IS THE STRING COUPLING CONSTANT INVARIANT UNDER T-DUALITY?

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ABSTRACT

It is well known that under T-duality the sigma model dilaton (which is normally thought to be related to the string coupling constant through the simple formula $\kappa = \exp <\phi>$), undergoes an additive shift. On the other hand, Kugo and Zwiebach, using a simplified form of string field theory, claim that the string coupling constant does not change under the T-duality. Obviously, what seems to happen is that two different coupling constants, associated to different dilatons, are used. In this contribution we shall try to clarify this, and related issues.

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1 Introduction

It is a well-known fact [1], [2] in the theory of closed bosonic strings propagating on a spacetime with a non-trivial group of isometries, that an equivalent dual formulation exists. The original and the dual models are related by Buscher’s formulas (which are not diffeomorphisms in general). For example, for an abelian isometry the dilaton undergoes a change

\[ \phi \rightarrow \phi' = \phi - \frac{1}{2} \log k^2. \]  

(1)

Here \( k \) is the norm of the Killing vector associated to the isometry. In adapted coordinates, in which the Killing vector is \( \frac{\partial}{\partial x^0} \), then \( k^2 = g_{00} \).

There are basically three different (and complementary) ways of studying the physics of strings: in first quantization, in the (target space) effective action formalism and in the string field theory (SFT) approach. Different dilatons exist in these three formulations with corresponding coupling constants. This means that we have several different couplings with slightly different meanings.

In the first quantization the string coupling constant \( \kappa \) is related to the dilaton through

\[ \kappa = \exp <\phi>. \]  

(2)

This coupling appears as the parameter of the expansion of the partition function of the string over different world-sheet topologies (in the closed case, this is equivalent to an expansion over Riemann surfaces). The relation above immediately implies a corresponding change in the (naive) string coupling constant under T-duality:

\[ \kappa \rightarrow \kappa' = \frac{\kappa}{\sqrt{g_{00}}}. \]  

(3)

There is further evidence for this transformation law, stemming from the explicit computation by one of us [2] of the free energy density of a string on a circle at arbitrary genus, an exact result.

In the SFT approach, as we discuss below, the coupling \( \lambda \) enters in a completely different way, as the coefficient of the interaction term. It was argued in Ref. [3] that T-duality is a symmetry of SFT and, as such, does not change the value of the string coupling constant \( \lambda \) (which, as has been repeatedly emphasized by the authors of Ref. [3], must be, in principle, an observable). Although a simplified version of SFT (HIKKO theory) is used, it is supposed to be powerful enough to tackle the problem under consideration.

In sections 2-4 we review the existing definitions of the dilaton in various formulations of the string theory and discuss the coupling constants associated to them as well as their mutual relationships. We consider for simplicity closed bosonic strings in the critical dimension \( n = 26 \) with \( d \) spatial dimensions compactified to a torus \( T^d \) with common radius \( R \), so that there are
n − d non-compact dimensions. We study in some detail the change of dilaton under T-duality separately in each formulation, and reach our conclusions in section 5. A conjecture is made, in particular, on the relationship between the ghost-dilaton field in SFT and the sigma-model dilaton (based on previous results in the linearized approximation). It is not clear to us, however, how this conjecture could be compatible with the implications of the ghost-dilaton theorem in SFT.

We denote, as usual, the intrinsic string tension dimensionful parameter by \(\alpha'\) when necessary. The couplings that we are going to discuss are associated to the vacuum expectation values of the dilaton. In that sense, they are characteristics of the background and not another fundamental parameters of the string theory.

## 2 First quantized bosonic string

In the standard worldsheet formulation of the free closed string moving in the 26-dimensional target space in a background characterized by the metric \(G_{\mu\nu}(X)\), the antisymmetric tensor \(B_{\mu\nu}(X)\) and the (sigma-model or curvature)-dilaton \(\Phi_\sigma(X)\) the action is given by

\[
S = \frac{1}{4\pi\alpha'} \int d^2\sigma \left[ \sqrt{\gamma} \gamma^{\alpha\beta} G_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu + i \epsilon^{\alpha\beta} B_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu + \alpha' \sqrt{\gamma} \Phi_\sigma R^{(2)} \right].
\]

(4)

In what follows we take \(2\alpha' = 1\). We consider the case when the background space-time has \(d\) spatial dimensions compactified to a torus of common radius \(R\) and \(26 − d\) non-compact dimensions, i.e. it is of the form \(M^{26−d} \times T^d\).

For further discussion it is important to know the dependence of the partition function \(Z(R)\) on the radius \(R\). Let us give a sketch of this calculation \[\text{[2], [3]}\]. The contribution \(Z_g(R)\) of the Riemann surface of genus \(g\) is

\[
Z_g(R) = \int_{\mathcal{F}_g} d\mu F_g(R; \tau) \Lambda(\tau, \bar{\tau}),
\]

(5)

where the integral is taken over the moduli space of the surface of genus \(g\), \(\tau\) is the period matrix of the world-sheet Riemann surface and \(\Lambda(\tau, \bar{\tau})\) is some function of \(\tau\) and \(\bar{\tau}\) only, whose explicit form is not important for us. \(F_g(R; \tau)\) is given by the functional integral over the string field \(X^\mu\) satisfying the conditions

\[
X^a(z + a_i, \bar{z} + a_i) = X^a(z, \bar{z}) + 2\pi R m_i^a, \quad \text{and} \quad X^a(z + b_i, \bar{z} + b_i) = X^a(z, \bar{z}) + 2\pi R n_i^a,
\]

(6) and (7)

where \(a_i\) and \(b_i\) \((i = 1, 2, \ldots g)\) are cycles generating the homology of the Riemann surface \(\Sigma_g\), \(a = 1, 2, \ldots, d\) and \(m_i, n_i\) are the winding numbers:

\[
F_g(R; \tau) = \int DX e^{-S(X)}.
\]

(8)
The action $S'$ is given by Eq. (4) but without the dilaton term. The integral in (8) can be calculated by summing over the contributions of the instanton solutions $X_{mn}^M$, satisfying Eqs. (6) and (7), and then integrating over the fluctuations $x$ around the instantons:

$$F_g(R; \tau) = \sum_{m,n} e^{-S'(X_{mn})} \int dq \int Dxe^{-S'(x)}$$

The integral over the fluctuations is independent of $R$. There are two points which are important for us. First, we indicate explicitly the integration over the zero mode $q$ of the instanton solutions. It appears because in each sector $(m,n)$ we have a continuous family of instanton solutions $X_{mn}^\mu + q^\mu$ with the same action $S'(X_{mn})$ parametrized by the 26-vector $q^\mu$. Integration over this zero mode gives the volume factor

$$\int dq = V_D(2\pi R)^d.$$  

The appearance of this factor is in accordance with the calculations in [6]. We would like to stress here that the corresponding factor in [5] and [7] differs from the one in (10), and is physically inequivalent to it.

Second, summation over $m$ and $n$ gives

$$\sum_{mn} e^{-S'(X_{mn})} = (\det I_n \tau)^{d/2} \left( \frac{1}{(R\sqrt{2})^d} \sum_{mn} \exp f \left( \sqrt{2}Rm, \frac{n}{\sqrt{2}R} \right) \right)^d,$$

where $f$ is a given symmetric function. Notice that the sum in the r.h.s. is obviously invariant under the $R \to 1/(2R)$ transformation. Tracing the dependence on the radius $R$ we obtain that

$$Z_g(R) = R^{d(1-g)} \tilde{Z}_g(R),$$

where $\tilde{Z}_g(R)$ is invariant under the T-duality transformation $R \to 1/(2R)$. From the formula above it follows that

$$Z_g \left( \frac{1}{2R} \right) = (2R^2)^{d(g-1)} Z_g(R).$$

If we add the dilaton term to the action (see Eq. (4)), the complete partition function for this background is given by the sum

$$Z(\Phi, R) = \sum_{g=1}^{\infty} e^{2(g-1)\Phi} Z_g(R),$$

where the exponent appears from the last term in the action (4) with a constant dilaton background $\Phi$ (recall that $\chi = (1/4\pi) \int d^2\sigma \sqrt{|g|} R(2)$ is the Euler characteristic of the two-dimensional manifold which for a compact Riemann surface of genus $g$ is equal to $\chi = 2(1-g)$). Because of Eq. (13) the requirement that the T-duality is a symmetry of the string theory implies that

$$Z \left( \Phi, \frac{d}{2} \log(2R^2), \frac{1}{R} \right) = Z(\Phi, R).$$
As it is known, all correlators can be obtained from the partition function, and thus automatically transform correctly under the T-duality transformation.

In this formulation the dimensionless string coupling constant is defined through the vacuum expectation value of the background field $\Phi_\sigma$, that is:

$$\kappa = e^{<\Phi_\sigma>}.$$  \hfill (16)

Then the relation (15) immediately leads to the advertised change of the coupling constant, associated to a given background, under the T-duality transformation:

$$\kappa \rightarrow \kappa' = \frac{\kappa}{(2R^2)^{d/2}}.$$  \hfill (17)

Let us also comment here that the change of the dilaton

$$\Phi_\sigma \rightarrow \Phi_\sigma - d \log(2R^2)/2,$$  \hfill (18)

which appears in Eq. (15) as a consequence of the T-duality invariance of the full partition function, coincides with the shift of the dilaton (1) due to the change of the measure under the T-duality transformation in Buscher’s construction, see [8].

Given the fact that the determinant of the spacetime metric is related to the one of its dual by the formula

$$\sqrt{G} = k^2 \sqrt{G'},$$  \hfill (20)

there is an obvious combination which is duality invariant, namely:

$$\phi_{inv} \equiv \Phi_\sigma - \frac{1}{4} \log G.$$  \hfill (21)

We would like to mention that the formulas (14), (15) and (17) were also verified in a discretization of the toroidally compactified string model in Ref. [11].

We conclude this section with discussion of the dilaton states. It appears that there are (at least) three states which qualify for the name ‘dilaton’ in string theory (4) (see, for example, [13], [12]). In particular, zero-momentum physical states correspond to

\footnote{At a fundamental level T-duality is essentially a canonical transformation, with the generating function

$$F = \frac{1}{2} \oint d\tilde{\theta} \wedge d\theta$$  \hfill (19)

where the two Killing vectors in the original and the dual models are $k = \frac{\partial}{\partial \sigma}$, and $\tilde{k} = \frac{\partial}{\partial \tilde{\sigma}}$ [8], [11], and from this point of view the dilaton transformation (1) is rooted in the integration over momenta to recover the lagrangian formulation.}

5
1. the *ghost dilaton*, defined by the operator

\[ D_g = \frac{1}{2}(c\partial^2 c - \bar{c}\partial^2 \bar{c}) \quad (22) \]

and

2. the *matter dilaton*, defined by:

\[ \eta_{\mu\nu} D_{\mu\nu} \equiv \eta_{\mu\nu} c\partial X^\mu \bar{\partial} X^\nu \quad (23) \]

The relevant linear combination of the states above is

3. the *zero momentum dilaton*, defined through:

\[ \mathcal{D} \equiv \eta_{\mu\nu} D_{\mu\nu} + D_g \quad (24) \]

This field transforms as a scalar under spacetime diffeomorphisms. Another relevant combination is the *Graviton Trace*, defined by:

\[ G \equiv \eta_{\mu\nu} D_{\mu\nu} + \frac{n}{2} D_g \quad (25) \]

(Here \(c\) and \(\bar{c}\) are ghost fields, as usual).

### 3 Low energy effective theory

The effective action, describing the interactions of the massless modes of the bosonic closed string in the lowest order in \(\alpha'\) is equal to (see [4]):

\[ S \equiv -\frac{1}{2\kappa_{26}^2} \int d^n x \sqrt{G} e^{-2\Phi (R(G) + 4\partial_{\mu} \Phi \partial^\mu \Phi - \frac{3}{4} H_{\mu\nu\rho} H^{\mu\nu\rho})} \quad (26) \]

In the formula above the kinetic term for the gravitational field, described by the (string) metric \(G_{\mu\nu}\), is not normalized in the standard way because of the exponential factor of the dilaton in front of it; this can be unraveled by the use of the *Einstein* metric, conformally related to the string metric as follows:

\[ g_{\mu\nu} \equiv e^{-\frac{8\Phi}{n-2}} G_{\mu\nu} \quad (27) \]

In terms of the Einstein metric the effective action reads:

\[ S \equiv -\frac{1}{2\kappa_{26}^2} \int d^n x \sqrt{g} (R^{(g)} - \frac{4}{n-2} \partial_{\mu} \Phi \partial^\mu \Phi - \frac{3}{4} e^{-\frac{8\Phi}{n-2}} H_{\mu\nu\rho} H^{\mu\nu\rho}) \quad (28) \]

This “target space dilaton” (which is called also sometimes the Fradkin-Tseytlin dilaton [14]), represents the same \(\sigma\)-model dilaton introduced in the previous section. It is important to keep in mind that, first, it is a scalar and, second, it transforms according to (18).
The coupling $\kappa_{26}$, which appears in front of the action plays the role of the gravitational constant in 26-dimensional space-time. As it is known there is no actually any new dimensional constant (apart from $\alpha'$). Indeed, similar to the first quantized formulation, $\kappa_{26}$ can be always absorbed into the vacuum expectation value $\Phi_0 \equiv <\Phi>$ of the dilaton. Thus, we can consider that for a given background characterized by $\Phi_0$ we have the gravitational coupling

$$\kappa_{26} = \kappa_{26}^{(0)} e^{\Phi_0}. \quad (29)$$

($\kappa_{26}^{(0)} \sim (\alpha')^6$). In the case of a space-time with $d$ dimensions toroidally compactified (with dilaton $\Phi$ not depending on these $d$ compact dimensions) the transformation $R \rightarrow 1/(2R)$ in the metric $G_{\mu\nu}$ with the corresponding change of the dilaton $\Phi \rightarrow \Phi' = \Phi - d \log(2R^2)/2$ (see Eq. (18)) leaves the action (26) invariant. In this we should assume that the coupling is $\kappa_{26}^{(0)}$ and does not change. Another point of view is to consider that in Eq. (26) $\Phi = \phi$ is the quantum fluctuation of the dilaton field and its vacuum expectation value is represented by the gravitational coupling $\kappa_{26}$. Then the T-duality transformation $R \rightarrow 1/(2R)$ does not affect the field $\phi$, but on the contrary the value of the coupling changes as

$$\kappa_{26} \rightarrow \kappa_{26}' = \frac{\kappa_{26}}{2R^2 d/2}, \quad (30)$$

the same as for $\kappa$, Eq. (17). It is easy to see from (26) that after the dimensional reduction to the spacetime of $26 - d$ non-compact dimensions the effective coupling constant becomes

$$\kappa_{26-d} = \frac{\kappa_{26}}{(2\pi R)^d/2}. \quad (31)$$

The dimensionally reduced theory in $M^{26-d}$ should not be affected by the change of the radius of the compactified dimensions of the initial 26-dimensional spacetime. Indeed, from the definition (31) and the transformation law (30) it follows that $\kappa_{26-d}$ is invariant.

### 4 Dilaton in string field theory

It is not unlikely that in the future the “string field theory” will be a quantum version of an eleven-dimensional M-theory, which is probably not itself a theory of strings. But in the meantime, some impressive achievements have been already made (see, for example,[15]-[18]) leading to a consistent (non-polynomial) string field theory for closed strings. In SFT the free action is given by

$$S_0 = ((\Psi|K|\Psi)), \quad (32)$$

where $K = Q + \tilde{Q}$ ,$Q$ and $\tilde{Q}$ are the BRST charges for the left-moving and right-moving modes, and the scalar product $((.,.)$) is defined in detail in [17]. The state $|\Psi>$, which describes the shift of the original $\sigma$-model away from the empty flat spacetime, can be written as

$$|\Psi> = \{... + h_{\mu\nu}(q)\alpha_{-1}^{\mu} \bar{\alpha}_{-1}^{\nu} + \phi (\bar{e}_{-1}b_{-1} + c_{-1} \bar{b}_{-1})\}} c(0)\bar{c}(0)|0>,$$
We write only the part of $|\Psi>$ which is of interest for us, and we use the standard notations for the coefficients of the mode expansion for the string field and ghosts, now understood as operators. Notice that the states which appears in the expansion (33), are the ghost dilaton, $|D_g > \equiv \phi D_g |0>$ with $D_g$ given by (22), and $h_{\mu \nu} D^{\mu \nu}$, where $D^{\mu \nu}$ can be read off from (23).

The symmetric part $h_{\mu \nu}^{(s)}$ of the wave function $h_{\mu \nu}$ can be shown to represent the graviton. This interpretation appears in various ways. First of all it is known that physical states are defined modulo a huge gauge invariance, generated by all exact states in the BRST cohomology, that is, all states of the form:

$$|\Psi> + (Q + \bar{Q})|\epsilon>$$

are physically equivalent. Thus, the gauge transformations, which also leave the action (32) invariant, have the form

$$\delta |\Psi> = (Q + \bar{Q})|\epsilon>.$$  

(35)

If we now choose

$$|\epsilon> = \epsilon_{\mu} (\partial X^\mu c(0) + \bar{\partial} X^\mu \bar{c}(0))|0>$$

we generate a gauge transformation on the “graviton” state

$$|\text{grav} > = h_{\mu \nu}^{(s)} D^{\mu \nu} |0>$$

(37)

of the following form:

$$\delta h_{\mu \nu} = \partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu$$

(38)

and in the “ghost-dilaton” state, $|D_g > \equiv \phi D_g |0>$,

$$\delta \Phi = \partial_\mu \epsilon^\mu.$$  

(39)

These formulas show that the gauge transformation (35), (36) lead to a general covariant transformation and $\phi$ is not a scalar under this transformation. If we make the field redefinitions

$$h_{\mu \nu}^{(s)} = \hat{h}_{\mu \nu}^{(s)} + \eta_{\mu \nu} \hat{\phi},$$

$$\phi = \hat{\phi} + \frac{1}{2} \hat{h}^{(s)}_{\mu} \hat{h}^{(s)}_{\mu},$$

(40)

then it is easy to check that the graviton field $\hat{h}^{(s)}_{\mu \nu}$ has the same transformation law as (38) and $\hat{\phi}$ is a scalar. More detailed considerations suggest that at least to the linear order the relation between the fields $g_{\mu \nu}$ and $\Phi$ in (28) and the string field (33) is given by

$$g_{\mu \nu} = \eta_{\mu \nu} + \hat{h}_{\mu \nu}^{(s)} + \ldots,$$

$$\Phi - \Phi_0 = \frac{n-2}{4} \hat{\phi} + \ldots = -\frac{1}{2} \left( \phi - \frac{1}{2} \hat{h}^{(s)}_{\mu} \hat{h}^{(s)}_{\mu} + \ldots \right)$$

(41)

where $\Phi_0$ is the constant background value of the dilaton. These relations are also verified by the properties of the fields under the $O(d) \times O(d)$ transformations [19].

In terms of these new fields the state (33) can be written as

$$|\psi> = \{ \ldots + (\hat{h}_{\mu \nu}^{(s)} - (1/n) \eta_{\mu \nu} \hat{h}^{(s)}_{\rho} \rho)(D^{\mu \nu} - (1/n)\eta^{\mu \nu} D_{\rho} \rho) + (1/n) \hat{h}^{(s)}_{\rho} \rho G + \hat{\phi} D \} |0>$$

(42)
with $D$ and $G$ are given by (24) and (25).

The formulas above show that the ghost-dilaton field $\phi$ fails to be invariant under (linearized) diffeomorphisms. On the other hand, the form of the variation immediately conveys the fact that the zero momentum dilaton

$$|D| = \hat{\phi}D|0>$$

is really invariant, $\delta\hat{\phi} = 0$. Further relations between different formulations of the theory of strings come from the calculation of the action (32). It was shown that this action reproduces the part of the effective action (28), containing the graviton and the dilaton, in the quadratic approximation provided the relations (41) are taken into account [20], [21] (see also [15]). Also in Ref. [22] it was demonstrated that the equation of motion $(Q + \bar{Q})|\Psi> = 0$ in string field theory coincides with the standard 1-loop beta-functions for a bosonic string in background metric and dilaton fields with the same identifications.

All this suggests that the $\sigma$-model dilaton and the dilaton field in (20)-(28) should be identified with the zero-momentum dilaton in string field theory (in fact, with $\frac{1}{2}\phi$), whereas the field $\hat{\phi}_{inv}$, Eq. (21), should be identified with the ghost dilaton, $-\frac{1}{2}\phi$. (This agrees with Polchinski in [12]).

Let us discuss now the T-duality transformation. According to the idea of the string field theory approach the general formalism is considered to be independent of the background [3], [23]. As in usual quantum field theory, the background appears when a general field $|\Psi>$ is presented as

$$|\Psi> = |\Psi'> + |S>,$$

where $|S>$ describes the classical background and $|\Psi'>$ represents quantum fluctuations. One goes from one background to another by making a shift of the field of this type. Now, the claim of Ref. [3] is that $O(d,d,Z)$ - transformations of the background in general and T-duality transformation in particular do not move the field $|\Psi>$ in the direction of the ghost dilaton, specified by the states proportional to $\phi$ in Eq. (33). This is in accordance with the T-duality transformations in the $\sigma$-model formulation and the relation between the ghost dilaton of the string field theory and $\phi_{inv}$ conjectured above.

Up to now we do not have any coupling constant in string field theory. In this approach it is introduced, mimicking ordinary quantum field theory, through the definition of the interaction. There are quite a few versions of the theory with interaction, starting from the first covariant formulation by Siegel [20], [24]. It is worth mentioning, in particular, the one proposed by Witten [25] for open strings, which is based on differential geometry constructions. A towering achievement has been the consistent formulation of a non-polynomial bosonic string field theory [27]. A particularly simple theory (stemming from Siegel’s picture) usually denoted as $\alpha = p^+$ HIKKO theory, [28] has been argued by Kugo and Zwiebach to serve the purpose of discussion of the T-duality.

In the $\alpha = p^+$ HIKKO theory the interaction term schematically is given by

$$S_{int} = \frac{\lambda}{3} <V||\Psi>|\Psi> |\Psi>,$$

(45)
where $< V \rangle$ is the 3-vertex and we emphasize that the string coupling constant $\lambda$ is defined through the three-point function (many technical details of the construction are omitted). In the functional integral a general shift of the type (44) followed by a change of the field, made in order to maintain the invariance of the kinetic term (32), leads to a change of the coupling constant $\lambda$. Moreover, there are general statements, known as ghost-dilaton theorems telling that the ghost dilaton is the only BRST-physical state changing the string field coupling [13], [29], [30].

In particular, if for the non-polynomial bosonic string the shift $|S \rangle$ in (44) is along the ghost-dilaton:

$$\delta |\Psi \rangle \equiv |S \rangle = \epsilon |D_g \rangle,$$

then the corresponding change of the coupling constant is

$$\delta \lambda = \epsilon \lambda \tag{47}$$

([29]). The situation remains essentially the same for the $\alpha = p^+$ HIKKO theory [3].

The key point of the article [3] is that the shift which corresponds to the T-duality transformation of the background in $\alpha = p^+$ HIKKO, does not involve states which change the coupling constant (in particular it does not involve ghost dilaton states, as it was said before).

With arguments, which still remain unclear to us, it was argued in [3] that the formula for the contribution $Z^{SFT}_g$ of a string diagram with $g$ loops and $V$ vertices is given by

$$Z^{SFT}_g = \lambda^V (\sqrt{G})^g \int_{x: \text{fixed}} \mathcal{D}X e^{-S}. \tag{48}$$

The functional integral corresponds to the $Z_g(R)$ but with the zero mode integration being carried out. For the interaction of the type (15) (and only for this type, in fact) $V = 2(g - 1)$ for vacuum diagrams, and we have the correct power of $\lambda$, in agreement with Eqs. (14), (16).

## 5 Conclusions

As we have discussed at some length the simplest combination out of the metric and the ghost dilaton which transforms as a scalar, is $\phi - \frac{1}{2} \eta^{\mu\nu} h_{\mu\nu}$, so that it is tempting to conjecture that, at the full nonlinear level, there is a relationship between the ghost-dilaton and the sigma model dilaton of the type [6]

$$\Phi_\sigma = -\frac{1}{2}(\phi - \frac{1}{2} \log G). \tag{49}$$

\footnote{We would like to stress that our rather primitive methods do not allow us to consider the full spacetime dependence of the observables involved; all conjectured relationships must be understood to be valid only up to T-invariant quantities}
This relationship has previously been also considered in Refs. [3], [19], and, although we have only been able to argue for it in the linear approximation, it would reconcile string field theory with sigma model results.

If the conjecture made in (49) is true, the coupling \( \kappa \) is associated with the zero-momentum dilaton. Provided the SFT coupling \( \lambda \) comes uniquely from the ghost dilaton, as it is suggested by the discussion of the ghost-dilaton theorem in the previous section,

\[
\lambda = \mu(\phi),
\]

where \( \mu \) is some function, we would find a relation of the type

\[
\lambda = \mu \left( -2 \log \frac{\kappa}{G_{1/4}} \right) = \mu \left( -2 \log \frac{\kappa}{R^{d/2}} \right).
\]

This implies that \( \lambda \) is indeed invariant under T-duality (cf. Eq. (17)). For Eq. (48) to be equivalent to (14) the function \( \mu \) in Eq. (50) must be chosen as

\[
\lambda = \mu(\phi) = \exp \left( -\frac{1}{2} \phi \right) = \frac{\kappa}{R^{d/2}},
\]

which is quite an expected form for a relation between the dilaton and the corresponding coupling constant. The discussion in the main body of the paper then would suggest that \( \lambda \) is associated with \( \phi_{\text{inv}} \) (cf. Eq. (21)), and with \( \kappa_{26-d} \) in (31).

There is, however, a fact which we do not understand, and which may point towards some inconsistency: the transformation of the ghost-dilaton \( \delta \phi = \frac{\phi}{\lambda} \) under the shift (46), and the corresponding change of the coupling constant (17), as given by the ghost-dilaton theorem, imply that

\[
\lambda = \mu(\phi) = \frac{1}{a - \phi} = \frac{1}{a + 2 \log \frac{\kappa}{R^{d/2}}},
\]

where \( a \) is a constant. The relation of this form is rather peculiar. In particular we have weak coupling regime \( \lambda \to 0 \) in string field theory both when \( \kappa/R^{d/2} \to +\infty \) and \( \kappa/R^{d/2} \to 0 \).

Thus, there is an obvious incompatibility between Eq. (52) and Eq. (53). We feel that it would be interesting to work out further details and clarify more this issue. After all, it seems that many string dualities are in a sense a consequence of T-duality in a different context (such as compactifications of M-Theory). It is then obviously of the utmost importance to gain a knowledge of T-duality in general settings as precise as possible.

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7 It is known that in the closed string field theory, an infinite number of (genus dependent) contact terms are needed to avoid overcounting of Riemann surfaces. Detailed non-trivial calculations are obviously neccessary to check whether all these terms conspire to give our conjectured result. We are grateful to Kelly Stelle for stressing this point.

8 Let us notice in passing that eq. (14) has a form compatible with Weinberg’s general unitarity arguments.
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