TIME DEPENDENT RELATIVISTIC MEAN-FIELD THEORY

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Abstract
The relativistic mean-field theory provides a framework in which the nuclear many-body problem is described as a self-consistent system of nucleons and mesons. In the mean-field approximation, the self-consistent time evolution of the nuclear system describes the dynamics of collective motion: double giant resonances, nuclear compressibility from monopole resonances, regular and chaotic dynamics of isoscalar and isovector collective vibrations.

1 Introduction and outline of the model

Relativistic mean-field (RMF) models have been successfully applied in calculations of nuclear matter and properties of finite nuclei throughout the periodic table. In the self-consistent mean-field approximation, detailed calculations have been performed for a variety of nuclear structure phenomena [1]. In the present work we review the applications of RMF to the dynamics of collective vibrations in spherical nuclei. In relativistic quantum hadrodynamics [2], the nucleus is described as a system of Dirac nucleons which interact through the exchange of virtual mesons and photons. The Lagrangian density of the model is

\[ \mathcal{L} = \bar{\psi} (i \gamma \cdot \partial - m) \psi + \frac{1}{2} (\partial \sigma)^2 - U(\sigma) \]

\[ - \frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu} + \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{4} \vec{R}_{\mu\nu} \vec{R}^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^2 - \frac{1}{4} m_\rho^2 \rho^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \]

\[ - g_\sigma \bar{\psi} \sigma \psi - \frac{1}{2} \bar{\psi} \gamma \cdot \omega \psi - \frac{1}{3} \bar{\psi} \gamma \cdot \rho \tau \psi - \frac{1}{2} \bar{\psi} \gamma \cdot A \frac{(1 - \tau_3)}{2} \psi . \]

The Dirac spinor \( \psi \) denotes the nucleon with mass \( m \). \( m_\sigma, m_\omega, \) and \( m_\rho \) are the masses of the \( \sigma \)-meson, the \( \omega \)-meson, and the \( \rho \)-meson, and \( g_\sigma, g_\omega, \) and \( g_\rho \) are the corresponding coupling constants for the mesons to the nucleon. \( U(\sigma) \) denotes the nonlinear \( \sigma \) self-interaction

\[ U(\sigma) = \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{3} g_2 \sigma^3 + \frac{1}{4} g_3 \sigma^4, \]

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and $\Omega^{\mu\nu}$, $\tilde{R}^{\mu\nu}$, and $F^{\mu\nu}$ are field tensors $[^2]$. The coupled equations of motion are derived from the Lagrangian density (1). The Dirac equation for the nucleons:

$$i\partial_t \psi_i = \left[ \alpha \left( -i \nabla - g_\omega \omega - g_\rho \vec{\tau}_\rho \rho + e \left( \frac{1 - \tau_3}{2} \right) A \right) + \beta (m + g_\sigma \sigma) 
+ g_\omega \omega_0 + g_\rho \vec{\tau}_\rho \rho_0 + e \left( \frac{1 - \tau_3}{2} \right) A_0 \right] \psi_i$$

(3)

and the Klein-Gordon equations for the mesons:

$$\left( \partial_t^2 - \Delta + m^2_\sigma \right) \sigma = -g_\sigma \rho_s - g_2 \sigma^2 - g_3 \sigma^3$$

(4)

$$\left( \partial_t^2 - \Delta + m^2_\omega \right) \omega_\mu = g_\omega j_\mu$$

(5)

$$\left( \partial_t^2 - \Delta + m^2_\rho \right) \vec{\rho}_\mu = g_\rho \vec{j}_\mu$$

(6)

$$\left( \partial_t^2 - \Delta \right) A_\mu = e j_{\mu}^{\text{em}}.$$

(7)

In the relativistic mean-field approximation, the nucleons described by single-particle spinors $\psi_i$ ($i = 1, 2, ..., A$) are assumed to form the $A$-particle Slater determinant $|\Phi\rangle$, and to move independently in the classical meson fields. The sources of the fields, i.e. densities and currents, are calculated in the no-sea approximation $[^3]$: the scalar density: $\rho_s = \sum_{i=1}^{A} \bar{\psi}_i \psi_i$, the isoscalar baryon current: $j^\mu = \sum_{i=1}^{A} \bar{\psi}_i \gamma^\mu \psi_i$, the isovector baryon current: $\vec{j}_\mu = \sum_{i=1}^{A} \bar{\psi}_i \gamma^\mu \vec{\tau} \psi_i$, the electromagnetic current for the photon-field: $j_{\mu}^{\text{em}} = \sum_{i=1}^{A} \bar{\psi}_i \gamma^{\mu \frac{1 - \tau_3}{2}} \psi_i$. The summation is over all occupied states in the Slater determinant $|\Phi\rangle$. Negative-energy states do not contribute to the densities in the no-sea approximation of the stationary solutions. It is assumed that nucleon single-particle states do not mix isospin.

The ground state of a nucleus is described by the stationary self-consistent solution of the coupled system of equations (3)–(7), for a given number of nucleons and a set of coupling constants and masses. The solution for the ground state specifies part of the initial conditions for the time-dependent problem. The dynamics of nuclear collective motion is analyzed in the framework of time-dependent relativistic mean-field model, which represents a relativistic generalization of the time-dependent Hartree-Fock approach. For a given set of initial conditions, i.e. initial values for the densities and currents, nuclear dynamics is described by the simultaneous evolution of $A$ single-particle Dirac spinors in the time-dependent mean fields. Frequencies of eigenmodes are determined from a Fourier analysis of dynamical quantities. In this microscopic model, self-consistent time-dependent mean-field calculations are performed for multipole excitations. An advantage of the time-dependent approach is that no assumption about the nature of a particular mode of vibrations has to be made. Retardation effects for the meson fields are not included in the model, i.e. the time derivatives $\partial_t^2$ in the equations of motions for the meson fields are neglected. This is justified by the large masses in the meson propagators causing a short range of the corresponding meson exchange forces. Negative energy contributions are included implicitly in the time-dependent calculation, since the Dirac equation is solved at each step in time for a different basis set $[^3]$. 


Negative energy components with respect to the original ground-state basis are taken into account automatically, even if at each time step the *no-sea* approximation is applied. The description of nuclear dynamics as a time-dependent initial-value problem is intrinsically semi-classical, since there is no systematic procedure to derive the initial conditions that characterize the motion of a specific mode of the nuclear system. The theory can be quantized by the requirement that there exist time-periodic solutions of the equations of motion, which give integer multiples of Planck’s constant for the classical action along one period [4]. For giant resonances the time-dependence of collective dynamical quantities is actually not periodic, since generally giant resonances are not stationary states of the mean-field Hamiltonian. The coupling of the mean-field to the particle continuum allows for the decay of giant resonances by direct escape of particles. In the limit of small amplitude oscillations, however, the energy obtained from the frequency of the oscillation coincides with the excitation energy of the collective state. In Refs. [3, 4, 5] we have shown that the model reproduces experimental data on giant resonances in spherical nuclei.

2 Dynamics of collective vibrations

2.1 Double Giant Resonances

The physics of giant resonances in nuclei has been the subject of extensive experimental and theoretical studies for many years. However, only recently substantial evidence has been reported for two-phonon states built with giant resonances. Resonant structures that were observed in heavy-ion inelastic scattering, have been interpreted as possible multiple excitations of the giant quadrupole resonance. Double giant dipole resonances have also been discovered in the neutron and in the $\gamma$-spectra of nuclei that have been Coulomb excited in relativistic heavy-ion collisions. In Ref. [4] we have performed time-dependent relativistic mean-field calculations and found evidence for modes which can be interpreted as double resonances, and which in a quantized theory correspond to two-phonon states.

As an example of double resonances in light nuclei, we consider the double isoscalar giant quadrupole resonance in $^{40}$Ca. The experimental spectrum exhibits a prominent structure, centered at $34 \pm 2$ MeV excitation energy, with a width of $9 \pm 2$ MeV. It is interpreted as the two-phonon state of the single isoscalar GQR at 17.5 MeV. Using the NL-SH parameter set for the effective Lagrangian, we have studied isoscalar quadrupole oscillations in $^{40}$Ca. The quadrupole mode of oscillations is excited by deforming the spherical solution for the ground-state. For a specific initial deformation, we follow the time-evolution of the collective variable, the quadrupole moment

$$q_{20}(t) = \langle \Phi(t) | \hat{Q}_{20} | \Phi(t) \rangle = \langle \Phi(t) | r^2 Y_{20} | \Phi(t) \rangle$$

(8)

The time-dependent quadrupole moment shown in Fig. 1 corresponds to an initial axial deformation of the baryon density $\beta = 0.38$. The resulting Fourier spectrum displays a strong peak at $18.5$ MeV, in reasonable agreement with the experimental data. We find evidence for excitation of a higher mode in the oscillations of the baryon density. The wave
function of the nuclear system is a Slater determinant at all times, and therefore can be expanded in the basis of the ground state $|\Phi_0\rangle$

$$|\Phi(t)\rangle = |\Phi_0\rangle + \sum_{mi} z_{mi}(t) a^+_m a_i |\Phi_0\rangle + \sum_{mim'i'} z_{mi}(t) z_{m'i'}(t) a^+_m a^+_m a_i a_{i'} |\Phi_0\rangle + \ldots$$  \hspace{1cm} (9)

If the total wave function contains collective $2p - 2h$ components, they will be observed in the Fourier spectrum of the time-dependent baryon density $\rho_B(\mathbf{r}, t) = \sum_{i=1}^A \psi_i^+(\mathbf{r}, t) \psi_i(\mathbf{r}, t)$. Because of axial symmetry and the isoscalar nature of the excitation, it is sufficient to consider oscillations in time of the baryon density on the positive $z$-axis. In Fig. 2 we display the Fourier transforms of the time-dependent baryon density for various values of the coordinate $z$. Two peaks are clearly observed. The first one at 18.5 MeV corresponds to the isoscalar quadrupole resonance. It gradually increases from the center toward the surface of the nucleus. If we plot the values of the Fourier transforms at 18.5 MeV as a function of $z$, the resulting curve corresponds to the transition density. The transition density for the first peak is typical for isoscalar quadrupole resonances. The second peak is at 37 MeV, twice the energy of the GQR. It has a maximum in the center of the nucleus, at first decreases with $z$, but then appears again on the surface. Compared to the GQR transition density, the curve for the 37 MeV peak displays an additional node.

Periodic solutions of the time-dependent Dirac equations can be used to construct the energies of the many-body system. The energy spectrum can be obtained from a semi-classical quantization procedure. One finds periodic solutions such that the mean-field action along a periodic orbit

$$I = \sum_i \int_{t_0}^{t_0+T} dt \left[ <\psi_i(t)|i\hbar \frac{\partial}{\partial t}|\psi_i(t)> - e_i \right]$$ \hspace{1cm} (10)

is equal to an integer multiple of the Planck constant $I = nh$. In Eq. (10) $T$ is the period of oscillations, $|\psi_i(t)>$ denotes time-dependent single-nucleon Dirac spinors, $e_i$ are the corresponding single-nucleon energies in the unperturbed ground-state, and the summation runs over the occupied states. Because of the coupling to the continuum in the mean-field description, giant resonances are not stationary states of the Hamiltonian. Consequently, a non-periodic dependence on time is obtained for dynamical quantities. If the damping is very strong, the giant resonance is not periodic even on the average, and the quantization condition cannot be applied. However, if the motion is nearly periodic, i.e. the damping is relatively weak, the quantization procedure can still be used to calculate the energies, and the effect of damping can be taken into account approximately.

In Fig. 3 we display the mean-field action as function of the excitation energy of the nucleons and of the initial deformation $\beta$. For the values of $\beta$ indicated by dots we have integrated the coupled system of Dirac and Klein-Gordon equations and calculated the action integral. The mean-field action is a quadratic function of the initial deformation $\beta$, and an almost perfect linear function of the excitation energy. Only above 60 MeV a slight deviation from a pure linear dependence is observed. The values of $<E^*>$ for which the action is an integer multiple of the Planck constant are: $I = 1\,h$ for 18.5 MeV, and $I = 2\,h$ for 37.1 MeV. A double giant dipole resonance has been observed in relativistic Coulomb excitation of $^{208}$Pb. The single GDR is found at $13.3 \pm 0.1$ MeV with a width of $4.1 \pm 0.1$ MeV.
The sum energy of coincident photon pairs displays a broad structure at 25.6 ± 0.9 MeV with a width of 5.8 ± 1.1 MeV. It is interpreted as the double GDR. In order to excite isovector dipole motion we define the initial conditions: at $t = 0$ (in the center of mass system) all protons start moving in the $+z$ direction with velocity $v_\pi$, and all neutrons start moving in the $-z$ direction with velocity $v_\nu = \frac{Z}{A} v_\pi$. For the NL-SH parameter set, the Fourier spectrum of the time-dependent dipole moment displays a strong peak at 12.9 MeV excitation energy, in good agreement with experimental data. We have found that the mean-field action is an integer multiple of the Planck constant: $I = 1 \ h$ for $< E^* > = 12.9$ MeV, and $I = 2 \ h$ for $< E^* > = 25.9$ MeV. Therefore, the energy of the one-phonon state, calculated from the mean-field action, coincides with the resonant energy of the mean peak in the Fourier spectrum, and the two-phonon state at 25.9 MeV is in excellent agreement with the experimental value for the excitation energy of the double GDR.

### 2.2 Monopole Giant Resonances and nuclear compressibility

The study of isoscalar monopole resonances in nuclei provides an important source of information on the nuclear matter compression modulus $K_{nm}$. This quantity is crucial in the description of properties of nuclei, supernovae explosions, neutron stars, and heavy-ion collisions. In principle the value of $K_{nm}$ can be extracted from experimental energies of isoscalar monopole vibrations in nuclei (giant monopole resonances GMR). However, the complete experimental data set on isoscalar GMR does not limit the range of $K_{nm}$ to better than $200 - 350$ MeV. Microscopic calculations of GMR excitation energies might provide a more reliable approach to the determination of the nuclear matter compression modulus. Modern non-relativistic Hartree-Fock plus RPA calculations, using both Skyrme and Gogny effective interactions, indicate that the value of $K_{nm}$ should be in the range 210-220 MeV. In relativistic calculations on the other hand, both time-dependent and constrained RMF results indicate that empirical GMR energies are best reproduced by an effective force with $K_{nm} \approx 250 - 270$ MeV.

In Ref. \cite{5} we have performed time-dependent and constrained RMF calculations for monopole giant resonances of a number of spherical closed shell nuclei, from $^{16}$O to the heavy nucleus $^{208}$Pb. For the effective Lagrangian we have used six parameter sets, which differ mostly by their prediction for $K_{nm}$, but otherwise reproduce reasonably well experimental data on nuclear properties. The idea is to restrict the possible values of the nuclear matter compression modulus, on the basis of the excitation energies of giant monopole states calculated with different effective interactions. In addition to the four non-linear sets NL1, NL3, NL-SH and NL2, we have also included two older linear parametrizations, HS and L1. The sets NL1, NL-SH and NL2 have been extensively used in the description of properties of finite nuclei \cite{1}. In order to bridge the gap between NL1 ($K_{nm} = 211.7$ MeV), and NL-SH ($K_{nm} = 355.0$ MeV), we have also included a new effective interaction NL3 \cite{6} ($K_{nm} = 271.8$ MeV). This new parameter set provides an excellent description not only for the properties of stable nuclei, but also for those far from the valley of beta stability. From the energy spectra and transition densities calculated with these effective forces, it has been possible to study the connection between the incompressibility of nuclear matter and the breathing...
mode energy of spherical nuclei. For the isoscalar mode we have found an almost linear relation between the excitation energy of the monopole resonance and the nuclear matter compression modulus. For the determination of $K_{\text{nm}}$ especially relevant are microscopic calculations of GMR excitation energies in heavy nuclei. The results of TD RMF calculations for $^{208}\text{Pb}$ are displayed in Fig. 4: time-dependent monopole moments $\langle r^2(t) \rangle = \frac{1}{2} \langle \Phi(t) | r^2 | \Phi(t) \rangle$ and the corresponding Fourier power spectra for the nonlinear effective interactions. As one would expect for a heavy nucleus, there is very little spectral fragmentation and a single mode dominates, at least for NL1 and NL3. The experimental excitation energy $13.7 \pm 0.3$ MeV is very close to the frequency of oscillations obtained with the NL3 parameter set: 14.1 MeV. The calculated excitation energy for the NL1 parameter set ($K_{\text{nm}} = 211.7$ MeV), is approximately 1 MeV lower than the average experimental value. For the linear effective forces HS and L1 the oscillations are more anharmonic, and the monopole strength is located well above the experimental GMR energy. The effective interactions NL1 and NL3 seem to produce GMR excitation energies which are quite close to the experimental values. For these two parameter sets we have calculated the isoscalar giant monopole resonances in a number of doubly closed-shell nuclei: $^{40}\text{Ca}$, $^{56}\text{Ni}$, $^{100,114,132}\text{Sn}$, $^{90,122}\text{Zr}$, $^{146}\text{Gd}$. The results are shown in Fig. 5. The energies of giant monopole states are determined from the Fourier spectra of the time-dependent monopole moments, and are displayed as function of the mass number. The NL1 excitation energies are systematically lower, but otherwise the two effective interactions produce very similar dependence on the mass number. The empirical curve $E_x \approx 80 A^{-1/3}$ MeV is also included in the figure, and it follows very closely the excitation energies calculated with the NL3 parameter set. Similar results are obtained from constrained RMF calculations. Both methods indicate that, in the framework of relativistic mean field theory, the nuclear matter compression modulus $K_{\text{nm}} \approx 250 - 270$ MeV is in reasonable agreement with the available data on spherical nuclei. This value is approximately 20% larger than the values deduced from recent non-relativistic density dependent Hartree-Fock calculations with Skyrme or Gogny forces.

### 2.3 Regular and chaotic dynamics of collective vibrations

The atomic nucleus has been used as a laboratory, both experimentally and theoretically, for investigating the transition from order to chaos in quantum dynamical systems. Most of these studies have concentrated on two major aspects: (i) generic signatures of chaos in local fluctuations and correlations of nuclear level distributions, and (ii) chaos in microscopic and collective dynamics of realistic many-body systems. Regular and chaotic dynamics in giant nuclear oscillations has been the subject of a number of studies. What has emerged as a very interesting result is that an undamped collective mode may coexists with chaotic single-particle motion. It appears that the slowly vibrating self-consistent mean field created by the same nucleons averages out the random components in their motion. In all investigations the motion of only one type of particles has been considered. That is, only the dynamics of isoscalar collective modes.

We have studied the difference in the dynamics of isoscalar and isovector collective modes.
In particular, we consider isoscalar and isovector monopole oscillations in spherical nuclei, but analogous considerations apply to higher multipolarities. In Fig. 6 results are shown of time-dependent relativistic mean-field calculations for isoscalar and isovector oscillations in $^{208}$Pb. The experimental isoscalar GMR energy in $^{208}$Pb is $13.7 \pm 0.3$ MeV, and the excitation energy of the isovector mode is $26 \pm 3$ MeV. Calculations have been performed for the NL1 effective interaction. In the isoscalar case both proton and neutron densities are radially expanded, while for the isovector mode the proton density is initially compressed by the same amount. Therefore, in both cases we follow the time evolution of the same system, just the initial conditions are different. In Fig. 6a we plot the time history of the isoscalar monopole moment $\langle r^2(t) \rangle$, and in Fig. 6b the corresponding isovector moment $\langle r_p^2(t) \rangle - \langle r_n^2(t) \rangle$ is displayed. The isoscalar mode displays regular undamped oscillations, while for the isovector mode we observe strongly damped anharmonic oscillations. On the right hand panels we plot the corresponding Fourier power spectra. The Fourier spectrum of the isovector mode is strongly fragmented. However, the main peaks are found in the energy region $25 - 30$ MeV, in agreement with the experimental data. For the isoscalar mode, the time history of the monopole moment and the Fourier spectrum show that the oscillations of the collective coordinate are regular. On the other hand, the appearance of a broad spectrum of frequencies seems to indicate that the isovector oscillations are chaotic.

A diagnosis of chaotic vibrations would imply that one has a clear definition of such motion. For a quantum system, however, the concept of chaos, especially in time-dependent problems, is not well defined. And although our description of nuclear vibrations is semi-classical, quantum effects like the Pauli principle are present in the initial conditions and during the dynamical evolution. There exists a number of tests that can help to identify chaotic oscillations in physical systems, and some of them can be applied in the present consideration. In Figs. 7 - 9 we display some additional qualitative measures which can be used to characterize the response of our nonlinear system. In Fig. 7 we have constructed the two-dimensional time-delayed pseudo-phase space for the isoscalar (a) and isovector (b) oscillations, shown in Fig. 6. Since information is available on the time evolution of just one variable, the collective coordinate, one plots the signal versus itself, but delayed or advanced by a fixed time constant $\langle r^2(t) \rangle, \langle r^2(t+\tau) \rangle$. The phase space trajectories for the isoscalar mode are closed ellipses, indicating regular oscillations. For the isovector oscillations on the other hand, the trajectories are completely chaotic. The strong damping results from one-body processes: (i) escape of nucleons into the continuum states and (ii) collisions of the nucleons with the moving wall of the nuclear potential generated by the self-consistent mean fields. In Fig. 8 we display the corresponding Poincaré sections constructed from three-dimensional time-delayed pseudo-phase space. The Poincaré map for the isoscalar mode consists of two sets of closely located points, and therefore confirms regular oscillations. For the isovector oscillations the Poincaré map appears as a cloud of unorganized points in the phase plane. Such a map indicates stochastic motion. Another measure that is related to the Fourier transform is the autocorrelation function

$$A(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T \langle r^2(t) \rangle \langle r^2(t+\tau) \rangle \, dt$$

(11)
When the signal is chaotic, information about its past origins is lost. This means that \( A(\tau) \to 0 \) as \( \tau \to \infty \), or the signal is only correlated with its recent past. The autocorrelation functions for isoscalar and isovector oscillations are shown in Fig. 9. For the isovector mode \( A(\tau) \) displays a rapid decrease, and the envelope appears as an irregular waveform. The oscillations of the collective coordinate can be characterized as regular for the isoscalar mode, and they become chaotic when initial conditions correspond to the isovector mode. In Ref. [7] we have shown how a regular collective mode can coexist with chaotic single-particle dynamics, a result which confirms the conclusions of a number of studies. However, we have also shown that this is the case only for isoscalar modes, that is, only if one considers the motion of a single type of particles. When protons and neutrons move out of phase, as it happens for isovector modes, the resulting dynamics of the collective coordinate exhibit chaotic behavior. This is explained by the fact that protons and neutrons effectively move in two self-consistent potentials that oscillate out of phase. For example, when neutrons move inward, they scatter on the potential wall with positive curvature that is created by protons moving outward. This will lead to pseudo-random motion of the nucleons and dissipation of collective oscillations.

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Figure Captions

**FIG. 1.** Time-dependent quadrupole moment and the corresponding Fourier spectrum for $^{40}\text{Ca}$.  

**FIG. 2.** Fourier spectra of the time-dependent baryon density for various values of the coordinate $z$, on the axis along which the initial densities are deformed.  

**FIG. 3.** Mean-field action as function of the excitation energy of the nucleons $E^* = <\Phi(t)|H_D(t)|\Phi(t)> - <\Phi_{GS}|H_D(0)|\Phi_{GS}>$ (a), and of the initial deformation $\beta$ (b).  

**FIG. 4.** Time-dependent isoscalar monopole moments $<r^2>(t)$ and the corresponding Fourier power spectra for $^{208}\text{Pb}$. The parameter sets are NL1, NL3, NL-SH and NL2.  

**FIG. 5.** Excitation energies of isoscalar giant monopole resonances in spherical nuclei as function of the mass number. The effective interactions are: NL1 (squares) and NL3 (circles). The solid curve corresponds to the empirical relation $\approx 80 A^{-1/3}$ MeV.  

**FIG. 6.** Results of time-dependent relativistic mean-field calculations for isoscalar and isovector oscillations in $^{208}\text{Pb}$.  

**FIG. 7.** Pseudo-phase space for isoscalar (a) and isovector (b) monopole oscillations in $^{208}\text{Pb}$.  

**FIG. 8.** Poincaré sections for isoscalar (a) and isovector (b) monopole oscillations in $^{208}\text{Pb}$.  

**FIG. 9.** Autocorrelation functions for isoscalar and isovector oscillations in $^{208}\text{Pb}$.
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Abstract
The relativistic mean-field theory provides a framework in which the nuclear many-body problem is described as a self-consistent system of nucleons and mesons. In the mean-field approximation, the self-consistent time evolution of the nuclear system describes the dynamics of collective motion: double giant resonances, nuclear compressibility from monopole resonances, regular and chaotic dynamics of isoscalar and isovector collective vibrations.

1 Introduction and outline of the model
Relativistic mean-field (RMF) models have been successfully applied in calculations of nuclear matter and properties of finite nuclei throughout the periodic table. In the self-consistent mean-field approximation, detailed calculations have been performed for a variety of nuclear structure phenomena [1]. In the present work we review the applications of RMF to the dynamics of collective vibrations in spherical nuclei. In relativistic quantum hadrodynamics [2], the nucleus is described as a system of Dirac nucleons which interact through the exchange of virtual mesons and photons. The Lagrangian density of the model is

\[
\mathcal{L} = \bar{\psi} \left( i \gamma \cdot \partial - m \right) \psi + \frac{1}{2} (\partial \sigma)^2 - U(\sigma) \\
- \frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu} + \frac{1}{2} m_{\omega}^2 \omega^2 - \frac{1}{4} \bar{R}_{\mu\nu} R^{\mu\nu} + \frac{1}{2} \bar{m}_\rho \rho^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\
- g_\sigma \bar{\psi} \sigma \psi - g_\omega \bar{\psi} \gamma \cdot \omega \psi - g_\rho \bar{\psi} \gamma \cdot \bar{\rho} \tau \psi - e \bar{\psi} \gamma \cdot A \frac{1 - \tau_3}{2} \psi .
\]

The Dirac spinor \( \psi \) denotes the nucleon with mass \( m \). \( m_\sigma \), \( m_\omega \), and \( m_\rho \) are the masses of the \( \sigma \)-meson, the \( \omega \)-meson, and the \( \rho \)-meson, and \( g_\sigma \), \( g_\omega \), and \( g_\rho \) are the corresponding coupling constants for the mesons to the nucleon. \( U(\sigma) \) denotes the nonlinear \( \sigma \) self-interaction

\[
U(\sigma) = \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{3} g_2 \sigma^3 + \frac{1}{4} g_3 \sigma^4,
\]

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and $\Omega^{\mu\nu}$, $\tilde{R}^{\mu\nu}$, and $F^{\mu\nu}$ are field tensors \[2\].

The coupled equations of motion are derived from the Lagrangian density (1). The Dirac equation for the nucleons:

$$i\partial_t \psi_i = \left[ \alpha \left( -i\nabla - g_\omega \omega - g_\rho \overline{\mathbf{r}}\mathbf{r} - e \left( \frac{1 - \tau_3}{2} \right) A \right) + \beta (m + g_\sigma \sigma) + g_\omega \omega_0 + g_\rho \overline{\mathbf{r}}\mathbf{r}_0 + e \left( \frac{1 - \tau_3}{2} \right) A_0 \right] \psi_i \tag{3}$$

and the Klein-Gordon equations for the mesons:

$$\left( \partial_t^2 - \Delta + m_\sigma^2 \right) \sigma = -g_\sigma \rho_s - g_2 \sigma^2 - g_3 \sigma^3 \tag{4}$$

$$\left( \partial_t^2 - \Delta + m_\omega^2 \right) \omega_\mu = g_\omega j_\mu \tag{5}$$

$$\left( \partial_t^2 - \Delta + m_\rho^2 \right) \rho_\mu = g_\rho \overline{\mathbf{r}}\mathbf{r}_\mu \tag{6}$$

$$\left( \partial_t^2 - \Delta \right) A_\mu = e j^{\text{em}}_\mu. \tag{7}$$

In the relativistic mean-field approximation, the nucleons described by single-particle spinors $\psi_i (i = 1, 2, ..., A)$ are assumed to form the A-particle Slater determinant $|\Phi\rangle$, and to move independently in the classical meson fields. The sources of the fields, i.e. densities and currents, are calculated in the no-sea approximation \[3\]: the scalar density: $\rho_s = \sum_{i=1}^A \overline{\psi}_i \psi_i$, the isoscalar baryon current: $j_\mu = \sum_{i=1}^A \overline{\psi}_i \gamma^\mu \psi_i$, the isovector baryon current: $\overline{\mathbf{r}}\mathbf{r}_\mu = \sum_{i=1}^A \overline{\psi}_i \gamma^\mu \overline{\mathbf{r}}\mathbf{r}_i$, the electromagnetic current for the photon-field: $\overline{\mathbf{r}}\mathbf{r}_\mu = \sum_{i=1}^A \overline{\psi}_i \gamma^\mu \overline{\mathbf{r}}\mathbf{r}_i$. The summation is over all occupied states in the Slater determinant $|\Phi\rangle$. Negative-energy states do not contribute to the densities in the no-sea approximation of the stationary solutions. It is assumed that nucleon single-particle states do not mix isospin.

The ground state of a nucleus is described by the stationary self-consistent solution of the coupled system of equations (3)–(7), for a given number of nucleons and a set of coupling constants and masses. The solution for the ground state specifies part of the initial conditions for the time-dependent problem. The dynamics of nuclear collective motion is analyzed in the framework of time-dependent relativistic mean-field model, which represents a relativistic generalization of the time-dependent Hartree-Fock approach. For a given set of initial conditions, i.e. initial values for the densities and currents, nuclear dynamics is described by the simultaneous evolution of $A$ single-particle Dirac spinors in the time-dependent mean fields. Frequencies of eigenmodes are determined from a Fourier analysis of dynamical quantities. In this microscopic model, self-consistent time-dependent mean-field calculations are performed for multipole excitations. An advantage of the time-dependent approach is that no assumption about the nature of a particular mode of vibrations has to be made. Retardation effects for the meson fields are not included in the model, i.e. the time derivatives $\partial_t^2$ in the equations of motions for the meson fields are neglected. This is justified by the large masses in the meson propagators causing a short range of the corresponding meson exchange forces. Negative energy contributions are included implicitly in the time-dependent calculation, since the Dirac equation is solved at each step in time for a different basis set \[3\].
Negative energy components with respect to the original ground-state basis are taken into account automatically, even if at each time step the no-sea approximation is applied. The description of nuclear dynamics as a time-dependent initial-value problem is intrinsically semi-classical, since there is no systematic procedure to derive the initial conditions that characterize the motion of a specific mode of the nuclear system. The theory can be quantized by the requirement that there exist time-periodic solutions of the equations of motion, which give integer multiples of Planck’s constant for the classical action along one period [4]. For giant resonances the time-dependence of collective dynamical quantities is actually not periodic, since generally giant resonances are not stationary states of the mean-field Hamiltonian. The coupling of the mean-field to the particle continuum allows for the decay of giant resonances by direct escape of particles. In the limit of small amplitude oscillations, however, the energy obtained from the frequency of the oscillation coincides with the excitation energy of the collective state. In Refs. [3, 4, 5] we have shown that the model reproduces experimental data on giant resonances in spherical nuclei.

2 Dynamics of collective vibrations

2.1 Double Giant Resonances

The physics of giant resonances in nuclei has been the subject of extensive experimental and theoretical studies for many years. However, only recently substantial evidence has been reported for two-phonon states built with giant resonances. Resonant structures that were observed in heavy-ion inelastic scattering, have been interpreted as possible multiple excitations of the giant quadrupole resonance. Double giant dipole resonances have also been discovered in the neutron and in the γ-spectra of nuclei that have been Coulomb excited in relativistic heavy-ion collisions. In Ref. [4] we have performed time-dependent relativistic mean-field calculations and found evidence for modes which can be interpreted as double resonances, and which in a quantized theory correspond to two-phonon states.

As an example of double resonances in light nuclei, we consider the double isoscalar giant quadrupole resonance in $^{40}$Ca. The experimental spectrum exhibits a prominent structure, centered at $34 \pm 2$ MeV excitation energy, with a width of $9 \pm 2$ MeV. It is interpreted as the two-phonon state of the single isoscalar GQR at 17.5 MeV. Using the NL-SH parameter set for the effective Lagrangian, we have studied isoscalar quadrupole oscillations in $^{40}$Ca. The quadrupole mode of oscillations is excited by deforming the spherical solution for the ground-state. For a specific initial deformation, we follow the time-evolution of the collective variable, the quadrupole moment

$$q_{20}(t) = \langle \Phi(t)|\hat{Q}_{20}|\Phi(t)\rangle = \langle \Phi(t)|r^2Y_{20}|\Phi(t)\rangle$$

The time-dependent quadrupole moment shown in Fig. 1 corresponds to an initial axial deformation of the baryon density $\beta = 0.38$. The resulting Fourier spectrum displays a strong peak at 18.5 MeV, in reasonable agreement with the experimental data.
We find evidence for excitation of a higher mode in the oscillations of the baryon density. The wave function of the nuclear system is a Slater determinant at all times, and therefore can be expanded in the basis of the ground state $|\Phi_0\rangle$

$$|\Phi(t)\rangle = |\Phi_0\rangle + \sum_{m} z_{m\uparrow}(t)a^+_m a_{\uparrow} |\Phi_0\rangle + \sum_{m'\uparrow \downarrow} z_{m\uparrow}(t)z_{m'\downarrow}(t) a^+_m a^+_m a_{\uparrow} a_{\downarrow} |\Phi_0\rangle + \ldots \quad (9)$$

If the total wave function contains collective $2p - 2n$ components, they will be observed in the Fourier spectrum of the time-dependent baryon density $\rho_B(r, t) = \sum_{i=1}^{A} \psi_i^+(r, t) \psi_i(r, t)$. Because of axial symmetry and the isoscalar nature of the excitation, it is sufficient to consider oscillations in time of the baryon density on the positive $z$-axis. In Fig. 2 we display the Fourier transforms of the time-dependent baryon density for various values of the coordinate $z$. Two peaks are clearly observed. The first one at 18.5 MeV corresponds to the isoscalar quadrupole resonance. It gradually increases from the center toward the surface of the nucleus. If we plot the values of the Fourier transforms at 18.5 MeV as a function of $z$, the resulting curve corresponds to the transition density. The transition density for the first peak is typical for isoscalar quadrupole resonances. The second peak is at 37 MeV, twice the energy of the GQR. It has a maximum in the center of the nucleus, at first decreases with $z$, but then appears again on the surface. Compared to the GQR transition density, the curve for the 37 MeV peak displays an additional node.

Periodic solutions of the time-dependent Dirac equations can be used to construct the energies of the many-body system. The energy spectrum can be obtained from a semi-classical quantization procedure. One finds periodic solutions such that the mean-field action along a periodic orbit

$$I = \sum_i \int_{t_0}^{t_0+T} dt \left[ <\psi_i(t)|i\hbar\frac{\partial}{\partial t}\psi_i(t)> - e_i \right] \quad (10)$$
is equal to an integer multiple of the Planck constant $I = n\hbar$. In Eq. (10) $T$ is the period of oscillations, $|\psi_i(t)\rangle$ denotes time-dependent single-nucleon Dirac spinors, $\epsilon_i$ are the corresponding single-nucleon energies in the unperturbed ground-state, and the summation runs over the occupied states. Because of the coupling to the continuum in the mean-field description, giant resonances are not stationary states of the Hamiltonian. Consequently, a non-periodic dependence on time is obtained for dynamical quantities. If the damping is very strong, the giant resonance is not periodic even on the average, and the quantization condition cannot be applied. However, if the motion is nearly periodic, i.e. the damping is relatively weak, the quantization procedure can still be used to calculate the energies, and the effect of damping can be taken into account approximately.

![FIG. 2. Fourier spectra of the time-dependent baryon density for various values of the coordinate $z$, on the axis along which the initial densities are deformed.](image)

In Fig. 3 we display the mean-field action as function of the excitation energy of the nucleons and of the initial deformation $\beta$. For the values of $\beta$ indicated by dots we have integrated the coupled system of Dirac and Klein-Gordon equations and calculated the action integral. The mean-field action is a quadratic function of the initial deformation $\beta$, and an almost perfect linear function of the excitation energy. Only above 60 MeV a slight deviation from a pure linear dependence is observed. The values of $<E^*>$ for which the action is an integer multiple of the Planck constant are: $I = 1\hbar$ for 18.5 MeV, and $I = 2\hbar$ for 37.1 MeV.
A double giant dipole resonance has been observed in relativistic Coulomb excitation of $^{208}$Pb. The single GDR is found at $13.3 \pm 0.1$ MeV with a width of $4.1 \pm 0.1$ MeV. The sum energy of coincident photon pairs displays a broad structure at $25.6 \pm 0.9$ MeV with a width of $5.8 \pm 1.1$ MeV. It is interpreted as the double GDR. In order to excite isovector dipole motion we define the initial conditions: at $t = 0$ (in the center of mass system) all protons start moving in the $+z$ direction with velocity $v_+$, and all neutrons start moving in the $-z$ direction with velocity $v_\nu = \frac{Z}{A}v_+$. For the NL-SH parameter set, the Fourier spectrum of the time-dependent dipole moment displays a strong peak at 12.9 MeV excitation energy, in good agreement with experimental data. We have found that the mean-field action is an integer multiple of the Planck constant: $I = 1 \ h$ for $<E^* >= 12.9$ MeV, and $I = 2 \ h$ for $<E^* >= 25.9$ MeV. Therefore, the energy of the one-phonon state, calculated from the mean-field action, coincides with the resonant energy of the mean peak in the Fourier spectrum, and the two-phonon state at 25.9 MeV is in excellent agreement with the experimental value for the excitation energy of the double GDR.

2.2 Monopole Giant Resonances and nuclear compressibility

The study of isoscalar monopole resonances in nuclei provides an important source of information on the nuclear matter compression modulus $K_{\text{nn}}$. This quantity is crucial in the description of properties of nuclei, supernovae explosions, neutron stars, and heavy-ion collisions. In principle the value of $K_{\text{nn}}$ can be extracted from experimental energies of isoscalar monopole vibrations in nuclei (giant monopole resonances GMR). However, the complete experimental data set on isoscalar GMR does not limit the range of $K_{\text{nn}}$ to better than $200 - 350$ MeV. Microscopic calculations of GMR excitation energies might provide
a more reliable approach to the determination of the nuclear matter compression modulus. Modern non-relativistic Hartree-Fock plus RPA calculations, using both Skyrme and Gogny effective interactions, indicate that the value of $K_{nm}$ should be in the range 210-220 MeV. In relativistic calculations on the other hand, both time-dependent and constrained RMF results indicate that empirical GMR energies are best reproduced by an effective force with $K_{nm} \approx 250 - 270$ MeV.

FIG. 4. Time-dependent isoscalar monopole moments $< r^2 > (t)$ and the corresponding Fourier power spectra for $^{208}$Pb. The parameter sets are NL1, NL3, NL-SH and NL2.

In Ref. [5] we have performed time-dependent and constrained RMF calculations for monopole giant resonances of a number of spherical closed shell nuclei, from $^{16}$O to the heavy nucleus $^{208}$Pb. For the effective Lagrangian we have used six parameter sets, which differ mostly by their prediction for $K_{nm}$, but otherwise reproduce reasonably well experimental data on nuclear properties. The idea is to restrict the possible values of the nuclear matter compression modulus, on the basis of the excitation energies of giant monopole states calculated with different effective interactions. In addition to the four non-linear sets NL1, NL3, NL-SH and NL2, we have also included two older linear parametrizations, HS and L1.
The sets NL1, NL-SH and NL2 have been extensively used in the description of properties of finite nuclei [1]. In order to bridge the gap between NL1 ($K_{nn} = 211.7$ MeV), and NL-SH ($K_{nn} = 355.0$ MeV), we have also included a new effective interaction NL3 [6] ($K_{nn} = 271.8$ MeV). This new parameter set provides an excellent description not only for the properties of stable nuclei, but also for those far from the valley of beta stability. From the energy spectra and transition densities calculated with these effective forces, it has been possible to study the connection between the incompressibility of nuclear matter and the breathing mode energy of spherical nuclei. For the isoscalar mode we have found an almost linear relation between the excitation energy of the monopole resonance and the nuclear matter compression modulus. For the determination of $K_{nn}$ especially relevant are microscopic calculations of GMR excitation energies in heavy nuclei. The results of TD RMF calculations for $^{208}$Pb are displayed in Fig. 4: time-dependent monopole moments $\langle r^2(t) \rangle = \frac{1}{A} \langle \Phi(t) | r^2 | \Phi(t) \rangle$ and the corresponding Fourier power spectra for the nonlinear effective interactions. As one would expect for a heavy nucleus, there is very little spectral fragmentation and a single mode dominates, at least for NL1 and NL3. The experimental excitation energy $13.7 \pm 0.3$ MeV is very close to the frequency of oscillations obtained with the NL3 parameter set: $14.1$ MeV. The calculated excitation energy for the NL1 parameter set ($K_{nn} = 211.7$ MeV), is approximately $1$ MeV lower than the average experimental value. For the linear effective forces HS and L1 the oscillations are more anharmonic, and the monopole strength is located well above the experimental GMR energy.

FIG. 5. Excitation energies of isoscalar giant monopole resonances in spherical nuclei as function of the mass number. The effective interactions are: NL1 (squares) and NL3 (circles). The solid curve corresponds to the empirical relation $\approx 80 \ A^{-1/3} \ MeV$.

The effective interactions NL1 and NL3 seem to produce GMR excitation energies which are quite close to the experimental values. For these two parameter sets we have calculated the isoscalar giant monopole resonances in a number of doubly closed-shell nuclei: $^{40}$Ca, $^{56}$Ni, $^{100,114,132}$Sn, $^{90,122}$Zr, $^{146}$Gd. The results are shown in Fig. 5. The energies of giant
monopole states are determined from the Fourier spectra of the time-dependent monopole moments, and are displayed as function of the mass number. The NL1 excitation energies are systematically lower, but otherwise the two effective interactions produce very similar dependence on the mass number. The empirical curve $E_x \approx 80 A^{-1/3} \text{MeV}$ is also included in the figure, and it follows very closely the excitation energies calculated with the NL3 parameter set. Similar results are obtained from constrained RMF calculations. Both methods indicate that, in the framework of relativistic mean field theory, the nuclear matter compression modulus $K_{nn} \approx 250 - 270 \text{MeV}$ is in reasonable agreement with the available data on spherical nuclei. This value is approximately 20% larger than the values deduced from recent non-relativistic density dependent Hartree-Fock calculations with Skyrme or Gogny forces.

2.3 Regular and chaotic dynamics of collective vibrations

The atomic nucleus has been used as a laboratory, both experimentally and theoretically, for investigating the transition from order to chaos in quantum dynamical systems. Most of these studies have concentrated on two major aspects: (i) generic signatures of chaos in local fluctuations and correlations of nuclear level distributions, and (ii) chaos in microscopic and collective dynamics of realistic many-body systems. Regular and chaotic dynamics in giant nuclear oscillations has been the subject of a number of studies. What has emerged as a very interesting result is that an undamped collective mode may coexists with chaotic single-particle motion. It appears that the slowly vibrating self-consistent mean field created by the same nucleons averages out the random components in their motion. In all investigations the motion of only one type of particles has been considered. That is, only the dynamics of isoscalar collective modes.

We have studied the difference in the dynamics of isoscalar and isovector collective modes. In particular, we consider isoscalar and isovector monopole oscillations in spherical nuclei, but analogous considerations apply to higher multipolarities. In Fig. 6 results are shown of time-dependent relativistic mean-field calculations for isoscalar and isovector oscillations in $^{208}\text{Pb}$. The experimental isoscalar GMR energy in $^{208}\text{Pb}$ is $13.7\pm0.3 \text{MeV}$, and the excitation energy of the isovector mode is $26 \pm 3 \text{MeV}$. Calculations have been performed for the NL1 effective interaction. In the isoscalar case both proton and neutron densities are radially expanded, while for the isovector mode the proton density is initially compressed by the same amount. Therefore, in both cases we follow the time evolution of the same system, just the initial conditions are different. In Fig. 6 we plot the time history of the isoscalar monopole moment $\langle r^2(t) \rangle$, and in Fig. 1b the corresponding isovector moment $\langle r^2_p(t) \rangle - \langle r^2_n(t) \rangle$ is displayed. The isoscalar mode displays regular undamped oscillations, while for the isovector mode we observe strongly damped anharmonic oscillations. On the right hand panels we plot the corresponding Fourier power spectra. The Fourier spectrum of the isovector mode is strongly fragmented. However, the main peaks are found in the energy region $25 - 30 \text{MeV}$, in agreement with the experimental data. For the isoscalar mode, the time history of the monopole moment and the Fourier spectrum show that the oscillations of the collective coordinate are regular. On the other hand, the appearance of a broad spectrum of frequencies
seems to indicate that the isovector oscillations are chaotic.

A diagnosis of chaotic vibrations would imply that one has a clear definition of such motion. For a quantum system, however, the concept of chaos, especially in time-dependent problems, is not well defined. And although our description of nuclear vibrations is semi-classical, quantum effects like the Pauli principle are present in the initial conditions and during the dynamical evolution. There exists a number of tests that can help to identify chaotic oscillations in physical systems, and some of them can be applied in the present consideration. In Figs. 7 - 9 we display some additional qualitative measures which can be used to characterize the response of our nonlinear system. In Fig. 7 we have constructed the two-dimensional time-delayed pseudo-phase space for the isoscalar (a) and isovector (b) oscillations, shown in Fig. 6. Since information is available on the time evolution of just one variable, the collective coordinate, one plots the signal versus itself, but delayed or advanced by a fixed time constant $\langle r^2(t) \rangle$, $\langle r^2(t+\tau) \rangle$. The phase space trajectories for the isoscalar mode are closed ellipses, indicating regular oscillations. For the isovector oscillations on the other hand, the trajectories are completely chaotic. The strong damping results from one-body processes: (i) escape of nucleons into the continuum states and (ii) collisions of the nucleons with the moving wall of the nuclear potential generated by the self-consistent mean fields. In Fig. 8 we display the corresponding Poincaré sections constructed from three-dimensional time-delayed pseudo-phase space. The Poincaré map for the isoscalar mode consists of two
sets of closely located points, and therefore confirms regular oscillations. For the isovector oscillations the Poincaré map appears as a cloud of unorganized points in the phase plane. Such a map indicates stochastic motion.

\[
\langle r^2 \rangle = 35 \text{ fm}^2
\]

\[
\langle r^2 \rangle = -9.5 \text{ fm}^2
\]

**FIG. 7.** Pseudo-phase space for isoscalar (a) and isovector (b) monopole oscillations in $^{208}\text{Pb}$.

**FIG. 8.** Poincaré sections for isoscalar (a) and isovector (b) monopole oscillations in $^{208}\text{Pb}$.

Another measure that is related to the Fourier transform is the autocorrelation function

\[
A(\tau) = \lim_{T \to \infty} \int_0^T \langle r^2(t) \rangle \langle r^2(t + \tau) \rangle \, dt
\]

When the signal is chaotic, information about its past origins is lost. This means that $A(\tau) \to 0$ as $\tau \to \infty$, or the signal is only correlated with its recent past. The autocorrelation functions for isoscalar and isovector oscillations are shown in Fig. 9. For the isovector mode $A(\tau)$ displays a rapid decrease, and the envelope appears as an irregular waveform.
The oscillations of the collective coordinate can be characterized as regular for the isoscalar mode, and they become chaotic when initial conditions correspond to the isovector mode. In Ref. [7] we have shown how a regular collective mode can coexist with chaotic single-particle dynamics, a result which confirms the conclusions of a number of studies. However, we have also shown that this is the case only for isoscalar modes, that is, only if one considers the motion of a single type of particles. When protons and neutrons move out of phase, as it happens for isovector modes, the resulting dynamics of the collective coordinate exhibit chaotic behavior. This is explained by the fact that protons and neutrons effectively move in two self-consistent potentials that oscillate out of phase. For example, when neutrons move inward, they scatter on the potential wall with positive curvature that is created by protons moving outward. This will lead to pseudo-random motion of the nucleons and dissipation of collective oscillations.

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$^{208}\text{Pb Isoscalar monopole}$

$^{208}\text{Pb Isovector monopole}$
isovector autocorrelation function

isoscalar autocorrelation function