Proposed Experiments to Test the Unified Description of Gravitation and Electromagnetism through a Symmetric Metric

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Abstract
If gravitation and electromagnetism are both described in terms of a symmetric metric tensor, then the deflection of an electron beam by a charged sphere should be different from its deflection according to the Reissner-Nordstrøm solution of General Relativity. If such a unified description is true, the equivalence principle for the electric field implies that the photon has a nonzero effective electric charge-to-mass ratio and should be redshifted as it moves in an electric field and be deflected in a magnetic field. Experiments to test these predictions are proposed.

Of all the unification schemes for gravitation and electromagnetism suggested so far, the simplest is the one through a symmetric metric tensor \( g_{\mu\nu} \) [1]. In this scheme gravitation and electromagnetism curve the spacetime in exactly the same way, as a result of which the interpretation of the metric tensor as the gravitational field proper must be given up. If this scheme of unified description does indeed correspond to reality, it must possess testable deviations from Einstein’s general relativity (hereafter GR) [2] as well as new physical phenomena. The purpose of this letter is, therefore, to propose experiments through which this new scheme can be tested. To this end, we shall discuss three topics and their experimental implications.

I. The Line Element for a Spherically Symmetric Distribution of Matter and Charge: In Einstein’s GR theory, the gravitational field around a spherical distribution of mass \( M \) and charge \( Q \) located at \( r = 0 \) is described by the field equation

\[
R^{\mu\nu} = \frac{8\pi G}{c^4} T_{EM}^{\mu\nu},
\]  

(1)
where \( T^\mu_\nu_{EM} \) is the usual traceless tensor of the electromagnetic field of the charge \( Q \). The spherically symmetric solution of eq.(1) for the line element (the invariant interval) is known as the Reissner-Nordstrøm solution [3,4]. It is given by

\[
ds^2 = -\left(1 - 2\frac{GM}{c^2r} + \frac{Gk_eQ^2}{c^4r^2}\right) c^2 dt^2 + \left(1 - 2\frac{GM}{c^2r} + \frac{Gk_eQ^2}{c^4r^2}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2,
\]

where \( G \) and \( k_e \) are the gravitational and electric constants, \( c \) is the speed of light.

In our scheme, the equation describing the dynamical effects of the gravitational as well as the electric field around such a mass and electric charge distribution on a test particle of mass \( m \) and electric charge \( q \) is

\[
R^\mu_\nu = 0.
\]

The solution of eq.(3) is similar to the Schwarzschild solution [6] and is easily found to be

\[
ds^2 = -\left(1 - 2\frac{GM}{c^2r} + 2\frac{q}{m} \frac{k_eQ}{c^2r}\right) c^2 dt^2 + \left(1 - 2\frac{GM}{c^2r} + 2\frac{q}{m} \frac{k_eQ}{c^2r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2.
\]

Comparison of the third terms in \( g_{00} \) of equations (2) and (4) reveal the philosophy of our unification. In eq.(1), the electric field of the charge distribution contributes to the gravitational field of the matter. Whereas in our scheme, there is an equivalence principle for the electromagnetic field as well [1], and the right-hand side of eq.(3) is zero, as opposed to eq.(1) of GR; the electric field does not contribute to the gravitational field, it asserts itself separately. To test which of the third terms in \( g_{00} \) of equations (2) and (4) reflects the physical reality, consider a positively charged metallic sphere of radius \( R \), mass \( M \), and electric charge \( Q \). The electric potential on the surface of the sphere is

\[
V(R) = \frac{k_eQ}{R},
\]

in terms of which the \( g_{00} \) are

\[
g_{00}^{RN} = -\left(1 - 2\frac{m_G}{r} + \frac{GR^2}{k_e c^4 r^2} V(R)^2\right);
\]

\[
g_{00}^{MGR} = -\left(1 - 2\frac{m_G}{r} + 2\frac{q}{m} \frac{R}{c^2r} V(R)\right),
\]

where the first one corresponds to the Reissner-Nordstrom (RN) solution and the second one to ours, which we call “modified general relativity” (hereafter MGR), and \( m_G = GM/c^2 \). Now, for a sphere of \( M = 1kg \), \( R = 5cm \), and an electric

\[1\text{We use the conventions of Misner, Thorne, and Wheeler [5] for metrics, curvatures, etc.} \]
potential of $10^3 V$ on the surface of the sphere, we have, for an electron just grazing the sphere:

\[
\begin{align*}
\gamma^{RN}_{00} &= - \left(1 - 1.48 \times 10^{-26} + 9.19 \times 10^{-49}\right) \approx -1; \\
\gamma^{MGR}_{00} &= - \left(1 - 1.48 \times 10^{-26} - 3.91 \times 10^{-3}\right) \approx -0.996. 
\end{align*}
\]

Thus, the space around such a charged sphere is extremely close to being flat in the Reissner-Nordstrøm case and is approximated perfectly by the metric of special relativity, the Minkowski metric. In our case, however, there is a great deal of deviation from flatness that can assert itself in the trajectory of an electron moving in the vicinity of the sphere. The trajectory of an electron ($q = -e$) moving in the gravitational and electric fields, however weak they are, of such a sphere is described by

\[
\frac{d^2x^\mu}{ds^2} + \Gamma_{\alpha\beta}^\mu \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} = \frac{q}{mc^2} F^\mu_\alpha \frac{dx^\alpha}{ds},
\]

in the Reissner-Nordstrøm case with $\Gamma_{\alpha\beta}^\mu$ calculated from eq.(2), and by

\[
\frac{d^2x^\mu}{ds^2} + \Gamma_{\alpha\beta}^\mu \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} = 0,
\]

in our scheme with $\Gamma_{\alpha\beta}^\mu$ calculated from eq(4).

To simplify the notation, let us, as usual, write the line element in the form

\[
ds^2 = -e^\eta c^2 dt^2 + e^{-\eta} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2.
\]

The nonzero components of

\[
\Gamma_{\alpha\beta}^\mu = \frac{1}{2} g^{\mu\nu} (g_{\nu\alpha,\beta} + g_{\nu\beta,\alpha} - g_{\alpha\beta,\nu}),
\]

that we need in our calculation are

\[
\Gamma^0_{01} = \Gamma^0_{10} = \frac{1}{2} \frac{d\eta}{dr}, \quad \Gamma^3_{13} = \Gamma^3_{31} = \frac{1}{r},
\]

Using

\[
A_\mu = \left(-\Phi_E, \vec{A}\right) = \left(-k_e \frac{Q}{r}, 0\right),
\]

the nonzero components of the electromagnetic field strength tensor

\[
F_{\mu\nu} = \frac{\partial A_\nu}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\nu},
\]

\footnote{Note that in the Reissner-Nordstrøm case, the contribution of the electric charge of the sphere to its gravitational field turns out to be much smaller than the mass term $2GM/c^2r$ for reasonable values of $r$.}

\footnote{The other nonzero components of $\Gamma_{\alpha\beta}^\mu$ that are not required in our calculation have not been quoted here.}
are
\[ F_{01} = -F_{10} = -k_e \frac{Q}{r^2}. \] (15)

Confining the motion of the electron in the \( \theta = \pi/2 \) plane not only simplifies the calculation a lot but also the experiment to be described later. We, then, obtain the following equations from eq.(8) for the coordinates \( x^0 = ct \) and \( x^3 = \phi \)

\[
\frac{d^2t}{ds^2} + \frac{d\eta}{dr} \frac{dt}{ds} = \frac{q}{mc^2} e^{-\eta} k_e Q \frac{dr}{r^2},
\]

\[
\frac{d^2\phi}{ds^2} + 2 \frac{dr}{r} \frac{d\phi}{ds} = 0,
\]

where we have put \( d\theta/ds = 0 \). A further simplification is achieved by trading the equation for the coordinate \( x^1 = r \) with the one that follows from the condition of timelike geodesics

\[ g_{\mu\nu} \frac{dx^\mu}{ds} \frac{dx^\nu}{ds} = -1, \]

which gives

\[ e^{-\eta} \left( \frac{dr}{ds} \right)^2 + r^2 \left( \frac{d\phi}{ds} \right)^2 - e^\eta c^2 \left( \frac{dt}{ds} \right)^2 + 1 = 0. \] (19)

Equations (16) and (17) can be integrated to yield, respectively

\[ \frac{dt}{ds} = e^{-\eta} \left( -\frac{q k_e Q}{mc^2} \frac{1}{r} + a \right), \]

\[ r^2 \frac{d\phi}{ds} = h, \]

where \( a \) and \( h \) are integration constants. Noting that \( dr/ds = (dr/d\phi)(d\phi/ds) \) and inserting equations (20) and (21) in eq.(19) and then dividing by \( e^{-\eta} \) we get

\[ \left( \frac{du}{d\phi} \right)^2 + u^2 e^\eta - \left( -\frac{q k_e Q}{mc^2} u + a \right)^2 \frac{1}{h^2} + \frac{e^\eta}{h^2} = 0, \] (22)

where we have set \( u = 1/r \). For the Reissner-Nordstrøm solution we now put \( e^\eta \approx 1 \). Differentiating eq.(22) with respect to \( \phi \) and removing the factor \( du/d\phi \) we get

\[ \frac{d^2 u}{d\phi^2} + u = \frac{m_E}{h^2} + \frac{m_E^2}{h^2} u \]

where we have set the constant \( a = 1 \) so that when \( h = l/mc \), with \( l = mr^2 \dot{\phi} \) being the ordinary angular momentum, the first term on the right-hand side of eq.(23) agrees with the Newtonian (hereafter N) expression

\[ \frac{d^2 u}{d\phi^2} + u = \frac{m^2 c^2}{l^2} m_E. \] (24)
Here
\[ m_E = -\frac{q}{m} \frac{k_e Q}{c^2} = -\frac{q}{mc^2} RV(R) \]  

(25)

has the dimension of length and corresponds to \( m_G = GM/c^2 \) in the Schwarzschild solution. Eq.(23) describes the trajectory of a charged test particle when \( g_{11} \approx -g_{00} \approx 1 \) in the Reissner-Nordstrøm solution. Hence, it also describes exactly the trajectory of a test charge in an electric field according to special relativity. The second term on the right-hand side of eq.(23) is a special relativistic correction to the Newtonian result.

As for eq.(9), we get
\[ \frac{d^2 t}{ds^2} + \frac{d\eta}{ds} \frac{dt}{ds} = 0 \]  

(26)

instead of eq.(16), and
\[ \frac{dt}{ds} = \frac{e^{-\eta}}{c} \]  

(27)

instead of eq.(20) with \( a = 1 \). Equations (17) and (21) do not change. Proceeding as before, we find
\[ \frac{d^2 u}{d\phi^2} + u = \frac{m_E}{\hbar^2} + 3m_E u^2. \]  

(28)

Recall that terms involving \( m_G \) on the right-hand sides of equations (23) and (28) have been dropped because \( m_G << m_E \) for the metallic sphere we are considering. It should be noted that when \( m_E \) is replaced with \( m_G \) in eq.(28), the equation of a neutral test particle of mass \( m \) moving in the Schwarzschild field of a spherical mass \( M \) is obtained. Since \( mch \) is the conserved angular momentum of the test charge in its rest frame, we need to express \( h \) in terms of \( l \), the ordinary angular momentum of the test charge in the laboratory frame (with respect to the coordinate time \( t \)).

In the Reissner-Nordstrøm case, equations (20) and (21), with \( e^{-\eta} = 1 \), yield
\[ h = \frac{l}{mc}(1 + m_E u)^{-1}, \]  

(29)

and in our scheme equations (21) and (27), with \( e^{-\eta} = (1 - 2m_E u)^{-1} \), yield
\[ h = \frac{l}{mc}(1 - 2m_E u)^{-1}. \]  

(30)

Equations (23) and (28) then reduce to
\[ \frac{d^2 u}{d\phi^2} + u = \frac{m_E}{l^2} \frac{m_E}{(1 + m_E u)}, \]  

(31)

which is the orbit equation for the Reissner-Nordstrøm solution, and
\[ \frac{d^2 u}{d\phi^2} + u = \frac{m_E}{l^2} m_E (1 - 2m_E u)^2 + 3m_E u^2, \]  

(32)
which is the orbit equation in our scheme.

We now propose the following experiment to distinguish between the two equations; (31) and (32): Consider a vacuum chamber in the shape of a rectangular metallic box. Let a metallic ball of radius $R$ positively charged to a potential of $V(R)$ be hanged freely from an insulating thread. Let an electron gun be located at angle $\alpha$ on the equatorial plane of the ball at a distance $r_i$ from the ball’s center. The point of emergence of the electrons may be taken to be on the negative $y$ axis and thus has $\phi = 3\pi/2$. Put a calibrated phosphorous screen on the positive $y$ axis at $\phi = 5\pi/2$. Make a large enough glass window on the side of the box facing the screen (or monitor the position of the electron beam on the screen electronically) to observe where the electron beam hits on the screen. Equations (31) and (32) can be solved numerically for $u$, and hence for $r$. The two initial conditions required are $u(\phi = 3\pi/2) = r_i^{-1}$ and $du/d\phi(\phi = 3\pi/2) = \sqrt{1 - \sin^2\alpha/(r_isina)}$, where as above $\alpha$ is the angle the initial velocity $v_i$ of the electrons makes with the positive $y$ axis. In obtaining the second initial condition we have made use of $dr/dt = \dot{r} = (dr/d\phi)(d\phi/dt) = r'\dot{\phi}$, $v^2 = \dot{r}^2 + r^2\dot{\phi}^2$, and $l = mr^2\dot{\phi} = mv_ib$, where $b = r_isina$ is the impact parameter of the electrons. The solutions of the equations (31) and (32) can thus be found numerically at any value of the angle $\phi$, and especially on the positive $y$ axis. We have tabulated some exemplary results in Tables 1 and 2. It is seen that in all cases the prediction of our scheme for the position of the electron beam on the screen is distinctly different from the Newtonian and Reissner-Nordström (RN) predictions. One may be curious as to why the dispersion (see the last columns in Tables 1 and 2), $r_N - r_{MGR}$, between the Newtonian and Modified General Relativistic trajectories decreases as the potential $V(R)$ of the sphere increases. For weak potentials the curvature of spacetime is small and the angle between the two trajectories is large, as a result of which the two trajectories disperse more from each other at large distances from the sphere. Therefore, by measuring the position of the electron beam on the screen the correct theory can be distinguished. In Figures 1-4, the trajectory of the electrons is drawn according to the three theories, where again the differences in the trajectories are seen with certainty. For an anticipated difference of about 3-5 cm between $r_N$ and $r_{MGR}$, a rectangular metallic box with dimensions $130cm \times 30cm \times 30cm$ with a circular lid near the top of one end, and a glass window on the side facing the screen may be built very easily. A rotary-diffusion pump system can easily obtain the desired

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4In our calculations we have used the relativistic expression $eV_{AC} = m_e c^2 / \sqrt{1 - v_i^2/c^2} - m_e c^2$ to calculate $v_i$, the initial velocity of the electrons. For an anode-cathode voltage of $V_{AC} = 1000V$ for the electron gun, this gives $c/v_i = 16.0077$, whereas the nonrelativistic expression gives $c/v_i = 15.9843$. The positions in the Tables are very sensitive to variations in $c/v_i$.

5The same phenomenon occurs in geometry between the Newtonian and general relativistic trajectories of a neutral test particle moving in the gravitational (Schwartzschild) field of a spherical mass. Replace $m_E$ with $m_G$ in equations (24) and (28) to get the gravitational equations.

6If evacuating the box is not a problem, a longer box can be built to obtain larger $r_N - r_{MGR}$ (see Table 1). Figures 3 and 4, on the other hand, suggest that a much smaller box could be used for very large V(R) and anode-cathode voltage for the electron gun. Mathematically this is true.
Table 1: Predicted positions, according to the three theories, of the electron beam at $\phi = 5\pi/2$ after deflected by a sphere of $R = 5\text{cm}$ and potential $V(R)$ for an anode-cathode voltage difference of $1000V$ for the electron gun located at $\phi = 3\pi/2$ and $r_i = 15\text{cm}$ from the center of the sphere.

| V(R) (Volt) | $\frac{m\alpha}{R}$ | $\alpha$ (degree) | $r_{RN}$ (cm) | $r_N$ (cm) | $r_{MGR}$ (cm) | $r_N - r_{MGR}$ (cm) |
|------------|---------------------|-------------------|----------------|------------|----------------|------------------------|
| 1325       | 2.59 x 10^{-3}      | 40                | 212.08         | 208.07     | 190.71         | 17.36                  |
| 1350       | 2.64 x 10^{-3}      | 40                | 165.02         | 162.48     | 151.51         | 10.97                  |
| 1375       | 2.69 x 10^{-3}      | 40                | 135.06         | 133.27     | 125.68         | 7.59                   |
| 1400       | 2.74 x 10^{-3}      | 40                | 114.31         | 112.97     | 107.37         | 5.60                   |
| 1425       | 2.79 x 10^{-3}      | 40                | 99.08          | 98.03      | 93.72          | 4.31                   |
| 1600       | 3.13 x 10^{-3}      | 45                | 219.53         | 214.91     | 198.36         | 16.55                  |
| 1625       | 3.18 x 10^{-3}      | 45                | 176.50         | 173.38     | 162.30         | 11.08                  |
| 1650       | 3.23 x 10^{-3}      | 45                | 147.57         | 145.31     | 137.34         | 7.97                   |
| 1700       | 3.33 x 10^{-3}      | 45                | 111.15         | 109.76     | 105.03         | 4.73                   |
| 1750       | 3.42 x 10^{-3}      | 45                | 89.15          | 88.19      | 85.03          | 3.16                   |
| 1875       | 3.67 x 10^{-3}      | 50                | 225.14         | 220.00     | 204.86         | 15.14                  |
| 1900       | 3.72 x 10^{-3}      | 50                | 185.61         | 181.99     | 171.39         | 10.60                  |
| 1925       | 3.77 x 10^{-3}      | 50                | 157.88         | 155.17     | 147.32         | 7.85                   |
| 1950       | 3.82 x 10^{-3}      | 50                | 137.37         | 135.25     | 129.18         | 6.07                   |
| 2000       | 3.91 x 10^{-3}      | 50                | 109.03         | 107.61     | 103.66         | 3.95                   |

vacuum required for the electron gun to work. Care must be taken to set the angles and the distances as precisely as possible because the solutions are very sensitive to variations in them.

One may wonder, if in scattering experiments of the Rutherford type a deviation in the cross-section should have been seen due to the electrical curvature of the spacetime. For the scattering of $\alpha$ particles off gold nuclei, the correction term $2(q/m)k_eQ_{Gad}/(c^2r) = 1.2 \times 10^{-16}/r$ to $g_{10}^{MGR}$ turns out to be between $10^{-3}$ and $10^{-16}$ for $10^{-13}m \leq r \leq 1m$, where $r$ is the position of the alpha-particle from the target nucleus. So, within the precision of these experiments, no deviation from the cross-section can be seen.

II. The Electrical Redshift of Light: If true, one immediate and dramatic consequence of the gravito-electromagnetic unified description in our scheme is that light should undergo a redshift as it travels against a uniform electric field. The existence of the electrical redshift can be inferred from the equivalence principle for the electric field [1]. Consider a cabin and two clocks separated by a horizontal distance $d$ in it, all with the same $q/m$ ratio. For definiteness, assume the charges are positive. Let the cabin be accelerating to the left at the rate $a = (q/m)E$ to
Figure 1: The trajectories of the electron beam according to the Reissner-Nordstrøm (the top curve), Newtonian (the middle curve), and the Modified General Relativity (the bottom curve) theories for an anode-cathode voltage of 1000\(V\) for the electron gun, and for a sphere of \(R = 2.5cm\) and \(V(R) = 2250V\).
Figure 2: Same as Figure 1, but $R = 5cm$ and $V(R) = 1750V$. 
Figure 3: Same as Figure 1, but the anode-cathode voltage for the electron gun is 10000V, $R = 2.5cm$ and $V(R) = 30000V$. 
Figure 4: Same as Figure 1, but the anode-cathode voltage for the electron gun is $10000\text{V}$, $R = 5\text{cm}$, and $V(R) = 15000\text{V}$. 
Table 2: Same as Table 1, but $R = 2.5\text{cm}$ and $r_i = 10\text{cm}$. \\

| V(R) (Volt) | $\frac{m_e}{R}$ | $\alpha$ (degree) | $r_{RN}$ (cm) | $r_N$ (cm) | $r_{MGR}$ (cm) | $r_N - r_{MGR}$ (cm) |
|-------------|-----------------|-------------------|--------------|------------|----------------|----------------------|
| 2100        | $4.11 \times 10^{-3}$ | 45                | 193.55       | 188.39     | 170.01         | 18.38                |
| 2125        | $4.16 \times 10^{-3}$ | 45                | 155.86       | 152.40     | 140.01         | 12.38                |
| 2150        | $4.21 \times 10^{-3}$ | 45                | 130.45       | 127.95     | 119.02         | 8.93                 |
| 2200        | $4.31 \times 10^{-3}$ | 45                | 98.38        | 96.87      | 91.56          | 5.31                 |
| 2250        | $4.40 \times 10^{-3}$ | 45                | 78.97        | 77.94      | 74.39          | 3.55                 |

simulate an electric field $E$ directed to the right. An inertial observer describes the following chain of events: The right and left-hand clocks are both accelerating to the left with acceleration $a$. The right-hand clock is sending photons to the left-hand clock at the rate $\nu_R$ photons per second. It takes time $t = \frac{d}{c}$ for a photon to reach the left-hand clock, during which time the velocity of the left-hand clock increases by $\Delta v = \left(\frac{q}{m}\right)Ed/c$. Therefore the rate $\nu_L$ that the photons are detected by the left-hand clock is decreased by a Doppler redshift

$$\nu_L = \nu_R \left(1 - \frac{\Delta v}{c}\right) = \nu_R \left(1 - \frac{q}{m} \frac{Ed}{c^2}\right).$$

(33)

This means that the frequency of a photon detected by the left-hand clock undergoes a Doppler shift exactly as in eq.(33). Therefore the fractional change in the frequency of the photons is

$$\frac{\Delta \nu}{\nu} = \frac{\nu_L - \nu_R}{\nu_R} = -\frac{q}{m} \frac{Ed}{c^2},$$

(34)

where now $\nu$ refers to the photon frequency. Then according to the equivalence principle, the same redshift must be observed as light travels to the left in a uniform static electric field $E$ directed to the right. Note, strange as it may sound though, that the above argument implies that the photon behaves in an electric field as if it has a nonzero “effective electric charge” and hence an electric charge-to-mass ratio $(q/m)$.\footnote{This is similar to the gravitational situation in which the photon has “effective” gravitational and inertial masses and $(m_g/m_i) = 1$. In the electrical case, however, we do not know the value of $(q/m_i)$. It must be determined from the experiment.} Note, however, that the above argument does not fix the sign of the effective charge of the photon. If the effective charge is negative, photons then would be redshifted as they moved in the same direction as the electric field. Hence,\footnote{Having found out that photons have a nonzero electric-charge-to-mass ratio, we point out that the cabin and the clocks then must have this very same ratio so that the equivalence principle for the electric field is applicable. However, one should not conclude from this that, in reality the atoms (clocks) emitting and absorbing the photons must have the same electric charge-to-mass ratio as the photons. This can be seen by excluding the clocks from the cabin in the above thought experiment, or from the conservation of energy argument as applied to a particle moving in a uniform electric field and converting to a photon. This argument does not involve any “clocks”.

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assuming a positive “effective electric charge” for the photon, the conservation of energy of a particle moving in an electric field and then converting to a photon, just like a particle falling in a gravitational field and then converting to a photon [7], yields the same redshift as in eq.(34).

An experiment of the Pound-Rebka-Snider type [8,9] can be done to verify the redshift and/or to put a limit on the \((q/m)\gamma\) of the photon. A \(\Delta\lambda/\lambda\approx 10^{-15}\) should be seen for a voltage difference of about 100\(V\) between the detection and emission points of the photons if \((q/m)\gamma = 1C/kg\)[9]. If, on the other hand, \((q/m)\gamma = 0.1C/kg\) or \(0.01C/kg\), the required voltage difference would be about \(10^3V\) or \(10^4V\), respectively.

Before we end this section, we would like to remark that a nonzero \((q/m)\gamma\) implies that light would be deflected or scattered off as it passes a charged spherical object just as it is deflected by a massive spherical object like the sun. The magnitude of the deflection, however, is so small, even for \((q/m)\gamma = 1C/kg\), that a laboratory experiment does not seem possible.

III. The Deflection of Light in a Magnetic Field: Another consequence of a nonzero electric charge-to mass ratio for the photon is that light would be deflected in a magnetic field. Consider a uniform static magnetic field \(B\) directed downward in the \(-z\) direction. Let a light beam be emitted from a point and travel in the \(xy\) plane so that the velocity of the light beam is perpendicular to the magnetic field. The light beam should travel in a counterclockwise circle of radius

\[
R = \frac{1}{(q/m)\gamma B} c, \tag{35}
\]

which follows from the equality of the centripetal and magnetic forces on a single photon. Let \(d\) be a straight distance that a photon would have travelled had it been not deflected by the magnetic field. Then the deflection \(\Delta\), the distance from the end of the distance \(d\) to the actual position of the photon on the circle, is

\[
\Delta = \frac{1}{(q/m)\gamma B} c \left[ 1 - \cos \left( \sin^{-1} \left( \frac{q}{m} \gamma B d c \right) \right) \right]. \tag{36}
\]

Tabulated in Table 3 are the deflections for \((q/m)\gamma = 1C/kg\) a light beam would suffer as a function of \(B\) and the straight distance \(d\), the distance light is allowed to travel when \(B = 0\). We see that a deflection of a tenth of a millimeter is expected for \(B = 1T\) and \(d = 250m\). A uniform magnetic field extending to a desired length can easily be obtained by placing a number of electromagnets end-to-end. The positions of a light beam on a “film” in the absence and presence of the magnetic field can be measured. The distance between the two positions would be the anticipated deflection.

\[\text{Note, as we have pointed out in [1], that in a different system of units the electric charge } q \text{ and the mass } m \text{ may be measured in the same unit. In such a system of units, } (q/m)_\gamma = \pm 0/0 \rightarrow \pm 1 \text{ seems more likely.}\]
Table 3: The deflections, $\Delta$, a light beam is expected to suffer for $(q/m)_\gamma = 1 \text{C/kg}$ as a function of the magnetic field $B$ and the distance $d$ the beam travels when $B = 0$.

| B (T) | d (m) | $\Delta$ (mm) |
|-------|-------|---------------|
| 1     | 250   | 0.104         |
| 1     | 500   | 0.417         |
| 1     | 1000  | 1.668         |
| 5     | 100   | 0.083         |
| 5     | 150   | 0.188         |
| 5     | 250   | 0.521         |
| 5     | 350   | 1.022         |
| 10    | 50    | 0.042         |
| 10    | 100   | 0.167         |
| 10    | 250   | 1.042         |

In this letter, we have proposed three experiments to test whether or not gravitation and electromagnetism have a unified description through a symmetric metric tensor. The experiment of the deflection of an electron beam by a positively charged sphere, which is to show if a distribution of electric charge curves the spacetime independently of its gravitational field, is the simplest one and should be done first. The other two experiments depend strongly on the predicted electric-charge-to-mass ratio for the photon. A negative result in these experiments would still be useful to place an upper limit on $(q/m)_\gamma$.

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