The bicycle model of a 4WS car lateral dynamics for lane change controller

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Abstract. The article shows the validity of four-wheel steering (4WS) bicycle model. At the beginning it is presented in which tasks, connected with 4WS vehicles, the bicycle model is applied. Then the equations in local coordinate system and their forms in global coordinate system are presented. The research tested the response of the model to a lane change manoeuvre, using transfer functions and a “bang-bang” control signal. The transfer functions was connected with static ratio characteristic to construct full four-wheel steering model. A reference signal generator and vehicle model were constructed in analytical form and checked in Matlab&Simulink using sample vehicle’s data from another literature item. One example of simulations was shown. The results presented led to interesting conclusions and indicated directions for further research. This research will include the issue of regulators derivation. This will allow for the construction of a fully functioning control system. The effects of this activities will be tested again in simulation environment.

Keywords: road safety, infrastructure, automotive safety

1. Introduction

It was decided to take up the above topic of the article due to the authors’ work on the topic of automatic control the lane change manoeuvre by a car with four steered wheels (4WS). The bicycle model of a 4WS car lateral dynamics is treated as the basic model for the synthesis of algorithms of the lane change controller. The idea behind this system was shown in the Figure 1.
The proposed control system consists of a reference signal generator and regulators [5]. The reference signal generator is based on the lane change process reference model. It generates a reference steering signal and corresponding reference output signals. According to the theory of control systems and the practice of drivers [7], the reference steering signal has a form of “bang-bang” type signal. This signal is then corrected by regulators which work on the signal errors between reference and measured (in real car) output signals. The regulators correct the reference control signal so that, after correction, the actual control signal acting on the car ensures the minimization of errors, and finally a good implementation of lane change process. The algorithms of the regulators are also based on a reference model describing the lane change process. The reference model is therefore crucial for the design of the lane change controller. As shown in the literature analyzes, the bicycle model turned out to be the correct reference model used in many solutions of other authors.

The bicycle model is one of the models that forms the basis for considering the control concept in cars (2WS as well as 4WS), which can be seen from several publications in which this model appears together with other solutions. In papers [1], [9], [10] and [21] the bicycle model is used together with the MPC (Model Predictive Control) technique for the process of lane change. A wider spectrum has even devoted several review papers to the bicycle model: [5], [6], [8], [16] and [17]. It has also been used in, among other things: various computer simulations [2], real-world studies [3], in the study of variable vehicle dynamics [4], vehicle performance [12], active steering [15], comparison of 2WS and 4WS vehicle control [18] and control system design [20].

2. The “bicycle model” in ODE form
The bicycle model is a plane model with three degrees of freedom. It assumes that the vehicle’s centre of mass is located on the plane of the road. The vehicle itself is symmetrical with respect to the longitudinal vertical plane. The axles are reduced to individual wheels. This model assumes that the left and right wheels of the vehicle generate the same lateral forces and depend only on the drift angle of the entire vehicle axis (Figure 2).

![Figure 2. Idea of bicycle model.](image)

Notation:
- \((X, Y)\) – global coordinate system,
- \((x, y)\) – local coordinate system,
- \(m\) – vehicle mass,
- \(J\) – mass moment of inertia \((J_{zz} \text{ to p. C})\),
- \(L_A, L_B\) – distances AC and BC between the center of mass to the front / rear axis,
\begin{itemize}
  \item $K_A, K_B$ – front and rear tyre cornering stiffness (yaw coefficients to points A and B),
  \item $\delta_A, \delta_B$ – front and rear steering angles,
  \item $\Omega$ – angular velocity of the vehicle,
  \item $V$ – longitudinal vehicle speed in the local coordinate system,
  \item $U$ – lateral vehicle speed in the local coordinate system
\end{itemize}

The considered issues concern the control of motion of a 4WS vehicle travelling at a steady speed on a straight, level road with good adhesion, so that the well-known "bicycle model" can be used to describe the dynamics. The vehicle is treated as a moving single mass planar object on elastic torsion pneumatics, symmetric about its longitudinal axis.

The road reaction forces are perpendicular to the wheel rim side planes and depend directly (functional dependence) on the drift angles, so the angles between the wheel rim side planes and the wheel tyre line marks [11]. The road reaction forces are related to the points defined by the vertical projections of the wheel axis centres. Using the assumptions, one obtains ordinary differential equations (ODE) form:

$$m\ddot{U}(t) + \frac{K_A + K_B}{V}U(t) + \frac{mV^2 + K_A L_A - K_B L_B}{V}\Omega(t) = K_A \delta_A(t) + K_B \delta_B(t)$$  \hspace{1cm} (1)

$$J\ddot{\Omega}(t) + \frac{K_A L_A^2 + K_B L_B^2}{V}\Omega(t) + \frac{K_A L_A - K_B L_B}{V}U(t) = K_A L_A \delta_A(t) - K_B L_B \delta_B(t)$$  \hspace{1cm} (2)

$$X(t) = \int_0^t \dot{X}(\tau)d\tau = \int_0^t (V \cos(\psi(\tau)) - U(\tau) \sin(\psi(\tau)))d\tau$$  \hspace{1cm} (3)

$$Y(t) = \int_0^t \dot{Y}(\tau)d\tau = \int_0^t (V \sin(\psi(\tau)) + U(\tau) \cos(\psi(\tau)))d\tau$$  \hspace{1cm} (4)

$$\psi(t) = \int_0^t \Omega(\tau)d\tau$$  \hspace{1cm} (5)

When $\psi(t)$ is small (for example during the lane change process) the linearisation of non-linear equations is possible and in this case equations (3), (4) obtain the linear form:

$$X(t) = \int_0^t \dot{X}(\tau)d\tau = Vt$$  \hspace{1cm} (6)

$$Y(t) = \int_0^t \dot{Y}(\tau)d\tau = \int_0^t (V \psi(\tau) + U(\tau))d\tau$$  \hspace{1cm} (7)

3. Transmittance formulation of the model
The analysis of the 4WS bicycle model will be based on the research of the 4WS vehicle lane change process through a suddenly appearing obstacle [13] and [14]. The driver or controller during this manoeuvre generates a "bang-bang" signal (a jerk of the steering wheel, a hold, a jerk in the other direction, a hold and a return to the zero position) (Figure 3).

![Figure 3. The concept of time decomposition of lane change control in 2WS vehicle [6].](image-url)
After the equations have been linearised, they can be subjected to the Laplace transform and then the model was determined as a transfer function forms (Figure 4):

\[
G_{Y\delta_A}(s) = \frac{K_Y\delta_A(T\delta_A^2+2\xi_Y\delta_AT\delta_A+1)}{s^2(T\delta_A^2+2\xi_0\delta_A+1)}
\]

\[
G_{Y\delta_B}(s) = \frac{K_Y\delta_B(T\delta_B^2+2\xi_Y\delta_BT\delta_B+1)}{s^2(T\delta_B^2+2\xi_0\delta_B+1)}
\]

\[
G_{\psi\delta_A}(s) = \frac{K_{\psi\delta_A}(T\psi\delta_A^2+1)}{s(T\delta_A^2+2\xi_0\delta_A+1)}
\]

\[
G_{\psi\delta_B}(s) = \frac{K_{\psi\delta_B}(T\psi\delta_B^2+1)}{s(T\delta_B^2+2\xi_0\delta_B+1)}
\]

where:

\[
K_Y\delta_A = KV_0 \quad K_{\psi\delta_A} = K_0 \quad K_{\psi\delta_B} = -K_0 \quad K_0 = \frac{K_AL_B(V+L_B)}{K_AL_B(V+L_B)^2-mV^2(K_AL_A-K_BL_B)}
\]

\[
T_0 = V \sqrt{\frac{m}{K_AL_A+K_BL_B}} \quad \xi_0 = \frac{m(K_AL_A+K_BL_B)}{2\sqrt{mK_AL_B(L_A+L_B)^2-mV^2(K_AL_A-K_BL_B)}}
\]

\[
T_{\psi\delta_A} = \frac{mL_AV}{K_AL_A+L_B} \quad T_{\psi\delta_B} = \frac{mL_BV}{K_BL_B+L_B} \quad T_Y\delta_A = \sqrt{\frac{J}{K_AL_A+L_B}} \quad T_Y\delta_B = \sqrt{\frac{J}{K_BL_B+L_B}}
\]

The above representation of the transfer functions show a very strong connection between the transfer functions parameters and the mechanical parameters. The transfer functions show the essence of the effects of steering angles on lateral and angular displacements.

In classic 4WS controls, the rear wheels are controlled by transforming the front wheel control accordingly (Figure 5).
With the assumed ratio characteristics, the transfer functions model appears as follows (Figure 6):

\[ Y(s) = G_{Y\delta}(s)\delta(s) \quad \text{where} \quad G_{Y\delta}(s) = G_{Y\delta A}(s) + P_{AB}(V)G_{Y\delta B}(s) \quad (24, 25) \]

\[ \psi(s) = G_{\psi\delta}(s)\delta(s) \quad \text{where} \quad G_{\psi\delta}(s) = G_{\psi\delta A}(s) + P_{AB}(V)G_{\psi\delta B}(s) \quad (26, 27) \]

from: \[ G_{Y\delta}(s) = \frac{K_{Y\delta}(T_0^2s^2+2\xi_0T_0s+1)}{s^2(T_0^2s^2+2\xi_0T_0s+1)} \]

\[ G_{\psi\delta}(s) = \frac{K_{\psi\delta}(T_0^2s^2+1)}{s(T_0^2s^2+2\xi_0T_0s+1)} \quad (28, 29) \]

where:

\[ K_{Y\delta} = (1 - P_{AB}(V))WK_0 \quad T_{Y\delta} = \frac{\int \frac{1}{K_B} \frac{P_{AB}(V)}{K_A}}{(L_A+L_B)(1-P_{AB}(V))} \quad \xi_{Y\delta} = \frac{L_B+P_{AB}(V)L_A}{2V(1-P_{AB}(V))} \quad (30-32) \]

\[ K_{\psi\delta} = (1 - P_{AB}(V))K_0 \quad T_{\psi\delta} = \frac{mv(\frac{L_A}{K_B} - P_{AB}(V))}{(L_A+L_B)(1-P_{AB}(V))} \quad (33, 34) \]

4. Theoretical and simulation based on analyses

The next step is determine and analyse the limes of the transfer functions. In particular, the "bang-bang" input waveforms (Figure 7) should be considered, as they approximate the real waveforms during lane change process.

\[ \delta(t) = \delta_0(1(t) - 2 \cdot 1(t - T) + 1(t - 2T)) \]

\[ \delta_\delta(t) = \frac{\delta(t)}{s} \]

\[ \lim_{t \to \infty}(\psi(t)) = \lim_{s \to 0}(s\psi(s)) = \lim_{s \to 0} \left( s(G_{\psi\delta A}(s)\delta_A(s) + G_{\psi\delta B}(s)\delta_B(s)) \right) = 0 \quad (40) \]

\[ \lim_{t \to \infty}(Y(t)) = \lim_{s \to 0}(sY(s)) = \lim_{s \to 0} \left( s(G_{Y\delta A}(s)\delta_A(s) + G_{Y\delta B}(s)\delta_B(s)) \right) = (1 - P_{AB}(V))\delta_\delta T^2VK_0 \quad (41) \]

It was decided to use also the model in a reduced version (members affecting only the transients are omitted), as this enables an analytical synthesis of the control of classical 4WS vehicles.
\[
G_{Y_R}(s) = \frac{k_{Y\delta}}{s^2} \quad G_{\psi_R}(s) = \frac{k_{\psi\delta}}{s}
\]

The same results of the boundary analysis are also obtained assuming a reduction of the initial transfer functions model. The limit defined by equation (39) is the steady-state displacement value \(Y_0\), also the magnitude of the lane change. It should be noted here that theoretically, according to formula (40), the steady-state yaw angle is zero, so the vehicle moves on a track parallel to the original track. However, equations (40) and (41) are not sufficient to determine the signal parameters \(\delta_0\) and \(T\). Therefore, an analogous study was performed here by updating the state after step excitation at time \(t = T\) using (43). In this case, the angular displacement \(\psi(T) = \psi_0\) reaches the maximum value \(\psi_0\) defining the assumed linearity of the model. Based on these analyses, formulas for the values of \(T\) i \(\delta_0\) can be calculated (Figure 8):

\[
T = \frac{Y_0}{\psi_0} \quad \delta_0 = \frac{\psi_0}{k_{\psi\delta}Y_0}
\]

Figure 8. Reference signals in the control system.

The form of the individual elements for the control process must be defined at the outset in the planned control concept. This mainly concerns the form of the reference signal generator and the vehicle, which will generate the real signals on the basis of the reference signals. In the feedback loop, so-called failures will arise as a result of the interaction of the vehicle’s individual parameters with the prevailing traffic situation. In order to check the operation of the analysed reference signal generator and 4WS vehicle, it was implemented into the Matlab&Simulink program. As a result of not having an real 4WS vehicle, this paper uses vehicle’s data from the article [22] (Table 1).

| Parameter | Unit       | Value |
|-----------|------------|-------|
| \(m\)    | \([kg]\)  | 1740  |
| \(J\)    | \([kg \cdot m^2]\) | 3048  |
| \(L_A\)  | \([m]\)   | 1.035 |
| \(L_B\)  | \([m]\)   | 1.655 |
| \(K_A\)  | \([N/rad]\) | 35000 |
| \(K_B\)  | \([N/rad]\) | 37500 |
| \(V_0\)  | \([m/s]\) | 15    |
| \(\Delta V\) | \([m/s]\) | 5     |
| \(P_{AB0}\) | \([-\] | 0.1   |
| \(Y_0\)  | \([m]\)   | 2     |
| \(\psi_0\) | \([rad]\) | 0.11  |
The tests were carried out for several speed variants $V$ with process time $t = 10\, s$ to check the functionality of the system. The graphs show reference (green color) and real (blue color) values. Example simulation result is presented in Figure 9.

![Simulation Result](image)

Figure 9. The result of simulation for $V = 50\, km/h$.

As a result of the simulations carried out, it can be seen what role the extended and reduced transfer functions have. Simulations were repeated for many different sets of data (not presented here). They had shown the sensitivity of the model, but in every cases the errors between the results of reduced and extended forms of transfer functions model were not high.

5. Conclusions

This study shows how important and significant role the bicycle model plays in the overall control concept of the 4WS cars. The linearisation of the model and its subsequent presentation in the form of operator calculus allowed the derivation of transfer functions, which greatly facilitated the process of analysing the model in terms of lane changes. Appropriate use of the "bang-bang" signal and a reduced model with the application of limes let to make reference signal generator. The bicycle model in reduced form can be used for analytical synthesis of the regulation system, which correct the reference steering signal. The proposed model in extended as well as in reduced form seems to be good proposition for synthesis and analyses the lane change control system algorithms for 4WS vehicles. The simulations, which were carried out, had shown that constructed model works as theoretical. This simulations show very well the effect of transition parameters on model. Graphs of extended transfer functions are not as ideal as graphs of reduced transfer functions. It is possible to construct fully regulation system. The next step for further research is to derivate regulators to proposed system on Figure 1. Everything will be checked again in Matlab&Simulink system for verification sensitivity of the model for different vehicles’ data.

6. References

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