Research Article

A Distributed Model Predictive Control for Multiple Mobile Robots with the Model Uncertainty

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In this paper, a distributed predictive control with the model uncertainty which uses the data-driven strategy and robust theory (data-driven RDMPC) is proposed for the formation control of multiple mobile robots. The robust performance objective minimization is applied to replace the quadratic performance objective minimization to establish the optimization problem, where the model uncertainty is considered in the distributed system. The control policy is derived by applying the data-driven strategy, and the future predictive value is obtained by employing the linear law in the historical data. Lyapunov theory is referred to analyze the stability of the mobile robot formation system. The effectiveness of the proposed method is proved by a set of simulation experiments.

1. Introduction

In recent years, the robot formation control has become a key technology of multiple mobile robots’ research studies which can be used in the military field, production field, and service field [1–4]. In contrast with a single mobile robot, multiple mobile robots have the advantages of coordination and cooperation, which could improve the efficiency of completing tasks. The multiple mobile robots also enhance the fault tolerance and robustness of the system and strengthen the ability of robots’ environment recognition. Comparing with the leader-follower method [5–8], behavior-based method [9–11], and virtual structure method [12, 13], the model predictive control (MPC) has attracted attention because of its ability of improving the robustness of the system and having better dynamic control performance in solving the problem of consensus protocol in distributed formation control of multiple mobile robots. Also, MPC is convenient to establish the system model.

To further improve the performance of MPC, researchers investigate MPC combining with robust control [14, 15], adaptive control [16, 17], and sliding model control [18]. Among them, there are lots of works focusing on overcoming the uncertainty problem of formation control of multiple mobile robots. In the research of multiagent [19], a fault-tolerant controller was designed to compensate the influence of the system uncertainty. By proposing a static nonsmooth event-triggered protocol [20], the influence of the uncertainty is inhibited. Robust and adaptive control algorithms were used [21] for each agent to deal with the system uncertainty associated with leader input and follower agent dynamics. In the study of MAVs [22], adaptive control algorithms were employed locally for each vehicle to solve the uncertainty associated with the system and flying environment. Although these methods mentioned above have disposed of the uncertainty problem of the formation control with MPC, they are not combined with a distributed control. As for the MPC applied in the distributed control (distributed model predictive control (DMPC)), a cooperative DMPC algorithm was reported [23] where the relaxed constraints are used to find the optimization problem including the global optimization objective. This algorithm can be utilized to control the UAV tracking the ground target for real-time path planning [24] and multiple cooperative autonomous ships forming a multiship formation system [25]. To transport the plants and keep the formation at the same time [26], an objective function including trajectory tracking error and formation measurement error was adopted to
meet the design requirements based on the DMPC. The DMPC is widely used in the research studies above, but the uncertainty is not considered. To the best of our knowledge, although lots of results have been available, the issue of simultaneous system uncertainty and DMPC has not been fully studied to date.

In practical application, the parameters in the model may not be accurate, and it is difficult to get an accurate model of system [27, 28]. Due to the uncertainty of the model, it is almost impossible to predict the future accurately, and the mismatch factors must be noticed. The model uncertainty of the system is ignored in the DMPC algorithm. In addition, the DMPC is too optimistic in the prediction process which would cause the prediction results deviating from the actual effect and lead to many restrictions on the decoupling of the distributed system [29, 30]. For this reason, it is necessary to study the DMPC with system uncertainty. A robust model predictive control approach [31] was suggested to achieve the path tracking of an omnidirectional robot with system uncertainty. The design of robust distributed MPC controller was cast as [32] into solving a linear matrix inequality (LMI) optimization problem. A global performance was achieved by using an iterative algorithm. However, the iterative algorithm would cause the jitter fluctuation of the convergence curve and a low computational efficiency [33, 34]. A novel model-free adaptive SMC approach for unknown discrete nonlinear processes was designed [35] by bringing the merits of data-driven control strategy and SMC scheme together, which made the system with model uncertainty of strong robustness. A data-driven hierarchical control (DHC) structure was introduced [36] to ameliorate the performance of systems under the effect of the system uncertainty. The above references demonstrate both robust model predictive control and data-driven strategy which are effective methods to work out the problem of system uncertainty so that it is very important to study the data-driven robust distributed model predictive control with system uncertainty.

In this paper, a data-driven robust distributed model predictive control (data-driven RDMPC) with model uncertainty is proposed for multiple mobile robots. The main control method in this paper has these advantages. (1) The data-driven strategy is applied in this method using the historical data representing the future predictive value so that data processing is not limited to the model and the conservatism of system can be reduced. (2) The robust performance objective not the quadratic performance objective is chosen in this method to solve the optimization problem. As a result, the system can consider the model uncertainty, the fluctuation of the convergence curve can be decreased, and also the efficiency and accuracy of the algorithm can be promoted.

This paper is arranged as follows. The control problem is described in Section 2. In Section 3, a data-driven distributed model predictive controller is designed with considering the system uncertainty, and the stability of the system is analyzed. Section 4 presents a simulation example to illustrate the effectiveness of the method. Section 5 puts forward some conclusions.

2. Problem Formulation

2.1. Notations and Graph Theory. Throughout the paper, the following notations will be used. The function \( X(k + \Delta k|k) \) means the predicted value of variables \( X(k + \Delta k) \) in time \( k \). \( X(k|k) = X(k) \) represents the variable \( x \) in current time and can be measured by sensors. And a quadric form function is given as \( x^T Q x \) where \( Q \geq 0 \).

This paper examines the mobile robot formation control with the distributed control method. A finite index is set to mark the mobile robot, making each robot node as 1, 2, ..., \( n \). A directed and fixed graph \( G \) is \( (V, E, A) \). \( V = \{ v_1, v_2, \ldots, v_n \} \) is a set of nodes, where \( v_i \) is referred to the \( i \)-th mobile robot. \( e \in V \times V \) is the edge set. \( e_{ij} = (v_i, v_j) \in e \) indicates that the \( i \)-th robot can obtain the information from \( j \)-th robot. \( A = [a_{ij}] \) is the weighted adjacency matrix of \( G \). If \( e_{ij} \in e \), we have \( a_{ij} = a_{ji} > 0 \) while \( a_{ii} = 0 \) and \( a_{ij} = 0 \).

2.2. Mathematical Model. The mobile robot \( i \) has \( \tilde{q}_i = [x_i, y_i, a_{ij}]^T \) on their path \( \Gamma_i, i = 1, 2, \ldots, n \). The statement of the robot \( j \) communicating with the robot \( i \) is \( \tilde{q}_j = [x_j, y_j, a_{ij}]^T \), and \( (x_i, y_i, a_i) \) and \( (x_j, y_j, a_j) \) are the position \((x, y)\) and orientation \( \alpha \) of robot \( i \) and \( j \), respectively. The desired relative statement of robot \( i \) and \( j \) is \( p_{ij} = \{x_{ij}, y_{ij}, a_{ij}\}^T \) where \( x_{ij}, y_{ij}, \) and \( a_{ij} \) represent the desired relative position and relative orientation. The kinematic models of \( i \) and \( j \) are established as follows:

\[
\begin{bmatrix}
\dot{x}_i \\
\dot{y}_i \\
\dot{\alpha}_i
\end{bmatrix} =
\begin{bmatrix}
\cos \alpha_i & 0 & v_i \\
\sin \alpha_i & 0 & \omega_i
\end{bmatrix},
\]

(1)

\[
\begin{bmatrix}
\dot{x}_j \\
\dot{y}_j \\
\dot{\alpha}_j
\end{bmatrix} =
\begin{bmatrix}
\cos \alpha_j & 0 & v_j \\
\sin \alpha_j & 0 & \omega_j
\end{bmatrix},
\]

(2)

where \( v_i \) and \( \omega_i \) represent the control input of the forward and angular velocities of robot \( i \) and \( \tilde{q}_j \) are the control input of the forward and angular velocities of robot \( j \), and then the relative statement between them can be expressed as follows:

\[
\begin{bmatrix}
x_{ij} \\
y_{ij} \\
\alpha_{ij}
\end{bmatrix} =
\begin{bmatrix}
\cos \alpha_i & \sin \alpha_i & 0 \\
-\sin \alpha_i & \cos \alpha_i & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
\tilde{q}_i - \tilde{q}_j - p_{ij}
\end{bmatrix}.
\]

(3)

By taking the derivative of (3) and combining with (1) and (2), the following system is obtained:

\[
\begin{align*}
\dot{x}_{ij} &= \omega_j (y_{ij} - y'_{ij}) - v_i + v_j \cos(\alpha_{ij} - \alpha'_{ij}), \\
\dot{y}_{ij} &= -\omega_i (x_{ij} - x'_{ij}) + v_j \sin(\alpha_{ij} - \alpha'_{ij}), \\
\dot{\alpha}_{ij} &= \omega_i - \omega_j - \omega_{ij},
\end{align*}
\]

(4)

where \( \omega_{ij} \) is the desired angular speed error between robot \( i \) and robot \( j \). System (4) between robot \( i \) and robot \( j \) can be linearized around \( x_{ij} = 0, U_{ij} = 0 \) as follows:
\[
\dot{X}_{ij} = A_{ii}X_{ij} + B_{ii}U_{ij},
\]
(5)

where \( A_{ii} = \begin{bmatrix} 0 & \omega_i & 0 \\ -\omega_i & 0 & v_j \\ 0 & 0 & 0 \end{bmatrix}, \) \( B_{ii} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \) and \( X_{ij} = [x_{ij} - x'_{ij}, \ y_{ij} - y'_{ij}, \ a_{ij} - a'_{ij}]^T \) are the statement and \( U_{ij} = [-v_j + v_j \cos(\alpha_{ij} - \alpha'_{ij}) \omega_j - \omega_j - a'_{ij}]^T \) is the control input. Considering the communication between robot \( i \) and other robots, the linearized system of robot \( i \) is written as follows:

\[
\dot{X}_i = A_{ii}X_i + B_{ii}U_i,
\]
(6)

where \( X_i = \sum_{m=1}^{m} a_{ij}X_{ij} \) and \( U_i = \sum_{m=1}^{m} a_{ij}U_{ij}, \) and the elements of \( X_i \) are defined as \( X_i = [X_{ii}, y_{ii}, a_{ii}]^T, \) therefore, \( x_{ii} = \sum_{m=1}^{m} a_{ij}(x_{ij} - x'_{ij}), \) \( y_{ii} = \sum_{m=1}^{m} a_{ij}(y_{ij} - y'_{ij}), \) and \( a_{ii} = \sum_{m=1}^{m} a_{ij}(a_{ij} - a'_{ij}). \) It should be noted that \( X_i \) is the sum of relative position errors between each mobile robot \( i \) and robot \( j \) and \( m \) is the number of robots communicating with robot \( i. \) After discretizing equation (6), the normative system of mobile robot \( i \) is as follows:

\[
X_{i\text{norm}}(k + 1) = A_{ii}X_i(k) + B_{ii}U_i(k),
\]
(7)

where \( A_{ii} = e^{A_{ii}T} \) and \( B_{ii} = \int_{t}^{T} e^{A_{ii}(t - \tau)}d\tau B_{ii}. \) \( T \) is the sampling period. Because of the uncertainty, the system will produce errors. Besides, the linearization process will also bring the errors to the system. To deal with these two errors concurrently, an uncertainty term \( \Delta X_i(k) \) related to control input \( U_i(k) \) and state variable \( X_i(k) \) is considered to describe the uncertainty of the system as follows:

\[
\Delta X_i(k) = \delta_i(\Delta C_i, X_i(k) + \Delta D_iU_i(k)),
\]
(8)

where \( \Delta C_i \) and \( \Delta D_i \) are known matrices. \( \delta_i \) is an unknown but bounded time-independent matrix and satisfies \( \delta_{ii} \equiv \sigma(\delta_{ii}) \leq 1. \) Then, considering the external disturbance of the mobile robot, the bounded disturbance term \( d_i(k) \in \mathbb{R}^{3 \times 1} \) is added. The uncertain but bounded system model of mobile robot \( i \) is shown as follows:

\[
x_i(k + 1) = x_{i\text{norm}}(k + 1) + \Delta X_i(k) + d_i(k) = (A_{ii} + \delta_i \Delta C_i)X_i(k) + (B_{ii} + \delta_i \Delta D_i)U_i(k) + d_i(k),
\]
(9)

and the parameter of this uncertain system model is \([ A_{ii}, B_{ii} ] = [ A_{ii} + \delta_i \Delta C_i, B_{ii} + \delta_i \Delta D_i ] \in \Omega. \)

In the control process, the actuator usually bears limited control force. Therefore, it is very necessary to consider the saturation constraints of the input and the hard constraints of the state variables in the controller design [37]. Otherwise, the control will be inaccurate because of the control input and state variables exceed the limits, and this situation will inevitably reduce the tracking performance of the control system [38]. Therefore, saturation constraint (10) to limit the control input and hard constraint (11) to limit the state variable are added as follows:

\[
|U_i(k)| \leq U_{i\text{cmax}}, \quad c = 1, 2,
\]
(10)

\[
|F_{ic}X_i(k)| \leq X_{i\text{cmax}}, \quad c = 1, 2, 3,
\]
(11)

where \( U_{i\text{cmax}} \) and \( X_{i\text{cmax}} \) are the maximum of \( c \)-th element of \( U_i(k) \) and \( X_i(k) \) and \( F_{ic} \) is the \( c \)-th row of the assigned matrix \( F_{ij} \in \mathbb{R}^{3 \times 3}. \)

### 3. Controller Design and Stability Analysis

#### 3.1. Controller Design

It is worth emphasizing that each mobile robot has its own independent MPC controller. By communicating with each other, these robots receive the information which can form the coupling conditions to solve the optimal control problem online on their respective controllers. For each robot MPC controller, in the absence of an accurate model, the performance of solving the optimization problem which uses the method of quadratic performance objective minimization will be very poor, and the computational load will be large. So, the method applying the robust performance objective minimization to replace the quadratic performance objective minimization to establish the optimization problem is considered for the controller design. A feedback gain \( K \) is set to adjust \( U(k) \) and is continuously updated by the optimization problem. The optimization problem is shown as follows:

\[
\min_{U_i} \max_{[A_{ii}, B_{ii}] \in \Omega} \int_{t_0}^{t_f} (k)\]

\[\text{s.t. (6) – (10).}\]

In order to propose an appropriate objective function, a trajectory parameter is needed to be included in the objective function of the normative system to construct the coupling relationship. A parameter \( \zeta_i \) is defined to describe the trajectory \( \Gamma_i(\zeta_i) = [x_{r_i}(\zeta_i), \ y_{r_i}(\zeta_i), \ a_{r_i}(\zeta_i)] \) of mobile robot \( i \), which is an iterative function related to control input, forward, and angular velocities:

\[
\zeta_i(k + 1) = \zeta_i(k) + M_iU_i(k) + N_i[v_i \omega_i]^T,
\]
(13)

where \( M_i \) and \( N_i \) are the given parameter matrix. Defining \( \zeta_{ij}^r \) as the desired value of the trajectory parameter difference between robot \( i \) and robot \( j \), it is an important parameter to predict the trajectory of robot \( j \) through robot \( i \) and then the trajectory of robot \( j \) can be described as \( \Gamma_j(\zeta_j) = [x_{r_j}(\zeta_j), \ y_{r_j}(\zeta_j), \ a_{r_j}(\zeta_j)] \). The information which robots interact includes their position states and trajectory parameters. The position states and trajectory parameters are used to calculate \( X_{ij} \) and form coupling conditions, respectively. The objective function of the normative system coupled with \( \zeta_i \) is as follows:
where $\psi_{ij} = \Theta_j(k + \Delta k[k] + H_j(U_j(k + \Delta k[k] + \upsilon_j)) + \Theta_i(k + \Delta k[k] - H_j\upsilon_j) \Theta_j = \text{diag}[Q_i, Q_i, Q_i]$, $Q_i > 0$, $R_i = \text{diag} \{R_i, R_i\}$, $R_i > 0$, $W_{ij} = \text{diag} \{w_i, w_i, w_i\}$, $w_i > 0$, $\Theta_i = [\zeta_i, \zeta_i, \zeta_i]^T$, $\Theta_j = [\zeta_j, \zeta_j, \zeta_j]^T$, $\Theta_i = [\zeta_i, \zeta_i, \zeta_i]^T$, $\Theta_j = [\zeta_j, \zeta_j, \zeta_j]^T$, $\psi_j = [\psi_j, \psi_j, \psi_j]^T$, and $H_i = [M_i, M_i, M_i]^T$.

For the uncertain system (9), if there exists a quadratic function $J_{11}(k)$ and $J_{12}(k)$ which can be divided into two parts as follows:

$$J_{11}^*(k) = \sum_{\Delta k=0}^{\infty} \left\| X_i(k + \Delta k[k]) \right\|_Q^2 + \left\| U_i(k + \Delta k[k]) \right\|_R^2 + \sum_{s=1}^{m} \left\| H_i U_i(k + \Delta k[k]) - \psi_s \right\|_W^2,$$

$$J_{12}^*(k) = \sum_{\Delta k=1}^{\infty} \left\| A_i X_i(k + \Delta k[k]) + B_i U_i(k + \Delta k[k]) \right\|_Q^2 + \left\| U_i(k + \Delta k[k]) \right\|_R^2 + \sum_{s=1}^{m} \left\| H_i U_i(k + \Delta k[k]) \psi_s \right\|_W^2 = J_{10}(k) + J_{11}^*(k).$$

The optimization problem is transformed into solving a set of state feedback laws $U_i = K_i X_i$. In the case, the controller objective minimizes the upper bound of the worst-case objective function. Let $\gamma_1$ and $\gamma_2$ be the upper bound of objective function $J_{10}(k)$ and $J_{11}^*(k)$, respectively.

**Theorem 1.** For the uncertain system (9), if there exists $\Lambda_1 = \text{diag}[\lambda_1, \lambda_2, \lambda_3] > 0$, then the system will be stable and $Z$ is an invariant set when following conditions are satisfied:

$$\begin{bmatrix} Q_i & * & * & * \\ A_0 Q_i + B_0 Y_i & * & * & * \\ \Delta C_i Q_i + \Delta D_i Y_i & 0 & \Lambda_1 & * \\ (A_0^T Q_i A_0)^{0.5} Q_i & 0 & 0 & \gamma_2 I \end{bmatrix} \geq 0, \quad \gamma_2 > 0, \quad Y_i \geq 0,$$

$$\begin{bmatrix} Q_i & * & * & * \\ d_i^T(k) y_i & * & * & * \\ \psi_i & 0 & 0 & 0 \end{bmatrix} \sum_{s=1}^{m} \psi_i W_{ij} \psi_j \geq 0,$$
stability criterion, $V(X_i)$ should meet the following inequality:

$$V(X_i(k + j + 1|k)) - V(X_i(k + j|k)) \leq \left[ \|A_{ii} X_i(k + \Delta k|k) + B_{ii} U_i(k + \Delta k|k)\|_F^2 + \|U_i(k + \Delta k|k)\|_F^2 \\
+ \sum_{s=1}^{m} \|H_s U_i(k + j|k) - \psi_{ij}\|_{W,ij}^2 \right].$$

(21)

Summing (21) from $i = 1$ to $i = \infty$ and requiring $X_i(\infty|k) = 0$ or $V(X_i(\infty|k)) = 0$, the result is as follows:

$$\max \{ A_{ii}, B_{ii} \} \in \Omega \quad \text{s.t.} \quad V(X_i(k + 1|k)) = X_i^T(k + 1|k) P_i X_i(k + 1|k) \leq \gamma_2.$$  

(22)

$$X_i^T(k + \Delta k|k) \left\{ (A_i + B_i K_i)^T P_i (A_i + B_i K_i) - P_i + (G_i + B_i K_i)^T Q_i (G_i + B_i K_i) + K_i^T R_i K_i + \sum_{s=1}^{m} (H_i K_i)^T W_{ij} (H_i K_i) \right\} X_i(k + \Delta k|k) + d_i^T(k) P_i d_i(k) - \sum_{s=1}^{m} \psi_{ij}^T W_{ij} \psi_{ij} \leq 0,$$

where (23) can be regarded as two parts, and they are as follows:

$$\left( A_i + B_i K_i \right)^T P_i (A_i + B_i K_i) - P_i + (G_i + B_i K_i)^T Q_i (G_i + B_i K_i) + K_i^T R_i K_i + \sum_{s=1}^{m} (H_i K_i)^T W_{ij} (H_i K_i) \leq 0,$$

(23a)

$$d_i^T(k) P_i d_i(k) - \sum_{s=1}^{m} \psi_{ij}^T W_{ij} \psi_{ij} \leq 0.$$

(23b)

In the above formula, the feedback gain is defined as $K_i = Y_i Q_i^{-1}$, and the solution of $Y_i$ is given by the optimization problem. Substituting $[A_i, B_i] = [A_{ii} + \delta_i, \Delta C_i B_{ii} + \delta_i \Delta D_i]$, feedback gain $K_i = Y_i Q_i^{-1}$ and $P_i = Y_2 Q_i^{-1}$ into (23a) and (23b), and transforming (23a) and (23b) into matrix inequality form, the results are (19) and (20), respectively. Systems (19) and (20) guarantee the positive invariance of $Z$. In the invariant set $Z$, the saturation constraint (10) and hard constraint (11) are expressed as (19) and (20) in matrix form, respectively, and the control input is limited by $G_i$, and the state variable is limited by $L_i$. □

**Remark 1.** In Theorem 1, equation (17) can get the constraint that the upper bound $\gamma_2$ of $f_{i}^{\infty}(k)$ is satisfied when the system is stable. After the upper bound $\gamma_1$ of $f_{i}(k)$ can also be expressed as some concrete constraints, the problem of minimizing the upper bound of the objective function in the worst case can be formulated and solved. And with the update of feedback gain, $X_i(k)$ is always limited in the ellipsoid $Z$ at any time in the optimization process.

**Theorem 2.** The upper bound $\gamma_1$ of $f_{i}(k)$ satisfies the following relation:

$$\left[ \gamma_1 \right] \left[ \psi_{ij}^* \right] \geq 0,$$

(24)

where $\gamma_1 = x_i^T(k) (A_{ii} Q_i A_{ii}) x_i(k) - \sum_{s=1}^{m} \psi_{ij}^T W_{ij} \psi_{ij}$.

**Proof.** of Theorem 2. The objective function of $f_{i}(k)$ is as follows:
\begin{align*}
J_{i0}(k) &= \|A_{i0}X_i(k)| + B_{i0}U_j(k)|k\|_2^2 + \|U_j(k)|k\|_2^2 + \sum_{s=1}^m \|H_{is}u_s(k) - \psi_s\|_{\bar{W}_i}^2 \\
&= x_i^T(k)(A_i^TQ_iA_i)x_i(k) + u_i^T\bar{S}_iu_i - \sum_{s=1}^m \psi_{ij}^T\bar{W}_{ij}\psi_{ij} \leq \gamma_{ii},
\end{align*}

where \(x_i(k)\) can be measured by sensors and exists in historical data. \(\psi_{ij}\) is the function related to \(\zeta_j\) and \(u_j\) can also be measured by sensors, and \(\zeta_i\) can interact with mobile robot \(j\) through wireless network or Bluetooth technology. Let \(u_i^T\bar{S}_iu_i \leq \gamma_{ii}\), equation (24) is obtained after Schur complement.

Theorem 2 gives the condition of upper bound \(\gamma_{ij}\) of \(J_{i0}(k)\), then, the upper bound of the objective function in the infinite time in the worst case can be expressed as follows:

\[
\max_{\left[\begin{array}{c}
A_{i0} \\
B_{i0}
\end{array} \right]} \left( \begin{array}{c}
J_{i0}(k) \\
\gamma_{ii}
\end{array} \right) \leq \gamma_{ii} + \gamma_{ii},
\]

As mentioned in the paper above, we focus on \(X_i(k) + 1|k\) not \(X_i(k)\) to control the system when solving the optimization problem. At this time, the difficulty is how to use the historical data from the previous time to represent the predicted value of the future time. So, consider the following linear law of historical data:

\[
X_i(k|k) = \varepsilon_{ik}(1)\bar{Si}_d(1,:) + \varepsilon_{ik}(2)\bar{Si}_d(2,:) + A_{i0}\Delta X_i(k) + B_{i0}\Delta U_i(k) + \delta_i(\Delta C_iX_i(k) + \Delta D_i\Delta U_i(k)).
\]

Substituting (30) into (15), we can get

\[
\left[\begin{array}{c}
\varepsilon_{ik}(1)\bar{Si}_d(1,:) + \varepsilon_{ik}(2)\bar{Si}_d(2,:) + A_{i0}\Delta X_i(k) \\
B_{i0}\Delta U_i(k) + \delta_i(\Delta C_iX_i(k) + \Delta D_i\Delta U_i(k)
\end{array} \right] \geq 0.
\]

After expanding (31), set \(\Lambda_2 = \text{diag} (\lambda_4, \lambda_5, \lambda_6) > 0\), and the result is shown as follows:

\[
\left[\begin{array}{c}
\varepsilon_{ik}(1)\bar{Si}_d(1,:) + \varepsilon_{ik}(2)\bar{Si}_d(2,:) + A_{i0}\Delta X_i(k) + B_{i0}\Delta U_i(k) \\
\Delta C_iX_i(k) + \Delta D_i\Delta U_i(k)
\end{array} \right] \geq 0.
\]
\[
\begin{bmatrix}
\gamma^*_i \\
\delta_k(1)\sigma_k(1,:) + \delta_k(2)\sigma_k(2,:) + \Delta U_i(k)S_i
\end{bmatrix} \geq 0.
\] (33)

Inequalities (32) and (33) are obtained from historical data, and then the data-driven optimization problem (27) can be expressed as follows:

\[
\min_{Q,Y,G,L,A_1,E_0\Delta X(k)\Delta U(k),y_{i1},y_{2i}} y^*_1 + y_{2i}
\quad \text{s.t. (17), (18), (19), (20), (32), (33).}
\] (34)

3.2. Stability Analysis

**Lemma 1.** When the system is inspired, the stored energy decreases with time and finally reaches a minimum, and then the system is gradually stable. If there is a positive definite scalar function \( V(X_i) \) with negative definite \( \dot{V}(X_i) \), the system is stable.

A positive definite Lyapunov function \( V(X_i) = X_i^TP_iX_i \) is defined in 3.1, where \( P_i > 0 \). If \( \dot{V}(X_i) \) is negative definite, the system is asymptotically stable. The condition for \( \dot{V}(X_i) \) to be negative definite is given in equation (21). By using the robust theory, the analysis of \( X_i \) is transformed into the analysis of state feedback law \( U_i = K_iX_i \), and the conditions (17) and (18) for state feedback gain \( K_i \) are derived. According to Lemma 1, when (17) and (18) are held, the system will tend to stable.

4. Results

In this part, two simulations with a group of mobile robots are conducted to verify the proposed approach.

4.1. Simulation 1. In this simulation, the lane-change trajectory is considered. The desired trajectory is considered as follows:

\[
\Gamma = \begin{cases} 
X^* = -\xi, \\
y^* = \frac{2}{1 + e^{-2t(t-2)}} 
\end{cases}
\] (35)

Let \( \xi_{12}^* = -10, \xi_{23}^* = -10, \xi_{34}^* = 15, \xi_{45}^* = -10, \) and \( \xi_{51}^* = 15 \). The trajectory error of every two mobile robots can be expressed as \( x_{ij}^* = \xi_{ij}^* \) and \( y_{ij}^* = 2 \). The communication topology is shown in Figure 1. The parameters of robots are given as follows: \( Q_1 = 100, S_1 = 15, w_{1j} = 4, Q_2 = 100, S_2 = 6, w_{2j} = 4, Q_3 = 120, S_3 = 9.6, w_{3j} = 4, Q_4 = 100, S_4 = 6, w_{4j} = 4, Q_5 = 120, S_5 = 9.6, \) and \( w_{5j} = 4 \). Set the sampling period \( T = 0.1 \text{s} \) and coefficients \( \Delta C_i = 0, \Delta d_i = B_g, \) and \( M_i = N_i = [-0.2 \ 0.1] \). Each robot's angle speed and line speed are \( \omega = 1 \) and \( v = 1 \), respectively. Taking the uncertainty set as \( \delta_i = \text{diag}[-1,-1,-1] \), the simulation is carried out for 50 s. The initial state values of each mobile robot are as follows:

\[
\begin{align*}
X_1(0) &= [0.25 \ -0.05 \ 0.1]^T, \\
X_2(0) &= [0.1 \ 0.03 \ 0.15]^T, \\
X_3(0) &= [0.15 \ -0.02 \ 0.1]^T, \\
X_4(0) &= [0.15 \ -0.02 \ 0.1]^T, \\
X_5(0) &= [0.15 \ -0.02 \ 0.1]^T.
\end{align*}
\] (36)

The bounded external disturbance of each mobile robot is as follows:

\[
\begin{align*}
d_1(k) &= \left[ \begin{array}{c}
\frac{0.2}{2k+1} + \frac{2}{1000} \\
\frac{0.1}{2k+1} + \frac{1}{1000}
\end{array} \right] \cos(0.1k), \\
d_2(k) &= \left[ \begin{array}{c}
\frac{0.1}{2k+1} + \frac{1}{1000} \\
\frac{0.2}{2k+1} + \frac{2}{1000}
\end{array} \right] \cos(0.2k), \\
d_3(k) &= \left[ \begin{array}{c}
\frac{0.1}{2k+1} + \frac{1}{1000} \\
\frac{0.15}{k+1} + \frac{1}{1000}
\end{array} \right] \sin(0.1k), \\
d_4(k) &= \left[ \begin{array}{c}
\frac{0.15}{k+1} + \frac{1}{1000} \\
\frac{0.2}{2k+1} + \frac{2}{1000}
\end{array} \right] \cos(0.2k), \\
d_5(k) &= \left[ \begin{array}{c}
\frac{0.15}{k+1} + \frac{1}{1000} \\
\frac{0.2}{2k+1} + \frac{2}{1000}
\end{array} \right] \sin(0.1k). \\
\end{align*}
\] (37)
The simulation is compared with the traditional DMPC. Figure 2 shows the state error $X_i$ of each mobile robot. Figure 3 illustrates the absolute value of state error $X_i$ of each mobile robot. Figure 4 displays $\zeta_{ij}$ and $\zeta_{ij}'$ of each mobile robot. Figure 5 presents the different values between $\zeta_{ij}$ and $\zeta_{ij}'$ of each mobile robot. Figure 6 demonstrates the trajectories of five mobile robots.

The result in Figure 2 shows that compared with the traditional DMPC, the optimization process of data-driven RDMPC uses the optimal solution instead of the iterative algorithm, which is more efficient and greatly reduces the fluctuation of the convergence curve. In Figure 3, the simulation results with the control target $X_i = [0 \ 0 \ 0]^T$ indicate that the simulation results of data-driven RDMPC are always smaller than those of the traditional DMPC in the process of gradual stabilization, which highlights the more accurate results of the data-driven RDMPC algorithm. From Figure 4, it is obvious that $\zeta_{ij}$ and $\zeta_{ij}'$ almost coincide. To see errors, although the simulation results fluctuate, the different values representing $\zeta_{ij} - \zeta_{ij}'$ of each mobile robot are always about $10^{-12}$ as shown in Figure 5, and such a small error indicates that the parameter differences among mobile robots converge to the expected value $\zeta_{ij}'$ between them. Figure 6 displays the trajectories of five robots, where the triangle, circle, and diamond represent the initial position, position at 30 s, and the position at 50 s of each mobile robot, respectively. It can be seen that five mobile robots start from their respective starting points and keep their formation shape in the process of advancing according to each ideal trajectory.

4.2. Simulation 2. In this simulation, we choose the sine function as the trajectory. The desired trajectory is considered as follows:

$$
\Gamma = \begin{cases} 
\begin{array}{l}
\xi = \zeta, \\
\eta = \sin(0.2\zeta).
\end{array}
\end{cases}
$$

Let $\zeta_{12} = -1, \zeta_{23} = -1, \zeta_{34} = -1, \zeta_{45} = -1, \text{ and } \zeta_{51} = 4$. The trajectory error between every two mobile robots can be expressed as $x_{ij}(t) = -\zeta_{ij}$ and $y_{ij} = 3$. The communication topology is shown in Figure 1. The parameters and the bounded external disturbances of five mobile robots are the same as simulation 1. Taking the uncertainty set as $\delta_i = \text{diag}[-0.5, -0.5, -0.5]$, the simulation is carried out for 50 s. The initial state values of each mobile robot are follows:

$$
\begin{align*}
X_1(0) &= [1 \ 1 \ 1]^T, \\
X_2(0) &= [1 \ -2 \ 1]^T, \\
X_3(0) &= [-1.5 \ 2 \ 1]^T, \\
X_4(0) &= [1.5 \ -2 \ 1]^T, \\
X_5(0) &= [1 \ 2 \ -1.5]^T.
\end{align*}
$$

Comparing with the traditional DMPC, Figure 7 reveals the state error $X_i$ of each mobile robot. Figure 8 shows the absolute value of state error $X_i$ of each mobile robot. The trajectory parameter difference $\zeta_{ij}$ and the desired trajectory parameter difference $\zeta_{ij}'$ of each mobile robot are shown in Figure 9. Figure 10 indicates the different values between $\zeta_{ij}$ and $\zeta_{ij}'$ of each mobile robot. Figure 11 displays the trajectories of five robots.

Through the simulation result, we can see that the fluctuation of the convergence curve using the data-driven RDMPC method is obviously been reduced than that using the traditional DMPC method in Figure 7. Taking the absolute value of $X_i$, the result of data-driven RDMPC is more accurate than that of traditional DMPC as indicated in Figure 8. The simulation result of path parameter differences $\zeta_{ij}$ of each mobile robot shown in Figure 9 is very close to $\zeta_{ij}'$ after analyzing the different values between them of each mobile robot in Figure 10. And the difference values are always about $10^{-14}$ or $10^{-15}$, and we can think that $\zeta_{ij}$ tracks $\zeta_{ij}'$ well. Figure 11 displays the trajectories of five robots, where the triangle, circle, and diamond represent the initial position, position at 30 s, and the position at 50 s of each mobile robot, respectively; these five mobile robots keep their formation shapes ideally.
Figure 2: The state error $X_i$ of each mobile robot: (a) Robot 1; (b) Robot 2; (c) Robot 3; (d) Robot 4; (e) Robot 5.
Figure 3: The absolute value of state error $X_i$ of each mobile robot: (a) Robot 1; (b) Robot 2; (c) Robot 3; (d) Robot 4; (e) Robot 5.
Figure 4: Continued.
Figure 4: Continued.
Figure 4: The trajectory parameter difference $\zeta_{ij}$ and the desired trajectory parameter difference $\zeta_{ij}^r$ of each mobile robot: (a) Robot 1; (b) Robot 2; (c) Robot 3; (d) Robot 4; (e) Robot 5.

Figure 5: Continued.
Figure 5: The different values between $\zeta_{ij}$ and $\zeta'_{ij}$ of each mobile robot: (a) Robot 1; (b) Robot 2; (c) Robot 3; (d) Robot 4; (e) Robot 5.
Figure 6: Trajectories of five mobile robots.

Figure 7: Continued.
Figure 7: The state error $X_i$ of each mobile robot: (a) Robot 1; (b) Robot 2; (c) Robot 3; (d) Robot 4; (e) Robot 5.

Figure 8: Continued.
Figure 8: The absolute value of state error $X_i$ of each mobile robot: (a) Robot 1; (b) Robot 2; (c) Robot 3; (d) Robot 4; (e) Robot 5.

Figure 9: Continued.
Figure 9: The trajectory parameter difference $\zeta_{ij}$ and the desired trajectory parameter difference $\zeta^r_{ij}$ of each mobile robot: (a) Robot 1; (b) Robot 2; (c) Robot 3; (d) Robot 4; (e) Robot 5.
Figure 10: The different values between $\zeta_{ij}$ and $\zeta_{r,ij}$ of each mobile robot: (a) Robot 1; (b) Robot 2; (c) Robot 3; (d) Robot 4; (e) Robot 5.
5. Conclusions

The traditional distributed model predictive control has been widely used in the field of control; however, it is difficult to handle the model uncertainty of the system. This paper discusses a data-driven robust distributed model predictive control with the model uncertainty. The accuracy of the algorithm is improved by applying the robust performance objective minimization to replace the quadratic performance objective minimization when establishing the optimization problem with considering the model uncertainty of system. And on this basis, the fluctuation of convergence curve and the computational efficiency are abated due to the way of substituting for the iterative algorithm with optimal solution. The system conservatism is reduced after the data-driven strategy that the future predictive value is represented by employing the linear law of historical data which is put to use in deriving the control policy. The stability of the system is proved by Lyapunov theorem, and the effectiveness of this method is illustrated by a set of simulation examples.

Data Availability

The data used to support this study are available from the corresponding authors upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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