Mathematical model on MHD oscillatory flow through a porous medium under the influence of heat and mass transfer

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Abstract
In this paper, the unsteady MHD Oscillatory viscous incompressible fluid flow on radiative heat and mass transfer under the influence of homogeneous magnetic field, through a porous medium in a circular pipe with time varying pressure gradient are investigated. The fundamental idea of our work is to study this aspect mathematically and the equation of momentum, energy and mass are solved analytically for the velocity, temperature, concentration, flow rate and shear stress. The effects of magnetic parameter, porosity parameter, Peclet number, Grashof number, modified Grashof number, radiation, chemical reaction, and Schmidt number on temperature, concentration, fluid velocity, shear stress and flow rate profiles are discussed and results are shown graphically.

Keywords
Heat and mass transfer, MHD, Oscillatory flow, porous medium, Time varying pressure gradient.

AMS Subject Classification
76S05.

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1. Introduction
The flow of conducting fluid, heat and mass transfer has important applications in many branches of engineering science such as petroleum transport, waste water treatment, oil recovery technique, power plant piping, engineering and heat storage, etc. Specially, the flow of heat and mass transfer in a circular pipe with magnetic field arises in MHD generators, pumps, nuclear reactors, accelerators, geothermal systems and others. El-Hakiem (2000) analyzed thermal radiation effects on transient, two-dimensional hydromagnetic free convection along a vertical surface in a highly porous medium using the Roseland diffusion approximation for the radiative heat flux in the energy equation, for the case where free-stream velocity of the fluid vibrates about mean constant value and the surface absorbs the fluid with constant velocity. Chmkha.J. (2000) has been investigated the unsteady laminar hydromagnetic fluid – particle flow and heat transfer in channels and circular pipes. Makinde and Mhone (2005) have considered heat transfer to MHD oscillatory flow in a channel filled with porous medium. Makinde and Osalusi (2006) have been discussed the effect of slip condition on MHD steady flow in a channel with permeable boundaries.

Cheng (2006) presented the free convection heat and mass transfer from a horizontal cylinder of elliptic cross section in a micro polar fluid. Thereafter, Panda et al., (2006) analyzed the free convection of conducting viscous fluid between two vertical walls filled with porous material. Mehmood and Ali (2007) discussed the effect of slip condition on unsteady MHD oscillatory flow of a viscous fluid in a planer channel. Mustafa et al., (2008) investigated unsteady MHD memory flow with oscillatory suction, variable free stream and heat...
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Archana Dixit and Sanjeev Kumar (2014) studied an analysis as under these assumptions, the momentum equation are given as:

\[ \frac{\partial \mathbf{C}'}{\partial t'} = D' \left( \frac{\partial^2 \mathbf{C}'}{\partial r'^2} + \frac{1}{r} \frac{\partial \mathbf{C}'}{\partial r'} \right) - \mathbf{E}' (\mathbf{C}' - \mathbf{C}_0') \quad (2.3) \]

Mass transfer equation are given as:

\[ \frac{\partial \mathbf{C}'}{\partial t'} = \frac{\partial^2 \mathbf{C}'}{\partial r'^2} + \frac{1}{r} \frac{\partial \mathbf{C}'}{\partial r'} \quad (2.1) \]

Energy equation are given as

\[ \rho' C_p \mathbf{v} \frac{\partial \mathbf{T}'}{\partial t'} = K \left( \frac{\partial^2 \mathbf{T}'}{\partial r'^2} + \frac{1}{r} \frac{\partial \mathbf{T}'}{\partial r'} \right) - \mathbf{q}' \quad (2.2) \]

Noreen and Nadeem et al., (2012) discussed the influence of heat transfer and chemical reactions on Williamson fluid model for blood flow through a tapered artery with a stenosis. Archana Dixit and Sanjeev Kumar (2014) studied an analysis to observe the effects of heat transfer on oscillatory blood flow in a constricted tube. Vasudev et al., (2011) analysed peristaltic flow of a Newtonian fluid through a porous medium in a vertical tube under the effect of magnetic field. Mallikarjuna et al., (2017) discussed the radiation effects on an oscillatory flow of a viscous fluid in a circular tube. The purpose of this paper is to obtain the influence of magnetic field, radiative heat and mass transfer on unsteady viscous incompressible fluid flow in a circular pipe through a porous medium. The impact of various pertinent parameters on the velocity, concentration and temperature, shear stress and flow rate profiles are discussed through graphically.

## 2. Mathematical Formulation

Consider the unsteady, laminar, hydromagnetic oscillatory fluid flow in a circular pipe through a porous medium under the influence of homogeneous magnetic field and radiative heat transfer and concentration. The fluid is assumed to have a low electrical conductivity, and the electromagnetic force generated is very small. Let \( \mathbf{x} \)-axis be the axis along the axis of the pipe and \( r \) be the radial distance from the centre. The only none zero component of velocity is \( w \) which will be a function of \( r \) and \( t \).

Under these assumptions, the momentum equation are given as:

\[ \frac{\partial \mathbf{w}'}{\partial t'} = -\frac{1}{\rho} \frac{\partial p'}{\partial z'} + \mu \left( \frac{\partial^2 \mathbf{w}'}{\partial r'^2} + \frac{1}{r} \frac{\partial \mathbf{w}'}{\partial r'} \right) - \frac{\sigma \mathbf{B}_0' \mathbf{w}'}{\rho} - \left( \frac{\mu}{k} \right) \mathbf{w}' + \left( \frac{\rho'}{\rho} \right) \mathbf{v}' (T' - T_0') + \left( \frac{\rho'}{\rho} \right) \mathbf{B}' (\mathbf{C}' - \mathbf{C}_0') \quad (2.1) \]

Energy equation are given as:

\[ \rho' C_p \mathbf{v} \frac{\partial \mathbf{T}'}{\partial t'} = K \left( \frac{\partial^2 \mathbf{T}'}{\partial r'^2} + \frac{1}{r} \frac{\partial \mathbf{T}'}{\partial r'} \right) - \mathbf{q}' \quad (2.2) \]

Our assumption that the radiative heat flux and varying pressure gradient are given below:

\[ \frac{\partial \mathbf{q}'}{\partial \mathbf{r}'} = 4\gamma' (T_0' - T') \quad (2.4) \]

\[ \frac{\partial \mathbf{p}'}{\partial \mathbf{z}'} = \lambda \mathbf{c}' \quad (2.5) \]

Where \( \gamma' \) - mean radiation absorption coefficient, \( n' \)-frequency of the oscillation.

And the boundary conditions are:

\[ w' - s' \frac{\partial w'}{\partial r'} = 0 \quad \text{at} \quad r' = a \quad (2.6) \]

\[ T' = T_0' + (T_n' - T_0') \cos n' t' \quad \text{at} \quad r' = a \quad (2.7) \]

\[ \mathbf{C}' = \mathbf{C}_0' + (\mathbf{C}_n' - \mathbf{C}_0') \cos n' t' \quad \text{at} \quad r' = a \quad (2.8) \]

\[ \frac{\partial \mathbf{w}'}{\partial r'} = \frac{\partial \mathbf{T}'}{\partial r'} = \frac{\partial \mathbf{C}'}{\partial r'} \quad \text{at} \quad r' = 0 \quad (2.9) \]

Where \( s' \) is the slip parameter.
We get the differential form as

\[ M^2 = \frac{a^2 \sigma B_0^2}{\rho \mu}, \quad \theta = \frac{T' - T_0'}{T_n' - T_0'}, \quad \phi = \frac{C' - C_0'}{C_n' - C_0'} \]

Where \( W \)- flow mean velocity, \( G_r \)- Grashof number, \( G_m \)- modified Grashof number, \( M \)- Magnetic parameter, \( N \)- radiation parameter, \( R_e \)- Reynolds number, \( P_e \)- Peclet number, \( D_r \)- Darcy parameter.

In terms of these variables, equations (2.1), (2.2) and (2.3) [non dimensional] becomes

\[ R_e \frac{\partial w}{\partial t} = \lambda e^{int} + \left( \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right) - \frac{1}{k} w - M^2 w + G_r \theta + G_m \phi \]  

(2.11)

\[ P_e \frac{\partial \theta}{\partial t} = \left( \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} \right) - N^2 \theta \]  

(2.12)

\[ R_e \frac{\partial \phi}{\partial t} = \left( \frac{1}{S_c} \left( \frac{C^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} \right) - E \phi \right. \]  

(2.13)

With boundary conditions are:

\[ w - s \frac{\partial w}{\partial r} = 0, \quad \theta = \text{cosnt}, \quad \phi = \text{cosnt} \quad \text{at} \quad r = 1 \]  

(2.14)

\[ \frac{\partial w}{\partial r} = 0, \quad \frac{\partial \theta}{\partial r} = 0, \quad \frac{\partial \phi}{\partial r} = 0, \quad \text{at} \quad r = 0 \]  

(2.15)

### 3. Method of Solution

To solve the temperature equation (2.12) for purely oscillatory flow, we assume that the solution of form

\[ \theta(r,t) = \theta_0(r)e^{int} \]  

(3.1)

We get the differential form as

\[ \frac{\partial^2 \theta_0}{\partial r^2} + \frac{1}{r} \frac{\partial \theta_0}{\partial r} + \beta_1 \theta_0 = 0 \]  

(3.2)

Where \( \beta_1 = -(N^2 - P_e \text{in}) \)

Which is a Bessel equation of zero order. We have the solution of above equation is

\[ \theta(r,t) = F_1 J_0(\sqrt{\beta_1} r)e^{int} \]  

(3.3)

After substituting boundary conditions, the expression for the temperature profile is

\[ \theta(r,t) = \frac{J_0(\sqrt{\beta_1} r)}{J_0(\sqrt{\beta_1})} e^{int} \]  

(3.4)

Let us assume that the solution of the concentration equation (2.13) is of the form

\[ \phi(r,t) = \phi_0(r)e^{int} \]  

(3.5)

The differential form as

\[ \frac{\partial^2 \phi_0}{\partial r^2} + \frac{1}{r} \frac{\partial \phi_0}{\partial r} + \beta_2 \phi_0 = 0 \]  

(3.6)

Where \( \beta_2 = -(E S_c + R_e \text{in} S_c) \)

Which is a Bessel equation of zero order. The solution of the above equation is

\[ \phi(r,t) = F_2 J_0(\sqrt{\beta_2} r)e^{int} \]  

(3.7)

After substituting boundary conditions, the expression for concentration profile is

\[ \phi(r,t) = \frac{J_0(\sqrt{\beta_2} r)}{J_0(\sqrt{\beta_2})} e^{int} \]  

(3.8)

We assume that the solution of velocity distribution equation (2.11) is of the form

\[ w(r,t) = w_0(r)e^{int} \]  

(3.9)

The differential form as

\[ \frac{\partial^2 w_0}{\partial r^2} + \frac{1}{r} \frac{\partial w_0}{\partial r} + \beta_3 w_0 = -(P + G_r \theta_0 + G_m \phi_0) \]  

(3.10)

Where \( \beta_3 = -(M^2 + inR_e + 1/k) \)

Converting the governing equation to the above equation form. Which is the ordinary differential equation. First we use the variation parameter method to calculate the solution of homogeneous differential equation. Finally, we use the definition of Bessel differential equation, we get

\[ w_0 = C_1 J_0(\sqrt{\beta_3} r) + C_2 Y_0(\sqrt{\beta_3} r) \]  

(3.11)

The Wronskian is

\[ W_1 = \left| \begin{array}{cc} J_0(\sqrt{\beta_3} r) & Y_0(\sqrt{\beta_3} r) \\ \sqrt{\beta_3} J_1(\sqrt{\beta_3} r) & -\sqrt{\beta_3} Y_1(\sqrt{\beta_3} r) \end{array} \right| = \frac{2}{\beta_3} \]  

(3.12)

The complete solution of the non-homogeneous equation is

\[ A = -\int \frac{Y_0(\sqrt{\beta_3} r) \left( P + G_r \theta_0 + G_m \phi_0 \right)}{W_1} \, dr \]  

(3.13)
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where

\[ B = \int \frac{J_0(\sqrt{B_3} r)}{W_1} dr \]  

(3.14)

Hence, the complete solution is

\[ w_0 = C_1 J_0(\sqrt{B_3} r) + C_2 Y_0(\sqrt{B_3} r) + AJ_0(\sqrt{B_3} r) + BY_0(\sqrt{B_3} r) \]  

(3.15)

Substituting equation (3.15) in (3.9), we get

\[ w(r,t) = [C_1 J_0(\sqrt{B_3} r) + C_2 Y_0(\sqrt{B_3} r) + AJ_0(\sqrt{B_3} r) + BY_0(\sqrt{B_3} r)]e^{int} \]  

(3.16)

Using boundary conditions, we get \( C_2 = 0 \), the expression of velocity profile is given by

\[ w(r,t) = [C_1 J_0(\sqrt{B_3} r) + AJ_0(\sqrt{B_3} r) + BY_0(\sqrt{B_3} r)]e^{int} \]  

(3.17)

Where

\[ C_1 = \frac{AJ_0(\sqrt{B_3} r) J_0(\sqrt{B_3} r) - J_1(\sqrt{B_3} r) J_0(\sqrt{B_3} r)}{J_0(\sqrt{B_3} r) - J_1(\sqrt{B_3} r)} \]  

(3.18)

**Shear stress:**

The expression for shear stress is given as

\[ \tau = -\mu \left( \frac{\partial w}{\partial r} \right)_{r=1} \]  

(3.19)

Hence, the required solution of shear stress is

\[ \tau = -[C_1 J_1(\sqrt{B_3} r) + AJ_1(\sqrt{B_3} r) + BY_1(\sqrt{B_3} r)]e^{int} \]  

(3.20)

**Flow rate:**

The expression for flow rate is given as

\[ Q = 2\pi \int_0^R wrdr \]  

(3.21)

Hence, the required solution of flow rate is

\[ Q = 2\pi \left[ C_1 \frac{J_1(\sqrt{B_3} r)}{\sqrt{B_3}} + AJ_1(\sqrt{B_3} r) + BY_1(\sqrt{B_3} r) \right]e^{int} \]  

(3.22)

### 4. Results and Discussion

The contemporary mathematical analysis gives some light on investigating the unsteady MHD oscillatory flow in a circular pipe with heat and mass transfer under the influence of magnetic field through porous medium. The impact of magnetic parameter, porosity, Peclet number, Grashof number, modified Grashof number, radiation, chemical reaction, Schmidt number on velocity, temperature, concentration profiles, shear stress and flow rate have been computed in following graphs.

We made use of the other values indicated \( \lambda = 1, t = 1, n = 1, N = 3, \rho = 0.87, E = 0.5, S_c = 0.5, R_c = 0.005, M = 0.5, k = 0.5, G_r = 2, G_m = 2. \)

![Figure 1. Profiles of fluid velocity (w) for different values of magnetic parameter (M).](image1)

![Figure 2. Profiles of fluid velocity (w) for different values of porosity parameter (k).](image2)

**Velocity Distribution:**

- Figure (1) and (2), the velocity profiles for different values of magnetic field parameter (M) and porosity parameter (k) are presented. It is observed that the increasing value of magnetic field parameter and porosity parameters, the fluid velocity decreases.
- Figure (3) and (4), the velocity profiles for different values of Grashof number (G_r) and modified Grashof number (G_m) are illustrated. It is shown that the Grashof number and...
modified Grashof number are increases the fluid velocity is also increases.

**Temperature:**
- Figure (5) and (6), the temperature profiles for different values of radiation parameter \((N)\) and Peclet number \((P_e)\) are indicated. It is seems that the ration parameter and Peclet number are increases the temperature profile is also increases.

**Concentration:**
- Figure (7) and (8), the concentration profiles for different values of chemical reaction parameter \((E)\) and Schmidt number \((S_c)\) are exhibited. It is examined that the parameter of chemical reaction and the number of Schmidt are increases the profile of concentration is also increases.

**Shear stress:**
- Figure (9) and (10), the shear stress profiles for different values of magnetic filed parameter \((M)\) and porosity parameter \((k)\) are described. It is clear that the magnetic field parameter, and porosity parameter are increases the shear stress profile is also increases.

**Flow rate:**
- Figure (11) and (12), the flow rate profiles for different values of chemical reaction parameter \((E)\).
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Figure 8. Profiles of concentration ($\phi$) for different values of Schmidt number ($S_c$).

Figure 9. Profiles of shear stress ($\tau$) for different values of magnetic parameter ($M$).

Figure 10. Profiles of shear stress ($\tau$) for different values of porosity parameter ($k$).

Figure 11. Profiles of flow rate ($Q$) for different values of magnetic parameter ($M$).

Figure 12. Profiles of flow rate ($Q$) for different values of porosity parameter ($k$).
values of magnetic filed parameter \((M)\) and porosity parameter \((k)\) are presented. It is point out the magnetic field and porosity parameter are increases the flow rate profile is also increases.

## 5. Conclusion

The unsteady viscous incompressible oscillatory fluid flow in a circular pipe through a porous medium under the impact of magnetic field, heat and mass transfer was investigated. We notice that the following disciplinary:

- The velocity profile decreases with increasing of magnetic field parameter and porosity parameter and the velocity profile increases with increasing of Grashof number and modified Grashof number.
- The temperature profile increases with increasing of radiation, Peclet number.
- The concentration profile increases with increasing of chemical reaction, Schmidt number.
- The shear stress profile increases with increasing of magnetic field and porosity parameter.
- Flow rate profile increases with increasing of magnetic field and porosity parameter.

The impact of magnetic field on a viscous incompressible conducting fluid is to suppress the velocity field which in turn causes the enhancement of the temperature and concentration field. In any process industry, heat and mass exchangers are one of the most useful and most common pieces of process equipment. I hope that our present investigation work to use a knowledge of how they operate and how to analyze their performance in an essential to your future success as a science and technical field.

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