Definition of Yield Seismic Coefficient Spectrum Considering the Uncertainty of the Earthquake Motion Phase

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Abstract: Earthquake engineers are typically faced with the challenge of safely and economically designing structures in highly uncertain seismic environments. Yield strength demand spectra provide basic information for the seismic design of structures and take nonlinear behavior into account. The designed structures, however, must be checked for seismic performance through dynamic analysis. Design-response spectra compatible earthquake motions (DRSCEM) are commonly used for this purpose. Because DRSCEM are strongly affected by the assigned phase characteristics, in this paper, we simulate realistic earthquake motion phase based on a stochastic process that modifies fractional Brownian motion (fBm). The parameters that control this process were determined via regression equations as functions of the earthquake magnitude and epicenter distance, which were obtained through a regression analysis that was performed on data from a database of recorded ground motions. After validating the efficiency of the modeled phase spectrum, large numbers of DRSCEMs were simulated with which several ductility demand spectra were obtained. By statistically analyzing these results, a rigorous yield seismic coefficient demand spectrum is proposed.

Keywords: yield seismic coefficient (YSC); design-response spectrum compatible earthquake motions (DRSCEM); fractional Brownian motion (fBm); phase spectrum; Hurst index; fluctuation

1. Introduction

1.1. Background

Due to the severe damage to infrastructure that has occurred during recent earthquakes, engineers have recognized that the use of only classical seismic design methods based on the strength capacity criterion are not appropriate, and that the dynamic behavior of a structural system, especially the nonlinear response characteristics, should be precisely evaluated. Therefore, evaluation of the seismic performance of new or existing structures is an important issue in earthquake engineering [1–3].

In the current design practice, seismic design forces are represented by smooth design-response spectra. These design-response spectra are convenient for calculating the maximum response of structures if the building is assumed to behave linearly during earthquakes. For severe ground motions, a building will exceed its elastic limit, in which case it is desirable to directly evaluate its displacement capacity through dynamic analysis of the structural system, and therefore, an input regarding the time history of ground acceleration is required [4,5]. Additionally, an observed accelerogram cannot provide the necessary pieces of information that are specific to the site where a structure is located, such as the
surficial topography, underground layer conditions, irregular profiles of primary layers and crustal conditions and existence of active faults and lineaments. Furthermore, the use of earthquake motion as an input for dynamic analyses may be affected by uncertainty in the location of the seismological sources and be controlled by fault rupture mechanisms and propagation path effects. Because of these reasons DRSCEMs are commonly used to analyze structural responses, not only to keep a kind of continuity between the codes based on elastic and inelastic design criteria but also to consider the effect of a high uncertainty characteristic of earthquake ground motion used for seismic design purpose [6–8].

International seismic design codes do not provide any details about the procedure to scale or simulate earthquake ground motion for design purposes, only impose the requirement that the recalculated response spectra obtained using the modeled earthquake motion should match the response spectrum of the target [9,10]. Consequently, simulations of DRSCEMs have been conducted by numerous researchers. However, a proper method to simulate DRSCEMs should be developed that can incorporate newly determined characteristics of earthquake ground motion [11]. Recently to simulate realistic earthquake motion time histories, many methods such as, Empirical Green’s Function [12], Stochastic Simulation [13] and numerical methods [14] are often used in engineering and research practices. It is a new trend in the field of seismic design of structures to take into account uncertainty of fault mechanisms, such as the uncertainty of the rupture starting point, uncertainty of asperity distribution and rupture propagation direction [15] to simulate earthquake motions to be used for dynamic analysis of the structural systems.

However, both the Fourier amplitude and phase spectra of earthquake motion are basic pieces of information that are needed to simulate a DRSCEM model. Following studies conducted by Ohsaki [16] to investigate phase characteristics show that the modeling of the phase spectrum plays an essential role in simulating earthquake ground motion. Much research effort has therefore been performed to identify the phase spectrum characteristics of earthquake motion. Izumi and Katsukura [17] used the concept of group delay time (GDT) and showed that the average arrival time of the earthquake energy and duration of the earthquake motion could be evaluated by the mean and standard deviation of the GDT. Yokoyama et al. [18] used phase difference distributions as functions of the magnitude, distance to the fault, and local soil conditions. Sato et al. [19] developed a method to model phase spectra near the source region, taking into account the source rupture mechanisms, propagation path, and local soil conditions. Moreover, Sato et al. [20,21] presented a simple method for modeling phase characteristics based on GDT and wavelet analyses. Boore [22] used the stochastic differential equation to simulate the GDT. Additionally, Zhang et al. [23] developed a method to apply the stochastic differential equation in relation to earthquake magnitude, epicenter distance, and local soil conditions. Chai et al. [24] used the GDT concept to model phase spectra according to the compact support of the Meyer wavelet. The non-stationary features of design ground motions synthesized using a stochastic model of the Fourier phase were presented by Liao and Jin [25]. Implication of the model in engineering application to synthesizing design accelerograms was then discussed. Also, there is other method can be used to generate a DRSCEM by selecting the phases of the ground motions from particular observed earthquakes [6,26,27].

Many statistical studies have been conducted over the years to evaluate the appropriate stochastic characteristics of earthquake motions. Some researchers used actual recorded ground motions [28–30]. Other researchers used simulated ground motions [31,32] to take into account the uncertainty of ground motion precisely. Because the uncertainty of the time history of earthquake motions affects the response characteristics of structural systems, strong fluctuations are commonly seen in response spectra calculated from observed earthquake motions. Due to this phenomenon, design-response spectra are usually defined through a stochastic evaluation. On the other hand, earthquake motion uncertainties are decomposed to uncertainties of the Fourier amplitude and phases. It is commonly understood that a design-response spectrum is a type of amplitude indicator for earthquake motion and it is used to control the amplitude of DRSCEMs. Although the uncertainty of Fourier amplitude
has already been taken into account in linear response spectra, the uncertainty of Fourier phases has not been precisely considered thus far.

1.2. Importance of Fourier Phases for Nonlinear Seismic Design

To demonstrate the importance of the phase spectrum for nonlinear seismic design, we simulated DRSCEMs by taking Fourier phases from different observed ground motions. Since a proper initial Fourier amplitude is needed to simulate DRSCEMs, the Fourier amplitude obtained from the acceleration record during the Northwest China earthquake was assigned as the initial Fourier amplitude. The information of this event is shown in Table 1 record number 1. The Fourier phases are taken from different recorded ground motions shown in Table 1 records number 2 to 7. Phases of different earthquake events with different magnitude and epicenter distance are used to investigate the phase effect on nonlinear response of a structure. Using these simulated DRSCEMs, we calculated the yield seismic coefficient (YSC) demand spectra. The YSC is defined by the yield strength of the structure $F_y$ divided by structural weight ($W$) as follows:

$$k_{by} = \frac{F_y}{W} = \frac{A_y}{g}$$

(1)

where $A_y$ is the yield acceleration and $g$ is gravitational acceleration. More details about how to calculate YSC are described at the Section 5.1.

Calculated results are shows in Figure 1. Figure 1a shows the examples of YSC demand spectra obtained by maintaining the ductility response of a structure equal to 4. It is clearly observed that the YSC demand spectrum is strongly affected by the assigned phase spectrum of earthquake motion. As an example, the value of the YSC demand spectra at the natural period of 0.5 s varies from 0.14 to 0.26, this difference increases the design force by more than 85%. On the other hand, it is commonly recognized that YSC demand spectra are also affected by the initially assigned Fourier amplitude. Several DRSCEMs are, therefore, simulated by different initial Fourier amplitudes taken from Table 1 records number (1, 3 and 7) with the same phase spectrum being fixed to the Kobe earthquake motion phase (record number 8). Figure 1b shows the YSC demand spectrum calculated by assigning different initial Fourier amplitudes but a fixed Fourier phase. From this figure, it is concluded that the effect of assigning different initial Fourier amplitudes is very minor. However, effort to reduce this type of minor effect is still necessary. Therefore, the average Fourier amplitude of several earthquake motions have been taken to reduce this effect. The reason we use the average Fourier amplitude of several earthquake motions as the initial Fourier amplitude is discussed at the beginning of Section 4. Regardless, a modeling the phase spectra of earthquake motion is an important issue for developing seismic design procedures that take into account the nonlinear behavior of structures.

![Figure 1](image-url)
Table 1. Main information of recorded earthquake ground motions used in the study. However, to conduct the regression analysis in Section 3 we added other 250 ground motions records.

| No | Number | Event, Year | Station | Magnitude | Epic. Distance (km) |
|----|--------|-------------|---------|-----------|---------------------|
| 1  | 1752   | Northwest China-03, 1997 | Jiashi | 6.10 | 19.11 |
| 2  | 6      | Imperial Valley-02, 1940 | El Centro Array #9 | 6.95 | 12.99 |
| 3  | 942    | Northridge-01, 1994 | Alhambra—Fremont School | 6.69 | 40.15 |
| 4  | 42     | Lytle Creek, 1970 | Cedar Springs Pumphouse | 5.33 | 22.51 |
| 5  | 101    | Northern Calif-07, 1975 | Cape Mendocino | 5.2 | 30.54 |
| 6  | 1151   | Kocaeli, Turkey, 1999 | Balikesir | 7.51 | 218.64 |
| 7  | 143    | Tabas, Iran, 1978 | Tabas | 7.35 | 55.24 |
| 8  | 1108   | Kobe, Japan, 1995 | Kobe University | 6.9 | 25.4 |
| 9  | 864    | Landers, 1992 | Joshua Tree | 7.48 | 13.67 |
| 10 | 855    | Landers, 1992 | Fort Irwin | 7.48 | 120.99 |
| 11 | 586    | New Zealand-02, 1987 | Maraenui Primary School | 6.60 | 72.62 |
| 12 | 825    | Cape Mendocino, 1992 | Cape Mendocino | 7.01 | 10.36 |
| 13 | 1749   | Northwest China-01, 1997 | Xiker | 5.90 | 51.75 |
| 14 | 688    | Whittier Narrows-01, 1987 | Riverside Airport | 5.99 | 59.59 |
| 15 | 142    | Tabas, Iran, 1978 | Sedeh | 7.35 | 177.9 |
| 16 | ~      | Off-Kushiro, Japan, 1993 | Ishino | 7.60 | ~ |
| 17 | ~      | Hyogoken-Nanbu, 1995 | Amagasaki | 7.20 | ~ |
| 18 | ~      | Tangshan, China, 1976 | Beijing Hotel Department | 7.60 | ~ |

1.3. Purpose of the Study

In this paper, we discuss how to define the phase spectrum and appropriate initial Fourier amplitude for simulating DRSCEMs that reflect site conditions and source mechanisms. Because the earthquake motion phase has a significant effect on the nonlinear response of structures (as described in the above section), much effort has been made to clarify the stochastic characteristics of the phases, but the uncertainty of the earthquake motion phase still cannot be evaluated properly. Recently, Sato [33] studied the fractal characteristic of the earthquake motion phase and showed that it is possible to simulate a phase with this characteristic by fBm. The fBm is a continuous stochastic process that has a long memory with respect to the circular frequency. Therefore, it is essential to detect the Hurst index and a parameter, which controls the strength of the power law defined by using an observed earthquake motion phase. One of our main research purposes is to provide regression equations for these two parameters as functions of the earthquake magnitude, epicenter distance, and bias index, which can be treated as a type of site-specific effect.

After the occurrence of severe damage to infrastructure systems caused by recent strong earthquakes in several countries, a movement to revise seismic design codes from the classical strength demand concept to a new displacement capacity demand concept has been initiated. To this end, it is essential to conduct nonlinear dynamic analyses of structural systems. The simulation of DRSCEMs then become a common topic for earthquake engineers to maintain continuity between classical and new design codes because the seismic forces used in the classical codes were usually defined based on response spectra. To simulate DRSCEMs, we must assign a proper phase spectrum. This requirement is one of the main topics of this paper. Using many simulated DRSCEMs, we conducted a comprehensive statistical study of nonlinear response spectra, termed “YSC demand spectra”. Based on our statistical analysis, we propose a method to define YSC demand spectra for seismic design purposes. To simulate DRSCEMs, the target response spectrum used here is defined in the Chinese seismic design code (GB50011-2010) as: earthquake intensity 8 (peak ground acceleration of 0.2 or 0.3 g), a rare earthquake event (the probability of exceedance during 50 years is 2–3% with a return period of 1640–2475 years), site class II (medium firm), design earthquake group 2 and a damping ratio of 0.05 [34].
2. Phase Spectrum Simulation by Modified Fractional Brownian Motion

2.1. Method for Calculating the Continuous Phase with Respect to the Circular Frequency

If the time history \( f(t) \) is an Riemann integrable function with respect to the time we can obtain the real and imaginary parts of Fourier transform of \( f(t) \) as \( R(\omega) \) and \( I(\omega) \) that are continuous functions with respect to the circular frequency \( (\omega) \). We usually calculate the Fourier phase by \( \tan \tilde{\phi}(\omega) = I(\omega)/R(\omega) \) but we could not distinguish the value of \( \tilde{\phi}(\omega) \) at where \( F(\omega) \) is located in the first and third quadrants as well as the second and the fourth quadrants. Therefore, the ordinary Fourier phase \( \tilde{\phi}(\omega) \) is given by

\[
\tilde{\phi}(\omega) = \text{Arg}(F(\omega))
\]  

where \( \text{Arg} \) is the function taking argument of the complex value \( F(\omega) \). \( \tilde{\phi}(\omega) \) is a partially continuous function with respect to the circular frequency within a principal value of \([-\pi, \pi]\). To obtain the continuous phase \( \phi(\omega) \) from \( \tilde{\phi}(\omega) \) we need the operation to unwrap a phase value near \( \pi \) to a value near \( -\pi \). Because \( F(\omega) \) is a multi-variable complex function for a fixed circular frequency and we must carefully identify the complex sheet number on which \( F(\omega) \) is located. This requests to define the value of \( n \) that is equal to the number of complex sheets being passed by to reach the current complex value of \( F(\omega) \). Due to this complexity, there is no rigorous algorithms for this purpose, so we used Equation (3) which was developed by Sato [33] to calculate the phase increment \( d\phi(\omega) \) of the continuous phase \( \phi(\omega) \) with respect to the circular frequency.

\[
d\phi(\omega) = \frac{RdI - IdR}{R^2 + I^2}
\]  

The continuous phase \( \phi(\omega) \) can be obtained by integrating \( d\phi(\omega) \) with respect to the circular frequency. The Fourier phase of earthquake motion can usually be de-convoluted into linear delay and fluctuation parts as follows:

\[
\phi(\omega) = -\omega t_0 + \psi(\omega)
\]  

where \( -\omega t_0 \) corresponds to the linear delay part and \( \psi(\omega) \) corresponds to fluctuation part. The linear delay expression only shifts the position of the time history and does not change its configuration, which is only influenced by the phase corresponding to the fluctuation part. Therefore, throughout this paper, we refer to this fluctuation part as the phase. Importantly, attention must be paid is an asymmetric (odd) characteristic of the phase with respect to the circular frequency, it is expressed as \( \phi(\omega) = -\phi(-\omega) \). It follows that the fluctuation part of phase \( \psi(\omega) \) and its increment \( d\psi(\omega) \) are of the same nature:

\[
\psi(\omega) = -\psi(-\omega), \quad d\psi(\omega) = -d\psi(-\omega)
\]  

After Sato [33] found the power law hidden in the earthquake motion phase, he recognized that the phase spectrum can also be expressed by the fBm. However, it was recognized that direct application could not generate a sample phase process because the phase process is an asymmetric function with respect to the circular frequency, as defined by Equation (5).

2.2. Calculation of the Hurst Index and Variance from the Phase Difference

The simplest way to identify the Hurst index \( H \) and variance \( \sigma^2_{\Delta\psi} \) from observed ground motion is to calculate the phase difference \( \Delta\psi \) for several circular frequency interval, \( \Delta\omega = kd\omega, \ (k = 2^m, \ m = 0, 1, 2, \ldots) \) as defined by Equation (6).

\[
\Delta\psi(\Delta\omega, \omega) = \psi(\omega + \Delta\omega) - \psi(\omega)
\]
By keeping $\Delta \omega$ constant, we can calculate the variance of $\Delta \psi$ using the hypotheses of the ergodic and stationary characteristics of the phase difference process as follows:

$$
\sigma^2_{\Delta \psi}(\Delta \omega) = \frac{1}{\omega_E - \Delta \omega} \int_0^{\omega_E - \Delta \omega} \Delta \Phi(\Delta \omega, \omega)^2 d\omega
$$

(7)

where $\omega_E$ is the upper bound of the circular frequency used to calculate the phase. Take $m$ from 0 to 25 and calculate the variance of phase difference for each step. We obtained 25 variances of phase difference $\sigma^2_{\Delta \psi}(\Delta \omega)$ from each circular frequency interval $\Delta \omega$. As an example, Figure 2 shows the relationship between the variance of the phase difference and circular frequency interval obtained from the acceleration record during the Landers earthquake. The information of this event is shown in Table 1 record number 9. In this figure we only show the calculated result with the red circles on a common logarithmic scale. A linear regression equation is obtained by a least-squares fitting for determining the values of $H$ and $\sigma^2_0$. From this regression line, we can obtain the following relationship between $\sigma^2_{\Delta \psi}(\Delta \omega)$ and $\Delta \omega$:

$$
\sigma^2_{\Delta \psi}(\Delta \omega) = \sigma^2_0(\Delta \omega)^{2H}
$$

(8)

where $\sigma^2_0$ is the value of $\sigma^2_{\Delta \psi}(\Delta \omega)$ at $\Delta \omega = 1$. From Figure 2 we can obtain $\sigma^2_0 = 36.413$ and $2H = 1.589$.

![Figure 2](image-url)

**Figure 2.** Relationship between the variance of the phase difference and circular frequency interval.

Based on the confidence interval analysis of identified parameters with one sigma the all parameters have accuracy of three-digit level, therefore we changed the effective digit of parameters are within the three digits.
2.3. Concept of Modified Fractional Brownian Motion

To simulate the phase difference with power law expressed by Equation (8) as a stochastic process we need probability density function of the phase difference $p(\Delta \psi)$. The simplest way to define $p(\Delta \psi)$ is applying the normal distribution expressed by

$$p(\Delta \psi) = \frac{1}{2\pi(\Delta \omega)^H c_0} \exp\left(-\frac{(\Delta \psi)^2}{2(\Delta \omega)^{2H} c_0^2}\right)$$  \hspace{1cm} (9)

Sato [33] discussed a possibility that the phase difference process with the probability density function expressed by Equation (9) can be simulated by fBm developed by Mandelbrot and Van Ness [35]. However, if we used the simple fBm to simulate an earthquake motion phase, we could not satisfy the constraint expressed by Equation (5), then, we modify the original fBm to satisfy this constraint. Because the basic concept for deriving a continuous formula of the modified fBm is given in Sato [36] we only express here the discretized formula of incremental modified fBm at the $j$th discrete point that is expressed by

$$d\psi_j (d\omega)^H = \gamma_b \left\{ \sum_{m=1}^{L} a_m Z_{j-m} + Z_j + \sum_{m=1}^{L} a_m Z_{j+m} \right\}$$  \hspace{1cm} (10)

where $d\psi$ is the phase difference, $d\omega$ is the circular frequency interval, $Z_j$ is random number sequence generated under the identically independently distributed assumption from $N(0, 1)$ and $L$ is a measurement of the weighting vector dimension and is obtained from the following equation:

$$L = \exp\left(\left(\frac{1}{2(1-H)}\right) \left[ -\ln(\epsilon) - \ln(3-2H) + \ln\left(H - \frac{1}{2}\right)^2 \right]\right)$$  \hspace{1cm} (11)

where $\epsilon$ is an index controlling the sum of terms in the equation. The proper value of $L$ can be defined by selecting a small value for $\epsilon$, such as 0.0002. $a_m$ is the weighting vector with $L$ dimensions defined as a sequence given by the following equation:

$$a_m = \frac{1}{2} \left\{ (m+1)^\beta - (m-1)^\beta \right\}$$  \hspace{1cm} (12)

where $m$ is an integer that increases from 1 to $L$ and $\beta$ is:

$$\beta = H - \frac{1}{2}$$  \hspace{1cm} (13)

$\gamma_b$ is defined by using the variance $c_0^2$ of the phase difference, for which a regression equation is defined in the next section, and uses the following relationship to be obtained:

$$\gamma_b = \frac{c_0}{A}$$  \hspace{1cm} (14)

where $A$ is defined by Equation (15):

$$A = \sqrt{2 \sum_{i=1}^{L} a_i^2 + 1}$$  \hspace{1cm} (15)

According to the above equations, the information needed to simulate a phase difference is the Hurst index $H$ and the variance $c_0^2$ that controls the amplitude of the modified fBm process as well as the discrete frequency interval $d\omega$. A flowchart for simulating the phase difference is shown in Figure 3.
Use $H$ to calculate the weighting dimension factor with Equation 11, then generate the weighting vector $a_0$ using Equation 12. Use $a_0$ and $\sigma_0^2$ calculate the standard deviation of the modified fBm ($\phi$) by Equation 14. Assign an initial random number to generate a random number series $\{Z\}$, taking into account $Z_j \sim \mathcal{N}(0,1)$. Calculate the phase difference $d\phi$ based on Equation 10. Change the initial random number. Run another $d\phi$? Start

**Figure 3.** Flowchart of the phase difference simulation.

### 2.4. The Validation of the Developed Method to Generate Earthquake Motion Phase

To demonstrate the efficiency of the algorithm to simulate earthquake motion phase, we used the Hurst index and the variance, which were obtained in Section 2.2 for Landers ground motion. Then we simulated an ensemble of Fourier phases according to the modified fBm process. Based on Fourier inverse analysis we can simulate an ensemble of time histories by using the simulated phases and the Fourier amplitude of Joshua Tree time history. After we have conducted studies of the elastic and inelastic response using original and simulated time histories, we find that the usage of the modified fBm gives almost similar elastic response over the wide range of natural periods, but in inelastic response, we can recognize many fluctuations. In Figure 4a, the full blue line shows the elastic response of the original Joshua Tree time history and the red dots lines show elastic response of 15 simulated time histories using phases of modified fBm process. Figure 4b shows the inelastic response spectra of the original and simulated time histories. Those fluctuations in the inelastic response are especially important to evaluate the nonlinear dynamic performance of a structure.

Furthermore, to show the efficiency of the proposed method, we simulated another an ensemble of time histories again by using recorded ground motions phases and the Fourier amplitude of Joshua Tree time history following to the previous studies [6,25,26]. Here we use ten different ground motions phases (Northwest China-03, Imperial Valley-02, Northridge-01, Tabas, Sedeh, Kobe, Kushiro-oki, and Whettier) detail information about these records is given in Table 1. We then calculate the elastic and inelastic responses with which we compare the results obtained by using phases simulated based on the modified fBm. Figure 5a shows the elastic response of the Joshua Tree and the ten simulated time histories, and the inelastic response in Figure 5b. Comparing Figures 4b and 5b, we can see that the fluctuation of nonlinear response calculated by using the phase from observed ground motions is higher than those of obtained by using phase simulated based on the modified fBm.

There is no doubt that reducing the fluctuations and uncertainty is a popular demand in the earthquake design field. Because the fluctuation in Figure 4b is lesser than that of Figure 5b, it seems better to use the results shown in Figure 4b for discussing the effect of earthquake motion phase uncertainty on the inelastic response spectrum. To deduce Figure 5b we assume that ten sample phases are chosen from the mother set of phase composed of whole time histories of observed earthquake motions with wide range of earthquake magnitudes and extensive epicenter distance ranges as well as the variety of local soil conditions. That means, we consider that each earthquake motion phase...
is extracted from a sample time history of the mother set in which as much as possible observed earthquake motion time histories are collected. This is the reason the fluctuation in Figure 5b is large. Moreover, we developed a method to extract the variance feature of phase with the power law from one sample time history and proposed an algorithm to simulate earthquake motion phases according to probability density function defined by Equation (9) with a power law. This is the reason that the fluctuation in Figure 4b becomes smaller than that in Figure 5b. Therefore, we can say that the usage of modified fBm is better than the classic methods to evaluate the uncertainty of Fourier phase, in which the uncertainty of phase is extracted by direct use of many observed earthquake motions. In our method we assume that each observed earthquake motion phase itself has an uncertainty and developed a method to evaluate this uncertainty in the Sections 2.1–2.3 and in Section 3. If we use the mother set composed of observed earthquake motions for evaluating the uncertainty of Fourier phase, the whole uncertainty becomes an overlapped Fourier phase uncertainty of each observed earthquake motion. Therefore, the effect of uncertainty is reduced when we use our proposed method.

**Figure 4.** (a) Elastic and (b) Inelastic response spectra. Full blue line is the result obtained by using original time history and red dots line are the results obtained by using simulated time histories according to the simulated phases based on the modified fBm process.

**Figure 5.** (a) Elastic and (b) Inelastic response spectra. Full blue line is the result obtained by using original time history and dots red lines are the results obtained by simulated time histories according to the different observed ground motions phases.
3. Estimating the Hurst Index and Variance of the Phase Differences from Observed Ground Motions

3.1. Regression Analysis of the Observed Ground Motions

As mentioned in Section 1.1, we only consider the effect of the Fourier phase uncertainty to evaluate the nonlinear seismic performance of a structural system. Therefore, we require numerous sample DRSCEMs to conduct a series of stochastic analyses that take into account the effect of earthquake motion phase uncertainty on the nonlinear response of a structural system. To do this, we need to assign the correct combination of parameters that control the modified fBm. For this purpose, we developed nonlinear regression equations to define the Hurst index and variance in the modified fBm process as functions of the earthquake magnitude $M$, epicenter distance $\Delta$ (km) and bias index. The regression analysis was conducted using 250 free-field acceleration time histories recorded at observation sites distributed throughout the world (the Peer NGA Strong Motion Database and a database offered by the Japanese Meteorological Agency) with different magnitudes $M$ and epicenter distances $\Delta$. The data were chosen from 17 different events for the regression analysis. Figure 6 shows the relationship between $M$ and $\Delta$ of the recorded accelerograms used in this analysis. Except of 6 data points (shown by red circles), all data are located within a distance of 7 to 800 km from the epicenter, which means the nonlinear regression equations are valid for use in this range because all of the data used in the present study are within this range. Additionally, most of the records selected are for earthquakes with magnitudes between 6 and 8.1. Earthquakes with a magnitude larger than 6 were chosen because our motivation is to use phase spectra to simulate DRSCEMs using the response spectra defined in the Chinese Seismic Design Code GB50011-2010 (targeting earthquakes of intensity 8, rare earthquake events, site class 2 and earthquake group 2) for engineering purposes. The Hurst index and variances for each observed earthquake motion were computed. Equation (16) is the form of the basic nonlinear regression equation used to develop the final form of Hurst index. Equation (17) is also used to determine the variance. We evaluate the final form of the regression equation based on the R-squared value, which is a measure of expressing the proximity of data to a regression line, and the Akaike Information Criterion (AIC), which is a method for selecting the best model. Table 2 shows some of the nonlinear regression results.

$$\hat{H} = \beta_1 M + \beta_2 \Delta + \beta_3 M\Delta + \beta_4 M^2 + \beta_5 \Delta^2 + \beta_6$$

(16)

$$\sigma_0^2 = 10^{\beta_1 M} \times \Delta^{\beta_2} \times 10^{\beta_3}$$

(17)

where $\beta_1, \beta_2, \beta_3, \beta_4, \beta_5$ and $\beta_6$ are regression coefficients.

Figure 6. Distribution of magnitudes and epicenter distances of observed ground motions used in this study.
were significantly correlated with each other (with each other with each other). The result obtained is a sample of the fluctuation part of the phase difference, described in Section 2.3. By adding these phase differences, we can obtain a cumulative phase at discrete circular frequency interval, the sample $M$ from Equations (18) and (19) with  

$$\hat{\psi}$$

As an example, we obtain

3.2. Modeling A Stochastic Phase Process with Modified Fractional Brownian Motion

The earthquake magnitude and epicenter distance at a site of interest are needed to generate a phase process based on the modified fBm. As an example, we obtain $\hat{H} = 0.89$ and $\hat{\sigma}_0^2 = 150$ from Equations (18) and (19) with $M = 6.2$ and $\Delta = 207$ km. With these two parameters and the discrete circular frequency interval, the sample $d\hat{\psi}$ can be simulated using the modified fBm algorithm described in Section 2.3. By adding these phase differences, we can obtain a cumulative phase at each discrete frequency; the result obtained is a sample of the fluctuation part of the phase difference, 

![Figure 7. (a) Linear association between the original and expected Hurst index values and (b) Common logarithm of original and expected variance values.](image)

Table 2. Nonlinear regression results.

| No | Nonlinear Regression Equations | $R^2 (H)$ | $AIC (H)$ |
|----|--------------------------------|-----------|-----------|
| 1  | $\hat{H} = \beta_1 M + \beta_2 M A + \beta_3 M^2 + \beta_4 M^3 + \beta_5 A^2 + \beta_6$ | 0.603 | -1110.395 |
| 2  | $\hat{H} = \beta_1 M + \beta_2 M A + \beta_3 M^2 + \beta_6$ | 0.573 | -1094.270 |
| 3  | $\hat{H} = \beta_1 M + \beta_2 M A + \beta_3 M^2 + \beta_6$ | 0.550 | -1080.937 |
| 4  | $\hat{H} = \beta_1 M + \beta_2 M A + \beta_3 M^2 + \beta_6$ | 0.533 | -1071.608 |
| 5  | $\hat{H} = \beta_1 M + \beta_2 M A + \beta_3 M^2 + \beta_6$ | 0.527 | -1068.614 |
| 6  | $\hat{H} = \beta_1 M + \beta_2 M A + \beta_3 M^2 + \beta_6$ | 0.535 | -1072.811 |
| 7  | $\hat{H} = \beta_1 M + \beta_2 M A + \beta_3 M^2 + \beta_6$ | 0.422 | -1020.627 |
| 8  | $\hat{H} = \beta_1 M + \beta_2 M A + \beta_3 M^2 + \beta_6$ | 0.517 | -1065.579 |
| 9  | $\hat{H} = \beta_1 M + \beta_2 M A + \beta_3 M^2 + \beta_6$ | 0.407 | -1013.858 |
| 10 | $\hat{H} = \beta_1 M + \beta_2 M A + \beta_3 M^2 + \beta_6$ | 0.516 | -1064.637 |
| 11 | $\hat{H} = \beta_1 M + \beta_2 M A + \beta_3 M^2 + \beta_6$ | 0.189 | -935.748 |
| 12 | $\hat{H} = \beta_1 M + \beta_2 M A + \beta_3 M^2 + \beta_6$ | 0.516 | -1064.917 |

According to the AIC, the best model has a minimum AIC value. Thus, the expected values of the Hurst index $\hat{H}$ and variance $\hat{\sigma}_0^2$ can be estimated using the following regression equation:

$$\hat{H} = 0.89M + 1.616 \times 10^{-3} A - 2.353 \times 10^{-4} MA - 5.749 \times 10^{-2} M^2 + 1.827 \times 10^{-7} A^2 - 2.547$$  (18)

$$\hat{\sigma}_0^2 = 10^{(0.3809M - 1.8964)} \times A^{0.5515}$$  (19)

The association between original Hurst index and expected Hurst index are significantly correlated with each other ($R = 0.603$), and the common logarithm of original variance and expected variance were significantly correlated with each other ($R = 0.563$), respectively as presented in Figure 7a,b.

3.2. Modeling A Stochastic Phase Process with Modified Fractional Brownian Motion

The earthquake magnitude and epicenter distance at a site of interest are needed to generate a phase process based on the modified fBm. As an example, we obtain $\hat{H} = 0.80$ and $\hat{\sigma}_0^2 = 150$ from Equations (18) and (19) with $M = 6.2$ and $\Delta = 207$ km. With these two parameters and the discrete circular frequency interval, the sample $d\hat{\psi}$ can be simulated using the modified fBm algorithm described in Section 2.3. By adding these phase differences, we can obtain a cumulative phase at each discrete frequency; the result obtained is a sample of the fluctuation part of the phase difference,
as shown in Figure 8. A sample of the earthquake motion phases is obtained by adding the linear delay part to the fluctuation part of the phase as expressed by Equation (4). By changing the initial random number for simulating $d\psi$, we can obtain a dataset of phase spectra.

![Figure 8](image-url)  
**Figure 8.** A Sample of the fluctuation part of the phase difference simulated by modified fBm process.

### 4. Simulation of DRSCEM

To simulate a DRSCEM, a fixed phase spectrum $\phi(\omega)$ and initial Fourier amplitude spectrum $A_{int}(\omega)$ are needed. Although the Fourier phase uncertainty has a strong influence on the fluctuation of YSC demand spectrum, the initial Fourier amplitude also has a minor effect on it because the frequency content of the initial Fourier amplitude differs from one observed earthquake Fourier amplitude to another. The difference in the initial Fourier amplitude level can be adjusted in the process of attaining the compatibility between the response spectrum calculated by DRSCEM and the design-response spectrum. Therefore, the initial Fourier amplitude spectrum should contain enough frequency components. To certain this, we assigned the initial Fourier amplitude spectrum to the direct averaging of several Fourier amplitude spectra of the recorded time histories in the frequency domain. To guarantee its efficiency we used the average of normalized Fourier amplitude spectra.

Figure 9a shows the amplitudes of ten recorded earthquake ground motions (Northwest China-01, Northwest China-03, Northridge-01, Kobe Japan, Off-Kushiro Japan, Imperial Valley-02, Tabas Iran-Tabas, New Zealand-02, Landers-Fort Irwin, and Cape Mendocino), and the black line is the average Fourier amplitude. Additionally, in Figure 9b shows the YSC demand spectra calculated from the DRSCEMs, which are simulated using the average initial Fourier amplitude of the ten recorded ground motions mentioned above. From Figure 9b, it is apparent that it is more appropriate to initially use the average initial Fourier amplitude than individual Fourier amplitudes because the YSC demand spectra obtained from the average Fourier amplitude are located within the fluctuation range and are smoother. It is commonly recognized that the response spectrum recalculated using DRSCEMs coincides with the target response spectrum regulated in the design code. Figure 10 is an example that demonstrates matching a simulated DRSCEM with the target design-response spectrum. The red line is the target response spectrum. The black line is the recalculated response spectrum using a DRSCEM simulated by taking the initial Fourier amplitude from the El Centro earthquake record and the phase spectrum simulated by fBm. The blue line is directly obtained from the El Centro record. By comparing the red and black lines, we confirm the commonly recognized fact except that rather lower fluctuations are seen at a lower natural period.
5. Proposition of YSC Demand Spectra for Design Purposes

The YSC demand spectrum is a commonly used concept that is included in several seismic design codes adopting performance design criteria. However, the YSC demand spectrum is strongly affected by the uncertainty of the assigned Fourier phase. We use the modified fBm to simulate Fourier phase samples, which enables us to investigate the effect of phase uncertainty on the simulated DRSCEMs, and as a result, we can investigate the uncertainty of the YSC demand spectra. From this analysis, we conduct a comprehensive statistical study to define a proper design YSC demand spectrum.

5.1. Method of Analysis and the Systems Considered

The YSC demand spectrum for a fixed response ductility is computed for each simulated DRSCEM by the iteratively obtained yield strength of SDOF systems (Clough type nonlinear constitutive equation). The elastic natural period of SDOF systems to calculate YSC demand spectra ranges from 0.1 to 5.0 s with an interval of 0.1 s and a 5% viscous damping ratio (all the results presented in this paper are for this damping ratio) is used. The elastic natural period is obtained from the initial stiffness
of SDOF and its mass by Equation (20). After computing the YSC demand spectra for all DRSCEMs, statistical studies were completed based on these results.

\[ T_n = \frac{1}{2\pi} \sqrt{\frac{m}{k_0}} \]  

(20)

where \( k_0 \) and \( m \) are the initial stiffness and the mass of the concerned SDOFs.

To perform statistical studies of YSC demand spectra, the mean and mean plus one standard deviation at each natural period were computed. Another two confidence levels at the 5th and 95th percentiles were also computed. Although this analysis process was conducted using response ductility factors of 1 (elastic), 1.5, 2, 3, 4, and 6, we here show only the results from using a response ductility factor of 4 as an example. Figure 11a shows the mean, mean plus and minus one standard deviation, 5th and 95th percentiles of YSC demand spectra using a response ductility factor of 4. Additionally, the thin gray lines show the scattering of the YSC demand spectra when keeping the response ductility factor set to 4. Also, we compared the results obtained using phase spectra simulated by the modified fBm with those obtained by taking the phase spectra from observed ground motion. Figure 11b shows the fluctuation of the YSC demand spectra for a response ductility factor of 4, which are calculated by simulated DRSCEMs using modified fBm and taking the phase from observed ground motion together. From this figure, it seems that the fluctuation of the YSC demand spectra calculated using only 6 DRSCEMs obtained from the observed grand motion phases covered the fluctuation range of YSC demand spectra calculated using 1000 DRSCEMs based on the modified fBm. Therefore, a lot of DRSCEM is become necessary to define a design YSC demand spectrum.

5.2. Fluctuation of the YSC Demand Spectra

Although the mean and mean plus one standard deviation of the YSC demand spectra are very useful as they represent what can be expected beyond the average, it is equally important to know the level of scatter about the mean. A common and effective way to quantify fluctuation is to use the coefficient of variance (CV), which is defined as the ratio of the standard deviation to the mean. Figure 12a shows the CV of a YSC demand spectrum corresponding to all DRSCEMs. Figure 12b shows the mean of YSC demand spectra for all ductility values. The CV, which shows the fluctuation of the YSC demand spectra, is not constant over the entire natural period ranges and depends on the natural period and ductility level selected. In general, fluctuation increases as the level of ductility increases and the natural period of the structural system increases. The CV for a ductility of 1 is almost constant and close to zero for all natural period ranges because it is an elastic response. Additionally,
we observe that the CV for all of the ductility values up to the natural period of 3 s are almost in the range of 10%, which is a very peculiar characteristic of the YSC demand spectra because the mean value of the YSC provides the variance of the YSC value within the natural period between 0.1 to and 5.0 s.

On the other hand, if the data obey a normal distribution at its mean, the statistic shows that over 68% of the data values are within one standard deviation, over 95% of the data values are within two standard deviations and over 99% fall within three standard deviations. Depending on these results, these confidence interval concepts are very useful for defining the design YSC demand spectra by taking into account safety and economic aspects of the structural design. For the purpose to define the design YSC demand spectrum we can use the statistic concept mentioned above. However, even if we use mean + one standard deviation criterion, more than 16% designed structure still drops in the unsafe region. We, therefore, propose to use the smoothed envelope YSC demand spectra shown in Figure 13a by a thick green dash dot line for design purposes. To calculate a smoothed envelope curve of YSC demand spectra we must simulate DRSCMs as many as possible. If we simulate more than 1000 DRSCMs and obtain the smoothed envelope curve the ratio that the designed structure drops in the unsafe region is less than 0.1% in all natural period range of structure systems.

![Figure 12](image1.png)  
**Figure 12.** For ductility values of 1, 1.5, 2, 3, 4, and 6 (a) Coefficients of variation and (b) Mean of YSC demand spectra.

![Figure 13](image2.png)  
**Figure 13.** Smoothed envelope curve and the fluctuation of YSC demand spectra obtained by (a) 1000 DRSCMs simulated by the modified fBm process and (b) 500 DRSCMs simulated by observed earthquake motions phases.
To verify the efficiency of the proposed method, we also simulated more than 500 DRSCEMs using the observed ground motion phases and investigated the fluctuations of YSC along the range of the concerned natural periods. We found that the maximum exceedance of the fluctuations above the smoothed envelope curve was 1 out of 500 samples, which was a 0.2% exceedance rate. Figure 13b shows the proposed YSC demand spectrum (the thick green dash dot line) and the distribution of the YSC demand spectra calculated using DRSCEMs simulated by taking the Fourier phase from observed earthquake motions. From Figure 13, it is apparent that the YSC demand spectra fluctuate randomly above the smoothed envelope curve by a maximum exceedance of 0.2%, thereby confirming that the smooth envelop curve is suitable for design purposes. Here, a method to define a design yield seismic coefficient demand spectrum is proposed, while its application to nonlinear seismic design of structures will be studied in the near future.

6. Conclusions

The primary purpose of this study was to investigate the effect of phase spectrum uncertainty on the nonlinear behavior of structural responses. We proposed to use the modified fBm process to simulate phase spectra with uncertainty features of observed ground motion phases. We also defined a rigorous nonlinear design spectrum, named the yield seismic coefficient demand spectrum, which was obtained through statistical analysis of the nonlinear analysis results using simulated DRSCEMs and taking into account the uncertainty of the earthquake motion phases. Using recorded ground motion observed from the world, a nonlinear regression analysis was conducted to calculate the Hurst index and variance parameters for simulating the modified fBm process. Two regression equations were developed to determine the Hurst index and variance as functions of the earthquake magnitude and epicenter distance, respectively.

Comparing the results obtained by this study with those of previous studies, the principal contents can be summarized as follows:

- Modification of the fBm process for modeling Fourier phase provides a good physical interpretation for phase uncertainty and led to a feasible and easy way to simulate design-response spectra compatible earthquake motions (DRSCEM).
- A method for generating DRSCEM was developed in this paper and was useful for generating an input ground motion for a seismic response analysis of building structures with nonlinear behavior at sites of interest.
- We developed regression equations of two parameters (the Hurst index and the variance) to define the power low of the modified fractional Brownian motion as the function of earthquake magnitude, epicenter distance, and bias index.
- Based on the assumption that even one observed earthquake motion phase has an uncertainty, we proposed a new concept to extract the phase uncertainty from one observed earthquake motion phase. In classical criterion many researchers believe that it is useful to collect many observed earthquake motion phases for evaluating phase uncertainty. Using this new criterion to evaluate uncertainty of phase we succeeded to reduce the effect of phase uncertainty compared to the classical criterion.
- We proposed a method to define a unique yield seismic coefficient demand spectrum for design purpose in China, but the same method can be used in all over the world only by changing the seismic design code that is effective at the concerned region.

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