Learning Structured Outputs from Partial Labels using Forest Ensemble

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Abstract

Learning structured outputs with general structures is computationally challenging, except for tree-structured models. Thus we propose an efficient boosting-based algorithm AdaBoost.MRF for this task. The idea is based on the realization that a graph is a superimposition of trees. Different from most existing work, our algorithm can handle partial labelling, and thus is particularly attractive in practice where reliable labels are often sparsely observed. In addition, our method works exclusively on trees and thus is guaranteed to converge. We apply the AdaBoost.MRF algorithm to an indoor video surveillance scenario, where activities are modelled at multiple levels.

1 Introduction

There has been a growing research interest in developing probabilistic temporal graphical models for recognising human activities from sensory data. In this paper we address an important aspect of the problem in that there are multiple levels of abstraction, that is, an activity is often composed of several sub-activities. A popular approach to deal with such a hierarchical nature is to build a cascaded model: each level is modelled separately, and the output of the lower levels is subsequently used as the input for the upper levels [20]. This approach is sub-optimal because the information at the higher level is often very discriminative to infer about the lower levels, but it is not modelled. Moreover, the layered approach often suffers from the so-called cascading error problem, as the error introduced from the lower level will propagate to higher tasks.

A better and more holistic approach is to build a joint representation at all layers. Emerging methods include generative/directed models such as abstract hidden Markov models (AH-MMs) [4], hierarchical HMMs [19], dynamic Bayesian networks [10], and their discriminative/undirected counterparts such as hierarchical conditional random field (HCRF) [17], and dynamic CRF (DCRF) [28]. In general, generative models are useful in jointly modelling the activity semantics and observed sensory data, and thus dealing well with hidden variables. On the contrary, discriminative models make less assumption about the nature of the sensory data, and thus can potentially achieve higher classification accuracy when the generative assumption is violated.

Our application of interest is the problem of activity recognition in an indoor environment. The solution to this problem will provide important technology for building intelligent surveillance systems. To this end, we propose in this paper the use of the DCRF for modelling activities at multiple scales. Such a problem is known to be difficult due to the hierarchical nature of the
activities. The DCRF used in this paper is a generalisation of a powerful probabilistic formulation known as conditional random field (CRF) [15]. It is an expressive representation scheme that seamlessly integrates domain knowledge, temporal regularities, and at the same time encodes complex interdependency between semantic levels. However, its expressiveness does come with great challenges, one of which is problem of parameter estimation under arbitrary structures. It is known that parameter estimation is generally intractable to perform exactly in a maximum likelihood setting for general MRFs, except for tree structures.

One of the earliest and most popular methods to deal with this intractability is to optimise the pseudo-likelihood instead [3], which achieves its efficiency through limiting to individual nodes in the graph and their neighbours. However, the method cannot handle missing variables, which unfortunately often happen in real situations. Sampling-based methods such as MCMC are theoretically attractive, but they are often impractical for extremely slow convergence. The state-of-the-art methods often involves approximate inference algorithms such as Pearl’s belief propagation (BP) [21] or a more recent method introduced by Wainwright, Jaakkola and Willsky (WJW) [39]. These methods are efficient but their biggest problem is that there is no guarantee on the convergence. In addition, the WJW inference has not been applied for learning in conditional MRFs, an issue that we will also explored in this paper.

Our main contribution in this paper is the introduction of AdaBoosted Markov Random Forests (AdaBoost.MRF), a learning method that operates only on Markovian trees at each step, but can be proved to achieve global optimum under some mild assumptions. Distinct from most existing work on discriminative models including the work of [15, 28], our AdaBoost.MRF can also handle partial labels) – an important enhancement for real-world applications.

In essence, our AdaBoost.MRF is based on a ranking-based multiclass boosting algorithm called AdaBoost.MR [24]. At each round, the AdaBoost.MRF selects the best trained spanning tree of the network based on the performance on weighted error. The data is adaptively re-weighted to address more hard-to-classify instances. Each selected spanning tree is weighted and combined to recover the original graph. Finally, the parameters of the graph are the convex combination of all selected tree parameters. Since our method works exclusively on trees, inference is very efficient and convergence is guaranteed. We also prove that under mild assumptions, the AdaBoost.MRF reaches the unique optimum. Furthermore, since the AdaBoost.MRF considers all the variables in the MRFs, the partial labels problem can also be effectively handled.

We evaluate AdaBoost.MRF on the activity data obtained in an indoor video monitoring scenario. We compare our AdaBoost.MRF with the maximum likelihood method, which used BP and WJW as inference engines. To evaluate the effectiveness of the discriminative DCRFs against generative methods, we implement a variant of the layered hidden Markov models (LHMMs) [20] which can handle partially observed state variables to make it comparable with the DCRFs considered in this paper. We also show that the multi-level DCRFs perform better than the flat-CRFs as more information is added.

The paper will continue with a related background in Section 2. Section 3 introduces the AdaBoost.MRF algorithm and discusses its convergence. Section 4 shows how we model and learn multi-level activities with DCRFs, followed by Section 5 to present the experimental results. Discussion is given in Section 6 and conclusion in Section 7.
2 Related Work and Background

Since the starting point of our work is the conditional random field and multiclass boosting, we shall provide a brief related work and background to these two problems respectively in this section.

2.1 Conditional random fields

Our work is based on conditional random fields (CRFs) [14, 15, 26, 35]. A Markov random field (e.g., see Figure 1) is an undirected graph $G = (V, E)$ which represents a joint distribution of state variables $\mathbf{x} = (x_1, x_2, ..., x_n) \in \mathcal{X}$, where each $x_r$ corresponds to a node $r \in \mathcal{V}$ in the graph. For simplicity we assume these variables have the same domain and receive assignments from a fixed set of discrete values $S = \{1, 2, ..., |S|\}$. In conditional MRF, the graph is further associated with a data observation $\mathbf{o}$ in which no knowledge of the structure within the observation is required. The edge-set $E$ of the graph specifies a set of cliques $C$, each of which supports a feature vector $f(x_c, \mathbf{o})$ that maps the observation and the clique-based state variable $x_c$. Let $F(x, \mathbf{o}) = \sum_{c \in C} f(x_c, \mathbf{o})$ be the global feature vector. A standard CRF defines a conditional distribution of the state variable given the observation $\mathbf{o}$ in an exponential family distribution as:

$$P(x|\mathbf{o}; \mathbf{w}) = \exp\{\langle \mathbf{w}, F(x, \mathbf{o}) \rangle - A(\mathbf{o})\}$$  \hspace{1cm} (1)

where $\mathbf{w}$ is the feature weight vector, $\langle \cdot, \cdot \rangle$ is the inner product, and $A(\mathbf{o}) = \log \sum_{x \in \mathcal{X}} \exp\{\langle \mathbf{w}, F(x, \mathbf{o}) \rangle\}$.

Parameter learning in CRFs is often based on the maximum likelihood criterion. Given a training set $\mathcal{D} = \{\mathbf{x}^{(i)}, \mathbf{o}^{(i)}\}_{i=1}^D$, the goal is to find the maximiser of the (log) likelihood with respect to $\mathbf{w}$:

$$L(\mathbf{w}) = \frac{1}{D} \sum_{i=1}^D \log P(x^{(i)}|o^{(i)}; \mathbf{w})$$ \hspace{1cm} (2)

$$= \frac{1}{D} \sum_{i=1}^D \left\{ \langle \mathbf{w}, F(x^{(i)}, o^{(i)}) \rangle - A(o^{(i)}) \right\}$$ \hspace{1cm} (3)

This function is concave in $\mathbf{w}$ thus it has a unique global maximum. Optimisation methods often require the gradient, which can obtained as in a standard exponential family case as:

$$\nabla L(\mathbf{w}) = \frac{1}{D} \sum_{i=1}^D \sum_{c \in C} \left\{ f(x_c^{(i)}, o^{(i)}) - \mathbb{E}_{x_c|o}[f(x_c, o^{(i)})] \right\}$$

where $\mathbb{E}_{x_c|o}[\cdot]$ is the expectation evaluated with respect to $P(x_c|o)$.

However, inference in general networks is known to be intractable except for trees with limited tree-widths. For tree-structures, a well-known method is a two-pass belief propagation procedure [21] that takes $O(2|\mathcal{V}||S|^2)$ time to compute all quantities needed for learning, where $|\mathcal{V}|$ is number of nodes in the tree.

For structures other than trees extract inference is intractable and approximate methods such as mean fields and loopy belief propagation (BP) are widely used. A more recently proposed method by Wainwright, Jaakkola and Willsky (WJW) [39] also offers an interesting alternative.
to compute the so-called pseudo-marginals based on minimising the upper bound of the log-partition function. Like BP, the WJW is an efficient message passing scheme. However, the BP is not guaranteed to converge and there has not been any formal proof nor extensive empirical evaluation for WJW found in literature. Our AdaBoost.MRF, in contrast, works on tree inference and thus is guaranteed to converge with known analytical complexity. Besides, to the best of our knowledge, we are the first to perform learning using WJW in the conditional MRFs setting. CRFs have also been applied to activity recognition recently [17, 34, 25, 30, 23, 37]. These works have reported promising results. However, due to the inherent intractability of the general structures, either simple chains and trees have been assumed or approximate inference methods have been used.

2.2 Learning Structured Output with Partial Labels

Conditional random fields are an example of structured output models [36, 29, 7]. Learning in structured output models can be based on principles other than maximum likelihood, for example, large-margin [36, 29], or search [7]. With the latter, the computation of feature expectation is replaced by finding the most probable labelling.

Learning with partial labels have been addressed in the past decade [33, 32, 34]. Related problems include weak supervision [38] and indirect supervision [5]. Partial labels arise when only some components of \( x \) are observed, i.e., \( x = \{v, h\} \) where \( v \) and \( h \) are observed and missing components respectively. In the CRF setting, parameter learning requires to maximise the conditional incomplete log-likelihood instead, which can be shown to be:

\[
L(w) = \frac{1}{D} \sum_{i=1}^{D} \log P(v^{(i)} | o^{(i)})
\]

(4)

\[
= \frac{1}{D} \sum_{i=1}^{D} A(v^{(i)}, o^{(i)}) - A(o^{(i)})
\]

(5)

where \( A(v, o) = \log \sum_h \exp(w F(v, h, o)) \). The gradient \( \nabla L(w) \) can now be derived as:

\[
\nabla L(w) = \frac{1}{D} \sum_{i=1}^{D} \sum_{c \in C} \left( \sum_{h_c} \mathbb{E}_{h_c, o}[F(v^{(i)}_c, h_c, o^{(i)})] - \mathbb{E}_{x_c, o}[f(x_c, o^{(i)})] \right)
\]

(6)

where \( h_c \) denotes the missing components associated with clique \( c \). Equations (4) and (6) reveal that learning depends on the inference ability to compute \( A(v, o) \), \( A(o) \) and local clique distributions \( P(x_c | o) \).

2.3 Ranking-based Multiclass Boosting

This section reviews a multi-class boosting algorithm known as AdaBoost.MR [24], based on which our work will be developed. We adopt the functional view of boosting from [18] in this paper.

Given a pool of weak learners \( \{h_m(x, o)\} \), in a boosting setting we seek to learn a subset \( \{h_k(x, o)\}_{k=1}^{K} \) and their corresponding weights \( \{w_k\}_{k=1}^{K} \). Denote \( H(x, o) = \sum_{k=1}^{K} w_k h_k(x, o) \)

\[4\]
to be a final classifier that outputs the prediction:

\[
\hat{x} = \arg \max_{x \in X} H(x, o).
\] (7)

\(H(x, o)\) is also known as strong learner in the boosting literature. For each training instance \((x^{(i)}, o^{(i)})\), we would expect that \(x^{(i)} = \hat{x}^{(i)}\), and thus we expect:

\[
H(x^{(i)}, o^{(i)}) \geq H(x, o^{(i)})
\] (8)

for all \(x \in X\) and \(i = 1, 2, ..., D\). Whenever there exists an observation \(o\) that invalidates this assertion, the system suffers a loss. The \textit{rank loss} is defined as

\[
L_{\text{rank}} = \frac{1}{D} \sum_{i=1}^{D} \sum_{x} \mathbb{I}\{H(x, o^{(i)}) - H(x^{(i)}, o^{(i)}) > 0\}
\] (9)

where \(\mathbb{I}\{\cdot\}\) is the indicator function. This rank loss is basically the number of possibilities where the system misclassifies the data. The loss vanishes if the system correctly classifies all the data instances.

However, the rank-loss in Equation 9 is difficult to minimise. Therefore, we resort to the exponential-loss, which is a smooth, convex upper-bound of the rank-loss:

\[
L_{\text{exp}} = \frac{1}{D} \sum_{i=1}^{D} \sum_{x} \exp\{H(x, o^{(i)}) - H(x^{(i)}, o^{(i)})\}
\] (10)

Term-by-term comparison of Equations 9 and Equation 10 it can easily verify that \(L_{\text{exp}}\) is indeed the upper-bound of \(L_{\text{rank}}\) up to a constant.

The learning process in boosting is iterative, in that at each step \(l\), we greedily seek for an update of the functional \(H(.)\) that best reduces the loss:

\[
H_l \leftarrow H_{l-1} + \alpha_l h_j \quad \text{where} \quad (\alpha_l, j) = \arg \min_{\alpha, k} L_{\text{exp}}(H_{l-1} + \alpha h_k)
\] (15)

\(\text{1To see the connection between the exponential loss and the log-likelihood, assume a conditional distribution}

\[
P(x|o) = \frac{1}{Z(o)} \exp\{H(x, o)\}
\] (11)

where \(Z(o) = \sum_x \exp\{H(x, o)\}\) is the normalisation constant. This assumption makes sense because the prediction is identical to the Maximum A Posteriori:

\[
\hat{x} = \arg \max_{x} H(x, o) = \arg \max_{x} P(x|o)
\] (12)

Substituting \(P(x|o)\) into Equation 10 yields

\[
L_{\text{exp}}(w) = \frac{1}{D} \sum_{i=1}^{D} \frac{1}{P(x^{(i)}|o^{(i)})}
\] (13)

This appears similar to the log-loss used in the maximum likelihood estimation

\[
L_{\text{log}}(w) = \frac{1}{D} \sum_{i=1}^{D} \log \frac{1}{P(x^{(i)}|o^{(i)})}
\] (14)

The difference between the exponential loss and the log-loss is about the numerical scale, because of the log function in the log-loss. However, in [16] the authors show that the two losses give very close results given enough data. This paper suggests that boosting can be regarded as an (approximate) alternative for the maximum likelihood estimation (MLE). From another related angle, boosting-style MLE algorithms are derived in [8].
The standard AdaBoost.MR addresses only simple classification, where the data of interest does not have any structure. For structured data such as CRFs, boosting has been applied in [31], but the algorithm relies on the BP for approximate inference and does not address the missing variables. Similarly, work in [1, 8] is limited to tractable CRFs.

3 Forest Ensemble Algorithm: AdaBoost.MRF

In this section we present a novel boosting algorithm for parameter estimation of general conditional random fields, termed as AdaBoost.MRF. We consider the general case where the state label $x$ may have a visible component $v$ and a missing component $h$, i.e. $x = \{v, h\}$.

3.1 Forest Ensemble

![Figure 1: An example of Markov network (left-most) and some spanning trees (right).](image)

We formulate an objective function based on the exponential loss of AdaBoost.MR. Recall from Section 2.3 that given training data pairs $(x^{(i)}, o^{(i)})$, the loss is $L_{exp} = \sum_i \sum_{x} \exp\{H(x, o^{(i)}) - H(x^{(i)}, o^{(i)})\}$. Since for an instance $i$, we are given only the visible part $v^{(i)}$ of $x^{(i)}$, we formulate the incomplete loss as:

$$L_{inco}(H) = \frac{1}{D} \sum_i \left( \sum_{v} \exp\{H(v, o^{(i)}) - H(v^{(i)}, o^{(i)})\}\right)^{\beta}$$  \hspace{1cm} (17)

Here apart from summing over only the visible component $v$ we also introduce an extra regularisation term $\beta \in (0, 1]$ for numerical stability\footnote{We note that a main difference from most of the previous boosting work is that the number of classes in our cases can be extremely large, e.g., $|X| = |S|^n$.}. In each round $l$ of boosting, the strong learner $H_l(v, o)$ is updated by adding a ‘weak-learner’ $h_l(v, o)$ to the previous $H_{l-1}(v, o)$ as $H_l(v, o) = H_{l-1}(v, o) + \alpha_l h_l(v, o)$, where $\alpha_l$ is the weight of each weak learner in the ensemble. The weak learner and its weight are chosen to minimise the loss in (17), i.e.

$$\left(h_l, \alpha_l \right) = \arg \min_{h, \alpha} L_{inco}(H_{l-1} + \alpha h)$$  \hspace{1cm} (18)

Since we are interested in the distribution $P(v|o)$, it is sensible to choose the weak learner as $h(v, o) = \log P(v|o)$. However, as stated before, if the distribution defined over the general Markov networks is used, the computation of the weak learner itself becomes intractable. To this end, we propose the use spanning trees over graph $G$ as weak learner serving as an approximation to the whole network. Thus, each learner is “weak” in the sense that it is an approximation of the...
true model, but with moderate and tractable complexity. Let \( P_{\tau}(v|o) \) be a conditional distribution of visible state variables \( v \) given the observation \( o \) with respect to a spanning tree \( \tau \), we define the weak learner to be:

\[
h(v, o) = \log P_{\tau}(v|o)
\] (19)

This choice also allows incorporation of the missing information since \( h(v, o) = \log P_{\tau}(v|o) = \log \sum_h P_{\tau}(v, h|o) \). Thus, the strong learner \( H \) is a collection of trees, and we term our boosting method AdaBoost.MRF (AdaBoosted Markov Random Forests). Figure 1 shows an example of a simple network and some spanning trees. We defer the discussion on the choice of tree-based distributions \( P_{\tau}(v|o) \) and their relationship to the model distribution \( P(v|o) \) to later sections.

3.2 Loss bound using Hölder’s inequality

Although the exponential loss in (17) is meaningful, it is unfortunately intractable to compute, let alone minimising it. In this subsection, we propose to replace the loss by a tractable upper bound using tree likelihood. Recall that the boosting procedure is incremental, by substituting (19) into (17) yields the following expression at step \( l \):

\[
L_{\text{inco}} = \frac{1}{D} \sum_i \left( \frac{\sum_v \prod_{j=1}^l P_{\tau_j}(v|o^{(i)})^{\alpha_j}}{\prod_{j=1}^l P_{\tau_j}(v^{(i)}|o^{(i)})^{\alpha_j}} \right)^{\beta}
\] (20)

The intractability comes from the sum over all visible variables in the numerator, except for a special case that all selected spanning trees are the same.

Fortunately, we can get around the summation in the numerator by applying the Hölder’s inequality [11, Theorem 11] (see the appendix for details) to the numerator

\[
\sum_v \prod_{j=1}^l P_{\tau_j}(v|o^{(i)})^{\alpha_j} \leq \prod_{j=1}^l \left[ \sum_v P_{\tau_j}(v|o^{(i)})^{\alpha_j r_j} \right]^{1/r_j}
\]

where \( \sum_{j=1}^l 1/r_j = 1 \) and \( r_j > 0 \). Under mild assumption that \( \alpha_j > 0 \) and \( \alpha_j r_j = 1 \) \( \forall j \), or \( \sum_{j=1}^l \alpha_j = 1 \) and note that \( 1/r_j = \alpha_j \), we obtain:

\[
L_{\text{inco}} \leq \frac{1}{D} \sum_i \left( \frac{\prod_{j=1}^l \left[ \sum_v P_{\tau_j}(v|o^{(i)})^{\alpha_j r_j} \right]^{\alpha_j r_j}}{\prod_{j=1}^l P_{\tau_j}(v^{(i)}|o^{(i)})^{\alpha_j}} \right)^{\beta}
\]

\[
= \frac{1}{D} \sum_i \left( \frac{1}{\prod_{j=1}^l P_{\tau_j}(v^{(i)}|o^{(i)})^{\alpha_j}} \right)^{\beta}
\]

\[
= \frac{1}{D} \sum_i \exp\{-\beta \sum_{j=1}^l \alpha_j \log P_{\tau_j}(v^{(i)}|o^{(i)})\}
\] (21)

Let \( L_H \) be the upper bound on the RHS and given (19) we can further simplify:

\[
L_H = \frac{1}{D} \sum_i \exp\{-\beta H_i(v^{(i)}, o^{(i)})\}
\] (22)
since \( \sum_v P_{\tau_j}(v|o^{(i)}) = 1 \), \( \forall i, j \). It can be seen that the new bound is tractable to evaluate, and since is also convex, a global minimum does exist. We thus use the new loss \( \mathcal{L}_H \) for parameter learning. It can be seen that the domain of \( \mathcal{L}_H \) is a linear space of functions \([18]\), which are \( \{h(v, o^{(i)}) = \log P_{\tau}(v|o^{(i)})\} \) in our case.

As we update one weak learner at a step, the requirement \( \sum_{j=1}^{l-1} \alpha_j = 1 \) can be met by defining the following ensemble

\[
H_l(v, o) = (1 - \alpha_l)H_{l-1}(v, o) + \alpha_l h_l(v, o)
\]

(23)

\[
s_l(v, o) = h_l(v, o) - H_{l-1}(v, o)
\]

(24)

Each previous weak learner’s weight is scaled down by a factor of \( 1 - \alpha_l \) as \( \alpha_j' \leftarrow \alpha_j(1 - \alpha_l) \), for \( j = 1, ..., l - 1 \), so that \( \sum_{j=1}^{l-1} \alpha_j' + \alpha_l = \sum_{j=1}^{l-1} \alpha_j (1 - \alpha_l) + \alpha_l = 1 \) since \( \sum_{j=1}^{l-1} \alpha_j = 1 \).

### 3.3 Weak learners, convergence and complexity

In the previous subsection, we have suggested to use \( \mathcal{L}_H \) as the loss to minimise. Recall that \( H(v, o) \) is a collection of spanning trees (or weak learners), we need to find the set of trees (and their parameters) that minimise \( \mathcal{L}_H \). In this subsection, we present an iterative procedure to select trees that guarantee to reduce the loss. We also provide an analysis of convergence and time complexity of the procedure.

#### 3.3.1 Selecting the best tree

We now show how to carry out the stepwise optimisation in \([18]\) with the incomplete loss replaced by the upper bound \( \mathcal{L}_H(H) \) in \((22)\).

The loss \( \mathcal{L}_H(H) \) as a function of \( H(v, o) \) can be minimised by moving in the gradient descent direction

\[
\nabla \mathcal{L}_H(v, o^{(i)}) = \begin{cases} 
-\beta \exp \{-\beta H_{l-1}(v^{(i)}, o^{(i)})\} & \text{if } v = v^{(i)} \\
0 & \text{otherwise}
\end{cases}
\]

(26)

However, as the functional gradient \( \nabla \mathcal{L}_H \) and and the functional direction \( s \) in \((24)\) may not belong to the same function space, direct optimisation may not apply. In \([18]\) the authors propose to find the best \( s_l \) pointing to the decreasing direction of \( \mathcal{L}_H \), i.e.,

\[
s_l = \arg \min_s \langle \nabla \mathcal{L}_H, s \rangle
\]

(27)

subject to

\[
\langle \nabla \mathcal{L}_H, s_l \rangle < 0
\]

(28)

The step size \( \alpha_l \) is determined using a line search or by setting it to a small constant between 0 and 1.

Let \( \lambda_{l-1}^{(i)} \propto \exp \{-\beta H_{l-1}(v^{(i)}, o^{(i)})\} \) be data weights, i.e., \( \sum_i \lambda_{l-1}^{(i)} = 1 \). Substituting \((26)\) into \((27)\), we have

\[
s_l = \arg \min_s \sum_i -\lambda_{l-1}^{(i)} s(v^{(i)}, o^{(i)})
\]

(29)
As $s(v^{(i)}, o^{(i)}) = h(v^{(i)}, o^{(i)}) - H^{t-1}(v^{(i)}, o^{(i)})$, minimising with respect to $s(v^{(i)}, o^{(i)})$ and $h(v^{(i)}, o^{(i)})$ is equivalent since $H^{t-1}(v^{(i)}, o^{(i)})$ is a constant. Recall from [19] that $h(v^{(i)}, o^{(i)}) = \log P_{\tau}(v^{(i)}|o^{(i)}; w_{\tau})$, this minimisation translates to selecting the best tree $t$ and its parameters $w_t$ as follows:

$$(t, w_t) = \arg \max_{t, w_t} \sum_i \lambda_t^{(i)} \log P_{\tau}(v^{(i)}|o^{(i)}, w_{\tau}) \tag{30}$$

Our final result has a satisfying interpretation: the functional gradient descent step tries to solve the maximum re-weighted log-likelihood problem (30) for each tree, and select the best tree with the largest re-weighted log-likelihood. As boosting proceeds, some trees may be more likely to be selected than others, so the accumulated weights of trees may be different.

As with the standard boosting [24], the data distribution is iteratively updated as

$$\lambda_t^{(i)} \propto \lambda_{t-1}^{(i)} \exp\{-\beta \alpha_t s_t(v^{(i)}, o^{(i)})\} \tag{31}$$

where $s_t$ is the new learner added to the ensemble in (24). The factor $\beta$ can be used to control the data weights, i.e., as $\beta \to 0$, the weights approach the uniform distribution.

Since $\alpha_t > 0$, the weight increases if $s_t = h_t - H_{t-1} < 0$. The new interpretation is that for a given data instance $i$, if the new weak learner $h_t$ is less likely than the average of previous weak learners $H_{t-1}$, the AdaBoost.MRF will increase the weight for that data instance. This is different from the usual boosting behaviour, where the data weight increases if the strong learner fails to correctly classify the instance. The AdaBoost.MRF seems to maximise data likelihood rather than to minimise the training error, and this is particularly desirable for density estimation.

### 3.3.2 Convergence property

We now provide a formal support for the convergence of the tree selection procedure in (30). The search direction $s$ satisfying the condition in Equation (27) is called gradient-related to $H_t$ [2] Proposition 1.2.3.

Given a Lipschitz continuity condition on $\nabla L_H$, i.e., $\|\nabla L_H(H) - \nabla L_H(H')\| \leq M \|H - H'\|$, for some $M > 0, \forall H, H' \in \mathcal{H}$, where $\mathcal{H}$ is the function space, a gradient-related search direction $s_t$, and a reasonably (positive) small step size $\alpha_t$ that satisfies

$$\epsilon \leq \alpha_t \leq (2 - \epsilon) \frac{\|\nabla L_H(H_{t-1})\|}{M \|s_t\|^2} \tag{32}$$

where $\epsilon$ is a fixed positive scalar. Then

$$\lim_{t \to \infty} H_t = \arg \min_H \mathcal{L}_H(H) \tag{33}$$

The Lipschitz continuity condition can be satisfied in our case because $L_H$ is twice differentiable, and the Hessian $\nabla^2 L_H$ is bounded [2] p. 48]. The constant $M$ is hard to find analytically, so in our implementation, we set the step size to a small constant $\alpha_t = 0.05$, and we found it is sufficient in our experiments. The algorithm terminates when we cannot find any weak learner $s$ that satisfies the condition in Equation (27)
3.3.3 Complexity

The running time of AdaBoost.MRF scales linearly in number of trees $R$, each of which (cf. section 2.1) takes $O(2|\mathcal{V}|^2|\mathcal{S}|^2)$ inference time. If we only consider limited spanning trees, just enough to cover the whole network, then $R$ can be quite moderate. For example, for a fully connected network, we just need $R = |\mathcal{V}|$, and in a grid-like network (e.g., Figure 3a), $R = 2$ is sufficient (e.g., Figure 4).

3.4 Combining the parameters

Up to this point, we have successfully estimated the parameters of individual trees, and thus the strong learner, which may be enough for classification purposes. However, our ultimate goal is to (approximately) estimate the parameters of the original network, which is a superimposition of individual trees. This subsection presents a method for such an approximate estimation.

Recall that $H(x, o) = \sum_l \alpha_l h_l(x, o)$ and $h_l(x, o) = \log P_{\tau_l}(x|o)$, and $H(x, o) = \sum_l \alpha_l \log P_{\tau_l}(x|o)$. Assume that the tree distribution also belongs to the exponential family in (1) with different parameters $w_l$ and the same global feature vector $F(x, o)$. We require that the parts of the parameters $w_l$, which correspond to cliques outside the trees, to be zero. We then can rewrite:

$$H(x, o) = \langle \sum_l \alpha_l w_l, F(x, o) \rangle - \sum_l \alpha_l A_l(o)$$

Thus, the label $\hat{x}$ returned by the strong learner in (7) becomes

$$\hat{x} = \arg \max_x \langle \sum_l \alpha_l w_l, F(x, o) \rangle$$

Obviously, $\hat{x}$ should also be the MAP assignment of the model defined by $x_{MAP} = \arg \max_x P(x|o) = \arg \max_w \{w, F(x, o)\}$, i.e., $\hat{x} = x_{MAP}$. One natural way is to set $w$ as the ensemble parameters $w = \sum_l \alpha_l w_l$, so that

$$P(x|o) \propto \exp \left( \sum_l \alpha_l w_l, F(x, o) \right)$$

$$\propto \prod_l P_{\tau_l}(x|o)^{\alpha_l}$$

$$P(x|o) = \frac{\prod_l P_{\tau_l}(x|o)^{\alpha_l}}{\sum_x \prod_l P_{\tau_l}(x|o)^{\alpha_l}}$$

The combined model turns out to be a Logarithmic Opinion Pool (LogOP) [12, 22], a special case of the more general ensemble framework. Each model $P_{\tau_l}(x|o)$ is an expert to provide an estimate of the true distribution $Q(x|o)$. The aggregator $P(x|o)$ is indeed a minimiser of the weighted sum of Kullback-Leibler divergences between the $Q(x|o)$ and each $P_{\tau_l}(x|o)$ [12].

$$P(x|o) = \arg \min_{Q(x|o)} \sum_l \alpha_l \sum_x Q(x|o) \log \frac{Q(x|o)}{P_{\tau_l}(x|o)}$$

A complete set of algorithmic steps for the proposed AdaBoost.MRF is summarised in Figure 8.
3.5 Relations to other works

In relation to other approaches, the work of [12] shows that $P(x|o)$ is closer to the true distribution $Q(x|o)$ than the average of all individual experts $P_{\tau}(x|o)$. Our boosting algorithm can be seen as an estimator of the weighting factors $\{\alpha_l\}$. [22] offers an interesting discussion on the relation between Markov networks and the LogOP and the properties of desirable aggregators which the LogOP satisfies. Our method is based on the idea of superimposition, or union of sub-networks, that is, if a node or an edge belongs to the aggregated network, it must belong to one of the individual sub-networks. In [27] the authors consider the combination of different models but they share the same underlying simple chain structure. Model are trained independently and then combined using the LogOP. The model weights $\{\alpha_l\}$ are then estimated by maximising the likelihood of the combined models. This approach is fine as long as the underlying structure is tractable. Another related idea is the product-of-experts [13], where all weights are unity. In [13] sampling is used to overcome the intractability, which may not converge within a limited time. By contrast, our method is efficient as it deals directly with trees.

4 Recognition of Multi-level Activities with Missing Data

Our application of interest is to model indoor activities of a person that are observed through two cameras mounted to the ceilings in a kitchen, as shown in Figure 2. In this environment, activities are naturally acted out in a hierarchical manner. We consider two levels of activity abstraction, which can be modelled using a two-layer dynamic conditional random field (DCRF) [28].

The bottom level presents primitive or atomic activities such as go-to-cupboard or at-the-fridge. Higher-order activities are captured at the higher level such as having-snack or short-meal. Differing from the original setting of the DCRF in [28], we allow some missing labels.
in our model, and thus we call the model the partially labelled DCRF (ph-DCRF) (Figure 3). We note that although the two-level DCRF is considered in this paper, the same construction can readily be generalized to model more complex semantics with richer levels of hierarchy and temporal interactions.

Given the training data, we first learn the parameters and then use it for annotating and segmenting unseen data. We now describe and compare some alternatives to the AdaBoost.MRF for parameter learning.

For the original ph-DCRF, exact estimation of marginals can be carried out by collapsing all the states at the current time into a mega-state (see Figure 3b) and performing a forward-backward procedure, which is infeasible for deep models. Approximate inference using the BP and WJW [39] methods has the complexity of $O(2|\mathcal{E}||S|^2)$, where $I$ is the number of message passing rounds, $|\mathcal{E}|$ is the number of edges in the network, and $S$ is the state size per node. However, the number of rounds $I$ until convergence if it does is not known analytically, and there has not been any theoretical estimate of it yet.

In our AdaBoost.MRF, inference in the trees takes $O(2|\mathcal{V}||S|^2)$ time, where $|\mathcal{V}|$ is the number of nodes in the network. Thus, for $D$ data instances, and $R$ trees, the AdaBoost.MRF costs $O(4DR|\mathcal{V}||S|^2)$ in total time for each gradient evaluation since we need to take both $A(\mathbf{v},\mathbf{o})$ and $A(\mathbf{o})$ into account. Similarly, the BP and WJW-based MLE requires $O(4DI|\mathcal{E}||S|^2)$ time. As for fully connected networks, $|\mathcal{E}|=\frac{1}{2}|\mathcal{V}|(|\mathcal{V}|+1)$ while for the grid DCRFs, $|\mathcal{E}| \approx 2|\mathcal{V}|$, if we take only $R=|\mathcal{V}|$ trees for the former case, and $R=2$ for the latter case, the total complexity per gradient evaluation of the BP and WJW-based MLE and the AdaBoost.MRF will be similar up to a constant $I$. We summarise the complexities in Table 1.

![Figure 3: (a) The partially labelled DCRF (ph-DCRF), and (b) The equivalent chain CRF. Filled circles and bars are data observations, empty circles are missing labels, shaded labels are the visible.](image)

| BP/WJW        | AdaBoost.MRF          |
|---------------|-----------------------|
| $O(4DI|\mathcal{E}||S|^2)$ | $O(4DR|\mathcal{V}||S|^2)$ |

Table 1: Complexity per gradient evaluation compared with existing message-passing methods.

5 Experimental Results

The dataset used in this experiment was collected in our previous work [19] using a system shown in Figure 2. We captured 45 video sequences for training and 45 sequences for testing. The observations are sequences of noisy coordinates of the actor walking in scene acquired using a background subtraction tracking algorithm. We consider 3 complex activities (states) at the
top level: short-meal (1), have-snack (2), normal-meal (3), and twelve primitive activities at the bottom level, which are summarised in Table 2.

Each complex activity is comprised of some primitive activities, and states at each level can freely transit to each other but generally we do not have this knowledge at hand for our experiment.

For evaluating the aspect of missing labels, we randomly provide half the labels for each level during training. For testing, the MAP assignments resulted from Pearl’s loopy max-product algorithm are compared against the ground-truth.

| No. | Activity       | No. | Activity      |
|-----|----------------|-----|---------------|
| 1   | Door→Cupboard  | 7   | Fridge→TV chair|
| 2   | Cupboard→Fridge| 8   | TV chair→Door |
| 3   | Fridge→Dining chair | 9   | Fridge→Stove |
| 4   | Dining chair→Door | 10  | Stove→Dining chair |
| 5   | Door→TV chair  | 11  | Fridge→Door   |
| 6   | TV chair→Cupboard | 12  | Dining chair→Fridge |

Table 2: Primitive activities used in the experiment.

5.1 Feature Extraction

With the data described above, the input to the DCRFs is simply sequences of coordinates. At each time slice $\gamma$, we extract a vector of five elements from the observation sequence $g(o, \gamma) = \{g_m(o, \gamma)\}_{m=1}^{5}$ where each $g_m(o, \gamma)$ corresponds to the $(X, Y)$ coordinates, the $X$ and $Y$ velocities, and the speed respectively. To fully specify the model, we consider three types of feature functions for the potentials of the network: (a) data-association corresponding to node potentials, (b) temporal-relation corresponding to state transition potentials at the same level, and (c) cross-semantic-relation corresponding to parent-child potentials across different levels.

Let $x^2_\gamma$ denotes the state variable at the level 2 (the bottom) and time $\gamma$. For the first feature set, we define the data-association features at the bottom level as:

$$f_{i,m,\epsilon}(x^2_\gamma, o) := g_m(o, \gamma + \epsilon)I\{x^2_\gamma = i\}$$

where $\epsilon = -s_1, \ldots, 0, \ldots, s_2$ is the amount of look-ahead or look-back, for some positive integers $s_1$, $s_2$, $x^2_\gamma = 1, 2, \ldots, 12$ is the state (at level 2). We choose $s_1 = s_2 = 2$ for reasonable computation, so that the current primitive activity $x^2_\gamma$ is correlated with five surrounding observation features $g_m(o, \gamma)$. At the top level, however, instant information such as velocities offer limited help since the complex activities often span long periods. Instead of using the real coordinates $(X, Y)$ for data association, we quantize them into 24 squares in the room. We also use much larger windows with $s_1 = s_2 = 20$. To avoid computational overhead, we take $\epsilon = -s_1, -s_1 + 5, \ldots, s_2 - 5, s_2$.

The second and third feature sets consist of simple indicator functions

$$f_{l_1,l_2}(x^d_{\gamma-1}, x^d_\gamma) := I\{x^d_{\gamma-1} = l_1\} I\{x^d_\gamma = l_2\}$$

for the second set, and
for the third set, where \( d = 1, 2 \) is the depth level.

5.2 Spanning trees for AdaBoost.MRF

The AdaBoost.MRF algorithm described in Figure 8 requires the specification of a set of spanning trees which will be used as the weak classifiers. Given the grid structure considered in this experiment, there are many spanning trees that can be extracted. However, since the nature of our problem is about temporal regularities where the slice structure is repeated over time, it is natural to decompose the network into trees in such a way that the structural repetition is maintained. With this hint, there are two most noticeable trees that stand out as shown in Figure 4, which roughly corresponds the top and bottom chains respectively.

![Two process view of the DCRF in activity modeling: (a) the complex activity, and (b) the primitive.](image)

Figure 5: Two-slice structure of a DCRF (left-most), and some spanning trees (the rest).

With the same method, the number of trees for dynamic models which respect the Markov assumption is reduced drastically. If we impose further restrictions that each state can only interact with the level right above and right below it, then the number of trees can be manageable (e.g. see Figure 5 for another example).

5.3 Segmentation and annotation results

For comparing with the AdaBoost.MRF for the DCRFs, we implement MLE learning methods based on BP, WJW and exact inference. We also evaluate the effectiveness of the DCRFs against the Layered HMMs (LHMMs) [20], where the output of the bottom HMM is used as the input for the top HMM. Since, it is difficult to encode rich feature information in the LHMMs without producing very large state space, we limit the LHMMs features to be the discretised positions and
the differences between current position and the previous and next ones. Our new implementation of LHMMs differs from the original in [20] for each HMM has been extended to handle the partially observed states. To test whether adding more layers can improve the performance of the model, we run a simple Flat-CRF on the data at the lower level. All learning algorithms are initialised uniformly. For segmentation purposes, we report the macro-averaged $F_1$ scores on a per-label basis.

![Figure 6: Macro-averaged $F_1$ scores at the bottom layer vs. training time.](image)

For parameter optimisation of the (re-weighted) log-likelihood, initially we used the limited memory quasi-Newton method (L-BFGS) as suggested in the CRF literature but it seems to be slower and it converges prematurely to poor solutions for the BP and the exact inference. The conjugate-gradient (CG) method works better in our experiments. For the Markov forests, we run for only two iterations of CG per boosting round with the initial parameters from the previously learned ones since we only need to meet the condition (27). The WJW inference loop is stopped if the messages have converged at the rate of $10^{-4}$ or after 100 rounds. It appears that the final performance of BP is sensitive to the choice of convergence rates, while it is fairly stable for the WJW. For example, the $F_1$ scores at the bottom level for BP are 0.84, 0.87 and 0.82 corresponding to the rates of $10^{-3}$, $10^{-4}$ and $10^{-5}$, respectively. Below we report only the case of $10^{-4}$, which appears to be the best both in terms of accuracy and speed. Learning algorithms for the DCRFs are stopped after 100 iterations if they have not converged at the rate of $10^{-5}$.

The performance of the AdaBoost.MRF and its alternatives is reported in Figure 6 and Table 3, respectively. Overall, after enough training time, the AdaBoost.MRF performs comparably with the MLE methods based on BP and WJW. The exact inference MLE method gives slightly better result as expected but at the cost of much slower training time. However, it should be stressed that inference in our AdaBoost.MRF always converges, while it is not guaranteed in the BP and WJW and it is generally intractable in the exact method. The complexity per evaluation of the log-likelihood gradient is known and fixed for the AdaBoost.MRF, while for the BP and the WJW, it is generally dependent on the convergence criteria and how much the distribution is different
| Algorithm     | Top-layer | Bottom-layer |
|---------------|-----------|--------------|
| AdaBoost.MRF  | 0.98      | 0.87         |
| BP            | 0.99      | 0.87         |
| WJW           | 0.98      | 0.87         |
| Exact         | 0.98      | 0.88         |
| LHMM          | 0.88      | 0.67         |
| Flat-CRFs     | -         | 0.78         |

Table 3: Macro-averaged $F_1$ scores for top and bottom layers.

from uniform (see Table[1]).

Table[3] also shows that the choice of discriminative model over the generative model in our activity recognition problem is justified. The LHMMs are worse than both the flat-CRFs at the bottom layer and the DCRFs at the top layer. Furthermore, the DCRFs variants are more consistently accurate than the flat CRFs. The result is consistent with that in [28]. This can be explained by the fact that more information is encoded in the DCRFs.

Figure 7 shows the AdaBoost.MRF segmentation details of 22 randomly selected sequences which are concatenated together.

![Figure 7](image-url)

Figure 7: The segmentation compared with the ground-truth at the bottom level.

6 Discussion

In this section we would like to further discuss the implications of the proposed AdaBoost.MRF as a guided search for estimation in maximum likelihood setting and as a method for feature selection.
6.1 AdaBoost.MRF as guided search for maximum likelihood estimation

As we rely on the boosting capacity to boost very weak learners to a strong one, we do not need to reach the maximum of the weighted log-likelihood in each round. We can simply run a few training iterations and take the partial results as long as the condition in (27) is met. To speedup the learning, we can initialise the parameters for each weak learner to the previously learned values. This procedure has an interesting interpretation for tree-structured graphs. As we do not have to select the best spanning trees anymore, the algorithm is simply to optimise the re-weighted log-likelihood in a stage-wise manner. We argue that this approach can be attractive because more information from the data distribution can be used to guide the maximum likelihood estimation (MLE), and it can create more diverse weak classifiers.

For non-tree graphs, the guided MLE can be derived by assuming that the tree mixing coefficients $\alpha^l$ are known in advance. Recall that we have assumed that parameters are not shared among trees. To see how we can relax this assumption by allowing trees to share some common parameters, let $w$ be the joint parameters of all the trees, we rewrite the loss in (22) as:

$$L_H(w) = \frac{1}{D} \sum_i \exp\{-\beta \alpha^l \log P^l(v^{(i)}|o^{(i)};w)\}$$

(39)

Taking derivative of $L_H(w)$ with respect to $w$ yields:

$$\nabla L_H(w) \propto -\sum_i \lambda^{(i)} \sum_t \alpha^l \nabla \log P^l(v^{(i)}|o^{(i)};w)$$

(40)

Thus we have arrived a parallel version of the AdaBoost.MRF in that all trees are updated at the same time. This version, however, loses the tree-selection property.

6.2 AdaBoost.MRF as embedded method for feature selection

It should be noted that in our formulation of AdaBoost.MRF, we do not require the unique existence of one distribution per tree. Instead, we have to freedom to choose as many distributions as we wish, provided that these distributions can be expressed in the log-linear form (cf. section 2.1). Thus, given a feature pool, we can create a set of distributions, each of which incorporates only a subset of features. As the AdaBoost.MRF proceeds, only one particular tree-based distribution is picked at a time, thus implicitly performing the feature selection capacity. It is reasonable to expect that bad feature combination will result in a poor likelihood, and thus will not be selected in Equation(30).

7 Conclusion

We have presented a novel method for using boosting in parameter estimation of the general Markov networks with partial labels. The algorithm AdaBoost.MRF offers an efficient way to tackle the intractability of the maximum likelihood method by breaking the model into tractable trees and combining them to recover the original networks. We apply the algorithm to the new problem of multi-level activity recognition and segmentation using the recently proposed DCRFs.

We would like to stress, however, that our AdaBoost.MRF is not limited to only DCRFs but can be readily applied to any arbitrary CRFs. In addition, not only it can discriminatively
approximately estimate the conditional distributions $P(x|o)$, but also it can generatively learn the joint distributions $P(x,o)$.

Furthermore, in our experiments, it appears that the AdaBoost.MRF exhibits a structure learning behaviour since it may selectively pick some trees more frequently than others, giving higher weights to those trees. An important issue we have left unanswered is how to automatically select the optimal tree at each round without knowing the set of trees in advance. We plan to investigate these aspects and the use of AdaBoost.MRF in a wider range of applications.

A General Hölder’s Inequality

Let us start with the elementary Hölder’s inequalities [11, Theorem 13]. For $r > 1$, $r' > 1$, $a_i \geq 0$, $b_i \geq 0$ and $1/r + 1/r' = 1$, the following inequality holds:

$$\sum_{i=1}^{n} a_i b_i \leq \left( \sum_{i=1}^{n} a_i^r \right)^{1/r} \left( \sum_{i=1}^{n} b_i^{r'} \right)^{1/r'}$$  \hspace{1cm} (41)

The sign of equality hold iff $a_i^r = \alpha b_i^{r'}$, \forall $i$. The case $\alpha = 0$ is trivial, thus we do not consider here. The Cauchy’s inequality is a special case if $r = r' = 2$.

By induction, we can obtain the following extension to this basic inequality [11, Theorem 11]. If $a_{ij} \geq 0$, for $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$, and if $r_j > 1$ with $\sum_{j=1}^{m} 1/r_j = 1$, denoting the vector $A_j = (a_{1j}, a_{2j}, \ldots, a_{nj})$, then

$$\sum_{i=1}^{n} \prod_{j=1}^{m} a_{ij} \leq \prod_{j=1}^{m} \left( \sum_{i=1}^{n} a_{ij}^{r_j} \right)^{1/r_j}$$  \hspace{1cm} (42)

and the sign of equality holding iff $A_j = \alpha_j A_k$ for some scalars $\alpha_j$ and $j \neq k$. In other words, the equality sign holds iff all vectors $A_j(s)$ are proportional.

Let us proceed by induction to prove the ‘inequality’ part.

(i) For $j = 1$, (42) holds trivially.

(ii) Assume (42) holds for any $1 \leq j \leq l$, we will prove that it also holds for $j = l + 1$. Using the basic Hölder inequality (41) for $r > 1; r' > 1; 1/r + 1/r' = 1$, we have:

$$\sum_{i}^{l+1} a_{i,j} = \sum_{i} a_{i,l+1} \prod_{j=1}^{l} a_{i,j} \leq \left( \sum_{i} a_{i,l+1} \right)^{1/r} \left( \prod_{j=1}^{l} a_{i,j} \right)^{1/r'}$$  \hspace{1cm} (43)

$$= \left( \sum_{i} a_{i,l+1}^{r} \right)^{1/r} \left( \prod_{i} a_{i,j}^{r'} \right)^{1/r'}$$  \hspace{1cm} (44)

Let $\beta_{i,j} = a_{i,j}^{r'}$, by applying the inductive assumption that (42) holds for $j \leq l$ to the second factor in the RHS of (43) yields:
\[
\left( \sum \prod_{j=1}^{l} \beta_{i,j} \right)^{1/r'} \leq \left( \prod_{j=1}^{l} \left[ \sum \beta_{i,j}^{r'j} \right]^{1/r'} \right)^{1/r'} \\
= \prod_{j=1}^{l} \left[ \sum \beta_{i,j}^{r'j} \right]^{1/r'j} \leq \prod_{j=1}^{l} \left[ \sum a_{i,j}^{r'j} \right]^{1/r'j} \tag{45}
\]

where \( \sum_{j=1}^{l} 1/r'_j = 1 \). Substituting (45) back into (44), we have:

\[
\sum \prod_{j=1}^{l+1} a_{i,j} \leq \left( \sum a_{i,l+1}^{r'j} \right)^{1/r} \prod_{j=1}^{l} \left[ \sum a_{i,j}^{r'j} \right]^{1/r'j} \tag{46}
\]

As \( \sum_{j=1}^{l} 1/r'_j = 1 \), we then have \( 1/r + \sum_{j=1}^{l} 1/r'_j = 1/r + 1/r' = 1 \). Now we change the notation as \( r'_{l+1} \leftarrow r \) and \( r'_j \leftarrow r'_j r' \), we have \( \sum_{j=1}^{l+1} 1/r'_j = 1 \). Thus (46) becomes

\[
\sum \prod_{j=1}^{l+1} a_{i,j} \leq \prod_{j=1}^{l+1} \left[ \sum a_{i,j}^{r'j} \right]^{1/r'_j} \tag{47}
\]

This means that the inequality (42) holds for \( j = l + 1 \). By the induction principle, the inequality (42) holds for all \( j \geq 1 \). This completes the proof. \( \diamond \)


\section*{B Pseudo-code for AdaBoost.MRF}

\begin{verbatim}
Input: i = 1, ..., D data pairs, graphs \{G^{(i)} = (V^{(i)}, E^{(i)})\}
and the regularisation term \(\beta \in (0, 1]\)

Output: learned parameter vector \(w\)

Begin

Select spanning trees for each data instance
Initialise \(\{\lambda_0^{(i)} = \frac{1}{D}\}\), and \(\alpha_1 = 1\)

For each boosting round \(l = 1, 2, \ldots\)

Train all trees given weighted data \(\{\lambda_l^{(i)}\}_{i=1}^D\)

/*Select the best tree distribution*/
\((t, w_t) = \text{arg max}_{\tau, w_{\tau}} \sum_{i=1}^D \lambda_l^{(i)} \log P_{\tau}(v^{(i)}|o^{(i)}, w_{\tau})\)

\(h_l = \log P_t(v|o, w_t)\)

\(s_l = h_l - H_{l-1}\)

If \(\sum_{i=1}^D \lambda_l^{(i)} s_l(v^{(i)}|o^{(i)}) \leq 0\) Then go to Output

If \(l > 1\) Then select the step size \(0 < \alpha_l < 1\)

/*Update the strong learner*/

\(H_l = (1 - \alpha_l)H_{l-1} + \alpha_l h_l\)

/*Scale down the previous learner weights*/

\(\alpha_j \leftarrow \alpha_j(1 - \alpha_l), \text{ for } j = 1, \ldots, l-1\)

/*Update the data weight*/

\(\lambda_l^{(i)} \leftarrow \lambda_{l-1}^{(i)} \exp\{-\beta \alpha_l s_l(v^{(i)}|o^{(i)})\}\)

\(\lambda_l^{(i)} \leftarrow \frac{\lambda_l^{(i)}}{\sum_{i=1}^D \lambda_l^{(i)}}\)

End

Output \(w = \sum_l \alpha_l w_l\)

End

Figure 8: AdaBoost.MRF - AdaBoosted Markov Random Forests.

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