Nonlinear Dynamics and Stability Analysis of Gas Turbine Rotor System Considering Eccentricity

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Abstract. According to the actual model of the gas turbine rotor system, a double-disc rotor-bearing system model is established. The appropriate parameters are selected, and the integral is solved by numerical integration method. The effects of rotational speed, unbalance and oil film thickness on the dynamics and stability of the rotor system are analyzed by combining illustration, axis trajectory and Poincaré. The results show that the rotor system experiences a single cycle, and the double cycle finally enters a chaotic state; with the increase of the unbalance, the oil film stiffness is improved, and the rotor system become much more stable; and the oil film thickness is too large to increase the oil film whirl and reduce the stability of the system. If the thickness is too small, whirl and oscillation will not occur. Finally, the experiment has been performed, and the experimental results well illustrate the influence of the increase of eccentricity on the stability.

1. Introduction
Rotor stability directly affects the safety of gas turbine rotor operation. As a fault of a rotor system, oil film instability is likely to cause rubbing and other regenerative faults. Therefore, it is of great significance to study the stability, improve the stability of the rotor system to ensure the normal and stable operation of the unit, and optimize the design system. Gas turbine rotor is a typical rotor-bearing system. The stability of sliding bearing-rotor system has been a hot topic in rotor dynamics research. In the retrieved articles, the investigations of stability of the rotor-bearing system are mainly focused on the nonlinear oil-film model. Tiago[1] compared the numerical results for rotors based on nonlinear with linear. Sun[2] et al considered the temperature influence, while temperature made great impacts on the abrasion. Li[3] study a tilting pad bearing, and meanwhile, because of the particularity of turbo-expander, temperature was also considered. Aiming at different locations of unbalance and considering the nonlinear oil film behavior of FRBs, Wang[4] investigate how the unbalance locations affect the nonlinear response characteristics of turbocharger. Sghiri[5] studied the nonlinear stability of a balanced flexible rotor considering the equilibrium point, and found that the progressive decrease of rotor stiffness reduced the stable speed range. Cao[6] et al analyzed the rub-impact and rotor focused on the effects of fractional order damping, while Okabe[7]analyzed the turbulence effect through a simulation of a Jeffcott/Laval rotor supported by tilting pad bearing. Tian[8]calculated the nonlinear bearing force by a new method, investigate the unbalance engine excitation effects. In this paper, a nonlinear oil film force model of the rotor system has been considered under the action of gas turbine rotor. According to this model, it chooses appropriate unbalance, and oil film thickness as the main research object, by the Runge-Kutta method to solve the function. Combined with the bifurcation diagrams and other graphic, it conducts a qualitative analysis of the gas turbine rotor system stability under nonlinear oil film force. It provides the design of rotor bearing system in practice.
2. Double Disc Rotor-bearing Model

Figure 1 is a schematic diagram of a rotor-bearing system of a gas turbine. The rotor is composed of a compressor, a turbine and sliding bearings at both ends.

![Double Disk Rotor-bearing System](image)

**Figure 1.** Double disk rotor-bearing system

2.1 Model of Oil-film Force

This paper studies the stability of gas turbine rotor under the oil film force. The dynamic pressure oil film model adopts Capone model based on the short bearing theory [9]. The data expression of this model is as follows. Herein, \( \mu \) is oil viscosity, \( L \) and \( R \) is the length and radius of bearing, while \( c \) is mounting clearance.

\[
\begin{align*}
F_x &= \sigma f_x \\
F_y &= \sigma f_y \\
\sigma &= \mu \omega RL \left( \frac{R}{c} \right)^2 \left( \frac{L}{2R} \right)^2
\end{align*}
\]

\[
\frac{f_x}{f_y} = -\left[ (x-2\dot{y})^2 + (y+2\dot{x})^2 \right]^{1/2} \times \left[ 3xV(x, y, \alpha) - \sin \alpha G(x, y, \alpha) - 2\cos \alpha S(x, y, \alpha) \right]
\]

\[
= \left[ 3xV(x, y, \alpha) + \cos \alpha G(x, y, \alpha) - 2\sin \alpha S(x, y, \alpha) \right]
\]

Where: \( f_x \) and \( f_y \) is the dimensionless nonlinear oil film force component in the \( x \) direction and \( y \) direction of the bearing, respectively, and \( \sigma \) is the number of sommerfeld.

2.2 Model of Rotor-Bearing System

The gas turbine rotor is a typical double-disk rotor system, where the concentrated masses of bearing are \( m_1 \) and \( m_4 \), and the concentrated masses of compressor and transparency are \( m_2 \) and \( m_3 \). The radial displacement of the bearing is \( x_1 \), \( y_1 \), \( x_4 \) and \( y_4 \), and the radial displacement of the compressor and turbine is \( x_3 \), \( y_3 \), \( x_4 \) and \( y_4 \). Ignoring the gyroscopic moment and torsional vibration. Equation (5) show the describes the nondimensional rotor dynamics equations. Herein, \( \tau = \omega t \), \( \bar{e} = e / c \), \( x_i = X_i / c \), \( y_i = Y_i / c \), \( i = 1, \ldots, 4 \), \( \zeta \) stand for the damping ratio, and \( F_{x1}, F_{y1}, F_{x4}, \text{and} F_{y4} \) is the forces witch produced by the bearing.
The basic parameters of a gas turbine rotor are as follows: the total mass of the rotor is about 16000kg, the mass of the compressor is 12000kg, the mass of the turbine is 4000kg, the working speed is 3000r/min, and the first-order critical speed is 1700r/min. Therefore, the parameters of the gas turbine system are selected, as shown in Table 1.

Table 1. Parameters of gas turbine system

| Parameters                  | Numbers                  |
|-----------------------------|--------------------------|
| Mass $m_i$ (i=1,2,3,4)kg    | 500,12000,4000,460       |
| Equivalent stiffness $K_i$ (i=1,2,3)N/m | 3.78e8,3.63e8,3.73e8     |
| Radius of bearing $R_i$ (i=1,2)m | 0.5,0.45                |
| Length of bearing $L_i$ (i=1,2)m | 0.7,0.63                |
| Mounting clearance c/mm     | 1.75                     |
| Oil viscosity $\mu$ (Pa·s)  | 0.0373                   |
| Damping ratio $\zeta$       | 0.036                    |

3. Nonlinear Analysis

In order to obtain the dynamic characteristics of the gas turbine rotor-bearing system, Runge-Kutta method should be used for numerical simulation to solve the ordinary differential equation (5), with the step size is $2\pi/100$.

3.1. Influence of Rotate Speed

Figure 2 shows the bifurcation diagram of the system response with time when the rotor-bearing system $e_2=0.23$, $e_3=0.25$ and the oil film gap is 1.75mm. As can be seen from the figure, the system mainly went through three stages as the rotation speed increased. When the system speed $\omega_r<1700$ r/min, the system maintains a stable single-period motion, which is shown as an isolated point in the Poincare cross section, and the system is in a stable state. With the increase of rotation speed, when the rotation speed is $1700$ r/min $< \omega_r < 2550$ r/min, the system moves from a single period to a double period, and the bifurcation occurs. At this point, the Poincare cross section is shown as two isolated points. There are vortex frequency, fundamental frequency and multiple frequency in the spectrum diagram, and the vortex frequency is about 0.5 times of the fundamental frequency, and the half-speed oil film vortex appears in the system. When the rotation speed $\omega_r = 2550$ r/min, oil film
vorticity develops into oil film oscillation through accumulation, and Hopf bifurcation occurs in the system. Poincare diagram shows an approximate closed circle, showing the quasi-periodic motion state.

3.2. Influence of Eccentricities

In practice, due to the rotor machining error and installation eccentricity, the rotor is inevitably
unbalanced. The results show that the magnitude of unbalance has a certain influence on the stability of rotor. Figure 6 is the bifurcation diagram when $e_2=0.46$ and $e_3=0.48$.

![Bifurcation diagram at $e_2=0.46$ and $e_3=0.48$](image)

**Figure 6.** Bifurcation diagram at $e_2=0.46$ and $e_3=0.48$

As can be seen from Figure 2 and Figure 6 of the bifurcation diagram, with the increase of eccentricity, the unbalanced force becomes larger. The larger unbalanced force simplifies the motion form of the system, and the bifurcation state of the system becomes simpler. The system is in a stable state under the second critical speed. In addition, it can be seen that after the increase of unbalance, the critical rotational speed of oil film vortexes increases, and the increase of unbalance makes the system instability lag, which indicates that the increase of unbalance improves the stability of the system. This is because when the unbalance increases, the clearance of the bearing oil film decreases, the stiffness of the oil film increases, and the bearing capacity of the bearing increases. This result corresponds to Capone short bearing oil film force model.

### 3.3 Influence of Clearance

The stability of sliding bearing system mainly depends on the stability of oil film, and the thickness of oil film is an essential factor to determine the stability of oil film. Figure 7 shows the bifurcation diagram of the rotor system with corresponding oil film thickness. When the oil film thickness is 1.25mm, the rotational speed of the generated oil film vortexes increases and is not obvious, and the oil film vortex rapidly turns to the oil film oscillation. This can be explained that when the oil film thickness is small, the oil film thickness is insufficient, thus affecting the formation of oil film vortexes. However, when the oil film thickness is 2.75mm, the dynamic characteristics of the rotor become complex, and both oil film vortexes and oil film oscillations occur in advance. And a similar phenomenon occurs when the thickness is 4.25mm. This phenomenon indicates that the larger oil film thickness will reduce the stability of the oil film, so that the oil film vortex intensifies, leading to the reduction of system stability.

![Bifurcation diagram with different clearances](image)

**Figure 7.** Bifurcation diagram with different clearances:
(a) $c=1.25\text{mm}$
(b) $c=2.75\text{mm}$
(c) $c=4.25\text{mm}$

### 4. Experimental Analysis

It is essential to do the experimental analysis. It is effective to correct the deficiency of the theoretical
model through experimental study. Figures 8 to 10 show the whole experimental system. The system consists of a rotor, motor control and the signal collecting. During the experimental research, the response of the system under different eccentricity and rotor speed at $\omega_r = 2000\text{r/min}$, $\omega_r = 4000\text{r/min}$ and $\omega_r = 6000\text{r/min}$ is tested respectively. Eccentricity is realized by the eccentricity mass installed on the disk.

![Motor and rotor system](image)

**Figure 8.** Motor and rotor system

![Signal collector](image)

**Figure 9.** Signal collector

![Motor controller](image)

**Figure 10.** Motor controller

![Response of expander Impeller at 2000r/min](image)

**Figure 11.** Response of expander Impeller at 2000r/min (a) signal of time, (b) frequency (c) trajectory of rotor axis, and (d) Poincaré diagram

![Response of expander Impeller at 4000r/min](image)

**Figure 12.** Response of expander Impeller at 4000r/min (a) signal of time, (b) frequency (c) trajectory of rotor axis, and (d) Poincaré diagram
Figure 13. Response of expander Impeller at 6000r/min (a) signal of time, (b) frequency, (c) trajectory of rotor axis, and (d) Poincaré diagram

Figures 11 to 13 show the system responses at different speeds in the process of speed rising of the non-eccentric rotor. When \( \omega_r = 2000 \text{r/min} \), due to the low rotation speed, stable oil film force has not yet been formed. The axis track diagram presents as a rough oval. The system is dominated by the fundamental frequency, with a small amount of double frequency. When \( \omega_r = 4000 \text{r/min} \), it can be found from the axis track that: after passing the first critical speed, the rotor gradually transits from non-stationary to stationary state, and the axis track presents as a more rounded oval. Poincare chart shows an isolated point. The frequency is based on the fundamental frequency, with a small amount of double frequency. When \( \omega_r = 6000 \text{r/min} \), P2 motion of the rotor can be seen from the axis track diagram. Poincare diagram shows two fuzzy points. In the spectrum diagram, in addition to the fundamental frequency, there are half-frequency and high-frequency multiples, indicating that the system has oil film vorticity at this time.

Figure 14. Response of expander Impeller at 2000r/min (a) signal of time, (b) frequency, (c) trajectory of rotor axis, and (d) Poincaré diagram

Figure 15. Response of expander Impeller at 4000r/min (a) signal of time, (b) frequency, (c) trajectory of rotor axis, and (d) Poincaré diagram
The eccentric mass block with a mass of 0.6351g was mounted on the disc for experiment. Figures 14 to 16 show the response of a rotor system with an eccentric mass of 0.6351g in the acoustic velocity process. Compared with Figure 11-13, it can be seen that with the increase of eccentricity, the dynamic characteristics of the rotor system change greatly. When $\omega_r=2000$ rpm, the axis trajectory is rougher, and the half-frequency amplitude in the frequency is significantly increased, indicating that the excessive eccentricity has inhibited the formation of the dynamic pressure oil film of the sliding bearing. When the speed rises to 4000 rpm, the system runs stably, but the motion amplitude increases significantly. When the rotation speed rises to 6000 rpm, an obvious beat vibration phenomenon appears in the time-domain waveform. Poincare chart now shows two isolated points. In the spectrum diagram, except the base frequency, there are half frequency and double frequency of higher amplitude. This result corresponds to the fact that the increase of unbalance in the simulation results will improve the stability of the rotor system and affect the formation of oil film vortexes. The results show that too much unbalance will seriously affect the formation of bearing dynamic pressure oil film, making the rotor movement more complex. Blind increase of unbalance will not improve the stability of the rotor, but affect the formation of bearing dynamic pressure oil film, and even cause damage to the rotor.

5. Conclusion
In this paper, a dual-disc rotor-sliding bearing model is adopted to analyze the influence of rotor unbalance and oil film thickness on the stability of the rotor subsystem in the process of rotor speed rise, and the following conclusions are drawn:

(1) In the process of rising speed, the gas turbine rotor system goes through a single period, and the period doubling finally enters a chaotic state. After the system passes the first critical speed, the oil film vortexes accumulate continuously until the oil film oscillates.

(2) The increase of unbalance will affect the stability of the system, and simplify its dynamic behavior. The appropriate increase of unbalance can improve the oil film stiffness of the bearing to enhance the stability of the system.

(3) Too much oil film thickness will reduce the stability of the system. Excessive oil film thickness will increase the oil film vorticity and reduce the stability of the system.

(4) The experimental results verify that the increase of eccentricity will improve the stability of the rotor system.

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