Diquark interactions in quark-gluon plasma and their role in diquark stars

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Abstract. Within the framework of a polynomial field theory the diquark interaction energies in quark-gluon plasma are explicitly calculated. In particular, two- and three-diquark interaction energies are computed using theory and terms in the effective Lagrangian and their role in the stability of diquark stars is pointed out.

1. Introduction
It is well known that a composite structure of two quarks, called ‘diquark’ has played an important role in the studies of quark structure of hadrons [1]. The diquark system has also played an important role in recent years [2, 3, 4] in the studies of quark-gluon plasma (QGP). While the context in which the diquarks are used in the studies of elementary particle physics is slightly different, the role of diquarks is investigated by Donogue and Sateesh (DS) mainly within the framework terms in the effective Lagrangian density

\[ L_{\text{eff}} = \frac{1}{2} \left( \partial_{\mu} \phi^+ \partial^{\mu} \phi - m^2_{\phi} \phi^2 \right) - \lambda (\phi^+ \phi)^2 - \beta (\phi^+ \phi)^3. \]  

(1)

In this description the role of a particular type of diquarks, namely, spin-zero, color antitriplet diquarks has been investigated. On the other hand this field-theoretic description of a system of diquarks, however, requires the involvement of all types of diquarks. In other words, it becomes rather essential to account for the plurality of color-spin states of two- and three-diquark states which, in fact, corresponds to terms in (1), respectively.

In this paper, we highlight the relative importance of these states by way of investigating the interaction energies between the quarks. For this purpose, we work within the framework of a constituent quark model and make use of generalized Pauli principle.

2. Single diquark and double-diquark interaction energies
There exist four possible states of a pair of quarks (i.e., diquark) with color and spin quantum numbers \( \{C, S\} = \{\overline{3}, 0\}, \{\overline{3}, 1\}, \{6, 0\} \) and \( \{6, 1\} \). Here, assuming that only (u, d) flavor \( I = 1/2 \) are present in QGP and taking the spatial configuration as symmetric, we write the interaction Hamiltonian in the spirit of constituent quark model as

\[ H_{\text{int}} = -A \sum_{i,j} (\tilde{\lambda}_i \tilde{\lambda}_j)(\tilde{\sigma}_i \tilde{\sigma}_j), \]  

(2)

where the indices \( i, j \) are used to label the interacting quarks and \( \tilde{\sigma} \) and \( \tilde{\lambda} \) are the usual Pauli spin and Gell-Mann color SU(3) matrices. In (2), the strength parameter \( A \) is determined to be 75/4 using the known values of \( N \) and \( \Delta \) masses. Using (2), we have computed the interaction energy corresponding to different color-spin states of single diquark and the results obtained are just half of the corresponding values given by DS [3]. The reason for this is that we have avoided the double counting of the terms while carrying out operation with \( H_{\text{int}} \).
For the computation of interaction energies between two-diquark composites the following irreducible representations in both the color and the spin-space of the dq-dq system are considered:

\[
| \overline{3} >_{dq_1} \otimes | \overline{3} >_{dq_2} \equiv | \overline{3}_a >_{dq_1dq_2} \otimes | 6_a >_{dq_1dq_2}, \tag{3a}
\]

\[
| 6 >_{dq_1} \otimes | 16 >_{dq_2} \equiv | 15_s >_{dq_1dq_2} \otimes | 15_s >_{dq_1dq_2} \otimes | 6_s >_{dq_1dq_2}, \tag{3b}
\]

\[
| \overline{3} >_{dq_1} \otimes | 16 >_{dq_2} \equiv | \overline{3}_s >_{dq_1dq_2} \otimes | 15_a >_{dq_1dq_2}, \tag{3c}
\]

(For color states);

\[
| 1, S = 0 >_{dq_1} \otimes | 1, S = 0 >_{dq_2} \equiv | 1_a, S = 0 >_{dq_1dq_2}, \tag{3d}
\]

\[
| 3, S = 1 >_{dq_1} \otimes | 3, S = 1 >_{dq_2} \equiv | 3_s, S = 1 >_{dq_1dq_2} \otimes | 1_s, S = 0 >_{dq_1dq_2}, \tag{3e}
\]

\[
| 1, S = 0 >_{dq_1} \otimes | 3, S = 1 >_{dq_2} \equiv | 3_s, S = 1 >_{dq_1dq_2}. \tag{3f}
\]

(For spin states).

Here the subscripts dq_i (i = 1, 2) and dq_1dq_2 denote the single and double diquark states respectively.

Note that such a labelling of color-spin states of two diquarks is done in order to account for their distinguishability. We further consider the following possible quark configurations in the two-diquark system:

**Case I:** Each diquark of the two-diquark composite consists of the same flavor of quarks (either u or d), viz., \(< u_1u_2u_3u_4 | H_{int} | u_1u_2u_3u_4 > or < d_1d_2d_3d_4 | H_{int} | d_1d_2d_3d_4 >.

**Case II:** One diquark of the two-diquark composite contains the same flavor of quarks (uu or dd) and the other diquark contains the different flavor of quarks (ud); thus the possible configurations turn out to be
\(< u_1u_2u_3d_4 | H_{int} | u_1u_2u_3d_4 >, < u_1u_2d_3u_4 | H_{int} | u_1u_2d_3u_4 >, < u_1d_3u_1u_4 | H_{int} | u_1d_3u_1u_4 >, < d_1u_2d_3u_4 | H_{int} | d_1u_2d_3u_4 >, < d_1d_2u_1u_4 | H_{int} | d_1d_2u_1u_4 >, < d_1d_2u_1u_4 | H_{int} | d_1d_2u_1u_4 >, < d_1u_2d_3u_4 | H_{int} | d_1u_2d_3u_4 >, < u_1d_3u_1u_4 | H_{int} | u_1d_3u_1u_4 >.

**Case III:** One diquark of the two-diquark composite contains both the quarks of the same flavor (say, uu) while the other one contains quark of different flavor (say, dd) or vice-versa. The possible configurations are \(< u_1u_2d_3d_4 | H_{int} | u_1u_2d_3d_4 > or < d_1d_2u_1u_4 | H_{int} | d_1d_2u_1u_4 >.

**Case IV:** Each diquark of the two-diquark composite is a hybrid diquark containing two quarks of different flavors. The possible configurations are \(< u_1d_3u_1u_4 | H_{int} | u_1d_3u_1u_4 >, < d_1u_2d_3u_4 | H_{int} | d_1u_2d_3u_4 >, < d_1d_2u_1u_4 | H_{int} | d_1d_2u_1u_4 > or < u_1d_3u_1u_4 | H_{int} | u_1d_3u_1u_4 >.

Out of several possible color-spin states (eq. (3a) - (3f)), the interaction energies corresponding to minimum energy state and those of the DS analogue state in cases I-IV are given in table I. To give a feeling for the contribution of the \(\phi^6\)-term in (1) we computed the interaction energy only for one three-diquark color-spin state, namely \{3, 0\}. This state is derived from (3a) and (3d). The three-diquark interaction energy corresponding to \(\phi^6\)-term in (1) turns out to be (-65A/16) in this state.

### 3. Equation of state (EOS) for diquark matter

For this purpose, we compute the parameter \(\lambda\) in (1) by comparing the results of section 2 with those obtained in the bag model (cf. Ref. (3)). Thus, we have \(\lambda = (\Delta E)_{d_1d_2} E^2 R^3 / 0.34\), where \((\Delta E)_{d_1d_2}\) is the two-diquark interaction energy. The values of \(\lambda\) corresponding to DS analogue and minimum interaction energy state of cases I-IV are given in table 1.

Next we compute the total energy of the diquark system from [2, 3]
\[ E_{dq} = \left[ d^3 k \left( k^2 + m_q^2 \right)^{1/2} n(k) + (3\lambda/2V) \right] \left[ d^3 k \left( n(k)/(k^2 + m_{dq}^2)^{1/2}) \right] \right]^2, \]

where for \( n(k) \) a Gaussian distribution (with width parameter \( a \)) namely, \( n(k) = (N/2(2\pi a^2 m_{dq}^2)^{3/2}) \exp(-k^2/2a^2m_{dq}^2) \) is used. The expression for \( E_{dq} \) in terms of quark mass \( m_q \) and quark number density \( \rho(=N/V) \) turns out to be

\[ E_{dq} = (1.49Nm_q/4\pi) \left[ 2\sqrt{2\pi a^3} I_1 + (3\lambda/(1.49)^3) a^{-6} (\rho/m_q^3) I_2^2 \right], \]

where \( I_1 = \int_0^\infty dkk^2(k^2 + 1)^{1/2} \exp(-k^2/2a^2) \) and \( I_2 = \int_0^\infty dkk^2(k^2 + 1)^{-1/2} \exp(-k^2/2a^2) \). In order to derive the EOS, we calculate the pressure for the diquark gas using \( p = (-dE_{dq}/dV)N \), and the resulting expression is given by

\[ p = -\sqrt{2/\pi a^3} I_1 \rho_m - (3\lambda/m_q^3) a^{-6} I_2^2 \rho_m^2, \]

where \( \rho_m(=Nm_{dq}/2V) \) is the diquark matter density. In fact the Gaussian width parameter \( a \) is fixed such that the energy of the diquark gas is minimum. For this purpose, one should have \((dE_{dq}/da) = 0\) and \((d^2E_{dq}/da^2) > 0\). The first condition yields

\[ \rho/m_q^3 = \frac{(3.4)\sqrt{2\pi a^3} (3a^2 I_1 - I_3) - (3\lambda)I_2(I_4 - 3a^2 I_2)}{(3\lambda)I_2(I_4 - 3a^2 I_2)}. \]

Corresponding to different values of \( \lambda \) of table 1 the pressure \( p \), as a function of matter density \( \rho_m \) is plotted in figure 1 for the diquark gas.

**Table 1.** Values of parameter \( \lambda \) corresponding to the DS analogue and the minimum interaction energy states.

| DS analogue state | Minimum interaction energy state |
|-------------------|----------------------------------|
| (\( \Delta E \)) dq_1 dq_2 | \( \lambda \) | (\( \Delta E \)) dq_1 dq_2 | \( \lambda \) |
| Case I | 15A/3 | 11.58 | 8A/3 | 6.18 |
| Case II | 5A/2 | 5.79 | 2A/3 | 1.54 |
| Case III | -(A/8) | -0.29 | 4A/3 | 3.08 |
| Case IV | 3A | 6.95 | 2A/3 | 1.54 |

4. **Discussion of results**

With a view to exploring the role of diquarks in the quark-gluon plasma via a new phase of diquark-gluon plasma we have computed the diquark interaction energies within the framework of a polynomial field theory. For this purpose, in addition to the \( \phi^4 \)-term which describes the two diquark interactions and is in accordance with earlier works [2, 3], an estimation of \( \phi^6 \)-term (corresponding to the three diquark interactions) is also carried out. We have worked in the spirit of constituent quark model and used the generalized Pauli principle. Corresponding to various allowed color-spin state of the diquark, the diquark interaction energies are computed [5]. The equations of state thus derived for various values of \( \lambda \), while conform to the trend of earlier works, however, are found somewhat more steep in general for the allowed values of the parameter \( a \) (cf. eq. (7)), viz., \( 0 < a < 0.074 \). For smaller values of both pressure and density, in fact, attain more or less constant values. However some interesting features in the EOS are expected in the range \( 0.06 < a < 0.074 \). Hopefully, these features in
the EOS, when used in the Tolman- Oppenheimer- Volkoff equations will give rise reasonable values of mass and radius for a stable configuration of a diquark star. Such studies are in progress.

**Figure 1.** The equation of state for diquark matter. Curves 1, 2 and 4 correspond to the DS analogue state of the two-diquark system in cases I, II, and IV respectively, whereas the curves 5-8, correspond the configurations of minimum energy in all the four cases.

5. References

[1] See, for example, Anselmino M *et al.* 1993 *Rev. Mod. Phys.* 65 1199 and the references therein

[2] Karn S K, Kaushal R S and Mathur Y K 2000 *Euro. Phys. J. C* 14 487; also see, 1996 *Z. Phys. C* 72 297

[3] Donoghue J F and Sateesh 1988 *Phys. Rev. D* 38 360; Kaster D and Traschen J 1991 *Phys. Rev. D* 44 3791

[4] Ekelin S and Fredriksson S 1986 *Phys. Rev. Lett.* 56 2428; Horvath J E *et al.* 1992 *Phys. Rev. D* 46 4754; Horvath J E *et al.* 2002 Quark-diquark matter equation of state and compact star structure *Preprint* astro-ph / 0203069 and the references therein

[5] Sisodiya A K, Bhasin V S and Kaushal R S On diquark clustering in quark-gluon plasma to be published