Generalized Reduced Gradient Approach for Solving Sustainable Production and Scheduling Planning of Crude Palm Oil Milling Industries

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Abstract. It is well known that crude palm oil (CPO) is obtained from the milling process of raw bunch, called fresh fruit bunch (FFB). The process usually flows through a series of stages producing and consuming intermediate products. It is assumed that the CPO industry considered in this paper does not own oil palm plantation, in such a way that the FFB are supplied by several public plantations. Due to the availability of FFB, it is then necessary to select from which plantations should be appropriate. This paper describes a mixed integer linear programming model to tackle the supply chain integrated problem, along with waste problem. The mathematical optimization model is solved using generalized reduced gradient search approach.

1. Introduction

Crude palm oil (CPO) production method had to go through the raw materials milling phase, called the fresh fruit bunch (FFB), into palm oil end products. The method generally involves a number of activities in the manufacture and consumption of intermediate products. These raw materials, intermediate and finished goods must inevitably be catalogued in such a way that they can be produced and consumed at different times and at different rates. Although the duration to store the raw material for palm oil is very short, each phase of the process may involve several input materials and one or more outputs may be produced. If there is no cultivation of palm oil in the CPO company, the FFB as raw material must be ordered from the neighbouring palm oil plantation.

Palm oil companies should have their production plan policies designed to meet market demand and remain viable. A plan concerning the essential procurement of resources and accessibility of raw materials, as well as the management of the manufacturing activities needed to convert products into finished products. The whole method is to meet consumer requirements in the most efficient or cost-effective manner, i.e. to minimize total costs [1]. Identifying the ideal quantity of production has become one of the main aspects of production planning [2].

Commonly, for each phase of the planning horizon, addressing the issues of production planning involves making choices about the magnitude of the production lots or the level of production. Such approaches may also cover decisions on the quantities of raw materials (components) to be acquired, ordered or processed, inventory rates for final products and components, the production cycle, and other factors related to those elements.

In production planning, it is vital to recognize the equations of inventory flow and the stability of inventories in a model using relative time discretization, as for example years, quarters, months or weeks [3]. Linear Programming (LP), Mixed Integer Linear Programming (MILP), and Mixed Integer
Non-Linear Programming (MINLP) models are often ideal and helpful for addressing these issues with a distinct objective function: net gain, contribution margin, expense, total revenue, total output, etc [3].

CPO milling industries can be considered as process industries in which the supply chain management system is to be described in the practice of production and distribution. The main ideal performance of the procedure is to achieve the lowest total operating costs from the processing of raw materials to the finished product and then to be sent to the distribution centre [4], [5]. [6] provided an optimization modelling of an integrated supply chain system linking raw resource vendors, various plants, distribution centres, distributors and consumers. The more complex situation of the integrated supply chain has been studied [7]. They modelled the performance of the supply chain containing a number of feed mills, plantations and products. [8] discussed the MILP model, which utilizes production, distribution and marketing, as well as integrated plants and sales sites. The aim of their article is to identify the applicable features desired by a multi-site production network for complete supply chain management. [9] develop a global MILP supply chain model for product – plant and customer – distribution centre assignments. There are several supply chains links in their paper and numbers and locations of distribution centre. [10] present a two-stage supply chain model to establish the optimum quantity of products to be processed at each plant and then transferred from each plant to each distribution centre. However, huge amounts of constraints and binary variables have emerged, and their range is increasing exponentially as product, plant and distribution centre statistics are increasing. [11] use a hybrid Taguchi-Immune technique to optimize the issue of supply chain design with various shipping possibilities, distributed consumer requirements and fixed lead times. Mixed integer linear programming models and computing solutions have been proposed to address the issue of uncertain multi-stage inventory supply chains [12]. [13] recognize a multi-period production system comprising load distribution and production planning. A three-phase heuristic method and Tabu Search were used to tackle the integrated model.

When we plan to implement production planning to complex manufacturing processes in a more complex manner, such applications created by MILP models are frequently encountered. This refers to the complexity of the decision variables for certain characteristics involved in issues such as set-up costs and times, start-up costs and times, machine assignment decisions, ordering costs and times, etc. These costs and times are fixed per batch and are not proportional to the batch size. It is therefore necessary to model binary or integer variables [1].

[14] create a continuous time model for the planning and scheduling to batch processing plants. The proposed model is initially a Mixed Integer Nonlinear Program (MINLP) and is then reconstituted using linearization techniques as a mixed-integer linear programming (MILP). The model intends to maximize the net profit from batch production in terms of batch assignment, operation and set-up time of the equipment and scheduling time intervals. To demonstrate the efficiency of the model, a multi-product batch paint processing plant is selected for real scenario execution. Another MILP model can be discovered [15] for integrated location-production-distribution planning.

[16] addresses the planning of palm oil production in conjunction with the planning of the palm oil processing plant capacity in Thailand. The aim of his article is to treat CPO as the core feedstock for the production of biodiesel and to propose a linear mixed integer programming model. The non-linear programming model for the planning of the production of CPO is discussed in [17] with a view to minimizing water usage during the milling phase. However, only the optimization method is mentioned in the CPO milling method. Another CPO production planning can be found in [18], which is considered only in the milling phase. Using a fuzzy logic approach, they provide a simpler system to achieve a link between processing factors and the amount of CPO and palm kernel losses. [19] establishes a model of optimization for CPO production planning in the supply and demand system. The aim of their model is to identify which sector of the CPO would be appropriate and also to measure how much demand should be provided in the specified market. [20] discusses a multi-objective optimization model for sustainable production planning for CPO, taking into account the reliability of financial risk.
2. Mathematical Formulation

The objective of the model is to decide optimally

1) The quantity of FFB needs to be bought from each palm oil plantation (ton)
2) The CPO to be produced (ton)
3) The CPO to be delivered to each customer (ton)
4) The amount of Waste will be produced in the milling process (ton)
5) To select from which FFB should be bought (binary)

This is to minimize the overall costs incurred.

Decision variables

- \( x'_{ij} \): The quantity of FFB (ton) to be bought from plantation I for milling j in period t
- \( y'_{ij} \): The number of CPO (ton) to be produced at milling j in period t
- \( z'_{jk} \): The number of CPO (ton) to be delivered from milling j to customer k in period t
- \( v'_{ij} \): The volume of liquid waste (ton) incurred after the milling process in milling j in period t
- \( w'_{ij} \): The number of solid waste (ton) occurred after the milling process in milling j in period t
- \( \delta_{ij} \): A binary variable equal to 1 if FFB for milling j is bought from plantation I and zero otherwise

Parameters.

- \( \alpha_{ij} \): Cost of transportation (Rp.) from plantation I to milling j per kilometre
- \( d_{ij} \): Distance (km) from oil palm plantation i to milling j
- \( \beta_{ij} \): Production cost (Rp.) at milling j
- \( \gamma_{jk} \): Cost for transportation (Rp.) to deliver CPO from milling j to customer k
- \( \lambda_{ij} \): Cost (Rp.) to process liquid waste at milling j
- \( \rho_{ij} \): Transportation cost (Rp.) for solid waste at milling j
- \( \tau_{ij} \): Overall price (Rp.) to buy FFB from plantation i for milling j in period t
- \( Cm^t_j \): Milling capacity (ton) at milling \( j \in J \), in period \( t \in T \)
- \( CP^t_j \): Production capacity (ton) at milling \( j \in J \), in period \( t \in T \)
- \( CA^t_i \): Quantity of FFB (ton) available at plantation \( i \in I \), in period \( t \in T \)

Sets.

- \( I \): set of oil palm plantations with index \( i \)
- \( J \): set of CPO milling with index \( j \)
- \( K \): set of customers with index \( k \)
- \( T \): set of time period with index \( t \)

From the structure of the problem, we can say that the problem can be formulated as a mixed integer linear programming (MILP) model.

Firstly, we build the objective function.

The operational costs can be written as follows.
\[ C_1 = \sum_{i \in I} \sum_{j \in J} (\alpha_j)(d_{ij})x^t_{ij} \] (1)

This is a transportation cost of FFB from plantation \( i \in I \) to milling \( j \in J \) in period \( t \in T \).

\[ C_2 = \sum_{j \in J} \sum_{i \in I} \beta_j \gamma_j^t \] (2)

Cost to produce CPO at milling \( j \in J \) in period \( t \in T \).

\[ C_3 = \sum_{j \in J} \sum_{k \in K} \sum_{i \in I} \gamma_{jk} z_{jk}^t \] (3)

The cost of transportation of CPO from milling \( j \in J \) to consumer \( k \in K \) in period \( t \in T \).

\[ C_4 = \sum_{j \in J} \sum_{i \in I} \lambda_j \gamma_j^t \] (4)

The cost to process liquid waste after the milling process at milling \( j \in J \) in period \( t \in T \).

\[ C_5 = \sum_{j \in J} \sum_{i \in I} \rho_j \omega^t_j \] (5)

Transportation cost for solid waste after the process at milling \( j \in J \) in period \( t \in T \).

\[ C_6 = \sum_{i \in I} \sum_{j \in J} \tau_{ij} \delta_{ij}^t \] (6)

The price to buy FFB from palm oil plantation \( i \in I \) for milling \( j \in J \) in period \( t \in T \), if plantation \( I \) is selected.

There are constraints must be met written as follows.

\[ \sum_{i \in I} x^t_{ij} \leq Cm^t_j \quad \forall j \in J, t \in T \] (7)

Eq. (7) represents that the amount of FFB to be bought from plantation \( i \in I \) for milling \( j \in J \) satisfy to the capacity of milling \( j \in J \) in period \( t \in T \).

\[ \sum_{j \in J} \sum_{i \in I} \gamma_j \leq \sum_{i \in I} \sum_{j \in J} C_{p_j}^t \] (8)

Eq. (8) states that the amount of processing FFB in milling \( j \in J \) meets the production capacity of milling \( j \in J \) in period \( t \in T \).

\[ \sum_{i \in I} x^t_{ij} \leq \sum_{i \in I} \sum_{j \in J} C_{A_i} \delta_{ij}^t \quad \forall j \in J, t \in T \] (9)

Eq. (9) describes the quantity of FFB to be bought from plantation \( i \in I \) for milling \( j \in J \) satisfies to the availability of FFB in the plantation \( i \in I \), if the plantation is selected, for period \( t \in T \).

\[ \sum_{j \in J} z_{jk}^t \leq \sum_{j \in J} \gamma_j^t \quad \forall k \in K, t \in T \] (10)

Eq. (10) formulates that the quantity of CPO to be transported to customer \( k \in K \) from \( j \in J \) should be less than the amount produced in milling \( j \in J \) in period \( t \in T \).

\[ \gamma_j^t + \omega_j^t \leq \sum_{i \in I} x^t_{ij} \quad \forall j \in J, t \in T \] (11)

Eq. (11) expresses that the amount of solid and liquid waste from milling \( j \in J \) are less than the quantity of FFB was processed in period \( t \in T \).

\[ \sum_{i \in I} \sum_{j \in J} \delta_{ij} = n \] (12)
Eq. (12) is formulated to be sure that there would be at least \( n \) plantation available to supply FFB for each milling \( j \in J \).

\[
x_{ij}, y_{ij}, z_{jk}, v_{ij}, w_{ij} \geq 0, \quad \delta_{ij} \in \{0,1\}, \quad \forall i \in I, j \in J, k \in K, t \in T
\]  

(13)

Eq. (13) is for the decision variables.

3. The Algorithm

This paper considers a class of algorithms in which the search direction along the surface of active constraints is characterized as being in the range of a matrix \( Z \) which is orthogonal to the matrix of constraint normal. Thus, if \( \tilde{A}x = \tilde{b} \) is the current set of \( (n - s) \) active constraints, then \( Z \) is an \( n \times s \) matrix such that

\[
\tilde{A}Z = 0.
\] 

(14)

The main steps to be performed at each iteration are as follows. (They generate a feasible descent direction \( p \).)

(A) Compute the reduced gradient \( g_A = Z^Tg \)

(B) Form some approximation to the reduced Hessian, viz.

\[
G_A \approx Z^T G Z
\]

(C) Obtain an approximate solution to the equations

\[
Z^T G Z p_A = -Z^T g
\]

by solving

\[
G_A p_A = -g_A.
\]

(D) Compute the search direction \( p = Z p_A \).

(E) Perform a linesearch to find an approximation to \( \alpha^* \), where

\[
f(x + \alpha^* p) = \min_{\alpha} f(x + \alpha p)
\]

\[
\text{subject to } x + \alpha p \text{ feasible}
\]

Apart from having full column rank, eq. (14) is (algebraically) the only constraint on \( Z \) and thus \( Z \) may take several forms. The particular \( Z \) corresponding to our own procedure is of the form

\[
Z = \begin{bmatrix}
-W & -b^{-1} S \\
 I & I \\
 0 & 0 \\
\end{bmatrix}
\] 

\[
\begin{array}{c}
|m \\
|s \\
|n - m - s \\
\end{array}
\] 

(16)

This is a convenient representation which can be referred to for exposition purposes in later sections, but we emphasize that computationally we work only with \( S \) and a triangular (LU) factorization of \( B \). However, the matrix \( Z \) itself is not computed.

For many good reasons we choose a \( Z \) whose columns are orthonormal \( (Z^T Z = I) \). The principal advantage is that transformation by such a \( Z \) does not introduce unnecessary ill-conditioning into the reduced problem (see steps A through D above, equation (20)). The approach has been implemented in programs in which \( Z \) is stored explicitly as a dense matrix. Extension to large sparse linear constraints would be possible via an LDV factorization of the matrix \( [BS] \):

\[
[BS] = [L O] DV
\]

where \( L \) is triangular, \( D \) is diagonal and \( D^{1/2} V \) is orthonormal, with \( L \) and \( V \) being stored in product form. However, if \( S \) has more than 1 or 2 columns, this factorization will always be substantially denser than an LU factorization of \( B \). Thus, on the grounds of efficiency we proceed with the \( Z \) in (16).

At the same time, we are conscious (from the unwelcome appearance of \( B^{-1} \)) that \( B \) must be kept as well-conditioned as possible.
An outline of the optimization algorithm is given in this section. Assume we have the following:

(a) A feasible vector $x$ satisfying $[BSN]x = b$, $l \leq x \leq u$.
(b) The corresponding function value $f(x)$ and gradient vector $g(x) = [g_B \ g_s \ g_N]^T$.
(c) The number of super basic variables, $s$ ($0 \leq s \leq n - m$).
(d) A factorization, LU, of the $m \times m$ basis matrix $B$.
(e) A factorization, $RTR$, of a quasi-Newton approximation to the $s \times s$ matrix $Z^T G Z$. (Note that $G$, $Z$ and $Z^T G Z$ are never actually computed.)
(f) A vector $\pi$ satisfying $B^T \pi = g_B$.
(g) The reduced-gradient vector $h = g_s - S^T \pi$.
(h) Small positive convergence tolerances $TOLR$ and $TOLDJ$.

Step 1. (Test for convergence in the current subspace). If $||h|| > TOLR$ go to step 3.

Step 2. ("PRICE", i.e., estimate Lagrange multipliers, add one superbasic).
   (a) Calculate $\lambda = g_s - N^T \pi$
   (b) Select $\lambda_q < -TOLDJ$ ($\lambda_q > +TOLDJ$), the largest elements of $\lambda$ corresponding to variables at their lower (upper) bound. If none, STOP; the Kuhn-Tucker necessary conditions for an optimal solution are satisfied.
   (c) Otherwise,
      (i) Choose $q = q_1$ or $q = q_2$ corresponding to $|\lambda_q| = \max(|\lambda_q|, |\lambda_{q_2}|)$;
      (ii) add $a_q$ as a new column of $S$;
      (iii) add $\lambda_q$ as a new element of $h$;
      (iv) add a suitable new column to $R$.
   (d) Increase $s$ by 1.
   (Note: MINOS also has a MULTIPLE PRICE option which allows more than one nonbasic variable to become superbasic.)

Step 3. (Compute direction of search, $p = Zp_s$).
   (a) Solve $R^T R p_s = -h$.
   (b) Solve $LU p_b = -Sp_s$.
   (c) Set $p = \begin{bmatrix} p_b \\ p_s \\ 0 \end{bmatrix}$

Step 4. (Ratio test, "CHUZR").
   (a) Find $\alpha_{\max} \geq 0$, the greatest value of $a$ for which $x + \alpha p$ is feasible.
   (b) If $\alpha_{\max} = 0$ go to step 7.

Step 5. (Linesearch).
   (a) Find $\alpha$, an approximation to $\alpha^*$, where
      $$F(x + \alpha^* p) = \min_{0 < \alpha \leq \alpha_{\max}} f(x + \alpha p)$$
   (b) Change $x$ to $x + \alpha p$ and set $f$ and $g$ to their values at the new $x$.

Step 6. (Compute reduced gradient, $\tilde{h} = Z^T g$).
   (a) Solve $U^T L^T \pi = g_B$.
   (b) Compute the new reduced gradient, $\tilde{h} = g_s - S^T \pi$. 
(c) Modify R to reflect some variable-metric recursion on $R^TR$, using $\bar{\alpha}$, $p_s$ and the change in reduced gradient, $\bar{h} - h$.
(d) Set $h = h$.
(e) If $\bar{\alpha} < \bar{\alpha}_{\text{max}}$ go to step 1. No new constraint was encountered so we remain in the current subspace.

**Step 7.** (Change basis if necessary; delete one superbasic). Here $\bar{\alpha} < \bar{\alpha}_{\text{max}}$ and for some $p (0 < p \leq m + s)$ a variable corresponding to the p-th column of $[B \; S]$ has reached one of its bounds.
(a) If a *basic* variable hit its bound ($0 < p \leq m$),
   (i) interchange the $p$-th and $q$-th columns of
   \[
   \begin{bmatrix}
   B \\
   x_p^T
   \end{bmatrix}
   \quad \text{and} \quad 
   \begin{bmatrix}
   S \\
   x_s^T
   \end{bmatrix}
   \]
   respectively, where $q$ is chosen to keep $B$ nonsingular (this requires a vector $\pi_p$ which satisfies $U^T \pi_p = e_p$);
   (ii) modify $L$, $U$, $R$ and $\pi$ to reflect this change in $B$;
   (iii) compute the new reduced gradient $h = g_s - S^T \pi$;
   (iv) go to (c).
(b) Otherwise, a *superbasic* variable hit its bound ($m < p \leq m + s$). Define $q = p - m$.
(c) Make the $q$-th variable in $S$ nonbasic at the appropriate bound, thus:
   (i) delete the $q$-th columns of
   \[
   \begin{bmatrix}
   S \\
   x_s^T
   \end{bmatrix}
   \quad \text{and} \quad 
   \begin{bmatrix}
   R \\
   h^T
   \end{bmatrix}
   \]
   (ii) restore $R$ to triangular form.
(d) Decrease $s$ by 1 and go to step 1.

4. **Conclusions**
This paper describes a generalized reduced gradient approach for solving an integrated of production – distribution planning encountered by a CPO industry without any plantation. Therefore, this company is necessarily to buy FFB from several plantations. Due to the availability of FFB then it is necessary for the management to decide from which plantations would be appropriate to buy such that to meet the capacity of the milling industry.

5. **References**
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