Metrics Admitting Killing Spinors In Five Dimensions

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Abstract

BPS black hole configurations which break half of supersymmetry in the theory of $N = 2 \ d = 5$ supergravity coupled to an arbitrary number of abelian vector multiplets are discussed. A general class of solutions comprising all known BPS rotating black hole solutions is obtained.
1 Introduction

Recently there has been lots of interest in the study of BPS black hole solutions of the low-energy effective action of compactified string and M-theory. These activities have been initiated mainly due to the realisation that the recent understanding of the non-perturbative structure of string theory provides the microscopic degrees of freedom, $D$-branes, which give rise to the Bekenstein-Hawking entropy. In particular, BPS saturated solutions of toroidally compactified string theory have been constructed and their entropy was microscopically calculated using “D-brane” technology. Later, the microscopic analysis was applied to near-extreme black hole solutions for both the static and the rotating case. However, in these cases, the arguments become more heuristic and less rigorous (for a review, see for example, [1]).

BPS saturated solutions in toroidal compactifications correspond to vacua with $N = 4$ and $N = 8$ supersymmetry. These are severely constrained by the large supersymmetry and corrections to these solutions and their entropies can only arise from higher loop corrections, as the lowest order corrections are known to vanish. In contrast, BPS saturated solutions of $N = 2$ string vacua can receive corrections even at the one loop level. In the context of string theories, $N = 2$ supergravity models in four and five dimensions with vector and hyper-multiplets arise, respectively, from type II string and M-theory compactified on a Calabi-Yau threefold. The presence of perturbative and nonperturbative corrections for these models makes $N = 2$ black holes more intricate and also more difficult to analyse. However, the analysis of these black holes can be considerably simplified by the rich geometric structure of the underlying four and five dimensional low-energy effective field theory provided, respectively, by special and very special geometry [2, 3].

For example, the metric in four dimensions can be expressed in terms of symplectic invariant quantities in which the symplectic sections satisfy algebraic constraints involving a set of constrained harmonic functions [6]. Like for Einstein-Maxwell theory [4], various types of solutions, such as rotating and TAUB-NUT spaces, depend very much on the choice of the harmonic functions as well as the prepotential defining special geometry.

Using the explicit static black hole solutions, one can calculate the entropy and the value of the scalar fields near the horizon. An important feature of these black holes is that, for those with non-singular horizons, the entropy can be expressed in terms of the extremum of the central charge and that
the scalar fields take fixed values at the horizon independent of their initial values at spatial infinity \( [3] \).

In five dimensions, static metrics admitting supersymmetry or Killing spinors were constructed for the case of pure \( N = 2 \) supergravity in \( [8] \). The metric in this case is of the Tanghelini form \( [9, 10] \). In the context of \( N = 2 \) supergravity in five dimensions with abelian vector multiplets, extreme black holes with constant scalars, the so-called double-extreme BPS black holes, were considered in \( [11] \). It was also shown that their entropy can be expressed in terms of the extremised central charge. Moreover, the Strominger-Vafa black hole \( [14] \) was reproduced as a double-extreme black hole of an \( N = 2 \) supergravity model with one vector multiplet.

Moreover, an extremal rotating back hole solution was constructed in \( [13] \). This solution was later embedded into \( N = 2 \) supergravity theory in five dimensions interacting with one vector multiplet whose scalar field is set to a constant \( [14] \). This solution was then shown to be supersymmetric by solving for the Killing spinor equations. Rotating black hole solutions for five-dimensional \( N = 4 \) superstring vacua were also constructed in \( [19, 20] \).

It is our purpose in this work to study general BPS black hole solutions which break half of supersymmetry of \( d = 5 \), \( N = 2 \) supergravity theory with an arbitrary number of vector multiplets. Static non-rotating solutions have been discussed recently in \( [17] \). This will be generalised here to allow for rotating solutions. The class of solutions obtained will include all known rotating BPS black hole solutions. This work is organised as follows. In the next section, the structure of \( d = 5 \), \( N = 2 \) supergravity is briefly reviewed, and we collect some formulae and expressions which will be relevant for our later discussion. In section three, we will present static black hole solutions and verify that they admit unbroken supersymmetry by solving for the supersymmetry transformation rules for the gravitino and the gauginos in a bosonic background.


2 \( \ d = 5 \ N = 2 \) Supergravity and Very Special Geometry

The theory of \( N = 2 \) supergravity coupled to an arbitrary number \( n \) of Maxwell’s supermultiplets was first considered in \( [7] \). In this work, it was es-
established that the real scalar fields of the vector supermultiplets parametrise a riemannian space. The classification of homogeneous symmetric spaces is related to that of Jordan algebras of degree three. These spaces can be expressed in the form

\[ \mathcal{M} = \frac{\text{Str}_0(J)}{\text{Aut}(J)}, \]

(1)

where \( \text{Str}_0(J) \) is the reduced structure group of a formally real unital Jordan Algebra, \( \text{Aut}(J) \) is its automorphism group. The scalar manifold can be regarded as a hyperspace, with vanishing second fundamental form of an \((n + 1)\)-dimensional riemannian space \( \mathcal{G} \) whose coordinates \( X \) are in correspondence with the vector multiplets including that of the graviphoton. The equation of the hypersurface is \( V = 1 \), where \( V \), the prepotential, is a homogeneous cubic polynomial in the coordinates of \( \mathcal{G} \),

\[ V = \frac{1}{6}C_{IJK}X^IX^JX^K. \]

(2)

More recent treatment of the bosonic part of \( N = 2 \) supergravity theory was given in [3] in terms of “very special geometry”. The construction of \( N = 2 \) supergravity arising from the compactification of 11 dimensional supergravity on a Calabi-Yau 3-folds was discussed more recently in [12].

The bosonic part of the effective supersymmetric \( N = 2 \) Lagrangian which describes the coupling of vector multiplets to supergravity is entirely determined in terms of the homogeneous cubic prepotential (2) defining very special geometry [3] and which in the case of Calabi-Yau compactification corresponds to the intersection form. This Lagrangian is

\[ e^{-1} \mathcal{L} = -\frac{1}{2} R - \frac{1}{4} G_{IJ} F_{\mu I} F^{\mu J} - \frac{1}{2} g_{ij} \partial_\mu \phi^i \partial^\mu \phi^j + \frac{e^{-1}}{48} \epsilon^{\mu \rho \sigma \lambda} C_{IJK} F_{\mu I} F_{\rho J} A^K_\lambda \]

(3)

where \( R \) is the scalar curvature, \( F_{\mu I} = 2 \partial_{[\mu} A^{I]}_\nu \) is the Maxwell field-strength tensor and \( e = \sqrt{-g} \) is the determinant of the F"unfbein \( e_{\mu a} \).

The fields \( X^I = X^I(\phi) \) are the special coordinates satisfying

\[ X^I X_I = 1, \quad \frac{1}{6} C_{IJK} X^IX^JX^K = 1, \]

(4)

\[ [ab] = \frac{1}{2}(ab - ba). \]

1In this paper, we shall be using the signature \((-++++)\) and for the indices we take: \( m, n, \cdots \) to denote curved indices whereas the indices \( a, b, \cdots \) are flat. Antisymmetrized indices are defined by: \([ab] = \frac{1}{2}(ab - ba)\).
where, $X_I$, the dual coordinate is defined by,

$$X_I = \frac{1}{6} C_{IJK} X^J X^K. \quad (5)$$

The gauge coupling metric $G_{IJ}$ which depends on the moduli, and the metric $g_{ij}$ are given in terms of the prepotential \( \mathcal{V} \) by

$$G_{IJ} = -\frac{1}{2} \frac{\partial}{\partial X^I} \frac{\partial}{\partial X^J} (\ln \mathcal{V}) \big|_{\mathcal{V}=1},$$

$$g_{ij} = G_{IJ} \partial_i X^I \partial_j X^J \big|_{\mathcal{V}=1}, \quad (\partial_i \equiv \frac{\partial}{\partial \phi^i}). \quad (6)$$

Now we list some useful relations which follow from very special geometry. Using the definition $\mathcal{V} = 1$, one can easily deduce that

$$\partial_i X_I = \frac{1}{3} C_{IJK} \partial_i X^J X^K, \quad X^I \partial_i X_I = X_I \partial_i X^I = 0. \quad (7)$$

Moreover, using the definition of (6), the gauge coupling metric can be expressed in terms of the special coordinates by

$$G_{IJ} = -\frac{1}{2} C_{IJK} X^K + \frac{9}{2} X_I X_J. \quad (8)$$

Also one can easily verify the following relations

$$X_I = \frac{2}{3} G_{IJ} X^J,$$

$$\partial_i X_I = -\frac{2}{3} G_{IJ} \partial_i X^J. \quad (9)$$

The supersymmetry transformation laws for the Fermi fields in a bosonic background are given by \[7, 11\]

$$\delta \psi_\mu = D_\mu \epsilon + \frac{i}{8} X_I \left( \Gamma_\mu^{\nu \rho} - 4 \delta_\mu^{\nu} \Gamma_\rho \right) F_{\nu \rho}^I \epsilon,$$

$$\delta \lambda_i = \frac{3}{8} \partial_i X_I \Gamma^{\mu \nu} \epsilon F_{\mu \nu}^I - \frac{i}{2} g_{ij} \Gamma^{\mu} \partial_\mu \phi^j \epsilon, \quad (10)$$

where $\epsilon$ is the supersymmetry parameter and $D_\mu$ the covariant derivative

$$D_\mu = \partial_\mu + \frac{1}{4} \omega_{\mu ab} \Gamma^{ab}. \quad (11)$$

Here, $\omega_{\mu ab}$ is the spin connection, $\Gamma^\nu$ are Dirac matrices and

$$\Gamma^{a_1 a_2 \ldots a_n} = \frac{1}{n!} \Gamma^{[a_1} \Gamma^{a_2} \ldots \Gamma^{a_n]}. \quad (12)$$
### 3 BPS Rotating Black Hole Solutions

We are interested in finding BPS black hole solutions which break half of the supersymmetry of the underlying \( N = 2 \ d = 5 \) supergravity in five dimensions. Our analysis is for a generic model with arbitrary number of vector multiplets and general values for \( C_{IJK} \). Motivated by the form of the non-rotating metric which admits supersymmetry in the case of pure supergravity with no vector multiplets \([8]\) as well as the static solutions found in \([17]\), we assume that the metric can be brought to the form

\[
ds^2 = -e^{-4U}(dt + w_m dx^m)^2 + e^{2U}(d\vec{x})^2
\]

where \( U = U(x) \) and \( w_m = w_m(x) \). Note that for \( w_m = 0 \) (no rotation), and for pure supergravity, the solution which admits Killing spinors is known to be given by \([8]\)

\[
ds^2 = -H^{-2}dt^2 + H(d\vec{x})^2,
\]

where \( H \) is a harmonic function.

The F"unfbeins for the metric in (13) are

\[
\begin{align*}
e^0_\tau &= e^{-2U}, & e^0_m &= e^{-2U}w_m \\
e^a_\tau &= 0, & e^a_m &= e^a U \\
e^0_\rho &= e^{2U}, & e^\rho_a &= -e^{-U}w_m \delta^m_a \\
e^m_0 &= 0, & e^m_a &= e^{-U} \delta^m_a
\end{align*}
\]

For the spin connections one obtains

\[
\begin{align*}
\omega_{\tau}^{a0} &= 2\partial_m U (e^{-3U}) \delta^m_a, \\
\omega_{\rho}^{a0} &= 2\partial_m U e^{-3U} w_n \partial_m^a + \frac{1}{2}e^{-3U} \delta^{am}(\partial_n w_m - \partial_m w_n) \\
\omega_{\rho}^{ab} &= -\frac{1}{2}e^{-6U} \delta^m_a \delta^m_b (\partial_n w_m - \partial_m w_n), \\
\omega_{\rho}^{ab} &= -\frac{1}{2}e^{-6U} w_p \delta^m_a \delta^m_b (\partial_n w_m - \partial_m w_n) - \partial_n U (\delta^{an} \delta^b_p - \delta^{nb} \delta^a_p)
\end{align*}
\]

We now turn to our Ansatz and try to determine the constraints imposed by unbroken supersymmetry on the functions \( U \) and \( w \). This is achieved by
solving the equations obtained by demanding the vanishing of the gravitino and gauginos supersymmetry transformation laws in a bosonic background. We first start with the equation corresponding to the vanishing of the time component of the gravitino supersymmetry transformation. For our metric this is given by

$$ \delta \psi_t = \partial_t \epsilon - \frac{1}{8} e^{-6U} \delta^{am} \delta^{bn} \Gamma_{ab} \left( \left( \partial_m w_n - \partial_n w_m \right) + e^{2U} X_I F_{mn}^I \right) \epsilon + i \partial_m U \delta^{am} e^{-4U} X_I F_{tn}^I \Gamma_{ab} \epsilon - i e^{-2U} X_I F_{tm}^I \Gamma^a \epsilon \delta^m_a. $$

Using the relation

$$ \Gamma^{ab} = -\frac{i}{2} \epsilon^{abcd} \Gamma_{cd} \Gamma_0, \quad (17) $$

and demanding that $\Gamma^0 \epsilon = -i \epsilon$, we obtain the following relations for the supersymmetry transformation parameter and the graviphoton field strength

$$ \partial_t \epsilon = 0, $$

$$ (X_I F_{mn}^I)^- = \left( \partial_m Q_n - \partial_n Q_m \right)^-, $$

$$ (X_I F_{tm}^I) = -\partial_m e^{-2U}. \quad (18) $$

where $Q_n \equiv e^{-2U} w_n$ and $F_{mn}^- = F_{mn} - F_{mn}^*$. Notice that the chirality constraint on the spinor $\epsilon$ reduces the $N = 2$ supersymmetry to $N = 1$. Next, we turn to the space-component of the gravitino supersymmetry transformation. Using our Ansatz, we obtain the following equation

$$ \delta \psi_m = \partial_m \epsilon + \frac{1}{4} \left( \omega_m^{ab} \Gamma^{ab} + 2 \omega_m^{a0} \Gamma_a \Gamma_0 \right) \epsilon + \frac{i}{8} X_I \left( \Gamma_m^{np} F_{np}^I + 2 \Gamma_m^{nt} F_{nt}^I - 4 \Gamma^n F_{mn}^I - 4 \Gamma^t F_{mt}^I \right) \epsilon = 0. \quad (19) $$

The $\Gamma_a$ dependent terms are independent of the other terms and have to vanish, this implies the condition

$$ 2(\partial_m U w_n - \partial_n U w_m) - \frac{1}{2}(\partial_m w_n - \partial_n w_m) + e^{2U} (X_I F_{mn}^I) - \frac{1}{2} e^{2U} (X_I F_{mn}^I)^* - (\partial_m U w_n - \partial_n U w_m)^* = 0. \quad (20) $$
The above equations together with (18) then gives
\[
\left( \partial_m w_n - \partial_n w_m \right) - X_I F^I_{mn} = (\partial_m Q_n - \partial_n Q_m).
\] (21)

Using the above relations, it can be easily shown that the $\Gamma_{ab}$ coefficient is identically vanishing. Therefore, the vanishing of the space-component of the gravitino transformation, with the supersymmetry breaking condition, amounts to the following simple differential equation,
\[
(\partial_m + \partial_m U)\epsilon = 0
\] (22)

which admits the solution
\[
\epsilon = e^{-U} \epsilon_0
\] (23)

where $\epsilon_0$ is a constant spinor satisfying $\Gamma^0 \epsilon_0 = -i \epsilon_0$. Finally we consider the supersymmetry transformation law of the gauginos given in (10). Using (6), this can be rewritten in the form
\[
\delta \lambda_i = -\frac{1}{4} \left( G_{IJ} \partial_i X^I \Gamma^{\mu \nu} F^J_{\mu \nu} - 3i \Gamma^\mu \partial_\mu X^I \partial_i X^I \right) \epsilon.
\] (24)

The vanishing of the gaugino transformation thus gives
\[
G_{IJ} \partial_i X^I (\Gamma^{mn} F^J_{mn} + 2 \Gamma^{mt} F^J_{mt}) \epsilon - 3i \partial_\mu X^I \partial_i \partial_j X^I \Gamma^m \epsilon = 0
\] (25)

This leads to the two equations corresponding to the vanishing of the coefficients of the $\Gamma^m$ and $\Gamma^{mn}$ terms
\[
\frac{3}{2} e^{-U} \partial_m X_I \partial_i X^I - G_{IJ} e^U \partial_i X^I F^J_{im} = 0,
\]
\[
\left( G_{IJ} \partial_i X^I F^J_{mn} + \frac{3}{2} e^{-U} (\partial_m X_I w_n - \partial_n X_I w_m) \right) = 0.
\] (26)

The first equation can be solved by
\[
F^I_{im} = -\partial_m (e^{-2U} X^I).
\] (27)

This can be verified by noticing that
\[
\partial_i X^I G_{IJ} F^J_{im} = -G_{IJ} \partial_i X^I \partial_m e^{-2U} X^J - G_{IJ} \partial_i X^I e^{-2U} \partial_m X^J
\]
\[
= \frac{3}{2} e^{-2U} \partial_m X_I \partial_i X^I
\] (28)
where we have made use of the relations in (18) and (19). Clearly the Ansatz (27) is consistent with (18),

\[ X^I \partial_m (e^{-2U} X^I) = -\partial_m (e^{-2U}) \] (29)

where we have made use of the relation \( X^I X_I = 1 \). Also one can easily verify that

\[ G_{IJ} F_{tm}^I = \frac{3}{2} e^{-4U} \partial_m (e^{2U} X_I) \] (30)

The second equation in (26) gives

\[ \partial_i X^I (F_{mn}^J)^- = -\frac{3}{2} e^{-2U} \partial_i X^I (\partial_m X_I Q_n - \partial_n X_I Q_m)^-. \] (31)

This together with (21) can be solved by

\[ F_{mn}^I = \partial_m (X^I Q_n) - \partial_n (X^I Q_m). \] (32)

In order to fix the various quantities in terms of space-time functions, we solve for the equations of motion for the gauge fields. From the Lagrangian (3), one can derive the following equation of motion for the gauge fields,

\[ \partial_\nu \left( eG_{IJ} g^{\mu\rho} g^{\nu\sigma} F_{\rho\sigma}^J \right) = \frac{1}{16} C_{IJK} e^{\mu\nu\rho\sigma} F_{\nu\rho}^J F^K_{\sigma\kappa}. \] (33)

After some lengthy calculation, one finds that for our Ansatz the equation of motion (33) gives

\[ e^{2U} X_I = \frac{1}{3} H_I. \] (34)

where \( H_I \) is a harmonic function. Using this algebraic equation together with (4) and (5), one can determine \( X^I \) and the metric in terms of a set of harmonic functions. The gauge fields are then determined by using their relations to the special coordinates given by (27) and (32). Before we discuss particular solutions, we will demonstrate that the BPS solution discussed can be expressed in terms of the geometry of the internal space, i.e., can be related to the cubic polynomial \( V \). If we define the rescaled coordinates

\[ Y_I = e^{2U} X_I, \quad Y^I = e^U X^I, \] (35)

\(^2\) notice that the Bianchi identities are trivially satisfied
then the underlying very special geometry implies that
\[ Y_I Y^I = e^{3U} X_I X^I = e^{3U} = V(Y) = \frac{1}{3} C_{IJK} Y^I Y^J Y^K \]  
(36)
and thus the metric takes the form
\[ ds^2 = -V^{-4/3}(Y)(dt + \omega_m dx^m)^2 + V^{2/3}(Y)(d\vec{x})^2 \]  
(37)
where
\[ \frac{1}{2} C_{IJK} Y^I Y^J Y^K = H_I. \]  
(38)

Using the above general Ansatz one can construct models with rotational symmetry. To proceed perhaps it is more convenient to work in spherical coordinates. If we choose the four spatial coordinates as
\[ x^1 + ix^2 = r \sin \theta e^{i\phi}, \quad x^3 + ix^4 = r \cos \theta e^{i\psi} \]  
(39)
and specialise to solutions with rotational symmetry in two orthogonal planes, i.e.,
\[ w_\phi = w_\phi(r, \theta), \quad w_\psi = w_\psi(r, \theta), \quad w_r = \omega_\theta = 0. \]  
(40)
then the self-duality condition of the field strength of \( w \) implies
\[ \partial_r w_\phi + \frac{\tan \theta}{r} \partial_\theta w_\psi = 0, \]
\[ \partial_\theta w_\phi - r \tan \theta \partial_r w_\psi = 0. \]  
(41)
One finds for a (decaying) solution,
\[ w_\phi = -\frac{\alpha}{r^2} \sin^2 \theta, \quad w_\psi = \frac{\alpha}{r^2} \cos^2 \theta, \]  
(42)
which in Cartesian coordinates give
\[ w_1 = \frac{\alpha x^2}{r^4}, \quad w_2 = -\frac{\alpha x^1}{r^4}, \quad w_3 = -\frac{\alpha x^4}{r^4}, \quad w_4 = \frac{\alpha x^3}{r^4}. \]  
(43)
Thus the angular momentum is given by
\[ J^{(12)} = J^\phi = -J^{(34)} = -J^\psi = \frac{\alpha \pi}{4G_N}. \]  
(44)
Therefore, the general form of solution with rotational symmetry in two orthogonal planes has the following form for the metric
\[
ds^2 = -e^{-4U} \left( dt - \frac{\alpha \sin^2 \theta}{r^2} d\phi + \frac{\alpha \cos^2 \theta}{r^2} \right)^2 + e^{2U} \left( dr^2 + r^2 d\Omega^2 \right) \tag{45}\]

where
\[
d\Omega^2 = (d\theta^2 + \sin^2 \theta d\phi^2 + \cos^2 \theta d\psi^2). \tag{46}\]

Let us examine the behaviour of our solution near the horizon, \((r \to 0)\). There \(V(Y)\) can be approximated as follows.
\[
V_{\text{hor}}(Y) = \frac{1}{3} (Y^I H_I)_{\text{hor}} = \frac{1}{3} (Y^I)_{\text{hor}} (\frac{q_I}{r^2}). \tag{47}\]

However, \(Y^I_{\text{hor}} = V_{\text{hor}}^{1/3}(Y) X^I_{\text{hor}}\), and thus
\[
V_{\text{hor}}^{2/3}(Y) = \frac{1}{3} \frac{Z_{\text{hor}}}{r^2} \tag{48}\]

where \(Z = q_I X^I\) is the central charge, and \(Z_{\text{hor}}\) is its value at the horizon. Also, equation (54) which defines the moduli over space-time, becomes near the horizon
\[
(Z X^I)_{\text{hor}} = q_I, \tag{49}\]

which is the equation obtained from the extremization of the central charge \(\Box_J, Z_{\text{hor}} = Z_{\text{ext}}\).

The ADM mass of black holes in five dimensions is given by
\[
g_{tt} = 1 - \frac{8 G_N M_{\text{ADM}}}{3 \pi r^2} + \cdots \tag{50}\]

where \(G_N\) is Newton’s constant. This implies for our metric
\[
V(Y) = Y^I Y_I = 1 + \frac{2 G_N M_{\text{ADM}}}{\pi r^2} + \cdots \tag{51}\]

If we expand \(Y^I\) as
\[
Y^I = Y^I_{\infty} + \frac{\beta^I}{r^2} + \cdots \tag{52}\]

and write the harmonic functions as \(H_I = h_I + \frac{q_I}{r^2}\), then one obtains
\[
1 + \frac{2 G_N M_{\text{ADM}}}{\pi r^2} + \cdots = \frac{1}{3} \left( h_I Y^I_{\infty} + \frac{h_I \beta^I + Y^I_{\infty} q_I}{r^2} + \cdots \right) \tag{53}\]
However, from the relation $Y_1 \partial_t Y^1 = \frac{1}{3} \partial_r \mathcal{V}(Y)$ one obtains $\beta^I h_I = \frac{2}{3} G_N M_{ADM}$, and thus the ADM mass is related to the central charge by

$$M_{ADM} = \frac{\pi}{4 G_N} Z_\infty,$$

where we have used $Y^I_\infty = X^I_\infty$. Therefore these black holes saturate the BPS bound as should be expected.

The Bekenstein-Hawking entropy $S_{BH}$, related to the area of the horizon ($r = 0$) $A$, is given by

$$S_{BH} = \frac{A}{4 G_N} = \frac{\pi^2}{2 G_N} \sqrt{\left(\frac{Z_{htr}}{3}\right)^3 - \alpha^2}.$$  \hfill (55)

If one assumes that the values of the moduli at the horizon are valid throughout the entire space-time, then one obtains the double-extreme black hole solution \cite{11} where the metric takes form

$$ds^2 = -\left(1 + \frac{Z}{3r^2}\right)^{-2} \left(dt - \frac{\alpha \sin^2 \theta}{r^2} d\phi + \frac{\alpha \cos^2 \theta}{r^2} \right)^2 + \left(1 + \frac{Z}{3r^2}\right) (dr^2 + r^2 d\Omega^2).$$  \hfill (56)

As a specific example, consider the so-called $STU = 1$ model \cite{18, 17}, $(X^1 = S, X^2 = T, X^3 = U)$. The equations one obtains from (34) are given by \cite{17}

$$e^{2U}TU = H_0,$$
$$e^{2U}SU = H_1,$$
$$e^{2U}ST = H_2,$$  \hfill (57)

where $H_0$, $H_1$ and $H_2$ are harmonic functions. Equation (57) together with the fact that $STU = 1$ implies the following solution for the metric and the moduli fields,

$$e^{2U} = (H_0 H_1 H_2)^\frac{1}{3}$$  \hfill (58)

and

$$S = \left(\frac{H_1 H_2}{H_0^3}\right)^\frac{1}{3},$$
$$T = \left(\frac{H_0 H_2}{H_1^3}\right)^\frac{1}{3},$$
$$U = \left(\frac{H_0 H_1}{H_2^3}\right)^\frac{1}{3},$$  \hfill (59)
If one writes the harmonic function as $H_I = 1 + \frac{q_I}{\mathcal{M}}$, for this model, the ADM mass and the entropy are

$$M_{ADM} = \frac{\pi}{4G_N}(q_0 + q_1 + q_2),$$

$$S_{BH} = \frac{\pi^2}{2G_N} \sqrt{q_0q_1q_2 - \alpha^2}. \quad (60)$$

The solution obtained is the one found in [20, 19] in a different context.

In conclusion, we have given an algorithm for obtaining general BPS black holes which breaks half of supersymmetry ($\Gamma^0\epsilon = -i\epsilon$) for the theory of $N = 2 \ d = 5$ supergravity coupled to an arbitrary number of vector multiplets. These solutions were expressed in terms of the rescaled cubic polynomial which in the case of Calabi-Yau compactification corresponds to the intersection form. For the solutions found, the gauge fields are related to the special coordinates $X^I$ via the relations (27) and (32). It should be emphasized that the unbroken supersymmetry of the BPS solutions does not fix the configuration completely but rather provide a relationship between the various physical fields (metric, gauge and scalar fields) as well as a constraint on $w$, the function that allows for rotating solution. The self-duality of the field strength of $w$ forces the angular momentum in the two orthogonal planes to be equal. Such a condition was also derived in the conformal sigma model approach, as arising from the requirement of conformal invariance [21]. The black hole solution can be fixed in terms of space-time functions by solving for the equations of motion for the gauge fields. It is of interest to generalise these solutions to the non-extreme case and also to obtain microscopically their entropies.

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