The Interference Term between the Spin and Orbital Contributions to $M_1$ Transitions

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(April 21, 2022)

Abstract

We study the cross-correlation between the spin and orbital parts of magnetic dipole transitions $M_1$ in both isoscalar and isovector channels. In particular, we closely examine certain cases where $\sum B(M_1)$ is very close to $\sum B(M_1)_{\sigma} + \sum B(M_1)_{l}$, implying a cancellation of the summed interference terms. We gain some insight into this problem by considering special cases approaching the $SU(3)$ limit, and by examining the behaviour of single-particle transitions at the beginning and towards the end of the $s-d$ shell.

I. INTRODUCTION

In a previous work [1], the magnetic dipole transitions from the ground states to excited states of several nuclei in the $s-d$ shell were calculated: $^{20}Ne$, $^{22}Ne$, $^{24}Mg$, $^{28}Si$, $^{32}S$, and $^{36}Ar$. The main focus of the work was on how the transition strengths were affected by the strengths of the spin-orbit and tensor interactions inside a nucleus, and near the end of the paper, it was briefly mentioned that a new topic would be of interest: "the cross-correlation between the spin and orbital parts of $B(M_1)$". In the present work, we wish to elaborate on this point.

As in the previous work, we define the $M_1$ transition strength as:

$$B(M_1)^\uparrow = \frac{1}{(2J_i + 1)} \sum_{M_{f},\mu,M_{i}} |\langle \Psi_{M_{f}}^{J_{f}} A_{\mu} \Psi_{M_{i}}^{J_{i}} \rangle|^2,$$

where
\[ \vec{A} = \sqrt{\frac{3}{4\pi}} \left[ \sum_{i=1}^{Z} (g_{l\pi} \vec{l}_i + g_{s\pi} \vec{s}_i) + \sum_{j=1}^{N} (g_{l\nu} \vec{l}_j + g_{s\nu} \vec{s}_j) \right] . \]  

(2)

We define three \( B(M1) \)'s: total, spin, and orbital with the following parameters (in units of \( \mu_N \))

Total \( B(M1) \): \( g_{l\pi} = 1, \ g_{s\pi} = 5.586, \ g_{l\nu} = 0, \ g_{s\nu} = -3.826 \)

Orbital \( B(M1)_l \): \( g_{l\pi} = 1, \ g_{s\pi} = 0, \ g_{l\nu} = 0, \ g_{s\nu} = 0 \)

Spin \( B(M1) \): \( g_{l\pi} = 0, \ g_{s\pi} = 5.586, \ g_{l\nu} = 0, \ g_{s\nu} = -3.826 \)

As the title of this work implies, we wish to study the interference terms between the spin and orbital parts. For a transition to an individual state, we can write

\[
B(M1) = (\sqrt{B(M1)}_l \pm \sqrt{B(M1)}_\sigma)^2 \\
= B(M1)_l + B(M1)_\sigma \pm 2\sqrt{B(M1)}_l \sqrt{B(M1)}_\sigma
\]

For some states, we would have a plus sign (constructive interference), and for some the minus sign (destructive interference). We can also consider the summed strength to all calculated states. It was already noted in our previous work \[1\] that for some cases “\( \sum B(M1) \) is very close to \( \sum B(M1)_\sigma + \sum B(M1)_l \)” . This would imply a cancellation of the summed interference terms. In this work, we will study this in a more quantitative manner.

II. CALCULATIONS

The interaction that was used was described in \[1\], so we will be brief. We used the \((x, y)\) interaction

\[ V(r) = V_c(r) + x \cdot V_{s,o} + y \cdot V_t \]  

(3)

where \( s.o. \) stands for the two-body spin-orbit interaction, \( t \) for the tensor interaction, and \( V_c(r) \) is everything else, especially the (spin-dependent) central interaction. We can vary the strength of the spin-orbit and tensor interactions by varying \( x \) and \( y \). The optimum fit to a free \( G \)-matrix is obtained with \( x = 1, \ y = 1 \) \[2\]. Arguments could be made that these parameters should be changed inside a nucleus.
As an example, we show results for $M1$ transitions in $^{28}\text{Si}$ with the $(x, y)$ interaction for $x = 1$, $y = 1$. In Table I we show results for isoscalar transitions from the ground state ($T = 0$, $J = 0^+$) to excited states ($T = 0$, $J = 1^+$) in units of $10^{-2}\mu_N^2$. In Table II we give results for isovector transitions in $^{28}\text{Si}$ in units of $\mu_N^2$. We show only the first ten states, but the sum $\sum B(M1)$ is over all states (around 500). We also show the sign of the interference term.

In Tables III and IV we show respectively the isoscalar and isovector summed $M1$ strengths for the $(x, y)$ interaction with $x = 1$, $y = 1$. We also show the deviation $\Delta$ which is equal to $\sum B(M1) - \sum B(M1)_x - \sum B(M1)_l$. We also introduce an angle $\theta$ in order to better describe the interference:

$$\sum B(M1) = \sum B(M1)_x + \sum B(M1)_l + 2\sqrt{\sum B(M1)_l \sqrt{\sum B(M1)_x \cos(\theta)}}$$

(4)

In order for the interference term to vanish, we must have $\theta = 90^\circ$.

By examining Tables III and IV, we see that there is a striking contrast between the isoscalar and isovector cases. In the former case, we have $\theta = 180^\circ$ for all cases. For the isovector case, $\theta$ is closer to $90^\circ$. When $\theta$ is exactly $90^\circ$, the sum of the interference terms over all states would vanish.

The isoscalar result is easy to understand. Consider the total angular momentum operator $\vec{J} = \vec{L} + \vec{S}$. Clearly the matrix element $\langle 1^+ \vec{J} 0^+ \rangle$ will vanish. Hence $\langle 1^+ \vec{L} 0^+ \rangle = -\langle 1^+ \vec{S} 0^+ \rangle$ for each $1^+$ state. Thus, if the isoscalar $M1$ operator is written as $a\vec{L} + b\vec{S}$, then a transition matrix element from the ground state to a state $1^+_i$ ($T = 0$) is given by:

$$\langle 1^+_i a\vec{L} + b\vec{S} 0^+ \rangle = (a - b)\langle 1^+_i \vec{L} 0^+ \rangle$$

(5)

The summed $B(M1)$ strength is $(a - b)^2 \sum_i |\langle 1^+_i \vec{L} 0^+ \rangle|^2$, and as long as $a$ and $b$ have the same sign, it is easy to see that $\theta$ is equal to $180^\circ$. Indeed, $a = (g_{t\pi} + g_{t\nu})/2 = 0.5$ and $b = (g_{s\pi} + g_{s\nu})/2 = 0.88$, and they do have the same sign.

For the isovector case, however, we do not have such a constraint, so that the signs of the interference terms are more random, and the angle $\theta$ is closer to $90^\circ$. We cannot help but notice that the deviation from $90^\circ$ increases as we increase the mass number in the $1s - 0d$ shell. For $^{20}\text{Ne}$, $\theta$ is equal to $90.00^\circ$, but it increases steadily to $90.45^\circ$, $92.31^\circ$, $93.33^\circ$ and $95.45^\circ$ for $^{24}\text{Mg}$, $^{28}\text{Si}$, $^{32}\text{S}$, $^{36}\text{Ar}$. This suggests that $\theta$ gets closer to $90^\circ$ as we get closer to Wigner’s U(4) limit [3], which includes the $SU(3)$ limit of Elliott [4]. The $SU(3)$ model holds much better in the lower half of the $s - d$ shell than in the upper half. In the extreme $SU(3)$ limit, the spin $M1$’s will vanish, and in that case $B(M1) = B(M1)_l$. The interference term will vanish trivially. We can see from Table II, however, that for $^{28}\text{Si}$ the summed isovector
spin and orbital strengths are almost the same, so that it is a non-trivial result that \( \theta \) is close to 90° in this case.

Another way of looking at this is to note that the results get better as the spin-orbit interaction strength is decreased.

In Table V we consider two cases where \( N \) does not equal \( Z \): \(^{10}\text{Be} \) and \(^{22}\text{Ne} \). We break up the contributions into two parts: \( J = 0^+ T = 1 \rightarrow J = 1^+ T = 1 \) and \( J = 0^+ T = 1 \rightarrow J = 1^+ T = 2 \). We then consider the total contribution. For both nuclei we see large deviations from 90°. For example, in \(^{22}\text{Ne} \) the value of \( \theta \) for \( T = 1 \rightarrow T = 1 \) is 85.965°, whilst for \( T = 1 \rightarrow T = 2 \) it is 102.488°. The combined result yields 88.748°. What this tells us is that when we consider all transitions, it appears that we are close to randomness of the sign of the interference term. However, if we consider each part separately, there is a large deviation from randomness. For \( T = 1 \rightarrow T = 1 \) the signs conspire so as to enhance the total \( B(M1) \), whilst for \( T = 1 \rightarrow T = 2 \) they act to diminish the total \( B(M1) \). In combination, the two effects oppose each other and yield a result close to randomness of the phase.

For a transition from \( j = l + 1/2 \) to \( j = l + 1/2 \) (e.g. \( d_{5/2} \rightarrow d_{5/2} \)), the isovector matrix element is proportional to \((l + \mu_p - \mu_n) = (l + 4.766)\). Thus the interference term will be positive corresponding to \( \theta = 0° \). This should be relevant to the lower part of the \( s-d \) shell. For the transition \( j = l - 1/2 \) to \( j = l - 1/2 \) (e.g. \( d_{3/2} \rightarrow d_{3/2} \)), the isovector matrix element is proportional to \((l + 1 - (\mu_p - \mu_n)) = (l + 1 - 4.706)\). In this case, the interference term will be negative, corresponding to \( \theta = 180° \). This will be most important near the end of the \( s-d \) shell e.g. for \(^{36}\text{Ar} \). The matrix element \( j = l \pm 1/2 \rightarrow j = l \mp 1/2 \) is proportional to \((g_l - g_s) = [l - 2(\mu_p - \mu_n)]\). Here again the interference term is negative, corresponding to \( \theta = 180° \).

In Table VI we vary the parameters of our \((x, y)\) interaction in order to see how the ‘interference angle \( \theta \)’ depends on the spin-orbit and tensor interactions. We use \(^{28}\text{Si} \) as an example. We see that the smaller the value of \( x \) (the strength of the spin-orbit interaction), the closer \( \theta \) is to 90°. For \( y = 0 \) (no tensor interaction present), the values of \( \theta \) for \( x = 0 \) and \( x = 1 \) are respectively 90.03° and 94.93°. With \( y = 1 \) (full strength interaction), the corresponding values are 89.93° and 92.33°. This behaviour is consistent with the well-known fact that the spin-orbit interaction destroys the \( SU(3) \) symmetry. Concerning the tensor interaction, the behaviour can be explained by the fact that in an open-shell nucleus this interaction behaves somewhat like a spin-orbit interaction but with sign opposite to that of the basic spin-orbit interaction \([3]\). Thus, for \( x = 1 \ y = 1 \), the value of \( \theta \) is smaller than for \( x = 1 \ y = 0 \). The values are 92.33° and 94.93° respectively. The effective spin-orbit
interaction for $x = 1 \ y = 1$ is weaker than for $x = 1 \ y = 0$, and so we are closer to the $SU(3)$ limit.

Whereas in free space the choice $x = 1 \ y = 1$ gives the best results in comparison with $G$ matrices obtained from realistic interactions, there is some evidence discussed in [1] that inside a nucleus the spin-orbit interaction should be stronger than in free space, and the tensor interaction weaker. We therefore also consider the case $x = 1.5 \ y = 0.5$. Because the spin-orbit interaction is stronger, we find that the interference angle deviates from $90^\circ$ i.e. $\theta = 98.31^\circ$ in this case.

III. CLOSING REMARKS

When we are close to the $U(4)$ limit of Wigner [3], we find that for isovector transitions in $N = Z$ nuclei the sum $\sum B(M1)$ is close to $\sum B(M1)_\sigma + \sum B(M1)_t$. More generally, we have defined an interference angle $\theta$ in Eq. (4). For $x = 0$ (i.e. no spin-orbit interaction present), $\theta$ is very close to $90^\circ$ and the sum of all the interference terms is almost zero (randomness). As we increase the spin-orbit splitting by increasing $x$, the angle $\theta$ becomes larger than $90^\circ$. Also, in nuclei where $SU(3)$ symmetry is not so good, $\theta$ becomes larger than $90^\circ$. For isoscalar transitions in $N = Z$ nuclei, we find that we have maximum destructive interference between the orbital and spin $M1$ amplitudes. This can be explained by the fact that the total angular momentum operator cannot induce $M1$ transitions. For $N \neq Z$ nuclei, like $^{22}$Ne, and if we consider all transitions, it looks like we are close to randomness ($\theta = 88.7^\circ$). However, if we look at each final isospin separately, the picture is changed e.g.: for $T = 1 \rightarrow T = 1$ transitions $\theta$ is $85.9^\circ$ (net constructive interference), whilst for $T = 1 \rightarrow T = 2 \ \theta$ is $102.7^\circ$ (considerable destructive interference).

This work was supported in part by a D.O.E. grant DE-FG02-95ER-40940.
REFERENCES

[1] M.S. Fayache, P. von-Neumann-Cosel, A. Richter, Y.Y. Sharon and L. Zamick, Nucl. Phys. A 627 14(1997).

[2] D.C. Zheng and L. Zamick, Ann. Phys. (NY) 206 106(1991).

[3] E.P. Wigner, Phys. Rev. 51 106 (1937); 51 947 (1937).

[4] J.P. Elliott, Proc. Royal Soc. A 245 128(1958); A245 562(1958).

[5] C.W. Wong, Nucl. Phys. A 108 481(1968).
TABLES

TABLE I. \(B(M1)\) Transitions in \(^{28}\text{Si}\): Isoscalar Case \(T = 0 \rightarrow T = 0\) in units of \(10^{-2} \mu_N^2\). The sign of the interference term is shown.

| Energy (MeV) | \(B(M1)\) | \(B(M1)_t\) | \(B(M1)_\sigma\) | Sign |
|-------------|------------|--------------|------------------|------|
| 9.394       | 0.4514     | 0.7814       | 2.4210           | -    |
| 9.817       | 0.1440     | 0.2493       | 0.7724           | -    |
| 10.424      | 0.0428     | 0.0742       | 0.2298           | -    |
| 12.237      | 0.0831     | 0.1438       | 0.4456           | -    |
| 12.709      | 0.2088     | 0.3615       | 1.1200           | -    |
| 13.350      | 0.1011     | 0.1749       | 0.5419           | -    |
| 13.557      | 0.0000     | 0.0001       | 0.0001           | -    |
| 13.706      | 0.0019     | 0.0033       | 0.0101           | -    |
| 14.219      | 0.0061     | 0.0106       | 0.0328           | -    |
| 14.497      | 0.0133     | 0.0231       | 0.0715           | -    |
| \(\sum B(M1)\) | 1.6876     | 2.9216       | 9.0511           | -    |

TABLE II. \(B(M1)\) Transitions in \(^{28}\text{Si}\): Isovector Case \(T = 0 \rightarrow T = 1\) in units of \(\mu_N^2\).

| Energy (MeV) | \(B(M1)\) | \(B(M1)_t\) | \(B(M1)_\sigma\) | Sign |
|-------------|------------|--------------|------------------|------|
| 10.310      | 1.3520     | 0.0552       | 0.8603           | +    |
| 11.768      | 0.4906     | 0.1224       | 0.1229           | +    |
| 12.660      | 0.0344     | 0.1761       | 0.5490           | -    |
| 12.970      | 0.3874     | 0.3591       | 0.0005           | +    |
| 13.318      | 0.0612     | 0.0296       | 0.0057           | +    |
| 13.622      | 0.0339     | 0.0769       | 0.0087           | -    |
| 13.901      | 0.0111     | 0.0137       | 0.0494           | -    |
| 14.050      | 0.0544     | 0.0670       | 0.0007           | -    |
| 14.535      | 0.3469     | 0.3989       | 0.0018           | -    |
| 15.044      | 0.1597     | 0.0013       | 0.1900           | -    |
| \(\sum B(M1)\) | 4.4449     | 2.2424       | 2.3911           |      |
TABLE III. Summed $B(M1)$ Strength: Isoscalar Case $T = 0 \rightarrow T = 0$ in units of $10^{-2} \mu_N^2$.

| Nucleus | $\sum B(M1)$ | $\sum B(M1)_l$ | $\sum B(M1)_\sigma$ | $\Delta^a$ | $\theta^b$ |
|---------|--------------|----------------|---------------------|------------|------------|
| $^{20}Ne$ | 0.3925 | 0.6794 | 2.1047 | -2.3916 | 180° |
| $^{24}Mg$ | 1.1810 | 2.0451 | 6.3341 | -7.1982 | 180° |
| $^{28}Si$ | 1.6876 | 2.9216 | 9.0611 | -10.2851 | 180° |
| $^{32}S$ | 1.9131 | 3.3137 | 10.2640 | -11.6646 | 180° |
| $^{36}Ar$ | 1.5189 | 2.6299 | 8.1467 | -9.2577 | 180° |

$^a\Delta = \sum B(M1) - \sum B(M1)_\sigma - \sum B(M1)_l$

$^b\cos(\theta) = \Delta / (2\sqrt{(\sum B(M1)_l)(\sum B(M1)_\sigma)})$

TABLE IV. Summed $B(M1)$ Strength: Isovector Case $T = 0 \rightarrow T = 1$ in units of $\mu_N^2$.

| Nucleus | $\sum B(M1)^a$ | $\sum B(M1)_l$ | $\sum B(M1)_\sigma$ | $\Delta$ | $\theta^b$ |
|---------|----------------|----------------|---------------------|---------|------------|
| $^8Be$ | 1.055 | 0.6701 | 0.3784 | 0.0065 | 89.60 |
| $^{20}Ne$ | 1.5326 | 0.9456 | 0.5817 | -0.00002 | 90.00 |
| $^{24}Mg$ | 3.7797 | 1.7302 | 2.0793 | -0.0298 | 90.40 |
| $^{28}Si$ | 4.4449 | 2.2424 | 2.3911 | -0.1886 | 92.30 |
| $^{32}S$ | 4.7569 | 2.7134 | 2.3181 | -0.2926 | 93.33 |
| $^{36}Ar$ | 3.3485 | 2.2480 | 1.4426 | -0.3421 | 95.40 |

$^a$In this table, we use the interaction given in Eq. (3), with $x = 1 \quad y = 1$. 

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TABLE V. Summed $B(M1)$ Strength ($N \neq Z$) in units of $\mu^2_N$.

| Nucleus | Transition | $\sum B(M1)$ | $\sum B(M1)_l$ | $\sum B(M1)_s$ | $\Delta$ | $\theta(\circ)$ |
|---------|------------|---------------|----------------|----------------|----------|----------------|
| $^{10}$Be | $T = 1 \rightarrow T = 1$ | 2.0930 | 0.1266 | 1.622 | 0.3444 | 67.64 |
|         | $T = 1 \rightarrow T = 2$ | 0.0597 | 0.1508 | 0.0932 | -0.1843 | 141.01 |
|         | Total      | 2.1527 | 0.2744 | 1.7152 | 0.1601 | 83.33 |
| $^{22}$Ne | $T = 1 \rightarrow T = 1$ | 2.2796 | 0.3597 | 1.8063 | 0.1134 | 85.90 |
|         | $T = 1 \rightarrow T = 2$ | 0.3398 | 0.2732 | 0.1559 | -0.0893 | 102.4 |
|         | Total      | 2.6194 | 0.6329 | 1.9621 | 0.0243 | 88.70 |

TABLE VI. The dependence of the interference angle $\theta$ on the details of the interaction for $^{28}$Si.

| $x$ | $y$ | $\sum B(M1)$ | $\sum B(M1)_l$ | $\sum B(M1)_s$ | $\Delta$ | $\theta(\circ)$ |
|-----|-----|---------------|----------------|----------------|----------|----------------|
| 0   | 0   | 2.838         | 2.465          | 0.372          | -0.001   | 90.03 |
| 1   | 0   | 5.616         | 2.014          | 4.096          | -0.494   | 94.93 |
| 0   | 1   | 3.112         | 2.502          | 0.607          | 0.003    | 89.93 |
| 1   | 1   | 4.445         | 2.242          | 2.391          | -0.189   | 92.33 |
| 1.5 | 0.5 | 7.870         | 1.619          | 7.209          | -0.988   | 98.31 |