Observational Constraints in Delta-gravity: CMB and Supernovae

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Abstract

Delta-gravity (DG) is a gravitational model based on an extension of general relativity given by a new symmetry called $\delta$. In this model, new matter fields are added to the original matter fields, motivated by the additional symmetry. We call them $\delta$ matter fields. This model predicts an accelerating universe without the need to introduce a cosmological constant. In this work, we study the scalar cosmic microwave background (CMB) temperature (TT) power spectrum predicted by DG using an analytical hydrodynamic approach. To fit the Planck satellite’s data with the DG model, we used a Markov Chain Monte Carlo analysis. We also include a study about the compatibility between Type Ia supernovae (SNe Ia) and CMB observations in the DG context. Finally, we obtain the scalar CMB TT power spectrum and the fitted parameters needed to explain both SN Ia data and CMB measurements. The results are in reasonable agreement with both observations considering the analytical approximation. We also discuss whether the Hubble constant and the accelerating universe are in concordance with the observational evidence in the DG context.

Unified Astronomy Thesaurus concepts: Dark energy (351); Cosmology (343); Cosmic microwave background radiation (322); Cosmological perturbation theory (341); Cosmological models (337)

1. Introduction

Cosmology is a very active area of study, where many observational data allow a better understanding of theoretical physics. The scientific community has evidence that most of the composition of the universe is unknown. This sector comprises two kinds of components called dark matter (DM) and dark energy (DE) (Riess et al. 1998; Perlmutter et al. 1999; Weinberg 2008; Caldwell & Kamionkowski 2009; Planck Collaboration et al. 2020). The DM was initially detected by Zwicky (1937), when he observed that some clusters were not principally made of stars or clusters of stars, but consist predominantly of matter that does not emit light. Then, Rubin et al. (1970, 1980) found that the DM is the principal component of galaxies in terms of mass. Today we know that DM dominates the galaxies (Beasley et al. 2016), and cosmological simulations such as those from Angulo et al. (2012), Vogelsberger et al. (2014, 2020), and Wang et al. (2020) show that DM plays an essential role as a source of the gravitational potential.

Regarding the DE, this is the main component of the universe, and it is strictly necessary to reproduce the universe’s acceleration in the standard cosmological model called $\Lambda$CDM (Riess et al. 1998; Perlmutter et al. 1999; Caldwell & Kamionkowski 2009; Planck Collaboration et al. 2020). Despite the observational evidence, the origin of the DE in the Einstein field equations or in the Einstein–Hilbert action is not clear (Martin 2012). In early times after the Big Bang, this constant is irrelevant, but at the later stages of the evolution of the universe, $\Lambda$ will dominate the expansion, explaining the acceleration. Such small $\Lambda$, which is commonly associated with the vacuum energy, is very difficult to generate in quantum field theory models because the predictions reach up to 120 orders of magnitude above the observed $\Lambda$ in cosmology (Frieman et al. 2008; Martin 2012). Moreover, in other attempts to obtain a better value, the result is about 54 orders of magnitude above the $\Lambda$ observed value (Martin 2012). This explanation is not satisfactory.

In the past decades, there have been various proposals to explain the observed acceleration of the universe. They involve the inclusion of some additional fields in approaches like quintessence, chameleon, vector DE, or massive gravity; the addition of higher-order terms in the Einstein–Hilbert action, like $f(R)$ theories and Gauss–Bonnet terms; and the introduction of extra dimensions for a modification of gravity on large scales (Tsujikawa 2010). Other interesting possibilities are the search for nontrivial ultraviolet fixed points in gravity (asymptotic safety; Weinberg 1979) and the notion of induced gravity (Zeldovich 1967; Sakharov 1968; Klein 1974; Adler 1982). The first possibility uses exact renormalization-group techniques (Litim 2004; Reuter & Saueressig 2010), together with lattice and numerical techniques such as Lorentzian triangulation analysis (Ambjørn et al. 2000). Induced gravity proposes that gravitation is a residual force produced by other interactions.

The cosmic microwave background (CMB) Planck data and its power spectrum provide important information to fit many cosmological parameters (Planck Collaboration et al. 2020). These cosmological fluctuations have been deeply studied and numerically solved in programs such as CMBFast (Seljak & Zaldarriaga 1996; Zaldarriaga et al. 1998) or CAMB (Lewis et al. 2000). From the CMB observations and the Type Ia supernova (SN Ia) data, the $\Lambda$CDM model indicates that the universe is composed of about 68% DE (Planck Collaboration et al. 2020).

The state of the art of cosmology is controversial. A measurement about the $H_0$ by Sorce et al. (2012) found a value of 75.2 ± 3.0 km Mpc$^{-1}$ s$^{-1}$. A few years later, Riess et al. (2016) found an observed value $H_0 = 73.24 ± 1.74$ km Mpc$^{-1}$ s$^{-1}$ using new parallaxes from Cepheids. This measurement is important because it is independent of cosmological models. This value is 3.4σ higher than 66.93 ± 0.62 km Mpc$^{-1}$ s$^{-1}$ predicted by $\Lambda$CDM with Planck. But the discrepancy reduces to 2.1σ with
respect to the prediction of $69.3 \pm 0.7 \text{ km Mpc}^{-1} \text{ s}^{-1}$ based on the comparably precise combination of WMAP+ACT+SPT+BAO (baryon acoustic oscillation) observations. This value was updated in Riess et al. (2018) using more precise parallaxes for Cepheids. The $H_0$ updated value at 2018 is $73.52 \pm 1.62 \text{ km Mpc}^{-1} \text{ s}^{-1}$. All the results from Riess et al. (2016, 2018, 2019) are incompatible with Planck Collaboration et al. (2020). This tension between both observations has been widely discussed. For instance, other researchers used methods independent of distance ladders and the CMB, and they found that the Hubble constant exceeds the Planck results (Pesci et al. 2020; Suyu et al. 2013). However, the errors calculated in the local measurements of the $H_0$ have been criticized (Efstathiou 2014; Zhang et al. 2017). Other measurements based on the tip of the red giant branch (TRGB) have found that $H_0$ is close to $69.6 \text{ km Mpc}^{-1} \text{ s}^{-1}$ (Freedman et al. 2019, 2020). On the other hand, Cardona et al. (2017) and Follin & Knox (2018) confirmed a high $H_0$ value, and recently Wong et al. (2020) used lensed quasars and found $H_0 = 73.3 \text{ km Mpc}^{-1} \text{ s}^{-1}$, which agrees with local measurements but is in tension with Planck observations.

Many solutions have been proposed to explain this tension, such as extended models based on $\Lambda$CDM (Guo et al. 2019), time-varying DE density models (Risaliti & Lusso 2019), or cosmography models (Benetti & Capozziello 2019). Others attempt modifications in the early-time physics, including a component of dark radiation (Bernal et al. 2016) or analyzing early physics related to the sound horizon (Aylor et al. 2019). Many efforts related to the recombination physics have been developed to solve the Hubble tension (Agrawal et al. 2019; Lin et al. 2019; Knox & Millea 2020).

This controversy opens a window for new alternative theories based on modifications or variations of $\Lambda$CDM, such as Camarena & Marra (2018), Huang & Wang (2016), Li et al. (2013), Cedeño et al. (2019), Xu et al. (2019), Deser & Woodard (2019), Anagnostopoulos et al. (2019), and Poulin et al. (2018); other proposals introduce modifications in the physics of neutrinos, for example, Battye & Moss (2014), Zhang et al. (2014), Bernal et al. (2016), Di Valentino & Bouchet (2016), Guo et al. (2017), Feng et al. (2017, 2018), Zhao et al. (2017, 2018), Guo & Zhang (2017), Benetti et al. (2017, 2018), Choudhury & Choubey (2019), Carneiro et al. (2019), and Nakamura et al. (2019); and others consider that DE can couple with DM, such as Salvatelli et al. (2013), Costa et al. (2014), Yang et al. (2017, 2018), Di Valentino et al. (2017b), and Feng et al. (2019).

Some independent studies support the idea that the tension is due more to the physics than to observational errors (Benetti & Capozziello 2019; Bonvin et al. 2016; Abbott et al. 2018; Lemos et al. 2018). Others have found tension in the CMB analysis (Addison et al. 2016; Di Valentino et al. 2019) or suggest errors in the values predicted by Planck CMB (Spergel et al. 2015). Also, it has been suggested as a solution to include modifications in the Planck analysis through more free parameters and varying the equation of state of DE (Di Valentino et al. 2016, 2017a).

The delta-gravity (DG) model (Alfaro & González 2019a) emerges as a model of gravitation that is very similar to classical general relativity (GR) but could make sense at the quantum level. DG could give clues about some incompatibilities in cosmology, eventually produced by the GR and $\Lambda$CDM models. This model has been studied as an alternative to the accelerating expansion because DG can fit the observational SN Ia data, and it does not require DE to explain the acceleration of the universe because it appears naturally from the equations (Alfaro et al. 2019). In this work we calculate the scalar temperature (TT) CMB spectrum and analyze the results and the physical implications. This spectrum is crucial evidence because it gives us information about the constituents of the universe and allows us to constrain the DG model. We do not include a joint analysis (CMB-BAO-SN) in this work because we have only developed a semianalytical approach to describe temperature anisotropies. We have written the perturbative equations, including the baryonic case. However, the hydrodynamic and thermodynamic description of fluids in DG is nontrivial since there are two sets of equations, those from the background associated with $g_{\mu\nu}$ and those from the delta sector associated with $g_{\mu\nu}^\delta$. Furthermore, considering that this work is extensive, we have only included the TT CMB spectrum and not the BAO analysis. However, we are currently working on a hydrodynamic approach to describe BAOs and unify all the cosmological observations in the DG cosmological model. In Section 2, we show a summary of DG and develop some critical definitions and characteristics of the model. In Section 3 we review some important concepts defined in Alfaro et al. (2019) and also include new definitions related to the physical densities and thermodynamics in DG. In Section 4, we show some new results related to SN Ia data. These results are slightly different from Alfaro et al. (2019), but they are vital to analyze if the CMB spectrum is in concordance with SNe Ia. In Section 5 we develop some aspects related to the CMB fluctuations and calculate the scalar TT CMB spectrum assuming a hydrodynamical approach. Finally, we discuss the results and the compatibility between the scalar TT CMB spectrum and the SN Ia data.

2. Delta-gravity Model

In a previous work, Alfaro & González (2019a) studied a model of gravitation that is very similar to classical GR but could make sense at the quantum level. In this construction, he considered two different points. The first is that GR is finite on shell at one loop (’t Hooft & Veltman 1974), and then renormalization is not necessary at this level. The second is a type of gauge theories, $\delta$ gauge theories (delta gauge theories; Alfaro 1997; Alfaro & Labrana 2002), whose main properties are as follows: (a) New kinds of fields are created, $\phi_I$, from the originals $\phi_f$. (b) The classical equations of motion of $\phi_I$ are satisfied in the full quantum theory. (c) The model lives at one loop. (d) The action is obtained by extending the original gauge symmetry of the model, introducing an extra symmetry that we call $\delta$ symmetry since it is formally obtained as the variation of the original symmetry. When we apply this prescription to GR, we obtain DG.

We studied the classical effects of DG at the cosmological level. For this, we assume that the universe is composed of nonrelativistic matter (DM and baryonic matter) and radiation (photons and massless particles), which satisfy a fluid-like equation $p = \omega \rho$. Matter dynamics are not considered, except by demanding that the energy-momentum tensor of the matter fluid is covariantly conserved. In Alfaro et al. (2019) we used the exact solution of the equations, corresponding to the above suppositions, to fit the SN Ia data, and we obtained an accelerated expansion of the universe in the model without DE. We have to redefine some important equations and introduce important modifications with respect to previous works.
These modified theories consist of the application of a variation represented by $\delta$. It has all the properties of a common variation, such as
\[
\begin{align*}
\tilde{\delta}(AB) &= \tilde{\delta}(A)B + A\tilde{\delta}(B), \\
\tilde{\delta}\Delta &= \tilde{\delta}\Delta, \\
\tilde{\delta}(\Phi,_{\mu}) &= (\tilde{\delta}\Phi)_{,\mu},
\end{align*}
\]
where $\delta$ is another variation. The particular characteristic with this variation is that, when we apply it on a field (function, tensor, etc.), it will give new elements that we define as $\tilde{\delta}$ fields, which are an entirely new independent object from the original, $\tilde{\delta} = \tilde{\delta}(\Phi)$. We use the convention that the new tensor is equal to the $\tilde{\delta}$ transformation of the original tensor when all its indices are covariant.

First, we need to apply the $\tilde{\delta}$ prescription to a general action. The extension of the new symmetry is given by
\[
S_0 = \int d^4x L_0(\phi, \partial_\mu\phi) \to S
= \int d^4x (L_0(\phi, \partial_\mu\phi) + \tilde{\delta}L_0(\phi, \partial_\mu\phi)),
\]
where $S_0$ is the original action and $S$ is the extended action in delta gauge theories. GR is based on Einstein–Hilbert action:
\[
S_0 = \int d^4x L_0(\phi) = \int d^4x \sqrt{-g} \left( \frac{R}{2\kappa} + L_M \right),
\]
where $L_M = L_M(\phi, \partial_\mu\phi)$ is the Lagrangian of the matter fields $\phi_I$ and $\kappa = \frac{8\pi G}{c^4}$. Then, the DG action is given by
\[
S = S_0 + \tilde{\delta}S_0 = \int d^4x \sqrt{-g} \left( \frac{R}{2\kappa} + L_M \right) - \frac{1}{2\kappa} (G^{\alpha\beta} - \kappa T^{\alpha\beta}) \tilde{g}_{\alpha\beta} + \tilde{L}_M,
\]
where we have used the definition of the new symmetry, $\tilde{\delta} = \tilde{\delta}\phi$, and the metric convention of Weinberg (2008) and
\[
\tilde{g}_{\mu\nu} = \tilde{\delta}g_{\mu\nu},
\]
\[
T^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}L_M)}{\delta g_{\mu\nu}},
\]
\[
\tilde{L}_M = \tilde{\phi}_I \left( \frac{\delta L_M}{\delta \phi_I} \right) + (\partial_\mu\tilde{\phi}_I) \left( \frac{\delta L_M}{\delta (\partial_\mu\phi)} \right),
\]
where $\tilde{\phi}_I = \tilde{\delta}\phi_I$ are the $\tilde{\delta}$ matter fields (also called called delta matter fields). Thus, the equations of motion are
\[
G^{\mu\nu} = \kappa T^{\mu\nu},
\]
\[
F^{(\mu\nu)(\alpha\beta)}\tilde{D}_\beta \tilde{g}_{\alpha\beta} + \frac{1}{2} g^{\mu\nu} R_{\alpha\beta} - \frac{1}{2} \tilde{g}^{\mu\nu} R = \kappa \tilde{T}^{\mu\nu},
\]
with
\[
F^{(\mu\nu)(\alpha\beta)} = p^{(\mu\nu)(\alpha\beta)} g^{\lambda\alpha} + p^{(\mu\nu)(\alpha\beta)} g^{\lambda\beta} - p^{(\lambda\alpha)(\beta\nu)} g^{\mu\beta} - p^{(\lambda\beta)(\alpha\nu)} g^{\mu\alpha},
\]
\[
p^{(\alpha\beta)(\mu\nu)} = \frac{1}{4} (g^{\alpha\mu} g^{\beta\nu} + g^{\alpha\nu} g^{\beta\mu} - g^{\alpha\beta} g^{\mu\nu}),
\]
\[
T^{\mu\nu} = \tilde{T}^{\mu\nu},
\]
where $(\mu\nu)$ denotes that $\mu$ and $\nu$ are in a totally symmetric combination. The DG equations are of second order in derivatives, which is needed to preserve causality and the equations (9)$_{\nu\mu} = \tilde{\delta}[(8)_{\nu\mu}]$. Also, there are two conservation rules given by
\[
D_{\nu} T^{\mu\nu} = 0
\]
\[
D_{\nu} T^{\mu\nu} = \frac{1}{2} T^{(\alpha\beta)} D^\mu D_\mu \tilde{g}_{\alpha\beta} - \frac{1}{2} T^{(\alpha\beta)} D_\nu \tilde{g}^{\alpha\beta} + D_\mu (\tilde{g}_{\alpha\beta} T^{\mu\alpha\beta}).
\]
It is easy to see that Equation (11) is $\tilde{\delta}(D_{\nu} T^{\mu\nu}) = 0$.

2.1. $T^{\mu\nu}$ and $T^{\mu\nu}$ for a Perfect Fluid

In DG, the energy-momentum tensors for a perfect fluid are
\[
T_{\mu\nu} = p(\rho) \tilde{g}_{\mu\nu} + (\rho + p(\rho)) U_\mu U_\nu,
\]
\[
T_{\mu\nu} = p(\rho) \tilde{g}_{\mu\nu} + \frac{\partial p}{\partial \rho}(\rho) \tilde{g}_{\mu\nu} + (\rho + p(\rho)) \left( \frac{1}{2} (U_\mu U_\nu \tilde{g}_{\alpha\beta} + U_\mu U_\nu \tilde{g}_{\alpha\beta}) + U_\mu U_\nu \tilde{T}_{\mu\nu} \right),
\]
where $U_{\mu} U_{\nu} = 0$, $\rho$ is the pressure, $\rho$ is the density, and $U^\mu$ is the four-velocity. For more details you can see Alfaro & González (2019a).

2.2. Geodesic Equation for Massless Particles

In DG, a massless particle behaves according to the following equation:
\[
g_{\mu\nu} \ddot{x}^\mu \ddot{x}^\nu = 0.
\]
Here the effective metric $g_{\mu\nu}$ is a linear combination given by the two tensors:
\[
g_{\mu\nu} = g_{\mu\nu} + \tilde{g}_{\mu\nu}.
\]
Thus, the massless particles follow null geodesic, like in the GR theory. We remark that massive particles do not follow geodesics (Alfaro & González 2013).

3. Cosmology in Delta-gravity

3.1. Effective Metric to Describe the Universe in a Cosmological Frame

The usual metric to describe the universe in the standard cosmology is the FLRW metric. We assume a flat universe ($k = 0$), and then the metric is
\[
ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -c^2 dt^2 + a(t)^2 (dx^2 + dy^2 + dz^2),
\]
where the scale factor is called $a(t)$.

5 Where $c = 1$ is the speed of light.
The objective is to build an effective metric for the universe; then, the equations need to explain the photon trajectories, because these particles are what we observe and provide us the information from the observables (such as the SN Ia data), showing us the expansion of the universe. As in the GR frame, we build the metric for the universe using the massless particle geodesic in DG. We have to include a “scale factor” in the space-metric component to explain the expansion of the universe. This factor must be space independent because we want to preserve the homogeneity and isotropy for the universe, and then it has to be time dependent. Therefore, we have to find \( \tilde{g}_{\mu\nu} \) from the \( g_{\mu\nu} \). We are going to do a change of variable in the standard metric tensor, \( t \rightarrow u \), where \( T(u) = \frac{dt}{du}(u) \):

\[
g_{\mu\nu}dx^\mu dx^\nu = -T^2(u)c^2du^2 + a^2(u)(dx^2 + dy^2 + dz^2).
\]

Now we add the new dependencies to the temporal and spatial components of the equation, building the most general metric without losing the homogeneity and isotropy of the universe:

\[
\tilde{g}_{\mu\nu}dx^\mu dx^\nu = -F_0(u)T^2(u)c^2du^2 + F_5(u)a^2(u)(dx^2 + dy^2 + dz^2);
\]

thus, we have to fix a gauge to delete the extra degrees of freedom. Fixing a harmonic gauge (described in Alfaro & González 2013), we obtain

\[
T(u) = T_0a^3(u),
\]

\[
F_0(u) = 3(F_0(u) + T_1),
\]

where \( T_0 \) and \( T_1 \) are gauge constants. Choosing \( T_0 = 1 \) and \( T_1 = 0 \), the gauge is fully fixed. Finally, we go back to the effective metric described by Equation (15) to substitute the fixed gauges. This defines the effective metric for the universe in DG:

\[
g_{\mu\nu} = g_{\mu\nu} + \tilde{g}_{\mu\nu} = -(1 + 3F(t))c^2dt^2 + a^2(t)(1 + F(t))(dx^2 + dy^2 + dz^2),
\]

(17)

where the proper time is determined by the original tensor \( g_{\mu\nu} \) (Alfaro 2012b).

### 3.2. Delta-gravity Equations of Motion

To apply this theory to cosmology, we assume that the universe has two components: matter and radiation. With the new symmetry, two kinds of new components appear: delta matter and delta radiation.

To calculate the equations that govern the universe, we assume that \( g_{\mu\nu} \) is expressed by Equation (16), and we calculate the first field equation given by Equation (8):

\[
\left( \frac{\dot{a}(t)}{a(t)} \right)^2 = \frac{\kappa c^4}{3}(\rho_r(t) + \rho_m(t)).
\]

(18)

If we solve Equation (18), we obtain the following expression:

\[
\dot{\rho}_r(t) = -\frac{3\dot{a}(t)}{a(t)}(\rho_r(t) + \rho_p(t)).
\]

(19)

Considering an equation of state, it is possible to relate \( \rho \) and \( p \) for each component \( i \). Assuming that there are only matter (baryonic matter and DM) and radiation (photons and other massless particles), we have (same as GR at this point) for matter

\[
\rho_m(a) = 0,
\]

and for radiation

\[
\rho_p(a) = \frac{1}{3}\rho_r(a).
\]

With these equations we can solve Equation (18) expressing \( \rho(t) \). Summarizing, we have

\[
\rho(a) = \rho_m(a) + \rho_r(a),
\]

(20)

\[
\rho_r(a) = \frac{1}{3}\rho_r(a),
\]

(21)

\[
t(Y) = \frac{2\sqrt{C}}{3H_0\sqrt{\Omega_{r0}}}Y,\]

(22)

\[
Y(t) = \frac{a(t)}{a_0},
\]

(23)

\[
a_0 \equiv a(t = t_0) \equiv 1,
\]

(24)

\[
\Omega_{r0} \equiv \frac{\rho_{r0}}{\rho_0},
\]

(25)

\[
\Omega_{m0} \equiv \frac{\rho_{m0}}{\rho_0},
\]

(26)

\[
\rho_{r0} \equiv \frac{3H_0^2}{8\pi G},
\]

(27)

\[
\Omega_{r0} + \Omega_{m0} \equiv 1,
\]

(28)

where \( t_0 \) is the age of the universe (today). We emphasize that \( t \) is the cosmic time, \( a_0 \) is the scale factor today, and \( C \equiv \frac{\Omega_{r0}}{\Omega_{m0}} \), where \( \Omega_{r0} \) and \( \Omega_{m0} \) are the density energies normalized by the critical density today, defined as the same as the standard cosmology. Furthermore, we have imposed that the universe must be flat (\( k = 0 \)), so we require that \( \Omega_{r0} + \Omega_{m0} \equiv 1 \). Note that \( \rho_i \) is not a physical density. They are only density parameters that are related to physical densities.\(^6\) We are going to discuss this aspect in the next pages.

Using the second continuity Equation (11), where \( \tilde{T}_{\mu\nu} \) is a new energy-momentum tensor, we define two new densities called \( \tilde{\rho}_m \) (delta matter density) and \( \tilde{\rho}_r \) (delta radiation density). They are associated with this new tensor. When we solve this equation, we find

\[
\tilde{\rho}_m(Y) = \frac{C_1 - 3\rho_{m0}F(Y)}{Y^3},
\]

(29)

\[
\tilde{\rho}_r(Y) = \frac{C_2 - 2\rho_{r0}F(Y)}{Y^4},
\]

(30)

where \( C_1 \) and \( C_2 \) are integration constants. It is crucial to clarify that \( \tilde{\rho}_m \) and \( \tilde{\rho}_r \) depend on the normalized scale factor \( Y \). We can note that both energy density parameters (remember that these parameters are not real physical densities, but they are related to the physical densities) have terms that behave like the standard cosmology densities \( \sim \frac{1}{Y^2} \) and \( \sim \frac{1}{Y^4} \) that also are

\(^6\) They are not energy per volume.
preserved in DG:

\[ \rho_r(Y) = \frac{\rho_{r,0}}{Y^4} \]  
\[ \rho_r(Y) = \frac{\rho_{m,0}}{Y^3}. \]  
(31)

(32)

If we preserve \( C_1 = 0 \) and \( C_2 = 0 \), we have equations that are considering two kinds of dependence: \( \frac{1}{Y^2} + \frac{F(Y)}{Y^3} \) and \( \frac{1}{Y^2} + \frac{F(Y)}{Y^4} \). This condition implies that the total energy density (proportional to the real physical densities) considers the standard energy density and the new dependence given by DG, in other words, this is equivalent to considering that \( \tilde{\rho}_r \) is the standard density radiation \( \rho_r \) plus the new DG dependence. We only want to consider the new dependence in the \( \tilde{\rho}_r \) term without the standard radiation contribution. This same reasoning is valid for the density of matter. Thus, defining \( C_1 = C_2 = 0 \), we obtain the following equations:

\[ \tilde{\rho}_m(Y) = -\frac{3\rho_{m,0} F(Y)}{2 Y^3}, \]  
\[ \tilde{\rho}_r(Y) = -\frac{2\rho_{r,0} F(Y)}{Y^4}. \]  
(33)

(34)

There is another reason to define \( C_1 \) and \( C_2 \) equal to 0. When \( Y \ll C \), the effective scale factor \( Y_{DG} \) (defined in Equations (37) and (36)) represents the evolution of the universe at the beginning. We know that an accelerated expansion appears at late times, and then the nonrelativistic matter and radiation must drive the expansion at early times; this means \( Y_{DG} = 1 + O(Y) \). We fix \( C_1 = 0 \) and \( C_2 = 0 \) to guarantee that the behavior of expansion seems like GR at early times. The full development of this idea can be found in Alfaro (2012a) and Alfaro & González (2013).

Using Equation (9) with the solutions from Equations (33) and (34), we found (and redefining with respect to \( Y \))

\[ F(Y) = -\frac{L_2}{3} Y \sqrt{Y + C}, \]  
where \( L_2 \) is an arbitrary constant.

3.3. Relation between the Effective Scale Factor \( Y_{DG} \) and the Normalized Scale Factor \( Y \)

The effective metric for the universe is given by Equation (17). From this expression, it is possible to define the DG scale factor as follows:

\[ a_{DG}(t) = Y(t) \sqrt{1 - \frac{L_2 Y}{3} \sqrt{Y + C}}. \]  
(36)

Furthermore, we define the effective scale factor as \( a_{DG} \) normalized by itself:

\[ Y_{DG}(t) = \frac{a_{DG}(t)}{a_{DG}(t_0)}. \]  
(37)

With the new definition of \( L_2 \), the delta densities are given by

\[ \tilde{\rho}_m(Y) = \frac{L_2}{2} \rho_{m,0} \sqrt{Y + C \over Y^2}. \]  
(38)

If we know \( C \) and \( L_2 \), it is possible to calculate the delta densities \( \tilde{\rho}_m \) and \( \tilde{\rho}_r \) as a function of the common densities. We emphasize that the denominator in Equation (37) is equal to zero when \( 1 = Y_L Y + C \). Taking into account that \( C = \Omega_{r,0}/\Omega_{m,0} \ll 1 \), if \( Y = 1 \) (current time), then the denominator goes to 0 when \( L_2 \approx 1 \). Furthermore, we have imposed that \( \tilde{\rho}_m > 0 \) and \( \tilde{\rho}_r > 0 \), and then \( L_2 \) must be greater than 0. Then, the valid range for \( L_2 \) is approximately 0 \( \leq L_2 < 1 \).

Regarding the \( C \) value, it must be a small positive number because the radiation is not dominant compared to matter. Then, we can analyze cases close to the standardly accepted value for \( \Omega_{r,0}/\Omega_{m,0} \sim 10^{-3} \) (we have assumed GR values to estimate an order of magnitude).

3.4. Useful Equations for Cosmology

Here we present useful equations to fit the SN Ia data and to obtain the cosmological parameters.

3.4.1. Redshift Dependence

DG preserves the relation between the cosmological redshift and the effective scale factor. The reason is straightforward: it is the same as in GR, but changing the scale factor \( a(t) \rightarrow a_{DG}(t) \) in the GR metric \( g_{\mu\nu} dx^\mu dx^\nu \rightarrow g_{\mu\nu} dx^\mu dx^\nu \) (Alfaro & González 2013). Thus, the dependence is given by

\[ Y_{DG}(t) = \frac{1}{1 + z}. \]  
(40)

It is important to consider that the current time is given by \( t_0 \rightarrow Y(t_0) \rightarrow Y_{DG}(Y = 1) = 1 \).

3.4.2. Luminosity Distance

The proof is the same as GR, because the main idea is based on the light traveling through a null geodesic described by the effective metric given by Equation (17) in DG. Then, the equation that describes the luminosity distance for DG is the same as in GR, but changing the scale factor \( a(t) \) by the \( a_{DG}(t) \), because \( a_{DG}(t) \) is the factor that is describing the observable expansion (or scaling) of the universe (Alfaro et al. 2019).

We remark that the relation between the luminosity distance \( d_L^{DG} \) and angular diameter distance \( d_A^{DG} \) in DG is the same as in GR (Etherington 1933). This relation is a direct consequence of the structure of the metric. This relation is given by

\[ d_L^{DG} = (1 + z)^2 d_A^{DG}. \]  
(41)

The luminosity distance was calculated in Alfaro et al. (2019) and is given by

\[ d_L^{DG}(z, L_2, C, h^2 \Omega_{m,0}) = c \int_{Y(t_0)}^{1} \frac{Y}{\sqrt{Y + C \over Y_{DG}(t)}}, \]  
(42)

where \( Y = 1 \) denotes today. To solve \( Y(t_1) \) at a given redshift \( z \), we need to solve Equations (37) and (40) numerically. Furthermore, the integrand contains the effective scale factor \( Y_{DG}(t) \) that can be expressed as a function of \( Y \) through Equation (37). Do not confuse \( c \) (speed of light) with \( C \). If the integration assumes \( Y \gg C \) (a good approximation for SNe Ia, because we are integrating in late times), this equation can be
approximated to
\[ d_L^{DG}(z, L, h) \approx \frac{1}{100h} \int_0^1 \frac{\sqrt{Y}}{Y_{DG}(t)} \, dY, \]
where \( d_L^{DG} \) is independent of \( C \). Also, if \( C \rightarrow 0 \), then \( \Omega_{m,0} = 1/(1 + C) \rightarrow 1 \). In this scenario, the only two free parameters are \( h \) and \( L_2 \).

We underline that it is impossible to know the \( C \) using the SN Ia data, but we can constrain this value with the CMB. In DG, \( C \) is a constant that is related to the physical densities, but it does not represent a ratio between physical densities.

3.5. Distance Modulus

This relation is fundamental because it lets us calculate the dependence between the apparent magnitude and the distance to the object. It is essential to consider that we need to know the value of the absolute magnitude \( M \) to avoid degeneration:
\[ \mu = m - M = 5 \log_{10} \left( \frac{d_L^{DG/GR}}{10 \, \text{pc}} \right). \]

3.6. Normalized Effective Scale Factor

In DG, the “size” of the universe is given by \( Y_{DG}(t) \), and then every cosmological parameter that in the GR theory was built up from the standard scale factor \( a(t) \) in DG will be built from \( Y_{DG}(t) \).

3.7. Hubble Parameter

The Hubble parameter (and also the Hubble constant) is defined in GR cosmology as
\[ H(t) = \frac{\ddot{a}(t)}{a(t)}. \]

Thus, in DG we define the Hubble parameter as follows:
\[ H^{DG}(t) = \frac{\dot{a}_{DG}(t)}{a_{DG}(t)}. \]

The Hubble constant is the Hubble parameter \( H^{DG}(t) \) evaluated today, in other words, when \( Y = 1 \). Therefore, the Hubble parameter is given by
\[ H^{DG}(t) = \frac{\dot{a}_{DG}(t)}{a_{DG}(t)} \frac{\ddot{Y}_{DG}(t)}{Y_{DG}(t)}. \]

Observe that all the DG parameters are written as a function of \( Y \).

3.8. Deceleration Parameter

In the standard cosmology the deceleration parameter is given by
\[ q(t) = -\frac{\ddot{a}}{a^2}. \]

Thus, in DG we define the deceleration parameter as follows:
\[ q^{DG}(t) = -\frac{\ddot{a}_{DG} a_{DG}}{a_{DG}^3}. \]

Thus,
\[ q^{DG}(t) = -\frac{\ddot{a}_{DG} a_{DG}}{a_{DG}^3}. \]

3.9. Dependence between Redshift and Cosmic Time

All the equations are parameterized as a function of \( Y \), and then we need to use Equations (22), (37), and (40) to relate redshift and cosmic time. In the matter-dominated universe until today, \( C \sim 10^{-4} \ll Y \), and then
\[ t(Y) = \frac{2}{3H_0} Y^{1/2}. \]

This \( H_0 \) constant is not the Hubble constant. In DG, this is an arbitrary constant that can be obtained from the SN Ia fit. It is different from \( H_0^{DG} \), the physical and observable constant. The age of the universe can be easily calculated from Equation (51), where \( L_2 \) does not play the role in the time evolution.

3.10. Nonphysical Densities of Common Components: \( \Omega_{m,0} \) and \( \Omega_{r,0} \)

We have imposed that \( \Omega_{m,0} + \Omega_{r,0} = 1 \) and \( C = \frac{\Omega_{r,0}}{\Omega_{m,0}} \), and then
\[ \Omega_{r,0} = \frac{C}{1 + C}; \quad \Omega_{m,0} = \frac{1}{1 + C}. \]

It is vital to consider that this equation only expresses a relation, or a proportion, between the nonphysical energy density for common matter and common radiation densities and does not express a real percentage of composition of the universe because in DG we also have delta matter and delta radiation.

This condition is imposed when we assumed that \( T^\mu_{\nu} \) only expresses a standard composition, and when we assumed that the DE does not exist at the level of either action or field equations.

3.11. Physical Densities in DG: A Thermodynamic Approach

This definition is essential to define any physical interaction that is related to the physical parameters, for example, the damping associated with fluids or collision probabilities between particles. Thus, this is essential to fit the CMB spectrum.

The physical element of volume in DG is \( dV = a_{DG}^3 dx dy dz \) (given by the effective metric), which is described by the DG scale factor \( a_{DG} \). Then, the density of any kind of matter in terms of energy per volume is
\[ \rho_{DG} = \frac{U}{c^2 V}, \]
where \( U \) is the internal energy, \( V \) is the volume, and \( \rho_{DG} \) is the physical density.\(^7\) Therefore, if we apply the first law of thermodynamics and assume that the evolution of the universe
is adiabatic as in GR (Padmanabhan 2002),
\[
\dot{\rho}_{\text{DG}} = -3H_{\text{DG}}\left(\rho_{\text{DG}} + \frac{P_{\text{DG}}}{c^2}\right). \tag{54}
\]

In the standard cosmology, the equations of state are written as \( P = \omega \rho \); thus, in DG we assume an equation as \( P_{\text{DG}} = \omega \rho_{\text{DG}} \) and replace it in Equation (54), and then we obtain
\[
\rho_{\text{DG}}\dot{\rho}_{\text{DG}}^{3(1+\omega)} = \rho_{\text{DG},0}\dot{\rho}_{\text{DG},0}^{3(1+\omega)}, \tag{55}
\]
where \( \rho_{\text{DG},0} \) is the density today. We can relate the physical and the background densities by the ratio between them:
\[
\frac{\rho_{\text{DG}}}{\rho} = \left(1 + F(t)\right)^{3(1+\omega)} = \text{constant}(\omega). \tag{56}
\]

The standard cosmological perturbations are defined as
\[
\delta_\alpha = \frac{\delta \rho_\alpha}{\bar{\rho}_\alpha + \bar{\rho}_\gamma}, \tag{57}
\]
where \( \alpha = \gamma, \nu, B, \) and \( D \) (photons, neutrinos, baryons, and DM, respectively). In the early universe, when \( Y \sim 10^{-3} \) (near the last scattering surface), the \( F \) factor tends to 1. This aspect is vital for the development of the perturbative equations, because at that moment the physical densities were proportional to the standard densities, and by definition, the physical perturbations are equal to the standard perturbation:
\[
\delta_{\text{phys,} \alpha}(t) = \delta_\alpha(t). \tag{58}
\]

This approximation is accurate, and it is valid from the beginning of the universe \((z \to \infty)\) to \( z \sim 10 \).

### 3.12. The Shape of the Blackbody Spectrum

We want to preserve the shape of the CMB blackbody spectrum because it is an observable, which is described by the number of photons
\[
n_T(\nu)d\nu = \frac{8\pi\nu^2d\nu}{e^{\nu/k_B T} - 1}. \tag{59}
\]

After the last scattering surface, the photons traveled decoupled from the baryons, and then the spectrum changes its frequency as \( \nu = \nu_{\text{DG}}(\nu_0) / \nu_{\text{DG}} \), but the volume also changes as \( V = V_{\text{DG}}^3(\nu_0) / V_{\text{DG}}^3 \); then, the conservation of the number of photons \( dN = n_T(\nu)d\nu dV \) implies that
\[
T = \frac{T_0}{Y_{\text{DG}}}, \tag{60}
\]
where \( T_0 \) is the CMB temperature. In other words, the temperature of the universe evolves with the effective scale factor described by \( Y_{\text{DG}} \) and not \( a \).

All these definitions and interpretations are essential to describe the CMB physics. The preservation of the relation given by Equation (60) is important because the deviation of \( T \) with \( z \) has been previously studied in Lima et al. (2000) as an arbitrary dependence in \( T \), where the results found by de Martino et al. (2012) and Avgoustidis et al. (2012) indicated that \( T = T_0(1 + z) \) is right.

We are interested in the viability of DG as a real alternative cosmology theory that could explain the accelerating universe without \( \Lambda \). The first section shows the SN Ia data and the equations, Section 2 shows the results, and the last section contains the analysis and the conclusions. This section is similar to the previous one, but the meaning of some parameters and their numerical values change. This change is relevant to be able to explain the CMB later.

### 4. SNe Ia and the Accelerated Expansion

The SNe Ia are very useful in cosmology because they can be used as standard candles, allowing us to fit a cosmological model (Riess et al. 1998). The main characteristic of the SNe Ia that makes them so useful is that they have a very standardized absolute magnitude close to \(-19\) (Betoule et al. 2014; Richardson et al. 2014; Uemura et al. 2015; Riess et al. 2016; Alsalehi & Murdin 2017).

DG is a model that produces an accelerated expansion. The denominator of the effective scale factor \( Y_{\text{DG}} \) can be very small, depending on the \( L_2, C, \) and \( Y \) values. When this denominator tends to 0, \( Y_{\text{DG}} \) goes to infinity, causing a big rip. This phenomenon modulates the rate of acceleration through the \( Y_{\text{DG}} \) factor. It is not evident that this type of scale factor can describe the accelerating expansion of the universe by itself. Then, it is necessary to fit the model using data from SNe Ia to know whether DG can describe the acceleration of the universe. The DG model has three free parameters, \( h, C, \) and \( L_2 \), but the acceleration depends strongly on \( L_2 \). On the other hand, \( C \) does not play an essential role in the acceleration description. \( C \) tends to be very small \((10^{-3})\) since it describes the ratio between radiation and matter at present (see physical densities in Section 4.1 and Equation (141) for more detail).

It is important to note that the SNe Ia have a standardized absolute magnitude \( M \), but this value is unknown (it must be calibrated). This value can be determined by cosmic ladders (Riess et al. 2019), causing it to appear degenerated with \( H_0 \). Therefore, the only way to measure \( H_0 \) using SNe is standardizing \( M \) (in the standard cosmology).

In the most recent SN calibration (using local Cepheids), \( H_0 \) and \( M \) were found through minimizing the error of the whole system of equations (from local Cepheids to SNe; Riess et al. 2019). We wish to emphasize two aspects of this analysis: first, the expansion of the luminosity distance as a function of the redshift until order 2 is the same as that of \( \Lambda \)CDM, and then the Riess analysis using \( \Lambda \)CDM would be the same as for DG. However, the observable Hubble constant is a function of the derivative of the effective scale factor and not of the background scale factor; thus, all the definitions in DG are self-consistent. Second, there is a degeneration between \( C \) and \( M \) (that appears in a specific range of \( C \) values); however, \( L_2 \) is independent of \( M \). In other words, the \( L_2 \) parameter is fixed by the expansion rate of the universe but not by the apparent magnitudes of SNe. This does not happen with \( C \) and \( h \), which have significant implications in this model.

With this behavior, we can assume two scenarios, one where \( C \) can take relatively large values since SNe do not restrict it (e.g., \( 10^{-3} \)), and another where \( C \) is arbitrarily small (e.g., \( 10^{-10} \)). Since \( C \) is approximately the ratio of the physical radiation density divided by the physical matter density, we assume \( C + \gamma \approx (C \approx 0) \) to fit SNe Ia. However, we need to know an accurate \( C \) value to work with the CMB because we have to describe the temperature anisotropies, where the physical radiation density and the physical matter density are essential to modulate the power spectrum.
We fitted the TT CMB spectrum, assuming \( C = 0 \) as a free parameter. Nevertheless, to be more restrictive with the observations, we decided that \( C \) must be compatible with the local Riess measurements. In other words, we decided to preserve the local and high-redshift results from SNe Ia, constraining the \( C \) value to be small enough such that the SN Ia observations are insensitive to \( C \) (see Figure 2).

In this section, we are not saying that DG predicts a high \( H_0 \). Instead, we find the parameters such that DG is compatible with the Riess results, assuming that \( C \) is negligible. Therefore, to study the compatibility between SNe Ia and the TT CMB spectrum, we calculate the TT CMB spectrum considering \( C \) as a free parameter. Therefore, in the best scenario, the TT CMB spectrum must be explained with a \( C \) value smaller than the \( C \) upper bound given by the local observations.

To be explicit and transparent, we have decided that DG describes the accelerated expansion of the universe, including all the SN Ia data and setting an absolute magnitude \( M \). However, we emphasize that \( L_2 \) is independent of \( M \), but \( h \) and \( C \) depend on \( M \).

In the following sections, we show the procedure and the results associated with the SN Ia analysis. Our objective is to show the following: (1) DG explains the accelerated expansion of the universe through the coefficient \( L_2 \), which is independent of \( M \); (2) if \( C \) is smaller than an upper bound, then it is negligible in the SN Ia analysis; and (3) \( M \) and \( h \) are degenerate, but we assume that Riess measurements are correct because we want to verify that DG can simultaneously explain the TT CMB power spectrum and Riess observations.9,10

4.1. SN Ia Data

To analyze the expansion of the universe, we used 1048 SNe from the SN Ia catalog from Scolnic et al. (2018). We only need to know the distance modulus \( \mu \) and the redshift \( z \) for every SN to fit the DG model using the luminosity distance given in Equation (44). We assume a scenario with a fixed \( M \) and a flat universe where the radiation is negligible (\( C = 0 \) for DG) and fit the DG model to find \( L_2 \) and \( h \). The \( M \) value was calculated using 210 SNe Ia from Riess et al. (2016) and corresponds to the absolute magnitude, which is independent of the model.11

Summarizing, in DG we fit \( L_2 \) and \( h \). We emphasize that \( h \) in the DG model is not the Hubble constant \( (H_0^G) \) can be calculated with Equation (47)). We used the least-squares method.

4.2. DG Fit

We present two figures associated with the DG model. Figure 1 assumes \( C = 0 \) and describes very well the SN Ia data; DG and GR are indistinguishable in describing the SN Ia data. The fitted DG parameters \( h \) and \( L_2 \) are shown in Table 1.

It is important to analyze the influence of \( C = 0 \) in the approximation that we used. Thus, we show the squared error associated with different \( C \) values in Figure 2. The results from SN Ia analysis indicate that DG explains the accelerating expansion of the universe without including \( \Lambda \) or anything like DE. The acceleration is naturally produced in DG, caused by a coefficient named \( L_2 \), which appears when we solve the differential equations that describe the cosmology. There are two crucial differences between these results and Alfaro et al. (2019). First, the fit assumes \( C = 0 \). Second, the

Figure 1. The fitted curve for the DG model assumes \( C = 0 \) and \( M = -19.23 \). The DG model can describe the acceleration of the universe without introducing a DE term. The \( L_2 \) parameter, which regulates the acceleration, is independent of the absolute magnitude.

Table 1

| Parameter | Value | Standard Error | Relative Error |
|-----------|-------|----------------|----------------|
| \( L_2 \) | 0.457 | 0.007          | 1.57%          |
| \( h \)   | 0.496 | 0.004          | 0.77%          |

9 \( h \) in DG is not equivalent to fixing the observable Hubble constant \( H_0^G \). In this theory, the parameter \( h \) does not describe the dimensionless Hubble constant. However, it is relevant to calculate distances. We emphasize that we are not reproducing the TT CMB spectrum with high accuracy. Instead of using a numerical method, we are using a semianalytic approximation. This is the main reason that we do not fit both observations at the same time.

10 We are not interested in study cases where \( C \) is greater than the upper bound \((10^{-2})\), because this would imply a very high proportion of radiation with respect to matter density. Then, assuming \( C = 0 \) in the SN Ia case appears to be reasonable.

11 This value is independent of the cosmological model. It was calculated through a distance ladder based on local Cepheids and Cepheids hosted in the nearest galaxies (these galaxies are also SN Ia host; Riess et al. 2016). They did not use any particular cosmological model.
The luminosity distance given by Equation (42) can be simplified assuming $C = 0$ and using the relation between the DG scale factor and redshift given by Equation (40). We obtain an expression around $z = 0$ up to second order in redshift given by:

$$d_{L}^{DG}(z, L_2, C) \approx \frac{c}{H_{0}^{DG}} \left( z + \frac{1}{2}(1 - q_{0}^{DG})z^2 \right).$$

At first order, the local expansion is exactly as in the $\Lambda$CDM model:

$$m = 5 \log \frac{c}{H_{0}^{DG}} + M + 25. \quad (62)$$

This expression is in concordance with the definition given in Equation (47).

### 4.4. $H_{0}^{DG}$ and $q_{0}^{DG}$

With the fitted DG parameters we can find $H_{0}^{DG}$ and $q_{0}^{DG}$. To evaluate the Hubble constant, we use $Y_{DG} = 1$. Note that we are not predicting the Hubble constant, but rather extending the local measurements to the full SN Ia observations in a simple procedure in order to determine whether DG fits the high-redshift SN Ia.

The $H_{0}^{DG}$ can be approximated assuming $C = 0$. This estimation is very precise:\[12\]

$$H_{0}^{DG} \approx 50h(-6 + 11L_2 - 7L_2^2 + 2L_2^3)/(9 - 3L_2)(1 + L_2)^3. \quad (63)$$

In Table 2 we present the DG results. We also include in the table other measurements of the Hubble constant. The similar values found by DG are a consequence of the excellent fit (we are only working with $h$ and $L_2$) and the series expansion of $d_{L}^{DG}$ in terms of $z$ (this term can be expanded as a $z$ series, with the same physical significance, such as the Hubble constant and the deceleration parameter, but the way these parameters depend on the scale factor is very different compared to the case of GR).\[13\]

Figure 3 shows the change in the Hubble parameter for both models. In the DG case, the Hubble parameter increases after $Y_{DG} \approx 1.2$, and the universe starts to increase its size to end with a big rip. In contrast, as we know, $\Lambda$CDM does not predict a big rip. The $H(a)$ tends to be constant when $a \rightarrow \infty$ (Alfaro et al. 2019).

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**Table 2**

| Model | $H_0$ (km s$^{-1}$ Mpc$^{-1}$) | Error (km s$^{-1}$ Mpc$^{-1}$) |
|-------|-------------------------------|-------------------------------|
| Planck 2015 Planck Collaboration et al. (2016) | 67.74 | 0.46 |
| Planck 2018 Planck Collaboration et al. (2020) | 67.4 | 0.5 |
| Riess 2016 Riess et al. (2016) | 73.24 | 1.74 |
| Riess 2018 Riess et al. (2018) | 73.52 | 1.62 |
| Riess 2019 Riess et al. (2018) | 74.03 | 1.42 |
| DG | 74.3 | 1.3 |
| DGapprox | 74.2 | … |

**Notes.** Furthermore, we tabulate Planck satellite’s data (Planck Collaboration et al. 2016, 2020), and Riess et al. (2018) $H_0$ values. DG is the value obtained in Section 4 using all the SN Ia data. DGapprox was calculated from Equation (63). The errors associated with DG values are purely statistical (related to the fit method) and do not include other effects.

---

\[12\] This equation is straightforward from the definition of Equation (47).

\[13\] “The direct measurement is very model-independent, but prone to systematics related to local flows and the standard candle assumption. On the other hand, the indirect method is very robust and precise, but relies completely on the underlying model to be correct. Any disagreement between the two types of measurements could in principle point to a problem with the underlying $\Lambda$CDM model” (Odderskov et al. 2014).
Figure 4 shows how the deceleration parameter depends on $C$ and $L^2$. In the regime of interest, where $C \to 10^{-4}$, $H_0^{DG}$ is independent of $C$ and it increases with $L^2$. In GR, the deceleration parameter is calculated from Equation (48) and the Friedmann equations

$$q_0 = \frac{1}{2} \left( \Omega_{m,0} - \Omega_{r,0} \right). \quad (64)$$

For DG, we used Equation (50). To evaluate the deceleration today, we evaluate $a = 1$ for GR and $Y = 1$ for DG.

We show the deceleration parameters for both models in Table 3. Both models have $q_0 < 0$; in other words, the universe is accelerating but with slightly different rates.

In Figure 5 we show how the deceleration parameter depends on $C$ and $L^2$. It is important to consider that acceleration depends on $L^2$ and is independent of $C$ (if $C$ is small).

The $L^2$ parameter is driving the acceleration, and it is describing the SN Ia data. If $L^2 \to 1$, then $q_0$ is more negative, and the universe has a higher acceleration.

4.4.1. Cosmic Time and Redshift

To calculate the cosmic time in DG, we used Equation (22). The redshift is obtained by numerical solution from Equation (40).

Meanwhile, for the GR model, we obtained the cosmic time integrating the first Friedmann equation and solving $t(\Omega_{r,0}, \Omega_{m,0}, H_0)$. Here we have included $\Omega_A = 1 - \Omega_{m,0} - \Omega_{r,0}$ (we chose a flat cosmology). The integral for the first Friedmann equation can be numerically solved:

$$t = \frac{1}{H_0} \int_0^a \frac{1}{\sqrt{\frac{\Omega_{r,0}}{x^2} + \frac{\Omega_{m,0}}{x^2} + (1 - \Omega_{m,0} - \Omega_{r,0})x^2}} dx, \quad (65)$$
Table 3

$q_0$ Values Were Found Using the Least-squares Method with SN Ia Data

| Model | $q_0$ | Error |
|-------|-------|-------|
| DG    | −0.700 | 0.001 |
| GR    | −0.58  | 0.02  |

Note. The errors are purely statistical.

where $t$ is the cosmic time for GR. The behavior of cosmic time dependence with redshift for both models is very similar (Alfaro et al. 2019).

The age of the universe in DG is calculated using Equation (22). $n(Y)$ depends on $h$ and $C$, but not on $L_0$. To calculate the age of the universe in DG, we evaluate $Y = 1 \gg C$, and then the age only depends on $h$. On the other hand, in GR we calculate the age of the universe using Equation (65). The age for the DG model is 13.1 Gyr and for GR is 13.2 Gyr.\(^{14}\)

The higher the Hubble constant, the lower the age of the universe. This relation is vital since if the local fit of SNe radically changes $H_0$, then the age of the universe changes.

The ages of the universe for DG and GR are small (13.1 Gyr for DG and 13.2 Gyr for GR) compared with the age calculated from Planck (13.8 Gyr). A crucial and precise estimation made by Pasquini et al. (2004) based on the ages of globular clusters in the Milky Way (which is independent of cosmology) indicates that the universe has to be older than 13.6 ± 0.8 Gyr. DG, assuming the results of SN Ia local measurements, is on the verge of this observational constraint. We emphasize that the problem goes beyond DG because this discrepancy is related to the local measurements and is due to the calibration made by Riess et al. (2016). As we discussed in Section 1, there are many different $H_0$ measurements, but in this work we are working assuming that this high $H_0$ value is correct.

5. TT CMB Scalar Spectrum

To fit the CMB power spectrum with DG, we have to use perturbation theory. The perturbation theory has been developed in previous work (Alfaro et al. 2020), where we have decomposed the perturbation terms as the standard scalar-vector-tensor method. Here we show a summary of the main equations required to obtain the CMB and fit the parameters. The metric is perturbed up to first order:\(^{15}\)

$$ g_{\mu\nu} = \tilde{g}_{\mu\nu} + h_{\mu\nu}. \quad (66) $$

$$ \tilde{g}_{\mu\nu} = \bar{g}_{\mu\nu} + \tilde{h}_{\mu\nu}. \quad (67) $$

In particular, we followed Weinberg's approach (Weinberg 2008; he developed this method in the synchronous gauge\(^{16}\)), which consists of two main aspects: The first one is the so-called hydrodynamic limit, which implies that near recombination time photons were in local thermal equilibrium with the baryonic plasma, and then photons can be treated hydrodynamically, like plasma and CDM. The second assumption is a sharp transition from thermal equilibrium to complete transparency at last scattering moment $t_L$.

In this context, the components of the universe are photons, neutrinos, baryons, and CDM and the delta sector. The approximation used here both neglected anisotropic energy-momentum tensors and assumed the usual equation of state for the components. Besides, as we treated photons and delta photons hydrodynamically, we used $\delta u_\gamma = \delta u_\gamma$ and $\delta u_\gamma = \delta u_\gamma$ (velocity perturbations). Moreover, as the synchronous scheme did not fully fix the gauge, the remaining degrees of freedom were used to fix $\delta u_D = 0$, which means that CDM evolves at rest with respect to the universe expansion. In our theory, the extended synchronous scheme also had an extra degree of freedom, which we used to put $\delta u_D = 0$ as its standard part.

It is useful to rewrite these equations in terms of the following dimensionless term:

$$ \delta_{\mu\nu} = \frac{\delta \rho_{\mu\nu}}{\bar{\rho}_\gamma + \bar{\rho}_b}. \quad (68) $$

where $\alpha$ can be $\gamma, \nu, B,$ and $D$ (photons, neutrinos, baryons, and DM, respectively) and $q$ is the mode. Also, we used $R = 3\bar{\rho}_\gamma /4\bar{\rho}_b$ and $\bar{R} = 3\bar{\rho}_D /4\bar{\rho}_b$. On the other side, in the delta sector we used a dimensionless fractional perturbation. However, this perturbation was defined as the delta transformation of Equation (68),\(^{17}\)

$$ \tilde{\delta}_{\mu\nu} = \delta \rho_{\mu\nu} = \bar{\rho}_\gamma + \bar{\rho}_b \delta_{\mu\nu}. \quad (69) $$

The equations for the GR sector are

$$ \frac{d}{dt}(\alpha^2 \Psi_\alpha) = - 4\pi G a^2 \left( p_\alpha \delta D_{\alpha} + p_\alpha \delta u_{\alpha} + \frac{8}{3} \bar{\rho}_\gamma \delta \gamma - \frac{8}{3} \bar{\rho}_b \delta \nu_{\alpha} \right), \quad (70) $$

$$ \delta_{\mu\nu} - (q^2 / a^2) \delta u_{\mu\nu} = - \Psi_\gamma, \quad (71) $$

$$ \delta_{\mu\nu} - (q^2 / a^2) \delta u_{\mu\nu} = - \Psi_\nu, \quad (72) $$

$$ \delta_{\mu\nu} - (q^2 / a^2) \delta u_{\mu\nu} = - \Psi_\gamma, \quad (73) $$

$$ \delta_{\mu\nu} - (q^2 / a^2) \delta u_{\mu\nu} = - \Psi_\nu, \quad (74) $$

$$ \frac{d}{dt} \left( \frac{(1 + R) \delta u_{\mu\nu}}{a} \right) = - \frac{1}{3a} \delta_{\mu\nu}, \quad (75) $$

$$ \frac{d}{dt} \left( \frac{\delta u_{\mu\nu}}{a} \right) = - \frac{1}{3a} \delta_{\mu\nu}. \quad (76) $$

\(^{17}\) We choose this definition because the system of equations now seems to be a homogeneous system exactly equal to the GR sector (where now the variables are the delta-fields) with external forces mediated by the GR solutions. Maybe the most intuitive solution should be

$$ \delta_{\mu\nu} = \frac{\delta \rho_{\mu\nu}}{\bar{\rho}_\gamma + \bar{\rho}_b}, $$

however, these definitions are related by

$$ \delta_{\mu\nu} = \bar{\rho}_\gamma + \bar{\rho}_b (\delta_{\mu\nu} - \delta_{\mu\nu}). $$
The equations for the DG sector are

\[
\begin{align*}
&\left[2F\frac{d}{a} + F\right]a^2\dot{\Psi}_q + \left[6F\frac{d}{a} + \frac{5}{2}F\right]a^3\ddot{\Psi}_q \\
&+ 3Fa^2\dot{\Psi}_q - \frac{d}{dt}(a^2\ddot{\Psi}_q) = \kappa\frac{a^2}{2}\left[\rho_D\delta_{Dq} + \rho_B\delta_{Bq} + \frac{8}{3}\dot{\rho}_B\delta_{Bq} + \frac{8}{3}\dot{\rho}_q\delta_{qq}ight] - \frac{F}{2}(\rho_D\delta_{Dq} + \rho_B\delta_{Bq}) - \frac{3F}{3}(\dot{\rho}_B\delta_{Bq} + \dot{\rho}_q\delta_{qq}),
\end{align*}
\]

where \(\Psi_q\) and \(\dot{\Psi}_q\) are a particular combination of the scalar perturbations in the metric (Alfaro et al. 2020).

5.1. Matter Era

In this era \(a \gg C (R = \bar{R} = 0)\) and the perturbative equations for GR can be approximated and solved. These solutions are given by\(^{18}\)

\[
\begin{align*}
&\delta_{Dq} = \frac{9q^2\kappa^2\mathcal{R}_{q}T(\kappa)}{10a^2}, \\
&\dot{\Psi}_q = -\frac{3q^2\kappa R_{q}T(\kappa)}{5a^2}, \quad \delta_{qq} = \frac{3R_{q}T(\kappa)}{5}\left[\frac{T(\kappa) - S(\kappa)\cos\left(q \int_0^t \frac{dt}{\sqrt{a}} + \Delta(\kappa)\right)}{\mathcal{S}(\kappa)}\right], \\
&\delta_{u_{qq}} = \frac{3R_{q}}{5}\left[-T(\kappa) + S(\kappa)\frac{a}{\sqrt{3}q}\sin \left(q \int_0^t \frac{dt}{\sqrt{3}a} + \Delta(\kappa)\right)\right],
\end{align*}
\]

where \(T(\kappa), S(\kappa),\) and \(\Delta(\kappa)\) are known as transfer functions. They only depend on \(\kappa \equiv \frac{q\sqrt{2}}{a_{\text{EQ}}H_{\text{EQ}}}\).

\(^{18}\) \(\mathcal{R}_q\) is defined as \(q^2\mathcal{R}_q \equiv -a^2H\dot{\Psi}_q + 4\pi G a^2\dot{\rho}_q + a^2H\dot{\rho}_q\). It is a gauge-invariant quantity, which takes a time independent value for \(q/a \ll H\) (Weinberg 2008).
Then, the perturbative equations given in the matter era for GR and DG can be rewritten as

\[
\delta_{DG} = \frac{\kappa^2 R_0^2 d(y)}{4}, \quad \delta_{GR} = \frac{\kappa^2 R_0^2 f(y)}{4}, \quad \dot{\delta}_{DG} = \frac{\kappa^2 H_0^2 f(y)}{4 \sqrt{2}} R_0^2 f(y),
\]

\[
\delta_{DG} = \frac{\kappa^2 R_0^2 g(y)}{4}, \quad \delta_{GR} = \frac{\kappa^2 R_0^2 f(y)}{4 \sqrt{2}} R_0^2 f(y),
\]

\[
\delta \delta_{DG} = \frac{\kappa^2 R_0^2 g(y)}{4}, \quad \delta \delta_{GR} = \frac{\kappa^2 R_0^2 f(y)}{4 \sqrt{2}} R_0^2 f(y).
\]

Then, the perturbative equations given in the matter era for GR and DG can be rewritten as

\[
\sqrt{1 + y \frac{d}{dy}(y^2 f(y))} = -\frac{3}{2} d(y) - \frac{4 f(y)}{y}, \quad (90)
\]

\[
\sqrt{1 + y \frac{d}{dy}(y^2 f(y))} = -y f(y), \quad (91)
\]

\[
\sqrt{1 + y \frac{d}{dy}(y^2 f(y))} = -y f(y), \quad (92)
\]

\[
\sqrt{1 + y \frac{d}{dy}(y^2 f(y))} = -\frac{r(y)}{3}, \quad (93)
\]

Then we have to include the damping effect in the fluid of baryons and photons. This effect is known as Silk damping and considers coefficients of shear viscosity, heat conduction, bulk viscosity, and Thomson scattering associated with the fluid (Silk 1968; Weinberg 1971; Kaiser 1983). Then, the full solutions for the photon density perturbations are

\[
\delta_{DG} = \frac{3 R_0^2}{5} \left[ T(\kappa_0) (1 + 3 R) \right.
\]

\[
- (1 + R)^{-1/4} e^{-\int_0^t \frac{a_q}{a_{DG}} dt} S(\kappa), \quad \times \cos \left( \int_0^t \frac{q dt}{\sqrt{3 (1 + R(t)) a_{DG}(t)}} + \Delta(\kappa) \right) \right], \quad (98)
\]

\[
\delta \delta_{DG} = \frac{3 R_0^2}{5} \left[ -t T(\kappa_0) \right.
\]

\[
+ \frac{a_{DG}}{\sqrt{3} (1 + R)^{3/4}} e^{-\int_0^t \frac{a_q}{a_{DG}} dt} S(\kappa), \quad \times \sin \left( \int_0^t \frac{q dt}{\sqrt{3 (1 + R(t)) a_{DG}(t)}} + \Delta(\kappa) \right) \right], \quad (99)
\]

where

\[
\Gamma = \frac{q^2 T}{6 a_{DG}(1 + R)} \left[ \frac{16}{15} + \frac{R^2}{1 + R} \right], \quad (100)
\]

Now we have to express the temperature perturbation as a function of the densities perturbations. This procedure is long and takes many pages. It is not the objective of this paper to show the steps to obtain this result (Alfaro et al. 2020). However, it is vital to understand the physics behind the equations, the approximations, and the numerical contributions behind every term. First of all, we show four essential functions called form factors that are the main contributions to the TT CMB spectrum,

\[
\mathcal{F}(q) = -\frac{1}{2} a_{DG}(t) \hat{B}_q(t_h) - \frac{1}{2} a_{DG}(t) \hat{B}_q(t_h) \beta_q(t_h)
\]

\[
+ \frac{1}{2} E_q(t_h) + \frac{\delta T_q(t_h)}{T(t_h)}, \quad (101)
\]

\[
\delta \mathcal{F}(q) = -\frac{1}{2} a_{DG}(t) \hat{B}_q(t_h) - \frac{1}{2} a_{DG}(t) \hat{B}_q(t_h) \beta_q(t_h), \quad (102)
\]

\[
\mathcal{G}(q) = q \left( \frac{1}{2} a_{DG}(t_h) \hat{B}_q(t_h) \right.
\]

\[
+ \frac{1}{(1 + 3 F(t_h)) a_{DG}(t_h)} \delta \mu_q(t_h), \quad (103)
\]

\[
\tilde{\mathcal{G}}(q) = q \left( \frac{1}{2} a_{DG}(t_h) \hat{B}_q(t_h) \right.
\]

\[
+ \frac{1}{(1 + 3 F(t_h)) a_{DG}(t_h)} \delta \tilde{\mu}_q(t_h), \quad (104)
\]
where the TT CMB spectrum is given by Equation (125). These formulae will be very useful.¹⁹

These form factors can be rearranged using many new definitions that introduce physics notation. Before doing that, it is important to define some physical concepts.

**Angular diameter distance** $d_A^{\text{DG}}$.—The relation between the luminosity distance and angular diameter distance expressed by Equation (41) in DG is preserved, and we used it to find the $d_A^{\text{DG}}$ at a given redshift. In the DG perturbative equations, the angular diameter distance appears naturally as $d_A(t_h) = r_B a_L DG(t_h)$. This equation has the same definition given here, evaluated at the last scattering surface. The angular diameter distance is crucial to define the physical meaning of the next equations.

In equations,
\[
d_A^{\text{DG}}(t_h) = c a_L DG(t_h) \int_{t_h}^{t_0} \frac{dt'}{a_L DG(t')} = c \frac{a_L DG(t_h)}{1 + z_h} \int_{t_h}^{t_0} \frac{dt'}{a_L DG(t')}
\]
\[
= c \frac{1}{1 + z_h} \int_{t_h}^{t_0} d\tau \frac{d\tau}{Y_{DG}(\tau)} = \frac{d_H^{\text{DG}}(t_h)}{(1 + z_h)^2}.
\]

**Horizon distance** $d_H^{\text{DG}}$.—We have to consider the effective metric. This will produce the same integrand as Equation (42) but substituting $a(t) \rightarrow Y_{DG}(Y)$. Note that $Y_{DG}$ depends on $Y(t)$. We have to apply the chain rule and also change the integral limits to $\int_0^{Y(t_h)}$. Finally, the horizon distance in DG is given by
\[
d_H^{\text{DG}}(z, L, C) = \sqrt{\frac{1 + C}{1 + z}} 100 h \int_0^{Y(t_h)} c_s \frac{dY}{Y + C Y_{DG}}.
\]

Note 1: The speed of light $c$ has been replaced by $c_s$, where the subscript $s$ represents the sound. This change is introduced because we want to use this equation to calculate the acoustic horizon distance. This acoustic horizon is the maximum distance that a fluid with speed $c_s$ has traveled between redshift $\in (\infty, z)$.

Note 2: Do not confuse $C$ in terms of GR densities that are not physical with physical densities labeled with $DG$ or $DG$. For example, $h^2 \Omega_{r,0}$ is not a physical density.

In the standard cosmology, the speed of sound is given by
\[
c_s^2 = \frac{\delta p}{\delta \rho} = \frac{1}{3} \frac{2}{(1 + R)}.
\]
where $R = \frac{4}{3} \frac{\rho}{\dot{\rho}}$ in GR. We emphasize that delta matter and delta radiation could change this equation. In the simplest case, delta particles do not affect the speed of sound of the fluid because we are assuming that delta particles behave like DM particles; they are noninteracting particles. Neither DM nor the delta particles appear in this equation. However, in DG we use the following definition:
\[
R = \frac{4c_s^2 h^2 \Omega_{DG}}{3h^2 \Omega_{\gamma}^{DG}}.
\]

¹⁹ The $B_\gamma, B_\rho$, and $E_\gamma$ are scalar perturbative terms that appear in the SVT decomposition. For more details see Alfaro et al. (2020).

Now, $R$ is a function of physical densities. We did not include the delta matter or the delta radiation.

Unfortunately, due to all the approximations we have used, we need to add one more correction to the GR sector’s solutions. We considered a sharp transition from the moment when the universe was opaque to when it was transparent. However, this was not instantaneous, yet it could be considered Gaussian. This normal distribution implies an effect known as Landau damping (Landau 1946), and it is related to the dispersion of the distribution of a wave front in a plasma. This consideration is relevant, and it is related to the standard deviation of temperature at the last scattering moment (labeled as $\Delta$). With these considerations, the solutions of the perturbations are given by
\[
\psi_h(t_h) = -\frac{3q^2 h^2 R^2 \delta_t(k)}{5a_L^{DG}(t_h)},
\]
\[
\delta_m(t_h) = \frac{3R^2}{5} \left[ \frac{T(k)(1 + 3R_h) - (1 + R_h)^{-1/2} e^{-q^2 H^2 t_h}}{a_L^{DG}(t_h)} \right],
\]
\[
\delta u_{\gamma X}(t_h) = \frac{3R^2}{5} \left[ -t_h T(k) + \frac{a_L^{DG}(t_h)}{\sqrt{3}(1 + R(t_h)) a_L^{DG}(t_h)} e^{-q^2 H^2 t_h} \right],
\]

where
\[
d_H^{DG}(t_h) = a_L^{DG}(t_h) \int_0^{t_h} \frac{dt}{\sqrt{3}(1 + R(t)) a_L^{DG}(t)} = \frac{a_L^{DG}(t_h)}{a_L^{DG}(t_h) + \Delta(t_h)}.
\]

In order to evaluate the Silk damping, we use
\[
\gamma = \frac{1}{n_e \sigma_T c},
\]

where $n_e$ is the number density of electrons and $\sigma_T$ is the Thomson cross section. On the other hand,
\[
q \int_0^{t_h} c_s dr = q \int_0^{t_h} dt \frac{1}{\sqrt{3}(1 + R(t)) a_L^{DG}(t)} = q \delta_t^{DG}(t_h),
\]
\[
= \frac{q}{a_L^{DG}(t_h)} \cdot (a_L^{DG}(t_h) T_h^{DG}) = \frac{q}{a_L^{DG}(t_h)} \cdot d_H^{DG}(t_h),
\]

where $c_s$ is the speed of sound, $T_h^{DG}$ is the sound horizon radial coordinate, and $d_H^{DG}$ is the horizon distance, and
\[ \kappa = q d_{\ell}^{DG}/a_{DG}(t_b) \] (defined in Equation (88)) implies

\[ d_{T}^{DG}(t_b) = c \frac{\sqrt{2} a_{DG}(t_b)}{a_{EQ}(t_b) H_{0} \Omega_{M}} \]

\[ = \frac{c a_{DG}(t_b)}{100 h \sqrt{C(C + 1)}}. \] (118)

We must include that, in \[ \tau_{\text{reion}} \approx 10 \] (reionization), the neutral hydrogen left over from the time of recombination becomes reionized by ultraviolet light from the first generation of massive stars (Weinberg 2008; Piattella 2018). The photons of the CMB have a small but nonnegligible probability \( 1 - \exp(-2\tau_{\text{reion}}) \) (where \( \tau_{\text{reion}} \) is the optical depth of the reionized plasma) of being scattered by the electrons set free by this reionization. The TT spectrum is a quadratic function of the temperature fluctuations, and then we have to weigh the spectrum by a factor \( \exp(-2\tau_{\text{reion}}) \). Also, we used a standard parameterization of \( R_{\ell}^{0} \) given by

\[ |R_{\ell}^{0}|^2 = N^2 q^{-3} \left( \frac{q}{R_{0}} \right)^{n_s - 1}, \] (119)

where \( n_s \) is the spectral index. It is usual to take \( \kappa_R = 0.05 \) Mpc\(^{-1}\).

All these definitions are consistent. Then, if we use \( q = \beta \ell/n_s \), we obtain

\[ |R_{\ell}^{0}/R_{\ell}|^2 = N^2 \left( \frac{\beta \ell}{R_{b}} \right)^{-3} \left( \frac{\beta \ell}{\kappa_R R_{b}} \right)^{n_s - 1} \]

\[ = N^2 \left( \frac{\beta \ell}{R_{b}} \right)^{-3} \left( \frac{\beta \ell a_{DG}(t_b)}{\kappa_R a_{DG}(t_b)} \right)^{n_s - 1} \]

\[ = N^2 \left( \frac{\beta \ell}{R_{b}} \right)^{-3} \left( \frac{\beta \ell d_{\ell}}{\ell_{b}} \right) \] (120)

Using similar calculations for the other distances, the final forms of the form factors are given by

\[ \mathcal{F}(q) = \frac{R_{\ell}^{0}}{5} [3 T(\beta \ell/\ell_{T}) R_{b} - (1 + R_{b})^{-1/4} e^{-\beta \ell \ell_{T} / \ell_{b}} \times S(\beta \ell/\ell_{T}) \cos(\beta \ell/\ell_{H} + \Delta(\beta \ell/\ell_{T}))], \] (122)

\[ \mathcal{G}(q) = \frac{\sqrt{3} R_{\ell}^{0}}{5 (1 + R_{b})^{1/4} e^{-\beta \ell \ell_{H} / \ell_{b}} S(\beta \ell/\ell_{T}) \times \sin(\beta \ell/\ell_{H} + \Delta(\beta \ell/\ell_{T}))}, \] (123)

where

\[ \ell_{R} = \frac{\kappa_R d_{\ell}^{DG}(t_b)}{a_{DG}(t_b)}, \quad \ell_{H} = \frac{d_{\ell}^{DG}(t_b)}{a_{DG}(t_b)} \]

\[ \ell_{T} = \frac{d_{\ell}^{DG}(t_b)}{d_{T}^{DG}(t_b)} \quad \ell_{\tilde{R}} = \frac{d_{\ell}^{DG}(t_b)}{\tilde{d}_{\ell}^{DG}(t_b)}. \] (124)

To summarize, for reasonably large values of \( \ell \), the CMB multipoles are given by

\[ \ell (\ell + 1) C_{\ell \ell}^{T} = \frac{4 \pi T_{0}^2 e^{3} \exp(-2\tau_{\text{reion}})}{2\pi} \int_{1}^{\infty} \beta d\beta \]

\[ \times \left[ \mathcal{F}(\ell \beta) + \mathcal{F}(\ell \beta / R_{b}) \right]^{2} \] (125)

We emphasize that the structure of Equation (125) considers that the delta sector contributes additively inside the integral. If we set all delta sectors equal to zero, we recover the result for the scalar temperature–temperature multipole coefficients in GR given by Weinberg (2008). Thus, Equation (125) is the main expression to implement the numerical analysis.

The DG contribution appears in many different forms in Equation (125). The most notorious contribution is given by the functions \( \mathcal{F} \) and \( \mathcal{G} \). These functions are given by the functions \( f, r, d, g \) and \( \tilde{f}, \tilde{r}, \tilde{d}, \tilde{g} \) through Equations (90)–(93) and (94)–(97). They are related to the evolution of the perturbation, and all these functions are coupled with the GR solutions.

The standard way to solve this problem is to obtain an analytical solution for the approximated Equations (84)–(86) and solve them for every \( \kappa \) (e.g., from 0 to 100). Finally, match both results numerically, and solve \( T, S, \) and \( \Delta \) as a function of \( \kappa \). These equations evolve the perturbations given by the \( f, r, d, g \) and \( \tilde{f}, \tilde{r}, \tilde{d}, \tilde{g} \) functions, and then they must be evaluated inside the matter regime. They start to evolve inside the matter era, but very close to the radiation era. This parameterization is given by \( y = \mathcal{F} \) and \( \mathcal{G} \). The solutions were obtained starting from \( y < 10^{-34} \) and stopping at \( y \approx 10^{2} \). If the solutions are evaluated after the equality time, they could change, but they are stable after \( y \approx 10^{2} \).

The TT CMB spectrum needs these solutions because they build the form factors, and they are evaluated in an arbitrary \( \kappa \) that is related to \( \beta \) and \( \ell \) through Equation (125). First, we find the results for the numerical solutions of \( f, r, d, g \) and \( \tilde{f}, \tilde{r}, \tilde{d}, \tilde{g} \), and then we solve the expressions \( T, S, \) and \( \Delta \). Then, we calculate the delta perturbations, and finally we obtain the delta form factors. Figures 6 and 7 show the form factors for the background \( \mathcal{F} \) and \( \mathcal{G} \) and for the delta contribution: \( \tilde{\mathcal{F}} \) and \( \tilde{\mathcal{G}} \). Numerically, the delta contribution is \( \approx 10^{-55} \) times smaller than the common form factors; thus, we neglect the \( \tilde{\mathcal{F}} \) and \( \tilde{\mathcal{G}} \) terms.

However, the DG contribution appears in other ways. The next stage is going to be divided into three parts. The first is about the \( \ell_{i} \) factors, the physics behind them, and the dependencies with physical processes. This is the biggest constraint that DG has. The second part is about the algorithm to include all the physical effects and the equations to obtain the TT CMB spectrum. The third and final part is about the results.

### 5.1.1. \( \ell_{R} \)

This coefficient depends on the angular diameter distance and the DG scale factor \( a_{DG} \) evaluated at the last scattering.
time. This term is associated with the $\mathcal{F}$ and $\mathcal{G}$ functions and depends on $n_s$, the spectral index of the primordial spectrum. In the case where the contribution to the delta form factors is $\sim 0$, then the coefficient given by Equation (121) appears as a number powered to $n_s - 1$. This factor appears in Equation (125) in front of the integral and regulates the entire spectrum amplitude. We decided to assume an arbitrary $n_s$ to include the $\ell_R$ coefficient. This assumption is important because, at first glance, these parameters appear to be correlated: $N$, $n_s$, and $\ell_R$. This idea is incorrect because the $\ell_R$ value depends on the last scattering moment, defined by $z_{ls}$, and this redshift appears in many other places of Equation (125). If $z_{ls}$ is not arbitrary, then the coefficient in Equation (121) is unique, and then $N^2$ has to compensate for the scale of the spectrum to fit the observable data. The $\ell_R$ parameter is a function of $z_{ls}$ and $C$.

5.1.2. $\ell_H$

We followed the notation introduced in Weinberg (2008), but the most known notation is $\theta = 1/\ell_H$. If we want to preserve the CMB TT spectrum, we must use a value close to the standard $\theta$, but not strictly the same. In this context, it is essential to remember that in the SN Ia analysis we worked with $C = 0$. This implies that there is no radiation and it is contradictory to the CMB procedure. Nonetheless, the SN Ia analysis is compatible with $C$ small values. Then, we can try to fit the TT CMB spectrum assuming a small $C$ value, where $M \approx -19.3$ and the $H_0$ local value is preserved. We are going to work only in this scenario. Then, the CMB fit assumes a fixed $L_2$ value from SNe Ia (we do not want to change this value) and a $C$ value close to 0. After this process, we have to check that the $C$ value found by this method is compatible with the SN Ia data.

The most notorious constraint from the CMB spectrum is the acoustic peak position. This parameter determines the TT CMB spectrum (in the $\ell$ scale) and fits the hydrodynamic approach to the $\ell$-axis. Also, another important property of $\theta$ is that it is obtained directly from the CMB spectrum. It’s not a derived parameter (Planck Collaboration et al. 2020):

$$100\theta_{\text{Planck}} = 1.0411 \pm 0.0003.$$  \hfill (126)

This value almost always appears in the literature as $\theta_{MC}$, where it was obtained by fitting the CMB data. However, in this work we calculate $\ell_H = 1/\theta$ as a function of $d_{h}^{DG}$ and $d_{A}^{DG}$. In our case, $\theta$ is not constraining the peak position by itself; we are constraining the $z_{ls}$, $C$, and $h^2\Theta_{b,0}$ values.

The physical meaning of this parameter is as follows: the angle that subtends the size of fluctuation with respect to the distance to this fluctuation. $d_H^{DG}$ is the horizon distance (size of the universe at a specific redshift given by when the photons were decoupled). $d_A^{DG}$ is the angular diameter distance between us and the TT CMB fluctuation. This relation must be corrected changing the speed of light $c$ by $c_s$ (the speed of sound) because it is the growing fluctuation speed (Planck Collaboration et al. 2016, 2020). The correction has been introduced in Equations (107) and (109).

The Fourier modes give an easy way to understand the dependence between $\theta$ and $\ell$. For simplicity, in a flat universe, the modes of wavelength $\lambda \sim 2\pi a(h_0)/k$ on the last scattering surface seen today under an angle $\theta = \lambda/d_A(h_0)/2\pi/\ell$ (the factor of 2 appears because, for a given multipole, $\pi/\ell$ gives the angle between a maximum and a minimum; this is half of the wavelength of the perturbation on the surface; Lesgourgues et al. 2013). This position of the peak is very well determined; then, this parameter is very well constrained. This condition imposes constraints over $C$ or $z_{ls}$ or $c_s$ (the speed of sound in a specific period; from $z = \infty$ to $z_{ls}$). In this analysis $L_2$ is fixed and is independent of any other value that we are changing.

From Equation (108) and knowing $R$, we can obtain the $d_{H}^{DG}(c)$ value in order to calculate $\theta$. As we have seen, $R$ is the baryon–photon relation. This factor considers particles that interact with the fluid, and then the physical phenomena are described as sound waves. We can change this parameter if we suppose that more components interact in the fluid. But we assume only the case where the photon–baryon relation determines the horizon distance.

The $R$ relation to calculate the speed of sound is determined with $h^2\Theta_{b,0}$ and $h^2\Theta_{c,0}^{DG}$ values. This is very important because these parameters are physical and not apparent magnitudes. First of all, they depend on $Y_{DG}$ and not directly on $Y$. Second, they are physical magnitudes, they represent the real density of energy per volume, and then the interactions determine the physical speed of sound.

The CMB radiation gives the physical density of photons: the blackbody spectrum has the associated $T_0$ temperature, where the real density is described as $\rho_b \propto T_0^4$ (Stefan–Boltzmann law). We know that the real physical densities in DG evolve with $Y_{DG}$, and then it is easy to evolve any physical
parameter as a function of $Y_{DG}$. The $\ell_T$ parameter is a function of $z_{DG}$, $C$, and $\hbar^2 \Omega_{DG,0}$.

5.1.3. $\ell_T$

The $\ell_T$ parameter appears also inside of $\cos$ and $\sin$ functions in Equations (122) and (123). Nevertheless, they move the $\cos$ and $\sin$ on the horizontal axis through the $\Delta$ transfer function. They also appear outside the sinusoidal solutions, regulating the amplitude of these oscillations. The role of these parameters is to convert the arguments of the transfer functions into the correct units. The origin of this normalization comes from Equations (88) and (118). Those definitions are important because it implies that $d\ell_T \propto a_{DG}(t_{ls})$, where $z_{ls}$ determines the DG scale factor at the moment of the last scattering. This normalization of the wavenumber appears until this step of the numerical evaluation.

To evaluate this function, first we solve $Y$ as a function of $z_{ls}$ and then evaluate $a_{DG}(t_{ls})$. Finally, it returns $d\ell_T^{DG}/d\ell_T^{DG}$ for that particular combination of $z_{ls}$ and $C$. Remember that the $\ell_T$ parameter modulates the position and the amplitude of the $\sin$ and $\cos$ functions. Thus, it is not trivial to know whether this parameter is degenerated with another. Also, this is the only parameter that appears as an argument for the transfer functions. Then, the result depends on the numerical solution of the transfer functions. The $T$, $S$, and $\Delta$ functions can be solved numerically from the differential equations given by Equations (90)–(93) and the $T$, $S$, and $\Delta$ definitions. The $\ell_T$ parameter is a function of $z_{ls}$ and $C$.

5.1.4. $\ell_D$

Finally, the fourth parameter includes many steps that are related to physical processes. This parameter appears as a result of the physical damping of the oscillations, which is related to both processes: Silk and Landau dampings. These effects only appear next to every $\cos$ and $\sin$ function in Equation (125) as an exponential. The TT CMB spectrum is very sensitive to this value because it changes the whole spectrum’s amplitude.

First, the Silk damping is described by a special relativistic nonperfect fluid. This approximation implies damping. The cosmology part appears when the damping effect acts on a range of time, and the effect must be integrated and corrected by the expanding universe. The expression that describes the Silk damping is Equation (114), where the cosmological correction appears with $Y_{DG}$.

Second, the calculation of Landau damping is challenging. Despite that Equation (115) is very short, its intrinsic relation with the dispersion of the temperature creates many calculations. $\sigma_T$ is the standard deviation of the temperature at the last scattering moment when the transparency is a normal distribution function centered around the $z_{ls}$. This is a good approximation, but it requires many calculations provided by interactions related to the free electrons and photons. In terms of the dispersion,

$$\sigma_T = \frac{\sigma_T}{TH_{DG}},$$

(127)

because

$$\sigma_T dt = 2\sigma_T dT \rightarrow \frac{dt}{dT} = \frac{dY}{dY_{DG}} \frac{dY_{DG}}{dT} \rightarrow \frac{dt}{dT} = \frac{1}{H_{DG}T}.$$}

With this transformation, we can express the time dispersion in terms of temperature. To obtain the temperature dispersion, first, we have to find the visibility function in DG, and before that, we have to define the opacity function. This function is (Weinberg 2008, p. 125)

$$O(T) = 1 - \exp \left(-\int_{t_1}^{t_0} c\sigma_{Thomson} n_e(t) dt\right).$$

(128)

Another essential physical definition is the visibility function given by $O(T)$, which describes the probability that the last scattering of a photon was at a temperature between $T$ and $T - dT$. It behaves like a probability distribution, and then we try to find a normal distribution and obtain an estimation of $\sigma_T$ using the visibility function calculated $O(T)$,

$$O_{fit}(T) \approx \frac{1}{\sigma_T \sqrt{2\pi}} e^{-\frac{(T - T_0)^2}{2\sigma_T^2}}.$$  \hspace{1cm} \text{(129)}$$

To obtain the $\sigma_T$ value, we evaluated the maximum of the distribution, where the $O'(T_{max}) \approx \frac{1}{\sigma_T \sqrt{2\pi}}$.

To calculate the opacity function, we have to know the physical electron density at that epoch. This is strictly related to the $H, e^-$, and $p$ abundances at that moment. These values can be easily correlated using an equation that describes the formation of the $H$. There are many methods to do this calculation. The most naive approximation is assuming an equilibrium through the Saha equation. The equilibrium involves only atomic parameters, and it does not depend on cosmological parameters. Then, any assumption and equation in this calculation are preserved in DG. We emphasize that the evolution is given in terms of $T$. Furthermore, the relation between $T$ and $z$ in DG is the same as in GR. Then, this procedure is totally preserved. In order to clarify any doubt, we are going to show the general scheme.

The naive approximation (Weinberg 2008, p. 113) begins at a time early enough so that protons, electrons, hydrogen atoms, and helium atoms were in thermal equilibrium at the radiation’s temperature. Then, the number density of any nonrelativistic nondegenerate particle of type $i$ is given by the Maxwell–Boltzmann distribution:

$$n_i = \frac{g_i}{(2\pi \hbar)^3} e^{\frac{\mu_i}{kT}} \int d^3q e^{-\frac{m_i^2 + q^2}{2m}},$$

(130)

where $m_i$ is the particle mass, $g_i$ is the number of its spin states, and $\mu_i$ is the chemical potential of particles of type $i$. $g_p = g_e = 2$ while the $1s$ ground state of $H$ has two hyperfine states with spins 0 and 1, so $g_{1s} = 1 + 3 = 4$. The most dominant reaction is given by $p + e \rightarrow H_{1s}$. The equilibrium is described by

$$\mu_p + \mu_e \equiv \mu_{1s}.$$  \hspace{1cm} \text{(131)}$$

Then, the relation between the density numbers is described by

$$\frac{n_{1s}}{n_p n_e} = \left(\frac{m_i k_B T}{2\pi \hbar^2}\right)^{3/2} \frac{\mu_i}{e^{\mu_i/kT}},$$

(132)
where $B_1 \equiv m_p + m_e - m_H = 13.6 \text{ eV}$ is the binding energy of the 1s ground state of hydrogen. Now, including that $n_e = n_p$ because the universe has to be neutral, and also considering that 76% of the baryons were neutral or ionized hydrogen, $n_p + n_{1s} = 0.76n_B$ (Weinberg 2008, p. 114), we can define the fractional hydrogen ionization as $X = n_p/(n_p + n_{1s})$, where the Saha equation is satisfied as

$$X(1 + SX) = 1.$$  \hspace{1cm} (133)

Finally, $S$ can be expressed as

$$S = \frac{(n_p + n_{1s})n_{1s}}{n_p^2} = 0.76n_B \left( \frac{m_e k_b T}{2 \pi h^2} \right)^{-3/2} e^{B_1/k_b T}.$$  \hspace{1cm} (134)

Note that $S$ can be expressed in terms of $T$ (K) and $h^2\Omega_{b,0}$ as

$$S = 1.747 \times 10^{-22} e^{(57804/T)^{3/2}} h^2\Omega_{b,0}.$$  \hspace{1cm} (135)

This dependence is significant for DG. First of all, the evolution is in terms of $T$ and not cosmic time, and also the fraction $S$ depends on the baryon density parameter $h^2\Omega_{b,0}$, and then it will appear as a free parameter in the TT CMB spectrum. In DG, as we have said, the effect of delta-fields does not affect the spectrum (they are minimal). Only the evolution in time, represented by distances, can be affected by DG.

To improve the calculation, it is possible to add more corrections, including the 2p and 2s levels of the H atom. The full discussion about the decay and the emission processes can be found in Weinberg (2008, p. 116).

The differential equation that describes this process with all those corrections is given by

$$\frac{dX}{dT} = \frac{\alpha n}{H^{DG} T} \left( 1 + \frac{\beta}{\Gamma_{2s} + \lambda_{2s}(1-X)} \right)^{-1} \left( X^2 - \frac{1 - X}{S} \right),$$  \hspace{1cm} (136)

where $n = n(h^2\Omega_{b,0}, T)$, $H^{DG} = H^{DG}(C, L_2, Y(T))$, and $\lambda = \alpha(T)$, $\beta = \beta(T)$ are functions related to the transitions of the $H$.\footnote{For more details, see Weinberg (2008).} This equation depends on the Hubble parameter: $H^{DG}$. This is important because in the derivation of this equation $H^{DG}$ appears in two different places: the first term $1/TH^{DG}$ is a coefficient that comes from changing $t$ to $T$ (to evolve the equations in temperature instead of time), and the second term (where $H^{DG}$ appears as $8\pi H^{DG}$) comes from the change of the frequency (or wavelength) produced by the cosmic expansion. Therefore, both of those corrections appear in DG as $H^{DG}$ and not like the standard $H$ (then, this equation looks similar, but it is different because the dependence between the variables is totally different; Weinberg 2008, p. 122).

In DG, this effect could be crucial because the evolution could change because the Hubble parameter is a function of the effective scale factor $Y_{DG}$, and this is a function of $Y(t)$. Furthermore, $T$ preserves the standard dependence with the effective scale factor $Y_{DG}$; in other words, in standard cosmology, we have $T = T_0(1 + z)$, and this relation is preserved in DG, but the dependence on $z$ in DG appears related to $\theta_{DG}(Y(t))$. Furthermore, the numerical solution with all those corrections changes the Saha approximation, and then it also changes the GR solution. It is also essential to note that the differential equations are solved in a high range of $T$, and DG tends to be very similar to the standard GR at the beginning. The scale factor tends to be the same because the delta-field contributions disappear when $Y \to 0$. Nevertheless, all these aspects must be taken into account to compute $X(T)$ in order to obtain an excellent numerical value to fix $z_{\alpha}$ and $n_e$ affecting the visibility function: the peak position in redshift ($z_{\alpha}$) and the standard deviation ($\sigma_z$). We remark that the $\alpha(T)$ and $\beta(T)$ are numerical functions of $T$ (Pequignot et al. 1991) and there is no cosmological influence here; hence, it does not affect the DG calculations. The visibility is a function of $C$ and $h^2\Omega_{b,0}$.

This function is essential to find the $z_{\alpha}$ because the peak is associated with the $z_{\alpha}$. The $\ell_D$ parameter is a function of $z_{\alpha}$, $C$, and $h^2\Omega_{b,0}$.

5.1.5. Algorithm to Obtain the CMB

The MCMC algorithm consists of a modified adaptive Metropolis MCMC algorithm. We used the TT spectrum from Planck Collaboration et al. (2020).\footnote{The data were obtained from https://pla.esac.esa.int/#cosmology.}

In our case, we want to find all the possible values that match, in the best way, the TT CMB spectrum. The algorithm works as follows: we propose an original distribution of values, called priors: $C$, $h^2\Omega_{b,0}$, $z_{\alpha}$, $n_e$, and $N$, which are all normally distributed. Then, we calculate the predicted TT CMB spectrum and compare with the TT CMB spectrum from Planck Collaboration et al. (2020). The likelihood is defined as usual, based on the squared error.

We introduced a modification to give more flexibility in the $z_{\alpha}$ fitting. We did not want to force the system to choose a $z_{\alpha}$ exactly in the peak position of the visibility function; therefore, we create a proposal distribution centered in the peak of the visibility function, and then MCMC takes that prior and moves it around the peak. With this method, we give more freedom to the $z_{\alpha}$ parameter, and the final posterior probability associated with this parameter could be slightly different from the peak of $O'(T)$. All the other parameters were found as the standard Metropolis MCMC.

6. Results

First of all, we clarify that all the chains always converged to the same values; all are independent of the prior distributions. Now, we present the results. This corresponds to a chain with 20,000 steps for every parameter.

The posterior distributions for every parameter are shown in Figure 8 in the diagonal. All the distributions show only one peak, but some of them are not normally distributed. We specify the case of $h^2\Omega_{b,0}$ and $n_e$. These parameters show multimodal distributions, but always with a clearly main peak. We fit in both cases a normal distribution, but the error was defined such that the $\sigma_\alpha$ includes the smallest multimodal distributions with its errors. Then, all the parameters have errors defined as $\pm 1\sigma_\alpha$ with the exception of the baryon density parameter, which is $h^2\Omega_{b,0.0.0.2\sigma}$, and the spectral index given by $n_{\alpha.2\sigma}$\footnote{The errors associated with the baryon density term and the spectral index term are different from $\pm 1\sigma$ because the distributions of these values are not normal. We define the errors such that more than 68% of the distribution is within the mean $\pm$ error.}.

Figure 8 shows all the combinations for the five free parameters. All the parameters are constrained to a normal-like...
distribution, and they are independent of each other. Then, the shape of the TT CMB spectrum constrains all the parameters to “accurate” values. The fitted curve is shown in Figure 9.

These results are good according to the approximation given by Weinberg (2008). This analytic and hydrodynamic approach shows a good fit for the most prominent three peaks, including the acoustic peak, but it is inaccurate at larger multipoles. The numerical results of the MCMC (Figure 8) are also shown in Table 4, where we included the averages and errors for the fitted parameters. Figure 9 shows that the DG prediction is very similar to the observable data, but the prediction is inaccurate from the third peak on. However, the precision of the approximation includes that error scale. In Weinberg (2008) the TT CMB spectrum has a similar error, and the differences also appear at larger multipoles. The DG TT CMB spectrum (green line) calculated with this approach is very similar to the spectrum calculated with the ΛCDM model (red line).

Two important aspects must be checked: the $C$ value and the visibility function peak compatibility with the $z_{ls}$ needed to fit the TT CMB spectrum. With respect to the $C$ value, the TT CMB spectrum fixes this value around $C = 4.6 \times 10^{-4}$. This result is completely in concordance with SN Ia results. The $C$ parameter is small enough that the SN Ia analysis cannot detect a difference between 0 and $\approx 10^{-4}$. Then, the $M$ and $H_0$ observables obtained from Riess et al. (2016, 2018, 2019) are in concordance with our results, assuming a standard error in the approximation of the hydrodynamic approach similar to GR.

Figure 8. Contour plot for all posterior probabilities associated with the DG parameters.
In the last scattering redshift case, we have to check whether $z_{ls}$ is close to the visibility function peak. Figure 10 shows how the fraction of free electrons $X$ depends on $T$ and $z$. At lower temperatures $X \rightarrow 0$, while at higher temperatures $X \rightarrow 1$. The $X$ function depends on $C$, $b^2\Omega_{b,0}^D$, and $T$, where the MCMC results have fixed the two first parameters. This case is shown in Figure 10. Then, the visibility function has a maximum close to $T_{294}^{max}K(z_{1078}^{max})$ with a temperature dispersion $\sigma_T \approx 244K$. This function is shown in Figure 11. Furthermore, we add a normal distribution centered at the same peak to show the similarity between the visibility function and a normal distribution.

The $\sigma_T$ was estimated from the height of the peak (not by fitting a distribution, FWHM, or any other method).

The GR case (Weinberg 2008) finds $T_{max} \approx 2941K$ with a $\sigma_T \approx 248K$, while the DG peak around $z \approx 1078$ is near the MCMC result $z_{ls} \approx 1075$. Despite that $z_{ls}$ was obtained varying the redshift around the peak estimation, the $z_{ls}$ is not exactly the peak associated with the visibility function.

Finally, the density of matter and radiation is related to the $C$ and $L_2$ values through the definition of the physical densities. In GR, the equality moment is vital because the hydrodynamic approach uses equality to match the equations when the universe was dominated by radiation and dominated by matter. In the case of GR, it naturally appears that

$$\frac{\rho_{GR,r}}{\rho_{GR,m}} = \frac{Y}{C},$$

where $C = \Omega_{r,0} / \Omega_{m,0}$ by definition. Then, the moment of equality in GR corresponds to $Y_{EQ} = C$. But for DG densities, the physical densities depend on $Y_{DG}$, and thus

$$\frac{\rho_{DG,m}}{\rho_{DG,r}} = \frac{Y_{DG}}{C_{DG}},$$

where $C_{DG} = \Omega_{r,0}^{DG} / \Omega_{m,0}^{DG}$. In DG, we imposed that the equality moment must occur in both sectors at the same time. In other words,

$$Y_{DG}(Y_{EQ}) = C_{DG} \rightarrow C_{DG} = C \sqrt{\frac{1 + F(C)}{1 + 3F(C)}},$$

From the MCMC results, we know that $C \ll 1$ and $L_2 \approx 0.45$, and then

$$C_{DG} \approx C \sqrt{\frac{1 - L_2}{1 - L_2/3}}.$$  

This result is useful because if we know the physical density of radiation, we can find the physical density of matter. Then,

$$C_{DG} \approx C \sqrt{\frac{1 - L_2}{1 - L_2/3}} \approx 0.80C \approx 3.7 \times 10^{-4}. \quad (141)$$

Note: To be clear, in the next calculations we emphasize the observable (physical) densities with a DG sub- or superscript.

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**Table 4**

| Parameter      | MCMC Fit Results for the DG Free Parameters | Standard Deviation |
|----------------|---------------------------------------------|-------------------|
| $z_{ls}$       | 1075.3                                      | 9.4               |
| $C$            | $4.6 \times 10^{-4}$                        | $0.3 \times 10^{-4}$ |
| $b^2\Omega_{b,0}^D$ | 0.026                                       | 0.002             |
| $n_s$          | 1.09                                        | 0.08              |
| $N$            | $1.34 \times 10^{-5}$                      | $0.04 \times 10^{-5}$ |

Note. These values are related to posterior distributions.
To calculate the physical densities, we can use the photon density given by the integrated blackbody spectrum (based on the TT CMB spectrum):

$$\rho_{\gamma,0} c^2 = a_B T_0^4,$$

where

$$a_B = \frac{8\pi^5 k_B^4}{15\hbar^3 c^3} = 7.56577 \times 10^{-16} \text{ J m}^{-3} \text{K}^{-4} \quad (143)$$

is the radiation energy constant. With $T_0 = 2.7255 \text{ K}$, we get the today density associated with the photons $\rho_{\gamma,0}^{DG} = a_B T_0^4 / c^2 = 4.64511 \times 10^{-31} \text{ kg m}^3$. This is a physical quantity.

The neutrino density (physical quantity) is related to the photon density as follows (Planck Collaboration et al. 2014):

$$\rho_{\nu,0}^{DG} = N_{\text{eff}}^{\text{Planck}} \left( \frac{4}{11} \right)^{4/3} \rho_{\gamma,0}^{DG},$$

where $N_{\text{eff}}^{\text{Planck}} = 3.04678$ (Planck Collaboration et al. 2020). The relation given by Equation (144) is based on statistical mechanics: photons and neutrinos are in thermal equilibrium,

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure10}
\caption{\(x(T)\) fraction as a function of temperature $T$ and redshift $z$ assuming $C$ and $h T_{0,0}^{\text{DG}}$ MCMC results. This fraction allows us to calculate the visibility function for every combination of $C$ and $h T_{0,0}^{\text{DG}}$.}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure11}
\caption{In blue color, the visibility function is associated with the $x(T)$ obtained from the MCMC results. The orange line is a normal distribution centered in the peak of the DG solution. Every combination of parameters changes the peak of the visibility function, affecting the redshift of the transition and the standard deviation of the distribution.}
\end{figure}
but neutrinos are fermions and photons are bosons. Thus,
\[ \rho_{r,0}^{DG} = 3.21334 \times 10^{-31} \text{ kg m}^{-3}, \]  \hfill (145)
and the total radiation density (physical quantity) is given by
\[ \rho_{r,0} = \rho_{r,0}^{DG} + \rho_{\nu,0} = 7.85846 \times 10^{-31} \text{ kg m}^{-3}. \]  \hfill (146)

Until now, we have assumed that neutrinos are relativistic particles and contribute to the radiation density. We can also write these values divided by the critical density given by
\[ \rho_{c,0} = \frac{3H_0^2}{8\pi G} = 1.87847h^2 \times 10^{-26} \text{ kg m}^{-3}, \]  \hfill (147)
where the GR Hubble constant has been expressed in terms of the dimensionless parameter \( h \), where \( H_0 = 100h \text{ km s}^{-1}\text{ Mpc}^{-1}. \) Therefore, the density parameters are (these are physical; we emphasize that the \( h \) constant is simplified, and thus these parameters are independent of \( h \))
\[ h^2\Omega_{r,0}^{DG} = \frac{\rho_{r,0}^{DG}}{\rho_{c,0}} = 2.47 \times 10^{-5}, \]  \hfill (148)
\[ h^2\Omega_{\nu,0}^{DG} = \frac{\rho_{\nu,0}^{DG}}{\rho_{c,0}} = 1.71 \times 10^{-5}, \]  \hfill (149)
and (where \( \text{cdm} \) is “cold dark matter”)
\[ h^2\Omega_{\text{cdm},0}^{DG} \equiv h^2\Omega_{\text{b},0}^{DG} + h^2\Omega_{\text{cdm},0} + (3 - N_{\text{eff}})h^2\Omega_{r,0}^{DG} \approx h^2\Omega_{\text{b},0}^{DG} + h^2\Omega_{\text{cdm},0}. \]  \hfill (150)
Finally, we assume that \( N_{\text{eff}} = 3 \) (we emphasize, again, that \( h^2\Omega_{r,0}^{DG} \) quantities are not related to \( H_0 \); they are related only to the physical density and \( 3 \times 10^{10}/8\pi G \)), and the quantities are
\[ h^2\Omega_{r,0} = 4.18 \times 10^{-5}, \]  \hfill (151)
\[ h^2\Omega_{\text{b},0} = 0.113, \]  \hfill (152)
\[ h^2\Omega_{\text{cdm},0} \equiv h^2\Omega_{\text{b},0} - h^2\Omega_{r,0}^{DG} = 0.087. \]  \hfill (153)

7. Conclusions

We have studied the cosmological implications for a modified gravity theory named DG. The results from SN Ia analysis indicate that DG explains the accelerating expansion of the universe without \( \Lambda \) or anything like DE. The DG equations naturally produce the acceleration. In this work we performed a fit to the SN Ia data considering three free parameters \( M, C, \) and \( L_2 \), finding that \( C \) is not relevant if it is small enough. Also, we found that \( L_2 \approx 0.457 \) and \( h \approx 0.496 \), where \( h \) is not the Hubble constant. Regarding \( L_2 \), this parameter establishes the acceleration of the universe and is independent of \( M \), where \( M \) is degenerated with \( h \). In this case, the universe is accelerating as a result of \( L_2 > 0 \) and implying that a new kind of densities called delta matter and radiation

must exist. These can be associated with the new delta-fields. It is not clear whether this delta composition is made of real particles or not. However, we propose two different interpretations. The first is that the universe only contains matter (baryonic matter and CDM) and radiation where the delta sector is only a geometric effect. The other scenario is that the universe also contains delta matter and delta radiation because they are particles. In both scenarios, the universe shows the same behavior, and it is accelerating, but the difference is that in the first case the delta sector could be invisible because the geometry provides the fundamental physics behind the delta sector and not the particles. This is part of the interpretation, and for now we cannot conclude more about this aspect.

Regarding the TT CMB spectrum, we used five free parameters to fit it: \( C, h^2\Omega_{\text{b},0}, n_s, \) and \( N \).

The first peak is very well determined in position and shape, but not the other two peaks. In the GR case, they tend to be modulated by the DM and baryon density (Lewis et al. 2000). Nevertheless, in the hydrodynamic approach (Weinberg 2008), the DM evolution is assumed as dominant considering that the entire gravitational potential is driven by DM. This approximation is useful because the equations are easy to solve; however, it is not accurate according to Weinberg (2008, p. 358): this approach introduced 10% errors or less in the GR case. In DG we used the same approximation and obtain a very similar result. Despite this approximation, the TT CMB spectrum is very well described, but the large multipoles show deviations from the observable data. It is vital to consider that the fitted values were obtained from an approximation called the hydrodynamic approach, and then the numerical values contain intrinsic errors associated with the approximations; hence, they are not accurate. Nonetheless, these values are very similar to the GR case.

The \( z_h \) obtained from the MCMC is compatible with the transition range shown in Figure 10 and the peak of the visibility function shown in Figure 11. The amount of baryonic matter given by \( h^2\Omega_{\text{b},0} = 0.026 \) is close to the GR case: 0.022. It is important to contrast this value with other measurements, especially because DG has a very different description of the universe, where the distances are calculated with other equations. Then, other observational constraints must be examined meticulously in order to determine whether DG fits those observations.

The parameters related to the primordial spectrum, \( A \) and \( n_s \), are close to the standard values; the spectral index is close to 1, and the amplitude is \( \sim 10^{-5} \).

An assumption that is essential for the entire CMB analysis is that the plasma fluid, which is described with the speed of sound \( c_s \) within the horizon radius, is only affected by baryons and radiation. This aspect could indicate that delta components do not interact with common radiation and matter, but it would be interesting to analyze all the changes that introduce a delta sector that interacts with common matter and radiation. This aspect may change many approximations and, hence, could affect enormously the TT CMB spectrum. This could be part of a future research.

The observable rate of expansion of the universe in DG is given by \( H_0^{DG} \). This parameter is determined by \( L_2 \) and \( h \). In the context of the TT CMB analysis, if \( C \) is very small, then the SN Ia observations can be compatible with the TT CMB spectrum.
The results show that $C \sim 10^{-4}$. In this regime, the SNe Ia are not affected, and the compatibility between both observations is possible. It is important to emphasize that there are two values that are different. One is $h$, which is provided from the GR background, and the second is $H_0^{DG}$, which is the observable Hubble constant in this model.

A relevant cosmological value that can be constrained from the observations is the age of the universe. The higher the Hubble constant, the lower the age of the universe. This relation is vital since if the local fit of SNe radically changes $H_0$, then the age of the universe changes. Therefore, they could conflict with some estimates of the age of the universe that are independent of cosmology. We note the fact that, according to local measurements of SNe, the ages of the universe for DG and GR are 13.1 and 13.0 Gyr, respectively. Instead, Planck’s data imply a larger age of the universe: 13.8 Gyr. A crucial and consistent with the Delta sector and GR are 13.1 and 13.0 Gyr, respectively. Instead, Planck’s data imply a larger age of the universe: 13.8 Gyr. A crucial and fundamental result, which is independent of cosmology, indicates that the universe has to be older than 13.6 ± 0.8 Gyr. DG and GR, assuming the results of the local measurements of SNe, are on the verge of this observational constraint. According to this, one wonders whether SNe can in conflict with the age of the universe. It is a very recent discussion, and we are only commenting on the problems when astrophysicists try to make SNe and CMB compatible. We emphasize that the problem goes beyond DG because a high Hubble constant causes it, and it also involves other types of measurements that yield high values of the Hubble constant. This discrepancy could be caused by the calibrations and methods used by Riess et al., but this tension between both observations has been widely discussed, and until now there is no agreement. Other researchers have even tried to measure the $H_0$ value using methods independent of distance ladders and the CMB. They found that the Hubble constant exceeds the Planck results, with the confidence of 95% (Pesce et al. 2020). However, other measurements based on the TRGB have found that $H_0$ is close to 69.6 km Mpc$^{-1}$ s$^{-1}$ (Freedman et al. 2019, 2020). Other methods based on lensed quasars found that $H_0 = 73.3$ Mpc km$^{-1}$ s$^{-1}$ agrees with local measurements but is in tension with Planck observations (Wong et al. 2020).

The entire TT CMB spectrum analysis was made in the DG context, were the delta contributions represented by $\delta G$ and $\delta \mathcal{G}$ can be neglected. This is an essential part of the development of the perturbation theory, and it implies many simplifications when we want to calculate the spectrum and creates more constraints on the spectrum fitting.

To summarize, DG requires more development to compare with other constraints such as the He produced at the Big Bang nucleosynthesis, or the BAO constraints, or even cosmological simulations. This last aspect could be relevant if the interpretation of the delta sector is given in terms of particles that create gravitational interactions. In fact, at the Newtonian limit, the dilaton appears as a new source of the gravitational potential (Alfaro & González 2019b). Finally, it is remarkable that DG finds a well-behaved TT CMB spectrum, where it is possible to constrain new parameters, even related to inflation. However, this analysis does not use all the numerical precision, because the equations are only an approximation, and even more, we are calculating only the scalar contributions to the total TT CMB spectrum. Furthermore, many other sources that contribute to the “spectrum” have been avoided to simplify the analytical solution, such as the Sachs–Wolfe effect or lensing. This is only a first-order approximation, and it shows that DG could fit the TT CMB spectrum, but it is essential to fit the spectrum with all the numerical precision without approximations because the conclusions drawn in that case could be different. Thus, these numerical results must be understood as values that are near the correct value, not as a final and undeniable result.

The incompatibility between the SNe Ia and CMB occurs when the $\Lambda$CDM model is constrained using BAOs and SNe Ia. Even when the model uses curvature, if all the parameters describe the same universe, the whole model must be compatible with only one geometry given by $\Omega_\Lambda$. For example, recently an article was published that shows a discrepancy between the Planck data (Planck Collaboration et al. 2020). These differences can be caused by the assumption that the universe is flat. Despite this curvature assumption in the $\Lambda$CDM model, the cosmological parameters are incompatible because some of them are compatible with a flat universe, but others indicate a closed universe (Di Valentino et al. 2019). Furthermore, regarding the SN Ia analysis, another article shows an anisotropy in the SN Ia distribution, and then the acceleration measurement could be wrong (Colin et al. 2019). The entire DG analysis could change because the $L_2$ value will be different, and all the distances would change (Kang et al. 2019). In this context, it is relevant to emphasize that there are many approximations in our procedure, and DG must be contrasted with other observations to determine with a good precision whether this model is a solution for today’s paradigm. BAOs could be an excellent option to verify the model, mainly because these observations are related to the angular diameter distance and could constrain the DG model and verify that DG can survive to describe SNe Ia and BAOs.

Despite these interpretations, problems, and approximations, DG can fit both SN Ia and TT CMB spectrum data, without DE, but it is very necessary to include the complete numerical solutions without approximations to be able to conclude with certainty that DG can explain both phenomena.

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