Probing Spectator Scattering and Annihilation Corrections in $B_s \to PV$ Decays

Qin Chang,¹,² Xiaohui Hu,¹ Junfeng Sun,¹ and Yueling Yang¹

¹Institute of Particle and Nuclear Physics, Henan Normal University, Xinxiang 453007, China
²State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing, China

Abstract

Motivated by the recent LHCb measurements on $B_s \to \pi^- K^{*+}$ and $B_s \to K^\pm K^{*\mp}$ decay modes, we revisit the $B_s \to PV$ decays within QCD factorization framework. The effects of hard-spectator scattering and annihilation corrections are studied in detail. After performing a $\chi^2$-fit on the end-point parameters $X_{A}^{i,j}$ ($p_{A}^{i,j}$, $\phi_{A}^{i,j}$) and $X_{H}$ ($\rho_{H}$, $\phi_{H}$) with available data, it is found that although some possible mismatches exist, the universalities of $X_{A}^{i,j}$ and $X_{H}$ in $B_s$ and $B_{u,d}$ systems are still allowed within theoretical uncertainties and experimental errors. With the end-point parameters gotten from $B_{u,d} \to PV$ decays, the numerical results and detailed analyses for the observables of $B_s \to \pi K^*$, $\rho K$, $\pi \rho$, $\pi \phi$ and $K \phi$ decay modes are presented. In addition, we have identified a few useful observables, especially the ones of $B_s \to \pi^0 \phi$ decay for instance, for probing hard-spectator scattering and annihilation contributions.

PACS numbers: 12.39.St 13.25.Hw 14.40.Nd
I. INTRODUCTION

In the past years, many experimental efforts have devoted to precisely measuring non-leptonic two-body B meson decays, which provide a fertile ground to explore the underlying mechanism of hadron weak decays. For instance, many $B_{u,d}$ decay modes with branching fractions $\gtrsim \mathcal{O}(10^{-6})$ are well measured by the B factories BABAR and Belle, and branching ratios of some $B_s$ decay modes are given with relatively high precision by CDF collaboration at Fermilab. Moreover, with the running of the Large Hadron Collider (LHC) and the upgrading Super-B factory Belle-II, not only the measurements on $B_{u,d}$ decays will be greatly refined, but also the $B_s$ decays are expected to be measured with extraordinary precision in the near future. Recently, using the data sample corresponding to $1.0 fb^{-1}$ of $pp$ collision, the $\bar{B}_s^0 \to K^{\pm} K^{*\mp}$ and $\pi^- K^{*+}$ decays are firstly observed by LHCb collaboration, and their branching fractions are \[1\]

$$
\mathcal{B}(\bar{B}_s^0 \to K^{\pm} K^{*\mp}) = (12.7 \pm 1.9 \pm 1.9) \times 10^{-6}, \quad (1) \\
\mathcal{B}(\bar{B}_s^0 \to \pi^- K^{*+}) = (3.3 \pm 1.1 \pm 0.5) \times 10^{-6}, \quad (2)
$$

with the total significances of $7.8\sigma$ and $3.4\sigma$, respectively. They are the first measurements of $\bar{B}_s^0$ meson decays to charmless $PV$ final states, where $P$ and $V$ denote the lightest pseudoscalar and vector $SU(3)$ meson, respectively.

Theoretically, based on the QCD factorization (QCDF) approach \[2\], the perturbative QCD (pQCD) approach \[3\] and the soft-collinear effective theory (SCET) \[4\], the latest predictions of such two quantities read (in the units of $10^{-6})^1$

\[
\begin{align*}
\text{QCDF} \ [5] & \quad \left\{ 
\begin{array}{l}
\mathcal{B}(\bar{B}_s^0 \to K^{\pm} K^{*\mp}) = (10.3^{+3.0+4.8}_{-2.2-4.2} + 11.3^{+7.0+8.1}_{-3.5-5.1}), \\
\mathcal{B}(\bar{B}_s^0 \to \pi^- K^{*+}) = 7.8^{+0.4+0.5}_{-0.7-0.7};
\end{array}
\right. \\
\text{pQCD} \ [6] & \quad \left\{ 
\begin{array}{l}
\mathcal{B}(\bar{B}_s^0 \to K^{\pm} K^{*\mp}) = (6.0^{+1.7+1.7+0.7}_{-1.5-1.2-0.3} + 4.7^{+1.1+2.5+0.0}_{-0.8-1.4-0.0}), \\
\mathcal{B}(\bar{B}_s^0 \to \pi^- K^{*+}) = 7.6^{+2.9+0.4+0.5}_{-2.2-0.5-0.3};
\end{array}
\right. \\
\text{SCET} \ [7] & \quad \left\{ 
\begin{array}{l}
\mathcal{B}(\bar{B}_s^0 \to K^{\pm} K^{*\mp}) = 18.2^{+6.3+3.3}_{-5.0-2.7}, \quad 19.7^{+5.0+2.6}_{-4.2-2.2}, \\
\mathcal{B}(\bar{B}_s^0 \to \pi^- K^{*+}) = 5.9^{+0.5+0.5}_{-0.5-0.5}, \quad 6.6^{+0.2+0.7}_{-0.1-0.7}.
\end{array}
\right.
\end{align*}
\]

\[1\] In Eqs. \[3\] and \[4\], the numerical results of $\mathcal{B}(\bar{B}_s^0 \to K^{\pm} K^{*\mp})$ mean $\mathcal{B}(\bar{B}_s^0 \to K^{*-} K^+) + \mathcal{B}(\bar{B}_s^0 \to K^{*+} K^-) [5, 6]$. 

2
It is obvious that (1) For the branching ratio $B(\bar{B}_s^0 \to \pi^- K^{*+})$, all of the theoretical results agree with each other, but are larger than the LHCb data. (2) For the observable $B(\bar{B}_s^0 \to K^\pm K^{*\mp})$, the pQCD’s predictions are consistent with the LHCb’s measurement within errors, while the default results of QCDF and SCET are about $3.3\sigma$ and $2.6\sigma$ larger than LHCb data, respectively. Here, it should be noted that the QCDF’s results in Eq.(3) correspond to a set of specified phenomenological annihilation parameters $(\rho_A, \phi_A) = (0.90, -65^\circ)$ for $B_s \to VP$ decays and $(0.85, -30^\circ)$ for $B_s \to PV$ decays [5]. It is well known that for the $\bar{B}_s^0 \to K^\pm K^{*\mp}$ decay, the tree amplitudes are suppressed by the CKM factor $|V_{ub}V_{us}^*| \sim \mathcal{O}(\lambda^4)$, while the penguin amplitudes proportional to the CKM factor $|V_{tb}V_{ts}^*| \sim \mathcal{O}(\lambda^2)$ are usually sensitive to nonfactorizable corrections and effects from new interactions beyond the standard model. It will be very interesting to scrutinize the above possible mismatches. Before searching for possible solutions from new physics, it is essential to examine carefully whether the disagreement could be accommodated within the standard model. In this paper, the effects of weak annihilation (WA) and hard spectator scattering (HSS) corrections in $\bar{B}_s^0 \to PV$ decays are studied in detail within the QCDF framework.

Our paper is organized as follows. After a brief review of WA and HSS corrections in QCDF approach and recent researches in section II, we present our numerical analyses and discussions in section III. Our main conclusions are summarized in section IV.

II. HSS AND WA CORRECTIONS IN THE QCDF APPROACH

Combining the hard-scattering approach [8] with the power counting rules in the heavy quark limit $m_b \gg \Lambda_{QCD}$, M. Beneke et al. developed the QCDF approach to deal with the hadronic matrix elements for the $B$ meson decays based on the collinear factorization scheme and colour transparency hypothesis [2, 9]. Up to power corrections of order $\Lambda_{QCD}/m_b$, the factorization formula for $B$ decaying into two light meson can be written as

$$\langle M_1M_2|O_i|B \rangle = \sum_j \left\{ F_{j \to M_1} \int dz T_{ij}(z)\Phi_{M_2}(z) + (M_1 \leftrightarrow M_2) \right\} + \int dx dy dz T^{II}_{ij}(x,y,z)\Phi_B(x)\Phi_{M_1}(y)\Phi_{M_2}(z),$$  \hspace{1cm} (6)$$

where $F_{j \to M}$ denotes a $B \to M$ transition form factor, and $\Phi_X(z)$ is the light-cone wave functions for the two-particle Fock state of the participating meson $X$. Form factors and light-cone wave functions in Eq.(6) are nonperturbative inputs. Form factors can be obtained
from lattice QCD or QCD sum rules. Light-cone wave functions are universal and process-
-independent. Both $T^I(z)$ and $T^{II}(x, y, z)$ are hard scattering functions, which could be
systematically calculable order by order with the perturbation theory in principle.

When the chirally enhanced corrections to HHS amplitudes are estimated with the second
line of QCDF formula Eq.(6), the twist-3 contributions involve the logarithmically divergent
integral

$$\int_0^1 \frac{dt}{1-t} \Phi^p_v(t) = X_H,$$

where the asymptotic forms of the twist-3 distribution amplitudes $\Phi^p_v(t) = 1$ are employed.

It is assumed that the endpoint singularity in Eq.(7) contains many poorly known soft contribu-
tions and hence unfortunately does not admit a perturbative treatment within the QCDF
framework. For an estimate, the divergent integral $X_H$ is phenomenologically parameterized
as

$$X_H = \ln \frac{m_B}{\Lambda_h} (1 + \rho_H e^{i\phi_H}),$$

with $\Lambda_h = 0.5$ GeV.

Moreover, it is found that although the WA amplitudes are power suppressed in the
heavy quark limit and hence disappear from Eq.(6), they are very important in practical
application of the QCDF approach to nonleptonic $B$ decays, especially for the pure annihi-
lation processes, such as $B_d \rightarrow K^+K^-$ and $B_s \rightarrow \pi^+\pi^-$ decays which have been observed experimentally [10–12]. The WA amplitudes are expressed in the terms of convolutions
of “hard scattering” kernels with light cone wave functions to estimate the importance of
annihilation contributions within the QCDF framework. A worse problem is that there is
endpoint singularities even with twist-2 light cone distribution amplitudes. Similar to the
case of the twist-3 hard scattering contributions parameterized by $X_H$ in Eq.(7), another
phenomenological quantity $X_A$ are introduced to parameterize the WA divergent endpoint
integrals, i.e.

$$\int_0^1 \frac{dt}{t} = X_A = \ln \frac{m_B}{\Lambda_h} (1 + \rho_A e^{i\phi_A}).$$

One cannot get any information on parameters $\rho_{H,A}$ and $\phi_{H,A}$ from the QCDF approach. Because that the hard scattering terms arise first at order $\alpha_s$ and that the annihilation
contributions are power suppressed, it is conservatively assumed that $\rho_{A,H} \leq 1$ because too
large value will give rise to numerically enhanced subleading contributions and put a question
to the validity of the $1/m_b$ power expansion of the QCDF approach. The strong phases $\phi_{A,H}$
can vary freely, which show the phenomenological importance of HSS and WA contributions to $CP$ asymmetric observables. In addition, different values of $X_{A,H} (\rho_{A,H}, \phi_{A,H})$ according to different types of final states $PP$, $PV$, $VP$ and $VV$ are introduced in phenomenological investigation. And it is traditionally assumed that $X_A = X_H$ and they were universal for each decay types, which have been thoroughly discussed in Refs. [5, 9, 13, 14].

Motivated by recent measurements on pure annihilation $\bar{B}_s^0 \to \pi^+\pi^-$ decay by CDF and LHCb collaborations [11, 12], many more detailed studies on HSS and WA contributions are performed, for instance, by Refs. [15–19], and some interesting findings are presented. In Ref. [18], the universality assumption of WA parameters is carefully tested, and some tensions in $B \to \pi K, \phi K^*$ decay modes with the standard model are presented. In Refs. [19, 20], a “new treatment” is proposed that complex parameters $X_{A}^i$ and $X_{A}^f$ corresponding to WA topologies with gluon emission from initial and final states, respectively, should be treated independently and that the flavor dependence of $X_{A,i}^i$ should be carefully considered. Following such “new treatment” of Refs. [19, 20], with available data of $B_{u,d,s} \to PP$ decays, comprehensive statistical $\chi^2$ analyses on parameters $X_{A,i}^i$ and $X_H$ are performed in our previous works [21, 22]. The findings could be briefly summarized as that: (1) parameters $X_A^i$ and $X_A^f$ should be treated individually, as presented in Ref. [20], and the simplification $X_H = X_A^i$ is allowed by data; (2) The flavor dependences of $X_A^i$ are hardly to be clarified due to large experimental errors and theoretical uncertainties, which implies that the universality of $X_A^i$ in $B_{u,d}$ and $B_s$ systems still persists. So, motivated by the forthcoming plenty data of $B_s$ decays provided by LHCb and Super-B experiments, it is worth to reevaluate the effects of HSS and WA corrections in $B_s \to PV$ decays in detail.

III. NUMERICAL RESULTS AND DISCUSSION

In the QCDF framework, the explicit expressions of the decay amplitudes and relevant formula of $B_s \to PV$ decays have been given in Ref. [9]. In this paper, the $CP$-averaged branching fractions and $CP$ asymmetries of $\bar{B}_s^0 \to \pi K^*, \rho K, \rho \pi, KK^*, \phi\pi$ and $\phi K$ decay modes are evaluated. The same definition for these observables as HFAG [23] are taken. The values of input parameters used in our calculations and analyses are summarized in Table I. In addition, in evaluating the $CP$ asymmetry parameters $C_{f f}$, $S_{f f}$, $\Delta C_{f f}$, $\Delta S_{f f}$ and $A_{CP}^{f f}$ of $\bar{B}_s^0 \to \pi^+\rho^-$, $K^\pm K^{*\mp}$ and $K^0\bar{K}^{*0}(\bar{K}^0K^{*0})$ decays, the conventions $\bar{f} = \pi^+\rho^-$,
$K^+K^{*-}$ and $K^0\bar{K}^{*0}$ are chosen.

TABLE I: The values of input parameters: Wolfenstein parameters, pole and running quark masses, decay constants, form factors and Gegenbauer moments.

| Parameter | Value |
|-----------|-------|
| $\rho$    | $0.1453^{+0.0133}_{-0.0073}$ |
| $\eta$    | $0.343^{+0.011}_{-0.012}$ |
| $\lambda$ | $0.22548^{+0.00068}_{-0.00034}$ |
| $m_c$     | $1.67 \pm 0.07$ GeV |
| $m_b$     | $4.78 \pm 0.06$ GeV |
| $m_t$     | $173.21 \pm 0.87$ GeV |
| $\bar{m}_s(2\text{GeV})$ | $95 \pm 5$ MeV |
| $\bar{m}_b$ | $4.18 \pm 0.03$ GeV |
| $f_{B_s}$ | $(227.6 \pm 5.0)$ MeV |
| $f_{\pi}$ | $(130.41 \pm 0.20)$ MeV |
| $f_K$     | $(156.2 \pm 0.7)$ MeV |
| $f_\rho$  | $(216 \pm 3)$ MeV |
| $f_{K^*}$ | $(220 \pm 5)$ MeV |
| $f_{\phi}$| $(215 \pm 5)$ MeV |
| $A_0^{B_s \to \phi}$ | $0.26 \pm 0.06$ |
| $A_0^{B_s \to K^*}$ | $0.22 \pm 0.06$ |
| $f_{\phi}^{B_s \to K}$ | $0.23 \pm 0.06$ |

$\alpha^2_1 = 0$, $a_2^2(1\text{GeV}) = 0.25$, $a_2^K(1\text{GeV}) = 0.06$, $a_2(1\text{GeV}) = 0.25$,
$\alpha^{||}_{1,\rho} = 0$, $a_{2,\rho}(1\text{GeV}) = 0.15$, $a^{||}_{1,K^*}(1\text{GeV}) = 0.03$, $a^{||}_{2,K^*}(1\text{GeV}) = 0.11$,
$\alpha^{||}_{1,\phi} = 0$, $a_{2,\phi}(1\text{GeV}) = 0.18$.

FIG. 1: The allowed spaces of parameters ($\rho^{i,f}_{A}, \phi^{i,f}_{A}$) at 68% C.L. with the combined constraints from $B(B^0_s \to K^0K^{*-})$ and $B(B^0_s \to \pi^-K^{*-})$ (red points). For comparison, the fitted results for $B_{u,d} \to PV$ decays at 68% C.L. [24] are also shown by blue points.

Firstly, with the simplification $(\rho_h, \phi_h) = (\rho^{i,f}_A, \phi^{i,f}_A)$ and input $\lambda_B = 0.19^{+0.09}_{-0.04}$ GeV favored by $B_{u,d,s} \to PP$ and $B_{u,d} \to PV$ decays [22, 24], we perform a $\chi^2$-fit on the parameters ($\rho^{i,f}_A$, $\phi^{i,f}_A$). The statistical fitting approach has been detailed in Appendix of Ref. [21]. Using the constraints from data on $B(B^0_s \to K^\pm K^{*\mp})$ and $B(B^0_s \to \pi^-K^{*-})$ in Eqs. (1) and (2), the decay mode $B^0_s \to K^0\bar{K}^{*0} + \bar{K}^{*0}K^{*0}$ is labeled as $B^0_s \to K^0\bar{K}^{*0}(\bar{K}^{*0}K^{*0})$ for convenience.

---

2 The decay mode $B^0_s \to K^0\bar{K}^{*0} + \bar{K}^{*0}K^{*0}$ is labeled as $B^0_s \to K^0\bar{K}^{*0}(\bar{K}^{*0}K^{*0})$ for convenience.
allowed spaces of the WA parameters at 68% C.L. are shown in Fig.1. The fitted results for $B_{u,d} \to PV$ decays [24] are also redisplayed for comparison. From Fig.1(a), it is found that the allowed space of $(\rho_A^i, \phi_A^i)$ in $B_{u,d} \to PV$ decays entirely overlaps with the one in $B_s \to PV$ decays, which implies that the same parameters $(\rho_A^i, \phi_A^i)$ for $B_{u,d}$ and $B_s$ decays are still allowed by now. For parameters $(\rho_A^f, \phi_A^f)$ in $B_s \to PV$ decays, the red pointed region in Fig.1(b) shows that the allowed values of $\rho_A^f$ are strongly suppressed around $\phi_A^f \sim 0^\circ$ compared with those with a larger $\phi_A^f$, which is interestingly similar to the situation in $B_{u,d} \to PV$ decays shown by blue points. Unfortunately, due to the large theoretical uncertainties and only few available experimental data with large errors, the spaces of both $(\rho_A^i, \phi_A^i)$ and $(\rho_A^f, \phi_A^f)$ in $B_s$ decays are hardly to be well restricted for now.

To clarify the effects of WA and HSS corrections, the dependences of $\mathcal{B}(\bar{B}_s^0 \to K^\pm K^{*\mp})$ and $\mathcal{B}(\bar{B}_s^0 \to \pi^- K^{*+})$ on parameter $\phi_{H,A}$ are presented in Fig.2. For the tree-dominated $\bar{B}_s^0 \to \pi^- K^{*+}$ decay, its branching fraction is very sensitive to the HSS contributions related to $(\rho_H, \phi_H)$ rather than the WA ones, as Fig.2(a) shows. So, the fitted result (red points) in Fig.1(a) in fact almost refers to the constraints on $(\rho_H, \phi_H)$ from $\mathcal{B}(\bar{B}_s^0 \to \pi^- K^{*+})$. Similarly, for the penguin-dominated $\bar{B}_s^0 \to K^\pm K^{*\mp}$ decay, its branching fraction is very sensitive to the factorizable WA contributions related to $(\rho_A^f, \phi_A^f)$ rather than others as Fig.2(b) shows. As a result, the fitting result (red points) in Fig.1(b) is mainly due to the constraints from $\mathcal{B}(\bar{B}_s^0 \to K^\pm K^{*\mp})$. It is expected that the future refined measurements on $\bar{B}_s^0 \to K^\pm K^{*\mp}$ and $\bar{B}_s^0 \to \pi^- K^{*+}$ decays would perform strong constraints on factorizable
TABLE II: The $CP$-averaged branching fractions (in units of $10^{-6}$) of $\bar{B}_s \to \pi K^*$, $\rho K$, $\pi \rho$, $\pi \phi$, $K \phi$ and $KK^*$ decays. The first and second theoretical errors of the “this work” column are caused by uncertainties of the CKM and the other parameters in Table I, respectively.

| Decay Modes          | Class | This work     | Cheng [5]     | S4 [9]   | FS [31] |
|----------------------|-------|---------------|---------------|----------|---------|
| $\bar{B}_s \to \pi^- K^{*+}$ | T     | $6.8^{+0.6+3.6}_{-0.6-2.8}$ | $7.8^{+0.4+0.5}_{-0.7-0.7}$ | $6.8$   | $7.92 \pm 1.02$ |
| $\bar{B}_s \to \pi^0 K^{*0}$ | C     | $1.4^{+0.1+0.2}_{-0.1-0.2}$ | $0.89^{+0.80+0.84}_{-0.34-0.35}$ | $0.33$   | $3.07 \pm 1.20$ |
| $\bar{B}_s \to K^+ \rho^-$ | T     | $16.0^{+1.3+9.4}_{-1.3-7.2}$ | $14.7^{+1.4+0.9}_{-1.9-1.3}$ | $19.8$   | $14.63 \pm 1.46$ |
| $\bar{B}_s \to K^0 \rho^0$ | C     | $1.4^{+0.1+0.2}_{-0.1-0.2}$ | $1.9^{+2.9+1.4}_{-0.9-0.6}$ | $0.68$   | $0.56 \pm 0.24$ |
| $\bar{B}_s \to \pi^0 \phi$ | C, $P_{EW}$ | $0.20^{+0.01+0.05}_{-0.02-0.04}$ | $0.12^{+0.02+0.04}_{-0.01-0.02}$ | $0.12$   | $1.94 \pm 1.14$ |
| $\bar{B}_s \to K^0 \phi$ | P     | $0.45^{+0.02+0.07}_{-0.03-0.07}$ | $0.6^{+0.5+0.4}_{-0.2-0.3}$ | $0.46$   | $0.41 \pm 0.07$ |
| $\bar{B}_s \to \pi^0 \rho^0$ | WA    | $0.017^{+0.001+0.001}_{-0.001-0.001}$ | $0.02^{+0.00+0.01}_{-0.00-0.01}$ | $0.017$  | $-$       |
| $\bar{B}_s \to \pi^0 \rho^0$ | WA    | $0.034^{+0.002+0.002}_{-0.002-0.002}$ | $0.04^{+0.00+0.02}_{-0.00-0.02}$ | $0.029$  | $-$       |
| $\bar{B}_s \to K^0 K^{*0}$ | P     | $17.3^{+0.7+3.1}_{-1.1-2.7}$ | $21.6^{+10.9+12.9}_{-5.7-9.3}$ | $22.7$   | $16.01 \pm 1.25$ |
| $\bar{B}_s \to K^0 K^{*0}$ | P     | $17.3^{+0.7+3.1}_{-1.1-2.7}$ | $20.6^{+10.9+12.8}_{-6.4-9.3}$ | $22.2$   | $15.65 \pm 1.22$ |

WA and HSS corrections, respectively.

Based on above analyses, in order to get the theoretical predictions for $B_s \to PV$ decays, we assume that the endpoint parameters are universal for $B_{u,d,s} \to PV$ decays, and so we take

$$(\rho_{A,H}^j, \phi_{A,H}^j) = (3.08, -145^\circ), \quad (\rho_A^j, \phi_A^j) = (0.83, -36^\circ),$$

(10)
gotten from $B_{u,d} \to PV$ decays [24] as inputs. Now, we present the theoretical results for the observables of $B_s \to PV$ decays in Tables II III and IV in which the previous results in QCDF framework and in flavor symmetry (FS) framework are also summarized for comparison. It could be found that most of our predictions agree with the other theoretical results except for few tensions, which will be discussed in detail below.

For the (color-suppressed) tree dominated $\bar{B}_s \to \pi K^*$ and $\rho K$ decays, we find that: (1) even though our prediction for the branching fraction of observed $\bar{B}_s \to \pi^- K^{*+}$ decay agrees with experimental data given in Eq. (2) due to large theoretical uncertainties and experimental errors, the default value is still about twice larger than experimental central data. In fact, as noted in Ref. [31], all current theoretical results have such similar situation, even
though these results are consistent with one another. So, more theoretical and experimental efforts are required to confirm or refute such possible mismatch. (2) The amplitudes of $\bar{B}_s \to \pi^0 K^0$ and $K^0 \rho^0$ decays are more sensitive to the HSS corrections related to large Wilson coefficient $C_1$. Due to a relative large $\rho_H$ is used in evaluations, our prediction for

| Decay Modes | This work | Cheng [5] | S4 [9] | FS [31] |
|-------------|-----------|-----------|--------|---------|
| $A_{CP}^{\text{dir}}(\bar{B}_s \to \pi^- K^{*+})$ | $-28^{+1+6}_{-1-9}$ | $-24.0^{+1.2+7.7}_{-1.5-3.9}$ | $-22.0$ | $-13.6 \pm 5.3$ |
| $A_{CP}^{\text{dir}}(\bar{B}_s \to K^+ \rho^-)$ | $11.5^{+0.3+4.8}_{-0.5-2.8}$ | $11.7^{+3.5+10.1}_{-2.1-11.6}$ | $6.2$ | $12.0 \pm 2.7$ |
| $A_{CP}^{\text{dir}}(\bar{B}_s \to \pi^0 K^{*0})$ | $-64^{+2+6}_{-1-6}$ | $-26.3^{+10.8+42.2}_{-10.9-36.7}$ | $15.4$ | $-42.3 \pm 15.8$ |
| $A_{CP}^{\text{dir}}(\bar{B}_s \to \rho^0 K^0)$ | $51.8^{+1.4+2.7}_{-1.8-3.0}$ | $28.9^{+14.6+25.0}_{-14.5-23.7}$ | $11.6$ | $-12.4 \pm 45.3$ |
| $A_{CP}^{\text{mix}}(\bar{B}_s \to \rho^0 K^0)$ | $43^{+7+2}_{-4-2}$ | $29^{+23+16}_{-24-21}$ | $-$ | $-34.8 \pm 28.5$ |
| $A_{CP}^{\text{dir}}(\bar{B}_s \to \pi^0 \phi)$ | $66^{+1+3}_{-2-5}$ | $82.2^{+10.9+9.0}_{-14.0-55.3}$ | $24.8$ | $7.3 \pm 20.1$ |
| $A_{CP}^{\text{mix}}(\bar{B}_s \to \pi^0 \phi)$ | $-73^{+1+7}_{-1-7}$ | $40^{+4+32}_{-10-53}$ | $-$ | $43.9 \pm 17.1$ |
| $A_{CP}^{\text{dir}}(\bar{B}_s \to K^0 \phi)$ | $-2.85^{+0.08+0.75}_{-0.12-0.73}$ | $-3.2^{+1.2+0.6}_{-1.4-1.3}$ | $-7.4$ | $0$ |
| $A_{CP}^{\text{mix}}(\bar{B}_s \to K^0 \phi)$ | $-70^{+2+0}_{-2-0}$ | $-69^{+1+1}_{-1-1}$ | $-$ | $-69.2 \pm 0.0$ |
| $A_{CP}^{\text{dir}}(\bar{B}_s \to \pi^0 \rho^0)$ | $0$ | $0$ | $-$ | $-$ |
| $A_{CP}^{\text{mix}}(\bar{B}_s \to \pi^0 \rho^0)$ | $-74^{+2+0}_{-2-0}$ | $-65^{+3+0}_{-3-0}$ | $-$ | $-$ |
\(B(\bar{B}_s \to \pi^0 K^0)\) is larger than the results of “Cheng” \cite{5} and “S4” \cite{9}, but much smaller than the one of “FS” \cite{31} where a large color-suppressed amplitude \(C_V\) have similar magnitude to, even larger than in Scheme C, the color-favored tree amplitude \(T_V\).

The color-suppressed tree and electroweak penguin dominated \(\bar{B}_s \to \pi^0 \phi\) decay is an important and interesting decay mode for exploring the HSS corrections, due to the fact that its amplitude,

\[
A(\bar{B}_s \to \pi^0 \phi) \propto V_{ub}V_{us}^*\alpha_2 + V_{tb}V_{ts}^* \frac{3}{2} \alpha_{3,EW},
\]  

(11)

is sensitive to \(\alpha_2\) related to possible large HSS corrections and irrelevant to the interference induced by WA corrections. Moreover, it plays an important role to judge the two direct ways through \(\alpha_2\) or \(\alpha_{3,EW}\) respectively to resolve the so-called “\(\pi K\) CP puzzle” \cite{32, 33}.

From Tables II and III it is found that our results for observables of \(\bar{B}_s \to \pi^0 \phi\) decay, especially the mixing-induced CP asymmetry, are more or less different from the other ones. To clarify the reason, we present the dependence of observables on \(\phi_H\) with different values of \(\rho_H\) in Fig. 3. It is found that the branching fraction is always about \(O(10^{-7})\), and hardly to be enhanced to \(O(10^{-6})\) level (predicted by “FS” \cite{31}) by HSS corrections as Fig.3(a) shows. From Fig.3(b), it can be seen that the direct CP asymmetry is much more sensitive to \(\phi_H\) than the other observables, and so very suitable for exploring the possible strong phase in HSS. For \(A_{CP}^{mix}(\bar{B}_s \to \pi^0 \phi)\), one may note from Table III that our prediction, even its sign, is significantly different from the others, which is an interesting finding and could be understood by the following reason. From Eq.(11), it can be found that whether \(\alpha_2\) related to CKM factor \(V_{ub}V_{us}^*\) or \(\alpha_{3,EW}\) related to \(V_{tb}V_{cs}^*\) (the part related to \(V_{ub}V_{us}^*\) is negligible) dominates amplitude \(A_{\bar{B}_s \to \pi^0 \phi}\) is crucial for evaluating the mixing-induced CP asymmetry. As a result, as Fig.3(c) shows, \(A_{CP}^{mix}(\bar{B}_s \to \pi^0 \phi)\) is very sensitive to \(\rho_H\), and
obviously negative when $\rho_H \gtrsim 2$. So, the future measurements on $A_{CP}^{mix}(B_s \to \pi^0\phi)$ will play an important role to probe the strength of HSS contribution.

For the (electroweak) penguin dominated $B_s$ decays, there are no significant difference between our predictions and the others within uncertainties. However, similar to the situation in $B_s \to \pi^- K^{*+}$ decay, the theoretical results for branching fraction of $B_s \to K^\pm K^{*\mp}$ decay listed in Table [I] is significantly larger than the LHCb data in Eq. (1). While, it is not a definite mismatch for now due to the large hadronic uncertainties. Besides of branching fraction, both $B_s \to \pi^- K^{*+}$ and $B_s \to K^0 K^{*0}(\bar{K}^0\bar{K}^{*0})$ decays involve five non-zero $CP$ asymmetry observables listed in Table [IV] which would perform much stronger constraints on WA contributions if measured, especially on the possible strong phases therein.

The pure annihilation $B_s \to \pi \rho$ decays are principally suitable for probing the WA contributions without the interferences induced by HSS corrections. However, their branching fractions are stable at about $O(10^{-8})$ level, and no significant $CP$ asymmetries are found theoretically. So these decay modes are hardly to be measured very soon and possibly hard to provide useful information for the WA strong phases.

IV. SUMMARY

In summary, motivated by recent LHCb measurements on $B_s \to \pi^- K^{*+}$ and $B_s \to K^\pm K^{*\mp}$ decays, we revisit the $B_s \to PV$ decays within the QCD factorization framework, and analysis the effects of HSS and WA corrections related to end-point parameters in detail. Our main findings and conclusions could be summarized as follows:

- By $\chi^2$ fitting, it is found that the WA parameters in $B_s \to PV$ decays could not be well bounded due to large theoretical uncertainties and rough experimental measurements, and the universalities of their values in $B_s$ and $B_{u,d}$ system are still allowed. Assuming the values of $(\rho^i_A, \phi^i_A)$ are universal for $B_s$ and $B_{u,d}$ decays and $(\rho_H, \phi_H) = (\rho^i_A, \phi^i_A)$, our theoretical predictions are presented in Tables [II], [III] and [IV] which will be tested in the near future.

- For the branching fractions of observed $B_s \to \pi^- K^{*+}$ and $K^\pm K^{*\mp}$ decays, even though the current theoretical results agree with each other and are consistent with data within errors, their default results are significantly much larger than the central values of data.
The refined measurements on such two observables will perform strong constraints on $(\rho_H, \phi_H)$ and $(\rho'_{f}, \phi'_{f})$, respectively, and further to check such possible mismatches.

- A detailed analysis for the other observables and decays modes are performed, and some interesting findings are presented in text. Especially, for instance, without the interference induced by annihilation corrections, $\bar{B}_s \to \pi^0 \phi$ decay plays an important role to exploring the HSS contributions, where the size and strong phase could be clearly determined by direct and mixing-induced CP asymmetries, respectively. In addition, the pure annihilation $\bar{B}_s \to \pi \rho$ decays will be suitable for probing the WA contributions, although their branching ratios are very small.

Moreover, more $B_s$ meson decay modes are urgently expected to be measured soon by LHCb and upgrading Belle II in the near future, which will exhibit exact picture of WA and HSS contributions in $B_s$ decays.

**Acknowledgments**

The work is supported by the National Natural Science Foundation of China (Grant Nos. 11475055, 11275057 and U1232101). Q. Chang is also supported by the Foundation for the Author of National Excellent Doctoral Dissertation of P. R. China (Grant No. 201317) and the Program for Science and Technology Innovation Talents in Universities of Henan Province (Grant No. 14HASTIT036).

[1] R. Aaij et al. (LHCb Collaboration), New J. Phys. 16 (2014) 123001.

[2] M. Beneke, G. Buchalla, M. Neubert and C. Sachrajda, Phys. Rev. Lett. 83 (1999) 1914; Nucl. Phys. B 591 (2000) 313.

[3] Y. Keum, H. Li and A. Sanda, Phys. Lett. B 504 (2001) 6; Phys. Rev. D 63 (2001) 054008.

[4] C. Bauer, S. Fleming and M. Luke, Phys. Rev. D 63 (2000) 014006; C. Bauer, S. Fleming, D. Pirjol and I. Stewart, Phys. Rev. D 63 (2001) 114020; C. Bauer and I. Stewart, Phys. Lett. B 516 (2001) 134; C. Bauer, D. Pirjol and I. Stewart, Phys. Rev. D 65 (2002) 054022.

[5] H. Cheng and C. Chua, Phys. Rev. D 80 (2009) 114026.

[6] A. Ali et al., Phys. Rev. D 76 (2007) 074018.
[7] W. Wang, Y. Wang, D. Yang and C. Lü, Phys. Rev. D 78 (2008) 034011.
[8] G. Lepage and S. Brodsky, Phys. Rev. D 22 (1980) 2157.
[9] M. Beneke, G. Buchalla, M. Neubert and C. Sachrajda, Nucl. Phys. B 606 (2001) 245; M.
    Beneke and M. Neubert, Nucl. Phys. B 651 (2003) 225; Nucl. Phys. B 675 (2003) 333.
[10] Y. Duh et al. (Belle Collaboration), Phys. Rev. D 87 (2013) 031103.
[11] R. Aaij et al. (LHCb Collaboration), JHEP 1210 (2012) 037.
[12] T. Aaltonen et al. (CDF Collaboration), Phys. Rev. Lett. 108 (2012) 211803.
[13] M. Beneke, J. Rohrer and D. Yang, Nucl. Phys. B 774 (2007) 64.
[14] H. Cheng and C. Chua, Phys. Rev. D 80 (2009) 114008.
[15] Q. Chang, X. Cui, L. Han and Y. Yang, Phys. Rev. D 86 (2012) 054016.
[16] Z. Xiao, W. Wang and Y. Fan, Phys. Rev. D 85 (2012) 094003.
[17] M. Gronau, D. London and J. Rosner, Phys. Rev. D 87 (2013) 036008.
[18] C. Bobeth, M. Gorbahn and S. Vickers, arXiv:1409.3252.
[19] G. Zhu, Phys. Lett. B 702 (2011) 408.
[20] K. Wang and G. Zhu, Phys. Rev. D 88 (2013) 014043.
[21] Q. Chang, J. Sun, Y. Yang and X. Li, Phys. Rev. D 90 (2014) 054019.
[22] Q. Chang, J. Sun, Y. Yang and X. Li, Phys. Lett. B 740 (2015) 56.
[23] Y. Amhis et al. (HFAG), arXiv:1207.1158 and online update at:
    http://www.slac.stanford.edu/xorg/hfag.
[24] J. Sun, Q. Chang, X. Hu and Y. Yang, arXiv:1412.2334.
[25] J. Charles et al. (CKMfitter Group), Eur. Phys. J. C 41 (2005) 1; updated results and plots
    available at: http://ckmfitter.in2p3.fr.
[26] K. Olive et al. (Particle Data Group), Chin. Phys. C 38 (2014) 090001.
[27] J. Laiho, E. Lunghi and R. Van de Water, Phys. Rev. D 81 (2010) 034503, updated results
    available at: http://www.latticeaverages.org.
[28] P. Ball, G. Jones and R. Zwicky, Phys. Rev. D 75 (2007) 054004.
[29] A. Al-Haydari et al. (QCDSF Collaboration), Eur. Phys. J. A 43 (2010) 107.
[30] P. Ball, V. Braun and A. Lenz, JHEP 0605 (2006) 004; P. Ball and G. Jones, JHEP 0703
    (2007) 069.
[31] H. Cheng, C. Chiang and A. Kuo, Phys. Rev. D 91 (2015) 014011.
[32] Q. Chang, X. Li and Y. Yang, J. Phys. G 41 (2014) 105002.
[33] L. Hofer, D. Scherer and L. Vernazza, JHEP 1102 (2011) 080.