Fault-related impulsive component detection for vibration-based diagnostics in the presence of random impulsive noise

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Abstract. In this paper, the authors propose the methodology for vibration-based diagnostics towards local damage detection in rolling element bearings in the presence of non-Gaussian noise. In real-life cases, the main problem making the analysis difficult is the non-Gaussianity of the high-energy noise present in the operational environment. Because of this fact, popular impulsiveness-related detection techniques cannot be used. In the presented article, a real-life data measured in an industrial scenario will be presented and a proposition of an approach to cyclic component extraction will be discussed. The proposed approach takes advantage of the Cyclic Spectral Coherence (CSC) map as multidimensional data representation. It can be very useful for indicating cyclic modulated components in the otherwise non-cyclic signal content. However, due to the limitations of statistics used in CSC map calculation impacting the quality of CSC map in the presence of non-cyclic impulsive behavior in the signal, Nonnegative Matrix Factorization idea is used as a method for component separation. The presented method allows for obtaining carrier-related and modulation-related features of the component of interest. The main advantage of the presented method is pairing very useful multidimensional data representation (CSC) with very potent decomposition technique (NMF), that allows to reconstruct separated components in the final stage of the analysis. As a consequence, the time series of the component can be reconstructed based on the carrier feature.

1. Introduction
Vibration-based diagnostics of machines with rotating elements is still an important subject nowadays. When local damage occurs on one of the elements, it typically produces the effects in the vibration signal in the form of cyclic, impulsive nonstationary components. Unfortunately, when besides the impulsive component related to the damage there exists another impulsive component in the signal, it is challenging to be able to detect the former in a correct way automatically.

Numerous approaches are taken into consideration when analyzing the vibration signal. The popular techniques include wavelet decomposition [1, 2], optimal filtering [3], statistical modeling [4, 5, 6], envelope analysis [7], time-frequency analysis [8], selectors [9, 10, 11] or cyclostationary analysis [12, 13, 14, 15, 16, 17]. Unfortunately, in the presence of random high-energy impulsive component that energetically overwhelms the signal of interest (SOI) and additionally overlaps it in terms of the frequency band, classical methods are insufficient, and the diagnostic process becomes much more difficult.
In this paper, authors utilize the bi-frequency representation called the Cyclic Spectral Coherence map (CSC), which describes modulation spectra related to different carrier frequencies [18, 19]. To be able to analyze such data structure, Nonnegative Matrix Factorization (NMF) is used for recognizing separate patterns of spectra that are connected to different processes occurring in the vibration signal [20, 21, 22, 23, 24].

2. Methodology
In this section, the details of the presented methodology are described. First, the CSC representation of the input time series is calculated (see section 2.1). Next, it is factorized using the NMF algorithm into two feature matrices (see section 2.2). When features are obtained, the component of interest is reconstructed as a new partial CSC map that is easier to be interpreted. It is also possible to obtain the time series of a component by filtering with its base feature.

2.1. Cyclic Spectral Coherence
The Cyclic Spectral Coherence (CSC) is the bi-frequency representation. It depends on the carrier frequency \( f \) and the modulation frequency \( \alpha \). It was introduced by Antoni in 2007 [25, 18] and used successfully by the authors in the past [19, 26]. The Cyclic Power Spectrum (CPS) \( S_X(f; \alpha) \) of the signal \( X \) can be described by the following formula:

\[
S_X(f; \alpha) = \lim_{L \to \infty} \frac{1}{L} E \left( \mathcal{F}_{x,L}(f + \frac{\alpha}{2}) \mathcal{F}_{x,L}(f - \frac{\alpha}{2}) \right),
\]

where \( \mathcal{F}_{x,L}(f) \) is Fourier transform of the signal \( x \) calculated on the interval of length \( L \). In the cyclostationary signal, for some of the modulation frequency \( \alpha \neq 0 \), the CPS is expected to meet the condition \( |S_X(f; \alpha)| > 0 \). Based on the Eq. (1), the formula of CSC can be introduced [25]:

\[
\text{CSC}(f; \alpha) = |\gamma_X(f; \alpha)|^2 = \frac{|S_X(f; \alpha)|^2}{S_X(f + \frac{\alpha}{2}; 0) S_X(f - \frac{\alpha}{2}; 0)}.
\]

The CSC is a useful tool to analyze the cyclostationarity of the signal. It allows determining the strength of the cyclic spectral autocorrelation of the signal. The CSC statistic is normalized and its values range between 0 and 1. When the \( |\gamma_X(f; \alpha)|^2 \) is significantly higher than 0, then the signal reveals the cyclostationarity property at carrier frequency \( f \) with modulation period equal to \( T = 1/\alpha \).

Following the Eq. (2) the CSC estimation is performed with an estimator of CPS. The estimator of SC is given by the formula:

\[
|\hat{\gamma}_X(f; \alpha)|^2 = \frac{|\hat{S}_X(f; \alpha)|^2}{\hat{S}_X(f + \frac{\alpha}{2}; 0) \hat{S}_X(f - \frac{\alpha}{2}; 0)},
\]

where \( \hat{S}_X(f; \alpha) \) is an estimator of the CPS [25].

2.2. Standard NMF with Euclidean objective
Let \( V = R^{n \times m} \) be a non-negative matrix of size \( n \times m \) (denoted \( V_{n \times m} \)). For this data matrix \( V \), Lee and Seung proposed the factorization into two components [27]. The considered NMF model can be defined as follows:

\[
V_{n \times m} \simeq W_{n \times r} \ast H_{r \times m}.
\]
The sought matrices $W$ and $H$ also have to be non-negative. It can be expressed by the restrictions:

$$
v_{ij} \geq 0, \quad w_{ik} \geq 0, \quad h_{kj} \geq 0, \quad i = 1, \ldots, n; \quad j = 1, \ldots, m; \quad k = 1, \ldots, r,
$$

where the $r$ parameter has to satisfy the inequality: $r \leq \min(n, m)$. In this paper, the $r$ parameter is corresponding to the expected number of components present in the analyzed signal.

In Eq. (4), the authors [21] consider the data matrix $V$ as the input data matrix. $V$ contains $m$ 'samples' (like face images, documents, etc.) in its columns. Each 'sample' is characterized by $n$ variables or parameters.

Based on the Eq. (4), the matrix $V$ is approximated by the product of real-valued lower rank matrices $W$ and $H$. The $W_{n \times r}$ is called the base matrix and the $H_{r \times m}$ as the encoding matrix. Both matrices have jointly fewer elements than $V$, which can be expressed by:

$$
(n + m) \ast r < n \ast m,
$$

where '$\ast$' means much smaller.

If indeed formula (4) occurs, it means that the product $W \ast H$ is a good approximation of $V$. Hence, the data matrix $V$ can be efficiently analyzed (and stored) by using only about $(n+m) \ast r$ instead of $n \ast m$ matrix entries.

To obtain as well as possible approximation $V \simeq W \ast H$, one should define the cost function to quantify the quality of the approximation. The three options for the cost function was proposed by the Lee and Seung, namely: Euclidean distance, divergence and Poisson error. In this implementation, we use one of the most common measures, namely the square of the Euclidean metric:

$$
\|V - WH\|^2.
$$

The square of the Euclidean metric (Eq.(6)) is the convex function in terms of $H$ or $W$ only, but not in both of them together. That is why it is typically impossible to find global minima. However, we can use many optimization techniques to obtain them.

Lee and Seung have stated that the multiplicative element-wise update rules presented below (for practical implementation presented in matrix forms by Hoyer [28]) are computationally efficient and straightforward to implement for minimizing the cost function.

**Theorem 1** The Euclidean metric $\|V - WH\|$ is non-increasing under the update rules

$$
H \leftarrow H \odot (W^T V) \odot (W^T WH),
W \leftarrow W \odot (VH^T) \odot (WHH^T),
$$

where $\odot$ denotes elementwise division and $\otimes$ denotes the elementwise multiplication [27].

Both matrices need to be updated simultaneously. Renormalization of matrices $W$ and $H$ by the norms of rows of matrix $H$ allows preserving the constant energy of the clusters. In principle, normalization is performed by the columns of matrix $W$ [29]:

$$
W \leftarrow WD_W,
$$

where

$$
D_W = \text{diag}\left(\frac{1}{\|w_1\|^{-1}}, \frac{1}{\|w_2\|^{-1}}, \ldots, \frac{1}{\|w_J\|^{-1}}\right).
$$
In this particular case, the input matrix $V$ is transposed from the beginning relative to the methodology described in [29]. Hence, the functionality of the output matrices of the NMF algorithm is inverted. Consequently, normalization is performed by the rows of matrix $H$, that for individual rows of matrix $H$ and columns of matrix $W$ is as follows:

$$W_k \leftarrow \frac{W_k}{\|H_k\|}, \quad H_k \leftarrow \frac{H_k}{\|H_k\|},$$

where $\|H_k\|$ is a Euclidean norm of the vector $H_k$.

### 3. Real data analysis

In this section, the application of the proposed method is demonstrated. The investigated machine is the real-life copper ore crusher operating in the mining industry (see Fig. 1). In Table 1 the characteristic frequencies of bearings are presented, with the fault on the inner race present with the frequency of 30.7 Hz. Data has been acquired with the sampling frequency $f_s = 25\,kHz$ using Endevco accelerometer and National Instruments data acquisition hardware.

| Description                                      | Value     |
|--------------------------------------------------|-----------|
| Rotational frequency of the inner ring            | 3 Hz      |
| Rotational freq. of the rolling element and cage assembly | 1.3 Hz    |
| Rotational freq. of a rolling element about its own axis | 10.6 Hz   |
| Over-rolling frequency of one point on the inner ring | 30.7 Hz   |
| Over-rolling frequency of one point on the outer ring | 23.3 Hz   |
| Over-rolling frequency of one point on a rolling element | 21.1 Hz   |

Raw vibration signal measured on the machine bearing is presented in the Fig. 2 (see [31]), and in the Fig. 3, one can see the CSC map calculated for this signal. The authors marked the informative frequency band (IFB) and harmonic frequencies of modulation related to the component of interest.

In the next step, the CSC matrix is factorized using the NMF algorithm described in section 2.2. The number of classes for the factorization process was set to three since this is the number of processes occurring in the signal: fault, random impulsive noise, and background machine-related process.

The results of the factorization are presented in Fig. 4. In practice, two matrices were produced, containing three vectors in each of them, but since the overall amount of those vectors is low, they are presented in the form of ordinary plots. Based on those feature vectors, the classes were labeled as DMG (damage-related component), ART (artifacts in the form of random impulses) and Noise (background machine-related behavior).

In terms of interpretation, base features describe the behavior in the carrier frequency domain and can be understood as selectors (filters). On the other hand, encoding features yield the meaning in the modulating frequency domain and are equivalent to IES (improved envelope spectra) of the components.

Following the NMF model (see Eq. 4) for the individual classes, it is possible to reconstruct the CSC maps for the separate components by multiplying their respective features. The example of the fault component is presented in Fig. 5. As one can see, the map remains almost empty, describing only the cyclic behavior in the narrow frequency band specified by its base feature, with modulating harmonic frequencies described by its encoding feature.
Additionally, since the base feature is effectively a selector, one could perform a conventional filtration to extract the time series of particular components. Such a case is presented in Fig. 6, where time series of the individual components, are shown.

4. Conclusions
In this paper, the authors propose a technique for vibration-based local damage detection in the presence of impulsive noise. The method assumes periodicity of the fault-related impulsive component and utilizes the combination of cyclostationary analysis and structural
decomposition. As a result, it is possible to describe the fault signature in terms of its cyclic properties. It is especially useful when other types of diagnostic methods fail due to the high energy of external components making the detection impossible using those methods (especially focusing on impulsiveness-related detection in the presence of high-energy non-cyclic impulsive components that are other than the fault). The presented method allows to describe the result in several different ways: as selector-modulator feature vectors obtained from NMF, as reconstructed partial CSC map for a resulting component, or as a time series obtained by filtering the input signal with the selector feature. Authors warn however that such an approach yields results imperfect from the point of view of utilized information since the data carried by the
Figure 5. Extracted features.

Figure 6. Components obtained by filtering the input signal with the base features.

encoding components is not used for the reconstruction. Unfortunately, there is no proper way to invert CSC map back to the form of time series, since a) time-related information is lost,
and b) CSC map is calculated only over the limited range of modulating frequencies and does not contain most of the information from the signal. It is also important to note that due to the assumptions about stationarity regarding CSC, the disadvantage arises when the carrier frequency band of the non-cyclic impulsive component significantly overlaps the frequency band of the cyclic component of interest.

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