A Loophole in an Argument for Interpreting Classical Physics Indeterministically

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An indeterministic interpretation of classical physics has been proposed recently, in which the argument relies on the claim that real numbers cannot represent physical reality. This paper aims at showing that the defenses for this claim are fallacious, and hence the argument has a loophole.

I. INTRODUCTION

An argument for an indeterministic interpretation of classical physics (i.e., Newton's mechanics and Maxwell's electrodynamics) was put forth by Gisin and Del Santo in [1] (see also [2] [3] [4] and [6]). They maintained that although classical physics has traditionally been construed as deterministic (i.e., the physical laws determine a unique definite future (and past) state of a physical system once its current state is fixed, as famously revealed in the scenario of "Laplace's Demon"), it is not necessarily the case. There are metaphysical assumptions behind the traditional deterministic interpretation, and it is possible to give an alternative indeterministic interpretation by revising those assumptions, they contended. In particular, the usual practice that real numbers are used to represent physical quantities was held to be problematic, because this would lead to the unacceptable consequence of "infinite information density" (as related to the infinite string of digits following the decimal point of a real number) in the relevant physical space, according to them. In this paper, I argue that their attempt to establish such a consequence of infinite information density has failed, and therefore their argument for interpreting classical physics indeterministically has a loophole.

II. THE LOOPHOLE

[1] has referred [2] and [5] for detailed explanations of the alleged consequence of infinite information density. Let us first examine the latter.

In [5], Dowek said:

In Newtonian theory, just like information travels at an infinite velocity because the motion of a mass induces an instantaneous modification of the gravitational field in the whole Universe, an object as simple as a pencil contains an infinite amount of information, because its length is a real number, containing an infinite number of digits. (p. 347)

This was supposed to cause trouble, namely, an infinite information density would result because a pencil occupies only a finite volume of space. But why is the length of a pencil a real number? According to Dowek:

This idea that a magnitude, such as the length of a pencil, is a real number comes from an idealization of the process of measurement. The measure of the length of a segment, for instance the length of a pencil, is defined as the number of times a yardstick fits in this segment. More precisely, this natural number is a lower approximation of the length of the segment, with an accuracy which is the length of the yardstick—or twice this length, if the last fit is uncertain. A more precise measure is obtained by dividing this yardstick in ten equal parts, and counting the number of tenths of yardsticks that fit in the segment. Dividing again this tenth of yardstick in ten parts, an even more precise measure is obtained, and so on. The result of each individual measurement is thus a rational number, and only the hypothetical possibility to repeat this process indefinitely leads to the idea that the measured magnitude, per se, is the limit of an bounded increasing sequence of rational numbers, that is a real number. (p. 348)

To Dowek's opinions, my first reply is that I find the saying that "an object as simple as a pencil contains an infinite amount of information, because its length is a real number, containing an infinite number of digits" problematic. First of all, the numerical value of the length of a pencil depends on what unit is used to measure that length. In principle, that unit can have an infinite number of possibilities, and for some units, that length would not be a number with an infinite number of digits (a trivial example: taking the pencil itself as the basic unit, then its length would be 1).

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Moreover, even if a unit, under which the pencil’s length would be a number with an infinite number of digits, is used, then would the pencil contain an infinite amount of information? In fact, it is weird to assume that the pencil contains that information. As I already pointed out, there is in principle an infinitely many possible numbers (under various units) associated with the length of the pencil, then should we say that the pencil contains all the information associated with all these numbers? Obviously, this is absurd: if we think so, then the pencil would contain an infinite amount of information anyway, regardless of whether those numbers have an infinite number of digits or not. So, then why assuming the pencil contains a particular piece of information associated with a particular unit? No reason, seemingly. Even if that unit is actually used to measure the length, then the actually measured value would not be a number with an infinite number of digits, on the one hand, and that value is not contained by the pencil, but, say, written on a piece of paper as it is produced, on the other hand.

Hence, there is some confusion here, which I think may be clarified by making the distinction between “potential information” and “actual information”. When we say that under certain unit, the pencil’s length may be a number with an infinite number of digits, the concrete value of the number is not yet measured and produced, and therefore there is no question of storing the actual information of that number in physical space. What we talk about is only a potential piece of information about that number, and potential information occupies no space. As a result, Dowek has failed to give us reason to think that “an object as simple as a pencil contains an infinite amount of information”.

Let us turn to examine the detailed explanation of the alleged consequence of infinite information density in [2]. In Section 4, “A Finite Volume in Space Contains at Most Finite Information”, of that paper, Gisin opened with these two remarks: “Here I present an argument supporting the claim that real numbers cannot represent anything physical. This argument is based on the assumption that no finite volume of space can contain an infinite amount of information.” I have no disagreement with the assumption in the second remark, but I don’t think Gisin has succeeded in defending the claim in the first remark. The following is his attempt to defend that claim:

Consider a small volume, a cubic centimetre let’s say, containing a marble ball. This small volume can contain but a finite amount of information. Hence, the centre of mass of this marble ball can’t be a real number (and even less 3 real numbers), since real numbers contain—with probability one—an infinite amount of information. Classical physics describes the centre of mass of the ball by 3 real numbers; and this is an extremely efficient description. But the assumption that a finite volume of space can’t contain more but a finite amount of information implies that the centre of mass of any object cannot be identified with mathematical real numbers. Real numbers are useful tools, but are only tools. [An original footnote is omitted here.] They do not represent physical reality.

Just like Dowek (wrongly) assumed that the information of the number representing the length of the pencil is somehow stored in the pencil, it is clear that Gisin here assumed that the marble ball (or the small volume of space containing it) somehow stores the information of the numbers representing the centre of mass of the marble ball; hence if these numbers contain infinite amount of information, then there would be infinite information density in that small space.

However, just like the above criticism towards Dowek’s case, it must be pointed out that it is weird and unreasonable to make such an assumption. Why? Because centre of mass is only a mathematical construction useful for theorizing in classical mechanics, but not something “physically real” in space—an extended body having a centre of mass is still in reality an extended body, but not a point mass. Moreover, the actual values of the numbers representing the centre of mass depend on which coordinate system is used for measurement. So, just like the infinite number of possible units in Dowek’s case, there is an infinite number of possible coordinate systems that can be used. Then does it mean that all the corresponding infinitely many different values of the center of mass are all stored in that physical space? Again, obviously absurd. In fact, when we talk about centre of mass in theory, as long as it is not yet actually measured, there is no question of storing that piece of potential information.

III. CONCLUDING REMARKS

Can real numbers represent physical reality? In particular, can they be used to represent the length of a pencil or its centre of mass? This short paper has not tackled these questions directly, but mainly attempted
to expose the fallacies in the negative answers provided by [2] and [5]. As a result, the argument in [1] for interpreting classical physics indeterministically has a loophole.

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