Rate Loss Mitigation for 60-GHz mmWave Massive MIMO Lens Antenna Array Systems

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Abstract—The data rate-loss is an important issue for the next generation wireless communications. Hence, we investigate the mitigation of the data rate-loss per-user considering a hybrid selection (HS) criterion with feedback-based hybrid zero-forcing precoding (FHZFP). Although a conventional beam selection (CBS) criterion with FHZFP significantly operate and decrease the dimension of the beamspace channel, this scheme unfortunately does not act on the data streams due to the optimal selected beams is equal to the number of the transmitted data streams. Moreover, the rate-loss effects depend on the phase shifting error, array and path gains error which lead as a function of the quantization error (QE). To completely mitigate this problem, we propose a HS criterion with FHZFP to operate the beams and streams simultaneously. The equivalent hybrid channel is quantized by the proposed HS criterion in FHZFP and feedback to the base station. The numerical result is verified by computer simulations in terms of the downlink millimeter-wave massive multiple-input-multiple-output channel.

Index Terms—Millimeter-wave massive MIMO, lens antenna array, CBS and HS criterion, FHZFP, QE, per-user data rate.

I. INTRODUCTION

MILLIMETER-WAVE (mmWave) massive multiple-input multiple-output (mMIMO) system is a promising technology for the fifth generation (5G) wireless communications, which have received a lot of attention due to the large bandwidth available [1], [2]. This system uses 30-300 GHz band, whereas the microwave wireless communications operate at carrier frequencies below 6 GHz. The principal advantage of mmWave is to support higher data rate because of larger bandwidth [1]. Consequently, the conventional MIMO processing may not be feasible in mmWave mMIMO systems [3]. The analog beamforming was implemented via phase shifters in [4]. In [5], a hybrid analog-digital precoding scheme was proposed to enable spatial multiplexing and decrease phase shifter error. The analog precoding includes in phase shifted array to produce random phases, whereas the digital precoder includes in baseband to generate the discrete phases. Particularly, the hybrid precoding requires a large number of phase shifters, antenna subset selection in [6] by replacing the phase shifters with switches.

The application of mmWave mMIMO is much more challenging due to unpredictable propagation environments, system efficiency, and power consumption. Several researchers investigated the prototype of the MIMO lens antenna array (LAA) systems [7]-[11]. Particularly, a lens antenna array consists of an electromagnetic (EM) lens [7] which is modeled approximately as a discrete Fourier transform (DFT) matrix and treated separately with the matching antenna array. In [8], authors proposed a path division multiple access method with full-dimensional LAA which follows two “sinc” functions reflecting the elevation and the azimuth angle resolutions. An overview of the state-of-art in signal processing for mmWave wireless systems was provided in [9]. In the most recent literatures [10], [11], authors implemented a low-dimensional beamspace channel considering a conventional beam selector (CBS) criterion with feedback-based hybrid zero-forcing precoding (FHZFP) for mmWave mMIMO LAA systems. A CBS criterion with FHZFP was proposed in [11] to reduce the data rate-loss per-user, where the rate-loss is dominated by the quantization error (QE). Although, most of the previous literatures have implemented CBS criterion with FHZFP to significantly operate the hybrid channel, this criterion cannot operate the transmitted data streams because of the optimal selected beams is equal to the number of transmitted data streams. In addition, the rate-loss effect depends on the phase shifting error, array and path gains error which lead as a function of the QE. In this circumstances, we propose a hybrid selection (HS) criterion with FHZFP to optate the beams and streams simultaneously. To the best of our knowledge, this problem has not been investigated yet in the literature.

This letter investigates a HS criterion with FHZFP for 60-GHz mmWave mMIMO LAA systems. A HS criterion is designed by a beam and a stream selector. A beam selector is included with an analog precoding based LAA system whereas a stream selector is enrolled with a digital precoding based LAA system. Developing a better selector which is the key factor to operate the beams and streams simultaneously and to mitigate the dimensional and rate-loss issues, is the main subject of this letter. The superiority of the proposed HS criterion over the CBS criterion for mmWave mMIMO LAA systems, in terms of data rate per-user is verified through computer simulations.

II. SYSTEM MODEL

We consider a mmWave mMIMO system with a base station (BS) and K single antenna users. The BS is equipped with M transmit antennas and M ≫ K. M is driven by a far smaller number of RF chains, namely NR, where K ≤ NR ≤ M. The system model is depicted in Fig.1. Let K ≤ l ≤ NR, where NR drives by l dimensional space to operate the transmitted data streams. Thus, the number of the total transmitted data streams is l ≤ L. Now, we adopt a mmWave mMIMO downlink channel vector h(k) ∈ ℂM×1 between the BS antennas and the k-th users. Therefore, the received signal r(k) can be modeled as

\[
r(k) = \sqrt{\frac{\rho}{K}} h(k)^H \mathbf{F}^H \mathbf{S}_n \mathbf{W} x + n(k),
\]

where h(k) = \sum_{i=1}^L h_k \alpha_{k,i} \mathbf{a}(\phi_{k,i}) [9], L is the maximum number of dominant paths of the k-th user which is typically small.
i.e., $L_k \ll N_{RF}$ (e.g., $L_k$ is no larger than 3 due to the multi-path sparsity of mmWave channels [12]), $\alpha_{k,i}$ denotes the complex valued $i$-th path gain at $k$-th user, $a(\phi_{k,i})$ is the $M \times 1$ steering vector of the $i$-th path at $k$-th user, $F = [a(0), a(\delta), \ldots, a(\delta(M - 1))]^H$ is the $M \times M$ discrete Fourier transform (DFT) matrix with $\delta = \frac{1}{M}$ [9]-[11], and $W = [w_1, w_2, \ldots, w_K]$ is the $l_d \times K$ zero-forcing (ZF) precoding matrix. Lenses are characterized by $\lambda_{s_i}$, where the wavelength $\lambda$ is the downlink channel vector in (1). The beamspace channel matrix $H_{eq}$ which is given by

$$H_{eq} = [S_h^H h_{b,1}, S_h^H h_{b,2}, \ldots, S_h^H h_{b,K}], \quad (4)$$

where $h_{eq,k} = S_h^H h_{b,k}$ is the $l_d \times 1$ equivalent channel vector. Hence, multi-stream digital precoding can be applied to $H_{eq}$, where simple $l_d \times K$ dimensional ZF precoding is executed as

$$W = H_{eq}^H (H_{eq} H_{eq}^H)^{-1} \Lambda, \quad (5)$$

where $\Lambda$ is a diagonal matrix, which is introduced for transmit power normalization at $k$-th user, $w_i = H_{eq}^{\dagger} (\cdot)\Lambda$ is the ZF precoding vector, $H_{eq} = [h_{eq,k}] \in \mathbb{C}^{l_d \times K}$ and the operator $(\cdot)^\dagger$ is called the Moore-Penrose pseudo-inverse.

### III. Hybrid Equivalent Channel Model

We consider $M \times K$ beamspace channel matrix $H_b$ which is given by

$$H_b = [F h_1, F h_2, \ldots, F h_K] = [h_{b,1}, h_{b,2}, \ldots, h_{b,K}], \quad (3)$$

where $H = [h_1, h_2, \ldots, h_K] \in \mathbb{C}^{M \times K}$, $h_{b,k} = F h_k \in \mathbb{C}^{M \times 1}$ is the beamspace channel vector between BS and the $k$-th user, $h_k$ is the downlink channel vector in (1). The beamspace channel matrix $H_b$ has a sparse nature [12], where the number of dominant elements of each beamspace channel vector $h_{b,k}$ is much smaller than $M$. This sparse nature can be utilized to construct a low-dimensional beamspace mmWave mMIMO system without undeniable performance loss by a beam selection scheme [10]. In particular, under the sparse beamspace channel matrix, only a few number of beams can be selected to simultaneously serve $K$ users. Therefore, we consider the multiuser low-dimensional $l_d \times K$ equivalent channel matrix $H_{eq}$ which is given by

$$H_{eq} = [S_h^H h_{b,1}, S_h^H h_{b,2}, \ldots, S_h^H h_{b,K}], \quad (4)$$

where $h_{eq,k} = S_h^H h_{b,k}$ is the $l_d \times 1$ equivalent channel vector. Hence, multi-stream digital precoding can be applied to $H_{eq}$, where simple $l_d \times K$ dimensional ZF precoding is executed as

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### IV. Rate-Loss Mitigation

In this section, we analyze the expected QE using the proposed HS criterion with FHZFP and qualify the rate-loss degradation as a function of the feedback rate.

**A. Rate Loss:** The rate-loss, $\Delta R(\rho)$, is expressed as

$$\Delta R(\rho) = R_{\text{perf.-CSI}}(\rho) - R_{f_d}(\rho), \quad (6)$$

where $R_{\text{perf.-CSI}}(\rho)$ and $R_{f_d}(\rho)$ are respectively given by

$$R_{\text{perf.-CSI}}(\rho) = \frac{1}{K} \log_2 \left[ 1 + \frac{\rho}{K} \|H_{eq,k} w_{\text{perf.-k}}\|^2 \right], \quad (7)$$

$$R_{f_d}(\rho) = \frac{1}{K} \log_2 \left[ 1 + \frac{\rho}{K} \|H_{eq,k} w_k\|^2 \right], \quad (8)$$

Note that $R_{\text{perf.-CSI}}(\rho)$ refers to the per-user rate considering perfect equivalent channel state information (PE-CSI) at BS and $R_{f_d}(\rho)$ denotes per-user rate under the proposed HS criterion. Now by substituting (7) and (8) into (6), and using [3, Theorem 1], the rate-loss $\Delta R(P)$ can be upper-bounded as follows

$$\Delta R(\rho) \leq \frac{1}{K} \log_2 \left[ 1 + \frac{\rho}{K} \|H_{eq,k} w_{\text{perf.-k}}\|^2 \right] - \frac{1}{K} \log_2 \left[ 1 + \frac{\rho}{K} \|H_{eq,k} w_k\|^2 \right] + \frac{1}{K} \log_2 \left[ 1 + \sum_{k \neq i} \frac{\rho}{K} \|H_{eq,k} w_k\|^2 \right], \quad (9)$$
where $\sum_{k \neq i} \frac{e^{-K_{eq,k}}} {K_{eq,k} |w_i|^2} \geq 0$ and $\log_2(\cdot)$ is a monotonically increasing function \cite{3}. Let the PE-CSI based expected rate is $E[\log_2 (1 + \frac{e^{-K_{eq,k}}} {K_{eq,k} |w_{perf,k}|^2})] = E[\log_2 (1 + \frac{e^{-K_{eq,k}}} {K_{eq,k} |w_k|^2})]$ where $w_{perf,k}$ and $w_k$ are isotropically distributed unit vectors, independent of $h_{eq,k}$. Then, the resultant rate-loss $\triangle R(\rho)$ is given by
\[
\triangle R(\rho) = E\left[\log_2 \left(1 + \sum_{k \neq i} \frac{\rho}{K} |h_{eq,k}^H w_i|^2\right)\right]. \tag{10}
\]
Applying Jensen's inequality to the upper bound in (10) and exploiting the independence of the channel norm and channel direction, we obtain
\[
\triangle R(\rho) \leq \log_2 \left\{1 + \gamma (K-1) E[Z]\right\}, \tag{11}
\]
where $\gamma = 10 \log_{10} \frac{E}{\|h_{eq,k}\|^2}$ is the signal-to-noise ratio (SNR) at the receiver and $E[Z]$ denotes the expected quantization error.

B. The Quantization Error: Let the quantization error, $Z = 1 - \cos^2 \left(\angle(h_{eq,k}, h_{eq,k})\right)$ where $h_{eq,k} = \frac{h_{eq,k}}{\|h_{eq,k}\|}$ is the directions of the equivalent channel, $h_{eq,k} = ||h_{eq,k}|| \xi_k, \xi_k$ is the quantization of $h_{eq,k}$, $\xi_k = S^d/\|S||, \forall k \neq i$ is the proposed HS based codebook which satisfies $\xi_k = \frac{\xi_k}{\|\xi_k\|}$ \cite{11} whereas the proposed selector $S_h$ can be designed based on the angle of departures (AoDs) and the angle of arrivals (AoAs), and $c_{k, i}$ is the large-dimensional channel subspace vector. Moreover, the codebook index $\xi_k$ is given by
\[
C_k = \text{arg } \min_{c_{1,2,...,2^B}} (1 - \cos^2 (\angle(h_{eq,k}, \xi_{k,i}))). \tag{12}
\]
Using \cite[Lemma 1]{13}, we can obtain the complementary cumulative distribution function (CCDF) as follows
\[
P_r(Z \geq z) = 1 - \left(1 - (1 - z^{L-1})^{2^B}\right) = (1 - z^{L-1})^{2^B}. \tag{13}
\]
Thus, the expected quantization error $E[Z]$ is given by
\[
E[Z] = E\left[1 - \cos^2 \left(\angle(h_{eq,k}, h_{eq,k})\right)\right] = E\left[\sin^2 \left(\angle(h_{eq,k}, h_{eq,k})\right)\right] \tag{14}
= \int_0^1 (1 - z^{L-1})^{2^B} \, dz.
\]
Let the integral $I_{eqe} = \int_0^1 (1 - z^{L-1})^{2^B} \, dz$ denotes the beta function and is given using \cite[p.5]{14}
\[
I_{eqe} = \frac{1}{L-1} \beta \left(2^B + 1, \frac{1}{L-1}\right) = \frac{\Gamma(2^B + 1) \Gamma \left(\frac{L-1}{L-1}\right)}{\Gamma(2^B + 1 + \frac{L-1}{L-1})} = \frac{2^B \Gamma(2^B) \Gamma \left(\frac{L-1}{L-1}\right)}{\Gamma(2^B + 1 + \frac{L-1}{L-1})}, \tag{15}
\]
where $\beta(\cdot)$ denotes the beta function which is defined in terms of the gamma function i.e., $\beta(x, y) = \frac{\Gamma(x) \Gamma(y)}{\Gamma(x+y)}$ in \cite{15}. If each user dominant $L = 2$ independent channel paths, then the integral $I_{eqe}$ is given by
\[
I_{eqe} = \frac{2^B \Gamma(2^B) \Gamma(2)}{\Gamma(2^B + 2)} = (2^B + 1)^{-1} < 2^{-B}. \tag{16}
\]
From (16), we observe that $I_{eqe}$ satisfy an ideal Voronoi region around a quantization error which is a spherical cap of area $2^{-B}$. In this scenarios, the beta function does not affect on the path gains. In contrast, if each user dominant $L > 2$ independent channel paths, then the integral $I_{eqe}$ is given by
\[
I_{eqe} = \frac{\Gamma(L-1)}{\Gamma(2^B + 1 + \frac{L-1}{L-1})}, \tag{17}
\]
where $\Gamma \left(\frac{L-1}{L-1}\right) \leq 1$ for $1 \leq \frac{L-1}{L-1} \leq 2$ and $\Gamma(1) = \Gamma(2) = 1$. By applying Kershaw's inequality for gamma function is given by \cite{16}
\[
I_{eqe} < \left(2^B + \frac{L-2}{2(L-1)} \right)^{-\frac{1}{2^B}}, \tag{18}
\]
(18) provides further upper bound of $2^{-\frac{B}{L-1}}$. This means that the quantization error function $(\cdot)^{-\frac{1}{2^B}}$ in (18) provides further upper bound of $2^{-\frac{B}{L-1}}$, which denotes the rate-loss mitigating nature of the function $(\cdot)^{-\frac{1}{2^B}}$ in (14). Hence, the integral $I_{eqe}$ is upper bounded by
\[
I_{eqe} < 2^{-\frac{B}{L-1}}. \tag{19}
\]
Using (16) and (19), the upper bound of the expected quantization error $E[Z]$ can be written as
\[
E[Z] = I_{eqe} = \begin{cases}
2^{-B}, & \text{when } L = 2 \\
2^{-\frac{B}{L-1}}, & \text{when } L > 2.
\end{cases} \tag{20}
\]
Finally, we incorporate (20) in (11) and the measured rate-loss per-user as follows.
\[
\triangle R(\rho) < \begin{cases}
\log_2 \left\{1 + \gamma (K-1) 2^{-B}\right\}, & \text{when } L = 2 \\
\log_2 \left\{1 + \gamma (K-1) 2^{-\frac{B}{L-1}}\right\}, & \text{when } L > 2.
\end{cases} \tag{21}
\]
Now, Using (21), the rate-loss $\triangle R(\rho)$ can be computed. In Table 1, we have provided a comparison of the rate-loss $\triangle R(\rho)$ between the proposed and \cite{11} considering $B \geq 2(L-1)/(L-1)\log_2(K-1)$ when $\triangle R(\rho) \leq 1$bps/Hz.

| Scheme | $\triangle R(\rho)$ [bps/Hz] with iters. = 10 |
|-------|----------------------------------|
| Proposed | 0.03 if $l_d = 16$, $B = 6$, $L = 2$ |
| HS (8) | 0.00 if $l_d = 24$, $B = 12$, $L = 3$ |

V. Simulation Results

In this section, we shows some simulation results to demonstrate the proposed HS criterion with FHZFP. We implemented the simulations of mmWave mMIMO LAA systems as computer simulations over a downlink mmWave mMIMO channel. A perfect equivalent channel feedback is considered in computer simulations. Throughout the simulations, we assumed
that $M = 128$, $N_{RF} = 24$, $K = 8$, $f_c = 60$GHz, $\lambda = 5$mm, $\Delta = \lambda/2$, and $L = \frac{N_{RF}}{K} = \max \left( \frac{N_{RF} - K}{L} \right) > \min \left( \frac{N_{RF} - l_d}{L} \right)$ whereas $l_d \leq N_{RF}$. $N_{RF}$ is selected by the analog beam selector $S_d$ and $l_d$ is selected by the digital data selector $S_d$, respectively. For example, if $N_{RF} > l_d$ and $l_d = 16$, then $L = 2$ and the feedback bit becomes $B = 6$ bits at 10dB SNR values which satisfies as a spherical cap of area $2^{-B}$ in (16). If $N_{RF} = l_d$ and $l_d = 24$, then $L = 3$ and the feedback bit becomes $B = 12$ bits at same SNR values which satisfies as in (19). Therefore, we observe the above example, the proposed HS criterion can easily dominant the channel paths 2 and 3 independently by every user where the CBS criterion can only dominate the channel path 3 independently, this means the CBS criterion cannot operate the transmitted data streams because of the constructional imperfection of the BS. In a fair comparison, the proposed HS criterion achieved an identical data rate performance per-user for both cases as shown in Table I and outperformed the CBS criterion [11] and unlike random vector quantization (RVQ) method in [3], [13].

Fig. 2, and Fig. 3 shows a comparison of data rate-loss per-user using various kinds of criterion at 10 dB SNR values. For ideal case, an error-free feedback is considered in (7). The CBS criterion is achieved 0.80 [bps/Hz] considering 10 iterations at $L = 3$ and 10dB SNR values. The proposed HS criterion shows a very small gap of rate-loss in Fig.2 using $L = 2$ due to the reduction of the transmitted data streams. In addition, the proposed HS criterion illustrates zero-gap of rate-loss in Fig.3 considering $L = 3$ in the same environments.

VI. Conclusions

We studied HS criterion in FHZFP for 60-GHz mmWave mMIMO LAA systems. The CBS criterion is illustrated worse data rate performance per-user due to the constructional imperfection of the BS. Moreover, the simulation results confirmed that the proposed HS criterion outperforms the CBS criterion, in terms of data rate per-user. Lastly, the HS criterion can be extended further to be applied in the next generation MIMO non-orthogonal multiple access (MIMO-NOMA) networks, which is subjected to future works.