Gauging $U(1)$ symmetries and the number of right-handed neutrinos

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Abstract

In this letter we consider that assuming: a) that the only left-handed neutral fermions are the active neutrinos, b) that $B - L$ is a gauge symmetry, and c) that the $L$ assignment is restricted to the integer numbers, the anomaly cancellation imply that at least three right-handed neutrinos must be added to the minimal representation content of the electroweak standard model. However, two types of models arise: i) the usual one where each of the three identical right-handed neutrinos has total lepton number $L = 1$; ii) and the other one in which two of them carry $L = 4$ while the third one carries $L = -5$.

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It is well known that it is possible to enlarge the representation content of the minimal electroweak standard model (ESM) by adding an arbitrary number of right-handed neutrinos. Since they are sterile under the interactions of that model they do not contribute to the anomaly cancellation of the gauge symmetries, then nothing determine their number. Until now, it has been a question of taste to consider a particular number of these fields in extensions of the model. It is also well known that within the ESM (no right-handed neutrinos) both, baryon ($B$) and total lepton ($L$) numbers, are conserved automatically up to anomaly effects: both global $U(1)_B$ and $U(1)_L$ are anomalous [1] (but their consequences are well suppressed at least at zero temperature) and only the combination $U(1)_{B-L}$ is a global anomaly free symmetry if right-handed neutrinos are added for cancelling the mixed gauge-gravitational anomaly [2].

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When the $B-L$ symmetry is gauged, it becomes anomaly free but only again if an appropriate number of right-handed neutrinos is added, but this time they also must cancel out other anomalies, like the cubic one, induced by the active left-handed neutrinos. For instance, adding one per generation solve again the problem.

In this Letter we will propose extensions of the standard model in which $B-L$ appears as a local symmetry. Many of the extension of the SM in which $B-L$ is a gauge symmetry are based on $SMG \otimes U(1)_{B-L}$ gauge symmetry [3]. However, in those models, since $SMG$ is the gauge symmetry of the SM, the usual Higgs doublet does not carry the $U(1)_X$ charge, and then the electric charge $Q$ is given in terms of the $SU(2)_L$ and $U(1)_Y$ generators alone. This implies important phenomenological differences with the models that we will consider below, in which the electric charge includes the $U(1)$ extra generators. Other models with extra $U(1)$ factors are based on grand unified scenarios [4,5]. There are also models with an extra $U(1)$ factor and a $Z'$ with non-universal couplings to fermions in which right-handed interactions single out the third generation [6]. The difference between models with additional $U(1)$ groups not inspired in unified theories is that the neutral current parameters in the latter case must satisfied some relations [7] that do not exist in the former. For this reason these parameters are more arbitrary in our models than in models like those in Refs. [4,5]. In these sort of model there is $Z-Z_X$ mixing in the mass matrix at the tree level. Of course, mixing in the kinetic term is possible [8], but we will assume that we are working in a basis in which the kinetic mixing vanishes. For a review of the phenomenology of the extra neutral vector boson see Ref. [9].

Hence, we will consider an extension of the $SMG$ based on the following gauge symmetry:

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_{Y'} \otimes U(1)_{B-L}$$

$$\downarrow \langle \phi \rangle$$

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$

$$\downarrow \langle \Phi \rangle$$

$$SU(3)_C \otimes U(1)_{em},$$

where $Y'$ is chosen to obtain the hypercharge $Y$ of the standard model, given by $Y = Y' + (B - L)$. Thus, in this case, the charge operator is given by

$$\frac{Q}{e} = I_3 + \frac{1}{2} [Y' + (B - L)].$$

The simplest possibility is adding three right-handed neutrinos with the same lepton number as that of the left-handed ones. In this case $B - L$ is anomaly
free. We also add a complex neutral scalar $\varphi$ that because of $\langle \varphi \rangle \neq 0$, breaks the $U(1)_{B-L}$ gauge symmetry. The quantum number of the fields in this model are shown in Table 1.

The model has three real neutral gauge bosons $W^3$, $A$, $B$ corresponding to the $SU(2)_L$, $U(1)_{Y'}$, and $U(1)_{B-L}$ factors respectively, are mixtures of the photon, $A$, and two massive neutral bosons, $Z_1 \approx Z$, and $Z_2 \approx Z'$, fields. The model introduces deviations of the $\rho$ parameter, at the tree level, that can be parameterized by the $T$ parameter defined, in absence of new charged $W$-like vector bosons, and neglecting the contributions of the Majorana neutrinos which contributions to the $T$-parameter may have either sign, as $\hat{\alpha}(M_Z)T \equiv -\Pi^{ew}_{ZZ}(0)/M^2_{Z_1}$, where $\Pi^{ew}_{ZZ}(0) = M^2_{Z_1} - (g^2 v^2 / 4 c^2_W)$, being $M^2_{Z_1}$ the exact mass of the lighter neutral vector boson that we are not showing here. We obtain $\Delta \rho = \hat{\alpha}(M_Z)T \approx (g^4/4)^2 \hat{v}^2$. This implies in the lower bound $u > (10^4 g^2) \text{GeV} > 4\pi (10^4 \alpha^2 s^2_W / c^2_W) \text{GeV}$, in order to be consistent with the experimental data [10]. The scalar singlet contributes less to the mass of the lighter vector boson as its VEV is higher, i.e., if $u \to \infty$ then $Z_1 \to Z$ and $Z_2$ decouples. Besides, since we are working in a basis where there is no kinetic mixing between the $U(1)_{Y'}$ and $U(1)_{B-L}$ gauge bosons, there are no tree level contributions to the $S$ and $U$ parameters [11].

Quark and charged lepton Yukawa interactions are the same as in the ESM. However, the neutrino mass terms are Dirac terms involving the left-handed

|   | $I_3$ | $I$ | $Q$ | $Y'$ | $B - L$ | $Y$ |
|---|---|---|---|---|---|---|
| $\nu_{eL}$ | 1/2 | 1/2 | 0 | 0 | −1 | −1 |
| $e_L$ | −1/2 | 1/2 | −1 | 0 | −1 | −1 |
| $e_R$ | 0 | 0 | −1 | −1 | −1 | −2 |
| $n_R$ | 0 | 0 | 0 | 1 | −1 | 0 |
| $u_L$ | 1/2 | 1/2 | 2/3 | 0 | 1/3 | 1/3 |
| $d_L$ | −1/2 | 1/2 | −1/3 | 0 | 1/3 | 1/3 |
| $u_R$ | 0 | 0 | 2/3 | 1 | 1/3 | 4/3 |
| $d_R$ | 0 | 0 | −1/3 | −1 | 1/3 | −2/3 |
| $\varphi^+$ | 1/2 | 1/2 | 1 | 1 | 0 | 1 |
| $\varphi^0$ | −1/2 | 1/2 | 0 | 1 | 0 | 1 |
| $\phi$ | 0 | 0 | 0 | −2 | 2 | 0 |

Table 1
Quantum number assignment in the model with three identical right-handed neutrinos.
leptons $\Psi = (\nu_l)^T$, and the scalar doublet $\Phi$, $\overline{\Psi} a_L G^D a_{\alpha} \Phi n_{\alpha R}$, and Majorana terms involving the singlet $\phi$, $\phi (n_{\alpha R})^c G^M_{ab} n_{bR}$, where $a = e, \mu, \tau$ and we have omitted summation symbols. If $\langle \Phi \rangle = v/\sqrt{2} \simeq 174 \text{ GeV}$ the neutrino Dirac masses are of the same order of magnitude (up a fine tuning in $G^D$). Hence, in this case for implementing the seesaw mechanism we have to have that $\langle \phi \rangle = u/\sqrt{2} \gg \langle \Phi \rangle$ and there is no natural possibility for having light right-handed neutrinos. However, if the doublet $\Phi$ is different from the doublet which gives masses for quarks and charged leptons, $\langle \Phi \rangle$ can be smaller than the electroweak scale, and $\langle \phi \rangle$ is not necessarily a large energy scale and could be constrained only by the phenomenological allowed value for the $Z'$ mass. More details of the phenomenology of this model at LHC and ILC energies and its comparison with other models with a $Z'$ will be given elsewhere [12].

One condition for having $B - L$ as a local anomaly free symmetry is that considered above. The number of right-handed neutrinos is $N_R = 3$, one per generation, and all of them carry $Y'(n_{\alpha R}) = -(B - L)(n_{\alpha R}) = -1, \forall \alpha$. However, it is possible to consider these quantum numbers as free parameters. In this case, in order to generate Dirac mass for neutrinos, it is necessary to introduce scalar doublets that carry also $Y'$ and $B - L$ charges. The quantum numbers of the new fields are shown in Table 2. Since the number of right-handed neutrinos and their $B - L$ assignment are free parameters, the only constraint is that they have to cancel the cubic and linear anomalies of the three active left-handed neutrinos altogether (not generation by generation) by having the appropriate $B - L$ attribution which is not necessarily an integer number.

The right-handed neutrinos contribute to the following anomalies:

$$\text{Tr} [U(1)_{B-L}]^2 U(1)_{Y'}$$, $\text{Tr} [U(1)_{Y'}]^2 U(1)_{B-L}$, $\text{Tr}[U(1)_{Y'}^3$, $\text{Tr}[U(1)_{B-L}]^3$; (3)

that imply the following equations:

$$\sum_{\alpha=1}^{N_R} Y'(n_{\alpha R})(B - L)^2(n_{\alpha R}) = 3, \quad \sum_{\alpha=1}^{N_R} Y'^2(n_{\alpha R})(B - L)(n_{\alpha R}) = -3,$$

$$\sum_{\alpha=1}^{N_R} Y'^3(n_{\alpha R}) = 3, \quad \sum_{\alpha=1}^{N_R} (B - L)^3(n_{\alpha R}) = -3,$$  \hspace{1cm} (4)

besides the two conditions for cancelling the gauge–gravitational anomaly:

$$\sum_{\alpha=1}^{N_R} Y'(n_{\alpha R}) = 3, \quad \sum_{\alpha=1}^{N_R} (B - L)(n_{\alpha R}) = -3.$$

However, the condition $[Y' + (B - L)](n_{\alpha R}) = 0$, for $\alpha$ fixed, has to be imposed
Table 2
Quantum number assignment in the model with three non-identical right-handed neutrinos. The number of doublet and singlet scalars depend on the values for $Y'_{1,2,3}$. The other fields have the quantum number given in Table 1.

In order to have right-handed neutrinos that are sterile with respect to the standard model interactions, so that the anomaly cancellation conditions in Eqs. (4) and (5) are reduced to the following equations:

$$
\sum_{n=1}^{N_{R}} Y'^{3}(n_{\alpha R}) = 3, \quad \sum_{n=1}^{N_{R}} Y''(n_{\alpha R}) = 3. \quad (6)
$$

In solving Eqs. (6), we will also assume that there is no vectorial neutral leptons, i.e., $Y'(N_{1L}) = Y'(N_{1R})$, and also that no neutral mirror leptons, i.e., $Y'(N_{1R}) = -Y'(N_{2R})$, are added. For Majorana fermions both cases are equivalent since $N_{1L}$ is related by CP to its right-handed conjugate. It means that having found a solution for the Eqs. (6), no extra terms vanishing among themselves are introduced: these sort of leptons would only cancel out their own anomalies, not the anomalies induced by the active left-handed neutrinos. They just add “0” to the left side of Eqs. (6) and, hence, are meaningless to our strategy.

Solving the constraint equations in Eq. (6), we have found that when $N_{R} = 1$ they have no solutions; when $N_{R} = 2$, there are only complex solutions. In the case of $N_{R} = 3$, we can only find two $Y'$ in terms of the third one, say, $Y'(n_{1R}) \equiv Y'_1$ and $Y'(n_{2R}) \equiv Y'_2$ in terms of $Y'(n_{3R}) \equiv Y'_3$, and the solutions are:

$$
2Y'_1 = 3 - Y'_3 \pm \frac{1 - Y'_3}{Y'_3 - 3} R(Y'_3), \quad 2Y'_2 = 3 - Y'_3 \mp \frac{1 - Y'_3}{Y'_3 - 3} R(Y'_3), \quad (7)
$$

where, $R(x) = [(x - 3)(x + 5)]^{1/2}$. 

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| $n_{1R}$ | $n_{2R}$ | $n_{3R}$ | $\varphi^0_i$ | $\varphi^-_i$ | $\phi_s$ |
|---------|---------|---------|--------------|--------------|---------|
| 0       | 0       | 0       | $Y'_1$       | $-Y'_1$      | 0       |
| 0       | 0       | 0       | $Y'_2$       | $-Y'_2$      | 0       |
| 0       | 0       | 0       | $Y'_3$       | $-Y'_3$      | 0       |
| 1/2 1/2 | 1 1     | 1       | $Y'_1 - Y'_1 - 1 - 1$ | $Y'_s - Y'_s - Y'_s - Y'_s$ | 0       |
| -1/2 1/2| 0 0     | 0       | $Y'_1 - Y'_1 - 1 - 1$ | $Y'_s - Y'_s - Y'_s - Y'_s$ | 0       |
From the last equations we obtain again the solution with identical right-handed neutrinos, i.e., all of them carrying \( Y' = 1 \) and \((B-L)_1 = (B-L)_2 = (B-L)_3 \equiv B-L = -1\), we have already studied above. However, there is also other solution concerning only integer values of \( Y' \) and \( B-L \) (we recall that these numbers are integer for charged leptons and active neutrinos): two right-handed neutrinos with, say, \( Y'_1 = Y'_2 = -(B-L)_1 = -(B-L)_2 = 4 \) and the third one with \( Y'_3 = -(B-L)_3 = -5 \). There are also real non-integer solutions but we will not consider them here. For \( N_R = 4 \) we have also found an infinite number of real (non-integer) solutions for the assignment of \( Y' = -(B-L) \) for the right-handed neutrinos, that we are not showing explicitly. The only integer solutions are those of the \( N_R = 3 \) but with the fourth neutrino carrying \( Y' = 0 \). However we are not considering right-handed neutrinos which are singlets of the new interactions. We have also worked the cases for \( N_R = 5,6 \) and found out that there are several solutions with \( Y' \) integer. For instance, \( Y'_i = (-11, -2, -1, 7, 10) \) for \( N_R = 5 \); and \( Y' = (-6, -6, 1, 3, 4, 7) \) for \( N_R = 6 \). In general for \( N_R \geq 5 \) it is possible that there exist an infinite set of solutions. Hence, only the case \( N_R = 3 \) has just two solutions of this sort: \( Y' = (1, 1, 1) \), which is the usual one, and the exotic \((-5, 4, 4)\) one. We will consider below a model based on the exotic solution for the case of three right-handed neutrinos.

In this model the analysis of the \( T \) parameter is more complicated than in the first model because, besides the Majorana neutrinos, there are additional Higgs doublets which, unlike the Dirac fermion case which are always positive, give contributions to the \( T \)-parameter with either sign [13,14]. We will shown these explicitly elsewhere. Here, we will give details only of the scalar and the Yukawa sectors.

The scalar sector of the theory is constituted by several doublets and singlets. For instance, the scalar sector which interacts in the lepton sector could be: the usual doublet with \( Y = +1 \), here denoted by \( \Phi_{SM} \), two doublets with \( Y = -1 \): one, denoted by \( \Phi_1 \), with \( Y' = -4 \), and \((B-L) = +3\), and the other, \( \Phi_2 \), with \( Y' = 5 \), and \((B-L) = -6\); and three complex scalar singlets \((Y = 0)\): \( \phi_1 \) with \( Y' = -(B-L) = -8 \), \( \phi_2 \) with \( Y' = -(B-L) = 10 \), and \( \phi_3 \) with \( Y' = -(B-L) = 1 \). Notice that whenever the scalar doublets carry a non-zero \( B-L \), it means that these doublets contribute to the spontaneous violation of this number, which is also induced by the complex scalar singlets.

This model is interesting for introducing three scales for the Majorana masses of the right-handed neutrinos. With these fields and the leptons we have the Yukawa interactions (omitting summation symbols)

\[
-L_{\text{Yukawa}}^c = \overline{\psi}_L G_{am}^D \phi_1 n_{mR} + \overline{\psi}_L G_{ad}^D \phi_2 n_{3R} + \phi_1 (\bar{n}_{mR})^c G_{mn}^M n_{nR} + \phi_2 (\bar{n}_{3R})^c G_{3N}^M n_{3R} + \phi_3 (\bar{n}_{mR})^c G_{m3}^M n_{3R} + H.c.,
\]

(8)
where $m, n = 1, 2$.

Not all of the Majorana mass terms, for the right-handed neutrinos, are necessarily too large since only one of the singlets has to have a large VEV so that the breaking of the $B − L$ symmetry occurs at a high energy scale. In fact, two of them can be light enough to implement the $3 + 2$ neutrino scheme, with $CP$ violation, as in Ref. [15]. If some singlet neutrinos are heavy but not too much, effects of them could be detectable at the LHC [16], linear [17] or $e−\gamma$ [18] colliders, or in low energy processes [19]. In particular lepton colliders would be appropriate for discovering these sort of neutrinos [20]. If $n_{\alpha R}$ are heavier than all the physical scalar fields which are almost doublets, the decays $n_{\alpha R} \to l^\pm h^\mp$ are kinematically allowed, and hence $h^\pm \to h^0 + W^{\pm*}$ or $h^\pm \to \bar{q} q'$, where $h^+(h^0)$ denotes any charged (neutral) physical scalar, $q, q'$ are quarks with different electric charge, and $W^{\pm*}$ is a virtual vector boson. Hence, in this model, only the lightest of the neutral almost scalar singlets would be a candidate for dark matter [21].

In the model with quantum number given in Table 1, the more general $SU(2)_L \otimes U(1)_{Y'} \otimes U(1)_{B−L}$ invariant scalar potential for the doublet $\Phi$ and the singlet $\phi$, is given by

$$V(\Phi, \phi) = \mu^2|\Phi|^2 + \mu^2|\phi|^2 + \lambda_1|\Phi^\dagger \Phi|^2 + \lambda_2|\phi^\dagger \phi|^2 + \lambda_3|\Phi|^2|\phi|^2. \quad (9)$$

Doing as usual the shifted as $\varphi^0 = \frac{1}{\sqrt{2}}(v + H + iF)$ and $\phi = \frac{1}{\sqrt{2}}(u + S + iG)$, so that the constraint equations are given by:

$$v \left( \mu_1^2 + \lambda_1 v^2 + \frac{\lambda_3}{2} u^2 \right) = 0, \quad u \left( \mu_2^2 + \lambda_2 u^2 + \frac{\lambda_3}{2} v^2 \right) = 0. \quad (10)$$

We will choose real $v, u \neq 0$ solutions for simplicity. We also must have $\lambda_1, \lambda_2 > 0$, in order to the scalar potential be bounded from below, and $\lambda_3^2 < 4\lambda_1\lambda_2$, to assure we have a minimum. The mass square matrix in the basis $(H, S)$, after the use of Eq. (10), is given by

$$M^2_S = \begin{pmatrix} 2\lambda_1 v^2 & \lambda_3 uv \\ \lambda_3 uv & 2\lambda_2 u^2 \end{pmatrix}, \quad (11)$$

with $\text{Det} M^2_S \neq 0$ by the above conditions. The exact eigenvalues for the mass square matrix are:

$$m_{1,2}^2 = \lambda_1 v^2 + \lambda_2 u^2 \pm \left[(\lambda_1 v^2 + \lambda_2 u^2)^2 - (4\lambda_1\lambda_2 - \lambda_3^2) u^2 v^2 \right]^\frac{1}{2}, \quad (12)$$
which can be approximate by considering $u \gg v$ (but still arbitrary),

$$m_1^2 \approx 2\lambda_1 \left(1 - \frac{\lambda_3^2}{4\lambda_2\lambda_1}\right)v^2, \quad m_2^2 \approx 2\lambda_2u^2 + \frac{\lambda_3^2}{2\lambda_2}v^2. \quad (13)$$

Notice that the heavier neutral boson has a mass square proportional to $u^2$, $m_2 > m_1$. The exact eigenvectors are give by

$$H_1 = -\frac{1}{\sqrt{N_1}} \left(\frac{a - \sqrt{a^2 + b^2}}{b} H + S\right), \quad H_2 = \frac{1}{\sqrt{N_2}} \left(\frac{a + \sqrt{a^2 + b^2}}{b} H + S\right), \quad (14)$$

where $a = \lambda_1v^2 - \lambda_2v^2$, $b = \lambda_3uv$, and $N_{1,2} = 1 + (\sqrt{a^2 + b^2} \mp a)/b^2$. We have maximal mixing when $\lambda_1/\lambda_2 = u^2/v^2$. The eigenvectors in Eq. (14) can be written as follows

$$\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} H \\ S \end{pmatrix}. \quad (15)$$

This implies a reduction on the value of the couplings of the Higgs to standard model particles, $h_1 = h \cos \theta$, and $h_2 = h \sin \theta$, where $h$ denotes any of the SM coupling constants for the Higgs scalar. Depending on the value of the angle $\theta$ we can suppress the Higgs decays making the SM Higgs invisible even at the LHC. This effect has been considered in literature when the added scalar singlet is real $[22]$. The would be Goldstone boson, $F$ and $G$ in the unitary gauge, are absorbed by the longitudinal components of $Z$ and $Z'$ respectively.

On the other hand, for the second model the most general $SU(2)_L \otimes U(1)_{Y'} \otimes U(1)_B$ invariant potential may be written as

$$V_{B-L} = V_{SM}(\Phi_{SM}) + \mu_1^2 |\Phi_1|^2 + \mu_2^2 |\Phi_2|^2 + \lambda_1 |\Phi_1|^4 + \lambda_2 |\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \lambda_4 |\Phi_1|^2 |\Phi_2|^2 + \lambda_{SM}\alpha_\alpha |\Phi_{SM\alpha}|^2 + \lambda_{SM\alpha} |\Phi_{SM\alpha}|^2 |\phi_\alpha|^2 + \lambda_{SM\alpha} |\phi_\alpha|^4 + \lambda_{\alpha\beta} |\phi_\alpha|^2 |\phi_\beta|^2$$

$$+ \mu_\alpha^2 |\phi_\alpha|^2 + \lambda_\alpha |\phi_\alpha|^4 + \lambda_{\alpha\beta} |\phi_\alpha|^2 |\phi_\beta|^2 + \lambda_{\alpha\beta} |\phi_\alpha|^2 |\phi_\beta|^2 + \lambda'_{SM\alpha} |\Phi_{SM\alpha}|^2 + \lambda'_{SM\alpha} |\Phi_{SM\alpha}|^2 |\phi_\alpha|^2 + \lambda''_{SM\alpha} |\phi_\alpha|^4$$

$$+ \lambda_{\alpha\beta} |\phi_\alpha|^2 |\phi_\beta|^2, \quad (16)$$

where $i, j = 1, 2$ and $\alpha = 1, 2, 3$ (we have omitted summation symbols), in the last term $\alpha < \beta$; and since $\Phi_{SM}$ is the usual Higgs doublet of the SM, $V_{SM}(\Phi_{SM})$ denotes the respective potential.

The constraint equations coming from the linear terms of the scalar potential in Eqs. (16) are:
with this potential if \( \lambda', \kappa, \kappa' \neq 0 \) all VEVs have to be different from zero and it is possible to give to all fermions masses with the correct values. This model has extra global \( U(1) \) symmetries as can be verified by the number of neutral Goldstone bosons: there are four of them. Notice that only the fields carrying exotic values of \( Y' \) and \( B - L \) can carry the charge of the extra global symmetries. Hence, these extra symmetries are restricted to the exotic scalars and neutrino singlets, and from Eqs. (8), we have the following equations:

\[
\begin{align*}
v_1[2\mu_{11}^2 + 2\lambda_1 v_1^2 + (\lambda_3 + \lambda_4) v_2^2 + \lambda_{SM1} v_{SM1}^2 + \lambda_{111} v_{s1}^2 + \lambda_{12} v_{s2}^2 + \lambda_{13} v_{s3}^2] & + v_2(\kappa v_{s1} v_{s2} \kappa' v_{s3}) = 0, \\
v_2[2\mu_{22}^2 + 2\lambda_2 v_2^2 + (\lambda_3 + \lambda_4) v_1^2 + \lambda_{SM2} v_{SM2}^2 + \lambda_{211} v_{s1}^2 + \lambda_{22} v_{s2}^2 + \lambda_{23} v_{s3}^2] & + v_1(\kappa v_{s1} v_{s3} \kappa' v_{s2}) = 0, \\
v_{SM}[2\mu_{SM}^2 + \lambda_{SM1} v_1^2 + \lambda_{SM2} v_2^2 + \lambda_{SM1} v_{SM1}^2 + \lambda_{SM2} v_{SM2}^2 + \lambda_{SM3} v_{SM3}^2] & = 0, \\
v_{s1}[2\mu_{s1}^2 + 2\lambda_{s1} v_{s1}^2 + \lambda_{s1} v_{s1}^2 + \lambda_{s2} v_{s2}^2 + \lambda_{s3} v_{s3}^2 + \lambda_{s11} v_{s1}^2 + \lambda_{s21} v_{s2}^2 + \lambda_{s32} v_{s3}^2] & + \lambda' v_{s2} v_{s3} + \kappa v_{s1} v_{s2} = 0, \\
v_{s2}[2\mu_{s2}^2 + 2\lambda_{s2} v_{s2}^2 + \lambda_{s1} v_{s1}^2 + \lambda_{s2} v_{s2}^2 + \lambda_{s3} v_{s3}^2 + \lambda_{s11} v_{s1}^2 + \lambda_{s21} v_{s2}^2 + \lambda_{s32} v_{s3}^2] & + \lambda' v_{s1} v_{s3} + \kappa v_{s1} v_{s2} = 0, \\
v_{s3}[2\mu_{s3}^2 + 2\lambda_{s3} v_{s3}^2 + \lambda_{s1} v_{s1}^2 + \lambda_{s2} v_{s2}^2 + \lambda_{s3} v_{s3}^2 + \lambda_{s11} v_{s1}^2 + \lambda_{s21} v_{s2}^2 + \lambda_{s32} v_{s3}^2] & + 2\lambda' v_{s1} v_{s2} + \kappa v_{s1} v_{s1} + \kappa' v_{s2} v_{s2} = 0,
\end{align*}
\]

and we have also used the VEVs as being real for the sake of simplicity.

From Eqs. (8), we have the following equations:

\[
\begin{align*}
\zeta(\Phi_1) + \zeta(n_{MR}) = 0, & \quad \zeta(\Phi_2) + \zeta(n_{MR}) = 0, \quad \zeta(\phi_1) + 2\zeta(n_{MR}) = 0, \\
\zeta(\phi_2) + 2\zeta(n_{MR}) = 0, & \quad \zeta(\phi_3) + \zeta(n_{MR}) = 0.
\end{align*}
\]

where \( \zeta(f) \) denotes the \( U(1)_\zeta \) charge of the field \( f \). Fermionic left-handed doublets, electrically charged right-handed singlets and the scalar doublet \( \Phi_{SM} \) do not carry this sort of new charges. There are two solutions for the equations above that we will denote \( \zeta = X, X' \): i) \( X(\Phi_1) = -X(n_{MR}) = 1, X(\Phi_2) = -X(n_{3R}) = 1, X(\phi_1) = X(\phi_2) = X(\phi_3) = 2; \) and ii) \( 2X'(\Phi_2) = X'(\phi_2) = 2X'(\phi_3) = -2X'(n_{3R}) = -2 \) and the other fields no carrying this charge.

It worth noting that extra Goldstone bosons arise in supersymmetric models with extra \( U(1) \) factors and several scalar singlets under the SM gauge symmetries \[23\]. However, in the present model, this is not a flaw because the extra Goldstone bosons, denoted by \( G_X \) and \( G_{X'} \), can be almost singlets: \( G_X \) can always be made almost singlet, \( G_X \sim \phi_1; G_{X'} \) may have its main projection on \( \phi_2 \) or \( \phi_3 \). Anyway, the extra Goldstone bosons are not a problem in this model also because they couple mainly to active and sterile neutrinos, hence its consequences may be important only on cosmological scales. In the scalar (CP even) sector all fields are massive.

Another possibility is to avoid the appearance of \( G_X \) and \( G_{X'} \). First, note that interactions that can break those symmetries are forbidden by the \( U(1)_Y \),
and $U(1)_{B-L}$ symmetries that in the present model are local symmetries. Hence, it is not allowed to break directly and softly the global $U(1)_\zeta$ symmetries. One way to solve this issue is to add non-renormalizable operators that are invariant under the gauge symmetry of the model. For instance $h(\phi_1^*\phi_1)(\phi_2^*\phi_2)(\phi_3^*\phi_3)/\Lambda^2$, where $\Lambda$ is an energy scale higher than the electroweak scale, and $h$ is a dimensionless constant. When the singlets get the VEVs they induce terms like $\mu_{123}\phi_1\phi_2\phi_3$, where $\mu_{123} = hv_1^*v_2^*v_3^*/\Lambda^2$. When terms like that are introduced they modified the last three constraint equations in (17) and the Goldstone bosons are reduced to just two: $G_X$ and $G_X'$. Notice that $Y'$ and $B-L$ are only hidden because the original dimension six operators are invariant under these symmetries.

It is interesting to note that the SM is anomalous with respect to the mixed global $(B-L)$-gravitational anomaly. It is canceled out if right-handed neutrinos are introduced. In this case the condition for cancelling that anomaly, for the three generation case, is $\sum_{a=1}^{N_R} (B-L)(n_{aR}) = -3$. For instance, if $N_R = 1$ the unique right-handed neutrino must carry $L = 3$; if $N_R = 2$ one of them can have $L = 4$ and the other $L = -1$, and so on. In particular $N_R = 3$, is the unique case that contains the usual solution with the three neutrinos having the same lepton number which is identical to the generation-by-generation case. However, there are infinite exotic solutions, say $L = (L_1, L_2, -L_1 - L_2 + 3)$. It means that even in the context of the model with the gauge symmetries of the SM, the addition of that sort of neutrinos is mandatory but their number remains arbitrary, i.e., $N_R = 1, 2, 3, \cdots$, since the constraint equation above has always solution in the global $(B-L)$ case for any $N_R$. We have extended this scenario when $B-L$ is gauged and contributes to the electric charge.

We have in these models that $\Delta(B-L) \equiv -\Delta L$ and the $(\beta\beta)_0$ occurs through the usual mechanism with massive neutrinos. On the other hand, the proton is appropriately stabilized because there is no dimension five operator $Q^cQ\overline{Q}^cL$ at the tree level. The lowest dimension effective operators, $B-L$ conserving, that contribute to its decay are dimension eight, for instance $\Lambda^4 Q^c\overline{Q}^cL|\phi|^2$ which induces, after the spontaneous symmetry breaking, interactions like $Q^c\overline{Q}^cL\mu^2_{\overline{u}}$ that are enough suppressed whenever $u \ll \Lambda$. A similar analysis can be made with other effective operators [24] including those that involve right-handed sterile neutrinos [25].

We have considered here the case of a local $U(1)_{B-L}$ symmetry. In the same way, it is also possible to build models with $U(1)_X$, where $X$ denotes any of the combinations $L_a - L_b, 2L_a - L_b - L_c$, with $a \neq b \neq c$, for $a, b, c = e, \mu, \tau$. In these cases right-handed neutrinos may carry non-standard values of $X$. 

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