Benefit of Delay on the Diversity-Multiplexing Tradeoffs of MIMO Channels with Partial CSI

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Abstract—This paper re-examines the well-known fundamental tradeoffs between rate and reliability for the multi-antenna, block Rayleigh fading channel in the high signal to noise ratio (SNR) regime when (i) the transmitter has access to (noiseless) one bit per coherence-interval of causal channel state information (CSI) and (ii) soft decoding delays together with worst-case delay guarantees are acceptable. A key finding of this work is that substantial improvements in reliability can be realized with a very short expected delay and a slightly longer (but bounded) worst-case decoding delay guarantee in communication systems where the transmitter has access to one bit per coherence interval of causal CSI. While similar in spirit to the recent work on communication systems based on automatic repeat requests (ARQ) where decoding failure is known at the transmitter and leads to re-transmission, here transmit side-information is purely based on CSI. The findings reported here also lend further support to an emerging understanding that information is purely based on CSI. The findings reported here also lend further support to an emerging understanding that information is purely based on CSI.

I. INTRODUCTION

It is well known that multiple antennas can substantially increase the capacity of a point-to-point fading channel and can also significantly improve the reliability of communications via space diversity. Interestingly, both these gains can be obtained without having any CSI at the transmitter [1], [2]. In fact, even with full non-causal transmit CSI, the ergodic capacity of multi-antenna links cannot be improved substantially at high SNR (specifically, the scaling law of the capacity with SNR remains the same). This motivates the question of whether any knowledge of the channel at the transmitter could improve the reliability of multi-antenna channels.

There is a large body of work in characterizing the reliability gains of multiple-input multiple-output (MIMO) block-fading channels [2]–[4]. When the transmitter has no CSI, Zheng and Tse [1] characterized the tradeoffs between the rate and the reliability exponent at high SNRs by analyzing what is referred to as the outage event and proving that the overall error probability is dominated by the probability of the outage event. Specifically, outage refers to the event that the channel realization during the code length interval is too poor to support the given rate [5]. It turns out that for a channel with coherence interval \( L \), the reliability exponent of a block code of blocklength \( l \) depends only on rate and the number of antennas and does not depend on the code length \( l \) (when \( l \) is larger than the total number of transmit and receive antennas). In this setting, the decoding delay is, of course, equal to the code blocklength.

Any side information which is correlated to the channel conditions when made available to the transmitter, including causal CSI [6] and/or decoder status [7]–[11], can potentially improve the rate and reliability. In particular, it is well known that feedback can substantially improve the error exponent of additive white Gaussian noise (AWGN) channels via variable length coding and power control assuming no strict delays and peak power constraints [7]–[11]. For block-fading channels, in [3] an automatic retransmission request (ARQ) is shown to substantially improve the diversity gain by allowing codeword retransmissions (random variable-length channel codes) with the aid of noiseless one-bit decision feedback and power control. Therefore, in a communication system with feedback, the decoding time \( T \) (\#channel uses) is a random variable depending on \( l \) and the state of the decoder. In order to capture this random decoding delay, Burnashev introduced the notion of error exponents for a feedback code which transmits one of \( M \) messages in expected time at most \( l \) [8].

In this paper, our focus is on the former type of transmitter side information, namely causal CSI. In particular, we assume that the transmitter has access to only one noiseless bit of causal side information per coherence interval from the receiver which describes the channel state as being "good" (above a predecided threshold) or bad (below the threshold). This type of channel state feedback has substantially less coding complexity than that of the ARQ systems as it only requires the knowledge of the channel state and not the status of the decoder. Following [8], we define the reliability function as the smallest error probability that can be achieved by a code of length \( l \) which transmits one of \( \rho^l \) messages in the expected time at most \( l + \epsilon \) where \( \rho \) is the multiplexing gain and \( \epsilon \) tends to zero as the SNR \( \rho \) goes to infinity. By leveraging this side information and the notion of an expected decoding delay, we show that the reliability function can be substantially improved in the high SNR regime. In our scheme,

\[ \text{1 Throughout the paper, we assume the code blocklength } l \text{ is less than or equal to the coherence interval } L \text{ of the channel. Results for } l \geq L \text{ are straightforward generalizations but are omitted.} \]

\[ \text{2 Technically, one distinguishes between causal CSI which is independent of the message and is available at the transmitter before the next transmission and decoder feedback which depends on the channel outputs after a transmission.} \]
transmission occurs only when the channel satisfies a certain condition which can be ascertained at the receiver. Therefore, similar to feedback channels, the decoding delay is random and rate may be reduced as we do not use the channel all the time [3], [9], [11]. However, it turns out that in the regime of high SNR, the channel most likely satisfies the conditions and thus the scaling law of the rate remains unchanged.

When there is no delay constraint and noiseless one-bit causal CSI feedback is available, we show that the outage events can be completely avoided and an error exponent which is as good as that of a pure multi-antenna AWGN channel without fading can be attained. In this case, in sharp contrast to the case where the transmitter has no CSI, the error probability approaches zero exponentially with the blocklength for all multiplexing gains. We further consider the case where the decoding delay has a strict deterministic deadline, that is, a finite maximum decoding delay, in which the receiver has to decode the message in at most $D$ coherence intervals. In this case, it is clear that outage events cannot be avoided and the effects of fading will reappear. However, we show that a substantial gain on the reliability function can be achieved even with a specified finite maximum decoding delay. For instance, in a channel with $M$ transmit antennas, and one receive antenna, $90\%$ of the diversity gain of an AWGN channel can be attained with a worst case delay of $D = \log_M (10/9)$. This shows when $l = 10$ and $M = 3$, we only need a delay of $D = 4$ to completely overcome the effects of fading and approach the performance of MIMO AWGN channels.

This paper is organized as follows: In Section II we present the channel model and introduce the problem set-up. In Section III we obtain the reliability exponent when there is no bound on the worst case delay. In Section IV we analyze the case where the maximum decoding delay is bounded and treat the scenarios with and without a finite peak to average power constraint.

II. BACKGROUND

A. Channel Model

We consider a frequency-flat block-fading MIMO channel with $M$ transmit and $N$ receive antennas. The channel is assumed to remain fixed for $L$ channel uses (the channel coherence interval) and change independently to another state in the next block. Therefore, within each coherence interval, the received signal at time $t$ can be expressed as,

$$y_t = \sqrt{\frac{\rho}{M}} H S_t + W_t, \quad t = 1, \ldots, L$$

where $\rho$ is the average transmit power, $H$ denotes the (fixed) $N \times M$ random fading channel matrix, and for each discrete time $t$, $S_t$ denotes the $M \times 1$ channel input symbol and $W_t$ the $N \times 1$ additive white Gaussian noise vector. All the entries of $H$ and $W_t$ are assumed to have a zero-mean, unit-variance, complex circular Gaussian distribution. Furthermore, the transmit message $S_t$ at each time $t$ is required to satisfy the average power constraint $E(S_t S_t^\dagger) \leq 1$. In all random coding bounds for the reliability function, we assume $S_t$ to be a random Gaussian codeword with blocklength $l \leq L$. The $l > L$ case can also be analyzed but is omitted for brevity.

B. Reliability, Rate: No CSI at the Transmitter

Rate versus reliability tradeoffs have been well studied for point-to-point AWGN channels through the notions of the error exponent (large blocklength) and the diversity gain (large SNR). For fading MIMO channels with no transmit CSI, Zheng and Tse studied the smallest error probability that can be achieved in the asymptotic of high SNR by a code of size $\rho^l$. In this regime, the error probability is characterized by the diversity gain defined as,

$$d(r) = \liminf_{\rho \to \infty} \frac{-\log P_e(R, \rho, l, M, N)}{\log \rho}$$

where $R = \frac{\log \rho^l}{l}$ is the code rate in bits per channel use, $l$ is the code blocklength, and $P_e$ is the smallest achievable error probability for a rate-$R$, blocklength $l$ code. It is further shown that when the transmitter has no CSI, $d(r)$ is given by a piecewise linear function connecting the points $(r, (M - r)(N - r))$ for $r = 0, 1, \ldots, \min(M, N)$. More explicitly,

$$d(r) = \alpha_k - r\beta_k, \quad k - 1 \leq r \leq k,$$

where $1 \leq k \leq \min(M, N)$, $\alpha_k = MN - 2k(M + N) + 3k^2 - k$, and $\beta_k = 2M + 2N - 2k + 1$.

It is interesting to note that unlike AWGN channels, the error probability of fading channels (without transmit CSI) tends to zero with an exponent independent of the code blocklength as long as $l \geq (M + N)$. This is due to the fact that the error probability is dominated by the so-called outage error event: the event of having a sequence of atypically poor channel fading gains over the duration of a codeword. In this setting, in [1] it is shown that the optimal diversity gain can be achieved by using a fixed-length blockcode and when the decoding delay is equal to the length of the codeword blocklength $l$.

C. Reliability, Rate, and Decoding Delay: Transmit CSI

Transmit side information in its broadest sense includes both CSI and the received signal at the decoder (feedback channels). It is folklore that while feedback cannot improve the capacity of a discrete memoryless channel, it can improve the reliability function (see [6], [7] and references therein). The gain in the reliability is achieved by leveraging (i) the possibility of retransmissions while keeping the codeword blocklength fixed, and (ii) using power control to boost the power when rare events happen [7]. This implies that the decoding time $T$ is random and no longer equals the codeword length. For general DMCs, in [8], Burnashev obtained tight bounds on the smallest error probability of a code which transmits one of the messages in expected time at most $l$, that is, $\mathbb{E}(T) \leq l$ and unbounded maximum delay [8], [9]. In the asymptotic of large SNR, Gamal, Caire, and Damen proposed an ARQ scheme and proved that a single noiseless bit of feedback per codeword block pertaining to the status of the decoder can substantially
improve the reliability even when a finite maximum decoding delay constraint is imposed, that is, with a finite total number of retransmissions.

Inspired by these results on feedback channels, we explore the high-SNR asymptotics of the channel reliability function with one noiseless bit of causal CSI at the transmitter. We show that with this limited transmit CSI and for a fixed code-word blocklength, the reliability function can be substantially improved even with a strict finite maximum decoding delay constraint.

Here, the reliability function is defined in terms of the smallest achievable error probability for a blockcode of blocklength \( l \), size \( \rho^l \), and maximum decoding time of \( lD \) (that is, \( T \leq lD \)). In particular, following [8], we define the diversity gain with CSI as

\[
d_{1-\text{bit}}(r, D) = \lim_{\rho \to \infty} \frac{-\log \mathbb{E}_H \{ P_e(R, \rho, l, M, N, D) \}}{\log \rho},
\]

where \( R = \frac{\log(\rho^l)}{2(lD)} \), \( T \) is decoding time, and the multiplexing gain \( r \) is defined as,

\[
r = \lim_{\rho \to \infty} \frac{R}{\rho \log \rho}.
\]

In this paper we assume that the transmitter knows, through one bit receiver feedback, whether or not the channel realization satisfies a predefined criterion before transmission. The transmitter leverages this side information to postpone the transmission up to the time that the channel is favorable. When \( D \) is bounded and the transmitter has delayed the transmission for \( (D-1) \) coherence blocks, the message will be sent in the next coherence block. In what follows, we investigate the behavior of \( d_{1-\text{bit}}(r, D) \) for different values of \( D \).

III. DIVERSITY-MULTIPLEXING TRADEOFF: BOUNDED EXPECTED DELAY, UNBOUNDED MAXIMUM DELAY

The reliability of MIMO fading channels without transmit CSI is dominated by the probability of the outage event [1]. However, in the asymptotic of large SNR, the outage event is a rare event with a small probability of \( \rho^{-d(r)} \) where \( d(r) \) is defined in [3]. Therefore knowing whether or not the channel is in outage, messages can be scheduled for transmission only during favorable conditions. This would eliminate the atypical outage events and may also lead to unbounded decoding delay, though the expected delay can still be bounded. This is due to the fact there is a nonzero probability, however small, of arbitrarily long sequence of outage events.

When there is no maximum decoding delay constraint, we assume that the receiver sends one bit to the transmitter at the beginning of each coherence interval indicating the occurrence/non-occurrence of the event:

\[ O_{r,\infty} : \log \det(I + \rho H^* H) \leq \min(M, N)(\log \rho - 2 \log \log \rho). \]

A message is transmitted only when the channel fails to satisfy the condition \( O_{r,\infty} \). The following theorem provides the reliability function defined in [3] with \( D \) infinite.

Theorem 1 (Lower bound on diversity gain) Consider the channel of Section II-A with noiseless one-bit causal transmit CSI confirming or denying condition \( O_{r,\infty} \). For any blocklength-\( l \), \( l \leq L \), there exists a block code with \( \rho^l \) codewords for which

\[
d_{1-\text{bit}}(r, \infty) \geq l(\min(M, N) - r),
\]

where \( d_{1-\text{bit}} \) is as defined in [2].

Remark: It is interesting to note that the error probability is now decreasing exponentially with the blocklength unlike the case where there is no transmit CSI. Comparing this error exponent with that of an AWGN channel with the same SNR, it is seen that the effect of channel fades can be completely removed. This substantial gain is obtained due to the relaxed (but practical) requirement on the decoding delay from being exactly equal to the codeword blocklength to being equal to the expected number of channel uses until codeword reception. It is straightforward to establish that the average decoding delay is equal to \( l(1 + \Theta(\frac{1}{\log \rho})) \approx l \).

Proof-sketch: By bounding the determinant of a matrix by its minimum eigenvalue and its trace, it readily follows that

\[ \Pr(O_{r,\infty}) = \Theta(\frac{1}{(\log \rho)^r}). \]

This implies that the decoding delay \( T \) has a geometric distribution with parameter \( 1 - \Pr(O_{r,\infty}) \). Therefore, the multiplexing gain as defined in [5] is equal to,

\[
\lim_{\rho \to \infty} \frac{R}{\rho \log \rho} = \lim_{\rho \to \infty} \frac{r l \log \rho}{(1 - \Pr(O_{r,\infty}))} = r.
\]

An achievable error probability can be found using the random coding bound. Following [2], we may write,

\[
\mathbb{E}_H \{ P_e \} \leq \rho^{-d} \int_{O_{r,\infty}} \frac{f_H(H)dH}{(\det(I + \rho H^* H))^l} \leq \rho^{-l(\min(M, N) - r)}(\log \rho)^{-2l},
\]

where \( O_{r,\infty} \) is the complement of the event \( O_{r,\infty} \).

In the following section, we investigate the scenario in which the message decoding delay is bounded.

IV. DIVERSITY-MULTIPLEXING-DELAY TRADEOFF: BOUNDED MAXIMUM DELAY

In the previous section, we observed that a substantial gain can be obtained in the diversity gain with little CSI at the transmitter at the cost of having the possibility of infinite decoding delays (albeit with arbitrarily small probability for large SNR). In this section, we further impose a worst case delay constraint of \( D \) for message decoding. In other words, we assume that the decoder has to decide on a message at most \( D \) coherence time instants after the message is sent. This is due to the fact that there is a non-zero probability for a sequence of really bad channel events.

When \( D = 1 \), the problem reduces to the problem considered in [1] with the only difference that the transmitter has 1-bit causal CSI. Since there is no possibility for postponing the transmission and power allocation optimization for one
coherence interval does not change the diversity gain, it is straightforward to show that \( d_{1\text{-bit}}(r, l) = d(r, l) \).

In this section, we prove that with only one bit causal CSI feedback at the transmitter, one can substantially improve the diversity gain of codes of length \( l \) by leveraging the possibility of postponing the transmission and/or exploiting the long-term average power constraint. This substantial gain is achieved by only transmitting when the channel is in a favorable condition or we have reached the maximum transmission delay. Since the event of having a sequence of unfavorable channels is very unlikely, one can, without violating a long-term average (over messages) power constraint, boost the transmit power when faced with greater delays. We also explore the potential gains for a (short-term) peak to average power constraint.

A. Short-Term Average Power Constraint

Here we assume that the receiver sends one bit feedback to the transmitter confirming or denying the following condition:

\[
O_{r, D} : \log \det(I + \rho H^* H) \leq f(r, D) \log \rho.
\]

(8)

where \( f(r, D) \) is defined as,

\[
f(r, D) = r + \frac{d(r) + (D - 1)M_{eq}(\min(M, N) - r)}{l + (D - 1)M_{eq}},
\]

(9)

where \( l \) is the code blocklength, \( M_{eq} = |M - N| + 1 \), and \( d(r) \) is as defined in (3). Of course, this is the outage event and has a probability of \( \rho^{-d(f(r,D))} \). In our transmission scheme, the transmitter postpones the transmission until the channel belongs to the set of channels that do not satisfy the condition \( O_{r, D} \) or reaches the maximum delay constraint of \( D \). In this transmission scheme, transmission always occurs with a transmit power \( \rho \).

**Theorem 2** For the channel of Section II-A, the diversity gain of a code of blocklength \( l \) and cardinality \( r \), with short-term power constraint, is \( \rho \), i.e.,

\[
d_{1\text{-bit}}(r, D) \geq l(f(r, D) - r) = \frac{l}{l + (D - 1)M_{eq}}(d(r) + (D - 1)M_{eq}(\min(M, N) - r))
\]

for any \((M + N) \leq l \leq L\), where \( M_{eq} = |M - N| + 1 \), \( 0 \leq r \leq \min(M, N) \), and \( d(r) \) is as defined in (3).

**Proof-sketch:** We use the random coding bound to obtain an upper bound on the error probability. We condition the error probability into two events, namely, i) \( A_1 \): the channel realization satisfies \( O_{r, D}^c \) in at least one out of the first \((D - 1)\) coherence intervals, ii) \( A_2^c \): the channel satisfies \( O_{r, D} \) during all \((D - 1)\) coherence intervals. The error probability in the event of \( A_1 \) can be easily bounded using the same approach as in Theorem 1 as,

\[
E(P_{e,A_1}) \leq \rho^{l(f(r, D) - r)}.
\]

(10)

The reason being that in the regime of large SNR, the exponent of the scaling law of the outage probability with SNR would not be changed by optimal power allocation over one coherence interval.

Also, note that the probability of the event \( A_2^c \) is the event that condition \( O_{r, D} \) (cf. (8)) is met \((D - 1)\) times. Therefore,

\[
\Pr(A_2^c) \leq \rho^{-(D-1)d(f(r,D))}.
\]

Furthermore, conditioned on the event \( A_2^c \), the error probability is given by the error probability for \( D = 1 \). Therefore,

\[
E(P_e) = E(P_{e,A_1}) + E(P_{e,A_2}) \leq \rho^{l(f(r,d))} + \rho^{-(D-1)d(f(r,D))}.
\]

(11)

In order to calculate the largest achievable diversity gain, one needs to minimize the upper bound on the error probability in (11), that is, match the exponents by finding \( f(r, D) \) which satisfies,

\[
l(r - f(r, D)) = -(D - 1)d(f(r, D)) - d(r).
\]

(12)

The expression in (9) solves (12) with \( d(f(r,D)) \) replaced by the smaller value \( (|M-N| - f(r, D)) \). This corresponds to minimizing a larger upper bound (than in (11)) on the error probability. Replacing \( f(r, D) \) in (11) completes the proof.

Specializing Theorem 2 to the case \( N = 1 \) gives,

\[
d_{1\text{-bit}}(r, D) \geq \frac{l(MD - 1)}{l + (MD - 1)}(1 - r),
\]

(13)

for any \( 0 \leq r \leq 1 \). This shows that when MD \( \gg 1 \), \( d_{1\text{-bit}} \gg l(1 - r) \) and when \( l \gg MD \), \( \frac{l}{MD - 1}(1 - r) \). Figure 1 shows the diversity multiplexing tradeoff for increasing values of \( D \). It is clear that with little side information and a small delay tolerance, the reliability function can be significantly improved. In the next section, we show that the gains with bounded delay can be further increased by exploiting a long-term power constraint, that is, by boosting the power when rare events happen.

B. Long-Term Average Power Constraint

In the regime of high SNR, the outage event has a very small probability and therefore the transmitter can boost the power in proportion to the corresponding probability when the
transmitter is subject to a long-term power constraint. In this section, we show that we can further improve the reliability function using power control.

Side information here is assumed to be one bit causal CSI that informs the transmitter whether the following condition is true or false:

\[ O_{r,D,i} : \log \det(I + p^{(r,D)} H^* H) \leq f_i(r, D) \log \rho. \]  

where \( i \) denotes the number of consecutive channels that satisfies the conditions \( O_{r,D,1}, \ldots, O_{r,D,i-1} \), respectively. Here \( f_i(r, D) \) and \( g_i(r, D) \) are defined as,

\[
f_i(r, D) = \frac{(l - \beta_k)r - \frac{M}{M-1} \left( \sum_{i=1}^{D} M_i^e - D \right) - \alpha_k}{l + \frac{M}{M-1} \left( \sum_{i=1}^{D} M_i^e - D \right) (1 + \alpha_k)},
\]

\[
g_i(r, D) = 1 + \sum_{j=1}^{i-1} \Pr(O_{r,D,j}).
\]

where \( g_1(r, D) = 1 \). Here \( g_i(r, D) \) is the transmit power when the previous \( i - 1 \) channels satisfy all the conditions of \( O_{r,D,1}, \ldots, O_{r,D,i-1} \). The next theorem provides an achievable upper bound on the error probability that implies a lower bound for the diversity gain.

**Theorem 3** For the channel of Section II-A, the diversity gain of a code of blocklength \( l \) and cardinality \( \rho^\gamma \), with long-term average power constraint \( \rho \), is at least equal to

\[
d_{1\text{-bit}}(r, D) \geq \frac{l}{l + D_{eq} M_{eq}^e r} \left( \frac{\alpha_k - (D_{eq} + \beta_k)r}{M_{eq}^e - D} \right),
\]

where for \( M \neq N \),

\[
D_{eq} = \frac{1}{M - 1} \left( \sum_{i=1}^{D} M_i^e - D \right) - 1,
\]

and for \( M = N \), we have \( D_{eq} = \frac{D(D+1)}{2} \).

**Proof-sketch:** The proof follows again using the random coding upper bound on the error probability. The only advantage here is that the probability of not transmitting until the \( D \)th coherence interval is much smaller as we can use power control. This can significantly improve the achievable diversity gain.

In the special case where \( N = 1 \), Theorem 3 implies that the achievable diversity gain is no smaller than

\[
d_{1\text{-bit}}(r, D) \geq \frac{l}{l + \frac{M}{M-1} \left( \sum_{i=1}^{D} M_i^e - D \right) - M} (1 - r).
\]

**Figure 2** compares the achievable diversity gains with and without power control as obtained in (17) and (13), respectively. It is clear that with only \( D = 4 \), we can obtain most of the gains achieved via infinite worst case delay obtained in (6). We can quantify this observation by computing the delay \( D \) required to achieve within \( \epsilon \) of the lower bound for

\[
d_{1\text{-bit}}(r, \infty), \text{ that is, } l(1 - \epsilon)(1 - r).
\]

It is straightforward to show that the delay required is equal to

1. \( D = \frac{1}{\epsilon M} \) with no power control,
2. \( D = \log_{\frac{\epsilon}{1-\epsilon}}(\frac{1}{D}) \) with power control.

Therefore with power control, we need an exponentially smaller delay to achieve the same performance as opposed to the case where no power control is employed.

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**References**

[1] L. Zheng and D. N. Tse, “Diversity and multiplexing: a fundamental tradeoff in multiple antenna channels,” IEEE Trans. Info. Theory, vol. IT–49, pp. 1073–1096, May 2003.

[2] V. Tarokh, N. Seshadri, and R. Calderbank, “Space-time codes for high data rate wireless communication: Performance analysis and code construction,” JEEE Trans. Info. Theory, vol. IT–44, pp. 744–765, Mar. 1998.

[3] H. E. Gamal, G. Caire, and M. O. Damen, “The MIMO ARQ Channel: Diversity-Multiplexing-Delay tradeoff,” IEEE Trans. Info. Theory, vol. IT–52, pp. 3601–3621, Aug. 2006.

[4] L. Weng, S. Pradhan, and A. Anastasopoulos, “Error exponent regions for Gaussian broadcast and multiple access channels,” submitted IEEE Trans. Info. Theory, 2005.

[5] G. Caire, G. Taricco, and E. Biglieri, “Optimum power control over fading channels,” IEEE Trans. Info. Theory, vol. IT–45, pp. 1468–1498, Jul. 1999.

[6] C. E. Shannon, “Channels with Side Information at the Transmitter,” IBM Journal of Research and Development, vol. 2, no. 4, pp. 289–293, Oct. 1958.

[7] J. P. M. Schalkwijk and M. E. Barron, “Sequential signaling under a peak power constraint,” IEEE Trans. Info. Theory, vol. IT–17, pp. 278–282, May 1971.

[8] M. V. Burnashev, “Data transmission over a discrete channel with feedback,” Probl. Inf. Trans., vol. 12, no. 7, pp. 10–30, 1976.

[9] H. Yamamoto and K. Inoh, “Asymptotic Performance of a modified Schalkwijk-Barron scheme for channels with noiseless feedback,” IEEE Trans. Info. Theory, vol. IT–25, pp. 729–733, Nov. 1979.

[10] A. Tchamkerten, E. Telatar, “Variable length coding over an unknown channel,” IEEE Trans. Info. Theory, vol. IT–52, May 2006.

[11] A. Sahai, “Why block length and delay are not the same thing,” submitted IEEE Trans. Info. Theory, 2006.