Study on Natural Vibration Frequency and Mode Shape of Deep Sea Top Tensioned Riser

Guanghai Gao¹, Yunjing Cui²*, Xingqi Qiu¹, Qi Shu³

¹College of Chemical Engineering, China University of Petroleum (East China), Qingdao, 266580, China
²College of Mechanical and Electronic Engineering, China University of Petroleum (East China), Qingdao, 266580, China
³Corresponding author. E-mail: cuiyj@upc.edu.cn.

Abstract. In order to analyse the natural vibration frequencies and mode shapes of deep sea top tensioned riser, an analysis model is established in the present study. The effect of in-line bending displacement on riser natural vibration is considered in the present model. The finite element method is adopted in this paper to calculate the natural vibration frequencies and mode shapes of the riser. The effect of top-tension force and sea current velocity on the riser vibration frequencies and mode shapes has discussed in the present study. Results show that the riser vibration frequency of the in-line direction is higher than the cross-flow direction and the vibration mode shapes of the riser are not standard sinusoidal curve. The riser mode shapes of the in-line direction are different from the cross-flow direction. Moreover, the change of top-tension force and sea surface current velocity has a great influence on the riser vibration frequencies and mode shapes.

1. Introduction

Top tensioned riser is the key equipment connecting subsea blow out preventer system and floating system in deep sea oil/gas drilling and production. With the exploitation and production of offshore oil/gas to deeper sea, the application of top tensioned riser is more and more extensive.

Accurately mastering the natural vibration frequency and mode shape of riser is the basis for the riser vortex-induced vibration analysis and precise design. Scholars have developed a number of analysis models and methods on riser natural vibration. Kim et al. adopted WKB method to analyse the structure natural vibration[1-3]. A direct method was used by Graves and Dareing to determine natural frequencies of marine risers in deep water[4]. Senjanovic et al. analysed natural vibration of tensioned risers by using segmentation method[5]. Chatjigeorgiou used perturbation approach to calculate natural modes of riser-type slender structures[6]. Chen et al. used differential transformation and variational iteration methods respectively to analyse natural frequencies and mode shapes of marine risers[7-8]. The finite element method was used by Li et al. to study natural vibration of marine risers[9-11].

The length/diameter ratio of deep sea top tensioned riser is very large and the support is simple. So, the riser has a large in-line bending displacement under the action of sea current velocity and this has a great effect on riser natural vibration. At present, the related research is insufficient. In view of this, an analysis model was established to study the riser natural vibration frequencies and mode shapes by taking into account the influence of in-line bending displacement on riser natural vibration. The effect...
of top-tension force and sea current velocity on the riser natural vibration frequencies and mode shapes has discussed in this paper.

2. Analysis model

2.1. Structure model

The deep sea top tensioned riser can be idealized as an Euler–Bernoulli beam located in the vertical plane. A Cartesian coordinate system with its origin at the bottom of the riser has been used, in which the \( x \) axis is parallel to the sea current (in-line direction), \( z \) coincides with the vertical axis of the riser in its un-deflected configuration and \( y \) (cross-flow direction) is perpendicular to both (see figure 1).

![Figure 1. Schematic model of deep sea top tensioned riser](image)

2.2. Quasi-static analysis control equation

Based on the theory of Euler-Bernoulli vertical and horizontal bending beam\(^{[12-13]}\), the quasi-static control equation of the riser can be written as:

\[
E I \frac{d^4 x(z)}{dz^4} - T(z) \frac{d^2 x(z)}{dz^2} - w \frac{dx}{dz} = 0.5 C_D \rho_w D u(z)^2
\]

where \( E I \) is the riser flexural stiffness; \( x(z) \) is the riser bending displacement; \( T(z) \) is the riser effective axial tension force; \( w \) is the wet weight of per unit length riser; \( D \) is the riser outer diameter; \( C_D \) is the mean drag force coefficient; \( u(z) \) is the sea current velocity at different water depth.

Considering the effect of the top-tension force, riser self-weight, buoyancy force, as well as the internal and external hydrostatic pressure acting on the riser\(^{[14]}\), the effective axial tension force of the riser can be written as:

\[
T(z) = T_{wp} - \int_0^H wd(z)\,dz
\]

where \( T_{wp} \) is the top-tension force acting on the riser, \( T_{wp} = f_{wp} w H \), \( f_{wp} \) is the top-tension force coefficient; \( H \) is the water depth.

The wet weight of per unit length riser can be expressed as:

\[
w = \frac{\pi}{4} \left[ \rho_r (D^2 - d^2) + \rho_f d^2 - \rho_s D^2 \right] g
\]

where \( \rho_r, \rho_f, \rho_s \) denote the density of the riser, internal fluid and seawater respectively; \( d \) is the riser inner diameter; \( g \) is the gravity acceleration.

Assuming that the sea current is linear sheared flow, the current velocity at different water depth can be written as:

\[
u(z) = u_b + (u_c - u_b) \frac{z}{H}
\]
where \( u_s \) and \( u_b \) denote the sea surface and bottom current velocity respectively.

2.3. Dynamic analysis control equation

Based on the theory of Euler-Bernoulli vertical and horizontal bending beam\(^{[12,15]}\), the dynamic control equation of the riser can be written as:

\[
\left\{ \begin{array}{l}
EI \frac{\partial^4 x(z,t)}{\partial z^4} - T(z) \frac{\partial^2 x(z,t)}{\partial z^2} - w \frac{\partial x(z,t)}{\partial z} \\
+ (m_r + m_i + m_a) \frac{\partial^2 x(z,t)}{\partial t^2} = f_r(z,t) \\
EI \frac{\partial^4 y(z,t)}{\partial z^4} - T(z) \frac{\partial^2 y(z,t)}{\partial z^2} - w \frac{\partial y(z,t)}{\partial z} \\
+ (m_r + m_i + m_a) \frac{\partial^2 y(z,t)}{\partial t^2} = f_r(z,t)
\end{array} \right. \tag{5}
\]

where \( \partial x(z,t)/\partial t, \partial y(z,t)/\partial t \) are the riser vibration velocities; \( \partial^2 x(z,t)/\partial t^2, \partial^2 y(z,t)/\partial t^2 \) are the riser vibration accelerations; \( m_r, m_i, m_a \) denote the mass of per unit length riser, internal fluid and added fluid; \( f_r(z,t), f_r(z,t) \) is the fluctuating hydrodynamic forces acting on the riser.

The mass of per unit length riser, internal fluid and added fluid can be expressed as:

\[
\left\{ \begin{array}{l}
m_r = \pi(D^2 - d^2) \rho_i / 4 \\
m_i = \pi d^2 \rho_i / 4 \\
m_a = C_a \pi D^2 \rho_i / 4
\end{array} \right. \tag{6}
\]

where \( C_a \) is the added fluid mass coefficient (\( C_a = 1.0 \)).

2.4. Boundary conditions

The riser model was pin-ended, so the displacements and curvatures were zero at each end of the riser. Hence, the boundary conditions can be expressed as:

\[
\left\{ \begin{array}{l}
x(0,t) = 0, y(0,t) = 0, x(H,t) = 0, y(H,t) = 0 \\
\frac{\partial^2 x(0,t)}{\partial z^2} = 0, \frac{\partial^2 y(0,t)}{\partial z^2} = 0 \\
\frac{\partial^2 x(H,t)}{\partial z^2} = 0, \frac{\partial^2 y(H,t)}{\partial z^2} = 0
\end{array} \right. \tag{7}
\]

2.5. Numerical solution method

The finite element method was applied to solve the quasi-static and dynamic governing equations of the riser. At the first time step, the Hermite cubic interpolation functions were used to obtain the system stiffness matrix and static load matrix. Then the Newton-Raphson iterative method was used to solve the non-linear quasi-static control equation. The riser in-line bending displacement and the system stiffness matrix and mass matrix were obtained at the static equilibrium position of the riser. Then the static equilibrium position of the riser was taken as the initial state of the riser natural vibration analysis. According to the initial and boundary conditions, the natural frequencies and mode shapes of the riser were obtained by subspace iteration method. The entire analysis process was carried out on the Matlab platform by compiling finite element calculation program.

3. Application and case study

3.1. Validity of the analysis model
In order to test the validity of the analysis model, comparison with experimental results is carried out in the present study. The experiment was carried out at Delft Hydraulics in the Delta Flume\cite{16-17}. The main parameters of the riser: total length 13.12m, outer diameter 28mm, submerged length 5.9m, bending stiffness 29.9N·m², unit length mass in water 1.85kg/m, mass ratio 3, length/diameter ratio 469, top tension force 1904N.

The experimental and numerical results are given in Table 1. It is found that the numerical results are highly consistent with the experimental results. The numerical results are slightly larger than experimental results. The slight difference between the numerical and experimental results may be attributed to computational errors.

Table 1. Comparison between experimental and numerical vibration frequencies of the riser

| Method     | Vibration Frequency(Hz) |
|------------|-------------------------|
|            | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  |
| Experimental| 1.037 | 2.046 | 3.084 | 4.099 | 5.127 | 6.222 | 7.298 | 8.371 |
| Numerical  | 1.042 | 2.065 | 3.119 | 4.156 | 5.237 | 6.341 | 7.435 | 8.504 |

3.2. Case study

In the present study, a real scale top-tension riser used in the South China Sea is taken as an example to analyse the riser natural vibration. The main parameters of the riser: total length 2000m, outer diameter 0.5334m, inner diameter 0.4826m, elastic modulus 210GPa, riser density 7850kg/m³, internal fluid density 865kg/m³, seawater density 1030kg/m³, top-tension force coefficient 1.1~1.6, sea surface current velocity 0.5~1.5m/s, sea bottom current velocity 0.2m/s.

When the sea surface current velocity is 1.5m/s and the top-tension force coefficient is 1.6, the riser vibration frequencies are shown in figure 2(a) and the 1st, 5th and 10th vibration mode shapes of the riser are shown in figure 2(b).

As is shown in figure 2(a), the vibration frequencies of the riser are very low and the frequency interval of adjacent mode numbers is very small. The in-line vibration frequency of the riser is higher than that of the cross-flow frequency. With the increase of the mode number, the difference is becoming more and more obvious. It can be seen from figure 2(b) that the mode shapes of the riser are not standard sinusoidal curve. The in-line mode shapes of the riser are different from the cross-flow mode shapes.

3.3. Influence of top-tension force

When the sea surface current velocity is 1.5m/s, three different top-tension force coefficients are selected in the present study and they are 1.2, 1.4 and 1.6. The vibration frequencies of the riser under three different top-tension forces are shown in figure 3.

It can be seen from figure 3 that the change of the top-tension force has a great influence on the
The riser vibration frequency. At the same mode number, with the increase of the top-tension force, the in-line vibration frequency of the riser decreases gradually and the cross-flow vibration frequency of the riser increases gradually. With the increase of mode number, the difference is becoming bigger.

Figure 4 illustrates the vibration mode shapes of the riser under three different top-tension forces. As is shown in figure 4, the change of the top-tension force changes the mode shapes of the riser. With the increase of the top-tension force, the in-line vibration amplitude of the riser increases gradually and the riser in-line vibration becomes more intense. There is a certain phase difference for all kinds of cross-flow mode shapes of the riser under different top-tension force.

![Figure 3. Vibration frequencies of the riser under different top-tension forces](image)

![Figure 4. Vibration mode shapes of the riser under different top-tension forces](image)

3.4. Influence of sea current velocity

When the top-tension force coefficient is 1.6, three different sea surface current velocities are selected in the present study and they are 0.5, 1.0 and 1.5m/s. The vibration frequencies of the riser under three different sea surface current velocities are shown in figure 5.
As is shown in figure 5, the variation of the sea current velocity has a great influence on the in-line vibration frequency of the riser and almost has no influence on the riser cross-flow frequency. At the same mode number, with the increase of the sea surface current velocity, the in-line vibration frequency of the riser increase gradually. With the increase of mode number, the difference is becoming bigger.

Figure 6 illustrates the vibration mode shapes of the riser under three different sea surface current velocities. It can be seen that the variation of the sea surface current velocity changes the in-line mode shapes of the riser. There is a certain phase difference for all kinds of in-line mode shapes of the riser under different sea surface current velocity. The variation of the sea surface current velocity almost has no effect on the riser cross-flow vibration mode shapes.

4 Conclusions
In this paper, an analysis model was proposed to study the natural vibration characteristics of a deep sea top tensioned riser used in South China Sea. The influence of in-line bending displacement on riser natural vibration was considered in the present model. The comparison between the numerical and experimental results has shown that the model is reasonable. Numerical results show that the riser vibration frequency of the in-line direction is higher than the cross-flow direction. The vibration mode shapes of the riser are not standard sinusoidal curve and the riser mode shapes of the in-line direction are different from the cross-flow direction. Moreover, the change of top-tension force has a great influence on the riser vibration frequencies and mode shapes. The variation of sea surface current velocity has a great influence on the riser in-line vibration and almost has no effect on the riser cross-flow vibration.
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