Large scale calculations for cluster structure of light nuclei with Skyrme interaction

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Abstract. We present a computational approach to describe structure of light nuclei including cluster states. Apart from the use of an empirical nuclear force, Skyrme interaction, our scheme does not utilize any a priori knowledge on the structure of nuclei. In our framework, we first generate a number of Slater determinants in a stochastic way. We then make projections of parity and angular momentum, and perform configuration mixing calculation. We show results for ¹²C and ¹⁶O nuclei. Our calculation provides a reasonable description for the ground state rotational band, Hoyle state, and low-lying negative parity states of ¹²C. We may also describe the ⁰⁺² rotational band of ¹⁶O, although excitation energies are slightly overestimated.

1. Introduction
In light nuclei, a variety of cluster structures appear in excited states. For a theoretical description of these states, microscopic cluster models such as resonating group method and generator coordinate method have been developed [1]. In these methods, existence of nuclear clusters are assumed from the beginning. To understand mechanisms of a formation of clusters, theoretical studies which do not assume existence of clusters are required. The anti-symmetrized molecular dynamics is one of the successful approaches for that purpose [2]. In its simplest version, however, the model wave function is rather simple, a Slater determinant composed of Gaussian wave packets.

In last two decades, ab-initio calculations starting from a realistic nucleon-nucleon force have been developed. They have been successful for description of the ground and low-lying excited states. However, at a moment, they are not able to describe typical clustering states, such as the second ⁰⁺ states in ¹²C and ¹⁶O [3, 4].

In this paper, we report an application of the configuration mixing approach which we have recently developed [5] for light nuclei, ¹²C and ¹⁶O. Superposing a number of Slater determinants which include both shell-model like and cluster configurations, we can obtain a converged excitation spectrum of these nuclei for a given Skyrme Hamiltonian. We will show that our approach is capable of describe both ground state and excited states with cluster structures simultaneously.
2. Formulation

We use a method developed by us [5]. The method is composed of the following three steps. First we prepare a number of Slater determinants with different correlation structures employing the imaginary-time method. The imaginary-time method is usually used to calculate self-consistent Hartree-Fock solutions. We use this method for different purpose, generating a number of Slater determinants with different correlation structures. In practice, we prepare an initial orbitals of a Slater determinant in a stochastic way. We then perform the imaginary time evolution. We pick up and store a number of Slater determinants in the course of iterations before reaching the self-consistent solution. In Fig. 1, we show density distributions of several Slater determinants which are generated in the above mentioned procedure. One can see that both shell-model-like and cluster-like configurations are included. We typically generate 50 Slater determinants. We then make projections of parity and three-dimensional angular momentum for the Slater determinants. Finally we achieve a configuration mixing calculation using the projected states. We use SLy4 parameter in our calculation. A real-space Cartesian grid representation is used with a grid spacing of 0.8 fm. Grid points inside a sphere of 8 fm are used.

3. Results

3.1. $^{12}\text{C}$

We first show results for $^{12}\text{C}$. We show the energy spectrum and the transition strength in Fig. 2. We also show density distributions of Slater determinants which are main components of $0^+_2$ state. The binding energy is reasonably reproduced, 95.3 MeV after configuration mixing. This consists of the Hartree-Fock energy of the ground state, 90.6 MeV, and the correlation energy, 4.7 MeV. The binding energy in our calculation is slightly larger than the measured value, 92.1 MeV. As seen from the figure, our calculation accurately reproduces the excitation energies of the ground rotational band. The transition strengths are also reproduced reasonably. The $0^+_2$ state, so-called the Hoyle state, appears somewhat high compared with the measurement. We show density distributions of two Slater determinants which are the main components of this state in the configuration mixing calculation. They show a well-developed 3-α cluster structure. We find it necessary to superpose a number of Slater determinants to describe the state, consistent with a picture of alpha-condensation [6].

3.2. $^{16}\text{O}$

We show results for $^{16}\text{O}$ in Fig. 3. The main component of the ground state is the Hartree-Fock solution which accounts for more than 80% in the configuration mixing calculation. Our calculation describes second $0^+$ rotational band, $0^+_2$, $2^+_1$, and $4^+_2$. The band head energy of the
Figure 2. Energy spectrum for $^{12}\text{C}$. Density distributions of Slater determinants which are main components of $0^+_2$ state is shown as well. The experimental data are taken from Refs. [7].

$0^+_2$ is 9.5 MeV, higher than the measured energy, 6.1 MeV. The B(E2) transition strengths of the $0^+_2$ band is underestimated in our calculation. We show the density distribution of a Slater determinant which is a major component of the $0^+_2$ band. It shows a cluster structure of $\alpha-^{12}\text{C}$.

4. Conclusion
We have presented configuration mixing calculations using Skyrme interaction for $^{12}\text{C}$ and $^{16}\text{O}$ nuclei superposing a number of Slater determinants which include cluster configurations as well as shell-model-like configurations. For $^{12}\text{C}$, our calculation reproduces the ground state rotational band and the $0^+_2$ Hoyle state simultaneously. For $^{16}\text{O}$, our calculation reproduces the $0^+_2$ rotational band having $\alpha+^{12}\text{C}$ cluster state, though excitation energies are slightly too high.

5. Acknowledgments
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References
[1] Ikeda K, Horiuchi H and Saito S 1980 Prog. Theor. Phys. Suppl. 68 1; Fujiwara Y et al 1980 Prog. Theor. Phys. Suppl. 68 29
[2] Horiuchi H 1991 Nucl. Phys. A 522 257; Kanada-En’yo Y and Horiuchi H 2001 Prog. Theor. Phys. Suppl. 142 205
[3] Wiringa R B, Pieper S C, Carlson J and Pandharipande V R 2000 Phys. Rev. C 62 014001
[4] Navrátil P, Vary J P and Barrett B R 2000 Phys. Rev. C 62 054311
[5] Shinohara S, Ohta H, Nakatsukasa T and Yabana K 2006 Phys. Rev. C 74 054315
[6] Funaki Y et al 2003 Phys. Rev. C 67 051306
[7] Ajzenberg-Selove F 1990 Nucl. Phys. A 506 1; Zimmerman W R et al 2011 Phys. Rev. C 84 054308; Freer M et al 2012 Phys. Rev. C 86 034320; Freer M et al 2011 Phys. Rev. C 83 034314
[8] Tilley D R et al 1993 Nucl. Phys. A 564 1