Applying Stochastic Network Calculus to 802.11 Backlog and Delay Analysis

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Abstract—Stochastic network calculus provides an elegant way to characterize traffic and service processes. However, little effort has been made on applying it to multi-access communication systems such as 802.11. In this paper, we take the first step to apply it to the backlog and delay analysis of an 802.11 wireless local network. We found the derived bounds are very loose in comparison to ns-2 simulation results especially for heavy arriving traffic, indicating that improvements are needed for the current version of stochastic network calculus.

I. Introduction

Network calculus provides an elegant way to characterize traffic and service processes of network and communication systems. Unlike traditional queueing analysis in which one has to make strong assumptions on arrival or service processes (e.g., Poisson arrival process, exponential service distribution, etc) so as to derive closed-form solutions, network calculus allows general arrival and service processes. Instead of getting exact solutions, network calculus derives network delay and backlog bounds. Stochastic network calculus was proposed to deal with stochastic arrival and service processes [1] [2] [3] [4].

There have been some applications of stochastic network calculus [5] [6] [7] [8]. However, little effort has been made on applying it to multi-access communication systems. In the paper, we take the first step to apply stochastic network calculus to an 802.11 wireless local network (WLAN). We found the derived bounds are very loose in comparison to ns-2 simulation results especially for heavy arriving traffic load. We discuss that the reasons are due to the relaxation problem in the current stochastic network calculus.

II. Stochastic Network Calculus Model of an 802.11 Node

We adopt virtual backlog centric (v.b.c) stochastic arrival curve and weak stochastic service curve in our analysis for they are the very recent results [4]. The utility is that if we can characterize the traffic by a v.b.c stochastic arrival curve and the server’s service process by a weak stochastic service curve, then we can calculate backlog and delay bounds of the network element.

Consider a wireless node. Let \( A(t) \) denote the total traffic arrived at the node from the application layer from the time 0 to \( t \). Suppose \( A \) is \((\sigma_A(\theta_1), \rho_A(\theta_1))\)-upper constrained. From Lemma 3.3 and Theorem 5.1 in [4], \( A \) has a v.b.c stochastic arrival curve \( A \sim_{vb} f, \alpha >, \) where

\[
\alpha(t) = r_A \cdot t
\]

\[
f(x) = \frac{e^{\theta_1(x)} \cdot \sigma_A(\theta_1)}{1 - e^{\theta_1(\rho_A(\theta_1) - r_A)}} \cdot e^{-\theta_1 x}
\]

for any \( r_A > \rho_A(\theta_1) \).

We model the transmission capability of a wireless node by a stochastic strict server. Let the channel capacity be \( c \) traffic unit per slot. The departure process \( A^* (s, t) = \beta(s, t) - I(s, t) \) during any backlogged period \( (s, t) \), where \( \beta(t) = c \cdot t \) is the ideal service curve and \( I \) is the impairment process due to backoff, channel sharing and transmission errors. Suppose \( I \) is \((\sigma_I(\theta_2), \rho_I(\theta_2))\)-upper constrained. Again, we have \( I \sim_{vb} g, \gamma >, \) where

\[
\gamma(t) = r_I \cdot t
\]

\[
g(x) = \frac{e^{\theta_2(x)} \cdot \sigma_I(\theta_2)}{1 - e^{\theta_2(\rho_I(\theta_2) - r_I)}} \cdot e^{-\theta_2 x}
\]

for any \( r_I > \rho_I(\theta_2) \).

From Lemma 4.2 in [4], the node provides a weak stochastic service curve \( S \sim_{ws} g, \beta >, \) where

\[
\beta(t) = (c - r_I) \cdot t,
\]

for any \( c > r_I \).

When \( \alpha(t) < \beta(t) \), from Theorem 3.5 in [4], the backlog bound at time \( t \) is

\[
P\{B(t) > x\} \leq f \otimes g(x).
\]

Otherwise, a trivial bound \( P\{B(t) > x\} \leq f \otimes g(-\infty) = \infty \) returns. The delay bound at time \( t \) \( D(t) \) often returns trivial result \( P\{D(t) > x\} \leq 1 \). This is a mistake found in this theorem.

III. Deriving 802.11 Backlog and Delay Bounds

For simplicity, we assume there are \( n \) identical 802.11 nodes sending packets to an access point. All nodes operate in Distributed Coordination Function (DCF) mode with RTS/CTS turned off. We consider an ideal channel of constant channel

\[\text{[Footnotes]}\]
capacity and transmission errors are only caused by collisions. Two packets are collided if their transmissions overlap in time. For simplicity, we assume that all DATA packets are of the same size.

A. Service Curve

We measure traffic amount in packets. For simplicity, we measure time duration (e.g. SIFS, DIFS, DATA and ACK) in idle slots and define that one slot (i.e., \( t = 1 \)) in the calculation of network calculus is equal to \( L \) idle slots, where

\[
L = DIFS + DATA + SIFS + ACK. \tag{5}
\]

In practice, it is difficult to calculate the impairment process \( I \) accurately since \( I \) depends on the complex interactions of traffic arrival and DCF. Here, we perform the worst case analysis and assume the backlog at each node is always nonzero (saturate condition). Based on a existing model of 802.11 [9]. Nevertheless, this model was shown be very accurate in predicting throughput when \( n \) is large. Let \( \tau \) denote the transmission attempt probability per idle slot by a node and let \( \gamma \) denote the conditional collision probability given that there is a transmission. We can solve \( \tau \) and \( \gamma \) by [9]. We define the following notations for an 802.11 node. The probability of no transmissions at an idle slot, denoted by \( P_{nt} \), is \((1-\tau)^n\). The probability of having at least one transmission at an idle slot, denoted by \( P_s \), is \(\tau(1-\gamma)\). For the given node, the probability of the other nodes starting transmissions at an idle slot, denoted by \( P_o \), is \( P_s-P_t = \gamma \).

An 802.11 node can be seen as a stochastic strict server. Clearly, the stochastic strict service curve \( \beta(t) = t \), which means that one packet is transmitted during one slot in the ideal case. In order to characterize the impairment process \( I \), we should first derive \( Ee^{\beta I(s,s+t)} \). Then we can get \( \rho(\theta) \) and \( \sigma(\theta) \) from the equation \( \frac{1}{\theta} \log(Ee^{\beta I(s,s+t)}) \leq \rho(\theta) t + \sigma(\theta) \) [4].

We consider the transmissions of a given 802.11 node during \( t \) slots. From Eq. (5), there are \( tL \) idle slots in \( t \) slots, indexed \( 1, 2, \ldots, tL \). At the first or the last idle slot, the transmission (if any) can be incomplete, that is, the transmission can start before the first idle slot or it can end after the last idle slot. We assume that at the first slot there is always a complete transmission by any of the other nodes except the given node, which actually overestimates \( Ee^{\beta I(s,s+t)} \). Suppose there are \( i \) complete transmissions and zero or one incomplete transmission within the remaining \((t-1)L \) idle slots. The incomplete transmission occupies the last \( k \) idle slots where \( 0 \leq k \leq L-1 \) (\( k = 0 \) means that the last transmission is actually complete). Thus, there are \((t-i-1)L-k \) idle slots of no transmissions within the \( t \) slots. The probability of \( i \) complete transmissions and the last \( k \) idle slots occupied by an incomplete transmission, denoted by \( p_{k,i} \), is \( C_{k}^{t-i-1}L-k-1 \cdot \frac{t-i-1}{t} \cdot \frac{k}{L} \cdot P_{nt}^{t-i-1}L-k \cdot P_{o}^{i} \). Furthermore, the probability of the given node having \( j \) successful complete transmissions on condition that there are \( i \) complete transmissions, denoted by \( p_{i,j} \), is \( C_{i}^{t} \cdot (P_{s}/P_{t})^{j} \cdot (P_{o}/P_{t})^{i-j} \). Finally, we have \( Ee^{\beta I(s,s+t)} \) is upper-bounded by

\[
P_{t} \cdot \sum_{k=1}^{L-1} \sum_{i=0}^{t-2} \sum_{j=0}^{t-i-1} p_{k,i} p_{i,j} e^{\beta(t-j)} + \sum_{i=0}^{t-1} \sum_{j=0}^{t-i} p_{o,i} p_{i,j} e^{\beta(t-j)}. \tag{6}
\]

The first term is for the case that the last transmission is incomplete and the second term is for the case that the last transmission is complete.

In general, we do not have an analytical form of Eq. (6), so we resort to numerical methods to get \( \sigma(\theta) \) and \( \rho(\theta) \) (see Appendix A-1 in [10], which is immediately inspired from the definition of \( \rho(\theta) \) and \( \sigma(\theta) \)).

B. Backlog and Delay Bounds

We can immediately calculate backlog bounds by applying Eq. (1)-(3) into Eq. (4). The only technical issue is to select proper \( \theta_1 \) and \( \theta_2 \) to minimize the bounds. We have the following optimization problem:

\[
P\{B(t) > x\} \leq \min[f \otimes g(x)]
\]

subject to

\[
r_A > \rho_A(\theta_1), r_i > \rho_i(\theta_2) \quad \text{and} \quad r_A \leq 1 - r_I, \tag{7}
\]

where \( r_A \leq 1 - r_I \) is because \( \alpha(t) \leq \beta(t) \) to give a non-trivial backlog bound (see Section II). We resort to numerical methods to solve it (see Appendix A-2 in [10]).

We proved in [10]: as long as the average arriving rate is less than the average service rate, we can always derive finite backlog bounds by stochastic network calculus.

The delay bound in Theorem 3.5 [4] often returns trivial results (see Section II). We propose the following way to estimate delay bound. By Little’s law and Markov’s inequality to the above equation,

\[
P\{\lim_{t \to \infty} D(t) \geq x\} \leq \lim_{t \to \infty} \frac{EB(t)}{\lambda x}. \tag{8}
\]

And \( EB(t) = \sum_{i=0}^{t} P\{B(t) > i\} \cdot (i + 1) \). We can use this way to estimate delay bound when \( t \) is sufficiently large.

IV. Performance Evaluation

In this section, we use ns-2 simulations to verify our derived backlog and delay bounds for Poisson traffic arrivals. We carry out all experiments for the scenario of 10 transmitting nodes. We set the payload of a DATA packet to be 256 bytes. Each simulation duration is 100 seconds which is long enough to let a node transmit thousands of packets. Each data point in the plots is averaged over 100 independent simulations.

Let \( \lambda \) be the average traffic rate (packets/slot). For Poisson traffic, we have

\[
Ee^{\beta A(s,s+t)} = \sum_{i=0}^{\infty} \frac{(\lambda t)^{i}}{i!} e^{-\lambda t} e^{\beta t} = e^{\lambda t(e^{\beta t}-1)}. \tag{9}
\]

So Poisson traffic is \((0, \frac{\lambda (e^{\beta t}-1)}{\beta})\)-upper constrained and we can obtain its arrival curve by Eq. (1).

We set \( \lambda = 0.04 \) \((\lambda = 0.07)\) to simulate low (high) traffic load. Fig. 1 and Fig. 2 show the comparison of simulation results and the bounds of network calculus at \( t = 50\)s which
is sufficiently large to get to the steady state. Note that we plug \( EB(t) \) of the simulation results into Eq. (8) in order to verify its effectiveness.

![Fig. 1. Experiment 1 (\( \lambda = 0.04 \)). When \( t = 50s \) (a) \( P\{B(t) > x\} \) by simulation (b) upper bound of \( P\{B(t) > x\} \) by network calculus (c) \( P\{D(t) \geq x\} \) by simulation (d) upper bound of \( P\{D(t) \geq x\} \) by Eq. (8)](image)

We attribute the main reason to the problem of stochastic network calculus itself, because we see the bounds in Experiment 2 is even looser when the system is almost saturated and the worst-case analysis is valid!

V. Conclusion and Future Work

In this paper, we presented a stochastic network calculus model of 802.11. We derived the backlog and delay bounds of an 802.11 node by using the theorems of stochastic network calculus [4]. And we carried out ns-2 simulations to verify these bounds. We observe that the derived bounds are very loose. We discuss the reason is due to the relaxations in the theory of stochastic network calculus, calling for further improvements on it.

Although network calculus is designed for the calculation of service guarantee in computer networks, we note that its idea is quite general: given a place (e.g., a warehouse), if we can characterize the upper bound of goods arrival and the lower bound of service capability of the place, we can predict the upper bound of backlog in the place. In the future, we want to expand the usage of network calculus to other applications in real life.

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