The flow of Giesekus fluid around a cylinder between two parallel plates

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Abstract. This paper presents the numerical results of the flow of the viscoelastic Giesekus fluid around a cylinder between two plates. An aqueous solution of polyacrylamide (PAA) is taken as the working fluid. A method for determining the nonlinear parameter of the Giesekus model to approximate the viscosity curve is proposed. The developed method is tested on the approximation of experimental data presented in the literature. The parameters of the four-mode Giesekus model for describing the rheological properties of an aqueous solution of polyacrylamide with concentrations from 2,500 to 10,000 ppm are determined based on a set of experimental data.

1. Introduction
The differential models, which include the Giesekus model, are the most widespread to describe the rheological properties of viscoelastic media [1].

\[
\begin{align*}
\sigma &= \sum_{i=1}^{m} \sigma_i + \sigma_N \cdot \sigma_N = 2\eta_N D, \\
\sigma_i + \gamma \sigma_i + \frac{\alpha_i \lambda_i}{\eta_i} \sigma_i \cdot \sigma_i &= 2\eta_i D, \quad (i = 1...m),
\end{align*}
\]

where \( \sigma_i \) is the stress tensor deviator; \( \sigma_N \) is the viscous and elastic components of the tensor \( \sigma \); \( D = \frac{\partial \sigma}{\partial t} - \sigma_i \cdot \nabla V^T - \nabla V \cdot \sigma_i = \frac{\partial \sigma}{\partial t} + \nabla \sigma \cdot V - \sigma_i \cdot \nabla V^T - \nabla V \cdot \sigma_i \) is the upper convective derivative of the tensor \( \sigma_i \); \( \gamma \) is the strain rate tensor; \( \eta_N, \eta_i \) are viscosities; \( \lambda_i, \alpha_i \) are the relaxation time and rheological parameter corresponding to the \( i \)-th mode; \( V \) is the velocity vector; \( m \) is the number of modes.

It is known that the Giesekus model describes the rheological behavior of linear low-density polyethylene, polypropylene, and polystyrene [2]; polyacrylamide [3]; guar gum [4]; polybutadienes [5] well. The works [6, 7, 8] show that the Giesekus model reflects the main features of the rheology of micellar solutions with long worm-like micelles. The Giesekus model includes a spectrum of
relaxation times \((\lambda_i, \eta_i)\) and a nonlinear parameter \(\alpha_i\). The most common method for determining the parameter \(\alpha_i\) is the approximation of the viscosity curve obtained experimentally [3]. Moreover, in the literature there are no specific dependences of shear stresses on shear rate for a Giesekus fluid arising in torsional flow when plotting a viscosity curve using plate-plate systems. The paper presents this equation. Based on experimental studies, the parameters of the four-mode Giesekus model describing the rheological properties of aqueous solutions of polyacrylamide are found. Numerical flow simulation of the considered fluids during flow around the cylinder between two parallel plates is performed.

2. Experimental

2.1.1. Materials and Instruments

The aqueous solutions of polyacrylamide with concentrations of 2500, 5000, and 10 000 ppm are used as a working fluid. Distilled water at a temperature of 40 degrees Celsius and FC XB Polymer brand polyacrylamide powder (Russia) were used to prepare the solutions. Before adding the powder, the required volume of water was preliminarily mixed using the Hei-TORQUE Value 100 overhead stirrer (Heidolph, Germany) with a VISCO JET mixing device (diameter 60 mm) at a speed of 1500 rpm to create a vortex. Then, polyacrylamide powder was added within 15 seconds. The weighing error of the required amount of water and powder did not exceed 0.06% and 0.33%, respectively. After 3 minutes of mixing, the speed of the stirrer was reduced to 1300 rpm/min the mixing process lasted 10 minutes. The finished sample was transferred into a jar with a tight lid and placed in a dark place for a day, which ensured the disappearance of air bubbles.

2.1.2. Determination of parameters of Giesekus model

Experimental studies were performed using a Physica MCR 102 rheometer manufactured by Anton Paar (Austria) using a PP50 “plate-plate” system. The diameter of the plate is 50 mm, the gap – 1 mm. The temperature of the test sample was maintained constant using the built-in temperature control system on Peltier elements and was equal to 20 degrees Celsius. The upper active casing was additionally sawn also on Peltier elements H-PTD 200 to exclude the occurrence of temperature gradients over the sample thickness.

It is known from the literature [2] that the spectrum of relaxation time \((\lambda_i, \eta_i)\) can be determined by the results of oscillation tests at small amplitudes. The nonlinear parameter \(\alpha_i\) for each mode is determined from the approximation of the viscosity curve obtained at a shear flow between two parallel plates. After simple transformations, the Giesekus model describing the dependence \(\tau_i = r\sigma_i^{\text{vol}}\) (shear stresses for each mode) on the shear rate \(\dot{\gamma}\) in implicit form takes the form:

\[
2b_i^2\lambda_i\dot{\gamma} - 2h_i\lambda_i\dot{\gamma}\sqrt{b_i^2 - 4\alpha_i^2\tau_i^2} + \left(\alpha_i\sqrt{b_i^2 - 4\alpha_i^2\tau_i^2} + 8\alpha_i h_i\lambda_i\dot{\gamma}\tau_i + \alpha_i\sqrt{b_i^2 - 4\alpha_i^2\tau_i^2}\right)\tau_i - 2\alpha_i\lambda_i\dot{\gamma} b_i^2 = 0,
\]

where \(r \dot{\chi}_i = \frac{d\varphi}{dt} r \lambda_i = \dot{\gamma} \lambda_i\) - the product of shear rate by relaxation time for each mode, \(\frac{d\varphi}{dt}\) - angular velocity, \(i\) - mode number.

Thus, unknown \(\alpha_i\) is calculated from the condition of minimizing the
\[ F(\alpha_1, \ldots, \alpha_m) = \sum_{k=1}^{m} \left( \tau_k^{(\text{experiment})} - \tau_k^{(\text{calc})} \right)^2 \rightarrow \min, \]

where \( \tau_k^{(\text{experiment})} \) - values obtained from experimental studies, \( \tau_k^{(\text{calc})} = \eta_N \dot{\gamma}_k + \sum_{i=1}^{m} \gamma_i^{(\text{calc})} \) - calculated data.

Note that the relative error of repeated tests in the mode of oscillation and shear flows did not exceed 5%.

The reliability of the developed method for determining the nonlinear parameter of the Giesekus model is confirmed by comparison with the experimental results for 10.000 ppm presented in [2]. Figures 1-2 present the calculation results, which indicate a satisfactory agreement between the experimental [2] and calculated data (where \( \omega \) is angular frequency).

Figure 1. Dynamic moduli (a) and viscosity curve (b): dots – experimental data from [2], solid line – our fit based on (2).

Figure 2 presents approximations of the experimental data of the dependence of the storage and loss moduli on angular frequency for aqueous solutions of polyacrylamide with concentrations of 2500, 5000, and 10000 ppm. As can be seen from the figure, the elastic and loss moduli can be approximated with the number of modes equal to 4. A further increase in the number of modes complicates the numerical implementation when solving applied problems. A small deviation of the elastic modulus at low angular frequencies has practically no effect on the approximation of the viscosity curve (Fig. 3). The average relative error between the experimental and calculated data obtained based on the four-mode Giesekus model does not exceed 2.6%. As can be seen from the figure, for the concentrations of 2500 and 5000 ppm, the Giesekus model describes the experimental results in the best way. Table 1 presents the parameters of the Giesekus model for the studied fluids.

Table 1. The parameters of 4 modes Giesekus constitutive equation for PAA.

| i  | \( \lambda_i \) | \( \eta_i \) | \( \eta_N \) | \( \alpha_i \) |
|----|-----------------|-----------------|-----------------|-----------------|
| 1  | 0.1184          | 0.137           | 0.039           | 0.99            |
| 2  | 0.9489          | 0.9004          | -               | 0.9             |
| 3  | 7.6671          | 6.0406          | -               | 0.99            |
| 4  | 72.3015         | 32.2598         | -               | 0.55            |
| i  | $\lambda_i$ | $\eta_i$ | $\eta_N$ | $\alpha_i$ |
|----|------------|--------|--------|------|
| 5000 ppm |
| 1  | 0.1043    | 0.2662 | 0.062  | 0.99 |
| 2  | 0.8716    | 1.9358 | -      | 0.97 |
| 3  | 7.294     | 13.436 | -      | 0.99 |
| 4  | 78.218    | 68.778 | -      | 0.47 |
| 10000 ppm |
| 1  | 0.1158    | 0.6573 | 0.114  | 0.99 |
| 2  | 1.0776    | 5.4452 | -      | 0.92 |
| 3  | 10.326    | 44.4336 | -    | 0.71 |
| 4  | 112.578   | 303.422 | -    | 0.94 |

Figure 2. Dynamic moduli and viscosity curve (our measurements and fit):
(a) 2500 ppm, (b) 5000 ppm, (c) 10000 ppm.

3. Numerical simulations

Let us consider the isothermal laminar flow around a cylinder between two parallel plates. The problem is stated in two dimensions. Geometrical characteristics of the simulated area: cylinder radius 1.1875 mm, channel height 4.95 mm. The ratio of length to the height of the channel is 8.08, and the ratio of height to diameter is 2.08. At the entrance and exit, a fully developed velocity profile is prescribed. The no-slip boundary conditions of fluid are accepted on the channel walls. Ansys Polyflow is used to solve the problem.

For a start, we will test the numerical implementation of this problem using as an example the flow of a commercial grade low-density polyethylene (LDPE) (DSM, Stamylan LD 2008 XC43) [9], where the average inlet flow velocity is 1.975 mm/s. The corresponding parameters of the four-mode
Giesekus model are taken from the literature [9], and the flow-averaged Weissenberg number $Wi = 2.896$ is calculated by the formula.

\[
Wi = \frac{\bar{\lambda}u_{1D}}{h}
\]  

(4)

where, $\bar{\lambda}$ denotes the viscosity-averaged relaxation time for the material ($\bar{\lambda} = \left( \sum_{i=1}^{m} \lambda_i \eta_i \right) / \left( \sum_{i=1}^{m} \eta_i \right)$), $u_{1D}$ - the two-dimensional mean velocity and $h$ – a characteristic length of the flow geometry.

Figure 3 presents the calculation results.

(a) (b)

Figure 3. Mesh (a) and dimensionless velocity profiles (b) in the planar flow around a cylinder over five cross-sections ($Wi=2.896$): solid line – AnsysPolyflow for LDPE, dots – experimental result for LDPE [9], dashed line – AnsysPolyflow for 5000 ppm PAA

The same figure shows the velocity profiles for PAA with a concentration of 5000 ppm. In order to observe the similarity of viscoelastic fluid flow and to compare the numerical results with experimental data, the velocity of PAA with a concentration of 5000 ppm is selected so that $Wi = 2.896$. As can be seen from the figure, the velocity profiles calculated for PAA are in satisfactory agreement with the experimental data, which confirms the adequacy of the parameters found for the four-mode Giesekus model for aqueous solutions of PAA.

**Conclusions**

The set of numerical studies of viscoelastic fluid flow around a cylinder between two plates agrees with experimental data. For the numerical implementation of the task, the parameters of the four-mode Giesekus model for aqueous solutions of polyacrylamide with concentrations of 2500, 5000 and 10 000 ppm are preliminarily determined. The use of the Weissenberg criterion made it possible to compare the results of the LDPE flow at 170 degrees Celsius and PAA with a concentration of 5000 ppm at a temperature of 20 degrees. The parameters obtained for aqueous solutions of polyacrylamide can be used for a physical experiment simulating the flow of polymer melt in an extruder.

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