ABSTRACT

Context. The discovery of extended gamma-ray emission toward a number of middle-aged pulsars suggests the possibility of long-lived particle confinement beyond the classical pulsar wind nebula (PWN) stage. How this emerging source class can be extrapolated to a Galactic population remains unclear.

Aims. We aim to evaluate how pulsar halos fit in existing TeV observations, under the assumption that all middle-aged pulsars develop halos similar to those observed toward the J0633+1746 or B0656+14 pulsars.

Methods. We modeled the populations of supernova remnants, PWNe, and pulsar halos in the Milky Way. The PWN-halo evolutionary sequence is described in a simple yet coherent framework, and both kinds of objects are assumed to share the same particle injection properties. We then assessed the contribution of the different source classes to the very-high-energy emission from the Galaxy.

Results. The synthetic population can be made consistent with the flux distribution of all known objects, including unidentified objects, for a reasonable set of parameters. The fraction of the populations predicted to be detectable in surveys of the Galactic plane with HESS and HAWC is then found to be in good agreement with their actual outcome, with a number of detectable halos ranging from 30 to 80% of the number of detectable PWNe. Prospects for CTA involve the detection of 250–300 sources in the Galactic Plane Survey, including 170 PWNe and up to 100 halos. The extent of diffusion suppression in halos has a limited impact on such prospects but its magnitude has a strong influence. The level of diffuse emission from unresolved populations in each survey is found to be dominated by halos and comparable to large-scale interstellar radiation powered by cosmic rays above 0.1–1 TeV.

Conclusions. Pulsar halos are shown to be viable counterparts to a fraction of the currently unidentified sources if they develop around most middle-aged pulsars. Yet, if the phenomenon is rare, with an occurrence rate of 5–10% as suggested in a previous work from the local positron flux constraint, the total number of currently known TeV sources including unidentified ones cannot be accounted for in our model from young PWNe only. This calls for continued efforts to model pulsar-powered emission along the full evolutionary path, including the late stages past the young nebula phase.

Key words. astroparticle physics – pulsars: general – gamma rays: general – cosmic rays

1. Introduction

Over the past decade, it has become clear that pulsars are major players in the very-high-energy (VHE) sky, and possibly even in the ultra-high-energy (UHE) sky (H.E.S.S. Collaboration 2018a; Albert et al. 2021; Breuhaus et al. 2022; de Oña Wilhelmi et al. 2022). While formal identification is still lacking for a large fraction of the constantly increasing population of known VHE and UHE gamma-ray sources, observational evidence is growing in support to this conclusion.

Emission components detected in the HESS Galactic plane survey (HGPS) are found to be significantly correlated in position with energetic pulsars, while this correlation is absent for less energetic pulsars (H.E.S.S. Collaboration 2018b). Out of the 78 sources detected in the HGPS, 42 can be positionally associated with an energetic pulsar (H.E.S.S. Collaboration 2018a); among those, 14 are clearly identified as pulsar wind nebulae (PWNe) produced by young and energetic pulsars, with characteristic ages of $\lesssim 50–100$ kyr, and an additional ten are solid candidates (H.E.S.S. Collaboration 2018b). At slightly higher energies and over a shifted portion of the plane, 15 out of 39 sources listed in the second catalog of HAWC sources (2HWC; Abeysekara et al. 2017) are spatially coincident with a pulsar (Linden et al. 2017), and an additional 14 positional associations are found among the 20 new sources with no previous TeV counterpart listed in the third catalog of HAWC sources (3HWC; Albert et al. 2020). At even higher energies, ten out of the 12 sources detected with LHAASO in the $\sim 0.1–1$ PeV range are positionally coincident with energetic pulsars (Cao et al. 2021).

In all cases, the association criteria are rather loose and cannot prevent chance coincidence or erroneous associations, in part due to the significant extension of most Galactic sources at these energies. On the other hand, the sample of known pulsars used in the correlations mostly consists of objects identified from their beamed emission and, for each $\sim 10–100$ kyr pulsar with a radio or X-ray beam crossing our line of sight, there are three to four misaligned pulsars that would go undetected in these bands and they could give rise to unbeamed gamma-ray emission (Linden et al. 2017).
Although the exact chain of processes by which this happens has not been completely revealed, pulsars seem to be able to convert a significant fraction of their rotational energy into the acceleration of electron-positron pairs with energies reaching 100 TeV and above (Kargaltsev et al. 2015; Amato 2020). Electron and positron pairs accelerated by pulsar systems are eventually released into the interstellar medium after some confinement close to the source. The conditions of this confinement – such as spatial extent, time evolution, and energy-dependent nature – have important observable consequences. They determine how these leptons will contribute to the overall nonthermal emission of the Galaxy, in spectral and angular distribution, and, for pulsars in our vicinity, they drive the amount and spectrum of energetic particles that can be detected at Earth, which has implications on the search for dark matter by-products in the local flux of cosmic rays (Hooper et al. 2017; Profumo et al. 2018; Manconi et al. 2020).

Accelerated pairs are efficiently trapped in a hot and highly magnetized shocked pulsar wind bubble over the first few $1−10$ kyr (Gaensler & Slane 2006). These PWNe are the most likely origin for a (possibly large) fraction of the sources detected in the Galactic plane, for instance with HESS (H.E.S.S. Collaboration 2018b,a). The recent discovery of extended emission around two nearby middle-aged pulsars, with characteristic ages $\geq$100 kyr, suggests the possibility of further confinement beyond the nebula stage (Abeysekara et al. 2017). Since then, a number of other candidates were suggested for these so-called TeV-halos or gamma-ray halos (Di Mauro et al. 2020; Albert et al. 2020; Aharonian et al. 2021), and the phenomenon was demonstrated to have a broadband signature, at least in the gamma-ray range from below 10 GeV up to above 100 TeV (Di Mauro et al. 2019; Aharonian et al. 2021). Theoretically, the question remains essentially open as to how exactly efficient confinement in the vicinity of the pulsar is achieved (Evoli et al. 2018; López-Coto & Giacinti 2018; Giacinti et al. 2020; Fang et al. 2019; Mukhopadhyay & Linden 2022) – in which medium, by which physical mechanism, and over which extent and duration – and how the most solid examples of this emerging source class could be extrapolated to a Galactic population of objects located in a variety of environments.

Even if the physics driving gamma-ray halos still remains to be elucidated for the most part, the source class holds promise for a better and more complete understanding of the VHE and UHE sky. If most middle-aged pulsars in the Galaxy happen to develop spatially extended and long-lived halos (still a bold assumption at that stage), the total contribution could end up being non-negligible. The brightest halos could already be present as unidentified or unassociated sources in the gamma-ray source catalogues; their identification, however, could turn out to be quite tricky owing to the complex morphology that can result from the combined effects of proper motion, injection history, and actual structure of the surrounding medium (Zhang et al. 2021). For those halos below the current detection threshold, they could account as unresolved sources for a fair fraction of the diffuse emission detected in regions of the Galactic plane (Linden & Buckman 2018) or toward the Galactic center (Hooper & Linden 2018, 2022).

In this paper, we aim to provide a quantitative assessment of how pulsar halos fit in the Galactic population of VHE sources, especially in relation to the PWNe population. Specifically, we want to evaluate: (i) whether it is conceivable that most middle-aged pulsars in the Galaxy develop halos similar to the few instances known today; (ii) the fraction of the currently unidentified or unassociated VHE sources that could actually be halos; (iii) how many halos could become detectable in forthcoming surveys of the Galactic plane; (iv) how the cumulated emission of unresolved halos compares to that resulting from cosmic rays (CRs) interacting with the interstellar medium (ISM), and to that of other unresolved source classes.

We start by introducing in Sect. 2 the frameworks used to model individual objects, and then present in Sect. 3 how these were combined to generate a mock population for the whole Milky Way. We discuss in Sect. 4 the properties of one statistical realization of the synthetic population and assess the prospects for observing them, either as individually resolved objects or as an unresolved diffuse emission. Last, we summarize the main findings of our study in Sect. 5.

2. Single SNR-PWN-halo models

We describe in this section the main assumptions and parameters underlying our models for an individual pulsar wind nebula (PWN), pulsar halo, or supernova remnant (SNR).

2.1. Pulsar wind nebulae

The modeling of the PWNe population is based on the individual PWN model introduced in Mayer et al. (2012) and subsequently corrected in H.E.S.S. Collaboration (2018b). Appendix A in the latter reference provides a complete summary of the formalism subsuming the model and here we just refer to the main equations.

The modeling starts from essential pulsar properties (synthetic or inferred from observations), such as spin period, magnetic field, and braking index, that determine the spin-down history of the compact object, hence the time evolution of the maximum power available for nonthermal particle injection into the nebula or halo. Following Eqs. (3) and (4) in Martin et al. (2022), the latter is assumed to occur with constant efficiency over the pulsar’s lifetime, and to have a constant broken power-law spectral shape for each pulsar, with a harder distribution below a break energy of 100 GeV, and a softer one above it (Zhang et al. 2008; Bucciantini et al. 2011; Torres et al. 2014). The minimum particle energy is set at 1 GeV, while the maximum is determined by an exponential cutoff randomly selected in a uniform distribution extending from 200 to 800 TeV. Such a prescription results in cutoff energies that are on average a factor 5–6 below the maximum possible particle energy under the assumption of ideal magnetohydrodynamics for spin-down powers in the $10^{35}−10^{37}$ erg s$^{-1}$ range (de Oña Wilhelmi et al. 2022). In the range $10^{34}−10^{36}$ erg s$^{-1}$ most relevant for pulsar halos, our randomly sampled cutoff energies are on average within $\pm 50\%$ of this physical limit. At low spin-down powers $10^{33}−10^{34}$ erg s$^{-1}$, the cutoff energies generally exceed the limit by a factor 4 on average, but this range contributes little to the VHE landscape. Increasing measurements at ultra high energies with LIHAASO will be instrumental in characterizing the maximum particle energies attained by pulsars in different regimes (Cao et al. 2021).

The PWN model prescribes the expansion of a spherical nebula over three consecutive dynamical stages: (i) rapid supersonic expansion in the unshocked supernova ejecta, powered by nearly constant spin-down, until the latter starts declining; (ii) constant-speed expansion, as the nebula receives little energy from the pulsar, until the reverse shock from the remnant crushes the nebula; (iii) subsonic expansion into the hot shocked ejecta (an alternative sequence occurs in the rarer cases where the reverse shock crushing takes place before the spin-down time scale; see
Table 1. Summary of parameters used in the modeling of PWNe, halos, and SNRs.

| Parameter                                      | Value                      |
|------------------------------------------------|----------------------------|
| **PWNe**                                       |                            |
| Nebular magnetic field initial strength $B_0$ (G) | $5 \times 10^{-5}$         |
| Nebular magnetic field evolution index $\delta_B$ | 0.6                        |
| Injection distribution $S_{PWNe}$               | BPLEC                      |
| Injection distribution index below break $\alpha_1$ | 1.5                       |
| Injection distribution index above break $\alpha_2$ | $\mathcal{U}(2.4, 0.4)$  |
| Injection distribution break energy $E_{brk}$ (GeV) | 100                        |
| Injection distribution cutoff energy $E_{cut}$ (TeV) | $\mathcal{U}(500, 300)$ |
| Injection efficiency $\eta_{PWNe}$ (%)         | $\mathcal{U}(70, 30)$     |
| PWN age limit $\tau_{PWNe}$ (yr)               | $10^3$                     |
| **Halos**                                      |                            |
| Suppressed diffusion region size $R_{SDR}$ (pc)  | 30 or 50 or 80             |
| Suppressed diffusion normalization at 100 TeV $D_{0}^{SDR}$ (cm$^2$ s$^{-1}$) | $4 \times 10^{33}$ or $4 \times 10^{38}$ |
| Average interstellar diffusion normalization at 100 TeV $D_{0}^{ISM}$ (cm$^2$ s$^{-1}$) | $2 \times 10^{30}$       |
| Diffusion rigidity scaling index $D_{R}$       | $1/3$                      |
| Injection distribution $S_{HALO}$              | $S_{HALO} = S_{PWNe}$     |
| Injection efficiency $\eta_{HALO}$              | $\eta_{HALO} = \eta_{PWNe}$ |
| Injection start time $\tau_{INI}$ (yr)         | $\tau_{INI} = \tau_{EXIT}$ |
| Halo age limit $\tau_{HALO}$ (yr)              | $4 \times 10^5$           |
| **SNRs**                                       |                            |
| Ejecta mass $M_{ej}$ ($M_\odot$)               | $1.4 M_\odot/5 M_\odot$ for SNe Ia/ccSNe |
| Ejecta energy $E_{ej}$ (erg)                    | $\mathcal{L}(51, 0.5)$    |
| Injection distribution $S_{SNR}$                | PLEC                      |
| Injection efficiency $\eta_{SNR}$ (%)          | $\mathcal{U}(10, 30)$     |
| Injection index $\alpha_{e,p}$                 | $\mathcal{U}(2.2, 2.4)$   |
| Injection electron-to-proton ratio $K_{ep}$     | $10^{-3}$                  |
| SNR age limit $\tau_{SNR}$ (yr)                | $3 \times 10^{4}$         |

Notes. $\mathcal{U}(\mu, \sigma)$ indicates a uniform distribution with mean $\mu$ and standard deviation $\sigma$. $\mathcal{L}(\mu, \sigma)$ indicates a log-normal distribution with mean of the logarithm $\mu$ and standard deviation of the logarithm $\sigma$. (B)PLEC stands for (broken) power law with exponential cutoff.

Eqs. (A.12) and (A.13) in H.E.S.S. Collaboration 2018b). Quantitatively, the size evolution of the PWN is set by the radius of the nebula at the pulsar spin-down time scale and the reverse shock interaction time scale, both of which are determined from Eqs. (22) and (29) presented in Reynolds & Chevalier (1984).

This model neglects the reverse shock crushing of the nebula and its subsequent reverberation, an evolutionary phase that can bring in some complexity to the dynamical and radiative evolution of the PWN, even in the simple case of spherical symmetry without the effects of pulsar motion and asymmetric remnant evolution (Bucciantini et al. 2003). A PWN population synthesis including reverberation in the dynamical evolution of the nebula, and a thorough discussion on the current knowledge and uncertainties on this evolutionary phase, can be found in Fiori et al. (2022).

The prescribed dynamics of the spherical nebula is used to compute the evolution of its nonthermal particles content. The latter results from the combined time-dependent injection by the pulsar, synchrotron, inverse-Compton and adiabatic energy losses, and diffusive escape from the nebula. For synchrotron radiation, the model includes a flux-conserving time evolution for the uniform nebular magnetic field, starting from an initial value treated as free parameter (see Eq. (A.14) in H.E.S.S. Collaboration 2018b).

In our joint model for a pulsar nebula and halo (to be introduced in more details below), we assume that particle injection in the nebula ends when the pulsar exits it as a result of its kick velocity. This marks the beginning of the pulsar halo phase, while the nebula continues its evolution as a relic structure, not fed anymore by freshly accelerated particles (and not powered anymore by the pulsar’s spin-down, although this is less important as exit from the nebula usually occurs at times $\gtrsim 10 \sim 20$ kyr, when the pulsar has gone relatively weak). The evolution of the PWN is computed until a maximum age $\tau_{PWNe} = 100$ kyr.

The main model parameters are summarized in Table 1. These include a number of free parameters that were set by comparison of the population synthesis predictions with the properties of the sample of objects observed at very high energies, starting from initial guesses informed by studies of individual objects (e.g., Zhang et al. 2008; Bucciantini et al. 2011; Mayer et al. 2012; Torres et al. 2014).

2.2. Pulsar halos

The detection with MILAGRO, HAWC, and now LHAASO of extended gamma-ray emission structures around some middle-aged pulsars (Abdo et al. 2009; Aheysekara et al. 2017; Aharonian et al. 2021), subsequently dubbed TeV halos or gamma-ray halos (Linden et al. 2017; Linden & Buckman 2018), has generated some debate about the exact physical setup at stake in these objects and its relation to the evolution of pulsar-PWN-SNR systems. A review of the phenomenon and its potential as a new and distinct source class in high-energy astrophysics can be found in López-Coto et al. (2022).

An important question is that of the physical state of the medium in which the pair halo develops, which is particularly...
relevant to any attempt to account for the efficient confinement of particles in the vicinity of the pulsar. The various possibilities envisioned so far include: (i) the undisturbed ISM, where standard magnetohydrodynamical turbulence happens to have the required properties for efficient scattering of ~100 TeV particles, for instance a small pc-scale turbulence correlation length (López-Coto & Giacinti 2018; Giacinti et al. 2020); (ii) the relic nebula, the specific magnetic topology and relatively high fields of which would cause the trapping of particles (Tang & Piran 2019); (iii) the parent SNR, the expansion and evolution of which is a source of fluid turbulence downstream of the forward shock, eventually responsible for the suppressed diffusion close to a pulsar that did not escape its parent remnant (Fang et al. 2019); (iv) a stellar wind-blown bubble, the interior of which features a high level of standard magnetohydrodynamical turbulence (Fang et al. 2019).

The above scenarios are all likely to be realized in nature, but they may have different consequences in terms of halo properties and evolution and extrapolation to a galactic population. It is beyond the scope of this work to explore all these alternatives and their combinations, and we instead adopted a physical setup for a halo in the spirit of the picture sketched in Giacinti et al. (2020): (i) the halo phase starts when the pulsar becomes supersonic in its surrounding medium and enters the bow-shock phase, which could happen either within the remnant after leaving the relic nebula or out of it when crossing the forward shock; (ii) relativistic pairs accelerated in the pulsar and its wind nebula can easily escape, almost unaffected by energy losses, and are injected isotropically in the surrounding medium; (iii) particles are free to propagate diffusively in the surrounding medium, either the remnant interior or the ISM essentially undisturbed, except for the possibility of a suppressed diffusion of unspecified origin.

The full model provides some coherence at the population level for the PWNe and halos classes, in particular because they share a common prescription for the injection and follow a physically motivated evolutionary sequence. Yet, it has a limited capability to capture the complexity of some individual objects, especially the transitional ones moving from nebula to halo after dispersion of the former by the reverse-shock crushing.

We note that the delayed injection scheme has two important consequences: (i) it reduces the impact on the results of uncertainties in the pulsar’s spin-down history, especially at early times when most of the rotational energy is lost; (ii) the morphology of the halo can be expected to be more compact and may display less complicated patterns than those considered for instance in Zhang et al. (2021).

In practice, the modeling of individual halos is based on the phenomenological two-zone model presented in Tang & Piran (2019) and Di Mauro et al. (2019) in the context of Geminga. We briefly describe here the main points of the model and refer the reader to these articles for the full formalism. The main equations of the model and a thorough discussion of the parameters adopted can be found in Martin et al. (2022).

Particle injection from a subparsec bow-shock nebula is treated as point-like in space and the effect of proper motion over the lifetime of the halo is not included because we are not interested here in detailed morphological aspects (that can be quite complex; see Zhang et al. 2021). Injection starts when the pulsar develops a bow-shock, typically 40–60 kyr after birth (Bykov et al. 2017; Evoli et al. 2021), and it proceeds with a power being a constant fraction of the declining spin-down luminosity of the pulsar, with typical values on the order of a few tens of percent (Bucciantini et al. 2011; Torres et al. 2014). The injection spectrum is assumed to be a broken power law, similar in shape and acceleration efficiency to that inferred for PWNe. Considering that high injection efficiencies close to 100% inferred for kyr-old objects could still hold for the 100 kyr-old pulsars powering halos may appear as a bold extrapolation. Yet, as demonstrated in Martin et al. (2022), the injection efficiencies required for the two canonical halos around J0633+1746 and B0656+14 are of that order.

Particles diffuse away spherically in a medium characterized by a two-zone concentric structure for diffusion properties, with an outer region representative of the large-scale average ISM in the Galactic plane, and an inner region with a radial extent of a few tens of pc where diffusion is suppressed. For both the inner and outer regions, we assumed a diffusion coefficient with power-law rigidity dependence with index 1/3, applicable for scattering in magnetic turbulence with a Kolmogorov spectrum. The normalization of the diffusion coefficient in the average ISM is set to $2 \times 10^{30} \text{cm}^2 \text{s}^{-1}$ at 100 TeV, in agreement with the values obtained in fits of diffusion reacceleration propagation scenarios to a large set of observables (Trotta et al. 2011; Orlando 2018).

Along their propagation, particles lose energy and radiate via the synchrotron and inverse-Compton scattering processes in interstellar magnetic and radiation fields, the distribution of which across the Galaxy will be described below. Inverse-Compton scattering emission is computed using the Naima python package (Zabalza 2015), in the isotropic seed photon field approximation (see Jóhannesson et al. 2019, for an example of anisotropic calculation in the case of the Geminga halo).

Injection parameters are mostly set by past studies of PWNe, and environmental parameters are mostly defined by large-scale models of the Galactic ISM. Eventually, the parameters most specific to halos are the extent of the suppressed diffusion region and the diffusion coefficient therein. Possible values for these were determined from gamma-ray observations of the halos around the J0633+1746 and B0656+14 pulsars. In Martin et al. (2022), various halo model setups for either object were jointly fit to the angular intensity profile inferred from HAWC (Abeysekara et al. 2017) and the integrated spectrum obtained with the Fermi-LAT (Di Mauro et al. 2019). An additional constraint in this exercise was to make sure that the escaping positron flux from either halo does not exceed the local positron flux measured with AMS-02 (Aguilar et al. 2014, 2019a). From this work, we retain as possible scenarios for diffusive halos: three possible sizes (30, 50 and 80 pc), and two levels of diffusion suppression (by a factor 500, as indicated by Geminga, and by a factor 50, in agreement with B0656+14 within uncertainties).

Table 1 summarizes the full list of parameters characterizing these scenarios in our halo population synthesis. By default, we assume that all middle-aged pulsars in the bow-shock phase develop a halo. We discuss below the implications of an occurrence rate for pulsar halos much smaller than 100%, as was suggested in Martin et al. (2022) from the realization that the local positron flux produced in a scenario where all nearby pulsars develop a halo may exceed the AMS-02 measurement. We compute the evolution of halos until a maximum age $t_{\text{HALO}} = 400 \text{ kyr}$. Depending on the actual processes driving the existence of pulsar halos, a minimum spin-down power could be a more relevant stopping criterion than a maximum age but our model setup does not link halo properties to pulsar properties. Adopting a 400 kyr age limit encompasses all known canonical halos and include in the population those objects providing a significant contribution to the detectability prospects, as illustrated in Sect. 4.1, while keeping computation time within reasonable limits.
2.3. Supernova remnants

Although not the main focus of this work, our population model also includes a component for supernova remnants (SNRs) that we briefly describe here, in particular because the evolution of the parent SNR determines the dynamics of the PWN it contains (for pulsar-producing core-collapse supernovae). The modeling of SNRs is based on the individual SNR model presented in Cristofari et al. (2013, 2017), and the full formalism can be found in that reference. We just recall here the working of the model and introduce the values considered for its main parameters.

The model implements analytical prescriptions for the dynamics of the forward shock in the remnant (Eqs. (1)–(4) in Cristofari et al. 2013). Different treatments are used depending on whether the SNR results from a thermonuclear or core-collapse explosion (hereafter SNe Ia or ccSNe): in the former case, the expansion occurs in a uniform circumstellar medium, while in the latter case it occurs in a layered wind-blown cavity shaped by the progenitor massive star.

Remnants of each type are assumed to have the same ejecta mass, $1.4 M_\odot$ for SNe Ia and $5 M_\odot$ for ccSNe, but their ejecta kinetic energies are sampled from a log-normal distribution with mean $\mu_{\log(E_0)} = 51$ and standard deviation $\sigma_{\log(E_0)} = 0.5$ for $E_0$ the supernova explosion energy in erg. Such a distribution is close to that inferred from the properties of X-ray SNRs in the Large Magellanic Cloud and the Milky Way (Leahy 2017; Leahy et al. 2020). For remnants of ccSNe, the same progenitor stellar wind properties were assumed (see the parameters in Cristofari et al. 2013), which is most likely not realistic, but variation of the dynamics over the population ensues nevertheless from the random-selected ejecta energy and outer interstellar density of each mock SNR (see below).

From the prescribed dynamics, the model defines a parameterized distribution of accelerated particles at the shock at each time step, under the assumptions that it has a power-law shape with exponential cutoff and fixed index and amounts to a given level of pressure at the shock. Accelerated particles are assumed to follow a power-law in energy starting from 1 GeV and extending up to the maximum allowed energy that results from upstream escape in the case of protons, and from energy losses in the case of electrons (Eqs. (7) and (15) in Cristofari et al. 2013). In the case of electrons, a cooling break can appear at an energy at which the loss time scale is smaller than the remnant’s age (Eq. (16) in Cristofari et al. 2013). The power-law index (below the break energy if there is one in the electron spectrum) is a free parameter and, in our population model, it was randomly selected for each SNR from a uniform distribution between 2.2 and 2.4. As acceleration efficiency parameter, we used the definition of Eq. (13) in Ellison et al. (2000), neglecting escape flux and upstream pressure (the latter point may need critical examination in the context of remnants from ccSNe, which expand in wind-blown bubbles).

When advancing in time, particles at the shock are advected downstream and a whole population of accelerated particles progressively builds up in concentric layers, filling the remnant from the center out to the forward shock, and contributing to the non-thermal emission of the remnant: inverse-Compton emission is computed in the radiation fields assumed to bath the system, while pion-decay emission is computed in the assumed radial density distribution of the remnant (Eq. (6) in Cristofari et al. 2013, combined with the continuity equation). The model is valid over the free expansion and Sedov-Taylor stages and breaks down as the forward shock becomes radiative. Adiabatic losses for spherical expansion are included for those particles trapped within the remnant and accompanying its expansion. We assumed a lifetime of $\tau_{\text{SNR}} = 30$ kyr for model SNRs but some do not even reach that limit as they become radiative before.

When calibrating our population model from Galactic observations of VHE objects (see below), acceleration efficiencies extending to relatively high values were required: we found that a uniform distribution of acceleration efficiencies between 0.1 and 0.3 and a fixed electron-to-proton number ratio of $10^{-3}$ provided an acceptable match to the observed flux distribution for Galactic SNRs in the TeV range. Several caveats should be mentioned though: (i) this does not come out of a formal fit over the full parameter space of the SNRs population, so it may well be that a range of lower values provides a good match; (ii) in particular, the previous point depends on the assumed spatial distribution for SNRs, and especially the description of our local environment, which is a shortcoming of the adopted spatial distribution model; (iii) the model does not describe remnants interacting with dense gas clouds, which are a non-negligible fraction of the observed population over the Galaxy, following the 2018 spiral arm model; the 0.1–0.3 efficiencies apply to the full range of particle energies, $\geq 1$ GeV, but the model was not tested against observations in lower-energy bands such as GeV, X-rays, or radio.

3. Galactic population model

In this section, we introduce the assumptions subtending the construction of a mock population of PWNe and pulsar halos from the above model for individual objects.

3.1. Young pulsar population

The modeling of the Galactic population of PWNe and halos starts with the random generation of a pulsar population. We followed an approach similar to that of Sudoh et al. (2019) for pulsar halos, or Johnston et al. (2020) for radio and GeV young pulsars.

The source population model starts with the random generation of core-collapse supernovae (ccSNe) over the last 400 kyr (the lifetime of the longest-lived objects in our population synthesis, pulsar halos): random generation of a number of supernovae (SNE) events from a Poisson distribution with mean $r_{\text{SN}} \times T_{\text{HALO}}$; random generation of an age in a uniform distribution, then random generation of a supernova (SN) type in a binomial distribution and finally, for ccSNe, random selection of those giving birth to pulsars again from a binomial distribution. We assumed an SN rate of $r_{\text{SN}} = 0.02$ SN yr$^{-1}$ (Tammann et al. 1994), constant over the past 400 kyr, with a ratio of core-collapse to thermonuclear SNE of 2 (van den Bergh 1991), and a pulsar-producing core-collapse SNe rate of $r_{\text{PSR}} = 0.01$ PSR yr$^{-1}$ (Johnston et al. 2020). The population of mock pulsars thus obtained is distributed over the Galaxy following the four-spiral-arm pattern used in Faucher-Giguère & Kaspi (2006), using the asympetrical radial distribution from Lorimer & Kramer (2004) and as vertical distribution that of molecular gas at the solar circle from Bronfman et al. (1988).

For each pulsar, an initial pulsar spin period is sampled from a normal distribution with $\mu_p = 50$ ms and standard deviation $\sigma_p = 35$ ms, in agreement with the observed population of young and energetic pulsars (Watters & Romani 2011; Johnston et al. 2020). Initial periods are truncated at the centrifugal breakup limit of 0.85 ms (Sudoh et al. 2019). Following Faucher-Giguère & Kaspi (2006) and Watters & Romani (2011), the initial magnetic field is sampled from a log-normal distribution with $\mu_{\log(B_0)} = 12.65$ and standard deviation

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environment. SNRs are then computed first because the properties of each object is needed for the following reasons: (i) gas density influences the development of parent SNRs, hence the dynamical evolution of PWNe; (ii) magnetic and radiation fields set the level of energy losses for nonthermal electrons and positrons; (iii) radiation fields shape the inverse-Compton spectrum.

As Galactic interstellar radiation field (ISRF), we adopted the axisymmetric model introduced in Popescu et al. (2017). In the absence of a robust physical model for the magnetic field on small scales, for instance in the vicinity of star clusters or in superbubbles, we considered that its strength in the close neighborhood of each PWN-halo is the large-scale interstellar one. We described the latter as a double exponential model (Strong et al. 2000), with a radial (vertical) scale length of 6 kpc (2 kpc) and a peak value of 12 μG. In practice, this yields magnetic field values of 4–7 μG in the molecular ring, where a large fraction of the pulsars is found, and of 3 μG at the solar circle.

Conversely, interstellar gas density is not described with a complete spatial model over the Galaxy, but as a statistical distribution relevant to the vicinity of parent SNRs. The latter is inspired by a systematic study of X-ray SNRs in the Large Magellanic Cloud and Milky Way (Leahy 2017; Leahy et al. 2020) and was assumed to have a log-normal form with mean $\mu_{\log(n)} = 0.0$ and standard deviation $\sigma_{\log(n)} = 0.9$ for $n_H$ in units of $\text{H cm}^{-3}$. The mean differs from the value inferred in Leahy (2017) or Leahy et al. (2020) and was actually adjusted to have our population of gamma-ray emitting SNRs match the observed flux distribution (see Sect. 3.4 for a discussion).

3.3. Population calculation

The calculation starts with the sampling of SNe events in space and time and the determination of interstellar conditions at their positions. SNRs are then computed first because the properties of the core-collapse ones sets the dynamics of PWNe (via the reverse-shock crushing time estimate).

For each mock pulsar of given age $\tau_{\text{PSR}}$, we compute the time $\tau_{\text{EXIT}}$ at which it escapes the spherical PWN that initially developed at the center of the parent SNR. The dynamics of the latter is determined by the random-selected explosion energy and circumstellar density of the system (ejecta mass being the same for all objects, $5 \, M_\odot$), which in turn sets the dynamics of the former via the reverse-shock crushing time. The calculations performed depend on the relative values of the different ages. If $\tau_{\text{PSR}} < \tau_{\text{PWN}}$, the evolution of a PWN is computed; it can be a relic one if $\tau_{\text{PSR}} > \tau_{\text{EXIT}}$, or an active one otherwise. If $\tau_{\text{EXIT}} < \tau_{\text{PSR}} < \tau_{\text{HALO}}$, the evolution of a halo is computed, with injection starting at $\tau_{\text{EXIT}}$: the halo can coexist with a relic PWN if $\tau_{\text{PSR}} < \tau_{\text{PWN}}$. The full time evolution of each mock PWN or halo is computed from the models introduced in Sects. 2.1 and 2.2 until age $\tau_{\text{PSR}}$.

With the assumptions exposed so far, a complete steady-state population features on average about 1000 PWNe and 2600 halos, fewer than expected from the product of pulsar birth rate and halo lifetime because, in a significant number of cases, the pulsar never exits its nebula and so the halo phase never starts. The latter result is the combined effect of a non-negligible fraction of PWNe growing to relatively large physical sizes, with radii in excess of 30–40 pc (in agreement with the limited population known today; see H.E.S.S. Collaboration 2018b), and more than 30% of pulsars having velocities below 200 km s$^{-1}$, owing to the low-velocity component in the pulsar velocity distribution from Verbunt et al. (2017).

3.4. Population calibration

The resulting predicted emission was compared to observations in order to validate or refine the main parameters of the full model, focusing in this exercise on the flux distribution in each source class. The flux distribution of real TeV objects was inferred from existing observations in parts of the Galactic plane (with most of the population statistics being contributed by the HGPS; see H.E.S.S. Collaboration 2018a), and compiled in a modified version of the gamma-cat catalog, developed for prospect studies of the future Galactic Plane Survey with Cherenkov Telescope Array (CTA) (Remy et al. 2022). The results for the final set of parameters will be extensively reviewed in the next sections, and we just discuss here those parameters that needed some tuning for the population synthesis to yield a satisfactory description of observations.

We first emphasize that the model optimization does not result from a formal fit over the full parameter space, which would have been prohibitive in terms of computing time. Instead, we proceeded by educated guess, starting from typical parameters for PWNe and SNRs. This means that the values and statistical distributions eventually adopted as final set of parameters for the population model are probably not a unique solution, nor are they guaranteed to be the best ones, especially if one adds more observational constraints. They fall however in the range of commonly-accepted values and do provide a decent description of emission properties at the population level, which was our main goal here.

Actually, only modest tuning of the initial model parameters was required to obtain a decent match of predicted and observed flux distributions. This is not surprising because our initial values were informed by previous studies of a subset of the same objects with very similar models.

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1. https://gamma-cat.readthedocs.io
Eventually, the most crucial parameters turned out to be: (i) the pulsar birth rate, which linearly sets the normalization of the whole population, but the related parameters are already quite constrained from other observations (exploited in population synthesis efforts such as Lorimer et al. 2006; Faucher-Giguère & Kaspi 2006; Johnston et al. 2020); (ii) the injection efficiencies in all objects, which determines the maxima in the flux distributions; in the framework of our model, rather high injection efficiencies in PWNe, at the level 40–100% and pretty efficient acceleration in SNRs, in the 10–30% range were required; in both cases, however, this is in line with commonly accepted values, although at the high end for SNRs; (iii) the initial nebular magnetic field strength in PWNe, which sets the fraction of the nonthermal particle energy channeled into gamma rays for the younger PWNe (the rest going into adiabatic losses and synchrotron radiation); to be complete, this should ideally be evaluated together with the time evolution prescription for the nebular field, but we did not engage into this; nevertheless, our adopted value of 50 G is right in the broad range of $\sim 10–100 \mu G$ values found in applications of a similar model to various real objects (Zhang et al. 2008; Mayer et al. 2012); (iv) the circumstellar density distribution in SNRs: this sets the level of hadronic gamma-ray emission, which is the dominant emission channel for most SNRs in our model; we ended up with a density distribution with a log-normal mean value of $1 \, \text{H cm}^{-3}$, which is the canonical value for the average ISM. This mean value is about ten times higher than what was inferred from X-ray SNRs (Leahy 2017; Leahy et al. 2020), which is not necessarily surprising since we are comparing two different sets of objects in two different emission bands (X-rays vs. gamma-rays), which is likely to lead to various selection biases (e.g., on age, environment, dynamical state...).

### 4. Synthetic population properties

In this section, we present the properties of our synthetic population of SNRs, PWNe, and pulsar halos, for the final set of parameters adopted (including several alternative scenarios for halos). The focus is, however, on pulsar-powered objects, PWNe and halos, because they are the dominant and most numerous sources in the VHE sky.

#### 4.1. Population properties

Ideally, the synthetic population properties should be described by statistical distributions obtained from a large number of realizations sampling their variance. This would however imply a computation time that is currently out of reach. We present below the results obtained for one realization of the population model that contains a total of 260 SNRs + 945 PWNe + 2613 halos out of 3824 objects in total. All quantitative prospects, for instance the number of detectable sources, should therefore not be taken as accurate predictions but rather as order of magnitude estimates, subject to statistical fluctuations from the actual sampling of the population for one given set of input parameters (and, maybe more importantly, to changes due to variations in the numerous input parameters).

The spatial distribution in the longitude-distance plane are shown in Fig. 1 for PWNe and halos. We overlaid in the same plane the positions of a set of pulsars selected from the ATNF database after the following filtering: distance $<20 \, \text{kpc}$, spin-down power $>10^{31} \, \text{erg s}^{-1}$, and characteristic age below the maximum age allowed for PWNe or halos in our model (ages $<100 \, \text{kyr}$ for PWNe and $<400 \, \text{kyr}$ for halos).

![Fig. 1. Spatial distribution of mock PWNe and halos in the Galactic plane, for halos with suppressed diffusion region of size 50 pc and diffusion suppression by a factor 500. Overlaid for comparison and cross-check are the positions of a set of pulsars from the ATNF database within the same age ranges (characteristic ages $<100 \, \text{kyr}$ for PWNe and $<400 \, \text{kyr}$ for halos).](image)

The comparison of the random positions of mock pulsars and actual positions of true pulsars is overall satisfactory, all the more so that the distance estimates for true pulsars are based on a free electron density model that can yield significant uncertainties (Yao et al. 2017). There is however a deficit of objects within $\lesssim 1 \, \text{kpc}$ from the Sun, which is most likely due to the local arm not being included in the spiral arm model from Faucher-Giguère & Kaspi (2006). This is particularly apparent in the case of halos, with about four times more known middle-aged pulsars than synthesized. Such a caveat may have important consequences for local observables related to halos, such as the positron flux (see Sect. 4.5).

The physical and angular sizes of PWNe and halos are displayed in Figs. 2 and 3 (see Sect. 4.3 for a discussion on the way halo size is computed). The majority of mock PWNe have physical radii of $\sim 5–50 \, \text{pc}$, which translate into angular extents $\sim 0.03–0.3^\circ$. The largest objects in the sky reach up to about 100 pc and the degree scale, similar to the values inferred for HESS J1825–137 (Principe et al. 2020).

With our definition for a halo size in the context of VHE analyses, the physical sizes of most halos is quite clustered in the 20–50 pc range for the halo model setup with 50 pc suppressed diffusion region. Adopting a more extended confinement region of 80 pc instead enlarge the size range only modestly, up to 60 pc. Using a less extended confinement region of 30 pc gives rise to
more subtle effects and a less compact size distribution: halos around weak and young pulsars remain contained in the suppressed diffusion region, with sizes of 20–30 pc; older and more powerful pulsars can develop halos extending beyond the suppressed diffusion region, reaching up to 80 pc. The latter effect is due to significant leakage of particles such that a halo of pairs in excess of the interstellar background can exist over large distances. Eventually, although halos tend to be larger than PWNe on average, even when allowing for variations of the suppressed diffusion region, there is significant overlap in the angular size distribution of halos and PWNe in the ~0.1–0.3° range.

Figure 4 displays the 1–10 TeV luminosities of the mock population as a function of age and distance. In the latter case, this is compared to the luminosities corresponding to various reference flux levels to provide a first sense of the accessible fraction of the population. The top panel shows how halos extend the VHE emission of pulsar-powered systems beyond the classical PWN stage. Interestingly, with the adopted criterion for the start of the halo phase (see Sect. 2.2), there is a significant overlap of the populations of halos and PWNe in the 10–100 kyr range. Such systems are expected to simultaneously harbour both a young relic nebula and a bright halo, most likely offset from each other because of the pulsar motion and reverse-shock crushing of the nebula, and it may be hard to disentangle both components. HESS J1825–137, a very extended source powered by a pulsar with a characteristic age of 21 kyr, may actually be an example of such a system (Principe et al. 2020). The bottom panel of Fig. 4 shows the luminosity distribution over the populations, together with power-law fits to their high-luminosity ends. Interestingly, these fits reveal distributions that are much flatter than those used or inferred in generic source population synthesis efforts like Steppa & Egberts (2020) or Vecchiotti et al. (2022). There is a number of reasons for that difference, such as the fact that these works assume that the luminosity and position distributions are independent, but we defer a complete discussion of that point to a subsequent work.

The flux distribution of the full population located in the footprint of the HGPS is presented in the top panel of Fig. 5, for a modeling of pulsar halos with a suppressed diffusion region of 50 pc and diffusion suppression by a factor 500. The population of mock SNRs reproduces pretty well the distribution of observed objects from the highest fluxes down to about 1% Crab, except maybe for the presence of two sources shining at the Crab level that this realization of our population model failed to produce. The population of mock PWNe also matches pretty well the distribution of observed objects from the highest values, at about the Crab level, down to ~10% Crab. Below this limit, completeness drops and the observed population becomes an increasingly smaller fraction of the mock population. At ~1% Crab, or about the sensitivity limit of current surveys, the number of firmly identified or solid candidate PWNe is only ~20–30% of the total number of objects at this flux in the mock population.

2 Throughout the paper, we used as reference spectrum for the Crab nebula the one from Meyer et al. (2010) as implemented in the gammapy library.
Interestingly, our mock population seems only marginally consistent with PWNe being the majority of currently unidentified sources above 1 TeV, thereby leaving space for another class of objects, possibly halos around a fraction of powerful middle-aged pulsars. The situation is however more contrasted when looking at flux distributions above 100 GeV, in the bottom panel of Fig. 5, where mock PWNe are distributed in flux almost like known PWNe+unidentified sources taken together. This may point to a shortcoming of our model in reproducing the broadband gamma-ray spectra of PWNe.

With a level of diffusion suppression such as that inferred for the halo around Geminga, the number of halos exceeds that of PWNe at high fluxes above 10% Crab. This is mostly a random sampling effect because halos are not expected to be more luminous than the brightest PWNe on average, as illustrated in Fig. 4. The flux distributions of halos and PWNe are similar over the 1–10% Crab flux range, which suggests that both classes of objects could be present in similar proportions in the current census of VHE sources, if halos are found around a majority of middle-aged pulsars, as assumed by default, and if they can be detected despite their large size. At lower fluxes, halos become more numerous, by about a factor of 2 at 1 mCrab; we see below, however, that in forthcoming surveys reaching that sensitivity, PWNe will dominate over halos in number of detectable sources.

The flux distribution of the population of halos depends on the extent of the suppressed diffusion region, as illustrated in Fig. 6. The trend is that larger halos tend to be brighter, all other things being equal. At the sensitivity limit of current surveys, about 10 mCrab, or at the level to be reached in future surveys, about 1 mCrab, the number of halos with large extents $\geq 80$ pc is 50–100% larger than the number of halos with small extents $\leq 30$ pc. This difference should in principle be discernible when the number of detectable sources becomes sufficient, as we discuss more quantitatively in the next section. The flux distribution of halos depends more strongly on the level of diffusion suppression. Modeling halos with shallower diffusion suppression,
consistent with the level inferred for PSR B0656+14 instead of that for PSR J0633+1746, results in 2–3 times fewer objects at any given flux. As discussed below, the prospects for detection are even more degraded because such halos also have a larger angular extent.

Overall, our population model does provide a satisfactory description of the currently observed population at fluxes above 5–10% Crab. In the next section, we demonstrate that the model can also account fairly well for the properties of observed sources at lower fluxes, by assessing the detectable fraction of the population in existing surveys and comparing it to the actual results. The predicted PWNe population does not saturate the flux distribution of known objects with >1 TeV fluxes above 5–10% Crab, which leaves room for another class of emitters as likely counterparts to the currently unidentified sources. Pulsar halos seem to be a likely possibility.

### 4.2. Detectable fraction of the population

We assessed the fraction of the mock population that should have been detected in existing or past surveys or ought to be detected in forthcoming surveys. For that purpose, we used very simple criteria for the detectability, typically that flux is above a certain threshold characteristic of the survey in question. Actual data analysis is otherwise more complex, as thoroughly exposed for instance in the case of HGPS (H.E.S.S. Collaboration 2018a).

We considered the HGPS (H.E.S.S. Collaboration 2018a), the HAWC 1523-day survey from which the 3HWC catalog was derived (Albert et al. 2020), and the future CTA Galactic Plane Survey, hereafter GPS (Cherenkov Telescope Array Consortium 2019), for which we adopted the following criteria for detectability:

1. **HESS**: integrated >1 TeV photon flux above 10 mCrab, for a source located within [−100°, 70°] in longitude and [−2°, 2°] in latitude; sensitivity degradation as a result of source extension was implemented following Eq. (28) in H.E.S.S. Collaboration (2018a), assuming a 0.08° point-spread function, with a limit of 0.7° in radius beyond which any source is considered undetectable.

2. **HAWC**: for a source located within [−20°, 60°] in declination, 7 TeV photon flux density above 3\times10^{-15} ph cm^{-2} s^{-1} TeV^{-1} at best, degrading up to 7 \times 10^{-15} ph cm^{-2} s^{-1} TeV^{-1} following a parabolic dependence on declination with minimum at 19°; sensitivity degradation as a result of source extension was implemented as above assuming a 0.2° point-spread function.

3. **CTA**: integrated >125 GeV photon flux above 1.8, 2.7, 3.8, 3.1, or 2.6 mCrab, for a source located in the longitude range [−60°, 60°], [60°, 150°], [150°, −150°], [−150°, −120°], and [−120°, −60°], defined respectively as Inner, Cygnus+Perseus, Anticenter, and Vela+Carina in Cherenkov Telescope Array Consortium (2019); the latitude range for all segments was taken to be [−3°, 3°], and sensitivity degradation as a result of source extension was computed as above assuming a 0.07° point-spread function, with a somewhat arbitrary limit of 2.0° in radius.

The threshold used for the HGPS is an average over the ~0.5–1.5% Crab values reached over most of the surveyed area, which however correspond to optimistic sensitivities to isolated point sources that are recognized overestimates of the actual sensitivities in H.E.S.S. Collaboration (2018a). Conversely, the ~2–3 mCrab sensitivities presented for the CTA GPS correspond to estimates that were most likely refined since the publication of Cherenkov Telescope Array Consortium (2019), as the final design of the instrument was settled and its performances were investigated more in depth (Remy et al. 2022). The Southern Wide-field Gamma-ray Observatory will no doubt advance the census and knowledge of extended TeV sources, including pulsar halos, but the design of the project is not yet sufficiently settled for quantitative prospects to be derived (Hinton 2022).

The angular extent of a source enters in the above criteria for detection, as it implies a mixing of the source signal with more background (instrumental or astrophysical in origin, although here we consider only the effect of instrumental background). Source extent is easily defined for SNRs and PWNe, as the for- ward shock and outer nebula radius, respectively, but is more complicated for halos. We describe in Sect. 4.3 how halo size is computed and how alternative definitions impact the number of detectable halos.

From one realization of the population model, and based on the above detectability criteria, the number of detectable sources we obtain are reported in Table 2. Out of a total of ~3800 objects, it is predicted that the HESS survey of the Galactic plane should have detected about 80, with little dependence on how pulsar halos are modeled in terms of extent. The majority of sources accessible to the survey would be PWNe (about 50), followed by halos (about 20), and then SNRs (about ten). This is consistent with the actual outcome of the survey (H.E.S.S. Collaboration 2018a) if pulsar-powered objects do constitute the majority of currently unidentified or unassociated sources (47 out of 78).

The predicted number of detectable sources in the HAWC observations used in the 3HWC catalog making is about 30–40, split almost evenly between PWNe and halos and with a low number of SNRs. This undershoots the actual number of detected sources, which is 65. The latter number however comprises 17 objects that are not well separated from neighboring sources and may just be secondary local maxima resulting from statistical fluctuations (Albert et al. 2020). The number of truly distinct sources in the 3HWC catalog could therefore be as low as 48, which becomes more consistent with our model prediction within Poisson fluctuations.

Last, prospects for detection with the upcoming CTA are very promising, with as many as 30 SNRs, 170 PWNe, and 40–100 pulsar halos accessible to the GPS (with an additional ~10–20% uncertainty on the latter figure owing to the choice of a typical extent for detectability assessment; see Sect. 4.3). If the survey sensitivities along the plane are better than those
published in Cherenkov Telescope Array Consortium (2019), by a factor of ~2 leading to ~1 mCrab sensitivity in the innermost regions, the number of detectable sources increases significantly, by about 50% for SNRs and PWNe, and by more than 100% for halos (the stronger increase for halos reflecting the fact that a larger fraction of the population resides at low fluxes).

It is interesting to compare these numbers with the prospects presented in Remy et al. (2022) from a simulation and analysis of CTA survey observations in conditions close to our assumed GPS+ scenario and with a different model for source populations (including more source classes like binaries and interacting SNRs, excluding pulsar halos, and using alternative prescriptions for PWNe). The numbers of detectable simulated sources is on the order of 290 PWNe, 40 young shell SNRs, and 100 already known objects that include mostly PWNe and unidentified sources. We obtain 260 PWNe, 40 SNRs, and 100–200 pulsar halos. This is rather consistent because the source model in Remy et al. (2022) was tuned to have the mock population of PWNe account for the large fraction of currently unidentified sources, whereas we did not enforce such a requirement.

In this assessment, the recent HESS and HAWC surveys are little sensitive to the extent of the suppressed diffusion region in halos. The small number of such objects accessible to these current instruments is too low to discern the effect that larger halos are on average brighter. In contrast, the better prospects for detection offered by CTA, with about 50–100 halos or more, should in principle make it possible to study such trends at a population scale. This illustrates the potential of CTA for a deeper investigation of pulsar halos, although it is unclear which fraction of these detectable halos could actually be identified as such, and which fraction could lead to solid insight into their physics.

Using a more moderate diffusion suppression in halos, with a factor 50 instead of 500 (that is more representative of what is inferred for the halo of PSR B0656+14 than for that of PSR J0633+1746), prospects for detection of this class of objects collapse. They fall down to a handful for the HESS and HAWC surveys, and less than 20 for CTA. The main reason is that the higher diffusion coefficient in the close vicinity of the pulsars results in much more extended emission structures, which thus evade detection. On the other hand, we are probably reaching here the limit of our simple method to assess detectability and a more appropriate treatment would be needed, taking into account the full spectro-morphological properties on the halo and the actual energy-dependent sensitivity of the instrument over a large band.

Figure 7 displays the flux distributions of detectable PWNe and halos in the existing or future surveys, compared to the distributions of all mock and known sources over the full sky. The plot nicely illustrates the step that will be made possible with CTA in probing with a high degree of completeness the populations of pulsar-powered sources over about two orders of magnitude in flux. Such prospects suggest or rather confirm that forthcoming surveys will have to deal with a high level of source confusion, especially when the complexities of realistic morphologies are folded in. The majority of the sources included in our population are found in the innermost 100–120° of the Galactic plane, which means for CTA an average 2–4 detectable sources per degree in longitude, with extensions typically of ~0.1–0.3° but reaching up to the degree scale.

4.3. Impact of size estimates for pulsar halos

While the physical extent of SNRs and PWNe can be unambiguously defined in the framework of our models, as the forward

| Survey          | SNRs | PWNe | Halos (30 pc) | Halos (50 pc) | Halos (80 pc) | Total |
|-----------------|------|------|---------------|---------------|---------------|-------|
| HESS (HGPS)     | 8    | 54   | 17            | 23            | 23            | 79, 85, or 85 |
| HAWC (3HWC)     | 2    | 19   | 10            | 15            | 16            | 31, 36, or 37 |
| CTA (GPS)       | 30   | 171  | 43            | 74            | 103           | 244, 275, or 304 |
| CTA (GPS+)      | 44   | 261  | 103           | 166           | 217           | 408, 471, or 522 |

Notes. The prospects for halos are given for three alternative modeling, with different radii for the suppressed diffusion regions of 30, 50, or 80 pc, and diffusion suppression by factor 500 (see Table 1). Prospects for CTA are also given for a variant of the survey, GPS+, with sensitivity improvement by a factor of 2 with respect to the values published in Cherenkov Telescope Array Consortium (2019).
shock and outer nebula radius, respectively, this is more problematic for halos since they do not have a well-defined external boundary and feature a strongly energy-dependent morphology (the latter being also true for PWNe, but our simple model does not allow us to properly describe this effect). Since halos can be quite extended emission structures, reaching beyond the angular resolution of VHE instruments, the way their size is defined can have strong consequences when assessing their detectability.

The only characteristic scale of a halo, the extent of the suppressed diffusion region, is not quite relevant. Depending on the parameters of the system (such as suppressed diffusion coefficient and injection efficiency, pulsar power and age), the resulting pair halo will constitute an overdensity on top of interstellar background up to a distance that varies with energy, and in some cases may not even yield any significant excess at all. Therefore, we defined the typical extent of a halo at any given energy as the radius where the pair density equals that of the interstellar background. This does not come without ambiguity, however, because the density of cosmic-ray leptons at very high energies \( \gtrsim 100 \text{GeV} \) can be expected to fluctuate significantly, in space and time, as illustrated in Porter et al. (2019). In the absence of information about its value at any position in the Galaxy, we used as typical interstellar lepton background that absence of information about its value at any position in the Galaxy, we used as typical interstellar lepton background that inferred in our local neighborhood, which we approximated in the \( \sim 0.1-10 \text{TeV} \) range as a power law with slope 3.18 and flux normalization \( E^3 \Phi_s(E) = 100 \text{GeV} \text{m}^{-2} \text{s}^{-1} \text{sr}^{-1} \) at 1 TeV (Aguiar et al. 2019b).

A second problem then is the choice of the typical extent to use when assessing detectability by a given instrument with the simple approach introduced in Sect. 4.2. Broadband detectability assessment should in principle exploit the complete spectro-morphological properties of each source, given the actual spectral sensitivity of the survey at its position in the plane. Such a dedicated approach is however intractable in the context of our population synthesis, with a few thousands objects. We need instead one effective extent that would yield a good estimate of the detectability over the few hundreds of GeV to few tens of TeV band. From a study of the detectability of halos with CTA that some of us are involved in (Eckner et al. in prep.), it seems that the optimum energy for the detection of halos is in the few TeV range, on average: at lower energies, the halo is brighter but it is also more extended and the instrumental sensitivity degrades; at higher energies, the halo is more compact but it is also dimmer and sensitivity starts degrading too.

Ultimately, we used two definitions for a halo angular size: the 68% or 95% containment radius of the 3 TeV flux, within the region where the halo pair density is above the local interstellar lepton background at 10 TeV. Variations of the containment fraction or reference energies result in changes by 20% at most, as illustrated in Table 3. The 68% containment fraction is applicable to detectability assessment, because it implies a higher surface brightness over a smaller patch of sky, which is more favorable to detection. The 95% containment fraction is applicable to a general purpose and permits a more consistent comparison with SNRs and PWNe (whose sizes encapsulate 100% of the emission).

### 4.4. Diffuse emission from the unresolved population

Objects that are not detectable individually in existing or future surveys can be expected to give rise to some diffuse emission. In this section, we assess the spectral properties of the total emission from the unresolved population of each class of objects, and compare it to a model for the interstellar emission powered by the large-scale population of Galactic CRs interacting with the ISM. For the latter, we use the so-called “Base Max” model from De la Torre Luque et al. (2022), which is representative of the conventional interstellar transport scenario. Among the different model setups explored in that paper, the “Base Max” variant yields the smallest intensities in the innermost regions of the Galaxy and over the 0.1–10 TeV range. This minimal prediction already seems disfavored at TeV energies by Fermi-LAT and ARGO-YBJ measurements, but diffuse emission from unresolved sources may have a non-negligible in that range, as we discuss below.

We first compare each population of sources to the interstellar emission, independently of the detectability by any instrument. For each object, we integrate the interstellar emission over its angular extent and over the 1–10 TeV band, and compare it to the predicted flux from the object itself, within the same angular region. We then sum up the contributions for all objects with emissions above or below the interstellar radiation. The result is presented in Fig. 8. Only one third of PWNe and one

| \( f_c \) | \( E_{\gamma} \) (TeV) | \( E^\gamma \) (TeV) | \( N_{\text{HALOS}} \) |
|---|---|---|---|
| 0.68 | 1 | 1 | 70 |
| 0.95 | 1 | 1 | 56 |
| 0.68 | 10 | 1 | 69 |
| 0.95 | 10 | 1 | 58 |
| 0.68 | 10 | 3 | 74 |
| 0.95 | 10 | 3 | 63 |
| 0.68 | 10 | 10 | 82 |
| 0.95 | 10 | 10 | 70 |
| 0.68 | 100 | 10 | 83 |
| 0.95 | 100 | 10 | 71 |

**Table 3.** Number of detectable halos in the CTA survey, \( N_{\text{HALOS}} \), depending on how their typical size is defined (lepton energy \( E^\gamma \) at which the extent of pair halo overdensity on top of interstellar background is evaluated, reference gamma-ray energy \( E^\gamma \) at which the flux profile is taken, and containment fraction \( f_c \) of the latter).

**Notes.** The numbers correspond to a population of halos with suppressed diffusion region of size 50 pc and diffusion suppression by a factor 500.

![Fig. 8. Cumulative emission from objects that are individually brighter or fainter than coincident interstellar radiation, in the 1–10 TeV range, compared with the “Base Max” interstellar emission model from De la Torre Luque et al. (2022) integrated over the whole sky (in black, labeled “IEM”).](image-url)
...fifth of halos are brighter than coincident interstellar emission in the TeV range. This emission integrated over the full sky is one to two orders of magnitude above that of fainter objects, for any class of sources. In the case of PWNe and halos, it is comparable to or exceeds interstellar radiation over 2–200 TeV. This can have interesting consequences on observations of external galaxies, such as M31, where the population of pulsar-powered sources may rival in intensity with interstellar emission and bias the interpretation of diffuse emission in terms of cosmic-ray (CR) transport.

We then assess the cumulative emission from all undetectable objects in the surveys considered above, using for detectability the methodology described in Sect. 4.2. The results are presented in Fig. 9 for the total spectra, and in Fig. 10 for the intensity profiles along the plane. In each panel, the interstellar emission spectrum displayed for comparison was integrated over the footprint of each survey (with a restriction to [0°, 180°] in longitude and [−6°, 6°] in latitude for the HAWC survey).

The HAWC survey probes less than half of the population owing to the location of the instrument in the northern hemisphere. Overall, over the footprint of the survey and in the core energy range of HAWC, the emission from unresolved halos is a factor 2–3 below that of interstellar radiation, while that of unresolved PWNe is fainter by about an order of magnitude. The actual distribution of these emission components along the portion of the inner plane that was best surveyed is however rather contrasted, as illustrated in the top panel of Fig. 10.

In the HESS survey, the emission from unresolved PWNe is at least a factor 5–6 fainter than interstellar emission over most of the relevant energy range, while that from unresolved halos is comparable to it, especially above 10 TeV. Overall, at core energies for HESS, the total emission from resolved or unresolved pulsar-powered sources and interstellar radiation are predicted to be of similar magnitude, while the contribution from SNRs is subdominant.

The CTA survey widens the gap between the total emission from resolved and unresolved sources, especially in the case of PWNe where the difference reaches more than an order of magnitude over most of the relevant energy range. Eventually, the CTA survey succeeds in pushing the emission from unresolved PWNe by a factor 20 or more below the level of interstellar radiation, leaving only unresolved halos as a comparable contribution at energies above 10 TeV.

We emphasize that the above statements come with the caveat that detectability was assessed from the simple criterion that the flux be above the survey sensitivity. In reality, source confusion and complicated emission morphologies will most likely tend to lower the detectable fraction and enhance the unresolved contribution. Even more important in the case of halos is the fact that their unresolved emission is estimated based on the limit assumption that all middle-aged pulsars do develop a halo. If only a small fraction of them do so, in the range 5–10% as suggested in Martin et al. (2022), the above results need to be rescaled accordingly. In such a case, the emission from halos as a whole would be subdominant compared to interstellar radiation (see Fig. 8) and their unresolved emission would be a minor component in all surveys (see Fig. 9). Both statements are all the more true that our reference model for interstellar radiation is a minimal prediction; improved models yielding better fits to gamma-ray observations result in emission levels higher by 20–30% in the 1–10 TeV range (De la Torre Luque et al. 2022).

### 4.5. Local positron flux from the halo population

Figure 11 shows the local flux of positrons from all mock halos, for the three suppressed diffusion region sizes 30, 50, or 80 pc and diffusion suppression by a factor 500, compared to the AMS-02 measurement from Aguilar et al. (2019a). As illustrated in the plot, most of the local positron flux is contributed by three dozens of nearby objects within 2 kpc and the corresponding spectrum peaks at an energy of about 2 TeV. Particles of higher energies are limited in range owing to strong energy losses,
while particle of lower energies cannot diffuse efficiently up to Earth within the age of their parent pulsars (set in our population model to a maximum of 400 kyr). An enhanced confinement enforced by a larger suppressed diffusion region results in a smaller positron flux and a more peaked spectrum.

The total predicted positron flux from the whole population is consistent with existing measurements below 1 TeV for all considered sizes of the suppressed diffusion region. Such a result complements the work presented in Martin et al. (2022), where the contribution of putative nearby halos within 1 kpc was studied. Indeed, as illustrated in Fig. 1, the spatial distribution adopted for pulsars yields a deficit of objects within ≤1 kpc, as a result of the local arm not being included in the model.

In Martin et al. (2022), it was demonstrated that known middle-aged pulsars within 1 kpc alone would saturate the measured positron flux if they were to all develop halos with intermediate sizes 30–80 pc, diffusion suppression levels like those around J0633+1746 or B0656+14, and injection efficiencies significantly smaller than those inferred for the canonical halos in J0633+1746 and B0656+14, and more generally with the values typical of younger PWNe. Conversely, if positrons from nearby pulsars besides J0633+1746 or B0656+14 are released in the ISM without any confinement around the pulsars, the total positron flux fits into the observed spectrum for similar injection efficiencies of a few tens of percent for all pulsars, from kyr-old objects powering PWNe to 100 kyr-old objects like J0633+1746 and B0656+14. This led to the suggestion that pulsar halos may be a rare phenomenon, with an occurrence rate as low as 5−10% among middle-aged pulsars, although the evidence supporting that depends on the exact properties of the local pulsar population and on the uncertain physics driving the formation and evolution of halos.

If halos are indeed rather rare, Martin et al. (2022) show that the local positron flux in the 0.1–1 TeV range can be attributed to 2−3 dozens nearby middle-aged pulsars within 1 kpc, releasing pairs into the ISM without confinement at the source. Particles escaping from the halo around J0633+1746 would contribute over part or most of the range, depending on the exact properties of the halo, while B0656+14 would have a maximum contribution at 10 TeV. The population synthesis presented here complements that conclusion by showing that the contribution from all pulsars at 1−2 kpc distances would be negligible in that picture.

5. Conclusions

We presented a modeling of the populations of SNRs, PWNe, and pulsar halos in the Milky Way, and assessed their contribution to the VHE emission of the Galaxy. For pulsar halos, we assumed by default that they develop around all middle-aged pulsars after pulsar exit from the nebula. We considered three
possible extents for the suppressed diffusion region, from 30 to 80 pc, and two diffusion suppression levels, by a factor of 500 and 50, representative of the values inferred for the halos around PSR J0633+1746 and B0656+14, respectively. The realization of the mock population we worked on features about 260 SNRs, 950 PWNe, and 2600 halos.

Focusing on pulsar-powered objects, expected to be the dominant emitters in the VHE range, the mock population seems to account satisfactorily for the properties of currently known objects. The TeV flux distribution is well reproduced from the highest fluxes down to 5–10% Crab. In this range, the predicted PWNe population does not saturate the flux distribution of all known Galactic objects, thus leaving room for another class of emitters as likely counterparts to the currently unidentified sources. Pulsar halos are shown to be a viable solution.

Assessing the detectability in existing surveys with HESS and HAWC or the planned survey of the Galactic plane with CTA yields the following prospects: ∼50 PWNe and HAWC or the planned survey of the Galactic plane with CTA, ∼50 PWNe and HAWC or the planned survey of the Galactic plane with CTA, and ∼50 PWNe and unidentified sources. Pulsar halos are shown to be a viable solution.

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Assessing the detectability in existing surveys with HESS and HAWC or the planned survey of the Galactic plane with CTA yields the following prospects: ∼50 PWNe and HAWC or the planned survey of the Galactic plane with CTA, ∼50 PWNe and HAWC or the planned survey of the Galactic plane with CTA, and ∼50 PWNe and unidentified Galactic sources taken together. The mock PWNe population cannot account for the properties of currently known objects. The TeV flux distribution is well reproduced from the highest fluxes down to 5–10% Crab. In this range, the predicted PWNe population does not saturate the flux distribution of all known Galactic objects, thus leaving room for another class of emitters as likely counterparts to the currently unidentified sources. Pulsar halos are shown to be a viable solution.

The large number of individually unresolved PWNe and halos in existing surveys feeds a significant diffusive emission compared to interstellar radiation powered by the large-scale population of CRs. In the HESS survey of the Galactic plane, the emission from unresolved halos is comparable to interstellar radiation above 10 TeV, while that from PWNe is at least a factor 5–6 fainter above 100 GeV. The planned CTA survey may help us reduce the emission from unresolved PWNe by a factor 20 or more below the level of interstellar radiation, leaving only unresolved halos as a comparable contribution at energies above 10 TeV. This underlines the importance of nailing down the commonness of the phenomenon in the Galaxy. If only a small fraction of middle-aged pulsars do develop a halo, ∼5–10% of them as suggested in another work from the local position flux constraint (Martin et al. 2022), the emission from halos as a whole becomes subdominant compared to interstellar radiation and their unresolved emission would be a minor component in all surveys.

If pulsar halos are rare, the total number of currently known VHE sources, including unidentified ones, cannot be explained within our model. The mock PWNe population cannot account for the number and flux distribution of established or candidate PWNe and unidentified Galactic sources taken together. This points either to the need for a better modeling of PWNe or to the possibility that another class of emitters not modeled in this work makes up the bulk of currently unidentified sources.

Interestingly, an alternative PWNe population synthesis, based on a more complete framework including the effect of reverberation in the dynamical evolution of the nebula, yields the same prediction: fewer synthetic PWNe than the total number of observed sources at intermediate fluxes (Fiori et al. 2022). Continued efforts to model PWNe along their full evolutionary path and assess their contribution to the VHE sky seem warranted. At the very least, the present work shows that middle-aged pulsars bear some potential to account for a significant fraction of currently unidentified TeV sources, but maybe not in the form of halos as they are described today.

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