Probing the supersymmetric grand unified theories with gravity mediation at the future proton–proton colliders and hyper-Kamiokande experiment

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Abstract With the grand desert hypothesis, we have proposed to probe the supersymmetric Grand Unified Theories (GUTs) at the future proton–proton (pp) colliders and Hyper-Kamiokande experiment previously. In this paper, we study the supersymmetric GUTs with gravity mediated supersymmetry breaking in details. First, considering the dimension-six proton decay via heavy gauge boson exchange, we point out that we can probe the supersymmetric GUTs with GUT scale $M_{\text{GUT}}$ up to $1.778 \times 10^{16}$ GeV at the Hyper-Kamiokande experiment. Second, for the supersymmetric GUTs with $M_{\text{GUT}} \geq 1.0 \times 10^{16}$ GeV and $M_{\text{GUT}} \geq 1.2 \times 10^{16}$ GeV, we show that the upper bounds on the universal gaugino mass are 7.2 TeV and 3.5 TeV, respectively, and thus the corresponding upper bounds on gluino mass are 15 TeV and 8 TeV, respectively. Also, we shall study the masses for charginos, neutralinos, squarks, sleptons, and Higgs particles in details. In particular, the supersymmetric GUTs with $M_{\text{GUT}} \leq 1.2 \times 10^{16}$ GeV can be probed at the Hyper-Kamiokande experiment, and the supersymmetric GUTs with $M_{\text{GUT}} \geq 1.2 \times 10^{16}$ GeV can be probed at the future 100 TeV pp collider experiments such as the FCC\textsubscript{hh}, SppC via gluino searches. Thus, the supersymmetric GUTs with gravity mediation can be probed by the FCC\textsubscript{hh}, SppC, and Hyper-Kamiokande experiments. In our previous study, we have shown that the supersymmetric GUTs with anomaly and gauge mediated supersymmetry breakings are well within the reaches of these experiments. Therefore, our proposal provides the concrete scientific goal for the FCC\textsubscript{hh}, SppC, and Hyper-Kamiokande experiments: probing the supersymmetric GUTs.

1 Introduction

It is well known that supersymmetry (SUSY) provides a natural solution to the gauge hierarchy problem in the Standard Model (SM). In the supersymmetric SMs (SSMs) with $Z_2$-parity, we can achieve the gauge coupling unification \cite{1–5}, have the Lightest Supersymmetric Particle (LSP) like neutralino as a dark matter (DM) candidate \cite{6}, and break the electroweak (EW) gauge symmetry radiatively due to the large top quark Yukawa coupling, etc. In particular, gauge coupling unification strongly suggests the Grand Unified Theories (GUTs) \cite{7–10}, and the SUSY GUTs can be constructed from superstring theory, which is the most competitive candidate for quantum gravity. Therefore, supersymmetry is not only the most promising new physics beyond the SM, but also a bridge between the low energy phenomenology and high-energy fundamental physics.

However, after accumulation of data from the LHC Run-1 and Run-2, we have no hints for the SSMs. Of course, with the help of these data now we have stronger bounds on the spectra of the supersymmetric particles (sparticles). For instance, the lower mass bounds on gluino, the first two generation of squarks, stop, and sbottom are 2.3 TeV, 1.9 TeV, 1.25 TeV, and 1.5 TeV, respectively \cite{11–15}. Thus, there might exist the EW fine-tuning problem, and some promising solutions have been proposed \cite{16–29}. These natural SSMs generically predicts some relatively light sparticles, for instance, Bino, Higgsino, stop, gluino, and sleptons, etc.
On the other hand, to probe the new physics beyond the SM, we have a few proposals for the future proton–proton (pp) colliders, for example, the FCC$_{hh}$ [30] and SppC [31]. The naive question is whether we can probe the supersymmetric GUTs with the GUT scale\( M_{\text{GUT}} \leq 1.0 \times 10^{16} \) GeV, the dimension-six proton decay via heavy gauge boson exchange can be probed at the Hyper-Kamiokande experiment. Also, for the GUTs with gravitation mediation, we showed that the supersymmetric GUTs with anomaly and gauge mediated supersymmetry breakings are well within the reach of the future Hyper-Kamiokande experiment. For the supersymmetric GUTs with gravity mediated supersymmetry breaking at the FCC$_{hh}$ and SppC, and the supersymmetric GUTs with gravity mediated supersymmetry breaking can be probed. Hence, the remaining interesting question is whether we can probe the supersymmetric GUTs with gravity mediated supersymmetry breaking at the Hyper-Kamiokande experiment.

This paper is organized as follows. In Sect. 2, we discuss the supersymmetric GUT searches at the Hyper-Kamiokande experiment. In Sect. 3, we study the supersymmetric GUTs with gravity mediated supersymmetry breaking in details, and their searches at the future proton–proton colliders. Our conclusion is given in Sect. 4.

2 Probing the supersymmetric grand unified theories at the hyper-Kamiokande experiment

In the GUTs, the well-know prediction is the dimension-six proton decay \( p \to e^+ \pi^0 \) via heavy gauge boson exchange, and the proton lifetime is given by [36–38]

\[
\tau_p(e^+\pi^0) \simeq 1.16 \times 10^{35} \times \left( \frac{2.5}{A_R} \right)^2 \times \left( \frac{0.04}{\alpha_{\text{GUT}}} \right)^2 \times \left( \frac{M_{X/Y}}{1.0 \times 10^{16} \text{ GeV}} \right)^4 \text{ years} ,
\]

(1)

where \( A_R \) is the dimensionless one-loop renormalization factor associated with anomalous dimension of the relevant baryon-number violating operators, \( \alpha_{\text{GUT}} \) is the unified gauge coupling, and \( M_{X/Y} \) is the mass for the heavy gauge bosons \( X_{\mu}/Y_{\mu} \). The current lower limit on the proton lifetime from the Super-Kamiokande experiment is \( \tau_p > 1.6 \times 10^{34} \) years [39]. Thus, we obtain \( M_{X/Y} \geq 1.0 \times 10^{16} \) GeV. At the future Hyper-Kamiokande experiment, we can probe the proton lifetime at least above \( 1.0 \times 10^{35} \) years [40]. Thus, the GUTs with \( M_{X/Y} \leq 1.778 \times 10^{16} \) GeV is within the reach of the future Hyper-Kamiokande experiment. For more detail related GUTs and \( M_{GUT} \) related bosons see [41–49].

To clarify the subtle point, we want to emphasize that the mass of the heavy gauge bosons \( X_{\mu}/Y_{\mu} \) is smaller than or equal to the GUT scale \( M_{GUT} \). Thus, the supersymmetric GUTs with GUT scale up to \( 1.778 \times 10^{16} \) GeV can be probed at the future Hyper-Kamiokande experiment.

3 Probing the supersymmetric grand unified theories with gravity mediation at the future proton–proton colliders

The supersymmetry searches at the 100 TeV pp colliders have been studied previously [30,31,50,52–54]. For the integrated luminosity \( 30 \text{ ab}^{-1} \), Wino via Bino decay, gluino \( \tilde{g} \) via heavy flavor decay, gluino via light flavor decay, first-two generation squarks \( \tilde{g} \), and stop can be discovered for their masses up to about 6.5 TeV, 11 TeV, 17 TeV, 14 TeV, and 11 TeV, respectively. Moreover, if the gluino and first-two generation squark masses are similar, they can be probed up to 20 TeV.

By the way, the correlations between the low energy SUSY spectra and the GUT scale have been studied via the one-loop renormalization group equations before, and it was found that the bound from dimension-six proton decay already excludes the gluinos and Winos heavier than about 120 TeV.
and 40 TeV, respectively, if their mass ratio $M_3/M_2$ is about 3 [55]. In our paper, we employ the ISAJET 7.85 package [56] to perform the scan, which will give us more precise results. To be concrete, for the supersymmetric GUTs with $M_{GUT} \lesssim 1.0 \times 10^{16}$ GeV, we find that the current bound from dimension-six proton decay excludes the gluinos and Winos heavier than 15 TeV and 6 TeV, respectively. Because Winos might decay via Higgsinos as the benchmark point 4 given in the following Sect. 3.2, we are not sure whether Wino is within the reach of the FCC$\ell$b and SppC experiments, which will be studied elsewhere. Moreover, we shall study the masses for charginos, neutralinos, squarks, sleptons, and Higgs particles as well.

3.1 Phenomenological constraints and scanning procedure

We employ package ISAJET 7.85 [56] for the scans of mSUGRA/CMSSM parameter space. Here we want to state that due to the unknown GUT-scale threshold corrections [59–61], we do not strictly demand gauge coupling unification conditions $g_1 = g_2 = g_3$ at $M_{GUT}$. It should be noted that we allow $g_3$ to deviate from $g_1 = g_2$ within 2%. In particular, in this study we note that most of our points have $g_1 = g_2$ and $g_3$ within about 0.75%. Moreover, we set $\mu > 0$ and use $m_t = 173.3$ GeV [62]. We also use $m_b(m_b) = 4.1999$ GeV which is given in ISAJET.

The fundamental parameters of mSUGRA/CMSSM are restricted as follows

\begin{align}
0 \leq m_0 & \leq 90 \text{ TeV}, \\
0 \leq M_{1/2} & \leq 30 \text{ TeV}, \\
-3 \leq A_0/m_0 & \leq 3, \\
2 \leq \tan \beta & \leq 60.
\end{align}

We would like to draw attention of the reader to the fact that the requirement of radiative electroweak symmetry breaking (REWSB) [63–67] puts an important theoretical constraint on the parameter space. Another important constraint comes from the limits on the cosmological abundance of stable charged particles [68]. They exclude the parameter space where the charged SUSY particles, such as $\tilde{t}_1$ or $\tilde{t}_1$, become the LSP [69].

The data points collected all satisfy the requirement of REWSB, with the neutralino being the LSP. In addition, after collecting the data, we impose the mass bounds on all the sparticles [70], and the constraints from rare decay processes $B_s \to \mu^+ \mu^-$ [71], $b \to s\gamma$ [72], and $B_u \to \tau \nu_\tau$ [73]. More explicitly, we set

\begin{align}
m_h = 122 - 128 & \text{ GeV}, \\
m_{\tilde{g}} & \geq 2.2 \text{ TeV}, \\
0.8 \times 10^{-9} & \leq \text{BR}(B_s \to \mu^+ \mu^-) \leq 6.2 \times 10^{-9} (2\sigma), \\
2.99 \times 10^{-4} & \leq \text{BR}(b \to s\gamma) \leq 3.87 \times 10^{-4} (2\sigma), \\
0.15 & \leq \frac{\text{BR}(B_u \to \tau \nu_\tau)_{	ext{MSSM}}}{\text{BR}(B_u \to \tau \nu_\tau)_{	ext{SM}}} \leq 2.41 (3\sigma). \quad (7)
\end{align}

To be general, we do not require the relic abundance of the LSP neutralino to satisfy the Planck 2018 bound within 5σ: $0.114 \leq \Omega_{\text{CDM}}h^2\langle\text{Planck}\rangle \leq 0.126$ [74].

3.2 Scan results

We shall discuss results from the systematical scans. In Fig. 1, we show plot $M_{GUT}$ as a function of $M_{1/2}$. Gray points are consistent with REWSB and LSP neutralino. Orange points satisfy the mass bounds including $m_h = 125 \pm 3$ GeV and the constraints from rare $B-$ meson decays. Blue points form a subset of orange points and satisfy $1 \lesssim M_{GUT} \lesssim 1 \times 10^{16}$ GeV, while red points form a subset of orange points and satisfy $M_{GUT} \gtrsim 1.2 \times 10^{16}$ GeV. Two horizontal blue and red lines represent $M_{GUT} = 1 \times 10^{16}$ GeV and $M_{GUT} = 1.2 \times 10^{16}$ GeV, respectively. The first vertical line shows the upper bound on $M_{1/2}$ for red points ($M_{1/2} = 3.5$ TeV), and the second vertical line shows the upper bound on $M_{1/2}$ for blue points ($M_{1/2} = 7.2$ TeV). From the upper bounds on gaugino masses $M_{1/2}$ given by two vertical lines, we obtain
that the upper bounds on gluino masses are 8 TeV and 15 TeV respectively for the red and blue points. Therefore, we clearly show that SUSY GUTs with $M_{GUT} \gtrsim 1.2 \times 10^{16}$ GeV for gravity mediated SUSY breaking scenario [33–35], i.e. the red points, can be probed by the future 100 TeV pp colliders such as the FCC$^h$ and SppC. Moreover, the blue points and orange points with $M_{GUT} \leq 1.0 \times 10^{16}$ GeV can be explored by the Hyper-Kamiokande experiment. In the latter part of the paper we see the impact of these bounds on the fundamental parameters of the mSUGRA/CMSSM and sparticle spectrum.

In Fig. 2, we display plots the mSUGRA/CMSSM fundamental parameters as function of gluino mass. The color coding is the same as in Fig. 1. Horizontal red, black and blue lines represent the gluino mass upper bounds of 8 TeV, 11 TeV, and 15 TeV corresponding to the red points, gluino discovery via heavy flavor decay at 100 TeV pp collider [50], and blue points, respectively. The first vertical line shows the upper bound on $M_{1/2}$ for red points ($M_{1/2} = 3.5$ TeV), and the second vertical line shows the upper bound on $M_{1/2}$ for blue points ($M_{1/2} = 7.2$ TeV).
Fig. 3: The gluino mass versus the left-handed slepton mass, light stau mass, light stop mass, and CP-odd Higgs boson mass. The color coding is the same as in Fig. 2.

as 64 TeV for red points as well as blue points. This has important implications. Because $m^2_{\tilde{q}} \approx m_0^2 + (6 - 7)M^2_{1/2}$, $m^2_{\tilde{e}_L} \approx m_0^2 + 0.5M^2_{1/2}$, and $m^2_{\tilde{e}_R} \approx m_0^2 + 0.15M^2_{1/2}$ [51], the large $m_0$ term will give the dominate contributions to the squark and slepton masses. Plot in $A_0/m_0 - m_{\tilde{g}}$ plane is shown in lower left panel. Here we see that for smaller values of $A_0/m_0$, gluino mass $m_{\tilde{g}}$ rises. Plot is almost symmetric along $A_0/m_0 = 0$, and $m_{\tilde{g}}$ decreases as $|A_0/m_0|$ increases. Plot in $\tan \beta - m_{\tilde{g}}$ plane is shown in the lower right corner. It is evident from the plot that the red points, blue points and orange points can have $\tan \beta$ from 2 to 60.

In Fig. 3, we present the gluino mass versus the left-handed slepton mass, light stau mass, light stop mass, and CP-odd Higgs boson mass. In the top upper panel, we display plot in $m_{\tilde{e}_L} - m_{\tilde{g}}$ plane. As we stated earlier, large $m_0$ term can give dominant contributions to the squark and slepton masses, so this plot is very much similar to $m_0 - m_{\tilde{g}}$. We do not show the plot for $m_{\tilde{e}_R} - m_{\tilde{g}}$ because it is similar to $m_{\tilde{e}_L} - m_{\tilde{g}}$ plot. Similarly, we depict $m_{\tilde{\tau}_1} - m_{\tilde{g}}$ plot in the top right panel in which also have similar feature to $m_{\tilde{e}_L} - m_{\tilde{g}}$ plot. Plot in $m_{\tilde{t}_1} - m_{\tilde{g}}$ plane is shown in lower left panel. Here we see the similar trend but the mass ranges are reduced. Plot in lower right panel is shown in $m_A - m_{\tilde{g}}$ plane. Here we notice that $m_A$ can be as heavy as 64 TeV for both red and blue points. Please note that the gaps present in sleptons masses and $m_A$ plots is just due to lack of data and do not reflect any exclusion phenomena. If we have done some more focused scans we would have populated these gaps.

In Fig. 4, we show the gluino mass versus the light chargino mass and lightest neutralino mass, the light chargino mass versus the lightest neutralino mass, and dark matter density versus the lightest neutralino mass. Plot in the top left
The gluino mass versus the light chargino mass and lightest neutralino mass, the light chargino mass versus the lightest neutralino mass, and dark matter density versus the lightest neutralino mass. The color coding is the same as in Fig. 2.

Panel shows a graph in $m_{\tilde{\chi}_1^\pm} - m_{\tilde{g}}$ plane. It shows that for red points the chargino mass $m_{\tilde{\chi}_1^\pm}$ can be in the range from 0.1 to 2 TeV but for the blue points the chargino mass reaches up to 3.4 TeV. Reference [75] reports the exclusion limits on $\tilde{\chi}_1^+ \tilde{\chi}_1^-$ and $\tilde{\chi}_1^+ \tilde{\chi}_1^0$ productions with $\tilde{t}$-mediated decays and the productions with the SM-boson-mediated decays, which require $m_{\tilde{\chi}_1^\pm} \gtrsim 700$ GeV. This means that most of our points can satisfy these bounds. Moreover, we will show later that for some of the lighter solutions $m_{\tilde{\chi}_1^\pm}$ and $m_{\tilde{\chi}_1^0}$ are mass-degenerate and then can evade these constraints as well. A plot in $m_{\tilde{\chi}_1^0} - m_{\tilde{\chi}_1}$ plane is shown in the top right panel. This plot also has similar features to the previous plot. Here, the neutralino mass can reached up to 1.1 TeV for red points and 2 TeV for blue points. In the lower left panel, we show plot in $m_{\tilde{\chi}_1^0} - m_{\tilde{\chi}_1^\pm}$ plane. The diagonal black line is just to show the region of chargino-neutralino coannihilation where chargino and neutralino masses are degenerate, and most of these points might have the Higgsino LSP. For red and blue points, the coannihilation region has the upper mass bounds on $m_{\tilde{\chi}_1^0}$ about 1 TeV and 1.9 TeV, respectively. The coannihilation region with lower mass range as stated before is safe from collider constraints. In the right lower panel, we displays the plot with the LSP neutralino mass versus dark matter relic density $\Omega_1 h^2$. Horizontal two black lines represent Planck 2018 5$\sigma$ bounds on cold dark matter relic density as shown above. It can be seen that some of the red and blue points can satisfy the relic density bounds. In the following, we present four benchmark points in Table 1 for the red and blue points, which are consistent with the dark matter relic density bounds. Here we want to draw attention of the
reader to an important observation that for point 1 and point 2 where $M_{GUT} \geq 1.2 \times 10^{16}$ GeV, the LSP neutralino is higgsino with admixture of bino and for point 3 and point 4 where $M_{GUT} \leq 1.2 \times 10^{16}$ GeV the LSP neutralino is almost pure higgsino.

4 Conclusion

We have studied the supersymmetric GUTs with gravity mediated supersymmetry breaking in details. First, considering the dimension-six proton decay via heavy gauge boson exchange, we pointed out that the supersymmetric GUTs with GUT scale $M_{GUT}$ up to $1.778 \times 10^{16}$ GeV can be probed at the Hyper-Kamiokande experiment. Second, for the supersymmetric GUTs with $M_{GUT} \geq 1.0 \times 10^{16}$ GeV and $M_{GUT} \geq 1.2 \times 10^{16}$ GeV, we showed that the upper bounds on the universal gaugino mass are $7.2$ TeV and $3.5$ TeV, respectively, and thus the corresponding upper bounds on gluino mass are $15$ TeV and $8$ TeV, respectively. In particular, the supersymmetric GUTs with $M_{GUT} \leq 1.2 \times 10^{16}$ GeV can be probed at the Hyper-Kamiokande experiment, and the supersymmetric GUTs with $M_{GUT} \geq 1.2 \times 10^{16}$ GeV can be probed at the FCC$_{hh}$ and SppC experiments via gluino searches. Thus, the supersymmetric GUTs with gravity mediation can be probed by the FCC$_{hh}$, SppC, and Hyper-Kamiokande experiments. In our previous study, we have shown that the supersymmetric GUTs with anomaly and gauge mediated supersymmetry breakings are well within the reaches of these experiments. Therefore, we propose the concrete scientific goal for the FCC$_{hh}$, SppC, and Hyper-Kamiokande experiments: probing the supersymmetric GUTs.

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