Preliminary Results of Evaluation of the Influence of Deformation of the Water Drop Growing in the Electromagnetic Field on Their Scattering Properties

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Abstract. This work is devoted to development of the method of calculation of back scattering cross sections for the water drops growing in the electromagnetic field. Calculations are carried out taking into account deformation of the drops under the influence of horizontal electric field and without this influence. It is shown that increasing of extent of deformation of the water drops growing in electric field leads values of back scattering cross section to deviate from the similar values for equivolumetric spheres of water more significantly. The more extent of deformation of the water drop, the more difference between horizontally and vertically polarized components of back scattering cross section.

Introduction

Development of the new methods of radar researches of microphysical parameters of convective clouds must include the study of the influence of electric activity of a cloud environment on processes of growth of the water drops and crystals. In special degree it is important for the correct physical interpretation of thunderstorms radar parameters on a stage of heavy rain and hail formation. It is known that growth of water drops in the electromagnetic field is followed by a number of the physical phenomena, detailed accounting of each of which is not always obviously possible. That is why the experimental works made in recent years where authors analyze growth of the water drops in the wind tunnel under the influence of the electromagnetic field are of great interest [1, 2]. The photo registration data of the growing water drops executed with high space-time resolution are presented in these publications. Such approach allows to estimate more precisely the sizes, a form and orientation of the water drops growing in the electromagnetic field. There is a unique opportunity to calculate the characteristics of scattering of the separate particles and radar parameters of a spectrum of the growing water drops taking into account their actual geometrical sizes. It is obvious that for assessment of the maximum values of radar reflectivity of cloud volume, the most important stage of the water drop growth is its deformation before the final breakup. At this stage the back scattering cross section of the water drop reaches the maximum value. We must take into account that geometrical sizes of the water drops growing in electric field can surpass considerably their maximum values in lack of the field. This factor is extremely important for the development of the radar methods of quantitative parameterization of the heavy rains and hail, especially in case of their simultaneous falling [3]. Calculations of the back scattering cross section of the water drop growing in the electromagnetic field at a stage when its form is close to spherical and also during the maximum deformation when its

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form approaches a flat spheroid, can determine a contribution of each of these values to radar reflectivity of cloud volume.

In this regard the purpose of the given work is:

• development of a method of calculation of the back scattering cross section of the water drop having the form of arbitrary spheroid and the extreme sizes as a result of growth under the influence of horizontal electric field;

• evaluation of the influence of horizontal electric field on scattering properties of the water drops at a stage of the maximum size before the final breakup.

A method of calculation of the back scattering cross section of the growing water drop

Remote methods of a research of atmospheric environments (clouds, rainfall, aerosols, interstellar dust, etc.) are widely used in various fields of science and have a great applied importance. All of these methods are based on exact solution of a problem of scattering of an electromagnetic wave by isolated particle offered by Mie. This solution was received by a classical method of division of variables. Expression for calculation of the back scattering cross section can be written in this form:

$$\sigma(r, m, \lambda) = \frac{\lambda^2}{4\pi} \sum_{n=1}^{\infty} (-1)^n (2n+1)(a_n - b_n)^2$$

(1)

where $r$ – particle radius; $\lambda$ – wavelength of electromagnetic radiation; $m$ – complex index of refraction; $a_n$ and $b_n$ the coefficients depending on cylindrical Bessel's functions of the first sort for the solution inside of sphere and Hankel's functions of the second sort for the solution outside of the sphere [4, 5]. However the real particles, as a rule, are aspheric, and Mie's theory still properly was not developed for aspheric particles. Only approximate methods which scope is very limited are known.

Researchers from the St. Petersburg University modified a method of division of variables for particles of arbitrary spheroid form [6, 7]. Use of spheroidal basis allowed to receive reliable results for particles with high degree of asphericity. According to the offered method, the spheroidal system of coordinates is introduced in such a way that the origin of coordinates coincide with the center of a spheroid. In this case the semimajor axis of a spheroid is designated through $a$, and the semiminor axis one through $b$. It is supposed that the flat polarized wave which can be presented in the form of the superposition of vertically polarized (TM mode) and horizontally polarized (TE mode) components is falling on a particle. For each of these waves electric and magnetic vector respectively vibrates perpendicular to the falling surface. Then expressions for the back scattering cross section are defined as follows [8]:

for TM mode

$$\sigma^{TM} = \frac{4\pi}{(2\pi / \lambda)^2} \sum_{l=1}^{\infty} i^l b_l^{(1)} S_{l1}(c, \cos \alpha) - \sum_{m=1}^{\infty} \sum_{l=0}^{\infty} i^{l+m} (k, a_m^{(1)} S_{ml}(c, \cos \alpha) - i b_m^{(1)} \frac{dS_{m1}(c, \cos \alpha)}{d \cos \alpha}) \sin \alpha \right] \right|^2,$$  

(2)

$$b_l^{(1)} = -2 i \begin{pmatrix} \xi^2_0 - 1 \end{pmatrix}^{1/2} R_{l1}^{(1)}(c, \xi_0) \begin{pmatrix} \xi^2_0 - 1 \end{pmatrix}^{1/2} R_{l1}^{(1)}(c, \xi_0) N_{l1}^{-2}(c) S_{l1}(c, \cos \alpha)$$

for TE mode

$$\sigma^{TE} = \frac{4\pi}{(2\pi / \lambda)^2} \sum_{l=1}^{\infty} i^l a_l^{(1)} S_{l1}(c, \cos \alpha) - \sum_{m=1}^{\infty} \sum_{l=0}^{\infty} i^{l+m} (k, a_m^{(1)} S_{ml}(c, \cos \alpha) - i b_m^{(1)} \frac{dS_{m1}(c, \cos \alpha)}{d \cos \alpha}) \sin \alpha \right] \right|^2,$$  

(3)

$$a_l^{(1)} = -2 i \begin{pmatrix} \xi^2_0 - 1 \end{pmatrix}^{1/2} R_{l1}^{(1)}(c, \xi_0) \begin{pmatrix} \xi^2_0 - 1 \end{pmatrix}^{1/2} R_{l1}^{(1)}(c, \xi_0) N_{l1}^{-2}(c) S_{l1}(c, \cos \alpha),$$

$$\xi_0 = \left( \frac{a}{b} \right)^{1+1/2} \left[ \left( \frac{a}{b} \right) - 1 \right]^{-1/2}$$
where parameter $\tilde{f} = 1$ – for the prolate spheroidal coordinates; parameter $\tilde{f} = -1$ – for oblate spheroidal coordinates; $\alpha$ – an angle of incident the electromagnetic wave to the rotation axis of the spheroid; $c$– dimensionless parameter; $\lambda$ – wavelength; $k_1$ – the wavenumber in vacuum; $S_{ml}(c, \eta)$ – the prolate spheroidal angular functions; $R_{ml}^{(1)(3)}(c, \xi)$ – the prolate spheroidal radial functions of the first and third order; $N_{ml}(c)$ – normalizing multiplier.

**Calculation results of back scattering cross section of the growing water drop without the influence of electric field**

It is known that in the conditions of free fall the water drop takes irregular shape. During initial growth the water drop has the sphere form. In process of growth in clouds under the influence of gravitation forces and air resistance the water drop gets a form of a flattened ellipsoid. The lower part of the water drops with a diameter more than 1 mm begins to bend. At a certain stage this drop begins to take the small volume of air before itself, getting a form of a parachute dome. Beginning from the diameter of 6 mm water drop becomes unstable and begins to breakup to separated fragments which sizes correspond to the sizes of rain drops which reach the surface.

Authors of work [9] presented results of laboratory researches (in the wind tunnel), natural experiments (with use of the polarimeter) and numerical modeling of the water drops form factor. The results of these researches allow to approximate a form of the falling water drop as a flattened spheroid. They recommend for use the following empirical formula representing a axes ratio as function of equivolumetric diameter of a water drop:

$$\frac{b}{a} = 1.012 - 0.144 \cdot D_v - 1.03 \cdot D_v^2,$$

(4)

where $D_v$ – equivolumetric sphere diameter.

Calculations for the water drop with a diameter from 0.1 to 8 mm falling in the lower part of a cloud where temperatures about 10°C prevail were carried out. The large water drops with diameter about 8 mm can be presented in this part of the cloud. Calculations are carried out taking into account a form factor according to Eq. 4. The results of the calculations carried out in accordance with Eq. 1, Eq.2 and Eq.3 are presented in Figure 1.
Figure 1. Dependence of the normalized back scattering cross section of the water drop on wavelength $\lambda = 5.5\, cm$ from radius. Mie – equivolumetric spherical drop, TM – horizontally and TE – vertically polarized components of the spheroidal drop.

As shown on Fig. 1 back scattering cross section of the water drop at various polarization of a signal depends on a form factor. The more water drop size (the form factor is more), the more difference between back scattering cross section of vertical (TM mode) and horizontal (TE mode) polarization. For sizes up to $r < 0.12\, mm$ (the Relay zone) three curves coincide. From sizes $r > 0.12\, mm$ the influence of a form-factor on back scattering cross section is beginning [10].

Calculation results of back scattering cross section of the water drops growing under the influence of horizontal electric field

It is known that the water drops growth in a real thunderstorm takes place under the influence of the electric field initiated by the natural lightning activity accompanying process of heavy rain and hail formation. In this case the form, orientation and size of the growing water drops can significantly differ from similar values for the water drops growing without the influence of the electric field. The results received in [1, 2] give to researchers the rare chance to calculate the scattering properties of the drops growing in horizontal electric field. At the same time the form, orientation and the size of the growing water drops are presented in these works with rather high space-time resolution. The approximate scheme of the calculation of back scattering cross section for the most typical case from [1] is shown in Figure 2. The photos of the growing water drop in various time points are shown in the left part of the Figure 2. The spheroids for which the back scattering cross sections were calculated are shown in the central part of this figure.

Figure 2. Approximate scheme of representation of the water drop growing under the influence of horizontal electric field in the form of arbitrary spheroids

As shown on Figure 2 at an early stage of growth without the influence of electric field the water drop has the form close to spherical (fragment 1). Approximation of a form of the water drop by the sphere in this case looks quite justified. Also by the sphere it is possible to approximate products of disintegration of the water drop of the maximum diameter as a result of breakup [1]. Further growth of the water drop and influence of aerodynamic effects leads to the beginning of deformation process and
since the moment $\Delta \tau = 0$ the form of a drop is better approximated by an ellipsoid (fragment 2). From this moment horizontal electric field with strength $500kV/m$ begins to affect the water drop growing in the wind tunnel. Under the influence of the electric field the horizontal size of the water drop significantly increase. As shown in Figure 2 at $\Delta \tau = 23m\mu c$ the semimajor axis of an ellipsoid approaches $20 mm$ (fragment 3) and at $\Delta \tau = 59m\mu c$ it significantly surpasses this value (fragment 4). At this moment the most suitable approximation of a form of the growing water drop is the combination of two ellipsoids as shown on Figure 2.

Calculation results of the back scattering cross section for the fragments shown in Fig. 2 and also for the drops with extreme extent of deformation from [1] are given in Table 1. It should be noted that the normalized values of back scattering cross section, i.e. back scattering cross section divided by $\pi R^2$ are given in Table 1.

**Table 1.** The normalized back scattering cross section of the water drops with various extent of deformation under the influence of horizontal electric field

| Equivolumetric sphere radius, $R_{[mm]}$ | Semimajor axis of a spheroid, $a$ | Semiminor axis of a spheroid, $b$ | Form-factor, $a/b$ | Normalized back scattering cross section of the water drops, $[dB]$ |
|-----------------------------------------|----------------------------------|----------------------------------|-------------------|--------------------------------------------------|
|                                        | 1                                | 2                                | 3                 | 4                | 5          | 6          | 7          |
|                                        | 3.3                              | 14.98                            | 4.39              | 3.4123           | -20.82     | -1.37      | -7.39     |
|                                        | 3.5                              | 15.8                             | 4.67              | 3.3833           | -19.58     | -0.5       | -6.04     |
|                                        | 3.625                            | 17.08                            | 4.73              | 3.6110           | -19.3      | 0.27       | -5.4      |
| Average deformation of the water drop  |                                  |                                  |                   |                   |            |            |            |
|                                        | 3.3                              | 17.7                             | 4.04              | 4.3812           | -21.46     | 0.06       | -7.39     |
|                                        | 3.5                              | 19.05                            | 4.25              | 4.4824           | -20.96     | 1.08       | -6.04     |
|                                        | 3.625                            | 20.07                            | 4.36              | 4.6032           | -20.68     | 1.63       | -5.4      |
| Maximum deformation of the water drop  |                                  |                                  |                   |                   |            |            |            |
|                                        | 3.5                              | 29                               | 3.44              | 8.4302           | -21.7      | 4.81       | -6.04     |
|                                        | 3.625                            | 42                               | 3.01              | 13.9535          | -22.50     | 7.68       | -5.4      |
| Extreme deformation of the water drop  |                                  |                                  |                   |                   |            |            |            |

Graphic interpretation of the results presented in Table 1 is given in Figure 3 where the dependences normalized back scattering cross section from the equivolumetric sphere radius $R$, for cases with various extent of deformation of the water drop under the influence of horizontal electric field are shown.
Figure 3. Dependences of the normalized back scattering cross section from the equivolumetric sphere radius of $R_v$ for the cases with various extent of deformation of the water drops under the influence of horizontal electric field. a), b), c) – average, maximum and extreme deformation respectively.

In two cases taken from [1] we deal with extreme deformation when the horizontal sizes of the water drops before breakup reached 29 and 42 mm respectively. It is obvious that the growing water drops can’t approach such sizes in the natural conditions without the influence of electric field. On the fragment (c) of Fig. 3 the results of calculation of back scattering cross section for these drops are given. As shown on the Fig. 3 the maximum distinction between horizontally and vertically polarized components is noted for these sizes. It is known that values of real radar reflectivity are defined by spectral characteristics of cloud volume which strongly depends on the largest particles of spectrum. In this case the existence of such drops can significantly change values, both radar reflectivity, and polarizing parameters of zones of heavy rains and a hail localization.

Summary

The method of calculation of back scattering cross section of the water drops offered in the given work allows finding horizontal and vertical polarizing components for particles with any extent of deformation. The experimental data obtained in [1] for the water drops growing in the wind tunnel under the influence of horizontal electric field were used for calculation of back scattering cross section of real particles with various size and form.

It is shown that increasing of extent of deformation of the water drops growing in electric field leads values of back scattering cross section to deviate from the similar values for equivolumetric spheres of water more significantly. The more water drop size (the form factor is more), the more difference between back scattering cross section of vertical (TM mode) and horizontal (TE mode) polarization. For sizes up to $r < 0.12 \text{mm}$ (the Relay zone) three curves coincide. From sizes $r > 0.12 \text{mm}$ the influence of a form-factor on back scattering cross section is beginning.

The difference between back scattering cross section of vertical (TM mode) and horizontal (TE mode) polarization for cases of average (a) and the maximum (b) deformation practically coincide. Though these values are considerably differ from values for particles with an identical equivolumetric spheres radius without electric field. The maximum distinction between horizontally and vertically polarized components is noted for the water drops of an ellipsoidal form with extreme (c) horizontal sizes (29 and 42 mm).

The received results can be useful for the development of radar methods for thunderstorms and hailstorms research. Including with use of DMRL-C (the Doppler meteorological radar with double polarization) which are installing by Roshydromet over the territory of Russia for the purpose of creation of the system of radar meteorological observations.
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