A Universe with a Ghost Dark Energy and van der Waals fluid interacting with a Fluid

Martiros Khurshudyan
CNR NANO Research Center S3, Via Campi 213a, 41125 Modena MO, Italy
and
Dipartimento di Scienze Fisiche, Informatiche e Matematiche,
Università degli Studi di Modena e Reggio Emilia, Modena, Italy
email:martiros.khurshudyan@nano.cnr.it

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Abstract
We consider a model of a Universe with Ghost Dark Energy and van der Waals fluid interacting with a fluid which was born as a result of interaction between original fluid and some other fluid existing in Universe. We suppose that Ghost Dark energy has its contribution to the model by an interaction term $Q$ and we suppose that $Q = 3Hb(\rho_{tot} - \rho_{GDe})$. 
Introduction

We consider a Universe composed of van der Waals fluid [1], [2]

\[ P_w = \frac{8\omega_w \rho_w}{3 - \rho_w} - 3\rho_w^2. \] (1)

interacting with a modified fluid model [3]

\[ \rho_m = \rho_m^0 a^{-3} f(a), \] (2)

\[ f(a) = 1 + \gamma a^5 \exp[-a^2/\sigma^2]. \] (3)

where \( a(t) \) is a scale factor, \( \gamma \) is an interaction coefficient. The origin of such fluid can be supposed to be an interaction or different types of interactions between an original fluid and other fluids in Universe, which are coded in \( \gamma \). Generally the origin of this fluid can be anything, but here we will not make assumption and will consider the Universe in the presence of this fluid. We will consider an interaction between last two fluids and we will suppose the presence of a Ghost Dark energy [4], [5] with energy density given as

\[ \rho_{GDe} = \alpha H, \] (4)

where \( H(t) = \dot{a}/a \) is the Hubble parameter. Concerning to this component we assume that Ghost Dark energy with [4] will appear and play a role only in interaction term. Dark Energy was born in GR to explain experimental data claiming, that the expansion of the Universe is accelerated. Dark Energy with Dark Matter are exotic and mysterious components of the Universe, capturing a lot of attention and not to be explained still. Generally, it is accepted that Dark Energy should be described by negative pressure and positive energy density, therefore implying negative EoS parameter \( \omega < 0 \). The simplest model is a cosmological constant \( \omega_\Lambda = -1 \) introduced by Einstein, but it suffers with the problem known as cosmological coincidence problem asking why are we living in an epoch in which the densities of dark energy and matter are comparable? One of the ways to solve the cosmological coincidence problem is to consider the interaction between the components on phenomenological level. Generally, interaction could be considered as a function of energy densities and their derivatives: \( Q(\rho_i, \dot{\rho}_i, \ldots) \). To make everything working with units, we only should consider the fact that unit of interaction term \( [Q] = [\text{energy density}]/[\text{time}] \) and assume that unit \( [\text{time}]^{-1} \) could be contributed, for instance, from Hubble parameter. Over years different models of interactions were proposed and considered, but everything was done on phenomenological level only. Very intensively were considered interactions \( Q = 3Hb\rho_m, Q = 3Hb\dot{\rho}_{de}, Q = 3Hb\dot{\rho}_{tot} \), where \( b > 0 \) is a coupling constant, other form of interactions, where question of time unit were solved with help of first order time derivative \( Q = \gamma\dot{\rho}_m, Q = \gamma\dot{\rho}_{de}, Q = \gamma\dot{\rho}_{tot} \). Interaction of the general form \( Q = 3Hb\gamma\rho_i + \gamma\dot{\rho}_i \), where \( i = \{m, de, tot\} \) also captured a lot of attention. Interaction between components arose as a result of splitting of the energy conservation, which mathematically can be described as follows:

\[ \dot{\rho}_1 + 3H(\rho_1 + P_1) = Q \] (5)

and

\[ \dot{\rho}_2 + 3H(\rho_2 + P_2) = -Q, \] (6)

which could be understood as: there is not energy conservation for the components separately, but due to interaction between all components, energy of whole mixture
conserves. This approach is correct for this moment only. Other type of interaction considered in literature and supported by experimental data is sign-changeable interaction \[6\], \[7\], \[8\]. Motivated by \[8\] we would like to consider an interaction where energy density of the Ghost Dark energy appears

\[ Q = 3Hb(\rho_{tot} - \rho_{GDe}). \]  

(7)

By this way we would like to make a possibility for other two components to feel an existence of other component which does not appear in field equations. The mixture of our consideration will be described by \( \rho_{tot} \) and \( P_{tot} \) given by

\[ \rho_{tot} = \rho_w + \rho_m \]  

(8)

and

\[ P_{tot} = P_w + P_m. \]  

(9)

Paper organized as follow: After Introduction, in next section we will give field equations and a strategy how to solve and find scale factor \( a(t) \). Then in two other sections we will work in case when there is not interaction between components and then we will switch on interaction. In section Discussion obtained results will be summarized.

**Field Equations and Mathematics**

Field equations that governs our model of consideration are

\[ R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = T^{\mu\nu} \]  

(10)

and with FRW metric (the metric of a spatially flat homogeneous and isotropic universe)

\[ ds^2 = dt^2 - a(t)^2 (dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2). \]  

(11)

they reduce to

\[ H^2 = \frac{\dot{a}^2}{a^2} = \frac{\rho_{tot}}{3} \]  

(12)

\[ \dot{H} = -\frac{1}{2}(\rho_{tot} + P_{tot}). \]  

(13)

Energy conservation condition reads as

\[ \dot{\rho}_{tot} + 3H(\rho_{tot} + P_{tot}) = 0. \]  

(14)

To introduce an interaction between Dark Energy and Matter, as already was mentioned in Introduction section, \( \[14\] \) splits into two following equations

\[ \dot{\rho}_m + 3H(\rho_m + P_m) = Q \]  

(15)

and

\[ \dot{\rho}_w + 3H(\rho_w + P_w) = -Q. \]  

(16)

In our case problem solving strategy will start with finding pressure of Modified fluid from \( \[15\] \)

\[ P_m = \frac{Q - \dot{\rho}_m}{3H} - \rho_m. \]  

(17)
Pressure of van der Waals fluid \[\text{with (12)}\] will be expressed by \(\rho_m\)

\[
P_w = \frac{8\omega_w(3H^2 - \rho_m)}{3 - (3H^2 - \rho_m)} - 3(3H^2 - \rho_m)^2.
\] \hspace{1cm} (18)

(12), (13), (17) and (18) together allow us to determine a scale factor. From the next section we will start from the simple case, when interaction between Modified Fluid and van der Waals fluid is absence and then we will present results accounting interaction (7).

**Non interaction case**

Non interacting case with \(Q = 0\) simplify equations under consideration and solution could be find very easily. Bellow we will present behavior of \(\ddot{a}\) and \(\omega_{tot}\) over time (Fig. 1-3).

**Figure 1:** \(\ddot{a}\) against \(t\) and \(\omega_{tot}\) against \(t\). Parameters: \(\gamma = 0.5, \omega_w = 1, \sigma = 1.5, \rho_{0m} = 1\)

**Figure 2:** \(\ddot{a}\) against \(t\) and \(\omega_{tot}\) against \(t\). Parameters: \(\gamma = 0.5, \omega_w = 0.1, \sigma = 1.5, \rho_{0m} = 1\)
Interaction case

In this section we will analyze the case when we account interaction between mixture components. Below we presented graphical analysis for several cases, with different values of parameters (Fig. 5-8).

Figure 3: $\ddot{a}$ against $t$ and $\omega_{tot}$ against $t$. Parameters: $\gamma = 0.5$, $\omega_{w} = 1$, $\sigma = 1$, $\rho_{0m} = 1$

Figure 4: $\ddot{a}$ against $t$ and $\omega_{tot}$ against $t$. Parameters: $\gamma = 0.5$, $\omega_{w} = 1$, $\sigma = 1.5$, $\rho_{0m} = 1$, $b = 0.4$, $\alpha = 1$
Discussion

We suppose a model of an Universe consists of Ghost Dark energy, van der Waals fluid interacting with a modified fluid, which could be result of some interaction (we could only suppose). Interaction between components were assumed to be \( Q = 3bH(\rho_{\text{tot}} - \rho_{\text{GDE}}) \). By this way the role of the Ghost Dark energy were seen by interaction term only. During the investigation we recover scale factor \( a(t) \) and present graphical analysis of \( \ddot{a} \) and EoS parameter \( \omega_{\text{tot}} \). Analysis shows that the effects concerning to the presence of a Ghost Dark Energy expressed in interaction term was observed in later stages of the evolution comparing with cases corresponding to absence and presence an interaction of the \( Q = 3bH\rho_{\text{tot}} \) form between components. From the graphical analysis we see, that having a situation proposed in this article it is not always true the assumption that dark energy could be the source of the accelerated expansion of the Universe. At least, for some values of the parameters of the model we observe that the mixture behaves as a dark energy with negative EoS parameter, while \( \ddot{a} < 0 \). For some cases we saw that dark energy really is responsible for accelerated expansion with \( \ddot{a} > 0 \). Almost the same result we obtain considering a modified fluid not described by the energy density of our consideration, but as a fluid described as \( P = -\rho \). Which corresponds to a cosmological constant model with \( \omega = -1 \). Here we have following possibilities, first that the modification of a fluid of our consideration could not work with van der Waals gas, or probably an interaction of the consideration between components is not realistic and other modification should be done with interaction term. There is other possibility, which from our opinion is also has right
Figure 7: $\ddot{a}$ against $t$ and $\omega_{tot}$ against $t$. Parameters: $\gamma = 0.5$, $\omega_w = 1$, $\sigma = 1$, $\rho_0m = 1$, $b = 0.4$, $\alpha = 1$

Figure 8: $\ddot{a}$ against $t$ and $\omega_{tot}$ against $t$. Parameters: $\gamma = 0.5$, $\omega_w = 1$, $\sigma = 1$, $\rho_0m = 1$, $b = 0.4$, $\alpha = 2.5$

to be, that dark energy is not always was and will be responsible for the accelerated expansion of the Universe.

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Figure 9: \( \ddot{a} \) against \( t \) and \( \omega_{\text{tot}} \) against \( t \). Parameters: \( \gamma = 0.5, \omega_w = 1, \sigma = 1.5,\rho_{\text{um}} = 1, \, b = 0.4, \, Q = 3Hb\rho_{\text{tot}} \)

Figure 10: \( \ddot{a} \) against \( t \) and \( \omega_{\text{tot}} \) against \( t \). Parameters: \( \gamma = 0.5, \omega_w = 1, \sigma = 1,\rho_{\text{um}} = 1, \, b = 0.4, \, Q = 3Hb\rho_{\text{tot}} \)