The Role of the Anomalous $U(1)_A$ for the Solution of the Doublet–Triplet Splitting Problem

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Abstract

The anomalous $U(1)_A$ symmetry provides a generic method of getting accidental symmetries. Therefore, it can play a crucial role in solving the doublet–triplet splitting and the $\mu$-problems via the pseudo-Goldstone mechanism: $U(1)_A$ can naturally uncorrelate the two grand unified Higgs sectors in the superpotential and simultaneously induce the desired expectation values through the Fayet–Iliopoulos $D$-term. The zero modes of the resulting compact vacuum degeneracy can be identified with the massless electroweak Higgs doublets. This automatically solves the doublet–triplet splitting and the $\mu$-problems to all orders in $M_P^{-1}$. No additional discrete or global symmetries are needed. $U(1)_A$ can also play the role of the ‘matter parity’ and suppress the baryon number violating operators. We present the simplest $SU(6)$ gauge model with a minimal Higgs sector and no doublet–triplet splitting problem. This model also relates the fermion mass hierarchy to the hierarchy of scales and predicts, for the generic non-minimal Kähler potential, approximately universal values for the tree-level electroweak Higgs mass parameters.
1. Introduction

One of the most difficult problems of the supersymmetric grand unified theories (GUTs) is the doublet–triplet splitting problem. It is difficult to understand how the theory, which knows only the very large scales $M_G \sim 10^{16}$ GeV and $M_P \sim 10^{19}$ GeV, arranges itself in such a way that a pair of essentially massless electroweak doublets $H, \bar{H}$ survive down to the low energies, not accompanied by their colour triplet partners. The natural logic is to attribute the lightness of the Higgs doublets to the smallness of the supersymmetry-breaking scale in the low energy sector $m_{3/2} \sim 100$ GeV. This requires a mechanism that would ensure masslessness of the doublets in the supersymmetric limit and at the same time guarantee that desired mass terms ($\mu$ and $B\mu$) of the right order of magnitude are generated by the SUSY breaking. As a guideline we will follow this strong criterion of naturalness, according to which the single mechanism must be responsible for both: i) vanishing doublet mass in the SUSY limit and ii) appearance of $\mu^2 \sim B\mu \sim m_{3/2}^2$ after its breaking. We also adopt the minimality requirement: both problems must be solved within the minimal set of the Higgs fields needed to break the GUT symmetry to the standard model. Besides the aesthetic problems, the non-minimal Higgs sector (additional adjoints etc.) usually creates difficulties with asymptotic freedom. As far as we know, the only approach that can satisfy the above criterion is the ‘pseudo-Goldstone’ idea \[1, 2, 3, 4, 5\]. The key point is that Higgs doublets can be identified as the zero modes of the compact vacuum degeneracy, which are massless to all orders in perturbation theory, because of supersymmetry. Once supersymmetry is broken, the flat directions are lifted and the doublets get masses of just the right order of magnitude: $\sim m_{3/2}$. On the way to constructing a realistic model along these lines there are a few potential difficulties: 1) flat direction should not be a result of the fine tuning, but rather be guaranteed by the exact symmetries of the theory; 2) unless it is protected by the gauge symmetries, the flat direction can be lifted by the $M_P$-suppressed operators in the superpotential, which can destroy the original solution; 3) colour-triplet partners must be heavy and decouple along the flat direction.

Closer to the realization of this program came the model of\[2, 3\]. The crucial observation was that the desired compact degeneracy, automatically satisfying condition (3) above, could result if the different Higgs fields that break GUT symmetry are not correlated (have no cross-couplings) in the superpotential. In this case, the vacuum has an accidental flat direction corresponding to the independent global rotations of the uncorrelated vacuum expectation values (VEVs). Since this rotation is not an exact symmetry of the theory (it is broken by the gauge and Yukawa couplings) the corresponding zero modes are not eaten up by any gauge field and are physical.

Thus, the central issue is to suppress the unwanted cross-couplings by exact symmetries. Here one can identify the following problems: first, the symmetries, which forbid the cross-couplings, also restrict the possible self-couplings of one of the fields, so that its VEV vanishes and the flat direction disappears; secondly these symmetries are anomalous and cannot be ordinary gauge symmetries. Thus, there is no reason why they should be respected by the Planck scale suppressed, operators which would generate an unacceptably large mass for the doublets.

The key point of the present paper is that the anomalous gauge $U(1)_A$ symmetry,
usually present in string theories \[6\], can provide a simultaneous solution to the above problems. Cancellation of the anomalies by the Green–Schwarz mechanism \[7\] requires non-zero mixed anomalies and thus, some of the GUT fields must transform under \(U(1)_A\). Since the symmetry is anomalous, the Fayet–Iliopoulos term (proportional to the sum of charges \(\text{Tr}Q\)) is always generated \[8\] and, in strings, is given by\[6\]

\[
\xi = \frac{g^2 \text{Tr}Q}{192\pi^2} M_p^2. \tag{1}
\]

Since it is a gauge symmetry, the anomalous \(U(1)_A\) can naturally uncorrelate the GUT VEVs in the superpotential to \textit{all orders} in \(M_p^{-1}\) and at the same time induce the desired VEV \(\sim \sqrt{\xi}\) through the Fayet–Iliopoulos \(D\)-term. This gives an exciting possibility of solving the doublet–triplet splitting and the \(\mu\)-problems in all orders in \(M_p^{-1}\) without any need of additional discrete or global symmetries, and within the minimal Higgs content. Incidentally it turns out that in this approach \(U(1)_A\) plays the role of the matter parity also and can suppress all the dangerous baryon number violating operators.

Previously the implications of the anomalous \(U(1)_A\) were considered for the fermion \[9\] and sfermion \[10\] masses, for mediating the supersymmetry breaking, and for the flavour problem \[11\]. Here we show that it is a new and crucial role that \(U(1)_A\) can play for the solution of the doublet–triplet splitting and the \(\mu\)-problems\[12\].

\section{The Problem and the Solution}

To illustrate the problem and our solution we will consider the model of ref.\[2,3\]. Consider the minimal supersymmetric \(SU(6)\) GUT. In order to break the symmetry down to the standard model group \(G_W = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y\), a minimum of two Higgs representations are necessary: an adjoint \(\Sigma^k\) and a fundamental–antifundamental pair \(\bar{\Phi}_i, \Phi_i\) \((i, k = 1, 2, ..., 6)\). The relevant \(D\)-flat VEVs are:

\[
\Sigma = \text{diag}(1, 1, 1, 1, -2, -2)\sigma, \quad \Phi_i = \bar{\Phi}^i = (\phi, 0, 0, 0, 0, 0), \tag{2}
\]

which leave unbroken \(G^\Sigma = SU(4)_c \otimes SU(2)_L \otimes U(1)\) and \(G^\Phi = SU(5)\) symmetries respectively, so that the intersection gives unbroken \(G_W\). Assume now that these two sectors have no cross-couplings in the superpotential

\[
W = W(\Sigma) + W(\Phi). \tag{3}
\]

Thus, it effectively has \(G_{gl} = SU(6)_\Sigma \otimes SU(6)_\Phi\) symmetry. Since for the VEVs given in Eq(2) this global symmetry is broken to \(G^\Sigma \otimes G^\Phi\), there are compact flat directions in the vacuum that do not correspond to any broken gauge generator; thus, the corresponding zero modes are physical fields. Note that the \(SU(6)\) \(D\)-terms cannot lift this degeneracy, since the contributions of \(\Phi, \bar{\Phi}\) and \(\Sigma\) are \textit{independently} zero. By a simple counting of the

\footnote{The possibility of suppressing the \(\mu\)-term in the superpotential via the anomalous \(U(1)_A\) and generating it through the couplings in the Kähler potential, via the Giudice–Masiero mechanism \[12\], has been considered \[13\]. This approach, however, requires an additional mechanism for the doublet-triplet splitting problem and will not be considered here.}
Goldstone states and of the broken gauge generators, we find that left-over zero modes are two linear combinations of the electroweak doublets from $\Sigma$ and $\Phi$ ($\bar{\Phi}$):

$$H = \frac{H_\Sigma \langle \phi \rangle - H_\Phi 3 \langle \sigma \rangle}{\sqrt{\langle \phi \rangle^2 + 9 \langle \sigma \rangle^2}}, \quad \bar{H} = \frac{H_\Sigma \langle \phi \rangle - H_\Phi 3 \langle \sigma \rangle}{\sqrt{\langle \phi \rangle^2 + 9 \langle \sigma \rangle^2}}.$$

(4)

All other states are heavy and the doublet–triplet splitting problem is solved. The main difficulty is to justify the absence of the possible cross-couplings in the superpotential up to a sufficiently high order in $M_P^{-1}$, by some exact symmetry. This is very difficult to do without also forbidding the possible self-couplings of the Higgs fields, so that usually one ends up either with one of the VEVs being zero, or with an enormous degeneracy of the vacuum, with many new, massless, coloured and charged superfields. More importantly, perhaps, the global symmetries under which the cross-coupling $\Sigma \bar{\phi}\phi$ is non-invariant are anomalous and need not be respected by $M_P^{-1}$ suppressed operators. Any such mixed operator with dimensionality less than 6 – 7 would destroy the solution completely. (Note, the higher operators are safe only if $\phi << M_P$, which is an additional input of the theory.)

This consideration indicates that we are naturally lead, in the problem of separating the two sectors, to the concept of anomalous gauge symmetry. As we now show, the $U(1)_A$ symmetry provides a natural loophole due to the simple reason that it is ‘anomalous’. It is enough to assume that $\Phi, \bar{\Phi}$ fields carry negative charges $q$ and $\bar{q}$ and all the other fields, and in particular quarks and leptons, carry non-negative charges so that the total trace $\text{Tr}Q > 0$. As we will see below, this assumption naturally fits in the structure of Yukawa couplings and also avoids dangerous charge and colour breaking flat directions. We also assume that $\Sigma$ carries zero charge. Then $\Phi$ and $\bar{\Phi}$ are simply left out of the most general $SU(6) \otimes U(1)_A$-invariant Higgs superpotential:

$$W_{Higgs} = \frac{M}{2} \Sigma^2 + \frac{h}{3} \Sigma^3 + \lambda_\nu \frac{\Sigma^\nu}{M_P^{-3}},$$

(5)

which fixes the VEV as in Eq(1) with $\sigma = M \approx (1 + O(M_G/M_P))$. The VEV of the $\phi$ is fixed from the $D$-terms

$$\frac{g^2}{2} \left[ \Phi^* T^a \Phi - \bar{\Phi} T^a \bar{\Phi}^* + [\Sigma^* \Sigma] T^a + \text{matter fields} \right]^2 + \frac{g_A^2}{2} \left[ q|\Phi|^2 + \bar{q} |\bar{\Phi}|^2 + \xi + q_i |S_i|^2 \right]^2,$$

(6)

where $T^a$ are $SU(6)$-generators and $q_i |S_i|^2$ is a sum over all the positively charged fields with $q_i > 0$. Minimization gives $\phi^2 = -\xi / (q + \bar{q})$ (the equality $\Phi = \bar{\Phi}$ is demanded from the $SU(6)$ $D$-term). The only allowed cross-couplings between $\Sigma$ and $\Phi$ sectors are the ones that involve positively charged matter field (see Yukawa couplings below). These couplings, however, can never affect the vacuum degeneracy, since all the positively charged fields have zero VEVs. Thus, the doublet–triplet splitting problem is solved in all order in $M_P^{-1}$ without need of any extra symmetries.

Since our discussion goes at the level of an effective field theory below $M_P$, we have simply presented the most general superpotential. Its precise form is not important for our purposes provided it can induce the desired VEV $\sigma \sim M_G$. This is just a necessary condition for having the GUT below the Planck scale. In reality we assume the scale $M \sim M_G$ to be dynamically generated from $M_P$ by some yet unexplored mechanism.
3 μ and Bμ

Assuming the conventional gravity-mediated hidden sector supersymmetry breaking, both $Bμ \sim μ^2$ of the desired magnitude are automatically generated in this scenario and we end up with the following tree-level relation among the electroweak Higgs doublet mass parameters

$$m_H^2 \simeq m_H^2 \simeq Bμ = μ^2 + m^2, \quad μ = (3A_{(3)}/h - 2A_{(2)}),$$

(7)

where $m$ is a soft mass of the Σ field and $A_{(3)}$ and $A_{(2)}$ are coefficients of the soft trilinear and bilinear couplings, respectively. The above relation is given at $M_P$ and holds up to the corrections of order $ε = \xi/M_P$. This is a standard pseudo-Goldstone relation of for the minimal soft terms and is due to the fact that for the minimal Kähler potential at the tree level there should be one exactly massless state

$$H_+ = \frac{H + H^*}{\sqrt{2}}$$

(8)

In our case this relation holds for the arbitrary non-minimal Kähler potential and essentially is a prediction of the model. This is because in our model the scale $φ \sim \sqrt{ξ}$ is predicted to be just half-way between $M_P$ and $M_G$ and the light pseudo-Goldstones predominantly reside in Σ. Due to this, both contributions from the non-universal soft terms of $Φ$ and $Φ^*$ and contributions from the cross couplings in the Kähler potential are suppressed. Thus, we have in this model one less free parameter than in minimal supergravity; hence it can predict, for instance, $tan\ β$ in terms of the other masses[1, 3, 15]

4 The Fermion Masses and Proton Stability

The fermion masses in the above scheme were analysed in more detail in [4], where it was shown that the model admits a realistic (within uncertainties in coupling constants of order 1) description of the fermion mass hierarchy in terms of the hierarchy of scales $M_P >> φ >> M_G$ without invoking flavour symmetries. The most interesting result is that only the top quark has a renormalizable Yukawa interaction at the tree level. This happens if besides the three chiral families in $15_α + 6̄_α + 6_α$ (the minimal anomaly-free set that accommodates 10 + 5 of $SU(5)$ per family) one assumes an odd number of real 20-plets with invariant $M_P$ mass terms. A decomposition of these multiplets in terms of $SU(5)$ representations gives

$$15_α = 10_α + 5_α, \quad 20 = 10 + 10$$

(9)

The important group-theoretical fact is that no invariant mass term can be formed from the symmetric product of two 20-plets; thus, a single 20-plet will survive as light and can get a mass only after $SU(6)$ symmetry breaking. Up to a field redefinition, the most general renormalizable couplings are (coupling constants are neglected)

$$Σ2020 + Φ15_320 + Φ15_α 6\̄_β.$$
The last coupling simply gives $SU(5)$-invariant masses to the extra heavy states $(5, 5')$ from $15$-s and $\bar{6}'$-s, mixing them with each other. The second term combines a $10_3$-plet from $15_3$ with $\bar{1}$ from $20$, and they become heavy as well. The remaining light $10$, predominantly residing in $20$, gets a tree-level Yukawa coupling with $H$ through the first term, giving mass to the top. The masses of lighter fermions are generated through the higher-dimensional operators

$$\frac{1}{M_{P}^{n+1}}\Phi \Sigma^n \Phi 1515 + \frac{1}{M_{P}^{n}}\bar{\Phi} \Sigma^n 15\bar{6} + ...$$

with different possible $\Sigma$ insertions. This gives us the possibility to account for the fermion mass hierarchy in terms of two ratios of the scales $\frac{M_G}{M_P}$ and $\frac{\phi}{M_P}$; for more details we refer the reader to [4]. The only new point in our case is that the necessary condition $M_G << \phi << M_P$, which was an additional input of the theory in the case of [4], is now a natural outcome since the scale $\phi$ is generated from the Fayet–Iliopoulos $D$-term. It is easy to show that our $U(1)_A$ charge assignment (necessary to solve the doublet–triplet splitting problem) is automatically compatible with the above structure of Yukawa couplings. The simplest possibility is not to invoke any flavour dependence in the spirit of ref [4]. Then the flavour-blind $U(1)_A$ charges are constrained as

$$q_{15} = -q, \quad q_{\bar{6}} = q - \bar{q}. \quad (12)$$

The additional constraint comes from the neutrino masses. For example, if we generate the right-handed neutrino masses from the operator

$$\frac{(\Phi \bar{\Phi})^2}{M_P^3}(\Phi \bar{\Phi}),$$

then charges are fixed as $q_{15} = -q, \quad q_{\bar{6}} = -4q, \quad \bar{q} = 5q$. This assignment automatically kills any baryon number violating operator trilinear in the matter fields $615\bar{6}$ to all orders in $M_P^{-1}$. Thus, $U(1)_A$ can play the role of the matter parity. Family-dependent charge assignment, along the lines of [3], is also possible without altering any of our conclusions. The novel feature in such a construction, not attempted here, will be that in contrast to [4] the Higgses that break $U(1)_A$ are not the GUT singlets. Thus their Yukawa couplings will be constrained both, by the GUT symmetry and by the anomalous $U(1)_A$. This can offer the possibility of generating specific (and hopefully predictive) textures for the fermion masses.

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