Nonperturbative QCD: confinement and deconfinement

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Abstract

After a short exposition of field correlators in the QCD vacuum and the recently discovered Casimir scaling phenomenon, the origin of confinement in QCD is discussed and two possible mechanisms are suggested, which can be checked by new lattice measurements. Screening of confinement due to sea quarks is discussed and quantitatively explained. Deconfinement is introduced via the colorelectric field evaporation and the transition temperature $T_c$ is found numerically in good agreement with lattice measurements. The $T_c$ dependence on $N_c$ and $n_f$ is also predicted and agrees with the recent lattice data.

1 Introduction

The confinement is known to be the most important QCD dynamics at large distances preserving stability of matter and existence of our world (for a review see [1]). The force of approximately 15 tons between quark and antiquark in mesons, or between quark and the string junction in proton is known from the lattice with excellent accuracy to be constant up to very small distances [2]. However theoretical understanding of this phenomenon still far from complete despite many efforts during the last decade. It is a purpose of this talk to describe in some detail what is understood now in the picture of confinement and deconfinement and where is the front line of
our present knowledge. The main instrument in what follows is the gauge-invariant Formalism of Field Correlators (FFC) [3, 4], however references to particular applications are done at appropriate places.

As it is understood now the nonperturbative structure of the QCD vacuum responsible for mass generation and confinement is adequately described by gauge-invariant field correlators

\[ D^{(n)}(1, ...n) \equiv \langle tr F_{\mu_1 \nu_1}(x_1) \Phi(x_1, x_2) F_{\mu_2 \nu_2}(x_2) ... F_{\mu_n \nu_n}(x_n) \Phi(x_n, x_1) \rangle \]  

(1)

where \( \Phi(x, y) = P \exp i g \int_y^x A_{\mu} dz_{\mu} \) is the parallel transporter, and \( \langle ... \rangle \) means the vacuum average with the standard QCD action.

The ensemble of \( D^{(n)} \), \( n = 2, ... \infty, (D^{(1)} \equiv 0) \) or equivalently, the ensemble of connected correlators \( \bar{D}^{(n)} \) (called cumulants) contains a complete information for the quark-antiquark dynamics in the limit of large \( N_c \) (at finite \( N_c \) also another set of correlators containing more color traces is necessary).

A recent lattice data [5] for Wilson loops in different representations of SU(3) color group have confirmed the notion of Casimir scaling with very high accuracy (around 1%), i.e. the fact that static \( Q \bar{Q} \) potentials are proportional to the quadratic Casimir operator. As it was argued in [6] it is the lowest field correlator \( D^{(2)} \) which has the property of Casimir scaling, while all higher correlators violate this property and hence are strongly suppressed in the vacuum. This remarkable observation makes the picture of the QCD vacuum rather simple, it can be called the Gaussian Stochastic Vacuum (GSV) and it also enables one to use the lowest correlator known from lattice calculations [7] for all nonperturbative dynamical calculations, including the study of QCD strings in mesons [8] and baryons [9], study of spectra of mesons, hybrids, glueballs and baryons (for a review and references see [10, 11]).

At the same time one should stress that to make the theoretical approach of FFC complete, one should calculate FC in the framework of the same method using the only QCD parameter, (string tension \( \sigma \) or \( \Lambda_{QCD} \)), as an input, and in addition to understand the dynamics of Casimir scaling, i.e. suppression of higher cumulants.

In this direction only first steps are done [12] with encouraging results, in particular the smallness of the gluonic correlation length \( T_g \), entering FC, is understood from the gluehump spectrum [13].

However, the understanding of the Casimir scaling is not yet complete. On one hand, the contribution of higher correlators \( D^{(n)} \) to the string tension
in general can be estimated as $\langle F \Phi F \rangle_n^{-1}$, where $\Phi$ is average field strength, estimated from the condensate to be $\Phi \sim 0.5$ GeV, and here we have not taken into account that cumulants are connected. If this is included, then the Casimir scaling violation can be shown to arise due to white exchanges between ”dipoles” $\langle F \Phi F \rangle$, while higher cumulants are suppressed as $1/N_c^2$ (see last ref. in [6]). Altogether one gets a suppression factor $\sigma(4)/\sigma(2) \approx (\Phi T^2 g)^2/N_c \sim 1 - 3\%$ which gives an order-of-magnitude agreement with the violation of Casimir scaling on the lattice [6].

Another set of relevant questions concerns the nature of confinement, i.e. which field configurations are responsible for confinement. In the GSV it is the lowest correlator $\mathcal{D}(2)$ which confines, therefore the question is about the nature of field configurations which saturate $\mathcal{D}(2)$. It is the purpose of this talk to discuss these points one by one, starting with confinement mechanism in the next section, and temperature deconfinement in section 3.

## 2 Mechanism of confinement

Since $\mathcal{D}(2)$ according to lattice data [5] ensures some 99% of confinement, one should look more carefully at its structure, namely

$$D_{\mu_1\nu_1,\mu_2\nu_2}(x_1, x_2) \equiv \frac{g^2}{N_c} \langle tr F_{\mu_1\nu_1}(x_1) \Phi(x_1, x_2) F_{\mu_2\nu_2}(x_2) \Phi(x_2, x_1) \rangle =$$

$$D(z)(\delta_{\mu_1\mu_2}\delta_{\nu_1\nu_2} - \delta_{\mu_1\nu_2}\delta_{\mu_2\nu_1}) + \frac{1}{2} [\partial_{\mu_1}(z_{\mu_2}\delta_{\nu_1\nu_2} - z_{\nu_2}\delta_{\nu_1\mu_2}) +$$

$$+ (\mu_i \leftrightarrow \nu_i)]D_1(z), \quad z \equiv x_1 - x_2, \quad \partial_\mu = \frac{\partial}{\partial z_\mu}. \quad (2)$$

Using the nonabelian Stokes theorem and the cluster expansion (see [1] for details and references) one has for a large Wilson loop

$$\langle W(C) \rangle = \frac{1}{N_c} \langle tr P \exp ig \int_C A_\mu dx_\mu \rangle = \exp(-\sigma S_{min}) \quad (3)$$

where

$$\sigma(2) = \frac{1}{2} \int D(u) d^2 u \quad (4)$$

and $S_{min}$ is the minimal area inside the contour $C$. From (4) it is clear that $D(z)$ plays the role of the order parameter for confinement (at least for
$N_c \to \infty$) and this is confirmed by lattice calculations of $D(z)$, where $D(z)$ vanishes abruptly above the critical temperature $T_c$. A further analysis can be done applying to both sides of \((3)\) the operator $\frac{1}{2} e_{\alpha\beta\mu_\nu} \frac{\partial}{\partial z^\alpha}$ which yields \([4]\)

$$z \frac{\partial D(z)}{\partial z^2} = f^{(1)}_\alpha + f^{(2)}_\alpha,$$

and

$$f^{(2)}_\alpha \equiv \frac{g^2}{24N_c} \left[ i g \int_0^z dy \alpha(y) \langle \text{tr} \tilde{F}_{\lambda\beta}(z) \Phi(z, y) F_{\lambda\delta}(y) \Phi(y, 0) \tilde{F}_{\alpha\beta}(0) \Phi(0, z) \rangle - h.c. \right].$$

We start with the Abelian case where one should replace $D_\gamma \to \partial_\gamma$, $\Phi \to 1$ in \((3)\) and the last two terms inside square brackets, coming from the contour differentiation, are absent:

$$z \frac{\partial D^{\text{abelian}}(z)}{\partial z^2} = f^{(1)}_\alpha = \text{const} \langle j^{(\text{mon})}_\beta(z) \tilde{F}_{\alpha\beta}(0) \rangle$$

and $j^{(\text{mon})}_\beta \equiv \partial_\gamma \tilde{F}_{\gamma\beta}(z)$ is the monopole current. Hence in the Abelian U(1) case confinement might be due to Abelian monopoles and this fact agrees with what is known about the Abelian mechanism of confinement on the lattice \([1]\).

In the nonabelian case, however, the corresponding Bianchi Identities (BI)

$$D_\mu \tilde{F}_{\mu\nu}(x) = J_\nu(x),$$

where $J_\nu(x) \equiv 0$, are usually assumed to hold. In the Abelian projection method (see \([1]\) for a discussion) one separates from $F_{\mu\nu}$ in some special gauge a singular term, the color-diagonal monopole-type configuration which violates both Abelian and nonabelian BI. As it is clear from \((4)\), \((5)\) this procedure clearly supplies a source $f^{(1)}_\alpha$ which makes $D(z)$ nonzero. One should, however, make two remarks in this connection. Firstly, when one tries to associate the confining configuration with some classical monopole, there appear no magnetic monopoles with finite selfenergy and stable \([15]\). Therefore in \([16]\) it was concluded that only quantum confining configuration can survive, which however not described analytically.

Secondly, and this is a more fundamental difficulty, associating confining configuration with necessary violation of BI, one usually abandons the connection of $F_{\mu\nu}$ with $A_\mu$, $F_{\mu\nu}(A) = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$ adding to this
expression extra terms (see e.g. in [15]). In this way one finds confinement not for the original QCD Lagrangian, but for a modified form.

In what follows we shall insist on keeping the original expression $F_{\mu\nu}(A)$ intact. In this case the nonabelian BI reduces to the equation

$$(\partial_{\mu}\partial_{\alpha} - \partial_{\alpha}\partial_{\mu})A_{\beta}(x) = \text{const} \; e_{\alpha\mu\beta\gamma} J_{\gamma}(x) = 0. \quad (8)$$

One can see that violation of BI, Eq. (8), requires a very special form of $A_{\beta}(x)$. One possible form of $A_{\beta}(x)$ is given by the path ordered exponents, e.g.

$$A_{\beta}(x) = P \exp[\int_{C} dz_{\mu} \lambda_{\mu}(z)] b_{\beta}(x) + h.c. \quad (9)$$

with noncommuting $\lambda_{\mu}, \lambda_{\nu}, \mu \neq \nu$ and the contour $C$, which belongs to a wide class of nondifferentiable contours; $x$ is the end point of the contour, and $P$ orders $\lambda_{\mu}(z)$ from say, right to left with $z$ approaching $x$. Such configurations, if possible, would make nonzero $J_{\gamma}(x)$ and contribute to the first term on the r.h.s. of (3), thus yielding nonzero $D(z)$ and string tension. In this way one would find the first possible source of confinement – due to the term $f_{\alpha}^{(1)}$ in (3). If this mechanism is proven, it would physically imply a monopole-like mechanism of confinement. One should stress that this does not mean real magnetic monopoles or magnetic fluxes present in the vacuum, since both violate Casimir scaling [6], while $D(z)$ supports it. The second source is due to the term $f_{\alpha}^{(2)}$ in (3), which is always present irrespectively of the violation of the BI. To make explicit the meaning of this term one can take the limit $z \to 0$ and obtains [14]

$$\left. \frac{dD(x^2)}{dx^2} \right|_{x^2 \to 0} = \frac{g^3}{96N_c} f_{abc} \langle F_{\alpha\beta}^{a}(0) F_{\beta\gamma}^{b}(0) F_{\gamma\alpha}^{c}(0) \rangle. \quad (10)$$

Since the triple correlator can be written as $e_{ijk} f_{abc} \langle E_{i}^{a}(0) E_{j}^{b}(0) B_{k}^{c}(0) \rangle$ one can view this second mechanism as the creation of magnetic fluxes from electric fluxes, i.e. the electric fluxes contained in the parallel transporter when shifted in the process of differentiation, serve as a source of magnetic flux, thus replacing magnetic monopoles.

To distinguish between these two possibilities one could measure on the lattice the correlator [17]

$$\Delta_{\nu\beta}(x, y) \equiv \frac{g^2}{N_c} tr(D_{\mu}\tilde{F}_{\mu\nu}(x) \Phi(x, y) D_{\alpha}\tilde{F}_{\alpha\beta}(y) \Phi(y, x)). \quad (11)$$
The nonzero answer for $\Delta_{\nu\beta}$ would mean that the violation of BI is indeed the source of confining configurations and writing

$$\Delta_{\alpha\beta}(x) = (\partial_\alpha \partial_\beta - \partial^2 \delta_{\alpha\beta})D(x) + \Delta^{(2)}_{\alpha\beta}, \quad (12)$$

where $\Delta^{(2)}_{\alpha\beta}$ is the contribution of $f^{(2)}_\alpha$, one can estimate the role of the first mechanism as compared to the second. Here $\Delta_{\alpha\beta}$ is measured via (11) and $D(x)$ is measured directly from (2).

One should note that the given above formulation of confinement mechanisms is fully gauge invariant and does not need gauge fixing.

It is worth mentioning that the FFC and the applicability of the method are not directly related to the question of the mechanism of confinement, since FFC exploits field correlators as input, but in the analytic calculation of $D(x)$ and $D_1(x)$ the problem of confining configurations becomes essential.

3 Deconfinement

Discussion of this phenomena can be done in 3 different directions: 1) when $N_c = 3$ and not infinite, the string connecting static quark antiquark breaks up at some distance $R_b$ (typically $R_b \approx 1.4$ fm) forming two heavy-light mesons. The same happens for a light $q\bar{q}$ pair; 2) when the temperature $T$ exceeds $T_c$ the color electric string between $Q$ and $\bar{Q}$ disappears; 3) when baryon density $\rho_B$ exceeds critical value $\rho_c$, confinement is believed to disappear. We shall discuss below only points 1) and 2).

1) For finite $N_c$, e.g. $N_c = 3$, the effect of sea quarks is given by the quark determinant $\det(m + \hat{D})$ present in the integral measure of averaging the Wilson loop in (3). This determinant is producing additional light-quark loops due to the heat-kernel representation

$$\langle \det(m + \hat{D})W(C) \rangle = \langle \exp\left[-\frac{1}{2} tr \int_0^\infty \frac{ds}{s} (Dz)_{xx} e^{-K} w_s(C_{xx})\right]W(C)\rangle. \quad (13)$$

Expanding the first exponential in (13) one obtains corrections to the static $Q\bar{Q}$ potential due to the sea-quark loops $w_s(C_{xx})$ where $C_{xx}$ is the closed contour passing through the point $x$, which is integrated over in (13).

The leading correction is proportional to the loop-loop correlator

$$\chi(C_1, C_2) = \langle W(C_1)W(C_2) \rangle - \langle W(C_1) \rangle \langle (W(C_2)) \rangle, \quad (14)$$
which was calculated in [18], and the energetically lowest configuration corresponds to the creation of a hole in the largest quark loop. These holes produce a partial screening of the $Q\bar{Q}$ static potential and consequently a significant decrease of highly excited meson masses. A detailed analysis of this situation was done in [19] where the form of the screened potential was found yield a perfect agreement of predicted light meson masses with experiment (for $L = 0, 1, 2, 3$ and $n_r = 0, 1, 2, 3, 4$).

Note that this mechanism of the screened confinement is different from the two-channel model, since in the former sea-quark loops are virtual and do not cause actual decay. Physically this difference results in the relative insensitivity of the meson mass depletion on the channel quantum numbers and on the proximity to the decay threshold in the screened confinement picture in contrast to the two-channel approach.

2) The temperature deconfinement

We start with gluodynamics and use background perturbation theory for $T > 0$ [20] which allows to separate in $A_\mu = B_\mu + a_\mu$ background gluon field $B_\mu$ from valence gluons $a_\mu$, both with periodic boundary conditions at $x_4 = n\beta = nT$.

The main effect of nonzero $T$ is that the correlators of electric fields, $g^2\langle E_i(z)E_k(0)\rangle = \delta_{ik}D^E(z) + O(D^E_1)$, and magnetic fields $g^2\langle H_i(z)H_k(0)\rangle = \delta_{ik}D^H(z) + O(D^H_1)$ are different, $D^E(z) \neq D^H(z)$, $D^E_1(z) \neq D^H_1(z)$. The same is true for the condensates, $\langle E^2 \rangle \neq \langle H^2 \rangle$, which define the vacuum energy density

$$\varepsilon_0 = -\frac{11}{3}N_c\frac{\alpha_s}{16\pi}\langle E^2 + H^2 \rangle.$$ (15)

It is clear that the confinement phase of gluodynamics with $\varepsilon = \varepsilon(1)$ consists of glueballs moving in the confining background $(E^2 + H^2)$, while the deconfinement phase with $\varepsilon = \varepsilon(2)$ represents valence gluons moving in the deconfined vacuum background $(H^2)$. It was understood already in [21] that it is advantageous (for the minimum of the Free Energy of the Vacuum (FEV)) to keep magnetic condensate intact at $T > T_c$, while at $T < T_c$ both electric and magnetic can be nonzero. This implies that $D^{(H)}$ and the spatial string tension $\sigma_{sp}$ stay constant across phase transition which was supported by lattice data [4]. One can define FEV in two phases including quarks as

$$F(1)/V_3 = \varepsilon(1) - \frac{\pi}{30}T^4 -(\text{higher mesons})$$
\[ F(2)/V_3 = \varepsilon(2) - (N_c^2 - 1) \frac{T^4 \pi^2}{45} \Omega_g - \frac{7\pi^2}{180} N_c T^4 n_f \Omega_g + O(N_c^0) \quad (16) \]

where \( \Omega_q, \Omega_g \) are perimeter contributions to the quark and gluon loops respectively. Assuming that magnetic part of \( \varepsilon(1) \) does not change for \( 0 \leq T \leq T_c \) and equal to the electric one, one has \( \varepsilon(1) = \varepsilon_0, \varepsilon(2) = \frac{1}{2}\varepsilon_0 \) and from \( F(1) = F(2) \) one obtains for the transition temperature [20]:

\[ T_c = \left( \frac{|\varepsilon_0|}{\frac{2\pi^2}{45} (N_c^2 - 1) \Omega_g + \frac{7\pi^2}{90} N_c n_f \Omega_g - \frac{\pi}{15} (n_f \geq 2)} \right)^{1/4}. \quad (17) \]

Neglecting the influence of magnetic fields on the quark and gluon motion yields \( \Omega_q = \Omega_g = 1 \), and one can compute \( T_c \) for different \( n_f \) taking \( \varepsilon_0 \) from standard gluon condensate estimate \( \frac{\alpha_s}{\pi} \langle (F_{\mu\nu}^a)^2 \rangle = 0.012 \text{ GeV}^4 \), for \( n_f = 2, 3, 4 \) and 3 times more for \( n_f = 0 \) [22]. One obtains in this way from (17), \( T_c = 240, 150, 141, 134 \text{ MeV} \) for \( n_f = 0, 2, 3, 4 \) which can be compared to the lattice data \( T_c = 270, 172, 154, 131 \text{ MeV} \) respectively. One can see a systematic 10-12% disagreement which can be removed taking standard gluonic condensate [22] 1.5 times larger.

Note that (17) gives for \( T_c \) the constant values for large \( N_c \), and \( T_c \) is only weakly dependent on \( N_c \).

Moreover, the phase transition is the first order for large \( N_c \). Both facts agree with recent lattice data [24]. Indeed the analysis of lattice data at \( n_f = 0 \) in [24] yields: \( \frac{T_c}{\sqrt{\sigma}} = 0.582 \) \( \frac{0.43}{N_c} \), and for \( \sigma = 0.18 \text{ GeV}^2 \) it gives \( T_c = (0.246 + 0.02 \frac{2}{N_c}) \text{ GeV} \), which agrees well with (17) for \( \Omega_q = \Omega_g = 1 \), where \( T_c = 0.243 \text{ GeV} \) and for \( n_f = 0 \) it does not depend on \( N_c \), so that (significant) \( N_c \) dependence appears only for \( n_f > 0 \). The simple picture described above is sometimes called "the Vacuum Evaporation Model" (VEM) and is actually the only known model successful in all these features. However, the latent heat for \( N_c = 3, n_f = 2 \) comes out in VEM too high, and to improve the situation one should take into account that \( \Omega_q, \Omega_g \neq 1 \) above \( T_c \) and the contribution of other than pion mesons in (16) below \( T_c \), which was done in [24]. Both factors make the phase transition more smooth and strongly decrease the specific heat.

In conclusion, it is clear that the present approach yields a very consistent picture of both confinement and deconfinement, which can be derived from the standard QCD Lagrangian using background perturbation theory and FFC. In this approach inputs are \( D(x), D_1(x) \), which are known from the lattice data [7] to be \( D(x) = D(0) \exp(-|x|/T_g), \quad D_1(x) = D_1(0) \exp(-|x|/T_g) \)
reducing input to 3 numbers, $D(0)$, $D_1(0)$ and $T_g$. Since $D_1(0) \ll D(0)$, one actually has $D(0)$ and $T_g$, or equivalently $\sigma$ and $T_g$, and most hadron spectrum data are computed through only $\sigma$ with 10% accuracy. Nevertheless the method can be considered as logically consistent, with the statement that everything including all field correlators is computed through the only QCD parameter: $\Lambda_{QCD}$ or $\sigma$. The work in this direction was partly done in [12], [13]. There the correlator length $T_g$ was expressed through the gluelump masses and $T_g$ was found in the range 0.13-0.17 fm.

Moreover the full correlator $D(x), D_1(x)$ can be computed through the gluelump Green’s function [27], if one assumes that the nonabelian BI are not violated. In the opposite case there appears the problem of identification and study of the BI-violating configurations, similar to the study of magnetic monopole ensemble, if they really exist in the QCD vacuum. Indeed the Casimir scaling phenomenon [4] strongly limits the admixture of any coherent configurations (like classical solutions for monopoles, dyons and instantons) with size larger than 0.2 fm [9], which makes the popular picture of the QCD vacuum with magnetic monopoles or central fluxes rather unrealistic. Therefore for confinement one has either the picture of the BI violating configurations with very small radius or an alternative picture with vanishing BI and the gluelumps saturating the field correlators.

Present lattice data cannot unfortunately distinguish between two alternatives since, as argued above, the popular abelian projection method or central vortex projection both measure the quantities proportional to $D(x)$ or its derivatives, while only direct measurement of the correlator $\Delta_{\alpha\beta}$ [14] can separate two possibilities, provided a clean lattice analog of $\Delta_{\alpha\beta}$ is formulated.

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