Evolution of Dark Energy Reconstructed from the Latest Observations

Yuting Wang1, Levon Pogosian2,3, Gong-Bo Zhao1,3,4, and Alex Zucca2

1 National Astronomy Observatories, Chinese Academy of Science, Beijing, 100101, People’s Republic of China; ytwang@nao.cas.cn
2 Department of Physics, Simon Fraser University, Burnaby, British Columbia, V5A 1S6, Canada
3 Institute of Cosmology and Gravitation, University of Portsmouth, Portsmouth PO1 3FX, UK
4 University of Chinese Academy of Sciences, Beijing, 100049, People’s Republic of China

Received 2018 October 1; revised 2018 November 18; accepted 2018 November 19; published 2018 December 5

Abstract

We reconstruct the evolution of the dark energy (DE) density using a nonparametric Bayesian approach from a combination of the latest observational data. We caution against parameterizing DE in terms of its equation of state as it can be singular in modified gravity models, and using it introduces a bias preventing negative effective DE densities. We find a $3.7\sigma$ preference for an evolving effective DE density with interesting features. For example, it oscillates around the $\Lambda$ cold dark matter prediction at $z \lesssim 0.7$, and could be negative at $z \gtrsim 2.3$; DE can be pressure-less at multiple redshifts, and a short period of cosmic deceleration is allowed at $0.1 \lesssim z \lesssim 0.2$. We perform the reconstruction for several choices of the prior, as well as a evidence-weighted reconstruction. We find that some of the dynamical features, such as the oscillatory behavior of the DE density, are supported by the Bayesian evidence, which is a first detection of a dynamical DE with a positive Bayesian evidence. The evidence-weighted reconstruction prefers a dynamical DE at a $(2.5 \pm 0.06)\sigma$ significance level.

Key words: dark energy

1. Introduction

Since becoming the working model of cosmology following the discovery of cosmic acceleration (Riess et al. 1998; Perlmutter et al. 1999), the $\Lambda$ cold dark matter ($\Lambda$CDM) model has withstood all of the tests using increasingly accurate and comprehensive cosmic microwave background (CMB; Bennett et al. 2013; Planck Collaboration et al. 2016), supernovae (SNe; Conley et al. 2011; Suzuki et al. 2012), galaxy clustering (Zehavi et al. 2011), and weak lensing (Heymans et al. 2013; Hildebrandt et al. 2017) data. There are, however, good reasons to keep an open mind regarding possible extensions and alternatives to $\Lambda$CDM. The two main ingredients of the model, $\Lambda$ and CDM, are not understood at a fundamental level. Direct searches of dark matter have so far failed, while the observed value of the vacuum energy implies a technically unnatural fine-tuning of $\Lambda$ in the context of the effective field theory (Burgess 2017). This is the framework for the present understanding of particle interactions.

The recent exquisite measurements of the CMB temperature and polarization by Planck (Planck Collaboration et al. 2016) significantly reduced the uncertainties in $\Lambda$CDM parameters. With this dramatic improvement in precision, it is perhaps not surprising that several $2–3\sigma$ level tensions have appeared between Planck and other data sets, as well as within the Planck data itself (Addison et al. 2016), when interpreted within the $\Lambda$CDM model. For instance, the locally measured value of the Hubble constant $H_0$ is off by $3.5\sigma$ from the Planck best fit (Riess et al. 2016). The expansion rate at $z = 2.34$, implied by the Baryon Oscillation Spectroscopic Survey (BOSS) baryonic acoustic oscillations (BAOs) measurement from the Ly$\alpha$ forest (Font-Ribera et al. 2014; Delubac et al. 2015), disagrees with the best-fit $\Lambda$CDM prediction at a $\sim 2.7\sigma$ level. These tensions are not at a significance level sufficient to rule out $\Lambda$CDM—they could simply be statistical fluctuations (Scott 2018). It is also possible that they are caused by unaccounted systematic effects in the measurements, or the modeling of the data. However, it is worth noting that these tensions have persisted and become stronger over the past three years, fueling significant interest in possible extensions of $\Lambda$CDM, such as dynamical dark energy (DE; Di Valentin et al. 2017a; Solà et al. 2017; Zhao et al. 2017; Capozziello et al. 2018; Ooba et al. 2018; Solà Peracaula et al. 2018), interacting DE and dark matter (Das et al. 2006; Wang et al. 2015; Di Valentin et al. 2017b; Yang et al. 2018), and other extensions of $\Lambda$CDM.

A reconstruction of the effective DE equation of state (EOS) $w_{\text{eff}}(a)$ from a combination of recent data was presented in Zhao et al. (2017), showing a clear preference for $w_{\text{eff}} \lesssim -1$, along with a possible crossing of the $w_{\text{eff}} = -1$ divide. Such behavior is impossible for quintessence—a minimally coupled scalar field with a canonical kinetic term (Ratra & Peebles 1988; Wetterich 1995, 2002; Caldwell et al. 1998; Steinhardt et al. 1999; Peebles & Ratra 2003), unless the sign of the kinetic term is changed by hand (Caldwell 2002; Carroll et al. 2003; Vikman 2005). This, however, leads to ghost instabilities, i.e., energy being unbounded from below, unless one postulates a non-Lorentz-invariant cutoff at an appropriately chosen scale (Cline et al. 2004). On the other hand, such “ghostly” behavior generically occurs in non-minimally coupled DE models, i.e., where DE and matter interact through an additional scalar force (Carroll et al. 2005; Das et al. 2006). There, the $w_{\text{eff}}$ measured in Zhao et al. (2017) would not be the EOS of the scalar field, but it would be an effective quantity that depends on the coupling to matter. Such a $w_{\text{eff}}$ is allowed to be phantom and to cross $-1$, with the effective DE density (defined below) allowed to change sign. In such cases, parameterizing the DE evolution via $w_{\text{eff}}$ is unnecessarily restrictive as, by design, it does not allow for negative effective DE densities. To address this, we will directly reconstruct the effective DE density.

We define the effective DE in a purely phenomenological way, by letting it describe all of the contributions to the standard Friedmann equation other than matter and radiation. Namely, the effective DE density $\rho_{\text{DE}}(a)$ is defined via

$$\frac{H^2}{H_0^2} = \frac{\Omega_{\text{DE}}}{a^2} + \frac{\Omega_{\text{M}}}{a^3} + \Omega_{\text{DE}}X(a),$$

(1)
where $a$ is the scale factor, and $X(a) \equiv \rho_{\text{DE}}^\text{eff}(a)/\rho_{\text{DE}}^\text{eff}(1)$. $\rho_{\text{DE}}^\text{eff}$ could include modifications of gravity, non-minimal interactions with matter, etc. We assume a flat universe, so that $\Omega_L + \Omega_M + \Omega_{\text{DE}} = 1$ and $H(a) = H_0$.

Most studies of DE dynamics (but not all, see, e.g., Wang & Tegmark 2004; Sahni & Starobinsky 2006; Sahni et al. 2014; Poulin et al. 2018) attempt to measure the DE EOS, $w_{\text{DE}} = \rho_{\text{DE}}/p_{\text{DE}}$. If DE is a conserved fluid with $\rho_{\text{DE}} > 0$, then specifying $w_{\text{DE}}(a)$ fully determines the dynamics of DE. In such a case, one can replace $p_{\text{DE}}$ with $w_{\text{DE}} \rho_{\text{DE}}$ in the conservation equation, $\dot{\rho}_{\text{DE}} + 3H \rho_{\text{DE}}(1 + w_{\text{DE}}) = 0$, and solve it to find $\rho_{\text{DE}}(a) = \rho_{\text{DE}}(1) \exp \left[ \int_a^1 da' 3(1 + w_{\text{DE}}(a'))/a' \right]$. Working with the EOS provides a simple test of the LCDM as $w_a = -1$ independent of the value of $\Lambda$. However, in theories where the DE field mediates a force between matter particles, the effective DE EOS, $w_{\text{DE}}^\text{eff} = \rho_{\text{DE}}/\rho_{\text{DE}}^\text{eff}$ can become singular, because $\rho_{\text{DE}}^\text{eff}$ can change sign (see Appendix A for details; Horndeski 1974; Deffayet et al. 2009a, 2009b, 2011; Gleyzes et al. 2015). Thus, parameterizing the expansion history $w_{\text{DE}}^\text{eff}$ in cosmological codes used for model-independent tests of gravity, such as MGCLAMB (Zhao et al. 2009; Hojjati et al. 2011), MFTCAMB (Hu et al. 2014; Raveri et al. 2014), and h1CLASS (Zumalacárregui et al. 2017), could lead to a bias, as it assumes $\rho_{\text{DE}} > 0$ at all times. This is particularly relevant because the data indicate a preference for $w_{\text{DE}} < -1$, which is prohibited for quintessence but can happen in modified gravity and brane-world models (Damour et al. 1990; Torres 2002; Chung et al. 2003; Faraoni 2006). For these reasons, looking directly at the evolution of the DE density is more appropriate. Avoiding the assumption of a positive $\rho_{\text{DE}}$ also allows one to constrain models such as “Everpresent $\Lambda$” (Ahmed et al. 2004; Ahmed & Sorkin 2013; Zwane et al. 2017), in which the observed cosmological “constant” fluctuates between positive and negative values.

### 2. Data and Method

In what follows, we use the correlated prior method of (Crittenden et al. 2012, 2009) to reconstruct the evolution of the DE density from the available data sets used to probe the background expansion. We start with a brief review of the data sets and the reconstruction method.

Our phenomenological definition of the effective DE (1) does not specify the underlying theory needed for calculating cosmological perturbations. To keep the analysis general, we only evaluate observables predicted by the Friedmann equation and consider data sets probing the background expansion history. They include the CMB distance information from Planck (Planck Collaboration et al. 2016), the “Joint Lightcurve Analysis” (JLA) supernovae (Betoule et al. 2014), BAO measurements from 6dF Galaxy Survey (6dFGS; Beutler et al. 2011), Sloan Digital Sky Survey (SDSS) data release (DR7) Main Galaxy Sample (MGS; Ross et al. 2015), tomographic BOSS DR12 (TomoBOSS; Wang et al. 2017), eBOSS DR14 quasar sample (DR14Q; Ata et al. 2018) and the Lyα forest (LyαFBAO) of BOSS DR11 quasars (Font-Ribera et al. 2014; Delubac et al. 2015), the local measurement of $H_0 = 73.24 \pm 1.74$ km s$^{-1}$ Mpc$^{-1}$ in Riess et al. (2016), and the Observational Hubble parameter Data (OHD; Moreseco et al. 2016).

To reconstruct $X(a)$, we parameterize it in terms of its values at $N$ points in $a$. Namely, we have bins $X_i = X(a_i)$, $i = 1, \ldots, N$, with $a_i$ distributed uniformly in the interval $a \in [1, 0.001]$. We take $N = 40$, which is large enough so that further increases do not affect the reconstruction results. If $X_i$ were assumed to be independent, fitting them to data would yield large uncertainties, rendering the reconstruction useless. Moreover, treating the bins as completely independent is an unreasonably strong assumption as, in any specific theory, the effective DE density would be correlated at nearby points in $a$. Motivated by these considerations, we use the method of Crittenden et al. (2009, 2012) and introduce a prior that correlates the neighboring bins. Specifically, we take $X(a)$ to be a Gaussian random variable with a given correlation $\xi$ between its values at $a$ and $a'$, i.e., $\xi((a - a')) \equiv \langle (X(a) - X^\text{fid}(a))(X(a') - X^\text{fid}(a')) \rangle$. Here, $X^\text{fid}(a)$ is a reference fiducial model, and the correlation function $\xi$ is chosen so that it is nonzero for $|a - a'|$ below a given “correlation length” $\sigma_a$, and approaches zero at larger separations. We adopt the CPZ form (Crittenden et al. 2009, 2012) for the correlation, namely, $\xi((a - a')) = \{ (X(a) - X^\text{fid}(a))(X(a') - X^\text{fid}(a')) \}$. The value of $\xi(0)$ sets the strength of the prior. The latter can be related to the expected variance in the mean value of $X$ as $\sigma_m \approx \xi(0) a_c/(\sigma_{a_{\text{max}}} - \sigma_{a_{\text{min}}})$. In practice, we set $\sigma_m$ and $a_c$, and derive the corresponding $\xi(0)$.

As our “standard” working prior we adopt $a_c = 0.06$ and $\sigma_m = 0.04$, which were the values used in Zhao et al. (2017) to reconstruct $w_{\text{DE}}^\text{eff}(a)$. Their physical meaning is, of course, different, as $w_{\text{DE}}$ and $X$ are related through an integral and, therefore, have different correlation properties. To better understand the impact of the prior, we have also performed reconstructions using different values of $a_c$ and $\sigma_m$. We find that our standard prior is rather weak in the sense that decreasing $a_c$, or increasing $\sigma_m$, increases the uncertainties but does not change the shape of the reconstruction. We show results with a few stronger priors in Appendix B. As expected, in the limit of $a_c \to 1$ or $\sigma_m \to 0$ the reconstructed $X$ becomes a constant.

Our binning scheme for $X$ includes eight bins in the redshift range $3 < z < 1000$, between the redshift of the LyαFBAO measurement and the CMB last scattering surface. The reason for keeping these eight bins, even though there are no data points in that range, was to simplify the evaluation of the prior covariance that can be performed analytically for uniformly spaced bins. However, physically no information can be gained from having these additional degrees of freedom at $z > 3$. To check the role that they play in the reconstruction, we replaced them with a single bin for $z \in [3.85, 1000]$. The value of $X$ in the wide bin was either fixed to 1, or allowed to vary freely, uncorrelated with the 32 lower redshift bins. As shown in Appendix B, the reconstruction of $X$ in the range probed by observations, $z \in [0, 3]$, is largely unaffected by what is assumed at higher redshifts.

We use PolyChord\footnote{The CMB distance prior used in this Letter was derived in Wang & Dai (2016) from the Planck 2015 data, which is largely consistent with the latest Planck 2018 result (Planck Collaboration et al. 2018).} (Handley et al. 2015a, 2015b), a nested sampling plug-in for CosmoMC\footnote{http://cosmologist.info/cosmomc/} (Lewis & Bridle 2002), which enables computation of the Bayesian evidence, to sample the parameter space $P = \{ \Omega_b h^2, \Omega_c h^2, \Theta_m, X_b, N \}$, where $\Omega_b h^2$ and $\Omega_c h^2$ are the physical densities of baryon, and $\Theta_m$ is the ratio of the baryon and CMB quadrupole moments.

---

[5] The CMB distance prior used in this Letter was derived in Wang & Dai (2016) from the Planck 2015 data, which is largely consistent with the latest Planck 2018 result (Planck Collaboration et al. 2018).

[6] https://cccpforge.cse.rl.ac.uk/gf/project/polychord/

[7] http://cosmologist.info/cosmomc/
and CDM, respectively, $\Theta_\lambda$ is the ratio of the sound horizon to the angular diameter distance at the decoupling epoch (multiplied by 100), $X_i$ ($i = 1, \ldots, 40$) are the binned DE density parameters, and $N\;'$ collectively denotes all of the data nuisance parameters that need to be marginalized over. All parameters are sampled from sufficiently wide flat priors. In particular, the range for the $X$ bins is set to $[1 - \Delta X, 1 + \Delta X]$ with $\Delta X = 4$ as the default value. Note that the value of $X$ in the first bin is fixed by definition in Equation (1): $X_1 = X(a = 1) = 1$. The range of nearby bins is then effectively reduced by the correlations induced by the prior. In addition, the sampling procedure guarantees $H^2(a) > 0$ at all times. To test how the reconstruction and Bayesian evidence for dynamical DE change with the strength of the prior, we additionally perform reconstructions for a set of $\Delta X$ values logarithmically spaced in the interval $\Delta X \in [0.01, 4]$. We also run a special case with $\Delta X = 0$, which represents the $\Lambda$CDM model, for a comparison. A modified version of CAMB\(^8\) (Lewis et al. 2000) is used to calculate theoretical predictions.

3. Results

Panel (A) of Figure 1 compares our reconstructed $X(z)$ to the one derived from $w_{\text{DE}}^\text{fit}(z)$ reconstructed in Zhao et al. (2017) using similar data, showing them as mutually consistent. The $X(z)$ derived from $w_{\text{DE}}^\text{fit}(z)$ is strictly positive, as using the EOS implicitly assumes the positivity of the DE density that can bias the reconstruction. Separate reconstructions from individual data sets are presented in the Appendix C, showing that the $H_0$ and the Ly$\alpha$FBAO data both drag $X(z)$ toward negative values at high $z$. The best-fit parameters of $\Lambda$CDM are given in Table 1. From the values of $X$ at various redshifts, we can see that the oscillatory features of energy density is favored by the combined data.

Figure 2 compares $H(z)$ and $D_A(z)$ derived from the reconstructed $X(z)$ to their observed values, with all quantities rescaled by their best-fit $\Lambda$CDM values. The oscillatory features in the derived $H(z)$ and $D_A(z)$ at $z \lesssim 0.7$, which are directly related to those in $X(z)$ shown in panel (A) of Figure 1, are driven by measurements of $H_0$, SNe and TomoBAO. The bump-and-damp feature in $H(z)$ at $z \gtrsim 0.7$ (also seen in the reconstruction performed in Poulin et al. 2018), on the other hand, is due to the Ly$\alpha$FBAO measurement and the integral constraint of the CMB. This can be read from the improved $\chi^2$ listed in Table 2, namely, the $\Lambda$CDM model reduces the $\chi^2$ of TomoBAO, Ly$\alpha$FBAO, $H_0$ and SNe by 4.9, 4.3, 4.1, and 4.1, respectively, meaning that it is these data sets that contribute the most to the features in $X$.

In Figure 1, in addition to $X$, we show the normalized effective DE pressure, $Y(z) \equiv \rho_{\text{DE}}^\text{eff}(z)/\rho_0^\text{eff}(0)^\dagger$ derived via $Y = -X + 1/2dX/dz(1 + z)$, and the deceleration parameter, $q(z)$. As shown in panel (B), $Y(z)$ oscillates around the $\Lambda$CDM prediction of $-1$, and, interestingly, DE is within $\sim 1\sigma$ of having zero pressure at $z \simeq 0.1, 0.5, 0.9$ and $z \gtrsim 3$. Panel (C) shows $q(z)$ oscillating around the prediction of the best-fit $\Lambda$CDM model (dashed–dotted line). Unlike in $\Lambda$CDM, where the acceleration starts at $z \sim 0.6$, the best fit XCDM universe would not be accelerating until $z \sim 0.45$, and would experience a short period of deceleration during $0.1 \lesssim z \lesssim 0.2$, although it is far from conclusive given the uncertainties.

The best-fit dynamical DE model reduces the $\chi^2$ by 13.9 compared to $\Lambda$CDM, which implies that it being preferred at a $3.7\sigma$ level. However, XCDM has more degrees of freedom, and the appropriate way to assess the significance of DE dynamics is to compare the Bayesian evidences. For our “standard” prior, the Bayes factor, which is the logarithm of the ratio of the evidences, is negative, indicating no evidence for the best-fit XCDM.

To see the extent to which these conclusions depend on the choice of the prior parameters, we perform reconstructions with different prior strength and show, in Figure 3, the significance, signal-to-noise ratio ($S/N$), $S/\sqrt{N} = \sqrt{[\Delta \chi^2]_i}$, and the evidence as functions of $\Delta X$ (the range of variation of $X$ in each bin). We see that, as expected, both the $S/N$ and the evidence approach zero when the prior is very strong. Interestingly, the evidence shows a trend of climbing toward positive values as $\Delta X$ increases, with a peak showing up at $\Delta X \sim 0.1$, and drops below zero for $\Delta X > 0.4$. This motivates us to consider an evidence-weighted reconstruction, which linearly combines reconstructions with different $\Delta X$, weighted by the Bayesian evidence, i.e.,

$$Z_w(z) = \frac{\sum [Z(z; \Delta X_i) e^{\Delta \ln E(\Delta X_i)}]}{\sum [e^{\Delta \ln E(\Delta X_i)}]} \quad (2)$$

\(^8\) http://camb.info

\(^9\) We divide the effective pressure by the energy density today instead of $\rho_{\text{DE}}^\text{eff}(z)$ to avoid a singularity when $\rho_{\text{DE}}^\text{eff}(z)$ changes sign.
Figure 2. Top panel: $H(z)$ derived from the reconstructed $X(z)$ rescaled by that of the best fit $\Lambda$CDM model. The data points with error bars show the measurements of $H$ as illustrated in the legend. Bottom panel: same as the top panel but for the angular diameter distance $D_{\theta}$.

Table 1
The Best-fit Values and the 68% CL Uncertainties of Cosmological Parameters in the $\Lambda$CDM and XCDM Models

| Parameter          | $\Lambda$CDM       | XCDM          |
|--------------------|---------------------|---------------|
| $\Omega_b h^2$    | $0.0224 \pm 0.00013$| $0.0223 \pm 0.00016$ |
| $\Omega_c h^2$    | $0.1189 \pm 0.00087$| $0.1203 \pm 0.00141$ |
| $\Omega_k$        | $1.0412 \pm 0.00029$| $1.0411 \pm 0.00032$ |
| $H(z=0.08)$       | 1                   | 0.926 $\pm$ 0.056 |
| $H(z=0.18)$       | 1                   | 1.042 $\pm$ 0.062 |
| $H(z=0.39)$       | 1                   | 0.835 $\pm$ 0.068 |
| $H(z=1.16)$       | 1                   | 1.251 $\pm$ 0.282 |
| $H(z=2.24)$       | 1                   | 0.076 $\pm$ 0.438 |
| $\Omega_m$        | $0.302 \pm 0.005$   | $0.288 \pm 0.008$ |
| $H_0$ [km s$^{-1}$ Mpc$^{-1}$] | 68.41 $\pm$ 0.387 | 70.30 $\pm$ 0.998 |

Table 2
Top Section: the Difference in $\chi^2$ between XCDM and $\Lambda$CDM Models, $\Delta \chi^2 = \chi^2_{\text{XCDM}} - \chi^2_{\text{$\Lambda$CDM}}$, for Individual Data Sets. Bottom Section: the Significance, $S/N = \sqrt{\Delta \chi^2}$, and the Bayesian Evidence $\Delta\ln E$ for Reconstructions using $\Delta x = 4$ and $\Delta x = 0.09$, and the Evidence-weighted Reconstruction

| Data Set          | TomoBAO | Ly$\alpha$BAO | SNe | $H_0$ | OHD | Prior |
|-------------------|---------|---------------|-----|-------|-----|-------|
| $\Delta \chi^2$   | $-4.9$  | $-4.3$        | $-4.1$ | $-4.1$ | $-1.2$ | +3.0 |

All data + standard correlated prior

| $\Delta x = 4$ | $\Delta x = 0.09$ | evidence-weighted |
|----------------|-------------------|--------------------|
| S/N            | $3.7\sigma$       | $2.8\sigma$        | $\left(2.5 \pm 0.06\right)\sigma$ |
| $\Delta \ln E$ | $-5.7 \pm 0.3$    | $0.77 \pm 0.28$    | N/A     |

Figure 3. Bayes factor relative to that of the $\Lambda$CDM ($\Delta \ln E$) and the significance level of deviation from $\Lambda$CDM ($S/N = \sqrt{\Delta \chi^2}$) for different values of $\Delta x$ that set the range of variation of $x$ in each bin.

where $Z = X, Y, q$. The evidence-weighted reconstructions are shown in panels (D)–(F) of Figure 1. They retain the key features of the best-fit XCDM shown in panels (A)–(C), but at a lesser significance. In particular, the overall significance of the deviation from $\Lambda$CDM reduces to $(2.5 \pm 0.06)\sigma$. Panels (D)–(F) show the results for $\Delta x = 0.09$, which corresponds to the maximal Bayesian evidence $\Delta \ln E = 0.77 \pm 0.28$. We find them to be quite similar, as expected, because the linear combination in the evidence-weighted reconstruction is dominated by the component with maximal weight.

4. Conclusion and Discussions

Different levels of tension among various kinds of observations within the framework of $\Lambda$CDM necessitates the exploration of extended cosmological models beyond $\Lambda$CDM. As was shown in an earlier study (Zhao et al. 2017), dynamical DE parameterized in terms of its EOS is able to release the tension, but is not favored over $\Lambda$CDM by the Bayesian evidence.

In this Letter, we take another route to investigating the evolution of DE; namely, we directly reconstruct the effective DE density $X$ from data. This is advantageous over reconstructing the EOS, as $X$ is more directly related to data, while the EOS is related to the derivative of $H(z)$. Thus, we caution against parameterizing DE in terms of its EOS as it generally biases toward positive values and smoother evolution of the DE density. This can affect the Bayesian evidence—one can obtain from the same data a positive Bayes while using $X(z)$, and a negative one when using $w_{\text{DE}}(z)$. Furthermore, $X$ is more physically relevant in modified gravity theories, where it can be negative and change sign. Such dynamics would be forbidden for $X$ derived from an EOS.

We find hints of DE dynamics at a significance of $3.7\sigma$ with interesting features. For example, $X$ oscillates around the $\Lambda$CDM prediction at $z \lesssim 0.7$, and can become negative at $z \gtrsim 2.3$; DE can be pressure-less at multiple redshifts during evolution, and a short period of cosmic deceleration is allowed by current data at $0.1 \lesssim z \lesssim 0.2$. We note that these features would have been missed if the DE density was parameterized using a simple polynomial (Lemos et al. 2018). Some of these dynamical features, including the oscillations, are supported by the Bayesian evidence (the Bayesian factor is positive at about...
2.8σ level for the case of Δχ = 0.09, for example), which is the first time a dynamical DE with a positive Bayesian evidence is detected. Furthermore, the evidence-weighted reconstruction prefers the dynamical DE at a (2.5 ± 0.06)σ significance level.

The new features of DE dynamics awaits scrutiny by forthcoming BAO measurements by Dark Energy Spectroscopic Instrument,10 Euclid,11 and Prime Focus Spectrograph.12 Gravitational wave sources accompanied by electromagnetic counterparts will also offer accurate independent estimates of H0 at very low redshifts (Abbott et al. 2017; Guidorzi et al. 2017; Hotokezaka et al. 2018). The methodology developed in this Letter will be useful in further studies of DE and modified gravity.

We benefited from valuable discussions and previous collaborations with Rob Crittenden. We thank George Efstathiou, Wenjuan Fang, and Marco Raveri for useful discussions. Y.W. and G.B.Z. are supported by NSFC grants 1171001024 and 11673025, and the National Key Basic Research and Development Program of China (No. 2018YFA0404503). Y.W. is also supported by the Nebula Talents Program of NAOC and the Young Researcher Grant of NAOC. L.P. and A.Z. are supported by the National Sciences and Engineering Research Council of Canada. This research used resources of the SCIAMA cluster supported by University of Portsmouth, and the ZEN cluster supported by NAOC.

Appendix A

The Effective DE Density in General Cosmologies

In what follows, we show that in modified gravity theories the effective DE density X(a), defined in Equation (1), is allowed to change sign. To see this, consider the class of generalized Brans–Dicke models described by the action (Bergmann 1968; Nordtvedt 1970; Wagoner 1970)

\[ S = \int d^4x \sqrt{-g} \left[ \frac{F(\phi) G}{16\pi G} - \frac{1}{2} \partial \mu \phi \partial \nu \phi - V(\phi) + L_M \right], \]

where \( L_M \) is the Lagrangian of all particle and radiation fields. The modified Einstein equation in this model is

\[ G_{\mu\nu} = 8\pi G F^{-1} T_{\mu\nu}^T + T_{\mu\nu}^0 + \nabla_\mu \nabla_\nu F - g_{\mu\nu} \Box F \]

\[ = 8\pi G \left( T_{\mu\nu}^M + (T_{\mu\nu}^{\text{DE}})_{\mu\nu} \right), \]

where, in the second line, we have defined the effective DE stress-energy by absorbing into it all of the terms on the right-hand side other than the usual matter term, i.e.,

\[(T_{\mu\nu}^{\text{DE}})_{\mu\nu} \equiv F^{-1} \left( T_{\mu\nu}^0 + \nabla_\mu \nabla_\nu F - g_{\mu\nu} \Box F \right) \left( 1 - F \right) T_{\mu\nu}^M. \]

Then, the effective DE density is

\[ \rho_{\text{DE}}^{\text{eff}} = F^{-1} \left( \dot{\phi}^2/2 + V(\phi) - 3HF + (1 - F)\rho_M \right), \]

while the effective DE pressure is

\[ p_{\text{DE}}^{\text{eff}} = F^{-1} \left( \dot{\phi}^2/2 - V(\phi) + 2HF + \dot{F} \right). \]

The μ = ν = 0 component of Equation (3) for a Friedmann–Robertson–Walker background metric gives the usual Friedmann equation

\[ H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \left[ \rho_M(a) + \rho_{\text{DE}}^{\text{eff}}(a) \right], \]

which can then be recast in the form of Equation (1).

The effective DE “fluid” is, by construction, conserved,

\[ \dot{\rho}_{\text{DE}} + 3H (\rho_{\text{DE}}^{\text{eff}} + p_{\text{DE}}) = 0, \]

but its EOS, 

\[ w_{\text{DE}}^{\text{eff}} = \frac{\dot{\phi}^2/2 - V(\phi) + 2HF + \dot{F}}{\dot{\phi}^2/2 + V(\phi) - 3HF + (1 - F)\rho_M}, \]

is not always well-defined because \( \rho_{\text{DE}}^{\text{eff}} \) in the denominator is allowed to change sign thanks to the new terms generated by the non-minimal coupling \( F(\phi) \). In the case of quintessence, \( F = 1 \), the effective DE EOS is the same as the EOS of the scalar field, \( w_{\phi} \geq -1 \). For a general \( F(\phi) \), the scalar field mediates a force between matter particles, coupling the matter fluid with DE so that they are no longer separately conserved. Thus, as articulated in Carroll et al. (2005) and Das et al. (2006), observing \( w_{\text{DE}}^{\text{eff}} < -1 \), or finding that \( \rho_{\text{DE}}^{\text{eff}} \) changes its sign, could be a smoking gun for new interactions in the dark sector.

Appendix B

Investigating Alternative Priors and High-z Parameterizations of X(z)

The reconstructed evolution of \( X(z) \) from the combination of all data using our “standard” prior is shown in Figure 4. To help interpret the reconstruction, we also show the 1σ uncertainties around the 40 bins from our Gaussian prior alone. The latter are obtained by running CosmoMC and letting it converge with using just the prior and no data. The fiducial model assumed by the prior is \( X(z) = 1 \), and the “best fit” to the prior alone is very close, although not identical to it, as expected. One can see that at lower redshifts the data significantly improves on the prior, while at high redshifts the reduction in uncertainties is relatively small.

There are no data points probing the expansion history between \( z = 2.34 \), where the Lyα forest provides a BAO measurement, and the epoch of last scattering at \( z \sim 1000 \) probed by CMB. Thus, having several bins of \( X(z) \) in that redshift range is not justified, except for the purpose of keeping the spacing between the bins uniform. To check that these “extra” bins do not affect the reconstruction of \( X(z) \), we try a couple of alternative choices of parameterizing \( X \) at \( z > 2.34 \). Specifically, we tried replacing the last eight bins in the \( a \in [0.001, 0.206] \) range (3.85 < \( z < 1000 \)) with a single wide bin and either fixed it to \( X = 1 \), or let it vary independently from other bins. As shown in Figure 4, the reconstructed dynamics and the size of uncertainties at \( z < 3 \) remains consistent in all three cases.

Finally, in Figure 5, we show the effect of using alternative prior parameters in our reconstructions. There are two types of prior parameters: \( (\alpha, \sigma_\alpha) \), which set the prior covariance of the bins, and \( \Delta \chi^2 \), which sets the range of allowed values of \( X \) in each bin. In each case, we also show the 1σ uncertainties on the bins from the prior alone. The significance of the DE dynamics detection in panels (A) and (B) is 3.4σ and 2.9σ, respectively. For the stronger priors, i.e., panels (E) and (F), the reconstructions are consistent with ΛCDM.

10 http://desi.lbl.gov/
11 https://www.euclid-ec.org
12 http://pfs.ipmu.jp/
Appendix C

The $X(z)$ Reconstructions using Different Data Combinations

To show how each particular data set affects the reconstruction result, we reconstruct $X(z)$ from various data combinations. We first combine a collection of data sets with no reported tensions among themselves into a “Base” data set, which includes the 2015 Planck distance priors, JLA supernovae, and BAO measurements from 6dFGS, MGS, and eBOSS DR14Q.$^{13}$ We then add other data sets, one at a time, to “Base” to form other data combinations, finally combining all of the data together to form the most constraining data set. The

---

$^{13}$ This Letter additionally uses the eBOSS BAO data, while excluding the galaxy power spectra, redshift space distortions, weak lensing, and CMB anisotropies, which require modeling of perturbations and were used in the EOS reconstruction (Zhao et al. 2017).
reconstructions of $X(z)$, i.e., the best-fit values and the 68% confidence level (CL) uncertainties for each bin, from these data sets are presented in panels (A)–(F) of Figure 6. One can see that $X(z)$ reconstructed from either Base or Base+OHD is consistent with that predicted by ΛCDM, while the reconstruction derived from the other four data combinations show different levels of dynamics in $X(z)$. For example, results derived either from Base+$H_0$ or Base+LyαFBAO prefer a lower $X(z)$ at higher redshifts. Specifically, Base+LyαFBAO diminishes $X$ at $z \sim 2.3$ (it even makes $X$ negative at $z \gtrsim 2.3$), which is due to the fact that the LyαFBAO measurement at $z \sim 2.3$ is lower than the theoretical prediction of ΛCDM at around 2.5σ (Font-Ribera et al. 2014; Delubac et al. 2015). On the other hand, Base+$H_0$ drags $X$ downward at $z \sim 1$. This is because the local measurement of $H_0$ prefers a much greater value than that extrapolated from the best-fit ΛCDM. The most effective way to fit a higher $H_0$ is to increase $X(z=0)$, but as $X(z=0)$ is fixed to unity by definition, $X(z)$ has to be reduced at higher redshifts, namely, at $z \sim 1$.\(^{14}\) Note that, these redshift-dependent reductions of $X$ caused by $H_0$ and LyαFBAO would be degenerate if one fit a constant $X$ instead, as both data sets “pull” $X$ in the same direction. When the tomographic BAO is added to Base, a statistically significant oscillatory feature shows up at $z \lesssim 0.6$ (see panel (B)). $X(z)$, in particular, tends to go below unity at $z \sim 0.1$ and $z \sim 0.3$ and above unity at $z \sim 0.2$. Such details would not be revealed without the high-redshift resolution BAO measurements (Wang et al. 2017; Zhao et al. 2017). The same oscillatory feature at $z \lesssim 0.6$ is present in the SNe data, and becomes more significant when all data are combined, as shown in panel (F). The decrease in $X$ at $z \gtrsim 1.5$ also becomes more pronounced. In addition, a new bump appears at $z \sim 1.3$ caused by the requirement to maintain a fixed distance to last scattering, set by CMB measurements, while compensating for the reduction in $X$ at high redshifts.

**ORCID iDs**

Yuting Wang @ https://orcid.org/0000-0001-7756-8479
Gong-Bo Zhao @ https://orcid.org/0000-0003-4726-6714

**References**

Abbott, B. P., Abbott, R., Abbott, T. D., et al. 2017, Natur, 551, 85
Addison, G. E., Huang, Y., Watts, D. J., et al. 2016, ApJ, 818, 132
Ahmed, M., Dodelson, S., Greene, P. B., & Sorkin, R. 2004, PhRvD, 69, 103523
Ahmed, M., & Sorkin, R. D. 2013, PhRvD, 87, 063515
Ata, M., Baungarten, F., Bautista, J., et al. 2018, MNRAS, 473, 4773
Bennett, C. L., Larson, D., Weiland, J. L., et al. 2013, ApJS, 208, 20
Bergmann, P. G. 1968, IJTP, 1, 25
Betoule, M., Kessler, R., Guy, J., et al. 2014, A&A, 568, A22
Beutler, F., Blake, C., Colless, M., et al. 2011, MNRAS, 416, 3017
Burgess, C. P. 2017, arXiv:1711.10592
Caldwell, R. R. 2002, Phil.B, 545, 23
Caldwell, R. R., Dave, R., & Steinhardt, P. J. 1998, PhRvL, 80, 1582
Capozziello, S., Ruchika, & Sen, A. A. 2018, arXiv:1806.03943
Carroll, S. M., de Felice, A., & Trodden, M. 2005, PhRvD, 71, 023525
Carroll, S. M., Hoffman, M., & Trodden, M. 2003, PhRvD, 68, 023509
Chung, D. J. H., Everett, L. L., & Riotto, A. 2003, PhLB, 556, 61
Clune, J. M., Jem, S., & Moore, G. D. 2004, PhRvD, 70, 043543
Conley, A., Guy, J., Sullivan, M., et al. 2011, ApJS, 192, 1
Crittenden, R. G., Pogosian, L., & Zhao, G.-B. 2009, JCAP, 12, 025
Crittenden, R. G., Zhao, G.-B., Pogosian, L., Samushia, L., & Zhang, X. 2012, JCAP, 2, 048
Damour, T., Gibbons, G. W., & Gundlach, C. 1990, PhRvL, 64, 123
Das, S., Corasaniti, P. S., & Khoury, J. 2006, PhRvD, 73, 083509
Deffayet, C., Deser, S., & Esposito-Farese, G. 2009a, PhRvD, 80, 064015

---

\(^{14}\) $X$ is not allowed by SNe, which have a strong constraining power, to deviate from unity at $z \lesssim 1$. On the other hand, DE becomes dynamically unimportant at $z \gg 1$. Thus, to compensate for a higher $H_0$, $X$ is reduced at $z \sim 1$. 

**Figure 6.** Panels (A)–(F): best-fit and the 68% CL uncertainty of $X(z)$ reconstructed from six different data combinations.
