Dynamic Strategy of Power Battery Closed-Loop Supply Chain Considering Cascade Utilization

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ABSTRACT Considering the effective utilization of power battery, the cascade utilization was introduced power battery closed-loop supply chain, the system decision-making problem of the power battery dual circulation closed-loop supply chain composed of a manufacturer, recycler and cascade utilization enterprise was the research object. Under the scenario of government subsidizing cascade utilization enterprise and manufacturer sharing the innovation cost of cascade utilization, this paper (1) constructs a differential game model of the closed-loop supply chain dynamic system; (2) studies the equilibrium strategy of each game participant under the dual mechanism of government subsidy and cost-sharing; (3) analyses the influence of each parameter on the decision-making of main bodies by numerical simulation. The results show that with the increase of government subsidies, the utility of cascade utilization efforts will increase, the cascade utilization enterprise will reduce the sale price of cascade utilization products, the recycler will increase the wholesale price of high energy density batteries, the manufacturer shares part of the innovation cost of cascade utilization enterprise, which helps to reduce the burden of cascade utilization enterprise, and the government subsidy and coordination mechanism of cost-sharing are conducive to improving the level of cascade utilization.

INDEX TERMS Cascade utilization, closed-loop supply chain, government subsidy, cost-sharing, differential game.

I. INTRODUCTION

With the global warming and the decline of fossil resources, the role of new energy in the field of production and life has become increasingly prominent. The new energy vehicle industry has become the fastest application scenario for the development of new energy industry [1]. As the driving force of new energy vehicles, the production and sales of power batteries have increased rapidly [2]. At present, the power battery of new energy vehicles in the market will soon enter the first peak of retirement. From 2018 to 2020, the total scrapped power battery in China will reach 25 GWh (200000 tons). According to the prediction of China Automotive Technology Research Centre, the annual waste of power battery will reach 116 GWh (about 780000 tons) in 2025. However, when the capacity of the power battery decays to 80% of the rated capacity, the power battery will no longer be suitable for electric vehicles. The battery capacity of electric vehicles accounts for only 50% of the total service life of the battery. Direct disassembly will cause 50% waste. Power battery recycling has become a hot topic in the extension of new energy vehicle industry, and cascade utilization of power battery has become an important measure to improve the level of resource utilization. Cascade utilization of power battery is to technically treat the retired power battery, reorganize it into cascade utilization products, and then apply it to fields with low requirements for battery performance, such as communication base station energy storage, peak shaving and valley filling, microgrid power regulation, etc. [3]. It can be seen that cascade utilization not only helps to improve the use value of power battery in the whole life cycle, expand the utilization level of resources and dilute the manufacturing cost, but also helps to promote the application of new energy vehicles and realize industrial closed loop. At the same time, it can properly dispose of heavy metals and organic pollutants in waste batteries and avoid environmental pollution [4], [5]. Therefore, how to make full and efficient cascade utilization to improve the level of resource utilization and reduce
environmental pollution has become an urgent problem to be solved.

II. LITERATURE REVIEW

The continuous improvement of electric vehicle market share and the issue of battery disposal after retirement are imminent, the closed-loop supply chain of power batteries has become a research hotspot at home and abroad. Gu et al. established a three-cycle closed-loop supply chain model for the recovery and reuse of power batteries for new energy vehicles, and found that the environmental and economic benefits of the supply chain can be effectively balanced [6]. Zhu et al. studied the impact of adverse selection and moral hazard of recyclers on the closed-loop supply chain of power batteries [7]. Xie et al. discussed the optimal strategy and the effectiveness of sharing contract of each enterprise in the three-level closed-loop supply chain for the two modes of single recycling and double recycling channels [8]. Zhang used the method of mixed integer nonlinear programming to study the dual-channel closed-loop supply chain [9]. Umangi et al. considered the dominance of different manufacturers and discussed the impact of self-owned and cross-channel recycling prices on demand, optimal pricing and recycling rates [10]. Ismail I. Almaraj et al. designed a multi-period and multi-level closed-loop supply chain model to study incomplete quality production under multiple uncertain factors [11]. Hao put forward countermeasures such as positive and negative combination, joint consultation and joint construction for the development of power battery recycling and reverse logistics under circular economy [12]. Xie et al. constructed a tripartite game model with vehicle companies as the main body of recycling responsibility, and studied the influence of the behavioral strategies of different subjects on the assumption of extended responsibility in the Pareto equilibrium [13]. Zhu et al. consider the value-at-risk design of joint contracts to realize the Pareto improvement of the dual-channel supply chain of risk-averse retailers [14]. Alamdar et al. studied a fuzzy closed-loop supply chain composed of manufacturer, retailer, and collector, established six game theory models, and compared different optimal solutions using game and fuzzy theory. The results showed that the cooperation between manufacturer and retailer is the optimal strategy of the two companies [15].

Research on the impact of government policies on closed-loop supply chain. Li Xin and others discussed the recovery decision-making and coordination mechanism of the three-level closed-loop supply chain of power battery under the recovery rate policy with or without government regulation [16]. Gu et al. studied the impact of government subsidies and battery recycling on the optimal production strategy of electric vehicles to avoid loss under uncertain market demand [17]. Qiu et al. analyzed the impact of government subsidies on the equilibrium strategies investment of all parties in the recycling process of power battery closed-loop supply chain based on evolutionary game [18]. Huang et al. constructed the pricing model of new energy vehicles in the two-way dual channel closed-loop supply chain, and analyzed the impact of government subsidies on the recovery price, the cost and total profit of each subject in the supply chain [19]. Considering cascade utilization enterprise and consumers, Liu et al. focused on impact of reverse subsidies and the scale effect of recyclers on the variables of supply chain nodes and profit distribution [20]. Fan et al. emphasized the coordination and optimization of forward sales and reverse recycling, and designed recycling reward and punishment contracts and forward franchising and reverse cost sharing contracts to coordinate the supply chain [21]. Based on the secondary reverse supply chain composed of manufacturers and retailers, Heydari et al. analyzed the effect of different incentive measures adopted by the government on supply chain coordination. The analysis shows that when the total profit of the supply chain is improved, it can effectively encourage each member to participate in recycling, and the way the government provides subsidies for manufacturers is more effective [22].

The current research on the closed-loop supply chain of power batteries considering the cascade utilization mainly focuses on the economic analysis of cascade utilization, the design of the recycling channel structure, the distribution of benefits in the system, and the game of various subjects. Among them, the optimization of supply chain structure, interest distribution and contract structure design have become the current research hotspot. Based on the research results of the existing closed-loop supply chain, from a dynamic perspective, considering the dual mechanisms of government subsidies and cost sharing and coordination, this paper studies and analyzes the equilibrium decision-making of power battery closed-loop supply chain enterprises introducing cascade utilization market. The contents are arranged as follows: firstly, under the scenario of government subsidy and cost sharing coordination, a closed-loop supply chain architecture of power battery with cascade utilization market is established, and a Stackelberg differential game model of dynamic system is constructed; Secondly, using differential game theory and dynamic optimization technology, the profit optimal value function and equilibrium strategy of each game participant are obtained; Finally, the sensitivity of each parameter is analyzed and verified from the perspective of theory and simulation, and the influence of each parameter on the equilibrium decision-making of the actors is determined.

III. PROBLEM DESCRIPTION AND MODEL CONSTRUCTION

A. PROBLEM DESCRIPTION

Cascade utilization of power battery is to apply the retired power battery to other fields with low requirements for battery performance after professional and technical treatment, which can avoid large-scale idling and abandonment of resources, effectively improve resource utilization, effectively improve resource utilization, dilute manufacturing costs and improve economy while saving energy and environmental protection. The double-cycle multi-stage
closed-loop supply chain of power battery considering cascade utilization is composed of manufacturers, electric vehicle consumers, recyclers, cascade utilization enterprise, cascade utilization consumers and the government. This system is composed of two competitive cycles of primary cycle “Manufacturer-Electric vehicle consumer-Third-party recycler” and secondary cycle “Manufacturer-Electric vehicle consumer-Third-party recycler-Cascade utilization enterprise-Government- Cascade utilization consumer”. In the secondary cycle, power battery cascade utilization needs to achieve breakthroughs in technology, safety and cost at the initial stage, in order to effectively and quickly promote the development of this field, the government provides cascade utilization enterprise certain cost subsidies. At the same time, manufacturers bear part of innovation costs of cascade utilization in order to strengthen coordination and cooperation among enterprises. The basic structure of power battery closed-loop supply chain is shown in Figure 1.

In this system, due to the existence of EPR (Extended Producer Responsibility), the manufacturer is in a leading position in the whole market, and the behaviors of various stakeholders are as follows:

1. The manufacturer is responsible for producing new power batteries and selling them in the electric vehicle market at retail price $p_n(t)$. At the same time, as a remanufacturer, all waste power batteries are recycled at a unified price $f(t)$ for product remanufacturing in both cycles, and manufacturer bear part of the effort utility cost of cascade utilization enterprise to improve the cascade utilization level in the secondary cycle.

2. Recyclers recycle the retired power batteries in the electric vehicle market at the recovery rate $\theta(t)$ and sort the recovered retired batteries into two categories: high energy density and low energy density. The waste batteries with low energy density are recycled and remanufactured by the manufacturer at a unified price $f(t)$ in the primary cycle, and the waste batteries with high energy density are sold to cascade utilization enterprise at a wholesale price $g(t)$ in the secondary cycle.

3. Cascade utilization enterprise process and reorganize the purchased high-energy density batteries in the secondary cycle, sell them to the cascade utilization market at retail price $p_t(t)$, and determine their effort level of cascade utilization $n(t)$ according to the market conditions. Cascade utilization enterprise recycle all cascade utilized batteries and give them to manufacturers for recycling and remanufacturing at a unified price $f(t)$.

4. The support of government subsidies can promote the cascade utilization of waste batteries to be accepted by consumers, and help to improve the recognition and profit margin of cascade utilization of waste batteries. The government subsidizes cascade utilization enterprise in the secondary cycle to support and encourage the healthy development of cascade utilization market and promote the steady improvement of cascade utilization level.

B. PARAMETER DESCRIPTION AND MODEL ASSUMPTION

In order to clearly describe the problem, the symbols in the model are defined and explained. Details are shown in TABLE 1.
According to the actual situation of power battery production, recycling, cascade utilization and remanufacturing process, and referring to the existing research, the following assumptions are made:

Assumption 1: All actors participate in a dynamic game with complete information, which is risk-neutral.

Assumption 2: The demand function for new power batteries in the electric vehicle market [23] is \( D(p_n(t)) = a - bp_n(t) \), and \( c_r < c_m < p_n(t) \).

Assumption 3: The number of retired power batteries recovered by the recycler is \( G(\theta(t)) = \theta(t) D(p_n(t)) \), and the fixed cost of recovery is \( I_r = 1/2(A\theta(t)^2) \). Where \( A \) is the difficulty coefficient of recovery [24].

Assumption 4: The demand function of the cascade utilization market for high energy density waste batteries [25] is \( D(p_h(t), n(t)) = (k - lp_r(t)) n(t) \), and \( g(t) < p_r(t) < p_h(t) \). The cost of cascade utilization effort utility [26] is \( I_r = 1/2(\alpha m(t)^2) \). The cascade utilization market is different from the ordinary reuse market, and the demand of the cascade utilization market does not affect the demand of the electric vehicle market, cascade utilization enterprise is responsible for recycling all the waste batteries after the cascade utilization, and the unit recycling cost is \( c_1 \).

Assumption 5: As an emerging market, the cascade utilization market has low demand. The number of high-energy density waste batteries recovered and sorted from the electric vehicle market can meet the demand of the cascade utilization market.

Assumption 6: The battery manufacturer also acts as a remanufacturer, recycling and remanufacturing all waste batteries at a unified price \( f(t) \), and \( 0 < h(t) < f(t) < g(t) \), \( f(t) < \Delta, \Delta = c_m - c_r \). Remanufactured power batteries and power batteries produced with new materials have the same sales price and consumer preferences.

Assumption 7: The forward sales, reverse recycling, cascade utilization and remanufacturing in the model are completed in one cycle. That is, the model only considers the power battery cycle process in a single cycle.

Assumption 8: \( n(t) \) is the efforts made by cascade utilization enterprise through technology research and development and other means. However, due to factors such as large investment, long profit cycle and the decline of consumers’ awareness of environmental protection, there is a natural decline in the level of cascade utilization efforts [27]. The cascade utilization effort level changes with time as \( dn(t) = \alpha m(t) - \delta n(t)\ dt \).

Assumption 9: Manufacturer, recycler and cascade utilization enterprise have the same discount factor at any time \( \rho (\rho > 0) \).

C. BASIC RELATION EXPRESSION AND DIFFERENTIAL GAME MODEL

Under the joint action of government subsidy and cost-sharing mechanism, the manufacturer’s profit function consists of the revenue from selling new power batteries, the cost of recovering power batteries, and the part of effort utility cost of sharing cascade utilization enterprise. The profit function of the recycler consists of the revenue from selling recycled power batteries and the fixed cost of the recovery. The profit function of cascade utilization enterprise consists of the revenue from selling power batteries (including government subsidies) for cascade utilization and the effort utility cost of cascade utilization. Subscripts \( m, r \), and \( t \) represent manufacturers, recyclers and cascade utilization enterprises respectively. In order to simplify writing, the time variables \( t \) is omitted in the analysis process.
In summary, the manufacturer’s profit function in the infinite time domain is:

\[
\pi_m = \int_0^\infty e^{-\rho t} \left( (p_n - c_m) D(p_n) + (c_m - c_r - \theta) G(\theta) - \mu_m I_t \right) dt
\]

\[
= \int_0^\infty e^{-\rho t} \left( (p_n - c_m) (a - b p_n) + (c_m - c_r - \theta) (\theta (a - b p_n)) \right) dt
\]

The profit function of the recycler in infinite time domain is:

\[
\pi_r = \int_0^\infty e^{-\rho t} \left\{ (p_t - c_1 + f - g + s) T(p_t, n) \right\} dt
\]

\[
= \int_0^\infty e^{-\rho t} \left\{ (p_t - c_1 + f - g + s) (k - l p_t) n \right\} dt
\]

The profit function in the infinite time domain of the cascade utilization enterprise is:

\[
\pi_t = \int_0^\infty e^{-\rho t} \left\{ (p_t - c_1 + f - g + s) D(p_t, n) \right\} dt
\]

\[
= \int_0^\infty e^{-\rho t} \left\{ (p_t - c_1 + f - g + s) (k - l p_t) n \right\} dt
\]

Assuming that the power battery closed-loop supply chain system adopts a manufacturer-led Stackelberg game, the Stackelberg differential game model is expressed as:

\[
\begin{align*}
\max_{[P_n,f]} & \quad \pi_m [P_n,f, \theta, g, P_t, m] \\
\text{s.t.} & \quad \max_{[g,\theta]} \pi_r [P_n,f, \theta, g, P_t, m] \\
& \quad \max_{[P_t,m]} \pi_t [P_n,f, \theta, g, P_t, m] \\
& \quad \delta t (t) = [\alpha (m(t)) - \delta n(t)] dt
\end{align*}
\]

IV. EQUILIBRIUM STRATEGY UNDER DUAL MECHANISM

In order to obtain the equilibrium strategy of manufacturer, recycler and cascade utilization enterprise under the dual mechanism of government subsidy and cost-sharing, the following contents are analyzed by using differential game theory. The manufacturer first determines the retail price of new power battery and the unified price of repurchasing waste batteries; Then, the recycler determines the recovery rate of waste batteries and the wholesale price of high-energy density waste batteries to cascade utilization enterprise; Finally, the cascade utilization enterprise determines the retail price of cascade utilization products in the cascade utilization market and its own effort utility.

Theorem 1: The optimal strategies of the game participants who pursue their own profit maximization are as follows:

\[
p_n^* = \frac{2A (a + bc_m) - ab (c_m - c_r - h)^2}{4Ab - b^2 (c_m - c_r - h)^2}
\]

\[
\theta^* = \frac{2}{4A - b (c_m - c_r - h)^2}
\]

\[
g^* = \frac{k + l (c_m - c_1 + c_r + h + s)}{4l}
\]

\[
p_t^* = \frac{3k + l (c_1 - s)}{4l}
\]

\[
m^* = \frac{\alpha \left( \frac{V_t^N}{o (1 - \mu_m)} \right)}{\delta n}
\]

Proof: Assuming that the profit optimal value functions of manufacturer, recycler and cascade utilization enterprise are \(V_m, V_r, V_t\) respectively, then the following equations are satisfied:

\[
\pi_m^* = e^{-\rho t} V_m^N; \quad \pi_r^* = e^{-\rho t} V_r^N; \quad \pi_t^* = e^{-\rho t} V_t^N
\]

At any given time \(t \in [0, \infty)\), according to the optimal control theory, the Hamilton-Jacobi-Bellman equations that should be satisfied by the optimal decision-making of each game participant are obtained as follows:

\[
\rho V_m^N = \max \left\{ (p_n - c_m) (a - b p_n) + (c_m - c_r - \theta) (\theta (a - b p_n)) \right\}
\]

\[
= \max \left\{ (g - h) (k - l p_t) n \right\}
\]

\[
\rho V_r^N = \max \left\{ (f - h) (\theta (a - b p_n) - (k - l p_t) n) \right\}
\]

\[
\rho V_t^N = \max \left\{ (p_t - c_1 + f - g + s) (k - l p_t) n \right\}
\]

To ensure that the model has an optimal solution, suppose \(8A - b(c_m - c_r - h)^2 > 0\). Following uses the reverse induction method to solve the model. First, solve the optimal decision-making of the cascade utilization enterprise, and respectively derive the sales price of the cascade utilization battery \(p_t\) and cascade utilization effort utility \(m\) at the right end of equation (14):

\[
\frac{\partial \rho V_t^N}{\partial m} = kn - fnl + g \ln -2p_t n + +lc_1 n - snl
\]

\[
\frac{\partial \rho V_t^N}{\delta m} = -m (1 - \mu_m) + \alpha V_t^N
\]

Make equation (15) and equation (16) equal to 0, and combine two equations, then obtain expressions of \(p_t, m\) into equation (13), then the derivative of \(g, \theta\) are obtained:

\[
\frac{\partial \rho V_r^N}{\partial g} = (k - l p_t) n - \frac{1}{2} n l (g - f)
\]

\[
\frac{\partial \rho V_t^N}{\delta \theta} = (f - h) (a - b p_n) - A \theta
\]
Make equation (17) and equation (18) equal to 0, and combine two equations, then obtain expressions of \( g, \theta \) and substitute the expressions of \( g, \theta \) into equation (12), then the derivative of \( p_n, f \) are obtained:

\[
\frac{\partial \rho}{\partial p_n} V^N_m = (a - 2bp_n) + bc_m + (c_m - c_r - f) \theta(-h) + (c_m - c_r - f) (a - bp_n) \frac{(f - h)(-h)}{A} \tag{19}
\]

\[
\frac{\partial \rho}{\partial f} V^N_m = -\theta(a - bp_n) + (c_m - c_r - f) \times (a - bp_n) \frac{(a - bp_n)}{A} \tag{20}
\]

Make equation (19) and equation (20) equal to 0, and combine two equations, then obtain expressions of optimal \( p_n, f \), and substitute the expressions of \( p_n^*, f^* \) into the expressions of \( p_n, m, g, \theta \), the optimal strategies of manufacturer, recycler and cascade utilization enterprise are obtained as follows:

\[
p_n^* = \frac{2A(a + bc_m) - ab(c_m - c_r - h)^2}{4Ab - b^2(c_m - c_r - h)^2} \tag{21}
\]

\[
f^* = \frac{(c_m - c_r + h)}{2} \tag{22}
\]

\[
\theta^* = \frac{(a - bc_m)(c_m - c_r - h)}{4A - b(c_m - c_r - h)^2} \tag{23}
\]

\[
g^* = \frac{k + l(c_m - c_1 - c_r + h + s)}{2l} \tag{24}
\]

\[
p_t^* = \frac{3k + l(c_1 - s)}{4l} \tag{25}
\]

\[
m^* = \frac{\alpha(V^N_t)}{\mu(1 - \mu_m)} \tag{26}
\]

In order to further clarify the equilibrium strategies of manufacturer, recycler and cascade utilization enterprise, the strategic formulas (21) \~ (26) of manufacturer, recycler and cascade utilization enterprise are substituted into formulas (12) \~ (14) respectively to solve their profit optimal value function \( V^N_m, V^N_r, V^N_t \). The profit optimal value functions of manufacturer, recycler and cascade utilization enterprise are obtained to meet the following requirements respectively:

\[
\rho V^N_m = \frac{A(a - bc_m)}{4Ab - b^2(c_m - c_r - h)^2} - \frac{a^2 \mu_m V^N_t V^N_m}{2o(1 - \mu_m)} + \frac{\alpha^2 V^N_m V^N_t}{o(1 - \mu_m)} - \delta V^N_m n \tag{27}
\]

\[
\rho V^N_r = \frac{[k - l(c_1 - s)]^2 n}{8l} + \frac{A(a - bc_m)^2}{2[4A - b(c_m - c_r - h)^2]} + \frac{\alpha^2 V^N_t V^N_r}{o(1 - \mu_m)} - \delta V^N_r n \tag{28}
\]

\[
\rho V^N_t = \frac{[k - l(c_1 - s)]^2 n}{16l} + \frac{\alpha^2 V^N_t V^N_r}{2o(1 - \mu_m)} - \delta V^N_t n \tag{29}
\]

According to the structure of the problem and the intrinsic function relationship, referring to the relevant research of Ma [27], it is suggested that the profit optimal value functions of manufacturer, recycler and cascade utilization enterprise are as follows:

\[
\begin{aligned}
V^N_m(n) &= f_1 n^2 + f_2 n + f_3 \\
V^N_r(n) &= i_1 n^2 + i_2 n + i_3 \\
V^N_t(n) &= j_1 n^2 + j_2 n + j_3
\end{aligned} \tag{30}
\]

Calculate the first derivative of the profit optimal value function of the manufacturer, recycler and cascade utilization enterprise, respectively, and obtain:

\[
\begin{aligned}
V^N_m'(n) &= 2f_1 n + f_2 \\
V^N_r'(n) &= 2i_1 n + i_2 \\
V^N_t'(n) &= 2j_1 n + j_2
\end{aligned} \tag{31}
\]

In order to make the profit optimal value function of each game participant suggested in equation (30) the solution of equation (27) \~ (29), the value of the coefficient \( f_1, f_2, f_3, i_1, i_2, i_3, j_1, j_2, j_3 \), in equation (30) should be determined. Therefore, substituting equation (30) and equation (31) into equation (27) \~ (29) respectively, and obtain:

\[
\begin{aligned}
\rho f_1 &= \frac{4a^2 j_1 f_1}{o(1 - \mu_m)} - \frac{4a^2 \mu_m f_1^2}{2o(1 - \mu_m)^2} - 2\delta f_1 \\
\rho f_2 &= \frac{2a^2 j_1 f_2}{o(1 - \mu_m)} + \frac{2a^2 j_2 f_1}{o(1 - \mu_m)} - \frac{4a^2 \mu_m f_1 f_2}{2o(1 - \mu_m)^2} - \delta f_2 \\
\rho f_3 &= \frac{A(a - bc_m)^2}{4Ab - b^2(c_m - c_r - h)^2} + \frac{\alpha^2 \mu_m f_1 f_2}{o(1 - \mu_m)} - \frac{4a^2 \mu_m f_1 f_2}{2o(1 - \mu_m)^2} - \delta f_3 \\
\rho i_1 &= \frac{4a^2 j_1 i_1}{o(1 - \mu_m)} - 2\delta i_1 \\
\rho i_2 &= \frac{8l}{o(1 - \mu_m)} + \frac{2a^2 j_2 i_2}{o(1 - \mu_m)} + \frac{8l}{o(1 - \mu_m)} + \delta i_2 \\
\rho i_3 &= \frac{A(a - bc_m)^2}{2[4Ab - b^2(c_m - c_r - h)^2]} + \frac{\alpha^2 j_2 i_2}{o(1 - \mu_m)} - \frac{4a^2 \mu_m f_1 f_2}{2o(1 - \mu_m)^2} - \delta i_3 \\
\rho j_1 &= \frac{-4a^2 j_1 f_1}{2o(1 - \mu_m)} - 2\delta j_1 \\
\rho j_2 &= \frac{8l}{o(1 - \mu_m)} + \frac{4a^2 j_2 f_1}{o(1 - \mu_m)} - 2\delta j_2 \\
\rho j_3 &= \frac{4a^2 j_2 f_1}{2o(1 - \mu_m)} - \delta j_3
\end{aligned} \tag{32} \tag{33} \tag{34}
\]
be equal. Solve the coefficients of the optimal value function \( f_1, f_2, f_3, i_1, i_2, i_3, j_1, j_2, j_3 \) as:

\[
\begin{aligned}
  f_1^* &= -\frac{\alpha \mu_m (\rho + 2 debtor)}{6\alpha^2} \\
  f_2^* &= \frac{\mu_m (\rho + 2 debtor) [k - l (c - s)]^2}{24l (2\rho + 3 debtor)^2 (1 - \mu_m)} \\
  f_3^* &= \frac{\rho [4Ab - b^2 (c - c - h)]}{\alpha^2 \mu_m [k - l (c - s)]^4} \\
  &+ \frac{512\rho l^2 (2\rho + 3 debtor)^2 (1 - \mu_m)^2}{(2\rho + 3 debtor)^3 (1 - \mu_m)^2} \\
  i_1^* &= 0 \\
  i_2^* &= \frac{[k - l (c - s)]^2}{8l (2\rho + 3 debtor)} \\
  i_3^* &= \frac{\alpha^2 [k - l (c - s)]^4}{128\rho l^2 (2\rho + 3 debtor)^2 (1 - \mu_m)} \\
  &+ \frac{A (a - bc_m)^2 (c - c - h)^2}{2\rho [4Ab - b^2 (c - c - h)]^2} \\
  j_1^* &= -\frac{\alpha (1 - \mu_m) \rho + 2 debtor}{2\alpha^2} \\
  j_2^* &= \frac{[k - l (c - s)]^2}{16l (2\rho + 3 debtor)} \\
  j_3^* &= \frac{\alpha^2 [k - l (c - s)]^4}{512\rho l^2 (2\rho + 3 debtor)^2 (1 - \mu_m)}
\end{aligned}
\]

Substituting \( m^* \) into the state equation of the cascade utilization effort level:

\[
dn(t) = \left[ -\frac{\alpha^2 V_{model}^n}{N_i} - \delta n(t) \right] dt
\]

Further obtained:

\[
n(t) = \left[ \frac{\alpha^2 j_2}{o (1 - \mu_m)} \right] (1 - e^{-(\rho + 3 debtor)t}) + e^{-(\rho + 3 debtor)t} n_0
\]

When \( n_0 = 0 \) and \( t \to \infty \), the stable value of the cascade utilization effort level is:

\[
n(t) = \frac{\alpha^2 j_2}{o (1 - \mu_m)}
\]

V. PARAMETER SENSITIVITY ANALYSIS

Analyze the impact of price sensitivity coefficient of cascade utilization market \( l \), price of battery recovered by recycler \( h \), cost saving of remanufactured products by manufacturer \( \Delta \), and government subsidy \( s \) on retail price of cascade utilization products \( p_1, \) effort utility of cascade utilization \( m, \) wholesale price of high energy density battery sold by recycler \( b, \) recovery rate of waste battery \( \theta, \) retail price of new power battery \( p_n, \) and unified price for manufacturer to buy waste battery \( f. \) The relevant conclusions are shown in TABLE 2 and inferences respectively.

| \( p_1^* \) | \( m^* \) | \( g^* \) | \( \theta^* \) | \( p_n^* \) | \( f^* \) |
|---|---|---|---|---|---|
| \( l \) | \( \downarrow \) | \( \downarrow \) | \( \downarrow \) | \( \to \) | \( \to \) | \( \to \) |
| \( h \) | \( \to \) | \( \uparrow \) | \( \downarrow \) | \( \uparrow \) | \( \uparrow \) | \( \uparrow \) |
| \( \Delta \) | \( \to \) | \( \uparrow \) | \( \uparrow \) | \( \downarrow \) | \( \uparrow \) | \( \uparrow \) |
| \( s \) | \( \downarrow \) | \( \uparrow \) | \( \uparrow \) | \( \to \) | \( \to \) | \( \to \) |

Inference 1: As the price sensitivity coefficient of cascade utilization market increases, the effort utility of cascade utilization will decrease. The cascade utilization enterprise will reduce \( p_1, \) the recycler will reduce \( g. \) The recovery rate of waste batteries, the retail price of new power batteries, and the unified price of manufacturer to buy waste batteries will remain unchanged.

Proof:

\[
\begin{align*}
  \frac{\partial m}{\partial l} &= -\frac{\alpha [k^2 - l^2 (c - s)]}{16l^2 (2\rho + 3 debtor) \alpha (1 - \mu_m)} < 0; \\
  \frac{\partial p_1}{\partial l} &= \frac{3k}{4l^2} < 0; \\
  \frac{\partial g}{\partial l} &= \frac{k}{2l^2} < 0 \\
  \frac{\partial n}{\partial l} &= \frac{\partial p_n}{\partial l} = 0
\end{align*}
\]

Inference 2: With the increase of the price of retired batteries recovered by recyclers, the recycler will increase \( g \) while reducing \( \theta. \) The manufacturer will increase \( p_n \) and \( f. \) The retail price of cascade utilization products and the effort utility of cascade utilization will remain unchanged.

Proof:

\[
\begin{align*}
  \frac{\partial g}{\partial h} &= \frac{1}{2} > 0; \\
  \frac{\partial \theta}{\partial h} &= -\frac{(a - bc_m) [4Ab - b^2 (\Delta - h)^2]^2}{[4Ab - b^2 (\Delta - h)^2]^2} < 0 \\
  \frac{\partial f}{\partial h} &= \frac{1}{2} > 0; \\
  \frac{\partial p_n}{\partial h} &= \frac{2ab (\Delta - h) [4Ab - b^2 (\Delta - h)^2]}{[4Ab - b^2 (\Delta - h)^2]^2} > 0 \\
  \frac{\partial p_1}{\partial h} &= \frac{\partial m}{\partial h} = 0
\end{align*}
\]

Inference 3: With the increase of cost saving of remanufactured products by manufacturer, the recycler will increase \( g \) and \( \theta. \) The manufacturer will reduce \( p_n \) and increase \( f. \) The retail price of cascade utilization products and the effort utility of cascade utilization will remain unchanged.
Proof:
\[
\frac{\partial g}{\partial \Delta} = \frac{1}{2} > 0; \\
\frac{\partial \theta}{\partial \Delta} = \frac{(a - bc_m) \left[4A + b (\Delta - h)^2\right]}{4A - b (\Delta - h)^2} > 0 \\
\frac{\partial f}{\partial \Delta} = \frac{1}{2} > 0; \\
\frac{\partial p_u}{\partial \Delta} = \frac{2ab (\Delta - h) \left[4Ab - b^2 (\Delta - h)^2\right]}{4Ab - b^2 (\Delta - h)^2} < 0 \\
\frac{\partial m}{\partial \Delta} = \frac{\partial p_t}{\partial \Delta} = 0
\]

Inference 4: As government subsidies increase, the effort utility of cascade utilization will increase. The cascade utilization enterprise will reduce \( p_t \), and the recycler will increase \( g \). The recovery rate of waste batteries, the retail price of new power batteries, and the unified price for manufacturer to buy waste batteries will remain unchanged.

Proof:
\[
\frac{\partial m}{\partial s} = \frac{\alpha [k - l (c_1 - s)]}{8 (2\rho + 3\delta) \alpha (1 - \mu_m)} > 0; \\
\frac{\partial p_t}{\partial s} = -\frac{1}{4} < 0; \\
\frac{\partial g}{\partial s} = \frac{1}{2} > 0 \\
\frac{\partial p_n}{\partial s} = \frac{\partial f}{\partial s} = \frac{\partial \theta}{\partial s} = 0
\]

VI. NUMERICAL EXAMPLE
According to the above theoretical derivation and analysis, the following is an intuitive analysis of the equilibrium strategy trajectory of the power battery closed-loop supply chain.
under the cost-sharing coordination mechanism based on government subsidies by simulating assignment parameters. With reference to the relevant research of Liu [20], Octave is used for numerical simulation. The system parameters are set as follows: $\rho = 0.3$, $A = 2$, $a = 70$, $o = 100$, $b = 0.5$, $c_m = 6$, $k = 35$, $c_r = 2$, $\delta = 0.3$, $c_1 = 1$, $s = 0.8$, $\alpha = 3$, $\mu_m = 0.3$, $h = 3$, $l = 0.6$.

A. EVOLUTION PATH ANALYSIS
The change trajectory of the profits of manufacturer, recycler and cascade utilization enterprise under the scenario of government subsidy and cost-sharing is shown in Fig. 2.

The profit curves of manufacturer, recycler and cascade utilization enterprise all show a non-linear upward trend with the increase of time, and the rising speed gradually decreases with the passage of time, and finally reach a stable profit state at the same time.

B. SENSITIVITY ANALYSIS
1) PRICE SENSITIVITY COEFFICIENT OF CASCADE UTILIZATION MARKET
Assuming that other parameters remain unchanged, setting $l = [0.1, 0.8]$, $s = [0.2, 0.6]$, the simulation results of price sensitivity coefficient of cascade utilization market are shown in Fig. 3.

With the increase of $l$ and $s$, the effort utility of cascade utilization, retail price of cascade utilization products, the wholesale price of high energy density batteries, and the overall profits of recycler and cascade utilization enterprise show a nonlinear downward trend, and only the profits of manufacturer show a nonlinear upward trend. The price sensitivity coefficient of cascade utilization market has a strong negative impact on the decision-making of cascade utilization enterprise and recycler, and has a more significant impact on cascade utilization enterprise. In contrast, the subsidy effect...
of the government on cascade utilization enterprise is relatively weak. At the same time, due to the reduction of cascade utilization effort utility, part of the effort utility cost shared by manufacturer decreases. Therefore, when other parameters remain unchanged, the market sensitivity of cascade utilization and government subsidies increase, only manufacturer can obtain more profits.

2) PRICE OF BATTERY RECOVERED BY RECYCLER
Assuming that other parameters remain unchanged, setting \( h = [1, 3] \), \( s = [0.2, 0.8] \), the simulation results of price of battery recovered by recycler are shown in Fig. 4.

With the increase of \( h \) and \( s \), the profit of manufacturer and recycler show a downward trend. The effort utility of cascade utilization, retail price of cascade utilization products, and the profit of cascade utilization enterprise have nothing to do with the recovery price of recycler, and are obviously affected by government subsidies. With the increase of government subsidies, the effort utility of cascade utilization and the profit of cascade utilization enterprise continue to rise, and the retail price of cascade utilization products decreases slightly.

3) COST SAVING OF REMANUFACTURED PRODUCTS BY MANUFACTURER
Assuming that other parameters remain unchanged, setting \( \Delta = [3, 5] \), \( s = [0.2, 1] \), the simulation results of cost saving of remanufactured products by manufacturer are shown in Fig. 5.

With the increase of \( \Delta \) and \( s \), the profit of cascade utilization enterprise continue to rise, and the retail price of cascade utilization products decreases slightly.

VII. CONCLUSION
This paper constructs a dual-cycle closed-loop supply chain system for power batteries that considers cascade utilization, studies the coordinated decision-making problem in the closed-loop supply chain by using the differential game method, and analyzes the influence of different parameters on the equilibrium strategies of each game subject.

It is found that the profit trajectories of manufacturer, recycler and cascade utilization enterprise show a non-linear upward trend with the increase of time, and finally reach a steady state; the price sensitivity coefficient of cascade utilization market has a more obvious negative impact on all parties. The government should actively encourage the development of the cascade utilization industry, increase public welfare publicity and improve public recognition. At the same time, relevant enterprises should also change the public consumption concept by strengthening the publicity of green consumption concept with the support of the government, so as to enhance consumers’ acceptance of the cascade utilization of retired power batteries, reduce the market price sensitivity of cascade utilization and help the cascade utilization level to steadily improve; the effectiveness of the government subsidy mechanism is remarkable. With the increase of government subsidies, the effort utility of cascade utilization is enhanced, and the cascade utilization enterprise will reduce the retail price of cascade utilization products, and the recycler will increase the wholesale price of high-energy density batteries. It can be seen that the government should vigorously support the weak links of the supply chain, provide subsidies, incentives and other policy benefits for relevant enterprises, and guide relevant node enterprises to recycle and reuse power batteries. In addition, manufacturer should actively carry out relevant technology R&D and innovation to increase the cost saving of remanufactured products, it is also an effective measure to improve the level of cascade utilization, and the recycling level of industrial resources. The contribution of this paper is to study and analyze the equilibrium decision-making of power battery closed-loop supply chain enterprises with cascade utilization market from a dynamic perspective and considering the dual mechanism of government subsidy and cost-sharing coordination. In addition, this paper provides ideas for improving and innovating the operation mechanism of power battery closed-loop supply chain, and the research results have important guiding significance for the government to guide relevant enterprises to make full and efficient cascade utilization, so as to improve the level of resource utilization and reduce environmental pollution.

In this paper, the characteristics of the attenuation of cascade utilization effort with time are considered, but the impact of uncertain factors on the power battery closed-loop supply chain is not considered. At the same time, the impact of different decision preferences on the decision-making of power battery closed-loop supply chain is the focus of further research in this field.

CONFLICTS OF INTEREST
The authors declare no conflicts of interest regarding the publication of this paper.

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REFERENCES
[1] L.-Y. He, L.-L. Pei, and Y.-H. Yang, “An optimised grey buffer operator for forecasting the production and sales of new energy vehicles in China,” Sci. Total Environ., vol. 704, Feb. 2020, Art. no. 135321.
[2] H. Ke and M. Cai, “Pricing decisions in closed-loop supply chain with collection-relevant demand,” J. Remanufacturing, vol. 9, no. 2, pp. 75–87, Jul. 2019.
[3] L. Ahmadi, S. B. Young, M. Fowler, R. A. Fraser, and M. A. Achachlouei, “A cascaded life cycle: Reuse of electric vehicle lithium-ion battery packs in energy storage systems,” Int. J. Life Cycle Assessment, vol. 22, no. 1, pp. 111–124, Jan. 2017.

[4] S. Tong, T. Fung, M. P. Klein, D. A. Weisbach, and J. W. Park, “Demonstration of reusing electric vehicle battery for solar energy storage and demand side management,” J. Energy Storage, vol. 11, pp. 200–210, Jun. 2017.

[5] W.-C. Lih, J.-H. Yen, F.-H. Shieh, and Y.-M. Liao, “Second use of retired lithium-ion battery packs from electric vehicles: Technological challenges, cost analysis and optimal business model,” in Proc. Int. Symp. Comput., Consum. Control, Jun. 2012, pp. 381–384.

[6] X. Gu, P. Ieromonachou, L. Zhou, and M.-L. Tseng, “Developing pricing strategy to optimise total profits in an electric vehicle battery closed loop supply chain,” J. Cleaner Prod., vol. 203, pp. 376–385, Dec. 2018.

[7] X. Zhu and L. Yu, “Screening contract excitation models involving closed-loop supply chains under asymmetric information games: A case study with new energy vehicle power battery,” Appl. Sci., vol. 9, no. 1, pp. 37–48, Mar. 2019.

[8] J. P. Xie, J. Li, and F. F. Yang, “Multi-level contract decision optimization of new energy vehicle closed loop supply chain,” J. Ind. Eng. Eng. Manage., vol. 34, no. 2, pp. 180–193, Jun. 2020.

[9] C. Chao, Q. Z. Guo, and M. S. Jian, “Remanufacturing network design for dual-channel closed-loop supply chain,” Proc. CIRP, vol. 83, pp. 73–80, Jul. 2019.

[10] P. Umangi, K. Ravi, and S. Ravi, Effect of Buyback Price on Channel’s Decision Parameters for Manufacturer-Led Close Loop Dual Supply Chain. Cham, Switzerland: Springer, 2019.

[11] I. I. Almaraj and T. B. Trafalis, “An integrated multi-echelon robust closed-loop supply chain under imperfect quality production,” Int. J. Prod. Econ., vol. 218, pp. 212–227, Dec. 2019.

[12] H. Hao, J. Zhang, and Q. Zhang, “Development countermeasures of reverse logistics of power battery recycling in China under circular economy,” Ecol. Economy, vol. 36, no. 1, pp. 86–91, Mar. 2020.

[13] J. Y. Xie, W. Yue, and B. H. Guo, “Pareto equilibrium of new energy vehicle power battery recovery based on extended producer responsibility,” Chin. J. Manage. Sci., vol. 28, no. 11, pp. 1–12, Nov. 2020.

[14] B. Zhu, B. Wen, S. Ji, and R. Qiu, “Coordinating a dual-channel supply chain with conditional value-at-risk under uncertainties of yield and demand,” Comput. Ind. Eng., vol. 139, Jan. 2020, Art. no. 106181.

[15] S. F. Alamdar, M. Rabbani, and J. Heydari, “Pricing, collection, and effort decisions with coordination contracts in a fuzzy, three-level closed-loop supply chain,” Expert Syst. Appl., vol. 104, pp. 261–276, Aug. 2018.

[16] X. Li and D. Mu, “Research on recovery pricing and coordination mechanism of power battery closed loop supply chain,” Soft. Sci., vol. 32, no. 11, pp. 124–129, Feb. 2018.

[17] H. Gu, Z. Liu, and Q. Qing, “Optimal electric vehicle production strategy under subsidy and battery recycling,” Energy Policy, vol. 109, pp. 579–589, Oct. 2017.

[18] Z. G. Qiu, Y. Zheng, and Y. Q. Xu, “Recycling subsidy strategy of power battery closed loop supply chain for new energy vehicles—Based on evolutionary game analysis,” J. Commercial, vol. 8, pp. 28–36, Oct. 2020.

[19] H. Huang, X. Zhou, and J. Zhang, “Pricing of two-way double channel closed loop supply chain for new energy vehicles under government subsidy,” Syst. Eng., vol. 38, no. 4, pp. 69–77, Sep. 2020.

[20] J. J. Liu and J. L. Ma, “Research on reverse subsidy mechanism of power battery closed loop supply chain considering cascade utilization,” Ind. Eng. Manage., vol. 26, no. 3, pp. 80–88, Jun. 2021.

[21] D. X. Fan, C. L. Li, and H. Bin, “Research on coordination of forward sales and reverse recovery contracts in dual channel closed loop supply chain,” Economy Manage., vol. 35, no. 1, pp. 85–92, May 2021.

[22] J. Heydari, K. Govindan, and A. Jafari, “Reverse and closed loop supply chain coordination by considering government role,” Transp. Res. D, Transp. Environ., vol. 52, pp. 379–398, May 2017.

[23] F. El Ouardighi, “Supply quality management with optimal wholesale price and revenue sharing contracts: A two-stage game approach,” Int. J. Prod. Econ., vol. 156, pp. 260–268, Oct. 2014.

[24] W. B. Wang, M. Zhang, and L. Zhao, “Closed loop supply chain decision model with fairness concerns of third-party Recyclers,” J. Syst. Eng., vol. 34, no. 3, pp. 409–421, Jun. 2019.

[25] L. M. Xu, S. Guo, and H. Y. Jian, “Closed loop supply chain decision making considering corporate social responsibility and advertising effect,” Chin. J. Manage., vol. 16, no. 4, pp. 615–623, Apr. 2019.

[26] H. P. Tian and C. X. Liu, “Dual objective mixed incentive model of sales channel under dual information asymmetry,” J. Manage. Sci. China, vol. 14, no. 3, pp. 34–47, Mar. 2011.

[27] D. Q. Ma and J. S. Hu, “Stochastic differential game model of closed loop supply chain with relative fairness for retailers,” Chin. J. Manage., vol. 15, no. 3, pp. 467–474, Mar. 2018.

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