Measurements of Partial Branching Fractions for $\bar{B} \to X_u \ell \nu$ and Determination of $|V_{ub}|$

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383 million $\Upsilon(4S)$ decays into $B\bar{B}$ pairs collected with the BaBar detector at the PEP-II $e^+e^-$ storage rings. We select events using either the invariant mass $M_X$ of the hadronic system, the invariant mass squared, $q^2$, of the lepton and neutrino pair, the kinematic variable $P_+$, or one of their combinations. We then determine partial branching fractions in limited regions of phase space: 
$$\Delta B = (1.18 \pm 0.09_{\text{stat}} \pm 0.07_{\text{syst}} \pm 0.03_{\text{ theor}}) \times 10^{-3} \quad (M_X < 1.55 \text{ GeV}/c^2),$$
$$\Delta B = (0.95 \pm 0.10_{\text{ stat}} \pm 0.08_{\text{ syst}} \pm 0.01_{\text{ theor}}) \times 10^{-3} \quad (P_+ < 0.66 \text{ GeV}/c),$$
$$\Delta B = (0.81 \pm 0.08_{\text{stat}} \pm 0.07_{\text{syst}} \pm 0.02_{\text{ theor}}) \times 10^{-3} \quad (M_X < 1.7 \text{ GeV}/c^2, \ q^2 > 8 \text{ GeV}^2/c^4).$$
Corresponding values of $|V_{ub}|$ are extracted using several theoretical calculations.

In the Standard Model the element $V_{ub}$ of the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix [1] plays a critical role in tests of the prediction of CP violation. Since the rate for charmless semileptonic decays, $B \to X_u \ell \bar{\nu}$ [2], is proportional to $|V_{ub}|^2$, and the hadronic and leptonic currents are factorizable, the best method to extract this quantity is to measure branching fractions for such decays [3]. Experimentally, the principal challenge is to separate the rare $\bar{B} \to X_u \ell \bar{\nu}$ decays from the approximately 50 times larger $B \to X_c \ell \bar{\nu}$ background. Given that the $u$ quark is much lighter than the $c$ quark, regions of phase space can be defined where the background is suppressed. To relate the decay rate of the $B$ meson to $|V_{ub}|$, parton level calculations have to be corrected for perturbative and non-perturbative QCD effects. A variety of QCD calculations are available to determine these corrections [4–6].

In this letter, we present a measurement of partial branching fractions for inclusive charmless semileptonic decays, $\bar{B} \to X_u \ell \bar{\nu}$ [7]. $\Upsilon(4S) \to B\bar{B}$ events are tagged by the full reconstruction of a hadronic decay of one of the $B$ mesons ($B_{\text{reco}}$). The semileptonic decay of the second $B$ meson ($B_{\text{recoil}}$) is identified by the presence of an electron or a muon. This technique results in a low event selection efficiency but allows the determination of the momentum, charge, and flavor of the $B$ mesons.

We use three kinematic variables to separate $\bar{B} \to X_u \ell \bar{\nu}$ decays from the dominant $\bar{B} \to X_c \ell \bar{\nu}$ background: $M_X$, the invariant mass of the hadronic system $X_{u,c}$; $q^2$, the invariant mass squared of the lepton-neutrino system; and $P_+ \equiv E_X - |\vec{P}_X|$ [4, 5], where $E_X$ and $\vec{P}_X$ are the energy and momentum of the hadronic system $X_{u,c}$ calculated in the $B$ rest frame. We measure the fraction of partial rates of charmless semileptonic decays $\Delta R_{u/c} = \Delta B(\bar{B} \to X_u \ell \bar{\nu})/B(\bar{B} \to X_l \ell \bar{\nu})$ in restricted phase space regions, corrected for resolution effects. The resulting partial branching fractions are used to calculate $|V_{ub}|$ following theoretical prescriptions.

The analysis uses a sample of 383 million $\Upsilon(4S)$ decays into $B\bar{B}$ pairs, corresponding to an integrated luminosity of 347.4 fb$^{-1}$, collected with the BaBar detector [8]. Charmless semileptonic $\bar{B} \to X_u \ell \bar{\nu}$ decays are simulated as a combination of three-body decays ($X_u = \pi, \eta, \eta', \rho, \omega, \ldots$) [9] and decays to non-resonant hadronic final states $X_u$ [10]. The motion of the $b$ quark inside the $B$ meson is modeled with the shape function parametrization given in Ref. [10]. The simulation of the $\bar{B} \to X_u \ell \bar{\nu}$ background uses an HQET parametrization of form factors for $B \to D^{(*)}\ell\bar{\nu}$ [11, 12], and models for $\bar{B} \to D\pi\ell\bar{\nu}, D^*\pi\ell\bar{\nu}$ [13], and for $\bar{B} \to D\pi, D^{(*)}\ell\bar{\nu}$ [9]. The simulation of the hadronization is performed by Jetset7.4 [14]. We use GEANT4 [15] to simulate the detector response.

To reconstruct a large sample of hadronically decaying $B$ mesons, $B_{\text{reco}} \to D^{(*)}\Upsilon^\pm$ are selected. Here, the system $T^\pm$ consists of hadrons with a total charge of $\pm 1$, composed of $n_1 \pi^\pm n_2 K^\pm n_3 K^0_s n_4 \pi^0$, where $n_1 + n_2 \leq 5$, $n_3 \leq 2$, and $n_4 \leq 2$. The kinematic consistency of $B_{\text{reco}}$ candidates is checked with two variables, $m_{ES} = \sqrt{s/4 - \vec{p}_B^2}$ and $\Delta E = E_B - \sqrt{s}/2$. Here $\sqrt{s}$ is the total energy in the $T(4S)$ center of mass frame, and $\vec{p}_B$ and $E_B$ denote the momentum and energy of the $B_{\text{reco}}$ candidate in the same frame. We require $\Delta E = 0$ within three standard deviations as measured for each decay mode. For each of the $B_{\text{reco}}$ decay modes, the purity $P$ is estimated using Monte Carlo (MC) simulation. $P$ is defined as the ratio of signal over background events with $m_{ES} \geq 5.27 \text{ GeV}/c^2$. Only modes for which $P$ exceeds 20% are used. On average, we reconstruct at least one $B$ candidate in 0.3% (0.5%) of the $B^0(B^-)$ events. For events with more than one reconstructed $B$ decay, the decay mode with the highest purity is selected.

We determine the number of $B_{\text{reco}}$ candidates from an unbinned maximum likelihood fit to the $m_{ES}$ distribution. The data are fit to the sum of three contributions: signal $B_{\text{reco}}$ decays, combinatorial background from $B\bar{B}$ events, and continuum ($e^+e^- \to q\bar{q}$, $q = u,d,s,c$) events. A Threshold function [16] is used to describe the combinatorial and continuum backgrounds. To obtain a good description of the signal $m_{ES}$ distribution, we adopt the modified Gaussian function used in Ref. [17], to account for energy losses of photons in the detector. Fits to the $m_{ES}$ distribution are shown in Fig. 1. Semileptonic decays $\bar{B} \to X_l \ell \bar{\nu}$ of the $B_{\text{recoil}}$ candidate are identified by an electron or muon with momentum, $p_\ell$, defined in the $\bar{B}$ rest frame, greater than 1 GeV/c. For charged $B_{\text{reco}}$ candidates, we require the charge of the lepton to be consistent with a prompt semileptonic $\bar{B}$ decay. For neutral $B_{\text{reco}}$ candidates, both charge-flavor combinations are retained and the known average $B^0(\bar{B})$ mixing rate [18] is used to extract the prompt lepton yield.
The hadronic system $X$ in the decay $B \to X\ell\nu$ is reconstructed from charged tracks and energy depositions in the calorimeter that are not associated with the $B_{\text{reco}}$ candidate or the identified lepton. We reconstruct $K^0_s$ by performing a mass-constrained fit to $\pi^+\pi^-$ pairs with an invariant mass in the range 0.473–0.523 GeV/c². The neutrino four-momentum $p_\nu$ is estimated from the missing momentum four-vector $p_{\text{miss}} = p_{\text{rec}} - p_B - p_X - p_\ell$, where all momenta are measured in the laboratory frame and $p_{\text{rec}}$ refers to the $Y(4S)$ meson.

To select $\bar{B} \to X_u\ell\nu$ candidates we require exactly one charged lepton with $p_\ell > 1$ GeV/c, charge conservation ($Q_X + Q_\ell + Q_{B_{\text{reco}}} = 0$), and a missing mass consistent with zero ($m^2_{\text{miss}} < 0.5$ GeV²/c⁴). These criteria suppress the dominant $\bar{B} \to X_u\ell\nu$ decays, many of which contain additional leptons or an undetected $K^0_s$ meson. We suppress the $B \to D^*\ell\nu$ background by reconstructing the low momentum $\pi^+$ from the $D^+ \to D^0\pi^+$ decay. Since the momentum of the $\pi^+$ is almost collinear with the $D^+$ momentum $p_{D^+}$, we can approximate the $D^+$ energy as $E_{D^+} \simeq m_{D^+} \times E_\pi/145$ MeV/c². The neutrino mass $m^2_{\ell\text{ veto}} = (p_B - p_{D^+} - p_\ell)^2$ is peaked at zero for background events. The requirement $m^2_{\ell\text{ veto}} < -3$ GeV²/c⁴ reduces the $B \to D^*\ell\nu$ background by about 36% while keeping more than 90% of signal events. We reject events with charged kaons or $K^0_s$ in the $B_{\text{reco}}$ to reduce the background from $\bar{B} \to X_u\ell\nu$ decays.

To extract the distribution in the variables $M_X$, $P_+$, and the combination of $M_X$ and $q^2$, we perform fits to the $B_{\text{reco}}$ $m_{\text{ES}}$ distributions for subsamples of events in individual bins for each of the variables, and subsequently separate the signal from the combinatorial and continuum backgrounds for the three distributions. The resulting distributions are presented in Fig. 2. To reduce the systematic uncertainties in the derivation of the branching fractions we determine the ratios of the partial branching fractions to the total semileptonic branching fraction. This is done for restricted regions of phase space, $M_X < 1.55$ GeV/c², $P_+ < 0.66$ GeV/c, and $(M_X < 1.7$ GeV/c², $q^2 > 8.0$ GeV²/c⁴). Specifically we define this ratio as

$$\frac{\Delta B(X_u\ell\nu)}{B(X\ell\nu)} = \frac{(N_u - N_{\text{out}} - B_{\text{BG}})}{(N_{\text{fit}} - B_{\text{BG}})} \times \frac{c^u_{\ell} c^\ell_{\nu}}{c^u_{\ell} c^\ell_{\nu}},$$

where $N_u$ refers to the number of observed events, $B_{\text{BG}}$ to the estimated number of background events, and $N_{\text{out}}$ to the signal events that migrate from outside the kinematic region into the signal region. The efficiency of the tag and lepton selection, $\epsilon_\ell$ and $\epsilon_\nu$, differ slightly for the signal and the semileptonic samples, due to differences in the lepton momentum distribution and the multiplicity of the recoiling $B$ meson. To convert the ratio in Eq. 1 to partial branching fractions, we use the total semileptonic branching fraction, $B(B \to X\ell\nu) = (10.75 \pm 0.15)\%$ [18]. The resulting partial branching fractions for the three selected kinematic regions, along with parameters in Eq. 1, are listed in Table I. The statistical correlations between the $M_X$ and $(M_X, q^2)$, $P_+$ analyses are 65%, 67%, 38% respectively.

We consider several sources of systematic uncertainties. Detector-related uncertainties take into account particle ($e$, $\mu$, $K$) identification (efficiency, mis-identification), charged particle tracking efficiency, photon reconstruction efficiency and $K^0_s$ interactions. We estimate the uncertainty due to signal and background modeling. The uncertainty on the signal modeling are due to the modeling of exclusive charmless semileptonic decays and gluon splitting into $s\bar{s}$-quark pairs. We also calculate the uncertainties due to the non-perturbative parameters and the functional form of the shape function. The background simulation depends on the $B$ and $D$ branching fractions and $B \to D^*\ell\nu$ form factors; the corresponding systematic uncertainties are calculated by varying all these quantities within their experimental errors. We estimate the error due to $m_{\text{ES}}$ fits, coming from the uncertainty in the parameterization ansatz. Finally, we estimate the error due to MC statistics. The fractional contribution of each uncertainty is shown in Table II together with the total error.
ties (see Table I). The hadronic input parameters, the $b$-quark mass $m_b$, and the kinetic energy expectation value $\mu^2$, are extracted from moment measurements in $B \to X_s \gamma$ and $\bar{B} \to X_l \ell \bar{\nu}$. Their values in the kinetic scheme [19] are $m_b = (4.59 \pm 0.04) \text{ GeV}/c^2$ and $\mu^2 = (0.40 \pm 0.04) \text{ GeV}^2/c^2$ [20] and are translated into values in different schemes, as needed [4–6]. The partial branching fraction $\Delta B(\bar{B} \to X_u \ell \bar{\nu})$ is related directly to $|V_{ub}|$ by the relation $|V_{ub}| = |\Delta B(\bar{B} \to X_u \ell \bar{\nu})/\tau_\bar{B} \Delta \zeta|^{1/2}$, where $\tau_\bar{B}$ is the average $B$ lifetime [18], and $\Delta \zeta$ is the prediction for the partial rate for $\bar{B} \to X_u \ell \bar{\nu}$ in the given phase-space region [4–6].

In summary, we have measured the branching fractions for inclusive charless semileptonic $B$ decays $\bar{B} \to X_u \ell \bar{\nu}$ in three overlapping regions of phase space. Relying on theoretical predictions, we extract values for the CKM matrix element $|V_{ub}|$ from our measured $\Delta B$.

We find that the determinations of $|V_{ub}|$ agree at 1 $\sigma$ level in the BNLP framework for the $M_X$ and combined $(M_X, q^2)$ analyses. The analysis based on $P_+$ differs from the two others at a 2.5 $\sigma$ level, as indicated also by other experiments [21]. The $M_X$ analysis captures the largest portion of phase space and gives the most precise determination of $|V_{ub}|$. Within their stated theoretical uncertainties, the results based on BLNP and DGE give consistent results. The result, based on the hadronic mass spectrum, supersedes our previously published measurement [3], reducing the relative uncertainty by 40%. These values are in good agreement with other inclusive $|V_{ub}|$ determinations and they are somewhat higher, though

### Table I: Summary of the fitted number of events and efficiencies, $\Delta B(\bar{B} \to X_u \ell \bar{\nu})$, and extracted $|V_{ub}|$ for the three kinematic cuts.

| Method | $N_u$ | $N_u^{\text{cut}}$ | $BG_u$ | $\epsilon_{u0} \epsilon_{\text{kin}}$ | $\frac{\epsilon_{u0}^{2} \epsilon_{\text{kin}}^{2}}{\sigma_{u0}^{2} \sigma_{\text{kin}}^{2}}$ | $\Delta B(\bar{B} \to X_u \ell \bar{\nu}) \times 10^{-3}$ | $|V_{ub}| \times 10^{-3}$ |
|--------|------|-----------------|------|---------------------|-------------------------------|-----------------------------|----------------|
| $M_X$  | 803  | 27 ± 2          | 923  | 0.331 ± 0.003       | 0.76 ± 0.02                   | 1.18 ± 0.09 ± 0.07 ± 0.01   | 4.27 ± 0.16 ± 0.13 ± 0.30 [4] |
| $P_+$  | 633  | 48 ± 5          | 1183 | 0.344 ± 0.003       | 0.81 ± 0.02                   | 0.95 ± 0.10 ± 0.08 ± 0.01   | 3.88 ± 0.19 ± 0.16 ± 0.28 [4] |
| $M_X, q^2$ | 562 | 32 ± 2          | 789  | 0.353 ± 0.005       | 0.79 ± 0.03                   | 0.81 ± 0.08 ± 0.07 ± 0.02   | 4.64 ± 0.23 ± 0.19 ± 0.25 [5] |

FIG. 2: Upper row: measured $M_X$ (a), $P_+$ (b) and $q^2$ with $M_X < 1.7$ GeV/$c^2$ (c) spectra (data points). The result of the fit to the sum of three MC contributions is shown in the histograms: $\bar{B} \to X_u \ell \bar{\nu}$ decays generated inside (no shading) and outside (dark shading) the selected kinematic region, and $\bar{B} \to X_l \ell \bar{\nu}$ and other background (light shading). Lower row: corresponding spectra for $\bar{B} \to X_u \ell \bar{\nu}$ after $\bar{B} \to X_l \ell \bar{\nu}$ and other background subtraction; they have been rebinned in order to show the shape of the kinematic variables.
TABLE II: Contributions to the systematic uncertainty on the measured $\Delta \mathcal{B} (B \to X_u \ell \bar{\nu})$, shown in percent (%) for the three kinematic cuts, from: detector, shape function (input parameters and functional form), exclusive $\mathcal{B} (B \to X_u \ell \bar{\nu})$, gluon splitting, exclusive $\mathcal{B} (B \to X_c \ell \bar{\nu})$, $B \to D^* \ell \tau$ form factors, $B(D)$, $m_{ES}$ fit, MC statistics. The last column gives the total systematic uncertainty.

| Method | Detector | Shape function $B (B \to X_u \ell \bar{\nu})$ | Gluon splitting $B (B \to X_c \ell \bar{\nu})$ | $B \to D^* \ell \tau$ form factors | $B(D)$ | $m_{ES}$ fit | Monte Carlo statistics | Total |
|--------|----------|---------------------------------|----------------------------------|---------------------------------|--------|--------------|----------------------|-------|
| $M_X$  | 1.92     | 0.90                           | 2.08                            | 1.62                            | 0.87   | 0.21         | 0.44                 | 3.71  |
| $P_t$  | 3.88     | 1.31                           | 2.22                            | 1.47                            | 2.80   | 0.39         | 0.73                 | 3.98  |
| $M_X, q^2$ | 3.83 | 2.43                           | 2.71                            | 1.02                            | 1.17   | 0.55         | 0.79                 | 5.17  |

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