Radiative corrections to the semileptonic Dalitz plot with angular correlation between polarized decaying and emitted hyperons: effects of the four-body region

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Abstract

We obtain a model-independent expression for the complete Dalitz plot of semileptonic decays of polarized hyperons, which includes both the three-body and the four-body regions. We calculate radiative corrections to order $\alpha$, neglecting terms of order $\alpha q/\pi M_1$, where $q$ is the four-momentum transfer and $M_1$ is the mass of the decaying hyperon. Our results exhibit explicitly the correlation between the emitted hyperon three-momentum and the spin of the decaying hyperon. This allows us to obtain the corresponding radiative corrections to the integrated emitted hyperon spin-asymmetry coefficient. Our formulas are valid for charged as well as for neutral decaying hyperons and are appropriate for model-independent experimental analysis whether the real photon is discriminated or not.

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I. INTRODUCTION

In previous works [1,2] we have obtained the radiative corrections (RC) to the Dalitz plot (DP) of hyperon semileptonic decays (HSD), \( A \rightarrow B \ell \nu \ell \), for the case of polarized decaying hyperons (\( A \) and \( B \) are hyperons and \( \ell \) and \( \nu \ell \) are the accompanying charged lepton and neutrino). In these calculations we have kept the \( \mathbf{s}_1 \cdot \mathbf{p} \) spin correlation displayed explicitly in the DP. Here \( \mathbf{s}_1 \) is the spin of \( A \) and \( \mathbf{p} \) is a unit vector along the direction of the three-momentum of either the emitted baryon \( \mathbf{p}_2 \) or the emitted charged lepton \( \mathbf{l} \). In the former case we can obtain the spin-asymmetry coefficient of the outgoing baryon [1], while in the latter we can obtain the spin-asymmetry coefficient of the charged lepton [2].

In Ref. [2] we have considered the complete DP including in our calculations the so-called three- and four-body regions of this DP (hereafter, these regions will be referred to as TBR and FBR, respectively.) The results obtained showed us the importance of the contribution of the FBR to the RC. This region is present when real photons cannot be discriminated in an experimental analysis of HSD. It is the purpose of this work to extend the calculations of Ref. [1] in order to incorporate the four-body contribution to the RC of the corresponding DP, in the same way as we did in Ref. [2]. The result will be suitable for model independent analysis of experiments where real photons cannot be discriminated. We will also determine the spin-asymmetry coefficient \( \alpha_B \) of the outgoing baryon.

The strategy we follow in order to incorporate the four-body contribution to the results of Ref. [1] is the same as the one presented in Ref. [2]. Accordingly, in Sec. II we summarize the main results concerning the RC to the Dalitz plot in the TBR [1], and we rearrange them in parallel to Ref. [2]. In Sec. III we obtain the RC to the complete DP in terms of the triple integrals over the photon bremsstrahlung three-momentum variables, which can be numerically evaluated. In Sec. IV we perform analytically these integrals and we give our second main result, namely, the complete analytical RC to the DP of decaying polarized hyperons to order \( \alpha \) with the \( \mathbf{s}_1 \cdot \mathbf{p}_2 \) correlation explicitly displayed. In Sec. V we obtain the RC to the spin-asymmetry coefficient \( \alpha_B \) of the emitted baryon with the three- and four-body contributions explicitly indicated. In Sec. VI we evaluate numerically, for the TBR, the percentage RC to \( \alpha_B \) at very-well defined points of the DP for the decays \( \Sigma^- \rightarrow n e \nu \) and \( \Lambda \rightarrow p e \nu \). We also evaluate for these two decays, the percentage ratio of the spin-dependent part of the DP to their spin independent part at different points of the FBR. The RC to the integrated spin-asymmetry coefficient \( \alpha_B \) are also evaluated for several decays. All these results are compared with those of Ref. [3] and we find that the agreement is acceptable. Finally, in Sec. VII we present our conclusions.

II. TBR RADIATIVE CORRECTIONS TO THE DP

In this section we shall first briefly review the results of Ref. [1], without repeating details that can be found there, and we shall introduce our notation. Second, it turns out that in order to study the FBR, it is convenient to follow the steps of Ref. [2]. This then requires to rearrange the expressions of Ref. [1], before the photon three-momentum is integrated, and cast them into a new form which keeps a close parallelism with Ref. [2]. Doing this will make the analysis of the FBR very expedient and transparent, as will be appreciated in the next section.
For definiteness, let us consider the HSD

$$A \rightarrow B + \ell + \overline{\nu}_\ell,$$  
(1)

where the lepton $\ell$ is negatively charged. How to extend our results to the case when $\ell$ is positively charged will be discussed in Sec. IV. Our notation is the same as before [1,2].

Thus $p_1 = (E_1, p_1), p_2 = (E_2, p_2), l = (E, l)$, and $p_\nu = (E_\nu, p_\nu)$ are the four-momenta of $A$, $B$, $\ell$, and $\overline{\nu}_\ell$, respectively. $M_1$, $M_2$, and $m$ are the non-zero masses of the first three particles. In the center-of-mass frame of $A$, the quantities $p_2$, $l$, and $p_\nu$ will also denote the magnitudes of the corresponding three-momenta. All other conventions and notation are given in Ref. [1].

The result for the virtual RC to process (1) is compactly given by Eq. (15) of Ref. [1], namely,

$$d\Gamma_v = d\Omega \left\{ A'_0 + \frac{\alpha}{\pi} (A'_1 \phi + A''_1 \phi') - \hat{s}_1 \cdot \hat{p}_2 \left[ A''_0 + \frac{\alpha}{\pi} (A'_2 \phi + A''_2 \phi') \right] \right\},$$  
(2)

where

$$d\Omega = \frac{G^2_F}{2} \frac{dE_2 dE d\Omega_2}{(2\pi)^4} 2M_1.$$  
(3)

There is no need to reproduce here the detailed expressions of the contributions in Eq. (2), so we only provide the guide to find them. Respectively, $A'_0$, $A'_1$, $\phi$, $A''_1$, $A''_0$, $A_2$, and $A''_2$ are given by Eqs. (16), (17), (8), (18), (9), (19), (20), and (21) of Ref. [1].

As for the bremsstrahlung contribution, the approach to compute RC to the DP is discussed in full in Refs. [1,2], so only a few salient facts will be repeated here. We need to consider the four-body decay

$$A \rightarrow B + \ell + \overline{\nu}_\ell + \gamma,$$  
(4)

where $\gamma$ represents a real photon with four-momentum $k = (w, k)$. The TBR of the DP is the region where the three-body decay (1) and the four-body decay (4) overlap completely. The FBR is where in process (4) neither of the energies of $\overline{\nu}_\ell$ and $\gamma$ can be made zero. The complete DP can be seen as the union of the TBR and the FBR. The bounds for the kinematical variables in both regions are defined in Ref. [2].

The differential decay rate for process (4), given by Eq. (32) of Ref. [1], reads

$$d\Gamma^{(s)\text{TRB}}_B = d\Gamma^{\text{TRB}}_B - d\Gamma^{(s)\text{FBR}}_B,$$  
(5)

where $d\Gamma^{\text{TRB}}_B$ and $d\Gamma^{(s)\text{TRB}}_B$ are the spin-independent and spin-dependent contributions of $d\Gamma^{\text{TRB}}_B$, respectively. Unlike Eq. (32) of Ref. [1], here we have added the superscript TRB to the several quantities in Eq. (5) to emphasize the fact that they are defined in the TBR only, a distinction that is now necessary. $d\Gamma^{\text{TRB}}_B$ is given in Eq. (33) of Ref. [1] with the explicit forms for its contributions in Eqs. (34), (37), and (38). Similarly, $d\Gamma^{(s)\text{TRB}}_B$ is given by Eq. (43) of this reference with Eqs. (51) and (57) for its corresponding explicit contributions. For our present purposes it is convenient to rearrange these equations into the forms introduced in Ref. [2]. Therefore, $d\Gamma^{\text{TRB}}_B$ becomes
\[
d\Gamma_B^{TBR} = \frac{\alpha}{\pi} d\Omega \left\{ A'_1 I_0(E, E_2) + \frac{p_2 l}{4\pi} \int_{-1}^{1} dx \int_{-1}^{1} dy \int_{0}^{2\pi} d\varphi_k \left[ |M'|^2 + |M''|^2 \right] \right\},
\]
which agrees with Eq. (27) of this Ref. [2]. Whereas \( d\Gamma_B^{(s)TBR} \) becomes
\[
d\Gamma_B^{(s)TBR} = \frac{\alpha}{\pi} d\Omega \hat{s}_1 \cdot \hat{p}_2 \left\{ A'_2 I_0(E, E_2) + \frac{p_2 l}{4\pi} \int_{-1}^{1} dx \int_{-1}^{1} dy \int_{0}^{2\pi} d\varphi_k \left[ |N''|^2 + |N^{IV}|^2 \right] \right\}.
\]
Here \( I_0(E, E_2) \), given by Eq. (52) of Ref. [1], fully contains the infrared divergence which will be canceled by its counterpart contained in the virtual RC to the differential decay rate, namely, Eq. (2). The quantities \( |M'|^2 \) and \( |M''|^2 \) are explicitly given by Eqs. (28) and (29) of Ref. [2]. Whereas \( |N''|^2 \) and \( |N^{IV}|^2 \) are new, namely,
\[
|N''|^2 = \frac{\beta^2}{p_2} \left[ D_3 \left( E'_\nu + \frac{p_2 l y}{D} \right) - D_4 E \left( 1 - \frac{p_2 \cdot \hat{k}}{D} \right) \right] \frac{1 - x^2}{(1 - \beta x)^2},
\]
and
\[
|N^{IV}|^2 = \frac{1}{ED(1 - \beta x)} \left[ -D_3 E'_\nu \left( w + E - \frac{m^2}{E(1 - \beta x)} \right) \hat{p}_2 \cdot \hat{k} - D_3 E_\nu l y \\
+ D_4 \left( w + E(1 + \beta x) - \frac{m^2}{E(1 - \beta x)} \right) (ly + p_2 + w \hat{p}_2 \cdot \hat{k}) \right].
\]
The counterparts of \( |N''|^2 \) and \( |N^{IV}|^2 \) in Ref. [2] are \( |M''|^2 \) and \( |M^{IV}|^2 \), respectively, given there by Eqs. (42) and (43).

In these last equations, \( y = \hat{l} \cdot \hat{p}_2, x = \hat{l} \cdot \hat{k}, D = E'_\nu + (1 + p_2) \cdot \hat{k}, \) and \( E_\nu = E'_\nu - w \), where \( E'_\nu \) is the neutrino energy available when the photon is not present in the decay, \( \beta = l/E \), and \( \varphi_k \) is the azimuthal angle of the real photon. The \( D_i \) are quadratic functions of the leading form factors, they are introduced in those Eqs. (42) and (43), and are explicitly given in Eqs. (B13) and (B14) of the same reference.

Adding Eqs. (2) and (4) we obtain the differential decay rate with RC for the TBR, corresponding to Eq. (100) of Ref. [1], but it is now rearranged in parallelism with Eq. (44) of Ref. [2]. This decay rate has the real photon three-momentum integrations ready to be performed numerically [see Eqs. (3) and (7)].

To conclude our short review of Ref. [1], we must mention that the photon three-momentum integrations can be performed analytically. The result is the one given in Eq. (96) of Ref. [1], namely,
\[
d\Gamma_B^{TBR} = \frac{\alpha}{\pi} d\Omega \left\{ (D_1 + D_2)(\theta' + \theta'') + D_2(\theta'' + \theta^{IV}) + A'_1 \theta_1 \right. \\
- \left. \hat{s}_1 \cdot \hat{p}_2 \left[ A'_2 \theta_1 + D_3(\rho_1 + \rho_3) + D_4(\rho_2 + \rho_4) \right] \right\}.
\]
There is no need to repeat here the detailed expressions of the quantities that appear in Eq. (100). \( \theta_1 = I_0(E, E_2) \), the \( \rho_i \) are given in Eqs. (75)–(78), and \( \theta' + \theta'' \) and \( \theta'' + \theta^{IV} \) are given in Eqs. (97) and (98) of this reference. However, an erratum was detected in \( \theta' + \theta'' \) and it was corrected in Ref. [2]. One should better use Eqs. (B39) and (B40) of this last reference for \( \theta' + \theta'' \) and \( \theta'' + \theta^{IV} \).
Collecting partial results, Eqs. (2) and (10), we obtain for Eq. (10) of Ref. [1] the analytical DP of HSD with non-zero polarization of the initial hyperon including RC to order $\alpha$ and restricted to the TBR. It is given by Eq. (101) of this reference and reads

$$d\Gamma_{\text{TBR}}(A \rightarrow B(\ell\nu)) = d\Omega \left\{ A'_0 + \frac{\alpha}{\pi} \Phi_1 - \hat{s}_1 \cdot \hat{p}_2 \left[ A''_0 + \frac{\alpha}{\pi} \Phi_2 \right] \right\},$$  \hspace{1cm} (11)$$

where $\Phi_1$ and $\Phi_2$ can be found in Eqs. (102) and (103) of this same reference.

III. FBR BREMSSTRAHLUNG AND COMPLETE RC

We now come to the main issue of this paper, to obtain the contributions of the FBR to the RC of the decay (1). It is here where the effort of the last section, putting the bremsstrahlung contributions of Ref. [1] in close parallelism with their counterparts in Ref. [2], comes to our advantage. The calculation can now be performed following the same steps of Sec. III-C of this last reference. It is not necessary to repeat here the details. The point is that there it is shown that the FBR bremsstrahlung differential decay rate has the same structure as the TBR one, Eq. (5). Namely,

$$d\Gamma_{\text{FBR}} = d\Gamma_{\text{FBR}}^\prime - d\Gamma_{\text{FBR}}^{(s)},$$  \hspace{1cm} (12)$$

where $d\Gamma_{\text{FBR}}^\prime$ and $d\Gamma_{\text{FBR}}^{(s)}$ are again the spin-independent and spin-dependent contributions.

Now, instead of Eqs. (6) and (7) we get explicitly

$$d\Gamma_{\text{FBR}}^\prime = \frac{\alpha}{\pi} d\Omega \left\{ A'_1 I_{0F}(E, E_2) + \frac{p_2 l}{4\pi} \int_{-1}^{1} dx \int_{-1}^{1} dy \int_{0}^{2\pi} d\varphi_k \left[ |M'|^2 + |M''|^2 \right] \right\},$$  \hspace{1cm} (13)$$

and

$$d\Gamma_{\text{FBR}}^{(s)} = \frac{\alpha}{\pi} d\Omega \left\{ A'_2 I_{0F}(E, E_2) + \frac{p_2 l}{4\pi} \int_{-1}^{1} dx \int_{-1}^{1} dy \int_{0}^{2\pi} d\varphi_k \left[ |N''|^2 + |N''|^4 \right] \right\}. \hspace{1cm} (14)$$

The changes between Eqs. (6)–(7) and Eqs. (13)–(14) are very simple. The upper limit $y_0$ of Eqs. (6)–(7) becomes one in Eqs. (13)–(14) and the infrared divergent $I_0 = (E, E_2)$ becomes the infrared convergent $I_{0F}(E, E_2)$, which is explicitly given in Eq. (37) of Ref. [2]. Everything else in these equations is the same. The result Eq. (12) exhibits only the angular correlation $\hat{s}_1 \cdot \hat{p}_2$. The other two angular correlations $\hat{s}_1 \cdot \hat{k}$ and $\hat{s}_1 \cdot \hat{l}$ were eliminated in favor of the former one using the replacement [3]

$$\hat{s}_1 \cdot \hat{p} \rightarrow (\hat{s}_1 \cdot \hat{p}_2)(\hat{p} \cdot \hat{p}_2),$$  \hspace{1cm} (15)$$

with $\hat{p} = \hat{l}, \hat{k}$. In Ref. [2], it was the angular correlation $\hat{s}_1 \cdot \hat{l}$ that was extracted. The counterpart of the present Eq. (12) is Eq. (39) of that reference. Both equations have the same form as mentioned above and the detailed changes are that $d\Omega'$, $|M'|^2$, and $|M''|^2$ of that Eq. (39) are now replaced by Eqs. (3), (8), and (9) in Eq. (12). Of course, the spin-independent $d\Gamma_{\text{FBR}}^\prime$ of that Eq. (39) and this Eq. (12) is the same, except for the change of $d\Omega'$ into $d\Omega$. 

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The complete RC to process (1) without the restriction of eliminating real photons (either by direct detection or indirect energy-momentum conservation) is given by the addition of Eqs. (2), (3), and (12). The result can be compactly written as
\[ d\Gamma (A \rightarrow B(\pi^0)) = d\Gamma^{TBR} + d\Gamma^{FBR}. \]

This equation is our first main result. The correlation \( \hat{s}_1 \cdot \hat{p}_2 \) is explicitly extracted and the integral over the photon variables (\( \varphi_k, y, \) and \( x \)) are ready to be performed numerically. However, all the photon integrals can be analytically performed. Those of the TBR were already computed before and we reviewed the result in Eq. (11) of Sec. II. In the next section we shall obtain the analytical result for the new photon integrals that appear in the FBR contributions. This will lead to our second main result.

IV. ANALYTICAL INTEGRATIONS

Let us now proceed to obtain the analytical expression of Eq. (12). Not all of the photon integrals of the FBR in Eqs. (13) and (14) are new. It turns out that those of \( d\Gamma^{FBR}_B \) were already performed in Ref. [2], which as explained in the last section was to be expected. All we have to do in this respect is to bring here the result of Ref. [2]. That is,
\[ d\Gamma^{(s)FBR}_B = \frac{\alpha}{\pi} d\Omega \hat{s}_1 \cdot \hat{p}_2 \left[ A_1' I_{0F}(E, E_2) + (D_1 + D_2) (\theta_F' + \theta_F'') + D_2 (\theta_F'' + \theta_F^{IV}) \right]. \]

On the other hand, the photon integrals in Eq. (14) are new. Their calculation is quite straightforward, albeit tedious. It is, however, important to invest some extra effort to rearrange their result so that the notation resembles and uses as much as possible expressions already defined. Without giving further details, the result is
\[ d\Gamma^{(s)FBR}_B = \frac{\alpha}{\pi} d\Omega \hat{s}_1 \cdot \hat{p}_2 \left[ A_2' I_{0F}(E, E_2) + D_3 (\rho_1F + \rho_3F) + D_4 (\rho_2F + \rho_4F) \right]. \]

The functions \( \rho_iF \) have the same structure as the \( \rho_i \) previously defined in Ref. [1] for the TBR. Their explicit expressions are
\[ \rho_{1F} = \frac{l}{2} \left[ 2E\nu^0_0 \theta_{0F} - \zeta_{10F} + 2\zeta_{11F} + (\beta^2 - 1)\zeta_{12F} \right], \]
\[ \rho_{2F} = \frac{E}{2} \left[ -2l\theta_{0F} - \chi_{10F} + 2\chi_{11F} + (\beta^2 - 1)\chi_{12F} \right], \]
\[ \rho_{3F} = \frac{\beta}{4} \left[ -2E\nu^0_0 \chi_{11F} + \zeta_{21F} \right] + \frac{1}{2} \left[ -E\nu^0_0 \chi_{11F} + \frac{E - E\nu^0_0}{2E} \chi_{21F} + \frac{\chi_{31F}}{4E} \right] \]
\[ + (1 - \beta^2)E\nu^0_0 \chi_{12F} - \frac{1}{2} (1 - \beta^2) \chi_{22F} \],
\[ \rho_{4F} = \frac{\beta}{2} \left[ -E\zeta_{10F} + 2E\zeta_{11F} + \frac{\zeta_{21F}}{2} - \frac{m^2}{E} \zeta_{12F} \right] + \frac{p^2\beta}{2} \gamma_{0F} \]
\[ + \frac{1}{4} \left[ -\chi_{20F} + 2\chi_{21F} + \frac{\chi_{31F}}{2E} - \frac{m^2}{E^2} \chi_{22F} \right]. \]
The structure of the functions $\chi_{iF}$ and $\zeta_{iF}$ is exactly the same as their counterparts for the TBR. To obtain them explicitly all one needs to do is replace the $\theta_i$ and $\eta_i$ that appear in the latter by $\theta_{iF}$ and $\eta_{iF}$. The $\theta_{iF}$ were already given in Appendix B of Ref. [2]. The $\eta_{iF}$ are new. Before performing the last analytical integration they are given by

$$\eta_{0F} = \int_{-1}^{1} dy,$$  \hfill (23)

$$\eta_{1F} = \int_{-1}^{1} dy \frac{1}{G(y)},$$  \hfill (24)

$$\eta_{(2+j)F} = \int_{-1}^{1} dy [G(y)]^{1/2-j} \ln \left[ \frac{E^0_\nu + [G(y)]^{1/2}}{E^0_\nu - [G(y)]^{1/2}} \right],$$  \hfill (25)

where $j = 0, 1, 2$ and

$$G(y) = E^0_\nu + 2p_2l(y - y_0).$$  \hfill (26)

After performing the $y$ integration, their explicit forms are

$$\eta_{1F} = \frac{1}{2p_2l} \ln \left[ \frac{(p_2 + l)^2}{(p_2 - l)^2} \right],$$  \hfill (27)

$$\eta_{2F} = \frac{1}{3p_2l} \left\{ 4E^0_\nu p_2l + E^0_\nu \ln \left[ \frac{y_0 - 1}{y_0 + 1} \right] + (p_2 + l)^3 \ln \left[ \frac{E^0_\nu + p_2 + l}{E^0_\nu - p_2 - l} \right] - (p_2 - l)^3 \ln \left[ \frac{E^0_\nu + p_2 - l}{E^0_\nu - p_2 + l} \right] \right\},$$  \hfill (28)

$$\eta_{3F} = \frac{1}{p_2l} \left\{ E^0_\nu \ln \left[ \frac{y_0 - 1}{y_0 + 1} \right] + (p_2 + l) \ln \left[ \frac{E^0_\nu + p_2 + l}{E^0_\nu - p_2 - l} \right] - (p_2 - l) \ln \left[ \frac{E^0_\nu + p_2 - l}{E^0_\nu - p_2 + l} \right] \right\},$$  \hfill (29)

$$\eta_{4F} = \frac{1}{E^0_\nu p_2l} \left\{ \ln \left[ \frac{(p_2 + l)^2}{(p_2 - l)^2} \right] - \ln \left[ \frac{y_0 - 1}{y_0 + 1} \right] + \frac{E^0_\nu}{p_2 - l} \ln \left[ \frac{E^0_\nu + p_2 - l}{E^0_\nu - p_2 + l} \right] - \frac{E^0_\nu}{p_2 + l} \ln \left[ \frac{E^0_\nu + p_2 + l}{E^0_\nu - p_2 - l} \right] \right\}.$$  \hfill (30)

At this point it is convenient to mention that all the $\rho_{iF}$ functions are convergent when $p_2 \to 0$. Therefore, the value of the bremsstrahlung RC when $E_2 \to M_2$ is finite and their numerical evaluations present no problems.

Following Eq. (12), the FBR analytical bremsstrahlung differential decay rate of decaying polarized hyperons reads
\[ d\Gamma_{FB}^T = \frac{\alpha}{\pi} d\Omega \left[ \Phi_1 - \mathbf{s}_1 \cdot \mathbf{p}_2 \Phi_2 \right], \quad (31) \]

with

\[ \Phi_1 = A'_{I0} \left( E, E_2 \right) + \left( D_1 + D_2 \right) \left( \theta'_F + \theta''_F \right) + D_2 \left( \theta''_F + \theta'''_F \right), \quad (32) \]
\[ \Phi_2 = A'_{I0} \left( E, E_2 \right) + D_3 \left( \rho_1 + \rho_3 \right) + D_4 \left( \rho_2 + \rho_4 \right). \quad (33) \]

The complete analytical RC to the DP of polarized decaying hyperons to order \( \alpha \), in the approximation of neglecting terms of order \( \alpha q/\pi M_1 \), is obtained by adding \( d\Gamma_{FB}^T \) and \( d\Gamma_{FB}^B \), Eqs. (11) and (31). The final expression is

\[ d\Gamma_{TOT} = \frac{G^2_F}{2} \frac{dE_2}{dE} \frac{d\Omega}{(2\pi)^4} 2M_1 \left\{ A'_0 + \frac{\alpha}{\pi} \left( \Phi_1 + \Phi_1 \right) - \mathbf{s}_1 \cdot \mathbf{p}_2 \left[ A''_0 + \frac{\alpha}{\pi} \left( \Phi_2 + \Phi_2 \right) \right] \right\}. \quad (34) \]

This is the analytical counterpart of the DP of Eq. (16) and our second main result. In the next section we will use this Eq. (34) in order to obtain the spin-asymmetry coefficient of the emitted baryon, \( \alpha_B \).

Equations (16) and (34) were obtained for the case when the emitted lepton \( \ell \) is negatively charged. The expressions for the case when \( \ell \) is positively charged are obtained \[4\] by changing the sign of each axial form factor \( g_i \) (\( i = 1, 2, 3 \)) and by reversing the sign in front of \( \mathbf{s}_1 \cdot \mathbf{p}_2 \), in these equations.

**V. SPIN-ASYMMETRY COEFFICIENT \( \alpha_B \)**

Here we will discuss the total RC of order \( \alpha \) to the spin-asymmetry coefficient \( \alpha_B \) of the outgoing baryon. For this purpose, we will use the complete DP with RC, Eq. (34), in order to calculate the quantities \( N^\pm \) in terms of which \( \alpha_B \) is defined, namely,

\[ \alpha_B = 2 \frac{N^+ - N^-}{N^+ + N^-}. \quad (35) \]

\( N^+ \) (\( N^- \)) denotes the number of baryons with momenta \( \mathbf{p}_2 \) emitted in the forward (backward) hemisphere with respect to the polarization of the decaying hyperon. Thus, \( \alpha_B \) can be written as

\[ \alpha_B^T = - \frac{B_2 + (\alpha/\pi) (a_2 + a_{2F})}{B_1 + (\alpha/\pi) (a_1 + a_{1F})}. \quad (36) \]

In this equation the superscript \( T \) (for total) on \( \alpha_B \) indicates that the contributions of both the TBR and the FBR are taken into account. \( B_2 \) and \( B_1 \) are given by Eqs. (109) and (108) of Ref. \[1\]. The RC to the spin-asymmetry parameter corresponding only to the TBR are obtained by setting \( a_{1F} = a_{2F} = 0 \) in Eq. (36). In this case the \( \alpha_B \) parameter is

\[ \alpha_B^R = - \frac{B_2 + (\alpha/\pi) a_2}{B_1 + (\alpha/\pi) a_1}, \quad (37) \]

where the superscript \( R \) (for restricted) denotes the TBR contribution only. The uncorrected angular spin-asymmetry coefficient of the emitted hyperon is simply given by
\[ \alpha_B^0 = -\frac{B_2}{B_1}. \] (38)

\( a_1 \) and \( a_2 \) are defined in Eqs. (112) and (113) of Ref. [4]. \( a_{1F} \) and \( a_{2F} \) are the new FBR contributions to the RC. Explicitly, they are defined using Eq. (31) as

\[ a_{iF} = \int_{E_m}^{E_B} \int_{E_2^{-}}^{E_2^{+}} \Phi_{iF} dE_2 dE, \] (39)

where \( i = 1, 2 \). The kinematical bounds of the FBR are \( E_B \) and \( E_2^{-} \) and they are given in Eqs. (20) and (16) of Ref. [2], respectively.

Equation (38) can be further expanded and can be rewritten in such a way that only terms of order \( \alpha \) appear. Neglecting terms of order \( \alpha q/\pi M_1 \) we rearrange \( \alpha_B^T \) as

\[ \alpha_B^T = \alpha_B^0 \left[ 1 + \frac{\alpha}{\pi} \left( \frac{a_2 + a_{2F}}{B_2(0)} - \frac{a_1 + a_{1F}}{B_1(0)} \right) \right], \] (40)

with

\[ B_i(0) = \int_{E_m}^{E_B} \int_{E_2^{-}}^{E_2^{+}} A_i' dE_2 dE. \] (41)

\( E_m \) is the maximum energy of the electron and \( E_2^{+} \) is the upper boundary of the TBR in the DP. Their explicit forms appear in Eqs. (17) and (16), respectively, of Ref. [2].

A word of caution is necessary here. Equation (40) may be employed provided \( |B_2 - B_2(0)| \ll |B_2(0)| \). It may happen that this condition is not met when certain values of the leading form factors are assumed. This anomalous situation occurs when \( f_1(0) \approx 0 \), as is the case in \( \Sigma^\pm \to \Lambda e\bar{\nu} \). In fact, one can show either analytically or numerically that \( B_2(0) \approx 0 \) when \( f_1(0) \approx 0 \) and, accordingly, Eq. (40) becomes ill-defined. When this occurs we should only use the unexpanded version Eq. (36). When \( f_1(0) \) is appreciably large, the results obtained with Eqs. (40) and (36) are acceptable within our approximations. In case of doubt it is safer to simply use Eq. (36).

In Ref. [1] we only obtained \( \alpha_B \) corresponding to the TBR. With the addition of the terms \( a_{1F} \) and \( a_{2F} \) we can now consider the photons of the FBR without assuming them to be experimentally discriminated and to appreciate the relevance of their contribution to the RC. In the next section we shall display several numerical evaluations to illustrate this, both at the level of the DP and at the level of \( \alpha_B^T \).

VI. NUMERICAL RESULTS

In this section we shall perform numerical evaluations of the RC. We have two purposes in mind. One is to make an internal check of our results and the other one is to compare with numerical results available in the literature [3].

The internal check consists of performing numerically the photon triple integrals of Eq. (16) and comparing them with the analytical result of Eq. (34). This comparison is made over a lattice of points \((E, E_2)\) of the complete DP. At the same time, the choice of this lattice is made so as to be able to compare with the numbers of Ref. [3]. These detailed
comparisons will be made specifically for the decays \( \Lambda \rightarrow pe\bar{\nu} \) and \( \Sigma^- \rightarrow ne\bar{\nu} \). It is then necessary that we adopt here the definitions introduced in Ref. [3] and to take the same values of the corresponding form factors. Accordingly, we introduce the two-dimensional function

\[
\delta \alpha^R_B(E, E_2) = \alpha^R_B(E, E_2) - \alpha^0_B(E, E_2). \tag{42}
\]

\( \alpha^R_B(E, E_2) \) and \( \alpha^0_B(E, E_2) \) are defined as in Eq. (35), but this time without integrating over \( E \) and \( E_2 \) (i.e. integrating only over \( d\Omega_2 \)). The upper index \( R \) has the same meaning as in Sec. V.

When the photon triple integration is to be performed numerically \( \alpha^R_B(E, E_2) \) is explicitly given, according to our discussions of Sec. II, by

\[
\alpha^R_B(E, E_2) = -\frac{A'_0 + (\alpha/\pi)\Phi_2}{A'_0 + (\alpha/\pi)\Phi_1}, \tag{43}
\]

where

\[
\Psi_1 = A'_1(\phi + \theta_1) + A''_1(\theta_1) + \frac{p_2 l}{4\pi} \int_{-1}^{1} dx \int_{-1}^{1} dy \int_{0}^{2\pi} d\varphi_k \left[ |M'|^2 + |M''|^2 \right], \tag{44}
\]

and

\[
\Psi_2 = A'_2(\phi + \theta_1) + A''_2(\theta_1) + \frac{p_2 l}{4\pi} \int_{-1}^{1} dx \int_{-1}^{1} dy \int_{0}^{2\pi} d\varphi_k \left[ |N''|^2 + |NIV|^2 \right]. \tag{45}
\]

The numerical values obtained from our analytical result use

\[
\alpha^R_B(E, E_2) = -\frac{A'_0 + (\alpha/\pi)\Phi_2}{A'_0 + (\alpha/\pi)\Phi_1}, \tag{46}
\]

where \( \Phi_1 \) and \( \Phi_2 \) are given in Eq. (11) of Sec. II. For the FBR, when the photon triple integration is to be performed numerically, we introduce the definition

\[
\delta \alpha^F_B(E, E_2) = -\frac{\Psi_2^F}{\Psi_1^F}, \tag{47}
\]

where

\[
\Psi_1^F = A'_I I_0^F(E, E_2) + \frac{p_2 l}{4\pi} \int_{-1}^{1} dx \int_{-1}^{1} dy \int_{0}^{2\pi} d\varphi_k \left[ |M'|^2 + |M''|^2 \right], \tag{48}
\]

and

\[
\Psi_2^F = A'_I I_0^F(E, E_2) + \frac{p_2 l}{4\pi} \int_{-1}^{1} dx \int_{-1}^{1} dy \int_{0}^{2\pi} d\varphi_k \left[ |N''|^2 + |NIV|^2 \right]. \tag{49}
\]

The numerical values obtained for the analytical result for the FBR use the definition

\[
\delta \alpha^F_B(E, E_2) = -\frac{\Phi_2^F}{\Phi_1^F}, \tag{50}
\]

where \( \Phi_2^F \) and \( \Phi_1^F \) are given in Eqs. (32) and (33), respectively.
For $\Lambda \to p e \bar{\nu}$ and $\Sigma^- \to n e \bar{\nu}$ the numerical results are displayed in Tables I and II, respectively. The lattices in these tables are given in terms of $\delta = E/E_m$ and $\sigma = E_2/M_1$. In Tables I(a) and II(a) we display the values obtained with Eqs. (43) and (47). In Tables I(b) and II(b) we display the values obtained with Eqs. (46) and (50). In Tables I(c) and II(c) the numbers of Ref. [3] are displayed.

The internal cross-check in Tables I(a)–II(a) and I(b)–II(b) is very good. The comparison with Ref. [3] is quite acceptable. Some minor differences can be observed, but they can reasonably attributed to the difference in approximations, i.e., within our approximations this last comparison with Ref. [3] is satisfactory.

As a last step, in Table III we display the values of the totally integrated spin-asymmetry coefficient $\alpha_B$ for several processes of interest, namely, $n \to p e \bar{\nu}$, $\Lambda \to p e \bar{\nu}$, $\Sigma^- \to n e \bar{\nu}$, $\Sigma^- \to \Lambda e \bar{\nu}$, $\Sigma^+ \to \Lambda e \bar{\nu}$, $\Xi^- \to \Sigma^0 e \bar{\nu}$, $\Xi^- \to \Sigma^+ e \bar{\nu}$, and $\Lambda_c^+ \to \Lambda e^+ \nu$. The values of the form factors used are those given in Ref. [2]. In the second column of this table we display the uncorrected coefficient $\alpha^0_B$. In the third column we list the correction to this coefficient defined as

$$\delta \alpha^R_B = \alpha^R_B - \alpha^0_B.$$  \hspace{1cm} (51)

In the next column we list the radiatively corrected $\alpha_B$ for the complete DP, which is analogously defined as

$$\delta \alpha^T_B = \alpha^T_B - \alpha^0_B.$$  \hspace{1cm} (52)

In order to compare with our results in the last column we display the values of $\delta \alpha^T_B$ reported in Ref. [3].

From Table III we can appreciate, by comparing the third and the fourth columns, that the inclusion of the FBR is important. In general it reduces the total radiative corrections. It may even be that the values in the third column are one order of magnitude larger than the corresponding ones of the fourth column. Therefore, there is an important difference between $\alpha^R_B$ and $\alpha^T_B$. From this Table III we can see that there is an acceptable agreement between our $\delta \alpha^T_B$ and the one of Ref. [3] for the two decays reported there.

**VII. CONCLUSIONS**

In this paper we have calculated the RC to the emitted baryon angular distribution w.r.t. the spin of the decaying baryon of HSD, without the restriction imposed in Ref. [1]. This restriction was, that bremsstrahlung photons be experimentally discriminated either by direct detection or indirectly by energy-momentum conservation.

It proved to be convenient to recast the results of Ref. [1] in close parallelism with Ref. [2], where the emitted charged-lepton angular distribution w.r.t. the spin of the decaying baryon was studied and the above restriction was not imposed either. This facilitated our task greatly in two respects. First, it gave us the differential decay rate with RC of the TBR of Ref. [1] in a form that it could be extended to incorporate the previously discriminated photons of the FBR by simply replacing the limits of integration over the real photon three-momentum. Second, it allowed us to express our analytical results using in as-much-as
possible expressions already obtained in Ref. \[2\], and thus considerably reducing the number of new analytical integrals.

Accordingly, our main result has two very compact forms, given in Eqs. (16) and (34). In the first one the integrations over the photon three-momentum are explicitly indicated and can easily be performed numerically. In the second one all such integrations were performed and a complete analytical result is obtained.

As an application we computed the RC to $\Lambda \rightarrow p e^+ \nu$ and $\Sigma^- \rightarrow n e^+ \nu$ over a detailed lattice of points covering the TBR and FBR of the DP of these decays. The results are displayed in Tables I and II, respectively. In these tables we exhibited an internal cross-check of Eqs. (16) and (34) and a comparison with numerical results published in the literature \[3\]. The comparisons are satisfactory. In addition, we calculated the RC to the total asymmetry-coefficient of the emitted baryon for nine decays, including the charm-baryon decay $\Lambda_c^+ \rightarrow \Lambda e^+ \nu$. The results are displayed in Table III. Here we separated the contributions of TBR from the RC including also the FBR contributions, and we also compared with the results for two decays given in Ref. \[3\]. This last is also satisfactory within our approximations. The contributions of the FBR photons to the RC are, generally speaking, quite appreciable and, in some cases, even reverse the sign of the total RC.

Our results are useful for a model-independent experimental analysis. They are reliable up to a precision of around 0.5% and, thus, are useful for experiments involving several thousands of events. For high statistics experiments with several hundreds of thousands of events or for decays involving charm baryons, such as $\Lambda_c^+ \rightarrow \Lambda e^+ \nu$, or even heavier quarks, our Eqs. (16) and (34) provide a good first approximation. To improve the precision of our formulas it becomes necessary to include terms of order $\alpha q/\pi M_1$. This can be done still in a model-independent way by extending the general analysis of Ref. \[5\] for the virtual RC and by use of the Low theorem \[6\] for the bremsstrahlung photons.

We should make a few more remarks. Our results are valid for both neutral and charged polarized decaying hyperons and whether the emitted-charged lepton is an electron or a muon. If this lepton is positively-charged our formulas are also applicable provided the sign of the $g_i$ form factors and the sign in front of the $\hat{s}_1 \cdot \hat{p}_2$ correlation are all reversed \[4\]. This rule applies equally well to total asymmetry coefficients $\alpha^T_B$, $\alpha^R_B$, and $\alpha^0_B$. Finally, let us mention that in a Monte Carlo analysis the analytical result Eq. (34) represents a considerable advantage, because the triple photon integration does not have to be repeated every time the values of $f_1$ and $g_1$, or $E$ and $E_2$ are changed. This leads to a considerable simplification of the experimental Monte Carlo simulation.

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### TABLE I. Percentage RC \( \delta \alpha_{B}(E, E_{2}) \) over the TBR and \( \delta \alpha_{F}(E, E_{2}) \) over the FBR in \( \Lambda \to p e\bar{\tau} \).

The entries corresponding to the latter are marked with bold-face characters. The energies \( E_{2} \) and \( E \) are replaced by \( \sigma = E_{2}/M_{1} \) and \( \delta = E/E_{m} \), respectively. (a) gives the results of the numerical integrations Eqs. (43) and (47), (b) gives the results of the analytical formulas Eqs. (46) and (50), and (c) gives the results of Ref. [3]. In each column we provide the kinematical limits on \( \sigma \) in the TBR in terms of \( \sigma_{\text{max}} \) and \( \sigma_{\text{min}} \).

| \( \sigma \) | \( \delta \) | \( \sigma_{\text{max}} \) | \( \sigma_{\text{min}} \) |
|------------|------------|-----------------|-----------------|
| 0.8529     | 0.2        | 0.0             | 0.0             |
| 0.8517     | 1.4        | 0.2             | 0.1             |
| 0.8504     | -89.8      | 0.5             | 0.3             |
| 0.8492     | -85.4      | 1.0             | 0.6             |
| 0.8479     | -80.3      | -93.1           | 0.9             |
| 0.8466     | -74.1      | -81.8           | 1.3             |
| 0.8454     | -66.6      | -73.2           | 1.9             |
| 0.8441     | -57.3      | -63.0           | -74.5           |
| 0.8429     | -45.1      | -49.7           | -57.8           |
| 0.8416     | -26.5      | -29.2           | -33.8           |
| \( \delta \) | 0.0500     | 0.1500          | 0.2500          |
| 0.8530     | 0.2        | 0.1             | 0.1             |
| 0.8518     | 1.9        | 0.4             | 0.3             |
| 0.8505     | -84.2      | 0.8             | 0.6             |
| 0.8493     | -77.6      | 1.4             | 0.9             |
| 0.8480     | -70.5      | 2.4             | 1.3             |
| 0.8467     | -62.8      | -72.1           | 1.7             |
| 0.8455     | -54.4      | -62.2           | 2.3             |
| 0.8442     | -45.2      | -51.7           | -64.6           |
| 0.8429     | -34.5      | -39.5           | -47.8           |
| 0.8417     | -20.2      | -23.1           | -27.1           |

| \( \sigma_{\text{max}} \) | \( \sigma_{\text{min}} \) |
|-----------------------------|-----------------------------|
| 0.8536                      | 0.8516                      |
| 0.8536                      | 0.8479                      |
| 0.8536                      | 0.8450                      |
| 0.8536                      | 0.8428                      |
| 0.8536                      | 0.8414                      |
| 0.8536                      | 0.8416                      |
| 0.8536                      | 0.8433                      |
| 0.8536                      | 0.8464                      |
| 0.8536                      | 0.8508                      |
TABLE II. Everything here is as explained in the caption of Table I, except that the decay studied is now $\Sigma^- \to n\nu$.

| $\delta$ | 0.0500 | 0.1500 | 0.2500 | 0.3500 | 0.4500 | 0.5500 | 0.6500 | 0.7500 | 0.8500 | 0.9500 |
|----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.8067   | 0.6    | 0.1    | 0.0    | 0.0    | 0.0    | 0.0    | 0.0    | 0.0    | 0.0    | 0.1    |
| 0.8044   | 66.0   | 0.3    | 0.1    | 0.0    | 0.0    | 0.0    | 0.0    | 0.0    | 0.1    | 0.3    |
| 0.8020   | 54.3   | 1.1    | 0.2    | 0.1    | 0.0    | 0.0    | 0.0    | 0.0    | 0.1    | 0.1    |
| 0.7997   | 55.0   | 31.1   | 0.6    | 0.3    | 0.2    | 0.1    | 0.1    | 0.1    | 0.1    | 0.1    |
| 0.7950   | 54.5   | 43.3   | 1.1    | 0.5    | 0.3    | 0.2    | 0.1    | 0.1    | 0.1    | 0.1    |
| 0.7912   | 53.1   | 45.4   | 20.3   | 0.9    | 0.4    | 0.2    | 0.1    | 0.1    | 0.1    | 0.1    |
| 0.7884   | 44.1   | 40.1   | 31.8   | -4.9   | 3.2    | 1.1    | 0.3    | 0.3    | 0.3    | 0.3    |
| 0.7858   | 29.4   | 27.3   | 23.2   | 15.2   | 9.7    | 2.2    | 0.2    | 0.2    | 0.2    | 0.2    |

| $\sigma_{\text{max}}$ | 0.8078 | 0.8078 | 0.8078 | 0.8078 | 0.8078 | 0.8078 | 0.8078 | 0.8078 | 0.8078 | 0.8078 |
| $\sigma_{\text{min}}$ | 0.8043 | 0.7978 | 0.7925 | 0.7884 | 0.7857 | 0.7846 | 0.7854 | 0.7884 | 0.7938 | 0.8023 |

(a)

(b)
TABLE III. Percentage RC to the total spin-asymmetry coefficient of the emitted baryon for nine HSD. RC of the TBR have been separated, in the third column, from the total RC including the FBR photons displayed in the fourth column. In these calculations the analytical result Eq. (34) was employed. The last column reproduces the numerical results of Ref. [3].

| Decay       | $\alpha^0_B$ | $\delta\alpha^R_B = \alpha^R_B - \alpha^0_B$ | $\delta\alpha^T_B = \alpha^T_B - \alpha^0_B$ | $\delta\alpha^T_B$ Ref. [3] |
|-------------|--------------|---------------------------------|---------------------------------|------------------|
| $n \to p$   | -47.92       | -0.28                           | -0.26                           | -0.1             |
| $\Lambda \to p$ | -58.60       | -0.20                           | -0.26                           | -0.0             |
| $\Sigma^- \to n$ | 66.73        | 0.12                            | -0.03                           | -0.0             |
| $\Sigma^- \to \Lambda$ | 7.24         | 0.12                            | -0.12                           |                  |
| $\Sigma^+ \to \Lambda$ | 6.59         | -0.05                           | 0.10                            |                  |
| $\Xi^- \to \Lambda$ | -54.72       | 0.04                            | -0.01                           |                  |
| $\Xi^- \to \Sigma^0$ | -45.87       | -0.01                           | -0.09                           |                  |
| $\Xi^0 \to \Sigma^+$ | -46.15       | -0.16                           | -0.23                           |                  |
| $\Lambda^+_c \to \Lambda$ | -31.06       | -0.93                           | 0.12                            |                  |