Reynolds number influence on preferential concentration of heavy particles in turbulent flows

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Résumé. We present a study of the preferential concentration and clustering in homogeneous and isotropic turbulence. Using Voronoï diagrams, we have formerly quantified preferential concentration as a function of the Stokes number in moderate turbulence conditions up to Reynolds number based on Taylor microscale of the order of $R_\lambda \sim 120$. Using an active grid recently implemented in our windtunnel, we investigate in the present study, the effect of Reynolds number on particles clustering, in the range $R_\lambda \sim 200 - 400$.

1. Introduction

A striking feature of turbulent flows laden with inertial particles is the so-called preferential concentration or clustering that leads to very strong inhomogeneities in the concentration field at any scale. This has now been widely observed in many experimental and numerical configurations including homogeneous and isotropic turbulence (see [9, 3, 8]) and a robust result is that the concentration field is more intermittent for particles whose Stokes number (defined as the ratio of the particle relaxation time to the Kolmogorov viscous time) are close to unity. Aliseda and co-workers [1] have suggested that the local enhancement of the concentration could be responsible for the measured enhancement of particles settling velocity in turbulent flows due to collective effects. It is therefore important to have a good quantification of the amount of clustering. In a recent work [5] we have shown that Voronoï tessellations are a very useful tool to analyze and quantify clustering properties of particles in a turbulent flow. Our previous study have mostly focused on the influence of varying particles properties (we used water droplet of various sizes, hence exploring Stokes number effects). Due to experimental limitations, we were not able in this first work to investigate satisfactorily the influence of the carrier flow Reynolds number (limited to $R_\lambda \sim 120$ in the previous experiment), which is the topic of this new investigation. Using our new active grid, we were able to explore a wide range of Reynolds numbers up to $R_\lambda \sim 400$. This article is organized as follows: in section 2 we present the experimental setup, in section 3 we briefly recall the voronoi tessellation analysis procedure before applying it for a quantitative analysis of Reynolds number effects on particles clustering in section 4. A brief discussion and conclusions are given in section 5.
2. Experimental setup

Experiments are conducted in a large wind tunnel with a square cross-section of 0.75 m × 0.75 m (see figure 1). Turbulence is generated with an active grid made of 16 rotating axes (eight horizontal and eight vertical) mounted with square blades. Such active grids are known to produce higher turbulence with still relatively good homogeneity and isotropy levels [4, 6, 7]. Each axis is driven independently with a step motor whose rotation rate and direction can be changed dynamically. In the present work the active grid was driven in a triple random mode: each axis has a random rotation rate (within a prescribed range), with random rotation and both (rate and direction) may change randomly in time (within a prescribed time-lag range). In the experiments presented here, the range of prescribed rotation rate was typically between 0.5 Hz and 3 Hz and the time lag between random changes of direction and/or rate was typically between 1 and 3 seconds. The mean velocity of the wind was varied from 3.4 m/s up to 7.6 m/s, corresponding to a range of Reynolds number $R_\lambda$ from 230 up to 400. Table 1 summarizes the main turbulence parameters of the flow generated at the measurement volume location (3 m downstream the active grid) for the 5 mean wind velocities investigated. Besides, figure 2a shows a typical turbulent spectrum measured with hot-wire anemometry where a well-developed inertial range of scales can be identified. Inertial particles are water droplets generated by 36 injectors (6x6 mesh with identical spacing than the grid) located in a transverse plane 15 cm downstream the grid. As injectors we use single phase industrial spray generators with 0.3 mm nozzles with filtered water pressurized at 40 bars. Figure 2b shows the size distribution (in volume) of the generated spray measured at the measurement volume location (3 m downstream the grid), with a Spraytech diffractometer from Malvern. We have checked that this size distribution is robust and does no depend significantly on the mean wind velocity. As it can be seen on the figure, the size distribution is peaked around a most probable droplet diameter of the order of $D_p \sim 60 \mu m$, but is relatively polydisperse. The water flow rate cumulated over all 36 nozzles is $\Gamma_w = 18$ L/min, giving a volume fraction of water in the range $\phi_w = [5 \cdot 10^{-5} - 2 \cdot 10^{-4}]$ (the lower the average wind velocity, the larger the volume fraction). We define the droplets Stokes number as the ratio $St = \tau_p/\tau_\eta$, with $\tau_\eta$ the carrier flow dissipation scale estimated from hot-wire anemmetry (see table 1) and $\tau_p$ the particles viscous relaxation item based on the most probable particle diameter $D_p$ estimated from the diameter distribution function (in figure 2b). The Stokes number can be equivalently estimated as $St = (D_p/\eta)^2(1 + 2\Gamma)/36$ with
Fig. 2. (a) Typical velocity spectrum generated 3 m downstream the active grid. The red solid line indicate a $k^{-5/3}$ power law. (b) Droplets diameter probability density function.

$\eta$ the dissipation scale of the carrier turbulence and $\Gamma = \rho_{\text{water}}/\rho_{\text{air}}$ the density ratio between the particles and the carrier flow, what in the present case (water particles carried in air) simply gives $St \approx 46(D_p/\eta)^2$. Therefore, as the most probable particles diameter $D_p$ is kept constant and does not depend on flow conditions, varying the carrier flow Reynolds number (and hence varying the dissipation scale $\eta$) also results in a variation of particles Stokes number. Table 1 also indicates the particles Stokes number corresponding to each Reynolds number investigated in the present study. Acquisitions are performed using a Phantom V12 high speed camera operated at 10 kHz and acquiring 12 bits images at a resolution of 1280 pixels $\times$ 488 pixels corresponding to a 125 mm (along $x$) $\times$ 45 mm (along $y$), though homogeneous illumination conditions (tested a posteriori during the post-processing) were actually limited to a smaller visualization window of 70 mm in the streamwise $x$-direction and 50 mm in the transverse $y$-direction, what is about half an integral scale. The camera is mounted with a 90 mm macro lens; the view angle with respect to the laser sheet is of the order of 60°, thuw a Scheimpflug mount is used to compensate loss of depth of field. At the given spatial resolution and repetition rate, the onboard memory of our camera (8 Gb) allows to record slightly more than $10^4$ images (hence slightly more than one second of recording), what corresponds already to a few integral time scales of the carrier flow. For each experimental configuration we record at least 15 such recordings, thus a set of more than $1.5 \cdot 10^5$ images are recorded for each experiment, though in the present work only a subsample of about $10^4$ statistically independent images have been processed at the moment.

3. Voronoi analysis of particles spatial distribution

Figure 3a shows a typical image of the particles spatial distribution (the image shown has been corrected for spatial perspective and distorsions). A structuration of the dispersed phase with clusters of droplets and depleted regions is qualitatively evident in this image. To go furhter in a quantitative analysis of clustering, we have used Voronoî diagrams that allows measurements of the concentration field at an intrinsic resolution. A Voronoî diagram is a decomposition of the 2D space into independent cells. Each cell is associated to one particle and is defined as the ensemble of points that are closer to this particle than to any other. Use of Voronoî diagrams is very classical in the study of granular systems and has also been
Fig. 3. (a) Typical raw image of particles. (b) Probability density function of the position of detected particles. (c) Voronoi diagram limited to the region of interest.

used to identify galaxies clusters. From their definition, the Voronoi cells areas $A$ are exactly the inverse of the concentration field around each particle. Figure 3c presents an example of a calculated Voronoi diagram from the experimental field corresponding to the central region of the image in figure 3a, after detection of the particles center. As already explained in the previous section, due to illumination inhomogeneity (resulting both from weak lateral blurring due to depth of field limitation, in spite of the use of the Scheimpflug mount, and laser sheet attenuation) we have limited the region of interest for the Voronoi analysis to a central region of the images (corresponding to the yellow rectangle in figure 3a) where we have checked that the detection of particles centers is statistically homogeneous over the entire set of $1.5 \cdot 10^5$ images for each experiment (figure 3b shows the spatial probability density function of particles detected position).

In a previous work [5], we have shown that a quantitative measure of clusering is given by the analysis of the probability distribution function (PDF) of Voronoi areas compared to that of an exactly random Poisson process (RPP). We recall that no analytical expression is known for the PDF of an RPP, though it is known to be well approximated by a Gamma function [2], however the standard deviation of Voronoi areas (normalized to the average Voronoi area) $\mathcal{V} = \mathcal{A}/\overline{\mathcal{A}}$ for an RPP has a known analytical value $\sigma_{\mathcal{V}}^{\text{RPP}} = 0.53$.

| $R_\lambda$ | $U$ (m/s) | $L$ (cm) | $\eta$ (\(\mu\)m) | $\epsilon$ (m\(^3\)s\(^{-3}\)) | $St$ |
|---------|---------|---------|----------------|-----------------|------|
| 234     | 3.4     | 13.0    | 280            | .69             | 2.1  |
| 264     | 4.0     | 13.2    | 240            | 1.2             | 3.3  |
| 295     | 4.8     | 13.5    | 208            | 2.0             | 4.3  |
| 331     | 5.7     | 13.8    | 178            | 3.4             | 5.8  |
| 357     | 6.4     | 14.0    | 160            | 4.7             | 6.6  |
| 400     | 7.6     | 14.3    | 140            | 7.7             | 9.9  |

Tab. 1. Experimental parameters: Reynolds number based on Taylor microscale ($R_\lambda$), mean wind velocity ($U$), dissipation scale ($\eta$), energy dissipation rate per unit mass ($\epsilon$), Stokes number ($St$).
4. Clustering evidence

In this section we present the statistical analysis of Voronoi areas for the different experiments reported in table 1. Figure 4a presents the probability density function (PDF) of the normalized Voronoi areas $V$ for all the experiments at different Reynolds numbers and Stokes numbers; we have also superimposed in the figure the distribution expected for a uniform random distribution. It can be clearly seen that the measured distribution are not that of a uniform random process. Large Voronoi areas are significantly over-represented compared to the RPP case, indicating the existence of large depleted regions. Similarly areas smaller than $V \sim 0.5$ are significantly over-represented, indicating the clustering phenomenon. More precisely Figure 4a shows that clustering results in a net increase of the probability of observing particles with voronoi areas between a few centesimals and half the average Voronoi areas. This corresponds to an increase of the porbability of finding regions with local concentration (given by the inverse of the Voronoi areas) between 2 and several tens of the average concentration. We note however that the probability density of very small normalized Voronoi areas (typically smaller than a few centesimals) drops below that of the RPP case (below a threshold which depends on the Reynolds number of the carrier flow and/or the Stokes of the particles). This suggests that local concentrations higher than a few tens of the average concentration are less probable for inertial particles than for the RPP case. This point will be discussed further in section ??.

Interestingly we also observe that only the left part of the PDF (corresponding to small Voronoi areas, hence highly concentrated regions) appears to be influenced by Reynolds/Stokes number effects while the right part (corresponding to the depleted regions) is extremly robust, what is consistent with our previous observation at lower Reynolds numbers.

In our previous study at moderate Reynolds number, we have observed that the Voronoi area PDFs for inertial particles in turbulence were well approximated by a lognormal distribution. This is confirmed as a robust characteristic also preserved in the higher Reynolds number regime explored in the present study, as it can be seen in figure 4b: the PDF of $\log(V)$ is approximately gaussian, at least within he range $\pm 3\sigma_{\log(V)}$. Deviation from lognormality is only observed for small values of $\log(V)$ which are slightly under-estimated. Therefore, the overall statistical distribution of Voronoi areas is almost characterized entirely by one single parameter, which we
choose to be the standard deviation $\sigma_V$ and whose dependence with experimental parameters can be used to quantify the evolution of particles clustering.

Figure 5a shows the evolution of $\sigma_V$ with the Reynolds number $R_\lambda$ of the carrier flow. As expected by the previous qualitative considerations, $\sigma_V$ exceeds significantly the RPP value $\sigma_V^{\text{RPP}} = 0.53$, what reveals the high level of clustering. Interestingly we also note that, though $\sigma_V$ changes by less than 10% over the different experiments, a maximum of clustering is reached for $R_\lambda \sim 300$.

As discussed previously, carrier flow Reynolds number and Stokes number of carried particles are related in our experiment, since particles are injected with constant size distribution. Therefore, we have plotted in figure 5b the evolution of $\sigma_V$ as a function of $St$. We have also artificially added the point for $St = 0$, which is expected to represent tracers with $\sigma_V^{\text{RPP}} = 0.53$. We have also reported on the same figure previous measurements at lower Reynolds number (from [5]). In this plot, solid lines connect measurements from this previous campaign performed at constant Reynolds number. For guide-eye purpose we have also connected (with a dashed-line) the measurements from the present study as though they are not at constant Reynolds, they all correspond to conditions at much higher Reynolds number than previously. This figure shows a consistent increase of the clustering level with the Reynolds number. It also shows that the maximum of clustering for $R_\lambda \sim 300$ observed if figure 5a may be interpreted as an optimal Stoked number around 3-4, consistently with observations at lower Reynolds number.

5. Discussion and conclusions

We have investigated the preferential concentration of particles of a given size transported in a turbulent flow with varying Reynolds number up to $R_\lambda \sim 400$. We find clear evidence of particles clustering as Voronoi areas PDF strongly deviates from that of a random distribution. Clustering is found to be Reynolds number dependent in highly concentrated regions (left part of Voronoi areas PDF), while large depleted regions appear to be mostly independent of Reynolds number. This is consistent with previous observations at lower Reynolds numbers [5], and can be interpreted as the fact that large depleted regions are mostly associated to large scale structures.
of the carrier flow, which are not much affected as the Reynolds number is increased. Table 1 shows indeed that the large scale of the turbulent flow generated with our active grid remains relatively constant regardless of the Reynolds number, and that as we increase the Reynolds number small scales are further developed and the dissipation scale does decrease.

The PDF of Voronoi areas also show that clustering phenomenon increases more specifically the probability of finding regions with local concentration between twice and a few tens the average concentration, while very high concentrations are less probable than a random process. Though a further analysis (in particular regarding to possible biases related to the particle detection procedure) are still required to be fully conclusive, this effect may be reminiscent of specific interactions between particles very close to each other. In our previous measurements at lower Reynolds number we have indeed pointed that clusters tend to be less over-concentrated when the average concentration increases. Though we have not identified yet the precise mechanism responsible for this phenomenon, it is very likely related to collective effects taking place in the clusters, which tend to repel or to redistribute particles (by a collective increase of their seeding density, as reported by [1], for instance). The low probability of finding very small Voronoi areas observed in the present study may be also reminiscent of a similar phenomenon. Further investigations, at varying seeding density will help understanding this issue (in the present study we only considered volume fractions of water larger than $3 \cdot 10^5$, experiments in more diluted conditions are planned in the coming months).

Our new measurements confirm the previously reported quasi-lognormal shape of Voronoi areas distribution, what is an interesting feature, as the standard deviation $\sigma_V$ of the Voronoi areas can therefore indeed be considered as a good indicator of clustering level. Based on this indicator, the set new measurements at high Reynolds number combined with the previous measurements at moderate Reynolds number show that increasing the Reynolds number at constant Stokes number results in a clear increase of clustering level. On the contrary, the new measurements show that when increasing the Reynolds number at constant particle size (and hence at increasing Stokes number) results an optimal Reynolds exists for which clustering level is maximal ($R_{\lambda}^{\text{max}} \sim 300$ in the present study with particles with average diameter around 60 $\mu$m). Optimal clustering is generally observed for Stokes number around unity (what is classically interpreted as an optimal response time of the particles to the turbulent solicitations). This is consistent with the observation that the maximum of clustering level observed at $R_{\lambda} \sim 300$ can be reinterpreted as a maximum of clustering for $St \sim 4$. Furthermore considering that clustering increases with $R_{\lambda}$, if we take in figure 4b the point at $St \sim 4$ and $R_{\lambda} \sim 300$ (corresponding to the maximum of clustering we have observed) as reference, one would expect that for the evolution of $\sigma_V$ as a function of Stokes at constant $R_{\lambda} = 300$ points at the right of the reference point (for $St > 4$) will be below the points (stars) in figure 4b, while points at the left of the reference point (for $St < 4$), will be above the points in the figure. Therefore, it is likely that the curve representing the evolution of $\sigma_V$ as a function of Stokes at constant $R_{\lambda} \sim 300$ should look more peaked than the dahed-line curve represented in figure 4b with a maximum for an optimal Stokes number between 2 and 4, in a range comparable to what we previously observed for $R_{\lambda} = 114$. Further experiments, with the active grid generated turbulence, varying particles Stokes number at constant Reynolds number will be performed soon to better address the question of a possible dependence of the optimal Stokes number as a function of Reynolds number.

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