Dual versions of extended supergravities

V. A. Tsokur, Yu. M. Zinoviev *

Institute for High Energy Physics
Protvino, Moscow Region, 142284, Russia

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Abstract

Recently, using the model of $N = 2$ supergravity — vector multiplets interaction with the scalar field geometry $SU(1, m)/SU(m) \otimes U(1)$ as an example, we have shown that even when the scalar field geometry is fixed, one can have a whole family of the Lagrangians, which differ by vector field duality transformation. In this paper we carry out the construction of such families for the case of $N = 3$ and $N = 4$ supergravities, the scalar field geometry being $SU(3, m)/SU(3) \otimes SU(m) \otimes U(1)$ and $SU(1, 1)/U(1) \otimes O(6, m)/O(6) \otimes O(m)$, correspondingly. Moreover, it turns out that these families contain, as a partial case, the models describing the interaction of arbitrary number of vector multiplets with our hidden sectors, admitting spontaneous supersymmetry breaking without a cosmological term.

*E-mail address: ZINOVIEV@MX.IHEP.SU
1 Introduction

As is well known (see e.g. [1]), scalar fields in extended supergravities usually describe some non-linear $\sigma$-models of the form $G/H$, where $G$ is some noncompact group, and $H$ — its maximal compact subgroup. In this, the group $G$ would be the global symmetry of the whole Lagrangian, but in many models it turns out that the realization of this group on the vector fields includes duality transformations. This leads to the fact that only a part of the group $G$ appears to be the symmetry of the Lagrangian, the rest being the symmetry of the equations of motion only. In turn, the invariance of the equations of motion leads to the existence of the whole family of the Lagrangians (we will call them dual versions) with the same scalar field geometry. In our recent paper [2], we have demonstrated this fact on the simplest example — $N = 2$ supergravity with vector multiplets and scalar field geometry $SU(1, m)/SU(m) \otimes U(1)$. Moreover, a deep connection between different dual versions and the problem of spontaneous supersymmetry breaking appeared.

In this paper we consider the generalization of such construction to the case of $N = 3$ and $N = 4$ supergravities. For completeness, in the next section we reproduce our $N = 2$ example, mentioned above. Then we show that it is easy to generalize this result to the case of $N = 3$ supergravity with arbitrary number of vector multiplets and scalar field geometry $SU(3, m)/SU(3) \otimes SU(m) \otimes U(1)$. Moreover, it turns out that the Lagrangian constructed contains, as a partial case, model [3], describing an interaction of ”matter” vector multiplets with the hidden sector [4], admitting spontaneous supersymmetry breaking without a cosmological term.

The $N = 4$ case appears to be more difficult. The reason is that the $N = 4$ supergravity itself [3, 4, 5] already contains two scalar fields, corresponding to the non-linear $\sigma$-model $SU(1,1)/U(1)$. So, we started with the pure $N = 4$ supergravity and found the most general Lagrangian for this theory. Then we have managed to construct the family of the Lagrangians, which gives the most general (to our knowledge) interaction of $N = 4$ supergravity with vector multiplets, scalar field geometry for the whole family being $SU(1,1)/U(1) \otimes O(6, m)/O(6) \otimes O(m)$. As in the $N = 3$ case, it turns out that as a partial case this family contains model [8], describing an interaction of vector multiplets with hidden sector [9].

2 $N = 2$ supergravity

In this section we give a brief description of our model for general interaction of $N = 2$ supergravity with vector multiplets, the scalar field geometry being $SU(1, m)/SU(m) \otimes U(1)$. The simplest way to describe this model is to introduce, apart from the graviton $e_{\mu \nu}$ and gravitini $\Psi_{\mu i}$, $i = 1, 2, m + 1$ vector multiplets $\{A_{\mu}^a, \Omega_{ia}, z^a\}$, $a = 0, 1, \ldots, m$. The following constraints correspond to the model with required geometry:

$$\bar{z}_a \cdot z^a = -2, \quad z^a \cdot \Omega_{ia} = 0. \quad (1)$$

In this, the theory has local axial $U(1)$ invariance with composite gauge field $U_\mu = (\bar{z}_a \partial_\mu z^a)$. The corresponding covariant derivatives look like, e.g.,

$$D_\mu z^a = \partial_\mu z^a + \frac{1}{2}(\bar{z} \partial_\mu z)z^a, \quad \bar{z}_a D_\mu z^a = 0.$$
\[
D_\mu \eta_i = D_\mu \eta_i - \frac{1}{4} (\bar{z} \partial_\mu z) \eta_i. \tag{2}
\]

The crucial point is that this local \(U(1)\) invariance and constraints \([1]\) do not determine the form of the Lagrangian and supertransformations unambiguously. Indeed, one can make the following general ansatz for the supertransformations:

\[
\begin{align*}
\delta \epsilon_{\mu \nu} &= i(\bar{\Psi}_\mu \gamma_\nu \eta), \\
\delta \Psi_{\mu i} &= 2D_\mu \eta_i - \frac{i}{2} \varepsilon_{ij} z^a E_{aa}(\sigma A)^a, \\
\delta A_m^a &= \varepsilon^{ij} (\bar{\Psi}_{\mu i} z^a K_a \eta_j) + i(\bar{\Omega}^a \gamma_\mu K_a \eta), \\
\delta \Omega_{ia} &= \frac{1}{2} \left[ E_{aa}(\sigma A)^a - g_{ab} \varepsilon^{b}(z^c E_{ca}(\sigma A)^a) \right] \eta_i - i\varepsilon_{ij} \partial z^a \eta_j, \\
\delta x^a &= \varepsilon^{ij} (\bar{\Omega}_{ia} \eta_j).
\end{align*}
\tag{3}
\]

where \(E_{aa}, K_a^a, \alpha = 0, 1, \ldots m\) and \(g_{ab}\) are constant matrices and then find the corresponding Lagrangian:

\[
L = -\frac{1}{2} R + i \varepsilon^{\mu \nu \rho \sigma} \bar{\Psi}_\mu \gamma_\nu \partial_\rho \Psi_\sigma + \frac{i}{2} \bar{\Omega}_{ia} \partial \Omega_{ia} + \frac{1}{2} D_\mu \bar{z}_a D_\mu z^a - \frac{1}{4} A^a A^a E_{aa} \bar{E}_\beta A^\beta \mu - \frac{1}{4} \left[ (z E A_{\mu \nu})(z E (A_{\mu \nu} + \gamma_5 \bar{A}_\mu)) \right] + h.c. - \frac{1}{2} \varepsilon^{ij} \bar{\Psi}_{\mu i} \bar{z}_a E_{aa}^a \left( A^a_{\mu \nu} - \gamma_5 A^a_{\mu \nu} \right) \eta_j - \frac{1}{2} \varepsilon^{ij} \bar{\Omega}_{ia} \gamma_\mu \gamma_\nu D_\nu z^a \eta_j + \frac{i}{4} \bar{\Omega}_{ia} \gamma_\mu \left[ E_{aa}(\sigma A)^a - g_{ab} \varepsilon^{b}(z^c E_{ca}(\sigma A)) \right] \eta_i. \tag{4}
\]

In this, the requirements of the closure of the superalgebra and the invariance of the Lagrangian give:

\[
\bar{K}^{\alpha \alpha} E_{a \beta} = \delta^{\alpha \beta}, \quad g_{ab} = K_a^a E_{ba}, \quad E_{a[a} \bar{E}_{b]}^{a} = 0. \tag{5}
\]

The first two equations allow one to express \(K_a^a\) and \(g_{ab}\) in terms of the \(E_{aa}\) while the last one turns out to be the only constraint on \(E_{aa}\). Note that in such model vector fields \(A_{\mu}^a\) and spinor \(\Omega_{ia}\) and scalar \(z^a\) fields carry different kind of indices exactly as in the general construction of \([1]\).

Thus, we have really obtained a whole family of the Lagrangians with the same scalar field geometry. In \([2]\) we have shown that as a partial case such family contains a model corresponding to the interaction of arbitrary number of vector multiplets with hidden sector \([1]\], admitting spontaneous supersymmetry breaking without a cosmological term. So, it is the choice of the dual version that determines the possibility of spontaneous supersymmetry breaking, while the scalar field geometry determines the pattern of such breaking, i.e. the structure of soft breaking terms that are generated after symmetry breaking had taken place. It is this close connection of the existence of dual versions and the problem of spontaneous supersymmetry breaking that makes an investigation of dual version for other extended supergravities interesting.
3 \( N = 3 \) supergravity

The \( N = 3 \) vector multiplets contain complex scalar fields \( z_i, \ i = 1, 2, 3 \), which are transformed under the triplet representation of \( SU(3) \) group. So the only natural candidate for the scalar field geometry, corresponding to the \( N = 3 \) supergravity — vector multiplet interaction is the non-linear \( \sigma \)-model \( SU(3, m)/SU(3) \otimes SU(m) \otimes U(1) \). Such a model was indeed constructed \cite{12, 4} some time ago. It turned out that due to reality of the vector fields only the real subgroup \( O(3, m) \) appeared to be the global symmetry of the Lagrangian, while the rest of the \( SU(3, m) \) group (containing vector field duality transformations) was the symmetry of the equations of motion only. But this means that there exists a whole family of the Lagrangians with the same scalar field geometry but with different vector fields coupling. It is this general coupling that we are going to construct in this section.

Let us introduce the following set of fields: graviton \( e_{\mu r} \), gravitini \( \Psi_{\mu i} \), Majorana spinor \( \rho \) and \( m + 3 \) vector multiplets \( \{ A^{a \mu} = A^{a \mu}_i \}, \lambda^a, z_i^a \} \), where \( a = 1, 2, \ldots m + 3 \) with the signature \((- - +, + + +)\). Then in order to have the model with the required scalar field geometry one has to impose the following constraints on the scalar and spinor fields:

\[
z_i^a z_a^j = -2\delta^j_i, \quad z_i^a \Omega_{ja} = \tilde{z}_a^i \lambda^a = 0. \tag{6}
\]

In this, the theory has local \( SU(3) \otimes U(1) \) invariance with composite gauge fields, e.g.

\[
D_{\mu} z_i^a = \partial_{\mu} z_i^a - \frac{1}{2} (\partial_{\mu} \tilde{z}) z_j^a, \quad \tilde{z}^a D_\mu z_j = 0. \tag{7}
\]

Now, by the analogy with the \( N = 2 \) case, we will try the following ansatz for the supertransformations, compatible with constraints (8) and \( SU(3) \otimes U(1) \) invariance:

\[
\begin{align*}
\delta e_{\mu r} &= i (\tilde{\Psi}_{\mu i} \gamma_\rho \eta_i), \\
\delta \Psi_{\mu i} &= 2D_\mu \eta_i + \frac{i}{2} \tilde{z}^{ijk} (g^{-1})^{ij} [z_k^a M^a_\alpha (\sigma A)_\alpha] \gamma_\mu \eta^k, \\
\delta A^{a \mu}_i &= -\tilde{z}^{ijk} (\tilde{\Psi}_{\mu j} K^a_\alpha \eta_k) + \frac{i}{\sqrt{2}} \rho z_i^a K^a_\alpha \gamma_\mu \eta_i + i (\dot{\Omega}_{ia} K^{aa}_\alpha \gamma_\mu \eta_i), \\
\delta \Omega_{ia} &= \frac{1}{2} (M^a_\alpha (\sigma A)_\alpha - g_{ab} z_j^b (g^{-1})^{jk} [z_k^a M^a_\alpha (\sigma A)_\alpha] \eta_i + i \varepsilon_{ijk} \gamma_\mu D_\mu z_j^a \eta_k, \\
\delta \rho &= -\frac{1}{\sqrt{2}} (g^{-1})^{ij} [z_j^a M^a_\alpha (\sigma A)_\alpha] \eta_i, \quad \delta \lambda^a = -i \gamma_\mu D_\mu \tilde{z}_a^i \eta_i, \\
\delta \varphi_i^a &= (\tilde{\lambda}^{a i} \eta_i) + \varepsilon_{ijk} (\tilde{\Omega}_{ja}^i \eta_k), \quad \delta \pi_i^a = -(\tilde{\lambda}^{a i} \eta_i) + \varepsilon_{ijk} (\tilde{\Omega}_{ja}^i \eta_k),
\end{align*}
\]

where \( K^a_\alpha, M^a_\alpha, \alpha = 1, 2, \ldots m + 3 \) and \( g_{ab} \) are constant matrices, while \( g_{ij} = z_i^a g_{ab} z_j^b \). Note, that a rather complicated structure for \( \delta \Omega_{ia} \) was chosen so that \( z_i^a \delta \Omega_{ja} = 0 \). In this, the requirement of the closure of the superalgebra on the bosonic fields leads to:

\[
\begin{align*}
\tilde{K}^a_\alpha M^a_\beta &= \delta^a_{\alpha \beta}, \\
K^a_\alpha &= g_{ab} \tilde{K}^{ba}.
\end{align*}
\tag{9}
\]

By rather long but straightforward calculations one can construct the corresponding Lagrangian:

\[
L = -\frac{1}{2} R + \frac{i}{2} \varepsilon^{\mu \nu \rho \sigma} \tilde{\Psi}_{\mu i} \gamma_5 \gamma_\rho D_\nu \Psi_{\sigma i} + \frac{i}{2} \bar{\rho} D \rho +
\]
I matrix $M$

peculiar features of the model). So, one has to make some kind of regularization for the two scalar fields in the supergravity multiplets. These fields parameterize non-linear model $SU(N)$ that one indeed can reproduce all the formulas from [3] in this way.

As we have already mentioned, the only difficulty arises from the fact that matrix $M$ allows one to express these relations together with ones from the requirement of the closure of the superalgebra on it.

The number of vector multiplets with hidden sector [4] (recall, that it is just the dual version physically more interesting) case is model [3], describing the interaction of the arbitrary number of vector multiplets with hidden sector [4] (recall, that it is just the dual version of the system $N = 3$ supergravity with three vector multiplets). The corresponding matrix $M$ looks like:

$$M = \left( \begin{array}{ccc} I_{3 \times 3} & \gamma_5 \times I_{3 \times 3} & 0_{3 \times (m-3)} \\ I_{3 \times 3} & -\gamma_5 \times I_{3 \times 3} & 0_{3 \times (m-3)} \\ 0_{(m-3) \times 3} & 0_{(m-3) \times 3} & I_{(m-3) \times (m-3)} \end{array} \right).$$  \hspace{1cm} (12)

The only difficulty arises from the fact that matrix $g_{ab}$ and hence $g_{ij}$ turns out to be degenerate in this case (this is the reason for the enhancement of the global symmetry and other peculiar features of the model). So, one has to make some kind of regularization for the matrix $M$ to avoid singularities, keeping only the terms which survive when regularization parameter goes to zero and making field rescaling if necessary. We have explicitly checked that one indeed can reproduce all the formulas from [4] in this way.

4 \hspace{1cm} N = 4 supergravity

As we have already mentioned, $N = 4$ case is more complicated due to the presence of two scalar fields in the supergravity multiplets. These fields parameterize non-linear $\sigma$-model $SU(1,1)/U(1)$, in this, the group $SU(1,1)$ fails to be the global symmetry of the
whole Lagrangian including vector field terms. This leads to the existence of different dual versions for such theory, the best known examples being so called $O(4)$ \[5, 6\] and $SU(4)$ \[7\] supergravities.

To describe the most general form of $N = 4$ supergravity let us introduce the following fields: graviton $e_{\mu r}$, gravitini $\Psi_{\mu r}$, $i = 1, 2, 3, 4$, vector fields $A_{\mu}^a$, $a = 1, 2, 3, 4, 5, 6$, Majorana spinors $\lambda_\alpha$, $\alpha = 0, 1$ and a couple of complex scalars $z^\alpha$. The scalars and spinors satisfy the usual constraints:

$$z^\alpha \bar{z}_\alpha = -2, \quad z^\alpha \lambda_\alpha = 0 \quad (13)$$

and the theory has local $U(1)$ invariance with the composite gauge field. Now one can choose the following general ansatz for the supertransformations:

$$\begin{align*}
\delta e_{\mu r} &= i(\bar{\Psi}_\mu \gamma_r \eta), \\
\delta \Psi_\mu &= 2D_\mu \eta - \frac{i}{4}(\sigma A)^m E_{ma} \tau^a \eta, \\
\delta A_\mu^m &= \bar{\Psi}_\mu z^\alpha K_{\alpha, ma} \bar{\tau}^a \eta - i\bar{\lambda}_\alpha \gamma_\mu K_{\alpha, ma} \bar{\tau}^a \eta, \\
\delta \lambda_\alpha &= -\frac{1}{4}\bar{\tau}^a z^\beta (\sigma A)^m E_{ma} \bar{\tau}^a \eta - i\gamma^\mu D_\mu z^\alpha \eta, \\
\delta \bar{z}_\alpha &= 2(\bar{\lambda}_\alpha \eta),
\end{align*} \quad (14)$$

where $K_{\alpha, ma}$ is a constant matrix, while $E_{ma}$ is a function of $z_\alpha$ with the axial charge equal to that of $\bar{z}_\alpha$. Besides, we introduced six antisymmetric matrices $(\tau^a)_{ij}$, such that

$$(\bar{\tau}^a)_{ij} = \frac{1}{2}\varepsilon^{ijkl}(\tau^a)_{kl}, \quad (\tau^a)_{ij}(\bar{\tau}^b)_{jk} + (a \leftrightarrow b) = -2\delta^i_k. \quad (15)$$

The requirement of the closure of the superalgebra on the bosonic fields leads to:

$$K_{\alpha, ma} = \varepsilon_{\alpha, \beta} \bar{K}_{\beta, ma}, \quad K_{ma} E_{na} = z^\alpha K_{\alpha, ma} E_{na} = \delta^m_n. \quad (16)$$

The corresponding ansatz for the Lagrangian looks like:

$$L = -\frac{1}{2}R + \frac{i}{2}\varepsilon^{\mu \nu \rho \sigma} \bar{\Psi}_\mu \gamma_5 \gamma_\nu D_\rho \Psi_\sigma + \frac{i}{2}\bar{\lambda}_\alpha \bar{D}_\alpha \lambda_\alpha + \frac{1}{2}D_\mu \tilde{z}^\alpha D_\mu \bar{z}_\alpha +$$

$$+\frac{1}{8}M_{ma} E_{na} A_{\mu \nu}^m (A_{\mu \nu} + \gamma_5 \bar{A}_{\mu \nu})^n + h.c. -$$

$$-\frac{1}{4}\bar{\Psi}_\mu (A_{\mu \nu} - \gamma_5 \bar{A}_{\mu \nu})^m \bar{E}_{ma} \bar{z}^a \Psi_\nu - \frac{1}{2}\bar{\lambda}_\alpha \gamma^\mu \gamma_\nu D_\nu \bar{z}^\alpha \Psi_\mu +$$

$$+\frac{i}{8}\bar{\lambda}_\alpha \gamma^\mu \varepsilon_{\alpha, \beta} \bar{z}^\beta (\sigma A)^m E_{ma} \bar{z}^a \Psi_\mu, \quad (17)$$

where $M_{ma}$ is also the function of $z^\alpha$. This Lagrangian will be invariant under the supertransformations provided:

$$\varepsilon^{\alpha, \beta} K_{\alpha, ma} K_{\beta, na} = 0, \quad M_{ma} \bar{K}_{na} + \bar{M}_{ma} K_{na} = -2\delta_{mn}. \quad (18)$$

If one chooses $M_{ma} = z^\alpha M_{\alpha, ma}$, where $M_{\alpha, ma}$ is a constant matrix, satisfying $M_{\alpha, ma} = -\varepsilon_{\alpha, \beta} \bar{M}_{\beta, ma}$, then the second relation gives:

$$\bar{M}_{ma} K_{\alpha, na} = \delta_{mn}. \quad (19)$$
Note, that this relation determines \( M_{ma} \) only up to the change \( M_{ma} \rightarrow M_{ma} + \omega \gamma_5 K_{ma} \), but the corresponding Lagrangians differ by the total divergency.

Now we are ready to consider the most general interaction of \( N = 4 \) supergravity with the arbitrary number of vector supermultiplets, the scalar field geometry being \( SU(1,1)/U(1) \otimes O(6,m)/O(6) \otimes O(m) \). For this purpose we shall use the same set of fields as before, but instead of six vector fields \( A_{\mu}^A \) we introduce now \((6+m)\) vector supermultiplets \((A_{\mu}^A, \Omega^A, \Phi^A)\), \( A = 1,2,...m+6 \). The scalar and spinor fields satisfy the following constraints:

\[
\Phi_a A \Phi_b A = -\delta_{ab}, \quad \Phi_a A \Omega_{IA} = 0 \tag{20}
\]

which correspond to the required geometry and lead to the local \( O(6) \) invariance, as usual. Taking into account all the scalar and spinor field constraints, we choose the following ansatz for the supertransformations, generalizing our results for pure \( N = 4 \) supergravity:

\[
\delta \Psi_\mu = 2 \mathcal{D}_\mu \eta - \frac{i}{4} (\sigma A)^M E_{MA} \Phi_a A (g^{-1})^{ab} \bar{\tau}^b \gamma_\mu \eta,
\]

\[
\delta A_\mu^M = (\Psi_\mu \gamma^a K_{\alpha}^{MA} \Phi^A_{\alpha} \bar{\tau}^a \eta) + i (\bar{\Omega}_{\alpha} \gamma_\mu z^\alpha K_{\alpha}^{MA} \eta) - i (\bar{\lambda}_{\alpha} \gamma_\mu \Phi_{\alpha}^{MA} A \bar{\tau}^a \eta),
\]

\[
\delta \Omega^A = -\frac{1}{2} (E_{MA} (\sigma A)^M - g_{AB} A \Phi_a B (g^{-1})^{ab} \bar{\tau}^b \eta) - i \gamma_\mu \mathcal{D}_\mu A \Phi_a A \bar{\tau}^a \eta,
\]

\[
\delta \lambda_\alpha = -\frac{1}{4} \varepsilon_{\alpha \beta z} \Phi^A (g^{-1})^{ab} \bar{\tau}^b \eta - i \gamma_\mu \mathcal{D}_\mu A \Phi_a A \bar{\tau}^a \eta, \quad \delta \bar{\lambda}_\alpha = 2 (\bar{\lambda}_\alpha \eta), \tag{21}
\]

where \( K_{\alpha}^{MA} \) are constant matrices, \( E_{MA} \) and \( G_{AB} \) are functions of \( z^\alpha \) to be determined, while \( g_{ab} = \Phi_a A g_{AB} \Phi_b B \). The requirement of the closure of the superalgebra on the bosonic fields leads to the following relations on them:

\[
K_{\alpha}^{MA} E_{NA} = \delta^M N, \quad K_{\alpha}^{MA} = g^{AB} \bar{K}_{MB}, \quad K_{\alpha}^{MA} = \varepsilon_{\alpha \beta} \bar{K}_{\beta,MA} \tag{22}
\]

where \( K_{\alpha}^{MA} = z^\alpha K_{\alpha}^{MA} \). It is not hard to construct the fermionic part of the corresponding Lagrangian:

\[
L_F = \frac{1}{2} \bar{\Omega}^\alpha \gamma_\mu \gamma_\nu \mathcal{D}_\nu A \Phi_a A \bar{\tau}^a \Psi_\mu - \frac{1}{2} \bar{\lambda}_\alpha \gamma_\mu \gamma_\nu \mathcal{D}_\nu z^\alpha \Psi_\mu - \frac{1}{4} \bar{\Psi}_\mu (A_{\mu \nu} - \gamma_5 \bar{A}_{\mu \nu})^M E_{MA} \Phi_a A (g^{-1})^{ab} \bar{\tau}^b \Psi_\nu + i \bar{\Omega}^\alpha \gamma_\mu [E_{MA} (\sigma A)^M - g_{AB} A \Phi_a B (g^{-1})^{ab} \bar{\tau}^b \eta] \Psi_\mu + i \bar{\lambda}_\alpha \varepsilon_{\beta} \bar{z}_\beta \gamma_\mu (\sigma A)^M E_{MA} \Phi_a A (g^{-1})^{ab} \bar{\tau}^b \Psi_\mu + \frac{1}{8} \bar{\Omega}^A [E_{MA} - g_{AB} A \Phi_a B (g^{-1})^{bc} \Phi_b C E_{MC}] (\sigma A)^M \bar{z}_\alpha \varepsilon_{\beta} \lambda_\beta - \frac{1}{4} \bar{\Omega}^A (\sigma A)^M E_{MB} \Phi_a B (g^{-1})^{ab} \bar{\tau}^b \Omega^A. \tag{23}
\]

The invariance of the whole Lagrangian can be achieved with the following form of vector fields kinetic terms:

\[
L_V = -\frac{1}{4} A_{\mu \nu}^M A_{\mu \nu}^N E_{MA} \bar{E}_{NA} - \frac{1}{4} \gamma_5 A_{\mu \nu}^M \bar{A}_{\mu \nu}^N [\bar{E}_{MA} \bar{F}_{NA} - E_{MA} F_{NA}] + \frac{1}{4} A_{\mu \nu}^M (A_{\mu \nu}^N + \gamma_5 \bar{A}_{\mu \nu}^N) [E_{MA} \Phi_a A (g^{-1})^{ab} \Phi_b B E_{NB}] + h.c., \tag{24}
\]
where $F_{MA}$ — one more function of $z^\alpha$, provided:

$$
g_{AB} = E_{NA} K^N_B \quad E_{MA} E_{NA} = E_{NA} E_{MA} \quad E_{MA} F_{NA} = E_{NA} F_{MA} \quad (25)$$

Thus we have managed to construct the most general (as far as we know) model for the $N = 4$–matter interaction, all previously known results [13, 14, 15, 16] being partial cases of our general formulas. Moreover, just as in the previous cases, one can obtain as one more interesting partial case our model [8], corresponding to the interaction of arbitrary number of vector multiplets with the hidden sector [9], admitting spontaneous supersymmetry breaking without a cosmological term.

Note, that another interesting result can be easily obtained from the formulas given above. Indeed, if one puts $i = 1, 2, a = 1, 2, A + 1, 2, \ldots m + 2$ and uses $\tau^a = (\varepsilon^{ij}, \gamma^5 \varepsilon^{ij})$ then the same formulas given one the most general dual version for the $N = 2$ supergravity interacting with vector multiplets with scalar field geometry $SU(1, 1)/U(1) \otimes O(2, m)/O(2) \otimes O(m)$!

## 5 Conclusion

Thus, we have seen that for all extended $N = 2, 3, 4$ supergravities there exist dual versions for the Lagrangian of supergravity — matter interaction having the same scalar field geometry but different vector fields couplings. This fact appears to be tightly connected with the problem of spontaneous supersymmetry breaking in such theories. The reason is that the choice of dual version determines the global symmetry of the Lagrangian, which in turn implies different possible gaugings and leads to the models with or without spontaneous supersymmetry breaking, cosmological term and so on. One more interesting question arises in the superstring context. Namely, if one has a four dimensional superstring having the extended $N > 1$ supersymmetry it is not enough to determine the geometry of the scalar fields to fix the effective low-energy Lagrangian. As we have seen, one has also to know the dual version it corresponds to.

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