A Study on Flexible Manufacturing Systems using Petri Net Reduction

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Abstract. Petrinets are the suitable tool for modeling and analysis of discrete event systems. The effective methods of analyzing the discrete event systems (DES) are exhibited by Petri nets. The entire structure of the system is mathematically expressed using Petrinets and also all the properties are analysed by different methods. The properties of Petrinets are different in various systems, a Flexible Manufacturing System (FMS) requires production activities in which raw materials, energy, labour, and equipment are combined together in the modification process for the production of good quality goods.” The word ”flexible ” explains the capability of the system to relate productively to make changes in the system. These changes can be internal failure of the system or classification problems such as a sample or external changes in the structure of the model as well as demand for the process. The inherent limitations such as undecidable properties, complexity in checking, automatic verification of the modeled system are existing in FMS. This paper explains a study on FMS using Petri net reduction. A PN underlies event structures with casuality and compatibility relations. Partial order methods and reduction methods reduce the complexity of constructing the reachability graph. The determination of PN system through reduction methods in recent years focused by many researchers. The significance of reduction methods are highlighted to execute the faster analysis of a system. Instead of coverability trees and the graphical representation the incidence matrix of the PNs are used for the reduction methods. Reduction methods are used to simplify large scale PNs into “equivalent” smaller nets. Boundedness, liveness, simplification of implicit places, shared resources and elimination of self-loops are checked through equivalent smaller nets. The structure of FMS is preserved.

1. Introduction

The determination of PN system through reduction methods in recent years focused by many researchers. Murata [1] explained six reduction rules for the petrinets. The application and analysis of the coverability tree are discussed in the work of Henry et al and also by Han Zandog (2003). In 2011 some changes are made in PN models of some particular discrete systems and it is verified by using four incidence matrix operations. With the continuation of the work properties of Petrinets such as boundedness and liveness of the original PN model is maintained. The application of reduction methods to the inhibitor arc of the Petrinet system by Verbeek, et. al. (2010), in addition these are used to represent cancellation and blocking. In a work of Gui-Yuig et al. verification of the reduction
method of PN is applied to check the accuracy of models of workflow system. Very simple rules of Shen helpful to the huge system which is digitally connected. Large PNs are reduced into many subnets with structural analysis and application designed by Chuanliang Xia (2011). Reduction method for reachability analysis of PN proposed in Han Zandog (2003) for any system consist of numerous places and transitions. In this paper instead of coverability trees and the graphical representation the incidence matrix of the PNs are used for the reduction methods.

Definition 1.1: A Petrinet is a bipartite directed graph represented by a quadruple. PN = (P, T, Pr, Po) where \( P = \{ p_1, p_2, ..., p_n \} \) is a finite set of places, \( T = \{ t_1, t_2, ..., t_m \} \) is a finite set of transitions. \( Pr(p, t) \) is a mapping \( P \times T \rightarrow \{0,1\} \) marked as a arrow diagram with directions from places to transitions. \( Po(t, p) \) is a mapping \( P \times T \rightarrow \{0,1\} \) marked as a arrow diagram with directions from transitions to places.

2. Reduction Rules
The reduction rules of places and transitions with graphical representation and its generalised incidence matrix operations are explained. The application of these rules to Petrinet of FMS with 4 places and 4 transitions are executed using incidence matrix operations.

\[ C = \begin{bmatrix}
  p_1 & p_2 & ... & p_i & ... & p_j & ... & p_m \\
  t_{11} & - & ... & i_1 & ... & j_1 & ... & - \\
  t_{21} & - & ... & - & ... & - & ... & - \\
  t_{31} & 0 & 0 & ... & -1 & 0 & 1 & 0 & 0 \\
  : & : & : & : & : & : & : & : & : \\
  : & : & : & : & : & : & : & : & : \\
  t_{n1} & - & - & i_n & ... & j_n & ... & - \\
\end{bmatrix} \]

Rule:1 - Fusion of Series Places

The transitions \( t_x \) and \( t_y \) will be combined by deleting the place \( p_i \) shown in figure given below. As an outcome of this fusion we have a transformed result \( t_{xy} \), whose incoming and outgoing paths are the combination of incoming and outgoing arrows of \( t_x \) and \( t_y \) respectively the arcs connecting from \( t_x \) to \( p_i \) and from \( p_i \) to \( t_y \).
Rule: 3 - Fusion of Parallel Places [FPP]

The notion of the rule is combining the places coinciding with the incomparable transition and output transition and only with one input arc and one output arc. To apply FPP rule on the incidence matrix, except only one place, all the parallel places are removed.

\[
C = \begin{bmatrix}
P_1 & P_2 & \cdots & P_i & \cdots & P_m \\
T_1 & \cdots & \cdots & 0 & \cdots & \cdots \\
\vdots & \cdots & \cdots & 0 & \cdots & \cdots \\
x_1 & \cdots & \cdots & 1 & \cdots & x_m \\
\vdots & \cdots & \cdots & 0 & \cdots & \cdots \\
y_1 & \cdots & \cdots & -1 & \cdots & y_m \\
\vdots & \cdots & \cdots & 0 & \cdots & \cdots \\
T_n & \cdots & \cdots & 0 & \cdots & \cdots
\end{bmatrix}
\]

Rule: 4 - Fusion of Parallel Transitions [FPT]

In this rule, except one of the row, the rows of combined transitions are deleted. The combination of two transitions occur which has same incoming and outgoing arc from one place therefore the position of resultant matrix row is the combination of same two transitions which has same incoming and outgoing arc. A Petrinet with equivalent transitions marked as the incidence matrix expressed diagrammatically in figure as the following:

\[
C = \begin{bmatrix}
P_1 & \cdots & P_i & \cdots & P_j & \cdots & P_m \\
T_1 & \cdots & T_i & \cdots & \hat{J}_i & \cdots & \cdots \\
\vdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
x_1 & \cdots & 1 & \cdots & 1 & \cdots & x_m \\
\vdots & \cdots & 0 & \cdots & 0 & \cdots & \cdots \\
y_1 & \cdots & -1 & \cdots & -1 & \cdots & y_m \\
\vdots & \cdots & \cdots & 0 & \cdots & \cdots & \cdots \\
T_n & \cdots & 0 & \cdots & 0 & \cdots & \cdots
\end{bmatrix}
\]
Rule :5 - Removal of Self-loop Places (RSP)

The incidence matrix of the Petrinet graph is marked with ‘0’ in the respective column for the Self-loop places. The aim of reduction method is to eliminate isolated places which prevent only zero entries on the matrix representation.

\[
C = \begin{bmatrix}
  p_1 & \cdots & p_i & \cdots & p_m \\
  t_1 & \cdots & 0 & \cdots & \cdots \\
  \vdots & \cdots & 0 & \cdots & \cdots \\
  t_i & \cdots & 0 & \cdots & \cdots \\
  \vdots & \cdots & 0 & \cdots & \cdots \\
  t_n & \cdots & 0 & \cdots & \cdots 
\end{bmatrix}
\]

Rule:6 - Removal of Self-loop Transitions [RST]

In this rule the transition with both input and output arc to the same place must be removed. In matrix representation this transition is entered as zero including an unreachable transition. In these cases row will be deleted from the matrix. The net described in figure has the following incidence matrix:

\[
C = \begin{bmatrix}
  p_1 & \cdots & p_i & \cdots & p_m \\
  t_1 & \cdots & i_1 & \cdots & \cdots \\
  \vdots & \cdots & \cdots & \cdots & \cdots \\
  t_i & \cdots & 0 & \cdots & 0 \\
  \vdots & \cdots & \cdots & \cdots & \cdots \\
  t_n & \cdots & i_n & \cdots & \cdots 
\end{bmatrix}
\]

Example 1:

The net considered here is taken from Murata [1].

\[ \text{Figure 1. Example Petrinet} \]
The incidence matrix of the example Petrinet is

\[
C = \begin{bmatrix}
1 & -1 & 0 & 0 \\
-1 & 1 & 0 & 0 \\
0 & -1 & -1 & 1 \\
0 & 0 & 1 & -1
\end{bmatrix}
\]

Places \( p_3 \) and \( p_4 \) are in series. As an initial stage we have to follow the steps explained as a FSP rule

1. Remove the row ‘4’ from initial matrix.
2. Add the elements of 3rd and 4th columns.
3. Restore the column ‘3’ with the elements formed in step 2 then delete column ‘4’. After these 3 operations the resultant matrix is given as:

\[
C_{fp} = \begin{bmatrix}
1 & -1 & 0 \\
-1 & 1 & 0 \\
0 & -1 & 0
\end{bmatrix}
\]

Now the rule FST can be applied to transitions \( t_1 \) and \( t_2 \) which are in series diagrammatically.

1. Remove the 1st column from the incidence matrix (Place \( p_1 \)).
2. Add the elements of 1st and 2nd row.
3. Restore the row ‘1’ with the elements formed in step 2 and remove row ‘2’.

The reduced one at the end will be as follows:

\[
C_{fa} = \begin{bmatrix}
0 & 0 \\
-1 & 0
\end{bmatrix}
\]

Figure 2(a). Reduced PNI
In this stage, the RSP rule can now be applied to this initial matrix of a net, because there is a second column of $C_{\text{fst}}$ with zero entries which represents a place with self-loop. The stages of applying rules of RSP to the matrix $C_{\text{fst}}$ are as given below. Remove the 2nd column from the reduced matrix.

The new incidence matrix is $C_{\text{rsp}} = \begin{bmatrix} 0 & \cdot \\ -1 & \cdot \end{bmatrix}$.

In the last stage, RST rule can now be applied on newly formed matrix $C_{\text{rsp}}$ the existence of the zero in first row shows a transition with self-loop.
Remove the first row from the reduced matrix $C_{r\alpha p}$. Applying the rule of RST to the last final matrix is as follows:

$$C_{r\alpha p} = [-1].$$

Based on the structure of a large Petrinet in example the existence of selfloop places are removed from the large PN by applying the rule of RSP and similarly the existence of selfloop transitions are removed by applying the rule of RST. These selfloop places and transitions which creates unbounded firing of transitions. The same way the set of parallel transitions and set of parallel places are also adjusted by its fusions leads to the changes in the structure of the PN.

Example 2:

\[ \text{PN Model: The PN (with 8 places and 8 transitions) used in this example is taken from (Zhou and Venkatesh 1999) the Flexible manufacturing system. The initial incidence matrix of the PN in the figure is given as} \]

\[
C = \begin{bmatrix}
t_1 & p_1 & p_2 & p_3 & p_4 & p_5 & p_6 & p_7 & p_8 \\
t_2 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\
t_3 & 1 & 0 & -2 & 1 & 0 & 0 & 0 & 0 \\
t_4 & 0 & 0 & 0 & -1 & 1 & 1 & 0 & 0 \\
t_5 & 0 & 0 & 0 & 0 & -1 & -1 & 1 & 0 \\
t_6 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\
t_7 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\
t_8 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\
\end{bmatrix}
\]

The places $p_2$ and $p_3$ can be combined by the application of FSP rule, in which the transition $t_2$ exist between these two places.

1. Remove the 2nd row ($t_2^t$) from the matrix $C$. 
Add the columns 2 and 3 of $C_{\text{rsp}}$.

$\begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

Restore the 2\textsuperscript{nd} column elements as addition of 2 columns obtained in step 2 and then delete the 3\textsuperscript{rd} column of $C_{\text{rsp}}$.

Next, by applying the rule FPT the transitions $t_6$ and $t_7$ which are in parallel transitions are reduced.

Delete the $t_7$ from the incidence matrix.
By FST rule the transitions $t_1$ and $t_3$ which are in series must be fused. Now $p_2$ is the only one output and input place from $t_1$ and $t_3$ respectively.

1. Remove column $p_2$ from the incidence matrix

$$C_{f_{st}} = \begin{bmatrix} P_1 & P_2 & P_4 & P_5 & P_7 & P_8 \end{bmatrix} = \begin{bmatrix} t_1 & -1 & 2 & -1 & 0 & 0 & 0 \\ t_3 & 1 & -2 & 1 & 0 & 0 & 0 \end{bmatrix}$$

2. Add rows $t_1$ and $t_3$ from $C_{f_{st}}$.

$$\begin{bmatrix} -1 & 1000 \end{bmatrix} + \begin{bmatrix} 11000 \end{bmatrix} = \begin{bmatrix} 00000 \end{bmatrix}$$

3. Restore the row in $t_1$ position which is the sum of rows $t_1$ and $t_3$ and then remove the row $t_3$ from $C_{f_{st}}$.

$$C_{f_{st}} = \begin{bmatrix} P_1 & P_4 & P_5 & P_7 & P_8 \end{bmatrix} = \begin{bmatrix} t_1 & 0 & 0 & 0 & 0 & 0 \\ t_4 & 0 & -1 & 1 & 0 & 0 \\ t_5 & 0 & 0 & -1 & 1 & 0 \\ t_6 & 0 & 0 & 0 & -1 & 1 \\ t_8 & 0 & 1 & 0 & 0 & -1 \end{bmatrix}$$

Next, applying FST rule to combine serial transitions $t_4$ and $t_5$. Place $p_5$ is between $t_4$ and $t_5$.

1. Remove column $p_5$ from the incidence matrix.

$$C_{f_{st'}} = \begin{bmatrix} P_1 & P_4 & P_7 & P_8 \end{bmatrix} = \begin{bmatrix} t_1 & 0 & 0 & 0 & 0 \\ t_4 & 0 & -1 & 0 & 0 \\ t_5 & 0 & 0 & 1 & 0 \\ t_6 & 0 & 0 & -1 & 1 \\ t_8 & 0 & 1 & 0 & -1 \end{bmatrix}$$
2. Add rows $t_4$ and $t_5$ from $C_{fst}^\prime$.

$$[0 \ -1 \ 0 \ 0] + [0 \ 0 \ 1 \ 0] = [0 \ -1 \ 1 \ 0]$$

3. Restore the row in $t_4$ position which is the sum of rows $t_4$ and $t_5$ and then remove the row $t_5$ from $C_{fst}^\prime$.

$$C_{fst}^\prime = t_4 \left[ \begin{array}{cccc} \ p_1 \ p_4 \ p_7 \ p_8 \ \\ 0 \ 0 \ 0 \ 0 \ \\ t_4 \ 0 \ -1 \ 1 \ 0 \ \\ t_6 \ 0 \ 0 \ -1 \ 1 \ \\ t_8 \ 0 \ 1 \ 0 \ -1 \ \end{array} \right]$$

Then, transitions $t_6$ and $t_8$ are in series combined by applying the rule of FST in which the place $p_8$ from $t_6$ and also the outgoing place to $p_8$.

1. Remove the column $p_8$ from the incidence matrix.

$$C_{fst}^\prime = t_4 \left[ \begin{array}{ccc} \ p_1 \ p_4 \ p_7 \ \\ 0 \ 0 \ 0 \ \\ t_4 \ 0 \ -1 \ 1 \ \\ t_6 \ 0 \ 0 \ -1 \ \\ t_8 \ 0 \ 1 \ 0 \ \end{array} \right]$$

2. Add rows $t_6$ and $t_8$ from $C_{fst}^\prime$.

$$[0 \ 0 \ -1] + [0 \ 1 \ 0] = [0 \ 1 \ -1]$$

3. Restore the row in the position of $t_6$ which is the sum of $t_6$ and $t_8$ and then remove the row $t_8$ from $C_{fst}^\prime$.

$$C_{fst}^\prime = t_4 \left[ \begin{array}{ccc} \ p_1 \ p_4 \ p_7 \ \\ t_1 \ 0 \ 0 \ 0 \ \\ t_4 \ 0 \ -1 \ 1 \ \\ t_6 \ 0 \ 0 \ -1 \ \\ t_7 \ 0 \ 1 \ 0 \ \\ \end{array} \right]$$

By the rule FST the transitions $t_4$ and $t_6$ which are in series can be added in which the place $p_7$ from $t_4$ is the incoming place to $t_6$.

1. Remove the column $p_7$ from the incidence matrix.
\[
C_{fst^*} = \begin{bmatrix}
P_1 & P_4 \\
0 & 0 \\
t_4 & 0 \\
t_6 & 1
\end{bmatrix}
\]

2. Add rows \( t_4 \) and \( t_6 \) from \( C_{fst^*} \).

\[
[0 \ -1] + [0 \ 1] = [0 \ 0]
\]

3. Restore the row in the position of \( t_4 \) which is the sum of rows \( t_4 \) and \( t_6 \) and then remove the row of \( t_6 \) from \( C_{fst^*} \).

\[
C_{fst^*} = \begin{bmatrix}
P_1 & P_4 \\
0 & 0 \\
t_4 & 0
\end{bmatrix}
\]

In the reduced matrix the zero entries indicates the presence of self-loop in a net which should be moved from \( C_{fst^*} \).

Therefore the place \( p_1 \) with zero entries eliminated from \( C_{fst^*} \).

\[
C_{rsp} = \begin{bmatrix}
P_4 \\
0 \\
t_4
\end{bmatrix}
\]

\( C_{rsp} \) is the matrix of the reduced Petrinet model. Figure 3 represents the reduced net after the application of reduction rules given in rule 1 to rule 6.

![Figure 3. Reduced Petrinet](image)

3. Conclusion

Analysing the properties of a Petrinet of big in size, it may be very lengthy and also complex for a PN with many reachable marking. Applying rules of linear algebra also becomes very difficult when the size of the Petrinet increases. Reduction methods are used to simplify large scale PNs into equivalent smaller nets. Boundedness, liveness, simplification of implicit places, shared resources and elimination of self-loops are checked though equivalent smaller nets. The structure of FMS is preserved. As a continuation of this work in future the process of reduction could be discussed for unbounded Petrinets.
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