High intermodulation gain in a micromechanical Duffing resonator

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In this work we use a micromechanical resonator to experimentally study small signal amplification near the onset of Duffing bistability. The device consists of a PdAu beam serving as a micromechanical resonator excited by an adjacent gate electrode. A large pump signal drives the resonator near the onset of bistability, enabling amplification of small signals in a narrow bandwidth.

To first order, the amplification is inversely proportional to the frequency difference between the pump and signal. We estimate the gain to be about 15dB for our device.

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Micro/Nanoelectromechanical resonators play a key role in microdevices for applications such as sensing, switching, and filtering [1,2]. Understanding nonlinear dynamics in such devices is highly important, both for applications and for basic research [3-10].

Generally, for resonators driven using a bias voltage (at 10⁻³ torr) and the fundamental mode resonance frequency ω₀/2π is in the range of 560 kHz to 630kHz.

The device has a quality factor of Q = ω₀/2µ ≈ 2000 (at 10⁻³ torr) and the fundamental mode resonance frequency ω₀/2π is in the range of 560 kHz to 630kHz.

To investigate nonlinear amplification, the resonator is driven by an applied force F(t) = f_p cos(ω_p t) + f_s cos(ω_s t + ϕ), composed of an intense pump with frequency ω_p = ω_0 + σ, amplitude f_p, and a small force (called signal) with frequency ω_s = ω_p + δ, relative phase ϕ, and amplitude f_s, where f_s << f_p and σ,δ << ω_p.

This is achieved by applying a voltage of the form V = V_{dc} + v_p cos(ω_p t) + v_s cos(ω_s t + ϕ) where V_{dc} is a dc bias and v_s << v_p << V_{dc}. The resonator’s displacement has spectral components at ω_p, ω_s, and at the intermodulations ω_p ± kδ where k is an integer. The one at frequency ω_i = ω_p − δ is called the idler component, as in nonlinear parametric terms [12,18]. In our case however, the prefactors of these parametric terms is at least one order smaller below threshold, thus negligible.

The device is mounted inside an SEM operated in a spot mode to detect the resonator’s displacement. The displacement signal is probed by a secondary electron detector and measured using a spectrum analyzer.

FIG. 1: The experimental setup. The inset shows an electron micrograph of the device, consisting of two suspended doubly clamped micromechanical resonators. Each resonator is of length l=100µm, width w=0.6µm, and thickness t=0.25µm, centered around a gate electrode with d = 4µm gap. The device is mounted inside an SEM operated in a spot mode to detect the resonator’s displacement. The displacement signal is probed by a secondary electron detector and measured using a spectrum analyzer.
In the slowly varying envelope method, the displacement $x$ is written as

$$x(t) = \frac{1}{2}A(t)e^{i\omega_p t} + c.c.,$$

where $A(t)$ is a slowly varying function (relative to the time scale $1/\omega_p$). Substituting Eq. 2 in the equation of motion and neglecting the $d^2A/dt^2$ term yields

$$\frac{dA}{dt} = -\left(\frac{\omega_0}{2Q} + i\sigma\right)A + i\frac{3}{8}\kappa\omega_0 A^2A^* + \frac{1}{2\kappa\omega_0}(f_p + f_s e^{i(\delta t + \varphi)}).$$

$A(t)$ can be written as

$$A(t) = a_p + a_se^{i\delta t} + a_i e^{-i\delta t},$$

where the complex numbers $a_p$, $a_s$ and $a_i$ are the pump, signal and idler components of $A(t)$ respectively, and $|a_s|, |a_i| \ll |a_p|$. Substituting Eq. 3 in Eq. 4 and keeping small terms up to first order, leads to

$$a_s = \frac{1}{2\kappa\omega_0}f_s e^{i\varphi} - \frac{3}{8}\kappa\omega_0 a_p^2 a_i^*,$$

$$a_i = \frac{3}{4}\kappa\omega_0 |a_p|^2 - \delta - \sigma + i\frac{\omega_0}{2Q},$$

and

$$a_i = \frac{3}{8}\kappa\omega_0 |a_p|^2 - \delta - \sigma + i\frac{\omega_0}{2Q}.$$  

The pump response $|a_p|$ in the absence of any additional signal is shown in Fig. 2 panels (a),(b) and (c). Above some critical driving amplitude $f_c$, the response becomes a multi-valued function of the frequency in some finite frequency range, and the system becomes bistable with jump points in the frequency response. We refer to the onset point of bistability (which is also a saddle-node bifurcation point) as the critical point. When the pump is tuned to the critical point ($\sigma = \sqrt{3}\omega_c/2Q$, $|a_p|^2 = \delta/3\sqrt{3}\kappa Q$) and $\delta \rightarrow 0$, we expect high amplification of both signal and idler in this limit

$$|a_s| \simeq |a_i| \simeq \frac{f_s}{2\omega_0 \delta}.$$  

Thus, in our model which assumes that $|a_s|$ and $|a_i|$ are small, and takes nonlinearity into account only to lowest order, the amplification diverges in the limit $\delta \rightarrow 0$. When $|a_s|$ and $|a_i|$ become comparable with $|a_p|$, however, the former assumptions are no longer valid and higher order terms have to be taken into account.

The pump, signal, and idler’s responses were calculated analytically and are shown in Fig. 2. For a small $f_p$, the signal’s response is nearly Lorentzian, while for $f_p > f_c$, both signal and idler response diverge near the jump points.

The resonators are fabricated using bulk-nanomachining process together with electron beam lithography. The experimental setup is shown in Fig.

FIG. 2: Calculation of the pump, signal, and idler responses ($(|a_p|, |a_s|, |a_i|)$ for vanishing offset frequency $\delta$, shown for sub-critical case $f_p = 0.5 f_c$ (a,d,g), critical case $f_p = f_c$ (b,e,h), and over-critical case $f_p = 2 f_c$ (c,f,i). The $y$-axis of the pump is shown in a linear scale while the signal’s and idler’s response are normalized to the signal’s excitation amplitude and are shown in a logarithmic scale. The signal’s and idler’s response diverge at the critical point and at the jump points. The parameters for this example are $\kappa = 10^{-2} \text{ m}^{-2}$, $\mu = 10^2 \text{ Hz}$, $\omega_c/2\pi = 1 \text{ MHz}$, and $\delta/2\pi = 10 \text{ Hz}$.

FIG. 3: Simultaneous measurement of the pump, signal and idler spectral components of the mechanical displacement. The excitation frequency is swept upward (blue line) and downward (green line). The arrows in the pump’s plot indicate the hysteresis loop. The excitation parameters are: pump ac voltage $v_p = 0.5V$, $v_p/v_s = 6$, frequency offset $\delta/2\pi = 1 \text{ kHz}$ and $V_{dc} = 5 \text{ V}$. The horizontal axis is the pump frequency for all three plots. The pump signal and idler exhibit simultaneous jumps, as expected.

A typical mechanical response is shown in fig. 3. The pump’s frequency is swept upward and then back down-
FIG. 4: Mesh plots showing the response of the pump, signal and idler. The horizontal axis is the pump’s frequency $\omega_p$, the diagonal axis is the pump’s ac voltage $v_p$, and the vertical axis is the response (displacement) axis in logarithmic scale. For each frequency, $v_p$ is scanned from 0 to 0.5V, $v_p/v_s = 6$, $\delta/2\pi = 100$ Hz, $V_{dc} = 5$ V. Note that the pump’s response undergoes a jump along a line in the ($v_p, \omega_p$) plane, starting from the bifurcation point. Along the same line, the spectral components of the signal and idler obtain their maximum value.

As expected, we find hysteretic response and simultaneous jumps for the pump, signal, and idler spectral components. In Fig. 4 the mechanical responses of the pump, signal and idler are depicted as a function of the pump’s frequency $\omega_p/2\pi$ and the pump’s ac voltage $v_p$. For each frequency, the voltage $v_p$ is scanned from 0 to 0.5 V. The results show a good agreement with theory. As expected, we observe high signal amplification near the jump points. The amplification can be quantified in logarithmic scale as

$$G = 20 \log\left(\frac{a_{s,pump\text{ on}}}{a_{s,pump\text{ off}}}\right).$$

The highest value of $G$, obtained near one of the jump points is 15dB. Note, however, that this value is an underestimation of the actual gain due to nonlinearity of our displacement detection scheme.

In conclusion, we have shown that a Duffing micromechanical resonator, driven into the bistability regime, can be employed as a high gain narrow band mechanical amplifier.

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