Mathematical Modelling and Operation Simulation of the Indirect Connection Scheme of the Heating Installation and of the Consumption Hot Water Preparation in an Accumulating Stage

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Abstract. This paper presents the mathematical model and simulation of a thermal system for heating and supplying hot water to industrial or residential consumers consisting of a heat exchanger on the heating circuit and a heat exchanger provided with an accumulation tank on the domestic hot water circuit, this scheme is generally adopted in the industrial thermal points and increasingly in module-type thermal points for residential consumers. The mathematical model is based on the mathematical equations describing this system and developed using the MATLAB - Simulink program. Thus, as a result of the simulations, we can see the evolutions in time of the water temperatures on the heating circuit and the domestic hot water circuit, as well as the quantity of heat delivered by them.

1. Introduction
The energy can be transmitted in the form of heat from a physical system that contains it to another physical system based on the difference between the temperature of the system that yields energy and the temperature of the system that receives energy [1, 2].

Worldwide, the energy is a product of great economic, social, political and strategic value. The heat supply plays an important role within the energy sector [3]. In an increasingly globalised economy, the energy strategy of a country is made in the context of developments and changes taking place globally. The total demand for energy in 2030 will be about 50% higher than in 2003, for example, and for oil, it will be about 46% higher. The proven known reserves of oil can support the current consumption level only until 2040, and the natural gas reserves by 2070, while the global coal reserves provide a period of over 200 years to even higher levels of exploration [2].

In this context, the optimisation of the heat supply systems is a topical field in the conditions in which the reduction of the costs of production, transport and transfer of thermal energy is made by optimization. Thus, many studies were conducted to solve functional problems and develop heat supply systems in a competitive market environment [3-10].
The heat exchangers are having a very important role in centralised heat supply systems, because they are thermal apparatus that make the heat exchange between a hot thermal agent, called primary heat agent, and a cooler thermal agent, called secondary heat agent.

The internal heating installations for the hot water transport and distribution are hydraulically separated through heat exchangers, and although expensive in terms of investment, they provide many advantages such as the possibility of modifying the operating parameters of the systems, temperatures, pressures.

Indirect connection schemes are used in most centralised heat supply systems, which have maximum safety in operation, perfectly adapt to the mixed adjustment of the heat supply and show elasticity in the variations of the operating modes.

From the point of view of the preparation of consumption hot water, there are often used the one stage preparation schemes in parallel with the heating installations with accumulation tanks, accumulation which reduces the unfavourable influence of the water flow variation imposed by consumption with large variations. There are many situations in which the consumption hot water shows large variations during the day when overlapping the effect of variation of the network water flow with the change of temperature on the departure pipe dictated by the climatic factors.

The need for rehabilitation has increasingly occurred over the last period of time within the large heat supply systems in all their compartments (sources, transport, distribution, thermal points, consumers). From this point of view, many of the centralised thermal points have been dismantled and replaced with thermal modules that feed a block entrance or a block. Typically, these thermal points are made with an indirect connection of the heating systems, and the scheme that is best suited for the hot water preparation is the one stage scheme, parallel to the heating and provided with the accumulation of hot water.

Figure 1 shows the scheme of a thermal system made up of heat exchanger used to heat the thermal agent and heat exchanger used to prepare the domestic hot water with a hot water accumulation tank.

**Figure 1.** Scheme of a system with indirect connection of heating and preparation with hot water accumulation.
2. Materials and Methods

The article deals with a mathematical model of a thermal system with indirect connection of heating and preparation with hot water accumulation.

The mathematical equations describing the mathematical model of a thermal point with indirect connection of the heating installation and the preparation of the consumption hot water in a stage in parallel with the heating system with accumulation are the following [8 - 10]:

\[ t_{RP, ACM} = \frac{(1-y) E_S}{1-y E_S} \cdot t_{TP} + \frac{(1-E_S)}{1-y E_S} \cdot t_{AC} \]  
(1)

\[ t_{TS, ACM} = \frac{(1-E_S) y}{1-y E_S} \cdot t_{TP} + \frac{(1-y)}{1-y E_S} \cdot t_{AC} \]  
(2)

\[ \frac{dt_{AC}}{d\tau} = \frac{g_{s, ACM} \cdot g_{cons}}{1-y E_S} \cdot t_{AC} + \frac{g_{s, ACM}}{1-y E_S} \cdot t_{TP} + \frac{g_{cons}}{v} \cdot t_{AR} \]  
(3)

\[ E_S = e^{-NTU_S(1-y)} \]  
(4)

\[ NTU_S = \frac{U \cdot A}{G_{P, ACM} \rho \cdot c} \]  
(5)

\[ y = \frac{g_{p, ACM}}{g_{s, ACM}} \]  
(6)

\[ t_{RP, INC} = \frac{(1-y_{INC}) E_{S, INC}}{1-y_{INC} E_{S, INC}} \cdot t_{TP} + \frac{1-E_{S, INC}}{1-y_{INC} E_{S, INC}} \cdot t_{RS} \]  
(7)

\[ t_{TS} = \frac{(1-E_{S, INC}) y_{INC}}{(1-E_{S, INC}) y_{INC} + (1-E_{I}) (1-y_{INC})} \cdot t_{TP} + \frac{(1-E_{I})(1-y_{INC})}{(1-E_{S, INC}) y_{INC} + (1-E_{I})(1-y_{INC})} \cdot t_{I} \]  
(8)

\[ E_{S, INC} = e^{-NTU_{S, INC}(1-y_{INC})} \]  
(9)

\[ NTU_{S, INC} = \frac{U_{S, INC} A_{S, INC}}{G_{P, INC} \rho \cdot c} \]  
(10)

\[ y_{INC} = \frac{g_{p, INC}}{g_{S}} \]  
(11)

\[ t_{RS} = E_I \cdot t_{TS} + (1 - E_I) \cdot t_{I} \]  
(12)

\[ E_I = e^{-NTU_I} \]  
(13)

\[ NTU_I = \frac{U_I \cdot A_I}{G_{S} \cdot \rho \cdot c} \]  
(14)

\[ t_{RP} = \frac{g_{p, INC}}{g_{P}} \cdot t_{RP, INC} + \frac{g_{p, ACM}}{g_{P}} \cdot t_{RP, ACM} \]  
(15)

\[ G_P = G_{P, INC} + G_{P, ACM} \]  
(16)

\[ Q_{INC} = G_{P, INC} \cdot \rho \cdot c \cdot (t_{TP} - t_{RP, INC}) = G_{S} \cdot \rho \cdot c \cdot (t_{TS} - t_{RS}) \]  
(17)

\[ Q_{ACM} = G_{CONS} \cdot \rho \cdot c \cdot (t_{AC} - t_{AR}) \]  
(18)
where:

\( t_{RP, ACM} \) – water temperature - primary return of the heat exchanger at domestic hot water [°C];
\( t_{AC} \) – hot water temperature [°C];
\( t_{RP, INC} \) – water temperature - primary return of the heat exchanger at heating [°C];
\( t_{RS} \) – water temperature - secondary return of the heat exchanger at heating [°C];
\( t_{TP} \) – water temperature - primary turn of the heat exchanger [°C];
\( t_{RP} \) – water temperature - primary return of the heat exchanger [°C];
\( t_{TS} \) – water temperature - secondary turn of the heat exchanger at heating [°C];
\( G_{P, ACM} \) – primary thermal agent flow to domestic hot water [m\(^3\)/s];
\( G_{cons} \) – flow of consumed domestic hot water [m\(^3\)/s];
\( G_{P, INC} \) – flow of primary thermal agent on heating [m\(^3\)/s];
\( G_{P, INC} \) – flow of secondary thermal agent at heating [m\(^3\)/s];
\( A \) – surface of the heat exchanger on domestic hot water [m\(^2\)];
\( A_{S, INC} \) – surface of the heat exchanger at heating [m\(^2\)];
\( A_I \) – surface of the heating bodies [m\(^2\)];
\( \rho \) - water density [kg/m\(^3\)];
\( c \) – water specific heat [J/kg·K];
\( U \) – global thermal transfer coefficient at domestic hot water [W/m\(^2\)-K];
\( U_{S, INC} \) – global thermal transfer coefficient at heating [W/m\(^2\)-K];
\( U_I \) – global thermal transfer coefficient of the heating bodies [W/m\(^2\)-K];
\( Q_{INC} \) – thermal power delivered to the heating installation [W];
\( Q_{ACM} \) – thermal power delivered to the consumer [W];
\( N_{TU} \) – number of heat transfer units [-];
\( \tau \) – time [s].

Starting from the above equations, the diagrams in figures 2, 3 and 4 were developed by which simulations can be made for various scenarios. For the different physical characteristics of heat exchangers and various temperatures and flows of primary and secondary thermal agent, it is possible to simulate the evolution in time of the temperatures of the thermal agents on the turn/return pipes in the
heating systems, the consumption hot water temperature but also the thermal powers delivered to consumers. The following have been defined in the diagrams: Subsystem 1 - consumption hot water preparation exchanger, Subsystem 2 - exchanger related to the heating circuit and Subsystem - consumption hot water accumulation tank.

The underlying assumption in the case of the accumulation tank is a uniform temperature in the tank. By the model taken into account, the temperature increase in the tank is infinitesimal, and the water temperature is variable in time depending on the tank stresses.

**Figure 2.** Simulink diagram for simulating a system for heating and preparing the consumption hot water with indirect connection of heating and preparation with hot water accumulation.

**Figure 3.** Defining the Subsystem and Subsystem 1 parameters: (a) Subsystem; (b) Subsystem 1
Figure 4. Simulink diagrams of Subsystem, Subsystem 1 and Subsystem 2: (a) Subsystem diagram; (b) Subsystem 2 diagram; (c) Subsystem 1 diagram.
3. Results and discussions

By using MATLAB, Simulink [11], which offers a special graphical interface for the achievement of the dynamic systems models represented in the block scheme, due to its extensive library, rapid and clear modelling of the indirect connection scheme with the preparation of the consumption hot water in an accumulating stage.

The set of initial values can be changed in the simulations that can be made, depending on the specific conditions of each case.

![Graphs showing water temperature evolution](image)

**Figure 5.** Evolution of water temperature in the simulated period: (a) primary turn of the heat exchanger, (b) hot water temperature, (c) internal temperature over the.

Following the 9-hour simulation, the evolution of the interest parameters for the studied system can be highlighted (Figures 6, 7 and 8).
Figure 6. The evolution in time of the hot water temperature, the temperature on the primary return of the heat exchanger and the temperature on the secondary circuit turn of the heat exchanger on the hot water circuit over the simulated period.

Figure 7. The evolution in time of the water temperature on the secondary turn of the heat exchanger, the temperature on the secondary return of the heat exchanger, the temperature on the primary return of the heat exchanger, all the heating circuit and the temperature on the primary return over the simulated period.

Figure 8. The evolution in time of the power delivered by the installation preparing the domestic hot water and the consumers’ heating installation.
The results of the system simulation can be even tracked in real time, with SIMULINK having advanced integration algorithms and analysis functions that provide quick and accurate simulation results. The following initial data were used in the simulations made: $U=3155 \text{ W/m}^2\cdot\text{K}$; $U_{\text{INC}}=3155 \text{ W/m}^2\cdot\text{K}$; $U_I=7 \text{ W/m}^2\cdot\text{K}$; $c=4180 \text{ J/kg\cdotK}$; $\rho=1000 \text{ kg/m}^3$; $A=100\cdot0.14 \text{ m}^2$; $A_{\text{INC}}=100\cdot0.14 \text{ m}^2$; $G_{\text{P\_ACM}}=1 \text{ m}^3/h$; $G_{\text{P\_INC}}=10 \text{ m}^3/h$; $G_{\text{cons}}=2 \text{ m}^3/h$; $G_S=7 \text{ m}^3/h$; $A_I=100\cdot1.51 \text{ m}^2$.

With regard to temperatures, a set of initial values for the water temperature on the primary turn of the heat exchanger $t_{\text{TP}}=90^\circ\text{C}$, the cold-water temperature $t_{\text{AR}}=10^\circ\text{C}$ and an internal temperature $t_i=23^\circ\text{C}$ (Figure 5) are considered. The $t_{\text{TS\_ACM}}$ curve shown in Figure 6, respects the initial set of values indicated in Figure 5 (a). Depending on these values and the variation of the cold water temperature $t_{\text{AR}}$ (Figure 5 b) the evolution of the temperature at the exit of the heat exchanger on the primary and secondary circuit is presented It is observed that between the primary and the secondary circuit, the temperature difference keeps the same simulation interval.

In the Figure 7, based on the variation of the temperature $t_{\text{TS}}$ of the secondary thermal agent on the heating supply circuit, shows the evolution of the temperature of the secondary thermal agent on the heating return circuit ($t_{\text{RS}}$) and the evolution of the temperature ($t_{\text{RP\_INC}}$) of the thermal agent on the output of the primary circuit of the heat exchanger. It is observed that all the curves have the same form keeping the same difference over the entire simulation period (9 hours).

From the point of view of the power delivered by the heat exchangers on the two installations (heating and preparation of hot water for domestic consumption) it is observed from Figure 8 that the heat exchanger for heating, based on the evolution of the temperature presented above, has a delivered power that oscillates around 65kW, while for domestic hot water consumption the delivered power has a variation of 9kW between the start and end time of the simulation period, oscillating in the range of 94-85kW.

4. Conclusions
The calculation program (figures 2, 3 and 4) developed by the authors of this article in MATLAB-Simulink [11], solves the mathematical equations (1÷18) describing the evolution in time of the water temperatures on the circuits of the heat exchangers used to prepare the hot water for heating and for preparing the domestic hot water and the thermal powers delivered by them. This program has immediate applicability and is useful in designing and operating the systems made up of such equipment. The evolutions of the primary and secondary thermal agent temperatures for indirect connection of the heating installations, the evolution of the thermal load supplied to the consumers over the time considered, but also the connection of the installations for the preparation of the consumption hot water can be obtained with the help of this program in the case of centralised heat supply systems. The simulations made can emphasise the evolution of temperatures and thermal loads supplied during the simulated period and for the conditions specific to each case.

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