BER DEGRADATION OF MC-CDMA AT HIGH SNR WITH MMSE EQUALIZATION AND RESIDUAL FREQUENCY OFFSET

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by
Harinath Reddy P
200431004
harinath@research.iiit.ac.in
Communications Research Center

INTERNATIONAL INSTITUTE OF INFORMATION TECHNOLOGY
GACHIBOWLI, HYDERABAD, A.P., INDIA - 500 032
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CERTIFICATE

It is certified that the work contained in this thesis, titled "BER Degradation of MC-CDMA at High SNR with MMSE Equalization and Residual Frequency Offset" by Harinath Reddy P, has been carried out under my supervision and it is fully adequate in scope and quality as a dissertation for the degree of Master of Science.

Date

Prof V U Reddy (Advisor)
Abstract

Multicarrier Code Division Multiple Access (MC-CDMA) is an attractive technique for high speed wireless data transmission in view of its advantages over orthogonal frequency division multiplexing. In this thesis, we analyze the performance of fully loaded downlink MC-CDMA systems with minimum mean square error (MMSE) equalizer in the presence of residual frequency offset (RFO) in multipath Rayleigh fading channels. We first show that as the SNR is increased beyond a value, referred as threshold SNR, the performance degrades. We then analyze the cause for this behavior and propose a remedy to prevent the degradation by regularizing the coefficient(s) of the equalizer, and use the regularized equalizer for SNRs beyond the threshold value.

The threshold SNR depends on the RFO and the profile of multipath channel. We suggest two methods for estimating this SNR, one gives close to the true value but requires the knowledge of RFO and the channel state information (CSI), while the other gives an approximate value but requires only CSI. We first show that if the actual value of RFO is less than the assumed, the threshold SNR estimate based on the assumed RFO will still be appropriate. Next, we show that the regularization based on the approximate value also prevents the degradation, but the performance at higher SNRs is slightly poorer compared to that with the better estimate. Numerical and simulation results are provided to support the analysis.
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Chapter 1

Introduction

Third Generation (3G) mobile communication systems allow us to have whole new ways to communicate and access information. They have already been deployed in several countries. 3G is a recent technology, and a lot of research is going on in this area. Further research efforts are especially focussed on systems that can provide much higher data rates and seamless connectivity. Such systems are categorized under Fourth Generation (4G). While wide-band systems are considered by many to be a natural choice to provide higher data rates, the spectrum required comes at a very high cost. Spectral efficiency is always a factor on the choice of any wireless technology. Wide-band systems usually require complex receivers as the channel is frequency selective due to the presence of large number of resolvable multipaths.

1.1 Multi-Carrier Transmission

The principle of multi-carrier transmission is to convert a high-rate data-stream into several parallel low-rate data streams. In other words, a wide band channel is divided into many parallel narrow band sub-channels. Since the symbol rate on each sub-carrier associated with each sub-channel is much less than the initial symbol rate, the effects of delay spread, i.e., ISI, significantly decrease, reducing the complexity of the equalizer.
1.1.1 OFDM Systems

OFDM is a low-complex technique to efficiently modulate multiple sub-carriers by using digital signal processing [1]. Several present and upcoming wireless communication standards adopted OFDM as the modulation format. The major reasons for choosing OFDM are:

1. A frequency selective channel is transformed into sum of frequency flat sub-channels.

2. It is very efficient in spectrum usage.

3. Modulator and demodulator are implemented very efficiently using IFFT and FFT.

4. A single tap frequency-domain equalizer (same as zero-forcing equalizer) is adequate at the receiver.

The OFDM transmitter and receiver block diagrams are shown in Fig. 1.1 and 1.2 respectively. At the transmitter, the data bits are first mapped to symbols and then sent to the serial-to-parallel converter. The output of serial-to-parallel converter is sent to IFFT block. In the IFFT block, the IFFT of the symbols is computed and the output of this block is sent to Add Guard Interval block which adds cyclic prefix (to prevent the inter-block interference[20] and make the linear convolution as circular convolution which helps in reducing the complexity of the equalizer at the receiver) and sends the resultant output to parallel-to-serial converter. The output of the parallel-to-serial converter is transmitted.

The received signal is first passed through a serial to parallel converter and then given to the FFT block after removing the guard interval (cyclic prefix). The output of the FFT block is fed to the equalizer block and its output is given to parallel to
CHAPTER 1. INTRODUCTION

Figure 1.1: Transmitter of an OFDM system

Figure 1.2: Receiver of an OFDM system
serial converter. The output of parallel to serial converter is sent to symbol de-mapper.

A frequency selective channel may result in some bins being very weak. The symbols loaded in these bins will experience a poor SNR at the receiver. Consequently, symbol error probability in these bins will be high even at high SNRs. Consider a mechanism whereby each symbol is carried by all the bins, and we use certain orthogonal spreading sequences such that decoding of each symbol is possible with minimal interference from other symbols. Then, symbol carried by all the bins can be combined to give a strengthened symbol if we have the perfect knowledge of the channel. The loss of orthogonality among the spreading sequences, caused by the frequency selective nature of the channel, introduces multi-code interference (MCI) [3] which can be minimized by applying minimum mean square error (MMSE) equalization. This method of spreading the symbols across the sub-carriers is called Multi Carrier Code Division Multiple Access (MC-CDMA) [4] and the systems which use this method are MC-CDMA systems.

1.1.2 MC-CDMA Systems

In MC-CDMA systems the symbols are spread across the sub-carriers. By spreading, we get frequency diversity. Combination of frequency diversity and an appropriate equalizer yields improved bit error rate (BER) performance in multipath channels compared to OFDM [2].

As MC-CDMA is implemented by spreading across the sub-carriers, we use Walsh-Hadamard codes for spreading. The Walsh-Hadamard matrix is given by

\[
W_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}
\]

(1.1)

\[
W_{2^n} = \begin{bmatrix} W_{2^{n-1}} & W_{2^{n-1}} \\ W_{2^{n-1}} & -W_{2^{n-1}} \end{bmatrix}
\]

(1.2)
for \( n \geq 2 \). We assume that the frequency spreading factor is same as the number of sub-carriers, \( N_f \), which is an integer power of 2. Let \( \mathbf{W} \) denote the Walsh-Hadamard matrix of size \( N_f \times N_f \)

\[
\mathbf{W} = \begin{bmatrix}
w_0 & w_1 & \cdots & w_{N_f-1}
\end{bmatrix}
\]

where \( w_k = [w_{0,k} \ w_{1,k} \ \cdots \ w_{N_f-1,k}]^T \) with the superscript \( T \) denoting transpose of a vector.

Let the symbols be \( a_k \), \( 0 \leq k \leq N_f - 1 \). The symbol \( a_k \) is spreaded in frequency by \( w_k \). The output of the frequency spreader block (see Fig. 1.3) is given by

\[
x = \sum_{k=0}^{N_f-1} w_k a_k
\]

where \( x \) is a vector of size \( N_f \times 1 \). From the above equation its clear that we are considering a fully loaded downlink MC-CDMA system. The output of this block is fed to the IFFT block whose output is given by

\[
y = \mathbf{F} \mathbf{x}
\]

where \( y = [y_0 \ y_1 \ \cdots \ y_{N_f-1}]^T \) and \( \mathbf{F} \) denotes normalized IFFT matrix given by

\[
\mathbf{F} = \frac{1}{\sqrt{N_f}} \begin{bmatrix}
1 & 1 & \cdots & \cdots & 1 \\
1 & u & u^2 & \cdots & u^{N_f-1} \\
1 & u^2 & u^4 & \cdots & u^{2(N_f-1)} \\
1 & \vdots & \vdots & \ddots & \vdots \\
1 & u^{N_f-1} & \cdots & \cdots & u^{(N_f-1)(N_f-1)}
\end{bmatrix}_{N_f \times N_f}
\]

where \( u = e^{\frac{2\pi i}{N_f}} \) and \( i = \sqrt{-1} \). The output \( y \) is sent to the Add Guard Interval block where the cyclic prefix of length \( (L-1) \) is added giving

\[
\tilde{y} = [y_{N_f-L+1} \ y_{N_f-L} \ \cdots \ y_{N_f-1} \ y_0 \ \cdots \ y_{N_f-1}]^T_{(N_f+L-1) \times 1}
\]
Figure 1.3: Transmitter of an MC-CDMA system

We add the cyclic prefix because it prevents the inter-block interference (provided the number of channel taps is less than or equal to $L$)\cite{20} and makes the linear convolution as circular convolution which helps in reducing the complexity of the equalizer at the receiver. $\tilde{y}$ is transmitted after parallel to serial conversion.

The receiver block diagram is shown in Fig. 1.4. The received signal is first passed through a serial to parallel converter and then given to the FFT block after removing the guard interval (cyclic prefix). The output of the FFT block is fed to equalizer block and its output is given to the frequency despreader block. The other blocks are parallel to serial converter and symbol de-mapper (see Fig. 1.4).

1.2 Effects of RFO

In OFDM systems, timing and frequency synchronization is very important \cite{5}. In particular, lack of frequency synchronization causes loss of orthogonality among the
Figure 1.4: Receiver of an MC-CDMA system
sub-carriers thereby introducing the inter carrier interference (ICI). Though several algorithms are proposed for estimating and correcting the frequency offset [6]-[10], there will always be some amount of residual frequency offset (RFO) left uncompensated. In [11], the authors analyze the BER performance of the OFDM in multipath fading channels in the presence of RFO.

As MC-CDMA is a combination of OFDM and CDMA, it is sensitive to RFO [12]-[15]. In [12] and [13], the authors analysed the performance of MC-CDMA in the presence of RFO using equal gain combining and maximal ratio combining equalizers. In [14], the authors compared the performance of maximal ratio combining (MRC) and equal gain combining (EGC) with synchronization errors over fading channels. In [15], the authors discussed the sensitivity of two-dimensional spreading schemes, such as orthogonal frequency code division multiplexing, to synchronization errors using zero-forcing (ZF) and minimum mean square error (MMSE) equalizers.

In this thesis we analysed the performance of fully loaded downlink MC-CDMA systems in the presence of RFO with ZF and MMSE equalizers. As an MMSE equalizer tends to behave like a ZF equalizer at high SNRs, we observed how the performance of MC-CDMA in the presence of RFO with MMSE equalizer is affected at high SNRs. We also looked into methods to improve the performance.

1.3 Contributions

The key contributions of this thesis are as follows: We analyse the performance of fully loaded downlink MC-CDMA systems in the presence of RFO with ZF and MMSE equalizers. We obtain a closed-form expression for the average signal-to-interference-plus noise ratio (SINR). We show that though the performance of the MMSE equalizer is significantly better than that of ZF at lower SNRs, it starts degrading beyond a SNR value, referred hereafter as threshold SNR, which depends on RFO and the profile of multipath channel, and tends towards that of ZF as SNR is increased further. We analyse the cause for this behavior and suggest a remedy to prevent degradation
by regularization of the coefficient(s) of the MMSE equalizer. The regularized equalizer is used for SNRs beyond the threshold value.

We suggest two methods for estimating this SNR value, one of them gives close to the true value but it requires the knowledge of RFO and channel state information (CSI), while the other gives an approximate value which needs the knowledge of only CSI. If the actual value of RFO is less than the assumed, the threshold SNR estimated based on the assumed RFO will still be appropriate. The regularization with approximate value of the threshold SNR also prevents degradation, but with a small loss in the BER performance at higher SNRs compared to that with the better estimate. Numerical and simulation results are provided to support the analysis.

1.4 Thesis Organization

This thesis is organized as follows. In Chapter 2, we obtain the expression for BER of MC-CDMA with MMSE and ZF equalizers and bring out the BER performance degradation with the former at high SNRs. In Chapter 3, we analyse the cause for the degradation beyond a threshold SNR and suggest a remedy as well as two methods for estimating the threshold SNR. In Chapter 4, we give simulation results for multipath Rayleigh fading channels. Chapter 5 concludes with a summary and future work.
Chapter 2

BER Performance of MC-CDMA

At the receiver, we perform timing and frequency synchronization, and channel estimation using a preamble. We assume perfect timing synchronization and perfect knowledge of the CSI. Also, we assume that some amount of RFO is left after correction with estimated carrier frequency offset, and normalized value of this (normalized with sub-carrier spacing) is of the order $10^{-2}$. Let this RFO be denoted as $\epsilon$.

We collect $(N_f + L - 1)$ samples of the received signal, remove the cyclic prefix and compute its FFT. The FFT output is given by [11]

$$
r = e^{i2\pi \epsilon \left(\sum_{n=0}^{N_f-1} n (N_f + L - 1)\right)} F^H \mathbf{H} \mathbf{y} + \eta
$$

(2.1)

where the superscript $^H$ denotes Hermitian transpose, $n$ refers to $n^{th}$ MC-CDMA symbol and the exponent is the phase accumulation term after removing the cyclic
prefix. $H$ is a circulant matrix

$$H = \begin{bmatrix}
h_0 & 0 & \ldots & h_{L-1} & h_{L-2} & \ldots & h_1 \\
h_1 & h_0 & \ldots & 0 & h_{L-1} & \ldots & h_2 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
h_{L-2} & h_{L-3} & \ldots & h_0 & 0 & \ldots & h_{L-1} \\
h_{L-1} & h_{L-2} & \ldots & h_1 & h_0 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & h_{L-1} & h_{L-2} & \ldots & h_0
\end{bmatrix}_{N_f \times N_f}$$ (2.2)
and \((m, l)\)th element of \(\tilde{T}\), for \(m, l = 0, 1, \ldots, N_f - 1\), is
\[
\tilde{T}(m, l) = \frac{\sin\left(\pi((l - m) + \epsilon)\right)}{N_f \sin\left(\pi((l - m) + \epsilon)\right)} e^{i\pi(N_f - 1)((l - m))} N_f
\]  
and \(\Lambda\) is a diagonal matrix with the diagonal elements as the eigenvalues of \(H\). Let these eigenvalues be \(\lambda_0, \lambda_1 \ldots \lambda_{N_f - 1}\). They represent the sub-channel gains. We may point out here that the \(N_f\) DFT coefficients of \(h_0, h_1, \ldots h_{L-1}\), computed using normalized DFT and scaled by \(\sqrt{N_f}\), represent the eigenvalues of \(H\). The term \(e^{i2\pi\left(\frac{n(N_f+L-1)}{N_f} + L-1 + \frac{(N_f-1)/2}{N_f}\right)}\) in (2.6) is the common phase error and we assume that the receiver is able to perfectly compensate this error for each block using pilot tones (see [11], [16]). After compensation with common phase error, we have
\[
\tilde{r} = \tilde{T}\Lambda x + \tilde{\eta}
\]  
In view of the model assumed for \(\eta\), the elements of \(\tilde{\eta}\) are also i.i.d complex Gaussian with zero mean and variance \(\sigma^2\).

### 2.1 MMSE and ZF Equalizers

The frequency selective nature of the multipath channel causes multi-code interference (MCI) while RFO causes inter carrier interference (ICI). If the value of RFO is known as well as the knowledge of CSI, we can design the equalizer to combat both MCI and ICI. However, in practice, we will not have the knowledge of the exact value of RFO, and hence, we design the equalizer only to combat MCI.

Let \(V_{eq}\) denote the equalizer matrix. Applying this to (2.8), we have
\[
d = V_{eq} \tilde{r} = \sum_{l=0}^{N_f-1} a_l V_{eq} \tilde{T} \Lambda w_l + V_{eq} \tilde{\eta}
\]  
where we replaced \(x\) with (1.4), and \(d\) is a vector of size \(N_f \times 1\). Next, we apply
frequency de-spreading to decode the transmitted symbol $a_m$

$$\hat{a}_m = \sum_{l=0}^{N_f-1} a_l w^T_m V_{eq} \hat{\Lambda} w_l + w^T_m V_{eq} \tilde{\eta}$$  \hspace{1cm} (2.10)

for $0 \leq m \leq N_f - 1$.

If we now design a MMSE equalizer only to combat MCI, then $V_{eq}$ is a diagonal matrix with diagonal elements as (see [17] and [2])

$$V_{eq,mmse}(m, m) = \frac{(\lambda_m)^*}{N_f |\lambda_m|^2 + \frac{\alpha^2}{P}}, \quad 0 \leq m \leq N_f - 1 \hspace{1cm} (2.11)$$

where $P = E(a_m a_m^*)$ with superscript $*$ denoting complex conjugate. The detailed derivation of MMSE equalizer is presented in Appendix A. On the other hand, if we choose a ZF equalizer, then the corresponding $V_{eq}$ is given by

$$V_{eq,zf} = \Lambda^{-1} \hspace{1cm} (2.12)$$

## 2.2 BER Performance of MC-CDMA with ZF and MMSE equalizers

To compute BER with a particular equalizer, we follow the analysis suggested in [12], [13] and [15]. We decompose the first term of (2.10) into three parts, $a^1_m$, $a^2_m$ and $a^3_m$, as follows.

$$\hat{a}_m = a^1_m + a^2_m + a^3_m + w^T_m V_{eq} \tilde{\eta}$$  \hspace{1cm} (2.13)

where

$$a^1_m = a_m w^T_m V_{eq} \Lambda w_m, \hspace{1cm} (2.14)$$
$a_m^2 = a_m w_m^T V_{eq} (\bar{T} - E) \Lambda w_m,$ \hspace{1cm} (2.15)

and

$a_m^3 = \sum_{l=0, l \neq m}^{N_f-1} a_l w_m^T V_{eq} \bar{T} \Lambda w_l.$ \hspace{1cm} (2.16)

The matrix $E$ is a diagonal matrix whose elements are the diagonal elements of $\bar{T}$. From the structure of $\bar{T}$ (see (2.7)), we note that $a_m^1$ and $a_m^2$ are the desired symbol multiplied by a real scalar and a complex scalar, respectively, and $a_m^3$ is the interference from other symbols caused by MCI and ICI. Though the second term contains the desired symbol, it can add to the first term constructively or destructively because of the associated complex scalar. We, therefore, treat it as the interference and following [13], we define it as self interference. The third term is the interference from other symbols.

In view of the assumption that the symbols are identical and independently distributed random variables with zero mean and variance $P$, and applying the central limit theorem, we model the second and third terms as zero mean and uncorrelated Gaussian variables. The fourth term in (2.13) is a zero mean Gaussian variable which is uncorrelated with the second and third terms. Further, in view of the i.i.d. nature of the elements of $\tilde{\eta}$, its variance is independent of which symbol is being decoded.

To find the BER, we first evaluate average SINR for each decoded symbol. Denoting the average SINR for the decoded symbol $a_m$ as $SINR_m$, we have from (2.14), (2.15), (2.16) and the fourth term in (2.13)

\begin{equation}
SINR_m = \frac{E(a_m^1 a_m^*)}{E(a_m^2 a_m^*) + E(a_m^3 a_m^*) + E(N_m N_m^*)} \hspace{1cm} (2.17)
\end{equation}

where

\begin{equation}
N_m = w_m^T V_{eq} \tilde{\eta} \hspace{1cm} (2.18)
\end{equation}
Assuming that the symbols are drawn from a 4-QAM constellation of average power $P$ and they are equally likely, and the mapping of data bits to symbols is based on Gray encoding, the BER is given by [19]

$$BER((A, \epsilon) \approx \frac{1}{N_f} \sum_{m=0}^{N_f-1} Q(\sqrt{\text{SNR}_m}) \quad (2.19)$$

The above expression can be evaluated using numerical integration for both MMSE and ZF equalizers choosing the corresponding equalizer coefficients in the SINR expression.

### 2.3 Degradation of MMSE Equalizer Performance at High SNRs

Note that at high SNRs (see (2.11)),

$$v_{eq, mmse}(m, m) \approx \frac{(\lambda_m)^*}{N_f|\lambda_m|^2} = \frac{1}{N_f} v_{eq, zf}(m, m) \quad (2.20)$$

The above relation, combined with the fact that the BER performance of MMSE equalizer is significantly better than that of ZF equalizer at low and moderate SNRs in multipath fading channels (see [2]) and also that ZF performance has a floor due to ICI, suggests that the MMSE equalizer performance degrades beyond a SNR value, referred as threshold SNR. This SNR depends on RFO and multipath channel profile.

To see if this is the case, we first considered a particular realization of a 6-tap Rayleigh fading channel model [18] given in Table 2.1, and evaluated the BER performance from (2.19) with both MMSE and ZF equalizers for two values of RFO, 0.05 and 0.03, choosing $N_f = 64$. We may mention here that the channel model considered here is slightly different from the one given in [18] in that we normalized each tap variance such that the total variance is one (The variances given in Table 2.1 are not normalized ones).
Figure 2.1 gives the BER plots which support our above remark regarding the performance of MMSE equalizer, i.e., the performance degrades beyond a threshold SNR. Note from the plots that for RFO=0.05, the threshold SNR is 28 dB while it is 33 dB for RFO=0.03. We may point out here that when we considered a particular channel realization in the analysis, we normalized the channel impulse response coefficients so as to make it a unit-norm channel. Note that $N_f P$ is the average transmitted signal power in each bin and this prompted us to plot the curves as a function of $\frac{N_f P}{\sigma^2}$.

Next, we considered 3 different realizations of the same 6-tap channel and Figure 2.2 shows the BER plots for RFO=0.05. Note that the degradation happens for the channel realizations with large values of $\frac{|\lambda_{\text{max}}|}{|\lambda_{\text{min}}|}$ (see Table 2.2) where $|\lambda_{\text{max}}|$ and $|\lambda_{\text{min}}|$ denote, respectively, the largest and smallest of $|\lambda_0|$, $|\lambda_1|$, $\ldots$, $|\lambda_{N_f-1}|$ and correspond to the strongest and weakest bin gains, respectively. Note that the threshold SNR in the case of CR-1 is 28 dB while it is 26 dB for CR-2. We will explain in the next chapter why the BER performance does not degrade at high SNRs in the case of channel realization CR-3.
Table 2.1: Channel model given in [18]

| Tap index | Average power in dB |
|-----------|---------------------|
| 0         | 0                   |
| 1         | -0.2                |
| 2         | -5.4                |
| 3         | -6.9                |
| 4         | -24.5               |
| 5         | -29.7               |

Table 2.2: Channel realizations used in the performance plots of Figs. 2.1 to 3.3 (CR=channel realization)

| Tap weight | CR-1                      | CR-2                      | CR-3                      |
|------------|----------------------------|----------------------------|----------------------------|
| $h_0$      | 0.0617 - i0.0084          | -0.7407 - i0.3066         | -0.6855 + i0.2442         |
| $h_1$      | 0.2688 - i0.6350          | -0.3272 - i0.3975         | -0.2038 + i0.4685         |
| $h_2$      | -0.1936 - i0.1330         | -0.0152 + i0.1807         | -0.0562 + i0.1787         |
| $h_3$      | -0.5929 + i0.3363         | 0.2067 + i0.1272          | -0.4068 - i0.0845         |
| $h_4$      | -0.0069 + i0.0209         | 0.0087 - i0.0029          | 0.0277 - i0.0279          |
| $h_5$      | -0.0057 + i0.0169         | -0.0211 - i0.0054         | -0.0037 - i0.0129         |

\[\lambda_{\text{max}}\] \begin{align*}
114 \text{ for } N_f=64 \\
67 \text{ for } N_f=64 \\
17 \text{ for } N_f=64
\end{align*}
Figure 2.1: BER Performance of MC-CDMA with MMSE and ZF equalizers, evaluated from (2.19), for RFO=0.05 (plots marked 1) and RFO=0.03 (plots marked 2) ($N_f=64$ and the channel realization is CR-1 given in Table 2.2, and the symbols are from 4-QAM constellation with $P = 1$)
Figure 2.2: BER Performance of MC-CDMA with MMSE and ZF equalizers, evaluated from (2.19), for three different channel realizations (RFO=0.05, $N_f=64$ and CR-1, CR-2, CR-3 refer to the channel realizations given in Table 2.2, and symbols are from 4-QAM constellation with $P = 1$ )
Chapter 3

Cause and Remedy for the Degradation

The results of the preceding chapter shows that the performance of MC-CDMA with MMSE equalizer degrades beyond a threshold SNR in multipath channels in the presence of RFO. In other words, the SINR decreases with increasing SNR beyond the threshold SNR. We now make an attempt to pinpoint the cause for such behavior.

3.1 Cause

Recall that the MMSE equalizer is not designed to combat the ICI which contributes to the term $a^3_m$ in the demodulated symbol $a_m$. To see the effect of this, we evaluated $E(a^2_m a^3_m)$ as a function of SNR with RFO=0.05 and for CR-1. Note from Fig. 3.1 that the $E(a^2_m a^3_m)$ begins to increase beyond about 28 dB SNR which is the threshold SNR for the plot 1 corresponding to MMSE in Fig. 2.1.
As $a^3_m$ is the interference from the symbols other than the symbol being decoded and carried by all the sub-carriers (see (2.16)), we express $a^3_m$ as

$$a^3_m = \sum_{k=0}^{N_f-1} a^3_{m,k}$$

(3.1)

with $a^3_{m,k}$ given by

$$a^3_{m,k} = \sum_{l=0, l \neq m}^{N_f-1} a_l w^T_m V_{eq,mmse}^k \bar{T}_l \Lambda_l$$

(3.2)

where

$$V_{eq,mmse}^k = diag [0 \cdots 0 V_{eq,mmse}(k, k) 0 \cdots 0]$$

(3.3)

Here, $a^3_{m,k}$ represents the amount of interference caused by the symbols other than the symbol being decoded, carried by $k^{th}$ sub-carrier. We now evaluate $a^3_{m,k}$ for three different sub-carriers, one corresponding to the strongest bin (i.e., sub-channel with largest gain), another corresponding to the weakest bin and the third corresponding the next weakest bin.

Figure 3.2 gives the plots of $E(a^3_{m,k}a^3_{m,k})$ corresponding to these three bins. We observe the following from the plots. The interference contribution from the strongest bin (Plot 3) is nearly independent of SNR, while the contribution from the weakest bin (Plot 1) increases monotonically with SNR, tending to a constant value only at very large values of SNR. The contribution from the next weakest bin (Plot 2) increases with SNR initially at a slower rate compared to that in the weakest bin case, and tends to a constant value quickly after the SNR exceeds the threshold value 28 dB. This behavior of the interference contribution from the sub-carriers with varying gains suggests that it is the weakest bin which essentially determines the degradation beyond the threshold SNR.

Now, consider the performance of MMSE equalizer in the case of CR-3. We note from the plots of Fig. 2.2 that the performance of MMSE is not significantly different from that of ZF at low and moderate SNRs, and consequently, there is no degradation
as in the cases of CR-1 and CR-2. To understand the reasons for this, consider (3.2). We note from (3.2) that the product of three matrices, $V^k_{eq,mmse} \Lambda$, is a matrix with all zeros except the $k^{th}$ row, and the elements of this row are $V_{eq,mmse}(k,k)T(k,0)\lambda_0$, $V_{eq,mmse}(k,k)T(k,1)\lambda_1$, ..., $V_{eq,mmse}(k,k)T(k,N_f-1)\lambda_{N_f-1}$.

Consider the term with $k$ corresponding to the weakest bin and $l$ corresponding to the strongest bin. This term is of the form \( \frac{(\lambda_{\text{min}})^r}{N_f|\lambda_{\text{min}}|^2+\sigma^2} \lambda_{\text{max}} T(k,l) \). When \( \frac{\sigma^2}{P} \) is small compared to \( N_f|\lambda_{\text{min}}|^2 \), we can approximate the term as \( \frac{\lambda_{\text{max}}}{N_f\lambda_{\text{min}}} T(k,l) \) which shows that its contribution depends on the ratio \( |\lambda_{\text{max}}|/|\lambda_{\text{min}}| \), suggesting that this ratio plays the role of magnification factor. Thus, the interference contribution from the symbols (other than the one being decoded) carried by the sub-channels depends on the spread of sub-channel gains. The low value of this ratio for the channel realization CR-3 (see Table 2.2) explains why the BER does not degrade as the SNR is increased.

### 3.2 Remedy

Since the weakest bin determines the degradation, we regularize the corresponding coefficient of the equalizer, i.e., $V_{eq,mmse}(k,k)$, $k$ corresponding to $\lambda_{\text{min}}$, as \( \frac{(\lambda_{\text{min}})^r}{N_f|\lambda_{\text{min}}|^2+\left(\frac{\sigma^2}{P}\right)_{\text{th}}} \) and use the regularized equalizer for the SNRs exceeding the threshold value. Here, \( \left(\frac{\sigma^2}{N_fP}\right)_{\text{th}} \) denotes the value of \( \left(\frac{\sigma^2}{N_fP}\right) \) at the threshold SNR. This implicitly assumes that we have the knowledge of the threshold SNR. Before addressing this issue, we first examine if the suggested regularization prevents the degradation.

Figure 3.3 gives the BER plots for CR-1 with the equalizer coefficients as given in (2.11) (plot 1) and with the regularization as suggested above (plot 1').

In this figure, we chose the value of RFO as 0.05 and $N_f = 64$, and applied the regularization with \( \left(\frac{\sigma^2}{N_fP}\right)_{\text{th}} \) corresponding to the threshold SNR 28 dB.
CHAPTER 3. CAUSE AND REMEDY FOR THE DEGRADATION

Note that, as predicted, the regularization prevents the degradation.

Use of above threshold SNR implicitly assumes that we have the knowledge of RFO value.

In practice, this will not be the case. However, from the system specifications and the synchronization algorithm, one will have an estimate of the maximum possible RFO which is of the order $10^{-2}$. It will then be of interest to know how the regularization, computed based on the assumed knowledge of maximum RFO value, will perform if the actual RFO is different from the assumed.

In Fig. 3.3, Plots 2 and 2' correspond to the equalizer as given in (2.11) and the regularized equalizer respectively, for RFO=0.03. We note that the suggested regularization prevents the degradation even though the actual RFO is different from the assumed based on which the regularized coefficient were computed. From these results, we are tempted to state that the knowledge of the actual value of RFO is not critical to the suggested method.

3.3 Estimating the Threshold SNR

In practical applications, we first perform synchronization and channel estimation using a pre-amble. From the estimated channel impulse response coefficients, we compute $\lambda_k$’s. From the knowledge of $\lambda_k$’s and assuming a maximum value for RFO, and for a given transmitted symbol constellation, we can evaluate BER as a function of $(\frac{N_f P}{\sigma^2})$ using (2.19). As the precise value of the threshold SNR is not crucial to the suggested regularization method, a good estimate of this is adequate. Evaluate the BER over a range of SNR values with a spacing of 2 dB, determine the SNR at which the BER starts increasing and take the immediate previous SNR value as the estimate of the threshold SNR $(\frac{N_f P}{\sigma^2})_{th}$). The range over which the BER is to be evaluated may be taken large enough, but not very large. Here, the value of
|λ_{\text{max}}|/|λ_{\text{min}}| can be used as a guideline. If this value is less than N_f, then there is no need of regularization, and hence, no search is required for the threshold SNR.

### 3.4 An Approximate Value of Threshold SNR

Recall that in arriving at the regularization coefficient, we assumed \((\sigma^2 / P)\) to be small compared to \(N_f|λ_{\text{min}}|^2\) and argued that major contribution to the term \(a^3_m\) comes from the weakest bin if |λ_{max}|/|λ_{min}| is large. This suggests that an approximate value of the \((N_f \sigma^2)^{th}\) can be obtained from

\[
\left(\frac{N_f P}{\sigma^2}\right)_{\text{th-approx}} \approx 3/|λ_{\text{min}}|^2
\]  

For CR-1 (3.4) gives nearly 23 dB. Note that only the knowledge of CSI is required in this case. To see how the regularization based on the approximate threshold SNR performs, we computed this value from (3.4) and regularized the equalizer coefficient corresponding to the weakest bin as \(\left(\frac{N_f P}{\sigma^2}\right)^{th-approx}\) and evaluated the corresponding BER curves using (2.19). Plots 1” and 2” of Fig. 3.3 show these results.

The regularization based on the approximate value of the threshold SNR prevents degradation independent of RFO value and spread in the bin gains. However, at higher SNRs, there is a small loss in the performance compared to that based on better estimate of the threshold SNR computed as described in the previous section.
Figure 3.1: $E(a_m^3 a_m^{3*})$ as a function of SNR for the channel realization CR-1 given in Table 2.2 ($N_f=64$ and RFO=0.05, and symbols are from 4-QAM constellation with $P = 1$)
Figure 3.2: $E(a_{m,k}^3 a_{m,k}^{3\ast})$ as a function of SNR for the channel realization CR-1 given in Table 2.2 ($N_f=64$, RFO=0.05, plot marked 1-weakest bin, plot marked 2-next weakest bin, plot marked 3-strongest bin. Symbols are from 4-QAM constellation with $P = 1$)
Figure 3.3: BER Performance of MC-CDMA with MMSE equalizer, evaluated using (2.19) (Plots 1 and 2 are without regularization, plots 1’ and 2’ are with regularization based on the threshold SNR estimated from plot 1 as described in Sec. 3.3, plots 1” and 2” are with regularization based on the threshold SNR computed from (3.4). Plots (1, 1’, 1”) and (2, 2’, 2”) correspond to RFOs=0.05 and 0.03, respectively. \( N_f=64 \) and channel realization is CR-1 given in Table 2.2, and symbols are from 4-QAM constellation with \( P = 1 \)
Chapter 4

Simulation Results

To illustrate how the proposed regularization performs in multipath Rayleigh fading channels, we conducted simulations using the following simulation set up.

We considered a burst communication in slow fading scenario. Here, we assumed perfect timing and channel estimate, and assumed a maximum value of RFO as 0.05. We considered $10^6$ realizations of the channel model given in [18], normalizing each tap variance such that the total variance is one. This results in the average received signal power in each bin same as the transmitted power which is $N_f P$. Thus, $N_f P/\sigma^2$ represents the received SNR in each bin. The data burst consisted of 100 OFDM symbols where each OFDM symbol was made up of $N_f$ 4-QAM symbols and mapping of data bits to symbols was based on Gray encoding. A complex Gaussian noise, with appropriate variance to give the required SNR, was added to the received signal. The noise corrupted received signal was pre-processed with i) MMSE equalizer (2.11)), ii) regularized equalizer based on threshold SNR estimated as described in Sec. 3.3 and iii) regularized equalizer based on the approximate threshold SNR computed from (3.4). In each case, for different values of $N_f P/\sigma^2$, the pre-processed received signal was decoded and the number of decoded symbols in error was noted. This was repeated for $10^6$ channel realizations, choosing a different sequence of transmitted 4-QAM symbols and a different noise sequence in each case, and the number of decoded symbols in error was noted. From the results so obtained, the average symbol error probability was computed for each value of $N_f P/\sigma^2$, and one half of this was taken
as the BER.

We first considered $N_f=64$. For each realization, we computed $\lambda_k$’s and regularized the equalizer coefficient corresponding to the weakest bin for every realization with $|\lambda_{\text{max}}|/|\lambda_{\text{min}}| \geq 64$, using two different values of threshold SNR as described above, and processed the noise corrupted received signal. The results of BER are given in Fig. 4.1. We note from the plots that the suggested regularization performs as predicted. Further, the regularization with approximate threshold SNR performs nearly as good as that with better estimate of the threshold SNR over a wide range of RFO values (0.01 to 0.05). This is very significant since the approximate threshold SNR is computed from the knowledge of CSI only, which is available at the receiver.

To verify if regularization with approximate threshold SNR performs well for other values of $N_f$, we considered $N_f=16$ and 256, and used the same simulation set up as given above.

$N_f=16$

In this case, we regularized the equalizer coefficient corresponding to weakest bin, using the threshold SNR computed from (3.4), for every realization with $|\lambda_{\text{max}}|/|\lambda_{\text{min}}| \geq 16$. The results are shown in Fig. 4.2. The plots show that the regularization based on approximate threshold SNR performs well.

$N_f=256$

In this case, it has been observed from the simulations that with regularized equalizer coefficients corresponding to 5 bins whose gains are least of the 256 $\lambda_k$’s, choosing the the threshold SNR from (3.4) replacing $\lambda_{\text{min}}$ with $\lambda_k$ of the corresponding bin, the degradation can be prevented. We applied this for every realization with $|\lambda_{\text{max}}|/|\lambda_{\text{min}}| \geq 128$ and the results are given in Fig. 4.3. Note that with regularization using the approximate threshold SNR, the BER reaches a floor instead of rising.
Figure 4.1: BER performance of MC-CDMA for $N_f=64$ with MMSE equalizer, averaged over $10^6$ realizations of the channel model given in [18] with tap variances normalized such that the total variance is one (Plots 1, 2, 3 are without regularization, 1’, 2’, 3’ are with regularization based on the threshold SNR estimated as given in Sec. 3.3 with RFO=0.05, plots 1”, 2”, 3” are with regularization based on the threshold SNR computed from (3.4). Plots (1,1’,1”), (2,2’,2”) and (3,3’,3”) correspond to RFOs=0.05, 0.03 and 0.01, respectively. Symbols are from 4-QAM constellation with $P=1$)
Figure 4.2: BER performance of MC-CDMA for $N_f=16$ with MMSE equalizer, averaged over $10^6$ realizations of the channel model given in [18] with tap variances normalized such that the total variance is one (Plots 1, 2 are without regularization, plots $1'$, $2'$ are with regularization based on the threshold SNR computed from (3.4). Plots $(1,1')$, $(2,2')$ correspond to RFOs=0.05 and 0.03, respectively. Symbols are from 4-QAM constellation with $P=1$)
Figure 4.3: BER performance of MC-CDMA for $N_f=256$ with MMSE equalizer, averaged over $10^6$ realizations of the channel model given in [18] with tap variances normalized such that the total variance is one (Plots 1, 2 are without regularization, plots 1”, 2” are with regularization corresponding to the 5 bins whose gains are least of the 256 bin gains, computing the threshold SNR from (3.4) by replacing $\lambda_{\min}$ with the corresponding bin gain. Plots (1,1”), (2,2”) correspond to RFOs=0.05 and 0.03, respectively. Symbols are from 4-QAM constellation with $P=1$)
Chapter 5

Conclusions

In this thesis, we have studied the BER performance of MC-CDMA with MMSE equalizer in the presence of RFO in multipath Rayleigh fading channels, and brought out the threshold effect, i.e., beyond certain SNR the BER deteriorates, and the value of this SNR depends on the value of RFO and multipath channel profile. An attempt has been made to pinpoint the cause for such behavior and a remedy has been suggested to prevent the deterioration in the BER values. To implement the remedy, knowledge of the threshold SNR is needed which in turn requires the knowledge of RFO and CSI. It is shown that with an approximate value of the threshold SNR, which can be computed from the knowledge of CSI only, deterioration in the BER performance can be prevented. Numerical and simulation results are provided to support the analysis.

5.1 Future Work

It would be interesting to investigate the performance of MC-CDMA systems with Gold codes which are preferred for uplink MC-CDMA systems over Walsh Hadamard codes and also study the MIMO MC-CDMA performance in the presence of RFO.
Appendix A

Derivation of MMSE Equalizer for MC-CDMA

In practice, we will not have the knowledge of the exact value of RFO, and hence, we design the equalizer only to combat MCI. With RFO as zero (i.e., $\epsilon = 0$) (2.8) can be written as

$$\tilde{r} = \Lambda x + \tilde{\eta} \quad (A.1)$$

For convenience, we repeat (1.4) here

$$x = \sum_{k=0}^{N_f-1} w_k a_k = Wa \quad (A.2)$$

where $a = [a_0 \ a_1 \ldots \ a_{N_f-1}]^T$. Combining (A.1) with (A.2), we have

$$\tilde{r} = \Lambda Wa + \tilde{\eta} \quad (A.3)$$

Let $V_{eq,mmse}$ denote the MMSE equalizer matrix. With $\epsilon = 0$, (2.10) can be written as

$$\hat{a}_m = w_m^T V_{eq,mmse} \Lambda Wa + w_m^T V_{eq,mmse} \tilde{\eta} \quad (A.4)$$
Denote the error in $a_m$ and $\hat{a}_m$ as

$$e_m = \hat{a}_m - a_m$$  (A.5)

Using the principle of orthogonality [24], we have

$$E(e_m^* \tilde{r}) = 0_{N_f \times 1}$$ (A.6)

The above equation can be re-written as

$$E(e_m \tilde{r}^H) = 0_{1 \times N_f}$$ (A.7)

Using (A.3) and (A.5), (A.7) can be expressed as

$$E \left( (w_m^T V_{eq,mmse} \Lambda W a + w_m^T V_{eq,mmse} \tilde{\eta} - a_m) (\Lambda W a + \tilde{\eta})^H \right) = 0_{1 \times N_f}$$ (A.8)

The symbols $a_k$’s are independent and identically distributed (i.i.d) and the $\tilde{\eta}$ is a circularly symmetric complex Gaussian noise vector of size $N_f \times 1$ with i.i.d elements, each having zero mean and variance $\sigma^2$. Simplifying (A.8) we get

$$w_m^T V_{eq,mmse} \left( N_f \Lambda \Lambda^H + \frac{\sigma^2}{P} I \right) - w_m^T \Lambda^H = 0_{1 \times N_f}$$ (A.9)

which can be re-written as

$$w_m^T V_{eq,mmse} = w_m^T \Lambda^H \left( N_f \Lambda \Lambda^H + \frac{\sigma^2}{P} I \right)^{-1}$$ (A.10)

From (A.10), the MMSE equalizer is obtained as

$$V_{eq,mmse} = \Lambda^H \left( N_f \Lambda \Lambda^H + \frac{\sigma^2}{P} I \right)^{-1}$$ (A.11)

Since $\Lambda$ is a diagonal matrix, $V_{eq,mmse}$ is a diagonal matrix with the diagonal
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elements given by

\[ V_{eq,mmse}(m, m) = \frac{(\lambda_m)^*}{N_f |\lambda_m|^2 + \frac{a^2}{(P)}} , \quad 0 \leq m \leq N_f - 1 \]  

(A.12)
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