Analysis of $D^+_s \rightarrow \phi \pi^+$ beyond naive factorization

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Abstract

We analyze the decay $D_s \rightarrow \phi \pi$ with QCD factorization in the heavy quark limit. The nonfactorizable contributions, including hard spectator contribution are discussed and numerical results are presented. Our predictions on the branching ratio of the decay are in agreement with the experiment. We also use a pure phenomenological method to estimate the branching ratio for $D_s \rightarrow \phi \pi$ with the existed $D^0 \rightarrow K^* \pi$ data.

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1 Introduction

Both CLEO [1] and BES [2] have reported their direct model-independent measurements for the $D_s \to \phi \pi$ branching fraction:

$$\text{Br}(D_s \to \phi \pi) = \begin{cases} (3.59 \pm 0.77 \pm 0.48) \times 10^{-2} & \text{CLEO}, \\ (3.9^{+5.1+1.8}_{-1.9-1.1}) \times 10^{-2} & \text{BES}. \end{cases}$$

The average branching ratio of $D_s \to \phi \pi$ is $(3.6 \pm 0.9) \times 10^{-2}$ [3].

The precise estimation of the branching ratio for the decay $D_s \to \phi \pi$ is very important. First, it is difficult to measure the absolute branching ratio of $D_s \to \phi \pi$ because we do not know the fraction of $D_s^+D_s^- \bar{D}_s^+ \bar{D}_s^-$ pairs production in $e^+e^-$ annihilation in comparison with $D\bar{D}$ pairs ( BES used $e^+e^- \to D_s^+D_s^-$ to obtain the first direct model-independent measurement of the $D_s \to \phi \pi$ branching fraction, however, with only two "double-tagged" events ). But we need to know the branching ratio for the study of B decays such as $B \to D_s X$ etc. Moreover, most of the measurements of the $D_s$ meson branching fractions are normalized to the clean $D_s \to \phi \pi$ channel. Second, theoretically, the decay of $D_s \to \phi \pi$ is dominated by spectator diagram with external emission of pion. This is easier to handle compared with other exclusive non-leptonic decay channels.

Previous calculations for the branching ratio $\text{Br}(D_s \to \phi \pi)$ are based on the naive factorization approach which is proposed by Bauer, Stech and Wirbel (BSW) [4]. But in BSW approach, non-factorizable effects can not be calculated, they have to be parameterized by an effective color number $N_{\text{eff}}^c$ which is treated as a free parameter. Moreover, results obtained with BSW approach still depend on renormalization scale and scheme. The authors in [3] examine the $D_s \to \phi \pi$ amplitude through a constituent quark-meson model. With this model, the calculated decay width $\Gamma(D_s \to \phi \pi)$ is larger than the experimental data. Paver and Riazuddin [6] studied $D_s \to \phi \pi$ in a valence quark triangle model, incorporating chiral symmetries, the result is compatible with the experimental data. In [7, 8], the authors considered the contribution from the color octet: $\langle \phi \pi | H^8_w | D_s^+ \rangle$, where, $H^8_w \equiv \frac{1}{2} \sum_a (\bar{u} \lambda^a c)(\bar{s} \lambda^a d)$. But they all introduced some new parameters, so they brought new theoretical uncertainties.

In the past years, Beneke, Buchalla, Neubert and Sachrajda developed QCD factorization (QCDF) approach [9] to calculate the hadronic matrix elements of B decays in the heavy quark limit. It has been used for many B decays modes [9, 10] with interesting results. In the present paper, we will follow this method to calculate the branching ratio for $D_s \to \phi \pi$. In the heavy quark limit $m_c \gg \Lambda_{QCD}$, non-factorizable contributions are considered from the first principle. In $D_s \to \phi \pi$ decay, the hadronic matrix elements can be represented as:

$$\langle \pi \phi | Q_1(\mu) | D_s \rangle = \langle \pi | J_1 | 0 \rangle \langle \phi | J_2 | D_s \rangle \cdot [1 + \sum r_n a^n + O(\Lambda_{QCD}/m_c)].$$

The naive factorization corresponds to neglecting the $O(\alpha_s)$ corrections and the power corrections in $\Lambda_{QCD}/m_c$. Although $m_c$ is not as large as $m_b$, we still hope that the QCD factorization approach in the heavy quark limit can also give a reasonable description of
$D_s$ meson hadronic decays. With this method, we analyze $D_s \rightarrow \phi \pi$ decay and compare the results with those obtained with naive factorization. Finally we use existed data of $D^0 \rightarrow K^* - \pi^+$ to estimate the branching ratio $Br(D_s \rightarrow \phi \pi)$ in a model independent way.

2 $D_s \rightarrow \phi \pi$ in QCD Factorization

The low energy effective Hamiltonian for $D_s \rightarrow \phi \pi$ can be expressed as follows:

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud} [C_1(\mu)Q_1(\mu) + C_2(\mu)Q_2(\mu)].$$

(2)

The four-quark local operators $Q_{1,2}$ are

$$Q_1 = (\bar{s}_\alpha c_\beta)_{V-A}(\bar{u}_\beta d_\alpha)_{V-A}$$
$$Q_2 = (\bar{s}_\alpha c_\alpha)_{V-A}(\bar{u}_\beta d_\beta)_{V-A},$$

(3)

where $\alpha, \beta$ are the color indices of $SU(3)_C$. Wilson coefficients $C_i(\mu)$ are universal, process-independent and calculable with the renormalization group improved perturbative theory, their $\mu$-dependence are expected to be cancelled by the hadronic matrix elements. The leading order (LO) and next-to-leading order (NLO) corrections to $C_i(\mu)$ have been presented in [11]. In the naive dimensional regularization (NDR) scheme, we give the numerical values for $C_i(\mu)$ at three renormalization scales in Tab.1. In Fig.1, we also display the dependence of $C_i (i = 1, 2)$ on $\mu$ in the LO and NLO approximation. We will take the values of $C_i (i = 1, 2)$ at NLO for our forthcoming calculations.

The decay constant and form factors are defined by [4] :

$$\langle \pi^+ | (\bar{u}d)^\mu_{(V-A)} | 0 \rangle = -if_{\pi} p_{\pi}^\mu,$$

$$\langle \phi | (\bar{s}c)_{\mu(V-A)} | D_s^+ \rangle = -i \left\{ (m_{D_s} + m_\phi)\varepsilon_\mu^* A_1^{D_s \phi}(q^2) - \frac{\varepsilon_\mu^* q}{m_{D_s} + m_\phi} (p_{D_s} + p_\phi)_{\mu} A_2^{D_s \phi}(q^2) \right\}$$

(4)
Table 1: Wilson coefficients in NDR scheme. The input parameters in numerical calculations are fixed: $\alpha_s(m_Z) = 0.1185$, $\alpha_{em}(m_W) = 1/128$, $m_W = 80.42$ GeV, $m_Z = 91.188$ GeV, $m_t = 168.2$ GeV, $m_c = 1.45$ GeV.

| $\mu$    | $C_1(\mu)$ | $C_2(\mu)$ |
|----------|-------------|-------------|
|          | LO | NLO | LO | NLO |
| $\mu = 1\text{GeV}$ | -0.650 | -0.500 | 1.356 | 1.272 |
| $\mu = m_c$ | -0.520 | -0.390 | 1.268 | 1.200 |
| $\mu = 2\text{GeV}$ | -0.435 | -0.314 | 1.214 | 1.153 |

$$- 2m_\phi \frac{\varepsilon^* \cdot q}{q^2} q_\mu A_3^{D\phi}(q^2) + \frac{\varepsilon^* \cdot q}{q^2} (2m_\phi)q_\mu A_0^{D\phi}(q^2) \right)$$

$$+ \frac{2}{m_{D_s} + m_\phi} \varepsilon_{\mu\rho\sigma\tau} \varepsilon^{\nu\rho} p_{D_s} p_\phi \rho_\tau V^{D_s\phi}(q^2),$$

$$A_0^{D_s\phi}(0) = A_3^{D_s\phi}(0),$$

$$2m_\phi A_3^{D_s\phi}(0) = (m_{D_s} + m_\phi) A_1^{D_s\phi}(0) - (m_{D_s} - m_\phi) A_2^{D_s\phi}(0).$$

The relations (6) - (7) ensure that there is no kinematical singularity in the matrix element at $q^2 = 0$.

Under naive factorization, using Eq. (4) - (7), the decay amplitude of $D_s \rightarrow \phi \pi$ reads

$$A(D_s \rightarrow \phi \pi) = \sqrt{2} G_F V_{cs}^* V_{ud} f_\pi m_\phi A_0^{D_s\phi}(m_\pi^2)(\varepsilon^* \cdot p_{D_s}) \cdot a_2,$$

where $a_2 = C_2 + \frac{1}{N_c^{eff}} C_1$, $N_c^{eff}$ is the number of colors. The form factor $A_0^{D_s\phi}$ is defined by (3). From Eq. (8) we can see that the amplitude depends on the renormalization scale $\mu$, because the Wilson coefficients $C_1(\mu)$, $C_2(\mu)$, and hence $a_1$, $a_2$ depend on $\mu$, whereas the decay constant and form factor are independent of $\mu$. So the amplitude $A(D_s \rightarrow \phi \pi)$ is $\mu$-dependent. On the other hand, it does not consider the nonfactorizable effects. If we calculate it with QCD factorization, take all the high order corrections into account, $a_i$ and the amplitude $A(D_s \rightarrow \phi \pi)$ will be $\mu$ independent. In our paper, we calculate it only to the order of $\alpha_s$, so $a_i$ and the amplitude $A(D_s \rightarrow \phi \pi)$ still depend on $\mu$, but the dependence is less sensitive to $\mu$. With these preliminaries, we now analyze $D_s \rightarrow \phi \pi$ with QCD factorization.

In $D_s \rightarrow \phi \pi$ decay, the emitted meson $\pi$ is light, the hadronic matrix elements can be written as:

$$\langle \pi | Q_i(\mu) | D_s \rangle = A_0^{D_s\phi}(0) \int_0^1 dx T_i^I(x) \Phi_\pi(x)$$

$$+ \int_0^1 dx dy T_i^{II}(\xi, x, y) \Phi_{D_s}(\xi) \Phi_\pi(x) \Phi_\phi(y).$$
$A_0^{D_s \phi}(0)$ denotes the $D_s \rightarrow \phi$ transition form factor, $\Phi_D(\xi)$, $\Phi_\pi(x)$ and $\Phi_\phi(x)$ label light-cone distribution amplitudes (LCDAs) of $D_s$, $\pi$ and $\phi$ meson respectively. $T_i^{I,II}$ denote hard-scattering kernels which are calculable in perturbative theory. Neglecting the $O(\Lambda_{QCD}/m_c)$ corrections, $T_i^{I,II}$ are hard gluon exchange dominant. Other non-perturbative contributions are contained in the LCDAs of mesons or the form factor. The second term in Eq. (9) represents the hard spectator contribution.

We next proceed to calculate the nonfactorizable effects in the $D_s^+ \rightarrow \phi\pi^+$ with QCDF approach. Then in heavy quark limit, for simplicity, we will neglect the masses of light quarks and $\pi$. We consider the vertex corrections and hard spectator interactions depicted in Fig.2. The technique is similar to that of the $B \rightarrow \pi\pi/K$ mode, readers can be referred to [9] for details. As in [9], we obtain the QCD coefficients $a_i$ ($i = 1, 2$) at NLO in NDR scheme. Then the coefficients $a_i$ are given as

$$
a_1 = C_1 + \frac{C_2}{N} + \frac{\alpha_s C_F}{4\pi N} C_2 F, \\
a_2 = C_2 + \frac{C_1}{N} + \frac{\alpha_s C_F}{4\pi N} C_1 F. \tag{10}
$$

Here $N = 3$ ($f = 4$) is the number of colors (flavors), and $C_F = \frac{N^2 - 1}{2N}$ is the factor of color. We define the symbols in the above expressions as the same as Beneke’s, which are

$$
F = -18 - 12 \ln \frac{\mu}{m_c} + f_I + f_{II}, \tag{11}
$$

$$
f_I = \int_0^1 dx g(x) \Phi_\pi(x), \tag{12}
$$

with the hard-scattering function

$$
g(x) = 3 \frac{1 - 2x}{1 - x} \ln x - 3i\pi.
$$

The hard spectator scattering contribution is given by

$$
f_{II} = \frac{4\pi^2}{N} \frac{f_\phi f_{D_s}}{A_0^{D_s \phi}(0)m_{D_s}^2} \int_0^1 d\xi \frac{\Phi_{D_s}(\xi)}{\xi} \int_0^1 dx \frac{\Phi_\pi(x)}{x} \int_0^1 dy \frac{\Phi_\phi(y)}{y}, \tag{13}
$$

where $f_\phi (f_{D_s})$ is the $\phi$ ($D_s$) meson decay constant, $m_{D_s}$ the mass of $D_s$ meson, $A_0^{D_s \phi}(0)$ the $D_s \rightarrow \phi$ transition form factor at zero momentum transfer, and $\xi$ the light-cone momentum fraction of the spectator quark in the $D_s$ meson, $f_{II}$ depends on the wave function $\Phi_{D_s}$ through the integral

$$
\int_0^1 d\xi \Phi_{D_s}(\xi)/\xi = m_{D_s}/\lambda_D. \tag{14}
$$

This introduces a new hadronic parameter $\lambda_D$, $\lambda_D$ is of order $\Lambda_{QCD}$, we take $\lambda_D = 335$ MeV here.
Figure 2: Order of $\alpha_s$ corrections to the hard scattering kernels $T_i^I$ and $T_i^{II}$. The two lines
directed upwards represent the two quarks that make up $\pi$. These diagrams are called vertex
corrections for Fig.(a)-(d) and hard spectator diagrams for Fig.(e)-(f) respectively.

From the expression (10) of the QCD coefficients $a_i(i=1,2)$, with the renormalization
group equation for Wilson coefficients $C_i(\mu)$ at leading order logarithm approximation
\cite{11}:
\[
\frac{dC_i(\mu)}{d\ln\mu} = \frac{\alpha_s}{4\pi} \gamma_{ij} T_i C_i(\mu),
\]
where $\gamma$ is the anomalous dimension matrix, we find $\frac{da_i}{d\ln\mu} = 0 (i=1,2)$ at the order of
$\alpha_s$, this makes the $\mu$-dependence of the decay amplitude calculated with QCDF approach
less sensitive than that calculated with naive factorization. This point can also be seen
roughly from the data in Tab.2 and Fig.3 - 4. But there are still uncertainties in the
 calculation, such as the form of wave functions and unknown form factor $A_0^{D_s\phi}$.

Notice that in the decay $D_s \to \phi\pi$, using the isospin analyses \cite{12, 13}, we find that the
final state involves only a single isospin, so there is no interference effects from the final
state interactions (FSI) when we calculate the branching ratio of $D_s \to \phi\pi$.

In the $D_s$ rest frame, the two body decay width is
\[
\Gamma(D_s \to \phi\pi) = \frac{1}{8\pi} |A(D_s \to \phi\pi)|^2 \frac{|p|}{m_{D_s}^2},
\]
where
\[
|p| = \sqrt{[m_{D_s}^2 - (m_\phi + m_\pi)^2][m_{D_s}^2 - (m_\phi - m_\pi)^2]} \frac{1}{2m_{D_s}}
\]
is the magnitude of the momentum of $\phi$ meson. With the approximation $m_{\pi}^2/m_{D_s}^2 \approx 0$,
the decay width is given by
\[
\Gamma(D_s^+ \to \phi\pi^+) = \frac{G_F m_{D_s}^5}{32\pi} |V_{cs}|^2 |V_{ud}|^2 |a_2|^2 \left( \frac{f_\pi}{m_{D_s}} \right)^2 \left( 1 - \left( \frac{m_\phi}{m_{D_s}} \right)^2 \right)^3 (A_0^{D_s\phi}(0))^2.
\]

The corresponding branching ratio is given by
\[
Br(D_s \to \phi\pi) = \frac{\Gamma(D_s \to \phi\pi)}{\Gamma_{total}}, \quad \Gamma_{total} = \frac{1}{\tau_{D_s}}.
\]
In our numerical calculations, we will take the following values for the relevant input parameters \cite{3}:

\[ V_{cs} = V_{ud} = 0.975, \ f_\pi = 131 \text{ MeV}, \ f_\phi = 233 \text{ MeV}, \ m_c = 1.45 \text{ GeV}, \ f_{D_s} = 280 \pm 19 \pm 28 \pm 34 \text{ MeV}. \]

As for the form factor \( A_{0}^{D_s}(0) \), for lack of experimental data, we use the value taken from the Ref. \cite{4} \( A_{0}^{D_s}(0) = 0.70 \). For mass of the mesons, we use \( m_{D_s} = 1968.6 \pm 0.6 \text{ MeV}, \ m_\phi = 1019.417 \pm 0.014 \text{ MeV}. \) If not stated otherwise, we shall use the central values as the default values in our later calculations.

For distribution amplitude of \( \pi \), two kinds of the wave functions are used, one is the asymptonic form \( \Phi_\pi(x) = 6x(1-x) \), the other is delta-function \( \Phi_\pi(x) = \delta(x - \frac{1}{2}) \).

In Tab.2 we list the values of \( a_1, a_2 \) and branching ratio(\( Br \)) at \( \mu = 1 \text{ GeV}, \ m_c, \) and \( 2 \text{ GeV} \) with different wave functions of \( \pi \). The numerical results which are calculated with BSW approach ( Where we take \( N_{e}^{eff} = \infty \) because the experimental data of MARK III for charm decays do not show color suppression \cite{14} ) are also listed for comparison.

### Table 2: The values of \( a_i \) and \( Br \) at \( \mu = 1 \text{ GeV}, \ m_c \) and \( 2 \text{ GeV} \) calculated with QCDF and BSW approach (\( N_{e}^{eff} = \infty \)). For QCDF, we calculate the spectator contribution with three forms: \( f_{II} = 0 \) and two different wave functions of \( \pi \). In the QCDF columns, the values in the parentheses are those with \( \Phi_\pi(x) = 6x(1-x) \), the values in the brackets are those with \( \Phi_\pi(x) = \delta(x - \frac{1}{2}) \).

| \( \mu \) | \( a_1 \) | \( a_2 \) | \( Br \) % |
| --- | --- | --- | --- |
| BSW | QCDF | BSW | QCDF | BSW | QCDF |
| \( \mu = 1 \text{ GeV} \) | -0.396 - 0.215 i | 1.272 | 1.231 + 0.084 i | 3.75 | 3.53 |
| \( \mu = m_c \) | -0.284 - 0.150 i | 1.200 | 1.165 + 0.049 i | 3.33 | 3.15 |
| \( \mu = 2 \text{ GeV} \) | -0.213 - 0.119 i | 1.153 | 1.126 + 0.033 i | 3.08 | 2.94 |

It is necessary to note that the QCDF approach gives \( a_i(i = 1, 2) \) an imaginary part, which comes from the gluon exchange of the quarks \( u \) and \( d \) in \( \pi \) with the \( s \) quark in \( \phi \) (see Fig.2 (c)-(d)). Moreover, the imaginary parts of \( a_i(i = 1, 2) \) have no relations with \( f_{II} \). From the numerical values summarized in Tab.2, we find that the vertex correction in Fig.2 (a)-(d) is about 5\% - 7\%, the hard-spectator diagrams can reduce over 10% of the values obtained with BSW approach. And the coefficients \( a_i(i = 1, 2) \) are less sensitive to the choice of the wave functions. In Fig.3, we depict the dependence of \( a_1, a_2 \) and \( Br \) on scale \( \mu \) when considering the vertex corrections (but neglecting the hard spectator.
contribution), we also show the results calculated by BSW approach for comparison. The horizontal solid lines in Fig.3(b) show the experimental branching ratio at 1σ level. It is clear that the scale dependence of the values calculated with QCDF approach are milder than that with BSW approach. But the μ dependence still exists, the reason is that we calculate a_i only at one-loop level, the source of μ dependence is from the high order effects. When considering the contributions from the high order corrections in α_s or Λ_QCD/m_c, the μ dependence of our predictions will be further reduced.

In Fig.4, we compare the results which are calculated with different wave functions of π when considering the hard-spectator contribution in QCDF. It shows again that a_1, a_2 and Br are less sensitive to the selection of the wave function of π, moreover, their μ-dependence is further reduced. From Fig.3 and Fig.4, we find that the results obtained with QCD factorization approach fall in the 1σ allowed region from the central experimental value 3.6×10^{-2}, regardless of the selection of the function of π. Though the branching ratios with BSW approach are also within the 1σ region, this approach takes N_{c eff} = ∞ in order to fit the experimental data, so it is more phenomenological in comparison with QCDF approach. From Fig.3 and Fig.4, we can see apparently that our predictions with QCDF approach are small compared with the values obtained with BSW approach.

3 Direct estimation of Br(D_s→φπ)

Now we estimate the Br(D_s→φπ) directly from the existed data of D^0→K^*−π^+. Assuming spectator diagram dominance, D_s^+→φπ^+ can go through quark decay diagram depicted in Fig.5(a), the decay width is (16). Using the experimental data listed in Sec.2, we get |p| = 0.720.

Consider the decay D^0→K^*−π which proceeds dominantly through diagram Fig.5(b), obviously, in Fig.5, diagram (a) and (b) are very similar, if 5 in (a) is replaced by π, we

![Figure 3: Dependence of a_1, a_2 and Br on the renormalization scale μ in BSW and QCDF(f_{II} = 0). The dotted and dashed lines correspond to the values obtained with BSW and QCDF approach.](image)
Figure 4: Dependence of $a_1$, $a_2$ and $Br$ on the renormalization scale $\mu$ in QCDF with different function of $\pi$. The dotted and dashed lines correspond to the values obtained with $\Phi_\pi(x) = 6x(1 - x)$ and $\Phi_\pi(x) = \delta(x - \frac{1}{2})$.

will get (b). In addition, the particle decay width of $D^0 \rightarrow K^*\pi$ is

$$\Gamma(D^0 \rightarrow K^*\pi) = \frac{1}{8\pi} |A(D^0 \rightarrow K^*\pi)|^2 \frac{|p'|}{m_{D^0}^2},$$

(19)

where $|p'| = 0.719$. The momentum of $K^*$ in the $D^0$ rest frame is almost the same as that of $\phi$ in $D^+_s \rightarrow \phi\pi^+$. So the Lorentz contraction effects of the wave function of $\phi$ and $K^*$ are nearly the same. We know that the decay amplitudes $A(D_s \rightarrow \phi\pi)$ and $A(D^0 \rightarrow K^*\pi)$ are proportional to the wave function overlap integrates of $D^+_s - \phi$ and $D^0 - K^*$ respectively. Moreover, $|p| = 0.720$ and $|p'| = 0.719$ mean that these overlap integrates are almost the same under the condition of SU(3) symmetry. The SU(3) symmetry breaking effects in the case of $D^+_s \rightarrow \phi\pi^+$ and $D^0 \rightarrow K^{*-}\pi^+$ should be fairly small. So in approximation, we can have

$$|A(D_s \rightarrow \phi\pi)| \approx |A(D^0 \rightarrow K^{*-}\pi^+)|.$$  \hspace{1cm} (20)

Using the experimental data [3]:

$$\tau(D^0) = (0.4126 \pm 0.0028) \times 10^{-12} s,$$
$$Br(D^0 \rightarrow K^{*-}\pi^+) = (5.0 \pm 0.4)\%,$$
$$\tau(D_s) = (0.496^{+0.010}_{-0.009}) \times 10^{-12} s,$$

with Eq.(18) - (20), we obtain $Br(D_s \rightarrow \phi\pi) \approx (5.40 \pm 0.45)\%$, where the error comes from that of the data of $\tau(D^0)$, $Br(D^0 \rightarrow K^{*-}\pi^+)$ and $\tau(D_s)$. It is a little outside the one $\sigma$ allowed region from the central experimental value $3.6 \times 10^{-2}$. 


4 Conclusions

We have analyzed the decay $D_s \to \phi \pi$ with QCD factorization in the heavy quark limit. We calculate the nonfactorizable contributions, including vertex correction, hard-spectator contribution. These nonfactorizable contributions can give over 10% corrections to naive factorization. Moreover, according to our calculations, the branching ratios with QCDF approach is not sensitive to the choice of the wave function of pion. Our predictions are in agreement with the present experimental data. The direct estimation of $Br(D_s \to \phi \pi)$ from $D^0 \to K^* \pi$ data gives a bit larger result comparing with the present data. But the measured data on $Br(D_s \to \phi \pi)$ are still rough, we need more data for drawing our final conclusion.

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