QCD field strength correlator at the next-to-leading order

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Abstract

The gauge invariant two-point correlation function of the gauge field strength tensor is calculated in perturbation theory at the next-to-leading order. Besides a direct calculation in perturbative QCD we also present a derivation of the correlation function in the heavy quark effective theory. Our results are briefly compared with recent determinations of the field strength correlator on the lattice.

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1 Introduction

The gauge invariant two-point correlation function of the QCD field strength tensor $F^{a}_{\mu\nu}(x)$ in the adjoint representation can be defined as

$$D_{\mu\nu\lambda\omega}(z) \equiv \langle 0 | T \{ F^{a}_{\mu\nu}(y) \mathcal{P} e^{g f^{abc} z^\tau \int_{0}^{1} d\sigma A^{c}_{\lambda\omega}(x + \sigma z) F^{b}_{\lambda\omega}(x) } \} | 0 \rangle \quad (1.1)$$

where the field strength $F^{a}_{\mu\nu} = \partial_{\mu} A^{a}_{\nu} - \partial_{\nu} A^{a}_{\mu} + g f^{abc} A^{b}_{\mu} A^{c}_{\nu}$, $z = y - x$ and $\mathcal{P}$ denotes path ordering of the exponential. In general, the gauge invariant field strength correlator could be defined with an arbitrary gauge string connecting the end points $x$ and $y$. To simplify the calculation, however, in this work we shall restrict ourselves to a straight line.

The field strength correlator $D_{\mu\nu\lambda\omega}(z)$ plays an important role in non-perturbative approaches to QCD [1–4]. It is the basic quantity in the stochastic model of the QCD vacuum [5–7] and in the description of high energy hadron-hadron scattering [8–11]. In the spectrum of heavy quark bound states it governs the effect of the gluon condensate on the level splittings [12–14] and it is useful for the determination of the spin dependent parts in the heavy quark potential [15].

Until today, only the leading non-perturbative contribution to the field strength correlator which is related to the gluon condensate has been included in the phenomenological analyses. More details on this subject as well as further references can be found in the review article by Dosch [4]. However, as yet, higher order perturbative corrections to the correlator $D_{\mu\nu\lambda\omega}(z)$ have only been considered in the context of QED [16]. Actually in this case it is possible to calculate the correlator to all orders in the fine structure constant as a function of the QED $\beta$-function. It is of clear interest for applications in non-abelian gauge theories to also have control over the radiative corrections in the regime where perturbation theory is applicable.

Our paper is organised as follows. In section 2 results for the field strength correlator up to next-to-leading order in perturbative QCD will be presented and some details of the calculation are discussed. An alternative derivation of the correlator from a hybrid quark gluon current in the framework of the heavy quark effective theory (HQET) is given in section 3. In section 4, we make a brief comparison of our results with recent determinations of the field strength correlator on the lattice [17, 18] and summarise our work.
Figure 1: Leading order diagram for the field strength correlator.

2 Field strength correlator

From the Lorentz structure of the field strength correlator it follows that the correlator can be parametrised in terms of two scalar functions $D(z^2)$ and $D_1(z^2)$:

$$D_{\mu\nu\lambda\omega}(z) = \left[ g_{\mu\lambda}g_{\nu\omega} - g_{\mu\omega}g_{\nu\lambda} \right] \left( D(z^2) + D_1(z^2) \right)$$

$$+ \left[ g_{\mu\lambda}z_{\nu}z_{\omega} - g_{\mu\omega}z_{\nu}z_{\lambda} - g_{\nu\lambda}z_{\mu}z_{\omega} + g_{\nu\omega}z_{\mu}z_{\lambda} \right] \frac{\partial D_1(z^2)}{\partial z^2}. \quad (2.1)$$

At the leading order the functions $D^{(0)}(z^2)$ and $D_1^{(0)}(z^2)$ are readily calculated from the diagram of fig. 1. Note that at this order the path ordered exponential in eq. (1.1) reduces to $\delta^{ab}$. For an arbitrary gauge group $SU(N)$, we then find

$$D^{(0)}(z^2) = 0,$$

$$D_1^{(0)}(z^2) = (N^2 - 1) \frac{\Gamma(2 - \epsilon)}{\pi^{2-\epsilon} z^{4-2\epsilon}} \epsilon \rightarrow 0 \quad (N^2 - 1) \frac{1}{\pi^2 z^4}. \quad (2.2)$$

Throughout this work, we use dimensional regularisation in $D = 4 - 2\epsilon$ dimensions. In eq. (2.2) we have also given $D_1^{(0)}$ for arbitrary $\epsilon$, because this result will be used for the renormalization of the next-to-leading order.

At the next-to-leading order, we have to calculate the diagrams of figs. 2 and 3. The graphs of fig. 2 correspond to diagrams where the string term does not contribute whereas in fig. 3 contractions with the string, denoted by the dashed line, arise. The separate results for all diagrams are presented in appendix A. A complication arises in the calculation of the diagram of fig. 3c). As can be seen from the logarithm in eq. (A.5) this diagram generates a divergence, and thus a logarithmic contribution to $D^{(1)}(z^2)$, although $D^{(0)}(z^2)$ vanishes and no immediate counterterm proportional to $D^{(0)}(z^2)$ is present. However, the vertex correction of a field strength tensor times the string exponential generates new operators which, when inserted in the two-point function, produces a mixing into $D^{(1)}(z^2)$. For a proper renormalization of the correlation function this operator mixing has to be taken into account.

The final results for the perturbative next-to-leading order corrections to the
Figure 2: Next-to-leading order diagrams for the field strength correlator without string contribution. The diagrams a) implicitly include the ghost contribution.

Field strength correlator are then found to be

\[ D^{(1)}(z^2) = N(N^2 - 1) \frac{1}{\pi^2 z^4} \alpha_s \left( -\frac{1}{4} L + \frac{3}{8} \right), \quad (2.3) \]

\[ D_1^{(1)}(z^2) = N(N^2 - 1) \frac{1}{\pi^2 z^4} \alpha_s \left( \frac{\beta_1}{2N} - \frac{1}{4} \right) L + \frac{\beta_1}{3N} + \frac{29}{24} + \frac{\pi^2}{3} \], \quad (2.4) \]

with \( L = \ln(e^{2\gamma_E} \mu^2 z^2/4) \), \( \mu \) being a renormalization scale in the \( \overline{MS} \) scheme and the first coefficient of the \( \beta \)-function \( \beta_1 = (11N - 2f)/6 \).

From the contributing diagrams it can be seen that \( D^{(1)}_1(z^2) \) has a logarithmic contribution which is proportional to \( \beta_1 \) and is related to the renormalization of the QCD coupling constant. This logarithm can be resummed by considering \( \alpha_s(\mu^2)D_{\mu\nu\lambda\omega}(z) \) and choosing as the renormalization scale in \( \alpha_s \mu^2 = 4e^{-2\gamma_E}/z^2 \). The coefficients of the remaining logarithms are equal for \( D^{(1)}(z^2) \) and \( D_1^{(1)}(z^2) \) such that

\[ \alpha_s(D_1(z^2) - D(z^2)) = (N^2 - 1) \frac{\alpha_s(4e^{-2\gamma_E}/z^2)}{\pi^2 z^4} \left\{ 1 + N \frac{\alpha_s}{\pi} \left[ \frac{\beta_1}{3N} + \frac{5}{6} + \frac{\pi^2}{3} \right] \right\} \quad (2.5) \]

is a scale independent quantity at the next-to-leading order.

Even though the scale of \( \alpha_s(\mu^2) \) in the next-to-leading order term is only unambiguously fixed at the next-next-to-leading order, for our numerical analysis let us assume \( \mu^2 \approx 1/z^2 \) to be a natural scale. From eq. (2.5) we obtain that at 0.2 fm, which corresponds to 1 GeV, the next-to-leading order correction is roughly 100% and even at 4 GeV or 0.05 fm, the radiative correction still amounts
Figure 3: Next-to-leading order diagrams for the field strength correlator with string contribution.

to about 50% of the leading term. This finding entails that the $O(\alpha_s)$ correction to the field strength correlator is large and that for phenomenological applications further understanding of this large correction should be achieved.

3 Field strength correlator from HQET

The field strength correlation function $D_{\mu\nu\lambda\omega}(z)$ can also be calculated indirectly within the framework of the HQET. Consider the two-point correlator of the hybrid current $(h^a F^a_{\mu\nu})(x)$,

$$\tilde{D}_{\mu\nu\lambda\omega}(z) \equiv \langle 0| T\{(h^a F^a_{\mu\nu})(y) (\bar{h}^b F^b_{\lambda\omega})(x)\}|0\rangle,$$

(3.1)

where $h^a(x)$ is an octet of heavy quark fields, interacting according to QCD in the adjoint representation. From a path integral representation of the field strength correlator it can be shown that

$$\tilde{D}_{\mu\nu\lambda\omega}(z) = S(z) D_{\mu\nu\lambda\omega}(z)$$

(3.2)

with $S(z)$ being the coordinate space propagator of a heavy quark field defined by $T\{h^a(y)\bar{h}^b(x)\} = \delta^{ab} S(z)$. In this derivation the heavy quark propagator replaces the path ordered exponential and serves to make the correlation function gauge invariant. A great advantage of this representation of the field strength correlator

\footnote{For a review on HQET as well as original references the reader is referred to [19].}
is the fact that for its perturbative evaluation we only have to deal with local operators.

The diagrams which have to be calculated in order to obtain $\tilde{D}_{\mu\nu\lambda\omega}(z)$ at the next-to-leading order are two-loop diagrams similar to the diagrams of figs. 2 and 3. In the diagrams of fig. 2 there is an additional heavy quark propagator connecting the external vertices and in the diagrams of fig. 3, the heavy quark propagator replaces the gauge string. All integrals which were needed in the course of this calculation were already known from HQET \[21,22\]. Analogously to the previous section, in the case of diagram 3c), we have to take into account that under renormalization the operator $(h^a F^a_{\mu\nu})(x)$ mixes into new operators.

Calculating all diagrams and performing the renormalization diagram by diagram, we found complete agreement with the direct QCD calculation of the previous section and the results presented in appendix A \[20\]. This strong check gives us confidence in the correctness of our results. In the next section we shall compare these results to most recent determinations of the field strength correlation function on the lattice.

## 4 Discussion

Quite recently, the gluon field strength correlator $D_{\mu\nu\lambda\omega}(z)$ has been measured on the lattice \[17,18\]. The quantity determined in refs. \[17,18\] was

$$C_{\mu\nu\lambda\omega}(z) = g^2 T_R D_{\mu\nu\lambda\omega}(z) \tag{4.1}$$

where in our conventions $T_R = 1/2$ and the corresponding functions $C(z^2)$ and $C_1(z^2)$ are defined analogously to eq. (2.1). Our central results of eqs. (2.3) and (2.4) for the field strength correlator show that it generally depends on the renormalization scale as well as the renormalization scheme. A direct comparison to the lattice results of refs. \[17,18\] is therefore difficult and a quantitative analysis would require a similar calculation of $C_{\mu\nu\lambda\omega}(z)$ in lattice perturbation theory to establish the relation between the two schemes. It is to be expected that the scheme dependence goes beyond merely replacing $\alpha_s^{\overline{MS}}$ by the lattice coupling constant and that there should be additional constant terms. Nevertheless, we shall attempt to present some qualitative observations.

\[2\] The authors would like to thank M. Neubert for providing an integral needed in the calculation of diagram 3c).
In ref. [17] the pure QCD correlator has been measured for four values of \( \beta = 6/g^2 \) between 6.6 and 7.2. It was found that an acceptable fit to \( C_1(z^2) \) can be obtained by a pure perturbative \( 1/z^4 \) behaviour. Dividing out \( g^2 T_R \) with an average value \( g^2 = 0.87 \), we then obtain \( D_1^{\text{lat}}(z^2) \approx 0.69/z^4 \). This can be compared with the leading order result of eq. (2.2), \( D_1^{(0)}(z^2) \approx 0.81/z^4 \) showing reasonable agreement at the 20% level. However, whereas for a natural \( \overline{MS} \) scale \( \mu^2 \approx 1/z^2 \) the next-to-leading order correction to \( D_1(z^2) \) turns out to be large and positive, the lattice results suggest a moderate, negative correction from higher orders. On the other hand in perturbation theory \( D(z^2) \) vanishes at the leading order and the next-to-leading order correction in the \( \overline{MS} \) scheme is found to be much smaller than the correction to \( D_1(z^2) \). Surprisingly, on the lattice \( C(z^2) \) was found to be larger than \( C_1(z^2) \). Hopefully, a perturbative calculation in the lattice regularisation scheme will clarify these issues in the future.

In QED the gauge string exponential is absent and higher order corrections are only due to the renormalization of the electric charge. Vainshtein and Zakharov calculated the quantity \( D_{\mu\nu}(z^2) \) in terms of the QED \( \beta \)-function [16]. However, the leading term for \( D_{\mu\nu}(z^2) \) in QED starts at order \( \alpha^2 \). Hence it is not possible to compare our results at the next-to-leading order to the QED case presented in ref. [16].

To summarise, in this work we have presented a calculation of the gauge invariant field strength correlator at the next-to-leading order in perturbation theory. It was found that the correlator depends on both the renormalization scale as well as the renormalization scheme. Constructing a scale independent quantity at this order, it turned out that the first order correction is large, namely roughly 50% to 100% for distances of \( z = 0.05 \) fm to 0.2 fm. Because of the scheme dependence of the field strength correlator, for a sound comparison with recent lattice data, an analogous calculation of the correlator in lattice perturbation theory would be required. We hope to return to this questions and phenomenological applications of this work in the future.

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Appendix

A  Results for $D^{(1)}$ and $D_1^{(1)}$

Below, we shall present our results for all diagrams separately. In order to check explicitly the gauge invariance of the field strength correlator, we have performed the calculation in a general covariant gauge. The next-to-leading order corrections to the functions $D(z^2)$ and $D_1(z^2)$ can be written as follows:

$$ D^{(1)}(z^2) = D_1^{(0)}(z^2) G^{(1)}(z^2) \quad \text{and} \quad D^{(1)}_1(z^2) = D_1^{(0)}(z^2) G^{(1)}_1(z^2), \quad (A.1) $$

with $D_1^{(0)}(z^2)$ given in eq. (2.2).

For the separate renormalised contributions to $G^{(1)}(z^2)$ we find:

$$ G^{(1)}_{2c} = N \frac{\alpha_s}{\pi} \left[ \frac{1}{8} + \frac{a}{8} \right], \quad (A.2) $$

$$ G^{(1)}_{3a} = N \frac{\alpha_s}{\pi} \left[ -\frac{3}{4} - \frac{a}{4} \right], \quad (A.3) $$

$$ G^{(1)}_{3b} = N \frac{\alpha_s}{\pi} \left[ \frac{3}{8} + \frac{a}{8} \right], \quad (A.4) $$

$$ G^{(1)}_{3c} = N \frac{\alpha_s}{\pi} \left[ -\frac{1}{4} L - \frac{1}{8} \right], \quad (A.5) $$

where $L = \ln(\pi e^{\gamma_E} \nu^2 z^2)$ and $\nu^2$ is a renormalization scale in the $\overline{MS}$-scheme [2].

We have already subtracted the corresponding counterterms diagram by diagram. To relate our expressions to the more conventional $\overline{MS}$-scheme [24] we have to use $\nu^2 = e^{\gamma_E} \mu^2/(4\pi)$ where now $\mu^2$ is a renormalization scale in the $\overline{MS}$-scheme.

All diagrams which have not been listed explicitly give a vanishing contribution. Summing all diagrams, we obtain:

$$ G^{(1)}(z^2) = N \frac{\alpha_s}{\pi} \left[ -\frac{1}{4} L + \frac{3}{8} \right]. \quad (A.6) $$

The separate contributions to $G^{(1)}_1(z^2)$ are found to be:

$$ G^{(1)}_{1,2a} = N \frac{\alpha_s}{\pi} \left[ \frac{\beta_1}{2N} - \frac{3}{8} - \frac{a}{8} \right] L + \frac{\beta_1}{3N} - \frac{23}{48} + \frac{a}{4} + \frac{a^2}{16}, \quad (A.7) $$

$$ G^{(1)}_{1,2b} = N \frac{\alpha_s}{\pi} \left[ -\frac{5}{8} - \frac{a}{8} \right] L - \frac{1}{4} - \frac{3a}{8} - \frac{a^2}{8}, \quad (A.8) $$
\begin{align}
G_{1,2c}^{(1)} &= N \frac{\alpha_s}{\pi} \left[ -\frac{1}{16} + \frac{a^2}{16} \right], \\
G_{1,3a}^{(1)} &= N \frac{\alpha_s}{\pi} \left[ -\frac{3}{4} + \frac{a}{4} \right], \\
G_{1,3b}^{(1)} &= N \frac{\alpha_s}{\pi} \left[ \left( \frac{3}{4} - \frac{a}{4} \right)L + \frac{11}{8} - \frac{a}{8} \right], \\
G_{1,3c}^{(1)} &= N \frac{\alpha_s}{\pi} \left[ \frac{a}{2} L + \frac{11}{8} + \frac{\pi^2}{3} \right].
\end{align}

The flavour dependent contributions have been rewritten in terms of the first coefficient of the QCD $\beta$-function, $\beta_1 = (11N - 2f)/6$. Again summing all diagrams, we obtain:

\begin{align}
G_1^{(1)}(z^2) &= N \frac{\alpha_s}{\pi} \left[ \left( \frac{\beta_1}{2N} - \frac{1}{4} \right)L + \frac{\beta_1}{3N} + \frac{29}{24} + \frac{\pi^2}{3} \right].
\end{align}
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