Hamiltonian Based nRules
– Time’s Arrow

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Abstract
The auxiliary rules of quantum mechanics can be written without the Born rule by using what are called the nRules. The nRules are understood in part by making certain modifications in the Hamiltonian. In this paper, those modifications are written directly into the nRules, reducing their number from four to three. It is shown that the nRules in either form provide for a definite direction in time, guaranteeing that a statistically irreversible interaction is absolutely irreversible.

Introduction
It is shown in another papers [1, 2] that the Born rule (relating probability with square modulus) and the other auxiliary rules of standard quantum mechanics are disposable. In their place two other rule-sets have been proposed (called the nRules and the oRules) that introduce probability into quantum mechanics through probability current alone. There is no attempt to explain these rules. The strategy has been to show that these two rule-sets work over a wide range of examples without attempting a theoretical justification. We assume that a theory will one day be found to cover the auxiliary rules of quantum mechanics, so it is important to have the right rules.

The Schrödinger solutions are continuous in all variables for the most part; but occasionally, there is a discontinuity in one or more variables (except time) that leads to a quantum jump. An example is the change that takes place when an electron drops from a higher atomic orbit to a lower one, discontinuously emitting a photon in the process. When a system encounters a quantum jump of this kind, the Hamiltonian in standard quantum mechanics is $H = H_0 + $
\[ H_{01} + H_1, \text{ where } H_0 \text{ drives the original system } S_0, \ H_1 \text{ drives the final system } S_1, \text{ and } H_{01} \text{ drives the interaction between the two.} \]  

The separation of the Hamiltonian into these parts is possible because the variables are indexed to distinguish systems \( S_0 \) and \( S_1 \). However, a truncated Hamiltonian equal to \( H = H_0 + H_{01} \) was found in Ref. 1 to be required by the nRules. This has the desired effect of ‘bridging’ the discontinuous quantum gap; and at the same time, forbidding any further evolution until there has been a stochastic hit on the original system. When that happens, the system acquires new boundary conditions that launch the new state \( S_1 \). This abrupt change of state (from \( S_0 \) to \( S_1 \)) is the “collapse” of the wave function.

Adopting the truncated Hamiltonian enforces nRule (4) – the fourth of the four nRules; so if the Hamiltonian were directly referred to in the auxiliary nRules, then nRule (4) would be unnecessary. When this is done (below), the total number of nRules is reduced from four to three. Where a distinction between the new three-rule rule-set and the previous four-rule rule-set is desirable, they will be designated nRules\(^3\) and nRules\(^4\) respectively.

Finally, it is shown that the nRules\(^3\) exhibited below insure that probability current can only flow from the low entropy side of an irreversible interaction to the high entropy side. Thermodynamics claims that a reverse process is very improbable, but the nRules\(^3\) insure that a current reversal of this kind will not occur across any irreversible quantum gap. Therefore, they guarantee the omnidirectional of time’s arrow. The same guarantee is shown (below) to follow from the original nRules\(^4\) that are developed in previous papers.

**The nRules\(^3\)**

They are:

**nRule\(^3\) (1):** If an irreversible interaction connects complete components that are discontinuous with each other in some variable, then the high entropy component will not be driven by its own Hamiltonian.

[**note:** Complete components contain all of the symmetrized objects in the universe. Each included object is itself complete in that it is not a partial expansion in some representation. The entropy change across a discontinuous gap is therefore the entropy change of the entire universe.]

[**note:** A complete component’s own Hamiltonian is that part of the Hamiltonian whose variables are exclusively those of the component. These variables only intermingle with the variables of another complete component in
the ‘interaction’ part of the Hamiltonian.]

\textbf{nRule}^3 (2): For a system of total square modulus $s$ that has $n$ launch components, a stochastic trigger will choose stochastically from among them. The probability per unit time of such a choice among $m$ of these components at time $t$ is given by $(\Sigma mJ_m)/s$, where the square modular current $J_m$ flowing into the $m$th component at that time is positive.

\textit{note:} A launch component is the high entropy component of an irreversible and discontinuous gap.

\textbf{nRule}^3 (3): If a launch component is stochastically chosen, it will be driven by the entire Hamiltonian of the new solution. All other components in the original superposition will be immediately reduced to zero.

\section*{Time’s Arrow}

Let probability current flow across the discontinuous gap between components $C_0$ and $C_1$ in the first row of Eq. 1, where component $C_0$ has a lower entropy than $C_1$.

\begin{align*}
\Phi(t \geq t_0) &= C_0(t_0) \rightarrow C_0(t > t_0) + C_1(t > t_0) \quad (1) \\
\Phi(t > t_{sc}) &= C_1(t = t_{sc}) \rightarrow C_1(t > t_{sc})
\end{align*}

The first row represents the time between $t_0$ and the time $t_{sc}$ of a stochastic hit, and the second row is the collapsed wave function after a stochastic hit on $C_1$. The higher of two entropy components is underlined in this treatment. The arrow represents a continuous evolution and the $+$ sign is a discontinuous evolution in some variable. According to thermodynamics, the direction of flow in Eq. 1 will most probably be from left to right.

Now consider how this equation would look if the direction of the current were reversed. First, suspend nRule$^3$ (1) to give

\begin{align*}
\Phi(t \geq t_0) &= C_0(t_{sc} > t) \leftarrow C_0(t > t_0) + C_1(t \geq t_0) \leftarrow C_1(t_0) \quad (2)
\end{align*}

Both $C_1$ components in this equation are the same component at different times. In that case there is no stochastic hit because there is no probability current flowing into $C_1$ as required by nRule$^3$ (2). If we now restore Rule$^3$ (1), then the continuous evolution $C_1(t > t_0) \leftarrow C_1(t_0)$ cannot occur because the component will not be driven by its own Hamiltonian; so Eq. 2 cannot occur. This means that probability current can only flow from left to right as shown in Eq. 1, going
from the lower to the higher entropy across the discontinuous gap. Thermo-
dynamics says that a reverse flow is very improbable, but the nRules\textsuperscript{3} say it is impossible. An unambiguous direction of time is thereby established in the direction of higher entropy in a quantum mechanical system in which stochastic choices occur.

If the system is governed by the auxiliary nRules\textsuperscript{4} found in Refs. 1 and 2, its forward evolution will be the same as that given in Eq. 1, where the underlined components are now called ‘ready’ components. The wording of nRule\textsuperscript{4} (1) is not optimal for our present purpose because it contains the word “predecessor” or “initial”. However, the high and low entropy language of nRule\textsuperscript{3} (1) can easily be substituted.

The backward evolution of the system is again described in Eq. 2 when nRule\textsuperscript{4} (4) is suspended. As before, both $C_1$ components are the same component at different times. If nRule\textsuperscript{4} (4) is restored, then Eq. 2 cannot exist because current cannot flow from $C_1(t_0)$ to $C_1(t > t_0)$, or from $C_1(t > t_0)$ across the gap to $C_0(t > t_0)$. Therefore, only the forward flow of current in Eq. 1 is possible. Again, an unambiguous direction of time is established in the direction of higher entropy in a quantum mechanical system in which stochastic choices occur.

References

[1] R. A. Mould, “Without the nRules”, arXiv: quant-ph/0507170

[2] R. A. Mould, “Auxiliary nRules of Quantum Mechanics”, arXiv: quant-ph/0505231