Traveling circumferential unstable wave of cylindrical flame front

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Abstract. The researches of stability of cylindrical front of deflagration combustion in an annular combustion chamber were made using phenomenological model. The flame front is described as discontinuity of gasdynamic parameters. It is considered that the combustion products are under chemical equilibrium. The combustible mixture and the combustion products are ideal gases. The velocity of deflagration combustion is determined using the Chapman-Jouget theory. It depends on the temperature of combustible mixture only. It is found that the combustible flame front is unstable for several types of small disturbances in the system. Mechanics of instabilities are examined using both the numeric and analytical methods. The cases of evolution of the unstable waves rotating in circular channel are presented.

1. Introduction

It was discovered in the 20s of XX century that there is a helical sharp trace \cite{1} on sooty inner surface of glass tube, produced by an unknown element of detonation front after the detonation wave (DW) propagation. This effect did not agree with the idealized models of planar detonation front proposed by earlier researchers. Such a wave was called spinning detonation, and the unknown rotating region was called “head of spin”. For a long time the nature of spinning DW was unknown and the spinning DW was an exotic phenomenon. Hypothesis of existing unknown element of the spinning DW (transverse wave – TW propagating in induction zone) was suggested in 1950 \cite{2}. Works of Schelkin and Troshin \cite{3} made a valuable contribution to the thorough analysis. It was shown experimentally by Voitsekhovskii \cite{4} that TW does exist.

Discovering of TW made it possible to explain the helical trace produced by spinning configuration that propagates along the tube. It also allowed to find out the inner structure of both the spinning and multifront DW. Thus, contemporary models of the multifront detonation structure with TW in induction zone differ from the idealized structure with planar front.

In order to explain the nature of the spinning detonation in \cite{5} the acoustic theory of the spinning detonation trace was suggested. Later the theory was improved in \cite{6}.

The idea of mixture combustion under detonation conditions was suggested many years ago. Voitsekhovskii was the first who suggested to combust a mixture under detonation condition using rotating TW (e.g. transverse wave of spinning configuration). He carried out this idea in practice \cite{7} of annular combustion chamber (circular V-shaped channel), the mixture being delivered radially from the center.

Investigations of rotating DW in annular combustion chamber revealed an important feature. Namely the propagation velocity of rotating DW never reaches a velocity of the ideal detonation...
but it belongs to the range between \( D_0 \) and sonic speed in detonation products \( c_s \) (for gas combustible mixtures \( c_s \approx 0.55 D_0 \)).

Nowadays there is no explanation for this phenomenon. The transverse detonation wave is called a quasidetonation wave due to this behavior. It is possible to expect that acoustic properties of hot detonation products play important role like it takes place in observing of TW in spinning detonation. The purpose of our research is to investigate the velocity of rotating quasidetonation wave and to corroborate the hypothesis about acoustic nature of this phenomenon. Numeric and analytical methods were used in our research to model several behavior cases of the system that consists of the initial mixture and the products of combustion. Main results are presented.

2. The statement of quasisteady problem

2.1. Modeling of combustion process for calculation

Velocity of gas in annular combustion chamber in quasisteady flow is subsonic and the transverse wave are forced initialized on a certain radius across the flow. The above mentioned facts and the theory of exothermic discontinuity [8] allow us to model the combustion front as a strong discontinuity of the gas-dynamic parameters with a subsonic radial combustion velocity (deflagration wave).

It is necessary to stipulate that the size and the scale of combustion zone is much less than the main geometric scale of the problem in many cases. Therefore it is conventional to replace the finite area of combustion with big gradients of the gas-dynamic parameters with the area of their step-wise changes. Moreover, replacing the line of discontinuity with a combustion zone is fully correct if considered wave length of disturbances are much more than width of combustion area. In other words, this approach is fully correct if processes inside the combustion zone steady much faster than essential changes of gas-dynamic parameters in the flow occur. It is considered that the combustion occurs instantly in the zone of zero width and all essential features of combustion may be reduced to concrete relations for the discontinuity line [9]. It should be emphasized, that the combustion zone is considered as both an area of chemical transformation of mixture to the products and an area of preheating in front of the reaction zone, in which there are gradients of parameters related with the heat conductivity, diffusion and other effects of propagation of the deflagration wave.

2.2. Quasisteady problem

Let us consider a planar radially diverging subsonic flow of combustible (premixed) mixture from a section with the initial radius \( R_{in} \) (Fig. 1). In order to describe the flow let’s choose the polar
coordinates \((r, \varphi)\). At initial moment at the output cross-section with the radius \(R_{in}\) the ejection velocity is defined by a drop of pressure between the receiver for the initial mixture and one for the products of combustion (the classical problem of break-up of discontinuity). The ejection velocity with the large drop of pressure reaches the critical velocity equal to the local sonic speed. The compression wave in the area \(r>R_{in}\) and the rarefaction wave in the area \(r<R_{in}\) cause a nonsteady ejection of mixture from the initial receiver. The velocity of ejection mixture decreases as the coordinate \(r\) grows. At some radius \(R\) it will reach a value equal to the velocity of combustion of the given combustible mixture. If one ignite the mixture against its movement on this radius, all flow will be divided on two areas: area 1 is the flow of the initial combustible mixture (cold) and area 2 is the area of the flow of high-temperature products of reaction. If the volume of the initial receiver is sufficiently large and the ejection is done in a large-scale volume, the flow will get quasisteady. Then parameters of the flow do not explicitly depend on time (all of derivatives of parameters of the flow with respect to time are equal to zero in equations), and the cylindrical flame front presented in Fig. 2 will rest on (in the laboratory reference system). We shall consider a nonviscous, nonheat-conducting and perfect gas with a constant specific heats (the initial mixture and combustion products). Then, the equations describing the quasisteady flows of combustible mixture and products in polar coordinates for radial ejection will take the form:

\[
\begin{align*}
\frac{\partial u_{1,2}}{\partial r} &= -\frac{1}{\rho_{1,2}} \frac{\partial p_{1,2}}{\partial r}, \\
\frac{1}{r} \frac{\partial}{\partial r} \left( r \rho_{1,2} u_{1,2} \right) &= 0, \\
p_{1,2} &= A_{1,2} \rho_{1,2}^{\gamma_{1,2}},
\end{align*}
\]

(2.1)

where \(u\) is the mass velocity, \(p\) is the pressure, \(\rho\) is the mass density, \(\gamma\) is the adiabatic ratio, \(A\) is the constant depending on thermodynamic parameters at the boundaries \(R_{in}\) and \(R\); indices 1 and 2 denote the flow parameters in areas 1 and 2, respectively.

It is well known that the relative changes of density in a flow is linked with the Mach number squared \([10]\). Thus, if the Mach number equals to 1/3, the relative changes of density of flow are no more than 4\%. Thus, they can be neglected. The Mach number in the flow described above steadily decreases as the coordinate \(r\) grows, therefore, it reaches 1/3 at some radius \(R_0\) (Fig. 1). Further, the radius \(R_0\) is assumed as the initial one, and the flow in the area \(1 (R_0<r<R)\) is assumed to have a constant density \(\rho_{01}\). The Mach number, thus, is a small parameter of our problem. Considering all this, equations (2.1) in the area 1 have the following solutions:

\[
(u_1(r) = \frac{u_{01} R_0}{r}; p_1(r) = \rho_{01} + \frac{\rho_{01} u_{01}^2}{2} - \frac{\rho_{01} u_{01}^2 R_0^2}{2 r^2}; p_1 = \rho_{01} T_1(r) = \mu \cdot p_1(r) R_0),
\]

(2.2)

where \(\mu\) is the molar mass, \(R_0\) is the universal gas constant, \(T\) is the temperature, \((u_{00}, p_{00}, \rho_{00})\) are the gasdynamic parameters on the radius \(R_0\) which can be calculated from gasdynamic solution of equations (2.1) \([10]\), specifying the critical parameters on the boundary \(R_{in}\) and supposing the Mach number equal 1/3. In turn, the critical parameters depend on the condition of combustible mixture in the initial receiver. On the combustion front \(R\) the conditions of exothermic discontinuity \((Q>0)\) hold. They are conservation laws of mass, momentum, energy and the condition of stationary front in the laboratory reference system:

\[
\begin{align*}
\rho_0 |_{k} &= \rho_2 u_2 |_{k}, \\
p_1 + \rho_0 u_1^2 |_{k} &= p_2 + \rho_2 u_2^2 |_{k}, \\
h_1 + \frac{u_1^2}{2} |_{k} &= h_2 + \frac{u_2^2}{2} |_{k}, \\
D_1 (p_1, T_1) |_{k} &= u_1 |_{k} = \frac{u_0 R_0}{R}.
\end{align*}
\]

(2.3)
where \( h \) is the enthalpy and \( D_f \) is the combustion velocity. If we know a velocity of combustion, we may calculate the location \( R \) of stationary front using the fourth equation in (2.3). If energy release \( Q \) is known, the velocity of combustion can be calculated from the well-known Chapman-Jouget condition. This condition means the tangency of Michelson-Rayleigh line to bottom, or deflagration, branch of adiabat of energy release [8] as the flow is subsonic. According to paper [11] the velocity of deflagration combustion linearly depends on temperature of mixture and practically does not depend on pressure of mixture if a chemical equilibrium between the initial stoichiometric mixture and products of combustion is assumed:

\[
D_f(T) = \alpha \cdot T,
\]

(2.4)

where \( \alpha \) is the constant that depends on a concrete combustible mixture. Using equation (2.4) and the Chapman-Jouget condition we may calculate \( Q \) if it is necessary. Using the Chapman-Jouget condition we also may calculate all gasdynamic parameters behind the front of combustion located on the radius \( R \) and then, using equations (2.1), we may construct a solution in the area 2. However, this solution will not be needed in the sequel as we shall see below.

The condition of equality of the local sonic speed behind the front of combustion to the mass velocity of the flow is one of the equivalent formulations of Chapman-Jouget condition:

\[
u_2(R) = c_2(R) = \frac{\gamma_2 p_2(R)}{\rho_2(R)}.
\]

(2.5)

This relation will be needed in the sequel in order to transform a boundary conditions for nonsteady parameters at the flame front.

3. The statement of nonsteady problem

Let us consider the nonsteady problem. Let add small disturbances harmonically varying according to time to all quasisteady parameters:

\[
\begin{align*}
\tilde{u}_{1,2}(r, \varphi, t) &= u_{1,2}(r) a_r + (\delta u_{1,2}(r, \varphi) a_r + \delta \tilde{t}_{1,2}(r, \varphi) a_\varphi) e^{-i\omega t} \\
\tilde{p}_{1,2}(r, \varphi, t) &= p_{1,2} + \delta p_{1,2}(r, \varphi) e^{-i\omega t} \\
\tilde{\delta p}_1(r, \varphi) e^{-i\omega t} &= c_1^2(r) \delta p_1(r, \varphi) e^{-i\omega t} \\
\delta p_2(r, \varphi) e^{-i\omega t} &= (c_2^2(r) \delta p_2(r, \varphi) + \delta \sigma_2(r, \varphi) p_2(r) / c_{v2}) e^{-i\omega t} \\
\tilde{\delta \sigma}(r, \varphi, t) &= R + A(\varphi) e^{-i\omega t}
\end{align*}
\]

(3.1)

The third equation in (3.1) between the equations for disturbances of density and pressure means that disturbances propagate isentropically because it is assumed that there is no disturbances of entropy on the boundary \( R_0 \). On the front of combustion a heat release occurs, therefore, the disturbances of entropy can appear behind the front and then in the area 2. Therefore, in equations (3.1) the expression for pressure disturbances in the area 2 is presented of the general form. The traveling circumferential transverse wave (along the azimuthal coordinate \( \varphi \)) is simulated by the function \( A(\varphi) \) (see the fifth equation in (3.1)).

Substituting equations (3.1) in the well-known conditions of strong discontinuity (the combustion front) [10] marked in the front reference system and neglecting the terms with disturbances of order higher than the first we obtained a conditions on the boundary \( R \) which are linear system of equations linking the amplitudes of disturbances in the areas 1 and 2:

\[
\begin{align*}
\left. \frac{1}{M_1} (\tilde{u}_1, + i \omega \cdot \tilde{A}(\varphi)) + \tilde{p}_1 \right|_{R} &= \left. \frac{1}{M_2} (\tilde{u}_2, + i \omega \cdot \tilde{A}(\varphi)) + \tilde{p}_2 \right|_{R} \\
\left. \frac{1}{K_1} (\tilde{t}_1, + i \omega \cdot \tilde{A}(\varphi)) + \tilde{\delta t}_1 \right|_{R} &= \left. \frac{1}{K_2} (\tilde{t}_2, + i \omega \cdot \tilde{A}(\varphi)) + \tilde{\delta t}_2 \right|_{R}
\end{align*}
\]

(3.2a)
\[ 2 \cdot (\mathbf{u}_r + i \mathbf{v} \cdot \mathbf{A}(\phi)) + \frac{\mathbf{p}_1}{M_1} (M_2^2 + \zeta_1^2) \mid_{\mathbf{r} = \mathbf{r}_0} = 2 \cdot (K_2 \mathbf{u}_r + i \mathbf{v} \cdot \mathbf{A}(\phi)) + \frac{\mathbf{p}_2}{M_2} (M_2^2 + 1) + \frac{\zeta_2^2}{M_2^2} \mid_{\mathbf{r} = \mathbf{r}_0} \]  
(3.2b)

\[ \mathbf{p}_{1} (\zeta_1^2 + \mathbf{Q}) + M_1 (\mathbf{u}_r + i \mathbf{v} \cdot \mathbf{A}(\phi)) \mid_{\mathbf{r} = \mathbf{r}_0} = \frac{\mathbf{p}_2}{M_2} K_2^2 + M_2 \zeta_2^2 (\mathbf{u}_r + 1) + \frac{\zeta_2^2}{M_2^2} \mid_{\mathbf{r} = \mathbf{r}_0} \]  
(3.2c)

\[ \mathbf{u}_r - \frac{\mathbf{p}_1}{\mathbf{c}_{01}} \frac{\alpha R c_1^2}{c_{01}} (\gamma_1 - 1) = -i \mathbf{v} \cdot \mathbf{A}(\phi) \mid_{\mathbf{r} = \mathbf{r}_0} \]  
(3.2d)

\[ M_1 \frac{R - R_0}{R} \frac{dA(\phi)}{d\phi} + \pi_{2\phi} \bigg|_{\mathbf{r} = \mathbf{r}_0} = M_2 K_2 \frac{R - R_0}{R} \frac{dA(\phi)}{d\phi} + K_2 \pi_{2\phi} \bigg|_{\mathbf{r} = \mathbf{r}_0} \]  
(3.2e)

\[ K_2 = c_2(R) \frac{\rho_0}{c_{01}} c_{01}^2 = c_2(R) \frac{\rho_0}{c_{01}} c_{01}^2 + \frac{\partial Q}{\partial \rho_{01}} \]  
(3.3)

where dimensionless quantities were introduced:

\[ \mathbf{u}_1 = \frac{\delta u_1}{c_{01}}; \mathbf{p}_1 = \frac{\delta p_1}{\gamma_1 \rho_0}; \mathbf{v}_1 = \frac{\delta v_1}{c_{01}}; \mathbf{v}_1 = \frac{c_s^2}{c_{01}}; \mathbf{T}_1 = \frac{\delta T_1}{\gamma_1 \rho_0}; \mathbf{c} = \frac{\delta c}{\partial \rho_0}; \mathbf{A}(\phi) = \frac{\Lambda(\phi)}{R - R_0} \]  
(3.4)

\[ \mathbf{u}_2 = \frac{\delta u_2}{c_2(R)}; \mathbf{p}_2 = \frac{\delta p_2}{\gamma_2 \rho_2(R)}; \mathbf{v}_2 = \frac{\delta v_2}{c_2(R)}; \mathbf{v}_2 = \frac{c_s^2}{c_2(R)}; \mathbf{T}_2 = \frac{\delta T_2}{\gamma_2 \rho_2(R)}; \mathbf{M}_1(\bar{\tau}) = \frac{u_2((R - R_0)\bar{\tau})}{c_2(R)} \]  
(3.5)

where \( c_{01} = \gamma_1 \rho_0 / \rho_0 \) is the sonic speed on the boundary \( R_0 \). In the system of equations (3.2) equations
(a), (b) and (c) are the linearized mass, momentum and energy laws, respectively; equation (d) is the linearized kinematic condition, namely the vector sum of the velocity of gas flow and flame velocity relative to the gas flow which equals to the velocity of flame front in laboratory reference system. In equation (e) the disturbances of temperature was expressed through the disturbances of the density using state equation of perfect gas and isentropic condition. If one expresses the disturbances in the area 2 only in terms of the disturbances in the area 1 and uses equation (2.5) in a dimensionless form (Mach number behind combustion front is equal to unit), equations (3.2) will essentially be simplified:

\[ \mathbf{p}_1 \mid_{\mathbf{r} = \mathbf{r}_0} = 0 \]  
(3.6)

\[ \mathbf{u}_r \bigg|_{\mathbf{r} = \mathbf{r}_0} = -i \mathbf{v} \cdot \mathbf{A}(\phi) \]  

\[ \mathbf{v}_2 \bigg|_{\mathbf{r} = \mathbf{r}_0} = 0 \]  

\[ \pi_{2\phi} \bigg|_{\mathbf{r} = \mathbf{r}_0} = \frac{1}{K_2} \pi_{2\phi} + \frac{M_2}{K_2} - 1 \frac{R - R_0}{R} \frac{dA(\phi)}{d\phi} \]  

Thus, we may see that the system of boundary conditions is splitted. Therefore, it is sufficient to solve the unsteady problem in the area 1 in order to find out the behavior of disturbed flame front. On the boundary \( R_0 \) we assume no vortical disturbances of velocity of quasisteady flow; therefore, the velocity disturbances can be described by only one function, namely by the potential. The fourth condition in the system of equations (3.6) needs to specify vortical disturbances of velocity behind the flame front and, thus, it will not be needed in the sequel. We may obtain the equations describing the potential disturbances in the area 1 from a general equations of acoustics of moving and inhomogeneous mediums [12]. Thus, we obtain the following equations taking into account that the
main quasisteady flow is vortex-free, isentropic and that the Mach number of the main flow is a small parameter:

\[
\ddot{f} + \left( \frac{1}{r} \frac{\partial}{\partial r} (r^2 \ddot{\phi}) + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} \right) = -2i \ddot{\omega} M_1 \frac{\partial \ddot{f}}{\partial \phi},
\]

(3.7)

\[
\ddot{p}_1 = \ddot{p}_1 = i \ddot{\omega} \cdot \ddot{f} - M_1 \frac{\partial \ddot{f}}{\partial \phi}.
\]

(3.8)

where \( \ddot{f} \) is the dimensionless potential of disturbances of velocity. In the equations (3.7) and (3.8) time has been already separated using a harmonic variation according to time. Thus, in the area 1 we obtained the following problem:

\[
\ddot{f} + \left( \frac{1}{r} \frac{\partial}{\partial r} (r^2 \ddot{\phi}) + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} \right) = -2i \ddot{\omega} M_1 \frac{\partial \ddot{f}}{\partial \phi},
\]

(3.9)

\[
\ddot{p}_1 = i \ddot{\omega} \cdot \ddot{f} - M_1 \frac{\partial \ddot{f}}{\partial \phi} \bigg|_{r=R_0} = 0
\]

\[
\ddot{u}_{1r} = \frac{\partial \ddot{f}}{\partial \phi} \bigg|_{r=R_0} = -i \ddot{\omega} \lambda (\phi)
\]

In order to determine the problem in area 1 completely, it is necessary to identify a condition on the boundary \( R_0 \). A form of this condition depends on combustible mixture supply. We shall consider a three following cases: mass flow is constant, pressure is constant and velocity is constant. Corresponding linearized conditions on the boundary \( R_0 \) are:

\[
i \ddot{\omega} \cdot M_1 \ddot{f} + \frac{\partial \ddot{f}}{\partial \phi} (1 - M_1^2) \bigg|_{r=R_0} = 0
\]

(3.10)

\[
\ddot{p}_1 = i \ddot{\omega} \cdot \ddot{f} - M_1 \frac{\partial \ddot{f}}{\partial \phi} \bigg|_{r=R_0} = 0
\]

(3.11)

\[
\ddot{u}_{1r} = \frac{\partial \ddot{f}}{\partial \phi} \bigg|_{r=R_0} = 0
\]

(3.12)

4. Results

Let us carry out calculations for the annular combustion chamber in which experiments for burning of stoichiometric acetylene-oxygen mixture were carried out [7]. Problems governed by equations (3.9) and (3.10), (3.9) and (3.11), (3.9) and (3.12) with the uniform boundary conditions has been solved using variable separation method (Fourier method). Consequently, we have obtained dimensionless eigenfrequencies \( \ddot{\omega} \) of cylindrical flame front oscillations and have carried out a modal stability analysis.

Table 1. Eigenfrequencies of modes of the problem with constant mass flow: \( l \) is the radial mode number, \( k \) is the azimuthal mode number

| \( k \) | \( l \) | 1     | 2     | 3     |
|-------|-------|-------|-------|-------|
| 0     | 1     | 1.82-0.144i | 4.73-0.161i | 7.76-0.164i |
| 1     | 2     | 2.11-0.122i | 4.84-0.158i | 7.82-0.163i |
| 2     | 3     | 2.79-0.094i | 5.14-0.149i | 8.00-0.160i |
| 3     | 4     | 3.59-0.080i | 5.63-0.135i | 8.29-0.155i |
Table 2. Eigenfrequencies of modes of the problem with constant pressure: \( l \) is the radial mode number, \( k \) is the azimuthal mode number

| \( k \) | \( L \) | \( 1 \)   | \( 2 \)   | \( 3 \)   |
|-------|-------|--------|--------|--------|
| 0     | 0     | 3.03+0.199i | 6.13+0.194i | 9.21+0.193i |
| 1     | 1     | 3.15+0.163i | 6.20+0.184i | 9.26+0.190i |
| 2     | 2     | 3.51+0.089i | 6.40+0.155i | 9.40+0.182i |
| 3     | 3     | 4.02-0.021i | 6.73-0.114i | 9.63+0.169i |

In Table 1 the frequencies for the problem with constant mass flow are presented. It can be seen from Table 1 that all frequencies have the negative imaginary part. We used \( e^{-i\tau} \); therefore, the modes with frequencies represented in Table 1 are stable to small harmonic disturbances.

In Table 2 the frequencies for the problem with constant pressure are presented. It can be seen from Table 2 that there are frequencies with the positive imaginary part, i.e. there are modes unstable to small harmonic disturbances. In the problem with constant velocity the unstable modes also exist.

Disturbed flame front of mode of order \( k-l \) is a closed curve which has \( k \) loops rotating together along the circumference of radius \( R \) with the angular velocity \( \Re(\tilde{\omega}_{kl}) \). We also may calculate a linear phase velocity of loops rotation:

\[
V_{kl} = \frac{c_2 \Re(\tilde{\omega})}{R - R_0} R
\]

Table 4. Linear phase velocity of loops for various boundary conditions on \( R_0 \)

| \( k \) | \( V_{1l}, \text{m/sec} \) | \( V_{2l}, \text{m/sec} \) | \( V_{3l}, \text{m/sec} \) | \( V_{4l}, \text{m/sec} \) |
|-------|-----------------|-----------------|-----------------|-----------------|
| 1     | 1138            | 1155            | 1702            |
| 2     | 1504            | 1526            | 1892            |
| 3     | 1940            | 1962            | 2171            |
| 4     | 2379            | 2395            | 2499            |

In Table 4 the phase velocities of rotation of disturbed flame front are presented for radial number \( l=1 \) and various boundary conditions on \( R_0 \). The sonic speed behind flame front, \( c_2 \), according to the Chapman-Jouget parameters equals 1126 m/sec.
In Fig. 3 the shapes of disturbed flame front are visualized.

5. Conclusion
The researches of stability of cylindrical front of deflagration combustion in annular combustion chamber were made using phenomenological model of the combustion and acoustic approximation. The Mach number of main flow was the small parameter of the problem.

It is shown that if the mass flow of combustible mixture supply is strictly constant and does not depend on the acoustic disturbances, the flame front is always stable.

It is found that if the disturbance of pressure or velocity in combustible mixture supply is negligible quantity, then there are discrete sets of frequencies and corresponding modes of oscillations for which the flame front is unstable. Spatial shapes of the oscillations were obtained using both the numeric and analytical technique. Rotating waves with a finite number of local loops were discovered. Linear phase velocities of rotation were calculated.

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