Low-energy $E1$ strength in select nuclei: Possible constraints on the neutron skins and the symmetry energy

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Correlations between low-lying electric dipole ($E1$) strength and neutron skin thickness are systematically investigated with the fully self-consistent random-phase approximation using the Skyrme energy functionals. The presence of strong correlation among these quantities is currently under dispute. We find that the strong correlation is present in properly selected nuclei, namely in spherical neutron-rich nuclei in the region where the neutron Fermi levels are located at orbits with low orbital angular momenta. The significant correlation between the fraction of the energy-weighted sum value and the slope of the symmetry energy is also observed. The deformation in the ground state seems to weaken the correlation.

PACS numbers: 21.10.Pc, 21.60.Jz, 25.20.-x

The isospin-dependent part of the nuclear equation of state (EOS), especially the symmetry energy, is receiving current attention [1, 2]. Although the symmetry energy at the saturation density $E_{sym}(\rho_0)$ is relatively well known, its values at other densities, which have a strong impact on the description of neutron stars and stellar explosions, are poorly determined at present. Information on the density dependence of the symmetry energy might be obtained from the neutron-skin thickness $\Delta r_{np}$, since the skin thickness was found to be strongly correlated with the slope $L$ of the symmetry energy: $L = 3\rho_0 E_{sym}^{\prime}(\rho_0)$ [3, 4]. However, the large uncertainties in measured neutron-skin thickness have practically prohibited us from making an accurate estimate on $L$.

The electric dipole ($E1$) response is a fundamental tool to probe the isovector property of nuclei. The giant dipole resonance (GDR), which is rather insensitive to the structure of an individual nucleus, provides information on the magnitude of the symmetry energy near the saturation density $\rho_0$. In contrast, the low-energy $E1$ modes, which are often referred to as pygmy dipole resonances (PDR), is sensitive to the nuclear structure, such as the existence of loosely bound nucleons. Thus, the PDR, which is currently of significant interest in physics of exotic nuclei, may carry information on the symmetry energy $E_{sym}(\rho)$ at densities away from $\rho_0$.

Among many issues on the PDR, the correlation between the PDR and neutron skin is one of important sub-issues currently under dispute. If the strong correlation exists, the PDR may constrain both $\Delta r_{np}$ and the slope parameter $L$. The calculation by Piekarewicz with the random-phase approximation (RPA) based on the relativistic mean-field model predicted a linear correlation for $L$ [3]. The correlation between the PDR strength and $\Delta r_{np}$ is very weak [10]. Recently, they have extended their studies to the $E1$ strength at finite momentum transfer $q$ [11]. It should be noted that these conclusions, which seemed to contradict each other, were given from RPA calculations for specific spherical nuclei using different ways of analysis.

Recently, we have performed a systematic RPA calculation on the PDR for even-even nuclei [12] using the finite amplitude method [13, 14]. The calculation is self-consistent with the Skyrme energy functional and fully takes into account the deformation effects. We found that the significant enhancement of the PDR strength takes place in regions of specific neutron numbers. The main purpose of the present paper is to show that the quality of the correlation between the PDR strength and $\Delta r_{np}$ are also sensitive to the neutron number of the isotopes. Namely, the strong correlation exists only in particular neutron-rich nuclei. This may provide a possible suggestion for future measurements to constrain $\Delta r_{np}$ and $L$.

Numerical calculations — We perform an analysis similar to Ref. [10] to investigate the Skyrme parameter dependence of the RPA results for nuclei of many kinds (mostly with $Z \leq 40$), including stable, neutron-rich, spherical, and deformed nuclei. The fully self-consistent RPA equation is solved using a revised version of the RPA code in Ref. [18]. The size of the RPA matrix is reduced by assuming the reflection symmetry of the ground state with respect to $x = 0, y = 0$, and $z = 0$ planes. We adopt the representation of the three-dimensional adaptive Cartesian grids [19] within a sphere of the radius $R_{\text{max}} = 15$ fm. The real-space representation has an advantage over other representations, such as harmonic oscillator basis, on the treatment of the continuum states.

The Skyrme functional of the SkM* parameter set [20] is used unless otherwise specified. The residual interaction in the present calculation contains all terms of the Skyrme interaction including the residual spin-orbit, the residual Coulomb, and the time-odd components. The
pairing correlation is neglected for simplicity, which has little impact on E1 modes [17].

Definition of PDR strength, PDR fraction, and correlation coefficient—We define the PDR strength as

\[ S_{\text{PDR}} \equiv \int_0^{\omega_c} S(E1;E)dE = \sum_n E_n < \omega_c B(E1;n), \]

with the PDR cutoff energy \( \omega_c \). The PDR fraction \( f_{\text{PDR}} \) is the ratio of the integrated photoabsorption cross section below \( \omega_c \) to the total integrated cross section.

\[ f_{\text{PDR}} = \frac{\int_0^{\omega_c} \sigma_{\text{abs}}(E)dE}{\int_0^{\omega_{\text{tot}}} \sigma_{\text{abs}}(E)dE} = \frac{\sum_n E_n < \omega_c E_n B(E1;n)}{\sum_n E_n B(E1;n)}, \]

In Eqs. (1) and (2), we fix the cutoff at \( \omega_c = 10 \) MeV. Many former works adopted the same definition [10, 12], because of its simplicity. In light spherical neutron-rich nuclei, the value of \( \omega_c = 10 \) MeV can reasonably separate the PDR peaks from the GDR. However, for heavier nuclei, the separation becomes more ambiguous. It is especially difficult for deformed nuclei. Later, we introduce another definition of the PDR strength using a variable \( \omega_c \), to check the validity.

To quantify the correlation between two quantities, we use the correlation coefficient \( r \). When we have data points for \( (x_i, y_i) \) with \( i = 1, \cdots, N_d \), it is defined by

\[ r = \frac{\sum_{i=1}^{N_d} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{N_d} (x_i - \bar{x})^2 \sum_{j=1}^{N_d} (y_j - \bar{y})^2}} \]

where \( \bar{x} \) and \( \bar{y} \) are the mean values of \( x_i \) and \( y_i \), respectively. The absolute value of \( r \) does not exceed the unity. A perfect linear correlation, \( y_i = a x_i + b \), corresponds to \( r = \pm 1 \) with the same sign as that of parameter \( a \). In the followings, the correlation with \( r > 0 \) (\( r < 0 \)) is referred to as “positive” (“negative”) correlation.

Neutron skin thickness in \(^{208}\text{Pb}\) — First, we confirm the result in Ref. [10]. Reference [10] reported that the \( S_{\text{PDR}} \) for \(^{132}\text{Sn}\) has only a weak correlation with the neutron skin thickness defined by \( \Delta r_{np} = \sqrt{r_{\text{p}}^2} - \sqrt{r_{\text{p}}^2} \) of \(^{208}\text{Pb}\). In Fig. 1 the \( S_{\text{PDR}} \) for \(^{132}\text{Sn}\) is shown as a function of the neutron skin thickness, \( \Delta r_{np} \), of \(^{208}\text{Pb}\). The plotted 21 points are obtained by calculating \( \Delta r_{np} \) with the SkM* functional, and with slightly modified values of 10 Skyrme parameters (\( t_{0.1,2.3}, 7.0_{1.2.3}, W_0, \) and \( \alpha \)). It seems to indicate some correlation, however, the calculated points are somewhat scattered.

Using these 21 sample values (\( N_d = 21 \)), the correlation coefficient \( r \) is calculated according to Eq. (3). In the present case of Fig. 1 we obtain the coefficient \( r = 0.55 \). The correlations between \( \Delta r_{np} \) and \( S_{\text{PDR}} \) in \(^{68,78,84}\text{Ni}\) are also weak with \( r = 0.5 - 0.6 \). Thus, the PDR strength in these spherical (magic) nuclei indicate a positive correlation with the skin thickness in \(^{208}\text{Pb}\), however, the correlation is weak. This is qualitatively consistent with the result in Ref. [10].

Correlation between \( S_{\text{PDR}} \) and \( \Delta r_{np} \) — Next, we discuss the same correlation, but between the \( \Delta r_{np} \) and \( S_{\text{PDR}} \) in the same nucleus. In Fig. 2 (a) we show the results for \(^{68}\text{Ni}\) (\( N = 40 \)), \(^{78}\text{Ni}\) (\( N = 50 \)), and \(^{84}\text{Ni}\) (\( N = 56 \)). The scattered data points in Fig. 2 (a) suggest a relatively weak correlation in \(^{68}\text{Ni}\), while the correlation becomes moderately strong for \(^{78}\text{Ni}\). The calculated correlation coefficients are \( r = 0.69 \) and 0.76 for \(^{68,78}\text{Ni}\), respectively.
respectively. In contrast, a very strong linear correlation with \( r = 0.94 \) for \(^{84}\text{Ni}\) is observed in Fig. 2(c). It is apparent that the linear correlation is qualitatively different among the isotopes.

The qualitative difference in \( S_{\text{PDR}} \) among the isotopes was previously observed in the PDR photoabsorption cross section \(^{12}\). In Ref. \(^{12}\), we systematically calculated, for even-even nuclei up to \( Z = 40 \), the PDR fraction \( f_{\text{PDR}} \). Then, we found that \( f_{\text{PDR}} \) significantly increases as a function of neutron number in regions where the neutron Fermi levels are located at the weakly-bound low-\( \ell \) shells, such as \( s \), \( p \), and \( d \) orbits. In Ni isotopes, this corresponds to the region with neutron number beyond 50, as illustrated in Fig. 2(d). Thus, the present result (Fig. 2(a)-(c)) indicates that the neutron shell effect also has a significant impact on the linear correlation between the neutron skin thickness and the PDR strength.

We confirm the same neutron shell effect in other light spherical isotopes; \(^{50}\text{O}\) and \(^{48}\text{Ca}\). For Ca isotopes, the PDR strength appears beyond \( N = 28 \) \(^{12}\). Accordingly, the strong linear correlation can be seen for \(^{52}\text{Ca}\) and \(^{54}\text{Ca}\), in Fig. 3. The calculated correlation coefficients are \( r = 0.91 \) and 0.96 for \(^{52,54}\text{Ca}\), respectively. These nuclei have neutrons more than 28 and the neutron Fermi level is located at the \( p \) shell. They are predicted to have the PDR peaks around \( E = 8 \text{ MeV} \) with \( f_{\text{PDR}} \approx 0.03 - 0.04 \) \(^{12}\). In contrast, nuclei with \( N \leq 28 \) have very small values of \( f_{\text{PDR}} < 0.01 \) and the linear correlation in \(^{48}\text{Ca} \) (\( N = 28 \)) indicates \( r = 0.78 \) which is much weaker than \(^{52,54}\text{Ca}\). For \( O \) isotopes, because of the neutron occupation of the \( 2s \) orbit, \(^{24}\text{O} \) (\( N = 16 \)) provides another example to show a significant jump in \( f_{\text{PDR}} \) from \(^{22}\text{O} \). This nucleus has the strongest linear correlation with \( r = 0.97 \).

We also calculate the correlation coefficient for \(^{132}\text{Sn}\). It indicates a relatively weak correlation with \( r = 0.68 \). Note that \(^{132}\text{Sn}\) corresponds to a kink point similar to \(^{78}\text{Ni}\) in Fig. 2. Namely, the PDR fraction in Sn isotopes will jump up beyond \( N = 82 \) \(^{21}\). The correlation coefficients are summarized in the second column of Table 1 for various nuclei.

**Deformed nuclei** — The deformation effect seems to somewhat weaken the correlation. Figure 4 shows two deformed nuclei, \(^{58}\text{Cr}\) with the quadrupole deformation of \( \beta_2 = 0.17 \) and \(^{110}\text{Zr}\) with a larger deformation of \( \beta_2 = 0.36 \). The \(^{58}\text{Cr}\) nucleus has the same number of neutrons as \(^{54}\text{Ca}\) carrying a comparable PDR strengths to \(^{52}\text{Ca} \). Nevertheless, the correlation in \(^{58}\text{Cr} \), \( r = 0.80 \), is significantly weaker than that in spherical \(^{52,54}\text{Ca}\). \(^{110}\text{Zr}\) has an even larger deformation and a weaker correlation, \( r = 0.74 \), although it has sizable PDR strength. The ground-state deformation is expected to produce a peak splitting both in the PDR and GDR. Due to the complicated characters in the \( E1 \) strength distribution, the PDR strength \( S_{\text{PDR}} \) may be contaminated by the low-energy tail of GDR strength.

**Universal behaviors** — The property of the linear correlation is very robust with respect to choice of the Skyrme energy functionals. In Fig. 5 we show the same correlation plot as Fig. 2 calculated with the parameter set of SkM* replaced by SIII \(^{22}\) and SGII \(^{23}\). All the three Skyrme functionals yield a relatively weak corre-
TABLE I: Calculated correlation coefficients $r$ between $\Delta r_{np}$ and $S_{PDR}$ for selected nuclei. The SkM$^*$ parameter set is adopted as the central values. The values of variable $\omega_c$ are also listed. Note that we cannot identify a prominent PDR peak for $^{48}$Ca. $r^{(v)}$ are obtained with the variable cutoff energies $\omega_c$ in the fourth row. The correlation coefficients larger than 0.9 are shown in boldface.

|       | $^{24}$O | $^{26}$Ne | $^{48}$Ca | $^{52}$Ca | $^{54}$Ca | $^{68}$Ni | $^{78}$Ni | $^{84}$Ni | $^{58}$Cr | $^{110}$Zr |
|-------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| $r$   | 0.97    | 0.83    | 0.78    | 0.91    | 0.96    | 0.69    | 0.76    | 0.94    | 0.80    | 0.74    |
| $r^{(v)}$ | 0.97    | 0.88    | -       | 0.92    | 0.94    | 0.77    | 0.92    | 0.96    | 0.80    | 0.84    |
| $\omega_c$ [MeV] | 8.29    | 9.95    | -       | 10.49   | 9.41    | 11.48   | 8.73    | 8.59    | 9.82    | 8.36    |

The slope of the straight line, obtained by linear fit, turns out to be universal too, with respect to different Skyrme energy functionals. All these three parameter sets (SkM*), SGII, and SIII) produce the similar slope, $dS_{PDR}/d(\Delta r_{np}) = 13 - 16 \text{e}^{2}\text{fm}$ for $^{84}$Ni. We observe the linear correlation of $f_{PDR}$ instead of $S_{PDR}$, as well, with respect to $\Delta r_{np}$. However, in this case, the slope obtained by the linear fit has a sizable dependence on functionals.

Correlation among different energy functionals — Instead of slightly modifying the Skyrme parameters, we next examine the correlation adopting many different Skyrme functionals corresponding to a variety of values of the $L$ parameter; SIII, SGII, SkM*, SLy4 [22], SkT4 [24], SkI2, SkI3, SkI4, SkI5 [27], UNEDF0, and UNEDF1 [28]. From these eleven different parameter sets, we estimate the correlation coefficient $r$ in Eq. (3) with $N_d = 11$. Again, we have found a weak correlation with $r = 0.47$ for $^{68}$Ni, and a strong correlation $r = 0.89$ for $^{84}$Ni.

We also examine the correlation between the slope parameter of the symmetry energy $L$ and the PDR fraction $f_{PDR}$, in $^{68}$Ni and $^{84}$Ni. This leads to the similar coefficients, $r = 0.37$ and 0.84 for $^{68}$Ni and $^{84}$Ni, respectively. Thus, to quantitatively constrain $\Delta r_{np}$ and $L$, the measurement of the PDR in the very neutron-rich $^{84}$Ni is more favored than in $^{68}$Ni.

The small correlation coefficient between $L$ and $f_{PDR}$ for $^{68}$Ni ($r = 0.37$) turns out to be due to the fact that the choice of $\omega_c = 10 \text{MeV}$ has different meaning for different functionals. Namely, the different energy functionals produce different PDR peak energies, some of which are below 10 MeV but some are above that. The tail of the GDR strength also depends on the choice of the energy functionals. Therefore, to make a more sensible analysis for this study, we should use the variable cutoff $\omega_c$. This will be discussed below.

Use of variable $\omega_c$ — The PDR strength [11] and PDR fraction [2] based on variable $\omega_c$ are hereafter referred to as $S^{(v)}_{PDR}$ and $r^{(v)}_{PDR}$ respectively. The variable $\omega_c$ is determined according to the following procedure: The calculated (discrete) $B(E1)$ values are smeared with the Lorentzian with the width of $\gamma = 1 \text{MeV}$. Plotting this smeared $E1$ strength $S(E1;E)$ as a function of energy, if we can find a distinguishable PDR peak and its energy $E_{\text{peak}}$, $\omega_c$ is defined as the energy corresponding to the minimum value of $S(E1;E)$ at $E > E_{\text{peak}}$. In Fig. 6, as an example, the determination of $\omega_c$ is shown for $^{84}$Ni. Since the determination of the variable $\omega_c$ requires a noticeable PDR peak structure, it is difficult to define $S^{(v)}_{PDR}$ for most of stable isotopes.

The values of $\omega_c$ varies from nucleus to nucleus within a range of $10 \pm 2 \text{MeV}$ for those listed in Table I. Note that $\omega_c$ may also change when we slightly modify the Skyrme parameters. Although the correlation is slightly enhanced by replacing $S_{PDR}$ by $S^{(v)}_{PDR}$ in most cases, they are approximately similar, $r^{(v)} \approx r$. In Table I there are a few exceptions; $^{78}$Ni ($r = 0.76 \rightarrow r^{(v)} = 0.92$), $^{68}$Ni ($r = 0.69 \rightarrow r^{(v)} = 0.77$), and deformed $^{110}$Zr ($r = 0.74 \rightarrow r^{(v)} = 0.84$). In these cases, we found that the separation between PDR and GDR is somewhat ambiguous, and the results depend on the choice of $\omega_c$. On the other hand, isotopes indicating $r > 0.9$ with fixed $\omega_c = 10 \text{MeV}$ show $r^{(v)} \approx 1$ with variable $\omega_c$ as well. In Ni isotopes, although the value of $r^{(v)}$ are slightly different from $r$, it is confirmed that the linear correlation is significantly stronger in $^{84}$Ni than in $^{68}$Ni.

For eleven different parameter sets, the correlation between $S^{(v)}_{PDR}$ and $\Delta r_{np}$ for $^{68,84}$Ni is shown in the upper part of Fig. 7. A strong positive correlation ($r^{(v)} > 0.9$) between the PDR strength $S^{(v)}_{PDR}$ and $\Delta r_{np}$ can be seen in $^{84}$Ni. In contrast, it is significantly weaker for $^{68}$Ni ($r^{(v)} = 0.48$). The bottom part of Fig. 7 shows correla-
between $f^{(v)}_{\text{PDR}}$ and $L$ (bottom) for $^{68}\text{Ni}$ (left) and $^{84}\text{Ni}$ (right), among eleven different Skyrme functionals.

The linear correlation is seen only in particular nuclei. The PDR strength has a very strong linear correlation with the neutron skin thickness in spherical neutron-rich nuclei with $14 < N \leq 16$, $28 < N \leq 34$, and $50 < N \leq 56$. In these regions, the neutron Fermi levels are located at the loosely-bound low-$\ell$ shells and the PDR strength significantly increase as the neutron number. Nuclei outside of these regions have weaker correlations. This linear correlation is robust with respect to the choice of the energy functional parameter set. This suggests that the experimental observation of PDR in properly selected neutron-rich nuclei could be a possible probe of the neutron skin thickness $\Delta r_{np}$ and a constraint on the slope parameter $L$ of the symmetry energy. The linear correlation seems to be weakened by the deformation due to the peak splitting of the PDR and the GDR. The present result may provide a solution for the controversial issue on the correlation between the PDR and the neutron skin, for which different conclusions were reported previously [5, 6, 8, 10].

**Acknowledgments**

This work is partly supported by HPCI System Research Projects (Project ID: hp120192 and hp120287), by Collaborative Interdisciplinary Program (Project ID: 13a-33, 12a-20 and 11a-21) at University of Tsukuba, and by JSPS KAKENHI Grant numbers 21340073, 24105006, 25287065, and 25287066. The numerical calculations were partially performed on the RIKEN Integrated Cluster of Clusters (RICC) as well.

**Summary** — We have studied the correlation of the PDR and the neutron skin thickness, for nuclei with $Z \leq 40$ and $^{132}\text{Sn}$. We have found that the strong linear correlation is seen only in particular nuclei.

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