Towards measuring variations of Casimir energy by a superconducting cavity

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We consider a Casimir cavity, one plate of which is a thin superconducting film. We show that when the cavity is cooled below the critical temperature for the onset of superconductivity, the sharp variation (in the far infrared) of the reflection coefficient of the film engenders a variation in the value of the Casimir energy. Even though the relative variation in the Casimir energy is very small, its magnitude can be comparable to the condensation energy of the superconducting film, and this gives rise to a number of testable effects, including a significant increase in the value of the critical magnetic field, required to destroy the superconductivity of the film. The theoretical ground is therefore prepared for the first experiment ever aimed at measuring variations of the Casimir energy itself.

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In recent years, new and exciting advances in experimental techniques have prompted a great revival of interest in the Casimir effect. As is well known, this phenomenon is a manifestation of the quantum zero-point fluctuations of the electromagnetic field (for recent reviews, see [1]). For the simple case of two plane parallel, perfectly conducting plates of area A, separated by a distance L, the Casimir energy is negative and equal to

\[ E^{(C)} = -\frac{\pi^2 \hbar c A}{720 \lambda^4}, \]

which corresponds to an attractive force between the plates.

All experiments on the Casimir effect performed so far, measured the Casimir force, in a number of different geometric configurations. In this Letter we find that by realizing a rigid cavity, including a superconducting plate, it might be possible for the first time to measure directly variations of the Casimir energy. The basic idea is very simple: since the Casimir energy depends on the reflective power of plates, there should be a variation in the Casimir energy (and force), as soon as the plate becomes superconducting, because the transition determines a sharp change in the reflective properties in the infrared (IR) region. Indeed, an attempt at modulating the Casimir force by changing the reflective power of mirrors has been made recently [2], with negative results. This may appear as very discouraging, especially if one considers that in this experiment, based on the technology of hydrogen switchable mirrors, there was a large modulation in the optical region of the spectrum, which is very relevant for typical submicron separations between the mirrors. The possibility of success with superconducting mirrors would seem even worse then, since the superconducting transition affects the reflective power only in the far IR region [2], which is clearly of little relevance for typical Casimir cavities. There is however a very important difference between our modulation scheme and the previous ones, which should make it possible to obtain very high sensitivities. Indeed, we do not mean to directly measure the relative variation of the Casimir energy (or force) accompanying the transition, which we indeed evaluate to be very small (typically, a few parts in hundred millions or less, in our conditions). In the experimental setting that we propose, aiming at a measurement of the critical field of a thin superconducting film included in a Casimir cavity, one is sensitive to variations of the Casimir energy as compared with the condensation energy of the film. This implies an enormous improvement in sensitivity, for the condensation energy is several orders of magnitude smaller than the Casimir energy, such that even a tiny fractional variation in the latter can produce significant effects on the critical field (see below). We observe another advantage of our setting, as the use of rigid cavities allows a large number of geometries, which will prove useful in the study of the dependence of the Casimir effect on geometry, indeed a distinctive feature of the Casimir effect, arising from the long-range character of retarded van der Waals forces.

To be definite, we consider a double cavity, consisting of two identical plane parallel mirrors, made of a non-superconducting and non-magnetic metal, between which a plane superconducting film of thickness \( D = 5 \text{ nm} \) is placed, separated by an empty gap of equal width \( L = 10 \text{ nm} \) from the two mirrors.

For any fixed temperature \( T \) below the critical temperature \( T_c \), we wish to determine the shift in the value of critical parallel field \( H_{\parallel}^{\text{c}}(T) \) of the film, determined by the Casimir energy of the cavity. Now, as is well known, \( H_{\parallel}^{\text{c}}(T) \) is determined by the difference between free energies \( \Delta F = F_n(T) - F_s(T) \) of the system (for zero field), in the normal (n) and in the superconducting (s) state. For a thin film \( (D \ll \lambda, \xi \text{ with } \lambda \text{ the penetration depth and } \xi \text{ the correlation length}) \), exploiting known formulae [3], we arrive at

\[ \left( \frac{H_{\parallel}^{\text{c}}(T)}{\rho} \right)^2 \frac{V}{8\pi} = \Delta F(T). \]
factor $\rho$ is introduced to take into account the incomplete expulsion of magnetic fields by a thin film, and the phenomenon of surface nucleation:

$$\rho = \frac{\sqrt{24\lambda}}{D} \left(1 + \frac{9D^2}{\pi^2\xi^2}\right).$$

(3)

For a film in a cavity, $\Delta F$ is the sum of the condensation energy $\mathcal{E}_{\text{cond}}(T)$ of the film, plus the variation $\Delta E^{(C)} = E^{(C)}_n - E^{(C)}_s$ of the Casimir (free) energy:

$$\Delta F = \mathcal{E}_{\text{cond}}(T) + \Delta E^{(C)}(T).$$

(4)

In writing these Equations, we have exploited the fact that all quantities referring to the film, like the penetration depth, condensation energy etc. are not affected by virtual photons in the surrounding cavity. This is a very good approximation, since the leading effect of radiative corrections is a small renormalization of the electron mass $\delta m_e = \frac{\alpha}{\pi} \frac{\hbar \omega_c}{mc^2}$ (up to logarithmic corrections), where $\alpha$ is the fine structure constant, $\omega_c = \epsilon/L$ is the typical angular frequency of virtual photons, and $m$ is the electron mass. For $L = 10$ nm, $\epsilon \approx 3 \times 10^{-7}$. The associated shift of critical field is of the same order of magnitude, and thus negligible with respect to that caused by $\Delta E^{(C)}$, which will turn out to be of some percent (see below).

We see from Eqs. [4, 1] that the change in the Casimir energy causes a shift $\delta H_{\parallel}/H_{\parallel} \approx \Delta E^{(C)}/(2\mathcal{E}_{\text{cond}}(T))$ of critical field, with respect to its value in a simple film, with same thickness and temperature. The magnitude of the effect depends on the relative magnitude of $\mathcal{E}_{\text{cond}}(T)$ and $\Delta E^{(C)}$, and below we show that these two quantities can indeed be comparable. Let us begin by the condensation energy $\mathcal{E}_{\text{cond}}(T)$. As is well known, it can be expressed in terms of the so-called thermodynamical field $H_{\text{therm}}(T)$, according to the formula $\mathcal{E}_{\text{cond}}(T) = H_{\text{therm}}^2(T) V/(8\pi)$. The temperature dependence of the thermodynamical field approximately follows the parabolic law $H_{\text{therm}}(T) \approx H_{\text{therm}}(0)[1 - (T/T_c)^2]$. We consider a film of Beryllium, which is a type I superconductor with a very low critical temperature ($T_c = 24$ mK) and a low critical field ($H_c(0) = 1.08$ Oe) [4]. Thin Be films possess a much higher critical temperature and a proportionally larger thermodynamical field. We take $D = 5$ nm, and then $T_c \approx 0.5$ K, which gives $H_c(0) \approx 22.5$ Oe. Thus, using the formula for $\mathcal{E}_{\text{cond}}(T)$ in terms of $H_{\text{therm}}(T)$, and the parabolic law for $H_c(T)$, we estimate that a Be film, with an area of $1 \text{ cm}^2$ (the area is not really important because both the condensation energy and the Casimir energy are proportional to the area), at a temperature $T = 0.97 \times T_c$ has a condensation energy $\mathcal{E}_{\text{cond}}(T) \approx 3.5 \times 10^{-8}$ erg. On the other hand, we see from Eq. [4] that a typical Casimir energy for a cavity with an area of $1 \text{ cm}^2$ and a width $L = 10$ nm, has a magnitude of 0.43 erg, i.e. over ten million times larger than the condensation energy of the film. We see then that a relative variation of Casimir energy as small as one part in $10^8$ would still correspond to more than 10% of condensation energy of the film, and would induce a shift of critical field of over 5%!

We now have to evaluate the difference $\Delta E^{(C)}$ among the Casimir free energies, for the two states of the film. The starting point of our analysis is the theory of the Casimir effect for dispersive media, developed long ago by Lifshitz [8]. In order to establish whether it is applicable to our superconducting cavity, we briefly recall what are its assumptions, and what is its range of applicability, as is obtained from the current literature. The main assumption of the theory is that, in the relevant range of frequencies and wave vectors, one can describe the propagation of electromagnetic waves inside the media forming the cavity, in terms of a complex permittivity, depending on the frequency $\omega$ and possibly on the wave vector $q$. Thus, provided that one takes into account the full dependence of the permittivity on the wave vector (besides the frequency), the Lifshitz theory retains its validity also in cases where space non-local effects become important (see the discussion in the first of Refs. [1]). It is important to stress that the theory includes also non-retarded effects [1], and hence it has as limiting cases both van der Waals forces (that become important at small distances, like those we consider) and Casimir forces. On this ground, it has been used recently to study van der Waals interactions among thin metal films (of thickness around 10 Å), till very small separations (a few Å) [2].

It is clear that non-local effects are important, in general, in superconductors and, for the small separations that we consider ($L = 10$ nm), also in normal metals (for an interesting discussion of non-local effects in the computation of dispersion forces in superconductors, see Ref. [12]). However, spatial dispersion is unimportant for the purpose of computing the difference between the Casimir energies in the two states of the film. The reason is that the optical properties of thin films (with a thickness $D$ much smaller than the skin depth or correlation length), in the normal and in the superconducting states, are indistinguishable for photon energies larger than a few times $kT_c$ ($k$ being the Boltzmann constant), as accurate measurements have shown [8]. This implies that, in the computation of $\Delta E^{(C)}$, the only relevant photon energies are those below roughly $10kT_c$ (corresponding to the far IR), which is where the optical properties of the film actually change when it becomes superconducting. In this region, the experiments show [8] that the transmittivity data for thin superconducting films can be well interpreted in terms of a complex permittivity that depends only on the frequency, and is independent of the film thickness.

Starting from the formulae in the first of Ref. [1], that provide a generalization of Lifshitz theory to multilayer systems, we have obtained the following expression for the variation of Casimir energy, in the limit of low tem-
permittivity of the normal mirrors at its sides. By using the long wavelength limit (\(\lambda \rightarrow 0\)), we can write the permittivity as

\[ \epsilon = \frac{\delta_{K} \epsilon_{q} \left(i \xi_{q}\right) - K_{l} \epsilon_{j} \left(i \xi_{j}\right)}{K_{j} \epsilon_{j} \left(i \xi_{j}\right) + K_{l} \epsilon_{j} \left(i \xi_{j}\right)} \]

where \(\xi_{j} = 1\), while \(\epsilon_{n/s}(i \xi_{j})\) denote the permittivities of the film in the \(n/s\) state, respectively, and \(\epsilon_{j}(i \xi_{j})\) is the permittivity of the normal mirrors at its sides. By using dispersion relations, the dielectric permittivities \(\epsilon_{j}(i \xi_{j})\) at imaginary frequencies \(i \xi_{j}\) can be expressed in terms of the imaginary part \(\epsilon''(\omega)\) of the dielectric permittivity \(\epsilon(\omega) = \epsilon'(\omega) + i \epsilon''(\omega)\) at real frequencies, i.e.

\[ \epsilon(i \xi_{j}) - 1 = \frac{2}{\pi} \int_{0}^{\infty} \frac{d\omega \epsilon''(\omega)}{\xi_{j}^{2} + \omega^{2}}. \]  

For the permittivities \(\epsilon_{d}\) and \(\epsilon_{s}\), we have used the Drude formula, which is very accurate at low frequencies:

\[ \epsilon_{d}(\omega) = 1 - \frac{\Omega_{j}^{2}}{\omega(\omega + \gamma_{j})}, \quad j = n, 2 \]

where \(\Omega_{j}\) is the (temperature-independent) plasmon frequency, and \(\gamma_{j}\) is the collision time of the metal. We have neglected the temperature variation of \(\gamma_{j}\), assuming that well before the transition temperature they reached their saturation values, as determined by the impurities present. We have taken \(\hbar \Omega_{n} = \hbar \Omega_{2} = 18.9\ \text{eV}\) (the values for Be) and \(\gamma_{n} = 2.4 \times 10^{-12}\ \text{sec}\) (the value for pure bulk Be samples [1]). As for \(\tau_{n}\), its actual value depends on the preparation procedure of the film, and as a rule it is much smaller than in bulk samples. We have considered for it three values, ranging from 10^{-13} sec to 10^{-12} sec.

The permittivity \(\epsilon_{s}(i \xi_{j})\) was computed by substituting into Eq. (5) the following formula for \(\epsilon_{e}''(\omega)\) [11], which is the long wavelength limit (\(q \rightarrow 0\)) of the ordinary Mattis-Bardeen complex permittivity of BCS theory:

\[ \epsilon_{s}''(\omega) = \frac{\hbar \Omega_{T}^{2}}{2 \omega^{2} \tau_{n}} \left[ \int_{-\Delta}^{\Delta} dE J_{T} + \theta(\hbar \omega - 2\Delta) \int_{\Delta-\hbar \omega}^{\Delta} dE J_{D} \right] \]

where \(\Delta\) is the (temperature dependent) gap and

\[ J_{T} := \left[ \tan \left( \frac{E + \hbar \omega}{2kT} \right) - \tan \left( \frac{E}{2kT} \right) \right] g(\omega, \tau_{n}, E) \]

\[ J_{D} := - \tan \left( \frac{E}{2kT} \right) g(\omega, \tau_{n}, E). \]

where \(\xi\) is an imaginary frequency, \(p\) is an auxiliary variable, \(\Lambda\) is some cutoff of order 10 or so (the final results are independent of its precise value) and

\[ Q_{n}^{TE/TM}(\xi, p) = \frac{1}{1 - \left( \Delta_{1}^{TE/TM} \right)^{2} \left( \Delta_{2}^{TE/TM} e^{-2\xi_{p} L/c} \right)^{2} - \left( \Delta_{1}^{TE/TM} - \Delta_{2}^{TE/TM} e^{-2\xi_{p} L/c} \right)^{2} e^{-2\xi_{p} D/c} \}}{1 - \left( \Delta_{1}^{TE/TM} \right)^{2} e^{-2\xi_{p} D/c} \}}. \]

The curves are computed for \(\omega_{c}/(\Omega_{n}^{2} \tau_{n})\), for \(T/T_{c} = 0.3\) (solid line), \(T/T_{c} = 0.9\) (dashed line) and \(T = T_{c}\) (point-dashed line). On the abscissa, the frequency \(\omega\) is in reduced units \(x_{0} = h\omega/(2\Delta(0))\)

Defining \(P_{1} := \sqrt{(E + \hbar \omega)^{2} - \Delta^{2}}\) and \(P_{2} := \sqrt{E^{2} - \Delta^{2}},\) the function \(g(\omega, \tau_{n}, E)\) is

\[ g := \frac{1 + \frac{E(\hbar \omega + \Delta^{2})}{P_{1} P_{2}}}{(P_{1} - P_{2})^{2} + (\hbar/\tau_{n})^{2}}, \quad \frac{1 - \frac{E(\hbar \omega + \Delta^{2})}{P_{1} P_{2}}}{(P_{1} - P_{2})^{2} + (\hbar/\tau_{n})^{2}}. \]

The expression for \(\epsilon''(\omega)\) includes also a singular contribution at zero frequency \(\delta(\omega)/\omega\), with a coefficient which is determined so as to satisfy the oscillator strength sum rule (see Eq. (14) in the first of Refs. [11]). Note that for \(T \rightarrow T_{c}\) \(\epsilon_{d}'' = \epsilon_{s}''\).

In Fig. 1, we show the plots of \(\omega_{c}''(\omega)/(\Omega_{n}^{2} \tau_{n})\), for \(T/T_{c} = 0.3\), \(T/T_{c} = 0.9\) and \(T = T_{c}\). The curves are computed for \(\tau_{n} = 5 \times 10^{-13}\ \text{sec}\). Frequencies are measured in reduced units \(x_{0} = h\omega/(2\Delta(0))\) (\(\Delta(0) = 7.6 \times 10^{-5}\ \text{eV}\). We have evaluated numerically Eq. (10). It turns out that the contribution of \(TM\) modes is completely negligible with respect to that of \(TE\) modes, in agreement with the findings of Ref. [3]. For fixed values of the impurity parameter \(y_{0} = h/(2\tau_{n} \Delta(0))\) (in the range 1 < \(y_{0} < 30\)), \(\Delta E(C)\) has the following approxi-
mate dependence on $L$, $T_c$ and $T/T_c$:

$$\Delta E^{(C)} \propto \frac{1}{L^{0.6}} \times T_c \times \left(1 - \frac{T}{T_c}\right). \quad (13)$$

In Table I (last three columns), we report the values of $\Delta E^{(C)}$, for three values of $\tau_n$, and for different temperatures close to $T_c$. Note that the values of $\Delta E^{(C)}$ are all positive, and hence the Casimir energy is smaller in the superconducting state of the film. This implies, according to Eq. 2, that the critical magnetic field is shifted towards larger values. In Fig. 2, we show the parallel critical field for a Be film placed in a Casimir cavity (solid line), as compared to that of a simple film (dashed line) of same thickness, in the interval $0.93 \leq T/T_c \leq 0.995$, for $\tau_n = 5 \times 10^{-13}$ sec. The curve for the Casimir cavity has been computed using a power-law fit to the numerical values of $\Delta E^{(C)}$. As we see from Fig. 2, the shift of critical field is larger near $T_c$.

Since close to $T_c$ the thermal fluctuations of the superconductor order parameter $\psi$ become sensible, one may wonder whether our results are altered when account is taken of these fluctuations. We find, however, that the shift of critical field resulting from this effect is negligible. The reason is that the order parameter is confined within the film, and hence it is not directly sensitive to the width $L$ of the empty gap at the sides of the film. The influence of the cavity width is only indirect, and arises from the coupling of electrons in the film to the virtual photons of the cavity. As we pointed out earlier (see considerations following Eq. 4), this radiative effect determines a small renormalization of parameters in the Ginzburg–Landau free-energy, of order $\varepsilon \approx 3 \times 10^{-7}$ to one-loop accuracy. Since the energy of thermal fluctuations of $\psi$ in the unperturbed film is at most of the same order as the condensation energy, we conclude that the energy shift caused by this effect is of order $\varepsilon \Delta E_{\text{cond}}(T)$, which in turn implies a shift of critical field $\delta H_c/H_c \approx 10^{-7}$, which is several orders of magnitude smaller than the shift resulting from the Casimir energy of virtual photons, that is of some percent.

In conclusion, we find that there is encouraging theoretical evidence in favor of suitable superconducting cavities being a promising tool for measuring variations of Casimir energy. We think that it would be very interesting to obtain an experimental verification of the effect of vacuum fluctuations on the critical field of a Casimir cavity [12].

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