Towards an Einstein–Podolsky–Rosen paradox between two macroscopic atomic ensembles at room temperature

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Abstract. Experiments have reported the entanglement of two spatially separated macroscopic atomic ensembles at room temperature (Krauter et al 2011 Phys. Rev. Lett. 107 080503; Julsgaard et al 2001 Nature 413 400). We show how an Einstein–Podolsky–Rosen (EPR) paradox is realizable with this experiment. Our proposed test involves violation of an inferred Heisenberg uncertainty principle, which is a sufficient condition for an EPR paradox. This is a stronger test of nonlocality than entanglement. Our proposal would enable the first definitive confirmation of quantum EPR paradox correlations between two macroscopic objects at room temperature. This is a necessary intermediate step towards a nonlocal experiment with causal measurement separations. As well as having fundamental significance, the realization of an atomic EPR paradox could provide a resource for novel applications in quantum technology.
1. Introduction

The Einstein–Podolsky–Rosen (EPR) paradox was presented in 1935 as an argument for the incompleteness of quantum mechanics [1]. From a modern perspective, this paradox reveals the inconsistency between local realism and the completeness of quantum mechanics. Based on the assumption of local realism, the argument was the first clear illustration of the nonlocality associated with an entangled state. This has motivated numerous fundamental studies as well as potential applications in quantum information [2–7]. In the original proposal, the quantum state comprised two spatially separated particles with perfectly correlated positions and momenta [1]. It is now understood that the demonstration does not require such perfect states, but can be inferred from correlations that violate an inferred Heisenberg inequality—thus signifying this paradoxical behaviour [8, 9].

Evidence for the EPR paradox has emerged in numerous experiments, including not only the correlations of single photon pairs, [2, 5, 10], but the amplitudes [6, 9, 11–15] and Stokes polarization observables [16] of twin optical beams and pulses with macroscopic particle numbers. By contrast, Bell inequality experiments that test local realism have largely been affected by the well-known efficiency loophole, and are limited to microscopic systems [7]. More recently, EPR experiments [12] have been motivated by the work of Wiseman and co-workers [17–19]. These workers established that the EPR paradox is a realization of ‘quantum steering’, a form of nonlocality identified by Schrödinger [20] whereby the measurements made by one observer at a location A can apparently ‘steer’ the state of another observer, at a different location B [21–24].

The task of measuring an EPR paradox between two spatially separated massive objects—as opposed to massless photons—remains an important challenge in physics. Despite the significant experimental advances in the detection of multi-particle entanglement [25, 26], evidence for an EPR paradox with massive particles is almost nonexistent. Strong EPR-type entanglement has been verified through the violation of Bell inequalities for ions [27] and Josephson phase qubits [28], but not for significant spatial separations. Entanglement has
also been investigated for cold atom and Bose–Einstein condensate (BEC) systems [29–32], neutrons [33] and between separated groups of atoms [34–39]. These observations do not connect directly with an EPR paradox, although proposals exist [29, 40–46]. In summary, previous experimental work has yet to detect an EPR paradox in the correlated macroscopic observables of two massive systems [47, 48].

Historically, a controversy exists as to the existence of EPR correlations at large space-like separations. Furry suggested that quantum correlations could decay with increasing distance between the two systems [49], thus removing the paradox for large distances. While this can now be ruled out for massless photons, the situation is not clear for massive objects. Tests of Furry’s hypothesis so far have focused only on the nonlocality between microscopic systems, each of one massless particle [50]. Could Furry’s hypothesis possibly be mass-dependent? Given the use of quantum models to explain the early universe, and known problems in unifying quantum mechanics with gravity, this question has an ever-increasing importance in current physics.

For macroscopic objects at room temperature, the observation of an EPR paradox is even more interesting. Here, the question is whether it is possible to confirm an entanglement involving superpositions of states that are macroscopically separated in state space for at least one of the sites. The existence of such superpositions could enhance our understanding of the issues raised in the Schrödinger’s cat paradox [51]. It is unknown whether an EPR paradox can be observed for large objects at room temperature. Our usual intuition is that strong quantum effects of this type will be masked by thermal motion. The fundamental question is whether macroscopic quantum paradoxes can be realized at all, or are simply made difficult by the known physics of decoherence [52].

In view of these outstanding questions, it is important to analyse the possibility of realizing an EPR paradox between two systems with a macroscopic mass. These effects may also have applications in high-precision interferometry, metrology, cryptography and for reduced quantum uncertainty for measurements in the presence of quantum memory [9, 53–59]. Polzik and co-workers have made a pioneering step in the direction of realizing an EPR paradox between massive objects, in work that experimentally confirmed the entanglement of two macroscopic ensembles of gaseous atoms at room temperature [60, 61, 72, 73]. The entanglement is signified by the correlation of position and momenta-like observables, thus giving a close analogy to the original paradox. The observables are macroscopic, in that the outcomes are for collective atomic spins, measured as a very large atom number difference. Similar experiments of Lee et al have succeeded in entangling room temperature macroscopic spins in diamond; however, in that work EPR observables are not identified [62].

In this paper, we examine the gas ensemble experiments of Julsgaard et al [60] and Krauter et al [61]. We show that the reported correlation is not yet strong enough to violate an inferred Heisenberg inequality. However, our detailed analysis reveals that this is achievable within the limits of the parameters reported for the experiments, provided the appropriate conditional variances are measured. This is the first prediction of an EPR paradox for room temperature atoms, using a model that accounts for thermal effects. We also explain how the measurement scheme must be modified to enable the local measurement of the relevant observables, and conclude with a discussion of macroscopic EPR experiments.

2. Entangling two macroscopic atomic ensembles

The experiments of Julsgaard et al [60], Krauter et al [61], Muschik et al [73] achieve entanglement of two macroscopic spatially separated atomic ensembles. The ensembles become
entangled when an ‘entangling’ light pulse propagates successively through the two ensembles. The method is based on the proposal of Duan et al [63].

We first summarize the theory developed by Duan et al and Julsgaard et al and Muschik et al. We will expand on that theory to give a prediction for the EPR paradox. Let us denote the two spatially separated atomic ensembles by $A$ and $B$. Schwinger collective spins, $J_{A/B}^X$, $J_{A/B}^Y$, $J_{A/B}^Z$ are defined for each atomic ensemble, assumed to contain $N$ atoms. The operators are defined with respect to two atomic levels which are denoted, for the $j$th atom in ensemble $A$, as $|1\rangle_j$ and $|2\rangle_j$.

Thus, we define

$$J_X^A = \sum_{j=1}^{N} (|2\rangle_j \langle 1|_j + |1\rangle_j \langle 2|_j) / 2,$$

$$J_Y^A = \sum_{j=1}^{N} (|2\rangle_j \langle 2|_j - |1\rangle_j \langle 1|_j) / 2i,$$

$$J_Z^A = \sum_{j=1}^{N} (|1\rangle_j \langle 1|_j - |2\rangle_j \langle 2|_j) / 2.$$

Similar operators $J_B^i$ are defined for two selected levels of the ensemble $B$. Each atomic ensemble is prepared initially, using a detuned pump laser pulse, in an atomic spin coherent state with a large mean spin $J_{A/B}^X$, so that $\langle J_X^{A/B} \rangle \sim N$. This implies the atoms are prepared in a superposition of states $|1\rangle$ and $|2\rangle$. The mean spins are equal and opposite for the ensembles $A$ and $B$, i.e. $\langle J_X^A \rangle = -\langle J_X^B \rangle$.

In order to observe an EPR paradox, the two ensembles must become entangled. In the experiments, entanglement is achieved via a detuned polarized laser pulse, which is called the ‘entangling pulse’. This laser field is described by another set of spin operators, called the Stokes operators $S_X$, $S_Y$, $S_Z$. In physical terms, $S_X$ is the difference between photon numbers in the orthogonal $X$ and $Y$ linear polarization directions; $S_Y$ is the difference in number for polarization modes rotated by $\pm \pi/4$; and $S_Z$ is the difference between the photon numbers in the two circular polarized modes defined relative to the propagation direction $Z$.

These Stokes operators are written in terms of $a_{1,2}^\dagger$, the operators for the circular polarized modes, using the Schwinger representation method. Thus,

$$S_X = \frac{1}{2}(a_2^\dagger a_1 + a_1 a_2^\dagger),$$

$$S_Y = \frac{-i}{2}(a_2^\dagger a_1 - a_1 a_2^\dagger),$$

$$S_Z = \frac{1}{2}(a_2^\dagger a_2 - a_1^\dagger a_1).$$

The light is polarized along the $x$ direction, so that $\langle S_X \rangle \sim N_p$, where $N_p$ is the number of photons in the optical pulse [60].

Before proceeding, we examine what strength of entanglement will be required to realize an EPR paradox. EPR’s original argument considered the positions and momentum of each of two particles, in an ideal state with perfect correlations. However, since this requires infinite momentum uncertainty for each particle—and therefore infinite energy—it cannot be achieved physically.
2.1. Einstein–Podolsky–Rosen (EPR) paradox inequality

More practically, the required paradox can be constructed with finite energy, by using two noncommuting observables that are defined for the systems at each location A and B, with finite correlations stronger than a critical value. These observables, which are analogous to position and momentum, will be \( J_A^Z \) and \( J_A^Y \) at A, and \( J_B^Z \) and \( J_B^Y \) at B. The Heisenberg uncertainty relation is \( \Delta^2 J_A^Z \Delta^2 J_A^Y \geq \frac{1}{4} |\langle J_A^X \rangle|^2 \), where \( \Delta^2 J_A^X \) is the variance for measurement of \( J_A^X \). As shown in [8], an EPR paradox is observed when a measurement at B collapses the wave-function at A, to such an extent that there is a violation of the inferred uncertainty relation.

The paradox is obtained when the uncertainty product of the conditional or ‘inference’ variances is less than a critical value, i.e. [8, 64, 65]

\[
\Delta_{\text{inf}}^2 J_A^Z \Delta_{\text{inf}}^2 J_A^Y < \frac{1}{4} |\langle J_A^X \rangle|^2. \tag{3}
\]

Here \( \Delta_{\text{inf}}^2 J_A^Z \) is the variance of the conditional distribution for a result of measurement \( J_A^Z \), given a measurement made on ensemble B, and \( \Delta_{\text{inf}}^2 J_A^Y \) is defined similarly. The uncertainty \( \Delta_{\text{inf}}^2 J_A^X \) can be viewed as the average error in the prediction of the result of \( J_A^X \) given a result for the measurement on ensemble B. A useful strategy is to estimate the result as \( g_Z j_B^Z \) where \( g_Z \) is a real constant and \( j_B^Z \) is the outcome of measurement \( J_B^Z \). Similarly, the result for measurement \( J_A^Y \) can be predicted as \( g_Y j_B^Y \), where \( g_Y \) is a real constant and \( j_B^Y \) is the result of the measurement of \( J_B^Y \).

Then, we can achieve an EPR paradox if (3) holds, where \( \Delta_{\text{inf}}^2 J_A^Z = \Delta^2 (J_A^Z + g_Z J_B^Z) \) and \( \Delta_{\text{inf}}^2 J_A^Y = \Delta^2 (J_A^Y + g_Y J_B^Y) \). Combining these equations, we obtain a gain-dependent criterion involving actual measured variances:

\[
\Delta^2 (J_A^Z + g_Z J_B^Z) \Delta^2 (J_A^Y + g_Y J_B^Y) < \frac{1}{4} |\langle J_A^X \rangle|^2. \tag{4}
\]

For Gaussian distributions, the inference variances \( \Delta^2 (J_A^Z + g_Z J_B^Z) \) and \( \Delta^2 (J_A^Y + g_Y J_B^Y) \) become the variances of the conditional distributions, for the optimal choice of \( g_Z \) and \( g_Y \) that will minimize \( \Delta^2 (J_A^Z + g_Z J_B^Z) \) and \( \Delta^2 (J_A^Y + g_Y J_B^Y) \) [9]. We mention here that EPR paradox criteria based on entropic uncertainty relations have more recently been derived by Walborn et al [66], and become useful in the non-Gaussian case.

We next wish to show that the atomic ensemble experiments can in principle enable a realization of the EPR paradox. In order to do this, it is helpful to first summarize the method used in the experiments in greater detail. Following the theory of Duan et al [63], when the detuned ‘entangling’ pulse propagates through the first atomic ensemble, the outputs are given in terms of the inputs according to

\[
\begin{align*}
S_{\text{out}}^Y &= S_{\text{in}}^Y + \alpha J_{\text{in}}^Z, \\
S_{\text{out}}^Z &= S_{\text{in}}^Z, \\
J_{\text{out}}^Y &= J_{\text{in}}^Y + \beta S_{\text{in}}^Z, \\
J_{\text{out}}^Z &= J_{\text{in}}^Z.
\end{align*} \tag{5}
\]

The \( \alpha \) and \( \beta \) are constants, and the subscripts out and in denote, for the field, the outputs and inputs to the ensemble, and, for the ensemble, the initial and final states. The \( \beta \) is reversed in sign for the second ensemble, so that after successive interaction with both ensembles, the final
output is given in terms of the first input as
\[
S_{\text{out}}^Y = S_{\text{in}}^Y + \alpha (J_{\text{in},A}^Z + J_{\text{in},B}^Z),
\]
\[
S_{\text{out}}^Z = S_{\text{in}}^Z,
\]
\[
J_{\text{out},A}^Y + J_{\text{out},B}^Y = J_{\text{in},A}^Y + J_{\text{in},B}^Y,
\]
\[
J_{\text{out}}^Z = J_{\text{in}}^Z.
\]
(6)

Here the subscripts \(A, B\) denote the spin operators for the \(A, B\) ensemble.

As described in the original theory of these experiments [60, 63], provided \(\alpha\) is large enough, the field output \(S_{\text{out}}^Y\) gives a measure of the collective spin \(J_{\text{collect}}^Z = J_{\text{in}}^Z\). This is a constant of the motion, and is not affected by the interaction. The Stokes parameter \(S_{\text{out}}^Y\) of the output field is measured using polarizing beam splitters and detectors. As a result, the atomic ensembles are prepared in a quantum state for which (ideally) the value of \(J_{\text{in}}^Z\) is known and constant. In practice, technical noise in the preparation implies a state with reduced noise level in \(J_{\text{in}}^Z\).

The collective spin \(J_{\text{collect}}^Z\) is also a constant of the motion, and hence the ensembles can be prepared via a second pulse (and by rotating the atomic spin) in a state with reduced fluctuation in \(J_{\text{collect}}^Z\).

2.2. Entanglement and EPR inequalities

One only has to show that there are reduced fluctuation in these noise levels according to the uncertainty sum criterion
\[
\Delta_{\text{ent}} = \frac{\Delta^2(J_{\text{in}}^Z + J_{\text{out}}^Z) + \Delta^2(J_{\text{in}}^Y + J_{\text{out}}^Y)}{|\langle J_{\text{in}}^Z \rangle| + |\langle J_{\text{in}}^Y \rangle|} < 1
\]
(7)
to signify entanglement between the two atomic ensembles [67, 68]. Details of how the \(\Delta_{\text{ent}}\) and the mean spin \(\langle J_{\text{in}}^X \rangle\) are measured are given in [60, 63]. For Gaussian, symmetric systems where the moments of ensemble \(A\) and ensemble \(B\) are equal, the entanglement criterion is necessary and sufficient for entanglement in two-mode systems [67].

The key point, from the perspective of this paper, is that the field–atom solutions equation (6) give no lower bound, in principle, to the amount of noise reduction in \(J_{\text{in}}^Z\) and \(J_{\text{in}}^Y\) that is possible. We comment that ultimately the amount of entanglement attainable will be limited by the uncertainty relation and the finite nature of the Schwinger spins [69, 70], but that, while significant in some BEC proposals [29, 42, 44], this limitation becomes unimportant here, because of the large numbers of atoms involved. Thus, a physical regime exists for which
\[
\Delta^2(J_{\text{in}}^Z + J_{\text{out}}^Z) \rightarrow 0, \quad \Delta^2(J_{\text{in}}^Y + J_{\text{out}}^Y) \rightarrow 0.
\]
(8)

In that case, the measurement of \(J_{\text{out}}^Y\) will imply the result for the measurement of \(J_{\text{in}}^Y\) with no uncertainty. Similarly, the result for \(J_{\text{in}}^Y\) will imply, precisely, the result for \(J_{\text{in}}^Z\). Therefore the variances of the conditional distributions \(P(J_{\text{in}}^Y|J_{\text{in}}^Z)\) and \(P(J_{\text{in}}^Z|J_{\text{in}}^Y)\) become zero, and the EPR condition (3) will be satisfied, with \(g_Z = g_Y = 1\). The solutions equation (5) on which the prediction is based are valid in the limit where damping effects can be neglected. Duan et al [63] analysed the full atomic solutions, and reported that this regime is achievable when \(N \sim N_p\) provided the field detunings are much greater than spontaneous emission rates.
For realistic systems, the variances $\Delta^2(J^Z_A + J^Z_B)$ and $\Delta^2(J^Y_A + J^Y_B)$ will not, however, be zero. The measure of entanglement given by $\Delta_{\text{ent}}$ can indicate the strength of correlation that is needed for an EPR paradox, when

$$\Delta_{\text{ent}} < 0.5$$

for symmetric ensembles. This follows because $\Delta_{\text{ent}} < 0.5$ implies $\Delta^2(J^Z_A + J^Z_B) + \Delta^2(J^Y_A + J^Y_B) < (\langle|J^Z_A|\rangle + \langle|J^Z_B|\rangle)/2$, which in turn implies that $\Delta(J^Z_A + J^Z_B)\Delta(J^Y_A + J^Y_B) < (\langle|J^Z_A|\rangle + \langle|J^Y_B|\rangle)/4$ is true [9]. If for the experiment, we measure that $\langle|J^Z_A|\rangle = \langle|J^Y_B|\rangle$, then the measured variances must also satisfy $\Delta(J^Z_A + J^Z_B)\Delta(J^Y_A + J^Y_B) < \langle|J^Z_A|\rangle/2$. This satisfies the condition (4), with the choice of $g_Z = g_Y = 1$. In general, it has been shown for Gaussian states (where losses and thermal noise are included) that the condition (3) for realizing the EPR paradox is more difficult to achieve than (7), that for entanglement.

We learn from this section that there are two important issues to be considered in realizing the EPR paradox. The first is that greater correlations are required for the EPR paradox than to realize simple entanglement. The EPR experiment will therefore be more sensitive to decoherence effects. We address this first issue in section 4, by analysing in detail the theoretical model presented by Muschik et al [73], that accounts for such effects in a recent experiment. Second, the EPR condition (3) also specifies that the measurements of the four observables $J^Z_A$, $J^Z_B$, $J^Y_A$ and $J^Y_B$ be made locally. We address this issue in the next section.

### 3. Measurement of the EPR paradox

The experiments [60, 61] detect entanglement using a ‘verifying pulse’ that propagates through the two ensembles (figure 1(a)). The Stokes observable $S^Y_{\text{out}}$ of the transmitted pulse is measured via the polarizing beam splitter. Using equations (6), the outcome for the Stokes observable allows the value of the collective spin $(J^Z_B + J^Z_A)$ to be inferred. The measurement of $(J^Y_B + J^Y_A)$ proceeds similarly, after rotation of the atomic spin. The measurement scheme thus establishes the collective spin by a final projective measurement at one location. The quality of the measurement improves as $\alpha$ becomes larger.

For smaller $\alpha$, the noise levels of the output become dominated by the vacuum noise $\Delta^2S^Y_{\text{in}}$ of the input pulse. In that case, quantum noise squeezing of the input Stokes observable would improve the signal to noise ratio of the measurement.

#### 3.1. Modification of the measurement strategy

To carry out an EPR experiment, it is necessary to make local measurements of $J^Z_B$ and $J^Z_A$ individually, and to then obtain the collective spin sum $(J^Z_B + J^Z_A)$ by addition of the measurement outcomes. Similarly, one should measure $(J^Y_B + J^Y_A)$ in a separate experimental procedure.

In order to achieve these goals, one possible strategy is to use two verifying pulses, defined with Stokes parameters $S^Y_A$ and $S^Y_B$, propagating through each ensemble $A$ and $B$ respectively (as shown in figure 1(b)). The outputs in terms of the inputs are given by the solutions equation (5), so that

$$S^Y_{\text{out},A} = S^Y_{\text{in},A} + \alpha J^Z_{\text{in},A},$$
$$S^Y_{\text{out},B} = S^Y_{\text{in},B} + \alpha J^Z_{\text{in},B}.$$

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Figure 1. Schematic diagram of how to measure an EPR paradox between two atomic ensembles. (a) The arrangement to measure the entanglement of the two ensembles, as in the experiments [60, 61, 73]. The variance of the collective spin $J^Z_A + J^Z_B$ (or $J^Y_A + J^Y_B$) is measured by passing a ‘verifying’ light pulse (with input Stokes operator $S^Y_{in}$) through both ensembles. Detection of the output pulse Stokes operator ($S^Y_{out}$) allows measurement of either $J^Z_A + J^Z_B$ or $J^Y_A + J^Y_B$, depending on the selection of rotation of the atomic spin of the ensemble. (b) An experiment to demonstrate the EPR paradox between two atomic ensembles. Independent local measurements are made on the atomic ensembles $A$ and $B$, by utilizing two verifying pulses. The choice of whether to measure $J^Z$ or $J^Y$ at each ensemble is made independently and after preparation of the EPR state (which is done by the entangling pulses). The variable gain factor $g$ is selected to enable a final optimal measurement of the conditional variance $\Delta^2(J^Z_A + gZ J^Z_B)$ (or $\Delta^2(J^Y_A + gY J^Y_B)$).

Measurement of $S^Y_{out,A}$ at one location, and $S^Y_{out,B}$ at the other location, enables local determination of $J^Z_A$ and $J^Z_B$, as required for a test of EPR nonlocality. The local measurements of $J^Y_A$ and $J^Y_B$ can be made similarly. The quality of the measurement of the atomic spins $J^Z_{A/B}$ improves when $\alpha$ is large, or if the input fields are ‘squeezed’, so that $\Delta S^Y_{in,A} \to 0$ and $\Delta S^Y_{in,B} \to 0$. 

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4. Detailed calculation for an engineered dissipative system

The recent experiment of Krauter et al [61] employs an engineered dissipative process to generate a long-lived entanglement between the atomic ensembles. The engineered dissipation approach offers a means to tailor dissipative processes, to enhance the generation of entangled or EPR states [45, 71]. Entanglement values of \( \Delta_{\text{ent}} \approx 0.9 \) were achieved, using this process, in reasonable agreement with the theory presented by Muschik, Polzik and Cirac (MPC) [72, 73] for this experiment. We therefore analyse that theory, to calculate whether the EPR paradox is predicted for a complete and realistic atomic model.

The MPC model introduces operators \( A = (\mu J^+ - \nu J^-)/\sqrt{N} \), \( B = (\mu J^- - \nu J^+)/\sqrt{N} \), where \( J_{A/B}^+ \) denote collective spin operators with \( J_{A/B}^+ = \sum_{j=1}^{N} \ upwards{\uparrow} \downarrow \downarrow \) and \( J_{A/B}^- = \sum_{j=1}^{N} \ downwards{\downarrow} \uparrow \uparrow \) such that \( J_{A/B}^+ = (J_{A/B}^+ + J_{A/B}^-)/2 \) and \( J_{A/B}^- = i(J_{A/B}^+ - J_{A/B}^-)/2 \), and \( \mu^2 - \nu^2 = 1 \). Here, the two-level states \( \uparrow \downarrow \) give in the definitions for the spins of ensemble \( A/ B \) refer only to the ensemble \( A/ B \) respectively. The \( \mu \) and \( \nu \) characterize a squeezed state with squeeze parameter \( r \), where \( \mu = \cosh (r) \) and \( \nu = \sinh (r) \). The \( \mu \) and \( \nu \) are functions of the Zeeman splitting of the two atomic energy levels and of the detuning of the laser that couples these levels to the excited states and to vacuum modes, as detailed in [73]. The work of Muschik et al [73] shows how the correlations are described by a master equation [61]

\[
\frac{d\rho}{dt} = \frac{d\Gamma}{2} \times (A\rho A^\dagger - A^\dagger A\rho + B\rho B^\dagger - B^\dagger B\rho + \text{H.c.)} + \mathcal{L}_{\text{noise}}\rho ,
\]

where \( \rho \) is the atomic density operator, \( d \) is the optical depth of an ensemble and \( \Gamma \) is the single atom radiative decay. The \( \mathcal{L}_{\text{noise}}\rho \) represents the detrimental processes such as single atom spontaneous emission noise, thermal effects and collisions which counter the development of the entangled state, but which are included in the analysis to give a realistic prediction. The Lindblad terms given in the parentheses arise from the engineered dissipative mechanism that drives the system into the EPR state [61]. For large optical depth \( d \), these entangling effects are enhanced. Other parameters are the number of atoms \( N_\uparrow \) and \( N_\downarrow \) in each of the two levels; and the normalized population \( P_2(t) = (N_\uparrow - N_\downarrow)/N_2(t) \) where \( N_2 = N_\uparrow + N_\downarrow \) is the number of atoms in the two-level system.

4.1. Correlation dynamics

MPC derive the following dynamical equations for the evolution of the atomic spin correlations:

\[
\begin{align*}
\frac{d}{dt} \left( \langle J_{A/B}^2 \rangle \right)^2 &= -[\tilde{\Gamma} + d\Gamma P_2(t)] \left( \langle J_{A/B}^2 \rangle \right)^2 + \frac{N}{4} \left[ \tilde{\Gamma} + d\Gamma P_2(t)^2 (\mu^2 + \nu^2) \right], \\
\frac{d}{dt} \left( \langle J_{A/B}^y \rangle \right)^2 &= -[\tilde{\Gamma} + d\Gamma P_2(t)] \left( \langle J_{A/B}^y \rangle \right)^2 + \frac{N}{4} \left[ \tilde{\Gamma} + d\Gamma P_2(t)^2 (\mu^2 + \nu^2) \right], \\
\frac{d}{dt} \langle J_A^x J_B^x \rangle &= -[\tilde{\Gamma} + d\Gamma P_2(t)] \langle J_A^x J_B^x \rangle + \frac{N}{2} \mu \nu d\Gamma P_2(t)^2, \\
\frac{d}{dt} \langle J_A^y J_B^y \rangle &= -[\tilde{\Gamma} + d\Gamma P_2(t)] \langle J_A^y J_B^y \rangle - \frac{N}{2} \mu \nu d\Gamma P_2(t)^2,
\end{align*}
\]

where \( \tilde{\Gamma} = \Gamma_{\text{cool}} + \Gamma_{\text{heat}} + \Gamma_d \), \( \Gamma_{\text{cool}}(\Gamma_{\text{heat}}) \) is the total single-particle cooling (heating) rate and \( \Gamma_d \) is the total dephasing rate. The entangling terms of equation (11) arise as those being
proportional to $d$ and will drive the system into an entangled EPR state. From this set of equations, in order to evaluate the predictions for the entanglement parameter $\Delta_{\text{ent}}$, MPC derive an equation describing the evolution of the variances $\Delta^2(J_{A}^{Y/Z} \pm J_{B}^{Y/Z})$. The results of solving these equations in the steady-state are given in figure 2, in the first panel, which plots the steady-state entanglement $\Delta_{\text{ent}} = \xi_{\infty}$. This is clearly adequate for demonstrating entanglement, since variance sums of this type are an entanglement witness, as explained in the previous sections. These types of correlations were measured in the original experiment, as illustrated in figure 1.

However, as we explain in greater detail below, these variance sums are not sufficient to demonstrate the stronger EPR requirement that we are interested in here. It has been shown that variance sums can be used as a witness, but only provided correlations are strong enough and causality requirements are satisfied [9]. The problem is that under the conditions of these experiments, with relatively imperfect correlations, variance sums are not an efficient witness. In simple terms, the use of this type of witness does not lead to evidence for an EPR paradox. The observed correlation strength in these experiments is not strong enough.

For the purpose of this paper, it is better, instead, to directly investigate the conditional variances which are at the heart of the inferred Heisenberg inequality.

4.2. Conditional variance calculation

To obtain the optimal prediction for the EPR paradox we need to modify the MPC analysis, to calculate the evolution of the conditional variances, $\Delta_{\text{inf}}^2(J_{A}^{Y}) = \Delta^2(J_{A}^{Y} \pm g Y J_{B}^{Y})$ and $\Delta_{\text{inf}}^2(J_{A}^{Z}) = \Delta^2(J_{A}^{Z} \pm g Z J_{B}^{Z})$, as given by equation (4). These EPR variances can be measured by the arrangement of figure 1(b). The equations are

\[
\frac{d}{dt} \Delta_{\text{inf}}^2(J_{A}^{Y}) = -[\tilde{\Gamma} + d\Gamma P_{2}(t)]\left\{ \Delta_{\text{inf}}^2(J_{A}^{Y}) \right\} + \frac{N(1 + g_{Y}^{2})}{4}[\tilde{\Gamma} + d\Gamma t^{2}(\mu^{2} + \nu^{2})]
\]

\[
= \pm N g_{Y} \mu \nu d\Gamma P_{2}(t),
\]

\[
\frac{d}{dt} \Delta_{\text{inf}}^2(J_{A}^{Z}) = -[\tilde{\Gamma} + d\Gamma P_{2}(t)]\left\{ \Delta_{\text{inf}}^2(J_{A}^{Z}) \right\} + \frac{N(1 + g_{Z}^{2})}{4}[\tilde{\Gamma} + d\Gamma P_{2}(t)^{2}(\mu^{2} + \nu^{2})]
\]

\[
= \pm N g_{Z} \mu \nu d\Gamma P_{2}(t).\]

The steady state solutions are given by

\[
\Delta_{\text{inf}}^2(J_{A}^{Y})_{\infty} = \frac{A_{Y}[\tilde{\Gamma} + d\Gamma P_{2,\infty}^{2}(\mu^{2} + \nu^{2})]}{[\tilde{\Gamma} + d\Gamma P_{2,\infty}]} = \pm N g_{Y} \mu \nu d\Gamma P_{2,\infty}^{2},
\]

\[
\Delta_{\text{inf}}^2(J_{A}^{Z})_{\infty} = \frac{A_{Z}[\tilde{\Gamma} + d\Gamma P_{2,\infty}^{2}(\mu^{2} + \nu^{2})]}{[\tilde{\Gamma} + d\Gamma P_{2,\infty}]} = \pm N g_{Y} \mu \nu d\Gamma P_{2,\infty}^{2},
\]

where $A_{Y} = N(1 + g_{Y}^{2})/4$ and $A_{Z} = N(1 + g_{Z}^{2})/4$, $P_{2,\infty} = \lim_{t\to\infty} P_{2}(t)$ and we have chosen the gains such that

\[
g_{Y} = -g_{Z} = \frac{\pm \mu \nu d\Gamma P_{2,\infty}}{[\tilde{\Gamma} + d\Gamma P_{2,\infty}^{2}(\mu^{2} + \nu^{2})]/2}
\]
Figure 2. Realizing a stable steady-state EPR paradox using the engineered dissipatively driven system of Muschik et al [73]. (a) The predictions for the steady-state entanglement $\Delta_{\text{ent}} = \xi_\infty$ versus $Z = (\mu - v)^{-1}$ for an optical depth $d = 30$ per ensemble. We use the parameters presented in [73]. $\xi_\infty < 1$ indicates entanglement of the two ensembles, and the observation of $\Delta_{\text{ent}} = \xi_\infty < 0.5$ would imply the correlations of an EPR paradox. (b) The predictions for the EPR paradox parameter (16) are given using the same parameters as in (a), but with optimal values of gain factor $g_Y$ and $g_Z$. $E_{\text{epr,}\infty} < 1$ indicates an EPR paradox. The inset shows the optimal gain factor $g_Y (-g_Z)$ used to minimize $E_{\text{epr,}\infty}$. Here, $\Gamma_{\text{cool}} = \mu^2 \Gamma$ and $\Gamma_{\text{heat}} = v^2 \Gamma$. The dephasing rate $\Gamma_d = \Gamma_{\text{rad}}^{\text{d}} + \Gamma_{\text{add}}^{\text{d}}$ consists of a radiative part $\Gamma_{\text{rad}}^{\text{d}} = 2(\mu^2 + v^2)\Gamma$ which is due to light-induced transitions, and an additional term $\Gamma_{\text{add}}^{\text{d}}$ which summarizes all nonradiative sources of dephasing. The violet line is for pure radiative damping, $\Gamma_{\text{add}}^{\text{d}} = 0$. $\Gamma$ is the single particle decay rate.
to minimize the conditional EPR variances. The steady state solution for $|\langle J_{A,B}^X \rangle|$ is given by Muschik et al [73], as

$$|\langle J_{A,B}^X \rangle|_\infty = \frac{N}{2} P_{2,\infty}. \quad (14)$$

These inferred variances correspond to quantities that would be measurable in the second type of experiment illustrated in figure 1. So far, this type of experiment with local measurements at each site has not been carried out. The important issue is to be able to measure the spin of each atomic ensemble individually, in order to calculate the inferred or conditional variance.

4.3. EPR paradox predictions

We can define from the EPR paradox condition (3) the normalized EPR paradox parameter

$$E_{epr}(A|B) = \frac{\Delta_{inf}(J_A^Z) \Delta_{inf}^2(J_A^X)}{\frac{1}{2} |\langle J_A^X \rangle|}$$

that gives an indication of the amount of EPR paradox. The EPR paradox is obtained when $E_{epr}(A|B) < 1$ and is strongest as $E_{epr}(A|B) \to 0$ [9]. We note the asymmetry of this definition, with relation to the subsystems $A$ and $B$.

An EPR paradox is also obtained when $E_{epr}(B|A) < 1$. Either condition ($E_{epr}(A|B) < 1$ or $E_{epr}(B|A) < 1$) is sufficient to demonstrate the paradox, and for asymmetric systems, where the parameters are not equal, this fact can become important and interesting [74–77]. Recent work identifies $E_{epr}(A|B) < 1$ as a criterion to verify an ‘EPR steering’ of Alice’s system $A$, by Bob’s measurements on system $B$ [17–19]. The asymmetry of definition is inherent in the original argument of EPR.

The steady state solution for the EPR parameter is given as

$$E_{epr,\infty} = \frac{\Delta_{inf}(J_A^Z) \Delta_{inf}^2(J_A^X)}{\frac{1}{2} |\langle J_A^X \rangle|_\infty} \quad (16)$$

and is plotted in the figure 2. Entanglement is verified if $\Delta_{g,ent} < 1$ where

$$\Delta_{g,ent} = \xi_{g,\infty} = \frac{\Delta_{inf}(J_A^Z) |\langle J_A^X \rangle|_\infty + \Delta_{inf}^2(J_A^X) |\langle J_A^X \rangle|_\infty + |g_y g_z |\langle J_B^X \rangle|_\infty} / 2 \quad (17)$$

and we introduce the notation $\Delta_{g,ent}$ to remind us that there is a dependence on the experimental parameters $g_y$ and $g_Z$ [78–80]. These criteria are more general than the symmetric criterion (7) of Duan et al [67]. We note that $\Delta_{ent} = \Delta_{g,ent}$ when $g_y = -g_Z = 1$. We introduce the notation $\xi_{\infty}$ used by Muschik et al: $\xi_{g,\infty}$ is the value of $\Delta_{g,ent}$ in the steady state, and $\epsilon_{\infty}$ is the steady state value of $\Delta_{ent}$.

An important point is that the choice for $g_X$ and $g_Y$ to minimize $E_{epr}$ is not the same choice that will minimize the entanglement parameter $\Delta_{g,ent}$. In fact, for ensembles symmetric under interchange $A \leftrightarrow B$, it is possible to show that $\Delta_{g,ent}$ is minimized by $g_y = -g_Z = 1$. In this case, $\Delta_{g,ent} = \Delta_{ent}$. The steady state value of the entanglement parameter $\Delta_{ent}$ denoted $\xi_{\infty}$ is plotted in figure 2(a), in agreement with Muschik et al [73]. As explained in section 3, the observation of $\Delta_{ent} = \xi_{\infty} < 0.5$ would imply the correlations of an EPR paradox. However, we see from the figure 2(a) that this cannot be achieved, for the parameter range chosen.
We plot the prediction for the EPR paradox in figure 2(b), using the optimal arrangement of figure 1(b), where the gain factors $g_Y$ and $g_Z$ are selected as in (13) different to 1. We use the same parameters as for figure 2(a), that are selected by Muschik et al [73] to model the ensembles realistically in the experimental regime. Values of $E_{\text{epr,} \infty} \sim 0.9$ are predicted for mainly pure radiative damping. In the same regime, values of $\Delta_{\text{ent}} \sim 0.6$ for the steady state entanglement are predicted. The EPR paradox parameter is more sensitive to dephasing than is entanglement. Given that the experiment has achieved a steady state entanglement of $\Delta_{\text{ent}} \sim 0.9$ however, a realization of an EPR paradox would seem feasible.

5. Discussion

The evidence for quantum nonlocality becomes compelling if the entangled systems are separated to the extent that the measurements made on the atomic ensembles by the verifying pulses are space-like separated events [24]. This consolidates the locality premise because then one can rule out a causal influence on the system $A$ due to measurements at $B$. The measurement events are made in the time taken for the verifying pulse to interact with the individual ensemble. We denote this time as $\Delta T$. Based on the description of the experiment of [60, 61], for that case, $\Delta T \sim 0.45 \text{ ms}$.

The entangling pulse (or engineered entanglement mechanism, as discussed above) must in the first instance be able to propagate through both ensembles to create the entanglement. It is also necessary that the entanglement lifetime exceed the measurement time, a condition that is clearly satisfied for the experiments of [61] which report entanglement times of up to 0.04 s.

The separation between the ensembles must then be at least $D > c\Delta T$, which gives a requirement of much larger than room-size separations for current experiments. We therefore propose that the set-up of [60, 61], which is spatially separated but not strictly causal, would be a first step in which one obtains correlations of enough strength to guarantee the EPR paradox. A necessary subsequent experiment using pulsed entanglement with causal separation would be needed for a full EPR paradox demonstration. We note that pulsed local oscillator measurements provide well-defined properties analogous to the quadrature variables defined here [81]. Such quadrature measurements have been carried out previously for purely optical systems [82]. They are not impossible in a future, fully causal experiment on correlated atomic ensembles.

A second potential criticism of the proposed experiment is the nature by which the macroscopic atomic spin of the atomic ensembles are measured. The value for the spin is inferred via the relationship given by equations (10), based on the measurement of the Stokes field observables. The validity of the measurement then depends on the correctness of the quantum based equations, which involves the question of Gaussian mode-coupling constants [83] and the corresponding noise calibration issues. In this respect, the proposals such as given in [29, 40] to realize an EPR paradox based on the spin observables of two groups of atoms of a BEC, by using four-wave mixing or molecular dissociation, provide an important alternative. Here, each group of atoms can be constrained spatially by the potential well of an optical lattice. While the spatial separations are therefore limited, the advantage is that atomic populations is measured directly by atomic imaging, which simplifies calibration issues. Other advantages that exist include the possibility of longer decoherence times [84] and reduced atomic dephasing which the theory summarized in section 4 indicates is detrimental to the EPR correlations.
Perhaps the most fascinating feature of the current proposal is the macroscopic nature of the EPR correlations. The EPR observables are the Schwinger atomic spins, which correspond to the difference in the numbers of atoms populating two specified atomic states. For these experiments, the total atom numbers are large ($N \sim 10^{13}$ atoms). The atom number differences do not need to be measured microscopically—that is, with a microscopic precision where single atoms are distinguished—in order to attain the EPR paradox. The reason for this is understood by examining the EPR paradox condition (3), and noticing that the right side depends on the total mean spin $\langle J_A^X \rangle$, which is given as $\langle J^X \rangle \sim N$ [85]. The quantum noise level of the uncertainty relation becomes a macroscopically measurable quantity, thus enabling the paradox to be realized based on the ratio (15) only.

6. Conclusion

In summary, we have examined the possibility of detecting an EPR paradox between two macroscopic atomic ensembles at room temperature, based on the experiments that have realized an entanglement between the ensembles. Although the realization of an EPR paradox is more difficult than for entanglement, detailed models that account for decoherence effects allow prediction of the EPR paradox, for room temperature atoms. We have shown that the measurement scheme must be modified, to enable local measurements on each ensemble to be performed. The proposed experiment would be a convincing demonstration of quantum nonlocality in the form of the EPR paradox and quantum steering for truly macroscopic objects.

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