On the growth of powers of operators with spectrum contained in Cantor sets

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Abstract

For \( \xi \in (0, \frac{1}{2}) \), we denote by \( E_\xi \) the perfect symmetric set associated to \( \xi \), that is

\[
E_\xi = \left\{ \exp \left( 2i\pi (1 - \xi) \sum_{n=1}^{+\infty} \epsilon_n \xi^{n-1} \right) : \epsilon_n = 0 \text{ or } 1 \ (n \geq 1) \right\}.
\]

Let \( s \) be a nonnegative real number, and \( T \) be an invertible bounded operator on a Banach space with spectrum included in \( E_\xi \). We show that if

\[
\|T^n\| = O(n^s), \ n \to +\infty
\]

and

\[
\|T^{-n}\| = O(e^{n^\beta}), \ n \to +\infty \text{ for some } \beta < \frac{\log \frac{1}{\xi} - \log 2}{2\log \frac{1}{\xi} - \log 2},
\]

then for every \( \varepsilon > 0 \), \( T \) satisfies the stronger property

\[
\|T^{-n}\| = O(n^{s+\frac{1}{2}+\varepsilon}), \ n \to +\infty.
\]

This result is a particular case of a more general result concerning operators with spectrum satisfying some geometrical conditions.

1 Introduction

We denote by \( \mathbb{T} \) the unit circle and by \( \mathbb{D} \) the open unit disk. We shall say that a closed subset \( E \) of \( \mathbb{T} \) is a \( K \)-set if there exists a positive constant \( c \) such that for any arc \( L \) of \( \mathbb{T} \),

\[
\sup_{z \in L} d(z, E) \geq c|L|,
\]

where \( |L| \) denotes the length of the arc \( L \) and \( d(z, E) \) the distance between \( z \) and \( E \). Let \( E \) be a \( K \)-set. We set

\[
\delta(E) = \sup \left\{ \delta \geq 0 : \int_{0}^{2\pi} \frac{1}{d(e^{it}, E)^\delta} dt < +\infty \right\}.
\]

We have \( \delta(E) \geq \frac{\log \frac{1}{1-c}}{\log \frac{1}{1-c}} \) (see [2] section 5, proof of lemma 2 and corollary). E. M. Dyn’kin showed in [2] that condition \( (K) \) characterizes the interpolating sets for \( \Lambda_s^+(\mathbb{T}) \), \( s > 0 \) (see
section 2 for the definition of $\Lambda^+_s(T)$. Let $s$ be a nonnegative real number, and let $T$ be an invertible operator on a Banach space. We show (theorem 2.3) that if the spectrum of $T$ is included in $E$ and if $T$ satisfies

$$
\|T^n\| = O(n^s), \ n \to +\infty
$$

and

$$
\|T^{-n}\| = O(e^{n\beta}), \ n \to +\infty
$$

for some $\beta < \frac{\delta(E)}{1 + \delta(E)}$,

then for every $\varepsilon > 0$, $T$ also satisfies the stronger property

$$
\|T^{-n}\| = O(n^{s+\frac{1}{2}+\varepsilon}), \ n \to +\infty.
$$

(1)

For $\xi \in \left(0, \frac{1}{2}\right)$, we denote by $E_\xi$ the perfect symmetric set associated to $\xi$, that is

$$
E_\xi = \left\{ \exp \left(2i\pi(1 - \xi) \sum_{n=1}^{+\infty} \epsilon_n \xi^{n-1} \right) : \epsilon_n = 0 \text{ or } 1 \ (n \geq 1) \right\}.
$$

We set $b(\xi) = \frac{\log \frac{1}{\xi} - \log 2}{2\log \frac{1}{\xi} - \log 2}$. We obtain (as a consequence of theorem 2.3) that if the spectrum of $T$ is included in $E_\xi$, $\|T^n\| = O(n^s), \ n \to +\infty$ and $\|T^{-n}\| = O(e^{n\beta}), \ n \to +\infty$ for some $\beta < b(\xi)$, then $T$ satisfies (1). Notice that J. Esterle showed in [5] that if $T$ is a contraction on a Banach space (respectively on a Hilbert space) with spectrum included in $E_{\frac{1}{q}}$ (respectively included in $E_\xi$) such that $\|T^{-n}\| = O(e^{n\beta}), \ n \to +\infty$ for some $\beta < b(\frac{1}{q})$ (respectively $\beta < b(\xi)$), then $\sup_{n \geq 0} \|T^{-n}\| < +\infty$ (respectively $T$ is an isometry). Here $q$ is an integer greater than or equal to 3.

## 2 Growth of powers of operators

Let $p$ be a non-negative integer. We denote by $C^p(T)$ the space of $p$ times continuously differentiable functions on $T$. We set

$$
A^p(\mathbb{D}) = \left\{ f \in C^p(T) : \hat{f}(n) = 0 \ (n < 0) \right\},
$$

$C^\infty(T) = \bigcap_{p \geq 0} C^p(T)$ and $A^\infty(\mathbb{D}) = \bigcap_{p \geq 0} A^p(\mathbb{D})$.

Let $s$ be a nonnegative real number, we denote by $[s]$ the nonnegative integer such that $[s] \leq s < [s] + 1$. We define the Banach algebra

$$
\Lambda_s(T) = \left\{ f \in C^s(T) : \sup_{z, z' \in T} \left| \frac{f([s])(z) - f([s])(z')}{z - z'[s]^{-s}} \right| < +\infty \right\};
$$

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equipped with the norm \( \|f\|_{\Lambda_s} = \|f\|_{C^s(\mathbb{T})} + \sup_{z,z' \in \mathbb{T}} \frac{|f^{(s)}(z) - f^{(s)}(z')|}{|z - z'|^{s-s}} \). We also define the subalgebra

\[
\lambda_s(\mathbb{T}) = \left\{ f \in C^s(\mathbb{T}) : |f^{(s)}(z) - f^{(s)}(z')| = o(|z - z'|^{s-s}), |z - z'| \to 0 \right\},
\]

which we equip with the same norm. We also set

\[
\Lambda_s^+ (\mathbb{T}) = \left\{ f \in \lambda_s(\mathbb{T}) : \hat{f}(n) = 0 \quad (n < 0) \right\}
\]

and \( \lambda_s^+ (\mathbb{T}) = \left\{ f \in \lambda_s(\mathbb{T}) : \hat{f}(n) = 0 \quad (n < 0) \right\} \).

We remark that if \( s \) is an integer, \( \lambda_s(\mathbb{T}) = C^s(\mathbb{T}) \) and so \( \Lambda_s^+ (\mathbb{T}) = \lambda_s^+ (\mathbb{T}) = A^s(\mathbb{D}) \). We define

\[
N_s(E) = \{ f \in \lambda_s(\mathbb{T}) : f|_E = \ldots = f|_E^{(s)} = 0 \},
\]

and set \( N_s^+(E) = N_s(E) \cap \Lambda_s^+ (\mathbb{T}) \).

**Lemma 2.1.** Let \( s \) be a nonnegative real number. Then for all \( \varepsilon > 0 \), we have the following continuous embedding

\[
A_{s + \frac{1}{2} + \varepsilon} (\mathbb{T}) \hookrightarrow A_s (\mathbb{T}).
\]

**Proof.** For \( s = 0 \), this is a result of Bernstein (see [9], p.13). The general case is obtained by the same arguments. Let \( \varepsilon > 0 \), and set \( \tilde{s} = s + \frac{1}{2} + \varepsilon \). Let \( f \in \Lambda_{\tilde{s}} (\mathbb{T}) \). For \( h > 0 \), define

\[
P(h) = \int_0^{2\pi} |f^{(\tilde{s})}(e^{i(t-h)}) - f^{(\tilde{s})}(e^{i(t+h)})|^2 dt.
\]

It follows from Parseval equality that

\[
P(h) = 8\pi \sum_{n = -\infty}^{+\infty} |f^{(\tilde{s})}(n)|^2 \sin^2 (nh). \quad (2)
\]

Let \( j_0 \) be the smallest integer such that \( \tilde{s} < 2^{j_0} \) and let \( j \geq j_0 \). It follows from the relation

\[
\hat{f}^{(\tilde{s})}(n) = \left( \prod_{k=1}^{\tilde{s}} (n+k) \right) \hat{f}(n + \tilde{s}) \quad (n \in \mathbb{Z})
\]

and from [2] that there exists a constant \( C_1 > 0 \) independent of \( f \) such that

\[
P(h) \geq \frac{4}{C_1} \sum_{|n| = 2^j} |\hat{f}(n + \tilde{s})|^2 (1 + |n|)^{2\tilde{s}} \sin^2 (nh). \quad (3)
\]

Using the Cauchy-Schwartz inequality, we have

\[
\sum_{|n| = 2^j} |\hat{f}(n + \tilde{s})|(1 + |n|)^{\tilde{s}} \leq \left( \sum_{|n| = 2^j} |\hat{f}(n + \tilde{s})|^2 (1 + |n|)^{2\tilde{s}} \right)^{\frac{1}{2}} \left( \sum_{n = 2^j} (1 + |n|)^{2s-2\tilde{s}} \right)^{\frac{1}{2}} \quad (4)
\]
Set \( h = \frac{\pi}{3.2^j} \). For all integers \( n \) such that \( 2^j \leq |n| \leq 2^{j+1} - 1 \), we have \( \frac{\pi}{3} \leq |nh| \leq \frac{2\pi}{3} \), and so \( \sin^2(nh) \geq \frac{1}{4} \). So, we deduce from (3) that

\[
\left( \sum_{|n| = 2^j}^{2^{j+1} - 1} |\hat{f}(n + [\tilde{s}])|^2 (1 + |n|)^{2[\tilde{s}] - 1} \right)^{\frac{1}{2}} \leq C_1 P \left( \frac{\pi}{3.2^j} \right)^{\frac{1}{2}}.
\]

Then, as \( f \in \Lambda \tilde{s}(\mathbb{T}) \), we have

\[
P \left( \frac{\pi}{3.2^j} \right)^{\frac{1}{2}} \leq (2\pi)^{\frac{1}{2}} \|f\|_{\Lambda \tilde{s}} \left( \frac{2\pi}{3.2^j} \right)^{\tilde{s} - [\tilde{s}]}.
\]

so that

\[
\left( \sum_{|n| = 2^j}^{2^{j+1} - 1} (1 + |n|)^{2[\tilde{s}] - 1} \right)^{\frac{1}{2}} \leq C_2 2^j \left( s - [\tilde{s}] + \frac{1}{2} \right).
\]

Furthermore, there exists a constant \( C_2 > 0 \) such that

\[
\left( \sum_{|n| = 2^j}^{2^{j+1} - 1} (1 + |n|)^{2[\tilde{s}] - 1} \right)^{\frac{1}{2}} \leq C_2 2^j \left( s - [\tilde{s}] + \frac{1}{2} \right).
\]\n
Finally we deduce from (4) and the inequalities (5) and (6) that there exists a constant \( C_3 > 0 \) independent of \( f \) such that for all \( j \geq j_0 \),

\[
\sum_{|n| = 2^j}^{2^{j+1} - 1} |\hat{f}(n + [\tilde{s}])|(1 + |n|)^s \leq 2^{j(s - [\tilde{s}] + \frac{1}{2})} C_3 \|f\|_{\Lambda \tilde{s}} = 2^{-\varepsilon j} C_3 \|f\|_{\Lambda \tilde{s}}.
\]

Summing over \( j \geq j_0 \) these inequalities, we get

\[
\sum_{|n| \geq 2^{j_0}} |\hat{f}(n + [\tilde{s}])|(1 + |n|)^s \leq \frac{C_3}{1 - 2^{-\varepsilon}} \|f\|_{\Lambda \tilde{s}},
\]

On the other hand, we have \( |\hat{f}(n)| \leq \|f\|_{\Lambda \tilde{s}} \) for every \( n \in \mathbb{Z} \). So, since \( j_0 \) is independent of \( f \), there exists a constant \( K > 0 \) (independent of \( f \)) such that

\[
\|f\|_{\tilde{s}} \leq K \|f\|_{\Lambda \tilde{s}}.
\]

Before giving the main theorem of the paper, we need the following lemma.
Lemma 2.2. Let $E$ be a closed subset of $\mathbb{T}$. We assume that there exists $\delta > 0$ for which 
\[ \int_0^{2\pi} \frac{1}{d(e^{it}, E)^{\delta}} dt < +\infty. \]
Let $\beta < \frac{\delta}{1+\delta}$ and let $T$ be an invertible operator on a Banach space with spectrum included in $E$ that satisfies 
\[ \|T^n\| = O(n^s), \quad n \to +\infty \quad (\text{for some nonnegative real } s) \]
and 
\[ \|T^{-n}\| = O(e^{n\beta}), \quad n \to +\infty, \]
Then there exists an outer function $f \in \mathcal{A}^\infty(\mathbb{D})$ which vanishes exactly on $E$ and such that 
\[ f(T) := \sum_{n=0}^{+\infty} \hat{f}(n) T^n = 0. \]

Proof. Let $\omega$ be the weight defined by 
\[ \omega(n) = \|T^n\| (n \in \mathbb{Z}). \]
Let $\Phi$ be the continuous morphism from $A_{\omega}(\mathbb{T})$ to $L(X)$ defined by 
\[ \Phi(f) = f(T) = \sum_{n=-\infty}^{+\infty} \hat{f}(n) T^n \quad (f \in A_{\omega}(\mathbb{T})). \]
Since the algebra $A_{\omega}(\mathbb{T})$ is regular, we have \{ $z \in \mathbb{T}: f(z) = 0$ \ (f \in Ker $\Phi$) \} \subset E (see [7], theorem 2.5), and so $J_{\omega}(E) \subset \text{Ker } \Phi$. Then the result follows from lemmas 7.1 and 7.2 of [5].

Theorem 2.3. Let $E$ be a $K$-set, and let $s$ be a nonnegative real number. Then, any invertible operator $T$ on a Banach space with spectrum included in $E$ that satisfies 
\[ \|T^n\| = O(n^s), \quad n \to +\infty \]
and 
\[ \|T^{-n}\| = O(e^{n\beta}), \quad n \to +\infty \text{ for some } \beta < \frac{\delta(E)}{1+\delta(E)}, \]
also satisfies the stronger property 
\[ \|T^{-n}\| = O(n^{s+\frac{1}{2}+\varepsilon}), \quad n \to +\infty, \]
for all $\varepsilon > 0$.

Proof. Let $\varepsilon > 0$ and set $\tilde{s} = s + \frac{1}{2} + \varepsilon$. Without loss of generality, we may assume that $\tilde{s}$ is not an integer. Let $t$ a real number, which is not an integer, and satisfies $s + \frac{1}{2} < t < \tilde{s}$ and $[t] = [\tilde{s}]$. According to lemma 2.1, we can define a continuous morphism $\Phi$ from $\lambda_t^+(\mathbb{T})$ to $L(X)$ by 
\[ \Phi(f) = f(T) = \sum_{n=0}^{+\infty} \hat{f}(n) T^n \quad (f \in \lambda_t^+(\mathbb{T})). \]
Let $I = \text{Ker } \Phi$, $I$ is a closed ideal of $\lambda_t^+(\mathbb{T})$. We denote by $S_I$ its inner factor, that is the greatest common divisor of all inner factors of the non-zero functions in $I$ (see [8] p.85), and
we set, for $0 \leq k \leq |t|$, $h^k(I) = \{z \in \mathbb{T} : f(z) = \ldots = f^{(k)}(z) = 0 \text{ } (f \in I)\}$.

F. A. Shamoyan showed in [12] that

$$ I = \left\{ f \in \Lambda^+(\mathbb{T}) : S(I)|S(f) \text{ and } f^{(k)} = 0 \text{ on } h^k(I) \text{ for all } 0 \leq k \leq |t| \right\}, $$

where $S(f)$ denotes the inner factor of $f$ and $S(I)|S(f)$ means that $S(f)/S(I)$ is a bounded holomorphic function in $\mathbb{D}$. Since $\beta < \frac{\delta(E)}{1+\delta}$, there exists $0 < \delta < \delta(E)$ such that

$$ \beta < \frac{\delta}{1+\delta}. $$

We have, by definition of $\delta(E)$,

$$ \int_0^{2\pi} \frac{1}{d(e^{it}, E)} dt < +\infty. $$

So we deduce from lemma [22] that there exists an outer function $f \in A^\infty(\mathbb{D})$ which vanishes exactly on $E$ and such that $f \in I$. Therefore, we have $S(I) = 1$ and $h^0(I) \subset E$, so that $N^+_s(E) \cap \lambda_s(\mathbb{T}) \subset I$.

Now, as $\Lambda^+_s(\mathbb{T}) \subset \lambda^+_s(\mathbb{T})$, we can define a continuous morphism $\Psi$ from $\Lambda^+_s(\mathbb{T})$ to $\mathcal{L}(X)$ by $\Psi = \Phi|_{\Lambda^+_s(\mathbb{T})}$. Using what precedes, we have

$$ N^+_s(E) \subset \text{Ker } \Psi. $$

So there exists a continuous morphism $\tilde{\Psi}$ from $\Lambda^+_s(\mathbb{T})/N^+_s(E)$ into $\mathcal{L}(X)$ such that $\Psi = \tilde{\Psi} \circ \pi^+_s$, where $\pi^+_s$ is the canonical surjection from $\Lambda^+_s(\mathbb{T})$ to $\Lambda^+_s(\mathbb{T})/N^+_s(E)$. Since $E$ is a $K$-set, by a theorem of E. M. Dyn’kin [2], it is an interpolating set for $\Lambda^+_s(\mathbb{T})$, so that the canonical imbedding $i$ from $\Lambda^+_s(\mathbb{T})/N^+_s(E)$ into $\Lambda^+_s(\mathbb{T})/N^+_s(E)$ is onto. We have, for $n \geq 0$,

$$ T^{-n} = \tilde{\Psi} \circ i^{-1} \circ \pi^+_s(\alpha^{-n}), $$

where $\pi^+_s$ denote the canonical surjection from $\Lambda^+_s(\mathbb{T})$ to $\Lambda^+_s(\mathbb{T})/N^+_s(E)$ and where $\alpha : z \to z$ is the identity map. So we have, for $n \geq 0$,

$$ \|T^{-n}\| \leq \|\tilde{\Psi} \circ i^{-1} \|\pi^+_s(\alpha^{-n})\|_{\Lambda^+_s} \leq \|\tilde{\Psi} \circ i^{-1}\| (1+n)^{\bar{s}}, $$

which completes the proof. \(\square\)

We give two immediate corollaries of this theorem.

**Corollary 2.4.** Let $\xi \in (0, \frac{1}{2}]$ and let $s$ be a nonnegative real number. Then, any invertible operator $T$ on a Banach space with spectrum included in $E_\xi$ that satisfies

$$ \|T^n\| = O(n^s), \text{ } n \to +\infty $$

and

$$ \|T^{-n}\| = O(e^{n^{\beta}}), \text{ } n \to +\infty \text{ for some } \beta < b(\xi), $$

also satisfies the stronger property

$$ \|T^{-n}\| = O(n^{s+\frac{1}{2}+\varepsilon}), \text{ } n \to +\infty, $$

for all $\varepsilon > 0$. 

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Proof. It is well known that $E_\xi$ is a $K$-set (see proposition 2.5 of \[5\]). Moreover, $E_\xi$ satisfies
$$\int_0^{2\pi} \frac{1}{d(e^{it}, E)^\delta} dt < +\infty$$
if and only if $\delta < 1 + \frac{\log 2}{\log \xi}$. Indeed, the condition $\int_0^{2\pi} \frac{1}{d(e^{it}, E)^\delta} dt < +\infty$ is equivalent to
$$\sum_{n=1}^{+\infty} \sum_{i=1}^{2^{n-1}-1} |L_{n,i}|^{1-\delta},$$
where $L_{n,i}$ are the arcs contiguous to $E_\xi$, and $|L_{n,i}|$ are their length, which is equal to $2\pi \xi^{n-1}(1-2\xi)$ (see \[10\] for further details). Then it is easily seen that the last series converges if and only if $\delta < 1 + \frac{\log 2}{\log \xi}$, so $\delta(E_\xi) = 1 + \frac{\log 2}{\log \xi}$.

Now, the result follows immediately from theorem 2.3.

Then we obtain an other immediate result, which generalizes theorem 4.1 of \[3\]. Indeed, the condition "\[\|T^{-n}\| = O(e^{n\beta}), n \to +\infty\]" which appears in the following corollary is weaker than the condition used by the authors of \[3\].

Corollary 2.5. Let $E$ be a $K$-set, and let $s$ be a nonnegative real number. Then, there exists a constant $\beta > 0$ independent of $s$ such that any invertible operator $T$ on a Banach space with spectrum included in $E$ that satisfies
$$\|T^n\| = O(n^s), n \to +\infty$$
and
$$\|T^{-n}\| = O(e^{n\beta}), n \to +\infty,$$
also satisfies the stronger property
$$\|T^{-n}\| = O(n^{s+\frac{1}{2}+\varepsilon}), n \to +\infty,$$
for all $\varepsilon > 0$.

Proof. As $E$ is a $K$-set, we deduce from \[2\] (section 5, corollary) that $\delta(E) > 0$. Then the result follows immediately from theorem 2.3 with any $\beta < \frac{\delta(E)}{1+\delta(E)}$. \qed

Remark 2.6:
1) Some results concerning operators with countable spectrum are obtained in \[13\] and in \[1\]. Let $E$ be a closed subset of $\mathbb{T}$ and let $s$, $t$ be two nonnegative reals. We denote by $P(s, t, E)$ the following property: every invertible operator $T$ on a Banach space such that $\mathrm{Sp} T \subset E$ and satisfies the conditions:
$$\|T^n\| = O(n^s), n \to +\infty$$
$$\|T^{-n}\| = O(e^{\varepsilon\sqrt{n}}), n \to +\infty,$$
for all $\varepsilon > 0$, also satisfies the stronger property
$$\|T^{-n}\| = O(n^{t}+\varepsilon), n \to +\infty.$$

M. Zarrabi showed in \[13\] (théorème 3.1 and remarque 2.a) that a closed subset $E$ of $\mathbb{T}$ satisfies $P(0, 0, E)$ if and only if $E$ is countable. Notice that $E$ is called a Carleson set if
$$\int_0^{2\pi} \log^+ \frac{1}{d(e^{it}, E)} dt < +\infty.$$ If $E$ is a countable closed subset of $\mathbb{T}$, we show in \[11\] that the
following conditions are equivalent:

(i) there exist two positive constants $C_1, C_2$ such that for every arc $I \subset \mathbb{T}$,

$$\frac{1}{|I|} \int_I \log^+ \frac{1}{d(e^{it}, E)} \, dt \leq C_1 \log \frac{1}{|I|} + C_2.$$ 

(ii) $E$ is a Carleson set and for all $s \geq 0$, there exists $t$ such that $P(s, t, E)$ is satisfied.

For contractions with spectrum satisfying the Carleson condition, we can see [11].

2) When $\xi = \frac{1}{q}$, the constant $b(\frac{1}{q})$ in corollary 2.4 is the best possible in view of [6], where the authors built a contraction $T$ such that $\lim_{n \to +\infty} \log \|T^{-n}\| = +\infty$, $Sp T \subset E_{\frac{4}{q}}$ and $\log \|T^{-n}\| = O(n^{b(\frac{1}{q})})$. According to theorem 6.4 of [11], $T$ doesn’t satisfy $\|T^{-n}\| = O(n^s)$ for any real $s \geq 0$.

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