Detecting fundamental fields with LISA observations of gravitational waves from extreme mass-ratio inspirals

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The Laser Interferometer Space Antenna, LISA, will detect gravitational wave signals from extreme mass-ratio inspirals, where a stellar mass compact object orbits a supermassive black hole and eventually plunges into it. Here we report on LISA’s capability to detect whether the smaller compact object in an extreme mass-ratio inspiral is endowed with a scalar field, and to measure its scalar charge—a dimensionless quantity that acts as a measure of how much scalar field the object carries. By direct comparison of signals, we show that LISA will be able to detect and measure the scalar charge with an accuracy of the order of per cent, which is an unprecedented level of precision. This result is independent of the origin of the scalar field and of the structure and other properties of the small compact object, so it can be seen as a generic assessment of LISA’s capabilities to detect new fundamental fields.

Such fields, and scalars in particular, are ubiquitous in extensions of general relativity (GR) or the standard model. Depending on the model, they might be low-energy classical remnants of quantum gravity; they might address the hierarchy problem or other naturalness problems in the standard model; or they may explain the accelerated expansion of the universe or account for dark matter. So far, searches for new fundamental fields in the vicinity of astrophysical objects with weak gravitational fields, or small spacetime curvatures, have yielded strong constraints rather than detections. However, gravitational wave (GW) observations are starting to probe for the first time strong gravitational fields, or larger spacetime curvature, and there is hope that new fields will be detected in this regime.

This hope is not based purely on pragmatism related to the strong constraint from weak-gravity experiments. From a theoretical standpoint, it is quite reasonable and intuitive to expect that deviations from GR, or in the interactions between gravity and new fields that are part of some extension of the standard model, will be more prominent at larger curvatures. Indeed, this is the behaviour one tends to find in models where black holes (BHs) or compact stars are known to exhibit different structure than their GR counterparts. Moreover, there are specific effects, such as super-radiance or spontaneous scalarization, that can render new fields detectable only in the vicinity of compact objects.

In this context, extreme mass-ratio inspirals (EMRIs) are perhaps somewhat special. Although both of their constituents are compact objects and one tends to associate a strong gravitational field to both of them, they have very different masses and, hence, very different characteristic curvatures. The central (primary) object is expected to be a supermassive BH (with mass above $M \approx 10^5$ solar masses, $M_\odot$) and the curvature near the horizon will thus be very small (it scales as one over the mass squared). The small (secondary) object, irrespective of what it is, has much larger characteristic curvature. Combined with the above, this implies that, at least for some sizeable subset of EMRIs, the primary object can be taken to be a Kerr BH to a high degree of precision and any potential deviation will only affect the smaller secondary object (Methods). This observation introduces an important simplification both in the modelling of the signal in the presence of new fundamental fields and in the conceptual understanding of how deviations in the signal arise.

Let us consider an EMRI in which the secondary object (which can be a neutron star, a black hole or something more exotic) moves in the spacetime generated by the primary body and is endowed with a massless scalar field configuration. In a local reference frame $\{x^\mu\}$ centred on the secondary body the scalar field has the form

$$\varphi = \varphi_0 + \frac{\mu d}{\tilde{r}} + O\left(\frac{\mu^2}{\tilde{r}^2}\right) \quad (1)$$

where $\tilde{r}$ is the radial coordinate, $\mu \ll M$ is the mass of the secondary body, $d$ is its dimensionless scalar charge, $\varphi_0$ is the asymptotic value of the scalar field, and we are using geometric units $G = c = 1$, with $G$ and $c$ being the gravitational constant and the speed of light, respectively. As the secondary body orbits the supermassive Kerr BH, it will emit GWs. Its motion can be approximated by that of a point particle moving in the ‘exterior’ Kerr spacetime, described by a set of global coordinates $\{x^\nu\}$ (for example, the Boyer–Lindquist coordinates; Methods). In this frame, the worldline is described by functions $\nu^\mu(\lambda)$, where $\lambda$ is the affine parameter. The flux for the polarizations of the emitted GWs will be virtually unaffected by the fact that it carries a scalar charge, as will be discussed in more detail in Methods. It can hence be computed using state-of-the-art EMRI models. However, the secondary body is accelerating due to gravity and accelerating charges emit radiation. Hence, on top of the standard GW emission, there will be also scalar radiation throughout the inspiral.
Due to its scalar charge, the secondary body acts as a source of the scalar field equation. In the setup described above, the latter takes the form

$$\Box \phi = -4 \pi \mu \int \frac{\delta^{(4)}(x - y_p(\lambda))}{\sqrt{-g}} \, d\lambda,$$

where $\Box$ is the D’Alambertian operator for the Kerr metric and $g$ is the metric determinant (Methods). This equation can be used to compute the scalar field flux, $E_{\text{scal}}$, which is proportional to the scalar charge squared. The orbital energy of the particle decreases due to the total energy emission:

$$\dot{E}_{\text{orb}} = -\dot{E}_{\text{grav}} - \dot{E}_{\text{scal}},$$

where $\dot{E}_{\text{grav}}$ is the gravitational flux. The scalar flux affects the orbital evolution of the secondary body. Since the GW phase is determined by the orbital evolution, the scalar-field emission contributes to the GW phase of the EMRI. LISA is expected to constrain the source parameters from the EMRI evolution with exquisite precision (see Supplementary Information for further details). Thus, it will also be able to measure the scalar charge of the secondary body, if present.

Hereafter, we study prototype EMRIs in which the primary body is a supermassive BH of $M = 10^6 M_\odot$ and dimensionless spin $\chi = J / M^2 = 0.9$, where $J$ is the BHs angular momentum. We take the secondary body to be a compact object with mass $\mu = 10 M_\odot$. We track the evolution of a binary on a circular equatorial orbit for one year before the plunge, that is, we choose the initial radial position $r_o$ such that, after a year, the secondary body is within a distance of 0.1$M$ from the innermost stable circular orbit (ISCO). Further details on the source parameters are discussed in Methods. We shall consider values of the signal-to-noise ratio (SNR) for LISA detection of EMRIs ranging from 30 to a few hundred, reflecting conservative or more optimistic expectations based on rather uncertain event-rate estimates for EMRIs.

A preliminary assessment of the detectability of the scalar charge can be made using the evolution of the phase of the GW signal. The fact that the phase difference between binaries with and without scalar charge exceeds a certain threshold is an indication that the scalar charge should be detectable by LISA after 12 months of observation. We have calculated the phase difference and it is indeed above the threshold even for values of the scalar charge as small as $d \approx 5 \times 10^{-4}$. More details are given in Methods.

A more quantitative analysis on LISA’s ability to detect a scalar charge is given in Fig. 1, which shows the faithfulness $F$ between two GW signals emitted by binaries with and without the charge. The faithfulness (see Methods for a precise definition) provides an estimate of how much two signals differ, weighted by the noise spectral density of LISA. Given the SNR $\rho$ of a signal, values of $F$ smaller than $-1 - D(2\rho^2)$, with $D$ dimension of the model (−10), indicate that the two waveforms are significantly different and don’t provide a faithful description of one another20–22. For $\rho = 30$ this requirement translates into $F \lesssim 0.988$. Figure 1 shows the values of $F$ for the chosen prototype of binary configuration. After one year the faithfulness is always smaller than the threshold set by $\rho = 30$, even for scalar charges as small as $d \gtrsim 0.01$. For the same configuration, on a period of just six months before the plunge, the faithfulness is already below the threshold for charges of $d \gtrsim 0.05$.

Fig. 1 | Faithfulness between the GW plus polarization computed with and without the scalar charge, as a function of the latter and for different signal durations. The signal duration is measured in months (6 or 12) before the plunge. The grey horizontal line identifies the threshold of distinguishability, $F \lesssim 0.988$, set up by an SNR of 30.

The dephasing and the faithfulness, however, don’t fully take into account possible degeneracies among the waveform parameters, which may jeopardize our ability to constrain the scalar charge. A more sophisticated study requires a joint investigation of the full parameter space, which includes correlations among $d$ and other quantities characterizing the EMRI GW emission. In the following, we perform such an analysis, assessing the capability of LISA to perform an actual measurement of the scalar charge.

Figure 2 shows the probability distribution obtained using a Fisher matrix approach (Methods) for the component masses, the spin of the primary and the scalar charge of the secondary for EMRIs observed one year before the plunge with $d = 0.05$ and an SNR of 150. This analysis shows that a single detection can provide a measurement of the scalar charge with a relative error smaller than 10%, with a probability distribution that does not have any support on $d = 0$ at more than 3σ. Off-diagonal panels, yielding 68% and 98% joint probability confidence intervals between the source parameters, also show that the charge is highly correlated with the secondary mass and anti-correlated with the spin parameter and the mass of the primary.

Figure 3 shows the error in the scalar charge as a function of the scalar charge itself, for EMRIs detected by LISA with SNR ranging from 30 to 150. The errors on $d$ can also be accurately fitted with a simple law of the form $\sigma_d^\text{fit} = \beta / d$, where $\beta = 2.09 \times 10^{-3}$ ($\beta = 4.18 \times 10^{-3}$) for an SNR of 30 (150). In the top panel we show the relative error $\sigma_d / d$ and the analytical fit; in the bottom panel we show the 3σ intervals around the injected values of the scalar charge.

Our analysis shows that one year of EMRI observation can pinpoint a scalar charge smaller than $\sim 0.3$ with per cent accuracy. For an SNR of 30 a charge $d \approx 0.1$ could be constrained to consistently exclude the value $d = 0$. For the louder signals we consider, LISA could constrain a scalar charge as small as $d \approx 0.05$ to be inconsistent with 0 at 3σ confidence level.

Detecting and measuring the scalar charge of a compact object would be of enormous importance, as first evidence of new physics, regardless of the origin of the charge or the nature of the compact object. Indeed, so far, our analysis and results have been theory-agnostic. However, it is worth pointing out that in many cases the scalar charge is uniquely determined by theoretical parameters that mark deviations from GR or the standard model. In such cases, a measurement of the scalar charge can be used to measure these parameters. LISA will provide impressive precision for that.
Let us demonstrate this point using a simple but characteristic example. Assume that the secondary body is a black hole and the scalar field is massless (shift-symmetric). No-hair theorems then dictate that there cannot be a scalar charge unless the scalar field couples to the Gauss–Bonnet invariant, 
\[ R_{GB} = R^2 - 8R_{\mu \nu}R^{\mu \nu} + R_{\mu \nu \alpha \beta}R^{\mu \nu \alpha \beta} \]
(where \( R_{\mu \nu \alpha \beta} \) and \( R_{\mu \nu} \) are the Riemann and the Ricci tensor, respectively, and \( R \) is the Ricci scalar), as follows \( \alpha \phi R_{GB} \), where \( \alpha \) is the new coupling constant. In this case, the relation between \( \alpha \) and the scalar charge \( d \) of a BH has the simple form \( \alpha \approx 2d\mu^2 - 73d\mu^2/240 \) (ref. 21).

To study the constraints on \( \alpha \) from LISA observations, we draw \( N = 10^5 \) samples of \((\mu, d)\) from the joint probability distribution of the secondary mass and scalar charge obtained from the Fisher analysis. We then compute \( N \) values of \( \alpha \) building the corresponding probability density functions. Figure 4 shows the probability distribution \( \mathcal{P}(\sqrt{\alpha}) \) for our prototype EMRIs, for \( d = 0.05 \) and \( d = 0.2 \). Vertical lines in each panel identify the 90% confidence intervals of the coupling constant. Even for \( d = 0.05 \) the probability distribution does not have support on \( \alpha = 0 \). This analysis demonstrates that, in theories where the scalar charge is determined by theoretical parameters, EMRI observations by LISA can be used to measure these parameters with unprecedented accuracy.

In summary, our results demonstrate that EMRI observations by LISA will be able to detect and potentially measure scalar charges to exquisite accuracy. Our analysis and results are independent of the origin of the charge and are hence theory-agnostic. We have also shown that a further analysis can allow one to measure the coupling parameters for specific theories.

This is the first attempt to perform a rigorous estimation of the measurability of beyond-GR effects with EMRIs. The EMRI template we developed here is only the starting point of a more refined analysis, which is necessary to assess LISA’s full potential to detect fundamental fields and new physics beyond GR. A number of constraints on \( \alpha \) were derived from \( \sqrt{\alpha} \), where \( \alpha \) is the coupling constant.
of improvements to this template would need to be made before it can be used for data analysis. These include using more generic (eccentric, non-equatorial, inclined) orbits\textsuperscript{19-22}, taking into account finite size effects and a full self-force treatment or considering further beyond-GR and standard model effects, such as dark matter spikes or superradiant clouds. Environmental effects, as produced by accretion disks, should also be considered, to study possible degeneracies with the scalar emission. However, such effects would in general carry a specific frequency content, allowing them to disentangle or partially alleviate correlations\textsuperscript{27}. On the data analysis side, the next step is to perform Bayesian inference\textsuperscript{28}. Although producing more realistic templates is a major technical challenge, we expect that such templates would break degeneracies between source parameters and could potentially enhance detection capabilities. Finally, data gaps, that is, interruptions in the interferometric measurements due to various astrophysical and instrumental factors, may provide important challenges to be overcome. Studies in this context are currently underway, showing how Bayesian methods can solve this problem and improve detection efficiency\textsuperscript{29}.

**Methods**

**Theoretical setup.** In this Letter, we are assuming that the primary BH object is described, with good accuracy, by the Kerr metric, and that the particle with scalar charge is a massless scalar field and \(\kappa\) is a coupling constant (or otherwise GR modifications would show up in the scale of the particle itself. Finite-size effects appear at higher order compared with the leading, dissipative contribution we consider, and thus can be neglected\textsuperscript{30}. This is an accurate approximation because the gravitational field of the secondary body is large only within a world-tube whose radius is much smaller than the characteristic length of the ‘exterior’ spacetime, that is, the spacetime generated by the primary body; thus, this world-tube can be treated as a world-line \(j_{\gamma}^{\mu}(\lambda)\) in the exterior spacetime. By integrating the matter action \(S_m\) over this world-tube, it reduces to the ‘particle action’

\[
S_{\gamma} = - \int m(\phi) d\lambda = - \int m(\phi) \sqrt{g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}} d\lambda .
\]

where \(d\lambda\) is the invariant line element. Assume now that the stationary BH solutions of the theory in equation (1) are continuously connected with the corresponding GR solution as \(\alpha \to 0\), and that \(S_{\gamma}\) is analytic in \(\phi\). Then the exterior metric reduces to the Kerr solution for \(\alpha \to 0\) and its only dimensionful parameter is the mass \(M\). The corrections to the Kerr metric depend on the dimensionless parameter

\[
\zeta \equiv \frac{\alpha}{M} = \frac{a}{\mu} \frac{\alpha}{\mu} .
\]

where \(q = \mu/M \ll 1\). Since \(\alpha/\mu < 1\) (otherwise GR modifications would show up in current astrophysical observations), \(\zeta \ll 1\) and the exterior spacetime can be approximated with the Kerr metric.

The gravitational field equations are

\[
G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{2M}{\lambda^2} + \alpha T_{\mu\nu} + \frac{1}{8\pi} \delta(\phi)^2 .
\]

where \(T_{\mu\nu}^{\text{scal}} = \frac{1}{2\kappa} \partial_{\alpha} \phi \partial^\alpha \phi - \frac{1}{4} g_{\mu\nu} (\delta \phi)^2\) is the stress-energy tensor of the scalar field, and

\[
T_{\mu\nu}^{\text{em}} = 8\pi \int m(\phi) \frac{\delta(\phi)(x - \gamma(\lambda))}{\sqrt{-g}} \frac{dy^\rho}{d\lambda} \frac{dy^\sigma}{d\lambda} d\lambda d\sigma
\]

is the stress-energy tensor of the particle. The scalar field equation is
where \( m'(\phi) = 4m(\phi)/d\phi \) and the \( \square \) operator is evaluated on the Kerr background.

In geometric units, \( S_\ell = (\text{mass})^2 \) and \( S_\ell = (\text{mass})^{-2} \). Since in the Kerr background the only dimensionful scale is the BH mass \( M \) (in addition to the angular momentum \( J \), which is anyway bounded by \( M^2 \)), we expect that \( S_\ell = M^2 S_0 \). Thus, \( \alpha T_\ell m \approx \zeta \gamma_\ell m < G_M \) and \( \alpha_0 S_\ell \approx \zeta_0 \gamma_\ell m \approx \square \phi \). The scalar field stress–energy tensor is also negligible. Indeed, in the interior Kerr spacetime, the scalar field (due to the aforementioned no-hair theorems) has to be a constant, \( \phi_0 \), and \( T^s_{\mu\nu} \) is quadratic in perturbations around \( \phi_0 \).

We can conclude that, under our approximations, the gravitational field equations are the same as in GR, while the scalar field equation acquires the source term on the right-hand side of equation (10). By replacing equation (1), we find \( m'(\phi)/\phi = -d/4 \), and since in the weak-field limit the stress–energy tensor of the particle reduces to its matter density, equation (9) leads to \( m(\phi) = \mu \). Therefore, the scalar field equation for an EMRI in the class of theories we consider reduces to equation (2).

Technically speaking, our analysis does not apply to EMRIs if the primary BH carries a appreciable scalar charge or deviates strongly from Kerr. However, it is important to emphasize that in both of these cases, which may already be difficult to reconcile with current observations, one expects deviations from GR to be even larger: orbital dynamics would be further affected by the fact that the secondary object is moving in a different spacetime. Hence, our results can certainly be seen as a conservative estimate of LISAs capabilities to detect scalar fields.

### Waveforms.

We study the adiabatic evolution of EMRIs within the framework of perturbation theory, in which the secondary orbits in the Kerr background are generated by the supermassive BH. Relativistic perturbations of the scalar and tensor sectors can be analysed by expanding the gravitational and scalar fields, together with the source term, in spin-weighted harmonics\(^{39}\)

\[
\psi^{(s)}(t, r, \theta, \phi) = \int d\omega \int \frac{d^2 \ell}{(2\pi)^2} R^{(s)}_{\ell m}(r, \theta) S^{(s)}_{\ell m}(\theta, \phi) \sin \theta \, e^{i m \phi - i \omega t}.
\]

where tensor and scalar perturbations correspond to \( \omega = -2 \) and \( \omega = 0 \), respectively, \((t, r, \theta, \phi)\) are the Boyer–Lindquist coordinates, \( R^{(s)}_{\ell m} \) is the radial part of the metric perturbations, and \( S^{(s)}_{\ell m} \) are the spherical functions, specified by the mode index \( \ell \) and the azimuthal number \( m \). For the scalar case, \( \psi^{(s)} \equiv \psi \). The expansion yields a decoupling of the field’s radial and angular dependence, with the latter being described by the spherical harmonics equation. The radial component satisfies the Teukolsky equation\(^{24}\)

\[
\square_r + \frac{\Delta}{r^3} + \frac{2\ell(\ell + 1)}{r^2} R^{(s)}_{\ell m} = 0.
\]

where \( \Delta = r^2 - 2Mr + \gamma_\ell^2M^2 \), \( K = (\gamma + \gamma_\ell^2M^2)/(\gamma_\ell M) \), and \( \gamma = J/M^2 \), \((s, \lambda)\) are the spin-weight of the perturbed field and the spin-weighted spherical eigenvalue and \( \omega \) is the mode frequency. For the gravitational field equation, the source term \( T^{(s)}_{\ell m} \) is a combination of the components of the stress–energy tensor, expanded in spin-weighted harmonics. For the scalar field equation, assuming that the particle with scalar charge \( d \) moves in equatorial circular motion

\[
r^{(s)}_{\ell m}(r, t) = -4\pi \mu_d \delta(\omega - m\Omega) \delta(r - r_p) \delta^{(s)}(t - t_p) + \psi^{(s)}_{\ell m}(r, \theta, \phi).
\]

the conjugate of the scalar gravitational harmonic computed in \( \theta = \pi/2, \ell \) stands for the derivative of the Boyer–Lindquist coordinate \( t \) with respect to the proper time \( r_p \) is the radius of the circular orbit and

\[
\frac{d\Phi}{dt} = \pm \frac{M^{\ell/2}}{r^3} \left( \ell M^{\ell/2} \right)^{\ell/2
\end{eqnarray}

is the orbital frequency, where the plus (minus) sign holds for prograde (retrograde) orbits. In the following, we will consider prograde orbits only. Solutions of equation (16) can be computed by first substituting

\[
\psi^{(s)}_{\ell m}(r, \theta, \phi) = \int d\omega \int \frac{d^2 \ell}{(2\pi)^2} \psi^{(s)}_{\ell m}(\omega, \ell, m, \theta, \phi) e^{i \omega t - i \ell \theta}.
\]

which satisfies the boundary condition of purely outgoing wave at the horizon, and \( \psi^{(s)}_{\ell m} \), which satisfies the boundary condition of purely ingoing wave at infinity. The full solutions at infinity and at the horizon are obtained by integrating over the source term \( T^{(s)}_{\ell m} \)
Parameter estimation. We consider EMRIs in which the secondary body is moving on equatorial circular orbits around the primary black hole. In the time domain, the GW signal, equation (22), is completely determined by 11 parameters $\theta = (ln M, ln \mu, \mu, ln D, \theta_s, \theta_c, \theta, r_\ast, \phi_s, a, \Psi_0)$. We have considered EMRIs with $M = 10^9 M_\odot, \psi = 0.9, \mu = 10 M_\odot$. Varying the scalar charge $d$ and fixing the source angles as $\theta_s = \phi_s = \pi/2$, $\theta = \phi = 0$. We neglect the spin of the secondary body.\(^{20,23}\)

The initial phase has been set to $\Phi_0 = 0$, while the initial orbital separation is adjusted, depending on $d$, to have one year of evolution before the plunge $r_{\text{inj}} = r_{\text{inj}}(d)$, where $r_{\text{inj}} = 2.32$ for a Kerr BH with $\psi = 0.9$. We have fixed the radius shift to $\delta r/M = 0.1$, which is more conservative than the transition region between the inspiral and the plunge as has been described elsewhere.\(^{46}\)

The luminosity distance $D$ is a scale factor for $h(t)$ and can be changed freely to vary the SNR of the signal (see below).

Given the waveform model we can now introduce the noise-weighted inner product between two templates

$$\langle h_1 | h_2 \rangle = 4\pi \int_{\mathbb{R}} h_1^*(f) h_2(f) S_N(f) d f,$$

where $h(f)$ is the Fourier transform of the time-domain signal, $h^*(f)$ is its complex conjugate, and $S_N(f)$ is the LISA noise spectral density.\(^2\)

We sample the signal, equation (22), in the time domain, and then apply a discrete Fourier transform, evaluating the integral, equation (23), between $f_{\text{low}} = 10^{-13} \text{Hz}$ and $f_{\text{high}} = f_{\text{Ny}}$, with $f_{\text{Ny}}$ being the Nyquist frequency. The component related to the latter has been set to zero, and only Fourier components with $|f| f_{\text{Ny}}$ have been included. Before passing to the frequency space, we taper $h(t)$ with a Tukey window with cosine fraction $\tau = 0.05$.

Equation (23) allows us to determine the irreducible domain

$$\mathcal{F} \left\langle h_1 | h_2 \right\rangle = \max_{(t, \phi)} \left( \frac{h_1^*(t, \phi) h_2(t, \phi)}{\sqrt{|h_1^*(t, \phi) h_2(t, \phi)|}} \right),$$

where $(t, \phi)$ are time and phase offsets.\(^2\)

The SNR for a specific choice of the source parameters reads $\rho = (tb(h))^2$. In the limit of large $\rho$, the posterior probability distribution ($\rho$) of the source parameters, assuming flat or Gaussian priors on $\theta$ and given a certain observation $b$, can be approximated by a multivariate Gaussian distribution centred around the true values,\(^2\) with covariance given by the inverse of the Fisher information matrix $\Gamma_i$:

$$\log p(\theta) \propto \log p_0(\theta) - \frac{1}{2} \Delta \Gamma_i \Delta,$$

where $\rho$ is the parameter's prior distribution, $\Delta = \hat{\theta} - \bar{\theta}$ is the shift between the measured and the true values $\bar{\theta}$ and

$$\Gamma_i = \frac{\partial h}{\partial \theta_i} \frac{\partial h}{\partial \theta_j} |_\theta = \hat{\theta}.$$

Statistical errors on $\theta$ and coefficients among parameters are provided by diagonal and off-diagonal components of the inverse of the Fisher matrix

$$\Sigma = \Gamma^{-1},$$

Given the two-interferometer configuration for the LISA detector, we can define a total SNR $\rho = \sqrt{\rho_1^2 + \rho_2^2}$ and a total covariance matrix of the binary parameters obtained by inverting the sum of the Fisher matrices $\sigma^2_{ij} = (\Gamma_1^{-1} + \Gamma_2^{-1})^{-1}$.

Since waveforms are generated fully numerically in the time domain, derivatives appearing in the Fisher matrix are also numerical. We use a centred 11- and 9-point stencil for $(M, \mu, \mu, r_\ast, \phi_s, \phi, a, \Psi_0)$ and $d$. For the luminosity distance $D$ and the initial phase $\Phi_0$, analytical expressions of $\partial h/\partial \theta_i$ can be computed.\(^\dagger\)

Integrals are performed through the Boole rule. In our 11 $\times$ 11 parameter space, inversion of the Fisher matrices may depend on the value of the numerical displacement chosen to compute finite difference of $h(t)$ for a specific parameter, due to numerical instability. Indeed, Fisher matrices for EMRIs are known to feature large condition numbers $\kappa = \max_j (\lambda_j/\min_j)$, that is, the ratio between the largest and the smallest eigenvalues.\(^2\)

We compute $\Gamma_i$ using high-precision numerics with fluxes obtained through the Black Hole Perturbation Toolkit (http://bhptoolkit.org/) with 300 digits of input precision, which lead to Fisher matrices of $\sim 185$ digits of final precision and $\kappa = \mathcal{O}(10^{14})$. Calculations of $\Gamma$ and its inverse are extremely stable, with respect to discrepancies among various Fisher matrices derived with different numerical shifts of $\pm 0.1\%$. Differences in the source parameters and in the correlation coefficients are also very small, $\lesssim 1\%$ (see Supplementary Information for further details).

**Data availability**

The data that support the plots within this paper and the other findings of this study are available from the corresponding author upon request.

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