Negative Hopping Magnetoresistance and Dimensional Crossover in Lightly Doped Cuprate Superconductors

Valeri N. Kotov,1 Oleg P. Sushkov,2 M. B. Silva Neto,3 L. Benfatto,4,5 and A. H. Castro Neto1,6

1Department of Physics, Boston University, 590 Commonwealth Avenue, Boston, MA 02215
2School of Physics, University of New South Wales, Sydney 2052, Australia
3Institut für Theoretische Physik, Universität Stuttgart, Pfaffenwaldring 57, 70550, Stuttgart, Germany
4Centro Studi e Ricerche “Enrico Fermi”, via Panisperna 89/A, 00184, Rome, Italy
5CNR-INFM and Department of Physics, University of Rome “La Sapienza”, 00185, Rome, Italy
6Department of Physics, Harvard University, Cambridge, MA 02138

We show that, due to the weak ferromagnetism of La2−xSrxCuO4, an external magnetic field leads to a dimensional crossover 2D → 3D for the in-plane transport. The crossover results in an increase of the hole’s localization length and hence in a dramatic negative magnetoresistance in the variable range hopping regime. This mechanism quantitatively explains puzzling experimental data on the negative magnetoresistance in the Néel phase of La2−xSrxCuO4.

I. INTRODUCTION

The physics of the high-temperature superconducting oxides is determined by the interplay between the charge and spin degrees of freedom, ultimately responsible for the superconductivity itself. A variety of interesting phenomena exists already at low doping when the oxide layers are insulating. In La2−xSrxCuO4 (LSCO), the insulating (spin-glass) region corresponds to doping x < 0.055, with incommensurate magnetism which exists down to the boundary with the antiferromagnetic phase (at x = 0.02), and even inside the Néel region (x < 0.02). A popular point of view favors an explanation of the incommensurate magnetism based on the tendency of the holes to form “stripes.” However, experimental data on variable range hopping (VRH) (see the review Ref. 1) unambiguously indicate localization of holes for x < 0.055 and therefore support an approach based on a purely magnetic scenario, where a spiral distortion of the spin background is generated by localized holes. The corresponding theory explains quantitatively the variety of magnetic and transport data in LSCO.

Magnetic phenomena in the low-doping region reflect, in addition to the Heisenberg exchange, the presence of anisotropies in the spin-spin interactions, such as Dzyaloshinsky-Moriya (DM) and XY terms. In the present paper we consider the Néel phase, x < 0.02. In this phase the anisotropies confine the spins to the (ab) plane and fix the direction of the Néel vector to the b-orthorhombic axis. Moreover, the DM interaction induces a small out-of-plane spin component that is ferromagnetic in the plane (weak ferromagnetism) but staggered in the out-of-plane c-direction. This component can be easily influenced by an external magnetic field applied in different directions, as it has been recently addressed both experimentally10,11,12,13,14 and theoretically.15,16 For example, a perpendicular field (H || c) can cause an alignment of the out-of-plane moments via a spin-flop transition at a critical field Hf, determined by the competition between the DM and inter-layer Heisenberg exchange (typically Hf ≈ 5 − 7 T).11,12,14 Perhaps most intriguingly, the in-plane resistivity (along with the c-axis resistivity) decreases by as much as 50% across such a transition.10,11 The magnitude of the magnetoresistance (MR) shows a rapid increase only below ≈ 50 K11 where LSCO exhibits VRH conduction.3,17 This implies that the MR is accumulated mostly in transitions between localized states. Therefore it is very natural to assume that the large negative MR is due to an increase of the hole’s localization length as it was suggested in the first experimental paper.20 From theoretical viewpoint the problem is why the localization length increases at the spin flop transition. The first model for the localization length increase, invoking a three-dimensional (3D) VRH mechanism, was proposed in Ref. 18. However, it is clear now that except for ultra-low temperatures (that we estimate to be below ∼ 50mK), the VRH conduction at zero magnetic field is dominated by two-dimensional (2D) physics.3,17 Because of this the 3D picture is not able to describe the most recent and detailed MR data, as we discuss below. Experiments are performed typically in the temperature range of a few Kelvin and higher where the out-of-plane resistivity anisotropy is large ρc/ρab ∼ 102 − 103. While we ultimately expect that at T → 0 VRH will become 3D, in the temperature range of experimental interest the 2D mechanism is the relevant one, as is clear from the analysis of the 2D-3D crossover temperature and the fits of the hopping conductivity presented in the next section.

In the present work we demonstrate that the large MR arises from a change of the effective dimensionality of the VRH mechanism with applied field. We support our conclusions by detailed comparison with recent experiments on magnetotransport which can be described by our theory with excellent accuracy. The main idea of the present work is that a dimensional crossover (2D → 3D) occurs at the spin flop, and this is conceptually and quantitatively different from the 3D picture of Ref. 18. In particular in our approach the increase of the MR (and the localization length) is not simply due to the change of the out-of-plane effective mass as in Ref. 18, but rather arises from a change in the shape of the (localized) wave-functions.
across the spin-flop. In the temperature regime that we keep in mind, 1K and higher, the change of the out-of-plane effective mass is a small, secondary effect (which can manifest itself only at ultra-low temperatures where the full 3D VRH mechanism is responsible for transport).

We show that the alignment of the weak ferromagnetic moments in neighboring planes with the field allows the inter-layer hopping of localized holes, which in turn leads to an increase of the hole’s in-plane hopping probability and thus negative MR. The presence of an inter-layer hopping channel across the spin-flop was already identified in Ref. [18]; however our analysis differs in the effects this additional channel can produce in VRH conduction. By investigating the evolution of the hole bound state at zero field. In the experimentally relevant temperature range the hopping turns out to be quasi-two-dimensional, leading to a negative MR in very good agreement with the most recent experiments.[11,12]

The paper is organized as follows. In Section II we analyze the effect of the magnetic field on the dispersion of the localized holes, through the inter-layer hopping. In Section III we present a detailed analysis of the change of the hole’s wave-function, due to the modified dispersion. In Sections IV and V we then use the wave-functions to calculate the magnetoresistance for out-of-plane and in-plane magnetic fields, and compare with experiment. Section VI contains our conclusions.

II. INTER-PLANE HOPPING AND SPIN-FLOP TRANSITION FOR MAGNETIC FIELD $H \parallel \hat{c}$

First we briefly summarize previous results related to the structure of the hole’s 2D bound state at zero field. As a starting point, we consider the hole dynamics in the antiferromagnetic background within the framework of the $t - t' - t'' - J$ model. In the absence of the Coulomb potential $V(r)$ of the Sr ion, a hole resides, in momentum space, near the nodal points $(\pm \pi/2, \pm \pi/2)$ and has dispersion $\epsilon_k \approx \frac{\Delta}{2} k_1^2 + \frac{\Delta}{2} k_2^2$, where $\beta_1 \approx \beta_2 = \beta \equiv m^{-1} \approx 2 J$ is the inverse 2D effective mass appropriate for LSCO ($m_{\parallel} \approx 2 m_e$ in absolute units). We measure energies in units of $J = 130$ meV, and the lattice spacing is set to unity. Due to $V(r)$, the hole is localized, and its wave function has the form $\psi(r) \sim e^{-\kappa_0 r}$, corresponding to binding energy $\epsilon_0 = \beta \kappa_0^2 / 2$. Here the inverse (2D) localization radius for LSCO is $\kappa_0 \sim 0.3 - 0.4 \AA$, giving $\epsilon_0 \approx 10$ meV. On a perfect square lattice the bound state is four-fold degenerate: the hole can reside on either up or down sub-lattices (pseudospin), and it can reside in either of the two pockets $(\pi/2, \pm \pi/2)$ (flavor). The orthorhombic distortion lifts the flavor degeneracy due to the presence of diagonal next-nearest neighbor hopping $t'$. Hence, the holes occupy only the pocket $(\pi/2, -\pi/2)$ $\AA$. This is also consistent with the fact that the spin structure becomes incommensurate along the $\hat{b}$ orthorhombic direction, as seen in neutron scattering for $x < 0.055$

Now let us consider the correction to the 2D dispersion $\delta \epsilon_k$ arising from the inter-layer hopping $t_\perp$. Without account of correlations we have $\delta \epsilon_k = -8t_\perp \cos(k_x/2) \cos(k_y/2) \cos(k_z)$, with $k$-dependent $t_\perp$, $t_\perp = \frac{1}{4} (\cos k_x - \cos k_y)^2$, where $t_\perp \approx 50$ meV. By averaging over the momentum distribution $(k_x, k_y)$ in the 2D bound state, we find the effective value of inter-layer hopping $t_\perp \rightarrow t_\perp \kappa_0^2 / 4 \approx 1$ meV. Since this is a crude, order of magnitude estimate, below we will use $t_\perp$ as a fitting parameter.

The $t - J$ model correlations change $\delta \epsilon_k$. First, the hopping matrix element should be replaced by $t_\perp \rightarrow Z t_\perp$, where $Z \approx 0.3$ is the quasiparticle residue. Second, direct hopping is allowed only between spins in the same sub-lattice. In LSCO a spin in a given plane (e.g. spin “1” in Fig. 1) interacts with four others in the plane above (and below) but at $H = 0$ (or $H < H_f$) only out-of-plane hopping allowed in the $\hat{b}$ direction is allowed, because this corresponds to ferromagnetic ordering of spins in neighboring planes (see Fig. 1). However, when a magnetic field is applied along the $\hat{c}$-axis the spins in the next layer reverse their signs across the spin-flop transition at $H_f$, so that for $H > H_f$ only hopping in the $\hat{a}$ direction contributes. This is schematically shown in Fig. 1, and reflects in the effective dispersion:

$$\delta \epsilon_k = -4Z t_\perp \cos(k_z) \cos\left(\frac{k_x \pm k_y}{2}\right),$$

where we define the z-direction $z \parallel \hat{c}$. Since in the occupied pocket $k_x \approx \pi/2, k_y \approx -\pi/2$, Eq. (1) reads

$$\delta \epsilon_k^{H < H_f} \approx 0, \quad H < H_f$$

$$\delta \epsilon_k^{H > H_f} = -4Z t_\perp \cos(k_z), \quad H > H_f.$$
III. EVOLUTION OF THE HOLE BOUND STATE ACROSS THE SPIN-FLOP TRANSITION

To qualitatively understand the effect of the change of dispersion \( \epsilon \) at \( H > H_f \) on the in-plane hole dynamics, we first estimate the change in the hole binding energy. From (2), after the spin flop the edge of the continuum \( (k_z = 0) \) decreases by \( 4Zt_\perp \), while the absolute energy of the bound state, to second order in the small parameter \( Zt_\perp /\epsilon_0 \ll 1 \), decreases only by amount \( \Delta E \sim (Zt_\perp)^2 /\epsilon_0 \ll t_\perp \). The binding energy, which is the magnitude of the difference between the absolute energy and the continuum limit, changes after the flop as

\[
\epsilon_0 \to \epsilon \approx \epsilon_0 - 4Zt_\perp + (Zt_\perp)^2 /\epsilon_0 \approx \epsilon_0 - 4Zt_\perp .
\]

Within the VRH picture the conduction is proportional to the hole’s hopping probability, which decays exponentially away from the donor site due to the hole localization. Thus, the decrease of the hole binding energy across the spin flop signals an increase of the localization length, and in turn an increase of the VRH conductivity. This is our central idea that explains the negative MR.

To make this argument more quantitative, we need to compute the change in the hole’s hopping probability across the spin-flop transition. Let us enumerate planes by the index \( n \) and assume for simplicity that Sr produces a local potential \( V(r) \) that acts only in the plane \( n = 0 \). For a shallow level the exact form of the local potential is not important\(^\text{2}\) and for simplicity we will use the \( \delta \)-function approximation: \( V(r) = -2\delta(r - r_0) \), where \( r_0 \) is assumed to be smaller than the localization length, \( r_0 \ll 1 /\kappa_0 \). The bound state is described by the wave function \( \psi_n(r) \) that depends on both \( n \) and \( r \). Before the spin flop, \( H < H_f \), it obeys the Schrödinger equation:

\[
\left( -\frac{\beta}{2}\Delta_r - \kappa_0^2 \delta(r - r_0) \right) \psi_n(r) = E_0 \psi_n(r),
\]

where \( E_0 = -\epsilon_0 = -\beta \kappa_0^2 /2 \). The solution for \( r < r_0 \) is given by \( I_0(\kappa_0 r) \), and for \( r > r_0 \) it reads

\[
\psi_n = 0, \quad n \neq 0,
\]

\[
\psi_0 = \frac{\kappa_0}{\kappa_0^2} K_0(\kappa_0 r) \approx \sqrt{\frac{\kappa_0}{2r}} e^{-\kappa_0 r}, \quad \kappa_0 \gg 1,
\]

\[
H < H_f,
\]

with \( I_0(r) \) and \( K_0(r) \) are the modified Bessel functions of the first and second kind respectively. For the inverse localization length we obtain \( \kappa_0 = 2^{\gamma - 1} \exp(-\beta/(2g)) \), where \( \gamma = 0.577 \) is Euler’s constant. However the exact dependence of \( \kappa_0 \) on the parameters of the potential is not important, since the energy scale that \( \kappa_0 \) determines, the binding energy \( \epsilon_0 = \beta \kappa_0^2 /2 \), was extracted from experiment in earlier work, \( \epsilon_0 \approx 10 \text{meV} \) (see Section II).

For \( H > H_f \) the Schrödinger equation becomes

\[
\left( -\frac{\beta}{2}\Delta_r + \delta_n \epsilon V(r) \right) \psi_n - 2Zt_\perp (\psi_n + \psi_{n-1}) = E\psi_n,
\]

with \( V(r) = -\frac{2}{2}\delta(r - r_0) \), and \( E = -\beta \kappa^2 /2 \). After the Fourier transform \( \psi_n(r) = \sum_p \phi_p(r) e^{ipn} \), we obtain

\[
-\frac{\beta}{2} \Delta_r \phi_p(r) + V(r) \psi_0(r) = (E + 4Zt_\perp \cos p) \phi_p(r).
\]

By solving this equation we find

\[
\kappa /\kappa_0 \approx 1 + 2(Zt_\perp /\epsilon_0)^2,
\]

\[
\phi_p(r) = \frac{\kappa_0}{\sqrt{\pi}} K_0(\kappa_0 r \sqrt{\kappa^2 - (8Zt_\perp /\beta) \cos p}) .
\]

Thus, as we have already pointed out before Eq. (3), after the spin flop the absolute energy \( E \) is shifted only in the second order in \( Zt_\perp /\epsilon_0 \) and hence this shift can be neglected. Consequently below we set \( \kappa = \kappa_0 \). However, in contrast to Eq. (5), the new wave function,

\[
\psi_n(r) = \frac{\kappa_0}{\sqrt{\pi}} \int dp e^{ipn} K_0(\kappa_0 r \sqrt{1 - (4Zt_\perp /\epsilon_0) \cos p}),
\]

\[
H > H_f,
\]

does not have a simple exponential decay.

IV. MAGNETORESISTANCE ACROSS THE SPIN-FLOP TRANSITION FOR FIELD \( H || \hat{c} \)

To evaluate the MR across the spin flop we compute now the change of the hole’s probability to propagate at the (large) VRH distance \( R_T \): \( \kappa_0 R_T = \frac{1}{2} (\frac{\epsilon_0}{2 \kappa_0})^{1/2} \). We can then identify two large-distance regimes for the wave function (5): (1) Very large distances, \( \kappa_0 r \gg \epsilon_0 / (2Zt_\perp) \), and (2) Intermediate large distances, \( \epsilon_0 / (2Zt_\perp) \gg \kappa_0 r \gg 1 \).

In the first regime the integral in (5) can be evaluated using the saddle-point approximation and the wave
function is spread over many transverse channels, $n \sim \sqrt{4 \kappa_0 r Z_{t\perp}/\epsilon_0} \gg 1$. The probability density at distance $r$, $P(r, t_{\perp}) = \sum_n |\psi_n(r)|^2$, is

$$P(r, t_{\perp}) = \frac{\kappa_0}{2r} \frac{1}{\sqrt{8\pi(Z_{t\perp}/\epsilon_0)\kappa_0 r}} e^{-2\kappa_0 r \sqrt{1-(4Z_{t\perp}/\epsilon_0)}},$$

for $\kappa_0 r \gg \epsilon_0/(2Z_{t\perp})$.

In this regime the wave function has a pure exponential decay with a localization length corresponding to the shift of the binding energy given in Eq. [8]. We then find that the ratio of conductivities after ($H > H_f$) and before ($H < H_f$) the flop is

$$\frac{\sigma_{H > H_f}}{\sigma_{H < H_f}} = \frac{P(R_f, t_{\perp})}{P(R_f, t_{\perp} = 0)} \sim \exp \left\{ \frac{4Z_{t\perp}}{3\epsilon_0} \left( \frac{T_0}{T} \right)^{1/3} \right\} .$$

Here $P(R_f, t_{\perp} = 0) = |\psi_0|^2$, with $\psi_0$ from Eq. [8]. This result is valid when $\kappa_0 R_f \sim (T_0/T)^{1/3} \gg \epsilon_0/(2Z_{t\perp})$, which happens in practice when the temperature is below 1K. In this case $\sigma_{H > H_f}/\sigma_{H < H_f} \gg 1$, i.e. the corresponding MR is large: $|\Delta \rho/\rho| = (\rho_{H > H_f}/\rho_{H < H_f}) - 1 \approx 1$, and a full 2D $\rightarrow$ 3D crossover is expected in the spin flop. Notice that the MR is always negative, $\Delta \rho/\rho < 0$.

In the Intermediate large distances regime, $\epsilon_0/(2Z_{t\perp}) \gg \kappa_0 r \gg 1$, the integral in [8] can be evaluated by direct expansion in powers of $\kappa_0 r Z_{t\perp}/\epsilon_0$. Only the layers $n = 0, \pm 1$ contribute in this case, $P(r, t_{\perp}) = \sum_{n=0,\pm 1} |\psi_n|^2$, leading to

$$P(r, t_{\perp}) = \frac{\kappa_0}{2r} e^{-2\kappa_0 r} \left( 1 + 4(\kappa_0 r)^2 (Z_{t\perp})^2/\epsilon_0^2 \right),$$

for $\epsilon_0/(2Z_{t\perp}) \gg \kappa_0 r \gg 1$.

From [11] we obtain ($r \rightarrow R_f$) across the spin flop

$$\frac{\sigma_{H > H_f}}{\sigma_{H < H_f}} = 1 + \frac{4}{9} \left( \frac{T_0}{T} \right)^{2/3} \frac{(Z_{t\perp})^2}{\epsilon_0^2} .$$

(12)

This formula corresponds to a crossover from a pure 2D case to an “intermediate dimension” (three transverse channels), and it is justified when ($\sigma_{H > H_f}/\sigma_{H < H_f}) - 1 \ll 1$. We have also performed a full numerical evaluation of $P(r, t_{\perp})$ and of the MR with $\psi_n(r)$ from Eq. [8], since the above considerations are based on asymptotic behavior. The exact numerical form was used in both Figures 2 and 3 below. We have found quite clearly that indeed in the temperature range where experimental data are available, $T > 10$K, the intermediate asymptotic formula [12] is the relevant one, since the MR is still relatively small there, e.g. $|\Delta \rho/\rho| \approx 0.35, T = 10$K. However as the temperature is lowered, $T < 10$K, the MR increases and the system enters a crossover region between the asymptotic formulas Eq. [12] and Eq. [10], which requires the use of the exact wave-functions. The calculated MR is plotted in Fig. 2 and we observe that the fitted value $Z_{t\perp}/\epsilon_0 = 0.057$ agrees quite well with the estimate $Z_{t\perp}/\epsilon_0 \sim 0.03$ from band structure calculations [12].

From the above, we obtain ($r \rightarrow R_f$) across the spin flop

$$\frac{\sigma_{H > H_f}}{\sigma_{H < H_f}} = 1 + \frac{4}{9} \left( \frac{T_0}{T} \right)^{2/3} \frac{(Z_{t\perp})^2}{\epsilon_0^2} .$$

(12)

This formula corresponds to a crossover from a pure 2D case to an “intermediate dimension” (three transverse channels), and it is justified when ($\sigma_{H > H_f}/\sigma_{H < H_f}) - 1 \ll 1$. We have also performed a full numerical evaluation of $P(r, t_{\perp})$ and of the MR with $\psi_n(r)$ from Eq. [8], since the above considerations are based on asymptotic behavior. The exact numerical form was used in both Figures 2 and 3 below. We have found quite clearly that indeed in the temperature range where experimental data are available, $T > 10$K, the intermediate asymptotic formula [12] is the relevant one, since the MR is still relatively small there, e.g. $|\Delta \rho/\rho| \approx 0.35, T = 10$K. However as the temperature is lowered, $T < 10$K, the MR increases and the system enters a crossover region between the asymptotic formulas Eq. [12] and Eq. [10], which requires the use of the exact wave-functions. The calculated MR is plotted in Fig. 2 and we observe that the fitted value $Z_{t\perp}/\epsilon_0 = 0.057$ agrees quite well with the estimate $Z_{t\perp}/\epsilon_0 \sim 0.03$ from band structure calculations [12].

In this case, due to the alignment of the DM-induced moments with the magnetic field, the spins rotate in the $\hat{b} - \hat{c}$ plane (in opposite directions on the two sub-lattices) [10,12,13,15,16,22] and align completely along $\hat{c}$ at a field $H_{c2}$. Thus, once we introduce the angle $\theta(H)$ that
the spins make with the $\hat{b}$-direction, our previous calculations leading to Eq. (12) remain valid with the replacement $t_{\perp} \rightarrow t_{\perp} \sin \theta$. Observe that while for the undoped LCO an intermediate in-plane spin flop is expected at $H_{c1} < H_{c2}^{\perp} \approx 19$ T [12] in LSCO the doped holes contribute to enhance the $b$-axis spin susceptibility and then to confuse the spins in the $\hat{b} - \hat{c}$ plane, as it has been discussed recently in Ref. [22]. We can then write the field dependence of $\sin \theta$ between $H = 0$ and $H = H_{c2}$ as

$$\sin \theta(H) = \frac{HD/\sigma_0}{\Delta_{out}^2 + 4\eta - (1 - x \chi_{imp})H^2}$$

In the above expression $D$ is the DM anisotropy, $\eta = 2JJ_\perp$ is the inter-layer exchange, $\Delta_{out}$ is the out-of-plane (or XY) anisotropy gap, $\sigma_0$ is the staggered order parameter, $x$ is the doping and $\chi_{imp}$ is a dimensionless measure of the holes-induced spin susceptibility. Here $H$ is measured in units of $g_b^2\mu_B H$, with $g_b^2 = 2.1$ and $\mu_B$ is the Bohr magneton. The parameter values can be extracted from the experiments and for $x = 0.01$ we take $D = 2.16$ meV, $\eta = 1$ (meV)$^2$, $\Delta_{out} = 3.2$ meV, $\chi_{imp} = 80$, and $\sigma_0 = 0.36$, which gives $H_{c2} \approx 19$ T. Using $Z_{t_{\perp}}/\epsilon_0 = 0.047$ as the only fitting parameter, we find a remarkable agreement with the experimental data of Ref. [12] at $T = 20$ K, as shown in Fig. 4.

We also note that in Oxygen-doped compounds the contribution of the localized holes to the longitudinal susceptibility is much smaller, with $\chi_{imp} \sim 1$. As a consequence, one expects to observe at a field $H_{c2} \approx 10$ T an intermediate flop which reflects in a kink in both the $\sin \theta(H)$ curve and in the MR curve, as it has been measured indeed in the earlier transport measurements in La$_2$CuO$_{4+y}$ [10].

VI. DISCUSSION AND CONCLUSIONS

In conclusion, we have shown that in the strongly localized VRH regime for $T < 50$ K, the in-plane MR is sensitive to the inter-layer hopping $t_{\perp}$ because an external magnetic field effectively changes the dimensionality of the problem, making the hopping quasi-2D. The MR reflects the physics of the spin-flop and is always negative; its value as well as temperature and field dependence are in excellent quantitative agreement with recent experiments in LSCO at $x = 0.01$ [11,12]. Orbital effects, typically causing positive MR, are negligible at this small doping because the hole’s localization length $1/k \sim 2 - 3$ is much smaller than the magnetic length at any reasonable fields.

Finally we comment that unlike LSCO where the role of magnetic anisotropies is very well established, in insulating YBa$_2$Cu$_3$O$_{6+x}$ (YBCO) such anisotropies are expected to be very weak, which is related to the absence of strong localization in this material [23]. Spin-related effects in the in-plane MR are absent for field in the $\hat{c}$ direction, while the in-plane field MR remains small and appears to be due to the dynamics of holes that are very weakly influenced by disorder [22]. The complete understanding of these phenomena in YBCO remains an open issue although it is clear that magnetotransport is not dominated by the local spin physics.

Acknowledgments

We are grateful to D. K. Campbell, O. K. Andersen, O. Jepsen, A. Lavrov, and Y. Ando for stimulating discussions. O.P.S. gratefully acknowledges support from the Alexander von Humboldt Foundation, and the hospitality of the Max-Planck-Institute for Solid State Research Stuttgart and Leipzig University. V.N.K. is supported by Boston University. A.H.C.N. is supported through NSF grant DMR-0343790.

---

1. M. Matsuda, M. Fujita, K. Yamada, R. J. Birgeneau, M. A. Kastner, H. Hiraka, Y. Endoh, S. Wakimoto, and G. Shirane, Phys. Rev. B 62, 9148 (2000); M. Matsuda, M. Fujita, K. Yamada, R. J. Birgeneau, Y. Endoh, and G. Shirane, Phys. Rev. B 65, 134515 (2002).
2. S. A. Kivelson, I. P. Bindloss, E. Fradkin, V. Oganesyan, J. M. Tranquada, A. Kapitulnik, and C. Howald, Rev. Mod. Phys. 75, 1201 (2003).
3. M. A. Kastner, R. J. Birgeneau, G. Shirane, and Y. Endoh, Rev. Mod. Phys. 70, 897 (1998).
4. N. Hasselmann, A. H. Castro Neto, and C. Morais Smith, Phys. Rev. B 69, 014424 (2004).
5. V. Juricic, L. Benfatto, A. O. Caldeira, and C. Morais Smith, Phys. Rev. Lett. 92, 137202 (2004).
6. O. P. Sushkov and V. N. Kotov, Phys. Rev. Lett. 94, 097005 (2005); V. N. Kotov and O. P. Sushkov, Phys. Rev. B 72, 184519 (2005).
7. V. Juricic, M. B. Silva Neto, and C. Morais Smith, Phys. Rev. Lett. 96, 077004 (2006).
8. A. Luscher, Gr. Misguich, A. I. Milstein, and O. P. Sushkov, Phys. Rev. B 73, 085122 (2006).
9. A. Luscher, A. I. Milstein, and O. P. Sushkov, Phys. Rev. Lett. 98, 037001 (2007).
10. T. Thio, T. R. Thurston, N. W. Preyer, P. J. Picone, M. A. Kastner, H. P. Jenssen, D. R. Gabbe, C. Y. Chen, R. J. Birgeneau, and A. Aharony, Phys. Rev. B 38, 905 (1988); T. Thio, C. Y. Chen, B. S. Freer, D. R. Gabbe, H. P. Jenssen, M. A. Kastner, P. J. Picone, N. W. Preyer, and R. J. Birgeneau, Phys. Rev. B 41, 231 (1990).
11. Y. Ando, A. N. Lavrov, and S. Komiyia, Phys. Rev. Lett. 90, 247003 (2003).
12. S. Ono, S. Komiyia, A. N. Lavrov, Y. Ando, F. F. Balakirev,
13. A. Gozar, B. S. Dennis, G. Blumberg, S. Komiya, and Y. Ando, Phys. Rev. Lett. 93, 027001 (2004).
14. M. Reehuis, C. Ulrich, K. Prokes, A. Gozar, G. Blumberg, S. Komiya, Y. Ando, P. Pattison, and B. Keimer, Phys. Rev. B 73, 144513 (2006).
15. L. Benfatto and M. B. Silva Neto, Phys. Rev. B 74, 024415 (2006); L. Benfatto, M. B. Silva Neto, A. Gozar, B. S. Dennis, G. Blumberg, L. L. Miller, S. Komiya, and Y. Ando, Phys. Rev. B 74, 024416 (2006).
16. A. Luscher and O. P. Sushkov, Phys. Rev. B 74, 064412 (2006), and cited references.
17. E. Lai and R. J. Gooding, Phys. Rev. B 57, 1498 (1998).
18. L. Shekhtman, I. Ya. Korenblit, and A. Aharony, Phys. Rev. B 49, 7080 (1994).
19. O. K. Andersen, private communication.
20. B. I. Shklovskii and A. L. Efros, *Electronic Properties of Doped Semiconductors*, (Springer-Verlag, Berlin, 1984).
21. L. D. Landau and E. M. Lifshitz, *Quantum Mechanics*, (Pergamon Press, Oxford, New York, 1965).
22. M. B. Silva Neto and L. Benfatto, Phys. Rev. B 75, 140501(R) (2007).
23. X. F. Sun, K. Segawa, and Y. Ando, Phys. Rev. B 72, 100502(R) (2005).
24. Y. Ando, A. N. Lavrov, and K. Segawa, Phys. Rev. Lett. 83, 2813 (1999); A. N. Lavrov, Y. Ando, K. Segawa, and J. Takeya, Phys. Rev. Lett. 83, 1419 (1999).