Arbitrary spin massless bosonic fields in d-dimensional anti-de Sitter space

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Abstract. Arbitrary spin free massless bosonic fields propagating in even \(d\)-dimensional anti-de Sitter spacetime are investigated. Free wave equations of motion, subsidiary conditions and the corresponding gauge transformations for such fields are proposed. The lowest eigenvalues of the energy operator for the massless fields and the gauge parameter fields are derived. The results are formulated in \(SO(d-1,2)\) covariant form as well as in terms of intrinsic coordinates. An inter-relation of two definitions of masslessness based on gauge invariance and conformal invariance is discussed.

Motivation. Some time ago a completely self-consistent interacting equations of motion for higher massless fields of all spins have been discovered (Vasiliev (1990)). First these equations have been formulated for the case of four dimensional \(d = 4\) AdS spacetime. Then because the equations allow very natural generalization to higher spacetime dimensions \(d > 4\) they have immediately been extended to such the dimensions (Vasiliev (1991)). These equations are formulated in terms of wavefunctions \(\Psi(x,Z)\) which depend on usual spacetime coordinates \(x^\mu\) and certain twistor like variables \(Z^\alpha\). Usual physical fields as well as certain auxiliary fields are obtainable by expanding \(\Psi(x,Z)\) in powers \(Z^\alpha\): \(\Psi(x,Z) = \sum_0^\infty Z^{\alpha_1}...Z^{\alpha_n}\Phi(x)_{\alpha_1}...\alpha_n\), where \(\Phi = \{\Phi_{phys}, \Phi_{aux}\}\). For the case of 4d theories it is well-established (Vasiliev (1991)) that \(\Phi_{phys}\) satisfy the equations of motion which at free level are equivalent to those investigated in (Fronsdal (1978)). As to \(d > 4\) theories, although such a statement is not proved, it is believed that equations of motion suggested also describe massless higher spin fields in a self-consistent way. Unfortunately in contrast to the completeness of description for \(d = 4\) little was known about the higher spin massless spin fields in arbitrary \(d > 4\) even at the level of free fields, unless considerations are restricted to totally symmetric tensor (or tensor-spinor) fields (Lopatin and Vasiliev (1988),Vasiliev (1988)). Filling this gap was a motivation of our investigation (Metsaev (1994)-Metsaev (1997)). Here we report summary of results.

Setting up the problem. First, let us remind the main fact about representations of anti-de Sitter algebra \(so(d-1,2)\)

\[
[J^{AB}, J^{CD}] = \eta^{BC} J^{AD} \pm 3 \text{ terms}, \quad \eta^{AB} = (-, -, +, \ldots, +),
\]

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As is usual, we split the generators $J = 4$ and corresponding field theoretical realization has been found in (Evans' simplify our expressions we will drop the metric tensor $\eta$; $A, B$ are raised and lowered by $\eta^{AB}$ and $\eta_{AB}$ respectively. In what follows to simplify our expressions we will drop the metric tensor $\eta_{AB}$ in scalar products. As is usual, we split the generators $J^{AB}$ into an orbital part $L^{AB}$ and a spin

\[ h = (h_1, \ldots, h_\nu), \quad h_1 \geq \ldots \geq h_\nu \geq 0, \quad \nu \equiv (d - 2)/2 \quad (1) \]

which is the highest weight of representation of the $so(d - 1)$ algebra. Since in the representations under considerations the energy is by definition bounded from below the $D(E_0, h)$ contains the vacuum $|\phi^{E_0}(h)\rangle$ annihilated by all those elements of $so(d - 1, 2)$ which decrease the energy. This vacuum forms a linear space which is invariant under the action of the energy operator $i\mathcal{J}^{00'}$ and elements of the $so(d - 1)$ algebra. In other words $|\phi^{E_0}(h)\rangle$ is (a) the eigenvalue vector of $i\mathcal{J}^{00'}$; (b) a weight $h$ representation of the $so(d - 1)$ algebra. To expose these properties of $|\phi^{E_0}(h)\rangle$ it is convenient a usage of the coordinates $(z, \bar{z}, y^I)$, $I = 1, \ldots, d - 1$, where $z = (y^0 + iy^d)/\sqrt{2}$, $\bar{z} = z^*$ and the $\eta^{AB}$ takes the nonvanishing elements $\eta^{zz} = -1, \eta^{IK} = \delta^{IK}$. In these coordinates the generators $J^{AB}$ split into $J^{zz}$-energy operator, $J^{zI}$-spin deboost operator, $J^{I\bar{I}}$-spin boost operator and $J^{IK}$- generators of the $SO(d - 1)$ group. Now the $|\phi^{E_0}(h)\rangle$ is defined by the relations

\[ J^{zz}|\phi^{E_0}(h)\rangle = E_0|\phi^{E_0}(h)\rangle, \quad J^{zI}|\phi^{E_0}(h)\rangle = 0. \]

Then the representation space $D(E_0, h)$ can be built by acting with boost operator $J^{zI}$ on the vacuum $|\phi^{E_0}(h)\rangle$:

\[ D(E_0, h) = \sum_{n=0}^{\infty} \oplus J^{zI_1} \ldots J^{zI_n}|\phi^{E_0}(h)\rangle. \]

It turns out that for certain value of $E_0$ there is singular vector on the first energy level. If we factorise whole space by space built on this singular vector then we get irreducible representations which is, by definition, massless representation. Now the problem solution to which we are going to provide can be formulated as follows. Find (i) $E_0$ corresponding to massless representation; (ii) second order relativistic equations of motion whose space of solution is a carrier for massless representations; (iii) corresponding gauge transformations. The relevant $E_0$ for $d = 4$ and corresponding field theoretical realization has been found in (Evans (1967)) and (Fronsdal (1978)) respectively (for review see Nicolai (1984)).

\textit{Summary of results.} We describe the AdS spacetime as a hyperboloid

\[ \eta_{AB}y^Ay^B = -1, \quad (2) \]

in $d+1$- dimensional pseudo-Euclidean space with metric tensor $\eta_{AB}$. The indices $A, B$ are raised and lowered by $\eta^{AB}$ and $\eta_{AB}$ respectively. In what follows to simplify our expressions we will drop the metric tensor $\eta_{AB}$ in scalar products. As is usual, we split the generators $J^{AB}$ into an orbital part $L^{AB}$ and a spin
part $M^{AB}$, $J^{AB} = L^{AB} + M^{AB}$. The realization of $L^{AB}$ in terms of differential operators defined on the hyperboloid (2) is:

$$L^{AB} = y^A \nabla^B - y^B \nabla^A, \quad \nabla^A \equiv \partial^{AB} \frac{\partial}{\partial y^B}, \quad \nabla^{AB} = \eta^{AB} + y^A y^B.$$ 

The tangent derivative $\nabla^A$ satisfies relations

$$[\nabla^A, y^B] = \theta^{AB}, \quad [\nabla^A, \nabla^B] = -L^{AB}, \quad y^A \nabla^A = 0, \quad \nabla^A y^A = d.$$ 

A form for $M^{AB}$ depends on the realization of the representation. We will use the tensor realization of representation. As the carrier for $D(E_0, \mathbf{h})$ we use of tensor field of the $SO(d - 1, 2)$ group

$$A^{C(h)} = A^{C_1, \ldots, C_{h_1}, \ldots, C_2, \ldots, C_{h_\nu}}$$

defined on the hyperboloid (2). By definition, $A^{C(h)}$ is a tensor field whose $SO(d - 1, 2)$ indices $C(h)$ have the structure of the Young tableaux ($YT$) corresponding to the irreps of the $SO(d - 1, 2)$ group labeled by $h$. In what follows we use the notation $YT(h)$ to indicate such $YT$. The $h_i$ indicates the number of boxes in the $i$-th row of $YT(h)$. To simplify our expressions we introduce $\nu$ creation and annihilation operators $a^A_l$ and $\bar{a}^A_l$, $l = 1, \ldots, \nu$, and construct a Fock space vector

$$|A\rangle = \prod_{i=1}^{\nu} \prod_{a_i=1}^{h_i} a^{C_{i}^1, \ldots, C_{i}^{h_i}} |0\rangle, \quad |\bar{a}^A_1, a^B_1\rangle = \eta^{AB} \delta_{ij}, \quad \bar{a}^A_l |0\rangle = 0.$$ 

For a realization of this kind, $M^{AB}$ has the form

$$M^{AB} = \sum_{l=1}^{\nu} (a^A_l \bar{a}^B_l - \bar{a}^B_l a^A_l).$$

Throughout of the paper, unless otherwise specified, the indices $i, j, l, n$ run over $1, \ldots, \nu$. For these indices we drop the summation over repeated indices. Because the $A^{C(h)}$ is associated with $YT(h)$ then the $|A\rangle$ should satisfy the constraints

$$(a_{ii} - h_i)|A\rangle = 0, \quad a^-_{ij}|A\rangle = 0, \quad \varepsilon^{ij} a_{ij}|A\rangle = 0,$$

where in (4) and below we use the notation

$$a_{ij} \equiv a_i^A a^A_j, \quad a^-_{ij} \equiv \bar{a}^A_i \bar{a}^A_j, \quad a^+_{ij} \equiv a^A_i a_j^A,$$

and $\varepsilon^{ij} = 1(0)$ for $i < j(i \geq j)$. The 1st equation in (4) tells us that $a_i$ occurs $h_i$ times in $|A\rangle$. Tracelessness of $A^{C(h)}$ is reflected in the 2nd equation in (4). The 3rd equation in (4) implies that the generic tensor field (3) is antisymmetric with respect to indices in columns. As a result the $|A\rangle$ is obtainable from $YT$ by making use of the following symmetrization rule: (i) first we perform alternating with respect to indices in all columns, (ii) then we perform symmetrization with
respect to indices in all rows. Note that usual one uses the symmetrization rule when first one performs (ii) and then (i). Such kind of $|A\rangle$ could be described by using generic tensor field (3) which is symmetric with respect to indices in columns and by using anticommuting oscillators in place of commuting ones.

Because, by assumption, the $|A\rangle$ is a carrier for $D(E_0, h)$ it should satisfy the equation

$$(Q - \langle Q \rangle)|A\rangle = 0,$$

where $Q$ is the second order Casimir operator of the $so(d - 1, 2)$ algebra while $\langle Q \rangle$ is its eigenvalue for $D(E_0, h)$

$$Q \equiv \frac{1}{2} f^{AB} f^{AB}, \quad \langle Q \rangle = -E_0(E_0 + 1 - d) - \sum_{l=1}^{\nu} h_l(h_l - 2l + d - 1).$$

In addition we impose on $|A\rangle$ the following subsidiary constraints

$$\nabla_n |A\rangle = 0 \quad (\text{divergencelessness}), \quad (6)$$

$$\bar{y}_n |A\rangle = 0 \quad (\text{transversality}). \quad (7)$$

Here and below we use the notation

$$\nabla_n \equiv \nabla^A \bar{a}_n^A, \quad \nabla_n \equiv \bar{a}_n^A \nabla^A, \quad \bar{y}_n \equiv \bar{a}_n^A y^A, \quad y_n \equiv a_n^A y^A,$$

$$a_n^{ij} \equiv \theta^{ij}_{AB} a_n^B, \quad \bar{a}_n^{ij} \equiv \theta^{ij}_{AB} \bar{a}_n^B.$$

The constraint (6) is a $SO(d - 1, 2)$ covariant analog of usual divergencelessness condition $\partial_j A^\mu \cdots = 0$. The $SO(d - 2, 1)$ tensor decomposes into the same rank tensor of Lorentz subgroup $SO(d - 1, 1)$ and a lower rank tensor of $SO(d - 1, 1)$. The constraint (7) implies that the lower rank tensor is set to zero. In other words we use a $SO(d - 1, 2)$ tensor which is irreducible when reducing to Lorentz subgroup. Taking into account the transversality (7) and the relations

$$a_{ij} = a_{ij} - y_i \bar{y}_j, \quad a_{\bar{i} \bar{j}} = a_{\bar{i} \bar{j}} - \bar{y}_i \bar{y}_j, \quad (8)$$

we transform the constraints (4) to form which is more convenient in practical calculations

$$(a_{ii} - h_i)|A\rangle = 0, \quad a_{\bar{i}\bar{j}}|A\rangle = 0, \quad \varepsilon^{ij} a_{ij} |A\rangle = 0. \quad (9)$$

Making use of constraints above the equations of motion may be simplified. To this end we rewrite the $Q$ as follows

$$Q = -\nabla^2 + M^{AB} L^{AB} + \frac{1}{2} M^{AB} M^{AB}, \quad \nabla^2 \equiv \nabla^A \nabla^A,$$

use then the relations

$$M^{AB} L^{AB} |A\rangle = 2 \sum_{l=1}^{\nu} h_l |A\rangle, \quad M^{AB} M^{AB} |A\rangle = -2 \sum_{l=1}^{\nu} h_l(h_l - 2l + d + 1) |A\rangle.$$
and get the desired form of equations of motion

\[
(\nabla^2 - m^2)|A| = 0, \quad m^2 \equiv E_0(E_0 + 1 - d). 
\]  

To define \( E_0 \) corresponding to massless representations we should construct gauge transformations and choose such the \( E_0 \) that the equations (10) to be invariant with respect to gauge transformations. In order to formulate gauge transformations we use the gauge parameters fields whose spacetime indices correspond to the \( YT \) which one can make by removing one box from the \( YT(h) \).

The most general gauge transformations we start with are

\[
\delta_{(n)}|A| \sim \nabla_n|A_n| + y_n|R_n|,
\]

where the gauge parameters fields \(|A_n\rangle\) and \(|R_n\rangle\) are associated with \( YT(h_{(n)}) \), and \( i\)-th component of the \( h_{(n)} \) is equal to \( h_{i(n)} = h_i - \delta_{in} \). The \( YT(h_{(n)}) \) is obtained by removing one box from \( n\)-th row of the \( YT(h) \). We assume that only those \(|A_n\rangle\) and \(|R_n\rangle\) are non-zero whose \( h_{(n)} \) satisfy the inequalities

\[
h_{1(n)} \geq \ldots \geq h_{\nu(n)} \geq 0.
\]  

Given the \( h \), the set of those \( n \) whose \( h_{(n)} \) satisfy (12) will be referred to as \( S(h) \). We impose on the \(|A_n\rangle\), \(|R_n\rangle\) and the constraints similar to those for \(|A\rangle\)

\[
\nabla_i|A_n| = 0, \quad \bar{y}_i|A_n| = 0
\]

and constraints obtained from (13) by replacing \( A \rightarrow R \). Since \(|A_n\rangle\), \(|R_n\rangle\) correspond to \( YT(h_{(n)}) \), they satisfy the constraints

\[
(a_{ii} - h_{i(n)})|A_n\rangle = 0, \quad a_{ij}|A_n\rangle = 0, \quad \varepsilon^{ij}a_{ij}|A_n\rangle = 0,
\]

and those which are obtainable from (14) by replacing \( A \rightarrow R \). In practical calculation it is convenient to rewrite (14) in the form

\[
(a_{ii} - h_{i(n)})|A_n\rangle = 0, \quad a^{ij}_n|A_n\rangle = 0, \quad \varepsilon^{ij}a_{ij}|A_n\rangle = 0.
\]

which can be obtained by using the constraints (13).

It turns out that the invariance requirement of constraints (7),(9) with respect to gauge transformations fixes the form of gauge transformations

\[
\delta_{(n)}|A| = D_n|A_n| , \quad D_n = \sum_{j=0}^{n-1} (-)^j \sum_{l_1, \ldots, l_{j+1}} \delta_{nl_{j+1}} \prod_{l=1}^j \varepsilon^{l_{l+1}} \lambda_{l,n}^{-1} a_{i+1}a_{l+1} D_{l_1}, \quad
\]

\[
D_n \equiv \nabla_n + \sum_{l=1}^n (-y_{ln}a_{ln} + a^+_{ln}y_{ln}), \quad \lambda_{ln} = h_l - h_n + n - l + 1.
\]

In (16) the \( a_{l+1}a_{r+1} \) are ordered as follows: \( a_{r+1}a_{l+1} \ldots a_{i+1} \). Then from the invariance requirement of (6) with respect to (16), i.e. \( \nabla_n\delta_{(n)}|A| = 0 \), we find the equation of motion for gauge parameter field

\[
(\nabla^2 - (h_n - n)(h_n - n - 1 + d))|A_n| = 0.
\]
Finally from the invariance requirement of equation of motion (10) with respect to gauge transformations (16), i.e. \((\nabla^2 - m^2)\delta_{(n)}|A\rangle = 0\), we get the equation for \(E_0\)
\[
E_0(E_0 + 1 - d) = (h_n - n - 1)(h_n - n - 2 + d), \quad n \in S(h).
\] (18)

Note that in deriving (18) the equations of motion for gauge parameter has been used. Solutions to the quadratic equation for \(E_0\) (18) read:
\[
E_0^{(1)}(n) = h_n - n - 2 + d, \quad E_0^{(2)}(n) = n + 1 - h_n.
\] (19)

As seen from (19) there exists an arbitrariness of \(E_0\) parametrized by subscript \(n\) which labels gauge transformations and by superscripts (1), (2) which label two solutions of equation (18). Because the values of \(E_0\) have been derived by exploiting gauge invariance we can conclude that the gauge invariance by itself does not uniquely determine the physical relevant value of \(E_0\). To choose physical relevant value of \(E_0\) we exploit the unitarity condition, that is: 1) hermiticity \((iJ^{AB})^\dagger = iJ^{AB}\); 2) the positive norm requirement. For details of resulting procedure we refer to (Metsaev (1995)) and now let us formulate the result.

Given \(\Upsilon T(h)\) let \(k, k = 1 \ldots \nu\), indicates maximal number of upper rows which have the same number of boxes. We call such Young tableaux the level-\(k\) \(\Upsilon T(h)\).

For the case of level-\(k\) Young tableaux the inequalities (1) can be rewritten as
\[
h_1 = \ldots = h_k > h_{k+1} \geq h_{k+2} \geq \ldots \geq h_{\nu} \geq 0.
\] (20)

Then making use of unitarity condition one proves (Metsaev (1995)) that for the level-\(k\) Young tableaux the \(E_0\) should satisfy the inequality \(^2\)
\[
E_0 \geq h_k - k - 2 + d.
\] (21)

Comparing (19) with (21) we conclude that only \(E_0^{(1)}(n=k)\) satisfies the unitarity condition. Thus anti-de Sitter bosonic massless particles described by level-\(k\) \(\Upsilon T(h)\) takes lowest value of energy equal to
\[
E_0 = h_k - k - 2 + d.
\] (22)

Note that it is gauge transformation with \(n = k\) (16) that leads to relevant \(E_0\), i.e. given level-\(k\) \(\Upsilon T(h)\) only the gauge transformation \(\delta_{(k)}\) respects the unitarity. Therefore only the \(\delta_{(k)}\) will be used in what follows. From now on we use letter \(k\) to indicate level of \(\Upsilon T(h)\). Thus the final form of gauge transformation is
\[
\delta_{(k)}|A\rangle = \sum_{j=0}^{k-1} (-)^j \sum_{l_1 \ldots l_{j+1}=1}^k \delta_{kl_1} \prod_{i=1}^{j} \frac{\varepsilon_{l_i l_{i+1}} a_{l_i+1} l_i}{k + 1 - l_i} D_{l_i}|A_k\rangle.
\] (23)

\(^2\) This bound for \(d = 4\) has been found by Evans (1967). For the case \(d = 5\) see Mack (1977) and references therein. Note that to extend (21) to odd \(d\) we should simply replace \(h_k \rightarrow |h_k|\). In view that for odd \(d\) all \(h_i \geq 0\) with exception of \(h_{(d-1)/2} \neq 0\) our result are valid also for odd \(d\) when \(h_{(d-1)/2} = 0\). At present time field theoretical description of representations for arbitrary \(h_{(d-1)/2}\) is absent.
Arbitrary spin massless bosonic fields in d-dimensional anti-de Sitter space

As an illustration of (23) we write down $\delta_k$ for $k = 1, 2, 3$:

$$
\delta(1) A = D_1 A_1, \quad \delta(2) A = (D_2 - \frac{1}{2} a_{21} D_1) A_2, \\
\delta(3) A = (D_3 - \frac{1}{2} a_{32} D_2 - \frac{1}{3} a_{31} D_1 + \frac{1}{6} a_{32} a_{21} D_1) A_3.
$$

Note that from the equation for gauge parameter field (17) we get the following lowest energy value for $A_k$: $E_0^A = E_0 + 1$, i.e.

$$
E_0^A = h_k - k - 1 + d. \quad (24)
$$

Comparing this relation with (22) we conclude that only for $k > 1$ the gauge parameters are massless fields while for $k = 1$ they are massive fields. Thus we have constructed equations of motion (10) which respect gauge transformations (23), where the gauge parameter fields $A_k$ satisfy the constraints (13), (15) and equations of motion (17). The relevant $E_0$ and $E_0^A$ are given by (22) and (24).

All things above have been done in $SO(d - 1, 2)$ covariant form. Because sometimes a formulation in terms of intrinsic coordinates is preferable let us transform our result to such the coordinates. Let $x^\mu$, $\mu = 0, 1, \ldots, d - 1$ be the intrinsic coordinates in AdS spacetime and let $y^A(x)$ be imbedding map, where $y^A(x)$ satisfy (2). The relationship between $SO(d - 1, 2)$ tensor field $A^{\mu_1 \cdots}$ and the usual tensor field $A^{\mu_1 \cdots}$ is given by

$$
A^{\mu_1 \cdots}(x) = y^\mu_1 \cdots A^{\mu_1 \cdots}(y), \quad y^\mu_C = g^{\mu \nu} \partial_\nu y_C,
$$

where the intrinsic geometry metric tensor is given by $g_{\mu \nu} = \partial_\mu y^A \partial_\nu y^A$ while its inverse is $g^{\mu \nu} = \nabla^A x^\mu \nabla^A x^\nu$. The $x^\mu = x^\mu(y)$ is a certain representation of intrinsic coordinates. There are useful relations

$$
\theta^{AB} = g^{\mu \nu} \partial_\mu y^A \partial_\nu y^B, \quad \partial_\mu y^A \nabla^A x^\nu = \delta_\mu^\nu,
$$

$$
\nabla^A x^\mu = g^{\mu \nu} \partial_\nu y^A, \quad \nabla^2 x^\mu = -\Gamma^\mu_{\rho \sigma} g^{\rho \sigma}, \quad D_\mu y^A = g_{\mu \nu} y^A.
$$

With these relation at hand and with the help of relations

$$
y^\mu_1 \cdots y^\mu_C \nabla^A C_1 \cdots C_s = D^2 A^{\mu_1 \cdots \mu_s} + s A^{\mu_1 \cdots \mu_s}, \quad D^2 A^{C_1 \cdots} = \nabla^2 A^{C_1 \cdots}
$$

where $D^2 = D_\mu D^\mu$, $D_\mu = \partial_\mu + \Gamma_\mu$, we can immediately transform equation of motion (10) to the desired form

$$
(D^2 - (h_k - k - 1)(h_k - k - 2 + d) + \sum_{l=1}^s h_l) A^{\mu_1 \cdots} = 0. \quad (25)
$$
It turns out that in order to write gauge transformation it is convenient to transform spacetime tensors into the tangent space tensors $A^\mu_1\cdots\equiv e^a_{\mu_1}\cdots A^{\mu_1}$ where the $e^a_{\mu}$ is a einbein of AdS geometry, introduce new creation and annihilation operators $a^a_i$ and $\bar{a}^a_i$, $l=1,\ldots,\nu$, $a=0,1,\ldots,d-1$, and construct Fock space vector

$|a\rangle = a^a_1\cdots A^{a_1} |0\rangle$, \quad $[\bar{a}^a_i,a^b_j] = \delta_{ij}\eta^{ab}$, \quad $\eta^{ab} = (-,+,\ldots,+)$.\]

Now the equation, constraints and gauge transformation take the form

\begin{equation}
(D_L^2 - (h_k - k - 1)(h_k - k - 2 + d) + \sum_{i=1}^\nu h_i)|a\rangle = 0,
\end{equation}

\begin{equation}
\delta_{(k)}|a\rangle = \sum_{j=0}^{k-1}(-)^j\sum_{l_1,\ldots,l_{j+1}=1}^k \delta_{k,l_{j+1}} \prod_{i=1}^j \frac{\varepsilon_{l_i+1}(a_{l_i+1}\bar{a}_{l_i})}{k+1-l_i} e^{a_{l_i}} D_{\mu L}|\lambda_k\rangle,
\end{equation}

\begin{equation}
\bar{a}^b_i e^a_b D_{\mu L}|a\rangle = 0, \quad (a^b_i \bar{a}^a_j - h_i)|a\rangle = 0, \quad \bar{a}^b_i a^a_i|a\rangle = 0, \quad \varepsilon^{ij} a^b_i a^b_j|a\rangle = 0,
\end{equation}

\begin{equation}
D_{\mu L} \equiv \partial_\mu + \frac{1}{2} \omega^{ab}_\mu M^{ab}, \quad M^{ab} \equiv \sum_{l=1}^\nu (a^a_l \bar{a}^b_l - a^b_l \bar{a}^a_l).
\end{equation}

The $\omega^{ab}_\mu$ is a Lorentz connection of AdS spacetime. The equation and constraints for the gauge parameter field $\lambda_k$ are obtainable from (26) and (27) by making there the substitutions $|a\rangle \rightarrow |\lambda_k\rangle$, $h_i \rightarrow h_i(k)$ and $k \rightarrow k - 2$. In order to demonstrate how our results are working let us consider some particular cases.

**Totally antisymmetric fields.** In this case $h = (1,\ldots,1,0)$ where unit occurs $s$-times in this sequence. Therefore we have $k = s$ and $h_k = \varepsilon^{ks+1}$. For this case $E_0 = d-1-s$ and we get the equations

\begin{equation}
(D^2 + s(d-s))A^{\mu_1\cdots\mu_s} = 0,
\end{equation}

where the relevant constraint is $D_{\mu}A^{\mu\nu_2\cdots\mu_s} = 0$. By making use of this gauge the equations above can be easily derived from well known equations

\begin{equation}
D_{\mu}F^{\mu\nu_2\cdots\nu_s} = 0, \quad F_{\mu_1\cdots\mu_n} = n\partial_{[\mu_1} A_{\mu_2\cdots\mu_n]}.
\end{equation}

**Totally symmetric fields.** In this case $h = (s,0,\ldots,0)$. Therefore we have $k = 1$ and $h_k = s\varepsilon^{k2}$, the $E_0$ is given by $E_0 = s+d-3$ and we get the equations

\begin{equation}
(D^2 - s^2 + (6-d)s + 2d - 6)A^{\mu_1\cdots\mu_s} = 0,
\end{equation}

where the relevant constraints are

\begin{equation}
A^{\mu\nu_2\cdots\nu_s} = 0, \quad D_{\mu}A^{\mu\nu_2\cdots\nu_s} = 0.
\end{equation}

For $d = 4$ these equations can be obtained from those discovered in (Fronsdal (1978)) by making use of the gauge (29). The graviton is a particular case when
s = 2. From (28) we get the equation \((D^2 + 2)h_{\mu\nu} = 0\) which should be supplemented by constraints like (29). This equation coincides with that obtained from Einstein equation for excitation of metric tensor

\[ R_{\mu\nu} = -(d-1)G_{\mu\nu}, \quad G_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu}, \quad R_{\mu\nu\rho\sigma}(g) = -(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}) \]

where \(g_{\mu\nu}\) is a metric tensor of AdS geometry. Note that the value \(s = 2\) is the only when the dependence on \(d\) in (28) is cancelled. Thus we have demonstrated that our results cover all previously known particular cases and solve problem for arbitrary spin \(h\) massless bosonic fields in \(d\) - dimensional AdS spacetime.

In conclusion let us discuss masslessness in \(d\) - dimensional AdS spacetime by using the requirement of conformal invariance. By conformal invariant representations we will understand those irreducible representations of anti-de Sitter group that can be realized as irreducible representations of the conformal group \(SO(d,2)\). It turns out (for details see Metsaev (1995)) that this requirement leads to representations whose \(h_i\) satisfy the constants \(h_1 = \ldots = h_\nu \equiv h\), while \(E_0 = h + \nu\). These \(E_0\) and \(h_i\) are in accordance with (22), i.e. conformal invariance respects the gauge invariance, but because of constraints above-mentioned the conformal representations constitute only a subset of all massless states for \(d > 4\), i.e. conformal group for \(d > 4\) cannot be used for defining all massless representations. In this respects the situation in AdS spacetime (Metsaev (1995)) is similar to that in Minkowski spacetime (Siegel (1989)).

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