Capacity of the State-Dependent Wiretap Channel: Secure Writing on Dirty Paper

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Abstract—In this paper we consider the State-Dependent Wiretap Channel (SD-WC). As the main idea, we model the SD-WC as a Cognitive Interference Channel (CIC), in which the primary receiver acts as an eavesdropper for the cognitive transmitter’s message. By this point of view, the Channel State Information (CSI) in SD-WC plays the role of the primary user’s message in CIC which can be decoded at the eavesdropper. This idea enables us to use the main achievability approaches of CIC, i.e., Gel’fand-Pinsker Coding (GPC) and Superposition Coding (SPC), to find new achievable equivocation-rates for the SD-WC. We show that these approaches meet the capacity under some constraints on the rate of the channel state. Similar to the dirty paper channel, extending the results to the Gaussian case shows that the GPC lead to the capacity of the Gaussian SD-WC which is equal to the capacity of the wiretap channel without channel state. Hence, we achieve the capacity of the Gaussian SD-WC using the dirty paper technique. Moreover, our proposed approaches provide the capacity of the Binary SD-WC. It is shown that the capacity of the Binary SD-WC is equal to the capacity of the Binary wiretap channel without channel state.

Index Terms—Equivocation rate, channel capacity, wiretap channel, channel state information.

I. INTRODUCTION

Secure communication from an information theoretic perspective attracts some attentions nowadays [1]–[5]. There are a lot of works to study the secrecy problem in different channel models, which are inspired of the wiretap channel as a basic physical layer model [6]. All these attempts are based on two principal elements: the Equivocation as a measurement to evaluate the secrecy level at the eavesdropper which is introduced by Shannon [7], and the Random Coding [6] as a coding scheme which leads to the secrecy condition in the wiretap channels (see [5] and the references therein).

Using the Channel State Information (CSI) in information theoretic communication models was initiated by Shannon [8] in which he considered the availability of CSI at the Transmitter (CSIT). Gel’fand and Pinsker obtained the capacity of the discrete memoryless channel with non-causal CSIT [9]. Their main result was based on a binning scheme named Gel’fand-Pinsker Coding (GPC), and it was shown that the capacity of the Gaussian state-dependent channel is equal to the capacity of a channel with no channel state [10]. This means that the transmitter, who knows the CSI non-causally, can adapt its signal to the channel state such that the receiver senses no interference [10], as if the receiver had knowledge of the interference and could subtract it out. The name of the Dirty Paper channel [10] is inspired of a spotted paper, one wish to write on such that the reader can easily find the text.

Despite of all these works which are trying to cancel out the channel state, the authors in [11] and [12] deal with the CSIT in an innovative manner. In these works, the CSIT assumed as the user’s signal which is wished to be sent through the channel. In [12], the transmitter wishes to mask the CSI at the legitimate receiver, whereas in [11] the transmitter wishes to forward the CSI to the legitimate receiver. The achievable rate in each case is derived and results to the trade-off between the rates of the transmitter’s information and the CSI.

The State-Dependent Wiretap Channel (SD-WC) was studied in [1]. In [1], the authors derived an achievable equivocation-rate for the SD-WC in general case, and in [2] the Gaussian SD-WC was considered. The main ideas of these papers were based on using a combination of GPC and random coding. In these works, it was assumed that the eavesdropper has no access to the CSI. Therefore, the CSI can be used potentially to improve the secrecy rate of the SD-WC. Specially in Gaussian case, under some constraints, the capacity was derived as the capacity of a channel [1], [2] with no state. Afterward, [13] studied the SD-WC from the secret-key sharing aspect. In this model, the channel state was assumed as a key to achieve the secrecy rate. El-Gamal et. al., improved the equivocation-rate for the SD-WC in the case that the channel state is known causally at both sides [14].

In this paper we study the SD-WC (see Fig. 1). We consider the channel state sequence as a random sequence with elements drawn from a finite alphabet, known non-causally at the transmitter. The transmitter wishes to keep its message secret from an external eavesdropper. A well known concept is to cancel out the CSI using the GPC and use the random coding to achieve the secrecy rate as [1]. We know from the random coding idea [6] that we should randomize a part of the message to confuse the eavesdropper which has less channel capacity than the main channel. In this paper, we model the CSI of SD-WC as the message of the primary user at CIC and we use it at the transmitter to randomize a part of the message. Our main idea relies on modeling the CSI with a (primary) message, since both are known at the (cognitive) transmitter [15], [16]. In this model, the transmitter of SD-WC plays the role of the cognitive transmitter of CIC and the CSI is considered as the message of the primary one. The transmitter, similar to the cognitive one in CIC, tries
Dirty Paper Coding rates for the SD-WC lead to the secrecy capacity using the we show that the derived equivocation-rates meet the capacity Then, we prove that these achievable equivocation-rates meet derive two new achievable equivocation-rates for the SD-WC. At first, using these two coding schemes, we Superposition Coding (SPC), to achieve the secrecy rate for cancelation schemes as CICs in [15], [16], i.e., GPC and message to the eavesdropper, i.e., the primary receiver in its destination with low enough information leakage of its state is assumed to be known non-causally at the transmitter.

Fig. 1. The State-Dependent Wiretap Channel (SD-WC) in which the channel model is introduced. In Section III the encoding strategy for the point to point state-dependent communication channel is explained. The main results on the achievable equivocation-rates, the capacity of the SD-WC and the binary example are presented in Section IV. In Section V, using DPC, the capacity of the Gaussian SD-WC is derived and the paper is concluded with some discussions in the last section.

II. CHANNEL MODEL, DEFINITIONS AND PRELIMINARIES

First, we clear our notation in this paper. Let $\mathcal{X}$ be a finite set. Denote its cardinality by $|\mathcal{X}|$. If we consider $\mathcal{X}^N$, its members are $x^N = (x_1, x_2, \ldots, x_N)$, where subscripted letters denote the components and superscripted letters denote the vector. A similar convention applies to random vectors and random variables, which are denoted by uppercase letters. Denote the zero mean Gaussian random variable with variance $P$ by $\mathcal{N}(0, P)$. A Bernoulli random variable $X$ with $P_r(X = 1) = P$ is denoted by $X \sim B(P)$.

Consider the channel model shown in Fig. 1. Assume that the state information of the channel, i.e., $S_i$, $1 \leq i \leq N$ is known at the transmitter non-causally. The transmitter sends the message $W$ which is uniformly distributed on the set $W \in \{1, \ldots, M\}$ to the legitimate receiver in $N$ channel uses. Based on the $w$ and $S^N$, the transmitter generates the codeword $x^N$ to transmit through the channel. The decoder at the legitimate receiver makes an estimation of the transmitted message $\hat{w}$ based on the channel output $y^N$; and $z^N$ is the corresponding output at the eavesdropper. The channel is memoryless, i.e.,

$$p(y^N, z^N | x^N, S^N) = \prod_{i=1}^{N} p(y_i, z_i | x_i, s_i).$$

The secrecy level of the transmitter’s message at the eavesdropper is measured by normalized equivocation:

$$R_e(N) = \frac{1}{N} H(W | Z^N, S^N),$$

in which we assume that the eavesdropper can decode the CSI. The average probability of error $P_e$ is given by

$$P_e = \frac{1}{M} \sum_{i=1}^{M} P_r(\hat{w}(Y^N) \neq w).$$

We define the rate of the transmission to the intended receiver to be

$$R = \frac{\log M}{N}.$$  

The rate-pair $(R, R_e)$ is achievable if for any $0 \leq \epsilon \leq 1$ there exists an $(M, N, P_e)$ code such that $P_e \leq \epsilon$, and the secrecy rate $R_e$ is

$$0 \leq R_e \leq \liminf_{N \to \infty} R_e(N).$$

We define the perfect secrecy condition as the case in which we have $R \leq R_e$. Thus, under this condition, the sent message to the legitimate receiver must be secure.

III. OVERVIEW OF ENCODING STRATEGY

Consider SD-WC in Fig. 1. Assume that the channel state $S$ plays the role of the codebook of rate $R_S = I(S; Y)$ interfering with the communication of message $W$ at the rate $R = I(X; Y)$. As proposed in [17], two coding strategy can be used for this scenario: GPC scheme and SPC depending on the interference’s rate $R_S$. When $R_S$ is small, it can be exploited for achieving the higher rates using the SPC. Using the GPC, the interference is considered as a codebook with rate $R_S$. The following lemma expresses the result of using these two coding schemes. This lemma is used to derive the achievable equivocation-rate for the SD-WC in the next section.

Lemma 1: [17] Lemma 1 The following rate is achievable for a point to point communicating system with non-causal CSIT

$$R \leq \max_{P_{U|S}, P(x(u, s))} \min_{P_{U|S}} \{I(X; Y | S), \max_{P_r(U; S, Y) - R_S, I(U; Y) - I(U; S)} \}.$$  

Outline Of The Proof: This lemma is proved in [17, Appendix B]. For the case $I(S; U, Y) \leq R_S \leq H(S)$, the second term of (6) is achievable using the GPC. For $R_S \leq I(S; U, Y)$, the first term in (6) is achievable using the SPC.

Remark 1: Interestingly, in the case $R_S \leq I(S; U, Y)$, the receiver decodes both the messages and the channel state, i.e., the interference. A related model in which both data and the channel state are decoded at the receiver was considered by [11].
IV. RESULTS ON THE CAPACITY OF THE SD-WC

In this section, we provide two achievable equivocation-rates for the SD-WC, shown in Fig. 1 in two cases, i. e., GPC and SPC.

A. Gel’fand-Pinsker Coding (GPC)

Assume that $V$ and $U$ are two random variables to construct the private and the common messages, respectively. In the case where $\min\{I(S;U,Y), I(S;V,Y|U)\} \leq R_S \leq H(S)$, using GPC and the rate splitting, we have the following lemma for the achievable equivocation-rate of the SD-WC.

\textbf{Lemma 2 (Achievable equivocation-rate using GPC):} The set of rates $(R, R_1, R_2, R_c)$ satisfying

\begin{align}
R_{GPC} = \max \left\{ R : \begin{array}{l}
R = R_1 + R_2 + R_c \\
R_1 \leq I(V;Y|U,T) - I(V;S|U,T), \\
R \leq I(V;U,Y|T) - I(V;U,S|T), \\
R_c \leq I(V;Y|U,T) - I(V;S,Z|U,T),
\end{array} \right\}.
\end{align}

is achievable for SD-WC, in which the equations in the right-hand-sides of (7) are non-negative and $T$ is a time-sharing random variable.

\textbf{Outline of the Proof:} The details of the proof are relegated to the Appendix A based on the result of [15]. As an outline, the message $W$ is split into two messages $W_1$ and $W_2$ with denoted rates $R_1$ and $R_2$, respectively. In our scheme, $W_2$ can be decoded at the eavesdropper and does not contribute to the secrecy level of $W$ at the eavesdropper. Therefore, $W_1$ may be hidden from the unintended receiver. We model SD-WC with a CIC with a confidential message [15], in which the message of the primary transmitter plays the role of CSI and the transmitter who knows CSI can be considered as the cognitive transmitter. By this setting, the proof of Lemma 2 is deduced from the proof of [15] Theorem 1 in which a cognitive radio tries to communicate with a related destination through a main channel which belongs to a primary transmitter-receiver pair. Therefore, the transmitter in SD-WC (similar to the cognitive transmitter in CIC), can cancel the channel state out at the corresponding receiver, using the known CSI. The transmitter uses rate splitting scheme and GPC by binning $U$ against the channel state $S$, and binning $V$ against $S$ conditioned on $U$. The GPC structure is shown in Fig. 2.

B. Superposition Coding (SPC)

The perfect secrecy condition using the GPC scheme is derived as follows.

\textbf{Theorem 1 (Perfect secrecy condition using GPC):} In the case that $\min\{I(S;U,Y), I(S;V,Y|U)\} \leq R_S \leq H(S)$, the perfect secrecy rate $R_{ps}^{GPC}$ satisfying

\begin{align}
R_{ps}^{GPC} = \max \left\{ R : \begin{array}{l}
R \leq \min\{I(V,U;Y) - I(V,U;S), \\
I(V;Y|U) - I(V;S,Z|U)\},
\end{array} \right\}, \tag{8}
\end{align}

is achievable for the SD-WC.

\textbf{Proof:} The proof of the theorem is strictly deduced from Lemma 2 applying perfect secrecy condition and using Fourier-Motzkin elimination [16].

\textbf{Corollary 1:} The following rate

\begin{align}
R = \max \left\{ R : \begin{array}{l}
R \leq \min\{I(V,U;Y) - I(V,U;S), \\
I(V;Y|S) \leq H(S|Y),
\end{array} \right\}, \tag{9}
\end{align}

is an achievable secrecy rate for the SD-WC.

\textbf{Proof:} Substituting $U = S$ in the proof of the Corollary 1 means that the transmitter sends the channel state $S$ as a part of its message. Thus, the channel state can be decoded at the legitimate receiver. This setting is similar to the one proposed by [11], in which the transmitter sends its data and the information about the channel state to the receiver, without secrecy issue.

\textbf{Remark 2:} The proof of the corollary is deduced directly from Theorem 1 by substituting $U = S$ and using Fourier-Motzkin elimination [16]. Note that the transmitter which knows CSI non-causally can use the channel state ransom variable $S$ as part of its message, i. e., the common one.

C. Superposition Coding (SPC)

Consider the case $R_S \leq \min\{I(S;U,Y), I(S;V,Y|U)\}$. From Lemma 1, we can superimpose $U^N$ and $V^N$ against $S^N$ instead of using GPC. To derive the perfect secrecy rate for the SD-WC in this case, first we present the following lemma.

\textbf{Lemma 3 (Achievable equivocation-rate using SPC):} In the case that $R_S \leq \min\{I(S;U,Y), I(S;V,Y|U)\}$, the

\begin{align}
&\text{Fig. 2. Gel’fand-Pinsker Coding (GPC) scheme for the case } \\
&\text{min}\{I(S;U,Y); I(S;V;Y|U)\} \leq R_S \leq H(S). \tag{7}
\end{align}

\begin{align}
&\text{Fig. 3. Superposition Coding (SPC) scheme for the case } R_S \leq \\
&\min\{I(S;U,Y); I(S;V;Y|U)\}. \tag{8}
\end{align}
Theorem 2 (Perfect secrecy condition using SPC): In the case that

\[ R_S \leq \min \{ I(S;U,Y), I(S;V,Y|U) \} \]  \hspace{1cm} (13)

or

\[ R_S \geq \max \left\{ \min \{ I(S;U,Y), I(S;V,Y|U) \}, I(V,U;S) - I(U;Y|S) - I(V;Z|S) \right\} \]  \hspace{1cm} (14)

the secrecy capacity \( C_S \) of the SD-WC is as follows

\[ C_S = \max \left\{ I(V;Y|U,S) - I(V;Z|S) \right\} \]  \hspace{1cm} (15)

**Proof:** The achievability of (15) is derived directly from Theorem 1 and Theorem 2. In more detail, in the case that \( R_S \leq \min \{ I(S;U,Y), I(S;V,Y|U) \} \), the rate (15) is achievable for the SD-WC by using SPC as Theorem 2. In the case that \( \max \{ I(S;U,Y), I(S;V,Y|U) \}, I(V,U;S) - I(U;Y|S) - I(V;Z|S) \} \leq R_S \), GPC achieves the rate (15), by using Theorem 1 and substituting \( U = (U,S) \). The converse proof is relegated to the Appendix C.

**Remark 3:** The capacity of the SD-WC in Theorem 3 in the case that \( U = S = \emptyset \), is reduced to the capacity of the wiretap channel without channel state information.

**Corollary 3:** In the case that \( Y \) is more capable than \( Z \), i.e., \( I(X;Y|U,S) \geq I(X;Z|U,S) \) for all \( p(x) \), and under the conditions (13)–(14), the secrecy capacity of the SD-WC is as follows

\[ C_S = \max \left\{ I(X;Y|S) - I(X;Z|S) \right\} \]  \hspace{1cm} (16)

**Proof:** The achievability of (16) is directly proved by substituting \( U = \emptyset, V = X \) in Theorem 3. To prove the converse, we have

\[ I(V;Y|U,S) - I(V;Z|S) = I(X,V;Y|U,S) - I(X,V;Z|U,S) \]

\[ = [I(X,Y|U,S,V) - I(X,Z|U,S,V)] \]

\[ = [I(X,Y|U,S,V) - I(X,Z|U,S,V)] + [I(V;Y|U,S,X) - I(V;Z|U,S,X)] \]

\[ \leq [I(X,Y|U,S,V) - I(X,Z|U,S,V)] \]

\[ \leq I(X,Y|U,S,V) - I(X,Z|U,S,V) \]  \hspace{1cm} (17)

where (a) is due to the Markov chain relationship \( (U,V) \rightarrow (X,S) \rightarrow (Y,Z) \) which implies that \( I(V;Y|U,S,X) - I(V;Z|U,S,X) = 0 \); and (b) is derived by using the more
capable condition. Now, we have
\[
I(X; Y|U, S) - I(X; Z|U, S) = I(U, X; Y|S) - I(U, X; Z|S)
\]
\[
- I(U, X; Y|S) - I(U, X; Z|S) = I(X; Y|S) - I(X; Z|S) + I(U; Y|S) - I(U; Z|S)
\]
\[
= I(U; Y|S) - I(U; Z|S)
\]
\[
- I(U, X; Y|S) - I(U, X; Z|S)
\]
\[
\leq I(X; Y|S) - I(X; Z|S),
\]
where \((c)\) is derived by using the Markov chain relationship and the more capable condition.

1) An Example (Binary SD-WC): As an example, consider the Binary SD-WC (BSD-WC), in which the channel outputs are described as
\[
Y = X \oplus S \oplus \eta_1, \quad \text{(19)}
\]
\[
Z = X \oplus S \oplus \eta_2, \quad \text{(20)}
\]
where \(\eta_1 \sim \mathcal{B}(N_1), \eta_2 \sim \mathcal{B}(N_2)\) and \(S \sim \mathcal{B}(Q)\). For this channel we have the following theorem.

**Theorem 4:** When
\[
R_S \leq I(S; Y), \quad \text{(21)}
\]
or
\[
R_S \geq \max\{I(S; Y), I(X; S) - I(X; Z|S)\}, \quad \text{(22)}
\]
the secrecy capacity of the BSD-WC is
\[
C_S^{BSD-WC} = [H(N_2) - H(N_1)]^+, \quad \text{(23)}
\]
in which \([x]^+ = \max\{0, x\}\).

**Proof:** Substituting \(U = 0, V = X\) in Theorem 3 leads to the secrecy capacity as following under the conditions (21)-(22)
\[
C'_S = \max_{P_x, P_{X|Y}, P_{Y|Z|x,S}} I(X; Y|S) - I(X; Z|S). \quad \text{(24)}
\]
Let \(X \sim \mathcal{B}(P)\) which is independent of \(S\). Note that the nature of the BSC forces us to choose Bernoulli distribution function for the channel input. Now, we have
\[
I(X; Y|S) - I(X; Z|S) = H(Y|S) - H(Y|X, S) - H(Z|S) + H(Z|X, S)
\]
\[
= H(X \oplus \eta_1) - H(\eta_1) - H(X \oplus \eta_2) + H(\eta_2)
\]
\[
= H(N_2) - H(N_1) - [H(P \ast N_2) - H(P \ast N_1)], \quad \text{(25)}
\]
in which \(P \ast u = P(1 - u) + (1 - P)u\) and \(H(N_i) = -N_i \log(N_i) - (1 - N_i) \log(1 - N_i), i \in \{1, 2\}\) is the binary entropy function. For \(P < \frac{1}{2}\), the function \(P \ast u\) is monotonically increasing in \(u \in [0, 1/2]\). Hence, setting \(P = 1/2\) we have \(H(P \ast N_2) - H(P \ast N_1) = 0\) which achieves the maximum of \(I(X; Y|S) - I(X; Z|S)\) as \(H(N_2) - H(N_1)\). In the case that \(N_1 \leq N_2\) the right hand side of (25) is negative. It means that the eavesdropper can decode any message intended for the receiver. The proof of the converse is directly derived from Corollary 3 which implies that the right-hand-side of (24) is an outer bound on the capacity of the model in which the legitimate receiver is more capable than the eavesdropper. This completes the proof.

**Remark 4:** The capacity of the BSD-WC under the conditions (21) and (22), is equal to the capacity of the binary wiretap channel without channel state. It means that the coding schemes used in the theorems 1 and 2 cancels the channel state out to meet the capacity.

V. GAUSSIAN SD-WC: DIRTY PAPER SCHEME

In this section, we extend the results of the theorems 1 and 2 to the Gaussian SD-WC (GSD-WC). First, consider the GSD-WC (Fig. 5) which is described as follows.

\[
Y = X + S + \eta_1, \quad \text{(26)}
\]
\[
Z = X + S + \eta_2, \quad \text{(27)}
\]
where \(Y\) denotes the channel input and \(Y\) and \(Z\) denote the channel outputs at the legitimate receiver and eavesdropper, respectively. \(\eta_i \sim N(0, N_i), i \in \{1, 2\}\) is Additive White Gaussian Noise (AWGN), and we assume the channel state random variable as \(S \sim N(0, Q)\). Now, we have the following theorem for the GSD-WC.

**Theorem 5:** The secrecy capacity of the GSD-WC is
\[
C_S^{GSD-WC} = [C(P/N_1) - C(P/N_2)]^+, \quad \text{(28)}
\]
in which \(C(x) = \frac{1}{2} \log(1 + x)\).

**Proof:** The proof the achievability of the rate (28) is derived from Theorem 1 and 2 as follows:

First, we assume the channel input as \(X \sim N(0, P)\) which is independent of the channel state sequence and the AWGNs. Then, we use the Dirty Paper approach [10] on theorems 1 and 2, directly. For this, we split the channel input as \(X = X_1 + X_2 + \sqrt{P/Q}\), in which \(X_1\) is related to the confidential message and \(X_2\) is related to the common message which reduces the interference at the legitimate receiver, and we have
\[
X_1 \sim N(0, \alpha \beta P), \quad \text{(29)}
\]
\[
X_2 \sim N(0, \alpha \beta P), \quad \text{(30)}
\]
where \(0 \leq \alpha, \beta \leq 1\) are the power coefficients for sending the confidential and the common messages in the transmitter, respectively. Also we define \(\tilde{\alpha} = 1 - \alpha, \tilde{\beta} = 1 - \beta\).
Let the auxiliary random variables $V$ and $U$ as
\[ V = X_1 + \lambda_1 S, \quad (30) \]
\[ U = X_2 + \lambda_2 S \quad (31) \]
in which the transmitter uses $0 \leq \lambda_1, \lambda_2 \leq 1$ to bin its message against the state of the channel as GPC. Note that in the SPC the transmitter does not forward the channel state, and the channel state is not contained in $V$. Thus, in SPC case, we should substitute $\lambda_1 = 0$. Now, we find the variables $\alpha, \beta, \lambda_1, \lambda_2$ which maximize the achievable secrecy rate of Theorem 1 leading to the capacity of GSD-WC.

Based on Theorem 1, for the GPC case, we can define
\[ R_{e_1} \triangleq I(V; U; Y) - I(V; U; S) \]
\[ = H(V; U|S) - H(V; U|Y), \]
\[ R_{e_2} \triangleq I(V; Y|U) - I(V; S, Z|U) \]
\[ = H(V|U) - H(V; Y, U) \]
\[ = -H(S, Z|U) + H(S, Z|V, U). \quad (32) \]

Now, we calculate each term of (32) using the standard approach yielding (33)-(38) in top of the next page. Taking derivatives of $R_{e_1}$ with respect to $\lambda_1$ and $\lambda_2$ and setting to zero yields
\[ \hat{\lambda}_1 = \frac{\alpha \beta P(K + 1)}{\alpha P + N_1}, \quad (39) \]
\[ \hat{\lambda}_2 = \frac{\alpha \beta P(K + 1)}{\alpha P + N_1}. \quad (40) \]

Then, we substitute these optimal variables in (32) to get
\[ R_{e_1}(\hat{\lambda}_1, \hat{\lambda}_2) = C\left(\frac{\alpha P}{N_1}\right), \quad (41) \]
\[ R_{e_2}(\hat{\lambda}_1, \hat{\lambda}_2) = C\left(\frac{\alpha P}{N_2}\right) - C\left(\frac{\alpha P}{N_2}\right). \quad (42) \]

Next, we optimize (41)-(42) with respect to $\alpha$ and $\beta$. In the case $N_1 > N_2$, the derived result of $R_{e_2}$ is a decreasing function with respect to $\alpha$ and $\beta$. Thus, this function is maximized at $\alpha = \beta = 0$, and $\lambda_1 = \lambda_2 = 0$. Therefore, the achievable rates are equal to zero for this case, i.e., $R_{e_1} = R_{e_2} = 0$.

In the case that $N_1 < N_2$, $R_{e_2}$ is an increasing function with respect to $\alpha$ and $\beta$. Thus, this function is maximized at $\alpha^* = \beta^* = 1$. Finally, the variables $\alpha^* = 1, \beta^* = 1, \lambda_1^* = \frac{P}{P+N_1}, \lambda_2^* = 0$ leads the secrecy achievable rate of Theorem
\[ R_e^* = \min\{R_{e_1}(\lambda_1, \lambda_2), R_{e_2}(\lambda_1, \lambda_2)\} \]
\[ = C\left(\frac{P}{N_1}\right) - C\left(\frac{P}{N_2}\right). \quad (43) \]

We conclude that for the GPC for all cases of $N_1$ and $N_2$ the secrecy achievable rate is as follows
\[ R_{e-GPC}^* = \left[C\left(\frac{P}{N_1}\right) - C\left(\frac{P}{N_2}\right)\right]^+. \quad (44) \]

Now, based on Theorem 2 for the SPC case, we can define
\[ V = X, \quad (45) \]
\[ U = \emptyset. \quad (46) \]

substituting these parameters in (11) we have
\[ I(V; Y|U, S) - I(V; Z|U, S) \]
\[ = I(X; Y|S) - I(X; Z|S) \]
\[ = H(Y|S) - H(Y|X, S) - H(Z|S) + H(Z|X, S) \]
\[ = H(X + \eta_1) - H(\eta_1) - H(X + \eta_2) + H(\eta_2) \]
\[ = C\left(\frac{P}{N_1}\right) - C\left(\frac{P}{N_2}\right), \quad (47) \]

which means that the chosen parameters in (45)-(46), lead Theorem 2 to the following secrecy achievable rate
\[ R_{e-SPC}^* = \left[C\left(\frac{P}{N_1}\right) - C\left(\frac{P}{N_2}\right)\right]^+. \]

Note that the necessary condition, in Theorem 2, under which the SPC gives the secrecy achievable rate, is satisfied and discussed in Remark 5.

For the converse proof, using the fact that the legitimate receiver is more capable than the wiretapper, we can use the result of Corollary 3. Thus, we should prove that choosing the Gaussian distribution for the channel input, maximizes the achievable rate to the capacity of the wiretap channel without channel state. Without loss of generality, we assume that the channel is physically degraded, i.e., $\eta_2 = \eta_1 + \eta_2$, in which $\eta_2 \sim N(0, N_2 - N_1)$. For the outer bound on the capacity of the channel we have
\[ I(X; Y|S) - I(X; Z|S) \]
\[ = I(X; X + \eta_1) - I(X; X + \eta_2) \]
\[ = H(X + \eta_1) - H(\eta_1) - H(X + \eta_2) + H(\eta_2) \]
\[ = \frac{1}{2} \log \left(\frac{N_2}{N_1}\right) - [H(X + \eta_2) - H(X + \eta_1)]. \quad (49) \]

Then, by substituting $M = X + \eta_1$ we have
\[ H(X + \eta_2) - H(X + \eta_1) \]
\[ = H(M + \eta_2') - H(M) \]
\[ \geq \frac{1}{2} \log(2^{2H(\eta_2')} + 2^{2H(M)}) - H(M) \]
\[ = \frac{1}{2} \log(2\pi e(N_2 - N_1) + 2^{2H(M)}) - H(M) \quad (50) \]

where (c) is derived by the entropy power inequality (EPI) [3]. Moreover, $\frac{1}{2} \log(2\pi e(N_2 - N_1) + 2^{2H(u)}) - H(u)$ is a monotonic increasing function with respect to $u$ and $H(M) \leq \frac{1}{2} \log(2\pi e(P + N_1))$. Thus, we have
\[ H(X + \eta_2) - H(X + \eta_1) \]
\[ \geq \frac{1}{2} \log(2\pi e(N_2 - N_1) + 2\pi e(P + N_1)) \]
\[ - \frac{1}{2} \log(2\pi e(P + N_1)) \]
\[ = \frac{1}{2} \log \left(\frac{P + N_2}{P + N_1}\right). \quad (51) \]
Finally, we have

\[ I(X; Y | S) - I(X; Z | S) \geq \frac{1}{2} \log \left( \frac{N_2}{N_1} \right) - \frac{1}{2} \log \left( \frac{P + N_2}{P + N_1} \right) = C \left( \frac{P}{N_1} \right) - C \left( \frac{P}{N_2} \right), \quad (52) \]

and the equality is attained by choosing \( X \sim \mathcal{N}(0, P) \).

**Remark 5:** We should note that GPC is used in the case that \( \min \{ I(S; U, Y), I(S; V, Y | U) \} \leq R_S \leq H(S) \). By substituting the optimal parameters \( \alpha^* = 1, \beta^* = 0, \lambda_1^* = \frac{P}{P + N_1}, \lambda_2^* = 0 \), this condition is reduced to \( I(S; Y) \leq R_S \leq H(S) \). On the other hand, SPC is used when \( R_S \leq \min \{ I(S; U, Y), I(S; V, Y | U) \} \), which by substituting R.V.s as \( (55), (56) \), we have \( R_S \leq I(S; Y) \). As shown in Fig. 6 our proposed coding schemes meet the capacity for any \( R_S \). Note that in the Gaussian case, \( I(S; Y) = C \left( \frac{P}{P + N_1} \right) \).

**Remark 6:** In GPC scheme, choosing \( \alpha^* = \beta^* = 1 \) and \( \lambda_1^*, \lambda_2^* \) results in

\[ X = \sqrt{\frac{P}{Q}} S, \]
\[ U = \emptyset, \]

and it is noticeable that the parameter \( \lambda_1^* = \frac{P}{P + N_1} \) is similar to the one chosen in dirty paper channel \( (57) \) to achieve the capacity in the state-dependent channel. This inspired the authors to name the proposed method Secure Dirty Paper Coding (SDPC).

**VI. DISCUSSIONS AND CONCLUSIONS**

In this paper we derived two equivocation-rates for the state-dependent wiretap channel in which the channel state information is assumed to be known non-causally at the transmitter. These equivocation-rates are derived from the equivocation-rate regions reported for the cognitive interference channel \([15], [16]\). Comparing our model to the cognitive interference channel, the channel state plays the role of the message of the primary user. The transmitter uses the cognitive interference channel and superposition coding. By this point of view, we derived new achievable equivocation-rates for the state-dependent wiretap channel. Then, we showed that the derived equivocation-rates meet the capacity of the state-dependent wiretap channel under some conditions. As an example, the secure capacity of a state-dependent binary symmetric channel was considered which confirms the general results. Afterward, the state-dependent Gaussian wiretap channel was studied, and our achievable equivocation-rates lead to the capacity in Gaussian case. It was shown that the capacity of the state-dependent Gaussian wiretap channel is equal to the capacity of the Gaussian wiretap channel without

![Fig. 6. The conditions under which GPC (by substituting R.V.s as V = X + (\frac{P}{P + N_1}) S) and SPC (by substituting R.V.s V = X; U = \emptyset) are used in Gaussian state-dependent wiretap channel.](image)
channel state. This result was derived using dirty paper coding approach \cite{10}, by maximizing the equivocation-rates. The authors called this coding scheme Secure Dirty Paper Coding.

To compare our model with the one presented in \cite{1}, we should note that in \cite{1}, the transmitter which non-causally knows the CSI, uses this information to increase its secrecy rate to $R_{\text{Chen- Vincek}} = I(V; Y) - \max\{I(V; S), I(V; Z)\}$. Therein, it is assumed that the CSI is not known at the eavesdropper. Thus, in this case the output of the channel at the eavesdropper assumed to be $\mathcal{W}$, when necessary to assume the channel output at the eavesdropper. Moreover, in our model it is not assumed that the CSI can be decoded at the eavesdropper, the capability of the eavesdropper to estimate the channel, which is related to the rate of the primary transmitter’s message (the channel state in SD-WC). It is noticeable that due to the following lemma.

**Lemma 4:** [15, Theorem 1] The set of the rates $(R_1, R_2, R_e)$ satisfying

\[
R_1 \leq I(X_1; U, Y_1|T), \quad R_2 \leq I(V; Y_2|U, T) - I(V; X_1|U, T), \quad R_e \leq I(U, V; Y_2|T) - I(U, V; X_1|T), \quad R_1 + R_2 + R_e \leq I(X_1, U; Y_1|T), \quad R_2 + R_e \leq I(V; Y_2|U, T) - I(V; X_1, Y_1|U, T),
\]

is achievable for CIC with a confidential message.

Now, comparing the SD-WC (Fig. 1) with the CIC (Fig. 7), we can derive a new achievable rate for the SD-WC. We should note that the message of the primary transmitter plays the role of the channel state in SD-WC. Since the eavesdropper is not forced to decode the channel state in SD-WC, we should relax the terms contain $R_1$ which is related to the rate of the primary transmitter’s message (the channel state in SD-WC). Thus, by setting

\[
W_1 = W, R_1 = R_S, R_2 = R_e, R_e = R_e, R_1 = R_1 + R_2, Y_1 = Z, Y_2 = Y, X_1 = S, X_2 = X,
\]

and relaxing the rates (48) and (51), we derive the equivocation-rate of Lemma 2. Finally, we remark that we can derive the equivocation-rate of Lemma 2 directly by introducing the codebook generation, encoding and decoding schemes, error analysis and equivocation computation similar to the one presented in [15].

**APPENDIX A**

**PROOF OF THE LEMMA 2**

The SD-WC is modeled with a CIC, i.e., the CSI is considered as a primary transmitter’s message and thus is transmitted through the channel; and the transmitter in SD-WC plays the role of a cognitive transmitter who has the channel state of the primary one non-causally. On the other hand, the transmitted message in SD-WC must be confidential at the primary receiver who acts as an eavesdropper for the cognitive transmitter’s message. First, note that in [15] two confidential messages are considered in CIC model, i.e., the primary and the cognitive receivers act as eavesdroppers for each other’s message. Here, we just consider the cognitive transmitter’s message to be confidential at the primary receiver. Hence, we reduce the rate region of [15] to the CIC with one confidential message by excluding extra secrecy condition. Thus, we have the following lemma.

**Lemma 4:** [15, Theorem 1] The set of the rates $(R_1, R_2, R_e)$ satisfying

\[
R_1 \leq I(X_1; U, Y_1|T), \quad R_2 \leq I(V; Y_2|U, T) - I(V; X_1|U, T), \quad R_e \leq I(U, V; Y_2|T) - I(U, V; X_1|T), \quad R_1 + R_2 + R_e \leq I(X_1, U; Y_1|T),
\]

is achievable for CIC with a confidential message.

**APPENDIX B**

**PROOF OF THE LEMMA 3**

Consider the CIC with one confidential message (Fig. 7). Using the SPC, the achievable equivocation-rate region for this channel is derived by [16] as follows.

**Lemma 5:** [16, Theorem 1] The set of the rates $(R_1, R_2, R_e)$ satisfying

\[
R_1 \leq \min\{I(U, X_1; Y), I(U, X_1; Z)\} \quad (60)
\]

\[
R_2 \leq I(U; V; Z|X_1), \quad (61)
\]

\[
R_1 + R_2 \leq \min\{I(U, X_1; Y), I(U, X_1; Z)\} + I(V; Z|U, X_1), \quad (62)
\]

\[
R_e \leq I(V; Z|U, X_1) - I(V; Y|U, X_1), \quad (63)
\]

is achievable for CIC with a secret message.

Now, by substituting $R_2 = R_e = R_1 = S$ and relaxing the rates (60) and (62) which correspond to the primary user (channel state in our setting) the achievability of the equivocation-rate of Lemma 3 is proved.

**APPENDIX C**

**THE CONVERSE PROOF OF THEOREM 3**

The converse proof of Theorem 3 is derived as follows. Consider the rate-pair $(R, R_e)$ to be achievable. Then, we have

\[
NR \leq I(W; Y^N) - I(W; Z^N, S^N) + \epsilon
\]

\[
\leq I(W; Y^N|S^N) - I(W; Z^N|S^N) + \epsilon
\]

\[
\leq \sum_{i=1}^{N} I(W; Y_i|Y^{i-1}, S^N) - I(W; Z_i|Z_{i+1}^N, S^N) + \epsilon
\]
\[
\begin{align*}
&\leq \sum_{i=1}^{N} I(W, Z_{i+1}^N; Y_{i}^i, S_N) - I(W, Y_{i}^{i-1}, Z_{i+1}^N; S_N) + \epsilon \\
&\leq \sum_{i=1}^{N} I(W; Y_{i}^{i-1}, Z_{i+1}^N, S_N) - I(W; Z_{i}^{i-1}, Z_{i+1}^N, S_N) + \epsilon \\
&\leq \sum_{i=1}^{N} I(V_i; Y_{i}|U_i, S_i) - I(V_i; Z_{i}|U_i, S_i) + \epsilon \\
&\leq I(V; Y|U, S) - I(V; Z|U, S) + \epsilon
\end{align*}
\]

in which (a) follows from the Fano’s inequality and the fact that \( I(W; Z^N, S^N) \) tends to zero for \( N \to \infty \); (b) and (c) follow from the Csiszár sum identity [3]; (d) is derived by substituting the random variables \( U_i = (Y_{i-1}, Z_{i+1}^N, S_{i+1}^N), V_i = (W, U_i) \), and (e) follows by defining a time-sharing random variable \( Q \) and defining \( U = (U_Q, Q), V = (V_Q, Q), Y = Y_Q \), and \( Z = Z_Q \). This completes the proof.

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