The wedge form of relativistic dynamics. II. The Gluons

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I derive expressions for various correlators of the gauge field and find propagators in a new gauge $A^\tau = 0$. This gauge is a part of the wedge form of relativistic dynamics suggested earlier [1] as the tool for the study of quantum dynamics in collisions of hadrons at extremely high energies and in ultrarelativistic heavy ion collisions. The new gauge puts the quark and gluon fields of the colliding hadrons in one Hilbert space and thus allows one to avoid factorization.

I. INTRODUCTION

In the previous paper [2] I explained physical motivation of the “wedge form of dynamics” as a promising tool to explore the processes which take place during the collision of two heavy ions. In compliance with the general definition of dynamics given by Dirac [3], the wedge form includes its specific definition of the quantum mechanical observables on the space-like surfaces, as well as the means to describe evolution of the observables from the “earlier” space-like surface to the “later” one. Unlike the other forms, the wedge form explicitly refers to the two main geometrical features of the phenomenon, strong localization of the initial interaction and, as a consequence, the absence of the translational invariance in the temporal and longitudinal directions.

Usually, every hadron (or nucleus) before the collision are considered separately, in their own infinite momentum frame. Thus, we always deal with two different Hamiltonian dynamics with their own definition of the time variable. This problem is seemingly resolved by the factorization scheme which replaces the true bound state by an artificial flux of free partons. However, the class of the physical processes which comply with this method is restricted to the inclusive production of jets with the high transverse momenta. Factorization is obviously violated already in the diffractive deep inelastic electron-proton scattering and, in fact, is not the case in any semi-inclusive process. The constraints imposed by the factorization become critical when one is compelled to consider interaction of the two bounded systems without appealing to the parton picture [2], which is the case of the heavy-ion collision. One cannot design this experiment in such a way that the scale of the “hard probe,” like, e.g., the dilepton mass in the Drell-Yan process, will become a measured parameter. In fact, the nuclei a priori probe each other at all scales and the expected factorization scale turns into ill-defined infra-red cut-off. Therefore, it is necessary to find a way to describe both colliding systems, the hadrons or the nuclei, using the same Hamiltonian dynamic. This requirement follows solely from the fact that the definition of the field states (particles) depends on how the observables are defined. The problem is most painful for gluons since the choice of the gauge is one of the elements of the Hamiltonian dynamic. It manifestly affects the definition of the physical states of the gauge fields. Indeed, since every dynamics has a specific definition of the time variable, Gauss’ law which defines the non-dynamical longitudinal fields, also looks differently.

Factorization succeeds to fragment the cross section of the hadronic process into the hard partonic cross section, which is gauge invariant because the partons are considered as the free on mass shell particles, and the gauge invariant structure functions. By the failure of the factorization, one is lead back to the generic field-theory approach. It strongly requires the gluon fields from the incoming hadrons be given within the same gauge.

The gluons are expected to be most abundant and active at the early stage of the heavy-ion collision. According to estimates of Shuryak and Xiong [4] most of the entropy should be produced due to the gluon interactions. However, before the collision, the quark and the gluon fields are assembled into the two coherent wave packets, the nuclei, and, therefore, the initial entropy equals zero. The coherence is lost and the entropy is created only due to interaction. Though it is tempting to rely on the general formula, $S = \text{Sp} \ln \rho$, which invariantly expresses the entropy via the density matrix $\rho$, at least one basis of states should be given explicitly. It is imperative to find what are the states that may be used for computing the entropy.

In the wedge form of dynamics, the states of the quark and gluon fields are defined on the space-like hypersurfaces of the constant proper time $\tau$, $\tau^2 = t^2 - z^2$. The states of scalar and fermion fields were discussed in Ref. [1]. In this paper, we continue the study for the gluons and augment our previous consideration by the gauge condition $A^\tau = 0$. This simple idea solves several problems. First, it becomes possible to treat two different light-front gauges which
describe gluons from each nucleus of the initial state separately, as the two limits of this single gauge. Therefore, the new approach keeps important connections with the theory of the deep inelastic e-p scattering. This fact is vital for the subsequent calculations since the e-p deep inelastic scattering is the only existing source of the data on the nucleon structure in the high-energy collisions. An alternative point of view is based on the classical model of the large nucleus in the infinite momentum frame. Second, after collision, this kind of a gauge becomes a local temporal axial gauge, thus providing a smooth transition to the Bjorken regime of the boost-invariant expansion.

Most part of this paper is technical, and any relevant physical discussion of the results always goes only after their mathematical derivation. In section II A, I derive equations of motion for the gauge field in the gauge \( A^\tau = 0 \), find Hamiltonian variables and normalization condition. Equations of motion are linearized and the modes of the free radiation field are obtained in section II B. In section II C the retarded propagator of the perturbation theory is found as the response function of the field on the external current. I carefully examine possibility to separate dynamics of the transverse and longitudinal fields and arrive at the negative conclusion. This part of calculation turned out to be the most time-consuming, since the gauge condition is inhomogeneous and none of the currently used methods is effective. The old-fusioned variation of parameters does work. To reassure its effectiveness, propagators of more familiar gauges, \( A^0 = 0 \) and \( A^\pm = 0 \), were computed in Ref. 6. In section II E I show that the previously obtained propagator solves the initial data problem for the gauge field.

Section III is devoted to the quantization of the vector field in the gauge \( A^\tau = 0 \). I begin in III A with computation of the Wightman functions and study the causal properties of the commutators in III B. The latter appears to be abnormal, the Riemann function is not symmetric and penetrate the exterior of the light cone. However, behavior of the observables is fully causal and procedure of the canonical quantization is accomplished in III C. Even though it is impossible to introduce transverse and longitudinal currents and thus fully separate dynamics of the corresponding fields, I found useful to classify various field patterns by the type of their propagation. Propagator of the transverse field is sensitive to the light cone boundaries while longitudinal and instantaneous parts of the field do not really propagate. These two fragments of the response function are derived in section IV. In section IV I study the limit behavior of the propagator in the central rapidity region and in the vicinity of the null-planes. I show that propagators of the gauges \( A^0 = 0 \) and \( A^\pm = 0 \), respectively are recovered. This result is practically important because it establishes connection of the new approach with the existing theory of the deeply inelastic processes at high energies.

II. THE CLASSICAL TREATMENT

A. Classical equations of motion

Here we consider the case of pure glue-dynamics. We denote \( A_\mu(x) = t^a A^a_\mu(x) \), the gluon field in the fundamental representation of the color group. Consequently, we have the field tensor,

\[
F_{\mu\nu} = t^a F^a_{\mu\nu} = D_\mu A_\nu - D_\nu A_\mu = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu],
\]

where \( D_\mu = \partial_\mu - ig[A_\mu(x), ...] \) is the covariant derivative on the local color group. The gauge invariant action of the theory looks as follows,

\[
S = \int \mathcal{L}(x)dt^4x = \int \left[ -\frac{1}{4} g^{\mu\lambda}(x)g^{\nu\sigma}(x)F_{\mu\nu}(x)F_{\lambda\sigma}(x) - j^\mu A_\mu \right] \sqrt{-g}dt^4x.
\]

Its variation with respect to the gluon field yields the Lagrangian equations of motion,

\[
\partial_\lambda [(-g)^{1/2}g^{\mu\lambda}g^{\nu\sigma}F_{\mu\nu}] - ig(-g)^{1/2}[A_\lambda, g^{\mu\lambda}g^{\nu\sigma}F_{\mu\nu}] = (-g)^{1/2}j^\sigma,
\]

where \( j^\mu \) is the color current of the fermion fields and \( g = \det|g_{\mu\nu}| \). The equations are twice covariant: with respect to the gauge transformations in color space and the arbitrary transformations of the coordinates. In what follows we shall employ the special coordinates associated with the constant proper time hypersurfaces inside the light cone of the collision point \( t = z = 0 \). The new coordinates parameterize the Minkowsky coordinates \((t, x, y, z)\) as \((\tau \cosh \eta, x, y, \tau \sinh \eta)\). In addition, we impose the gauge condition \( A_\tau = 0 \). The corresponding gauge transformation is well defined. Indeed, let \( A_\mu(x) \) be an arbitrary field configuration and \( A'_\mu(x) \) its gauge transform with the generator

\[
U(\tau, \eta, r^\perp) = P_\tau \exp\left\{-\int_0^\tau A_\tau(\tau', \eta, r^\perp) d\tau'\right\}
\]

Then the new field, \( A'_\mu = UA_\mu U^{-1} + \partial_\mu UU^{-1} \), obey the condition \( A'^\tau = 0 \). Imposing this gauge condition we arrive at the system of four equations:
\[ C(x) = \frac{1}{\tau} \partial_\eta \partial_\tau A_\eta + \tau \partial_\tau \partial_\tau A_\tau - ig \left\{ \frac{1}{\tau} [A_\eta, \partial_\tau A_\eta] + \tau [A_\tau, \partial_\tau A_\tau] \right\} - \tau j^\tau = 0, \quad (2.4) \]

\[ - \partial_\tau \tau \partial_\tau A_\tau + \frac{1}{\tau} \partial_\eta (\partial_\eta A_\tau - \partial_\tau A_\eta) + \tau \partial_\eta (\partial_\eta A_\tau - \partial_\tau A_\eta) \]

\[ - ig \left\{ \frac{1}{\tau} \partial_\eta [A_\eta, A_\tau] + \tau \partial_\eta [A_\tau, A_\eta] + \frac{1}{\tau} [A_\eta, F_{\eta \tau}] + \tau [A_\tau, F_{s \tau}] \right\} - \tau j^\tau = 0, \quad (2.5) \]

\[ - \partial_\tau \frac{1}{\tau} \partial_\tau A_\eta + \frac{1}{\tau} \partial_\tau (\partial_\tau A_\tau - \partial_\eta A_\eta) - ig \left\{ \frac{1}{\tau} \partial_\eta [A_\tau, A_\eta] + \frac{1}{\tau} [A_\tau, F_{\tau \eta}] \right\} - \tau j^\eta = 0 \quad (2.6) \]

Here, we use the latin indices from \( r \) to \( w \) for transverse \( x \) - and \( y \) -components, \( r, ..., w, = 1, 2 \). We shall also use the arrows over the letters to denote the two-dimensional vectors, like \( \vec{k} = (k_x, k_y) \), \( |\vec{k}| = k_\perp \). The latin indices from \( i \) to \( n \), \( i, ..., n = 1, 2, 3 \), will be used for the three-dimensional internal coordinates \( u^i = (x, y, \eta) \) on the hypersurface \( \tau = \text{const.} \)

The metric tensor has only diagonal components \( g_{\tau \tau} = -g_{xx} = -g_{yy} = 1 \), \( g_{\eta \eta} = -\tau^2 \). The first of these equations contains no second order time derivates and is a constraint rather than a dynamical equation. The constraint weakens to zero in classical Hamiltonian dynamics and serves as the condition on physical states in quantum theory. The canonical momenta of the theory are as follows:

\[ p^\tau = 0, \quad p^\eta = \frac{1}{\tau} F_{\tau \eta} = \frac{1}{\tau} \dot{A}_\eta, \quad p_\tau = \tau F_{\tau \tau} = \tau \dot{A}_r. \quad (2.7) \]

Hereon, the dot above the letter denotes derivative with respect to the Hamiltonian time \( \tau \). Because of the gauge condition, the canonical momenta do not contain the color commutators. After excluding the velocities, the Hamiltonian can be written down in the canonical variables:

\[ H = \int d\eta d\tau \frac{1}{2} p^\eta p^\eta + \frac{1}{2\tau^2} p^\tau p^\tau + \tau \frac{1}{2} F_{\tau \eta} F_{\tau \eta} + \frac{1}{4} F_{rs} F_{rs} + j^\eta A_\eta + j^\tau A_\tau \quad (2.8) \]

Then the equations (2.3) and (2.6) are immediately recognized as the Hamiltonian equations of motion. The Poisson bracket of the constraint \( C \) with the Hamiltonian vanishes thus creating the generator of the residual gauge transformations which are tangent to the hypersurface. Conservation of the constraint is a direct consequence of the Lagrange (or Hamiltonian) classical equations of motion as well.

The normalization condition for the one-particle solutions is obviously derived from the charge conservation law. For the gauge field this is impossible. Therefore we shall accept the condition which supports self-adjointness of the homogeneous system after its linearization. This leads to a natural definition for the scalar product of the states of the vector field in the gauge \( A^\tau = 0 \):

\[ (V, W) = \int_{-\infty}^{\infty} d\eta \int d^2\tau \sqrt{g} \delta^{ik} V_{i}^{*} W_{k} \quad (2.9) \]

where \( g^{ik} \) is the metric tensor of the three-dimensional internal geometry of the hypersurface \( \tau = \text{const.} \). This norm of the one-particle states prevents them from the flow out of the interior of the past and future light wedges of the interaction plane.

**B. Modes of the free radiation field. Field of the static source**

As a tool for the future development of the perturbation theory, we need to find the propagators and Wightman functions when the nonlinear self-interaction of the gluon field is switched off. In this case the system of the equations for the nonvanishing components of the vector potential and the constraint look as follows:

\[ \left[ \partial_\tau \partial_\tau - \frac{1}{\tau} \partial_\eta^2 - \tau \partial_\tau^2 \right] A_\tau + \partial_\tau \left[ \tau \partial_\eta A_\eta + \frac{1}{\tau} \partial_\eta A_\eta \right] = -\tau j^\tau, \quad (2.10) \]

\[ \left[ \partial_\tau \frac{1}{\tau} \partial_\tau - \frac{1}{\tau} \partial_\eta^2 \right] A_\eta + \frac{1}{\tau} \partial_\eta \partial_\tau A_\eta = -\tau j^\eta, \quad (2.11) \]
where $j^{\mu}$ includes all kinds of the color currents. An explicit form of the solution for the homogeneous system is found in the Appendix 1. In compliance with the gauge condition which explicitly eliminates one of four field components we find three modes $V^{(\lambda)}$ of the free vector field. Two transverse modes obey Gauss law without the charge and have the unit norm (see Appendix 1) with respect to the scalar product (2.9):

$$V^{(1)}_{k,\nu}(x) = \frac{e^{-\pi\nu/2}}{2^{5/2}k_{\perp}} \begin{pmatrix} k_y \\ -k_x \\ 0 \end{pmatrix} H^{(2)}_{-i\nu}(k_{\perp}\tau)e^{i\nu\eta+ik_{\perp}},$$

and

$$V^{(2)}_{k,\nu}(x) = \frac{e^{-\pi\nu/2}}{2^{5/2}k_{\perp}} \begin{pmatrix} \nu k_x R^{(2)}_{-1,-i\nu}(k_{\perp}\tau) \\ \nu k_y R^{(2)}_{-1,-i\nu}(k_{\perp}\tau) \\ -R^{(2)}_{-1,-i\nu}(k_{\perp}\tau) \end{pmatrix} e^{i\nu\eta+ik_{\perp}}. \quad (2.13)$$

The mode $V^{(2)}$ is constructed from the functions $R^{(j)}_{-i\nu}(k_{\perp}\tau) = R^{(j)}_{i\nu}(k_{\perp}\tau|s)$ corresponding to the boundary condition of vanishing gauge field at $\tau = 0$. This guarantee continuous behavior of the field at $\tau = 0$. Indeed, at $\tau \rightarrow 0$, the normal and the tangent directions become degenerate. As long as $A^\tau = 0$ is the gauge condition, the continuity requires that $A^\gamma \rightarrow 0$ at $\tau \rightarrow 0$.

In order to simplify some subsequent calculations it is useful to write down explicitly the physical components of electric and magnetic fields of these modes, $E^m = \sqrt{-g}g^{mn}\hat{A}_n$ and $B^m = -2^{-1}e^{m\nu}F_{mn}$:

$$E^{(1)m}_{\nu\nu}(x) = iB^{(2)m}_{\nu\nu}(x) = \frac{e^{-\pi\nu/2}}{2^{5/2}k_{\perp}} \begin{pmatrix} k_y \\ -k_x \\ 0 \end{pmatrix} H^{(2)}_{-i\nu}(k_{\perp}\tau)e^{i\nu\eta+ik_{\perp}},$$

and

$$E^{(2)m}_{\nu\nu}(x) = iB^{(1)m}_{\nu\nu}(x) = \frac{e^{-\pi\nu/2}}{2^{5/2}k_{\perp}} \begin{pmatrix} \nu k_x \\ \nu k_y \\ -k^2_{\perp} \end{pmatrix} H^{(2)}_{-i\nu}(k_{\perp}\tau)e^{i\nu\eta+ik_{\perp}}. \quad (2.14)$$

Therefore, the mode $V^{(2)}$ can be obtained from the mode $V^{(1)}$ by simple interchange of its electric and magnetic fields. Using the standard wave-guide terminology, one may call mode $V^{(1)}$ as the “transverse electric mode” and the mode $V^{(2)}$ as the “transverse magnetic mode”.

An equivalent full set of the transverse modes carries instead of the boost $\nu$, the quantum number $\theta$, rapidity: $k_0 = k_{\perp}\cosh\theta, \quad k_3 = k_{\perp}\sinh\theta$. These functions can be obtained by means of the Fourier transform,

$$v^{(\lambda)}_{k,\nu}(x) = \int^{+\infty}_{-\infty} \frac{d\nu}{(2\pi)^{1/2}} e^{i\nu\theta} V^{(\lambda)}_{k,\nu}(x), \quad (2.15)$$

and have the following form,

$$v^{(1)}_{k,\theta}(x) = \frac{1}{4\pi^{1/2}k_{\perp}} \begin{pmatrix} k_y \\ -k_x \\ 0 \end{pmatrix} e^{-ik_{\perp}\tau\cosh(\theta-\eta)+ik_{\perp}},$$

$$v^{(2)}_{k,\theta}(x) = \frac{1}{4\pi^{3/2}k_{\perp}} \begin{pmatrix} k_x f_1 \\ k_y f_1 \\ -f_2 \end{pmatrix} e^{+i\theta}, \quad (2.16)$$

where

$$f_1(\tau, \eta) = k_\perp \sinh(\theta-\eta) \int^\tau_0 e^{-ik_{\perp}\tau'} \cosh(\theta-\eta)d\tau' = i \tanh(\theta-\eta)(e^{-ik_{\perp}\tau\cosh(\theta-\eta)} - 1),$$

$$f_2(\tau, \eta) = k_\perp^2 \int^\tau_0 e^{-ik_{\perp}\tau'} \cosh(\theta-\eta) d\tau' = \frac{e^{-ik_{\perp}\tau\cosh(\theta-\eta)} - 1}{\cosh^2(\theta-\eta)} + ik_{\perp} \frac{e^{-ik_{\perp}\tau\cosh(\theta-\eta)}}{\cosh(\theta-\eta)}, \quad (2.17)$$

The norm of the Coulomb mode $V^{(3)}$, as defined by Eq. (2.3), equals zero, and it is orthogonal to $V^{(1)}$ and $V^{(2)}$. Though this solution obeys equations of motion without the current, it does not obey the Gauss law without a charge. Therefore, it should be discarded in decomposition of the radiation field. However, it should have been kept if we considered the radiation field in the presence of the static source with the $\tau$-independent density $\rho(\vec{k}, \nu) = \tau j^{\tau}_{k\nu}(\tau) = const(\tau)$. In this case its definition can be completed using the Gauss law:

$$V^{(3)}_{k,\nu}(x) = \frac{\rho(\vec{k}, \nu)}{(2\pi)^{3/2}k_{\perp}} \begin{pmatrix} k_x \xi_{-1,\nu}(k_{\perp}\tau) \\ k_y \xi_{1,\nu}(k_{\perp}\tau) \end{pmatrix} e^{i\nu\eta+ik_{\perp}}. \quad (2.18)$$
The coordinate form of this solution is noteworthy. The physical components, $E^m = \sqrt{-g^m_\nu} A^\nu$, of the electric field of the “τ–static” source can be written down in the integral form,

$$E^i_{\text{(stat)}}(\tau, \vec{r}, \eta) = \int d^2\vec{r} d\eta K_i(\tau; \vec{r} - \vec{r}_2, \eta_1 - \eta_2) \rho(\vec{r}_2, \eta_2) ,$$

(2.19)

with the kernel

$$K_i(\tau; \vec{r}, \eta) = \frac{1}{(2\pi)^3} \frac{e^{i\vec{k}\cdot\vec{r}}}{ik_\perp^2} \left( \frac{k_\perp s_{1,\perp}(k_\perp \tau)}{\nu k_\perp^2 s_{1,\nu}(k_\perp \tau)} \right) = \frac{-\theta(\tau - \tau_\perp)}{4\pi} \left( \frac{\tau \cosh \eta (\partial/\partial \tau^\nu)}{\partial/\partial (\tau \sinh \eta)} \right) \frac{1}{R_{12}} ,$$

(2.20)

where

$$R_{12} = (\tau_\perp^2 + \tau^2 \sinh^2 \eta)^{1/2} = [(\vec{r}_1 - \vec{r}_2)^2 - \tau^2 \sinh^2 (\eta_1 - \eta_2)]^{1/2} ,$$

(2.21)

is the distance between the points $\vec{r}_1, \eta_1$ and $\vec{r}_2, \eta_2$ in the internal geometry of the surface $\tau = \text{const}$. The technical details of derivation of the last expression will be adduced in Sec. IV. Eq. (2.20) is an analogue of the Coulomb law meets several problems. Three methods are commonly used in the field theory. One of them strongly appeals to the Fourrier analysis in the plane Minkowsky space which is not applicable now because the metric itself is coordinate-dependent. The second method uses the path-integral formulation which is also ineffective because of the explicit coordinate dependence of the gauge-fixing term in the Lagrangian. One could also try to study the spectrum of the matrix differential operator, to find its eigen-functions and to use the standard expression for the resolvent. However, the extension of the system for the non-zero eigenvalues leads to the unwieldy equations. On the other hand, the Green function of the perturbation theory must coincide with the one which solves the problem of the gauge field interaction with the classical “external” current. For this reason, we shall compute the Green function in a most straightforward way: we shall look for the partial solution of the inhomogeneous system using the method of “variation of parameters”.

Let us start derivation of the propagator in the gauge $A^\tau = 0$ with obtaining the separate differential equations for the $\eta$-component of the magnetic field, $\Psi = \partial_\eta A_\tau - \partial_\tau A_\eta$; the transverse divergence of the electric field, $\varphi = \tau(\partial_x \dot{A}_x + \partial_y \dot{A}_y)$; and the $\eta$-component of the electric field, $a = \dot{A}_\eta/\tau$ . In terms of the Fourier components with respect to the spacial coordinates, these equations read as

$$[\partial^2_\tau + \frac{1}{\tau} \partial_\tau + \frac{k^2}{\tau^2} + k^2_\perp] \Psi_{\vec{k},\nu}(\tau) = - j^\psi(\vec{k}, \nu, \tau) ,$$

(2.22)

$$[\partial_\tau \partial_\tau + \frac{\nu^2}{\tau^2}] \varphi(\vec{k}, \nu, \tau) - i\tau \nu k^2_\perp a(\vec{k}, \nu, \tau) = - \partial_\tau [\tau^2 j^\varphi(\vec{k}, \nu, \tau)] ,$$

(2.23)

$$[\partial_\tau \partial_\tau + \tau k^2_\perp] a(\vec{k}, \nu, \tau) - \frac{i\nu}{\tau} \varphi(\vec{k}, \nu, \tau) = - \partial_\tau [\tau^2 j^a(\vec{k}, \nu, \tau)] ,$$

(2.24)

where $j^\psi = \partial_y \dot{j}_x - \partial_x \dot{j}_y$, $j^\varphi = \partial_y \dot{j}_x + \partial_x \dot{j}_y$. Using the constraint conservation, which may be explicitly integrated to

$$\varphi(\vec{k}, \nu, \tau) + i\nu a(\vec{k}, \nu, \tau) - \tau j^\varphi(\vec{k}, \nu, \tau) = -\rho_0(\vec{k}, \nu) = \text{const}(\tau) ,$$

(2.25)

one easily obtains two independent equations for $\varphi(\vec{k}, \nu, \tau)$ and $a(\vec{k}, \nu, \tau)$:

$$[\partial^2_\tau + \frac{1}{\tau} \partial_\tau + \frac{k^2}{\tau^2} + k^2_\perp] \varphi(\vec{k}, \nu, \tau) = k^2_\perp [\rho(\vec{k}, \nu, \tau) - \rho_0(\vec{k}, \nu)] - \frac{1}{\tau} \partial_\tau (\tau^2 j^\varphi(\vec{k}, \nu, \tau)) \equiv f^\varphi ,$$

(2.26)
\[ \partial_r^2 + \frac{1}{r} \partial_r + \frac{\mu^2}{r^2} + k_r^2] a(\vec{k}, \nu, \tau) = \frac{-i\nu}{r^2} [\rho(k, \nu, \tau) - \rho_0(k, \nu)] - \frac{1}{r} \partial_r (\tau^2 j^y(\vec{k}, \nu, \tau)) = f', \quad (2.27) \]

The constant of integration \( \rho_0(\vec{k}, \nu) \) has meaning of the arbitrary static charge density and it should be retained until the Gauss law is explicitly imposed on the solution. In what follows, we shall not write it down explicitly assuming that it is included in the true charge density \( \rho(\vec{k}, \nu, \tau) \). Equations (2.22), (2.27) and (2.28) can be solved by the method of “variation of parameters”:

\[ \mathcal{F}(\tau) = \frac{\pi i}{4} \int_0^\tau \tau_2 d\tau_2 \mathcal{H}(\tau, \tau_2) f(\tau_2), \quad (2.28) \]

where \( \mathcal{F} \) stands for anyone of the unknown functions in these equations, and \( f \) for the corresponding right hand side. The kernel

\[ \mathcal{H}(\tau, \tau_2) = H_{iv}^{(1)}(\vec{k}_\perp \tau) H_{iv}^{(2)}(\vec{k}_\perp \tau_2) - H_{iv}^{(2)}(\vec{k}_\perp \tau) H_{iv}^{(1)}(\vec{k}_\perp \tau_2). \]

is a usual bilinear form built from the linearly independent solutions of the homogeneous equation. [ The Wronskian of these solutions is exactly \( 4/i\pi \tau_2. \) ] Taking \( \mathcal{F} = \Psi \), we obtain the first equation for the components \( A_x(\vec{k}, \nu, \tau) \) and \( A_y(\vec{k}, \nu, \tau) \) of the vector potential:

\[ \Psi(\vec{k}, \nu, \tau_1) \equiv i[-k_y A_x + k_x A_y] = \frac{\pi i}{4} \int_0^\tau \tau_2 d\tau_2 \mathcal{H}(\tau, \tau_2) \ i [-k_y j^x(\tau_2) + k_x j^y(\tau_2)], \quad (2.29) \]

In order to find the second equation for the \( x \)- and \( y \)-components and the equation for \( A_\eta(\vec{k}, \nu, \tau) \) we must integrate twice:

\[ \Phi(\vec{k}, \nu, \tau_1) \equiv i[k_x A_x + k_y A_y] = \frac{\pi i}{4} \int_0^\tau d\tau' \int_0^{\tau'} \mathcal{H}(\tau', \tau_2) \tau_2 d\tau_2 \left[ -k_\perp^2 \rho(\vec{k}, \nu, \tau_2) + \frac{1}{\tau_2} \partial_{\tau_2} \left( \tau_2^2 j^y(\vec{k}, \nu, \tau_2) \right) \right], \quad (2.30) \]

\[ A_y(\vec{k}, \nu, \tau_1) = \frac{\pi i}{4} \int_0^\tau d\tau' \int_0^{\tau'} \mathcal{H}(\tau', \tau_2) \tau_2 d\tau_2 \left[ \frac{i\nu}{\tau_2} \rho(\vec{k}, \nu, \tau_2) + \frac{1}{\tau_2} \partial_{\tau_2} \left( \tau_2^2 j^y(\vec{k}, \nu, \tau_2) \right) \right]. \quad (2.31) \]

Here, the integration over \( \tau_2 \) recovers the electric fields at the moment \( \tau' \), whilst the integration over \( \tau' \) gives the vector potential at the moment \( \tau_1 \). It is convenient to start with the latter one which has the limits \( \tau_2 < \tau' < \tau_1 \). Let us consider the main line of calculations in detail, using the \( \eta \)-component as an example. The first integration follows the formula (A2.1),

\[ k_\perp^{\mu+1} \int_{\tau_2}^{\tau_1} (\tau')^\mu H_{iv}^{(j)}(\vec{k}_\perp \tau') d\tau' = R_{\mu, iv}^{(j)}(k_\perp \tau_1) - R_{\mu, iv}^{(j)}(k_\perp \tau_2), \quad (2.32) \]

and the terms emerging from the lower limit \( \tau_2 \) can be conveniently transformed according to the relation (see Appendix 2),

\[ R_{\mu, iv}^{(1)}(k_\perp \tau_2) H_{iv}^{(2)}(k_\perp \tau_2) - R_{\mu, iv}^{(2)}(k_\perp \tau_2) H_{iv}^{(1)}(k_\perp \tau_2) = \frac{4}{i\pi} s_{\mu, iv}(k_\perp \tau_2) \quad (2.33) \]

As a result, one obtains, e.g., the following formula for \( A_\eta(\vec{k}, \nu, \tau_1) \):

\[ A_\eta(\vec{k}, \nu, \tau_1) = \frac{\pi i}{4 k_\perp^{\mu+1}} \int_{\tau_2}^{\tau_1} \tau_2 d\tau_2 \left[ R_{1, iv}^{(1)}(k_\perp \tau_1) H_{iv}^{(2)}(k_\perp \tau_2) - R_{1, iv}^{(2)}(k_\perp \tau_1) H_{iv}^{(1)}(k_\perp \tau_2) - \frac{4}{i\pi} s_{1, iv}(k_\perp \tau_2) \right] \times \left[ \frac{i\nu}{\tau_2} \rho(\vec{k}, \nu, \tau_2) + \frac{1}{\tau_2} \partial_{\tau_2} \left( \tau_2^2 j^y(\vec{k}, \nu, \tau_2) \right) \right]. \quad (2.34) \]

In order to eliminate the charge density \( \rho \) from the integrand and to separate the transverse and the longitudinal parts of the propagator, all the terms of this formula should be integrated by parts with explicit account for the charge conservation which reads as

\[ i\tau [k_x j^x(\vec{k}, \nu, \tau) + k_x j^y(\vec{k}, \nu, \tau) + \nu j^y(\vec{k}, \nu, \tau)] + \partial_\tau \rho(\vec{k}, \nu, \tau) = 0. \quad (2.35) \]
We have in sequence:

\[
iv \int_0^{\tau_1} \frac{d\tau_2}{\tau_2} \rho(\tau_2) H^{(j)}_{iv}(k_\perp \tau_2) = iv \int_0^{\tau_1} \frac{dR^{(j)}_{-1,iv}(k_\perp \tau_2)}{d\tau_2} \rho(\tau_2)d\tau_2
\]

\[
= ivR^{(j)}_{-1,iv}(k_\perp \tau_1)\rho(\tau_1) - \nu \int_0^{\tau_1} \tau_2 d\tau_2 R^{(j)}_{-1,iv}(k_\perp \tau_2)[k_x j^x(\tau_2) + k_y j^y(\tau_2) + \nu j^\eta(\tau_2)] ,
\]

\[
(2.36)
\]

\[
iv \int_0^{\tau_1} d\tau_2 H^{(j)}_{iv}(k_\perp \tau_2) \partial_\tau_2 (\tau_2^2 j^\eta(\tau_2)) = \tau_1^2 j^\eta(\tau_1) H^{(j)}_{iv}(k_\perp \tau_1) + \int_0^{\tau_1} \tau_2 d\tau_2 [R^{(j)}_{1,iv}(k_\perp \tau_2) + \nu^2 R^{(j)}_{-1,iv}(k_\perp \tau_2)] j^\eta(\tau_2).
\]

\[
(2.37)
\]

In a similar way we have,

\[
iv \int_0^{\tau_1} \frac{d\tau_2}{\tau_2} \rho(\tau_2)s_{1,iv}(k_\perp \tau_2) = iv \int_0^{\tau_1} \frac{dQ_{-1,iv}(k_\perp \tau_2)}{d\tau_2} \rho(\tau_2)d\tau_2
\]

\[
= ivQ_{-1,iv}(k_\perp \tau_1)\rho(\tau_1) - \nu \int_0^{\tau_1} \tau_2 d\tau_2 Q_{-1,iv}(k_\perp \tau_2)[k_x j^x(\tau_2) + k_y j^y(\tau_2) + \nu j^\eta(\tau_2)] ,
\]

\[
(2.38)
\]

\[
\int_0^{\tau_1} d\tau_2 s_{1,iv}(k_\perp \tau_2) \partial_\tau_2 (\tau_2^2 j^\eta(\tau_2)) = \tau_2^2 j^\eta(\tau_2)s_{1,iv}(k_\perp \tau_1) + \nu^2 \int_0^{\tau_1} \tau_2 d\tau_2 [Q_{-1,iv}(k_\perp \tau_2) - Q_{1,iv}(k_\perp \tau_2)] j^\eta(\tau_2) .
\]

\[
(2.39)
\]

Assembling these pieces together and repeating the same calculations for the function \( \Phi \) one obtains three different terms which contribute to the field \( A \) produced by the current \( j \): \( A = A^{(tr)} + A^{(L)} + A^{(inst)} \).

The transverse field \( A^{(tr)} \) is defined by the integral terms in the R.H.S. of Eqs. (2.36) and (2.37). It can be conveniently written down in the following form:

\[
A^{(tr)}_l(x_1) = \int d^4 x_2 \theta(\tau_1 - \tau_2) \Delta^{(tr)}_{lm}(x_1, x_2) j^m(x_2),
\]

\[
(2.40)
\]

where

\[
\Delta^{(tr)}_{lm}(x, y) = -i \int_{\infty}^\infty dv \int d^2 \mathbf{k} \sum_{k=1,2} \left[ V^{(\lambda)}_{\nu k l}(x) V^{(\lambda)}_{\nu k m}(y) - V^{(\lambda)}_{\nu k l}(x) V^{(\lambda)}_{\nu k m}(y) \right],
\]

\[
(2.41)
\]

can be easily recognized as the Riemann function of the original homogeneous hyperbolic system. The Riemann function solves the boundary value problem for the evolution of the free radiation field. It is obtained immediately generated via integration by parts. It depends on a single time variable \( \tau_1 \). Using two functional relations, (2.33) and

\[
R^{(1)}_{1,iv}(x)R^{(2)}_{-1,iv}(x) - R^{(2)}_{1,iv}(x)R^{(1)}_{-1,iv}(x) = -\frac{4}{i\pi} \frac{ds_{1,iv}(x)}{dx} = -\frac{4}{i\pi} [Q_{1,iv}(x) - Q_{-1,iv}(x)],
\]

\[
(2.44)
\]
(see Appendix 2) its Fourier transform can be presented in the form

\[ A_l^{(\text{inst})}(\vec{k}, \nu; \tau_1) = \frac{\rho(\vec{k}, \nu; \tau_1)}{(2\pi)^3} \frac{1}{ik^2_\perp} \left[ \frac{k_r Q_{-1,iv}(k_\perp \tau_1)}{\nu Q_{1,iv}(k_\perp \tau_1)} \right]_m, \quad (2.45) \]

which leads to the Poisson-type integral,

\[ A_m^{(\text{inst})}(\tau_1, \eta_1, \vec{r}_1) = \int d\vec{r}_2 d\eta_2 K_m(\tau_1; \vec{r}_1 - \vec{r}_2, \eta_1 - \eta_2) \rho(\tau_1, \vec{r}_2, \eta_2), \quad (2.46) \]

with the instantaneous kernel,

\[ K_m(\tau; \vec{r}, \eta) = \int \frac{d\nu d\vec{k} e^{i\nu \eta + i\vec{k}\cdot\vec{r}}}{(2\pi)^3} \left[ \frac{k_r Q_{-1,iv}(k_\perp \tau)}{\nu Q_{1,iv}(k_\perp \tau)} \right]_m. \quad (2.47) \]

Therefore, this term represents instantaneous distribution of the potential at the moment \( \tau_1 \), corresponding to the charge density taken at the same moment. Recalling that the charge density \( \rho(\vec{k}, \nu, \tau_1) \) in Eq. (2.45) still includes the arbitrary constant \( \rho_0(\vec{k}, \nu) \), we see that imposing the constraint indeed affects only the potential of static charge distribution and puts it in agreement with the Gauss law. If \( \rho_0 \) is set to zero, then the gauge is completely fixed and all calculations with this propagator will be gauge invariant.

For the practical calculation, it is easier to work with the components \( j^\nu \) of the current rather than to keep the charge density \( \rho \) in its original form. Otherwise, e.g., the expression for the vertex function will be unwieldy. Replacement follows the prescription:

\[ \rho(\tau_1, \eta_2, \vec{r}_2) = \int_0^{\tau_1} d\tau_2 \frac{\partial \rho}{\partial \tau_2} = -i \int_0^{\tau_1} \tau_2 d\tau_2 [k_r s^x(\tau_2, \eta_2, \vec{r}_2) + \nu j^y(\tau_2, \eta_2, \vec{r}_2)] , \]

and restores an extraneous “initial” configuration of the static charge which has been previously removed from Eq. (2.45). Being \( \tau \)-dependent, its vector potential cannot be eliminated by the residual gauge transformation. However, this artificial contribution corresponds to the easily recognizable static pattern in the longitudinal part of the propagator and is under the full control. Keeping this fragment in mind, we arrive at the standard form of the representation,

\[ A_l^{(\text{inst})}(\tau_1, \eta_1, \vec{r}_1) = \int_0^{\tau_1} d\tau_2 d^2 \vec{r}_2 \Delta_{lm}^{(\text{inst})}(\tau_1; \eta_1 - \eta_2, \vec{r}_1 - \vec{r}_2) j^m(\tau_2, \eta_2, \vec{r}_2), \quad (2.48) \]

with the kernel given by the formula,

\[ \Delta_{lm}^{(\text{inst})}(\tau_1; \eta_1 - \eta_2, \vec{r}_1 - \vec{r}_2) = -\int \frac{d\nu d^3 \vec{k}}{(2\pi)^3 k^2_\perp} \left[ \frac{k_r Q_{-1,iv}(k_\perp \tau_1)}{\nu Q_{1,iv}(k_\perp \tau_1)} \right]_l \left[ \frac{k_s}{\nu} \right]_m e^{i\nu (\eta_1 - \eta_2) + i\vec{k} \cdot (\vec{r}_1 - \vec{r}_2)}. \quad (2.49) \]

Eqs. (2.40)–(2.43) and (2.48), (2.49) present the propagator in a split form. Different constituents of this form are preliminary identified as transverse, longitudinal and instantaneous parts of the propagator. It would be useful to learn if the same kind of splitting is possible for the current itself. An affirmative answer (as in the cases of the Coulomb and radiation gauges) would be helpful for the design of the perturbation theory. To answer the question, one should substitute the different pieces of the solution into the left hand side of the original system of differential equations. This leads to the following expressions for the Fourier components of the three currents:

\[ \tau j^m_{(\text{tr})}(\vec{k}, \nu; \tau) = \tau j^m_{(\text{tr})}(\vec{k}, \nu; \tau) + \frac{1}{ik_\perp^2} \left[ k_r s_{1,iv}(k_\perp \tau) + \nu Q_{1,iv}(k_\perp \tau) \right]_m \left[ \frac{\partial \rho}{\partial \tau} \right] \left( Q_{-1,iv}(k_\perp \tau)(k_r s^x + k_y j^y) + Q_{1,iv}(k_\perp \tau) j^y \right) \right]_m, \quad (2.50) \]

\[ \tau j^m_{(L)}(\vec{k}, \nu; \tau) = \frac{1}{k_\perp^2} \frac{\partial}{\partial \tau} \left[ k_r s_{1,iv}(k_\perp \tau) + \nu Q_{1,iv}(k_\perp \tau) \right]_m \left[ Q_{-1,iv}(k_\perp \tau)(k_r s^x + k_y j^y) + Q_{1,iv}(k_\perp \tau) j^y \right] \right]_m, \quad (2.51) \]

\[ \tau j^m_{(\text{inst})}(\vec{k}, \nu; \tau) = -\frac{1}{ik_\perp^2} \frac{\partial}{\partial \tau} \left[ k_r Q_{-1,iv}(k_\perp \tau) + \nu Q_{1,iv}(k_\perp \tau) \right]_m \left[ \frac{\partial \rho}{\partial \tau} \right] \left( k_r s_{1,iv}(k_\perp \tau) + \nu Q_{1,iv}(k_\perp \tau) \right)_m, \quad (2.52) \]
Providing the current is conserved, these three currents, added together, give the full current from the right hand side of the system. Therefore, the solution is correct. However, none of these three currents carries any signature of being longitudinal or transversal in a usual sense. None of them has zero divergence, since the operator of the divergence does not commute with the differential operator of the system. No desired simplification is possible in our case.

In fact, even the above splitting of the potential has no real physical meaning. To see it explicitly, let us find the divergence of the electric field, \( \text{div} \mathbf{E} = \partial_m \mathbf{E}^m \)
[again, for brevity, in the Fourier representation]:

\[
\text{div} \mathbf{E}^{(tr)}(\vec{k}, \nu; \tau) = i(Q_{-1, \nu}(k_\perp \tau) - Q_{1, \nu}(k_\perp \tau)) \left( \nu \tau^2 j^\eta - \frac{\nu^2}{k_\perp^2}(k_x j^x + k_y j^y) \right),
\]

(2.53)

\[
\text{div} \mathbf{E}^{(L)}(\vec{k}, \nu; \tau) = i(\tau^2 + \frac{\nu^2}{k_\perp^2})[(k_x j^x + k_y j^y)Q_{-1, \nu}(k_\perp \tau) + \nu j^\eta Q_{1, \nu}(k_\perp \tau)],
\]

(2.54)

\[
\text{div} \mathbf{E}^{(\text{inst})}(\vec{k}, \nu; \tau) = \rho(\vec{k}, \nu; \tau) - i(\tau^2 Q_{-1, \nu}(k_\perp \tau) - \frac{\nu^2}{k_\perp^2} Q_{1, \nu}(k_\perp \tau))[(k_x j^x + k_y j^y) + \nu j^\eta].
\]

(2.55)

Only the divergence of the true retarded component of the field \( \mathbf{E}^{(tr)} \) turns out to be zero. The term which prevents \( \text{div} \mathbf{E}^{(tr)} \) from being zero is due to non-symmetry of the propagator, \( \Delta^\eta \neq \Delta^\eta_0 \). It appears when the \( \theta \)-function in Eq. (2.40) is differentiated with respect to Hamiltonian time \( \tau \). This term is vital for obtaining the expression that obeys Gauss law constraint, \( \text{div} \mathbf{E}(\vec{k}, \nu; \tau) = \rho(\vec{k}, \nu; \tau) \). We remind that \( \rho(\vec{k}, \nu; \tau) \) still includes an arbitrary constant \( \rho_0(\vec{k}, \nu) \).

The known examples, when the transverse and the longitudinal fields are separated at the level of equations of motion, are related to the narrow class of homogeneous gauges. Impossibility to perform a universal separation of the transverse and longitudinal fields thus appears to be a rule rather than an exception. It reflects a general principle: the radiation field created at some time interval has the preceding and the subsequent configurations of the longitudinal field as the boundary condition. Dynamics of the longitudinal field falls out of any scattering problem in its \( S \)-matrix formulation. However, this dynamics is, in fact, a subject of the QCD evolution in deep inelastic scattering.

D. Initial data problem in the gauge \( A^\tau = 0 \)

We obtained the expression for the (retarded) propagator as the response function between the “external” current and the potential of the gauge field. We must verify that the same propagator solves the Cauchy problem for the gauge field. This can be easily done by presenting the initial data at the surface \( \tau = \tau_0 \) in the form of the source density at the hypersurface \( \tau = \tau_0 \),

\[
\sqrt{-g} J^\eta(\tau_2) = \sqrt{-g(\tau_0)} g^{nm} (\tau_0) [\delta'(\tau_2 - \tau_0) \tilde{A}_m(\vec{r}, \eta) + \delta(\tau_2 - \tau_0) \tilde{A}_m'(\vec{r}, \eta)] ,
\]

(2.56)

where \( \tilde{A}_m(\vec{r}, \eta) \) and \( \tilde{A}_m'(\vec{r}, \eta) \) are the initial data for the potential and its normal derivative on the hypersurface \( \tau = \tau_0 \). Usually, it is assumed that the real currents vanish at \( \tau < \tau_0 \). Substituting this source into the Eqs. (2.40), (2.42) and (2.46), and taking the limit of \( \tau_1 \to \tau_0 \), we may verify that the standard prescription for the solution of the initial data problem,

\[
A_1(x_1) = \int_{(\tau_2 = \tau_0)} d^2 \vec{r}_2 \, dh_2 \, \Delta_{lm}(x_1, x_2) \frac{\partial}{\partial \tau_2} A^m(x_2) ,
\]

(2.57)

holds with the same propagator \( \Delta_{lm}(x_1, x_2) \) that was used to solve the emission problem. For example, in the limit of \( \tau \to \tau_0 \), the \( \eta \)-component of the vector potential is a sum of three terms,

\[
A^{(tr)}_\eta(\tau_0 + 0) = \frac{i \pi}{4k_\perp^2} \left\{ [R^{(2)}_{1, i\nu}(k_{\perp} \tau_0) R^{(1)}_{i\nu}(k_{\perp} \tau_0) - R^{(1)}_{1, i\nu}(k_{\perp} \tau_0) R^{(2)}_{i\nu}(k_{\perp} \tau_0)] [\nu \tilde{A}_\phi - k_\perp^2 \tilde{A}_\eta] \\
- \tau_0 \nu [R^{(2)}_{1, i\nu}(k_{\perp} \tau_0) R^{(1)}_{-1, i\nu}(k_{\perp} \tau_0) - R^{(1)}_{1, i\nu}(k_{\perp} \tau_0) R^{(2)}_{-1, i\nu}(k_{\perp} \tau_0)] \tilde{A}_\phi \right\} ,
\]

(2.58)

\[
A^{(L)}_\eta(\tau_0 + 0) = \frac{-\nu}{k_\perp^2} \left\{ -s_{1, i\nu}(k_{\perp} \tau_0) \tilde{A}_\phi + \tau_0 Q_{-1, i\nu}(k_{\perp} \tau_0) \tilde{A}_\phi - \frac{\nu k_\perp^2}{\tau_0} s_{-1, i\nu}(k_{\perp} \tau_0) \tilde{A}_\eta + \frac{\nu}{\tau_0} Q_{1, i\nu}(k_{\perp} \tau_0) \tilde{A}_\eta \right\} ,
\]

(2.59)
where we have denoted: \( \tilde{A}_\phi = k_x \tilde{A}_x + k_y \tilde{A}_y \). Here, Eq. (2.60) follows from Eq. (2.46) and takes care of the consistency between the charge density at the moment \( \tau_0 \) and the initial data for the gauge field. Using relations (2.33) and (2.45) and adding up Eqs. (2.58)–(2.60) we come to a desired identity, \( A_\phi(\tau_0 + 0) = \tilde{A}_\phi \).

When the initial data \( A_m(r, \eta) \) and \( \tilde{A}_m'(r, \eta) \) correspond to the free radiation field, then only the part \( \Delta^{(tr)}_{lm}(x_1, x_2) \) of the full propagator “works” here, and only Eq. (2.58) may be retained. The other two equations acquire status of the constraints imposed on the initial data. Since the current is absent, we have \( A^{(L)} = 0 \) on the left hand side of the Eqs. (2.59). Then the right hand side confirms that the kernel \( K \) is orthogonal to the free radiation field modes. Since the charge density \( \rho \) vanishes, we have \( A^{(inst)} = 0 \), which is equivalent to the Gauss law for the free gauge field. The two transverse modes already obey these constraints. This fact provides a reliable footing for canonical quantization of the free field in the gauge \( A^\tau = 0 \). Indeed, the Riemann function induces commutation relations for the gauge field. It can be found via its bilinear decomposition over the physical modes. Thus one can avoid technical problems of inverting the constraint equations. (See Sec. [III]) The longitudinal part of the propagator will be studied in details in Sec. [IV].

E. Gluon vertices in the gauge \( A^\tau = 0 \)

The terms proportional to the first and the second powers of the coupling constant in the classical wave equations may be viewed as the external current and allow one to define the explicit form of the 3- and 4-gluon vertices. One should start from the solution of the Maxwell equations,

\[
A^{g'}_{ \vec{k}}(z_1) = \int d^4x \Delta^{g'}_{ \vec{k} \vec{k}'}(z_1, x) \sqrt{-g(x)} J_{ \vec{k}}^g(x) .
\]  

with the color current of the form

\[
\sqrt{-g(x)} J_{ \vec{k}}^g(x) = -gf_{abc} \sqrt{-g(x)} g^{kn}(x) \{ g^{ml}(x) [ \partial_m (A^g_{ \vec{k}}(x) A^c_{ \vec{n}}(x)) + A^b_{ \vec{k}}(x) \partial_m A^c_{ \vec{n}}(x) + A^b_{ \vec{m}}(x) \partial_n A^c_{ \vec{k}}(x)] - g^2 \sqrt{-g(x)} f_{abc} f_{cde} g^{kn}(x) g^{ml}(x) A^d_{ \vec{k}}(x) A^e_{ \vec{m}}(x) \} .
\]

In perturbation calculations, every field \( A(x) \) in the RHS of this expression is a part of some correlator \( \Delta(x, z_N) \). The components of the metric depend only on the time \( \tau \) while the derivatives affect only the spacial directions \( u^a = (r, \eta) \). Moreover, in these directions, all the gluon correlators depend only on the differences of the coordinates and can be rewritten in terms of their spacial Fourier components. After symmetrization over the outer arguments \( z_N \), one immediately obtains,

\[
V_{abc}(p_1, p_2, p_3; \tau) = -i \tau f_{abc} \delta(p_1 + p_2 + p_3) \{ g^{ln}(p_2 - p_3)^k + g^{nk}(p_3 - p_1)^l + g^{li}(p_1 - p_2)^n \} ,
\]

where \( p^a = g^{nk} p_k \), and the components of the momentum in the curvilinear coordinates are equal to \( p_k = (p_x, p_y, \nu) \). The four-gluon vertex has no derivatives and is the same as usually.

III. QUANTIZATION

The second quantization of the field has several practical goals. We would like to have an expansion of the operator of the free gluon field like

\[
A_i(x) = \sum_{\lambda = 1, 2} \int d^2 \vec{k} dv [c_{\lambda}(\nu, \vec{k}) V_{\nu \vec{k} i}^{(\lambda)}(x) + c_{\lambda}^\dagger(\nu, \vec{k}) V_{\nu \vec{k} i}^{(\lambda)*}(x)] ,
\]

with the creation and annihilation operators which obey the commutation relations

\[
[c_{\lambda}(\nu, \vec{k}), c_{\lambda'}^\dagger(\nu', \vec{k}')] = \delta_{\lambda\lambda'} \delta(\nu - \nu') \delta(\vec{k} - \vec{k}') , \quad [c_{\lambda}(\nu, \vec{k}), c_{\lambda'}(\nu', \vec{k}')] = [c_{\lambda}^\dagger(\nu, \vec{k}), c_{\lambda'}^\dagger(\nu', \vec{k}')] = 0 .
\]

Once obtained, commutation relations (3.2) allow one to find various correlators of the free gluon field as the averages of the binary operator products over the state of the perturbative vacuum and express them via the solutions \( V_{\nu \vec{k} i}^{(1)}(x) \) and \( V_{\nu \vec{k} i}^{(2)}(x) \). For example, the Wightman functions,
\[ i\Delta_{10,ij}(x,y) = \langle 0|A_i(x)A_j(y)|0\rangle = \sum_{\lambda=1,2} \int dv d^2k V^{(\lambda)}_{\nu k;i}(x)V^{(\lambda)*}_{\nu k;i}(y) = i\Delta_{01,jj}(y,x) \quad , \quad (3.3) \]

serve as the projectors onto the space of the on-mass-shell gluons and should be known explicitly in order to have a good definition for the production rate of the gluons in the final states. With these two Wightman functions at hand, one immediately obtains the expression for the commutator of the free field operators,

\[ \Delta_{0,ij}(x,y) = -i\langle 0| [A_i(x), A_j(y)] |0 \rangle = \Delta_{10,ij}(x,y) - \Delta_{01,ij}(x,y) \quad , \quad (3.4) \]

which should coincide with the Riemann function of the homogeneous field equations. The program of secondary quantization do not reveal any technical problems if we give preference to the holomorphic quantization which starts with equations (3.2) as the consequence.

The way to obtain the canonical commutation relations in cases of the scalar and the spinor fields is quite straightforward. For the vector gauge field we meet a well known problem, an excess of the number of the components of the vector field over the number of the physical degrees of freedom. For example, in the so-called radiation gauge, \( A^0 = 0 \) and \( \text{div} \mathbf{A} = 0 \), we write the canonical commutation relations in the following form [3],

\[ [A_i(x,t), E_j(y,t)] = \delta_{ij}^r(x-y) = \int \frac{d^2k}{(2\pi)^3} \left( \delta_{ij} - \frac{k_i k_j}{k^2} \right) e^{-ik(x-y)}, \]

\[ [A_i(x,t), A_j(y,t)] = [E_i(x,t), E_j(y,t)] = 0 \quad , \quad (3.5) \]

thus eliminating the longitudinally polarized photons from the dynamical degrees of freedom. The function \( \delta_{ij}^r \) plays a role as the unit operator in the space of the physical states. Here, \( i, j = 1, 2, 3 \) and the number of relations postulated by equations (3.5) apparently exceeds the actual number required by the count of the independent degrees of freedom, \( \lambda = 1, 2 \), of the free gauge field. The Fourier transform of the function \( \delta_{ij}^r \) is easily guessed because the basis of the plane-wave solutions is very simple [3], and it can be obtained rigorously by solving the system of constraint equations [10,11]. A similar guess or procedure in our case is not so obvious. We have the gauge condition \( \lambda^2, \eta \sum_{\perp} \lambda^2 = 0 \) as the primary constraint and the Gauss law as the secondary one. The latter can be resolved in a way which allows one to exclude the \( \eta \)-components of the potential and the electric field from the set of independent canonical variables. Thus, only \( x \)- and \( y \)-components are subject for the canonical commutation relations. To resolve the constraints, one anyway needs the integral operators with the kernels built from the solutions of the Maxwell equations in the gauge \( A^r = 0 \). Therefore we shall proceed in two steps. In section III A we shall sketch the results for the Wightman functions (3.3). These, will be used for the explicit calculation of the free field commutator (3.4) in section III B and for the study of its causal behavior.

### A. Gluon correlators in the gauge \( A^r = 0 \)

Here, we shall write down components of the field correlator \( \Delta_{10,ij}(x,y) \) in the curvilinear coordinates \( u = (\tau, \eta, r) \). We shall denote their covariant components as \( \Delta_{10,ik}(u_1, u_2) \). Later we shall transform them to the standard Minkowsky coordinates and find the correlators of the temporal axial and the null-plane gauges as their limits in the central rapidity region and in the vicinity of the null-planes, respectively. The most convenient for this purpose basis consists of the transverse modes \( v^{(\lambda)} \). The mode \( v^{(1)} \) gives the following contribution to the correlator \( \Delta_{10,ik} \):

\[ i\Delta^{(1)}_{10,rs}(1,2) = \int_{-\infty}^{\infty} \frac{d\theta}{\pi} \int \frac{d^2k}{(2\pi)^3} \frac{\epsilon_{ru} \epsilon_{sv} k_u k_v }{k^2_i} e^{i\vec{k}(\vec{r}_1 - \vec{r}_2)} e^{-ik_\perp (\tau_1 \cosh(\theta - \eta_1) + ik_\perp \tau_2 \cosh(\theta - \eta_2))} \quad . \quad (3.6) \]

Realising that \( d\theta /2 = dk^3 / 2k^0 \), we recognize a standard representation of this part of the correlator in terms of the on-mass-shell plane waves decomposition.

The second part of the correlator is determined by the mode \( v^{(2)} \) and has the following components:

\[ \Delta^{(2)}_{10,rs}(1,2) = -i \int_{-\infty}^{\infty} \frac{d\theta}{\pi} \int \frac{d^2k}{(2\pi)^3} \frac{k_r k_s}{k^2_i} f_1(\theta, \tau_1, \eta_1) f_1^*(\theta, \tau_2, \eta_2) \quad , \quad (3.7) \]

\[ \Delta^{(2)}_{10,rs}(1,2) = -i \int_{-\infty}^{\infty} \frac{d\theta}{\pi} \int \frac{d^2k}{(2\pi)^3} \frac{k_r k_s}{k^2_i} f_1(\theta, \tau_1, \eta_1) f_2^*(\theta, \tau_2, \eta_2) = i\Delta^{(2)}_{10,rs}(2,1) \quad , \quad (3.8) \]
\[ \Delta^{(2)}_{10,\eta}(1, 2) = -i \int_{-\infty}^{\infty} \frac{d\theta}{2} \int \frac{d^2\vec{k}}{(2\pi)^3} \frac{e^{i\vec{k}(\tau_1 - \tau_2)}}{k^2_\perp} f_2(\theta, \tau_1, \eta_1)f_2^*(\theta, \tau_2, \eta_2). \]  
\[ (3.9) \]

One may easily see that all components of \( \Delta_{10}(1, 2) \) vanish when either \( \tau_1 \) or \( \tau_2 \) go to zero.

### B. Causal properties of the field commutators in the gauge \( A^r = 0 \)

Causal properties of the radiation field commutator may be studied starting from the representation (3.4). Using Eqs. (3.6) and (3.7) we may conveniently write contribution of two transverse modes in the following form:

\[ i\Delta^{(1)}_{0,r,s}(1, 2) = -i \int \frac{d^2\vec{k}}{(2\pi)^3} \frac{\epsilon_{rs}k_\tau k_\sigma}{k^2_\perp} e^{i\vec{k}\vec{r}} \int_{-\infty}^{\infty} d\theta \sin k_{\perp}\Phi, \]
\[ (3.10) \]

\[ i\Delta^{(2)}_{0,r,s}(1, 2) = -i \int \frac{d^2\vec{k}}{(2\pi)^3} \frac{k_\tau k_\sigma}{k^2_\perp} e^{i\vec{r}'} \int_{-\infty}^{\infty} d\theta \left[ 1 - \frac{\cosh 2\eta}{\sinh^2 \theta - \cosh^2 \eta} \right] (\sin k_{\perp}\Phi - \sin k_{\perp}\Phi_1 + \sin k_{\perp}\Phi_2), \]
\[ (3.11) \]

where we have introduced the following notation: \( 2\eta = \eta_1 - \eta_2 \), \( \vec{r} = \vec{r}_1 - \vec{r}_2 \), \( \Phi_i = \tau_1 \cosh(\theta - \eta_i) \), \( \Phi = \Phi_1 - \Phi_2 \). The sum of (3.10) and (3.11) can be rearranged to the following form,

\[ i\Delta_{0,r,s}(1, 2) = i \int \frac{d^2\vec{k}}{(2\pi)^3} \frac{d\theta}{2\pi} e^{i\vec{k}\vec{r}} \left[ -\delta_{rs} \sin k_{\perp}\Phi + \frac{k_\tau k_\sigma}{k^2_\perp} [\sin k_{\perp}\Phi_1 - \sin k_{\perp}\Phi_2] \\
+ k_\tau k_\sigma \cosh(\eta_1 - \eta_2) \int_0^{\tau_1} d\tau' \int_0^{\tau_2} d\tau'' \sin[k_{\perp}\tau' \cosh(\theta - \eta) - k_{\perp}\tau' \cosh(\theta + \eta)] \right]. \]
\[ (3.12) \]

Joining the integration \( d^2\vec{k} \) \( d\theta \) into the three dimensional integration \( d^3\vec{k}/|\vec{k}| \) in the Cartesian coordinates, the first integral in (3.12),

\[ D_0(1, 2) = \int \frac{d^2\vec{k}}{(2\pi)^3} \frac{d\theta}{2\pi} e^{i\vec{k}\vec{r}} \sin k_{\perp}\Phi = \frac{\text{sign}(t_1 - t_2)}{2\pi} \delta[(t_1 - t_2)^2 - (r_1 - r_2)^2], \]
\[ (3.13) \]

is easy to calculate, and to recognize it as the commutator of the massless scalar field. It differs from zero only if the line between the points \( x_1 \) and \( x_2 \) has the light-like direction. In this way we integrate the first and the third terms in the integrand of the Eq. (3.12). To reduce two integrals in the second term to the same type, we must exclude the factor \( 1/k^2_\perp \) using the fundamental solution of the two-dimensional Laplace operator,

\[ \frac{k_\tau k_\sigma}{k^2_\perp} e^{i\vec{k}\vec{r}} = \partial_\tau \partial_s \int \frac{d^2\vec{\xi}}{2\pi} \ln |\vec{\xi} - \vec{r}| e^{i\vec{\xi}\vec{r}}. \]
\[ (3.14) \]

After that we arrive at the final result,

\[ \Delta_{0,r,s}(1, 2) = -\delta_{rs} D_0(1, 2) - \cosh(\eta_1 - \eta_2) \partial_\tau \partial_s \int_0^{\tau_1} d\tau' \int_0^{\tau_2} d\tau'' D_0(1, 2) \\
\quad + \partial_\tau \partial_s \int \frac{d^2\vec{\xi}}{(2\pi)^2} \ln |\vec{\xi} - \vec{r}| \left[ \delta(\tau_1^2 - \vec{\xi}^2) - \delta(\tau_2^2 - \vec{\xi}^2) \right]. \]
\[ (3.15) \]

From this form it immediately follows that the commutator of the potentials vanishes at \( \tau_1 = \tau_2 \). Even more strong result takes place for the commutator of the two electric fields,

\[ [E_r(1), E_s(2)] = \partial^2_{\tau_1 \tau_2} i\Delta_{0,r,s}(1, 2) = \left[ -\delta_{rs} \partial^2_{\tau_1 \tau_2} - \cosh(\eta_1 - \eta_2) \frac{\partial^2}{\partial x^r \partial x^s} \right] iD_0(1, 2). \]
\[ (3.16) \]

This commutator vanishes everywhere except for the light cone, in full compliance with the microcausality principle for the electric field which is an observable. However, this does not happen for the commutator of the potentials since they are defined nonlocally. It does not vanish neither at space-like nor at space-like separation, because the line of integration which recovers the potential at the point \( x_2 \), in general, intersects (e.g. at some point \( x_3 \)) with the
light cone which has its vertex at the point \( x_1 \), and the commutator of the electric fields at the points \( x_1 \) and \( x_3 \) is not zero.

Similar results take place for the commutator of the \( \eta \)-components of the potential and the electric field. The field commutator,

\[
[E_\eta(1), E_\eta(2)] = \frac{\partial^2}{\partial \tau_1 \partial \tau_2}i\Delta_{0,\eta}(1, 2) = -i\nabla_2^2 D_0(1, 2),
\]

is entirely causal, while the commutator of the potentials,

\[
[A_\eta(1), A_\eta(2)] = i\Delta_{0,\eta}(1, 2) = -i\nabla_2^2 \int_0^{\tau_1} \tau_1 d\tau_1 \int_0^{\tau_2} \tau_2 d\tau_2 D_0(1, 2),
\]

does not vanish at space-like distances, except for \( \tau_1 = \tau_2 \). Finally, the formally designed commutator between the \( r \)- and \( \eta \)-components of the electric field, the two observables,

\[
[E_r(1), E_\eta(2)] = \frac{\partial^2}{\partial \tau_1 \partial \tau_2}i\Delta_{0,r\eta}(1, 2) = -i\nabla_2^2 \frac{\tau_2}{\tau_1} D_0(1, 2),
\]

is entirely confined to the light cone, while the commutator of the potentials, which are not the observables,

\[
[A_r(1), A_\eta(2)] = i\Delta_{0,r\eta}(1, 2) = -\int_0^{\tau_1} \int_0^{\tau_2} \frac{\partial^2}{\partial x^r \partial \eta} D_0(1, 2),
\]

does not vanish at space-like distance, even at \( \tau_1 = \tau_2 \). This result, however, is not a subject for any concern since the potentials are defined nonlocally and commutation relations for electric and magnetic [cf. \( (2.14) \)] fields are reproduced correctly. Moreover, we have argued above that the \( \eta \)-components of \( A \) and \( E \) are not the canonical variables since the constraints express them via \( x \)- and \( y \)-components.

The “acausal” behavior of the Riemann function \( \Delta^{\mu
u}_{\alpha\beta}(1, 2) \) may cause doubts if the gauge \( A^r = 0 \) allows for meaningful retarded and advanced Green functions which, by causality, should vanish at space-like distances. Fortunately, this anomalous behavior appears only for the gauge–variant potential; the response functions for observable electric and magnetic fields are causal. This can be easily seen, e.g., from Eqs. \( (2.23) \), \( (2.26) \) and \( (2.27) \), the usual inhomogeneous relativistic wave equations for various physical components of the field strengths \( \mathcal{E} \) and \( \mathcal{B} \).

### C. Canonical commutation relations in the gauge \( A^r = 0 \)

A proof of the commutation relations \( (3.3) \) for the Fock operators follows the standard guidelines \( [9] \). The creation and annihilation operators are defined via relations,

\[
c_{\lambda}(\mu, \vec{k}) = (V^{(\lambda)}_{\nu \vec{k}}, A) = ig^{ij} \int d^3x [V^{(\lambda)*}_{\nu \vec{k};j}(x) A_i(x, \tau) - V^{(\lambda)*}_{\nu \vec{k};j}(x) A_i(x, \tau)],
\]

\[
c_{\lambda}^\dagger(\mu, \vec{k}) = (A, V^{(\lambda)}_{\nu \vec{k}}) = ig^{ij} \int d^3x [A_i(x, \tau) V^{(\lambda)}_{\nu \vec{k};j}(x) - A_i(x, \tau) V^{(\lambda)}_{\nu \vec{k};j}(x)].
\]

The latter result in the following expression for the commutator,

\[
[c_{\lambda}(\mu, \vec{k}), c_{\lambda}^\dagger(\mu', \vec{k}')] = \int d^3x d^3y g^{ij}(x)g^{lm}(y) \left\{ [A_i(x, \tau), A_l(y, \tau)] \left( V^{(\lambda)*}_{\nu \vec{k};j}(x) V^{(\lambda)}_{\nu' \vec{k}';l}(y) - V^{(\lambda)*}_{\nu' \vec{k}';l}(x) V^{(\lambda)}_{\nu \vec{k};j}(y) \right) + [A_i(x, \tau), A_l(y, \tau)] V^{(\lambda)*}_{\nu \vec{k};j}(x) V^{(\lambda)}_{\nu' \vec{k}';l}(y) + [A_i(x, \tau), A_l(y, \tau)] V^{(\lambda)*}_{\nu' \vec{k}';l}(x) V^{(\lambda)}_{\nu \vec{k};j}(y) \right\}.
\]

Most of the terms in the second line vanish due to the commutation relations. Next, we rely on the following guess about the form of the commutator,

\[
[A_i(x), A_j(y)] = \sum_{\lambda=1,2} \int d\nu d^2k (V^{(\lambda)}_{\nu \vec{k};i}(x) V^{(\lambda)*}_{\nu \vec{k};j}(y) - V^{(\lambda)*}_{\nu \vec{k};j}(x) V^{(\lambda)}_{\nu \vec{k};i}(y)),
\]

which leads to the proper equal-time commutation relations for the independent canonical variables. Finally, explicitly using the orthogonality relations for the eigenmodes \( V^{(\lambda)} \), we immediately obtain the commutation relations \( (3.2) \).
IV. LONGITUDINAL PROPAGATOR AND STATIC FIELDS

In this section we shall find the explicit expressions for the kernels \([n\,1]_m(\tau_2, \vec{r}, \eta)\) and \([n\,2]_m(\tau_2, \vec{r}, \eta)\) which represent the longitudinal and instantaneous components of the gauge field produced by the “external” current \(j^\mu\). The calculations are lengthy and their details are adduced in Appendix 3. Here, we present only the final answers.

The components of the longitudinal propagator \(\Delta_{\mu\nu}^{(L)}(\tau_2, \vec{r}, \eta)\) are already obtained in the form of the three-dimensional integrals \([n\,1]_m(\tau_2, \vec{r}, \eta)\). \(\Delta_{\mu\nu}^{(L)}\) depends on the differences of the curvilinear spatial coordinates, \(\vec{r} = \vec{r}_1 - \vec{r}_2\) and \(\eta = \eta_1 - \eta_2\), but not on the difference of the temporal arguments \(\tau_1\) of the field and \(\tau_2\) of the source. Introducing the shorthand notation for the distance in the \((x_0)\)-plane, \(r_\perp = |\vec{r}|\), and for the full distance \(R_2 = R(\tau_2) = [(\vec{r}_1 - \vec{r}_2)^2 + \tau_2^2 \sin\tau^2(\eta_1 - \eta_2)]^{1/2}\) between the two points of the surface \(\tau_2 = \text{const}\), we obtain:

\[
\Delta_{\mu\nu}^{(L)} = -\frac{\theta(\tau_2 - r_\perp)}{4\pi} \frac{\partial^2}{\partial \eta \partial x^\nu} \left[ L_2 \coth|\eta| - \ln \frac{\tau_2}{r_\perp} \right],
\]

\[
\Delta_{\eta\eta}^{(L)} = -\frac{\theta(\tau_2 - r_\perp)}{4\pi} \frac{\partial^2}{\partial \eta \partial x^\nu} \left[ L_2 \coth|\eta| \right],
\]

\[
\Delta_{\eta\eta}^{(L)} = -\frac{\tau_2^2}{2}\delta(\vec{r})\delta(\eta) + \frac{\theta(\tau_2 - r_\perp)}{4\pi} \left[ 2\frac{\eta \coth \eta - 1}{\sinh^2 \eta} + \frac{\nabla^2}{2} \left( -\frac{r_\perp \cosh \eta}{\sinh^3 |\eta|} L_2 + \frac{\tau_2 R_2 \cosh \eta}{\sinh^2 \eta} \right) \right].
\]

where \(L_2 = L(\tau_2) = \ln[(\tau_2 \sinh |\eta| + R_2)/r_\perp]\). After the derivatives are evaluated, most of the logarithms here vanish:

\[
\Delta_{\eta\eta}^{(L)} = -\frac{\theta(\tau_2 - r_\perp)}{4\pi} \frac{\partial^2}{\partial \eta \partial x^\nu} \left[ 1 - \frac{\tau_2 \cosh \eta}{R_2} \right] \left( \partial_{x^\nu} - \frac{2x^\nu x^\nu}{r_\perp^2} \right) - \frac{2x^\nu x^\nu \tau_2 \cosh \eta}{r_\perp^2 R_2^2},
\]

\[
\Delta_{\eta\eta}^{(L)} = -\frac{\theta(\tau_2 - r_\perp)}{4\pi} \frac{\partial^2}{\partial \eta \partial x^\nu} \left[ 1 - \frac{\tau_2 \cosh \eta}{R_2} \right] \left( \partial_{x^\nu} - \frac{2x^\nu x^\nu \tau_2 \cosh \eta}{r_\perp^2 R_2^2} \right),
\]

\[
\Delta_{\eta\eta}^{(L)} = -\frac{\theta(\tau_2 - r_\perp)}{4\pi} \frac{\partial^2}{\partial \eta \partial x^\nu} \left[ \frac{2\eta \coth \eta - 1}{\sinh^2 \eta} + \frac{\tau_2 \cosh \eta}{R_2 \sinh^2 \eta} \right],
\]

\[
\Delta_{\eta\eta}^{(L)} = -\frac{\theta(\tau_2 - r_\perp)}{4\pi} \frac{\partial^2}{\partial \eta \partial x^\nu} \left[ \frac{3 - r_\perp^2}{r_\perp^2 R_2^2} \right] - \frac{2\cosh \eta}{\sinh^3 |\eta|} L_2.
\]

By examination of Eq. \([n\,1]_m(\tau_2, \vec{r}, \eta)\), one may see that after replacement of \(\tau_2\) by \(\tau_1\) the same kernel, \(\Delta_{\mu\nu}^{(L)}(\tau_1, \vec{r}, \eta)\), determines the components \(E_{\mu\nu}^{(L)}(\tau_1)\) of the longitudinal part of the electric field via the components \(j^\nu(\tau_1)\) of the current at the same time.

Similar calculations have led to the expression \([2.20]\) for the electric field of the static Coulomb source. The kernel \(K_{\mu\nu}\) of the instantaneous potential has the following components,

\[
K_r = -\frac{\theta(\tau_1 - r_\perp)}{4\pi} \frac{\partial L_1}{\partial x^\nu} = \frac{\theta(\tau_1 - r_\perp)}{4\pi} \frac{x^\nu \tau_2 \cosh |\eta| \sinh |\eta|}{R_1},
\]

\[
K_\eta = -\frac{\theta(\tau_1 - r_\perp)}{4\pi} \left( \frac{\tau_1}{R_1 \sinh |\eta|} + \frac{|\eta| - \tanh |\eta|}{\sinh^2 |\eta|} - L_1 \right),
\]

where \(R_1 = R(\tau_1)\) and \(L_1 = L(\tau_1)\). These propagators do not respect the light cone, but have a remarkable property that the longitudinal fields at the surface of the constant proper time \(\tau\) do not exist at the distances \(r_\perp\) from their sources that exceed \(r\). This establishes the upper limit for the possible dynamical correlations between the longitudinal fields in the \((x_0)\)-plane.

V. GLUON CORRELATORS IN THE CENTRAL RAPIDITY REGION AND NEAR THE LIGHT WEDGE

Our next step is to compare the correlators of the gauge \(A_\mu = 0\) with the similar correlators in three other gauges, \(A^0 = 0, A^+ = 0\) and \(A^- = 0\). We shall start with the simplest on-mass-shell Wightman function \(\Delta_{\mu\nu}^{(0)}\). This type correlators, \(\Delta_{\eta\eta}^{(0)}\), \(\Delta_{\mu\nu}^{(0)}\) and \(\Delta_{(1)}^{(0)}\) share the same polarization sum of the free gauge field. They correspond to the densities of the final states of the radiation field and are important for various calculations. The same polarization sum appears in expressions for the transverse part of the propagators, \(\Delta_{\tau\tau}^{(1\eta)}, \Delta_{\tau\tau}^{(1\nu)}, \Delta_{\tau\tau}^{(0\eta)}\) and \(\Delta_{\tau\tau}^{(1\nu)}\). For our immediate purpose we shall include the projector \(d_{\mu\nu}\) of the gauge \(A_\mu = 0\) into the formal Fourier representation,
with the “extraneous” dependence of the Fourier transform on the time and spacial coordinates. This dependence disappear in some important limits. Therefore, we discover the domains where the wedge dynamic simplifies and describe the processes which are approximately homogeneous in space and time. These domains are: (i) the central rapidity region, \( \eta_{1,2} \ll 1 \) (or \( x_{1,2}^3 \sim 0 \)), where the projector in the integrand of Eq. (5.4) is

\[
d_{\mu\nu}(k, u) = -g_{\mu\nu} + \frac{k_\mu u_\nu + u_\mu k_\nu}{ku} - \frac{k_\mu k_\nu}{(ku)^2} \tag{5.2}
\]

with the gauge-fixing vector, \( u_\mu = (1, 0, 0, 0) \), which approximately coincide with the local normal to the hypersurface \( \tau = const \); and (ii), the vicinities of two null-planes, \( \eta \to \pm \infty \) (or \( x^- \to 0 \)) where

\[
d_{\mu\nu}(n_{\pm}, k) = -g_{\mu\nu} + \frac{k_\mu n_\nu + k_\nu n_\mu}{(kn_{\pm})} \tag{5.3}
\]

with the null-plane vectors \( n_{\pm}^\mu = (1, 0, 0, \mp 1) \).

Eqs. (5.3)-(5.9) almost fit our needs. In all three cases, \( x^3 \to 0 \), \( k_0 x^0 \gg 1 \), as well as \( x^- \to 0 \), \( k^- x^+ \gg 1 \) and \( x^\pm \to 0 \), \( k^\pm x^\mp \gg 1 \), the functions \( f_1 \) and \( f_2 \) can be approximated by the following expressions

\[
f_1 \approx i \tanh(\theta - \eta) e^{-ik_\perp \tau \cosh(\theta - \eta) + i \vec{k} \vec{r}} = \frac{k_0 x^3 - k_3 x^0}{k_0 x^0 - k_3 x^3} e^{-ikx} = \frac{k^+ x^- - k^- x^+}{k^+ x^- + k^- x^+} e^{-ikx}. \tag{5.4}
\]

\[
f_2 \approx ik_\perp \tau \frac{e^{-ik_\perp \tau \cosh(\theta - \eta) + i \vec{k} \vec{r}}}{\cosh(\theta - \eta) - ik x} = \frac{k^2 \tau^2}{k_0 x^0 - k_3 x^3} e^{-ikx} = \frac{2ik_\perp \tau^2}{k^+ x^- + k^- x^+} e^{-ikx}. \tag{5.5}
\]

(We have omitted the time independent terms in \( f_1 \) and \( f_2 \) which set the potentials of the mode \( \nu(2) \) to zero at \( \tau = 0 \). This kind of terms would correspond to the residual gauge symmetries and is not kept in the axial and the null-plane gauges as well.)

Transformation of the correlator \( \Delta^{lm}(1, 2) \) to the Minkowsky coordinates is carried out according to the formula,

\[
D_{\mu\nu}(x_1, x_2) = a_l^\mu(x_1)g_{ij}(x_1)\Delta^{lm}(u_1, u_2)g_{nk}(x_2)a_m^n(x_2) \tag{5.6}
\]

where the matrix of the transformation is defined in a standard way,

\[
a_l^\mu(x) = \frac{\partial x^\mu}{\partial u^l}, \quad a_0^0(x) = x^3, \quad a_0^3(x) = x^0, \quad a_0^\tau = \delta^\tau_0. \tag{5.7}
\]

These are the only components of the tensor \( a_l^\mu(x) \) which participate the transformation. In this way we obtain

\[
D^{00}(1, 2) = x_1^3 x_2^0 \Delta^{00}(1, 2); \quad D^{03}(1, 2) = x_1^3 x_2^0 \Delta^{03}(1, 2); \\
D^{00}(1, 2) = x_1^3 x_2^0 \Delta^{00}(1, 2); \quad D^{33}(1, 2) = x_1^0 x_2^3 \Delta^{00}(1, 2); \\
D^{03}(1, 2) = x_1^0 x_2^3 \Delta^{03}(1, 2); \\
D^{00}(1, 2) = x_1^0 x_2^0 \Delta^{00}(1, 2); \\
D^{03}(1, 2) = x_1^0 x_2^3 \Delta^{03}(1, 2); \\
D^{03}(1, 2) = \Delta^{\tau \tau}(1, 2). \tag{5.8}
\]

Every additional factor \( g_{\eta \eta} = r^{-2} \) finds a counterpart which prevents a singular behavior at \( \tau = 0 \). In the above approximation, the expression for the \( \Delta^{\eta \eta}(x_1, x_2) \) component of the correlator has the form:

\[
\Delta^{\eta \eta}(x_1, x_2) = \int \frac{d^4k}{(2\pi)^32k^0} \frac{k_\perp^2 e^{-ik(x_1-x_2)}}{k_0^+ x^-_1 + k^- x^+_1} = \int \frac{d^4k}{(2\pi)^32k^0} \frac{4k_\perp^2 e^{-ik(x_1-x_2)}}{(k^+ x^- + k^- x^+)(k^+ x^- + k^- x^+)} \tag{5.9}
\]

Therefore, in the limit of \( x_{1,2}^3 \sim 0 \) we obtain that \( D^{00}, D^{0i} \to 0 \), while \( d_{\mu\nu}(k, u) \to k_\perp^2/k_0^2 \), thus reproducing the corresponding components of the gauge \( A^\eta = 0 \). The other components are reproduced one by one as well, and one can expect smooth transition between the gauge of the the wedge dynamic and the local temporal axial gauge of the reference frame co-moving with the dense quark-gluon matter created in the collision.

In the limits of \( x_{1,2}^\tau \to 0 \) we obtain that
the QCD evolution equations are adequate only when most hard back scattered quark is localized. This qualitative picture is in compliance with the well known fact that before the collision the whole packet is strongly localized in the vicinity of the null-plane entire surface of the light wedge, the phases and amplitudes of this decomposition are balanced in such a way that interaction in the vertex in order to break down this tiny balance and filter out an unusual field configuration.
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APPENDIX 1. Modes of the free gauge field

Here, we shall obtain the complete set of the one-particle solutions to the homogeneous system of the Maxwell equations with the gauge $A^\tau = 0$, that is Eqs. (2.10)-(2.11). This gauge condition explicitly depends on the coordinates, thus introducing effective non-locality in the path integral that represents the action. Therefore it becomes impossible to invert the differential operators using the standard symbolic methods. The knowledge of the one-particle solutions becomes necessary in order to find the Wightman functions of the free vector field, to establish the the explicit form of the field commutators, and to separate the propagators of the transverse and the longitudinal fields. It is natural to look for the solution in the form of the Fourier transform with respect to the spacial coordinates:

$$A_i(x) = \int_{-\infty}^{\infty} d\nu \int d^2\vec{k} \ e^{i \nu \eta} e^{i \vec{k} \vec{r}} \ A_i(\vec{k}, \nu, \tau)$$  \hspace{1cm} (A1.1)

Then the system of the second order ordinary differential equations for the Fourier transforms becomes as follows:

$$[\tau \partial_\tau^2 + \partial_\tau + \frac{\nu^2}{\tau} - \tau k_x k_y A_x(\vec{k}, \nu, \tau) - \tau k_x k_y A_y(\vec{k}, \nu, \tau) - \frac{\nu k_x}{\tau} A_\eta(\vec{k}, \nu, \tau) = 0 , \hspace{1cm} (A1.2)$$

$$- \tau k_x k_y A_x(\vec{k}, \nu, \tau) + \tau \partial_\tau^2 + \partial_\tau + \frac{\nu^2}{\tau} + \tau k_x^2 A_y(\vec{k}, \nu, \tau) - \frac{\nu k_y}{\tau} A_\eta(\vec{k}, \nu, \tau) = 0 , \hspace{1cm} (A1.3)$$

$$- \frac{\nu k_x}{\tau} A_x(\vec{k}, \nu, \tau) - \frac{\nu k_y}{\tau} A_y(\vec{k}, \nu, \tau) + \frac{1}{\tau} \partial_\tau^2 - \frac{1}{\tau^2} \partial_\tau + \frac{1}{\tau} k_x^2] A_\eta(\vec{k}, \nu, \tau) = 0 , \hspace{1cm} (A1.4)$$

In this form the system is manifestly symmetric and self-adjoint. An additional equation of the constraint reads as

$$C(\vec{k}, \nu, \tau) = \frac{1}{\tau} \nu \partial_\tau A_\eta + \tau \partial_\tau [k_x A_x(\vec{k}, \nu, \tau) + k_y A_y(\vec{k}, \nu, \tau)] = 0 . \hspace{1cm} (A1.5)$$

Let us rewrite the homogeneous system of the Maxwell equations in terms of the variables

$$\Phi = \partial_x A_x + \partial_y A_y, \hspace{0.5cm} \Psi = \partial_y A_x - \partial_x A_y, \hspace{0.5cm} \text{and} \hspace{0.5cm} A = A_\eta.$$  \hspace{1cm} (A1.6)

One immediately sees that the equation for the Fourier component $\Psi(\vec{k}, \nu, \tau)$ of the longitudinal magnetic field $\Psi(x)$ decouple:

$$[\partial_\tau^2 + \frac{1}{\tau} \partial_\tau + \frac{\nu^2}{\tau^2} + k_x^2] \Psi(\vec{k}, \nu, \tau) = 0$$ \hspace{1cm} (A1.7)

Then the other two equations of motion take shape

$$[\tau^2 \partial_\tau^2 + \tau \partial_\tau + \nu^2] \Phi_{\vec{k}, \nu}(\tau) - i \nu k^2 \Phi_{\vec{k}, \nu}(\tau) = 0 , \hspace{1cm} (A1.8)$$

$$[\partial_\tau^2 - \frac{1}{\tau} \partial_\tau + k_x^2] A_{\vec{k}, \nu}(\tau) + i \nu A_{\vec{k}, \nu}(\tau) = 0 , \hspace{1cm} (A1.9)$$

The additional constraint equation can be conveniently rewritten as

$$C(\vec{k}, \nu, \tau) = \frac{i \nu}{\tau} \partial_\tau A_{\vec{k}, \nu}(\tau) + \tau \partial_\tau \Phi_{\vec{k}, \nu}(\tau) = 0 . \hspace{1cm} (A1.10)$$
This is an independent equation. However, the conservation of the constraint along the Hamiltonian time \( \tau \) is a consequence of the equations of motion, and it can be employed to obtain the independent equations for the components of the vector field. This is easily done in terms of the auxiliary functions,

\[
\varphi_{\vec{k},\nu}(\tau) = \tau \Phi_{\vec{k},\nu}(\tau) \quad \text{and} \quad a_{\vec{k},\nu}(\tau) = \tau^{-1} \Phi_{a,\nu}(\tau),
\]

which are directly connected to the “physical” components of the electric field, \( \mathcal{E}^m = \sqrt{-g^m_{\mu \nu}} A_l \):

\[
\partial_\tau [\frac{1}{\tau^2} \partial_\tau + \frac{\nu^2}{\tau^2} + k_1^2] \varphi_{\vec{k},\nu}(\tau) = 0 ,
\]

\[
\partial_\tau [\tau^2 \frac{1}{\tau^2} \partial_\tau + \tau \partial_\tau + \nu^2 + k_1^2] a_{\vec{k},\nu}(\tau) = 0 ,
\]

As a result, we obtain that the functions \( \varphi_{\vec{k},\nu}(\tau) \) and \( a_{\vec{k},\nu}(\tau) \) obey inhomogeneous Bessel equations,

\[
[\partial_\tau^2 + \frac{1}{\tau} \partial_\tau + \frac{\nu^2}{\tau^2} + k_1^2] \varphi_{\vec{k},\nu}(\tau) = -k_1^2 c_\varphi ,
\]

\[
[\partial_\tau^2 + \frac{1}{\tau} \partial_\tau + \frac{\nu^2}{\tau^2} + k_1^2] a_{\vec{k},\nu}(\tau) = \tau^{-2} c_a ,
\]

where \( c_\varphi \) and \( c_a \) are arbitrary constants. Now we may cast the solution of these equations in the form of the sum of the partial solution of the inhomogeneous equation and a general solution of the homogeneous equation,

\[
\Psi_{\vec{k},\nu}(\tau) = a H^{(2)}_{-\nu \nu}(k_\perp \tau) + a^s H^{(1)}_{-\nu \nu}(k_\perp \tau) ,
\]

\[
\varphi_{\vec{k},\nu}(\tau) = c H^{(2)}_{-\nu \nu}(k_\perp \tau) + c^s H^{(1)}_{-\nu \nu}(k_\perp \tau) + c \varphi s_{1,\nu}(k_\perp \tau) ,
\]

\[
a_{\vec{k},\nu}(\tau) = \gamma H^{(2)}_{-\nu \nu}(k_\perp \tau) + \gamma^s H^{(1)}_{-\nu \nu}(k_\perp \tau) + c a s_{-1,\nu}(k_\perp \tau) ,
\]

where \( s_{\mu,\nu}(x) \) is the so-called Lommel function.

Furthermore, it is useful to notice that the system of the Maxwell equations \( (2.10)-(2.11) \) also has an infinite set of the \( \tau \)-independent solutions of the form

\[
W_i(\eta, \vec{r}) = \partial_\eta \chi(\eta, \vec{r}) ,
\]

where \( \chi \) is an arbitrary function of the spacial coordinates \( \eta \) and \( \vec{r} \). Thus, they are the pure gauge solutions of the Abelian theory, compatible with the gauge condition.

In order to find the coefficients one should integrate Eqs. \( (A1.17) \) and \( (A1.18) \) with respect to the Hamiltonian time \( \tau \), thus finding the functions \( \Phi \) and \( A \). Next, it is necessary to solve Eqs. \( (A1.6) \) for the Fourier components of the vector potential and to substitute them into the original system of Eqs. \( (A1.2)-(A1.5) \). Using functional relations from Appendix 2, one obtains that \( c + \nu^2 \gamma = 0 \) and \( c_a - \nu c_\varphi = 0 \).

One of the solutions, (already normalized according to Eq. \( (2.9) \)) is found immediately:

\[
V^{(1)}_{\vec{k},\nu}(x) = \frac{e^{-\nu \gamma/2}}{2^{\nu/2} \pi k_\perp} \begin{pmatrix} k_y \\ -k_x \\ 0 \end{pmatrix} H^{(2)}_{-\nu \nu}(k_\perp \tau) e^{i\nu \eta + i \vec{k} \vec{r}} .
\]

Initially, the components of the vector mode \( V^{(2)} \) appear in the following form required by the convergence of the integral,

\[
\begin{pmatrix} \nu k_r R^{(2)}_{1,\nu}(k_\perp \tau) |S| \\ -R^{(2)}_{1,\nu}(k_\perp \tau) |S| - i \nu [e^{\pi \nu / 2} / \sinh(\pi \nu / 2)] \end{pmatrix} e^{i\nu \eta + i \vec{k} \vec{r}} ,
\]

However, it can be gauge transformed to the more compact form,
\[ V_{\vec{k},\nu}^{(2)}(x) = \frac{e^{-\pi \nu/2}}{2^{5/2} \pi k_{\perp}} \left( k_{\nu} R_{1,-\nu}^{(2)}(k_{\perp} \tau|s) - R_{1,-\nu}^{(2)}(k_{\perp} \tau|s) \right) e^{i\nu \eta + ik_{\tau}} \]  

(A1.21)

The third solution, the last one by the count of the nonvanishing components of the vector potential in the gauge \( A^\tau = 0 \), has the following form,

\[ V_{\vec{k},\nu}^{(3)}(x) = \left( \frac{k_{\nu} Q_{1,-\nu}(k_{\perp} \tau)}{\nu Q_{1,\nu}(k_{\perp} \tau)} \right) e^{i\nu \eta + ik \tau} . \]  

(A1.22)

The modes \( V^{(1)} \) and \( V^{(2)} \) are the normalized solutions of the Maxwell equations. They are orthogonal and obey the normalization condition,

\[ (V_{\vec{k},\nu}^{(1,2)}, V_{\vec{k}',\nu'}^{(1,2)}) = \delta(\nu - \nu') \delta(\vec{k} - \vec{k}'), \quad (V_{\vec{k},\nu}^{(1)}, V_{\vec{k},\nu}^{(2)}) = 0 . \]  

(A1.23)

which can be easily verified by means of Eq. (A2.5). The norm of these solutions is given by the Eq. (2.9). A normalization coefficient of the mode \( V^{(3)} \) is not defined as this mode has a zero norm. It is also orthogonal to \( V^{(1)} \) and \( V^{(2)} \):

\[ (V_{\vec{k},\nu}^{(3)}, V_{\vec{k}',\nu'}^{(3)}) = (V_{\vec{k},\nu}^{(1)}, V_{\vec{k},\nu}^{(3)}) = (V_{\vec{k},\nu}^{(2)}, V_{\vec{k},\nu}^{(3)}) = 0 . \]  

(A1.24)

Thus this mode drops out from the decomposition of the free gauge field.

The conservation of the constraint can be obtained as a consequence of the Eqs. (A1.12) and (A1.13) in the form,

\[ \tau \partial_\tau [\varphi_{\vec{E},\nu}(\tau) + \nu a_{\vec{E},\nu}(\tau)] \equiv \tau \partial_\tau C_{\vec{E},\nu}(\tau) = 0 , \]  

(A1.25)

which reassures us in consistency between the dynamic equations and conservation of the Gauss law constraint.

One can explicitly check that the modes \( V^{(1)} \) and \( V^{(2)} \) obey the constraint equation (A1.10), which expresses the Gauss law. The mode \( V^{(3)} \) does not. This mode corresponds to the longitudinal field which cannot exist without the source.

**APPENDIX 2. Mathematical miscellany**

This appendix contains a list of mathematical formulae for the functions which appear in various calculations in the body of the paper and Appendix 1. The components of the vector field are expressed via two types of integrals. The first of them was studied in Ref. [7,8]:

\[ R_{\mu,\nu}(x|S) = \int x^\mu H_{\nu}^{(j)}(x) dx = x[(\mu + \nu - 1)H_{\nu}^{(j)}(x)S_{\mu-1,\nu-1}(x) - H_{\nu}^{(j)}(x)S_{\mu,\nu}(x)] , \]  

(A2.1)

where \( S_{\mu,\nu} \) stands for any of the two Lommel functions, \( s_{\mu,\nu} \) or \( S_{\mu,\nu} \). Whenever we omit the indicator \( |S| \), the function \( R_{\mu,\nu}(x|S) \) is assumed.\] The second type of integrals,

\[ Q_{\mu,\nu}(x) = \int_0^x x^\mu d x s_{-\mu,\nu}(x) , \]  

(A2.2)

is a new one. The functions \( R_{\mu,\nu}(x|S) \) are introduced as the indefinite integrals. The preliminary choice of the lower limit and, consequently, between \( s_{\mu,\nu} \) and \( S_{\mu,\nu} \), is motivated by the requirement of the convergence and regular behavior. One can easily prove that

\[ R_{1,\pm\nu}^{(2)}(x|S) - R_{1,\pm\nu}^{(2)}(x|s) = \mp ie^{\pi \nu/2} \nu \sinh(\pi \nu/2) / \nu ^2 \sinh(\pi \nu/2) \]  

(A2.3)

We often use the following relation between the Lommel functions

\[ S_{1,\pm\nu}(k_{\perp} \tau) = 1 - \nu^2 S_{1,\pm\nu}(k_{\perp} \tau) . \]  

(A2.4)
From the integral representations (A2.1) and (A2.3), it is straightforward to derive the functional relations

\[ R_{-1,iv}^{(j)}(k_\perp \tau) + \frac{1}{\nu^2} R_{1,iv}^{(j)}(k_\perp \tau) = - \frac{\tau}{\nu^2} \frac{\partial}{\partial \tau} H_{iv}^{(j)}(k_\perp \tau) \]  

(A2.5)

\[ Q_{-1,iv}(k_\perp \tau) - Q_{1,iv}(k_\perp \tau) = - \frac{\tau}{\nu^2} \frac{\partial}{\partial \tau} s_{1,iv}(k_\perp \tau) = \frac{\tau}{\nu^2} s_{-1,iv}(k_\perp \tau) \]  

(A2.6)

The Wronskian of the Hankel and Lommel functions,

\[ W\{s_{1,iv}(x), H_{iv}^{(j)}(x)\} = - \frac{1}{x} R_{1,iv}^{(j)}(x) \]  

(A2.7)

is necessary to prove orthogonality of \( V^{(2)} \) and \( V^{(3)} \). To prove (A2.7), one should use the following representation for the Lommel function,

\[ s_{1,iv}(x) = \frac{\pi}{4i} [H_{iv}^{(1)}(x)R_{1,iv}^{(2)}(x) - H_{iv}^{(2)}(x)R_{1,iv}^{(1)}(x)] \]  

(A2.8)

which follows from Eq. (A2.1) and its consequence,

\[ s_{1,iv}^{(j)}(x) = \frac{\pi}{4i} [H_{iv}^{(1)}(x)R_{1,iv}^{(2)}(x) - H_{iv}^{(2)}(x)R_{1,iv}^{(1)}(x)] \]  

(A2.9)

In order to prove relation (2.33) one should use representation (A2.1) for the functions \( R_{iv}^{(j)} \) and the Wronskian of two independent Hankel functions. The proof of relation (2.44) begins with replacing the functions \( R_{-1,iv}^{(j)} \) by \( R_{1,iv}^{(j)} \) by means of Eq. (A2.5). The final result follows from Eq. (A2.9) and (A2.6).

**APPENDIX 3. Calculation of the longitudinal part of the propagator**

The kernels (2.43) and (2.47) of the longitudinal and instantaneous parts of the propagator are given in the form of the three–dimensional Fourier integrals \( dvdu^2 k \). Here, we describe major steps of calculations which lead to Eqs. (4.1) and (4.3).

We permanently use the following integral representation for the Hankel functions,

\[ e^{-\pi \nu/2} e^{\pm iv \eta} H_{iv}^{(2)}(k_\perp \tau) = \frac{\pm i}{\pi} \int_{-\infty}^{\infty} e^{\pm ik_\perp \tau \cosh(\theta - \eta)} e^{\pm iv \theta} d\theta \]  

(A3.1)

which allows one to calculate many integrals by changing the order of integration. The Lommel function \( S_{1,iv} \) has a similar representation,

\[ S_{1,iv}(x) = x \int_0^\infty \cosh u \cos \nu u e^{-x \sinh u} du \]  

(A3.2)

Integrating it by parts, and using Eq. (A2.4), we find the integral representation for \( S_{-1,iv} \),

\[ \nu S_{-1,iv}(x) = \int_0^\infty \sin(\nu u) e^{-x \sinh u} du \]  

(A3.3)

We start with integral representation (A2.2) of the functions \( Q_{\pm 1,iv} \) and perform integration over \( \nu \). To compute the integrals from the function \( s_{1,iv} \) it can be conveniently decomposed in the following way,

\[ s_{1,iv}(x) = S_{1,iv}(x) - h_{iv}(x) \]  

(A3.4)

which allows one to find

\[ \int_{-\infty}^{\infty} s_{1,iv}(k_\perp \tau)e^{i\nu \eta} d\nu = \pi k_\perp \tau \cosh \eta e^{-k_\perp \tau \sinh |\eta|} \]  

(A3.5)
The similar Fourier integrals from the function $h_{ij}$ are calculated using the representation (A3.1) for the Hankel functions and the integral,

$$\frac{\pi \nu/2}{\sinh(\pi \nu/2)} = \frac{1}{2} \int_0^\infty d\theta \frac{e^{i\nu \theta}}{\cosh^2 \theta}.$$  \hspace{1cm} \text{(A3.6)}

This yields, for example,

$$\int_{-\infty}^\infty dv e^{i\nu \eta} h_{ij}(k_\perp \tau) = \int_{-\infty}^\infty \frac{d \theta}{\cosh^2 \theta} \sin[k_\perp \tau \cosh(\theta - \eta)] .$$ \hspace{1cm} \text{(A3.7)}

After integration over $\nu$ we obtain the following integral for the $\Delta^{(L)}_{rs}$

$$\Delta^{(L)}_{rs} = \int \frac{d^2 \vec{k}}{(2\pi)^3} \frac{k_r k_s}{k_\perp^2} e^{i\vec{k} \cdot \vec{r}_\perp} \int_0^{\tau_z} \frac{d \tau}{\tau} \left( \pi k_\perp \tau \cosh \eta e^{-k_\perp \tau \sinh |\eta|} - \int_{-\infty}^\infty \frac{d \theta}{\cosh^2 \theta} \sin[k_\perp \tau \cosh(\theta - \eta)] \right).$$ \hspace{1cm} \text{(A3.8)}

and similar integrals for the other components. The first term in this formula is calculated in the following way. After integration over $\tau$ we continue:

$$\Delta'_{rs} = -\frac{\partial_r \partial_s}{8\pi^2} \coth |\eta| \int \frac{d^2 \vec{k}}{k_\perp^2} e^{i\vec{k} \cdot \vec{r}_\perp} \left[ 1 - e^{-k_\perp \tau_2 \sinh |\eta|} \right]$$

$$= -\frac{\partial_r \partial_s}{4\pi} \coth |\eta| \int_0^{\tau_z} \frac{d \tau}{\tau} \int \frac{d^2 \vec{k}}{k_0^2} \frac{e^{i\vec{k} \cdot \vec{r}_\perp}}{k_\perp^2} \left[ \frac{\tau_2 \sinh |\eta| + \sqrt{\tau_2^2 + \tau_2^2 \sinh^2 \eta}}{\tau_2^2 + \tau_2^2 \sinh^2 \eta} \right].$$ \hspace{1cm} \text{(A3.9)}

To work out the second term, one should introduce $k_z = k_\perp \sin \theta$ and $k_0 = k_\perp \cosh \theta = |\vec{k}|$ and join $d^2 \vec{k} d\theta$ in one three-dimensional integration $d^2 \vec{k}$. With $t = \tau \cosh \eta$, $\vec{r} = (x, y, \tau \sin \eta)$, this leads to

$$\Delta''_{rs} = \frac{\partial_r \partial_s}{2\pi} \int_0^{\tau_z} \frac{d \tau}{\tau} \int \frac{d^2 \vec{k}}{k_0^2} \frac{e^{i\vec{k} \cdot \vec{r}_\perp}}{k_\perp^2} \sin k_0 t \frac{\partial_r \partial_s}{4\pi} \int_0^{\tau_z} \frac{d \tau}{\tau} \frac{\tau \cosh \eta}{\sqrt{\tau_\perp^2 + \tau^2 \sinh^2 \eta}}$$

$$= \frac{\partial_r \partial_s}{4\pi} \left( \frac{\tau_\perp \sinh |\eta| + \sqrt{\tau_\perp^2 + \tau_\perp^2 \sinh^2 \eta}}{\tau_\perp^2 + \tau_\perp^2 \sinh^2 \eta} \right) + \tau_\perp \sinh |\eta| + \sqrt{\tau_\perp^2 + \tau_\perp^2 \sinh^2 \eta} \frac{\tau_\perp}{r_\perp}.$$ \hspace{1cm} \text{(A3.10)}

Adding (A3.1) and (A3.10), we obtain the first of the equations (4.1).

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