Investigation of $^9$Be from nonlocalized clustering concept

Mengjiao Lyu,$^{1,2}$ Zhongzhou Ren,$^{1,3}$ Bo Zhou,$^{1,4,5}$ Yasuro Funaki,$^5$ Hisashi Horiuchi,$^{2,6}$ Gerd Röpke,$^7$ Peter Schuck,$^{8,9}$ Akihiro Tohsaki,$^2$ Chang Xu,$^1$ and Taiichi Yamada$^{10}$

$^1$School of Physics and Key Laboratory of Modern Acoustics, Institute of Acoustics, Nanjing University, Nanjing 210093, China
$^2$Research Center for Nuclear Physics (RCNP), Osaka University, Osaka 567-0047, Japan
$^3$Center of Theoretical Nuclear Physics, National Laboratory of Heavy-Ion Accelerator, Lanzhou 730000, China
$^4$Meme Media Laboratory, Hokkaido University, Sapporo 060-0810, Japan
$^5$Nishina Center for Accelerator-Based Science, The institute of Physical and Chemical Research (RIKEN), Wako 351-0198, Japan
$^6$International Institute for Advanced Studies, Kizugawa 619-0225, Japan
$^7$Institut für Physik, Universität Rostock, D-18051 Rostock, Germany
$^8$Institut de Physique Nucléaire, Université Paris-Sud, IN2P3-CNRS, UMR 8608, F-91406, Orsay, France
$^9$Laboratoire de Physique et Modélisation des Milieux Condensés, CNRS-UMR 5493, F-38042 Grenoble Cedex 9, France
$^{10}$Laboratory of Physics, Kanto Gakuin University, Yokohama 236-8501, Japan

(Dated: August 25, 2014)
Abstract

The nonlocalized aspect of clustering, which is a new concept for self-conjugate nuclei, is extended for the investigation of the $N \neq Z$ nucleus $^9$Be. A modified version of the THSR (Tohsaki-Horiuchi-Schuck-Röpke) wave function is introduced based on the container picture. It is found that the constructed negative-parity THSR wave function is very suitable for describing the cluster states of $^9$Be. Namely the nonlocalized clustering is shown to prevail in $^9$Be. The calculated binding energy and radius of $^9$Be are consistent with calculations in other models and with experimental values. The squared overlaps between the THSR wave function and the Brink+GCM wave function for the $3/2^-$ rotational band of $^9$Be are found to be near 96%. Furthermore, by showing the density distribution of the ground state of $^9$Be, the $\pi$-orbit structure is naturally reproduced by using this THSR wave function.

PACS numbers: 21.60.Gx, 27.20.+n
I. INTRODUCTION

The clustering phenomena is one of the fundamental problems in nuclear physics. Despite of its long history since the discovery of the $\alpha$-cluster, the clustering structure in nuclei is still under active investigation [1-6]. To describe the $\alpha$-cluster condensation in self-conjugate nuclei $^{12}$C and $^{16}$O, the Tohsaki-Horiuchi-Schuck-Röpke (THSR) wave function was proposed and provided a successful treatment of the famous Hoyle ($0^+_2$) state in $^{12}$C [1, 2]. The THSR wave function has been applied to different aggregates of $\alpha$-clusters including $^8$Be, $^{12}$C and $^{16}$O [1, 3]. It is also extended to study systems composed of general clusters such as $^{20}$Ne treated as a combination of an $\alpha$-cluster and $^{16}$O [7]. The THSR wave function describes nonlocalized clustering dynamics which has been shown to be an essential property of low energy cluster states in light nuclei [4]. To understand this nonlocalized dynamics, the container picture has recently been proposed where the clusters are considered to occupy the lowest orbit of the cluster mean-field potential [8]. This mean-field potential, which is called container, is characterized by the size parameter in the THSR wave function. Following the success in describing self-conjugate nuclei, a natural question is whether we can apply this container picture to general nuclei which consist not only of $\alpha$-clusters but have also extra nucleons, such as $^{13}$C studied with the interaction of $\alpha$-condensation and an extra neutron probe [9]. Because the extra nucleons can also be considered as small clusters, it is promising to extend the THSR wave function to $N\neq Z$ nuclei.

The nucleus $^9$Be is a typical $N\neq Z$ nucleus with $\alpha + \alpha + n$ cluster structure, which is most suitable for the extension of the THSR wave function. It has been studied with the antisymmetrized molecular dynamics free from cluster assumptions [10]. It is also investigated with the nuclear molecular orbit (MO) model in which the extra neutron occupies nuclear molecular orbits [11-13]. In the nuclear molecular orbit model, the wave function of the extra neutron is assumed to be a linear combination of cluster orbitals and provides a successful description of $^9$Be. However, the dynamics of the clusters is not explicitly given in these calculations. To have a clear view of the nonlocalized clustering dynamics in the $^9$Be nucleus, we need a new wave function in which this characteristic is intrinsically included. In the present work, $^9$Be is investigated with the new container picture in which two $\alpha$-clusters and an extra nucleon are making nonlocalized motion within containers. One container is used for the two-$\alpha$-clusters subsystem and another one is used for the extra neutron, and a
modified version of THSR wave function is proposed for the description of this structure.

The outline of this paper is as follows. In Section II we formulate the THSR wave function with intrinsic parity for $^9\text{Be}$. Then in Section III we give the results of the calculation for $3/2^-$ rotational band of $^9\text{Be}$ and analyze the structure of the ground state obtained from the THSR wave function. The last Section IV contains the conclusions.

II. FORMULATION OF THSR WAVE FUNCTION FOR $^9\text{Be}$

The THSR wave function of $^9\text{Be}$ is constructed with creation operators as

$$\langle \Phi \rangle = (C^\dagger_{\alpha})^2 c^\dagger_n |\text{vac}\rangle,$$

where $C^\dagger_{\alpha}$ and $c^\dagger_n$ are creation operators of $\alpha$-particle and neutron, respectively. Here we take the same $\alpha$-creator $C^\dagger_{\alpha}$ as used in previous deformed THSR wave functions [3],

$$C^\dagger_{\alpha} = \int d^3R \exp(-\frac{R_x^2}{\beta^2_{\alpha,xy}} - \frac{R_y^2}{\beta^2_{\alpha,xy}} - \frac{R_z^2}{\beta^2_{\alpha,z}}) \int d^3r_1 \cdots d^3r_4$$

$$\times \psi(r_1 - R) a^\dagger_{\sigma_1,\tau_1}(r_1) \cdots \psi(r_4 - R) a^\dagger_{\sigma_4,\tau_4}(r_4),$$

where $a^\dagger_{\sigma,\tau}(r)$ is the creation operator of nucleon with spin $\sigma$ and isospin $\tau$ at position $r$. $\psi(r) = (\pi b^2)^{-3/4} \exp(-r^2/2b^2)$ is the single-nucleon harmonic oscillator shell model wave function of $\alpha$ clusters, where $b$ is a size parameter. $\beta_{\alpha,xy}$ and $\beta_{\alpha,z}$ are parameters for the nonlocalized motion of two $\alpha$-clusters. The subsystem of two $\alpha$-clusters is supposed to have rotational symmetry around the $z$-axis, so the same size parameters $\beta_{\alpha,xy}$ are used in $x$ and $y$ direction. For the extra neutron, we use a creation operator $c^\dagger_n$ which is similar to $C^\dagger_{\alpha}$ but has a new introduced phase factor,

$$c^\dagger_n = \int d^3R_n \exp(-\frac{R_{n,x}^2}{\beta^2_{n,xy}} - \frac{R_{n,y}^2}{\beta^2_{n,xy}} - \frac{R_{n,z}^2}{\beta^2_{n,z}}) e^{im\phi_{R_n}} \int d^3r$$

$$\times (\pi b^2)^{-3/4} e^{-\frac{(r-R_n)^2}{2\beta_n^2}} a^\dagger_{\uparrow,n}(r),$$

where $\phi_{R_n}$ is the azimuthal angle in spherical coordinates $(R_{R_n}, \theta_{R_n}, \phi_{R_n})$ of vector $R_n$, and $a^\dagger_{\uparrow,n}(r)$ creates a neutron at position $r$ with spin up. $\beta_{n,xy}$ and $\beta_{n,z}$ are parameters for the nonlocalized motion of the extra neutron. The spatial wave function of the extra neutron $\Phi_n(r)$ can be written as,

$$\Phi_n(r) = \int d^3R \exp(-\frac{R_x^2}{\beta^2_{n,xy}} - \frac{R_y^2}{\beta^2_{n,xy}} - \frac{R_z^2}{\beta^2_{n,z}}) e^{im\phi_{R_n}}$$

$$\times (\pi b^2)^{-3/4} e^{-\frac{(r-R)^2}{2\beta_n^2}}.$$
To demonstrate its intrinsic parity, $\Phi_n(r)$ is analytically written in a simple form,

$$\Phi_n(r) \propto \sum_{L=0}^{\infty} C_L(r) Y^m_L(\hat{r}), \quad (5)$$

where

$$C_L(r) = \sum_{l=0}^{\infty} \int_0^\infty f_{2l,L}(r,R)dR \times \int_0^\pi P^0_{2l}(\cos \theta_R) P^m_L(\cos \theta_R) \sin \theta_R d\theta_R. \quad (6)$$

Here the $f_{2l,L}$ is a coefficient, function of the radial distances $r$ and $R$. The first Legendre polynomial $P^0_{2l}(\cos \theta_R)$ in $C_L(r)$ comes from the spherical harmonics expansion of the exponent $\exp(-R_x^2/\beta_{n,xy}^2 - R_y^2/\beta_{n,zy}^2 - R_z^2/\beta_{n,z}^2)$ and the second Legendre polynomial $P^m_L(\cos \theta_R)$ comes from the expansion of $e^{im\phi_R} (\pi/2)^{-3/4} e^{-(r/R)^2/2b^2}$. In Eq. (6), $P^0_{2l}(\cos \theta_R)$ is an even function of $\cos \theta_R$, and, when $m = \pm 1$ and $L$=even, $P^m_L(\cos \theta_R)$ is an odd function of $\cos \theta_R$. Therefore, when $m = \pm 1$, $C_L$ with even $L$ will vanish because of the odd integrand function in the integration and $\Phi_n(r)$ will become a superposition of $C_L(r) Y^m_L(\hat{r})$ with odd $L$. Thus the negative parity of $\Phi_n$ is demonstrated. Another proof of the negative parity of $\Phi_n$ with $m = \pm 1$ is as follows: When we change the integration variables $(R_x, R_y, R_z)$ to $(-R_x, -R_y, -R_z)$, in the integral representation of $\Phi_n(-r)$ by Eq. (4), we obtain $\Phi_n(-r) = -\Phi_n(r)$. It is because the azimuthal angle of $(-R_x, -R_y, -R_z)$ is $(\pi + \phi_R)$ and $\exp(i(\pi + \phi_R)) = -\exp(i\phi_R)$. When $m = 0$, Eq. (4) is a standard THSR wave function and has a positive parity as already known before in Ref. [4]. So the total parity of $^9\text{Be}$ is now determined by $m$,

$$\pi = \pi^{(1)}_\alpha \times \pi^{(2)}_\alpha \times \pi_n = \begin{cases} + & (m = 0) \\ - & (m = \pm 1). \end{cases} \quad (7)$$

From Eq. (5) we can also see that the $z$ component of angular momentum $l_{z,n} = m$ is a good quantum number for the extra neutron. Because of the rotational symmetry of the two-$\alpha$-cluster subsystem about $z$-axis, we have $l_{z,\alpha} = 0$ for $\alpha$ clusters. Thus $l_z$ of the total system is

$$l_z = l^{(1)}_{z,\alpha} + l^{(2)}_{z,\alpha} + l_{z,n} = m. \quad (8)$$

In order to eliminate effects from spurious center-of-mass (c.o.m.) motion, the c.o.m. part of $|\Phi\rangle$ is projected onto a $(0s)$ state [11],

$$|\Psi\rangle = |(0s)\text{c.o.m.}\rangle \langle (0s)\text{c.o.m.}|\Phi\rangle. \quad (9)$$
Here \( (0s) \) represents the wave function of the c.o.m. coordinate \( X_G \), which is the ground state of harmonic oscillator. For the THSR wave function of \(^9\)Be, this projection can be accomplished simply by the transformation of coordinates \( r_i \) in \( |\Phi\rangle \) as

\[
r_i \to r_i - X_G.
\]

(10)

Then the spurious center-of-mass motion in the wave function can be separated and eliminated analytically.

We also apply the angular-momentum projection technique \( \hat{P}_{IK} |\Psi\rangle \) to restore the rotational symmetry \([14]\),

\[
|\Psi^{IM}\rangle = \hat{P}_{IK} |\Psi\rangle = \frac{2I + 1}{8\pi^2} \int d\Omega D_{IK}^{I*} (\Omega) \hat{R}(\Omega) |\Psi\rangle,
\]

(11)

where \( I \) is the total angular momentum or the orbital angular momentum without spin part.

The Hamiltonian of \(^9\)Be system can be written as

\[
H = \sum_{i=1}^{9} T_i - T_{c.m.} + \sum_{i<j}^{9} V_{ij}^N + \sum_{i<j}^{9} V_{ij}^C + \sum_{i<j}^{9} V_{ij}^{ls},
\]

(12)

where \( T_{c.m.} \) is the kinetic energy of the center-of-mass motion. Volkov No. 2 \([15]\) is used as the central force of nucleon-nucleon potential,

\[
V_{ij}^N = \{V_1 e^{-\alpha_1 r_{ij}^2} - V_2 e^{-\alpha_2 r_{ij}^2}\} \{W - M \hat{P}_\sigma \hat{P}_\tau + B \hat{P}_\sigma - H \hat{P}_\tau\},
\]

(13)

where \( M = 0.6, \ W = 0.4 \) and \( B = H = 0.125 \). Other parameters are \( V_1 = -60.650 \) MeV, \( V_2 = 61.140 \) MeV, \( \alpha_1 = 0.980 \) fm\(^{-2}\), and \( \alpha_2 = 0.309 \) fm\(^{-2}\).

In traditional THSR calculations of 4-n nuclei, the spin-orbit interaction cancels out and can be safely neglected. However, for the THSR calculation of \(^9\)Be, the spin-orbit interaction plays a key role because of the existence of extra neutron. The G3RS term \([16]\), which is a two-body type interaction, is taken as the spin-orbit interaction,

\[
V_{ij}^{ls} = V_0^{ls} \{e^{-\alpha_1 r_{ij}^2} - e^{-\alpha_2 r_{ij}^2}\} L \cdot S \hat{P}_{31},
\]

(14)

where \( \hat{P}_{31} \) projects the two-body system into triplet odd state. Parameters in \( V_{ij}^{ls} \) are taken from Ref. \([17]\) with \( V_0^{ls} = 2000 \) MeV, \( \alpha_1 = 5.00 \) fm\(^{-2}\), and \( \alpha_2 = 2.778 \) fm\(^{-2}\).
III. RESULTS AND DISCUSSIONS

Here we calculated the ground state properties of $^9$Be, which is a stable bound state and the only state that exists below the $\alpha + \alpha + n$ threshold. The excited states $5/2^-$ and $7/2^-$, which belong to the rotational band of the $3/2^-$ ground state, are also calculated. All of these states are constructed with parallel coupling of the orbital angular momentum $|I = J - 1/2, I_z = J - 1/2\rangle$ and the spin angular momentum $|s = 1/2, s_z = 1/2\rangle$. In order to obtain a correct orbital angular momentum, the parameter $I$ and parameter $M$ in Eq. (11) are taken as $I = M = 1$, $I = M = 2$, and $I = M = 3$ for state $3/2^-$, state $5/2^-$ and state $7/2^-$ respectively. In the phase factor $e^{im\phi_R}$ in Eq. (3) $m = 1$ is taken to guarantee the negative parity of the THSR wave function. Because $l_z = m$ is a good quantum number of the wave function as shown in Eq. (8), $K = 1$ is used in Eq. (11) for a proper description of the intrinsic wave function.

The binding energy of $^9$Be can be obtained by a variational calculation with respect to parameters $b, \beta_{\alpha,xy}, \beta_{\alpha,z}, \beta_{n,xy}$, and $\beta_{n,z}$ as,

$$E(b, \beta_{\alpha,xy}, \beta_{\alpha,z}, \beta_{n,xy}, \beta_{n,z}) = \frac{\langle \Psi^{JM} | \hat{H} | \Psi^{JM} \rangle}{\langle \Psi^{JM} | \Psi^{JM} \rangle}.$$  \hspace{1cm} (15)

The Monte Carlo method is used for the numerical integration of Euler angle $\Omega$ in the angular momentum projection and coordinates $\{R\}$ in creation operators.

In our calculations, we treat $b$ as a variational parameter and get as the optimum value $b = 1.35$ fm, which is the same as that obtained for $^8$Be. This shows that the size of each $\alpha$ clusters in $^9$Be is similar to the ones in $^8$Be. This also shows a relatively weak influence of the extra neutron on $\alpha$-clusters. The remaining optimum parameters in the THSR wave function are $\beta_{\alpha,xy} = 0.1$ fm, $\beta_{\alpha,z} = 4.2$ fm, $\beta_{n,xy} = 2.5$ fm, and $\beta_{n,z} = 2.8$ fm.

In Table I we show the calculated results of the $3/2^-$ rotational band. Binding energies calculated with both THSR wave function and the Brink+GCM technique are included. The Brink+GCM results are calculated with the same interactions as used in the THSR calculations. We use 72 Brink wave functions of $^9$Be with 6 different $\alpha-\alpha$ distances and 12 different positions of the extra neutron as the bases in the Brink+GCM calculation. For all three states in the rotational band, the calculated results from THSR wave function agree well with the Brink+GCM results. The excitation energies of the two excited states are also reasonable compared with the experimental values. The differences between theoretical
results and experimental results is due to the choice of interactions. We also show the squared overlap between the THSR wave function and the Brink+GCM wave function. The calculated squared overlap for the ground state state is about 96% and the results for the excited states are similar. As the Brink+GCM wave function is generally considered as the exact wave function of the system, these overlaps show that the THSR wave function provides a good description for these states.

TABLE I. Calculation results of the $3/2^-$ rotational band. G.S. denotes the ground state and E.S. denotes the excited state. $E^{\text{THSR}}$ and $E^{\text{GCM}}$ are calculated binding energy with the THSR wave function and the Brink+GCM wave function respectively. $E^{\text{Exp}}$ is the experimental result. Values in parentheses are corresponding excitation energies. $|\langle \Psi^{\text{THSR}} | \Psi^{\text{GCM}} \rangle|^2$ is the squared overlap between the THSR wave function and the Brink+GCM wave function. All units of energies are MeV.

| State       | $E^{\text{THSR}}$ (Excited) | $E^{\text{GCM}}$ (Excited) | $E^{\text{Exp}}$ (Excited) [18, 19] | $|\langle \Psi^{\text{THSR}} | \Psi^{\text{GCM}} \rangle|^2$ |
|-------------|-----------------------------|-----------------------------|-----------------------------------|---------------------------------|
| $7/2^-$ (E.S.) | -49.4 (6.6)                 | -50.4 (6.2)                 | -51.8 (6.4)                        | 0.93                            |
| $5/2^-$ (E.S.) | -54.2 (1.8)                 | -55.0 (1.8)                 | -55.8 (2.4)                        | 0.97                            |
| $3/2^-$ (G.S.) | -56.0                      | -56.8                      | -58.2                              | 0.96                            |

To compare the THSR wave function with the nuclear molecular orbit model, we use $b = 1.46$ fm which is the same value as in Ref. [12]. The calculated binding energy of the ground state $3/2^-$ with different models but same interaction and parameter $b$ are listed in Table II. With the THSR wave function, we get a value of -54.8 MeV for the binding energy of the ground state, which is the same as the result of the Molecular Orbit (MO) Model without GCM technique in Ref. [12]. This agreement shows that the motion of the valence neutron in $^9$Be is well treated with the THSR wave function. Comparison with the result of MO+GCM method in Ref. [12] shows that our result is about 1.3 MeV higher. This is acceptable because the results of MO+GCM method should be compared with results of THSR+GCM model rather than a single THSR wave function.

The RMS radius is also calculated for the ground state of $^9$Be with,

$$r_{\text{RMS}} = \sqrt{\frac{\langle \Psi^{JM} | \frac{1}{9} \sum_{i=1}^{9} (r_i - X_G)^2 | \Psi^{JM} \rangle}{\langle \Psi^{JM} | \Psi^{JM} \rangle}}.$$  \hspace{1cm} (16)
TABLE II. Comparison of results form the THSR wave function and the nuclear Molecular Orbit model. $E$ is the binding energy in MeV. “THSR” denotes the result calculated with THSR wave function. “MO” denotes the result of Molecular Orbit model and “MO+GCM” is the result of Molecular Orbit model plus GCM technique. Parameter $b = 1.46$ fm as used in Ref. [12]. Other parameters are variationally optimized.

| Model               | $b$ (fm) | $E(3/2^-)$ (MeV) |
|---------------------|----------|------------------|
| THSR               | 1.46     | -54.8            |
| MO [12]            | 1.46     | -54.8            |
| MO + GCM [12]      | 1.46     | -56.1            |

With all parameters variationally optimized, the THSR wave function gives a RMS radius of 2.55 fm for the ground state, which agrees well with the experimental value 2.45 fm [20].

FIG. 1. Contour maps of binding energy surface with the $\beta$ parameters in the THSR wave function. Left part (a) is the contour map of parameters $\beta_{\alpha,xy}$ and $\beta_{\alpha,z}$ and the right part (b) is the contour map of parameters $\beta_{n,xy}$ and $\beta_{n,z}$. The optimum value is marked on each map labeled with coordinates. Parameter $b = 1.35$ fm.

Fig. 1 (a) and (b) are contour maps of the ground state binding energy surface. The optimum parameters for the ground state are labeled in the map. The very large difference
between $\beta_{\alpha,xy}$ and $\beta_{\alpha,z}$ indicates a long prolate shape of $\alpha$ cluster distribution which is surrounded by the less deformed distribution of the extra neutron. This configuration indicates a structure of nuclear molecular orbit for the ground state of $^9$Be. In order to illustrate this structure in detail, the density distribution of $^9$Be is calculated as the expectation value of the density operator,

$$\rho(\mathbf{r}') = \langle \Psi | \frac{1}{g} \sum_{i=1}^{9} \delta(\mathbf{r}_i - \mathbf{X}_G - \mathbf{r}') | \Psi \rangle.$$  \hspace{1cm} (17)

where $\Psi$ is the normalized intrinsic THSR wave function of $^9$Be. Fig. 2 shows the density distribution in the $y = 0$ cross section. Similar to the case of $^8$Be, a clear structure of two $\alpha$ clusters is displayed. Due to the Pauli blocking effect, the two $\alpha$ clusters can not get too close to each other and a neck structure appears. The distance between two $\alpha$-clusters in $^9$Be is about 3.4 fm while the inter-cluster distance for the ground state of $^8$Be is about 4.6 fm. This shows a more compact structure and a stronger binding effect of two $\alpha$-clusters in $^9$Be because of the existence of the extra nucleon.

FIG. 2. Density distribution of the intrinsic ground state of $^9$Be. The gray scale of each point in the figure stands for the nucleon density on $x - z$ plane of the $y = 0$ cross section. The unit of the density is fm$^{-3}$.

To get a clear view of the binding effect of the extra neutron and the structure of the ground state, we also calculate the density distribution $\rho(\mathbf{r}'_n)$ of the extra nucleon. The intrinsic wave function $\Psi$ can be written in the form of,

$$\Psi = C \hat{A}[\Phi^{\text{THSR}}(2\alpha)\phi_n(\mathbf{r})],$$  \hspace{1cm} (18)
where $\hat{A}$ is the antisymmetrizer, and $C$ is the normalization constant. Then we could define the density distribution $\rho(r'_n)$ of the extra neutron as

$$\rho(r'_n) = \sqrt{9} \langle \Phi_{THSR}^{2\alpha}(r)|\delta(r - X_G - r'_n)|\Psi \rangle,$$

(19)

where $\sqrt{9}$ comes from the normalization constant [21]. As shown in Fig. 3, distribution which consists of two parts is displayed on the $y = 0$ cross section and a ring style distribution can be seen on the $z = 0$ cross section. The distribution of the extra nucleon spreads more than 6 fm in $z$ direction, which is about double size of each $\alpha$ cluster. It also has overlaps with distributions of both two $\alpha$ clusters, which is known as the $\pi$ orbit in nuclear Molecular Orbit model. This is interesting because, at variance with previous works in Ref. [13] and Ref. [12], no molecular orbit is presumed in our wave function. In the THSR wave function, the extra nucleon is only assumed to make a nonlocalized motion inside the nucleus. The $\pi$-orbit emerges naturally from the antisymmetrization which cancels out nonphysical distributions. This reproduction of nuclear molecular orbit structure provides another support for our extension of the nonlocalized clustering concept to $^9$Be.

FIG. 3. Density distribution $\rho(r'_n)$ of the extra neutron of the intrinsic ground state of $^9$Be. The gray scale of each point in left part (a) of the figure stands for the nucleon density on $x - z$ plane of the $y = 0$ cross section. The gray scale of each point in right part (b) of the figure stands for the nucleon density on $x - y$ plane of the $z = 0$ cross section. The unit of the density is fm$^{-3}$. 

11
IV. CONCLUSION

We extended the nonlocalized clustering concept inherent to the THSR wave function to the N≠Z nucleus $^9\text{Be}$, in which the $\alpha$-clusters and extra neutron make nonlocalized motion inside the nucleus. We introduce a modified version of THSR wave function that includes a creation operator of the extra neutron. With the introduced phase factor $e^{im\phi_R}$, our wave function has intrinsic negative parity for $m = \pm 1$. Binding energies are calculated for the $3/2^-$ rotational band of $^9\text{Be}$ by the variational method. The calculated binding energy from THSR wave function fits well with the Brink+GCM results. The excitation energies of two excited states are also reasonable compared with the experimental results. The squared overlap between the THSR wave function and the Brink+GCM wave function are found to be close to 96%. This means that the THSR wave function provides a good description of the $3/2^-$ rotational band of $^9\text{Be}$. With the same parameter $b$, our result for the binding energy of the ground state is consistent with the Molecular Orbital (MO) model but higher than in the MO+GCM model. The calculated RMS radius of the ground state also agrees well with the experimental value. By calculating density distributions of the ground state of $^9\text{Be}$, the $\pi$-orbit structure is naturally reproduced by the THSR wave function without prior assumption. The calculation of $^9\text{Be}$ provides support for the extension of the nonlocalized clustering concept to N≠Z nuclei. It also shows to possess the flexibility to describe other structures such as nuclear molecular orbital structure with the THSR wave function. Though with our technique we essentially have not found anything for the 9Be structure which was not known before, we think that it is interesting to see that the THSR wave function also works with adding valence neutrons to the $\alpha$ particles. This is because it is shown that the geometrical cluster structures arise only from kinematical reasons which is a new aspect of cluster physics. Otherwise clusters and extra neutrons are free in their motion.

ACKNOWLEDGMENTS

This work is supported by the National Natural Science Foundation of China (grant nos 11035001, 11375086, 11105079, 10735010, 10975072, 11175085 and 11235001), by the 973 National Major State Basic Research and Development of China (grant nos 2013CB834400 and 2010CB327803), by the Research Fund of Doctoral Point (RFDP),
grant no. 20100091110028, and by the Science and Technology Development Fund of Macao under grant no. 068/2011/A.

[1] A. Tohsaki, H. Horiuchi, P. Schuck, and G. Röpke, Phys. Rev. Lett. 87, 192501 (2001).
[2] T. Yamada and P. Schuck, Eur. Phys. J. A 26, 185 (2005).
[3] Y. Funaki, H. Horiuchi, A. Tohsaki, P. Schuck, and G. Röpke, Prog. Theor. Phys. 108, 297 (2002).
[4] B. Zhou, Y. Funaki, H. Horiuchi, Z. Ren, G. Röpke, P. Schuck, A. Tohsaki, C. Xu, and T. Yamada, Phys. Rev. Lett. 110, 262501 (2013).
[5] C. Xu and Z. Ren, Phys. Rev. C 73, 041301 (2006).
[6] Y. Ren and Z. Ren, Phys. Rev. C 85, 044608 (2012).
[7] B. Zhou, Z. Ren, C. Xu, Y. Funaki, T. Yamada, A. Tohsaki, H. Horiuchi, P. Schuck, and G. Röpke, Phys. Rev. C 86, 014301 (2012).
[8] B. Zhou, Y. Funaki, H. Horiuchi, Z. Ren, G. Röpke, P. Schuck, A. Tohsaki, C. Xu, and T. Yamada, Phys. Rev. C 89, 034319 (2014).
[9] A. Tohsaki, Int. J. Mod. Phys. E 17, 2106 (2008).
[10] Y. Kanada-En’yo, H. Horiuchi, and A. Ono, Phys. Rev. C 52, 628 (1995).
[11] S. Okabe, Y. Abe, and H. Tanaka, Prog. Theor. Phys. 57, 866 (1977).
[12] N. Itagaki and S. Okabe, Phys. Rev. C 61, 044306 (2000).
[13] W. von Oertzen, Z. Phys. A 354, 37 (1996).
[14] P. Ring and P. Schuck, The Nuclear Many-Body Problem (Springer-Verlag, New York, 1980), p. 474.
[15] A.B. Volkov, Nucl. Phys. 74, 33 (1965).
[16] N. Yamaguchi, T. Kasahara, S. Nagata, and Y. Akaishi, Prog. Theor. Phys. 62, 1018 (1979).
[17] S. Okabe and Y. Abe, Prog. Theor. Phys. 61, 1049 (1979).
[18] D. R. Tilley, J. H. Kelley, J. L. Godwin, D. J. Millener, J. E. Purcell, C. G. Sheu, and H. R. Weller, Nucl. Phys. A 745, 155 (2004).
[19] G. Audi, F. G. Kondev, M. Wang, B. Pfeiffer, X. Sun, J. Blachot, and M. MacCormick, Chin. Phys. C 36, 1157 (2012).
[20] I. Tanihata, H. Hamagaki, O. Hashimoto, Y. Shida, N. Yoshikawa, K. Sugimoto, O. Yamakawa, T. Kobayashi, and N. Takahashi, Phys. Rev. Lett. 55, 2676 (1985).

[21] H. Horiuchi, Prog. Theor. Phys. Supple. 62, 90 (1977).