Finding consensus in speech recognition:
word error minimization and other applications of confusion networks

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Abstract
We describe a new framework for distilling information from word lattices to improve the accuracy of speech recognition and obtain a more perspicuous representation of a set of alternative hypotheses. In the standard MAP decoding approach the recognizer outputs the string of words corresponding to the path with the highest posterior probability given the acoustics and a language model. However, even given optimal models, the MAP decoder does not necessarily minimize the commonly used performance metric, word error rate (WER). We describe a method for explicitly minimizing WER by extracting word hypotheses with the highest posterior probabilities from word lattices. We change the standard problem formulation by replacing global search over a large set of sentence hypotheses with local search over a small set of word candidates. In addition to improving the accuracy of the recognizer, our method produces a new representation of the set of candidate hypotheses that specifies the sequence of word-level confusions in a compact lattice format. We study the properties of confusion networks and examine their use for other tasks, such as lattice compression, word spotting, confidence annotation, and reevaluation of recognition hypotheses using higher-level knowledge sources.

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1. Introduction

In the standard maximum \textit{a posteriori} probability (MAP) decoding approach to speech recognition (Bahl, Jelinek & Mercer, 1983) the recognizer outputs the string of word hypotheses corresponding to the path with the highest posterior probability given the acoustics and a language model. However, even given optimal models, the MAP decoder does not necessarily minimize the commonly used performance criterion for recognition, the word error rate (WER). Intuitively, one should maximize \textit{word} posterior probabilities instead of whole sentence posteriors to minimize WER.

Prior work (Stolcke, Konig & Weintraub, 1997) has shown how WER can be explicitly minimized in an \(N\)-best rescoring algorithm. That approach is suboptimal because it restricts hypothesis choice to a rather small set compared to the search space of the recognizer. In this paper we present a new word error minimization algorithm that is applicable to word lattices, or partially ordered networks of word hypotheses. Word lattices are used by many large vocabulary recognizers as a compact intermediate representation of alternative hypotheses and contain orders of magnitude more hypotheses than typical \(N\)-best lists. Word error minimization applied to word lattices promises better performance than \(N\)-best lists for two reasons. First, lattices provide a larger set of hypotheses from which to choose; second, the more accurate representation of the hypothesis space gives better estimates for word posterior probabilities and, consequently, of expected word error. However, as we will see below, the lattice representation also leads to new computational problems: it is no longer feasible to compute word errors between hypotheses exactly. The solution will be to minimize a slightly modified word error function that can be computed efficiently and is empirically shown to closely approximate the standard word error.

In Section 2 we motivate our approach on theoretical and empirical grounds, and survey prior work. Section 3 describes the algorithm itself. Section 4 gives an experimental evaluation of our method in terms of recognition accuracy, followed by further diagnostic experiments in Section 5. In Section 6 we describe various properties of \textit{confusion networks}, a specialized form of lattice generated by our method, and show how they can be employed for tasks other than word recognition. Section 7 compares our approach to a recently developed alternative method for lattice-based word error minimization. Section 8 concludes.

2. Motivation and prior work

2.1. Theoretical motivation

We can motivate our approach from a theoretical point of view by the mismatch between the standard scoring paradigm (MAP) and the commonly used performance metric (WER). In the standard approach to speech recognition (Bahl \textit{et al.}, 1983), the goal is to find the sentence hypothesis that maximizes the posterior probability \(P(W|A)\) of the word sequence \(W\) given the acoustic information
We call this the sentence MAP approach. Sentence posteriors are then usually approximated as the product of a number of knowledge sources, and normalized. For example, given a language model $P(W)$ and acoustic likelihoods $P(A|W)$, we can approximate

$$P(W|A) \approx \frac{P(W)P(A|W)}{\sum_k P(W^{(k)})P(A|W^{(k)})}$$

where $k$ ranges over the set of hypotheses generated by the recognizer.

Bayesian decision theory [e.g. Duda & Hart (1973)] tells us that maximizing sentence posteriors minimizes the sentence level error (the probability of having at least one error in the sentence string). However, the commonly used performance metric in speech recognition is word error, i.e. the Levenshtein (string edit) distance $WE(W, R)$ between a hypothesis $W$ and the reference string $R$. $WE(W, R)$ is defined as the number of substitutions, deletions, and insertions in $W$ relative to $R$ under an alignment of the two strings that minimizes a weighted combination of these error types. The string edit distance is a more forgiving (and for many applications more relevant) error metric because it gives partial credit for correctly recognized portions of sentences.

Sentence error and word error rates are assumed to be highly correlated, so minimizing one would tend to minimize the other. However, as our empirical results will show, there is a significant difference between optimizing for sentence vs. word error rate. To gain an intuitive understanding of this difference it is helpful to examine an example.

| Hypothesis ($H$) | $w_1$ | $w_2$ | $w_3$ | $P(H|A)$ | $P(w_1|A)$ | $P(w_2|A)$ | $P(w_3|A)$ | $E[\text{correct}]$ |
|------------------|------|------|------|---------|---------|---------|---------|-----------------|
| I DO INSIDE      | 0.16 | 0.34 | 0.29 | 0.16    | 0.79    |
| I DO FINE        | 0.13 | 0.34 | 0.29 | 0.28    | 0.11    |
| BY DOING FINE    | 0.11 | 0.45 | 0.49 | 0.28    | 1.22    |
| BY DOING WELL    | 0.11 | 0.45 | 0.49 | 0.11    | 1.05    |
| BY DOING SIGHT   | 0.10 | 0.45 | 0.49 | 0.10    | 1.04    |
| BY DOING BYE     | 0.07 | 0.45 | 0.49 | 0.07    | 1.01    |
| BY DOING THOUGHT | 0.05 | 0.45 | 0.49 | 0.07    | 0.99    |
| I DOING FINE     | 0.04 | 0.34 | 0.49 | 0.28    | 1.11    |
| I DON’T BUY      | 0.01 | 0.34 | 0.01 | 0.01    | 0.36    |
| BY DOING FUN     | 0.01 | 0.45 | 0.49 | 0.01    | 0.95    |

The first column in Table I shows a 10-best list of hypotheses that are produced by the recognizer, and the second column shows the corresponding joint posterior.
probabilities $P(H|A)$ for these hypotheses. Columns 3, 4 and 5 give the posterior probabilities $P(w|A)$ for individual words. These posterior word probabilities follow from the joint posterior probabilities by summing over all hypotheses that share a word in a given position. Column 6 shows the expected number of correct words $E[\text{correct}]$ in each hypothesis, under the posterior distribution. This is simply the sum of the individual word posterior probabilities, since

$$E[\text{words correct}(w_1w_2w_3)|A] = E[\text{correct}(w_1)|A] + E[\text{correct}(w_2)|A] + E[\text{correct}(w_3)|A]$$

As can be seen, although the hypothesis “BY DOING FINE” does not have the highest posterior, it has the highest expected number of correct words, i.e. the minimum expected word error. The correct answer for this example is “I’M DOING FINE”, which means that the MAP hypothesis “I DO INSIDE” has misrecognized all the words ($WER = 3$), whereas the new hypothesis recognized incorrectly only one word ($WER = 1$). Thus, we have shown that optimizing overall posterior probability (sentence error) does not always minimize expected word error. This happened because words with high posterior probability did not have high posterior probability when combined.

2.2. Word error minimization

Given word error as our objective function, we can replace the MAP approach with a new hypothesis selection approach based on minimizing the expected word error under the posterior distribution:

$$E_{P(R|A)}[WE(W, R)] = \sum_r P(R|A) \cdot WE(W, R)$$

This equation provides a general recipe for computing expected word-level error from sentence-level posterior estimates.

A direct algorithmic version involves two iterations: a summation over potential references $R$ and a minimization over hypotheses $W$. Therefore, the amount of work grows quadratically with the number of hypotheses. Furthermore, the computation of the word error function is nontrivial: it involves a dynamic programming alignment of $W$ and $R$ and takes time proportional to the square of the hypotheses lengths. This raises the question of whether explicit word error minimization can be carried out feasibly for large vocabulary recognition. The next section briefly reviews a prior approach based on an $N$-best approximation. The remainder of the paper is devoted to an improved approach considering the hypotheses in a word lattice.
2.3. An $N$-best approximation

In previous work (Stolcke et al., 1997), word error minimization was implemented by limiting the search space and the expected word error computation to an $N$-best list, i.e. letting both $W$ and $R$ range over the $N$ best hypothesis output by a recognizer:

$$W_c = \arg\min_{i=1,N} \sum_{k=1}^{N} P(R^{(k)}|A) \cdot WE(W^{(i)}, R^{(k)})$$  \hspace{1cm} (4)

We refer to the hypothesis $W_c$ thus obtained as the *center hypothesis*. An optimized algorithm that finds the center hypothesis and scales linearly with $N$ in practice is given in Appendix A.

Goel, Byrne & Khudanpur (1998) have noted that the brute-force optimization of Equation (3) and its $N$-best approximation (4) generalizes to objective functions other than word error. For example, they show (Goel & Byrne, 2000) that the named-entity tagging performance on automatically recognized speech can be improved by explicitly optimizing an appropriate tagging score (e.g. F-score).

2.4. Lattice-based word error minimization

Lattices represent a combinatorial number of sentence hypotheses, presenting the potential to improve the $N$-best approach through both more accurate error estimates [the summation in Equation (3)] and a larger search space for minimization.

From a practical point of view, lattices are often generated as a preliminary step to $N$-best lists in a multipass recognition system, and are thus obtained with less computational overhead. Furthermore, efficient lattice generation techniques that are essentially no more burdensome than simple 1-best recognition have recently been developed (Ljolje, Pereira & Riley, 1999). Therefore, being able to carry out word error minimization directly from lattices also represents a practical simplification and efficiency enhancement of the overall recognition system.

In moving to lattice-based hypothesis selection, we are faced with a computational problem. The number of hypotheses contained in a lattice is several orders of magnitude larger than in $N$-best lists of practical size, making a straightforward computation of the center hypothesis as in Equation (4) infeasible. A natural approach to this problem is to exploit the structure of the lattice for efficient computation of the center hypothesis. Unfortunately, there seems to be no efficient (e.g. dynamic programming) algorithm of this kind. The fundamental difficulty is that the objective function is based on pairwise string distance, a nonlocal measure. A single word difference anywhere in a lattice path can have global consequences on the alignment of that path to other paths, preventing a decomposition of the objective function that exploits the lattice structure.

To work around this problem, we decided to replace the original pairwise string
alignment (which gives rise to the standard string edit distance $WE(W,R)$) with a modified, multiple string alignment. The new approach incorporates all lattice hypotheses into a single alignment, and word error between any two hypotheses is then computed according to that one alignment. The multiple alignment thus defines a new string edit distance, which we will call $MWE(W,R)$. While the new alignment may in some cases overestimate the word error between two hypotheses, as we will show in Section 5 it gives very similar results in practice.

The main benefit of the multiple alignment is that it allows us to extract the hypothesis with the smallest expected (modified) word error very efficiently. To see this, consider an example. Figure 1 shows a word lattice and the corresponding hypothesis alignment. Each word hypothesis is mapped to a position in the alignment (with deletions marked by “-”). The alignment also supports the computation of word posterior probabilities. The posterior probability of a word hypothesis is the sum of the posterior probabilities of all lattice paths of which the word is a part. Given an alignment and posterior probabilities, it is easy to see that the hypothesis with the lowest expected word error is obtained by picking the word with the highest posterior at each position in the alignment. We call this the consensus hypothesis.

In practice we apply some pruning of the lattice to remove low probability word hypotheses (see Section 3.4).
3. The algorithm

Having given an intuitive idea of word error minimization based on lattice alignment, we can now make these notions more precise and describe the algorithm in detail.

Pseudo-code for the crucial steps and information on algorithmic complexity is given in Appendix B.

3.1. Lattice alignment

As we saw, the main complexity of the approach is in finding a good multiple alignment of lattice hypotheses, i.e. one that approximates the pairwise alignments. Once an alignment is found we can determine the minimizing word hypothesis exactly. Finding the optimal alignment itself is a problem for which no efficient solution is known (Gusfield, 1992). We therefore resort to a heuristic approach based on lattice topology, as well as time and phonetic information associated with word hypotheses.

Let $E$ be the set of links (or edges) in a word lattice, each link $e$ being characterized by its starting node $Inode(e)$, ending node $Fnode(e)$, starting time $Itime(e)$, ending time $Ftime(e)$, and word label $Word(e)$. From the acoustic and language model scores in the lattice, we can also compute the posterior probability $p(e)$ of each link, i.e. the sum of posteriors of all paths through $e$. Furthermore, for a given link subset $F \subset E$, let

$$\text{Words}(F) = \{w | \exists e \in F : Word(e) = w\}$$

be its set of words, and

$$p(F) = \sum_{e \in F} p(e)$$

its total posterior probability.

Formally, an alignment consists of an equivalence relation over the word hypotheses (edges) in the lattice, together with a total ordering of the equivalence classes, such that the ordering is consistent with that of the original lattice. Each equivalence class corresponds to one “position” in the alignment, and the members of a class are those word hypotheses that are “aligned to each other,” i.e. represent alternatives. We use $[e]$ to denote the equivalence class of which $e$ is a member.

The lattice defines a partial order $\leq$ on the links. For $e, f \in E$, $e \leq f$ iff

- $e = f$ or
- $Fnode(e) = Inode(f)$ or
- $\exists e' \in E$ such that $e \leq e'$ and $e' \leq f$.

Informally, $e \leq f$ means that $e$ “comes before” $f$ in the lattice.

Now let $\mathcal{E} \subset 2^E$ be a set of equivalence classes on $E$, and let $\preceq$ be a partial
order on $\mathcal{E}$. We say that $\preceq$ is consistent with the lattice order $\leq$ if $e_1 \leq e_2$ implies $[e_1] \preceq [e_2]$, for all $e_1 \in [e_1], e_2 \in [e_2], [e_1], [e_2] \in \mathcal{E}$. Consistency means that the equivalence relation preserves the temporal order of word hypotheses in the lattice.

Given a lattice, then, we are looking for an ordered link equivalence that is consistent with the lattice order and is also a total (linear) order, i.e. for any two $e_1, e_2 \in E$, $[e_1] \preceq [e_2]$ or $[e_2] \preceq [e_1]$. Many such equivalences exist; for example, one can always sort the links topologically and assign each link its own class. However, such an alignment would be very poor: it would vastly overestimate the word error between hypotheses.

We initialize the link equivalence such that each initial class consists of all the links with the same starting and ending times and the same word label. Starting with this initial partition, the algorithm successively merges equivalence classes until a totally ordered equivalence is obtained.

Correctness and termination of the algorithm are based on the following observation. Given a consistent equivalence relation with two classes $E_1$ and $E_2$ that are not ordered ($E_1 \not\preceq E_2$ and $E_2 \not\preceq E_1$), we can always merge $E_1$ and $E_2$ to obtain a new equivalence that is still consistent and has strictly fewer unordered classes (a formal proof is given in Appendix C). We are thus guaranteed to create a totally ordered, consistent equivalence relation after a finite number of steps.

Our clustering algorithm has two stages. We first merge only equivalence classes corresponding to instances of the same word (intra-word clustering), and then start grouping together heterogeneous equivalence classes (inter-word clustering), based on the phonetic similarity of the word components. At the end of the first stage we are able to compute word posterior probabilities, but it is only after the second stage that we are able to identify competing word hypotheses.

### 3.2. Intra-word clustering

The purpose of this step is to group together all the links corresponding to same word instance. Candidates for merging at this step are all the equivalence classes that are not in relation and correspond to the same word. The cost function used for intra-word clustering is the following similarity measure between two sets of links:

$$\text{SIM}(E_1, E_2) = \max_{e_1 \in E_1} \max_{e_2 \in E_2} \text{overlap}(e_1, e_2) \cdot p(e_1) \cdot p(e_2)$$

where $\text{overlap}(e_1, e_2)$ is defined as the time overlap between the two links normalized by the sum of their lengths. The temporal overlap is weighted by the link posteriors so as to make the measure less sensitive to unlikely word hypotheses. At each step we compute the similarity between all possible pairs of equivalence class candidates, and merge those that are most similar. At the end of this iterative process we obtain a link equivalence relation that has overlapping instances
of the same word clustered together (although not all such instances necessarily end up in the same equivalence class due to ordering constraints).

### 3.3. Inter-word clustering

In this step we group together equivalence classes corresponding to different words. Candidates for merging are any two classes that are not in relation. The algorithm stops when no more candidates are available, i.e. a total order has been achieved.

The cost function used for inter-word clustering is the following similarity measure based on phonetic similarity between words:

\[
\text{SIM}(F_1, F_2) = \frac{\text{avg}}{w_1 \in \text{Words}(F_1)} \frac{\text{avg}}{w_2 \in \text{Words}(F_2)} \text{sim}(w_1, w_2) \cdot p_{F_1}(w_1) \cdot p_{F_2}(w_2)
\]

where \(p_F(w) = p\{e \in F : \text{Word}(e) = w\}\) and \(\text{sim}(\cdot, \cdot)\) is the phonetic similarity between two words, computed using the most likely phonetic base form. In our implementation we defined phonetic similarity to be 1 minus the edit distance of the two phone strings, where phone edit distances are normalized by the sum of their lengths. Other, more sophisticated definitions are conceivable; for example, the similarity function could be sensitive to the phonetic features (e.g. vowel/consonant, voiced/unvoiced) of the phones in the pronunciations.

### 3.4. Pruning

Typical word lattices contain links with very low posterior probability. Such links are negligible in computing the total posterior probabilities of word hypotheses, but they can have a detrimental effect on the alignment. This occurs because the alignment preserves consistency with the lattice order, no matter how low the probability of the links imposing the order. For example, in Figure 2 we see the words “BE” and “ME”, which are phonetically similar and overlap in time, and should therefore be mutually exclusive. However, even a single path with “BE” preceding “ME”, no matter how low in probability, will prevent “BE” and “ME” from being aligned.

To help eliminate such cases we introduce a preliminary pruning step. Lattice pruning removes all links whose posteriors are below an empirically determined threshold. The equivalence class initialization and subsequent merging only considers links that survive the initial pruning. Section 5.2 gives results showing the effect of lattice pruning on the overall effectiveness of our algorithm. Experiments show that word recognition accuracy indeed improves with lattice pruning, and that results are not very sensitive to the exact value of the pruning threshold.

### 3.5. Confusion networks

The total posterior probability of an alignment class can be strictly less than 1. That happens when there are paths in the original lattice that do not contain a
word at that position; the missing probability mass corresponds precisely to the probability of a deletion (or null word). We explicitly represent deletions by a link $e_-$ with the corresponding empty word $\text{Word}(e_-) = \text{"-"}$. For example, in the lattice in Figure 1(a) there are some hypotheses having “I” as the first word, while others have no corresponding word in that position. The final alignment thus contains two competing hypotheses in the first position: the word “I” (with posterior equal to the sum over all hypotheses starting with that word), and the null word (with posterior equal to the sum over all other hypotheses).

As illustrated in Figure 1(b), the alignment is itself equivalent to a lattice, which we refer to as a confusion network. The confusion network has one node for each equivalence class of original lattice nodes (plus one initial/final node), and adjacent nodes are linked by one edge per word hypothesis (including the null word).

We can think of the confusion network as a highly compacted representation of the original lattice with the property that all word hypotheses are totally ordered. As such, the confusion network has other interesting uses besides word error minimization, some of which will be discussed in Section 6.

### 3.6. The consensus hypothesis

Once we have a complete alignment it is straightforward to extract the hypothesis with the lowest expected word error. Let $C_i, i = 1, \ldots, L$ be the final link equivalence classes making up the alignment. We need to choose a hypothesis $W = w_1 \ldots w_L$ such that $w_i = \text{"-"}$, or $w_i = \text{Word}(e_i)$ for some $e_i \in C_i$. It is easy to see that the expected word error of $W$ is the sum total of word errors for each
position in the alignment. Specifically, the expected word error at position \( i \) is

\[
1 - \sum_{e \in C_i, \text{Word}(e)=w_i} p(e) \quad \text{if} \quad w_i \neq "-" \\
1 - \sum_{e \in C_i} p(e) \quad \text{if} \quad w_i = "-"
\]  

(7)

In other words, the best hypothesis is obtained by picking the links in the confusion graph that have the highest posterior probability among all links at a given position. This is equivalent to finding the path through the confusion graph with the highest combined link weight.

### 3.7. Score scaling

Posterior probability estimates are based on a combination of recognizer acoustic and language model scores [Equation (1)]. In practice it is necessary to adjust the dynamic ranges of the two scores relative to each other to obtain satisfactory results. This is usually accomplished by multiplying the logarithm of the language model score by a constant, the language model weight. The absolute scale of the scores does not matter in MAP decoding since a maximization is performed.

In computing posterior word probabilities or expected word error, however, the choice of scale for log scores becomes crucial. Specifically, multiplying scores on the log scale controls the peakedness of the posterior probability distribution and greatly affects the additive combination of posteriors. An overly peaked posterior concentrates all its mass on the MAP word hypothesis, causing the consensus or center hypothesis to converge to that MAP hypothesis.

For these reasons, we need to optimize the scaling of the recognizer scores empirically. We did so using data that was independent of the test sets used in later experiments, and found the best scaling constant to be unity for the language model log probabilities, and the inverse of the language model weight for the acoustic log likelihoods. We thus arrive at the following expression for the posterior estimate:

\[
\log P(W|A) = \log P(W) + \log P(Q|W) + \frac{1}{\lambda} \log P(A|W, Q) - C
\]  

(8)

where \( \lambda \) is the language model weight, \( P(Q|W) \) is the aggregate pronunciation probability, and \( C \) is a normalization constant that makes \( P(W|A) \) sum to unity over all hypotheses \( W \).

What is noteworthy about this finding is that it presents empirical support for a commonly given rationalization for the need of the language model weight in current recognizers. According to that explanation, the language model weight

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8The constant \( C \) corresponds to the logarithm of the denominator in Equation (1), except for the fact that the former equation did not include pronunciation probabilities for simplicity. We did not find a benefit from using a word insertion penalty for hypothesis selection, but if a recognition system uses such a parameter it needs to be adjusted to be compatible with the unscaled language model score \( \log P(W) \).
compensates for the frame-independence assumption in HMM-based acoustic models, which underestimate the joint likelihoods of correlated, nearby acoustic observations. It follows that if it were not for the invariance to constant factors, the correct language model weight is close to 1, whereas the acoustic log likelihoods should be scaled with a weight less than 1. Our experiments show that this is indeed the proper weighting.

If we do not use the aggregate pronunciation model $P(Q|W)$ in the posterior probability computation we favor the words with multiple baseforms. In our method we add the posterior probabilities of all the links corresponding to a word in a confusion set regardless of the pronunciation variant hypothesized for that particular link. Because all pronunciation variants for a word have probability 1 in the recognizer, and because words with multiple baseforms tend to occur more frequently, we need to adjust the likelihood computation so as to penalize these words. The simplest way to do this is to apply a uniform pronunciation model in which all pronunciation variants of a word receive the same probability and sum to 1. This is the model we employed in all our experiments, though more elaborate, nonuniform or even context-dependent pronunciation probabilities are certainly possible.

4. Word recognition experiments

We now report experiments investigating the effort of the new lattice-based word error minimization algorithm on word recognition accuracy. We experimented with two widely used, very different sets of data: the Switchboard conversational telephone speech corpus (Godfrey, Holliman & McDaniel, 1992) and the Broadcast News corpus of radio and television news programs (Graff, 1997).

4.1. Results on the Switchboard corpus

The system that generated the lattices used in our experiments was built using HTK (Young, 1997). It is an HMM-based state-clustered, cross-word triphone system, trained with about 60 hours of Switchboard speech. The system uses about 6700 HMM state clusters, each of which refers to a mixture of 12 Gaussian densities as the state output distribution. The front end uses MF-PLP derived coefficients with cepstral means removed per conversation side. The language model is a backoff trigram model trained on about 2.2 million words of Switchboard transcripts.

The first set of experiments compares the consensus hypothesis with the sentence MAP baseline hypothesis. We also report experiments comparing the lattice-based approach to the $N$-best word error minimization approach.

4.1.1. Comparison to MAP approach

The first column in Table II (Set I) shows results on a test set used in the 1997 Johns Hopkins University LVCSR Workshop (WS97) (Jelinek, 1998). This test
set consists of 2427 utterances from 19 conversations comprising about 18,000
words.

Table II: Comparison of consensus hypotheses and baseline MAP hypotheses on two Switch-
board test sets.

| Hypothesis | WER (%) | Set I  | Set II |
|------------|---------|--------|--------|
| MAP        | 38.5    | 42.9   |        |
| Consensus  | 37.3    | 41.6   |        |
| ∆ WER      | -1.2    | -1.3   |        |

Two parameters of our algorithm were optimized on a separate set of Switch-
board lattices that was disjoint from those used in the recognition test. The
threshold for pruning the original lattices (cf. Section 3.4) was set such that
word hypotheses with posteriors less than $10^{-3}$ were eliminated. Section 4.2
examines how variations of this threshold affected the results. Another optimized
parameter was the scale of the log posterior probabilities (cf. Section 3.7). As
mentioned earlier, we found that posterior scaling with the inverse of the lan-
guage model weight (12 in our recognizer) gave best results.

The consensus hypothesis results in an absolute WER reduction of 1.2% over the baseline, the standard MAP approach. This difference is statistically
significant at the 0.0001 level. To verify the consistency of the improvement
we ran a similar experiment on another set of lattices. Set II consists of lattices
 corresponding to a different set of utterances, generated using the same acoustic
models as used previously. The baseline WER on this set was more than 4%
higher than that of Set I. Set II results showed a very similar WER reduction
over the baseline (1.3%).

4.1.2. Comparison with $N$-best list approach

We also compared the lattice-based consensus hypothesis to the $N$-best based
center hypothesis (cf. Section 2.3). The maximum number of hypotheses per
utterance was 300. We found that increasing the number of $N$-best hypotheses
to 1000 did not give significant error reductions, and therefore concluded that
$N = 300$ was a safe cutoff for this experiment. Table III shows the WER results
for the two methods on the sets of lattices from the previous experiment.

We see that there are significant differences between the two methods, the
lattice-based approach being consistently better. For example, on Set I the $N$-
best center hypothesis results in 0.6% improvement over the baseline, whereas
the lattice consensus hypothesis doubles the gain.

9The scoring software used throughout this paper is the one provided by NIST.
10The significance test used throughout this paper is a matched-pairs sign test.
11The significance level was 0.001 on Set I and 0.0001 on Set II.
Table III: Comparison of N-best (center) and lattice-based (consensus) word error minimization on two Switchboard test sets.

| Hypothesis         | WER (%) |   |   |
|--------------------|---------|---|---|
|                    | Set I   | Set II |
| MAP                | 38.5    | 42.9 |
| N-best (Center)    | 37.9    | 42.3 |
| Lattice (Consensus)| 37.3    | 41.6 |

Table IV shows that under the new decoding scheme the sentence error rate (SER) in fact increases. This is to be expected given that, unlike for MAP hypothesis selection, the objective function for the center and the consensus hypothesis is word error, rather than sentence error.

Table IV: Word error rate (WER) and sentence error rate (SER) results on Switchboard test set I.

| Hypothesis          | WER | SER |
|---------------------|-----|-----|
| MAP                 | 38.5| 65.3|
| N-best (Center)     | 37.9| 65.6|
| Lattice (Consensus) | 37.3| 65.8|

4.2. Results on the Broadcast News corpus

We also ran experiments on a set of 1280 lattices corresponding to the 1996 DARPA Hub-4 development test set, drawn from the Broadcast News corpus. We did not reoptimize the posterior probability scaling or the link pruning threshold for this experiment. Instead, we applied what we had learned in the Switchboard experiments: we set the value for the scale to be the language model weight (which is 13 in this system) and used the pruning value optimized for Switchboard.

Table V: WER on Broadcast News showing the breakdown by focus condition. The conditions are F0 (clean read speech), F1 (conversational speech), F2 (telephone speech), F3 (speech with background music), F4 (noisy speech), F5 (non-native speech), and FX (all other conditions).

| Hypothesis          | F0  | F1  | F2  | F3  | F4  | F5  | FX  | Overall |
|---------------------|-----|-----|-----|-----|-----|-----|-----|---------|
| MAP                 | 13.0| 30.8| 42.1| 31.0| 22.8| 52.3| 53.9| 33.1    |
| N-best (Center)     | 13.0| 30.6| 42.1| 31.1| 22.6| 52.4| 53.9| 33.0    |
| Lattice (Consensus) | 11.9| 30.5| 42.1| 30.7| 22.3| 51.8| 52.7| 32.5    |

Overall WER results are presented in the last column of Table V. We see that on this corpus the N-best approach results in almost no improvement, whereas
the lattice-based approach significantly reduces the WER (0.6% absolute, 1.8% relative).

The Broadcast News corpus classifies speech into several different focus conditions, corresponding to different acoustic conditions and speaking styles. These are labeled as F1 through FX in Table V. The consensus hypothesis results in improvements in accuracy across almost all conditions (with no change in F2). While there is no obvious correlation between the nature of the focus conditions and the magnitude of the WER differences, we observed that the largest improvements (more than 1% absolute) were obtained on the conditions with lowest and highest WER (F0 and FX, respectively).

One significant difference between the Broadcast News corpus and the Switchboard data is that the former contains much longer utterances on average. Longer utterances can be expected to show more of a difference between sentence error and word error minimization (in the extreme case of a one-word sentence, both measures are identical). This suggests the following diagnostic experiment: we divided the test utterances into two sets, one containing “long” utterances (number of words > 25) and one containing “short” utterances (all others), and measured the WER reduction for the consensus hypothesis on each. As shown in Table VI, we found indeed that the larger gains over the MAP hypothesis come from the longer utterances. The absolute WER reduction was 0.7% on the long utterances and only 0.3% on the short utterances.

| Hypothesis       | WER (%) |          |          |
|------------------|---------|----------|----------|
|                  | Short utt. | Long utt. | Total    |
| MAP              | 33.3     | 31.5     | 33.1     |
| Lattice (Consensus) | 33.0     | 30.8     | 32.5     |

The longer utterances in Broadcast News also provide a likely explanation for the poor performance of the $N$-best algorithm on this test set. The sentence hypothesis space grows roughly exponentially with the length of an utterance, so given $N$-best lists of fixed length, any method based on $N$-best processing will be ignoring a larger and larger portion of the true posterior distribution of hypotheses. Lattices, on the other hand, by their combinatorial nature, should capture a fraction of the true posterior distribution that is roughly constant.

---

12 The result is significant at the 0.005 level.

13 The mean utterance length in the Broadcast News test set is about 17 words, and about 8 words in the Switchboard test set.
5. Detailed analyses

Here we report several diagnostic experiments and associated empirical analyses of our algorithm. All analyses here and in the following section are based on the WS97 Switchboard test set (Set I from Section 4.1.1).

5.1. Multiple alignment word error and true word error

In Section 2.1 we showed that the key to our lattice-based word error minimization approach was approximating the word error (WE) between two hypotheses with a new string distance MWE, computed on the multiple alignment of the entire set of hypotheses. A diagnostic experiment was designed to quantify the difference between MWE and WE. We sampled a large number of pairs of hypotheses from the posterior distribution represented by our lattices, and compared $WE$ and $MWE$ for each pair. The total number of errors per utterance (sum of substitutions, deletions, and insertions) under the two types of alignment differed by only 0.15 on average. This suggests that the suboptimal nature of the alignment is a small factor in practice, and more than justified by the computational advantages it affords.

5.2. Effect of lattice pruning

As explained in Section 3.4, removing low-probability word hypotheses from the original lattices can improve the quality of the multiple alignment of hypotheses. Because poor alignments distort the MWE metric away from the true WE metric, pruning can actually improve the accuracy of the consensus hypothesis.

Figure 3 plots the WER of the consensus hypothesis as a function of percentage of links retained from the original lattice. The original lattices have an average link density of 1350 and an average node density of 370. The lattice WER, i.e. the best WER that can be achieved by choosing a path in the lattice, is 9.5%. We observe that with less than 2% of the links in the original lattice we obtain a hypothesis with the same accuracy as the baseline MAP hypothesis, and with only 5% of the links in the original lattice we obtain a hypothesis that results in 1.2% improvement over the baseline (see Table II).

5.3. Lattices vs. N-best lists

The WER reductions obtained with lattices compared to N-best lists can be attributed to two factors: exploring more hypotheses in the search for the minimizer and using more evidence when computing the expected word error. The contributions of these two factors are quantified in the following diagnostic experiment.

$N$-best sentence hypotheses were generated as described in Section 4.1.2. Also, the average link/node density is the ratio between the total number of links/nodes in the lattice and the number of words in the reference transcription.
Figure 3: Switchboard WER of the consensus hypothesis when varying the percentage of links remaining after the pruning step. The horizontal line marks the baseline WER of the MAP approach.

Posteriors for individual word hypotheses were computed from the corresponding lattices as usual, using the consensus method. However, instead of allowing any hypothesis from the confusion network, we limited the choice of the best sentence hypothesis to those in the \(N\)-best list. Thus, this result has the benefit of the improved word posterior estimates based on lattices, but not of the expanded hypothesis selection space.

Table VII: Comparison of \(N\)-best (center), \(N\)-best (consensus), and lattice-based (consensus) word error minimization on the Switchboard corpus.

| Hypothesis                | WER (%) |
|---------------------------|---------|
| MAP                       | 38.5    |
| \(N\)-best (Center)       | 37.9    |
| \(N\)-best (Consensus)    | 37.6    |
| Lattice (Consensus)       | 37.3    |

The diagnostic result is shown as “\(N\)-best (Consensus)” in Table VII. Comparing the results for the baseline \(N\)-best approach, the lattice consensus approach, and the diagnostic experiment, we conclude that about half of the overall WER reduction in the full lattice approach comes from improved word posterior estimates, leaving the other half due to greater freedom in hypothesis selection.
5.4. Alternative clustering metrics

In Section 3 we introduced two similarity metrics [cf. Equations (5) and (6)] that form the basis of the clustering procedure. Here we examine some variations on these metrics.

5.4.1. The importance of time information

If the input lattice does not have information about the start and end times of word hypotheses, we must eliminate the time overlap term in the similarity metric (5) for the intra-word clustering stage (Section 3.2). In other words, we compute the similarity between two clusters based solely on the posterior probabilities. We ran experiments using this modified metric and found just a slight increase in WER (0.15% absolute).

An alternative is to estimate word time using the available information. For example, we can compute for each lattice node the length of the longest path from the initial node. The path lengths can be determined either in terms of the number of words or, more accurately, the number of phones, both of which we expect to be reasonably correlated with actual times. The overlap(.,.) term in Equation (5) is then computed based on these approximate time marks. We carried out experiments using word times estimated from phone counts, and obtained exactly the same WER as in the original experiments with time information.

5.4.2. The importance of phonetic similarity

We now investigate the importance of phonetic similarity in the inter-word clustering stage (Section 3.3). We ran experiments with a modified similarity function based entirely on word posterior probabilities, i.e. we removed the sim(.,.) term from Equation (6):

\[
SIM(F_1, F_2) = \frac{\text{avg}_{w_1 \in \text{Words}(F_1)} p_{F_1}(w_1) \cdot p_{F_2}(w_2)}{\text{avg}_{w_2 \in \text{Words}(F_2)} p_{F_2}(w_2)}
\]

We were surprised to find no change in WER as a result of this change, and conclude that the topology of the lattice alone constrains the alignment process sufficiently, as long as equivalence classes with high posterior probability are merged first.

We might still want to use phonetic similarity in clustering if our goal is to post-process confusion networks. For example, while low probability hypotheses do not seem to affect the quality of the best hypothesis as defined by our present algorithm, phonetic similarity might improve the alignment of hypotheses with

15 In some systems, lattices contain only word identities and likelihoods; such lattices are typically used as restricted language models in multipass speech recognizers.
low probability. This in turn might be important once we move to more sophisticated methods for choosing among the words of one equivalence class, as suggested in Section 6.

Furthermore, we found that phonetic similarity does become important when the lattices do not contain time information. Table VIII shows results where time information was not used during intra-word clustering (cf. the previous section). We see that phonetic similarity does improve the accuracy of the consensus hypotheses by about 0.2% absolute in this case. This suggests that time information and phonetic similarity are somewhat complementary for the purpose of word alignment.

Table VIII: Effect of phonetic similarity when no time information is used in clustering (Switchboard test set I).

| Method                                      | WER (%) |
|---------------------------------------------|---------|
| MAP                                         | 38.5    |
| Consensus (no times, with phonetic similarity) | 37.3    |
| Consensus (no times, w/o phonetic similarity)  | 37.5    |

5.4.3. The role of the posterior probabilities

The clustering procedure is a greedy algorithm. Whenever we merge two clusters, we add new constraints to the partial order. Consequently, some equivalence classes that could have been merged earlier are no longer candidates for merging. For this reason it is very important to have a robust similarity measure that does the right thing first and foremost on the high probability words. Here we investigate the importance of posterior probabilities as weights in computing cluster similarities.

Figure 4 shows one situation that was encountered quite often in our experiments. In this example we have to choose between merging “BE” and “BEEN” or “BE” and “THIN”. If “BE” and “BEEN” are the actual competitors and we prefer to merge “BEEN” and “THIN”, then we might end up with a consensus hypothesis in which both “BEEN” and “BE” are deleted because the deletion has a high posterior in both classes. However, the fact that both “BE” and “BEEN” have high posteriors, and at the same time there is no path containing both of them, suggests precisely the fact that they are candidates for the same word.

The above scenario is consistent with the results we obtained when we experimented without posterior weighting in the similarity metrics. As shown in Table IX we observed a large increase in the number of deletion errors, and a moderate increase in the overall error rate.
Table IX: WER of the consensus hypothesis when no posterior probabilities are used in clustering (Switchboard test set I).

| Method                        | WER (%) | Substitutions | Deletions | Insertions |
|-------------------------------|---------|---------------|-----------|------------|
| Similarity with posteriors    | 37.3    | 4365          | 1837      | 547        |
| Similarity w/o posteriors     | 37.6    | 4143          | 2165      | 494        |

6. Confusion networks revisited

Our method produces two main outputs: the consensus hypothesis and the confusion network. Up to this point we have focused on the properties of the consensus hypothesis. Now, we study the properties of confusion networks and show how they can be used for tasks other than word error minimization.

6.1. Correct hypothesis rank

Confusion networks can serve as a compact representation of the hypothesis space for post-processing speech recognizer output with other knowledge sources. A relevant question for such applications is how word hypotheses ranked by the confusion network compare to the truth.

Words in each confusion set in the confusion network can be ranked based on their posterior probabilities. If we align the reference transcription to the network and compute the rank of the correct word in each confusion set we obtain the histogram in Figure 5. We see that very rarely is the rank of the correct word greater than 10, and rarely greater than 7. This suggests limiting the confusion set size before using the confusion networks for some other applications. The results also show that if we find a method for better discriminating between the best two candidates in each confusion set, we can improve the recognizer’s accuracy by 10%. This suggests that there is space for potential improvement.
over the current choice, but we should note that the task of finding the right discriminator is, of course, not trivial.

Analyzing the properties of the confusion sets further we extracted some other interesting statistics:

- When a confusion set has only one candidate, this candidate is correct 97% of the time; 25% of the confusion sets have this property.
- When a confusion set has only two candidates, the highest-scoring candidate is correct 90% of the time; 25% of the confusion sets have this property.

This means that in 50% of the cases we can predict the correct word with high accuracy. These words could therefore be used as features for disambiguating the rest of the confusion sets. We also found that in confusion sets containing the best two candidates with close scores, choosing the one with the highest score is almost as good as picking randomly. This suggests that even little additional information could help the disambiguation procedure.

In summary, these statistics suggest that confusion networks are an excellent representation if one aims to narrow down the large search space associated with speech recognition to a small set of mutually exclusive, single-word candidates. By constructing confusion networks one can start to apply machine learning techniques based on discrete k-way classifiers to enhance the quality of the recognition hypothesis, for example, by incorporating more sophisticated linguistic knowledge.
6.2. Hypothesis space size

In Section 5.2 we discussed the effect of lattice pruning (as defined in Section 3.4) on the accuracy of the consensus hypothesis. We now ask a related question: How does lattice pruning affect the hypothesis space represented in the confusion network? To investigate the issue we first look at the accuracy of confusion networks, defined as for general lattices, i.e. 1 minus the WER of the best path through the graph. Figure 6 shows the relationship between lattice pruning and the word accuracy of confusion networks. As before, we quantify the degree of pruning by the percentage of links retained from the original lattices.

As shown, with only 5% of the links in the original lattice we obtain word graphs with the same accuracy. Given that 5% of the links in the original lattices represent only 20% of the word types, this suggests that only a small percentage of the word hypotheses in the original lattice are relevant to further processing. Figure 6 also shows that if we retain more than 5% of the original links we obtain networks with even better accuracy than the original lattices. This is a consequence of the fact that the confusion network connects high-probability words that were disconnected in the original, partially ordered representation.

One might suspect that despite the comparatively small number of word hypotheses, pruned confusion networks still contain a larger number of sentence hypotheses, since all word hypotheses may be combined (except those in the same confusion set). This might present a problem for post-processing algorithms, such as a parser, that are concerned with sentence-level analyses.

Figure 6: Confusion network accuracy on Switchboard test set I as a function of the percentage of links in the original lattice remaining after the pruning step.
We explicitly computed the number of paths in both our original WS97 lattice set and the pruned confusion networks. For clustering purposes, the lattices were pruned to 5% of the original links, so that the resulting confusion networks had an accuracy equal to that of the original full lattices. We found that the number of total paths in the 2427 original lattices was $10^{81}$, compared to $10^{80}$ paths in the pruned confusion networks. Furthermore, in 90% of the utterances the confusion network contained fewer paths than the original lattice.

We conclude that, based on the number of hypotheses alone, confusion networks should not be less well suited to sentence-level post-processing than recognition lattices. Of course it might still be the case that other properties of the sentence hypotheses allowed by confusion network cause problems in certain natural language processing applications. In the next section we examine a different pruning scheme that guarantees that the final set of sentence hypotheses is a proper subset of the set of original sentence hypotheses.

### 6.3. Consensus-based lattice pruning

Word lattices are typically used as intermediate representations of the hypothesis space prior to further recognition passes or more sophisticated hypothesis evaluation techniques. Two conflicting goals for lattice generation are therefore compactness (to save computation in further processing) and accuracy (to preserve the correct hypothesis as often as possible). The standard method to control the tradeoff between lattice size and accuracy is likelihood-based pruning: paths whose overall score differs by more than a threshold from the best-scoring path are removed from the word graph.

Several techniques have been developed explicitly to reduce the size of lattice representations while preserving all hypotheses in the lattice, e.g. by determinizing and minimizing the lattice as a finite state network (Mohri, 1997) or by node merging in nondeterministic lattices (Weng, Stolcke & Sankar, 1998). In the previous section we saw that confusion networks can serve as a compacted lattice representation that can also improve lattice accuracy by adding plausible paths.

Here we propose an alternative pruning approach that uses confusion networks merely as a filter on the hypothesis space, i.e. without adding paths to the original lattice. In this approach we first construct a confusion network, prune it as discussed previously, and then intersect the original lattice with the pruned confusion network. In other words, we preserve only original lattice paths that are also in the pruned confusion network. Final lattice size is controlled by the pruning threshold as applied to the confusion network. The effect of this consensus-based pruning is that paths are preserved, even if they have overall low probability, if they consist of pieces that have individually high posteriors.

Figure 7 shows the size of lattices (in terms of node and link densities) as a function of lattice accuracy, for both the traditional likelihood-based pruning and consensus-based pruning. We see that consensus-pruned lattices are three to four times smaller at equivalent accuracy levels. We can conclude that while...
Figure 7: Effectiveness of consensus-based pruning in terms of average node and link density reduction at different lattice accuracy levels when compared with the standard likelihood-based pruning. The original lattices had an average link density of 1350, an average node density of 370, and an oracle accuracy of 90.5%.
consensus-based pruning involves the extra processing step of confusion network construction and filtering, this effort could be worthwhile when a good size-accuracy tradeoff is desired for expensive post-processing algorithms.

6.4. Other applications and related work

6.4.1. Confidence annotation

The word-level posterior probabilities included in the confusion network provide valuable information for several tasks that require assessing the validity of individual word recognition hypotheses. The most obvious such task is confidence annotation of recognition output, i.e. estimating the probability of correctness of each word. The posterior probability according to a recognizer’s scoring function (as incorporated in the confusion network) is itself a confidence measure. However, the posterior probabilities thus computed tend to overestimate the word accuracy since they are based on only a subset of the infinite word hypothesis space.

A common technique uses an additional estimator that converts N-best or lattice-based word posterior probabilities to unbiased word correctness probabilities. Logistic regression (Siu, Gish & Richardson, 1997), decision trees (Evermann & Woodland, 2000b), and neural networks (Weintraub, Beaufays, Rivlin, König & Stolcke, 1997) have been used for this purpose. These techniques also allow miscellaneous other features and knowledge sources to be incorporated into word confidence estimation. However, it has been found that the recognizer-based word posteriors are usually among the most informative features predicting word correctness (Siu et al., 1997; Weintraub et al., 1997). Still, confusion networks could provide several additional input features for confidence estimators, such as the difference in posteriors between candidates, their posterior entropy, or the number of candidates in each confusion set.

Several papers investigate the usefulness of lattice-based features for confidence prediction (Kemp & Schaaf, 1997; Wessel, Macherey & Ney, 1999). Kemp & Schaaf (1997) show that the features obtained from a word graph are the most important predictors, although in their study the predictive feature considered is the posterior probability computed for lattice links rather than for words. Wessel et al. (1999) describe methods to compute word posterior probabilities by summing over the posterior probabilities of the links associated with a common time frame. This purely time-based alignment approach contrasts with ours, which relies primarily on lattice topology. Evermann & Woodland (2000b) investigate both time-dependent posteriors and posteriors based on confusion networks for confidence estimation. Their results show both approaches to give comparable estimates, with one or the other at a slight advantage depending on the acoustic models and lattice sizes used.
6.4.2. Word spotting

Another application of word posterior estimation is word spotting. As shown by Weintraub (1995), high-performance word spotting can be achieved by estimating word posteriors from the \(N\)-best output of a large vocabulary recognizer. In that approach, a time-based criterion was used to identify word hypotheses across \(N\)-best lists that correspond to the same word. An estimate of the word’s probability of occurrence was then obtained by adding the posterior probabilities for the corresponding sentence hypotheses. The confusion network algorithm is a lattice-based generalization of that approach, and we would therefore expect improved word spotting accuracy based on confusion network posterior probabilities.

6.4.3. System combination

Fiscus (1997) developed a popular algorithm known as ROVER (Recognizer Output Voting Error Reduction) for combining the outputs of several recognizers to yield higher-accuracy hypotheses. In the standard ROVER algorithm the 1-best outputs from multiple recognizers are aligned to “vote” on a new hypothesis that is generated by splicing together pieces of the original hypotheses. ROVER is similar to the confusion network construction algorithm in that it uses multiple alignments and word confidences for voting, although its task is simpler since only linear input hypotheses are considered. Its benefit is also limited by the fact that only a single word hypothesis is available from each recognizer. This suggests that an improved ROVER algorithm can be obtained by using the full confusion networks and associated posterior probabilities from each recognizer. For example, if there is a tie in one of the alignment columns, it might help to know that one of the words was a close second choice of some of the other recognizers (see Figure 8).

The idea of a generalized ROVER algorithm based on alignment of confusion networks was recently developed and implemented independently by Evermann & Woodland (2000a). The technique was applied in the NIST Hub-5 conversational speech recognition evaluation and found to give a 0.3% absolute reduction in WER compared to the standard ROVER algorithm. An improvement of the same magnitude was found with a different implementation based on combining \(N\)-best hypotheses lists (Stolcke et al., 2000).

7. Other lattice-based approaches

To make word error minimization on lattices feasible we took the approach to slightly modify the word error function (to a close approximation) so as to allow an efficient, exact determination of the best hypothesis. An alternative approach recently developed by Goel & Byrne (2000) takes a different approach and searches the lattice directly using the original word error cost function. To keep the search feasible an \(A^*\) heuristic search is employed, along with various
approximations and pruning strategies. Results show a consistent word error reduction beyond that achieved with the $N$-best approximation. Based on results reported in Section 4.1 and by Goel & Byrne (2000), the algorithm described here compares favorably with the A* approach.

The two lattice-based approaches to word error minimization each have distinct advantages. The A* method can be generalized to certain loss functions other than standard word error, in particular to weighted variants of word error. Practical limits to generalizability arise from the need to compute lower and upper bounds on the expected loss. The main strength of our algorithm is that it also produces a confusion network representation of the hypothesis space together with associated word posterior probabilities.

8. Conclusion

We have developed a new method for extracting from a recognition word lattice hypotheses that minimize expected word error, unlike the standard MAP scoring approach that minimizes sentence error. The core of the method is a clustering procedure that identifies mutually supporting and competing word hypotheses in a lattice, constructing a total order over all word hypotheses. Together with word posterior probabilities computed from recognizer scores, this allows an efficient extraction of the hypothesis with a minimum expected number of errors. Experiments on two speech corpora show that this approach results in significant WER reductions and significant lattice compression. Our method is also an estimator of word posterior probabilities and as such it can be used for other tasks, such as confidence annotation, word spotting, and system combination.

10Test set I used in our experiments and the MAP baseline accuracy coincide with that used by Goel & Byrne (2000).
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References

Bahl, L. R., Jelinek, F. & Mercer, R. L. (1983). A Maximum Likelihood Approach to Continuous Speech Recognition. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 5(2), 179–190.

Duda, R. O. & Hart, P. E. (1973). *Pattern Classification and Scene Analysis*. John Wiley, New York.

Evermann, G. & Woodland, P. (2000a). Posterior Probability Decoding, Confidence Estimation, and System Combination. In *Proceedings NIST Speech Transcription Workshop*, College Park, MD.

Evermann, G. & Woodland, P. C. (2000b). Large Vocabulary Decoding and Confidence Estimation Using Word Posterior Probabilities. In *Proceedings of the IEEE Conference on Acoustics, Speech, and Signal Processing*, volume III, pp. 1659–1662, Istanbul.

Fiscus, J. G. (1997). A Post-Processing System to Yield Reduced Word Error Rates: Recognizer Output Voting Error Reduction (ROVER). In *Proceedings IEEE Automatic Speech Recognition and Understanding Workshop*, pp. 347–352, Santa Barbara, CA.

Godfrey, J. J., Holliman, E. C. & McDaniel, J. (1992). SWITCHBOARD: Telephone speech corpus for research and development. In *Proceedings of the IEEE Conference on Acoustics, Speech, and Signal Processing*, volume 1, pp. 517–520, San Francisco.

Goel, V. & Byrne, W. (2000). Minimum Bayes-Risk Automatic Speech Recognition. *Computer Speech and Language*, 14(2), 115–135.

Goel, V., Byrne, W. & Khudanpur, S. (1998). LVCSR Rescoring with Modified Loss Functions: A Decision Theoretic Perspective. In *Proceedings of the IEEE Conference on Acoustics, Speech, and Signal Processing*, volume I, pp. 425–429, Seattle, WA.

Graff, D. (1997). The 1996 Broadcast News Speech and Language-Model Corpus. In *Proceedings DARPA Speech Recognition Workshop*, pp. 11–14, Chantilly, VA. Morgan Kaufmann.

Gusfield, D. (1992). Efficient Methods for Multiple Sequence Alignment with Guaranteed Error Bounds. *Bulletin of Mathematical Biology*, 54, 141–154.

Jelinek, F. (1998). 1997 Large Vocabulary Continuous Speech Recognition Summer Research Workshop Technical Reports. Research Note 30, Center for Language and Speech Processing, Johns Hopkins University, Baltimore.

Kemp, T. & Schaal, T. (1997). Estimating Confidence using Word Lattices. In G. Kokkinakis, N. Fakotakis & E. Dermatas, eds., *Proceedings of the 5th European Conference on Speech Communication and Technology*, volume 2, pp. 827–830, Rhodes, Greece.
Ljolje, A., Pereira, F. & Riley, M. (1999). Efficient General Lattice Generation and Rescoring. In *Proceedings of the 6th European Conference on Speech Communication and Technology*, volume 3, pp. 1251–1254, Budapest.

Mangu, L. & Brill, E. (1999). Lattice Compression in the Consensual Post-Processing Framework. In *Proceedings of the Third World Multiconference on Systemics, Cybernetics and Informatics joint with the Fifth International Conference on Information Systems Analysis and Synthesis*, volume 5, pp. 246–252, Orlando, Florida.

Mangu, L., Brill, E. & Stolcke, A. (1999). Finding Consensus Among Words: Lattice-based Word Error Minimization. In *Proceedings of the 6th European Conference on Speech Communication and Technology*, volume 1, pp. 495–498, Budapest.

Mohri, M. (1997). Finite-State Transducers in Language and Speech Processing. *Computational Linguistics*, 23(2), 269–311.

Siu, M.-H., Gish, H. & Richardson, F. (1997). Improved Estimation, Evaluation, and Applications of Confidence Measures for Speech Recognition. In G. Kokkinakis, N. Fakotakis & E. Dermatas, eds., *Proceedings of the 5th European Conference on Speech Communication and Technology*, volume 2, pp. 831–834, Rhodes, Greece.

Stolcke, A., Bratt, H., Butzberger, J., Franco, H., Rao Gadde, V. R., Plauché, M., Richey, C., Shriberg, E., Sönmez, K., Weng, F. & Zheng, J. (2000). The SRI March 2000 Hub-5 Conversational Speech Transcription System. In *Proceedings NIST Speech Transcription Workshop*, College Park, MD.

Stolcke, A., Konig, Y. & Weintraub, M. (1997). Explicit Word Error Minimization in N-best List Rescoring. In G. Kokkinakis, N. Fakotakis & E. Dermatas, eds., *Proceedings of the 5th European Conference on Speech Communication and Technology*, volume 1, pp. 163–166, Rhodes, Greece.

Weintraub, M. (1995). LVCSR Log-Likelihood Ratio Rescoring for Keyword Spotting. In *Proceedings of the IEEE Conference on Acoustics, Speech, and Signal Processing*, volume 1, pp. 297–300, Detroit.

Weintraub, M., Beaufays, F., Rivlin, Z., Konig, Y. & Stolcke, A. (1997). Neural-Network Based Measures of Confidence for Word Recognition. In *Proceedings of the IEEE Conference on Acoustics, Speech, and Signal Processing*, volume 2, pp. 887–890, Munich.

Weng, F., Stolcke, A. & Sankar, A. (1998). New Developments in Lattice-Based Search Strategies in SRI’s Hub4 System. In *Proceedings DARPA Broadcast News Transcription and Understanding Workshop*, pp. 138–143, Lansdowne, VA. Morgan Kaufmann.

Wessel, F., Macherey, K. & Ney, H. (1999). A comparison of word graph and N-best list based confidence measures. In *Proceedings of the 6th European Conference on Speech Communication and Technology*, volume 1, pp. 315–318, Budapest.

Young, S. (1997). HTK 2.1. Entropic Cambridge Research Laboratory Ltd., Cambridge.
A. Optimized N-best rescoring algorithm

Here we give the pseudo-code for an optimized version of the N-best word error minimization algorithm, implementing Equation 4. The N-best hypotheses are denoted by $W^{(1)}, \ldots, W^{(N)}$. $P(\cdot|A)$ are the posterior probabilities estimated as described in Section 3.7, and $WE(\cdot, \cdot)$ is the word error function.

```
BestHyp := 0;
BestError := \infty;
for i := 1, \ldots, N do
    ThisError := 0;
    for k := 1, \ldots, N do
        ThisError := ThisError + P(W^{(k)}|A)WE(W^{(k)}, W^{(i)});
        if ThisError \geq BestError then
            (*)
            goto next;
        end
    end
    if ThisError < BestError then
        BestError := ThisError;
        BestHyp := i;
    end
next:
/* BestHyp contains index of best hypothesis */
```

The test and loop exit marked by (*) are crucial for efficiency. With it, the worst-case complexity of the algorithm is still $O(N^2)$; however, in practice the inner loop is exited after a few iterations whose average number is independent of $N$. On Switchboard N-best lists we measured runtimes (excluding constant overhead) that scale perfectly linearly with $N$. In practice, the optimized algorithm is quite fast: The Switchboard 300-best lists were processed in about $0.1 \times$ real time using a 400 MHz Pentium-II CPU.
B. Lattice alignment algorithm

Here we give a concise description of the algorithm that constructs the edge equivalence relation that results in a complete lattice alignment.

As outlined in Section 3, the algorithm proceeds in three stages. The initial link equivalence classes are formed by word identity and start and end times:

\[ L_{w,t_1,t_2} = \{ e \in E | \text{Word}(e) = w, \text{Itime}(e) = t_1, \text{Ftime}(e) = t_2 \} \]

The initial partial order \( \preceq \) is given as the transitive closure of the edge order \( \leq \) defined on the lattice (see Section 3.1).

The next step performs intra-word clustering, i.e., merging of classes containing the same words:

\[
\begin{align*}
\text{do} \\
\text{MaxSim} &:= 0; \\
\text{for all } L_1 \in \mathcal{E} \\
&\text{for all } L_2 \in \mathcal{E} \\
&\quad \text{if Words}(L_1) = \text{Words}(L_2) \text{ and } L_1 \not\preceq L_2 \text{ and } L_2 \not\preceq L_1 \text{ then} \\
&\quad \quad \text{sim} := \text{SIM}(L_1, L_2); \\
&\quad \quad \text{if sim} > \text{MaxSim} \text{ then} \\
&\quad \quad \quad \text{MaxSim} := \text{sim}; \\
&\quad \quad \quad \text{bestSet}_1 := L_1; \\
&\quad \quad \quad \text{bestSet}_2 := L_2; \\
&\quad \text{end} \\
&\text{end} \\
&\text{end} \\
\text{end}
\end{align*}
\]

\[ L_{\text{new}} := \text{bestSet}_1 \cup \text{bestSet}_2; \]

\[
\begin{align*}
\text{for all } L_i \in \mathcal{E} \\
&\quad \text{if } L_i \preceq \text{bestSet}_1 \text{ or } L_i \preceq \text{bestSet}_2 \text{ then} \\
&\quad \quad \text{let } L_i \preceq L_{\text{new}}; \\
&\quad \text{end} \\
&\text{end} \\
\text{for all } L_i \in \mathcal{E} \\
&\quad \text{if } \text{bestSet}_1 \preceq L_i \text{ or } \text{bestSet}_2 \preceq L_i \text{ then} \\
&\quad \quad \text{let } L_{\text{new}} \preceq L_i; \\
&\quad \text{end} \\
&\text{end} \\
\text{for all } L_i \in \mathcal{E} \\
&\quad \text{for all } L_j \in \mathcal{E} \\
&\quad \quad \text{if } (L_i \preceq \text{bestSet}_1 \text{ and } \text{bestSet}_2 \preceq L_j) \text{ or } (L_i \preceq \text{bestSet}_2 \text{ and } \text{bestSet}_1 \preceq L_j) \text{ then} \\
&\quad \quad \quad \text{let } L_i \preceq L_j; \\
&\quad \text{end} \\
&\text{end} \\
\end{align*}
\]
$\mathcal{E} := \mathcal{E} \cup \{L_{\text{new}}\} \setminus \{\text{bestSet}_1, \text{bestSet}_2\}$;

\textbf{while} $\text{MaxSim} > 0$

The notation used is described in Section 3. In this stage we use the similarity metric $\text{SIM}(\cdot, \cdot)$ described in Section 3.2. The partial order relation $\preceq$ is updated minimally upon merging of equivalence classes so as to keep it consistent with the previous order; an inductive definition of $\preceq$ is given as part of the correctness proof in Appendix C.

The final step, inter-word clustering, uses the same algorithm as the previous step, but we replace the $\text{SIM}$ metric with one that is based on phonetic similarity as described in Section 3.3 and eliminate the condition $\text{Words}(L_1) = \text{Words}(L_2)$ that the words corresponding to two equivalence class candidates should be the same.

The complexity of the entire confusion network construction algorithm is dominated by the hypothesis alignment, which is of order $O(T^3)$, where $T$ is the number of links in the word lattice. As shown in Figure 3, we can prune about 95% of the links in the original lattices without affecting the accuracy of the consensus hypothesis. Hence, $T$ is kept small and runtimes are very reasonable in practice: on a 400 MHz Pentium-II processor, the Switchboard lattices were processed in about 0.55× real time on average.
C. Proof of correctness

To complete a formal proof of the correctness of the lattice alignment algorithm we need to show that successive merging of equivalence classes is possible while maintaining a consistent partial order on the classes. To demonstrate termination of the algorithm, we need to establish that classes suitable for merging can be found as long as any two classes are unordered. (Since the initial number of classes is finite and is decremented in each merging step, this guarantees that the algorithm terminates with a total order of classes after a finite number of steps.)

Here we give a constructive proof by induction that a consistent partial order can be defined for the initial equivalence partition, and maintained for each merging step. Recall that a relation \( \preceq \) is a partial order if and only if it has the following properties:

- Reflexivity: For all \( x \), \( x \preceq x \).
- Transitivity: For all \( x, y, z \), \( x \preceq y \) and \( y \preceq z \) imply \( x \preceq z \).
- Antisymmetry: For all \( x, y \), \( x \preceq y \) and \( y \preceq x \) imply \( x = y \).

The concept of consistency was defined in Section 3.1.

For the first part of the proof, we must show that the relation we define on the initial set of equivalence classes is a consistent partial order. We form the initial partition \( E_0 = \{L_{w_1, t_1}, t_2\} \) as follows. If \( \leq \) is the partial order on links introduced in Section 3.1 define the relation \( R \) on \( E_0 \) as follows:

\[
E_1 \mathrel{R} E_2 \iff \text{there exists } e \in E_1 \text{ and } f \in E_2 \text{ such that } e \leq f.
\]

and define \( \preceq_0 \) as the transitive closure of \( R \). Reflexivity of \( R \) holds, since \( e \leq e \) for any link \( e \in E \). We claim that \( R \) is an antisymmetric relation. Let \( E_1 \) and \( E_2 \) be two sets of links in \( E_0 \) such that \( E_1 \neq E_2 \), \( E_1 \mathrel{R} E_2 \) and \( E_2 \mathrel{R} E_1 \). Consequently there exist \( e_{11} \in E_1, e_{21} \in E_2 \) with \( e_{11} \leq e_{21} \) and there exist \( e_{22} \in E_2, e_{12} \in E_1 \) with \( e_{22} \leq e_{12} \). Given that all the links in the same equivalence class have the same start and end times, and that if two links in the lattice are \( \leq \) ordered then the start and end times are ordered accordingly, we obtain the following:

\[
Ftime(e_{11}) \leq Itime(e_{21}) = Itime(e_{22}) < Ftime(e_{22}) \leq Itime(e_{12}) = Itime(e_{11})
\]

which is a contradiction. Thus, \( R \) is a reflexive and antisymmetric relation, which makes its transitive closure \( \preceq_0 \) a partial order. The consistency of \( \preceq_0 \) comes directly from the definition of \( R \).

In the second part of the proof we have to show that if we start with a consistent partial order and at each step merge only equivalence classes that are not ordered (under the current order), we still have a consistent partial order on the new set of equivalence classes. The proof proceeds by induction over the merging steps.

Let \( E_i \) be the set of equivalence classes and \( \preceq_i \) the consistent partial order on \( E_i \) at step \( i \), and \( L_1 \) and \( L_2 \) the two candidate equivalence classes that can be merged at step \( i \), hence having the property \( (L_1 \not\preceq_i L_2 \text{ and } L_2 \not\preceq_i L_1) \). We define \( L_{\text{new}} = L_1 \cup L_2 \) as the new merged equivalence class.

The set of equivalence classes at step \( i + 1 \) is

\[
E_{i+1} = E_i \cup \{L_{\text{new}}\} \setminus \{L_1, L_2\}
\]
and the relation $\preceq_{i+1}$ is derived from $\preceq_i$ as follows:

- if $U \preceq_i V$ then $U \preceq_{i+1} V$ for any $U, V \in \mathcal{E}_{i+1}$, $U \neq L_{\text{new}}$ and $V \neq L_{\text{new}}$

- if $U \preceq_i L_1$ or $U \preceq_i L_2$ then $U \preceq_{i+1} L_{\text{new}}$

- if $L_1 \preceq_i V$ or $L_2 \preceq_i V$ then $L_{\text{new}} \preceq_{i+1} V$

- if $(U \preceq_i L_1$ and $L_2 \preceq_i V)$ or $(U \preceq_i L_2$ and $L_1 \preceq_i V)$ then $U \preceq_{i+1} V$

- $L_{\text{new}} \preceq_{i+1} L_{\text{new}}$

We claim that $\preceq_{i+1}$ as defined above is a partial order.

**Reflexivity** holds, because $\preceq_i$ is reflexive and $L_{\text{new}} \preceq_{i+1} L_{\text{new}}$.

**Antisymmetry** can be shown by contradiction. Let us assume that there exist $U, V \in \mathcal{E}_{i+1}, U \neq V$ such that $U \preceq_{i+1} V$ and $V \preceq_{i+1} U$.

*Case 1*: $U \neq L_{\text{new}}$ and $V \neq L_{\text{new}}$

Given that $U \neq V$, the fact that $U \preceq_{i+1} V$ comes either from $U \preceq_i V$ or $(U \preceq_i L_1$ and $L_2 \preceq_i V)$ or $(U \preceq_i L_2$ and $L_1 \preceq_i V)$. The same holds for $V \preceq_{i+1} U$. We would then have to consider all nine combinations, but the proofs for many of them are very similar, and therefore we spell out only the most interesting cases:

- If $U \preceq_i V$ and $V \preceq_i U$ then $U = V$ (antisymmetry of $\preceq_i$). Contradiction with the assumption $U \neq V$.

- If $U \preceq_i V$ and $V \preceq_i L_1$ and $L_2 \preceq_i U$ then $L_2 \preceq_i L_1$ (transitivity of $\preceq_i$). This contradicts the condition that $L_1$ and $L_2$ be unordered to be candidates for merging.

- If $U \preceq_i L_1$ and $L_2 \preceq_i V$ and $V \preceq_i U$ then $L_2 \preceq_i L_1$. Same contradiction as in the previous case.

- If $U \preceq_i L_1$ and $L_2 \preceq_i V$ and $V \preceq_i L_2$ and $L_1 \preceq_i U$ then $U = L_1$ and $V = L_2$ (antisymmetry of $\preceq_i$). Contradiction with $U, V \in \mathcal{E}_{i+1}$.

*Case 2*: $U = L_{\text{new}}$

By definition, $L_{\text{new}} \preceq_{i+1} V$ only if either $L_1 \preceq_i V$ or $L_2 \preceq_i V$, and similarly for $V \preceq_{i+1} L_{\text{new}}$. Therefore we have to consider four cases, but only two of them are structurally distinct:

- If $L_1 \preceq_i V$ and $V \preceq_i L_1$ then $V = L_1$ (antisymmetry of $\preceq_i$). Contradiction with $V \in \mathcal{E}_{i+1}$.

- If $L_1 \preceq_i V$ and $V \preceq_i L_2$ then $L_1 \preceq_i L_2$ (transitivity of $\preceq_i$). Contradiction with the condition that $L_1$ and $L_2$ be unordered.

*Case 3*: $V = L_{\text{new}}$ is similar to Case 2.

**Transitivity** Let us consider $U, V, W \in \mathcal{E}_{i+1}$ such that $U \preceq_{i+1} V$ and $V \preceq_{i+1} W$. We have to show that $U \preceq_{i+1} W$.

*Case 1*: $U \neq L_{\text{new}}$, $V \neq L_{\text{new}}$ and $W \neq L_{\text{new}}$

Using the previous remark concerning the reasons for $U \preceq_{i+1} V$ and $V \preceq_{i+1} W$, we have the following cases:

- If $U \preceq_i V$ and $V \preceq_i W$, then $U \preceq_i W$ (transitivity of $\preceq_i$). Consequently, $U \preceq_{i+1} W$.  

If $U \preceq_i V$ and $V \preceq_i L_1$ and $L_2 \preceq_i W$, then $U \preceq_i L_1$ (transitivity of $\preceq_i$) and $L_2 \preceq_i W$. Thus, $U \preceq_{i+1} W$ (definition of $\preceq_{i+1}$).

If $U \preceq_i L_1$ and $L_2 \preceq_i V$ and $V \preceq_i W$, then $U \preceq_i L_1$ and $L_2 \preceq_i W$. Consequently, $U \preceq_{i+1} W$.

It is not possible to have $U \preceq_i L_1$ and $L_2 \preceq_i V$ and $V \preceq_i L_1$ and $L_2 \preceq_i W$ because this would imply $L_2 \preceq_i L_1$. Also, if we had $U \preceq_i L_1$ and $L_2 \preceq_i V$ and $V \preceq_i L_2$ and $L_1 \preceq_i W$ we could infer $L_2 = V$, which contradicts $V \in \mathcal{E}_{i+1}$.

Case 2: $U = L_{new}$

The only possible combinations are

- $L_1 \preceq_i V$ and $V \preceq_i W$, in which case $L_1 \preceq_i W$, therefore $U \preceq_i W$ (definition of $\preceq_{i+1}$)
- $L_2 \preceq_i V$ and $V \preceq_i W$, in which case $L_2 \preceq_i W$, therefore $U \preceq_i W$ (definition of $\preceq_{i+1}$)

The other combinations are not possible. For example, if $L_1 \preceq_i V$ and $V \preceq_i L_1$ and $L_2 \preceq_i W$ then $V = L_1$ which contradicts $V \in \mathcal{E}_{i+1}$.

Case 3: $W = L_{new}$ is similar to Case 2.

Case 4: $V = L_{new}$. Given that $U \preceq_{i+1} L_{new}$ only if $U \preceq_i L_1$ or $U \preceq_i L_2$, we have the following cases:

- If $U \preceq_i L_1$ and $L_1 \preceq_i W$ then $U \preceq_i W$ (transitivity of $\preceq_i$), and so $U \preceq_{i+1} W$.
- If $U \preceq_i L_2$ and $L_1 \preceq_i W$ then $U \preceq_i W$ (definition of $\preceq_{i+1}$), hence $U \preceq_{i+1} W$.

We have shown that the new relation is a partial order. We still need to show that it is consistent, i.e., if $e \le f$ then $[e]_{i+1} \preceq_{i+1} [f]_{i+1}$.

- If $[e]_{i+1} \neq L_{new}$ and $[f]_{i+1} \neq L_{new}$ then $[e]_{i+1} = [e]_i$ and $[f]_{i+1} = [f]_i$. Given that $[e]_i \preceq_i [f]_i$ (consistency of $\preceq_i$) we obtain $[e]_{i+1} \preceq_{i+1} [f]_{i+1}$ (definition of $\preceq_{i+1}$).
- If $[e]_{i+1} = L_{new}$ (which implies $[f]_{i+1} \neq L_{new}$) then $e \in L_1$ or $e \in L_2$, and so $L_1 \preceq_i [f]_i$ or $L_2 \preceq_i [f]_i$ (consistency of $\preceq_i$). Consequently, $L_{new} \preceq_{i+1} [f]_i$. Given that $[f]_i = [f]_{i+1}$ we obtain $[e]_{i+1} \preceq_{i+1} [f]_{i+1}$.
- Same for $[f]_{i+1} = L_{new}$.