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Is market liquidity less resilient after the financial crisis? Evidence for US Treasuries

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ABSTRACT
Understanding market liquidity resilience, i.e. the capacity of liquidity to absorb shocks, of United States Treasuries is crucial from a financial stability standpoint. The conventional resilience measure has limitations due to the use of the liquidity level. We propose a new complementary approach to analyze resilience based on liquidity volatility. For this purpose, we focus on the link between returns volatility and liquidity volatility, which is a relatively unexplored field. We fit a bivariate conditional correlation (CC-) GARCH model for the 10-year bond returns and five liquidity indicators from January 2003 to June 2016 to analyze persistence and spillovers between these variables in a parsimonious way. We find that after the crisis, spillovers between liquidity volatility and returns volatility are higher, feedback loops are more likely and volatility persistence is lower, which is consistent with a lower resilience. Our results help to explain recent episodes of high volatility in this market.

1. Introduction
Market liquidity is the ease with which market participants can buy or sell securities without affecting their prices (Elliot, 2015) and plays a central role from a financial stability standpoint. First, impaired market liquidity makes securities trading more difficult by increasing funding costs. Also, a lack of liquidity is a well-known shock amplifier. In recent years, market liquidity has received growing attention given its apparent decline in some markets, such as the United States (US) Treasury debt market (Fender and Lewrick, 2015). Some authors link this likely liquidity deterioration to recent significant structural changes in several markets, including Treasuries (IMF, 2015). Indeed, some recent episodes of heightened volatility in the US Treasury markets such as the so-called “taper tantrum” in the second quarter of 2013, the October 2014 “flash crash”, or even the volatility spike linked to the coronavirus crisis in March 2020 have been associated with liquidity strains in this bond market. Those events demonstrated that liquidity strains also affect the most liquid markets, even under benign market conditions, and increased the fear of further volatility spikes (IMF, 2015; Nguyen et al., 2020).

A better understanding of the US Treasury debt market liquidity is crucial, since a severe deterioration of this market could pose a threat to financial stability. As it is the most liquid fixed income asset, liquidity strains would likely feed through to other markets. Besides, illiquidity bouts affect critical functions of US Treasury debt, traditionally perceived as a safe haven for investors, a source of high-quality collateral and a key instrument of monetary policy (Anderson et al., 2015; Thornton, 2018) and distort the functioning of the financial markets.

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1 Market liquidity differs from monetary liquidity, which is related to central banks’ monetary aggregates, or from funding liquidity, which is the ability to obtain funding for a risky asset (Brunnermeier and Pedersen, 2009).

2 During upturns, agents perceive that liquidity will remain abundant, which encourages them to gain exposure to apparent liquid assets. When market sentiment changes, liquidity conditions worsen and agents convert securities that were actually illiquid into cash (Nesvetailova, 2008; Houben et al., 2015).

3 The rise of electronic platforms and new trading techniques, the profound transformation of intermediation, together with some of the new regulations, could have discouraged market-making activities. Besides, unconventional monetary policies and asset purchases under quantitative easing (QE) programs also have a potential negative impact on market liquidity. See Bao et al. (2016), Bekaert et al. (2013) or Lagos et al. (2011) for further discussion.

4 Investors price in this high liquidity in the flight-to-liquidity premium of Treasury bond prices (Longstaff, 2004).
Our objective is to analyze if liquidity resilience, rather than the liquidity level, of the US Treasury debt has deteriorated after the great financial crisis (GFC) of 2007–2009. From a financial stability perspective, the main concern about liquidity is not the level itself but its resilience. The concept of liquidity resilience is linked to the ability of market liquidity to withstand shocks. In a non-resilient market, a state of high liquidity can suddenly transform into one of low liquidity in response to a shock (IMF, 2015). Therefore, the evolution of liquidity level can be different from that of liquidity resilience. Even apparently sound markets with ample liquidity such as US Treasuries can be prone to evaporation. Thus, although low liquidity is a leading indicator of low liquidity resilience during periods of financial stress, in normal times, higher liquidity levels can support the illusion of resilient liquidity and induce excessive risk taking (Clementi, 2001).

The conventional liquidity resilience measure is the speed of recovery of the liquidity level after one shock (Degryse et al., 2005; IMF, 2015). Although this approximation is intuitive and easy to fit by VAR-type models, it has limitations due to the fact that it relies on the liquidity level. To begin with, the market liquidity level is not easy to quantify. In fact, liquidity is an unobservable variable that embodies several heterogeneous characteristics (Sarr and Lybek, 2002). Accordingly, a large number of indicators have been proposed to monitor this multidimensionality. This plethora of indices sometimes provide different signals and do not allow for an unequivocal assessment of how liquidity conditions evolve (Broto and Lamas, 2016). The lack of empirical evidence on the existence of the recent liquidity strains based on the main liquidity level indicators of US Treasuries (Adrian et al., 2017a) illustrates this weakness. Besides, the standard resilience measure depends on the equilibrium liquidity level to be recovered after one shock. In the case of the Treasury bond market, recent structural changes have surely modified this fundamental level to be reached by liquidity (Joint Staff Report, 2015). In other words, the level that ensures that the market is resilient affects the standard measure based on the mean-reverting speed of the liquidity level.

Given these limitations of the liquidity level in the standard analysis of resilience, we propose a new complementary approach where liquidity volatility, rather than liquidity level, becomes the key variable to characterize resilience. To this end, we fit a bivariate model of the family of constant conditional correlation (CC-) GARCH models by Bollerslev (1990) as proposed by Nakatani (2010). We use this model to relate the volatility of five liquidity measures to returns volatility of the 10-year on-the-run US Treasury note. This model approach allows to analyze the ability of liquidity to absorb shocks through the study of the impact of market volatility on liquidity volatility (and vice versa).

The economic intuition behind our methodology is as follows. Volatile dynamics in liquidity lead to lower resilience. If liquidity is resilient, its volatility will scarcely react to shocks. This lack of reaction after shocks will lead to non-significant spillover coefficients, which is related to the fact that liquidity maintains certain capacity to absorb shocks. On the contrary, in a non-resilient scenario, liquidity volatility will deteriorate and spike hand in hand with the shock, which would create illiquidity bouts. Therefore, the empirical results will exhibit significant spillover coefficients. As the model simultaneously fits liquidity volatility, returns volatility and their spillovers, shocks can be either return shocks or liquidity shocks. Thus, in practice, when both spillover coefficients are significant the model allows to empirically characterize if there is a feedback-loop between both variables. Apart from the analysis of spillovers, the use of second order moments also permits us to reach conclusions on the volatility persistence of shocks. Thus, a low volatility persistence estimated by the model will be consistent with the occurrence of liquidity and financial spikes that tend to go back to normal soon.

Our new approach complements the conventional measure of liquidity resilience. Since our focus is on liquidity volatility rather than on its level, the limitations of this latter indicator do not affect our model. In our setting, spikes in liquidity volatility, resulting from past shocks in liquidity or in returns, simply expose investors to potential adverse liquidity environments, raising liquidity risks accordingly. It is worth clarifying that our bivariate GARCH models should not be interpreted as a new liquidity resilience measure, but as a tool to better understand resilience. For this purpose, our approach contributes to the literature on the link between returns’ volatility and liquidity volatility, instead of that of liquidity level, which is a relatively unexplored field.

Our results are consistent with a lower liquidity resilience of the 10-year note than before the GFC. Returns of the 10-year note and liquidity volatility measures are more volatile than before the crisis. Spillover estimates, which quantify if financial volatility is fed by liquidity volatility, and vice versa, have also increased in recent years. Thus, liquidity volatility reacts in a more acute way to shocks and this heightened liquidity volatility also impacts on financial volatility. We also show stronger feedback loop effects between liquidity volatility and returns volatility after GFC. On a more positive note, volatility recoveries are faster than before the crisis given its lower persistence. This means that, after the GFC, more shifting market conditions in the US Treasury market can be explained by a less resilient market liquidity with a lower capacity to absorb shocks, although other factors might be contributing to this trend. Our results help to explain the occurrence of recent episodes of heightened volatility in the markets, which have been disruptive but short-lived in nature, and accompanied by liquidity strains (IMF, 2015).

The rest of the paper is organized as follows. After the Introduction, Section 2 briefly reviews the literature on the link between liquidity and financial volatility. Section 3 describes the five US Treasuries liquidity indicators that we model in the resilience analysis. In Section 4 we present our empirical model to analyze the relationship between both variables. Section 5 summarizes the main results. Finally, section 6 concludes.

2. Market liquidity and financial volatility: a literature overview

The close ties between the level of market liquidity and financial volatility are well documented in the literature. There are two main theoretical explanations. On the one hand, according to the mixture of distribution hypothesis, price volatility and liquidity, proxied by trading volumes, should be positively correlated because of their dependence on the rate of information flow. This explanation justifies the positive correlation between price volatility and volumes found empirically by Clark (1973) and Tauchen and Pitts (1983). On the other hand, the sequential information arrival hypothesis of Copeland (1976) states that there is a positive correlation between trading volume and price volatility in a sequential manner, as once new information arrives at the markets, intermediate equilibriums occur prior to the final equilibrium, which leads to significant lead-lag relationships between the information flow and the returns.

Regarding the empirical literature, most studies that link returns volatility and liquidity levels fit GARCH-type models. The pioneering work by Lamoureux and Lastrapes (1990) analyze the volatility of 20 traded stocks through a univariate GARCH model where the daily trading volume is an exogenous regressor in the variance equation. Since then, univariate and multivariate GARCH models alike have been broadly used to study the link between market liquidity and volatility in different asset classes, while model specifications have been enriched over time—see Frank et al. (2008), Chuang et al. (2012), Ribeiro et al. (2017), Chundakkadan and Sasidharan (2019) or Nguyen et al. (2020),
among others. As in these empirical works on the link between returns and liquidity level, we also fit a GARCH-type model.

Although the literature on the link between the liquidity level and financial volatility is extensive, previous studies on the relationship between liquidity volatility and returns volatility are scarcer. However, these few works illustrate that liquidity volatility also matters for financial volatility. For instance, Akbas et al. (2011) conclude that there is a positive correlation between the volatility of liquidity and expected stock returns. If liquidity is very volatile and fluctuates within a wide range, investors may be exposed to a relatively higher probability of low liquidity at the time of selling the asset. Therefore, risk-averse investors will require a premium for holding stocks with high liquidity volatility. In this vein, Fall et al. (2019) argue that investors ask larger premiums for holding illiquid portfolios. On the contrary, Chordia et al. (2001) document a negative relationship between both variables using volumes as liquidity indicators. Additionally, Nguyen et al. (2020) propose a joint model of liquidity and volatility for US Treasuries. They develop a “liquidity-at-risk” measure and conclude that extreme illiquidity events are more correlated with extremely high volatility episodes than the usual liquidity level–volatility correlation. Likewise, Gong et al. (2018) document tail dependence of returns and bid-ask spreads in a stock index futures market. Finally, Jiang et al. (2011) explore unexpected returns in the US Treasury market and link these “jumps” to shocks in market liquidity, or variations in liquidity indicators. These papers use volumes or alternative liquidity measures in their specifications. We contribute to this little explored branch of the literature as we exploit liquidity volatility to analyze liquidity resilience.

3. Market liquidity measures

3.1. Selected market liquidity indicators

Next, we analyze the market liquidity level of US 10-year Treasuries. This analysis poses at least two difficulties. First, market liquidity is not an observable variable. Second, the concept of market liquidity entails several dimensions, so that various proxies are needed to capture all the relevant features. According to Sarr and Lybek (2002), a liquid market should have five characteristics, namely tightness, immediacy, efficiency, depth and breadth. Tightness refers to transaction costs, which are low in liquid markets, whereas immediacy characterizes those markets where trades are executed quickly and in an orderly manner. In an efficient market, prices rapidly adjust to new equilibrium levels once unbalances or new information arrive. Finally, depth is linked to the number of orders, while breadth allows orders to flow with a minimal impact on prices, even if they are large.

Given the heterogeneous characteristics behind the definition of market liquidity, a large number of indicators have been proposed to monitor these aspects (see Sarr and Lybek, 2002). We focus on five indicators, one per each feature of market liquidity, to cover the five dimensions. Three of them are based on prices, while the remaining two measures consist of quantiles or a combination of prices and quantities. Table 1 provides further details about their construction and interpretation.

We analyze tightness, immediacy and market efficiency with metrics based on prices. First, we proxy tightness with the estimation of the effective bid-ask spread proposed in the seminal work by Roll (1984), which follows this expression,

$$L_R = 2 \sqrt{-\text{Cov}(r_t, r_{t-1})}.$$  \hspace{1cm} (1)

7 Chundakkadan and Sasidharan (2019) demonstrate through EGARCH models that liquidity has even significant predictive power on the volatility of government bond yields in India.

8 To avoid confusion with our definition of resilience, which is linked with the ability of liquidity to absorb shocks, we use the term “efficiency” to denote the same concept that Sarr and Lybek (2002) call “resilience”.

| Market liquidity measures | Definition | Aspect of Liquidity | Source |
|---------------------------|------------|---------------------|--------|
| Roll (1984) | Estimation of the effective bid – ask spread based on the covariance of returns over two consecutive days. | Tightness | Bloomberg |
| Volatility | Market efficiency coefficient (MEC) | Efficiency | Bank of America Merrill Lynch |
| Volume | Average daily transactions in USD | Depth | Federal Reserve Bank of New York |
| Absorption spread | Market liquidity measure of US 10-year Treasuries. | Breadth | Bloomberg FINRA |
| Daily range | Variance of weekly returns to variance of daily returns. | Efficiency | Bank of America Merrill Lynch |
| Daily range | Variance are computed over sample period of three months. | Depth | Federal Reserve Bank of New York |
| Daily range | Average daily transactions, in USD | Breadth | Bloomberg FINRA |
| Daily | Absolute return to trading volume. | Efficiency | Bank of America Merrill Lynch |
where \( r_t \) is the percentage return on the US 10-year Treasuries, which is given by,

\[
r_t = 100 \times (\Delta \ln p_t),
\]

where \( p_t \) is the bond price in \( t \) and \( \Delta \) is the difference operator. The intuition behind \( L_R \) is based on the fact that in an efficient market the underlying value of an asset fluctuates randomly, whereas trading costs introduce negative serial dependence in market price changes. This indicator is more representative of real transaction costs in a market than other widely used indicators like the quoted bid-ask spread. The reason for this is that the measure by Roll (1984) is an estimation of the effective spread, defined as the execution price and the midpoint of bid and ask quotes. Nevertheless, the covariance of price changes is frequently positive, so that \( L_R \) might become a noisy estimator even in relative large samples (Harris, 1990). Subsequently, several authors have proposed alternative measures that outperform \( L_R \) estimation of transaction costs, such as the high and low prices spread estimator of Corwin and Schultz (2012) or the indicator by Abdi and Ranaldo (2017) calculated from daily close, high and low prices.\(^9\) After several proofs with Corwin and Schultz (2012) and Abdi and Ranaldo (2017) measures for the trading costs, we discard them due to convergence problems. Besides, the measure by Abdi and Ranaldo (2017) provides more accurate estimates of trading costs than Roll (1984) for the less liquid securities. For highly liquid assets, such as US Treasuries, the new measure barely improves the Roll (1984) estimator.

Second, we use the daily range, denoted as \( L_{LR} \), as a measure of immediacy. We calculate this as the difference between the highest and lowest price in a day. This a dispersion measure that contains information on market liquidity. Thus, the daily range widens when bonds are thinly traded in the markets and orders execution is poor, that is, when liquidity is lower. The use of the daily range or similar measures as liquidity proxies is not new in the literature (see, for instance Pu (2009) or Schestag et al. (2016)).\(^10\) Third, we analyze market efficiency with the Market Efficiency Coefficient (MEC), \( L_{MEC} \). The MEC is a ratio between the variances of two returns with different time spans, that is,

\[
L_{MEC} = \frac{\text{Var}(R_t)}{(\text{Var}(r_t) \times 5)},
\]

where \( R_t \) are weekly returns, whereas \( r_t \) are daily returns. This indicator is close to one in efficient, liquid markets, whereas substantial departures from unity reveal lower liquidity. The intuition behind \( L_{MEC} \) is that price movements are more continuous in liquid markets. That is to say, if new information affects equilibrium prices, the transitory changes that price are minimal in efficient markets. See Sarr and Lybek (2002) or Gabrielsen et al. (2011) for further details on the interpretation of the MEC coefficient.

With regard to measures that require quantities, we use the trading volume, \( V_t \), to analyze market depth. The volume is the amount of securities traded per period (in US dollars). Finally, we study breadth through the indicator proposed by Amihud (2002), given by,

\[
L_A = \frac{r_t}{V_t},
\]

which is the ratio of the absolute return in the market to the trading volume, \( V_t \). Sudden spikes in \( L_A \) suggest that the market is not able to suitably absorb a given amount of trading orders, so that market breadth will be impaired. By construction, this measure is more volatile than the trading volume, \( V_t \).\(^11\)

\(^9\) See Corwin and Schultz (2012) and Schestag et al. (2016) for a comparison of trading costs estimates.

\(^10\) Pu (2009) constructs a ratio between the daily range and the trading volume to proxy liquidity and Schestag et al. (2016) analyze liquidity in the US corporate market by means of the interquartile range.

\(^11\) We drop observations of the last week of each year for \( V_t \) and \( L_A \) since trading activity is usually abnormally low in these periods.

3.2. The data

Fig. 1 represents the five liquidity measures, together with the daily returns of the 10-year on-the-run Treasury note. Due to data constraints, \( L_R, L_{LR}, \) and \( L_{MEC} \) are daily, whereas volumes and the Amihud (2002) ratio are weekly. Price data are provided by Bloomberg Generic dataset, which is Bloomberg’s market consensus price for government bonds. We obtain volumes from the Primary Dealers’ release of the Federal Reserve Bank of New York (FRBNY). The release compiles volume data for US Treasuries with a remaining term to final maturity from seven to eleven years, so that the trading volumes include not only the 10-year note but also other securities.\(^12\) However, our measure is representative, as the bulk of trading activity is carried out in the 10-year on-the-run note.\(^13\) With regard to the Amihud (2002) ratio, \( L_A \), we use trading volumes in the denominator, whereas we observe absolute returns from an index by Bank of America that summarizes prices of all outstanding Treasury issues within a similar maturity sector for the numerator.\(^14\) Table 1 summarizes data sources.

The sample period runs from January 2003 to end-June 2016, so that the sample size is 3465 trading days and 692 weeks for daily and weekly indicators, respectively. We also focus on the pre-crisis period, from January 2003 to May 2007, and the post-crisis period, from April 2009 to June 2016, to gain an insight into liquidity dynamics in periods of calm. We date the beginning of the crisis in June 2007 coinciding with the first evidence on problematic subprime mortgages and mortgage-related securities, more specifically, when two Bear Stearns hedge funds with large holdings of subprime mortgages run large losses.\(^15\) We consider the end of the crisis in April 2009, after the QE program was launched and expanded in March 2009, which also coincided with an agreement of the G20 on a global stimulus package, which helped to stabilize the markets.

According to Fig. 1, all indices but the MEC exhibit a severe deterioration of market liquidity during the GFC, around 2008:Q3 and 2009:Q1. Since then, market liquidity has recovered, but liquidity indicators fluctuate in a different way than in the pre-crisis period. By construction, price-based measures, i.e., \( L_R, L_{LR} \), and \( L_{MEC} \), are more erratic and exhibit more volatility spikes than volume-based indicators. These bouts of illiquidity are more frequent after the financial crisis. Volumes rapidly recovered after the GFC, but since mid-2013 they have steadily declined. All in all, in the post-crisis period, the five liquidity indices have fluctuated in a non-homogeneous way, so that it is difficult to clearly characterize a market liquidity recovery through Fig. 1.\(^16\) This problem is a result of the inherently diverse nature of market liquidity

\(^12\) We calculate the covariance of \( L_R \) and the variances of \( L_{MEC} \) over a moving window of 66 days.

\(^13\) From March 2013 to September 2015 the average volume of the 10-year on-the-run Treasury note was 84.7% of the total trading volume of US Treasuries (Brain et al., 2018).

\(^14\) This index, the BofA Merrill Lynch 7–10 Year US Treasury Index, also excludes TIPS and comprises securities with a remaining term to final maturity greater than or equal to seven years and less than ten years.

\(^15\) Namely, in June 2007 S&P and Moody’s downgraded over 100 bonds backed by second-lien subprime mortgages.

\(^16\) Our liquidity level description is in line with Adrian et al. (2017a), who also offer a non-conclusive picture of liquidity conditions in US Treasury securities when comparing market liquidity before and after the financial crisis. However, contrary to our evidence, they do not identify a liquidity decline in the aftermath of the crisis. Our analysis is not fully comparable with Adrian et al. (2017a), who focus on the on-the-run 2-, 5- and 10-year Treasury notes.
Fig. 1. Daily returns of the US 10-year Treasury note and five market liquidity measures.
To gain greater insight into liquidity dynamics after the crisis, Table 2 reports the mean and standard deviation of the five indicators in the pre-crisis and post-crisis periods. The average transaction costs and daily range are slightly higher in the post-crisis period, which suggests that market liquidity conditions could have worsened. In this same vein, average daily trading volume dropped below pre-crisis levels, and the Amihud ratio, $L_A$, stabilized at higher levels. The MEC ratio, however, shows more benign liquidity conditions in the most recent period, as it stands closer to the unity. All averages during the pre-crisis period and the post-crisis period are significantly different, as shown by the t-test for equal means. Standard deviations of the five indicators are significantly higher post-crisis than pre-crisis. Finally, Table 3 shows the pairwise correlations of the five liquidity indicators. Since correlations are generally low, our measures illustrate the multidimensional nature of the concept of market liquidity.

4. Market liquidity resilience: empirical model

To study liquidity resilience in the US Treasury market we propose a bivariate model to analyze simultaneously both liquidity volatility and returns volatility. Liquidity volatility serves as a proxy for the uncertainty of the liquidity level. We also incorporate US Treasuries returns volatility in the model to analyze its interlinkages with liquidity volatility. If liquidity is resilient to stressed conditions, it will scarcely respond to shocks and more volatile markets will barely impact liquidity volatility. On the contrary, in a non-liquid scenario, higher market volatility will affect liquidity volatility, which could also react in a feedback loop.

Therefore, we model two dependent variables,

$$Y_t = \begin{pmatrix} \Delta r_t \\ \Delta L_t \end{pmatrix},$$

(5)

where $r_t$ are daily returns and $\Delta L_t$ is the first difference of each of the five liquidity indicators, calculated on a daily basis for $L_R$, $L_{DR}$ and $L_{MGE}$, and on a weekly basis for $V$ and $L_A$.\(^{16}\) Treasuries’ returns of the models for $V$ and $L_A$ are also weekly, $\sigma_{rt}$.\(^{19}\) Table 4 summarizes the main statistics for these variables. All series are asymmetric and/or have excess kurtosis. Box-Pierce Q-statistics for higher order serial correlation suggest the presence of conditional heteroskedasticity in both returns and first differences of liquidity indicators, which evidences the suitability of a GARCH-type model.

Specifically, our baseline model is a bivariate ECCC-GARCH (Extended Constant Conditional Correlation GARCH) model as proposed by Nakatani (2010) and Jeantheau (1998), which belongs to the family of conditional correlation (CC-) GARCH models by Bollerslev (1990). The main advantage of the specification by Nakatani (2010) for our resilience analysis is that it allows to analyze volatility transmission among the variables of the model. That is to say, whereas in a standard CCC-GARCH specification the conditional volatility would be modeled as a combination of its squared innovations and volatility, Nakatani (2010) considers that conditional volatility also depends on those squared innovations and volatilities of other equations, while keeping the conditional correlation structure constant. Therefore, our proposed bivariate model enables to study the simultaneous impact of higher liquidity volatility on financial volatility and that of financial volatility on liquidity volatility. This approach is more realistic than previous models where the level of market liquidity affects financial volatility exogenously (see, for instance, Lamoureux and Lastrapes, 1990).

We choose the CCC-GARCH, which forces conditional correlation to be constant over time, instead of DCC-GARCH (Dynamic Conditional Correlation GARCH) as in our case there is no gain in the use of a DCC model. The DCC-GARCH model is estimated in two steps. In the first step parameters in the variance equations are estimated, while in the second step dynamic correlations are fitted. As our main focus is on volatility spillovers, which are estimated from the variance equations in the first stage, we rule out a DCC-GARCH specification. Besides, as shown in Table 5, tests based on Engle and Sheppard (2001) suggest that our bivariate dataset has constant conditional correlations rather than time-varying ones since the null hypothesis of constant conditional correlation cannot be rejected for almost all liquidity indicators and sample periods.

As usual in the empirical finance literature, we first prewhiten the dependent variables with AR and VAR filters,\(^{20}\)

$$Y_t = A_0 + A_1 Y_{t-1} + \cdots + A_4 Y_{t-4} + \epsilon_t,$$

(6)

where $\epsilon_t = [\epsilon_{1t}, \epsilon_{2t}]'$ and $\epsilon_t \sim N(0, \Omega)$. Conditional variance equations follow this expression,

$$\epsilon_t \mid L_{t-1} \sim N(0, D_t R D_t)$$

(7)

$$D_t = \text{diag}(\sqrt{h_t})$$

$$h_t = C + A_1 \epsilon_{1t-1} + B h_{t-1}$$

(8)

where $R$ is the constant conditional correlation matrix, $D_t$ is a diagonal matrix with the conditional variance of returns, $h_{t1}$, and liquidity, $h_{t21}$. $C$ is a $(2 \times 1)$ vector, $A$ and $B$ are $(2 \times 2)$ matrices, and $\epsilon_{t}^{(2)} = \begin{pmatrix} \epsilon_{1t}^{2t} \\ \epsilon_{2t}^{2t} \end{pmatrix}'.$

If both $A$ and $B$ are diagonal, this ECCC-GARCH model collapses into the CCC-GARCH model of Bollerslev (1990). Specifically, our conditional variances follow the proposal by Nakatani (2010) and are given by,

$$h_{t1} = c_1 + a_{11} \epsilon_{1t-1}^2 + a_{12} \epsilon_{2t-1}^2 + b_{11} h_{t-11} + b_{12} h_{t2t-1}\)$$

$$h_{t21} = c_2 + a_{22} \epsilon_{2t-1}^2 + a_{21} \epsilon_{1t-1}^2 + b_{22} h_{t-11} + b_{21} h_{t2t-1}\)$$

where, apart from the determinants of the conditional variance of univariate GARCH models, this specification enables us to describe the impact of liquidity volatility on returns volatility through coefficients $a_{12}$ and $b_{12}$, and that of returns volatility on liquidity volatility, through coefficients $a_{21}$ and $b_{12}$. Spillovers to financial volatility, $h_{t11}$, can be the result of past shocks of both liquidity measures, $a_{12}$, and/or of higher liquidity volatility in $(t-1), h_{t12}$. In the same vein, shocks in returns and returns volatility might influence liquidity volatility through $h_{t21}$.

That is, since the model simultaneously fits returns volatility and liquidity volatility and their spillovers, shocks can be either return shocks or liquidity shocks.

The estimated spillover coefficients serve us to infer conclusions on liquidity resilience. If liquidity is resilient, it will withstand shocks and spillovers to liquidity volatility will not be significant. On the contrary, if liquidity is non-resilient, liquidity volatility will react and spike in response to shocks and spillover coefficients will be significant. These spillover coefficients can also characterize feedback loops between both

\(^{17}\) There are synthetic indicators that summarize the information of a set of liquidity measures, which enables to disentangle the conflicting signals between indicators (see, for instance, Broto and Lamas (2016), who propose a synthetic liquidity index for government and corporate US fixed income based on the methodology for financial stress indicators).

\(^{18}\) We take the first difference of the five liquidity measures as they are non-stationary. Dickey-Fuller tests for null hypothesis of unit root in the liquidity levels are available upon request.

\(^{19}\) Weekly returns, $\Delta r_t^w$, are calculated as $100 \times (\Delta \ln p_{t}^w)$, where $p_{t}^w$ is a weekly series of Wednesday’s bond prices, which is the day of the week when the Federal Reserve releases the data on volumes.

\(^{20}\) We use Granger-causality tests to choose between AR or VAR filters. We use VAR filters when the tests identify feedback between liquidity measures and returns. Otherwise, we employ AR filters. The number of lags is determined using the Akaike information criterion. We have performed all the prewhitening process with E-Views.
variables, as a return shock could translate into more uncertain liquidity conditions (and vice versa) simultaneously. If present, these feedback effects are consistent with a lower liquidity resilience, as they will cause further difficulties for liquidity to absorb shocks. The model disentangles whether these interlinkages between volatility and liquidity volatility have grown after the GFC as well as the causality direction.

As usual in GARCH-type models, the estimates of $\alpha_{11} + b_{11}$ and $\alpha_{22} + b_{22}$ provide a measure of volatility persistence, so that if this sum is close to one, high volatility tends to be followed by high volatility. In our analysis of resilience, a lower persistence after the crisis as proxied by this sum of coefficients has a positive interpretation as this fact is consistent with the occurrence of liquidity and financial spikes that tend to go back to normal soon.

We estimate all parameters simultaneously in $\mathbb{R}$ with the ccgarch package of Nakatani (2014) by maximizing the log-likelihood function of this model. As initial values in the estimation process we choose univariate GARCH estimates for diagonal elements of $\textbf{A}$ and $\textbf{B}$ and values slightly above zero for the non-diagonal elements. The modulus of the largest eigenvalue of $\textbf{A} + \textbf{B}$ matrix is constrained to be strictly less than one to ensure positive and stationary variances (Nakatani, 2010). \footnote{Following the notation in Nakatani (2010), $\lambda(\Gamma_C) < 1$ denotes the stationarity condition, whereas the fourth order moment condition is fulfilled if $\lambda(\Gamma^4_{CcG}) < 1$. See Nakatani (2010) for further details.}

### Table 2
Descriptive statistics of market liquidity indicators.

|          | $l_R$ | $l_{DR}$ | $l_{MRC}$ | $V$ | $l_A$ |
|----------|-------|----------|-----------|-----|-------|
| Mean     | 0.281 | 0.553    | 0.828     | 11.591 | 2.967 |
| Pre-crisis | 0.244 | 0.487    | 0.812     | 11.621 | 2.287 |
| Crisis   | 0.399 | 0.795    | 0.729     | 11.699 | 4.537 |
| Post-crisis | 0.274 | 0.532    | 0.863     | 11.567 | 2.990 |
| t-test   | -2.889*** | 3.932*** | -2.441**  | 2.336** | -3.692*** |
| (0.004)  | (0.000) | (0.015)  | (0.020)  | (0.000) |
| SD       | 0.305 | 0.343    | 0.533     | 0.281  | 3.085 |
| Pre-crisis | 0.246 | 0.287    | 0.482     | 0.230  | 1.858 |
| Crisis   | 0.432 | 0.477    | 0.404     | 0.312  | 5.889 |
| Post-crisis | 0.290 | 0.306    | 0.585     | 0.300  | 2.475 |
| F-test   | 0.714*** | 0.881**  | 0.678***  | 0.587*** | 0.563*** |
| (0.000)  | (0.018) | (0.000)  | (0.000)  | (0.000) |
| Observations | 3465 | 3465 | 3465 | 692 | 692 |
| Pre-crisis | 1132 | 1132 | 1132 | 227 | 227 |
| Crisis   | 471  | 471      | 471       | 93    | 93   |
| Post-crisis | 1862 | 1862 | 1862 | 372 | 372 |

Notes: $l_R$ is the transaction costs indicator by Roll (1984), $l_{DR}$ is the daily range, $l_{MRC}$ denotes the market efficiency coefficient, $V$ is trading volume expressed in logarithms and $l_A$ stands for the Amihud (2002) ratio. The pre-crisis period runs from January 2003 to May 2007, whereas the post-crisis period covers the time from April 2009 to June 2016. t-test and F-test denote a test for the equality of means and variances, respectively, in the pre-crisis and in the post-crisis period. $p$-values in parentheses. *** and * refer to significance at 1%, 5% and 10% level.

### Table 3
Correlation matrix of five market liquidity indicators.

| Total sample | $l_R$ | $l_{DR}$ | $l_{MRC}$ | $V$ | $l_A$ | $l_R$ | $l_{DR}$ | $l_{MRC}$ | $V$ | $l_A$ |
|--------------|-------|----------|-----------|-----|-------|-------|----------|-----------|-----|-------|
| $l_R$        | 1     |          |           |     |       | 1     |          |           |     |       |
| $l_{DR}$     | 0.211*| 1        |           |     |       |       |          |           |     |       |
| $l_{MRC}$    | -0.306* | -0.601  | 1         |     |       |       |          |           |     |       |
| $V$          | 0.012 | 0.103*   | 0.012     | 1   |       | 0.052 | 0.149*   | 0.014     | 1   |       |
| $l_A$        | 0.134* | 0.627*   | -0.030    | -0.260* | 1 | 0.073 | 0.477* | 0.042     | -0.223* | 1 |

| Pre-crisis period | $l_R$ | $l_{DR}$ | $l_{MRC}$ | $V$ | $l_A$ | $l_R$ | $l_{DR}$ | $l_{MRC}$ | $V$ | $l_A$ |
|-------------------|-------|----------|-----------|-----|-------|-------|----------|-----------|-----|-------|
| $l_R$             | 1     |          |           |     |       | 1     |          |           |     |       |
| $l_{DR}$          | 0.174*| 1        |           |     |       |       |          |           |     |       |
| $l_{MRC}$         | -0.148* | -0.087  | 1         |     |       |       |          |           |     |       |
| $V$               | 0.052 | 0.149*   | 0.014     | 1   |       | 0.073 | 0.477* | 0.042     | -0.223* | 1 |
| $l_A$             | 0.073 | 0.477*   | 0.042     | -0.223* | 1 |

| Crisis period | $l_R$ | $l_{DR}$ | $l_{MRC}$ | $V$ | $l_A$ | $l_R$ | $l_{DR}$ | $l_{MRC}$ | $V$ | $l_A$ |
|---------------|-------|----------|-----------|-----|-------|-------|----------|-----------|-----|-------|
| $l_R$         | 1     |          |           |     |       | 1     |          |           |     |       |
| $l_{DR}$      | 0.234*| 1        |           |     |       |       |          |           |     |       |
| $l_{MRC}$     | -0.540* | -0.020  | 1         |     |       |       |          |           |     |       |
| $V$           | -0.040 | -0.136  | -0.299*   | 1   |       | 0.016 | 0.239* | 0.108     | 1   |       |
| $l_A$         | 0.199 | 0.763*   | 0.019     | -0.411* | 1 |

| Post-crisis period | $l_R$ | $l_{DR}$ | $l_{MRC}$ | $V$ | $l_A$ | $l_R$ | $l_{DR}$ | $l_{MRC}$ | $V$ | $l_A$ |
|--------------------|-------|----------|-----------|-----|-------|-------|----------|-----------|-----|-------|
| $l_R$              | 1     |          |           |     |       | 1     |          |           |     |       |
| $l_{DR}$           | 0.112*| 1        |           |     |       |       |          |           |     |       |
| $l_{MRC}$          | -0.344* | -0.009  | 1         |     |       |       |          |           |     |       |
| $V$                | 0.016 | 0.239*   | 0.108     | 1   |       | 0.038 | 0.463* | 0.067     | -0.224* | 1 |
| $l_A$              | 0.038 | 0.463*   | 0.067     | -0.224* | 1 |

Notes: $l_R$ is the transaction costs indicator by Roll (1984), $l_{DR}$ is the daily range, $l_{MRC}$ denotes the market efficiency coefficient, $V$ is trading volume expressed in logarithms and $l_A$ stands for the Amihud (2002) ratio. The pre-crisis period runs from January 2003 to May 2007, whereas the post-crisis period covers the time from April 2009 to June 2016. * refer to significant pair-wise correlations at 5% level.
Table 4
Descriptive statistics of US 10-year bonds’ returns and market liquidity indicators.

|                       | Total sample | Pre-crisis period | Post-crisis period |
|-----------------------|--------------|-------------------|--------------------|
|                       |              |                   |                    |
| **r_t**               | 0.014        | 0.016             | 0.015              |
| **r_{t+1}**           | 0.009        | 0.010             | 0.011              |
| **Δr_t**              | -0.009       | 0.006             | -0.008             |
| **Δt_{LR}**           | 0.008        | 0.005             | 0.006              |
| **Δt_{LMEC}**         | 0.009        | 0.007             | 0.008              |
| **Δt_{Δ}**            | 0.009        | 0.007             | 0.008              |
| **Δt_{V}**            | 0.009        | 0.007             | 0.008              |
| **Δt_{Δt}**           | 0.009        | 0.007             | 0.008              |
| **Mean**              | 0.005        | 0.010             | 0.011              |
| **SD**                | 0.889        | 0.107             | 0.216              |
| **Maximum**           | 3.743        | 1.760             | 3.844              |
| **Minimum**           | -2.176       | -3.321            | -0.928             |
| **Skewness**          | 0.112**      | 0.291***          | 0.093              |
| **Kurtosis**          | 3.378**      | 3.378**           | 3.378**            |
| **Observations**      | 471          | 91                | 471                |
| **Q(10)**             | 394.790***   | 201.650***        | 756.220***         |

Notes: \( r_t \) and \( r_{t+1} \) are the daily and weekly 10-year bond returns, respectively. \( L_R \) is the transaction costs indicator by Roll (1984), \( L_{LR} \) is the daily range, \( L_{LMEC} \) denotes the market efficiency coefficient, \( V \) is trading volume expressed in logarithms and \( L_{Δ} \) stands for the Amihud (2002) ratio. All liquidity indicators are expressed in first differences, \( Δ \). \( Q(10) \) is the Ljung-Box Q-statistic (with 10 lags) and \( Q^2(10) \) is the Ljung-Box Q-statistic (with 10 lags) for the squared returns. ***, **, and * refer to significance at 1%, 5% and 10% level.
Table 5
Tests of constant conditional correlation (Engle and Sheppard, 2001).

|               | Full sample | Pre-crisis period | Post-crisis period |
|---------------|-------------|-------------------|--------------------|
| $\Delta I_{10}$ | 5.734*      | 0.000             | 0.000              |
| $\Delta I_{20}$ | 1.657       | (0.437)           | 0.455              |
| $\Delta I_{M}$  | 0.010       | (0.995)           | 0.064              |
| $\Delta V$     | 0.065       | (0.968)           | 0.005              |
| $\Delta I_{A}$  | 0.519       | (0.772)           | 1.895              |

Notes: Engle and Sheppard (2001) test for the null hypothesis of constant correlation in a DCC-GARCH model for 10-year US Treasury returns (log difference of prices) and the first difference, denoted by $\Delta$, of each of the five liquidity indicators (namely, $I_{10}$ is the transaction costs indicator by Roll (1984), $I_{20}$ is the daily range, $I_{M}$ the Market efficiency coefficient, $V$ is trading volume expressed in logarithms and $I_{A}$ stands for the ratio proposed by Amihud (2002)). $p$-values in parentheses. * refers to significance at 10% level.

Table 6
Estimates of the model for the pre-crisis period, from January 2003 to May 2007.

|               | $\Delta I_{10}$ | $\Delta I_{20}$ | $\Delta I_{M}$  | $\Delta V$ | $\Delta I_{A}$ |
|---------------|-----------------|-----------------|-----------------|------------|----------------|
| $c_1$         | 0.000           | 0.000           | 0.009           | 0.000      | 0.000          |
| $c_2$         | 0.001           | 0.000           | 0.086***        | 0.006      | 0.062**        |
| $a_{11}$      | 0.001           | 0.000           | 0.004           | 0.000      | 0.247**        |
| $a_{22}$      | 0.180***        | 0.000           | 0.005           | 0.104***   | 0.001          |
| $a_{12}$      | 0.011           | 0.013           | 0.001           | 0.017      | 0.011          |
| $a_{21}$      | 0.009           | 0.029           | 0.003           | 0.000      | 0.247**        |
| $b_{11}$      | 0.950***        | 0.980***        | 0.752***        | 0.901***   | 0.932***       |
| $b_{22}$      | 0.273**         | 0.753***        | 0.002           | 0.468      | 0.928***       |
| $b_{12}$      | 0.510***        | 0.024           | 0.255           | 0.961***   | 0.000          |
| $b_{21}$      | 0.008           | 0.056**         | 0.005           | 0.013      | 0.001          |
| $LogI$        | -757.872        | -479.1963       | -801.2718       | -143.8205  | -648.3826      |
| $T$           | 1131            | 1131            | 1131            | 226        | 226            |
| $Q(10)$       | 3.745           | 4.124           | 3.279           | 7.992      | 4.679          |
| $Q(20)$       | 5.352           | 5.517           | 3.125           | 3.675      | 9.739          |
| $Q(30)$       | 6.038           | 3.955           | 10.823          | 4.897      | 5.007          |
| $Q(40)$       | 1.689           | 7.274           | 0.339           | 12.752     | 13.171         |
| $\hat{\alpha}(I_{10})$ | 0.995 | 0.999 | 0.827 | 0.976 | 0.984 |
| $\hat{\alpha}(I_{M})$ | 0.993 | 0.998 | 0.694 | 0.956 | 0.968 |

Estimation results of the conditional variances of a bivariate ECC-GARCH model (Nakatani, 2010):

\[ h_{11} = c_1 + a_{11} \varepsilon_{t-1}^2 + a_{12} \varepsilon_{2t-1}^2 + b_{11} I_{10,t-1} + b_{12} I_{20,t-1} \]
\[ h_{22} = c_2 + a_{22} \varepsilon_{2t-1}^2 + a_{21} \varepsilon_{1t-1}^2 + b_{22} I_{20,t-1} + b_{21} I_{10,t-1} \]

where $h_{11}$ is the conditional volatility of 10-year US Treasury returns (log difference of prices) and $h_{22}$ the conditional volatility of the first difference, denoted by $\Delta$, of each of the five liquidity indicators (namely, $I_{10}$ is the transaction costs indicator by Roll (1984), $I_{20}$ is the daily range, $I_{M}$ the Market efficiency coefficient, $V$ is trading volume expressed in logarithms and $I_{A}$ stands for the ratio proposed by Amihud (2002)). See Section 2 for further details on these indexes; $LogI$ denotes the value of the log likelihood function; $Q(10)$ denotes the Ljung-Box Q-statistic (with 10 lags) for the standardized residuals; $Q^2(10)$ denotes the Ljung-Box Q-statistic (with 10 lags) for the squared standardized residuals; $\hat{\alpha}(I_{10})$ and $\hat{\alpha}(I_{M})$ denote the stationarity and the fourth-order moment conditions, respectively; Standard errors in brackets; ***, **, and * refer to significance at 1%, 5% and 10% level; Estimates of the equations for trading volumes and for the Amihud (2002) ratio are based on weekly data.

5. Results

Next we analyze whether market liquidity has become less resilient after the GFC. We fit the model in equations (6)-(8) for the returns, $r_t$, and the first differences of the five liquidity measures, $I_{10}$, $I_{20}$, $I_{M}$, $V$ and $I_{A}$. We estimate the model for the pre-crisis period, from January 2003 to May 2007, for the post-crisis period, from April 2009 to June 2016, as well as for the full sample. Tables 6-8 report the estimates of the model for the three periods, respectively. Our main interest is in the two calm periods before and after the GFC, as we do not want the outcome of our models to be driven by the financial crisis episode.

Spillovers from financial volatility to liquidity volatility, as approximated by $b_{21}$, have increased substantially. As stated in Table 6, this coefficient is significant only for $LR_{20}$ in the pre-crisis period, whereas Table 7 indicates that after the crisis $b_{21}$ also becomes significant for the models with $I_{M}$ and $I_{A}$. Therefore, after the GFC liquidity volatility spikes hand in hand with financial shocks, which implies that liquidity
period this coefficient is only significant for the models with
less, the Amihud ratio, and vice versa. These increasing simultaneous spillovers between
around, so that the more unstable liquidity is, the more volatile returns are, and vice versa. These increasing simultaneous spillovers between both variables imply a lower liquidity resilience after the GFC.

Besides, according to Table 7, the estimates for $b_{12}$ tend to be larger and more significant than those for $b_{21}$, except for the model for the Amihud ratio, $L_{A}$, which suggests that liquidity volatility dominates these dynamics. Thus, liquidity volatility not only reacts to returns volatility, but also plays a major role in exacerbating financial volatility. This result supports our approach based on a simultaneous analysis of returns volatility and liquidity volatility, as a univariate model approach to characterize resilience through the reaction of liquidity volatility to financial shocks would have ignored these coinciding effects. This outcome complements that of Fleming and Remolona (1999) and Fleming and Piazzesi (2005), who state that depth, which we approximate by volumes, tends to disappear prior to economic news announcements.

Table 8 reports the estimates for the full sample. In line with the post-crisis outcome, spillovers from financial volatility to liquidity volatility, which are identified in the post-crisis period though $b_{12}$, have the model specifications with $L_{DR}$, $L_{MFC}$ and $L_{A}$, are also significant for the complete sample. However, and as in the results for the pre-crisis period in Table 6, the coefficient for $b_{21}$ is only significant for the models with $L_{DR}$ and $V$. That is, some of the feedback loop effects between liquidity volatility and price uncertainty that we identify for the post-crisis period in Table 7 would not hold for the entire sample, as they would be hidden by the dynamics of the series during the pre-crisis and the crisis periods.

Moreover, according to the estimates of $a_{11}$ and $b_{11}$ in Tables 6 and 7, returns volatility of the US Treasuries became less persistent after the financial crisis. Persistence of financial volatility, which is approximated by $\hat{\phi}_{11} + \hat{\theta}_{11}$, is lower in the post-crisis period for all liquidity indicators except for the estimates with $L_{MFC}$; although these coefficients are not significant. These results suggest that after the GFC calm periods last less than before the crisis. We reach similar conclusions on the persistence of liquidity volatility as $\hat{\phi}_{21} + \hat{\theta}_{21}$ also diminish after the crisis for all liquidity indicators, except for the model with $L_{MFC}$; also

| $\Delta L_{E}$ | $\Delta L_{DR}$ | $\Delta L_{MFC}$ | $\Delta V$ | $\Delta L_{A}$ |
|---------------|---------------|----------------|----------|--------------|
| $c_1$ | 0.003 | 0.000 | 0.011 $^*$ | 0.003 | 0.224 |
| $c_{2}$ | 0.002 | 0.011 | 0.023 | 0.022 | 0.168 |
| $a_{11}$ | 0.032 | 0.053 $^*$ | 0.037 $^*$ | 0.114 | 0.033 $^*$ |
| $a_{22}$ | 0.338 $^*$ | 0.012 | 0.003 | 0.122 $^*$ | 0.030 |
| $a_{12}$ | 0.315 | 0.001 | 0.008 | 0.729 | 0.002 |
| $a_{21}$ | 0.013 | 0.032 | 0.352 $^*$ | 0.002 | 0.000 |
| $b_{11}$ | 0.921 $^*$ | 0.874 $^*$ | 0.849 $^*$ | 0.823 $^*$ | 0.542 $^*$ |
| $b_{21}$ | 0.057 | 0.606 $^*$ | 0.010 | 0.147 | 0.844 $^*$ |
| $b_{12}$ | 0.477 $^*$ | 0.232 $^*$ | 0.145 $^*$ | 0.816 $^*$ | 0.002 |
| $b_{22}$ | 0.008 | 0.040 $^*$ | 0.025 $^*$ | 0.000 | 0.908 $^*$ |

Estimation results of the conditional variances of a bivariate ECC-GARCH model (Nakatani, 2010):

$$h_{i1} = c_1 + a_{11} x_{i-1}^2 + a_{12} x_{i-1}^2 + b_{11} h_{i-1} + b_{12} h_{i-1}$$

$$h_{i2} = c_2 + a_{21} x_{i-1}^2 + a_{22} x_{i-1}^2 + b_{21} h_{i-1} + b_{22} h_{i-1}$$

where $h_i$ is the conditional volatility of 10-year US Treasury returns (log difference of prices) and $h_{i1}$ the conditional volatility of the first difference, denoted by $\Delta$, of each of the five liquidity indicators, namely, $L_{DR}$ is the transaction costs indicator by Roll (1984), $L_{DR}$ is the daily range, $L_{DR}$ denotes the Market efficiency coefficient, $V$ is trading volume expressed in logarithms and $L_{A}$ stands for the ratio proposed by Amihud (2002). See Section 2 for further details on these indexes; $Logl_t$ denotes the value of the log likelihood function; $Q(10)$ denotes the Ljung-Box Q-statistic (with 10 lags) for the squared standardized residuals; $\hat{\alpha}(1)$ and $\hat{\alpha}(C_{EG})$ denote the stationarity and the fourth-order moment conditions, respectively; Standard errors in brackets; $^*$, $^*$, and $^*$ refer to significance at 1%, 5%, and 10% level; Estimates of the equations for trading volumes and for the Amihud (2002) ratio are based on weekly data.
the US and the euro area by means of time-varying estimates of GARCH market volatility after the announcement of asset purchase programs in on the changing pattern in volatility after the crisis in different mar-
crisis.

metric GARCH models for UK equity and credit markets to conclude future additional research. There are two main types of tests for structural in the pre-crisis and the post crisis periods given that the choice of a proper GFC in both returns and liquidity volatility indicate that volatility will

intermsofliquidityresilienceasthislowerpersistencethanbeforethe with non-significant coefficients. That is to say, this is a positive result in terms of liquidity resilience as this lower persistence than before the GFC in both returns and liquidity volatility indicates that volatility will return to low levels more quickly than in the past.22

Our results are in line with previous works by some central banks on the changing pattern in volatility after the crisis in different markets. For instance, the ECB (2016) observes a lower persistence of stock market volatility after the announcement of asset purchase programs in the US and the euro area by means of time-varying estimates of GARCH (1,1) parameters, whereas the BoE (2015) estimates a range of asymmetric GARCH models for UK equity and credit markets to conclude that financial volatility has become more sensitive to news after the crisis.

Important financial stability considerations arise from our findings. Higher feedback effects between financial volatility and market liquidity in the aftermath of the GFC imply that financial shocks coexist with liquidity strains, which in turn exacerbate market reaction and limits liquidity capacity to absorb shocks. On a more positive note, episodes of high volatility are less prolonged, as implied by lower volatility persistence, potentially reducing the severity of financial strains, which is positive for financial stability. In any event, our results suggest that the market liquidity of US Treasuries has become less resilient even with apparently sound liquidity. Our assessment on liquidity resilience is helpful to explain the occurrence of the volatility episodes that this key market has witnessed in recent years, such as the October 2014 “flash crash”, which have been intense but short-lived in nature, and characterized by severe liquidity strains.

Poorer liquidity resilience might be related to some recent structural changes in the markets, such as the lower presence of market makers, or the existence of large and concentrated holdings by institutional investors (IMF, 2015). Moreover, the rise of electronic platforms and new trading strategies, such as automatic trading and high-frequency trading, could have promoted more unstable liquidity conditions as well (Joint Staff Report, 2015). From a regulatory perspective, some specific features of the Basel III capital framework, such as the introduction of the leverage ratio, could have contributed to the decline of liquidity provision by bank-dealers (Adrian et al., 2017b). Besides suffering from serious data limitations, the quantification of the impact of these drivers on the link between market liquidity and financial volatility exceeds the

| $c_1$ | $c_2$ | $a_{11}$ | $a_{22}$ | $b_1$ | $b_2$ |
|------|------|--------|--------|------|------|
| 0.012*** | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 |
| (0.003) | (0.001) | (0.008) | (0.002) | (0.146) | (0.013) |
| 0.000 | 0.001 | 0.028 | 0.040 | 0.040 | 0.040 |
| (0.017) | (0.017) | (0.002) | (0.028) | (0.040) | (0.028) |
| 0.043 | 0.032*** | 0.022** | 0.001 | 0.001 | 0.001 |
| (0.079) | (0.010) | (0.002) | (0.001) | (0.002) | (0.001) |
| 0.316*** | 0.009*** | 0.000 | 0.023** | 0.054 | 0.054 |
| (0.001) | (0.004) | (0.006) | (0.001) | (0.006) | (0.006) |
| 0.000 | 0.008 | 0.028 | 0.028 | 0.028 | 0.028 |
| (0.006) | (0.001) | (0.001) | (0.002) | (0.002) | (0.002) |
| 0.017 | 0.038 | 0.000 | 0.000 | 0.000 | 0.000 |
| (0.032) | (0.115) | (0.103) | (0.074) | (0.074) | (0.074) |
| 0.842*** | 0.957*** | 0.960*** | 0.844*** | 0.909*** | 0.909*** |
| (0.004) | (0.019) | (0.014) | (0.002) | (0.002) | (0.002) |
| 0.265*** | 0.407 | 0.963** | 0.802* | 0.915** | 0.915** |
| (0.067) | (0.351) | (0.141) | (0.426) | (0.426) | (0.426) |
| 1.383*** | 0.013 | 0.008 | 0.036*** | 0.000 | 0.000 |
| (0.004) | (0.108) | (0.052) | (0.005) | (0.005) | (0.005) |
| 0.005 | 0.151*** | 0.011*** | 0.000 | 0.110** | 0.110** |
| (0.074) | (0.017) | (0.002) | (0.002) | (0.002) | (0.002) |
| −1.188359 | −2616.47 | −3233.109 | −626.485 | −2355.404 | −2355.404 |
| TLogL | −3464 | −3464 | −3464 | −3464 | −3464 |
| Q(10) | 7.948 | 6.931 | 7.948 | 6.931 | 7.948 |
| Q(20) | 5.693 | 10.203 | 5.693 | 10.203 | 5.693 |
| Q(20) | 11.436 | 10.541 | 11.436 | 10.541 | 11.436 |
| Q(20) | 4.456 | 3.873 | 4.456 | 3.873 | 4.456 |
| $J(h_{E})$ | 0.964 | 0.996 | 0.964 | 0.937 | 0.980 |
| $J(h_{C})$ | 0.940 | 0.995 | 0.940 | 0.896 | 0.965 |

Estimation results of the conditional variances of a bivariate ECC-GARCH model (Nakatani, 2010):

$$h_{1t} = c_1 + a_{11}^2_{t-1} + a_{12}^2_{t-1} + b_{11} h_{1t-1} + b_{12} h_{2t-1}$$

$$h_{2t} = c_2 + a_{21}^2_{t-1} + a_{22}^2_{t-1} + b_{21} h_{2t-1} + b_{11} h_{1t-1}$$

where $h_t$ is the conditional volatility of 10-year US Treasury returns (log difference of prices) and $b_t$ the conditional volatility of the first difference, denoted by $\Delta$, of each of the five liquidity indicators (namely, $L_{b0}$ is the transaction costs indicator by Roll (1984)), $L_{b1}$ is the daily range, $L_{b2}$ denotes the Market efficiency coefficient, $V$ is trading volume expressed in logarithms and $L_{b3}$ stands for the ratio proposed by Amihud (2002). See Section 2 for further details on these indexes; $LogL$ denotes the value of the log likelihood function; $Q(10)$ denotes the Ljung-Box Q-statistic (with 10 lags) for the standardized residuals; $J(h_{E})$ and $J(h_{C})$ denote the stationarity and the fourth-order moment conditions, respectively; Standard errors in brackets: ***, **, and * refer to significance at 1%, 5% and 10% level; Estimates of the equations for trading volumes and for the Amihud (2002) ratio are based on weekly data.

22 We provide no formal test for the difference of $a_{11} + b_{11}$ and $a_{22} + b_{22}$ in the pre-crisis and the post crisis periods given that the choice of a proper structural break test on GARCH parameters is not straightforward and deserve future additional research. There are two main types of tests for structural breaks in GARCH type models—see, for instance, Smith (2008) for a survey—First, Lagrange Multiplier (LM) tests are useful to identify persistence variations across the sample but can only identify one break. In our case, these tests are not useful as the sample consists of three periods (two breaks). Second, CUSUM tests, such as that of Inclan and Tiao (1994), are able to detect more than one break but they only identify them in the unconditional volatility, but not in the conditional volatility. That is, tests for changes in persistence related parameters cannot be performed either.
scope of this paper.

6. Conclusions

In this paper, we analyze market liquidity resilience, i.e. its ability to absorb shocks, of the US 10-year Treasury bonds given its relevance from a financial stability perspective. The conventional liquidity resilience measure, defined as the mean-reverting speed of the liquidity level after one shock, has limitations due to the fact that it relies on the liquidity level measurement. We propose a new complementary approach where liquidity volatility, rather than the liquidity level, becomes the key variable to characterize resilience.

First, we choose five market liquidity indicators, which do not show an unequivocal drop in liquidity in the aftermath of the GFC. Then, we fit a bivariate CC-GARCH model as proposed by Nakatani (2010) for US 10-year Treasury returns and the first difference of the liquidity measures from January 2003 to June 2016. This model approach allows to evaluate liquidity resilience, as it is useful for understanding the dynamics of returns volatility and liquidity volatility, their interactions, as well as volatility persistence. Our bivariate GARCH models should not be interpreted as a new liquidity resilience measure, but as a tool to better understand resilience through the relatively unexplored link between liquidity volatility and financial volatility that complements the standard quantification.

According to our results, despite apparently sound liquidity, market liquidity resilience has deteriorated after the GFC. Spillovers from liquidity volatility to returns volatility and vice versa have become more intense and feedback loops more likely, which may aggravate turmoil episodes and signals a lower capacity of liquidity to absorb shocks. That being said, the volatility of both returns and liquidity is less persistent than in the past. Therefore, while liquidity volatility and returns volatility could suddenly increase, they will also tend to recover faster than in the past. These results help to explain some of the crash-type episodes that the US Treasury bond market has witnessed in recent years, short-lived in nature and characterized by liquidity strains.

Our results have important financial stability implications regarding the pivotal position of US Treasury debt as a safe asset. These new market liquidity dynamics are also relevant for the liquidity buffer of banks and other institutions. Further research to understand the drivers of this lower liquidity resilience, which are probably related to ongoing structural changes in this market, is needed to guide policy responses that reinforce the market liquidity resilience.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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