We propose a novel method for studying the production of anticentauro events in high energy heavy ion collisions utilizing Chebyshev expansion coefficients. These coefficients have proved to be very efficient in investigating the pattern of fluctuations in neutral pion fraction. For the anticentauro like events, the magnitude of first few coefficients is strongly enhanced (≈3 times) as compared to those of normal HIJING events. Various characteristics of Chebyshev coefficients are studied in detail and the probability of formation of exotic events is calculated from the simulated events.

1. Introduction

In ultra-relativistic heavy ion collisions the rapid expansion of collision debris leads to the production of vacuum states with anomalous chiral order parameters. As a result of this, interior vacuum straighten out and radiate away its pionic orientation. If the deflection of vacuum orientation is in the $\pi^0$ direction all the condensate radiation will be $\pi^0$. On the other side, if deflection is orthogonal to the $\pi^0$ direction all the emitted pions will be charged. The former case is called “Anti-centauro” behavior and the latter is “Centauro”. Both of these type of events are detected in the cosmic ray experiments. Various models have been developed to understand the characteristics of these events and numerous signatures for identification of such events are suggested.

The LHC facility at CERN will soon be operational and it would be interesting to search for anomalous events of centauro/anticentauro nature at LHC energies. Further, it is expected that at these high energies the production of exotic events will be enormous. So, it is useful to perform a theoretical simulation to identify the specific variables and mathematical techniques that may be informative in this study. The primary and widely accepted signal for observing these events is to look for anomalous fluctuations in neutral pion fraction (correspondingly, large fluctuations in the energy ratio, electromagnetic to hadronic). In this paper, we introduce a technique of Chebyshev coefficients to study the fluctuations in the
neutral pion fraction.

2. Characteristics of Chebyshev polynomials

Chebyshev polynomials are actively used in the field of mathematics for computing the derivative/integral of a non-singular function in the finite domain and in solving various differential equations [6]. In the High Energy Physics area, this method is applied in the track reconstruction procedure [7], vertex calculation and in studying the pattern of fluctuations [8]. The advantage of Chebyshev expansion technique is that it can be extended to any variable and it can be easily applied to any experimental data. In this paper, we have used this novel technique to study the fluctuations in the azimuthal distribution of neutral pions to the total number of pions produced in an event. The neutral pion fraction, \( f(\phi) \), can be expanded in terms of sum of Chebyshev polynomials as:

\[
f(\phi) = \frac{C_0}{2} + \sum_{k=1}^{N} C_k T_k(\phi) \quad (1)
\]

where \( T_k(\phi) \) is the Chebyshev polynomial of degree \( k \). The coefficients \( C_j, j=0, 1, 2, \cdots, N-1 \), of Chebyshev expansion are defined as

\[
C_j = \frac{2}{N} \sum_{k=1}^{N} f(\phi_k) T_{j-1}(\phi_k) \quad (2)
\]

\[
C_j = \frac{2}{N} \sum_{k=1}^{N} f[\cos(\frac{\pi(k - \frac{1}{2})}{N})] \cos(\frac{\pi(j - 1)(k - \frac{1}{2})}{N}) \quad (3)
\]

The coefficients of Chebyshev polynomial carry all the information on the fluctuation of \( f(\phi) \). The useful measures are provided by ranking the coefficients, \( C_k \), in order of the magnitude of the absolute values, i.e.,

\[
| C_0 | > | C_1 | > | C_2 | > | C_3 | \cdots \quad (4)
\]

The magnitude and range of these coefficients define the strength and level of fluctuations present in the data sample.

3. Simulation

The cosmic ray evidence on “anti-centauro events” serve as a great motivation for performing an exotic event search at Large Hadron Collider (LHC). Therefore, for any study, in this direction, it is essential to follow the hints and inferences provided by the cosmic ray experiments. We have taken the spectacular JACEE prototype [9] as a reference for generating the anticentauro events artificially. The standard Monte Carlo program, HIJING, is used to simulate minimum biased (0-20 fm) Pb-Pb collisions at LHC energies. Default values are taken for all the parameters. Two ensembles of 10 K events, (i) the normal HIJING events and (ii) the HIJING events...
with anticeptuauro type fluctuations embedded in it, are generated. The anticeptuauro event is modeled according to the $(1/2\sqrt{f})$ distribution. For the present simulation, a domain of size $60^\circ$ is injected into the normal HIJING event. This is established by flipping the charged pions into neutral pions unless they attain a value of the neutral pion fraction, $f \geq 0.9$.

### 4. Analysis

We calculate the neutral pion fraction, $f$, for the generated data sets (HIJING as well as “anticeptuauro like events”) in the $\Delta \eta - \Delta \phi$ phase space as,

$$f = \frac{n_{\pi^0}}{n_{\pi^0} + n_{\pi^\pm}}$$  \hspace{1cm} (5)

This calculation is performed on an event-by-event basis for various azimuthal bins, viz, $\Delta \phi = 2\pi/18, 2\pi/9, 2\pi/6, 2\pi/4, 2\pi/3, \pi$ and $2\pi$ and in the broad pseudorapidity region ($\eta = -10$ to $10$). The Chebyshev coefficients corresponding to the computed neutral pion fraction are determined according to the equation 3, as described in section II. Various characteristics of Chebyshev polynomials are investigated in view of their ability to pick up the events with anomalous fluctuations in the neutral pion fraction. The details are described below:

#### 4.1. First order coefficients

The first Chebyshev coefficient, $C_0$, for $\Delta \phi = 2\pi/18$, for normal events ($f = 0.33$) is plotted in Fig. 1. This distribution is Gaussian (dotted line) and it is chosen as ref-
Table 1. Mean value of the first coefficient, \( C_0 \), for the exotic and normal HIJING events.

| Azimuthal size | Mean \( C_0 \) Anticentauro like event | Mean \( C_0 \) HIJING event |
|----------------|----------------------------------------|-----------------------------|
| \( 2\pi/18 \)  | 124.8                                  | 45.0                        |
| \( 2\pi/9 \)   | 63.2                                   | 23.0                        |
| \( 2\pi/6 \)   | 42.7                                   | 15.4                        |
| \( 2\pi/4 \)   | 28.5                                   | 10.2                        |
| \( 2\pi/3 \)   | 21.4                                   | 7.7                         |
| \( \pi \)      | 14.3                                   | 5.2                         |
| \( 2\pi \)     | 7.1                                    | 2.6                         |

reference for studying the pattern of fluctuations in the generated anti-centauro type events. This is an ideal choice for comparison as there are no hidden fluctuations present in this sample and it purely represents the true nature of the system. In the same figure, we have plotted the first order coefficient (\( C_0 \)), for the anticentauro like events (solid line). The absolute value of the magnitude of this coefficient is large as compared to that of normal events. Similar behavior is observed for other order coefficients, i.e., for the exotic events the value of coefficients is significantly enhanced as compared to the normal events. Table 1. describes the detailed results of the variation of absolute magnitude of first order coefficient, \( C_0 \), with azimuth, for the top 5% central events. The mean value of the coefficient, \( C_0 \), for anticentauro like events is approximately thrice the mean value of normal HIJING events. It is observed that with increase in size of domain the magnitude of coefficients decreases. This behavior is seen for both HIJING and anticentauro like events but the magnitude of the coefficients for exotic events is much more than normal events for all the azimuthal bins.

4.2. Higher order coefficients

The coefficients of higher degree Chebyshev polynomial, \( C_k \) for \( 0 \leq k \leq N-1 \) contain all the information of fluctuations. In our study, we have calculated 100 coefficients, \( C_0 \) to \( C_{99} \). All these coefficients, \( C_k \) (\( k = 0, 99 \)) are pure Gaussian (for normal HIJING events), however, the absolute values of their magnitude decreases very rapidly. Fig. 2 shows the distribution of some higher order coefficients, \( C_{30}, C_{40}, C_{50} \) and \( C_{90} \). It is observed that higher order coefficients gradually attain the null value. Therefore, the Chebyshev expansion series can be truncated at some order \( m < N \) and the extreme higher order coefficients are neglected. It is important to note that the absolute value of the magnitude of higher order coefficients for the “anticentauro like events” is still much higher than the corresponding HIJING events as observed for the first order coefficient.
4.3. Correlation of Chebyshev coefficients

The magnitude and range of the Chebyshev coefficients is very useful in predicting whether or not the fluctuations in the neutral pion fraction are of non-statistical nature. A more critical criteria can be evolved by considering the two coefficients simultaneously. For this, it is necessary to study the correlation between various coefficients. Fig 3. shows the relationship between the two consecutive coefficients \( C_1 \) and \( C_2 \). It is seen that the coefficients are correlated i.e., with increase in \( C_1 \), the coefficient \( C_2 \) also increases. Similar behavior is observed for other coefficients. The corresponding magnitude of coefficients for HIJING events is very less.

4.4. Probability function: An estimate from simulation

A more quantitative way to analyze fluctuations is to estimate the production of anticentauro events from simulation. For this, we define a probability function \( P \), which is the fraction of events which have the magnitude of first order coefficient, \( C_0 \), greater than the threshold set by the sum of mean first order coefficient, obtained from HIJING events and five times the dispersion \( \langle C_0 \rangle + 5\sigma \). For all the anticentauro events, the absolute value of the magnitude of first order coefficient is much greater than the set threshold but for the HIJING events we do not observe any event which satisfies the above criteria. As an example, for the azimuthal bin, \( \Delta \phi = 2\pi/6 \), all the artificially generated anticentauro events have magnitude of the first order coefficient \( C_0 = 42.7 \) which is much higher than the threshold value.
The corresponding statistical probability obtained from HIJING events is

\[ P(C_0 > 17.21) = 0 \]  

(6)

For the semi-central events the probability is

\[ P(C_0 > 16.61) = 2.2 \times 10^{-3} \]  

(7)

Similar behavior is observed for the other azimuthal bins. Thus, for the generated exotic events, high value of neutral pion fraction in the \( \eta - \phi \) phase space is clearly exhibited by high magnitude of the Chebyshev coefficient whereas statistically the probability of formation of exotic event is either 0 or very small.

5. Summary

In this study, we have described a method of Chebyshev polynomials to study the fluctuations in the neutral pion fraction of artificially generated anticentauro events for Pb+Pb collisions at LHC energies. This method is proved to be very efficient to measure the strength of fluctuations. Our analysis shows that the magnitude of higher range first order coefficients for exotic events is approximately 3 times greater than those of normal events. This trend is seen for other higher range coefficients. The detailed analysis is carried out by estimating the probability of formation of exotic events from the normal events.

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