Shining Light on Polarizable Dark Particles

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Abstract

We investigate the possibilities of searching for a self-conjugate polarizable particle in the self-interactions of light. We first observe that polarizability can arise either from the exchange of mediator states or as a consequence of the inner structure of the particle. To exemplify this second possibility we calculate the polarizability of a neutral bosonic open string, and find it is described only by dimension-8 operators. Focussing on the spin-0 case, we calculate the light-by-light scattering amplitudes induced by the dimension-6 and 8 polarizability operators. Performing a simulation of exclusive diphoton production with proton tagging at the LHC, we find that the imprint of the polarizable dark particle can be potentially detected at $5\sigma$ significance for mass and cutoff reaching values above the TeV scale, for $\sqrt{s} = 13$ TeV and 300 fb$^{-1}$ of integrated luminosity. If the polarizable dark particle is stable, it can be a dark matter candidate, in which case we argue this exclusive diphoton search may complement the existing LHC searches for polarizable dark matter.

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1 Introduction

Among the speculations about what lies beyond the Standard Model (SM) of particles, there is the intriguing possibility of particles that are electrically neutral but can still slightly interact with photons. The existence of such “almost-dark” particles is theoretically well-motivated. These could for example be the hadrons created by a hidden strongly-interacting gauge force, binding together electrically charged constituents (for recent scenarios featuring such bound states, see for instance Stealth dark matter [1] and Vectorlike confinement [2]). Models where the dark particle has an electromagnetic coupling have been investigated in the scope of explaining Dark Matter (DM) of the universe. When interpreted as DM, the dark particle is assumed to be stable. In the present work this assumption of stability will not be needed.

In general, a particle with no electric charge may still interact with one photon through a dipole operator and/or a charge radius operator - in which case the particle couples directly to the photon field strength $F^{\mu\nu}$. Such scenarios for dark particles have been investigated in [3–20] in the context of dark matter. However, if the neutral particle can be described as a self-conjugate field in a low-energy theory, these operators vanish.  

The main interaction of the dark particle with light is then controlled by its polarizability, i.e. its tendency to interact with two photons. Such scenarios have also been investigated [4, 16, 21–28], still in the context of dark matter. Such polarizable dark particles are the topic of the present paper.

The interactions of our focus are bilinear in both the dark particle and in $F^{\mu\nu}$. In the case of a scalar, a linear coupling of the form $\phi(F)^2$ may also exist in principle. We will assume this coupling is either negligible or forbidden by a symmetry.  

By electroweak (EW) gauge invariance, a dark particle with electromagnetic polarizability should also be polarizable with respect to the $W$ and $Z$ bosons. This aspect will play little role in our analysis, but is relevant when it comes to comparisons with the literature.

To the best of our knowledge of the literature, most of the searches for a dark particle polarizable by EW gauge bosons are done within the assumption that the dark particle is stable. This is true by definition for direct and indirect detection, and is also the case for collider searches [22, 23, 26, 28–37], where the search strategies always involve detection of large missing energy. If this kind of searches turned out to be successful, it would provide a striking signature for the existence of dark matter.

In this paper we would like to adopt a slightly different strategy. Instead of readily testing the existence of a stable dark particle, we propose to rather test the existence of a dark particle, whether it is stable or not. In such approach, the assessment of stability is postponed to the post-discovery era, together with the characterization of the other properties of the new particle such as spin and mass. A consequence of this approach is...
that observing a large missing energy is not required anymore. Rather, one can set up a search which is independent of the hypothesis of stability. Adopting this slightly different viewpoint naturally leads to consider searches for the effects of virtual polarizable dark particles.

From a theoretical viewpoint, one advantage of looking for virtual dark particles is that, if a full dark sector is present, all the dark polarizable particles contribute to the signal, and not only the stable ones. This implies that the signal is enhanced with respect to searches only focussed on stable dark particles (e.g. usual DM searches). In the following we will consider the case of a single dark particle, unless stated otherwise.

As a first step, we will classify the CP-even polarizability operators up to dimension-8 and discuss their possible microscopic origin in Sec. 2. As an example, the intrinsic, dimension-8 polarizability of the neutral bosonic open string case is calculated in Sec. 3. These preliminary studies are needed to establish under which conditions the virtual search we propose is relevant. The virtual process we will focus on in this paper is photon-photon scattering. The amplitudes in the spin-0 case are given in Sec. 4. Moreover we will consider the exclusive channel, where outgoing protons remain intact and are detected. The simulation and its results are presented in Sec. 5, and Sec. 6 contains our conclusions.

2 Polarizability operators

We use a low-energy effective field theory (EFT) approach. Here the set of CP-even polarizability operators up to dimension-8 is classified. One writes down the operators featuring two photon field strengths and two dark particles of a given spin. It will be claimed below that the dimension-6 operators can be vanishing depending on the UV origin of polarizability, hence the dimension-8 operators can potentially be the dominant ones. The cutoff scale is denoted $\Lambda$, and validity of the effective description of polarizability by local operators requires that the dark particle mass and the energy flowing through the polarizability vertices be smaller than $\Lambda$ (see also Sec. 4.1).

The effective Lagrangian describing the spin-$s$ polarizable dark particle has the form
\begin{equation}
\mathcal{L}^s = \mathcal{L}_{\text{kin}}^s + \mathcal{L}_6^s + \mathcal{L}_7^s + \mathcal{L}_8^s + O\left(\Lambda^{-9}\right),
\end{equation}
with $\mathcal{L}_{4+n}^s = \sum_I c_I^s \Lambda^{-n} \mathcal{O}_I^s$.\footnote{We use a metric with $(+, -, -, -)$ signature, except in Sec. 3.} One introduces the dual electromagnetic field strength $F_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} F^{\alpha\beta}$, and one defines
\begin{equation}
(F)^2 = F^{\mu\nu} F^{\mu\nu}, \quad (F F)^{\mu\nu} = F^{\mu\alpha} F^{\rho\nu}, \quad (F F) = F^{\mu\nu} F^{\mu\nu}, \quad (F F) = F^{\mu\nu} F^{\mu\nu}. \quad (2.2)
\end{equation}
The coefficients of the operators of Eq. (2.1) should be understood as given at the EFT matching scale $\Lambda$. These coefficients should be in general written as $c_i(\Lambda)$. However, only the coefficients defined at the $\Lambda$ scale will effectively appear in our results, hence we will simply refer to them as $c_i$ in the following.

The dark particle will be called $\phi$, $\psi$, $X^\mu$ for spin-0, $1/2$, $1$ respectively, and its mass will be denoted $m$. The operators allowed by EW gauge invariance and inequivalent under
Table 1. CP-even polarizability operators for a self-conjugate particle of spin 0, 1/2 and 1. The notations for the field strength contractions of $X$ are the same as for $F$.

The coefficients of the $\hat{O}_s^6$ operators will be written $\hat{c}_s^6$, and similarly for $\hat{O}_s^7$.

Finally, we remark that the dimension-6 and 7 operators can be naturally suppressed with respect to the dimension-8 ones if the dark particle has an approximate shift-symmetry.
When this happens, the dark particle mass should be suppressed similarly. One can parametrize the explicit breaking of the shift-symmetry using the dark particle mass, and the dimension-6 (-7) operators are then respectively suppressed by $m^2/\Lambda^2$, $m/\Lambda$ and can thus be identified as the hatted operators of Eq.(2.4). This situation occurs for instance if the dark particle is the Nambu-Goldstone particle of a spontaneously broken approximate global symmetry, for example a $U(n)$ symmetry or supersymmetry, respectively giving a Nambu-Goldstone scalar and a Nambu-Goldstini (see [39, 40] for a related analysis in the context of dark matter).

2.1 Microscopic origin

Even though we simply listed the polarizability operators in an effective theory approach, some aspects of the UV origin of these operators can already be deduced. We identify two mechanisms. Either polarizability could arise from the exchange of heavy virtual particles, referred to as “mediator”. Or the polarizable particle may actually be an extended object in the UV, and polarizability could then originate from the inner structure of the particle. We shall refer to these two scenarios as mediated polarizability and intrinsic polarizability.

2.1.1 Mediated polarizability

Here we consider the case of polarizability induced by heavy mediators. First, we notice that no operator in Tab. 1 can be generated via the tree-level exchange of a particle in a fully renormalizable theory. It may seem possible in the case of the $O_{16}^{\mu}$ operator, starting from a dipole operator

$$O_{XY}^{1} = X_{\mu}Y_{\nu}F^{\mu\nu}$$  \hspace{1cm} (2.5)

and integrating out the heavy spin-1 mediator $Y$. However, renormalisability requires $X$ and $Y$ to arise as massive gauge fields of a spontaneously broken gauge symmetry $G$ containing the electroweak group $G_{EW}$ in its unbroken sector. This $O_{XY}^{1}$ operator can then arise from the kinetic term of the $G$ gauge field. Inspecting the broken sector (see [41]), it turns out that $O_{XY}^{1}$ is controlled by the broken constant structures $f^{abc}$, where the hatted (unhatted) indexes label the broken (unbroken) generators. These same constant structures determine the coupling of the massive gauge fields to the electroweak gauge fields. One concludes that the $X$ and $Y$ fields have to be charged in order for $O_{XY}^{1}$ to be non-zero. This is in contradiction with the hypothesis of a self-conjugate $X$, therefore polarizability of $X$ cannot be induced by tree-level exchange of a spin-1 mediator in a renormalizable theory.

Possibilities for tree-level exchange of heavy mediators arise in case of non-renormalizable interactions. The $O_{6a}^{0}$, $O_{8b}^{0}$, $O_{7a}^{1/2}$, $O_{8b}^{1/2}$, $O_{6a}^{1}$, $O_{8b}^{1}$ operators can be generated by a CP-even spin-0 mediator, such as a radion/dilaton. The $O_{7b}^{1/2}$, $O_{8d}^{1/2}$ could be induced by the exchange of an axion-like CP-odd scalar. The $O_{7a}^{1/2}$, $O_{7b}^{1/2}$ operators can also be generated together if a Majorana fermion $\Psi$ shares a dipole operator with another, heavier Majorana fermion $\Psi^*$, $\Psi\sigma_{\mu\nu}\Psi^*F^{\mu\nu}$ [21]. A similar possibility is that the components of $\Psi$ and $\Psi^*$ be part of a single Dirac fermion, with a mass splitting induced by a Majorana mass [42]. Finally, the
\( \mathcal{O}_{0a}, \mathcal{O}_{0a}^{1/2}, \mathcal{O}_{1a} \) can be generated by a spin-2 mediator (such as a Kaluza-Klein graviton), together with \( \mathcal{O}_{0b}, \mathcal{O}_{0b}^{1/2}, \mathcal{O}_{1b} \) terms coming from tracelessness of the spin-2 representation.

All of the operators of Tab. 1 could in principle be generated at loop-level, in particular by loops of charged mediators. In such case, the coefficient \( c_i^a \) of the polarizability operators must come with a factor \( e^2/16\pi^2 \), and \( \Lambda \) is identified with the mass of the particle in the loop.

### 2.1.2 Intrinsic polarizability

Here, we consider the possibility that polarizability arises from the inner structure of the dark particle. Let us consider a generic 4-point amplitude with two dark particles and two photons in external legs. We focus on the scalar case \( \gamma\gamma\phi\phi \) for concreteness, but the same reasoning applies to spin 1/2 and 1 similarly. The scattering amplitude has the form

\[ \mathcal{M} = \epsilon_a(p_a)\epsilon_b(p_b)V^{\alpha\beta}, \quad (2.6) \]

where \( V^{\mu\nu} \) is a function of the momenta and of the intrinsic scale of the dark particle \( \Lambda \). Using Ward identities and the fact that the photon does not couple to the dark particle through covariant derivatives by definition, we readily know that the \( V^{\alpha\beta} \) tensor has the form

\[ V^{\alpha\beta} = \frac{1}{m^2}R^{\alpha\mu\nu\alpha}(p_a)R^{\beta\mu\nu\beta}(p_b)F^{\mu\nu\alpha\beta\mu\nu\beta}, \quad (2.7) \]

where one introduces the projector \( R^{\mu\nu\alpha\beta}(p) = \mathcal{P}^\mu\nu - \mathcal{P}^\alpha\beta \). The dimensionless tensor \( F^{\mu\nu\alpha\beta\mu\nu\beta} \) is the general form factor of the dark particle, that encodes the information about its inner structure. In the low-energy domain \( s, t, u, m^2 < \Lambda^2 \), where \( s, t, u \) are the Mandelstam variables, the lower order Lorentz structures can then be written as

\[ F^{\mu\nu\alpha\beta\mu\nu\beta} = F_0(s, t, u, \Lambda)g^{\mu\nu\alpha\beta}g^{\nu\mu\beta} + \frac{1}{\Lambda^2}F_1(s, t, u, \Lambda)\mathcal{P}_1^{\mu\nu\alpha\beta}g^{\nu\mu\beta} + O\left(\frac{1}{\Lambda^4}\right). \quad (2.8) \]

We assume that a massless polarizable dark particle can exist, and thus ask for the amplitude to remain finite in the massless limit. This implies that the form factors should decrease at least as \( F_{0,1} \sim m^2/\Lambda^2 \) at small \( m/\Lambda \) in order to compensate the \( m^{-2} \) in Eq. (2.7). This can also be checked taking the massless limit of the amplitudes of Sec. 4.3. Expanding the form factors for large \( \Lambda \) and using the symmetries of the diagram \(^4\), one gets that the leading terms should be given by \( F_0 = \frac{m^2}{\Lambda^2}\tilde{F}_0 \), \( F_1 = \frac{m^2}{\Lambda^2}\tilde{F}_1 \) and \( \tilde{F}_0(s, t, u, \Lambda) = A + (Bp_1.p_2 + Cm^2)/\Lambda^2 + O(\Lambda^{-4}), \tilde{F}_1(s, t, u, \Lambda) = D + O(\Lambda^{-2}) \) where \( A, B, C, D \) are constants. The general form factor reads

\[ F^{\mu\nu\alpha\beta\mu\nu\beta} = \frac{m^2}{\Lambda^2}(A + \frac{Bp_1.p_2 + Cm^2}{\Lambda^2})g^{\mu\nu\alpha\beta}g^{\nu\mu\beta} + \frac{m^2}{\Lambda^4}Bp_1^{\mu\nu\alpha\beta}p_2^{\nu\mu\beta}g^{\nu\mu\beta} + O\left(\frac{1}{\Lambda^6}\right). \quad (2.9) \]

All the terms vanish in the pointlike limit \( \Lambda \to \infty \), as expected from effects arising from compositeness. The \( A, B, C, D \) constants are in direct correspondence with the spin-0 effective operators of Tab. 1. Identifying the Lorentz structures, one has simply

\[ A = c_0^0, \quad B = c_0^0, \quad C = c_0^0, \quad D = c_0^0. \quad (2.10) \]

\(^4\)Because of \( t \leftrightarrow u \) symmetry one can expand with respect to \( s/\Lambda^2 \) and \( (t + u)/\Lambda^2 \). One uses then \( s + t + u = 2m^2 \).
We can deduce some physical features of the polarizability operators by studying the non-relativistic limit \((p_i)^2 \ll m^2\), where \(p_i\) is the three-momentum. Here we limit our discussion to spin-0 and 1/2, for which one can recognize familiar electromagnetic features.\(^5\) Let us first remark that in the non-relativistic limit the operators satisfy
\[
\mathcal{O}^{0}_{8a} \propto (E_i)^2, \quad \mathcal{O}^{0}_{6a}, \quad \mathcal{O}^{0}_{8b} \propto (E_i)^2 - (B_i)^2, \quad \mathcal{O}^{0}_{7,6a}, \quad \mathcal{O}^{0}_{8c,d} \rightarrow 0, \quad \mathcal{O}^{0}_{8d} \propto (B_i)^2, \quad \mathcal{O}^{0}_{8e} \propto (B_i)^2.
\]
where \(E_i, B_i\) are the standard electric and magnetic fields. One also has
\[
\mathcal{O}^{0}_{8a} \propto (E_i)^2, \quad \mathcal{O}^{0}_{7a}, \quad \mathcal{O}^{0}_{8b} \propto (E_i)^2 - (B_i)^2, \quad \mathcal{O}^{0}_{7,8a}, \quad \mathcal{O}^{0}_{8c,d} \rightarrow 0, \quad \mathcal{O}^{0}_{8e} \propto (B_i)^2.
\]
The \((E_i)^2\) and \((B_i)^2\) term that appear in the non-relativistic Lagrangian correspond respectively to the static electric and magnetic susceptibilities of the inner structure of the dark particle. We can see that the \(\mathcal{O}^{0,1/2}_{8a}(\mathcal{O}^{0,1/2}_{8b})\) operators describe respectively a polarizability with purely electric (respectively magnetic) origin. These properties can in turn be used to infer some features of the polarizability operators for a given object.

In the case of dark hadrons, made of electrically charged fermions glued by a hidden strong interaction, we certainly expect an electric polarizability, as these constituents form an electronic density that can be deformed by an external electric field. Also, as the constituents carry intrinsic spin, a magnetic polarizability should exist, however both theoretical arguments \([43]\) and observations \([44]\) suggest that it is suppressed with respect to the electric one, thus one may expect \(\mathcal{O}^{0,1/2}_{8a}(\mathcal{O}^{0,1/2}_{8b})\) operators describe respectively a polarizability with purely electric (respectively magnetic) origin. These properties can in turn be used to infer some features of the polarizability operators for a given object.

Finally, one may wonder how electro-magnetic duality applies to the arguments above. From a macroscopic viewpoint, electromagnetic duality exchange the susceptibilities \(\alpha_E \leftrightarrow \alpha_B\), and thus exchanges the \(\alpha_E \neq 0, \alpha_M = 0\) case with \(\alpha_E = 0, \alpha_M \neq 0\) in the string case. Microscopically, such object would be a sort of open string with magnetic monopoles attached at endpoints. Such objects, called \(D\)-strings, do exist in string theories, and are related by \(S\)-duality to the original strings (see \([46]\)). From a low energy point of view, the polarizability of such objects should be expected to be described by the \(\mathcal{O}_{8d}\) operator, while the combinations \(\mathcal{O}^{0}_{8b} + \mathcal{O}^{0}_{6a} / m^2 \mathcal{O}^{0}_{6a} + (c_{8d} + \mathcal{O}^{0}_{6a} / m^2 \mathcal{O}^{0}_{6a}) = 0\), and similarly for spin-1/2. These coefficients will be calculated in next section for the neutral bosonic string.

\(^5\)The case of a massive non-relativistic spin-1 particle is not straightforward to analyze, for example one has \(\mathcal{O}_{8a} \propto X_i X_j (E_i E_j + B_i B_j - \delta_{ij} B^2)\).

\(^6\)The dipole operators associated with the quantum states of the open bosonic and super strings have been evaluated in Ref. \([45]\) and are vanishing if the string is neutral.
3 Polarizability of the neutral bosonic string

To give a concrete example of an object with intrinsic polarizability, we work out the case of a neutral open string \( (i.e. \text{a string with charges } q_0, q_1 \text{ at ends, satisfying } q_0 = -q_1 = q) \). For the sake of describing polarizability of the string states, there is no need to assume that spacetime has critical dimension. In fact, being ultimately interested in the 4D case, the string we consider cannot be considered as a fundamental one. Instead, it may for example be taken as a QCD-like string, \( i.e. \) an effective description of the binding between a quark and an antiquark arising in a gauge theory with large number of colors. A mostly-plus signature \( (−, +, +, +) \) is used for \( g_{\mu \nu} \) in this section.

The action of an open string with length scale \( l_s \equiv \sqrt{\alpha'} \) in an electromagnetic background is given by

\[
S = \int_{−\infty}^{\infty} d\tau \int_0^{\pi} d\sigma \left[ \frac{1}{4\pi l_s^2} \left( \dot{X}^\mu \dot{X}_\mu - \dot{X}'^\mu \dot{X}'_\mu \right) - A^\mu \dot{X}_\mu \left( q_0 \delta(\sigma) + q_1 \delta(\sigma - \pi) \right) \right],
\]  

(3.1)

where \( A_\mu \) is the canonically normalized electromagnetic field. Propagation of a bosonic open string in an abelian background gauge field has been worked out in Ref. [47], and canonical quantization is done in details in Ref. [48]. Our calculation follows closely [48], details are given in App.A.

Computing the solutions of the equation of motion, defining an orthogonal basis for the oscillator and zero modes, and asking for canonical commutators between the position, momentum and Fourier operators \( x_\mu, p_\mu, a_n^{(1)} \), the string decomposition over orthonormal modes is

\[
X_\mu = x_\mu + 2l_s^2 \left( g - 4\pi^2 l_s^4 q^2 F.F \right)^{−1/2} \left( \tau g^{\mu \rho} + \frac{\pi}{2} q F^{\mu \rho} \right) p_\rho + i\sqrt{2l_s} \left( \sum_{n=1}^{\infty} a_n \psi_n(\tau, \sigma) - \sum_{n=1}^{\infty} a_n^\dagger \psi^\dagger_{-n}(\tau, \sigma) \right).
\]

(3.2)

It turns out that the background field does not affect the oscillator modes, only the zero mode gets deformed. \(^7\) The \( L_0 \) Virasoro operator of the open string is then given by

\[
L_0 = \frac{1}{2} p^{\mu} \left( g - (2\pi)^2 l_s^4 q^2 F.F \right)^{−1} p^{\nu} + \frac{1}{2} N
\]

(3.3)

where \( N = \sum_{n=1}^{\infty} \alpha_n \alpha_n \alpha_n \alpha_n \) is the usual number operator, using \( \alpha_n = \sqrt{n} a_n, \alpha_{-n} = \sqrt{n} a_n^\dagger \). The states of the string are built from a ground state \(|0\rangle\) using creation operators,

\[
\Phi^{\mu_1 \mu_2 \ldots \mu_s}(x^\mu) = \int \frac{d^4k}{(2\pi)^4} e^{ikx^\mu} \prod_{i=1}^{s} \alpha_{-m_i}^{\mu_i} |0\rangle.
\]

(3.4)

The \( L_0 \) operator satisfies the condition

\[
(L_0 + a)\Phi = 0,
\]

(3.5)

\(^7\) There is a freedom in normalising the \( x^\mu \) and \( p^\mu \) operators inside the zero mode. It is convenient to let the position operator unchanged and to incorporate all the effect of the background field into the momentum term.
where \( a \) is a constant from normal ordering which is left unspecified and is irrelevant regarding the property of polarizability. \(^8\) Equation (3.5) gives the equation of motion for the string states,

\[
\left( \partial^\mu \left( g - (2\pi)^2 l_s^4 q^2 F.F \right)^{-1} \partial^\nu - m^2 \right) \Phi^{\mu_1 \mu_2 \cdots \mu_s} = 0, \tag{3.6}
\]

where the mass is given by \( m = l_s^{-1}(\sum_i m_i + a)^{1/2} \). Retaining the leading term in power of \( l_s \) gives

\[
\left( \partial^\mu \left( g_{\mu\nu} + (2\pi)^2 l_s^4 q^2 (F.F)_{\mu\nu} + O(l_s^8) \right) \partial^\nu - m^2 \right) \Phi^{\mu_1 \mu_2 \cdots \mu_s} = 0. \tag{3.7}
\]

This equation of motion describes the polarizability of a string state of any integer spin \( s \).

Going back to the mostly-minus metric used for the effective Lagrangian of Eq. (2.1), we can deduce the Lagrangian giving rise to the equation of motion Eq. (3.7) in case of spin 0 and 1. We conclude that polarizability of the spin-0 state and spin-1 state is respectively described by the operators \( \mathcal{O}_{0a}^0, \mathcal{O}_{1a}^1 \). Identifying \( \Lambda \) with the inverse string length, \( \Lambda = l_s^{-1} \), the operator coefficients are \((2\pi)^2 q^2\), so that the effective Lagrangian is

\[
\mathcal{L} \supset \frac{4\pi^2 q^2}{\Lambda^4} \mathcal{O}_{0a}^0 + \frac{4\pi^2 q^2}{\Lambda^4} \mathcal{O}_{1a}^1. \tag{3.8}
\]

Establishing the consistent Lagrangian for a neutral polarizable state of higher spin is probably more challenging conceptually and technically, and lies outside the scope of this study. In particular, the electromagnetic interactions of the auxiliary fields present in the higher-spin Lagrangian would have to be determined. \(^9\)

4 Four-photon amplitudes from polarizable dark particles

Polarizable dark particles automatically induce loops with four external photon legs (see Fig. 1). Following our strategy of focussing on virtual processes (see Sec. 1), we propose to use such anomalous photon couplings as a probe for the existence of a dark particle. For a first analysis of this proposal, we focus on the case of a dark particle of spin-0. The spin-1/2 and spin-1 cases would deserve to be treated similarly, but lie outside the scope of this paper.

4.1 Consistency of the approach

A necessary condition for our proposal to make sense is that the dark particle produces the main contribution to the four-photon coupling. While in principle the complete UV picture is needed to answer this question, the considerations on the microscopic origin of polarizability made in Sec. 2 already provide a useful constraint. Indeed, in the case where polarizability is induced by mediators, the mediators themselves can form diagrams with four external photons. The contributions from dark particles are expected to be smaller

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\(^8\)We will assume \( a \geq 0 \) whenever discussing the spin-0 state, otherwise it is tachyonic.

\(^9\)These aspects might be treated in a further work.
Figure 1. Four-photon interaction induced by a virtual polarizable dark particle.

than the ones from mediators by at least a loop factor. This happens in both the cases of loop and tree diagrams, induced respectively by charged mediators and mediators with non-renormalizable couplings. The four-photon search then essentially probes the existence of these mediators. The sensitivity for such particles has already been estimated [49, 50], irrespective of the existence of a dark particle. In contrast, if polarizability originates from the inner structure of the dark particle, the dark particle loop can in principle be the dominant contribution to the anomalous four-photon vertex.

Some consistency constraints also come from the validity of the EFT approach. The validity of the low-energy expansion requires that

\[ s, |t|, |u|, m^2 < \Lambda^2, \]

otherwise the form factor from UV physics becomes important, and the description of polarizability of the dark particle by local operators is not valid anymore. The partonic center-of-mass energy for exclusive photon scattering is typically of \( \sqrt{s} \sim 1 \) TeV at the 13 TeV LHC. Moreover, tree-level unitary of photon - dark particle scattering imposes the conditions

\[ |c_{0a}|^2 s^2/\Lambda^4 < 16\pi, \quad |c_{0b}|^2 s^2/\Lambda^4 < 8\pi, \quad |c_{0a}|^2 s^2/\Lambda^2 < 8\pi, \quad |\hat{c}_{0a}|^2 m^2 s/\Lambda^4 < 8\pi. \]

For \( \sqrt{s} \sim \Lambda \), the bound translates as \( c_i < 8\pi \). It is worth noticing that for \( \Lambda > \sqrt{s} \), the \( c_i \) are allowed to be larger than \( 8\pi \). In our estimations of LHC sensitivity of Sec. 5 we will use \( |c_i| = 10 \). We emphasize that these unitarity constraints are qualitatively equivalent to requiring perturbativity of the effective interactions in the EFT. Constraints similar to those of Eq. (4.2) can be obtained by requiring that a diagram with \( n + 1 \) loops be smaller or of same order of magnitude than a diagram with \( n \) loops (when using dimensional regularization).

4.2 Consistency of the calculation

An important subtlety is that the four-photon loop diagrams we consider come from higher-dimensional operators and are thus more divergent than the four-photon diagrams from the UV theory. This implies that four-photon local operators (i.e. counter-terms) are also present in the effective Lagrangian to cancel the divergences which are not present in the UV theory. The finite contribution from these local operators is fixed by the UV theory
at the matching scale, and is expected to be of same order as the coefficient of the log $\Lambda$ term in the amplitude by naive dimensional analysis (this situation is analog to renormalisation of the non-linear sigma model, see Ref. [51]). This implies that the amplitudes obtained from calculating the loop graphs should only be considered as estimates of the complete amplitudes, the latter being determined only once the UV theory is specified. Concretely, for four-photon interactions induced by loops with dimension-8 operators, local four-photon operators of dimension-12 are present in the Lagrangian. Four-photon interactions induced by loops of dimension-6 operators imply the presence of dimension-8 operators, corresponding to the two Lorentz structures shown in Eq. 4.19.

Cutoff regularisation in an effective theory is very difficult because it breaks the expansion with respect to $\Lambda^{-1}$, as loops from operators of arbitrarily high dimension contribute at same order to the amplitudes (see [51]). A much simpler scheme is dimensional regularisation, in which case power-counting is respected and it is thus consistent to include only operators of lower dimension (up to dimension-8 in our case). The matching of the effective theory with the UV theory being done at the scale $\Lambda$, we can readily identify the divergent integrals as (see [52, 53])

\[
\int \frac{d^4 l}{(2\pi)^4} \frac{l^2}{(l^2 - \Delta)^2} \to -\frac{2i}{(4\pi)^2} \Delta \log(\Delta/\Lambda^2), \tag{4.4}
\]

\[
\int \frac{d^4 l}{(2\pi)^4} \frac{1}{(l^2 - \Delta)^2} \to -\frac{3i}{(4\pi)^2} \Delta^2 \log(\Delta/\Lambda^2). \tag{4.5}
\]

As a final remark, we note that in the limit of heavy mass, $m^2 \gg s,t,u$, the loops reduce to local effective interactions. The amplitudes from these local interactions are given in [50], and have the Lorentz structure

\[
\mathcal{M}_{++++} \propto s^2, \quad \mathcal{M}_{++-} \propto s^2 + t^2 + u^2, \quad \mathcal{M}_{+++-} = 0 \tag{4.6}
\]

and will be given below.

4.3 Helicity amplitudes

Focussing on the case of a spin-0 dark particle, we calculate the four-photon amplitudes induced by the dimension-8 polarizability operators $\mathcal{O}_{8a}^0$, $\mathcal{O}_{8b}^0$, $\hat{\mathcal{O}}_{6a}^0$, which are theoretically well-motivated as discussed in Sec. 4.1. We limit ourselves to cases where one of these operators is dominant and do not calculate diagrams involving two different operators.

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10The running of the $c_0^\mu(\mu)$ coefficients is taken into account at leading-log order with this method.
Helicity amplitudes are given under the form \( \mathcal{M}_{\lambda_a \lambda_b \lambda_1 \lambda_2}(s, t, u) \), where \( \lambda_{a,b} = \pm \) denotes the polarization of two ingoing photons and \( \lambda_{1,2} \) denotes the polarization of two outgoing photons. Due to the relations \( \mathcal{M}_{++--}(s, t, u) = \mathcal{M}_{+---}(u, t, s) \), \( \mathcal{M}_{++--}(s, t, u) = \mathcal{M}_{++++}(t, s, u) \), only the \( \mathcal{M}_{++++} \), \( \mathcal{M}_{+---} \), \( \mathcal{M}_{++++-} \), \( \mathcal{M}_{+++++} \) configurations have to be calculated (see Ref. [54]). Full amplitudes and details of the calculation are given in App. B. The \( \mathcal{M}_{++++} \) amplitude is found to be exactly zero in all cases. Here below we display only the helicity amplitudes in the high energy limit \( m^2 \ll s, t, u \) and in the low-energy limit \( s, t, u \ll m^2 \), where in both cases \( s, t, u, m^2 < \Lambda^2 \).

- \( \mathcal{O}_{8a}^0 \) operator

  If \( m^2 \ll s, t, u \),

  \[
  \mathcal{M}_{++++} \approx -\frac{(c_{8a})^2}{32\pi^2 \Lambda^8} s^2 \left[ -\left( \frac{68}{75} s^2 + \frac{47}{300} (t^2 + u^2) \right) + i\pi \left( \frac{3}{5} s^2 + \frac{1}{30} (t^2 + u^2) \right) + \left( \frac{3}{5} s^2 \log \left( \frac{s}{\Lambda^2} \right) + \frac{1}{30} (t^2 \log \left( \frac{t}{\Lambda^2} \right) + u^2 \log \left( \frac{u}{\Lambda^2} \right) \right) \right],
  \]

  \[
  \mathcal{M}_{+---} \approx -\frac{(c_{8a})^2}{32\pi^2 \Lambda^8} \left( \frac{-68}{75} + i \frac{3\pi}{5} \right) \left( s^4 + t^4 + u^4 \right) + \frac{3}{5} \left( s^4 \log \left( \frac{s}{\Lambda^2} \right) + t^4 \log \left( \frac{t}{\Lambda^2} \right) + u^4 \log \left( \frac{u}{\Lambda^2} \right) \right).
  \]  

  If \( m^2 \gg s, t, u \),

  \[
  \mathcal{M}_{++++} \approx -\frac{(c_{8a})^2}{32\pi^2 \Lambda^8} 5 s^2 m^4 \log \left( \frac{m^2}{\Lambda^2} \right),
  \]

  \[
  \mathcal{M}_{+---} \approx -\frac{(c_{8a})^2}{32\pi^2 \Lambda^8} 3 (s^2 + t^2 + u^2) m^4 \log \left( \frac{m^2}{\Lambda^2} \right).
  \]

- \( \mathcal{O}_{8b}^0 \) operator

  If \( m^2 \ll s, t, u \),

  \[
  \mathcal{M}_{++++} = -\frac{(c_{8b})^2}{8\pi^2 \Lambda^8} s^4 \left[ -\frac{157}{225} + i\pi \frac{7}{15} \right] + \frac{7}{15} \log \left( \frac{s}{\Lambda^2} \right),
  \]

  \[
  \mathcal{M}_{+---} = -\frac{(c_{8b})^2}{8\pi^2 \Lambda^8} \left[ \left( \frac{157}{225} + i\pi \frac{7}{15} \right) \left( s^4 + t^4 + u^4 \right) + \frac{7}{15} \left( s^4 \log \left( \frac{s}{\Lambda^2} \right) + t^4 \log \left( \frac{t}{\Lambda^2} \right) + u^4 \log \left( \frac{u}{\Lambda^2} \right) \right) \right].
  \]

  If \( m^2 \gg s, t, u \),

  \[
  \mathcal{M}_{++++} = -3 \frac{(c_{8b})^2}{2\pi^2 \Lambda^8} s^2 m^4 \log \left( \frac{m^2}{\Lambda^2} \right),
  \]

  \[
  \mathcal{M}_{+---} = -3 \frac{(c_{8b})^2}{2\pi^2 \Lambda^8} (s^2 + t^2 + u^2) m^4 \log \left( \frac{m^2}{\Lambda^2} \right).
  \]
Finally, in the $m^2 \gg s, t, u$ case, it is well-known that four-photon interactions can be represented by two independent dimension-8 operators

$$\mathcal{L} = \frac{b_1}{\Lambda^4} F^{\mu\nu} F_{\mu\nu} F^{\rho\sigma} F_{\rho\sigma} + \frac{b_2}{\Lambda^4} F^{\mu\nu} F_{\mu\rho} F^{\rho\sigma} F_{\sigma\mu},$$

(4.19)

and the helicity amplitudes as a function of the $b_{1,2}$ coefficients have been given in Ref. [49].

Matching these amplitudes to the low-energy limit of the ones from loops of polarizable particles Eqs. (4.9), (4.10), (4.13), (4.14), (4.17), (4.18) gives

$$b_1 = -\left(\frac{c_{8a}}{8\pi^2 \Lambda^4}\right)^2 m^4 \log \left(\frac{m^2}{\Lambda^2}\right), \quad b_2 = -\left(\frac{c_{8b}}{128\pi^2 \Lambda^4}\right)^2 m^4 \log \left(\frac{m^2}{\Lambda^2}\right).$$

(4.20)

from the $\mathcal{O}_{8a}$ operator,

$$b_1 = -3\left(\frac{c_{8b}}{16\pi^2 \Lambda^4}\right)^2 m^4 \log \left(\frac{m^2}{\Lambda^2}\right), \quad b_2 = 0$$

(4.21)

from the $\mathcal{O}_{8b}$ and

$$b_1 = -\left(\frac{c_{6a}}{16\pi^2 \Lambda^4}\right)^2 m^4 \log \left(\frac{m^2}{\Lambda^2}\right), \quad b_2 = 0$$

(4.22)

from the $\hat{\mathcal{O}}_{6a}$ operator.

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11To adapt the amplitudes given in the conventions of [49] to the ones of the present paper, the amplitudes in [49] have to be multiplied by a factor $-8$. 
5 Light-by-light scattering as a probe for polarizable dark particles

We propose to focus on photon-photon scattering in the exclusive channel, where the two protons remain intact after the collision,

\[ pp \to \gamma\gamma pp. \quad (5.1) \]

These intact protons can be detected and characterised using forward proton detectors along the beam pipe, that are scheduled by both ATLAS [55] and CMS/TOTEM [56] collaborations. The interest of the exclusive diphoton channel with proton characterization is that there is enough kinematic information to eliminate most of the background. The sensitivity of this measurement to new physics has been studied in details in [49, 50, 57], where the residual background rate after all cuts has been estimated to \(3 \cdot 10^{-4} \text{ fb}\). This background comes from inclusive diphoton events occurring simultaneously with the tagging of two intact protons from pileup. \(^{12}\)

5.1 Sensitivity at 13 TeV and \(L = 300 \text{ fb}^{-1}\)

In order to obtain a realistic estimation for the discovery potential of the dark particle, we implemented the four-photon amplitudes induced by dark particles in the Forward Physics Monte Carlo generator (\texttt{FPMC} [68]). The model of photon flux of Ref. [69] is assumed. We reproduce the acceptance of the forward detectors by constraining the fractional momentum loss of both protons to be \(0.015 < \xi < 0.15\). \(^{13}\)

We set a cut of \(|p_T| > 150 \text{ GeV}\) on the transverse momentum of each photon. Like in Ref. [50], the main impact on the signal rates is expected to come from these cuts. We include the effect of the other cuts on the signal with a global efficiency of \(\epsilon_s = 90\%\).

The average sensitivities for a signal induced by the \(\mathcal{O}_{8a}^0\), \(\mathcal{O}_{8b}^0\), \(\mathcal{O}_{6a}^0\) operators are shown in Figs. 2 and 3. We simply set that 3 \(\sigma\) and 5 \(\sigma\) statistical significance for the existence of a signal roughly correspond to \(\hat{n} = 3\) and 5 observed events. More evolved statistical analysis give similar conclusions. The uncertainty on the cross-section is expected to blow up when approaching the \(m = \Lambda\) limit. In fact, the cross-sections used for the figures are probably under-estimated in this region because we did not included the local 4\(\gamma\) operators arising from matching, that should dominate in this region has the \(\log(m/\Lambda)\) term becomes small (see also discussion in Sec. 4.2).

We observe that for the chosen values of \(c_i^0\), the sensitivity regions can go above the TeV. It turns out that the sensitivity for the \(\mathcal{O}_{8b}^0\) operator is better than for the \(\mathcal{O}_{6a}^0\) operator, which itself is better than for the \(\mathcal{O}_{8a}^0\) operator. The regions for each operators

\(^{12}\) Other studies using proton-tagging at the LHC for New Physics searches can be found in Refs. [41, 58–66]. We refer to [67] for a study of light-by-light scattering at the LHC without proton tagging.

\(^{13}\) For CMS, these expectations have recently been updated to be 0.037 < \(\xi\) < 0.15 [70]. We checked that our results are essentially the same with this new range, the sensitivity regions decrease only slightly.

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Figure 2. Sensitivity of the exclusive diphoton channel to a spin-0 dark particle with dimension-8 polarizability with coefficient \( c_{8a,8b}^0(\Lambda) = 10 \), represented in the mass-cutoff plane, and assuming \( \sqrt{s} = 13 \) TeV, \( L = 300 \) fb\(^{-1} \). The dashed lines correspond to the 5 \( \sigma \) sensitivity in presence of \( N = 5 \) copies of the dark particle. The two dotted lines corresponds to the 5 \( \sigma \) sensitivities for the spin-0 state of the neutral string assuming \( q = 1 \) and \( q = 2 \), and taking \( \Lambda = l_s^{-1} \).

have sensibly different shapes. In particular, a sensitivity remains at low mass for the \( O_{8a}^0 \), \( O_{8b}^0 \) operators, while it vanishes for the \( \hat{O}_{8a}^0 \) operator. These estimations are for a single self-conjugate scalar. An important point to keep in mind is that the search we propose is multiplicity-sensitive: The more the dark sector is populated by polarizable dark particles, the more the sensitivity regions improve. For illustration we show how the regions grow when assuming \( N = 5 \) particles with same mass and couplings. In this case the photon-photon cross-section is enhanced by a \( N^2 \) factor.

Although our present study is limited to the case of one operator turned on at a time, some conclusions can already be drawn regarding some realizations of the spin-0 dark particle. In the case of a dark bosonic string computed in Sec. 3, we only have a \( O_{8a}^0 \) polarizability, for which the photon-photon search is the less sensitive. However, if one identifies \( \Lambda = l_s^{-1} \), the coefficient of the operator is large, \( c_{8a}^0 = 4\pi^2 q^2 \) (see Eq. (3.8)). For a charge of \( q = 1 \), the sensitivity reaches \( m \sim 2.5 \) TeV and \( \Lambda \sim 3 \) TeV, as shown in Fig. 2.

Regarding the dark spin-0 baryon of the Stealth DM scenario [27], only a polarizability of \( O_{8a}^0 \) has been considered. However, to the best of our understanding, the \( O_{8b}^0 \), \( \hat{O}_{8a}^0 \) operators do not need to be zero, provided that the sum of their coefficients is small (see Sec. 2). This may make an important difference in the prospects for the diphoton search, as the sensitivity to \( \hat{O}_{8a}^0 \) and particularly \( O_{8b}^0 \) is much better than for the \( O_{8a}^0 \) coefficient. Finally, for a pNGB dark particle, we expect all of the three operators to be non-zero.\(^{14}\)

The present study, as a proof of principle, is limited to the spin-0 case and to turning on one operator at a time. Given these encouraging first results, it would be worthwhile

\(^{14}\)For the pNGB dark particle, it is not clear to us if the operators should enter in a specific combination.
to go further by computing the loops in presence of all operators at a time. Also, it would be certainly interesting to similarly analyze the spin-1/2 and spin-1 cases.

5.2 Interplay with other searches for a stable dark particle

Here we briefly discuss the case where the dark particle is stable and identified as dark matter. We recall that, compared to DM searches, a general drawback of the diphoton search is that it does not detect stability, while a general advantage is its sensitivity to the entire spectrum of polarizable dark particles.

Comparison with collider searches. A quantitative comparison with the reach of missing-energy searches obtained in the literature (see e.g. [22, 26]) would require to take into account the nature of the dark particle, the assumed luminosity and center-of-mass energy, assumptions on the couplings, normalization of the operators and statistical criteria. Here we will remain at a qualitative level. We observe that in the prospects for missing-energy based searches at the 13/14 TeV LHC, the sensitivity drops quickly above $m > 1$ TeV. While in our case, one can see from Figs. 2, 3 that the sensitivity goes over regions with masses above $\sim 1$ TeV. This can be understood from the kinematics of the two kinds of process: The cross-section for producing two on-shell dark particles plus other states drops faster with the center-of-mass energy than for a photon-photon final state. There is thus a complementarity between the two kind of searches.

One can also notice that if the stable dark particle has a multiplicity $N$, the diphoton cross-section grows with $N^2$, but the cross-section for pair production grows only with $N$. Thus a large multiplicity for the stable dark particle favours the diphoton search, as the photon-photon production is enhanced by $N$ with respect to pair-production.  

15 A roughly similar conclusion is expected for $N$ particles which are non-degenerate, as the decay chains of unstable particles end up with the stable one and thus contribute to missing energy signatures.
For these reasons we conclude that, qualitatively, the proposed diphoton search seems to compete with and sometimes complement missing-energy searches at the LHC.

**Comment on indirect detection.** A strong constraint on stable polarizable dark particles naturally comes from indirect detection bounds on photons. If the annihilation rate is not velocity-suppressed, these bounds are expected (see [26] and references therein) to dominate over collider and direct searches. As velocity-suppression annihilation is a crucial aspect, we compute the annihilation rate induced simultaneously by the $O_{8a}^0 \ O_{8b}^0 \ O_{6a}^0$ polarizabilities. None of these operators alone lead to a suppressed annihilation rate. However, it turns out that the full squared matrix element takes the form of a complete square

$$|M|_{\phi\phi\rightarrow\gamma\gamma}^2 = \frac{32m^8}{\Lambda^8}(c_{8a} - 4(c_{8b} + \hat{c}_{6a}))^2 + O\left(\frac{(p_i)^2}{m^2}\right). \quad (5.4)$$

Thus there exists a combination of coefficients for which the annihilation rate is velocity-suppressed. Interestingly, this happens in particular for $\hat{c}_{6a} = 0$, $c_{8b} = c_{8a}/4$, which corresponds precisely to coupling the traceless part of $F_{\rho\nu} \ F^\nu_{\rho}$ to $\partial_\mu \phi \partial_\nu \phi$.\footnote{This is consistent with the velocity-suppressed rate found in [39], Tab. 4.}

Such operator appears in particular when integrating out a heavy spin-2 particle, like a KK graviton. It would be interesting to further investigate this effective scenario of a “spin-2 portal”. From the point of view of the diphoton search, the spin-2 particle is a mediator, thus the loop of the polarizable scalar is subdominant with respect to the spin-2 induced four-photon loop. It would be interesting to investigate whether the combination of Eq. (5.4) can vanish in a scenario with intrinsic polarizability.

6 Conclusions

We propose to test the existence of a self-conjugate polarizable particle by searching for the virtual effects it induces. We focus on the process of photon-photon scattering, occurring via loops of this “almost dark” particle. The method does not depend on whether the particle is stable. Thus if there is a dark sector with many polarizable dark particles, the search is sensitive to the cumulative effect of the whole spectrum.

As a preliminary step we classified the CP-even polarizability operators up to dimension 8 for particles with spin 0, 1/2, 1. We further identified two possible scenarios for the microscopic nature of polarizability: mediated and intrinsic polarizability. We illustrate intrinsic polarizability in the case of a neutral bosonic open string and find it is described by dimension-8 operators.

The scenario of a dark particle with intrinsic polarizability is the relevant one for the search we propose. Focussing on the spin-0 case, we evaluate the four-photon helicity amplitudes induced by the dimension-8 polarizability operators. The matching of this effective interaction onto local four-photon operators for $s \ll m^2$ is also provided.

We then evaluate the prospects of a $pp \rightarrow \gamma\gamma pp$ search at the 13 TeV LHC using forward detectors to characterize the intact protons. This channel is known for being sensitive to new physics searches. For operator coefficients equal to 10, it turns out that
the sensitivity in mass and cutoff can go beyond the TeV. For the string with unit charge, mass and inverse string length can be probed up to roughly 1.5 TeV. The center-of-mass energy of the process is typically of $\sim 1$ TeV, hence the EFT expansion is roughly valid unless the coefficients of the operators get too small.

In case the dark particle is stable, it is a DM candidate. In this context we qualitatively compare DM collider searches with our diphoton search. It turns out that these two methods are fairly complementary, as the diphoton search tends to have a sensitivity to higher masses and is multiplicity-enhanced. The annihilation rate of two dark particles into photons is found to be suppressed if the $c_{0a}^0 - 4(c_{0a}^0 + \tilde{c}_{0a}^0)$ combination vanishes. This happens in case of mediated polarizability from a spin-2 particle, and it would be interesting to find a UV completion of intrinsic polarizability in which this cancellation occurs.

We emphasize that the present study of the spin-0 case should be taken as a proof of concept, used to get a rough idea of the sensitivities that can be reached. As the first conclusions seem encouraging, it would be interesting to further analyze the spin-0 case, and to investigate the cases of polarizable self-conjugate particles of spin-1/2 and 1.

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**A Neutral open string in an electromagnetic background**

In this appendix one sets $l_s^2 = \frac{1}{2}$. The string equations of motions in the background field following from Eq. (3.1) are given by

$$\ddot{X}_\mu - X''_\mu = 0,$$  

(A.1)

$$X'_\mu = q F_{\mu\nu} \dot{X}^\nu \text{ if } \sigma \in \{0, \pi\},$$  

(A.2)

with $q_0 = -q_1 = q$. The background field being antisymmetric, it can be brought into a $2 \times 2$ block diagonal form by orthogonal transformations, and it is thus enough to focus on two dimensions, taken to be space dimensions with $\mu = 1, 2$. One has

$$F_{\mu\nu} = \begin{pmatrix} 0 & f \\ -f & 0 \end{pmatrix}, \text{ with } \mu = 1, 2.$$  

(A.3)

It is further convenient to rotate space coordinates as

$$X_+ = \frac{1}{\sqrt{2}}(X_1 + iX_2), \quad X_- = \frac{1}{\sqrt{2}}(X_1 - iX_2).$$  

(A.4)

The boundary conditions become simply

$$X'_+ = -iq f \dot{X}_+ \text{ if } \sigma \in \{0, \pi\},$$  

(A.5)
The oscillator modes are

\[ \psi_n(\sigma, \tau) = \frac{1}{\sqrt{|n|}} \cos (n \sigma + \gamma) e^{-in\tau}, \]  

(A.6)

where \( \gamma = \tan^{-1}(qf) \). The oscillators and the zero mode shown in Eq. (3.2) are orthogonal according to the inner product

\[ \langle \psi_m | \psi_n \rangle = \int_0^\pi d\sigma \pi \psi_m^\dagger(\sigma \tau + \dot{\sigma} + qf) \psi_n = i\delta_{mn} \operatorname{sgn}(n). \]  

(A.7)

One then introduces the canonical momentum \( P_- = \partial L / \partial \dot{X}_+ \), giving

\[ P_- = \frac{1}{\pi} \dot{X}_+ + qA_+(\delta(\sigma) - \delta(\sigma - \pi)) \]  

(A.8)

To go further, one uses the approximation that the background field is constant. The potential is then linear in \( X_{1,2} \), and one can make the following gauge choice as in [48],

\[ A_\mu = \frac{1}{2} f \left( -X_2 \ X_1 \right), \]  

(A.9)

which reproduces well the background field Eq. (A.3) when using the definition \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \). This provides the canonical momentum

\[ P_- = \frac{1}{\pi} \dot{X}_+ + i \frac{q}{2} X_+ \left( \delta(\sigma) - \delta(\sigma - \pi) \right). \]  

(A.10)

We have then everything to express the operators \( x_\pm, p_\pm, a_n^{(\dagger)} \), in terms of \( X_\pm \) and \( P_\pm \), using the inner product and Eq. (A.10). Using the canonical equal-time commutators for \( X_\pm \) and \( P_\pm \)

\[ [X_u(\tau, \sigma), X_v(\tau, \sigma')] = 0, \quad [P_u(\tau, \sigma), P_v(\tau, \sigma')] = 0, \]

\[ [X_u(\tau, \sigma), P_v(\tau, \sigma')] = i\delta_{uv} \delta(\sigma - \sigma'), \]  

(A.11)

we can check that all the operators satisfy well canonical commutation relations. Finally, the \( L_0 \) operator of the Virasoro algebra is given by

\[ L_0 = \frac{1}{2} \sum_{\mu=1}^2 (\dot{X}_\mu + X'_\mu)^2 = (\dot{X}_+ + X'_+) (\dot{X}_- + X'_-), \]  

(A.12)

which gives Eq. (3.3) using

\[ \dot{X}_+ + X'_+ = e^{-i\tau} \left[ \frac{p_+}{\sqrt{1 + q^2f^2}} + \sum_{n=1}^{\infty} \left( a_n e^{-in(\tau + \sigma)} - a_n^\dagger e^{in(\tau + \sigma)} \right) \right], \]  

(A.13)

after rotating back to \( X_{1,2} \) coordinates and putting together all block matrices to restore all dimensions of spacetime.
B Four-photon amplitude calculations

We define $\Delta = m^2 - x(1 - x)q^2$. After loop integration, all the $x$-dependence of the numerators appears via powers of $x(1 - x)$ after combination of all terms. Thus we introduce a basis of loop functions

$$f_n(q^2, m, \Lambda) = \int_0^1 dx (x(1 - x))^n \log \left( \frac{\Delta(q^2)}{\Lambda^2} \right), \quad (B.1)$$

over which all amplitudes decompose. One further introduces the combinations

$$A(q^2, m, \Lambda) = (m^4 f_0 - 2m^2 q^2 f_1 + q^4 f_2), \quad (B.2)$$
$$X(q^2, m, \Lambda) = (3m^4 + 2m^2 q^2) f_0 - (30m^2 q^2 + 2q^4) f_1 + 28q^4 f_2, \quad (B.3)$$
$$C(q^2, m, \Lambda) = (12m^4 + 2q^2 m^2) f_0 - (32m^2 q^2 + 2q^4) f_1 + 24q^4 f_2. \quad (B.4)$$

The helicity amplitudes are then given by

- **$O_{8a}$ operator**

$$M_{++++} = -\frac{(c_{8a}^0)^2}{32\pi^2 A^8} s^2 (X(s, m, \Lambda) + A(t, m, \Lambda) + A(u, m, \Lambda)), \quad (B.5)$$

$$M_{+++-} = -\frac{(c_{8a}^0)^2}{32\pi^2 A^8} (s^2 X(s, m, \Lambda) + t^2 X(t, m, \Lambda) + u^2 X(u, m, \Lambda)), \quad (B.6)$$

$$M_{++++} = 0. \quad (B.7)$$

- **$O_{8b}$ operator**

$$M_{++++} = -\frac{(c_{8b}^0)^2}{8\pi^2 A^8} s^2 C(s, m, \Lambda), \quad (B.8)$$

$$M_{+++-} = -\frac{(c_{8b}^0)^2}{8\pi^2 A^8} (s^2 C(s, m, \Lambda) + t^2 C(t, m, \Lambda) + u^2 C(u, m, \Lambda)), \quad (B.9)$$

$$M_{++++} = 0. \quad (B.10)$$

- **$O_{6a}$ operator**

$$M_{++++} = -\frac{(c_{6a}^0)^2 s^2}{2\pi^2 A^4} f_0(s, m, \Lambda), \quad (B.11)$$

$$M_{+++-} = -\frac{(c_{6a}^0)^2 s^2}{2\pi^2 A^4} (s^2 f_0(s, m, \Lambda) + t^2 f_0(t, m, \Lambda) + u^2 f_0(u, m, \Lambda)), \quad (B.12)$$

$$M_{++++} = 0. \quad (B.13)$$

The unpolarized $\gamma\gamma \rightarrow \gamma\gamma$ cross-section is given by

$$\frac{d\sigma}{dt} = \frac{1}{16\pi s^2} \left( |M_{++++}|^2 + |M_{+++-}|^2 + |M_{+++-}|^2 + |M_{++++}|^2 + 4|M_{++++}|^2 \right). \quad (B.14)$$
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