Research on Mission Assignment Assurance of Remote Rocket Barrage Based on Stackelberg Game

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Abstract: Aiming at the problem of task assignment of maintenance and repair for long-range rocket launchers, based on systematic review of the basic concepts and basic principles of Stackelberg game, Through the basic assumption’s analysis, the Stackelberg model is designed to allocate far-fire tasks. Finally, the results of running MATLAB are calculated. It shows that Stackelberg model of far-fire task allocation can effectively solve the problem of long-range rocket launcher maintenance and guarantee task assignment, which has strong practical significance.

Key words: Stackelberg game; long range rocket artillery; task assignment

1. Introduction
The use of artillery fire, as an important means in information warfare, can effectively kill the enemy's living force, destroy the enemy's defense facilities and make the enemy lose its ability to fight. The long-range rocket artillery (hereinafter referred to as "far fire") assault group will be an important part of the campaign artillery, and it is the main force to carry out the deep fire assault and the long distance precision strike. The direct guarantee of the maintenance of the people will directly reduce the intermediate storage and transfer links, and save the waste of resources brought by the intermediate redundancy. One of the core aims is to improve the supply and timeliness of the equipment and equipment. In the process of military self-protection, far fire equipment and equipment support and timeliness are affected by many aspects, such as intermediate link and security level, and under the guarantee of local units, the reduction of intermediate links accelerates the speed of information and material circulation, and has become the external factor to promote the improvement of equipment support and time of far fire. The enthusiasm, initiative and sense of responsibility of the unit are the internal factors that affect the timeliness of the equipment for the remote fire equipment. Effective incentives can fully promote the initiative and enthusiasm of local units, and make up for the losses caused by the increase in the stock of local units. However, excessive incentives will affect the own interests of the military. In order to maximize the interests of the military, and to ensure the timeliness of the remote fire equipment and equipment support, it is necessary to design a reasonable incentive strategy for local units.

In the field of remote fire equipment maintenance support research, some scholars have put forward many methods. From the shortcomings of the effectiveness evaluation of the far fire equipment, Huang Shaoluo[1] analyzed the two aspects of the mission success and the readiness, established the functional composition model, the maintenance support system model, the operational task model and the
effectiveness evaluation model respectively, and found out the prominent problems that existed in the former by the sensitivity analysis; Jin Shuchang[2] passed through the analysis. This paper analyzes the failure mode of communication control machine for remote fire equipment, gives the structure analysis, function analysis, failure influence analysis and failure mode analysis of the communication control machine, and finally obtains the influence analysis report. Shen Hong Bao[3] is a qualitative analysis of the training situation and Countermeasures of the remote rocket artillery command system. The song Guo He[4] is a simulation training platform for the fire control system of the far fire, and discusses its design idea, the system physical structure, the function structure of the system and the overall design of the system software.

Although there are many researches on remote fire equipment, there is no research on the maintenance support incentive model for remote fire equipment contractor. In this paper, by analyzing and assuming the basic condition of the maintenance support of the current far fire equipment contractor, this paper establishes an incentive model based on the two stage Stackelberg game. By calculating the cost objective function of the military and the expected income of the people, and solving the model through the genetic algorithm, the maintenance task of the far fire equipment contractor is realized. Match. Finally, the effectiveness is verified. The example shows that the method is effective.

2. Basic principles

2.1. Stackelberg game theory

Stackelberg game is a game problem that conforms to the principal and subordinate relationship among the decision-makers, that is, the position of the game party that is divided into 2 decision-making layers is unequal, and the game square in the upper level is more influential than the lower one, which is called the leader and the followers respectively [5]. The leader always takes the lead in making decisions, while the follower makes the best decisions based on the strategy of the leader and other followers at the same level.

A game model usually consists of 3 elements: game, strategy and profit. Because Stackelberg game has two different decision-making levels, the players are divided into two categories, and each has its own strategy space and income.

The game side: the definition of $m$ leader and $n$ follower, respectively, using 2 sets of $M = \{1, 2, \cdots, m\}$ and $N = \{1, 2, \cdots, n\}$ to express. Policy: The leader's policy combination is defined as $x = \{x_1, x_2, \cdots, x_m\}$, the set as $X$; the follower's policy combination is defined as $y = \{y_1, y_2, \cdots, y_n\}$, and the set as $Y$.

Earnings: the earnings of the leader $i$ are $U_{si}$, and the followers' earnings are $U_{oj}$, $i \in M$, $j \in N$.

The above game model is defined as a game with multiple leaders and multiple followers. Assuming that the leader has made a decision, each participant in the game plays a non-cooperative game under the decision to maximize its own income [6], then the best strategy of the followers under the dominant strategy can be expressed as:

$$ S(x) = \{y^* = (y_1^*, y_2^*, \cdots, y_n^*): U_{oj}(x, y_1^*, y_j^*) \geq U_{oj}(x, y_i^*, y_j^*) \}$$ (1)

Among them, $y^*$ represents the best strategy of the follower $j$; $y_{-j}^*$ represents the best set of strategies for other followers other than $j$. In the case of the dominant strategy, each follower has a best strategy, and it does not gain greater returns by unilaterally adjusting its own strategy, when all follow The optimal policy space $y^* = (y_1^*, y_2^*, \cdots, y_n^*)$ is the Nash equilibrium point of the non-cooperative game.
The whole Stackelberg game process consists of two stages: the dominant decision-making stage and
the follower decision-making stage, in which the game problem is represented as
\( G = \{ x, y, U_{si}(x, y), U_{ij}(x, y) \} \), where \( i \in M, j \in N \). First, the leader takes the lead in making the
strategy, then the followers make decisions according to the leader's strategy, and then the leader adjusts
the strategy according to the followers' strategy. The 2 phases alternately will not change as the leader
strategy, and the Nash equilibrium of the game is achieved when the follower's strategy is adjusted to
the best. The process can be expressed as formula: solving \((x^*, y^*) \in X \times Y\) and establishing formula
(2).

\[
U_{si}(x^*_i, x^*_i, y^*) \geq U_{si}(x_i, x^*_i, y^*)
\]  

(2)

Among them, \( i \in M, y^* \in S(x) \). If any strategy \((x^*, y^*) \in X \times Y\) satisfies the upper form, then
\((x^*, y^*)\) is called the subgame perfect Nash equilibrium point of the Stackelberg game [7].

2.2. Genetic algorithm
In order to verify the proposed game model, genetic algorithm is used to solve it. Genetic Algorithm
(GA) is more and more used in the fields of pattern recognition, robot control, signal processing and
combinatorial optimization by learning from the genetic mechanism of the survival of the fittest.

The processing object of the genetic algorithm is not the parameter itself, but the individual that
encodes the parameter set. The genetic algorithm uses the target function value as the search information,
and uses the fitness function value worth coming from the target function. It can determine the direction
of the next search and the search range, by adopting the adaptive probability search technique. It
increases the flexibility of its search process. According to the basic principle of genetic algorithm, the
better degree of the optimal solution is measured by the fitness degree, the stronger the adaptability of
the individual, the higher the corresponding adaptability, the better the representative solution.

3. Problem description and basic hypothesis
The relationship between the general and the local units in the incentive decision is regarded as a two
stage Stackelberg game with complete information. Both sides expect the minimum expected cost to be
the objective function in the far fire equipment support activities and achieve the overall optimal. The
focus of our concern is the cost arising from changes, so the model does not consider the order price of
the product.

Assume 1: no advance delivery, local units will produce a part of the inventory cost, the cost of
completion of the order, including the completion time to the supply time inventory cost
\( C_h = \int_0^T (t-T) f(t) dt \), \( T \) for the military and local units to supply time, \( t \) is the generation of far
fire equipment demand, \( f(t) \) for far fire equipment demand time The probability density, which obeys
the Weibull distribution, \( f(x) = 1 - e^{-\frac{(x-h)}{3}}, h(h \geq 0) \), is the unit time inventory cost for local units;
similarly, if the local unit fails to finish the order in time, it will increase the cost of the labor, and the
the corresponding cost is \( C_g = g \int_0^T (T-t) f(t) dt \), in which \( g(g > 0) \) is the cost for the local
enterprises to drive the work per unit time.

Hypothesis 2: Local units in order to deliver on time and choose a certain degree of effort \( e \), it is a
one-dimensional continuous variable, the larger \( e \) means the more effort. This will lead to the effort
cost \( C_e = \frac{1}{2} \eta e^2 \), in which the effort cost coefficient \( \eta > 0 \), the greater the negative effect of the same
effort level \( e \), the greater the negative effect of the same level of effort; and at the same time, the
$R(e) = \varphi e + \phi \epsilon$ will have output gains, in which, \(\varphi\) is an effort to produce a coefficient, and \(\epsilon\) is an exogenous variable that is not influenced by the external factors controlled by the military and local units, and it is assumed to be \(\epsilon \sim N(0, \sigma^2)\). \(\phi\) is the influence coefficient of exogenous variables.

Hypothesis 3: The incentive policy implemented by the military takes the linear form used in most studies, namely \(C_i = \Gamma R(e)\), where \(\Gamma\) is the military's incentive coefficient.

Hypothesis 4: The military is the risk neutral, the local unit uses the Arrow-Pratt way of risk aversion, then the local unit's risk cost is \(C_r = \frac{1}{2} \rho \Gamma \phi^2 \sigma^2\), and \(\rho\) is the risk aversion coefficient.

4. Model establishment

In order to make the self-interest of the army unaffected and to increase the enthusiasm of the local units as much as possible, the objective function should minimize the cost function of the army. At the same time, the expected income of the local units is the most, and the objective function is to set up the following functions as follows:

\[
\begin{aligned}
\max C_s \\
\min C_m
\end{aligned}
\] (3)

The objective function of obtaining the income of local units is:

\[
\begin{aligned}
\max C_s &= C_i - (C_h + C_g + C_e + C_r) \\
&= \Gamma(\varphi e + \phi \epsilon) - [h \int_0^T (t - T) f(t)dt + g \int_0^T (T - t) f(t)dt + \frac{1}{2} \eta e^2 + \frac{1}{2} \rho \Gamma \phi^2 \sigma^2]
\end{aligned}
\] (4)

The unit time loss coefficient of the equipment support for remote fire equipment is \(k\) because of the shortage, and the change costs that the military needs to consider include the incentive costs resulting from the implementation of the incentive and the cost of the loss caused by the delay in the supply of local units. The task of the military's incentive is to determine a reasonable incentive factor to minimize the expected cost of its own, and to make local units work hard to achieve the goal of timely guarantee. When the local unit's actual income is greater than the retained earnings \(\overline{C}\), the guarantee contract will be accepted. Therefore, the military cost objective function can be transformed to solve the following optimization problems:

\[
\begin{aligned}
\min C_m (\Gamma) &= \min \left[\Gamma(\varphi e + \phi \epsilon) + k \int_0^T (T - t) f(t)dt\right] \\
&= \left[\Gamma(\varphi e + \phi \epsilon) - (h \int_0^T (t - T) f(t)dt + g \int_0^T (T - t) f(t)dt + \frac{1}{2} \eta e^2 + \frac{1}{2} \rho \Gamma \phi^2 \sigma^2)\right] \geq \overline{C}
\end{aligned}
\] (5)

In this study, we do not consider the so-called "moral hazard" problem, that is, the local unit is rational and will make a hard choice when the marginal cost is equal to the actual income. The degree of effort observed is the same as that of the reality. Using the first order condition method, the extreme value of \(e\) in the constraint condition (5) can be obtained by the incentive compatibility constraint. For \(\Gamma \varphi - \eta e = 0\). The above optimization problem can be transformed into:
The first order condition is satisfied when \( \Gamma = \phi^2 (\phi^2 + \eta \phi^2 \sigma^2)^{-1} > 0 \).

The above derivation shows that the military incentive coefficient depends on the local unit effort cost coefficient, the effort output coefficient, the risk aversion coefficient, the influence coefficient and the variance of the exogenous variables. Further analysis of formula (6) can draw the following conclusions:

1) the greater the degree of effort of local units, the greater the incentive reward for the military. This is in line with the actual situation, but this improvement or impact does not change in magnitude. Therefore, the military should formulate appropriate incentives based on the time distribution of the demand for far fire equipment, so that both the military and the military can gain the greatest benefit and the cooperation relationship between each other can also win for a long time.

2) the stronger the risk aversion ability of local units, the smaller the incentive reward of the military. In order to avoid the risk of business loss, local units will maintain a certain degree of effort even if the military gives a small incentive. Correspondingly, by increasing incentives to compensate for the local units of inventory costs, rush costs and military shortage losses caused by the cost increase. Therefore, in order to guarantee some important long-range fire equipment with large loss of out-of-stock, the military needs to consider a higher incentive to maximize the incentive for local units to supply on time.

3) the greater the degree of influence of exogenous variables, the smaller the incentive reward of the military. In making the best incentive contract, the military needs to consider the efforts of local units and its ability level, in order to design effective long-term incentive mechanism.

5. Example analysis

Taking a long distance rocket battalion as an example, through the field visit and expert consultation, the relevant parameters are as follows: the local enterprise unit time labor cost \( g \) is 1200 yuan, the unit time inventory cost of local units \( h \) is 3000 yuan, the equipment supply time agreed by the military and local units \( T \) is 10 days, the exogenous variable \( \phi \) is 7, the degree of effort \( \varphi \) is 30. The coefficient of real estate is \( A \), the influence coefficient of exogenous variable \( \phi \) is 7, the unit time loss coefficient \( k \) is 0.5, the external variable variance \( \sigma^2 \) is 0.4, the local unit effort cost coefficient \( \eta \) is 0.7, the exogenous variable \( \varepsilon \) is 2000, the local unit risk evasion coefficient \( \rho \) is 0.2, and the above parameter generation formula is obtained:

\[
\min \{ h \int_{0}^{\infty} (t-T) f(t)dt + (k+g) \int_{0}^{T} (T-t) f(t)dt + \frac{\Gamma^2 \phi^2}{2\eta} + \frac{1}{2} \rho \Gamma \phi^2 \sigma^2 - C \} \tag{6}
\]

The cross probability of setting GA is 0.5, the mutation probability is 0.007, the population size is 100, the evolution algebra is 50, and the threshold value is 0.001. The evolutionary process of the algorithm is shown in Figure 1.
The operating system used in simulation calculation is windows XP, and the machine hardware CPU is Pentium Dual-core 3.0Ghz, 2GB memory, and Matlab R2014a programming is used, the calculation time is not more than 3 seconds. By using the GA toolbox in MATLAB, the game equilibrium point parameter set and the corresponding income cost are as follows: $e = \frac{\Gamma \varphi}{\eta} = 21.4286, \Gamma = 5.0544$, the military minimum target cost $\min C_M(\Gamma) = 73150$ yuan, the local expected profit $C_S = 54930$ yuan, which is in line with the actual distribution of the maintenance support task of the remote rocket artillery.

After a comparative analysis, the curve of cost and profit changes is shown in Figure 2.
6. Summary
In this paper, based on the analysis of the current situation of the remote fire equipment support, this paper establishes the maintenance support model of the contract merchant of the long-range rocket launcher based on Stackelberg game, establishes the objective function, and finally carries out an example analysis, and uses the genetic algorithm to solve the model. The calculation results show that the method can effectively predict the cost of remote fire equipment and has certain reference value.

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