Explicit Numerical Model of Solar Cells to Determine Current and Voltage

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Abstract: This paper deals with the extraction of the physical parameters of solar cells (a single diode model) from the equivalent circuit of the cell. The extraction is carried out by three different numerical methods with a comparison between them. The first and second methods are a difference of the Newton Raphson algorithm (NRM) and Aitken’s extrapolation algorithm (AEM), respectively. The roots of the nonlinear equation of this cell have been described and solved using the three methods. The proposed method is tested to solve the output voltage; current and power of this cell from the roots of this equation with the various values of load resistance $R_L$.

Keywords: Aitken's extrapolation algorithm; three step method; iterations; load resistance; parameters of solar cell.

1. Introduction

Numerical methods are a class of methods used to solve a wide range of mathematical problems whose origins can be mathematical models of physical conditions. These methods are unique in that they only use calculations and logic, which can be used directly on a digital computer. Numerical analysis can solve many kinds of nonlinear equations such as differential and partial equations; linear systems; Taylor series; integral equations; optimal control problems and physical problems like solar cells [1-5]. A photovoltaic cell is a specialized semiconductor diode that converts light into direct current electricity (DC). Depending on the optical band gap in the light absorbing range, photovoltaic cells can also convert low-energy, infrared (IR) or high-energy, ultraviolet (UV) photons into DC electricity. These cells are made of semiconductor materials such as silicon. PV cells can be described corresponding to its manufacturing technology and the material used such as monocrystalline, polycrystalline, and amorphous solar cells; in addition; thin films solar cells [9-27].

This work aims to propose and characterize a new numerical method in order to find the real roots of the single-diode nonlinear equation of the solar cells based on three different techniques with the comparison between them. It is organized as follows: section 2 characterizing the analytical model of a single-diode design of the solar cell; Section 3 establishing the root-finding Newton Raphson Method (NRM); Aitken's extrapolation algorithm (AEM) and three step method (TSM); section 4 results and discussion; section 5 conclusions of the obtained results. MATLAB program is achieved for all the acquired results.

2. Characteristics of Single-Diode Solar Cells Equation

The simple equivalent electric circuit of a solar cell is shown in Figure 1.
Kirchhoff’s current law is applied for single-diode model to calculate the current $I$, the equation is given by

$$I = I_{ph} - I_D$$

(1)

$$I_D = I_0 \left( e^{\frac{V_{PV}}{nV_T}} - 1 \right)$$

(2)

$$I = I_{ph} - I_0 \left( e^{\frac{V_{PV}}{nV_T}} - 1 \right)$$

(3)

where:

$I_{ph}$ is the photocurrent (A); $I_0$ is reverse saturation current of the diode (A); $I$ and $V_{PV}$ are the delivered current and voltage, respectively. $V_T = \frac{kT}{q} = 0.0259$ V is thermic voltage = 27.5 $\pm$ 26 mV at ($T=25\,^\circ C$ Air-Mass $n=1.5$); $m$ is the recombination factor closeness to an ideal diode ($1 < m < 2$, $k$ is Boltzmann constant=1.38$\times$10$^{-23}$ J/K; $T$ is P-n junction temperature (K); $q$ is the electron charge=1.6$\times$10$^{-19}$ C.

$I_{ph}$=$I_{source}$

(4)

$$I_D = I_s \times \left( e^{\frac{V_D}{nV_T}} - 1 \right)$$

(5)

Substituting Eq. 4 in Eq. 5 we get

$$\left( I_{source} \right) = 10^{-12} \left( e^{\frac{-V}{1.2\times10^{-26}}} - 1 \right)$$

(6)

where $I_s$ reverse saturation current= 10$^{-12}$ A, suppose n is ideality factor=1.2 in our case (normally n between 1 and 2). In parallel, $V_D = V_{PV} = V$

According to Eq. 6 one can calculate $V$ of the cell numerically based on the first derivative of this equation.

3. Newton Raphson Method

The following algorithm suggestion for solving Eq. 5 by using NRM (see Figure 3)

INPUT initial approximate solution $x_0$, tolerance $\varepsilon$, maximum number of iterations $N$.

OUTPUT approximate solution $x_{n+1}$

Step 1: Set $x = 0$

Step 2: while $i \leq x_0$

Step 3: Calculate

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \text{ for } n = 0, 1, 2, \ldots$$

Step 4: if $|x_i - x_{i-1}| < \varepsilon$; then OUTPUT $x_{n+1}$ and stop.

Step 5: Set $n = n + 1; i = i + 1$ and go to Step 2.

Step 6: OUTPUT

4. Aitken's Extrapolation Algorithm (AEM)

Given: $x_0$, $\varepsilon = 10^{-9}$, $N$, $f$, $df$
Step 1: For \( i = 1 \) to 2

Step 2: Calculate \( \bar{E}_n = \bar{E}_{n+2} - \frac{(\bar{E}_{n+2} - \bar{E}_{n+1})^2}{\bar{E}_{n+2} - 2 \times \bar{E}_{n+1} + \bar{E}_n} \) for \( n = 0, 1, 2, \ldots \).

Step 3: If \( f(x_i) = 0 \) or \( f(x_i) < \varepsilon \), then go to Step 6

Step 4: Set \( \bar{E}_{n+1} = \bar{E}_n \)

Step 5: \( n = n + 1, i = i + 1 \), go back to Step 2.

Step 6: OUTPUT \( x_{n+1} \) and stop iteration.

5. Three Step Method (TSM)

Six-order convergences with three steps are investigated. Let \( f(x) = 0 \) is a nonlinear equation, suppose \( x_0 \) as an initial value, so the iteration results \( x_{n+1} \) can be calculated using the following scheme

\[
\begin{align*}
\gamma_n &= x_n - \frac{f(x_n)}{f'(x_n)} \\
z_n &= x_n - \left[ \frac{f(x_n) + f(y_n)}{f(x_n)} \right] \\
x_{n+1} &= z_n - \frac{f(x_n)x_n}{f'(x_n)[f(x_n) + f(y_n)] - f(x_n)x_n} \tag{7}
\end{align*}
\]

Eq. 7 has a six-order convergence called a three-step method (TSM); the proposed method.

6. Results and Discussion

Consider the Eq. 6 is modeled in the form single-diode solar cell; has obtained the following approximate solutions and three numerical methods are applied: first, Newton-Raphson methods (NRM) with initial value \( x_0 \), second, the Aitken's extrapolation algorithm (AEM) with the initial values from (NRM) \( x_0 \) and \( x_1 \); third the proposed method three-step method (TSM) with the initial value from (AEM). In Table 1 the Aitken's extrapolation algorithm (AEM) and three-step method (TSM) of the solution results (voltage \( V_{pv} \); current \( I_{pv} \) and power \( P_{pv} \) of the solar cell) are given and listed in the last columns of this table when the load resistance \( R = 1 \).

| Iterations | \( R \) | \( X_n \) | \( V_{pv}\) - AEM | \( I_{pv}\) - AEM | \( P_{pv}\) - AEM | \( V_{pv}\) - TSM | \( I_{pv}\) - TSM | \( P_{pv}\) - TSM |
|------------|--------|-----------|----------------|----------------|----------------|----------------|----------------|----------------|
| 1          | 1      | \( x_0 \) | 0.947037857    | 0.947037857    | 0.896880703    | 0.922220699    | 0.922220699    | 0.850491018    |
| 2          | 1      | \( x_1 \) | 0.930012729    | 0.930012729    | 0.864923676    | 0.921919557    | 0.921919557    | 0.84993567    |
| 3          | 1      | \( x_2 \) | 0.923271149    | 0.923271149    | 0.852429615    | 0.922412266    | 0.922412266    | 0.850844388    |
| 4          | 1      | \( x_3 \) | 0.922434357    | 0.922434357    | 0.850885144    | 0.922423132    | 0.922423132    | 0.850864435    |
| 5          | 1      | \( x_4 \) | 0.922423136    | 0.922423136    | 0.850864433    | 0.922423135    | 0.922423135    | 0.850864439    |
| 6          | 1      | \( x_5 \) | 0.922423135    | 0.922423135    | 0.850864439    | 0.922423135    | 0.922423135    | 0.850864439    |
| 7          | 1      | \( x_6 \) | 0.922423135    | 0.922423135    | 0.850864439    | 0.922423135    | 0.922423135    | 0.850864439    |

Figure 2 presents the obtained solutions of the study result.
Figure 2. Obtained solutions of the study result at the load resistance $R = 1$.

In Table 2 the Aitken's extrapolation algorithm (AEM) and three-step method (TSM) of the solution results (voltage $V_{pv}$; current $I_{pv}$, and power $P_{pv}$ of the solar cell) are given and listed in the last columns of this table when the load resistance $R = 2$.

| Iterations | $R$ | $x_n$ | $V_{pv}$-AEM | $I_{pv}$-AEM | $P_{pv}$-AEM | $V_{pv}$-TSM | $I_{pv}$-TSM | $P_{pv}$-TSM |
|------------|-----|-------|---------------|--------------|--------------|--------------|--------------|--------------|
| 1          | 2   | $x_{10}$ | 0.945750417   | 0.472875208  | 0.447221925  | 0.917699118  | 0.458849559  | 0.421085836  |
| 2          | 2   | $x_{1}$  | 0.927013023   | 0.463506512  | 0.429676573  | 0.916335587  | 0.458167793  | 0.419835454  |
| 3          | 2   | $x_{2}$  | 0.918476227   | 0.459238113  | 0.421799289  | 0.91700519   | 0.458502595  | 0.420449259  |
| 4          | 2   | $x_{3}$  | 0.917067904   | 0.458539352  | 0.42050677   | 0.917035365  | 0.458517683  | 0.420476931  |
| 5          | 2   | $x_{4}$  | 0.917035399   | 0.4585177    | 0.420476962  | 0.917035382  | 0.458517691  | 0.420476946  |
| 6          | 2   | $x_{5}$  | 0.917035382   | 0.458517691  | 0.420476946  | 0.917035382  | 0.458517691  | 0.420476946  |
| 7          | 2   | $x_{6}$  | 0.917035382   | 0.458517691  | 0.420476946  | 0.917035382  | 0.458517691  | 0.420476946  |

Figure 3 presents the obtained solutions of the study result.
Obtained solutions of the study result at the load resistance $R = 2$.

Table 3. The $V_{PV}$, $I_{PV}$, $P_{PV}$ and $\varepsilon$ values using AEM and TSM

| Iterations | $R$ | $X_n$ | $V_{PV}$-AEM | $I_{PV}$-AEM | $P_{PV}$-AEM | $V_{PV}$-TSM | $I_{PV}$-TSM | $P_{PV}$-TSM |
|------------|-----|-------|--------------|--------------|--------------|--------------|--------------|--------------|
| 1          | 3   | $x_0$ | 0.944437431  | 0.472218715  | 0.44598103   | 0.91268674   | 0.45634337   | 0.41649854   |
| 2          | 3   | $x_1$ | 0.92381119   | 0.461905595  | 0.426713557  | 0.90952263   | 0.454761319  | 0.413615714  |
| 3          | 3   | $x_2$ | 0.912938978  | 0.456469489  | 0.416728789  | 0.910316762  | 0.455158381  | 0.414338303  |
| 4          | 3   | $x_3$ | 0.910504334  | 0.455252167  | 0.414509071  | 0.910403208  | 0.455201604  | 0.414417     |
| 5          | 3   | $x_4$ | 0.910403537  | 0.455201768  | 0.4144173    | 0.910403374  | 0.455201687  | 0.414417152  |
| 6          | 3   | $x_5$ | 0.910403374  | 0.455201687  | 0.41441752   | 0.910403374  | 0.455201687  | 0.414417152  |
| 7          | 3   | $x_6$ | 0.910403374  | 0.455201687  | 0.41441752   | 0.910403374  | 0.455201687  | 0.414417152  |

Figure 4 presents the obtained solutions of the study result.
In Table 4 the Aitken's extrapolation algorithm (AEM) and three-step method (TSM) of the solution results (voltage $V_{pv}$; current $I_{pv}$, and power $P_{pv}$ of the solar cell) are given and listed in the last columns of this table when the load resistance $R = 4$.

Table 4. The $V_{pv}$, $I_{pv}$, $P_{pv}$ and $\varepsilon$ values using AEM and TSM

| Iterations | $R$ | $X_0$ | $V_{pv}$-AEM | $I_{pv}$-AEM | $P_{pv}$-AEM | $V_{pv}$-TSM | $I_{pv}$-TSM | $P_{pv}$-TSM |
|------------|-----|-------|---------------|--------------|--------------|--------------|--------------|--------------|
| 1          | 4   | $x_0$ | 0.943098312   | 0.235774578  | 0.222358607  | 0.907097754  | 0.226774439  | 0.205706584  |
| 2          | 4   | $x_1$ | 0.92038679    | 0.23096697   | 0.21177961   | 0.9010338    | 0.22525845   | 0.202965477  |
| 3          | 4   | $x_2$ | 0.90644763    | 0.226611907  | 0.205411826  | 0.901608561  | 0.22540214   | 0.203224499  |
| 4          | 4   | $x_3$ | 0.90208766    | 0.225521915  | 0.203440537  | 0.901753616  | 0.225438404  | 0.203289896  |
| 5          | 4   | $x_4$ | 0.901742565   | 0.225435641  | 0.203249413  | 0.90174069   | 0.225435173  | 0.203284068  |
| 6          | 4   | $x_5$ | 0.901740602   | 0.225435151  | 0.203248028  | 0.901740602  | 0.22543515   | 0.203284028  |
| 7          | 4   | $x_6$ | 0.901740602   | 0.22543515   | 0.203284028  | 0.901740602  | 0.22543515   | 0.203284028  |

Figure 5 presents the obtained solutions of the study result.
Figure 5. Obtained solutions of the study result at the load resistance $R = 4$.

In Table 5 the Aitken’s extrapolation algorithm (AEM) and three-step method (TSM) of the solution results (voltage $V_{pv}$; current $I_{pv}$, and power $P_{pv}$ of the solar cell) are given and listed in the last columns of this table when the load resistance $R = 5$.

| Iterations | $R$ | $X_n$  | $V_{pv}$-AEM | $I_{pv}$-AEM | $P_{pv}$-AEM | $V_{pv}$-TSM | $I_{pv}$-TSM | $P_{pv}$-TSM |
|------------|-----|--------|-------------|-------------|-------------|-------------|-------------|-------------|
| 1          | 5   | $x_0$  | 0.941732458 | 0.188346492 | 0.177372004 | 0.900883039 | 0.180176608 | 0.16231805 |
| 2          | 5   | $x_1$  | 0.916716819 | 0.183343364 | 0.168073945 | 0.889778347 | 0.177955669 | 0.158341101|
| 3          | 5   | $x_2$  | 0.898705719 | 0.179741144 | 0.161534394 | 0.8884201  | 0.17768402  | 0.157858055|
| 4          | 5   | $x_3$  | 0.890512633 | 0.178102527 | 0.15860255  | 0.889061297 | 0.177812259 | 0.158085998|
| 5          | 5   | $x_4$  | 0.889126783 | 0.177825357 | 0.158109287 | 0.889092694 | 0.177818539 | 0.158097164|
| 6          | 5   | $x_5$  | 0.889092735 | 0.177818547 | 0.158097178 | 0.889092715 | 0.177818543 | 0.158097171|
| 7          | 5   | $x_6$  | 0.889092715 | 0.177818543 | 0.158097171 | 0.889092715 | 0.177818543 | 0.158097171|
| 8          | 5   | $x_7$  | 0.889092715 | 0.177818543 | 0.158097171 | 0.889092715 | 0.177818543 | 0.158097171|

Figure 6 presents the obtained solutions of the study result.
Figure 6. Obtained solutions of the study result at the load resistance $R = 5$.

The obtained solution plot in the (no. of iteration)-$\xi$-plane and the initial-output values proves that the proposed method (TSM) has seven iterations indicated a fast behaviour. Parallel to this feature, it is noticed that the proposed method (TSM) has a behaviour of the solution in the initial values $x_0$-(AEM) has the smallest error tolerance compared with (NRM) and (AEM) with initial value $x_0 = 1$.

Results of tables 1 to 5 are showing that the suggested method (TSM) have low error after relatively view iterations are computed and this in turn is demonstrating their efficiency

7. Conclusion

This paper, give three numerical solutions for single-diode for PV cells mathematical model. The basic advantages of the proposed method (TSM) are simplicity and high accurate approximate solution which was achieved using a few numbers of iterations. The obtained numerical results were compared with two other methods (NRM and AEM).

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