The photo-assisted heat current and its Peltier coefficient in a metal/dot/metal junction

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Abstract
The photo-assisted heat current through a metal/dot/metal junction and its associated Peltier coefficient are computed in the framework of the time-dependent out-of-equilibrium Keldysh formalism in the presence of a dot energy modulation. When the frequency of the modulation is much larger than the amplitude of the modulation, the heat current follows the sinusoidal time evolution of the dot energy. This is no longer the case when the modulation frequency becomes of the order of or smaller than the amplitude of the modulation. To characterize this non-sinusoidal behavior, we have calculated the harmonics of the photo-assisted heat current. The zero-order harmonic can be expressed as an infinite sum of dc heat currents associated with a dot with shifted energies. It exhibits a devil’s staircase profile with non-horizontal steps, whereas it is established that the steps are horizontal for the zero-order harmonic of the photo-assisted electric current. This particularity is related to the fact that the dot heat is not a conserved quantity due to energy dissipation within the tunnel barriers.

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1. Introduction
The photo-assisted electric current through an insulating barrier is the topic of a large number of studies both theoretically [1–4] and experimentally [5–8] in a wide range of systems, such as quantum dots, normal or superconducting tunnel junctions and Luttinger liquids. In the presence of a voltage modulation, the electrons can emit or absorb photons when they travel across the barrier and the resulting current is thus a superposition of dc currents. If the current–voltage characteristic is nonlinear, it leads to a specific type of behavior for the current [6].

It is only recently that the study of photo-assisted heat current has emerged. On the theoretical side, the heat flow generated by an adiabatic quantum pump through a mesoscopic sample [9, 10], as well as the photo-assisted heat flow in a normal metal/superconductor junction [11], has been studied. Non-adiabatic pumping heat in an asymmetric double quantum dot has also been considered [12]. Even more recently, the chiral heat transport in driven quantum Hall edge states [13] and the microwave-mediated heat transport and thermoelectric effect in a quantum dot in the presence of Coulomb interaction [14] have been calculated. On the experimental side, measurements of the Seebeck voltage have been carried out in magnetic tunnel junctions in the presence of a frequency-modulated laser used to heat up the device [15].

The formalism that has been developed [16] for calculating the time-dependent heat current with the help of out-of-equilibrium Green’s functions allows us to treat any type of time-dependent voltage, in particular a modulated one. It is therefore the appropriate formalism that has to be used when one considers time-dependent excitation in the quantum regime. It enables us to calculate not only the photo-assisted electric and heat currents but also, as done here for the first time, the photo-assisted Peltier coefficient, since the last quantity is defined as the ratio between the heat current and
the electric current in the absence of a temperature gradient. However, this formalism does not allow for calculation of the photo-assisted Seebeck coefficient. Indeed, whereas this quantity is generally measured in an open circuit, it is not straightforward in such a calculation to ensure the cancelation of the electric current, except in the linear regime [17], which is not the one we consider here.

Even though several theoretical works are devoted to the photo-assisted heat current in quantum dots [12, 14, 18–20], they are restricted to the calculation of its time average value, which corresponds to the zero-order harmonic. A detailed study of higher-order harmonics is still missing. The main objective of this paper is to fill this need. This will allow us to know if the heat current through the junction will follow the time evolution of the modulated gate voltage, i.e. if it is sinusoidal or if it has a more complicated profile. A careful study of the heat current and Peltier coefficient according to the amplitude and frequency of the modulation, as done here, is thus needed.

This paper is organized as follows. In section 2, we give the model we have used for calculating the time-dependent heat current. In sections 3 and 4, we discuss the photo-assisted heat current and photo-assisted Peltier coefficient. Next, in section 4, we study in more detail the harmonics of the ac heat current. We conclude in section 5.

2. The model

We consider a single-level non-interacting quantum dot connected to the left (L) and right (R) reservoirs with chemical potentials $\mu_{L,R}$ at temperatures $T_{L,R}$ (see the upper panel of figure 1). We define as well the source–drain voltage: $eV = \mu_L - \mu_R$, and the average temperature of the reservoirs: $T = (T_L + T_R)/2$. The energy level of the quantum dot can be modulated in time by tuning the gate voltage; we denote it as $\dot{\epsilon}_{\text{dot}}(t)$. We use the following Hamiltonian to describe this system:

$$H = \sum_{k \in L,R} \epsilon_k c_k^\dagger c_k + \dot{\epsilon}_{\text{dot}}(t) d^\dagger d + \sum_{k \in L,R} V_k c_k^\dagger d + \text{h.c.},$$

(1)

where $\epsilon_{k \in L,R}$ is the energy band of the reservoir L or R. The notations $c_k^\dagger (d^\dagger)$ and $c_k (d)$ refer to the creation and annihilation operators associated with the reservoirs (dot). $V_{k \in L,R}$ is the hopping amplitude between the reservoir L or R and the dot.

The time-dependent heat current through the left (L) or right (R) reservoirs, $I_{L,R}^h(t) = -\dot{Q}_{L,R}$, is related to the energy current, $I_{L,R}^E = -E_{L,R}$, and to the electric current, $I_{L,R}^E$, by the identity:

$$I_{L,R}^h(t) = I_{L,R}^E(t) - \frac{\mu_{L,R}}{e} I_{L,R}^E(t),$$

(2)

which is obtained from the thermodynamic relation

$$d\dot{Q}_{L,R} = dE_{L,R} - \mu_{L,R} dN_{L,R},$$

(3)

since the reservoirs are at equilibrium. The quantities $Q_{L,R}$, $E_{L,R}$ and $N_{L,R}$ correspond, respectively, to the heat, the energy and the number of particles in the left (L) or right (R) reservoirs. Thus, by calculating the electric and energy currents, one can deduce the heat current with the help of equation (2). To determine these quantities, we use the fact that $i\hbar N_{L,R} = [N_{L,R}, H]$ and $i\hbar E_{L,R} = [E_{L,R}, H]$ with

$$N_{L,R} = \sum_{k \in L,R} c_k^\dagger c_k,$$

(4)

$$E_{L,R} = \sum_{k \in L,R} \epsilon_k c_k^\dagger c_k,$$

(5)

Note that the definitions of the energies of the reservoirs, $E_{L,R}$, given above implicitly assume that we consider only the electronic contribution to the energy current (i.e. we neglect a possible phononic contribution).

Within this model and with the help of Kelysh out-of-equilibrium Green’s functions, the expression for the time-dependent heat current through the reservoir $p$ in the wide-band limit reads as [16]

$$I_p^h(t) = -\frac{\Gamma_p}{\hbar} \left[2 \int_{-\infty}^{\infty} (\epsilon - \mu_p) f_p(\epsilon) \text{Im}[A(\epsilon, t)] d\epsilon \right. + \left. \sum_{p' \neq L,R} \Gamma_{p'} \int_{-\infty}^{\infty} (\epsilon - \mu_p) f_p(\epsilon) |A(\epsilon, t)|^2 d\epsilon \right],$$

(6)

where $f_p$ is the Fermi–Dirac distribution function of the reservoir $p$. The enlargement of the dot energy level due to its coupling with the reservoir $p$ is defined as $\Gamma_p = 2\pi \rho_p |V_p|^2$, where $\rho_p$ is the density of states of the reservoir $p$ and $V_p \equiv V_{k = 0}$. In the wide-band limit, it is assumed to be energy independent. Note that this is the energy current contribution, $I_p^E = -E_p$, which is responsible for the presence of the $\epsilon$ term in the heat current.

The quantity $A(\epsilon, t)$ in equation (6) is the spectral function defined as [21]

$$A(\epsilon, t) = \int_{-\infty}^{\infty} G'(t, t_1) e^{i(\epsilon - \epsilon_1)/\hbar} dt_1.$$

(7)
The retarded Green’s function \( G' \) is that of the dot connected to the reservoirs, which is given here by

\[
G'(t, t_1) = g'(t, t_1) e^{\Gamma(t_1-t)/2\hbar},
\]

where \( g'(t, t_1) = -i\Theta(t-t_1) e^{-i\int_0^t dt' \omega_{\text{dot}}(t')/h} \) is the retarded Green’s function of the isolated dot, \( \Gamma = \Gamma_L + \Gamma_R \) and \( \Theta \) is the Heaviside function.

In this work, we consider the following time modulation for the dot energy level:

\[
\varepsilon_{\text{dot}}(t) = \varepsilon_0 + \varepsilon_1 \cos(\omega t).
\]

In that case, the spectral function of equation (7) becomes

\[
A(\varepsilon, t) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} J_n \left( \frac{\varepsilon_1}{\hbar \omega} \right) J_m \left( \frac{\varepsilon_1}{\hbar \omega} \right) \frac{e^{i(n-m)\omega t}}{E_{\text{a}}(\varepsilon) + i \Gamma},
\]

where \( E_{\text{a}}(\varepsilon) = \varepsilon - \mu_{0} - n \hbar \omega \) and \( J_n \) is Bessel’s function of order \( n \) which appears in the expression for the spectral function through the identity \( e^{i \omega \sin(\omega t)} = \sum_{n=-\infty}^{\infty} J_n(x) e^{i n \omega t} \).

### 3. Photo-assisted heat current

In this section, we look at the time evolution of the heat current \( I_p^h(t) \), which has been obtained by inserting equation (10) into (6) and integrating over energy numerically. We discuss its principal characteristics and compare them to those of the time-dependent electric current \( I_p^e(t) \) that has a similar form as equation (6) except for the factors \((\varepsilon - \mu_p)\) which are not present [21].

It is important to note that there are several characteristic energies in this system that are related to: the temperatures, \( k_B T_L,R \), the source–drain voltage, \( eV \), the modulation frequency, \( \hbar \omega \), the amplitude of the modulation, \( \varepsilon_1 \), the dot energy level, \( \varepsilon_0 \), and the coupling strength between the dot and the reservoirs, \( \Gamma \). The heat current depends on relative values of all these energies. Here, we focus on the change in its behavior according to the value of the ratio \( \varepsilon_1/\hbar \omega \) that appears in the argument of Bessel’s functions present in the expression for the heat current. Figure 2 shows the time evolution of the heat and electric currents, at a fixed \( \varepsilon_1 \) and at temperatures \( k_B T_L,R \) much smaller than all the other energies, for several values of the modulation frequency \( \omega \) during a full period.

We see that when \( \hbar \omega \gg \varepsilon_1 \), the left and right currents are both sinusoidal (see the solid red lines) and oscillate almost in phase around their corresponding dc values (see the black straight lines), whereas when \( \hbar \omega \ll \varepsilon_1 \), the signals are much more complex (see the dotted green lines). Indeed, when the ratio \( \varepsilon_1/\hbar \omega \) is smaller than 1, the dominant contributions in the heat current come from the terms associated with \( n, m \in \{-1, 0, 1\} \) in the sums that appear in the spectral function of equation (10) because the value of the product of Bessel’s functions, \( J_n(x) J_m(x) \), with argument \( x \) smaller than 1 decreases quickly with increasing \( n \) and \( m \) (see figure 3).

As a consequence, we have

\[
I_p^h(t) \approx I_p^{h(0)} + 2 \text{Re}\{I_p^{h(1)} e^{i \omega t}\},
\]

where \( I_p^{h(0)} \) is the zero-order harmonic (or average value) of the time-dependent heat current and \( I_p^{h(1)} \) its first-order harmonic (see section 5). This result explains the sinusoidal behavior we have obtained in the limit \( \hbar \omega \gg \varepsilon_1 \) (i.e. the red solid lines of figure 2). In contrast, when the ratio \( \varepsilon_1/\hbar \omega \) is close to or higher than 1, a large number of terms in the sums over \( n \) and \( m \) in equation (10) contribute and \( I_p^h(t) \) is a superposition of several harmonics (see section 5), which leads to a non-sinusoidal signal (i.e. the green dotted lines of figure 2).

### 4. The photo-assisted Peltier coefficient

We have all the ingredients to calculate the time evolution of the Peltier coefficient of the junction which is defined as [16]

\[
\Pi(t) = \frac{I_p^h(t) - I_p^e(t)}{I_p^e(t)} \bigg|_{t_L=T_R}.
\]
Figure 4. Peltier coefficient as a function of $\omega t$. The parameters are the same as those of figure 2: $\varepsilon_0 = 1$, $\varepsilon_1 = 0.5$, $\Gamma_{1,L,R} = 0.5$, $k_B T_{L,R} = 0.01$ and $eV = 1$. The modulation frequency is $h\omega = 10$ (solid red line) and $h\omega = 0.5$ (dotted green line). The black straight line corresponds to the Peltier coefficient in the absence of modulation (i.e., $\varepsilon_1 = 0$). The unit for energies is $\Gamma$.

By looking at figure 2, we note that whereas the left and right heat currents can be equal at some specific times (compare the thick and thin lines in the upper panel of figure 2), this is not the case for the left and right electric currents which take distinct values at any time (compare the thick and thin lines in the lower panel of figure 2). This property is general and not limited to the parameters we have chosen for plotting figure 2. Thus, we can conclude that the Peltier coefficient of the junction will never be divergent. However, it vanishes every time that the left and right heat currents are equal, i.e. $I_{L,R}^h(t) = I_{R,L}^h(t)$.

Figure 4 shows the time evolution of the Peltier coefficient at low temperature (i.e. at $k_B T_{L,R} \ll \Gamma$). As was the case for the heat and electric currents, the Peltier coefficient is sinusoidal in the limit $h\omega \gg \varepsilon_1$ (see the solid red line): it follows the time evolution of the imposed gate voltage modulation. When the modulation frequency is reduced and becomes of the order of amplitude of the modulation $h\omega \sim \varepsilon_1$, the Peltier coefficient is no longer sinusoidal (see the dotted green line).

Another quantity which characterizes the thermoelectricity is the Seebeck coefficient defined as the ratio between the voltage gradient and the temperature gradient in an open circuit: $S = \Delta V / \Delta T |_{V=0}$. However, this definition applies only in the linear response regime, as well as the direct relation between the Peltier and the Seebeck coefficients: $\Pi = ST$. Since here we are not necessarily in the linear response regime, calculation of the photo-assisted Seebeck coefficient is a more difficult task that is beyond the scope of the present study.

5. Harmonics of the heat current

We now turn our attention to the harmonics, $I_{L,R}^{h(N)}$, of the left and right heat currents which are defined through the relation

$$I_{L,R}^{h(N)}(t) = \sum_{N=-\infty}^{\infty} I_{L,R}^{h(N)} e^{iN\omega t}.$$  \hspace{1cm} (13)

The fact that $I_{L,R}^{h(N)}(t)$ is a real quantity imposes that $I_{L,R}^{h(N)}$ and $I_{L,R}^{h(-N)}$ are complex conjugates.

Reporting equation (10) in equations (6) and (13) and performing integration over time, we obtain

$$I_{L,R}^{h(N)} = \frac{\Gamma_{L,R}}{2\hbar} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ \Gamma_{R,L}(f_{L,R}(\epsilon) - f_{R,L}(\epsilon)) \mp 2i f_{L,R}(\epsilon) E_{n\pm N}(\epsilon) \right] \frac{e - \mu_{L,R}}{(E_n(\epsilon) \mp i\Gamma/2)(E_{n\pm N}(\epsilon) \pm i\Gamma/2)} \, d\epsilon, \hspace{1cm} (14)$$

where we have used the identity $\sum_{m} J_{m}(x) J_{m}(y) = J_{0}(x-y)$.

In the following, we study separately the zero-order harmonic of the heat current and its harmonics of higher orders.

5.1. Harmonic of order zero

The zero-order ($N = 0$) harmonic of the heat current corresponds to the time average heat current. From equation (14), we obtain

$$I_{L,R}^{h(0)} = \sum_{n=-\infty}^{\infty} J_{n} \left( \frac{\varepsilon_1}{\hbar \omega} \right) I_{L,R}^{h(0)}(\varepsilon_0 + n\omega), \hspace{1cm} (15)$$

where $I_{L,R}^{h(0)}$ is the dc heat current, defined as

$$I_{L,R}^{h(0)}(z) = \frac{\Gamma_{L} \Gamma_{R}}{h} \int_{-\infty}^{\infty} \frac{(e - \mu_{L,R})(f_{L,R}(\epsilon) - f_{R,L}(\epsilon))}{(e - z)^2 + \Gamma^2/4} \, d\epsilon. \hspace{1cm} (16)$$

The description of equation (15) is as follows: the zero-order harmonic of the ac heat current is equal to an infinite sum over $n$ of dc heat currents associated with shifted dot energies: $\varepsilon_0 + n\omega$, times the square of Bessel's function of order $n$ (see the lower panel of figure 1 for a schematic representation of those shifted energy levels). A similar kind of relation linking the zero-order harmonic of the electric current, $I_{p}^{e(0)}$, and the dc electric currents, $I_{p}^{e(0)}(\varepsilon_0 + n\omega)$, holds [2]:

$$I_{L,R}^{e(0)} = \sum_{n=-\infty}^{\infty} J_{n} \left( \frac{\varepsilon_1}{\hbar \omega} \right) I_{L,R}^{e(0)}(\varepsilon_0 + n\omega), \hspace{1cm} (17)$$

where

$$e I_{L,R}^{e(0)}(z) = \frac{\Gamma_{L} \Gamma_{R}}{h} \int_{-\infty}^{\infty} f_{L,R}(\epsilon) - f_{R,L}(\epsilon) \frac{e - \mu_{L,R}}{(e - z)^2 + \Gamma^2/4} \, d\epsilon. \hspace{1cm} (18)$$

Even though the expressions for the zero-order harmonics of the heat and electric currents look similar, i.e. they contain an infinite sum of dc contributions, they differ on an essential point: whereas $I_{p}^{e(0)} = -I_{p}^{e(0)}$, we have $I_{L,R}^{h(0)} \neq -I_{L,R}^{h(0)}$ because of the $(e - \mu_{L,R})$ factor which is present in equation (16) but not in equation (18). This leads to a rather different profile for the zero-order harmonics of the heat and electric currents as detailed below.

Figure 5 shows the zero-order harmonic of the left and right heat currents at a fixed frequency modulation, $h\omega = 10\Gamma$, and voltage, $eV = -10\Gamma$, as a function of $\varepsilon_0$. We observe...
The other parameters are \( \varepsilon_1 = 10, eV = -10 \) and \( \hbar \omega = 10 \). The unit for energies is \( \Gamma \).

Figure 5. Zero-order harmonic of the left heat current \( I_L^{(0)} \) (upper panel) and the right heat current \( I_R^{(0)} \) (lower panel) as a function of \( \varepsilon_0 \). The temperatures are \( k_B T_L = 0.01 \) (solid black lines), \( k_B T_L = 1 \) (dashed blue lines) and \( k_B T_L = 2 \) (dotted orange lines). The other parameters are \( \varepsilon_1 = 10 \), \( eV = -10 \) and \( \hbar \omega = 10 \). The unit for energies is \( \Gamma \).

Figure 6. Zero-order harmonic of the electric current \( I_e^{(0)} = -I_R^{(0)} \) as a function of \( \varepsilon_0 \). The parameters and the legends are the same as those of figure 5. The unit for energies is \( \Gamma \).

Figure 7. Amplitude of the zero-order (solid black lines), first-order (dashed red lines) and second-order (dotted purple lines) harmonics of the left heat current (upper panel) and right heat current (lower panel). The parameters are \( k_B T_L = 0.01 \) \( \varepsilon_0 = 1 \), \( eV = 1 \) and \( \varepsilon_1 = 0.5 \). The unit for energies is \( \Gamma \).

5.2. Harmonics of order \( N \)

With the help of equation (14), we have calculated numerically the harmonics of the heat currents. In figure 7 is plotted the modulus of the harmonics of order 0, 1 and 2 as a function of the modulation frequency. The modulus of the higher-order harmonics of the heat currents, i.e. \( |I_L^{h(N>2)}| \), is not shown on the graphs because they are much smaller in amplitude in comparison to \( |I_L^{h(2)}| \).

Crucial information that we extract from figure 7 is the fact that when \( \hbar \omega \lesssim \varepsilon_1 \), the moduli of the first and second harmonics are of the same order in magnitude: \( |I_L^{h(1)}| \sim |I_L^{h(2)}| \), whereas when \( \hbar \omega \gg \varepsilon_1 \), the first harmonic dominates over the second harmonic (compare the dashed red lines to the dotted purple lines). This last result justifies the approximation that we have made in order to write equation (11) and explain the sinusoidal behavior of the time-dependent heat current obtained in that regime.

6. Conclusions

We have highlighted several interesting features in the photo-assisted heat current through a metal/dot/metal junction: (i) its zero-order harmonic exhibits a devil’s staircase with non-horizontal steps, which results from the fact that the dot heat is not a conserved quantity even in the stationary regime because of energy dissipation within the tunnel barriers; (ii) the time evolution of the heat current is non-sinusoidal when the modulation frequency of the gate voltage is of the order of the amplitude of the modulation; and (iii) the time evolution of the associated Peltier coefficient is strongly dependent on the energy scales which characterize the junction.

\[ \varepsilon_0 \]
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