Investigation of students’ mathematical reasoning ability in solving open-ended problems

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Abstract. The aim of this study was to analyse the reasoning ability in solving open-ended mathematics problems in the area of circle material. Reasoning ability is an important component of education as it necessary for understanding mathematics in particular and presents an important means of developing ideas. The importance of reasoning ability presented in solving open-ended mathematics problems that is a problem that has multiple solutions and alternatives or settlement. The study was conducted in April at the 2015/2016 academic year. Subjects in this study were 8th-grade students of SMP Negeri 1 Singaraja. The study used a case study with qualitative research techniques to investigate how students’ reasoning ability in solving open-ended mathematics problems. The data was collected through an essay test, interview, observation, and documentation. The study was use reasoning aspect that described by Trends in International Mathematics and Science Study (TIMSS) 2008. From the data obtained, it was identified that the students lacked of reasoning ability, specifically in aspect of think mathematically and describe a new the results obtained through problem solving and expand on these solutions, provide evidence for the validity of an action or the truth of a mathematical expression, illustrating an appropriate geometric figure, and identifying and use interrelations between variables in mathematical situations. However, students also demonstrate a creative thinking ability when solving the problems using their knowledge in a real life situation to present a unique solution that the author never predicted before.

1. Introduction
Mathematics instruction is already developing in various significant ways. The developments are closely related to the students’ thinking ability. Today’s paradigms of education require that students get used to mathematical thinking and reasoning performance in solving problems.

Reasoning ability is an ability that demonstrated during problem-solving processes and which represents high-order mathematical thinking. Mansi [5] defines, “Reasoning as the ability to think coherently and logically and draw inferences or conclusions from facts known or assumed”. NCTM stated that reasoning ability is an important component of education as it necessary for understanding mathematics in particular and present an important means of developing ideas [6].

Kilpatrick, Swafford, & Findell stated that “Mathematical reasoning ability refers to the ability to formulate and represent a given mathematics problem, and to explain and justify the solution or argument” [3]. Students must be able to evaluate conjectures and assertions, to reason deductively and inductively by formulating mathematical assertions, and to develop and maintain their reasoning ability. If their reasoning ability remains underdeveloped, students will come to view mathematics as an aggregate of specific rules, and an ensemble of thoughtlessly executed calculations and drawings.
In addition, studies conducted by Toole [3] indicate that sufficient mathematical reasoning ability is imperative to proof-writing performance, have emphasised that a direct relation exists between reasoning ability and success in mathematics, where individuals who demonstrate better reasoning ability display good problem-solving profiles with the interrelations they are able to identify, while also having better communication ability.

Trends in International Mathematics and Science Study (TIMSS) [3] has stated that student with reasoning ability must be able to perform the following: (1) identify and use interrelations between variables in mathematical situations; dissociate geometric shapes in order to facilitate the resolution of a geometrical problem, draw the expansion of an object, visualise the transformation of three dimensional objects, and deduce valid results based on the provided information (Analyse); (2) think mathematically and describe anew the results obtained through problem solving and expand on these solutions (Generalize); (3) use mathematical operations in combination and combine the results in order to obtain more advanced solutions (Synthesize); (4) use mathematical results or properties to provide evidence for the validity of an action or the truth of a mathematical expression (Justify); and (5) solve non-routine problems by applying his/her geometrical knowledge and appropriate mathematical processes (Solve Non-routine Problems).

Reasoning ability is an ability that becomes based on mathematics problem solving, especially problems that require a high-order thinking ability. In order to improve student ability in solving high-order thinking problems, students’ reasoning ability need to be developed. One of the most related problems to reasoning ability is open-ended mathematical problems.

According to Shimada [8], “An open-ended problem (open-ended mathematical problem) is a problem that has a lot of solutions and alternatives or settlement”. Through open-ended mathematical problems, students are taught to think differently and not singularly. Students are conditioned to perceive a problem widely by identifying various possibilities. Learning with open-ended approach begins by providing open-ended problems to students. Learning activities should lead and bring the students in answering a problem with many ways and with many [correct] answers, so as to stimulate the intellectual ability and the experience of students in the process of discovering something new. Foong [2] states that there are three basic criteria for open-ended mathematical problems. This criterion is that open-ended mathematical problems must: (1) give all students the opportunity to demonstrate some of the knowledge, abilities, and understanding of mathematics they have, (2) rich enough to challenge students to think and reason and go beyond what they think can do, and (3) can be applied in a wide scope in terms of approaches or strategies used to find solutions. It is clear that to solve open-ended mathematical problems need sufficient reasoning ability.

Importance of reasoning ability in solving open-ended mathematical problems is not materialize well in learning activities. Research conducted by Clarke, D. Sullivan, S., & Spandel, U [1] indicated that the student’s understanding of open-ended mathematical problems still lacked. Most students were less familiar with open-ended mathematical problems which cause them to find difficulty in handling the problems. This research also revealed that students tend to suffer difficulty to find more than one way as well as answers to respond to mathematical problems. Furthermore, research conducted by Lowrie, T. Francis, R., & Rogers, G. [4] showed that students were accustomed to answering routine problems that usually only lead to one way and one solution causing the students’ ability in solving open-ended mathematical problems to be limited. The students are hard to manage and receive open-ended mathematical problems because of their initial mind-set concerning with problems that lead to single solutions. These studies are some of the research that shows that students’ ability in handling the open-ended mathematical problems is still lacking.

In addition, students’ problems in solving open-ended mathematical problems are also advanced by Sullivan, P., Warn, E., & White, P. [10]. In this research, it was found that most students assume an open-ended mathematical problem is much more difficult than the normal problems and the students are hard to handle it. Payadnya [7] found that most students use trial-error ways in solving open-ended mathematical problems and less able to provide the appropriate reasoning. For this reason, it is important to investigate students’ reasoning skills in solving open-ended mathematical problems. As
described by Payadnya [7], in order to determine the difficulties encountered by students in solving open-ended mathematical problems, it is necessary to identify the cognitive processes that underlie open-ended processes. The reasoning skills demonstrated by a student whilst problem solving will allow us to observe the way in which the student associates open-ended mathematical problems and by what means they reach a solution.

Based on the explanation above, the author considers it necessary to conduct an investigation of students’ mathematics reasoning ability in solving open-ended problems. The importance of this research is to be able to understand how students' reasoning ability in solving open-ended mathematical problems, understand the various problems faced by students while demonstrating reasoning ability in solving open-ended mathematical problems that will be the basis for finding the appropriate solution. The results of this study are important for assessing the way in which students think when solving open-ended mathematical problems as well as for observing the way in which they demonstrate reasoning skills. Additionally, the study will help teachers to assess whether the students' performance is reflected in their reasoning skills. Furthermore, the results also provide for the teachers' self-assessment and encourage them to reconsider the current techniques used in learning processes.

2. Method
In this study, a qualitative research method was used. According to Sugiyono [9] “Qualitative research method is a research method based on the philosophy of positivism, used to examine the condition of natural objects, where researchers are as key instruments”. The type of qualitative method that used in this study is Case Study that is a series of scientific activities carried out intensively, in detail and in depth about a program, event, and activity, both at the individual level, a group of people, institutions, or organizations to obtain in-depth knowledge of the event. The author tries to find out how student reasoning ability in solving open-ended mathematical problems.

Because of various limitations, this study was conducted in the 8th-grade students of SMPN 1 Singaraja with the area of the circle material. Using purposive sampling, the class chosen to be the subject of the study was class VIII A8 SMP Negeri 1 Singaraja with 28 students. Research time was Semester II 2015/2016 Academic Year.

Data collection methods in this study were a test, observation, and interviews. According to Sugiyono [9], a test can be interpreted as a method of psychological research to obtain information about various aspects of behavior with the use of measurements that produce a quantitative description of the aspects studied. An interview is a meeting of two or more people to exchange information and ideas through question and answer so that the meaning can be constructed in a particular topic. Observations are intended to make direct observations of the subject that are carefully investigated, then proceed with recording the things that are considered important to strengthen the accuracy of the data. The research instrument relates to information/data collection and processing activities. The instrument used in this study is a diagnostic test, interview guidelines, and observation guidelines. Triangulation method is used in this study to ensure internal validity. According to Sugiyono [9], triangulation is defined as checking data from various sources in various ways, and various times including source triangulation, technical triangulation, and time triangulation. Triangulation used was triangulation of sources which refers to the suitability of the data obtained from various sources, namely researcher, teacher, and student; and triangulation of techniques which refers to various methods of data collection conducted in this study in the form of observation, interviews, and the results of analysis of students’ answers.

To find out the difficulty of students in using students’ reasoning abilities in solving open-ended mathematical problems, an open-ended mathematical problem diagnosis test was given in the form of a description with the area of circle material. Test results were then analysed in depth in terms of students’ reasoning abilities according to the indicators described by TIMMS 2008. To find out the constraints faced by students and strengthen the results of test analysis interviews were conducted with teacher and students related to the difficulties faced by students and constraints in learning. To find out the implementation of mathematics learning in the class which is used as the research subject,
observations were made to the mathematics subject teachers, to find out what the teacher did when teaching mathematics in class. Test results, interviews, and observations were then combined to draw conclusions.

3. Results and Discussion

Author conducted tests for students consisting of 5 backgrounds that are different from the area of circle material. This test was carried out by Grade VIII A8 students of SMP Negeri 1 Singaraja. The results of student answers were then collected and analysed to find difficulties that involve students. Here are the results of an analysis of students’ reasoning ability in the context of solving open-ended mathematical problems. In this case, the area of the circle material was chosen require the sufficient thinking ability on representation and analysis to be able to understand the concept and solve the problems.

Previously, the researcher had first introduced about open-ended problems related to the form of the problem and how to answer it. The researcher and the teacher carried out this during the material learning process the area of the circle takes place. After the area of the circle learning was completed, students are given a diagnostic test regarding the open ended matter of the area of the circle area. Students worked on the test individually and are convinced that the results of the test will not affect the value of the report card but will be used as an additional value and assessment material for the teacher on student development. This is important to prevent students from pressure and fear when answering questions but can still keep the students’ focus and seriousness. From the students’ answer, author analyse the mathematical reasoning ability achievement of students’ based on TIMSS aspect, as shown in the table below.

| Aspect               | Number of students who achieve the aspect | Number of students who do not achieve the aspect | Aspect achievement percentage |
|----------------------|------------------------------------------|-----------------------------------------------|-------------------------------|
| Aspect 1 (Analyse)   | 2                                        | 26                                            | 7.14%                         |
| Aspect 2 (Generalize)| 4                                        | 24                                            | 14.28%                        |
| Aspect 3 (Synthesize)| 12                                       | 16                                            | 42.85%                        |
| Aspect 4 (Justify)   | 1                                        | 27                                            | 3.5%                          |
| Aspect 5(Solve non-routine problems) | 5                                        | 23                                            | 17.85%                        |

Problem No. 1
"Draw two circles with an area between 0 cm² and 200 cm²!"

In answering this problem, most students do not presented appropriate reasoning or more specific to the aspect of providing evidence for the validity of an action or truth of a mathematical expression. Students draw a circle with a certain radius or diameter. However, it cannot calculate which can be the desired reason. This is caused the students were accustomed to making procedural adjustments. Students were confused when they need to make reasoning in solving open-ended mathematical problems. Example of students answer is:

![Example of student answer in problems no. 1.](image)
In addition to the examples above, there were many students who only describe two circles without the size of the radius or diameter or just write down the area of the two circles to meet the desired conditions. This illustrated the lacked ability of students in the aspect of ‘identify and use interrelations between variables in mathematical situations’. The following figure is the example of the student's answer.

**Figure 2.** Another example of student answer in problems no. 1.

Problem No. 2
"Anton has a circular cardboard with a radius of 10 cm, if Andi wants to have a carton with an area is greater than or equal to the area of Anton's carton, what is the cardboard radius of Andi?"

In problem no. 2, the problem focuses on determining the radius of the new circle with the area requirement of the new circle is greater than or equal to the area of the old circle. In answering the problem number 2, most students did not understand the problems presented. In this problem students were free to choose the desired area of the circle as long as it is greater than or equal to the area of the old circle. However, in reality almost all students chose the area of the new circle from the area of the old circle. Students did not realize the possibility of being able to choose the area of the new circle with the value or area of the old circle. This showed a lack of students' ability in ‘thinking mathematically and describing the results obtained through problem solving and expanding on these solutions (Generalize)’. Examples of student answers are as follows.

**Figure 3.** Example of student answer in problems no. 2.

In addition, there were a lot of lacked of reasoning in student answers. Many students chose the size of the radius of a particular circle without including the reason why the measure is chosen which shows the students’ lacked of ability in providing evidence for the validity of the action or the truth of a mathematical expression. Examples of the intended student answers are presented in the following picture.
Problem No. 3
"A sheet of zinc is a square measuring 50 cm x 40 cm. The zinc will form circular can lids of the same size. To make as many can lids as possible, the area of zinc that is not used is."

In problem no. 3, there are problems related to the application of the concept of the area of the circle and the area of other flat building in daily life. In this case, students are asked to determine the area of the remaining square shaped zinc used to make a circular can lid. In answering this problem, students' creativity and precision are needed in determining the possible and appropriate size of the can lids. Basically, students are free to determine the size of the can lid as long as it is rational. However, determining the size of the diameter and radius of the can lid must be included with supporting evidence and exact explanation of how many can lids can be made. In this case, most students originally determined the amount of can lids that can be made after firstly determining its size. Almost all students thought, the amount of can lids is determined by merely dividing the area of zinc with the total area of the can lids. Whereas if implemented in real terms, there were limitations to the amount of can lids that can be made because it follows the size of the zinc. It could be that the remaining zinc remains for some can lids but can no longer form one. This characterizes the lack of students' ability to 'think mathematically and describe the results obtained through problem solving and expand on these solutions' and 'identify and use interrelations between variables in mathematical situations'. Examples of student answers in question are as follows.

![Figure 5. Example of student answer in problems no. 3.](image)
Question without including an appropriate representation that causes errors in students' answers. This showed the lacked of ability of students in aspects of ‘illustrating an appropriate geometric figure’.

Problem No. 4
"A circular wooden stick rolled a full turn and almost reaches the distance of 88 cm, what is the area of the circular cross section of the wooden stick?"

Problem no. 4 focus on the relationship and application of the concept of circle around the area of the circle in daily life. In problem 4, it was clear that there is a lack of students' ability in aspects of ‘identifying and using interrelations between variables in mathematical situations’. All students did not answer this problem correctly. Almost all students did not even have the right idea to answer this problem. In answering this question, students just stare and stop until they get the radius or diameter of the wooden stick and do not look for what they want, namely the cross-sectional area of the circular wooden stick. Students also did not show the aspect of ‘think mathematically and describing aspects of the results obtained through problem solving and expand on these solutions’ because almost all students searched for the radius and diameter of the circle by using a circle circumference of 88 cm. Whereas what should be around the circle is less than 88 cm because it has been confirmed in the question that the circular wooden stick almost reached 88 cm after rolling a full rotation. Examples of the intended student answers are presented in the following figure.

![Figure 6. Example of student answer in problems no. 4.](image)

In problem no. 4, there were many students' answers that are far from the desired expectations or answers. There were many errors in students' understanding of the problem and many students did not answer.

Problem 5
"A farmer placed a goat in a field full of grass with a length of 21 m and a width of 7 m as shown below. The farmer has 2 ropes with a length of 1 m and 3.5 m. By utilizing one or both of these ropes, what is the maximum area of grass can the goat eat? Describe the dining area and the move that the farmer made to the goat!"

In Problem 5, many students did not use critical or creative thinking skills that show students' lacked of ability in mathematical reasoning and ‘describe the results obtained through problem solving and expand on these solutions’. In answering this problem, students did not use both ropes properly and couldn’t describe the use of the rope to the maximum so that the goat can eat grass as much as possible. This shows the lacked of students' ability in the aspect of reasoning that is ‘illustrating an appropriate geometric figure’. Students tend to only used one rope and only make one move so that the remaining grass that is not eaten by goats is very broad. Examples of the answers in question are as follows.
Figure 7. Example of student answer in problems no. 5.

There were also unique answers from students that author have never predicted before. The answer in question is where students take advantage of the two ropes by connecting the two ropes to obtain a 4.5 m long rope. However, there were still many errors in student answers, especially in terms of their use and depiction. Examples of these student answers are as follows.

Figure 8. Another example of student answer in problems no. 5.

After students were given a test, interviews are conducted with the teacher and students to find out the reasoning ability of students in solving open-ended mathematical problems. The results of interviews conducted on mathematics subject teachers showed that the problem faced by teachers in teaching mathematics in class was the lacked of students' thinking ability in dealing with mathematical problems. The teacher explains that when students were given non-routine problems or problems that require high-order thinking skills, students become confused even though the concept used is still the same. Meanwhile students stated, the obstacles faced were the difficulty of understanding the purpose of the mathematical problems presented so that the students had difficulty in determining the right method to solve the problem. In dealing with open-ended mathematical problems, students state that it was difficult to reason and understand problems.

In addition, class observations were also conducted by author to see how mathematics learning has been going on so far. Author find that mathematics learning that has been running is still focused on procedural aspects, namely emphasizing the understanding and use of formulas and not emphasizing the aspects of critical and open thinking. This explains why students often had difficulty when given a question that utilizes high-level thinking skills such as open-ended mathematical problems and also causes a lack of reasoning ability in students.
4. Conclusion
From the tests, interviews, and observations made, there are some conclusions that the students still lacked of reasoning ability in solving open-ended mathematical problems specifically in the aspects of: 1) think mathematically and describe the results obtained through problem-solving and expanding on these solutions, 2) provide evidence for the validity of action or the truth of a mathematical expression, 3) appropriate geometric figure illustration, 4) and identifying and using interrelations between variables in mathematical situations. Students experience difficulties when using their reasoning ability in solving open-ended mathematical problems. In addition, the results are also obtained where students are still experiencing procedural mathematics learning so that students find it difficult to develop their thinking skills and reasoning.

5. Suggestion
Suggestions that can be given by the author is that teachers of mathematics subjects can emphasize aspects of understanding concepts and aspects of developing reasoning skills and high-level thinking in conducting classroom learning. The teacher must focus on how mathematics can improve students’ thinking ability and avoid procedural learning. This will enable students to handle more complex problems that will become demands in the present and future such as open-ended mathematical problems better.

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