A PolSAR despeckling method based on Wishart gradient and anisotropic diffusion

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The anisotropic diffusion filtering is a classical tool in image processing due to its simplicity and effectiveness. In this letter, we propose a new diffusion-like filter based on the polarimetric synthetic aperture radar (PolSAR) ratio gradients. The PolSAR ratio gradients combine the information of different polarisation channels and describe the local directional changes in polarimetric characteristics. The proposed filter is then built by utilising the new gradients to derive the diffusion coefficients. The experimental results on synthetic data and real PolSAR images demonstrate the superiority and effectiveness of the new method.

Introduction: Despeckling filters for polarimetric synthetic aperture radar (PolSAR) are active research areas in remote sensing. The despeckling effect has a great influence on later tasks, such as object detection [1] and land use classification [2]. At present, the typical PolSAR filter can mainly be divided into three categories. The first is local filters that utilise local statistical characteristics [3]. The second is non-local mean filters, which involve similarities between image patches to estimate the weights of a box filter [4–7]. The third, which is mainly focused on in this study, is to minimise an energy model of the whole image and solve it by a diffusion partial differential equation (PDE) [8, 9]. Terebes et al. [8] proposed to calculate the diffusion coefficient by the multiplicative gradient on the span image without involving all the information on different polarisation channels of PolSAR image. Santana et al. [9] introduced a structure tensor based on Wishart distance. The diffusion coefficient is obtained from the structure tensor instead of directly calculating from the PolSAR image. In this letter, we propose a new anisotropic diffusion-like despeckling filter. PolSAR ratio gradients, which measures the local changes in polarimetric characteristics, can be simply calculated on PolSAR images. Diffusion coefficients in the diffusion PDE are a certain function of the proposed gradients. Then the despeckling results can be obtained by numerically solving the PDE iteratively. Experimental results show that the proposed method can reduce speckles effectively while preserving textures such as edges and small targets.

PolSAR ratio gradient: We propose to calculate PolSAR ratio gradients to describe the local changes of PolSAR images. Gradients are calculated in both horizontal and vertical directions by

\[ \nabla I = \log \left( \frac{\text{Tr} (T_{1,1}^T T_{1,1})}{\text{Tr} (T_{2,1}^T T_{2,1})} \right), \quad \nabla V I = \log \left( \frac{\text{Tr} (T_{1,1}^T T_{H V,1})}{\text{Tr} (T_{2,1}^T T_{H V,1})} \right) \] (1)

where \( I \) is a PolSAR image, \( \nabla I \) and \( \nabla V I \) are gradients in vertical and horizontal directions, respectively. \( \text{Tr} (\cdot) \) denotes the trace of matrix. \( T_{1,1} \) and \( T_{2,1} \) denote the local average coherency matrices on opposite sides of the centre pixel along the vertical direction. \( T_{1,1} \) and \( T_{2,1} \) denote those along the horizontal direction. Then the gradient magnitude of PolSAR image \( I \) can be expressed as

\[ |\nabla I| = \sqrt{(\nabla V I)^2 + (\nabla I)^2}. \] (2)

Despeckling with anisotropic diffusion: The anisotropic diffusion filtering is a classical tool introduced initially by Perona and Malik [10] in image processing. We propose a novel anisotropic diffusion filter for PolSAR images based on the PolSAR ratio gradient. Suppose \( I_0 : \Omega \to \mathbb{H}_{3}^+ \) to be the origin PolSAR image where \( \Omega \) denotes the domain of the image, and \( \mathbb{H}_{3}^+ \) denotes the set of 3 \( \times \) 3 Hermitian positive definite matrices. Then the filtered image is the solution of the following optimisation problem:

\[ \min \epsilon (I) = \int_{\Omega} f (|\nabla I (x)|^2) \, dx, \quad \text{s.t.} \int_{\Omega} |I (x) - I_0 (x)| \, dx \leq \varepsilon, \] (3)

where \( f (\cdot) \) is an energy function, \( \nabla I \) denotes the PolSAR ratio gradients, and \( \varepsilon \) is a parameter to control the filtering degree. The function \( f (\cdot) \) usually has a shape that will saturate at large values, such as sigmoid and arctangent functions. The derivative of such a function \( f (|\nabla I (x)|^2) \) will decrease to a very small value at large \( |\nabla I (x)|^2 \). Hence, blurring a pixel \( x \) with large \( |\nabla I (x)|^2 \) may cause a little decrease in the energy \( E \), but blurring a pixel \( x \) with small \( |\nabla I (x)|^2 \) can indeed cause a decrease in \( E \) and contribute to the minimisation of the total energy. Therefore, to minimise the total energy \( E \) under the restriction in Equation (3), the solution of Equation (3) will, to some extent, preserve the pixels with large \( |\nabla I (x)|^2 \) and blur the pixels with small \( |\nabla I (x)|^2 \). In the case of PolSAR image filtering, the textures needed to be preserved are generally considered to be edges and small targets, where the gradients \( |\nabla I| \) is usually very large, so the despeckling result of Equation (3) can reduce speckle while preserve edges and small targets.

We utilise a PDE to solve the above minimisation problem as proposed in [10]:

\[ \frac{\partial I (x, t)}{\partial t} = \epsilon (x) \nabla^2 I (x, t), \] (4)

where \( R (x, t) \) denotes the filtered image at time \( t \) with initial value \( I (x, 0) = I_0 \) and \( g (\cdot) \) denotes the diffusion coefficients, which is usually set to \( g (x) = (1 + (\lambda^2 / \varepsilon^2)^2)^{-1} \). The final filtered image is the result at \( t \), when \( \int_{\Omega} |I (x, t) - I_0 (x)| \, dx = \varepsilon \). It is seen that each channel of the PolSAR image is filtered by the same diffusion coefficient, so the cross-talk between polarisation channels is avoided and the polarimetric characteristics can be preserved.

In practical applications, the PDE in Equation (4) is solved by the finite difference method, which is

\[ I (x, t_{n+1}) = I (x, t_{n}) + \delta t \cdot \sum_{y \in N (x)} \left[ g (x, t_{n}) + g (y, t_{n}) \right] \cdot I (y, t_{n}) - I (x, t_{n}), \] (5)

where \( \delta t \) is the time step of each iteration and \( N (x) \) is the four-point neighbourhood stencil. We empirically set the time step as \( \delta t = 0.05 \), and set the total iteration number as \( n = 100 \), instead of setting a specific value of \( \varepsilon \).

Experimental results: We evaluated the performance of the proposed method by comparing it with the filter based on stochastic distance non-local means (SDNLM) [7] and the diffusion filter based on the multiplicative gradient (MGPD) [8]. The experiments were carried on both synthetic data and real datasets.

Figure 1(a) shows the 250 \( \times \) 250 phantom for synthetic data with five classes. The ground-truth coherency matrix for each class is sampled from real PolSAR data. We simulated the speckled image (Figure 1(b)) with a number of looks \( L = 4 \) by the simulation method reported in [11]. The despeckling results of SDNLM, MGPD, and the proposed method on the simulated data are shown in Figure 2. It can be seen that all filters provide notable despeckling results. However, in the result from the proposed filter, the edges are better preserved as observed in the closeups within the red squares.

The filtering results on synthetic data are quantitatively evaluated by the equivalent number of looks (ENL) and the edge-preservation degree based on the ratio of average (EPD-ROA) [12] as shown in Table 1. ENL is calculated on the whole coherency matrix by the trace-moment estimate [13]. In the case of synthetic data, the ENL of different classes is calculated separately. EPD-ROA is calculated along the horizontal and vertical directions (HD and VD) on three polarisation channels (HH, HV, HV, etc.).
and VH). It is shown in Table 1 that all three methods obtained relatively large ENL on synthetic data. However, the proposed method provided better EPD-ROA than the other two methods.

For the real case, we used two L-band AIRSAR PolSAR 4-look images acquired over Flevoland, the Netherlands, and over San Francisco, the United States. For the Flevoland data, a region of interest (ROI) of size 500 × 500 is sampled from the raw image. The pseudocolour image with the Pauli basis is shown in Figure 3(a). The results of the three methods are shown in Figures 3(b)–(d), respectively. It is shown that SDNLM can reduce speckles notably, but the two diffusion-based filters provide more smoother homogeneous areas. Among the diffusion-based methods, the proposed method obtained better preservation of edges, especially in areas within the red and blue frames. The reason is that the proposed method utilises all polarisation channels to filter the images, while only span information is considered in MGPDE.

The quantitative results for Flevoland data are compared and shown in Table 2. In addition to the ENL and EPD-ROA mentioned above, we also recorded the time cost of different filters. It is seen that SDNLM reached the best edge-preserving effect and a modest despeckling effect at the cost of huge time-consuming. Due to the low time complexity in one iteration, the proposed method and MGPDE are much faster than the SDNLM, although these methods solve a PDE by finite difference. Also, we can see that the proposed method obtained larger ENL and EPD-ROA than the MGPDE.

For the San Francisco data, we sampled an ROI of size 400 × 400. Figure 4(a) shows the pseudocolour Pauli image of the ROI, and Figure 4(b)–(d) show the despeckling results of the three comparison methods. As observed from the magnified areas, all of the three filters can obtain good despeckling effects, but the proposed filter provides better preservation of the small targets (red frame) and the edges (blue frame). Also, we evaluated the quantitative results in Table 3. Table 3 follows the same structure as Table 2. As in the case of the Flevoland data, the proposed method provides better ENL and EPD-ROA than the
**Fig 4** Original PolSAR image and comparison results on San Francisco. (a) Original image, (b) SDNLM, (c) MGPDE, (d) the proposed method

**Table 3. Quantitative evaluation of different filtering methods on San Francisco data**

| Methods       | SDNLM | MGPDE | The proposed |
|---------------|-------|-------|--------------|
| ENL           | 12.68 | 9.59  | 11.01        |
| EPD-ROA       | 0.7345| 0.6786| 0.7043       |
| HH-HD         | 0.7329| 0.6557| 0.6715       |
| HV-HD         | 0.7462| 0.6809| 0.7111       |
| VV-HD         | 0.7793| 0.792  | 0.7941       |
| HH-VD         | 0.7609| 0.7594| 0.7807       |
| HV-VD         | 0.7790| 0.7779| 0.8026       |
| VV-VD         |       |       |              |
| Time cost(s)  | 112.53| 13.56 | 12.88        |

MGPE method. Compared to the SDNLM filter, the proposed filter obtained almost evenly quantitative results at a low time cost.

The implementation details are as follows. All filters are coded in MATLAB R2019b. The programmes are run on an Intel(R) Core i7-4790 CPU 3.60 GHz (24.0 GB RAM) computer.

**Conclusion:** In this study, we propose a PolSAR despeckling method based on PolSAR ratio gradients and anisotropic diffusion. PolSAR ratio gradients can be simply calculated to describe local directional changes in polarimetric characteristics. The proposed filter based on the new gradients can reduce speckles effectively and also preserve textures, such as edges and small targets. Comparison experiments were carried on synthetic data and real PolSAR datasets. Both visual effects and quantitative results demonstrate the superiority of the proposed method.

**Acknowledgements:** This work was supported by the National Key Research and Development Program of China Grant number: 2017YFB0502703. The authors would like to thank the anonymous reviewers for their constructive suggestions.

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Received: 13 November 2020    Accepted: 5 January 2021

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