Collective Neutrino Flavor Instability Requires Spectral Crossing

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We prove that a spectral crossing is necessary for any collective instability in the flavor evolution of ultra-relativistic Standard Model neutrinos. Our argument applies for any number of flavors, for both slow and fast instabilities, and also in the presence of damping due to collisions. This result provides a simple but rigorous condition for collective flavor transformations that are believed to be important for stellar dynamics, nucleosynthesis, and neutrino phenomenology.

I. INTRODUCTION

Supernovae and neutron star mergers produce enormous numbers of neutrinos that carry away a bulk of energy. These neutrinos travel through the dense material of the star, and crucially influence stellar dynamics and nucleosynthesis [1–6]. The precise impact can depend on the flavor states of the neutrinos, because the different flavors interact with the background medium with unequal interaction rates. Naturally, a characterization of the neutrino flavor evolution in such environments is of interest and importance.

The flavor evolution of dense neutrino clouds can be very complex [7–9]. This complexity arises because neutrinos forward scatter off each-other, and influence each-other’s flavor evolution. As a result, the flavor evolution of such neutrinos and antineutrinos is nonlinear, and remains to be fully understood. Neutrinos produced in the core of these sources initially remain trapped due to frequent collisions. They leak out via diffusion, before eventually free-streaming. As the density of background matter is high, flavor-mixing is suppressed due to forward and non-forward scatterings [10–12]. However, owing to neutrino-antineutrino interactions, large collective flavor conversion can occur even for vanishingly small mixing because they occur due an instability.

In the last two decades, much insight has been obtained into the origin and impact of a variety of collective flavor transformations. Collective flavor transformations stem from neutrino-antineutrino forward scatterings [13, 14], and come in three variants. The simplest type are synchronized, occurring for all neutrino energies with the average oscillation frequency $\langle \omega_E \rangle$, but not seeded by an instability [15]. These are usually suppressed in dense matter, though there is also the possibility of synchronized resonance that can cause large effects [16]. The next type are slow instabilities, leading to evolution with a frequency proportional to $(\omega_E G_F n_\nu)^{1/2}$ [17–22], where $n_\nu$ is the neutrino density. At heart, these are analogous to the tipping of an inverted pendulum [22–24]. Despite being dubbed slow, they are faster than the usual neutrino oscillations in vacuum/matter or even the synchronized oscillations because $n_\nu \gg G_F^{-1} \langle \omega_E \rangle$ deep in the star. As the neutrino density drops below $G_F^{-1} \langle \omega_E \rangle$, typically at a radius of a few $\times 100$ km in a supernova, these slow instabilities tend to produce a spectral swap of two flavors across broad range of energies [23, 24]. The edges of these swaps could appear as sharp spectral splits in the energy spectrum and potentially observable in the signal reaching Earth. Finally, there are fast instabilities that cause flavor evolution with a very high frequency proportional to $G_F n_\nu$ [25–31]. These can occur very deep in a star at radii of few $\times 10$ km or so [32–34], thus possibly impacting stellar heating and nucleosynthesis in a more nontrivial fashion. Fast instabilities correspond to rolling down a quartic potential [35, 36]. The eventual impact of these fast instabilities is not fully established yet, but a number of studies hint that they cause partial flavor equilibration in some range of neutrino velocities for all energies [37–39]. This is called depolarization [37], and may be a key observable of fast instability.

A fundamental problem is to determine the conditions that lead to collective instability. A belief, held at least by this author, is that instabilities occur only if the difference of the phase space distribution of two neutrino flavors changes sign at some momentum [24, 28]. The importance of such spectral crossings was first pointed out in a study of multiple spectral splits [24]. Several subsequent investigations have further strengthened this notion for slow [40] and fast [30, 41–43] instabilities. Recently, a proof was proposed for necessary and sufficient condition for a fast instability [44].

In this paper we show that spectral crossings are necessary for any collective instability. The argument is agnostic to whether the instability is slow or fast, to the number of neutrino flavors, and to whether damping due to collisions are present. Our hope is that this provides a simple yet rigorous foundation for studies of collective flavor transformations that may be important for stellar dynamics [45, 46], nucleosynthesis [47–49], and neutrino phenomenology [50]. The paper is organized as follows: In Sec. II, we first derive a linearized version of the neutrino flavor evolution equation including collisions. Using the linearized equation, in Sec. III we present a proof of the above claim. We conclude with a summary and some remarks in Sec. IV.
II. DISPERSION RELATION WITH DAMPING

We consider scenarios where the flavor-dependent occupation matrix for a neutrino evolves as

\[ \nu^\alpha \partial_\alpha \psi_p = -i [\mathbf{H}_p, \psi_p] + C_p, \]

where a summation over the spacetime indices \( \alpha = 0, \ldots, 3 \) is implied. \( \psi_p \) and \( \bar{\psi}_p \) are the \( 3 \times 3 \) occupation number matrices for neutrinos and antineutrinos, and \( \mathbf{H}_p \) and \( C_p \) are the Hamiltonian and collision matrices, respectively. The problem is nonlinear despite appearances because \( \mathbf{H}_p \) contains terms involving \( \psi_p \) and \( \bar{\psi}_p \), as does \( C_p \). The EOM for the antineutrino matrices \( \bar{\psi}_p \) is the same except for a sign-change in the mass-mixing term in \( \mathbf{H}_p \), i.e., the first term on right hand side (rhs) of equation (4). A detailed derivation stating the underlying assumptions and approximations can be found in refs. \([51, 52]\).

In the following, we will derive a linearized equation for the off-diagonal elements \( \bar{\psi}_{ij}^\mu \) in the flavor basis, where \( i, j \in \{e, \mu, \tau\} \) in the usual three-flavor scenario. Our derivation remains essentially unchanged from ref. \([53]\), except for the inclusion of collisions.

We take the collision term \( C_p \) to have the form \([54, 55]\)

\[ C_p = -\frac{G_F^2}{2} \sum_a \left( \{ C_{a,p}, \psi_p - \bar{\psi}_p^{eq} \} - 2 C_{a,p} (\psi_p - \bar{\psi}_p^{eq}) C_{a,p} \right), \]

where \( \bar{\psi}_p^{eq} \) is a diagonal equilibrium occupation matrix and \( C_{a,p} \) is a diagonal matrix encoding the collision rate for the interaction channel \( a \). For any pair of neutrino flavors, say \( e \) and \( \mu \), one finds explicitly

\[ C_p^{\mu} = -|\Delta_p^{\mu}| \bar{\psi}_p^{\mu}, \]

where the damping rate \( |\Delta_p^{\mu}| = c^{\mu} G_F^2 |E|^{\delta} \) with \( c^{\mu} \) being a non-negative numerical constant \([56, 57]\), and \( |E| = |p| \) for ultra-relativistic neutrinos that we shall assume. Similarly for any other pair of flavors. In this approximation, collisions lead to damping of the off-diagonal elements, as would be the case for charged-current production and annihilation, etc. See also \([58–60]\). We say more about other collisions in Sec. IV.

The Hamiltonian matrix \( \mathbf{H}_p \) has the usual contributions from neutrino mass-mixing as well as the refractive effects of other neutrinos and background leptons

\[ \mathbf{H}_p = \frac{M^2}{2E} + \mathbf{H}_p^{\nu} + H_p^{\text{bkg}}. \]

Explicitly, the neutrino-neutrino refractive term has the form \( \mathbf{H}_p^{\nu} = \sqrt{2} G_F \nu \bar{\nu} F_\nu^\nu \) with the neutrino flux matrix \( F_\nu^\nu = \int dp u^\nu (\psi_p - \bar{\psi}_p) \), where \( u^\nu = (1, \nu) \) is the neutrino four-velocity with \( \nu = \nu/|E| \). The ordinary matter contribution is \( H_p^{\text{bkg}} = \sqrt{2} G_F v_n F_{\text{bkg}} \), which is diagonal and has the elements \((F_{\text{bkg}})^{\alpha\beta} = \int 2 dp u^\alpha (f_i,p - f_i,p)\), for the \( i \text{th} \) charged lepton with phase space distribution \( f_i,p \) and a four-velocity \( u_i^\alpha = (1, p/(p^2 + m_i^2))^{1/2} \). With only at-rest electrons in the background, one finds the familiar matter potential diagonal \((2\sqrt{2} G_F v_n, 0, 0)\) for three flavors. The expression used here is more general and includes other (anti)leptons as well as their currents. The mass-mixing term does not depend on \( \nu \) and the refractive term does not depend on \( E \), but only on \( \nu \). We may define an overall matter effect caused by both neutrinos and charged leptons as

\[ \mathbf{H}_{\text{matter, eff}} = \nu_0 \Lambda^\alpha, \]

where \( \Lambda^\alpha = \text{diag}(\Lambda^e, \Lambda^\mu, \Lambda^\tau) \) to represent the diagonal part of \( \sqrt{2} G_F (F_{\text{bkg}} + F_\nu)^\alpha \).

In the limit of vanishing neutrino mixing, the linearized EOMs for the off-diagonal elements of \( \psi_p \) (and their complex conjugates) decouple, leading to equations of the form

\[ i \nu^\alpha \partial_\alpha \bar{\psi}_p^{\mu} = -\frac{M_{\nu e}^2 - M_{\nu \mu}^2}{2E} \bar{\psi}_p^{\mu} + i |\Delta_p^{\mu}| + \nu_0 (\Lambda_e - \Lambda_\mu)^\alpha \bar{\psi}_p^{\mu} - \sqrt{2} G_F (f_{\nu e,p} - f_{\nu \mu,p}) \int dp \nu_0 \left( \bar{\psi}_p^{\mu} - \bar{\psi}_p^{\mu(\bar{\psi}_p^{\mu})} \right), \]

and analogous for the other pairs of flavors.

In this approach the three-flavor system corresponds to three independent two-flavor cases. There are three nontrivial cases only if the distributions of the three flavors are different, as recently considered \([61]\). Extension to more than three flavors is obvious.

All flavor coherence effects depend only on the difference of the original neutrino distributions and the diagonal parts of all matrices in flavor space drop out. In particular, we may write the effective two-flavor neutrino matrices of occupation numbers in the form

\[ \bar{\rho}_p^{\mu} = \frac{f_{\nu e,p} + f_{\nu \mu,p}}{2} \pm \frac{f_{\nu e,p} - f_{\nu \mu,p}}{2} \left( S_p^{\mu} - S_p \right), \]

whose off-diagonal element equals \( \bar{\rho}_p^{\mu} \), where \( S_p \) is a real number, \( S_p \) a complex one, and \( S_p^2 + |S_p|^2 = 1 \). To linear order in \( |S_p| \), one has \( s_p = 1 \), so in our linearized study we focus on the space-time evolution of \( S_p \) alone which holds all the information concerning flavor coherence.

Defining the two-flavor matter effect through \( \Lambda^\alpha = (\Lambda_e - \Lambda_\mu)^\alpha \), the vacuum oscillation frequency through \( \omega_E = (M_{\nu e}^2 - M_{\nu \mu}^2)/(2E) \), and the damping as \( |\Delta_p| \), the EOM in equation (6) becomes

\[ i \nu^\alpha \partial_\alpha S_p = (\omega_E + i |\Delta_p|) S_p - \nu^\alpha \int dp \nu_0 \left( S_p^{\mu} g_p^{\nu} - S_p^{\nu} g_p^{\mu} \right). \]

An analogous equation applies to the antineutrino flavor coherence \( S_{\bar{\nu}} \) with a sign-change of \( \omega_E \). Here we use the spectrum \( g_p = \sqrt{2} G_F (f_{\nu e,p} - f_{\nu \mu,p}) \) and \( \bar{g}_p = \sqrt{2} G_F (f_{\nu e,p} - f_{\nu \mu,p}) \), where we have absorbed \( \sqrt{2} G_F \) for notational convenience.
These equations become more compact and physically transparent in a convention where we interpret antiparticles as particles with negative energy and describe their spectrum with negative occupation numbers. Thus the modes are labeled by $-\infty < E < +\infty$ and their direction of motion $v$ with $p = |E|v$. The two-flavor spectrum is

\[ g_r = \sqrt{2G_F} \left\{ \int f_{\nu,\mu} - f_{\bar{\nu},\bar{\mu}} \right\} \text{ for } E > 0, \]
\[ \int f_{\nu,\mu} - f_{\bar{\nu},\bar{\mu}} \right\} \text{ for } E < 0, \]

with $\Gamma = \{E, v\}$. There is no sign-change in the definition of $S$. The EOM thus reads

\[ \left( v^\alpha (i \partial_\alpha - \Lambda_\alpha) - \omega_E + i|\Delta_\Gamma| \right) S_{\Gamma,\kappa} = -v^\alpha \int d\Gamma' v'^\alpha g_{\Gamma'\Gamma} S_{\Gamma'}, \]

where the phase-space integration is over

\[ \int d\Gamma = \int_{-\infty}^{+\infty} \frac{E^2 dE}{2\pi^2} \int \frac{dv}{4\pi}, \]

with $\int dv$ an integral over the unit sphere, i.e., over all polar angles of $p$.

The vacuum oscillation frequency $\omega_E$, in this convention, automatically changes sign for antineutrinos. For positive $E$, it is positive for inverted mass ordering ($M^2_{ee} > M^2_{\mu\mu}$) and negative for the normal mass ordering ($M^2_{ee} < M^2_{\mu\mu}$).

As usual, for a linear EOM we search for space-time dependent solutions of equation (10) in terms of its independent Fourier components

\[ S_{\Gamma,\kappa} = \sum_K Q_{\Gamma,K} e^{-i(K\kappa - \kappa \cdot r)}, \]

where $\Gamma = \{E, v\}$, $R^\mu = (t, r)$ and $K^\mu = (K^0, \mathbf{K})$. The quantity $Q_{\Gamma,K}$ is the eigenvector in $\Gamma$-space for the eigenvalue $K$. To find the eigenmodes we insert the ansatz of equation (12) into equation (10) and find

\[ (v^\alpha k^\alpha - \omega_E + i|\Delta_\Gamma|) Q_{\Gamma,K} = v^\alpha A^\alpha_k, \]

where $A^\alpha_k = -\int d\Gamma v^\alpha g_{\Gamma\Gamma} Q_{\Gamma,K}$ and $k^\alpha = K^\alpha - \Lambda^\alpha$. Fully analogous to the fast-flavor case, we have shifted the original four-vector, $K^\alpha$, to the redefined four-vector, $k^\alpha = (k^0, \mathbf{k})$, by subtracting the matter-effect four-vector $A^\mu$. Solving the EOMs in Fourier space allows the diagonal parts of all matter effect to be included as an origin-shift in the four-vector space.

In the absence of neutrino-neutrino interactions, the rhs of equation (13) vanishes and nontrivial solutions require $v^\alpha k^\alpha - \omega_E + i|\Delta_\Gamma| = 0$, i.e., the propagation relation $Re k^0 - v \cdot k = \omega_E$ and the damping $Im k^0 = -|\Delta_\Gamma|$, where each neutrino mode labelled by $\{E, v\}$ evolves independently. In the presence of neutrino-neutrino interactions, collective oscillations become possible where this dispersion relation changes. Therefore, we consider solutions with $v^\alpha k^\alpha - \omega_E + i|\Delta_\Gamma| \neq 0$ for any $\{E, v\}$ so that equation (13) implies

\[ Q_{\Gamma,K} = \frac{v^\alpha A^\alpha_k}{v^\gamma k^\gamma - \omega_E + i|\Delta_\Gamma|}. \]

Inserting this form on both sides of equation (13) yields

\[ v^\alpha A^\alpha_k = -v^\alpha A^\alpha_k \int d\Gamma' v'^\gamma k'^\gamma - \omega_E' + i|\Delta_\Gamma'|. \]

In more compact notation this can be written in the form

\[ v^\alpha \Pi^\alpha_k A_k = 0 \]

where

\[ \Pi^\alpha_k = h^\alpha + \int d\Gamma v^\alpha v^\beta (\omega_E + i|\Delta_\Gamma|), \]

with $h^\alpha = \text{diag}(+, - , - , -)$ being the metric tensor. This equation must hold for any $\alpha$ and thus consists of four independent equations $\Pi^\beta A_k = 0$. Nontrivial solutions require

\[ \mathcal{D}(k) \equiv \text{det} \Pi^\alpha_k = 0, \]

establishing a connection between the components of $k = (k^0, \mathbf{k})$, i.e., the dispersion relation of the system. It depends only on the neutrino flavor spectrum $g_r$, which itself contains the neutrino density, the vacuum oscillation frequency $\omega_E$, and the damping rate $|\Delta_\Gamma|$.

If the imaginary part of $k^0$ is positive, for any $k$ that satisfies equation (18), equation (12) tells us that it leads to exponential growth of the off-diagonal flavor coherence between the two flavors under consideration, i.e., $S^{\alpha\beta} \sim e^{\text{i}Im k^0}$. In the limit of vanishing flavor-mixing, as relevant in dense matter, such flavor conversion is surprising and called a collective instability.

### III. CROSSINGS ARE NECESSARY

Now we prove that collective instabilities can arise only if there is a spectral crossing. Technically, our proposition is that if any solution of the dispersion relation $\mathcal{D}(k) = 0$ has $\text{Im} k^0 \equiv \sigma > 0$ and $k \in \mathbb{R}^3$, then the spectrum $g_r$ cannot have the same sign everywhere. We will prove the proposition by contradiction, following Morinaga [44], but with a treatment of the singular case.

In the following, we omit explicitly noting the $k$-dependence of the matrix $\Pi_k$ and its eigenvector $A_k$. Also we separate the real and imaginary parts of $k^0 = \kappa + i\sigma$, where $\kappa, \sigma \in \mathbb{R}$, and write the $\Pi$ matrix as

\[ \Pi^\alpha = M^\alpha_k - iN^\alpha_k, \]

where $M$ and $N$ are real-symmetric matrices

\[ M^\alpha_k = h^\alpha + \int d\Gamma v^\alpha (\kappa - v \cdot k - \omega_E) v^\beta v^\beta, \]
\[ N^\alpha_k = \int d\Gamma v^\alpha (\sigma + |\Delta_\Gamma|) v^\beta v^\beta. \]

The matrix $N$ can be diagonalized by a real orthogonal matrix $O$ as

\[ O^\alpha O^\beta N^{\alpha\nu} = D^{\alpha\beta}, \]

where $D^{\alpha\beta}$ is the diagonal matrix with $D^{\alpha\beta} = \delta^{\alpha\beta}$.
where \( D \) is a diagonal matrix whose components are

\[
D^{\alpha \alpha} = \int d\Gamma \, g_\Gamma \frac{(\sigma + |\Delta_\Gamma|) (O_{\mu \nu}^\alpha)^2}{(\kappa - \mathbf{v} \cdot \mathbf{k} - \omega_E)^2 + (\sigma + |\Delta_\Gamma|)^2} .
\]  

(22)

In this basis, where \( N \) becomes diagonal, the matrix \( M \) becomes \( M_i \) and the dispersion relation \( D(k) = 0 \) becomes \( \text{det}(M - iD) = 0 \) which implies that there exists a nontrivial four-eigenvector \( A \) such that

\[
\tilde{M}^{\alpha \beta} A_\beta = iD^{\alpha \beta} A_\beta .
\]  

(23)

Note that \( \tilde{M} - iD \) is a complex-symmetric matrix, so in general \( A \) is a complex vector. We multiply the above equation by \( A_\alpha^* \) and sum over \( \alpha \) to get

\[
\tilde{M}^{\alpha \beta} A_\alpha^* A_\beta = iD^{\alpha \beta} A_\alpha^* A_\beta ,
\]  

(24)

whose complex conjugate is given by

\[
\tilde{M}^{\alpha \beta} A_\alpha A_\beta^* = -iD^{\alpha \beta} A_\alpha A_\beta^* .
\]  

(25)

Using the fact that \( \alpha \) and \( \beta \) are dummy indices and can be renamed \( \beta \) and \( \alpha \), respectively, and that \( \tilde{M} \) is symmetric, i.e., \( \tilde{M}^{\alpha \beta} = \tilde{M}^{\beta \alpha} \), we get

\[
\tilde{M}^{\alpha \beta} A_\alpha A_\beta^* = -iD^{\alpha \beta} A_\alpha A_\beta^* .
\]  

(26)

Subtracting equation (26) from equation (24) gives

\[
\sum_\alpha D^{\alpha \alpha} |A_\alpha|^2 = 0 .
\]  

(27)

In the equation (27) above, \( |A_\alpha|^2 \) are non-negative and not all of them vanish. As proposed, we have \( \sigma > 0 \) and \( g_\Gamma \) has the same sign everywhere, so equation (22) dictates that all \( D^{\alpha \alpha} \) have the same signature as \( g_\Gamma \).

There would appear to be two possibilities for equation (27). First, the singular case where \( D^{\alpha \alpha} = 0 \) for all \( \alpha \) for which \( |A_\alpha|^2 \neq 0 \). However, in that case equation (22) requires that the integral of \( (O_{\mu \nu}^\alpha)^2 \) times an everywhere-same-sign function vanishes, for some \( \alpha \). This is possible only if \( (O_{\mu \nu}^\alpha)^2 = 0 \) for all points in \( \Gamma \) or if \( g_\Gamma = 0 \). That is, the same \( O \) makes the \( \alpha \)-component of an arbitrary \( v \) vanish or that there are no collective effects at all, respectively. These are either impossible or trivial, and therefore excluded. Second is the non-singular case, where \( D^{\alpha \alpha} \neq 0 \) for some \( A_\alpha \neq 0 \). In this case, in equation (27) at least one term is nonzero and all terms are non-negative. But then the equation (27), which algebraically followed from our original assumptions, cannot be satisfied! The only resolution is that \( g_\Gamma \) must change sign if there exists a \( \sigma > 0 \). This completes the proof of the proposition. As a corollary, setting \( \omega_E \to 0 \) and \( \Delta_\Gamma \to 0 \), one recovers the necessary condition for collisionless fast instability [44].

IV. REMARKS

Observation of signatures of collective neutrino flavor instabilities, such as spectral splits, depolarization, and their impact on stellar heating and nucleosynthesis, in neutrinos from dense astrophysical environments will provide information on their distributions deep inside the star. Our result gives a foundation for this physical expectation. It also makes it eminently sensible to search for collective flavor instabilities in supernova simulations by simply looking for spectral crossings [62–64].

One may wonder if the proof survives in more general scenarios. For collisions involving neutrinos, other terms proportional to \( \int d\epsilon d_p p \cdot \epsilon p - d_\epsilon p \) and/or \( \Phi p \) arise in the EOM; cf. Sec.III of ref. [52]. The former shift \( v^\alpha v^\beta \) in equation (17) to \( v^\alpha v^\beta - i v^\alpha v^\beta d_{\Gamma_1 \Gamma_2} \), with \( u^\beta \) being the medium four-velocity and \( d_{\Gamma_1 \Gamma_2} \) a real function of the momenta. The latter couple equations for \( \Phi_\mu \) and \( \Phi_{\nu} \). In general, none of these are damping-only terms. Including helicity/spin changing operators, etc., will also obviously expand the dimension of the flavor space [65–67]. Beyond the Standard Model, even certain kinds of forward interactions render the linearized EOMs to not remain an eigenvalue equation [68]. These need a dedicated treatment, perhaps extending the stability analysis strategy. Going beyond mean-field requires more thought [19, 69, 70].

Further, one could ask if a spectral crossing is a sufficient condition for a collective instability? In the fast limit, it has been proposed that if there is a crossing in the energy-integrated spectrum, then there must exist some solution \( k \) of the dispersion relation \( D(k) = 0 \) with a complex \( k^0 \) and real \( k \) [44]. Does the same proof extend to all collective effects? For now, the answer is unclear. Clearly, more secrets of collective neutrino flavor transformations remain to be discovered.

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