QUANTUM RIEMANN SURFACES, 2D GRAVITY AND
THE GEOMETRICAL ORIGIN OF MINIMAL MODELS

Marco Matone

Department of Physics “G. Galilei” - Istituto Nazionale di Fisica Nucleare
University of Padova
Via Marzolo, 8 - 35131 Padova, Italy

ABSTRACT

Based on a recent paper by Takhtajan, we propose a formulation of 2D quantum gravity whose basic object is the Liouville action on the Riemann sphere $\Sigma_{0,m+n}$ with both parabolic and elliptic points. The identification of the classical limit of the conformal Ward identity with the Fuchsian projective connection on $\Sigma_{0,m+n}$ implies a relation between conformal weights and ramification indices. This formulation works for arbitrary $d$ and admits a standard representation only for $d \leq 1$. Furthermore, it turns out that the integrerness of the ramification number constrains $d = 1 - 24/(n^2 - 1)$ that for $n = 2m + 1$ coincides with the unitary minimal series of CFT.

*Partly supported by the European Community Research Programme ‘Gauge Theories, applied supersymmetry and quantum gravity’, contract SC1-CT92-0789

*e-mail: matone@padova.infn.it, mvxpd5::matone
1. Recently in [1] it has been developed an approach to quantum Liouville theory based on the original proposal by Polyakov [2]. The basic object in this theory is the ‘partition function of $\Sigma_{0,n}$’ with $\Sigma_{0,n}$ the Riemann sphere punctured at $z_1, \ldots, z_{n-1}$ and $z_n = \infty$

$$\langle \Sigma_{0,n} \rangle = \int_{\mathcal{C}(\Sigma_{0,n})} \mathcal{D}\phi e^{-\frac{1}{\pi\hbar} S^{(0,n)}(\phi)}, \quad (1)$$

where the measure is defined with respect to the scalar product $||\delta\phi||^2 = \int_{\Sigma_{0,n}} e^\phi |\delta\phi|^2$, and the integration is performed on the $\phi$’s such that $e^\phi$ be a smooth metric on $\Sigma_{0,n}$ with asymptotic behaviour at the punctures given by the Poincaré metric $e^{\phi_{cl}}$ (see (8)). The functional $S^{(0,n)}$ denotes the Liouville action

$$S^{(0,n)}(\phi) = \lim_{r \to 0} S^r_{(0,n)}(\phi) = \lim_{r \to 0} \left[ \int_{\Sigma_r} \left( \partial_z \phi \partial_{\bar{z}} \phi + e^\phi \right) + 2\pi (n \log r + 2(n - 2) \log |\log r|) \right] \langle \Sigma_{0,n} \rangle, \quad (2)$$

where $\Sigma_r = \Sigma_{0,n} \setminus \left( \bigcup_{i=1}^{n-1} \{ z | |z - z_i| < r \} \cup \{ z | |z| > r^{-1} \} \right)$ and $z$ is the global coordinate on $\Sigma_{0,n}$. An important remark in [1] is that by $SL(2,\mathbb{C})$-symmetry one gets the exact result

$$\langle \Sigma_{0,3} \rangle = \frac{c}{|z_1 - z_2|^{1/\hbar}}, \quad \Sigma_{0,3} = \mathbb{C}\setminus\{z_1, z_2\}, \quad c = \langle \mathbb{C}\setminus\{0, 1\} \rangle, \quad (3)$$

which can be interpreted as correlation function of puncture operators $e^{\phi/2\hbar}$ of conformal weight $\Delta = \Delta = 1/2\hbar$. In [1], after fixing the standard normalization $z_{n-2} = 0, z_{n-1} = 1, z_n = \infty$, it is assumed that the theory defined by (1) satisfies the conformal Ward identity

$$\langle T(z) \Sigma_{0,n} \rangle = \left[ \sum_{i=1}^{n-1} \frac{\Delta}{(z - z_i)^2} + \sum_{i=1}^{n-3} \left( \frac{1}{z - z_i} + \frac{z_i - 1}{z} - \frac{z_i}{z - 1} \right) \frac{\partial}{\partial z_i} \right] \langle \Sigma_{0,n} \rangle, \quad (4)$$

where $T = (\phi_{zz} - \frac{1}{2} \frac{\partial^2}{\partial z^2})/\hbar$ is the Liouville stress tensor. Eq.(4) is verified at the tree level where $\Delta_{cl} = 1/2\hbar = \Delta$ implying that

$$\Delta_{loops} = 0. \quad (5)$$

Remarkably, in considering the tree level of (4) one uses [1-3] the well-known relation between the accessory parameters and the classical Liouville action [4] $c_i = -\frac{1}{2\pi} \partial S^{(0,n)}_{cl}/\partial z_i$. Expanding around the Poincaré metric $e^{\phi_{cl}}$, we obtain the semiclassical approximation

$$\log \langle \Sigma_{0,n} \rangle = -\frac{1}{2\pi \hbar} S^{(0,n)}_{cl} - \frac{1}{2} \log \det (2\Delta + 1) + \mathcal{O} (\hbar), \quad (6)$$

\footnote{Note that in getting the second term in (6) one identifies $\{ \phi_{cl} + \sum_{k} a_k \psi_k | a_k \in \mathbb{R} \}$, where the $\psi_k$’s are the eigenfunctions of $\Delta$, with the space $\mathcal{C}(\Sigma_{0,n})$.}
with $\Delta = e^{-\phi_{cl}} \partial_z \partial_{\bar{z}}$ the scalar Laplacian on $\Sigma_{0,n}$. Eq.(6) implies [1] that the Ward identity works up to one loop if $\Delta_{1\text{ loop}} = 0$ in agreement with (5).

2. It is natural to formulate a generalization of (1) in order to understand what is the geometrical analogous of the correlators of Liouville vertices with conformal dimension $\neq 1/2h$. To do this we first consider some facts about Poincaré metric.

Near to an elliptic point the behaviour of the Poincaré metric is

$$e^{\gamma \phi_{cl}} \sim \frac{4q_k^2 r_k^{-2}}{(1 - r_k^2)^2},$$

where $q_k^{-1}$ is the ramification index of $z_k$ and $r_k = |z - z_k|$, $k = 1, \ldots, n - 1$, $r_n = |z|$. Taking the $q_k \to 0$ limit we get the parabolic singularity (puncture)

$$e^{\gamma \phi_{cl}} \sim \frac{1}{r_k^2 \log^2 r_k}.$$ (8)

Let $\Sigma_{h,m+n}$ be a Riemann surface of genus $h$ with $m$ elliptic points $\{z_1, \ldots, z_m\}$ with ramification indices $\{q_1^{-1}, \ldots, q_m^{-1}\}$ and $n$ parabolic points ($p = m + n$). Outside the elliptic points (the parabolic ones do not belong to $\Sigma_{h,m+n}$) the Poincaré metric satisfies the Liouville equation $R \gamma \phi_{cl} = -1$, that is $\partial_z \partial_{\bar{z}} \gamma \phi_{cl} = e^{\gamma \phi_{cl}}/2$. Let $\Sigma = \Sigma_{h,m}$ be the compactification of $\Sigma_{h,m+n}$ (filling in the punctures). The scalar curvature of $e^{\gamma \phi_{cl}}$ on $\Sigma_{h,m+n}$

$$R \gamma \phi_{cl} = -1 + 4\pi e^{-\gamma \phi_{cl}} \sum_{k=1}^m (1 - q_k) \delta(2)(z - z_k),$$

extends on $\Sigma$ to $R \gamma \phi_{cl} = R \gamma \phi_{cl} + 4\pi e^{-\gamma \phi_{cl}} \sum_{k=m+1}^{m+n} \delta(2)(z - z_k)$. Therefore on $\Sigma$

$$\partial_z \partial_{\bar{z}} \gamma \phi_{cl} = \frac{1}{2\gamma} e^{\gamma \phi_{cl}} - \frac{2\pi}{\gamma} \left[ \sum_{k=1}^m (1 - q_k) \delta(2)(z - z_k) + \sum_{k=m+1}^{m+n} \delta(2)(z - z_k) \right].$$ (9)

Note that Gauss-Bonnet formula implies that $e^{\gamma \phi_{cl}}$ is not an admissible metric on $\Sigma$.

Let us now consider the $p$-point function in the standard approach to 2D gravity

$$\langle \prod_{k=1}^p e^{\alpha_k \phi(z_k)} \rangle = \int_{C(\Sigma_{h,0})} D\phi e^{-\frac{S^{(h,0)}}{2\pi} - \sum_{k=1}^p \alpha_k \phi(z_k)}.$$ (10)

Here we do not care about the explicit form of the Liouville action. We only assume $S^{(h,0)}$ be defined on a compact Riemann surface $\Sigma_{h,0}$, and that the associated equation of motion be $\partial_z \partial_{\bar{z}} \phi = \frac{1}{2\gamma} e^{\gamma \phi}$. In the saddle-point approximation the leading term reads

$$e^{-\frac{S^{(h,0)}(\phi)}{2\pi} + \sum_{k} \alpha_k \phi(z_k)},$$

\footnote{Here and in section 3 we consider the rescaled field: $\phi \to \gamma \phi$, $\gamma = 2h$.}
where \( \tilde{\phi} \) satisfies the equation (note that \( \tilde{\phi} \notin C(\Sigma_{h,0}) \))

\[
\partial_z \partial_{\bar{z}} \tilde{\phi} = \frac{e^{\gamma \tilde{\phi}}}{2\gamma} - 2\pi \sum_{k=1}^{p} \alpha_k \delta^{(2)}(z - z_k),
\]

(12)

that for \( \alpha_k = \frac{1-q_k}{\gamma} \), coincides with eq. (11). Eq. (12) defines a \((1,1)\)-differential \( e^{\gamma \tilde{\phi}} \) which is not an admissible metric on \( \Sigma_{h,0} \). Nevertheless the previous discussion shows that eq. (12) can be considered as the Liouville equation on the compactification \( \Sigma = \Sigma_{h,m} \) of a Riemann surface \( \Sigma_{h,m+n} \) with \( n \)-punctures (where \( n \) is the number of \( \alpha_k \)'s equal to \( 1/\gamma \)) and \( m \)-elliptic points where \( e^{\gamma \tilde{\phi}} \) coincides with the Poincaré metric (for a discussion on admissible metrics in this framework see [6]). This investigation suggests to extend the approach (1) by considering as basic object the Liouville action for Riemann surfaces with \( \gamma \) ramified points. In particular we will still have the same classical limit as (10) but without the constraint \( \alpha_k = 1/2h \). As a consequence we will get a purely geometrical definition of conformal weight in Liouville gravity. We recall that usually one defines conformal weights by assuming validity of the free field representation in order to perform the OPE.

Eqs. (7,8) imply that the classical term (11) is divergent so that \( e^{\alpha \phi} \) must be regularized. The regularization is precisely the same that one considers in defining the regularized Liouville action (2). The crucial point is that, as eq. (3) shows, the regularization term fixes the scaling properties of Liouville vertices. Similar aspects have been discussed in [6].

3. Here we shortly discuss the null vector equation arising in the CFT approach to Liouville theory. The correctness of this approach needs to be proved, nevertheless the following analysis will suggest a relationship between conformal weights and ramification indices.

In [7] it was pointed out that the uniformization equation for the punctured sphere is related to the classical limit of the null vector equation for the \( V_{2,1} \) field \( \psi \)

\[
\frac{\partial^2 \psi(z)}{\partial z^2} + \frac{\gamma^2}{2} : T(z) \psi(z) : = 0.
\]

In the CFT approach to Liouville theory one has \( T(z) = \frac{1}{2} Q_{\psi z z} - \frac{1}{2} \phi_z^2 \). In [8] it has been proposed to compare the classical limit \( c_{\text{Liouv}} \to \infty \) of the decoupling equation for the null vectors in the \( V_{2,1} \) Verma module

\[
\left( \frac{\partial^2}{\partial z^2} + \frac{\gamma^2}{2} \sum_{i=1}^{n} \frac{\Delta_i}{(z - z_i)^2} + \frac{\gamma^2}{2} \sum_{i=1}^{n} \frac{1}{(z - z_i)^2} \frac{\partial}{\partial z_i} \right) \langle V_{2,1}(z) \prod_{i=1}^{n} V_i(z_i) \rangle = 0,
\]

(13)

\(^3\)By abuse of language by ‘ramified points’ we mean both parabolic and elliptic points.
\[ \gamma = \left( Q - \sqrt{Q^2 - 8} \right)/2, \quad c_{\text{Liouv}} = 1 + 3Q^2, \]

with the uniformization equation

\[ \left( \partial_z^2 + \frac{1}{2} T^F(z) \right) \psi(z) = 0, \]  

(14)

where \( T^F = hT_d \) is the Fuchsian connection on the punctured Riemann sphere. This gives

\[ \gamma^2 \Delta_i = \frac{1}{2} = \Delta_i^{(c)}, \quad i = 1, \ldots, n, \]  

(15)

where \( \Delta_{p,q} = \frac{\alpha_{p,q}}{2}(Q - \alpha_{p,q}), \alpha_{p,q} = \frac{1-p}{2} \gamma + \frac{1-q}{2}, \) and \( \Delta_i^{(c)} \equiv \lim_{\gamma \to 0} \gamma^2 \Delta_{p,q} = (1 - q^2)/2. \) Eq.(15) implies the constraint \( \Delta_i = \Delta_{1,0}, \forall i, \) so that \( \langle V_{2,1}(z) \prod_{i=1}^n V_i(z_i) \rangle = 0. \)

Instead of ‘changing uniformization’ as proposed in [8], we compare eq.(13) with the uniformization equation

\[ \left( \partial_z^2 + \frac{1}{2} T^{\{q_k\}}(z) \right) \psi(z) = 0, \]

where \( T^{\{q_k\}} \) denotes the Fuchsian connection on the Riemann sphere whose points \( \{z_1, \ldots, z_{n-3}, 0, 1, \infty\} \) have ramification indices \( \{q_1^{-1}, \ldots, q_n^{-1}\}. \)

The important point is that now the coefficient of the second order pole of \( T^{\{q_k\}} \) at the elliptic points is modified by a factor \( 1 - q_k^2 \) with respect to the parabolic case, that is

\[ \frac{1}{2(z - z_k)^2} \rightarrow \frac{1 - q_k^2}{2(z - z_k)^2}. \]

(16)

4. By comparing eqs.(13-15) with eq.(16) we have

\[ \Delta_{cl}(q) = (1 - q^2)/2. \]

(17)

Furthermore, the analysis in sect.2 shows that to a point of index \( q^{-1} \) we can associate a Liouville vertex of charge

\[ \alpha = (1 - q)/2h. \]

(18)

In the following we will show the correctness of eq.(17) and will see that \( \Delta(q) = \Delta_{cl}(q). \)

Let us introduce the following ‘partition function of \( \Sigma_{0,m+n} \)’

\[ \langle \Sigma_{0,m+n}\{q_k\} \rangle = \int_{C(\Sigma_{0,m+n})} \mathcal{D}\phi e^{-\frac{i}{2\pi} S^{(0,m+n)}(\phi)}. \]

(19)

The functional \( S^{(0,m+n)} \) denotes the Liouville action on \( \Sigma_{0,m+n} \) whereas the domain of integration consists of smooth metrics on \( \Sigma_{0,m+n} \) with asymptotics given by (7) and (8) at the points \( \{z_1, \ldots, z_m\} \) and \( \{z_{m+1}, \ldots, z_p\} \) respectively. For each ramified point the regularization term in \( S^{(0,m+n)} \) reads

\[ -2\pi \left( (q - 1) \log r + 2 \log \left| \frac{2q}{1 - r^2q} \right| \right). \]

(20)
Let $\Sigma_{0,1+2}$ be a Riemann sphere with a puncture at $z_3 = \infty$ and two elliptic points at $z_1, z_2$ with ramification numbers $q_{1,2}^{-1} = q^{-1}$. By $SL(2, \mathbb{C})$-symmetry we have

$$\langle \Sigma_{0,1+2} \rangle = \frac{c}{|z_1 - z_2|^{1+2\alpha}},$$

so that we have the exact result

$$\Delta(q) = \frac{1 - q^2}{2h}.$$  (22)

Let us set $z_{p-2} = 0, z_{p-1} = 1$ and $z_p = \infty (q_p = 0)$ and denote by $T^{(m+n)}(z) = (\phi_{zz} - \frac{1}{2}\phi_z^2)/h$ the stress tensor associated to (19). Still in this case we assume the validity of the conformal Ward identity

$$\langle T^{(m+n)}(z) \Sigma_{0,m+n} \rangle = \int_{\mathcal{C}(\Sigma_{0,m+n})} D\phi T^{(m+n)}(z)e^{-\frac{1}{2\pi h}S^{(0,m+n)}(\phi)}. $$

The tree level of (23) reads

$$T^{(m+n)}_{cl}(z) = \sum_{i=1}^{p-1} \frac{1 - q_i^2}{2h(z - z_i)^2} - \frac{1}{2\pi h} \sum_{i=1}^{p-3} \left( \frac{1}{z - z_i} + \frac{z_i - 1}{z} - \frac{z_i}{z - 1} \right) \partial S_{cl}^{(0,m+n)}. $$

By (11) it follows that $-2\pi c_i = \partial z_i S_{cl}^{(0,m+n)}$, where now the $c_i$'s are the accessory parameters of $\Sigma_{0,m+n}$. In this case the classical limit (23) reduces to the Fuchsian projective connection $T^{(q_k)}$ (times $1/h$). As before the semiclassical approximation of $\langle \Sigma_{0,m+n} \rangle$ implies that the Ward identity works up to one loop if $\Delta_{\text{loop}}(q) = 0$, in agreement with (17) and (22). The result in [1] concerning the evaluation of the Liouville central charge extends to (19), that is

$$c_{\text{Liouv}} = 1 + \frac{12}{h}. $$

In bosonic string theory $h = 12/(25 - d)$, so that $c_{\text{Liouv}} = 26 - d$ and

$$\Delta_k \equiv \Delta(q_k) = \frac{(1 - q_k^2)(25 - d)}{24}. $$

In order to interpret $\langle \Sigma_{0,m+n} \rangle$ in terms of Liouville correlators we first recall that in the DDK model [3] the modified Liouville action has the term $\sim \int \sqrt{g} e^{\alpha \sigma}$ which is well-defined only for $\Delta(e^{\alpha \sigma}) = 1$. Such a Liouville vertex can be represented by a ramified point. However, by (27), a necessary condition for the existence of this representation is

$$\Delta(q) = 1 \rightarrow q^2 = \frac{1 - d}{25 - d}. $$
On the other hand, since $0 \leq q^2 \leq 1$, it follows that the DDK model has a geometrical counterpart only for
\[ d \leq 1. \]  
(29)
This result furnishes a geometrical framework to consider the $d = 1$ barrier arising in the standard approach \[9, 10\] to 2D gravity coupled to conformal matter. Furthermore, since $q^{-1} \in \mathbb{N}$, we get
\[ d = 1 - 24/(n^2 - 1), \quad n = q^{-1}, \]  
(30)
that for $n = 2m + 1$ is the unitary minimal series of CFT
\[ d = 1 - 6/m(m + 1). \]
Note that by (28) it follows that $d = 1$ is related to a puncture. In this case (30) gets a $\log |\log r|$ term which is reminiscent of the log correction to $\gamma_{str}$ for $d = 1$.

By (18) and (28) it follows that
\[ \alpha = \frac{\sqrt{25 - d} \left( \sqrt{25 - d} - \sqrt{1 - d} \right)}{24}, \]  
(31)
which should be compared with the rescaled value given in \[9, 10\]. Note that positivity of $q$ implies not sign ambiguity in getting (31). The relation (30) between ramification index and central charge is analogous to the relation arising in the $k$th-matrix model where $d = 1 - 3(2k - 3)^2/(2k - 1)$. The value of $k$ fixes the possible values of the deficit angle in the triangularization.

5. We now discuss the origin of the $d = 1$ barrier in the DDK model. To do this we first consider the split
\[ Dg = D[\tilde{m}]D_g v^z D_g v^\bar{z} D_g \sigma \det \nabla^z \det \nabla^{\bar{z}}. \]
Since $||v, v||^2_{g=e^\sigma \tilde{g}} = \int_\Sigma \sqrt{\tilde{g}} g_{ab} e^{2\sigma} v^a v^b$, it follows that $Vol_g(\text{Diff}(\Sigma))$ depends on $\sigma$. In critical string theory one usually assumes that this dependence can be absorbed into $D_g \sigma$ and then drop the $D_g v^z D_g v^{\bar{z}}$ term. However for $d \neq 26$ this procedure is not correct. The question is to understand whether the DDK assumption in finding the form of the Jacobian $J(\sigma, \tilde{g}) = e^{-S}$ still works when the term $D_g v^z D_g v^{\bar{z}}$ is included. A possibility to overcome this question is to consider the partition function $Z = \int_{M_h} Z$ of non critical strings by investigating its properties by the point of view of the theory of moduli spaces of Riemann surfaces $M_h$. 
Of course $\mathcal{Z}$ must be a well-defined volume form on $\mathcal{M}_h$. An important result about $\mathcal{M}_h$ is the Mumford isomorphism

$$\lambda_n \cong \lambda_1^c, \quad c_n = 6n^2 - 6n + 1,$$

where $\lambda_n = \det \text{ind} \bar{\mathcal{F}}_n$ are the determinant line bundles. The fact that the metric measure cannot depend on the background choice implies that $c_{tot} = 0$. It follows that by the Mumford isomorphism $\mathcal{Z}$ is (essentially) the modulo square of a section of the bundle

$$\Lambda = \prod_{k=1}^l \lambda_k^{d_k}, \quad \sum_{k=1}^l c_k d_k = 0,$$  \hspace{1cm} (32)

where $-2c_j d_j$ is the central charge of the sector $j$. In the Polyakov string the matter and ghosts sectors have $d_1 = -d/2$ and $d_2 = 1$ respectively, thus (32) gives for the Liouville sector $c_{\text{Liouv}} = 26 - d$.

Eq.(32) suggests to extend to the non critical case the Belavin-Knizhnik conjecture [11] (based on the GAGA principle [12]) concerning the algebraic properties of multiloop amplitudes.

A way to represent CFT matter of central charge $d$ is to use a $b$-$c$ system of weight $n$, such that $-2c_n = d$ [13]. Notice that, since the maximum of $-2c_n$ is 1, this approach works for $d \leq 1$ only. The model is exactly a CFT realization of the Feigin-Fuchs approach where semi-infinite forms can be interpreted in terms of $b$-$c$ system vacua. Of course one can use the bosonized version of the $b$-$c$ system which is equivalent to the Coulomb gas approach.

For $d > 1$ it is not possible to represent the conformal matter by a $b$-$c$ system. In this case one can consider the $\beta$-$\gamma$ system of weight $n$ whose central charge is $2c_n$. However the representation of the $\beta$-$\gamma$ system in terms of free fields is a long-standing problem which seems related to the $d = 1$ barrier.

Let us go back to eq.(32). The question is to find the line bundle on $\mathcal{M}_h$ representing the Liouville sector. The fact that $e^\sigma$ is positive definite suggests possible mixing between Liouville, matter and ghost sectors. In this context it is useful to recall that the Liouville action defines a Hermitian metric on moduli space [14].

References

[1] L.A. Takhtajan, *Liouville Theory: Quantum Geometry Of Riemann Surfaces*, preprint hep-th/9308125.
[2] A.M. Polyakov, *Lectures At Steklov Institute*, Leningrad, 1982, unpublished.

[3] L.A. Takhtajan, in ‘New Symmetry Principles In Quantum Field Theory’, Eds. J. Fröhlich et al., Plenum Press, 1992.

[4] P.G. Zograf and L.A. Takhtajan, Math. USSR Sbornik, **60** (1988) 143.

[5] I. Kra, *Automorphic Forms And Kleinian Groups*, Benjamin 1972.

[6] M. Matone, *Uniformization Theory And 2D Gravity. I. Liouville Action And Intersection Numbers*, preprint IC-MATH/8-92, DFPD/92/TH/41, hepth/9306150.

[7] A.A. Belavin, A.M. Polyakov and A.B. Zamolodchikov, Nucl. Phys. **B241** (1984) 333.

[8] C. Gomez and G. Sierra, Phys. Lett. **255B** (1991) 51.

[9] J. Distler and H. Kawai, Nucl. Phys. **B321** (1989) 509.

[10] V. Knizhnik, A.M. Polyakov and A.B. Zamolodchikov, Mod. Phys. Lett. **A3** (1988) 819.

[11] A. Belavin and V. Knizhnik, Phys. Lett. **168B** (1986) 201; Sov. Phys. JEPT **64** (2) (1986) 214.

V. Knizhnik, Sov. Phys. Usp. **32** (1989) 945.

[12] J.P. Serre, Ann. Inst. Fourier **6** (1950) 1.

[13] L. Bonora, M. Matone, F. Toppan and K. Wu, Phys. Lett. **224B** (1989) 115; Nucl. Phys. **B334** (1990) 717.

[14] P.G. Zograf, Leningrad Math. J. **1** (1990) 941.