THE EFFECTS OF POLARIZED FOREGROUNDS ON 21 cm EPOCH OF REIONIZATION POWER SPECTRUM MEASUREMENTS

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ABSTRACT

Experiments aimed at detecting highly-redshifted 21 cm emission from the epoch of reionization (EoR) are plagued by the contamination of foreground emission. A potentially important source of contaminating foregrounds may be Faraday-rotated, polarized emission, which leaks into the estimate of the intrinsically unpolarized EoR signal. While these foregrounds’ intrinsic polarization may not be problematic, the spectral structure introduced by the Faraday rotation could be. To better understand and characterize these effects, we present a simulation of the polarized sky between 120 and 180 MHz. We compute a single visibility, and estimate the three-dimensional power spectrum from that visibility using the delay spectrum approach presented in Parsons et al. Using the Donald C. Backer Precision Array to Probe the Epoch of Reionization as an example instrument, we show the expected leakage into the unpolarized power spectrum to be several orders of magnitude above the expected 21 cm EoR signal.

Key words: cosmology: observations – instrumentation: interferometers – instrumentation: polarimeters

1. INTRODUCTION

A significant amount of thought has gone into the problem of foreground removal allowing for the detection of the power spectrum of neutral hydrogen during the epoch of reionization (EoR; e.g., Bowman et al. 2009; Morales et al. 2006; Liu et al. 2009; Liu & Tegmark 2011; Dillon et al. 2013). Essential to nearly all these techniques is the spectral smoothness of these foregrounds, which allows for a separation in k space of the foreground emission and the signal from the EoR. Various mechanisms will leak polarized emission into the best estimate of the EoR signal, which then may introduce spectral structure to an EoR experiment’s measurement.

In contrast to unpolarized foregrounds, comparatively little work has addressed the problem of detecting and removing polarized foregrounds. Pen et al. (2009) provided some of the first relevant upper limits at small angular scales using the Giant Metrewave Radio Telescope. They found, for spherical harmonic multipoles $0 \leq \ell \leq 5000$, an angular power spectrum upper limit of around $C_\ell \lesssim 100 \text{ mK}^2$. More recent work by Bernardi et al. (2010) detected polarized power at the same level at $\ell \lesssim 1000$ using the Westerbork Synthesis Radio Telescope, with no significant detection above $\ell$ of 1000. Bernardi et al. (2010) did not detect emission directly attributable to polarized point sources. Pen et al. (2009) present their upper limit in terms of the three-dimensional power spectrum, but Bernardi et al. (2010) only present a $C_\ell$ spectrum which has been integrated along the frequency direction. Figure 1 gives a summary of the low-frequency measurements of polarized power spectra. It is clear that the angular power spectrum of the unpolarized sky must be scaled by a mean polarization fraction of about 0.3% to agree with the Bernardi et al. (2010) measurements. This requires a significant degree of depolarization of synchrotron emission from ordered magnetic fields.

Jelić et al. (2010) attempted to further constrain the problem by fully simulating a full-Stokes realization of galactic synchrotron emission over a $10^\circ \times 10^\circ$ field of view. They present a realistic spectrum of the mean temperature of polarized emission, but do not extend their analysis into the power spectrum.

Their analysis also predicts a polarization fraction from diffuse emission much higher than the limited measurements available. Geil et al. (2011) also investigate the issue, proposing the use of the RMCLEAN algorithm (Heald et al. 2009) to mitigate the effects of these polarized foregrounds. While these papers provide detailed descriptions of the polarized sky and removal strategies in the image plane, they provide little discussion of the line-of-sight direction.

This paper aims to steer the discussion of polarized foregrounds toward the terra incognita of the third, frequency dimension. We begin in Section 2 by reviewing the basics of polarized interferometry, and the delay spectrum approach to estimating the power spectrum, presented in Parsons et al. (2012b). We discuss the relevance of polarized foregrounds to measuring the 21 cm power spectrum. In Section 3, we discuss the design and implementation of a simulation of the polarized sky and its results. Finally, in Section 4 we briefly discuss prospects of polarized source removal and leakage mitigation.

Throughout the paper, we will use the Donald C. Backer Precision Array to Probe the Epoch of Reionization (PAPER; Parsons et al. 2010) as a model instrument. The results are not specific to that instrument, nor are they specific to the Delay Spectrum analysis used in this paper (Parsons et al. 2012b). Any 21 cm EoR power spectrum detection experiment with linear feeds, including the Murchison Widefield Array (Tingay et al. 2013) or the Low Frequency ARray (Röttgering 2003) could fall subject to the leakages described here without a perfect calibration.

2. PRELIMINARIES

2.1. Definition of Stokes Visibilities

Two prominent ways in which polarized sky emission can leak into an interferometric estimate of Stokes $I$ are leakages due to non-orthogonal and rotated feeds and beam ellipticity—an asymmetry in the two linear polarizations of a primary beam which causes unpolarized signals to appear polarized, and vice versa. The first is a well-understood question, discussed in length in the series of papers by Hamaker et al. (1996). This type of
leakage can be corrected by the proper linear combination of visibilities. Hence, we will focus on the latter issue. To begin, we will examine the contents of an interferometric spectrum, and relate them to the intrinsic Stokes parameters.

As a reminder, we present the measurement equation for an interferometer in the flat-sky limit:

\[ V_{ab}(u, v, v) = \int A_{ab}(l, m)I_{ab}(l, m)e^{-2\pi i (lu+vm)} \, dldm. \]  

(1)

Here the polarization indices \(a, b\) indicate the polarization state of the measurement. \(A_{ab}\) is the primary beam, \(I\) and \(m\) are direction cosines of the celestial sphere (with their Fourier components \(u\) and \(v\)), and \(I_{ab}\) is the sky emission projected along the polarization state of the measurement. Henceforth, we will write all visibility equations in the flat-sky limit. We can do this without loss of generality since the polarization properties of a visibility are unaffected by this assumption.

It is worth noting that each linear feed measures a one-dimensional projection of the incident electric field. This causes correlations with different feed orientations to contain information about different polarization states of the incident radiation. A convenient short-hand notation for the polarization content of each visibility is

\[
\begin{pmatrix}
V_{xx} & V_{xy} \\
V_{yx} & V_{yy}
\end{pmatrix}
= J_I \cdot \begin{pmatrix}
I + Q & U - iV \\
U + iV & I - Q
\end{pmatrix} \cdot J_J.
\]

(2)

The Jones matrices \(J_{I,J}\) (see Hamaker et al. 1996) relate the sky emission (\(I, Q, U, V\)) to an interferometric measurement. Each antenna’s Jones matrix is dependent on nearly all instrumental parameters, but for the purposes of this paper, we will investigate the effects of direction-dependent gains, or the primary beam.

In the flat-sky approximation, an interferometer natively measures the two-dimensional spatial Fourier transform of the sky. Ideally, this would allow an observer to estimate the three-dimensional power spectrum by simply transforming the frequency, line of sight direction, and cross-multiplying measurements without imaging. Relaxing the imaging requirement provides an incentive to make estimates of the Stokes parameters—\(I\) for instance—in the visibility domain, where the Stokes parameters are not defined. A naive addition between the linearly polarized, \(xx\) and \(yy\) visibilities should estimate the total power of sky emission, Stokes \(I\). Similar operations can be performed for all polarization states. A sensible method of estimating the four Stokes parameters in the visibility domain is to add visibilities as images are typically added. Hence, we define Stokes visibilities (for linear feeds) as:

\[
\begin{pmatrix}
V_I \\
V_Q \\
V_U \\
V_V
\end{pmatrix}
= \frac{1}{2} \begin{pmatrix}
1 & 0 & 0 & 1 \\
1 & 0 & 0 & -1 \\
0 & 1 & 1 & 0 \\
0 & i & -i & 0
\end{pmatrix}
\begin{pmatrix}
V_{xx} \\
V_{xy} \\
V_{yx} \\
V_{yy}
\end{pmatrix}.
\]

(3)

Explicitly writing the expression of \(V_I\) by substituting Equation (1) into Equation (3), we find that

\[
V_I = V_{xx} + V_{yy}
= \int (A_{xx} + A_{yy})Ie^{-2\pi i (lu+vm)} \, dldm
+ \int (A_{xx} - A_{yy})Qe^{-2\pi i (lu+vm)} \, dldm.
\]

(4)

Writing the visibility this way highlights one source of polarized leakage: that due to beam ellipticity. If the \(xx\) and \(yy\) beams are not equal, the last term of Equation (4) would be non-zero. Polarized emission then enters the estimate of \(I\), weighted by the differenced beam. This equation also points out the difference between Stokes parameters and the Stokes visibilities defined in Equation (3). They estimate their respective Stokes parameters, but still include terms from other polarization states.

Thus, if an instrument’s linearly polarized beam is not symmetric under a 90° rotation, all the intrinsic linear polarization will not cancel, corrupting the visibility estimate of Stokes parameters. Figure 2 shows two slices through the model beam for PAPER, through constant azimuth and through zenith, and a cut through the same beam, rotated by 90°. We note that these
slices are not identical, and show the differenced beam, which provides the mechanism for leakage.

In an image-based analysis, these effects may be corrected by a simple reweighting of the image, but these reconstructions are subject to the accuracy of the primary beam model. The results of this paper indicate the leakages due to the primary beam shown in Figure 2, but could equally be applied to an image-based analysis whose primary beam is only known to the level of the differences in that figure.

Once more, we note that the Stokes visibilities are only the best guess at the true Stokes parameters. Without an exact characterization of the primary beam or dense \( u-v \) sampling, one does not have the ability to fully correct for the leakages mentioned. Hence, it is imperative to inspect the Stokes I signal’s corruption.

2.2. The Delay Spectrum Approach

The delay-spectrum approach to measuring the 21 cm power spectrum (Parsons et al. 2012b) requires no imaging, again providing an incentive for adding raw visibilities. The delay-spectrum approach embraces the natural units of an interferometer by sampling the power spectrum in baseline and frequency dimensions, native to an interferometer. With this approach, each baseline can be individually transformed into an estimator of the power spectrum of the incident temperature. This method also prevents the small-scale structure introduced by Fourier transforming over gain calibration errors (Morales et al. 2012).

Perhaps the greatest advantage of the delay spectrum approach is its ability to isolate smooth-spectrum foregrounds on a per-baseline basis. This relaxes the burden of isolating sources in the image plane, and allows for a more sparsely sampled array (e.g., a redundantly sampled one, as in Parsons et al. 2012a). The 21 cm EoR’s extent to super-horizon delays depends precisely upon its spectral non-smoothness. It is imperative, then, to identify and characterize spectrally non-smooth foregrounds that may corrupt the EoR measurement.

The delay spectrum approach is particularly useful for simulating the effects of a systematic error on the power spectrum. Because each visibility estimates the power spectrum, a simulation needs to only create one visibility, which encapsulates much of the power spectral information needed for analysis. The computational cost is additionally reduced by the ability to calculate a power spectrum by Fourier transforming only one visibility, rather than gridding several visibilities into a \( (u, v, \nu) \) cube and Fourier transforming along each \( u, v \) pixel.

It is important to note that the results of this paper are not limited to delay-spectrum-based analyses. All effects mentioned in this paper are native to the sky and its expected spectral structure. Any analysis technique will incur all the same issues; in this paper are native to the sky and its expected spectral limited to delay-spectrum-based analyses. All effects mentioned

\[
\tilde{V}(\tau) \equiv \int_{\Delta \nu} V(u, v, \nu) e^{-2\pi i \nu \tau} d\nu,
\]

where \( \nu \) represents the frequency, \( \tau \) is the delay or the Fourier transform pair to frequency, whose physical meaning is given in Parsons et al. (2012b), and the tilde denotes a delay-transformed visibility. We treat this as an estimator of the Fourier transform of the temperature field, squaring it to derive an estimator of the three-dimensional power spectrum \( P(k) \),

\[
\tilde{V}^2(\tau) = |\tilde{V}(\tau)|^2 \propto P(k),
\]

where \( k \) is the wavemode corresponding to the measurement. To better mimic actual measurements, we divide the band into ten 6 MHz sub-bands and perform the delay transform on each individual band. The sub-bandwidth is chosen to sample the maximum cosmological distance over which the 21 cm signal is expected to be cotemporal (e.g., Wyithe & Loeb 2004; Furlanetto et al. 2006a). Once these transforms are computed, we multiply each spectrum by the appropriate factors to obtain the power spectrum in units of temperature squared. We write the expression converting the squared, delay-transformed visibility \( \tilde{V}(\tau) \) into a “unitless,” cosmological power spectrum \( \Delta^2(k) \) as

\[
\Delta^2(\tau) \approx \left( \frac{2k_B}{\lambda^2} \right)^2 \frac{\Omega B}{X^2 Y} \frac{2\pi^2}{k^3} \Delta^2(k),
\]

where \( k_B \) is the Boltzmann constant, \( \lambda \) is the observing wavelength, \( \Omega \) is the solid angle of the primary beam, \( B \) is the observed bandwidth, and \( X \) and \( Y \) are cosmological scalars which convert observed angles and frequencies in to \( h \) Mpc\(^{-1} \), appropriately derived from Equations (3) and (4) of Furlanetto et al. (2006b).

Parsons et al. (2012b) offer a much more detailed discussion of this approach, which is beyond the scope of this paper.

2.3. Sparse \( u-v \) Sampling and Wide-field Polarimetry

Another advantage of the delay spectrum approach is that it relaxes the requirement of gridding in the \( u-v \) plane. Each baseline is assigned a position in the \( u-v \) plane ab initio, and visibilities from similar baselines may be coherently added without imaging. This allows for sparse sampling in the \( u-v \) plane without damaging effects from sidelobes or missing data, problems other methods may experience. Since the delay spectrum rotates a power-spectrum estimate into the native coordinate system of an interferometer, there are no inherently missing-frequency data. Parsons et al. (2012a) present the sensitivity benefits of a sparse, redundant array configuration, but other techniques aim to uniformly sample the \( (u, v, \nu) \) cube, in order to mitigate the systematic effects of computing a Fourier Transform across unevenly sampled data.

An obvious disadvantage of having sparsely-sampled data is poor imaging. Not only does sparse sampling provide a highly irregular synthesized beam, but it also limits the available information for a full reconstruction of the image. Without adjacent \( u-v \) samples, a full, accurate deconvolution by a wide beam simply has insuffcient information. As we will see, the inability to correct for beam effects will provide a significant source of systematic error via polarized leakage.

By choosing to wield the full power of the delay spectrum approach and redundant sampling, an observer is forced to add visibilities with no beam weighting. The beam information supplied by adjacent \( u-v \) samples simply does not exist, and without transforming into the image plane is unrecoverable. Hence, the imperative to investigate the implications of a lack of beam-weighting, the naive construction of the I visibility, arises.

Together, redundant sampling and the delay spectrum approach give a 21 cm EoR experiment incentive to add raw visibilities, subjecting it to potential leakage. An elliptical primary beam gives a mechanism whereby polarized emission can corrupt an estimate of the total power. To what degree does polarized emission corrupt an estimate of the 21 cm EoR signal? We will begin answering this question by characterizing the
spectral non-smoothness that will possibly arise from the rotation measure (RM) structure of polarized leakages.

2.4. Faraday Rotation

Faraday Rotation affects the polarization properties of an electromagnetic wave traveling through a plasma containing a magnetic field (Rybicki & Lightman 1979). The circular polarization oriented in a right-handed fashion to the direction of the incident field will be slowed by the plasma. This causes a rotation of the E-field’s polarization angle by

$$\Delta \phi = \lambda^2 \frac{e^3}{(m_e c^3)^2} \int B_i(s) n_e(s) ds \equiv \lambda^2 \text{RM}. \quad (8)$$

Equation (8) defines the rotation measure (RM) by which we characterize this phase wrapping. Since the Stokes parameters characterize the square of the electric field, the phase of the polarization vector is shifted by twice the angle defined in Equation (8). After a signal passes through a Faraday screen, we measure a rotated polarization angle,

$$(Q + iU)_{\text{meas}} = (Q + iU)_{\text{int}} e^{2\text{RM} \lambda^2}, \quad (9)$$

where the subscript “meas” denotes the measured $Q$ and $U$ measurements, and “int” denotes the intrinsic $Q$ and $U$ signal, as would be measured behind the Faraday screen.

3. SIMULATION

3.1. Single-source Power Spectrum

We begin our investigation of the effects of polarized foregrounds on the 21 cm EoR signature by examining the power spectrum of a single source at zenith, whose signal has the structure of a single RM. In doing so, we can develop an intuition for the RMs that affect cosmologically interesting $k$ modes of the power spectrum. By looking at what is effectively the impulse response of a Faraday screen on the power spectrum, it will be easier later to interpret a more complicated model.

Figure 3 shows the real part of the spectra of a few linearly polarized sources behind Faraday screens, $S(\nu) = \exp(-2i \text{RM} \lambda^2)$. Each spectrum contains one source with 1 Jy of polarized flux, located at zenith (delay of zero). Note that at the highest RM shown, the spectrum is not critically sampled at the lowest frequencies. This is due to the uneven sampling of $\lambda^2$ across the band: as $\Delta \lambda^2 \approx d\lambda^2 / d\nu \Delta \nu \propto \Delta \nu / \nu^3$ increases, the sensitivity to large RMs decreases.

Figure 4 shows the Fourier transform over frequency of the spectra in Figure 3. While this does not exactly represent a delay-spectrum of a visibility—there is no beam-weighting, and no $\exp(-2\pi i \mathbf{b} \cdot \hat{s})$ component, which essentially defines the delay spectrum—we interpret it as the delay-structure introduced by a polarized source behind a Faraday screen. The results of these transforms over the full simulated band are shown in Figure 5. The most important feature of this plot is this: there is a $k$ mode associated with each RM at each redshift. We can construct an analytic estimate of this $k$ mode by setting the argument of the exponents of a delay mode and a RM mode to sum to zero. First, we approximate the cosmological $k$ mode sampled by a delay mode as $\tau \approx k || dr || / d\nu$. Next, we recall the cosmological scaling from frequency into $h \text{Mpc}^{-1}$,

$$\frac{dr}{d\nu} = \frac{dr}{dz} \frac{dz}{d\nu} = - c (1 + z) H(z) \nu. \quad (10)$$

Finally, we set $k || \cdot dr || / d\nu \cdot \nu + 2\text{RM} \lambda^2 = 0$. Substituting Equation (10) for the derivative, we derive an expression for the $k$-mode most affected by a RM:

$$k || \approx \frac{2 H(z)}{c (1 + z)} \cdot \text{RM} \lambda^2, \quad (11)$$

where $H(z)$ is the Hubble parameter. Figure 6 shows a plot of the most culpable RM versus frequency and redshift.

3.2. Full-sky Simulation

To better grasp the effects of Faraday leakage into the 21 cm signal, we generate several random realizations of the sky,
Figure 5. Power spectra of the visibilities plotted in Figure 3, computed for the median redshift bin of the PAPER band ($z \sim 8$). This plot demonstrates the coupling between rotation measure and $k$-modes, and confirms that Faraday-rotated spectra scatter power to higher $k$ modes than would be contained within the horizon. The gray, vertical lines show the maximally-infected $k$-mode, predicted by Equation (11). The rise in power at high $k$ is due to the $\sim 10^{-9}$ sidelobe of the Blackman-Harris filter. The units of the $y$-axis are tuned so the total power integrates to one.

Figure 6. The rotation measure most affecting $k \approx 0.15 \ h \text{Mpc}^{-1}$, calculated from Equation (11). The shaded region indicates the range of rotation measures affecting $0.1 \ h \text{Mpc}^{-1} \lesssim k \lesssim 0.2 \ h \text{Mpc}^{-1}$.

Rather than creating an exact simulation of the physical sky, we create a simulation whose statistical properties are physically motivated. This choice reflects a desire for simple, easily tunable parameters for the simulation. In that same spirit, we model all sources simply as point sources with a Poisson distribution. The simulation’s primary concern with the spectral information of polarized foregrounds allows us to justify neglecting the angular terms. This is equivalent to assuming for all the relevant $k$ modes, $k_{||} \gg k_{\perp}$. Emphasis on the $k_{||}$ or spectral modes also motivates our decision to model the sky as numerous point sources. For a more detailed discussion of these effects, we direct the reader to Jelić et al. (2010).

Source positions are distributed uniformly over the sphere. A single source’s altitude $\theta$ is drawn from a distribution in which $\cos \theta$ is uniform on $[0, 1]$. A source’s azimuthal angle $\phi$ is drawn independently from $\cos \theta$ from a distribution uniform on $[0, 2\pi]$. This choice of source position distributions conserves the density per area of sources across the sky, and is equivalent to drawing both direction cosines, $(l, m) = (\sin \theta \cos \phi, \sin \theta \sin \phi)$, from a uniform distribution on $[-1, 1]$.

In order to achieve realistic source fluxes and source counts, we base the distributions from which we draw various parameters on previous radio surveys. For the source fluxes, we aim to agree with VLSS (Cohen et al. 2007), NVSS surveys (Condon et al. 1998), and the 6C survey (Hales et al. 1988). For the polarization information, we aim to agree with polarized measurements taken by the NVSS survey, particularly, their nearly full sky RM map (Taylor et al. 2009), as well as the polarized survey of the VLSS.

We present two scenarios for the source counts. First, we draw from the 6C distribution (Hales et al. 1988), taken in the PAPER band at 151 MHz. Second, we extrapolate VLSS source counts (Cohen et al. 2007) from 74 MHz to 150 MHz, using a spectral index of $-0.79$, following the work of Cohen et al. (2004). The latter source counts provide more sources at higher flux, which infect the power spectrum from the $I$ visibilities.
as we will show in the following sections. Both number counts are consistent with the recent measurements by the Murchison Widefield Array (Williams et al. 2012), an instrument similar in many regards to PAPER.

The differential number counts \( \frac{dN}{dS} \) found by Hales et al. (1988) may be characterized by two power laws, turning over at some knee flux \( S_o \)

\[
\frac{dN}{dS} = \begin{cases} 4000 \ S_o^{-0.76} S^{-1.75} \ Jy^{-1} \ sr^{-1} & S_{\text{min}} \leq S < S_o \\ 4000 \ S^{-2.81} \ Jy^{-1} \ sr^{-1} & S_o \leq S \end{cases}
\]  

(12)

Following the 6C survey, we choose the turning point, \( S_o \) to be 0.88 Jy. The number of sources simulated (14,855) is chosen by the size of the PAPER beam at 151 MHz (0.76 sr) and a flux range over which to integrate. We choose to include those sources in between 100 mJy and 10 Jy. This choice provides a reasonable dynamic range of sources. Below the lower limit, the 6C sources are unreliable, and we assume sources above 10 Jy may be easily identified and removed.

If we were to blindly extrapolate the 6C source counts below the lower limit of the catalog, we would add a negligible amount of power. Integrating \( S^2 dN/dS \) down to some minimum flux estimates the contribution of the sources above that flux to the total variance of flux. By inserting the 6C source counts, we find that we are including \( \sim 70\% \) of the estimated total variance. Extending the minimum flux would add more power to the simulation, but would not drastically alter the results of this paper.

VLSS source counts follow a single power law, given by Cohen et al. (2004) as

\[
\frac{dN}{dS} = 4865 \ S^{-2.3} \ Jy^{-1} \ sr^{-1}.
\]  

(13)

For these source counts, we choose minimum and maximum fluxes of 0.8 Jy and 100 Jy, respectively, rejecting sources well below the lower limit of the catalogue, and providing a reasonable dynamic range for the included sources. Integrating over the PAPER beam provides 6,995 sources. These source counts are not qualitatively different from the 6C counts at low fluxes, but they do differ substantially in that they provide many more bright sources.

For both source counts, we also assign a spectral index to each individual source spectrum, drawn from a normal distribution, mean \( -0.8 \), standard deviation 0.1, which roughly agrees with the findings of Helmbold et al. (2008).

Each source is also assigned a random polarization angle \( \chi \), uniformly sampled from \([0, \pi]\). Total flux is multiplied by a polarization fraction, chosen to reflect the studies of Tucci & Toffolati (2012). We sample the polarized fraction \( (\Pi) \) from a log-normal distribution whose mean is 2.01% and whose standard deviation is 3.84%. Because the log-normal distribution is not upper-bounded, we reject any drawing over 30%. Following the aforementioned study, we do not impose any correlation between source flux and polarization fraction. It has been noted that, among other effects, bandwidth depolarization causes the polarized fraction to decrease at lower frequencies (Law et al. 2011). This along with measurements from Pen et al. (2009) indicates that these distributions, taken at 1.4 GHz, may overestimate the distribution at 150 MHz. We neglect these effects, taking the 1.4 GHz distribution at face value, since the mean polarization fraction can be thought of as a scale factor to the overall power spectrum.

We base our distribution of RMs on the map presented in Oppermann et al. (2012). To mimic the effects of depolarization due to a finite spatial resolution (e.g., Law et al. 2011), we apply a low-pass filter to the RM map. We project the map into a spherical-harmonic basis, and keep only those modes below the resolution of our simulated instrument. In the case of this simulation, we choose to keep only \( k \leq 100 = 2 \pi |u| \). This averages the polarization vectors in much the same way as a synthesized beam, and its effect is to essentially remove outliers in the RM distribution, to which instruments like PAPER may not be sensitive. We then randomly draw RMs from the computed cumulative distribution function of RM’s given in the Oppermann et al. (2012) data. Aside from low-pass filtering, no spatial information from the data is used. Section 3.4 briefly discusses the negligible consequences of spatially correlating RM.

Histograms of the distributions of RM, polarized fraction, and source counts can be found in Figure 7. Overplotted on all is the distribution from which they are drawn.

We model all sources as point sources, neglecting the effects of any diffuse emission. This choice reflects the desire for simple, easily tunable parameters in the simulation. While diffuse emission certainly is present, its spectral structure is qualitatively the same as that of a point source, so the results in the frequency direction will not change. Were we to add a diffuse model to the simulation, it would widen a source’s response in delay-space, preventing it from being localized. This will not affect the line-of-sight power spectrum, at high \( k \), since the spectrally-smooth, diffuse emission still falls within the horizon.

Table 1 summarizes the three treatments of the simulation we will be using. Simulation A, with 6C source counts and the Oppermann RM distribution is likely the most accurate. Simulation B steepens the source counts, providing fewer, brighter sources, and simulation C doubles the width of the RM distribution.

| Name | Source Counts | \( N_{\text{bc}} \) | RM Distribution |
|------|---------------|-----------------|----------------|
| A    | 6C            | 14,855          | Oppermann       |
| B    | VLSS          | 6,995           | Oppermann       |
| C    | 6C            | 14,855          | 2\times Oppermann |

To check if the results of this simulation are consistent with the measurements in Bernardi et al. (2010), we compare the two-dimensional \( C_\ell \) power spectrum with that presented in their paper. Figure 8 shows the two-dimensional power spectrum of one realization of the simulation, with the constraints from Figure 20 of Bernardi et al. (2010) plotted over it. We see qualitatively that our simulation well obeys the upper-limit imposed by the Bernardi measurement, and proceed with the results.

We calculate the visibilities for a 32 m, east–west baseline. This choice reflects the most common spacing of the maximum-redundancy configuration presented in Parsons et al. (2012a). The choice of baseline orientation is arbitrary, and since we are modeling only point sources, the choice of baseline length will only set the horizon limit of the power spectrum. Since the delay affected by a RM is independent of a choice of baseline (Equation (11)), choosing a relatively short baseline will isolate the foregrounds at lower \( \tau \), and highlight the Faraday leakage.
Figure 7. Distributions of simulated parameters. Top left: $S^{-3/2}N(> S)$ source counts of unpolarized flux. In black, the 6C source counts from Hales et al. (1988), and in gray, the VLSS counts from Cohen et al. (2007). Top right: normalized histogram of the log-normal distribution of polarized fraction, taken from Tucci & Toffolatti (2012). Bottom: normalized histograms of the two distributions of rotation measures. Black is taken from Oppermann et al. (2012), and gray is that with a doubled standard deviation. The RM distribution extends to several hundred m$^{-2}$, but we restrict the extent of the x-axis to highlight the distribution, rather than the width of its tails. In the upper two panels, the dashed line is the distribution from which sources are drawn, and the bins or points are the values of one realization of the simulation.

Figure 8. $\ell$ vs. $C_\ell$ for one realization of the simulation (solid line), alongside the results from Bernardi et al. (2010; crosses). This roughly demonstrates the agreement of the simulation presented in Section 3.2 with current observations. The simulation is consistently lower than the observed values due to the lack of extended structure.

The full measurement equation used in this simulation is

$$V(u, v, \nu) = \sum_{j=1}^{N_{src}} A(l_j, m_j, \nu) \Pi_j S_j^{150} \left(\frac{150 \text{ MHz}}{\nu}\right)^{\alpha_j} \times \exp \left\{-i[2\pi \nu(u l_j + v m_j) + 2\text{RM}_j \lambda^2 + 2\chi_j]\right\},$$

(14)

where each source $j$ is assigned a flux ($S_j$), polarization fraction ($\Pi_j$), spectral index ($\alpha_j$), a position ($l_j, m_j$), RM, polarization angle ($\chi_j$), and is weighted by the model primary beam ($A$). A sample $Q$ visibility is shown in Figure 9.

We choose not to include the parallactic rotation of $Q$ into $U$, implying that the $Q$ we label in this paper are fixed to topecentric, azimuth and altitude coordinates. This choice clarifies equations and allows for an ease of understanding which would be obfuscated by writing both $Q$ and $U$. 
Figure 10. Power spectral measurements for the three treatments of the simulation shown in Table 1. The left column shows the power spectra of the $I$ visibilities, and the right shows the spectra of $Q$ visibilities. The top row shows simulation treatment A, the second row shows treatment B, and the third row shows treatment C. Line styles depict different redshift bins: 7.25 (solid), 8.33 (dashed), 9.73 (dot-dashed). The gray line gives a toy model of the expected 21 cm emission from Furlanetto et al. (2006b). Both visibilities include contributions from both the (intrinsic) Stokes $I$ and $Q$. The simulated levels of $Q$ emission indicate that polarized leakage into $I$ needs to be less than four or five orders of magnitude in mK$^2$.

3.3. Simulated Power Spectra

Figure 10 show the power spectra for source counts from the extrapolated VLSS and 6C surveys, as well as the spectrum of 6C source counts with a widened RM distribution. We interpret the power spectrum of the $I$ visibility as the amount of polarized leakage corrupting the EoR signal (henceforth called the $Q \rightarrow I$ leakage), and the $Q$ visibility’s power spectrum is our best representation of the polarized signal. These plots show the median power in each $k$ bin for 1000 realizations of the simulation, with error bars show the 1σ extent of the bandpowers for these realizations. These power spectra confirm the prediction made in Section 3.1 that $\lambda^2$ phase wrapping extends the foreground cutoff presented in Parsons et al. (2012b) to higher $\tau$ bins, corrupting some of the most sensitive regions of $k$ space for 21 cm EoR analysis. They also demonstrate the prediction in that section that high-redshift bins will be most affected.

The severity of the leakage can be inferred from the power in the most EoR-sensitive $k$ bins ($0.2 \ h\text{Mpc}^{-1} \lesssim k \lesssim 0.3 \ h\text{Mpc}^{-1}$). Figure 11 shows $\Delta^2(k)$ in these bins as a function of redshift. The leaked power ranges in the thousands of mK$^2$, increasing from high frequency/low redshift to low frequency/high redshift. These estimates are about two orders of magnitude above level of the expected 21 cm signal (Lidz et al. 2008). If we take this simulation as an accurate prediction of the low-frequency sky’s polarized emission, these results imply that naively adding $V_{xx}$ and $V_{yy}$, formed with an approximately 10% asymmetric primary beam, incorporates enough bias from polarized leakage to completely obscure the 21 cm signal. The levels of leakage in our simulations demands a strategy to model and remove polarized sources.

We note that simply adding $V_{xx}$ and $V_{yy}$ will also remove a negligible component of the EoR signal via the same mechanism. In a sense, the $Q \rightarrow I$ leakage can be thought of as a...
rotation of power between the two Stokes parameters. Hence, for precision measurements of the EoR signal, this simple estimate may not be ideal. However, the effect of the $I \rightarrow Q$ leakage is small (compare the levels of $V_Q$ and high-$k$ modes of $V_I$) and should not provide a significant hinderance to detection.

3.4. Correlated Polarization Vectors

The results of the previous section were intended to extrapolate previous measurements to low fluxes and investigate the effects of an unresolved forest of dim, polarized point sources. It neglects the known spatial correlations of the RM distribution (Kronberg & Newton-McGee 2011). Furthermore, the random drawing of polarization angles could have a canceling effect on the visibilities. This neglect could potentially suppress our estimation of polarized leakage into the power spectrum.

To investigate the possible effects of correlating the polarization vector, we include a treatment of the simulation where we choose RMs from the Oppermann map (Oppermann et al. 2012), with a pointing center at the Galactic south pole—a reasonable field for EoR analysis. We then set all polarization angles to zero, maximally correlating polarization vectors, while still including information of the polarized sky. All other simulation parameters are identical to simulation A of Table 1. Figure 12 compares the results of this treatment with simulation A of the previous section. The power spectrum of this treatment agrees with simulation A at all redshifts and values of $k$ for both polarizations. This agreement indicates that spatial correlations in RM and polarization angle do not significantly affect polarized leakage into the power spectrum. Thus the assumption of the previous section that the polarization vectors are spatially uncorrelated does not affect the results of this paper.
We conclude our discussion of the simulation results by noting the large variance in the simulated power. The results shown are the median band-powers in \( \Delta^2(k) \) for 500 realizations of the simulation. Taking so many realizations into account essentially maps out the posterior distribution of the \( \Delta^2(k) \) bandpowers. The 1\( \sigma \) width covers nearly an order of magnitude, which indicates the level of Faraday leakage is highly sensitive to the exact parameters drawn in any realization. The actual level of leakage measured will thus be highly dependent on a choice of field, and on cosmic variance.

6. CONCLUSION

We have predicted the three-dimensional power spectrum of polarized emission around 150 MHz to be in the range of \( 10^3 - 10^6 \) mK\(^2\) at \( k_\parallel \sim 0.15 \) h Mpc\(^{-1}\). These predictions were based on simulations motivated by current observations
of the polarized sky at 150 MHz and 1.4 GHz. An elliptical beam provides one mechanism for this power to leak into a measurement of the unpolarized signal. Using a fiducial model of the PAPER beam, we estimated this leakage to be in the thousands of mK², several orders of magnitude above the expected 21 cm EoR signal. Modeling and removing polarized sources may eliminate much of this leakage, but these simulations suggest the amount of removal required far exceeds reasonable capabilities of current instruments.

Work is currently underway to measure frequency structure of polarized power, to investigate the amount of its leakage into the I power spectrum, and to better characterize the polarized radio sky using existing PAPER data.

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