Mixed controller (IRC + NSC) involved in the harmonic vibration response cantilever beam model

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Abstract
This manuscript aims for improving the vibrational behaviors of a cantilever beam model through an intermediate lumped mass via offering a new control methodology to suppress for such high oscillations of the system. The equation of the considered cantilever beam structure is gained applying Euler–Lagrange technique. Accordingly, the considered model is modified by mixing Integral Resonant Control (IRC) along with the Nonlinear Saturation Controller (NSC) as a new controller to the considered system. Due to the recommended control technique, the modified system model is studied and analyzed by the perturbation technique. Time histories figures of the measured system plus the new controller are involved to display the response before and after control. The frequency response figures of the modified model before and after new controller near simultaneous condition \( \Omega \approx \omega_1, \, \omega_1 \approx 2\omega_2 \) are gained. Each frequency-response curves have stable and unstable regions are determined numerically. Numerical results show the vibrations of the system are eliminated when adding combined IRC and NSC controllers. Finally, numerical outcomes are performed that illustrated an excellent agreement with the analytical ones. Comparison between this paper and recent papers of the cantilever beam are done.

Keywords
Cantilever beam, perturbation technique, simultaneous resonance, soft spring

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Introduction
Robotic manipulators, high-speed apparatus, parts of basic structures and different various models¹–⁵ which are popular common applications of the cantilever beam through a middle of the road lumped mass inside base excitation. Hamdan et al.⁶,⁷ diminished huge free vibration sufficiency of a uniform bar per a lumped mass. Al-Qaisia and Hamdan⁸,⁹ examined bifurcations and the stability of a nonlinear oscillator excited harmonically. Feng et al.¹⁰ measured the stochastic hop and bifurcations of a cantilever bar within a suffered mass by perturbation process analytically and numerically. Qian et al.¹¹ studied the asymptotic solutions for the cantilever beam within a lumped mass by homotopy analysis method (HAM). Herisanu and Marinca¹² carried out the free nonlinear vibration of the uniform cantilever beam by rotary inertia and an intermediate lumped mass by optimal homotopy asymptotic method (OHAM). Flexural–extensional huge vibration fullness of non-uniform cantilever pillars loud transversely and pivotally mass is premeditated in Malaek and Moeenfard.¹³ Ekici and Boyaci¹⁴ studied the vibration amplitudes of the micro-beams for super-harmonic and sub-harmonic resonances by perturbation technique. They build up that non-perfect limit conditions affect the vibrations of the framework. Mehran et al.¹⁵ premeditated a nonlinear cantilever pillar conveying a halfway lumped mass under harmonic force near different resonances. They originated that the recurrence reaction of the model remains powerfully prejudiced by the damping and force heights. The control of various nonlinear systems has been investigated recently¹⁶–²⁰ involved to cantilever beam within intermediate lumped mass. Time delay and various active control methods are public ways in controlling

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nonlinear structures, numerous researchers are employed on these main various controllers.\textsuperscript{21–25} Additionally, Liu et al.\textsuperscript{26} examined the displacement plus velocity time-delay feedback in a cantilever beam model conveying a lumped mass. Besides, the displacement feedback gain constant makes top sufficiency moving to the short

\[ L = \frac{ml^2}{2} \left[ c_1 \left( \frac{du}{d\tau} \right)^2 + c_2 u^2 \left( \frac{du}{d\tau} \right)^2 + c_3 u^2 \left( \frac{du}{d\tau} \right)^2 + c_4 u^2 \left( \frac{du}{d\tau} \right)^2 + c_5 u^2 \left( \frac{du}{d\tau} \right)^2 + c_6 u^2 \left( \frac{du}{d\tau} \right)^2 \right] \]

recurrence only, however velocity feedback and their time-delays can be utilized in vibrational stability and reducing the nonlinear vibrations of the model. Alhazza et al.\textsuperscript{22,27,28} considered the nonlinear vibrations of a cantilever pillar under outer, parametric excitations through directly and nonlinearly time-postpone criticism controllers. They set up that time-postpone criticism controllers are diminished high vibrations framework. The recent paper, introduced a new controller via IRC and NSC together for eliminating the nonlinear vibration and resonance of the considered beam within harmonic excitations. The perturbation procedure is useful to acquire the frequency-response conditions near simultaneous resonance. The numerical solutions and reaction amplitude of this framework are measured and examined. The stability analysis observed via frequency response equations and phase plane procedure at the considered worst resonance case. We determined vibrational graphs and stability study using MAPLE and MATLAB algorithms. The analytical and numerical solutions are in excellent agreement. Comparison between this paper and recent papers of the cantilever beam are prepared.

**Measured model**

Figure 1 presented a cantilever beam model loud an intermediate lumped mass. The model with a length \( l \) and a mass for each unit length \( m \) has a lumped mass situated at a distance \( d \) from its base. The model is related to a joint rotation spring of stiffness \( K_r \) at the base and is exposed to a harmonic distribution load of amplitude \( P \). We apply the total kinetic and potential energy of the system to achieve the resultant Lagrangian

\[ u(\tau) = \text{time dependence of the beam displacement.} \]
\[ I \] and \( E \) symbolise the moment of inertia and the Young’s modulus of elasticity, respectively. The constant values \( c_1 - c_8 \) are defined as follows:

\[ c_1 = \int_0^1 \phi^2 d\zeta + \rho \beta^2 \phi(\eta); \quad c_2 = 1 \int_0^1 \phi d\zeta; \]
\[ c_3 = \int_0^1 \left( \frac{d\phi}{d\zeta} \right)^2 d\zeta; \quad c_4 = \rho \int_0^1 \left( \frac{d\phi}{d\zeta} \right)^2 d\zeta; \]
\[ c_5 = \rho \int_0^1 \left( \frac{d\phi}{d\zeta} \right)^2 \left( \frac{d\phi}{d\zeta} \right) d\zeta; \]
\[ c_6 = \int_0^1 \left( \frac{d\phi}{d\zeta} \right)^2 d\zeta; \]
\[ c_7 = \int_0^1 \left( \frac{d\phi}{d\zeta} \right)^2 d\zeta; \]
\[ c_8 = 1 \int_0^1 \phi d\zeta \]

where \( \phi(\zeta) \) is the eigenfunction of the beam, which written as\textsuperscript{22} \( \phi(\zeta) = \sin \beta \zeta - U \sin \beta z - V \cos \beta \zeta - \cos \beta \zeta \)

Here, \( r (r = \phi(1)) \) is the scaling factor, and \( \beta \) is the non-dimensional frequency parameter defined as \( \beta = \frac{m \omega_0^2}{I} \) where \( \omega_0 \) is the natural frequency of the beam. The coefficients \( U \) and \( V \) are the weighting constants, which are defined as:

\[ U = \frac{IK_r - 2EI\beta \sin \beta \cos \beta + \cosh \beta}{IK_r + 2EI\beta \sinh \beta \cos \beta + \cosh \beta}; \]
\[ V = \frac{\sin \beta + U \sinh \beta}{\cosh \beta + \cos \beta}. \]

The dimensionless parameters \( \eta = \frac{d}{l}, \xi = \frac{z}{l}, \rho = \frac{m}{ml} \) denote the distance, span length, and mass ratio, respectively. \( \rho \) stands as the mass density per unit length. By utilizing the Euler-Lagrange equation, the dimensionless administering equation of movement for the elements is as per the following\textsuperscript{13}

\[ \ddot{u} + \mu \dot{u} + \alpha_1 u + \alpha_2 u^3 + \beta_1 u^2 \dot{u} + \beta_2 u^4 \ddot{u} = F \cos(\Omega t) \]
Where \( m \) is the damping coefficient. The parameters \( a_1, a_2, b_1, \) and \( b_2 \) are defined as
\[
a_1 = \frac{2a_1}{c_2}, \quad a_2 = \frac{2a_2}{c_2}, \quad b_1 = \frac{c_1 a_1}{\beta c_1}, \quad b_2 = \frac{c_1 a_2}{\beta c_1},
\]
\[
F_0 = \frac{P}{\int_{0}^{\infty} \xi d\xi}, \quad \text{and} \quad \Omega = \frac{m}{\omega_0} \text{is the dimensionless excitation frequency.}
\]

**Mixed (IRC + NSC) controller design**

The suggested mixed control is designed to reduce the vibration of a cantilever beam model conveying a lumped mass with a collocated piezoelectric actuator and sensor pair as shown in Figure 2.

By integrating the new controller into system (*) as shown in Figure 3, we have the following equations:

\[
\ddot{u} + \omega_1^2 u = -\epsilon \left[ \alpha_1 \dot{u} \dot{u} + \alpha_2 \dot{u}^2 + \beta_1 \dot{u}^2 + \beta_1 \dot{u} \dot{v} + \beta_2 \dot{u} \dot{v} - F_0 \cos(\Omega t) \right]
\]
\[
- k_1 \dot{u}^2 - k_1 \dot{v}^2
\]

\[
\dot{v} + \delta_1 v = k_2 u
\]

\[
\ddot{w} + \omega_2^2 w = -\epsilon [ \mu_2 \dot{w} - k_2 w] \quad (3)\]

Where \( u, v, \) and \( w \) are the amplitudes of the chief system, IRC and NSC, respectively, \( \alpha_1 \) and \( \mu_1 \) are the linear damping coefficients of the chief system and NSC, \( F_0, \) and \( \Omega \) are the forcing amplitude and the frequency of the chief system, \( \omega_1 \) and \( \omega_2 \) are natural frequencies of the chief system and NSC, \( \alpha_1, \alpha_2, \beta_1, \) and \( \beta_2 \) are non-linear parameters, \( \delta_1 \) is the integrator’s frequency parameter of IRC, \( k_1, k_2, k_3 \) are control signal gains of NSC and IRC, respectively, \( k_2, k_3 \) is feedback signal gains of NSC and IRC, respectively and \( \epsilon \) is the perturbation coefficient.

**Perturbation procedure**

By employing the standard multiple scales perturbation technique (MSTP),\(^{29,30}\) we can get the expansion of \( u(t), v(t) \) and \( w(t) \) in the following form:

\[
u(t, \epsilon) = u_0(T_0, T_1) + \epsilon u_1(T_0, T_1) + O(\epsilon^2) \quad (4a)
\]

\[
v(t, \epsilon) = v_0(T_0, T_1) + \epsilon v_1(T_0, T_1) + O(\epsilon^2) \quad (4b)
\]

\[
w(t, \epsilon) = w_0(T_0, T_1) + \epsilon w_1(T_0, T_1) + O(\epsilon^2) \quad (4c)
\]

where, \( T_0 = t \) and \( T_1 = \epsilon t \) are two time scales. Mathematically, the time derivatives can be written in terms of \( T_0 \) and \( T_1 \) as follows:

\[
\frac{d}{dt} = D_0 + \epsilon D_1 + ..., \quad \frac{d^2}{dt^2} = D_0^2 + 2\epsilon D_0 D_1 + ...
\]

Substituting equations (4)-(5) into (1)-(3), then comparing the parameters of the similar power of \( \epsilon \), the
resulting expressions of the 2nd order differential equations are derived as

\[ e^0: (D_0^2 + \omega_1^2)u_0 = 0 \]  
\[ (D_0 + \delta_0)v_0 = 0 \]  
\[ (D_0^2 + \omega_2^2)w_0 = 0 \] 

\[ e^1: (D_0^2 + \omega_1^2)u_1 = -2D_0D_1u_0 - \mu_1D_0u_0 - \alpha_1u_0^2 \] 
\[ -\alpha_2u_0^2 - \beta_1u_0^2D_0^2u_0 - \beta_2u_0^2(D_0u_0)^2 \]
\[ + \frac{F_0}{2}(c.d + c.c) + k_1w_0^2 + k_1v_0 \]
\[ (D_0 + \delta_1)v_1 = -D_1v_0 + k_2u_1 \]
\[ (D_0^2 + \omega_2^2)w_1 = -2D_0D_1w_0 - \mu_2D_0w_0 + k_2u_0w_0 \] 

(7a) \hspace{1cm} (7b) \hspace{1cm} (7c)

Where, \( c.c. \) represents the complex conjugate coefficients. From equation (7) one could get \( u_0, v_0 \) and \( w_0 \) as:

\[ u_0 = A_1(T_1)e^{i\omega T_0} + c.c. \]
\[ v_0 = C_1(T_1)e^{i\delta T_0} + \frac{k_2(\delta_0 - i\omega_1)}{\delta_0^2 + \omega_1^2}A_1(T_1)e^{i\omega_1 T_0} + c.c. \]
\[ w_0 = A_2(T_1)e^{i\omega_2 T_0} + c.c. \]

(8a) \hspace{1cm} (8b) \hspace{1cm} (8c)

Substituting equation (8) into (7) yields:

\[ (D_0^2 + \omega_1^2)u_1 = \left[ -2i\omega_1D_1A_1 - \mu_1i\omega_1A_1 - 3\alpha_1A_1^2A_1 \right. \]
\[ -10\alpha_2A_1^2A_1^2 + 2\beta_1\omega_1^2A_1^2A_1 \]
\[ + 8\beta_2\omega_1^2A_1^3A_1 + \frac{k_1k_2(\delta_0 - i\omega_1)}{\delta_0^2 + \omega_1^2} \left. \right] e^{i\omega_1 T_0} \]
\[ + \left[ -\alpha_1A_1^3 - 5\alpha_2A_1^4A_1 \right. \]
\[ + 2\beta_1\omega_1^2A_1^2A_1^2 \]
\[ + \left[ -\alpha_2A_1^5 + 2\beta_2\omega_1^2A_1^3A_1 \right. \]
\[ + \frac{F_0}{2}(c.d + c.c) + k_1A_1^2e^{i\delta_0 T_0} + k_1A_2A_2 \]
\[ + k_1C_1e^{i\delta T_0} + c.c. \]
\[ (D_0 + \delta_1)v_1 = -D_1C_1e^{i\delta T_0} - \frac{k_2(\delta_0 - i\omega_1)}{\delta_0^2 + \omega_1^2} \]
\[ \]
\[ D_1A_1e^{i\omega_1 T_0} + k_2u_1 + c.c. \]
\[ (D_0^2 + \omega_2^2)w_1 = \left[ -2i\omega_2D_1A_2 - \mu_2i\omega_2A_2 \right. \]
\[ + k_2A_2A_1^2e^{i\omega_1 T_0} \]
\[ + \left[ k_2A_2A_1^2 \right. \]
\[ e^{i(\omega_1 - \omega_2)T_0} + c.c. \]

(9a) \hspace{1cm} (9b) \hspace{1cm} (9c)

Non-dimensional excitation frequency \( \Omega \) and internal resonance \( \omega_2 \) can be presented by:

\[ \Omega = \omega_1 + \Delta \omega_1, \quad \omega_1 = 2\omega_2 + \Delta \omega_2 \]

(10)

Taking into consideration equations (10) and (9), one can obtain the secular terms as:

\[ 2i\omega_1D_1A_1 = -\mu_1i\omega_1A_1 - 3\alpha_1A_1^2A_1 - 10\alpha_2A_1^2A_1^2 \]
\[ + 2\beta_1\omega_1^2A_1^2A_1 + 8\beta_2\omega_1^2A_1^3A_1 \]
\[ + \frac{1}{2}F_0e^{i\delta T_1} + \frac{k_1k_2(\delta_0 - i\omega_1)}{\delta_0^2 + \omega_1^2}A_1 + \left[ k_1A_2^2 \right] e^{i\delta T_1} \]

(11a)

\[ 2i\omega_2D_1A_2 = -\mu_2i\omega_2A_2 + k_2A_2e^{i\delta T_1} \]

(11b)

The common term must be wiped out since it brings about an unbounded extension of the reaction of the model, which doesn’t struggle with the physical framework. The complex amplitude \( A_n(n = 1, 2) \) can be supposed as:

\[ A_n = \frac{1}{2}a_ne^{i\eta_n} \]

(12)

in which \( a_n \) and \( \eta_n \) are real functions of \( T_1 \).

Taking into consideration \( \theta_1 = \sigma_1T_1 - \eta_1 \), \( \theta_2 = \sigma_2T_1 - 2\eta_2 + \eta_1 \), then isolating the real and imaginary phases of equation (11) we get:

\[ \dot{a}_1 = \left[ -\frac{\mu_1}{2} - \frac{k_1k_2}{2(\delta_0^2 + \omega_1^2)} \right] a_1 + \frac{F_0}{2\omega_1} \sin \theta_1 \]
\[ + \left[ -\frac{k_1}{4\omega_1} \right] a_2^2 \sin \theta_2 \]

(13a)

\[ \dot{\theta}_1 = \left[ \sigma_1 + \frac{k_1k_2\delta_0}{2\omega_1(\delta_0^2 + \omega_1^2)} \right] \]
\[ + \left[ -\frac{3\alpha_1}{8\omega_1} - \frac{\beta_1\omega_1^2}{4\omega_1} \right] a_1^2 \]
\[ + \frac{F_0}{2\omega_1 a_1} \cos \theta_1 + \left[ \frac{k_1}{4\omega_1 a_1} \right] a_2^2 \cos \theta_2 \]

(13b)

\[ \dot{a}_2 = -\frac{\mu_2}{2} a_2 + \frac{k_2}{4\omega_2} a_1a_2 \sin \theta_2 \]

(13c)

\[ \dot{\theta}_2 = \left[ \sigma_2 - \frac{k_1k_2\delta_0}{2\omega_1(\delta_0^2 + \omega_1^2)} \right] \]
\[ + \left[ -\frac{3\alpha_1}{8\omega_1} - \frac{\beta_1\omega_1^2}{4\omega_1} \right] a_1^2 \]
\[ + \frac{5\alpha_2}{16\omega_1} - \frac{\beta_2\omega_1^2}{4\omega_1} a_1^2 \]
\[ - \frac{F_0}{2\omega_1 a_1} \cos \theta_1 + \left[ -\frac{k_1}{4\omega_1 a_1} \right] a_2^2 \cos \theta_2 \]

(13d)

Putting the vibration amplitudes \( a_n \) and the modified phases \( \theta_n \) would equal zero into equation (13), we obtain four nonlinear algebraic equations. By solving these nonlinear algebraic equations (Frequency-response equations) by using MATLAB program, we get the steady-state solution equation (13). Furthermore, for investigating the stability examination, start with the subsequent procedures:
\[ a_1 = a_{11} + a_{10}, \quad a_2 = a_{21} + a_{20}, \quad \theta_1 = \theta_{11} + \theta_{10}, \quad \theta_2 = \theta_{21} + \theta_{20} \]

Substituting equation (14) into (13) and expanding for little \( a_{11}, \ a_{21}, \ \theta_{11}, \ \text{and} \ \theta_{21} \) to carry on linear terms only, introduce the following linearized system:

\[
\begin{bmatrix}
\dot{a}_{11} \\
\dot{\theta}_{11} \\
\dot{a}_{21} \\
\dot{\theta}_{21}
\end{bmatrix} =
\begin{bmatrix}
r_{11} & r_{12} & r_{13} & r_{14} \\
r_{21} & r_{22} & r_{23} & r_{24} \\
r_{31} & r_{32} & r_{33} & r_{34} \\
r_{41} & r_{42} & r_{43} & r_{44}
\end{bmatrix}
\begin{bmatrix}
a_{11} \\
\theta_{11} \\
a_{21} \\
\theta_{21}
\end{bmatrix}
\]

Wherever the over matrix remains the Jacobian matrix of the model and its coefficients entries \( r_{ij} (i, j = 1, 2, 3, 4) \) are prearranged in the Appendix.

Therefore, the solution of the nonlinear framework equation (13) is asymptotically stable if and only if the real piece of every eigenvalue of equation (15) is negative; otherwise, the solution is unstable.

**Outcomes and conversation**

In this section, we presented all the curves which happened in the considered model before and after addition a new controller.

Figure 4(a) portrays the frequency–response curve of the cantilever beam mode before the control is on at \( a_1 \neq 0, a_2 = 0 \). In this diagram, the frequency response curvature bends away to the left give rise to soft spring. Arrows indicate the motion-related oscillation mode. We explain the jump phenomenon as shown from the figure, whereas \( \sigma_1 \) is increased progressively from negative value, the oscillation amplitude of the cantilever beam has a small value corresponding to reaching point \( A \). As \( \sigma_1 \) is increased further, a jump with an observable rate takes from the point \( B \) to \( C \); at that time, this is decreased slowly until \( \sigma_1 \) reaching point \( D \). As well, we note that there is an unstable region between \( B \) and \( D \); while for the other regions, they are stable regions.

Figure 4(b) portrays the phase sketch of the non-controlled cantilever beam system at \( \sigma_1 = -0.5 \). The phase trajectories illustrated the stable spiral point and the unstable saddle point allowing the eigenvalues which classified from the critical points which obtained from equations (3)–(4) at \( a_2 = 0 \).

Figure 5 exhibits the difference between the response curves for the system and new control before (blue line)
and after (red line) the controller commences in action. It is noticeable that the new controller (IRC + NSC) has a perfect vibration reduction job in the high amplitude’s region (from $\sigma_1 = -0.077$ to $0.0609$) as shown in Figure 5(a) (see black dashed line). Further, after the controller is on, the unstable regions are becoming completely non-existent in the cantilever beam model.

Figure 6 portrays the performances of the linear damping coefficient $\mu_1$ on the model and new controller. It is obvious from Figure 6(a) and (b), once $\mu_1$ is increased, a slight increase, the supreme productivity of the vibration decrease is high by reducing the amplitude of the system. In addition, the unstable zone in the response curve of the new controller mode is shrunk.

Following, we will discuss the act on varying the nonlinear parameters $\alpha_1$ and $\alpha_2$ of the cantilever beam and the new controller amplitudes. As seen in Figures 7 and 8, increasing the nonlinear parameters $\alpha_1$ and $\alpha_2$ make the vibration amplitude for the system displacement $\sigma_1$ is saturated at the value reached. Also, the controller amplitude $\alpha_2$ decreases when raising $\sigma_1$ from zero. The opposite happens when reducing $\sigma_1$ from zero.

As a sequel to the curves discussion, at $\sigma_1 < 0$, the frequency-response amplitudes of the model and the new control show that, the unstable region for the controller are decreasing when increases the values of the parameters $\beta_1$ and $\beta_2$ as presented in Figures 9 and 10.
But the opposite happens only in the controller amplitude when increasing $\sigma_1$ from zero.

Figure 11(a) evidences that the amplitude of the system is saturated such the parameter $k_1$ increased. Further, the amplitude of new control is a monotonic decreasing function of the parameter $k_1$ as shown in Figure 11(b).

From Figures 12 and 13, the response-amplitudes of the model and new controller are monotonic growing when the force amplitude $F_0$ and lossy integrator’s frequency parameter $\delta_v$ are increased.

Figures 14 and 15 illustrates that for increasing the values of $k_{v1}$ and $k_{v2}$, the amplitudes of the modified system besides new controller are decreased. Furthermore, bandwidth for the steady-state amplitude of the model around $\sigma_1 = 0$ isn’t change.

At $\sigma_1 = 0$, the amplitude of the framework is increasing which exposed in Figure 16(a), also the amplitude for the new controller is monotonically lessening as exposed in Figure 16(b). Both are discussed at increasing the linear damping coefficient for the controller ($\mu_2$).
Figure 17, presented the influence of the feedback signal gain $k_2$ arranged both the model and new controller. For large values of $k_2$, the effective amplitude range of the model and new controller wider around $s_1 = 0$.

We have explained the detuning between the excitation frequency $\Omega$ and the natural frequency of new controller ($\omega_2$) with $s_1$ and $s_2$, respectively. Hither, we are going to study the effect of varying these parameters ($s_1$ and $s_2$) on response curves as shown in Figure 18. We can notice that the lowest amplitude of the system occurs at $s_2 = -s_1 = \{-0.05, 0, 0.05\}$. Such a case is called perfect tuning between the cantilever beam and the new controller.

In this part, equations (1)–(3) which exhibited the measured framework before control plus after joined with different kinds of controls (NSC – new control) are reenacted numerically dependent on MATLAB® software (solver ODE45) to select the chief controller which diminish the vibration with a short time. This study is proposed based on the values of the coefficients presented in the illustrated Table 1:

Figure 19(a) portrays the vibration amplitude $u(t)$ is about 500% of the external force amplitude $F_0$, at the time history, for the excited cantilever beam system before the controller on (i.e.: without control) and the initial conditions are zeros. The phase plane of the
Figure 15. Effect of varying $k_2$ on: (a) the model, and (b) new controller.

Figure 16. Effect of varying $\mu_2$ on: (a) the model, and (b) new controller.

Figure 17. Effect of varying $k_2$ on: (a) the model, and (b) new controller.

Figure 18. Effect of varying $\sigma_2$ on: (a) the model, and (b) new controller.
system is stable with slight chaotic multi-limit cycles as shown in Figure 19(b).

As well, Figure 20(a) clarified the reduction for the system displacement $u(t)$ after applied the NSC controller $w(t)$ in equilibrium (time approximately equal 550) with the initial conditions which presented as $u(0) = \dot{u}(0) = \ddot{u}(0) = 0$, $w(0) = 0.05$.

Herein, the best possible efficiency in reducing system displacement was developed when creating a new controller by adding the IRC controller $v(t)$ to the NSC controller $w(t)$ at $u(0) = \dot{u}(0) = \ddot{u}(0) = v(0) = 0$, $w(0) = 0.05$. This efficiency was evidenced when time is shortened to approximately 300 as presented in Figure 20(b). This means that the new controller is one of the most superb controllers to repress the vibration at a few times.

Figure 21(a) shows the system amplitude is about 50% of the external force amplitude $F_0$ when a new controller (IRC + NSC) is connected to one of the worst resonances. Figure 21(b) shows the steady state amplitudes of the new control $w$ (for NSC); $v$ (for IRC) are about 600% of the excitation force amplitude $F_0$ and 10% of the excitation force amplitude $F_0$, respectively. Also, Figure 21(c) shows frequency spectra affecting system vibration and what happened to the system amplitude before and after the controller was turned on (i.e.: with and without new control).

In addition, Figure 22 presented the verification of the result, which obtained from the frequency response equation, as presented in Figure 5 and numerically result of equations (1)–(3) using Runge-Kutta method and plotted on the same figure as the following:

Table 1. The parameters values of the measured framework.

| A cantilever beam parameter | Value | IRC controller parameter | Value | NSC controller parameter | Value |
|-----------------------------|-------|--------------------------|-------|--------------------------|-------|
| $\omega_1$                  | 2.4   | $\delta$                 | 2.8   | $\omega_2$              | $\approx \omega_1/2$ |
| $\mu_1$                     | 0.024 | $k_{v1}$                  | 0.7   | $\mu_2$                 | 0.0008 |
| $\alpha_1$                  | 0.3331| $k_{v2}$                  | 0.7   | $k_1$                    | 0.5   |
| $\alpha_2$                  | 0.1299|                          |       | $k_2$                    | 0.5   |
| $\beta_1$                   | 0.3338|                          |       |                          |       |
| $\beta_2$                   | 0.1319|                          |       |                          |       |
| $F_0$                       | 0.1   |                          |       |                          |       |
| $\Omega$                    | $\approx \omega_1$ |                          |       |                          |       |

Figure 19. The cantilever beam system without any control.

Figure 20. The cantilever beam system with different controllers.
1. Blue line with yellow circles for the system amplitude before new control and red line with green circles for the system amplitude after new control as shown in Figure 22(a).
2. Blue line with yellow circles for the control amplitude before the new control is connected and red line with green circles for the control amplitude after new control is connected as shown in Figure 22(b), to display outstanding agreement between the logical and arithmetical solutions.

Figure 23 declares the comparison between the first order perturbation method given by equation (13) and the numerical solution of equation (1)–(3). Furthermore, the dashed lines appear the modulation $a_1$ and $a_2$ of the amplitudes for the generalized coordinates $u(t)$ and $w(t)$, as presented in Figure 23(a) and Figure 23(b), respectively. Whereas the continuous lines denote the time history of vibrations which gotten numerically as solutions of equations (1) and (3). We obtained the good harmony between this approximate against the numerical result.

Comparison with different controllers
The authors have made a comparison through another controller design’s to verify the “high-performance” of the suggested vibration attenuation strategy and to show the amount of vibration reduction produced by the system using mixed (IRC + NSC) controller at small values of control and feedback signal gains is preferable than that of Integral Resonant Control (IRC), Negative velocity feedback (NVF), Negative
displacement feedback (NDF) control and Nonlinear saturation control (NSC) as presented in Figure 24.

**Conclusion**

In this work a cantilever beam model which loud an intermediate lumped mass under harmonic excitation was studied and derived. Then, we investigate a new control effectiveness of the measured system. The control currents in the cantilever beam system are designed to become a combination of two control signals: One of them is generated via the controller IRC, and the other one comes from the controller NSC. The technique of perturbation is useful to approximate the solution of the measured controlled system. The new controller consists of IRC plus NSC, which is excellent in reduction the vibrations of the cantilever beam near worst simultaneous resonance case. The numeric stability study was performed to acquire the unstable – stable regions for each frequency response curves. The variety of the amplitude of the model within frequency response curves under new controller was thought and clarified numerically. The outcomes display that the new measured controller is excellent in reduction the vibrations of the cantilever beam system. The influence effects of all coefficients over the modified controlled system within the frequency response curves were fully detailed illustrated in section “Outcomes and
conversation.” In this manner, sensible choice of the control framework parameters can viably improve the degree of vibration control for the framework.

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**Appendix**

\[
\begin{align*}
    r_{11} &= \left[ -\frac{\mu_1}{2} - \frac{k_1 k_{12}}{2(\delta_1^2 + \omega_1^2)} \right],
    r_{12} = \left[ \frac{F_0}{2\omega_1} \cos \theta_{10} \right],
    r_{13} = \left[ -\frac{k_1}{2\omega_1} a_{20} \sin \theta_{20} \right],
    r_{14} = \left[ -\frac{k_1}{4\omega_1} a_{20}^2 \cos \theta_{20} \right],
    r_{21} &= \left[ \frac{\sigma_1}{a_{10}} - \frac{3a_{10}}{8\omega_1} \left( 3\alpha_1 - \beta_1 \omega_1 \right) \right] - 5a_{10}^2 \left( \frac{5\alpha_2}{16\omega_1} - \beta_2 \omega_1 \right) + \frac{k_1 k_{12} \delta_v}{2a_{10} \omega_1 (\delta_v^2 + \omega_1^2)},
    r_{22} &= \left[ -\frac{F_0}{2a_{10} \omega_1} \sin \theta_{10} \right],
    r_{23} = \left[ \frac{k_1}{2a_{10} \omega_1} a_{20} \cos \theta_{20} \right],
    r_{24} = \left[ -\frac{k_1}{4a_{10} \omega_1} a_{20}^2 \sin \theta_{20} \right],
    r_{31} &= \left[ \frac{k_2}{4\omega_2} a_{20} \sin \theta_{20} \right],
    r_{32} = 0, \quad r_{33} = \left[ -\frac{\mu_2}{2} + \frac{k_2}{4\omega_2} a_{10} \sin \theta_{20} \right],
    r_{34} = \left[ \frac{k_2}{4\omega_2} a_{10} a_{20} \cos \theta_{20} \right],
    r_{41} &= \left[ \frac{k_2}{2\omega_2} \cos \theta_{20} - \frac{a_1}{a_{10}} + 3a_{10} \left( 3\alpha_1 - \beta_1 \omega_1 \right) \right] - 5a_{10}^2 \left( \frac{5\alpha_2}{16\omega_1} - \beta_2 \omega_1 \right) - \frac{k_1 k_{12} \delta_v}{2a_{10} \omega_1 (\delta_v^2 + \omega_1^2)},
    r_{42} &= \left[ \frac{F_0}{2a_{10} \omega_1} \sin \theta_{10} \right],
    r_{43} = \left[ (\alpha_2 + \sigma_1) + \frac{k_2}{a_{20} a_{10}} a_{10} \cos \theta_{20} - \frac{k_1}{2a_{10} \omega_1} a_{20} \cos \theta_{20} \right],
    r_{44} &= \left[ -\frac{k_2}{2\omega_2} a_{10} \sin \theta_{20} + \frac{k_1}{4a_{10} \omega_1} a_{20}^2 \sin \theta_{20} \right].
\end{align*}
\]