Simulation of Spin-orbit Dynamics in Storage Rings

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Abstract. In the article a mapping approach based on nonlinear matrix integration for long-term spin-orbit dynamics simulation is briefly described. Using this technique the nonlinear effects of spin dynamics in an electrostatics storage ring are investigated. Namely, the fringe fields, the energy conservation law and the random field errors are considered. The necessity of examination of such effects arises, for example, in the storage ring design for search the Electrical Dipole Moment of proton and deuteron. The EDM ring is proposed to measure EDM using the spin transformation of polarized particle in the magneto-electrostatic elements of the ring. The article consists of short description of the spin-orbit simulation results based on the nonlinear model.

1. Introduction
Simulation of spin-orbit dynamics plays a crucial role in a storage ring design for search of EDM [1]. Such nonlinear effects as the fringe fields, the energy conservation law and the random field errors should be carefully investigated. The main problem is that one has to distinguish the fake EDM signal from presumably existing EDM signal. For example there were fears that fringe fields may change reference orbit dramatically in sense of spin dynamics and lead to unexpected spin behaviour. The electrostatic fields which are going to be used induce additional problems. In particular, consideration of the energy conservation law means we should take into account accordance between the particle energy and its coordinates in electrostatic fields. To investigate such effects on spin dynamics the numerical analysis can be used only. In this section we will briefly describe our mapping approach for nonlinear spin-orbit dynamics simulation. Then we will present results of simulation and indicate further directions of numerical method analysis.

For numerical integration of differential equation of spin-orbit dynamics the nonlinear matrix integration is used. This mapping approach allows describing particle beam dynamics for whole ring (see [2])

$$X = R_0 + R_1 X_0 + R_2 X_0^{[2]} + \ldots + R_k X_0^{[k]},$$

(1)

where $X_0$ means initial state of a particle (space coordinates, energy and spin) and $R_i$ are 2 dimensional matrices that describe nonlinear map of particle dynamics for a ring. In general case this map depends on lattice of storage ring and can also include effects of fringe fields and random errors.

1.1. Simulation
For building the map (1) we have developed the MODE software that allows creating the parametric nonlinear matrix map for arbitrary field distribution in indicated lattice (see [3]). Aside from the

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standard magnets and lenses we have implemented additional elements, such as fringe fields and fields with random errors. The MODE allows building the parametric map, and the simulation can be performed just varying set of parameters (such as field distribution characteristic, strength and length of fringe field, etc.) Spin-orbit dynamics investigation

1.2. Fringe fields
Modeling of fringe fields shows that there are no dramatic influences on spin dynamics at all. Fringe fields change reference orbit and energy. Nevertheless one can optimize parameters of a lens or magnet to achieve the same spin behaviour as in case without fringe field.

1.3. Energy conservation low
The spin-orbit dynamics of charged particles in electrostatic fields has more complicated behaviour than in magnetic ones. If in the magnetostatic field the force acting on the particle is always perpendicular to the motion direction and the kinetic energy remains constant, in the electrostatic field the kinetic energy is continuously transferred into the potential energy and vice versa. This means that the velocity of a particle $v$ is not more a constant value, but a function of coordinates. More precisely one can write $v = v(u(x, y, z))$, where $u$ is a scalar potential of an electrostatic field and $x, y, z$ are the space coordinates of a particle.

As shown in [4] the low-order electrostatic fields introduce the high orders in actual force that effect on particle. For instance, in the electrostatic quadrupoles (first order field) there are the second order aberrations in spin dynamics.

![Figure 1](image1.png)

**Figure 1.** (a) spin aberrations without lattice optimization; (b) with quadrupoles optimization.

1.4. Random fields errors
Spin dynamics due to the beam dynamics nonlinearity has complex behaviour with respect to the random errors. Various types of error fields in the elements lead to systematic errors, limiting the possibility of the EDM experiment. One way is to decrease the systematic errors it is the method of direct and reverse direction of the beam in the storage ring, the so-called clockwise and contra-clockwise method.

Image that we have error field $E_{er}$ in electrostatic storage ring that belongs to normal distribution. Then the frequency of spin precision

$$w(E_{er}) \in N, \langle w(E_{er}) \rangle \neq 0, w_{cw} \neq w_{ccw},$$

where $<a>$ denotes mean value of random variable $a$. So it is not possible to easy use clockwise ($w_{cw}$) and contra-clockwise ($w_{ccw}$) beams to measure EDM. Moreover spin tune now depends on beam position and it is not clear how to correct spin aberration without impact on EDM.

2. Conclusion
Thus we have developed a mathematical method of the spin-orbital dynamics simulation, which is necessary for the design of the storage ring to search for EDM. This method allows taking into account all the necessary properties of the system, affecting the spin. The method became the basis for writing the program MODE, which was tested by other codes.
References

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