Results for the η’ Mass from Two-Flavor Lattice QCD

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1. Introduction

An accurate lattice calculation of \( m_{\eta'} \) would distinguish between QCD and quark models, and provide information relevant to the U(1) problem. Those attempts carried out so far [1], however, have employed the quenched approximation or used only one value of lattice spacing \( a \) for full QCD. In this work we make a step forward by using \( N_L = 2 \) full QCD at several \( a \), and evaluating \( m_{\eta'} \) in the continuum limit.

A technical difficulty which is important to overcome is the poor signal-to-noise ratio of the \( \eta' \) propagator \( G_{\eta'}(t) \). Its error increases quickly with the time separation \( t \) so that an effective mass plateau is not in general observed when a point quark source is used. One sometimes fits \( m_{\eta'} \) from the ratio \( G_{\eta'}(t)/G_{\pi}(t) = 1 - A \exp((m_{\eta'} - m_{\pi})t) \), hoping for cancellation of effects of excited states. However there is no rationale behind this expectation. We improve upon this by employing a smearing method which enables us to obtain ground state signals in the \( \eta' \) channel for all simulation parameters.

After a description of our method in Sec. 2, we evaluate in Sec. 3 \( m_{\eta'} \) including systematic error from chiral and continuum extrapolations. A correlation between topology and \( m_{\eta'} \) which we have observed is presented in Sec. 4.

2. Calculational Method

We use \( N_f = 2 \) gauge configurations previously generated for our study of the spectrum of ordinary hadrons [2], with an RG-improved gluon action and clover quark action with tadpole-improved \( c_{sw} \). We use three couplings \( \beta = 1.8, 1.95 \) and 2.1 corresponding to \( a \approx 0.22, 0.16, 0.11 \) fm, and four hopping parameters \( \kappa \) matched to \( m_{\pi}/m_{\rho} \approx 0.8, 0.75, 0.7, 0.6 \). The box size \( L_s a \approx 2.5 \) fm has also been matched between \( \beta \). Valence and sea quark masses are identical. We take \( (\bar{u}u + d\bar{d})/\sqrt{2} \) as an operator for the \( \eta' \).

\( G_{\eta'}(t) \equiv G_1(t) - G_2(t) \) is calculated on 400–800 stored configurations for each \( (\beta, \kappa) \) pair, where \( G_1(t) \equiv G_{\pi}(t)G_2(t) \) is the contribution of the one- (two-) loop diagram. We use an exponential-like smearing function given in Ref. [2] for both \( G_1 \) and \( G_2 \). One or two quark
sources are smeared, while sink is always local. Errors are estimated by the jackknife method using bins of 50 trajectories. The lattice scale is fixed from $m_\rho$.

After fixing configurations to the Coulomb gauge, we estimate $G_2(t)$ using a noisy source method with $U(1)$ random fields. In order to enhance signals, inversions for the quark propagator are carried out independently for each (color, spin) index. Fig. 1 obtained for the coarsest lattice at $\beta = 1.8$ shows that increasing the number of noisy samples from $N_{\text{noise}} = 3$ to 20 gives no substantial gain in precision in $m_{\eta'}$. We thus choose $N_{\text{noise}} = 3$ for finer lattices at $\beta = 1.95$ and 2.1. At $\beta = 1.8$, we use results with $N_{\text{noise}} = 8$, giving the smallest error in the chiral limit.

Fig. 2 compares effective masses of $m_{\eta'}$ obtained from one-quark smeared and local sources. (Local results plotted are calculated on the fly by the volume source method without gauge fixing, and have smaller errors than those calculated from noisy sources; these have already been reported in Ref. [3].) Smearing has the clear effect of ensuring a plateau from $t \approx 1.5$ for all parameter values. Therefore $m_{\eta'}$ is determined by fitting smeared $G_{\eta'}$ for $t_{\text{min}} \geq 1$ using a single hyperbolic cosine function. We note for this smeared data that fitting the ratio $G_{\eta'}/G_\pi$ gives a result consistent with the direct fit as shown in Fig. 3. The deviation remains within 8%.

3. $\eta'$ Meson Mass in the Continuum Limit

We test two functional forms for chiral extrapolation, 1) $(am_{\eta'})^2 = A + B(am_\pi)^2$ (Goldstone
boson type: ‘GB’) and 2) $a m_{\eta'} = A + B (a m_\pi)^2$ (non-GB type) and find that both reproduce data equally well (see Fig. 4 for example) with comparable $\chi^2/df = 0.9\,1.1$ (GB) and $0.4\,2.2$ (non-GB). GB fit is used to determine central values, while non-GB fit is used for error estimation.

Fig. 5 shows $m_{\eta'}$ as a function of $a$. Continuum extrapolation is carried out linear in $a$, since $O(a)$ error remains in our action combination. We also try a constant plus quadratic form, since most $O(a)$ effects may be removed by use of the improved action. Finally, since $m_{\eta'}$ hardly changes over the finest two lattices, we make a constant fit removing the coarsest point. The latter two are used to estimate systematic error.

Systematic errors arising from chiral and continuum extrapolations are added in quadrature, separately for upper and lower sides. The final result reads

$$m_{\eta'} = 0.960(87)^{+0.036}_{-0.286} \text{ GeV}$$

(1)

4. Correlation with Topological Charge

The large $\eta'$ mass compared to the pion octet may originate from the topological structure of QCD. Using results for topological charge [4] $|Q_{\text{ topo}}|$, we split the measurements of the $\eta'$ propagator into two equally sized bins of high and low $|Q_{\text{ topo}}|$, and measure $m_{\eta'}$ on each bin. As shown in Fig. 6, configurations with high $|Q_{\text{ topo}}|$ give large values of $m_{\eta'}$ for lighter quarks, though interpretation of results remains as open problem.

Figure 5. $m_{\eta'}$ vs. $a$ with linear, quadratic and constant continuum extrapolations.

Figure 6. $m_{\eta'}$ from configurations with high and low $|Q_{\text{ topo}}|$ normalized by the value for all configurations.

5. Conclusions

Our value $m_{\eta'}$ for $N_f = 2$ full QCD in the continuum limit, Eq. 1, turns out to be consistent with the experimental value, though there still remains considerable numerical uncertainty, particularly on the lower side ($-30\%$), coming from the continuum extrapolation. Additionally, our result contains systematic error from quenching effects of the strange quark and from neglecting mixing with the $\bar{s}s$ state. Calculations are currently being performed analysing mixing between quark-based states and mass eigenstates for $N_f = 2$ QCD.

This work is supported in part by Grants-in-Aid of the Ministry of Education (Nos. P01182, 11640294, 12304011, 12640253, 13640259, 13640260, 14740173). VIL is a JSPS Research Fellow.

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