QCD SUM RULE STUDY FOR A POSSIBLE CHARMED PENTAQUARK $\Theta_c(3250)$

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We use QCD sum rules to study the possible existence of a $\Theta_c(3250)$ charmed pentaquark. We consider the contributions of condensates up to dimension-10 and work at leading order in $\alpha_s$. We obtain $m_{\Theta_c} = (3.21 \pm 0.13) \text{ GeV}$, compatible with the mass of the structure seen by BaBar Collaboration in the decay channel $B^- \to \bar{p} \Sigma^{++} \pi^- \pi^-$. The proposed state is compatible with a previous proposed pentaquark state in the anti-charmed sector.

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I. INTRODUCTION

Recently, the BaBar Collaboration has reported [1] the observation of unexplained structures in the $B^- \to \bar{p} \Sigma^{++} \pi^- \pi^-$ decay channel. In particular, they observed three enhancements in the $\Sigma^{++} \pi^- \pi^-$ invariant mass distribution at 3.25 GeV, 3.80 GeV and 4.20 GeV [1]. We shall refer to these signals $\Theta_c(3250), \Theta_c(3800)$ and $\Theta_c(4200)$, respectively. There are already theoretical calculations interpreting the $\Theta_c(3250)$ enhancement as a possible $D^*_0(2400) N$ molecular state [2, 3]. In this note we follow a different approach, and we use the QCD sum rules (QCDSR) [4–6] to try to interpret $\Theta_c(3250)$ enhancement as a charmed pentaquark.

There are already some calculations for charmed pentaquarks. Based on simple theoretical considerations, Diakonov has predicted the masses of the exotic anti-decapenta-plet of charmed pentaquarks [7]. In his model, the lightest members of this multiplet are explicitly exotic doublets, $cuud\bar{s}$ and $cudd\bar{u}$, with mass about 2.42 GeV. The crypto-exotic $cudd\bar{u}$ pentaquark should have a mass around 140 MeV heavier. Since the accuracy of this prediction is $\sim 150$ MeV, Diakonov’s prediction for the mass of the $cudd\bar{u}$ pentaquark is $\sim 50$ MeV smaller than the observed enhancement. Using the Skyrme soliton model Wu and Ma have studied the exotic pentaquark states with charm and anti-charm [8]. In their approach, they obtained a mass around 2.70 GeV for both $cudd\bar{u}$ and $uudd\bar{c}$ states.

The first QCDSR calculation for a possible anti-charmed pentaquark was done in Ref. [9]. The authors have found a mass around 3.10 GeV, supposing that the anti-charmed pentaquark can be described by a current with two-light diquarks and one anti-charm quark. Since for a charmed $cudd\bar{u}$ pentaquark one needs a light diquark, a heavy-light diquark and a light antiquark to describe it, and since light diquarks are supposed to be very bound states [10] and heavy-light diquarks less bound [11], we expect the mass of the charmed pentaquark to be bigger than the mass of the anti-charmed pentaquark and, therefore, compatible with the observed $\Theta_c(3250)$ enhancement.

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II. TWO-POINT CORRELATION FUNCTION

A possible current describing a charmed neutral pentaquark with quark content $[cudd\bar{u}]$, which we call $\Theta_{1c}$, is given by:

$$\eta_{1c} = e^{abc}(e^{aef} u^c_e C \gamma_5 d_f) (e^{bgh} c^T g C \gamma_5 d_h) C \gamma_5 \bar{u}^T_c,$$  \hfill (1)

where $a, b, \ldots$ are color indices, $C$ is the charge conjugation matrix and in bold letters are the respective quark fields. We have considered two scalar diquarks since they are supposed to be more bound than the pseudoscalars $\Theta$. However, since the study presented in Ref. \[10\] is related with the light diquarks, one could also have a current describing another pentaquark, $\Theta_{2c}$, with a scalar light-diquark and a pseudoscalar heavy-light-diquark as follows:

$$\eta_{2c} = e^{abc}(e^{aef} u^c_e C \gamma_5 d_f) (e^{bgh} c^T g C d_h) C \bar{u}^T_c,$$  \hfill (2)

like the current used in Ref. \[8\] for $\Theta$. The sum rule for both currents (1) and (2) is constructed from the two-point correlation

$$\Pi(q) = i \int d^4 x e^{i q \cdot x} \langle 0 | T [\eta_c(x) \bar{\eta}_c(0)] | 0 \rangle = \Pi_1(q^2) + \frac{1}{2} \Pi_2(q^2),$$  \hfill (3)

where $\Pi_1$ and $\Pi_2$ are two invariant independent functions. In the phenomenological side, we parametrize the spectral function using the standard duality ansatz: "one resonance" + "QCD continuum". The QCD continuum starts from a threshold $s_0$ and comes from the discontinuity of the QCD diagrams. Transferring its contribution to the standard duality ansatz: 

$$|\lambda_{\eta_c}|^2 m_{\eta_c} e^{-m_{\eta_c}^2/M_B^2} = \int_{m_c^2}^{s_0} ds e^{-s/M_B^2} \rho_1(s),$$

$$|\lambda_{\eta_c}|^2 e^{-m_{\eta_c}^2/M_B^2} = \int_{m_c^2}^{s_0} ds e^{-s/M_B^2} \rho_2(s),$$  \hfill (4)

where $\rho_i = \frac{1}{\pi} \text{Im} \Pi_i(s)$ are the spectral densities whose expressions are given in the Appendix. In Eq. (4), $\lambda_{\eta_c}$ and $m_{\eta_c}$ are the pentaquark residue and mass, respectively; $M_B^2$ is the sum rule variable. One can estimate the pentaquark mass from the following ratios

$$\mathcal{R}_i = \frac{\int_{m_c^2}^{s_0} ds e^{-s/M_B^2} \rho_i(s)}{\int_{m_c^2}^{s_0} ds e^{-s/M_B^2} \rho_1(s)}, \quad i = 1, 2,$$

$$\mathcal{R}_{12} = \frac{\int_{m_c^2}^{s_0} ds e^{-s/M_B^2} \rho_1(s)}{\int_{m_c^2}^{s_0} ds e^{-s/M_B^2} \rho_2(s)},$$  \hfill (5)

where at the $M_B^2$-stability point, we have

$$m_{\eta_c} \simeq \sqrt{\mathcal{R}_i} \simeq \mathcal{R}_{12}.$$  \hfill (6)

III. NUMERICAL RESULTS

For a consistent comparison with the results obtained for other pentaquark states using the QCDSR approach, we have considered the same values used for the heavy quark mass and condensates as in Ref. \[10\], listed in Table I. It is worth mentioning that, for both currents $\eta_{1c}$ and $\eta_{2c}$, we have found a substantial $M_B^2$-instability in the $\mathcal{R}_{12}$ sum rule evaluation. Therefore, in this work, we will only consider the results from $\mathcal{R}_i$.

A. $\Theta_{2c}$ Pentaquark State

We start our analysis with the current $\eta_{2c}$. As mentioned above, we calculate the mass related to this current using only the results from the $\mathcal{R}_1$ and $\mathcal{R}_2$ sum rules.
considering the $R_2$ sum rule, we show in Fig. (a) the relative contributions of the terms in the OPE, for $\sqrt{s_0} = 4.70$ GeV. From this figure, we see that the contribution of the dimension-10 condensate is smaller than 20% of the total contribution for values of $M_B^2 \geq 2.7$ GeV$^2$, which indicates the starting point for a good OPE convergence. In Fig. (b), we also see that the pole contribution is bigger than the continuum contribution only for values $M_B^2 \leq 3.1$ GeV$^2$. Therefore, we can fix the Borel window as: $(2.7 \leq M_B^2 \leq 3.1)$ GeV$^2$. From Eq. (6), we can estimate the ground state mass, which is shown, as a

\begin{table}[h]
\centering
\caption{QCD input parameters.}
\begin{tabular}{|c|c|}
\hline
Parameters & Values \\
\hline
$m_c$ & $(1.23 - 1.47)$ GeV \\
$\langle \bar{q}q \rangle$ & $-(0.23 \pm 0.03)^3$ GeV$^3$ \\
$\langle g_s^2 G^2 \rangle$ & $(0.88 \pm 0.25)$ GeV$^4$ \\
$\langle g_s^3 G^3 \rangle$ & $(0.58 \pm 0.18)$ GeV$^6$ \\
$m_0^2 \equiv \langle \bar{q}Gq \rangle/\langle \bar{q}q \rangle$ & $(0.8 \pm 0.1)$ GeV$^2$ \\
\hline
\end{tabular}
\end{table}
function of $M^2_B$, in Fig. 1). We conclude that there is a very good $M^2_B$-stability in the determined Borel window, which is indicated through the parenthesis.

Varying the value of the continuum threshold in the range $\sqrt{s_0} = 4.70 \pm 0.10$ GeV, and other parameters as indicated in Table I, we get

$$m_{\Theta^{c}_{2}} = 4.15 \pm 0.11 \text{GeV}.$$  \hspace{1cm} (7)

This mass is surprisingly compatible with one of the unexplained structures observed by BaBar Collaboration [1] at 4.2 GeV. Therefore, from a sum rule point of view, such a $\Theta^{c}_{2}$, pentaquark state with an internal structure composed by a scalar light-diquark and a pseudoscalar heavy-light-diquark could be a good candidate to explain the $\Theta^{c}(4200)$ enhancement.

For completeness, we evaluate the $R_1$ sum rule for the current $\eta^{c}_{2}$. We would naively expect to obtain a mass in accordance with Eq. (7). The comparison between the two sum rules is shown in Fig. 2, considering the Borel range $(2.0 \leq M^2_B \leq 6.0)$ GeV$^2$ and $\sqrt{s_0} = 4.70$ GeV. As one can see, the $R_2$ sum rule presents a better $M^2_B$-stability than $R_1$. Besides, the Borel window for the $R_1$ sum rule lies on the range $(2.5 \leq M^2_B \leq 3.4)$ GeV$^2$ which does not contain $M^2_B$-stability. Thus, we conclude that the results extracted from the $R_1$ can be ruled out, while the $R_2$ provides a more reliable sum rule calculation and the mass found in Eq. (7) must be settled as the optimized estimation for the $\Theta^{c}_{2}$ pentaquark mass.

B. $\Theta^{c}_{1c}$ Pentaquark State

In the case of the current $\eta^{c}_{1c}$, we also retain only the results from the $R_2$ sum rule, according to the previous analysis. The results for the pole dominance and OPE convergence are shown in Fig. 3 a) and b), respectively. From these figures, we can fix the Borel window as: $(2.3 \leq M^2_B \leq 2.5)$ GeV$^2$, for $\sqrt{s_0} = 3.90$ GeV. As one can see, from the Fig. 3 c), we obtain $M^2_B$-stability only for a narrow Borel window. However, it is still possible to extract reliable results from this sum rule. Varying the continuum threshold in the range $\sqrt{s_0} = 3.90 \pm 0.10$ GeV, and the other parameters as indicated in Table I, we get

$$m_{\Theta^{c}_{1c}} = 3.21 \pm 0.13 \text{GeV}.$$  \hspace{1cm} (8)

This value for the mass is compatible with the first signal observed in Ref. [1] at 3.25 GeV. Therefore, we conclude that the $\Theta^{c}_{c}(3250)$ state also can be described by a pentaquark containing two scalar diquarks in its internal structure. It is very interesting to notice that we get a smaller mass with the current with two scalar diquarks, when compared with the current with one scalar and one pseudoscalar diquarks. Although we have one light and one light-heavy diquarks, our results follow the phenomenology obtained by Shuryak [10], for the light diquarks.
FIG. 3: $\mathcal{R}_2$ sum rule analysis using the pentaquark current $\eta_1 c$. We have considered contributions up to dimension-10 in the OPE, using $m_c = 1.23$ GeV. a) OPE convergence in the region $(1.8 \leq M_B^2 \leq 3.2)$ GeV$^{-2}$ for $\sqrt{s_0} = 3.90$ GeV. We plot the relative contributions starting with the perturbative contribution and each other line represents the relative contribution after adding of one dimension in the OPE expansion. b) The relative pole and continuum contributions for $\sqrt{s_0} = 3.90$ GeV. c) The mass as a function of the sum rule parameter $M_B^2$, for different values of $\sqrt{s_0}$. For each line, the region bounded by parenthesis indicates a valid Borel window.

It is interesting to compare our results with the result in Ref. [3], where the author evaluates the sum rule for the $D^*_0(2400)$ $N$ molecule, since such a molecular current can be rewritten in terms of a sum over pentaquark type currents, by using Fierz transformations [13]. Indeed, the result found in Ref. [3] is in agreement with our result in Eq. (8), which was obtained with the current in Eq. (1). However, there are some points in the analysis done in Ref. [3] that deserve consideration. In particular, to obtain a mass compatible with the 3.25 GeV enhancement observed by BaBar, the author of Ref. [3], had to release the criteria of pole dominance and the usual good OPE convergence. In doing so, the analysis inevitably led to a misleading definition of the Borel window, fixed as $(2.0 \leq M_B^2 \leq 3.0)$ GeV$^2$ for the $D^*_0(2400)$ $N$ molecule. Besides, one can see that there is also no $M_B^2$-stability in such Borel window. Therefore, we believe that if the author of Ref. [3] had imposed pole dominance, good OPE convergence and Borel stability in his analysis he would have obtained a bigger value for the mass of the $D^*_0(2400)$ $N$ current.

IV. CONCLUSIONS

In conclusion, we have presented a QCDSR calculation for the two-point function of two possible pentaquark states, whose internal structure is composed of two scalar diquarks, for $\Theta_1 c$, and a scalar light-diquark plus a pseudoscalar heavy-light-diquark, for $\Theta_2 c$. As expected from phenomenology [10], we
get a smaller mass with the current \( \eta_{1c} \) containing two scalar diquarks, in comparison with the current \( \eta_{2c} \) containing one scalar and one pseudoscalar diquarks. Also, we get a bigger mass for the \( \Theta_{2c} \) pentaquark state when comparing with the one studied in Ref. \( [9] \), where the authors considered for the \( \Theta_c \) state a current with two-light diquarks and one anti-charm quark. Indeed, this result is in agreement with the expectation that heavy-light diquarks are less bound than light diquarks \( [11] \). Our findings strongly suggest that at least two enhancements observed by BaBar Collaboration, with a peak at 3.25 GeV and 4.20 GeV, decaying into \( \Sigma_c^{++} \pi^- \pi^- \), could be understood as being such pentaquarks.

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Appendix A: Spectral Densities

The spectral densities expressions for the charmed neutral pentaquarks, \( \Theta_{1c} \) and \( \Theta_{2c} \), described by the currents in Eq. (1) and (2) respectively, have been calculated up to dimension-10 condensates, at leading order in \( \alpha_s \). To keep the heavy quark mass finite, we use the momentum-space expression for the heavy quark propagator. We calculate the light quark part of the correlation function in the coordinate-space, and we use the Schwinger parameters to evaluate the heavy quark part of the correlator. To evaluate the \( d^4x \) integration in Eq. (3), we use again the Schwinger parameters, after a Wick rotation. Finally we get integrals in the Schwinger parameters. The result of these integrals are given in terms of logarithmic functions, from where we extract the spectral densities and the limits of the integration. The same technique can be used to evaluate the condensate contributions.

For the \( \bar{q}q \)-structure of the correlation function (3), we get:

\[
\rho_{2}^{\text{pert}}(s) = -\frac{1}{5^2 \cdot 3^2 \cdot 2^{15}} \int_0^\Lambda d\alpha \frac{\alpha^3 H_2^5}{(1-\alpha)^4},
\]

\[
\rho_{2}^{(\bar{q}q)}(s) = (1)^{j+1} \frac{m_c \langle \bar{q}q \rangle}{3^2 \cdot 2^{11}} \int_0^\Lambda d\alpha \frac{\alpha^4 H_2^3}{(1-\alpha)^3},
\]

\[
\rho_{2}^{(G^2)}(s) = -\frac{\langle g^2 G^2 \rangle}{5 \cdot 3^2 \cdot 2^{21}} \int_0^\Lambda d\alpha \frac{\alpha^3 H_2^2}{(1-\alpha)^4} \left[ 32 m_c^2 \alpha^2 + 5 H_2 (1-\alpha)(52-33\alpha) \right],
\]

\[
\rho_{2}^{(\bar{q}Gq)}(s) = (1)^{j+1} \frac{m_c \langle \bar{q}Gq \rangle}{3 \cdot 2^{15}} \int_0^\Lambda d\alpha \frac{\alpha^3 H_2^2}{(1-\alpha)^3} (19-23\alpha),
\]

\[
\rho_{2}^{\bar{q}q^2}(s) = \frac{\langle \bar{q}q \rangle^2}{3 \cdot 2^7} \int_0^\Lambda d\alpha \frac{\alpha^2 H_2^2}{1-\alpha},
\]

\[
\rho_{2}^{(G^3)}(s) = -\frac{\langle g^3 G^3 \rangle}{5 \cdot 3^2 \cdot 2^{20}} \int_0^\Lambda d\alpha \frac{\alpha^4 H_2}{(1-\alpha)^4} \left[ 4 m_c^2 (95-91\alpha) + H_2 (285-281\alpha) \right],
\]

\[
\rho_{2}^{(\bar{q}G^2)}(s) = (1)^{j+1} \frac{m_c \langle \bar{q}G^2 \rangle}{3 \cdot 2^{15}} \int_0^\Lambda d\alpha \frac{\alpha^2 H_2^2}{(1-\alpha)^3} \left[ 4 m_c^2 \alpha^2 + 3 H_2 (49-\alpha(119-74\alpha)) \right],
\]

\[
\rho_{2}^{(\bar{q}Gq)}(s) = \frac{\langle \bar{q}q \rangle \langle Gq \rangle}{3 \cdot 2^{12}} \int_0^\Lambda d\alpha \frac{\alpha H_2}{(1-\alpha)} (70-73\alpha),
\]
\[ \rho_2^{(gq)} (s) = (-1)^{j+1} \frac{m_c \langle \bar{q} q \rangle}{3^2 \cdot 2^7 \pi^3} \int_0^\Lambda d\alpha \frac{\alpha^3 H_\alpha}{(1-\alpha)^3}, \]

\[ \rho_2^{(G^2) (\bar{q} G q)} (s) = (-1)^{j+1} \frac{m_c \langle g^2 G^2 \rangle \langle \bar{q} G q \rangle}{3^3 \cdot 2^7 \pi^6} \left\{ \int_0^\Lambda d\alpha \frac{3\alpha}{(1-\alpha)^2} (39-\alpha(89-66\alpha)) - \int_0^\Lambda d\alpha \frac{16m_c^2 \alpha^3}{(1-\alpha)^3} \delta(s-m_c^2) \right\}, \]

\[ \rho_2^{(\bar{q} G q) (G^3)} (s) = (-1)^{j+1} \frac{m_c \langle \bar{q} q \rangle \langle \bar{q}^3 G^3 \rangle}{3^3 \cdot 2^{13} \pi^6} \left\{ \int_0^\Lambda d\alpha \frac{\alpha^3}{(1-\alpha)^2} (9-8\alpha) - \int_0^\Lambda d\alpha \frac{m_c^2 \alpha^3}{(1-\alpha)^2} (6-5\alpha) \delta(s-m_c^2) \right\}, \]

\[ \rho_2^{(\bar{q} G q)^2} (s) = \frac{\langle \bar{q} G q \rangle^2}{3^3 \cdot 2^{13} \pi^4} \int_0^\Lambda d\alpha (57-70\alpha), \]

\[ \rho_2^{(G^2) (\bar{q} G q)^2} (s) = \frac{\langle g^2 G^2 \rangle \langle \bar{q} q \rangle^2}{3^3 \cdot 2^{13} \pi^4} \left\{ \int_0^\Lambda d\alpha (91-61\alpha) - \int_0^\Lambda d\alpha \frac{16m_c^2 \alpha^2}{(1-\alpha)^2} \delta(s-m_c^2) \right\} \]

where the integration limit is given by \( \Lambda = 1 - m_c^2 / s \). We also have used the definition \( H_\alpha = m_c^2 - (1-\alpha)s \), and \( j = 1, 2 \) for the currents \( \eta_{1c} \) and \( \eta_{2c} \), respectively.

For the 1-structure, we get:

\[ \rho_1^{pert} (s) = 0, \]

\[ \rho_1^{(gq)} (s) = (-1)^{j+1} \frac{\langle \bar{q} q \rangle}{3^2 \cdot 2^{11} \pi^6} \int_0^\Lambda d\alpha \frac{\alpha^3 H_\alpha^4}{(1-\alpha)^3}, \]

\[ \rho_1^{(G^2)} (s) = 0, \]

\[ \rho_1^{(\bar{q} G q)} (s) = (-1)^{j+1} \frac{\langle \bar{q} G q \rangle}{3 \cdot 2^{11} \pi^6} \int_0^\Lambda d\alpha \frac{\alpha^2 H_\alpha^3}{(1-\alpha)^2}, \]

\[ \rho_1^{(\bar{q} q)^2} (s) = -\frac{m_c \langle \bar{q} q \rangle}{3 \cdot 2^7 \pi^4} \int_0^\Lambda d\alpha \frac{\alpha^2 H_\alpha^2}{(1-\alpha)^2}, \]

\[ \rho_1^{(G^2)} (s) = 0, \]

\[ \rho_1^{(\bar{q} q) (G^2)} (s) = (-1)^{j+1} \frac{\langle \bar{q} q \rangle \langle g^2 G^2 \rangle}{3^3 \cdot 2^{17} \pi^6} \int_0^\Lambda d\alpha \frac{\alpha H_\alpha}{(1-\alpha)^3} \left[ 64m_c^2 \alpha^2 + 3H_\alpha (142 - 85\alpha) \right], \]

\[ \rho_1^{(\bar{q} q) (\bar{q} G q)} (s) = -\frac{m_c \langle \bar{q} q \rangle \langle \bar{q} G q \rangle}{3 \cdot 2^{11} \pi^4} \int_0^\Lambda d\alpha \frac{\alpha H_\alpha}{(1-\alpha)^2} (35 - 43\alpha), \]

\[ \rho_1^{(\bar{q} q)^3} (s) = (-1)^{j+1} \frac{\langle \bar{q} q \rangle^3}{3^2 \cdot 2^9 \pi^2} \int_0^\Lambda d\alpha H_\alpha, \]

\[ \rho_1^{(G^2) (\bar{q} G q)} (s) = (-1)^{j+1} \frac{\langle g^2 G^2 \rangle \langle \bar{q} G q \rangle}{3^3 \cdot 2^{17} \pi^6} \int_0^\Lambda d\alpha \frac{1}{(1-\alpha)^2} \left[ 16m_c^2 \alpha^2 + 3H_\alpha (13 + 6\alpha) \right], \]

\[ \rho_1^{(\bar{q} q) (G^3)} (s) = (-1)^{j+1} \frac{\langle \bar{q} q \rangle \langle g^3 G^3 \rangle}{3^3 \cdot 2^{15} \pi^6} \int_0^\Lambda d\alpha \frac{\alpha^2}{(1-\alpha)^3} \left[ 2m_c^2 (57 - 53\alpha) + H_\alpha (171 - 167\alpha) \right], \]

\[ \rho_1^{(\bar{q} G q)^2} (s) = -\frac{m_c \langle \bar{q} G q \rangle^2}{3 \cdot 2^{13} \pi^4} \int_0^\Lambda d\alpha \frac{19 - 35\alpha}{1 - \alpha}, \]
\rho_1^{(G^2)\langle \bar{q}q \rangle^2}(s) = \frac{-m_c \langle g^2 G^2 \rangle \langle \bar{q}q \rangle^2}{3^3 \cdot 2^{12} \pi^4} \left\{ \int_0^\Lambda d\alpha \frac{65 - \alpha(193 - 152\alpha)}{(1 - \alpha)^2} - \int_0^1 d\alpha \frac{8m_c^2 \alpha^2}{(1 - \alpha)^3} \delta \left( s - \frac{m_c^2}{1 - \alpha} \right) \right\}.

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