Multi-Objective Shortest Path Model for Optimal Route between Commercial Cities on America

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Abstract
The traditional shortest path problem is mainly concerned with identifying the associated paths in the transportation network that represent the shortest distance between the source and the destination in the transportation network by finding either cost or distance. As for the problem of research under study it is to find the shortest optimal path of multi-objective (cost, distance and time) at the same time has been clarified through the application of a proposed practical model of the problem of multi-objective shortest path to solve the problem of the most important 25 commercial US cities by travel in the car or plane. The proposed model was also solved using the lexicographic method through package program Win-QSB 2.0 for operational research applications.

Keywords: shortest path problem, multi-objective optimization, multi-objective shortest path problem, lexicographic method

استخدام نموذج المسار الأقصر متعدد الأهداف لإيجاد أمثل طريق بين المدن التجارية في أمريكا

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الخلاصة
تتم مشكلة أقصر طرق تقليدية أساساً بتحديد الطرق المتصلة في شبكة النقل والتي تتمثل أقصر مسافة بين المصدر، ومكان الوصول في شبكة النقل عن طريق إيجاد أقصر طرق إما الكلفة أو الميزة. أما فيما يخص مشكلة البحث في الدراسة فهي تمثل بإيجاد أقصر طرق أشتر متعدد الأهداف لكل من (الكلفة، المسافة والوقت) في أن واحد وقد تم توضيح ذلك من خلال تطبيق نموذج عملي مفتوح لمشكلة أقصر طرق متعدد الأهداف لحل مشكلة أهم 25 مدينة تجارية أمريكية عن طريق السفر بالسواقة أو بالطائرة، كما تم حل النموذج المقترح باستخدام الطرق المعجمية من خلال البرنامج الجاهز (Win–QSB) الخاص بتطبيقات بحوث العمليات.

1. Introduction and Reference Review
Multi-objective combinatorial optimization MOCO, also called multi-criteria optimization, is a well-studied branch of optimization, where the goal is to find optimal solutions based on multi-objective. Many real-life problems can be represented as networks, such as transportation networks, communication networks, pipeline distribution networks, and neural networks. The primary aim of these network models is to optimize the performance with respect to predefined objectives. Multi-objectives such as optimization of cost, distance, time, delay, risk, reliability, quality of service and environment impact etc. may arise in such problems. This problem can be formulated as a multi-objective shortest path problem MOSPP. There are many studies to treating the problem of the multi-

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objective shortest path, where White [1] used the multi-objective linear programming model to identify the optimal path with an effective set of efficiently policies and paths. Arthur [2] also studied a range of approximate methods of Pareto optimal paths in multi-objective shortest path problems. John et al. [3] proposed an interactive method to generate an approximation of the non-inferior solution set for two objective shortest path problems, the goal of this approach is to assist the decision maker DM in selecting the preferred or best compromise solution from among the noninferior solutions. Coutinho-Rodrigues, Climaco, and Current [4] presented a proposal for a new interactive approach to search for unsupported non-dominated solutions (lying inside duality gaps) based on the k-shortest path procedure. Guerriero and Musmanno [5] presented a method for finding an optimal solution for a multi-criteria shortest path problem by using a class of labeling methods to generate a complete set of Pareto optimal path-length POPL vectors from an origin node (source) to all other nodes in a multi-criteria network. Tarapata [6] compared the effectiveness of solving the shortest multi-objective path problem defined as a mathematical programming problem by using the CPLEX 7.0 solver, and the multiple-weighted graph problems using the modified Dijkstra's algorithm. Raith and Ehrgott [7] were able to make an effective comparison of the different strategies to solve the bi-objective shortest path problem of complex networks (i.e., many nodes and arcs). Paixão and Santos [8] studied traditional labeling technology to solve the multi-objective shortest path problem, taking into account the presence of more than a cost of arcs. Duque, Lozano, and Medaglia [9] presented a precise iterative method based on the implicit census to solve a bi-objective shortest path problem to a network consist of 1.2 million nodes and 2.8 million arcs. Thomas, Twan, and Wilcovanden [10] presented a new fully polynomial time approximation scheme for the multi-objective shortest path problem with non-negative and integer arc costs.

1.1. Research Problem
The problem of this paper is how to choose the (DM or businessman) for the shortest possible way to travel by car or plane among the most important 25 US commercial cities from Los Angeles to New York City, taking into consideration achieve three objectives, (cost, distance and time. The goal of this research is to find an optimal multi-objective solution to the problem of the shortest path to the most important US commercial cities, while making a comparison between the preference of travel by car or plane, taking into account the minimization of the objectives of cost, distance and time together.

2. Multi-Objective Linear Programming Problem MOLPP
MOLPP is one of the most important mathematical optimization models and is an extension of traditional one-objective programming, where consideration is given more than one objective is sought by the DM to achieve [11]. Often these objectives are conflicting with each other, and these ones of the most important difficulties facing us in the multi-objective optimization models. The concept of the optimal solution is no longer logical. In general, there is no possible solution that can optimize all objective functions at the same time, where Zeleny [12] has demonstrated in a very expressive manner the fundamental differences between traditional single-objective optimization models and multi-objective optimization models. The multi-objective linear programming model can now be represented as follows [13]:

\[ MOLP = \min \ Z(x) = (z_1(x), z_2(x), \ldots, z_n(x)) \]

\[ s.t. \quad x \in D \]  

(2.1)

Where the represents: \( n \geq 2 \) the number of objective functions; \( x = (x_1, x_2, \ldots, x_r) \) the vector of decision variables; \( D = Ax \geq b \) a space of feasible solution; and \( Z(x) \) vector of objective functions, respectively. The solution obtained from the multi-objective linear programming model is a set of non-dominant solutions called the Pareto set PS.

2. Shortest Path Problem SPP
Let's consider that we have a flow network that has a certain starting point and a certain endpoint and that the arrows that connect the network points take many paths to reach the starting point and the access point. We try through the shortest path problem, to get the shorter of these arrows or lines that connect the starting point and the end point. For example, this model is used to obtain the shortest distance or path that can be taken between one city and another through a network of paths. The length of each arc is a function of that distance, travel time, cost of travel or any other measure. In other words, we try to find the shortest path between the start and access point. One of the most important issues in network flow is that of assigning the shortest path between the source node and the
destination node. Consider that we have the network composed of n the node (1, 2, ..., n) so that each arc \((i, j)\) corresponds to a non-negative number \(d_{ij}\), called distance, or transit time from node i to node j. If there is no direct route between i and j, the distance is: \(d_{ij} = +\infty\). The distance \(d_{ij}\) can vary from a distance \(d_{ji}\) (i.e. \(d_{ij} \neq d_{ji}\)). The problem lies in how to find the length of the shortest path, and the shorter route, from the source node 1, to the destination node n. As one way to solve this problem, we can interpret it as a network represented by a guided graph \(G(V, E)\), where V represents the set of headers (nodes) and E represents the set of links (arcs). The link between node i and node j is expressed by \((i, j)\) \(\in\) E; and \(d_{ij}\) is the cost of the link between i and j; as well as \(x_{pq}^{ij}\) (whereas \(0 \leq x_{pq}^{ij} \leq 1\)) represents the amount of traffic from node \(p \in V\) to node \(q \in V\) by routing it through \((i, j)\) \(\in\) E [14].

2.1. A General Model of Single Objective Shortest Path Problem MOSPP

Generally, the problem of finding the shortest path can be formulated as a linear programming problem as shown below:

\[
\text{Min } Z(x) = \sum_{(i,j)\in E} d_{ij} x_{ij} \tag{3.1.1}
\]

\[
s.t. \quad \sum_{j:(i,j)\in E} x_{ij} - \sum_{j:(j,i)\in E} x_{ji} = 1, \quad \text{if } i = p \tag{3.1.2}
\]

\[
\sum_{j:(i,j)\in E} x_{ij} - \sum_{j:(j,i)\in E} x_{ji} = 0, \quad \forall i \neq p, q \in V \tag{3.1.3}
\]

\[
x_{ij} = 0 \text{ or } 1, \quad \forall (i, j) \in E. \tag{3.1.4}
\]

\(x_{ij}\) and \(d_{ij}\) is the decision variable and the link cost \((i, j)\), respectively [14]. The equation (3.1.1) represents the objective function that reduces the cost of the path from the node p to node q and \(x_{ij}\) is the amount flow from node p to node q through \((i, j)\). Equations (3.1.2) to (3.1.4) are constraints of the model, where equations (3.1.2) - (3.1.3) are represented the conditions for maintaining flow. And Equation (3.1.2) represents the maintenance of flows in the source node, node p. The difference between the amount of traffic received and the amount of traffic issued, \(\sum_{j:(i,j)\in E} x_{ij} - \sum_{j:(j,i)\in E} x_{ji}\), is equal to one. Here, the amount of outgoing traffic at node p equals one. Equation (3.1.3) maintains flows at the intermediate node i, such that \(i \neq p, q\). The amount of traffic issued at the node i, \(\sum_{j:(i,j)\in E} x_{ij}\), equal to the amount of incoming traffic at the node i, \(\sum_{j:(j,i)\in E} x_{ji}\).

The equation (3.1.4) represents the range of decision variables \(x_{ij}\). At the destination node, node q, the condition to maintain flows is,

\[
\sum_{j:(i,j)\in E} x_{ij} - \sum_{j:(j,i)\in E} x_{ji} = -1, \quad \text{if } i = q \tag{3.1.5}
\]

The equation (3.1.5) must be met. However, Equation (3.1.5) is discounted using equations (3.1.2)-(3.1.3). Therefore, Equation (3.1.5) is guaranteed by Equations (3.1.2) and (3.1.3) [14].

4. Multi-Objective Shortest Path Problem MOSPP

The MOSPP is one of the most important and common problems. The problem is how to find the shortest path of any network based on a certain set of objectives, like cost, time, distance, etc. Consider that the DM looks at how to choose the feasible shortest path to travel by car or plane to the most important 25 US cities, taking into consideration three main objectives: cost, distance and time. It can be said that the main objective of the DM is to find an efficient solution for MOSPP to find an of the most important US commercial cities.

4.1. The Proposed Mathematical Model to Solve MOSPP

In this paragraph, a proposed mathematical model will be formulated to solve MOSPP based on the original model of SOSPP mentioned above in paragraph 3.1. Let’s consider that there are three objectives that the DM seeks to achieve together of travel using the car (the first, second and third objectives) is to minimization (cost \(Z_1\), distance \(Z_2\), and time \(Z_3\), respectively. There are also three other objectives that the DM seeks to achieve in order to travel with the aircraft (Target 1, 2 and 3):
minimization (cost \( W_1 \), distance \( W_2 \), and time \( W_3 \)), respectively. The mathematical model can now be formulated for the MOSPP as shown below:

Minimized of all objective functions

\[
\begin{align*}
\text{Cost function by car} & : \quad Z_1 = \sum_{\text{all arcs}} c_{ij} x_{ij} \\
\text{Distance function by car} & : \quad Z_2 = \sum_{\text{all arcs}} d_{ij} x_{ij} \\
\text{Time function by car} & : \quad Z_3 = \sum_{\text{all arcs}} t_{ij} x_{ij}
\end{align*}
\]

(4.1.1)

\[
\begin{align*}
\text{Cost function by airplane} & : \quad W_1 = \sum_{\text{all arcs}} \alpha_{ij} x_{ij} \\
\text{Distance function by airplane} & : \quad W_2 = \sum_{\text{all arcs}} \beta_{ij} x_{ij}
\end{align*}
\]

(4.1.3)

\[
\begin{align*}
\text{Time function by airplane} & : \quad W_3 = \sum_{\text{all arcs}} \delta_{ij} x_{ij}
\end{align*}
\]

(4.1.5)

\[s.t. \quad \sum_{\text{arcs out}} x_{ij} - \sum_{\text{arcs in}} x_{ji} = 1, \quad \text{Origin node (i)}\]  

(4.1.7)

\[\sum_{\text{arcs out}} x_{ij} - \sum_{\text{arcs in}} x_{ji} = 0, \quad \text{Intermediate nodes}\]  

(4.1.8)

\[\sum_{\text{arcs in}} x_{ij} - \sum_{\text{arcs out}} x_{ji} = -1, \quad \text{Destination node}\]  

(4.1.9)

\[0 \leq x_{ij} \leq 1.\]  

(4.1.10)

There are many approaches used to solve the above model. We will use the lexicographic method that described in the below.

4.2. Lexicographic Method

In this method, the (DM) has to arrange the objective functions according to its importance. The preferred solution of the problem (2.1), in this case, is the solution that minimizes the vales of the most important objective functions as \( [z_1(x), z_2(x), ..., z_n(x)] \) which is a vector of objective functions which is arranged according to the importance of the functions of the (DM), such that \( z_1(x) \) is the most important objective function among the other and \( z_n(x) \) is the objective function followed by the importance and so on [15]. The first problem to be solved is: Find a vector that minimizes,

\[
\begin{align*}
\{ z_1(x) \} \\
\text{s.t. } x \in D
\end{align*}
\]

(4.2.1)

Where \( x \in D \) represents a feasible region solution, and \( x^*_1 \) is a solution of this problem as well \( z_1(x^*_1) = z_1^* \); if this solution is unique then \( x^*_1 \) will be a solution to a problem (2.1), but if there is more than one solution, the second problem that must be solved is: Find a vector \( x \) that minimizes,

\[
\begin{align*}
\{ z_2(x) \} \\
\text{s.t. } x \in D : z_1(x^*_1) = z_1^*
\end{align*}
\]

(4.2.2)

Let \( x^*_2 \) represents a solution to this problem and \( z_2(x^*_2) = z_2^* \). Now, if \( x^*_2 \) is a unique solution, the problem (2.1) has this solution, otherwise, this frequency is repeated until we get a unique solution to one of these problems or complete all the objective functions and \( x_n^* \), which is the solution to the problem \( n \), is the solution to the problem (2.1). In general, the problem that must be resolved are: Find a vector \( x \) that minimizes,

\[
\begin{align*}
\{ z_j(x) \} \\
\text{s.t. } x \in D : z_{\ell}(x) = z_{\ell}^*, \quad \ell = 1, 2, ..., j - 1
\end{align*}
\]

(4.2.3)

And since the steps of the solution are stopped when we obtained a unique solution of the problem \( j \), this solution is the solution to the problem (2.1), and the functions which have the ranks are less than \( j \) neglected.

5. Application for Real Case Study

Consider that, there is a businessman who wants to take the shortest path by using the car or plane of the most important 25 US commercial cities to reach New York City from the hometown city Los Angeles, taking into account the minimization (cost, distance and time) together. Cost, distance and time data were collected among the major US commercial cities (https://www.distance-cities.com/).
Figure 1 illustrates the network diagram of the link among the most important 25 US commercial cities. Table 1 shows the names of the most important US commercial cities; Table 2 shows the cost, distance and time of travel by car, and finally, Table 3 shows the cost, distance and time of travel by plane.

**Figure 1-** Map of networking among the top 25 US commercial cities

**Table 1-** Names of the most important of (25) American commercial city

| Node | City                        | Node | City                        |
|------|-----------------------------|------|-----------------------------|
| 1    | Los Angeles, California     | 14   | St Louis, Missouri          |
| 2    | Riverside, California       | 15   | Chicago, Illinois           |
| 3    | San Diego, California       | 16   | Detroit, Michigan           |
| 4    | Phoenix, Arizona            | 17   | Atlanta, Georgia            |
| 5    | San Jose, California        | 18   | Tampa, Florida              |
| 6    | San Francisco, California   | 19   | Miami, Florida              |
| 7    | Portland, Oregon            | 20   | Charlotte, N.C.             |
| 8    | Seattle, Washington         | 21   | Washington, D.C.            |
| 9    | Washington, State           | 22   | Pittsburgh, Pennsylvania    |
| 10   | Denver, Colorado            | 23   | Philadelphia, Pennsylvania  |
| 11   | Dallas, Texas               | 24   | Boston, Massachusetts       |
| 12   | Houston, Texas              | 25   | New York, N.Y.              |
| 13   | Minneapolis, Minnesota      |      |                             |

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Table 2: Cost (C) in dollars ($), distance (D) in miles (Miles) and time (T) in minutes to travel among the most important (25) American commercial cities by using the car

| Start - Destination node | Travel using the car | Start - Destination node | Travel using the car |
|--------------------------|----------------------|--------------------------|----------------------|
|                          | Cost  | Distance | Time |                          | Cost  | Distance | Time |
| 1 – 2                    | 7.15  | 57.3     | 60   | 11 – 17                  | 72.01 | 782      | 672  |
| 1 – 3                    | 14.67 | 120      | 124  | 12 – 14                  | 69.90 | 779      | 726  |
| 1 – 5                    | 41.33 | 340      | 324  | 12 – 17                  | 72.93 | 793      | 673  |
| 1 – 7                    | 113.17| 963      | 882  | 13 – 14                  | 52.92 | 559      | 505  |
| 2 – 3                    | 12.24 | 101      | 115  | 13 – 15                  | 40.64 | 408      | 371  |
| 2 – 4                    | 34.83 | 326      | 287  | 14 – 15                  | 28.16 | 297      | 280  |
| 2 – 10                   | 108.47| 984      | 859  | 14 – 17                  | 50.92 | 554      | 505  |
| 3 – 4                    | 38.28 | 355      | 323  | 14 – 22                  | 60.64 | 602      | 545  |
| 4 – 8                    | 153.10| 1421     | 1306 | 15 – 16                  | 28.53 | 283      | 257  |
| 4 – 10                   | 83.41 | 821      | 759  | 16 – 17                  | 69.82 | 733      | 674  |
| 4 – 11                   | 98.24 | 1065     | 910  | 16 – 20                  | 64.44 | 639      | 621  |
| 4 – 12                   | 108.75| 1176     | 994  | 16 – 22                  | 31.07 | 289      | 270  |
| 5 – 6                    | 5.82  | 48.4     | 58   | 17 – 18                  | 43.42 | 456      | 380  |
| 5 – 10                   | 142.94| 1273     | 1122 | 17 – 19                  | 62.89 | 663      | 561  |
| 6 – 7                    | 74.67 | 635      | 599  | 17 – 15                  | 69.48 | 716      | 629  |
| 6 – 13                   | 217.21| 1966     | 1740 | 17 – 20                  | 23.33 | 244      | 222  |
| 7 – 8                    | 19.90 | 173      | 198  | 18 – 19                  | 26.74 | 280      | 247  |
| 7 – 9                    | 22.66 | 269      | 289  | 18 – 20                  | 55.71 | 580      | 520  |
| 8 – 9                    | 5.12  | 103      | 122  | 20 – 21                  | 40.25 | 399      | 380  |
| 9 – 13                   | 175.27| 1585     | 1401 | 20 – 22                  | 46.56 | 447      | 419  |
| 10 – 9                   | 140.04| 1299     | 1148 | 21 – 23                  | 15.49 | 139      | 148  |
| 10 – 11                  | 74.48 | 791      | 724  | 22 – 21                  | 26.72 | 242      | 238  |
| 10 – 13                  | 90.75 | 914      | 774  | 22 – 23                  | 33.78 | 304      | 283  |
| 10 – 14                  | 80.30 | 850      | 718  | 22 – 25                  | 40.65 | 371      | 354  |
| 10 – 15                  | 99.78 | 1004     | 858  | 23 – 24                  | 32.84 | 310      | 321  |
| 11 – 12                  | 21.51 | 239      | 214  | 23 – 25                  | 10.30 | 96.7     | 109  |
| 11 – 13                  | 93.71 | 942      | 840  | 24 – 25                  | 22.67 | 215      | 229  |
| 11 – 14                  | 56.69 | 631      | 582  |
### Table 3- Cost (C) in dollars ($), distance (D) in miles (Miles) and time (T) in minutes to travel among the top 25 US commercial cities by using the plane

| Start Destination node | Travel using the airplane | Start Destination node | Travel using the airplane |
|------------------------|--------------------------|------------------------|--------------------------|
|                        | Cost | Distance | Time |                        | Cost | Distance | Time |
| 1 – 2                  | 300  | 49       | 36   | 11 – 17                 | 126  | 721      | 99   |
| 1 – 3                  | 182  | 112      | 31   | 12 – 14                 | 239.5| 679      | 96   |
| 1 – 5                  | 121.5| 306      | 50   | 12 – 17                 | 103  | 702      | 95   |
| 1 – 7                  | 142  | 825      | 117  | 13 – 14                 | 235.5| 466      | 68   |
| 2 – 3                  | 385.5| 86       | 40   | 13 – 15                 | 79.5 | 355      | 66   |
| 2 – 4                  | 153.5| 309      | 50   | 14 – 15                 | 190.5| 262      | 45   |
| 2 – 10                 | 216  | 794      | 105  | 14 – 17                 | 64.5 | 467      | 67   |
| 3 – 4                  | 124  | 299      | 50   | 14 – 22                 | 121.5| 559      | 97   |
| 4 – 8                  | 232  | 1114     | 156  | 15 – 16                 | 122  | 238      | 45   |
| 4 – 10                 | 135  | 586      | 80   | 16 – 17                 | 134.5| 596      | 90   |
| 4 – 11                 | 107.5| 886      | 115  | 16 – 20                 | 229.5| 504      | 78   |
| 4 – 12                 | 222.5| 1016     | 128  | 16 – 22                 | 211  | 205      | 37   |
| 5 – 6                  | 190.5| 42       | 35   | 17 – 18                 | 100.5| 416      | 62   |
| 5 – 10                 | 155.5| 929      | 122  | 17 – 19                 | 121.5| 604      | 88   |
| 6 – 7                  | 118  | 535      | 76   | 17 – 15                 | 123  | 588      | 88   |
| 6 – 13                 | 221  | 1585     | 186  | 17 – 20                 | 214.5| 226      | 42   |
| 7 – 8                  | 152.5| 145      | 42   | 18 – 19                 | 101.5| 205      | 39   |
| 7 – 9                  | 143.5| 139      | 40   | 18 – 20                 | 243  | 510      | 78   |
| 8 – 9                  | 117  | 39       | 35   | 20 – 21                 | 159  | 329      | 54   |
| 9 – 13                 | 231  | 1360     | 161  | 20 – 22                 | 265.5| 363      | 58   |
| 10 – 9                 | 178.5| 983      | 140  | 21 – 23                 | 297  | 124      | 34   |
| 10 – 11                | 81   | 662      | 96   | 22 – 21                 | 108  | 190      | 40   |
| 10 – 13                | 112  | 700      | 93   | 22 – 23                 | 210.5| 258      | 46   |
| 10 – 14                | 111  | 797      | 100  | 22 – 25                 | 216  | 316      | 55   |
| 10 – 15                | 105  | 920      | 117  | 23 – 24                 | 133.5| 271      | 49   |
| 11 – 12                | 171.5| 225      | 42   | 23 – 25                 | 208  | 81       | 23   |
| 11 – 13                | 96.5 | 862      | 116  | 24 – 25                 | 110  | 190      | 48   |
| 11 – 14                | 205.5| 548      | 78   |                          |      |          |      |

5.1. MOSPP by Using Car or Airplane

Based on the model proposed in paragraph 4.1 and using the above Tables-(2-3) that mentioned (2 and 3), a multi-objective linear programming model will be built to solve the shortest path problem among the most important 25 US commercial cities by traveling either by car or by plane as shown below:

Minimization (Travel by Car)

\[
\begin{align*}
Z_1(x) &= 7.15x_{1,2} + 14.67x_{1,3} + 41.33x_{1,5} + 113.17x_{1,7} + \ldots + 22.67x_{24,25} \\
Z_2(x) &= 57.3x_{1,2} + 120x_{1,3} + 340x_{1,5} + 963x_{1,7} + \ldots + 215x_{24,25} \\
Z_3(x) &= 60x_{1,2} + 124x_{1,3} + 324x_{1,5} + 882x_{1,7} + \ldots + 229x_{24,25}
\end{align*}
\]
Subject to:
\[ x_{1,2} + x_{1,3} + x_{1,5} + x_{1,7} = 1 \quad (5.1.7), \quad x_{1,2} = x_{2,3} + x_{2,4} + x_{2,10} \quad (5.1.8) \]
\[ x_{1,3} + x_{2,3} = x_{3,4} \quad (5.1.9), \quad x_{2,4} + x_{3,4} = x_{4,8} + x_{4,10} + x_{4,11} + x_{4,12} \quad (5.1.10) \]
\[ x_{1,5} = x_{5,6} + x_{5,10} \quad (5.1.11), \quad x_{5,6} = x_{6,7} + x_{6,13} \quad (5.1.12) \]
\[ x_{1,7} + x_{6,7} = x_{7,8} + x_{7,9} \quad (5.1.13), \quad x_{4,8} + x_{7,8} = x_{8,9} \quad (5.1.14) \]
\[ x_{7,9} + x_{10,9} = x_{9,13} \quad (5.1.15), \quad x_{2,10} + x_{4,10} + x_{5,10} = x_{10,9} + x_{10,11} + x_{10,13} + x_{10,14} + x_{10,15} \quad (5.1.16) \]
\[ x_{4,11} + x_{10,11} = x_{11,12} + x_{11,14} + x_{11,15} + x_{11,17} \quad (5.1.17), \quad x_{4,12} + x_{11,12} = x_{12,14} + x_{12,17} \quad (5.1.18) \]
\[ x_{6,13} + x_{9,13} + x_{10,13} + x_{11,13} = x_{13,14} + x_{13,15} \quad (5.1.19) \]
\[ x_{10,14} + x_{11,14} + x_{12,14} + x_{13,14} = x_{14,15} + x_{14,17} + x_{14,22} \quad (5.1.20) \]
\[ x_{10,15} + x_{13,15} + x_{14,15} + x_{17,15} = x_{15,16} \quad (5.1.21), \quad x_{15,16} = x_{16,17} + x_{16,20} + x_{16,22} \quad (5.1.22) \]
\[ x_{11,17} + x_{12,17} + x_{14,17} + x_{16,17} = x_{17,18} + x_{17,19} + x_{17,20} \quad (5.1.23), \quad x_{17,18} = x_{18,19} + x_{18,20} \quad (5.1.24) \]
\[ x_{16,20} + x_{17,20} + x_{18,20} = x_{20,21} + x_{20,22} \quad (5.1.25), \quad x_{20,21} + x_{22,21} = x_{21,23} \quad (5.1.26) \]
\[ x_{14,22} + x_{16,22} + x_{20,22} = x_{22,21} + x_{22,23} + x_{22,25} \quad (5.1.27), \quad x_{21,23} + x_{22,23} = x_{23,24} + x_{23,25} \quad (5.1.28) \]
\[ x_{23,24} = x_{24,25} \quad (5.1.29), \quad x_{22,25} + x_{23,25} + x_{24,25} = 1 \quad (5.1.30) \]

All decision variables \((\geq 0 \text{ and } \leq 1)\) \quad (5.1.31)

Where: \(Z_1(x), Z_2(x), Z_3(x)\); represents the function of (cost, distance and time) respectively, when businessman traveling by car, and \(W_1(x), W_2(x), W_3(x)\); represents the function of (cost, distance and time) respectively, when businessman traveling by plane. The equations from (5.1.7) to (5.1.31) represent the constraints of the problem to which we will refer briefly \(x \in D\).

5.2. Solve the Model of MOSPP by Using the Car or Airplane

In this paragraph, the above model will be resolved by using the package program (Win-QSB 2.0) [16]. Firstly, we will solve each objective function individually with the problem constraints as a standard linear programming model to obtain the values of \(Z_1(x), Z_2(x)\) and \(Z_3(x)\), where the travel model by using the car; and the values of \(W_1(x), W_2(x)\) and \(W_3(x)\) for the travel model by using the aircraft, respectively. So that we can use the Lexicographic method that mentioned above in paragraph 4.2 to find the final optimal solution for the travel either by car or by plane and as follows:

\[
\begin{align*}
\text{Min } Z_1(x) & \quad (5.2.1) \quad \text{Min } Z_2(x) & \quad (5.2.2) \quad \text{Min } Z_3(x) & \quad (5.2.3) \\
\text{s.t. } x \in D & \quad \text{s.t. } x \in D & \quad \text{s.t. } x \in D
\end{align*}
\]

- After we solved the models (5.2.1-5.2.3), were obtained the preliminary results shown below,

\[
\begin{align*}
\text{Min } W_1(x) & \quad (5.2.4) \quad \text{Min } W_2(x) & \quad (5.2.5) \quad \text{Min } W_3(x) & \quad (5.2.6) \\
\text{s.t. } x \in D & \quad \text{s.t. } x \in D & \quad \text{s.t. } x \in D
\end{align*}
\]

- After we solved the models (5.2.4-5.2.6), were obtained the preliminary results shown below,

\[
\begin{align*}
W_1(x) = 964.500; W_2(x) = 2515; W_3(x) = 393.
\end{align*}
\]

We will now use the Lexicographic method to obtain the optimal solution of travel by using the car or plane as follows:

- (The final model of travel by car)
Min \( Z_1(x) = 7.15x_{1,2} + 14.67x_{1,3} + 41.33x_{1,5} + 113.17x_{1,7} + \ldots + 22.67x_{24,25} \)

\[ s.t. \quad Z_2(x) = 57.3x_{1,2} + 120x_{1,3} + 340x_{1,5} + 963x_{1,7} + \ldots + 215x_{24,25} = 2864.3 \]

\[ Z_3(x) = 60x_{1,2} + 124x_{1,3} + 324x_{1,5} + 882x_{1,7} + \ldots + 229x_{24,25} = 2536 \]

\( x \in D \)

After we solving the model (5.2.7), the final optimal solution was obtained for the three objectives as shown below,

\[ Z_1^*(x) = 297.21/ \$; Z_2^*(x) = 2864.30/ \text{Miles}; Z_3^*(x) = 2536/ \text{Minutes} / 42.27/ \text{Hour}; \]

\[ \{x_{1,2}^*, x_{2,10}^*, x_{10,14}^*, x_{14,22}^*, x_{22,25}^*\} = 1. \]

- (The final model of travel by plane)

Min \( W_1(x) = 300x_{1,2} + 182x_{1,3} + 121.5x_{1,5} + 142x_{1,7} + \ldots + 110x_{24,25} \)

\[ s.t. \quad W_2(x) = 49x_{1,2} + 112x_{1,3} + 306x_{1,5} + 825x_{1,7} + \ldots + 190x_{24,25} = 2515 \]

\[ W_3(x) = 36x_{1,2} + 34x_{1,3} + 50x_{1,5} + 117x_{1,7} + \ldots + 48x_{24,25} = 393 \]

\( x \in D \)

After solving the model (5.2.8), the final optimal solution was obtained for the three objectives as shown below,

\[ W_1(x) = 964.5/ \$; W_2(x) = 2515/ \text{Miles}; W_3(x) = 393/ \text{Minutes} / 6.55/ \text{Hour}; \]

\[ \{x_{1,2}^*, x_{2,10}^*, x_{10,14}^*, x_{14,22}^*, x_{22,25}^*\} = 1. \]

The above results show that, if the DM is looking for minimization cost only, the traveling by car is the optimal solution; and if the DM is looking to minimize the distance and time, the travel by plane is optimal. The optimal path is represented in Table (4).

**Table 4** The optimal path has the minimization of the objective functions simultaneously, for travel by car or plane

| City of America                     | Optimal path | Travel using the car | Travel using the airplane |
|-------------------------------------|--------------|----------------------|--------------------------|
|                                     |              | Cost     | Distance | Time | Cost     | Distance | Time |
| Los Angeles, California - Riverside, California | 1 → 2     | 7.15    | 57.3     | 60   | 300      | 49       | 36   |
| Riverside, California - Denver, Colorado | 2 → 10   | 108.47  | 984      | 859  | 216      | 794      | 105  |
| Denver, Colorado - St Louis, Missouri | 10 → 14  | 80.30   | 850      | 718  | 111      | 797      | 100  |
| St Louis, Missouri - Pittsburgh, Pennsylvania | 14 → 22 | 60.64   | 602      | 545  | 121.5    | 559      | 97   |
| Pittsburgh, Pennsylvania - New York, N.Y. | 22 → 25  | 40.65   | 371      | 354  | 216      | 316      | 55   |
| Total                               |             | 297.21  | 2864.3   | 2536 | 964.5    | 2515     | 393  |

6. Conclusions

In this paper, we have focused on how to solve the problem of the shortest path problem of the most important 25 commercial cities in the United State of America with real data analysis, also how to find a final optimal solution that satisfies the DM ambition and obtain the maximum budget achieved in terms of the minimization cost, distance and time. Therefore, a multi-objective linear programming model was proposed to solve the shortest path problem. This model proved the efficiency and effectiveness of solving the problem, were taken into account the objective functions (cost, distance and time) the minimization cost only, is the optimal solution when traveling is by car; and minimization the distance and time is optimal when the travel is by plane. The lexicographic method provided a definitive solution for all objective functions with one optimal path. The use of this
type of mathematical model is a new technique that will raise the quality of rational decision making in institutions that are interested in logistics transportation and looking for real solutions to the problems of multi-objective optimization.

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