$\alpha_s(M_Z^2)$ and $R_b$ discrepancy with nonuniversal interactions

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Abstract

We implement global fits to LEP data with the nonuniversal interactions. Consistent $R_b$ with experimental value and consistent $\alpha_s(M_Z^2)$ with that from low energy experiments are obtained. We also find that the $\chi^2$ is better than the Standard Model. And we argue that other kinds of new physics are needed to explain the difference between the values of $\alpha_s(M_Z^2)$ from low energy experiments and from the 3-jet ratio.
Recently the Collider Detector of Fermilab (CDF) Collaboration presented evidence for a top quark with a mass \( m_t \sim 175 \text{ GeV} \) \[1\]. Such a heavy top affects the partial width of \( Z \rightarrow b\bar{b} \) and recent analysis indicates that the experimentally measured value for the ratio \( R_b \equiv \frac{\Gamma(Z \rightarrow b\bar{b})}{\Gamma(Z \rightarrow \text{hadron})} \) is higher than the Standard Model (SM) prediction at a 2.5\( \sigma \) level \[2,3\]. This discrepancy may be the first signal for new physics beyond the SM if it will be confirmed by future experiments. A number of possible scenarios of new physics are being suggested to explain this \( R_b \) discrepancy.

The nonuniversal interaction acting on only the third generation attracts us as a candidate for the new physics since the SM predictions for other flavours should not be disrupted by the new physics. Models of this type are motivated by the idea that the top quark has a mass of the order of the weak scale and would play a key role in electroweak symmetry breaking via top quark condensation \[4\]. Considering the general approach, the anomalous nonuniversal interaction terms are \( \text{SU}(2)_L \times \text{U}(1)_Y \) invariant and the \( b \)–quark will take part in top quark interactions when the left–handed doublet is involved. This can result in a modification of the \( Z \rightarrow b\bar{b} \) vertex. We parametrize the nonuniversal interaction effects in the \( Z \rightarrow b\bar{b} \) vertex by introducing the parameters \( \kappa_{L,R} \). These parameters shift the SM tree level couplings of the neutral currents \( g_{L,R} \) to the effective couplings \( g_{L,R}^{\text{eff}} \)

\[
g_{L,R}^{\text{eff}} = g_{L,R}(1 + \kappa_{L,R})
\]  

where

\[
g_L = \frac{1}{2} + \frac{1}{3} \sin^2 \theta_W, \quad g_R = \frac{1}{3} \sin^2 \theta_W .
\]

It was shown that \( R_b \) could be fitted to the LEP data within 1\( \sigma \) with nonuniversal interactions in the ref. \[5\].

Another possible deviation of the LEP/SLC data from the SM is being proposed. Shifman \[6\] has pointed out that the value of the strong coupling constant \( \alpha_s(M_Z^2) \simeq 0.126 \) determined by global fits to the \( Z \)–line shape variables at the \( Z \)–peak shows much discrepancy with \( \alpha_s(M_Z^2) \simeq 0.112 \) extracted from low energy experiments, which is scaled to \( M_Z \).
scale. And we note that the value of $\alpha_s(M_Z^2) \simeq 0.119$ from events shape variables also shows difference from that from low energy experiments. Kane et al. noticed this point in relation to the $R_b$ discrepancy. They reanalyzed the LEP/SLC data in minimal supersymmetric standard model (MSSM) scheme with light superpartners and found that the global fit with low $\alpha_s(M_Z^2) = 0.112$ yields better fit to the data than that of the SM. Several authors have noted that if $R_b$ is explained by new physics, then in general $\alpha_s(M_Z^2)$ will decrease.

In this paper we study the model with nonuniversal interactions to explain the $\alpha_s$ problem and $R_b$ discrepancy. We do not construct a specific model but use the effective lagrangian technique. We take the $Z \to b\bar{b}$ vertex to be given phenomenologically by the expression

$$\mathcal{L} \sim Z^n (\bar{b} \gamma_{\mu} (g_V^{\text{eff}} + g_A^{\text{eff}}) b)$$

(2)

where $g_V^{\text{eff}}$ and $g_A^{\text{eff}}$ are the effective vector and axial coupling constants given by

$$g_V^{\text{eff}} = 2(g_R^{\text{eff}} + g_L^{\text{eff}})$$

$$g_A^{\text{eff}} = 2(g_R^{\text{eff}} - g_L^{\text{eff}}).$$

(3)

We used ZFITTER with the function minimizing program MINUIT to perform the $\chi^2$ fit for the LEP observables. By $\chi^2$ fitting to the LEP observables with nonuniversal interactions, we find that the value of $\alpha_s(M_Z^2) = 0.103$ lies at the global $\chi^2$ minimum.

Alternatively we consider the extraction of $\alpha_s$ from 3–jet ratio. We observe that this jet variable is very insensitive to the modification of the $Z \to b\bar{b}$ vertex given in eq. (1). So we find that this jet variable can be used to extract $\alpha_s(M_Z^2)$ independently of such kinds of new physics that effectively change $g_L$ and $g_R$.

For completeness, we implement the $\chi^2$ fit to the data in the SM framework at first. The SM value of $\alpha_s(M_Z^2)$ from the $Z$ line shape variables has been reported to be $\alpha_s(M_Z^2)=0.126\pm0.005$ by LEP Electroweak working group. We use the set of following 12 variables in our fitting procedure: $M_W$, $\Gamma_Z$, $\sigma_{\text{tot}}$, $R_l \equiv \Gamma_{\text{had}}/\Gamma_{\text{lepton}}$, $A^{\text{lep}}_{FB}$, $A_r$, $A_e$, $R_b$, $R_c$, $A^b_{FB}$, $A^c_{FB}$, $\sin^2 \theta^\text{lep}_W$. The Higgs mass is fixed to be 100 GeV. Our $\chi^2$ fit is not sensitive to the values of Higgs mass in the region $m_H=100–1000$ GeV. As fitting parameters, we use $t$–quark mass $m_t$ and $\alpha_s$. We obtain followings:
\[ m_t = 162.67 \pm 8.98 \text{GeV}, \quad \alpha_s(M_Z^2) = 0.121 \pm 0.004. \]

These results are consistent with the fits obtained by the LEP Electroweak working group.

The deviation of \( \Gamma_b \) from the SM by the effects of \( \kappa_{L,R} \) is expressed by

\[
\frac{\delta \Gamma_b}{\Gamma_b} \sim 2 \frac{g_L^2 \kappa_L + g_R^2 \kappa_R}{g_L^2 + g_R^2}. \tag{4}
\]

Since \( g_L^2 \gg g_R^2 \), \( \kappa_R \) does not affect much on \( \Gamma_b \) and we can neglect the second term. Therefore we fix \( \kappa_R = 0 \) in our analysis.

With a nonzero parameter \( \kappa_L \), we implement the \( \chi^2 \) fit to the same set of LEP observables. We found the much better \( \chi^2 \) than the SM, well–agreed \( R_b \) within 1\( \sigma \) range of experimentally measured value and the lower \( \alpha_s(M_Z^2) \) than that of the SM. We obtain the values:

\[
m_t = 165.33 \pm 8.70 \text{ GeV}, \\
\alpha_s(M_Z^2) = 0.103 \pm 0.009, \\
\kappa_L = 0.013 \pm 0.006.
\]

The results of our \( \chi^2 \) fit to LEP observables are summarized in Table 1 compared with those of the SM. In Fig. 1, we plot \( R_b \) as a function of \( \kappa_L \) for these values of \( m_t \) and \( \alpha_s(M_Z^2) \).

Because we take a model–independent approach, we do not explicitly describe the parameter \( \kappa_L \) by specific physical quantities here. We know, however, that \( \kappa_L \) is related to the new physics scale \( \Lambda \). For example, if we take the relevant term of the effective lagrangian as the 4–fermion coupling

\[
L_{\text{eff}} \sim -\frac{1}{\Lambda^2} \bar{b} \gamma_\mu b \bar{t} \gamma^\mu (g_V + g_A \gamma_5) t, \tag{5}
\]

\( \kappa_L \) is computed by \( t \)–quark correction to the \( Z \rightarrow b \bar{b} \) vertex as follows

\[
\kappa_L = \frac{g_A N_c m_t^2}{g_L 8 \pi^2 \Lambda^2} \ln \left( \frac{\Lambda^2}{m_t^2} \right). \tag{6}
\]

Our fit result \( \kappa_L \sim 0.013 \) yields \( \Lambda \sim 1.5 \text{ TeV} \) from eq. (6).

The value of \( \alpha_s(M_Z^2) \) at the \( Z \)-peak can also be extracted from jet event shape variables. There are several jet variables; thrust, jet mass, energy–energy corelation, oblateness,
C–parameter, jet multiplicity and 3–jet ratio etc.. Here we explore the effects of the nonuniversal interactions on 3–jet ratio and determination of $\alpha_s(M^2_Z)$.

Jets are defined as a bunch of particles based on jet–clustering algorithms. For example, with a jet–clustering algorithm in the EM scheme [10], two particles are regarded as belonging to the same jet if their momenta satisfy the condition

$$y_c > y_{ij} = \frac{2p_i \cdot p_j}{s}$$

where $\sqrt{s}$ is the total energy of collision and $y_c$, so–called $y$–cut, is a given resolution parameter.

We used the 3–jet decay width at the Z peak formula derived by Bilenky et al., which is calculated up to the order of $\alpha_s$ and $r_b \equiv m_b^2/m_Z^2$. Their analytic expressions are found in ref. [11]. We calculate the ratio of $\Gamma_{3jet}^b$ to $\Gamma_b$ with the nonuniversal interactions given in eq. (1) for the values of the parameter $\kappa_L = 0, 0.02, 0.08$.

$$R_{3j} = 0.2450 \quad \text{for} \quad \kappa_L = 0 ,$$

$$R_{3j} = 0.2449 \quad \text{for} \quad \kappa_L = 0.02 ,$$

$$R_{3j} = 0.2448 \quad \text{for} \quad \kappa_L = 0.08 .$$

We used $\alpha_s = 0.119$ which is reported by LEP Electroweak working group for event shape variables [3]. Each value of $\kappa_L$ corresponds to the Standard Model, $\Lambda \sim 1$ TeV and $\Lambda \sim 300$ GeV if we assume the effective lagrangian such as eq. (5). The change of 3–jet ratio with varying $\kappa_L$ is very slight and it cannot change the value of $\alpha_s(M^2_Z)$. We conclude that this variable is very insensitive to the change of the parameter $\kappa_L$ and the value of $\alpha_s(M^2_Z)$ extracted from this variable is not lowered by introduction of the new physics effects such as eq. (1), contrary to the case of the line shape variables.

When one introduce the new physics beyond the SM to cure the $R_b$ discrepancy, the value of $\alpha_s(M^2_Z)$ is usually known to be lower than that extracted from the SM. This fact can be the answer of the problem that the value of $\alpha_s(M^2_Z)$ emerging from the global fits on the data at the Z–peak is almost $3\sigma$ deviations higher than the value stemming from
the low energy phenomenology. With a generic nonuniversal correction given in eq. (1), we implemented the global fits to the observables of LEP and found that $\alpha_s (M_Z^2) \approx 0.103$ gives the best fit. All the data including $R_b$ are consistent with our model predictions.

We also found that the 3-jet ratio is very insensitive to this nonuniversal correction. If we predict this jet variable more exactly, therefore, we can extract $\alpha_s (M_Z^2)$ from the jet ratio independently of new physics as eq. (1). If an exact determination of $\alpha_s (M_Z^2)$ from the jet ratio still shows discrepancy with $\alpha_s$ from low energy value, it may mean the existence of other kinds of new physics different from that described by eq. (1).

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TABLES

TABLE I. Our global fit results to LEP observables in the Standard Model framework and with the nonuniversal interactions.
FIGURES

FIG. 1. Plot of $R_b$ as a function of the parameter $\kappa_L$. The solid line denotes the prediction with nonzero $\kappa_L$ and the dashed line the Standard Model prediction. The hatched area represents the LEP data within $1\sigma$ error.
Fig. 1

$R_b$ vs $\kappa_L$

- Prediction with Nonzero $\kappa_L$
- SM Prediction

$R_b^{(LEP)}$
| Observables          | Experiment      | SM results  | $\chi^2$ | New Physics | $\chi^2$ |
|----------------------|-----------------|-------------|----------|-------------|----------|
| $M_W$(GeV)           | 80.33 ± 0.18    | 80.3225     | 0.002    | 80.3393     | 0.003    |
| $\Gamma_Z$(GeV)      | 2.4971 ± 0.0033 | 2.4971      | 0.000    | 2.4976      | 0.023    |
| $\sigma_{tot}$(nb)   | 41.492 ± 0.081  | 41.396      | 1.397    | 41.399      | 1.321    |
| $R_l$                | 20.815 ± 0.033  | 20.801      | 0.184    | 20.799      | 0.247    |
| $A_{lep}^{FB}$       | 0.0172 ± 0.0013 | 0.0155      | 1.628    | 0.0157      | 1.337    |
| $A_r$                | 0.140 ± 0.008   | 0.144       | 0.244    | 0.145       | 0.341    |
| $A_e$                | 0.137 ± 0.009   | 0.144       | 0.596    | 0.145       | 0.726    |
| $R_b$                | 0.2204 ± 0.0020 | 0.2161      | 4.536    | 0.2205      | 0.002    |
| $R_c$                | 0.1606 ± 0.0095 | 0.1710      | 1.189    | 0.1700      | 0.983    |
| $A_{FB}^b$           | 0.1015 ± 0.0036 | 0.1010      | 0.019    | 0.1017      | 0.003    |
| $A_{FB}^c$           | 0.0760 ± 0.0089 | 0.0720      | 0.206    | 0.7214      | 0.168    |
| $\sin^2 \theta_W^{lep}$ | 0.2320 ± 0.0016 | 0.2320      | 0.000    | 0.2319      | 0.003    |
| total                |                 | 10.0        |          | 5.2         |          |

Table 1: