Disk Planet Interactions and Early Evolution in Young Planetary Systems

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ABSTRACT

We study and review disk protoplanet interactions using local shearing box simulations. These suffer the disadvantage of having potential artefacts arising from periodic boundary conditions but the advantage, when compared to global simulations, of being able to capture much of the dynamics close to the protoplanet at high resolution for low computational cost. Cases with and without self sustained MHD turbulence are considered. The conditions for gap formation and the transition from type I migration are investigated and found to depend on whether the single parameter $M_p R^3/(M_* H^3)$, with $M_p$, $M_*$, $R$, and $H$ being the protoplanet mass, the central mass, the orbital radius and the disk semi-thickness respectively exceeds a number of order unity. We also investigate the coorbital torques experienced by a moving protoplanet in an inviscid disk. This is done by demonstrating the equivalence of the problem for a moving protoplanet to one where the protoplanet is in a fixed orbit which the disk material flows through radially as a result of the action of an appropriate external torque. For sustainable coorbital torques to be realized a quasi steady state must be realized in which the planet migrates through the disk without accreting significant mass. In that case, although there is sensitivity to computational parameters, in agreement with earlier work by Masset & Papaloizou (2003) based on global simulations, the coorbital torques are proportional to the migration speed and result in a positive feedback on the migration, enhancing it and potentially leading to a runaway. This could lead to fast migration for protoplanets in the Saturn mass range in massive disks and may be relevant to the mass period correlation for extrasolar planets which gives a preponderance of sub Jovian masses at short orbital periods.

Subject headings: accretion disks, MHD, planetary formation, migration

1. Introduction

The ongoing discovery of extrasolar planets has resulted in increased investigation of theories of planet formation (e.g. Mayor & Queloz 1995; Marcy, Cochran, & Mayor 2000; Vogt et al. 2002).
Possible formation scenarios for gas giant planets are either through direct gravitational instability occurring in a young protostellar disk (e.g. Boss 2001) or through the accumulation of a solid core that undergoes rapid gas accretion once its mass has reached a critical value \( \simeq 15 \) Earth masses (e.g. Pollack et al. 1996). In either case planetary formation is likely to be initiated at significantly greater orbital radii than those currently observed implying that a process of orbital migration has brought them closer to the central star. Disk protoplanet interaction provides a natural migration mechanism.

A protoplanet exerts torques on a protostellar disk through the excitation of spiral density waves (e.g. Goldreich & Tremaine 1979; Papaloizou & Lin 1984; Lin & Papaloizou 1993). These waves carry away either positive or negative angular momentum which is deposited in the disk at the locations where the waves are damped. As a result of this, a negative torque is exerted on the protoplanet by the outer disk and a positive torque is exerted on it by the disk interior to its orbit.

In recent years much numerical work on disk protoplanet interactions has been undertaken for protoplanets with a range of masses in both laminar and turbulent disks in which the turbulence is maintained by the magnetorotational instability (eg. Bryden et al. 1999; Kley 1999; Lubow, Seibert, & Artymowicz 1999, D’Angelo, G., Henning, & Kley, W., 2002, Nelson & Papaloizou 2003, Winters Balbus & Hawley 2003a, Papaloizou Nelson & Snellgrove 2004; Nelson & Papaloizou 2004). This work indicates that a sufficiently massive protoplanet can open up an annular gap in the disk centred on its orbital radius. For typical protostellar disk models gap formation at 5 AU starts to occur for protoplanet masses of around a Saturn mass with the gap becoming deep for a Jovian mass.

For low mass disks gap formation, or the transition from linear to non linear disk response, is associated with the transition between type I migration (eg. Ward 1997) and type II migration (eg. Lin & Papaloizou 1986). For both of these regimes the time scale of the migration in standard disk models, with mass comparable to that of the minimum mass solar nebula, is shorter than the disk lifetime making it a threat to the survival of protoplanets (see eg. Terquem Papaloizou & Nelson 2000 for a review) as well as a mechanism for producing close orbiting giant planets.

More recently Masset & Papaloizou (2003) considered a form of potentially very fast migration induced by the action of coorbital torques acting on disk material as it passes through the coorbital region of a migrating protoplanet. This is in contrast to torques produced by waves dissipating in the disk away from the orbit. They found using two dimensional global simulations at rather low resolution that such torques could lead to a positive feedback and a fast migration for typically Saturn mass protoplanets in massive disks. This may provide a mechanism for bringing sub Jovian mass objects close to the star which, if they are then slowed down and undergo some additional accretion may become hot Jupiters.

In this paper we explore some calculations of disk planet interactions and related migrational torque calculations using local shearing box simulations. These focus on a local patch on the disk and so may be done at high resolution for relatively low cost. We consider both non turbulent disks
and disks with turbulence driven by the magnetorotational instability or MRI (Balbus & Hawley 1991). This is the most likely mechanism producing angular momentum transport and thus the 'viscous' evolution of the disk that ultimately drives type II migration (Lin & Papaloizou 1986).

We discuss the noisy type I migration in turbulent disks and the condition for gap formation. We also explore at length coorbital torques associated with migrating protoplanets and find that the local box simulations are consistent with the lower resolution global ones.

The plan of the paper is as follows. In section 2 we give the basic equations and describe the local shearing box model in frames that may be either uniformly rotating or migrating with an angular velocity which is a function of time. In section 3 we describe the numerical procedure and models simulated. In section 4 we go on to describe results of simulations with protoplanets and the calculation of the forces acting on them that lead to migration. In particular we examine the coorbital torque generated by material as it passes through the coorbital region from one side of the protoplanet to the other and present numerical results for two different flow through speeds. Finally in section 5 we discuss our results and their implications for extrasolar planetary configurations.

2. Basic Equations and Model set up

In this paper we describe local simulations of protoplanets interacting with a Keplerian disk flow with and without MHD turbulence driven by the MRI. We consider simulations for which the magnetic field, when present has zero net flux so that an internally generated dynamo is maintained. Simulations with turbulence require high resolution with usually not less than 32 grid cells per scale height $H$. They are greatly facilitated by considering a local shearing box rather than a complete disk (Goldreich & Lynden-Bell 1965).

The governing equations for ideal MHD written in an inertial frame are:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (1)$$

$$\rho \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p - \rho \nabla \Phi + \frac{1}{4\pi} (\nabla \times \mathbf{B}) \times \mathbf{B}, \quad (2)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}). \quad (3)$$

where $\mathbf{v}, P, \rho, \mathbf{B}$ and $\Phi$ denote the fluid velocity, pressure, density and magnetic field respectively. The gravitational potential is $\Phi$. It contains contributions from a central mass $M_*$ and an orbiting secondary mass or planet $M_p$. Thus adopting a cylindrical coordinate system $(r, \varphi, z)$ centred on $M_*$,

$$\Phi = -\frac{GM_*/r}{\sqrt{r^2 + R^2 - 2rR \cos(\varphi - \varphi_s) + z^2 + b^2}}, \quad (4)$$

where $(R, \varphi_s, 0)$ are the coordinates of the secondary, $M_p$ is the secondary mass and $G$ the gravitational constant.
2.1. The Local Shearing Box

Following Goldreich & Lynden-Bell (1965) we consider a small Cartesian box centred on a point at which the Keplerian angular velocity is $\Omega_p$. Normally one takes the centre of the box to be at a fixed point in the underlying disk flow which corresponds to the motion of a secondary mass in circular orbit. However, it is possible to consider a box centred on a point with changing radius $R(t)$ and varying angular velocity $\Omega_p(t) = \sqrt{GM_\ast/R(t)^3}$, $M_\ast$ being the central mass and $G$ being the gravitational constant. For a box centred on the secondary mass this corresponds to the situation when the secondary migrates. We also suppose that as the point moves the disk semi-thickness is $H(t)$ and that this determines the radial scale of the flow.

We use the local Cartesian coordinates $x = r - R(t)$, $z$, and the mutually orthogonal coordinate $y$, to define local dimensionless Cartesian coordinates $(x', y', z') = (x/H(t), y/H(t), z/H(t))$ with associated unit vectors $(i, j, k)$. The direction $i$ is outwards along the line joining the origin to the central object while $k$ points in the vertical direction. The direction $j$ is that of the unperturbed Keplerian shear flow. In this non-inertial frame the flow velocity is $u$. It is also convenient to use a rescaled time $t'$ such that $dt' = \Omega_p(t)dt$.

The idea behind using the local box is that there is a small parameter $h = H/R$, measuring the aspect ratio of the disk. We adopt a spatially isothermal equation of state $P = \rho c_s^2$, with $c_s$ being the sound speed which may depend on time, such that the aspect ratio is also equal to the ratio of sound speed to orbital velocity so that $h = c_s/(R\Omega_p)$.

For a moving centre we suppose that there is another small parameter $|\epsilon| = |dR/dt/\Omega_p R| \sim |dH/dt/(\Omega_p H)|$. This measures the ratio of the orbital time to the time for the centre to migrate significantly in the radial direction.

We shall assume that $|\epsilon|/h = O(h^k)$, for some $1 > k > 0$. This enables us to retain terms of order $\epsilon/h = dR/dt/(\Omega_p H)$ but neglect terms of order $\epsilon$ and higher.

2.2. Dimensionless Variables

We introduce a dimensionless velocity, sound speed and planet mass through $u' = u/(\Omega_p H)$, $c'_s = c_s/(\Omega_p H)$, and $q = M_p R^3/(M_\ast H^3)$ respectively. As we do not include the self-gravity of the disk in calculating its response, the magnitude of the disk density may be arbitrarily scaled. All the simulations presented here start with a uniform density $\rho_0$ which, together with $H$, may be used to specify the mass scale. We define a dimensionless density and magnetic field through $\rho' = \rho/\rho_0$ and $B' = B/(\Omega_p H\sqrt{4\pi\rho_0})$. The equation of motion may be written in terms of these dimensionless variables, after taking the limit $\epsilon \to 0$, and $h \to 0$ simultaneously but retaining the ratio $\epsilon/h$ in
the form

\[ \frac{\partial \mathbf{u}'}{\partial t'} + \mathbf{u}' \cdot \nabla \mathbf{u}' + 2\mathbf{k} \times \mathbf{u}' - 3x'\mathbf{i} = -\frac{\mathbf{j}}{2h} - \frac{\nabla(\rho' c'_s^2)}{\rho'} - \nabla \Phi'_p + \left( \nabla \times \mathbf{B}' \right) \times \mathbf{B}', \tag{5} \]

where

\[ \Phi'_p = -\frac{q}{\sqrt{x'^2 + y'^2 + z'^2 + b'^2}}, \tag{6} \]

with dimensionless softening parameter \( b' = b/H \) and of course the spatial derivatives are with respect to the dimensionless coordinates. Similarly the dimensionless induction equation becomes

\[ \frac{\partial \mathbf{B}'}{\partial t'} = \nabla \times (\mathbf{u}' \times \mathbf{B'}). \tag{7} \]

and the dimensionless continuity equation is

\[ \frac{\partial \rho'}{\partial t'} + \nabla (\cdot \rho' \mathbf{u}') = 0. \tag{8} \]

We note that the term \( \propto x' \) in equation (5) is derived from a first order Taylor expansion about \( x = 0 \) of the combination of the gravitational acceleration due to the central mass and the centrifugal acceleration (Goldreich & Lynden-Bell 1965). However, in order to reduce computational requirements, in the work presented here, we have neglected the \( z' \) dependence of the gravitational potential due to the secondary given by equation (6) and also of that due to the central mass. Therefore vertical stratification is neglected. Simulations of unstratified boxes are often undertaken in MHD because they contain the essential physics and ease computational requirements (eg. Hawley, Gammie & Balbus 1995).

In the steady state box with a fixed centre and no planet or magnetic field, the equilibrium velocity is due to Keplerian shear and thus

\[ \mathbf{u}' = (0, -3x' / 2, 0). \tag{9} \]

### 2.3. Steady and Moving Frames

When the centre is fixed so that \( \Omega_p, R \) and \( H \) are constant, equations (5-8) are the standard equations for a shearing box (Goldreich & Lynden-Bell 1965).

When the centre migrates with \( \Omega_p, R \) and \( H \) being functions of time, in the limit of small, \( \epsilon \) and \( h \) but \( \epsilon/h \) retained, the equations are the same but for one added dimensionless acceleration \(-\epsilon j/(2h)\). This corresponds to the dimensionless torque required to make the disk gas move with speed \(-dR/dt\) in the absence of the secondary. This situation means that a migrating planet moving through non migrating gas with speed \( dR/dt \) is equivalent to a non migrating planet immersed in a disk with gas given a torque which results in it migrating with speed \(-dR/dt\). This symmetry has already been exploited without justification in two dimensional global simulations by Masset &
Papaloizou (2003). However, the approximation scheme requires $\epsilon$ and $h$ to be small. This means that the migration time to move through $R$ measured by $1/\epsilon$ has to be very long compared to the orbital time. But the time to migrate through $H$, which is measured by $h/\epsilon$ can be significantly faster.

2.4. Boundary Conditions

The shearing box is presumed to represent a local patch of a differentially rotating disk. Thus the appropriate boundary conditions on the bounding faces $y = \text{constant} = \pm Y$ and $z = \text{constant} = \pm Z$ come from the requirement of periodicity in the local Cartesian coordinate directions normal to the boundaries. On the boundary faces $x = \text{constant} = \pm X$, the boundary requirement is for periodicity in local shearing coordinates. Thus for any state variable $F(x, y, z)$, the condition is that $F(X, y, z) = F(-X, y - 3Xt', z)$. This means that information on one radial boundary face is communicated to the other boundary face at a location in the azimuthal coordinate $y$ shifted by the distance the faces have sheared apart since the start of the simulation (see Hawley, Gammie & Balbus 1995).

In dimensionless coordinates the boundaries of the box are $x' = \pm X/H$, $y' = \pm Y/H$, $z' = \pm Z/H$. Also if the magnetic field has zero net flux and is dynamo generated and thus a spontaneous product of the simulation, given that the only parameter occurring in equations (5-8) is the dimensionless mass $q = M_p R^3 / (M_* H^3)$, we should be able to consider the dependence of the time averaged outcome of a simulation to be only on $X/H, Y/H, Z/H$, and $q = M_p R^3 / (M_* H^3)$.

Note that the above discussion implies that for a box of fixed dimension, the only distinguishing parameter is $q = M_p R^3 / (M_* H^3)$. This parameter may also be interpreted as the cube of the ratio of the Hill radius to disk scale height and the condition that $q > \sim 1$ leads to the thermal condition of Lin & Papaloizou (1993) that the Hill radius should exceed the disk scale height for gap formation to occur. From Korycansky & Papaloizou (1996) this condition is also required in order that the perturbation due to the protoplanet be non linear.

3. Numerical Procedure and Models Simulated

The numerical method is that of characteristics constrained transport MOCCT (eg. Hawley & Stone 1995). The code used has been developed from a version of NIRVANA originally written by U. Ziegler (see Ziegler & Rüdiger 2000 and references therein). It has been used in a number of simulations of disk planet interactions in two and three dimensions with and without magnetic fields (eg. Kley 1999; Nelson et al. 2000; Steinacker & Papaloizou 2002; Papaloizou Nelson & Snellgrove 2004).

All of the simulations were for shearing boxes with $X = 4H, Y = 2\pi H$, and $Z = H/2$. These
Fig. 1.— The volume averaged stress parameter $\langle \alpha \rangle$, is plotted as a function of a dimensionless time for no planet, $q = 0$ at all times and for the case when the planet corresponding to $q = 0.1$ was inserted at time 353. Although both curves are independently very noisy after this time they cannot be separated.
Fig. 2—Typical mid plane density contour plots near the end of the simulations for upper left panel, $q = 0.1$ and no magnetic field (model B), upper right panel, $q = 0.1$ with magnetic field (model C), lower left panel, $q = 0.3$ with magnetic field (model D) and lower right panel, $q = 1.0$ with magnetic field (model E). The ratio of maximum to minimum densities in these plots were 1.42, 2.9, 3.27, and 11.22 respectively. In the case of the upper left panel, the ratio was reduced to 1.25 if the wakes produced by the protoplanet were excluded.
boxes are larger than the standard one often used in MHD simulations (eg. Hawley Gammie & Balbus 1995) which has $X = H/2, Y = \pi H$ and $Z = H/2$. Such a larger box is required to cover the scale of the disk planet interaction (Papaloizou Nelson & Snellgrove 2004; Nelson & Papaloizou 2004). As in those works, the simulations were carried out with 261, 200, and 35 grid points in the $x', y'$ and $z'$ directions respectively. At this resolution, MHD turbulence and associated dynamo can be maintained for beyond one hundred orbits of the box centre. Some parameters associated with the simulations are given in table I.

In order to investigate conditions for gap formation we have run models with a variety of values of $q = M_p R^3/(M_* H^3)$ with and without magnetic fields. We have also studied the torques acting on a planet moving outwards by applying a torque to the disk material to make it flow inwards (see discussion above). Values of $|\epsilon|/h$ are given in table I. But note that because of the symmetry of the box, equivalent results are obtained if the flow is in the opposite direction.

The softening parameter should be chosen so as to represent the effects of vertical stratification and the expected value is expected to be $\sim H$ (eg. Papaloizou 2002; Masset 2001; Nelson & Papaloizou 2004). The value adopted here was usually $b = 0.3H$.

At this point a cautionary note should be inserted. For sustainable coorbital torques to be realized a quasi steady state must be realized in which the planet migrates through the disk with a mass consistent with that assumed when calculating the tidal interaction. If a very small softening parameter is used, the diverging point mass potential coupled with a fixed low temperature isothermal equation of state results in arbitrarily large amounts of mass being deposited on the planet. Apart from preventing states with a steady flow through the coorbital region being attained, large amounts of accretion onto the planet cannot be handled correctly in calculations such as those performed here and elsewhere for disk planet interactions that neglect self-gravity of the disk and the proper physics of protoplanetary structure. Steady coorbital torques require a planet that self-consistently does not increase its mass significantly through disk mass accretion as can be modeled with a softening parameter $b \sim H$ such as adopted here.

But note that there is some expected sensitivity of torques to the value of the softening parameter (eg. Artymowicz 1993; Papaloizou 2002; Nelson & Papaloizou 2004). Accordingly for purposes of comparison some models were run with $b = 0.6H$. We found negligible mass accumulation on the protoplanet for $b = 0.6H$ but a more noticeable effect when $b = 0.3H$.

Because they are not dimensionless and can be scaled away the following numerical values have no significance, but for convenience we adopted $h = 0.01$, $p_0 = 0.001$, $GM_* = 1$ and $R = 1$. All our results below are obtained using these values.

The gravitational potential was flattened (made to attain a constant value in a continuous manner) at distances exceeding $3H$ from the protoplanet in order to satisfy the periodic boundary conditions. Tests with larger flattening distance have indicated indicated insensitivity to the precise choice.
Fig. 3.— Running time averages of the force acting on the planet in the direction of orbital motion plotted as a function of dimensionless time. These are for, the upper left panel, \( q = 0.1 \) and no magnetic field (model B), upper right panel, \( q = 0.1 \) with magnetic field (model C), lower left panel, \( q = 0.3 \) with magnetic field (model D) and lower right panel, \( q = 1.0 \) with magnetic field (model E). For each panel the upper curve represents the force due to material exterior to the protoplanet, the lower curve represents the force due to material interior to the protoplanet while the central curve gives the total contribution.
Fig. 4.— Typical mid plane density contour plots for left panel $q = 2.0$ with torque induced fast flow through (model G) and right panel $q = 2.0$ with a torque induced slower flow through of disk material (model H). The softening parameter was $b = 0.3H$. In these cases the magnitude of the torque applied to the disk material was chosen to produce expected inflow speeds of $(3/(8\pi))c_s$ and $(3/(64\pi))c_s$ respectively. These inflow speeds correspond to $|\epsilon|/h = 3/(8\pi)$ (fast flow through) and $|\epsilon|/h = 3/(64\pi)$ (slower flow through). These models had no magnetic field.
Fig. 5.— The mean surface density averaged over $y$ and $z$ as a function of radial coordinate $x/H$ for $q = 2.0$ with no flow through (dotted curve, model F) and with flow through (full curve, model G). These models had $b = 0.3H$ and have no magnetic field.
Models with magnetic fields all had conserved zero net flux. For these the initial field was taken to be vertical, independent of \(y\) and \(z\) and to vary sinusoidally in \(x\) with a wavelength of \(H\). The amplitude was chosen to make the initial ratio of the total magnetic energy to volume integrated pressure to be 0.0025. The initial velocity in the \(x\) direction at each grid point was chosen to be the product of a random number between \(-1.0\) and \(1.0\) and \(1 c_s\). Without a perturbing secondary, these models reach a turbulent state in which the ratio of volume mean magnetic energy to volume mean pressure is in the range 0.002 – 0.02 and the radially or volume averaged Shakura & Sunyaev \(\alpha\) parameter is in the range 0.003 – 0.02. The time averaged value of the volume averaged \(\alpha\) is typically 0.008 (see eg. Winters Balbus & Hawley 2003b, Papaloizou Nelson & Snellgrove 2004 for details). A plot of the evolution of the volume averaged \(\alpha\) for a simulation with no secondary mass which begins with the above initial conditions for the magnetic field is presented in figure 1. Models with magnetic fields and protoplanets were initiated by inserting the protoplanet into this model with established MHD turbulence after 353 time units.

Models without magnetic fields all had perturbing protoplanets so that these could be initiated from rest.

3.0.1. Vertical and Horizontal Averages

We find it useful to consider quantities that are vertically and azimuthally averaged over the \((y, z)\) domain \(\mathcal{D}\). Sometimes an additional running time average may be adopted. The vertical and azimuthal average of some quantity \(Q\) is defined by

\[
\overline{Q} = \frac{\int_\mathcal{D} \rho Q dz dy}{\int_\mathcal{D} \rho dz dy}.
\]  

The disk surface density is given by

\[
\Sigma(x, t) = \frac{1}{2Y} \int_\mathcal{D} \rho dz dy.
\]  

The vertically and azimuthally averaged Maxwell and Reynolds stresses, are given by:

\[
T_M = 2Y \Sigma \overline{\frac{B_x B_y}{4\pi \rho}}
\]  

and

\[
T_{Re} = 2Y \Sigma \overline{\delta v_x \delta v_y}.
\]
Fig. 6.— The mean surface density averaged over $y$ and $z$ as a function of radial coordinate $x/H$ for $q = 2.0$ with fast flow through and $b = 0.3H$ (model G, full curve), with slower flow through and $b = 0.3H$ (model H, dotted curve), with fast flow through and $b = 0.6H$ (model I, dashed curve), with slower flow through and $b = 0.6H$ (model J, dot-dashed curve). These models have no magnetic field.
Fig. 7.— Velocity vectors in the mid plane coorbital region for $q = 2.0$ no magnetic field and no flow through (model F)
Here the velocity fluctuations $\delta v_x$ and $\delta v_y$ are defined through,

$$\delta v_x = v_x - \overline{v_x},$$

$$\delta v_y = v_y - \overline{v_y}.$$  \hfill (14)

The horizontally and vertically averaged Shakura & Sunyaev (1973) $\alpha$ stress parameter appropriate to the total stress is given by

$$\alpha = \frac{T_{Re} - T_M}{2Y \Sigma (P/\rho)},$$

(16)

The angular momentum flow across a line of constant $x$ is given by

$$F = 2Y R \Sigma \alpha P/\rho.$$  \hfill (17)

4. Results of Simulations with Protoplanets

We have performed simulations with fixed protoplanets with $q = 0, 0.1, 0.3, 1.0$ and $2.0$ with and without magnetic fields. As $q$ increases, a gap is formed once $q$ exceeds a number around unity. If magnetic fields are present this is also the value beyond which perturbations from the protoplanet dominate the turbulence.

In figure 1 we plot the volume averaged stress parameter $\langle \alpha \rangle$, as a function of dimensionless time for no planet, $q = 0$, together with the same quantity for the run in which a protoplanet with $q = 0.1$ was inserted at time 353. Although both curves are independently very noisy after this time they cannot be separated verifying the fact that a protoplanet with $q = 0.1$ is not strong enough to perturb the turbulence significantly.

Nonetheless all the simulations show the prominent density wake associated with it. Typical mid plane density contour plots near the end of the simulations for upper left panel, model B ($q=0.1$), upper right panel, model C ($q=0.1$), lower left panel, model D ($q=0.3$) and lower right panel, model E ($q=1$) are given in in figure 2. The ratio of maximum to minimum densities was 1.42, 2.9, 3.27, and 11.22 respectively. When the dominant wakes produced by the protoplanet are excluded in model B, this ratio is reduced to 1.25. This ratio measures the effect of waves excited by ghost protoplanets in neighbouring boxes which are there on account of the periodic boundary conditions in shearing coordinates. It indicates that the fluctuations due to ghost neighbours are relatively small compared to the effects due to either magnetic fields or dominant perturbations close to the protoplanet due to its own gravity.

The simulations with $q < 1$ all have embedded protoplanets while for $q \geq 1$ a prominent gap is formed (see also Papaloizou Nelson & Snellgrove 2004). This occurs with and without a magnetic field. The condition for gap formation that $q > 1$ is equivalent to requiring that The size of the Hill sphere exceed the disk thickness, $H$. The additional requirement discussed by
Fig. 8.— Velocity vectors in the mid plane coorbital region for $q = 2.0$, $b = 0.3H$, with no magnetic field and fast flow through by induced torqued disk material (model G).
Fig. 9.— Running time averages of the force acting on the planet in the direction of orbital motion as a function of dimensionless time. These are for $q = 2.0$ with fast flow through and $b = 0.3H$ (model G, upper left panel), $q = 2.0$ with fast flow through and $b = 0.6H$ (model I, upper right panel), $q = 2.0$ with slower flow through and $b = 0.3H$ (model H, lower left panel), and $q = 2.0$ with slower flow through and $b = 0.6H$ (model J, lower right panel). These models have no magnetic field. For each panel the upper curve represents the force due to material exterior to the protoplanet, the lower curve represents the force due to material interior to the protoplanet while the central curve gives the total contribution.
Lin & Papaloizou (1993) is that tidal forces should be enough to overcome viscous diffusion is expressed by \( q > 40\alpha(R/H) \). To compare with the box simulations considered here, we should set \( 2R = L_y/\pi = 4H \) and the value of \( \alpha \) has to be related to the MHD turbulence when present. We adopt the value \( \alpha \sim 0.005 \) corresponding to a mean value of the Maxwell stress. Thus we see that for the MHD cases the viscous criterion for gap formation is satisfied for the boxes when \( q > 0.4 \). Thus \( q \geq 1 \) gives the expected condition for gap formation in all the box simulations. An extrapolation to make the azimuthal extent of the box equal to \( 2\pi R \) would give \( q > 0.2(R/H) \). Thus it should be borne in mind that the tendency for the gap to fill in a full circle may be underestimated for thin disks with MHD in a local box of the type we have used.

4.0.2. Torques and Migration

An important aspect of the disk protoplanet interaction is that it produces torques on the protoplanet that can lead to orbital migration. For a box simulation the torque is \( RF_y \) where \( F_y \) is the force in the \( y \) direction. We consider the contributions from the regions exterior to and interior to the protoplanet separately. One can see from eg. figure 2 that the density enhancement in the wake exterior to the protoplanet tends to drag it backwards while the density enhancement interior to the protoplanet tends to accelerate it. By the symmetry of the box these contributions cancel on average. However, features which break the symmetry such as a radial flow through can result in a non zero net torque/force.

Running time averages of the forces acting on the planet in the direction of orbital motion are plotted as a function of dimensionless time in figure 3. These are for, the upper left panel, model B, upper right panel, model C, lower left panel, model D, lower right panel, model E. We normalize these using the fiducial form

\[
F_{y0} = q^2 \Omega_p^2 H^3 \Sigma.
\]

For our units, this gives

\[
F_{y0} = 10^{-11} q^2.
\]

For \( q = 0.1 \) and \( q = 0.3 \) a gap does not form and a reasonable fit to the one sided outward force produced by regions exterior to the secondary is

\[
F_y = 0.5F_{y0}.
\]

We remark that the time averaged forces from exterior and interior region are very similar with and without turbulence due to a magnetic field. Because of the symmetry of the shearing box, contributions from the regions exterior to the protoplanet orbit must eventually balance the contributions from the interior regions. However, this may take a long time to achieve in the MHD case where noise can produce a ten percent bias for embedded protoplanets even after fifty orbits. Effects not yet modelled such as long timescale turbulent fluctuations in global disks where there is bias introduced both by the geometry, initial conditions and external boundary conditions may
have significant consequences for the net migration of embedded objects (see Nelson & Papaloizou 2004).

To compare with the expected rates of type I migration in a disk without a magnetic field, (eg. Ward 1997) we assume there is an asymmetry between outward and inward forces introduced by global geometrical effects of magnitude $C_1 h$, where $C_1$ is a constant between 1 and 10 (Ward 2000). This is taken to be in favour of a net drag force and vanishes in the limit $h \to 0$ as required. The net force is then

$$F_y = -0.5 F_{y0} C_1 h.$$  

This leads to an expected inflow time

$$\tau_{\text{mig}} = -\frac{R}{dR/dt} = \frac{M_*}{M_p C_1 \Sigma R^2} h^2 \Omega_p^{-1}.$$  

A physical explanation for why the net force amounts to a drag can be based on the fact that the wakes originate as a density wave launched starting from the location where the disk shears past the protoplanet with the sound speed. In a constant aspect ratio disk geometrical effects (not present in the box) make this location closer to the protoplanet in the outer regions resulting in a larger contribution to the net force. The resulting asymmetry is of order $h$.

The migration time of a non gap forming protoplanet embedded in a three dimensional disk has been calculated using a linear analysis by Tanaka,Takeuchi,& Ward (2002). They derive the following expression for the migration time of a protoplanet embedded in a locally isothermal disk with a uniform surface density profile:

$$\tau_{\text{mig}} = \frac{M_*}{2.7 M_p \Sigma R^2} h^2 \Omega_p^{-1}$$  

These authors comment that in general this expression gives a migration time that is a factor of between 2 – 3 times slower than similar expressions derived for flat, two–dimensional disks (e.g. Ward 1997). Interestingly both (22) and (23) agree for $C_1 = 2.7$ and correspond to a rather fast inward migration time of $8 \times 10^5 y$ at 5AU for $1 M_\oplus$ in a gas disk with surface density chosen so that there is two Jupiter masses within 5AU. But note that the magnitude is sensitive to the softening parameter which needs to be comparable to the vertical thickness or $\sim H$.

### 4.1. Coorbital Torques

By coorbital torque we mean a torque exerted on the protoplanet by disk material flowing through the orbit. This has been discussed recently by Masset & Papaloizou (2003). We have shown that in the local approximation adopted here the torque on a slowly radially migrating protoplanet is the same as that due to disk material radially migrating with the same speed in the opposite direction flowing past a protoplanet in fixed orbit.
As mentioned above, in the absence of migration the symmetry of the shearing box results in zero net torque on the protoplanet. However, introducing a migration direction breaks the symmetry and results in a non-zero torque on the protoplanet which changes sign with the direction of migration. From very general considerations we expect the net torque to be proportional to the migration velocity. This can result in either positive feedback when the torque assists the protoplanet migration or negative feedback when it acts in reverse.

Masset & Papaloizou (2003) argue for positive feedback because for an outwardly migrating protoplanet material traverses the coorbital zone from outside to inside. As it does so it moves along the outer boundary of the coorbital zone occupied by material librating around the coorbital equilibrium (Lagrange points) passing close to the protoplanet. As this passage occurs angular momentum is transferred to the protoplanet, a process which acts to assist the migration of the protoplanet and which accordingly gives a positive feedback. We now look at this process in more detail.

The expected force exerted on the secondary by material flowing through the coorbital region is estimated as follows. The mass flow through the box is at rate $2Y \Sigma dR/dt$. The outward momentum per unit mass imparted to the secondary when the disk matter moves across the coorbital region is $w \Omega_p/2$, where $w$ is it's width. For this we adopt $w = 2H$ (see below). Then the rate of transfer of outward momentum to the planet is $F_{cr} = 2Y \Omega_p H \Sigma (dR/dt)$. This can be expressed as

$$F_{cr} = 2Y \Omega_p^2 H^2 \Sigma ((dR/dt)/c_s).$$

(24)

This may also be written in the form

$$F_{cr} = \frac{1}{2} M_d \Omega_p (dR/dt),$$

(25)

where $M_d = 4Y \Sigma H$ is the disk mass that would fill the coorbital zone of width $2H$ were it to do so at the background surface density.

However, torques on the protoplanet do not only arise from material passing through the coorbital zone. Material that is forced to comove with the protoplanet, either because it has been accreted by it, or because it librates about the coorbital equilibrium points has to be supplied with the same force per unit mass as the protoplanet, by the protoplanet so as to maintain its migration. This results in an additional force acting on the protoplanet given by

$$F_{crb} = -\frac{1}{2} M_b \Omega_p (dR/dt),$$

(26)

where $M_b$ is the coorbital bound mass. The total force so far is thus

$$F_{cr} = \frac{1}{2} (M_d - M_b) \Omega_p (dR/dt).$$

(27)
However, there are other forces. Because of the breaking of the symmetry of the box, the forces acting due to density waves from the two sides do not necessarily cancel. In this context note that these is an asymmetry in the surface density profile. Thus we should expect a further wave torque component which is proportional to the migration speed. The indication from our simulations is that this acts as a drag on the protoplanet as does $M_b$. Accordingly we shall consider the effects of asymmetric wave torques as modifying the value of $M_b$ and continue to use (27).

Suppose now the protoplanet is acted on by some external torque $T_{ext}$. The equation of motion governing the migration of the protoplanet of mass $M_p$ with speed $dR/dt$ obtained by considering the conservation of angular momentum is

$$\frac{1}{2}M_pR\Omega_p(dR/dt) = \frac{1}{2}(M_d - M_b)R\Omega_p(dR/dt) + T_{ext}.$$  \hspace{1cm} (28)

Accordingly we can consider the planet to move with an effective mass

$$M_{eff} = M_p - (M_d - M_b).$$  \hspace{1cm} (29)

The quantity $(M_d - M_b)$ has been called the coorbital mass deficit by Masset & Papaloizou (2003). When this is positive there is an effective reduction in the inertia of the protoplanet. If there were no asymmetry in wave torques, the coorbital mass deficit is seen to be the amount of mass evacuated in the gap region were it to be initially filled with the background density. Gap filling accordingly reduces the coorbital mass deficit. It is also clear that because at least a partial gap is required, the protoplanet must be massive enough to produce a non linear response in the disk. Hence we have considered $q = 2$.

Masset & Papaloizou (2003) indeed found the coorbital mass deficit to be positive resulting in positive feedback from coorbital torques. They also found that in some circumstances the coorbital mass deficit could become as large as the planet mass so reducing the effective inertia to zero. Under these circumstances a very fast or runaway migration may occur.

We have considered four simulations with torqued disk material giving flow through the coorbital region which correspond to a migrating protoplanet. These are model G with $q = 2.0, b/H = 0.3$ and $\epsilon/h = 3/(8\pi)$, model H with $q = 2.0, b/H = 0.3$ and $\epsilon/h = 3/(64\pi)$, model I with $q = 2.0, b/H = 0.6$ and $\epsilon/h = 3/(8\pi)$, and model J with $q = 2.0, b/H = 0.6$ and $\epsilon/h = 3/(64\pi)$. All of these models are inviscid with no magnetic field. Results for models with flow through and magnetic fields will be presented elsewhere. They all had $q = 2.0$ which, for $h = 0.05$, corresponds to a mass ratio of $2.5 \times 10^{-4}$ which is close to a Saturn mass for a central solar mass. In each case the breakdown of the box symmetry resulted in a net torque acting on the protoplanet.

Typical density contour plots for model G left panel and model H right panel obtained after a quasi steady state had been attained are given in figure 4. In these cases the magnitude of the torque applied to the disk material was chosen to produce expected inflow speeds of $(3/(8\pi))c_s$ and $(3/(64\pi))c_s$ respectively. The effective introducing a flow through the coorbital region is to tend
to fill in the gap and produce an asymmetry in the density profile, these effects being larger for a faster flow. In particular there is a density enhancement leading the planet that leads to a positive torque on the protoplanet assisting its outward migration.

The mean surface density averaged over \( y \) and \( z \) as a function of radial coordinate \( x/H \) for \( q = 2.0 \) with no flow through (model F) and for model G are plotted in figure 5. It is seen that the effect of the flow is to partially fill the gap and to produce a density asymmetry that gives a relative density increase just beyond the outer gap edge. This is responsible for an increase in the torque coming from these regions that acts against the sense of protoplanet migration and can be viewed as reducing the coorbital mass deficit.

Mean surface density averaged over \( y \) and \( z \) as a function of radial coordinate \( x/H \) for models G, H, I, and J are plotted in figure 6. These plots show an increasing amount of gap filling as the flow increases. But note that this effect is less pronounced for the smaller softening parameter because of the more effective gap production in that case.

Theoretical considerations (Masset & Papaloizou 2003) suggest the importance of trapped librating coorbital material. Velocity vectors in the coorbital region for model F with no flow through are plotted in figure 7. This indicates trapped librating trajectories in the coorbital zone essentially symmetrically placed with respect to the protoplanet and occupying a region of width \( w \sim 2H \). For comparison velocity vectors in the coorbital region for model G with fast flow through induced by torqued disk material are plotted in figure 8. This shows a trapped region of librating orbits predominantly leading the protoplanet and shifted inwards.

Running time averages of the force acting on the planet in the direction of orbital motion as a function of dimensionless time for models G, H, I and J are plotted in figure 9. For comparison with theoretical expectations, equation (25) gives in our arbitrary units for the original background density and for the fast flow through rate with \( (dR/dt)/c_s = 3/(8\pi) \), \( F_{cr} = 3.0 \times 10^{-11} \). Similarly for the slow flow through rate with \( (dR/dt)/c_s = 3/(64\pi) \) we get \( F_{cr} = 4.0 \times 10^{-12} \). We see that the latter net running means are consistent with these values. After allowing for adjustment of the background value, model G is a factor of two smaller, model H is a factor of two smaller, model I is a factor of eight smaller while model J is a factor of six smaller. We see therefore that, consistently with their more pronounced gaps, the cases with smaller softening give larger coorbital torques. This appears to be because gap clearance is more effective leading to a larger coorbital mass deficit. In addition the faster flow rates show indication of gap filling and consequent tendency towards torque saturation. An interesting aspect of these net torques is that in the faster flow through cases, the final net torque is comparable to the initial one sided torque. This means that the resulting net torque is comparable to the extrapolated type I value assuming no gap (see also Masset & Papaloizou 2003).
4.2. Occurrence of Fast or Runaway Migration

Fast or runaway migration occurs when the effective mass becomes very small or zero (Masset & Papaloizou 2003). Then the migrating planet loses inertia and can migrate rapidly in response to even small external torques. This requires $M_p = (M_d - M_b)$. If we set $(M_d - M_b) = fM_d$, $f$ is given above as 0.5 for models G, and H, 0.17 for model J, and 0.125 for model I respectively. The smaller cases apply for the larger softening parameter. To obtain $M_{e\text{ff}} = 0$, one requires $M_d = M_p/f$. This means the amount of background disk material required to fill the gap region should be $M_p/f$. One might expect this to be achievable for a massive enough disk. For example taking the case with the weakest coorbital torque with $f = 0.125$, $M_p = 2.5 \times 10^{-4} M_\odot$ corresponding to $q = 2$ for $h = 0.05$, one requires a disk about 20 times more massive than the minimum mass solar nebula in the 5AU region. This is essentially consistent with Masset & Papaloizou (2003) but cautionary remarks need to be inserted regarding the dependence on softening and the extrapolation of the behaviour in the local box to the whole disk annulus.

5. Discussion

In this paper we have reviewed local disk simulations performed using a local shearing box. Such simulations have an advantage of giving higher resolution for the same computational cost as a global simulation, but with their associated boundary condition of periodicity in shearing coordinates possibly introduce artefacts due to neighbouring ghost boxes and protoplanets. However, we saw no evidence that such features, although they can never be removed entirely, were very significant.

We considered protoplanets interacting with both quiescent disks and disks undergoing MHD turbulence with zero net flux fields. The latter case results in angular momentum transport with an effective Shakura & Sunyaev (1973) parameter $\alpha$ related to the effective kinematic viscosity $\nu$ by $\nu = \alpha H^2 \Omega_p$, given by $\alpha \sim 5 \times 10^{-3}$.

In both cases, when a protoplanet is present, there exists a natural scaling indicating that results depend only on the parameter $q = M_pR^3/(M_*H^3)$ and the condition for gap formation is $M_pR^3/(M_*H^3) \sim 1$, being the thermal condition of Lin & Papaloizou (1993).

The symmetry of a shearing box for a non migrating protoplanet ensures cancellation of the torque contributions from exterior and interior to the orbit. In a quiescent disk a natural bias occurs when non local curvature effects are introduced. The interaction is then stronger in the region exterior to the planet leading to a net inward migration that can be estimated using the expression of Tanaka, Takeuchi, & Ward (2002) for the migration time

$$\tau_{\text{mig}} = \frac{M_* M_p}{2.7 M_p \Sigma R^2 h^2 \Omega_p^{-1}}.$$  

(30)

However, the torques acting in a turbulent disk from either side of the orbit are very noisy to the
extent that a ten percent difference between the contributions can remain even after fifty orbits. The noise makes type I migration stochastic and evaluation of the final outcome will require long time global simulations which can simulate the fluctuations that occur on the longest timescales associated with the disk (see Nelson & Papaloizou 2004).

We then considered the coorbital torques experienced by a moving protoplanet in an inviscid disk. This was done by demonstrating, in an appropriate slow migration limit, the equivalence of the problem for a moving protoplanet to one where the protoplanet is in a fixed orbit which the disk material flows through radially as a result of the action of an appropriate external torque.

By considering the torques exerted by material flowing around and through the coorbital region, we showed that we could regard the protoplanet as moving with an effective mass

\[ M_{\text{eff}} = M_p - (M_d - M_b). \]  

(31)

The quantity \((M_d - M_b)\) was called the coorbital mass deficit by Masset & Papaloizou (2003). When this is positive there is an effective reduction in the inertia of the protoplanet by this amount. It is related to, but not exactly equivalent to, the amount of mass evacuated in the gap region were it to be initially filled with the background density. Because at least a partial gap is required the protoplanet must be large enough to produce a non linear response in the disk.

In addition in order to obtain measurable quasi steady coorbital torques a quasi steady state must be realized in which the planet does not accrete significant mass. We found as did Masset & Papaloizou (2003) that the coorbital torques are proportional to the migration speed, when that is sufficiently small, and result in a positive feedback on the migration, enhancing it and potentially leading to a runaway. This could lead to fast migration for protoplanets in the Saturn mass range in massive disks and may be relevant to the mass period correlation for extrasolar planets which gives a preponderance of sub Jovian masses at short orbital periods (see Zucker & Mazeh 2002).

The fast migration is limited in that the time to migrate through the coorbital region of \(w \sim 2H\), should be less than the time it takes material to circulate around the trapped coorbital region. When this is taken to extend for the full \(2\pi\), this limit gives \(R/(dR/dt) > 2\pi(R/H)^2/(3\Omega_p)\) which corresponds to a time on the order of a hundred orbits. Thus a migration stopping or slowing mechanism is required. This may be as a result of the radial density profile of the disk such as for example a reduction of the density at smaller radii and/or a transition to type II migration which operates on the longer viscous time scale (see Masset & Papaloizou 2003 for more discussion).

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Table 1: The first column gives the model label, the second the value of $q$, the third gives the softening parameter $b/H$, the fourth gives $|\epsilon|/h$ the magnitude of which gives the ratio of the induced disk flow through speed to the sound speed and the fifth column indicates whether a magnetic field was present. Model A was a reference model used to check that a box with no protoplanet and no magnetic field did not show any evolution or instabilities.
