ON SUPERFLUID PHASES OF COLD DECONFINED QCD MATTER AT MODERATE BARYON DENSITY

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Abstract

We present an overview of the arguments which lead to the picture that at low temperatures the QCD matter not far above a critical confinement-deconfinement density exists in one of several distinct superfluid phases exhibiting the quantum behavior on macroscopic scales.
I. INTRODUCTION

In QCD we trust [1]. Consequently, not far above a critical confinement-deconfinement baryon density $n_c \sim 5n_{nuc.matter}$ and at low temperatures $T$ the deconfined QCD matter should be a rather strongly interacting quantum many-colored-quark system. Its detailed actual behavior in the considered region of the QCD phase diagram depends solely upon the details of the effective interactions relevant there.

It is natural to assume that the strong gluon interactions dress the tiny quark masses $m_u$ and $m_d$ (we restrict our discussion to the case of two light flavors) into a common larger effective mass $m_*$, and become weak. Residual interaction between the effective massive (quasi)quark excitations $\psi^a_{\alpha A}$ ($a$-color, $\alpha$-Dirac, $A$-flavor $SU(2)$ indices) can bona fide be described by appropriate short-range (approximately contact) four-fermion interactions $\mathcal{L}_{int}$. Both $m_*$ and $\mathcal{L}_{int}$ are to be fixed experimentally. Some parts of $\mathcal{L}_{int}$ have, however, a solid theoretical justification: (i) The instanton-mediated interaction of t’Hooft $^2$

\[ \mathcal{L}^I = K_I [ (\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2 - (\bar{\psi}\vec{\tau}\psi)^2 - (\bar{\psi}i\gamma_5\psi)^2] \]  

(1)

(ii) The Debye-screened chromoelectric one-gluon exchange interaction

\[ \mathcal{L}^D = K_D (\bar{\psi}\gamma_0 \frac{1}{2}\lambda_\alpha \psi)^2, \]  

(2)

The resulting effective Lagrangian

\[ \mathcal{L}_{eff} = \bar{\psi}(i\gamma^\mu D_\mu - m_* + \mu\gamma_0)\psi - \frac{1}{4} F_{a\mu\nu}F^{a\mu\nu} + \mathcal{L}_{int} \]  

(3)

in which the gluon interactions are treated perturbatively (and neglected in the lowest approximation) defines a relativistic version of the Landau Fermi-liquid concept.

By assumption, $\mathcal{L}_{eff}$ is exactly $SU(3)_c \times SU(2)_I \times U(1)_V \times O(3)$ invariant. There is no approximate chiral $SU(2)$ symmetry of (3) which could be broken spontaneously. Hence, there should be no light Nambu-Goldstone (NG) pions in dense and cold deconfined phase(s) of QCD. It would be also misleading to think of $\psi$ and $m_*$ as of the constituent quark and of the constituent mass. There is nothing they might constitute.

At present, there are no experimental data, either real or the lattice ones which would check our assumption, although they are both heavily needed. For our at best semi-quantitative considerations the assumption is not, however, essential. An alternative picture is that in the cold deconfined QCD matter the $u, d$ quarks stay approximately massless at the Lagrangian level as they were in the confined phase. Discussion of the superfluid phases presented below applies also to this case. On top of that it is, however, mandatory to ask (and answer) how the (approximate) chiral symmetry is realized in this case.

The cold and dense deconfined QCD matter should exist in the interiors of the neutron stars, and optimistically also in the early stages of the relativistic heavy-ion collisions studied...
experimentally with much effort at present. In the following we will present arguments that the matter governed by (3) should exist below certain critical temperature \( T_c \) of the order of 100 MeV in some of several distinct superfluid phases exhibiting the quantum behavior on macroscopic scales.

II. COOPER INSTABILITY

When scaling the fermion momenta in dense, low-T quantum many-fermion systems, both non-relativistic [4] and relativistic [5], towards the Fermi surface, all interactions but one become irrelevant. This implies that such systems should behave thermodynamically as a corresponding noninteracting gas of fermions with the effective mass. For example, the specific heat of such systems should grow linearly with temperature. Such a behavior is indeed observed in the non-relativistic low-\( T \) electron gas in metals, and in the liquid \(^3\)He. We are not aware of any experimental data in the relativistic systems.

The four-fermion interaction attracting fermions with the opposite momenta at the Fermi surface is the only exception: Even if arbitrarily small, it causes the (Cooper) instability of the perturbative ground state with respect to spontaneous condensation of the fermion Cooper pairs with opposite momenta into a more energetically favorable ground state. The new ground state, being by construction and by definition translationally invariant, has in the simplest case of the ordinary Bardeen-Cooper-Schrieffer (BCS) superconductor [3] the property

\[
\langle \psi_\alpha^+(x)(\sigma_2)_{\alpha\beta}\psi_\beta^+(x) \rangle = \Delta \neq 0 \tag{4}
\]

It exhibits clearly the spontaneous breakdown of the \( U(1) \) phase symmetry generated by the operator of the particle number, \( N = \int d^3x \psi_\alpha^+ \psi_\alpha \).

In the pre-BCS era the properties of superconductors were successfully described by beautiful phenomenological theories: (i) The Ginzburg-Landau (GL) theory [3] associated to the ordered phase an order parameter - doubly charged (\( e^* = 2e \), \( m_{\text{eff}} = 2m_\ast \)) complex scalar field developing a nonzero ground-state expectation value, \( \langle \Phi \rangle \neq 0 \). The supercurrent follows easily from the gauge-invariant kinetic term in the free energy

\[
F_{\text{GL}}^{\text{kin}} = \frac{1}{2m_{\text{eff}}}[(-i\nabla + e^* \vec{A})\Phi]^+(-i\nabla + e^* \vec{A})\Phi \tag{5}
\]

A derivation of GL from BCS provided by Gor’kov [3] implies in particular \( \langle \Phi \rangle \sim \Delta \). (ii) Long before the GL theory F. London [6] postulated his famous equations (see Eqs.(7), (8) below) arguing as follows: "... the equations at which we have arrived are distinguished by their simplicity and symmetry in such a way that we could hardly avoid writing them down." Their form follows from the GL theory by simple replacement \( \Phi \rightarrow \langle \Phi \rangle \). Lorentz-invariant quantum-field theory version of the GL theory is notoriously known as the Higgs (H) model.
The BCS ground state acts as the Fock's vacuum of the Bogolubov-Valatin (BV) \[3\] true fermionic quasiparticles having the specific dispersion law

$$E_{\mathbf{k}} = \sqrt{\xi_{\mathbf{k}}^2 + \Delta^2}$$

in which the energy gap $\Delta$ and $\xi_{\mathbf{k}}$ are self-consistently fixed by the interaction. The quantities $\xi_{\mathbf{k}}$ are simply related to the perturbative form of the electron dispersion law $\mathbf{k}^2/2m$. The (super)current $\mathbf{j}_s$ acquires the form "$\mathbf{j}_s = -(e^*^2/m_{eff}) |< \Phi >|^2 \mathbf{A}$" where $\mathbf{A}$ is the gauge potential, in contrast with the normal conductivity current $\mathbf{j}_n = \sigma \mathbf{E}$ ($\sigma$ is the conductivity). The quotation marks abbreviate a shorthand notation for the gauge-invariant London equations

$$\frac{\partial \mathbf{j}_s}{\partial t} = (e^*^2/m_{eff}) |< \Phi >|^2 \mathbf{E}$$  \hspace{1cm} (7)

$$\text{rot} \mathbf{j}_s = -(e^*^2/m_{eff}) |< \Phi >|^2 \mathbf{B}$$  \hspace{1cm} (8)

Below a critical temperature $T_c \sim 1K$ at which the spontaneous condensation of Cooper pairs sets in the quasifermion dispersion law (6) results in an abrupt change of the behavior of the specific heat on $T$ from linear to the exponential one. Another robust manifestation of superconductivity characterized by the condensate $\Delta$ is the Meissner effect: Magnetic field penetrates into the superconductor only within a London penetration length $\lambda_L = (m_{eff}/e^*^2) |< \Phi >|^2 1/2$. Operationally this phenomenon follows immediately from the Maxwell equation $\text{rot} \mathbf{B} = \mathbf{j}_s$ combined with Eq.(8): $(\nabla^2 - \lambda_L^{-2}) \mathbf{B} = 0$. The equation (7) accounts for perfect conductivity i.e., for stationary electric currents in absence of electric fields.

Analogous, but more sophisticated phenomena take place below $1mK$ in electrically neutral Landau Fermi liquid of $^3He$ \[7\]. We do not sharply distinguish between superconductivity and superfluidity in many-fermion systems. The superconductor is a superfluid having the perturbative long-range gauge interactions switched on. Superfluidity in many-boson systems (for example in $^4He$) is, however, an entirely different story.

### III. ISOTROPIC COLOR-TRIPLET SUPERCONDUCTOR

In the cold and dense deconfined QCD matter governed by (3) the situation is physically very much the same as that in an electron Fermi liquid. The differences are rather technical: (i) Characteristic energies given by the chemical potential of the order of hundreds of $MeV$ require the relativistic description. (ii) The quarks carry, besides spin, also the flavor and the color. Consequently, in comparison with (4) there are more possibilities for the local superfluid condensates in accord with the Pauli principle. (iii) The gauge fields are both
Abelian (photon) and non-Abelian (gluons). (iv) The origin of the effective interactions is different.

The most energetically favorable superfluid phase is for the realistic interactions $\mathcal{L}_{\text{int}}$ characterized by the condensate

$$\langle \bar{\psi}_{\alpha a} A(x) \epsilon^{\alpha \beta} (\tau_2)_{AB} (\gamma_5 C)_{\alpha \beta} \bar{\psi}_{\beta b} B(x) \rangle = \Delta \quad (9)$$

In a self-explanatory notation $C$ is the matrix of charge conjugation. The condensate $\Delta$ corresponds to the ground-state expectation value $\langle \Phi^3 \rangle$ of a GLH order parameter $\Phi^c(x)$ which transforms as a color triplet, isospin singlet, spin zero complex field. Properties of this phase (and of its 3-flavor relative) are elaborated in the literature in most detail with several different interactions taken into account. Of interest are in particular: (i) form of the quasiquark dispersion law; (ii) properties of the NG excitations which must be present due to the fact that the ground state in (9) breaks the global $SU(3)_c$ down to $SU(2)_c$; (iii) the fate of gluons if their interaction is switched on.

(i) It follows from (9) that only the quarks of two colors participate in Cooper pairing, while the third one remains intact. Consequently, its dispersion law remains $\epsilon_{\vec{p}}^2 = (|\vec{p}| \pm \mu)^2 + m^2_\ast$. The form of the dispersion law of the BV quasiquarks is

$$E(\vec{p})^2 = (\epsilon(\vec{p}) \pm \mu)^2 + |\Delta|^2 \quad (10)$$

where $\epsilon(\vec{p}) = \sqrt{\vec{p}^2 + m^2_\ast}$, and $\Delta$ is the energy gap fixed by the ”gap equation”

$$\Delta = \Delta K \int \frac{d^3p}{(2\pi)^3} \left[ \frac{1}{\sqrt{(\epsilon(\vec{p}) + \mu)^2 + |\Delta|^2}} + \frac{1}{\sqrt{(\epsilon(\vec{p}) - \mu)^2 + |\Delta|^2}} \right] \quad (11)$$

In Eq.(11) $K$ is proportional to the coupling constant of the four-fermion interaction responsible for pairing. The UV divergence of the integral is eliminated by introducing a formfactor which mimics the asymptotic freedom. This phase is certainly interesting from the phenomenological point of view: In the alternative scenario the instanton- mediated interaction (II) gives rise simultaneously to the numerically acceptable spontaneous chiral symmetry breakdown, and to the phenomenologically interesting gaps $\Delta$ of the order of 100 MeV. The corresponding critical temperature is given by the universal BCS formula $\Delta/k_BT_c = \pi \exp(-\gamma)$, where $\gamma \approx 0.5772$ is the Euler’s constant.

(ii) It also follows from (9) that this condensate breaks down spontaneously the $SU(3)_c \times U(1)_V$ global symmetry of the Lagrangian (II) down to $SU(2)_c$ (the gauge interactions switched off). According to the Goldstone theorem there must exist 5+1 gapless collective excitations in the spectrum. Clearly, they can only be excited by the quark bilinears. Their quantum numbers can be found with the help of the Goldstone commutator $Q$ abbreviates the corresponding generators

$$[Q, \bar{\psi} \cdots \bar{\psi}] = \bar{\psi}_a \epsilon^{\alpha \beta \gamma} \gamma_5 C \psi_b \quad (12)$$
Not surprisingly, the NG quark bilinear combinations have the form ($A = 2, 5, 7, \epsilon^{ab3} = i(\lambda_2)^{ab}$)

\[ \bar{\psi} \lambda_A \tau_2 \gamma_5 C \bar{\psi} \pm H.c. \] (13)

(iii) If the gauge fields are switched on, and if they couple via the quasiquark loops to the NG bosons, the originally massless gauge bosons acquire masses by the Schwinger-Anderson mechanism. Corresponding calculations are rather involved, and they were done only recently [9]. In the GLH effective-field theory description of this color-superconducting phase the corresponding Higgs effect with a complex color-triplet scalar field is elementary:

The gauge fields $A^a_\mu$ stay massless for $a = 1, 2, 3$, and they acquire masses $m^2_a = g^2 |\Delta|^2$ and $m^2_a = \frac{4}{3} g^2 |\Delta|^2$ for $a = 5, 6, 7$ and $a = 8$, respectively.

IV. THE REMAINING THREE CONDENSATES

Phase II is characterized by the condensate

\[ \langle \bar{\psi}_{aaA}(x)(\tau_3 \tau_2)_{AB}(\gamma_5 C)_{\alpha\beta} \bar{\psi}_{\beta bB}(x) \rangle = \Delta_a \delta_{ab} \] (14)

As a previous one, also the condensate (14) characterizes a relativistic isotropic superfluid: in both cases the quark bilinears look like the Lorentz scalars. Equation (14) corresponds to the ground-state expectation value of a GLH order parameter $\Phi_{Iab}(x)$ which transforms as a color sextet, isospin triplet, spin zero complex field.

It is clearly possible to invent such an effective interaction which favors namely this phase. In real QCD nobody knows. (This statement does not apply to the case of very high densities where the one-gluon exchange dominates, and its color-sextet channel is known to be repulsive.) To the best of our knowledge the details of this phase were not elaborated, although the generic form of its excitations it is easy to guess. The phase is interesting mainly by spontaneous breakdown of the isospin symmetry. Since there is nobody who might "eat" them, the two corresponding gapless collective NG excitations remain in the physical spectrum, and become thermodynamically important.

Phase III is characterized by the condensate

\[ \langle \bar{\psi}_{aaA}(x)(\tau_2)_{AB}(\gamma_0 \gamma_3 C)_{\alpha\beta} \bar{\psi}_{\beta bB}(x) \rangle = \Delta_a \delta_{ab} \] (15)

It corresponds to the ground-state expectation value of a GLH order parameter $\Phi_{ab\mu
u}(x)$ which transforms as a color sextet, isospin singlet, antisymmetric tensor field describing spin 1.

Phase IV is characterized by the condensate
\[ \langle \psi_{aaA}(x) \rangle = \Delta \] (16)

In its GLH description the order parameter \( \Phi_{\mu \nu}(x) \) transforms as a color triplet, isospin triplet, antisymmetric tensor field describing spin 1. Remark: We speak of the order parameters as transforming as the Lorentz scalars or antisymmetric tensors, respectively. It is, however, tacitly assumed that the derivative terms in their corresponding GLH Lagrangians are only \( O(3) \) invariant.

Phases III and IV are the anisotropic color superconductors, and they are most probably interesting primarily from the theoretical point of view. They exhibit spontaneous breakdown of the rotational symmetry like ferromagnets. In relativistic systems this is certainly not a very frequent phenomenon. It is possible only at finite density which itself breaks explicitly the Lorentz invariance. The less symmetry the more fragile the ordered phase is expected to be. Therefore, these phases should have very low \( T_c \). For an illustration we present just the form of the BV quasiquark dispersion laws \( (i = 1,2) \) exhibiting nicely spontaneous breakdown of the rotational symmetry \( \Pi \):

\[ E_i^2(\vec{p}) = \epsilon^2(\vec{p}) + |\Delta|^2 + \mu^2 \pm 2\sqrt{\epsilon^2(\vec{p})\mu^2 + (p_1^2 + p_2^2 + m_i^2)|\Delta|^2} \] (17)

V. CONCLUSIONS

During the past two years so many interesting features of the color superconductors were pointed out and elaborated on that it is impossible to even mention them all. With apologies to those omitted in conclusion I consider appropriate to discuss briefly two beautiful theoretical ideas:

(i) Color-flavor locking (CFL) \( \Pi \). Variational analysis of a model with \( SU(3)_c \) and exact global \( SU(3) \) chiral symmetry reveals the following pattern of spontaneous symmetry breaking: There is a superfluid condensate corresponding to the lowest energy which 'locks' three groups \( SU(3) \) i.e., c, L, R together in such a way that only their diagonal \( SU(3) \) subgroup \( (c + L + R) \) remains unbroken. This is interesting by itself for there is a spontaneous chiral symmetry breakdown without \( \langle \bar{\psi} \psi \rangle \) condensate. Properties of the low-energy excitations in this deconfined phase are quite peculiar for they closely resemble hadrons of the confined phase: (1) There are 8+1 baryons excited by the quasiquark fields, all having the chiral symmetry breaking gap in their spectrum, and the integer charges. (2) There are 8+1 massive vector particles excited by the gauge fields, all having integer charges. (3) There are 8+1 massless pseudoscalar collective excitations with integer charges. (4) The electric charge corresponds to a particular linear combination of the diagonal \( SU(3) \) generators in flavor and color spaces.

(ii) There is a new type of solution of the gap equation in the case of very high densities. In the standard case of moderate densities considered in Sect.3 the generic form of the
dependence of $\Delta$ on the effective coupling constant $K$ which is related to the gauge coupling constant $g$ as $K = g^2/\Lambda^2$ is

$$\Delta \sim \mu \exp\left(-\frac{3\Lambda^2 \pi^2}{2g^2 \mu^2}\right)$$  \hspace{1cm} (18)

This is the characteristic BCS result.

As pointed out by Son [12] and confirmed by others the case of very high densities is different. The exchange of unscreened magnetic gluons yields the dependence of $\Delta$ upon the small coupling constant $g$ in the form

$$\Delta \sim \mu g^{-5} \exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right)$$  \hspace{1cm} (19)

This effect can provide numerically larger gaps than in previous estimates $\Delta \sim 100MeV$ based on (13).

It is gratifying to observe that our beloved QCD has a corner in its phase diagram full of new phenomena associated with the existence of ordered quantum phases of the superfluid type. Moreover, there are good reasons to expect that these phases can be described theoretically from the first principles.

Let us assume optimistically that some sort of color superconductive or superfluidity does exist in the interiors of the neutron stars, and also in some events in the relativistic heavy-ion collisions. Due to the macroscopic quantum nature of these phases it is perhaps justified to speculate that their experimental signatures might be brighter than those of the 'ordinary' quark-gluon plasma above a superfluid $T_c$.

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