Lorentz and CPT Violating Chern-Simons Term in the Derivative Expansion of QED

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Abstract

We calculate by the method of dimensional regularization and derivative expansion the one-loop effective action for a Dirac fermion with a Lorentz-violating and CPT-odd kinetic term in the background of a gauge field. We show that this term induces a Chern-Simons modification to Maxwell theory. Some related issues are also discussed.

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Violation of Lorentz symmetry, if it exists, will have a significant impact in our understanding of Nature and its symmetries. There have been some recent attempts \([1–3]\) to analyze the possible consequences of actual breaking of Lorentz invariance. One of the proposals was a Lorentz-CPT non-invariant modification of electromagnetism by a Chern-Simons term \([4]\). More recently, a Lorentz violating extension of the standard model was presented, and possible consequences including relation to QED were explored in detail \([3]\).

In this Report, we study QED with a Lorentz-violating CPT-odd term in the fermion sector, and show that this term induces a finite Chern-Simons modification to Maxwell theory in the one loop effective action. Recall that Carroll, Field, and Jackiw \([1]\) first proposed the following Lorentz and CPT violating Maxwell-Chern-Simons theory

\[
\mathcal{L} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - j_\mu A^\mu - \frac{1}{2} l_\mu \ast F^{\mu\nu} A^\nu ,
\]

and this model was reexamined in the context of Lorentz violating extensions of the standard model in Ref. \([3]\). In the above Lagrangian, \(l_\mu\) is a constant 4-vector that picks out a preferred direction in space-time, thereby violating Lorentz invariance. CPT symmetry is absent as well. Possible origin of such a Chern-Simons term is that it can be induced by radiative corrections \([5]\) from a Lorentz and CPT violating fermion sector. With this in mind, let us consider the Dirac fermion propagating in Lorentz and CPT non-invariant manner in the background of a photon field;

\[
\mathcal{L} = \bar{\psi} \left[ i \partial / - m - \gamma_5 b / - eA / \right] \psi .
\]

Here \(b_\mu\) is a constant 4-vector, and \(b_\mu \bar{\psi} \gamma^\mu \gamma_5 \psi\) is the Lorentz violating, CPT-odd term.

Such a term was considered as a possible extension to the standard model by Colladay and Kostelecký \([3]\). They also calculated by the diagrammatic method the radiative corrections to vacuum polarization involving the \(b_\mu\) term. The computation of vacuum polarization two-point diagram with an extra insertion of the factor \(-ib_\mu \gamma_5 \gamma^\mu\) on one internal fermion line leads to the triangular anomaly \([6,7]\) diagram in which there is zero momentum transfer to the axial-vector leg and the axial vector is replaced with a vacuum expectation value. The result is finite, but there is an ambiguity due to linear divergence of momentum integration. However, a more careful analysis \([8]\) shows that not only such superficial linear divergences cancel each other, but also a definite value can be calculated for the vacuum polarization diagram involving the \(b_\mu\) term.

Here, we directly compute the effective action of the model in Eq. (2). Using dimensional regularization \([9]\) and the derivative expansion method of Ref. \([10]\), we show that the Chern-Simons term of Eq. (1) is induced in the effective action with a fixed coefficient. The one loop effective action \(S_{\text{eff}}\) is given by

\[
S_{\text{eff}} = \int d^4 x \mathcal{L}_{\text{eff}} = -i \text{Tr} \ln [i \partial / - m - \gamma_5 b / - eA /] .
\]

To carry out the trace calculation, we first use the trace identity;

\[
\text{Tr} \ln [i \partial / - m - \gamma_5 b / - eA /] = \text{Tr} \ln [i \partial / - m - \gamma_5 b /] - \int_0^1 dz \frac{1}{i \partial / - m - \gamma_5 b / - zeA /} eA(x) .
\]
Note that $\partial_\mu$ and $A_\nu(x)$ do not commute, and to perform the momentum space integration of the second term in Eq. (4), we use the prescription of Ref. [10] in the denominator of Eq. (4);

$$i\partial \rightarrow \not{p}, \quad A(x) \rightarrow A(x - i\frac{\partial}{\partial p}).$$

Using this, we rewrite $S_{\text{eff}}$ as

$$S_{\text{eff}} = -i\text{Tr} \ln \mathcal{D}_0 + i \int_0^1 dz \int d^4x \int \frac{d^4p}{(2\pi)^4} \text{tr} \left( \frac{1}{\not{p} - m - \gamma_5 \not{b} - zeA(x - i\frac{\partial}{\partial p})} eA(x) \right),$$

where

$$\mathcal{D}_0 = i\partial - m - \gamma_5 \not{b}.$$

$\mathcal{D}_0$ propagator has been analyzed in detail in the second paper of Ref. [3]. We study the second term of $S_{\text{eff}}$, and look for first order derivative terms which are linear in $\not{b}$ and quadratic in $A_\mu$’s. Using the operator expansion

$$\begin{align*}
A - B &= A^{-1} + A^{-1}BA^{-1} + \cdots,
\end{align*}$$

we can extract these to be

$$\begin{align*}
\int d^4x \mathcal{L}^{(b)}_{\text{eff}} &= -i\frac{e^2}{2} \int d^4x \int \frac{d^4p}{(2\pi)^4} \text{tr} \left[ \frac{1}{\not{p} - m} \frac{i\partial_\mu A}{\partial p_\mu} \frac{1}{\not{p} - m} \gamma_5 \not{b} \frac{1}{\not{p} - m} A 
+ \frac{1}{\not{p} - m} \gamma_5 \not{b} \frac{1}{\not{p} - m} i\partial_\mu A \frac{1}{\partial p_\mu} \frac{1}{\not{p} - m} A \right]. 
\end{align*}$$

(5)

Using

$$\frac{\partial}{\partial p_\mu} \frac{1}{\not{p} - m} = -\frac{1}{\not{p} - m} \gamma^\mu \frac{1}{\not{p} - m},$$

we first rewrite Eq. (5) into

$$\begin{align*}
\int d^4x \mathcal{L}^{(b)}_{\text{eff}} &= -\frac{e^2}{2} \int d^4x \int \frac{d^4p}{(2\pi)^4} \text{tr} \left[ \gamma_5 \not{b} \frac{1}{\not{p} - m} \left( A \frac{1}{\not{p} - m} \partial_\mu A \frac{1}{\not{p} - m} \gamma^\mu \frac{1}{\not{p} - m} 
+ \gamma^\mu \frac{1}{\not{p} - m} A \frac{1}{\not{p} - m} \partial^\nu A \frac{1}{\not{p} - m} \gamma^\nu \frac{1}{\not{p} - m} + \partial_\mu A \frac{1}{\not{p} - m} \gamma^\mu \frac{1}{\not{p} - m} A \frac{1}{\not{p} - m} \right) \right]. 
\end{align*}$$

(6)

The momentum integral has terms which diverge logarithmically, and in order to regularize the expression, we use the dimensional regularization [9]. We use the rule [11] that we first perform the loop integrals in arbitrary $d$ dimensions, and identify the potential divergences as $d \rightarrow 4$. Then, we are left with traces of $\gamma$-matrices containing $\gamma_5$. We will take these traces directly in 4 dimensions. The traces are basically of three types,

$$\begin{align*}
\text{tr}(\gamma_5 \gamma_\alpha \gamma_\beta \gamma_\gamma \gamma_\delta), \quad \text{tr}(\gamma_5 \gamma^\alpha \gamma_\beta \gamma_\gamma \gamma_\delta \gamma_\mu \gamma_\nu), \quad \text{tr}(\gamma_5 \gamma^\alpha \gamma_\beta \gamma_\gamma \gamma_\delta \gamma_\alpha \gamma_\nu).
\end{align*}$$

(7)
The second and third terms contain $\gamma$ matrices contracted, and we eliminate such contractions by using the identities

$$\gamma^\alpha \gamma^\beta = -2 \gamma^\beta, \quad \gamma^\alpha \gamma^\gamma \gamma^\delta \gamma^\alpha = -2 \gamma^\delta \gamma^\gamma \gamma^\beta.$$ 

Then, all the trace computations involving $\gamma_5$ reduce to terms of the first type in Eq. (7), which is equal to $-4i \epsilon^{\alpha\beta\gamma\delta}$. Under this regularization scheme, potential divergences in the momentum integration come from the following type of integral:

$$\int \frac{d^d p}{(2\pi)^d (p^2 - m^2)^4} = \frac{i (g_{\alpha\beta} g_{\gamma\delta} + g_{\alpha\gamma} g_{\beta\delta} + g_{\alpha\delta} g_{\beta\gamma})}{(4\pi)^{d/2}} \Gamma(2 - d/2) \frac{1}{24} \left( \frac{1}{m^2} \right)^{2-d/2}. \quad (8)$$

But the leading logarithmic divergences cancel in each of the three terms of Eq. (8) separately, and we obtain a completely finite result. The remaining integrals converge like $1/p^2$ as $p \to \infty$ and $d \to 4$. By using the following momentum integration in Minkowski space,

$$\int \frac{d^4 p}{(2\pi)^4} \frac{1}{(p^2 - m^2)^4} = \frac{i}{96 \pi^2 m^4}, \quad \int \frac{d^4 p}{(2\pi)^4} p_\alpha p_\beta = -\frac{ig_{\alpha\beta}}{192 \pi^2 m^2},$$

we obtain the following effective Lagrangian

$$L^{(b)}_{\text{eff}} = \frac{3e^2}{16\pi^2} b_\mu F^{\mu\nu} A_\nu. \quad (9)$$

Comparing this effective Lagrangian with Eq. (1), we find that the Lorentz violating CPT odd term $\bar{\psi} \gamma_5 b / \psi$ produces the Chern-Simons term with coefficient

$$l_\mu = -\frac{3e^2}{8\pi^2} b_\mu. \quad (10)$$

Note that the momentum integral in our derivative expansion of Eq. (3) has leading logarithmic divergences, but the result is finite and it is consistent with the computation of the vacuum polarization diagram with the fermion propagator $S_F = i(\not{p} - m - \gamma_5 b)^{-1}$[2]. Also, Eq. (9) is independent of the mass term and the result is highly reminiscent of the finite anomaly contribution to the divergence of axial vector current [6,7].

If we include all the fermion species of the standard model, with each having electromagnetic coupling strength $q_i^2$, the induced Chern-Simons coefficient becomes

$$l_\mu = -\frac{3}{8\pi^2} \sum_f q_i^2 b^f_\mu, \quad (11)$$

where the sum over $f$ extends over all the leptons and quarks. If the coupling coefficient $b^f_\mu$ is indeed generated as the vacuum expectation values of an effective axial vector $<A^{a}_{\mu\delta}>$ which multiplies the axial-vector bilinears of fermions, as was suggested in Ref. [3], we would have $b^f_\mu = \sum_a g^a_f <A^{a}_{\mu\delta}>$, where the index $a$ ranges over the species of fermion which have such axial vector coupling, and $g^a_f$ is the associated coupling. Then the above equation for $l_\mu$
is equivalent to $\sum f q^2 g^a$, which must vanish according to the anomaly cancellation condition \[3\]. However, it is also possible that the Lorentz-violating term involving $b^f_\mu$ may not couple to fermions through the vacuum expectation value of an axial vector.

In conclusion, we found that the derivative expansion method for effective action with the dimensional regularization yields a finite Chern-Simons modification to Maxwell theory, if the standard Lagrangian is augmented by a Lorentz-violating CPT-odd term in the fermion sector. We used the dimensional regularization method, and the coefficient of the induced Chern-Simons term is fixed and non-vanishing in this scheme. But in the Pauli-Villars regularization, the coefficient is zero, because Eq. \[3\] is mass independent \[\[3,8\]. Hence the result depends on the regularization scheme one chooses to adopt. Chern-Simons modification of Maxwell theory predicts a preferred direction of space-time and birefringence of photon propagation, which was compared with experiment \[1\]. The currently available experimental data \[12\] rules out the occurrence of this Lorentz violating term in Nature. But this does not necessarily mean that $b^f_\mu$ vanishes separately for each $f$ in Eq. \[11\]. The details of the fermion sector will be determined by the ultimate theory.

NOTE ADDED

1. Shortly after the completion of this work, a no-go theorem by Coleman and Glashow which states that $l_\mu$ vanishes to first order in $b_\mu$ for any gauge invariant CPT-odd interaction appeared \[13\]. The issue of evading the no-go theorem for the induced Chern-Simons term has been subsequently discussed by Jackiw and Kostelecký in Ref. \[8\]. They calculate nonperturbatively the vacuum polarization tensor. Upon differentiation of their formula with respect to the external momentum which is then set zero, one arrives at our expression \[1\] and our final answer agrees with theirs.

2. There appeared several other computations on the subject \[14\] indicating that the induced coefficient is finite but undetermined perturbatively, and actual determination of the coefficient needs experimental input \[15\]. We briefly mention connection with axion physics in this context. Note that \[9\] can be regarded as an effective interaction $\sim a F_{\mu\nu} F^{\mu\nu}$ between photon and infinitely thick axion domain wall $a(x) = b_\mu x^\mu$ which has uniform space derivative. This axion-photon interaction can be induced by derivative axion-fermions interaction of the form $\sim \partial_\mu a \bar{\psi} \gamma^\mu \gamma_5 \psi$ through anomaly \[16\], and the induced coefficient depends on the details of the particular model employed and its experimental result \[17\]. However, since current experiment \[12\] rules out the induced Chern-Simons coefficient, such infinite thick domain wall configuration is unrealistic. We acknowledge the referee for raising this issue.

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REFERENCES

[1] S. Carroll, G. Field, and R. Jackiw, Phys. Rev. D 41, 1231 (1990); R. Jackiw, hep-ph/9811322.
[2] S. Coleman and S. L. Glashow, Phys. Lett. B 405, 249 (1997).
[3] D. Colladay and V. A. Kostelecký, Phys. Rev. D 58, 116002 (1998); ibid. 55, 6760 (1997);
See also R. Bluhm, V. A. Kostelecký, and N. Russell, Phys. Rev. D 55, 3932 (1998);
V. A. Kostelecký, Phys. Rev. Lett. 80, 1818 (1998); R. Bluhm, V. A. Kostelecký, and N. Russell, ibid. 79, 1432 (1997); O. Bertolami, D. Colladay, V. A. Kostelecký, and R. Potting, Phys. Lett. B 395, 178 (1997); V. A. Kostelecký and R. V. Kooten. Phys. Rev. D 54, 5585 (1996); V. A. Kostelecký and R. Potting, Phys. Lett. B 381, 89 (1996);
[4] R. Jackiw and S. Templeton, Phys. Rev. D 23, 2291 (1981); J. Schonfeld, Nucl. Phys. B 185, 157 (1981); S. Deser, R. Jackiw, and S. Templeton, Ann. Phys. (N. Y.) 140, 372 (1982); 185, 406 (E) (1988).
[5] A. N. Redlich and L. C. R. Wijewardhana, Phys. Rev. Lett. 54, 970 (1985); A. J. Niemi and G. W. Semenoff, ibid. 54, 2166 (1985).
[6] J. S. Bell and R. Jackiw, Nuovo Cim. 60A, 47 (1969).
[7] S. L. Adler, Phys. Rev. 177, 2426 (1969); S. L. Adler and W. A. Bardeen, Phys. Rev. 182, 1517 (1969).
[8] R. Jackiw and V. A. Kostelecký, Phys. Rev. Lett. 82, 3572 (1999).
[9] G. ’t Hooft and M. Veltman, Nucl. Phys. B 44, 189 (1972).
[10] L.-H. Chan, Phys. Rev. Lett. 54, 1222 (1985); L.-H. Chan, ibid. 55, 21 (1985); O. Cheyette, ibid. 55, 2394 (1985); J. A. Zuk, Phys. Rev. D 32, 2653 (1985).
[11] P. Ramond, Field Theory, 2nd ed. (Addison-Wesley, 1990).
[12] M. Goldhaber and V. Trimble, J. Astrophys. Astr. 17, 17 (1996); S. M. Carroll and G. B. Field, Phys. Rev. Lett. 79, 2394 (1997).
[13] S. Coleman and S. L. Glashow, Phys. Rev. D 59, 116008 (1999).
[14] W. F. Chen, hep-th/9903258; J. -M. Chung, hep-th/9904037; C. D. Fosco and J. C. Le Guillou, hep-th/9904138; J. -M. Chung, hep-th/9905095; M. Perez-Victoria, hep-th/9905061.
[15] R. Jackiw, hep-th/9903043.
[16] D. B. Kaplan, Nucl. Phys. B 260, 215 (1985); M. Srednicki, Nucl. Phys. B 260, 689 (1985); See also W. A. Bardeen and S.-H. H. Tye, Phys. Lett. B 74, 229 (1978).
[17] J. E. Kim, Phys. Rept. 150, 1 (1987); H. Y. Cheng, Phys. Rept. 158, 1 (1988); M. S. Turner, Phys. Rept. 197, 1 (1990); G. G. Raffelt, Phys. Rept. 198, 1 (1990), and Stars as Laboratory for Fundamental Physics (Univ. of Chicago Press, 1996).