.25$ |
| used fluid (Air) | $\rho = 1006$ J/kg K |
| specific heat capacity | $0.026$ W/mK |
| thermal conductivity | $1.17$ kg/m$^3$ |
| density | $1.15 \times 10^{-3}$ kg/ms |

**Figure 18.** Sectional view of temperature distribution, liquid saturation, and pressure after 2 h of the vibrating process.

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NOMENCLATURE

- $C_a$: specific heat of the air [kJ/kg K]
- $C_p$: specific heat at constant pressure [kJ/kg K]
- $C_v$: specific heat of the vapor [kJ/kg K]
- $C_w$: specific heat of the water [kJ/kg K]
- $D_{AB}$: diffusion coefficient [m$^2$/s]
- $D$: holes diameter [m]
- $g$: gravitational acceleration [m/s$^2$]
- $h_m$: convective mass transfer coefficient [m/s]
- $h_t$: convective heat transfer coefficient [W/m$^2$ °C]
- $K$: intrinsic permeability [m$^2$]
- $L_c$: characteristic length of brick [m]
- $M_a$: molar mass of air [kg/mol]
- $M_v$: molar mass of vapor [kg/mol]
- $m$: evaporation rate [kg/s]
- $P$: pressure [Pa]
- $P_c$: capillary pressure [Pa]
- $P_{vs}$: partial pressure of saturated vapor [Pa]
- $R$: gas constant [kJ/kmol K]
- $S$: liquid saturation [%]
- $T$: temperature [K]
- $t$: time [s]
- $w_a$: air velocity [m/s]

GREEK SYMBOLS

- $\epsilon$: porosity
- $\epsilon_l$: volume fraction of liquid phase
- $\mu$: dynamic viscosity [kg/ms]
- $\delta$: kinematic viscosity [m$^2$/s]
- $\rho$: density [kg/m$^3$]
- $\lambda$: conductive transfer coefficient [W/m °C]
- $\sigma$: surface tension [N/m]
- $\Delta H_{vap}$: vaporization latent heat [J/kg]

SUBSCRIPTS

- 0: initial condition

- air: air
- eff: effective
- g: gas
- l: liquid
- v: vapor
- vs: vapor saturated

DIMENSIONLESS GROUPS

- $Re$: Reynolds number
- $Pr$: Prandtl number
- $Sc$: Schmidt number

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