Weinberg angle, current coupling constant, and mass of particles as properties of culminating-point filters – consequences for particle astrophysics

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Culminating-point filter construction for particle points is distinguished from torus construction for wave functions in the tangent objects of their neighborhoods. Both constructions are not united by a general manifold diffeomorphism, but are united by a map of a hidden conformal $S^1 \times S^3$ charge with harmonic (Maxwell) potentials into a physical space formed by culminating points, tangent objects, and Feynman connections. The particles are obtained from three classes of eigensolutions of the homogeneous potential equations on $S^1 \times S^3$. The map of the $u(2)$ invariant vector fields into the Dirac phase factors of the connections yields the electro-weak Lagrangian with explicit mass operators for the massive leptons. The spectrum of massive particles is restricted by the small, manageable number of eigensolution classes and an instability of the model for higher mass values. This instability also defines the huge numbers of filter elements needed for the culminating points. Weinberg angle, current coupling constant, and lepton masses are calculated or estimated from the renormalization of filter properties. Consequences for particle astrophysics follow, on the one hand, from the restriction of particle classes and, on the other hand, from the suggestion of new particles from the three classes e.g. of dark matter, of a confinon for the hadrons, and of a prebaryon. Definitely excluded are e.g. SUSY constructions, Higgs particles, and a quark gluon plasma: three-piece phenomena from the confinons are always present.

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2. dark matter
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1. INTRODUCTION

A paradigm change in astroparticle physics is often not believed to be necessary or even possible by most colleagues discussing the progress in confirmation of the experimentally accessible part of the Standard Model of particle physics. Moreover, two sociological problems bother the communities. (1). The information between the old and a new paradigm breaks down (Kuhn), if the new one contains too many new conceptions and new links between them. (2). The power of communities is increased with the number of active members and with their intention to solve the problems during their time. The second problem forms a sharper hierarchy occasionally supporting ignorance and even arrogance against the bare announcement of a new paradigm.

To avoid Kuhn’s breakdown of information, first paradigmatic attempts should start from one or only a few key concepts which can understandably be explained. Suppose that a new paradigm with many new concepts and links truly exists which is consistent with the confirmed part of Standard Models. Then, to neutralize the negative power of the communities, the key concepts could be followed by a plausible, intuitive way to the new paradigm as a whole.

The physical key concept of the first attempt (this paper, later referred to as [I]) is the difference between mass and energy behind the \( E = mc^2 \) formula. The key concept of a second attempt (referred to as [II]) is a thermodynamic analysis of an initial liquid hypothesis: Two phenomena known from the dynamic heterogeneity in the dynamical glass transition are supposed to exist also in an initial liquid before the cosmological inflation: \( G \) defects (Glarum Levy defects) as seeds for later galaxies, and \( F \) speckles (Fischer speckles) claiming the largest causal regions forming a finite universe.

The mathematical tool of the first attempt is the filter [I] (section 2). Pedagogically, we may start in the Introduction from an Euclidean space \( E \) and ask about the physical use of its points \((x)\). In the frame of physical elementarity, these points \( x \) are to locate "pointlike" particles in \( E \), e.g. electrons. In quantum theory, the particles have signatures (charge, spin, parity, ...) that are taken from the wave function \( \psi(x) \) in the neighborhood. Additionally, the particles at \( x \) have numerical parameters (mass, coupling constant, Weinberg angle...) which are not taken from \( \psi \): they are, from the beginning, parameters in the wave equation determining \( \psi \) as a function of \( x \). In such terms, the difference between mass and energy is physically noted as \( M/E \) separation and geometrically as \( x/\psi \) separation.

Compare the \( x/\psi \) separation with a general mathematical topological space of a set \( E \). Then each point \( x \) of \( E \) is connected with a filter \( \mathcal{F}(x) \); all the elements of \( \mathcal{F}(x) \) contain the point \( x \) and are neighborhoods of \( x \in E \). Our physical space is therefore not such a topological space with its identification of neighborhoods and filters, because these two get different functions: The wave function \( \psi \) belongs to the neighborhood that cannot give the parameters of the particle. These parameters, however, may belong to a filter \( \mathcal{F} \), whose limit to some culminating point can be the basis for their calculation. We could e.g. use some kind of a Cauchy filter: having elements that can be assigned to arbitrary small diameters needed for a culminating point construction of physical point particles. We need, therefore, a physical space construction, that distinguishes between the neighborhoods as \( \psi \) carriers (later called tangent objects), and the points as culminating points (whose filters allow the calculation of parameters). This construction excludes the general use of a space manifold with its metric diffeomorphism as e.g. used for the gauge theory prior of the standard model.

The preliminary stage of the above attempts [II], of course, does not allow an exact prediction of the parameters for particle astrophysics. In attempt [II], only a qualitative, plausible sketch of cosmological development before the primordial stage is tried. – The suggested "hidden charge model" of [II] (shortly, the model) indicates a new theoretical way to the parameters of the Standard Model for elementary particles, especially to the reason why the model is so as observed. Moreover, the way leads to some consequences beyond the Standard Model physics. The stock of particles is strongly restricted, and new phenomena become plausible (dark matter in [II], and primary flatness instead of dark energy, a hadronic inflation, a secondary role of gravitation, and a phenomenological, thermodynamic reason for the desired primordial fluctuations in [II]).

The details and literature of the cosmological revolution can be learned from the reports of the Particle Data
Group\cite{2,3,4,5}, with their authentic, rational reviews, the methods for the establishment of cosmological parameters from Spergel's et al reports\cite{6,7}, and the development of cosmology from\cite{8,9}. The main work on the hidden charge model is published before the cosmological revolution: A relevant analysis of the Spin Statistics Theorem\cite{10}, the hidden charge model\cite{11,12}, the construction of tangent objects\cite{13}, the analysis of the space problem\cite{14}, the estimation of some parameters\cite{15}, and the classification of eigensolutions of the hidden charge\cite{16}. A systematic, detailed discussion of the whole construction is in a report\cite{17}, to be published later.

2. FILTERS VS NEIGHBORHOODS

2.1 Filters

The mathematics of filters (chapter 8 there) starts from a filter basis. This is a non-empty family $\mathfrak{B}$ of subsets of a set $E$, if 1) the intersection of two subsets belonging to the family $\mathfrak{B}$ also belongs to the family $\mathfrak{B}$, and 2) the empty set does not belong to the family $\mathfrak{B}$. Examples are a set of rectangles, all containing the point $A$ (figure 1a); or all sets containing twodimensional disks $D^2$ with the center $A$ (figure 1b). If $\mathfrak{B}$ is a filter basis on $E$, then the set of all subsets of $E$ that contain at least one of the sets of family $\mathfrak{B}$, is called a filter $\mathfrak{F}$ generated by the basis $\mathfrak{B}$ on $E$. In the example of figure 1a, the basis generates a filter consisting of all sets of rectangles containing the point $A$ (the later culminating point).

![Fig. 1. Examples for filter constructions (details see text)](image)

Three Remarks should help to sort our physical model into such mathematics:

1. A topological space is a set $E$ each element $x$ (point $x$) of which is connected with a filter $\mathfrak{F}(x)$; all sets of these filters must contain the point $x$, i.e. they are neighborhoods of the point $x$. In addition, there is an analogous condition for the neighborhoods\cite{11}. This filter $\mathfrak{F}(x)$ is called filter of neighborhoods of the point $x$.

2. Be $f(x)$ a map of a set $E$ into a topological space $F$, and $\mathfrak{F}$ a filter on $E$. A point $y \in F$ is called limit of the map $f$ via the filter $\mathfrak{F}(x)$ ($y = \lim_{\mathfrak{F}} f(x)$) if the image $\mathfrak{B}$ of the filter $\mathfrak{F}$ gets its culminating point $y$ by the map $f$. A filter on a separable metric space is called Cauchy filter, if it contains sets of arbitrary small diameters; a Cauchy filter is a converging filter. A Cauchy filter has at most one culminating point. If it has one, so the filter converges to this point.

3. Filters that are related to the set of complements to natural numbers $(n)$ are called Fréchet filters. A sequence of points $(x_n)$ is a Cauchy sequence, if for any $\varepsilon > 0$ exists such an integer $N$, that the inequalities $n_1 > N$, $n_2 > N$ imply the inequality for the distance $\rho(x_{n_1}, x_{n_2}) < \varepsilon$. The Fréchet filter associated with $x_n$ is, therefore, a Cauchy filter.
2.2 Charge model

What are the hidden things that connect, in the sense of elementarity, the culminating point $x$ with the wave function $\psi$? For this purpose, the existence of a hypothetical hidden charge is assumed (figure 2): (1) The hidden charge is an omnipresent compact "electrodynamic" $S^1 \times S^3$ manifold in four dimensions (geometrically called $H$ space; if completed by an $A$ potential: hidden charge). (2) Homogeneous wave fields for 1-form $A$ potentials on $H$ from Maxwell's equations (harmonic potentials) are assumed. This construction is conformally invariant, i.e. the coordinates are angles and do not have the meaning of lengths and their "wave metric" has no constant $c$ in m/s. The omnipresent hidden charge suits therefore for the whole universe as well as for the particles therein, from $R \approx 10^{-35}$ m to $R \approx 10^{+26}$ m, say. (Bell's theorem excludes only local hidden parameters.) (3) Three torus coordinates $(\tau, \varphi_1, \varphi_2, 0...2\pi)$ can carry potential components $(A_0, A_1, A_2)$ whereas the one coordinate segment $(\vartheta, 0...\pi/2)$ does not carry a potential component, $A_\vartheta \equiv 0$. Any kind of electrical charges on the $H$ space are excluded by $\text{div}_4 A = 0$. (4) Culminating points are generated by an existential instability. The many elements needed for the filters of culminating points come from the $\vartheta$ segment and are called vacuum elements: $z = \cos^2 \vartheta$ (or $\zeta = 1 - z = \sin^2 \vartheta$). (5) The $A$ potentials on the torus coordinates are mapped on the wave functions $\psi$ in the tangent objects, the parameters of the points $x$ come from the $\vartheta$ filter. (6) The culminating points and their tangent objects form the physical or reality space $M$, the filter belongs to the $H \mapsto M$ map. The whole construction is called (hidden) charge model.

FIG. 2. Illustration of two methodical steps for the $x/\psi$ separation. (a). Culminating point and tangent object. (b). Feynman fabric of points and connections over tangent objects. The set of reality $M$ space elements {points, tangent objects, connections} forms a "crumbled space" (section 4.1, below). This space is not so "tight" as a manifold.

The physical picture resulting from the model is not complicated. We can use prosy and heuristic physical argu-
ments: It is not necessary first to be lost in the abyss of deep new mathematics.

The culminating point / tangent object: the $x/\psi$ separation of the $H \mapsto M$ map is illustrated by two methodical steps. First, the filter of the $\vartheta$ segment vacuum elements is depicted by a segment cone of the $H \mapsto M$ map. This is e.g. for mass at $x$ (figure 2a). Second, motions in the reality space $M$ are generated by "rotations" (invariant vector fields) of the hidden $H$ space giving $\psi$. This leads to Dirac phase factors for Feynman path integrals of the electroweak interaction. This is e.g. for energy (figure 2b, cf. also section 4).

The key question for the $x/\psi$ separation is: What is the $x$ in the wave function $\psi(x)$, if there is no manifold for $x$? We need (1) some construction, where $x$ can get a firm geometric base that would be measurable by e.g. photons. This is the tangent object around any point $x$. We need (2) some construction that defines $\psi$. This is the "Feynman fabric" of random points and Dirac phases between (figure 3) defining path integrals.

![FIG. 3. The Feynman propagator can do without a manifold diffeomorphism. We have a Feynman fabric of • random culminating points and ||| Dirac phase factors between. This fabric is over the tangent objects of the points.](image)

Remark to the tangent objects (appendix C). An electrodynamical (physical) space structure is separately defined for each tangent object. The tangent object is an "electrodynamical neighborhood" of each culminating point. A common mathematical Minkowski structure for "before" and "after" a particle collision, however, is not a prior of the hidden charge model.

Each physical tangent object can methodically be stratified into layers which contain different physical aspects of the potentials from the different eigensolutions on the $H$ space. Examples. The first layer is an algebraic construction of a biquaternion space, and the second layer is specified by "physical coordinates" marked by the potential components of the eigenfunctions for different classes. This results in an electrodynamical Minkowski structure $M_4$ for the tangent object for 1-class eigensolutions (e.g. leptons) and in a fourdimensional Euclidean structure $E^4$ for 2-class eigensolutions (e.g. dark matter particles or the prebaryon). The common space is a mix of tangent objects with different specification, e.g. a mix of $M_4$ and $E^4$ tangent objects (crumbled space, section 4.1). This is not a manifold.
Remark to the path integrals (figure 3): particle collisions result in Feynman diagrams from the elements of the Feynman fabric over the tangent objects of the particles.

A prior of the conventional gauge-theory approach to the Standard model ($SU(2)$, $SU(3)$, ..., GUT unification) is based on absolutizing the space metric: the real space be a manifold with its diffeomorphism for the metric, also for physics without gravitation. This diffeomorphism would generate a topological space unifying points and filters as neighborhoods. This would destroy the $x/\psi$ separation of figure 2.

The $x/\psi$ separation maintains the chance of our model to be consistent with the confirmed part of the Standard Model: 1). The Feynman propagator (figure 3) can do without the diffeomorphism of a mathematical space manifold. 2). Electrodynamics ($dF = 0$, $d^*F = e^*j$) and invariant vector fields of a group space need only external differentials $d$ for our physical applications. 3). As mentioned above, the conformal electromagnetic wave metric on $H$ (equation 21 below) has no intrinsic length, but only angles. 4). The quantum mechanical relation (from the Feynman path integral) between mass $m_0$ (e.g. for an exchange boson) and length $l$, $m_0 \sim 1/l$, can do with the mass definition from the culminating point and a length $l$ mediated by the tangent objects. This $m_0 \sim 1/l$ relation does not need the diffeomorphism.

Once again in other words: The massless model photon alone cannot define a metric tensor on the tangent object. Nevertheless, electromagnetic properties are essentially used to construct the tangent object (as discussed in appendix C). The mathematical diffeomorphism corresponds physically to the motion of the model graviton. This graviton is fundamentally connected to large space scales and is “very weak” (a shadow with factor $10^{-40}$) in comparison to the “stronger” electromagnetic properties of our flat $M_4$ tangent object in the empty space.

What things (besides a strong charge symmetry) can we obtain from the model?

Section 3: Although there are no particles and no parameters in the hidden charge, we get a simple and clear system of particles in the reality space from the only three classes of eigensolutions.

Section 4: The conventional electroweak Lagrangian in $M$ is obtained from the $H \mapsto M$ map of the invariant vector fields of $H$. Instead of Higgs (no relevant eigensolutions for Higgs particles and no energetic cause for mass), a mass operator with action on the $\vartheta$ segment for the filter is detected. Quantum field theory is explicitly obtained via Feynman path integrals; quantum theory seems possible also for 2-class particles with $E_4$ tangent objects, if they are isolated and imbedded in only $M_4$ tangent objects of the crumbled space.

Section 5: From the filter construction we get numerical values for the Weinberg angle $\theta_W$, the current coupling constant $\alpha(\sin^2 \theta_W)$, and the mass curve of charges leptons ($e, \mu, \tau$) with an existential instability for higher families.

2.3 Wherefrom do the many elements for the filters come?

The existential instability in the whole model comes from the maximum of the mass curve of charged leptons (figure 6, below, cf. also figure 2 in [13]). Existential stability includes that the mass of a corresponding particle in the next family is larger. An instability would enter, if the mass in the next but one and the following families become smaller. Then, a particle in the next family cannot exist. The boundary defines the number of existing charged lepton families; e.g. $\tau = 3$ would be the last existing, if eka $\tau = 4$ would have a larger mass, but eka $\tau = 5, 6, 7, ... \rightarrow \infty$ successively smaller ones. Existential instability is sharper than the quantum mechanical one: Such particles cannot exist even as virtual particles in Feynman diagrams. Example: The tauon $\tau = 3$ is quantum-mechanical instable but existentially stable.
The mass operator contains only $\partial/\partial \vartheta$ (besides a Hopf fiber for imbedding the particle point in its tangent object). The filter concerns, therefore, only the $\vartheta$ segment coordinate. This coordinate is contained in the harmonic potentials by the "harmonic polynomials" $y_{mk}(z)$, $z = \cos^2 \vartheta$, with $m$ for charge and $k$ the family number (section 3.2). For the vacuum ($k \to \infty$, $m \approx 0$) the number of zeros $= k$, i.e. the size of the segment part is of order $0(1/k) \to 0$. It is suggested that the convergence of segment parts mediates some kind of a Cauchy filter with numbers for $z$ vacuum elements. From the polynomials we get huge numbers (section 3.2) that facilitate the handling of the filter convergence for coupling constant and masses.

How get the other particles their masses? If they (as common particles from several eigensolutions, e.g. hadrons, or with $E^3$ tangent objects, e.g. dark matter) are imbedded in $M_4$ tangent objects of the crumbled space, I expect, seen from outside, similar methods as for charged leptons and, therefore, similar limitations from existential stability. Example: Since the $z$ structure of the dark matter particle is similar to the $z$ structure for the model electroweak exchange bosons, a dark matter particle gets an estimated mass of order 100 GeV. The $z$ structures are: for $(1,1)$ dark matter $\sqrt{z/(1-z)}$, for $W^\pm$ exchange bosons $(\sqrt{z}, \sqrt{1-z})$, and for the $Z$ exchange boson $\sqrt{z(1-z)}$. The two latter structures lead to $m_{0}(W^\pm)/m_{0}(Z) = \cos \theta_W$ as expected for the mass ratio of the exchange bosons.

2.4 Hidden charge model vs pristine string model

String particles do not have physical elementary as defined in section 1. String particles are excitations of (in the pristine form: onedimensional) objects consisting of "many" points of a manifold; the string particles are much larger than the points. In the hidden charge model, however, the particles are elementary points in the reality space, generated by the converging filters of culminating points. The space points have physical elementarity in the above sense by being possible locations of elementary particles.

In addition, in the string theory, space-time and physics are separated in conventional manner. The prior of a higher-dimensional Riemann-Klein-Kaluza space-time manifold consistent with physics carries a physical Lagrangian and a physical Higgs mechanism. Complications by e.g. warping and sequesting the space are accepted in the string community. In the hidden charge model, the borderline between space-time and physics is complicated from the beginning. E.g., the tangent object is not only a geometrical object but is physically motivated and stratified in different layers (see table 2 in appendix C of section 7, below).

Moreover, the number of dimensions for string theory is 10 or 11, i.e. 6 or 7 of them are compactified, i.e. hidden at small lengths corresponding, in the pristine versions, to high (Planck) energy: $10^{-20}$ fermi and $10^{19}$ GeV, respectively. The hidden charge model remains in four dimensions. The hidden $H$ space is a compact $S^1 \times S^3$ manifold with conformal invariance. This ensures omnipresence and application (the map of figure 2) to e.g. the very large universe as well as to Compton wave lengths of hadrons. The map between $H$ space and reality $M$ space is initiated by existential instability for higher particle families.

Additionally, the number of eigensolutions in the string theory is usually very large, and the separation of an experimental range ($< 1000$ GeV, say) is a thankless task. In the hidden charge model, the number of particles is restricted by only three classes of eigensolutions, and to low energies due to existential instability, so that a clear picture can be developed from the very beginning.

String theory delegates crumbling of space to the small Planck length scale. The hidden charge model sees a crumbled space from the $x/\psi$ separation as a general companion for quantum theory in all length scales. Conventional quantum theory (so far successful for electroweak interaction) is, as seen from our model, a trial to define the circumstances on the basis of a manifold.

String theory includes not only a unification of electroweak and strong theory at the GUT scale ($3 \cdot 10^{16}$ GeV), but also the gravitation can be included. This suggests a "hot" unification at the above Planck scale ($10^{19}$ GeV) and a hot start of the universe. The hidden charge model, however, allows a cold unification ($\lesssim 10^3$ GeV) and a cold start,
mainly because of the unusual properties of our dark matter particles\textsuperscript{[11]} and because a metric diffeomorphism \( g(x) \) is not assumed as a prior.

3. STOCK OF PARTICLES FROM MAXWELL-TYPE EIGENSOLUTIONS FOR A POTENTIALS ON THE HIDDEN \( S^1 \times S^3 \) CHARGE

The hidden \( H \) space suits well to electrodynamics. It has four dimensions and allows a Minkowski (wave) metric in angles (though with no \( c \) constant in meters/second for conformal invariance of wave fields). Biharmonic coordinates (appendix A in section 7, below): \( S^1 \) with \( 0 \leq \tau \leq 2\pi \), \( S^3 \) with \( 0 \leq \varphi_1, \varphi_2 \leq 2\pi \), \( 0 \leq \vartheta \leq \pi/2 \), allow a simple definition of a \( \oplus \leftrightarrow \ominus \) charge symmetry \( C \) on the basis of only wave fields (\( \text{div}_4 A = 0 \)), reflecting the equivalence of the two Heegaard tori in \( S^3 \) (figure 7 in appendix B of section 7, below):

\[
C : (\oplus, \tau, (\varphi_1, m_1), (\varphi_2, m_2), z = \cos^2 \vartheta) \leftrightarrow (\ominus, \tau, (\varphi_2, m_2), (\varphi_1, m_1), \zeta = \sin^2 \vartheta).
\]  

(1)

The three onedimensional tori \( 0 \ldots 2\pi \) come\textsuperscript{[16]} from the Cartan subalgebra of \( u(2,2) \). The latter is the algebra for the conformal group \( \text{Conf}(M_4) \) with 15 parameters; the Lie algebra \( u(2,2) \) is equivalent to \( so(4,2) \). A Cartan subalgebra is the maximal commutative subalgebra and has here three dimensions: Maxwell’s: equations in \( A \) on \( S^1 \times S^3 \) (appendix B in section 7, below) have three such tori with corresponding torus factors in the potentials,

\[
\partial_\tau A_\tau = i\omega A_\tau \text{ with } e^{i\omega \tau}, \\
\partial_{\varphi_1} A_1 = im_1 A_1 \text{ with } e^{im_1 \varphi_1}, \text{ and } \partial_{\varphi_2} = im_2 A_2 \text{ with } e^{im_2 \varphi_2}.
\]

(2)

These tori are generally connected with the signatures of wave functions \( \psi \) on the tangent objects in the reality M space. The \( \vartheta \) segment is separated by \( (A_\vartheta = 0, \text{no torus}) \) and is used for culminating points \( x \). The potential \( A \) instead of the Maxwell field tensor \( F, F \sim dA \), was chosen because of its occurrence in the Dirac Phase factors (Aharonov Bohm effect of quantum mechanics).

3.1 Three classes of eigensolutions

A complete classification\textsuperscript{[16]} can then be based on properties of the \( S^1 \) torus component \( A_0 = A_\tau \); there are three classes (table 1) whose mathematical details for 1-class are in\textsuperscript{[12]} (harmonics with harmonic polynomials), and for 2-class and 3-class in\textsuperscript{[16]}. The verbal terms for the\textsuperscript{[16]} model use are partially redefined after the cosmological revolution\textsuperscript{[17]}.

The eigensolutions suggest leptons, dark matter particles, prebaryons, baryon charges, and confinons used for hadronic confinement (section 5.4), with a model approach\textsuperscript{[12]} to strong interaction instead of the conventional \( SU(3) \) gauge approach. The classification of table 1 excludes: Any independent, fluctuating (and therefore quantizable) vacuum field (i.e. no Higgs field and no Higgs particle in the model, cf. section 3.2); any supersymmetry (details in appendix C, section 7, tangent object construction), and isolated quarks with third charges (the model quarks are confined leptons, the third charges come from the confining confinon), there is also no quark gluon plasma at high temperatures in the model. The mass scale is limited by the order \( 1 \text{ TeV} = 1000 \text{ GeV} \), if the existential instability is transmittable to all kinds of isolated particles; we have vacuum elements suitable for filter constructions only from 1-class vacuum solutions. The culminating point / tangent object \( x/\psi \) separation is maintained for all isolated particles.

3.2 Vacuum elements. No Higgs

Be \( k \) the family number, \( \kappa, 0 \leq \kappa \leq k \), the dummy integer, and \( m \geq 0 \) the integer charge number for 1-class solutions. Then for the potential \( A_1 \) (analogously for \( A_2 \) of the antiparticle from charge symmetry \( C \), equation \( 1 \))
TABLE I. $S^1$ Classification of eigensolutions $[16,17]$

| class | potential components (physical coordinates) | tangent objects (wave equation) | typical functions | specification | model use |
|-------|------------------------------------------|----------------------------------|------------------|--------------|-----------|
| 1-class. | $A_{1}(\varphi_2, \vartheta, \tau) =$ | $M_{4}$ | harmonic polynomials (equation (3.3)), $y_{mk}(z)$, $z = \cos^2 \vartheta$. | neutral: $m_2 = 0$ | neutrinos. |
|        | $\frac{\partial A_{k}}{\partial \tau} = 0$, $A_0 \equiv 0$ | $\omega = 2k + m$, $(\varphi_1)$ $FN^a$ | charged ($m_2 \neq 0$) | $\tau$, $\varphi_2$ tori | charged leptons. |
|        | $A_{0} = e^{i \sum (m_1 \varphi_1 + m_2 \varphi_2)} \times A_{0}(\vartheta)$. | $E^1$ (elliptic) | $A_{0}(\vartheta)$ $FN^b$ | prebaryon. | (baryon charge $m_2 = 1$) |
|        | $\frac{\partial A_{k}}{\partial \tau} = 0$, $A_0 \neq 0$ | $(\tau)$ | hypergeometric | one-charge $\varphi_2$ tori ($m_2 \neq 0$), | prebaryon. |
|        | $A = (A_0, A_1, A_2, 0)$ | $M_{4}$ | all $|\text{charges}| \equiv 4/3$ | $|m_1| = |m_2| > 0$ | dark matter |
| 3-class. | $\frac{\partial A_{k}}{\partial \tau} = 0$, $A_0 \neq 0$ | $(\tau, \varphi_1, \varphi_2)$ | the one special solution with 8 possibilities for $\varphi_1, \varphi_2 \in \{(+1, -1)\}$ | $|m_1|$, $|m_2| > 0$ | confinons for hadrons. |
|        | $A = e^{i(\omega \vartheta - \sum m_i ^2 \varphi_i)} \times \sum \varphi_0, \varphi_1 \in \{(+1, -1)\}$ | $(\varphi_1, \varphi_2)$ | | | $FN^d$ |

$FN^a$: Correspondingly for $A_2$ from charge symmetry $C$, equation (3.1). The polynomials are in equation (2.3), and the vacuum elements in section 3.2. $FN^b$: Examples for the amplitudes $A_0(\vartheta)$ are the prebaryon, $z^{1/2} F^{(1, 1, 2, z)}$, dark matter $(1, 1), (z/(1 - z))^{1/2}, (m, m), (z/(1 - z))^{m/2}$ for $\oplus$ particles. $FN^c$. 2-class particles can be measured as usual in the isolated pure state (e.g. the mass of a dark matter particle) or in a caged state (e.g. the energy contributions to the baryon from the prebaryon), if the outside is the $M_4$ tangent object, e.g. in the "vacuum". $FN^d$. One hadron consists of two (for mesons) or three (for baryons) leptons captured by one confinon; the captured leptons are then called quarks. For baryons, additionally one prebaryon is captured introducing the baryon charge (section 5.4). 

we get the polynomial for the 1-class solution $[11,12]$

$$y_{mk}(z) = z^{m/2} \sum_{k=0}^{k} \frac{k}{k} \left( \begin{array}{c} k + \kappa + m - 1 \\ k - 1 \end{array} \right) z^{k};$$

$$A_{1}^{0k}(z) = \exp(2k\tau)y_{ok}(z),$$

$$y_{ok}(z) = \sum_{k}( -1) \left( \begin{array}{c} k \\ k \end{array} \right) \left( \begin{array}{c} k + \kappa - 1 \\ k - 1 \end{array} \right) z^{k}$$

with $z = \cos^2 \vartheta$. The $m = 0$ solutions, $A_{1}^{0k}(z)$ with $y_{0k}(z)$, are for neutrinos. Formally, a vacuum solution ($m = 0$ or $m \ll k$, $k \to \infty$) may be called "\( \infty \) neutrino".

If, however, existential instability can also be established for the neutrinos (as discussed in section 5.3), then large numbers of $z$ in $(z)^\kappa$, called vacuum elements, are always obtained, especially when the decay goes down to the single $z = (z)^1$ (this is called electronic vacuum in the filter). The number of vacuum elements is then a
The huge number is given by:

\[
\text{huge number} = \binom{k}{\kappa} \binom{k + \kappa - 1}{k - 1},
\tag{6}
\]

To get some kind of a Cauchy filter, we assume that a "distance" \(d\) can be defined by the \(\vartheta\) angle distance between the \(y_{mk}(z)\) zeros in the \(\vartheta\) segment. As then \(d = 0(1/k)\), "smaller" vacuum elements are obtained for larger \(k\), their number hugely increases with \(k\). If, therefore, the hidden charge with the conformal wave metric (7.1)-(7.3) is taken as a primary basis for the filter, then the filter is a (Fréchet) Cauchy filter (section 2.1): Since the filter is a part of the \(H \mapsto M\) map, the vacuum elements contain, as image constraints, also some information about lengths in the \(M\) space. This leads to the concept of filter elements, explained in \([\text{II}]\) (at the beginning of section 5.1 there).

The vacuum elements are no eigensolutions, because an individual torus \(S^1_\vartheta\) is missing for them: \(A_1^{0k} \neq (A_1^{01})^k\) for \(k > 1\). This means no independent particle can be connected with vacuum elements alone. The huge numbers prevent any kind of fluctuation, i.e. there is no independent and general vacuum field that could be quantizable. Physically, the vacuum elements form the (non-quantizable) filter of the mass point; neither Higgs fields nor Higgs particles are constituents of the model.

### 4. ElectroWeak Lagrangian in Physical Space from Invariant Vector Fields in the Hidden Space. The Mass Operator

The physical and mathematical roots in the "layers" of tangent objects are listed in the appendices C, D, and E: In C the construction of a biquaternion \(M\) space for each tangent object from wave metric and charge symmetry of the \(H\) space, the way to \(E^4\) or \(M_4\) tangent objects from physical coordinates, and the construction of exchange bosons; in D the \(u(2)\) invariant vector fields; resulting in the empirical (Weinberg Salam) electroweak sector of the Lagrangian recapitulated in E.

The main result is obtained from a term-by-term comparison of the invariant vector fields with the empirical (conventional) Lagrangian in the corresponding exponentials. No rests are left in D and E, i.e. neither in the fields nor in the Lagrangian. The Lagrangian is obtained from the fields if

1. The Weinberg angle \(\theta_W\) is a definite value of the \(\vartheta\) segment coordinate used for the filter.
2. The mass operator is a differentiation in the direction of the filter coordinate \(\vartheta\) and substitutes the Higgs construction of the conventional Lagrangian:

\[
\text{Weinberg angle : } \theta_W = \vartheta,
\tag{7}
\]

\[
\text{Mass operator : } \partial = \begin{pmatrix} \sin \varphi \\ \cos \varphi \end{pmatrix} \partial_3 \text{ with } \partial_3 = \partial/\partial \vartheta,
\tag{8}
\]

where \(\varphi = \varphi_1 + \varphi_2\) is the fiber of the chosen Hopf bundle card \(S^3 \to S^2\).

A full proof was announced in 1991 and will be published. The matter is complicated from two aspects.

1. The exchange bosons are binary torus constructions (table 2 and 3 in appendix C): We need rules for torus handling, e.g. which operators of the invariant vector fields activate the tori (torus induction, table 3).

2. Image constraints: In the comparison, seen as a map, the primary set of \(M\) space elements (points, connections, tangent objects) is the origin (figure 2) and the conventional Minkowski \(M_4\) manifold used for the formulation of the conventional Lagrangian is (part of) the image. If the primary set is finer than (and not so "dense" as) the conventional \(M_4\) manifold, then the comparison is constrained by the harder properties of the coarse tight manifold used for the conventional Lagrangian (section 4.1, below).
The results confirm two things of our model: A clear distinction of mass and energy, and a quantum theory from our "classical" hidden charge. The mass generation is ascribed to the coordinate of the filter construction for the culminating point. The energy, on the other hand, is a construction from the wave functions of the tangent objects. The wave equation for contains the mass values of the point particles, the Weinberg angle, the coupling parameters and (all the) other parameters from the filter, having in this way influence on the energies, when the quantum mechanical wave equation is solved. [According to the model, the field of appendix E (for the conventional Yukawa mass terms) is not an independent Higgs field, but at the best some symbolic trial to connect the filter with a field.]

The quantum theory of the model starts from the charge with their tori, and not from the Planck constant \(\hbar\). The partition into the above primary set of space elements and their bond by the hidden charge evaluates the model to be some realization of Pauli's vision of such a quantum field theory which also determines its numerical parameters. (Let us repeat, that there are no parameters in the conformal hidden charge, even no speed of light in meters/second). The probability aspects are introduced by the randomness of tori coordinates \((\tau, \varphi_1, \varphi_2)\) where the \(\vartheta\) segment for the filter is pinned at them. After the map, the randomness is regulated by the Lagrangian from the invariant vector fields determining the probability density \(|\psi(r)|^2\) and the transition probabilities. The electroweak Lagrangian is not put from gauge principles, but is derived solely from the \(u(2)\) symmetry of the hidden \(H\) space. This leads directly to the Dirac phase factors for finite "rotations", introducing \(\hbar\) and the quantum field theory from Feynman path integrals. The generality of quantum theory is restricted to a Minkowski \(M_4\) environment and is more detailedly discussed near the end of Appendix H).

4.1 Examples for image constraints. Crumbled space

The mix of tangent objects with different specifications (e.g. spin directions, \(E^4/M_4\) fractions) is called a crumbled space. This space has no metric diffeomorphism. Its map into conventional physical manifolds (e.g. \(M_4\) for electroweak interactions, \(g(x)\) for general relativity) is characterized by image constraints, because the crumbled space is "finer", not so restricted than the "coarse" manifold:

fine origin (crumbled space) \(\to\) coarse image (manifold). (9)

Examples: (1) The crumbled space allows two \(\vartheta\) directions (figure 4, section 5.1 below), but the \(M_4\) manifold must do with one Weinberg angle \(\theta_W\). For the neutral current \(e_L\) (7.28) e.g. we must use the internal \(\vartheta\) orientation and obtain \(\tan \theta_W = \cot \theta_W\), for the neutral current \(e_R\) (7.31), however, we must use the external \(\vartheta\) orientation and obtain \(\tan \theta_W + \tan \theta_W\) from the operator \(\cot \vartheta - \partial_1 - \tan \vartheta_2\) of the vector fields, because in the standard model \(e_L\) is only one component of the pair \((\vartheta_\perp, \vartheta_\parallel)\), and \(e_R\) is alone. (2) Pauli’s spin statistics theorem follows from a "projection" aspect of the map, since, for each pair of spins with opposite spins, a common 180°rotation of the pair is equivalent to the exchange of the partners. Crumbling means here that the pairs have different common spin directions. This construction excludes any SUSY construction from the particle list of our model (table 1, also not containing any SUSY partners). (3) A nontrivial mixture of \(E^4\) (e.g. dark matter) / \(M_4\) (others) tangent objects is used for an initial liquid before and during the cosmological inflation. This allows a cold big bang (\(\lesssim 1000\) GeV, section 5.2 of Appendix H).

The image constraints are an important methodological tool in our model.

5. CALCULATION / ESTIMATION OF PARAMETERS

This section is to calculate or estimate the Weinberg angle, \(\theta_W\), the current coupling constant of the electroweak sector, \(\alpha(\sin^2 \theta_W)\), and the mass values of the charged leptons \((m_{e\tau}, m_{\mu\tau}, m_{\tau\tau})\). The estimation becomes more complicated in this succession, because e.g. the mass estimation supposes the action of the mass operator \((\hbar)\) in the filter. We use some kind of a huge-number approximation, that the estimation can be based on the increasingly huge numbers of vacuum elements \((\hbar)\) alone. This means that the amplitudes, the details of segment partition etc. will be neglected. We must expect that the precision of estimations decreases in the above succession.
5.1 Weinberg angle

The Weinberg angle of the low-energy limit can be estimated\textsuperscript{[14]} from the consistency of an external and an internal view of the image constraint for $\theta \mapsto \theta_W$ (figure 4a and 4b).

![Graph showing internal and external orientations of the $\theta$ segment coordinate.](image)

**FIG. 4.** Two orientations of the $\theta$ segment coordinate.

The internal orientation is based on the strong charge symmetry $C$\textsuperscript{[11]} of the hidden space. The coordinate values of the $\theta$ segments must be the same for $\oplus$ and $\ominus$ charges. The internal measure for $\theta$ is defined by the Haar measure for the $u(2)$ group algebra, $d\sqrt{g} \sim d\sin^2 \theta$ (appendix A, wave metric equation\textsuperscript{[23]}). The external orientation distinguishes the $\oplus$ and $\ominus$ charges; the two corresponding cards of the $S^3 \rightarrow S^2$ Hopf map with corresponding fibers $\varphi_1 \pm \varphi_2$ have the external measure $d\theta$. The low-energy limit allows to consider only the lowest vacuum elements: $z = \zeta^2 = \cos^2 \theta$ or $x = \xi^2 = \sin^2 \theta$, for the two cards. Defining a mean value of a function $\tilde{f}(z)$ by

$$\left\langle \tilde{f}(z) \right\rangle^\pm = \frac{\pi}{2} \int_0^{\pi/2} d\theta f(\theta^\pm), \quad \theta^\pm = \theta \pm \theta_0,$$

we get from $\tilde{f}(z) \sim z$ (or $x$) for the low-energy limit of $y_0 = \sin^2 \theta_0$ the equation: "internal" = "external"\textsuperscript{[11],[14]}

$$y_0 = \frac{1}{2} - \frac{2}{\pi} [y_0(1 - y_0)]^{1/2},$$

with the solution

$$y_0 = \frac{\sqrt{\pi^2 + 4} - 2}{2\sqrt{\pi^2 + 4}} = 0.231458363927\ldots.$$  

The approximate result $0.23146$ is in a good correspondence to the experimental values of the effective Weinberg angle $0.23149(13)\textsuperscript{[15]}$. The value $\sin^2 \theta' \approx 0.2315$ was quoted in 1986\textsuperscript{[12]}. The $y_0$ value is a general parameter of the electroweak sector of our model and does not depend on the family number.

5.2 Current coupling constant

We try a connection between Feynman diagram renormalization with the huge-number approximation\textsuperscript{[5]} of the filter renormalization: $\kappa \rightarrow \infty$, $k \rightarrow \infty$, $y = \kappa/k = $ finite. The order factor of the diagram, $\alpha^\kappa$, is substituted by the filter prefactor

$$\alpha^\kappa \rightarrow g^\kappa \left(\frac{k}{\kappa}\right) \left(\frac{k + \kappa - 1}{k - 1}\right) = \text{finite}.$$  

Finiteness is required for any kind of experiment for a general observable $X$. From equation\textsuperscript{[13]} we obtain a finite current coupling constant, $\alpha(y)$, from the finiteness of the experimental result $\langle X \rangle$, \ldots.
\[ \langle X \rangle \propto g^\kappa \binom{k\kappa}{k\kappa} (k+\kappa-1) (k-1) = \begin{cases} 
 0 & \text{for } g < \alpha(y) \\
 \text{finite} & \text{for } g = \alpha(y) \\
 \infty & \text{for } g > \alpha(y) 
\end{cases} \] (14)

and put \( y = \sin^2 \theta_W \) as the simplest internal measure. From (14) we calculate for huge numbers

\[ \alpha(y) = y^2(1-y)^{1-y}(1+y)^{-1+y} \] (15)

which function is graphed in figure 5.

FIG. 5. Reciprocal current coupling constant \( 1/\alpha \) as function of a current Weinberg angle \( y = \sin^2 \theta_W \) from the renormalization in the huge-number approximation (14) for the filter.

Special values from the approximation are the low energy limit \( \alpha^{-1}(y_0) = 135.411 \) 830 525 for \( y_0 \) of equation (12), \( \alpha^{-1}(1) = 4 \), \( \alpha^{-1}(1/2) = 27 \), and \( \alpha^{-1}(3/8) \approx 50.0319 \).

5.3 Mass of charged leptons

To use the huge-number approximation similar to equation (14), the \( z \) components for the charged leptons must now be taken into consideration (equation (3) with charge \( m = 1 \) for low families \( k_0 = 1, 2, 3, \ldots \)). We assume that mass renormalization via the filter can be described by numbers for the possibilities of mistaking; also the \( z \) values of the individual leptons with the huge numbers (6) of general vacuum elements \( z \) in (3). There are three possibilities

- I individual - individual : particle factor \( \Pi(k_0) \), \( k_0 = 1, 2, 3, \ldots \)
- II general - general : vacuum factor \( P_V(\kappa, k) \)
- III individual - general : particle-vacuum factor \( P_L(k_0, \kappa, k) \)

\[
\begin{array}{ll}
\text{I individual - individual} & : \quad \text{particle factor} \quad \Pi(k_0), \ k_0 = 1, 2, 3, \ldots \\
\text{II general - general} & : \quad \text{vacuum factor} \quad P_V(\kappa, k) \\
\text{III individual - general} & : \quad \text{particle-vacuum factor} \quad P_L(k_0, \kappa, k)
\end{array}
\] (16)

The factor I is a damping factor, and II and III are amplification factors for mass. Putting the Compton wave length \( \lambda_0 \sim 1/m_0 \), with \( m_0 \) the mass value, we have now

\[ [\lambda_0\Pi(k_0)]^\kappa \cdot P_V(\kappa, k) \cdot P_L(k_0, \kappa, k) = \text{finite}. \] (17)

The analogy to equation (14) assures the scaling of the mass spectrum by the coupling constant \( \alpha \).

The result is a maximum of the mass curve \( m_0(k_0) \), because the damping factor varies stronger than the amplification factors, approximately
\[
\exp \Pi \sim k_0 \ln k_0 \cdot P_V \cdot P_L \sim \text{sums of polynomials.} \tag{18}
\]

The maximum was firstly published in 1988\(^{[13]}\) and supports the concept of existential instability of section 2.3 (figure 6).

FIG. 6. Estimated mass curve for charged leptons. A. "Thermodynamic variant" equation (19), adjusted to the two known experimental ratios for \(\mu/e\) and \(\tau/e\). B. "Statistical variant" of the text. C. Ad hoc example that would give the existential stability of the three observed families.

In a "thermodynamic variant", equation (19) is used for the estimation,

\[
\ln m_0 = G + k_0 F - k_0 \ln k_0 U,
\]

where the "potentials" \(G, F\) and \(U\) are assumed to depend slowly on the family number \(k_0\): We put them constants. After adjustment at the known experimental values the curve A is obtained, allowing four existentially stable families.

In a "statistical variant" with a mass operator differentiation \(\partial/\partial \vartheta\) in one component of a bilinear product of the polynomials, with an optimization \((dQ/du = 0)\) for \(u\) in the common amplification factor, \(P = P_V \cdot P_L\), assumed to be \(P(z, k_0) = z^{k_0} \exp[kQ(u)]\), and using \(\alpha \cdot z = 1\), the curve B is obtained, allowing only two existentially stable families. The start value of the mass curve, however, is obtained without adjustment as \(\ln(m_{0\mu}/m_{0e}) = 5.34\), near the experimental value 5.33.

A mass curve with three families seems possible after refinement of the methods (ad hoc hand waving curve C in figure 6).

In the huge-number approximation, the neutrinos get zero masses, because no Hopf fiber \(\varphi\) (or \(\psi = \varphi_1 - \varphi_2\)) can directly be constructed for their mass operator. The polynomials for a damping factor, however, would exist, say, for indirect higher-order approximations with borrowed fibers from other collision partners. – No Higgs fields or Higgs particles are necessary for our general mass-curve estimations, based only on the decay into 1-class vacuum elements (section 3.2) due to existential instability.

5.4 Outlook for strong interaction

The standard model of elementary particles and our hidden charge model have different, alternative principles for the construction of a strong interaction: The former is based on a special extension of the gauge theory beyond the
electroweak group: \(U_Y(1) \times SU_L(2) \times SU_C(3)\), the latter on the confinement into the frame of the three classes of particles from table 1.

Let us assume, that more complicated particles can be constructed by filters for a common particle from eigensolutions of different classes.

The general construction for hadrons is described in the footnote FN\(^4\) of table 1 in section 3.1. For baryons, the following "chemical" lepton capture reaction can be considered:

\[
\begin{align*}
1\text{-class leptons } \psi + 2\text{-class prebaryon } \chi' + 3\text{-class shell confinon } \varphi_3 &\rightarrow \\
\text{quarks } q \text{ in confinement} + \Delta U
\end{align*}
\]

The l.h.s. of the reaction is for the high temperature state with 1-class leptons \(\psi\), a single-charged 2-class prebaryon \(\chi'\), and a 3-class confinon \(\varphi_3\) (using a spatial interpretation as some "shell" cage from its three tori). The r.h.s. of the reaction is for low temperatures: a baryon (or meson) from quarks captured inside of the cage from the shell plus some binding energy \(\Delta U\) partly supported, for baryons, by the baryon charge from the prebaryon. Energy has its part in the baryon mass: In the low temperature state we get an additional elliptic binding potential from the 2-class \(E^4\) tangent object of the prebaryon in the crumbled space inside. The quarks have a limited independence, and the gluons may be some kind of exchange bosons inside the shell.

The conventionally expected phase transition is substituted by states with different reaction partners after a shift of temperature (or pressure, ...). We do not obtain a quark gluon plasma at high temperatures, but a mixture of leptons, prebaryons, and confinons. With the latter we find always three-piece animals, even at the high temperature state: further only the ordinary leptons, charged or not (no quarks), and the prebaryon with \(E^4\) tangent objects in the crumbled space, isolated or not. The existential instability of higher leptons ensures an upper mass limit of particles with strong interaction. The hidden charge model delivers numbers that are well known from the \(SU(3)\) gauge field construction: three physical coordinates, three tori, neutralized third charges, and eight \(\sigma\) combination possibilities for the confinon of table 1.

6. CONSEQUENCES FOR PARTICLE ASTROPHYSICS

I. The quaternion structure of the crumbled reality space results definitively in \(M_4\) tangent objects with spinor structure for 1-class and 3-class particles and in \(E^4\) tangent objects for 2-class particles (appendix C). Supersymmetric particles are excluded in our hidden charge model (section 4.1).

II. The vacuum elements for the filter construction are not eigensolutions of an independent field. This excludes Higgs particles that are, by the way, also not necessary for mass generation by filters of our model (section 3.2).

III. The lepton capture reaction by the 3-class particle (confinon) does not allow a quark gluon plasma at high temperature, we find three-piece animals also there and no independent quarks (section 5.4).

IV. Dark matter particles are identified as neutral \((m, m')\) 2-class eigensolutions (table 1). The simplest dark matter particle with \((1, 1)\) charge tori gets a mass of order 100 GeV from the filter construction (end of section 2.3).

V. The lepton capture of our model (alternatively to gauge theory extrapolation for conventional GUT) allows a cold cosmology\([\text{II}]\) that is limited by existential stability to energies of order 1000 GeV. [The quarks as captured leptons introduce damping factors like equation (16) in the cage, the Hopf fiber can be borrowed from \(\varphi_i\) coordinates of the confinons or, for baryons, from the prebaryon.]
7. APPENDICES

A. Biharmonic coordinates on $S^3$

Biharmonic coordinates distinguish by $(\varphi_1, \varphi_2)$ the two equivalent Heegaard tori of $S^3$ that are used for a mathematical characterization of the basic charge symmetry $C$ according to equation (11) of the text (figure 7). Its conformal "wave metric" $g_{ik}$ corresponds to the Haar metric of the group $U(2) = (S^1 \times S^3)/\mathbb{Z}_2$. Minkowski signature is allowed, because the Euler number is zero.

\[ g_{ik} = \text{diag} \left( 1, -\cos^2 \vartheta, -\sin^2 \vartheta, -1 \right), \]  
\[ ds^2 = d\tau^2 - (\cos^2 \vartheta d\varphi_1^2 + \sin^2 \vartheta d\varphi_2^2 + d\vartheta^2), \]  
\[ \sqrt{-g} = \cos \vartheta \cdot \sin \vartheta. \]  

The Hopf map $S^3(\varphi_1, \varphi_2, \vartheta) \longrightarrow S^2(\phi, \vartheta)$ uses two cards each of which can be connected with one Heegaard torus,

\[ 2\vartheta = \theta, \varphi_1 = 0, \varphi_2 = -\psi = +\phi \text{ for } S^2 \setminus \{S\}, \]  
\[ 2\vartheta = \theta, \varphi_2 = 0, \varphi_1 = +\psi = +\phi \text{ for } S^2 \setminus \{N\}; \]  

the corresponding fibers are $\psi = \varphi_1 - \varphi_2$ or $\varphi = \varphi_1 + \varphi_2$.

B. Maxwell equations for potentials on $S^1 \times S^3$

We start formally with an arbitrary metric $g^{ik}$ (Einstein’s sum convention)

\[ (-g)^{-1/2}(\partial/\partial x^i)((-g)^{1/2}g^{kl}g^{im}(A_{k,m} - A_{m,l})) = 0 \]  

with the comma for differentiation of the potential components,
\[ A_{t,m} = \partial A_i / \partial x^m. \] (26)

The non-torus component is excluded. The transversality is defined by a zero divergence. (This also excludes any charge on the hidden space),

\[ A_\vartheta \equiv 0, \quad \text{div}_4 A = \frac{1}{\sqrt{-g}} \partial / \partial x^\vartheta \left( \sqrt{-g} A_k \right) = 0. \] (27)

Explicitly,

\[ 0 = -\frac{1}{\cos^2 \vartheta} \cdot \partial / \partial \varphi_1 \left( \frac{\partial}{\partial \varphi_1} A_0 - \frac{\partial}{\partial \tau} A_1 \right) - \frac{1}{\sin^2 \vartheta} \cdot \partial / \partial \varphi_2 \left( \frac{\partial}{\partial \varphi_2} A_0 - \frac{\partial}{\partial \varphi_2} A_2 \right) - \frac{1}{\sin \vartheta \cos \vartheta} \cdot \partial / \partial \vartheta \left( \sin \vartheta \cos \vartheta \left( \frac{\partial}{\partial \vartheta} A_0 \right) \right), \] (28)

\[ 0 = -\frac{1}{\cos^2 \vartheta} \cdot \frac{\partial}{\partial \tau} \left( \frac{\partial}{\partial \varphi_1} A_0 - \frac{\partial}{\partial \tau} A_1 \right) + \frac{1}{\cos^2 \vartheta} \cdot \frac{1}{\sin^2 \vartheta} \cdot \frac{1}{\cos^2 \vartheta} \cdot \partial / \partial \varphi_2 \left( \frac{\partial}{\partial \varphi_2} A_0 - \frac{\partial}{\partial \varphi_2} A_2 \right) - \frac{1}{\sin \vartheta \cos \vartheta} \cdot \frac{\partial}{\partial \vartheta} \left( \cos \vartheta \sin \vartheta \left( \frac{\partial}{\partial \vartheta} A_1 \right) \right), \] (29)

\[ 0 = -\frac{1}{\sin^2 \vartheta} \cdot \frac{\partial}{\partial \varphi_2} \left( \frac{\partial}{\partial \varphi_2} A_0 - \frac{\partial}{\partial \varphi_2} A_2 \right) + \frac{1}{\sin^2 \vartheta} \cdot \frac{1}{\cos^2 \vartheta} \cdot \frac{1}{\cos^2 \vartheta} \cdot \frac{\partial}{\partial \varphi_1} \left( \frac{\partial}{\partial \varphi_1} A_1 - \frac{\partial}{\partial \varphi_1} A_2 \right) - \frac{1}{\sin \vartheta \cos \vartheta} \cdot \frac{\partial}{\partial \vartheta} \left( \sin \vartheta \cos \vartheta \left( \frac{\partial}{\partial \vartheta} A_2 \right) \right), \] (30)

\[ 0 = -\frac{\partial}{\partial \tau} \left( \frac{\partial}{\partial \tau} A_0 \right) + \frac{1}{\cos \vartheta} \cdot \frac{\partial}{\partial \varphi_1} \left( \frac{\partial}{\partial \varphi_1} A_1 \right) + \frac{1}{\sin^2 \vartheta} \cdot \frac{\partial}{\partial \varphi_2} \left( \frac{\partial}{\partial \varphi_2} A_2 \right), \] (31)

\[ 0 = \text{div}_4 A = \frac{\partial}{\partial \tau} A_0 - \frac{1}{\cos^2 \vartheta} \frac{\partial}{\partial \varphi_1} A_1 - \frac{1}{\sin^2 \vartheta} \frac{\partial}{\partial \varphi_2} A_2. \] (32)

Not only the wave metric (21)-(23) in angles, but also the equations (28)-(32) are conformally invariant\[^{16}\]. The 3-class broken 4/3 charges of the confion follow from a decoupling scheme\[^{16}\] for the eigensolution of the \( A_0 \) amplitude,

\[ A_0(\vartheta) = (\sin \vartheta \cos \vartheta)^{-4/3}, \quad \omega^2 = m_1^2 = m_2^2 = 16/9. \] (33)

### C. Tangent objects and their methodical physical stratification

As mentioned in section 2.2, the construction of an underlying tangent object around any particle is necessary for the geometric location of points \( x \) in \( \psi(x) \), which \( x \) values are connected by the Feynman fabric of figure 3. This construction\[^{13}\],\[^{14}\] has a physical input from the use of physical coordinates. Different physical aspects may then methodically be used for some formal stratification. The effects in the reality \( M \) space can then be sorted into such layers. Formal means that the tangent object remains an indivisible whole.
TABLE II. Formal stratification of the tangent objects

| Layer name         | Formal specification | Physical specification | FN$^a$ |
|--------------------|----------------------|-------------------------|--------|
| 5. Mach’s principle layer | Dollhouse universum from spots of galaxies by a map from large filter elements | Gravitation, flatness vs. dark energy |        |
| 4. Partial chirality layer | Double hidden asymmetry | Baryon survival |        |
| 3. Exchange boson layer | Bilinear boson construction, | Torus handling, Brauer Weyl theorem, Feynman diagrams | FN$^a$ Examples and details in the text |
| 2. $M_4/E^4$ layer; Spinor $M_4$ space or $E^4$ space. | Tangent objects for different particles | Physical coordinates from nonzero potential components, wave functions from $H$ potentials |        |
| 1. Biquaternion layer | Tangent object from biquaternions | Charge symmetry of the hidden space |        |

The **biquaternion layer** is the algebraic basis for the tangent objects. They are constructed from a van der Waerden (“vdW”) algebraic equivalence-class construction. Examples: $uu$ with no additional condition gives tensors, $uu = 0$, $uv + vu = 0$ gives Grassmannians, and so on up to the Clifford algebra used here.

We start from the coordinate vectors $u_i$ in the hidden $H$ space (i.e. from its ”tangent objects” = tangent space in $H$),

$$u_1 = \partial/\partial \varphi_1, \quad u_2 = \partial/\partial \varphi_2, \quad u_3 = \partial/\partial \vartheta, \quad u_0 \text{ or } u_4 = \partial/\partial \tau. \quad (34)$$

To preserve the complete charge symmetry $C$ [14] and the wave metric signature from equations (21)-(23), we use a bilinear two-$u$ construction: one $u$ from the $\varphi_1$ Heegaard torus and the other $u(= v)$ from $\varphi_2$ torus (figure 7):

$$uv + vu = B(u, v) = Q(uv) - Q(u) - Q(v) \quad (35)$$

with restriction to the quadratic form $Q(u)$,

$$u_iu_i = q_i, \quad u_iu_j + u_ju_i = q_{ij} = \text{diag} (-, -, +). \quad (36)$$
and \( B = 0 \). Excluding all odd combinations of the algebra (also because of charge symmetry \( C \)) gives the Clifford algebra \( C_+ \). This algebra partitions into two biquaternions, written by complex-number indicators \((l, i, k)\) as

\[
C_+ = \{ \mathbb{H}, \tilde{\mathbb{H}} \}, \quad \mathbb{H} = \{1, l, i, k\}, \quad \tilde{\mathbb{H}} = u_5 \mathbb{H} = \mathbb{H} u_5
\]

with \( u_5 = u_1 u_2 u_3 u_4 \). Using physical coordinates, we get for 1-class eigensolutions exactly the vdW spinor spaces defining a flat Minkowski structure \( M_4 \) on the tangent object (see the next layer).

The physics of the \( M_4 \) tangent object is of electromagnetic nature, but cannot be reduced to the photon properties alone. Instead, the above equivalence-class construction uses three physical concepts: (a) the signature \((36)\) left from the orthogonality of the conformal angle (wave) metric in the \( H \) space \((21) - (23)\); (b) the antisymmetry \((36)\) as for all forms in the \( H \) space; and (c) the bilinearity from the two-\( u \) construction emerging from the charge symmetry \( C \) of equation (1). [Using the next layer aspects, the main point can be worded as: the photon product-construction, suggested in the fabric by equation \((41)\), is quenched in the tangent object by the orthogonality \( u_1 u_2 \sim 1 \) of equation \((39)\).]

In the \( M_4/E^4 \) layer, we introduce physical coordinates \( \{\beta\} \). They are defined as the coordinates that carry potential components (e.g. \( \varphi_1 \) for \( A_1 \neq 0 \)). The general purpose is to give the wave function \( \psi \) in the \( M \) space a physical basis by the \( A_i \) potentials in the \( H \) space \([13],[14]\). Physics is mainly ascribed to these \( \{\beta\} \). The tangent object is then specific to the class of eigensolutions (table 1, above). It is required that the physical coordinates are linearly transferred from the tangent vectors of the hidden \( H \) space \((u_j \) of equation \((34)) \) to the quaternion coordinates of the tangent objects of the crumbled \( M \) space. This \( H \to M \) transfer is called linear \( \beta \) transfer. Example 1 (1-class solution). The physical coordinates are called \( \beta \), the other are designed by 1, which means no transfer of physical aspects. We have in the leptonic sector (allowing e.g. pair creation)

\[
\beta_1 \sim A_1, \quad \beta_2 \sim A_2, \quad \beta_3 \sim 1, \quad \beta_4 \sim 1.
\]  

Orthogonality of the biharmonic coordinates is then expressed by

\[
\begin{aligned}
\beta_1 & \text{ for } 1 \text{-class solutions } \psi^{(1)} \\
\beta_2 & \text{ for } 1 \text{-class solutions } \psi^{(2)} \\
1 & \text{ for products like a photon } A
\end{aligned}
\]

This yields, as mentioned above, the two non trivial vdW spinor spaces giving the usual bilinear Minkowski \( M_4 \) structure for the \( M_4 \) tangent objects for 1-class particles.

Example 2 (2-class solution). The linear \( \beta \) transfer results here in a situation, that the physical coordinates can be collected in one quaternion, e.g. \( \mathbb{H} \); we do not obtain two non trivial spinor spaces. In \([12]\) is shown: If only one quaternion component (i.e. the quaternion \( \mathbb{H} \) or \( \tilde{\mathbb{H}} \)) is physically relevant for the \( M \) space tangent object, then its total biquaternion metric is Euclidean, \( E^4 \). This leads to an elliptic "wave equation" for non-isolated dark matter particles and prebaryons. (For isolated objects with inside \( E^4/M_4 \) crumbling, the outside is Minkowskian flat, only \( M_4 \).)

In the exchange boson layer, the bosons are constructed by the Feynman fabric of figure 3. The electroweak exchange bosons are formed from the bilinear charge-conserving construction

\[
(A, W^\pm, Z) \sim (A_1 \text{ type}) \ast (A_2 \text{ type}).
\]

The fabric is reflected in the tangent object by the \( \ast \) which symbolizes a junction between the two spinor spaces. We obtain
TABLE III. Torus content and torus handling for electroweak exchange bosons

| Particle content | Charge torus | Torus induction |
|------------------|-------------|----------------|
| (m = 1)          | in M4 Lagrangian | (section 4) FN^a |
| A                | no          | LA = 0         |
| Z                | two: e^{im\varphi_1}, e^{-im\varphi_2} | LZ = \cot \vartheta \cdot \partial_1 - \tan \vartheta \cdot \partial_2 |
| W^+              | e^{-im\varphi_2} | LW^+ = (\partial_1 = 0, - \tan \vartheta \cdot \partial_2) |
| W^-              | e^{im\varphi_1} | LW^- = (\cot \vartheta \cdot \partial_1, \partial_2 = 0) |
| \nu_e, \nu_\mu, \nu_\tau | no          | L\nu = 0 |
| e, \mu, \tau     | e^{im\varphi_1} | L^e^- = (\cot \vartheta \cdot \partial_1 = 0) etc. |
| \bar{e}, \bar{\mu}, \bar{\tau} | e^{-im\varphi_2} | L^e^+ = (\partial_1 = 0, - \tan \vartheta \cdot \partial_2) etc. |

\[ A = \begin{pmatrix} 0 \\ v \end{pmatrix} \ast \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad Z = \begin{pmatrix} + \\ - \end{pmatrix} \ast \begin{pmatrix} - \\ + \end{pmatrix}, \quad W^+ = \begin{pmatrix} + \\ - \end{pmatrix} \ast \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad W^- = \begin{pmatrix} 0 \\ v \end{pmatrix} \ast \begin{pmatrix} - \\ + \end{pmatrix}, \quad (41) \]

where the (0, 0) defines the photon particle by the \( \tau \) torus, \((+, -)\) are the two electrical charges, and \( v \) is some indication of the vacuum filters for the bosons. Equations (41) are the linear \( \beta \)-transfer roots for the conventional Brauer-Weyl theorem

\[ \gamma^\mu \partial_\mu \rightarrow \{ \gamma^\mu \partial_\mu, \gamma^\mu A_\mu, \gamma^\mu Z_\mu, \gamma^\mu W^\pm_\mu \} \quad (42) \]

Remark: The relation of these boson construction to the vectors (operators) in the hidden charge is sketched in table 3. The \( L \) symbol means a so called torus exchange vector, constructed from the \((L_1, L_2)\) invariant vector fields of appendix D,

\[ \begin{pmatrix} L_1 \\ L_2 \end{pmatrix} = \begin{pmatrix} \cos \varphi & \sin \varphi \\ -\sin \varphi & \cos \varphi \end{pmatrix} \begin{pmatrix} L \end{pmatrix}, \quad (43) \]

\[ \begin{pmatrix} L_1 \\ L_2 \end{pmatrix} = \begin{pmatrix} \cos \vartheta \cdot \partial_1 - \tan \vartheta \cdot \partial_2 \end{pmatrix} \begin{pmatrix} \sin \varphi \\ \cos \varphi \end{pmatrix} \partial_3, \quad (44) \]

with \( \varphi \) the fiber coordinate of the Hopf map for the \( L_1 \) vector fields and \( \partial \) the mass operator [5]. Torus handling means the knowledge how to operate with the tori of the hidden charge to get the electroweak Lagrangian and the Feynman diagrams in the reality space. Torus induction is a part of this handling: which differentiations of (44) generate the exchange bosons.

In the partial chirality layer, a twofold hidden charge asymmetry is formulated. Twofold means hidden in the \( H \) space and hidden behind the charge symmetry \( C \) by means of the \( C_+ \) algebra. In this hypothetical layer we ask how the physical coordinates and the wave functions react on the charge exchange \( C: u_1 \leftrightarrow u_2 \). This chirality is defined by hidden nontrivialities with respect to the Heegaard tori, expressed by all possible \( u_i \) combination (including also \( u_1, u_2, u_1 u_2 u_3 \), e.g. by the differences between the \{ \( u_1, u_1 u_2, u_1 u_2 u_3 \) \} components. This chirality is e.g. different for 1-class spinors in \( M_4 \) and constructions for 2-class particles in \( E^4 \) (prebaryons, baryon charge, dark matter). Partial chirality cannot be defined on the basis of a \( M_4 \) manifold. The partial chirality \( (P') \) is conserved for 1-class and broken for 2-class particles. Requiring \( P'CT = 1 \), then \( P' \) properties can be transferred to time reversal violation \( T \). The participation of the prebaryon in the lepton capture reaction \( (20) \) for baryons can qualitatively explain their cosmological surviving [21].

In the Mach’s principle layer, a map of galaxies from the whole universe into spots on each tangent object (dollhouse universe) is considered, that is mediated by the large filter elements for the culminating-point particle in the center of any tangent object (details see [14]). The corresponding elements of the reality space are specific with respect to number and "direction" of gravitons that take share. These species define an additional shadow metric \( g(x) \) over an otherwise flat Minkowski space from isolated \( M_4 \) tangent objects without a metric diffeomorphism.
D. $u(2)$ invariant vector fields

These fields transport the local $u(2)$ symmetry over the whole $U(2)$ group space, and therefore over the whole hidden $S^1 \times S^3$ space because of the $U(2) = (S^1 \times S^3)/Z_2$ covering. These fields correspond to the "rotations" of figure 2 that are assumed to induce motions in the reality space defining the electroweak Lagrangian in the context of Feynman connections. There are 7 different fields, four left ones ($L_1, ..., L_4$) and four right ones ($R_1, ..., R_4$) with $L_4 = R_4$. (Such fields are called Killing vectors in general relativity.) We restrict ourselves to the left ones (left invariant multiplication) and have the following four fields, corresponding to the $u(2)$ matrices as indicated:

\[
\begin{pmatrix}
0 & i \\
i & 0
\end{pmatrix}: L_1 = \cos \varphi \cdot \cot \theta \cdot \partial / \partial \varphi_1 - \cos \varphi \cdot \tan \theta \cdot \partial / \partial \varphi_2 + \sin \varphi \cdot \partial / \partial \vartheta,
\]

\[
\begin{pmatrix}0 & 1 \\ -1 & 0 \end{pmatrix}: L_2 = -\sin \varphi \cdot \cot \theta \cdot \partial / \partial \varphi_1 + \sin \varphi \cdot \tan \theta \cdot \partial / \partial \varphi_2 + \cos \varphi \cdot \partial / \partial \vartheta,
\]

\[
\begin{pmatrix}i & 0 \\0 & -i \end{pmatrix}: L_3 = -\partial / \partial \varphi_1 - \partial / \partial \varphi_2,
\]

\[
\begin{pmatrix}0 & i \\i & 0 \end{pmatrix}: L_4 = \partial / \partial \tau,
\]

with the Hopf map fiber $\varphi = \varphi_1 + \varphi_2$.

E. Empirical electroweak Lagrangian

The conventional Lagrangian is written in a form that especially well fits to a comparison with the above vector fields:

\[
L_C/e_0 = \bar{e} \gamma_{\mu} e A_{\mu}
\]

\[
+ \frac{1}{2} \{ \tan \theta_W - \cot \theta_W \} \bar{e} L \gamma_{\mu} e_L
\]

\[
+ \{ \tan \theta_W + \cot \theta_W \} \bar{\nu} \gamma_{\mu} \nu
\]

\[
+ \{ \tan \theta_W + \tan \theta_W \} \bar{\nu} \gamma_{\mu} e_R Z_{\mu}
\]

\[
+ \cot \theta_W (2 \sqrt{2})^{-1} \bar{\nu} \gamma_{\mu} (1 + \gamma_5) e W_{\mu}^+ + h.c.
\]

\[
+ \frac{1}{2} \bar{\nu} \gamma_{\mu} \partial_{\mu} (1 + \gamma_5) \nu + \bar{\nu} \gamma_{\mu} \partial_{\mu} e
\]

\[
+ h_e \left( \bar{\nu} \gamma_{\mu} \partial_{\mu} e + ... \right)
\]

with $h_e$... the adjustable Yukawa mass parameters and $\phi$ the Higgs field.

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