Optimal Control Analyze the Bulk arrival Queueing system using Estimation Theory

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Abstract: In this paper, we analyze the system performance in FM/FM(k)/∞ using Estimation theory. Initially, we convert Erlang queueing system into fuzzy Erlang system, using Alpha-cut method and obtain different possible values of α. Finally, using Robust ranking method it’s helpful to measure the defuzzification values and analyzes the unique optimization of system performance.

Keywords: Fuzzy Sets; Mixed integer nonlinear programming; k-phase Erlang Distribution; Reduced Chi-square Distribution; Alpha-cut membership function

1. Introduction

In general, the management would expect the arrival without blocking due to the behavior of the service. If the system provides worst service, then the level of queue size will increase. As the management likes to avoid this kind of behaviour, we estimate the queue parameters by using statistical interference. Any statistical method apply to analyze the general queueing mainly focus on the estimation of the queue parameters and/or distribution of observed data. In practice, we regularly use the observable data to decide we arrival and service patterns of queueing system it’s extremely important to utilize the data to fullest extent possible. Since there are many statistical problems associated with the simulation modelling in queue analyses, the statistical procedures can help in making the best use of existing data, or in determining what and how much new data should be taken. That is an important fact of real queueing studies. The earliest work on the statistics of queues did, in fact, assume that the subject queue was fully observed over a period of time and therefore that complete information was available in the form of the arrival instants and the points from the beginning and each of the services of each customer. As would be expected, the queue was assumed to be a Markov chain in the process. Clarke (1957) began a sequence of papers on this and related topics by obtaining the maximum-likelihood estimators for the arrival and service parameters of an M/M/I queue, in addition to the variance –covariance matrix for the two statistics. The process of queueing system only depends on the time (steady state or transient state), but due to the nature of calamities it’s uncertain. In this regard, we consider the entire system fuzzy environment and analysis the system performance using statistical manner. [3, 4, 7, 10, 16, 21] have already analyzed the fuzzy queueing theory and declared important results based, on Alpha-level of membership functions, especially [2,
analyzed the queueing parameter via possibility theory used in fuzzy set theory and to determine the optimized fuzzy numbers. [5, 9] obtained the minimal queue waiting time in possibility theory. [6, 12, 13] has minimized the queue waiting time in fuzzy environment. [1, 22] compared various types of queueing model and to analyze the minimal cost but the result is complexity. [15, 17] considered the parameters are TFN and analyzed the system performance. [18] Analyzed the system performance through testing of hypothesis method. The article follows: In (2), the model under the steady state balance equation of Generalized Erlang K-phase service distribution. In (3), proposed a solution methodology. In (4), we analyze the MTNLP using the statistical interference. In (5), Example discussed

2. Generalized Erlang K-phase service distribution in two dimensional spaces

Consider k-phase Erlang system and the service rate of each phase denoted by $k\mu$. Let $p_{n,i}(t)$ be the probability that “n” arrival “i” service of the customer ($i=1, 2, 3, 4 \ldots k$). Here, we use the number of phases in backward process, so $k$ is the first phase of service and 1 is the last (a customer leaving phase 1 actually leaves the system). The two dimensional balanced equation is as:

$$p_{n,i}(t + \Delta t) = p_{n,i}(t)(1-\lambda\Delta t - k\mu\Delta t) + p_{n,i+1}(t)(k\mu\Delta t) + p_{n-1,i}(t)(\lambda\Delta t) \quad n \geq 2, 1 \leq i \leq k$$

$$p_{n,k}(t + \Delta t) = p_{n,k}(t)(1-\lambda\Delta t - k\mu\Delta t) + p_{n,k+1}(t)(k\mu\Delta t) + p_{n-1,k}(t)(\lambda\Delta t) \quad n \geq 2, 1 \leq i \leq k$$

$$0 = -(\lambda + k\mu)p_{n,i} + k\mu p_{n,i+1} + \lambda p_{n-1,i} \quad (n \geq 2, 1 \leq i \leq k-1)$$

$$0 = -(\lambda + k\mu)p_{n,k} + k\mu p_{n+1,1} + \lambda p_{n-1,k} \quad (n \geq 2)$$

$$0 = -(\lambda + k\mu)p_{1,i} + k\mu p_{1,i+1} \quad (1 \leq i \leq k-1)$$

$$0 = -(\lambda + k\mu)p_{1,k} + k\mu p_{2,1} + \lambda p_{0}$$

$$0 = -\lambda p_{n} + k\mu p_{n+1} \quad n > 0$$

Using the above equation, we obtained the total phase of waiting queue length is:

$$G(z) = \sum_{n=1}^{\infty} \sum_{i=1}^{k} p_{n,i} z^{k(n-1)+i} + p_{0} = \frac{p_{0}(1-z)}{1-\lambda(z+1)+r}$$

$$W_{q} = \sum_{n=1}^{\infty} \sum_{i=1}^{k} \frac{k(n-1)+i}{k\mu} p_{n,i} = \frac{1}{k\mu} G'(z)|_{z=1}$$

$$E(\frac{1}{\mu}) + E(\frac{1}{k\mu})$$

More specifically, the state of the system (n,i) transformed into there are (n-1) k+i phases system. In other words, there are n-1 customers in the queue (each requiring k phases) and the customer in service requires ‘i’ more phases in service. In the Erlang queue, the phases immediate completed of service do not correspond to departure of the customers, whereas in the bulk queue each individual customer departs the queue upon completion of service. It is helpful to determine the Wq for the M/E^\lambda/1 queue. From (1) convert to a single variable system using the transformation (n,i) ↔ (n-1)k+i the system of balanced equation is:

$$0 = -(\lambda + k\mu)p_{(n-1)k+i} + k\mu p_{(n-1)k+i+1} + \lambda p_{n-2k+i} \quad (n \geq 1, 1 \leq i \leq k) \quad -(2)$$

$$0 = -\lambda p_{0} + k\mu p_{1}$$
Where any p turning out to have a negative subscript is assumed to be zero, from (2) starting at n=1, i=1 gives a simplified set of equations:

\[
0 = -\lambda p_n + k\mu p_{n+1} + \lambda p_{n-k} \quad n \geq 1
\]

\[
0 = -\lambda p_0 + k\mu p_1 - \cdots - \lambda p_{n-k} - \cdots
\]

Where ‘k’ denotes the constant service rate, it is denoted \( k\mu \). If \( P^{(p)}_n \) denote the probability of n in the bulk-input system defined by (3), then it follows that the probability of n in the Erlang service system, \( P_n \) is given by

\[
P_n = \sum_{j=\max(n-k,0)}^{\min(n,\infty)} P^{(p)}_j
\]

Use z-transform and \( G(z) = 1 \), at \( z = 1 \), we find that

\[
G(z) = \sum_{n=0}^{\infty} p_n z^n, \quad p_0 = 1 - \rho, \quad G(z) = \frac{(1-\rho)(1-z)}{1 - z(r+1) + rz^{r+1}}, \quad \rho = \frac{\lambda}{k\mu}, \quad r = \frac{\lambda}{k\mu}.
\]

Denominator of \( G(z) \) has one zero at \( z = 1 \) and k zeros outside \( |z| = 1 \) for convergence. The Erlangen type \( k \) service model is equivalent to Bulk input model where \( c_k = 1, c_x = 0, x \neq k, p_0 = 1 - \rho \).

Using partial fraction, we get

\[
G(z) = (1-\rho) \sum_{j=1}^{\infty} \frac{A_j}{1 - (z/l)} \quad \text{where } A_j = \prod_{z_l \neq z_j} 1/(1-(z_j/l_z))
\]

And then \( P^{(p)}_j = (1-\rho) \sum_{j=1}^{\infty} A_j(z_j)^{-1} \) and \( P_n = \sum_{j=\max(n-k,0)}^{\min(n,\infty)} P^{(p)}_j \)

The performance measures

\[
W_q = E(T_q) = E(N_q) \frac{1}{\mu} + E(I) = \frac{1}{k\mu} \sum_{i=1}^{n} \sum_{j=0}^{\infty} k(n-i+1) \hat{p}_{n,i} + 0 \cdot p_n = \frac{1}{k\mu} G(z)|_{z=1}
\]

\[
W_q = \frac{(k+1)\rho}{2k\mu(1-\rho)} = \frac{k+1}{2k} \frac{\lambda}{\mu - \lambda}
\]

\( G(z) \) - Denotes the average total phase in the system, By Little formula \( L_q = \lambda W_q \quad (6) \)

**3. Fuzzy Queues with k-phase infinite capacity**

Consider the k-phases Erlang system with infinite capacity and parameter of the is discussed in fuzzy environment denoted as \( \overline{\lambda} \) and \( \overline{\mu} \) and it’s defined as \( \overline{\lambda} = \{(x, \mu_\lambda(x)) / x \in X\} \) and \( \overline{\mu} = \{(y, \mu_\mu(y)) / y \in Y\} \) where X and Y represent as the universal sets and \( \mu_\lambda(x), \mu_\mu(y) \), are the corresponding membership functions. Let \( P(x,y) \) represents the performance of the system, \( P(\overline{\lambda}, \overline{\mu}) \) is also fuzzy number. [23] Performance measure functions defined as

\[
\mu_{p(x, y)}(z) = \text{sup min} \left\{ \mu_{\lambda}(x), \mu_{\mu}(y) / z = p(x,y) \right\}
\]

From (5) Using Membership function, we get

\[
\mu_{\lambda}(z) = \text{sup min} \left\{ \mu_{\lambda}(x), \mu_{\mu}(y) / z = \frac{k+1}{2k} \frac{\rho}{\mu(1-\rho)} \right\}
\]

\[
\mu_{\mu}(z) = \text{sup min} \left\{ \mu_{\lambda}(x), \mu_{\mu}(y) / z = \frac{\lambda W_q}{\mu} \right\}
\]

Now, the idea is establishing the mathematical programming technique, a pair of nonlinear programs is developed and to find the degree of certainty of the system performance analyze, estimated the parameter using statistical manner.
4. Solution Procedure

[14, 19] Discussed the interval confidence of system performance, construct the membership function on the basis of \( \alpha \)-cut derivation as follows:
\[
\lambda^c_a = [x_a^c, x_a^d] = \left[ \min \{ x / \mu_a(x) \geq \alpha \}, \max \{ x / \mu_a(x) \geq \alpha \} \right] \quad \text{and} \quad \mu^c_a = [y_a^c, y_a^d] = \left[ \min \{ y / \mu_a(y) \geq \alpha \}, \max \{ y / \mu_a(y) \geq \alpha \} \right].
\]

Consequently; FM/FE\(^k\)/\(\infty\) can be reduced to a family of crisp M/E\(^k\)/\(\infty\) queues with different \( \alpha \)-level sets \([\lambda_a, \mu_a] / 0 < \alpha \leq 1\). [5] Discussed the relation between ordinary sets and fuzzy sets. The interval function of \( \alpha \) defined as \( x_a^c = \min(\mu_a^{-1}(\alpha)) \), \( x_a^d = \max(\mu_a^{-1}(\alpha)) \), \( y_a^c = \min(\mu_a(\alpha)) \) and \( y_a^d = \max(\mu_a(\alpha)) \). Consequently, we can use its \( \alpha \)-cut to construct its membership function. According to (i) \( \mu_{\lambda_a}^{-1}(z) \) \& \( \mu_{\mu_a}^{-1}(z) \) is the minimum of \( \mu_a(x) \) \& \( \mu_a(y) \). We require to either \( \mu_a(x) = \alpha \) and \( \mu_a(y) > \alpha \) or \( \mu_a(x) \geq \alpha \) and \( \mu_a(y) = \alpha \), such that \( L_a(z) = \lambda \mu_a \) and \( W_a \) to satisfy that \( \mu_{L_a}(z) = \alpha \) and \( \mu_{W_a}(z) = \alpha \). According from (i & ii) \( y \in \mu(\alpha) \) and \( x \in \lambda(\alpha) \) can be replaced by \( x \in [x_a^c, x_a^d] \) and \( y \in [y_a^c, y_a^d] \) respectively. Thus, based on (ii & iii) to find the membership function of \( \mu_{L_a}(z) \) \& \( \mu_{W_a}(z) \) it suffices to find the lower and upper bound \([L_{x_a}, L_{y_a}] \) \& \([W_{x_a}, W_{y_a}] \) of the \( \alpha \)-cuts of \( \mu_{x_a}(z) \) \& \( \mu_{w_a}(z) \) which can be rewritten as:
\[
W_{x_a}(z) = \min \left\{ \left( \frac{k+1}{2k} \right) \frac{\rho}{\mu(1-\rho)} \right\} \quad \text{---(iv)} \quad \text{and} \quad W_{w_a}(z) = \max \left\{ \left( \frac{k+1}{2k} \right) \frac{\rho}{\mu(1-\rho)} \right\} \quad \text{---(v)}
\]

Such that \( x_a^c \leq x \leq x_a^d \) and \( y_a^c \leq y \leq y_a^d \). There are several effective and efficient methods for solving these problems [20]. Moreover, analyze how to change the optimal solution as \( x_a^c, x_a^d, y_a^c \) and \( y_a^d \) where \( \alpha \in [0,1] \); they fall into the category of NLP [21]. If \( L_a = [L_{x_a}, L_{y_a}] \) \& \( W_a = [W_{x_a}, W_{y_a}] \) are invertible. Then shape of the function is \([L(z), R(z)] = L^{-1} \) \& \([L(z), R(z)] = W^{-1} \) constructed as follows:
\[
\mu_{x_a}(z) \& \mu_{w_a}(z) = \begin{cases} 
L(z) & z \leq z_2 \\
1 & z = z_2 \\
R(z) & z_2 \leq z \leq z_3 
\end{cases}
\]

Otherwise, logically the solution is some difficulty. So, using Alpha –cut method (iv & v) to get different possible values. But, the shape of the function value is not explicit and also the system performance. In this regard, we estimate the queue parameter in a statistical manner. It can be derived in the next section.

5. Estimating parameters of an FM/FE\(^k\)/\(\infty\) Queue:

Consider estimating the parameters of a homogeneous Markov chain, such as a simple birth-death process with rate \( \lambda \) and \( \mu \). To estimate the arrival rate \( \lambda \), we observe that the arrival process is Poisson. If the time required observing “n” arrival denoted by \( \varphi(n) \) then the MLE of \( \lambda \) being \( \hat{\lambda} = (n / s_n) \).

Also, sine \( 2\lambda \varphi(n) \) has a chi-square distribution with \( 2n \) degrees of freedom, a 100(1-\( \alpha \))% confidence interval for \( \lambda \) is given by \( \frac{\chi^2_{2n-1,\alpha/2}}{2s_n}, \frac{\chi^2_{2n+1,\alpha/2}}{2s_n} \). The service times \( X_1, X_2, ..., X_m \) have been observed, and if we get \( \phi(m) = \sum_{i=1}^{m} X_i \) the MLE of \( \mu \) is given by \( k = (m / \phi(m)) \) where \( \phi(m) \) may also be interpreted as the total busy time of the server during the observation period. Noting that \( 2\mu \phi(m) \) has \( \chi^2_{2m} \) distribution, we get a 100(1-\( \alpha \))% confidence interval for \( \mu \) as \( \left[ \frac{\chi^2_{2m,1-\alpha/2}}{2y_n}, \frac{\chi^2_{2m,\alpha/2}}{2y_n} \right] \). Server Utilization
\( \rho \) is now estimated by \( \frac{\phi(m)}{m} \). Now, since \( \phi(m) \) and \( \phi(n) \) have independent, and

\[ \frac{2\rho^2}{m} \] and \[ \frac{2\rho^2}{n} \] are both chi-square distributed, it follows that \( \frac{\rho}{\rho} \) has an “F” distribution with 2m and 2n degrees of freedom. To obtain a 100(1 - \( \alpha \))% confidence interval for \( \rho \).

6. Example
A clinic has a doctor examining every patient brought in for a general check-up. If each patient goes through three phases in the check-up and the arrivals to the clinic are Poisson with rate \( \lambda \). The service time of the patients is assumed to have an exponential distribution with parameter \( \mu \). The arrival rate and service rate are assumed as triangular fuzzy numbers represented by \( \lambda \) and \( \mu \) as \( \lambda = [1, 5, 7] \) and \( \mu = [9, 10, 11] \). It is clear that in this example the steady-state condition \( \rho = (\frac{\lambda}{\mu}) < 1 \) is satisfied, thus the performance measures of interest can be constructed by using the approach stated in solution procedure following (iv & v), two pairs of MINLP models for deriving the membership function of \( L_q \) can be formulated, whose solutions are as follows:

\[ l_{L_q}(\alpha) = \frac{32\alpha^2 + 16\alpha + 2}{15\alpha^2 - 195\alpha + 330} \quad \text{and} \quad u_{L_q}(\alpha) = \frac{8\alpha^2 - 56\alpha + 98}{9\alpha^2 + 87\alpha + 54} \]

The inverse functions of \( u_{L_q}(\alpha) \) and \( l_{L_q}(\alpha) \) exist which give the membership function \( \mu_{L_q}(z) \) as

\[ \mu_{L_q}(z) = \begin{cases} \frac{(95z + 16)\pm\sqrt{1225z^2 + 4860z}}{30z - 64} & \text{for } \frac{1}{165} \leq z \leq \frac{1}{3} \\ 1 & \text{for } \frac{1}{3} \leq z \leq 49 \frac{27}{27} \\ \frac{-(87z + 56)\pm\sqrt{625z^2 + 1500z}}{18z - 16} & \text{for } 49 \frac{27}{27} \leq z \leq 3 \end{cases} \]

Similarly, we derived,

\[ l_{w_q}(\alpha) = \frac{8\alpha + 2}{15\alpha^2 - 195\alpha + 330} \quad \text{and} \quad u_{w_q}(\alpha) = \frac{14 - 4\alpha}{9\alpha^2 + 87\alpha + 54} \]

The Inverse function of is \( l_{w_q}(\alpha) \) and \( u_{w_q}(\alpha) \) exist which give the membership function \( \mu_{w_q}(z) \) as

\[ \mu_{w_q}(z) = \begin{cases} \frac{(95z + 8)\pm\sqrt{1225z^2 + 3240z + 64}}{30z} & \text{for } \frac{1}{165} \leq z \leq \frac{13}{150} \\ 1 & \text{for } \frac{13}{150} \leq z \leq \frac{1}{15} \\ \frac{-(87z + 4)\pm\sqrt{625z^2 + 1200z + 16}}{18z} & \text{for } \frac{1}{15} \leq z \leq \frac{7}{27} \end{cases} \]

Table 1: The \( \alpha \) cuts of the performance measures at 11 \( \alpha \) values are as follows

| \( \alpha \) | \( l_{L_q}(\alpha) \) | \( l_{w_q}(\alpha) \) | \( u_{L_q}(\alpha) \) | \( u_{w_q}(\alpha) \) | \( L_q \% \) | \( W_q \% \) |
|---|---|---|---|---|---|---|
| 0.1 | 1.18148 | 0.0061 | 0.2593 | 1.0061 | 91 | 13 |
| 0.2 | 1.4728 | 0.0091 | 0.2166 | 1.4728 | 74 | 11 |
| 0.3 | 1.2140 | 0.0123 | 0.1839 | 1.2140 | 61 | 9 |
| 0.4 | 1.0125 | 0.0161 | 0.1582 | 1.0125 | 52 | 8 |
| 0.5 | 0.8520 | 0.0204 | 0.1374 | 0.8520 | 45 | 7 |
| 0.6 | 0.7218 | 0.0254 | 0.1203 | 0.7218 | 39 | 7 |
| 0.7 | 0.6148 | 0.0311 | 0.1060 | 0.6148 | 36 | 6 |
| 0.8 | 0.5254 | 0.0378 | 0.0939 | 0.5254 | 33 | 6 |
| 0.9 | 0.4508 | 0.0458 | 0.0835 | 0.4508 | 32 | 6 |
| 1 | 0.3874 | 0.0552 | 0.0745 | 0.3874 | 32 | 6 |

Using (sec 4) we analyze the system performance optimality relation between the average number of customers waiting in the queue and customer has to spend the time in queue.
7. Conclusion:

In this study, we analyze to optimize the system performance in the service status only. But, consider the parameter of queue in uncertain value applying the \( \alpha \)-cuts. Using Zadeh’s extension principle, transform to the fuzzy queue that can be described a pair of NLP models. We optimize the length of queue waiting time. If, the degree of certainty \( \alpha =0 \) then \( \bar{L}_q = [0.0061, 1.8148] \) and \( \bar{W}_q = [0.0061, 0.2593] \). If, the degree of certainty \( \alpha =1 \) then \( \bar{L}_q = 0.3333 \) & \( \bar{W}_q =0.0667 \). The fuzziness values are converted to crisp value using Robust ranking function the value of \( \bar{W}_q \) & \( \bar{L}_q \) are 0.1327 & 0.91945. The degree of uncertainty is analyzed the chi-square distribution \( \alpha = [0.4 \text{ to } 0.6] \) in this range performance of the system is not optimized. After that we analyze which customer has to suffer in service time using the statistical interference. Using our proposed method reduced the degrees of freedom eleven into seven we get the system performance is 0.1518. Suppose we analyze entire system
the error value of $\overline{W}_q$ & $\overline{L}_q$ are 0.03776 & 5.2947 and to adjust the chi-square value is 0.97295. Our proposed methodology is most helpful for the system operation (simulation) studies and then we quickly identified the blocking to the system.

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