CHIRAL EFFECTIVE THEORIES

205. WE-Heraeus-Seminar
Physikzentrum Bad Honnef, Bad Honnef, Germany
November 30 — December 4, 1998

Johan Bijnens
Department of Theoretical Physics 2, Lund University
Sölvegatan 14A, S22362 Lund, Sweden

Ulf-G. Meißner
FZ Jülich, IKP(Th), D-52425 Jülich, Germany

ABSTRACT
These are the proceedings of the workshop on “Chiral Effective Theories” held at the Physikzentrum Bad Honnef of the Deutsche Physikalische Gesellschaft, Bad Honnef, Germany from November 30 to December 4, 1998. The workshop concentrated on Chiral Perturbation Theory in its various settings and its relations with lattice QCD and dispersion theory. Included are a short contribution per talk and a listing of some review papers on the subject.
1 Introduction

The field of Chiral Perturbation Theory is a growing area in theoretical physics. We therefore decided to organize the next topical workshop. This meeting followed the series of workshops in Ringberg (Germany), 1988, Dobogókő (Hungary), 1991, Karrebæksminde (Denmark), 1993 and Trento (Italy), 1996. All these workshops shared the same features, about 50 participants, a fairly large amount of time devoted to discussions rather than presentations and an intimate environment with lots of discussion opportunities.

This meeting took place in late fall 1998 in the Physikzentrum Bad Honnef in Bad Honnef, Germany and the funding provided by the Dr. Wilhelm Heinrich Heraeus und Else Heraeus–Stiftung allowed us to provide for the local expenses for all participants and to support the travel of some participants. The WE-Heraeus foundation also provided the administrative support for the workshop in the person of the able secretary Jutta Lang. We extend our sincere gratitude to the WE-Heraeus Stiftung for this support. We would also like to thank the staff of the Physikzentrum for the excellent service given us during the workshop and last but not least the participants for making this an exciting and lively meeting.

The meeting had 53 participants whose names, institutes and email addresses are listed below. 43 of them presented results in presentations of various lengths. A short description of their contents and a list of the most relevant references can be found below. As in the previous two of these workshops we felt that this was more appropriate a framework than full-fledged proceedings. Most results are or will soon be published and available on the archives so this way we can achieve speedy publication and avoid duplication of results in the archives.

Below follows first the program, then the list of participants and a subjective list of review papers, lectures and other proceedings relevant to the subject of this workshop.

Johan Bijnens and Ulf-G. Meißner
2 The Program

Monday
12.30 Lunch
14.00 U.-G. Meißner
   E. Dreisigacker Opening and Welcome
14.20 G. Ecker Introduction to CHPT
14.35 G. Ecker The generating functional at next-to-next-to-leading
   order in CHPT
15.25 P. Talavera Pion form factors at $p^6$
15.50 Coffee
16.20 M. Golterman Quenched and Partially Quenched Chiral Logarithms
17.00 E. Pallante The Generating Functional for Hadronic Weak
   Interactions and its Quenched Approximation
17.40 H. Neufeld A Super-Heat-Kernel Representation for the One-Loop
   Functional of Boson-Fermion systems
18.10 End of Session
18.30 Dinner followed by a social evening

Tuesday
9.00 S. G"usken Some Low-Energy constants from lattice QCD
9.40 A. Vladikas Quark masses and the chiral condensate
   from the lattice
10.10 J. Donoghue Dispersive Calculation of Weak Matrix Elements
10.55 Coffee
11.25 J. Prades $\Delta S = 1$ Transitions in the $1/N_c$ Expansion
12.05 H.-C. Kim Effective $\Delta S = 1$ Chiral Lagrangian in the Chiral
   Quark Model
12.35 G. D’Ambrosio Vector Meson Dominance in the Delta S=1
   Weak Chiral Lagrangian
13.00 End of Session
14.20 M. Knecht Electromagnetic Corrections to Semi-Leptonic Decays
   of Pseudoscalar mesons
15.00 B. Moussalam An estimate of the $p^1$ EM contribution to $M_{\pi^+} - M_{\pi^0}$
15.30 S. Peris Matching Long and Short Distances in large $N_c$ QCD
16.00 Coffee
16.40 E. de Rafael QCD at large-$N_c$ and Weak matrix Elements
17.25 A. Rusetsky Bound States with Effective Lagrangians
17.55 End of session
**Wednesday**

| Time  | Speaker     | Title                                                                 |
|-------|-------------|----------------------------------------------------------------------|
| 9.00  | B. Borasoy  | Long Distance Regularization and Chiral Perturbation Theory           |
| 9.30  | J. Kambor   | Generalized Heavy Baryon Chiral Perturbation Theory and the Nucleon Sigma term |
| 10.10 | T. Hemmert  | Baryon CHPT and Resonance Physics                                     |
| 10.50 | Coffee      |                                                                      |
| 11.20 | M. Mojzic   | Nucleon properties to $O(p^4)$ in HBCHPT                              |
| 11.50 | N. Fettes    | Pion-Nucleon scattering in Chiral Perturbation theory                 |
| 12.20 | G. Höhler   | Relations of Dispersion Theory to Chiral Effective Theories for $\pi N$ Scattering |
| 13.00 | End of Session |                                               |
| 14.20 | S. Steininger | Isospin violation in the Pion Nucleon System            |
| 14.50 | M. Sainio   | Goldberger-Miyazawa-Oehme Sum Rule Revisited                      |
| 15.20 | E. Epelbaoum | Low-Momentum Effective Theory for Two Nucleons                    |
| 15.50 | Coffee      |                                                                      |
| 16.20 | M. Savage   | Effective Field Theory in the Two-Nucleon Sector                    |
| 17.00 | N. Kaiser   | Chiral Dynamics of NN Interaction                                    |
| 17.35 | S. Beane    | Compton scattering off the Deuteron in HBCHPT                       |
| 18.10 | End of session |                                               |

**Thursday**

| Time  | Speaker     | Title                                                                 |
|-------|-------------|----------------------------------------------------------------------|
| 9.00  | H. Leutwyler | $1/N_c$ Expansion in CHPT and Lorentzinvvariant HBCHPT: Introduction |
| 9.30  | T. Becher   | HBCHPT in a Lorentzinvariant Form                                    |
| 10.10 | R. Kaiser   | Chiral $SU(3) \times SU(3)$ at large $N_C$: The $\eta - \eta'$ system |
| 10.50 | Coffee      |                                                                      |
| 11.30 | D. Diakonov | Derivation of the Chiral Lagrangian from the Instanton Vacuum        |
| 12.15 | J. Stern    | Theoretical Study of Chiral Symmetry Breaking: Recent Developments   |
| 13.00 | End of Session |                                               |
| 14.20 | G. Colangelo | Numerical Solutions of Roy Equations                                |
| 15.00 | J. Gasser   | The One-Channel Roy Equation                                        |
| 15.35 | Coffee      |                                                                      |
| 16.05 | G. Wanders  | How do the Uncertainties on the Input Affect the Solution of the Roy Equations? |
| 16.40 | J. Schechter | Low Energy $\pi K$ Scattering and a Possible Scalar Nonet             |
| 17.20 | A. Pich     | CHPT Phenomenology in the Large-$N_C$ Limit                         |
| 18.00 | End of session |                                               |
Friday
9.00  A. Nyffeler  Gauge Invariant Effective Field Theory for a Heavy Higgs Boson
9.35  J. Goity  The Goldberger-Treiman Discrepancy in the Chiral Expansion
10.10 L. Girlanda  Tau decays and Generalized CHPT
10.35 Coffee Break
11.15 F. Orellana  Different Approaches to Loop Calculations in ChPT Exemplified with the Meson Form Factors
11.40 D. Toublan  From Chiral Random Matrix Theory to Chiral Perturbation Theory
12.10 J. Bijnens  Farewell

3 Participants and their email

S. Beane  Maryland  sbeane@physics.umd.edu
T. Becher  Bern  becher@itp.unibe.ch
V. Bernard  Strasbourg  bernard@lpt1.u-strasbg.fr
J. Bijnens  Lund  bijnens@theplu.se
B. Borasoy  Amherst  borasoy@het.phast.umass.edu
P. Buettiker  FZ Jülich  p.buettiker@fz-juelich.de
G. Colangelo  Zürich  gilberto@physik.unizh.ch
G. D’Ambrosio  Naples  dambrosio@na.infn.it
D. Diakonov  Nordita  diakonov@nordita.dk
J. Donoghue  Amherst  donoghue@phast.umass.edu
G. Ecker  Wien  ecker@merlin.pap.univie.ac.at
E. Epelbaum  FZ Jülich  e.epelbaum@fz-juelich.de
N. Fettes  FZ Jülich  n.fettes@fz-juelich.de
M. Franz  Bochum  mario.franz@tp2.ruhr-uni-bochum.de
J. Gasser  Bern  gasser@itp.unibe.ch
L. Girlanda  Orsay  girlanda@ipno.in2p3.fr
K. Goeke  Bochum  goeke@hadron.tp2.ruhr-uni-bochum.de
J. Goity  Jefferson Lab  goity@cebaf.gov
M. Golterman  St.Louis  maarten@aapje.wustl.edu
S. Gütschn  Wuppertal  guesken@theorie.physik.uni-wuppertal.de
T. Hemmert  FZ Jülich  th.hemmert@fz-juelich.de
G. Höhler  Karlsruhe  gerhard.hoechler@physik.uni-karlsruhe.de
N. Kaiser  TU München  nkaiser@physik.tu-muenchen.de
R. Kaiser  Bern  kaiser@itp.unibe.ch
J. Kambor  Zürich  kambor@physik.unizh.ch
H.-C. Kim  Pusan  hchkim@hyowon.pusan.ac.kr
M. Knecht  Marseille  knecht@cpt.univ-mrs.fr
4 A short guide to review literature

Chiral Perturbation Theory grew out of current algebra, and it soon was realized that certain terms beyond the lowest order were also uniquely defined. This early work and references to earlier review papers can be found in [1]. Weinberg then proposed a systematic method in [2], later systematized and extended to use the external field method in the classic papers by Gasser and Leutwyler [3],[4], which, according to Howard Georgi, everybody should put under his/her pillow before he/she goes to sleep. The field has since then extended a lot and relatively recent review papers are: Ref.[5] with an emphasis on the anomalous sector, Ref.[6] giving a general overview over the vast field of applications in various areas of physics, Ref.[7] on mesons and baryons, and Ref.[8] on baryons and multibaryon processes. In addition there are books by Georgi [9], which, however, does not cover the standard approach, including the terms in the lagrangian at higher order and a more recent one by Donoghue, Golowich and Holstein [10].

There are also the lectures available on the archives by H. Leutwyler [11], E. de Rafael [12], A. Pich [13], G. Ecker [14] as well as numerous others (the single nucleon sector is covered in most detail in [15]). The references to the previous meetings are [16],[17],[18],[19]. There are also the proceedings of the Chiral Dynamics meetings at MIT (1994) [20] and in Mainz (1997) [21]. The DAΦNE handbook [22] also contains useful overviews. The NN sector is covered in the proceedings of the Caltech workshop [23].

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[23] M. Savage, R. Seki and U. van Kolck (eds), Proceedings of the Workshop on Nuclear Physics with Effective Field Theories, Pasadena, CA, 26-27 Feb 1998, to be published.
The generating functional for Green functions of quark currents receives contributions from tree, one-loop and two-loop diagrams at $O(p^6)$. After a classification of these diagrams, I describe the construction of the effective chiral Lagrangian of $O(p^6)$ for a general number $N_f$ of light flavours. Special matrix relations reduce the number of independent terms substantially for $N_f = 3$ and 2. The $SU(3)$ Lagrangian is compared with the Lagrangian of Fearing and Scherer.

The one- and two-loop contributions to the generating functional are divergent. In a mass independent regularization scheme like dimensional regularization, the divergent parts are polynomials in masses and external momenta. The cancellation of nonlocal divergences is an important consistency check for the renormalization procedure. The double and single poles in $d - 4$ in the coefficients of the local divergences are then cancelled by the divergent parts of the coupling constants of $O(p^6)$. The dependence of the renormalization procedure on the choice of the chiral Lagrangian of $O(p^4)$ is discussed.

The divergence structure of the generating functional can be used to check specific two-loop calculations in chiral perturbation theory. Moreover, it provides the leading infrared singular pieces of Green functions, the chiral double logs. These double logs ($L^2$) come together with terms of the form $L \times L_i^r$ and products $L_i^r \times L_j^r$, involving also the low-energy constants $L_i^r(\mu)$ of $O(p^4)$. It is natural to include all such terms in the analysis especially because they are often comparable to or even bigger than the proper double-log terms. All contributions of this type to the generating functional can be given in closed form. This generalized double-log approximation is applied to several quantities of interest such as mesonic decay constants and form factors where complete calculations to $O(p^6)$ are not yet available. The results indicate where large $p^6$ corrections are to be expected.

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Pion Form Factors at $p^6$

J. Bijnens$^1$, G. Colangelo$^2$ and P. Talavera$^1$

$^1$Dep. Theor. Phys. 2, Lund University, Sölvegatan 14A, S22362 Lund, Sweden

$^2$Inst. Theor. Phys., Univ. Zürich, Winterthurerstr. 190, CH–8057 Zürich–Irchel, Switzerland.

We compute the vector and scalar form factors of the pion to two loops in CHPT and compare carefully with the existing data.

For the scalar form factor this involves a comparison with the form factor derived using dispersion theory and chiral constraints from the $\pi\pi$ phase shifts[1]. The CHPT formula fits well over the entire range of validity. Moreover, we show that using the “modified Omnès representation” which exponentiates the unitarity correction, the chiral representation improves and follows the exact form factor up to about 700 MeV.

For the vector form factor we collected all available data and performed the standard simple fits. We fit the CHPT formula at two loops together with a phenomenological higher order term to obtain the pion charge radius and $c_V^\pi$:

$$\langle r^2 \rangle_V^\pi = (0.437 \pm 0.016) \text{ fm}^2, \quad c_V^\pi = (3.85 \pm 0.60) \text{ GeV}^{-4}.$$  

The error is a combination of theoretical and experimental errors.

By comparing to the Taylor expansions of the measured form factors, we have been able to better determine some of the LEC’s: $\bar{l}_4$ and $\bar{l}_6$. $\bar{l}_6$ together with results from $\pi \to \nu \gamma$[2] lead then to $\bar{l}_5$.

$$\bar{l}_4 = 4.4 \pm 0.3, \quad \bar{l}_6 = 16.0 \pm 0.5 \pm 0.7 \quad \text{and} \quad \bar{l}_5 = 13.0 \pm 0.9.$$  

The other two LEC’s we determined are $\mathcal{O}(p^6)$ constants, that contribute to the quadratic term in the polynomial of the scalar and vector form factors. We found

$$r_{S3}^\pi(M_\rho) \simeq 1.5 \cdot 10^{-4}, \quad r_{V2}^\pi(M_\rho) \simeq 1.6 \cdot 10^{-4},$$  

with a substantial uncertainty. These values are rather close to those obtained with the resonance saturation hypothesis, supporting the idea that this hypothesis works also at order $p^6$. The full discussion can be found in [3].

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Quenched and Partially Quenched Chiral Logarithms

Maarten Golterman

Department of Physics, Washington University,
St. Louis, MO 63130, USA

In this talk, I reported on the recent quenched hadron spectrum results of the CP-PACS collaboration [1]. I discussed in particular the evidence for quenched chiral logarithms in the pion mass [2],[3]. In the second part, I gave an introduction to partially quenched chiral perturbation theory [4],[5],[6],[7], and discussed its relevance for the analysis of future results from lattice QCD.

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The Generating Functional for Hadronic Weak Interactions and its Quenched Approximation

Elisabetta Pallante

1Facultat de Física, Universitat de Barcelona,
Av. Diagonal 647, 08028 Barcelona, Spain.

Chiral Perturbation Theory (ChPT) combined with lattice QCD is a promising tool for computing hadronic weak matrix elements at long distances. Here I discuss the derivation of the one loop generating functional of ChPT in the presence of weak interactions with $|\Delta S| = 1, 2$ and the modifications induced by the quenched approximation adopted on the lattice. The framework I use is known as quenched ChPT \([1]\), while its extension to hadronic weak interactions can be found in \([2]\) and refs. therein. The advantage of deriving the generating functional is twofold: first, it allows for a systematic control on the quenched modifications (i.e. how much the coefficients of the chiral logs are modified by quenching) and second, it gives in one step the coefficients of all chiral logarithms for any Green’s function or S-matrix element, full and quenched. The main relevant modification induced by quenching is the appearance of the so called quenched chiral logs both in the strong and weak sector. They can be accounted for via a redefinition of the leading order parameters associated to the mass-like terms (the usual mass term and the weak mass term). As an immediate application, the full and quenched behaviours of the chiral logarithms which appear in $K \rightarrow \pi\pi$ matrix elements can be studied both for $\Delta I = 1/2$ and $3/2$. The numerical analysis shows that the modification induced by quenching follows a pattern that tends to suppress the $\Delta I = 1/2$ dominance.

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A Super-Heat-Kernel Representation for the One-Loop Functional of Boson–Fermion Systems

Helmut Neufeld

Institut für Theoretische Physik der Universität Wien,
Boltzmannngasse 5, A-1090 Wien, Austria.

The one-loop functional of a general quantum field theory with bosons and fermions can be written in terms of the super-determinant of a super-matrix differential-operator. This super-determinant is then further evaluated by using heat-kernel methods. This approach [1] corresponds to a simultaneous treatment of bosonic, fermionic and mixed loop-diagrams. The determination of the one-loop divergences is reduced to simple matrix manipulations, in complete analogy to the familiar heat-kernel expansion technique for bosonic or fermionic loops.

Applications to the renormalization of the pion–nucleon interaction [2] and to chiral perturbation theory with virtual photons and leptons [3] demonstrate the efficiency of the new method. The cumbersome and tedious calculations of the conventional approach are now reduced to a few simple algebraic manipulations. The presented computational scheme is, of course, not restricted to chiral perturbation theory, but can easily be applied or extended to any (in general non-renormalizable) theory with boson–fermion interactions.

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Some Low Energy Constants from Lattice QCD: Recent Results

Stephan Güsken
Dep. Phys., University of Wuppertal,
42097 Wuppertal, Germany

Hadronic properties at low energies are sensitive to non-perturbative contributions from quantum fluctuations. In particular, flavour singlet quantities like the pion-nucleon-sigma term $\sigma_{\pi N}$ and the flavour singlet axial coupling of the proton $G_A^1$ might be determined largely by so called disconnected insertions. These are given by the correlation of the nucleon propagator with a quark-antiquark vacuum loop.

Recently the SESAM collaboration has performed a full QCD lattice simulation with $n_f = 2$ dynamical fermions on 4 different values of the sea quark mass, and with a statistics of 200 independent gauge configurations per sea quark [1]. In this talk we present the results of the analysis of these gauge configurations with respect to $\sigma_{\pi N}$ [2] and $G_A^1$ [3].

SESAM finds a quite low value for the pion-nucleon-sigma term, $\sigma_{\pi N} = 18(5)\text{MeV}$. Its smallness is directly related to the apparent decrease of light quark masses when unquenching lattice QCD simulations [4],[5].

For the flavor singlet axial coupling of the proton, SESAM estimates $G_A^1 = 0.20(12)$, consistent with the experimental result and with previous findings from quenched simulations [1].

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Quark Masses and Chiral Condensate from the Lattice

Anastassios Vladikas

INFN c/o Department of Physics, Università di Roma “Tor Vergata”,
Via della Ricerca Scientifica 1, I-00133 Rome, Italy

Ward Identities can be used in order to measure, from first principles, the light quark masses and the chiral condensate in lattice QCD. Particular attention is paid to the problem of chiral symmetry breaking by the Wilson action and its restoration in the continuum limit [1]. The main sources of systematic errors in computations are: (1) quenching; (2) finite mass extrapolation to the chiral limit; (3) scaling violations; (4) renormalization of lattice operators in 1-loop perturbation theory. Scaling violations can be reduced by applying Symanzik-improvement to Wilson fermions, as reviewed in [2]. Operator renormalization can be carried out non-perturbatively; see [3].

All results are in the $\overline{MS}$ scheme at renormalization scale $\mu = 2\text{GeV}$:

$$\langle \bar{\psi} \psi \rangle = -[245 \pm 15 \text{MeV}]^3 \quad [4]$$
$$\langle \bar{\psi} \psi \rangle = -[253 \pm 25 \text{MeV}]^3 \quad [3]$$
$$m_{u,d} = 5.7 \pm 0.8 \text{MeV} \quad [4]$$
$$m_{u,d} = 4.5 \pm 0.4 \text{MeV} \quad [5]$$
$$m_{u,d} = 4.6 \pm 0.2 \text{MeV} \quad [6]$$

Important open questions remain: (1) the dependence of the strange quark mass on the bare mass calibration from the $\phi$- or the $K$-meson [6]; (2) current unquenched quark mass results appear to be smaller by about 30%.

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I described a project with Eugene Golowich in which we provide a rigorous calculational framework for certain weak non-leptonic matrix elements, valid in the chiral limit. This involves relating the weak operators to the vacuum polarization functions of vector and axial-vector currents. These functions obey dispersion relations and the inputs to these are largely known from experiment, and there are firm theoretical constraints. Certain aspects of this program were accomplished a few years ago [1], and we explored the use of data for the Weinberg sum rules and for this weak calculation [2]. What is new now is the understanding of how this fits into the operator product expansion, and the separate determination of two local operators (those related to $O_7$ and $O_8$ in the usual basis). We define these matrix elements at a scale $\mu$ in the $\overline{\text{MS}}$ scheme, and verify the renormalization group running and the OPE structure. An updated phenomenological analysis incorporating new data was described and will be given in the upcoming publication [3].

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\( \Delta S = 1 \) Transitions in the \( 1/N_c \) Expansion

Johan Bijnens\(^1\) and Joaquim Prades\(^2\)

\(^1\)Department of Theoretical Physics 2, Lund University,
Sölvegatan 14A, S-22362 Lund, Sweden

\(^2\)Departamento de Física Teórica y del Cosmos, Universidad de Granada,
Campus de Fuente Nueva, E-18002 Granada, Spain

In this talk we present the results obtained in a recent work on the \( \Delta I = 1/2 \) rule in the chiral limit \([1]\). In particular, we discuss the matching between long- and short-distance contributions at next-to-leading in a \( 1/N_c \) expansion and show how the scheme-dependence from the two-loop renormalization group running can be treated. We then use this method to study the three \( O(p^2) \) couplings modulating the terms contributing to non-leptonic kaon decays, namely the usual octet and 27-plet derivative terms as well as the weak mass term. We use the Extended Nambu–Jona-Lasinio model as the low energy approximation.

The known unsatisfactory high energy behaviour at large \( N_c \) of this model we treat as explained in \([1]\). For attempts to avoid this problem see the talks by Santi Peris and Eduardo de Rafael. At present, their method cannot be applied to the quantities considered here.

Reasonable matching for the three \( O(p^2) \) couplings introduced before is obtained. We predict them within ranges and obtain a huge enhancement of the \( \Delta I = 1/2 \) amplitude with respect to the \( \Delta I = 3/2 \) one. This we identify to come from \( Q_2 \) and \( Q_6 \) Penguin-like diagrams. These predictions are parameter free and agree within the uncertainties with the experimental values.

We also show how the factorizable contributions from the \( Q_6 \) operator are IR divergent. This divergence cancels only when the non-factorizable contributions are added consistently. This makes the \( B_6 \) parameter not well defined.

We believe that this work presents some advances towards the mastering of non-leptonic kaon decays. This will be pursued in forthcoming works determining the \( \Delta S = 1 \) non-leptonic couplings at \( O(p^4) \) and \( \varepsilon'/\varepsilon \) within the Standard Model \([2]\).

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Effective $\Delta S = 1$ Weak Chiral Lagrangian in the Instanton-induce Chiral Quark Model

Mario Franz$^1$, Hyun-Chul Kim$^2$, and Klaus Goeke$^1$

$^1$Inst. f. Theo. Phys. II, Ruhr-Universität Bochum, D-44780 Bochum, Germany

$^2$Dep. of Phys., Pusan National University, 609-735 Pusan, The Republic of Korea

In this talk, we present the recent investigation of the effective $\Delta S = 1$ weak chiral Lagrangian within the framework of the instanton-induced chiral quark model. Starting from the effective four-quark operators, we derive the effective weak chiral action by integrating out the constituent quark fields. Employing the derivative expansion, we are able to obtain the effective $\Delta S = 1$ weak chiral Lagrangian to order $\mathcal{O}(p^4)$. The resulting $\mathcal{O}(p^4)$ low energy constants are derived as follows [1]:

$$N_{1(27)}^{(27)} = N_{3(27)}^{(27)} = N_{20}^{(27)} = 0,$$

$$N_{1(27)}^{(27)} = N_{3}^{(27)} = N_{2}^{(27)} = N_{21}^{(27)} = \frac{N_c}{60\pi^2} \left(1 + \frac{1}{N_c}\right) \times \left(-3c_1 - 3c_2 - \frac{9}{2}c_9 - \frac{9}{2}c_{10}\right).$$

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Rare kaon decays: $K^+ \rightarrow \pi^+ \ell^+ \ell^-$ and $K_L \rightarrow \mu^+ \mu^-$

G. D’Ambrosio
INFN, Sezione di Napoli I–80134 Napoli, Italy.

$K^+ \rightarrow \pi^+ \gamma^*$ Rare kaon decays are an important tool to test the chiral theory and establish the Standard Model and/or its extensions. $K^+ \rightarrow \pi^+ \gamma^*$ starts at $\mathcal{O}(p^4)$ in $\chi$PT with loops (dominated by the $\pi\pi-$cut) and counter-term contributions [1]. Higher order contributions ($\mathcal{O}(p^6)$) might be large, but not completely under control since new (and unknown) counter-term structures will appear. Experimentally the $K^+ \rightarrow \pi^+ \ell^+ \ell^-$ ($\ell=e, \mu$) widths are known while the slope is known only in the electron channel. An interesting question is the origin of the $2.2 \sigma$ discrepancy of the ratio of the widths $e/\mu$ from the $\mathcal{O}(p^4)$ prediction. In Ref. [2] we have parameterized, very generally, the $K^+ \rightarrow \pi^+ \gamma^*(q)$ form factor as

$$W_+(z) = G_F M_K^2 (a_+ + b_+ z) + W_{\pi\pi}^+(z),$$

with $z = q^2/M_K^2$, and where $W_{\pi\pi}^+(z)$ is the loop contribution given by the unitarity cut of $K^+ \rightarrow \pi^+ \pi^- \pi^-$. The two unknown parameters $a_+$ and $b_+$ can be fixed from rate and the slope in the electron channel to predict the muon rate and consequently the ratio of the widths $e/\mu$, which still comes out $2.2\sigma$ away from the exp. result, pointing out either an experimental problem in the muon channel or a violation of lepton universality. Also we speculate on the prediction of Vector Meson Dominance (VMD) for this channel.

$K_L \rightarrow \mu^+ \mu^-$. To fully exploit the potential of $K_L \rightarrow \mu^+ \mu^-$ in probing short-distance dynamics it is necessary to have a reliable control on its long-distance amplitude. The branching ratio can be generally decomposed as $B(K_L \rightarrow \mu^+ \mu^-) = |\text{Re} A|^2 + |\text{Im} A|^2$ and $\text{Re} A = \text{Re} A_{\text{long}} + \text{Re} A_{\text{short}}$. The recent measurement of $B(K_L \rightarrow \mu^+ \mu^-)$ is almost saturated by the absorptive amplitude leaving a very small room for the dispersive contribution : $|\text{Re} A_{\text{exp}}|^2 = (-1.0 \pm 3.7) \times 10^{-10}$. Within the Standard Model the NLO short-distance amplitude gives the possibility to extract a lower bound on $\bar{\rho}$, once we have under control the dispersive contribution generated by the two–photon intermediate state. In order to saturate this lower bound we propose [3] a low energy parameterization of the $K_L \rightarrow \gamma^* \gamma^*$ form factor that include the poles of the lowest vector meson resonances. Using experimental slope from $K_L \rightarrow \gamma \ell^+ \ell^-$ and QCD constraint we predict $\bar{\rho} > -0.38$ (90%CL.). This bound could be very much improved if the linear and quadratic slope of the $K_L \rightarrow \gamma^* \gamma^*$ form factor were measured with good precision and a more stringent bound on $|\text{Re} A_{\text{exp}}|$ is established.

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Electromagnetic Corrections to Semi-leptonic Decays of Pseudoscalar Mesons

M. Knecht\textsuperscript{1}, H. Neufeld\textsuperscript{2}, H. Rupertsberger\textsuperscript{2} and P. Talavera\textsuperscript{3}

\textsuperscript{1}Centre de Physique Théorique, CNRS-Luminy, Case 907, F-13288 Marseille Cedex 9, France
\textsuperscript{2}Inst. f. Theor. Phys. der Universität Wien, Boltzmanngasse 5, A-1090 Wien, Austria.
\textsuperscript{3}Dept. Theor. Phys., Lund University, Sölvegatan 14A, S-22362 Lund, Sweden.

In order to extract the information on hadronic matrix elements of QCD currents made of light quarks from high-statistics semi-leptonic decays of pseudoscalar mesons, a quantitative understanding of electromagnetic corrections to these processes is necessary. Virtual photons spoil the factorization property of the effective Fermi theory, so that the description of radiative corrections must be done within an extended framework, which includes also the leptons. An effective theory for the interactions of the light pseudoscalar mesons with light leptons and with photons, and which respects all the properties required by chiral symmetry, has been constructed. It is based on a power counting consistent with the loop expansion. The renormalization of this effective theory has been studied at the one-loop order. The divergences arising from the loops involving a lepton require, besides the usual mass and wave function renormalizations, only three additional nontrivial counter-terms. Applications to the $\pi \ell^2$ and pion beta decays are being completed. Further work will also consider the semi-leptonic decays of the kaons, in view of the forthcoming high-precision data from the KLOE experiment at the DAPHNE $\phi$–Factory. The case of the $K_{\ell 4}$ decays of charged kaons with two charged pions in the final state is of particular importance, since they allow to access the $\pi-\pi$ phase-shifts at low energies.
An estimate of the $O(p^4)$ E.M. contribution to $M_{\pi^+} - M_{\pi^0}$

Bachir Moussallam
Groupe de Physique Théorique, IPN,
Université Paris-Sud, 91406 Orsay

We reanalyze a sum rule due to Das et al.\cite{Das}. This sum rule is interesting not as an approximation to the physical $\pi^+ - \pi^0$ mass difference but as an exact result for a chiral low-energy parameter. A sufficiently precise evaluation provides a model independent estimate for a combination of $O(p^4)$ electromagnetic chiral low-energy parameters recently introduced by Urech\cite{Urech}. Three ingredients are necessary in order to reach the required level of accuracy: firstly one must use a euclidian space approach, secondly one must use accurate experimental data such as provided recently by the ALEPH collaboration on $\tau$ into hadrons decays\cite{Aleph}. Finally, it is necessary to extrapolate to the chiral limit $m_u = m_d = 0$. We show how a set of sum rules allows to perform this extrapolation in a reliable way\cite{Moussallam}.

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20
Matching Long and Short Distances

in Large-\(N_c\) QCD

Santiago Peris\(^1\), Michel Perrottet\(^2\) and Eduardo de Rafael\(^2\)

\(^1\)Grup de Fisica Teorica and IFAE,
Universitat Autonoma de Barcelona, 08193 Barcelona, Spain

\(^2\)Centre de Physique Theorique, CNRS-Luminy, Case 907
F-13288 Marseille Cedex 9, France

It is shown, with the example of the experimentally known Adler function, that there is no matching in the intermediate region between the two asymptotic regimes described by perturbative QCD (for the very short-distances) and by chiral perturbation theory (for the very long-distances). We then propose to consider an approximation to large-\(N_c\) QCD which consists in restricting the hadronic spectrum in the channels with \(JP\) quantum numbers \(0^-\), \(1^-\), \(0^+\) and \(1^+\) to the lightest state and in treating the rest of the narrow states as a perturbative QCD continuum; the onset of this continuum being fixed by consistency constraints from the operator product expansion. We show how to construct the low-energy effective Lagrangian which describes this approximation. A comparison of the corresponding predictions, to \(\mathcal{O}(p^4)\) in the chiral expansion, with the phenomenologically known couplings \(L_i\) is also made in terms of a single unknown, namely \(f_\pi/M_V\) \([1]\):

\[
6L_1 = 3L_2 = \frac{-8}{7}L_3 = 4L_5 = 8L_8 = \frac{3}{4}L_9 = -L_{10} = \frac{3}{8} \frac{f_\pi^2}{M_V^2},
\]

where \(f_\pi, M_V\) are the pion decay constant and the \(1^-\) state’s mass, respectively.

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Large $N_c$ QCD and Weak Matrix Elements

Eduardo de Rafael

Centre de Physique Théorique
CNRS-Luminy, Case 907
F-13288 Marseille Cedex 9, France

The first part of my talk was an overview of the progress made and the problems which remain in deriving an effective Lagrangian which describes the non–leptonic weak interactions of the Nambu–Goldstone degrees of freedom ($K$, $\pi$ and $\eta$) of the spontaneous $SU(3)_L \times SU(3)_R$ symmetry breaking in the Standard Model. I showed, with examples, how the coupling constants of the $|\Delta S| = 1$ and $|\Delta S| = 2$ effective low energy Lagrangian are given by integrals of Green’s functions of QCD currents and density currents, while those of the strong sector (QCD only) are more simply related to the coefficients of the Taylor expansions of QCD Green’s functions. The study of these issues within the framework of the $1/N_c$ expansion, and in the lowest meson dominance approximation described in ref. [1], seems a very promising path.

The second part of my talk was dedicated to a review of the properties of the $\Pi_{LR}(Q^2)$ correlation function in the large–$N_c$ limit as recently discussed in ref. [2]. This is the correlation function which governs the electroweak $\pi^+ - \pi^0$ mass difference. Following the discussion of ref. [3], I showed how the calculation of this observable, which requires non–trivial contributions from next–to–leading terms in the $1/N_c$ expansion, provides an excellent theoretical laboratory for studying issues of long– and short– distance matching in calculations of weak matrix elements of four–quark operators.

The third part of my talk was dedicated to recent work reported in ref [4], where it is shown that the $K \rightarrow \pi\pi$ matrix elements of the four–quark operator $Q_7$, generated by the electroweak penguin–like diagrams of the Standard Model, can be calculated to first non–trivial order in the chiral expansion and in the $1/N_c$ expansion. I compared the results to recent numerical evaluations from lattice–QCD.

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Bound States with Effective Lagrangians: Energy Level Shift in the External Field

Vito Antonelli¹, Alex Gall¹, Jürg Gasser¹ and Akaki Rusetksy¹,²,³

¹Institute for Theoretical Physics, University of Bern, Sidlerstrasse 5, CH-3012, Bern, Switzerland
²Laboratory of Theoretical Physics, JINR, 141980 Dubna, Russia
³IHEP, Tbilisi State University, 380086 Tbilisi, Georgia

Recent growth of interest in both the experimental and theoretical study of the properties of hadronic atoms is motivated by the possibility of direct determination of the strong hadronic scattering lengths from the atomic data. The detailed analysis of the $\pi^+\pi^-$ atom decay within ChPT has been carried out under the assumption of locality of strong interactions at the atomic length scale [1].

Two conceptual difficulties arise in the theory beyond this approximation:

• Relativistic approaches to the bound-state problem deal with the off-shell Green’s functions. It is at present unclear whether the ambiguity of the off-shell extrapolation in the effective theory affects the bound-state observables.

• The bound-state observables in nonrenormalizable theories contain additional UV divergences in the matrix elements of strong amplitudes between the bound-state wave functions. It is at present unclear whether these divergences can be canceled by the same LEC’s which render finite the amplitudes itself.

Addressing these problems, a simple model of a heavy massive scalar particle - bound in an external Coulomb field - is considered within the nonrelativistic effective Lagrangian approach [2]. Radiative corrections to the bound state energy levels due to the interaction with a dynamical scalar ”photon”, are calculated. In the model studies it is demonstrated that [3]:

• The ambiguity in the off-shell extrapolation of the Green’s function in the relativistic theory does not affect the bound-state spectrum.

• Bound-state observables are made finite by the same counter-terms which render finite Green’s functions, even in effective nonrenormalizable theories.

• UV divergences in the nonrelativistic bound-state matrix elements are correlated by matching and cancel.

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SU(3) Baryon Chiral Perturbation Theory and Long Distance Regularization

Buğra Borasoy

Department of Physics and Astronomy
University of Massachusetts
Amherst, MA 01003, USA

Baryon chiral perturbation theory as conventionally applied has a well-known problem with the SU(3) chiral expansion: loop diagrams generate very large SU(3) breaking corrections and greatly upset the subsequent phenomenology. This problem is due to the portions of loop integrals that correspond to propagation at short distances for which the effective theory is not valid. One can reformulate the theory just as rigorously by regulating the loop integrals using a momentum-space cutoff which removes the spurious short distance physics [1], [2]. The chiral calculations can now provide a model independent realistic description of the very long distance physics. In [3] and [4] this scheme is applied to the sigma-terms and baryon axial currents. The results are promising and show that this development may finally allow realistic phenomenology to be accomplished in SU(3) baryon chiral perturbation theory.

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The scenario of spontaneous chiral symmetry breakdown with small or vanishing quark condensate [1] and its phenomenological consequences for the $\pi N$-system are investigated. [2] Standard Heavy Baryon Chiral Perturbation Theory is extended in order to account for the modified chiral counting rules of quark mass insertions. The effective lagrangian is given to $\mathcal{O}(p^2)$ in its most general form and to $\mathcal{O}(p^3)$ in the scalar sector. As a first application, mass- and wave-function renormalization as well as the scalar form factor of the nucleon is calculated to $\mathcal{O}(p^3)$. The result is compared to a dispersive analysis of the nucleon scalar form factor [3], adopted to the case of a small quark condensate. In this latter analysis, the shift of the scalar form factor between the Cheng-Dashen point and zero momentum transfer is found to be enhanced over the result assuming strong quark condensation by up to a factor of two, with substantial deviations starting to be visible for $r = m_s/\hat{m} \leq 12$. [2] As a result, the nucleon sigma term as determined from $\pi N$-scattering data decreases with decreasing quark condensate. [4] On the other hand, the sigma term can also be determined from the baryon masses. To leading order in the quark masses, $\sigma_N$ is in proportion to $(r - 1)^{-1}(1 - y)^{-1}$, i.e. strongly increasing with decreasing quark condensate. If the the strange quark content of the nucleon, $y$, were known, strong bounds on the light quark condensate would follow.

We also consider the so called backward sum rule for the difference of electric and magnetic polarizabilities of the nucleon. Although the effect of a small quark condensate is less pronounced here, this observable has the advantage of being directly experimentally accessible. Detailed numerical study of this sum rule is under way.

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Heavy Baryon ChPT and Nucleon Resonance Physics

Thomas R. Hemmert
FZ Jülich, IKP (Th), D-52425 Jülich, Germany

Several calculations [1] have appeared since the “small scale expansion” (SSE) has been presented to the chiral community at the Trento workshop in 1996 [2],[3]. The idea is to incorporate the effects of low lying nucleon resonances via a phenomenologically motivated power counting in $\mathcal{O}(\epsilon^n)$ with $\epsilon = \{m_\pi, p, \Delta\}$, which supersedes the standard $\mathcal{O}(p^n)$ power counting of HBChPT. The new scale $\Delta$ corresponds to the energy difference between the mass of a resonance and the mass of the nucleon and in SSE is counted as a small parameter$^{1}$ of $\mathcal{O}(\epsilon^1)$. SSE therefore not only allows for calculations with explicit resonance degrees of freedom but also resums the resonance effects into lower orders of the perturbative expansion—for example see the discussion regarding the spin-polarizabilities of the nucleon in [4]. The complete $\mathcal{O}(\epsilon^2)$ SSE lagrangians for $NN$, $N\Delta$ and $\Delta\Delta$ are now worked out and published [5]. Progress has also been achieved at $\mathcal{O}(\epsilon^3)$ where the complete lagrangian for single nucleon transitions has been worked out [6]. The $\mathcal{O}(\epsilon^3)$ divergence structure was found to be quite different from the corresponding $\mathcal{O}(p^3)$ one in HBChPT. In addition to modifications in the beta-functions of the 22 HBChPT counter terms (c.t.s) [7] one needs 10 extra c.t.s to account for additional divergences proportional to the new scale $\Delta^n$, $n \leq 3$. The finite parts of these 10 c.t.s are utilized to guarantee a smooth transition from $\mathcal{O}(\epsilon^n)$ SSE to $\mathcal{O}(p^n)$ HBChPT for any process in the decoupling limit $m_\pi/\Delta \to 0$.

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$^{1}$In ChPT the nucleon-delta mass splitting counts as $\mathcal{O}(p^0) = \mathcal{O}(1)$, in contrast to SSE.
Nucleon properties to $O(p^4)$ in HBCHPT

Martin Mojžiš

$^1$Dep. Theor. Phys., Comenius University, Mlynská dolina F2, SK-84215 Bratislava, Slovakia

Complete one-loop calculations of nucleon properties in CHPT require
1. construction of the effective Lagrangian up to the 4th order
2. renormalization of $m_N$, $Z_N$ and $g_A$ (and higher order LECs)
3. calculation of nucleon form-factors, cross-sections, ...

A brief summary of progress made in these three points:

1. A *Mathematica* program for the construction of the effective Lagrangian was developed. It reproduces successfully the known results in the 2nd and 3rd orders, but does not eliminate all the dependent terms yet (e.g. in the 3rd order it gives two terms more than it should). The number of the 4th order terms produced by the program is 155 so far, this number will be probably slightly decreased in the final version.

2. Nucleon wave-function renormalization is known to be a tricky issue already at the 3rd order of HBCHPT. At the 4th order yet some new subtleties enter, but all of them have become well understood recently. The discussion of general aspects of this topic, as well as explicit calculation of $m_N$, $Z_N$ and $g_A$, is to be found in [1] and references therein.

3. Work in progress.

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Pion–nucleon scattering in heavy baryon chiral perturbation theory

Nadia Fettes\textsuperscript{1}, Ulf-G. Mei\ss ner\textsuperscript{1} and Sven Steininger\textsuperscript{2}

\textsuperscript{1}Forschungszentrum Jülich, Institut für Kernphysik (Theorie)
D-52425 Jülich, Germany

\textsuperscript{2}Universität Bonn, Institut für Theoretische Kernphysik
Nussallee 14-16, D-53115 Bonn, Germany

We discuss in detail pion–nucleon scattering in the framework of heavy baryon chiral perturbation theory to third order in small momenta. In particular we show that the $1/m$ expansion of the Born graphs calculated relativistically can be recovered exactly in the heavy baryon approach without any additional momentum-dependent wave function renormalization. Since the normalization factor of the nucleon spinors, appearing in the relativistic calculation, enters the heavy baryon amplitude via the wave function renormalization\cite{1}, we do not expand this factor.

The pion–nucleon scattering amplitude is given in terms of four second order LECs and five combinations of LECs from $\mathcal{L}_{\pi N}^{(3)}$. In order to fix these constants, we fit various empirical phase shifts for pion laboratory momenta between 50 and 100 MeV. As input we use the phase shifts of the Karlsruhe group\cite{2} (KA85), from the analysis of Matsinos\cite{3} (EM98) and the latest update of the VPI group\cite{4} (SP98). This leads to a satisfactory description of the phase shifts up to momenta of about 200 MeV. The two S-waves are reproduced very well, whereas in the P-waves the tail of the Delta is strongly underestimated and the bending from the Roper resonance cannot be accounted for. We also predict threshold parameters, which turn out to be in good agreement with analyses based on dispersion relations. Finally we consider sub-threshold parameters and give a short comparison to other calculations\cite{5,6}.

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\end{itemize}
Relations of Dispersion Theory
to Chiral Effective Theories for $\pi N$ Scattering

Gerhard Höhler

Institut für Theoretische Teilchenphysik der Universität
D-76128 Karlsruhe, Germany

Predictions for $\pi N$ scattering amplitudes within the Mandelstam triangle and near its boundaries were recently derived from chiral perturbation theory (CHPT)\cite{1},\cite{2}. The results were compared with predictions from partial wave analyses or from analytic continuations using various dispersion relations.

Mandelstam analyticity is assumed for two steps:
i) for constraints which lead to a unique result of partial wave analysis. E. Pietarinen’s expansion method (references in Ref.\cite{3}) made it possible to include all data available at that time in the Karlsruhe-Helsinki solution KH80. The solution SP98 of R.A. Arndt et al.\cite{4} is based on an empirical parametrization which ignores well known nearby l.h. cuts. It includes new data, but covers only the range up to 2.1 GeV. The attempt to apply approximately the much weaker constraint from fixed-t analyticity was successful for all invariant amplitudes only up to 0.6 GeV, so above this energy the solution is questionable.

ii) For the continuation into the unphysical region we have used many single variable dispersion relations, which follow from Mandelstam analyticity\cite{3}. The compatibility of these relations with KH80 supports the assumptions on the analytic properties.

The comparison of the predictions from CHPT and dispersion relations should be made not only for the numerical results but mainly for the theoretical expressions. Plots of the integrands of fixed-t dispersion relations give an important information on sub-threshold coefficients. A paper with many details can be obtained from the author\cite{1}.

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\textsuperscript{1}E-mail gerhard.hoehler@physik.uni-karlsruhe.de
Isospin Violation in Pion–Nucleon Scattering

Nadia Fettes¹, Ulf-G. Meißner¹ and Sven Steininger¹, ²

¹IKP(Theorie), Forschungszentrum Jülich, D-52425 Jülich, Germany.
²ITKP, Uni Bonn, Nussallee 14-16, D-53115 Bonn, Germany.

We construct the complete effective chiral pion-nucleon Lagrangian in the presence of virtual photons to one loop. Taking only the charged to neutral pion and the proton to neutron mass differences into account, we calculate the scattering lengths of all physical $\pi N$–scattering processes. Furthermore we construct six independent relations among the physical scattering amplitudes, which vanish in the case of perfect isospin symmetry. If we now take these relations at threshold, we find large violation in the isoscalar ones:

$$R_1 = 2 \frac{T_{\pi^+p \to \pi^+p} + T_{\pi^-p \to \pi^-p} + 2T_{\pi^0p \to \pi^0p}}{T_{\pi^+p \to \pi^+p} + T_{\pi^-p \to \pi^-p} - 2T_{\pi^0p \to \pi^0p}} = 36\%$$

$$R_6 = 2 \frac{T_{\pi^0p \to \pi^0p} - T_{\pi^0n \to \pi^0n}}{T_{\pi^0p \to \pi^0p} + T_{\pi^0n \to \pi^0n}} = 19\%$$

In 1977 Weinberg has already pointed out the possible large value for $R_6$ but there isn’t (and probably will never be) any experimental data to verify this relation. On the other hand there is hope to measure the $\pi^0p$ scattering length so that one could check the prediction of the huge violation in the comparison of elastic scattering of charged with neutral pions. The not mentioned remaining four relations involving the isoscalar amplitudes are small (less than 2%).

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Goldberger-Miyazawa-Oehme Sum Rule Revisited

M.E. Sainio

Dept. of Physics, University of Helsinki,
P.O. Box 9, FIN-00014 Helsinki, Finland

There has been recently quite a lot of activity attempting to determine the value of the pion-nucleon coupling constant with high precision. The results vary roughly in the range $f^2 = 0.075 - 0.081$ which covers the previous standard value, $f^2 = 0.079$, but the tendency is to move the central value downwards.

The Goldberger-Miyazawa-Oehme sum rule [1], the forward dispersion relation for the $C^-$ amplitude at the physical threshold, provides a relationship between the isovector $s$-wave pion-nucleon scattering length, the total cross section data and the pion pole term. The coupling constant can be extracted ($x = \mu/m$, where $\mu$ and $m$ are the masses for the charged pion and the proton)

$$f^2 = \frac{1}{2}[1 - \left(\frac{x}{2}\right)^2] \times [(1 + x)\mu a_{0+} - \mu^2 J^-] \text{ with } J^- = \frac{1}{2\pi^2} \int_0^\infty \frac{\sigma^- (k)}{\omega} dk,$$

where $\sigma^- = \frac{1}{2}(\sigma_- - \sigma_+)$ in terms of the $\pi^- p$ and $\pi^+ p$ total cross sections.

The isovector scattering length is accessible in pionic hydrogen experiments. Also, there are plenty of cross section data up to about 350 GeV/c, which allows for a determination of the integral $J^-$. However, the data around the $\Delta$-resonance will have a significant influence on the $f^2$ value [2]. The precision of the input information is not, at present, high enough to be able to compete with other methods of determining the pion-nucleon coupling constant, but in principle the GMO approach can relate more directly the uncertainty in $f^2$ to the uncertainties in experimental quantities.

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Low–momentum effective theory for two nucleons

E. Epelbaoum$^{1,2}$, W. Glöckle$^1$ and Ulf-G. Meißner$^2$

$^1$Ruhr–Universität Bochum, Institut für theoretische Physik II
D–44870 Bochum, Germany

$^2$Forschungszentrum Jülich, Institut für Kernphysik (Theorie)
D–52425 Jülich, Germany

For the case of a Malfliet–Tjon type model potential [1] we show explicitly that it is possible to construct a low–momentum effective theory for two nucleons. To that aim we decouple the low and high momentum components of this two–nucleon potential using the method of unitary transformation [2]. We find the corresponding unitary operator for the $s$–wave numerically [3]. The $S$–matrices in the full space and in the subspace of low momenta are shown to be identical. This is also demonstrated numerically by solving the corresponding Schrödinger equation in the small momentum space.

Using our exact effective theory we address some issues related to the effective field theory approach of the two–nucleon system [4]. In particular, we consider the $np$ $^3S_1$ and $^1S_0$ channels. Expanding the heavy repulsive meson exchange of the effective potential in a series of local contact terms we discuss the question of naturalness of the corresponding coupling constants. We demonstrate that the quantum averages of the local expansion of the effective potential converge. This indicates that our effective theory has a systematic power counting. However terms of rather high order should be kept in the effective potential to obtain an accurate value of the binding energy (scattering length) in the $^3S_1$ ($^1S_0$) channel.

We hope that this study might be useful for the real case of NN–forces derived from chiral Lagrangians in the low–momentum regime.

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Effective Field Theory in the Two-Nucleon Sector

Martin J. Savage

Dept. of Physics, University of Washington, Seattle, WA 98915, USA.

The two-nucleon sector contains length scales that are much larger than one would naively expect from QCD. The s-wave scattering lengths $a^{(S_0)} = -23.7$ fm and $a^{(S_1)} = 5.4$ fm are much greater than both $1/\Lambda \sim 0.2$ fm and $1/f_\pi \sim 1.5$ fm. The lagrange density describing such interactions consists of local four-nucleon operators and nucleon-pion interactions. Weinberg[1] suggested expanding the NN potential in powers of the quark mass matrix and external momenta. Several phenomenological applications of this counting have been explored (e.g. [2]). However, one can show that leading order graphs require counter-terms that occur at higher orders in the expansion. A power counting was suggested[3] that does not suffer from these problems. The leading interaction is the four-nucleon interaction, with pion exchange being sub-leading order. Recently, Mehen and Stewart[4] have suggested that the scale for the breakdown of the theory is $\sim 500$ MeV. The deuteron can be simply incorporated into the theory and its moments, form factors[5] and polarizabilities[6] have been computed. The cross section for $\gamma$-deuteron Compton scattering[7] has been computed in this theory and agrees well with the available data.

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Peripheral NN-Scattering and Chiral Symmetry

Norbert Kaiser
Physik Department T39, TU München, D-85747 Garching, Germany.

We evaluate in one-loop chiral perturbation theory all $1\pi$- and $2\pi$-exchange contributions to the elastic NN-interaction. We find that the diagrams with virtual $\Delta(1232)$-excitation produce the correct amount of isoscalar central attraction as needed in the peripheral partial waves ($L \geq 3$). Thus there is no need to introduce the fictitious scalar isoscalar $\sigma$-meson. We also compute the two-loop diagrams involving $\pi\pi$-interaction (so-called correlated $2\pi$-exchange). Contrary to common believe these graphs lead to a negligibly small and repulsive NN-potential. Vector meson ($\rho$ and $\omega$) exchange becomes important for the F-waves above $T_{\text{lab}} = 150$ MeV. Without adjustable parameters we reproduce the empirical NN-phase shifts with $L \geq 3$ and mixing angles with $J \geq 3$ up to $T_{\text{lab}} = 350$ MeV. Further details on the subject can be found in refs. [1], [2].

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Compton Scattering on the Deuteron in HBChPT

Silas R. Beane

Department of Physics,
University of Maryland,
College Park, MD 20742 USA

There exists a systematic procedure for computing nuclear processes involving an external pionic or electromagnetic probe at energies of order $M_\pi \frac{k}{1}$. A perturbative kernel is calculated in baryon HBChPT and folded between phenomenological nuclear wavefunctions. A computation of $\pi^0$ photoproduction on the deuteron has been performed to $O(q^4)$ in HBChPT $\frac{1}{2}$. This process is of interest because an accurate measurement of the deuteron electric dipole amplitude (EDA) allows a model independent extraction of the neutron EDA, to this order in HBChPT. Recent experimental results from SAL indicate a value for the deuteron EDA which is very close to the HBChPT prediction, which in turn implies a large neutron EDA. In similar spirit, experimental information about the $\pi$-nucleus scattering lengths can be used to learn about $\pi$-N scattering lengths $\frac{1}{2}$. A recent accurate measurement of the $\pi$-deuteron scattering length constrains HBChPT counter-terms which contribute to the isoscalar S-wave $\pi$-N scattering length, which is a particularly problematic observable.

Compton scattering on the deuteron has been computed to order $O(q^3)$ in HBChPT $\frac{1}{2}$. Our predictions at low energies are in agreement with old Illinois data. This process has been measured at SAL at higher photon energies ($95\text{MeV}$) and data is currently being analyzed. An ingredient of the deuteron process is the neutron polarizability, which cannot be directly measured. Thus our calculation provides a systematic means of learning about the neutron polarizability.

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(a) Baryon $\chi$PT and (b) Mesons at Large $N_c$

H. Leutwyler

Institute for Theoretical Physics, University of Bern,
Sidlerstr. 5, CH-3012 Bern, Switzerland

I gave a brief introduction to some of the work being carried out at Bern:

(a) Thomas Becher and I have shown that the approach of Tang and Ellis [1] can be developed into a coherent, manifestly Lorentz invariant formulation of B$\chi$PT that preserves chiral power counting. The method avoids some of the shortcomings of HB$\chi$PT. In particular, it allows us to analyze the low energy structure also in cases where the straightforward nonrelativistic expansion of the amplitudes in powers of momenta and quark masses fails. As an illustration, I discussed the application of the method to the $\sigma$-term and to the corresponding form factor. A preliminary version of a paper on the subject is available [2].

(b) I then briefly described the work on the large $N_c$ limit in the mesonic sector, done in collaboration with Roland Kaiser. We use a simultaneous expansion of the effective Lagrangian in powers of derivatives, quark masses and $1/N_c$ and account for the terms of first non-leading order, as well as for the one loop graphs – although these only occur at next-to-next-to leading order, they are not irrelevant numerically on account of chiral logarithms. I drew attention to related work by Feldmann, Kroll and Stech [3]. We are currently grinding out the numerical implications for the various observables of interest. Some of the work is described in ref. [4]. A more extensive report is under way.

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1see also the reports of Thomas Becher and Roland Kaiser in these mini-proceedings.
Baryon $\chi$PT in Manifestly Lorentz Invariant Form

T. Becher and H. Leutwyler

Universität Bern, Institut für theoretische Physik,
Sidlerstr. 5, CH-3012 Bern, Switzerland

The effective theory describing the interaction of pions with a single nucleon can be formulated in manifestly Lorentz invariant form [1], but it is not a trivial matter to keep track of the chiral order of graphs containing loops within that framework: The chiral expansion of the loop graphs in general starts at the same order as the corresponding tree graphs, so that the renormalization of the divergences also requires a tuning of those effective coupling constants that occur at lower order.

Most of the recent calculations avoid this complication by expanding the baryon kinematics around the nonrelativistic limit. This method is referred to as heavy baryon chiral perturbation theory [2]. It keeps track of the chiral power counting at every step of the calculation at the price of manifest Lorentz invariance, but suffers from a deficiency: The corresponding perturbation series fails to converge in part of the low energy region. The problem is generated by a set of higher order graphs involving insertions in nucleon lines – this sum diverges. The problem does not occur in the relativistic formulation of the effective theory.

The purpose of this talk was to present a method [3] that exploits the advantages of the two techniques and avoids their disadvantages. We showed that the infrared singularities of the one-loop graphs occurring in $B\chi$PT can be extracted in a relativistically invariant fashion and that this result can be used to set up a renormalization scheme that preserves both Lorentz invariance and chiral power counting. The method we have presented follows the approach of Tang and Ellis [4], but we do not expand the infrared singular part in a chiral series, because that expansion does not always converge.

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In the large $N_c$ limit the variables required to analyze the low energy structure of QCD in the framework of an effective field theory necessarily include the degrees of freedom of the $\eta'$. In a previous analysis [1] we demonstrated that the effective Lagrangian prescription of the pseudoscalar nonet, pions, kaons, $\eta$ and $\eta'$, yields results consistent with nature. The calculation relies on a simultaneous expansion in powers of momenta, quark masses and $1/N_c$ which is truncated at first non-leading order. In particular we were able to calculate the decay constants of the $\eta$ and the $\eta'$ using experimental data on the decay rates of the neutral mesons into two photons. The main effect generated by the corrections to the well known leading order results concerns $\eta - \eta'$ mixing: at this order of the low energy expansion we need to distinguish two mixing angles.

The purpose of the present talk is to report on the results obtained when the above analysis is carried further on to systematically include the non-leading corrections to the decay amplitudes [2] (In ref. [1] we disregarded the effect of SU(3) breaking in the electromagnetic decay rates). Furthermore we include the effects of the one loop graphs: although algebraically these contributions are suppressed by one power of $1/N_c$, some of these are enhanced by logarithmic factors. The points of main interest are the following: (a) is the hypothesis that the $1/N_c$ corrections are small valid for the world we live in and (b) what happens to the low energy theorem for the difference between the two mixing angles if we include the one loop graphs. Unfortunately, the analysis now involves a larger number of unknown low energy constants, so that the numerical analysis yields ranges for the observables rather than definite values.

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Derivation of the Chiral Lagrangian from the Instanton Vacuum

Dmitri Diakonov
NORDITA, Blegdamsvej 17, 2100 Copenhagen Ø, Denmark

The QCD vacuum is populated by strong fluctuations of the gluon field carrying topological charge; these fluctuations are called instantons. A theory of the QCD instanton vacuum has been suggested in the 80’s, based on the Feynman variational principle [1]. In the last years it has been strongly supported by direct lattice simulations [2].

Instantons lead to a theoretically beautiful and phenomenologically realistic microscopic mechanism of chiral symmetry breaking (for a review see [3]), which has been also recently checked directly on the lattice [2]. Chiral symmetry breaking manifests itself in quarks acquiring a momentum-dependent dynamical mass, sometimes called the constituent mass, and in the appearance of pseudo-Goldstone pions interacting with the dynamically massive quarks. The strength and the formfactors of pion-quark interactions are fixed unambiguously by the basic parameters of the instanton medium, and agree nicely with phenomenology [4].

Integrating out quarks, one gets the chiral lagrangian, whose low-momentum expansion and the asymptotic form have been investigated [4].

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Theoretical Study of Chiral Symmetry Breaking in QCD: Recent Development

Jan Stern
Groupe de Physique Theorique, IPN, 91406 Orsay, France

If forthcoming experiments confirmed the actual value of $\pi\pi$ scattering length $a_0^\pi = 0.26 \pm 0.05$ with a smaller error bar, we would have to conclude that the condensate $\langle \bar{q}q \rangle$ is considerably smaller than one usually believes. I have presented few theoretical speculations inspired by this possibility: 1) Chiral symmetry breaking is due to the accumulation of small eigenvalues of the Euclidean Dirac operator in the limit of a large volume $V$. Since the fermion determinant reduces the weight of small eigenvalues, order parameters $F_\pi^2$ and $\langle \bar{q}q \rangle$ are suppressed for large $N_f/N_c$ and the symmetry will be restored for $N_f/N_c > n_0$, (likely, $n_0 = 3.25$). The condensate could disappear more rapidly than $F_\pi^2$, since it is merely sensitive to the smallest eigenvalues behaving as $1/V$. It is even conceivable that for $n_1 < N_f/N_c < n_0$, there is a phase in which $\langle \bar{q}q \rangle = 0$ but $F_\pi \neq 0$, implying symmetry breaking and the existence of Goldstone bosons$[1]$. Close to the critical point $n_1$, it would then be natural to expect a tiny quark condensate. 2) Kogan, Kovner and Shifman$[2]$ have pointed out that due to Weingarten’s inequality, the bare, cutoff dependent condensate cannot vanish unless $F_\pi = 0$. This, however, does not exclude vanishing of the renormalized condensate (assuming QCD-like sign of its anomalous dimension) and the existence of the critical point $n_1 < n_0$ remains an open possibility. 3) Below but close to $n_1$ one should expect important Zweig rule violation and huge corrections to large $N_c$ predictions$[3]$. 4) The critical point $n_1$ could be seen through the volume dependence of Leutwyler-Smilga sum rules for Dirac eigenvalues$[4]$. The latter could be investigated numerically, diagonalizing the discretized square of the continuum massless Dirac operator, thereby avoiding the well known difficulties with massless fermions on the lattice.

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NUMERICAL SOLUTIONS OF ROY EQUATIONS

Gilberto Colangelo
Institut für Theoretische Physik, Universität Zürich
Winterthurerstr. 190, CH–8057 Zürich

Roy equations [1] are a system of coupled integral equations for the partial wave amplitudes of $\pi\pi$ scattering that incorporate the properties of analyticity, unitarity and crossing symmetry. Besides the dispersive integrals containing the Roy kernels and the partial wave amplitudes, the equations contain polynomial terms which depend on two constants only, i.e. the two $S$-wave scattering lengths. Solutions of Roy equations depend on the input values of these two constants.

I have reported about recent work [2] in solving numerically the Roy equations, describing both the method and the results. Our aim is to extract information on the low-energy behaviour of the $\pi\pi$ scattering partial wave amplitudes. For what concerns the high energy part, we use all the available information (coming both from experimental data and, where these are not available, from theoretical modelling) as input in the Roy equations, and solve them numerically only in the low-energy region. Since unitarity, analyticity and crossing do not constrain in any way the two $S$-wave scattering lengths on which the solution depends, we need additional information on those. This information comes from chiral symmetry, and to provide it we match the chiral representation of the amplitude (now available at the two loop level [3]) to the dispersive one, at low energy.

I have detailed how this matching is done, and have shown that the outcome of this combination of two different theoretical tools, together with the experimental information available on this process, is a very precise representation for the $\pi\pi$ scattering amplitude at low energy. New $K_{l4}$ data on the $\pi\pi$ phase shifts near threshold, expected in the near future from the E865 collaboration at BNL and from KLOE at DAΦNE (the Frascati Φ factory), will test the validity of this representation with high accuracy. A different experimental test will also come from the measurement of the pionic atoms lifetime made by the DIRAC collaboration: this measurement will give direct access to the difference of the two $S$-wave scattering lengths.

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One-channel Roy equation revisited

J. Gasser\(^1\) and G. Wanders\(^2\)

\(^1\) Institut für Theoretische Physik, Universität Bern, Sidlerstrasse 5, CH–3012 Bern, Switzerland

\(^2\) Institut de Physique Théorique, Université de Lausanne, CH–1015 Lausanne, Switzerland

The Roy equation in the single channel case amounts to a nonlinear, singular integral equation for the phase shift \(\delta\) in the low–energy region,

\[
\frac{1}{2\sigma(s)} \sin[2\delta(s)] = a + \frac{(s - 4)}{\pi} \int_{4}^{\infty} \frac{dx}{x} \frac{\omega(x)}{x - 4 x - s},
\]

\[
\omega(x) = \begin{cases} 
\sigma(x)^{-1} \sin^2[\delta(x)] & ; 4 \leq x \leq s_0 \\
A(x) & ; x \geq s_0 
\end{cases}.
\]

(1)

Here, \(\sigma(s) = (1 - 4/s)^{1/2}\) is the phase space factor, and \(A(x)\) denotes the imaginary part of the partial wave above the matching point \(s_0\). The integral is a principal value one. The problem consists in solving (1) in the interval \([4, s_0]\) at given scattering length \(a\) and given imaginary part \(A\). Investigating the infinitesimal neighborhood of a solution, the following proposition was established some time ago:

Let \(\delta\) be a solution of equation (1) with input \((a, A)\). It is an isolated solution if \(-\frac{\pi}{2} < \delta(s_0) < \frac{\pi}{2}\). If \(\delta(s_0) > \frac{\pi}{2}\), the infinitesimal neighborhood of \(\delta\) is an \(m\)-parameter family of solutions \(\delta'\) with \(\delta'(s_0) = \delta(s_0)\), where \(m\) is the integral part of \(2\delta(s_0)/\pi\) for \(\delta(s_0) > \pi/2\), and zero otherwise.

In [3], we recall the derivation of this proposition – in particular, we detail its connection with the homogeneous Hilbert problem on a finite interval. In addition, we construct explicit expressions for amplitudes that solve the full, nonlinear Roy equation exactly. These amplitudes contain free parameters that render the non–uniqueness of the solution manifest. The amplitudes develop, however, in general an unphysical singular behaviour at the matching point \(s_0\). This singularity disappears and uniqueness is achieved if one uses analyticity properties of the amplitudes that are not encoded in the Roy equation.

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How do the uncertainties on the input affect the solution of the Roy equations?

Gérard Wanders
Institut de physique théorique, Université de Lausanne,
CH-1015 Lausanne, Switzerland
E-mail: Gerard.Wanders@ipt.unil.ch

An update of previous work \cite{1,2} on the possible non-uniqueness of the solution of \(S\)- and \(P\)-wave Roy equations \cite{3} and its sensitivity to the uncertainties in the input is under way. The Roy equations determine the \(S\)- and \(P\)-wave phase shifts in a low-energy interval \([2M_\pi, E_0]\). The \(S\)-wave scattering lengths and absorptive parts above \(E_0\) are part of the input.

The Roy equations are used nowadays for the extrapolation of the available data down to threshold. Colangelo et al. \cite{4} take \(E_0 = 800\) MeV, a value for which the solution is unique [there is a continuous family of solutions if \(E_0 > 860\) MeV]. The result of a variation of the input has been derived in \cite{2} for \(E_0 = 1.1\) GeV and this is redone now for \(E_0 = 800\) meV. The linear response to small variations of the scattering lengths has been determined using a model with \(a_0^0 = 0.2\) and \(a_0^2 = −0.041\). The effect is strongest on the \(P\)-wave. The phase shift \(\delta_1^1\) is changed into \(\delta_1^1 + \Delta_1^1\) and the ratio \(\Delta_1^1/\delta_1^1\) is of the same order of magnitude as \(\delta a_0^0/a_0^0\) or \(\delta a_0^2/a_0^2\) over the whole interval \([2M_\pi, E_0]\) when \(a_0^0\) and \(a_0^2\) are varied separately. According to the previous talk \cite{5}, \(\Delta_1^1/\delta_1^1\) exhibits a cusp at \(E_0\). This cusp is extremely sharp and plays a role in the numerical analysis \cite{5}. In conformity with a universal curve in the \((a_0^0, a_0^2)\)-plane, \(\Delta_1^1 \sim (\delta a_0^0 − 4.9\delta a_0^2)\) over \([2M_\pi, E_0]\). The effects on the \(S\)-waves are less spectacular and the cusps are practically invisible.

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Properties of a possible scalar nonet

Deirdre Black\(^1\), Amir Fariborz\(^1\), Francesco Sannino\(^2\)
and Joseph Schechter\(^1\)

\(^1\)Physics Dept, Syracuse University,
Syracuse NY 13244-1130, USA

\(^2\)Physics Dept, Yale University,
New Haven CT 06520-8120

It was found that a light sigma-type meson and a light kappa-type meson are needed to preserve unitarity in, respectively, models of \(\pi\pi\) \(^1\) and \(\pi K\) \(^2\) scattering. These models are based on a chiral Lagrangian and yield simple approximate amplitudes which satisfy both crossing symmetry and the unitarity bounds. Together with the well established \(f_0(980)\) and \(a_0(980)\) mesons these would fill up a low lying nonet. We investigate the "family" relationship of the members of this postulated nonet. We start by considering Okubo’s original formulation\(^3\) of "ideal mixing". It is noted that the original mass sum rules possess another solution which has a natural interpretation describing a meson nonet made from a dual quark and a dual anti-quark. This has the same structure as a model for the scalars proposed\(^4\) by Jaffe in the context of the MIT bag model. However, our masses do not exactly satisfy the alternate ideal mixing sum rule. For this reason, and also to obtain the experimental pattern of decay modes, we introduce additional terms which break ideal mixing. Then two different solutions giving correct masses but characterized by different scalar mixing angles emerge. The solution which yields decay widths agreeing with experiment has a mixing angle about \(-17^\circ\) while the other has a mixing angle about \(-90^\circ\). For comparison, ideal mixing for a quark anti-quark nonet is \(\pm 90^\circ\) while ideal mixing for a dual quark, dual anti-quark nonet is 0\(^\circ\). Hence the dual picture is favored. A more detailed description of this work is given in the report\(^5\).

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ChPT Phenomenology in the Large–$N_C$ Limit

A. Pich

Department of Theoretical Physics, IFIC, Univ. Valencia – CSIC
Dr. Moliner 50, 46100 Burjassot (Valencia), Spain.

Chiral symmetry provides strong low–energy constraints on the dynamics of the Goldstone bosons. However, we need additional input to analyze physics at higher energy scales. The following two examples show that very useful information can be obtained from the large–$N_C$ limit of QCD:

- Using our present knowledge on effective hadronic theories, short–distance QCD information, the $1/N_C$ expansion, analyticity and unitarity, we derive an expression for the pion form factor [1] in terms of $m_\pi$, $M_\rho$ and $f_\pi$. This parameter–free prediction provides a surprisingly good description of the experimental data up to energies of the order of 1 GeV. A similar analysis of the $K\pi$ scalar form factor, needed for the determination of the strange quark mass, is in progress.

- The dispersive two–photon contribution to the $K_L \rightarrow \mu^+\mu^-$ decay amplitude is analyzed using chiral perturbation theory techniques and large–$N_C$ considerations. A consistent description [2] of the decays $\pi^0 \rightarrow e^+e^-$, $\eta \rightarrow \mu^+\mu^-$ and $K_L \rightarrow \mu^+\mu^-$ is obtained. Moreover, the present data allow us to derive a useful constraint on the short–distance $K_L \rightarrow \mu^+\mu^-$ amplitude, which could be improved by more precise measurements of the $\eta \rightarrow \mu^+\mu^-$ and $K_L \rightarrow \mu^+\mu^-$ branching ratios. This offers a new possibility for testing the flavour–mixing structure of the Standard Model. As a by–product one predicts $B(\eta \rightarrow e^+e^-) = (5.8 \pm 0.2) \times 10^{-9}$ and $B(K_L \rightarrow e^+e^-) = (9.0 \pm 0.4) \times 10^{-12}$.

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Gauge-invariant Effective Field Theory for a Heavy Higgs Boson

Andreas Nyffeler\textsuperscript{1} and Andreas Schenk\textsuperscript{2}

\textsuperscript{1}DESY, Platanenallee 6, D-15738 Zeuthen, Germany; nyffeler@ifh.de
\textsuperscript{2}Hambacher Straße 14, D-64625 Bensheim-Gronau, Germany.

The method of effective field theory has repeatedly been used to analyze the symmetry breaking sector of the Standard Model and to parametrize effects of new physics. Comparing the theoretical predictions for the low energy constants in the corresponding effective Lagrangian for different models with experimental constraints might help to distinguish between different underlying theories before direct effects become visible. At low energies the Standard Model with a heavy Higgs boson in the spontaneously broken phase, which serves as a reference point for a strongly interacting symmetry breaking sector, can adequately be described by such an effective field theory. Moreover, the low energy constants can explicitly be evaluated using perturbative methods by matching the full and the effective theory at low energies. Several groups have performed such a calculation for a heavy Higgs boson in recent years \cite{1}. In gauge theories there are, however, some subtleties involved when this matching is performed with gauge-dependent Green’s functions. We therefore proposed a manifestly gauge-invariant approach in Ref. \cite{2} which deals only with gauge-invariant Green’s functions.

In this talk we presented the extension of our method to the non-abelian case. Using a generating functional of gauge-invariant Green’s functions for the bosonic sector of the Standard Model which was discussed recently \cite{3}, we evaluated the effective Lagrangian for a heavy Higgs boson at the one-loop level and at order $p^4$ in the low-energy expansion by matching corresponding gauge-invariant Green’s functions at low energies. A detailed description of our calculation which preserved gauge invariance throughout the matching procedure and a comparison of our results with those obtained by the other groups \cite{1} can be found in Ref. \cite{4}.

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The Goldberger-Treiman Discrepancy in SU(3)

J. L. Goity, R. Lewis, M. Schvellinger, and L. Zhang

1Dept. Phys., Hampton University, Hampton, VA 23668, USA, and Jefferson Lab, Newport News, VA 23606, USA.

2Dept. of Phys., University of Regina, Regina, Canada.

3Dept. of Phys., Universidad Nac. de La Plata, La Plata, Argentina.

We studied the Goldberger-Treiman discrepancy (GTD) in the baryon octet in the framework of HBChPT. We confirm the previous conclusion that at leading order (order $p^2$) the discrepancy is entirely given by contact terms in the baryon effective Lagrangian of order $p^3$, and demonstrate that the subleading corrections are of order $p^4$. At leading order and in the isospin symmetry limit there are only two terms that affect the GTD, namely,

$$L_{GTD}^{(3)} = F_{19} \text{Tr}(\bar{B} S_v^\mu [\nabla_\mu \chi_-, B]) + D_{19} \text{Tr}(\bar{B} S_v^\mu \{\nabla_\mu \chi_-, B\}),$$

where the notation is standard. We analyze the discrepancies that can be determined from the available meson-nucleon couplings, namely $\pi NN$, $K \Lambda$ and $K \Sigma$, and conclude that only the smaller values of the $g_{\pi NN}$ coupling lead to a consistent picture where the $p^4$ corrections to the discrepancies would have natural size. We also check that with the smaller values of $g_{\pi NN}$ the Dashen-Weinstein relation is well satisfied, while it is badly violated for the larger values.

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The hadronic matrix element of the decays $\tau \to 3\pi \nu_{\tau}$ contains a large spin-1 part, dominated by the $a_1$ resonance, and a small spin-0 part, which is proportional to the divergence of the axial current and then to the light quark mass. The latter, involving the explicit chiral symmetry breaking sector of the theory, is expected to be very sensitive to the size of the quark anti-quark condensate $\langle \bar{q}q \rangle$. In order to investigate this sensitivity we use the generalized version of $\chi$PT.[1]

The $SU(2) \times SU(2)$ generalized chiral lagrangian is constructed and renormalized at one-loop level. We then compute the nine structure functions (see Ref. [2]) for both the charge modes $\tau \to \pi^-\pi^-\pi^+\nu_{\tau}$ and $\tau \to \pi^0\pi^0\pi^-\nu_{\tau}$. In the limit of a large condensate we recover the results of standard $\chi$PT, found in Ref. [3].

The spin-1 contribution is kinematically suppressed, at threshold, leading to sizeable azimuthal asymmetries which depend strongly on the size of $\langle \bar{q}q \rangle$, up to rather high values of the hadronic invariant mass $Q^2$. The integrated left-right asymmetry for the all-charged mode, for instance, varies from $(28 \pm 4)\%$ up to $(60 \pm 6)\%$ at $Q^2 = 0.35$ GeV$^2$ if the condensate decreases from its standard value down to zero.[1]

Unfortunately the branching ratio in this kinematical region is quite small: the largest statistics up to now has been collected by the CLEO Collaboration, with about $10^7 \tau$-pairs [4], corresponding to about 100 events from threshold to $Q^2 = 0.35$ GeV$^2$.

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These numbers correspond to taus produced at rest, and thus they are relevant only for the $\tau$-charm factories. Minor modifications in the analysis are needed to account for ultrarelativistic taus, currently produced in the accelerators.
Automatized ChPT Calculations

Frederik Orellana

1Inst. Theor. Phys., University of Zürich,
Winterthurerstr. 190, 8057 Zürich, Switzerland

In recent years much effort has been put into automatizing Feynman diagram calculations in the Standard Model and in the Minimal Super Symmetric Standard Model (see [1], [2], [3] and references therein). The reason is the increasing complexity of calculations due to more loops, more couplings and intermediate particles and more particles in the final state. These complications are motivated by the increasing energy and precision of the experiments.

It is argued that despite the different nature of ChPT [4], it is interesting and feasible also here to apply some automatization. Some of the different features of ChPT are due to the fact that it is a non-renormalizable effective theory with an expansion in the momentum, multi-leg vertices and new couplings at each order in the expansion, expressed in a rather compact notation.

An overview of existing computer programs is given. A framework is proposed for automatizing ChPT calculations [5], [6] and a new program is presented [7].

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From Chiral Random Matrix Theory to Chiral Perturbation Theory

J.C. Osborn, D. Toublan and J.J.M. Verbaarschot
Department of Physics and Astronomy, SUNY, Stony Brook, New York 11794.

We study the spectrum of the QCD Dirac operator by means of the valence quark mass dependence of the chiral condensate in partially quenched Chiral Perturbation Theory in the supersymmetric formulation of Bernard and Golterman [1]. We consider valence quark masses both in the ergodic domain \( m_v \ll E_c \) and the diffusive domain \( m_v \gg E_c \). These domains are separated by a mass scale \( E_c \sim F^2/\Sigma_0L^2 \) (with \( F \) the pion decay constant, \( \Sigma_0 \) the chiral condensate and \( L \) the size of the box) [2]. We perform a finite volume analysis of partially quenched Chiral Perturbation Theory along the same lines as the one done in Chiral Perturbation Theory [3].

In the ergodic domain the effective super-Lagrangian reproduces the microscopic spectral density of chiral Random Matrix Theory [4],[5].

In the diffusive domain we extend the results for the slope of the Dirac spectrum first obtained by Smilga and Stern [6]. We find that the spectral density diverges logarithmically for nonzero (sea) quark masses. We study the transition between the ergodic and the diffusive domain and identify a range where chiral Random Matrix Theory and partially quenched Chiral Perturbation Theory coincide [5].

We also point out some interesting analogies with mesoscopic disordered systems and with chaotic ones [7].

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