Abstract

Background/Objectives: The necessity of ownship strategies is essential during the attack of enemy torpedo. Methods/Statistical Analysis: Torpedo tacking using advanced particle filter is proposed in bearings-only tracking environment for underwater applications. The observer platform has to perform evasive maneuver as well as perform target motion analysis to track the incoming hostile torpedo. Particle Filter (PF) combined with Modified Gain Bearings-Only Extended Kalman Filter (MGBEKF) is proposed technique. Findings: Monte-Carlo simulation is carried out for performance evaluation of the algorithm and the obtained results are presented which agree that PF-MGBEKF is most suitable as a part of ownship strategy.

Keywords: Bearing Measurements, Estimation, Simulation, Sonar, Torpedo, Tracking

1. Introduction

In bearings-only target motion analysis, 2D approach is an ideal choice. The observer platform monitors the sequence of bearing measurements form the radiating target and the kinematics of the target is obtained from these measurements. There are no constraints on observer motion with both target as well as observer on same plane. The foreign and Indian researchers have widely explored the area of BOT and various estimation algorithms are investigated1-4.

The problem of divergence in case of EKF is overcome by modifying the ownship gains which is called modified gain bearings-only extended Kalman filter (MGBEKF)3,4. MGBEKF is based on the algorithm for the EKF, where the gain is a function of previous measurements. PF are widely used for non-Gaussian and nonlinear applications5-10. PF use a set of weighted state samples, called particles, to approximate the posterior probability distribution in a Bayesian setup. PF with MGBEKF is method investigated in this paper.

The task is to estimate the torpedo motion parameters, while ownship is in attack by a torpedo. After getting the first contact of the torpedo, ownship tries to escape by doing a certain maneuver. The maneuver carried out is 70° RB method, which is being used by Navy11-14. The ownship’s subsequent escape maneuvers can be carried out properly if target kinematics are well known which are obtained by PF-MGBEKF. Range must decrease to get more bearing rate with increase in time. With this constraint, ownship tries to estimate the torpedo motion parameters to calculate proper evasive maneuvers using Closest Path of Approach (CPA) at various time instants and escape from torpedo attack12-17.

2. Mathematical Modeling

Each particle of the PF is updated every instant and re-sampled. This is equivalent to N Kalman filters and then adding a resampling step after each measurement. $X_{(k+1/k)}$ is updated to $X_{(k+1/k+1)}$ as shown below18.

$$P(k+1/k) = \varphi(k+1/k)P(k/k)\varphi^T(k+1/k) + \Gamma Q(k+1)\Gamma^T$$

(1)
G(k+1) = P(k+1/k)H^T(k+1) 
\quad [\sigma^2 + H(k+1)P(k+1/k)H^T(k+1)]^{-1} \tag{2}

X_i(k+1/K+1) = X_i(k+1/k) + G(k+1)B_m(k+1) -h(k+1,X_i(k+1/k)) \tag{3}

P(k+1/k+1) = [I -G(k+1)g(B_m(k+1),X_i(k+1/k))] 
\quad *P(k+1/k)[I -G(k+1)g(B_m(k+1),X_i(k+1/k))]
\quad + \sigma^2G(k+1)G^T(k+1) \tag{4}

g = [0 0 \cos B_m / (\hat{R}_x \sin B_m + \hat{R}_y \cos B_m) 
- \sin B_m / (\hat{R}_x \sin B_m + \hat{R}_y \cos B_m)] \tag{5}

2.1 Resampling

In every update of PFMGBEKF, the necessity of resampling of target state and its covariance matrix is checked. Resampling is required when \( N_{\text{eff}} < N/3 \).

\[
N_{\text{eff}} = \frac{1}{\sum_{i=1}^{N} q_i^2}
\tag{6}
\]

Whenever resampling is required, the following procedure based on weights of particles is adopted. In this method, weights are sorted in descending order with pre-sorted indices are stored in memory followed by \( T \) replication of particles\(^{16-20}\).

2.2 Closest path of Approach

At certain point of time the target and the observer move through a point at which minimum distance will be there between them which is called Closest Path of Approach (CPA). Once torpedo motion parameters are estimated using PFMGBEKF, CPAs are calculated for all possible ownship evasive courses (say 0 to 360 in step of 1 deg)\(^{20-26}\). Ownship will do evasive maneuver in the course at which maximum CPA is generated. CPA is calculated as follows.

Let the ownship & target courses be \( \phi \) and \( \psi \) respectively.

\[
x_i = R \sin B + (V_t \sin \psi - V_0 \sin \phi) t 
\tag{7}
\]
\[
y_i = R \cos B + (V_t \cos \psi - V_0 \cos \phi) t 
\tag{8}
\]

where \( V_t \) and \( V_0 \) are the speeds of target and ownship respectively.

To simplify the eqn. (20)

Let

\[
p = R \sin B \\
q = R \cos B \\
m = (V_t \sin \psi - V_0 \sin \phi) \\
n = (V_t \cos \psi - V_0 \cos \phi)
\]

Then eqn.(19) & eqn. (20) become

\[
x_i = (p + mt) 
\tag{9}
\]

**Figure 1.** Ownship and target encounter.
The distance \( R_t \) between ownship and target is given by

\[
R_t = \sqrt{(p + m)^2 + (q + n)^2}
\]

After necessary modifications and

\[
CPA = \begin{bmatrix} \frac{R^2 - (pm + qn)^2}{m^2 + n^2} \end{bmatrix}
\]

The initial estimate of target state vector and initial covariance matrix is given by

\[
X(0|0) = [15 \ 15 \ 10000 \sin B_m \ 10000 \cos B_m]^T
\]

\[
P(0/0) = Diag \begin{bmatrix} 4* \hat{x}\hat{x}(0/0) \ 4* \hat{y}\hat{y}(0/0) \ 4* \hat{r}\hat{r}(0/0) \ 12 \end{bmatrix}
\]

As PF is combined with MGBEKF, 1000 particles are used to estimate target motion parameters.

For the purpose of presentation, three scenarios as shown in Table 1 are considered for evaluation of the algorithm. The results obtained for the scenarios 1 to 3 are shown in Figure 2 to 4 respectively.

### Table 1

| S No | Initial Range (meters) | Initial Bearing (deg) | Target Speed (m/sec) | Target Course (deg) | Ownship Speed (m/sec) | Ownship Course (deg) | Convergence time (sec) |
|------|------------------------|-----------------------|----------------------|---------------------|----------------------|----------------------|------------------------|
| 1    | 4500                   | 90                    | 15.45                | 293 (0°)            | 6.18                 | 0°                   | 145                    |
| 2    | 6000                   | 270                   | 15.45                | 66.42 (0°)          | 6.18                 | 0°                   | 128                    |
| 3    | 5000                   | 320                   | 15.45                | 125 (0°)            | 6.18                 | 0°                   | 124                    |

**Figure 2(a).** Error in Range.

**Figure 2(b).** Error in Course.

**Figure 2(c).** Error in speed.

**Figure 3(a).** Error in Range.
3. Conclusion

PF-MGEBKF is proposed to estimate target motion parameters. The performance of the PF-MGEBKF is greatly superior to the standard extended Kalman filter which is evident from the results which consider three tactical geometries along with errors in target motion parameters.

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