A new method for reasonable injection production ratio calculation of considering external flow volume

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Abstract. The phenomenon of high injection production ratio and low formation pressure maintenance level are common in peripheral oil fields of Daqing oilfield. It is considered that the outflow of water injection exists in the reservoir. In this paper, the optimal mathematical model is established with the maximum oil production as the objective function and the injection pressure and flowing pressure as the constraints. Based on the material balance equation considering the overflow water, the calculation formula of formation pressure changing with time, and other parameters changing such as injection production ratio are derived. The reasonable interval injection production ratio in a block can be obtained by the iterative method, which provides a theoretical basis for reasonable production and injection allocation in the middle-later stage of water drive reservoir development.

1. Introduction
Injection production ratio represents the injection withdrawal balance in the process of waterflooding. It is a comprehensive index reflecting the relationship between liquid production, water injection and formation pressure. Injection production ratio is an important basis for planning and designing oilfield water injection. In Daqing peripheral oilfields, the reservoir physical property of reservoir is poor, the pressure transmission is slow, and the pressure around the water wells is serious; high injection production ratio is a common phenomenon. Analysis from the perspective of material balance, when the cumulative injection production ratio is greater than 1.0, the formation pressure should be kept above the initial formation pressure. But the actual reservoir formation pressure level is not high, so it is judged that there is water overflow in the process of water injection in peripheral oilfields[1]. Reasonable injection production ratio can not only maintain high formation pressure, but also stabilize oil and control water cut and save money. The material balance method is one of the main methods to calculate the reasonable injection production ratio because it needs fewer data and the calculation method and program is relatively simple. However, this method does not consider the influence of factors such as overflow and injection pressure on injection production ratio[2-5]. In this paper, starting from the material balance equation considering the overflow water, taking the oil production rate as the objective function, the maximum water injection pressure and the minimum bottom hole flowing pressure as the constraint conditions, and using the iterative method, the reasonable staged injection production ratio in block can be obtained.
2. Establishment of the Objective Function of Reasonable Injection Production Ratio

"Reasonable injection production ratio" is a dynamic, multi-objective, nonlinear constrained optimization problem. This kind of optimization problem cannot be completely solved, so it needs to be transformed into "dynamic, single objective, nonlinearly constrained optimization problem". In this paper, when the injection production ratio optimization model is established, the maximum oil production is selected as the optimization objective. The relationship equations of water injection, oil production, water cut and formation pressure are established, and the minimum water cut and maximum formation pressure are taken as constraints. By using the optimization theory and solving the optimization model, the reasonable injection production ratio is obtained.

Objective function:

\[ N_p = \max \sum_{j=1}^{m} Q_{oj} \]  

Constraint condition:

\[
\begin{align*}
 f_{s_j} &= f(R) \\
 p_{1j} &\geq p_{\min} \geq p_{b} \\
 p_{\text{inj}} &< a \cdot p_{b} \\
 q_{\text{inj}} &< q_{\text{inj,max}}
\end{align*}
\]  

Where: \( N_p \)—Cumulative oil production, t; \( Q_{oj} \)—Oil production in stage \( j \), t; \( f_{s_j} \)—Water cut in stage \( j \), \%; \( p_{1j} \)—Formation pressure in stage \( j \), MPa; \( p_{\min} \)—Minimum formation pressure, MPa; \( p_{b} \)—Bubble point pressure, MPa; \( p_{\text{inj}} \)—Injection pressure in stage \( j \), MPa; \( p_{b} \)—Fracture pressure, MPa; \( q_{\text{inj}} \)—Water injection volume in stage \( j \), m³; \( q_{\text{inj,max}} \)—Stage maximum water injection, m³.

In the material balance equation, it is assumed that the reservoir is a closed underground container for storing oil and gas. However, during the process of water injection, the injected water may overflow along large fractures, shattered fault zone and other channels, or enter the mudstone section for storing oil and gas.

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\end{align*}
\]  

Both sides of the formula (4) are derived at the same time:

\[
\frac{d(p-p_i)}{dt} = \frac{q_{j} \cdot \rho_{w} \cdot \rho_{ao} - Q_{w} \cdot \rho_{ao} \cdot \rho_{w} \cdot Q_{o} \cdot \rho_{ao}}{w_{j} \cdot \rho_{w} \cdot \rho_{ao} \cdot C_{t}}
\]  

Due to \( \rho_{e} \approx \rho_{ao} \), rearranging the above formula:

\[
\frac{d(p-p_i)}{dt} = \frac{q_{j} \cdot \rho_{o} \cdot Q_{w} - Q_{o} \cdot \rho_{w} \cdot Q_{o} \cdot \rho_{e} \cdot Q_{o} \cdot \rho_{e}}{w_{j} \cdot \rho_{w} \cdot N_{e} \cdot \rho_{ao} \cdot C_{t}}
\]  

Where: \( W_{j} \)—Cumulative water injected, 10⁴ m³; \( W_{o} \)—Cumulative water produced, 10⁴ m³; \( \rho_{w} \)—Water density at reservoir condition, t/m³; \( N_{p} \)—Cumulative oil produced, 10⁴ t; \( B_{e} \)—Oil volume factor, f; \( \rho_{e} \)—Oil density at reservoir condition, t/m³; \( N_{e} \)—Original oil in place, 10⁴ t; \( B_{ao} \)—Initial oil volume factor, f; \( \rho_{ao} \)—Original oil density at reservoir condition, t/m³; \( C_{t} \)—System compressibility, MPa⁻¹; \( p \)—Formation pressure, MPa; \( p_{i} \)—Initial formation pressure, MPa; \( W_{c} \)—Cumulative outflow, 10⁴ m³; \( Q_{w} \)—Injection volume, 10⁴ m³; \( Q_{o} \)—Oil production, 10⁴ t; \( Q_{c} \)—Outflow volume, 10⁴ m³.

According to the injection production ratio formula:

\[ R_{IP} = \frac{Q_{j} \cdot \rho_{o}}{Q_{w} \cdot \rho_{o} + Q_{o} \cdot \rho_{o}} \]  

\[ Q_{j} \cdot \rho_{o} = R_{IP} \left( Q_{w} \cdot \rho_{o} + Q_{o} \cdot \rho_{o} \right) \]

Where: \( R_{IP} \)—Injection production ratio.

Substituting (6) into (4):
\[
\frac{d(p-p_1)}{dt} = \frac{(R_{IP}-1) \cdot Q_w \rho_o + (R_{IP}-p_{w}) \cdot Q_o B_o - Q_e \rho_o}{\rho_w N B_{oi} c_t}
\]

(9)

Because \( \rho_o \approx 1 \), Equation (7) can be simplified as:

\[
\frac{d(p-p_1)}{dt} = \frac{(R_{IP}-1) \cdot (Q_w \rho_o + Q_o B_o) - Q_e \rho_o}{N B_{oi} c_t}
\]

(10)

Set water injection index \( J_w \) is:

\[ J_w = \frac{Q_i}{p_{inf} - p} \]

Fluid productivity index is \( J_L \):

\[ J_L = \frac{Q_L}{p - p_{wf}} \]

Outflow index is \( J_e \):

\[ J_e = \frac{Q_e}{p_{inf} - p} \]

Where: \( J_w \) — Injectivity index, \( m^3/(m:\text{MPa}) \); \( J_L \) — Fluid productivity index, \( m^3/(m:\text{MPa}) \); \( J_e \) — Outflow index, \( m^3/(m:\text{MPa}) \); \( p_{inf} \) — Injection pressure, MPa; \( p_{wf} \) — Bottom hole flowing pressure of oil well, MPa.

Then (8) can be rewritten as:

\[
\frac{d(p-p_1)}{dt} = \frac{J_L \rho_o \cdot (R_{IP}-1) \cdot (p-p_{wf}) - J_e \rho_o \cdot (p_{inf}-p)}{N B_{oi} c_t}
\]

(11)

Rearranging the above formula:

\[
\frac{d(p-p_1)}{dt} = \frac{[J_L \rho_o (R_{IP}-1) + J_e \rho_o] \cdot p - J_L \rho_o (R_{IP}-1) \cdot p_{wf} + J_e \rho_o p_{inf}]}{N B_{oi} c_t}
\]

(12)

By solving equation (10), the analytical solution of formation pressure with time is obtained:

\[
p(t) = \left[p_1 - \frac{J_L \rho_o (R_{IP}-1) \cdot p_{wf} + J_e \rho_o p_{inf}}{J_L \rho_o (R_{IP}-1) + J_e \rho_o} \right] e^{\frac{J_L \rho_o (R_{IP}-1) + J_e \rho_o}{N B_{oi} c_t} t} + \frac{J_L \rho_o (R_{IP}-1) \cdot p_{wf} + J_e \rho_o p_{inf}}{J_L \rho_o (R_{IP}-1) + J_e \rho_o}
\]

(13)

3. Determination of Constraint Conditions of Objective Function

3.1. Formation Pressure

Minimum bottom hole flowing pressure during oil well development:

\[
P_{a,min} = \frac{1}{1-n} \left[ \sqrt{n^2 p_h^2 + (1-n) n p_b p - n p_b} \right]
\]

(14)

Where: \( n = \frac{\alpha T}{B_s} (1 - f_w) \)

Theoretical production pressure difference:

\[
\Delta p = \frac{10000 V_L N}{t \cdot J_L}
\]

(15)

Then the minimum formation pressure:

\[
P_{a,min} = P_{a,min} + \Delta p
\]

(16)

Where: \( P_{a,min} \) — Minimum bottom hole flowing pressure, MPa; \( \alpha \) — Natural gas solubility coefficient, \( m^3/(m^3:\text{MPa}) \); \( T \) — Bottom hole reservoir temperature, \( K \); \( V_L \) — Liquid recovery rate, f.

The formation pressure calculated in each stage must meet the requirements: \( p_i \geq P_{a,min} \geq p_b \).

3.2. Prediction of Stage Liquid Production

Stage liquid production is:
Using Simpson integral formula, we can get:

\[ Q_{lj} = \int_{t_{lj-1}}^{t_{lj}} J_t (p - p_{wt}) \, dt \]  \hspace{1cm} (17)

Using Simpson integral formula, we can get:

\[ Q_{lj} = n_o J_L \frac{\Delta t}{6} \left[ \left( p_{j-1} - p_{wt} \right) + 4 \left( p_j - p_{wt} \right) + \left( p_j - p_{wt} \right) \right] \]  \hspace{1cm} (18)

Where: \( Q_{lj} \) —— Liquid production in stage \( j, t \); \( n_o \) —— Number of production wells.

3.3. Prediction of Flow Pressure of Injection Well at the End of Stage

Flow pressure of injection well at the end of stage:

\[ p_{iwfj} = R_{IPj} J_L \left( p_j - p_{wf} \right) \]  \hspace{1cm} (19)

Maximum bottom hole injection pressure:

\[ p_{iwfmax} = a \cdot p_F \]  \hspace{1cm} (20)

The flowing pressure of water injection well in each stage must be satisfied: \( p_{iwf} < p_{iwfmax} \).

Where: \( p_{iwf} \) —— Bottom hole pressure of injection well in stage \( j, f \), MPa; \( R_{IPj} \) —— Injection production ratio in stage \( j, f \).

3.4. Prediction of the Stage Water Cut

Due to the different geological conditions, fluid properties and wettability of reservoir rocks in different oilfields, the driving mode of reservoir is non piston type, and there are five types of relationship curves between water cut and recovery degree. According to the data of water cut and recovery degree, the type of water cut rising law is determined, and then the formula of water cut changing with recovery degree is regressed according to the fitting coefficient \( A \) and \( B \).

(1) Convex type:

\[ R = A + B \ln(1 - f_w) \]

\[ f_w = 1 - e^{\frac{R-A}{B}} \]  \hspace{1cm} (21)

(2) Convex S type:

\[ \ln(1 - R) = A + B \ln(1 - f_w) \]

\[ f_w = 1 - e^{\frac{\ln(1-R) - A}{B}} \]  \hspace{1cm} (22)

(3) S type:

\[ R = A + B \ln(\frac{f_w}{1-f_w}) \]

\[ f_w = \frac{e^{\frac{R-A}{B}}}{1 + e^{\frac{R-A}{B}}} \]  \hspace{1cm} (23)

(4) Concave S type:

\[ \ln R = A + B f_w \]

\[ f_w = \frac{\ln R - A}{B} \]  \hspace{1cm} (24)

(5) Concave type:

\[ \ln R = A + B \ln f_w \]

\[ f_w = e^{\frac{\ln R - A}{B}} \]  \hspace{1cm} (25)

Where: \( R \) —— Recovery factor, %.

3.5. Stage Oil Production Forecast

Using formula (17) to calculate the stage liquid production and using the water cut regression function to calculate the stage water cut, the stage oil production formula is as follows:

\[ Q_{oj} = Q_{lj} (1 - f_{wj}) \]  \hspace{1cm} (26)

Where: \( Q_{oj} \) —— Oil production in stage \( j, t \).

According to the above model programming, adjust the stage injection production ratio, make the stage production maximum under all constraints, and get the best stage injection production ratio.
4. Application Examples
Oilfield A is an ultra-low permeability oilfield with reservoir permeability of 5mD. At present, the monthly injection production ratio is 3.0. Other basic parameters are shown in Table 1. The proportion of water injection outflow is about 65% and the monthly overflow index is 362 m³ / MPa. Under the condition that the cause of injection outflow is uncertain and no stimulation measures are taken, the overflow index remains unchanged when calculating the reasonable injection production ratio. The minimum formation pressure calculated by formula (16) is 9.15 MPa, and the maximum water injection pressure calculated by formula (20) is 21.3 MPa. Take 24 months as a stage, adjust the monthly injection production ratio (range 1.0 ~ 5.0), under all constraints, when the injection production ratio is 3.4 in the current month, the cumulative oil production in 24 months is the largest, so it is considered that the reasonable injection production ratio of the oilfield is 3.4.

According to the calculation results of reasonable injection production ratio, the monthly injection water was increased to make the monthly injection production ratio reach about 3.4. After adjustment, the monthly oil production of the oilfield was increased from 6708 t to 6812 t. It is proved that calculating reasonable injection production ratio and adjusting injection production relationship according to reasonable value in different development stages of oilfield are effective means to maintain stable production and increase production.

Table 1 Basic parameters of Oilfield A

| Item                        | Symbol | Numerical value   | Item                        | Symbol | Numerical value   |
|-----------------------------|--------|-------------------|-----------------------------|--------|-------------------|
| Dissolved gas coefficient of crude oil | α      | 2.5 m³/(m³·MPa)   | Original dissolved gas oil ratio | Rs     | 21 m³/ m³         |
| Oil formation volume factor | Bo     | 1.076             | Initial formation pressure  | pi     | 8.4 MPa           |
| Deviation coefficient of natural gas | Z      | 0.9319            | Original oil in place       | N      | 215.2×10⁴ t       |
| Bottom hole reservoir temperature | T      | 323.15K           | Number of production well   | n      | 27 wells          |
| Bubble point pressure       | pb     | 6.82 Mpa          | Middle depth of reservoir   | H      | 1028.4 m          |
| Formation pressure          | pR     | 11.52 Mpa         | System compressibility      | C_t    | 15×10⁻⁴ /MPa      |

5. Conclusion
(1) Taking the maximum oil production as the optimization objective and the maximum water injection pressure and the minimum bottom hole flowing pressure as the constraints, the optimization model of reasonable injection production ratio is established;

(2) Combined with the fact that there is water outflow in Daqing peripheral oilfields, the analytical formula of formation pressure changing with time and injection production ratio is derived from the material balance equation considering the overflow water. By using the water cut rising curve to predict the water cut and adjust the stage injection production ratio, the optimal stage injection production ratio meeting the constraint conditions can be obtained;

(3) Case study shows that injection water outflow exists in ultra-low permeability reservoirs. It is an effective method to adjust the injection production ratio to a reasonable value without any stimulation measures.
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