Thermal expansion of two-dimensional itinerant nearly ferromagnetic metal

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Abstract. Thermal expansion of two-dimensional itinerant nearly ferromagnetic metal is investigated according to the recent theoretical development of magneto-volume effect for the three-dimensional weak ferromagnets. We particularly focus on the $T^2$-linear thermal expansion of magnetic origin at low temperatures, so far disregarded by conventional theories. As the effect of thermal spin fluctuations we have found that the $T$-linear thermal expansion coefficient shows strong enhancement by assuming the double Lorentzian form of the non-interacting dynamical susceptibility justified in the small wave-number and low frequency region. It grows faster in proportional to $y^{-1/2}$ as we approach the magnetic instability point than two-dimensional nearly antiferromagnetic metals with $\ln(1/y_s)$ dependence, where $y$ and $y_s$ are the inverses of the reduced uniform and staggered magnetic susceptibilities, respectively. Our result is consistent with the Grüneisen’s relation between the thermal expansion coefficient and the specific heat at low temperatures. In 2-dimensional electron gas we find that the thermal expansion coefficient is divergent with a finite $y$ when the higher order term of non-interacting dynamical susceptibility is taken into account.

1. Introduction

Low dimensional itinerant electron magnets have attracted interest because of the presence of large magnetic fluctuations that inhibit the occurrence of magnetic orders at finite temperatures. In our previous studies, we found that nearly antiferromagnetic itinerant magnets show enhancement of their $T$-linear thermal volume expansion coefficients proportional to $y_s^{-1/2}$ and $\ln(1/y_s)$, for one-dimensional (1D) and two-dimensional (2D) systems, respectively, towards the instability $y_s \rightarrow 0$, where $y_s$ is the reduced staggered susceptibility at $T = 0$ [¹]. The purpose of this paper is to extend our studies to 2D nearly ferromagnetic metal and to discuss the temperature dependence of the thermal expansion at low temperatures. Their magnetic properties have been already studied by Hatatani et. al. [²] for 2D and by Takahashi [³] for quasi 2D cases, respectively.

The effects of spin fluctuations on the magneto-volume expansion have been conventionally analysed based on the following equation by Moriya and Usami [⁴],

$$\omega = pKC[M^2(T) + \xi^2(T)], \quad \xi^2(T) = \sum_q \langle \delta \mathbf{M}_q \cdot \delta \mathbf{M}_{-q} \rangle,$$  \hspace{1cm} (1)
where the magneto-volume strain is defined by $\omega = \delta V/V$. $C$ is a magnetovolume coupling constant. $\rho = N_0/V$ where $N_0$ is the number of magnetic atoms in the crystal. $K$ is a compressibility. It was obtained by straightforward extension of the Wohlfarth’s formula [5], the first term proportional to the squared spontaneous magnetic moment $M(T)$, by including the second term from the effect of thermal spin fluctuation amplitudes. The above formula, however, suffers from the thermodynamical inconsistency between the thermal expansion coefficient and the specific heat, known as the Grüneisen’s relation in the case of lattice vibrations. The impediment has been now resolved by Takahashi and Nakano [6] from the different approach by analysing the explicit volume dependence of the free energy due to spin fluctuations. They have then succeeded in deriving the thermal expansion different from Eq.(1) that satisfies the relation.

In the next section, we will shortly explain how to derive the volume thermal expansion for 2D nearly ferromagnetic metal. The temperature dependence at low temperatures is discussed in Sec. 3. The thermal expansion of 2D electron gas is also studied here. Sec. 4 is devoted to conclusions.

2. Thermal expansion of 2D nearly ferromagnetic metal

We begin with the following free energy [6],

$$F_m(T, V) = \frac{3}{\pi} \sum_q \int_0^\infty d\nu \left[ T \ln \left(1 - e^{-\nu/T}\right) \right] \frac{\Gamma_q}{\nu^2 + \Gamma_q^2} + \Delta F(T, V),$$

$$\Gamma_q = \Gamma_0 q_B^2 g \left( y + \frac{q^2}{q_B^2} \right)$$

where $y$ and $q_B$ are the inverse of the reduced uniform magnetic susceptibility and the Brillouin zone vector, respectively. The first term represents the contribution of collective thermal spin fluctuations with the wave-vector dependent damping $\Gamma_q$. The effects of zero-point fluctuations and the additional constraint on the amplitudes are included in the second term, $\Delta F$. This is the same expression used in order to obtain the temperature dependence of the specific heat [7]. Volume derivative of the first term gives the dominant $T$-linear temperature dependence of the thermal expansion coefficient at low temperatures. Its slope therefore shows the same enhancement as the specific heat in consistent with the Grüneisen’s relation. Moriya and Usami [4], on the other hand, assumed the different Landau expansion form of the free energy, in contradiction to the treatment of the magnetic specific heat.

According to the thermodynamics, the thermal expansion $\omega$ is given by $-K\partial F_m/\partial V$. As the volume dependence of the free energy, let us introduce the magnetic Grüneisen parameters, $\gamma_0 = -d \ln T_0/d \omega$ ($T_0 = \Gamma_0 q_B^2/2\pi$), that causes the volume dependence of the damping constant, i.e. the spectral width of magnetic excitations. It corresponds to the same definition for lattice vibrations. The thermal volume expansion for 2D systems is now derived from the explicit $\omega$-derivative of $F_m$,

$$\omega_l(T) = 6\rho \gamma_0 T \int_0^1 dx x \ln [1/2u - \psi(u)], \quad u = T_0 x (y + x^2)/T \quad (3)$$

where $\psi(u)$ is the digamma function. The temperature dependence of $\omega_l(T)$ comes from both the explicit $T$-dependence as well as the implicit dependence through $y(T)$, the dependence of which is determined by the stability condition of the free energy. Therefore no $y$-derivative terms are present in Eq.(3). The linear thermal expansion coefficient is given by the $T$-derivative of $\omega_l(T)$. We define its reduced value by

$$\alpha_l(T) = \frac{\partial \omega_l(T)}{\partial T}.$$  \quad (4)
**Figure 1.** The reduced temperature dependence \((t = T/T_0)\) of thermal expansion coefficient over the reduced temperatures where the symbols \(*\), \(\square\), and \(\diamond\) indicate \(y_0 = 0.01\), \(y_0 = 0.02\), \(y_0 = 0.03\) when \(\gamma_0 = 0.1\).

### 3. Thermal expansion coefficients in the low-\(T\) limit

The temperature dependence of the thermal expansion \(\omega_t(T)\) and its coefficient \(\alpha_t(T)\) are numerically evaluated based on Eqs.(3) and (4), and their results are shown in Fig.1. The parameter \(y_0\) in the caption is the reduced inverse magnetic susceptibility at \(T = 0\). Particularly in the low temperature limit, the thermal expansion, \(\omega_t(T)\), shows the following \(T^2\)-linear dependence, i.e.

\[
\omega_t(T) \sim \rho K \times \frac{\gamma_0 T^2}{2T_0} \frac{1}{\sqrt{y}} \arctan \frac{1}{\sqrt{y}},
\]

(5)

obtained with the use of the asymptotic expansion of the digamma function,

\[
\ln u - \frac{1}{2u} - \psi(u) \sim \frac{1}{12u^2}, \quad \text{for } u \gg 1.
\]

It shows the stronger enhancement of the \(T\)-linear slope of the thermal expansion coefficient, \(\alpha_t(T)/T \propto y^{-1/2}\), than the case of 2D nearly antiferromagnets as we approach the magnetic instability point.

As another example, let us next discuss the thermal expansion of 2D nearly ferromagnetic electron gas systems, a model of liquid \(^3\)He films, by taking into account the higher order terms of the wave-vector dependence of non-interacting dynamical susceptibility \([8]\). Because of the non-linear wave-vector dependence of the damping constant, the thermal expansion is given by the following similar expression,

\[
\omega_t(T) = 6\rho K \gamma_0 T \int_0^1 dx x \ln u - 1/2u - \psi(u),
\]

(6)

where \(x = q/2k_F\), \(u = T_0 x (1 - x^2)^{\frac{3}{4}} (y + x^2)/T\), and \(k_F\) is the Fermi wave number. The above integral shows singular behavior because its \(T^2\)-linear coefficient shows the \(\log(1/T)\) divergence independent of the value of \(y\).
4. Conclusions
We have investigated the thermal expansion coefficient of 2D nearly ferromagnetic metal and the 2D electron gas model. We find that the $T$-linear slope of the thermal expansion coefficient is strongly enhanced near the magnetic instability point. The result depends on the new contribution of the thermal expansion derived by Takahashi and Nakano [6]. Its presence is required by the thermodynamic Gruneisen’s relation between the thermal expansion coefficient and the specific heat. On the other hand, the thermal expansion of a 2D electron gas model shows singular behavior. We have therefore to be careful of the result including the applicability of our approach to such a system.

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5. References
[1] Konno R, Takahashi Y and Nakano H 2007 J. Appl. Phys. 101, 09G517
[2] Hatatani M and Moriya T 1995 J. Phys. Soc. Jpn. 64, 3434
[3] Takahashi Y 1997 J. Phys. Condens. Matter 9, 10359
[4] Moriya T and Usami K 1980 Solid State Commun. 34, 95
[5] Wohlfarth E P 1977 Physica B91, 305
[6] Takahashi Y and Nakano H 2006 J. Phys. Condens. Matter 18, 521
[7] Takahashi Y and Nakano H 2004 J. Phys. Condens. Matter 16, 4505
[8] Theumann A and Ben-Monod T M 1984 Phys. Rev. B 29, 2567