Modifications of FastICA in Convolutive Blind Source Separation

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Abstract—Convolutional blind source separation (BSS) is intended to recover the unknown components from their convolutive mixtures. Contrary to the contrast functions used in instantaneous cases, the spatial-temporal prewhitening stage and the para-unitary filters constraint are difficult to implement in a convolutive context. In this paper, we propose several modifications of FastICA to alleviate these difficulties. Our method performs the simple prewhitening step on convolutive mixtures prior to the separation and optimizes the contrast function under the diagonalization constraint implemented by single value decomposition (SVD). Numerical simulations are implemented to verify the performance of the proposed method.

Index Terms—Convolutional blind source separation (BSS), FastICA, diagonalization constraint, singular value decomposition (SVD).

I. INTRODUCTION

CONVOLUTIONAL blind source separation (BSS) has received much attention in recent years. It has been successfully applied in many signal processing problems, such as sonar array processing, seismic exploration, and the "cocktail party problem". In the convolutive BSS, n dimensional observation \( x(k) = (x_1(k), \ldots, x_n(k))^T \) is the convolutive mixtures of the \( m \) dimensional independent source \( s(k) = (s_1(k), \ldots, s_m(k))^T \). The unknown mixing process \( A \) can be described by a Multi-Input Multi-Output (MIMO) linear time invariant (LTI) system \( (A(k)_{k \in \mathbb{Z}}) \),

\[
x(k) = A(k) * s(k) = \sum_{l \in \mathbb{Z}} A(l)s(k-l) \tag{1}
\]

Given \( N \) samples of observation \( x(k) \), recovering the sources’ estimation \( y(k) = (y_1(k), \ldots, y_m(k))^T \) from these mixtures is equivalent to find a filter banks \( (B(k)_{k \in \mathbb{Z}}) \) to inverse the mixing system \( (A(k)_{k \in \mathbb{Z}}) \):

\[
y(k) = B(k) * x(k) = \sum_{l \in \mathbb{Z}} B(l)x(k-l) \tag{2}
\]

each \( y_i(k) \) is a scaling and filtering version of the unique source’s component \( s_i'(k) \).

Although the ordering and scaling ambiguity in the instantaneous BSS can be efficiently handled by the prewhitening on \( x(k) \) and the orthogonal constraint of unmixing matrix \( B \), the filtering ambiguity caused by the time delay is difficult to deal with in a convolutive context and these strategies (used in instantaneous BSS) become invalid. These ambiguities lead to much more extreme points in convolutive BSS than in instantaneous BSS, making the convolutive BSS a tough problem to deal with.

In this paper, we shall assume that:

1) The source signals \( s(k) \) are real-valued, zero-mean, and mutually statistically independent, at most one of them is Gaussian.
2) The filter banks \( (A(k)_{k \in \mathbb{Z}}), (B(k)_{k \in \mathbb{Z}}) \) are stable, causal, and finite impulse response (FIR).

Many Methods\[3\][4\] have been proposed to solve the convolutive BSS. Most of them can be classified into two groups: the frequency domain approaches and the time domain approaches. In frequency domain\[5\][6\], convolutive BSS can be considered as instantaneous BSS for each frequency bin, where each bin has own scaling and ordering indeterminacy as mentioned before. Complex value after discrete Fourier transform (DFT) and circularity problem happen in frequency domain.

Time domain approaches include density matching methods and contrast function methods. Density matching methods apply the well-known InfoMax\[7\] proposed in ICA to the convolutive case\[8\]. The performance of density matching approaches is highly dependent on the prior knowledge on the unknown density distributions of \( s_i \). It’s important to determine whether the source \( s_i(k) \) is super-Gaussian or sub-Gaussian beforehand. Density matching methods linearly transform the observed mixtures \( x(k) \) with the demixing system \( (B(k)_{k \in \mathbb{Z}}) \), forcing the \( p_y(y_i) \) close to a selected density \( p_s(s_i) \). Most of these methods are based on the gradient optimization, requiring appropriate choice of learning rate and step direction. Natural gradient\[9\] and relative gradient\[10\] have been proposed to alleviate the drawbacks of stability and convergence above. Contrast function methods make the advantage of the statistical independence or non-Gaussianity of the components of source to recover the unknown \( s_i \). In addition, High order statistics (HOS)\[11\] of source’s estimation \( y_i(k) \) like cumulants, cross-cumulants, and cross-moments can work as contrast function in separation. Prewhitenising stage and coefficients constraints are required to guarantee the uniqueness of the extracted components during the contrast function methods. For instantaneous case, prewhitening stages and orthogonal constraint can be implemented by Gram–Schmidt orthogonalization or singular value decomposition (SVD). While in the convolutive mixtures, the corresponding spatial-temporal prewhitening stage and the para-unitary filters constraint are more difficult to realize.

FastICA\[2\] is a contrast function method in instantaneous BSS by maximizing the approximation of negentropy\[14\]. It works well in instantaneous case for its fast convergence.
and robustness. Unfortunately, it cannot directly be adapted to the convolutive BSS due to the filtering ambiguity caused by the time delay. Several convolutive extensions of FastICA have been proposed in [12] in recent years. The method proposed in [12] conducts a convolutive prewhitening to the transformed observation $x(k)$ at first, then removes the extracted estimations $y_i$ at previous steps from the mixtures $x$ in a deflation mode, leading to an accumulation of estimation errors which may become excessive after a certain number of source extractions. In [13], a spatial-temporal prewhitening stage and the para-unitary constraint is implemented at the cost of high computation burden. The existing FastICA extensions in convolutive BSS are difficult and inefficient.

In this paper, we propose a novel extension of FastICA, under a simpler framework. The proposed modifications combine a convolutive prewhitening stage for observations with the diagonalization constraint in both deflation or symmetric mode.

The rest of the paper is organized as follows. In Section II we state several assumptions of our method and rearrange the convolutive mixtures into instantaneous mixtures. An optimization problem based on the FastICA is described in Section III. We describe the prewhitening strategy along with the diagonalization constraint in Section IV. Experiments’ results are presented to verify our modifications in Section V. We conclude our method in Section VI.

II. PROBLEM STATEMENT

We describe the mixing system $A$ by the following FIR filter equation,

$$x_i(k) = \sum_{j=1}^{m} \sum_{l=0}^{P-1} a_{ij}(l)s_j(k-l) \quad i = 1, \cdots, n$$

where the $a_{ij}$ are the mixing filters. Without loss of generality, we assume all the mixing filters have the same filter order $P$. The estimation of the demixing system $B$ shares the same form,

$$y_i(k) = \sum_{j=1}^{m} \sum_{l=0}^{Q-1} b_{ij}(l)x_j(k-l) \quad i = 1, \cdots, m$$

where the $b_{ij}$ are the demixing filters and we assume all the demixing filters have the same filter order $Q$. $y_i(k)$ is a unique scaled, permuted, and filtered version of $s_i(k)$. To reconstruct the contributions in each observation $x_i(k)$, another assumption is considered:

1) Each signal source $s_i(k)$ is produced by an innovation process $u_i(k)$ via a stable FIR filters $F_i(k)$, where $u_i(k)$ is zero-mean, mutually independent and non-gaussian random process.

Then, signal source $s_i(k)$ can be expressed in the following form,

$$s_i(k) = \sum_{l=-R+1}^{R-1} F_i(l)u_i(k-l) \quad i = 1, \cdots, m$$

The order of the non causal FIR $F_i$ is $2R - 1$, all the filter banks $F(k) = diag(F_1(k), \cdots, F_m(k))$ are connected with innovation process $u(k) = (u_1(k), \cdots, u_m(k))^T$ respectively. In order to individually extract $s_i$, we regard the $F(k)$ as the coloring filters for the innovation process $u(k)$.

To rearrange the convolutive BSS to instantaneous BSS, we created several variables as follow,

$$s(k) = (s_1^T(k), s_2^T(k), \cdots, s_m^T(k))^T$$

$$s_i(k) = (s_i(k), s_i(k-1), \cdots, s_i(k-P+1))^T$$

where $s(k)$ is a $mP \times 1$ column vector, and $s_i(k)$ is a $P \times 1$ column vector. The mixing system $A$ can be expressed in the matrix form,

$$A = \begin{pmatrix} a_{11}^T & a_{12}^T & \cdots & a_{1m}^T \\ a_{21}^T & a_{22}^T & \cdots & a_{2m}^T \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1}^T & a_{n2}^T & \cdots & a_{nm}^T \end{pmatrix}$$

$$a_{ij} = (a_{ij}(0), a_{ij}(1), \cdots, a_{ij}(P-1))^T$$

where $A$ is a $n \times mP$ matrix, and $a_{ij}$ is $P \times 1$ column vector. The mixing system (3) becomes,

$$x(k) = As(k)$$

The demixing process can be transformed in the same way.

$$x(k) = (x_1^T(k), x_2^T(k), \cdots, x_n^T(k))^T$$

$$x_i(k) = (x_i(k), x_i(k-1), \cdots, x_i(k-Q+1))^T$$

where $x(k)$ is a $nQ \times 1$ column vector, and $x_i(k)$ is a $Q \times 1$ column vector.

$$B = \begin{pmatrix} b_{11}^T & b_{12}^T & \cdots & b_{1m}^T \\ b_{21}^T & b_{22}^T & \cdots & b_{2m}^T \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1}^T & b_{n2}^T & \cdots & b_{nm}^T \end{pmatrix}$$

$$b_{ij} = (b_{ij}(0), b_{ij}(1), \cdots, b_{ij}(Q-1))^T$$

where $B$ is a $m \times nQ$ matrix, and $b_{ij}$ is $Q \times 1$ column vector.

$$y(k) = Bx(k)$$

Owing to the concise expressions of the mixing process (10) and demixing process (15), we have converted the $m$ signals $n$ observations convolutive BSS into a $m$ signals $nQ$ observations instantaneous BSS problem. For the sake of uniqueness in extraction, a prewhitening stage is required in contrast function methods:

$$v(k) = Hx(k)$$

we represent the prewhitening stage in function $H()$, and the prewhitening output $v(k)$ should comply with some constraint. In instantaneous case, the particular step can be conducted via eigenvalue decomposition or PCA.

$$v(k) = Hx(k)$$
Fig. 1. Flow chart of the convolutive BSS.

where \( v(k) \) conforms to the below constraint.

\[
E\{v(k)v^T(k)\} = I_{n \times n} \tag{18}
\]

Unfortunately, the above prewhitening strategy fails in the convolutive case, more details are provided in Section IV.

\[
y(k) = Bx(k) = WHx(k) = Wv(k) \tag{19}
\]

After prewhitening stage, our method \((19)\) adjusts coefficients in separation matrix \( W \) to recover the \( y_i(k) \) as an estimation of a delayed scaled innovation process \( u_i(k-l) \). The complete routine of the convolutive BSS can be concluded in Fig 1. The source’s component \( s(k) \) is the output of the innovation process \( u(k) \) filtered by the coloring filters \( F \). Unknown mixing system \( A \) mixes the \( s(k) \) both in time and space. Given the observation \( x(k) \), the proposed method conducts a demixing procedure \( B \) to recover \( y(k) \). The prewhitening \( H \) stages and seperating \( W \) stages are the keys in the demixing system.

III. FastICA Extension

In the instantaneous case, FastICA looks for a sequence of orthogonal projections to maximize the negentropy \( J(y_i) \) \([14]\), which amounts to seek components as independent as possible.

\[
J(y_i) = H(z_i) - H(y_i) \\
\approx c [E\{G(y_i)\} - E\{G(z_i)\}]^2 \tag{20}
\]

where \( J(y_i) \) is the negentropy, \( H() \) is the random variable’s entropy, \( c \) is an irrelevant constant, \( G() \) is any non-quadratic function, \( z_i \) is a Gaussian random variable with the same variance as \( y_i \). Mutual information \( I(y) \) between the components of the random variable vector \( y(k) \) is a nature measure of dependence. It is always non-negative and becomes zero only when the components are statistically independent.

\[
I(y) = \sum_{i=1}^{m} H(y_i) - H(y) \\
= \sum_{i=1}^{m} H(y_i) - H(x) - \log |\text{det}(B)| \tag{21}
\]

\[
C - \sum_{i=1}^{m} J(y_i)
\]

where \( C \) is an irrelevant constant, the minimization of the mutual information \( I(y) \) for independence (under the constraint of decorrelation) is equivalent to the maximization of the sum of the negentropies of the components \( \sum_{i=1}^{m} J(y_i) \), and the original FastICA in instantaneous can be modeled as the optimization problem \((22)\) below,

\[
\begin{align*}
\max & \quad \sum_{i=1}^{m} J(y_i) \\
\text{s.t.} & \quad E\{y_i(k)y_j(k)\} = \delta_{ij}, \quad i,j = 1,2,\ldots,m.
\end{align*} \tag{22}
\]

where \( \delta_{ij} \) is the item in identity matrix \( I_{m \times m} \). The objective function in \((22)\) aims at the maximization of independence. In both the deflation and symmetric mode, the optimization problem in \((22)\) is divided as single maximization of \( J(y_i) \), and there are \( 2m \) extreme points to this problem due to the ordering and scaling ambiguities in instantaneous case. The constraint in \((22)\) is designed to avoiding extracting the same solution more than once.

While in the convolutive case, the filtering ambiguity introduces much more extreme points as the increasing of the demixing filters \( B \)’s filter order \( Q \), resulting in the failure of the same strategies in instantaneous BSS. It’s straightforward to change the equation \((22)\) in the convolutive context,

\[
\begin{align*}
\max & \quad \sum_{i=1}^{m} J(y_i) \\
\text{s.t.} & \quad E\{y_i(k)y_j(k)\} = 1, \quad i = 1,2,\ldots,m, \\
E\{y_i(k)y_j(k-l)\} = 0, \quad i \neq j, -\infty < l < +\infty. \tag{23}
\end{align*}
\]

The first constraint in \((23)\) is for the scaling ambiguity, and the second constraint is designed to tack the ordering and filtering ambiguities. In order to simplify the second constraint, we choose \(-L \leq l \leq L\), where \( L \) is a positive large enough integer. Prewhitening stage and diagonalization constraints in our modifications are used to satisfy these constraints.

IV. Prewhitening and Diagonalization Constraints

Many contrast function methods for convolutive mixtures have a prewhitening stage \((16)\) in space and time \([3][13]\).

\[
E\{v(k)v^T(k-l)\} = \delta_l I_{n \times n} \quad \forall l \in \mathbb{Z} \tag{24}
\]

The whitening filter \( H \) is not unique and hard to determine, the contrast functions are required to be optimized under the constraint of complicate para-unitary filters \( W \). Although several methods have been proposed to alleviated these difficulties \([15][16]\), they are both difficult and demands mass computing. We conduct a same prewhitening stage in instantaneous case \([5]\) on \( z(k) \) to produce the \( nQ \times 1 \) column vector \( v(k) \), which is regarded as the convolutive prewhitening.

\[
E\{v(k)v^T(k)\} = I_{nQ \times nQ} \tag{25}
\]

After the above prewhitening stage, the main effort is to adjust the seperating filters \( W \) to optimize equation \((23)\),

\[
\begin{align*}
\max & \quad \sum_{i=1}^{m} J(w_i^T v(k)) \\
\text{s.t.} & \quad w_i^T w_i = 1, \quad i = 1,2,\ldots,m, \\
w_i^T E\{v^T(k)v^T(k-l)\} w_j = 0, \quad i \neq j, -L \leq l \leq L. \tag{26}
\end{align*}
\]
where \( w_i^T \) is the \( i \)th row of \( m \times nQ \) matrix \( W \). In pursuit of particular \( w_i \), the following iteration routine is carried out until convergence.

\[
\begin{align*}
    w_i &= E\{g(k)(w_i^Tv(k))\} - E\{g'(w_i^Tv(k))\}w_i \\
    w_i &= w_i/\|w_i\|_2
\end{align*}
\]  

(27)

where \( g() \) is the derivative of particular non-quadratic function \( G() \), coefficients constraints are required during the process of (27) in deflation and symmetric mode.

The validity of constraint in (23)-(26) can also supported from [51]. The fact: If the source signals have unique temporal structures or non-stationary, simultaneous diagonalization of output correlation matrices over multiple time lags can separate the independent sources from convolutive mixtures.

\[
\begin{align*}
    R_y(\tau) &= E\{y(k)y^T(k-(\tau))\} \\
    &= WE\{y(k)y^T(k-(\tau))\}W^T
\end{align*}
\]  

(28)

The output correlation matrix at time lag \( \tau \) is represented as \( R_y(\tau) \), it is required to be a diagonal matrix, particularly, \( R_y(0) \) is the identity matrix \( I_{m\times m} \). For more intuitive explanation, we define several column vectors.

\[
y(k) = \begin{pmatrix} y_1^T(k), y_2^T(k), \ldots, y_m^T(k) \end{pmatrix}^T
\]

(29)

\[
y_i(k) = (y_i(k+L), y_i(k+L-1), \ldots, y_i(k-L))^T
\]  

(30)

where \( y(k) \) is a \((2L+1)\times 1\) column vector, and \( y_i(k) \) is a \((2L+1)\times 1\) column vector, the equivalent expression in (28) can be described below.

\[
\begin{pmatrix}
    R_{y_1y_1} & R_{y_1y_2} & \cdots & R_{y_1y_m} \\
    R_{y_2y_1} & R_{y_2y_2} & \cdots & R_{y_2y_m} \\
    \vdots & \vdots & \ddots & \vdots \\
    R_{y_my_1} & R_{y_my_2} & \cdots & R_{y_my_m}
\end{pmatrix}
\]  

(31)

\[
    R_{y_iy_j} = E\{y_i(k)y_j^T(k)\}
\]  

(32)

where \((2L+1)\times(2L+1)\) matrix \( R_{y_iy_j} \) is the combination of correlation matrix concerning different time lags. According to the constraint in (23) and the nonstationarity property of source signals, the \((2L+1)\times(2L+1)\) matrix \( R_{y_iy_j} \) becomes nonzero only in the diagonal position of \( R_{y_iy_j} \). We consider the coefficients constraints above as diagonalization constraints.

Compared with the existing para-unitary constraint, the diagonalization constraints are relaxed, it can be efficiently imposed via singular value decomposition in both deflation and symmetric mode.

A. deflation mode

When one of the sources’ estimation \( y_i(k) \) has been extracted, it’s necessary to subtract its contribution from the observations to obtain the mixtures of \( m-1 \) sources, then we repeat this procedure to extract the remaining sources one by one, until all the sources have been extracted.

Considering the constraints in (26), we construct the block matrix \( O \) during the extraction of the \( y_i(k) \) to guarantee uniqueness.

\[
O = (R_o(-L)w_1, R_o(-L+1)w_1, \ldots, R_o(L)w_{i-1})
\]  

(33)

\[
w_i^TO = 0
\]  

(34)

The \( w_i \) in the \( i \)th extraction is required to be orthogonal to the column space of the block matrix \( O \), we represent the column space of \( O \) as \( span\{O\} \).

If the block matrix \( O \) is overdetermined, the column space \( span\{O\} \) is not full column rank, it’s easy adjust the \( w_i \) via least square solution.

\[
w_i = w_i - (O^TO)^{-1}O^Tw_i
\]  

(35)

Unfortunately, \( O \) is often designed to be underdetermined due to choice of large \( L \), so the strategy in (35) cannot work any longer. The proposed method draws lessons from the principal component analysis based on singular value decomposition, only taking the directions with most variation in \( O \) into consideration.

\[
O = U\Sigma V^T
\]  

(36)

Here \( U \) and \( V \) are orthogonal matrices, with the columns of \( U \) spanning the column space of \( O \), and the columns of \( V \) spanning the row space. \( \Sigma \) is a diagonal matrix, with diagonal entries in decreasing order.

\[
\sigma_{11} \geq \sigma_{22} \geq \cdots \geq 0
\]  

(37)

The algorithm picks the first \( r \) columns vectors in \( U \) to obtain the most variance in original \( span\{O\} \) based on the effective rank of \( O \), \( r \) is the minimum integer when \( \mu(r) \) is greater than particular threshold \( \alpha \) (such as 0.99995).

\[
\mu(r) = \sqrt{\sigma_{11}^2 + \cdots + \sigma_{r}^2}/\|\Sigma\|_F
\]  

(38)

The most variant \( r \) column vectors in \( U \) is represented as \( U_{(r)} \), and each column vector in \( U_{(r)} \) is orthogonal with each other, \( U_{(r)} \) is always high matrix, the (35) can be described in simple form.

\[
w_i = w_i - U_{(r)}U_{(r)}^Tw_i
\]  

(39)

The FastICA convolutive algorithm in deflation mode is summarized in Alg.1 where \( tol \) is the threshold in iteration (such as \( 10^{-7} \)).

B. symmetric mode

Every extracted source has different priority in deflation mode, leading an accumulation of estimation errors. Symmetric mode is proposed to extract \( w_i \) equally. Supposing only the \( w_i \) is in process, other extracted sources are fixed. It’s nature to construct the block matrix from (26)-(33).

\[
O = (R_o(-L)w_1, \ldots, R_o(L)w_{i-1}, R_o(-L)w_{i+1}, \ldots)
\]  

(40)

The FastICA convolutive algorithm in symmetric mode is similar to the deflation mode, and it can be summarized in Alg2.
Algorithm 1 convolutive FastICA: deflation mode

\[ W = 0_{m \times nQ} \]

\[ i = 1 \]

\[ \text{repeat} \]

\[ w_i = 0_{nQ \times 1} \]

\[ O = (R_u(-L)w_1, R_u(-L+1)w_1, \ldots, R_u(L)w_{i-1}) \]

\[ \text{calculate } U_{(r)} \text{ from } \frac{36}{38} \text{ with threshold } \alpha \]

\[ \text{repeat} \]

\[ w_i = w_i \]

\[ w_i = E\{v(k)g(w_i^T v(k))\} - E\{g'(w_i^T v(k))\}w_i \]

\[ w_i = w_i - u_{(r)}U_{(r)}^T w_i \]

\[ w_i = w_i/\|w_i\|_2 \]

\[ \text{until } \|w_i^T w_i - 1.0\| \leq tol \]

\[ W[i,:] = w_i \]

\[ i = i + 1 \]

\[ \text{until } i > m \]

Algorithm 2 convolutive FastICA: symmetric mode

\[ W = 0_{m \times nQ} \]

\[ \text{repeat} \]

\[ W' = W \]

\[ i = 1 \]

\[ \text{repeat} \]

\[ w_i = W[i,:] \]

\[ O = (R_u(-L)w_1, \ldots, R_u(L)w_{i-1}, R_u(-L)w_{i+1}, \ldots) \]

\[ \text{calculate } U_{(r)} \text{ from } \frac{36}{38} \text{ with threshold } \alpha \]

\[ w_i = E\{v(k)g(w_i^T v(k))\} - E\{g'(w_i^T v(k))\}w_i \]

\[ w_i = w_i - u_{(r)}U_{(r)}^T w_i \]

\[ w_i = w_i/\|w_i\|_2 \]

\[ W[i,:] = w_i \]

\[ i = i + 1 \]

\[ \text{until } i > m \]

\[ \|W'W'^T - I_{m \times m}\|_2 \leq tol \]

C. sources’ reconstruction

After the extraction of \(v(k)\) in deflation or symmetric mode, we get the estimation of the ordered, scaled, and delayed version of innovation process \(u(k)\). Then we conduct a reconstruction process to recover the signals \(s(k)\)’s contributions in the observation mixtures \(x(k)\). In this part, for the convenience of discussing, the ordering ambiguity is ignored.

For the purpose of achieve innovation \(s_i(k)\)’s contributions in observation \(x_j(k)\), a \((2L + N) \times (2L + 1)\) high matrix \(T\) is built to represent the column space via zero padding, including all the possible forward and backward time shift of \(y_i\).

\[
T = \begin{pmatrix}
y_i(0) & 0 & \cdots & 0 & 0 \\
y_i(1) & y_i(0) & \cdots & 0 & 0 \\
y_i(2) & y_i(1) & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & y_i(N - 2) & y_i(N - 3) \\
0 & 0 & \cdots & y_i(N - 1) & y_i(N - 2) \\
0 & 0 & \cdots & 0 & y_i(N - 1)
\end{pmatrix}
\]  

(41)

\(N \times 1\) column vector \(x_j\) is padding with \(L\) zeros both forward and backward.

\[
x_j^{(2L+N)} = (0, 0, \cdots, x_j(0), x_j(1), \cdots, 0, 0)^T
\]  

(42)

The reconstruction is based on the regression opinion: regress \(x_j^{(2L+N)}\) on the column space of \(T\), finding the closest \(s_{ij}\) in the \(\text{span}\{T\}\) in least square sense. We describe the \(s_i\) contribution to observation \(x_j\) as \(\hat{s}_{ij}\).

\[
\hat{s}_{ij} = (T^T T)^{-1}T^T x_j^{(2L+N)}
\]  

(43)

V. Experiment results

We conducted two experiments in this section to explore the performance of the proposed algorithm.

The first experiment mixed the given innovation process \(u_1\) and \(u_2\) in Fig.\ref{fig:2} both satisfying the referred assumptions. After the convolutive prewhitening stage, the algorithm in symmetric mode recovered the \(y_1\) and \(y_2\), and calculated their contributions on each observation.

In Fig.\ref{fig:3} the recovered estimation \(y_1\) and \(y_2\) are similar to the original innovation process, while there are little deviations in the original zero amplitude regions. The contributions in Fig.\ref{fig:5} happened to be the same as innovation process. In this simulation, the algorithm accomplished the mission of convolutive BSS.

A more difficult task was considered in real recorded source signals from the public data of Salk Institute\cite{13}.

1) \(s_1\) : speaker says the digits from one to ten in English.
2) \(s_2\) : loud music in the background.

We conducted deflation mode in Alg.\ref{alg:1} to produce the estimations and contributions. The results in Fig.\ref{fig:5} showed the similarity between the calculated contributions and original source signals.

VI. Conclusion

In this paper, we have derived a novel extension of the FastICA for convolutive mixtures that enforces the diagonalization constraints on the separating filters for uniqueness. Our
algorithm also has simple convolutive prewhitening stages and contributions’ reconstruction procedure. Experiments are given to illustrate the performance of the proposed algorithm.

Our algorithm enjoys the robustness and fast convergence due to its fix-point iterations, no particular parameter tuning is required during the optimization. Compared with spatial-temporal prewhitening stages and para-unitary filter constraints in other contrast function methods [3], the corresponding procedures in our algorithm are much more straightforward and simpler.

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