Role of non-coplanarity in nuclear reactions using the Wong formula based on the proximity potential

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Introduction

Recently [1], we assessed Wong’s formula [2] for its angular momentum \( \ell \)-summation and “barrier modification” effects at sub-barrier energies in the dominant fusion-evaporation and capture (equivalently, quasi-fission) reaction cross-sections. For use of the multipole deformations (up to \( \beta_4 \)) and (in-plane, \( \Phi=0^0 \)) orientations-dependent proximity potential in fusion-evaporation cross-sections of \( ^{58}\text{Ni}+^{58}\text{Ni}, \; ^{64}\text{Ni}+^{64}\text{Ni} \) and \( ^{100}\text{Mo} \), known for fusion hindrance phenomenon in coupled-channels calculations, and the capture cross-sections of \( ^{48}\text{Ca}+^{238}\text{U}, \; ^{244}\text{Pu} \) and \( ^{248}\text{Cm} \) reactions, forming superheavy nuclei, though the simple \( \ell=0 \) barrier-based Wong formula is found inadequate, its extended version, the \( \ell \)-summed Wong expression fits very well the above noted capture cross-sections at all center-of-mass energies \( E_{c.m.'s} \), but require (additional) modifications of the barriers to fit the fusion-evaporation cross-sections in the Ni-based reactions at below-barrier energies. Some barrier modification effects are shown [1] to be already present in Wong expression due to its inbuilt \( \ell \)-dependence via \( \ell \)-summation.

In this paper, we study for the first time the dynamics of fission reactions, such as \( ^{11}\text{B}+^{235}\text{U} \) and \( ^{14}\text{N}+^{232}\text{Th} \) forming \( ^{246}\text{Bk}^* \) [3], on the basis of the extended, \( \ell \)-summed Wong formula, including also the non-coplanarity (\( \Phi \neq 0^0 \)) degree-of-freedom for all the three types of reactions, the fusion-evaporation, capture and fission cross-sections.

The extended Wong model

Wong’s expression for fusion cross-section due to colliding two deformed and oriented nuclei (orientations \( \theta_i \)), lying in two different planes (azimuthal angle \( \Phi \) between the planes), in terms of \( \ell \) partial waves, is

\[
\sigma(E_{c.m.}, \theta_i, \Phi) = \frac{\pi}{R^2} \sum_{\ell=0}^{\ell_{max}} (2\ell+1) P_\ell(E_{c.m.}, \theta_i, \Phi),
\]

with \( k = \sqrt{\frac{2\mu E_{c.m.}}{\hbar^2}} \), and \( \mu \), the reduced mass, \( P_\ell \) is the transmission coefficient for each \( \ell \), describing, in Hill-Wheeler approximation, the penetration of barrier \( V_f(R, E_{c.m.}, \theta_i, \Phi) \).

Instead of solving Eq. (1) explicitly, which require the complete \( \ell \)-dependent potentials \( V_f(R, E_{c.m.}, \theta_i, \Phi) \), Wong summed it up approximately, using only \( \ell=0 \) quantities, which on replacing the \( \ell \)-summation in [1] by an integral, gives the Wong formula [2]

\[
\sigma(E_{c.m.}, \theta_i, \Phi) = \frac{R_{\ell=0}^2 \hbar \omega_i}{2E_{c.m.}} \ln \left[ 1 + \exp \left( \frac{2\pi \hbar \omega_i (E_{c.m.} - V_B^0)}{\mu R^2} \right) \right].
\]

Integrating (2) over \( \theta_i \) and \( \Phi \), we get the fusion cross-section \( \sigma(E_{c.m.}) \).

For an explicit summation over \( \ell \) in Eq. (1), the \( \ell \)-dependent interaction potential \( V_f(R) \) is a sum of Coulomb and nuclear proximity and centrifugal potentials, as

\[
V_f(R) = V_p(R, A_1, \beta_{\lambda_1}, T, \theta_i, \Phi) + \frac{\hbar^2 \ell (\ell + 1)}{2\mu R^2} + V_C(R, Z_i, \beta_{\lambda_i}, T, \theta_i, \Phi),
\]

where, the \( \ell \)-summation in Eq. (1) is then carried out for the \( \ell_{max} \) determined empirically for a best fit to measured cross-section. This procedure of explicit \( \ell \)-summation works very well for \( \Phi=0^0 \) case [1] in capture reactions \( ^{48}\text{Ca}+^{238}\text{U}, ^{244}\text{Pu} \) and \( ^{248}\text{Cm} \), but require further modification of the barrier for Ni-based reactions at sub-barrier energies, which could be carry out empirically [1] by either (i) keeping the curvature \( \hbar \omega_i \) same and modifying the
sections. Fig. 1(a) shows that the capture cross-section given in Fig. 1 for all the three types of reactions. Fig. 1(b) and (c) $^{64}$Ni+$^{100}$Mo, and (d) to (f) $^{246}$Bk$^*$ due to $^{14}$N+$^{232}$Th and $^{11}$B+$^{235}$U channels.

barrier height $V_B^\ell$, as

$$V_B^\ell (modified) = V_B^\ell + \Delta V_B^{emp},$$

or (ii) keep the barrier height $V_B^\ell$ same and modify the curvature $h_{\omega \ell}$. We use here the method of modifying the barrier height.

Calculations and results

The results of $\ell$-summed Wong expression \[1\] for both the cases of $\Phi=0^0$ and $\Phi \neq 0^0$ are given in Fig. 1 for all the three types of reactions. Fig. 1(a) shows that the capture cross-section in $^{48}$Ca+$^{238}$U is fitted nicely even after giving a small deformation ($\beta_{21}=0.183$) to $^{48}$Ca for carrying out $\Phi$-integration. The fitted $\ell_{max}(E_{c.m.})$ increase by one-to-two units. Similarly, the $^{64}$Ni+$^{100}$Mo reaction is fitted for both $\Phi=0^0$ and $\Phi \neq 0^0$ by allowing empirically a small increase in “barrier lowering” $\Delta V_B^{emp}$ (Figs. 1(b) and 1(c)). On the other hand, there is a strong entrance channel dependence in the case of fission reaction: whereas a nice fit is obtained for both $\Phi=0^0$ and $\Phi \neq 0^0$ cases in $^{14}$N+$^{232}$Th channel (Fig. 1(d)), a large disagreement in cross-sections at higher energies (Fig. 1(e)) and hence a large “barrier lowering” $\Delta V_B^{emp}$ (Fig. 1(f)) is obtained for $\Phi=0^0$ in $^{11}$B+$^{235}$U channel, which reduces to zero for $\Phi \neq 0^0$ case. In other words, for fission of $^{246}$Bk$^*$, the inclusion of non-coplanarity gives a complete fit to data for both the reaction channels, without introducing $\Delta V_B^{emp}$.

References

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