Neutron-skin thickness of $^{208}$Pb from the energy of the anti-analogue giant dipole resonance

A Krasznahorkay$^1$, N Paar$^2$, D Vretenar$^2$ and M N Harakeh$^3$

$^1$ Institute of Nuclear Research, Hungarian Academy of Sciences (MTA Atomki), Debrecen, Hungary
$^2$ Department of Physics, Faculty of Science, University of Zagreb, Croatia
$^3$ Kernfysisch Versneller Instituut, University of Groningen, Groningen, The Netherlands

E-mail: kraszn@mta.atomki.hu

Received 30 October 2012
Accepted for publication 25 December 2012
Published 2 May 2013
Online at stacks.iop.org/PhysScr/T154/014018

Abstract

The energy of the charge-exchange anti-analogue giant dipole resonance (AGDR) has been calculated for the $^{208}$Pb isotope using the state-of-the-art fully self-consistent relativistic proton–neutron quasiparticle random-phase approximation based on the relativistic Hartree–Bogoliubov model. It is shown that the AGDR centroid energy is very sensitively related to the corresponding neutron-skin thickness. The neutron-skin thickness of $^{208}$Pb has been determined very precisely by comparing the theoretical results with the available experimental data on $E(AGDR)$. The result $\Delta R_{pn} = 0.161 \pm 0.042$ agrees nicely with the previous experimental results.

PACS numbers: 24.30.Cz, 21.10.Gv, 25.55.Kr, 27.60.+j

(Some figures may appear in color only in the online journal)

1. Introduction

A precise measurement of the thickness of neutron skin is important not only because it represents a basic nuclear property, but also because it constrains the symmetry-energy term of the nuclear equation of state [1–6]. A detailed knowledge of the symmetry energy is essential for describing the structure of neutron-rich nuclei, and for modeling properties of neutron-rich matter in nuclear astrophysics applications. The Pb radius experiment (PREX) using parity-violating elastic electron scattering at JLAB [4] has initiated a new method for determining the neutron-skin thickness of nuclei.

Presently, the most precise value for the neutron-skin thickness has been obtained from a high-resolution study of electric dipole polarizability $\alpha_D$ in $^{208}$Pb [2], and a respective correlation analysis of $\alpha_D$ and $\Delta R_{pn}$ [6]. Measuring the strength of the giant dipole resonance in the whole energy range turned out to be an excellent tool for determining $\alpha_D$.

The excitation of the isovector giant dipole resonance by an isoscalar probe, in particular inelastic $\alpha$ scattering, was also used to extract the neutron-skin thickness of nuclei [7, 8]. The cross-section of this process depends strongly on $\Delta R_{pn}$. Another tool used earlier for studying the neutron-skin thickness is the excitation of the isovector spin giant dipole resonance (IVSGDR). The $L = 1$ strength of the IVSGDR is sensitive to the neutron-skin thickness [9, 10].

Vretenar et al [11] suggested another new method for determining the $\Delta R_{pn}$ by measuring the energy of the Gamow–Teller resonance (GTR) relative to the isobaric analogue state (IAS). Constraints on the nuclear symmetry energy and neutron skin were also obtained recently from studies of the strength of the pygmy dipole resonance [12, 13].

In this paper, we suggest a new method for determining the thickness of the neutron skin, based on the measured energy of the anti-analogue of the giant dipole resonance (AGDR) [14].

2. On the energy of the AGDR

We have used two sum rules for calculating the energy of the AGDR. The non-energy-weighted sum rule (NEWSR), which
we used in an earlier study based on the IVSGDR [9, 10], is valid (apart from a factor of 3) also for the giant dipole resonance excited in charge-exchange reactions and predicts an increase in strength as a function of the neutron-skin thickness:

\[ S^- - S^+ = \frac{9}{2\pi} (N(r^2)_n - Z(r^2)_p), \tag{1} \]

where \( S^- (S^+) \) denotes the integrated \( \beta^- (\beta^+) \) strengths, \( N \) and \( Z \) are the neutron and proton numbers and \( (r^2)_n \) and \( (r^2)_p \) represent the mean-square radii of the neutron and proton distributions, respectively.

Auerbach et al [15] derived an energy-weighted sum rule (EWSR) also for the dipole strength excited in charge-exchange reactions. The corresponding energies are measured with respect to the random-phase approximation (RPA) ground-state energy (IAS state) in the parent nucleus. The result of this EWSR is almost independent of the neutron-skin thickness [15],

\[ \int (E^- s^-) dE + \int (E^+ s^+) dE = \frac{3}{4\pi} (\hbar^2/m) A(1 + \kappa + \eta), \tag{2} \]

where \( s^- (s^+) \) denotes the energy-dependent \( \beta^- (\beta^+) \) strengths, \( m \) is the nucleon mass, \( A \) is the atomic number and \( \kappa \) is the usual dipole enhancement factor, which for a Skyrme force is equal to

\[ \kappa = ((\hbar^2/m)A)^{-1} t_1 + t_2 \int (\rho_n(r)\rho_p(r)) d^3r, \tag{3} \]

where \( \rho_n(r) \) and \( \rho_p(r) \) are the neutron and proton densities, respectively. The correction term \( \eta \) is

\[ \eta = ((\hbar^2/2m)A)^{-1} \frac{1}{8} t_1 + t_2 \int (\rho_n(r) - \rho_p(r))^2 d^3r \]

\[ + \frac{1}{3} \int r^2 V_c(r)((\rho_n(r) - \rho_p(r)) d^3r), \tag{4} \]

where \( t_1 \) and \( t_2 \) are parameters of the Skyrme potential [16] and \( V_c \) is the Coulomb potential.

If one neglects the \( S^+ \) strength as compared to the \( S^- \) one, and assumes that the whole \( S^- \) strength is concentrated in one single transition, then the mean energy of the dipole state should decrease with increasing dipole strength and therefore with increasing neutron-skin thickness in consequence of the sum rule:

\[ E_{AGDR} = \frac{3\hbar^2 A}{8\pi N m R_p} \frac{1}{\Delta R_p + \sqrt{\langle r^2 \rangle (N - Z)}/2N}. \tag{5} \]

The strong sensitivity of the AGDR energy on \( \Delta R_p \) was noted also by Krmpotić et al [17] in a study that used the RPA.

In this work, we perform systematic calculations for the centroid energy of the AGDR using the framework of relativistic nuclear energy-density functionals. Effective interactions that span a wide range of the symmetry energy at saturation density are used to demonstrate the sensitivity of AGDR in constraining the neutron-skin thickness. Model calculations are carried out using the fully self-consistent relativistic proton–neutron quasiparticle random-phase approximation (pn-RQRPA) based on the relativistic Hartree–Bogoliubov model (RHB), and the results are compared with available data.

![Figure 1.](image-url)
GTR, with an angular distribution characteristic of $\Delta L = 1$ transfer.

4. Theoretical analysis

The theoretical analysis is performed using the fully self-consistent pn-RQRPA based on the RHB model [26]. The RQRPA was formulated in the canonical single-nucleon basis of the RHB model in [27] and extended to the description of charge-exchange excitations (pn-RQRPA) in [28]. The RHB + pn-RQRPA model is fully self-consistent: in the particle–hole channel, effective Lagrangians with density-dependent meson–nucleon couplings are employed, and pairing correlations are described by the pairing part of the finite-range Gogny interaction [29].

For the purpose of the present study, we employ a family of density-dependent meson-exchange (DD-ME) effective interactions, for which the constraint on the symmetry energy at saturation density was systematically varied: $a_4 = 30, 32, 34, 36$ and $38$ MeV, and the remaining model parameters were adjusted to reproduce empirical nuclear-matter properties (binding energy, saturation density, compression modulus), and the binding energies and charge radii of a standard set of spherical nuclei [30]. These effective interactions were used to provide a microscopic estimate of the nuclear-matter incompressibility and symmetry energy in relativistic mean-field models [30], and in [12] to study a possible correlation between the observed pygmy dipole strength in $^{130,132}$Sn and the corresponding values for the neutron-skin thickness. In addition to the set of effective interactions with $K_{\text{sym}} = 250$ MeV (this value reproduces the excitation energies of giant monopole resonances) and $a_4 = 30, 32, 34, 36$ and $38$ MeV, the relativistic functional DD-ME2 [31] is used here to calculate the excitation energies of the AGDR with respect to the IAS, as a function of the neutron skin. Pertinent to the present analysis is the fact that the relativistic RPA with the DD-ME2 effective interaction predicts for the dipole polarizability [2]

$$\alpha_D = \frac{8\pi}{9} e^2 m^{-1}$$

(directly proportional to the inverse energy-weighted moment $m_{-1}$) of $^{208}$Pb the value $\alpha_D = 20.8$ fm$^3$, in very good agreement with the recently measured value: $\alpha_D = (20.1 \pm 0.6)$ fm$^3$ [2].

5. Determination of the neutron-skin thickness of $^{208}$Pb

To explore the sensitivity of the centroid energy of the AGDR to the neutron-skin thickness of $^{208}$Pb, we have performed RHB + pn-RQRPA calculations using a set of the effective interactions with different values of the symmetry energy at saturation: $a_4 = 30, 32, 34, 36$ and $38$ MeV (and correspondingly different slopes of the symmetry energy [5]) and, in addition, the DD-ME2 effective interaction ($a_4 = 32.3$ MeV). In figure 2, the resulting energy differences $E(\text{AGDR}) - E(\text{IAS})$ are plotted as a function of the corresponding neutron-skin thickness $\Delta R_{pn}$ predicted by these effective interactions.

![Figure 2](image-url)
energy range (±5 MeV around the position of the peak) and an energy shift of 0.15 MeV was obtained for the AGDR. Thus, the corrected energy of the AGDR is $E_{\text{AGDR}} - E_{\text{IAS}} = 24.14 - 15.17 + 0.15 = 9.12 \pm 0.2$ MeV.

We plan to measure the $E_{\text{AGDR}} - E_{\text{IAS}}$ energy difference more precisely by observing the $\gamma$-transition from the AGDR to the IAS. This transition is expected to be as strong as the $\gamma$-decay of the well-known GDR to the ground state.

The two parallel solid lines in figure 2 delineate the region of theoretical uncertainty for the set of effective interactions with $\alpha_2 = 30, 32, 34, 36$ and 38 MeV. An uncertainty of 10% was used for the differences between the neutron and proton radii for the nuclei $^{116}\text{Sn}$, $^{124}\text{Sn}$ and $^{208}\text{Pb}$ in adjusting the parameters of these interactions [30, 31]. They were also used to calculate the electric dipole polarizability and neutron-skin thickness of $^{208}\text{Pb}$, $^{132}\text{Sn}$ and $^{48}\text{Ca}$, in comparison with the predictions of more than 40 non-relativistic and relativistic mean-field effective interactions [6]. From the results presented in that work, one can also assess the accuracy of the present calculations.

By comparing the experimental result of $E(\text{AGDR}) - E(\text{IAS})$ with the theoretical calculations (see figure 2), we deduce the value of the neutron-skin thickness in $^{208}\text{Pb}$: $\Delta R_{np} = 0.161 \pm 0.042$ fm (including theoretical uncertainties). In table 1 the value for $\Delta R_{np}$ determined in the present analysis is compared with previous results obtained with a variety of experimental methods. Very good agreement has been obtained with previous data, thus reinforcing the reliability of the present method.

### 6. Conclusion

Using the experimental results from [14] for $^{208}\text{Pb}$ and the RHB+pn-RQRPA model, we deduce the following values of neutron skin: $\Delta R_{np} = 0.161 \pm 0.042$ fm for $^{208}\text{Pb}$. The agreement between the $\Delta R_{np}$ determined using measurements of the AGDR–IAS and previous methods is very good. In particular, the present study supports the results from a very recent high-resolution study of electric dipole polarizability $\alpha_D$ in $^{208}\text{Pb}$ [2], respective correlation analysis of $\alpha_D$ and $\Delta R_{np}$ [6], as well as the PREX using parity-violating elastic electron scattering at JLAB [4]. The method we have introduced provides not only stringent constraints to the neutron-skin thicknesses in the nuclei under consideration, but also offers new possibilities for measuring $\Delta R_{np}$ in rare-isotope beams, which was tested recently [35].

### Acknowledgments

This work was supported by the Hungarian OTKA Foundation no. K106035, by the MZOS—project no. 1191005-1010 and the Croatian Science Foundation.

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