Monopole spectra in non–Abelian gauge theories.

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Abstract

We study the continuum limit of the length spectrum of magnetic monopole structures found after various Abelian projections of pure gauge SU(2), including the maximally Abelian gauge. We comment on Gribov copies, and measurements of the string tension.
1 Introduction.

The idea of ’t Hooft [1] that the long range degrees of freedom in non–Abelian gauge theories might be monopoles in suitable Abelian projections (AP’s) of the fields may be realised in the maximally Abelian gauge (MAG) [2], where the non–Abelian string tension appears to be reproduced by the monopole currents (‘monopole dominance’, e.g. [3]).

Such behaviour seems limited to a small class of AP’s; why this includes the MAG is unresolved. The MAG is plagued by Gribov copies which differ in long–range quantities such as the string tension [4, 5] and whose proper treatment is not known.

We seek to address the above issues by examining the continuum limit of the conserved magnetic currents identified [6] after various AP’s; although AP is strictly only concerned with reproducing the long distance physics, monopole dominance can only be taken seriously if the monopoles behave as physical objects on most length scales. We use this as a criterion for favouring the MAG (Sect. 2) over other gauges (Sect. 3), where we also comment on the Gribov ambiguity.

Lattice units are used, unless physical ones are specified which are in terms of the (non–Abelian) string tension [7]. The latter is an unambiguous, non–perturbative definition.

2 The Monopole Loop Spectrum.

In four dimensions, magnetic 4–current is conserved, and occurs in closed ‘clusters’. Typically only a few % of the dual links carrying magnetic current carry current greater than unity, so small clusters routinely take the form of a single, non–intersecting ‘loop’ of current.

Larger clusters can be resolved into a set of intersecting loops, but there is no unique solution to this. In this work, one such solution was selected randomly. The effect of this will be discussed in Sect. 2.3.

Fig. 1 is a typical example of a ‘loop spectrum’, a plot of the ensemble–average number of monopole loops, $N(l)$, of length, $l$ in the MAG.

The spectrum can be split into two parts; small loops follow a clear power law dependence. There is an identifiable point, $l_{br}$, where this breaks down and there is an increase in statistical errors. The number of large loops in the ‘tail’ is enhanced above the power law value.
2.1 The Power Law

Loops larger than the plaquette have lengths well described by a power law:

\[ N(l) = \frac{C}{l^\gamma} \]

where \( C \) is independent of \( l \). Assuming that the power law holds for loops of arbitrary size (see Sect. 2.3), \( \gamma \geq 2 \) ensures a finite total current density. To generate a string tension there must be sufficient large monopole loops to disorder the largest Wilson loops, and \( \gamma \leq 3 \). (A more model dependent argument \([8]\) requires \( \gamma = 3 \), based on the correspondence between instantons and monopole loops, e.g. \([9]\).)

Fig. 2 plots \( \gamma \) against the lattice side length in physical units for three values of \( \beta_{SU(2)} \); a factor two change in lattice spacing. \( \gamma \) is within the above bounds and consistent with a single value \( \overline{\gamma} = 2.87(1) \).

As we approach the continuum limit the number of monopole loops of a given physical length, \( l \), per unit physical volume \( n(l) \)

\[ \frac{n(l)}{L^4(a\sqrt{\sigma})^{4-\gamma}} \]

also scales and \( c_p \) remains near constant in Fig. 3 against changes in the lattice size, \( L \), and the string tension, \( a\sigma \), with a mean value \( \overline{c_p} = 1.27(2) \). Note \( \gamma \) implies a non–integer dimension for \( c_p \).

2.2 Breakdown of the Power Law.

On a given lattice the power law breaks down for the long loops at \( l \approx l_{br} \), which we find to be proportional to the lattice side length in Fig. 4.

The largest lattices are usually considered reasonable for extracting continuum physics in terms of finite volume and spacing effects, and the proportionality shows no sign of ceasing on larger lattices, suggesting the linear relation continues and the entire monopole loop spectrum follows the power law in the infinite volume limit.

2.3 The Tail of the Spectrum.

The total current density scales less well \([10]\) with the string tension than does the power law describing the smaller loops containing around 50 % of the current. The non-scaling behaviour comes from the very largest loops. There are two extreme possibilities:

1. The infinite volume spectrum is described exactly by the power law, and loops of all sizes scale well. Finite volume effects distort this above some multiple of the lattice size.
In this picture, the ambiguities encountered in resolving a cluster of monopole current into a set of intersecting loops are of limited importance.

Small loops, however, appear to make no contribution to the ‘asymptotic’ string tension measured on large length scales \[3\]. It is unlikely that the correct string tension is given by finite–size noise, which prejudices against this picture.

2. The infinite volume spectrum is composed of two components. In addition to the power law spectrum, there is also at least one large current cluster of near constant current density which when placed in a finite volume will give a current proportional to the volume. The increase in the statistical errors for this portion comes from there being no natural way of resolving this into closed loops.

This also poses questions. The large clusters, which apparently generate the string tension, appear to recede with the lattice size and have a current density which does not scale well. The string tension arises from disordering of the Wilson loop by monopole structures. The monopole loop must have an extent that is comparable to, or larger than, the Wilson loop if it is to disorder it. Only the largest monopole loops will contribute to the ‘asymptotic’ string tension, but loops of intermediate size described by the power law also would naively be expected to contribute to the smaller Creutz ratios. This contribution appears to be missing; the monopole contribution to the inter-quark potential is extremely linear; \(2 \times 2\) Creutz ratios differ from the asymptotic string tension by only a few per cent.

Figure 1: The monopole loop spectrum at \(\beta = 2.4\) on a \(14^4\) lattice.
Figure 2: Scaling of the spectrum exponent $\gamma$. Key: $+ \beta = 2.3$, $\times \beta = 2.4$, $\star \beta = 2.5$

One possible explanation is that the loops $l \leq l_{br}$ are extremely crumpled and their extent is not proportional to their length but less (one-fourth power in the extreme case). Loops of length 50 would then form objects that are only 3 units across. This is supported by the rapid loss of all these loops under $U(1)$ cooling, without significant decrease in the string tension [11].

3 Other gauges and Gribov copies.

We may AP to gauges other than the MAG which diagonalise operators such as; the Polyakov loop, the $U_{12}$ plaquette, $\sum U_{\mu\nu}U_{\mu\nu}$, or ‘no gauge’ where no diagonalisation is carried out.

The monopole spectrum is qualitatively the same; a power law breaking off at long loops. The power law exponents are consistent with the MAG. This is true even in ‘no gauge’, where we expect no physics, suggesting the value is of general, possibly combinatorial, origin.

There are more monopoles after projection to these gauges than to the MAG. The values of $C$ are correspondingly higher and although scaling well with lattice volume, scaling with physical length scales is poor; $C$ is approximately constant across different $\beta$, suggesting there is no contact with the long range physics.
Figure 3: Scaling of the spectrum density $c_p$. Key: + - $\beta = 2.3$, × - $\beta = 2.4$, * - $\beta = 2.5$

Preferring a gauge where monopole structures behave physically will cause us to favour the MAG, but may merely arise from the MAG operator alone requiring iterative gauge fixing.

This iterative fixing allows Gribov copies. There is a negative correlation between the value of the functional that is locally maximised when gauge fixing and $c_p$. Scaling, however, remains good and no motivation for the selection of Gribov copies is apparent from this work so far.

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Figure 4: The breakdown point of the power law. Key: $+ \beta = 2.3$, $\times - \beta = 2.4$, $\star - \beta = 2.5$

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