Mathematical modeling of power consumption based on rank analysis

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Abstract. In the article, based on the application of the rank analysis of the technocenosis, mathematical modeling of electricity consumption by consumers for the production of hard alloys is carried out. Two methodological levels of research in the field of energy-saving and power consumption are presented. At the same time, it was found that rank analysis allows you to streamline information, effectively predict power consumption by individual objects to identify dynamics, and visually represent objects with abnormal power consumption. The analysis of the results obtained confirms the efficiency of using the H-distribution for obtaining mathematical models of power consumption of Noah and Pointer castes in the production of hard alloys. For the first time, the regularity of changes in electricity consumption by technological equipment was established, which is described by an adequate mathematical model in the form of a damped harmonic function. The resulting model is recommended for use in an automated dispatching control system for electricity consumption for the purpose of short-term forecasting of electricity consumption.

1. Introduction

Mathematical modeling of power consumption is based on the study of issues related to expert, cluster analyzes, and rank distribution of technocenosis [1–7]. Scientists have recently made a great contribution to solving various aspects of this problem as applied to industrial enterprises in various countries [18-19, 24, 25].

Industrial enterprises are a complex technological complex consisting of a set of thousands of jointly functioning technological objects that randomly generate the power consumption of the enterprise [20-23]. To assess the power consumption of large industrial enterprises, a new systemic technocenological approach has recently been applied. A great contribution to the development of the method of rank H-distribution for calculating electrical loads was made by [8].

This approach is based on the concept of technocenosis, which reflects the specificity of connections between technical elements – individuals. It can be considered proven that technical systems, such as large industrial enterprises, live and develop according to the same laws as biological systems (biogeocenoses).
2. Materials and methods
To describe the structure of technocenoses and identify trends in their development and change, a special mathematical apparatus has been developed for hyperbolic $H$-distributions, which have the form:

$$W = \frac{a}{r^\beta},$$  \hspace{1cm} (1)

where $a$ – coefficient; 
$r$ – rank; 
$\beta$ – exponent.

Hyperbolic rank $H$-distributions are a decreasing sequence of parameter values, ordered in such a way that with increasing rank (ordinal number), each subsequent number is less than the previous one.

The general research methodology in the field of energy consumption and energy saving of an industrial enterprise can be represented by two levels, shown in Figure 1.

**Figure 1.** Methodological levels of research in the field of energy-saving and energy consumption.

The first level corresponds to research aimed at specific technical developments that contribute to the reduction of specific power consumption. The methodology is based on computer simulation based on the axiomatics of Gaussian distributions. This makes it possible to widely use probabilistic convolutions in determining the laws of functioning and quasi-parallel algorithms in modeling.

The second level corresponds to the study of the field of energy-saving. Here, the power consumption of an industrial enterprise is optimized as a whole [9, 10]. Optimization of energy consumption at the second level is carried out within the framework of a related methodology in two stages.

1. At the stage of analyzing the power consumption of an industrial enterprise according to specially developed request forms - expert assessments, data is collected on all electricity consumers. This allows you to prepare a sample of energy consumption data for further multivariate analysis.

2. At the stage of development of an information and analytical complex, a representative sample of data on the power consumption of enterprise facilities is processed, including a bank and a data management system, as well as calculation and graphic modules. The complex can be successfully used in planning and forecasting, and also allows online monitoring of information about electricity consumers, updating the initial data for analysis in almost real-time. The complex uses expert assessments and cluster analysis to optimize specific power consumption. At the request of the operator, any information on electricity consumers can be obtained from the database with the required degree of detail and generalization based on the information and analytical complex of the automated dispatch control system for electricity consumption (ADCSEC).

Rank analysis based on the technocenological approach, Zipf mathematical statistics, and the theory of hyperbolic infinitely divisible distributions is widely used as a methodological basis at this level. The rank analysis allows you to streamline information, effectively predict power consumption...
by individual objects and the enterprise as a whole, to identify dynamics and visualize objects with abnormal power consumption.

3. Brief description of the rank analysis of the technocenosis

The rank analysis consists in dividing the technocenosis objects into three groups of ranks: Noah, Pointer, and locust casts of distribution.

Noah caste is a group of the most energy-intensive objects occupying the first ranks of the rank $H$-distribution and forming the first point of the approximating rank $H$-distribution. The procedure for determining the first caste is formalized by two conditions: 1) a sequence of rank $H$-distributions is constructed, where in each subsequent distribution the largest object is excluded until the trajectory of the structural-topological dynamics of the discarded objects on the considered time interval coincides in time with the trajectory of the first rank; 2) if the first condition is met when the largest object is excluded, the rank indicator $\beta$ does not change when approximating the rank $H$-distribution. An excluded object or a group of the largest objects, if two conditions are met simultaneously, form the first Noah caste.

Pointer caste is a group of objects connected by the function of coenological influence, a number of which determine the numerical value of the rank indicator $\beta$. Pointer caste objects have a high coefficient of concordance, so there is a formalized rule for determining the boundaries of the Pointer caste. The function of coenological influence on each object of the rank $H$-distribution is formalized: the probability function of attendance of each rank by various objects in terms of structural and topological dynamics. The function reflects the strength of competition for a specific rank. When moving from the “head” of the rank $H$-distribution to the “tail”, the degree of coenological influence on the formation of the dynamics of each object on the structural-topological surface increases. Pointer border is determined by the rank with the maximum of the coenological influence function.

Locust caste (virtual) – a group of objects located beyond the Pointer border. This is a group of small objects corresponding to large ranks (“long tail” of the rank $H$-distribution) and practically indistinguishable in terms of the parameter.

For operational control, management, analysis, and forecasting of power consumption, it is advisable to carry out research using ADCSEC.

4. Mathematical modeling of calculation and forecasting of energy consumption by casts of rank $H$-distribution of technocenosis

Experimental studies were carried out at one of the largest non-ferrous metallurgy enterprises of the Russian Federation for the production of hard alloys. The main stages of rank analysis are as follows:

1. Isolation of technocenosis (industrial enterprise).
2. Determination of the list of types (electricity consumers).
3. Setting species-forming parameters.
4. Parametric description of the technocenosis (caste).
5. Construction of tabulated and graphical rank parametric distributions.
6. Approximation of distributions.
7. Optimization of technocenosis.

Energy-intensive production, the presence of a large number of consumers of different capacities and operating modes of consumers predetermines the enterprise in question as a worthy object for conducting a serious energy audit at a high level.

The most power-consuming equipment of the plant are electrolysers, welding machines, and pumps. They account for about 47% of the total power consumption of the enterprise (rank 1–6). The equipment that makes up the Pointer and the locust caste of distribution (consumers with a rank of 7–11 and 12–31, respectively) is defined in a similar way.
For each of the castes, using the least squares method (LSM), mathematical models were obtained in the form of approximating dependences of changes in power consumption. Figure 2 shows, as an example, the dependences for the Noah caste of distribution.

The regression equations, the coefficients of determination (R2) and least squares predicted values of electricity consumption for individual distribution castes for 2021 are presented in Table 1.

The analysis of the results obtained confirms the efficiency of using the H-distribution for obtaining mathematical models of power consumption of Noah and Pointer castes in the production of hard alloys. The H-distribution is described by a hyperbolic expression of the form: \( W = \frac{q}{\beta^3} \) with a stable value of the exponent \( \beta = 0.55 \) and \( \beta = 0.33 \) for Noah and Pointer, respectively. For the locust caste, due to its abundance and low power, the most adequate is the linear regression equation: \( W = a \cdot r + b \) (\( R^2 = 0.98 \)).

Based on the results of experimental studies on the basis of representative samples \( \{W\} \), the confidence intervals for the power consumption of technological equipment of individual distribution castes were determined. In the course of experimental studies, it was found that the form of the mathematical model (approximating functional) is an exponentially decaying harmonic function of the form:

\[
W(t,u) = \cos(u_0 t + u_1) e^{u_2 t} + u_3
\]

where \( t \) – time, year;
\( u_0, u_1, u_2, u_3 \) – constant coefficients.

For the obtained forecasting model, a retrospective assessment of the relative model error was carried out. The following are taken as comparative values: the actual consumption of electricity and the predicted one obtained by the model (2). Comparison of these values showed that the relative estimate of the forecast is 1.34%, which confirms the adequacy and high accuracy of the obtained forecasting model in various industrial-natural systems [11–17].
Table 1. Mathematical models of approximating curves and values of determination coefficients ($R^2$) of power consumption of individual castes of distribution.

| Years | Distribution caste (the number of communities in the caste $n_i$) | The equation of the approximating hyperbolic curve, coefficient of determination ($R^2$) |
|-------|---------------------------------------------------------------|-----------------------------------------------------------------------------------|
|       | Noah ($n = 6$) | Pointer ($n = 5$) | Locust ($n = 20$) |
| 2015  | $W = \frac{1.21 \times 10^7}{r^{0.55}}$ | $W = \frac{36.9 \times 10^6}{r^{0.33}}$ | $W = -0.63 \cdot 10^5 \cdot r + 1.21 \cdot 10^6$ |
|       | $R^2 = 0.9$ | $R^2 = 0.8$ | $R^2 = 0.98$ |
| 2016  | $W = \frac{1.14 \times 10^7}{r^{0.55}}$ | $W = \frac{3.48 \times 10^6}{r^{0.33}}$ | $W = -0.59 \cdot 10^5 \cdot r + 1.14 \cdot 10^6$ |
|       | $R^2 = 0.9$ | $R^2 = 0.8$ | $R^2 = 0.98$ |
| 2017  | $W = \frac{8.99 \times 10^6}{r^{0.55}}$ | $W = \frac{2.74 \times 10^6}{r^{0.33}}$ | $W = -0.47 \cdot 10^5 \cdot r + 0.9 \cdot 10^6$ |
|       | $R^2 = 0.9$ | $R^2 = 0.8$ | $R^2 = 0.98$ |
| 2018  | $W = \frac{1.24 \times 10^7}{r^{0.55}}$ | $W = \frac{3.79 \times 10^6}{r^{0.33}}$ | $W = -0.65 \cdot 10^5 \cdot r + 1.24 \cdot 10^6$ |
|       | $R^2 = 0.9$ | $R^2 = 0.8$ | $R^2 = 0.98$ |
| 2019  | $W = \frac{1.39 \times 10^7}{r^{0.55}}$ | $W = \frac{4.24 \times 10^6}{r^{0.33}}$ | $W = -0.72 \cdot 10^5 \cdot r + 1.39 \cdot 10^6$ |
|       | $R^2 = 0.9$ | $R^2 = 0.81$ | $R^2 = 0.98$ |
| 2020  | $W = \frac{1.33 \times 10^7}{r^{0.55}}$ | $W = \frac{4.04 \times 10^6}{r^{0.33}}$ | $W = -0.69 \cdot 10^5 \cdot r + 1.33 \cdot 10^6$ |
|       | $R^2 = 0.9$ | $R^2 = 0.8$ | $R^2 = 0.98$ |
| Forecast 2021 | $W = \frac{1.21 \times 10^7}{r^{0.55}}$ | $W = \frac{3.48 \times 10^6}{r^{0.33}}$ | $W = -0.63 \cdot 10^5 \cdot r + 1.27 \cdot 10^6$ |
|       | $R^2 = 0.9$ | $R^2 = 1$ | $R^2 = 1$ |

5. Conclusion

For the first time, the regularity of changes in electricity consumption by technological equipment was established, which is described by an adequate mathematical model in the form of a damped harmonic function:

$$W(t,u) = \cos(u_0t + u_1) e^{-u_2t} + u_3$$

A retrospective check of the relative forecasting error of electricity consumption showed that, for the resulting model $W(t,u)$, it does not exceed 2%, which is much lower than the relative forecast errors of a number of other types of models. The resulting model is recommended for use in ADCSEC for short-term forecasting of electricity consumption.

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