Predictions for the beauty meson spectrum

Mohammad H. Alhakami

Department of Physics and Astronomy, College of Science,
King Saud University, P. O. Box 2455, Riyadh 11451, Saudi Arabia;
Nuclear Science Research Institute, KACST, P.O. Box 6086, Riyadh 11442, Saudi Arabia;
and School of Physics and Astronomy, University of Glasgow, Glasgow, G12 8QQ, United Kingdom
(Dated: July 1, 2020)

We predict the spectrum of the four 1S and eight 1P nonstrange and strange states in the beauty meson family in the context of effective field theory. By the union of heavy quark effective theory and chiral perturbation theory, the mass formalisms for the heavy-light mesons are defined. Our analysis uses mass expressions involve, for the first time, the full leading self-energy corrections and leading power corrections to the heavy quark and chiral limits. The counterterms present in these expressions are fitted using available experimental and theoretical information on the charmed meson masses and couplings. The results from charm sector are used to make theoretical expectations on the analog beauty meson spectrum. The observed spectrum of the ground state, \( B(D) \), and excited, \( B(D)_1 \) and \( B(D)_2 \), beauty mesons are well reproduced in our theoretical calculations. The excited scalar, \( B(D)_{0} \), and axial-vector, \( B(D)_{1} \), beauty mesons have not yet been discovered. Hopefully our predictions may provide valuable clues to further experimental exploration of these missing resonances.
I. INTRODUCTION

The physics of heavy-light mesons is well described by heavy quark symmetry. In the heavy quark (HQ) limit, which is denoted either by $m_Q \to \infty$ or by $m_Q \gg \Lambda_{QCD}$, the spin of the heavy quark, $s_Q$, decouples from the spin of the light degrees of freedom (light antiquarks and gluons), $s_l$, and both separately become conserved in the strong interaction processes. The light degrees of freedom, in this limit, become blind of heavy quark spin and flavor; accordingly charmed and beauty mesons, as heavy-light meson systems, become degenerate. Heavy-light mesons can be organized in doublets of two states with total angular momentum $J_{\pm} = s_l \pm s_Q$ and parity $P = (-1)^{J_{\pm}+1}$, where $s_l = l \pm \frac{1}{2}$ and $l$ is the orbital angular momentum of the light degrees of freedom. Here, our focus is on the heavy meson doublets corresponding to $l = 0, 1$. For the ground state, $l = 0$ ($S$-wave in the quark model), the heavy mesons with $J^P = 0^-, 1^-$ are degenerate and form members of the ground state $J^P = 0^-$-doublet. For the low-lying excited states, $l = 1$ ($P$-wave in the quark model), there are two cases for $s_l$; it could be $\frac{1}{2}$ or $\frac{3}{2}$. For the $\frac{1}{2}^+$-doublet, the degenerate states are $0^+$ and $1^+$. The other $P$-wave states, which form members of the $\frac{3}{2}^+$-doublet, are $1^+$ and $2^+$. Although $1^+$ states of $\frac{1}{2}^+$ and $\frac{3}{2}^+$ doublets can mix, they can be distinguished by their strong decays. In the strict HQ limit, the state $1^+$ of the $\frac{3}{2}^+$-doublet can only decay to ground state by $D$-wave pion emission, which can be discriminated from $1^+$ of the $\frac{1}{2}^+$-doublet which decays by $S$-wave.

The measured masses of charmed and beauty mesons are provided in the diagrams given in Figs. 1-3. The degeneracy between charm and beauty systems, which is realized in the HQ limit, is in fact lifted by the finiteness of charm and beauty quark masses. As $m_c > m_u$, the kinetic energy of the heavy quark in the beauty system is much reduced compared to charm one. This, in turn, significantly reduces the splittings between different doublets in beauty meson system more than their corresponding splittings in charm system; e.g., $m_{D_{s2}^{*+}} - m_{D_{s1}^{*+}} = 600.76(80)$ MeV whereas $m_{B_{s2}^{+}} - m_{B_{s1}^{+}} = 472.97(21)$ MeV, which indicates the breaking of heavy quark flavor symmetry. Additionally, the members of each doublet, i.e., $\frac{1}{2}^-$, $\frac{3}{2}^-$, and $\frac{5}{2}^-$, in both charm and beauty sectors, are no longer degenerate, which implies the breaking of heavy quark spin symmetry. The size of such mass splitting, which is called hyperfine splitting, is of order $\Lambda_{QCD}^2/m_Q$, where $m_Q$ is mass of heavy quark. Consequently, one can relate hyperfine splittings in charm and beauty sectors by a universal factor, which is the ratio of heavy quark masses.

All the $1S$ and $1P$ charmed mesons are well established, as shown in Fig. 4 and hence completing one $S$-wave doublet, $s_{l0}^c = \frac{1}{2}^-$, and two $P$-wave doublets, $s_{l1}^c = \frac{1}{2}^+$ and $s_{l2}^c = \frac{3}{2}^+$. By examining splitting patterns within charmed meson states, one finds that the mass splittings between strange and nonstrange charmed mesons in the $s_{l0}^c = \frac{1}{2}^-$ and $s_{l2}^c = \frac{3}{2}^+$ doublets are compatible with theoretical expectations of...
analyze the low-energy strong interactions of heavy mesons with the light Goldstone particles, for a review perturbation theory (HMChPT); an effective field theory framework that provides a systematic tool to particles. For this, HQET and chiral perturbation theory (ChPT), where the latter describes the low-energy is insufficient in studying strong decays of heavy mesons, which involve emission of soft light Goldstone studying masses and semileptonic decays of mesons containing a single heavy quark. HQET alone, however, symmetries is obtained by constructing effective field theories (EFTs). The heavy quark symmetry is used to study the low-energy dynamics of heavy-light meson system. The formal approach to employ these from the charm sector, only the 1S beauty states have been well established, completing one S-wave doublet, $s_1^P = \frac{1}{2}^-$. For the 1P beauty family, the states belonging to $s_1^P = \frac{1}{2}^+$ doublet are well established. However, the other excited beauty mesons, which belong to $s_1^P = \frac{3}{2}^+$ doublet, have not yet been observed. The current paper is concerned to make model independent predictions for the 1S and 1P beauty meson spectrum using effective QFT.

The approximate chiral and heavy quark symmetries of quantum chromodynamics (QCD) allows one to study the low-energy dynamics of heavy-light meson system. The formal approach to employ these symmetries is obtained by constructing effective field theories (EFTs). The heavy quark symmetry is used to build heavy quark effective theory (HQET). This effective theory is potentially a very useful tool in studying masses and semileptonic decays of mesons containing a single heavy quark. HQET alone, however, is insufficient in studying strong decays of heavy mesons, which involve emission of soft light Goldstone particles. For this, HQET and chiral perturbation theory (ChPT), where the latter describes the low-energy dynamics of the Goldstone particles, are combined in a single framework by introducing heavy meson chiral perturbation theory (HMChPT): an effective field theory framework that provides a systematic tool to analyze the low-energy strong interactions of heavy mesons with the light Goldstone particles, for a review see Ref. [4].

Within HMChPT framework, the masses for the ground state $s_1^P = \frac{1}{2}^-$ and first excited state $s_1^P = \frac{1}{2}^+$ charmed mesons are derived in \[5\]. The expressions include the leading power corrections to the heavy quark and chiral limits. The leading self-energy corrections, which only neglect virtual loops effect from the $s_1^P = \frac{3}{2}^+$ states to $s_1^P = \frac{1}{2}^+$ states, are also included in these mass formulas. According to the power counting rules introduced in \[5\], the missing virtual loop effects are important to the physics of the scalar and axial-vector charmed and beauty mesons. Recently, these missing corrections are calculated in our work in \[6\]. There we also have derived the mass formalisms for the excited $s_1^P = \frac{3}{2}^+$ states, including full one-loop corrections and corrections due to chiral and heavy quark symmetry breaking terms.

In \[6\], we have used the mass formalisms of \[3\] to predict the beauty meson spectrum. There, the experimental data on charm sector has been employed to fix the counterterms of these one-loop mass expressions. Then, charm results are used to make predictions for the analog beauty meson spectrum. The predicted $s_1^P = \frac{1}{2}^-$ masses are found to be in a very good agreement with the experiments, which without a doubt reflects the power of HMChPT. As the excited $s_1^P = \frac{3}{2}^+$ states have not yet been observed, the corresponding theoretical results cannot be justified. It is worth mentioning that the calculations undertaken in \[6\] assuming heavy quark flavor independence; i.e., masses neglect other leading power corrections $O(\Lambda_{QCD}/m_c - \Lambda_{QCD}/m_b)$ to the HQ limit, which are needed to get the correct splittings between doublets in the (predicted) beauty meson spectrum.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{The spectrum of beauty mesons. The members of the excited $s_1^P = \frac{1}{2}^+$ doublet have not yet been observed. All masses are taken from the Particle Data Group \[1\]. For the nonstrange vector meson, we use the same mass for both $B_s^0$ and $B^-$, as given in \[3\]. The notation is the same as in Fig. \[1\].}
\end{figure}
Our main motivation is to extend the applications of HMChPT developed in [5-7] to predict the spectrum of the ground state and excited state beauty mesons. Our approach takes into account, for the first time, the full leading self-energy corrections and also properly include the leading power corrections to the HQ limit, which are required to get the correct mass splittings among the different doublets (fine splittings) in the beauty meson sector. This paper is organized as follows. The mass expressions for charmed and beauty mesons we use are briefly presented in Sec. II. Section III explains the fitting and predicting methods. It also presents our results for the beauty meson spectrum. The summary is given in Sec. IV.

II. MASS FORMALISMS

The residual masses for charmed mesons that form members of the \( s^P_1 = \frac{1}{2}^- \) (H-sector), \( s^P_2 = \frac{1}{2}^+ \) (S-sector), and \( s^P_3 = \frac{3}{2}^+ \) (T-sector) doublets have been derived within HMChPT framework including one loop chiral corrections [5, 7]. In compact forms, these masses are given by [5, 7]

\[
\begin{align*}
  m^r_{D_{(s)}} &= \eta_H - \frac{3}{4} \xi_H + \alpha_{(s)} L_H - \beta_{(s)} F_H + \Sigma_{D_{(s)}}, \\
  m^r_{D'_{(s)}} &= \eta_H + \frac{3}{4} \xi_H + \alpha_{(s)} L_H + \frac{1}{3} \beta_{(s)} F_H + \Sigma_{D'_{(s)}}, \\
  m^r_{D_{(s)0}} &= \eta_S - \frac{3}{4} \xi_S + \alpha_{(s)} L_S - \beta_{(s)} F_S + \Sigma_{D_{(s)0}}, \\
  m^r_{D_{(s)1}} &= \eta_S + \frac{3}{4} \xi_S + \alpha_{(s)} L_S + \frac{1}{3} \beta_{(s)} F_S + \Sigma_{D_{(s)1}}, \\
  m^r_{D_{(s)1}} &= \eta_T - \frac{5}{8} \xi_T + \alpha_{(s)} L_T - \frac{5}{6} \beta_{(s)} F_T + \Sigma_{D_{(s)1}}, \\
  m^r_{D_{(s)2}} &= \eta_T + \frac{5}{8} \xi_T + \alpha_{(s)} L_T + \frac{1}{2} \beta_{(s)} F_T + \Sigma_{D_{(s)2}},
\end{align*}
\]

where \( \alpha_{(s)} \) and \( \beta_{(s)} \) are \( \alpha = -1/3, \alpha_s = 2/3, \beta = -1/4, \beta_s = 1/2 \); the subscript \( s \) stands for the strange charmed meson. The \( \eta \) and \( L \) (\( \xi \) and \( F \)) parameters in \( H, S, \) and \( T \) sectors respect (violate) heavy quark spin-flavor symmetry. Above masses do not contain parameters that break heavy quark flavor symmetry. The self-energy corrections, which are represented by \( \Sigma_D \), are nonlinear functions of the mass difference of charmed mesons and masses of the light pseudoscalar mesons \( \pi, \eta, \) and \( K \). The explicit expressions of self energies for the excited \( s^P_1 = \frac{1}{2}^+ \) and \( s^P_2 = \frac{1}{2}^+ \) charmed meson states are given in the Appendix of [5]. For the \( s^P_3 = \frac{3}{2}^+ \) ground state, we use expressions given in Appendix of [5]. There are five \( (g, g', g'', h, \) and \( h' \) couplings entering loop corrections. These couplings describe strong transitions between heavy charmed mesons; for an illustration see Fig. [5]. The one-loop masses depend quadratically on these couplings [5, 7].

The chiral loop functions describing the interaction of heavy mesons with same (opposite) parity are denoted by \( K_1 \) \( K_2 \). They are given, in the \( \overline{MS} \)-scheme, by [5, 7]

\[
\begin{align*}
  K_1(\omega, m_i, \mu) &= \frac{1}{16\pi^2} \left[ -2\omega^3 + 3m_i^2\omega \ln \left( \frac{m_i^2}{\mu^2} \right) - 4(\omega^2 - m_i^2)F(\omega, m_i) + \frac{16}{3}\omega^3 - 7\omega m_i^2 \right], \\
  K_2(\omega, m_i, \mu) &= \frac{1}{16\pi^2} \left[ -2\omega^3 + m_i^2\omega \ln \left( \frac{m_i^2}{\mu^2} \right) - 4\omega^2 F(\omega, m_i) + 4\omega^3 - \omega m_i^2 \right],
\end{align*}
\]

where the arguments \( \omega, m_i, \) and \( \mu \) represent the charmed mass differences, the masses of the Goldstone bosons, and the renormalization scale, respectively. The function \( F(\omega, m_i) \) is given by

\[
F(\omega, m_i) = \begin{cases} 
  -\sqrt{m_i^2 - \omega^2} \cos^{-1}(\frac{\omega}{m_i}), & m_i^2 > \omega^2, \\
  \sqrt{\omega^2 - m_i^2} i\pi - \cosh^{-1}(\frac{-\omega}{m_i})], & \omega < -m_i, \\
  \sqrt{\omega^2 - m_i^2} \cosh^{-1}(\frac{\omega}{m_i}), & \omega > m_i.
\end{cases}
\]
MS masses, Eqs. (4) and (5), one can define the difference of spin averaged masses in the beauty sector, \( \bar{\lambda} \). The spin averaged mass, \( \bar{\lambda} \), is independent of spin symmetry violating parameters; i.e., quark spin symmetry intact. However, the last term, which has \( \bar{\lambda} \), where \( A \) has the same value for all particles in a given sector. As kinetic energy is a positive quantity, the sign of \( \bar{\lambda} \) in Eq. (4) should be negative.

In Eq. (4), the third term, which contains \( \lambda_{X,1} \), breaks heavy quark flavor symmetry, but leaves the heavy quark spin symmetry intact. However, the last term, which has \( \lambda_{X,2} \), breaks both heavy quark flavor and spin symmetries. The spin averaged mass, \( \bar{m}_X \), weighted by the number of helicity states

\[
\bar{m}_X^{(Q)} = \frac{n_- m_{X,-}^{(Q)} + n_+ m_{X,+}^{(Q)}}{n_+ + n_-},
\]

is independent of spin symmetry violating parameters; i.e., \( \lambda_{X,2} \) (\( \xi \) and \( F \)) in Eq. (4) [Eq. (1)]. By using Eqs. (4) and (5), one can define the difference of spin averaged masses in the beauty sector,

\[
\bar{m}_{A(s)}^{(b)} - \bar{m}_{H(s)}^{(b)} = \bar{m}_{A(s)}^{(c)} - \bar{m}_{H(s)}^{(c)} + \delta_{AH}^{(s)}.
\]

where \( A \in \{S, T\} \), and

\[
\delta_{AH}^{(s)} = (\lambda_{A,1} - \lambda_{H,1})^{(s)} \left( \frac{1}{2m_c} - \frac{1}{2m_b} \right),
\]

The HMChPT results for charmed mesons, Eq. (1), can be used to obtain the predictions for the analog beauty meson spectrum. For this, the heavy quark spin violating (\( \xi \) and \( F \)) parameters should be rescaled by \( \frac{m_c}{m_b} \) and the \( O(\Lambda_{QCD}/m_c - \Lambda_{QCD}/m_b) \) corrections have to be included to some parameters. Following Ref. [7], we use \( \overline{\text{MS}} \) masses, \( m_b = 4.18 \) GeV and \( m_c = 1.27 \) GeV, to define the rescaling factor \( m_c/m_b = 0.304(50) \), where an extra uncertainty of \( O(\Lambda_{QCD}) \) is added to cover the spread of \( b \) and \( c \) masses resulting from different schemes [1]. As mentioned, masses in Eq. (4) do no include parameters that break heavy quark flavor symmetry. So, one cannot successfully reproduce the correct splittings between doublets in beauty meson system using these masses. To incorporate such missing contributions to our masses, let us first recall heavy meson masses in HQET. In a compact form, the mass of a heavy meson \( X \) containing a single heavy quark flavor \( Q \) can be expressed up to the leading power corrections to the HQ limit as [3]

\[
m_{X,Q} = m_Q + \bar{\Lambda}^X - \frac{\lambda_{X,1}}{2m_Q} \pm n_\mp \frac{\lambda_{X,2}}{2m_Q},
\]
represents the leading $O(\Lambda_{QCD}/m_c - \Lambda_{QCD}/m_b)$ corrections. Such corrections to the HQ limit are missing in HMChPT formalisms [Eq. (1)]. Therefore, one has to add them, with proper inclusion of corrections due to chiral symmetry breaking, to HMChPT masses for the beauty sector. This is because the HMChPT masses given in Eq. (1) do not only involve effects to first order in the inverse heavy quark masses, $1/m_Q$, as HQET in Eq. (1), but also involve effects due to the light quark mass, $m_q$, and $m_q/m_Q$ terms. These terms, which are buried in the $(\eta, \xi, L, F)$ parameters of Eq. (1), scale as $\Lambda_{QCD}^2/m_Q \sim \Delta \sim Q$ and $m_q \sim Q^2$, where $Q \sim m_\pi, m_K, m_\eta$ and $\Delta$ is hyperfine splitting operator; for technical details see Refs. [6, 7]. As HMChPT masses are defined up to third order, we add the factor,

$$\Delta^{(s)}_{AH} = \delta^{(s)}_{AH} \left( 1 + \frac{M}{\Lambda_\chi} + \frac{M^2}{\Lambda_\chi^2} \right),$$

(8)

to the excited $s^P_1 = \frac{3}{2}^-$ and $s^P_2 = \frac{3}{2}^+$ beauty meson masses, see Eq. (9) below, where $\Lambda_\chi \approx 1.5$ GeV and $M$ is $m_\pi$ ($m_b$) for nonstrange (strange) splittings. In light of the foregoing, one can express the HMChPT masses for the beauty mesons as

$$m^P_{B(s)} = \eta_H - \frac{3}{4} \xi^b_H + \alpha(s)L_H - \beta(s)F^b_H + \Sigma_{B(s)},$$

$$m^P_{B'_{(s)}_p} = \eta_H + \frac{1}{4} \xi^b_H + \alpha(s)L_H + \frac{1}{3} \beta(s)F^b_H + \Sigma_{B'_{(s)}_p},$$

$$m^P_{B'_{(s)}_1} = \Delta^{(s)}_{SH} + \eta_S - \frac{3}{4} \xi^b_S + \alpha(s)L_S - \beta(s)F^b_S + \Sigma_{B'_{(s)}_1},$$

$$m^P_{B'_{(s)}_2} = \Delta^{(s)}_{TH} + \eta_T - \frac{5}{8} \xi^b_T + \alpha(s)L_T - \frac{5}{6} \beta(s)F^b_T + \Sigma_{B'_{(s)}_2},$$

(9)

where $\xi^H_A = m_c \xi_A$ and $F^b_A = m_b F_A$. The one-loop corrections, $\Sigma_B$, are new functions of the beauty meson mass differences and masses of the light Goldstone particles.

To proceed, we need to extract the values of $(\Lambda_{A,1} - \lambda H_{1,1})^{(s)}$ and $\delta^{(s)}_{AH}$ using Eqs. (6) and (7). From the spectroscopy of the $s^P_1 = \frac{1}{2}^-$ and $s^P_2 = \frac{3}{2}^+$ charmed and beauty mesons, one finds $\lambda_{T,1} - \lambda H_{1,1} = -0.197$ GeV$^2$ ($\delta_{T H} = -54$ MeV) for nonstrange particles and $\lambda_{s,1}^s - \lambda H_{1,1} = -0.174$ GeV$^2$ ($\delta_{s H} = -47.7$ MeV) for strange ones. The negative sign shows that the kinetic energy of the heavy quark in the excited $s^P_1 = \frac{3}{2}^+$ mesons is larger than that in the $s^P_0 = \frac{1}{2}^-$ ground state. From Eq. (8), the corrections are found to be $\Delta_{TH} = -60$ MeV for nonstrange beauty sector and $\Delta_{s H} = -69$ MeV for strange one. The extracted values for $\Delta^{(s)}_{T H}$ amount to lowering the masses of the excited $s^P_1 = \frac{3}{2}^+$ beauty mesons, see Eq. (9). The nonperturbative parameter $\lambda_{s,1}^{(s)}$ is unknown. So, we cannot extract $\Delta^{(s)}_{sH}$. It is plausible to consider that the kinetic energy of heavy quark in the $s^P_1 = \frac{1}{2}^-$ states lying between those of the $s^P_1 = \frac{3}{2}^-$ and $s^P_2 = \frac{3}{2}^+$ states. Thus, we take $(\lambda_{s,1} - \lambda H_{1,1})^{(s)} = -0.09(6)$ GeV$^2$, where the large uncertainty measures our ignorance of $\lambda_{s,1}^{(s)}$. From Eqs. (7) and (8), the beauty nonstrange (strange) $s^P_1 = \frac{1}{2}^+$ masses in Eq. (9) are lowered by $\Delta_{s H} = -35(24)$ MeV. At our level of precision, the uncertainties due to experimental masses and higher order $O(\Lambda_{QCD}/m_c - \Lambda_{QCD}/m_b)$ corrections are negligible.

III. RESULTS AND DISCUSSION

The one loop mass formalisms in Eq. (9) can be used to obtain predictions for the beauty meson. For this, we need to fix $\eta, \xi, L, F$ appear in Eq. (1) using charm spectrum. Here we follow the approach employed in [7] to fit such parameters and predict the beauty meson masses. The fitting method is essentially
based on using experimental data on masses and couplings to evaluate the chiral loop functions. This makes fit linear and hence helps to extract unique values for the \( \eta, \xi, L, \) and \( F \) parameters, which are then used in Eq. \( 11 \) to predict the analog beauty meson spectrum. In the fit, we use twelve masses of strange and nonstrange charmed mesons; see Fig. \( 1 \). We work in the isospin limit. We only average the masses of the well-determined charmed nonstrange mesons. The \( \frac{1}{2}^+ \) charmed nonstrange mesons, however, are poorly determined. As the mass of \( D_0^{\pm} \) is higher than the corresponding strange meson, which conflicts the predictions of \( SU(3) \) symmetry, we disregard it in our theoretical treatments and instead use the masses of the excited \( D_0^{\pm} \) and \( D_0^{*} \) mesons. For Goldstone particles, the following physical values are used: \( m_{\pi} = 140 \) MeV, \( m_K = 495 \) MeV, and \( m_{\eta} = 547 \) MeV. Our results smoothly change with the normalization scale \( \mu \); consequently, performing calculations at any other values of the normalization scale will not make much difference. In our numerical calculations, we set the normalization scale to the average of pion and kaon masses, \( \mu = 317 \) MeV, as in \( 7 \). The numerical values for the couplings can be extracted using available data on strong decays of charmed mesons. The coupling constant \( g \) at tree-level can be extracted using the measured width of \( D_{s}^{\pm} \); this gives \( g = 0.5672(80) \). For the \( h \) coupling, we use \( h = 0.514(17) \), which is extracted from the width of \( D_0^{\pm} \). However, the \( g', h', \) and \( g'' \) couplings are unknown experimentally. We, therefore, use lattice QCD result for \( g' = -0.122(8)(6) \) and restrict \( h' \) and \( g'' \) to lie between 0 and 1; so, one can study the variation of the calculated masses with the \( h' \) and \( g'' \) couplings. By confronting our resulting masses against experiments, these unmeasured couplings will be constrained to lie in a narrow range making our theory much reliable.

Now we want to use charm spectrum to fit the tree-level parameters in Eq. \( 11 \). As our calculations are performed at different \( h' \) and \( g'' \), we will only show the fitting method considering \( h' = 0 \) and \( g'' = 0.03 \). In Eq. \( 11 \), \( m_A^r \) represents the residual mass of the charmed meson \( A \), which is taken to be the difference between the experimental mass and an arbitrarily chosen reference mass of \( O(m_{\pi}) \). Here we choose \( m_{D^*} \) as a reference, which yields the following central values for charmed meson residual masses,

\[
\begin{align*}
    m_{D}^r &= -141.32 \text{ MeV}, \\
    m_{D_s}^r &= 0 \text{ MeV}, \\
    m_{D_s^*}^r &= -40.22 \text{ MeV}, \\
    m_{D_{s0}^*}^r &= 103.65 \text{ MeV}, \\
    m_{D_{s1}^*}^r &= 418.45 \text{ MeV}, \\
    m_{D_{s2}^*}^r &= 309.25 \text{ MeV}, \\
    m_{D_{s3}^*}^r &= 450.95 \text{ MeV}, \\
    m_{D_{s4}^*}^r &= 526.56 \text{ MeV}, \\
    m_{D_{s5}^*}^r &= 454.5 \text{ MeV}, \\
    m_{D_{s6}^*}^r &= 560.55 \text{ MeV}.
\end{align*}
\]

By fitting the mass expressions in Eq. \( 11 \) to the corresponding empirical masses in Eq. \( 11 \), one obtains

\[
\begin{align*}
    \eta_H &= 104(5), & \xi_H &= 149(3), & L_H &= 212(6), & F_H &= -44(7), \\
    \eta_S &= 357(18), & \xi_S &= 135(27), & L_S &= -8(27), & F_S &= 12(41), \\
    \eta_T &= 476(1), & \xi_T &= 39(1), & L_T &= 109(1), & F_T &= -7(1),
\end{align*}
\]

which are given in MeV units. The fit results are obtained by computing the chiral loop functions in Eq. \( 11 \) using physical masses and couplings, as mentioned above. The errors use to get this fit are the experimental errors on masses and couplings and the LQCD error on \( g' \). The uncertainty in the nonstrange \( D_0^{*0} \) and \( D_0^{*+} \) masses gives rise to the large uncertainties seen in \( \eta_S, \xi_S, L_S, \) and \( F_S \) parameters.

The charm sector results on HMChPT parameters, Eq. \( 11 \), can be used in Eq. \( 9 \), in which hyperfine operators \( \xi's \) and \( F's \) are rescaled, to make predictions for the analog beauty meson spectrum. Following \( 7 \), one can choose the ground state, \( B \), as a reference mass to define eleven independent mass splittings, \( \Delta m_A = m_A - m_B \), where \( A \in \{ B^+, B_s, B_s^+, B_s^0, B_s^1, B_s^{*0}, B_s^{*1}, B_1, B_2, B_{s1}, B_{s2}\} \). As the self-energies represent nonlinear functions of the \( B \) meson mass differences, these independent splittings form nonlinear equations. An iterative method is utilized to solve them starting from the tree-level masses. Adding the mean value of the observed mass \( m_B \), which is chosen as a reference mass in our calculations, to the predicted mass splittings, yields

\[
\begin{align*}
    m_{B^+} &= 5325(8), & m_{B^0} &= 5369(5), & m_{B^{*+}} &= 5415(9), \\
    m_{B_s^+} &= 5683(23)(18), & m_{B_s^0} &= 5721(24)(18), & m_{B_s^{*0}} &= 5711(29)(24), & m_{B_s^{*+}} &= 5755(29)(24), \\
    m_{B_s} &= 5727(11), & m_{B_s^{*}} &= 5739(12), & m_{B_{s1}} &= 5828(11), & m_{B_{s2}} &= 5838(12),
\end{align*}
\]

in MeV units. The errors (first for the \( s_f^D = \frac{1}{2}^+ \) masses) come from the uncertainties in the charmed masses, coupling constants, lattice QCD computation on \( g' \), and rescaling factor. The first error on the predicted
TABLE I. Comparison of higher $B_1$, $B_{s1}$, $B'_2$, and $B''_s$ meson masses (units in MeV). As we work in the isospin limit, the listed values for the observed nonstrange masses are obtained by averaging the masses of the two isospin states, $\bar{b}u$ and $\bar{b}d$.

| $J^P$ | Meson | 0.00  | 0.10  | 0.30  | 0.50  | 0.80  | Experiment |
|-------|-------|-------|-------|-------|-------|-------|------------|
| $1^+$ | $B_1$ | 5726.49 | 5727.71 | 5737.57 | 5758.48 | 5836.88 | 5726 |
| $2^+$ | $B'_2$ | 5738.96 | 5738.18 | 5731.71 | 5717.64 | 5685.84 | 5738.35 |
| $1^+$ | $B_{s1}$ | 5827.53 | 5830.3 | 5853.09 | 5904.08 | 6134.2 | 5828.7 |
| $2^+$ | $B''_s$ | 5837.86 | 5836.07 | 5820.95 | 5786.54 | 5697 | 5839.86 |

TABLE II. Comparison of the experimental data and our theoretical results. We take the isospin average of $B_{-1}$ and $B_{01}$ ($B'_{-2}$ and $B''_{02}$) to obtain the mass of nonstrange excited state $B_{1}$ ($B'_{2}$). In our calculations, we fix $g'' = 0.03$ and use different values for the $h'$ coupling. Masses are in units of MeV.

| $j^P$ | $J^P$ | Meson | 0.00  | 0.30  | 0.50  | 0.90  | Experiment |
|-------|-------|-------|-------|-------|-------|-------|------------|
| $\frac{1}{2}^-$ | $1^+$ | $B^*$ | 5325(8) | 5325(8) | 5325(8) | 5325(8) | 5324.70(21) |
| $\frac{1}{2}^-$ | $0^-$ | $B_s$ | 5369(5) | 5368(5) | 5368(5) | 5366(5) | 5366.88(14) |
| $\frac{1}{2}^-$ | $1^+$ | $B^*_s$ | 5415(9) | 5415(9) | 5414(9) | 5412(9) | 5415.4(2.3) |
| $\frac{1}{2}^+$ | $0^+$ | $B^*_0$ | 5683(23)(18) | 5681(23)(18) | 5678(23)(18) | 5666(23)(18) | unseen |
| $\frac{1}{2}^+$ | $1^+$ | $B^*_1$ | 5721(24)(18) | 5720(24)(18) | 5717(24)(18) | 5708(24)(18) | unseen |
| $\frac{1}{2}^+$ | $0^+$ | $B^*_{00}$ | 5711(29)(24) | 5710(29)(24) | 5707(29)(24) | 5697(29)(24) | unseen |
| $\frac{1}{2}^+$ | $1^+$ | $B^*_{10}$ | 5755(29)(24) | 5754(29)(24) | 5753(29)(24) | 5746(29)(24) | unseen |
| $\frac{3}{2}^+$ | $1^+$ | $B_1$ | 5727(11) | 5726(11) | 5726(11) | 5724(11) | 5726(2) |
| $\frac{3}{2}^+$ | $2^+$ | $B^*_2$ | 5739(12) | 5739(12) | 5738(12) | 5737(12) | 5738.35(49) |
| $\frac{3}{2}^+$ | $1^+$ | $B_{s1}$ | 5828(11) | 5826(11) | 5822(11) | 5808(11) | 5828.7(2) |
| $\frac{3}{2}^+$ | $2^+$ | $B''_s$ | 5838(12) | 5836(12) | 5833(12) | 5821(12) | 5839.86(12) |

masses of the excited $s^P_{1} = \frac{1}{2}^+$ beauty mesons is dominated by the uncertainty in $D^0_0$ and $D^0_1$ masses and the second is the uncertainty in $\lambda_{S1}$. To investigate the influence of virtual loop effects on the predicted $B$ meson spectrum, our calculations are performed considering different input values for $h'$ and $g''$. The spectrum of the $s^P_{1} = \frac{1}{2}^-$ and $s^P_{1} = \frac{3}{2}^+$ beauty mesons depend only on the $h'$ coupling. Both $h'$ and $g''$ couplings, however, affect the excited $s^P_{1} = \frac{3}{2}^+$ beauty meson masses. The results can be justified by confronting them with the experiments.

We first look at the dependence of the higher excited $s^P_{1} = \frac{3}{2}^+$ states on $g''$. To neglect loops effect from
the virtual excited \( s_i^P = \frac{1}{2}^+ \) states, the \( h' \) coupling is set to zero. As errors associated with our calculations are relatively large, e.g., see Eq. (12), it would be instructive to compare the mean values of the observed spectrum to our results. The data and calculated masses are given in Table IV. As shown, the self-energy effects within \( s_i^P = \frac{3}{2}^+ \) states, which are parameterized by \( g'' \), have a strong impact on the predicted masses. The difference between the data and the calculated masses using small values for the coupling \( (g'' \lesssim 0.10) \) is very small, i.e., of order few MeV. But, this is not the case for the masses extracted using large values for the coupling \( (g'' > 0.30) \); which in turn implies that the predictions are unreliable in this limit. Accordingly, \( g'' \) should be constrained to lie between 0 and 0.30. In the following, we will use \( g'' = 0.03 \).

The variation of the predicted beauty meson masses with \( h' \) is presented in Table I and compared with the experimental data. As shown, the \( s_i^P = \frac{1}{2}^- \) and nonstrange \( s_i^P = \frac{1}{2}^+ \) (\( s_i^P = \frac{3}{2}^- \) and strange \( s_i^P = \frac{3}{2}^+ \)) masses show a weak (strong) dependence on the \( h' \) coupling. For \( B^0, B_s, B_s^*, B_1, \) and \( B_2^* \) states, the difference between the mean values of the observed and predicted masses is very small, i.e., of order few MeV. However, this is not the case for the \( s_i^P = \frac{1}{2}^+ \) and strange \( s_i^P = \frac{3}{2}^+ \) beauty mesons, where the virtual loop effects lower their masses by an amount of \( \sim O(15) \) MeV.

By analyzing the predicted and observed spectrum in Table I, it is clear that the \( s_i^P = \frac{1}{2}^- \) and \( s_i^P = \frac{3}{2}^+ \) states are well reproduced. However, the best values for the strange excited \( s_i^P = \frac{3}{2}^+ \) masses are those extracted considering \( h' < 0.50 \), which have mean values that are very close to the observed ones. The \( s_i^P = \frac{1}{2}^+ \) beauty mesons have not yet been observed. It is worth remarking that the predicted excited \( s_i^P = \frac{1}{2}^+ \) beauty nonstrange (strange) masses are well above (below) the threshold for decays to ground state \( B \) mesons and pions (kaons), and therefore these mesons are expected to be very broad (narrow) like \( s_i^P = \frac{1}{2}^+ \) charmed nonstrange (strange) mesons. Additionally, the \( SU(3) \) mass splittings in the predicted \( s_i^P = \frac{1}{2}^+ \) beauty sector, i.e., \( m_{B_d^0} - m_{B_s^0} \sim O(30) \) MeV and \( m_{B_{s1}^0} - m_{B_{s1}^0} \sim O(40) \) MeV, are far below theoretical expectations like those in the charm sector. Thus, our results are consistent with the expectations of heavy quark spin-flavor symmetry. Our predictions for the not yet discovered \( B_{d0}^* \) and \( B_{s1}^* \) states are remarkably close to the lattice QCD results in [11],

\[
m_{B_{d0}^*} = 5711(13)(19) \text{ MeV}, \quad m_{B_{s1}^*} = 5750(17)(19) \text{ MeV}.
\] (13)

IV. SUMMARY

The spectroscopy of the ground-state \( (s_i^P = \frac{1}{2}^-) \) and lowest-excited \( (s_i^P = \frac{1}{2}^+ \) and \( s_i^P = \frac{3}{2}^+ \)) beauty mesons were analyzed within HMChPT framework. The mass expressions used in our study include all leading contributions from one-loop corrections and those due to chiral and heavy quark symmetry breakings. The charmed spectrum was used to fix the unknown parameters that appear in the mass formulas. Then, we used charm results to make predictions for the analog beauty meson spectrum. Our calculations were performed at different values for the experimentally unknown \( (h' \) and \( g'') \) couplings, which helped to examine the influence of virtual loops effect on the calculated masses. It was found that the data is more consistent with the predicted \( s_i^P = \frac{3}{2}^+ \) masses when \( g'' < 0.30 \). For self-energy corrections parameterized by \( h' \), the calculated masses for the \( s_i^P = \frac{1}{2}^- \) and nonstrange \( s_i^P = \frac{3}{2}^+ \) (\( s_i^P = \frac{1}{2}^+ \) and strange \( s_i^P = \frac{3}{2}^+ \)) beauty mesons were found to have a weak (strong) dependence on \( h' \). The \( s_i^P = \frac{1}{2}^+ \) and strange \( s_i^P = \frac{3}{2}^+ \) beauty mesons were pushed down by nearly \( O(15) \) MeV. The resulting masses for the \( s_i^P = \frac{1}{2}^- \) and \( s_i^P = \frac{3}{2}^+ \) beauty mesons are consistent with the observed values. However, the \( s_i^P = \frac{1}{2}^+ \) beauty mesons have not yet been discovered; so, our findings could provide useful information for experimentalists investigating such states.
V. ACKNOWLEDGMENTS

The author extends his appreciation to the Deanship of Scientific Research at King Saud University for funding this work through research group No. RG-1441-537.

[1] P. A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020).
[2] K. Abe et al. (Belle Collaboration), Phys. Rev. D 69, 112002 (2004).
[3] H. X. Chen, W. Chen, X. Liu, Y. R. Liu, and S. L. Zhu, Rep. Prog. Phys. 80, 076201 (2017).
[4] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio, and G. Nardulli, Phys. Rep. 281, 145 (1997).
[5] T. Mehen and R. Springer, Phys. Rev. D 72, 034006 (2005).
[6] M. H. Alhakami, Phys. Rev. D 101, 016001 (2020).
[7] M. H. Alhakami, Phys. Rev. D 93, 094007 (2016).
[8] A. V. Manohar and M. B. Wise, Cambridge Monogr. Part. Phys. Nucl. Phys. Cosmol. 10, 1 (2000).
[9] H. Y. Cheng and F. S. Yu, Eur. Phys. J. C 77, 668 (2017).
[10] B. Blossier, N. Garron, and A. Gerardin, Eur. Phys. J. C 75, 103 (2015).
[11] C. Lang, D. Mohler, S. Prelovsek, and R. Woloshyn, Phys. Lett. B 750, 17 (2015).