Probing New Physics
with $\bar{B} \to \rho(770) \ell^- \bar{\nu}_\ell$ and $\bar{B} \to a_1(1260) \ell^- \bar{\nu}_\ell$

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Abstract
The $B$ meson semileptonic modes to $\rho(770)$ and $a_1(1260)$ are useful to pin down possible non Standard Model effects. The 4d differential $\bar{B} \to \rho(\pi\pi) \ell^- \bar{\nu}_\ell$ and $\bar{B} \to a_1(\rho\pi) \ell^- \bar{\nu}_\ell$ decay distributions are computed in SM and in extensions involving new Lepton Flavour Universality violating semileptonic $b \to u$ operators. The Large Energy limit for the light meson is also considered for both modes. The new effective couplings are constrained using the available data, and several observables in $\bar{B} \to \rho(\pi\pi) \ell^- \bar{\nu}_\ell$ in which NP effects can be better identified are selected, using the angular coefficient functions. The complementary role of $\bar{B} \to \rho(\pi\pi) \ell^- \bar{\nu}_\ell$ and $\bar{B} \to a_1(\rho\pi) \ell^- \bar{\nu}_\ell$ is discussed.

1 Introduction
The anomalies recently emerged in the flavour sector challenge both the experimental analyses and the theoretical interpretations. In the tree-level $b \to c \ell^- \bar{\nu}_\ell$ process, deviations of the ratios $R(D) = \frac{B(B \to D^\tau \pi^- \nu_\tau)}{B(B \to D^\ell \ell^- \bar{\nu}_\ell)}$ (with $\ell = e, \mu$) from the Standard Model (SM) expectations have been observed by BABAR $^{[1,2]}$, Belle $^{[3-6]}$ and LHCb $^{[7-9]}$. The measurements can be summarized as $R(D)_{\text{exp}} = 0.407 \pm 0.039 \pm 0.024$ to be combined with the new Belle result $R(D)_{\text{exp}} = 0.307 \pm 0.037 \pm 0.016$ $^{[10]}$, and $R(D^*)_{\text{exp}} = 0.295 \pm 0.011 \pm 0.008$. These measurements are $3.1 \sigma$ away from the SM values quoted by the Heavy Flavour Averaging Group (HFLAV) $^{[11]}$: $R(D)_{\text{SM}} = 0.299 \pm 0.003$ and $R(D^*)_{\text{SM}} = 0.258 \pm 0.005$. The tension, noticed in $^{[12]}$, is significant since the hadronic uncertainties largely cancel out in the ratios of branching fractions $^{[13]}$. The LHCb measurement $R(J/\psi) = \frac{B(B_c^+ \to J/\psi \tau^+ \nu_\tau)}{B(B_c^+ \to J/\psi \mu^+ \nu_\mu)} = 0.71 \pm 0.17(\text{stat}) \pm 0.18(\text{syst})$ $^{[14]}$ also exceeds the SM expectation, however in these modes the hadronic uncertainties are sizable $^{[15-17]}$.

Other anomalies have been detected in neutral current $b \to s$ semileptonic transitions, in the ratios $R_{K^{(*)}} = \frac{\int_{q_{\text{min}}^2}^{q_{\text{max}}^2} \frac{dq^2}{dq^2} (B^+ \to K^{(*)} \mu^+ \mu^-) dq^2}{\int_{q_{\text{min}}^2}^{q_{\text{max}}^2} \frac{dq^2}{dq^2} (B^+ \to K^{(*)} e^+ e^-) dq^2}$ measured by LHCb and Belle. The updated
result for $R_K$ is $R_{K^+} = 0.846^{+0.060}_{-0.054}(\text{stat})^{-0.016}_{+0.014}(\text{syst})$ for $[q_{\text{min}}^2, q_{\text{max}}^2] = [1.1 \text{ GeV}^2, 6 \text{ GeV}^2]$ [18]. For $R_{K^*}$, the measurements $R_{K^*0} = 0.66^{+0.11}_{-0.07}(\text{stat})\pm 0.03(\text{syst})$ for $q^2$ in $[0.045 \text{ GeV}^2, 1.1 \text{ GeV}^2]$ and $R_{K^*0} = 0.69^{+0.11}_{-0.07} (\text{stat})\pm 0.05(\text{syst})$ for $q^2$ in $[1.1 \text{ GeV}^2, 6 \text{ GeV}^2]$ have been reported by LHCb [19]. Recent Belle measurements, averaged over the neutral and charged modes, are affected by larger errors: $R_{K^*} = 0.52^{+0.36}_{-0.26} (\text{stat})\pm 0.05(\text{syst})$ for $q^2$ in $[0.045 \text{ GeV}^2, 1.1 \text{ GeV}^2]$, $R_{K^*} = 0.90^{+0.27}_{-0.23} (\text{stat})\pm 0.10(\text{syst})$ for $q^2$ in $[0.1 \text{ GeV}^2, 8 \text{ GeV}^2]$, and $R_{K^*} = 1.18^{+0.52}_{-0.32} (\text{stat})\pm 0.10(\text{syst})$ for $q^2$ in $[15 \text{ GeV}^2, 19 \text{ GeV}^2]$ [20]. For all the ratios the SM predictions are close to one.

The anomalies in $b \rightarrow c$ and $b \rightarrow s$ semileptonic modes seem to point to violation of lepton flavour universality (LFU). This accidental SM symmetry is only broken by the Yukawa interactions, while the lepton couplings to the gauge bosons are independent of the lepton flavour. \footnote{For a review on LFU tests see [21].} It is unclear if the deviations emerged in angular observables in $B \rightarrow K^*\mu^+\mu^-$ and in the rate of $B^0_s \rightarrow \phi\mu^+\mu^-$ can have a connected origin.

In addition to these tensions, the long-standing difference in the determination of the CKM matrix element $|V_{cb}|$ from exclusive modes, in particular $\bar{B} \rightarrow D^*\ell^-\bar{\nu}_\ell$, and from inclusive $B \rightarrow X_c\ell^-\bar{\nu}_\ell$ observables (width and moments) still persists in new BABAR [25] and Belle analyses [26], with $|V_{cb}|_{\text{excl}} < |V_{cb}|_{\text{incl}}$. As an alternative to solutions to the puzzle within SM \cite{27,30}, a connection within a LFU violating framework has been proposed with the other $b \rightarrow c$ anomalies \cite{13,31}. The related experimental signatures have been studied, in particular the 4d differential $\bar{B} \rightarrow D^*(D\pi, D\gamma)\ell^-\bar{\nu}_\ell$ decay distributions for the three lepton species have been scrutinized \cite{32}, following analyses that have pointed out the relevance of such distributions \cite{33,36}.

It is worth wondering if similar deviations can appear in semileptonic $b \rightarrow u$ transitions. These modes are CKM suppressed with respect to the $b \rightarrow c$ ones, nevertheless high precision measurements are foreseen in the near future by LHCb and Belle II. At present, there is a tension between the exclusive measurement of $|V_{ub}|$, mainly from the $\bar{B} \rightarrow \pi\ell^-\bar{\nu}_\ell$ decay width, and the inclusive determination from $\bar{B} \rightarrow X_u\ell^-\bar{\nu}_\ell$ observables. New information is available on the purely leptonic and on the semileptonic $B \rightarrow \pi$ mode, while analyses of possible deviations from SM have been carried out \cite{37,44}.

The purpose of our study is to identify suitable observables for precision measurements, mainly of LFU observables, in these transitions testing the sensitivity to possible deviations from SM. We extend the SM $b \rightarrow u$ effective Hamiltonian with the inclusion of New Physics (NP) semileptonic operators weighted by complex couplings, and focus on the impact of such operators on two exclusive modes, $\bar{B} \rightarrow \rho(770)\ell^-\bar{\nu}_\ell$ and $\bar{B} \rightarrow a_1(1260)\ell^-\bar{\nu}_\ell$, with the vector and axial-vector light mesons decaying to $\pi\pi$ and $\rho\pi$, respectively. We show how the analysis of both the channels can pin down effects deviating from SM, starting from the full angular distributions. We constrain the new couplings using data, and provide examples of the deviations that can show up in various observables.

In Sect.3 we extend the semileptonic $b \rightarrow u$ effective Hamiltonian including scalar, pseudoscalar and tensor operators. We describe the $\bar{B}$ transitions to two leptons and to $\pi\ell\bar{\nu}$ incorporating the non-SM contributions, to bound the effective couplings. In Sect.4 we construct 4d differential decay distributions for the $\bar{B} \rightarrow \rho(\pi\pi)\ell^-\bar{\nu}_\ell$ and $\bar{B} \rightarrow a_1(\rho\pi)\ell^-\bar{\nu}_\ell$ modes, defining the sets of angular coefficient functions. We also consider the Large Energy limit for
the light mesons, which allows to express the angular functions in terms of a small number of hadronic form factors. In Sect. 4 we analyze several observables in $\bar{B} \to \rho(\pi\pi)\ell^-\bar{\nu}_\ell$ at a benchmark point in the parameter space of the new couplings, to scrutinize their sensitivity to the different new operators, and in Sect. 5 we elaborate on the $a_1$ mode. The last Section contains the conclusions. In the Appendices we collect the definitions of the hadronic matrix elements and the expressions of the angular coefficient functions.

2 Effective $b \to u\ell^-\bar{\nu}_\ell$ NP Hamiltonian

New Physics contributions to beauty hadron decays can be analysed within the Standard Model Effective Field Theory. If the NP scale $\Lambda_{NP}$ is much larger than the EW scale, all the new massive degrees of freedom can be integrated out, obtaining an effective Hamiltonian in which only the SM fields appear and which is invariant under the SM gauge group. This Hamiltonian contains additional operators with respect to SM, suppressed by increasing powers of $\Lambda_{NP}$. The contribution $O\left(\frac{1}{\Lambda_{NP}^2}\right)$ includes dimension-six four-fermion operators [45].

To describe the modes $\bar{B} \to M_u\ell^-\bar{\nu}_\ell$ with $M_u$ a light meson comprising an up quark we consider the effective Hamiltonian

$$
H_{\text{eff}}^{b \to u\ell\nu} = \frac{G_F}{\sqrt{2}} V_{ub} \left\{ (1 + \epsilon_{V}^\ell) (\bar{u}\gamma_\mu(1 - \gamma_5)b) (\bar{\ell}\gamma^\mu(1 - \gamma_5)\nu_\ell) + \epsilon_{S}^\ell (\bar{u}(1 - \gamma_5)\nu_\ell) + \epsilon_{P}^\ell (\bar{u}\gamma_5b) (\bar{\ell}(1 - \gamma_5)\nu_\ell) + \epsilon_{T}^\ell (\bar{u}\sigma_\mu\nu(1 - \gamma_5)b) (\bar{\ell}\sigma^{\mu\nu}(1 - \gamma_5)\nu_\ell) \right\} + h.c.,
$$

consisting in the SM term and in NP terms weighted by complex lepton-flavour dependent couplings $\epsilon_{V,S,P,T}^{\ell}$. We assume a purely left-handed lepton current as in SM, an extensively probed structure. We exclude the quark right-handed vector current, since the only four-fermion operator of this type, invariant under the SM group, is non-linear in the Higgs field [46–48].

The couplings of the NP operators in (1) are constrained by the measurements, in particular on the purely leptonic $B^- \to \ell^-\bar{\nu}_\ell$ and semileptonic $\bar{B} \to \pi\ell^-\bar{\nu}_\ell$ modes. The $B^- \to \ell^-\bar{\nu}_\ell$ decay width obtained from $H_{\text{eff}}^{b \to u\ell\nu}$ in Eq.(1) reads

$$
\Gamma(B^- \to \ell^-\bar{\nu}_\ell) = \frac{G_F^2 V_{ub}}{8\pi} \left| f_B m_B^3 \right|^2 \left(1 - \frac{m_\ell^2}{m_B^2}\right)^2 \left(\frac{m_\ell}{m_B}\right)^2 \left(1 + \epsilon_{V}^\ell + \frac{m_B}{m_b + m_u} \epsilon_{P}^\ell\right)^2,
$$

with the decay constant $f_B$ defined as

$$
\langle 0 | \bar{u}\gamma_\mu\gamma_5 b | \bar{B}(p) \rangle = i f_B p_\mu. \tag{3}
$$

The ew correction to (2) is tiny. The mode is insensitive to the NP scalar and tensor operators, and the inclusion of the pseudoscalar operator removes the helicity suppression, which is effective for light leptons. This provides stringent constraints for the couplings $\epsilon_{P}^{\ell,\mu}$.

Right-handed currents are investigated in [38, 39, 41].
The semileptonic $\bar{B} \to \pi \ell^- \bar{\nu}_\ell$ decay distribution in the dilepton mass squared $q^2$, obtained from Eq.(1) parametrizing the weak matrix element in terms of the form factors $f_i(q^2) = f_i^{B \to \pi}(q^2)$ defined in Appendix A is:

$$\frac{d\Gamma}{dq^2}(\bar{B} \to \pi \ell^- \bar{\nu}_\ell) = \frac{G_F^2 |V_{ub}|^2 \lambda^{1/2}(m_B^2, m_\pi^2, q^2)}{128 m_B^3 \pi^3 q^2} \left(1 - \frac{m_\ell^2}{q^2}\right)^2 \left(m_\ell(1 + \epsilon_\ell^f) + \frac{q^2 \epsilon_S^f}{m_b - m_u}\right) \left(m_B^2 - m_\pi^2\right)^2 f_0^2(q^2) +$$

$$+ \lambda(m_B^2, m_\pi^2, q^2) \left[\frac{1}{3} m_\ell(1 + \epsilon_\ell^f) f_+(q^2) + \frac{4q^2}{m_B + m_\pi} \epsilon_T f_T(q^2)\right]^2 \frac{2q^2}{3} \left(1 + \epsilon_\ell^f\right) f_+(q^2) + 4 \frac{m_\ell}{m_B + m_\pi} \epsilon_T f_T(q^2)^2 \right] \right\} .$$

(4)

$\lambda$ is the triangular function. In the large energy limit of the emitted pion, using Eq.(A.13) for the weak matrix element, the decay distribution is expressed in terms of a single form factor $\xi_\pi$:

$$\frac{d\Gamma}{dE}(\bar{B} \to \pi \ell^- \bar{\nu}_\ell) = \frac{G_F^2 |V_{ub}|^2 \lambda^{1/2}(m_B^2, m_\pi^2, q^2)}{64 m_B^2 \pi^3 q^2} \left(1 - \frac{m_\ell^2}{q^2}\right)^2 \xi_\pi^2(E) \left(m_\ell(1 + \epsilon_\ell^f) + \frac{q^2 \epsilon_S^f}{m_b - m_u}\right) \left(m_B^2 - m_\pi^2\right)^2 \left(1 + \frac{2q^2}{m_B^2} \epsilon_T\right)^2$$

$$+ \lambda(m_B^2, m_\pi^2, q^2) \left[\frac{1}{3} m_\ell(1 + \epsilon_\ell^f) + \frac{4q^2}{m_B} \epsilon_T\right]^2 + \frac{2q^2}{3} \left(1 + \epsilon_\ell^f\right) + \frac{4m_\ell}{m_B} \epsilon_T\right]^2 \right] ,$$

(5)

with $q^2 = m_B^2 + m_\pi^2 - 2m_B E$. While the full kinematical range for $E$ is $m_\pi \leq E \leq m_B \left(1 + \frac{m_\pi^2}{m_B^2} - \frac{m_\ell^2}{m_B^2}\right)$, Eq.(5) only holds for large $E \approx \frac{m_B}{2}$. This expression is useful if the distribution is independently measured for the three charged leptons, since the ratios

$$\frac{dR(\pi)^{\ell^f}}{dE} = \frac{d\Gamma}{dE}(\bar{B} \to \pi \ell^- \bar{\nu}_\ell) / \frac{d\Gamma}{dE}(\bar{B} \to \pi \ell^- \bar{\nu}_\ell)$$

are free of hadronic uncertainties and only involve combinations of the lepton flavour-dependent couplings $\epsilon_{V,S,T}^f$.

### 3 Angular distributions for $\bar{B} \to \rho(\to \pi \pi) \ell^- \bar{\nu}_\ell$ and $\bar{B} \to a_1(\to \rho \pi) \ell^- \bar{\nu}_\ell$

The main sensitivity to the new operators in (1), in the modes $\bar{B} \to \rho(\to \pi \pi) \ell^- \bar{\nu}_\ell$ and $\bar{B} \to a_1(\to \rho \pi) \ell^- \bar{\nu}_\ell$, is in the $4d$ differential decay distribution in the variables $q^2$ and in the angles $\theta, \theta_V$ and $\phi$ specified in Fig.1. The distributions can be written as:

$$....$$
Figure 1: Kinematics of the decay mode $B \to \rho(\pi\pi) \ell^- \bar{\nu}_\ell$.

\[
\frac{d^4\Gamma(B \to \rho(\pi\pi) \ell^- \bar{\nu}_\ell)}{dq^2 \, d\cos\theta \, d\phi \, d\cos\theta_V} = \mathcal{N}_\rho |\tilde{p}_\rho| \left(1 - \frac{m_\ell^2}{q^2}\right)^2 \left\{ I_{1a}^\rho \sin^2 \theta_V + I_{1c}^\rho \cos^2 \theta_V \\
+ \left( I_{2a}^\rho \sin^2 \theta_V + I_{2c}^\rho \cos^2 \theta_V \right) \cos 2\theta \\
+ I_{3}^\rho \sin^2 \theta_V \sin^2 \theta \cos 2\phi + I_{4}^\rho \sin 2\theta_V \sin 2\theta \cos \phi \\
+ I_{5}^\rho \sin 2\theta_V \sin \theta \cos \phi + \left( I_{6s}^\rho \sin^2 \theta_V + I_{6c}^\rho \cos^2 \theta_V \right) \cos \theta \\
+ I_{7}^\rho \sin 2\theta_V \sin \theta \sin \phi \right\},
\]

and

\[
\frac{d^4\Gamma(B \to a_1(\rho) \ell^- \bar{\nu}_\ell)}{dq^2 \, d\cos\theta \, d\phi \, d\cos\theta_V} = \mathcal{N}_{a_1} |\tilde{p}_{a_1}| \left(1 - \frac{m_\ell^2}{q^2}\right)^2 \left\{ I_{1a}^{a_1} \sin^2 \theta_V + I_{1c}^{a_1} \cos^2 \theta_V \left(3 + \cos 2\theta_V \right) \\
+ \left( I_{2a}^{a_1} \sin^2 \theta_V + I_{2c}^{a_1} \left(3 + \cos 2\theta_V \right) \right) \cos 2\theta \\
+ I_{3}^{a_1} \sin^2 \theta_V \sin^2 \theta \cos 2\phi + I_{4}^{a_1} \sin 2\theta_V \sin 2\theta \cos \phi \\
+ I_{5}^{a_1} \sin 2\theta_V \sin \theta \cos \phi \\
+ \left( I_{6s}^{a_1} \sin^2 \theta_V + I_{6c}^{a_1} \left(3 + \cos 2\theta_V \right) \right) \cos \theta \\
+ I_{7}^{a_1} \sin 2\theta_V \sin \theta \sin \phi \right\}. \tag{8}
\]

In the $\rho$ mode, the factor $\mathcal{N}_\rho$ is $\mathcal{N}_\rho = \frac{3G_F^2 |V_{ub}|^2 B(\rho \to \pi\pi)}{128(2\pi)^4 m_B^2}$. In the $a_1$ channel we separate the modes where the final $\rho$ is transversely $(\rho \perp)$ or longitudinally $(\rho \parallel)$ polarized, specified in \footnote{Other angular structures appear in the differential distributions if a quark right-handed vector current is included in [1].}.
Appendix B, with $X^\perp$.

Appendix B, with $N^\perp$. In Eq.(8) the subscripts $\perp, \parallel$ refer to these two $\rho$ polarizations. The differential distributions are computed in the narrow width approximation, factorizing the production and decay amplitude of the intermediate vector and axial-vector meson. This is connected to the procedure adopted in the experimental analyses to select the contributions of the intermediate resonances $[51]$.

The angular coefficient functions $I_i^P$ and $I_i^{P_1}$ in Eqs.(7) and (8) can be written as

$$I_i = [1 + \epsilon_V]^2 I_i^{SM} + |\epsilon_X|^2 I_i^{NP,X} + |\epsilon_T|^2 I_i^{NP,T} + 2 \text{Re} [\epsilon_X(1 + \epsilon_V^*]) I_i^{INT,X}$$

$$+ 2 \text{Re} [\epsilon_T(1 + \epsilon_V^*]) I_i^{INT,T} + 2 \text{Re} [\epsilon_X \epsilon_T^*] I_i^{INT,XT}, \quad (i = 1, \ldots 6), \quad (9)$$

$$I_7 = 2 \text{Im} [\epsilon_X(1 + \epsilon_V^*]) I_7^{INT,X} + 2 \text{Im} [\epsilon_T(1 + \epsilon_V^*]) I_7^{INT,T} + 2 \text{Im} [\epsilon_X \epsilon_T^*] I_7^{INT,XT},$$

with $X = P$ in case of $\rho$, and $X = S$ in case of $a_1$. The coefficient functions $I_i^{SM}$, $I_i^{NP}$ and $I_i^{INT}$, expressed in terms of helicity amplitudes, are collected in Tables 20 of Appendix B, together with the relations of the helicity amplitudes to the hadron form factors.

Considering the angular coefficient functions and their expressions, several remarks are in order.

1) With the exception of $I_7$, all angular coefficient functions do not vanish in SM and are sensitive to $\epsilon_V$. Apart from such a dependence, we can identify structures useful to disentangle the effects of the other $S$, $P$ and $T$ operators. In $B \to \rho \ell \bar{\nu}_\ell$ the functions $I_{1s}^P$, $I_{2s}^P$, $I_{2c}^P$, $I_{4}^P$, $I_{6s}^P$ do not depend on $\epsilon_P$, as it can be inferred from Table 3, and are sensitive only to the tensor operator. We denote these structures as belonging to set $A$, while set $B$ comprises the remaining ones. An analogous situation occurs for the corresponding quantities in $B \to a_1(\rho \pi) \ell \bar{\nu}_\ell$, which do not depend on $\epsilon_S$ (Table 6), while in $B \to a_1(\rho \pi) \ell \bar{\nu}_\ell$ the functions $I_{1c}^{a_1}$, $I_{2s}^{a_1}$, $I_{2c}^{a_1}$, $I_{4}^{a_1}$, $I_{6s}^{a_1}$ are insensitive to the scalar operator (Table 7).

2) In the absence of the tensor operator, the $\rho$ and $a_1$ modes give complementary information on the pseudoscalar $P$ (in the $\rho$ channel) and scalar $S$ (in $a_1$) operators, together with the purely leptonic mode (sensitive to $P$) and $B \to \pi$ mode (sensitive to $S$).

3) There are angular coefficient functions that depend only on the helicity amplitudes $H_\pm$, not on $H_0$ and $H_\ell$. These affect observables corresponding to the transversely polarized $W$, hence to transverse $\rho$ in $B \to \rho \ell \bar{\nu}_\ell$ and transverse $a_1$ in $B \to a_1 \ell \bar{\nu}_\ell$. Such observables depend on $\epsilon_T$, not on $\epsilon_P$ (in the $\rho$ mode) or $\epsilon_S$ (in the $a_1$ mode).

4) In the Large Energy Limit of the light meson, the form factors parametrizing the $B \to \rho(a_1)$ weak matrix elements can be written in terms of two form factors, $\xi_\perp^{\rho}(\xi_\perp^{a_1})$ and $\xi_\parallel^{\rho}(\xi_\parallel^{a_1})$ defined by the relations (A.14), (A.15). In this limit, several angular coefficients depend only on the form factor $\xi_\perp$, others involve both $\xi_\perp$ and $\xi_\parallel$. The coefficients depending only on $\xi_\perp^{a_1}(E)$ are:

- in $B \to \rho(770)$ mode: $I_{1s}^P$, $I_{2s}^P$, $I_{3}^P$ and $I_{6s}^P$,
We want to present examples of the possible effects of the NP operators in Eq. (1) in the future, will modify the ranges of the couplings, but the strategy and the measurements or more accurate theoretical determinations of the hadronic quantities, when couplings using the available data and a set of hadronic quantities. More precise experimental comparison of the NP terms in (1), allowing to determine the new couplings efficient functions it is possible to define sets of observables particularly sensitive to different q values.

When a single form factor is involved, ratios of coefficient functions are free of hadronic uncertainties (in the Large Energy Limit).

In conclusion, measuring the full angular distribution and reconstructing the angular coefficient functions it is possible to define sets of observables particularly sensitive to different NP couplings.

4 Observables in $\bar{B} \to \rho \ell^- \bar{\nu}_\ell$

We want to present examples of the possible effects of the NP operators in Eq. (1) in $\bar{B} \to \rho \ell^- \bar{\nu}_\ell$, identifying the most sensitive observables. For that, we constrain the space of the new couplings using the available data and a set of hadronic quantities. More precise experimental measurements or more accurate theoretical determinations of the hadronic quantities, when available in the future, will modify the ranges of the couplings, but the strategy and the overall picture we are presenting will remain valid.

The couplings $\epsilon_\rho^\mu$, $\epsilon_\rho^\tau$, $\epsilon_\tau^\rho$ are constrained by the measurements $\mathcal{B}(\bar{B}^0 \to \pi^+ \ell^- \bar{\nu}_\ell) = (1.50 \pm 0.06) \times 10^{-4}$ and $\mathcal{B}(\bar{B}^0 \to \rho^+ \ell^- \bar{\nu}_\ell) = (2.94 \pm 0.21) \times 10^{-4}$ [52], together with $\mathcal{B}(B^- \to \mu^- \bar{\nu}_\mu) = (6.46 \pm 2.2 \pm 1.60) \times 10^{-7}$ (and 90% probability interval $[2.0, 10.7] \times 10^{-7}$) [53]. For $e$ and $\tau$, the results for the purely leptonic modes are $\mathcal{B}(B^- \to e^- \bar{\nu}_e) < 9.8 \times 10^{-7}$ and $\mathcal{B}(B^- \to \tau^- \bar{\nu}_\tau) = (1.09 \pm 0.24) \times 10^{-4}$ [52]. The upper bound $\mathcal{B}(\bar{B}^0 \to \pi^+ \tau^- \bar{\nu}_\tau) < 2.5 \times 10^{-4}$ has also been established [54]. We use the $B \to \pi$ form factors given in Appendix C, obtained interpolating the Light-Cone sum rule results at low $q^2$ computed in Refs. [55, 56] with the lattice QCD results at large values of $q^2$ averaged by HFLAG [57]. For the $B \to \rho$ transition we use the form factors in Ref. [58], which update previous Light-Cone sum rule computations [59] and extrapolate the low $q^2$ determination to the full kinematical range.

In the case of $\mu$, the parameter space for the NP couplings, displayed in Fig. 2, is found imposing that the purely leptonic BR is in the range $[2.0, 10.7] \times 10^{-7}$, and that the semileptonic $B \to \pi$ and $B \to \rho$ branching fractions are compatible within $2\sigma$ with measurement. The benchmark point shown in Fig. 2 is chosen in the region of the smallest

$$\chi^2 = \sum_i^3 \left( \frac{B_{i}^{th} - B_{i}^{exp}}{\Delta B_{i}^{exp}} \right)^2$$

for the three modes, varying $|V_{ub}|$ in $[3.5, 4.4] \times 10^{-3}$.

For the $\tau$ modes, due to the smaller number of experimental constraints, we consider a limited parameter space setting $\epsilon_\tau^\nu = 0$ and $\epsilon_\tau^\rho = 0$ from the beginning. The region for $\epsilon_\tau^\rho$ in Fig. 3 (left panel) is constrained imposing the compatibility of $\mathcal{B}(B^- \to \tau^- \bar{\nu}_\tau)$ with measurement. We have checked that $\frac{\mathcal{B}(B^- \to \mu^- \bar{\nu}_\mu)}{\mathcal{B}(B^- \to \tau^- \bar{\nu}_\tau)}$ lies within the experimental range when $\epsilon_\rho^\mu, \epsilon_\rho^\tau$.
4 Observables in $\bar{B} \rightarrow \rho \ell^- \bar{\nu}_\ell$

Figure 2: Allowed regions for the couplings $\epsilon^\mu_V$, $\epsilon^\mu_P$, $\epsilon^\mu_S$ and $\epsilon^\mu_T$. The stars correspond to the benchmark points, chosen in the region of minimum $\chi^2$: $(\text{Re} \{\epsilon^\mu_V\}, \text{Im} \{\epsilon^\mu_V\}) = (0, 0)$, $(\text{Re} \{\epsilon^\mu_P\}, \text{Im} \{\epsilon^\mu_P\}) = (-0.03, -0.02)$, $(\text{Re} \{\epsilon^\mu_T\}, \text{Im} \{\epsilon^\mu_T\}) = (0.12, 0)$ and $(\text{Re} \{\epsilon^\mu_S\}, \text{Im} \{\epsilon^\mu_S\}) = (-0.04, 0)$, with $|V_{ub}| = 3.5 \times 10^{-3}$.

are varied in their ranges. The region for $\epsilon^\tau_T$ (right panel) is obtained imposing the experimental upper bound for $\mathcal{B}(\bar{B}^0 \rightarrow \pi^+ \tau^- \bar{\nu}_\tau)$ together with the limit for $R_\pi = \frac{\mathcal{B}(\bar{B}^0 \rightarrow \pi^+ \tau^- \bar{\nu}_\tau)}{\mathcal{B}(\bar{B}^0 \rightarrow \pi^+ \mu^- \bar{\nu}_\mu)}$. In the wide resulting region we set the range for $\epsilon^\tau_T$, with the parameters for the muon fixed at their benchmark values, then we fix a benchmark point to provide an example of NP effects.

Figure 3: Allowed regions for the couplings $\epsilon^\tau_P$ and $\epsilon^\tau_T$. The stars correspond to the benchmark points chosen setting $\epsilon^\tau_V = 0$ and $\epsilon^\tau_S = 0$: $(\text{Re} \{\epsilon^\tau_P\}, \text{Im} \{\epsilon^\tau_P\}) = (0.01, 0)$ and $(\text{Re} \{\epsilon^\tau_T\}, \text{Im} \{\epsilon^\tau_T\}) = (0.12, 0)$. 
Let us consider the ratios (11) has a zero in the NP, not in SM, whose position $q^2_{0,\rho}$ has a weak form factor.

Figure 4: $\bar{B} \to \rho(\pi\pi)\mu^-\bar{\nu}_\mu$ mode: angular coefficient functions $I_i^\rho(q^2)$ in set A, for SM and NP at the benchmark point. A zero in $I_{2s}^\rho(q^2)$ appears in NP.

We can now compare observables in SM and NP. The angular coefficient functions $I_{1s}^\rho$, $I_{2s}^\rho$, $I_{2c}^\rho$, $I_{5s}^\rho$, and $I_{6s}^\rho$, independent of $\epsilon_P$, are shown in Fig.4, setting $\epsilon_T^\mu$ at benchmark point. The zero in $I_{2s}^\rho(q^2)$ is absent in SM and appears in NP. The other coefficient functions are drawn in Fig.5, and also in this case there is a zero in $I_{6s}^\rho(q^2)$ which is absent in SM. The function $I_{7s}^\tau$ vanishes in SM, and is only sensitive to the imaginary part of the NP couplings; it is shown in Fig.6. The angular functions for the $\tau$ modes are in Fig.7 and 8; $I_7^\tau$ vanishes since at the chosen benchmark point all the NP couplings $\epsilon^\tau$ are real. Also in this mode the coefficient $I_{6c}^\rho$ has a zero not appearing in SM.

The measurement of the angular coefficients functions allows to determine the new couplings. Let us consider the ratios

$$R_{2s/1s}^\rho(q^2) = \frac{I_{2s}^\rho(q^2)}{I_{1s}^\rho(q^2)},$$

$$R_{2s/1s}^{\alpha_1,\parallel}(q^2) = \frac{I_{2s,\parallel}^{\alpha_1}(q^2)}{I_{1s,\parallel}^{\alpha_1}(q^2)},$$

and $R_{2s/1s}^{\alpha_1,\perp} = R_{2c/1c}^{\alpha_1,\perp}$. In SM $R_{2s/1s}^\rho$ is form factor independent. In NP it is still form factor independent in the Large Energy limit, where $I_{2s}^\rho$ and $I_{1s}^\rho$ depend on $\xi_{1s}^\rho$. As shown in Fig.9, the ratio (11) has a zero in the NP, not in SM, whose position $q^2_{0,\rho}$ has a weak form factor.
Observables in $\bar{B} \rightarrow \rho \ell^- \bar{\nu}_\ell$

Figure 5: $\bar{B} \rightarrow \rho(\pi\pi) \mu^- \bar{\nu}_\mu$ mode: angular coefficient functions (set B) $I_{\rho c}^\rho(q^2)$ (left), $I_\rho^\rho(q^2)$ (middle) and $I_{6c}^\rho(q^2)$ (right) for SM and NP at the benchmark point.

The effect and depends only on $|\epsilon_T^\mu|$. In the Large Energy Limit we have

$$|\epsilon_T^\mu|^2 = \frac{q_{0,\rho}^2 \lambda(m_B^2, m_\rho^2, q_{0,\rho}^2) + 2m_B^2 m_\rho^2}{16m_B^2 \lambda(m_B^2, m_\rho^2, q_{0,\rho}^2) + 2q_{0,\rho}^2 m_\rho^2}. \quad (13)$$

Analogously, for the $(a_1)_\parallel$ mode (and for $(a_1)_\perp$ considering $R_{2c/1c}$) we have:

$$|\epsilon_T^{a_1}|^2 = \frac{q_{0,a_1}^2 \lambda(m_B^2, m_{a_1}^2, q_{0,a_1}^2) + 2m_B^2 m_{a_1}^2}{16m_B^2 \lambda(m_B^2, m_{a_1}^2, q_{0,a_1}^2) + 2q_{0,a_1}^2 m_{a_1}^2}. \quad (14)$$

The positions of the zeros in two modes are related, see Fig.[10] and their independent measurement would provide a connection with the tensor operator.

Another suitable quantity is the angular coefficient function $I_6^\rho$ shown in the right panel of Fig.[5] in SM and NP, which is sensitive to $\epsilon_V, \epsilon_P, \epsilon_T$. At our benchmark point $\epsilon_V \simeq 0$, hence

Figure 6: $\bar{B} \rightarrow \rho(\pi\pi) \mu^- \bar{\nu}_\mu$ mode: angular coefficient function $I_7^\rho(q^2)$ in NP with the pseudoscalar operator at the benchmark point.
Observables in $\bar{B} \to \rho \ell^- \bar{\nu}_\ell$

we keep only the $\epsilon_P$ and $\epsilon_T$ dependence:

\[
(I_{6c}^\rho)_{\epsilon V \approx 0} = (-2H_{6c}^\rho) \left[ 4H_0^\rho m_\ell^2 - \text{Re}[\epsilon_T] H_L^{NP,\rho} m_\ell \sqrt{q^2} + 4\text{Re}[\epsilon_P] H_0^\rho \frac{m_\ell}{m_b + m_u} q^2 - H_L^{NP,\rho} \text{Re}[\epsilon_P M] \frac{(q^2)^{3/2}}{m_b + m_u} \right].
\]

Figure 8: $\bar{B} \to \rho(\pi\pi) \tau^- \bar{\nu}_\tau$ mode: angular coefficient functions (set B) $I_{6c}^\rho(q^2)$ (left), $I_5^\rho(q^2)$ (middle) and $I_{6c}^\rho(q^2)$ (right) for SM and NP at the benchmark point.
4 Observables in $\bar{B} \rightarrow \rho \ell^- \bar{\nu}_\ell$

Figure 9: Ratio $R_{2s/1s}^0$ in (11) for the modes $\bar{B} \rightarrow \rho(\pi\pi) \mu^- \bar{\nu}_\mu$ (left) and $\bar{B} \rightarrow \rho(\pi\pi) \tau^- \bar{\nu}_\mu$ (right), in SM and NP with tensor operator at the benchmark point. The dashed lines correspond to the Large Energy limit result (extrapolated to the full $q^2$ range).

Considering the $q^2$-dependence of the helicity amplitudes in Appendix B we have the following possibilities:

- No NP, i.e. $\epsilon_P = \epsilon_T = 0$. In this case, $I_{6c}^\rho = -8H_t^\rho H_0^\rho m_\ell^2$ does not have a zero, as shown in Fig. 5 (right panel).

- NP with $\epsilon_T = 0$ and $\epsilon_P \neq 0$. This gives:

$$\left. (I_{6c}^\rho) \right|_{\epsilon_T \approx 0} = (-8H_t^\rho H_0^\rho m_\ell) \left[ m_\ell + \text{Re}[\epsilon_P] \frac{q^2}{m_b + m_u} \right],$$

with a zero at

$$q_0^2 = -\frac{m_b + m_u}{m_\ell} \frac{1}{\text{Re}[\epsilon_P]}.$$

This position is form factor independent, its measurement would result in a determination of $\text{Re}[\epsilon_P]$. In the left panel of Fig. 11 we show $I_{6c}^\rho$ enlarging the region where the zero is present for the benchmark $\text{Re}[\epsilon_P]$, and in the middle panel we display $q_0^2$ versus $\text{Re}[\epsilon_P]$ in the whole range for the coupling.

Figure 10: Relation between the position of the zeroes $q_0^2$ of the ratios (11) and (12) for the $B \rightarrow \rho$ and $B \rightarrow a_1$ modes, respectively.
Integrating the 4d differential decay distribution several observables can be constructed.

- **q^2-dependent forward-backward (FB) lepton asymmetry**

  \[ A_{FB}(q^2) = \frac{1}{6I_{1c}^0 + 12I_{1s}^0 - 2I_{2c}^0 - 4I_{2s}^0} \left( 3I_{6c}^0 + 2I_{6s}^0 \right) \]

  which is given in terms of the angular coefficient functions as

  \[ A_{FB}(q^2) = \frac{3(I_{6c}^0 + 2I_{6s}^0)}{6I_{1c}^0 + 12I_{1s}^0 - 2I_{2c}^0 - 4I_{2s}^0}. \]

- **Transverse forward-backward (TFB) asymmetry**, the FB asymmetry for transversely polarized \( \rho \), reading in terms of the angular coefficient functions as

  \[ A_{TFB}(q^2) = \frac{3I_{6s}^0}{6I_{1s}^0 - 2I_{2s}^0}. \]

For \( \ell = \mu \) the asymmetries \( A_{FB} \) and \( A_{TFB}^T \) are shown in Fig.12 for \( \ell = \tau \) they are in Fig.14. In case of NP the zero of \( A_{FB} \) in the \( \tau \) mode is shifted. Moreover, \( A_{TFB}^T \) is very sensitive to the new operators, and in the case of \( \tau \) it has a zero not present in SM. This is related to \( I_{6s}^0 \), with a zero in NP and not in SM.

- Observables sensitive to the \( \rho \) polarization. We consider the differential branching ratio for longitudinally (L) and transversely (T) polarized \( \rho \) as a function of \( q^2 \) or of one of the two angles \( \theta, \theta_V \): \( dB_{L(T)/dq^2}, dB'_{L(T)/d\cos\theta} \) and \( dB_{L(T)/d\cos\theta_V} \). These observables are depicted for \( \ell = \mu \) and for \( \ell = \tau \) in Fig.13 and Fig.15, respectively.
Figure 12: $\bar{B} \rightarrow \rho \mu^- \bar{\nu}_\mu$ mode: forward-backward lepton asymmetry \(17\) and \(19\) in SM and NP at the benchmark point.

Among all these quantities, the ones corresponding to transversely polarized $\rho$ depend only on $\epsilon_T$, as stressed in the legendae of the corresponding figures.

Integrating the distributions, we obtain in SM the longitudinal and transverse polarization fractions and the branching fractions:

$$F_L(\bar{B} \rightarrow \rho \mu^- \bar{\nu}_\mu)|_{SM} = 0.52 \pm 0.15$$

$$F_T(\bar{B} \rightarrow \rho \mu^- \bar{\nu}_\mu)|_{SM} = 0.48 \pm 0.11$$

$$\mathcal{B}(\bar{B}^0 \rightarrow \rho^+ \mu^- \bar{\nu}_\mu)|_{SM} = (3.37 \pm 0.52) \times 10^{-4} \times \left( \frac{|V_{ub}|}{0.0035} \right)^2,$$  \hspace{1cm} (20)

$$F_L(\bar{B} \rightarrow \rho \tau^- \bar{\nu}_\tau)|_{SM} = 0.50 \pm 0.13$$

$$F_T(\bar{B} \rightarrow \rho \tau^- \bar{\nu}_\tau)|_{SM} = 0.50 \pm 0.12$$

$$\mathcal{B}(\bar{B}^0 \rightarrow \rho^+ \tau^- \bar{\nu}_\tau)|_{SM} = (1.80 \pm 0.25) \times 10^{-4} \times \left( \frac{|V_{ub}|}{0.0035} \right)^2.$$  \hspace{1cm} (21)

For the $B \rightarrow \pi$ mode we have:

$$\mathcal{B}(\bar{B}^0 \rightarrow \pi^+ \mu^- \bar{\nu}_\mu)|_{SM} = (1.5 \pm 0.1) \times 10^{-4} \times \left( \frac{|V_{ub}|}{0.0035} \right)^2$$

$$\mathcal{B}(\bar{B}^0 \rightarrow \pi^+ \tau^- \bar{\nu}_\tau)|_{SM} = (0.92 \pm 0.06) \times 10^{-4} \times \left( \frac{|V_{ub}|}{0.0035} \right)^2.$$ \hspace{1cm} (22)

The ratios

$$R_\pi = \frac{\mathcal{B}(\bar{B} \rightarrow \pi \tau^- \bar{\nu}_\tau)}{\mathcal{B}(\bar{B} \rightarrow \pi \ell^- \bar{\nu}_\ell)}$$

$$R_\rho = \frac{\mathcal{B}(\bar{B} \rightarrow \rho \tau^- \bar{\nu}_\tau)}{\mathcal{B}(\bar{B} \rightarrow \rho \ell^- \bar{\nu}_\ell)}$$ \hspace{1cm} (23)
are modified by the New Physics operators in (1). The results in SM and NP are collected in Table 1. The deviations are correlated when the new operators are included in the effective Hamiltonian and, as shown in Fig. 16, large effects are possible in corners of the parameter space of the new effective couplings.

Concerning $R_\pi$ in SM, the value $R_\pi = 0.641(17)$ is obtained using lattice form factors at large $q^2$ [60], the range $[0.654, 0.764]$ is found in [61], $R_\pi = 0.7$ together with $R_\rho \simeq 0.573$ is found using form factors computed in pQCD [62], $R_\pi \simeq 0.731$ and $R_\rho \simeq 0.585$ are quoted in [63]. The effect of a new charged Higgs reduces the SM result for $R_\pi$ and $R_\rho$ [64]. Considering a single NP operator per time, values for $R_\pi$ up to $\simeq 4$ are obtained in [61], the range $[0.5, 1.38]$ is found in [62], while the inclusion only of the pseudoscalar and scalar operators in the effective Hamiltonian gives $R_\pi \in [0.5, 1.2]$ [48].

|       | SM       | NP (benchmark point) |
|-------|----------|----------------------|
| $R_\pi$ | $0.60 \pm 0.05$ | $0.75 \pm 0.06$ |
| $R_\rho$ | $0.53 \pm 0.11$ | $0.49 \pm 0.06$ |

Table 1: Ratios $R_\pi$ and $R_\rho$ in Eq. (23) in SM and in NP at the benchmark point.
The numerical analysis of $\bar{B} \to a_1(\rho\pi)\ell^-\bar{\nu}_\ell$, in SM and in NP extensions for the same values of the couplings $\epsilon_{V,S,T}$, using the expressions for the angular coefficient functions and their relation with the form factors. From an experimental viewpoint, exclusive hadronic $B$ decays involving $a_1(1260)$ have been analyzed at the B factories, considering the dominant mode $a_1 \to \rho\pi$ [65]. The semileptonic mode is within the reach of the new experiments. From the theoretical viewpoint, the study of the semileptonic mode requires an assessment of the accuracy of the hadronic quantities. The $\bar{B} \to a_1$ form factors have been evaluated by different methods [66–74], however no comparative evaluation of their uncertainties has been carried out so far. We defer this study to a dedicated work. Here it is important to stress the synergy between the $\bar{B} \to \rho$ and $\bar{B} \to a_1$ modes in providing possible evidences of NP and in characterizing its realization. We outline the steps for such an analysis.

- The presence of the tensor structure in the effective Hamiltonian can be established independently of the presence of the other operators, looking at deviations of the observables that depend only on $\epsilon_T$. These are the observables involving transversely polarized $\rho$ and $a_1$. Moreover, it is possible to tightly constrain $|\epsilon_T|$ looking at the zero of the ratios defined in Eqs. (11), (12). A correlation between the position of the zero in the $\rho$ and $a_1$ modes should be observed, as in Fig. [10].

- If a pseudoscalar operator is present, without other NP structures, deviations should be observed in leptonic $B$ decays and in the semileptonic decay to $\rho$, not in semileptonic decays to $\pi$ and $a_1$. Determining the position of the zero in $I^\rho_{6c,||}$ allows to constrain Re[$\epsilon_P$]. Zeros should not be present in $I^{a_1}_{6c,||}$.

- If a scalar operator is present, without additional NP structures, deviations should be
6 Conclusions

The questions presented by the anomalies in \( b \to c \) semileptonic modes must be faced also in the suppressed \( b \to u \) modes, for which precise measurements are expected in the near future. We have enlarged the SM effective Hamiltonian including additional operators, and considered the effects in \( \bar{B} \to \rho(\pi\pi)\ell^-\bar{\nu}_\ell \) and \( \bar{B} \to a_1(\rho\pi)\ell^-\bar{\nu}_\ell \). We have constructed the 4d differential distribution for both the modes, finding that they are sensitive to different NP operators: the two modes provide complementary information about the SM extension. We have constrained the parameter space of the new coupling constants, and considered the impact on \( \bar{B} \to \rho\ell^-\bar{\nu}_\ell \). Among the various observables, we have found that a few angular coefficients have zeroes absent in SM, whose observation would provide evidence of the NP

observed in semileptonic \( B \) decays to \( \pi \) and \( a_1 \). In particular, a zero would be present in \( I_{6c,||}^{a_1} \), not in \( I_{6c}^{\rho} \).

- The simultaneous presence of all the operators would manifest in a more involved pattern of deviations. However, such deviations are correlated in the two modes, and the pattern of correlation can be used to clarify the role of the various new terms in \([1]\).

- Improved data on modes with final \( \tau \) provide new tests of LFU. For example, the measurement of \( R_\rho \) and \( R_\pi \) would give a hint on the relative sign of \( \text{Re}[\epsilon_T^\mu] \) and \( \text{Re}[\epsilon_T^\tau] \), as shown in Fig.16. In case of \( a_1 \) the reconstruction of the modes with final \( \tau \) is challenging.

Figure 15: \( \bar{B} \to \rho^- \bar{\nu}_\ell \) mode: distributions \( d\bar{B}/dq^2 \), \( d\bar{B}/d\cos\theta \) and \( d\bar{B}/d\cos\theta_V \) (first line) and \( d\bar{B}_T/dq^2 \), \( d\bar{B}_T/d\cos\theta \) and \( d\bar{B}_T/d\cos\theta_V \) (second line), with \( \bar{B} = \mathcal{B}(\rho \to \pi\pi) \), in SM and NP at the benchmark point.
A Hadronic matrix elements

For $M_u = \pi^+$ meson, the weak matrix elements are written in terms of form factors as follows:

\[
\langle \pi(p')|\bar{u}\gamma_\mu b|\bar{B}(p)\rangle = f_{B\rightarrow\pi}^B(q^2) \left[p_\mu + p'_\mu - \frac{m_B^2 - m_\pi^2}{q^2} q_\mu\right] + f_{0B\rightarrow\pi}^B(q^2) \frac{m_B^2 - m_\pi^2}{q^2} q_\mu
\]

\[
\langle \pi(p')|\bar{u}b|\bar{B}(p)\rangle = f_{S}^{B\rightarrow\pi}(q^2)
\]

\[
\langle \pi(p')|\bar{u}\sigma_{\mu\nu} b|\bar{B}(p)\rangle = -i \frac{2f_{T}^{B\rightarrow\pi}(q^2)}{m_B + m_\pi} [p_\mu p'_\nu - p_\nu p'_\mu]
\]

\[
\langle \pi(p')|\bar{u}\sigma_{\mu\nu}\gamma_5 b|\bar{B}(p)\rangle = -\frac{2f_{P}^{B\rightarrow\pi}(q^2)}{m_B + m_\pi} \epsilon_{\mu\nu\alpha\beta} p^\alpha p'^{\beta},
\]

where $\epsilon_{0123} = +1$. The relation $f_{S}^{B\rightarrow\pi}(q^2) = \frac{m_B^2 - m_\pi^2}{m_b - m_u} f_{0B\rightarrow\pi}^B(q^2)$ holds.

For $M_u = \rho^+$ the various matrix elements, expressed in terms of form factors (with $\epsilon$ the

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Figure 16: Correlation between $R_\rho$ and $R_\pi$ in Eq. (23) with only the tensor operator added to the SM effective Hamiltonian. The colors correspond to the different signs of $\text{Re}(\epsilon_\rho^\mu)$ and $\text{Re}(\epsilon_\pi^\tau)$ in the full range of the parameter space. The red and brown points are the SM and NP result at the benchmark point, respectively.
ρ polarization vector), read:

\[ \langle \rho(p', \epsilon) | \bar{u} \gamma_\mu (1 - \gamma_5) b \mid B(p) \rangle = -\frac{2V^{B\rightarrow\rho}(q^2)}{m_B + m_\rho} i \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} p^\alpha p'^\beta \]

\[ - \left\{ (m_B + m_\rho) \left[ \epsilon^*_\mu - \frac{(\epsilon^* \cdot q)}{q^2} q_\mu \right] \right. \left. A_1^{B\rightarrow\rho}(q^2) \right. \]

\[ - \frac{(\epsilon^* \cdot q)}{m_B + m_\rho} \left[ (p + p')_\mu - \frac{m_B^2 - m_\rho^2}{q^2} q_\mu \right] A_2^{B\rightarrow\rho}(q^2) \]

\[ + (\epsilon^* \cdot q) \frac{2m_\rho}{q^2} q_\mu A_0^{B\rightarrow\rho}(q^2) \right\} \text{ ,} \quad (A.2) \]

with the condition \( A_0^{B\rightarrow\rho}(0) = \frac{m_B + m_\rho}{2m_\rho} A_1^{B\rightarrow\rho}(0) - \frac{m_B - m_\rho}{2m_\rho} A_2^{B\rightarrow\rho}(0) \), and

\[ \langle \rho(p', \epsilon) | \bar{u} \sigma_{\mu\nu} b \mid B(p) \rangle = -\frac{2m_\rho}{m_B + m_\mu} (\epsilon^* \cdot q) A_0^{B\rightarrow\rho}(q^2) \quad (A.3) \]

\[ \langle \rho(p', \epsilon) | \bar{u} \sigma_{\mu\nu} \gamma_5 b \mid B(p) \rangle = T_0^{B\rightarrow\rho}(q^2) \frac{\epsilon^* \cdot q}{(m_B + m_\rho)^2} \epsilon_{\mu\nu\alpha\beta} p^\alpha p'^\beta \]

\[ + T_1^{B\rightarrow\rho}(q^2) \epsilon_{\mu\nu\alpha\beta} p^\alpha p'^\beta A_2^{B\rightarrow\rho}(q^2) \epsilon^{*\nu} \] \( \text{,} \quad (A.4) \)

\[ \langle \rho(p', \epsilon) | \bar{u} \sigma_{\mu\nu} \gamma_5 b \mid B(p) \rangle = i T_0^{B\rightarrow\rho}(q^2) \frac{\epsilon^* \cdot q}{(m_B + m_\rho)^2} (p_\mu p'_\nu - p_\nu p'_\mu) \]

\[ + i T_1^{B\rightarrow\rho}(q^2) (p_\mu \epsilon^*_\nu - \epsilon^*_\mu p_\nu) + i T_2^{B\rightarrow\rho}(q^2) (p'_\mu \epsilon^*_\nu - \epsilon^*_\mu p'_\nu) \text{ .} \quad (A.5) \]

For \( M_u = a_1^+ \) we use the decomposition:

\[ \langle a_1(p', \epsilon) | \bar{u} \gamma_\mu (1 - \gamma_5) b \mid B(p) \rangle = \frac{2A^{B\rightarrow a_1}(q^2)}{m_B + m_{a_1}} i \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} p^\alpha p'^\beta \]

\[ + \left\{ (m_B + m_{a_1}) \left[ \epsilon^*_\mu - \frac{(\epsilon^* \cdot q)}{q^2} q_\mu \right] V_1^{B\rightarrow a_1}(q^2) \right. \]

\[ - \frac{(\epsilon^* \cdot q)}{m_B + m_{a_1}} \left[ (p + p')_\mu - \frac{m_B^2 - m_{a_1}^2}{q^2} q_\mu \right] V_2^{B\rightarrow a_1}(q^2) \]

\[ + (\epsilon^* \cdot q) \frac{2m_{a_1}}{q^2} q_\mu V_0^{B\rightarrow a_1}(q^2) \right\} \text{ .} \quad (A.6) \]
with the condition $V_0^{B \to a_1}(0) = \frac{m_B + m_{a_1}}{2m_{a_1}} V_1^{B \to a_1}(0) - \frac{m_B - m_{a_1}}{2m_{a_1}} V_2^{B \to a_1}(0)$, and

$$\langle a_1(p', \epsilon)|\bar{u}b|\bar{B}(p)\rangle = \frac{2m_{a_1}}{m_b - m_u} (\epsilon^* \cdot q) V_0^{B \to a_1}(q^2) \quad (A.7)$$

$$\langle a_1(p', \epsilon)|\bar{u}\sigma_{\mu\nu}b|\bar{B}(p)\rangle = i T_0^{B \to a_1}(q^2) \frac{\epsilon^* \cdot q}{(m_B + m_{a_1})^2} (p_\mu p'_\nu - p_\nu p'_\mu) + i T_1^{B \to a_1}(q^2) (p'_\mu \epsilon^* - \epsilon^* p'_\mu) \quad (A.8)$$

$$\langle a_1(p', \epsilon)|\bar{u}\sigma_{\mu\nu}\gamma_5 b|\bar{B}(p)\rangle = T_0^{B \to a_1}(q^2) \frac{\epsilon^* \cdot q}{(m_B + m_{a_1})^2} \epsilon_{\mu\nu\alpha\beta} p^\alpha p^\beta + T_1^{B \to a_1}(q^2) \epsilon_{\mu\nu\alpha\beta} p^\alpha \epsilon^{*\beta} \quad (A.9)$$

In the large energy (large recoil) limit for the light meson the weak matrix elements can be expressed in terms of a smaller number of form factors. We define $E = \frac{m_B^2 + m^2 - q^2}{2m_B}$ the light meson energy in the $B$ rest-frame, and $m$ the light meson mass. The $B$ four-velocity is defined from $p = m_B v$, and $n_-$ is a light-like four-vector along $p'$: $p' = E n_-$. In the large recoil configuration, for $E \approx \frac{m_B}{2}$, the light quark $u$ carries almost all the momentum of the light meson: $p'_u = E (n_-)_\mu + k_\mu$, with the residual momentum $k \ll E$. Using, e.g., a eikonal formulation of the weak current, this allows to express the form factors in terms of universal functions $\xi_i(E)$ \cite{49,50}. For $B \to \pi$, a single form factor $\xi_\pi(E)$ parametrizes the matrix elements,

$$\langle \pi(p')|\bar{u}\gamma_\mu b|\bar{B}(p)\rangle = 2E \xi_\pi(E)(n_-)_\mu$$

$$\langle \pi(p')|\bar{u}\sigma_{\mu\nu}q^\nu b|\bar{B}(p)\rangle = 2iE \xi_\pi(E) \left[ (m_B - E)(n_-)_\mu - m_B v_\mu \right]. \quad (A.10)$$

For $B \to \rho$ there are two independent form factors, $\xi_\perp^\rho(E)$ and $\xi_\parallel^\rho(E)$,

$$\langle \rho(p', \epsilon)|\bar{u}\gamma_\mu b|\bar{B}(p)\rangle = 2iE \xi_\perp^\rho(E) \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu}(n_-)^\alpha v^\beta$$

$$\langle \rho(p', \epsilon)|\bar{u}\gamma_\mu\gamma_5 b|\bar{B}(p)\rangle = 2E \left \{ \xi_\perp^\rho(E) \left[ \epsilon^*_\mu - (\epsilon^* \cdot v)(n_-)_\mu \right] + \xi_\parallel^\rho(E) \epsilon^* \cdot v \right \}$$

$$\langle \rho(p', \epsilon)|\bar{u}\sigma_{\mu\nu}q^\nu b|\bar{B}(p)\rangle = 2Em_B \xi_\perp^\rho(E) \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} v^\alpha (n_-)^\beta \quad (A.11)$$

$$\langle \rho(p', \epsilon)|\bar{u}\sigma_{\mu\nu}\gamma_5 q^\nu b|\bar{B}(p)\rangle = -2iE \left \{ \xi_\perp^\rho(E) m_B \left[ \epsilon^*_\mu - (\epsilon^* \cdot v)(n_-)_\mu \right] + \xi_\parallel^\rho(E) \epsilon^* \cdot v \right \}.$$
and two independent $\xi_{1}^{a_1}(E)$ and $\xi_{1}^{a_1}(E)$ for factors are also involved for $a_1$:
\[
\begin{align*}
\langle a_1(p', e)|\bar{u}\gamma_{\mu}\gamma_5 b|\bar{B}(p)\rangle &= -2i E \xi_{1}^{a_1}(E)\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}(n_-)^{\alpha\beta} \\
\langle a_1(p', e)|\bar{u}\gamma_{\mu}b|\bar{B}(p)\rangle &= -2E \left\{ \xi_{1}^{a_1}(E)\left[ \epsilon_{\mu} - (\varepsilon^* \cdot v)(n_-)_{\mu} \right] + \xi_{1}^{a_1}(E)(\varepsilon^* \cdot v)(n_-)_{\mu} \right\} \\
\langle a_1(p', e)|\bar{u}\sigma_{\mu\nu}q'r_{\gamma}b|\bar{B}(p)\rangle &= 2Em_{B}\xi_{1}^{a_1}(E)\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}v^{\alpha}(n_-)^{\beta} \\
\langle a_1(p', e)|\bar{u}\sigma_{\mu\nu}q'b|\bar{B}(p)\rangle &= -2iE \left\{ \xi_{1}^{a_1}(E)m_{B}\left[ \epsilon_{\mu} - (\varepsilon^* \cdot v)(n_-)_{\mu} \right] + \xi_{1}^{a_1}(E)(\varepsilon^* \cdot v)(n_-)_{\mu} \right\}.
\end{align*}
\]
\]

Comparing Eqs. (A.1)-(A.8) with (A.10)-(A.12), the relations among the form factors and their large energy limit expressions can be worked out. For $B \to \pi$ they are:
\[
\begin{align*}
f_{+}^{B\to\pi}(q^2) &= \frac{m_{B}}{2E} f_{0}^{B\to\pi}(q^2) = \frac{m_{B}}{m_{B} + m_{\pi}} f_{T}^{B\to\pi}(q^2) = \xi_\pi(E), \quad (A.13)
\end{align*}
\]
for $B \to \rho$:
\[
\begin{align*}
\frac{m_{B}}{m_{B} + m_{\rho}} V_{B\to\rho}(q^2) &= \frac{m_{B} + m_{\rho}}{2E} A_{1}^{B\to\rho}(q^2) = \xi_{1}^{\rho}(E) \\
\frac{m_{\rho}}{E} A_{0}^{B\to\rho}(q^2) &= \frac{m_{B} + m_{\rho}}{2E} A_{1}^{B\to\rho}(q^2) - \frac{m_{B} - m_{\rho}}{m_{B}} A_{2}^{B\to\rho}(q^2) = \xi_{1}^{\rho}(E) \\
T_{1}^{B\to\rho}(q^2) &= 0 \\
T_{2}^{B\to\rho}(q^2) &= 2\xi_{1}^{\rho}(E) \\
T_{0}^{B\to\rho}(q^2) &= 2\xi_{\parallel}^{\rho}(E), \quad (A.14)
\end{align*}
\]
and for $B \to a_1$:
\[
\begin{align*}
\frac{m_{B}}{m_{B} + m_{a_1}} A_{B\to a_1}(q^2) &= \frac{m_{B} + m_{a_1}}{2E} V_{1}^{B\to a_1}(q^2) = \xi_{1}^{a_1}(E) \\
\frac{m_{a_1}}{E} V_{0}^{B\to a_1}(q^2) &= \frac{m_{B} + m_{a_1}}{2E} V_{1}^{B\to a_1}(q^2) - \frac{m_{B} - m_{a_1}}{m_{B}} V_{2}^{B\to a_1}(q^2) = \xi_{\parallel}^{a_1}(E) \\
T_{1}^{B\to a_1}(q^2) &= 0 \\
T_{2}^{B\to a_1}(q^2) &= 2\xi_{1}^{a_1}(E) \\
T_{0}^{B\to a_1}(q^2) &= 2\xi_{\parallel}^{a_1}(E). \quad (A.15)
\end{align*}
\]

The functions $\xi_{\pi}$, $\xi_{\parallel}^{\rho}$ and $\xi_{\parallel}^{a_1}$ have been determined by light-cone QCD sum rules within the Soft Collinear Effective Theory, using $B$ meson light-cone distribution amplitudes [75–77].
B Angular coefficient functions

Here we collect the expressions of the angular coefficient functions in Eqs. (7-11). The general form of the $\bar{B} \rightarrow V \ell^- \bar{\nu}_\ell$ decay amplitude, with $V = \rho$ and $a_1$,

$$A(\bar{B} \rightarrow V \ell^- \bar{\nu}_\ell) = \frac{G_F}{\sqrt{2}} V_{ub} \left[ (1 + \epsilon_V) H^{SM}_\mu L^{SM\mu} + \epsilon^\rho_S H^{NP,S} L^{NP,S} + \epsilon^\rho_P H^{NP,P} L^{NP,P} + \epsilon^\rho_T H^{NP,T} L^{NP,T\mu\nu} \right],$$

is given in terms of the quark current matrix elements

$$H^{SM}_\mu(m) = \langle p_{\nu}, \epsilon(m) | \bar{u} \gamma_\mu (1 - \gamma_5) b | \bar{B}(p_B) \rangle = \epsilon^{*\alpha}(m) T_{\mu\alpha}$$

$$H^{NP,S}(m) = \langle p_{\nu}, \epsilon(m) | \bar{u} b | \bar{B}(p_B) \rangle = \epsilon^{*\alpha}(m) T_{\mu\alpha}^{NP,S}$$

$$H^{NP,P}(m) = \langle p_{\nu}, \epsilon(m) | \bar{u} \gamma_5 b | \bar{B}(p_B) \rangle = \epsilon^{*\alpha}(m) T_{\mu\alpha}^{NP,P}$$

$$H^{NP,T}(m) = \langle \bar{D}^*(p_{D\gamma}, \epsilon(m)) | \sigma^{\mu\nu} (1 - \gamma_5) b | \bar{B}(p_B) \rangle = \epsilon^{*\alpha}(m) T_{\mu\alpha}^{NP,T}$$

and of the lepton currents

$$L^{SM\mu} = \bar{\ell} \gamma_\mu (1 - \gamma_5) \nu_\ell$$

$$L^{NP,S} = L^{NP,P} = \bar{\ell} (1 - \gamma_5) \nu_\ell$$

In SM one can relate the helicity amplitudes for the $V$ polarization states to the polarizations of the virtual $W(q, \bar{\epsilon})$. In the lepton pair rest-frame they are:

$$\bar{\epsilon}_\pm = \frac{1}{\sqrt{2}} (0, 1, \pm i, 0) , \quad \bar{\epsilon}_0 = (0, 0, 0, 1) , \quad \bar{\epsilon}_t = (1, 0, 0, 0).$$

This allows to define the amplitudes

$$H_m = \bar{\epsilon}_m^{*\alpha} \epsilon_m T_{\mu\alpha} , \quad (m = 0, \pm)$$

$$H_t = \bar{\epsilon}_t^{*\alpha} \epsilon_0 T_{\mu\alpha} , \quad (m = t) ,$$

which can be expressed in terms of the form factors in (A.2) and (A.6):

$$H_0^\rho = \frac{(m_B + m_\rho)^2 (m_B^2 - m_\rho^2 - q^2) A_1(q^2) - \lambda(m_B^2, m_\rho^2, q^2) A_2(q^2)}{2m_\rho (m_B + m_\rho) \sqrt{q^2}}$$

$$H_\pm^\rho = \frac{(m_B + m_\rho)^2 A_1(q^2) \mp \sqrt{\lambda(m_B^2, m_\rho^2, q^2)} V(q^2)}{m_B + m_\rho}$$

$$H_t^\rho = -\sqrt{\lambda(m_B^2, m_\rho^2, q^2)} A_0(q^2)$$
The expressions for $H_0^{a_1}$ in terms of the couplings $m_B$ and $m_\rho$, where $|B\rangle$ is the Angular coefficient functions.

No new definitions are needed in the case of $S$ and $P$ operators, since their matrix elements involve the same form factors as in SM. For the NP tensor operator one defines [32]:

$$H_{NP,\rho}^{a_1} = \frac{1}{\sqrt{q^2}} \left\{ \begin{array}{l} m_B^2 - m_\rho^2 + \lambda^{1/2}(m_B^2, m_\rho^2, q^2) \left( T_1^{B\to\rho} + T_2^{B\to\rho} \right) + q^2 \left( T_1^{B\to\rho} - T_2^{B\to\rho} \right) \\ 0 \end{array} \right\}$$

$$H_{NP,\rho}^{a_1} = \frac{1}{\sqrt{q^2}} \left\{ \begin{array}{l} m_B^2 - m_\rho^2 - \lambda^{1/2}(m_B^2, m_\rho^2, q^2) \left( T_1^{B\to\rho} + T_2^{B\to\rho} \right) + q^2 \left( T_1^{B\to\rho} - T_2^{B\to\rho} \right) \\ 0 \end{array} \right\}$$

$$H_{L}^{NP,\rho} = 4 \left\{ \begin{array}{l} \lambda(m_B^2, m_\rho^2, q^2) \left( T_0^{B\to\rho} + 2m_B^2 + m_\rho^2 - q^2 \right) T_1^{B\to\rho} + 4m_\rho T_2^{B\to\rho} \\ 0 \end{array} \right\} .$$

The expressions for $H_{NP,\rho}^{a_1}$ are obtained replacing $m_\rho \to m_{a_1}$ and $T_i^{B\to\rho} \to T_i^{B\to a_1}$.

For the decay $\bar{B} \to a_1(\rho\pi)\ell^-\bar{\nu}_\ell$ we define the $\rho$ helicity amplitudes $A_1$, $A_{-1}$, $A_0$ for $\lambda = +1, -1, 0$. Writing the matrix element

$$\langle \rho(p_\rho, \eta)(p_\pi)|a_1(p', \epsilon)\rangle = g_1(\epsilon \cdot \eta^*)(p' \cdot p_\rho) + g_2(\epsilon \cdot p_\rho)(p' \cdot \eta^*)$$

in terms of the couplings $g_1$ and $g_2$, we have:

$$\Gamma(a_1 \to \rho\pi) = \frac{|\vec{p}_\rho|}{24\pi m_{a_1}^2} \left( \tilde{\Gamma}_\perp + \tilde{\Gamma}_\parallel \right) ,$$

where $|\vec{p}_\rho| = \lambda^{1/2}(m_{a_1}, m_\rho, m_{\pi})$ and

$$\tilde{\Gamma}_\perp = 2|A_1|^2 = 2g_1^2 m_{a_1}^2 (m_\rho^2 + |\vec{p}_\rho|^2)$$

$$\tilde{\Gamma}_\parallel = |A_0|^2 = \frac{m_{a_1}^2}{m_\rho^2} \left[ (m_\rho^2 + |\vec{p}_\rho|^2) g_1 + |\vec{p}_\rho|^2 g_2 \right] .$$

The branching ratios for $\rho$ longitudinally and transversely polarized, appearing in the factors $N_{a_1}^{\rho,(\perp)}$ in Eq.[8], read:

$$\mathcal{B}(a_1 \to \rho_{(\perp)}\pi) = \frac{1}{\Gamma(a_1)} \frac{|\vec{p}_\rho|}{24\pi m_{a_1}^2} \cdot \tilde{\Gamma}_\parallel .$$
Table 2: Angular coefficient functions in the 4d $\bar{B} \to \rho(\pi\pi)\ell^-\bar{\nu}_\ell$ decay distribution, Eq. (7), in SM.

| $I^i_i$ | $I^\text{SM}_i$ |
|---------|----------------|
| $I^0_{1s}$ | $\frac{1}{2}(H_+^2 + H_-^2)(m_\ell^2 + 3q^2)$ |
| $I^0_{1c}$ | $4m_\ell^2H_0^2 + 2H_0^2(m_\ell^2 + q^2)$ |
| $I^0_{2s}$ | $-\frac{1}{2}(H_+^2 + H_-^2)(m_\ell^2 - q^2)$ |
| $I^0_{2c}$ | $2H_0^2(m_\ell^2 - q^2)$ |
| $I^0_3$ | $2H_+H_- (m_\ell^2 - q^2)$ |
| $I^0_4$ | $H_0(H_+ + H_-) (m_\ell^2 - q^2)$ |
| $I^0_5$ | $-2H_+(H_+ + H_-)m_\ell^2 - 2H_0(H_+ - H_-)q^2$ |
| $I^0_{6s}$ | $2(H_+^2 - H_-^2)q^2$ |
| $I^0_{6c}$ | $-8H_0H_0m_\ell^2$ |
| $I^0_7$ | 0 |

**C $B \to \pi$ form factors and other parameters**

For the $B \to \pi$ form factors defined in (A.1) we use the parametrization\(^{78}\)

$$f_{+,T}(t) = \frac{1}{1 - \frac{q^2}{m_{\text{pole}}^2}} \sum_{n=0}^{N-1} a_n \left[ z(t)^n - \frac{n}{N} (-1)^{n-N} z(t)^N \right]$$

$$f_0(t) = \sum_{n=0}^{N-1} a_n z(t)^n , \quad \text{(C.1)}$$

expressed as a truncated series in the variable

$$z(t) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}. \quad \text{(C.2)}$$

In this expression $t_+ = (m_B + m_\pi)^2$, and $t_0$ is chosen at the value $t_0 = (m_B + m_\pi) \left( \sqrt{m_B} - \sqrt{m_\pi} \right)^2$. For $\bar{B} \to \pi \mu^- \bar{\nu}_\mu$ the kinematic range is $0.279 \leq z \leq 0.283$, for $\bar{B} \to \pi \tau^- \bar{\nu}_\tau$ it is $-0.279 \leq z \leq 0.257$. The mass of the pole in $f_{+,T}$ is $m_{\text{pole}} = m_{B^*}$. The parameters $a_n$ for $f_+, f_0$ and $f_T$, with the condition $f_+(0) = f_0(0)$, are obtained fitting the Light-Cone QCD sum rule results in the range $m_\ell^2 \leq q^2 \leq 12 \text{ GeV}^2$\(^{55,56}\) and the lattice QCD results for $16 \text{ GeV}^2 \leq q^2$ in the recent FLAG report\(^{57}\): they are in Table 10. The other parameters used in the analysis are the quark masses $m_u = 2.16^{+0.49}_{-0.26}$ MeV (in the $\overline{MS}$ scheme at $\mu = 2$ GeV), $m_c(m_b) = 4.18^{+0.04}_{-0.03}$ GeV\(^{52}\), and the $B$ decay constant $f_B = 188 \pm 7$ MeV\(^{57}\).
Table 3: Angular coefficient functions for $\bar{B} \to \rho(\pi\pi)\ell^-\bar{\nu}_\ell$: NP term with P operator, interference term SM-NP with P operator, and NP-NP interference terms between P and T operators, Eq. (9).

| $i$ | $I_{i}^{NP,P}$ | $I_{i}^{INT,P}$ | $I_{i}^{INT,PT}$ |
|-----|----------------|----------------|----------------|
| $I_{1s}^{P}$ | 0 | 0 | 0 |
| $I_{1c}^{P}$ | $4H_{i}^{2} \frac{q^{4}}{(m_{b}+m_{u})^{2}}$ | $4H_{i}^{2} \frac{m_{q}q^{2}}{m_{b}+m_{u}}$ | 0 |
| $I_{2s}^{P}$ | 0 | 0 | 0 |
| $I_{2c}^{P}$ | 0 | 0 | 0 |
| $I_{3}^{P}$ | 0 | 0 | 0 |
| $I_{4}^{P}$ | 0 | 0 | 0 |
| $I_{5}^{P}$ | $-H_{t}(H_{+}+H_{-}) \frac{m_{q}q^{2}}{m_{b}+m_{u}}$ | $2H_{t}(H_{+}^{NP}+H_{-}^{NP}) \frac{(q^{2})^{3/2}}{m_{b}+m_{u}}$ | 0 |
| $I_{6s}^{P}$ | 0 | 0 | 0 |
| $I_{6c}^{P}$ | $-4H_{t}H_{0} \frac{m_{q}q^{2}}{m_{b}+m_{u}}$ | $H_{t}H_{NP}^{LN} \frac{(q^{2})^{3/2}}{m_{b}+m_{u}}$ | 0 |
| $I_{7}^{P}$ | $-H_{t}(H_{+}-H_{-}) \frac{m_{q}q^{2}}{m_{b}+m_{u}}$ | $2H_{t}(H_{+}^{NP}-H_{-}^{NP}) \frac{(q^{2})^{3/2}}{m_{b}+m_{u}}$ | 0 |

Table 4: Angular coefficient functions for $\bar{B} \to \rho(\pi\pi)\ell^-\bar{\nu}_\ell$: NP term with T operator and interference term SM-NP with T operator, Eq. (9).

| $i$ | $I_{i}^{NP,T}$ | $I_{i}^{INT,T}$ |
|-----|----------------|----------------|
| $I_{1s}^{P}$ | $2[(H_{+}^{NP})^{2}+(H_{-}^{NP})^{2}](3m_{\ell}^{2}+q^{2})$ | $-4(H_{+}^{NP}H_{+}+H_{-}^{NP}H_{-})m_{\ell}\sqrt{q^{2}}$ |
| $I_{1c}^{P}$ | $\frac{1}{8}(H_{NP}^{L})^{2}(m_{\ell}^{2}+q^{2})$ | $-H_{L}^{NP}H_{0}m_{\ell}\sqrt{q^{2}}$ |
| $I_{2s}^{P}$ | $2[(H_{+}^{NP})^{2}+(H_{-}^{NP})^{2}](m_{\ell}^{2}-q^{2})$ | 0 |
| $I_{2c}^{P}$ | $\frac{1}{8}(H_{NP}^{L})^{2}(q^{2}-m_{\ell}^{2})$ | 0 |
| $I_{3}^{P}$ | $8H_{+}^{NP}H_{-}^{NP}(q^{2}-m_{\ell}^{2})$ | 0 |
| $I_{4}^{P}$ | $\frac{1}{2}H_{NP}^{L}(H_{+}^{NP}+H_{-}^{NP})(q^{2}-m_{\ell}^{2})$ | 0 |
| $I_{5}^{P}$ | $-H_{L}^{NP}(H_{+}^{NP}-H_{-}^{NP})m_{\ell}^{2}$ | $\frac{1}{4}[H_{L}^{NP}(H_{+}-H_{-})+8H_{+}^{NP}(H_{t}+H_{0})$ $+8H_{-}^{NP}(H_{t}-H_{0})]m_{\ell}\sqrt{q^{2}}$ |
| $I_{6s}^{P}$ | $8[(H_{+}^{NP})^{2}-(H_{-}^{NP})^{2}]m_{\ell}^{2}$ | $-4(H_{+}^{NP}H_{+}-H_{-}^{NP}H_{-})m_{\ell}\sqrt{q^{2}}$ |
| $I_{6c}^{P}$ | 0 | $H_{L}^{NP}H_{0}m_{\ell}\sqrt{q^{2}}$ |
| $I_{7}^{P}$ | 0 | $\frac{1}{4}[H_{L}^{NP}(H_{+}-H_{-})-8H_{+}^{NP}(H_{t}+H_{0})$ $+8H_{-}^{NP}(H_{t}-H_{0})]m_{\ell}\sqrt{q^{2}}$ |
Table 5: Angular coefficient functions in the 4d $\bar{B} \to a_1(\rho \pi)\ell^-\bar{\nu}_\ell$ decay distribution, Eq.(8), in SM.

| $i$ | $I_{i,\parallel}^{SM}$ | $I_{i,\perp}^{SM}$ |
|-----|----------------------|------------------|
| $I_{1s}^{a_1}$ | $\frac{1}{2}(H_t^2 + H_t')(m_\ell^2 + 3q^2)$ | $2H_t^2m_\ell^2 + H_t^2(m_\ell^2 + q^2) + \frac{1}{4}(H_t^2 + H_t')(m_\ell^2 + 3q^2)$ |
| $I_{1c}^{a_1}$ | $4H_t^2m_\ell^2 + 2H_0^2(m_\ell^2 + q^2)$ | $\frac{1}{2}(H_t^2 + H_t')(m_\ell^2 + 3q^2)$ |
| $I_{2s}^{a_1}$ | $-\frac{1}{2}(H_+^2 + H_-^2)(m_\ell^2 - q^2)$ | $[H_0^2 - \frac{1}{4}(H_t^2 + H_t')](m_\ell^2 - q^2)$ |
| $I_{2c}^{a_1}$ | $2H_0^2(m_\ell^2 - q^2)$ | $-\frac{1}{2}(H_t^2 + H_t')(m_\ell^2 - q^2)$ |
| $I_3^{a_1}$ | $2H_+H_-(m_\ell^2 - q^2)$ | $-H_+H_-(m_\ell^2 - q^2)$ |
| $I_4^{a_1}$ | $H_0(H_+ + H_-)(m_\ell^2 - q^2)$ | $-\frac{1}{2}H_0(H_+ + H_-)(m_\ell^2 - q^2)$ |
| $I_5^{a_1}$ | $-2H_t(H_+ + H_-)m_\ell^2 - 2H_0(H_+ - H_-)q^2$ | $H_t(H_+ + H_-)m_\ell^2 + H_0(H_+ - H_-)q^2$ |
| $I_{6s}^{a_1}$ | $2(H_+^2 - H_-^2)q^2$ | $-4H_tH_0m_\ell^2 + (H_t^2 - H_-^2)q^2$ |
| $I_{6c}^{a_1}$ | $-8H_tH_0m_\ell^2$ | $2(H_+^2 - H_-^2)q^2$ |
| $I_7^{a_1}$ | 0 | 0 |

Table 6: Angular coefficient functions for $\bar{B} \to a_1(\rho \pi)\ell^-\bar{\nu}_\ell$: NP term with S operator, interference SM-NP with S operator, and NP-NP interference with S and T operators, Eq.(9).

| $i$ | $I_{i,\parallel}^{NP,S}$ | $I_{i,\parallel}^{INT,S}$ | $I_{i,\parallel}^{INT,ST}$ |
|-----|----------------------|----------------|------------------|
| $I_{1s}^{a_1}$ | 0 | 0 | 0 |
| $I_{1c}^{a_1}$ | $4H_t^2 \frac{q^4}{(m_b - m_u)^2}$ | $4H_t^2 \frac{m_\ell q^2}{m_b - m_u}$ | 0 |
| $I_{2s}^{a_1}$ | 0 | 0 | 0 |
| $I_{2c}^{a_1}$ | 0 | 0 | 0 |
| $I_3^{a_1}$ | 0 | 0 | 0 |
| $I_4^{a_1}$ | 0 | 0 | 0 |
| $I_5^{a_1}$ | 0 | $-H_t(H_+ + H_-) \frac{m_\ell q^2}{m_b - m_u}$ | $-2H_t(H_+^{NP} + H_-^{NP}) \frac{(q^2)^{3/2}}{m_b - m_u}$ |
| $I_{6s}^{a_1}$ | 0 | 0 | 0 |
| $I_{6c}^{a_1}$ | 0 | $-4H_tH_0 \frac{m_\ell q^2}{m_b - m_u}$ | $-H_t H_L^{NP} \frac{(q^2)^{3/2}}{m_b - m_u}$ |
| $I_7^{a_1}$ | 0 | $-H_t(H_+ - H_-) \frac{m_\ell q^2}{m_b - m_u}$ | $-2H_t(H_+^{NP} - H_-^{NP}) \frac{(q^2)^{3/2}}{m_b - m_u}$ |
Table 7: Angular coefficient functions for $\bar{B} \to a_1(\rho\pi)\ell^-\bar{\nu}_\ell$: NP term with S operator, interference SM-NP with S operator, and NP-NP interference with S and T operators, Eq. (9).

| $i$  | $I_{i,\perp}^{NP,S}$ | $I_{i,\perp}^{INT,S}$ | $I_{i,\perp}^{INT,ST}$ |
|------|----------------------|-----------------------|------------------------|
| $I^0_{1s}$ | $2H_t^2 \frac{q^2}{(m_b-m_u)^2}$ | $2H_t^2 \frac{m_q q^2}{m_b-m_u}$ | 0 |
| $I^0_{1c}$ | 0 | 0 | 0 |
| $I^0_{2s}$ | 0 | 0 | 0 |
| $I^0_{2c}$ | 0 | 0 | 0 |
| $I^1_{3}$ | 0 | 0 | 0 |
| $I^1_{4}$ | 0 | 0 | 0 |
| $I^1_{5}$ | $\frac{1}{2} H_t (H_+ + H_-) \frac{m_q q^2}{m_b-m_u}$ | $H_t (H_+^{NP} + H_-^{NP}) \frac{(q^2)^{3/2}}{m_b-m_u}$ | 0 |
| $I^0_{6s}$ | $-2H_t H_0 \frac{m_q q^2}{m_b-m_u}$ | $-H_t H_0^{NP} \frac{(q^2)^{3/2}}{2(m_b-m_u)}$ | 0 |
| $I^0_{6c}$ | 0 | 0 | 0 |
| $I^1_{7}$ | $\frac{1}{2} H_t (H_+ - H_-) \frac{m_q q^2}{m_b-m_u}$ | $H_t (H_+^{NP} - H_-^{NP}) \frac{(q^2)^{3/2}}{m_b-m_u}$ | 0 |

Table 8: Angular coefficient functions for $\bar{B} \to a_1(\rho\pi)\ell^-\bar{\nu}_\ell$: NP term with T operator and interference SM-NP with T operator.

| $i$  | $I_{i,||}^{NP,T}$ | $I_{i,||}^{INT,T}$ |
|------|------------------|-------------------|
| $I^0_{1s}$ | $2[ (H_+^{NP})^2 + (H_-^{NP})^2 ] (3m_\ell^2 + q^2)$ | $4(H_+^{NP} H_+ + H_-^{NP} H_-) m_\ell \sqrt{q^2}$ |
| $I^0_{1c}$ | $\frac{1}{8} (H_+^{NP})^2 (m_\ell^2 + q^2)$ | $H_0^{NP} H_0 m_\ell \sqrt{q^2}$ |
| $I^0_{2s}$ | $2[ (H_+^{NP})^2 + (H_-^{NP})^2 ] (m_\ell^2 - q^2)$ | 0 |
| $I^0_{2c}$ | $-\frac{1}{8} (H_+^{NP})^2 (m_\ell^2 - q^2)$ | 0 |
| $I^0_{3}$ | $-8H_+^{NP} H_-^{NP} (m_\ell^2 - q^2)$ | 0 |
| $I^0_{4}$ | $-\frac{1}{2} H_+^{NP} (H_+^{NP} + H_-^{NP}) (m_\ell^2 - q^2)$ | 0 |
| $I^0_{5}$ | $-H_+^{NP} (H_+^{NP} - H_-^{NP}) m_\ell$ | $-\frac{1}{4} [ H_+^{NP} (H_+ - H_-) + 8H_+^{NP} (H_t + H_0) + 8H_-^{NP} (H_t - H_0) ] m_\ell \sqrt{q^2}$ |
| $I^0_{6s}$ | $8[ (H_+^{NP})^2 - (H_-^{NP})^2 ] m_\ell^2$ | $4(H_+^{NP} H_+ - H_-^{NP} H_-) m_\ell \sqrt{q^2}$ |
| $I^0_{6c}$ | 0 | $-H_+^{NP} H_0 m_\ell \sqrt{q^2}$ |
| $I^0_{7}$ | 0 | $-\frac{1}{4} [ H_+^{NP} (H_+ + H_-) - 8H_+^{NP} (H_t + H_0) + 8H_-^{NP} (H_t - H_0) ] m_\ell \sqrt{q^2}$ |
Table 9: Angular coefficient functions for $\bar{B} \to a_1(\rho\pi)\ell^−\bar{\nu}_\ell$: NP term with T operator and interference SM-NP with T operator, Eq. (9).

| $i$ | $I_{i,\perp}^{\text{NP,T}}$ | $I_{i,\perp}^{\text{INT,T}}$ |
|-----|-----------------------------|-----------------------------|
| $I_{1s}^{a1}$ | $((H_+^{\text{NP}})^2 + (H_-^{\text{NP}})^2)(3m_\ell^2 + q^2)$ | $\frac{1}{2}[4(H_+^{\text{NP}}H_+ + H_-^{\text{NP}}H_-)]$ |
|     | $+ \frac{1}{16}(H_L^{\text{NP}})^2(m_\ell^2 + q^2)$ | $+ H_L^{\text{NP}}H_0]m_\ell\sqrt{q^2}$ |
| $I_{1c}^{a1}$ | $2[(H_+^{\text{NP}})^2 + (H_-^{\text{NP}})^2](3m_\ell^2 + q^2)$ | $4(H_+^{\text{NP}}H_+ + H_-^{\text{NP}}H_-)m_\ell\sqrt{q^2}$ |
| $I_{2s}^{a1}$ | $[((H_+^{\text{NP}})^2 + (H_-^{\text{NP}})^2)(m_\ell^2 - q^2)$ | $0$ |
|     | $- \frac{1}{16}(H_L^{\text{NP}})^2(m_\ell^2 - q^2)$ |     |
| $I_{2c}^{a1}$ | $2[(H_+^{\text{NP}})^2 + (H_-^{\text{NP}})^2](m_\ell^2 - q^2)$ | $0$ |
| $I_3^{a1}$ | $4H_+^{\text{NP}}H_-^{\text{NP}}(m_\ell^2 - q^2)$ | $0$ |
| $I_4^{a1}$ | $\frac{1}{4}H_L^{\text{NP}}(H_+^{\text{NP}} + H_-^{\text{NP}})(m_\ell^2 - q^2)$ | $0$ |
| $I_5^{a1}$ | $\frac{1}{2}H_L^{\text{NP}}(H_+^{\text{NP}} - H_-^{\text{NP}})m_\ell^2$ | $\frac{1}{8}[H_L^{\text{NP}}(H_+ - H_-) + 8H_+^{\text{NP}}(H_+ + H_0)$ |
|     |     | $+ 8H_-^{\text{NP}}(H_t - H_0)]m_\ell\sqrt{q^2}$ |
| $I_6s^{a1}$ | $4[(H_+^{\text{NP}})^2 - (H_-^{\text{NP}})^2]m_\ell^2$ | $-\frac{1}{2}[4(H_+^{\text{NP}}H_+ - H_-^{\text{NP}}H_-) + H_L^{\text{NP}}H_0]m_\ell\sqrt{q^2}$ |
| $I_6c^{a1}$ | $8[(H_+^{\text{NP}})^2 - (H_-^{\text{NP}})^2]m_\ell^2$ | $4(H_+^{\text{NP}}H_+ - H_-^{\text{NP}}H_-)m_\ell\sqrt{q^2}$ |
| $I_7^{a1}$ | $0$ | $\frac{1}{8}[H_L^{\text{NP}}(H_+ + H_-) - 8H_+^{\text{NP}}(H_t + H_0)$ |
|     |     | $+ 8H_-^{\text{NP}}(H_t - H_0)]$ |
Table 10: $B \to \pi$ form factor parameters in Eq. (C.1).

|       | $f_{+}^{B \to \pi}$ | $f_{0}^{B \to \pi}$ | $f_{T}^{B \to \pi}$ |
|-------|----------------------|----------------------|----------------------|
| $a_{0}$ | 0.416 (20)           | 0.492 (20)           | 0.400 (21)           |
| $a_{1}$ | −0.430               | −1.35                | −0.50                |
| $a_{2}$ | 0.114                | 2.50                 | 0.00076              |
| $a_{3}$ |                      | 0.534                |                      |

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