On-shell action and the Bekenstein-Hawking entropy of supersymmetric black holes in $AdS_6$

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Abstract

Recently, entropy of static supersymmetric black holes in $AdS_6$ was counted by topologically twisted index of five-dimensional superconformal field theories. However, the AdS/CFT dictionary states that the partition function of the boundary field theory is dual to the on-shell action of the bulk quantum gravity. In this paper, we aim to explain the microscopic counting of black hole entropy by topologically twisted index. We calculate the renomalized on-shell action and show that the on-shell action equals minus the Bekenstein-Hawking entropy of the supersymmetric black holes in $AdS_6$. 

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1 Introduction and conclusions

Recently, via the AdS/CFT correspondence \[1\], there has been great success in microscopic counting of supersymmetric black hole entropy in AdS4 \[2\] due to the localization technique for calculating topologically twisted index in three-dimensional quantum field theories \[3, 4\]. More recently, progress has been made in microscopic counting of entropy of supersymmetric black holes in AdS6 \[5, 6, 7\] by topologically twisted index for five-dimensional field theories \[8, 9\]. In AdS4 and AdS6, the Bekenstein-Hawking entropy of static supersymmetric black holes was shown to be counted by topologically twisted index.

However, the AdS/CFT dictionary \[1\] states that the partition function of the boundary field theory is dual to the on-shell action of the bulk quantum gravity. In this paper, we aim to explain the microscopic counting of AdS6 black hole entropy via topologically twisted index. We will show that the on-shell action equals the Bekenstein-Hawking entropy of supersymmetric black holes after holographic renormalization \[10, 11\]. We closely follow the parallel calculation performed for supersymmetric black holes in AdS4 \[12\].

We make some comments on our results. First, mainly, we will consider the supersymmetric black hole solution found in \[5\] in F(4) gauged supergravity in six dimensions \[18\]. The full black hole solution is an interpolating geometry between the asymptotic AdS6 boundary at \(r_0 = 0\) and the \(AdS_2 \times H^2 \times H^2\) horizon at \(r_h = \infty\). However, we do not need the explicit form of the black hole solution, and the boundary asymptotics will do enough. Second, however, the

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1 Note the title changes for \[5, 7\] soon.
2 See also \[13, 14\] for related works in AdS4. In \[15\], renormalized on-shell action was also calculated as an entropy function for five-dimensional rotating black holes in relation with \[10, 11\].
supersymmetry of our black hole solution is essential. We heavily depend on the supersymmetry equations in the course of holographic renormalization to obtain the on-shell action.

In section 2, we review $F(4)$ gauged supergravity in six dimensions and the supersymmetric $AdS_6$ black hole solution found in [3]. In section 3, we calculate the renormalized on-shell action and prove that it equals the Bekenstein-Hawking entropy of the black holes.

2 Supersymmetric black holes in $AdS_6$

2.1 $F(4)$ gauged supergravity in six dimensions

We review $SU(2) \times U(1)$-gauged $\mathcal{N} = 4$ supergravity in six dimensions [18]. The bosonic field content consists of the metric, $g_{\mu\nu}$, a real scalar, $\phi$, an $SU(2)$ gauge field, $A_I^\mu$, $I = 1, 2, 3$, a $U(1)$ gauge field, $A_\mu$, and a two-form gauge potential, $B_{\mu\nu}$. The fermionic field content is gravitinos, $\psi_\mu^i$, and dilatinos, $\chi_i$, $i = 1, 2$. The field strengths are defined by

\[
\begin{align*}
F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu, \\
F_I^{\mu\nu} &= \partial_\mu A_I^\nu - \partial_\nu A_I^\mu + g\epsilon^{IJK} A_J^\mu A_K^\nu, \\
G_{\mu\nu\rho} &= 3\delta_{[\mu} B_{\nu\rho]}, \\
\mathcal{H}_{\mu\nu} &= F_{\mu\nu} + m B_{\mu\nu}.
\end{align*}
\]

The bosonic Lagrangian is given by

\[
e^{-1} L = -\frac{1}{4} R + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{8} \left( g^2 e^{\sqrt{2} \phi} + 4 g m e^{-\sqrt{2} \phi} - m^2 e^{-3\sqrt{2} \phi} \right) \\
- \frac{1}{4} e^{-\sqrt{2} \phi} \left( \mathcal{H}_{\mu\nu} \mathcal{H}^{\mu\nu} + F_I^{\mu\nu} F_I^{\mu\nu} \right) + \frac{1}{12} e^{2\sqrt{2} \phi} G_{\mu\nu\rho} G^{\mu\nu\rho} \\
- \frac{1}{8} \epsilon^{\mu\nu\rho\sigma\tau\kappa} B_{\mu\nu} \left( F_{\rho\sigma} F_{\tau\kappa} + m B_{\rho\sigma} B_{\tau\kappa} + \frac{1}{3} m^2 B_{\rho\sigma} B_{\tau\kappa} + F_I^{\rho\sigma} F_I^{\tau\kappa} \right),
\]

where $g$ is the $SU(2)$ gauge coupling constant and $m$ is the mass of the two-form gauge potential. Described by the above Lagrangian, there are five inequivalent theories: $\mathcal{N} = 4^+$ ($g > 0, m > 0$), $\mathcal{N} = 4^-$ ($g < 0, m > 0$), $\mathcal{N} = 4^g$ ($g > 0, m = 0$), $\mathcal{N} = 4^m$ ($g = 0, m > 0$), $\mathcal{N} = 4^0$ ($g = 0, m = 0$). The $\mathcal{N} = 4^+$ theory admits a supersymmetric $AdS_6$ fixed point when $g = 3m$.

2.2 Supersymmetric black holes in $AdS_6$

We review the static supersymmetric black holes in $AdS_6$ in [5]. We consider the metric,

\[
ds^2 = e^{2f(r)} (dt^2 - dr^2) - e^{2g_1(r)} (d\theta_1^2 + \sin^2 \theta_1 d\phi_1^2) - e^{2g_2(r)} (d\theta_2^2 + \sin^2 \theta_1 d\phi_2^2),
\]

(2.3)
for the $S^2 \times S^2$ background, and

$$ds^2 = e^{2f(v)}(dt^2 - dr^2) - e^{2g_1(v)}(d\theta_1^2 + \sinh^2 \theta_1 d\phi_1^2) - e^{2g_2(v)}(d\theta_2^2 + \sinh^2 \theta_2 d\phi_2^2),$$

(2.4)

for the $H^2 \times H^2$ background. The only non-vanishing component of the non-Abelian $SU(2)$ gauge field, $A^I_{\mu}$, $I = 1, 2, 3$, is given by

$$A^3 = -a_1 \cos \theta_1 d\phi_1 - a_2 \cos \theta_2 d\phi_2,$$

(2.5)

for the $S^2 \times S^2$ background, and

$$A^3 = a_1 \cosh \theta_1 d\phi_1 + a_2 \cosh \theta_2 d\phi_2,$$

(2.6)

for the $H^2 \times H^2$ background, where the magnetic charges, $a_1$ and $a_2$, are constant. We also have a non-trivial two-form gauge potential,

$$B_{tr} = -\frac{2}{m^2}a_1 a_2 e^{\sqrt{2}\phi + 2f - 2g_1 - 2g_2}.$$ 

(2.7)

We turn off the Abelian $U(1)$ gauge field, $A_{\mu}$. The supersymmetry equations are given by

$$f'e^{-f} = -\frac{1}{4\sqrt{2}}\left(ge^{\frac{\phi}{2}} + me^{-\frac{3\phi}{2}}\right) - \frac{\lambda}{2\sqrt{2}}e^{-\frac{\phi}{2}}(a_1 e^{-2g_1} + a_2 e^{-2g_2}) - \frac{3}{\sqrt{2}m}a_1 a_2 e^{\frac{\phi}{2} - 2g_1 - 2g_2},$$

$$g'_1 e^{-f} = -\frac{1}{4\sqrt{2}}\left(ge^{\frac{\phi}{2}} + me^{-\frac{3\phi}{2}}\right) + \frac{\lambda}{2\sqrt{2}}e^{-\frac{\phi}{2}}(3a_1 e^{-2g_1} - a_2 e^{-2g_2}) + \frac{1}{\sqrt{2m}}a_1 a_2 e^{\frac{\phi}{2} - 2g_1 - 2g_2},$$

$$g'_2 e^{-f} = -\frac{1}{4\sqrt{2}}\left(ge^{\frac{\phi}{2}} + me^{-\frac{3\phi}{2}}\right) + \frac{\lambda}{2\sqrt{2}}e^{-\frac{\phi}{2}}(3a_2 e^{-2g_2} - a_1 e^{-2g_1}) + \frac{1}{\sqrt{2m}}a_1 a_2 e^{\frac{\phi}{2} - 2g_1 - 2g_2},$$

$$\frac{1}{\sqrt{2}}\phi' e^{-f} = \frac{1}{4\sqrt{2}}\left(ge^{\frac{\phi}{2}} - 3me^{-\frac{3\phi}{2}}\right) + \frac{\lambda}{2\sqrt{2}}e^{-\frac{\phi}{2}}(a_1 e^{-2g_1} + a_2 e^{-2g_2}) - \frac{1}{\sqrt{2m}}a_1 a_2 e^{\frac{\phi}{2} - 2g_1 - 2g_2}.$$ 

(2.8)

We also obtain the twist conditions on the magnetic charges,

$$a_1 = -\frac{k}{\lambda g}, \quad a_2 = -\frac{k}{\lambda g},$$

(2.9)

where $\lambda = \pm 1$, $k = +1$ for the $S^2 \times S^2$ background and $k = -1$ for the $H^2 \times H^2$ background. We will consider the $\mathcal{N} = 4^+$ theory, $g > 0$, $m > 0$. We find an $AdS_2$ fixed point solution for the $H^2 \times H^2$ background with $k = -1$,

$$e^f = \frac{2^{1/4}}{g^{3/4}m^{1/4}} r, \quad e^{g_1} = e^{g_2} = \frac{2^{3/4}}{g^{3/4}m^{1/4}}, \quad e^{\frac{\phi}{2}} = \frac{2^{1/4}m^{1/4}}{g^{1/4}}.$$ 

(2.10)

When we consider the $S^2 \times S^2$ background with $k = +1$, $AdS_2$ fixed point does not exist. The full black hole solution is an interpolating geometry between the asymptotic $AdS_6$ boundary at $r_0 = 0$ and the $AdS_2 \times H^2 \times H^2$ horizon at $r_h = \infty$. 

3
3 On-shell action

3.1 The bulk and boundary terms

In this section, we calculate the on-shell action. The calculation of the on-shell action is quite parallel to the calculation for $AdS_4$ black holes in [12]. The Einstein equations for the ansatz in the previous section can be presented by

\[ 0 = e^{-2f} R_{tt} + V + F + 3B, \]
\[ 0 = e^{-2f} R_{rr} + 4\Phi - V - F - 3B, \]
\[ 0 = \frac{1}{2} \left( e^{-2g_1} R_{\theta_1 \theta_1} + e^{-2g_2} R_{\theta_2 \theta_2} \right) - V + F + B, \]

(3.1)

where we defined

\[ V = -\frac{1}{8} \left( g^2 e^{\sqrt{2}\phi} + 4gmc^{-\sqrt{2}\phi} - m^2 e^{-3\sqrt{2}\phi} \right), \]
\[ F = -\frac{1}{2} e^{-\sqrt{2}\phi} \left( a_1^2 e^{-4g_1} + a_2^2 e^{-4g_2} \right), \]
\[ B = -\frac{2}{m^2} a_1^2 a_2^2 e^{\sqrt{2}\phi - 4g_1 - 4g_2}, \]
\[ \Phi = -\frac{1}{2} e^{-2f} \phi' \phi', \]

(3.2)

and $V$ is the scalar potential. With the Einstein equations, the bosonic Lagrangian can be expressed in terms of the metric functions,

\[ e^{-1}\mathcal{L} = -\frac{1}{4} R - \frac{1}{2} e^{-2f} \phi' \phi' - V - \frac{1}{2} e^{-\sqrt{2}\phi} \left( a_1^2 e^{-4g_1} + a_2^2 e^{-4g_2} \right) - \frac{2}{m^2} a_1^2 a_2^2 e^{\sqrt{2}\phi - 4g_1 - 4g_2}, \]
\[ = -\frac{1}{4} \left( R + e^{-2f} \left( R_{tt} + R_{rr} \right) + 2 \left( e^{-2g_1} R_{\theta_1 \theta_1} + e^{-2g_2} R_{\theta_2 \theta_2} \right) \right) \]
\[ = -\frac{1}{2} e^{-2f} \left( f'' + 2f \left( g_1' + g_2' \right) \right) \]
\[ = -\frac{1}{4} e^{-2f - 2g_1 - 2g_2} \left( e^{-2f + 2g_1 + 2g_2} \left( e^{2f} \right)' \right)' . \]

(3.3)

Then the bulk action and the Gibbons-Hawking term [20] are, respectively,

\[ S_{\text{bulk}} = \frac{1}{4\pi G_N} \int d^6x E^{Euc} \]
\[ = \frac{\beta \text{vol}(\Sigma_{g_1}) \text{vol}(\Sigma_{g_2})}{8\pi G_N} \int_{r_0}^{r_h} dr \frac{1}{2} \left( e^{-2f + 2g_1 + 2g_2} \left( e^{2f} \right)' \right)' , \]

(3.4)

\[ S_{\text{GH}} = -\frac{1}{8\pi G_N} \int d^5x \sqrt{h} K \]
\[ = -\frac{\beta \text{vol}(\Sigma_{g_1}) \text{vol}(\Sigma_{g_2})}{8\pi G_N} e^{2g_1 + 2g_2} \left( f' + 2g_1' + 2g_2' \right) \bigg|_{r=r_0} , \]

(3.5)
where $L^{\text{Euc}}$ is the Euclidean continuation of the Lagrangian in (2.2), $h_{ij}$ is the induced metric, and the extrinsic curvature, normal vector, and trace of the extrinsic curvature are, respectively,

$$K_{ij} = \frac{1}{2} n^k \partial_k g_{ij}, \quad (3.6)$$

$$n^j = \frac{1}{\sqrt{g_{rr}}} \left( \frac{\partial}{\partial r} \right)^j, \quad (3.7)$$

$$K = h^{ij} K_{ij}. \quad (3.8)$$

We denoted the six-dimensional Newton’s gravitational constant by $G_N$. Also the radius of the horizon and the location of the boundary were denoted by $r_h$ and $r_0$, respectively. We also obtained the inverse temperature by

$$\beta = \frac{1}{T} = \frac{4\pi}{e^2 f (-e^{-2f})'}. \quad (3.9)$$

Then the sum of the bulk action and the Gibbons-Hawking term is

$$S_{\text{bulk}} + S_{\text{GH}} = -A \frac{4G_N}{4\pi G_N} \left[ \frac{\beta \text{vol}(\Sigma g_1) \text{vol}(\Sigma g_2)}{e^{2g_1+2g_2}} \right]_r^{r_h}, \quad (3.10)$$

where the area of the horizon is

$$A = e^{2g_1+2g_2} \text{vol}(\Sigma g_1) \text{vol}(\Sigma g_2)|_{r=r_h}. \quad (3.11)$$

Employing the supersymmetry equations in (2.8), we can rewrite

$$(e^{2g_1+2g_2})' = 2e^{2g_1+2g_2}(g'_1 + g'_2)$$

$$= 4e^{f+2g_1+2g_2} \left[ -\frac{1}{2} W + \frac{\lambda}{2\sqrt{2}} e^{-\frac{\phi}{\sqrt{2}}} (a_1 e^{-2g_1} + a_2 e^{-2g_2}) + \frac{1}{\sqrt{2}m} a_1 a_2 e^{\frac{\phi}{\sqrt{2}} e^{2g_1+2g_2}} \right]. \quad (3.12)$$

The superpotential is defined by

$$W = \frac{1}{2\sqrt{2}} \left( g e^{\frac{\phi}{\sqrt{2}}} + me^{-\frac{3\phi}{2}} \right), \quad (3.13)$$

and it gives the scalar potential by

$$V = \frac{1}{2} \left( \frac{\partial W}{\partial \phi} \right)^2 - \frac{5}{2} W^2. \quad (3.14)$$

Using the expression in (3.12), the sum of the bulk action and the Gibbons-Hawking term is

$$S_{\text{bulk}} + S_{\text{GH}} = -A \frac{4G_N}{4\pi G_N} \left[ \frac{\beta \text{vol}(\Sigma g_1) \text{vol}(\Sigma g_2)}{e^{f+2g_1+2g_2} W} \right]_{r=r_0}$$

$$- \frac{\beta \text{vol}(\Sigma g_1) \text{vol}(\Sigma g_2)}{2\pi G_N} e^{f+2g_1+2g_2} \left[ \frac{\lambda}{2\sqrt{2}} e^{-\frac{\phi}{\sqrt{2}}} (a_1 e^{-2g_1} + a_2 e^{-2g_2}) + \frac{1}{\sqrt{2}m} a_1 a_2 e^{\frac{\phi}{\sqrt{2}} e^{2g_1+2g_2}} \right]_{r=r_0}. \quad (3.15)$$
As we discussed in the introduction, the full black hole solution is an interpolating geometry between the asymptotic $AdS_6$ boundary at $r_0 = 0$ and the $AdS_2 \times H^2 \times H^2$ horizon at $r_h = \infty$. At the boundary, $r_0 = 0$, the metric and the scalar field asymptote to $AdS_6$ spacetime,

$$e^f = e^{g_1} = e^{g_2} \cong \frac{R_{AdS_6}}{r}, \quad e^{\phi_2} \cong 1. \quad (3.16)$$

Hence, we observe that the superpotential term, the term with $a_1$ and $a_2$, the term with $a_1 a_2$ in (3.15) are quintic, cubic, and linear divergences, respectively, at the boundary, $r = r_0$. Therefore, in the next subsection, we will introduce counterterms to eliminate the divergent terms in the on-shell action.

### 3.2 The counterterms for pure gravity in $AdS_{n+1}$

In the case of pure gravity theories in $AdS_{n+1}$ spacetimes with $n < 7$, the counterterms to eliminate the divergent terms arising from on-shell action at the boundary were systematically obtained long ago in [27]. In this subsection, we briefly review.

The counterterms for generic pure gravity theories in $AdS_{n+1}$ spacetimes with $n < 7$, are presented in [27],

$$S_{ct} = \frac{1}{8\pi G_N} \int_\partial d^n x \sqrt{h} \left[ \frac{n-1}{l} + \frac{l}{2(n-2)} R + \frac{l^3}{2(n-4)(n-2)^2} \left( R_{ij} R^{ij} - \frac{n}{4(n-1)} R^2 \right) + \cdots \right], \quad (3.17)$$

where $h$, $R$, $R_{ij}$ are the determinant of the metric, the Ricci scalar, and the Ricci tensor of the boundary metric, respectively, and $l$ is the radius of $AdS_{n+1}$. According to [27], the first term first appeared in [28], the second term in [29], and the third term in [27]. As $n < 7$, it is enough to determine the counterterms up to $AdS_7$ spacetimes. In odd dimensions, some denominators vanish, and they produce the logarithmic corrections, e.g., [20, 21]. In $AdS_4$, as the third term is not divergent and vanishes at the boundary, there are counterterms only up to $R$, e.g., [19, 22]. On the other hand, as we are working in $AdS_6$, we will encounter the $R^2$ counterterms, as they appeared in [25, 23].

### 3.3 The counterterms

In the previous subsection, we reviewed the counterterms in pure gravity theories in $AdS_{n+1}$. In our case, we also have additional fields, and the counterterms also come dressed with the additional fields. Accordingly, we introduce the superpotential counterterm and the counterterms

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3 See [30, 31, 14] for more formal and systematic approaches.
from the curvature of the boundary, respectively.

\[ S_{SUSY} = -\frac{1}{8\pi G_N} \int_{\partial} d^5x \sqrt{h} (2W) \]

\[ = -\frac{\beta \text{vol}(\Sigma_{g_1})\text{vol}(\Sigma_{g_2})e^{f+2g_1+2g_2}W}{4\pi G_N} \bigg|_{r=r_0}, \]

\[ S_R = -\frac{1}{8\pi G_N} \int_{\partial} d^5x \sqrt{h} \left( \frac{1}{\sqrt{2g}} e^{-\frac{\phi}{\sqrt{2}}} \mathcal{R} \right) \]

\[ = -\frac{\beta \text{vol}(\Sigma_{g_1})\text{vol}(\Sigma_{g_2})e^{f+2g_1+2g_2} k}{2\pi G_N} e^{-\frac{\phi}{\sqrt{2}}} (e^{-2g_1} + e^{-2g_2}) \bigg|_{r=r_0}, \]

\[ S_{R^2} = -\frac{1}{8\pi G_N} \int_{\partial} d^5x \sqrt{h} \left( \frac{2}{\sqrt{2g^2m}} e^{\frac{\phi}{\sqrt{2}}} \left( \mathcal{R}_{ij} \mathcal{R}^{ij} - \frac{5}{16} \mathcal{R}^2 \right) \right) \]

\[ = \frac{\beta \text{vol}(\Sigma_{g_1})\text{vol}(\Sigma_{g_2})e^{f+2g_1+2g_2} \frac{1}{\sqrt{2g^2m}} e^{\frac{\phi}{\sqrt{2}}} - 4G}{2\pi G_N} \bigg|_{r=r_0, G=g_1=g_2}, \]

where the Ricci scalar of the induced metric is

\[ \mathcal{R} = 2k \left( e^{-2g_1} + e^{-2g_2} \right). \]

In (3.19) we set \( k = -1 \) and in (3.20) we identified, \( G \equiv g_1 = g_2 \), which are indeed the case for supersymmetric solutions. In the following, we will keep this identification and employ \( G \) for \( g_1 \) and \( g_2 \). Combining the counterterms to the bulk action and the Gibbons-Hawking term, we have

\[ S_{\text{on-shell}} = S_{\text{bulk}} + S_{GH} + S_{SUSY} + S_R + S_{R^2} \]

\[ = -\frac{A}{4G_N} - \frac{\beta \text{vol}(\Sigma_{g_1})\text{vol}(\Sigma_{g_2})e^{f+4G} \left[ \frac{1}{\sqrt{2m}_a} a_1 a_2 e^{\frac{\phi}{\sqrt{2}}} - 4G \right]}{4\pi G_N} \bigg|_{r=r_0}. \]

Therefore, the counterterms we introduced have eliminated the quintic and cubic divergences and half of the linear divergence in the on-shell action. We still have the second term in (3.22) which is linearly divergent at the boundary.

If we set the scalar field to vanish, \( \phi = 0 \), and use \( g = 3m \), we obtain

\[ S_{SUSY} + S_R + S_{R^2} = \frac{1}{8\pi G_N} \int_{\partial} d^5x \sqrt{h} \left[ \frac{4}{3\sqrt{2}} g + \frac{1}{\sqrt{2g}} \mathcal{R} + \frac{6}{\sqrt{2g^3}} \left( \mathcal{R}_{ij} \mathcal{R}^{ij} - \frac{5}{16} \mathcal{R}^2 \right) \right], \]

\[ = \frac{1}{8\pi G_N} \int_{\partial} d^5x \sqrt{h} \left[ \frac{4}{l} + \frac{l}{6} \mathcal{R} + \frac{l^3}{18} \left( \mathcal{R}_{ij} \mathcal{R}^{ij} - \frac{5}{16} \mathcal{R}^2 \right) \right], \]

where, in the second line, we reparametrized \( g \to 3\sqrt{2}/l \). This is indeed the counterterms introduced for pure gravity in \( AdS_6 \) in (3.17).

\footnote{We are grateful to Ioannis Papadimitriou for informing us the existence of \( R^2 \) counterterms in \( AdS_6 \).}
In order to eliminate the second term in (3.22), we should introduce an additional counterterm which is due to the presence of two-form gauge potential,

$$S_B = \frac{1}{16\pi G_N} \int_\partial d^5x \sqrt{\det g} e^{-\frac{2}{3}m} |B|,$$

(3.24)

where

$$|B|^2 = g^{\mu\nu} g^{\rho\sigma} B_{\mu\rho} B_{\nu\sigma}.$$ \hspace{1cm} (3.25)

We, then, finally, show that the renormalized on-shell action equals minus the Bekenstein-Hawking entropy of the supersymmetric black holes in $AdS_6$,

$$S_{\text{on-shell}} = S_{\text{bulk}} + S_{\text{GH}} + S_{\text{SUSY}} + S_R + S_{R^2} + S_B = -\frac{A}{4G_N}.$$ \hspace{1cm} (3.26)

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