Acceleration of Solar Energetic Particles at a Fast Traveling Shock in Non-uniform Coronal Conditions

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Abstract. Time-dependent solar energetic particle (SEP) acceleration is investigated at a fast, nearly parallel spherical traveling shock in the strongly non-uniform corona by solving the standard focused transport equation for SEPs and transport equations for parallel propagating Alfvén waves that form a set of coupled equations. This enables the modeling of self-excitation of Alfvén waves in the inertial range by SEPs ahead of the shock and its role in enhancing the efficiency of the diffusive shock acceleration (DSA) of SEPs in a self-regulatory fashion. Preliminary results suggest that, because of the highly non-uniform coronal conditions that the shock encounters, both DSA and wave excitation are highly time-dependent processes. Thus, DSA spectra of SEPs strongly deviate from the simple power-law prediction of standard steady-state DSA theory and initially strong wave excitation weakens rapidly. Consequently, the ability of DSA to produce high energy SEPs in the corona of ~1 GeV, as observed in the strongest gradual SEP events, appears to be strongly curtailed at a fast nearly parallel shock, but further research is needed before final conclusions can be drawn.

1. Introduction
We report preliminary simulation results of time-dependent solar energetic particle (SEP) acceleration at a fast, nearly parallel spherical traveling shock in the corona by taking into account the strong radial dependence in background coronal plasma conditions that the shock encounters during the first hour of shock propagation. For this purpose a simple empirical coronal plasma model \[1\] is used as a first step. The model involves solving a set of coupled equations for SEPs and parallel propagating Alfvén waves. The standard focused transport equation is solved to simulate SEP acceleration at the traveling shock when SEPs are scattered by Alfvén waves that form a set of coupled equations. This enables the modeling of self-excitation of Alfvén waves in the inertial range by SEPs ahead of the shock and its role in enhancing the efficiency of the diffusive shock acceleration (DSA) of SEPs in a self-regulatory fashion. Preliminary results suggest that, because of the highly non-uniform coronal conditions that the shock encounters, both DSA and wave excitation are highly time-dependent processes. Thus, DSA spectra of SEPs strongly deviate from the simple power-law prediction of standard steady-state DSA theory and initially strong wave excitation weakens rapidly. Consequently, the ability of DSA to produce high energy SEPs in the corona of ~1 GeV, as observed in the strongest gradual SEP events, appears to be strongly curtailed at a fast nearly parallel shock, but further research is needed before final conclusions can be drawn.
steady-state DSA theory. Furthermore, strong initial wave excitation weakens rapidly, thus hampering DSA in producing SEPs with high energies close to ~1 GeV at the nearly parallel shock as observed in the strongest SEP events. The two main factors that contribute to these results are the strong radial decrease, as inferred from the coronal model, in both the SEP source function when modeled as a kappa distribution, and in the SEP parallel diffusion coefficient that the traveling shock encounters in the corona. Thus, a key reason for our results is that both these factors, although in different ways, strongly reduce the efficiency of SEP injection into DSA with time at the traveling shock.

2. The Model

We solve numerically the standard focused transport equation to model time-dependent SEP acceleration at a fast spherical, nearly parallel traveling shock propagating radially outward at a constant speed in the corona. For this purpose it is convenient to transform the equation to the traveling shock frame. Then, the equation can be expressed (shown here for a purely parallel shock for simplicity) as [10, 12]

\[
\frac{df}{d\phi_{sh}} + \left( U - V_{sh} + v' \mu' \right) \frac{df}{dr} - \left[ \mu'^2 \frac{dU}{dr} + \left( 1 - \mu'^2 \right) \frac{dU}{dr} + \mu' \left( \frac{1}{v' \dot{r}_{sh}} - \frac{qE_{sh}}{p'} \right) \right] \frac{df}{d\mu'} + \left( \frac{1}{\mu'^2} \right) \frac{d\mu'}{d\mu} - \left( \frac{d}{d\mu} \left( \frac{D_{\mu\phi}}{D_{\mu'\phi}} \right) \right) + Q
\]

when neglecting small relativistic particle factors. In equation (1), both the radial solar wind speed \(U_{sh}\) and shock normal distance \(x_{sh}\) in the shock frame were mapped back to the observer frame using the Galilean transformations \(U_{sh} = U - V_{sh}\) and \(r = x_{sh} + V_{sh} t_{sh}\), and by applying \(\partial / \partial x_{sh} = \partial / \partial r\) valid for a constant shock speed (\(U\) is the radial solar wind speed in the observer frame, \(V_{sh}\) is the radial shock speed, \(r\) is heliocentric distance in the observer frame, and \(t_{sh}\) is time measured in the shock frame). Thus the SEP distribution \(f(r(x_{sh}, t_{sh}), p', \mu', t_{sh})\) is in a mixed coordinate system where particle momentum \(p'\) (or particle speed \(v'\)) and cosine of the particle pitch angle \(\mu'\) are both measured in the solar wind frame. Furthermore, the acceleration of the solar wind in the shock frame can be expressed as \(dU_{sh} / dt_{sh} = \partial U / \partial t_{sh} + (U - V_{sh}) \partial U / \partial r\) for a constant shock speed, while \(D_{\mu\phi}\) represents the standard pitch-angle scattering coefficient for gyro-resonant interaction of SEPs with parallel propagating Alfvén waves in the inertial range [13], modified with an ad-hoc-resonance broadening model [7]. The coefficient describes SEP interaction with forward and backward propagating left-hand, and forward and backward propagating right-hand circularly polarized Alfvén waves, amounting to four wave possibilities. Also, \(E_{sh}\) is the cross-shock electric field and \(Q\) is the SEP proton source term ahead of the shock determined from specifying a kappa distribution as a source function.

The SEP transport equation is coupled to the standard transport equation for Alfvén waves propagating parallel to the magnetic field in a radially varying radial solar wind outflow based on wave action conservation in a weakly non-uniform plasma medium [7, 8]. The wave transport equation, which is solved numerically in the spherical traveling shock frame for all four above-mentioned Alfvén wave possibilities cases, is given by

\[
\frac{DW^m}{dt_{sh}} + \left( U - V_{sh} + jV \right) \frac{DW^m}{dr} \left[ \frac{\partial}{\partial r} \left( U + jV \right) \right] k' \frac{\partial W^m}{\partial k'} = -\frac{1}{2} \frac{\partial}{\partial r} \left[ r^2 U \right] W^m + 2 \gamma^m W^m
\]

where \(W^m(r(x_{sh}, t_{sh}), k', t_{sh})\) is total spectral Alfvén wave energy density (sum of kinetic and magnetic energy) as a function of parallel wave number \(k'\) in the solar wind frame, \(j = \pm 1\) denotes forward \((j = +1)\) and backward \((j = -1)\) wave propagation directions, \(m = \pm 1\) indicates left-hand \((m = +1)\) and right-hand \((m = -1)\) circularly polarized Alfvén waves, and \(\gamma^m\) is the rate of Alfvén wave excitation/damping by SEPs ahead of the traveling shock following standard quasi-linear theory [7].
Because $\gamma'' = jV_a \partial f / \partial \mu'$, and SEPs undergoing DSA tend to be beam-like upstream at lower energies so that $\partial f / \partial \mu' > 0$, forward propagating Alfvén waves ($j = +1$) are generated ahead of the shock by these SEPs. This increases the rate of pitch-angle scattering of SEPs upstream that enhances the efficiency of DSA, and weakens the beam-like SEP distributions and rate of wave generation in a self-regulatory manner [10]. Consistent with shocks driven by coronal mass ejections in the largest SEP events, the shock is modeled to propagate for 1 hour at a high speed of $V_{sh} = 2400 \text{ km s}^{-1}$ through the corona. The shock, which has a compression ratio of ~4, is initialized at 3.5 solar radii to reach ~16 solar radii during this period, a sufficient distance to sample highly non-uniform coronal plasma conditions in accordance with the coronal model discussed below.

3. Radial Variation of Coronal Parameters

As a first step to model the strong radial variation of background plasma parameters in the corona that the traveling shock encounters, we simply apply the empirical model of Köhnlein [1] which smoothly connects remote sensing observations of the corona with in situ measurements in the heliosphere. Future work will involve further refining the coronal model by looking for consistency between the remote sensing observations and coronal models based on first principles, which is beyond the scope of the current paper. In Figure 1(a) and (b) we show the radial decrease in coronal density and temperature, respectively. The vertical lines delineate the radial interval in which we consider the variation of these parameters in our simulations corresponding to the 1 hour shock propagation interval between 3.5 and ~16 solar radii. Using the density and temperature variations as input, we calculate a strong radial decrease of ~4 orders of magnitude in the background SEP proton source function ahead of the traveling shock in the radial domain of shock propagation (see Figure 1(c)). The source function is modeled as a kappa distribution with $\kappa \approx 3$ [14]. We find the background Alfvén speed to be considerably larger than the background solar wind speed for most of the radial interval of shock propagation (Figure 1(d)). However, the assumed high shock speed ensures that, in the shock frame, both forward and backward propagating Alfvén waves upstream are rapidly advected toward the shock, and downstream both waves are advected away from the shock (the Alfvén speed decreases by a factor of ~2 across the strong, nearly parallel traveling shock, thus becoming less than the downstream solar wind speed).
In Figure 2(a) we illustrate the modeled coronal radial decrease in the background Alfvén wave energy density $E_w$. In the calculation we employed the radial Alfvén wave energy density transport model of Jacques [15] (based on conservation of wave action in a weakly non-uniform plasma medium) by inserting the radial dependence of the solar wind speed and Alfvén speed shown in Figure 1 into the expression for $E_w$. Upon combining the radial dependence of $E_w$ with that of the background magnetic field (assumed to be a radial field with strength $B \propto r^{-2}$) in the expression for $D_{\parallel}$, we find that in the corona the SEP parallel diffusion coefficient $\kappa_{\parallel}$ decreases by almost 2 orders of magnitude in the considered domain of shock propagation (see Figure 2(b)). As is discussed below in reference to Figure 6, both this decrease and the strong decrease in the SEP source function encountered by the fast traveling shock causes DSA of SEPs to be highly time-dependent in the corona, resulting in accelerated spectra that strongly differ from the predictions of standard steady-state DSA theory.

4. Simulation Results

In Figure 3 (right panel) simulation results are shown to illustrate the excitation of parallel propagating right-hand circularly polarized Alfvén waves in the inertial range just upstream of a fast spherical shock by SEPs undergoing DSA. The evolution of the wave generation is followed for Alfvén waves propagating in the forward direction (away from the shock) during the first hour of shock propagation in the non-uniform corona as modeled above. The initial background inertial range spectrum (dotted curve) is assumed to be a Kolmogorov $k^{-5/3}$ spectrum. After 10 minutes of shock propagation the wave spectrum (dashed curve) develops a strong enhancement in spectral energy density with an amplification factor of as much as ~10,000 relative to the background spectrum due to Alfvén wave generation by upstream SEPs undergoing DSA. After 60 minutes, however, the spectral bump (solid curve) is strongly diminished with a maximum amplification factor of only ~65 occurring at smaller wave numbers. The shift is a consequence of DSA producing an increasing number of higher energy SEPs that can resonate with Alfvén waves with longer wavelengths, thus increasing their excitation...
The rapid weakening in Alfvén wave generation can be understood in terms of the rate of Alfvén wave generation \( \gamma' = \partial f / \partial \mu \) and how this gradient decreases ahead of the traveling shock. The main contributions to the decrease are: (i) The initially large SEP pitch-angle anisotropy upstream reduces with time due to ongoing pitch-angle scattering of SEPs by Alfvén waves. (ii) The modeled radial decrease of the SEP source function in the corona implies that the propagating shock encounters rapidly increasingly less SEPs that can generate Alfvén waves upstream. (iii) When higher energy SEPs generate Alfvén waves at later times, there are simply less of them available for this purpose compared to when lower energies SEPs generate Alfvén waves during earlier times (negative SEP spectral slope).

In Figure 4 we display simulation results confirming a rapid decrease in the SEP pitch-angle anisotropy ahead of the traveling shock. The results in Figure 4(a) refer to upstream SEP pitch-angle distributions for selected particle energies after 10 minutes of shock propagation in the corona.

![Figure 3](image)

**Figure 3.** (right panel) Upstream inertial range spectral wave energy density in MeVcm\(^{-2}\) as a function of wave number in AU\(^{-1}\) for forward parallel propagating, right-hand circularly polarized Alfvén waves in a non-uniform corona. The dotted line is the initial background spectral wave energy density without a shock, the dashed curve is the spectral wave energy density after 10 minutes of shock propagation, and the solid curve is the spectral energy density after 60 minutes of shock propagation. (left panel) The same forward propagating Alfvén waves as right panel, but downstream of shock.

Especially at lower SEP energies (see solid curve for 0.3 MeV protons) these initial distributions are beam-like with most SEPs propagating upstream with 0° degree pitch angle away from the shock as SEPs with larger pitch angles find it difficult to stay ahead of the fast traveling shock. This explains the strong initial forward propagating Alfvén wave growth ahead of the shock at larger wave numbers as shown in Figure 3 (dashed curve). Compared to Figure 4(a), Figure 4(b) indicates rapid weakening of the initial beam-like SEP pitch-angle distributions upstream after 1 hour of shock propagation in the corona. The beam-like distributions in the forward hemisphere \((0 < \mu \leq 1)\) largely dissipated in response to ongoing pitch-angle scattering of SEPs by Alfvén waves. However, still some wave generation takes place (solid curve in Figure 3) due to the significant positive \( \partial f / \partial \mu \)'-gradient in the \( \mu' \)-interval around \( \mu' = 0 \) (transition between \( \mu' > 0 \) (forward propagating), and \( \mu' < 0 \) (backward propagating) SEPs ahead of the shock). The rapid dissipation of the beam-like SEP distributions by
pitch-angle scattering benefitted from the strong initial wave growth and the radial increase in the background pitch-angle scattering coefficient (decrease in $\kappa_j$) encountered by the traveling shock.

Displayed in Figure 5 is the simulated evolution of the radial variation in the total Alfvén energy density across the traveling shock during the first hour of shock propagation in the corona. The energy density is calculated by integrating over all wave numbers in the inertial range and by summing up energy densities for all four Alfvén wave possibilities. After 10 minutes of shock propagation, there is a clear radial bump in the wave energy density ahead of the shock as a consequence of strong initial wave excitation by upstream SEPs undergoing DSA due to their beam-like distributions. After 20 minutes or more, the radial bump disappears as wave excitation contributes negligibly to the total energy density in the inertial range due to the rapid weakening in upstream wave generation as discussed above. The upstream wave energy density keeps decreasing with time as the traveling shock encounters less energy density due to the modeled negative radial gradient in the background wave energy density in the corona (Figure 2(a)). The shock propagates much faster than Alfvén waves so that both forward and backward propagating waves get compressed across the shock. We find that the total Alfvén wave energy density jumps by a factor of ~20 across the shock initially. At later times the cross-shock jump levels off to a factor of ~10. It should be noted that the Alfvén wave transport equation that we solve across the shock (equation (2)) is simplified in the sense that wave reflection and conversion into different wave modes, such as magnetosonic waves, are not included. Nonetheless, the simulated jump in wave energy density across the shock is reasonable because it is of the same order of magnitude as observed across fast traveling shocks at 1 AU in ground level enhancement SEP events [16]. The jump can be roughly approximated as $s^2$, where $s$ is the shock compression ratio. Behind the nearly parallel traveling shock we see growth in the radial extent of the compressed wave energy density as downstream Alfvén waves, with an Alfvén speed reduced by a factor of 2 compared to upstream, cannot keep pace with the shock. Notice formation of a peak in

Figure 4. (a) Simulated SEP distributions $f(\mu')$ just ahead of the traveling shock as a function of $\mu' (\mu' = \cos \theta')$ where $\theta'$ is the particle pitch angle. The distributions are normalized to a value of 1 at $\mu' = 1$. SEPs with $\mu' = 1$ indicate particles propagating with 0° pitch angle away from the shock whereas $\mu' = -1$ refer to SEPs propagating with 180° pitch angle toward the shock. The distributions are shown after 10 minutes of shock propagation in the corona for different SEP kinetic energies ranging from 0.3-2 MeV as indicated in the legend in (b). (b) As (a), but after 60 minutes of shock propagation.
The total Alfvén wave energy density in erg cm\(^{-3}\), integrated over all wave numbers in the inertial range and summed up for all 4 parallel propagating wave mode cases (forward and backward left and right hand circularly polarized modes), as a function of radial distance relative to the spherical traveling shock normalized to the shock width in the simulation. Positive relative radial distances indicate positions ahead of the shock whereas negative values represent positions behind the shock. The dashed curve shows the radial decrease in the wave energy density in the absence of a shock, whereas the solid curves indicate the evolution of the wave energy density across the shock in 10 minute time intervals.

Simulated SEP proton spectra just behind the traveling shock during the first hour of shock propagation in the non-uniform corona are presented in Figure 6(a). For reference, the initial upstream proton source function (kappa distribution with \(\kappa = 3\)) is included as the dotted curve. The dashed curve is the downstream spectrum after 10 minutes of shock acceleration and the solid curve illustrates its evolution after 1 hour of shock acceleration. At lower energies the SEP spectra downstream have positive slopes with a peak at \(\sim 40\) keV. Comparison with the upstream source function (dotted curve), reveals strong pre-acceleration of lower energy SEPs by the shock. This is predominantly a consequence of the shock-frame acceleration of the plasma flow across the shock \(dU_{sh}/dt_{sh}\) (see expression below equation (1)) that acts as a non-inertial force on SEPs in the downstream direction. Thus SEPs crossing the shock to downstream will be heated whereas those SEPs returning back upstream will be cooled. Since SEPs experience pitch-angle scattering back and forth across the shock, energy gain from this mechanism is stochastic in nature and can be thought of as a second order Fermi pre-heating mechanism [20]. This pre-acceleration mechanism is especially strong at low energies for a fast traveling shock [12], thus dominating the heating effects of the cross-shock electric field and adiabatic compression across the shock which are also included in the simulation. Beyond \(\sim 40\) keV, the dashed SEP spectrum cuts off abruptly because the \(dU_{sh}/dt_{sh}\)-pre-heating mechanism \((1/p')dp'/dt_{sh} \propto (1/v')dU_{sh}/dt_{sh}\) and after 10 minutes there is little sign of DSA of SEPs at higher energies. This is consistent with Figure 2(a) where after 10 minutes upstream SEP distributions are
still strongly beam-like, thus showing little sign of the pitch-angle scattering needed for DSA to be efficient. After 60 minutes of shock acceleration (solid curve in Figure 6(a)), the DSA spectrum is in the process of unfolding at higher energies above \( \sim 40 \) keV. However, different from the prediction of standard steady-state DSA theory, there is no sign of a power law. In fact, the spectrum tends to be concave, and is harder than expected compared to the prediction of standard steady-state DSA theory at a fast traveling shock with a compression ratio of \( \sim 4 \). Our analysis shows that the spectral shape is a direct result of the strongly non-uniform coronal conditions that the shock encounters in two ways: (i) The DSA spectrum evolves to become concave because the traveling shock encounters a decreasing \( \kappa_i \) upstream with time in response to the radial decrease of \( \kappa_i \) as modeled in the corona (see Figure 2(b)). Thus, the injection speed for DSA increases with time as increasingly higher SEP energies are required to enable SEPs to diffuse ahead of the shock. Since less SEPs are available for injection at higher energies, the rate of SEP injection decreases during the injection threshold shift to higher SEP energies with time. (ii) The DSA spectrum also becomes harder than predicted by standard theory. This occurs because the shock encounters a strongly decreasing upstream proton source function in response to the estimated radial decrease of the source function in the corona (see Figure 1(c)). Accordingly, the rate of SEP injection into DSA decreases with time at a given injection threshold.

We conclude that DSA at a fast traveling, nearly parallel acceleration shock in a strongly non-uniform corona might be highly time-dependent so that the resultant SEP spectrum potentially strongly deviates from the predictions of steady-state DSA theory early on in the SEP event. We also note that the maximum energy that SEP protons reach after 1 hour of DSA is \( \sim 10-20 \) MeV in our simulation. This is considerably lower than in typical strong SEP events that can generate significant amounts of SEPs above 1 GeV within a few hours [21]. These results form a contrast to a previous focused transport simulation that produced 1 GeV SEPs within 10 minutes [10]. We note that recently Afansiev et al. [22], with the aid of a self-consistent Monte Carlo simulation model, reached maximum SEP energies of \( \sim 4 \) MeV after 10 minutes. However, linear extrapolation of their results assuming higher SEP injection efficiencies into DSA suggest that maximum energies of \( \sim 300 \) MeV in 10 minutes.
minutes should be possible. A main factor in failing to reach 1 GeV energies in our case is the rapid decline in the rate of Alfvén wave excitation by upstream SEPs as discussed above. Nonetheless, self-excitation of Alfvén waves by SEPs ahead of the shock considerably improves the efficiency of DSA at a fast, nearly parallel traveling shock in our simulations. Without wave excitation, the maximum SEP energy reached in our simulations is only ~100 keV.

Consider the large step-decrease transition in the downstream SEP spectrum between lower-energy SEPs experiencing the effect of the cross-shock acceleration of the solar wind flow, and higher energy SEPs undergoing DSA in the solid curve in Figure 6(a). We think the size of the step is a reflection of the dominance of the acceleration of the flow as an energization mechanism at lower energies that enforces a strong cutoff in the spectrum close to the minimum injection threshold (minimum energy SEPs need to stay ahead of the shock which is ~20-30 keV). We expect this step decrease to become considerably weaker when the shock travels beyond the corona where the cross-shock acceleration of the solar wind will weaken as the shock continues to slow down. Furthermore, the pressure in SEPs accelerated by the cross-shock solar wind flow is considerable, thus raising the question of modification of the shock structure by their pressure gradient. It could well be that a self-consistent calculation will produce weaker cross-shock acceleration of the solar wind, thus leading to a smaller step transition. Also note that in our simulations particle momentum is measured in the solar wind flow frame. Thus, the SEP spectrum will look quite different in the vicinity of step the transition when transformed to the observer frame.

Finally, we show simulation results in Figure 7 to illustrate the radial variation of the SEP differential intensity at selected SEP energies ahead of the fast, nearly parallel traveling shock after 60 minutes of shock propagation. It is interesting that especially the curves for 1 and 10 MeV SEPs indicate a power-law decrease in intensity with increasing radial distance ahead of the shock before their respective rollovers to the background proton distribution. The rollover occurs further ahead of the shock for 10 MeV SEPs than at 1 MeV, thus illustrating the more efficient diffusion of higher energy SEPs upstream. The power-law decay is in contrast to the familiar prediction of standard steady-state DSA theory of an exponential intensity decrease ahead of the shock. The 1 MeV curve upstream agrees well with the relationship for SEP intensity $\propto 1/(r - r_{sh})$ when $r > r_{sh}$ sufficiently ($r_{sh}$ is the shock position). This relationship was predicted by Bell [23] in a steady-state theory that includes self-generation of Alfvén waves assuming strong resonance broadening. Thus, it appears that a power-law
intensity decrease upstream is a direct consequence of Alfvén wave excitation by SEPs undergoing DSA at quasi-parallel shocks (see also Afanasiev [19] for a similar discussion). Therefore, at quasi-parallel shocks upstream wave excitation provides an alternative explanation for such power laws thought by some to be produced by anomalous diffusion of energetic particles across traveling shocks [24].

5. Summary and Conclusions
Results were presented to illustrate simulations of SEP acceleration at a fast spherical, nearly parallel shock propagating with a constant speed \( V_{sh} = 2400 \text{ km s}^{-1} \) in the corona taking into account (i) Alfvén wave excitation upstream by SEPs undergoing DSA, and (ii) the highly non-uniform radial plasma conditions that the shock may encounter. The non-uniform corona was modelled using a simple empirical coronal model [1] as a first step, so that our results should be considered as preliminary. The main idea was to investigate the interplay between DSA and upstream wave excitation during the first hour of fast shock propagation in a strongly radially dependent corona. For this purpose a coupled system of spherically-symmetric time-dependent equations involving the focused transport equation and wave transport equations for parallel propagating Alfvén waves were solved. The main results were: (i) Low-energy SEPs below the injection threshold for DSA was efficiently pre-accelerated stochastically across injection threshold. Above the injection threshold this acceleration mechanism became inefficient rapidly with increasing SEP energy. (ii) The simulated DSA spectrum for SEPs downstream became concave and harder than expected in comparison to the power-law prediction of standard steady-state DSA theory after 1 hour of acceleration. These differences occurred predominantly because the traveling shock encountered both a SEP source function upstream, modeled as a kappa distribution, and a SEP parallel diffusion coefficient upstream that decreased strongly with increasing radial distance in the corona. Both these factors contributed in different ways to a rapid decrease in the rate at which SEPs were injected into DSA, thus resulting in a distorted accelerated spectrum. We concluded that DSA at a fast traveling acceleration shock in the strongly non-uniform corona might be highly time-dependent and thus quite different from the predictions of standard steady-state DSA theory. (iii) The upstream wave excitation by SEPs, initially strong, became weaker rapidly during the first hour of shock propagation in response to the fast decreasing SEP injection rate as explained above, combined with ongoing pitch-angle scattering that reduced the magnitude of the SEP pitch-angle anisotropy upstream responsible for wave excitation. Consequently, during the first hour of shock acceleration, maximum energies of 10-20 MeV, considerably below maximum SEP energies \( > 1 \text{ GeV} \) possible in the strongest SEP events, were reached. Nonetheless, upstream wave excitation strongly enhanced the efficiency of the DSA of SEPs at the nearly parallel traveling shock because without it the maximum energy of SEPs was found to be \( \sim 100 \text{ keV} \). (iv) The boost in the total Alfvén energy density upstream resulting from strong initial wave excitation upstream, ended up later during the first hour of shock propagation as a prominent peak in total wave energy density downstream after advection and compression of the waves across the shock. This raised interesting questions for further research such as (a) to what extent can shock accelerated SEPs experience additional 2\(^{nd}\) order Fermi acceleration by the enhanced Alfvén wave intensity downstream? [17] (b) Assuming a “twin” shock scenario [18], how much can the leading shock contribute to a pre-accelerated SEP population that improves injection of SEPs into DSA at the trailing shock, and (c) to what extent can the enhanced wave turbulence behind the leading shock speed up DSA at the trailing shock by reducing \( \kappa_{\perp} \) upstream of the trailing shock [19]? (v) A consequence of Alfvén wave excitation upstream was that the SEP radial distribution ahead of a quasi-parallel shock became a power law [22,23] instead of an exponential as predicted by standard DSA theory. Therefore, at quasi-parallel shocks upstream wave excitation provided an alternative explanation for such power laws thought to by some to be produced by anomalous diffusion of energetic particles across traveling shocks [24].

6. References
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