A new braneworld in the sourced-Taub background is proposed. The gravity field equations in the internal source region and external vacuum region are investigated, respectively. We find that the equation of state for the effective dark energy of a dust brane in the source region can cross the phantom divide $w = -1$. Furthermore, there is a drop on $H(z)$ diagram, which presents a possible mechanism for the recent direct data of $H(z)$.

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I. INTRODUCTION

The brane world scenario is an impressive progress in high energy physics and cosmology in recent years. The basic idea of this scenario is that the standard model particles are confined to the 3-brane, while the gravitation can propagate in the whole spacetime [1]. Cosmology in the brane world scenario has been widely investigated. In the braneworld cosmology the bulk spacetime is assumed to be Schwarzschild-Anti-de Sitter (AdS)[2] or just Minkowski [3].

However, such bulks are not necessary for cosmology. In principle, we only require that the bulk space admits a 3-dimensional maximally symmetric spacelike submanifold, which serves as our space. Besides Schwarzschild-AdS (dS) and Minkowski, a 5-dimensional vacuum Taub space and the source of Taub space also admit 3-dimensional maximally symmetric spacelike submanifold, which are proper backgrounds for brane cosmology. We will study the cosmology in the background of a sourced-Taub bulk.

The source of Taub space is a long standing problem [4]. In a recent series of works [5, 6, 7], we successfully find the source of the Taub space both in 4-dimension and higher dimensions. The components of the source region are also preliminarily studied. These results strengthen the physical foundation of Taub space.

In the next section we briefly review the former results of the source of the Taub solution. In section III, we shall explain our set up of the sourced Taub system with a moving brane. We study the cosmology of this set up in section IV. In section V we present our conclusion.

II. THE SOURCE OF TAUB SPACE

A new class of static plane symmetric solution of Einstein field equation sourced by a perfect fluid was successfully found in 4-dimensional spacetime in [5]. This solution is identified as the source of the Taub space since there is a special family in this solution which can perfectly match to the Taub space.

In [6], we generalize the 4-dimensional source of Taub solution to a higher dimensional one, which reads,

$$
\text{ds}^2 = -e^{2az}dt^2 + dz^2 + e^{2[a_2z + be^{az/(n-3)}]}d\Sigma^2,
$$

where

$$
d\Sigma^2 = (dx^1)^2 + (dx^2)^2 + ... + (dx^{n-2})^2.
$$

The above metric [1] is an exact solution of Einstein field equation sourced by a perfect fluid,

$$
T = (\rho(z) + p(z))U \otimes U + p(z)g_n,
$$

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where $T$ denotes the energy momentum tensor of the fluid, $U$ stands for 4-velocity of the fluid, $g_n$ denotes the n-dimensional metric tensor, and,

$$
\rho = -\frac{a^2}{2} \frac{n-2}{(n-3)^2} \left[(n-3)^2(n-1) + 2(n-2)^2 be^{az/(n-3)} + (n-1)b^2 e^{2az/(n-3)}\right], \quad (4)
$$

$$
\rho = \frac{a^2}{2} \frac{n-2}{(n-3)} \left[(n-3)(n-1) + 2(n-2) be^{az/(n-3)} + b^2 e^{2az/(n-3)}\right]. \quad (5)
$$

In 4-dimensional case, the above energy momentum can be a phantom $\psi$ with dust and photon. The Lagrangian for the source is

$$
\mathcal{L}_{\text{source}} = \frac{1}{2} \partial_{\mu} \psi \partial^{\mu} \psi - U(\psi) + \mathcal{L}_{\text{dust}} + \mathcal{L}_{\text{photon}}, \quad (6)
$$

where the potential $U(\psi)$ is rather complicated, see [6] for details, $\mathcal{L}_{\text{dust}}$ denotes the Lagrangian for the dust and $\mathcal{L}_{\text{photon}}$ represents the Lagrangian of the photon. The substance of n-dimensional source is almost the same as that of the 4-dimensional case. The reason of this similarity roots in the following fact: the only reasonable $U(\psi)$ is only a function of $z$ [6].

It is found that a family in the solution \(1\) can perfectly match to the n-dimensional Taub solution, and thus is the source of a n-dimensional Taub. We write the vacuum Taub metric as follows,

$$
ds^2 = -z^2 a k^2 dt^2 + dz^2 + z^{2\beta} l^2 d\Sigma^2, \quad (7)
$$

where $\alpha = -\frac{n-3}{n-1}$, $\beta = \frac{2}{n-1}$. $k, l$ are two constants. If the vacuum region resides in $z > z_0$ and the source region inhabits $z < z_0$, the matching condition at the boundary $z = z_0$ requires,

$$
k = \pm \frac{e^{az_0}}{z_0}, \quad (8)
$$

$$
l = \pm \frac{e^{az_0 + be^{az_0/(n-3)}}}{z_0^\beta}, \quad (9)
$$

and

$$
a z_0 = -\frac{n-3}{n-1}. \quad (10)
$$

For detailed discussions, see our original Letter [7].

### III. THE SET UP

We consider a 3-brane imbedded in a 5-dimensional bulk. The action includes the action of the bulk and the action of the brane,

$$
S = S_{\text{bulk}} + S_{\text{brane}}. \quad (11)
$$

Here

$$
S_{\text{bulk}} = \int_{\mathcal{M}} d^5 X \sqrt{- \det (g_5)} \left( \frac{1}{2 \kappa^2} R_5 + \mathcal{L}_{\text{bulk}} \right), \quad (12)
$$

where $X = (t, z, x^1, x^2, x^3)$ is the bulk coordinate, $x^1, x^2, x^3$ are the coordinates of the maximally symmetric space. $\mathcal{M}$, $\det (g_5)$, $\kappa$, $R_5$, $\mathcal{L}_{\text{bulk}}$, denote the bulk manifold, the determinant of the bulk metric, the 5-dimensional Newton constant, the 5-dimensional Ricci scalar, and the bulk matter Lagrangian, respectively. Note that the bulk matter Lagrangian $\mathcal{L}_{\text{bulk}}$ can be a phantom with dust and photon.

The action of the brane can be written as,

$$
S_{\text{brane}} = \int_{\mathcal{M}} d^4 x \sqrt{- \det (g)} \left( \kappa^{-2} K + \mathcal{L}_{\text{brane}} \right), \quad (13)
$$
where \( M \) indicates the brane manifold, \( \det(g) \) denotes the determinant of the brane metric, \( L_{\text{brane}} \) stands for the Lagrangian confined to the brane, and \( K \) marks the trace of the second fundamental form of the brane. \( x = (\tau, x^1, x^2, x^3) \) is the brane coordinate. Note that \( \tau \) is not identified with \( t \) if the the brane is not fixed at a position in the extra dimension \( z = \text{constant} \). We will investigate the cosmology of a moving brane along the extra dimension \( z \) in the bulk, and such that \( \tau \) is different from \( t \).

We set the Lagrangian confined to the brane as follows,

\[
L_{\text{brane}} = -\lambda + L_m, \tag{14}
\]

where \( \lambda \) is the brane tension and \( L_m \) denotes the ordinary matter, such as dust and radiation, located at the brane. We see that our set up is a brane with tension and ordinary matter imbedded in a 5-dimensional vacuum bulk. Further we assume the bulk space is a sourced-Taub space. In this case, the n-dimensional metric in section II degenerates to,

\[
g_5 = e^{2f(z)} dt^2 - dz^2 - e^{2l(z)} \left[ (dx^1)^2 + (dx^2)^2 + (dx^3)^2 \right], \tag{15}
\]

where

\[
f(z) = az, \tag{16}
\]

\[
l(z) = az + be^{\frac{a}{2}z}, \tag{17}
\]

in the source region, and

\[
e^{2f(z)} = k^2 z^{-1}, \tag{18}
\]

\[
e^{2l(z)} = m^2 z, \tag{19}
\]

in the vacuum region, where \( a, b, k, m \) are constants. Correspondingly, the matching condition (8, 9, 10)

\[
k = \pm e^{\frac{az_0}{\alpha_0}}, \tag{20}
\]

\[
m = \pm e^{\frac{az_0 + be^{az_0/2}}{\beta_0}}, \tag{21}
\]

and

\[
az_0 = -\frac{1}{2}, \tag{22}
\]

where the subscript 0 denotes the value at the boundary of a quantity, and \( \alpha = -1/2, \beta = 1/2 \). So there are only two free parameters, \( a \) and \( b \), in this set-up (source and external vacuum space), just as the same of the source. This is natural since the source should uniquely determine the external vacuum metric. Under this matching condition, the metric is only \( C^1 \) at the boundary.

By considering a brane is moving in such a bulk, we investigate the cosmology of the brane in the next section.

**IV. COSMOLOGY**

We take a method developed in [8]. We suppose that a 3-brane is moving along \( z \) direction, whose velocity is

\[
u^\mu = (u^0, u^1) = (\dot{t}, \dot{z}), \tag{23}
\]

where the other 3 components of \( u \) have been omitted, since they are just 0, and a dot stands for the derivative with respective to the proper time of the orbit of the brane in the bulk, \( \tau \). The normalization condition of \( u \) requires,

\[
g_5(u, u) = 1, \tag{24}
\]
which implies,
\[ e^{2f} \dot{t}^2 - \dot{z}^2 = 1. \]  
(25)

The normal of the velocity satisfies,
\[ g_5(n, u) = 0, \]  
(26)

and
\[ g_5(n, n) = -1, \]  
(27)

which yield,
\[ e^{2f} n^0 \dot{t} - n^1 \dot{z} = 0, \]  
(28)

and
\[ e^{2f} (n^0)^2 - (n^1)^2 = -1, \]  
(29)

respectively. Associating (26), (27) and (24), we obtain
\[ (n^1)^2 = \dot{z}^2 + 1, \]  
(30)

\[ (n^0)^2 = e^{-4f} (\dot{z}^2 + 1) \frac{\dot{z}^2}{\ell^2}. \]  
(31)

\( n \) can be determined up to a sign, since direction of \( n \) has not been uniquely selected. The induced metric on the brane reads,
\[ g = g_5 + n \otimes n = d\tau^2 - e^{2f(z)} \left[ (dx^1)^2 + (dx^2)^2 + (dx^3)^2 \right]. \]  
(32)

The second form of the brane is the Lie derivation of the induced metric along the normal direction,
\[ K = \frac{1}{2} L_{\vec{n}} g, \]  
(33)

whose spatial components read,
\[ K_{ij} = \frac{dL}{dz} n^1 g_{ij}, \]  
(34)

where \( i, j \) denote the spatial index of the brane universe. By imposing a \( Z_2 \) symmetry on the two sides of the brane, the matching condition across the brane yields,
\[ K_{\mu\nu} - K g_{\mu\nu} = -\kappa^2 s_{\mu\nu}, \]  
(35)

where lower case of Greeks denotes the index of the quantity of the brane, which runs from 0 to 3, and \( s_{\mu\nu} \) represents the energy momentum confined to the brane, which is defined as,
\[ s_{\mu\nu} \triangleq \frac{2}{\sqrt{-\det(g)}} \frac{\delta (\sqrt{-\det(g)} L_{\text{brane}})}{\delta g_{\mu\nu}}, \]  
(36)

where \( L_{\text{brane}} \) is given by (14). The spatial components of (35) imply the Friedmann equation on the brane,
\[ \left( \frac{dl}{dz} \right)^2 (1 + \dot{z}^2) = \frac{\kappa^4}{36} \rho_{br}^2, \]  
(37)

where \( \rho_{br} \) is the density of the brane, which is defined as,
\[ \rho_{br} \triangleq s^0_0. \]  
(38)
Here, the final form of the Friedmann equation is independent of the form of \( f(z) \). We note that \( \rho_{br} \) carries the effect of all the matters confined to the brane, including the vacuum energy \( \lambda \). Only the Friedmann equation (37) is not enough to determine the evolution of the brane universe. The other essential equation is the Bianchi identity (continuity equation),

\[
\rho_{br} + 3H(\rho_{br} + p_{br}) = 0,
\]

where \( H \) is the Hubble parameter, which is defined as

\[
H \triangleq \sqrt{\frac{\dot{z}}{z}},
\]

\( p_{br} \) denotes the pressure of the brane for a comoving observer,

\[
p_{br} \triangleq s_1^1 = s_2^2 = s_3^3.
\]

The continuity equation (39) also can be derived from the time component of the matching condition (35). From (40) we see that the spatial coordinate \( \sqrt{z} \) plays the role of the scale factor of the brane universe. The Friedmann equation (37) is a special case of the most general form of braneworld model [9]. In this article, we first study a brane imbedded in the sourced Taub space.

In the source region of the bulk, \( l(z) = az + be^{az^2} \), hence the Friedmann equation (37) becomes,

\[
\frac{1}{4z^2} + H^2 = \frac{\kappa^2 \rho_{br}}{144a^2z^2(1 + \frac{a}{4}e^{az^2})^2}.
\]

In the external vacuum region, \( e^{2l(z)} = m^2 z \), the Friedmann equation (37) becomes,

\[
\frac{1}{4z^2} + H^2 = \frac{\kappa^2 \rho_{br}}{576}.
\]

Two comments on the Friedmann equation are listed as follows:

1. A term \( 1/z^2 \), which is very alike the radiation term, but it is not the radiation, appears. This term is often called dark radiation in the literatures, which is the contraction of bulk Weyl tensor. In a spherically symmetric bulk, the physical sense of this term is the gravitational mass of the bulk space. In the present set up, it is a free term since \( z \) is free adjusting coordinate. This confirms the fact that the Taub space has no proper definition of gravitational mass since it is not asymptotically flat.

2. The present model only admits a spatially flat universe since Taub space does not admit other type of splittings with a 3 dimensional space-like submanifold. It is clear that the 5-dimensional sourced-Taub (15) admits 6 space-like Killing vectors, including 3 translational vectors and 3 rotational ones, which span a 6 dimensional Euclidean group \( E_6 \).

We expect the present braneworld model can say something on the dark energy problem. First, we try to obtain the condition for cosmic acceleration in this model. Since the Friedmann equation seems fairly complicated, we deduce the acceleration equation by a very general function of the density of \( \rho_{br} \) and scale factor \( R \triangleq \sqrt{z} \),

\[
H^2 = \phi(\rho_{br}, R).
\]

Associating with continuity equation (39), one derives

\[
\frac{2\dot{R}}{R} = -3(\rho_{br} + p_{br}) \frac{\partial \phi}{\partial \rho_{br}} + R \frac{\partial \phi}{\partial R} + 2\phi.
\]

One sees that the acceleration of the universe is only determined by the density and pressure iff

\[
\phi(\rho_{br}, R) = \psi(\rho_{br}) + \frac{C}{R^2},
\]

where \( C \) is a constant, otherwise the scale factor \( R \) will explicitly appear in the representation of the acceleration. It is evident that \( \frac{C}{R^2} \) is just the spatial curvature term. Both the Friedmann equations in the source region (42) and in
the vacuum region cannot be written in the form of \( \text{(46)} \), thus the acceleration has to be an explicit function of \( R \). In the source region, the acceleration becomes
\[
\ddot{R} = -6(1 + \frac{\rho_{br}}{\rho_{br}})\left(H^2 + \frac{1}{4R^4}\right) + 2H^2 + R \frac{\partial \phi}{\partial R},
\]
(47)
where
\[
R \frac{\partial \phi}{\partial R} = \frac{1}{R^4} \left(1 - \frac{\kappa^4 \rho_{br}^2}{144a^2(1 + \frac{b^2}{2}e^{aR^2/2})^3} \left[4 + b(2 + aR^2)e^{aR^2/2}\right]\right).
\]
(48)
In the vacuum region, the acceleration becomes
\[
\ddot{R} = -\frac{\kappa^4}{144a^2}(2\rho_{br} + 3p_{br})\rho_{br} + \frac{1}{4R^4}.
\]
(49)
At the late-time, the dark radiation term is reasonably committed. So, if the brane is moving in the vacuum region of the bulk, the acceleration condition becomes
\[
2\rho_{br} + 3p_{br} < 0,
\]
(50)
if we require \( \rho_{br} > 0 \) (However, for an asymptotic anti-de Sitter brane, it is incorrect since the total density may be negative in the late time universe. Here we do not consider this case.). It is a stronger condition than that of the standard case \( \rho + 3p < 0 \) in the sense that it needs a smaller pressure for the same density.

By using (14) and (38), we obtain
\[
\rho_{br} = \rho_m + \lambda,
\]
(51)
where \( \rho_m \) is the density of matter confined to the brane. In the following text, we consider the case \( \rho_m \sim 1/R^3 \), which means that we consider a dust brane with tension, but without exotic matter. Substituting (51) into (42), we reach
\[
\frac{H^2}{R^4} = -\frac{1}{R^4} + \frac{\kappa^4 \rho_m^2}{144a^2R^4(1 + \frac{b^2}{2}e^{aR^2/2})^2} + \frac{\lambda \kappa^4}{72a^2R^4(1 + \frac{b^2}{2}e^{aR^2/2})^2} + \frac{\lambda^2 \kappa^4}{144a^2R^4(1 + \frac{b^2}{2}e^{aR^2/2})^2}.
\]
(52)
However, the Friedmann equation should recover to the standard one at the time when CMB decoupling and structure formation occur. (52) cannot recover to the standard one at the early time, so it is proper to consider a brane moving in a vacuum bulk in the early time, where the tension of the brane is required. In the vacuum region, by using (51) the Friedmann equation (43) becomes,
\[
H^2 = -\frac{1}{R^4} + \frac{\kappa^4}{576} \left(\rho_m^2 + 2\rho_m\lambda + \lambda^2\right).
\]
(53)
Then, as usual in the brane world model, we define,
\[
8\pi G_{eff} = \frac{\kappa^4 \lambda}{288}.
\]
(54)
Comparing with the previous brane models, we do not include a bulk cosmological constant since we do not find a proper source of Taub-(A)dS space yet. This definition of \( G_{eff} \) is the same as the former definition in brane world model up to a factor \( 2 \). It has been investigated in detail except a dark radiation term, which is unimportant in the late time universe.

Our scenario is that the brane moves in a vacuum bulk in the high redshift region and enters in the source region in some low redshift region. And the accelerating universe is driven by the source matter in the bulk. For recovering the standard cosmology in the middle redshift region and high redshift region, we need the brane tension.

For convenience of numerical calculation, we write the Friedmann equation in the form of evolution with respect to the redshift \( \xi \). In the source region,
\[
\frac{H^2}{H_p^2} = -\Omega_{dr}(1 + \xi)^4 + \frac{1}{2\Omega_\Lambda} \left[ \Omega_{mp}(1 + \xi)^3 + \Omega_\Lambda \right]^2,
\]
(55)
where

\[ \Omega_{dr} = \frac{1}{4H^2_p R^4_p}, \]

\[ \Omega_{mp} = \frac{8\pi B \rho_p}{3H^2_p}, \]

and

\[ \Omega_\lambda = \frac{8\pi B \lambda}{3H^2_p}, \]

where the subscript \( p \) labels the present epoch of a physical quantity, \( B \) is defined as

\[ \frac{8\pi B}{3} = \frac{\rho_m \kappa^4}{144a^2 R^4(1 + \frac{b}{2}e^{aR^2/2})^2}. \]

We should not confuse the symbol of the present value \( p \) with the value of the quantity at the boundary, for which we use \( 0 \) to denote.

In the vacuum region, the Friedmann equation keeps the same form. But \( B \) is replaced by \( G_{eff} \) in (54). In contrast to the previous brane models, we find an interesting property of this brane world model: The equation of state (EOS) of the effective dark energy (defined as the ratio of pressure to energy density) can cross the phantom divide \( w = -1 \) on a dust-brane with tension.

The crossing \(-1\) behavior of EOS is a deep problem and a big challenge to the fundamental physics, which is aroused by more accurate data: the recent analysis of the type Ia supernovae data indicates that the time varying dark energy gives a better fit than a cosmological constant, and in particular, the (EOS) \( w \) may cross \(-1\) \[11\]. The dark energy with \( w < -1 \) is called phantom dark energy \[12\], for which all energy conditions are violated. Here, it should be noted that the possibility that the dark energy behaves as phantom today is yet a matter in debate: the observational data, mainly those coming from the type Ia supernovae of high redshift and cosmic microwave background, may lead to different conclusions depending on what samples are selected, and what statistical analysis is applied \[13\]. By contrast, other researches imply that all classes of dark energy models are comfortably allowed by those observations \[14\]. Presently all observations seem not to rule out the possibility of the existence of matter with \( w < -1 \). Even in a model in which the Newton constant is evolving with respect to the redshift \( z \), the best fit \( w(z) \) crosses the phantom divide \( w = -1 \) \[15\]. Hence the phenomenological model for phantom dark energy should be considered seriously. To obtain \( w < -1 \), scalar field with a negative kinetic term, may be a simplest realization. The model with phantom matter has been investigated extensively \[10\], and a test of such matters in solar system, see \[17\]. However, the EOS of phantom field is always less than \(-1\) and can not cross \(-1\). It is easy to understand that if we put two scalar fields into the model, one is an ordinary scalar and the other is a phantom: they dominate the universe by turns, under this situation the effective EOS can cross \(-1\) \[18\]. It is worthy to point out that there exist some interacting models, in which the effective EOS of dark energy crosses \(-1\) \[19\]. More recently it has been found that crossing \(-1\) within one scalar field model is possible, the cost is that the action contains higher derivative terms \[20\] (see also \[21\]). Also it is found that such a crossing can be realized without introducing ordinary scalar or phantom component in a Gauss-Bonnet brane world with induced gravity, where a four dimensional curvature scalar on the brane and a five dimensional Gauss-Bonnet term in the bulk are present \[22\].

To explain the observed evolving EOS of effective dark energy, we calculate the equation of state \( w \) for the effective “dark energy” caused by the tension and term representing brane world effect by comparing the modified Friedmann equation in the brane world scenario and the standard Friedmann equation in general relativity, because all observed features of dark energy are “derived” in general relativity. Note that the standard Friedmann equation in a four dimensional spatially flat universe can be written as

\[ H^2 = \frac{8\pi G}{3}(\rho_m + \rho_{de}), \]

where the first term in RHS of the above equation represents the dust matter and the second term stands for the effective dark energy, and \( G \) is the Newton constant. Comparing (60) with (57), one obtains the density of effective dark energy,

\[ \rho_{de} = -\frac{1}{4R^4} + \frac{\kappa^4 \rho_m^2}{144a^2 R^4(1 + \frac{b}{2}e^{aR^2/2})^2} - \frac{8\pi G}{3} \rho_m. \]
FIG. 1: The evolution of the density for the effective dark energy $\rho_{de}$ with respect to the redshift $\xi$, where $\rho_{cri}$ denotes the present critical density. The left part (low redshift region) is the source region, and the right part (high redshift region) is the vacuum region. In this figure $a = -1.2$. The boundary of the source region inhabits at $R_0 = 0.645$. The brane sweeps across the boundary when $\xi = 0.549$.

Since the dust matter obeys the continuity equation and the Bianchi identity keeps valid, dark energy itself satisfies the continuity equation

$$\frac{d\rho_{de}}{dt} + 3H(\rho_{de} + p_{\text{eff}}) = 0,$$

where $p_{\text{eff}}$ denotes the effective pressure of the dark energy. And then we can express the equation of state for the dark energy as

$$w_{de} = \frac{p_{\text{eff}}}{\rho_{de}} = -1 + \frac{1}{3} \frac{d\ln\rho_{de}}{d\ln(1+\xi)}.$$  

Clearly, if $\frac{d\ln\rho_{de}}{d\ln(1+\xi)}$ is greater than 0, dark energy evolves as quintessence; if $\frac{d\ln\rho_{de}}{d\ln(1+\xi)}$ is less than 0, it evolves as phantom; if $\frac{d\ln\rho_{de}}{d\ln(1+\xi)}$ equals 0, it is just cosmological constant. In a more intuitive way, if $\rho_{de}$ decreases and then increases with respect to redshift (or time), or increases and then decreases, which implies that EOS of dark energy crosses phantom divide. The more important reason why we use the density to describe property of dark energy is that the density is more closely related to observables, hence is more tightly constrained for the same number of redshift bins used.

We find a concrete example in which the EOS of the effective dark energy crosses $-1$. For the numerical improvement, we should nail down the parameters in this model, which satisfy the requirements of theory and observation. For the theoretical aspect, the boundary condition of the source region yields

$$b = -4e^{-\frac{a_0}{2}},$$

and (22). We only consider the large branch in $\xi$, hence $\rho_{cri}$ is fixed by (22) and (64). Therefore, to this brane model imbedded in sourced Taub background itself, we have only one free parameter. It is $a$ or $z_0$, which implies the thickness of the source region. The partition of dark radiation should be smaller than that of cosmic microwave background (CMB), otherwise the universe will bounce at some high redshift, thus $\Omega_{dr} < 10^{-4}$. Here we set $\Omega_{dr} = 10^{-5}$, $R_p = 1$, $\Omega_{mp} = 0.27$ and $\Omega_\Lambda = 1.408$. The present part of baryon density is approximately 4% [25], and we have several evidences of the existence of the non-baryon dark matter, which are independent of $\Lambda$CDM model [26]. Here we take $\Omega_{mp}$ as the same value of the best fit of $\Lambda$CDM model. $\Omega_\Lambda$ is derived from (55) at $\xi = 0$. Figure 1 illustrates the evolution of the effective density. It is clear that the EOS of the dark energy crosses $-1$ at $\xi = 0.2 \sim 0.3$. This point is also confirmed by the direct plot of the EOS of effective dark energy in figure 2, which well explains the observations [11].

The most significant parameters from the viewpoint of observations is the Hubble parameter $H(z)$, which carries the total effects of cosmic fluids. Except the indirect data of $H(z)$, such as the luminosity distances of supernovae, the direct $H(z)$ data appear in recent years, which can be used to explore the fine structures of the Hubble expansion history [27]. There is an important new feature of $H(z)$ data which is not implied by the previous indirect observations.
FIG. 2: The evolution of $w_{de}$ with respect to $\xi$. The point of crossing $-1$ (C-point) appears at $\xi = 0.2 \sim 0.3$. The parameters are the same as Figure 1.

FIG. 3: The evolution of the Hubble parameter with respect to redshift. The unit of $H$ is $km \ s^{-1} \ Mpc^{-1}$. The parameters are the same as Figure 1.

of Hubble parameter: It decreases with respect to the redshift $\xi$ at redshift $\xi \sim 0.15$ and $\xi \sim 1.5$, which means that the total fluid in the universe behaves as phantom. We find that the present brane model also can partly realize this property with the same parameters of the example in which the EOS of the effective dark energy crosses $-1$. Figure 3 illustrates the evolution of the Hubble parameter. At $\xi \sim 0.15$, there is a clear drop on $H(z)$ diagram in Figure 3.

Figure 1 and Figure 3 illuminate that there are cusps at the boundary of vacuum and source region, which is related to the fact that the metric on the boundary is only $C^1$, while second derivatives with respective to coordinates are involved in Einstein equation.

V. CONCLUSION

We investigate the brane world model in the sourced-Taub background. The Friedmann equations are obtained both in the source region and the vacuum region.

The recent observations imply that the EOS of dark energy may cross $-1$ and there may be some drops on the Hubble diagram. These two phenomena are serious challenges to the physical cosmology. In the braneworld model with a sourced Taub background, for a reasonable parameter set the EOS of the effective dark energy can cross $-1$ in when the brane is moving in the source region. And furthermore, the Hubble parameter has a drop in the low redshift region (the source region).

In the present work, we do not include a bulk cosmological constant and thus the brane tension can not be too
large. When the brane is moving in the vacuum region, the quadratic term of $\rho$ will dominate the linear term $\rho$ before long. In the high redshift region of the vacuum bulk, the model is treated as a toy model. Our main aim of this article is to study the behavior in the source region. The detailed evolution of the brane universe in sources Taub background in the high redshift region needs to be investigated further in the future.

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[1] L. Randall and R. Sundrum, arXiv:hep-th/9906064; G. Dvali, G. Gabadadze, M. Porrati, Phys. Lett. B485 (2000) 208 [hep-th/0004249].
[2] D. Langlois, Prog. Theor. Phys. Suppl. 148, 181 (2003) arXiv:hep-th/0209261.
[3] A. Riotto, arXiv:hep-ph/0210162.
[4] A. H. Taub, Ann. Math. 53 (1951) 472; Phys. Rev. 103 (1956), 454.
[5] R. Lazkoz and G. Leon, Phys. Lett. B638, 303 (2006) [arXiv:astro-ph/0602590]; W. Wang, X. Y. Gui and Y. Shao, Chin. Phys. Lett. 23 (2006) 762; X. Zhang, Phys. Lett. B641, 101 (2006) [arXiv:astro-ph/0602590].
[6] Z. K. Guo, Y. S. Piao, X. Zhang, Phys. Lett. B641, 101 (2006) [arXiv:astro-ph/0602590].
[7] Z. K. Guo, Y. S. Piao, X. Zhang, Phys. Lett. B641, 101 (2006) [arXiv:astro-ph/0602590].
[8] Z. K. Guo, Y. S. Piao, X. Zhang, Phys. Lett. B641, 101 (2006) [arXiv:astro-ph/0602590].
[9] Z. K. Guo, Y. S. Piao, X. Zhang, Phys. Lett. B641, 101 (2006) [arXiv:astro-ph/0602590].
[10] Z. K. Guo, Y. S. Piao, X. Zhang, Phys. Lett. B641, 101 (2006) [arXiv:astro-ph/0602590].
[24] Y. Wang, and P. Garnavich, ApJ, 552, 445 (2001); M. Tegmark, Phys. Rev. D66, 103507 (2002); Y. Wang, and K. Freese, Phys.Lett. B632, 449 (2006); astro-ph/0402208.

[25] E. Komatsu et al. [WMAP Collaboration], arXiv:0803.0547 [astro-ph].

[26] D. Hooper, arXiv:0901.4090.

[27] R. Jimenez, L. Verde, T. Treu and D. Stern, Astrophys. J. 593, 622 (2003) astro-ph/0302560; R. Jimenez and A. Loeb, Astrophys. J. 573, 37 (2002) astro-ph/0106145; R. G. Abraham et al. [GDDS Collaboration], Astron. J. 127, 2455 (2004) astro-ph/0402436; T. Treu, M. Stiavelli, S. Casertano, P. Moller and G. Bertin, Mon. Not. Roy. Astron. Soc. 308, 1037 (1999); T. Treu, M. Stiavelli, P. Moller, S. Casertano and G. Bertin, Mon. Not. Roy. Astron. Soc. 326, 221 (2001) astro-ph/0104177; T. Treu, M. Stiavelli, S. Casertano, P. Moller and G. Bertin, Astrophys. J. Lett. 564, L13 (2002); J. Dunlop, J. Peacock, H. Spinrad, A. Dey, R. Jimenez, D. Stern and R. Windhorst, Nature 381, 581 (1996); H. Spinrad, A. Dey, D. Stern, J. Dunlop, J. Peacock, R. Jimenez and R. Windhorst, Astrophys. J. 484, 581 (1997); L. A. Nolan, J. S. Dunlop, R. Jimenez and A. F. Heavens, Mon. Not. Roy. Astron. Soc. 341, 464 (2003) astro-ph/0103450; J. Simon, L. Verde and R. Jimenez, Phys. Rev. D 71, 123001 (2005) astro-ph/0412269; Ruth A. Daly, S. G. Djorgovski, Kenneth A. Freeman, Matthew P. Mory, C. P. ODea, P. Kharb, and S. Baum, Astrophys. J. 677 (2008) 1; H. Wei and S. N. Zhang, Phys. Lett. B 644, 7 (2007) arXiv:astro-ph/0609507; Z. L. Yi and T. J. Zhang, astro-ph/0605596, MPLA in press; Hongsheng Zhang and Zong-Hong Zhu, JCAP03(2008)007; Hongsheng Zhang, Heng Yu, Hyerim Noh and Zong-Hong Zhu, Phys. Lett. B665, 319 (2008) arXiv:0806.4082.