**Reaching Fermi degeneracy in two-species optical dipole traps**

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We propose the use of a combined optical dipole trap to achieve Fermi degeneracy by sympathetic cooling with a different bosonic species. Two far-detuned pairs of laser beams focused on the atomic clouds are used to confine the two atomic species with different trapping strengths. We show that a deep Fermi degeneracy regime can be potentially achieved earlier than Bose-Einstein condensation, as discussed in the favorable situation of a \(^6\)Li–\(^23\)Na mixture. This opens up the possibility of experimentally investigating a mixture of superfluid Fermi and normal Bose gases.

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The study of ultracold atomic gases has witnessed in the last decade an impressive development mainly originated from the successful application of new experimental techniques such as laser cooling \(^1\) and evaporative cooling \(^2\). The interplay of these two cooling mechanisms has led to the degeneracy regime for Bose gases \(^3\), and to the related rich phenomenology recently explored \(^4\). However, these cooling techniques cannot be straightforwardly extended to reach degeneracy in Fermi dilute gases, and the status of their exploration is by far less advanced than for Bose systems. One limitation of current efforts to reach Fermi degeneracy arises from the use of magnetic trapping techniques, where spin-polarized Fermi gases are obtained. The Pauli principle limits the efficiency of direct evaporative cooling among fermions in the same hyperfine state, and also inhibits scattering among fermions in different hyperfine states (Pauli blocking) \(^5\). An alternative technique - sympathetic cooling of fermions through coupling to an ultracold bosonic reservoir \(^6\) - is limited by the decreased efficiency of the elastic scattering between fermions and bosons expected when the latter enter a superfluid regime \(^7,8\), and ultimately by Pauli blocking. Moreover, the use of magnetic trapping strongly limits the kind of hyperfine states suitable for obtaining Fermi degeneracy, and interferes with the use of tunable homogeneous magnetic fields aimed at modulating the elastic scattering length via Feshbach resonances \(^9\). Due to the possibility to unveil new phenomena, like a BCS transition to a fermionic superfluid state expected at very low temperatures \(^10,11,12,13\), it is crucial to further explore efficient cooling techniques for fermions.

More flexible trapping tools are offered by optical dipole traps \(^14\). These allow both trapping of different hyperfine states and the application of an arbitrary magnetic field. While preliminary studies on optical trapping of degenerate Bose gases were performed by transferring the condensate from a magnetic trap \(^21\), Bose-Einstein condensation (BEC) \(^21\) and Fermi degeneracy \(^22\) have been achieved directly in all-optical traps, and Feshbach resonances for fermions have been studied in an optical dipole trap loaded from a magnetic trap \(^23\). It is the purpose of this Letter to discuss a further advantage of using an optical dipole trap to achieve Fermi degeneracy, based on the possibility of trapping different species with selective confinement strengths. We discuss the case of an optical dipole trap obtained with the combination of two laser beams resulting in different trapping potentials for the two species of a mixture of Bose and Fermi gases. The trapping potentials can be engineered to make the Fermi gas more strongly confined than the Bose gas and, therefore, the Fermi temperature higher than the BEC critical temperature. In this way, Fermi degeneracy may be reached well before BEC occurs, leading to a promising avenue to look for superfluidity in dilute fermions in the presence of a normal Bose gas. This would provide an unprecedented situation as compared to the already available \(^3\)He–\(^4\)He mixtures.

A limitation of the efficiency of sympathetic cooling arises from the suppression of elastic scattering of fermions by the bosonic reservoir below the Bose-Einstein transition temperature \(^12\). This can be circumvented if the Fermi temperature \(T_F\) is made significantly higher than the BEC transition temperature \(T_c\) of the bosons. Both temperatures scale in the same manner for Fermi and Bose atomic gases confined in the same harmonic potential, just differing by a numerical factor,

\[
T_F = 1.82 \hbar \omega_f N_f^\frac{5}{3} k_B^{-1} \tag{1}
\]

\[
T_c = 0.94 \hbar \omega_b N_b^\frac{5}{3} k_B^{-1} \tag{2}
\]

with \(\omega_f = (\omega_{f1} \omega_{f2} \omega_{f3})^{1/3}\) (\(\omega_b = (\omega_{b1} \omega_{b2} \omega_{b3})^{1/3}\)) being the geometrical average of the angular trapping frequencies in the three directions for fermions (bosons), \(N_f\) and \(N_b\) the number of atoms of the Fermi and Bose gases, and \(\hbar\) and \(k_B\) the Planck and Boltzmann constants, re-
spectively. In a magnetic trap, since the magnetic moments of the alkali-metal atoms are very similar, the only difference in the trapping frequencies of different atomic species is due to the difference in their mass. In an optical dipole trap instead, the trapping frequencies for different atomic species can be made different by orders of magnitude by also properly choosing the laser beams.

The trap we discuss in the following is sketched in Fig. 1. A pair of laser beams is focused on the center of the trapping potential in a crossed-beam geometry and provides, if red-detuned with respect to the atomic transitions, an effective attractive potential for both the atomic species. Another pair of blue-detuned laser beams is added, for instance along the same directions or rotated by 45° in the same plane formed by the first two beams, and focused on the same point. This second pair is used to weaken the attractive potential created by the first one in a selective way. The basic idea is that, due to the different detunings experienced by the two species, the combination of the effective potentials resulting from the laser beams will give rise to a weaker confinement for one species with respect to the other one. If the confinement is stronger for the fermionic species, the degeneracy condition for it will be met earlier than for the more weakly confined bosonic species. The situation seems particularly favorable in the case of a \(^6\)Li–\(^{23}\)Na mixture (see Fig. 1), which has been recently brought to a degenerate regime in a magnetic trap. The two laser wavelengths are chosen at \(\lambda_1 = 1064\ \text{nm}\) and \(\lambda_2 = 532\ \text{nm}\), for instance by using a Nd:YAG laser and a frequency-doubling crystal, respectively. The relevant atomic transition for sodium is at \(\lambda_b = 589\ \text{nm}\) while for lithium is at \(\lambda_l = 671\ \text{nm}\). With respect to the sodium atoms, the lithium atoms are closer to the (attractive) red-detuned laser at 1064 nm and farther from the (repulsive) blue-detuned laser at 532 nm.

With the coordinate system chosen in Fig. 1, the effective potential energy felt by an atom of species \(\alpha\) (\(\alpha = b\) for \(^{23}\)Na and \(\alpha = f\) for \(^6\)Li) and due to the laser beams \(i\) \((i = 1, 2)\) is

\[
U_i^\alpha(x, y, z) = \frac{\hbar \Gamma_\alpha^2}{8 I_i^{\text{sat}}} \left( \frac{1}{\Omega_\alpha - \Omega_i} + \frac{1}{\Omega_\alpha + \Omega_i} \right) \times I_i(x, y, z),
\]

where \(\Gamma_\alpha\) is the atomic transition linewidth, \(\Omega_\alpha = 2\pi c/\lambda_\alpha\), \(\Omega_i = 2\pi c/\lambda_i\), \(I_i\) is the laser intensity, and \(I_i^{\text{sat}}\) is the saturation intensity for the atomic transition. The incoherent sum (obtained by proper polarization or a relative detuning of the orthogonal beams) of the intensities of the two beams propagating along orthogonal directions in the \(xy\) plane and focused at \((x, y, z) = (0, 0, 0)\). According to Fig. 1, we assume that the red-detuned beams propagate along the axes \(x\)-\(y\) while the blue-detuned ones along the axes \(\xi\)-\(\eta\) rotated with respect to \(x\)-\(y\) by an angle \(\theta, \xi = x \cos \theta + y \sin \theta\) and \(\eta = y \cos \theta - x \sin \theta\) with \(0 \leq \theta \leq \pi/4\). In both cases, we can write (with \(\theta = 0\) for the red-detuned beams)

\[
I_i(x, y, z) = \frac{2 P_i}{\pi w_i^2 \left(1 + \frac{\xi^2}{w_i^2}\right)} \exp \left[ -\frac{2(\eta^2 + \zeta^2)}{w_i^2 \left(1 + \frac{\xi^2}{w_i^2}\right)} \right]
\]

\[
+ \frac{2 P_i}{\pi w_i^2 \left(1 + \frac{x^2}{w_i^2}\right)} \exp \left[ -\frac{2(\xi^2 + \zeta^2)}{w_i^2 \left(1 + \frac{x^2}{w_i^2}\right)} \right],
\]

where \(P_i\) is the beam power, \(w_i\) is the 1/e\(^2\) beam waist radius, and \(R_i = \pi w_i^2 / \lambda_i\) is the Rayleigh range.

The total potential experienced by the fermions (bosons) is \(U_f = U_1^f + U_2^f\) \((U_b = U_1^b + U_2^b)\). For \(P_1 / P_2\) large enough, both the potentials \(U_1^f\) and \(U_b^f\) present a minimum at \((x, y, z) = (0, 0, 0)\). According to Fig. 1 and 2, the Fermi and the BEC critical temperatures are determined by the small oscillation frequencies around this minimum. Neglecting the terms \((\lambda_\alpha/\pi w_i)^2\) with respect to unity, we find

\[
\omega_{\alpha x} = \omega_{\alpha y} = \frac{\omega_{\alpha z}}{\sqrt{2}} = \sqrt{\frac{\hbar}{2 \pi m_\alpha} \left( \frac{k_\alpha^2 P_1}{w_1^2} + \frac{k_\alpha^2 P_2}{w_2^2} \right)},
\]

where \(k_\alpha\) is the wave vector for the atomic transition of species \(\alpha\).
where $m_\alpha$ is the mass of an atom of the species $\alpha$ and

$$k_i^\alpha = \frac{\Gamma_i^\alpha}{I_i^\text{sat}} \left( \frac{1}{\Omega^\alpha - \Omega_i} + \frac{1}{\Omega^\alpha + \Omega_i} \right).$$

(6)

Note that the trapping angular frequencies in (6) do not depend on the rotation angle $\theta$ between the blue- and red-detuned laser beams, as a consequence of the rotational invariance of the potential around the local minimum.

In Fig. 2 we show the ratio between the average angular frequencies for the fermionic and the bosonic species, $\omega_i = (\omega_{iz}\omega_{iy}\omega_{ix})^{1/3}$ and $\omega_b = (\omega_{bz}\omega_{by}\omega_{bx})^{1/3}$, as a function of the beam power ratio between the blue- and red-detuned lasers. The relative confinement becomes progressively stronger for the fermion species as the beam power ratio approaches the critical value $P_2/P_1 \approx 0.326$ at which $\omega_b = 0$. By using (7), we find the following expression for the critical power ratio

$$\left. \frac{P_2}{P_1} \right|_{\text{crit}} = \frac{\Omega^2_b - \Omega^2_i}{\Omega^2_i - \Omega_i^2} \left( \frac{w_2}{w_1} \right)^4. \quad (7)$$

The strong dependence of the critical power ratio upon the beam waists can be used to reduce the amount of blue-detuned light necessary to completely deconfine the boson gas. In terms of absolute values, the trapping frequency in the $z$-direction for the bosons in the presence of the red-detuned laser beam alone ($P_2/P_1 = 0$) and a beam waist $w_1 = 10\mu m$ is $\nu_{bz} = \omega_{bz}/2\pi \approx 15.87 P_1^{1/2} \text{ KHz}$, with $P_1$ in Watts. Thus, the optical dipole trap allows for large absolute degeneracy temperatures comparable or superior to the largest values obtained with ingenious designs of magnetic traps [24], and also mitigates any loss of efficiency in the evaporative cooling attributable to the differential gravitational sagging of the two trapped species. Moreover, the presence of a stiffer confinement for the fermions also increases the efficiency of evaporative cooling near Fermi degeneracy, because it always maintains a large overlap with the less confined bosonic species and the latter, unlike the fermion cloud, progressively shrinks in size.

To check for the confinement feature corresponding to various beam power ratios, we have also studied the minimal potential energy depth (confining energy) $\Delta U$ of the exact potentials $U_1$ and $U_b$. The behavior of $\Delta U$ for the fermionic and bosonic species is shown in Fig. 3 for four possible angles $\theta$ between the red-detuned and the blue-detuned beam pairs. We see that bosons are always less confined than fermions, and that the confinement is weaker for both species when $\theta \neq 0$. Thus evaporative cooling of $^{23}\text{Na}$ does not affect significantly the confinement of $^6\text{Li}$.

Based on Fig. 3 we can imagine the following evaporative cooling dynamics. First, the red-detuned power $P_1$ is decreased as typically done in an optical dipole trap [24]. When the temperature is approaching $T_F$, the blue-detuned laser is turned on with the ratio $P_2/P_1$ kept constant during the following stage of evaporation. There is a trade-off in choosing the final ratio $P_2/P_1$ since the shallower confinement of the bosons also results in a smaller peak density of this species, hence a smaller elastic scattering rate, affecting both evaporative cooling...
tive cooling and the subsequent sympathetic cooling of the fermions. With a power ratio $P_2/P_1 \simeq 0.31$, equal waists of the laser beams, and $N_1 = N_b$, according to \ref{eq:1} and \ref{eq:2}, a minimum temperature $T \simeq 5 \times 10^{-2} T_F$ can be achieved before the bosons condense. Deviations from the ideal Fermi-Bose mixture are negligible at temperatures above $T_F$, due to the weak interatomic interactions \ref{eq:24}. Thus one can assume ideal density profiles for fermions and bosons and estimate the elastic collision rate at the final stage of evaporation. For sodium atoms we have $\Gamma_{el} \simeq 30 \text{ Hz}$ for $P_1 = 10 \text{ mW}$, $w_1 = 10 \mu\text{m}$, and $N_b = 10^6$ \ref{eq:27}. This rate should be compared to the residual Rayleigh scattering from the blue-detuned beams which limit the atom lifetimes, provided that other heating sources such as laser intensity, frequency and beam-pointing stabilities are properly optimized. An estimate of the Rayleigh scattering rates for $^6\text{Li}$ and $^{23}\text{Na}$ gives $\gamma_2^{Na} = 5.7 \times 10^{-11} \text{ Hz}$, $\gamma_2^{Na} = 4.5 \times 10^{-11} \text{ Hz}$, $\gamma_2^{23\text{Li}} = 7.7 \text{ Hz}$, and $\gamma_2^{23\text{Na}} = 38 \text{ Hz}$ due to the red-detuned and blue-detuned beams, respectively, all referred to a laser power of 1 W and evaluated at the peak laser intensity. The biggest contribution comes from the blue-detuned beam acting on sodium atoms, due to the proximity of the corresponding frequencies. However, this is of less concern when one considers that the blue-detuned beams are only turned on in the latest stage of the evaporation, when approaching the Fermi degeneracy regime. Moreover, their powers are progressively increased to less than 1/3 of the simultaneous value of the power of the red-detuned beams (with $w_2 < w_1$ this ratio can be made even smaller according to Eq. \ref{eq:6}) while the latter undergo a continuous decrease. For a power $P_2 = 3.1 \text{ mW}$, in the same conditions as above we obtain a Rayleigh scattering rate $\gamma_2^{Na} = 1.2 \times 10^{-11} \text{ Hz} \simeq 1/250 \Gamma_{el}$. Thus atom lifetimes in excess of about 10 s should be achievable. These are long enough to perform experiments requiring mechanical stirring of the fermion cloud. The above analysis can be repeated by considering a CO$_2$ laser as red-detuned source. In this case, we should expect advantages in terms of larger and more stable available power, superior beam-pointing stability, and smaller residual Rayleigh scattering. \ref{eq:28} We plan to discuss this point in detail elsewhere.

In conclusion, we have outlined a novel strategy to reach a deep Fermi degenerate regime based upon a proper engineering of a two-species optical dipole trap. The case of a $^6\text{Li}$-$^{23}\text{Na}$ mixture has been discussed in detail also due to the very favorable properties of this mixture recently reported in \ref{eq:1}. The strategy can be also applied to other mixtures such as $^{40}\text{K}$-$^{87}\text{Rb}$ \ref{eq:2} or $^6\text{Li}$-$^{87}\text{Rb}$ for which $\lambda_0 > \lambda_1$, by choosing a blue-detuned beam wavelength such that $\lambda_1 < \lambda_0 < \lambda_0$. Our proposal could pave the road to the experimental study of a novel phase consisting of a superfluid Fermi gas and a normal Bose gas, a situation precluded in the $^3\text{He}$-$^4\text{He}$ Fermi-Bose mixtures. Besides providing access to a new system interesting in itself, this could considerably simplify signatures for fermion superfluidity based on the direct imaging of the density profile of the trapped fermions \ref{eq:18}.

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