Reducing post-surgery recovery bed occupancy through an analytical prediction model

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Abstract

Operations Research approaches to surgical scheduling are becoming increasingly popular in both theory and practice. Often these models neglect stochasticity in order to reduce the computational complexity of the problem. In this paper, historical data is used to examine the occupancy of post-surgery recovery spaces as a function of the initial surgical case sequence. We show that the number of patients in the recovery space is well modelled by a Poisson binomial random variable. A mixed integer nonlinear programming model for the surgical case sequencing problem is presented that reduces the maximum expected occupancy in post-surgery recovery spaces. Given the complexity of the problem, Simulated Annealing is used to produce good solutions in short amounts of computational time. Computational experiments are performed to compare the methodology here to a full year of historical data. The solution techniques presented are able to reduce maximum expected recovery occupancy by 18% on average. This reduction alleviates a large amount of stress on staff in the post-surgery recovery spaces and improves the quality of care provided to patients.

Keywords

OR in health services; stochastic; surgical case sequencing; operating room scheduling;

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Conflict of Interest

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1. Introduction

In recent years, a great deal of focus has been placed on improving the performance of the operating theatre (OT) through use of Operations Research approaches. Given an increasing number of surgical requests, and a fixed number of resources, it is necessary to increase surgical throughput in order to keep up with demand. For a variety of reasons the OT is often scheduled as a stand-alone entity, with no regard for the impact on the rest of the hospital.

Surgical schedules can have a large impact on downstream wards (e.g. patient recovery spaces). Not only is it necessary to ensure that patient demand on downstream wards does not exceed capacity, but it is also necessary to ensure that there are sufficient staff available at times of peak demand. As such, we present an analytical model for predicting recovery bed occupancy based on the assumption of lognormally distributed surgery and recovery durations. We then show that this model can be implemented in conjunction with standard Operations Research techniques. This is done to improve the surgical case sequences (SCSs) produced by a large Australian public hospital.

The remainder of the paper is organised as follows. In Section 2 we provide a review of the relevant literature. In Section 3 the motivating case study is discussed and a description of the problem is provided. Section 4 contains the details of the derivation of a Poisson Binomial model of recovery bed occupancy. This leads to the Mixed Integer Nonlinear Programming (MINLP) formulation in Section 5. Due to the complexity of the problem, we implement Simulated Annealing (SA) to produce good feasible solutions (cf. Section 6). Results are presented and discussed in Section 7. Concluding remarks are made in Section 8.

2. Literature Review

OT planning and scheduling is often classified into three main levels: strategic, tactical, and operational. These problems are addressed on long, medium, and short term planning horizons respectively (Cardoen, Demeulemeester, & Beliën, 2010). There is often a hierarchical approach to OT planning and scheduling. Strategic decisions influence tactical decisions which then, in turn, influence operational decisions. Furthermore, the surgical department has the highest impact on hospital workload (Vanberkel et al., 2011). For a thorough review of OT planning and scheduling, see Cardoen, et al. (2010). A review of the use of Operations Research in other hospital wards is provided by van de Vrugt, Schneider, Zonderland, Stanford, and Boucherie (2018).
The master surgical scheduling problem (MSSP) is the tactical problem of allocating surgeons or surgical specialties to OT time blocks. Motivated by the effect of OT department schedules on the entirety of the hospital, a number of authors have considered the impact of the MSS on downstream wards. Vanberkel, et al. (2011) provide a number of distributions including an exact form of ward occupancy based on an initial MSS. More recently, Fügener, Hans, Kolisch, Kortbeek, and Vanberkel (2014) solve the MSSP under downstream unit capacity constraints. In their model, Fügener, et al. (2014) produce probability distributions for the number of patients in the ICU and in the ward on a given day based on a cyclical MSS and aim to minimise downstream costs associated with post-surgical patient flow.

Whilst understanding the patient occupancy distributions caused by the choice of MSS is useful in medium term staff planning, it is important to understand how downstream occupancy changes throughout the day. To capture the daily variability in downstream ward occupancy caused by surgical throughput, operational level OT planning and scheduling decisions must be considered. As such, we focus on the operational level of OT planning and scheduling to predict recovery bed occupancy.

Chow, Puterman, Neda, Wenhai, and Derek (2011) implement a surgical scheduling model based on mixed integer programming (MIP) and Monte Carlo simulation to reduce peak bed occupancies. The MIP model is used to assign both surgeons and patient types to OT time blocks, whilst the Monte Carlo simulation is utilised to approximate recovery occupancy.

Min and Yih (2010) provide a closed form model for the expected number of patients in the surgical intensive care unit (SICU) based on the assumption of normally distributed surgical durations and uniformly distributed patient length of stay (LOS) in the SICU. The authors use this model to minimise patient related costs whilst assigning patients to OT time blocks. The closed form model provided by Min and Yih (2010) is inappropriate whenever these distribution assumptions are violated. In reality, surgical durations are closer to a lognormal distribution Strum, May, and Vargas (1998). Recovery durations are also well-modelled by the lognormal distribution (Spratt, Kozan, & Sinnott, in press). As such, a new model is required to predict post-surgical recovery occupancy.

Recently, a great deal of focus has been placed on ensuring patient throughput does not result in resource shortages in downstream wards. Latorre-Núñez et al. (2016) simultaneously schedule the OT and post anaesthesia recovery spaces when solving the surgical case sequencing problem (SCSP). When producing their schedules, the authors consider only
average durations (both surgical and recovery). The authors indicate that using deterministic surgery and recovery durations is a limitation and that the inclusion of stochasticity should be considered in future.

The contribution of this paper is two-fold. Firstly, we present an analytical model for the distribution of post-surgery recovery bed occupancy based on lognormally distributed surgical and recovery durations (cf. Section 4). Secondly, we present a simple MINLP formulation of the SCSP where the objective is to minimise the maximum expected recovery bed occupancy level (cf. Section 5). In doing so, we show that the analytical model presented in Section 4 can be implemented to produce surgical sequences that are less erratic, thus reducing the stress on post-surgery recovery staff and providing more information to hospital planning and rostering staff members.

3. Case Study and Problem Description

The work presented in this paper is based on a case study of an Australian public hospital. The hospital has a large surgical department consisting of 21 operating rooms (ORs). There are around twenty bed spaces available in the surgical care unit (SCU) and post-anaesthesia care unit (PACU).

The hospital utilises a modified block scheduling policy when generating a four week, rotating MSS. This means that they may reallocate OR time in the case that a surgical team is unavailable or does not require a particular OR block. In this case, other specialties are able to request this time based on the patient demand. The MSS is not modified often, and has remained largely unchanged since the hospital opened.

Nurse and anaesthetist scheduling is performed more regularly and is based on the current MSS. When scheduling nurses and anaesthetists, the hospital must consider their preference or experience regarding the different surgical specialties. Adjustments to the MSS are made without consideration of the availability of nurses and anaesthetists as there are usually sufficient staff available.

At the case study hospital, surgeons work in conjunction with case managers and other administrative staff members to plan and schedule the OT department. Each week, surgeons produce lists of patients that they would like to see during their allocated surgical time. The hospital holds an elective bookings meeting every Thursday to discuss potential cases in the

1 Note: The OT is the set of ORs.
upcoming week. In some cases, surgeons do not provide a list for a particular OR block as they predict non-elective arrivals will occur in the meantime.

On the day of surgery, elective patients are required to arrive on time for their surgery. If the patient is an inpatient (expected to stay for more than a single day) they are allocated to a ward for their stay. The patient is then sent either to their ward or to prepare in the SCU depending on the time until surgery and whether any other procedures are required beforehand.

Most patients recover from surgery in the PACU, with recovery times depending on the type of surgery. Any surgical transfers from the intensive care unit (ICU) are returned directly to the ICU following surgery. Once a patient has recovered sufficiently in the PACU, day patients continue recovery in the SCU whereas inpatients are sent back to the ward or to the ICU. The PACU forms a bottleneck in the system due to its limited capacity. A surgery cannot proceed if there is overcrowding in the PACU. Similarly, a patient may be held in the OR until a space becomes available in the PACU. This, in turn, results in a delay in subsequent surgeries.

In the past, OT planning and scheduling at this hospital has been performed without consideration of downstream resources. Here, we propose that patients are sequenced so as to reduce the strain on the PACU and create a more efficient surgical department by relieving the bottleneck in the system.

4. Predicting Recovery Bed Occupancy

The first step in sequencing patients to reduce PACU strain is to understand how the sequence of patients affects the occupancy of the PACU. Spratt, et al. (in press) show that both surgery and recovery durations at the case study hospital are well-modelled by the lognormal distribution. Figure 1 shows a histogram of the historical surgery and recovery durations of elective patients at the case study hospital. A fitted lognormal pdf is shown in grey.

Whilst there appear to be some deviations from the fitted lognormal distribution, a better fit is obtained when patients are further categorised by their specialty and ASA (American Society of Anesthesiologists) physical status classification code. The ASA code identifies the health of a patient prior to their surgery and ranges from one (healthy) to six (brain-dead).

In order to predict the occupancy of recovery beds in the PACU, we assume that patient surgery and recovery durations are lognormally distributed, with parameters dependent on
surgical specialty and ASA code. The remainder of this section is organised as follows. First, we derive the probability an individual patient is in recovery at any given time. We then discuss the distribution, expectation, and variance of the total number of patients in recovery at any given time. A Normal approximation is used to identify 95% predictive intervals. We then compare our prediction to historical data.

**Figure 1:** Historical surgery and recovery durations compared to lognormal distributions.

### 4.1. Predicting an Individual’s Recovery

Here, patient $p$’s surgical duration and recovery duration are assumed to be lognormally distributed, and are denoted $S_p$ and $R_p$ respectively. As the sum of lognormal distributions can be approximated by a lognormal distribution, $T_p$ is the random variable representing the total duration of patient $p$’s surgery and recovery.

\[
S_p \sim \text{Lognormal}(\mu_p, \sigma_p^2) \\
F_{S_p}(t) = \frac{1}{2} + \frac{1}{2} \text{erf} \left( \frac{\ln(t) - \mu_p}{\sqrt{2}\sigma_p} \right), \quad t \geq 0
\]

\[
R_p \sim \text{Lognormal}(\tilde{\mu}_p, \tilde{\sigma}_p^2) \\
T_p \approx S_p + R_p
\]

\[
T_p \sim \text{Lognormal}(\tilde{\mu}_p, \tilde{\sigma}_p^2) \\
F_{T_p}(t) = \frac{1}{2} + \frac{1}{2} \text{erf} \left( \frac{\ln(t) - \tilde{\mu}_p}{\sqrt{2}\tilde{\sigma}_p} \right), \quad t \geq 0
\]

The parameter values for the random variable $T_p$ are determined through moment matching.

\[
\tilde{\mu}_p = \ln \left( \frac{M_p^2}{\sqrt{V_p + M_p^2}} \right)
\]
\[
\hat{\sigma}_p^2 = \ln \left( \frac{V_p + M_p^2}{M_p^2} \right)
\]
\[
M_p = e^{\mu_p + \frac{\sigma_p^2}{2}} + e^{\bar{\mu}_p + \frac{\bar{\sigma}_p^2}{2}}
\]
\[
V_p = (e^{\sigma_p^2} - 1)e^{2\mu_p + \sigma_p^2} + (e^{\bar{\sigma}_p^2} - 1)e^{2\bar{\mu}_p + \bar{\sigma}_p^2}
\]

A binary variable, \(X_p(t)\) can be used to indicate whether patient \(p\) is in recovery at time \(t\).

It is assumed that the start time of patient \(p\)’s surgery is known and takes the value \(Z_p\).

\[
X_p(t) = \begin{cases} 
1, & \text{patient } p \text{ is in recovery at time } t \\
0, & \text{otherwise}
\end{cases}
\]

\[
\Pr(X_p(t) = 1) = \Pr(Z_p + S_p \leq t \leq Z_p + T_p)
= \Pr(t - Z_p \leq T_p) - \Pr(t - Z_p < S_p)
= \Pr(S_p \leq t - Z_p) - \Pr(T_p < t - Z_p)
= F_{S_p}(t - Z_p) - F_{T_p}(t - Z_p)
= \frac{1}{2} \text{erf} \left( \frac{\ln(t - Z_p) - \mu_p}{\sqrt{2} \sigma_p} \right) - \frac{1}{2} \text{erf} \left( \frac{\ln(t - Z_p) - \hat{\mu}_p}{\sqrt{2} \hat{\sigma}_p} \right),
\]

\[
Z_p \leq t \leq Z_p + \exp \left( \frac{\hat{\sigma}_p \mu_p - \sigma_p \hat{\mu}_p}{\hat{\sigma}_p - \sigma_p} \right)
\]

### 4.2. Predicting the Number of Patients in Recovery

Let \(N(t)\) be the number of patients in recovery at time \(t\). Here, in order to simplify remaining calculations, it is assumed that patient surgery durations and recovery times are independent.

Based on this assumption, \(N(t)\) is distributed according to a Poisson binomial distribution. It is possible to determine the expectation and variance of the number of patients in recovery at time \(t\).

\[
E[N(t)] = \sum_{p \in P} \Pr(X_p(t) = 1)
= \sum_{p \in P} \left( \frac{1}{2} \text{erf} \left( \frac{\ln(t - Z_p) - \mu_p}{\sqrt{2} \sigma_p} \right) - \frac{1}{2} \text{erf} \left( \frac{\ln(t - Z_p) - \hat{\mu}_p}{\sqrt{2} \hat{\sigma}_p} \right) \right)
\]

\[
\text{Var}[N(t)] = \sum_{p \in P} \Pr(X_p(t) = 1) (1 - \Pr(X_p(t) = 1))
= \sum_{p \in P} \left( \frac{1}{2} \text{erf} \left( \frac{\ln(t - Z_p) - \mu_p}{\sqrt{2} \sigma_p} \right) - \frac{1}{2} \text{erf} \left( \frac{\ln(t - Z_p) - \hat{\mu}_p}{\sqrt{2} \hat{\sigma}_p} \right) \right)
\times \left( 1 - \frac{1}{2} \text{erf} \left( \frac{\ln(t - Z_p) - \mu_p}{\sqrt{2} \sigma_p} \right) + \frac{1}{2} \text{erf} \left( \frac{\ln(t - Z_p) - \hat{\mu}_p}{\sqrt{2} \hat{\sigma}_p} \right) \right)
\]
The cumulative distribution function (CDF) of $N(t)$ is determined according to Hong (2013).

$$F_{N(t)}(k) = \frac{1}{n+1} \sum_{l=0}^{n} \{1 - \exp[-i\omega(l + 1)]\} x_l,$$

where $\omega = \frac{2\pi}{n+1}$,

and $x_l = \prod_{j=1}^{n}(1 - p_j + p_j \exp(i \omega l)].$

The above CDF must be evaluated at every value of $t$, as such it is too computationally expensive to be implemented in real-world situations. Here, in order to estimate 95% prediction intervals, $N(t)$ are approximated with a normal distribution, $Y(t)$.

$$N(t) \approx Y(t) \sim N(\text{E}[N(t)], \text{Var}[N(t)])$$

Historical data indicates that this approximation is suitable when examining the 95% prediction intervals.

$$\Pr(L(t) \leq Y(t) \leq U(t)) = 0.95,$$

where $L(t) = \text{E}[N(t)] - 1.96\sqrt{\text{Var}[N(t)]},$

and $U(t) = \text{E}[N(t)] + 1.96\sqrt{\text{Var}[N(t)]}.$

### 4.3. Model Validation

We validate the model presented above by comparing predictions to historical PACU occupancy levels through the duration of 2016. In particular, we are interested in the mean, variance, and estimated 95% prediction intervals. Figure 2 shows the actual recovery occupancy, the expected recovery occupancy, and the estimated 95% prediction interval for a week in February 2016.

Figure 2 indicates that there is a good agreement between the model and historical data. The peaks and troughs in historical PACU occupancy align well with the peaks and troughs in expected occupancy. The historical recovery occupancy is within the shaded 95% prediction interval for most of the week.

When validating the model against historical data, the historical and expected occupancies are calculated every 0.1 hours for the duration of 2016. On average, there was a difference of 0.07 in the historical and expected occupancy levels. As such, the model presented here slightly underestimates occupancy levels. The model overestimates occupancy 49.5% of the time and underestimates 49.6% of the time. Approximately 0.9% of the time, the model and historical
data were in exact agreement. Historical data showed actual occupancy exceeded the upper bound on the approximate 95% prediction interval 3.3% of the time. Actual occupancy was below the lower bound on the approximate 95% prediction interval only 0.2% of the time.

![Figure 2: Comparison of model to historical data.](image)

In future it may be worth considering modelling durations with truncated distributions rather than standard lognormal distributions. It may also be possible to refine the prediction model by incorporating real-time schedule realisations to assist in the recovery occupancy predictions. This is not included in the model as we focus instead on start-of-day schedules, prior to any schedule realisations.

5. Surgical case sequencing model

In this section, a multi-objective MINLP formulation for the surgical case sequencing problem is presented. The maximum expected number of patients in recovery at any one time is minimised. As such, this model can be used to level the occupancy of the PACU.

**5.1. Scalar Parameters**

$\overline{H}$: the number of surgeons scheduled for the day.

$\hat{P}$: the number of patients scheduled for the day.

$\overline{R}$: the number of ORs.

$\lambda$: the number of hours that each OR is open for during a standard working day.
\( \lambda^* \): the number of hours in a day.

### 5.2. Index Sets

- \( H \): the set of surgeons that practice at the hospital. \( H = \{1, \ldots, \bar{H}\} \)
- \( P \): the set of patients that are scheduled for the day. \( P = \{1, \ldots, \bar{P}\} \)
- \( R \): the set of ORs. \( R = \{1, \ldots, \bar{R}\} \)
- \( P_h \): the set of patients treated by surgeon \( h \), \( \forall h \in H \).
- \( P_r \): the set of patients treated in OR \( r \), \( \forall r \in R \).

### 5.3. Indices

- \( h \): index for surgeon in set \( H \).
- \( p \): index for patient in set \( P \).
- \( q \): alternative index for patient in set \( P \).
- \( r \): index for OR in set \( R \).

### 5.4. Vector Parameters

- \( \beta_h \): the setup time surgeon \( h \) requires when starting in a new OR, \( \forall h \in H \).
- \( \gamma_p \): 1 if patient \( p \) requires post-surgery recovery in the recovery space, 0 otherwise, \( \forall p \in P \).
- \( \tau_p \): the expected duration of patient \( p \)'s surgery, \( \forall p \in P \).
- \( \mu_p \): the mean of the natural logarithm of patient \( p \)'s surgical duration, \( \forall p \in P \).
- \( \sigma_p^2 \): the variance of the natural logarithm of patient \( p \)'s surgical duration, \( \forall p \in P \).
- \( \tilde{\mu}_p \): the mean of the natural logarithm of patient \( p \)'s recovery duration, \( \forall p \in P \).
- \( \tilde{\sigma}_p^2 \): the variance of the natural logarithm of patient \( p \)'s recovery duration, \( \forall p \in P \).
- \( \hat{\mu}_p \): the mean of the natural logarithm of the total duration of patient \( p \)'s surgery and recovery, \( \forall p \in P \).
- \( \hat{\sigma}_p^2 \): the variance of the natural logarithm of the total duration of patient \( p \)'s surgery and recovery, \( \forall p \in P \).
- \( \rho_h \): the start of shift time of surgeon \( h \), \( \forall h \in H \).
- \( T_p^+ \): the setup time (in hours) before patient \( p \)'s surgery, \( \forall p \in P \).
- \( T_p^- \): the clean-up time (in hours) after patient \( p \)'s surgery, \( \forall p \in P \).

### 5.5. Decision Variables

- \( V_{pq} \): 1 if patient \( q \)'s surgery starts during patient \( p \)'s surgery, 0 otherwise, \( \forall p, q \in P \).
$U_{pq}$: 1 if patient $q$’s surgery starts after patient $p$’s surgery starts, 0 otherwise, $\forall p, q \in P$.

$Z_p$: the expected start time of patient $p$’s surgery, $\forall p \in P$.

$Z_p^*$: the expected end time of patient $p$’s surgery, $\forall p \in P$.

$\Omega_h$: 1 if surgeon $h$ will require overtime, 0 otherwise, $\forall h \in H$.

$O_h$: the expected overtime associated with surgeon $h$, $\forall h \in H$.

### 5.6. Objective Function

The main objective is to minimise the maximum expected number of patients in recovery at any one time throughout the surgical department opening hours. In doing so, the demand on PACU staff is levelled. This not only assists in staff planning but may result in increased staff satisfaction and better care for patients in recovery. An example occupancy profile is provided in Figure 3.

**Figure 3:** An example occupancy profile over a single day before and after optimisation.

**Figure 3** includes both the original (historical) expected occupancy profile, and the improved expected occupancy profile. Whilst the historical occupancy profile exhibits peaks and troughs in demands, by reducing the maximum expected occupancy (MEO) throughout OT opening hours, the improved occupancy profile is steadier and predicts level demand throughout opening hours.

The objective function is formulated based on the work presented in Section 4.

Minimise

$$\max_{0 \leq t \leq \lambda} \sum_{p \in P} \gamma_p \left\{ \frac{1}{2} \text{erf} \left( \frac{\ln(t - Z_p) - \mu_p}{\sqrt{2}\sigma_p} \right) - \frac{1}{2} \text{erf} \left( \frac{\ln(t - Z_p) - \tilde{\mu}_p}{\sqrt{2}\tilde{\sigma}_p} \right) \right\} , \quad Z_p \leq t \leq Z_p^* + \exp \left( \frac{\tilde{\sigma}_p \mu_p - \sigma_p \tilde{\mu}_p}{\tilde{\sigma}_p - \sigma_p} \right)$$

This objective is equivalent to
Minimise
\[
\max_{0 \leq t \leq \lambda} \sum_{p \in P} \frac{\gamma_p}{2} \left[ \text{erf} \left( \frac{\ln(\max(t - Z_p, 0)) - \mu_p}{\sqrt{2}\sigma_p} \right) - \text{erf} \left( \frac{\ln(\max(t - Z_p, 0)) - \hat{\mu}_p}{\sqrt{2}\sigma_p} \right) \right],
\] (1)
when \(Z_p + \exp \left( \frac{\sigma_p(\mu_p - \mu_p) - \sigma_p}{\sigma_p} \right) > \lambda^*\). This condition holds for the parameters of interest in the case study.

5.7. Constraints

Surgeon \(h\)’s surgeries must be expected to start after the start of their shift.

\[
Z_p \geq \rho_h, \forall \ h \in H, p \in P_h
\] (2)

Constraint (2) is used to ensure that surgeries are completed before the end of surgeon \(h\)’s shift.

\[
Z_p + \tau_p \leq \rho^*_h + O_h, \forall \ h \in H, p \in P_h
\] (3)

In some instances, overtime may be required (as seen in constraint (2)). For each surgeon, the amount of overtime available is limited by the expected duration of their patient’s surgeries, with provision for setup and cleanup times. In most cases, surgeons will not be allocated overtime.

\[
O_h \leq \Omega_h \left( \sum_{p \in P_h} (\tau_p + T^+_p + T^-_p) - \rho_h + \rho^*_h \right), \forall \ h \in H
\] (4)

Constraints (5) and (6) determine whether patient \(q\) ends on or after the time patient \(p\) starts.

\[
Z_p - Z^*_q > -\lambda^*(U_{pq} - 1), \forall p, q \in P, p \neq q
\] (5)

\[
Z^*_q - Z_p \geq \lambda^*(U_{pq} - 1), \forall p, q \in P, p \neq q
\] (6)

Constraints (7) and (8) determine whether the treatment of patients overlaps. Although this could be done with a single constraint, that constraint would be nonlinear.

\[
V_{pq} \geq U_{pq} + U_{qp} - 1, \forall p, q \in P, p \neq q
\] (7)

\[
2V_{pq} \leq U_{pq} + U_{qp}, \forall p, q \in P, p \neq q
\] (8)

Each surgeon must not be assigned to treat more than one patient at a time.

\[
V_{pq} = 0, \forall h \in H, p, q \in P_h, p \neq q
\] (9)

If two patients’ surgeries overlap, then they must not be treated in the same OR.
\[ V_{pq} = 0, \forall r \in R, p, q \in P_r, p \neq q \]  

Constraint (11) is used to calculate the expected finish time of each surgery.

\[ Z_p^* = Z_p + \tau_p, \forall p \in P \]  

If two patients are treated by the same surgeon, or in the same OR, then they require clean-up and setup time between surgeries.

\[ Z_q \geq Z_p^* + T_q^+ + T_p^- - \lambda(1 - U_{pq}), \forall h \in H, p, q \in P_h, p \neq q \]  

\[ Z_q \geq Z_p^* + T_q^+ + T_p^- - \lambda(1 - U_{pq}), \forall r \in R, p, q \in P_r, p \neq q \]

Overtime must be non-negative.

\[ O_h \geq 0, \forall h \in H \]

### 5.8. Model Assumptions

A number of assumptions are made in order to simplify the model presented. The following assumptions are made:

- Surgical and recovery durations are independent.
- Non-elective patients are not considered as the hospital currently has other reservation techniques for their surgeries.
- Each patient’s surgeon and OR is known in advance based on the surgical case assignments made on a weekly basis.
- There is no delay between a patient’s surgery and recovery.
- Overtime may be unavoidable depending on the predetermined surgical case assignments.

### 6. Solution Approach

The SCSP is NP-hard as it is reducible to a machine scheduling problem. In addition to this, the objective function presented in Section 5.6 is nonlinear, nonconvex, and not continuously differentiable. Commercial solvers are inadequate.

Simulated Annealing (SA) (Kirkpatrick, Gelatt, & Vecchi, 1983) is a simple metaheuristic based on the cooling of metals. By occasionally accepting worsening solutions, SA is able to escape local optima in the search of global optima. Given the simplicity, ease of
implementation, and computational efficiency, SA is applied to solve the SCSP presented in Section 5. The metaheuristic is implemented in MATLAB R2017b on a desktop computer with an Intel® Core™ i7-6700 CPU with 16GB of RAM.

6.1. Constructive Heuristic and Local Search

At each iteration, a schedule can be constructed by defining the order of patient’s surgeries, and determining the earliest start and latest completion of each patient’s surgery based on this order. These surgeries are then allocated such that they start after their earliest start, and finish before their latest completion times, unless overtime is necessary. The pseudocode is provided in Algorithm 1.

**Algorithm 1:** Constructive Heuristic.

Late\text{st Completion (LC)} \leftarrow \text{OR closing time}

Earliest Start (ES) \leftarrow \text{OR opening time}

\textbf{FOR} p in reverse patient sequence

Successors (S) \leftarrow the set of patients after patient p in the same OR or treated by the same surgeon.

\textbf{IF} Successors exist

\hspace{1em} LC(p) \leftarrow \text{min} (\text{LC}(S) - \text{Duration}(S) - \text{Setup}(S)) - \text{Cleanup}(p)

\textbf{END}

\textbf{END}

\textbf{FOR} p in patient sequence

Predecessors (P) \leftarrow the set of patients before patient p in the same OR or treated by the same surgeon.

\textbf{IF} Predecessors exist

\hspace{1em} ES(p) \leftarrow \text{max} (\text{ES}(P) + \text{Duration}(P) + \text{Cleanup}(P) + \text{Setup}(p))

\textbf{END}

\hspace{1em} Start(p) \leftarrow ES(p) + \max (0, \text{rand} \times (LC(p) - ES(p) - \text{Duration}(p)))

\hspace{1em} ES(p) \leftarrow Start(p)

\textbf{END}

When implementing SA, it is necessary to define local search heuristics to explore solution neighbourhoods. Here, we exploit the problem structure in order to find local neighbours. At
each iteration, two patients are randomly selected and their order in the sequence is swapped. A schedule is then randomly generated using Algorithm 1.

### 6.2. Parameter Tuning

To obtain good solutions through SA, metaheuristic parameter tuning is required. A parameter sweep was performed and suitable parameter combinations were considered (see Table 1). Parameter tuning was performed in parallel using MATLAB® R2017b on the university’s High Performance Computing (HPC) facility.

**Table 1:** Set of metaheuristic parameters tested in parameter sweep.

| Parameter | Description                               | Values       |
|-----------|-------------------------------------------|--------------|
| **alpha** | The proportional decrease in temperature. | 0.85, 0.9, **0.95** |
| **Between** | The number of iterations between each temperature decrease. | 50, 100, **200** |
| **MaxIters** | The maximum number of iterations. | 1000, 1500, 2000, **2500**, 3000 |

To simplify the parameter tuning, the starting temperature was set to one. SA was run ten times under each parameter combination on historical data from February 2016. The average of the sum of MEO on each day was used to compare against other parameter combinations. Based on these computational experiments, the best performing metaheuristic parameters are highlighted in bold (Table 1).
The average results obtained under the best performing parameter combination are displayed in Figure 4. These results are compared to the historical MEO on each weekday in February 2016. The mean MEO for each day is shown in teal.

7. Results and Discussion

In this section, we perform computational experiments to compare the actual (historical) MEO to the improved MEO for each day in 2016. The metaheuristic parameters were selected based on the parameter tuning performed in Section 6.2. Computational experiments were performed using MATLAB® R2017b on a desktop computer with an Intel® Core™ i7 processor @ 3.40GHz with 16GB of RAM.

For each working day in 2016 (i.e. any day that elective surgeries were performed), SA was run ten times. Figure 5 shows the historical MEO compared to improved occupancy for each day in 2016. The teal line shows a cubic spline through the data.
From Figure 5 it can be seen that whilst there are a few instances where the historical MEO was not improved, in most instances, the MEO was decreased significantly. On average, there was an 18% reduction in MEO. In some instances this reduction was as large as 47%. Such large reductions in MEO have the effect of levelling the workload of staff in the PACU. This, in turn, may result in improved patient care and reduced levels of stress in staff members.

Figure 6 shows how the throughput of the OT (in number of elective patients treated) affects the MEO of the PACU. In black, historical throughput on each day in 2016 is compared to the MEO of the PACU on that day. The improved maximum occupancy is plotted in teal. A cubic spline is fitted to each set of data to show the general trend. Larger relative reductions in MEO are seen as the number of elective patients treated increases. It appears that in either the historical or improved case, the MEO is approximately proportional to the number of elective patients treated. The exception to this is the slight downward trend in improved MEO seen at around 50 patients. It is unclear whether this is a true trend, or if this is an artefact of having few observations at those values of throughput.
Figure 6: Throughput compared to MEO.

The metaheuristic approach presented in Section 6 runs in time $O(n)$, where $n$ is the number of elective patients to be sequenced on a given day. Computational experiments were performed using MATLAB R2017b on a desktop computer with an Intel® Core™ i7-6700 CPU with 16GB of RAM. Under the parameters reported in Table 1, the metaheuristic approach requires under six seconds to run on a standard desktop computer. This ensures that the solution methodology is accessible to hospital planning staff. Given such a short runtime, it would be reasonable to produce several schedules for the day and allow staff members to use their expert knowledge to select the most appropriate schedule.

8. Concluding Remarks

Hospital processes are subject to a wide range of uncertainty. The surgical department is often one of the main sources of uncertainty in the hospital. To ensure that resources are used efficiently, Operations Research techniques may be implemented to improve planning and scheduling. Given the highly random nature of the surgical department, it is necessary to consider stochasticity in a variety of forms, but particularly in task durations.

In this paper we presented an analytical model to predict the occupancy of the post-surgery recovery spaces. This model is based on the assumption of lognormally distributed surgery and recovery durations. The only inputs required to predict recovery bed occupancy are the expected start times of each patient’s surgery, and the parameters associated with the patients’
surgery and recovery duration distributions. Validation on historical data shows that this model is effective at capturing the mean recovery bed occupancy levels.

Based on the recovery occupancy model, a MINLP formulation is provided for the SCSP. The objective is to minimise the maximum expected recovery bed occupancy. Given the complexity of the SCSP, and the intricacies of the objective function, it is necessary to implement metaheuristic approaches to produce good feasible solutions. We show that, through the solution techniques implemented here, the maximum expected recovery bed occupancy is reduced by 18% on average, compared to historical data at the case study hospital.

The work presented in this paper will ensure more level demand on the PACU. This may have the effect of reducing stress on PACU staff, improving the quality of care for patients, and reducing bottlenecks associated with this limited resource.

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