Multibaryons with heavy flavors in the Skyrme model

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We investigate the possible existence of multibaryons with heavy flavor quantum numbers using the bound state approach to the topological soliton model and the recently proposed approximation for multiskyrmion fields based on rational maps. We use an effective interaction lagrangian which consistently incorporates both chiral symmetry and the heavy quark symmetry including the corrections up to order $O(1/m_Q)$. The model predicts some narrow heavy flavored multibaryon states with baryon number four and seven.

PACS number(s): 12.39.Dc, 12.39.Fe, 14.20.Lq, 14.20.Mr.

I. INTRODUCTION

In recent years multibaryons have received a considerable amount of attention. A reason for this is that their hypothetical existence could provide valuable information about the nature of the strong interactions at low energies. Perhaps the most celebrated example is that of the H-dibaryon predicted by Jaffe more than twenty years ago. Since then the possible existence of some other exotic states has been investigated in various models. Of particular interest are those containing flavor (i.e. $S$, $C$, $B$) quantum numbers. In fact, it has been speculated that strange matter could be stable. This has lead to numerous investigations of the properties of strange matter in bulk and in finite lumps (for a recent review see Ref. [3]). Moreover, with the advent of heavy ion colliders there is now the possibility of producing strange and even charmed multibaryonic states with rather low baryon number in the laboratory. These new developments provide further motivation for the study of the properties of multibaryons with heavy flavor quantum numbers. Most of the known predictions come from MIT bag model (see for example Refs. [6]) or non-relativistic quark model based calculations. Here, we will adopt a different point of view. We will assume that the heavy flavor multibaryons are formed by an SU(2) multiskyrmion with some heavy mesons bound to it. This is basically an extension of the bound state approach to strange hyperons originally introduced by Callan and Klebanov and later shown to describe heavier flavor baryons as well. A study of strange multibaryons within this approach has been recently presented in Ref. [9]. There, a chiral lagrangian written in terms of pseudoscalar meson fields with some chiral symmetry breaking terms has been used as the effective lagrangian. As well known by now, although adequate for the light (up and down) and strange sectors, such type of effective lagrangian has to be modified when heavier flavors (e.g. charm) are incorporated. In that case, heavy quark symmetry has to be imposed. This symmetry requires that both the heavy pseudoscalar and the heavy vector fields appear explicitly in the effective lagrangian. Lagrangians which have both chiral symmetry and heavy quark symmetry have been described in the literature. In our calculation we will adopt such type of lagrangian. As already mentioned, in our description the baryon number of the system comes from a non-trivial soliton configuration in the light sector. Until very recently only few multiskyrmion configurations (i.e. those with $B \leq 4$) were known. In 1997, however, after some demanding numerical work Battye and Sutcliffe were able to identify those which are believed to be the lowest energy configurations with baryon number up to $B = 9$. Interestingly, all these configurations have the symmetries corresponding to the regular polyhedra. Even more important for our purposes, Houghton, Manton and Sutcliffe have exploited the similarities between the BPS monopoles and skyrmions to propose some ansätze based on rational maps. They have shown that such configurations approximate very well the numerically found lowest energy solutions with $B \leq 9$. In our investigations we will make use of these approximate ansätze. An interesting feature of these configurations is that for $B > 1$ the derivative of the radial soliton profile vanishes at the origin. Since to leading order in the inverse of the heavy quark mass $m_Q$ the heavy meson-soliton interaction is proportional to this quantity we do not expect any bound state in that approximation. However, next-to-leading order corrections in $1/m_Q$ are required even to describe the spectrum of $B = 1$ heavy baryons. In the present work these corrections will be properly taken into account.
This article is organized as follows. In Sec.II we introduce the effective lagrangian together with the ansatz for the multiskyrmon configurations. In Sec.III we present our numerical results. Finally, in Sec.IV our main conclusions are given.

II. THE LAGRANGIAN

To describe the dynamics of the light and massive mesons interacting with each other we will consider an effective lagrangian of the following form \[12\]

\[
L = L_l + (D_\mu \phi) \dagger D^\mu \phi - m_\pi^2 \phi \dagger \phi - \frac{1}{2} \psi_\mu \dagger \psi^\mu + m_\psi \psi_\mu \dagger \psi^\mu + if \left( \phi_\mu a^\mu + \psi_\mu (a^\mu \phi) + \frac{i}{2} g \epsilon^{\mu\nu\rho\sigma} (\psi_{\mu\rho} \phi_{\nu\sigma} + \psi_{\nu\sigma} \phi_{\rho\mu}) \right),
\]

where \(L_l\) is the effective light meson lagrangian. In the present work we will consider pions as the only explicit light degree of freedom and choose \(L_l\) to be simply the Skyrme lagrangian. Consequently, the effective lagrangian \(\hat{L}_l\), written in terms of the chiral field \(U = \exp(i \vec{\tau} \cdot \vec{R} / f_\pi)\), reads

\[
\hat{L}_l = \frac{f_\pi^2}{4} \text{Tr} [\partial_\mu U^\dagger \partial^\mu U] + \frac{1}{32\pi^2} \text{Tr} \left[ (U^\dagger \partial_\mu U) (U^\dagger \partial_\nu U) \right]^2 + \frac{f_\pi^2 m_\pi^2}{4} \text{Tr} [U^\dagger + U - 2].
\]

Here, \(f_\pi\) is the pion decay constant and \(\epsilon\) is the so-called Skyrme parameter. In Eq.(1), \(\phi\) represents the heavy pseudoscalar doublet and \(\psi_\mu\) the corresponding vector doublet. For example, for charmed mesons

\[
\phi = \begin{pmatrix} D^0 \ CD^+ \end{pmatrix}; \quad \psi = \begin{pmatrix} D^{\ast 0} \\ D^+ \end{pmatrix}.
\]

Moreover, \(f\) and \(g\) are the \(\phi\psi_\mu \pi\) and \(\psi_\mu \psi_\pi\) coupling constants, respectively, and

\[
D_\mu = \partial_\mu + v_\mu, \quad (4)
\]

\[
\psi_{\mu\nu} = D_\mu \psi_\nu - D_\nu \psi_\mu. \quad (5)
\]

Finally, in terms of the chiral field the currents \(v_\mu\) and \(a_\mu\) read

\[
v_\mu = \frac{1}{2} \left( \sqrt{U} \partial_\mu \sqrt{U} + \sqrt{U} \partial_\mu \sqrt{U} \right), \quad (6)
\]

\[
a_\mu = \frac{1}{2} \left( \sqrt{U} \partial_\mu \sqrt{U} - \sqrt{U} \partial_\mu \sqrt{U} \right). \quad (7)
\]

As usual in the bound state model we should first determine the static skyrmion background. For this purpose we introduce the rational map ansatz for the pion field. It reads \([14]\)

\[
\tilde{\pi} = f_\pi F(r) \hat{n}.
\]

Here, \(F(r)\) is the (multi)skyrmion profile which depends on the radial coordinate only and \(\hat{n}\) is a unit vector given by

\[
\hat{n} = \frac{1}{1 + |R|^2} \left( 2 \Re(R) \hat{i} + 2 \Im(R) \hat{j} + (1 - |R|^2) \hat{k} \right), \quad (9)
\]

where \(R\) is the rational map corresponding to a certain winding number \(B\) which is identified with the baryon number. Such map is usually written as a function of the complex variable \(z\) which is related to the usual spherical coordinates \(\theta, \phi\) via stereographic projection, namely \(z = \tan(\theta/2) \exp(i \phi)\). For example, the map corresponding to the \(B = 1\) hedgehog ansatz is the identity map \(R = z\). The explicit form of the maps corresponding to the other baryon numbers \(B \leq 9\) can be found in Ref. [14]. Using Eq.(8) it is possible to obtain the expressions of the \(a_\mu\) and \(v_\mu\) currents to leading order in \(N_c\). The time components vanish at this order while the space components result

\[
a^i = -\frac{i}{2} \left[ F' \cdot \hat{n} \cdot \hat{n} + s \hat{r} \cdot \nabla \hat{n} \right],
\]

\[
v^i = -i \hat{k} \left( \hat{n} \times \nabla \hat{n} \right) \cdot \hat{r},
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\]

\[
v^i = -i \hat{k} \left( \hat{n} \times \nabla \hat{n} \right) \cdot \hat{r}, \quad (11)
\]
where we have introduced the short hand notation $s = \sin F$, $\hat{s} = \sin(F/2)$. The radial profile function $F(r)$ is determined by minimizing the soliton energy. Details of this procedure as well as plots of these functions for different baryon numbers are given in Ref. [14].

To order $N_c^0$, we have a system of heavy mesons moving in the static soliton background. To derive the explicit form of the relevant heavy meson-soliton lagrangian we need some consistent ansätze for the heavy meson fields. For the pseudoscalar field we use [3]

$$\phi(r, t) = \frac{1}{\sqrt{4\pi}} \phi(r, t) \vec{n} \cdot \hat{\chi} ,$$

where $\chi$ is a two-component spinor. To obtain the corresponding ansatz for the heavy vector meson field it is convenient to analyze the coupling terms in the effective lagrangian. They are the last two terms in Eq. (1). From their structure and the form of the $a_\mu$ and $v_\mu$ currents in the static limit it is possible to see that the ansatz should have the form

$$\psi^0(r, t) = \frac{i}{\sqrt{4\pi}} \psi_0(r, t) \chi ,$$

$$\psi^j(r, t) = \frac{1}{\sqrt{4\pi}} [\psi_1(r, t) \hat{\tau}^j + i r \psi_2(r, t) (\hat{n} \times \nabla) \hat{n} \hat{\tau}^j] \chi .$$

Replacing Eqs. (8), (12), (13) and (14) in the effective lagrangian, Eq. (4), we obtain that the heavy meson-soliton lagrangian $L_{HM-sol}$ is

$$L_{HM-sol} = \frac{1}{2} \int_0^\infty dr r^2 \left\{ \frac{g^2}{r^2} \psi_0^2 \psi_0 + \frac{\psi_0^4}{r^2} \psi_2 - \left( \frac{\psi_0^2}{r} + \psi_2 - \frac{\hat{s}^2}{r^2} \right) \right\} \frac{1}{2} \left( \psi_2 - \frac{\hat{s}^2}{r^2} \right) + 2i \frac{\hat{s}^2}{r^2} \psi_2 + 4 \int_0^\infty dr r^2 \left\{ \frac{g^2}{r^2} \psi_1^2 \psi_1 + \frac{\psi_1^4}{r^2} \psi_2 - \left( \frac{\psi_1^2}{r} + \psi_2 - \frac{\hat{s}^2}{r^2} \right) \right\} \frac{1}{2} \left( \psi_2 - \frac{\hat{s}^2}{r^2} \right) + 2i \frac{\hat{s}^2}{r^2} \psi_2 + + \text{h.c.}$$

where $s$ and $\hat{s}$ have been already defined, $\phi = \cos(F/2)$ and $\mathcal{I}$ is the angular integral

$$\mathcal{I} = \frac{r^4}{16\pi} \int d\Omega \left( \nabla \cdot \vec{n} \right)^2 .$$

The diagonalization of the hamiltonian obtained from $L_{HM-sol}$ leads to a set of eigenvalue equations for the heavy meson field. They are

$$\phi'' + \frac{2}{r} \phi' - \left( m_\phi^2 - \omega^2 + 2B \frac{d^4}{r^2} \right) \phi - \frac{1}{2} \frac{f^2}{r^2} \psi_1 + f \frac{B}{r^2} \psi_2 = 0 ,$$

$$\psi_0'' + \frac{2}{r} \psi_0' - \left( m_\psi^2 + 2B \frac{d^4}{r^2} \right) \psi_0 + \left( \frac{2\omega}{r} + 2g B \frac{\hat{s}^2}{r^2} \right) \psi_1 + \omega \psi_1'$$

$$- \left( \frac{2B}{r} \left( g f' + \omega \right) \frac{\hat{s}^2}{r^2} + \frac{g}{r} \right) \psi_2 - 2g B \frac{s^2}{r^2} \psi_2' = 0 ,$$

$$2g B \frac{\hat{s}^2}{r^2} \psi_0 - \omega \psi_0' + \left( m_\psi^2 - \omega^2 + 2B \frac{d^4}{r^2} \right) \psi_1 + \frac{1}{2} \frac{f^2}{r^2} \phi + \frac{2B}{r} \left( g \omega - \frac{\hat{s}^2}{r^2} \right) \psi_2 - 2B \frac{\hat{s}^2}{r^2} \psi_2' = 0 ,$$

$$\psi_2'' + \frac{2}{r} \psi_2' + 2 \frac{s^2}{r^2} \phi + \frac{f^2}{r^2} \left( g f' + \omega \right) \psi_0 - 2g B \frac{s}{r} \psi_0' - \frac{s}{r} \left( f \omega + 2g \omega \right) \psi_2 - \frac{\hat{s}^2}{r^2} \psi_2'$$

$$- \left( m_\psi^2 - \omega^2 - g f' + \frac{2\mathcal{I}}{B} \frac{d^4}{r^2} \right) \psi_2 = 0 .$$


The numerical solution of this set of coupled equations supplemented with the appropriate boundary conditions provides the heavy meson energy $\omega$ for the different baryon numbers $B$. The corresponding results are discussed in the following section.

III. NUMERICAL RESULTS

In our numerical calculations we use two sets of parameters in the $SU(2)$ sector. Set A corresponds to the case of massless pions and Set B to the case where the pion mass takes its empirical value $m_\pi = 138$ MeV. In both cases, $f_\pi$ and $\epsilon$ are adjusted so as to reproduce the empirical nucleon and $\Delta$ masses. The fitted values are $f_\pi = 64.5$ MeV, $\epsilon = 5.45$ for Set A and $f_\pi = 54$ MeV, $\epsilon = 4.84$ for Set B. With these parameters fixed and using for each baryon number $B$ the rational map given in Ref. [14], we obtain the profile $F(r)$ that minimizes the mass of the soliton.

We proceed to solve the bound state eigenvalue equations (17)-(20), using the values shown in Table I for the parameters that appear in the heavy meson lagrangian, Eq. (1). For the pseudoscalar and vector meson masses we use the empirical values. On the other hand, since little is known about the heavy meson coupling constants, we use the heavy quark symmetry relation [12]

$$ f = 2 m_\psi g $$

as a guideline in order to estimate $f$. We take $g$ to be the value given by the non-relativistic quark model: $g = -0.75$. As discussed in a recent analysis [18], this value is compatible with the upper bound $g^2 \lesssim 0.5$ established by the experimental upper limit for the decay width $\Gamma(D^{++}) < 131$ keV set by the ACCMOR Collaboration [19].

Our results for the heavy meson binding energies $\varepsilon_B$ defined by

$$ \varepsilon_B = m_\psi - \omega_B $$

are shown in Table II. Also listed in Table II are the soliton masses per baryon number $M_{\text{sol}}$ taken from Ref. [18] for the strangeness case. For the heavy flavors we also find that the binding energy decreases with increasing baryon number, except for the crossings that take place at $B = 4$ and $B = 7$. This general behaviour is the opposite to what was found in Ref. [17]. It should be pointed out that, due to the absence of explicit vector mesons in the corresponding effective action, the approach used in [17] is expected to be less accurate than the one followed in the present work.

From Table II we also notice that, for heavy flavors, the binding is stronger for Set A than for Set B. This can be understood as follows. In the $m_Q \rightarrow \infty$ limit the heavy flavored meson would be concentrated at the origin [5], wrapped by the soliton. Thus, it would only probe the potential at this point. For $B = 1$ such potential is basically proportional to $|gF'(0)|$ and attractive. As well known (see, e.g., Fig.1 of Ref. [20]), $|F'(0)|$ is larger in the massless case. That leads to the observed behaviour. Similar analysis can be done for higher values of $B$. For the strangeness case the behaviour of the profile function at medium distances becomes important and the situation is reversed [5].

In order to study the stability of the heavy multiskyrmions we will only consider the mass of the background multiskyrmion and the binding energy of the bound mesons. This should be a good approximation since the non-adiabatic corrections are expected to be small as a consequence of the rather large values of the moments of inertia involved [20]. In the following we will focus on those states which have flavor number equal to their baryon number. These states are of particular interest since in the strange sector the analog states, namely those with $Y = 0$, have been found to be stable for some values of $B$ [22].

Using

$$ I_B = M_1 + M_{B-1} - M_B $$

with $M_B = B(M_{\text{sol}} + m_\psi - \varepsilon_B)$ and the values given in Table II, we get the ionization energies $I_B$ shown in Table III. We observe that the only heavy flavored states that may be stable are those with $B = 4$ and $B = 7$. In Table IV we summarize the energies for the other possible strong decays of these states. We observe that, although these states are stable against strong decays into two fragments, some decays into a larger number of fragments are allowed. However, since the usual phase space factors tend to suppress the decay rates as the number of fragments $n$ in the final state increases we expect them to be quite narrow. For instance, in the case of the charmed heptalambda the decay width will be very small since the only allowed decay mode is the one that has seven $\Lambda_c$ in the final state (Set A).

Therefore the present model predicts, both for charm and bottom, narrow heavy multibaryon states with baryon number four and seven. The main reason for the unstability of these particles can be traced back to the rather large difference that exists between the $B = 1$ and $B > 1$ binding energies. This can be easily understood noting that,
to leading order in $1/m_Q$, the only non-vanishing binding energy would be that corresponding to $B = 1$ since for $B > 1$ the radial derivative of the soliton profile function vanishes at the origin. On the other hand, in the case of strangeness the meson wavefunction is wider and, therefore, much less sensitive to the value of the potential at the origin. Consequently, the gap between the binding energies of $B = 1$ and $B > 1$ is much smaller and the corresponding multibaryon states with baryon number four and seven turn out to be absolutely stable against strong decays.

IV. CONCLUSIONS

In this work we have studied the masses of heavy multibaryon configurations using the bound state approach to the Skyrme model. This is a natural extension of previous work done in the strange sector. In order to consider the heavier flavors of charm and bottom in a consistent way, however, it is important to take into account the heavy quark symmetry. This is accomplished by the lagrangian given in Ref. [12]. An important feature of this effective heavy meson lagrangian is that it contains the degrees of freedom of the heavy scalar meson and the heavy vector meson explicitly, which leads to a system of four coupled equations for the bound state problem instead of just one equation as in Ref. [9]. The baryon number is carried by the soliton configuration of the light background fields, for which we have used the expressions in terms of the rational maps given in Ref. [14].

We obtained solutions for the bound state equations for $B \leq 9$. As in the strangeness case we find that the binding energy decreases with the baryon number and that the $B = 4$ and $B = 7$ states are the most stable against strong decays. However, for the charm and bottom flavors these states are not absolutely stable. Still, they are expected to be quite narrow since only decays into final states with three or more fragments are energetically allowed. We do not expect that the collective quantization of the soliton-meson bound system will change this picture. It would be important, however, to estimate the zero point energy of these multiskyrmion configurations. As discussed in Ref. [24] this contribution may be the cause for the H particle, which appears almost at threshold, to be unbound. In any case, since the predicted tetralambda and heptalambda are more strongly bound against two particle decays than the H, this contribution is not expected to be so important so as to open those leading decay channels.

ACKNOWLEDGMENTS

The authors wish to thank the warm hospitality of the organizers and staff members of the INT-98-3 program of the Institute of Nuclear Theory at the University of Washington where this work was initiated. NNS is supported in part by a grant from Fundación Antorchas, Argentina, the grant PICT 03-00000-00133 from ANPCYT, Argentina, and CONICET, Argentina. C.L.S. thanks the CNPq-Brasil for financial support through a CLAF fellowship.

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TABLE I. Masses and coupling constants for the heavy meson lagrangian.

|        | charm          | bottom        |
|--------|----------------|---------------|
| $m_{\phi}$ | 1867 MeV      | 5279 MeV      |
| $m_{\psi}$ | 2010 MeV      | 5325 MeV      |
| $f$     | -3016 MeV     | -7988 MeV     |
| $g$     | -0.75         | -0.75         |

TABLE II. Meson binding energies $\varepsilon_B$ (in MeV) for the case of massless pions (Set A) and massive pions (Set B) as a function of the baryon number B. Also listed are the soliton masses per baryon unit $M_{sol}$ (in MeV) which are taken from Ref. [9].

| B | $M_{sol}$ (charm) | $\varepsilon_B$(charm) | $\varepsilon_B$(bottom) |
|---|------------------|------------------------|-------------------------|
|   | Set A    | Set B     | Set A    | Set B     | Set A    | Set B     |
| 1 | 863      | 864       | 383      | 328       | 554      | 474       |
| 2 | 847      | 848       | 321      | 272       | 438      | 374       |
| 3 | 830      | 832       | 300      | 255       | 408      | 351       |
| 4 | 797      | 798       | 301      | 256       | 407      | 350       |
| 5 | 804      | 808       | 287      | 245       | 391      | 339       |
| 6 | 797      | 802       | 283      | 243       | 386      | 336       |
| 7 | 776      | 780       | 288      | 247       | 392      | 341       |
| 8 | 784      | 790       | 280      | 242       | 383      | 335       |
| 9 | 787      | 796       | 275      | 239       | 378      | 332       |

TABLE III. Ionization energies $I_B$ (in MeV) of the charm and bottom multilambdas in the case of massless pions (Set A) and massive pions (Set B).

| B | $I_B$(charm) | $I_B$(bottom) |
|---|--------------|---------------|
|   | Set A    | Set B    | Set A   | Set B   |
| 2 | -92      | -80      | -200    | -168    |
| 3 | -58      | -43      | -139    | -105    |
| 4 | 86       | 99       | 15      | 41      |
| 5 | -121     | -111     | -196    | -163    |
| 6 | -19      | -3       | -92     | -61     |
| 7 | 148      | 159      | 87      | 113     |
| 8 | -136     | -117     | -211    | -177    |
| 9 | -96      | -93      | -164    | -146    |
TABLE IV. Energy balance (in MeV) for the multilambda strong decays in the case of massless pions (Set A) and massive pions (Set B). The first column indicates the number of fragments $n$ in the final state.

| $n$ | charm | Set A | Set B | bottom | Set A | Set B |
|-----|-------|-------|-------|--------|-------|-------|
| 2   | $M_{4\Lambda} - 2M_{2\Lambda}$ | -120  | -136  | -104   | -76   | -104  |
|     | $M_{4\Lambda} - (2M_{\Lambda} + M_{2\Lambda})$ | -28   | -56   | 64     | 124   | 64    |
| 4   | $M_{4\Lambda} - 4M_{\Lambda}$ | 64    | 24    | 232    | 324   | 232   |
| 2   | $M_{7\Lambda} - (M_{2\Lambda} + M_{3\Lambda})$ | -221  | -236  | -220   | -195  | -220  |
|     | $M_{7\Lambda} - (M_{3\Lambda} + M_{4\Lambda})$ | -158  | -168  | -162   | -138  | -162  |
| 3   | $M_{7\Lambda} - (M_{3\Lambda} + 2M_{2\Lambda} + M_{4\Lambda})$ | -100  | -125  | 1      | 1     | 1     |
|     | $M_{7\Lambda} - (M_{4\Lambda} + 2M_{3\Lambda})$ | -244  | -267  | -153   | -203  | -153  |
|     | $M_{7\Lambda} - (2M_{\Lambda} + M_{3\Lambda} + M_{4\Lambda})$ | -129  | -156  | 5      | 5     | 5     |
|     | $M_{7\Lambda} - (2M_{2\Lambda} + M_{3\Lambda})$ | -278  | -304  | -214   | -266  | -214  |
| 4   | $M_{7\Lambda} - (M_{\Lambda} + 3M_{2\Lambda})$ | -128  | -261  | -161   | -75   | -161  |
|     | $M_{7\Lambda} - (2M_{\Lambda} + M_{2\Lambda} + M_{3\Lambda})$ | -186  | -224  | -14    | -98   | -14   |
|     | $M_{7\Lambda} - (3M_{\Lambda} + M_{4\Lambda})$ | -8    | -45   | 201    | 111   | 201   |
| 5   | $M_{7\Lambda} - (3M_{\Lambda} + 2M_{2\Lambda})$ | -128  | -181  | 125    | 6     | 125   |
|     | $M_{7\Lambda} - (4M_{\Lambda} + M_{3\Lambda})$ | -94   | -144  | 186    | 70    | 186   |
| 6   | $M_{7\Lambda} - (5M_{\Lambda} + M_{2\Lambda})$ | -36   | -101  | 325    | 175   | 325   |
| 7   | $M_{7\Lambda} - (7M_{\Lambda})$ | 56    | -21   | 525    | 343   | 525   |