Oscillating modulation to B-mode polarization from varying propagating speed of primordial gravitational waves

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Abstract

In low-energy effective string theory and modified gravity theories, the propagating speed $c_T$ of primordial gravitational waves may deviate from unity. We find that the step-like variation of $c_T$ during slow-roll inflation may result in an oscillating modulation to the B-mode polarization spectrum, which can hardly be imitated by adjusting other cosmological parameters, and the intensity of the modulation is determined by the dynamics of $c_T$. Thus provided that the foreground contribution is under control, high-precision CMB polarization observations will be able to put tight constraint on the variation of $c_T$, and so the corresponding theories.

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I. INTRODUCTION

Inflation, as the paradigm of the early universe, has not only solved a lot of fine-tuning problems of the Big-Bang theory, but also predicted the primordial scalar and tensor perturbations. The primordial tensor perturbations, i.e. primordial gravitational waves (GWs) \[1\] \[2\] \[3\], have arisen great attentions after the BICEP2 collaboration’s announcement of the detection of B-mode signal in the CMB (around \(l \sim 80\)) \[4\], which was interpreted by them as the imprint of the primordial GWs, though this result is doubtful due to the foregrounds of polarized dust emissions \[5\], see also \[6\].

The detection of primordial GWs would verify general relativity (GR) and strengthen our confidence in inflation, and also put more constrains on inflation models and modified gravity at the same time. Besides the CMB experiments which mainly aimed at detecting low frequencies \((10^{-17} \sim 10^{-15} \text{ Hz})\) GWs, many experiments relate to higher frequencies based on other methods, such as pulsar timing array \((10^{-9} \sim 10^{-8} \text{ Hz})\), laser interferometer detectors \((10^{-4} \sim 10^{4} \text{ Hz})\), will be carried out in the coming decades. However, since the amplitude of the GWs would stay constant after they are stretched outside the horizon, and decrease with the expansion of the universe after they reenter the horizon, the primordial GWs with longer wavelength provide the most of opportunities for the detection \[7\]. Therefore, the CMB observations, especially the CMB B-mode detections, are still the most promising experiments to detect the primordial GWs if they actually exist.

Einstein’s GR is the most accepted theory of gravity. However, it might be required to modify when dealing with the inflation in primordial universe and the accelerated expansion of current universe. During matter and radiation dominated era, modified gravity has several effects on the CMB spectra, such as the lensing contribution to B-modes \[8\] and the variation of propagating speed \(c_T\) of primordial GWs \[9\] \[10\], we are especially interested in the latter in this paper, see e.g. \[11\] for the case with the scalar perturbation. In GR, graviton is massless and propagates along the null geodesics, so the propagating speed \(c_T\) of GWs is naturally set to be unity, i.e. the speed of light. But in modified gravity, e.g. the low-energy effective string theory with high-order corrections \[12\] \[13\] \[14\] \[15\] \[16\], and also modified Gauss-Bonnet gravity \[17\], and generalized Galileon (Horndeski theory \[18\]) \[19\] \[20\] \[21\] \[22\] and also \[23\], \(c_T\) might deviate from unity. Because the value of \(c_T\) determines the time of horizon crossing of GWs, during the recombination the change of \(c_T\) can result in a shift of the peak position...
of the primordial B-modes, see \cite{9,10}, for example.

In this paper, we focus on the effect of the variation of $c_T$ during slow-roll inflation on the CMB B-mode polarization, and show how it offers a distinct way to test the modified gravity theories. We find that the step-like variation of $c_T$ may result in an oscillating modulation to the B-mode polarization spectrum, which can hardly be imitated by adjusting other cosmological parameters. The intensity of the modulation is determined by the ratio of $c_T$ before and after the variation, which depends on the dynamics of theoretical models. Thus provided that the foreground contribution is under control, high-precision CMB polarization observations will be able to put tight constraint on the variation of $c_T$, and so the corresponding theories.

II. OSCILLATING SPECTRUM OF PRIMORDIAL GWs

We begin with the action for the GWs mode $h_{ij}$, e.g. \cite{15,21}

$$S_{(2)} = \int d\tau d^3x \frac{a^2 Q_T}{8} \left[ h_{ij}^2 - c_T^2 (\nabla h_{ij})^2 \right],$$

where $h_{ij}$ obeys $\partial_i h_{ij} = 0$ and $h_{ii} = 0$, $Q_T$ is regarded as effective Planck scale $M_{P,eff}^2(\tau)$, $c_T$ is the propagating speed of primordial GWs, and the prime is the derivative with respect to the conformal time $\tau$, $d\tau = dt/a$. During slow-roll inflation, the slow-roll parameter $\epsilon = -\dot{H}/H^2 \ll 1$, as well as

$$\epsilon_Q = \frac{\dot{Q}_T}{HQ_T} \ll 1, \quad s = \frac{\dot{c}_T}{Hc_T} \ll 1$$

are required, e.g. see \cite{24} for a recent study.

Here, we will mainly focus on the effect of varying $c_T$, i.e. the condition $s \ll 1$ might be broke at some point, on primordial GWs spectrum, and not get entangled with the details of \cite{11} and the evolution of background. We will assume that the background is the slow-roll inflation, and is not affected by the variation of $c_T$, and set $Q_T$ constant and $M_P^2 = 1$. We will discuss a possibility of such a case in Appendix A. In addition, there may be a mass term \cite{25,26} in \cite{11}, which might also be time-dependent \cite{27}, but we will not involve it.

We can expand $h_{ij}$ into Fourier series as $h_{ij}(\tau, x) = \int \frac{d^3k}{(2\pi)^3} e^{-ik \cdot x} \hat{h}_{ij}(\tau, k)$, where

$$\hat{h}_{ij}(\tau, k) = \sum_{\lambda=+,\times} \left[ \hat{h}_\lambda(\tau, k) a_\lambda(k) + \hat{h}_\lambda^*(\tau, -k) a_\lambda^*(k) \right] \epsilon_{ij}^{(\lambda)}(k),$$

with $\hat{h}_\lambda(\tau, k)$ the Fourier modes of $h_{ij}(\tau, x)$ and $a_\lambda(k)$ the Fourier modes of $h_{ij}(\tau, x)$

$$\epsilon_{ij}^{(\lambda)}(k) = \frac{\epsilon_{ij}(k)}{2\pi^2}$$

the 2D Fourier transform of the polarization. This latter term is included to account for the possibility of a 2D Fourier transform of the polarization.
\(\epsilon^{(\lambda)}_{ij}(k)\) are the transverse and traceless polarization tensors, \(k_j \epsilon^{(\lambda)}_{ij}(k) = 0, \epsilon^{(\lambda)}_{ii}(k) = 0\), and satisfy \(\epsilon^{(\lambda)}_{ij}(k)\epsilon^{*(\lambda)}_{ij}(k) = \delta_{\lambda\lambda'}, \epsilon^{*(\lambda)}_{ij}(k) = \epsilon^{(\lambda)}_{ij}(-k)\), the commutation relation for the annihilation and creation operators \(a_\lambda(k)\) and \(a^{\dagger}_\lambda(k')\) is \([a_\lambda(k), a^{\dagger}_\lambda(k')] = \delta_{\lambda\lambda'}\delta^{(3)}(k-k')\). We define \(h_\lambda(\tau, k) = u_k(\tau)/z_T\) and \(z_T = a\sqrt{Q_T/2}\). Thus we get the equation of motion for \(u_k\) as

\[
u'' + \left(\frac{C_T^2 k^2 - \frac{\nu''}{z_T}}{z_T^2}\right) u_k = 0. \tag{4}\]

To phenomenologically investigate the effect on primordial GWs spectrum induced by varying \(c_T\), we assume that the variation of \(c_T\) can be described by a step-like function

\[
C_T = \begin{cases} 
C_{T1} & (\tau < \tau_0) \\
C_{T2} & (\tau > \tau_0)
\end{cases}, \tag{5}
\]

where \(\tau_0 < 0\) is the transition time.

We take the background evolution to be the slow-roll inflation. Of course, it is also provided that the sudden change of \(c_T\) won’t affect the background evolution. Then, we have \(z_T''/z_T \equiv a''/a \approx (2 + 3\epsilon)/\tau^2\), and the equation of motion \((4)\) becomes

\[
u'' + \left(\frac{C_T^2 k^2 - \frac{\nu''}{z_T}}{\tau^2}\right) u_k = 0, \tag{6}\]

where \(\nu = \sqrt{\frac{2}{3} + 3\epsilon} \approx \frac{3}{2} + \epsilon\). The solution to Eq.\((6)\) is familiar, we can write it as

\[
u_{k1} = \sqrt{-c_{T1} k^2} \left[ C_{1,1} H^{(1)}_\nu(-c_{T1} k \tau) + C_{1,2} H^{(2)}_\nu(-c_{T1} k \tau) \right], \quad \tau < \tau_0, \\
u_{k2} = \sqrt{-c_{T2} k^2} \left[ C_{2,1} H^{(1)}_\nu(-c_{T2} k \tau) + C_{2,2} H^{(2)}_\nu(-c_{T2} k \tau) \right], \quad \tau > \tau_0, \tag{7}\]

where \(H^{(1)}_\nu\) and \(H^{(2)}_\nu\) are the first and second kind Hankel functions of \(\nu\)-th order, respectively. And these coefficients \(C\) are functions of the comoving wave number \(k\), but constants with respect to conformal time \(\tau\). \(C_{1,1}\) and \(C_{1,2}\) are determined by the initial condition.

Here, we set the initial condition as the standard Bunch-Davies(BD) vacuum. Therefore, when \(c_{T1} k \gg \frac{a''}{a}\), which corresponds to perturbations deep inside the horizon,

\[
u_k \sim \frac{1}{\sqrt{2c_{T1} k}} e^{-ic_{T1} k \tau}. \tag{8}\]

When \(c_{T1} k \gg \frac{a''}{a}, u_{k1}\) in Eq.\((7)\) should approximate to Eq.\((5)\). Allow for the Hankel function \(H^{(1)}_\nu(\xi) = \sqrt{\frac{2}{\pi \xi}} e^{i(\xi - \frac{\pi}{2} - \frac{\pi}{4})}\) and \(H^{(2)}_\nu(\xi) = \sqrt{\frac{2}{\pi \xi}} e^{-i(\xi - \frac{\pi}{2} - \frac{\pi}{4})}\) when \(|\xi| \to \infty\), we get

\[
C_{1,1} = \frac{\sqrt{\pi}}{2\sqrt{c_{T1} k}}, \quad C_{1,2} = 0. \tag{9}\]
The coefficients $C_{2,1}$ and $C_{2,2}$ are determined by requiring $u_k$ and $u_k'$ to be continuous at $\tau = \tau_0$, i.e., the matching condition. Then we obtain

$$C_{2,1} = \frac{i \pi^\frac{3}{2} \tau_0 k^\frac{1}{2}}{16 \sqrt{c_{T2}}} \left[ c_{T1} \left( H_{-1+\nu}^{(1)}(-c_{T1} k \tau_0) - H_{1+\nu}^{(2)}(-c_{T1} k \tau_0) \right) H_{\nu}^{(2)}(-c_{T2} k \tau_0) ight. \\ + c_{T2} \left( -H_{-1+\nu}^{(2)}(-c_{T2} k \tau_0) - H_{1+\nu}^{(2)}(-c_{T2} k \tau_0) \right) H_{\nu}^{(1)}(-c_{T1} k \tau_0) \right],$$

$$C_{2,2} = \frac{i \pi^\frac{3}{2} \tau_0 k^\frac{1}{2}}{16 \sqrt{c_{T2}}} \left[ c_{T1} \left( -H_{-1+\nu}^{(1)}(-c_{T1} k \tau_0) + H_{1+\nu}^{(1)}(-c_{T1} k \tau_0) \right) H_{\nu}^{(1)}(-c_{T2} k \tau_0) \\ + c_{T2} \left( H_{-1+\nu}^{(1)}(-c_{T2} k \tau_0) - H_{1+\nu}^{(1)}(-c_{T2} k \tau_0) \right) H_{\nu}^{(1)}(-c_{T1} k \tau_0) \right].$$ (10)

The spectrum of primordial GWs is defined by $P_T = (k^3/2\pi^2) \langle 0 | \hat{h}_{ij}(\tau, -k) \hat{h}_{ij}(\tau, k) | 0 \rangle$ with $\tau \to 0$, which is only a function of comoving wave number $k$. After neglecting the slow-roll parameter, from Eq.(7) with $\nu = 3/2$, we have

$$|u_{k2}| = \frac{\sqrt{2}}{-c_{T2} k \tau \sqrt{\pi}} |C_{2,1} - C_{2,2}|.$$ (11)

Therefore, we obtain the power spectrum of primordial GWs as

$$P_T = \frac{k^3}{2\pi^2} \sum_{\lambda=+,-,i} |h_\lambda(\tau, k)|^2 = P_{T}^{inf} \frac{4k}{Q_T \pi c_{T2}^2} |C_{2,1} - C_{2,2}|^2,$$ (12)

where

$$P_{T}^{inf} = 2H_{inf}^2/\pi^2$$ (13)

is that of standard slow-roll inflation without modified gravity, i.e. $Q_T = 1$ and $c_{T1} = c_{T2} = 1$, and $H_{inf}$ is the Hubble parameter during inflation, which sets the scale of inflation.

The effect of varying $c_T$ is encoded in $C_{2,1}$ and $C_{2,2}$. We set $x = c_{T2}/c_{T1}$ and defined a new function

$$f(k, k_0, x) = \frac{4c_{T1} k}{\pi x^2} |C_{2,1} - C_{2,2}|^2,$$ (14)

where $k_0 = -1/(c_{T1} x \tau_0)$. Then, the GWs spectrum (12) may be rewritten as

$$P_T = P_{T}^{inf} \cdot \frac{f(k, k_0, x)}{c_{T1} Q_T},$$ (15)

where $f(k, k_0, B)$ is obtained as

$$f(k, k_0, x) = \frac{1}{x^2} \sin^2 \left( \frac{k}{k_0} \right) + \frac{1}{x^4} \left[ \cos \left( \frac{k}{k_0} \right) - (1 - x^2) \frac{k_0}{k} \sin \left( \frac{k}{k_0} \right) \right]^2.$$ (16)
FIG. 1: The function $f(k, k_0, x)$, where $x = c_{T2}/c_{T1}$. We have set $x = 0.9$ on the left panel and $x = 1.1$ on the right panel, respectively.

We plot $f(k, k_0, x)$ with respect to $k/k_0$ in Fig. 1, in which we set $x = 0.9$ on the left panel and $x = 1.1$ on the right panel, respectively. Here, the transition time $\tau_0 = -1/(c_{T1}xk_0)$ sets a character scale $1/k_0$. When $k \ll k_0$, i.e. the perturbation mode has longer wavelength than $1/k_0$, we have $f(k, k_0, x) \approx 1$, and $P_T = P_{infT}/(c_{T1}^2Q_T)$ is scale-invariant, which is the result of slow-roll inflation with almost constant $c_T$ and $Q_T$ [21][28]. When $k \gg k_0$, we have

$$f(k, k_0, x) \approx \frac{1}{x^2} \left[ 1 + \left( \frac{1}{x^2} - 1 \right) \cos^2 \left( \frac{k}{k_0} \right) \right],$$

(17)

thus $f(k, k_0, x)$ oscillates between $1/x^2$ and $1/x^4$, and $P_T$ oscillates correspondingly, just as we can see from Fig. 1.

In Refs. [29][30], the effects of varying sound speed $c_S$ of primordial scalar perturbations on the scalar power spectrum have been investigated. The sudden change of $c_S$ may lead to the oscillating modulation to the primordial scalar spectrum, as well as the CMB TT-mode spectrum, was found in [29]. In addition, the oscillation in the primordial scalar spectrum can also be attributed to some other effects, such as inflaton potential with a small oscillation [31][32], a sudden change in inflaton potential or its derivative, e.g. [33][34][35][36].

However, it should be commented that the oscillation in the primordial scalar spectrum may be implemented without modified gravity, but that in the primordial GWs spectrum can only be attributed to the modified gravity. When the gravity is GR, $P_{infT}$ is that in (13). Thus the oscillation of $P_{infT}$ certainly requires $H_{inf}$ is oscillating, which is impossible, unless the null energy condition is violated periodically. Though the particle production may also modify the GWs spectrum [37][38][39], it only leads to a bump-like contribution, which is entirely different from the behavior of oscillation.
FIG. 2: Theoretical CMB BB and TT-mode power spectra for our oscillating GWs spectrum (brown line in left panel and red solid line in right panel) and the power-law GWs spectrum for reference (blue dashed line in left panel and green dashed line in right panel). The inset of right panels are the TT-mode spectra for our oscillating GWs spectrum (the yellow solid lines) and the power-law GWs spectrum (the blue dashed lines) for reference. We set $r = 0.05$ and $k_0 = 1/30000Mpc^{-1}$.

The primordial GWs is imprint in CMB as the B-mode polarization. Thus the oscillation in the primordial GWs spectrum will inevitably affect B-mode polarization spectrum.

III. CMB B-MODE POLARIZATION SPECTRUM

To see such effects, we plot the CMB BB and TT-mode correlations in Fig.2, in which $P_{T}^{\text{inf}}$ in (15) is parameterized as

$$P_{T}^{\text{inf}} = rA_{R}^{\text{inf}} \left( \frac{k}{k_s} \right)^{n_{T}^{\text{inf}}}.$$  \hspace{1cm} (18)

Here, we assume that the scalar spectrum is hardly affected by the modified gravity, which will be clarified in Appendix A. Thus the scalar perturbation spectrum is set as $P_{R}^{\text{inf}} = A_{R}^{\text{inf}} (k/k_s)^{n_{R}^{\text{inf}}-1}$, in which $A_{R}^{\text{inf}}$ is the amplitude of the scalar perturbations. In addition, we also assume that after inflation the propagating speed $c_T$ is unity, so that the spectrum of B-mode polarization is not affected by relevant evolution at late time, or see [9], [10].

In the left panels of Fig.2, we see some obvious enhancements or suppressions to the reion-
FIG. 3: BB-mode spectra at low multipoles for our oscillating GWs spectrum [15] with different $x$ (solid lines) and the power-law GWs spectrum with different $\tau_{ri}$ (dashed lines).

The height of the reionization bump can be estimated roughly as [40],

$$C_{T,l}^{BB} \approx \frac{1}{100} \left(1 - e^{-\tau_{ri}}\right)^2 C_{T,l}^{TT},$$  \hspace{1cm} (19)

where $C_{T,l}^{TT}$ stands for the TT-mode spectrum from the primordial GWs without the reionization and $\tau_{ri}$ is the optical depth to the beginning of reionization. The enhancements or suppressions of the reionization bump is a reflection of the oscillations of primordial GWs spectrum on large scales. In addition, we can also see some obvious oscillations around the recombination peak at $l \sim 80$.

In the right panels of Fig. 2, we see that the TT-mode spectrum is hardly affected by the oscillating primordial GWs power spectrum, since the contribution of GWs to TT-mode spectrum is negligible, compared with the scalar perturbations. The case of EE-mode polarization spectrum is actually also similar. Thus the mainly effect of varying speed of primordial GWs is on the B-mode polarization, which makes the B-mode polarization spectrum shows its obvious enhancements or suppressions to the reionization bump and oscillations around the recombination peak.

However, it might be concerned whether such a B-mode polarization spectrum can be imitated by adjusting other cosmological parameters or not. In Eq. (19), the optical depth $\tau_{ri}$ is relevant with the height of the reionization bump. We show the BB-mode spectrum with different $\tau_{ri}$ in Fig. 3. We see that the change of $\tau_{ri}$ can only alter the overall amplitude of BB-mode spectrum at low multipoles, but hardly create the oscillation. This indicates that
although the BB-mode spectrum may be modified by other ways, the oscillating modulation leaded by varying speed of primordial GWs is difficult to be imitated. Thus the measure of B-mode polarization spectrum provides a suited way for testing the corresponding gravity physics in primordial universe.

IV. DISCUSSION

In low-energy effective string theory and modified gravity theories, the propagating speed $c_T$ of primordial GWs may deviate from unity. We calculated the spectrum of primordial GWs, assuming that $c_T$ has a step-like variation and the background of slow-roll inflation is not affected by it. We found the spectrum of primordial GWs acquires an oscillating modulation, which makes the B-mode polarization spectrum shows its obvious enhancements or suppressions to the reionization bump and oscillations around the recombination peak. The intensity of the modulation is determined by $c_{T2}/c_{T1}$. And the frequency of the modulation is determined by $k_0 = -1/(c_{T2} \tau_0)$. Both depend on the dynamics of theoretical models.

The oscillating behavior of the B-mode polarization can only be attributed to the effect of modified gravity, since it can hardly be imitated by adjusting other cosmological parameters. The upcoming CMB experiments, such as CMBPol, B-Pol, will provide us with some clues of the primordial GWs, which will be able to put tight constraint on the propagating speed of primordial GWs, and so the corresponding theories, provided that the foreground contribution is under control. In certain sense, our work again highlights the significance of B-mode polarization measures in exploring the fundamental physics of primordial universe.

Here, we only postulate a simple variation of $c_T$, which, however, might be far complicate in some modified gravity models, as well as accompanied by the variation of $Q_T$. The effect could be non-trivial. And when we focus on the B-mode polarization spectrum, the assumption we adopted is that after inflation the propagating speed $c_T$ is unity, which may also be relaxed. In addition, the varying $c_T$ and $Q_T$ will also affect the non-Gaussianities of primordial perturbations. The relevant issues are open.

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Appendix A:

In this Appendix, we will argue how to realize the change of $c_T$ and the changeless of $Q_T$ in (1) in low-energy effective string theory and modified gravity theories.

In low-energy effective action of string theory, the simplest extension of the lowest-order action is e.g.\[15\]
\[
L_{\text{correction}} \sim -\frac{\alpha'\lambda\xi(\varphi)}{2}\left(c_1 R_{GB}^2 + c_2 G^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\right) \tag{20}
\]
where $G^{\mu\nu} = R^{\mu\nu} - g^{\mu\nu}R/2$, and $R_{GB}^2 = R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho} - 4R_{\mu\nu}R^{\mu\nu} + R^2$ is the Gauss-Bonnet term, $\alpha'$ is the inverse string tension, $\lambda$ is a parameter allowing for different species of string theories, $c_1$ and $c_2$ are coefficients. We have neglected the terms with $\Box \varphi$ and $(\partial_\mu\varphi\partial^\mu\varphi)^2$, since both do not contribute to GWs.

The introducing of (20) will affect not only the GWs, but also the adiabatic scalar perturbations, of course, it is interesting to check its effect on the latter. However, for our purpose, we will regard $\varphi$ as the curvaton, so that the effects of (20) on the background and the scalar perturbations are negligible.

The action for GWs is (1), and [15]
\[
Q_T = 1 - \frac{\alpha'\lambda}{2}\left(8c_1\dot{\xi}H_{\text{inf}} - c_2\xi\dot{\varphi}^2\right),
\]
\[
c_T^2 = \frac{1}{Q_T}\left[1 - \frac{\alpha'\lambda}{2}\left(c_2\xi\dot{\varphi}^2 + 8c_1\dot{\xi}\right)\right], \tag{21}
\]
see also Ref.[41] for that with $c_1 = 0$. Here, it is required that $8c_1\dot{\xi}H_{\text{inf}} \equiv c_2\xi\dot{\varphi}^2$ is always imposed, which sets $Q_T = 1$. Thus with slow-roll approximation $\ddot{\xi} = \xi\varphi\dot{\varphi}^2 + \xi\varphi\ddot{\varphi} \ll \xi\dot{\varphi}^2$,
\[
c_T^2 \simeq 1 - \frac{\alpha'\lambda}{2}c_2\xi\dot{\varphi}^2 \tag{22}
\]
is obtained. The step-like variation of $c_T^2$ requires that $\xi(\varphi)$ or $\dot{\varphi}^2$ has the step at $\tau = \tau_0$. Of course, the variation of $c_T^2$ could be more complicate than a step-like evolution, which would be harder to deal with. However, as it may, $Q_T > 0$ and $c_T^2 > 0$ should be required to avoid ghost and gradient instabilities.

The action (20) is actually equivalent to the Horndeski theory, see Ref.[21], and so above analysis is also similar for corresponding theory.

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