Many-body effects cause unusual transport properties (TPs) deviating from the Fermi liquid (FL) [1]. For example, the T-linear in-plane resistivity, $\rho_{ab}$, and Curie-Weiss (CW) like T dependence of the Hall coefficient, $R_H$, are observed in a quasi-2D single-orbital system near an antiferromagnetic (AF) quantum-critical point (QCP) [2]. Also, such unusual TPs are observed in ruthenates (i.e., Ru oxides), quasi-2D $t_{2g}$-orbital systems: Sr$_2$RuO$_4$, located near an AF QCP, shows the T-linear $\rho_{ab}$ [3]; Ca$_{2-x}$Sr$_x$RuO$_4$ around $x = 0.5$, located near a ferromagnetic QCP, shows the $T^{1.4}$ dependence of $\rho_{ab}$ and CW like T dependence of $R_H$ [4]. Note that Sr$_2$RuO$_4$ shows the FL behaviors [5, 6].

The origins of these unusual TPs of the ruthenates are unclear, although understanding their mechanisms gives deeper insight into roles of electron correlation and orbital degrees of freedom in TPs of multi-orbital systems.

To clarify these origins, we should understand roles of electron correlation and each $t_{2g}$ orbital. In particular, it is highly desirable to reveal effects of the self-energy (SE) of electrons and electron-hole (eh) four-point vertex function (VF) due to electron correlation. One of the reasons is that for the single-orbital Hubbard model on a square lattice near an AF QCP [7] (hereafter referred to as the single-orbital case), the characteristic T and $k$ dependence of quasiparticle (QP) damping causes the T-linear $\rho_{ab}$, and the characteristic T and $k$ dependence of Maki-Thompson (MT) current vertex correction (CVC), a correction of the current due to MT four-point VF [8], the CW like T dependence of $R_H$; these T and $k$ dependence arise from the CW like T dependence of the spin susceptibility at $k = (\pi, \pi)$. The other is that these will give considerable effects even for multi-orbital systems.

In this Letter, I reveal the roles of electron correlation and each $t_{2g}$ orbital in $\rho_{ab}$ and $R_H$ of the ruthenates near and away from the AF QCP. I show the emergence of the orbital-dependent transport due to orbital-cooperative spin fluctuation (SF) around $Q_{IC-AF} \approx (0.66\pi, 0.66\pi)$ and the conditions for realizing the similar orbital-dependent transport in other systems. The obtained results agree with experiments [3, 5, 6, 9] qualitatively.

To describe the electronic structure of the ruthenates, I use the $t_{2g}$-orbital Hubbard model on a square lattice,

$$
\hat{H} = \sum_{\mathbf{k}, a, b} \sum_{s=\uparrow, \downarrow} \epsilon_{ab}(\mathbf{k}) \hat{c}_{a\mathbf{k}s}^\dagger \hat{c}_{b\mathbf{k}s} + U \sum_{\mathbf{j}} \sum_a \hat{n}_{ja\uparrow} \hat{n}_{ja\downarrow} + U' \sum_{\mathbf{j}} \sum_{a>b} \hat{n}_{ja\uparrow} \hat{n}_{jb\downarrow} - J_H \sum_{\mathbf{j}} \sum_{a>b} (2\hat{s}_{ja\uparrow} \cdot \hat{s}_{jb\downarrow} + \frac{1}{2} \hat{n}_{ja\uparrow} \hat{n}_{jb\downarrow}) + J' \sum_{\mathbf{j}} \sum_{a>b} \epsilon_{a\mathbf{j}03} \hat{c}_{a\mathbf{j}03}^\dagger \hat{c}_{b\mathbf{j}03} \hat{c}_{a\mathbf{j}03} \hat{c}_{b\mathbf{j}03},
$$

with $\epsilon_{11/22}(\mathbf{k}) = -\frac{\Delta_{2s}}{\tilde{\alpha}} - 2t_1 \cos k_x y - 2t_2 \cos k_y x - \mu$, $\epsilon_{12/21}(\mathbf{k}) = 4t' \sin k_x \sin k_y$, $\epsilon_{33}(\mathbf{k}) = -\frac{2\Delta_{2s}}{\tilde{\alpha}} - 2t_3 \cos k_x + \cos k_y - 4t_4 \cos k_x \cos k_y - \mu$, $\epsilon_{13/23/31/32}(\mathbf{k}) = 0$, $J' = J_H$, and $U' = U - 2J_H$. Hereafter, I label the $d_{xz}$, $d_{yz}$ and $d_{xy}$ orbitals 1, 2 and 3, respectively, fix the energy unit at eV, and set $\hbar = c = e = \mu_B = k_B = 1$.

The parameters in $\epsilon_{ab}(\mathbf{k})$ are chosen so as to reproduce the electronic structure of Sr$_2$RuO$_4$ obtained in local-density approximation (LDA) [10]. I set $(t_1, t_2, t_3, t_4, t' , \Delta_{0g}) = (0.675, 0.09, 0.45, 0.18, 0.03, 0.13)$ and choose $\mu$ so that the total occupation number is four; the obtained Fermi surface (FS) and density-of-states (DOS) are shown in Fig. 1. In this choice, the total bandwidth is about 4, being twice as large as the experimentally estimated value of $U$ [11], and the occu-
pation numbers of the $d_{zx}/yz$ and $d_{xy}$ orbitals, $n_{zx}/yz$ and $n_{xy}$, are 1.38 and 1.25, respectively. The inconsistency of $n_{zx}/yz$ and $n_{xy}$ with the experimental values (12) ($n_{zx}/yz = n_{xy} = 1.33$) arises from the quantitative difference that the FS sheet of the $d_{xy}$ orbital in the LDA [10] is closer to the inner sheet in $k_x = k_y$ line.

The interaction term is treated by fluctuation-exchange (FLEX) approximation [13, 14] that bubble and ladder diagrams only for el-h scattering processes are considered. This approximation is suitable for describing properties at low $T$ near a QCP since this is a perturbation theory (PT) based on a conserving approximation [15] beyond mean-field approximations (MFAs) and can treat spatial correlation appropriately [16]. By using the procedure [14] for a paramagnetic phase and taking 64$^2$ meshes of the Brillouin zone and 2048 Matsubara frequencies, I solve a set of the self-consistent equations by iteration until the relative error of the SE becomes $10^{-4}$.

I believe the FLEX approximation is suitable to analyze the TPs of the ruthenates since the magnetic and electronic properties of Sr$_2$RuO$_4$ can be well described. First, enhancing the spin susceptibility at $Q_{IC-AF}$ [Figs. 2(a) and 2(c)] agrees with experiment [16]; the main contribution comes from the $d_{xy}$ orbital in contrast to the results of MFAs [10, 17]. This enhancement arises from both the SE of electrons beyond MFAs and orbital-cooperative SF: the SE causes merging of the nesting vectors for the $d_{zx}/yz$ and $d_{xy}$ orbitals around $Q_{IC-AF}$ due to the FS deformation for the $d_{xy}$ orbital [Figs. 2(a) and 2(d)]; this merging leads to enhancing the non-diagonal term of SF at $Q_{IC-AF}$ between these orbitals; this and diagonal terms cause orbital-cooperative enhancement of SF at $Q_{IC-AF}$. Second, the larger mass enhancement [12] of the $d_{xy}$ orbital than that of the $d_{zx}/yz$ orbital is naturally reproduced due to the stronger (non-local) SF of the $d_{xy}$ orbital [Figs. 2(c) and 2(f)]. This agreement is better than for dynamical-mean-field theory (DMFT) [18]. Third, the values of $n_{zx}/yz$ and $n_{xy}$ are improved in comparison to the LDA values [10]; e.g., at $(T, U, J_R) = (0.006, 2.1, 0.35)$ or $(0.006, 1.8, 0.3)$, $(n_{zx}/yz, n_{xy})$ is $(1.36, 1.29)$ or $(1.36, 1.28)$, respectively. This improvement is similar to that of the DMFT [18].

Then, I derive $\rho_{ab}$ and $R_H$ in the weak-field limit by using the Kubo formulae and considering only the most divergent terms [19] with respect to the QP lifetime [20]. This treatment is correct in the FL and remains reasonable in the metallic phases where a PT works, although

\[
\sigma_{xx} = \frac{2}{N} \sum_{k} \sum_{(a)\neq1} \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi} \frac{\partial f(\epsilon)}{\partial \epsilon} \frac{\partial G^{A}(\epsilon)}{\partial k_{x}} G^{R}(\epsilon) G_{ab}(\epsilon) \Lambda_{x,dc}(k),
\]

and

\[
\sigma_{xy} = -\frac{1}{N} \sum_{k} \sum_{(a)\neq1} \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi} \frac{\partial f(\epsilon)}{\partial \epsilon} \frac{\partial G^{A}(\epsilon)}{\partial k_{x}} G^{R}(\epsilon) G_{ab}(\epsilon) \times \Lambda_{x,ba}(k) \Lambda_{y,dc}(k) \Lambda_{y,dc}(k).\]

Here $\sum_{\{k\}} \equiv \sum_{a,b,c,d} \text{and} \ k \equiv (k, \epsilon)$ are used, $G_{ab}^{R/A}(\epsilon) k$ is retarded/advanced Green’s function, $f(\epsilon)$ Fermi function, $\Lambda_{x,dc}^{(0)}(k)$ renormalized band velocity,

\[
\Lambda_{x,dc}^{(0)}(k) = \frac{\partial G_{ab}^{(0)}(\epsilon)}{\partial k_{x}} + \frac{\partial G_{bc}^{(0)}(\epsilon)}{\partial k_{y}},
\]

and $\Lambda_{y,dc}(k)$ renormalized current,

\[
\Lambda_{y,dc}(k) = \frac{\partial G_{ab}^{(0)}(\epsilon)}{\partial k_{y}} + \Delta \Lambda_{y,dc}^{(AC-A)}(k),
\]

where $\sum_{\{k\}} \equiv \text{the retarded SE, and} \ \Lambda_{x,dc}(k)$ renormalized current,

\[
\Lambda_{x,dc}^{(0)}(k) = \frac{\partial G_{ab}^{(0)}(\epsilon)}{\partial k_{x}} + \frac{\partial G_{bc}^{(0)}(\epsilon)}{\partial k_{y}},
\]

and $\Lambda_{y,dc}(k)$ renormalized current,

\[
\Lambda_{y,dc}^{(0)}(k) = \frac{\partial G_{ab}^{(0)}(\epsilon)}{\partial k_{y}} + \Delta \Lambda_{y,dc}^{(AC-A)}(k),
\]

where $\sum_{\{k\}} \equiv \text{the retarded SE, and} \ \Lambda_{x,dc}^{(0)}(k)$ renormalized current,

\[
\Lambda_{x,dc}^{(0)}(k) = \frac{\partial G_{ab}^{(0)}(\epsilon)}{\partial k_{x}} + \frac{\partial G_{bc}^{(0)}(\epsilon)}{\partial k_{y}},
\]

and $\Lambda_{y,dc}(k)$ renormalized current,

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\Lambda_{y,dc}^{(0)}(k) = \frac{\partial G_{ab}^{(0)}(\epsilon)}{\partial k_{y}} + \Delta \Lambda_{y,dc}^{(AC-A)}(k),
\]

where $\sum_{\{k\}} \equiv \text{the retarded SE, and} \ \Lambda_{x,dc}^{(0)}(k)$ renormalized current,

\[
\Lambda_{x,dc}^{(0)}(k) = \frac{\partial G_{ab}^{(0)}(\epsilon)}{\partial k_{x}} + \frac{\partial G_{bc}^{(0)}(\epsilon)}{\partial k_{y}},
\]

and $\Lambda_{y,dc}(k)$ renormalized current,

\[
\Lambda_{y,dc}^{(0)}(k) = \frac{\partial G_{ab}^{(0)}(\epsilon)}{\partial k_{y}} + \Delta \Lambda_{y,dc}^{(AC-A)}(k),
\]
We turn to results of $\rho_{ab}$ and $R_H$. Several quantities as a function of $\epsilon$ are calculated by the Padé approximation\(\text{[25]}\) using the data for the lowest four Matsubara frequencies. The $\epsilon$ and $\epsilon'$ integrations are done by discretizing the interval 0.0025 and replacing the upper and lower values by 0.1 and $-0.1$; these are sufficient to obtain results without a few percent error. $\Lambda_{c\delta c}(k)$ is calculated by iteration until its relative error is less than $10^{-4}$; the singularity of the principal integral for the term of $\coth \frac{\omega_{\delta c}}{2T}$ is removed by the $\epsilon'$ derivatives of its numerator and denominator by using $\text{Im} \left[ V_{ab}(k) \right] = 0$.

We first compare $\rho_{ab}$ at $U = 2.1$ and 1.8 in Figs. 3(a) and 3(b); I consider $U = 2.1$ (1.8) case near (away from) the AF QCP since $\chi^S(Q_{IC-AF}, 0)$ shows the CW like (Pauli paramagnetic) $T$ dependence [Fig. 3(c)]. $\rho_{ab}$ with or without the MT CVC is roughly proportional to $T^2$ at $U = 1.8$ and to $T$ at $U = 2.1$. Thus, the power of the $T$ dependence of $\rho_{ab}$ is determined by the SE and becomes one near the AF QCP.

To reveal role of each $t_{2g}$ orbital in $\rho_{ab}$, orbital components of $\sigma_{xx}$ without or with the MT CVC at $U = 2.1$ are shown in Fig. 3(d) or 3(e); the component of the $d_{xz}$ and $d_{yz}$ orbitals or $d_{xy}$ orbital is calculated from the equation that $\sum_{\{a\}=1}$ in Eq. (2) is replaced by $\sum_{\{a\}=1}$ or $\sum_{\{a\}=3}$, respectively. The main contribution to $\sigma_{xx}$ ($\sigma_{yy}$) comes from the $d_{xz}$ ($d_{yz}$) orbital in contrast to that to the magnetic fluctuation. This result arises from the smaller QP damping and larger renormalized band velocity of the $d_{xz}/yz$ orbital than those of the $d_{xy}$ orbital.

I should remark that the FL description for the $d_{xz}$ orbital is violated at $(T, U) = (0.006, 2.1)$ since $\gamma_{d_{xz}}^\ast(k)$ at $k \approx (0.66\pi, 0)$ and $(0.66\pi, 0.66\pi)$ are not much smaller than $T = 0.006$ [Fig. 4(a)]; in contrast, the FL description for that orbital is possible at $(T, U) = (0.006, 1.8)$ [Fig. 4(b)]. (The FL description is possible if the QP damping on the Fermi level is much smaller than temperature considered\(\text{[24]}\). Thus, the origin of the $T$-linear $\rho_{ab}$ near the AF QCP is the non-FL like QP damping of the $d_{xz}$ orbital near the Fermi level. I emphasize that this $T$-linear $\rho_{ab}$ is not due to a breakdown of PTs.

We next compare $R_H$ at $U = 2.1$ and 1.8 in Fig. 5(a). Four properties should be noted. First, the difference between $R_H$ without the MT CVC at $U = 2.1$ and 1.8 is small, although the QP dampings are different. This results from the small effects of the QP damping since its effects on $\sigma_{xy}/H$ and $\sigma_{zz}^2$ are nearly cancelled out. Second, the values of these $R_H$ are nearly zero. Third, the MT CVC causes peak at low $T$ and negative enhancement towards low $T$ at $U = 2.1$ and 1.8. Fourth, the shift of the peak position to higher $T$ and larger negative enhancement are obtained at $U = 2.1$.

To understand the second, third, and fourth properties, I present orbital components of $\sigma_{xy}/H$ without or with the MT CVC at $U = 2.1$ in Fig. 5(b) or 5(c); these are calculated in a similar way to $\sigma_{xx}$. There are four remarks. (i) The sign of the component of the $d_{xz}$ and $d_{yz}$ orbitals is minus, and that of the $d_{xy}$ orbital is plus except the case with the MT CVC at $(T, U) = (0.006, 2.1)$. (ii) The components of the $d_{xz}$ and $d_{yz}$ orbitals and $d_{xy}$ orbital without the MT CVC are nearly the same in magnitude. Thus, the nearly zero $R_H$ without the MT CVC arises from the comparable and opposite-sign components of the $d_{xz}$ and $d_{yz}$ orbitals and $d_{xy}$ orbital. (iii) The component of the $d_{xy}$ orbital with the MT CVC shows the similar peak to that of $R_H$. (iv) The magnitude decrease for the $d_{xy}$ orbital due to the MT CVC is larger. These imply the peak and negative enhancement of $R_H$ are related to the different effects of the MT CVC on the currents of the $t_{2g}$ orbitals.

Furthermore, I analyze how the MT CVC affects the current of each orbital. Since its kernel contains the
FIG. 6: (Color online) (a)–(d) ImV_{ab}(q) at q = Q_{IC-AF} and Q'_{IC-AF}. (e) schematic picture of the currents connected by the MT CVC, (i) σ_{xy}(k)/H with the MT CVC at (T, U) = (0.006, 2.1), T dependence of (g)–(j) several orbital components of partially integrated σ_{xy}(k)/H at U = 2.1 and of (k) σ_{xy}/H for several special cases whose data are obtained by using part of ImV_{abc}(q) as the CVC, and (l) schematic picture of the most important el-h scattering process in determining R_H.

QP damping and ImV_{abc}(q). I present ImV_{abc}(q) at q = Q_{IC-AF} and Q'_{IC-AF} ≈ (π, 0.66π) in Figs. (a), (d). (The other orbital components are much smaller.) The largest, second largest, and third largest terms are ImV_{xy}(Q_{IC-AF}, ω), ImV_{xy}(Q'_{IC-AF}, ω), and ImV_{xy}(Q''_{IC-AF}, ω), respectively. Thus, the main effects of the MT CVC are (i) the magnitude decrease of the current of the d_{xy} orbital around Q_{IC-AF} due to ImV_{xy}(Q_{IC-AF}) around q = Q_{IC-AF}, (ii) the angle changes of the currents of the t_{2g} orbitals around Q_{IC-AF} due to ImV_{xy}(Q_{IC-AF}) around q = Q_{IC-AF}, and (iii) the magnitude decreases and angle changes of the currents of the d_{xz} and d_{yz} orbitals around Q_{IC-AF} due to ImV_{xy}(Q_{IC-AF}) around q = Q_{IC-AF} [Fig. (e)]; that angle change for the d_{xy} orbital is zero at k_x = k_y due to the cancellation of the contributions from the d_{xz} and d_{yz} orbitals. Also, ImV_{xy}(Q_{IC-AF}) [ImV_{xy}(Q''_{IC-AF})] around q = Q_{IC-AF} ≈ (0.66π, π) (symmetrically equivalent to Q'_{IC-AF}) causes the angle change of the current of the d_{xy} [d_{xz}] orbital around k ≈ (0.88π, 0) [(0.72π, 0)] [Fig. (e)].

Among these effects, the angle change of the current of the d_{xy} orbital around Q_{IC-AF} is most important in discussing R_H. One of the evidences is that the most drastic effect of the MT CVC on σ_{xy}/H is a sign change and negative enhancement of d_{xy} component of \( \frac{1}{N} \sum_{k_x, k_y \leq 2\pi q} \sigma_{xy}(k)/H \) (and symmetrically equivalent ones) [Figs. (f), (i)]. The other is that the T dependence of d_{xy}/H with the MT CVC is almost reproduced by the MT CVC of ImV_{xy}(Q=Q_{IC-AF}) [Fig. (k)]; the small difference comes from an increase due to the MT CVC of ImV_{aaaa}(q). Thus, the MT CVC arising from orbital-cooperative SF around Q_{IC-AF} between the d_{xz/yz} and d_{xy} orbitals [Fig. (l)] is most important.

The less considerable effect of the angle change of the current of the d_{xz/yz} orbital around Q_{IC-AF} than that of the d_{xy} orbital results from the smaller value of the above MT CVC for the d_{xz/yz} orbital than that for the d_{xy} orbital; the former (latter) is proportional to ImV_{xy}(Q_{IC-AF}, Q''_{IC-AF}) around Q_{IC-AF} and magnitude of the current and inverse of the QP damping of the d_{xy} (d_{xz/yz}) orbital. This difference is due to a multi-orbital effect since the currents connected by the diagonal terms of CVCs are always the same in magnitude.

Combining these results, we find the origins of the peak and negative enhancement of R_H with the MT CVC: the former arises from the large negative enhancement of σ_{xy}(k)/H of the d_{xy} orbital around k = Q_{IC-AF} compared with the positive enhancement of that around k ≈ (0.88π, 0), and the latter the larger negative enhancement of that at low T due to increasing ImV_{abc}(q) and decreasing the QP damping. Also, the shift of the peak position to higher T and larger negative enhancement at U = 2.1 arise from increasing ImV_{xy}(Q_{IC-AF}, Q''_{IC-AF}), which is more important in enhancing the MT CVC than decreasing the QP damping.

I should note that the more drastic effects of the MT CVC on R_H than those on ρ_{ab} result from the fact that within the leading order of the angle change, Δφ_{ab}(k) = φ_{ab}(k) − φ_{ab}^{(0)}(k), σ_{xz} and σ_{xy}/H contain, respectively, \( |Δ_{ba}(k)| \cos φ_{ba}^{(0)}(k) |Δ_{dc}(k)| \cos φ_{dc}^{(0)}(k) [1 − 0.5Δφ_{dc}(k)]^2 \) and \( |Δ_{ba}(k)| \cos φ_{ba}^{(0)}(k) |Δ_{dc}(k)| \cos φ_{dc}^{(0)}(k) \frac{∂φ_{dc}(k)}{∂k_y} + |Δ_{ba}(k)| \sin φ_{ba}^{(0)}(k) \frac{∂φ_{ba}(k)}{∂k_y} |Δ_{dc}(k)| \sin φ_{dc}^{(0)}(k) \); for R_H (ρ_{ab}), the angle (magnitude) change is more important than the magnitude (angle) change.

We now compare the results with experiments. ρ_{ab} and R_H with the MT CVC at U = 1.8 agree with experiments of SrRuO_3 [6, 7]; the T-square ρ_{ab}, monotonic increase of R_H in 0.007 ≤ T ≤ 0.02, and peak of R_H at T ∼ 0.007 are reproduced. Since the quantitive difference in R_H (e.g., the value of T where R_H is zero, T ∼ 0.014 in an experiment [8]) is related to the difference in n_{xz/yz} and n_{xy}, an analysis by the model having the same occupation numbers is a future work. Note that although a phenomenological theory [22] in relaxation-time approximation (which neglects the effects of the SE and el-h four-point VF on the current) can reproduce the experimental
results of Sr_2RuO_4 by choosing some parameters of the QP damping, I do not use any such parameters. Then, \( \rho_{ab} \) and \( R_H \) with the MT CVC at \( U = 2.1 \) agree with experiments of Sr_2Ru_{0.075}Ti_{0.025}O_4 \[8,9\]: the \( T \)-linear \( \rho_{ab} \), minus-sign \( R_H \) at low \( T \), and larger \(|R_H|\) at low \( T \) than for Sr_2RuO_4 are reproduced.

Finally, we discuss the possibility of the similar orbital-dependent transport in other multi-orbital systems. First, the difference between the main orbitals of \( \rho_{ab} \) and magnetic fluctuation will be realized in metallic phases since this results from the facts that \( \sigma_{xx} \) is inversely proportional to the QP damping, and that the strong magnetic fluctuation enhances the QP damping. Then, the enhancement of \( R_H \) due to a multi-orbital effect will be realized in some metallic phases (e.g., Ca_{2−x}Sr_2RuO_4 \[4\] around \( x = 0.5 \) and CeCoIn_5 \[20\]) satisfying three conditions: the dimensionalities of the orbitals are different; several FS sheets are located near each other; electron correlation is strong. If the first and second conditions are satisfied, there are several nesting vectors being nearly the same. Then, strong electron correlation causes merging of these nesting vectors due to the FS deformation, resulting in enhanced orbital-cooperative SF whose \( T \) dependence is CW like near a QCP. Since the MT CVC of the non-diagonal term of this SF connects the currents of the orbitals whose dimensionalities are different and causes the angle changes of the currents, \( R_H \) is enhanced. A peak of \( R_H \) is also induced when these conditions are satisfied and opposite-sign contributions exist.

In summary, I have studied \( \rho_{ab} \) and \( R_H \) of the ruthenates near and away from the AF QCP by the \( t_{2g} \)-orbital Hubbard model on a square lattice in the FLEX approximation including the MT CVC. I have shown that \( \rho_{ab} \) is determined mainly by the component of the \( d_{x^2−y^2} \) orbital and shows the \( T \)-linear dependence near the AF QCP. Also, I have shown that \( R_H \) shows the peak at low \( T \) and negative enhancement towards low \( T \) as the result of the sign change and negative enhancement of \( \sigma_{xy}(k)/H \) for the \( d_{xy} \) orbital around \( k = Q_{1C-AF} \) due to the MT CVC of the non-diagonal term of orbital-cooperative SF around \( Q_{1C-AF} \) between the \( d_{x^2−y^2} \) and \( d_{xy} \) orbitals; near the AF QCP, the peak position is located at higher \( T \) and the negative enhancement is larger. These observations clarify the mechanism for the orbital-dependent transport of the ruthenates and the conditions for realizing the similar orbital-dependent transport in other systems.

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