THE VARIATION OF INTEGRATED STAR INITIAL MASS FUNCTIONS AMONG GALAXIES

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ABSTRACT

The integrated galaxial initial mass function (IGIMF) is the relevant distribution function containing the information on the distribution of stellar remnants, the number of supernovae, and the chemical enrichment history of a galaxy. Since most stars form in embedded star clusters with different masses, the IGIMF becomes an integral of the assumed (universal or invariant) stellar IMF over the embedded star cluster mass function (ECMF). For a range of reasonable assumptions about the IMF and the ECMF we find the IGIMF to be steeper (containing fewer massive stars per star) than the stellar IMF, but below a few solar masses it is invariant and identical to the stellar IMF for all galaxies. However, the steepening sensitively depends on the form of the ECMF in the low-mass regime. Furthermore, observations indicate a relation between the star formation rate of a galaxy and the most massive young stellar cluster in it. The assumption that this cluster mass marks the upper end of a young-cluster mass function leads to a connection of the star formation rate and the slope of the IGIMF above a few solar masses. The IGIMF varies with the star formation history of a galaxy. Notably, large variations of the IGIMF are evident for dE, dIrr, and LSB galaxies with a small to modest stellar mass. We find that for any galaxy the number of supernovae per star (NSNS) is suppressed relative to that expected for a Salpeter IMF. Dwarf galaxies have a smaller NSNS than massive galaxies. For dwarf galaxies the NSNS varies substantially depending on the galaxy assembly history and the assumptions made about the low-mass end of the ECMF. The findings presented here may be of some consequence for the cosmological evolution of the number of supernovae per low-mass star and the chemical enrichment of galaxies of different mass.

Subject headings: galaxies: abundances — galaxies: evolution — galaxies: general — galaxies: star clusters — galaxies: stellar content — galaxies: structure

1. INTRODUCTION

Over the last several years it has become clear that star formation takes place mostly in embedded clusters, each cluster containing a dozen to many millions of stars (Kroupa 2005). Within these clusters stars appear to form following a universal initial mass function (IMF) with a Salpeter power-law slope or index (α = 2.35) for stars more massive than 1 $M_\odot$, $\xi (m) \propto m^{-\alpha}$, where $\xi dm$ is the number of stars in the mass interval $m, m + dm$.

This has been found to be the case for a wide range of different conditions in the Milky Way (MW), the Large and Small Magellanic Clouds, and other galaxies (Massey & Hunter 1998; Sirianni et al. 2000, 2002; Parker et al. 2001; Massey 2002, 2003; Wyse et al. 2002; Bell et al. 2003; Piskunov et al. 2004). For studies based on well-resolved stellar populations, the observational scatter around the Salpeter value above 1 $M_\odot$ is large but constant as a function of stellar mass range and consistent with a Gaussian distribution around this value (Fig. 1). This scatter can be explained by statistical fluctuations and stellar dynamical evolution of the clusters (Elmegreen 1999; Kroupa 2001). The latter changes the mass function on short and long timescales, making it extremely difficult to measure the IMF. Careful studies taking mass segregation into account often find Salpeter indices in young clusters (e.g., in the LMC and SMC; Gouliermis et al. 2004). Furthermore, a Salpeter IMF is found in very young clusters such as the starburst cluster R136 in the LMC (Brandl et al. 1996), NGC 1805 (de Grijs et al. 2002), NGC 2004, NGC 2100 (Gouliermis et al. 2004), and M82-F (McCrady et al. 2005). The often-observed flattening of the IMF below a few solar masses (e.g., Sirianni et al. 2000) can be explained by taking mass segregation into account (Sirianni et al. 2002; de Marchi et al. 2005). Especially, Figure 2 from Massey 2003 demonstrates the remarkable universality of the IMF over a factor of 4 in metallicity and 200 in stellar density.

On the other hand, several observations (e.g., Prisinzano et al. 2001, 2003; Sagar et al. 2001; Sanner & Geffert 2001; Kalirai et al. 2003) show steeper slopes for clusters with ages of 100–500 Myr and for stars ranging up to 4–15 $M_\odot$. For the Pleiades cluster, which has an age of about 100 Myr, Moraux et al. (2004) find that the IMF may be steeper than Salpeter ($\alpha > 2.35$) for $m \gtrsim 2 M_\odot$. While these are important constraints, it is clear that such clusters are already heavily dynamically evolved (Kroupa et al. 2001). Guandaliris et al. (2004) show for the very young Trapezium cluster that two OB runaway stars as far away as about 250 pc can be traced back to it. Furthermore, about 40% of O stars and 5%–10% of B stars are runaway stars most probably ejected from star-forming regions (Gualandris et al. 2004) and found up to several kiloparsecs away from their birthplaces. Obviously, such stars need to be included in order to reconstruct an IMF from a present-day mass function (PDMF), which has not been done. But also stars “evaporating” from a cluster after gas expulsion would travel 100–500 pc for clusters that are 100–500 Myr old even if they leave with a velocity of only 1 km s$^{-1}$! Thus, dynamical modeling on a cluster-to-cluster basis would be needed to affirm possible non-Salpeter IMFs above 1 $M_\odot$.

Thus, the case can be made that the IMF differs from cluster to cluster (Scalo 1998, 2005; Elmegreen 2004). However, the absence of any trends with physical conditions, together with the Gaussian distribution about the Salpeter value above $\sim 1 M_\odot$ and the similar although somewhat larger theoretical spread obtained for model clusters (Fig. 1), leads us to assume for now that the stellar IMF is invariant and universal in each cluster. In
adoption is a single-slope power-law ECMF with a lower limit \( M_{\text{min}} \) of 5\( M_\odot \), but different assumptions in the low cluster mass regime are investigated as well. Several studies show that star clusters also seem to be distributed according to a power-law embedded cluster mass function (ECMF), \( \xi_{\text{ecl}} = k_{\text{ecl}} M_{\text{ecl}}^{-\beta} \), where \( dN_{\text{ecl}} = \xi_{\text{ecl}}(M_{\text{ecl}})dM_{\text{ecl}} \) is the number of embedded clusters in the mass interval \( M_{\text{ecl}} \), \( M_{\text{ecl}} + dM_{\text{ecl}} \) and \( M_{\text{ecl}} \) is the mass in stars. For embedded stellar clusters in the solar neighborhood with masses between 50 and 1000 \( M_\odot \), Lada & Lada (2003) find a slope \( \beta = 2 \), while Hunter et al. (2003) find \( 2 \leq \beta \leq 2.4 \) for \( 10^3 \leq M_{\text{ecl}}/M_\odot \leq 10^4 \) in the SMC and LMC, and Zhang & Fall (1999) find \( 1.95 \pm 0.03 \) for \( 10^4 \leq M_{\text{ecl}}/M_\odot \leq 10^6 \) in the Antennae galaxies. Our default assumption is a single-slope power-law ECMF with a lower limit of 5 \( M_\odot \), but different assumptions in the low cluster mass regime are investigated as well.

Below roughly 100 stars (\( \sim 30 M_\odot \)) embedded clusters dissolve within a few Myr (Adams & Myers 2001; Kroupa & Bouvier 2003), but for our model only the initial distribution of \( M_{\text{ecl}} \) is of interest, not the evolution of the clusters. The detailed distribution of such small systems is not well known. Observational and theoretical evidence is contradictory, as Soares et al. (2005) conclude that they are not major contributors to the field, while Adams & Myers (2001) suggest that 90% of star formation takes place in such small systems. This would be roughly equivalent to a power-law function with \( \beta \approx 2.35 \) down to about 5 \( M_\odot \), which corresponds to a group of about 20 stars. From a careful study of 580 open clusters in the Milky Way de la Fuente Marcos & de la Fuente Marcos (2004) find that 90% of the open clusters are formed with less than 150 stars. They are able to fit a power-law cluster mass function as steep as \( \beta = 2.7 \) to the distribution down to a cluster mass of a few solar masses. In their sample 65% of the clusters have reconstructed masses less than 20 \( M_\odot \). They also conclude that 80% of newly formed open clusters will dissolve in less than 20 Myr.

The universality of the ECMF slope is not an established result, as a number of works based on \( \text{H}_\text{ii} \) region luminosity functions (Kennicutt et al. 1989; Youngblood & Hunter 1999; Alonso-Herrero et al. 2002; Thilker et al. 2002), direct cluster counts (Cresci et al. 2005), and giant molecular cloud counts (Blitz & Rosolowsky 2005) indicate that it may vary with galaxy type or even rather erratically. The situation may be resolved once early cluster evolution is taken into account in more detail; Kroupa & Boily (2002) have already made the important point that the cluster MF evolves rapidly within the first few \( 10^6 \) yr as a result of revirialization after significant residual-gas expulsion.

Under the assumption that (1) the stellar IMF is universal and canonical and (2) the ECMF is also universal, we showed in Kroupa & Weidner (2003) that the integrated galaxial initial stellar mass function (IGIMF) must be steeper than the individual canonical IMFs in the actual clusters. But this steepening is critically dependent on the assumptions regarding the low-mass end of the ECMF.

In this contribution we develop a tool to calculate the time-dependent IGIMF for different types of galaxies. This is possible by combining the above-mentioned results with a recently discovered relation between the maximum cluster mass formed in a star formation epoch and the star formation rate (SFR) of a galaxy (Weidner et al. 2004). The varying star formation histories (SFHs) of galaxies therefore leave their fingerprint in the distribution of the stellar content and the resulting chemical evolution of galaxies through a highly variable IMF.

However, this ought to be difficult to observe because variations of the SFR also imply changes of the relative number of young and old stars, even for an invariant IGIMF.

In the next section (§ 2) we introduce the method of calculating the IGIMF from a universal IMF, an ECMF, and an SFH before we present the results in § 3, where we construct standard, maximal, and minimal models that span the range of parameters characterizing the stellar IMF and the ECMF. The results are discussed in § 4. Before proceeding we note that throughout this paper IMF means the stellar IMF, which we take to be invariant.

2. THE METHOD

2.1. The Star Formation Rate–Maximal Cluster Mass Relation

In Weidner et al. (2004) we derived a relation between the maximal cluster mass in a galaxy and the current SFR of the galaxy,

\[
\log M_{\text{ecl,max}} = \log k_{\text{ML}} + (0.75 \pm 0.03) \log \text{SFR} + (6.77 \pm 0.02),
\]

(1)
where $k_{\text{ML}}$ is the mass-to-light ratio, typically 0.0144 for young (<6 Myr) clusters. This relation connects the properties of clustered star formation with the SFR of a galaxy. Of interest for investigating the sensitivity of the results on the SFR is also the use of a relation that is steeper than the default equation (1) but still within the 3 $\sigma$ uncertainty range of the data,

$$\log M_{\text{c},\text{max}} = \log k_{\text{ML}} + 0.84 \log \text{SFR} + 6.71. \quad (2)$$

For a given SFR this relation gives larger $M_{\text{c},\text{max}}$ values than equation (1).

Weidner et al. (2004) show that equation (1) can be reproduced theoretically for a power-law ECMF provided the entire population of star clusters with masses ranging from $M_{\text{c},\min} \approx 5 M_\odot$ to $M_{\text{c},\text{max}}$ is born within a time span of $\delta t \approx 10$ Myr, independently of the SFR. We refer to this time span as a “star formation epoch” of a galaxy. As these epochs are very short independently of the SFR. We refer to this time span as a “star formation epoch” of a galaxy. As these epochs are very short independently of the SFR.

Weidner & Kroupa (2004) interpret $m_{\text{max}}$, to be a fundamental upper stellar mass. Intriguingly, the same stellar mass limit is noted by Figer (2005) for the metal-rich MW nuclear cluster Arches suggesting that perhaps $m_{\text{max}} \approx 150 M_\odot$ may be quite independent of metallicity, and Oey & Clarke (2005) find such an upper mass limit based on statistical examinations of several clusters. Numerical simulations of star formation in clusters (Bonnell et al. 2004) also indicate that the mass of the most massive star scales with the system (cluster) mass and is not a purely random value but a conditional one.

### 2.3. The Integrated Galaxial Initial Mass Function

Under the assumption of an invariant canonical stellar IMF in clusters and an invariant ECMF (§1), the integrated galaxial initial mass function (IGIMF) is calculated by adding all stars in all clusters (as already noted by Vanbeveren 1982, 1983),

$$\xi_{\text{IGIMF}}(m, t) = \int_{M_{\text{c},\text{min}}}^{M_{\text{c},\text{max}}(\text{SFR})} \xi(m \leq m_{\text{max}}) \xi_{\text{c}}(M_{\text{c}}) dM_{\text{c}}. \quad (5)$$

Thus, $(m \leq m_{\text{max}}) \xi_{\text{c}}(M_{\text{c}}) dM_{\text{c}}$ is the stellar IMF contributed by $\xi_{\text{c}}(M_{\text{c}}) dM_{\text{c}}$ clusters with mass near $M_{\text{c}}$. While $M_{\text{c},\text{max}}$ follows from equation (1), $M_{\text{c},\text{min}} = 5 M_\odot$ is adopted. The stellar mass in an embedded cluster is $M_{\text{c}}$ so that the mass in stars and gas of the whole embedded cluster amounts to $M_{\text{c}}/\epsilon$ for a star formation efficiency of $\epsilon \approx 4/3$; Lada & Lada 2003). Note that in Kroupa & Weidner (2003) we referred to equation (5) as the “field star IMF,” $\xi_{\text{field}}$. This is strictly speaking not correct, because the IGIMF includes all stars in all clusters and the galaxial field, which consists of already dispersed clusters. However, as long as the surviving and newly born star clusters do not constitute a significant stellar contribution to the whole galaxy and ignoring issues arising from a flux limit, $\xi_{\text{IGIMF}} \approx \xi_{\text{field}}$ for the time-averaged galaxy. Another way of looking at equation (5) is to consider joint probabilities: the joint probability for finding a star of mass $m$ in a cluster of mass $M_{\text{c}}$ that has an upper stellar mass limit $m_{\text{max}}$ is $P(m, M_{\text{c}}) = P(m|M_{\text{c}}) P(M_{\text{c}})$, where $M_{\text{c}} = M_{\text{c},\text{max}}(m_{\text{max}})$. Integrating over all cluster masses and scaling to the correct number of stars then recovers equation (5).

The complete procedure is implemented in the following way: After the specification of an SFR for an individual star formation epoch, the resulting $M_{\text{c},\text{max}}$ (eq. [1]) is used to construct an ECMF with a predefined slope $\beta$. Each individual cluster in the ECMF is constructed from a predefined IMF up to a stellar mass limit determined by the mass of the individual cluster [$m_{\text{max}}(M_{\text{c}})$]. Then the separate IMFs of the clusters are added up to give the IGIMF for this epoch (eq. [5]). This is repeated with different SFRs to account for a varying SFH until the desired mass of the galaxy is reached at the desired age. The IGIMFs of the individual epochs are added to give the final IGIMF of the model galaxy.

It should be noted here that the IGIMF is not the PDFM, as stellar evolution is not included, but it is the galaxy-wide IMF (galaxial IMF), which may be used to estimate certain properties of galaxies (as in §3.1.2). The final IGIMF is, strictly speaking, only a theoretical construct because it counts all massive

The resulting equation, $m_{\text{max}}(M_{\text{c}})$, cannot be solved analytically. Weidner & Kroupa (2004) solve it numerically and show that an upper bound on $m_{\text{max}} \leq m_{\text{max}} \approx 150 M_\odot$ must exist, as otherwise there would be too many stars with masses larger than about 100 $M_\odot$ in the LMC starburst cluster R136. Weidner & Kroupa (2004) interpret $m_{\text{max}}$, to be a fundamental upper stellar mass.
stars irrespective of when they are formed. To quantify the stellar population at any given moment we would need to include stellar evolution.

3. RESULTS

Here we discuss the implications of our model for three different cases. In the first scenario (§3.1) so-called standard parameters are used (stellar IMF slope above $0.5 \, M_\odot$ being $\alpha_2 = \alpha_3 = 2.35$, and an ECMF slope $\beta = 2.35$ for $5 \, M_\odot \leq M_{\text{cut}}$, taking $\beta = \alpha_3$ for simplicity). In §3.2 we investigate a parameter set within the allowed range but which maximizes the effect on the IGIMF ($\alpha_3 = 2.70, \beta = 2.35$ for $5 \, M_\odot \leq M_{\text{cut}}$). Finally, a set of parameters out of the allowed ones that minimize the effect is studied in §3.3 ($\alpha_3 = 2.35$ and a two-part power-law ECMF with $\beta_1 = 1$ for $5 \leq M_{\text{cut}}/M_\odot \leq 50, \beta_2 = 2$ for $50 \, M_\odot \leq M_{\text{cut}}$), with some results also given for an ECMF truncated at $50 \, M_\odot$ with $\beta = 2$ for $M_{\text{cut}} \geq 50 \, M_\odot$.

3.1. The “Standard” Scenario

3.1.1. The IGIMF

Figure 2a shows the large differences of the final IGIMF for a galaxy with $M_{\text{gal}} \leq 10^3 \, M_\odot$ in stars. Depending on the SFH such a galaxy can be a dwarf spheroidal (dSph), a dwarf elliptical (dE), a dwarf irregular (dIrr), or a low surface brightness galaxy (LSB). We refer to such galaxies as “dwarf” galaxies. While in a single SF burst resulting in a dE system the IGIMF is populated up to the highest stellar masses, in the case of an LSB galaxy with a constant SFR the slope is much steeper and only stars up to $\sim 25 \, M_\odot$ ever form. Low-mass LSB galaxies thus appear chemically very young.

Considering more massive galaxies ($M_{\text{gal}} > 10^{10} \, M_\odot$ in stars; Fig. 2b), the variation of the IGIMF with the SFR is not as pronounced as before. The weak dependence on the SFH comes about because even the continuous SFR is high enough to sample the ECMF to massive clusters such that the massive stars end up being well represented. Note, however, that the resulting IGIMF is always significantly steeper than the canonical IMF, and equal to the canonical IMF below a few solar masses (Kroupa & Weidner 2003).

The variation of the IGIMF is best illustrated by comparing the slope, $\alpha_{IGIMF}$, of the resulting IGIMFs (Fig. 3). In order to fit IGIMF slopes consistently, the number of stars above $m_{\text{min}}$ (see Table 1) is calculated for each IGIMF up to the maximal mass of the corresponding model. This number, $N_1$, is compared with the corresponding number $N_2$ calculated for a representative IGIMF with a single slope $\alpha_{IGIMF}$ above $m_{\text{min}}$ up to the same mass limit. That $\alpha_{IGIMF}$ is chosen to represent the IGIMF for which $N_1 = N_2$. Figure 4 shows an example of the results of the automated fitting routine.

For dwarf galaxies a large difference is seen between models with continuous star formation (upper bound of the shaded area in Fig. 3) and a single, initial burst of star formation (lower bound of the shaded area). All other SFHs would produce results in between these two extremes.

3.1.2. Number of Supernovae of Type II

The steeper IGIMF slopes imply a less frequent occurrence of Type II supernovae (SNe II) in galaxies. To quantify this we calculate from the IGIMFs of each galaxy model the total number of stars formed above $8 \, M_\odot$ over a period of 14 Gyr. We then divide this number by the total number of stars in the galaxy in order to get the number of supernovae per star, $\text{NSNS}_{\text{IG}}$ (integrated galaxial NSNS). The $\text{NSNS}_{\text{IG}}$ is then divided by the NSNS for models in which the same mass in stars is distributed according to the canonical IMF ($\alpha_3 = 2.35$ for $m > 1.0 \, M_\odot$) to get the relative NSNS for the IGIMF models in comparison to the NSNS expected from applying the (incorrect) canonical IMF containing stars between 0.01 and 150 $M_\odot$. For this (incorrect) model in which the stellar upper mass limit does not depend on the SFR, the $\text{NSNS}_{\text{can}} = 0.003374$ SNe II per star. Thus, we have

$$\text{NSNS}_{\text{IG}} = \frac{\int_{m_\text{min}}^{m_{\text{max}}} \xi_{\text{IGIMF}}(m) \, dm}{\int_{0.01}^{m_{\text{max}}} \xi_{\text{can}}(m) \, dm},$$

and likewise for $\text{NSNS}_{\text{can}}$, where $\xi_{\text{IGIMF}}$ is replaced by $\xi(m)$.

The relative NSNS is then given by

$$\eta = \frac{\text{NSNS}_{\text{IG}}}{\text{NSNS}_{\text{can}}}. $$

The resulting $\eta$-values are shown in Figure 5 for $\beta = 2$ and 2.35. Two main effects are visible. First, the $\eta$ are always smaller than for models with a constant canonical IMF ($\eta < 1$). For example, for galaxies with a stellar mass of $10^7 \, M_\odot$ the actual
NSNS would be only 10% of the NSNS expected traditionally by adopting a canonical IMF. Second, there is a strong dependence on galaxy mass: \( \eta \) decreases substantially with decreasing galaxy mass, given that present-day galaxies are built up from dwarfs through hierarchical merging.

Note that the traditional calculation based on an invariant Salpeter IMF leads to a constant NSNS independent of the SFR or galaxy mass. Often a Salpeter IMF is used for stars between 0.1 and 100 \( M_\odot \). Such an IMF has the constant \( \text{NSNS}_{\text{Salp}} = 0.002512 \), which is 1.3 times smaller than the above constant NSNS, calculated for a constant canonical IMF (Salpeter above 0.5 \( M_\odot \)).

### 3.2. The “Maximal” Scenario

In this subsection we explore our so-called maximum scenario, in which we adopt a slope of \( \alpha_3 = 2.7 \) for the stellar IMF above 1.0 \( M_\odot \). Again we find that the effect is larger for less-massive galaxies.

#### Table 1

| \( \alpha_3 \) | \( \beta_1 \) | \( \beta_2 \) | \( M_{\text{d1,min}} \) (\( M_\odot \)) | \( m_{\text{knick}} \) (\( M_\odot \)) |
|-----------------|-------------|-------------|-----------------|-----------------|
| 2.35            | 2.35        | 2.35        | 5               | 1.307           |
| 2.70            | 2.35        | 2.35        | 5               | 1.120           |
| 2.35            | 1.00        | 2.00        | 5               | 1.307           |
| 2.35            | 2.00        | 2.00        | 5               | 1.307           |
| 2.35            | 2.35        | 2.35        | 50              | 5.560           |
| 2.35            | 2.35        | 100         | 8.823           |

**Note.**—For e.g., \( m_{\text{knick}} = 1.307 \) for the canonical models shown in Fig. 2; \( \beta_1 \) and \( \beta_2 \) are the power-law indices of the ECMF below and above 50 \( M_\odot \), respectively.

![Fig. 4.—Two examples of how \( \alpha_{\text{IGMF}} \) is fitted. The straight dotted line represents the canonical IMF (with \( \alpha_3 = 2.35 \)), while the other dotted lines show resulting IGIMFs for two models with \( M_{\text{gal}} = 10^7 \) \( M_\odot \). The left one has an input \( \alpha_3 = 2.70 \) and continuous star formation over 14 Gyr, and the right one has an input \( \alpha_3 = 2.35 \) and is formed in a single 100 Myr burst of star formation. In both cases the ECMF has a slope of \( \beta = 2.35 \). The solid lines show the fits derived from our algorithm based on the assumption that the number of stars beyond \( m_{\text{knick}} \) is equal for the corresponding solid and dotted lines.](https://example.com/f4.png)

![Fig. 5.—Relative NSNS in dependence of the stellar mass of the galaxy. The upper shaded area represents models with an ECMF slope \( \beta = 2 \), and for the lower area \( \beta = 2.35 \). The upper limit for each shaded region corresponds to a single-burst model, while the lower limit is for continuous SF models. The (input) IMF slope for stars above 1 \( M_\odot \) is \( \alpha_3 = 2.35 \) (canonical), and the \( \eta \) plotted here are relative to the NSNS, calculated for a constant canonical IMF (Salpeter above 0.5 \( M_\odot \)).](https://example.com/f5.png)
massive galaxies and that the dependence of the IGIMF on the SFR is weak for massive galaxies. Figure 6 illustrates the slopes of the IGIMF above $m_{\text{knick}}$ in the maximal scenario and the standard scenario introduced in § 3.1. Because of the steeper input slope, all resulting slopes are shifted to larger values by about 0.5 dex and the spread between single initial burst models and continuous SF models is somewhat larger for dwarf galaxies than in the standard scenario.

3.2.1. Number of Supernovae of Type II

The relative NSNS for the maximal scenario are shown in Figure 7, again for two cases of the ECMF slope $\beta$. As expected, $\eta$ drops in comparison to the standard scenario (Fig. 5), and in the case of low-mass galaxies $\eta$ can even become zero, meaning that there would be no SNe II in such galaxies if the SFR is low enough.

3.3. Other ECMF and the “Minimal” Scenario

In order to investigate the sensitivity of our results on a non-default ECMF at the lower mass end, three different assumptions for the ECMF are used: First, we flatten the slope below $50 M_\odot$ to $\beta_1 = 1$, with $\beta_2 = 2.35$ above $50 M_\odot$, and second, we use a lower mass cutoff in the ECMF at $50 M_\odot$ with $\beta = 2.35$ above. As a result, the downturn, $m_{\text{knick}}$, of the galaxial IMF is shifted to higher masses, but the deviations from the Salpeter value above $1 M_\odot$ remain large. Finally, we use a lower mass slope for the ECMF of $\beta_1 = 1$ below $50 M_\odot$ and additionally change the slope above to $\beta_2 = 2$ in order to explore a minimal case. Because of the smaller $\beta_2$ value of 2 in comparison with our other models, the deviation from the canonical value is small in both panels of Figure 8 for the initial burst models and for massive galaxies in general. Nevertheless, for dwarf galaxies the difference between continuous SF and an initial burst remains substantial.

3.3.1. Number of Supernovae of Type II

In Figure 9 $\eta$ is shown for a flat ($\beta_1 = 1$) ECMF below $50 M_\odot$ (Fig. 9a) and for a cutoff below $50 M_\odot$ (Fig. 9b). While in the first case $\eta$ is still very low (20%–40% of the corresponding canonical value), in the second case $\eta$ increases up to 60%, thus reducing the effect. In Figure 10 is $\eta$ shown for the minimal model with an ECMF slope of $\beta_1 = 1$ below $50 M_\odot$ and $\beta_2 = 2$ above $50 M_\odot$. Here the deviations from the invariant canonical model are only about 20% for massive galaxies but are still about 50% for dwarf galaxies. Thus, we find that even for assumptions that minimize the effects due to clustered star formation, $\eta < 0.8$ for all galaxies, with smaller values for less massive galaxies.
3.4. Comparison of the Scenarios

For the efficient construction of models (IGIMF vs. SFH) and for a useful comparison with observations we also compute the upper stellar mass limit and the IGIMF slope as functions of the galaxial SFR per epoch of 10 Myr duration (\(x = 2.1\)) for all three scenarios. The results are presented in Figure 11. The upper mass limits (Fig. 11a) are equal for the minimal and the standard scenarios. The slopes of the IGIMF (Fig. 11b) span a large range between the different scenarios, but in all cases steeper slopes are to be expected for low SFRs. The data plotted in Figure 11 allow the construction of IGIMFs without the need to perform the detailed modeling described in this paper. It is important to note here that the SFRs in Figure 11 are not the averaged SFRs of the galaxy but the SFRs during the star formation epoch. Thus, for example, a galaxy with \(M_{\text{gal}} = 10^7 M_\odot\) produced continuously over 1 Gyr has the same SFR of \(10^{-1} M_\odot/\text{yr}\) and the same IGIMF and \(\xi_{\text{IGIMF}}(m, t)\). The shaded areas enclosed by the dashed curves are the standard values from Fig. 5 for \(\alpha_3 = 2.35\) and \(\beta = 2.35\).

4. CONCLUSIONS AND DISCUSSION

We show that it is possible to explain varying stellar populations in galaxies by a simple mechanism based on universal
principles for all galaxies. Our assumptions are (1) that all stars are born in clusters following a universal embedded cluster mass function that is populated up to a maximum cluster mass, which depends on the star formation rate of a galaxy, and (2) that within these clusters all stars are born from a universal stellar IMF. But the magnitude of the effect strongly depends on the shape of the ECMF for low-mass clusters. The combination of a varying SFR with the empirical $M_{\text{cl, max}}(\text{SFR})$ relation, together with the stellar mass being limited by the cluster mass, naturally leads to a time-dependent integrated galaxial initial stellar mass function. This IGIMF also depends on the mass of the galaxy by virtue of the average level of the SFR being proportional to $M_{\text{gal}}$. The steep IGIMF and the variations—especially for dwarf galaxies—may have implications for the chemodynamical evolution of galaxies. But we note that this effect occurs in addition to standard (i.e., IMF-invariant) variations of the relative number of young and old populations as a result of varying SFRs, making the empirical detection of IGIMF variations challenging. We document in Figure 11 all quantities needed to construct, without additional computation, IGIMFs for galaxies of all morphological types and for a standard, a maximal, and a minimal scenario. This ought to be useful for future chemodynamical research within a hierarchical cosmological structure formation scenario.

In summary, given our assumptions the most important conclusions are

1. Chemical enrichment histories and SN rates calculated with an invariant Salpeter IMF may not be correct for any galaxy;
2. The number of supernovae is lower, and possibly significantly lower over cosmological times than for an invariant canonical IMF;
3. Irrespective of how old a galaxy is it will always appear less chemically evolved than a more massive, equally old galaxy as a result of the steeper IGIMF;
4. The scatter in chemical properties must increase with decreasing galaxy mass;
5. A steeper (input) IMF above $1 M_\odot$ would aggravate the systematic differences in galaxy properties when compared to a Salpeter IMF, as well as increasing the variations with galaxy mass.

Here galaxy mass refers only to the assembled mass in stars. Of future interest would be a direct observational test of this model. This may be possible by using deep luminosity functions of nearby galaxies. But these functions will reflect the PDMF, so stellar evolution and the SFH need to be taken into account in order to extract the IGIMF. The difficulty associated with such work is seen by several studies of starburst galaxies having found (IG)IMFs truncated at the lower mass end near a few solar masses (e.g., Wright et al. 1988), but more recent studies (Elmegreen 2005; Gouliermis et al. 2005 and references therein) showed that this seems not to be the case. In the same paper Elmegreen points out that a Salpeter IMF with a flattening below about 0.5–1 $M_\odot$ gives a good approximation for starburst galaxies. This does not necessarily contradict our result of a variable galaxy-wide IGIMF, as the observable part of the IGIMF in a starburst galaxy is dominated by a few young and massive clusters for which we adopt a Salpeter IMF in the high mass range. In our model, variations come from the fainter less massive clusters.

It should be noted here that for the more conservative approach of the minimal scenario our results still predict considerable differences in comparison with a constant canonical IMF. For example, Goodwin & Pagel (2004) make assumptions rather similar to our minimal scenario and predict a reduction of the general metal yield by a factor of about 1.8.

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