Role of Charm Factory in Extracting CKM-Phase Information via $B \rightarrow D \bar{K}$

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Abstract

In this paper we study the impact of data that can be obtained from a Charm Factory on the determination of the CKM parameter $\gamma$ from decays of the form $B \rightarrow D^0 K$ where the $D^0$ decays to specific inclusive and exclusive final states. In particular, for each exclusive final state $f$, the charm factory can determine the strong phase difference between $D^0 \rightarrow f$ and $\bar{D}^0 \rightarrow f$ by exploiting correlations in $\psi(3770) \rightarrow D^0 \bar{D}^0$. This provides crucial input to the determination of $\gamma$ via the interference of $B^\pm \rightarrow K^\pm D^0 \rightarrow f$ with $B^\pm \rightarrow K^\pm \bar{D}^0 \rightarrow f$. We discuss how the method may be generalized to inclusive final states and illustrate with a toy model how such methods may offer one of the best means to determine $\gamma$ with $O(10^{8-9})$ B-mesons.
1 Introduction

The B factories at KEK and SLAC have made remarkable progress in many areas of B-physics, in particular in the extraction of Cabibbo Kobayashi Maskawa (CKM) [1] parameters crucial to testing the standard model. Indeed, the determination of $\sin 2\beta$ via $B \to J/\psi K_S$ in such a way that there is no dependence on theoretical assumptions promises to usher in a new era of precision tests of the CKM paradigm[2, 3].

The determination of the other two unitarity angles, $\alpha$ and $\gamma$, without theoretical errors still presents a considerable experimental challenge. Even though CP violation itself may prove to be easy to observe in channels sensitive to $\alpha$ and $\gamma$, the effect tends to be modified by CP conserving effects such as strong phases. To obtain a truly model independent determination of these angles, these effects must be algebraically eliminated between several sets of observables or measured in an independent experiment.

Final states containing $D^0, \bar{D}^0$ in decays of charged and neutral B's provide methods for clean extraction of the angles of the unitarity triangle (UT)[4]. Direct CP violation in $B^\pm$ decays leads to a clean determination of $\gamma$. Time dependent CP asymmetry measurements in decays such as $B^0$ or $\bar{B}^0 \to K^0 D^0(\bar{D}^0)$ lead to a determination of $\delta \equiv \beta - \alpha + \pi = 2\beta + \gamma$[5, 6, 7, 8] and also in fact can give $\beta$[6, 8]. In these methods, for charged and neutral B's, the crucial role is played by those final states of $D^0$ and $\bar{D}^0$ that are common to both. Indeed, in addition to the exclusive modes of $D^0, \bar{D}^0$ that are common to them, as briefly discussed in[8] even multibody [9, 10] and inclusive decays can be used. The effectiveness of these $B \to K D$ methods for extracting angles of the UT can be vastly improved if charm factory can provide some of the information for the relevant D decays.

The primary focus of this paper will be to consider the case of $\gamma$ from modes such as $B^\pm \to K^\pm(K^{\mp\pm})D^0, \bar{D}^0$. Recall that there are a number of different final states which are common to $D^0$ and $\bar{D}^0$ decays that can be used here. Such final states may be (1) CP eigenstates (CPES) as first discussed in[11] (GLW) or CP non-eigenstates (CPNES) as discussed in[12] (ADS). The CPNES may be (2) doubly Cabibbo suppressed (DCS) decays of $D^0$ (e.g. $K^+\pi^-$) or (3) singly Cabibbo suppressed (SCS) states (e.g. $K^{*+}K^-$) as recently considered in [13]. Amongst these, CP asymmetries are expected to be large for case (2) and small for (1) and (3); on the other hand, the relevant Br's tend to be larger for the latter two cases.

In the ADS method, the results from the interference through each $D^0$ channel depend on: (a) The CKM angle $\gamma$; (b) The strong phase of the $B$ decay; (c) The strong phase of the $D^0$ decay and (d) The decay rate of $B^- \to K^- \bar{D}^0$. The phase $\gamma$ of course is the quantity that is of interest, and therefore, enough measurements, via sufficient number of decay modes,
must be made so that the dependence on parameters (b) through (d) can be eliminated.

It has previously been suggested [12, 14] that charm factory data could provide a useful additional input to the strong phase determination which is part of the ADS method. More generally, there are a number of ways in which a charm factory [15] can help in CKM-extractions.

1. Measurements of the Br’s of some of the D decay modes that enter the analysis; specifically determinations of the DCS D decays such as $K^+ (K^{*+}) [\pi^-, \rho^-, a_1^-]$.

2. Improving the constraints on the $D_0 - \bar{D}_0$ mixing parameters, $x_D, y_D$ can be very helpful. [9, 16, 17].

3. More relevant to this paper is the use of a charm factory to determine two parameters needed for being able to use (exclusive and) inclusive decays of $D_0, \bar{D}_0$, as briefly mentioned in[8].

In particular this last application of charm factory data offers the potential advantage of increasing the statistics available in comparison to the original ADS method which only allows an analysis based on exclusive states. Furthermore, it allows a way to integrate 3 or 4 body modes into the analysis without making any assumption concerning the decay distribution in phase space.

In section 2 we discuss a general formalism for strong phases in inclusive states. For exclusive states $f$ there is a single strong phase difference $\zeta(f)$ between $D^0 \to f$ and $\bar{D}^0 \to f$. We show that for an inclusive state $F$ we can introduce a net “coherence coefficient” $R_F$ together with a mean strong phase difference $\zeta(F)$ which fully describes the relation between $D^0 \to F$ and $\bar{D}^0 \to F$. In section 3 we discuss how these parameters can be extracted from correlations at a $\psi(3770)$ factory. In section 4, for illustration, we define a toy model for the inclusive states $D^0 \to K^\pm + X$ and discuss the extraction of $\zeta$ and $R$. In section 5 we discuss using this information to extract $\gamma$ from $B^\pm \to K^{\pm} D^0$ with the subsequent decay of $D^0$ to inclusive states along with the $\psi(3770)$ factory determination of $\zeta$ and $R$ and we give some numerical results obtained with the toy model. A brief summary appears in section VI.

2 Inclusive States

In the following discussion we will define an exclusive decay of the $D^0$ to be any decay which is governed by a single quantum mechanical amplitude. Thus, a decay such as $D^0 \to K^-\pi^+$ would be an exclusive state while for
a three body decay such as \( D^0 \rightarrow K^-\pi^+\pi^0 \), each point on the Dalitz plot would be considered a distinct exclusive state.

Conversely, an inclusive state is any state which is a set of exclusive states. For instance \( D^0 \rightarrow K^-\pi^\pm\pi^0 \) integrated over all or part of the Dalitz plot would be an inclusive state as sets of states with different particle content such as \( D^0 \rightarrow K^- + n\pi \). Inclusive states defined in this way may either be composed of a collection of discrete states (e.g. \( \{K^- + \pi^+, K^- + \rho^+\}\)) of states which are a continuum (e.g. \( K^-\pi^+\pi^0 \) integrated over the Dalitz plot) or of states which are a combination of both (e.g. \( K^- + n\pi \)). In the discussion below we will treat an inclusive state as being composed of a discrete set of exclusive states although the generalization to continuous sets of states is straightforward. Thus if \( F \) is an inclusive final state we will write:

\[
F = \{f_i\}
\]  

where \( f_i \) are the exclusive states which make up \( F \).

Primarily we are interested in final states which are common to \( D^0 \) and \( \bar{D}^0 \) decay. Two categories of such final states of particular interest are DCS states such as \( K^-\pi^- \) and CP eigenstates such as \( K_S\pi^0 \). For each \( f_i \in F \), let us denote:

\[
A(f_i) = \mathcal{M}(D^0 \rightarrow f_i) \\
\bar{A}(f_i) = \mathcal{M}(\bar{D}^0 \rightarrow f_i) \\
\zeta(f_i) = \arg(A^*(f_i)\bar{A}(f_i))
\]

so \( \zeta(f_i) \) is the strong phase difference between \( D^0 \) and \( \bar{D}^0 \) decay to \( f_i \).

Because both the channels \( B^- \rightarrow K^-D^0 \) and \( B^- \rightarrow K^-\bar{D}^0 \) can contribute to the overall process \( B^- \rightarrow K^-F \) for the inclusive state \( F \), we can regard the object that decays into \( F \) do be a quantum mechanical mixture of \( D^0 \) and \( \bar{D}^0 \).

Denoting this state by \( |I\rangle \), then

\[
|I\rangle = a|D^0\rangle + be^{i\lambda}|\bar{D}^0\rangle
\]

where \( a \) and \( b \) are real.

We can thus expand the decay rate for the mixed state \( |I\rangle \) to \( F \) as:

\[
\Gamma(I \rightarrow F) = \sum_i \left\{ a^2|A(f_i)|^2 + b^2|\bar{A}(f_i)|^2 + 2ab|A(f_i)||\bar{A}(f_i)|\cos(\zeta(f_i) + \lambda) \right\}
\]
\[ a^2 A(F)^2 + b^2 \bar{A}(F)^2 + 2R_F ab A(F) \bar{A}(F) \cos(\zeta(F) + \lambda) \] \hspace{1cm} (4)

where

\[ A(F)^2 = \sum |A(f_i)|^2 \]
\[ \bar{A}(F)^2 = \sum |\bar{A}(f_i)|^2 \]
\[ R_F e^{i\zeta(F)} = \frac{\sum |A(f_i)||\bar{A}(f_i)|e^{i\zeta(f_i)}}{A(F)\bar{A}(F)} \] \hspace{1cm} (5)

The key point to note is that \( R_F \) and \( \zeta(F) \) are independent of \( a, b \) and \( \lambda \). The decay rate of \( I \) thus depends only on four parameters of \( F \), namely \( A(F) \), \( \bar{A}(F) \), \( R_F \) and \( \zeta(F) \), regardless of how many states make up \( F \). We can think of \( A(F) \) and \( \bar{A}(F) \) as the average amplitudes of \( D \) and \( \bar{D} \) decay to \( F \) while \( \zeta(F) \) is the average strong phase difference for \( F \) and \( R_F \) is a measure of the coherence of \( F \). Note that \( 0 \leq R_F \leq 1 \).

In the case where \( F = \{f_1\} \) consists of a single quantum state, then \( R_F = 1 \). In this case the decay rate of \( I \) is only determined by three parameters: the amplitudes of \( D^0 \) and \( \bar{D}^0 \) decay and the strong phase difference, \( \zeta(F) = \zeta(f_1) \). If \( f_1 \) is a CP eigenstate then, assuming that \( D^0 \) decay is CP conserving [18], \( A(F) = \bar{A}(F) \), \( R_F = 1 \) and \( \zeta(F) = 0 \) or \( \pi \) depending on whether \( f_1 \) is CP=+1 or CP=−1.

More generally if \( F \) is a set of states such that \( CP : F \rightarrow F \) then it follows that \( A(F) = \bar{A}(F) \) and \( \zeta(F) = 0 \) or \( \pi \) (depending on whether \( F \) is predominantly CP=+1 or CP=−1) but \( R_F \) depends on the makeup of \( F \) and indicates the purity of \( F \). Thus, if \( F \) is made up of CP eigenstates with the same CP=±1 eigenvalue, \( R_F = 1 \) and \( F \) behaves as a single CP=±1 eigenstate. In [9, 10] the extraction of \( \gamma \) using the detailed analysis of 3 and 4 body states of this form is considered via an analysis of the amplitude structure in phase space.

Some examples of inclusive states to which we can apply this approach are:

1. \( F = \{K^-\pi^+\} \): This is a single CP non eigenstate (CPNES) so therefore \( R_F = 1 \) while \( A(F) \), \( \bar{A}(F) \) and \( \zeta(F) \) need to be determined experimentally. Current measurements [19] of the branching ratios give \( Br(D^0 \rightarrow K^-\pi^+) = 3.80\% \) and \( Br(D^0 \rightarrow K^+\pi^-) \approx 1.5 \times 10^{-4} \) thus \( \bar{A}(F)/A(F) \approx 0.05 \) in this case.

2. \( F = \{K^-\pi^+\pi^0\} \) This is an inclusive CPNES if one integrates over the Dalitz plot; therefore \( A(F) \), \( \bar{A}(F) \), \( \zeta(F) \) and \( R_F \) need to be experimentally determined. It is useful to break down the Dalitz plot into
a number of sub-regions $F = F_1 \cup F_2 \cup \ldots \cup F_n$ each of which will be characterized by $A(F_i)$, $\bar{A}(F_i)$, $\zeta(F_i)$ and $R_{F_i}$.

3. $F = \{K^- + X\}$: This case is an even more inclusive CPNES than the previous one. Again it may be useful to decompose $F$ into a number of subsets determined either by the composition of $X$ (e.g. number of pions) or by the energy of the $K^-$.  

4. $F = \{K_s\pi^0\}$: This is a single CP eigenstate (CPES) with CP=$-1$ therefore $A(F) = \bar{A}(F)$, $R_F = 1$ and $\zeta(F) = \pi$.

5. $F = K_s + X$: This is a CP invariant inclusive state (CPIIS) so $A(F) = \bar{A}(F)$ and $\zeta_F = 0$ or $\pi$. Which value of $\zeta_F$ applies and the value of $R_F$ need to be determined experimentally. Again, it may be useful to decompose this set of states according to the composition of $X$ or the energy of the $K_S$. Note that changing the $K_S$ to a $K_L$ changes $\zeta(F) \to \pi - \zeta(F)$ but keeps the other parameters unchanged.

6. $F = K_S\pi^+\pi^-$: Again this is a CPIIS which can be decomposed on the basis of the energy of the $K_S$.

3 Extracting Phases and Coherence from $\psi(3770)$

The ability to use inclusive and exclusive decays of $D^0$ in order to obtain CP violation phases will be enhanced if a separate determination of $A(F)$, $\bar{A}(F)$, $R_F$ and $\zeta(F)$ can be made.

We will assume that $A(F)$ and $\bar{A}(F)$ can be determined from the $D^0$ branching ratios. A determination of $\zeta(F)$ and $R_F$ may be made at a $\psi(3770)$ charm factory.

This follows from the fact that $\psi(3770)$ is a spin-1 state and therefore the decay $\psi \to D^0\bar{D}^0$ is an antisymmetric wave function:

$$\langle |D^0\rangle |D^0\rangle - |D^0\rangle |D^0\rangle \rangle / \sqrt{2}$$

This entangled state gives us access to strong phase information for final states in a number of different ways. Here we will discuss the case where we assume $D^0\bar{D}^0$ oscillation is small.

We can take advantage of this entanglement by observing various correlations between the decay of the two mesons which arise from $\psi(3770)$ decay. For a given inclusive state $F$, it is useful to distinguish between 4 different kinds of correlations

1. The correlation of $F$ with another inclusive state $G$ by measuring the branching ratio $\psi(3770) \to [F][G]$
2. The correlation of $F$ with its charge conjugate by measuring the branching ratio $\psi(3770) \rightarrow [F][\bar{F}]$ as previously discussed in the case of exclusive states in [14, 17].

3. The correlation of $F$ with a CP eigenstate by measuring the branching ratio $\psi(3770) \rightarrow [F][CP \text{ eigenstate}]$

4. The correlation of $F$ with itself by measuring the branching ratio $\psi(3770) \rightarrow [F][F]$

Let us consider first the most general case where the $\psi(3770)$ decays overall to the final state $FG$ where $F$ and $G$ are inclusive final states. The decay rate to this final state is thus

$$\Gamma(FG) = \Gamma_0 \sum_{ij} |A(f_i)\bar{A}(g_j) - \bar{A}(f_i)A(g_j)|^2$$

$$= \Gamma_0 \sum_{ij} \left[ |A(f_i)|^2 |\bar{A}(g_j)|^2 + |\bar{A}(f_i)|^2 |A(g_j)|^2 - 2|A(f_i)A(g_j)\bar{A}(f_i)\bar{A}(g_j)| \cdot \cos(\zeta(f_i) - \zeta(g_j)) \right]$$

(7)

where $\Gamma_0 = \Gamma(\psi(3770) \rightarrow D^0\bar{D}^0)$. Summing this over $i$ and $j$ we thus obtain an expression in terms of the exclusive quantities:

$$\Gamma(FG) = \Gamma_0 \left[ A_F^2 \bar{A}_G^2 + \bar{A}_F^2 A_G^2 - 2R_F R_G A_F \bar{A}_F A_G \bar{A}_G \cos(\zeta(F) - \zeta(G)) \right]$$

(8)

A special case of the above is where $G = \bar{F}$ in which case this expression reduces to:

$$\Gamma(FF) = \Gamma_0 \left[ A_F^4 + \bar{A}_F^4 - 2R_F^2 A_F^2 \bar{A}_F^2 \cos(2\zeta(F)) \right]$$

(9)

If, on the other hand, $G$ is a CP eigenstate, or indeed a set of CP eigenstates with the common eigenvalue $\lambda_{CP} = \pm 1$, then $\zeta_G = 0$ or $\pi$ respectively and $R_G = 1$ while $A(G) = \bar{A}(G)$. In this case then,

$$\Gamma(FG) = \Gamma_0 \lambda_{CP} \left[ A_F^2 + \bar{A}_F^2 - 2\lambda_{CP} R_F A_F \bar{A}_F \cos(\zeta(F)) \right]$$

(10)
Finally, in the special case where $F = G$ then

$$\Gamma(FF) = \Gamma_0 A_F^2 \bar{A}_F(1 - R_F^2)$$  \hspace{1cm} (11)

Note that this expression gives 0 if $F$ is an exclusive state as expected by Bose symmetry [16]. In each of these cases, one can specialize to the case where $F$ (or $G$) is an exclusive case where we then have $R_F = 1$ (or $R_G = 1$).

From the above relations, it is clear that if we have a number of inclusive or exclusive states we can solve for the various values of $\zeta(F)$ and $R_F$. To make this clearer, for a given set of inclusive final states $\mathcal{F} = \{F_1, \ldots, F_n\}$ let us define $n_o$ to be the number of observables, $n_p$ the number of parameters and $\delta n = n_o - n_p$. Thus, $\delta n \geq 0$ is a necessary condition for the determination of the parameters.

If $\mathcal{F}$ contains $k$ inclusive states and $\ell$ exclusive states, $n = k + \ell$, then the total number of free parameters is $n_p = 2k + \ell$, i.e. $\zeta$ for each inclusive and exclusive state and $R$ for each inclusive state.

For each pair of distinct members (exclusive or inclusive) of $\mathcal{F}$ there are two distinct observables, $Br(FG) = Br(\bar{F}\bar{G})$ and $Br(F\bar{G}) = Br(\bar{F}G)$. This gives a total contribution to $n_o$ of $n(n-1)$.

For each inclusive member of $\mathcal{F}$, there are the two observables $Br(FF) = Br(\bar{F}\bar{F})$ and $Br(F\bar{F})$ giving a contribution to $n_o$ of $2k$ while for each exclusive state $Br(FF) \equiv 0$, hence the contribution is $\ell$ from correlations of the form $F\bar{F}$.

Finally, for each member of $\mathcal{F}$ one can observe the correlation with a set of CP eigenstates giving an additional contribution of $n$ to $n_o$. Thus,

$$n_p = 2k + \ell = k + n$$
$$n_o = k + n + n^2$$
$$\delta n = n^2$$  \hspace{1cm} (12)

where $n_o$ is the sum of $n(n-1)$ correlations of the form $FG$ and $\bar{F}\bar{G}$; $n$ of the form $F\bar{F}$; $n$ of the form $F + CPES$ and $k$ of the form $FF$ (for inclusive states only).

Clearly, using one state “in isolation” Eqn. (11) and Eqn. (9) give us just enough information to determine $R$ and $\zeta$ for an inclusive state while Eqn. (9) will give us information to determine $\zeta$ for an exclusive state. Observing the cross correlations with other states and CP eigenstates thus gives us a degree of over determination $\delta n = n^2$.

Of course if the branching ratio to some of the final states is small, the statistical error on some of these correlations may be large. To consider how well this program may be carried out, we will construct a toy model of certain $D$ decays.
4 Toy Models

The effectiveness of various methods in determining $\delta$ depends to some extent on the properties of various $D$ decays. Since these decays have not been fully characterized, particularly in the DCS modes, we will use a “toy model” in which we will endeavor to capture the known properties of these decays. This should allow us to obtain a general idea of the values of $R$ and $\zeta$ which will be obtained for various classes of inclusive states. In particular we will construct a toy model for $K^\pm + X$ final states. We will attempt to model at least part of the rate in each of these channels as the sum of exclusive states. Each exclusive final state may either be produced through a continuum process, in which case we will assume that the amplitude is constant over phase space, or it may be produced through resonance channels which we will describe by a Breit-Wigner distribution.

4.1 Model for $D^0 \rightarrow K^- + X$ and $K^+ + X$

In this section we consider a model for $F$, the set of $K^- + X$ where $X$ contains at most one $\pi^0$. We will model such decays by considering decays of the form $D^0 \rightarrow K^- \pi^+; D^0 \rightarrow K^- \pi^+ \pi^0; D^0 \rightarrow K^- \pi^- \pi^+ \pi^+ \pi^0$. $D^0 \rightarrow K^- \pi^- \pi^+ \pi^+ \pi^0$.

In Table 1 we give the decomposition of these modes into resonance channels which we consider and their branching ratios from [19]. Note that the sum of all these branching ratios is 29% out of the total 58% for all $K^- + X$ states. In [20] the experimental data for $D^0 \rightarrow K^- \pi^+ \pi^0$ was fit to a model with $K^{0*}, \rho^+$ and $K^{++}$ channels and a 3-body continuum. We will use this model to describe these 3-body decays.

For each of the quasi-two body modes, we will assume that the resonances (ie. $\rho, a_1, K^*$) is modeled by a Breit-Wigner amplitude while the continuum 3 and 4-body states we will assume have a constant amplitude over phase space. We will assume that the strong phase difference between the different channels that lead to $K + 3\pi$ and $K + 4\pi$ final states are 0 although we find that the results considered below do not depend greatly if arbitrary phase differences are used.

For the corresponding DCS decays $\bar{D}^0 \rightarrow K^- + X$ we apply SU(3) to the Cabibbo-allowed decays, in particular we exchange $d \leftrightarrow s$ in the final state. We rescale the amplitudes so that the total DCS rates match the results in [19].

Using this model, we find that for $F$ in total, $R_F = 0.51$ and $\zeta_F = -11^\circ$. In Fig. 1, we show $\zeta$ as a function of the energy fraction $E_K/E_{max}$ of the $K^-$ where $E_{max} = (m_D^2 + m_K^2 - m_\pi^2)/(2m_D)$. Likewise in Fig. 2 we show $R$ as a function of the energy fraction.

From Fig. 2 we see that intermediate values of $E_K$ have smaller values of
$R$ than low energy and high energy $K$’s. We therefore consider dividing $F$ into three subsets $F = F_1 \cup F_2 \cup F_3$ where for $F_1$, $E_K/E_{\text{max}} \leq 0.65$ for $F_2$ $0.65 \leq E_K/E_{\text{max}} \leq 0.9$ and for $F_3$ $0.9 \leq E_K/E_{\text{max}}$.

Using the model, we find that for $F_1$, $\zeta_{F_1} = -34^\circ$ and $R = 0.74$ while $Br(F_1)/Br(F) = 20.5\%$; for $F_2$, $\zeta_{F_2} = -86^\circ$ and $R = 0.29$ while $Br(F_1)/Br(F) = 42.9\%$ and for $F_3$, $\zeta_{F_3} = 30^\circ$ and $R = 0.91$ while $Br(F_1)/Br(F) = 36.4\%$.

Likewise we can consider subsets of $F$ determined by particle content. For example if we define $F_{3\text{bdy}}$ to be final states of the form $K^-\pi^0\pi^-$ and $F_{4\text{bdy}}$ to be final states of the form $K^-\pi^-\pi^+\pi^+$ then $\zeta_{F_{3\text{bdy}}} = -2.1^\circ$; $R_{3\text{bdy}} = 0.60$ while $\zeta_{4\text{bdy}} = -81^\circ$; $R_{4\text{bdy}} = 0.13$.

The degree of CP violation in $B^- \to K^- [D^0 \to F]$ is proportional to $R_F$ so a larger value of $R$, in general, indicates greater utility in terms of extracting $\gamma$. The strong phase $\zeta$ will be combined with the strong phase difference between $B^- \to K^- D^0$ and $B^- \to K^- \bar{D}^0$.

\subsection{Phase and Coherence Determination for Toy Model}

Let us consider now the determination of $\zeta$ and $R$ at a $\psi$ factory for toy model for $K + X$. In the following, we will use the above decomposition of the events into $F_1$, $F_2$ and $F_3$ according to the $K^\pm$ energy.

The CP eigenstates which we will consider will consist of $(CP = -1)$ 2 body final states which do not contain a $K_L$. The branching ratios to such final states is $\sim 5\%$. Clearly some improvement could be obtained if more general final states were considered.

To determine the 6 parameters $R_{1-3}$ and $\zeta_{1-3}$ we thus have the modes displayed in Table (2) which, according to Eqn. (12) gives us $n_o = 15$ independent observables for $n_p = 6$ parameters giving $\delta n = n^2$.

In order to estimate how much data is required to determine the parameters $\zeta_i$ and $R_i$, we suppose that there are $\bar{N}_{DD} = 10^7$ events [15] of the form $\psi(3770) \to D^0\bar{D}^0$. In Fig. (3) the solid curve indicates the result for $\bar{F}$ taken as a whole so that the data which is used is the correlation of $\bar{F}$ with $\bar{F}$, $\bar{F}$ with $\bar{F}$ and $\bar{F}$ with CPES-. The other three curves indicate the results for $F_1$ (dashed), $F_2$ (dotted) and $F_3$ (dash-dotted).

The resulting curves are invariant under the transformation $\zeta_i \to -\zeta_i$ and $\zeta_i \to \pi + \zeta_i$ since those transformations clearly leave the correlations unchanged. The angles $\zeta_i$ can only therefore be determined up to a 4 fold ambiguity.

From these curves it is clear that modulo the ambiguity, the angles are determined to $O(2^\circ)$ with $\bar{N}_{DD} = 10^7$ events.

In Fig. (4) a similar plot of the minimum value of $\chi^2$ is given as a function of $R_i$ for $F_i$ given $N_{DD} = 10^7$. In this case $R_i$ can be determined to $\sim 2\%$ with $N_{DD} = 10^7$ while with $N_{DD} = 10^6$ the 3-sigma bound on $R$ is about

\[10\]
10%. We will find that applying this determination of $\zeta$ and $R$ is more than adequate for the determination of $\gamma$ at the $B$ factory.

5 Extracting $\gamma$ from Direct CP Violation

Let us now turn our attention to the case of $B^\pm \to D^0 K^\pm$. In this case the two contributing amplitudes are $b \to c\bar{u}s$ and $b \to u\bar{c}s$ and so the sensitivity is only to the single CKM angle $\gamma$ and indeed this process has been discussed extensively [11, 12, 13] as a means to measure this angle. The crucial factor in an effective determination of $\gamma$ is the final state chosen for the $D^0$ decay.

The method proposed in [11] (GLW method) requires the following measurements, two of which (eqn. (15) and eqn. (16)) are the same:

\[
Br(B^+ \to K^+ [D^0 \to FES^-]) \quad (13) \\
Br(B^- \to K^- [D^0 \to FES^+]) \quad (14) \\
Br(B^+ \to K^+ [D^0 \to FES^+]) \quad (15) \\
Br(B^- \to K^- [D^0 \to FES^-]) \quad (16) \\
Br(B^- \to K^- [D^0 \to CPES]) \quad (17) \\
Br(B^+ \to K^+ [D^0 \to CPES]) \quad (18)
\]

where $FES\pm$ is a flavor eigenstate of charm=$\pm1$ and $D^0$ generically means a mixture of $D^0\bar{D}^0$. One expects an $O(10\%)$ CP violating difference between (17) and (18) from this data; it is possible to reconstruct $\gamma$ up to an 8-fold ambiguity.

There is, however, a practical problem with the observation of FES states. The only states which are pure FES’s are semi-leptonic decays such as $D^0 \to \ell^+\nu\ell K^-$. In the case of the reaction Eqn. (16) there is the potential backgrounds from semileptonic decay of the parent $B$ [12]. There is perhaps some prospect of overcoming this in the case of the $D^{*0}$ analog [21] but this also may not be easy.

Cabibbo allowed hadronic final states cannot be used at all because in all such cases reaction (14) followed by a DCS decay to the same final state will quantum mechanically interfere. In fact such interference is $O(100\%)$ and thus provides another route to the determination of $\gamma$.

In particular, if $f$ and $g$ are exclusive final states (one of them may be a CP eigenstate) then $\gamma$ may be determined [12] (ADS method) up to a 8-16 fold ambiguity from:

\[
Br(B^- \to K^- [D^0 \to f]) \quad (19) \\
Br(B^+ \to K^+ [D^0 \to \bar{f}]) \quad (20)
\]
where we assume that the branching ratios of $D$ and $\bar{D}$ to $f$ and $g$ are known and that $Br(B^- \to D^0 K^-)$ is known but we must fit for the branching ratio $Br(B^- \to \bar{D}^0 K^-)$ which seems difficult to measure experimentally. For the method to be effective, $f$ should be chosen such that $D^0 \to f$ is a DCS transition in which case the CP violating difference between reaction (19) and (20) is expected to be $O(100\%)$.

To obtain a good determination of $\gamma$ it is best to add additional modes which will therefore over determine $\gamma$ and reduce the ambiguity to 4-fold (i.e. a unique determination of $\sin^2 \gamma$).

Let us now generalize this method by considering inclusive final states. As we discussed above, each inclusive set, $F$, carries with it an additional parameter $R_F$ compared to an exclusive final state. With such final states, one can never hope to determine $\gamma$ since the reactions

$$Br(B^- \to K^- [D^0 \to F])$$
$$Br(B^+ \to K^+ [D^0 \to \bar{F}])$$

provide two observables but introduce the two additional degrees of freedom $\{\zeta_F, R_F\}$. One must therefore have additional information bearing on these parameters. Clearly then, data from a $\psi(3770)$ charm factory can directly determine $R_F$ and $\zeta_F$ for each inclusive final state which can provide the additional information required.

### 5.1 Inclusive ADS

Following [12] let us introduce the following notation for the various branching ratios:

$$a = Br(B^- \to k^- D^0) \quad \bar{a} = Br(B^+ \to k^+ \bar{D}^0)$$
$$b = Br(B^- \to k^- \bar{D}^0) \quad \bar{b}(k) = Br(B^+ \to k^+ D^0)$$
$$c(F) = Br(D^0 \to F) \quad \bar{c}(F) = Br(\bar{D}^0 \to F)$$
$$c(\bar{F}) = Br(D^0 \to \bar{F}) \quad \bar{c}(\bar{F}) = Br(\bar{D}^0 \to \bar{F})$$

$$d(k, F) = Br(B^- \to k^- [D^0 \to F])$$
$$\bar{d}(k, \bar{F}) = Br(B^+ \to k^+ [D^0 \to \bar{F}])$$

Here $k^\pm$ represents either $K^\pm$ or $K^{*\pm}$ (or indeed one may consider any other kaonic resonance or system of strangeness=$-1$ and well defined CP).
In the standard model, it is expected that $a(k) = \bar{a}(k)$, $b(k) = \bar{b}(k)$ and $c(X) = c(\bar{X})$ all of which we will assume from here on [18]. The value of the quantities $d$, $\bar{d}$ may be expressed in terms of $a$, $b$ and $c$ as:

$$d(k, F) = a(k)c(F) + b(k)c(\bar{F}) + 2R_F \sqrt{a(k)b(k)c(F)c(\bar{F})} \cdot \cos(\zeta_k + \zeta_F + \gamma)$$

$$\bar{d}(k, F) = a(k)c(F) + b(k)c(\bar{F}) + 2R_F \sqrt{a(k)b(k)c(F)c(\bar{F})} \cdot \cos(\zeta_k + \zeta_F - \gamma)$$

(26)

where $\zeta_k$ is the strong phase difference between $B^- \to k^- D^0$ and $B^- \to k^- \bar{D}^0$; $\zeta_F$ is the strong phase difference between $D \to F$ and $D \to \bar{F}$ and $\gamma$ is the CP violating weak phase difference between $B^- \to k^- D^0$ and $B^- \to k^- \bar{D}^0$.

To illustrate the procedure for finding $\gamma$, let us first consider the use of exclusive modes via the ADS method and then supplementing it with information obtained from a $\psi(3770)$ factory. The exclusive modes we will consider are

1. $D^0 \to K^+ \pi^-$
2. $D^0 \to K^{*+} \pi^-$
3. $D^0 \to CP=\bar{1}$ eigenstates (CPES-).

The branching ratios for $\bar{D}^0 \to K^+ \pi^-$ is 3.80% while the branching ratio for $D^0 \to K^+ \pi^-$ is $1.48 \times 10^{-4}$. The branching ratio for $\bar{D}^0 \to K^{*+} \pi^-$ is 6.0% while the corresponding branching ratio for the $D^0$ decay is unknown. For our calculation we will assume that it is the latter multiplied by the double Cabibbo suppression factor $\sin^2 \theta_c$. The branching ratio to $CP = -1$ states we will take to be about 5%. Of course the strong phases and $\gamma$ are totally unknown but for the purposes of illustration, we will take as the real value of $\gamma = 60^\circ$, with $\zeta(K^+ \pi^-) = 120^\circ$; $\zeta(K^{*+} \pi^-) = 60^\circ$ and $\zeta_B = -50^\circ$.

In the ADS method, two modes are sufficient, in principle, to determine $\gamma$ with some ambiguity and this is illustrated in Fig. (5). In this figure, we assume that $\hat{N}_B$, the number of $B'$s times the acceptance, is $10^9$. We assume that the actual number of events of each type detected is equal to the theoretical value; the horizontal axis covers various possible values of $\gamma$ and for each hypothesized value of $\gamma$ a minimum value of $\chi^2$ is obtained\footnote{Thus, if we assume that acceptance is 10% and there are $10^9$ $B'$s, a $3\sigma$ criterion for eliminating values of $\gamma$ translates to $\chi^2 > 90$ on this graph.}.
The dotted curve assumes that data is taken for just the two modes: $B^- \rightarrow K^-\overline{D^0} \rightarrow K^* \pi^-$ and $B^- \rightarrow K^-\overline{D^0} \rightarrow \text{CPES}^- (+$ charge conjugates). It can be readily seen that the ambiguities of that method tend to interfere with a clean determination of $\gamma$.

Of course three modes will improve the situation so if we now add in the other $K^+\pi^-$ mode, we obtain the solid curve and clearly there is some improvement. Note that in all cases (including what we discuss below) there is a residual 4-fold ambiguity between $\gamma$, $\pi^- \gamma$, $\pi^+ \gamma$ and $2\pi^- \gamma$ as discussed in [12].

To improve the situation still further, let us consider adding information from a $\psi(3770)$ factory. Here we will take the number of $D^0\overline{D^0}$ pairs times acceptance to be $\hat{N}_D = 10^7$. The information from the correlations of the above two modes with each other and with the CP eigenstates thus serve to determine $\zeta(K^+\pi^-)$ and $\zeta(K^{*+}\pi^-)$ with a degree of over determination $\delta n = 4$. in Fig. (6) we show $\chi^2$ as a function of $\zeta$ for these two modes. It is clear that except for the discrete ambiguities, the values of these angles are relatively well determined.

Returning now to Fig. (5) the dashed-dotted line shows the improvement obtained by adding this information from the charm factory.

Let us now consider the case where we use inclusive final states, in particular we will consider the sets of inclusive states $\mathcal{F} = \mathcal{F}_1 \cup \mathcal{F}_2 \cup \mathcal{F}_3$ discussed in our toy model together with the CPES- states. As discussed above, the determination of $\zeta$ and $R$ using $\psi(3770)$ factory data is shown in figures Fig. (3) and Fig. (4).

We can not use inclusive modes to find $\gamma$ via the ADS method because of the extra degree of freedom $R$ associated with each mode. With the data from a $\psi(3770)$ charm factory such a determination becomes possible. This is illustrated in Fig. (7). The solid curve shows the result obtained from $\mathcal{F}$ with CPES- while the dashed curve shows the result using $\mathcal{F}_{1-3}$ separately with CPES-. Clearly there is considerable gain in segregating the data into different subsets.

In Table (3) we summarize the 3-σ errors in the determination of $\gamma$ which follow from the calculations in Figs. (5) and Figs. (7). The $\psi(3770)$ data clearly improves the error in the case of exclusive states. In the case of inclusive states where $\psi(3770)$ data is essential, we see that dividing the inclusive state into several sub modes can lead to an improved determination of $\gamma$.

Finally, it is important to note that these methods are not limited to the case where the $D^0$ decay is inclusive, one can also use $R$ and $\zeta$ to characterize inclusive decays of the parent $B$. For instance, as pointed out by [22] the decay $B^- \rightarrow D^0K^-\pi^0$ has the advantage that there is no color suppression in the $b \rightarrow u$ transition; hence larger branching ratios would be expected.
Using our methods, this mode integrated over a portion of the $B$ Dalitz plot would be characterized by two parameters $R_B$ and $\zeta_B$ as opposed to just $\zeta_B$ for the two body $B$ decay.

If exclusive $D$ decays are used alone then at least three modes would be required to determine $\gamma$. If, however exclusive and inclusive modes of $D^0$ decay are used where $R$ and $\zeta$ are supplied by the charm factory then two modes would be sufficient to determine $\gamma$. Again, this method makes no apriori assumptions concerning the structure of the $B$ decay amplitude and may be generalized to a broad class of inclusive decays such as $B^- \rightarrow D^0K^- + n\pi, D^0K^*\rho$ etc.

A special case of the above would be to consider the two $D^0$ decays to the exclusive states $K^+\pi^-$ and CPES where the $D^0$ is produced via $B^0 \rightarrow D^0K^-, B^0 \rightarrow D^0K^-\pi^+\pi^-$ and $B^0 \rightarrow D^0K^-\pi^+\pi^-$. It is likely that these four modes have similar strong phases so $R_B$ due to the summation over these $K$ and $D$ resonances is likely to not be $<< 1$. With a charm factory determination of $\zeta(K^+\pi^-)$ there would be enough information to determine $\gamma$. The advantage is that all the final states here are relatively clean experimentally and the ADS mode $K^+\pi^-$ is likely to have relatively large CP violation so this combination could provide an early determination of $\gamma$.

6 Summary

In this paper we have generalized the analysis of direct CP violation from states which are single quantum states to states which are inclusive either because they are integrated over phase space or include states with different particle content. In particular, we study the determination of $\gamma$ using decays of the form $B^- \rightarrow K^-D^0$ where the $D^0$ subsequently decays to various inclusive final states.

We have shown that for a given inclusive state $F$ the phase relation between $D^0 \rightarrow F$ and $\bar{D}^0 \rightarrow F$ can be expressed in terms of a strong phase $\zeta$ and a coherence coefficient $R$. These quantities can be extracted at a $\psi(3770)$ charm factory by observing correlations of the form $\psi(3770) \rightarrow DD \rightarrow F_i\bar{F}_j$, $\psi(3770) \rightarrow DD \rightarrow F_i\bar{F}_j$ and $\psi(3770) \rightarrow DD \rightarrow F_i$ CPES where $F_i$ are various inclusive states.

This information then allows the extraction of $\gamma$ via CP violating effects in the reaction $B^- \rightarrow K^- [D^0 \rightarrow F_i]$ in a model independent way. To illustrate the method we construct a toy model for inclusive decays of the form $K^-\pi^-X$. We find that if this is decomposed into three subsets according to the energy of the $K^-$, then a determination of $\gamma$ with a $3\sigma$ error of $2.3^\circ$ can be made if $\hat{N}_B = 10^9$. 

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Figure 1: $\zeta$ is shown as a function of the energy fraction of the $K^-$ for our model of $\mathcal{F}$.

Figure 2: $R$ is shown as a function of the energy fraction of the $K^-$ for our model of $\mathcal{F}$.

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Figure 3: $\chi^2$ is plotted as a function of $\zeta$ using correlations at a $\psi(3770)$ factory for the inclusive modes $F_1$, $F_2$ and $F_3$ defined in the text. Using the 15 correlations of the form $F_i F_j$, $F_i \bar{F}_j$ and $F_i$ with CPES-, the dashed curve gives $\chi^2$ as a function of $\zeta(F_1)$, the dotted curve gives $\chi^2$ as a function of $\zeta(F_2)$ and the dash-dotted curve gives $\chi^2$ as a function of $\zeta(F_3)$. The solid curve gives $\chi^2$ as a function of $\zeta(F)$ using only the correlations of $F F$, $F \bar{F}$ and $F$ with CPES-.

Figure 4: $\chi^2$ is plotted as a function of $R$ using correlations at a $\psi(3770)$ factory for the same final states as in Fig. (4).
Figure 5: $\chi^2$ is plotted as a function of $\gamma$ for input generated assuming $\gamma = 60^\circ$ and $\zeta_k = -50^\circ$ using various combinations of exclusive states with $\hat{N}_B = 10^9$. The dotted curve uses only data from two $D$ decay modes: CPES- and $K^{*+}\pi^-$; the solid curve includes data from three modes: CPES-, $K^{*+}\pi^-$ and $K^-\pi^+$. The dot-dashed curve uses the same modes but correlation data from a $\psi(3770)$ factory with $\hat{N}_D = 10^7$ is also included which helps determine the strong phase differences for each of the modes. The values of the strong phase differences used are $120^\circ$ for $K^+\pi^-$ and $60^\circ$ for $K^{*+}\pi^-$. 

Figure 6: $\chi^2$ is plotted as a function of $\zeta$ using correlations at a $\psi(3770)$ factory using correlations between the exclusive modes considered in Fig. (5). The solid line is for the mode $K^+\pi^-$ while the dashed line is for the mode $K^{*+}\pi^-$. 

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Figure 7: $\chi^2$ is plotted as a function of $\gamma$ for input generated assuming $\gamma = 60^\circ$ and $\zeta_k = -50^\circ$ using various combinations of inclusive states with $N_B = 10^9$ together with $\psi(3770)$ factory data. The solid curve uses only $\mathcal{F}$ and CPES-. The dashed line uses information from $\mathcal{F}_{1-3}$ together with CPES-.

| Mode                  | Sub-mode | Br  |
|-----------------------|----------|-----|
| $K^- \pi^+$           |          | 3.8%|
| $K^- \pi^+ \pi^0$     |          | 13.1%|
| $\pi^+ [\rho^+ \to \pi^+ \pi^0]$ |          | 8.64%|
| $\pi^0 [K^*^- \to K^- \pi^0]$ |          | 5.02%|
| $\pi^0 [\bar{K}^0 \to K^- \pi^0]$ |          | 1.46%|
| $K^- \pi^- \pi^+ \pi^+$ |          | 7.46%|
| $K^- \pi^- [\rho^0 \to \pi^+ \pi^-]$ |          | 4.7%|
| $K^*^0 \to K^- \pi^+$ |          | 0.97%|
| $K^- [a_1^+ \to \pi^+ \pi^+ \pi^-]$ |          | 3.6%|
| $K^- (1270) \to K^- \pi^- \pi^+$ |          | 0.37%|
| 4-body continuum      |          | 1.74%|
| $K^- \pi^- \pi^+ \pi^0$ |          | 4.0%|

Table 1: The modes used in the toy model for $D^0 \to K^- X$ where $X$ contains at most one $\pi^0$.

| Mode  | Br  | $\zeta$  | R  |
|-------|-----|-----------|----|
| $\mathcal{F}_1$ | 11.0% | $-34^\circ$ | 0.74 |
| $\mathcal{F}_2$ | 24.9% | $-86^\circ$ | 0.29 |
| $\mathcal{F}_3$ | 19.3% | $30^\circ$ | 0.91 |
| CP eigenstates | 5% |             | 1  |

Table 2: The parameters for the three different inclusive modes resulting in our toy model.
\[
\gamma = 60^\circ; \quad \zeta_k = -50^\circ
\]

\[
\mathcal{K}^* + \pi^+ - \pi^- \quad \text{with CPES-10.0^\circ}
\]

\[
\mathcal{K}^* + \pi^+ - \pi^- \quad \text{with CPES-9.1^\circ}
\]

\[
\mathcal{K}^* + \pi^+ - \pi^- \quad \text{with CPES-3.4^\circ}
\]

\[
\mathcal{F} \quad \text{with CPES-12.0^\circ}
\]

\[
\mathcal{F}_1, \mathcal{F}_2 \text{ and } \mathcal{F}_3 \quad \text{with CPES-2.3^\circ}
\]

| Input                                      | $\gamma = 60^\circ; \quad \zeta_k = -50^\circ$ |
|--------------------------------------------|-----------------------------------------------|
| $K^{++} \pi^-$ with CPES-                  | 10.0°                                         |
| $K^{++} \pi^-$ and $K^+ \pi^-$ with CPES-  | 9.1°                                          |
| $K^{++} \pi^-$ and $K^+ \pi^-$ with CPES- using $\psi(3770)$ | 3.4°                                          |
| $\mathcal{F}$ with CPES-                  | 12.0°                                         |
| $\mathcal{F}_1, \mathcal{F}_2 \text{ and } \mathcal{F}_3 \text{ with CPES-}$ | 2.3°                                          |

Table 3: The 3-σ error in degrees in the determination of $\gamma$ with $\hat{N}_B = 10^9$ for the various toy models considered in the text for $\gamma = 60^\circ$ and $\zeta_k = -50^\circ$. The first three rows refer to exclusive states where we take $\eta(K^+ \pi^-) = 120^\circ$ and $\zeta(K^+ \pi^-) = 60^\circ$ corresponding to the curves in Fig. (5) while the last two rows refer to the inclusive states $\mathcal{F}_i$ corresponding to the curves in Fig. (7).