Robust finite-horizon filtering for nonlinear time-delay Markovian jump systems with weighted try-once-discard protocol

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ABSTRACT
In this paper, the network-based robust $H_{\infty}$ filtering problem is investigated for a class of nonlinear delayed Markovian jump systems subject to packet dropouts. To regulate the transmission order of multiple sensor nodes, the Weighted Try-Once-Discard (WTOD) protocol is adopted to avoid data collision and improve transmission accuracy and efficiency. Via the scheduling of WTOD protocol, only one sensor node is allowed to get access to the shared communication network at each step to send data. The purpose of the addressed problem is to design a network-based filter such that, the prespecified $H_{\infty}$ disturbance attenuation level is guaranteed with the help of scheduling of WTOD protocol. Sufficient conditions are established for the existence of desired filter where the filter parameters are obtained by solving the proposed series of recursive linear matrix inequalities (RLMIs). Finally, an illustrative example is given to demonstrate the effectiveness of the presented filter design method.

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1. Introduction
As a class of stochastic hybrid systems, Markovian jump systems (MJSs) have been extensively utilized to model the dynamic systems with random abrupt variations of system structure (Boukas, 2006; Shi, Mahmoud, Nguang, & Ismail, 2006; Wu, Shi, & Gao, 2010). Consequently, the control and filtering problems for Markovian jump systems have attracted persistent research attentions and fruitful results have been so far available in the literature, see, e.g. Shi, Boukas, and Agarwal (1999), Zhang and Boukas (2009), Chen, Niu, and Zou (2013), Chen, Niu, and Zou (2014), Xu, Lam, and Mao (2007) and Ma, Wang, Han, and Liu (2017). Among various of filter design algorithms, the $H_{\infty}$ filtering approach has been exploited to handle systems subject to external noise signals with bounded energy but unknown statistics (Ma, Wang, Han, & Lam, 2018; Wang, Wang, Han, & Wei, 2018; Zhang et al., 2017; Zhang, Wang, Ding, & Liu, 2015). For instance, in Chen et al. (2013), the adaptive sliding mode control problem has been investigated for a class of Markovian jump systems with actuator degradation. The mode-dependent $H_{\infty}$ filter design problem has been solved in Zhang and Boukas (2009) for Markovian jump linear systems with partly unknown transition probabilities.

In recent years, networked systems have stirred much attention from both theoretical research and industrial application due to the advantages of low cost, wireless transmission and high reliability, and accordingly, the relevant control and filtering issues have become hot research directions (Chen, 1990; Ding, Wang, Ho, & Wei, 2017; Ding, Wang, Shen, & Dong, 2015b; Ding, Wang, Shen, & Wei, 2015; Ma, Wang, & Lam, 2017; Ma, Wang, Lam, & Kyriakoulis, 2017; Shen, Wang, & Qiao, 2017; Sun, Xie, & Xiao, 2008; Yuan, Wang, & Guo, 2017; Yuan, Yuan, Wang, Guo, & Yang, 2017; Zhu, Hua, & Wang, 2008). In a networked system, the components (e.g. sensors, filter, and controller) are connected through a shared network transmission channel. In comparison to the traditional point-to-point connection systems, the introduction of the transmission network inevitably brings a lot of network-induced phenomena, such as packet dropouts (Dong, Lam, & Gao, 2011; Sun, Xie, Xiao, & Soh, 2008; Wen, Wang, Hu, Liu, & Alsaadi, 2018), signal quantization (Chen, Wang, Qian, & Alsaadi, 2018; Dong, Wang, Ding, & Gao, 2015b; Gao & Chen, 2007; Geng, Wang, Liang, Cheng, & Alsaadi, 2017b; Tian, Yue, & Peng, 2008; Zhang, Ma, & Liu, 2016; Zhang, Wang, & Ma, 2016), communication delay (Ding et al., 2015b; Zhu et al., 2008), fading channels (Ding, Wang, Shen, & Dong, 2015a; Dong, Wang, Ding, & Gao, 2015a; Geng, Wang, Liang, Cheng, & Alsaadi, 2017a; Tian, Yue, & Peng, 2008; Zhang, Ma, & Liu, 2016), to name...
but a few. These network-induced phenomena lead to additional difficulties in analysis/synthesis problems and therefore have attracted considerable research interest. Based on the event-triggered communication mechanism, in Dong et al. (2015a), the $H_\infty$ filtering problem is researched for a class of nonlinear time-varying systems with fading channels and multiplicative noises.

It should be pointed out that most of existing results concerning the filtering problems of networked systems are based on the assumption that all sensor nodes could simultaneously send measurement signals via the shared network channel. However, in real-world communication network, due to the limited network bandwidth, the simultaneous transmission of measurement signals from multiple sensor nodes to the shared transmission network will inevitably cause data collisions (Zhang, Yu, & Feng, 2011; Zou, Wang, Gao, & Alsaadi, 2017; Zou, Wang, Hu, & Gao, 2017). Therefore, the aforementioned assumption is unreasonable and communication protocols are needed to determine the transmission order of multiple nodes, with which only one node is permitted to send signal to the shared network at each transmission instant. So far, a lot of communication protocols have been proposed and widely used in practical engineering, such as the Round-Robin protocol (Ugrinovskii & Fridman, 2014), the Weighted Try-Once-Discard (WTOD) protocol (Donkers, Heemels, van de Wouw, & Hetel, 2011; Walsh, Ye, & Bushnell, 2002; Zou, Wang, & Gao, 2016a) and stochastic communication protocol (Tabbara & Nesic, 2008; Zou, Wang, & Gao, 2016b). Compared with the other two transmission protocols, WTOD protocol is relatively complex and difficult to be analyzed and designed. Nevertheless, as a dynamic protocol, it could choose the most valuable measurement signal from multiple sensor nodes. Recently, the control and filtering problems of networked systems under the scheduling of WTOD protocol have appeared in some initial results (Donkers et al., 2011; Zou et al., 2016a). For instance, in Donkers et al. (2011), under the scheduling of WTOD protocol, a switched linear system approach has been proposed to analyze the stability of networked control systems. The set-membership filter has been designed in Zou et al. (2016a) for linear mixed time-delay system under WTOD protocol and Round-Robin protocol. Unfortunately, when WTOD protocol is considered, the network-based $H_\infty$ filtering problem for Markovian jump systems is still an open and challenging issue. As such, to shorten such a gap, the WTOD protocol is utilized in this research for the networked filtering problem of Markovian jump systems.

On the other hand, as is well known, packet dropout is a common network-induced phenomenon (Dong et al., 2011), which may degrade the filtering performance of networked systems. As such, taking the phenomenon of packet dropouts into consideration will make the research result more practical. However, the coupling between WTOD protocol and packet dropout is a key issue, which will make the networked filtering problem become complex. This motivates our research interest and the coupling problem will be solved in the following work.

It is well known that, time-delays and nonlinearities widely exist in actual system and have great influence on system performance (Shen, Wang, & Tan, 2018). The control and filtering problems for networked systems with time-delays and nonlinearities have been studied in a large amount, see e.g. Cai, Wang, Xu, Liu, and Alsaadi (2015), Ma, Wang, and Lam (2017), Dong, Wang, Shen, and Ding (2016), Ma, Wang, Liu, and Alsaadi (2017) and Lyu and Bo (2017) and the references therein. Summarizing the above discussions, an interesting and challenging research problem is clear, that is, investigating the $H_\infty$ filtering problem for nonlinear Markovian jump systems with packet dropout under the scheduling of WTOD protocol.

Motivated by the above discussions, the network-based $H_\infty$ finite-horizon filtering problem is investigated for a class of discrete time-varying nonlinear Markovian jump systems with packet dropouts and time-delay under the scheduling of WTOD protocol. Sufficient conditions are established for the existence of the designed filter in terms of the feasibility of a series of recursive linear matrix inequalities (RLMIs). A simulation example is presented to show the effectiveness of the proposed method. The main contributions of this paper are highlighted as follows:

1. The $H_\infty$ filtering problem is, for the first time, researched for discrete-time nonlinear Markovian jump systems under the scheduling of WTOD protocol.
2. For the purpose of describing the transmission of measurement signals from multiple sensor nodes to the remote filter, a unified network transmission framework combining the WTOD protocol and packet dropouts is constructed and described in a mathematical way.
3. The impact from the WTOD protocol on the filter parameters is considered during the process of filter design.
4. The filter design algorithm is addressed and the filter gain matrices can be obtained by solving the corresponding RLMIs.

The rest of this paper is organized as follows. In Section 2, the networked $H_\infty$ filtering problem for a class
of nonlinear Markovian jump systems with time-delay under the scheduling of WTOD protocol is introduced and formulated. In Section 3, several useful lemmas are listed and sufficient conditions guaranteeing the existence of the desired $H_{\infty}$ filter are derived. The filter design algorithm under the scheduling of WTOD protocol is solved in Section 4 and a numerical example is given in Section 5 to demonstrate the proposed method. Finally, we conclude in Section 6.

**Notation.** The notation used here is fairly standard except where otherwise stated. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $I_2[0, N - 1]$ is the space of square-summable vector functions over $[0, N - 1]$. $\mathbb{E}[x]$ and $\mathbb{E}[x \mid y]$, respectively, stand for the expectation of the stochastic variable $x$ and expectation of $x$ conditional on $y$. $\|x\|$ describes the Euclidean norm of a vector $x$. $A^T$ represents the transpose of $A$. $I_n$ denotes the identity matrix of $n$ dimensions. The notation $X \geq Y$ (respectively, $X > Y$) where $X$ and $Y$ are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). diag($F_1, F_2, \ldots$) stands for a block-diagonal matrix whose diagonal blocks are given by $F_1, F_2, \ldots$. The Kronecker delta function $\delta(l)$ is a binary function that equals 1 if $l = 0$ and 0 otherwise. The symbol $*$ in a matrix means that the corresponding term of the matrix can be obtained by symmetric property.

**2. Problem formulation and preliminaries**

**2.1. The weighted try-once-discard (WTOD) communication protocol**

The communication network under the scheduling of WTOD protocol is described as follows. Consider a networked system with $L$ nodes and the set of nodes is labeled as $\mathcal{G} = \{1, 2, \ldots, L\}$. In this system, the shared communication network is utilized to transmit signals in which only one node is allowed to get access to the network at each transmission instant. Let $\zeta(k) \in \mathcal{G}$ be the selected node which could send signals to the communication network at time instant $k$.

Under the scheduling of WTOD protocol, the value of $\zeta(k)$ is determined as follows:

$$\zeta(k) := \arg\max_{1 \leq l \leq L} (y_l(k) - y^*_l(k))^T W_l (y_l(k) - y^*_l(k)) \tag{1}$$

where $y^*_l(k)$ represents the last successfully transmitted signal before time instant $k$ of node $l$ and $W_l(l \in \mathcal{G})$ is a known positive definite weight matrix of the $l$th node. In practical engineering, $W_l(l \in \mathcal{G})$ can be determined on the basis of the physical meaning of measurement output.

Define $y(k) \triangleq [y_1^T(k), y_2^T(k), \ldots, y_L^T(k)]^T$ and $y^*(k) \triangleq [(y^*_1(k))^T, (y^*_2(k))^T, \ldots, (y^*_L(k))^T]^T$. We rewrite (1) as

$$\zeta(k) = \arg\max_{1 \leq l \leq L} (y(k) - y^*(k))^T \tilde{W}_l (y(k) - y^*(k)) \tag{2}$$

where $\tilde{W} = \text{diag}[W_1, W_2, \ldots, W_L]$, $\tilde{W}_l = \tilde{W}\Phi_l$, $\Phi_l = \text{diag} \{\delta(l - 1), \delta(l - 2), \ldots, \delta(l - L)\}$ $(1 \leq l \leq L)$ and $\delta(\cdot) \in \{0, 1\}$ is the Kronecker delta function.

**Remark 2.1:** Considering the existence of the network-induced packet dropouts, the WTOD protocol is modified accordingly. It is assumed that the information of packet dropouts could be transmitted from the filter end to the protocol end. Therefore, in the scheduling rule (2) of WTOD protocol, $y^*(k)$ is not the last transmitted signal before time instant $k$, but the last successfully transmitted signal before time instant $k$, which is different from Zou et al. (2016a).

**2.2. Problem formulation**

Let $r(k) \in [0, N]$ be a Markov chain taking values in a finite state space $S = \{1, 2, \ldots, M\}$ with transition probability matrix $\Phi = [\lambda_{ij}]$ given by

$$\text{Prob}[r(k + 1) = j \mid r(k) = i] = \lambda_{ij}, \quad \forall i, j \in S$$

where $\lambda_{ij} \geq 0 (i, j \in S)$ is the transition probability from $i$ to $j$ and $\sum_{j=1}^M \lambda_{ij} = 1 (i \in S)$.

In this paper, we consider the following class of stochastic nonlinear time-delay Markovian jump system defined on $k \in [0, N]$:

$$x(k + 1) = (A(k, r(k)) + \Delta A(k, r(k)))x(k) + (A_d(k, r(k)) + \Delta A_d(k, r(k))) \times x(k - d) + \alpha(k)g(x(k)) + B(k, r(k))w(k)$$

$$y(k) = C(k, r(k))x(k) + D(k, r(k))v(k)$$

$$z(k) = L(k, r(k))x(k)$$

where $x(k) \in \mathbb{R}^{n_x}$ represents the state vector, $y(k) \in \mathbb{R}^{n_y}$ is the process output, $z(k) \in \mathbb{R}^{n_z}$ is the signal to be estimated, $w(k) \in \mathbb{R}^{n_w}$ and $v(k) \in \mathbb{R}^{n_v}$ are the external disturbance signals that belong to $I_2[0, N - 1]$ and $g(\cdot, \cdot) : \mathbb{R}^+ \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ is nonlinear vector function. $d > 0$ is a known positive scalar. For fixed system mode, $A(k, r(k))$, $A_d(k, r(k)), C(k, r(k)), B(k, r(k)), D(k, r(k))$ and $L(k, r(k))$ are known real-valued time-varying matrices with appropriate dimensions. For simplicity, for fixed system mode $r(k) = i$, a matrix $U(i, k)$ will be denoted by $U_i(k)$, $U = \{A, A_d, C, B, D, L\}$. 

The real-valued matrices $\Delta A_i(k)$ and $\Delta A_{di}(k)$ represent the norm-bounded parameter uncertainties of the following structure

$$[\Delta A_i(k) \quad \Delta A_{di}(k)] = H_i(k)F(k)[E_{Ai}(k) \quad E_{di}(k)]$$  \hspace{1cm} (4)

where $H_i(k), E_{Ai}(k)$ and $E_{di}(k)$ are known real time-varying matrices and $F(k)$ is an unknown matrix satisfying the following condition

$$F^T(k)F(k) \leq I$$  \hspace{1cm} (5)

The stochastic variable $\alpha(k)$ in (3) is introduced to account for the random nature of the occurrence of the nonlinearity, which is a Bernoulli distributed white sequence taking values on 0 or 1 with

$$\text{Prob}[\alpha(k) = 1] = \mathbb{E}\{\alpha(k)\} = \bar{\alpha},$$

$$\text{Prob}[\alpha(k) = 0] = 1 - \bar{\alpha}$$  \hspace{1cm} (6)

where $\bar{\alpha} \in [0, 1]$ is a known constant.

Similar to Dong, Wang, and Gao (2012) and Zhang, Ma, et al. (2017), the nonlinear function $g(k,x(k))$ is assumed to satisfy $g(k,0) = 0$ and, for arbitrary $\sigma(k) \in \mathbb{R}^n,$

$$\|g(k,x(k) + \sigma(k)) - g(k,x(k))\| \leq \|G(k)\sigma(k)\|$$  \hspace{1cm} (7)

where $G(k)$ is a known matrix.

Next, the signal transmission of the measurement output $y(k)$ via the protocol-based network will be introduced. To this end, we divide the sensors of the system into $L$ sensor nodes according to their spatial distribution. Therefore, the measurement output $y(k)$ can be rewritten as

$$y(k) = [y^T_1(k) \quad y^T_2(k) \quad \cdots \quad y^T_L(k)]^T$$

where $y_l(k) (l \in \mathbb{N})$ is the measurement of the $l$th sensor node before transmission. The communication network is scheduled by WTOD protocol, where $\zeta(k) \in \mathbb{N}$ is the selected sensor node obtaining access to the communication network at time instant $k$. The value of $\zeta(k)$ is determined by the scheduling of WTOD protocol.

As is well known, owing to various reasons, the broadcast innovation could face packet dropouts. In view of this, the stochastic variable $\beta(k)$ is introduced to cater for the phenomenon of packet dropouts. $\beta(k)$ is a Bernoulli distributed white sequence which is uncorrelated to $\alpha(k)$ and takes values on 0 or 1 with

$$\text{Prob}[\beta(k) = 1] = \mathbb{E}\{\beta(k)\} = \bar{\beta},$$

$$\text{Prob}[\beta(k) = 0] = 1 - \bar{\beta}$$  \hspace{1cm} (8)

where $\bar{\beta} \in [0, 1]$ is a known constant.

The measurement output after transmission through the network is denoted as

$$\tilde{y}(k) = [\tilde{y}^T_1(k) \quad \tilde{y}^T_2(k) \quad \cdots \quad \tilde{y}^T_L(k)]^T$$

The initial state of $\tilde{y}(k)$ is assumed to be $\tilde{y}(j) = \tilde{y}_0$ for $j < 0$ where $\tilde{y}_0$ is a known vector. When WTOD protocol and the phenomenon of packet dropouts are considered, the updating rule of $\tilde{y}_l(k) (l \in \mathbb{N}^+)$ is set to be

$$\tilde{y}_l(k) = \begin{cases} \begin{array}{l} (1 - \beta(k))\tilde{y}_l(k-1) + \beta(k)y_l(k), \quad \text{if } l = \zeta(k) \\ \tilde{y}_l(k-1), \quad \text{otherwise} \end{array} \end{cases}$$  \hspace{1cm} (9)

According to (9), we have

$$\tilde{y}(k) = \beta(k)\Phi_{\zeta(k)}y + (I_{ny} - \beta(k)\Phi_{\zeta(k)})\tilde{y}(1 - \beta(k)), \quad j < 0$$  \hspace{1cm} (10)

**Remark 2.2:** For time-varying Markovian jump system, the dynamic scheduling characteristics of WTOD protocol may bring effective filtering performance. The WTOD protocol will be executed before the data transmission and the phenomenon of packet dropouts is occurred during the process of measurement transmission. In (10), the unified measurement transmission model for a networked system with WTOD protocol and packet dropouts is established, where the phenomenon of packet dropouts is described by the commonly used Bernoulli sequence $\beta(k)$ and the scheduling rule of WTOD protocol is explained in (2). Based on this measurement transmission model, the $H_{\infty}$ filtering problem under the scheduling of WTOD protocol will be investigated in the following part.

Letting $\eta(k) = [x^T(k) \quad \tilde{y}^T(k - 1) - \tilde{y}(k)]^T$ and $\tilde{w}(k) = [w^T(k) \quad \nu^T(k)]^T$, system (3) with WTOD protocol scheduling can be reformulated as follows:

$$\begin{align*}
\eta(k + 1) &= (\bar{A}_i(k) + \bar{\beta}(k)\bar{A}_i(k))\eta(k) \\
&\quad + \bar{A}_{di}(k)\eta(k - d) + \alpha(k)\tilde{g}(k,\eta(k)) \\
&\quad + (\bar{B}_i(k) + \bar{\beta}(k)\bar{B}_i(k))\tilde{w}(k) \\
\tilde{y}(k) &= (\bar{C}_i(k) + \bar{\beta}(k)\bar{C}_i(k))\eta(k) \\
&\quad + (\bar{D}_i(k) + \bar{\beta}(k)\bar{D}_i(k))\tilde{w}(k) \\
z(k) &= \bar{L}(k)\eta(k)
\end{align*}$$  \hspace{1cm} (11)
where

\[
\begin{align*}
\tilde{A}_i(k) &= \begin{bmatrix} A_i(k) + \Delta A_i(k) & \tilde{\beta} \Phi \xi(k) C_i(k) \\ \Phi \xi(k) C_i(k) & I_n - \tilde{\beta} \Phi \xi(k) \end{bmatrix}, \\
\tilde{A}_d(k) &= \begin{bmatrix} A_{di}(k) + \Delta A_{di}(k) & 0 \\ 0 & 0 \end{bmatrix}, \\
\tilde{B}_i(k) &= \begin{bmatrix} B_i(k) & 0 \\ 0 & \tilde{\beta} \Phi \xi(k) D_i(k) \end{bmatrix}, \\
\tilde{B}_d(k) &= \begin{bmatrix} 0 & 0 \\ 0 & \Phi \xi(k) D_i(k) \end{bmatrix}, \\
\tilde{B}_i(k) &= \begin{bmatrix} \tilde{B}_i(k) \\ 0 \end{bmatrix}, \\
\tilde{C}_i(k) &= \begin{bmatrix} \tilde{\beta} \Phi \xi(k) C_i(k) \\ \Phi \xi(k) C_i(k) \end{bmatrix}, \\
\tilde{C}_d(k) &= \begin{bmatrix} 0 \\ \tilde{\beta} \Phi \xi(k) D_i(k) \end{bmatrix}, \\
\tilde{D}_i(k) &= \begin{bmatrix} 0 \\ \Phi \xi(k) D_i(k) \end{bmatrix}, \\
\tilde{L}_i(k) &= \begin{bmatrix} L_i(k) & 0 \end{bmatrix}.
\end{align*}
\]

\[
\tilde{g}(k, \eta(k)) = \begin{bmatrix} g(k, x(k)) \\ 0 \end{bmatrix},
\]

where

\[
\begin{align*}
\tilde{\beta}(k) &= \beta(k) - \tilde{\beta}, \\
\tilde{C}_i(k) &= \begin{bmatrix} \tilde{\beta} \Phi \xi(k) C_i(k) \\ \Phi \xi(k) C_i(k) \end{bmatrix}, \\
\tilde{C}_d(k) &= \begin{bmatrix} 0 \\ \tilde{\beta} \Phi \xi(k) D_i(k) \end{bmatrix}, \\
\tilde{D}_d(k) &= \begin{bmatrix} 0 \\ \Phi \xi(k) D_i(k) \end{bmatrix}, \\
\tilde{D}_i(k) &= \begin{bmatrix} 0 \\ \tilde{\beta} \Phi \xi(k) D_i(k) \end{bmatrix}, \\
\tilde{L}_i(k) &= \begin{bmatrix} \tilde{L}_i(k) & -\tilde{L}_d(k) \end{bmatrix}.
\end{align*}
\]

2.3. Time-varying filter

In this paper, a mode-dependent filter based on the signal \(\tilde{y}(k)\) is considered for the augmented system (11), which is of the form

\[
\begin{align*}
\dot{x}(k+1) &= A_i(k) \dot{x}(k) + B_i(k) \tilde{y}(k) \\
\tilde{z}(k) &= L_i(k) \dot{x}(k)
\end{align*}
\]

(12)

where \(\tilde{z}(k) \in \mathbb{R}^n\) represents the estimate of \(z(k)\), and \(\dot{x}(k) \in \mathbb{R}^n\) is the estimate of the output \(z(k)\). For fixed system mode \(r(k) = i\), the time-varying matrices \(A_i(k), B_i(k)\) and \(L_i(k)\) are filter gain matrices with appropriate dimensions to be determined.

By letting \(\tilde{\eta}(k) = \begin{bmatrix} \eta^T(k) \tilde{x}^T(k) \end{bmatrix}^T, \tilde{z}(k) = z(k) - \tilde{z}(k)\), we subsequently obtain the following augmented system to be investigated:

\[
\begin{align*}
\tilde{\eta}(k+1) &= (A_i(k) + \tilde{\beta}(k), A_i(k)) \tilde{\eta}(k) \\
&\quad + \alpha(k) \tilde{g}(k, \tilde{\eta}(k)) \\
&\quad + A_{d}(k) E \tilde{\eta}(k) - d \\
&\quad + (B_i(k) + \tilde{\beta}(k), B_i(k)) \tilde{w}(k) \\
\tilde{z}(k) &= L_i(k) \tilde{\eta}(k)
\end{align*}
\]

(13)

where

\[
\begin{align*}
\tilde{A}_i(k) &= \begin{bmatrix} \tilde{A}_i(k) & 0 \\ B_i(k) C_i(k) & A_i(k) \end{bmatrix}, \\
\tilde{A}_d(k) &= \begin{bmatrix} \tilde{A}_d(k) & 0 \\ 0 & 0 \end{bmatrix}, \\
\tilde{g}(k, \tilde{\eta}(k)) &= \begin{bmatrix} \tilde{g}(k, \eta(k)) \\ 0 \end{bmatrix}, \\
\tilde{A}_{d}(k) &= \begin{bmatrix} \tilde{A}_{d}(k) \\ 0 \end{bmatrix}, \\
E &= \begin{bmatrix} I & 0 \end{bmatrix}, \\
B_i(k) &= \begin{bmatrix} B_i(k) D_i(k) \end{bmatrix}, \\
\tilde{B}_i(k) &= \begin{bmatrix} \tilde{B}_i(k) \\ B_i(k) D_i(k) \end{bmatrix}, \\
\tilde{L}_i(k) &= \begin{bmatrix} \tilde{L}_i(k) & -\tilde{L}_d(k) \end{bmatrix}.
\end{align*}
\]

Our aim in this paper is to design a time-varying filter of the form (12) such that, under the scheduling of WTOD protocol, for the given positive scalar \(\gamma\), the matrices \(U_i(0) > 0 (i \in S), U(j) > 0 (j = -1, -2, \ldots, -d)\) and the initial state \(x_0\) and \(\tilde{y}_0\), system (13) satisfies the following \(H_\infty\) filtering performance requirement:

\[
J : = \mathbb{E}[\frac{1}{2} \|\tilde{z}(k)\|^2 - \gamma^2 \|\tilde{w}(k)\|^2] - \gamma^2 \tilde{\eta}^T(0) U(0) \tilde{\eta}(0) \\
- \gamma^2 \sum_{j=-d}^{-1} \tilde{\eta}^T(j) E^T U(j) E \tilde{\eta}(j) \\
\leq 0, \quad \forall (\tilde{w}(k), x_0) \neq 0
\]

(14)

for all stochastic nonlinearities, where

\[
\|\tilde{z}(k)\|^2 = \sum_{k=0}^{N-1} \|\tilde{z}(k)\|^2, \quad \|\tilde{w}(k)\|^2 = \sum_{k=0}^{N-1} \|\tilde{w}(k)\|^2.
\]

3. Analysis of \(H_\infty\) performances

First of all, we introduce the following lemmas which will be used in this paper.

Lemma 3.1: (Schur complement Boyd, Ghaoui, Feron, & Balakrishnan, 1994): Given constant matrices \(S_1, S_2\) and \(S_3\), where \(S_1 = S_1^T\) and \(0 < S_2 = S_2^T\), then

\[
\begin{bmatrix}
S_1 & S_2^T \\
S_3 & -S_2
\end{bmatrix} < 0
\]

or

\[
\begin{bmatrix}
-S_2 & S_3 \\
S_2^T & S_1
\end{bmatrix} < 0
\]

(15)

Lemma 3.2 (S-procedure Boyd et al., 1994): Let \(N = N^T, H\) and \(E\) be real matrices of appropriate dimensions, and \(F^T F \leq I\). Then inequality \(N + HE + (HE)^T < 0\) if and
only if there exists a positive scalar $\varepsilon$ such that $N + \varepsilon HH^T + \varepsilon^{-1} E^T E < 0$ or, equivalently,

$$
\begin{bmatrix}
N & \varepsilon H & E^T \\
\varepsilon H^T & -\varepsilon I & 0 \\
E & 0 & -\varepsilon I
\end{bmatrix} < 0
$$

(16)

**Theorem 3.1:** Consider system (3), the WTOD protocol given by (1) and the filter (12). Let the disturbance attenuation level $\gamma > 0$, the filter gain matrices $\{A_i(k)\}_{k \in [0,N-1]}$, $\{B_i(k)\}_{k \in [0,N-1]}$ and $\{L_i(k)\}_{k \in [0,N-1]}$ be given. For given $U_i(0) > 0$ $(i \in S)$ and $U_j(0) > 0$ $(j = -1, -2, \ldots, -d)$, the $H_\infty$ performance requirement defined in (14) is achieved for all nonzero $\tilde{w}(k)$ if there exist sequences of positive definite matrices $\{P_i(k)\}_{k \in [0,N]}$, $\{Q_i(k)\}_{k \in [0,N-d]}$ and positive scalars $\{\varepsilon_i(k)\}_{k \in [0,N-1]}$ and $\{\alpha_i(k)\}_{k \in [0,N-1], i \in \mathbb{B}}$ satisfying the following recursive matrix inequalities:

$$
\begin{align*}
\hat{\Gamma}_{11}(k) & = \begin{bmatrix}
\hat{\Gamma}_{11i}(k) & \hat{\Gamma}_{12}(k) & \hat{\Gamma}_{13}(k) \\
\ast & \hat{\Gamma}_{22}(k) & 0 \\
\ast & \ast & \ast
\end{bmatrix} \\
& \leq 0
\end{align*}
$$

(3a^2 + 3\alpha)P_i(k + 1) \leq 1 \leq \varepsilon_i(k)I \quad (\forall i \in S)

(19)

with the initial condition

$$
\tilde{\eta}^T(0)(P_i(0) - \gamma^2 U_i(0))\tilde{\eta}(0)
+ \sum_{l=-d}^{-1} \tilde{\eta}^T(l)E^T Q(l) - \gamma^2 U(l)E\tilde{\eta}(l) \leq 0
$$

(20)

where

$$
\hat{\Gamma}_{11i}(k) = 
\begin{bmatrix}
-P_i(k) + \varepsilon_i(k)G_i^T(k)\hat{G}(k) + E^T Q(k)E \\
- \sum_{l=1}^L \rho_i(l) V_i^T(l) \tilde{W}_{i,k}(l) V_i(l) \\
0 \\
- \sum_{l=1}^L \rho_i(l) V_i^T(l) \tilde{W}_{i,k}(l) \hat{D}_i(k) \\
Q(k - d) \\
- \gamma^2 I
\end{bmatrix}
$$

(21)

**Proof:** Define the following function for system (13)

$$
\begin{align*}
V(k, r(k)) & := V_1(k, r(k)) + V_2(k, r(k)) \quad (21)
\end{align*}
$$

where

$$
\begin{align*}
V_1(k, r(k)) & = \tilde{\eta}^T(k)P_{r(k)}(k)\tilde{\eta}(k) \\
V_2(k, r(k)) & = \sum_{j=k-d}^{k-1} \tilde{\eta}^T(j)E^T Q(j)E\tilde{\eta}(j)
\end{align*}
$$

(22)

with $P_{r(k)}(k)$ and $Q(k)$ satisfying (18) and (19). Then, along the trajectory of system (13), we have

$$
\mathbb{E}\{\Delta V | r(k) = i\} = \mathbb{E}\{V(k + 1, r(k + 1)) | r(k) = i\} - V(k, r(k))
$$

(22)

Calculating $\mathbb{E}\{\Delta V | r(k) = i\}$ along the trajectory of system (13), we have

$$
\mathbb{E}\{\Delta V | r(k) = i\}
$$

(22)
\[ + \eta^T (k - d) A_{dl}^T (k) \tilde{P}_1 (k + 1) A_{dl} (k) \eta (k - d) \]
\[ + 2 \bar{\alpha} \eta^T (k - d) A_{dl}^T (k) \tilde{P}_1 (k + 1) \tilde{g}(k, \tilde{h}(k)) \]
\[ + 2 \eta^T (k - d) A_{dl}^T (k) \tilde{P}_1 (k + 1) \tilde{w}(k) \]
\[ + \hat{\alpha} \tilde{g}^T (k, \tilde{h}(k)) \tilde{P}_1 (k + 1) \tilde{g}(k, \tilde{h}(k)) \]
\[ + 2 \bar{\alpha} \tilde{g}^T (k, \tilde{h}(k)) \tilde{P}_1 (k + 1) B_i (k) \tilde{w}(k) \]
\[ + \tilde{\omega}^T (k) (B_i^T (k) \tilde{P}_1 (k + 1) B_i (k) \]
\[ + \hat{\beta} (1 - \hat{\beta}) B_i^T (k) \tilde{P}_1 (k + 1) B_i (k) \tilde{w}(k) \]
\[ \leq \mathbb{E} \{ \bar{\eta}^T (k) (2 A_i^T (k) \tilde{P}_1 (k + 1) A_i (k) + \hat{\beta} (1 - \hat{\beta}) \}
\times A_i^T (k) \tilde{P}_1 (k + 1) A_i (k) - P_i (k) \bar{\eta}(k) \]
\[ + 2 \bar{\eta}^T (k) A_i^T (k) \tilde{P}_1 (k + 1) A_i (k) \eta (k - d) \]
\[ + 2 \bar{\eta}^T (k - d) A_i^T (k) \tilde{P}_1 (k + 1) A_i (k) \eta (k - d) \]
\[ + 2 \bar{\eta}^T (k - d) A_i^T (k) \tilde{P}_1 (k + 1) \tilde{w}(k) \]
\[ + 2 \tilde{\omega}^T (k) (2 B_i^T (k) \tilde{P}_1 (k + 1) B_i (k) \]
\[ + \hat{\beta} (1 - \hat{\beta}) B_i^T (k) \tilde{P}_1 (k + 1) B_i (k) \tilde{w}(k) \} \]  \( (23) \)

Considering inequality (19) and the nonlinear constraint condition (7), we have

\[ (3 \bar{\alpha}^2 + \bar{\alpha}) \tilde{g}^T (k, \tilde{h}(k)) \tilde{P}_1 (k + 1) \tilde{g}(k, \tilde{h}(k)) \]
\[ \leq \varepsilon_1 (k) \tilde{g}^T (k, \tilde{h}(k)) \tilde{g}(k, \tilde{h}(k)) \]
\[ \leq \varepsilon_1 (k) \bar{\eta}^T (k) \tilde{g}^T (k, \tilde{h}(k)) \tilde{g}(k, \tilde{h}(k)) \]  \( (24) \)

Next, it can be derived that

\[ \mathbb{E} \{ \Delta V | r (k) = i \} \]
\[ = \mathbb{E} \{ V_2 (k + 1, r (k + 1)) | r (k) = i \} - V_2 (k, r (k)) \]
\[ = \mathbb{E} \{ \eta^T (k) Q (k) \eta (k) - \eta^T (k - d) Q (k - d) \eta (k - d) \} \]  \( (25) \)

Letting

\[ \xi (k) = \begin{bmatrix} \bar{\eta}^T (k) & \eta^T (k - d) & \tilde{w}^T (k) \end{bmatrix}^T \]

and combining (23), (24) and (25) result in

\[ \mathbb{E} \{ \Delta V | r (k) = i \} \leq \mathbb{E} \{ \xi^T (k) \hat{\Omega}_i (k) \xi (k) \}. \]  \( (26) \)

where

\[ \hat{\Omega}_i (k) = \begin{bmatrix} \hat{\Omega}_{11i} (k) & A_{di}^T (k) \tilde{P}_1 (k + 1) A_{di} (k) \]
\[ \ast \]
\[ - Q (k - d) \]
\[ \hat{\Omega}_{13i} (k) \]
\[ A_{di}^T (k) \tilde{P}_1 (k + 1) A_{di} (k) \]
\[ \ast \]
\[ \hat{\Omega}_{33i} (k) \]  \( \gamma^2 I \)

Moreover, it follows from the above inequality (28) that

\[ J = \mathbb{E} \{ \| \tilde{z} (k) \|^2 - \gamma^2 \| \tilde{w} (k) \|^2 \} \]
\[ - \gamma^2 \tilde{\eta}^T (0) U (0) \tilde{h} (0) \]
\[ - \gamma^2 \sum_{j=-d}^{-1} \tilde{\eta}^T (j) E^T U (j) E \tilde{h} (j) \]
According to the initial condition (20), it is easy to obtain that
\[ V(0, r(0)) - \gamma^2 \bar{\eta}_T(0) U_l(0) \bar{\eta}(0) \]
which can be written in terms of \( \bar{\eta}_T(0) U_l(0) \bar{\eta}(0) \) (30)

Now, to prove \( J \leq 0 \), we are in the position to derive \( \mathbb{E}\{ \sum_{k=0}^{N-1} \xi^T(k) \tilde{\Omega}_i(k) \xi(k) \} \leq 0 \). By analyzing the scheduling mechanism of packet dropouts-constrained WTOD protocol, we obtain that, for any \( l \in \mathbb{P} \)
\[ (y(k) - y^*(k))^\top (\bar{W}_i - \bar{W}_\xi(k)) (y(k) - y^*(k)) \leq 0 \] (31)
which can be written in terms of \( \bar{\eta}_T(0) U_l(0) \bar{\eta}(0) \) (32)

where
\[ \bar{U}_l(k) = [V_l(k) \ 0 \ \tilde{D}_l(k)] \]

According to Lemma 3.3, if there exist \( \rho_1(k), \rho_2(k), \ldots, \rho_L(k) > 0 \) such that
\[ \tilde{\Omega}_i(k) - \sum_{l=1}^L \rho_l(k) \bar{U}_l(k)^\top (\bar{W}_i - \bar{W}_\xi(k)) \bar{U}_l(k) \]
\[ = \Omega_i(k) \leq 0 \] (33)

where
\[ \Omega_i(k) = \begin{bmatrix} \tilde{\Omega}_{11}(k) + L^T_i(k) L_i(k) \\ - \sum_{l=1}^L \rho_l(k) V_l^T(k) \bar{W}_{\xi}(k) V_l(k) \\ * \\ A^T_{dl}(k) \tilde{\rho}_l(k+1) A_{dl}(k) \\ A^T_{dl}(k) \tilde{\rho}_l(k+1) B_{dl}(k) \\ \tilde{\Omega}_{13}(k) - \gamma^2 l \\ - \sum_{l=1}^L \rho_l(k) V_l^T(k) \bar{W}_{\xi}(k) \tilde{D}_l(k) \\ A^T_{dl}(k) \tilde{\rho}_l(k+1) B_{dl}(k) \\ \tilde{\Omega}_{33}(k) - \gamma^2 l \\ - \sum_{l=1}^L \rho_l(k) D^T_l(k) \bar{W}_{\xi}(k) \tilde{D}_l(k) \end{bmatrix} \]

then, inequality (32) implies \( \mathbb{E}\{ \sum_{k=0}^{N-1} \xi^T(k) \tilde{\Omega}_i(k) \xi(k) \} \leq 0 \).

Furthermore, according to Lemma 3.1, it is easy to see that inequality (33) is equivalent to (18).

Summarizing the above derivation, we obtain \( J \leq 0 \). Therefore, the \( H_\infty \) index defined in (14) is guaranteed. This proof is completed. \( \blacksquare \)

**Remark 3.1:** In Theorem 3.1, the impact from the WTOD protocol on the \( H_\infty \) filtering problem for the addressed discrete time-varying Markovian systems has been analyzed. The sufficient conditions have been derived that guarantee the existence of desired filter under the scheduling of WTOD protocol. However, due to the cross coupling of matrices in (18), it is difficult to directly design the desired filter by Theorem 3.1. To eliminate the coupling terms, similar to Zhang and Boukas (2009), the slack matrix variables will be introduced and the corresponding result is shown in Theorem 3.2.

**Theorem 3.2:** Consider system (3). Let the disturbance attenuation level \( \gamma > 0 \), the filter gain matrices \( \{ \tilde{A}_l(k) \}_{k \in [0, N-1]} \), \( \{ \tilde{B}_l(k) \}_{k \in [0, N-1]} \), \( \{ \tilde{C}_l(k) \}_{k \in [0, N-1]} \), and \( \{ \tilde{L}_l(k) \}_{k \in [0, N-1]} \) be given. For given \( U_l(0) > 0 \) \((i \in S)\), \( U_l(0) > 0 \) \((j = -1, -2, \ldots, -d)\), the \( H_\infty \) performance requirement defined in (14) is achieved for all nonzero \( \bar{w}_i(k) \) if there exist sequences of positive definite matrices \( \{ \tilde{P}_l(k) \}_{k \in [0, N-1]} \), sequences of positive scalars \( \{ \tilde{\rho}_l(k) \}_{k \in [0, N-1]} \), and \( \{ \tilde{r}_l(k) \}_{k \in [0, N-1]} \), satisfying the following recursive matrix inequalities:

\[ \Gamma_l(k) = \begin{bmatrix} \tilde{\Gamma}_{11}(k) & \tilde{\Gamma}_{12}(k) & \tilde{\Gamma}_{13}(k) \\ * & \tilde{\Gamma}_{22}(k) & 0 \\ * & * & -I \end{bmatrix} \leq 0 \] (34)

\[ (3\bar{a}^2 + \bar{a})\tilde{P}_l(k+1) \leq \tilde{e}_l(k)_l \] \((\forall i \in S)\) (35)

with the initial condition
\[ \bar{\eta}_T(0)(\tilde{P}_l(0) - \gamma^2 U_l(0)) \bar{\eta}(0) + \sum_{l=-d}^{-1} \tilde{\eta}_T(l) E(l) - \gamma^2 U(l) E \tilde{\eta}_l(l) \leq 0 \] (36)

where
\[ \Gamma_{12}(k) = \begin{bmatrix} \tilde{A}^T_{dl}(k) \tilde{r}_l(k) & \tilde{A}^T_{dl}(k) \tilde{r}_l(k) & 0 \\ \tilde{A}^T_{dl}(k) \tilde{r}_l(k) & \tilde{A}^T_{dl}(k) \tilde{r}_l(k) & \tilde{A}^T_{dl}(k) \tilde{r}_l(k) \\ 0 & \sqrt{\beta(1 - \tilde{\beta})} \tilde{A}^T_{dl}(k) \tilde{r}_l(k) \\ 0 & \sqrt{\beta(1 - \tilde{\beta})} \tilde{A}^T_{dl}(k) \tilde{r}_l(k) \\ \tilde{B}^T_{dl}(k) \tilde{r}_l(k) & \tilde{B}^T_{dl}(k) \tilde{r}_l(k) \end{bmatrix} \].
\[ \mathcal{H}_i(k) = \tilde{P}_i(k+1) - R_i(k) - R_i^T(k), \]
\[ \Gamma_{22}(k) = \text{diag}\{\mathcal{H}_i(k), \ldots, \mathcal{H}_i(k)\} \]

**Proof:** By Theorem 3.1, the augmented system (13) satisfies the $H_\infty$ performance defined in (14) if inequalities (18) and (19) hold with the initial condition (20). Compared with Theorem 3.2, we just need to verify that inequality (18) can be derived from inequality (34).

In fact, from inequality (34), we know that $\tilde{P}_i(k+1) - R_i(k) - R_i^T(k) \leq 0$. According to $\tilde{P}_i(k+1) > 0$, we have $R_i(k) + R_i^T(k) > 0$, which means that $R_i(k)$ are nonsingular matrices.

On the other hand, for arbitrary $R_i(k)$, the following is true:
\[ (\tilde{P}_i(k+1) - R_i(k))^T \tilde{P}_i^{'-1}(k+1)(\tilde{P}_i(k+1) - R_i(k)) \geq 0 \]
which is equivalent to
\[ \tilde{P}_i(k+1) - R_i(k) - R_i^T(k) \geq - R_i^T(k+1) \tilde{P}_i^{'-1}(k+1) R_i(k+1) \]
It follows from inequality (34) that
\[ \tilde{\Gamma}_1(k) = \begin{bmatrix} \hat{\Gamma}_{11}(k) & \hat{\Gamma}_{12}(k) & \hat{\Gamma}_{13}(k) \\ * & \hat{\Gamma}_{22}(k) & 0 \\ * & * & -I \end{bmatrix} \leq 0 (39) \]
where
\[ \mathcal{H}_i(k) = - R_i^T(k+1) \tilde{P}_i^{'-1}(k+1), \]
\[ \Gamma_{22}(k) = \text{diag}\{\mathcal{H}_i(k), \ldots, \mathcal{H}_i(k)\} \]

Performing now a congruence transformation on (39) using \( \text{diag}(I, I, I, \tilde{P}_i^{'-1}(k) \tilde{P}_i(k+1), \tilde{P}_i(k+1), \tilde{P}_i(k+1), \tilde{P}_i(k+1), I, I) \), we can conclude that inequality (18) holds. This proof is completed.  

**Theorem 4.1:** Consider system (3). For the given disturbance attenuation level $\gamma > 0$, $U_i(0) > 0$ ($i \in S$), $U_i(j) > 0$ ($j = -1, -2, \ldots, -d$), the $H_\infty$ performance requirement defined in (14) is achieved for all nonzero $\nu_i(k)$ if there exist a sequence of positive definite matrices $\{Q(k)\}_{k \in \{-d, N-1\}}$ and $\mathcal{H}_i(k) = \{ \epsilon_{1i}(k) \}_{k \in \{0, N-1\}}$, $\mathcal{H}_i(k) = \{ \epsilon_{2i}(k) \}_{k \in \{0, N-1\}}$, and families of real-valued matrices $\{P_{1i}(k)\}_{k \in \{0, N\}}$, $\{P_{2i}(k)\}_{k \in \{0, N\}}$, $\{P_{3i}(k)\}_{k \in \{0, N\}}$, $\{R_{1i}(k)\}_{k \in \{0, N\}}$, $\{R_{12i}(k)\}_{k \in \{0, N\}}$, $\{R_{22i}(k)\}_{k \in \{0, N\}}$, $\{X_i(k)\}_{k \in \{0, N-1\}}$, $\{Y_i(k)\}_{k \in \{0, N-1\}}$ and $\{L_i(k)\}_{k \in \{0, N-1\}}$ satisfying the following recursive matrix inequalities:
\[ P_i(k) = \begin{bmatrix} P_{1i}(k) & P_{3i}(k) \\ * & P_{2i}(k) \end{bmatrix} > 0 (40) \]
\[ \Pi_i(k) = \begin{bmatrix} \Pi_{11i}(k) & \Pi_{12i}(k) & \Pi_{13i}(k) \\ * & \Pi_{22i}(k) & \Pi_{23i}(k) \\ * & * & \Pi_{33i}(k) \end{bmatrix} \leq 0, (41) \]
\[ \begin{bmatrix} (3\bar{\alpha} + \bar{\alpha}) \tilde{P}_1(k+1) - \epsilon_{1i}(k)I \\ * \\ (3\bar{\alpha} + \bar{\alpha}) \tilde{P}_2i(k+1) - \epsilon_{1i}(k)I \end{bmatrix} \leq 0, (\forall i \in S) (42) \]
with the initial condition
\[ \tilde{\eta}^T(0) (P_i(0) - \gamma^2 U_i(0)) \tilde{\eta}(0) \]
\[ + \sum_{l = -d}^{-1} \tilde{\eta}^T(l) E(l) \gamma^2 U(l) E\tilde{\eta}(l) \leq 0 (43) \]
where
\[ \Pi_{11i}(k) = \begin{bmatrix} \Pi^{(1, 1)}_{11i}(k) & -P_{3i}(k) & 0 \\ * & -P_{2i}(k) & 0 \\ * & * & -Q(k-d) \end{bmatrix} \]
\[ \begin{bmatrix} - \sum_{l = 1}^{L} \rho_i(k) \tilde{C}_i^T(l) \tilde{W}_l(k) \tilde{D}_i(k) \\ 0 \\ 0 \end{bmatrix}, \]
\[ \Pi_{12i}(k) = \begin{bmatrix} \Pi^{(1, 1)}_{12i}(k) & 0 & \Pi^{(1, 3)}_{12i}(k) \\ \Pi^{(2, 1)}_{12i}(k) & \Pi^{(2, 2)}_{12i}(k) & \Pi^{(2, 3)}_{12i}(k) \end{bmatrix}, \]
\[ \Pi_{13i}(k) = [\epsilon_{2i}(k) \Pi^{(1, 3)}_{13i}(k) 0], \]
\[ \Pi_{22i}(k) = \text{diag}\{P_i(k), \ldots, P_i(k), I, \ldots, I\}, \]
\[ \Pi_{23i}(k) = [0 \ldots, P_i(k), I, \ldots, I], \]
\[ \Pi_{33i}(k) = \text{diag}\{[-\epsilon_{2i}(k), \ldots, -\epsilon_{2i}(k)]I, \ldots, [-\epsilon_{2i}(k)]I, \ldots, [-\epsilon_{2i}(k)]I\} \]
\[ \Pi_{11}^{(1,1)}(k) = -P_1(k) + \varepsilon_1(k) \tilde{G}_T(k) \tilde{G}(k) + Q(k), \]

\[ -\sum_{l=1}^L \rho_l(k) \tilde{C}_l(k) \tilde{W}_l(k) \tilde{C}_l(k), \]

\[ \Pi_{12}^{(1,1)}(k) = \begin{bmatrix} \tilde{A}_l(k) R_{11}(k) + \tilde{C}_l(k) Y_l^T(k) U \\ \tilde{A}_l(k) R_{12}(k) + \tilde{C}_l(k) Y_l^T(k) U \end{bmatrix}, \]

\[ \Pi_{12}^{(1,3)}(k) = \begin{bmatrix} \sqrt{\beta(1-\beta)}(\tilde{A}_l(k) R_{11}(k) + \tilde{C}_l(k) Y_l^T(k) U) \\ \sqrt{\beta(1-\beta)}(\tilde{A}_l(k) R_{12}(k) + \tilde{C}_l(k) Y_l^T(k) U) \end{bmatrix}, \]

\[ \Pi_{12}^{(2,1)}(k) = \begin{bmatrix} \tilde{A}_l(k) R_{11}(k) + \tilde{C}_l(k) Y_l^T(k) U \\ \tilde{A}_l(k) R_{12}(k) + \tilde{C}_l(k) Y_l^T(k) U \end{bmatrix}, \]

\[ \Pi_{12}^{(2,2)}(k) = \begin{bmatrix} \tilde{A}_l(k) R_{11}(k) + \tilde{C}_l(k) Y_l^T(k) U \\ 0 \\ 0 \end{bmatrix}, \]

\[ \Pi_{12}^{(2,3)}(k) = \begin{bmatrix} \sqrt{\beta(1-\beta)}(\tilde{B}_l(k) R_{11}(k) + \tilde{D}_l(k) Y_l^T(k) U) \\ \sqrt{\beta(1-\beta)}(\tilde{B}_l(k) R_{12}(k) + \tilde{D}_l(k) Y_l^T(k) U) \end{bmatrix}, \]

\[ \Pi_{13}^{(1,1)}(k) = \begin{bmatrix} \bar{\tilde{E}}_A(k) \\ \bar{\tilde{E}}_A(k) \end{bmatrix}, \]

\[ \Pi_{23}^{(1,2)}(k) = \begin{bmatrix} R_{11}(k) \\ R_{22}(k) \end{bmatrix}, \]

\[ \Pi_{23}^{(2,1)}(k) = \begin{bmatrix} R_{11}(k) \tilde{H}_l(k) & R_{12}(k) \tilde{H}_l(k) \\ R_{11}(k) \tilde{H}_l(k) & R_{12}(k) \tilde{H}_l(k) \end{bmatrix}, \]

\[ \Pi_{23}^{(2,2)}(k) = \begin{bmatrix} \tilde{A}_l(k) R_{11}(k) + \tilde{C}_l(k) Y_l^T(k) U \\ \tilde{A}_l(k) R_{12}(k) + \tilde{C}_l(k) Y_l^T(k) U \end{bmatrix}, \]

\[ \Pi_{23}^{(2,3)}(k) = \begin{bmatrix} \tilde{A}_l(k) R_{11}(k) + \tilde{C}_l(k) Y_l^T(k) U \\ \tilde{A}_l(k) R_{12}(k) + \tilde{C}_l(k) Y_l^T(k) U \end{bmatrix}, \]

\[ \bar{P}_1(k) = \begin{bmatrix} \tilde{P}_1(k) - R_{11}(k) - R_{11}^T(k) \\ \tilde{P}_1(k) - R_{12}(k) - U^T R_{22}^T(k) \\ \tilde{P}_1(k) - R_{22}(k) - R_{22}^T(k) \end{bmatrix}, \]

\[ \bar{P}_2(k) = \begin{bmatrix} \tilde{P}_2(k) - R_{12}(k) - U^T R_{22}^T(k) \\ \tilde{P}_2(k) - R_{22}(k) - R_{22}^T(k) \end{bmatrix}, \]

\[ \bar{A}_l(k) = \begin{bmatrix} A_l(k) \\ \beta \Phi_1(k) A_l(k) \\ \beta \Phi_1(k) A_l(k) \end{bmatrix}, \]

\[ \bar{H}_l(k) = \begin{bmatrix} H_l(k) \end{bmatrix}, \]

\[ \bar{E}_l(k) = \begin{bmatrix} E_l(k) \end{bmatrix}, \]

Moreover, if (41) and (42) have feasible solutions, then the system matrix \( L_{fi}(k) \) of the admissible filter in the form of (12) can be directly obtained, and \( A_0(k) \) and \( B_0(k) \) can be obtained by means of the matrices \( X_l(k), Y_l(k) \) and \( R_{22}(k) \) as follows:

\[ A_0(k) = R_{22}^T(k) X_l(k), \]

\[ B_0(k) = R_{22}^T(k) Y_l(k), \]

\[ \textbf{Proof:} \] First, we assume that \( P_i(k) \) and \( R_i(k) \) have the following forms:

\[ P_i(k) = \begin{bmatrix} P_{11}(k) & P_{12}(k) & * \\ P_{21}(k) & P_{22}(k) & \end{bmatrix}, \]

\[ R_i(k) = \begin{bmatrix} R_{11}(k) & R_{12}(k) & * \\ R_{21}(k) & R_{22}(k) & \end{bmatrix}, \]

where \( U \) is a given matrix with appropriate dimension. Then, by defining new matrix variables \( X_i(k) = R_{22}^T(k) A_0(k) \) and \( Y_i(k) = R_{22}^T(k) B_0(k), \) (34) can be written as follows:

\[ \Pi_{11}(k) < 0 \]

where

\[ \Pi_{12}(k) = \begin{bmatrix} \Pi_{12}^{(1,1)}(k) & 0 & \Pi_{12}^{(1,3)}(k) \\ \Pi_{12}^{(2,1)}(k) & \Pi_{12}^{(2,2)}(k) & \Pi_{12}^{(2,3)}(k) \end{bmatrix}, \]

\[ \Pi_{12}^{(1,1)}(k) = \begin{bmatrix} \tilde{A}_l(k) R_{11}(k) + \tilde{C}_l(k) Y_l^T(k) U \\ \tilde{A}_l(k) R_{12}(k) + \tilde{C}_l(k) Y_l^T(k) U \end{bmatrix}, \]

\[ \Pi_{12}^{(2,1)}(k) = \begin{bmatrix} \tilde{A}_l(k) R_{11}(k) + \tilde{C}_l(k) Y_l^T(k) U \\ \tilde{A}_l(k) R_{12}(k) + \tilde{C}_l(k) Y_l^T(k) U \end{bmatrix}, \]

\[ \Pi_{12}^{(2,2)}(k) = \begin{bmatrix} \tilde{A}_l(k) R_{11}(k) + \tilde{C}_l(k) Y_l^T(k) U \\ \tilde{A}_l(k) R_{12}(k) + \tilde{C}_l(k) Y_l^T(k) U \end{bmatrix}, \]

\[ \Pi_{12}^{(2,3)}(k) = \begin{bmatrix} \tilde{A}_l(k) R_{11}(k) + \tilde{C}_l(k) Y_l^T(k) U \\ \tilde{A}_l(k) R_{12}(k) + \tilde{C}_l(k) Y_l^T(k) U \end{bmatrix}, \]

\[ \Pi_{12}^{(2,3)}(k) = \begin{bmatrix} \tilde{A}_l(k) R_{11}(k) + \tilde{C}_l(k) Y_l^T(k) U \\ \tilde{A}_l(k) R_{12}(k) + \tilde{C}_l(k) Y_l^T(k) U \end{bmatrix}, \]

\[ \Pi_{12}^{(3,3)}(k) = \begin{bmatrix} \tilde{A}_l(k) R_{11}(k) + \tilde{C}_l(k) Y_l^T(k) U \\ \tilde{A}_l(k) R_{12}(k) + \tilde{C}_l(k) Y_l^T(k) U \end{bmatrix}, \]

\[ \Pi_{12}^{(3,3)}(k) = \begin{bmatrix} \tilde{A}_l(k) R_{11}(k) + \tilde{C}_l(k) Y_l^T(k) U \\ \tilde{A}_l(k) R_{12}(k) + \tilde{C}_l(k) Y_l^T(k) U \end{bmatrix}, \]
In order to eliminate the parameter uncertainty $\Delta A_i(k)$ and $\Delta A_{di}(k)$ in (46), we rewrite it in the following form:

$$
\begin{bmatrix}
\Pi_{11}(k) & \Pi_{12}(k) \\
\Pi_{21}(k) & \Pi_{22}(k)
\end{bmatrix} + N_i(k)F(k)\mathcal{E}(k) + (N_i(k)F(k)\mathcal{E}(k))^T < 0
$$

(47)

where

$$
N_i(k) = \begin{bmatrix}
\Pi_{13i}(k) \\
0
\end{bmatrix},
\mathcal{E}(k) = \begin{bmatrix}
0 \\
(\Pi_{23i}(k))^T
\end{bmatrix}
$$

$$
F(k) = \text{diag}(F(k), F(k))
$$

According to Lemma 3.2, inequality (47) holds if and only if (41) holds.

On the other hand, it is easy to see that (35) and (36) are equivalent to (42) and (43), respectively. Thus, according to Theorem 3.2, under the scheduling of WTOD protocol, the $H_\infty$ performance requirement of the filtering error system (13) is satisfied with initial conditions (43). This completes the proof.

Remark 4.1: So far, we have solved the filter design problem for a class of discrete time-varying nonlinear Markovian systems under the scheduling of WTOD protocol. For the filtering error dynamics (13), the sufficient conditions have been derived in Theorem 3.1, guaranteeing the existence of the desired $H_\infty$ filter. Furthermore, to eliminate the matrix product, the slack matrix variables have been introduced in Theorem 3.2. Then, the protocol-based filter design method for a class of Markovian jump systems has been proposed in terms of a set of RLMIs in Theorem 4.1.

By means of Theorem 4.1, we can summarize the Protocol-Based Robust $H_\infty$ Filter Design Algorithm (PBRHFDA) as follows.

5. Numerical simulations

In this section, a numerical simulation example is presented to illustrate the effectiveness of the proposed filter design method. The system parameters are given as follows:

**Mode 1:**

$$
A_1(k) = \begin{bmatrix}
0.5 & 0 & 0.1 \\
-0.2 & 0.2 & +0.1 \sin(2k) \\
0 & -0.3 & 0.3
\end{bmatrix}
$$

$$
A_{d1}(k) = \begin{bmatrix}
0.2 & 0.1 & 0 \\
0 & 0.1 + 0.02 \cos(2k) & 0.05 \\
0.06 & 0 & -0.2
\end{bmatrix}
$$

$$
B_1(k) = \begin{bmatrix}
0.2 \\
0.2 + 0.1 \sin(k) \\
0.1
\end{bmatrix}
$$

$$
C_1(k) = \begin{bmatrix}
0.3 & 0.2 & 0.5 + 0.2 \sin(3k) \\
0.2 & 0.15 & 0.3
\end{bmatrix}
$$

$$
D_1(k) = \begin{bmatrix}
0.5 \\
0.2 + 0.1 \sin(k)
\end{bmatrix}
$$

$$
L_1(k) = \begin{bmatrix}
0.1 & 0.15 + 0.05 \sin(3k) & 0.1
\end{bmatrix}
$$

$$
H_1(k) = \begin{bmatrix}
0.1 \\
0.2 \\
0.5
\end{bmatrix},
E_{A1}(k) = \begin{bmatrix}
0.2 & -0.3 & 0.5
\end{bmatrix}
$$

**Mode 2:**

$$
A_2(k) = \begin{bmatrix}
0.5 & 0 & 0.1 \\
0.2 & 0.5 & +0.1 \sin(k) \\
-0.2 & -0.1 & 0.4 + 0.2 \cos(2k)
\end{bmatrix}
$$

$$
A_{d2}(k) = \begin{bmatrix}
0.1 + 0.02 \cos(2k) & 0.1 & -0.1 \\
0 & 0.25 & 0.2 \\
0.1 + 0.02 \cos(2k) & 0.1 & -0.15
\end{bmatrix}
$$

$$
B_2(k) = \begin{bmatrix}
0.2 \\
-0.1
\end{bmatrix},
D_2(k) = \begin{bmatrix}
0.3 \sin(3k) \\
0.2
\end{bmatrix}
$$

$$
C_2(k) = \begin{bmatrix}
0.7 & 0.15 & 0.5 + 0.2 \sin(3k) \\
0.2 & 0.35 & 0.3
\end{bmatrix}
$$

$$
L_2(k) = \begin{bmatrix}
0.2 & 0.15 + 0.1 \sin(3k) & 0.2
\end{bmatrix}
$$

$$
H_2(k) = \begin{bmatrix}
0.2 \\
0.2 \\
0.1
\end{bmatrix},
E_{A2}(k) = \begin{bmatrix}
0.2 & 0.1 & 0.3
\end{bmatrix}
$$

$$
E_{d2}(k) = \begin{bmatrix}
0.2 & 0.1 & 0.1
\end{bmatrix}
$$
Algorithm PBRHFD

Step 1. Given the disturbance attenuation level $\gamma > 0$, the positive definite matrices $U_l(0)i \in S$ and $U_l(i = -d, -d + 1, \ldots, -1)$. Choose initial positive definite matrices $P_l(0) i \in S$ and $Q_l(i = -d, -d + 1, \ldots, -1)$ to satisfy the condition (43) and set $k = 0$.

Step 2. Obtain the values of matrices $\{P_{l0}(k + 1), P_{l1}(k + 1), P_{l2}(k + 1), Q_l(k), R_{10}(k + 1), R_{11}(k + 1), R_{12}(k + 1), X_l(k), Y_l(k)\}$ and the desired filter parameters $A_l(k), B_l(k), L_l(k)$, for the sampling instant $k$ by solving (41) and (42).

Step 3. Set $k = k + 1$ and then $P_{l0}(k) = P_{l0}(k + 1), P_{l1}(k) = P_{l1}(k + 1), P_{l2}(k) = P_{l2}(k + 1), Q(k - d) = Q(k - d + 1)$.

Step 4. If $k < N$, then go to Step 2, else go to Step 5.

Step 5. Stop.

Mode 3:

$$A_3(k) = \begin{bmatrix} 0.5 & 0 & 0.3 \\ 0.1 & 0.35 & 0.3 \\ 0.2 & -0.3 & 0.2 \\ +0.1 \sin(2k) & -0.2 \cos(2k) \end{bmatrix},$$

$$A_{d3}(k) = \begin{bmatrix} 0.12 + 0.02 \cos(2k) & 0.1 & -0.1 \\ 0 & 0.15 & 0.2 \\ 0.1 & 0 & -0.2 \end{bmatrix},$$

$$B_3(k) = \begin{bmatrix} 0.3 \\ -0.1 \\ 0.2 \end{bmatrix}, \quad D_3(k) = \begin{bmatrix} 0.3 + 0.2 \sin(2k) \\ -0.1 \end{bmatrix},$$

$$C_3(k) = \begin{bmatrix} 0.3 & 0.25 & 0.5 + 0.2 \sin(3k) \\ 0.35 & 0.15 & 0.3 \end{bmatrix},$$

$$L_3(k) = \begin{bmatrix} 0.1 & 0.15 & 0.2 + 0.1 \sin(3k) \end{bmatrix},$$

$$H_3(k) = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, \quad E_{a3}(k) = \begin{bmatrix} 0.2 & 0.1 & 0.3 \end{bmatrix},$$

$$E_{d3}(k) = \begin{bmatrix} 0.2 & 0.1 & 0.1 \end{bmatrix}.$$

The nonlinear function $g(k, x(k))$ and the external disturbance signals $w(k), v(k)$ are chosen as

$$g(k, x(k)) = \begin{bmatrix} 0.2x_1 \cos(x_3) \\ 0.5x_2 \\ \frac{x_1^3 + 1}{x_1^3 + 1} \\ 0.1x_3 \sin(2x_2) \end{bmatrix},$$

$$w(k) = \exp(-0.5k) \sin(k),$$

$$v(k) = 2 \exp(-0.4k) \cos(2k)$$

The statistical information of stochastic variable $\alpha(k)$ and $\beta(k)$ are taken as $\bar{\alpha} = 0.5$ and $\bar{\beta} = 0.9$, respectively. Set $d = 2$. The transition probability matrix of the Markov process and matrix $U$ in $R_l(k)$ are chosen as follows:

$$\Phi = \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.3 & 0.5 & 0.2 \\ 0.5 & 0.3 & 0.2 \end{bmatrix},$$

$$U = \begin{bmatrix} -1 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$
Figure 2. Output \( z(k) \) and its estimate.

Figure 3. Estimation error \( z(k) - \hat{z}(k) \).

scheduling is plotted in Figure 4, while ‘1’ represents the measurement output \( y_1(k) \) of the first sensor node is sent to the filter via the shared communication network, and ‘2’ represents the measurement output \( y_2(k) \) of the second sensor node is sent through the shared communication network. Figure 5 plots the realization of the Markovian jump mode \( r(k) \). The \( H_\infty \) performance index is \( J_1 = -1.9795 \). All the simulation results confirm the effectiveness of the proposed protocol-based filter design method that could well achieve the desired filtering requirement.

6. Conclusions

In this paper, we have investigated the robust \( H_\infty \) finite-horizon filtering problem for a class of discrete-time-varying nonlinear Markovian jump systems with packet dropouts and time-delay under the scheduling of WTOD protocol. Considering limited network bandwidth, the WTOD protocol has been introduced, with which only one node is permitted to send measurement signal to the shared communication network at each instant in order to avoid data collisions. Sufficient conditions for the existence of the desired finite-horizon filter satisfying the \( H_\infty \) performance requirement have been presented in terms of the feasibility of a series of RLMIs. Finally, an illustrative example has been set up to confirm the effectiveness of
the networked filtering method under the scheduling of WTOD protocol.

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No potential conflict of interest was reported by the authors.

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