Tau polarization in quasielastic charged-current neutrino(antineutrino)-nucleus scattering

Krzysztof M. Graczyk

July 23, 2004

Institute of Theoretical Physics, University of Wroclaw
pl. M. Borna 9, 50 – 204 Wroclaw, Poland
Phone: (+48-71) 375-9408
Fax: (+48-71) 321-4454

PACS codes: 13.15+g, 25.30Pt, 13.88+e
Keywords: tau polarization, neutrino-nucleus scattering, ring random phase approximation

Abstract

The quasielastic charged-current (CC) tau neutrino (antineutrino)-nucleus scattering is considered. The dependence of tau polarization on nuclear-structure effects is discussed in detail. The description of the nucleus is based on the Mean-Field Theory (MFT). The ground state of nucleus is described using the Relativistic Fermi Gas model (FG). The effective mass is introduced as well as the ring Random Phase Approximation (RPA) effects are taken into account in the framework of relativistic meson-nucleon model. The Local Density Approximation (LDA) is used for the argon nucleus, having in mind possible application to the ICARUS experiment. The discussion concentrates on the threshold region where the $\tau^-$ can be unpolarized and the nuclear effects play an important role.

1 Introduction

The oscillation of $\nu_\mu$ into $\nu_\tau$ is a possible explanation of the deficit of the atmospheric muon neutrinos which has been measured in SuperKAMIOKANDE. New projects such as ICARUS and OPERA will
enable a more precise study of the $\nu_\mu \rightarrow \nu_\tau$ oscillations phenomena by the detection of tau neutrinos. In the case of the CNGS beam and the ICARUS detector with five T600 modules about 12 events of $\nu_\tau$ are expected (after kinematical selection procedure) during five years of data taking (assuming $\delta m^2 = 2.5 \times 10^{-3} eV^2$) \[1\]. In the case of the OPERA 1.8kT detector about 11 events are expected. This small statistic requires detailed analysis which should be done based on precise theoretical predictions for the considered process. In the case of tau neutrino-matter interaction the produced lepton has short lifetime, so only its decay products can be observed. The large mass of the tau in contrast to electron and muon implies that it can be partially polarized \[2, 3\]. The degree of polarization of tau is one of the parameters which describe its decay products distributions \[4, 5\]. Therefore, the discussion the tau polarization in the neutrino-matter scattering can play an important role in the analysis of the experimental data and is also of theoretical interest.

Polarization properties of taus produced in charged-current neutrino-matter interactions have been considered by several groups \[6, 7, 8\]. In papers \[6, 7\] the polarization vector of the tau was calculated by using the spin density matrix \[9, 5\]. Authors discuss neutrino-matter interaction in the three regions namely quasielastic neutrino-nucleon scattering, neutrino-nucleon scattering with single pion production and deep inelastic scattering. As was shown by K. Hagiwara et al. \[6\] the produced leptons are characterized by high polarization degree. In the laboratory frame (LAB) the low energy $\tau^-$ can be right-handed (positive sign of the longitudinal polarization) and the high energy $\tau^-$ are always left-handed (have negative helicity). For $\tau^+$ there exists a similar effect. This effect can be explained by considering the $\nu - n$ scattering in the center of mass frame (CM) where the produced $\tau$s can be scattered in both forward and backward directions. In the CM frame the produced $\tau^-$ is always left-handed (helicity has negative sign). However, the left-handed taus which are scattered in backward directions in the CM frame usually become right-handed and are scattered in the forward directions after performing the boost to the LAB frame.

A covariant way of calculation of the spin density matrix was presented by K.S. Kuzmin et al. in \[7\]. In the description of the single pion production they used the Rein-Sehgal model. They also considered quasielastic neutrino-nucleon scattering using nonstandard vector and axial currents.
The aim of this paper is the investigation of how the nuclear matter affects the tau polarization.

The relativistic mean-field theory formalism is used to describe a nucleus \([10]\). This description is based on relativistic Fermi gas (FG) model \([11]\). The nucleons inside the nucleus do not interact with each other. Their momenta are uniformly distributed in the Fermi sphere which of the radius given by the Fermi momentum \(k_F\). There is a direct relation between the \(k_F\) and the nuclear matter density.

To make the description of the nucleus more realistic the effective mass of nucleons is considered and the ring random phase approximation (RPA) is done based on the residual interaction \(\pi + \rho + g'\) \([12]\). The RPA corrections are calculated by taking into account the infinite sum of one particle – one hole diagrams (p-h). The effective mass is introduced following Walecka (\(\sigma - \omega\) model) \([10]\). In the simplest version of the model the mean-field approximation leads to the modifications of the nucleon four-momentum and its mass (in the fermion propagator). One of the main results of the mean-field approximation of the \(\sigma - \omega\) model is a self-consistency equation relating the effective mass with the Fermi momentum. Solving this equation for \(k_F = 225\) MeV yields the effective mass equal to 638 MeV. The effective mass is then introduced into the fermion propagator instead of the free mass \((M \rightarrow M^*)\).

Having in mind application of calculations to the ICARUS experiment we discuss also neutrino-argon scattering. The argon nucleus is described in the local density approximation (LDA) \([14]\). The Fermi momenta of nucleons are local and given by the experimental charge density distributions \([15]\).

The paper is organized as follows. In the first part the degree of polarization and the technical details of the description of the nucleus model are given. The calculation of the polarization vector is based on the algebraic decomposition of the nuclear polarization tensor \([13]\). In the second part of the article the numerical results are presented and discussed. In general we get results similar to those of K. Hagiwara et al. \([6]\). The tau leptons are characterized by high degree of polarization for almost all energies and scattering angles with the exception for the zero scattering angle where the \(\tau^-\) can be partially polarized.

In the discussion of the nuclear-structure effects we focus on how the polarization of the tau is affected by introducing to the Fermi gas model the effective mass of the nucleon and the RPA corrections. We show that in the case of forward scattering (zero scattering angle) the mean value of degree of polarization of \(\tau^-\) has a minimum around
the neutrino energy of 4.5 GeV. The minimal value of \( \langle P_{\theta=0,E} \rangle \) for the basic FG model is 0.2 so the tau is almost unpolarized. The introduction of the effective mass and the RPA corrections increases this minimum to about 0.4.

2 Theoretical description

2.1 Polarization

We focus on the following quasielastic processes:

- \(\nu_\tau (k) + n(p) \rightarrow \tau^- (k', s^\mu) + p(p')\),
- \(\bar{\nu}_\tau (k) + p(p) \rightarrow \tau^+ (k', s^\mu) + n(p')\).

The produced \(\tau^-\) and \(\tau^+\) are polarized, its spin vector \(s_\mu\) satisfies (in any frame) the relations:

\[ k'_\mu s^\mu = 0, \quad s^2_\mu = -1. \]

The differential cross section is expressed by the following sum of two contributions:

\[ d\sigma(k, q, s) \sim \left( \sum_\lambda \sum_s L_{\mu\nu}(\lambda, s) + \sum_\lambda L_{\mu\nu}(\lambda, s) \right) W^{\mu\nu}. \]

The hadron tensor is defined as:

\[ W^{\mu\nu} = \sum_f (2\pi)^4 \delta^4(q - p_f + p_i) \langle i|J^\mu(0)|f\rangle \langle f|J^\nu(0)|i\rangle, \tag{1} \]

where \(J_\mu\) – electroweak current, \(|i\rangle, |f\rangle\) – initial and final hadronic states. The \(L^{\mu\nu}(\lambda, s)\) denotes the lepton tensor, \(\lambda\) – neutrino helicity.

Summing over helicities leads to the expressions:

\[ \sum_\lambda \sum_s L_{\mu\nu}(\lambda, s) = 8 \left( k_{\mu} k'_{\nu} + k_{\nu} k'_{\mu} - g_{\mu\nu} k^\alpha k^{\alpha} \mp i\epsilon_{\mu\nu\alpha\beta} k^{\alpha} k^{\beta} \right) \equiv L^{0}_{\mu\nu}, \]

\[ \sum_\lambda L_{\mu\nu}(\lambda, s) = 8m s^{\alpha} \left( k_{\nu} g_{\alpha\mu} + k_{\mu} g_{\nu\alpha} - g_{\mu\nu} k_\alpha \mp i\epsilon_{\mu\nu\beta\alpha} k^{\beta} \right) \equiv L^{s}_{\mu\nu}, \]

where sign \(\mp\) corresponds to scattering of neutrino/antineutrino. The second expression is linear in the lepton mass \(m\) and the spin four-vector \(s_\mu\). That is why in the case of electron and muon the contribution to the cross section due to polarization is most often neglected.
The polarization of the tau lepton measured in the direction of the four-vector \( s_\mu \) is given by the formula \([16]\):

\[
\mathcal{P}_{s_\mu} = \frac{d\sigma(k, q, s) - d\sigma(k, q, -s)}{d\sigma(k, q, s) + d\sigma(k, q, -s)} = \frac{L^s_{\mu\nu}W^{\mu\nu}}{L^0_{\mu\nu}W^{\mu\nu}} = P_\mu s_\mu, 
\]

which defines \( P_\mu \) - the polarization vector of the tau lepton.

We introduce the four-vectors \( e^\mu_l, e^\mu_t, e^\mu_p \) such that:

- they are orthogonal and in the rest frame of the tau
  \( e^\mu_l = (0, e^\mu_l), \quad e^\mu_t = (0, e^\mu_t), \quad e^\mu_p = (0, e^\mu_p) \)
  \(|e^\mu_l| = |e^\mu_p| = |e^\mu_t| = 1\);
- in the laboratory frame their spacelike parts satisfy:
  \( e^\mu_l \sim k', \quad e^\mu_t \sim k \times k', \quad e^\mu_p \sim e^\mu_l \times e^\mu_t \).

It follows that in the laboratory frame have the form:

\[
e^\mu_l = \frac{1}{m} \left( |k'|, E_\tau \frac{k'}{k'} \right), \quad e^\mu_t = (0, e^\mu_t), \quad e^\mu_p = (0, e^\mu_p).
\]

The decomposition of polarization four-vector in the above basis:

\[
P^\mu = \alpha k'^\mu + e^\mu_l \mathcal{P}_l + e^\mu_p \mathcal{P}_p + e^\mu_t \mathcal{P}_t
\]
defines its longitudinal \( \mathcal{P}_l \), perpendicular \( \mathcal{P}_p \) and transverse \( \mathcal{P}_t \) components.

To define the degree of polarization it is useful to go into the rest frame of the tau, where the four-vector \( s_\mu \) is spacelike

\[
s_\mu = (0, \hat{s}), \quad \hat{s}^2 = 1.
\]

Thus the polarization which is measured in the direction of \( \hat{s} \) is equal to:

\[
\mathcal{P}_s = -\mathbf{P} \cdot \mathbf{s} = -|\mathbf{P}| \cos(\beta),
\]

\( \beta \) being the angle between \( \mathbf{P} \) and \( \hat{s} \).

The quantity

\[
\mathcal{P} \equiv |\mathbf{P}|
\]

is called the degree of polarization.

Because \( \hat{s} \) is spanned (in the rest frame of \( \tau \)) by \( e_\mu, e_l, e_p \), hence the degree of polarization is given by the longitudinal, transverse and
perpendicular polarizations. The Lorentz boost does not change this properties in any frame:

\[ P = \sqrt{P_t^2 + P_p^2 + P_l^2}. \] (5)

Further consideration will be performed in the laboratory frame because the description of the nucleus (in particular distributions of the Fermi momenta) is established in the LAB frame.

### 2.2 Model of the nucleus

In the model of the nucleus adopted in this paper the crucial role is played by the polarization tensor \( \Pi^{\mu\nu} \) which is directly related to the hadronic tensor \( W^{\mu\nu} \). The inclusive cross section (normalized per one nucleon) for the neutrino-nucleus scattering is proportional to the imaginary part of the contraction of the polarization tensor with the lepton tensor \[12]:

\[ \frac{d^2\sigma}{d\cos(\theta)\,dE_\tau} = -\frac{G_F^2\cos^2\theta_c|k'|}{16\pi^2\rho_F E} \text{Im} (L_\mu^\nu\Pi_\nu^\mu). \] (6)

Here \( \rho_F = k_F^3/3\pi^2 \) is neutron or proton density in nucleus. \( E \) is the neutrino(antineutrino) energy and \( \theta \) – the scattering angle. We fix the Fermi momentum at 225 MeV in the presented calculations.

In general, the polarization tensor is defined by the chronological product of many body currents:

\[ \Pi^{\mu\nu}(q_\alpha) = -i \int d^4x e^{iq_\alpha x^\alpha} \langle 0 | T (J^\mu(x) J^\nu(0)) | 0 \rangle. \]

Usually to simplify the problem a one body current is considered instead of a many body one. In the case of the charged-current electroweak interaction the one body current is given by:

\[ J^\mu(x) = \bar{\Psi}(x)\Gamma^\mu \Psi(x), \]

\[ \Gamma^\alpha(q_\mu) = F_1(q_\mu^2)\gamma^\alpha + F_2(q_\mu^2)\frac{i\sigma^{\alpha\mu}q_\mu}{2M} + G_A(q_\mu^2)\gamma^\alpha\gamma^5 + F_p(q_\mu^2)q^\alpha\gamma^5 (7) \]

where the form factors \( F_1, F_2, G_A, \) and \( F_p \) have well known dipole form and are described by the following set of parameters \[17]: \( M_A = 1.0 \) GeV, \( g_A = -1.26, \mu = 4.71, M_F^2 = 0.71 \) GeV².
The ground state of the nucleus is the relativistic Fermi gas. One can easily obtain the polarization tensor in this case:

\[ \Pi^{\mu\nu}_{\text{free}}(q_\alpha) = -i \int \frac{d^4p}{(2\pi)^4} \text{Tr} \left( G(p + q)\Gamma^\mu(q)G(p)\Gamma^\nu(-q) \right), \]  

(8)

where \( G(p) \) is the nucleon propagator in the Fermi sea.

\[ G(p) = (p + M) \left( \frac{1}{p_\alpha^2 - M^2 + i\epsilon} + \frac{i\pi}{E_p} \delta(p_0 - E_p)\theta(k_F - p) \right) \]  

(9)

As was mentioned, the mean-field approximation in the \( \sigma - \omega \) model leads to the modifications of nucleon four-momentum and its mass which become effective [13]:

\[ p^*_\mu = (p^*_0 - g^N_V V_0, p), \quad M \to M^*, \]

where \( g^N_V \) is the coupling constant for the \( \omega \)-nucleon interaction. The effective mass is given by the self-consistency equation. In the simplest approach the potential \( V_0 \) is constant and can be eliminated by the change of variables in the integral which gives \( \Pi^{\mu\nu} \). Hence only the effective mass appears in further calculations.

The considered residual interaction \( \pi + \rho + g' \) is described by the effective lagrangian [10] which is given by nucleon-meson \( \rho \) and nucleon-pion interaction terms. Following H. Kim et al. [12] a short-range correlation effect is taken into account by introducing the Landau-Migdal parameter \( (g' = 0.7) \) into the pion propagator. Using the interaction introduced above the ring random phase approximation is performed [19]. It is calculated by taking into account the infinite sum of contributions from ring diagrams (one particle – one hole excitations). It leads to the correction of the free polarization tensor:

\[ \Pi^{\mu\nu} = \Pi^{\mu\nu}_{\text{free}} + \Delta \Pi^{\mu\nu}_{\text{RPA}}. \]  

(10)

To simplify the analysis, it is convenient to choose the coordinate system which in the four-momentum transfer \( q_\mu \) has the form \((q_0, q, 0, 0)\). Then the polarization tensor as a matrix can be decomposed in four independent parts:

\[ \Pi^{\mu\nu}_\mu = e_L \Pi^L + e_T \Pi^T + e_{VA} \Pi^{VA} + e_A \Pi^A, \]  

(11)

where \( \Pi_{L,T,A,VA} \) are complex functions which are called longitudinal, transverse, mixed and axial. \( e_{L,T,A,VA} \) are \( 4 \times 4 \) matrices which form
a closed algebra \[13\]. Properties of this algebra allow us to obtain analytical solution for each component of polarization tensor separately:

\[
\Pi_{L,T,A,V} \rightarrow \Pi_{L,T,A,V} + \Delta \Pi_{L,T,A,V}^{RPA}.
\]

In presented calculations only density dependent part of the polarization tensors is taken into account \[12\]. Contributions from the divergent Feynman parts are omitted.

Decomposition of the \(\Pi_{\mu\nu}\) leads to the following decomposition of the scattering amplitude:

\[
L_{\mu\nu}\Pi_{\mu\nu} = L_L\Pi^L + L_T\Pi^T \pm L_V^{\nu}\Pi_V^A + L_A^{\nu}\Pi_A^A,
\]

where:

\[
\begin{align*}
L_L &\equiv L_\mu^\nu e_\nu L^\mu = -\frac{q_\mu^2}{q_\mu^2}L_{00} + \frac{q_0 q_\mu}{q_\mu^2}(L_{01} + L_{10}) - \frac{q_0^2}{q_\mu^2}L_{11}, \\
L_T &\equiv L_\mu^\nu e_\nu T^\mu = -(L_{22} + L_{33}), \\
L_V^{\nu} &\equiv L_\mu^\nu e_\nu V^{\nu} = 2iL_{23}, \\
L_A^{\nu} &\equiv L_\mu^\nu e_\nu A^\mu = L_\mu^\nu,
\end{align*}
\]

are longitudinal, transverse, mixed and axial components of lepton tensor.

As was shown, calculations of the degree of polarization requires the knowledge of the longitudinal, transverse and perpendicular components of polarization:

\[
\mathcal{P}_{i\mu} = \frac{d\sigma(E_\tau, \theta, i\mu)}{d\sigma(E_\tau, \theta, -i\mu)} = \frac{\text{Im}(L_{i\mu\nu}\Pi_{\mu\nu})}{\text{Im}(L_{0\mu\nu}\Pi_{\mu\nu})},
\]

\[
= \frac{\text{Im}(L_{L}^i\Pi^L + L_{T}^i\Pi^T \pm L_{V}^{i\nu}\Pi_V^{A} + L_{A}^{i\nu}\Pi_A^{A})}{\text{Im}(L_{L}^0\Pi^L + L_{T}^0\Pi^T \pm L_{V}^{0\nu}\Pi_V^{A} + L_{A}^{0\nu}\Pi_A^{A})},
\]

where \(i\mu = e_\mu^l, e_\mu^t, e_\mu^p\).

Performing the corresponding decomposition of the lepton tensor we obtain:

- longitudinal:

\[
L_L^i = \frac{8m}{q_\mu^2} \left( q_0^2 q_0 (q_0 e_0^l - q e_1^l - 2E e_0^l) + 2q_0 q (E e_1^l + k_1 e_0^l) \right) - \frac{8q_0^2 m}{q_\mu^2} \left( q_0 e_0^l - q e_1^l + 2k_1 e_1^l \right),
\]
\[ L'_{T} = -16m \left( Ee_{0} - k_{1}e_{1} \right), \]
\[ L'_{A} = -16m(q_{0}e_{0} - q_{1}e_{1}), \]
\[ L'_{V,A} = 16m \left( Ee_{1} - k_{1} e_{0} \right); \]

- perpendicular:
  \[ L_{p} = \frac{8m e_{1}^{p}}{q_{\mu}^{2}} \left( 2E q_{0} q - q_{0}^{2} q - 2q_{0}^{2} k_{1} \right), \]
  \[ L_{T} = 16m e_{1}^{p} k_{1}, \]
  \[ L_{A} = 16m q e_{1}^{p}, \]
  \[ L_{V,A} = 16m E e_{1}^{p}. \]

The contribution from the transverse polarization vanishes which means that the polarization vector lies in the scattering plane. One can notice that \( P_{p} \) vanishes for zero scattering angle because it is proportional to \( \sin(\theta) \). In the zero lepton mass limit we get:

\[ (P_{l}, P_{t}, P_{p}) \rightarrow (\mp 1, 0, 0), \]

where sign \( + (-) \) corresponds to right-handed, and left-handed lepton helicity respectively.

It can be shown that the components of lepton tensor satisfy the following "triangle relations":

\[ (L'_{L})^{2} + (L'_{P})^{2} = (L'_{L})^{2}, \quad (15) \]
\[ (L'_{A})^{2} + (L'_{P})^{2} = (L'_{A})^{2}, \quad (16) \]
\[ (L'_{T})^{2} - (L'_{T})^{2} - (L'_{P})^{2} = (L'_{V,A})^{2} + (L'_{V,A})^{2} - (L'_{V,A})^{2}, \]
\[ = -\frac{64m^{2}}{q^{2}} \left( 4q_{\mu}^{2} E_{\tau} E + 4(E q_{0} - q_{\mu}^{2} m^{2} + (m^{2} + q_{\mu}^{2})^{2} \right), \quad (17) \]

which make numerical calculations easier.

### 3 Numerical results and discussion

We begin the discussion of the numerical results with the presentation of the total cross sections. In the Fig. the cross sections for quasielastic scattering on nucleus for both neutrino and antineutrino
are compared to the scattering on a free nucleon. The scattering on
the nucleus is described by the Fermi gas model. The Pauli blocking
reduces the total cross section by about 8% compared to the free nu-
cleon case while the Fermi motion shifts the cutoff energy from ∼ 3.4
to ∼ 3 GeV. The cutoff energy for tau production is equal to
\[ \frac{m(m + 2M)}{2(E_F + k_F)} \]

Then we enrich the model by using the effective mass. As a result the
cutoff energy is shifted to ∼ 3.2 GeV and the total cross section further
reduced by about 7%. The we take into account the RPA corrections
which results in a slight reduction of the total cross section noticeable
mainly for higher energies.

Fig. 2 illustrates how the introduction of the RPA corrections influ-
ences the differential cross section obtained within the Fermi gas model
with the effective mass for neutrino (antineutrino) energy of 7 GeV.
The main contribution is located in the region of small scattering an-
gles where the RPA corrections reduce the peak of the differential
cross section by several percent.

The mean value of the degree of polarization of tau is defined as
\[ \langle P \rangle = \frac{1}{\sigma(E)} \int dE_\tau d\theta d\sigma(E_\tau, \theta, E) P(E_\tau, \theta, E). \] (18)

Its dependence on the neutrino (antineutrino) energy is shown in Figs.
3(a) and 3(b) for three cases: the Fermi gas, the Fermi gas with the
effective mass, and the Fermi gas with the effective mass and the RPA
corrections.

In the case of τ− the mean value of degree of polarization is almost
constant (∋ ⟨P⟩ ∼ 0.85 without and about 0.82 with the effective mass)
up to 4 GeV then it gradually rises to saturate at about 9 GeV. In the
case of τ+ the mean value of degree of polarization raises more rapidly
saturating already at about 6 GeV. In both cases the application of
the effective mass to the free FG model decreases ⟨P⟩ by a few percent
and shifts the plots to the right. It is interesting that the result of
introducing the RPA corrections into the Fermi gas with effective mass
is opposite in the case of τ+ where it lowers the ⟨P⟩ than in the case
of τ− where we observe a slight enhancement.

The effects of the introduction of the effective mass and then the
RPA corrections are most visible in the Figs. 3(c) and 3(d) where the
mean value of the degree of polarization of $\tau^-$ at zero scattering angle is plotted.

\[
\langle P_{\theta,E} \rangle = \frac{1}{d\sigma(E, \theta)} \int dE_{\tau} d\sigma(E_{\tau}, \theta, E) P(E_{\tau}, \theta, E).
\] (19)

The minimal value of the degree of polarization is obtained for neutrino energy of about 4.5 GeV. It is equal to 0.2 for the Fermi gas model, to 0.3 after the effective mass is used, and to 0.4 when the RPA corrections are also included.

In Figs. 4, 5, 6, 7 the dependence of the degree of polarization on the tau energy is presented. Calculations are done for three scattering angles ($0^\circ$, $3^\circ$, $6^\circ$). We compare the plots of polarization with the appropriate differential cross sections given by equation (6).

In general, for a given scattering angle, there exist two kinematically allowed regions for tau energy. One is placed close to the tau mass, and the other is placed near the neutrino(antineutrino) energy. The Fermi motion widens these regions and for the beam energy of 4 GeV they join each other. For higher neutrino (antineutrino) energies, these two regions are separated by a large forbidden area. The plots of $P$ and $d^2\sigma/d\cos(\theta)dE_{\tau}$ for $E= 7$ GeV presented in Figs. 5, 7 are confined to the area of energy close to the neutrino (antineutrino) energy because in the other area the cross section is negligible.

One can notice that the degree of polarization strongly depends on the scattering angles and the produced leptons have high degree of polarization for almost all angles, apart from $\theta = 0$, where the tau degree of polarization can be small.

Figs. 4 and 6 show the results for scattering of 4 GeV neutrinos and antineutrinos. It can be seen that the polarization plots for $\theta = 0^\circ$ have sharp minimum where $P$ reaches zero. In this point the longitudinal polarization changes its sign from positive to negative. This effect is clearly explained in [6]. The helicity of $\tau^-$ in the LAB frame changes from right-handed to left-handed (for $\tau^+$ there is an analogical effect). For higher angles we can still see a local minimum in this point but it is much higher and is smooth because of the non zero contribution from the perpendicular polarization. It is interesting that this point is shifted after the introduction of the effective mass and also after the inclusion of the RPA corrections (it can be clearly seen in Figs. 5 and 6).

For the neutrino(antineutrino) energy of 7 GeV the two kinematically allowed regions correspond to opposite signs of the longitudinal
polarization of the tau (the minimum region is kinematically forbidden) when $M^* = M$. However, when $M^* = 638$ MeV, it is possible that both signs of $\mathcal{P}_l$ may be present in the same kinematical region which makes the minimal value of $\mathcal{P} = 0$ available for zero scattering angle. Lowering of the effective mass stretches the allowed kinematical regions reducing the gap between them.

Finally, we apply the Fermi gas model with the RPA corrections and the LDA to the case of the ICARUS detector and the CNGS beam. We fix the tau neutrinos energy at 7 GeV – the most probable energy of the tau neutrinos resulting from the oscillations of the CNGS beam muon neutrinos.

In the description of the argon nucleus (the target in the ICARUS detector) we use the LDA approach. We apply the charge density profile from atomic data tables [21] to describe nucleon density $\rho(r)$ in nucleus (see appendix). The Fermi momenta for protons and neutrons are the following:

$$k_F^p(r) = \sqrt[3]{\frac{3\pi^2}{2}} \rho_p(r), \quad \rho_p(r) = \frac{Z}{A}\rho(r),$$

(20)

$$k_F^n(r) = \sqrt[3]{\frac{3\pi^2}{2}} \rho_n(r), \quad \rho_n(r) = \frac{A-Z}{A}\rho(r).$$

(21)

where the atomic number $A = \int d^3r \rho(r) = 40$, and $Z = 18$. The cross section is given by the integral:

$$d\sigma(E_\tau, \theta) = \frac{1}{A} \int d^3r \rho_{n,p}(r) d\sigma(k_{F_n,p}(r), E_\tau, \theta).$$

(22)

The results are shown in Fig. 8 where we compare the degree of polarization of tau and the appropriate differential cross section obtained with and without the RPA corrections. As in the case of the global Fermi momentum (see Fig. 5) the introduction of the RPA is the most visible for the zero scattering angle. The plot of the degree polarization (in the case of the FG) is a straight line rising from 0.4 to 0.8. The application the RPA corrections yields much deeper minimum (less than 0.1) and a maximum reaching 0.9.

Summary
The tau leptons produced in neutrino, antineutrino-nucleus scattering are characterized by high degree of polarization. Nevertheless, the $\tau^-$ can be almost unpolarized at zero scattering angle for small neutrino energies. In particular, for the neutrino energy of 4.5 GeV, the mean degree of polarization reaches its minimum. The minimal
value of \( \langle P_\theta, E \rangle \) is very sensitive to the details of the model such as the effective mass and the RPA corrections. Introducing the effective mass increases it by about 50\% (from about 0.2 to 0.3) and including the RPA corrections causes an additional 33\% rise (from 0.3 to 0.4).

It should also be mentioned that the RPA corrections influence the energy for which the tau’s helicity changes the sign, whereas the introduction of the effective mass strongly affects the kinematics by broadening the kinematically allowed regions and causing the gap between them to disappear for some neutrino energy values.

The results of this paper are quite similar to those obtained for the neutrino-nucleon deep inelastic scattering in [6, 20]. It suggests that it is the kinematics that plays the main role in the calculations of the polarizations.

Acknowledgments

I would like to thank J. Sobczyk for stimulating discussions and useful remarks which helped to improve this paper. I thank D. Kielczewska and E. Rondio, as well as K. Kurek for interesting discussions. I also would like to thank C. Justczak for reading the manuscript and interesting comments.

This work was supported by the KBN grant 105/E-344/SPB/ICARUS/P-03/DZ 211/2003-2005.

The author is a Max Born Scholarship fellow.

A Basis Vectors

The four-momenta of the neutrino and the tau have the form:

\[ k_\mu = (E, k_1, k_2, k_3) \]

and

\[ k'_\mu = (E - q_0, k_1 - q, k_2, k_3). \]

Appropriate calculations lead to the expressions for basis vectors:

\[
e^t_\mu = \frac{\chi E_\tau}{m} \left( \frac{|k'|}{E_\tau} k_1 - q, \frac{k_2}{|k'|}, \frac{k_3}{|k'|} \right),
\]

\[
e^t_\mu = \frac{1}{\sqrt{k_2^2 + k_3^2}} (0, 0, -k_3, k_2),
\]
\[ e_\mu^0 = -\frac{1}{|k'|\sqrt{k_2^2 + k_3^2}} (0, k_2^2 + k_3^2, (k_1 - q)k_2, (k_1 - q)k_3), \]

where \( \chi = \mp 1 \) describes helicity of \( \tau^\mp \).

## B  Density profile of Argon

Charge density profile of \(^{18}\text{Ar}^{40}\) is the following \[^{21}\]:

\[ \rho(r) = \frac{\rho(0)}{1 + \exp \left( \left( r - C \right)/C_1 \right)}, \]

\[ \rho(0) = 0.176 \text{ fm}^{-3}, \ C = 3.530 \text{ fm}, \ C_1 = 0.542 \text{ fm}. \]

### References

[1] M. Komatsu, NuInt01 Conference, Nucl. Phys. B (Proc. Suppl.) 112, (2002) 15, I. Gil Botella, Oulu 2002, Beyond the desert, 373.

[2] C. H. Llwellyn Smith, Phys. Rep. 3, (1972) 261.

[3] C. H. Albright, C. Jarlskog, Nucl. Phys. B84, (1975) 467.

[4] L. B. Okun, Leptons and Quarks, North-Holland 1987.

[5] E. Leader, Spin in particle physics, Cambidge University Press 2001.

[6] K. Hagiwara, K. Mawatari, H. Yokoya, Nucl. Phys. B668, (2003) 364.

[7] K. S. Kuzmin, V. V. Lyubuskin and V. A. Naumov, contribution to 10th Int. Workshop on High Energy Spin Physics (SPIN03), JINR Dubna, Russia, 16-20 Sept. 2003, hep-ph/0312107, 8th Scientifc Conference for Young Scientists and Specialists (SCYSS-04), Dubna, Russia, 2-6 Feb 2004. e-Print Archive: hep-ph/0403110.

[8] C. Burrely, J. Soffer, O.V. Teryaev, Phys. Rev. D 69, (2004) 114019.

[9] L.D. Landau, E.M. Lifshitz, Quantum mechanics: non-relativistic theory, (1977).

[10] B.D. Serot, J.D. Walecka, Advances in Nuclear Physics, edited by J.W. Negele, E. Vogt, Plenum, New York 1986, vol 16.
[11] R.A. Smith, E.J. Moniz, Nucl. Phys. B43, (1972) 605.

[12] H. Kim, J. Piekarewicz, C.J. Horowitz, Phys. Rev. C 51, (1995) 2739.

[13] K. M. Graczyk, J. T. Sobczyk, Eur. Phys. J. C 31, 177 (2003).

[14] S.K. Singh, E. Oset, Nucl. Phys. A542, (1992) 587, Phys. Rev. C 48, (1993) 1246, T. S. Kosmas, E. Oset, Phys. Rev C 53, (1996) 1409, E. Paschos, L. Pasquali, J.Y. Yu, Nucl. Phys. B588, (2000) 263.

[15] E.J. Moniz et al. Phys. Rev. Lett. 26 (1971) 445, R.R. Whitney, et. Phys. Rev. C 9, (1974) 2230.

[16] J.D. Bjorken, S.D. Drell, Relativistic Quantum Mechanics, McGrawHill Book Company, New York (1965).

[17] H. Budd, A. Bodek, J. Arrington, International Workshop on Neutrino-Nucleus Interactions in the Few GeV Region (NUINT 02), Irvine, California, 12-15 Dec 2002, hep-ex/0308005.

[18] K. Wehrberger, Phys. Rep. 225, (1993) 273

[19] A.L. Fetter, J.D. Walecka, Quantum Theory of Many-Particle System, (McGraw-Hill, New York 1971).

[20] K. M. Graczyk, Poster Session, NuInt04 Conference, will be published in Nucl. Phys. B (Proc. Suppl.).

[21] H. De Vries, C.W. de Jager and C. De Vries, Atomic Data and Nuclear Data Tables 36, (1987) 495.
Figure 1: Comparison of the total cross sections (per one nucleon) for charged-current quasielastic neutrino and antineutrino-nucleus scattering. The model of nucleus is the Fermi gas (dashed line – FG), the Fermi gas with the effective mass $M^* = 638$ MeV (short dashed line – FG+$M^*$) and the Fermi gas with the effective mass $M^* = 638$ MeV and the RPA corrections (dotted line – RPA+$M^*$). The total cross sections calculated for neutrino and antineutrino-free nucleon scattering are shown (solid line – QE).
Figure 2: Comparison of the differential cross sections for charged-current quasielastic neutrino(antineutrino)-nucleus scattering. The nucleus is described by the Fermi gas with the effective mass – FG+M* (neutrino-nucleus scattering – solid line and antineutrino-nucleus – short dashed line), and by the Fermi gas with the effective mass and the RPA corrections – RPA+M* (neutrino-nucleus scattering – dashed line, antineutrino-nucleus scattering – dotted line). Calculations are done for neutrino(antineutrino) energy of 7 GeV.
Figure 3: In the first row (Figs. a, b) the mean values (given by equation (18)) of the degree of polarization of $\tau^\pm$ leptons are presented. The charts in the second row (Figs. c, d) present the mean values (given by equation (19)) of the degree of polarization of $\tau^\pm$ leptons calculated for zero scattering angle. The tau leptons are produced in charged-current quasielastic neutrino(antineutrino)-nucleus scattering. The nucleus is described by the Fermi gas (solid line – FG), the Fermi gas with the effective mass $M^* = 638$ MeV (dashed line – FG+$M^*$) and the Fermi gas with the effective mass $M^* = 638$ MeV and the RPA corrections (dotted line – RPA+$M^*$).
Figure 4: The charts in the first row present the differential cross sections calculated for three scattering angles: $\theta = 0^\circ, 3^\circ, 6^\circ$. The second row shows dependence of the degree of polarization of the tau on its energy. The tau lepton is produced in charged-current neutrino-nucleus scattering (for neutrino energy 4 GeV). The nucleus is described by the Fermi gas (solid line – FG), the Fermi gas with the effective mass $M^* = 638$ MeV (dashed line – FG+$M^*$) and the Fermi gas with the effective mass $M^* = 638$ MeV and the RPA corrections (dotted line – RPA+$M^*$).
Figure 5: The same as Fig. 4 but neutrino energy is 7 GeV.
Figure 6: The same as Fig. 4 but for antineutrino.
Figure 7: The same as Fig. 5 but for antineutrino.
Figure 8: The charts in the first row present the differential cross sections calculated for three scattering angles: $\theta = 0^\circ, 3^\circ, 6^\circ$. The second row shows dependence of the degree of polarization of the tau on its energy. The tau lepton is produced in charged-current neutrino-nucleus scattering (for neutrino energy 7 GeV). The nucleus is described using the LDA approach for argon and by the Fermi gas (solid line – FG) and the Fermi gas with the RPA corrections (dotted line – FG+RPA).