Abstract

We argue that features of hadron production in relativistic nuclear collisions, mainly at CERN-SPS energies, may be explained by the existence of three forms of matter: Hadronic Matter, Quarkyonic Matter, and a Quark-Gluon Plasma. We suggest that these meet at a triple point in the QCD phase diagram. Some of the features explained, both qualitatively and semi-quantitatively, include the curve for the decoupling of chemical equilibrium, along with the non-monotonic behavior of strange particle multiplicity ratios at center of mass energies near 10 GeV. If the transition(s) between the three phases are merely crossover(s), the triple point is only approximate.
1 Introduction

The SPS heavy ion program at CERN resulted in some of the first experimental data on heavy ion collisions at ultrarelativistic energies, see, e.g., [1]. A summary of these data and implications for the possible formation of a new state of matter were announced in a CERN press release [2]. In this paper we consider some generic features discovered in heavy ion experiments at the SPS. This gives us a general overview of how the collisions of heavy ions evolve in going from low energies, as studied at the SIS (GSI) and the AGS (BNL), to higher energies, at RHIC (BNL) and soon at the LHC (CERN) [3,4,5,6,7,8].

In particular, we concentrate on hadron abundances in heavy ion collisions. These have been widely studied using resonance gas models. By assuming that the observed particle yields are generated at a common surface at which all particles decouple, values of the baryon chemical potential, $\mu_B$, and temperature, $T$, on this surface, can be extracted. Fitting these two parameters, $\mu_B$ and $T$, together with the volume parameter gives values for the particle abundances which are in close agreement with experiment [9,10,11,12,13,14,15,16,17,18,19,20,21,22].
The resulting values of $\mu_B$ and $T$ are shown in Fig. 1 as functions of center-of-mass energy per nucleon pair.

We note that, near 10 GeV center of mass energy, the temperature saturates with increasing beam energy, reaching an asymptotic value of about 160 MeV, while the baryon chemical potential decreases smoothly.

![Figure 1: The decoupling temperatures and chemical potentials extracted by Statistical Model fits to experimental data. The freeze-out points are from Refs. [15] and [20,23,24]. The open points are obtained from fits to mid-rapidity whereas the full-points to 4\pi data. The inverse triangle at $T = 0$ indicates the position of normal nuclear matter. The lines are different model calculations to quantify these points [22,25,26]. The shaded lines are drawn to indicate different regimes in this diagram (see text).](image)

Plotting these temperature-chemical potential pairs for all available energies results in a phase diagram-like picture as is illustrated in Fig. 2. In the $\mu_B$ region from 800 to 400 MeV, as $T$ increases from 50 to 150 MeV, the experimental points rise approximately linearly. In contrast, below $\mu_B \sim 400$ MeV, the temperature is approximately constant, $T \simeq 160$ MeV. The highest collision energies studied to date at RHIC are those for which $\mu_B \sim 25$ MeV. Also shown on this plot are lines of fixed energy per particle and fixed entropy density per $T^3$; also shown is a line of hadron percolation (see below).

These experimental results can be compared to phase transition points computed on the lattice [27,28]. Numerical simulations in lattice QCD can be
performed at nonzero temperature, and small values of \( \mu_B \) without running into problems of principle. At \( \mu_B = 0 \), these simulations indicate that there is no true phase transition from Hadronic Matter to a Quark-Gluon Plasma, but rather a very rapid rise in the energy density at a temperature \( T_c \) which lines in 160 – 190 MeV within the systematic errors. Further, studies using the lattice technique imply that \( T_c \) decreases very little as \( \mu_B \) increases, at least for moderate values of \( \mu_B \).

![Diagram](image)

Fig. 3. Energy dependence of hadron yields relative to pions. The points are experimental data from various experiments. Lines are results of the Statistical Model calculations. The figure is taken from [21,20]).

With the parametrizations of \( T \) and \( \mu_B \) from Fig. [1] one can compute the energy dependence of the production yields of various hadrons relative to pions, shown in Fig. [3]. Important for our purposes is the observation that there are peaks in the abundances of strange to non-strange particles at center of mass energies near 10 GeV. In particular, the \( K^+ / \pi^+ \) and \( \Lambda / \pi \) ratios exhibit rather pronounced maxima there. We further note that in the region near 10 GeV, there is also a minimum in the chemical freeze-out volume [29,18] obtained from the Statistical Model fit to particle yields [18,21], as well as in
the volume obtained from the Hanbury-Brown and Twiss (HBT) radii of the fireball \cite{30}. The energy dependence of the volume parameters is shown in Fig. 4.

![Graph showing the energy dependence of the volume for central nucleus-nucleus collisions.](image)

Fig. 4. Energy dependence of the volume for central nucleus-nucleus collisions. The chemical freeze-out volume $dV/dy$ for one unit of rapidity (boxes) taken from Ref. \cite{18} is compared to the kinetic freeze-out volume $V_{\text{HBT}}$ (filled circles and triangles) from Ref. \cite{30}. The line is the Statistical Model calculations with thermal parameters from Fig. 1.

These experimental observations have long resisted interpretation in terms of a transition between Hadronic Matter and a Quark-Gluon Plasma. The general structures observed in the data are well reproduced only by the most recent model calculations \cite{20}. There, it is argued that these structures arise due to the interplay between the limit in hadronic temperature (see Fig. 1) due to the QCD phase transition and the rapid decrease of $\mu_B$ with increasing energy, thereby establishing a connection between Hadron Gas and Quark-Gluon Plasma. The possible existence of a critical endpoint is, however, not relevant for these considerations.

The above described structures seem puzzling if the corresponding energies would probe a critical endpoint in the QCD phase diagram \cite{33}. Near a critical point, lighter particles, such as pions, should be affected more than heavier particles, such as kaons; HBT radii should also increase. Both of these features are not easily linked to the trends in the data.

\footnote{We note the interpretation given in \cite{32}, obtained within a schematic 1st order phase transition model.}
We will discuss the relationship between the above Statistical Model descriptions of the transition to both the Quark-Gluon Plasma and Quarkyonic Matter, the triple point where three phases of matter coexist, and the underlying contribution to the spectrum of strange particles below, and argue that generic features of these curves may be explained in this context.

2 Quarkyonic Matter and the QCD Phase Diagram

In the following we show that by considering Quarkyonic Matter, which was recently proposed [34,35,36,37,38], the two regimes observed in the phase diagram and described above can be understood as arising from a triple point where Hadronic Matter, the Quark-Gluon Plasma, and Quarkyonic Matter all coexist. This triple point is located where the temperature is reaching its limiting value and, hence, is naturally also situated in the vicinity of the peaks in the observed hadron production ratios. A sketch of a possible phase diagram for QCD is shown in Fig. 5.

![Phase Diagram](image)

Fig. 5. The phase diagram of strongly interacting matter.

There are hadrons in the lower, left-hand corner of this phase diagram, at low temperatures and $\mu_B$. There are two, qualitatively distinct, phase boundaries by which one can leave Hadronic Matter. The first, is to increase the temperature at low $\mu_B$ until it is beyond $T_c$. This is the usual transition from a meson-dominated phase\(^2\) to a Quark-Gluon Plasma. This phase boundary

\(^2\) We note that, at chemical freeze-out, the density of baryons and anti-baryons, $n_B$, is similar in this regime to that at large-$\mu_B$ ($n_B \simeq 0.12$ fm\(^{-3}\)) [18].
is probed by collisions at high SPS energies, and by collisions at RHIC and the LHC. The second way is to increase $\mu_B$ at low temperatures, $T < T_c$, going from Hadronic Matter to Quarkyonic Matter. We suggest that this phase boundary is studied by heavy ion collisions at moderate and low energies, such as those at the AGS, SIS, and at low energies at the SPS, and in the future at FAIR and NICA [39].

At a special value of the baryon chemical potential and temperature, there is a triple point where Hadronic Matter, the Quark-Gluon Plasma, and Quarkyonic Matter all coexist. From experiment, Fig. 2, we estimate that this occurs for

$$\mu_B^{\text{triple pt}} \approx 350 - 400 \text{ MeV} , \ T^{\text{triple pt}} \approx 150 - 160 \text{ MeV} . \quad (1)$$

This point is presumably near where the linear and the flat temperature regime in Fig. 2 intersect. We argue in the following how this arises from a triple point.

In thermodynamics a triple point is the point in a phase diagram where three lines of first order phase transitions meet. A common example is where a gas, liquid, and solid coexist at a given value of the pressure and temperature. Since there are only first order phase transitions, no correlation length diverges at the triple point. For example, in the phase diagram of water, the phases of vapor, water, and ice all coexist at the triple point. There is also a critical point in the phase diagram of water, but it is situated far from the triple point, at much higher temperature and pressure.

The properties of strongly interacting matter at large density are characterized by several order parameters. One is the thermal Wilson or Polyakov loop, which measures the degree of deconfinement reached. This is strictly an order parameter in theories without quarks, or in the limit of a large number of colors, $N_c \to \infty$, if the number of flavors, $N_f$, is kept fixed. The second is the chiral condensate as an order parameter for chiral symmetry breaking. Chiral symmetry is an exact symmetry when there are two (or more) flavors of massless quarks. The last is the density of baryons, which is an order parameter even in the large $N_c$ limit, when $N_f$ grows with $N_c$ [35].

Hadronic Matter is confined and exhibits chiral symmetry breaking. It is technically difficult to define confinement for finite $N_c$ for a finite number of quark flavors, since the potential that separates quarks is never linear at large distances. This argument has a precise meaning at zero $N_f$ or infinite $N_c$, or for zero temperature. Nevertheless, there should be a well defined region of low baryon density and low temperature where the physical degrees of freedom are mesons. This phase is also to a good approximation free of baryons since their densities, $n_B/M_B^3 \sim e^{(\mu_B-M_B)/T} \leq 10^{-2}$ for typical values of $\mu_B$ and $T$ not too close to the phase boundary.

The Quark-Gluon Plasma is deconfined with restored chiral symmetry, and
has nonzero baryon number density when $\mu_B \neq 0$. It is composed of quarks and gluons, although we note that lattice simulations indicate that the transition to a deconfined state is rapid, but not discontinuous \cite{28}. This means that, for a range of temperatures above $T_c$, there is a “semi” Quark-Gluon Plasma, in which the theory is only partially deconfining \cite{40,41,42}. We neglect here the effects of the semi Quark-Gluon Plasma, since lattice simulations \cite{27,28} indicate that the energy density rises quickly to values close to the ideal gas value near $T_c$, and this is the main quantity which will concern us. This is unlike the pressure, which does not approach the ideal gas value until several times $T_c$, and for which the semi Quark-Gluon Plasma is important.

Quarkyonic Matter is (approximately) confined, but has a large baryon number density, and also a large energy density. Whether chiral symmetry is restored in Quarkyonic Matter is not yet fully understood. Even at very high densities, there could be residual chiral symmetry breaking from pairing effects near the Fermi surface. For the present discussion it does not matter when and how chiral symmetry is restored in the Quarkyonic phase.

We remark that studies of the Sakai-Sugimoto model at nonzero quark density serve as one realization of Quarkyonic matter \cite{43}.

At the outset we concede that, in the strict thermodynamic sense, the QCD phase diagram might or might not have a true triple point. After all, the deconfining transition at low $\mu_B$, and nonzero temperature, appears not to be of first order, but a rapid crossover. If the deconfinement transition remained a crossover for all $\mu_B$ values, then the triple point would not be a true point, since it would not connect matter separated by a true first order phase transition. It might happen that there is a second order critical end point along the deconfinement line, in which case the triple point might truly reflect three different phases connected by first order phase transitions.

We do suggest that there is a true triple point in the limit of an infinite number of colors \cite{34,35}. In this limit, the deconfinement transition is of first order \cite{44}, and the Quarkyonic transition may exist \cite{35}. Thus the behavior for QCD may be reminiscent of that for a large number of colors, and exhibit an approximate triple point.

For the present discussion, it is not important whether the triple point is exact, or only approximate. What is important is that, in going from Hadronic Matter to either the Quark-Gluon Plasma, or Quarkyonic Matter, there is a large increase in the number of degrees of freedom. Hadronic Matter is dominated by Goldstone bosons. In QCD, the hadronic phase has three types of pions, and a relatively small amount of kaons; for $N_f$ flavors, there are $N_f^2 - 1$ Goldstone bosons in the hadronic phase. These Goldstone bosons dominate bulk properties of the system for temperatures and quark chemical potentials,
\[ \mu_Q = \mu_B/N_c \] much smaller than \( \Lambda_{QCD} \). As one gets close to a transition temperature, massive degrees of freedom become important, eventually becoming so numerous that a transition to a new phase of matter is induced.

As is well known, there are many more degrees of freedom in the Quark-Gluon Plasma: for \( N_c \) colors, there are \( 2(N_c^2 - 1) \) bosonic and \( 4N_cN_f \) fermionic, or 16 bosonic and 24(36) fermionic degrees of freedom in QCD with 2(3) flavours. We note that for the pressure and energy density, ideal fermions contribute \( 7/8 \) of a boson.

While Quarkyonic Matter is confined, the principal point of Ref. 34 is that the energy density, or equivalently the number of degrees of freedom, can be counted as for deconfined quarks. While near the Fermi surface the degrees of freedom are confined baryons, most of the energy density is due to quarks, deep in the Fermi sea. This is a coarse description of what is surely a much more complicated reality. If we assume that chiral symmetry remains broken in the Quarkyonic phase, Quarkyonic Matter then has \( N_f^2 - 1 \) bosonic, and \( 2N_cN_f \) fermionic, degrees of freedom. The number of fermionic degrees of freedom is half that of the Quark-Gluon Plasma, since in Quarkyonic Matter, only quarks, but not anti-quarks, contribute. In QCD, there are 3(8) bosonic degrees of freedom, plus 12(18) fermionic degrees of freedom for 2(3) flavours. The number of degrees of freedom is smaller for Quarkyonic Matter than for the Quark-Gluon Plasma, but significantly larger than the number of Goldstone degrees of freedom of Hadronic Matter.

Thus, we argue, that while there may be no true phase transitions from either Quarkyonic Matter, or a Quark-Gluon Plasma, to Hadronic Matter, there is a rapid decrease in the number of degrees of freedom and so in the energy density. This rapid decrease could well cause the matter to decouple, and so define, experimentally, the surfaces for chemical equilibrium. This is approximately true for the transition from the Quark-Gluon Plasma to the hadronic phase, as observed at RHIC energies 45.

At RHIC energies, chemical freeze-out was shown 45 to take place very close (within less than about 10 MeV) to the phase boundary, driven by the rapid density change across the phase transition. Further it was argued that freeze-out ends when the system is fully hadronized, i.e. at low density in the hadronic phase. Were this not the case 46, one would also expect different freeze-out parameters for each hadron species due to widely different hadronic cross sections. This is not observed. We believe this argument to be generic 45; to ensure simultaneous (within a very small interval in temperature and chemical potential) freeze-out of all hadrons, the freeze-out curve has to be very close to a line with a rapid density change. An immediate consequence of this would be that the chemical freeze-out curve delineates phase boundaries, not only for small values of \( \mu_B \) but everywhere. But what provides the phase boundary
for large values of $\mu_B$, where the deconfinement transition seems far away, at least if one follows the guidance from lattice QCD calculations? As already indicated above we believe that the transition from Hadronic to Quarkyonic Matter provides the missing link.

Across the Quarkyonic line, we would expect that the transition takes place in a range of baryon chemical potentials of order $\delta\mu_B \sim k_F^2/2M_B \sim 35$ MeV in width, for $\Lambda_{QCD} \sim 200$ MeV as a typical baryonic mass scale in QCD and for $k_F = 0.263$ MeV. This width is parametrically of order $1/N_c$ which accounts for its anomalously small size compared to typical hadronic energy scales.

3 A Simple Hagedorn Model for the Quarkyonic Transition

In this section, we explore a very simple model of the Quarkyonic transition. This model only counts the number of degrees of freedom of baryonic resonances, and ignores effects due to the strong nucleonic interactions. It assumes that the resonance spectrum “turns on” in a very narrow window of $\mu_B$, as suggested by the large $N_c$ arguments of the previous section. Interaction effects should not therefore change the position of the phase boundary. Nevertheless in realistic computations interactions should be taken into account, and a realistic spectrum of baryons should be used. These modifications will be discussed in the next section.

Resonance formation is the dominant feature for mesonic interactions, and the most detailed model of hadron dynamics, the dual resonance model \[47\], in fact describes all scattering amplitudes in terms of resonance poles in the different kinematic channels. The number of states of mass $m$, the degeneracy $\rho(m)$, is found to increase as

$$\rho(m) \sim m^{-a} \exp\{2\pi \sqrt{2\alpha'/3m}\}$$

(2)

where $\alpha' \simeq 1$ GeV$^{-2}$ is the universal Regge resonance slope and $a$ a positive constant \[48\]. A basic result in the study of interacting systems is that if the interactions are resonance-dominated, the system can be replaced by an ideal gas of all possible resonances \[49,50\]. The partition function determining the thermodynamics of an ideal resonance gas \[51\] becomes

$$\ln Z(T, V) = \text{const.} \, V T^{3/2} \int_{m_0}^{\infty} dm \, m^{(3/2) - a} e^{-m[(1/T) - (1/T_H)]},$$

(3)

where $T_H^{-1} = 2\pi \sqrt{2\alpha'/3}$. It is seen that this partition function has a singular point at $T_H \simeq 190$ MeV, indicating that the system cannot exist at higher temperatures. Previous work assuming self-similar resonance formation, the so-called Statistical Bootstrap Model \[52\], had also led to an exponentially
increasing level density, and for some time it was assumed that \( T_H \) was the ultimate temperature of matter. Subsequently it was noted \[53\] that \( T_H \) marks a critical point, with a possible new state of matter at \( T > T_H \), which presumably is the Quark-Gluon Plasma.

An alternative approach is based on the intrinsic size of hadrons \[54\]. With increasing temperature, the hadron density increases, and - assuming again a mesonic system - the individual constituents will overlap more and more. At a certain density, the system will percolate, i.e., form a connected network spanning the entire system. The spanning cluster consists of overlapping mesons, so that it ceases to be meaningful to speak of the existence of individual mesons within this cluster. The density of mesons in the cluster is at the percolation point approximately

\[
n_p \approx \frac{1.2}{V_0},
\]

where \( V_0 \approx (4\pi/3)R_0^3 \) and \( R_0 \approx 0.8 \text{ fm} \). We can now ask for the temperature at which an ideal resonance gas, with all resonances having size \( V_0 \), attains this density. It is found to be \[55\]

\[
T_p \approx 180 \text{ MeV},
\]

so that such geometric percolation considerations lead to a limit of Hadronic Matter very much like that obtained from resonance dynamics.

The “mesonic” arguments used up to now continue to be valid also in the presence of baryons, as long as the baryon density is well below the point of dense packing; we will elaborate on this below. As a result, we conclude that resonance formation or percolation lead to a temperature limit \( T_H \) approximately independent of the baryon density \[3\]. Our “phase diagram” thus is so far a straight horizontal line \( T_H(\mu) = \text{const.} \) in the \( T - \mu \) plane, as shown in Fig. 6.

The nature of the limit depends on the conceptual basis. An ideal resonance gas with an exponentially growing mass spectrum results in a genuine thermal critical line, corresponding to continuous transitions; the associated critical exponents can be determined in terms of the space dimension \( d \) and the coefficient \( a \) in Eq. (2) \[51,56\]. Percolation is in general a geometric critical phenomenon, with singular behavior and corresponding critical exponents for cluster variables. It does not imply singular behavior of the partition function and could thus from a thermal point of view correspond to a rapid cross-over \[57\].

\[3\] The existence of strange baryons does lead to a slight decrease of \( T_H(\mu) \) with baryochemical potential \( \mu \) \[55\]; we ignore this here for simplicity.
We now turn to the other extreme, dense baryonic matter at low temperature. For baryochemical potential $\mu \simeq 0$, the contribution of baryons/antibaryons and baryonic resonances is relatively small, but with increasing baryon density, they form an ever larger fraction of the species present in the medium, and beyond some baryon density, they become the dominant constituents. Finally, at vanishing temperature, the medium consists essentially of nucleons.

For vanishing or low baryon number density, when the interactions are resonance dominated, the system could be described as an ideal gas of all possible resonance species. At high baryon density, however, the dominant interaction is non-resonant. Nuclear forces are short-range and strongly attractive at distances of about 1 fm; but for distances around 0.5 fm, they become strongly repulsive. The former is what makes nuclei, the latter (together with Coulomb and Fermi repulsion) prevents them from collapsing. The repulsion between a proton and a neutron shows a purely baryonic “hard-core” effect and is connected neither to Coulomb repulsion nor to Pauli blocking of nucleons. As a consequence, the volume of a nucleus grows linearly with the sum of its protons and neutrons. With increasing baryon density, the conceptual basis of a resonance gas thus becomes less and less correct, so that eventually one should encounter a regime of quite different nature. At high baryon density, the most striking effect is the onset of a “jamming” of nucleons: the mobility of baryons in the medium becomes strongly restricted by the presence of other baryons, leading to a jammed state [58], as shown in Fig. 7. The inverse mobility $s$ of a nucleon here plays the role of an order parameter: up to a certain density, it is zero, and beyond this point, it remains finite.

Baryonic matter thus becomes again a medium of extensive hadrons of radius $R_h$, but these now contain a hard core of a smaller radius $R_{hc} < R_0$. The overlap of such hadrons in percolation studies is thus restricted; nevertheless, the percolation onset can still be determined [59], and it is found [55] that the

![Fig. 6. Limits of Hadronic Matter, (a) meson percolation or resonance formation, (b) hard core baryon percolation.](image-url)
density of a spanning cluster now becomes

\[ n_{hc} \simeq \frac{2}{V_0}, \]  

assuming \( R_{hc} = R_0/2 \). With \( R_0 = 0.8 \) fm, this leads at \( T = 0 \) to a limit of about 5.5 times standard nuclear density. Requiring the baryon density (baryons minus antibaryons) in an ideal resonance gas to attain this limit as function of \( T \) and \( \mu \) then defines a critical curve based on baryon percolation. In the simplest model,

\[ \mu_p \simeq 1.12 \text{ GeV} \]  

becomes the limiting baryochemical potential \( T = 0 \). The general curve is included in Fig. 6 [55].

In the case of hard core percolation, a connection to thermodynamic critical behaviour has also been discussed [59]. If a system with hard core repulsion between its constituents is in addition subject to a density-dependent negative background potential, first order critical behaviour can appear, ending in a second order critical point specified by the background potential strength and the hard core volume.

The interpretation of the situation illustrated in Fig. 6 allows different interesting possibilities. In Ref. [55] it is assumed that the state outside the Hadronic Matter region is a deconfined Quark-Gluon Plasma. It is, however, also conceivable that below the meson percolation/resonance curve confined mesonic states survive, while baryons enter into the new phase. Such Quarkyonic Matter [34,35] is dealt with in detail in this work.

One can get some insight into the nature of the transition in the various regions of \( \mu_B \) and \( T \) by plotting the entropy density inferred from resonance model descriptions as a function of center of mass energy of the collision, as shown in Fig. 8. For low energies, below the hypothetical critical point, the matter is baryonic, consistent with a transition from Hadronic Matter to Quarkyonic Matter. For higher energies, it is largely mesonic matter, and consistent with a transition from Hadronic Matter to a Quark-Gluon Plasma. Turning this
Fig. 8. The baryon number and mesonic contributions to the entropy density as a function of center of mass energy for the collisions of heavy nuclei. The values of \( \mu_B \) and \( T \) used to make this plot arise from Statistical Model parameterization of the chemical abundance of produced particles. [63]

As a simple model that embodies some of the features discussed above, we suggest that, for small values of the chemical potential, \( \mu_B < \mu_B^{\text{triple pt}} \), the transition between a Hadronic phase, and the Quark-Gluon Plasma, is controlled by a single Hagedorn temperature \( T^M_H \) for mesons, \( T^M_H [51,52,53,60,61] \). Assuming that this transition is controlled entirely by mesons, we obtain a line which is independent of \( \mu_B \). Of course we do not believe that this behavior is exact, but it seems to be not a bad approximation in QCD. Numerical simulations of lattice QCD imply that \( T_c \) decreases very slowly, by only about 10\%, for \( \mu_B \) from 0 to 400 MeV [27,28]. The \( \mu_B \) independence of \( T^M_H \) is also in accord with arguments at large \( N_c \) and small \( N_f \), which imply that the critical temperature is independent of the baryon chemical potential (As we discuss below, this is true as long as \( \mu_B/N_c \) is of order one, and does not grow with \( N_c \).

We suggest that this horizontal line intersects with a second line, which is controlled by a Hagedorn temperature for baryons, \( T^B_H \). If there is such a

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4 The precise relation between the QCD phase boundary and the Hagedorn temperature is not well understood at the moment. Our schematic construction, leading to Eq. [10] below, implies asymptotically \( T^M_H < T^B_H \). Note, however, that, from the presently-known hadron spectrum up to 2 GeV, the effective \( T_H \) for mesons appears to be larger than for baryons [61].
Hagedorn temperature, the density of states of baryons grows like
\[ \rho_B(M_B) \sim \exp\left(+M_B/T_{H}^B\right), \ M_B \to \infty \] (8)

We assume, as is typical for a Hagedorn spectrum, that this balances against
the usual Boltzmann factor, \( \exp((\mu_B - M_B)/T) \). Then for a given value of
\( \mu_B \), there is a phase transition at a “Quarkyonic” temperature \( T_{Qk} \),
which is \( \mu_B \)-dependent. In the plane of \( \mu_B \) and temperature, this dependence is just a
straight line:
\[ T_{Qk}(\mu_B) = \left(1 - \frac{\mu_B}{M_0^B}\right) T_{H}^B. \] (9)

We have made a gross approximation in this formula, which is represented
by the parameter \( M_0^B \). The Hagedorn mass spectrum in Eq. (8) is only valid
asymptotically, as \( M_B \to \infty \). Thus strictly speaking, the transition temperature
from a Hagedorn spectrum is independent of \( \mu_B \). (For this reason, in string
models the Hagedorn temperature is common to all particles, determined only
by a single parameter, which is the string tension \[52,60].) Instead, in Eq. (9)
we introduce a new parameter, \( M_0^B \), by hand. This is meant to represent a
finite mass scale at which a Hagedorn spectrum appears. Here \( M_0^B \) is entirely
a phenomenological parameter, meant to illustrate how the transition tempera-
ture \( T_{Qk} \) to Quarkyonic Matter might depend upon \( \mu_B \). Clearly \( M_0^B \)
cannot be less than the mass of the lightest baryon; it could well be much larger.

As one decreases \( \mu_B \), eventually there will be a temperature at which this line
crosses that for deconfinement. We assume that, when this happens, the line
for the Quarkyonic transition ends and that the transition to a Quark-Gluon
Plasma, which has a much larger energy density, dominates. The point at
which these two lines cross defines the position of a triple point:
\[ T_{\text{triple pt}} = T_{H}^M = \left(1 - \frac{\mu_{\text{triple pt}}}{M_0^B}\right) T_{H}^B. \] (10)

We stress that our approximations are very crude, and are only meant to
illustrate how a triple point might arise.

The transition temperature line of Eq. (9) intersects the axis of \( T = 0 \) when
\( \mu_B = M_0^B \). This formula does not apply at arbitrarily low temperatures, how-
ever. A Quarkyonic phase is defined to be one in which both the baryon and
energy densities are large, but at small temperatures, when \( \mu_B \) is close to the
nucleon mass, one probes not Quarkyonic, but dilute nuclear matter. At large
\( N_c \), though, the region in which nuclear matter is dilute is a narrow window
in \( \mu_B \) \[34\]. This suggests that \( M_0^B \) is near the nucleon mass; fitting to Fig. 2
gives \( M_0^B \approx 1 \text{ GeV} \). The value of the baryonic Hagedorn temperature can be
read off from where \( T_{Qk}(\mu_B) \) intersects the axis of \( \mu_B = 0 \). Again from Fig. 2
this gives \( T_{H}^B \approx 250 \text{ MeV} \).
These values for mesonic and baryonic Hagedorn temperatures should only be taken as illustrative. Even at \( \mu_B = 0 \), experiment gives us the results at chemical freeze-out. This value is certainly lower than the temperature for the true transition (or crossover), and is lower still than that for the Hagedorn temperature. One might, however, expect that these values are close to one another. This is indicated by results from the lattice in pure gauge theories [62].

The limit of a large number of colors shows that the introduction of the parameter \( M_B^0 \) is not quite as contrived as might first appear. In the limit of large \( N_c \) and small \( N_f \) the transition from a hadronic to a Quarkyonic phase is a straight line along \( \mu_B = m_N \), where \( m_N \) is the mass of the lightest baryon (up to small effects from nuclear binding) [34]. This is just the usual mass threshold for a chemical potential.

In the limit in which both \( N_c \) and \( N_f \) are large, one cannot speak of deconfinement rigorously, and there is only a phase transition for the condensation of baryons, which is a straight line in the \( \mu_B - T \) plane [35]. This is because the density of states, for even the lowest baryon multiplet, grows exponentially. Note that this is analogous, but not identical, to a Hagedorn temperature, since the exponential growth is for the lightest multiplet, and not for asymptotically large masses. There are several effects which will act to modify this naive prediction, however. First, even in the Hadronic phase, baryons interact strongly with the numerous mesons. This will modify the baryon mass, and so shift the threshold at which they condense. Second, baryon baryon interactions are strong. In ordinary nuclear matter it is well known that baryons have a large hard core repulsion between them, and this surely persists when both \( N_c \) and \( N_f \) are large. Such a hard core repulsion between baryons will act to cut off the singularity in the free energy, which otherwise would be generated by an exponential growth in the degeneracy of states.

Of course in QCD the degeneracy of the lowest baryons does not grow exponentially. But \( M_B^0 \) can then be viewed as a way of characterizing when the growth of baryonic states starts to take off. For example, this could be estimated more accurately in resonance gas models. Consider, alternately, the result of [35]:

\[
T_{Qk}(\mu_B) = \frac{M_B - \mu_B}{\log(N_{\text{deg}})}
\]  

In this equation, \( N_{\text{deg}} \) is the number of approximately degenerate baryonic states. Extrapolating the formula for large \( N_c \) and \( N_f \) down to small values, for three colors and two flavors, \( T_{Qk} \approx 160 \) MeV; for three flavors, 140 MeV for \( \mu_B \approx 400 \) MeV [35]. In QCD we can also estimate \( N_{\text{deg}} \) directly. Including nucleons and the \( \Delta \) resonance, \( N_{\text{deg}} = 20 \); including all strange baryons, \( N_{\text{deg}} = 56 \). Further, by including higher resonances, in Eq. (11) we should take not the nucleon mass for \( M_B \), but some heavier state, which can then be used
to define $M_B^0$. By fitting the data in Fig. 2, with $M_B^0 \sim 1$ GeV, one finds that \( \log(N_{\text{deg}}) \approx 2 - 3 \), which is not too far from the extrapolation from large $N_c$. Of course, the precise tradeoff between the increasing masses of various states and their abundances is a tricky issue. It is not clear how much to include of the flavor and spin excitations of the lowest mass nucleon states, and hence the uncertainty. This may be addressed more directly within Statistical Model computations.

A natural question is what happens to the two phase transition lines beyond the triple point. Consider first the transition between the Hadronic phase and the Quark-Gluon Plasma, to the right of the triple point at approximately constant temperature, with $\mu_B > \mu_B^{\text{triple pt}}$. At large $N_c$, this line is of first order, and remains a boundary for a true phase transition. The lattice QCD results show that the rapid rise in the energy density is relatively independent of $\mu_B$; thus we suggest that this line delineates an approximate phase transition for $\mu_B > \mu_B^{\text{triple pt}}$. At large $N_c$, when $\mu_B/M_N \sim N_c^{1/2}$, eventually deconfinement is washed out by the quarks, and there is a critical endpoint for deconfinement. (This is the value of chemical potential where the Debye screening length becomes less than the confinement size scale.) In QCD, since there isn’t a first order transition to deconfinement, we expect that eventually the large increase in the energy density, seen in a narrow region in temperature, is just washed out by the contribution of dense quarks.

It is also possible to consider continuing the phase boundary for the Quarkyonic phase at chemical potentials below the triple point, that is, for $\mu_B < \mu_B^{\text{triple pt}}$. One might imagine that there is then a line for the Quarkyonic transition above that for deconfinement, with $T_{Qk} > T_c$ when $\mu_B < \mu_B^{\text{triple pt}}$. Even at large $N_c$, such a line of Quarkyonic “transitions” can only reflect the properties of some metastable state in the (semi-)Quark-Gluon Plasma. Numerical simulations on the lattice do find that like the energy density, quark number susceptibilities approach the ideal gas values very near $T_c$, by 20% above $T_c$ [27]. Thus perhaps this change in the quark number susceptibilities reflects the remnants of the Quarkyonic “transition” in the deconfined phase.

We conclude by discussing the relationship to the chiral phase transition. It is possible that the triple point coincides with a critical end point for a line of first order (chiral) transition [33]. However, as we noted above, the experimentally observed properties of the triple point do not seem indicative of a critical point. It is possible that QCD matter behaves similar to water, with a critical end point for the chiral transition which is distinct from the triple point. If so, it probably exists in the Quarkyonic phase. It is also possible that there is no well defined chiral transition; i.e., that because of the nonzero quark masses, the chiral transition is only a crossover. There would then be no critical end point for the chiral phase transition.
4 Strangeness along the phase boundary

We have already discussed and shown in Fig. 3 that there are abrupt changes in the abundances of various ratios of strange to non-strange particles at \( \sqrt{s} \) around 10 GeV\(^5\). The reason for such behaviour may be linked with the appearance of the Quarkyonic phase. Along the Quarkyonic line, the temperature changes substantially. The fraction of strange particles should increase as the temperature increases. Along the Quark-Gluon Plasma line, the temperature is constant and we expect the strange quark relative abundance to be roughly unchanged. When these two boundaries meet, we would expect a change in the strange quark density near the triple point. This is most easily seen approaching the triple point along the Quarkyonic curve since as one approaches the deconfinement transition, there should be a rapid increase in the energy density, favoring a higher relative abundance of strange quarks. The strange quark relative abundance increases rapidly as one approaches the triple point, but then slowly decreases beyond it due to the decreasing \( \mu_B \) at the almost constant temperature.

Some of the strange to non-strange particle ratios are very sensitive to small variations in \( T \) and/or \( \mu_B \) as demonstrated in Fig. 9-left showing the \( K^+/\pi^+ \) ratio as contour lines in the \( T-\mu_B \) plane. If in the region of the triple point the freeze-out would happen at somewhat higher temperatures then especially this ratio will increase. Fig. 9-left illustrates that in the Statistical Model the \( K^+/\pi^+ \) ratio can never exceed the value of \( \sim 0.25 \).

While different strange to non-strange particle ratios exhibit different trends, the relative strangeness content quantified by the Wróblewski factor\(^6\), similarly as \( K^+/\pi^+ \), exhibits a well pronounced peak as seen in Fig. 9-right.

The peak in the strangeness abundance naturally arises in the Statistical Model due to the presence of the phase boundaries between QGP-Quarkyonic Matter and Hadronic Matter, for the reasons stated above. It is nevertheless an indirect measure of the singularity associated with a triple point. If the triple point region is somewhat spread out, we might expect that the peaks in various particle ratios might not appear at the same point. If the critical point region is very narrow, there should be approximately discontinuous behavior at the critical point, but this does not necessarily imply a maximum in ratios of strange to non-strange particles near to the critical point. The relationship between strangeness abundance, the triple point, and experimental data is certainly worth more detailed and precise experimental and theoretical study.

\(^{5}\) Note that the energy axis in Figs. 3 is in logarithmic scale so the variation at higher energies is indeed quite slow.

\(^{6}\) The Wróblewski factor, \( 2N_s\pi/(N_u\pi + N_d\pi) \), determines the relative abundance of the initially produced strange to light quarks multiplicities.
5 Quarkyonic Matter and Chiral Symmetry Breaking

So far chiral symmetry played little role in our argument; the hadron resonance gas description assumes no explicit modification on the hadron masses in a hot and dense medium. In principle, however, it would be conceivable to anticipate a substantial change in the hadron spectrum depending on whether chiral symmetry is (partially) restored or not. There are in fact several theoretical and experimental indications that chiral symmetry is affected in a medium [65]. For instance, the leading-order of the virial expansion suggests that the chiral condensate receives, at normal nuclear density, a 30-40% reduction, which is shifted back by around 10% by higher order corrections in the in-medium chiral perturbation theory [66]. Of course, the chiral condensate itself is not a direct experimental observable, but useful information is available from the spectroscopy of deeply-bound pionic atoms and the experimentally deduced in-medium pion decay constant at normal nuclear density is reduced by 36% compared to its vacuum value [67].

Although chiral perturbation theory gives a fairly model independent statement on chiral properties at finite temperature and baryon density, forming a productive research area together with experimental measurements, its validity is strictly limited to low-energy regimes. As one tries to go beyond low-energy regimes to explore the phase diagram of strongly interacting QCD matter, it is extremely difficult to make any statement in a model-independent way. It is notable that the in-medium condensate strongly relies on the pion
mass coming from two-pion exchange correlations with virtual $\Delta (1232)$ excitations which stabilizes the dropping of the condensate for physical pion mass with increasing density \[66\]. Particularly, a deviation from the result in the linear density approximation is remarkable for symmetric nuclear matter. This might indicate that the chiral symmetry restoration would take place at much higher density as compared to the critical density given in the mean-field models. This also could suggest that in-medium correlations might weaken a phase transition and eventually a first-order phase transition might disappear from the phase diagram.

In the context of the chiral phase transition the most frequently used models are the Nambu–Jona-Lasinio (NJL) model and the quark meson (QM) model, which are sometimes improved by the introduction of partial gauge degrees of freedom, namely the Polyakov loop, and promoted to the PNJL and PQM \[68\] models, respectively. Crucial points in this sort of model treatment are that a description in terms of quasi-quarks is assumed and the effect of the confinement is totally neglected. Such chiral quark models as well as another non-perturbative approach using the Schwinger-Dyson equation \[69\] favor a first-order chiral phase transition at high density. This suggests a termination point of the first-order phase boundary, which defines a critical endpoint (or often called the QCD critical point). Results from finite-density lattice simulations are far from conclusive yet and thus the existence of the critical endpoint is still under extensive dispute. In a description in terms of quarks, the driving force to induce a chiral transition is the density contribution to the pressure. Therefore, a chiral phase transition in this region of low-$T$ and high-$\mu_B$ is always accompanied by a significant jump in the quark number density. So, if the correct degrees of freedom are quarks rather than baryons, the quarkyonic transition is naturally close to the chiral phase transition \[37\]. In these kinds of models there is a general tendency that the critical endpoint is found not far from the triple point region. This is because the quarkyonic transition boundary tends to stay along the chiral phase transition where the quark number density jumps discontinuously or increases rapidly. One must, however, bear in mind that the above-mentioned model indications on the critical endpoint are strongly dependent on neglected effects. These include the unknown model parameters and their dependence on $T$ and $\mu_B$ in the NJL and QM models, other possibilities of ground states such as the color superconducting phase, inhomogeneous states like the crystalline color superconducting phase and chiral density wave \[70, 71\], an unconventional pattern of chiral symmetry breaking \[72\], etc. Of course, in addition to them, the baryon degrees of freedom may change the whole picture completely.

Let us consider how the quark model results are affected by the baryons. To this end, we must consider how the baryon belongs to the representation of chiral symmetry. There are two possible assignments; one is just the same as the quark field which is called a naive assignment, and another is the so-called
mirror assignment \cite{73}. The important point is that one can construct a mass term which is chiral invariant in the case of the mirror assignment. This means that the baryons need not be lighter associated with chiral restoration in this case, so there is no jump in the baryon number density across the chiral phase transition if any. Thus the baryon number density need not necessarily exhibit a clear indication of either Quarkyonic or chiral transitions. On the other hand, if the assignment is naive, the situation becomes more or less similar to what we have seen for the quark model studies at the qualitative level.

Finally we shall comment on what the chiral model suggests for the triple point. In the PNJL model at small chemical potential ($\mu_q \ll T \sim T_c$), the behavior of the Polyakov loop as a function of $\mu_q$ and $T$ has a deviation from that of the chiral condensate. Such a possibility of unlocking of the deconfinement and chiral transitions was already pointed out in the first paper on the PNJL model \cite{41}. Together with the fact that the quark number density has a strong correlation with the chiral condensate in the quark-based model, this observation of separate deconfinement and chiral crossovers may well suggest that there appears a triple point region on the phase diagram. At least within the uncertainty of the model which can easily move the critical endpoint, it seems that the appearance of the triple point is a robust feature of the model output. It is notable that the anomaly matching condition may well imply that the chiral phase transition takes place later than the deconfinement \cite{74}, but strictly speaking, because of the violation of Lorentz symmetry in the presence of matter, the same argument as in vacuum cannot be directly applied to constrain the ordering of the phase transitions \cite{75}.

While the arguments that chiral symmetry should be approximately restored in the high baryon density region are strong, the arguments that it is completely restored are less so. It might turn out for example that chiral symmetry remains broken in the Quarkyonic phase due to non-perturbative effects at the Fermi surface. Such effects would be proportional to powers of $\Lambda_{\text{QCD}}/T$, and would be small but nevertheless non-negligible. Presumably these effects would disappear when confinement also disappears. For example, effects at the Fermi surface might make the chiral condensate of order $\Lambda_{\text{QCD}}^3$. While this would be small compared to the baryon number density, $\mu_Q^3$, and might be ignored for many purposes, its magnitude would be parametrically unchanged from its value in the confined phase. Although we have very little to say which is strongly compelling about the nature of chiral symmetry breaking and its relation to Quarkyonic Matter, the questions that arise are of fundamental interest for our understanding of the nature of mass generation in QCD. As such, these issues must eventually be understood in an absolutely compelling and simple way.
The possible relation between the chiral and deconfining phase transitions, discussed above, can be more transparent when referring to properties of an effective Lagrangian \[76\].

Consider the interaction of the (renormalized) Polyakov loop, \(\ell\), which is the trace of the renormalized Wilson line, \(L, \ell = \text{tr} L / N_c\). The Polyakov loop couples to the chiral field, \(\Phi\), as

\[
L_{\text{eff}} = c_1 \ell \text{tr} \Phi^\dagger \Phi .
\]

This term is chirally invariant. It is not invariant under the global \(Z(N_c)\) symmetry of the pure glue theory, under which \(\ell \rightarrow \exp(2\pi i / N_c) \ell\), but this symmetry is broken by the presence of quarks. While the Polyakov loop \(\ell\) is dimensionless, for the purposes of power counting, let us assume that like ordinary scalar fields, it has dimensions of mass. This implies that the coupling \(c_1\) has dimensions of mass. It is the dominant coupling of the Polyakov loop to quarks.

The observation of Ref. \[76\] is that the sign of \(c_1\) controls how the chiral and deconfining transitions are related. Assume that chiral symmetry is broken in the vacuum, so the expectation value of \(\text{tr} \Phi^\dagger \Phi\) is nonzero. If \(c_1\) is positive, \(L_{\text{eff}}\) is positive, so that this term resists the Polyakov loop from developing an expectation value until chiral symmetry is restored. That is, positive \(c_1\) links the deconfining and chiral symmetry phase transitions together. Conversely, if \(c_1\) is negative, the transitions tend to repel one another.

Clearly a special point occurs when \(c_1\) vanishes. At this point, it is natural for the deconfining and chiral phase transitions to split apart from one another. We suggest, then, that the chiral phase transition may split from the deconfining line at the triple point. This assumes that the triple point also coincides with the critical endpoint for the chiral phase transition \[33\].

About the triple point, it is then natural to ask what the next leading term is. There are many such terms. One involves the mass matrix of the chiral fields, \(M\), which is proportional to the current quark masses:

\[
L'_{\text{eff}} = c_2 \ell \text{tr} M \Phi = c_2 \ell \left( m_\pi^2 \pi^2 + m_K^2 K^2 + \ldots \right) / f_\pi .
\]

Like \(c_1\), it has dimensions of mass. It is chirally suppressed, however, and so is less important except when \(c_1\) is small.

Assuming that \(c_1\) vanishes at the triple point, the coupling \(c_2\) dominates in the region where \(c_1 \approx 0\). In this region, the coupling is reversed from the usual
expectation: in particular, the coupling to the heavier Goldstone bosons, such as kaons, is larger than that to the lighter Goldstone bosons, the pions.

The phenomenological implications of this term are interesting for experiment. About the point where $c_1$ vanishes, even though they are heavier, the coupling of kaons to the Polyakov loop is larger, by a factor of $m_K^2/m_\pi^2 \sim 13$. This might explain the enhancement of strangeness observed at the triple point.

7 Summary and Conclusions

In this work, we have presented an interpretation of the experimental data on particle production obtained in heavy ion collisions form SIS up to RHIC energies in the context of a new structure of the QCD phase diagram with Quarkyonic Matter. We have shown that by considering Quarkyonic Matter, the two regimes of chemical freeze-out with meson and baryon dominance observed phenomenologically can be understood as arising from a triple point where Hadronic Matter, the Quark-Gluon Plasma, and Quarkyonic Matter all coexist. This triple point is located where the freeze-out temperature is reaching its limiting value and were different strange to non-strange particle ratios exhibit non-monotonic behavior.

We have presented a set of qualitative and semi-quantitative arguments that the observed statistical properties of experimental data are naturally explained when assuming the existence of Quarkyonic Matter and of a triple point in the QCD phase diagram. We have also discussed in the context of different models the possible role of the chiral symmetry restoration and the interplay between triple and critical points.

Our findings and interpretation can be justified and/or verified in the near future with more data expected from ultrarelativistic heavy ion collisions. New data are soon to be available from the RHIC low energy runs and from the CERN NA61 experiment which aim at a scan of energy and system size dependence in the vicinity of the triple point discussed here. While these experiments will, most likely, pierce into the triple point region coming from higher energies, where the phase transition is a crossover between Hadronic Matter and Quark Plasma, the dedicated exploration of the phase border between the hadronic and the Quarkyonic Matter will be a task for the future experiments with highly compressed baryonic matter such as CBM@FAIR Darmstadt and NICA@JINR Dubna. If chemical equilibration will be substantiated with more precise data at those energies our quarkyonic phase boundary argument will gain a dramatic support as a viable explanation for equilibration.
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