Method of Measuring the Mismatch of Parasitic Capacitance in MEMS Accelerometer Based on Regulating Electrostatic Stiffness

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Abstract: For the MEMS capacitive accelerometer, parasitic capacitance is a serious problem. Its mismatch will deteriorate the performance of accelerometer. Obtaining the mismatch of the parasitic capacitance precisely is helpful for improving the performance of bias and scale. Currently, the method of measuring the mismatch is limited in the direct measuring using the instrument. This traditional method has low accuracy for it would lead in extra parasitic capacitive and have other problems. This paper presents a novel method based on the mechanism of a closed-loop accelerometer. The strongly linear relationship between the output of electric force and the square of pre-load voltage is obtained through theoretical derivation and validated by experiment. Based on this relationship, the mismatch of parasitic capacitance can be obtained precisely through regulating electrostatic stiffness without other equipment. The results can be applied in the design of decreasing the mismatch and electrical adjusting for eliminating the influence of the mismatch.

Keywords: MEMS accelerometer; mismatch of parasitic capacitance; electrostatic stiffness

1. Introduction

An accelerometer is a key device in inertial navigation and control systems for measuring the acceleration information of a carrier. With the progress of MEMS technology, the MEMS accelerometer has been rapidly developed and is widely used in military, industry, medicine, and consumer electronics fields for its small volume, light weight, small power consumption, and low cost. Among MEMS accelerometers, the closed-loop capacitive accelerometer based on electrostatic force balance is an important form for its relatively good performance [1,2].

The MEMS capacitive accelerometer measures the acceleration through electrically detecting the changed differential capacitance of sensor caused by the movement of proof-mass under acceleration. As is known to all, parasitic capacitance is a serious problem in MEMS capacitive accelerometers [3–5]. Its mismatch between electrodes including in the sensor, package, and circuit would produce an offset and deteriorate the performance of bias and scale. The mismatch of effective capacitance due to process variation during sensor fabrication can be eliminated by the closed-loop system, but the mismatch of parasitic capacitance remains. Some research has been carried out for eliminating the influence of the parasitic capacitance [6–8], but these methods are either unsolved completely or lead to extra questions. Reducing the mismatch of parasitic capacitance is more direct and effective, and another solution is compensating the mismatch through electrical adjusting or adding an extra capacitor which
is widely used [9,10]. Either reducing or compensating the mismatch of parasitic capacitance should be measured accurately.

Currently, the method of measuring the parasitic capacitance is limited in the direct measuring using the instrument or the capacitive measuring circuit [11,12]. This method has low accuracy for it would lead to extra parasitic capacitance and the measuring result is the state of off-power, moreover, some equivalent parasitic capacitance cannot be obtained and it cannot be implemented in some occasions. This paper proposes a novel method of measuring the mismatch of parasitic capacitance in MEMS accelerometer based on the mechanism of a closed-loop system. Through regulating the electrostatic negative stiffness and obtaining the curve between the output of electric force and the square of pre-load voltage, the mismatch can be obtained according to the coefficient of linear fitting. This method can be applied in the design for reducing the mismatch and electrical adjusting for eliminating the influence of mismatch, and the research for the characteristics of the mismatch influenced by the temperature and the self-calibrating technique of eliminating the mismatch can be further studied with this method.

2. Method of Measuring the Mismatch of Parasitic Capacitance

2.1. Influence of Parasitic Capacitance

Figure 1 shows the schematic of effective and parasitic capacitances in MEMS capacitive accelerometer interfaced with a C/V converting circuit. Obviously, there are several parasitic capacitances and the mismatch of parasitic capacitances $\Delta C_{m1}$ between $C_{p1}$ and $C_{p2}$—including in the sensor, package, and circuit—will confuse the differential effective capacitances $\Delta C$ between $C_{\text{top}}$ and $C_{\text{bottom}}$ that would produce an offset. The mismatch $\Delta C_{m2}$ between $C_{p3}$ and $C_{p4}$ will also have an influence on the output. Besides, the parasitic capacitances, $C_{p5}$ and $C_{p6}$, can affect the influence of $\Delta C_{m1}$ and $\Delta C_{m2}$ on the output.

![Figure 1. Schematic of capacitance in system of MEMS accelerometer.](image)

Generally, the sensitivity of effective capacitance is about 100 fF/g or even smaller and the mismatch of parasitic capacitance can be up to 100 fF that will result in an offset of 1 g. This large offset would severely deteriorate the performance of the accelerometer. Therefore, it is necessary to study the mismatch and do some work for reducing the influence. Measuring the mismatch accurately is a basic step. Though there are many discrete parasitic capacitances, we only need to obtain the total equivalent mismatch.

2.2. Theory of Measuring the Mismatch

In the closed-loop system of a MEMS capacitance accelerometer, there is electrostatic force between fixed plates and proof mass that balances the inertial force caused by acceleration [13], and the proof mass is not at the geometrical center for the mismatch of parasitic capacitance. Figure 2 shows a working diagram of the sensor.
Considering the process variation and parasitic capacitance, the electrostatic force $F_e$ of the proof mass is:

$$F_e = F_{e1} - F_{e2} = \frac{\varepsilon_r \varepsilon_0 A \times \left( V_d + V_{fb} - V_{ref} \right)^2}{2(d_0 - \Delta d - x)^2} - \frac{\varepsilon_r \varepsilon_0 A \times \left( -V_d - V_{fb} - V_{ref} \right)^2}{2(d_0 + \Delta d + x)^2}$$  (1)

where $\varepsilon_r$ and $\varepsilon_0$ are the relative and absolute dielectric constant respectively, $A$ is the overlapped area of capacitance, $V_d$ is the modulated voltage, $V_{fb}$ is the feedback voltage, $V_{ref}$ is the pre-load voltage, $d_0$ is the average gap between electrodes, $\Delta d$ is the gap deviation due to process variation, and $x$ is the bending value of the beam due to the mismatch of effective and parasitic capacitance.

In general, $x$ and $\Delta d$ are far smaller than $d_0$, and then, Equation (1) can be simplified to:

$$F_e = \frac{2\varepsilon_r \varepsilon_0 A \times V_{ref} V_{fb}}{d_0^2} - \frac{2\varepsilon_r \varepsilon_0 A \times \left( V_{ref}^2 + V_{fb}^2 + V_{m}^2 \right)}{d_0^3} \times (x + \Delta d)$$  (2)

where the bending value $x$ consists of $x_1$ brought by the mismatch of effective capacitance and $x_2$ brought by the mismatch of parasitic capacitance, so $x = x_1 + x_2 = -\Delta d + x_2$. Substituting this equation to Equation (2), the electrostatic force $F_e$ can be expressed as:

$$F_e = \frac{2\varepsilon_r \varepsilon_0 A \times V_{ref} V_{fb}}{d_0^2} - \frac{2\varepsilon_r \varepsilon_0 A \times \left( V_{ref}^2 + V_{fb}^2 + V_{m}^2 \right)}{d_0^3} \times x_2$$  (3)

where $2\varepsilon_r \varepsilon_0 A \times \left( V_{ref}^2 + V_{fb}^2 + V_{m}^2 \right) / d_0^3 = k_e$ is called electrostatic stiffness.

In the closed-loop system, there is the force balance for the proof mass:

$$F_e + kx + ma + F_r = 0$$  (4)

where $k$ is the stiffness of the beam, $m$ is the inertial mass of the proof mass, $a$ is the external acceleration, and $F_r$ is the residual stress. Replacing Equation (3) into Equation (4), the formula of force balance can be expressed as:

$$\frac{2\varepsilon_r \varepsilon_0 A \times V_{ref} V_{fb}}{d_0^2} - \frac{2\varepsilon_r \varepsilon_0 A \times \left( V_{ref}^2 + V_{fb}^2 \right)}{d_0^3} \times x_2 = B_0$$  (5)
where $B_0 = 2\varepsilon\varepsilon_0 A \times V_{ref}^2 \times x_2 / d_0^3 - kx - ma - F_g$. When the input acceleration is unchanged, the parameter $B_0$ can be considered as a fixed value. When the input acceleration and offset are small, $V_{ref}^2$ is far smaller than $V_{ref}^2$, so Equation (5) can be simplified to:

$$
\frac{2\varepsilon\varepsilon_0 A \times V_{ref} V_{fb}}{d_0^2} = \frac{2\varepsilon\varepsilon_0 A \times x_2}{d_0^2} \times V_{ref}^2 + B_0
$$

(6)

For the digital acquisition system, the left portion in Equation (6) can be transformed to $F'_x = 2\varepsilon\varepsilon_0 A \times V_{ref} V_{fb} / d_0^2 = U_{out} / K_1 \times m \times g_L$ where $U_{out}$ is digital output which unit is LSB, $K_1$ is the scale of accelerometer which unit is LSB/g and $g_L$ is local gravity acceleration. Then, Equation (6) can be transformed to:

$$
\frac{U_{out}}{K_1} \times m \times g_L = \frac{2\varepsilon\varepsilon_0 A \times x_2}{d_0^3} \times V_{ref}^2 + B_0
$$

(7)

Equation (7) can be transformed to:

$$
Y = B_1 \times X + B_0
$$

(8)

where $Y = U_{out} / K_1 \times m \times g_L$ is dependent variable, $X = V_{ref}^2$ is independent variable, $B_1 = 2\varepsilon\varepsilon_0 A \times x_2 / d_0^3$ is linear coefficient and $B_0$ is intercept which is a fixed value.

Equation (8) shows that the relationship between output of electrostatic force $F'_x = U_{out} / K_1 \times m \times g_L$ and the square of pre-load voltage $V_{ref}^2$ is linear. Thus, we can make a curve with $F'_x$ as y-axis and $V_{ref}^2$ as x-axis, and then, a linear fitting of the curve is made. Lastly, the mismatch of the parasitic capacitance can be obtained from the linear coefficient $B_1$ through the equation:

$$
\Delta C_p = \frac{\varepsilon\varepsilon_0 A}{d_0 - x_2} - \frac{\varepsilon\varepsilon_0 A}{d_0 + x_2} \approx \frac{2\varepsilon\varepsilon_0 A \times x_2}{d_0^3} \times d_0 = B_1 \times d_0,
$$

(9)

where $d_0$ can be calculated through the obtained scale of the closed-loop system. Meanwhile, we can get the offset and the deviation from geometrical center due to the mismatch of parasitic capacitance.

3. Measurement Results and Discussion

Measuring tests have been done with closed-loop MEMS accelerometer to verify this novel method and two applications with this method are present. The measuring work were implemented on a printed circuit board (PCB) with discrete component, interfaced with a packaged sensor using ceramic shell and bond wire. The senor is fabricated with bulk silicon process and the structure is comb finger. The control system is achieved by analogue circuit and the analogue output is digitally acquired through Analog to Digital Convert (ADC) and Field Programmable Gate Array (FPGA) chip. The full-scale range of the accelerometer is 30 g, and the noise is $10\mu g/\sqrt{Hz}$. In this system, the parasitic capacitances originate from the sensor, the ceramic shell, the bond wire and the PCB circuit. In our designed accelerometer, this mismatch commonly leads in an offset of several hundred mg that severely deteriorates the performance of accelerometer.

3.1. Measurement Results

3.1.1. Verification Experiment and Results

In the verification experiment, the accelerometer is placed on the marble platform and the input acceleration is about 0 g which purpose is to make the external acceleration stable and the output very small. This step can improve the accuracy of the measurement. Because the pre-load voltage goes through voltage follower and resistance, and then reaches the node of proof-mass, so, the pre-load voltage does not directly connect to this C/V node. We draw out a line from the node of pre-load voltage that did not change the output. Then, the pre-loaded voltage of the accelerometer is changed,
and the scale is tested through turning the accelerometer. The changed pre-loaded voltage, the digital output and the scale are record. Table 1 contains the measuring data with different pre-loaded voltage.

3.1.2. Applications and Results

Using these recorded data, we make a figure by taking \( V_{ref}^2 \) as x-axis and \( F_e' \) as y-axis as shown in Figure 3, and a linear fitting of the curve is made.

Figure 3. Relationship between \( V_{ref}^2 \) and \( F_e' \).

The \( R^2 \) of the linear fitting is 0.9999 which shows highly linear correlation between \( V_{ref}^2 \) and \( F_e' \). The strong linear relationship validates the theory of formula deduction. From the linear fitting formula, the linear coefficient can be obtained which is \( -1.98205 \times 10^{-8} \). Through calculation according to this number, the bending value \( x_2 \) of the beam owing to the mismatch of parasitic capacitance which is also the deviation from the geometrical center is \(-13.48 \text{ nm}\). It should be noted that the bending value of the beam is a vector. That is to say it can be positive or negative. The bending direction of the beam depends on the sum of \( x_1 \) and \( x_2 \), and the minus sign of this \( x_2 \) indicates that the beam bends to the bottom plate, owing to the mismatch of parasitic capacitance. Correspondingly, the mismatch of parasitic capacitance is \(-69.372 \text{ fF}\) and the offset caused by the mismatch is \(219 \text{ mg}\).

Table 1. The measuring data with different \( V_{ref} \).

| \( V_{ref}(V) \) | \( U_{out}(\text{LSB}) \) | \( K_1(\text{LSB/g}) \) | \( V_{ref}^2(V^2) \) | \( F_e'(\text{N}) \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| 1.00            | 5058            | 137,837         | 1.00            | \( 7.33 \times 10^{-8} \) |
| 2.00            | 1526            | 68,051          | 4.00            | \( 1.96 \times 10^{-8} \) |
| 3.00            | −49             | 45,092          | 9.00            | \( -8.48 \times 10^{-8} \) |
| 4.00            | −1079           | 33,768          | 16.00           | \( -2.29 \times 10^{-7} \) |
| 5.00            | −1946           | 26,993          | 25.00           | \( -4.09 \times 10^{-7} \) |
| 6.00            | −2651           | 22,491          | 36.00           | \( -6.22 \times 10^{-7} \) |
| 7.00            | −3329           | 19,253          | 49.00           | \( -8.73 \times 10^{-7} \) |
| 8.00            | −4043           | 16,811          | 64.00           | \( -1.18 \times 10^{-6} \) |
| 9.00            | −4682           | 14,942          | 81.00           | \( -1.51 \times 10^{-6} \) |

The charge amplifier and diode ring are the common used C/V converting circuit. Because the charge amplifier is based on current measurement, the parasitic capacitance \( C_p3 \) and \( C_p4 \) in figure1 has little influence on the output of charge amplifier. However, in our design the diode ring detecting circuit is adopted for its simple structure. In diode ring detecting circuit, the principle of C/V converting is based on charge-discharge of capacitance. The capacitance \( C_p3 \) and \( C_p4 \) would affect the charge–discharge process of demodulating capacitance, so, it has an effect on the output. We carried
out an experimental test to study the influence on output of capacitance to ground (GND) previously. A 1 pF difference between $C_{p3}$ and $C_{p4}$ was made in MEMS accelerometer using diode ring detecting circuit and a change of 0.5 g on output was observed, so it is necessary to study the influence of the parasitic capacitance between the fixed plate and GND. It should be noted that the effect of this equivalent mismatch on output is not equal to the effective differential capacitance, so its equivalent mismatch cannot be measured using the direct measuring method. The experiment for measuring the equivalent mismatch of the parasitic capacitance between the fixed plate and GND is carried out.

Another application using this method is improving the design of circuit to reduce the mismatch of parasitic capacitive. Table 2 shows the mismatch of parasitic capacitive for different sensors on same circuit board. For these six sensors, the average bending value $x_2$ is $-11.0$ nm and the average mismatch is $-56.44$ fF, which causes an offset of 179 mg. It can be seen that the values of the mismatch are near that indicates the mismatch is mainly from the circuit board for the mismatch of different sensors would have large discreteness.

### Table 2. Mismatch of different sensors on same board.

| Sensor | $x_2$ (m) | Mismatch/ff |
|--------|-----------|-------------|
| 1      | $-1.12 \times 10^{-8}$ | -57.64 |
| 2      | $-0.99 \times 10^{-8}$ | -50.95 |
| 3      | $-0.98 \times 10^{-8}$ | -50.43 |
| 4      | $-1.14 \times 10^{-8}$ | -58.67 |
| 5      | $-1.17 \times 10^{-8}$ | -60.21 |
| 6      | $-1.18 \times 10^{-8}$ | -60.73 |
| average | $-1.10 \times 10^{-8}$ | -56.44 |

The design of the circuit should be improved to reduce the mismatch of parasitic capacitance on the circuit board. An improved circuit was fabricated and the mismatch is measured with the same
sensor welded on different circuit boards. Figure 5 is the contrast of mismatch on different circuit boards. The mismatch of parasitic capacitance is $-69.372$ fF on the before-optimization circuit board, and it is $+22.332$ fF on the after-optimization circuit board. It can be seen that through optimizing the circuit design, the mismatch of parasitic capacitance is reduced by 69% and the sign of the mismatch is changed.

![Figure 5](image_url)

**Figure 5.** Mismatch of different circuit design: (a) result of before-optimization circuit; (b) result of after-optimization circuit.

### 3.2. Discussion

The linear relationship between output of electrostatic force and the square of pre-load voltage is validated by the experiment. In an ideal system with no mismatch, the force $F'_e$ is a fixed value for the feedback and pre-load voltage are changed at inverse proportions. However, due to the existence of the mismatch of parasitic capacitance in real system, the force $F'_e$ will be changed in proportion to $x_2$ following the changed force $k_x x_2$ when regulating the electrostatic stiffness through changing the pre-load voltage. The novel method exploits this characteristic to obtain the mismatch of parasitic capacitance.

It should be pointed out that the curve deviates from the straight line when the pre-load voltage is small, especially when the mismatch is small. This is because the force $k_x x_2$ has little change with a small pre-load voltage or a small mismatch that makes the linear relationship disturbed by the feedback voltage. Nevertheless, the mismatch of parasitic capacitance can be obtained precisely through regulating electrostatic stiffness with relatively high pre-loaded voltage.

The measured results show the mismatch of capacitance parasitic is fF level. The mismatch is so small that requires testing equipment of very high precision. Different from the traditional methods, in this novel method a line is just drawn out from the pre-loaded node which does not interfere with any electrical node of the C/V frond-end circuit, so it does not introduce additional parasitic capacitance. Moreover, the measured result is the equivalent mismatch of all parasitic capacitance when the accelerometer is in an operating state. Therefore, the mismatch result is that we want.

### 4. Conclusions

This paper describes a novel method for measuring the mismatch of parasitic capacitance in MEMS capacitive accelerometer. The strong linear relationship between output of electrostatic force and the square of pre-load voltage is validated by the theory and experiment. The total equivalent mismatch of parasitic capacitance can be obtained precisely and conveniently through regulating electrostatic stiffness with changing the pre-loaded voltage. The results can be used in the design and electrical adjusting for decreasing the influence of the mismatch that is helpful for improving the performance of accelerometer, and the temperature characteristics of the mismatch and the self-calibrating technique of eliminating the mismatch can be further studied with this method.
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