Disorder induced hysteresis in diluted kinetic Ising model

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Abstract. Dynamic phase transition in diluted kinetic Ising model in two dimensions has been studied in presence of external time dependent oscillating magnetic field applying Glauber Monte Carlo spin flip dynamics. A dynamic phase transition from a dynamically ordered to dynamically disordered ferromagnet is identified studying dynamic order parameter against temperature for different concentrations of disorder. For a given temperature, field strength and frequency for which there was no hysteresis, it is observed that disorder is able induce hysteresis in the system. Effect of increasing concentration of disorder on hysteresis loop area has also been studied for different amplitude and time period of the external field.

1. Introduction

Disordered soft magnetic materials have a wide range of industrial applications because of the property of fast magnetization reversal with minimal magnetic loss [1, 2]. Important desirable technical properties for soft magnetic materials include high permeability, low hysteresis loss, large saturation and remnant magnetization and high Curie temperature. Manipulation of chemical composition and optimization of microstructure are required to achieve all such properties in a single soft magnetic material. Though large amount of experimental effort has been devoted in the recent past to develop such materials [3], theoretical understanding of the magnetic properties of these system remain incomplete. Disordered magnetic materials under time independent magnetic field provide a rich static critical phenomena [4]. Dynamic phase transition and hysteresis effect in spin-1/2 Ising model without disorder have been studied using mean-field theory as well as employing Monte Carlo (MC) simulation technique and reported in [5–7]. However dynamic phase transition in disordered magnetic materials are rarely reported. Disordered magnetic material can be modeled by replacing magnetic elements randomly by non-magnetic elements and their magnetic properties under static as well as dynamic conditions can be obtained as function of disorder concentration.

In this paper, dynamic phase transition and hysteresis effect in two dimensional site diluted (or disordered) spin-1/2 Ising system using Glauber Monte Carlo stochastic spin flip dynamics are reported for different frequencies, amplitudes of the external magnetic field and concentrations of disorder.
2. Model
A diluted kinetic Ising model (DKIM) is developed on a two dimensional square lattice of size \( L \). A site is called disordered site if it is not occupied by a magnetic ion. Disordered sites are assigned randomly throughout the system with disorder concentration \( q \) by calling a random number \( r \) for each lattice site. If \( r \leq q \) the site is marked disordered. Otherwise the site is regular. All the regular lattice sites are assigned with a Ising spin \( S_i = \pm 1 \). Each spin can interact with its nearest neighbour spin, if available, under periodic boundary condition with ferromagnetic interaction strength \( J \). The system is studied under an external time varying oscillating magnetic field \( H(t) \) given by

\[
H(t) = h_0 \sin(2\pi t/\tau)
\]

where \( h_0 \) and \( \tau \) are the amplitude and time period of oscillating magnetic field. The Hamiltonian of the DKIM under external field \( H(t) \) is given by,

\[
\mathcal{H} = -J \sum_{\langle ij \rangle} c_i c_j S_i S_j - \mu H(t) \sum_i c_i S_i
\]

where the occupation index \( c_i = 0 \) corresponding to a disordered site and \( c_i = 1 \) represents a site that is occupied with a spin.

In order to equilibrate the system with the heat bath at temperature \( T \), the system is evolved without magnetic field following Glauber Monte Carlo (MC) single spin flip dynamics. The spin flip acceptance ratio \( W \) according to the Glauber MC dynamics is given by

\[
W(S_i \rightarrow -S_i) = \begin{cases} 
\frac{e^{-\Delta E/k_B T}}{1 + e^{-\Delta E/k_B T}} & \text{if } \Delta E > 0 \\
1 & \text{Otherwise}
\end{cases}
\]

where \( \Delta E \) is the change in energy due to spin flip. One MC step includes checking and updating of all the spins in the lattice. The equilibrium states for different temperatures \( T \) and disorder concentrations \( q \) with zero external field are obtained after \( 10^5 \) MC time steps of iteration starting from all spin up states. As the system achieves a constant magnetization in equilibrium at a given temperature \( T \) for a given \( q \), the external time dependent magnetic field \( H(t) \) is switched on. A spin is chosen randomly and updated using Glauber acceptance ratio \( W \) in which \( \Delta E \) is calculated using the full Hamiltonian. A MC step is taken as one unit of time. One cycle of the magnetic field of time period \( \tau \) then consists of \( \tau \) MC steps. To equilibrate the system with external magnetic field \( H(t) \), the system is evolved further over 256 cycles. Data collection starts only after the dynamical equilibrium in the system is achieved.

3. Results and Discussions
Simulations are performed on a square lattice of size \( L = 256 \). All the dynamical quantities are averaged over 1024 cycle for 8 different initial disorder configurations for a given \( q \) and other macroscopic conditions. Any average mentioned below then corresponds to cycle average as well as configuration average. In the simulation \( J = 1 \) has been taken as the simplest choice. The temperature is measured in the unit of \( J/k_B \) where \( k_B \) is the Boltzmann constant. Magnetic field intensity is measured in the unit of \( J/\mu \) where \( \mu \) is the magnetic moment of an individual spin. For a given disorder concentration \( q \), the magnetic state of the system is defined by the values of field amplitude \( h_0 \), field time period \( \tau \) and temperature \( T \).

The average instantaneous magnetization \( \langle m(t) \rangle \) of the system under a time varying magnetic field \( H(t) \) can be estimated as

\[
\langle m(t) \rangle = \frac{1}{N_s} \sum_{k=1}^{N_s} \left\{ \frac{1}{N_c} \sum_{j=1}^{N_c} m_{jk}(t) \right\}
\]

where \( m_{jk}(t) = \frac{1}{L^2} \sum_i c_i S_i \),

(4)
the instantaneous magnetization during $j$th cycle on $k$th configuration and $N_s = 8$ is the number of configurations, $N_c = 1024$ are cycles per configuration. Variation of $\langle m(t) \rangle$ over a complete cycle ($\tau = 128$) for different values of $q$ are shown in Fig.1 at two different temperatures (a) $T = 0.6$ and (b) $T = 2.0$. It is found that at $T = 0.6$ and low disorder concentration $q = 0$ or 0.1, $\langle m(t) \rangle$ has almost a constant value and remain unaffected over the cycle of $H(t)$. From $q \geq 0.2$, $\langle m(t) \rangle$ follows $H(t)$ with certain phase lag. Due to slow relaxation, $\langle m(t) \rangle$ lags behind $H(t)$. As $q$ increases $\langle m(t) \rangle$ gets more and more influenced by $H(t)$. At a fixed $T$, $h_0$ and $\tau$ for any nonzero value of $q$, some of the spins in the system lose one or more of their neighboring spins. As a result a spin with one or more disordered site in its neighbour becomes weakly coupled. In such a situation thermal energy can randomize the ferromagnetic order easily where it was not possible at zero or little disorder. Hence for higher disorder, $\langle m(t) \rangle$ shows better response to the field. On the other hand, at a higher temperature $T = 2.0$ for any $q$ the thermal energy wins over ferromagnetic coupling and randomize the system. The magnetization $\langle m(t) \rangle$ follows the external field $H(t)$ and oscillate symmetrically around zero with certain phase lags depending on disorder concentration $q$. With increase in $q$, the maximum of $\langle m(t) \rangle$ decreases as total number of spins in the system decreased.

Depending on temperature $T$, the system is found to belong to two distinct dynamical phases, one with a positive finite $\langle m(t) \rangle$ and other with $\langle m(t) \rangle$ oscillating about zero. To study the dynamical phase transition in the system, a dynamical order parameter $Q$ is defined and studied as a function of temperature $T$ for various values of $q$. The order parameter $Q$ over a full cycle of the magnetic field is defined as,

$$\langle Q \rangle = \frac{1}{\tau} \int_{0}^{\tau} m(t) dt$$

where $\langle Q \rangle$ is function of $q$, $T$, $h_0$ and $\tau$. In Fig.2(a), $\langle Q \rangle$ is plotted against $T$ for different values of $q$ taking $h_0 = 0.5$ and $\tau = 128$. For a given $q$, there exists a transition temperature $T_c$ at which the order parameter vanishes [8]. Though the qualitative behavior of $\langle Q \rangle$ versus $T$ remains same for all $qs$, the values of the saturation order parameter and the critical temperature $T_c$ are found to decrease as $q$ is increased. The decrease in saturation value of $\langle Q \rangle$ is expected as the magnetization per lattice site depends on total number of magnetic spins present in the system. However, the decrease in $T_c$ is due to the fact that some of the spins nearby the disordered sites becomes weakly coupled with the rest of the spins. A dynamical phase transition is then found to occur in diluted spin systems too. Below $T_c$ it corresponds to dynamically ordered phase (DOP) and above $T_c$ it corresponds to dynamically disordered phase (DDP). The nature of the

![Figure 1](image-url)
transition is asserted calculating Binder Cumulant $U$ defined as,

$$U = 1 - \frac{\langle Q^4 \rangle}{3\langle Q^2 \rangle}$$

(6)

where $\langle Q^2 \rangle = \langle \int_0^\tau m^2(t)dt/\tau \rangle$ and $\langle Q^4 \rangle = \langle \int_0^\tau m^4(t)dt/\tau \rangle$ are the second and the fourth moment of magnetization. In Fig.2(b) Binder Cumulant $U$ is plotted with temperature $T$. Around the transition temperature $T_c$, there is a sudden dip in $U$ for all values of $q$s which is a signature of a first order phase transition in contrast to the fact that $U$ decreases continuously with temperature $T$ and goes to zero in second order phase transition [9, 10].

Since appearance of hysteresis loop is often associated with a first order phase transition, a through study of the variation of hysteresis loop is performed over a wide range of parameter values. Since in DDP, $\langle m(t) \rangle$ varies symmetrically about zero value with $H(t)$, the hysteresis loop is also expected to be symmetric about $\langle m \rangle = 0$ line [11]. On the other hand, in DOP, $m(t)$ does not vary symmetrically about zero value with $H(t)$. Hence, the loop is expected to be asymmetric about $\langle m \rangle = 0$ line [11]. In Fig.3, $\langle m(t) \rangle$ is plotted against $H(t)$ over a complete cycle of $H(t)$ for different parameter values. Keeping $h_0 = 0.5$, $\tau = 128$ and $q = 0.1$ fixed, the variation in the hysteresis loop with $T$ is shown in Fig.3(a). As expected, the nature of the hysteresis loop changes from asymmetric to symmetric about $\langle m \rangle = 0$ line as $T$ passes through $T_c$. Similar variations in the hysteresis loop with $h_0$ in Fig.3(b), with $q$ in Fig.3(c) and with $\tau$ in Fig.3(d) are shown. Apart from $T_c$ for fixed values of $h_0$, $\tau$ and $q$, critical values of $h_0$, $\tau$ and $q$ (for fixed values of other parameters) can also be identified monitoring the change in the shape of the hysteresis loop. As $T$ the thermal energy is responsible to destroy the ferromagnetic order. However, as $h_0$ is increased for a fixed $T$ is increased, the system acquires energy from the external field to overcome the ferromagnetic order. At fixed $T$ and $h_0$, as the value of $\tau$ increases the hysteresis loop area is also increases resulting more and more loss of magnetic energy which eventually allow the thermal energy to randomize the spin orientation. On the other hand, as $q$ is increased, the system becomes more and more ferromagnetically weakened. Hence a transition to DDP is possible increasing $q$ at a fixed $T$ and $h_0$ or $\tau$.

Beside the change in the shape of the hysteresis loop, the loop area $A$ is also found to vary with system parameters. The hysteresis loop area $A$ is defined as

$$A = -\int m(t)dH = \frac{-2\pi h_0}{\tau} \int_0^\tau m(t)\cos(2\pi t/\tau)dt,$$

(7)
which is a measure of the loss in magnetic energy over a cycle. In Fig. 4(a), $A$ is plotted against $T$ for different values of $q$. For a given $q$, $A$ increases from zero to a maximum value and then decreases to a small finite value as $T$ is increased starting from a low value. At low temperature $m(t)$ does not follow $H(t)$ and remains constant at its saturation value leading to no hysteresis loop. As $T$ increases, $m(t)$ starts following $H(t)$ and loop area starts increasing. At the dynamical phase transition temperature $T_c$, a maximum change in $A$ [11] occurs as shown in Fig. 4(b). The loop area increases further with the increase of temperature as the system becomes more susceptible with external field. However, further increase of temperature beyond the temperature corresponding to maximum loop area the system becomes more and more paramagnetic and reduces the loop area. For the present field strength, a complete paramagnetic phase is not achieved in the temperature range used. In presence of disorder, the ferromagnetic order can be overcame at lower temperature and the peak of $A$ shifts towards lower values of $T$. At the same time, the maximum value of $A$ decreases as $q$ is increased due to scarcity of spins in the system.

4. Conclusion
Dynamical phase transition in a diluted kinetic Ising model is studied for various different values of dilution of spins. Dynamical order parameter $Q$ vanishes at the transition point from DOP to DDP. The critical temperature $T_c$ of the transition found to depend not only on $h_0$ and $\tau$ but also is found to depend strongly on $q$, amount of spins dilution in the system. Spin dilution makes the system magnetically softer and a dynamical transition occurs at a lower and lower $T$ as dilution increases. Hysteresis loop is found to change its shape from asymmetric to symmetric about zero.
magnetization line around $T_c$ along with vanishing dynamical order parameter. Beside decrease in $T_c$, dilution is found to have strong effect on magnetic hysteresis. For a given $T$, $h_0$ and $\tau$, not only hysteresis loop area is found to increase with $q$ but also the remnant magnetization is found to increase with $q$.

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