Enhancement of “CP-odd” Higgs Boson Production in the Minimal Supersymmetric Standard Model with Explicit CP Violation

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Abstract
We calculate the production cross section of the “CP-odd” Higgs boson via gluon fusion in the minimal supersymmetric standard model with explicit CP violation in the stop sector. We show that there is a parameter region in which the cross section is enhanced by a factor of about 1000, as compared to the case without CP violation in the stop sector. In the parameter region where the “CP-odd” Higgs boson can decay into a stop pair, the stop pair events will be the important signature of the enhanced “CP-odd” Higgs boson. In the case where the “CP-odd” Higgs boson cannot decay into any superparticles, the $\gamma\gamma$ and $\tau\tau$ decay channels could become important for discovering the “CP-odd” Higgs boson. We also discuss the constraints from electric dipole moments of electron, neutron and mercury on the viable parameter space mentioned above.
Low energy supersymmetry (SUSY) is one of the most promising candidates of physics beyond the Standard Model (SM). SUSY gives an elegant solution to the naturalness problem of the stability of the weak scale by canceling quadratically divergent radiative corrections.

One of the most important predictions of the minimal supersymmetric standard model (MSSM) is the upper bound of the lightest Higgs boson mass. At tree level, the MSSM predicts the lightest Higgs boson mass to be less than the Z boson mass. However, after including loop corrections, the contributions from top and stop loops are so important that the upper bound of the lightest Higgs boson mass can be increased to around 130 GeV \[1\]. This upper bound should be compared with the current lower limit of 89.8 GeV from the MSSM Higgs search at LEP \[2\]. If the lightest Higgs boson is discovered and its mass turns out to be less than 130 GeV, it is a strong hint for the MSSM.

If the MSSM is true, the CERN Large Hadron Collider (LHC) is expected to probe the Higgs sector by copiously producing the Higgs bosons. The Higgs sector in the MSSM has a rich structure; there are two CP-even Higgs bosons, one CP-odd Higgs boson and one (complex) charged Higgs boson. Their production and decay properties depend on various parameters in the MSSM including the SUSY breaking parameters. Therefore, to study the properties of the Higgs bosons at the LHC, a precise knowledge of the production cross section of the Higgs bosons is extremely important.

It has been shown that CP-violation in the Higgs sector could significantly affect the production and decay properties of the Higgs bosons \[3, 4, 5\]. In order to prepare for the discoveries of the MSSM Higgs bosons at the LHC in any case, further detailed studies on the MSSM with CP-violation would be important. The aim of this letter is to present our findings on the production cross section of the “CP-odd” Higgs boson in the MSSM with CP-violation.\(^1\) We show that the production cross section of the “CP-odd” Higgs boson can be enhanced by a factor of about 1000 compared to the case without CP-violation, and discuss some important decay signatures of the “CP-odd” Higgs boson.\(^2\) We also discuss some constraints on our CP-violating scenarios. The strongest constraint comes from the electric dipole moments (EDMs) of electron and neutron. Since there are possibilities that cancellations among many contributions to EDMs could happen, the searches for the “CP-odd” Higgs boson at the current and future colliders could provide important information on the CP-violation mechanism in the MSSM, which is generally independent of those from the EDM searches.

The MSSM has two Higgs doublets, \(H_1\) and \(H_2\). The neutral components \(H_1^0\) and \(H_2^0\) of the Higgs bosons develop vacuum expectation values (VEVs), which trigger the electroweak symmetry breaking (EWSB). After EWSB, there are three neutral Higgs bosons and a pair of charged Higgs bosons. If CP is a good symmetry in the Higgs sector, we can label the neutral Higgs bosons in

\(^1\)Strictly speaking, when CP is violated, we cannot define a “CP-odd” Higgs boson because all three neutral Higgs bosons are mixed with each other. As we will discuss later, however, in the parameter sets we consider, CP-violating Higgs boson mixing is small. Therefore, we still use the terminology “CP-odd” Higgs boson even in the CP-violating case.

\(^2\)Although in this letter we concentrate on the “CP-odd” Higgs production via gluon fusion at hadron colliders, we note that the same enhancement of the “CP-odd” Higgs production is also possible at a \(\gamma \gamma\) collider.
terms of CP properties as two CP-even Higgs bosons \( h^0 \) and \( H^0 \), and a CP-odd Higgs boson \( A \). In general, if CP is violated in the sfermion sector, CP-violating mixing among the three Higgs bosons is induced through radiative corrections. In this letter, we consider the CP violation in the Higgs sector radiatively induced by the trilinear coupling of stop \( A_t \), which is defined as

\[
\mathcal{L} = - \left( \frac{\sqrt{2} m_t}{v \sin \beta} A_t H_2^* \bar{q}_L + \text{h.c.} \right),
\]

where \( H_2 \) is the Higgs doublet that generates top quark mass \( m_t \) via Yukawa interaction, \( \bar{q}_L \) is the third generation squark doublet, and \( t_R \) is the right-handed stop. In our notation, \( \phi_1 \) and \( \phi_2 \) (\( a_1 \) and \( a_2 \)) are the real (imaginary) components of \( H^0_1 \) and \( e^{-i\xi} H^0_2 \), respectively, which are explicitly given by

\[
H^0_1 = \frac{1}{\sqrt{2}} (\phi_1 + v_1 + i a_1), \quad H^0_2 = \frac{e^{i\xi}}{\sqrt{2}} (\phi_2 + v_2 + i a_2).
\]

The VEV \( v_1 \) is relevant to the masses of down-type quarks and leptons, and \( v_2 \) is responsible for the up-type quark masses. The ratio of the two VEVs is parametrized by \( \tan \beta \equiv v_2/v_1 \), and \( v \) is defined as \( v \equiv \sqrt{v_1^2 + v_2^2} \), which is about 246 GeV. In general the relative phase \( \xi \) of the VEVs can be non-zero. For simplicity, in this letter we do not consider the effect of non-vanishing \( \xi \) and set \( \xi = 0 \) in the following. One of the linear combinations (\( G \)) of the CP-odd components \( a_1 \) and \( a_2 \) is eaten by the Z boson (\( G = a_1 \cos \beta - a_2 \sin \beta \)), and the other linear combination (\( A \)) becomes the physical “CP-odd” Higgs boson (\( A = a_1 \sin \beta + a_2 \cos \beta \)). Once we allow the \( A_t \) parameter to be complex, it induces CP-violating mixing among the neutral Higgs bosons. The CP-violating elements of the mass-squared matrix \( M^2_H \) at one-loop level are given as

\[
M^2_H \big|_{A_{\phi_1}} = \frac{3}{16\pi^2 \sin \beta} \frac{m^2_t}{m^2_{t_1} - m^2_{t_2}} \text{Im}(A_t \mu) F_t, \quad M^2_H \big|_{A_{\phi_2}} = \frac{3}{16\pi^2 \sin \beta} \frac{m^2_t}{m^2_{t_2} - m^2_{t_1}} \text{Im}(A_t \mu) G_t,
\]

where the explicit forms of the dimensionless quantities \( F_t \) and \( G_t \) were given in Ref. [6]. In the equations above, \( M^2_H \big|_{A_{\phi_1(2)}} \) is the \((A, \phi_1(2))\) element of the mass-squared matrix \( M^2_H \). \( m_{t_1} \) and \( m_{t_2} \) are the lighter and the heavier stop masses, respectively. In general, the Higgsino mass parameter \( \mu \) as well as \( A_t \) can have a CP violating phase. For simplicity, we assume that only the trilinear coupling \( A_t \) is complex and \( \mu \) is real. Because of the mixing induced by the CP-violating coupling \( A_t \), mass eigenstates of neutral Higgs bosons (\( h_1, h_2, h_3 \)) are linear combinations of the three neutral Higgs bosons \( \phi_1, \phi_2 \) and \( A \):

\[
\begin{pmatrix}
  h_1 \\
  h_2 \\
  h_3
\end{pmatrix}_i = O_{i\alpha} \begin{pmatrix}
  \phi_1 \\
  \phi_2 \\
  A
\end{pmatrix}_\alpha,
\]

\footnote{The complex trilinear coupling of sbottom \( A_b \) could also induce an important effect similar to the one discussed in this letter. For simplicity, however, we assume \( A_b \) to be a real parameter.}
Figure 1: The Feynman diagrams which contribute to $gg \to A$ in the MSSM with CP-violation when CP violating mixing among Higgs bosons are neglected. If the trilinear coupling $A_t$ is complex, there is a finite contribution from the diagrams (a) and (b) to the total production cross section. If there is no CP-violations in the sfermion sector, the diagrams (a) and (b) do not contribute to the total cross section. The contribution from the diagram (c) is always there, even in the CP-conserving case.

where $O_{i\alpha}$ is the orthogonal matrix which diagonalizes $M_{H_i}^2$, and the label of the mass eigenstates is determined in such a way that the masses $m_{h_1}, m_{h_2}, m_{h_3}$ satisfy $m_{h_1} \leq m_{h_2} \leq m_{h_3}$. It has been pointed out [3, 4, 5] that in some parameter regions the induced mixing can be large and play an important role in Higgs physics. However, in this letter, we focus on the regions of the SUSY parameter space in which the mixing with “CP-odd” Higgs boson is small and the second lightest Higgs boson $h_2$ is almost a “CP-odd” Higgs boson (typically $|O_{23}|^2 > 0.9$). Therefore, in the qualitative discussion below, we neglect the mixing effects and we still use the terminology “CP-odd” Higgs boson. However, in our numerical results to be shown below, we include the mixing effects, and we call the second lightest Higgs boson $h_2$ the “CP-odd” Higgs boson $A$.

Now we are ready to discuss the Higgs boson production cross section. For the lightest Higgs boson $h^0(=h_1)$, it is known that the radiatively induced CP-violation can significantly change the cross section of $gg \to h^0$ [4, 5]. In this letter we consider the production of the “CP-odd” Higgs boson $A$. This is motivated by the following reason.

If CP is not violated, the most important contribution to $gg \to A$ comes from the diagram (c) in Fig. 1. In the language of effective Lagrangian, this diagram is described by the CP-even operator,

$$\mathcal{L} = c_{i/b}^A G^{a\mu\nu} \tilde{G}_a^{\mu\nu}, \quad (5)$$

where the coefficient $c_{i/b}^A$ is obtained by integrating out the top and the bottom loops. $G_{\mu\nu}$ is the field strength tensor for gluon with $a$ being a color index ($a = 1, \ldots, 8$), and $\tilde{G}_a^{\mu\nu}$ is its dual, $\tilde{G}_a^{\mu\nu} \equiv \epsilon_{\mu\nu\rho\sigma} G^{a\rho\sigma}/2$. Note that the stop diagrams shown in Fig. 1 (a) and (b) do not contribute to $gg \to A$ simply because the couplings of the $\tilde{t}_i^* \tilde{t}_i A$ ($i = 1, 2$) interactions vanish due to the CP

$^4$Similar analyses had been done in Refs. [5, 7]. The authors of those articles performed the analyses for the parameter sets different from those discussed here.
Therefore, the leading order (LO) parton-level cross section of \( gg \to A \) in the CP-conserving (CPC) case, \( \sigma^{\text{LO}}(gg \to A)_{\text{CPC}} \), is given by the top/bottom contributions alone:

\[
\sigma^{\text{LO}}(gg \to A)_{\text{CPC}} \propto |c_{t/b}^A|^2
\]

On the other hand, in the CP-violating (CPV) case, the couplings \( \tilde{t}_i^* \tilde{t}_i A \) \((i = 1, 2)\) are not zero. Hence, the stop diagram contributes to the Higgs boson production \( gg \to A \). An important point is that the effective operator induced by the diagrams \((a)\) and \((b)\) of Fig. 1 is CP-odd,

\[
L = c_A^A \tilde{t} A G^{\mu\nu} G_{\mu\nu},
\]

where the coefficient \( c_A^A \) is determined from the stop loop contribution. Since the CP-properties of the operators in Eqs. (5) and (7) are opposite and the total cross section is a CP-even quantity, these two contributions do not interfere with each other in the total cross section. Hence, the LO total cross section \( \sigma^{\text{LO}}(gg \to A)_{\text{CPV}} \) in the CP-violating case is proportional to the sum of the squares of the contributions from these diagrams:

\[
\sigma^{\text{LO}}(gg \to A)_{\text{CPV}} \propto \left(|c_{t/b}^A|^2 + |c_{t/b}^A|^2\right).
\]

Note that in the case of the CP-even Higgs boson production, both the top/bottom and the stop/sbottom loops contribute to \( gg \to h \) even when CP is conserved, and generate the same effective operator,

\[
L = (c_{t/b}^h + c_{t/b}^h) h G^{\mu\nu} G_{\mu\nu},
\]

so that they could interfere with each other. Here, \( h \) represents the “CP-even” Higgs bosons, \( h^0 \) and \( H^0 \). When CP is violated, the induced operator is the same as the one in Eq. (9) (with a different coefficient) at the leading order, and the interference indeed can significantly affect the production cross section. Therefore the effect of CP-violation on the “CP-odd” Higgs boson production is quite different from that on the “CP-even” Higgs bosons, and the cross section of “CP-odd” Higgs boson in the CP-violating case is always enhanced by the stop contribution, compared to the one in the CP-conserving case. Thus, it is interesting to study the “CP-odd” Higgs boson production in the CP-violating case in order to see how large enhancement can be induced by the CP-violating interaction \( A_t \).

Our numerical results on the ratio \( \sigma^{\text{LO}}(gg \to A)_{\text{CPV}} / \sigma^{\text{LO}}(gg \to A)_{\text{CPC}} \) are shown as a function of \( |A_t| \) and \( \mu \) in Fig. 2. In the figure we have taken the sample parameter set as,

\[
m_A = 250 \text{ GeV}, \quad m_{\tilde{t}_1} = 120 \text{ GeV}, \quad \tan \beta = 6, \quad m_{\tilde{t}_L} = m_{\tilde{t}_R}, \quad A_t = i|A_t|, \quad \mu = |\mu|.
\]

In other words, this can be understood by the cancellation between diagrams of left- and right-handed stop loop contributions in the weak eigenstate basis.
Figure 2: The contour plot of the ratio of the LO parton-level cross sections in the CP-violating (CPV) case and the CP-conserving (CPC) case $\sigma^{\text{LO}}(gg \to A)_{\text{CPV}} / \sigma^{\text{LO}}(gg \to A)_{\text{CPC}}$ as a function of $|A_t|$ and $\mu$. The SUSY parameters are fixed as in Eq. (10).

where $m_{\tilde{t}_L}(m_{\tilde{t}_R})$ is the soft SUSY breaking mass for the left-handed (right-handed) stop. We see that the cross section can be enhanced by a factor of about 1000, compared to the case without CP violation. This huge enhancement can be understood in the following way. If we neglect the CP-violating mixing among Higgs bosons, the ratio $\sigma^{\text{LO}}(gg \to A)_{\text{CPV}} / \sigma^{\text{LO}}(gg \to A)_{\text{CPC}}$ can be written as

$$\frac{\sigma^{\text{LO}}(gg \to A)_{\text{CPV}}}{\sigma^{\text{LO}}(gg \to A)_{\text{CPC}}} = \left| \frac{c_A^2}{c_{t_1/b}} - 1 \right|^2, \tag{11}$$

for the same $m_A$ and $\tan \beta$ in both cases. Explicitly calculating the top/bottom loop and the stop loop diagrams, we obtain

$$\frac{\sigma^{\text{LO}}(gg \to A)_{\text{CPV}}}{\sigma^{\text{LO}}(gg \to A)_{\text{CPC}}} = \frac{m_t^2 |\mu A_t|^2 (1 + \cot^2 \beta)^2}{m_A^4 |A_t|^2 + |\mu \cot \beta|^2} \frac{|m_{t_2}^2 C_0(m_{t_2}^2, m_A^2) - m_{t_2}^2 C_0(m_{t_2}^2, m_A^2)|^2}{m_t^2 C_0(m_t^2, m_A^2) \cot \beta + m_b^2 C_0(m_b^2, m_A^2) \tan \beta} + 1, \tag{12}$$

where, for simplicity, we have assumed that $A_t$ is pure imaginary, $\mu$ is real, and the mixing between stops is maximal, i.e., $m_{\tilde{t}_L}^2 = m_{\tilde{t}_R}^2$ where $m_{\tilde{t}_L}^2$ and $m_{\tilde{t}_R}^2$ are $(\tilde{t}_L, \tilde{t}_L)$ and $(\tilde{t}_R, \tilde{t}_R)$ elements of the
stop mass matrix, respectively. The function $C_0$ is a one-loop function. For our particular case here, we define it as

$$C_0(m^2, m_A^2) = \frac{1}{i\pi^2} \int \frac{d^4q}{(q^2 - m^2)((q + p_1)^2 - m^2)((q + p_1 + p_2)^2 - m^2)},$$  \hspace{1cm} (13)$$

where $p_1^2 = p_2^2 = 0$ and $(p_1 + p_2)^2 = m_A^2$. If $m_A < 2m_{\tilde{t}_i}$, $|m_{\tilde{t}_i}^2 C_0(m^2, m_A^2) - m_{\tilde{t}_2}^2 C_0(m_{\tilde{t}_2}^2, m_A^2)|^2$ term in Eq. (12) is the square of a subtraction of a real number from another real number, where a GIM-like cancellation happens. When $2m_{\tilde{t}_i} < m_A < 2m_{\tilde{t}_2}$, which is satisfied for our sample parameters, the function $C_0(m_{\tilde{t}_i}^2, m_A^2)$ develops an imaginary part (when crossing the mass threshold for producing a light stop pair) and the factor is a subtraction of a real number from a complex number, which means the cancellation tends to be less severe. Since in our sample parameter set $m_A < 2m_{\tilde{t}_i}, C_0(m_{\tilde{t}_i}^2, m_A^2)$ in the denominator does not have an imaginary part, which also makes the ratio larger. (For moderate $\tan \beta$, the $C_0(m_{\tilde{t}_i}^2, m_A^2)$ term is not very important.) In addition, when $|A_t| \gg \mu \cot \beta$, the ratio in Eq. (12) behaves like $|\mu|^2$, as can be seen in Fig. 2. Therefore large $|A_t|$ and $\mu$ also induce large enhancement in the ratio.\footnote{A large $|A_t|$ may be dangerous because it could develop a color breaking VEV. Here, we have checked that the large part of our parameter space ($|A_t| \lesssim 950$ GeV) satisfies the condition $|A_t|^2 < 3(m_{\tilde{t}_k}^2 + m_{\tilde{t}_l}^2 + m_{\tilde{t}_h}^2 + |\mu|^2)$, which guarantees to avoid a color breaking VEV in a $D$-flat direction $|\tilde{t}_L| = |\tilde{t}_R| = |H^0_2|$ at the tree level potential.}

In Eq. (12), we have not included the effect from the mixing among the Higgs bosons although we have included that effect in the numerical results shown in Fig. 2. We have checked that the second lightest Higgs boson $h_2$ is almost a “CP-odd” Higgs boson for our sample parameter sets. In fact, $|O_{23}|^2 > 0.9$ for $2.3|A_t| - \mu \gtrsim 100$ GeV, and $|O_{23}|^2 > 0.7$ for $5|A_t| - \mu \gtrsim 350$ GeV in the range shown in the figure.

In Fig. 3 we also show the ratio $\sigma^{\text{LO}}(gg \to A)_{\text{CPV}}/\sigma^{\text{LO}}(gg \to A)_{\text{CP}}$ as a function of $m_{\tilde{t}_i}$ while fixing $m_A$ and $\tan \beta$. Here, we took the same parameter values as given in Eq. (10) except that we set $|A_t|$ and $\mu$ to be 700 GeV and 1 TeV, respectively. As can be seen from Fig. 3, as $m_{\tilde{t}_i}$ gets larger than $m_A/2$, the ratio rapidly drops off because of the GIM-like cancellation in the $|m_{\tilde{t}_i}^2 C_0(m_{\tilde{t}_i}^2, m_A^2) - m_{\tilde{t}_2}^2 C_0(m_{\tilde{t}_2}^2, m_A^2)|^2$ term in Eq. (12). However, due to the enhancement by large $|A_t|$ and $\mu$, the ratio can still be of $\mathcal{O}(100)$ if the stop mass is near the threshold $m_{\tilde{t}_i} \sim m_A/2$.

In Table 1 we summarize our results. In the table, we list the LO hadronic-level cross sections of the “CP-odd” Higgs boson $A$ via gluon fusion ($\sigma^{\text{LO}}(A)$) at the Tevatron ($\sqrt{s} = 1.96$ TeV) and the LHC ($\sqrt{s} = 14$ TeV), the branching ratios $\text{BR}(A \to \tilde{t}_{1}\tilde{\nu}_1)$, $\text{BR}(A \to \gamma\gamma)$, and $\text{BR}(A \to \tau\tau)$ in various cases discussed in this letter. The LO cross sections are calculated using the CTEQ6L parton distribution functions\footnote{The QCD corrections to the production cross section of the CP-odd Higgs boson are known up to and including the next-to-next-to-leading order (NNLO) in the CP-conserving MSSM. When we parametrize the hadron-level higher order (HO) production cross section $\sigma^{\text{HO}}(pp \to A)$ of the CP-odd Higgs boson using the LO hadron-level cross}, and the branching ratios of the “CP-odd” Higgs boson $A$ are computed using a publicly available code “CPsuperH”\footnote{We use a publicly available code “CPsuperH” to compute the branching ratios of the “CP-odd” Higgs boson $A$.}.
Figure 3: The ratio of the LO parton-level cross sections in the CP-violating (CPV) case and the CP-conserving (CPC) case, $\sigma^{\text{LO}}(gg \to A)_{\text{CPV}}/\sigma^{\text{LO}}(gg \to A)_{\text{CPC}}$, as a function of $m_{\tilde{t}_1}$. Here we took $\tan \beta = 6$, $m_A = 250$ GeV, $|A_t| = 700$ GeV, $\phi_{A_t} = \pi/2$ and $\mu = 1$ TeV. The LO hadron-level cross sections of the “CP-odd” Higgs boson via gluon fusion in the CP-conserving case are 0.8 fb and 0.2 pb at the Tevatron and the LHC, respectively, for $m_A = 250$ GeV and $\tan \beta = 6$.

|              | $\sigma^{\text{LO}}(A)$ | $\text{BR}(A \to \tilde{t}_1^* \tilde{t}_1^*)$ | $\text{BR}(A \to \gamma \gamma)$ | $\text{BR}(A \to \tau \tau)$ |
|--------------|--------------------------|-------------------------------------------|----------------------------------|-------------------------------|
| **Tevatron ($\sqrt{s} = 1.96$ TeV)** |                           |                                           |                                 |                               |
| CPC case     | 0.8 fb                   | 0                                         | $\sim 10^{-4}$                   | $\sim 0.05$                   |
| CPV case ($m_{\tilde{t}_1} = 120$ GeV) | $\sim 110 - 1200$ fb     | $\sim 1$                                 | $\mathcal{O}(10^{-5})$         | $\mathcal{O}(10^{-3})$       |
| **LHC ($\sqrt{s} = 14$ TeV)** |                           |                                           |                                 |                               |
| CPC case     | 0.2 pb                   | 0                                         | $\sim 10^{-4}$                   | $\sim 0.05$                   |
| CPV case ($m_{\tilde{t}_1} = 120$ GeV) | $\sim 30 - 300$ pb      | $\sim 1$                                 | $\mathcal{O}(10^{-5})$         | $\mathcal{O}(10^{-3})$       |
| CPV case ($m_{\tilde{t}_1} = 130$ GeV) | $\sim 10 - 90$ pb       | 0                                        | $\mathcal{O}(10^{-4})$         | $\mathcal{O}(10^{-1})$       |

Table 1: The leading order (LO) hadron-level cross sections of the “CP-odd” Higgs boson production via gluon fusion ($\sigma^{\text{LO}}(A)$) at the Tevatron ($\sqrt{s} = 1.96$ TeV) and the LHC ($\sqrt{s} = 14$ TeV) and the decay branching ratios of $A$ into $\tilde{t}_1^* \tilde{t}_1$, $\gamma \gamma$, $\tau \tau$ are shown in the CP-conserving (CPC) case and the CP-violating (CPV) case discussed in this letter. Here for the CPV case we took $m_A = 250$ GeV, $\tan \beta = 6$, $400$ GeV $< \mu < 1300$ GeV and $300$ GeV $< |A_t| < 1000$ GeV. For the calculation of the branching ratios in the CPC case we took $m_A = 250$ GeV, $m_{\tilde{t}_1} = 130$ GeV, $\tan \beta = 6$, $\mu = 700$ GeV and $A_t = 700$ GeV as an example.
The LO cross sections of the “CP-odd” Higgs boson via gluon fusion in the CP-conserving case are 0.8 fb and 0.2 pb at the Tevatron and the LHC, respectively, for \( m_A = 250 \) GeV and \( \tan \beta = 6 \). These cross sections are not large enough to allow us to discover the CP-odd Higgs boson at the 5\( \sigma \) level even at the LHC [13]. On the other hand, in the CP-violating case with \( m_A = 250 \) GeV, \( \tan \beta = 6 \), and \( m_{\tilde{t}_1} = 120 \) GeV, we can read from Fig. 2 that the LO cross section can be as large as \( 110 - 1200 \) fb at the Tevatron, and \( 30 - 300 \) pb at the LHC for \( 400 \text{ GeV} < \mu < 1300 \text{ GeV} \) and \( 300 \text{ GeV} < |A_t| < 1000 \text{ GeV} \). In the CP-violating case with \( m_A = 250 \) GeV and \( m_{\tilde{t}_1} = 120 \) GeV, the “CP-odd” Higgs boson can decay into a stop pair. Since the coupling of the “CP-odd” Higgs boson to stops is large, we found that the branching ratio \( \text{BR}(A \to \tilde{t}_1^* \tilde{t}_1) \) is almost one. Therefore, the stop pair production via the “CP-odd” Higgs boson production can be one of important signatures of the “CP-odd” Higgs boson in the CP-violating case. At the Tevatron, \( \sigma \times \text{BR}(A \to \tilde{t}_1^* \tilde{t}_1) \) can be \( \sim 110 - 1200 \) fb in the LO calculation. This stop production cross section via \( A \)-decay is smaller than the normal stop production cross section which is about 10 pb [14]. At the LHC, \( \sigma \times \text{BR}(A \to \tilde{t}_1^* \tilde{t}_1) \) can be as large as \( \sim 30 - 300 \) pb. Thus, it might be possible to detect the “CP-odd” Higgs boson \( A \) in the stop pair channel, although a detailed study for this process is needed. When \( m_A < 2m_{\tilde{t}_1} \), the “CP-odd” Higgs boson is not kinematically allowed to decay into a stop pair (and into any SUSY particle pairs if \( 2m_{\text{LSP}} > m_A \), where \( m_{\text{LSP}} \) is the lightest superparticle mass), though the production cross section of \( A \) can still be large. For example, in the case with \( m_A = 250 \) GeV and \( m_{\tilde{t}_1} = 130 \) GeV the LO cross section is about \( \sim 10 - 90 \) pb. As shown in Table 1 \( \sigma \times \text{BR}(A \to \gamma \gamma) \) can be \( \mathcal{O}(10) \) fb at the LHC in the leading order calculation. Comparing this result with the one analyzed in the ATLAS TDR [13], the LHC with an integrated luminosity of 100 fb\(^{-1}\) or more may be able to discover the “CP-odd” Higgs boson \( A \) via the diphoton mode. Also the \( A \to \tau \tau \) mode would be important, for its decay branching ratio is much larger than the diphoton mode. From Table 1 \( \sigma \times \text{BR}(A \to \tau \tau) \) can be \( \mathcal{O}(10) \) pb which is large enough to be detected at the LHC [13, 15]. Although the branching ratio of \( A \to \mu \mu \) is suppressed by a factor of \( (m_\mu/m_\tau)^2 \) compared to the branching ratio of \( A \to \tau \tau \), the \( A \to \mu \mu \) channel could also be useful for studying the “CP-odd” Higgs boson in some parameter regions. The branching ratio of \( A \to Zh \) is not large (at most 1-2 \% for our parameter sets). This can be understood by the fact that in the decoupling limit \( m_A \gg m_Z \), \( \text{BR}(A \to Zh) \) is zero in the CP-conserving case, and for the parameter sets studied in this letter in the CP-violating case, the “CP-odd” Higgs boson is heavy enough that the decoupling limit also holds. In summary, in the presence of CP-violation in the Higgs sector, the discovery potential for the “CP-odd” Higgs boson at the Tevatron and the LHC could be strongly modified.

Finally we would like to discuss some constraints on the CP-violating cases discussed in this letter. The first one is the lightest Higgs boson mass bound. Since in our CP-violating scenarios
the heavier Higgs bosons are heavy enough so that the coupling of the $ZZh$ interaction is not very different from the one in the SM, the lower limit on the SM Higgs boson mass $m_h > 114$ GeV would still apply. Using “CPsuperH” [12], we have checked the lightest Higgs boson mass limit is satisfied for $500$ GeV $< |A_t| < 900$ GeV. The second constraint is from electroweak precision measurement. Since the stop is light and its trilinear coupling $A_t$ is large, it induces non-decoupling effects on electroweak observables (such as the $W$ boson mass $M_W$, the effective weak mixing angle $\sin^2 \theta_{\text{eff}}$, and the leptonic decay width of the $Z$ boson $\Gamma_Z$, etc). We have estimated the stop-sbottom oblique corrections to $M_W$, $\sin^2 \theta_{\text{eff}}$, and $\Gamma_Z$ and found that a large left-right mixing of sbottoms with a light sbottom (close to the current experimental mass bound) is preferred in order to compensate the effects from the stop in the scenarios under consideration. The presence of light sbottom does not strongly modify the above results, though it could lead to interesting phenomenology at current and future colliders. The third one comes from EDMs of electron, neutron and mercury. When $A_t$ has a CP-violating phase and the stop and Higgs bosons are relatively light, two-loop diagrams through stops and Higgs boson mediation can induce large contributions to the EDMs [16]. The two-loop contributions to the electron and neutron EDMs have been given in Ref. [16]. From that we found those contributions are typically larger than the current experimental bounds in the parameter space discussed in this letter. Therefore, if these two-loop contributions are the only contributions to the EDMs, the possibilities we have discussed above would have been excluded. In order to avoid the EDM constraints, one can increase the stop and the “CP-odd” Higgs boson masses and still find the same effect discussed above. However, the production cross section of “CP-odd” Higgs boson will become smaller (for a larger mass), and hence it will be difficult to find the “CP-odd” Higgs boson even at the LHC. In the general MSSM, however, we cannot exclude a possibility that cancellations happen [17] among many contributions to the EDMs (not only two-loop contributions induced by stop and Higgs boson but also one-loop contributions and/or other two-loop contributions to the EDMs) since many other CP-phases in the first and second generation squarks and sleptons can contribute largely to the EDMs but very little to Higgs boson physics. Therefore, the searches for the large enhancement in the “CP-odd” Higgs boson production may provide an important information on the origin of CP-violation, independently of the EDM searches. Other possible constraints will come from B- and K-physics, which, however, depend strongly on the flavor structure in supersymmetry breaking. For example, our scenarios with a light stop will not contradict the $b \rightarrow s\gamma$ data if there is extra flavor violation in the squark sector. Therefore, we do not consider the constraints from B- and K-physics in our analysis.

In this letter, we have discussed the effect of CP-violating interaction in the stop sector on the “CP-odd” Higgs boson production via gluon fusion. We found that the cross section can be enhanced by a factor of about 1000 when $A_t$ and $\mu$ are large, especially when $m_A > 2m_{\tilde{t}_1}$. When the “CP-odd”

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8If the sbottom sector has an additional CP-violating phase, the light sbottom can play an important role in the “CP-odd” Higgs boson production when $\tan \beta$ is large.
Higgs boson can decay into a pair of stops, the stop pair production will be an interesting signature of CP-violation. When the “CP-odd” Higgs boson is not kinematically allowed to decay into any superparticles, the $A \rightarrow \gamma\gamma$ and $\tau\tau$ modes can be important discovery modes at the LHC. Although in order to avoid the EDM constraints one needs some unnatural fine tunings in the EDMs or needs to make the Higgs boson and the stop heavier, the searches for the “CP-odd” Higgs boson in the CP-violating case will give us an important information on the nature of CP-violation.

In the decoupling limit ($\alpha \sim \beta - \pi/2$), the interactions of the heavier “CP-even” Higgs boson $H^0$ with $\tilde{t}_L$ and $\tilde{t}_R$ take a similar form as those of the “CP-odd” Higgs boson $A$. Therefore, we expect that similar enhancement would also apply to $H^0$ production when $A_t$ and $\mu$ are large even in the case without CP violation in the stop sector [13].

We thank A. Belyaev, K. Hagiwara and P. Zerwas for useful comments and C.-R. Chen for collaboration at the early stages. This work was supported in part by the U. S. National Science Foundation under awards PHY-0244919 and PHY-0354838. C.-P. Y. is grateful for the hospitality of National Center for Theoretical Sciences in Taiwan, R. O. C., where part of this work was performed. K. T. gratefully acknowledges the hospitality of Osaka University, University of Tokyo, Tohoku University and Summer Institute 2005 at Fujiyoshida, where part of this work was done.

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