Direct generation of three-photon polarization entanglement

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Non-classical states of light are of fundamental importance for emerging quantum technologies. All optics experiments producing multi-qubit entangled states have until now relied on outcome post-selection, a procedure where only the measurement results corresponding to the desired state are considered. This method severely limits the usefulness of the resulting entangled states. Here, we show the direct production of polarization-entangled photon triplets by cascading two entangled downconversion processes. Detecting the triplets with high-efficiency superconducting nanowire single-photon detectors allows us to fully characterize them through quantum state tomography. We use our three-photon entangled state to demonstrate the ability to herald Bell states, a task that was not possible with previous three-photon states, and test local realism by violating the Mermin and Svetlichny inequalities. These results represent a significant breakthrough for entangled multi-photon state production by eliminating the constraints of outcome post-selection, providing a novel resource for optical quantum information processing.

Quantum optical technologies promise to revolutionize fields as varied as computing, metrology and communication. In most cases these applications require entangled states of light, but because photons are notoriously weakly interacting, creating entanglement between photons after they have been produced is challenging. Consequently, the ability to generate entanglement during the production process is of crucial importance for photons. New capabilities of quantum sources are thus critical for the advancement of quantum optical implementations.

The production of high-quality multi-photon entanglement, such as Greenberger–Horne–Zeilinger (GHZ) states, is particularly demanding. Currently, the most established method for producing photonic entanglement is spontaneous parametric downconversion (SPDC). This is a process that naturally produces photons in pairs, making it simple to entangle the various degrees of freedom of two photons. On the other hand, experiments with three or more entangled photons have thus far relied on combining photons from two or more different pair sources using linear optics and employing outcome post-selection—selecting only a specific subset of measurement outcomes while ignoring others. With this approach, the action of observing the photons both creates and destroys the state at the same time. Although this post-selection may be acceptable for some applications, it restricts the usefulness of the resulting entangled states for others. One example is heralding Bell states, also known as event-ready entanglement, which is the ability to know that a maximally entangled two-photon state is present before it is destroyed4,5,15. This task, known to be useful for applications such as quantum repeaters6, loophole-free Bell tests7 and optical quantum computing8,9, is in theory easily achieved with an appropriate three-photon state, but does not work if this state is created with SPDC and outcome post-selection. Creating three-photon entanglement directly, without the need for such post-selection, would therefore represent a significant advance in photonic quantum information processing.

This goal can be achieved through cascaded downconversion2,20,21, a process where one of the photons from a primary SPDC process is used to pump a secondary downconversion source. Specifically, if the primary source produces polarization-entangled photon pairs, and one of those photons is used to pump a secondary polarization-entangled source22, the resulting three-photon state will be a GHZ entangled state (Fig. 1a). In this Article, we use cascaded downconversion to produce entangled photon triplets directly, without relying on outcome post-selection. We fully characterize the entangled photon triplets with quantum state tomography, and use them to perform local realism tests and to generate heralded Bell states.

Cascaded downconversion naturally produces photon triplets that are entangled in energy and time. Recently, this was verified experimentally23, under the assumption that downconversion conserves energy. However, the creation of more useful polarization-entangled photon triplets with cascaded downconversion, and verifying it conclusively, is significantly more challenging. To produce the state, it is necessary to create a coherent superposition of two orthogonally polarized cascaded downconversion processes (Fig. 1a), where the photons must be indistinguishable in their spectral, timing and spatial characteristics. In addition, the phase between the two processes has to be stable. Meeting these requirements is especially demanding because of the properties of the high-efficiency downconverters used here to make cascaded downconversion possible.

Fully characterizing a three-qubit state with quantum state tomography using projective measurements, provided that both outcomes of the measurements are detected, requires at least 27 local measurement settings24. Performing this number of measurements with sufficient event statistics would be unfeasible with the highest previously reported detection rates of seven triplets per hour25, where an important limiting factor was the low single-photon detection efficiency. Here, we use newly developed superconducting
nanowire single-photon detectors (SNSPDs), which have a high system detection efficiency of over 90% at 1,550 nm (ref. 25), promising a hundredfold increase in detected triplet rates. This dramatic improvement enables us to perform quantum state tomography and other demanding tests and applications of the three-photon entangled state.

State production and characterization
The three-photon states we aim to produce are GHZ states of the form

\[ |\text{GHZ}^3\rangle = \frac{1}{\sqrt{2}}(|\text{HHH}\rangle \pm |\text{VVV}\rangle) \]

Here \(|H\rangle\) and \(|V\rangle\) represent horizontally and vertically polarized photons, respectively. The set-up (Fig. 1b) can be understood as a cascade of two sources of entangled photon pairs\(^\text{26,27}\). First, a Sagnac source\(^\text{26,27}\) produces non-degenerate polarization-entangled photon pairs with wavelengths of 776 nm and 842 nm into modes 0 and 1, respectively. These are in the Bell state 

\[ |\psi\rangle = \frac{1}{\sqrt{2}} (|H\rangle_0|H\rangle_3 + e^{i\theta}|V\rangle_0|V\rangle_3) \]

where \(\theta\) can be controlled by tuning the tilt angle \(\theta\) of the quarter-wave plate (QWP) in the pump beam. The 776 nm photon is used to pump the second entangled photon pair source, a polarizing Mach–Zehnder interferometer with a downconversion crystal in each arm\(^\text{26,27}\). If it downconverts, a pair of photons at 1,530 nm and 1,570 nm is created in modes 2 and 3 with a polarization state depending on the pump photon, according to \(|H\rangle_0 \rightarrow |H\rangle_2|H\rangle_3\) or \(|V\rangle_0 \rightarrow e^{i\phi}|V\rangle_2|V\rangle_3\), where \(\phi\) is the phase difference between the paths. This phase is kept constant using active stabilization. The quantum state describing the photon triplets is given by

\[ |\psi\rangle = \frac{1}{\sqrt{2}} (|H\rangle_1|H\rangle_3 + e^{i\theta}|V\rangle_1|V\rangle_3) \]
events are recorded as a set of time stamps. To analyse the data we use a coincidence window of 1.25 ns, which is larger than the combined timing jitters of the detectors and the timing electronics. We observe the phase dependence of the GHZ states by changing the pump QWP tilt angle $\theta$ and measuring the three-photon correlation in the diagonal polarization basis, $E(\sigma_x, \sigma_x, \sigma_x)$, with $\sigma_x = |H\rangle\langle V| + |V\rangle\langle H|$. Quantum mechanics predicts that for the state in equation (1), $E(\sigma_x, \sigma_x, \sigma_z) = (\sigma_x \sigma_x \sigma_z) = \cos(\phi + \theta(\theta))$. The high-visibilty sinusoidal dependence of the correlation on the phase $\theta(\theta)$ (Fig. 2) is a first signature of GHZ entanglement. For subsequent measurements, the QWP tilt angle is set such that the correlation is either at a minimum or a maximum, resulting in $|\text{GHZ}\rangle$ or $|\text{GHZ}\rangle$, respectively.

We use quantum state tomography to fully characterize the three-qubit state. Each qubit is independently measured using $\sigma_x$, $\sigma_y = -i[H\rangle\langle V| + |V\rangle\langle H]$ or $\sigma_z = [H\rangle\langle H| - |V\rangle\langle V]$), yielding 27 different measurement settings $\sigma_x \sigma_x \sigma_x, \sigma_x \sigma_y \sigma_y, \ldots, \sigma_z \sigma_z \sigma_z$. Because each measurement setting has eight different possible outcomes, these settings contain a total of 216 single-outcome projective measurements and form an over-complete set for quantum state tomography. Each setting is measured for 16 min, over a total of 7.2 h. A histogram of time differences for all three-photon detections (Fig. 3) shows the tight temporal correlations of the triplets. Using the coincidence window of 1.25 ns around the peak yields a total of 4,798 threefold coincidences, which are used for tomographic state reconstruction. This corresponds to an average detection rate of 11.1 triplets per minute, in good agreement with the expected value of $12.4 \pm 2.5$ triplets per minute based on measured downconversion efficiency, coupling and detection losses (see Methods). The state is reconstructed using a semi-definite-programming algorithm implementation of the maximum likelihood method. The reconstructed density matrix $\rho$ (Fig. 4) has a fidelity with the state $|\text{GHZ}\rangle$ of $F(\rho| |\text{GHZ}\rangle) = 86 \pm 1\%$, a fidelity with $|\text{GHZ}\rangle$ of $11 \pm 1\%$ and a purity $P = \text{Tr}(\rho^2)$ of $0.77 \pm 0.02$. Error bars are calculated from a 500-run Monte Carlo simulation of the tomographic reconstruction with Poissonian noise added to the individual count rates for each run. The most likely sources of imperfection are an incomplete overlap of the downconversion spectra of the two crystals in the second entangled pair source and an imperfect phase stabilization in the Mach-Zehnder interferometer. Nonetheless, our GHZ state fidelity is, to the best of our knowledge, the highest reported fidelity for a three-photon GHZ state measured with tomography, surpassing the value of 84% reported in previous experiments.

Local realism tests

The original motivation behind the introduction of GHZ states was that multipartite entanglement allows for a striking demonstration of the incompatibility of local realism and quantum mechanics, through inequalities such as those of Mermin or Svetlichny. These inequalities can also be recast as entanglement criteria, and here we use them as a further demonstration of the quality of our GHZ state. The Mermin inequality is derived by imposing locality and realism for all three particles. Here, we look at the following three-particle version of the inequality:

$$S_{\text{Mermin}} = |E(a', b', c) + E(a', b, c') + E(a, b, c') - E(a, b, c)| \leq 2$$

The inequality holds for any local hidden variable theory. It can be violated with a GHZ state by applying the measurements $a = b = c = \sigma_y$ and $a' = b' = c' = \sigma_x$, in the ideal case reaching the arithmetic limit of 4. These measurements are a subset of those used for three-photon tomography. Of the 4,798 triplet counts from the tomography, 674 correspond to measurements for the Mermin inequality, leading to
the correlation values shown in Table 1. Combining these correlations yields a Mermin parameter of $S_{\text{Mermin}} = 3.04 \pm 0.10$, which violates the local hidden variable limit by 10 standard deviations. Because we use Pauli measurements in the Mermin inequality, its violation is also a confirmation that the state is genuinely tripartite entangled. The same conclusion can be reached even if we do not implement ideal Pauli measurements, because a Mermin parameter larger than $2\sqrt{2}$ is a device-independent test of genuine tripartite entanglement.

What the Mermin inequality cannot do is confirm the presence of tripartite nonlocality, as it can be maximally violated with models allowing for arbitrarily strong correlations between two of the particles. The Svetlichny inequality addresses this problem by allowing for arbitrarily strong correlations between any pair of particles, but otherwise enforcing locality and realism. A violation of the Svetlichny inequality thus guarantees the presence of multipartite nonlocality and rules out a large class of nonlocal hidden variable theories, which Mermin’s inequality cannot.

The Svetlichny inequality for three particles is given as

$$S_{\text{Svet}} = |E(a, b, c) + E(a, b', c') - E(a, b', c) - E(a', b, c)| \leq 4$$

This inequality can be violated with a GHZ state, but the Pauli measurements from the three-qubit tomography are no longer sufficient. To test the Svetlichny inequality we perform another experiment using the measurement settings $a = b = \sigma_x$, $a' = b' = \sigma_y$, $c = \frac{1}{\sqrt{2}}(\sigma_x + \sigma_y)$ and $c' = \frac{1}{\sqrt{2}}(\sigma_x - \sigma_y)$. In the ideal case, these measurements would result in $S_{\text{Svet}} = 4\sqrt{2}$, which is the maximum value allowed by quantum mechanics. For this experiment, 1,960 threefold coincidences were measured over a period of 3.2 h. The values of the correlation are shown in Table 1. We find a Svetlichny parameter of $S_{\text{Svet}} = 4.88 \pm 0.16$, violating the bound by five standard deviations. To the best of our knowledge, this is the strongest measured violation of the three-particle Svetlichny inequality to date.

**Heralded Bell states**

Our cascaded downconversion method of generating GHZ states allows us to directly herald Bell states. This important task cannot be done with sources based on two independent SPDC sources and linear optics, including those that have been used to post-select GHZ correlations. Indeed, either a minimum of six photons from SPDC or nonlinear interactions, or nonlinear interactions, are required. This is an example of the superiority of directly created quantum states over those produced only in post-selection.

To illustrate how our set-up can be used as an event-ready source of two-photon entanglement, we rewrite the GHZ state as

$$|\text{GHZ}'\rangle = \frac{1}{\sqrt{2}} (|\Phi^+\rangle |D\rangle + |\Phi^-\rangle |A\rangle)$$

where $|D\rangle = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$ and $|A\rangle = \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)$ represent diagonal and anti-diagonal polarizations respectively, and $|\Phi^\pm\rangle = \frac{1}{\sqrt{2}}(|HH\rangle \pm |VV\rangle)$ are Bell states. By projecting one of the photons in the diagonal basis, we can herald the presence of one of two Bell states in the other two modes. The heralding detection should come from one of the photons at telecom wavelengths so that the conversion efficiency of the second downconversion does not affect the overall heralding efficiency. In our experiment, we chose the 1.530 nm photon to act as the herald. For this measurement the phase is set to prepare a $|\text{GHZ}'\rangle$ state. We measure the 1.530 nm photon in the diagonal basis and perform quantum state tomography on the other two photons. The density matrices resulting from each of the heralding outcomes (Fig. 5a–d) have a fidelity of 89.3% with $|\Phi^-\rangle$ when heralding with $|D\rangle$ and 90.4% with $|\Phi^+\rangle$ when heralding with $|A\rangle$. Ignoring the outcome of the heralding measurement results in an incoherent mixture of $|HH\rangle$ and $|VV\rangle$ (Fig. 5e,f). The fidelity with an equally weighted incoherent mixture is 96.6%.

We measured a rate of heralded two-photon pairs of 450 per hour. From this we can extract the heralding efficiency of the system, defined as the probability of detecting a Bell state given a heralding signal. The signal at the heralding detectors is dominated by dark counts, of which there are $\sim 330$ per second. The resulting heralding efficiency is $1.9 \times 10^{-4}$. However, this is not a fundamental limit; it is dominated by the ratio of signal photons to dark counts at the heralding detector. In cases where the heralding signal is not
caused by a dark count but by a signal photon, the heralding efficiency is as high as 0.06, limited by the coupling efficiency of the other two photons.

The measured heralding efficiency could be improved by using detectors with a lower dark count rate, by increasing the triplet production rate or by optimizing the efficiency of single-mode fibre coupling. Alternatively, switching the pump to a pulsed laser would provide additional timing information on the expected arrival time of the triplets, which could be exploited to remove a significant portion of the dark counts at the heralding detectors.

It is interesting to compare the performance of our source to previous experiments related to heralded Bell states. Triggered entangled two-photons states have been produced from quantum dot systems with rates and fidelities similar to those of our set-up43,44. However, these experiments have much lower heralding efficiencies; to the best of our knowledge, the best reported heralding efficiency for these systems is $3.3 \times 10^{-9}$, five orders of magnitude lower than what we measure here45.

Experiments based on six-photon schemes have resulted in two-photon states with a fidelity of 84% (ref. 40) and 87% (ref. 41). The measured heralding efficiency of $\sim 10^{-1}$ (including coupling and detection losses) reported by such six-photon experiments is higher, but, with the changes discussed above, our measured heralding efficiency would approach or even surpass this value. In terms of detection rates, however, cascaded downconversion has a significant advantage; the six-photon experiments detected at most four photons per 3.6 h.

Figure 5 | Real and imaginary parts of the reconstructed density matrices of the heralded two-photon states. The density matrices are reconstructed from 1,632 triplets, which were measured in 3.6 h. a,b, Heralding with $|D\rangle$ results in a state close to $|\Phi^+\rangle$. c,d, Heralding with $|A\rangle$ results in a state close to $|\Phi^-\rangle$. e,f, When heralding with $|D\rangle$ and $|A\rangle$ but ignoring the measurement outcomes, the coherent terms vanish, resulting in an incoherent mixture of $|HH\rangle$ and $|VV\rangle$. 

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Table 2 | CHSH correlations and $S_{\text{CHSH}}$ when heralding Bell states with $|D\rangle$ and $|A\rangle$.

|                | $|D\rangle$ Herding | $|A\rangle$ Herding |
|----------------|---------------------|---------------------|
| $E(a, b)$      | $0.71 \pm 0.07$     | $0.77 \pm 0.06$     |
| $E(a, b')$     | $0.66 \pm 0.08$     | $0.65 \pm 0.07$     |
| $E(a', b)$     | $0.57 \pm 0.08$     | $0.59 \pm 0.10$     |
| $E(a', b')$    | $0.68 \pm 0.07$     | $0.69 \pm 0.07$     |
| $S_{\text{CHSH}}$ | $2.62 \pm 0.16$     | $2.70 \pm 0.19$     |

Heralded Bell states per hour, a rate that is two orders of magnitude less than the rate we measure with our cascaded SPDC source. Moreover, the six-photon schemes have an inherent trade-off between trigger rates and heralding efficiency. In the present setup there is no such trade-off. The heralding efficiency is entirely limited by experimental imperfections, and would in fact be improved by higher trigger rates.

Because of the high rate of heralded Bell states with cascaded SPDC we are able to accumulate enough statistics to violate a Bell inequality with our heralded two-photon states. We use the CHSH inequality \(^{16}\):

$$S_{\text{CHSH}} = |E(a, b) - E(a, b') + E(a', b) - E(a', b')| \leq 2$$

where $a = \sigma_x$, $a' = \sigma_y$, $b = \frac{1}{\sqrt{2}}(\sigma_x + \sigma_y)$ and $b' = \frac{1}{\sqrt{2}}(\sigma_x - \sigma_y)$. Quantum mechanics predicts that the inequality can be violated up to a maximum of $S_{\text{CHSH}} = 2\sqrt{2}$. The results of our measurements are shown in Table 2. We find $S_{\text{CHSH}} = 2.62 \pm 0.16$ when heralding with $|D\rangle$, and $S_{\text{CHSH}} = 2.70 \pm 0.19$ when heralding with $|A\rangle$. Both correspond to violations by over three standard deviations of the local hidden variable limit.

An important feature of our method of heralding two-photon entangled states is that the amount of entanglement of the resulting two-qubit state can be tuned based on the heralding measurement. For example, by projecting the second photon of a $|\text{GHZ}\rangle$ state onto $|\psi(\beta)\rangle = \cos \beta |H\rangle_2 + \sin \beta |V\rangle_2$, we obtain states of the form

$$|\psi(\beta)\rangle = \cos \beta |H\rangle_1 |H\rangle_2 + \sin \beta |V\rangle_1 |V\rangle_2$$

To verify this we vary the projection angle $\beta$ and perform tomography on the resulting two-photon state. The fidelities of the measured states with the predicted states for $\beta = \pi/4$, $\beta = \pi/8$ and $\beta = 0$ are 78.4, 87.8 and 96.4\%, respectively (82.0, 85.6 and 94.8\% for the orthogonal projections). The density matrices for these states are shown in Supplementary Fig. 2.

Discussion

In the present experiment we demonstrate the direct generation of three-photon polarization entanglement with cascaded downconversion. This method does not rely on the interference of independently produced photon pairs, or on outcome post-selection of detected photons; every photon triplet produced in our source is in the desired GHZ state. The unique properties of this source enable a multitude of photonic quantum information tasks. As a first such demonstration, we have shown that our source can herald high-fidelity Bell states. It could also be used as a source of multipartite entanglement for quantum communications protocols, such as quantum secret sharing\(^{47}\). We expect that our photon triplets are entangled in energy–time\(^{29}\), opening the door to a demonstration of hyper-entangled photon triplets\(^{48}\). With improved coupling efficiency out of the secondary downconversion, our method could be used for photon precertification\(^{17}\) to mitigate the impact of loss inherent to sending photons over long distances; this would allow for an extended range of quantum communication, device-independent quantum key distribution\(^{49}\) and loophole-free Bell tests\(^{17}\). In addition, with further improvements in conversion efficiency through the use of novel materials or pumped third-order nonlinearities\(^{50}\), it may be possible to add more stages to the downconversion cascade. This provides an avenue to the direct generation of entangled states of four or more photons, and consequently the heralding of GHZ states.

Methods

Production of photon triplets. A 25 mW, 404 nm fibre-coupled grating-stabilized laser diode was used to pump a Sagnac source of entangled photons. Downconversion occurred in a 30 mm periodically poled potassium titanyl phosphate (PPLN) crystal. Phase matching in the crystal was temperature-tuned to produce entangled photon pairs at 776 nm and 842 nm. The 842 nm photons were measured according to analyser A1. The 776 nm photons were sent into a polarizing Mach–Zehnder interferometer, which contained a 30 mm periodically poled lithium niobate (PPLN) waveguide in each arm. The PPLN waveguides were also temperature-controlled, and phase-matched to produce photons centred at 1,530 nm and 1,570 nm. After the Mach–Zehnder interferometer, the telecom photons were split by a dichroic mirror. The photons at 1,530 nm and 1,570 nm were measured at analysers A2 and A3, respectively. The combined coupling and detection efficiency of the 842 nm photons was $\eta_1 = 0.23$, and for the 1,530 nm and 1,570 nm photons it was $\eta_2 = \eta_3 = 0.31$, as measured from the ratio of photon detections to coincident photon detections.

Stabilization of the Mach–Zehnder interferometer. The phase in the interferometer was kept constant by active stabilization. A piezoelectric positioning stage controlled the position of the polarizing beamsplitter at the exit of the Mach–Zehnder, based on a feedback signal provided by a 778 nm stabilization laser. The range of the piezoelectric stage alone was insufficient to stabilize the interferometer over long periods of time, so it was mounted on a motorized linear stage, which was activated whenever the piezoelectric stage approached the limits of its range of motion.

Projective measurements. The projective measurements on each photon were controlled using one half-wave plate (HWOP) and one QWP, each held in a computer-controlled motorized rotation stage placed in front of a polarizing beamsplitter. Photons were detected at both outputs of the polarizing beamsplitter. The correlation of a measurement was obtained by calculating the difference in relative frequency of events with a positive and negative product of outcomes. For example, for the $\sigma_x\sigma_x\sigma_x$ measurement, the correlation value is explicitly given by

$$E(\sigma_x, \sigma_x, \sigma_x) = \frac{N_{\text{HHH}} - N_{\text{HHV}} - N_{\text{VHH}} - N_{\text{VHV}}}{N_{\text{HHH}} + N_{\text{HHV}} + N_{\text{VHH}} + N_{\text{VHV}}}$$

where $N$ is the number of counts for each outcome combination, and $h$ and $v$ represent the positive and negative eigenvalue outcomes of the $\sigma_x$ measurement.

To mitigate the effect of any imbalance in coupling or detection efficiency, we set the wave plates to alternate which output represents the positive outcome of any given measurement. For example, the $\sigma_x\sigma_x\sigma_x$ measurement was performed in eight different ways, with all three photons being transmitted by their respective polarizing beam splitters, corresponding to a projection onto the states $|\text{HHH}\rangle$, $|\text{HHV}\rangle$, $|\text{VHH}\rangle$, $|\text{VHV}\rangle$, $|\text{HHV}\rangle$, $|\text{VHH}\rangle$, $|\text{VHV}\rangle$ and $|\text{VVV}\rangle$.

Photon detection. Two types of detector were used for the experiment. The 842 nm photons were detected with free-running silicon avalanche photodiodes (APDs), which have an efficiency of $\sim 40\%$ at that wavelength. The photons at 1,530 nm and 1,570 nm were detected using free-running tungsten silicide SNSPDs with $90\%$ detection efficiency. The SNSPDs were operated at a temperature of $\sim 330$ mK inside a compact, sealed, two-stage sorption-pumped \(^{35}\)He refrigerator with a sorption-pumped \(^{35}\)He stage for heat-sinking of the wiring and optical fibres. The sorption refrigerator stages were cooled by a closed-cycle Gifford–McMahon cryocooler with a nominal cooling power of 100 mW at 4.2 K. For these experiments, the complete cycle lasted 6.5 h, of which $\sim 1$ h was spent with the detectors at operating temperature. Time stamps of all events were recorded when three photons were detected within 15 ns of each other using a time-tagger with a resolution of 156 ps.

Expected triplet rate. The main factors that determine the rate of threefold coincidences, $R_{\text{3ph\text{t}}}$, are the number of photon pairs that can be detected from entanglement photon source 1 (EPS 1) ($R_{\text{EPS1}}$) and the PPLN downconversion efficiency ($\eta_{\text{PPLN}}$). Accounting for the various efficiencies and coupling ratios, the expected triplet rate is given by

$$R_{\text{3ph\text{t}}} = R_{\text{EPS1}} \eta_{\text{PPLN}}\eta_{\text{3ph\text{t}}} \eta_{\text{EPS1}} \eta_{\text{PPLN}} \eta_{\text{3ph\text{t}}}$$

where $\eta_{\text{3ph\text{t}}}$ is the efficiency of the detector used to measure the 776 nm photons from EPS 1 for the measurement of photon pairs, $\eta_{\text{EPS1}}$ is the coupling efficiency of photons into the PPLN waveguide, $\eta_{\text{PPLN}}$ accounts for the reduction of downconversion efficiency due to the bandwidth of the 776 nm photons\(^{51}\), and $\eta_{\text{3ph\text{t}}}$ is the detector efficiency of the 776 nm photons and $\eta_{\text{EPS1}}$ is the coupling efficiency of photons into the secondary downconversion.
and $\eta$ are the combined coupling and detection efficiencies of the 1.530 mm and 1.570 mm photons, respectively.

In the present experiment, $R_{\text{triple}} = 420,000 \pm 10,000$ pairs per second. By pumping EPN 2 with a weak laser, we extract $\eta_2 = 0.31 \pm 0.02$ from the coincidence to single ratios. For $P_{\text{SPDC}}$, Hübel et al. give three values, obtained from different measurements. Here, we use a weighted mean of these three values, with the inverse of the squares of the respective uncertainties as the weighting factors. This yields a value of $P_{\text{SPDC}} = (6.9 \pm 0.7) \times 10^{-8}$. For the remainder of the parameters we use the estimates given by Hübel et al.: $\eta_0 = 0.45 \pm 0.05$, $\eta_0 = 0.50 \pm 0.05$ and $\eta_0 = 0.67 \pm 0.05$. The result is an expected triple detection rate of $R_{\text{triple}} = 12.4 \pm 2.5$ triplets per minute.

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