Radiative $\beta$ Decay for Studies of CP Violation

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Abstract

A triple-product correlation in the radiative $\beta$ decay rate of neutrons or of nuclei, characterized by the kinematical variable $\xi \equiv (l_\nu \times l_e) \cdot k$, where, e.g., $n(p) \rightarrow p(p') + e^- (l_e) + \bar{\nu}_e (l_\nu) + \gamma (k)$, can be generated by the pseudo-Chern-Simons term found by Harvey, Hill, and Hill as a consequence of the baryon vector current anomaly and SU(2)$_L \times$U(1)$_Y$ gauge invariance at low energies. The correlation probes the imaginary part of its coupling constant, so that its observation at anticipated levels of sensitivity would reflect the presence of sources of CP violation beyond the standard model. We compute the size of the asymmetry in $n \rightarrow p e^- \bar{\nu}_e \gamma$ decay in chiral effective theory, compare it with the computed background from standard-model final-state interactions, and consider the new physics scenarios which would be limited by its experimental study.
Introduction — The first B-factory era, with key input from the Tevatron, established that both CP and flavor violation in flavor-changing processes are dominated by the Cabibbo-Kobayashi-Maskawa (CKM) mechanism [1]. The CKM mechanism, however, cannot explain the observed value of the baryon asymmetry of the universe, so that the problem of the missing antimatter still weighs upon us. A path to its resolution could lie in the discovery of non-zero values for observables which are inaccessibly small if calculated in the standard model (SM). Permanent electric dipole moments (EDMs) of nondegenerate systems, which violate T and P, are specific examples of such “null” tests [2]. In this paper we consider a different sort of null test, the pseudo-T-odd correlations of β decay, so-called because they can only be motion-reversal odd [3]. Consequently they can be mimicked by CP-conserving final-state interactions (FSI) in the SM, though these can be computed.

The triple-product correlations observable in ordinary neutron or nuclear β decay are all T violating in that they are motion-reversal odd and connect, through an assumption of CPT invariance, to constraints on sources of CP violation beyond the standard model (BSM). They are also spin dependent. In this context the study of radiative β decay opens a new possibility, namely, of constructing a triple-product correlation from momenta alone. Consequently its measurement would constrain new spin-independent sources of CP violation. Harvey, Hill, and Hill have found that interaction vertices involving the nucleon N, photon γ, and weak gauge bosons at low energies emerge from gauging the axial anomaly of QCD under the full electroweak symmetry of the SM [4, 5]. Such interactions can yield a triple-product momentum correlation in the radiative β decay of neutrons and nuclei. The correlation is both P and pseudo-T-odd, and it vanishes in the SM save for effects induced by FSI. Nevertheless, the correlation can be generated by sources of CP violation BSM, and such couplings, being spin-independent, are not constrained by the nonobservation of permanent EDMs. We discuss, in turn, the physical origins of a triple momentum correlation in the decay rate, its possible size in different systems, and its comparison to the asymmetry induced by electromagnetic FSI in the SM.

Anomalous Interactions at Low Energies — Radiative corrections in gauge theories need not respect all the symmetries present in a massless Dirac theory; in particular, the axial vector current is no longer conserved and becomes anomalous. This physics is also manifest in effective theories of QCD at low energies, in which the pseudoscalar mesons, interpreted as the Nambu-Goldstone bosons of a spontaneously broken chiral symmetry, are the natural degrees of freedom. In this context the nonconservation of the axial current is captured through the inclusion of the Wess-Zumino-Witten (WZW) term [6, 7], so that the chiral Lagrangian can then describe processes such as $K\bar{K} \to 3\pi$ and $\pi^0 \to \gamma\gamma$ [8]. If we study the gauge invariance of the WZW term in vector-like gauge theories such as QED, then the vector current is conserved [9]. Harvey, Hill, and Hill have observed, however, that the gauging of this term under the full electroweak gauge group SU(2)$_L \times$U(1)$_Y$ makes the baryon vector current anomalous and gives rise to pseudo-Chern-Simons contact interactions, containing $\varepsilon^{\mu\nu\rho\sigma}$, at low energy [4, 5]. Such structures are also found in a chiral effective theory in terms of nucleons, pions, and a complete set of SM electroweak gauge fields, where the impact of the use of the full electroweak gauge structure of the SM is illuminated through use of the limit in which the Higgs vacuum expectation value $v_{\text{weak}} \gg f_\pi$ with the SU(2)$_L$ coupling $g_2$ small [10]. Thus the $W^\pm$ and $Z$ appear explicitly in the low-energy effective theory, and the requisite terms appear at N$^2$LO in the chiral expansion. Namely,

$$\mathcal{L}^{(3)} = \ldots + \frac{c_5}{M^2} \bar{N} i\gamma_\mu \gamma_5 \gamma_\sigma \tau^a \text{Tr} \left( \hat{A}_\mu \left[ i\bar{D}_\nu, i\bar{D}_\rho \right] \right) N + \ldots,$$  

(1)
where we report the charged-current term only and note $\tilde{A}_\mu$ is a SU(2) matrix of axial-vector gauge fields, $\tilde{D}_\mu$ is a covariant derivative which contains a SU(2) matrix of vector gauge fields, $N$ is a nucleon doublet, and $M$ is nominally the nucleon mass. We refer to Ref. [10] for all details. Restoring the $W^\pm$ mass to its physical value, we remove the $W^\pm$ from the effective theory to find for neutron beta decay, e.g.,

$$\frac{4c_5}{M^2} \frac{eG_F V_{ud}}{\sqrt{2}} \varepsilon_{\sigma\mu\rho\nu} \bar{p}_e^{-} \gamma^\sigma n \bar{\psi}_e \gamma_\mu \psi_{L} L \gamma_5 \gamma_\nu F_{\nu\rho},$$

(2)

where $2\bar{\psi}_e L = (1 - \gamma_5)\bar{\psi}_e$ and $F_{\nu\rho}$ is the electromagnetic field strength tensor. Thus the baryon weak vector current can mediate parity violation on its own, through the interference of the leading vector amplitude mediated by

$$\frac{G_F V_{ud}}{\sqrt{2}} \varepsilon_{\mu\nu\rho} \bar{p}_e \gamma^\mu n \bar{\psi}_e \gamma_\mu (1 - \gamma_5)\psi_{\nu},$$

(3)

dressed by bremsstrahlung from the charged particles, with the $c_5$ term. An analogous interference term is possible in neutral weak current processes. The T-odd momentum correlation probes the imaginary part of $g_V c_5$ interference. Existing constraints on $c_5$ are poor and come directly only from the measured branching ratio in neutron radiative $\beta$ decay [11, 12], as we shall consider explicitly. The best constraint on $\text{Im}\ g_V$ comes from the recent $D$ term measurement [13, 14], to yield $\text{Im}\ g_V < 7 \times 10^{-4}$ at 68% CL [14]. Thus a first limit on $\text{Im}(g_V c_5)$ would limit $\text{Im}(c_5)$. A triple-product momentum correlation is also possible in theories BSM which do not strictly obey the $V - A$ law; we recall the general parametrization of Lee and Yang [15], updated to use the metric and conventions of Ref. [16] using Ref. [17]:

$$H_{\text{int}} = (\bar{\psi}_p \gamma_\mu \psi_{\nu})(C_S \bar{\psi}_e \gamma_\nu - C'_S \bar{\psi}_e \gamma_5 \gamma_\nu) + (\bar{\psi}_p \gamma_\mu \psi_{\nu})(C_V \bar{\psi}_e \gamma_\nu \gamma_5 - C'_V \bar{\psi}_e \gamma_\nu \gamma_5)$$

$$+ (\bar{\psi}_p \gamma_5 \psi_{\nu})(C_P \bar{\psi}_e \gamma_\nu - C'_P \bar{\psi}_e \gamma_\nu) + (\bar{\psi}_p \gamma_5 \psi_{\nu})(C_A \bar{\psi}_e \gamma_\nu \gamma_5 - C'_A \bar{\psi}_e \gamma_\nu \gamma_5)$$

$$+ \frac{1}{2} \bar{\psi}_p \gamma_{\mu\nu} \psi_{\nu}(C_T \bar{\psi}_e \gamma_{\mu\nu} \psi_{\nu} - C'_T \bar{\psi}_e \gamma_{\mu\nu} \psi_{\nu}),$$

(4)

where in the SM $C_V = C'_V \neq 0$, $C_A = C'_A \neq 0$, and all other $C_i^{(t)}$ vanish. There is an one-to-one map between these coefficients and those derived using modern effective field theory techniques at leading power in the new-physics scale, incorporating the exact gauge symmetry of the SM [18]. If the operators are dressed by bremsstrahlung from the charged particles, they can also contribute to radiative $\beta$ decay and generate a triple-product momentum correlation.

**T-odd Correlation in Radiative $\beta$ Decay** — In $n(p_n) \rightarrow p(p_p) + e^- (l_e) + \nu_e (l_\nu) + \gamma (k)$ decay the interference of the $c_5$ term with the leading $V - A$ terms [19, 21] yields the following contribution to the decay rate

$$|\mathcal{M}|^2_{c_5} = 256e^2 G_F^2 |V_{ud}|^2 \text{Im}(c_5 g_V) \frac{E_e}{l_e \cdot k} (l_e \times k) \cdot l_\nu + \ldots ,$$

(5)

where we neglect corrections of radiative and recoil order. The pseudo-T-odd interference term is finite as $\omega \equiv k^0 \rightarrow 0$, so that its appearance is compatible with Low’s theorem [29]. Alternatively, if we employ Eq. (4), we find

$$|\mathcal{M}|^2_{T-\text{odd,LY}} = 16e^2 G_F^2 |V_{ud}|^2 M l_\nu \cdot (l_e \times k) \frac{1}{l_e \cdot k} \text{Im}[\tilde{C}_T (\tilde{C}'_S + \tilde{C}'_P) + \tilde{C}'_T (\tilde{C}_S + \tilde{C}_P)]$$

(6)
TABLE I: T-odd asymmetries arising from Eq. (5), in units of $\text{Im}C_{HHH}\ [\text{MeV}^{-2}]$, for neutron, $^{19}\text{Ne}$, and $^{35}\text{Ar}$ radiative beta decay as a function of the minimum photon energy $\omega_{\text{min}}$. The branching ratios are reported as well.

| $\omega_{\text{min}}$ (MeV) | $A_{HHH}(n)$ | BR(n) | $A_{HHH}(^{19}\text{Ne})$ | BR($^{19}\text{Ne}$) | $A_{HHH}(^{35}\text{Ar})$ | BR($^{35}\text{Ar}$) |
|-----------------|-------------|-------|-----------------|-------------|-----------------|-------------|
| 0.01            | $-5.61 \times 10^{-3}$ | 3.45 $\times 10^{-3}$ | $-3.60 \times 10^{-2}$ | 4.82 $\times 10^{-2}$ | -0.280 | 0.0655 |
| 0.05            | $-1.30 \times 10^{-2}$ | 1.41 $\times 10^{-3}$ | $-6.13 \times 10^{-2}$ | 2.82 $\times 10^{-2}$ | -0.431 | 0.0424 |
| 0.1             | $-2.20 \times 10^{-2}$ | 7.19 $\times 10^{-4}$ | $-8.46 \times 10^{-2}$ | 2.01 $\times 10^{-2}$ | -0.556 | 0.0328 |
| 0.3             | $-5.34 \times 10^{-2}$ | 8.60 $\times 10^{-5}$ | -0.165 | 8.86 $\times 10^{-3}$ | -0.943 | 0.0185 |

to leading radiative and recoil order, noting $C_i^{(j)} \equiv G_F V_{ud} \tilde{C}_i^{(j)} / \sqrt{2}$ for convenience; our result is compatible with that of Braguta et al. [30] in kaon radiative $\beta$ decay, $K^+ \rightarrow \pi^0 l^+ l^- \gamma$, though they have employed a less general effective Hamiltonian.

Defining $\xi \equiv (l_e \times k) \cdot l_\nu$, we partition phase space into regions of definite sign, so that we form an asymmetry:

$$A(\omega_{\text{min}}) \equiv \frac{\Gamma_+(\omega_{\text{min}}) - \Gamma_-\omega_{\text{min}}}{\Gamma_+(\omega_{\text{min}}) + \Gamma_-\omega_{\text{min}}},$$

where $\Gamma_\pm$ contains an integral of the spin-averaged $|\mathcal{M}|^2$ over the region of phase space with $\xi \geq 0$, respectively, neglecting corrections of recoil order. We compute the branching ratio (BR) as a function of $\omega_{\text{min}}$, the minimum detectable photon energy, ignoring terms of $\mathcal{O}(e^2)$, as well as the BSM contributions of Eq. (4), and employing the inputs of Ref. [24], noting $e^2 = 4\pi\alpha$ with $\alpha \approx 1/137$ the fine-structure constant. As examples of nuclear radiative $\beta$ decays, we consider $^{19}\text{Ne} \rightarrow ^{19}\text{F} + e^+ + \nu_e + \gamma$ and $^{35}\text{Cl} \rightarrow ^{35}\text{Ar} \rightarrow ^{35}\text{Cl} + e^+ + \nu_e + \gamma$, namely, decays involving nuclear mirror transitions. In our evaluations, we employ the nuclear masses of Ref. [22], noting that the maximum positron energy, which is determined in leading recoil order by the nuclear mass difference $Q_{\text{EC}}$, is $3.23883 \pm 0.00030$ MeV for $^{19}\text{Ne}$ decay and $5.96614\pm 0.00070$ MeV for $^{35}\text{Ar}$ decay, and the half-lives of the compilation of Ref. [23], noting $t_{1/2}^{[19]\text{Ne}} = 17.248 \pm 0.029$ s and $t_{1/2}^{[35]\text{Ar}} = 1.7752 \pm 0.0010$ s. The asymmetries are also sensitive to the Gamow-Teller to Fermi mixing parameter, $\rho$ [23], which can be determined from either the measured decay rates [23, 25, 26] or the measured decay correlations [27, 28] in these $\beta$ decays. The $\rho$ values from the two methods are in agreement, except for the most recent $^{19}\text{Ne}$ results, which are only marginally so. The first method is more precise — we use the $\rho$ values of Ref. [23], namely, $\rho^{[19]\text{Ne}} = -1.5933 \pm 0.0030$ and $\rho^{[35]\text{Ar}} = 0.2841 \pm 0.0025$ as per the conventions of Eq. (4). For the neutron we note $\rho/\sqrt{3} = \lambda = -1.2701$ [24].

In Table I we display these results and the asymmetries associated with Eq. (5), reported in units of $\text{Im}C_{HHH} = \text{Im}[g_{\nu\nu}(c_5/M^2)]$, where we refer to Ref. [21] for all details. All nuclear calculations are in the impulse approximation computed in leading recoil order. As for the asymmetry $A_{LY}$, Eq. (6) shows that the contribution exists only at second order in the recoil expansion. In specific, noting $\text{Im}C_{LY} \equiv \text{Im}[(\hat{C}_T^{\nu_\nu} + \hat{C}_S^{\nu_\nu}) + \hat{C}_T^{\nu_s} + \hat{C}_S^{\nu_s} + \hat{C}_P]$, we have for $\omega_{\text{min}} = 0.3$ MeV, in units of $\text{Im}C_{LY}$

$$A_{LY}(n) = 5.21 \times 10^{-6} \ ; \ A_{LY}^{[19]\text{Ne}} = 4.53 \times 10^{-7} \ ; \ A_{LY}^{[35]\text{Ar}} = 8.63 \times 10^{-7}$$

All the asymmetries grow larger as $\omega_{\text{min}}$ increases. The asymmetries associated with Eqs. (5) and (6) appear of grossly dissimilar size; however, if the $M$ associated with $\text{Im}(c_5/M^2)$ is
FIG. 1: Processes which could give rise to the $c_5$-dependent interaction of Eq. (1). We use “$N^*$” to denote a nucleon resonance, and “⊗” for $\rho - $ $\rho'$ mixing.

set by the nucleon mass, as in the SM, then $\mathcal{A}^{\text{HHF}}$ is suppressed significantly — though we already know the asymmetry vanishes in the SM. Since the experimental figure of merit is determined by $\mathcal{A}^2\mathcal{B}R$, the use of larger values of $\omega_{\text{min}}$ would be more suitable for empirical studies. Moreover, the analytic structure of Eqs. (5) and (6) show that the T-odd correlation also increases if the energy released in the decay increases. We discuss the criteria for choosing optimal nuclear systems later.

We consider existing empirical constraints on the coefficients of Eqs. (5) and (6). As for $\text{Im}(c_5/M^2)$, the best and perhaps only constraint comes from the precision measurement of the branching ratio of neutron radiative $\beta$ decay, which has a contribution which goes as $|c_5|^2$. We note $|\text{Im}(c_5/M^2)| < 12\text{MeV}^{-2}$ at 68% CL from the most recent measurement of the branching ratio for neutron radiative $\beta$ decay [12], for which $\omega \in [15, 340]$ keV. The constraint is poor because the radiative decay rate is driven by the contributions from the lowest photon energies, for which $|\mathcal{M}|^2$ is proportional to $\omega^{-2}$ [20]. If one could measure the photon energy spectrum, e.g., close to its endpoint, then the constraint could be much stronger. That is, in the event that one could measure the BR to within 1% of its SM value for $\omega_{\text{min}} = 100$ keV, or for $\omega_{\text{min}} \approx \omega_{\text{max}} = 782$ keV, one would find at 68% CL the limits $|\text{Im}(c_5/M^2)| < 0.88\text{MeV}^{-2}$ and $|\text{Im}(c_5/M^2)| < 0.15\text{MeV}^{-2}$, respectively. In contrast the empirical limits on the couplings which appear in Eq. (6) are already sufficiently severe [31, 32], that any measured asymmetry would necessarily be attributed to the coefficients of Eq. (5).

Interpreting a Limit on the T-odd Asymmetry— The T-odd asymmetry is sensitive to the product $\text{Im}(g_Vc_5)$. The value of $\text{Im}(g_V)$ can be bounded from the deviation of the empirical CKM unitarity test, namely, $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.99995 \pm 0.00061$ [24] from unity, to yield $\text{Im}(g_V) < 0.024$ at 68% CL. The limit from the $D$ term is much sharper, as we have noted: $\text{Im}g_V < 7 \times 10^{-4}$ at 68% CL [14]; a measurement of the T-odd asymmetry would limit $\text{Im}(c_5)$. The $c_5$ coefficient of Eq. (1) can be generated in different ways, and we illustrate some possibilities in Fig. 1 which include mixing with new degrees of freedom, such as a “hidden sector” $\rho'$, as well as possible complex phases associated with the production of known nucleon resonances, or $N^*$’s. We now develop a rudimentary model in which the $\rho'$ helps mediate a difference in the radiative $n$ and $\bar{n}$ $\beta$ decay rates. The notion of a hidden sector of strongly coupled matter is of some standing [33, 34], and has more recently been discussed in the context of models which provide a common origin to baryons and dark matter [35, 36], though the mechanism need not be realized through strong dynamics [37, 38] — we note Ref. [39] for a recent review. Intriguing astrophysical anomalies have prompted the study of hidden sector models which permit couplings to SM leptons; specifically, the visible and hidden sectors are connected through the kinetic mixing of the gauge bosons.
of their respective U(1) symmetries, notably through a SM hypercharge U(1) Y portal [40–43]. Constraints on long-range interactions between dark-matter particles are sufficiently severe [44] that in such models the dark gauge symmetries are also broken through some dark Higgs sector [43]. In this paper we follow a different path. We consider a non-Abelian portal, mediated, e.g., by heavy scalars Φ which transform under the adjoint representation of the group, such an interaction can also be realized through kinetic mixing, generalizing from Ref. [33], through \( \text{tr}(\Phi F_{\mu \nu})\text{tr}(\tilde{\Phi} F^{\mu \nu}) \), as well as \( \epsilon^{\mu \nu \rho \sigma} \text{tr}(\Phi F_{\mu \nu})\text{tr}(\tilde{\Phi} F^{\rho \sigma}) \), where \( F^{\alpha \mu} \) is the SM SU(3)c field strength, and \( \tilde{\Phi}^a \) and \( \Phi^a_{\mu \nu} \) are fields and field strengths of a hidden strongly-coupled sector, nominally based on SU(3)c. We anticipate that the dark matter candidate is a color singlet, so that there are no dark long-range forces to negate. The connector is not a marginal operator, but the appearance of QCD-like couplings should make it more important in the infrared. To build a pertinent model at low energies we recall the hidden local symmetry model of QCD [47, 48], in which the ρ mesons function as effective gauge bosons of the strong interaction. Upon including electromagnetism this becomes a vector-meson dominance model, noting “VMD1” of Ref. [49], which we adapt to this case as

\[
\mathcal{L}_{\text{mix}} = -\frac{1}{4} \rho^{a \mu} \rho^{a \mu} - \frac{1}{4} \rho^{a \mu} \rho^{a \mu} + \frac{\epsilon}{2} \rho^{a \mu} \rho^{a \mu} + \frac{m^2}{2} \rho^{a \mu} \rho^{a \mu} + g_\rho J^{a \mu} \rho^{a \mu} \tag{9}
\]

where \( J^{a \mu} \) denotes the baryon vector current and \( \rho^{(l) a} \) are the gauge bosons of a hidden local SU(2) symmetry — though \( \rho^{(l) a} = \partial_\mu \rho^{(l) a} - \partial_a \rho^{(l) a} \) [49]. Our model resembles those in Refs. [40] but contains two massive vector fields. With \( J^\pm = J^1_\mu \pm i J^2_\mu \) and \( \rho^\pm = (\rho^1_\mu \mp i \rho^2_\mu)/\sqrt{2} \), the charged current pieces, dropping the mass terms, become

\[
\mathcal{L}_{\text{mix}} = -\frac{1}{4} \rho^{+ \mu \nu} \rho_{\mu \nu} - \frac{1}{4} \rho^{- \mu \nu} \rho_{\mu \nu} + \frac{\epsilon}{2} \rho^{+ \mu \nu} \rho_{\mu \nu} + \frac{m^2}{2} \rho^{+ \mu \nu} \rho^{+ \mu \nu} + g_\rho \left( \rho^+ J^\mu + \rho^- J^- \right). \tag{10}
\]

The kinetic mixing term can be removed through the field redefinition \( \tilde{\rho}^\pm = \rho^\pm - \epsilon \rho^\mp \), thus yielding a coupling of the baryon vector current to \( \rho' \), as illustrated in the first panel of Fig. 1 mimicking the role of the “dark photon” in fixed target experiments [50]. The \( \rho'^\pm \) does not couple to photons; indeed, the particles of the hidden sector couple only to strongly interacting particles — we refer to Ref. [39] for discussion of models with generalized conserved charges. We consider \( m_\rho \sim O(m_\rho) \) but with confinement scales \( \Lambda' < \Lambda \) so that \( m_\rho < m_\rho \), noting that dark and baryonic matter can have a common origin even if the dark matter candidate is lighter than the proton in mass [52]. Unlike related “quirk” models [51], the collider signatures of our scenario are minimal and are hidden within hadronization uncertainties. However, if \( m_\rho < 1 \text{ MeV} \) it can be constrained by other low-energy experiments and observations; e.g., it can appear as a mismatch in the value of the neutron lifetime inferred from counting surviving neutrons from that inferred from counting SM decay products. It is also possible to build a model with additional hidden-sector portals. With a U(1) Y portal, e.g., the hidden quarks are allowed to have a milli-electric charge if the dark-matter particle is an electrically neutral composite [53]. This possibility is illustrated in the “mixed basis” in the central panel of Fig. 1. Limits on the SU(2)_L and U(1)_{em} couplings follow, e.g., from studies of the \( W^\pm \) width and the running of \( \alpha \) and are significant; for simplicity we set this possibility aside. Thus limits on the T-odd asymmetry, for which a statistical error of \( O(10^{-3}) \) could be achievable [54], limits \( \text{Im}(e c_5/M^2) = 2\epsilon \text{Im} g_{\rho}\rho/(16\pi^2 m^2_{\rho}) \) with \( g_{\rho} \sim 3.3 \) [55].
TABLE II: Asymmetries from SM FSI in various weak decays. The range of the opening angle between the outgoing electron and photon is chosen to be $-0.9 < \cos(\theta_{e\gamma}) < 0.9$.

| $\omega_{\text{min}}$ (MeV) | $A^{\text{FSI}}(n)$ | $A^{\text{FSI}}(^{19}\text{Ne})$ | $A^{\text{FSI}}(^{35}\text{Ar})$ |
|---------------------|----------------------|----------------------|----------------------|
| 0.01                | $1.76 \times 10^{-5}$ | $-2.86 \times 10^{-5}$ | $-8.35 \times 10^{-4}$ |
| 0.05                | $3.86 \times 10^{-5}$ | $-4.76 \times 10^{-5}$ | $-1.26 \times 10^{-3}$ |
| 0.1                 | $6.07 \times 10^{-5}$ | $-6.40 \times 10^{-5}$ | $-1.60 \times 10^{-3}$ |
| 0.3                 | $1.31 \times 10^{-4}$ | $-1.14 \times 10^{-4}$ | $-2.55 \times 10^{-3}$ |

*SM Background*— CP-conserving FSI in the SM can induce T-odd decay correlations [50, 57]. A triple momentum correlation has been previously studied in $K^+ \rightarrow \pi^0 l^+ \nu \gamma$ decay [58, 59], for which both electromagnetic and strong radiative corrections enter, but the electromagnetic FSI effects are orders of magnitude larger [60]. The small energy release associated with neutron and nuclear radiative $\beta$ decay imply that only electromagnetic radiative corrections can mimic the T-odd effect. The induced T-odd effects in this case have never been studied before, and we describe our calculation in Refs. [21, 61]. We neglect effects of recoil order, which incurs corrections of $\mathcal{O}(Q_{\text{EC}}/M)$, and the nuclear computations are realized in the impulse approximation, so that the effect of meson-exchange currents, estimated to yield corrections of $\mathcal{O}(5–10\%)$ [62], have been neglected. Some numerical results are shown in Table II.

The asymmetry from SM FSI is controlled by $(1 - \rho^2/3)/(1 + \rho^2)$ [21], so that the best choice of nuclear target is determined by noting that (i) the Im $c_5$-induced T-odd asymmetry is bigger when the total energy release is bigger and (ii) the FSI effects can be suppressed if $\rho \sim 1.7$ and the daughter nucleus $Z$ is not too large. Thus lighter nuclear candidates with allowed transitions of large energy release and $\rho \sim 1.7$ are the most useful for the BSM studies we suggest. Furthermore, the radiative $\beta$ decay branching ratio grows as the total energy release grows, making it easier to accrue statistics.

*Summary*— The radiative $\beta$ decay of neutrons and nuclei admits the study of a triple-product correlation in the decay-product momenta. This decay correlation is both parity and motion-reversal odd but spin-independent, making it sensitive to sources of CP violation beyond the SM which are not constrained by searches for permanent EDMs. The appearance of the correlation is controlled by Im $(g_V c_5)$, where $c_5$ is the low-energy constant of the pseudo-Chern-Simons operator first noted by Harvey, Hill, and Hill [4, 5, 10]. The size of the SM background is small relative to the anticipated experimental sensitivity. Empirical limits on the triple-product correlation can be interpreted as limits on the CP-violating kinetic mixing of the gauge bosons of QCD with a strongly coupled hidden sector, possibly giving new insights on the origin of baryons and dark matter.

We acknowledge partial support from the U.S. Department of Energy under contract no. DE-FG02-96ER40989, and we thank Oscar Naviliat–Cuncic and Jeffrey Nico for information regarding experimental aspects of radiative $\beta$ decay.

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