Modeling and optimal control of motion of cotton harvesting machines MX-1.8 and hitching systems of picking apparatus under vertical oscillations

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Abstract. Equations of motion of cotton harvesting machines (CHM) MX-1.8 and hitching system of harvesting devices under vertical oscillations are derived in the paper. On the basis of obtained equations of motion the models and algorithms of optimum control of cotton harvesting machine CHM MX-1.8 are developed. The necessary conditions for optimal control of CHM MX-1.8 motion are investigated by applying the Pontryagin maximum principle. The values of vertical oscillation of CHM MX-1.8 and vertical and torsion oscillations of the hitching system of harvesting machines in the process of motion over the roughness on the headland of cotton fields are determined. The drawbacks of the design of hitching systems of CHM MX-1.8, i.e. the non-uniform distribution of force in the shaft of the swinging mechanism of the hitching are revealed.

1. Introduction

Construction of mathematical models of complex objects, diagnostics and design are associated, in many ways, with the qualification and erudition of the researcher or the person making the decision. At present engineering models, do not describe the real dynamic processes that occur when machines operate. The real dynamic processes with which an engineer or designer deals are very complex, and they are difficult to analyze in full extent. The whole complexity lies in the fact that to obtain a solution, the model must be simple enough and at the same time it should reflect the essence of the problem, so that the results obtained would have a real physical meaning.

In work [1], the main results and experimental data of studies on the stabilization of the position of harvesters during the movement of a cotton picker in a furrow groove are presented. In [2] the main characteristics of the rolling of an individual wheel, the relationship between the factors determining the rolling condition, the rolling kinematics of the rigid wheel of wheeled tractors are considered. In [3] the laws of motion of a wheeled vehicle as a mechanical system, the dependencies characterizing these laws, and the operational properties of wheeled vehicles are set forth. Based on the obtained equations of motion [4] and [5], models and algorithms for optimal control of the steering wheels of a cotton picker MX-1.8 are proposed. In work [6] methods of modeling and optimal control of movement of a cotton picker MX-1.8 and rolling conditions are given. Horizontal and vertical oscillations, parameters of transient processes with optimal traffic control by unevenness of the road are determined, and load distributions for the cotton-picking machine wheels are also given. Necessary optimality conditions in the problems considered by us are obtained using the Pontryagin maximum principle, studied in [7]. The paper [8] contains the basic numerical methods for solving extremely
problems, the theoretical justification and brief characteristics of these methods are given. We consider the formulation of the Pontryagin maximum principle, its proof, and the methods for solving the boundary-value problem of the maximum principle.

2. Main part

Taking into account the above and in accordance with the design scheme presented in Fig. 1, a general mathematical model of vertical oscillations of cotton harvesting machine (CHM) MX-1.8 in the course of motion along the roughness on the headland of a cotton field is compiled in the form of Lagrange equations of the second kind, where $b_i$, $c_i$ are the coefficients of viscous resistance and stiffness of the tire of machine wheel, of the shaft of the swinging mechanism of harvesting machines hitching; $m_i$ the distributed mass on the supports of machine and mechanism of harvesting machines hitching; $h_n$ is the height of road roughness; $F_{E_{aq}}$ is the force in the hydraulic cylinder of the hitching mechanism of harvesting machines; $l_1$, $l_2$, $l_3$, $l_4$ and $l_5$ are the distances between supports and roughness; $l_6$, $l_7$ are the length of the lever and the levers of the harvester hitching; $j_{E_{aq}}$ and $j_a$ are the moments of inertia of the levers for connecting the hydraulic cylinder and the hitching of harvesting machines (1).

Where

$b_i$, $c_i$ are the coefficients of viscous resistance and stiffness of the tire of machine wheel, of the shaft of the swinging mechanism of harvesting machines hitching;

$m_i$ the distributed mass on the supports of machine and mechanism of harvesting machines hitching;

$h_n$ is the height of road roughness;

$F_{E_{aq}}$ is the force in the hydraulic cylinder of the hitching mechanism of harvesting machines;

$l_1$, $l_2$, $l_3$, $l_4$ and $l_5$ are the distances between supports and roughness;

$l_6$, $l_7$ are the length of the lever and the levers of the harvester hitching;

$j_{E_{aq}}$ and $j_a$ are the moments of inertia of the levers for connecting the hydraulic cylinder and the hitching of harvesting machines.

To solve the problem, the theory of optimal systems is used. The statement of the problem of optimal control is given.

At the initial time, the test object is in the state

$$q_i(0) = q_{0i}(0), \quad \dot{q}_i(0) = \dot{q}_{0i}(0), \quad V_i(0) = V_{0i}(0).$$

It is required to select a control $u(t)$ that will transfer the test object to a predetermined final state.
\[ q_i(t) = q_0(t), \quad \dot{q}_i(t) = \dot{q}_0(t), \quad V_i(t) = V_0(t) \quad (i = 1, n), \quad 0 \leq t \leq T. \tag{3} \]

**Figure 1.** Design scheme (CHM) of the dynamic model MX-1.8: 2 is lever for connecting hydraulic cylinder; 3 is swinging shaft; 4 is levers for hitching the harvesting machines; 5 is harvesting machines.

**Figure 2.** Top view of the hitching mechanism of harvesting machine: 1 is hydraulic cylinder.
In this case (figure 1), it is required that the time of the transfer process be of the least value \[4-8\]. Then the goal of control is reduced to minimizing the functional with \[q=x_i, \quad q=y_i\]

\[J(q_0,u(t),q(t)) = \int_{t_0}^{T} f^0(q(t),u(t),t)dt + g^0(q_0,q(T)) .\]  

(4)

Under conditions (2)-(5)

\[\dot{q}(t) = f(q(t),u(t),t).\]  

(5)

Let the functions be

\[g^i(q_0,q(T)) \leq 0, \quad i=1,...,m; \quad g^i(q_0,q(T)) = 0, \quad i = m+1,...,s,\]  

(6)

\[u \in U, \quad t_0 \leq t \leq T,\]  

(7)

where \(f(q(t),u(t),t)\) are continuously differentiable functions with their derivatives; \(u(t)\) is a piecewise-continuous function on the interval \([t_0, T]\).

In the conditions of machines testing under given operating conditions, the speed performance can be a quality criterion. When investigating the necessary conditions for optimal control, the Pontryagin maximum principle is used \[7,8\]. To formulate the maximum principle, the Hamilton-Pontryagin function is introduced

\[H = (q,u,t,\psi_1,\psi_0) = -f^0(q,u,t) + \langle \psi, u \rangle\]  

(8)

and conjugate system

\[\begin{align*}
\frac{d\psi_1}{dt} &= -\frac{\partial H}{\partial y_1} = -m^1_y(c_1 - c_2)\psi_2, \\
\frac{d\psi_2}{dt} &= -\frac{\partial H}{\partial y_2} = -\psi_1 + m^0_y(b_1 - b_2)\psi_2, \\
\frac{d\psi_3}{dt} &= -\frac{\partial H}{\partial y_3} = -(m_1 + m_2)^{-1}c_1\psi_2, \\
\frac{d\psi_4}{dt} &= -\frac{\partial H}{\partial y_4} = -\psi_1 + (m_1 + m_3)^{-1}b_2\psi_2, \\
\frac{d\psi_5}{dt} &= -\frac{\partial H}{\partial y_5} = -(m_2 - m_3)^{-1}c_2\psi_2, \\
\frac{d\psi_6}{dt} &= -\frac{\partial H}{\partial y_6} = -\psi_1 + (m_2 - m_3)^{-1}b_2\psi_2.
\end{align*}\]  

(9)

with restriction on control \(|u| \leq 1\).

To solve the problem under consideration the following necessary condition must be satisfied:

\[H(q_i(t),u(t),t,\psi_y,\psi_0) = \max_{u \in U} H(q_i(t),u(t),\psi_y(t),\psi_0).\]  

(10)

Turning to the definition of optimal control on the basis of (8), we formulate the function

\[\begin{align*}
y_1 &= y_1, \quad \dot{y}_1 = y_2, \quad \dot{y}_2 = u - m^1_y[b_1(\dot{y}_u - \dot{y}_1) - c_1(y_u - y_1)] - b_2(\dot{y}_u - \dot{y}_2) - c_2(y_u - y_2), \\
y_2 &= y_3, \quad \dot{y}_3 = y_4, \quad \dot{y}_4 = (m_1 + m_2)^{-1}[b_1(\dot{y}_u - \dot{y}_1) + c_1(y_u - y_1)] - u, \\
y_3 &= y_5, \quad \dot{y}_5 = y_6, \quad \dot{y}_6 = (m_2 - m_3)^{-1}[b_2(\dot{y}_u - \dot{y}_2) + c_2(y_u - y_2)] - u.
\end{align*}\]  

(11)
If \( f^0 = 1, \ g^0 = 0 \), then \( J(q_0, u(t), q(t)) = T - t_0 \) — in this case, the problem is called the problem of speed performance. The object under consideration is a stationary system and problem (4) means that \( f \text{ и } U \) do not depend explicitly on time, i.e.

\[
f(t, q, u) = f(q, u), \quad U(t) = U.
\]

If stationary problems (4), (12) have an optimal control \( u(t) \) and an optimal trajectory \( q_0(t) \), then there exists a nonzero vector of conjugate variables \((\psi_1(t), \psi_2(t))\), \(\psi(t) \in \mathbb{R}^n\), satisfying the conditions (2), i.e., the condition of the maximum is satisfied (10)

\[
\psi_0(t) = \text{const} \leq 0.
\]

Since the conjugate system (9) is homogeneous with respect to \( \psi_i \), one can arbitrarily choose a constant in equation (13) so that

\[
\psi_0(t) = -1, \quad 0 \leq t \leq T.
\]

From condition \( \max H \) it follows that \( u = \text{sign} \psi_2 \) at \( \psi_2 \neq 0 \). Then, the boundary-value problem of the maximum principle can be written in the following form

\[
\begin{align*}
\dot{y}_2 &= \text{sign} \psi_2 - m_u^{-1}[b_1(\dot{y}_u - \dot{y}_1) - c_1(y_u - y_1) - b_2(\dot{y}_u - \dot{y}_2) - c_2(y_u - y_2)] \\
\dot{y}_4 &= (m_1 + m_2)^{-1}[b_1(\dot{y}_u - \dot{y}_1) + c_1(y_u - y_1)] - \text{sign} \psi_1 \\
\dot{y}_6 &= (m_2 - m_1)^{-1}[b_2(\dot{y}_u - \dot{y}_2) + c_2(y_u - y_2)] - \text{sign} \psi_2
\end{align*}
\]

The boundary-value problem of the maximum principle in these cases will consist of system (15), boundary conditions (2) and (3), following from (10), and condition (14). Draw up the Hamilton-Pontryagin function, which has the following form [4–8]:

\[
\begin{align*}
H_1 &= \psi_s + \psi_y y + \psi_{\dot{y}} \dot{y} \\
H_1 &= \psi_s + \psi_y y + \psi_{\dot{y}} \dot{y} \\
H_1 &= \psi_s + \psi_y y + \psi_{\dot{y}} \dot{y}
\end{align*}
\]

From condition (10) it is clear that the function \( u = \text{sign} \psi_2 \), \( \psi_2 \neq 0 \). is selected. The boundary value problem (15) in this case consists of

\[
H_i = -f^0 u + \psi_2(t)u_0.
\]

Proceed to the investigation of (9), (17) in domain

\[
u_k = \text{sign} \psi_2(t) = \begin{cases} 1, & \psi_2(t) > 1 \\ -1, & \psi_2(t) < 1, \quad k = 2, 4, ..., 2n, \end{cases}
\]

that is, the control \( u_k(t) \) can have only one switching point.

Thus, from the Pontryagin maximum principle, the structure of optimal control of the motion of guiding wheels of a cotton picker is obtained.
To determine auxiliary functions (9) by a numerical method, the conjugate system with variation of
design parameters \( b_i, c_i, j_i \) is studied. The systems (1), (9), (15) are solved using the numerical Runge-Kutta methods. The control \( u_k(t) \), which delivers the maximum of function (10), is defined in domain (18). Computational experiment was carried out with the following values of the parameters:

- \( c_1 = 1672402.9 \text{ N/m} \);
- \( b_1 = 127826.974 \text{ Nf/m} \);
- \( c_2 = 850075.086 \text{ N/m} \);
- \( b_2 = 64973.9 \text{ Nf/m} \);
- \( c_3 = 419679.144 \text{ Nm/rad} \);
- \( b_3 = 64195.86 \text{ Nf/m} \);
- \( m = 7714 \text{ kg} \);
- \( m_1 = 5114 \text{ kg} \);
- \( m_2 = 2600 \text{ kg} \);
- \( m_3 = 1262 \text{ kg} \);
- \( m_a = 500 \text{ kg} \);
- \( j_{\text{гц}} = 42.05 \text{ Nmf}^2 \);
- \( j_{\text{а}} = 204.8 \text{ Nmf}^2 \);
- \( r_1 = 0.7835 \text{ m} \);
- \( r_2 = 0.415 \text{ m} \);
- \( h_{\text{п}} = 0.07 \text{ m} \);
- \( h_{\text{п}} = 0.03 \text{ m} \);
- \( V_{\text{м}} = 1.21 \text{ m/s} \);
- \( F_0 = 17065 H \);
- \( F_y = F_w \sin \frac{2\pi V_{\text{м}}}{l_5} t = 14842 H \);
- \( F_{\text{г}} = 1140 H \).

As a result, graphical dependences of velocities and accelerations for horizontal oscillations of a
cotton picker are obtained, maximum values of H-function (Figure 2-4,5).

![Figure 3. Graphs of transient processes: 1,3,5,7,9,11 are velocities \( \dot{y}_1, \dot{y}_2; \) 2,4,6,8,10,12,14 are accelerations \( \ddot{y}_1, \ddot{y}_2; \) auxiliary functions 13,15 are \( \psi_1, \psi_2; \) 17,19 are \( \psi \); 14,16 are \( \psi \); 18,20 are \( \psi \); 21,22 are \( H \) functions under vertical oscillations of cotton harvesting machine MX-1.8 at 1,2,5,6,9,10,13,14,17,18,21 - \( u(t) = +1 \); at 3,4,7,8,11,12,15,16,19,20,22 - \( u(t) = -1 \).]
3. Conclusions
Thus, the uniformity of the machine's motion depends on the mass and parameters of the driven axles, the values of which are determined by numerical solution of system (1) and conjugate system (9) with the variation of motion parameters $F_i$, $M_i$ and design parameters $F_i$, $M_i$ for given road roughness.

Physical meaning of the results obtained can be formulated as follows. If conditions (2) - (7) are satisfied at the initial moment of time, then the optimal speed performance is achieved with the
following controls. On the time interval \([t_0, t]\), the transfer force \(u_\varphi(t) = +1\) has a maximum value. Hence, on the segment \([t_0, t]\) the mode is a full-forward and the speed of the machine will increase to \(V_m = 1\text{m/s}\), and at this moment the wheels of the machine rise to the upper edge of the roughness. On the interval \([t, T]\) the machine descends, and at this moment the transfer force switches to \(u_\varphi(t) = -1\), i.e. the mode is a full-back, providing uniformity of motion of guiding wheels of a cotton harvester.

The results obtained by solving mathematical models of vertical oscillations of CHM MX-1.8 and hitch systems in the course of motion along the roughness on the headland of a cotton field are in satisfactory agreement with the experimental data [1]. It is determined that the left-side and right-side harvesters oscillate unevenly under vertical oscillations of machine. The main reason for the uneven oscillation of harvesters is an installation of a lever that connects the hydraulic cylinder, on the left-side edge, and not on the middle of swinging shaft.

4. References

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