Destabilization of inertio-elastic mode via spatiotemporal intermittency in a Couette-Taylor viscoelastic flow

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Abstract. We have characterized the transition to turbulence in a flow with semi-dilute solutions (concentration of 0.07%) of polyethylene oxide (PEO) in the Couette-Taylor system with rotating only the inner cylinder. The first instability mode occurs via a supercritical Hopf bifurcation in form of interacting right and left spirals. For higher values of the rotation velocity, turbulent spots appear in the flow and coexist with the spirals. This spatio-temporal intermittency (STI) regime has been characterized by the turbulent fraction and the statistical properties of the laminar domains.

1. Introduction

Since 1949 when Toms B. A. discovered the drag reduction in turbulent flows by the addition of small quantity of the polymer [1, 2] with high molar mass, the viscoelastic flows have attracted much attention of the scientific community. In fact, these flows have many industrial applications such as the increase of the rate flow of oil in the long-distance pipelines or the improvement of thermal properties of some liquids. In viscoelastic flows, when the polymer concentration exceeds a critical value, the elastic shear stress can destabilize the flow and lead to turbulence even for very weak values of the Reynolds number [3-5]. To investigate the elastic effects, the Couette-Taylor system has been proved to be the best candidate because of its geometrical simplicity (a closed flow system) and the availability, for the Newtonian flows, of many results that can serve as reference for comparison [6].

When only the inner cylinder is rotating, the Couette-Taylor flow of a polymer solution can be characterized by 3 control parameters: the Taylor number $Ta$, the elasticity number $E = \lambda / \tau$ and the viscosity ratio $\eta_p/\eta_s$. Here $\lambda$ and $\tau$ are the relaxation time of the viscoelastic solution and its viscous diffusion time respectively. The elasticity number represents the relative importance between elastic and inertia effects. It is the property of the flow system and can be considered as the analog of the Prandtl number for thermal convection. For small values of the elasticity $E < E_{c1}$, the elasticity effects are weak compared to the inertial effects and the first two instability modes are the same as for a Newtonian liquid [7]. For a large elasticity $E > E_{c2}$, the elasticity effects dominate the inertia and the circular Couette flow can bifurcate to instationary large rolls at small values of $Ta$ [3-5, 8]. For intermediate values of elasticity $E_{c1} < E < E_{c2}$, the elastic and inertial effects are comparable and the flow becomes unstable to inertio-elastic modes that appears in form of either spirals or ribbons [5,7,8]. For a viscoelastic solution of polyethylene oxide (PEO) with concentration $c \in [500, 700]$ ppm (parts...
per million in mass) which corresponds to $0.01 < E < 0.1$, the circular Couette flow bifurcates to counterpropagating spirals (SPI) [8]. The increase of the rotation speed of the inner cylinder reinforces the inertio-elastic effect leading to the spatiotemporal intermittency (STI) characterized by the coexistence of turbulent spots and laminar zones (spirals) for the same value of the control parameter. When we continue to increase the speed rotation of the inner cylinder, the spirals disappear and the flow becomes quasi-turbulent. To our best knowledge, this regime has not been reported so far in the case of polymer solution in the Couette-Taylor system with a fixed outer cylinder. In Newtonian flows, spatio-temporal intermittency has been observed in the Couette-Taylor system with counter-rotating cylinders [9, 10]. In this paper, we present the results of an experiment performed in the Couette-Taylor system with a fixed outer cylinder for a moderate concentration $c = 700$ ppm = 0.07% corresponding to a moderate value of elasticity number $E = 0.076$. We describe in details the properties of spatiotemporal intermittency by providing the data obtained from measurements of the turbulent fraction, the characteristic length and time of the laminar zones and turbulent spots. In the next section we describe the experimental setup. The results are presented and discussed in the section 3 and the last section is the conclusion.

2. Experimental setup
The experimental setup consists of two coaxial horizontal cylinders that may rotate independently. The inner cylinder is made of black Delrin with a radius $a = 4.46$ cm. The outer cylinder is made of Plexiglass with a radius $b = 5.05$ cm. The gap size between the cylinders is $d = b - a = 0.59$ cm and has a working length $L = 27.5$ cm. The radius ratio is $a/b = 0.883$ and the aspect ratio is $\Gamma = L/d = 46.6$. In our experiment, the outer cylinder is fixed, while the inner cylinder is driven by a DC servomotor at the angular frequency $\Omega = 2\pi f$ where $f$ is varied by step of 0.001 Hz each 20 minutes. We will use the average shear rate in the flow $\dot{\gamma} = \Omega a/d$. Polymer solutions were prepared by mixing an initial suspension of polyethylene oxide (PEO, Aldrich, $8 \cdot 10^6$ g/mol) in 760 ml of water with 40 ml of isopropyl alcohol. The concentration of PEO in the resulting solution is 700 ppm per weight. The solution is maintained at rest 5 days at 4°C and then 14 h at room temperature. A final homogenization is performed with a magnetic shaker. Viscosity measurements were performed with a rheometer AR2000 (TA Instruments). The viscosity curves for 3 concentrations of polymer solutions (figure 1) exhibit a shear-thinning behavior, i.e. the viscosity decreases when the shear rate increases. For flow visualization, 2% of Kalliroscope AQ 1000 (a dilute suspension of highly anisotropic reflective platelets in a mixture of propylene glycol and water) has been added to the solution. From viscosity measurements, we have not observed any significant change of the solution viscosity which would have been induced by Kalliroscope platelets. A linear 1024-pixels CCD camera recorded the reflected light intensity $I(x)$, from a line of 24.2 cm of length along the axial direction, with 8-bits sampling. Recorded lines at regular intervals of 0.2 s were stacked together to provide space-time diagrams $I(x,t)$ of flow patterns. Total acquisition time can last 27 minutes.

| $c$ (ppm) | $\eta_0$ (mPa.s) | $\tau$ (s) | $\lambda$ (s) | $N$ | $S$ |
|-----------|-----------------|------------|-------------|----|----|
| 500       | 5.13            | 0.12       | 0.09        | 0.156 | 1.12 |
| 600       | 7.40            | 0.16       | 0.12        | 0.196 | 1.54 |
| 700       | 10.21           | 0.29       | 0.23        | 0.21  | 1.56 |

Table 1. Carreau characteristics of the PEO solutions obtained from viscosity curves (figure 1).
The flow is characterized by two dimensionless parameters: the viscosity ratio \( S = \eta_p(\dot{\gamma}) / \eta_s = (\eta_p(\dot{\gamma}) - \eta) / \eta_s \), the Taylor number given by \( Ta = \frac{\rho \dot{\gamma} d^2}{\eta(\dot{\gamma})} \left( \frac{d}{a} \right)^{1/2} \) and the elasticity number \( E = \lambda / \tau_v = \lambda \eta(\dot{\gamma}) / \rho d^2 \). Here \( \rho \) is the density of the polymer solution, \( \lambda \) is its relaxation time and \( \tau_v \) is its viscous diffusion time. The relaxation time of the polymer solution was determined from the viscosity curve fitted using Carreau law \( \eta(\dot{\gamma}) = \eta_0 \left[ 1 + (\dot{\gamma} \tau)^n \right]^{\lambda} \) where \( \eta_0 \) is the shear viscosity for vanishing shear rate, \( \tau \) is the characteristic time and \( n \) the power law index for shear-thinning. The relaxation time \( \lambda \) of the solution is determined from the viscosity curve parameters as follows \[ 2, 11 \] \( \lambda = \tau \sqrt{(0.95)^{2/n} - 1} \). The data from rheology measurements are given in Table 1. The elasticity number varies very weakly with the values of the shear rate used in our experiments, so that it will be considered constant for a given concentration.

3. Results
The different states observed in our experiment with a concentration of 700 ppm of PEO corresponding to \( E = 0.076 \) and \( S = 1.56 \) are shown in figure 2. We should mention that the elasticity number has a negligible variation with the shear rate in the range of the values of \( \dot{\gamma} \) used in our experiment. The base flow becomes unstable for \( Ta = 35 \) leading to a pattern of travelling waves at the edges of the flow system. When \( Ta \) is increased, these travelling waves fill the whole system and results in a pattern of counter-propagating spirals or ribbons (figure 3-a). For larger values of \( Ta \), the pattern may exhibit spatio-temporal-propagating defects resulting from the local disappearance or creation of one or a pair of spirals. For \( Ta = 42 \), the number of these defects has increased and the turbulent spots appear in some area of the flow pattern (figure 3-b). The number of spots increases with increasing \( Ta \) as long as there is no connection between them. The coexistence, for the same value of the control parameters, of these turbulent spots with area of regular laminar pattern is known as the spatio-temporal intermittency (STI) \[ 12 \]. A further increase of \( Ta \) leads to a completely chaotic state that has not been characterized in our experiment (figure 3-c).
**Figure 2.** State diagram for the solution with polyethylenoxide of 700 ppm ($E = 0.076$) for different values of $Ta$.

| Couette flow | Ribbons | Ribbons+Spots = STI | Unexplored |
|--------------|---------|---------------------|------------|
|              | 35      |                     | 102        |

**Figure 3.** Space-time Diagrams of spirals pattern of inertio-elastic instability for a solution with 700 ppm; (a): $Ta = 36$, (b): $Ta = 47$, (c): $Ta = 95$.

**Figure 4.** Space time diagram of the pattern for $Ta = 47$: a) raw data, amplitude of the pattern extracted by complex demodulation; (c) digitized amplitudes (white: laminar, black: turbulent).
In order to investigate the dynamics of the turbulent spots, we first have performed the complex demodulation of the space-time diagram of the whole pattern (figure 4), then the obtained amplitude has been digitized. The digitization process was performed using a threshold between 0.3 and 0.46 of the maximum amplitude. Below the threshold, all the pixels were set to zero (black pixels) and represent the turbulent spot. The pixels with amplitude above the threshold take the value 1 and represent the laminar zone (white pixels). So, for each value of the control parameter $Ta$, we have measured the number of turbulent spots and the turbulent fraction i.e. the ratio of the area $S_t$ of the turbulence spots to the total area $S$ of the space-time diagram: $F = S_t/S$.

The number of turbulent spots increases for $Ta \in [42; 49]$ and then decreases (figure 5-a). The increase of turbulent spots is due to the multiplication of the turbulent spots with a very weak connection between them. For $Ta > 49$, the number of turbulent spots decreases, because of the strong connection between them. The turbulent fraction (figure 5-b) increases for $Ta \in [42; 49]$ and then reaches a saturation value of about 0.6 for $Ta > 50$. We have fitted the dependence of the turbulent fraction on the control parameter $Ta$ in the growth zone with a power law:

$$F = F_0 \left( (Ta - Ta_i)^{\beta} \right)$$

where $Ta_i = 32.5 \pm 0.5; \beta = 2.41 \pm 0.01; F_0 = 1.54 \times 10^{-2}$. The spatiotemporal temporal fraction $F$ plays the role of the order parameter of the spatiotemporal intermittency.

Figure 5. Variation with the Taylor number $Ta$ of: (a) the number of the turbulent spots, (b) the spatiotemporal turbulent fraction.

We have performed a statistical analysis of the laminar domains distribution in size and duration and we have found that:

- Near the onset of the STI regime i.e. $Ta \in [42; 49]$, the number of laminar domains of size $l$ and duration $t$ decreases with $l$ and $t$ following power laws: $N \sim t^a$ and $N \sim l^b$. Such a regime is called algebraic regime and it can be related to the increase of the number of turbulent spots with a weak connection between them.
- For $Ta \in [50; 60]$, the number of laminar domains of size $l$ or duration $t$ decreases with the size or the duration following an exponential law. So we have $N = N_0 \exp(-l/L_c)$ for the size of laminar domains or $N = N_0 \exp(-t/T_c)$ of their duration. This exponential regime can be explained by the diminution of number of turbulent zones due to their connection [13].
- The characteristic length $L_c$ of the laminar domains can be considered as a coherence length of the laminar domains, it decreases with increasing $Ta$ (for 50 < $Ta < 60$) following a power law (figure 6-a): $L_c = L_0(Ta - Ta_i)^{\alpha_1}$ where $\alpha_1 = 1.96 \pm 0.10, \ Ta_i = 42\pm0.5$ and $L_0 = 176.32$. For 60 < $Ta < 102$, the coherence length of the laminar domains has an asymptotic value of about 1 (figure 6-a).
- For $50 < Ta < 60$, the characteristic time of laminar domains as function of the Taylor number $Ta$ follows a power law: $T_c(Ta, Ta_i) = T_0(Ta - Ta_i)^\alpha$ where $\alpha = 1.49 \pm 0.10$; $Ta_i = 42 \pm 0.5$ and $T_0 = 65$ (figure 6-b).

For $60 < Ta < 102$, $T_c(Ta)$ has an asymptotic value about 1 (figure 6-b).

4. Discussion

The observed scenario to turbulence in viscoelastic flow of the polyethylene oxide with high molar mass is due to the coupling between hydrodynamic nonlinearities and elastic nonlinearities. The values of Weissenberg number given by $Wi = (0.209 \times Ta) \in [8.8, 21.3]$ lie in the region where the elastic effects are significant ($Wi > 1$). To quantify the relative effects of these inertio-elastic coupling, it is convenient to introduce the parameter $K = Wi(d/a)^{1/2} S^{1/2} = E_{Ta} S^{1/2}$. The first instability mode corresponds to $K = 3.53$ and the spatio-temporal instability to $K = 4.23$. The Taylor number accounting for the hydrodynamic nonlinear effects is large enough so that they cannot be neglected. The spatiotemporal intermittency occurs not far from the onset of the primary instability, in fact the corresponding value of the criticality is $\varepsilon = (Ta - Ta_c)/Ta_c = 0.2$. It is worthwhile to remind that for a Newtonian solution in the Couette-Taylor system, the spiral pattern and the spatiotemporal intermittency have been observed only in the case of counter-rotating cylinders [6, 9, 10].

The spatiotemporal intermittency observed in our experiment has critical exponents that are different of those obtained in experiments with Newtonian solutions in different hydrodynamic systems [13-15] or in thermal convection [16-17]. In particular, the turbulent fraction grows faster than that of Newtonian flows.

![Figure 6](image)

**Figure 6.** (a) Coherence length $L_c$ (in units of $d$) and (b) coherence time $T_c$ (in units of $d^2/\nu$) of laminar domains as function of Taylor number $Ta$.

5. Conclusion

In this work, we have characterized the transition to the turbulence in the viscoelastic flow of the PEO solution with a concentration of 700 ppm in the Couette-Taylor system with rotating only the inner cylinder. The primary instability mode of the base Couette flow is formed by pattern of counterpropagating spirals. The increase of the angular velocity of the inner cylinder leads to the spatiotemporal intermittency (STI). The origin of the apparition of the STI regime lies in the interplay between nonlinear mechanisms due to inertial and elastic effects of the solution. The understanding of the mechanism of the STI generation remains an open problem.
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