BULGE FORMATION FROM SUPER STAR CLUSTERS
IN A RESPONDING CUSPY DARK MATTER HALO

YAN-NING FU
Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing 210008, China; and National Observatories,
Chinese Academy of Sciences, Beijing 100012, China; fyn@pmo.ac.cn

WEN-HAO LIU
Department of Astronomy and Physics, University of California,
Irvine, CA 92617; wenhaol@uci.edu

JIE-HAO HUANG
Department of Astronomy, Nanjing University, Nanjing 210093, China; jhh@nju.edu.cn

AND

ZU-GAN DENG
College of Physical Science, Graduate School of the Chinese Academy of Sciences,
Beijing 100049, China; dgz@vega.pku.bac.ac.cn

Received 2006 September 1; accepted 2007 February 18

ABSTRACT

We simulate the bulge formation in very late type dwarf galaxies from circumnuclear super star clusters (SSCs) moving in a responding cuspy dark matter halo (DMH). The simulations show that (1) the response of a DMH to the sinking of SSCs is detectable only in the region interior to about 200 pc. The mean logarithmic slope of the responding DM density profile over that area displays two different phases: a very early decrease followed by an increase until it approaches 1.2 at the age of 2 Gyr. (2) The detectable feedbacks of the DMH response on the bulge formation turned out to be very small, in the sense that the formed bulges and their nuclear cusps in the fixed and the responding DMH are basically the same; both are consistent with HST observations. (3) The yielded mass correlation of bulges to their nuclear (stellar) cusps and the time evolution of the cusps’ mass are in accordance with recent findings for relevant relations. In combination with the consistency of effective radii of nuclear cusps with observed quantities of nuclear clusters, we believe that the bulge formation scenario that we proposed could be a very promising mechanism for forming nuclear clusters.

Subject headings: dark matter — galaxies: bulges — galaxies: kinematics and dynamics — galaxies: nuclei — galaxies: star clusters — galaxies: structure

1. INTRODUCTION

In our previous works (Fu et al. 2003, hereafter FHD03; Huang et al. 2003, hereafter HDF03), we managed to construct a set of models for simulating the dynamical evolution of SSCs in a DMH. Simulations with vastly different settings consistently yielded bulges similar to those observed. Two main similarities are that (1) the derived surface density can be well fitted by an exponential profile with an additional nuclear cusp, which is consistent with Hubble Space Telescope (HST) observations (Carollo 1999); and (2) there is a causal connection between the masses of the bulge and its nuclear cusp, which is compatible with the observational evidence for a tight luminosity correlation between the two (Balcells et al. 2003).

However, the DMH response to SSCs sinking was not considered in both FHD03 and HDF03. The DMH response, in addition to its feedback on the bulge formation, is also interesting in the context of the inner slope of the DMH profile. As is well known, numerical simulations show that the density profiles of virialized ΛCDM DMHs have central cusps (e.g., Navarro et al. 1997, 2004; Moore et al. 1999; Jing & Suto 2002). However, high-resolution observations of dwarfs and low surface brightness galaxies (de Blok et al. 2003; Swaters et al. 2003; Simon et al. 2005) indicate that the Burkert profile (Burkert 1995) with a constant density core instead of the cuspy Navarro-Frenk-White (NFW) one is more suitable to these galaxies. This apparent contradiction has drawn much concern in the astronomical community. While some groups were checking the conflict by performing refined simulations or conducting higher resolution observations (e.g., Power et al. 2003; Navarro et al. 2004; Simon et al. 2005; Graham et al. 2006), others were trying to find a solution to this problem for CDM by considering additional physical processes involving luminous matter. In particular, El-Zant et al. (2001, 2004) showed that the DM distribution flattens when gas clumps in a galaxy or galaxies in clusters spiral inward.

On the other hand, Kazantzidis et al. (2004) have shown that the DMH initialized using the local Maxwellian approximation can result in spurious evolution. It was indeed what were we faced with during our early investigation of bulge formation in a DMH responding to sinking SSCs. We found false evolution of the DMH, due to the fact that the DMH is not in equilibrium under the local Maxwellian approximation. To ensure a real response of the host DMH to sinking SSCs, we will start with building an equilibrium DMH using an exact distribution function (Kazantzidis et al. 2004; Liu et al. 2005) in this work. We then consider the DMH response to the SSCs sinking, namely, the effects of gravitational contraction and heating expansion caused by the SSCs spiraling inward and the transferring orbital energy. Taking the above two effects into consideration, we simulate the dynamical evolution of SSCs in an initially cuspy DMH as a more sophisticated model of bulge formation in very late type galaxies. The properties of the formed bulges and their nuclear (stellar) cusps are basically the same; both are consistent with HST observations. (3) The yielded mass correlation of bulges to their nuclear (stellar) cusps and the time evolution of the cusps’ mass are in accordance with recent findings for relevant relations. In combination with the consistency of effective radii of nuclear cusps with observed quantities of nuclear clusters, we believe that the bulge formation scenario that we proposed could be a very promising mechanism for forming nuclear clusters.
will be compared with, in addition to those already mentioned, the most recent observations (Rossa et al. 2006), where the authors found a correlation between nuclear cluster (NC) mass and bulge luminosity for their 40 sample spiral galaxies.

The structure of this paper is as follows. In § 2, we describe our models. Sections 3 and 4 are devoted respectively to presenting and discussing our simulation results about DMH evolution and bulge formation. Conclusions are given in § 5.

2. MODELS

In this work, the bulge formations in a set of 40 sample galaxies are simulated. Apart from the DMH-related models, which will be described in §§ 2.1–2.3, the other settings are the same as those of HDF03 and listed in Table 1. The basis of these settings is summarized below for completeness.

The radial distribution of SSCs is obtained from the projected radial number distribution of bright star-forming complexes in a sample of very late type galaxies (Parodi & Binggeli 2003), $R \exp(-R/R_t)$, with a median scale length of $R_t \approx 0.6$ kpc and a median number of bright star-forming complexes of 17. These SSCs are all modeled as a truncated isothermal sphere. The initial mass function of SSCs is assumed to be a lognormal mass function, written as

$$\log_{10} \left( \frac{M}{M_{\odot}} \right) \sim N \left( \exp(\text{mean}) = 2 \times 10^6 M_{\odot}, \text{ variance} = 0.08 \right).$$  \hspace{1cm} (1)

We assume that the stellar mass outside a sphere of radius $R_t$, which corresponds to the instantaneous Hill stability region around the SSC center, will be stripped. On average, the stars stripped when the SSC goes from $r$ to $r - dr$ contribute to a region radially bounded by $r + R_t(r)$ and $r - dr - R_t(r - dr)$. In our simulations, tidal stripping is not allowed when the mass of the stripped SSC is less than 1 $M_{\odot}$.

2.1. Numerical DMH

2.1.1. Simulation Methods

In order to consider the effects of the interaction between the DMH and the SSCs on the bulge formation, we will use a particle representation of the DMH. When one uses a limited number of particles to represent a gravitational system and performs a usual $N$-body simulation, he in fact assumes that each particle is actually a representative of the physical ones with similar three-dimensional velocity in its three-dimensional neighborhood. Since our model is assumed to be spherically symmetric, each particle in our simulation is reasonably used to represent the physical particles with similar radial and transverse velocities in a shell. It can be understood that the radial position of a shell corresponds to the three-dimensional position of the above-mentioned neighborhood in the usual $N$-body simulation. In doing the numerical integration, one can follow El-Zant et al. (2001) and assume that the gravitation on a given particle at $r$ is $-GM(\frac{r}{r})^2$, where $r$ is the position vector starting from the center, $r = |r|$, and $M(r)$ is the total mass interior to $r$. This force model ensures automatic maintenance of the spherical symmetry. Based on this perspective (e.g., Henon 1971; El-Zant et al. 2001), we believe that the way we use to integrate the numerical DMH is well adapted to nearly spherically symmetric systems like ours.

One basic requirement for this representation is that it must be statistically meaningful at 5 pc from the center, so that the simulations could be compared with the aforementioned observations with linear resolution of about 5 pc. Simple calculation shows that, even if this requirement is so weak as to have only about 10 particles in the central globe of radius 5 pc, one needs about $10^8$ particles for representing the whole DMH interior to $r_{200}$. This would make it extremely time-consuming to complete our simulations.

To work around this difficulty, we neglect the DMH response outside 1 kpc. Indeed, our preliminary simulations show that the DMH response is hardly detectable at locations more than about 500 pc from the center (FHD04). Physically, this phenomenon has a connection with a reasonable general belief; that is, a low-mass intruder should not induce global evolution in a high-mass intruded system. In our case, the DMH mass of 3.4 $\times 10^8 M_{\odot}$ initially confined to the central globe of radius 1 kpc is more than 10 times larger than any single SSC mass. In fact, the SSC mass initially inside 1 kpc in our sample galaxies never exceeds 3.7 $\times 10^7 M_{\odot}$, about 10 times smaller than the DM mass in the same region. Also, as can be inferred from our previous simulations and verified ex post facto by the present one, only a small number of SSCs will spiral down to locations less than 1 kpc from the galactic center on the timescale of bulge formation. Therefore, the simplifying assumption that there is no DMH response outside 1 kpc should not have any severe influence on the simulation results.

A practical problem of using full $N$-body simulation for our investigation is that one is not able to use particles with a small enough mass to represent the DMH. This means that, when masses of SSCs become not much more massive than the representative particle of the DMH due to tidal stripping, the effect of dynamical friction cannot be correctly reproduced in practice. For example, if we set $2 \times 10^7$ particles, each representative particle of about 10 $M_{\odot}$ inside of 1 kpc for our simulations, the effect of dynamical friction would not be properly reproduced for stripped SSCs with less than about 100 $M_{\odot}$. For a large number of SSCs, however, the mass of the remaining SSCs at around 5 pc from the center would be a few tens of solar masses or less. Therefore, $10^8$ or more particles are needed in order to properly include the DMH heating in the inner 5 pc globe, as is required for the proposed bulge formation processes to be simulated with the required resolution of 5 pc.

It follows that even using a full $N$-body code for our investigation, one has to add the dynamical friction by hand. On the other hand, the gravitational response of the DMH is calculated using the full $N$-body code, in the form of the force model described above.

| Item | Description |
|------|-------------|
| DM DMH: initial density profile | $NFW$ with $M_{200} = 10^{11} M_{\odot}$ |
| SSC: initial mass function | Lognormal with $\exp(\text{mean}) = 2 \times 10^6 M_{\odot}$, variance = 0.08 |
| SSC: projected radial number distribution | $N(R) \sim R \exp(-R/0.6 \text{ kpc})$ |
| SSC: total number in a single galaxy | 17 |
| SSC: initial velocity | Local circular speed |

We assume the stellar mass outside a sphere of radius $R_t$, which corresponds to the instantaneous Hill stability region around the SSC center, will be stripped. On average, the stars stripped when the SSC goes from $r$ to $r - dr$ contribute to a region radially bounded by $r + R_t(r)$ and $r - dr - R_t(r - dr)$. In our simulations, tidal stripping is not allowed when the mass of the stripped SSC is less than 1 $M_{\odot}$.

2.1.1. Simulation Methods

In order to consider the effects of the interaction between the DMH and the SSCs on the bulge formation, we will use a particle representation of the DMH. When one uses a limited number of particles to represent a gravitational system and performs a usual $N$-body simulation, he in fact assumes that each particle is actually a representative of the physical ones with similar three-dimensional velocity in its three-dimensional neighborhood. Since our model is assumed to be spherically symmetric, each particle in our simulation is reasonably used to represent the physical particles with similar radial and transverse velocities in a shell. It can be understood that the radial position of a shell corresponds to the three-dimensional position of the above-mentioned neighborhood in the usual $N$-body simulation. In doing the numerical integration, one can follow El-Zant et al. (2001) and assume that the gravitation on a given particle at $r$ is $-GM(\frac{r}{r})^2$, where $r$ is the position vector starting from the center, $r = |r|$, and $M(r)$ is the total mass interior to $r$. This force model ensures automatic maintenance of the spherical symmetry. Based on this perspective (e.g., Henon 1971; El-Zant et al. 2001), we believe that the way we use to integrate the numerical DMH is well adapted to nearly spherically symmetric systems like ours.

One basic requirement for this representation is that it must be statistically meaningful at 5 pc from the center, so that the simulations could be compared with the aforementioned observations with linear resolution of about 5 pc. Simple calculation shows that, even if this requirement is so weak as to have only about 10 particles in the central globe of radius 5 pc, one needs about $10^8$ particles for representing the whole DMH interior to $r_{200}$. This would make it extremely time-consuming to complete our simulations.

To work around this difficulty, we neglect the DMH response outside 1 kpc. Indeed, our preliminary simulations show that the DMH response is hardly detectable at locations more than about 500 pc from the center (FHD04). Physically, this phenomenon has a connection with a reasonable general belief; that is, a low-mass intruder should not induce global evolution in a high-mass intruded system. In our case, the DMH mass of 3.4 $\times 10^8 M_{\odot}$ initially confined to the central globe of radius 1 kpc is more than 10 times larger than any single SSC mass. In fact, the SSC mass initially inside 1 kpc in our sample galaxies never exceeds 3.7 $\times 10^7 M_{\odot}$, about 10 times smaller than the DM mass in the same region. Also, as can be inferred from our previous simulations and verified ex post facto by the present one, only a small number of SSCs will spiral down to locations less than 1 kpc from the galactic center on the timescale of bulge formation. Therefore, the simplifying assumption that there is no DMH response outside 1 kpc should not have any severe influence on the simulation results.

A practical problem of using full $N$-body simulation for our investigation is that one is not able to use particles with a small enough mass to represent the DMH. This means that, when masses of SSCs become not much more massive than the representative particle of the DMH due to tidal stripping, the effect of dynamical friction cannot be correctly reproduced in practice. For example, if we set $2 \times 10^7$ particles, each representative particle of about 10 $M_{\odot}$ inside of 1 kpc for our simulations, the effect of dynamical friction would not be properly reproduced for stripped SSCs with less than about 100 $M_{\odot}$. For a large number of SSCs, however, the mass of the remaining SSCs at around 5 pc from the center would be a few tens of solar masses or less. Therefore, $10^8$ or more particles are needed in order to properly include the DMH heating in the inner 5 pc globe, as is required for the proposed bulge formation processes to be simulated with the required resolution of 5 pc.

It follows that even using a full $N$-body code for our investigation, one has to add the dynamical friction by hand. On the other hand, the gravitational response of the DMH is calculated using the full $N$-body code, in the form of the force model described above.
hand, a number of investigations performed with N-body simulations have provided the basis for taking the semianalytical approach (e.g., Cora et al. 1997; Velazquez & White 1999). These studies indicated that "the numerical (i.e., full N-body) results of low-mass satellites showed very good agreement with theoretical predictions obtained from straightforward application of the Chandrasekhar dynamical friction equation" (Cora et al. 1997), and they found "Chandrasekhar's dynamical friction formula works well provided a suitable value is chosen for the Coulomb logarithm and the satellite mass is taken to be the mass still bound to the satellite at each moment" (Velazquez & White 1999). In view of this, the Coulomb logarithm in our simulations is calculated each time the formula is applied, according to the relevant updated quantities, and by taking account the stripping effect, only the mass still bound to the SSC is used.

But we must point out that the above-mentioned numerical experiments did not follow the satellite all the way in toward the center of its host galaxy, where the semianalytical equation (2) could become invalid because the size of the satellite is not always adequately small as compared to the scale length of the background density variation. While this possible inadequacy could have some influences on the bulge formation process, the overall influence should not be severe because, on its way toward the center, an SSC also becomes smaller and smaller due to tidal stripping. In fact, most SSCs will be destroyed before they are close to the center, and the remaining ones are very compact.

Apart from computation time, the two methods, N-body and semianalytical approaches, are expected to be equivalent in regard to dealing with the dynamical friction, except for the unfavorable region very close to the galactic center. Considering the time-saving advantage of taking the semianalytical experiment as described in this paper, especially for simulating at least several tens of sample galaxies, we tend to adopt the later approach, especially for simulating at least several tens of sample galaxies, we tend to adopt the latter approach for our study.

However, it is surely important to reveal some potential inadequacies induced by the semianalytical treatment of dynamical friction and other physical processes to be introduced in §2.3, by simulating the proposed bulge formation processes with a full N-body code, such as GADGET, when a sufficiently powerful computer facility is available. This will help one to decide to what extent the semianalytical approximations can be accepted with full confidence.

2.1.2. Reflecting Sphere

Under the above assumption, we model our DMH as follows. The DMH content interior to the spherical surface of radius 1.1 kpc is represented by a system of 106 particles. These particles are randomly generated according to the isotropic stable distribution function corresponding to the NFW density profile but with an outer exponential cutoff (Kazantzidis et al. 2004; Liu et al. 2005). During the numerical integration of the particle system, a reflecting boundary condition will be applied on the above-mentioned spherical surface. To be specific, whenever an outgoing particle reaches this surface, its velocity will be reversed.

In other words, the inflow halo mass across the sphere of radius 1.1 kpc is accounted for by the reflected particles at the inner boundary of the same sphere. This is based on the assumption that this sphere is situated in the nearly stable region of the halo, which, in particular, means that each inflowing halo particle should be compensated by an outflowing one with the same velocity. In practice, the boundary condition cannot be applied exactly on the sphere; there are inward particles lie slightly outside 1.1 kpc. These particles are allowed to go inside freely.

For a given sample particle system, the part inside 1 kpc will be referred to as a numerical DMH, of which the particle positions will be used in calculating the gravitation of the DMH at distances interior to 1 kpc. For particles outside 1 kpc, the gravitation will be calculated by using the analytical NFW model. This is because we discuss the evolution of the DMH profile only inside 1 kpc; outside the halo is assumed to be in equilibrium.

To see to what extent our numerical DMHs can be claimed as stable systems, we integrate them in the absence of SSCs. The volume density profiles of the numerical DMHs at various times are illustrated in Figure 1. Based on this figure, we are convinced that the numerical DMHs are sufficiently stable for detecting a significant DMH response to SSCs sinking.

2.2. Dynamical Friction

In agreement with Kazantzidis et al. (2004), Figure 2 shows the one-dimensional velocity distributions of the numerical DMHs. We can see that the velocity distributions at distances not far from the center are significantly steeper than the Gaussian distribution. According to Liu et al. (2005), this could make it inadequate for estimating the dynamical friction with the formula we used in FHD03, HDF03, and FHD04 (see also Binney & Tremaine 1987; El-Zant et al. 2001). Instead, the Chandrasekhar dynamical friction formula, valid for any isotropic velocity distribution, should be used (Chandrasekhar 1943; Binney & Tremaine 1987). This latter formula reads

$$\frac{dV_M}{dt} = -16\pi^2 \ln \Delta G^2 m (M + m) \int_0^{V_M} \int f(v_m) v_m^2 \, dV_m \, V_M$$

$$\approx -4\pi \ln \Delta G^2 M \rho(V_M) \frac{dV_M}{dt},$$

(2)

where $f(v_m)$ is the phase-space number density of the background composed of particles of mass $m$, moving with speed $v_m$; $M(\gg m)$ and $V_M$ are the mass and velocity of the cluster experiencing the dynamical friction, respectively; $\Delta$ is the Coulomb logarithm; and $\rho(V_M)$ denotes the mass density of background matter moving slower than the cluster.

However, the dynamical friction of the stripped stars from SSCs is roughly estimated using the Maxwellian approximation, as we did in our previous simulations.

2.3. DMH Responses

There are two competing effects on the numerical DMH in response to the SSCs sinking, namely, gravitational contraction and heating expansion. The former is automatically included in the above-mentioned force model. We believe that the way to account for the halo heating by El-Zant et al. (2001, 2004) is a reasonable approach for us to take, because the densely distributed DMH matter should also be efficient in distributing, at least locally as is required, the energy obtained from the coupling with the SSCs.

In practice, at the end of each integration step, "the Cartesian velocity components of the DMH particles are updated through an additive term chosen from a normal distribution with zero mean and variance" (El-Zant et al. 2001) $2E_b/3m$, where $E_b$ is the energy gained by a DMH particle from all SSCs and $m$ is the mass of the particle. We know that an SSC spiraling inward will lose orbital energy mainly to DMH particles near its trajectory. And as these particles with isotropic velocities radially oscillate around their respective mean distances from the galactic center,
the energy lost by the SSC will be redistributed among all DMH particles around $\bar{r}$, where $\bar{r}$ is the distance of the SSC averaged over the last integration step. This implies that DMH particles around $\bar{r}$ should gain more energy than those far from it. Based on the above arguments, we distribute the energy lost by an SSC during the last integration step into several bins, within each of which the particles (including the involved DM not actually represented by particles. The corresponding number of “particles” that accounts for this part of the DM can be calculated from the fixed NFW density profile) will be assigned with the same energy increment due to the considered SSC. These SSC-dependent bins are obtained by dividing the radial distance interval $[0, \bar{r} + b]$, where $b = 2$ kpc is the effective impact parameter used in estimating the Coulomb logarithm. The bin nearest to the SSC always covers the region explored by the SSC in the last integration step, and the boundaries of the other bins are equally spaced in $\log d$, where $d$ is the radial distance from the SSC. The energy lost by an SSC is distributed among the bins according to the following fact: the dynamical friction from the DMH content within the distance $D$ from the SSC is proportional to $\ln(1 + \Lambda_D)$, where $\Lambda_D = D|V_M|^2/GM$ (e.g., Binney & Tremaine 1987). It should be pointed out that

![Diagram of volume density profiles of the numerical DMHs in the absence of SSCs moving in them.](image1)

**Fig. 1.** Volume density profiles of the numerical DMHs in the absence of SSCs moving in them. (We call them unpolluted DMHs in the following figures.)

![Diagram of mean radial velocity distribution over the numerical unpolluted DMHs.](image2)

**Fig. 2.** Mean radial velocity distribution over the numerical unpolluted DMHs. Also shown are Gaussian distributions with respective standard deviations.
the above simplifying semianalytic prescription for halo heating effectively neglects possible non–spherically symmetric evolution of the DMH, which can only be studied with full N-body simulations.

3. DMH EVOLUTION

For simplicity, the DMH used in this work will be referred to as the responding DMH, while the DMH with NFW density profile and Maxwellian velocity distribution used in HDF03 will be referred to as the fixed DMH. In a responding DMH, the evolution of the DM density profile depends on the redistribution of luminous matter induced by the sinking of SSCs. Figures 3 and 4 illustrate, respectively, the variation of SSCs radial distribution with time and the resulting evolution of the DM density profiles.

The upper panels of Figure 3 depict the early stage of SSCs’ sinking until 10 Myr, where we can find very few SSCs deposited to the central region. At that time, heating of the halo induced by the dissipated orbital energy of SSCs spiraling inward dominates. The DM in the central region has been puffed outward (also see Tasitsiomi 2003), leading to decreasing density in a region of several tens of parsecs from the center. At later times, more and more SSCs have sunk into the nuclear region, shown in the lower panels of Figure 3. The effect of gravitational contraction then overwhelms that of heating expansion, resulting in more DM distributed over that region. The steeper central cusps with respect to typical NFW profiles are obviously indicated in the lower panels of Figure 4.

We have noticed that the DMH response is detectable only in regions less than ~200 pc from the center. Adopting the logarithmic slope defined as \( \beta(r) = -d \log(\rho)/d \log(r) \) (Navarro et al. 2004), we can show the time evolution of the mean responding DMH in Figure 5 by use of the mean logarithmic slope over the inner 200 pc region. For comparison, the logarithmic slope of the analytical NFW and the mean unpolluted DMH profiles are also indicated in this figure. What we can see from this figure is an early decreasing phase for the inner slope of the mean responding DM profile, followed by a gradual increasing trend with time. At 2 Gyr, the inner slope of the mean responding DM profile is about 1.2. A similar phenomenon was recently obtained by Lin et al. (2006), who found that the concentration parameters of DMHs increase due to the influence of baryonic matter on them.

4. BULGE FORMATION

4.1. Feedback of DMH Response on Bulge Formation

We have shown in § 3 that the DM distribution experiences two different phases induced by the influence of sinking SSCs, an early decrease and a later increase of density over the inner central region. In return, the time evolution of the DMH will certainly affect the dynamical evolution of SSCs and the bulge formation histories. To compare our present simulations (in a responding DMH) with those in the fixed DMH (HDF03) would clarify the feedback of DMH response on bulge formation, shedding light on galaxy formation.

Following what we did in the fixed DMH (HDF03), we display the bulge formation processes and the mean surface density profiles in Figures 6 and 7, respectively. In general, they are about the same as what we obtained in HDF03, i.e., the formed bulges are characterized by the general presence of a central cusp on top of an exponential bulge. For this reason, we take the same fitting models as we used in HDF03 (Carollo & Stiavalli 1998),

\[
\sigma(R) = \sigma_0 \exp\left(-1.678 \frac{R}{R_c}\right) + \sigma_1 \left(1 + \frac{R_c}{R}\right)^\gamma \exp\left(-\frac{R}{R_c}\right),
\]

where the first term represents an exponential bulge and the second one an additional nuclear cusp. As shown by circles in

![Figure 3](image-url)
Figure 7, the mean profiles obtained from the present simulations are well fitted to this model. The stars in Figure 7 denote the derived mean profiles in HDF03. They are just slightly higher than the formed bulges in the responding DMH. It follows that the histogram of the scale length, $R_e$, for the simulated bulges and that of the FWHM for nuclear cusps derived in the responding DMH, displayed in Figure 8, would be about the same as those obtained in a fixed DMH (HDF03), although the later results are acquired at the age of 3 Gyr.

The disparity, although small, of mean surface density profiles in different DMHs should have something to do with the feedback of the DMH response on bulge formation. In fact, most SSCs take more than several hundred megayears to spiral down to several hundred parsecs from the center, shown in Figure 3. On the other hand, apart from the very early time the DMH response leads to an increasing density of DM over the central region. It would cause stronger tidal stripping on SSCs outside the bulge region, resulting in a lower surface density distribution than that derived in the case of a fixed DMH, which looks like what we illustrate in Figure 7.

The continuously stronger tidal stripping induced by the DMH response for most of the evolution time would make it harder for an SSC to survive as an off-center star cluster than in the fixed DMH. Nevertheless, there is one such cluster at about 6 pc from the center in the snapshot at 100 Myr in Figure 3. The surface density profile of the bulge hosting this cluster is indicated by squares in the lower left panel of Figure 6. Clearly, the outer part of this bulge is very weak. This simulated sample provides a possible explanation for the existence of off-center nuclear star clusters hosted in the so-called bulgeless spiral galaxies (Matthews & Gallagher 2002). Indeed, observational searches for bulges in very late type galaxies started only very recently, and it is possible that some very weak bulges remain undetected (see, e.g., Böker et al. 2003; Carollo 1999).

Figure 9 presents the bulge formation fractions in the responding and the fixed DMHs. Generally speaking, bulges form earlier in the former DMH. This is mainly because of its steeper velocity distribution than the Gaussian one. In fact, for two DMHs having the same volume density, the one with the steeper velocity distribution has a stronger effect of dynamical friction (Binney & Tremaine 1987; Liu et al. 2005). Besides, we have discussed above that the DMH density response should also favor a stronger effect of dynamical friction in most SSCs. However, the opposite situation is possible for SSCs initially very close to the central...
region where the DMH density undergoes a transitory, early decreasing phase, seen in Figure 4. This explains why the bulge formation fraction at a very early stage is lower in the responding DMH than in the fixed DMH.

In summary, the DMH response to SSCs sinking does have detectable feedbacks on bulge formation. However, the feedbacks discussed above are actually not very significant. The formed bulges in both fixed and responding DMHs are basically the same, at least in the view of the present-day observations. It suggests that the fixed DMH could still be an acceptable simplifying model for DM in galaxies, as far as the bulge formation is concerned.

4.2. Mass Correlation between the Bulges and their Nuclear Cusps

The occurrence of compact massive star clusters in the nuclei of spiral galaxies is now believed to be a common phenomenon (e.g., Börker et al. 2002, 2003; Matthews & Gallagher 2002; Rossa et al. 2006). The great progress made by Rossa et al. (2006) is to derive important properties of these NCs, such as ages, masses, and their composition of stellar populations. According to these investigations, we have a basic understanding to the physical properties of NCs hosted by most spiral galaxies. For example, the NC effective radii are typically in the range of 2–10 pc, and their average masses ($\log M$) and mass-weighted ages ($\langle \log \tau \rangle_M$) are, respectively, 6.25 versus 7.63 and 9.07 versus 9.89 for late-versus early-type spiral galaxies.

The most instructive result from Rossa et al. (2006) is the tight correlation of the NC mass with the luminosity of its bulge, similar to the striking luminosity correlation of bulges and their nuclear cusps obtained by Balcells et al. (2003). Both of these studies strongly imply a causally connected formation processes between the two components. In our previous work in the fixed DMH (HDF03), we have shown clearly that the mass relation between the simulated bulges and their nuclear (stellar) cusps coincides well with what Balcells et al. (2003) obtained.

In the case of a responding DMH, the mass relation between the nuclear (stellar) cusps and their bulges would also hold, as expected from the connected formation processes of these two components, as shown in Figure 10. The dashed line in the figure illustrates the linear fitting for these simulated data. The data point (square) derived from Balcells et al. (2003) (see HDF03) is also added in this figure for comparison. The thin solid line is extracted from Rossa et al. (2006), based on a typical value of the mass-to-light ratio.

The major problem for comparing our simulations to what Rossa et al. (2006) derived lies in their important finding, i.e., NCs are composed of mixed populations of different ages. It means that NCs form via at least more than one starburst event. On the contrary, our proposal for forming bulges and nuclear (stellar) cusps through dynamical evolution of SSCs in DM-dominated galaxies is only for very late type spiral galaxies experiencing one
starburst event, triggered by galactic harassment. This inherent difference causes the disparity of our simulations from the observed thin line shown in Figure 10.

According to the hierarchical processes of galaxy formation, the very late type spiral galaxies would be those undergoing the least merger/interaction events. That is to say, two Sm-type spiral galaxies among the 40 sample galaxies that Rossa et al. (2006) observed, NGC 428 and NGC 2552, are the sources most suitable for comparing with our simulations. The stars shown in Figure 10 are the data points of these two galaxies. Although two sample galaxies are really not enough for us to reach a definite conclusion, the close slopes of the two (observed) data points and the dashed (simulation) lines is very encouraging indeed.

The same situation that we see in Figure 10 also occurs in the time dependence of the nuclear (stellar) cusp mass, shown in Figure 11. The solid line extracted from Rossa et al. (2006) illustrates the NC mass in relation to the mass-weighted ages. The mass-weighted ages are referred to the older population of NCs, which contains most of the NCs’ mass. Obviously, it is this case that does match our simulations. The most impressive thing in this figure is the very close slopes of our simulations to the two galaxies with a Hubble type of Sm, NGC 428 and NGC 2552, which is just the same as we see in Figure 10.

The same closeness appearing in two different statistical correlations gives us a very strong hint that our hypothesis for bulge/

5. CONCLUSIONS

More and more observational data increasingly indicate that galactic harassments, even minor mergers, such as the case of NGC 3310 (de Grijs et al. 2003), can trigger the formation of a set of SSCs in these galaxies. The subsequent dynamical evolution of SSCs in a configuration of DM-dominated systems would be a general phenomenon, which motivated our investigation of this process as a model for bulge formation (FHD03).
As a basic part of this model, the evolution of the DMH induced by the influence of sinking SSCs was a major step that we studied. Understanding the resulting variation of the inner slopes of the DM density profiles in that case is also of interest. The simulations performed in this work have indicated that the DMH response to the sinking of SSCs does cause the inner slope of an initial NFW density profile to steepen, approaching 1.2 at 2 Gyr in mean logarithmic slope over the responding region.

However, compared with the region traversed by the sinking SSCs, the above-mentioned responding region is small. As a result, the feedback of the DMH response on the bulge formation turned out to be very small, in the sense that the formed bulges in both the fixed (HDF03) and the responding DMH (this work) are basically the same; both are consistent with HST observations.

One very instructive result obtained from our consecutive investigations (FHD03; HDF03; this work) is that nuclear (stellar) cusps are formed, no matter what kind of DM density profile was adopted, the NFW or the Burkert profile, and no matter whether the interaction between the dark and luminous matter was considered or not. The derived median mass and effective radii of the cusps are in accordance with the observed corresponding NC quantities in most spiral galaxies.

The more important point on this matter would be what we demonstrated in this work on the mass correlation of nuclear (stellar) cusps to the bulges, which is consistent with the similar observed relation for relevant sample spiral galaxies. The same situation appears in diverse statistics of the time dependence of NC/(nuclear cusp) mass.

No matter how complicated the formation processes are for NCs of spiral galaxies, the work that we have done indicates that forming nuclear cusps through sinking of SSCs would be a very promising mechanism for NC formation in very late type spiral galaxies. Also, the larger NCs in late-type spiral galaxies could be the product of the smaller seed NCs in very late-type spiral galaxies, which Rossa et al. (2006) observed.

Obviously, more work needs to be done in both observations and simulations so that we can reach positive conclusions. In particular, observations of more galaxies with Hubble type of Sm and simulations of bulge formation or growth originating from more than one galactic harassment event are desperately needed.

The anonymous referee is thanked for his critical comments that clarified and strengthened the reasoning of the present work. This work is supported by NSFC 10373008. Y.-N. F. and Z.-G. D. are also supported by NSFC key programs 10233020 and 10333060, respectively.