Testing the Swampland: $H_0$ tension

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The de Sitter Swampland conjecture compels us to consider dark energy models where $\lambda(\phi) \equiv |\nabla_\phi V|/V$ is bounded below by a positive constant. Moreover, it has been argued for Quintessence models that the constant $\lambda$ scenario is the least constrained. Here we demonstrate that increasing $\lambda$ only exacerbates existing tension in the Hubble constant $H_0$. The identification of dark energy models that both evade observational bounds and alleviate $H_0$ tension constitutes a robust test for the Swampland program.

INTRODUCTION

The de Sitter Swampland conjecture\cite{1} claims that de Sitter vacua belong to the Swampland\cite{2,3} of low-energy effective theories coupled to gravity. More concretely, it has been proposed that the potential for scalar fields must satisfy the universal bound,

$$|\nabla_\phi V| \geq \frac{c}{M_{\text{pl}}} \cdot V,$$

(1)

where $M_{\text{pl}}$ is the Planck mass and $c$ denotes a constant of order 1. Despite being controversial - it questions $\Lambda$CDM - the conjecture can be motivated from the distance conjecture\cite{3} and the Bousso covariant entropy bound\cite{2}, which places it on firmer footing\cite{6} (see also\cite{7–9}).

The cosmological implications of the conjecture were initially studied in\cite{11} where it was suggested that Quintessence models\cite{11} with an exponential potential $V = V_0 e^{-\lambda \phi}$ constituted valid dark energy models, valid in the sense that they satisfy the bound\cite{11} and are consistent with data. The constraints from data were subsequently tightened in\cite{12–14} (also\cite{15,16}). Throughout, the potential elephant in the room has been "Hubble tension", a discrepancy between a local determination of the Hubble constant due to Riess et al.\cite{17–19} and Planck CMB analysis based on $\Lambda$CDM\cite{20}. Indeed, in the lifetime of the de Sitter Swampland conjecture, we have witnessed the statistical importance of the difference slowly tick up from $3.8 \sigma$\cite{12} to $4.4 \sigma$\cite{13}. At some point this becomes difficult to ignore.

In the immediate aftermath of the conjecture\cite{1}, we suggested it was natural if $H_0$ tension and the Swampland were connected\cite{21}. More precisely, in order to reconcile the Riess result with other cosmological determinations of Hubble at low redshift $z < 2$, we floated the idea of a turning point in $H(z)$. Explaining the tension remains an open problem, but in this letter we take aim at the models proposed in\cite{10} and take them to their logical conclusion by asking, do they reduce $H_0$ tension? The answer seems NO! Note, there are various recent studies of Quintessence in the literature and the tension cannot be relieved\cite{22–25}, so this conclusion is not overly surprising.

Having said that, our analysis here is simple and to the point. It emphasises the need to consider $H_0$ tension as a further litmus test for good dark energy models satisfying $\mathbb{I}$. Otherwise, the Swampland may be jeopardy. The simplicity of our analysis comes from the fact that we can piggy-back on\cite{10} and integrate a single additional equation to identify the Hubble parameter as a function of redshift $H(z)$. Once this is done, we allow the Hubble constant $H_0 \equiv H(z = 0)$ and dark energy density today $\Omega_\phi(z = 0)$, alternatively the matter density today $\Omega_m \equiv 1 - \Omega_\phi(z = 0)$, to be free parameters, which we determine through fits to a well-known compilation of cosmological measurements of $H(z)$ at low redshift\cite{20}. We will see that any deviation from $\lambda = 0$ ($\Lambda$CDM) reduces the best-fit value of $H_0$ and in the process only increases $H_0$ tension.

QUINTESSENCE

Here we quickly review Quintessence following\cite{27}. Consider the scalar-gravity action

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_{\text{pl}}^2 R - \frac{1}{2} \partial_{\mu}\phi \partial^{\mu}\phi - V(\phi)\right] + S_m,$$

where $S_m$ denotes the matter sector. Consider also the Ricci-flat Friedmann-Lemaître-Robertson-Walker (FLRW) background with scale factor $a(t)$. Assuming non-relativistic matter, the equation of state $w_m$ for the matter sector becomes $w_m = 0$, so we can regard matter as pressure-less $p_m = 0$. With this restriction the equations of motion of the above action can be succinctly recast in terms of the following dynamical system,

$$x' = -3x + \frac{\sqrt{6}}{2} \lambda y^2 + \frac{3}{2} x(1 + x^2 - y^2),$$

(2)

$$y' = -\frac{\sqrt{6}}{2} \lambda xy + \frac{3}{2} y(1 + x^2 - y^2),$$

(3)

$$\frac{H'}{H} = -3x^2 - \frac{3}{2} (1 - x^2 - y^2),$$

(4)

where we have defined the variables:

$$x \equiv \frac{\dot{\phi}}{\sqrt{6} M_{\text{pl}} H}, \quad y \equiv \frac{\sqrt{V(\phi)}}{\sqrt{3} M_{\text{pl}} H}. $$

(5)

Note that dots denote the usual time derivatives and primes denote derivatives with respect to $N \equiv \ln a$. In
addition, we have defined $\lambda \equiv -M_\phi \nabla_\phi V/V$. In terms of these new variables the scalar density $\Omega_\phi \equiv \rho_\phi/(3M^2_{\text{pl}}H^2)$ describing dark energy is

$$\Omega_\phi = x^2 + y^2,$$

which sums to unity with the matter density $\Omega_m \equiv \rho_m/(3M^2_{\text{pl}}H^2)$, i.e. $\Omega_m + \Omega_\phi = 1$.

As explained in [27], when $\lambda$ is constant, the system has four fixed points where $x' = y' = 0$. When $w_m = 0$ and $\lambda^2 < 2$, the interpolating flows from the vicinity of unstable saddle point $(x, y) = (0, 0)$, where $\Omega_\phi = 0$, to the stable fixed point $(x, y) = (\lambda/\sqrt{6}, \sqrt{1 - \lambda^2/6})$, with $\Omega_\phi = 1$, provide us with one of the simplest models to describe a transition from a matter-dominated regime to one of cosmic acceleration.

Following [10, 12], we focus on this trajectory. To be more precise, we integrate [2] and [3] in $N$ from a point close to $(x, y) = (0, 0)$, where $N = 0$, to a large enough value of $N$, where $\Omega_\phi \approx 1$. We then employ the redefinition $N - N_s = -\log(1 + z)$, so that $N = N_s$ corresponds to redshift $z = 0$ today. To further identify $H(z)$, we change variables from $N$ to $z$ and integrate [4] in $z$ subject to the condition that $H(0) = H_0$, where $H_0$ and $N_s$ are constants we will determine through fits to the cosmological data. From $N_s$, we can infer the dark energy density $\Omega_\phi(z = 0) = \Omega_\phi(N = N_s)$ today, which in turn gives us the matter density $\Omega_m$. For presentation purposes, essentially to mirror the analysis of [10,12], we restrict our data fitting to $z < 1$, but one can check that extending this out to $z = 2.36$ does not change conclusions.

**COSMOLOGICAL DATA**

The data we use in this study comprises cosmological measurements of the Hubble parameter $H(z)$ compiled by Farooq et al. [26] based on cosmic chronometric and baryon acoustic oscillations (BAO) techniques [28–37]. As mentioned, we restrict the data to $z < 1$. To get oriented, we recall that $\Lambda$CDM can be described at low redshift by the expression

$$H(z) = H_0 \sqrt{1 - \Omega_m} + \Omega_m(1 + z)^3,$$

where $H_0$ is the Hubble constant and $\Omega_m$ is the matter density today. Performing an error-weighted least squares fit to the $z < 1$ data returns the best-fit values [5d]

$$H_0 = 67.64 \pm 1.79, \quad \Omega_m = 0.31 \pm 0.04,$$

where throughout we allow for a $1\sigma$ error. Recalling that $1 - \Omega_\phi(z = 0) = \Omega_m$, we will use this fit as a consistency check on our numerical solution. Setting $\lambda = 0$ and performing a two-parameter error-weighted best-fit of the numerical solution $H(z)$ to the data, we recover the above fit [8]. This agreement is illustrated in Figure 1, where we also include the data for comparison. It is worth noting that in contrast to CMB, the error bars are quite large, but this only gives more wiggle room for any deviation from $\Lambda$CDM at low redshift. We do not include the Riess et al. determination in the data fitting.

Now that we have confidence in the numerical solution for $\lambda = 0$, namely that it recovers $\Lambda$CDM, we turn our attention to non-zero values of $\lambda$. From the perspective of the de Sitter Swampland conjecture, it is important that $\lambda > 0$, but as $\lambda$ gets larger the constraints from datasets become more stringent [10,12,14], so we do not probe above $\lambda = 2$.

However, the point we want to make here is different and this brings us to the crux of this short note. Failing some unanticipated systematics, there is growing consensus in the community that the higher Riess et al. local determination of the Hubble constant [10] is here to stay. This provides some hint of a new cosmology beyond $\Lambda$CDM, potentially one without a de Sitter attractor, thus fitting the key premise of the de Sitter Swampland program. That being said, if $H_0$ tension is real, then deviations from $\Lambda$CDM within this framework should not lower the best-fit value of $H_0$ and therefore only increase the tension. They should not make the situation worse.

| $\lambda$ | $H_0$ | $\Omega_m$ |
|-----------|-------|------------|
| 0         | 67.64 ± 1.79 | 0.31 ± 0.04 |
| 0.2       | 67.58 ± 1.78 | 0.31 ± 0.04 |
| 0.4       | 67.41 ± 1.75 | 0.30 ± 0.04 |
| 0.6       | 67.11 ± 1.71 | 0.29 ± 0.04 |
| 0.8       | 66.68 ± 1.64 | 0.28 ± 0.04 |
| 1         | 66.09 ± 1.55 | 0.25 ± 0.04 |

**TABLE I**: Best-fit values of $(H_0, \Omega_m)$ with $\lambda$.

In the above table we record the best-fit values of $H_0$ and $\Omega_m$ as $\lambda$ is increased and confirm that $H_0$ is traveling in the wrong direction relative to $\Lambda$CDM. In other words, the tension is increasing. We provide a graphical representation of the same tendency in Figure 2, where we include the current Riess et al. determination $H_0 = 74.03 \pm 1.42$ km s$^{-1}$ Mpc$^{-1}$ [19] for reference. As is evident from the plot, as $\lambda$ is increased, the low redshift data pulls the best-fit $H_0$ to lower values and thus further away from Riess et al. This behaviour can be contrasted...
with the two-parameter model presented in \cite{21}, based on \cite{38}, which favours the higher value against the same data.

![Figure 2: We illustrate the best-fit values of $H_0$ as $\lambda$ is varied. As is evident from the plot, larger values of $\lambda$ lead to lower values of $H_0$. We include the current Riess et al. value $H_0 = 74.03 \pm 1.42$ km s$^{-1}$ Mpc$^{-1}$ \cite{14} for comparison.](image)

**SUMMARY**

To date studies of the de Sitter Swampland conjecture in the context of late-time cosmology have largely overlooked the tension in the Hubble constant. Following up on comments made initially in \cite{21}, the point we wish to drive home here is that the simple Quintessence models considered in \cite{10}, and followed up elsewhere, appear to inadvertantly make the problem worse.

To support this claim, we made use of cosmological measurements of the Hubble parameter and used this data to infer the Hubble constant. Given the current status of the data, for any monotonically increasing function of the Hubble parameter, essentially a feature that is expected to come from good EFTs where the null energy condition (NEC) is satisfied, one should arrive at similar conclusions, for example \cite{39,46}. In this sense, we believe that Quintessence is representative, but data may favour more dramactic reconciliations for $H_0$ tension e. g. \cite{17}. Note, although data may naively point to a violation of the NEC, this can in principle be masking further physics with no violation of the NEC. See \cite{18,49} for work in the context of dark energy.

Since $H_0$ tension may be expected to persist in the near future, it is imperative to identify dark energy models that satisfy the Swampland constraints \cite{11}, evade observational bounds, but to be prudent, it is best they do not increase $H_0$ tension. This provides a further litmus test for good dark energy models motivated by the Swampland.

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