Hysteresis Modeling for Power Magnetic Devices Based on Magnetic Circuit Method

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This paper introduces the latest research achievements on the hysteresis modeling for power magnetic devices based on the magnetic circuit method. First, the magnetic circuit model considering the magnetic hysteresis behavior is derived. Next, this magnetic circuit model is developed to the reluctance network model, to analyze more complicated devices such as an electric motor. Furthermore, this magnetic circuit model is improved to take the dynamic hysteresis characteristics including the skin effect into account. Finally, the prediction method of the deteriorated hysteresis loops in core material due to machining process is established.

Key words: magnetic circuit method, Landau Lifshitz-Gilbert (LLG) equation, play model, reluctance network analysis (RNA), Cauer circuit

1. Introduction

In recent years, quantitative analysis of the iron loss taking the magnetic hysteresis behavior into account is essential to the development of high-efficiency electric machines. In many cases of machine design and analysis based on numerical analysis using the finite element method (FEM), the iron loss is generally calculated by using approximate formulas, such as the Steinmetz' equation, in the post-processing of a main electromagnetic field analysis (1-3). However, their calculation accuracies are insufficient when the exciting voltage waveform is significantly distorted. To improve the calculation accuracy, the parameters in the formulas must be identified for each waveform, which is a huge practical drawback. Thus, various kinds of high-accuracy hysteresis models have been presented so far (4-8). However, most of the general-purpose FEM programs neglect the magnetic hysteresis since it generally takes a lot of calculation time and computer memory. Therefore, it is strongly required to establish a simple and practical iron loss calculation method considering the magnetic hysteresis.

To solve the above problem, the magnetic circuit method is one of the practical solutions because an analytical model is very simple, the calculation accuracy is relatively high, and it is easy to combine with an electric circuit and a motion equation. Therefore, this paper introduces the latest research achievements on the hysteresis modeling for electric machines based on the magnetic circuit. The Chapter 2 describes the magnetic circuit model considering the magnetic hysteresis behavior. The Chapter 3 describes that this magnetic circuit model is improved to the reluctance network model, to analyze more complicated devices such as an electric motor. The Chapter 4 describes that the magnetic circuit model, presented in the Chapter 2, is improved to take the dynamic hysteresis characteristics including the skin effect into account. The Chapter 5 describes the prediction method of the deteriorated hysteresis loops in core material due to machining process.

2. Magnetic Circuit Model Considering Magnetic Hysteresis Behavior by Incorporating Play Model

In this Chapter, the magnetic circuit model considering the magnetic hysteresis behavior by incorporating the play model (9, 10), and the derivation method of the play model by using the simplified Landau Lifshitz Gilbert (LLG) equation (11, 12), which are the bases of the analyses in the following chapters, are described.

2.1 Basis of magnetic circuit method

Fig. 1(a) illustrates a ring core with a winding of \( N \) turns. A cross section, magnetic path length, and permeability of a core are \( S_c \), \( l \), and \( \mu \) respectively. When current \( i \) flows in the winding, a relationship between magnetomotive force (MMF) \( Ni \) and flux \( \phi \) can be expressed in the following equation (14):

\[
Ni = R_e \phi, \text{ where } R_e = \frac{1}{\mu S_c}.
\]  

In Eq. (1), letting \( Ni \) and \( \phi \) correspond to voltage \( v \) and current \( i \) in an electric circuit, it is understood that the same relationship as the Ohm’s low exists between \( Ni \) and \( \phi \). Accordingly, a ring core in Fig. 1(a) can be expressed in the form of a magnetic circuit shown in Fig. 1(b). In this figure, \( R_m \), which corresponds to a resistance in an electric circuit, is called “reluctance”.

As mentioned above, there is a high degree of similarity between magnetic and electric circuits. However, actual magnetic material has the magnetic nonlinearity due to the magnetic saturation and hysteresis and causes the iron loss due to the magnetic hysteresis and eddy current. Therefore, it is necessary to derive the magnetic circuit model taking these complicated magnetic phenomena into consideration, to
obtain more accurate calculation result.

2.2 Derivation of magnetic circuit model considering magnetic hysteresis behavior

The iron loss of soft ferromagnetic material \( W \) can be approximately calculated as the sum of hysteresis loss, classical eddy current loss, and anomalous eddy current loss, by using the following Bertotti’s equation \(^2\):

\[
W = A_i f B_m^2 + A_f i^2 B_m^2 + A_m f^{1.5} B_m^{1.5},
\]

where a frequency is \( f \), maximum magnetic flux density is \( B_m \), and coefficients are \( A_i, A_f, \) and \( A_m \), respectively. Here, Fig. 2 shows a schematic diagram of the hysteresis loop under ac excitation. The magnetic field \( H \) at the operating point \( c \) in this figure can be represented by the sum of the component between a and b determined by the dc hysteresis and the component between b and c which changes depending on the time derivative of the magnetic flux density \( B \). Therefore, a relationship between \( H \) and \( B \) can be obtained as follows:

\[
H = \begin{cases} 
H_a + \frac{\gamma_1}{S_1} \frac{dB}{dt} + \frac{\gamma_2}{S_1} \frac{dB}{dt}^{0.5}, & (dB > 0) \\
H_a + \frac{\gamma_1}{S_1} \frac{dB}{dt} - \frac{\gamma_2}{S_1} \frac{dB}{dt}^{0.5}, & (dB < 0)
\end{cases}
\]

where the dc field is \( H_a \), and coefficients are \( \gamma_1 \) and \( \gamma_2 \), respectively. In Eq. (3), each of the first, second, and third terms represents the dc hysteresis, classical eddy current loss, and anomalous eddy current loss, respectively. This equation can be transformed into a relationship between \( Ni \) and \( \phi \) as follows:

\[
Ni = H_a + \frac{\gamma_1}{S_1} \frac{d\phi}{dt} + \frac{\gamma_2}{S_1} \frac{d\phi}{dt}^{0.5}.
\]

Eq. (4) can be expressed as the magnetic circuit model shown in Fig. 3. In this model, several kinds of representation methods of the dc hysteresis have been proposed, such as the look-up table \(^{15}\), simplified LLG equation \(^{13, 13}\), and play model \(^{8, 10}\), while the classical and anomalous eddy current losses are simply denoted by the inductance element and dependent source of flux, respectively.

Among the above dc hysteresis models, when using the look-up-table in the magnetic circuit model, it is possible to calculate the hysteresis loops under sinusoidal and square voltage excitations, while the minor loops cannot be simulated under PWM excitation. On the other hand, when using the simplified LLG equation (the principle and simplification process are explained in the Section 2.4), although the minor loops can be accurately simulated, the calculation time tends to be longer because repeat convergence calculation is indispensable. On the contrary, it is possible to calculate the hysteresis loop with not only high accuracy but also high speed by using the play model (the principle is explained in the Section 2.3).

Next, the identification method of parameters \( \gamma_1 \) and \( \gamma_2 \) is described. Assuming that a flux density waveform is sinusoidal, \( W \) can be obtained from Eq. (3) as follows:

\[
W = \frac{1}{T} \int_{0}^{T} S_i dB + \frac{2\pi^2 \gamma_1 B_m^2}{q_i} f + 8.763 \frac{\gamma_2 B_m^{1.5}}{q_i} f^{1.5}.
\]

where a period is \( T \) and mass density is \( q_i \), respectively. Dividing both sides of Eq. (5) by \( f \), the following equation is given:

\[
\frac{W}{f} = g(B_m) + \frac{2\pi^2 \gamma_1 B_m^2}{q_i} f + 8.763 \frac{\gamma_2 B_m^{1.5}}{q_i} f^{1.5}.
\]

where the constant 8.673 is obtained by numerical integration. Here, \( \gamma_1 \) is given by the following equation based on the classical eddy current theory:

\[
\gamma_1 = \frac{\sigma d^2}{12},
\]

where conductivity is \( \sigma \) and thickness of a steel sheet is \( d \), respectively. Then, \( \gamma_2 \) can be determined by approximating the core loss curves of core material by using the least squares method based on Eq. (6), as shown in Fig. 4.
2.3 Principle of play model

Fig. 5(a) illustrates a schematic diagram of the play model \(^9\), which is one of phenomenological hysteresis models. In this model, each play hysteron \(p(x)\), which has different value of width \(2\zeta\) and depends on not only the input \(x\) but also the past hysteresis as shown in Fig. 5(b), is defined as follows:

\[
p_i(x) = \max\left(\min\left(p_{0i}, x + \zeta\right), x - \zeta\right) \quad (i = 1 \sim N),
\]

where a value of \(p(x)\) in the previous calculation step is \(p_{0i}\). Then, a single-valued shape function \(f(p(x))\), which reflects magnetic properties of arbitrary core material, takes each of \(p(x)\), and the output \(y\) is obtained by the following equation:

\[
y = \sum_{i=1}^{N} f_i(p_i(x)).
\]

Here, the shape function is identified from a large number of measured dc hysteresis loops with various maximum flux densities.

The play model can simulate arbitrary hysteresis behavior including complicated minor loops under PWM excitation. Besides, this model has the advantage of fast calculation because no convergence calculation is required. However, the acquisition of a lot of measured data described above has been a large obstacle in practical use.

To overcome the above problem, the efficient method, which obtains the play model from the dc hysteresis loops calculated by the simplified LLG equation, was presented (the specific procedure is explained in the Section 2.5). In this way, only two or three dc hysteresis loops have to be measured \(^{10}\).

2.4 Principle and simplification process of LLG equation

Micromagnetics is one of physical models based on the LLG equation \(^{11}\), which can simulate the hysteresis behavior and magnetization distribution inside a magnetic substance. When an analytical region is divided into \(n\) elements, the behavior of each magnetization can be represented by the following equation:

\[
\frac{dm_i}{dt} = -\gamma |m_i| \times H_{ani} + \alpha |m_i| \times \frac{dm_i}{dt} \quad (i = 1 \sim n),
\]

where the normalized magnetization vector in each element is \(m_i\), the gyromagnetic ratio of electron is \(\gamma\), the damping constant is \(\alpha\), and the effective field in each element is \(H_{eff_i}\), respectively. \(H_{eff_i}\) is given by the following equation:

\[
H_{eff_i} = H_{app_i} + H_{ani_i} + H_{exc_i} + H_{mag_i},
\]

where the applied field is \(H_{app_i}\), the anisotropy field is \(H_{ani_i}\), the exchange field is \(H_{exc_i}\), and the magnetostatic field is \(H_{mag_i}\) (including the demagnetizing field), in each element, respectively.

Though the original LLG equation can express micro magnetic phenomena in detail, including the magnetic anisotropy, domain wall motion, and interaction between magnetizations, the micromagnetics model is too large and complicated to be applied to analysis of electric machines. To overcome this issue, Ref. 11) presented a simplified method for approximately expressing the magnetic hysteresis by providing several assumptions.

The first assumption is that originally multi-domain structure in each crystal grain is regarded as single-domain one, to significantly reduce the number of elements and thereby the computer memory. Here, \(H_{exc_i}\), which expresses the domain wall motion, is neglected. On the other hand, since the magnetization reversal due to the domain wall motion cannot be expressed, each magnetization is made easier to rotate by making the coefficient representing strength of the anisotropy field smaller than the actual physical constant.

Next, \(H_{mag_i}\), which expresses the interaction between magnetizations, are approximated as the function of the average normalized magnetization, which is called “the field generated by the magnetoelastic energy”, to shorten the calculation time to be practical.

Furthermore, the magnetoelastic effect, internal stress, lattice defect, etc. are approximately considered by determining the coefficients of \(H_{ani_i}\) and \(H_{exc_i}\), so as to match the measured hysteresis loop.

From the above, \(H_{eff_i}\) is given by the following equation in the simplified method:
\[ H_{dc,i} = H_{mag,i} + H_{an,i} + H_{sat,i}, \]

where the field generated by the magnetostatic energy in each element is \( H_{sat,i} \). In Eq. (12), \( H_{an,i} \) is given by the following equation:

\[ H_{an,i} = -\frac{\partial}{\partial m} \left[ \frac{1}{2} \sum_{j=1}^{n} b_{ij} M_i \right], \]

where the coefficient of the anisotropy field is \( h_{ani,i} \), and direction cosines of magnetization vectors with respect to \( x_i, y_i \), and \( z_i \) axes (easy axes) of each grain are \( a_{1i}, a_{2i}, a_{3i} \), and \( a_{4i} \), in each element, respectively. Here, \( h_{ani,i} \) has a normal distribution with the average value of \( h_{ani} \) and standard deviation value of \( \sigma_{ani} \), and easy axes of each magnetization are randomly distributed. \( h_{ani} \) and \( \sigma_{ani} \) are determined by the following empirical formulas:

\[ h_{ani} = 2.5 \sqrt{\frac{M_s H(B = 0)}{B_i}}, \]

\[ \sigma_{ani} = 0.25 h_{ani}, \]

where the spontaneous magnetization is \( M_s \). Besides, \( h_{ani} \) is smaller than the actual physical constant as mentioned above. On the contrary, \( H_{mag,i} \) is given by the following equation:

\[ H_{mag,i} = -\frac{\partial}{\partial m} \left[ \sum_{j=1}^{n} b_{ij} M_i \right], \]

where the generic term of coefficients of Taylor expansion is \( b_{2i} \), and the average normalized magnetization is \( \vec{m} \), respectively. Here, \( b_{2i} \) is determined by using the Gauss-Newton method so that an error between the measured and calculated hysteresis loops is the smallest. Fig. 6 illustrates the relationship between the parameters \( h_{ani} \) and \( b_{2i} \), and a shape of the hysteresis loop. As shown in these figures, the coercive force is proportional to \( h_{ani} \), and the magnetic nonlinearity due to the saturation is represented by \( b_{2i} \), which is determined from a shape of the rising curve in the first quadrant.

### 2.5 Derivation method of play model by using simplified LLG equation

As described in the Section 2.5, the play model can calculate the dc hysteresis loop with high accuracy and high speed, though a large number of measured dc hysteresis loops with different maximum flux densities are required to derive it. On the other hand, as mentioned in the Section 2.4, the simplified LLG equation can calculate the dc hysteresis loop with arbitrary maximum flux density at high accuracy despite relatively long time. In view of these advantages and disadvantages of both methods, Ref. 10) presented the practical method for deriving the play model by using the simplified LLG equation, and the magnetic circuit model incorporating the obtained play model.

The specific procedure of the proposed method is described below. First, two or three dc hysteresis loops are measured, and the parameters of the simplified LLG equation are determined. Next, a number of dc hysteresis loops with different maximum flux densities are calculated by the simplified LLG equation. Table 1 shows the parameters of the simplified LLG equation, which are obtained from the measured dc hysteresis loop with \( B_m = 1.6 \) T. Fig. 7(a) shows the comparison of measured and calculated dc hysteresis loops. From this figure, it is clear that measured and calculated results are in good agreement. Fig. 7(b) shows a lot of dc hysteresis loops calculated by the simplified LLG equation, which are used to obtain the play model instead of measured ones.

### 2.6 Simulation results by magnetic circuit model incorporating play model

In Ref. 10, the magnetic circuit model incorporating the play model was proposed and coupled with an electric circuit, as shown in Fig. 8, to calculate the hysteresis loops when a ring core is excited by sinusoidal and PWM voltage. Fig. 9(a) and (b) show the comparison of measured and calculated hysteresis loops under sinusoidal voltage excitation, when \( B_m = 0.4 \) T, 0.8 T, and 1.2 T and \( f = 50 \) Hz and 300 Hz, respectively. From these figures, it can be seen that measured and calculated results are in good agreement with various maximum flux densities and frequencies. Fig. 10(a) and (b) show the comparison of measured and calculated current waveform and hysteresis loop under PWM voltage excitation. As shown in these figures, the complexly distorted current waveform and the hysteresis loop including the minor loops are almost accurately simulated.

The above-described magnetic circuit model can relatively easily analyze the magnetic hysteresis with...
high accuracy and high speed. However, this method can be applied only for the objects with simple shapes such as a ring core, since it is generally difficult to apply the magnetic circuit method to analysis of devices with complicated structure or flux distribution, including an electric motor. Hence, a more detailed model, namely the reluctance network analysis (RNA) model, is necessary. Therefore, in the Chapter 3, the magnetic circuit model incorporating the play model is extended to the RNA, to analyze various kinds of electric machines considering the iron loss caused by the magnetic hysteresis behavior.

3. RNA Model Considering Magnetic Hysteresis Behavior by Incorporating Play Model

The RNA expresses an analytical object by one reluctance network. All the reluctances can be determined by $B$-$H$ curve of material and dimensions.\(^\text{17}\)

(a) Measured and calculated dc hysteresis loops of a non-oriented silicon steel with a thickness of 0.35 mm.

(b) Calculated dc hysteresis loops of a non-oriented silicon steel with a thickness of 0.35 mm from $B_m = 0.04$ T to $1.6$ T at intervals of $0.04$ T.

Fig. 7 Calculated dc hysteresis loops by using the simplified LLG equation.

Fig. 8 Magnetic circuit model incorporating the play model.

Fig. 9 Calculated hysteresis loops by using the magnetic circuit model incorporating the play model under sinusoidal voltage excitation.

(a) Current waveform.

(b) Hysteresis loop.

Fig. 10 Calculated current waveform and hysteresis loop by using the magnetic circuit model incorporating the play model under PWM voltage excitation.
The RNA has some advantages such as a simple model, fast calculation, and easy coupling with an external electric circuit and motion equation. In addition, the RNA can express flux distribution inside the analytical object. Thus, the RNA has been applied to the analyses of various kinds of electric machines, including an electric motor \(^{18-20}\).

In this Chapter, the basis of the RNA is explained first, and then the derivation method of the RNA model considering the magnetic hysteresis behavior is described. Ref. 21) and Ref. 22) previously proposed the RNA models incorporating the Preisach model \(^{23}\) and Loss Surface (LS) model \(^{24}\), respectively, which are ones of phenomenological hysteresis models, and these RNA models are applied to the analyses of permanent magnet (PM) motors. However, these hysteresis models essentially require high computer memory. Moreover, neither RNA models are insufficiently validated because measured and calculated results have never been compared. Therefore, this chapter introduces the RNA model incorporating the above-described play model and some of its analysis examples \(^{25-29}\).

3.1 Basis of RNA

In this section, a conventional method for deriving a two-dimensional (2-D) RNA model is described \(^{17}\), taking a cut-core made of a non-oriented silicon steel with a thickness of 0.35 mm shown in Fig. 11(a) as an example.

First, an analytical object, which consists of a core and a peripheral air region, is divided into multiple elements, as shown in Fig. 11(b). Each divided element is represented by a unit magnetic circuit composed of four reluctances, as shown in Fig. 11(c).

Among them, reluctances in a rolling direction are determined in consideration of the magnetic nonlinearity as follows:

\[
H = \alpha B + \alpha_m B^m, \tag{17}
\]

where coefficients are \(\alpha\) and \(\alpha_m\). Also, order \(m\) is determined by strength of the nonlinearity of the \(B-H\) curve. From Eq. (17), a relationship between MMF \(f_m\) and flux \(\phi\) in each reluctance can be expressed by the following equation:

\[
f_m = \left(\frac{\alpha l}{S} + \frac{\alpha_m l}{S} \phi^{m-1}\right) \phi, \tag{18}
\]

where an average cross-sectional area and magnetic path length of each element are \(S\) and \(l\), respectively. Therefore, the nonlinear reluctance \(R_{nl}\) is given by the following equation:

\[
R_{nl} = \frac{\alpha l}{S} + \frac{\alpha_m l}{S} \phi^{m-1}. \tag{19}
\]

Next, reluctances perpendicular to a rolling direction are needed to be determined in consideration of flux passing through nonmagnetic layers between steel sheets. In general, the reluctance \(R_{nl}\) is given by the following equation using the effective permeability \(\mu'\):

\[
\mu S
\]

Fig. 11 Conventional derivation method of the RNA model of a cut-core.

\[
R_{nl} = \frac{l}{\mu S}. \tag{20}
\]

Here, a core is composed of laminated steel sheets with the permeability \(\mu\), and nonmagnetic layers with the vacuum permeability \(\mu_0\) at a ratio of \(d_f : (1-d_f)\), where a space factor of a core is \(d_f\). Hence, \(\mu'\) is given by the following equation:

\[
\mu' = \frac{1}{\mu} + \frac{1-d_f}{\mu_0}. \tag{21}
\]

If flux flowing perpendicular to laminated steel sheets is so small that the saturation does not occur, \(\mu\) is sufficiently larger than \(\mu_0\). Thus, Eq. (21) can be approximated as follows:

\[
\frac{1}{\mu'} \approx \frac{1-d_f}{\mu_0}. \tag{22}
\]

Therefore, \(R_{nl}\) is given by the following equation:

\[
R_{nl} \approx \frac{(1-d_f)l}{\mu_0 S}. \tag{23}
\]

Moreover, the reluctance \(R_{nu}\) in an air region surrounding a core is simply given by the following equation:

\[
R_{nu} = \frac{l}{\mu_0 S}. \tag{24}
\]

Fig. 12 shows an example of the conventional RNA model. In this figure, MMF generated by winding current
is placed in the position where a winding is applied. As shown in this figure, the conventional RNA model consists of only reluctances and MMF. Hence, the magnetic hysteresis behavior is not taken into consideration. Therefore, in the Section 3.2, the play model described in the Chapter 2 is applied to the RNA model.

3.2 RNA model incorporating play model

Fig. 13 shows the RNA model incorporating the play model. In this model, the reluctances in a rolling direction, which are conventionally given by Eq. (19), are represented by the play model and magnetic circuit elements. On the contrary, the reluctances in a perpendicular direction and an air region are given by Eq. (23) and Eq. (24) in the same way as the conventional RNA model, respectively.

Fig. 14(a) and (b) show the comparison of measured and calculated hysteresis loops under sinusoidal voltage excitation, when $B_m = 0.4 \text{ T}$, $0.8 \text{ T}$, and $1.2 \text{ T}$ and $f = 50 \text{ Hz}$ and $300 \text{ Hz}$, respectively. From these figures, it is understood that measured and calculated results are in good agreement.

In addition, Fig. 15 indicates the calculated hysteresis loop in a certain divided element of the RNA model. As shown in this figure, the magnetic hysteresis inside a core, which is generally difficult to measure and calculate, can be drawn, though its validity should be experimentally proved.

3.3 Analysis of switched reluctance motor

In the previous research, the RNA model incorporating the play model, which is described in the

Section 3.2, has been applied to the analyses of a PM motor, three-phase laminated-core variable inductor, and switched reluctance (SR) motor. Among them, the analysis result of the SR motor is reported as a representative in this section.

Fig. 16 shows specifications of the SR motor with 4-slots and 2-poles used in the examination. The core material is a non-oriented silicon steel with a thickness of 0.35 mm.

Fig. 17 shows a schematic diagram of the RNA model of the SR motor incorporating the play model. The reluctances in stator and rotor cores are given by the play model and magnetic circuit elements. The reluctances between a stator pole and yoke, and between adjacent stator poles, are simply given by Eq. (24). MMF at each stator pole is generated by winding current.
Specifications of the SR motor used in the examination.

Schematic diagram of the RNA model of the SR motor incorporating the play model.

In addition, since an SR motor has salient pole structure, flux distribution around stator and rotor pole tips dynamically changes with rotation \(^{18}\). Hence, the magnetic circuit around pole tips and air gap is represented by the variable reluctances depending on the rotor position angle as shown in Fig. 18, so that the local saturation and fringing flux can be taken into consideration.

The characteristics of the SR motor are calculated by using the above-described magnetic circuit and RNA model coupled with an electric circuit, as shown in Fig. 19. Fig. 20 and Fig. 21 show the comparison of measured and calculated iron loss and winding current density versus torque characteristics, respectively. Here, the calculated iron loss is obtained by subtracting machine output, and copper and mechanical losses from electrical input. From these figures, it can be seen that measured and calculated values are almost in good agreement. However, it is not commonly seen in an SR motor that the calculated iron loss is reduced due to increasing the torque. The cause of this tendency is considered to be that the machine output, which is the product of torque and rotational speed, is two orders larger than iron loss so that a slight error of the torque can subordinately affect the iron loss value.

Furthermore, as shown in Fig. 22, this RNA model can draw such distorted or dc-biased hysteresis loops in certain divided elements, even though they should be validated by the experiment.

4. Magnetic Circuit Model Considering Dynamic Hysteresis Characteristics by Incorporating Play Model and Cauer Circuit

The above-described magnetic circuit and RNA model...
methods incorporating the play model can analyze the magnetic hysteresis with high accuracy and high speed by using relatively simple models. However, they cannot consider the influence of the skin effect because the classical eddy current loss is denoted by just a single inductance element based on the classical eddy current theory. Therefore, there is a possibility for further improvement in the calculation accuracy of the hysteresis loop at a high frequency, such as when including the minor loops generated from carrier harmonics under PWM excitation, as shown in Fig. 23 (a) and (b). Here, a ring core made of a grain-oriented silicon steel with a thickness of 0.23 mm is used in the examination. In this chapter, it should be noted the play model is derived from only the measured dc hysteresis loops, to more accurately verify the calculation accuracy by eliminating errors due to the simplified LLG equation.

To practically analyze taking the skin effect into account, some kind of approximate calculation is required, since it is not realistic to perform a three-dimensional (3-D) analysis by modeling each laminated steel sheet from the viewpoint of calculation time and computer memory. So far, various kinds of methods for calculating the classical eddy current loss considering the influence of the skin effect have been proposed for the FEM.

One of typical methods is a one-dimensional (1-D) eddy current analysis in the thickness direction for each mesh as the post-processing based on the result of the main 2-D FEM, to obtain loss distribution inside a steel sheet. Further, this post 1-D analysis can be implemented by simultaneously using the play model to express the dc hysteresis (this method is called “P1D method” in the following).

Another representative model of the skin effect is the Cauer circuit, which is a ladder-type electric equivalent circuit composed of multiple resistance and inductance elements connected in series and in parallel as shown in Fig. 24. Using the Cauer circuit, the 1-D electromagnetic field analysis can be performed by considering the frequency characteristic of the complex permeability. In Ref. 33, the 1-D analysis is conducted by combining the play model and Cauer circuit, when a ring core is excited by PWM voltage. It was clear that the hysteresis loop including the minor loops and iron loss can be calculated with high accuracy. Moreover, in Ref. 34, the post 1-D analysis is implemented by using the play model and Cauer circuit for each mesh, after the main 2-D analysis incorporating the play model for a PM motor. Compared with the above-described P1D method,
this method has the same calculation accuracy and can significantly reduce the calculation time. This is because the Cauer circuit requires low computer memory since each mesh of the main FEM model is just given by a simple electric equivalent circuit, while each mesh must be further divided into multiple elements in the thickness direction according to the skin depth in the P1D method.

As described above, various studies for modeling the skin effect have been conducted. However, none of those methods has been sufficiently studied because there are few examples of application to the 3-D analysis. Besides, since these methods have been used only for the post-processing, it is desired to establish a more practical method which can take the skin effect into account during the electromagnetic field analysis.

Therefore, this chapter introduces the magnetic circuit model considering the dynamic hysteresis characteristics including the skin effect, by incorporating the Cauer circuit in addition to the play model in Ref. 30. In the following, it is revealed that this model is improved in the calculation accuracy of the minor loops, compared to the previous one.

4.1 Derivation of magnetic circuit model incorporating play model and Cauer circuit

Fig. 24 shows a schematic diagram of the Cauer circuit, as explained above. The dc magnetization is represented by the first inductance $L_0$, and the frequency characteristic of the complex permeability due to the classical eddy current is represented by the part of the first resistance $R_0$ and after. For practical use, the theoretically infinite stages of the ladder circuit should be truncated to finite stages. The frequency characteristic including the skin effect at a higher frequency can be expressed as the number of circuit stages is larger $[31]$, $[32]$. The parameters $L$ and $R$ in the circuit are given by the following equations:

$$L = \mu,$$

$$R = \frac{4}{\sigma d},$$

where the dc permeability is $\mu$. Here, the units of $L$ and $R$ are H/m and $\Omega$/m, respectively. From Eq. (7) and Eq. (26), it is understood that $R_0 = R/\sigma$ is equivalent to the inverse of $y_1$. Although the Cauer circuit cannot express the dc hysteresis and anomalous eddy current loss in general, the improved model shown in Fig. 25 can take them into consideration $[30]$

In the magnetic circuit model incorporating the play model, which is described in the Chapter 2, the relationship between $H$ and $B$ is given by Eq. (3). On the other hand, in the Cauer circuit presented in Ref. 33), the relationship between $H$ and $B$ is given by the following equations in case of the number of circuit stages is one and more:

$$H = H_a + \frac{1}{3R} \frac{dB}{dt} \pm C_a \frac{dB^3}{dt} \quad (n = 0), \quad (27)$$

$$H = H_a + \sum_{i} \frac{B_i}{L} \left( \frac{1}{R} \frac{dB_i}{dt} \right) \pm C \frac{dB^3}{dt} \quad (n \geq 1). \quad (28)$$

Comparing Eq. (3), Eq. (27), and Eq. (28), the first terms, which are represented by the play model, and the third ones, which are given by the product of $|\partial B/\partial t|^3$ and the coefficient determined by the core loss curves, are equivalent, respectively. Next, the second terms are equivalent in Eq. (3) and Eq. (27) because they both represent the classical eddy current loss caused by the main flux and are given by the product of $dB/dt$ and the coefficient based on the classical eddy current theory. On the other hand, not only the classical eddy current loss due to the main flux but also the influence of the skin effect are expressed in Eq. (28). Hence, the calculation accuracy of the magnetic circuit model at a high frequency can be improved by making it equivalent to Eq. (28). However, it is difficult to express this with just a single magnetic circuit element.

Therefore, Ref. 35) presented the magnetic circuit model incorporating the Cauer circuit theory as well as the play model, as shown in Fig. 26. In this model, the classical eddy current loss including the influence of the skin effect is represented by coupling the magnetic circuit with the ladder circuit, which is equivalent to the part of $R_0$ and after in the Cauer circuit.

4.2 Simulation results by magnetic circuit model incorporating play model and Cauer circuit

Fig. 27(a) and (b) show enlarged views of the comparison of measured and calculated hysteresis loops under PWM voltage excitation at carrier frequencies $f_c = 1$ kHz and 2 kHz, respectively. Here, the ladder circuit is terminated by the second stage, that is, the second resistance $R_t$. From these figures, it is understood that the calculation accuracy of the minor loops is further improved compared to the calculation result shown in Fig. 23.
Fig. 27 Enlarged views of measured and calculated hysteresis loops under PWM voltage excitation (the classical eddy current is represented by the ladder circuit).

Fig. 28 Measured and calculated iron loss normalized by measured values.

Fig. 29 Measured and calculated frequency characteristics of the complex permeability.
5. Modeling Method of Deteriorated Magnetic Properties in Core Material due to Machining Process by Simplified LLG Equation

Magnetic properties of core material are often deteriorated in machining process, which leads to the performance worsening of electric machines. Therefore, the establishment of an analytical method taking deterioration of magnetic properties due to machining process into consideration and the optimization of processing method of core material are essential to development of high-efficiency electric machines.

So far, various kinds of analytical methods considering the deteriorated properties caused by mechanical stress and shrink fitting have been proposed mainly in Japan \[36-38\]. However, these methods have a practical problem that a large number of experimental data should be acquired by using several cores to which different magnitudes of stress is applied. Moreover, most methods only change the relutivity and loss coefficient of core material depending on the magnitude of stress, and none of them can accurately simulate considering change in a shape of the hysteresis loop. On the contrary, in Ref. 40, magnetic properties of core material are predicted when arbitrary compressive stress is applied, by formulating the simplified LLG equation, described in the Chapter 2, considering the influence of stress. In this method, only the measured data of core material, when no stress is applied, is required. On the other hand, in this method, although the tendency of the iron loss to increase with stress can be expressed, the calculation accuracy is not sufficiently high (e.g., up to about 1.5 times of larger calculated values of iron loss than measured ones).

To resolve the above problems, it is necessary to establish a practical and accurate prediction method of deteriorated magnetic properties under arbitrary stress from the minimum essential amount of measured data. Therefore, this chapter introduces that the simplified LLG equation is used for the prediction of deteriorated magnetic properties of arbitrary rolled non-oriented silicon steels from a small amount of measured data of the dc hysteresis loops \[41\].

5.1 Deterioration prediction method by simplified LLG equation

Table 2 and Fig. 30 show specifications and dimensions of tested samples used for measurement. The sample No. 0 is an unrolled one, and the ones No. 1~4 are rolled by applying stresses with different magnitudes so that each one has a different thickness. The rolled ratio, shown in Table 2, represents a reduction rate of a thickness of each rolled sample relative to the unrolled one. Fig. 31 shows the measured dc hysteresis loops with \(B_n = 1.0 \, \text{T}\) for each sample. As shown in these figures, a shape of the hysteresis loop remarkably changes from the original state in machining process.

In Ref. 41, the relationship between the rolled ratio and the parameters of the simplified LLG equation \(h_m\) and \(b_2\), which determines a shape of the hysteresis loop as shown in Fig. 6, is focused on. Table 3 shows the calculated parameters of the simplified LLG equation, and Fig. 32(a) – (e) show enlarged views of the calculated dc hysteresis loops using the parameters shown in this table. From these table and figure, it is understood that measured and calculated results are in good agreement for each sample.

First, the prediction method of \(h_m\) is explained by using the values shown in Table 3. Fig. 33 shows the relationship between the rolled ratio \(x\) and \(h_m\) of each sample. From these figures, it seems that \(h_m\) monotonically increases as \(x\) increases. Here, to predict \(h_m\) at the arbitrary rolled ratio, \(h_m(x)\), which is the ratio

\[
\begin{align*}
\text{Table 2 Specifications of test samples.} \\
| Sample No. | 0 | 1 | 2 | 3 | 4 | 5 |
\end{align*}
\]

| Rolled ratio | 0 | 1 | 2 | 3 | 4 |
|-------------|---|---|---|---|---|
| Density     | 7600 |

\[
\text{Table 3 Parameters of the simplified LLG equation of samples No. 0~4.} \\
\begin{align*}
\begin{array}{cccccc}
\text{No.} & 0 & 1 & 2 & 3 & 4 \\
 h_m/n & 1.09 & 1.04 & 1.05 & 1.07 & 1.08 \\
 b_2 & 1.65 & 1.80 & 1.95 & 2.10 & 2.25 \\
 b_4 & 2.00 & 2.25 & 2.50 & 2.75 & 3.00 \\
 b_6 & 2.50 & 2.75 & 3.00 & 3.25 & 3.50 \\
 b_8 & 3.00 & 3.25 & 3.50 & 3.75 & 4.00 \\
 b_{10} & 3.50 & 3.75 & 4.00 & 4.25 & 4.50 \\
 b_{12} & 4.00 & 4.25 & 4.50 & 4.75 & 5.00 \\
 b_{14} & 4.50 & 4.75 & 5.00 & 5.25 & 5.50 \\
 b_{16} & 5.00 & 5.25 & 5.50 & 5.75 & 6.00 \\
 b_{18} & 5.50 & 5.75 & 6.00 & 6.25 & 6.50 \\
 b_{20} & 6.00 & 6.25 & 6.50 & 6.75 & 7.00 \\
\end{array}
\end{align*}
\]

\[
\begin{align*}
\text{Fig. 30 Dimensions of test samples.} \\
\text{Fig. 31 Measured dc hysteresis loops of each samples with } B_n = 1.0 \, \text{T.} \\
\text{Fig. 32(a) – (e) show enlarged views of the calculated} \\
\text{dc hysteresis loops using the parameters shown in this} \\
\text{Table. From these table and figure, it is understood that} \\
\text{measured and calculated results are in good agreement} \\
\text{for each sample.} \\
\text{Fig. 33 shows the relationship between the rolled ratio } x \\
\text{and } h_m \text{ of each sample. From these figures, it seems that} \\
\text{ } h_m \text{ monotonically increases as } x \text{ increases. Here, to predict} \\
\text{ } h_m \text{ at the arbitrary rolled ratio, } h_m(x), \text{ which is the ratio}
\end{align*}
\]
Fig. 32 Measured and calculated dc hysteresis loops of each sample.

Fig. 33 Relationship between $x$ and $h_{ani}$.

of $h_{ani}$ of rolled and unrolled samples, is defined as the function of $x$ as follows:

$$h_{ani}(x) = \frac{h_{ani}(x)}{h_{ani}(0)}.$$  \hspace{1cm} (29)

Fig. 34 shows the relationship between $x$ and $h_{ani}(x)$. As shown in this figure, $h_{ani}(x)$ changes almost linearly at the rolled ratio of from 3% to 12%. Therefore, can be approximated by the following linear function:

$$h_{ani}(x) = \alpha x + \beta.$$  \hspace{1cm} (30)

where coefficients are $\alpha$ and $\beta$. It may be possible to predict $h_{ani}$ at the arbitrary rolled ratio by using Eq. (30), though it is necessary to consider how to function $h_{ani}(x)$ at the rolled ratio of from 0% to 3% in the future.

Next, the prediction method of $b_{2j}$ is explained. Fig. 35(a) - (e) show the relationships between $x$ and $b_{10}$ of each sample. From these figures, it is revealed that $b_{2j}$ do not monotonically change with respect to $x$, unlike the case of $h_{ani}$. Hence, since it is difficult to give the relationship between $x$ and $b_{2j}$ by a simple relational expression, it is necessary to predict $b_{2j}$ in another way. Here, $b_{2j}$ express the nonlinearity of the rising curve in the first quadrant as described above, that is to say, $b_{2j}$ of the rolled samples can be determined by predicting the nonlinearity of their hysteresis loops. Thus, $g(B)$, which is the function of $B$, is defined as follows:

$$g(B) = \frac{1}{h_{ani}} \times \frac{H_{pre}(B) - H_{nonpre}(B)}{H_{pre}(B) - H_{nonpre}(B)}.$$  \hspace{1cm} (31)

where the magnetic field on the ascending curve of the hysteresis loops of rolled and unrolled samples, where $B$ is positive, is $H_{pre}(B)$ and $H_{nonpre}(B)$, respectively. Fig. 36 shows $g(B)$ derived for each sample and average values.
Fig. 35  Relationship between $x$ and $b_2^2$.

Fig. 36  Relationship between $B$ and $g(B)$.

Fig. 37  Measured and predicted dc hysteresis loops of each sample.
From these figures, it is understood that \( g(B) \) of each sample has almost same values regardless of \( x \). Hence, \( g_{av}(B) \), which is the average of \( g(B) \) of each sample, can be approximated as the polynomial function as follows:

\[
g_{av}(B) = aB^5 + bB^4 + cB^3 + dB^2 + eB + 1, \tag{32}
\]

where coefficients are \( a, b, c, d, \) and \( e \). Substituting Eq. (32) into Eq. (31), the following equation can be obtained:

\[
H_{av}(B) = h_{av} \times H_{magn}(B) \times g_{av}(B). \tag{33}
\]

Therefore, the nonlinearity of the rising curve of the hysteresis loop of the arbitrary rolled sample can be predicted by calculating \( H_{av}(B) \) using Eq. (33).

The predicted dc hysteresis loops are compared with the measured ones, to confirm the validity of the proposed method, as shown in Fig. 37(a) – (d). These figures clarify that measured and predicted results are almost in good agreement for each sample.

Moreover, in Ref. 42, the prediction method in the case of higher rolled ratio is examined and its validity is indicated, though that is omitted in this paper.

6. Conclusion

This paper presented various studies on the hysteresis modeling for electric machines based on the magnetic circuit method. It was clear that the iron loss including the magnetic hysteresis behavior can be calculated at high accuracy with a relatively simple model based on the RNA, even for machines with complex shapes such as an electric motor. In addition, it was found that the magnetic circuit analysis considering the skin effect and the deterioration prediction of machined core’s magnetic properties are possible, so that further improvement in the calculation accuracy of the RNA is expected by incorporating them. Furthermore, the proposed method does not require a high-performance computer for conducting a large-scale numerical analysis and special experimental equipment for measuring a lot of hysteresis loops with high accuracy, hence it will be one of the practical solutions for design and analysis of electric machines.

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