Microscopic Description of $^{170}\text{Er}$, $^{172}\text{Yb}$, $^{174}\text{Hf}$, and $^{176}\text{W}$ Isotones

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Abstract

The properties of $^{170}\text{Er}$, $^{172}\text{Yb}$, $^{174}\text{Hf}$, and $^{176}\text{W}$ isotones have been studied and their energy states calculated. To identify the properties of each isotone, the values of the first excited states, $E2^+$, and the ratio of the second excited states to the first excited states, $R_{4/2} = E4^+_2/E2^+_1$, for all nuclei under consideration were adopted. To determine the properties of each nucleus, the relationship between the moment of inertia $2\hbar^2/\omega^2$ and the square of the angular frequency, $\hbar^2/\omega^2$, the relationship between successive excited states to those preceding them $r(\frac{i+2}{i})$ and the $\Delta I = 1$ staggering between the GSB and the NPB states were studied for all states of $^{170}\text{Er}$, $^{172}\text{Yb}$, $^{174}\text{Hf}$, and $^{176}\text{W}$ isotones. After identifying the properties of each isotone, the rotational limit in the interacting boson model IBM-1 and the IVBM model was used to calculate the energy states for each isotone and the results were compared with the experimental values. and good agreement was observed with some exception. The inaccuracy of some calculations in the IBM-1 results from the lying of some high states out the range of the rotational properties that were used.

Keywords: $^{170}\text{Er}$, $^{172}\text{Yb}$, $^{174}\text{Hf}$, $^{176}\text{W}$ isotones properties, IBM-1, IVBM.
1. Introduction

An even-even nucleus has an even number of protons and even numbers of neutrons. These protons and neutrons, called nucleons, are inside the nucleus. These nucleons are distributed among many energy levels with certain numbers; there are some numbers of occupation called a magic number (2, 8, 20, 28, 50, 82, 126). Those nuclei which have a magic number of protons or neutrons or both, behave differently in comparison with their neighbouring [1].

Many models were introduced to study the nuclei. In a collective model, the nuclei were classified into vibration, rotational, and transitional with a certain feature of equation to describe their movement [2]. In even-even nuclei, the nucleons were distributed on different states, each group of states forming a band [3]. The first band is the ground states band (GSB) with $I^\pi = 0^+, 2^+, 4^+, 6^+, ...$. A similar band to the GSB called the $\beta$-band with $I^\pi_{2,3} = 0^+, 2^+, 4^+, 6^+, ...$ was formed at a distance from the GSB. Another band formed close to the $\beta$-band, called the $\gamma$-band, with $I^\pi_{2,3} = 2^+, 3^+, 4^+, 5^+, 6^+, ...$. Another form of band with negative parity states is formed at high excited states called the negative parity band (NPB) with $I^\pi = 1^-, 3^-, 5^-, 7^-, ...$. The interweaving of GSB and NPB forms a single octupole band with $I^\pi = 0^+, 1^-, 2^+, 3^-, 4^+, 5^-, 6^+, ...$[4–7]. A classification of nuclei depending on the first excited state $E2^1_1$ and the ratio of the second to the first excited states $R_{4/2} = E4^+_1/E2^+_1$, were introduced with $E2^+_1 \approx 100, 300, 500$ and $1000$ keV for rotation, $\gamma$-soft, vibration, and magic nuclei, respectively [8], along with $3 < R_{4/2} < 3.3$ for rotational nuclei, $2.7 < R_{4/2} \leq 3$ for transitional nuclei, $2.4 < R_{4/2} \leq 2.7$ for $\gamma$-soft, $2 < R_{4/2} \leq 2.4$ for vibration nuclei, and $R_{4/2} < 2.0$ for magic nuclei [9].

Many other methods were presented to study the properties of the nuclei, the behaviour of the moment of inertia $2\theta/\hbar^2$ because the excited state of GSB as a function of the square of the frequency $\hbar^2\omega^2$ of these states is one of the important methods to determine the change of the properties of nuclei, observing the smooth or sudden change of $2\theta/\hbar^2$[10, 11]. Good information of the evolution of the energy states of the nuclei are achieved from the ratio of the successive energy states $r (l + 2/l)$, where $r$ changes from 0.1 to 1.0 for different nuclei: $0.1 \leq r \leq 0.35$ for vibration nuclei, $0.4 \leq r < 0.6$ for transitional nuclei, and $0.6 \leq r \leq 1.0$ for rotational nuclei [12, 13].

The octupole band consists of odd-spin states with negative parity and even-spin with positive parity. The difference in energy of these negative and positive parity states form a $\Delta I = 1$ staggering from high to low $-ve$ and $+ve$ values, and may reach zero, then growing again to high values. If the staggering reaches a zero value, we can say a change of phase occurred [14-16]. Many models were introduced to calculate the energy states of the nuclei. The collective model depends on the degree of freedom of the nucleus as a whole. In the collective model, the nuclei were classified as vibrational and rotational systems with suitable features of equations for each type [17, 18].

Arima and Iachello introduced the interacting bosons model (IBM-1) [19, 20]. In IBM-1, each couple of nucleons are considered one boson, and there are s and d bosons with $L=0$ and $L=2$. These s and d bosons form a unitary group, SU (6). In this model, the nuclei were classified into the vibration, U (5), the rotation, SU (3), and the $\gamma$-soft O (6) symmetries [21-23].

There is another type of interacting boson model called the interacting vector boson model (IVBM). IVBM depends on two types of bosons, the proton and the neutron bosons. These two types of bosons determine the shape of the collective excitations of the nucleus [24].
This study analyses the properties of $^{170}\text{Er}$, $^{172}\text{Yb}$, $^{174}\text{Hf}$, and $^{176}\text{W}$ isotones. The relation of the moment of inertia as a function of the square of the angular frequency of each state was drawn and studied, the characteristics of the ratio of successive energy states were calculated and studied, and the staggering $\Delta I = 1$ was also drawn and studied. Knowing the properties of each nuclei under consideration prepared us to use the suitable equations of IBM-1 to calculate the energy states of different bands. The IVBM were also used to calculate the energy states of the GSB and NPB, and the results were compared with the experimental data.

2. Method of Calculations

The relation between the moment of inertia $\frac{2\theta}{h^2}$ and the square of the angular frequency $\hbar^2 \omega^2$ of a state of a nucleus gives important information about the behaviour of this nucleus. Drawing $\frac{2\theta}{h^2}$ as a function of $\hbar^2 \omega^2$ and observing the behaviour of $\frac{2\theta}{h^2}$, and whether it makes a sudden or a smooth change, one can determine if the nucleus changes its phase or not. The square of the angular frequency $\hbar^2 \omega^2$ is given by [25]:

$$\hbar^2 \omega^2 = \frac{E^2}{\sqrt{I(I+1)} - \sqrt{(I-2)(I-1)}}$$

and the moment of inertia $\frac{2\theta}{h^2}$ is given by [10]:

$$\frac{2\theta}{h^2} = \frac{4I - 2}{E(I) - E(I-2)} = \frac{4I - 2}{E_y}$$

The ratio between the successive states $r \left( \frac{l+2}{l} \right)$ in the GSB gives a numeric value from which we can determine the characteristics of the nucleus at each state; thus, we can determine the properties of the nucleus. The $r \left( \frac{l+2}{l} \right)$ values are given by [12]:

$$r \left( \frac{l+2}{l} \right) = \left( R \left( \frac{l+2}{l} \right)_{\text{exp}} - \frac{l+2}{l} \right) \times \frac{l(l+1)}{2(l+2)}$$

where $R \left( \frac{l+2}{l} \right)_{\text{exp}}$ is the measured energy ratio between the two states $l + 2$ and $l$.

The interwoven between GSB and NPB in the octupole band gives an oscillatory pattern called $\Delta I = 1$ staggering. $\Delta I = 1$ staggering takes alternatively positive and negative zigzagging values and may reach zero, followed by other increases. This $\Delta I = 1$ staggering is given by [14]:

$$\Delta E_{1,\gamma}(I) = \frac{1}{16} \left( 6E_{1,\gamma}(I) - 4E_{1,\gamma}(I-1) - 4E_{1,\gamma}(I+1) + E_{1,\gamma}(I+2) \right)$$

In IBM-1, the general Hamiltonian is given by [26]:

$$H = \sum_{i=1}^{N} \epsilon_i + \sum_{i<j}^{N} V_{ij}$$

where $\epsilon_i$ is the intrinsic boson energy and $V_{ij}$ is the interaction strength between bosons $i$ and $j$.

The multipole form of the Hamiltonian is [26-30]:

$$H = \epsilon n_d + a_0 P + a_1 L + a_2 Q + a_3 T_3 + a_4 T_4$$

where $a_0, a_1, a_2, a_3$ and $a_4$ are the strengths of pairing, angular momentum, quadrupole, octupole, and hexadecapolar interactions of each term.

In multipole expansion, the Hamiltonian tends to reduce for the three forms of the three symmetry limits— the vibrational U (5), the rotational SU (3), and the $\gamma$－soft O (6). The effective parameter in the U (5) limit is $\epsilon$, the dominating parameter in the SU (3) limit is $a_2$ and
the predominate parameter in the O (6) limit, is $a_0$. The eigenvalues of these three limits are [31-33]:

\begin{align}
U(5): E &= \epsilon n_d + K_1 n_d (n_d + 4) + K_4 (\nu + 3) + K_5 L (L + 1) \\
SU(3): E &= K_2 (\lambda^2 + \mu^2 + 3(\lambda + \mu) + 3\mu) + K_5 L (L + 1) \\
O(6): E &= K_3 (N(N + 4) - \sigma (\sigma + 4)) + K_4 \tau (\tau + 3) + K_5 L (L + 1)
\end{align}

where $K_1, K_2, K_3, K_4$ and $K_5$ are other forms of the strength parameters and $N$ is the total boson number. Many nuclei have a transition property of two or three of the above limits. The eigenvalues in the IVBM model for the GSB and NPB states are given as [28,34,35]:

\begin{align}
E(I) &= \beta I(I + 1) + \gamma I \\
E(I) &= \beta I(I + 1) + (\gamma + \eta) I + \zeta
\end{align}

The values of $\beta$ and $\gamma$ can be determined from fitting to the positive GSB, while $\eta$ and $\zeta$ are estimated from fitting to the NPB.

3. Results and discussion

The $^{170}$Er, $^{172}$Yb, $^{174}$Hf, and $^{176}$W isotones have 68, 70, 72, and 74 protons, respectively, in the middle of 50 and 82 magic numbers. They also have 102 neutrons in the middle of 82 and 126 magic numbers, which gives the rotational properties. The first excited states for these isotones are 78.59, 78.74, 90.99, and 108.3, and the ratios $R_{4/2} = E^{4+}_{4/2}/E^{2+}_{2/2}$ are 3.31, 3.305, 3.268, and 3.215 for $^{170}$Er, $^{172}$Yb, $^{174}$Hf, and $^{176}$W isotones, respectively, which gives the rotational properties for those.

To insure the rotational properties and to test the phase change that may occur in these isotones, we drew the relationships in the moment of inertia $2\Theta/\hbar^2$ against the square of the angular frequency $\hbar^2 \omega^2$ of $^{170}$Er, $^{172}$Yb, $^{174}$Hf, and $^{176}$W for the GSB, and they are shown in Figure 1. No back-bending was observed in any isotope under consideration. In other words, there was no sudden change in the moment of inertia, which means no phase change was shown in these nuclei.

The ratios of the successive energy states of GSB for $^{170}$Er, $^{172}$Yb, $^{174}$Hf, and $^{176}$W were drawn and are shown in Figure 2. Figure 2 shows that all values of $r \left(\frac{142}{l}\right)$ are in the range of the rotational limit $0.6 \leq r \leq 1.0$ except for the last state considered for $^{174}$Hf and $^{176}$W, which are close to 0.6, which ensures the rotational properties for $^{170}$Er, $^{172}$Yb, $^{174}$Hf, and $^{176}$W isotones.

Figure 3 shows the $\Delta l = 1$ staggering of $^{170}$Er, $^{172}$Yb, $^{174}$Hf, and $^{176}$W isotones, which allowed the observation of the oscillatory pattern in the octupole band for each isotope. Positive and negative zigzagging was observed with no reach to zero values, which indicates that no phase change occurred for any of the isotones under consideration.
Figure 1. The moment of inertia as a function of the square of angular frequency of $^{170}\text{Er}, ^{172}\text{Yb}, ^{174}\text{Hf}, \text{and} ^{176}\text{W}$ isotone states.

Figure 2. The ratio of the successive energy states of GSB for $^{170}\text{Er}, ^{172}\text{Yb}, ^{174}\text{Hf}, \text{and} ^{176}\text{W}$ isotone states.
Figure 3. The $\Delta I = 1$ staggering of $^{170}Er$, $^{172}Yb$, $^{174}Hf$, and $^{176}W$ isotones.

The methods used above helped us to validate the rotational properties of the $^{170}Er$, $^{172}Yb$, $^{174}Hf$, and $^{176}W$ isotones for all states with no phase change. Using the rotational limit of IBM-1 and IVBM, we calculated the energy states of these isotones. A fitting to the experimental data [36-39] with the IBM-1 has been done to the GSB, $\beta – band$ and $\gamma – band$. Table 1 shows the fitted parameters of the rotational limit of IBM-1. A fitting of the experimental data with IVBM was done for both the GSB and NPB. Table 2 shows the best fitted parameters of the experimental data with the IVBM equation for both GSB and NPB of $^{170}Er$, $^{172}Yb$, $^{174}Hf$, and $^{176}W$ isotones.

Table 1. The IBM-1 parameters of $^{170}Er$, $^{172}Yb$, $^{174}Hf$, and $^{176}W$ isotones.

| Nucleus | GSB | $\beta – band$ | $\gamma – band$ |
|---------|-----|-----------------|-----------------|
| $^{170}Er$ | $N$ | $K_2 * 10^{-2}$ | $K_5$ | $K_2$ | $K_5$ | $K_2$ | $K_5$ |
| 17     | 0.99 | 12.32 | 1.20 | 7.72 | 1.07 | 12.26 |
| $^{172}Yb$ | 16 | 1.22 | 12.25 | 1.56 | 9.58 | 1.59 | 12.46 |
| $^{174}Hf$ | 15 | 1.83 | 13.53 | 1.49 | 9.55 | 1.85 | 17.67 |
| $^{176}W$ | 14 | 3.68 | 15.01 | 1.79 | 11.04 | 1.82 | 19.64 |
Table 2. The IVBM parameters of $^{170}$Er, $^{172}$Yb, $^{174}$Hf, and $^{176}$W isotones.

| Nucleus | GSB       | NPB       |
|---------|-----------|-----------|
| $^{170}$Er | $\beta$ 11.73, $\gamma$ 7.99, $\eta$ -85.3, $\zeta$ 1499.7 |
| $^{172}$Yb | $\beta$ 11.94, $\gamma$ 5.95, $\eta$ -62.2, $\zeta$ 1242.6 |
| $^{174}$Hf | $\beta$ 12.27, $\gamma$ 14.39, $\eta$ -87.9, $\zeta$ 1455.9 |
| $^{176}$W | $\beta$ 12.83, $\gamma$ 24.83, $\eta$ -83.5, $\zeta$ 1308.5 |

While using the best fitted parameters of IBM-1 and IVBM, we recalculated the energy of various states for both IBM-1 and IVBM. Tables 3 and 4 show the experimental data [28, 29, 30, 31] and the calculated values of IBM-1 for GSB, $\beta$ – band and $\gamma$ – band, as well as the error, where the error is given as: $\Delta\% = \left| \frac{\text{exp.data} - \text{cal.value}}{\text{exp.data}} \right| \times 100\%$. The results of the calculations of $^{170}$Er and $^{172}$Yb are shown in Table 3. We observed good agreement with the experimental data, with errors not exceeding 10.34 for all states of GSB, $\beta$ – band and $\gamma$ – band for $^{170}$Er. Table 4 shows the comparison between the experimental data and the calculated results of $^{174}$Hf and $^{176}$W. The results were in good agreement, except for the 16th and 18th states of GSB for $^{174}$Hf, with maximum errors 15.27 and 20.46, while the maximum error is 13.59 and 20.03 for the 14th and 16th states of GSB. For $^{176}$W, and we suggest that the reason for these high errors is due to the lie of these states out the range of the rotational limit, as shown in Figure 2.

Tables 5 and 6 show the experimental data, the IVBM calculations, and the errors for both the GSB and NPB of $^{170}$Er, $^{172}$Yb, $^{174}$Hf, and $^{176}$W. Table 5 shows the comparison of the experimental data with the calculated values for $^{170}$Er, $^{172}$Yb. The results of GSB are in good agreement with the experimental data, while the results of the NPB had some high error values for the first, 13th, and 15th states, and we suggest that these high errors result from the two configurations of the NPB states [29]. Table 6 shows good agreement between the experimental data and the calculated values for the $^{174}$Hf and $^{176}$W of GSB and NPB.
Table 3. The experimental [36-39] and IBM-1 calculated states (keV) of $^{170}$Er and $^{172}$Yb.

| Band | States | $^{170}$Er | $^{172}$Yb | \(\Delta\) % | $^{170}$Er | $^{172}$Yb | \(\Delta\) % |
|------|--------|-----------|-----------|-------------|-----------|-----------|-------------|
| GSB  |        |           |           |             |           |           |             |
| 2    |        | 78.59     | 86.4      | 9.42        | 78.74     | 87.2      | 10.34       |
| 4    |        | 260.14    | 259.0     | 0.39        | 260.27    | 258.6     | 0.53        |
| 6    |        | 540.68    | 530.1     | 2.01        | 539.98    | 528.1     | 2.21        |
| 8    |        | 914.97    | 899.8     | 1.66        | 912.12    | 895.5     | 1.81        |
| 10   |        | 1376.6    | 1368.1    | 0.64        | 1370.07   | 1360.8    | 0.67        |
| 12   |        | 1918.6    | 1935.0    | 0.84        | 1907.48   | 1924.2    | 0.90        |
| 14   |        | 2537.2    | 2600.5    | 2.50        | 2518.7    | 2585.0    | 2.64        |
| 16   |        | 3225.7    | 3.3646    | 4.30        | 3198.4    | 3344.8    | 4.59        |
| 18   |        | 3978.4    | 4227.3    | 6.27        |           |           |             |
|      | 0      | 890.88    | 977.4     | 9.69        | 1042.9    | 1100.6    | 5.52        |
|      | 2      | 959.99    | 1023.7    | 6.63        | 1117.9    | 1158.0    | 3.58        |
|      | 4      | 1103.36   | 1131.7    | 2.60        | 1286.54   | 1292.1    | 0.39        |
|      | 6      | 1350.48   | 1301.5    | 3.59        | 1537.5    | 1502.7    | 2.29        |
|      | 8      | 1677.3    | 1533.1    | 8.58        | 1853.46   | 1790.0    | 3.40        |
|      | 10     | 1963.9    | 1826.4    | 7.01        | 2212.52   | 2153.9    | 2.67        |
|      | 12     | 2080.7    | 2181.4    | 4.82        | 2607.2    | 2594.3    | 0.48        |
|      |        |           |           |             |           |           |             |
| \(\beta\) band |        |           |           |             |           |           |             |
| 2    |        | 934.02    | 945.1     | 1.19        |           |           |             |
| 3    |        | 1010.53   | 1018.6    | 0.75        | 1172.39   | 1201.0    | 2.47        |
| 4    |        | 1127.29   | 1116.7    | 0.92        | 1263.03   | 1275.7    | 1.01        |
| 5    |        | 1236.68   | 1239.3    | 0.1828      | 1375.8    | 1375.4    | 0.045       |
| 6    |        | 1401.92   | 1386.4    | 1.1158      | 1510.18   | 1500.0    | 0.66        |
| 7    |        | 1556.72   | 1558.0    | 0.0621      | 1666.12   | 1649.4    | 0.99        |
| 8    |        | 1773.10   | 1754.1    | 1.0664      | 1841.84   | 1823.8    | 0.98        |
| 9    |        | 1963.9    | 1974.7    | 0.5466      | 2039.38   | 2023.2    | 0.77        |
Table 4. The experimental [36-39] and IBM-1 calculated states (keV) of $^{174}$Hf and $^{176}$W.

| Band | States | $^{174}$Hf | | $^{176}$W | |
|------|--------|------------|--------|--------|--------|
|      | Exp.   | Cal.       | ∆%    | Exp.   | Cal.   | ∆%    |
| GSB  | 2      | 90.98      | 99.3   | 9.17   | 108.3  | 122.0  | 12.93 |
|      | 4      | 297.38     | 288.8  | 2.76   | 348.2  | 332.1  | 4.56  |
|      | 6      | 608.26     | 586.5  | 3.53   | 699.4  | 662.3  | 5.24  |
|      | 8      | 1009.6     | 992.5  | 1.73   | 1139.7 | 1112.6 | 2.40  |
|      | 10     | 1485.9     | 1506.7 | 1.39   | 1648.5 | 1683.0 | 2.06  |
|      | 12     | 2020.5     | 2129.2 | 5.35   | 2206.3 | 2373.5 | 7.59  |
|      | 14     | 2597.5     | 2859.9 | 10.08  | 3026.3 | 3184.1 | 13.59 |
|      | 16     | 3208.9     | 3698.9 | 15.27  | 3427.6 | 4114.7 | 20.03 |
|      | 18     | 3857.3     | 4646.2 | 20.46  |        |        |       |
| β - band |        |            |        |        |        |        |       |
| 0    | 828.13 | 903.5      | 9.12   | 843.3  | 912.2  | 8.21  |
| 2    | 900.24 | 960.8      | 6.76   | 930.0  | 978.4  | 5.21  |
| 4    | 1062.17| 1094.5     | 3.06   | 1117.0 | 1133.0 | 1.43  |
| 6    | 1307.4 | 1304.7     | 0.18   | 1396.2 | 1375.9 | 1.44  |
| 8    | 1630.5 | 1591.2     | 2.44   | 1759.0 | 1707.2 | 2.95  |
| 10   | 2026.3 | 1954.2     | 3.54   | 2189.7 | 2126.7 | 2.89  |
| 12   | 2489.35| 2393.6     | 3.83   | 2830.6 | 2634.6 | 6.94  |
| γ - band |        |            |        |        |        |        |       |
| 2    | 1226.7 | 1223.2     | 0.31   | 1040.2 | 1048.5 | 0.82  |
| 3    | 1336.48| 1329.2     | 0.51   | 1179.2 | 1166.3 | 1.07  |
| 4    | 1448.85| 1470.5     | 1.48   | 1321.3 | 1323.4 | 0.18  |
| 5    | 1658.41| 1647.2     | 0.65   | 1518.0 | 1519.8 | 0.12  |
| 6    | 1859.2 |           |        | 1755.4 |        |       |
| 7    | 2106.5 |           |        | 2030.3 |        |       |
| 8    | 2389.2 |           |        | 2344.5 |        |       |
| 9    | 2707.2 |           |        | 2697.9 |        |       |
Table 5. The experimental [36-39] and IVBM calculated states (keV) of $^{170}$Er and $^{172}$Yb.

| Band | States | $^{170}$Er | $^{172}$Yb |
|------|--------|-------------|-------------|
|      |        | $I^+$ Exp. | $I^+$ Cal. | $\Delta\%$ | $I^+$ Exp. | $I^+$ Cal. | $\Delta\%$ |
| GSB  | 2      | 78.59      | 86.4        | 9.42        | 78.74      | 83.6        | 5.76 |
|      | 4      | 260.14     | 266.6       | 2.53        | 260.27     | 262.7       | 1.02 |
|      | 6      | 540.68     | 540.6       | 0.07        | 539.98     | 537.3       | 0.50 |
|      | 8      | 914.97     | 908.5       | 0.71        | 912.12     | 907.5       | 0.49 |
|      | 10     | 1376.6     | 1370.2      | 0.49        | 1370.07    | 1373.2      | 0.24 |
|      | 12     | 1918.6     | 1925.8      | 0.35        | 1907.48    | 1934.5      | 1.44 |
|      | 14     | 2537.2     | 2575.2      | 1.50        | 2518.7     | 2591.4      | 2.87 |
|      | 16     | 3225.7     | 3318.4      | 2.86        | 3198.4     | 3344.7      | 4.56 |
|      | 18     | 3978.4     | 4155.4      | 4.46        |            |            |      |
|      | $I^-$  | 1266.63    | 1445.8      | 14.11       | 1154.94    | 1210.3      | 4.79 |
|      | 3      | 1340.18    | 1408.5      | 5.11        | 1221.72    | 1217.3      | 0.39 |
|      | 5      | 1483.75    | 1465.1      | 1.27        | 1352.95    | 1319.8      | 2.45 |
|      | 7      | 1704.84    | 1615.4      | 5.25        | 1557.58    | 1517.9      | 2.57 |
|      | 9      | 1990.8     | 1859.6      | 6.60        | 1839.8     | 1811.5      | 1.55 |
|      | 11     | 2434.2     | 2197.7      | 9.71        | 2199.47    | 2200.7      | 0.08 |
|      | 13     | 2973.2     | 2629.5      | 11.55       | 2636.1     | 2685.4      | 1.88 |
|      | 15     | 3583.1     | 3155.3      | 22.15       |            |            |      |

Table 6. The experimental [29, 30, 31, 32] and IVBM calculated states (keV) of $^{174}$Hf and $^{176}$W.

| Band | States | $^{174}$Hf | $^{176}$W |
|------|--------|-------------|------------|
|      | $I^+$  | $I^+$ Exp. | $I^+$ Cal. | $\Delta\%$ | $I^+$ Exp. | $I^+$ Cal. | $\Delta\%$ |
| GSB  | 2      | 90.98      | 86.4       | 9.42        | 108.3      | 83.6       | 5.76 |
|      | 4      | 297.38     | 266.6      | 2.53        | 348.2      | 262.7      | 1.02 |
|      | 6      | 608.26     | 540.6      | 0.07        | 699.4      | 537.3      | 0.50 |
|      | 8      | 1009.6     | 908.5      | 0.71        | 1139.7     | 907.5      | 0.49 |
|      | 10     | 1485.9     | 1370.2     | 0.49        | 1648.5     | 1373.2     | 0.24 |
|      | 12     | 2020.5     | 1925.8     | 0.35        | 2206.3     | 1934.5     | 1.44 |
|      | 14     | 2597.5     | 2575.2     | 1.50        | 2802.6     | 2591.4     | 2.87 |
|      | 16     | 3208.9     | 3318.4     | 2.86        | 3472.6     | 3344.7     | 4.56 |
|      | 18     | 3857.3     | 4155.4     | 4.46        |            |            |      |
|      | $I^-$  | 1321.0     | 1382.7     | 4.67        | 1197.1     | 1248.4     | 4.30 |
|      | 5      | 1442.66    | 1456.7     | 0.95        | 1400.7     | 1331.1     | 4.99 |
|      | 7      | 1650.6     | 1628.9     | 1.30        | 1673.2     | 1512.0     | 9.62 |
|      | 9      | 1943.9     | 1899.3     | 2.30        | 2007.7     | 1791.1     | 10.80 |
|      | 11     | 2319.2     | 2267.8     | 2.21        | 2409.3     | 2168.3     | 9.99 |
4 Conclusion

Knowing the number of protons and neutrons and their distribution on different shells and subshells, as well as the positions of their first excited states and the ratio of the second to the first excited states of $^{170}$Er, $^{172}$Yb, $^{174}$Hf, and $^{176}$W isotones are not enough to determine their properties among all their excited states. The relationship between the moment of inertia $2\Theta/h^2$ and the square of the angular frequency $\hbar^2\omega^2$ shows a smooth change in $2\Theta/h^2$, which means no phase change occurs. The relationship between successive excited states to those preceding them, $r\left(\frac{I+2}{I}\right)$, proved the rotational properties for $^{170}$Er, $^{172}$Yb, $^{174}$Hf, and $^{176}$W isotones, and $\Delta l = 1$ staggerings between the GSB and the NPB states for all isotones studied did not reach zero values. This means that there are no phase changes observed in these nuclei. Using IBM-1 to calculate the GSB, $\beta - band$ and $\gamma - band$, and using IVBM to calculate the GSB and NPB, the IBM-1 and IVBM calculations were in good agreement with the experimental data with some exception in the accuracy at high states of GSB for $^{174}$Hf and $^{176}$W as these states lie out the range of rotational properties which were used.

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