Could There Be Something Rather Than Nothing?

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Abstract

There is increasing evidence that the universe may have a small cosmological constant. We suggest a scheme for naturally generating a small cosmological constant. Our idea requires the presence of a discrete accidental symmetry which is spontaneously broken by vacuum expectation values of the fields, and explicitly broken by high dimensional operators in the Lagrangian.

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1. Introduction

Does the universe have an appreciable cosmological constant \([\Lambda]\)? The cosmological constant \(\Lambda\) is defined by

\[
H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \rho_m + \frac{\Lambda}{3} - \frac{k}{a^2},
\]

(1.1)

where \(H\) is the Hubble parameter, \(a\) is the scale factor of the universe, \(\rho_m\) is the energy density contained in the matter in the universe, and \(k = -1, 0, \) or \(+1\) depending on whether the universe is open, flat, or closed, respectively. It is useful to think of \(\Lambda\) as an energy density by defining

\[
\rho_\Lambda = \frac{\Lambda}{8\pi G},
\]

(1.2)

so that

\[
H^2 = \frac{8\pi G}{3} \rho_{\text{tot}} - \frac{k}{a^2},
\]

(1.3)

where \(\rho_{\text{tot}} = \rho_m + \rho_\Lambda\).

Since it is essentially impossible to measure \(\rho_\Lambda\) directly, it is necessary to measure various observables in the universe such as the Hubble parameter, the density parameter \(\Omega = \frac{8\pi G \rho_{\text{tot}}}{3 H^2}\), and the deceleration parameter \(q_0 = -\ddot{a}/\dot{a}^2\). Measurement of these parameters can demonstrate the presence or absence of an appreciable cosmological constant.

What, if anything, does particle physics predict for the cosmological constant? The naive value predicted by integrating the zero-point energies of all possible modes in the vacuum with a cutoff at the Planck scale disagrees with observations by some 120 orders of magnitude \([3]\). Hence, to a particle physicist, the relevant question is not what the actual value of \(\Lambda\) is, but why it is so inconceivably small. The fact that it is so small has led many to conclude that it must be zero, and that this value of zero must be enforced by some, as yet unknown, mechanism.

In addition to the standard assumption that \(\Lambda\) is precisely zero, it is common to assume that \(\Omega\) is exactly equal to one. This is due to the fact that, since \(\Omega = 1\) is an unstable fixed point in the evolution of the universe, it is very unlikely that it would be close to one today unless it started out at a value that is almost precisely one. Since dynamical evidence indicates \(\Omega \gtrsim 0.2\), and the age of the universe implies \(\Omega \lesssim 1–2\), \(\Omega\) must have been so close to unity in the early universe that many cosmologists are convinced it must be...
precisely one. In addition, if inflation provides the correct explanation for the horizon and flatness problems \cite{3}, then $\Omega$ must be one today.

However, it is becoming increasingly apparent that one of these assumptions ($\Lambda = 0$ and $\Omega = 1$) may have to be discarded. There are several reasons for this, and we will briefly discuss the most compelling of these, the so-called “age problem”. The age of the universe today $t_0$ can be written in terms of the Hubble constant today $H_0$ and the fraction of critical density $\Omega_i = \frac{8\pi G \rho_i}{3H_0^2}$ in the matter and vacuum as

$$t_0 = H_0^{-1} \int_0^1 \frac{dw}{\sqrt{1 + \Omega_m (w^{-1} - 1) + \Omega_\Lambda (w^2 - 1)}},$$

(1.4)

where $w = a/a_0$, the ratio of the size of the universe to its current size. This formula assumes all the matter in $\rho_m$ is nonrelativistic; if some of it is relativistic, the age will decrease somewhat. It is easy to see that the universe gets older as $\Omega_m$ decreases or $\Omega_\Lambda$ increases. For the case where $\Omega = \Omega_m = 1$ (the “standard” assumption), we have the simple result $t_0 = \frac{2}{3}H_0^{-1}$. In figure fig. \[ is plotted the age when we either drop the assumption $\Omega = 1$ while retaining $\Lambda = 0$ or drop the assumption $\Lambda = 0$ while retaining $\Omega = 1$. The point here is that, if it can be shown conclusively that there exist objects in the universe older than $\frac{2}{3}H_0^{-1}$, then we will be forced to conclude that, either $\Omega < 1$, or there exists some form of energy density in the universe other than matter or radiation. Indeed, if the universe is older than $H_0^{-1}$, then we must include a cosmological constant even if $\Omega_m < 1$.

This is significant, because it turns out that the best measurements for $H_0$ seem to be clustering around values that are inconsistent with the ages of the oldest stars in the galaxy assuming $\Omega = 1$ and $\Lambda = 0$. The best fit ages of the oldest globular clusters are $15 - 18$ Gyr, with lower limits in the range of $12 - 14$ Gyr \cite{4}. These conclusions are extremely difficult to arrive at, and the final word may not be in, but these results should be taken seriously. Since the universe must be older than the oldest stars, we will assume the universe must be older than 12 or 15 Gyr.

It is beyond the scope of this paper to undertake a general review of the various methods of measuring $H_0$ \cite{5}. Roughly speaking, one can divide methods for determining $H_0$ into two categories: methods that measure brightness, assume objects are standard candles, and determine the distance by calibrating using nearby objects whose distance can be found by other means; and methods that attempt to measure distance directly using some physical phenomenon which, it is hoped, is reasonably well understood. The
most reliable methods for determining $H_0$ tend to fall into the first category, and those discussed in [2] give values for $H_0$ of either $80 \pm 11$ or $73 \pm 11$ (in units of km s$^{-1}$Mpc$^{-1}$), depending on how the distance to the Virgo cluster is calculated.

The methods that attempt to measure distances directly tend to be less well developed than the brightness methods, and thus are generally thought to be less reliable at this time. However, several of these methods do tend to give lower values for $H_0$. One example is the Sunyaev-Zeldovich effect, which Birkinshaw, Hughes, and Arnaud [6] used to measure the distance to one cluster, from which they found $H_0 = (40 - 50) \pm 12$ (the range given is due to systematic uncertainties, while the error is due mainly to random effects). In addition, gravitational lens time delays can, in principle, be measured to give the distance to lensing galaxies, and this also has been applied to one lens system [7]. Again, this gives a (slightly) low value for $H_0$ (the best value seems to be $H_0 = 61 \pm 7$, although this assumes $\Omega = 1$). However, there is a great deal of uncertainty in this determination coming from uncertainties about the lensing mass distribution, and uncertainties about the geometry of the universe [8].

The one example of a method which attempts to measure distances directly in which there are significant statistics and which is thought to be fairly reliable at this time is the expanding photosphere method, used on type II supernovae [9]. This method gives a value of $H_0 = 73 \pm 9$ [10], which is consistent with the values derived from methods using brightness determinations.

Although the results are inconclusive, it is fair to say that, with the exception of a few techniques, measurements of $H_0$ seem to indicate $H_0$ has a relatively high value (in the neighborhood of $70 - 80$). Figure fig. [1] shows, assuming $H_0 > 65$ km s$^{-1}$Mpc$^{-1}$, that there is a conflict between the age of the oldest stars and the assumptions $\Lambda = 0$ and $\Omega = 1$. In all cases, $H_0 t_0 = \frac{2}{3}$ is well outside the allowed ranges. Interestingly, for the best fit globular cluster ages, if $H_0$ lies in the given range, even the case $\Omega = \Omega_m$, $\Omega_\Lambda = 0$ is ruled out, implying the necessity that $\Omega_\Lambda \neq 0$. It is only when one uses the lowest allowed values for $t_0$ that one is not forced into a model with $\Omega_\Lambda \neq 0$. Thus the experimental case for nonzero $\Omega_\Lambda$ is compelling, if not overwhelming.

In this paper, we examine the question of whether a small value for the cosmological constant can arise, in any sense, naturally. We find that, making certain seemingly reasonable assumptions, it can. In section 2, we outline the basic scheme. In sections 3 and 4, we discuss some toy models in which this scheme is realized, and, in section 5, we briefly discuss the issues of vacuum stability and domain walls arising from this scheme.
2. How a Small, Non-zero Cosmological Constant Might Arise

Because there is no viable quantum theory of gravity, it is impossible to predict the energy density of the vacuum. Its measured smallness implies that it may well be zero. In this paper, we assume that the true minimum of the particle potential will have exactly zero energy density. Although we do not promote a specific mechanism for achieving this, we note that Coleman [11] has argued that, if wormholes exist, they would have the effect of requiring the vacuum energy density of the ground state to be zero. However, note that this is only a statement about the ground state. There may be other false vacua which would have non-zero energy density. This is explicitly stated in the work by Coleman [11], but, indeed, if we are to preserve the idea of inflation (a motivation for maintaining $\Omega = 1$), then we are forced to imagine false vacua with non-zero energy density. Hence we assume that there is no mechanism for zeroing the energy density in false vacua, only in the true vacuum. We will not attempt to explain the zero energy density of the true vacuum, but simply assume it. In other words, we are not trying to understand why the universe should have zero cosmological constant, but rather, given that it should have zero cosmological constant, why it doesn’t.

Our second assumption is that the universe is, in fact, in a false vacuum state. This false vacuum state must have an energy density very close to the true vacuum energy. Our goal is to explain the smallness of the splitting.

In addition to the above assumptions, we also assume that the full low-energy particle theory must include all possible particle interactions; that is, all interactions that aren’t ruled out by any gauged or discrete symmetries of the full theory, independent of whether or not they are renormalizable. This is just an acknowledgment that we don’t know what the full particle theory should be, and that what we really have is a low-energy effective theory valid up to some energy scale at which we’ve integrated out all of the heavy physics. Thus, when writing down possible operators in the effective theory, we expect the coefficients of the non-renormalizable operators to be suppressed by the appropriate power of the mass scale at which the effective theory becomes invalid (e.g. a dimension seven operator would have a coefficient of the form $O(1)/M^3$).

Why have we restricted ourselves to discrete and gauge symmetries? Gauge symmetries are known to exist, and it seems likely that they are preserved by any non-renormalizable interactions. Furthermore, discrete symmetries can arise naturally as unbroken subgroups of spontaneously broken gauge symmetries. In contrast, it is believed that global symmetries must always be broken by quantum gravity effects. Hence
gauge and discrete symmetries may naturally be imposed on both renormalizable and non-renormalizable interactions.

Given these assumptions, the scheme works as follows. It is often the case that, when one enforces a certain set of symmetries on operators in a theory, certain subsets of the operators exhibit additional “accidental” symmetries. It is our conjecture that the Lagrangian of the universe will exhibit just such an accidental symmetry—a symmetry which is only broken by a fairly high dimensional operator. This symmetry must be discrete. Then, when the scalar fields in the theory acquire vacuum expectation values (VEV’s), this term will make different contributions to the vacuum energy corresponding to transformations under the (broken) discrete symmetry. If the operator is sufficiently suppressed, the difference in vacuum energy coming from the contribution in different vacua could provide the cosmological constant.

3. An Example Involving Discrete Symmetries

An extremely simple example which illustrates the scheme we are proposing is as follows. Assume that we have a set of $N$ real scalar fields $\phi_i$, $i = 1, \ldots, N$, which transform under the permutation group $S_N$, with the additional symmetry $\phi_i \rightarrow -\phi_i$, $\phi_j \rightarrow -\phi_j$ for $i \neq j$. The renormalizable terms in the scalar potential consistent with these symmetries are

$$V(\phi) = -m^2 \sum_i \phi_i^2 + \lambda_1 \left( \sum_i \phi_i^2 \right)^2 + \lambda_2 \sum_{i<j} (\phi_i^2 - \phi_j^2)^2.$$  (3.1)

Clearly, these terms also exhibit the symmetry $\phi_i \rightarrow -\phi_i$, even though we didn’t explicitly enforce this symmetry. If we assume that all three of the parameters $m^2$, $\lambda_1$, and $\lambda_2$ are positive, the minima of the potential will be given by

$$\langle \phi_i \rangle = \pm \frac{m}{\sqrt{2N\lambda_1}}.$$  (3.2)

These $2^N$ vacua are not all guaranteed to be truly degenerate. They break into two sets of $2^{N-1}$ vacua which are related among themselves by the exact symmetry, and to each other by the accidental symmetry. The degeneracy between these two sets is broken by the first term one can write down that breaks this accidental symmetry, which is the term

$$\frac{\lambda_0}{M^{N-4}} \phi_1 \phi_2 \cdots \phi_N.$$  (3.3)
Thus, in this model, there exist two discrete sets of minima where the difference in the vacuum energy between the two sets of minima is given, at lowest order in the VEV’s, by the term proportional to (3.3).

It is easy to calculate how large $N$ has to be in order to get specific values for the cosmological constant. If the $\phi_i$ get a VEV that is of order the weak scale ($\sim 100$ GeV), then this term would make a contribution of $\sim 10^8(100 \text{GeV}/M)^N - 4$ GeV$^4$ to the vacuum energy density. Thus, if we want a cosmological constant with $\rho_{\text{vac}} \sim 10^{-46}$ GeV$^4$, and if we assume that the heavy mass scale is the Planck mass ($\sim 10^{19}$ GeV), then we require $N = 7$. Alternatively, if the $\phi_i$ get a VEV of $\sim 10^{16}$ GeV (that is, something like the GUT scale), then, again using the Planck mass as our heavy mass scale, we require $N = 40$.

4. An “Accidental” Symmetry in $SO(N)$

An alternative approach to using discrete symmetries would be to use gauge symmetries, for example, $SO(N)$. For this example, we have again chosen a relatively simple gauge group where one can, in some sense, intuitively understand the accidental symmetry and how it arises. For the moment, let us restrict ourselves to a theory containing an arbitrary number of scalar fields $\Psi^i$ that transform under the vector representation of $SO(N)$. If we label the components of $\Psi^i$ by $\psi^i_{\mu}$, then, in general, any potential one could write down would have an accidental symmetry as we have described under which $\psi^i_{\mu} \rightarrow -\psi^i_{\mu}$ for all $i$ (more generally, any odd number of components of the $\Psi^i$ can change sign under this symmetry). It should be apparent that this symmetry (which we will call “parity”) is not, in fact, a symmetry of the theory since the determinant of the operator that gives this transformation is equal to $-1$.

How does this parity symmetry arise? Recall that, in the tensor representations of $SO(N)$, the only invariant tensors are the delta symbol $\delta_{\mu\nu}$, and the fully anti-symmetric epsilon symbol $\epsilon_{\mu_1\mu_2\cdots\mu_N}$, along with any other tensors formed by multiplying these tensors together in various combinations. Because the $\delta$ tensor is invariant under parity while the $\epsilon$ tensor changes sign, only terms involving the $\epsilon$ tensor can be parity noninvariant. The smallest such term will be one in which $\epsilon$ is contracted with $N$ distinct $\Psi^i$’s. Note that, in order for this term to actually break the degeneracy of the vacua, we require that there be at least $N$ $\Psi^i$’s in the theory, and that their VEV’s span the $SO(N)$ space.
To be more specific, let us imagine an SO(2\(n\)) symmetric theory with one scalar field \(\Phi\) in the adjoint (antisymmetric tensor) representation with components \(\phi_{\mu\nu}\). The renormalizable portion of the potential is given by

\[
V(\Phi) = m^2 \frac{1}{2n} \text{Tr}(\Phi^2) + \lambda_1 \left( \frac{1}{2n} \text{Tr}(\Phi^2) \right)^2 + \lambda_2 \left[ \frac{1}{2n} \text{Tr}(\Phi^4) - \left( \frac{1}{2n} \text{Tr}(\Phi^2) \right)^2 \right], \tag{4.1}
\]

where we have used a matrix notation for \(\Phi\). If \(m^2\), \(\lambda_1\), and \(\lambda_2\) are all positive, the minimum of the potential will be given by

\[
\langle \phi^0_{\mu\nu} \rangle = \frac{m}{\sqrt{2\lambda_1}} \begin{pmatrix}
0 & \mp 1 & 0 & 0 & \cdots & 0 & 0 \\
\pm 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & 1 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \cdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & 0 & -1 \\
0 & 0 & 0 & 0 & \cdots & 1 & 0
\end{pmatrix}, \tag{4.2}
\]

where \(\mp\) and \(\pm\) occur in only the first row and column. These minima are related by the parity symmetry and degenerate to this level.

Using this, let us now look at the parity violating contributions to the scalar potential (the extension of our definition of parity to tensor reps of SO(2\(n\)) is simply that any component \(\phi_{\mu_1\mu_2\cdots\mu_m}\) where an odd number of the \(\mu_i\) equal 1 changes sign under parity). The lowest dimensional operator that we can write using \(\Phi\) that violates parity has the form

\[
V_{\text{break}} = \frac{\lambda_0}{n! M^{n-4}} \varepsilon_{\mu_1\cdots\mu_{2n}} \phi_{\mu_1\mu_2} \phi_{\mu_3\mu_4} \cdots \phi_{\mu_{2n-1}\mu_{2n}}. \tag{4.3}
\]

Assuming, once again, that the vacuum expectation value is of order the weak scale, and the large mass is of order the Planck scale, we can get a cosmologically interesting vacuum energy density for \(n = 7\).

5. Domain Walls and Vacuum Stability

With this scheme in mind, one might wonder whether we must, of necessity, have domain walls separating regions in the true vacuum from regions in the false vacuum. Similarly, one might wonder whether high energy interactions in a region that is in the false vacuum might cause bubbles of true vacuum to nucleate, thus erasing any cosmological constant.
The first problem is easily solved if the universe undergoes a period of inflation after the universe “chooses” its vacuum state. The inflationary epoch then inflates the domain walls out beyond the observable horizon, and the problem is solved in the same way that similar problems with, for example, monopoles are solved [2].

As for the question of vacuum stability, it is easy to see that the false vacuum will, in fact, be stable. Let us assume that the universe is currently in the false vacuum, and calculate the critical radius at which a bubble of the true vacuum would expand and fill the universe. We do this by considering the energy associated with a bubble of true vacuum of a given radius $R$. This is given by

$$E_{\text{bubble}} = -\frac{4}{3} \pi R^3 \rho_\Lambda + 4\pi R^2 \sigma,$$

(5.1)

where $\rho_\Lambda$ is the energy difference between the true vacuum and the false vacuum and $\sigma$ is the surface tension, typically of order $v^3$, where $v$ is the VEV which breaks the accidental symmetry. If $dE_{\text{bubble}}/dR$ is negative, the bubble will grow; for smaller bubbles, it will shrink. Setting $dE_{\text{bubble}}/dR = 0$ we find that the critical bubble size is given by

$$R_{\text{crit}} = \frac{2\sigma}{\rho_0} = \left(\frac{75 \text{ km s}^{-1} \text{Mpc}^{-1}}{H_0}\right)^2 \Omega_\Lambda^{-1} \left(\frac{\sigma}{(100 \text{ GeV})^3}\right) 10^{21} \text{ Lt} - \text{yr}.$$

(5.2)

In other words, if the scale of the VEV separating the true vacuum from the false vacuum is around the weak scale, then, for a bubble of the true vacuum to nucleate and grow, it would have to form with a size that is at least $10^{10}$ times the size of the observable universe! Clearly, the possibility of bubble nucleation is not a problem for the cosmological constant in this scheme (i.e. the difference between the false and true vacua is irrelevant as far as particle physics is concerned).

6. Conclusions

A small cosmological constant is not as difficult to generate as might be imagined. The appearance of a spontaneously broken accidental discrete symmetry, which is broken by non-renormalizable terms, naturally leads to a small splitting between two nearly degenerate vacua. These accidental symmetries may occur in these models due to imposed symmetries, either discrete or gauged.

If the lower of the two minima of the potential has a naturally zero cosmological constant, then a universe stuck in the other minimum will have a very small apparent
cosmological constant. Such a constant may help resolve the universe’s “age problem.” To avoid a universe where nearby regions are in distinct minima, implying the existence of domain walls, it is necessary to assume inflation occurred at or after the time of spontaneous symmetry breaking. Because of the smallness of the splitting between the true and false vacua, there is no danger of the universe preferring one vacuum to the other, nor is there any risk of tunneling to the true vacuum from the false vacuum.

Unfortunately, the specific models we have proposed are unconstrained, since the scale of the symmetry breaking and the scale of the non-renormalizable terms, are completely unconnected to anything we know. A natural scale for the appearance of non-renormalizable terms is the Planck scale, but the scale of symmetry breaking is still free. We view this as an unattractive feature of our models. This problem can be removed in both of the models proposed simply by promoting the fields to SU(2) × U(1) doublets with appropriate hypercharge so that they can acquire a neutral VEV. Then the scale for this VEV can be shown to be at or slightly below the weak scale. Unfortunately, this introduces a huge number of additional doublets into the standard model. This may cause problems with oblique corrections to the standard model. Preliminary estimates indicate that the $S_N$ model (which introduces only 7 new doublets for $N = 7$) is probably allowed, but the SO(14) model (with 91 doublets) is in trouble. Such a theory would be more testable than the one proposed here, but would still be little more than a toy model.

Much more promising is the idea of having the symmetry group we impose be a grand unified theory (GUT) symmetry. Several complications ensue, such as the necessity of using a high power of $M_{\text{GUT}}/M_P$ to get an interesting cosmological constant. But we still feel this is the most promising direction for further development.

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References

[1] For a recent review of the status of the cosmological constant, see S.M. Carroll, W.H. Press, and E.L. Turner, Annu. Rev. Astron. Astrophys., 30, 499 (1992).

[2] For a discussion of this point, see, for example, S. Weinberg, Rev. Mod. Phys., 61, 1 (1989), or, for a nontechnical discussion, see L. Abbott, Sci. Am., 258(5), 106 (1988), or D.H. Freedman, Discover, 11(7), 46 (1990).

[3] A.H. Guth, Phys. Rev. D, 23, 347 (1981).

[4] For a reasonably concise introduction to this topic, see [1].

[5] See, for example, G.H. Jacoby, D. Branch, R. Ciardullo, R.L. Davies, W.E. Harris, M.J. Pierce, C.H. Pritchet, J.L. Tonry, and D.L. Welch, PASP, 104, 599 (1992).

[6] M. Birkinshaw, J.P. Hughes, and K.A. Arnaud, Ap. J. 379, 466 (1991).

[7] For the calculation of the time delay in this system, see W.H. Press, G.B. Rybicki, and J.N. Hewitt, Ap. J., 385, 416 (1992).

[8] For a discussion of these points and a general review of gravitational lensing, see R.D. Blandford and R. Narayan, Annu. Rev. Astron. Astrophys., 30, 311 (1992).

[9] For a description of this method, see B.P. Schmidt, R.P. Kirshner, and R.G. Eastman, Ap. J., 395, 366 (1992).

[10] B.P. Schmidt, private communication.

[11] S. Coleman, Nucl. Phys. B310, 643 (1988)

[12] K. Sato, Phys. Lett., 99B, 66 (1981).
Figure Captions

Fig. 1. $H_0 t_0$ versus $\Omega_m$ for two cases: $\Omega = 1$, $\Omega_\Lambda = 1 - \Omega_m$ (long dashes); and $\Omega = \Omega_m$, $\Omega_\Lambda = 0$ (short dashes). Note that, if $H_0 > 65 \text{ km s}^{-1}\text{Mpc}^{-1}$ and $t_0 > 12 \text{ Gyr}$, then the universe cannot have $\Omega = \Omega_m = 1$, $\Omega_\Lambda = 0$. Indeed, if $H_0 > 65 \text{ km s}^{-1}\text{Mpc}^{-1}$ and $t_0 > 15 \text{ Gyr}$, then even the values $\Omega = \Omega_m$, $\Omega_\Lambda = 0$ aren’t allowed.