The MuLan Experiment: A New Measurement of the Fermi Constant at the Level of 1 ppm

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Abstract. The Fermi constant, $G_F$, is best determined through measurements of the positive muon lifetime, $\tau_\mu$. Until about 10 years ago, incomplete theoretical calculations limited the precision with which $G_F$ could be extracted from experiment - the publication of those results paved the way for a series of new measurements of $\tau_\mu$. The MuLan collaboration published its first result in 2007 (11 ppm on $\tau_\mu$, 5.5 ppm on $G_F$) and will soon publish the final results from large production runs in 2006 and 2007. In my talk, I presented a preliminary result for $\tau_\mu$ on behalf of the collaboration. We expect that the overall error on $\tau_\mu$ will be about 1 ppm. I have also tried to explain how this very simple measurement is made and describe our strategies for controlling the systematic errors.

1. Introduction

Three of the most important input parameters of the symmetry-breaking section of the Standard Model (SM) are the fine structure constant $\alpha$, the mass of the $Z$ boson and Fermi constant, $G_F$. The fine structure constant was recently measured to a precision of 0.3 parts per billion (ppb) [1]. The mass of the $Z$ was determined to a precision of 22 parts per million (ppm) through studies of its production lineshape at LEP [2].

Figure 1. Effective four fermion interaction
The Fermi constant is best determined through measurements of muon decay, which is described in terms of a non-renormalizable four-fermion interaction, where V-A couplings are assumed. See Fig. 1. The decay rate is related to $G_F$ through the equation

$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5 (1 + \delta)}{192 \pi^3}$$

(1)

where $\delta$ encapsulates QED and hadronic radiative corrections. $G_F$, in turn, is related to the fundamental parameters of the standard model through the equation

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} (1 + \Delta r(m_t, m_H, ...)),$$

(2)

where $\Delta r$ contains all the weak interaction loop corrections. In the early 1990s, before the discovery of the top quark at FNAL, Eq. 2 was used in conjunction with precision electroweak measurements to constrain its mass. The most important constraint was provided by the diagram shown in Fig. 2, a loop correction which is quadratic in the (very large) top quark mass.

At the time of the top quark discovery, our knowledge of the $G_F$ was limited not by measurements of the muon lifetime but by uncertainties in the radiative correction term $\delta$, in Eq. 1 [3]. Then in 1999, van Ritbergen and Stuart calculated the missing two loop QED corrections [4], reducing the theoretical systematic error on the extraction of $G_F$ to 0.3 ppm. See also [5]. The dominant remaining error, about 18 ppm, was that of the muon lifetime itself [6].

2. Experimental Considerations

By the time the van Rithbergen and Stuart result was published, several major new measurements of the muon lifetime were underway: one at the Rutherford Appleton Laboratory and two others at the Paul Scherrer Institut, FAST [7] and MuLan [8]. In all three cases, the proposed sensitivity was on the order of a few parts per million. Since several trillion muon decays are required for a ppm level measurement, PSI was a natural site for such an experiment - high-flux low-energy pion and muon beams were readily available.

However, high flux beams are by themselves insufficient. If muon decays are measured one-at-a-time, as in many of the experiments from the 1980s, the maximum average data rate cannot exceed about 20 kHz, which makes the time required for a 1 ppm measurement prohibitively
long. The idea of the MuLan experiment is to create a pulsed surface muon beam, so that of order 20 muon decays can be measured per data-taking cycle of 30 microseconds.

The MuLan beamline, located in \( \pi E^3 \) at PSI, is shown in Fig. 3. The MuLan kicker [9] consists of a pair of parallel plates, separated by 13 cm. When uncharged, surface muons of momentum 29 MeV/c pass undeflected. When the electrodes are charged, to \( \pm 12.5 \) kV, their electric field deflects the muon beam onto a downstream collimator. The extinction factor, the ratio of muon flux with the kicker off to the flux with the kicker on, was about 800. Our typical data cycle consisted of a 5 \( \mu \text{sec} \) accumulation period, with the kicker off and a 22 \( \mu \text{sec} \) measurement period with the kicker on. Just downstream of the kicker (they are reversed in the figure) is an electrostatic separator, which is used to eliminate the large positron contamination in the beam. The beam it passes to the focusing elements contains roughly equal numbers of muons and positrons.

Following the separator are two more pairs of focusing triplets. The beam which arrives at the target (see Fig. 4) is spread over a roughly elliptical area 2 cm wide by 1 cm tall. In our 2004 commissioning run, there was a helium bag between the entrance to the MuLan detector and the target at its center. For the 2006 and 2007 data production runs, as shown in the figure we installed a thin-walled vacuum pipe between the entrance and the target, with a corresponding pipe on the other side to preserve front-back symmetry. In 2004, a wire chamber at the ball’s entrance served as a beam profile monitor. In 2006 and 2007, that wire chamber was moved to the far end of exit pipe: once per 8 hour shift, the target was swung up and out of the way, and a brief measurement of the beam profile was made.

Several different kinds of targets were used for production running and systematics tests. In 2006, our production target was made of the ferromagnetic alloy, Arnokrome-3 (AK-3) [10], which has a large internal magnetic field. With a 5 \( \mu \text{sec} \) accumulation time, the residual polarization is less than 0.1%. In 2004, we also used a target made of pressed sulfur, whose residual polarization is a few percent.

In 2007, our production target consisted of a single quartz crystal, 12 cm in radius and 2 mm thick. \( \mu \text{SR} \) studies reveal that 90 percent of the positive muons produce muonium [11]. The other 10 percent, remain free, produce large \( \mu \text{SR} \) effects, even with the dephasing which results from a \( 5 \mu \text{sec} \) accumulation time. We also used non-magnetic metallic targets in which the muon polarization is preserved. In particular, we did extensive and very important \( \mu \text{SR} \) tests with a silver target, to cross check results from the quartz target.
The MuLan detector consists of two layer scintillators arranged in a soccer-ball configuration. Each of the hexagons is divided into six triangular tile pairs and each of the pentagons is divided into five. With two of the pentagons removed for the beam pipe, there are 170 tile pairs in all. The two layers are read out through UVA lightguides, into a pair of photomultiplier tubes (PMTs). Signals from photomultipliers are sampled at 450 MHz with an 8-bit flash ADC. For the entire detector, typical data rates to tape were about 30 Mbytes/sec.

We collected more than a trillion muon decays in 2006 and 2007 year but because we ran fewer systematic checks in 2006, more of the former group made it into our final sample. The final statistical errors were 1.1 and 1.7 for 2006 and 2007 respectively.

A through-going positron typically generates about 80 photoelectrons. The spectrum of pulse amplitudes from each tile is well-described by a Landau response function convoluted with a gaussian resolution. We set a threshold for a “hit” at about one half the minimum ionizing...
particle (MIP) peak, which is well above the noise floor. A spectrum of coincidences (which largely removes the noise) is shown in Fig. 5. The timing and amplitude stability of the detectors with respect to time in the measurement period is monitored with a nitrogen laser system, in which a narrow light pulse is divided among 24 of the detector PMTs and a special reference PMT which is kept well away from the beam line and target region. The stability of the amplitude spectrum can also be monitored by examining the data itself.

3. Systematic Errors

The heart of the measurement is evaluation of systematic errors. While high resolution measurements of time and pulse height are very helpful, the most critical issue is whether those measurements change from early to late in the measurement period. Changes in either can distort the pure exponential (plus background) shape of the decay-time spectra. For targets which preserve significant muon polarization, muon spin precession and depolarization can also distort the measured lifetime, particularly since it is often hard to find a good mathematical description which separates the effects of the decay and $\mu$SR.

Another systematic issue is pileup, when two positrons strike a single tile pair within a narrow time interval. The separation of the pulses may be so small that the two cannot be resolved. Or at slightly wider separations, the two pulses may be readily resolved but the time or amplitude measurement may still be compromised. The loss of counts or mismeasurement of time and amplitude from pileup is not by itself dangerous. Pileup affects the measured lifetime because it arises more often at early times than late.

3.1. Results on gain and timing shifts

Timing shifts are measured by comparing the time of a laser pulse as measured by a detector PMT with that measured by the reference PMT. Gain shifts can be measured (without the reference PMT) by examining the most probable value (MPV) of the laser pulse amplitude spectrum as a function of time in the measurement period. The same can be done with much...
greater statistical precision using the amplitude spectrum from the experimental data.

Analysis of the laser data shows that there are no significant timing shifts in the experiment. The measured shifts are consistent with 0 and the associated systematic error is much less than 0.1 ppm. Nor does it reveal any systematic shifts in detector gain. However, the limit set by the laser system is uncomfortably large, about 0.5 ppm, chiefly for lack of statistics. The MPV from the experimental data is far more revealing. Fig. 6 shows the background-subtracted MPV vs. time for the two kinds of PMTs used in the detector and, in the middle, their average. The units on the y-axis are ADC counts, in this case about 0.004 V.

There is an artifact from the global start signal (an unfortunate side effect of a well-intentioned electronic upgrade) in both tubes. The upper curve, which corresponds to the majority of the tubes is very flat for $t > 1.5 \mu s$. The lower curve, which corresponds to a small minority, shows a noticeable sag over the same region. The response curves are used to correct the data. The residual error from gain shifts is less than 0.3 ppm.

3.2. Pileup
Pulse pileup is one the largest systematic errors. Left uncorrected, a dead time of 5 c.t. (roughly 11 ns) would result in a 100 ppm error! One can fit for the effects of pileup by including a term proportional to $\exp(-2t/\tau_p)$ but at the cost of increasing the error on the lifetime by a factor of 2. We have addressed pileup losses by imposing an artificial dead time($ADT$) after each pulse, using the data itself to replace any lost pulses. Specifically, if there is a pulse in measurement period $i$ at time $t$, we look for replacement pulses in measurement period $i + 1$ between $t$ and $t + ADT$. Longer $ADT$s provide a good test of the method. For large $ADT$, the effect of the first pulse on the tube’s response should be minimal. Of course, there are many details: we take coincidences with a time window of $\pm ADT$; there is timing jitter between coincident pulses in the inner and outer layer; for longer $ADT$s, multipulse pileup is significant. We tested our pileup construction procedure over a large range of $ADT$s (from 5 to 70 c.t.). There is a small (1 ppm) change in the lifetime, as if some small fraction (0.1 %) of pileup were uncorrected.

Figure 6. Effective gain vs. time for both kinds of PMT
Gain changes or timing shifts after a pulse, and flaws in the pulse reconstruction cannot explain the discrepancy. The most likely explanation is fluctuations in beam intensity. A detailed Monte Carlo simulation yields a lifetime which is flat vs. ADT.

3.3. Muon spin resonance

In 2006, with the AK-3 target, the effects of $\mu$SR were minimal. Decay lifetime spectra could be fit successfully to a pure exponential plus background:

$$N(t) = N_0 e^{-t/\tau} + B. \quad (3)$$

A small residual polarization from the highly-polarized surface muon beam produces a small forward-backward asymmetry in a plot of lifetime vs. $\cos \theta$. See Figure 7. If the data from front-back pairs is added together, the resulting distribution of lifetime with $|\cos \theta|$ is flat. The mean of all the fits by detector is the same as that of a simple fit to a sum of all the decay spectra.

However in 2007, with a significant residual polarization in the quartz target, it was more difficult to demonstrate that the data was fully consistent. Small misalignments of the target/magnet-detector system led to large asymmetries vs. detector element in the measured lifetime. To fit the data, we needed a mathematical description which included the effects of muon precession, as well as longitudinal and transverse relaxation. The flux of decays seen by a detector is modeled as

$$N(t) = N_0 e^{-t/\tau} \left(1 + \frac{1}{3} [\hat{P}_\parallel(0) \cdot \hat{e}_D e^{-t/T_\parallel} + \hat{P}_\perp(t) \cdot \hat{e}_D e^{-t/T_\perp}]\right) + B \quad (4)$$

$\hat{e}_D$ is a unit vector from the muon decay point to the detector element in question. The factor $1/3$ comes from the integration of an energy-dependent asymmetry term over all detected positron energies. In the experiment, where the lowest energy positrons do not reach the detector, this factor is slightly larger. $\hat{P}_\parallel$ is the component of the polarization along the magnetic field and $\hat{P}_\perp$ is the component perpendicular to the B-field, which is a function of time. $T_\parallel$ and $T_\perp$ are the corresponding lifetimes. The exponential decay of the polarization is only a (serviceable) approximation. In practice, to help with convergence, the fit procedure is modified slightly:
several parameters are fixed from preliminary studies. However, almost all of the variation in the measured lifetime which results from a naive fit to Eq. 3, can be explained in terms of the physics in Eq. 4. The lifetime results listed at the top of figure 8, the “Golden runs”, with the target rotated left and right are consistent. Moreover, a simple lifetime fit (using Eq. 3) to the sum of all detector spectra, and an average of the all the lifetimes derived from Eq. 4 are also consistent. The lifetimes listed below, in which the target was moved along the $z$ axis, or rotated around the $z$ axis and even rotated out of the $xy$ plane, are consistent with the golden runs and with each other.

4. Results
The final error tables for the 2006 and 2007 runs are shown in table 1. The kicker stability error is estimated by combining measurements of the high voltage stability and the extinction factor vs. voltage. Gain stability is taken from the data itself and timing stability from the laser system. The pileup correction systematic is based on uncertainties in extrapolating our result to the ideal case of $ADT = 0$ c.t. The total systematic error is about 0.55 ppm in each year, much less than the statistical uncertainties. The analyses were done blind. The clock frequency for each year’s production running, nominally 450 MHz, was detuned by a different amount. These offsets were unknown to the analyzers and all active members of the collaboration. When the analyses from both years were far advanced, their results were mapped into a common blind space. But only when the analyses were deemed final was the true clock frequency revealed. Remarkably, the lifetime results differ by only 0.3 ppm. Our new result and a few of the more recent muon lifetime results, including the MuLan result from 2004, are shown in figure 9.

5. Conclusions
Using two very different experimental techniques, we have made two consistent measurements of the positive muon lifetime and thus of the Fermi constant. Those measurements our consistent with our previous (2007) result and those of other groups. Our statistical errors are distinctly larger than the systematic errors, which are, in turn, slightly larger than current theoretical uncertainties. Our preliminary value for the muon lifetime is $2196981.3 \pm 2.3$ ps. Our value for $\tau_\mu^+$ leads to a new, preliminary determination of the Fermi constant:

$$G_F(\text{MuLan}) = 1.1663818(7) \times 10^{-5} \text{ GeV}^{-2} \ (0.6 \text{ ppm}),$$

(5)
Figure 8. Lifetime results for 2007 run configurations
Figure 9. Recent measurements of the muon lifetime

assuming the standard model Michel parameter $\eta \equiv 0$. The positive muon lifetime is also used to obtain ordinary muon capture rates in hydrogen [13] or deuterium [14] by the lifetime difference method, $\Gamma_{\text{cap}} = 1/\tau_{\mu^-} - 1/\tau_{\mu^+}$.

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