Pure Spin Currents and associated electrical voltage

T. P. Pareek
Max-Planck-Institute für Mikrostrukturphysik, Weinberg 2, D-06120 Halle, Germany

We present a generalize Landauer-Buttiker transport theory for multi-terminal spin transport in presence of spin-orbit interaction. It is pointed out that the presence of spin-orbit interaction results in equilibrium spin currents, since in presence of spin-orbit interaction spin is not a conserved quantitative. Further we illustrate the theory by applying it to a three terminal Y-shaped conductor. It is shown that when one of the terminal is a potential probe, there exist nonequilibrium pure spin currents without an accompanying charge current. It is shown that this pure spin currents causes a voltage drop which can be measured if the potential probe is ferromagnetic.

PACS numbers: 72.25-b,72.25.Dc, 72.25.Mk

Producing and measuring spin currents is a major goal of spintronics. The standard way is to inject spin currents from a Ferromagnet into a semiconductor in a two terminal geometry [1]. However this has a drawback, due to conductivity mismatch, the polarization of injected current is rather small and it always has an accompanying charge current [2]. Also for any spintronics operation spin orbit interaction plays an important role, for e.g., in Datta-Das spin-transistor [3].

In light of these development it would be interesting and highly desirable if one can produce spin currents intrinsically. One such possibility is provided by intrinsics spin-orbit interaction. Presence of impurity atom or defects gives rise to spin-orbit interaction of the form [4] [5], [6],

$$H_{so} = \lambda (\nabla U(r) \times \mathbf{k}) \cdot \sigma$$  \hspace{1cm} (1)

where $\sigma$ is a vector of Pauli matrices, $U(r)$ is potential due to defects or impurity atoms and $\mathbf{k}$ is the momentum wave vector of electrons and $\lambda$ is spin-orbit interaction strength. For strictly two dimensional case for which the potential $U(r)$ depends on $x$ and $y$ coordinates only the Hamiltonian commutes with $\sigma_z$, hence $z$ component of the spin is good quantum number. As is well know that this kind of spin-orbit interaction has a polarizing effect on particle scattering [7], i.e., when an unpolarized beam is scattered it gets polarized perpendicular to the plane of scattering. Further scattering of this polarized beam causes asymmetry in scattering processes, i.e. electrons with one particular spin direction, e.g., spin-up electrons have a larger probability to be scattered to the right compared to spin-down electrons [6], [7]. This property of spin-orbit scattering gives rise to novel effects like spin hall effect [4].

Here in this article we show that the the above discussed property of spin-orbit scattering can be used to generate and measure spin-currents [5]. Consider a three terminal, two dimensional Y shaped conductor shown in Fig.1. The plane of conductor is $xy$. Since the conductor is two dimensional which fixes the scattering plane, the scattered electrons will be polarized along $z$ axis (perpendicular to the scattering plane). However the polarization for the two branches of Y junction will be opposite [4]. Hence a three terminal structure would create spin currents from an unpolarized current in presence of spin-orbit interaction [6], [8]. Moreover a three terminal device provides an important possibility of generating nonequilibrium pure spin currents without an accompanying charge current. This is the case when one of the terminal acts as a voltage probe. For e.g., say the terminal 3 is a voltage probe as shown in Fig.1, i.e., the voltage $V_3$ at third probe is adjusted such that the total charge current flowing in terminal 3 is zero, i.e. $I_3=0$ [9], [10]. Physically it implies that the charge current flowing in (which is polarized as argued above) is same as charge current flowing out. However the polarization of charge current flowing out need not to be same as the polarization of charge current flowing in (see Fig. 1). Hence there will be a net spin current flowing without the accompanying charge current. This is a pure nonequilibrium spin current.

We support our prediction by generalize Landauer-Buttiker charge transport for multi-terminal spin transport. We provide unambiguous definition of spin currents. Using this theory we discusses the possibility of generating and detecting nonequilibrium pure spin currents and point out the existence of equilibrium spin current.

The possibility of injecting pure spin currents were first discussed in Ref. [12] for a three terminal device where two of the terminals were ferromagnetic. Also direct optical injection of pure spin currents in GaAs/AlGaAs quantum wells was demonstrated in Ref. [13]. We would stress that in our case spin current is not injected rather generated intrinsically due to the spin-orbit interaction without any magnetic element in the system, which is not the case in Ref. [12]. Thus we avoid the problem of spin injection altogether. Further, since the effect discussed relies on the general scattering properties due to the spin-orbit interaction. Hence it will be observable with any kind of spin-orbit interaction, e.g., Rashba spin-orbit interaction [11].
We first briefly outline the spin transport theory for multi-terminal devices. Let us consider the two dimensional Y-shaped structure shown in Fig.1. The plane of structure is $xy$ and a perpendicular to it defines the coordinate system. Let us choose the spin quantization axis to be along $\hat{u}$ pointing along $(\theta, \phi)$ where $\theta$ and $\phi$ are usual spherical angles (In other words we choose the spin basis to be eigen states of operator $\sigma \cdot \hat{u}$). This is essentially since a charge current flowing along a spatially direction can be polarized along a direction which need not coincide with the direction of flow of charge current. Also in presence of spin-orbit interaction the rotational invariance in spin space is lost [14], hence any theory for spin transport should take this fact into account. With this definition we can generalize Landauer-Büttiker theory for spin transport. Let $V_m$ be potential at terminal $m$ measured from the minima of lowest band, where $m$ can take values 1, 2 and 3 corresponding to the three terminals of Fig. 1. $T_{nm}^{\alpha \sigma}$ is spin resolved transmission probability of electrons incident in lead $m$ in spin channel $\sigma$ to be transmitted into lead $n$ in spin channel $\alpha$. We point out that $\sigma$ and $\alpha$ need not to be same in presence of spin-orbit interaction, since SO interaction will make spin flip transmission probability non zero. The spin current $I_m^\sigma$ flowing into terminal $m$ is, (here $\sigma$ can be either $\uparrow$ or $\downarrow$)

$$I_m^\sigma = \frac{e^2}{h} \sum_{n \neq m, \alpha} (T_{nm}^{\alpha \sigma} V_m - T_{mn}^{\sigma \alpha} V_n)$$

where $\alpha$ and $\sigma$ are indices labeling the two spin eigenstates for a chosen quantization axis. In writing above equations we have made an assumption that the spin resolved transmission coefficient are energy dependent. A generalization of the above equation when the spin resolved transmission coefficient are energy dependent is straight forward. It amounts to replacing $T_{nm}^{\alpha \sigma}$ by $\int T_{nm}^{\alpha \sigma}(E)$. Since SO interaction preserves time reversal symmetry, which lead to the following constrains on the spin resolved transmission coefficient,

$$T_{nm}^{\alpha \sigma} = T_{mn}^{-\alpha -\sigma}$$

(3)

Using eq. 1 we can immediately write down the net charge and spin current flowing through terminal $m$,

$$I_m^q = I_m^\sigma + I_m^{-\sigma} = \frac{e^2}{h} \sum_{n \neq m, \alpha} \{ (T_{nm}^{\alpha \sigma} V_m - T_{mn}^{\sigma \alpha} V_n) \}$$

(4)

$$I_m^\sigma = I_m^\sigma - I_m^{-\sigma} = \frac{e^2}{h} \sum_{n \neq m, \alpha} \{ (T_{nm}^{\alpha \sigma} - T_{mn}^{-\alpha -\sigma}) V_m + (T_{mn}^{-\sigma \alpha} - T_{nm}^{\sigma \alpha}) V_n \}$$

(5)

where $I_m^q$ is charge current and $I_m^\sigma$ is spin current. We stress that eq. (5) correctly determines spin current generated by presence of spin orbit interaction. Since in the absence of spin-orbit interaction and any magnetic element in the device, spin resolved transmission coefficient obey a further rotational symmetry in spin space i.e. $T_{nm}^{\alpha \sigma} = T_{mn}^{-\alpha -\sigma}$, which implies that spin currents are identically zero for all terminals, i.e., $I_m^\sigma = 0$. Equilibrium spin current : To discusses equilibrium spin currents let us consider the case when all the potential are equal i.e., $V_m = V_0 \forall m$. In this situation charge current flowing in any terminal should be zero ($I_m^q = 0$) which leads to the following sum rule (from eq. (4))

$$\sum_n T_{nm} = \sum_n T_{mn}$$

(6)

where $T_{nm} = \sum_{\alpha} T_{nm}^{\alpha \sigma}$ is total transmission probability(summed over all spin channels) from terminal $m$ to $n$. This sum rule is robust and should be satisfied irrespective of the detailed physics [15]. This is a well known gauge invariance condition. Charge conservation implies $\sum_{m} I_{m}^q = 0$ which follows from symmetry of spin resolved transmission coefficient,eq. (3), and the gauge invariance condition, eq. (6). So in equilibrium there are no charge currents flowing. However this is not the case for the spin currents. This point can be appreciated if we look closely at the equation (5) for spin current. Since in general the transmission coefficient, $T_{nm}^{\alpha \sigma} \neq T_{mn}^{-\alpha -\sigma}$, which occurs in eq. (5). Hence even when all the potential are equal, the spin current given by eq.(5) is non zero. This is equilibrium spin current. Notice that this is consistent with time reversal invariance (eq. (3)) and the gauge invariance condition given by eq. (6). We would like to point out that this equilibrium spin current would exist even in two terminal setup. The equilibrium spin current are carried by all the occupied state at a given temperature.
This is a non-linear response and is different from linear response which give rise to non-equilibrium spin currents and is a Fermi surface property. So strictly speaking for the equilibrium spin currents one should take into account the energy dependence of spin-resolved transmission coefficient. A detailed study of the equilibrium spin currents would be presented in a separate article [16]. Here in this study we concentrate more on the non-equilibrium pure spin currents and the related electrical effects. Slonczewski has shown in Ref. [17] for magnetic multilayers equilibrium spin currents causes non-local exchange coupling. The important difference in our case is, we do not need Ferromagnetic contact to have equilibrium spin currents, which was the case in Ref. [17]. Rather we do not need Ferromagnetic contact to have equilibrium pure spin currents, which was the case in Ref. [17].

Non-equilibrium spin currents: To study non-equilibrium spin currents , let us consider the case where the voltages at terminal 1 and 2 are respectively $V_1=0$ and $V_2$ and terminal third is a voltage probe, i.e, $I^3_i = 0$. With this condition one can determine the voltage, $V_3$, at third terminal using the set of equation (4) and is given by [10],

$$\frac{V_3}{V_2} = \frac{T_{32}}{T_{13} + T_{23}}$$

(7)

The spin current flowing through terminal 3 is

$$\frac{I^3_i}{\hbar} = \sum_\alpha \left\{ (T_{13}^{\alpha\sigma} - T_{13}^{\alpha-\sigma} + T_{23}^{\alpha\sigma} - T_{23}^{\alpha-\sigma})V_3 + (T_{32}^{\alpha-\sigma} - T_{32}^{\alpha\sigma})V_2 \right\}.$$

(8)

From above equation (8) it is clear that $I^3_i$ is non-zero while $I^3_i$ is zero by definition. Hence in terminal 3 there is a net spin currents flowing in the absence of any net charge current. This is pure spin current and is intrinsically generated by the spin-orbit interaction in the absence of any magnetization as discussed in the introduction.

To obtain quantitative results we perform numerical simulation on a Y-shaped conductor shown in Fig. 1. We model the conductor on a square tight binding lattice with lattice spacing $a$ and we use the corresponding tight binding model including spin orbit interaction given by eq.(1) [6]. For the calculation of spin resolved transmission coefficient we use the recursive green function method. Details of this can be found in Ref. [6], [14].

The numerical result presented are exact and takes the quantum effect and multiple scattering into account. For the model of disorder we take Anderson model, where on-site energies are distributed randomly within $[-U/2, U/2]$, where U is the width of distribution. All the calculation were performed on Y-shaped device of width $d=20a$, where a is lattice spacing. Other parameters are given in figure captions.

In Fig.2 we show the spin currents $I^3_i$ flowing through the terminal 3 (Right panel) and the corresponding voltage $V_3$ when all the three terminal are non-magnetic. In Fig.2 , $\theta=0$ corresponds to $z$ axis and $\theta=90$ corresponds to $y$ axis, we have kept fixed $\phi = 90$. We see that the maximum amount of spin currents flow along $z$ axis. This is understandable since for strictly two dimensional case, the spin-orbit coupling given by eq. (1) conserves $z$ component of spin. Hence the asymmetric scattering produced by spin orbit interaction causes a pure spin currents along $z$ axis, as discussed in introduction. For the ballistic case (curve for $U/E_F = 0$), spin currents is zero since there is no spin-orbit interaction in this case, as can be seen from eq. (1) by putting the potential $U(r) = 0$.

Also for strong disorder spin current changes sign (curve for $U/E_F = 2$) due to multiple scattering. We would like to stress that by definition charge current flowing in terminal 3 is zero, i.e, $I^3_i = 0$. Now from the right panel in Fig.2 we see that the voltage $V_3$ measured is different although there is no charge current flowing and the magnitude of $V_3$ is directly proportional to the $z$ component of spin current. As is seen , with the increase of disorder strength the magnitude of spin currents increases and accordingly the potential $V_3$ also increases. However potential $V_3$ is independent of quantization axis since the voltage probe is non-magnetic. Hence with a non-magnetic voltage probe one can detect the spin current , but can not measure it. To measure the spin currents one would need a ferromagnetic voltage probe. An intuitive understanding of this can be gained as follows. From Fig.2 (right panel) we notice that the spin currents depends on the quantization axis. Thus if the probe is a ferromagnetic, electrons which are polarized parallel to the ferromagnet would be transmitted easily than the electrons polarized anti-parallel to the ferromagnet. Since the voltage at the probe is determined by the ration of transmission coefficient (eq.(7)), hence the probe voltage should show variation in phase with the spin currents.

This is confirmed in Fig.3. Where we have plotted
spin current (left panel) and voltage (right panel) for the case when the third terminal is a ferromagnetic. Left panel shows the spin currents and the corresponding voltage is shown in right panel. The quantization axis is given by the direction of magnetization. We see that as the spin current changes the corresponding voltage measured also changes in phase. Hence by having a ferromagnetic voltage probe one can measure the pure spin current. We would like to mention that in our numerical simulation voltage probe is an invasive one, i.e. it is strongly coupled to the system, hence one sees the quantitative difference between the results of Fig.2 and Fig.3. In Fig.3 we see that spin currents are non zero for the ballistic case ($(U/E_F = 0)$) while for non-magnetic case shown in Fig.2 spin current for ballistic case is zero. This is so because the Ferromagnetic probe is strongly coupled (invasive probe), it essentially injects a polarized current. However this is not a hindrance for measuring spin currents generated by spin-orbit interaction. Since as is seen from Fig.3, it only gives rise to a constant shift compared to the non-magnetic case (Fig2). Recently it was pointed out in ref. [18] that in magnetic bilayer systems dynamic exchange coupling arises due to the injection of polarized charge current. In the said work effect of spin-orbit interaction were not taken into account. Since here we point out the existence of spin currents (equilibrium and nonequilibrium) due to the spin-orbit interaction, hence such currents in principle would modify quantatively proposed dynamic exchange coupling. Charge transport for the Y-shaped mesoscopic junction have been studied in past experimentally as well theoretically. In view of this we hope the study presented here for the spin transport would open up new opportunity in the field of spintronics.

Author acknowledges fruitful discussion with P. Bruno, G. Bouzerar, Y. Utsumi and A. M. Jayannavar. Author also acknowledge discussion with Gerrit. E. W. Bauer and for pointing out Ref. [17].

This work was financially supported by the German Federal Ministry of Research (BMBF)

![Diagram](image-url)  
**FIG. 3.** spin current flowing through the Ferromagnetic terminal 3 (voltage probe) versus quantization axis (left panel) and right panel shows the corresponding voltage at the terminal third. Different curves corresponds to different strength of disorder. Inset shows the strength of Anderson disorder. Ferromagnet is modeled as exchange split with exchange splitting ($\Delta$) given as $\Delta/E_F = 0.5$. Other parameters are same as for Fig. 2.

[1] S. A. Wolf, D. D. Awschalom, R. A. Buhrman, J. M. Daughton, S. Von. et al., Science 294, 1488 (2001) and references therein.
[2] L. W. Molenkamp et al., Phys. Rev. B. 64, R121202 (2001).
[3] S. Datta and B. Das, Appl. Phys. Lett. 56, 665(1990).
[4] J. E. Hirsch, Phys. Rev. Lett. 83, 1834 (1999), Shufeng Zhang, Phys. Rev. Lett. 85, 393 (2000).
[5] G. Bergman, Phys. Rev. B. 63, 193101-1 (2001).
[6] T. P. Pareek and P. Bruno, Phys. Rev. B. 63, 165424-1 (2001). P. Bruno Phys. Rev. Lett. 79, 4593 (1997).
[7] L.D. Landau and E. M. Lifshitz Quantum Mechanics Vol. 3, pp. 583-588.
[8] A. A. Kiseleve and K. W. Kim cond-mat 0203261.
[9] M. Büttiker, IBM J. Res. Develop. 32 63 (1988).
[10] T. P. Pareek, Sandeep K. Joshi and A. M. Jayannavar 57, 8809 (1998).
[11] Yu. A. Bychkov and E. I. Rashba, Sov. Phys. JETP Lett. 39, 78 (1984).
[12] A. Brataas, Yu. V. Nazarov and G.E.W. Bauer, Phys. Rev. Lett 84, 2481 (2000).
[13] M. J. Stevens, A. L. Smirl, R. D. R. Bhat, A. Najmaie, J. E. Sipe, and H. M. van. Driel, Phys. Rev. Lett. 90, 136603-1 (2003).
[14] T. P. Pareek, Phys. Rev. B. 66, 193301 (2002). T. P. Pareek and P. Bruno, Phys. Rev. B. 65, 241305 (2002).
[15] S. Datta,Electronic transport in mesoscopic systems, (Cambridge University press, Cambridge, 1997).
[16] T. P. Pareek, manuscript under preparation.
[17] J. C. Slonczewski, Journal of Magnetism and Magnetic Materials. 126, 374 (1993).
[18] B. Heinrich, Y. Tserkovnyak, G. Woltersdorf, A. Brataas, R. Urban and Gerrit E. W. Bauer, Phys. Rev. Lett. 90, 187601-1 (2003)