Radiative deflection by spin effect
in the quantum radiation-reaction regime

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The colliding between ultra-relativistic electrons and an ultra-intense laser pulse is a powerful approach to test the physics in strong-field QED regime. By considering spin-dependent radiation-reaction during laser-electron collision we find anti-symmetric deflection of electrons with different spin states. We revealed that such deflection is induced by the non-zero work done by radiation-reaction force along field polarization-direction in a half-period of phase, which is larger for spin-anti-parallelled electrons and smaller for spin-parallelled electrons. The spin-projection on the magnetic field of an electron gets inversed in adjacent half-periods due to oscillating magnetic field and therefore the deflection due to spin-dependent radiation is accumulated rather than vanishing. The new mechanism provides an extra dimension to observe quantum radiation-reaction effect in the strong-field QED regime by measuring the anti-symmetric distribution.

The rising laser intensity enables us of investigating the dynamics of electrons in intense field, e.g. in fast ignition, laser-driven particle acceleration and bright X/gamma-ray sources. When laser intensity approaches $10^{23}$ W/cm$^2$ electron dynamics is strongly coupled with photon radiation and consequent recoil effect, which is usually referred to as the radiation-reaction (RR) effect. Such phenomenon can be further enhanced by colliding the laser with high energy electrons where RR becomes dominant as the field strength in the rest frame of the electron is boosted by a factor of $\sim \gamma$ (the gamma factor of a relativistic electron) [1].

In the QED perspective, when the spin state of an electron is considered it has been found out that the spin-anti-parallelled electron tends to radiate more energy than the spin-parallelled electron [2, 3]. In the radiation-reaction regime, spin-flip along with “quantum-jump” process were proposed to polarize an unpolarized electron beam in the collision with an elliptically polarized laser pulse, which is not possible for linearly polarized laser [4]. However, in cases where spin-flip rate is negligible, when passing through the oscillating laser field the spin projection on the magnetic field axis in the rest frame of electron oscillates with the field and the net contribution from spin is averaged to zero. Therefore it seems that no sign of spin-dependent dynamics would emerge in the laser-electron collision, considering that the Stern-Gerlach effect is too weak to play a role [2] (will be discussed later).

In this article, we will show that when radiation-reaction is coupled with spin dynamics, spin-dependent radiation causes accumulative deflection of polarized electrons and causes inhomogeneous polarity after colliding laser with an unpolarized electron bunch. The distinctive spin-dependent signature in electron dynamics can be testified in the next-generation 10-100 PW laser facilities [6-11] to account for quantum RR in the strong-field QED regime, as the change of polarity provides a new degree of freedom in addition to gamma photon emission [12], electron energy reduction [13, 14] and electron motion [15, 16]. To our knowledge the mechanism of the spin-dependent radiative deflection has not been revealed previously.

We mimic the collision process via a QED Monte-Carlo (MC) modelling as well as the classical equation of motion. The radiation power for spin-parallelled/-anti-parallelled electrons was given in Ref. [2]. The theory for arbitrarily polarized electrons is further developed in Ref. [3]. For electron polarization vector of $s$ in its rest frame, the radiation probability rate is [3]

$$\frac{dP}{d\delta \psi} = -\frac{\alpha}{6} \left[ A_{i1}(z) + g \frac{2A_{i1}'(z)}{z} + s_{\zeta} \delta A_{i1}(z) \right]$$

where $\alpha$ is the fine structure constant, $b = \hbar k \cdot p/m^2 c^2$, $A_{i1}(z)$ the Airy function, $\delta = \hbar k \cdot \hbar k' / \hbar k \cdot p$, $\psi$ the laser phase, $\hbar k$, $\hbar k'$ and $p$ the four-momentum of laser, photon and electron, $z = [\frac{\delta}{(\delta - \delta_{\max})^{2/3}} - 1 + \delta^2/[2(1 - \delta)]$, $\chi_c = e\hbar |F| \cdot p/m^3 c^4$, $F$ the electromagnetic tensor, $e$ the electron charge, $h$ the reduced Planck constant, $m$ the electron mass at rest, $c$ the speed of light and $s_{\zeta} = s \cdot \hat{B}_{\text{rest}}$, respectively. Here $\hat{B}_{\text{rest}}$ is the direction of the magnetic field in the electron’s rest frame. The dependence on spin-orientation vanishes in the collision with a symmetric laser field. For spin-flip events, the radiation probability for $dP_{\text{flip}}/d\delta \psi$ and $dP_{\text{nonflip}}/d\delta \psi$ are given in Ref. [3] where the electron undergoes $s \rightarrow -s$ in a spin-flip process.

We numerically solve the equation of motion $dp/dt =$
\(-e(E + p \times B/\gamma m)\) between photon emission events, where the latter is modelled via QED-MC algorithm [17]. Random numbers are generated to sample the radiation spectrum at each time step to determine an emission event and the photon energy. The first random number defines the probability of radiation; the second random number determines whether the radiation happens; then the spin-flip event is determined by the weights of flip and non-flip probabilities.

For classical description, we treat RR as the average of quantum photon emission. The classical RR is described by the Landau-Lifshitz equation [1] which is quantum-corrected by multiplying the classical radiation power with the spin-dependent Gaunt factor [18]

\[
g^s = \left[ 1 + 2.54(\chi_c^2 - 1.28s\chi_c) + (4.34 + 2.58s)(1 + \chi_c) \times \ln((1 + (1.98 + 0.11s)\chi_c)) \right]^{-2/3} (2)
\]

where the spin is in parallel or anti-parallel to the chosen axis \(B_{\text{rest}}\). In ultra-relativistic limit, \(F_{\text{RR}}^s \sim g^s a^2 \gamma^2\). The contribution from spin-flip process is ignored here due to low flip-rate which will be discussed later.

When QED radiation is averaged to classical radiation, the stochastic quantum effect is then omitted. In QED, electrons do not necessarily radiate the same amount of energy, larger field strength can separate electrons of different spin by the divergence of magnetic field in a laser-electron collision. However, the difference between the deflection angle of spin \(\pm\) is at the order of \(10^{-7}\) rad when \(\gamma_0 = 100, a_0 = 200\) [3], which is negligible comparing to the radiative deflection by spin effect. Therefore it is not considered throughout this article.

In our modelling, the laser propagates along the \(z\)-axis, where \(\hat{x}\) is the E-field direction and \(\hat{y}\) is the B-field direction. Electron spin is defined in this coordinate with a unit vector, e.g. \((0,\pm 1,0)\) indicates the spin is parallel/anti-parallel to the \(y\)-axis.

Head-on collision between polarized electrons and a strong laser pulse. The electron propagates along \(-z\) direction; the laser propagates along \(z\) direction with polarization along \(x\). Electrons of different initial spin polarization, i.e. parallel/anti-parallel, get deflected to opposite direction due to spin-dependent radiation-reaction. (b) The deflection angle for different field strength with \(\gamma_0 = 1000\). Curves on the right are the angular distribution of \(\pm\) polarized electrons after collision for \(a_0 = 150\) and the gray area is the difference between spin \(\pm\). Electron records are normalized in all cases. (c-d) The difference between \(\theta\) for \(w_0 = 2\) and Inf in the \(a_0 - \gamma_0\) parametric space. The white line in (d) is the average along \(\gamma_0\). (e) The standard deviation of deflection angles which reflects the spread of the scattered electron shown by the double-arrow in (b).

We find that electrons of opposite spin-orientation \((\pm)\) tend to be scattered to opposite directions (Fig. 1(a)) via QED-MC. For reasons to be shown later, one will find more electrons of parallel/anti-parallel polarization in the upper \((\theta > 0)\) / lower \((\theta < 0)\) region, resulting in slight asymmetry of electron distribution along \(\theta\) in Fig. 1(b). We qualify such asymmetric distribution by defining the averaged deflection angle \(\langle \theta \rangle\). The phenomenon is investigated in a large parameter regime as shown in Fig. 1(b-e). We will focus on the \(\chi_e < 1\) region, e.g. Fig. 1(b) where pair-production [23] is suppressed [24]. The averaged deflection angle is calculated by repeating the collection for \(10^6\) times in Fig. 1(b) and \(10^5\) times in Fig. 1(c-e) for each set of parameters. For fixed electron energy, larger field strength can separate electrons of different initial spin-orientation to larger angles, as shown in Fig. 1(b), while the deflection angle of spin-free electrons stays near zero as \(\gamma_0 \gg a_0\). The deflection an-
The ponderomotive scattering and pulse shape contribute to the deflection angle. To reveal the net effect by spin-dependent radiation, we perform the collision for a flat-top pulse without beam waist. We focus on the plateau region where the electron flips and spin-dependent Gaunt factor is employed so that the rising/falling edge. Here the classical equation of motion with spin-dependent Gaunt factor is employed so that the time-resolved dynamics are not influenced by stochastic effects from QED-MC.

The mechanism shown by Fig. 2(a) is directly proved in Fig. 2(d) by colliding electron of $\gamma_0 = 1000$ with flat-top pulse of $a_0 = 150$ and infinite beam size, where the difference between $\Delta p^+_x$ in each half-period are presented (black and red-dotted bars). One can see $\Delta p^+_x$ becomes relative to spin-free electron can be defined by $s \times k$ in the plane-wave approximation.

FIG. 2. An electron collides with a flat-top laser pulse where the plateau region is $-5 < \psi/2\pi < 5$ for spin $\pm$ (red/blue) and spin-free (black). (a) Electron trajectories (curves) and momentum change due to RR in $x$-direction in half-period ($\Delta p_x$ and arrows). The lengths of arrows indicate the length of the vector. (b-c) The variation of vector $\Delta p_{RR}^x$ along the collision time steps for adjacent half-periods (“×” for (b), “·” for (c)). Integrate the vectors along $x$ and one will get $\Delta p_x^\pm$ shown in (a). (d) The electron trajectories of initially $+/-$ polarized electron (red/blue lines) and the difference between $\Delta p_x^\pm$ accumulated in half-periods for classical radiation (black bars) and QED-MC radiation (red-dotted bars).
The difference between $p_x^\pm$ is shown by the black line. (b, d) Accumulated momentum change from RR of ± polarized electrons, i.e. $p_x^{RR\pm}$ (red/blue line). The difference between $p_x^{RR\pm}$ is shown by the cyan line in (a) & (c).

greater than $\Delta p_x^-$ after the plateau begins ($\psi/2\pi > -5$) and the electron is deflected due to such momentum change. The deflection direction is along the $s \times k$ axis, which is exactly as predicted with our model in Fig. 2(a) in the plane-wave approximation. We note that in the rising edge of the pulse ($-10 < \psi/2\pi < -5$), the momentum change from RR is inversed, $\Delta p_x^+ < \Delta p_x^-$, as shown by the negative black bars. In this case, the increase of the field strength in the rising edge dominates over the correction from spin-dependent emission in terms of the RR force, therefore $F_{RR}^{\text{fall}}$ overrides $F_{RR}^{\text{rise}}$. The net work in $x$-direction is opposite to that in the plateau region $\int_{\text{fall}} df_{RR}^{\pm} \hat{\mathbf{x}} > \int_{\text{rise}} df_{RR}^{\pm} \hat{\mathbf{x}}$. Accordingly, in the falling edge we have $\Delta p_x^+ > \Delta p_x^-$ again. The counter-play between the field strength variation and the damping of $\gamma$ can be clearly seen from $F_{RR} \sim a^2 \gamma^2$.

For further analysis of the ponderomotive effect, the electron momentum and momentum change from radiation are presented in Fig. 3 by colliding the electron with a flat-top laser of beam waist $w_0 = 2\lambda$ and infinite beam waist. In Fig. 3(a) and (c), one can see that $p_x$ oscillates with constant amplitude during the plateau while the difference between $+(\text{red})$ and $-(\text{blue})$ gradually builds up (black). Such difference grows much quicker with the help of ponderomotive force ($\sim -\nabla E^2$) by comparing Fig. 3(a) to (c). Fig. 3(b) and (d) shows the accumulated momentum change due to $F_{RR}$ along the $x$-direction, i.e. $p_x^{RR\pm}$; the differences between $+(\text{red})$ and $-(\text{blue})$ are presented in Fig. 3(a) and (c) (cyan).

The coincidence between the black and cyan lines in Fig. 3(a) suggests the deflection is further amplified by ponderomotive scattering. The deflection angle becomes large enough due to ponderomotive enhancement such that an electron radiates more energy upwards/downwards if it is going up/down, as seen from the flipped momentum loss in Fig. 3(b).

Now that the electron with opposite spin-orientation tend to be scattered to opposite directions in a collision with laser pulse, the averaged spin polarization therefore becomes inhomogeneous as a function of the scattered angle. We consider an unpolarized electron bunch of $\gamma_0 = 1000$ with transverse size of $4\lambda$ and $12\lambda$ at $e^{-1}$ with energy spread of $\sim 1\%$ and angular divergence of $10\%$, then the polarization of electrons is measured along $y$-direction for different scattering angle. The radiation is treated in QED-MC manner without spin-flip due to the low flip-rate; we use a focused laser pulse $\mathbf{E}$ with FWHM of $27\text{fs}$ and beam waist of $2\lambda$ at $e^{-1}$. In a focused pulse the spin precession needs to be considered. Spin-vector precession is calculated by solving the Thomas-Bargmann-Michel-Telegdi equation [27] $ds/dt = \frac{\gamma}{mc} \mathbf{s} \times [(\hat{\mathbf{s}} \cdot \mathbf{B}) - (\hat{\mathbf{r}} - 1 + \frac{1}{\gamma^2})B - (\hat{\mathbf{r}} - 1 + \frac{1}{\gamma^2})\mathbf{B} \cdot \mathbf{E}]$, where $\alpha_e = \frac{e^2}{2m_e c^2} \approx 1.16 \times 10^{-3}$ is the anomalous magnetic moment of electron [28]. The inhomogeneous polarization along the $y$-axis is presented in Fig. 4. The bunch get polarized aligned $y$ because the electrons with positive $s_y$ tend to be scattered upwards while those with negative $s_y$ downwards. Experimental measurement of this antisymmetric phenomenon could provide an extra dimension to observe quantum RR effect in addition to QED gamma photon emission [12], electron energy reduction [13, 14] and quantum electron motion [15, 16]. Such phenomenon is not significantly disturbed by the energy spread and angular divergence of the electron bunch at
large angles (> angular divergence). One can always observe the inhomogeneous polarization by comparing the red-dot-dashed to the black-solid lines in Fig. 4. However, large transverse size of the electron bunch could wipe out the signal of $\langle s_y \rangle$ at small angles (∼ angular divergence) shown by the blue-dashed line in Fig. 4 because the electrons not interacting with the laser lower the $\langle s_y \rangle$ value at small angles.

In conclusion, we investigate the collision of initially polarized electron and laser pulse and find the deflection due to spin-dependent radiation-reaction. We model such deflection with non-zero momentum loss during half-period where the momentum losses divert between parallel and anti-parallel spin-state. Such momentum shift is non-vanishing during the collision because if an electron radiating more upwards during some half-period, it will radiate less downwards during next half-period, which induces electron to be deflected downwards, and visa versa. The spin-dependent deflection causes inhomogeneous polarity after collision between an unpolarized electron bunch and an intense laser pulse, which could provide an alternative to observing the quantum radiation-reaction in the strong-field QED regime.

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1. L. D. Landau and E. M. Lifshitz, The Classical Theory of Fields (Pergamon Press, Oxford, 1971).
2. I. M. Ternov, Physics-Uspekhi 38, 409 (1995).
3. D. Seipt, D. Del Sorbo, C. P. Ridgers, and A. G. R. Thomas, Phys. Rev. A 98, 023417 (2018).
4. Y.-F. Li, R. Shaisultanov, K. Z. Hatsagortsyan, F. Wan, C. H. Keitel, and J.-X. Li, (2018), arXiv:1812.07229 [physics.plasm-ph].
5. M. Wen, C. H. Keitel, and H. Bauske, Physical Review A 95 (2017), 10.1103/PhysRevA.95.042102.
6. B. Shen, Z. Bu, J. Xu, T. Xu, L. Ji, R. Li, and Z. Xu, Plasma Physics and Controlled Fusion 60, 044002 (2018).
7. Extreme Light Infrastructure European Project, www.eli-laser.eu
8. Exawatt Center for Extreme Light Studies, www.xcelsiaprasu
9. Apollon multi-PW laser Users Facility, www.polytech.nique.edu
10. The Vulcan 10-PW project, www.clf.stfc.ac.uk
11. R. Li, X. Liang, Y. Leng, and Z. Xu, in 1st AAPPS-DPP meeting (2017) 1st AAPPS-DPP meeting.
12. T. N. Wistisen, A. Di Piazza, H. V. Knudsen, and U. I. Uggerhoj, Nature Communications 9, (2018).
13. J. M. Cole, K. T. Behm, E. Gerstmayr, T. G. Blackburn, J. C. Wood, C. D. Baird, M. J. Duff, C. Harvey, A. Iderton, A. S. Joglekar, K. Krushelnick, S. Kuschel, M. Marklund, P. McKenna, C. D. Murphy, K. Poder, C. P. Ridgers, G. M. Samarin, G. Sarri, D. R. Symes, A. G. R. Thomas, J. Warwick, M. Zepf, Z. Najmudin, and S. P. D. Mangles, Physical Review X 8, 11 (2018).
14. K. Poder, M. Tamburini, G. Sarri, A. Di Piazza, S. Kuschel, C. D. Baird, K. Behm, S. Bohlen, J. M. Cole, D. J. Corvan, M. Duff, E. Gerstmayr, C. H. Keitel, K. Krushelnick, S. P. D. Mangles, P. McKenna, C. D. Murphy, Z. Najmudin, C. P. Ridgers, G. M. Samarin, D. R. Symes, A. G. R. Thomas, J. Warwick, and M. Zepf, Phys. Rev. X 8, 031004 (2018).
15. C. Harvey, A. Gonoskov, A. Iderton, and M. Marklund, Physical Review Letters 118, 105004 (2017).
16. X. S. Geng, L. L. Ji, B. F. Shen, B. Feng, Z. Guo, Q. Yu, L. G. Zhang, and Z. Z. Xu, (2018), arXiv:1811.04741 [physics.plasm-ph].
17. A. Gonoskov, S. Bastrakov, E. Efimenko, A. Iderton, M. Marklund, I. Meyerov, A. Muravev, A. Sergeev, L. Surmin, and E. Wallin, Phys Rev E Stat Nonlin Soft Matter Phys 92, 023305 (2015).
18. D. Del Sorbo, D. Seipt, A. G. R. Thomas, and C. P. Ridgers, Plasma Physics and Controlled Fusion 60 (2018), 10.1088/1361-6587
19. N. Neitz and A. Di Piazza, Phys Rev Lett 111, 054802 (2013).
20. M. Vranic, T. Grismayer, R. A. Fonseca, and L. O. Silva, New Journal of Physics 18, (2016).
21. J. X. Li, Y. Y. Chen, K. Z. Hatsagortsyan, and C. H. Keitel, Sci Rep 7, 11556 (2017).
22. C. P. Ridgers, T. G. Blackburn, D. Del Sorbo, L. E. Bradley, C. Slade-Lowther, C. D. Baird, S. P. D. Mangles, P. McKenna, M. Marklund, C. D. Murphy, and A. G. R. Thomas, Journal of Plasma Physics 83, (2017).
23. G. Breit and J. A. Wheeler, Physical Review 46, 1087 (1934).
24. V. Ritus, J. Russ. Laser Res. 6, 497 (1985).
25. $\Delta p$ is used for convenience.
26. Y. I. Salamin and C. H. Keitel, Phys Rev Lett 88, 095005 (2002).
27. J. D. Jackson, Classical Electrodynamics (3rd ed.) (John Wiley & Sons, New York, 1999).
28. J. Schwinger, Phys. Rev. 73, 416 (1948).