New measurements of galaxy clustering and background radiations provide improved constraints on the isotropy and homogeneity of the Universe on large scales. In particular, the angular distribution of radio sources and the X-Ray Background probe density fluctuations on scales intermediate between those explored by galaxy surveys and the Cosmic Microwave Background experiments. On a scale of $\sim 100 \, h^{-1} \text{Mpc}$ the rms density fluctuations are at the level of $\sim 10\%$ and on scales larger than $300 \, h^{-1} \text{Mpc}$ the distribution of both mass and luminous sources safely satisfies the ‘Cosmological Principle’ of isotropy and homogeneity. The transition with scale from clumpiness to homogeneity can be phrased in terms of the fractal dimension of the galaxy and mass distributions.

Most cosmologists believe that on the very large scales the Universe closely obeys the equations of General Relativity for an isotropic and homogeneous system—this is the so-called ‘Cosmological Principle’[1, 2]. However, on scales much smaller than the horizon the distribution of luminous matter (i.e.
galaxies) is clumpy (see Figure 1). This is commonly attributed to gravity, which amplifies the tiny initial density contrasts in the mass distribution as the universe expands. The Cosmological Principle was first adopted when observational cosmology was in its infancy; it was then little more than a conjecture. Observations could not then probe to significant redshifts, the ‘dark matter’ problem was not well-established and the Cosmic Microwave Background (CMB) and X-Ray Background (XRB) were still unknown. If the Cosmological Principle turned out to be invalid then the consequences to our understanding of cosmology would be dramatic, for example the conventional way of interpreting the age of the universe, its geometry and matter content would have to be revised. Therefore it is important to revisit this underlying assumption in the light of new galaxy surveys and measurements of the background radiations. The question of whether the universe is isotropic and homogeneous on large scales can also be phrased in terms of the fractal structure of the universe. A fractal is a geometric shape that is not homogeneous, yet preserves the property that that each part is a reduced-scale version of the whole. If the matter in the universe were actually distributed like a pure fractal on all scales then the Cosmological Principle would be invalid, and the standard model in trouble.

Past attempts to address the above questions have confronted two gaps: (i) Most of the gravitating material is dark, and it is still unclear how to relate the distributions of light and mass, in particular how to match the clustering of galaxies with the CMB anisotropies, which tell us about the mass fluctuations. (ii) Little is known about fluctuations on intermediate scales between those of local galaxy surveys (∼ 100 h⁻¹ Mpc, where h is the Hubble constant in units of 100 km/sec/Mpc) and the scales probed by the Cosmic Background Explorer (COBE) satellite (∼ 1000 h⁻¹ Mpc).

In this review we summarize what can be learnt from various recent observations and techniques for studying clustering on large scales. We consider in particular probes such as radio sources and the XRB, at median redshift $\bar{z} \sim 1$, which seem to cover effectively these intermediate scales. CMB experiments on smaller angular scales than those probed by COBE are also helping to bridge the gap, and future big redshift surveys of more than one million galaxies will probe to median redshift $\bar{z} \sim 0.1$ (which roughly corresponds to a comoving distance of ∼ 300 h⁻¹ Mpc). Current data already strongly constrain any non-uniformities in the galaxy distribution (as well as the overall mass distribution) on scales $\gtrsim 300 h^{-1}$ Mpc.
Any quantitative discussion of the large scale structure in the universe actually depends on the unknown cosmological parameters (defined only for a homogeneous and isotropic universe): the density parameter $\Omega$, the cosmological constant $\Lambda$ and the Hubble constant $H_0$. For simplicity we present the observational results interpreted for the Einstein-de Sitter model ($\Omega = 1$ and $\Lambda = 0$), but the main conclusions are not altered for other models.

**Local galaxies are strongly clustered**

The clumpiness of matter in the Universe was initially studied by measuring the clustering of bright galaxies[1, 2]. Figure 1 shows the distribution of 2 million optically selected galaxies[3] projected on the sky. The distribution is evidently not uniform: galaxies are arranged in clusters and ‘filaments’, and avoid certain regions termed ‘voids’. Figure 2 shows data from the largest redshift survey to date, Las Campanas[4], which illustrates (insofar as the redshift of each galaxy indicates its distance) the three-dimensional clustering of galaxies. Although clustering is seen on scales of tens of Mpc’s, on larger scales the distribution seems more homogeneous.

More quantitatively, it is well established that the probability of finding a galaxy $\sim 5 \, h^{-1}\text{Mpc}$ away from another galaxy is twice the probability expected in a uniform distribution (see Box 1). The clustering of optical galaxies[1, 2] is illustrated in Figure 3. The plot shows the fluctuations $\langle (\delta \rho / \rho)^2 \rangle$ as a function of characteristic length scale $\lambda$. The solid and dashed lines correspond to two variants of the Cold Dark Matter (CDM) model for mass density fluctuations[5], which is widely used as a ‘template’ for comparison with data. We see that the fluctuations drop monotonically with scale (although not as a pure power-law). On a scale of $\lambda \sim 100 \, h^{-1}\text{Mpc}$ the rms fluctuation is only 10%. This is the key evidence that on larger and larger scales the fluctuations become negligible.

As mentioned above it is most unlikely that luminous galaxies trace perfectly the mass distribution. Galaxies can only form in dense regions, and their formation may be affected by other physical conditions and local environment. Hence the clustering of galaxies is likely to be ‘biased’ relative to the mass fluctuations (see Box 1). Indeed the galaxy distribution could in principle display conspicuous features on very large scales even if the mass did not — for instance, a long cosmic string could ‘seed’ galaxy formation in
its wake. So the galaxies could be arrayed in a fractal structure, even if the mass distribution is non-fractal. It is important therefore to understand the biasing in order to match the fluctuations in galaxies to the fluctuations in mass in Figure 3.

Other (biased) probes at large distances are clusters of galaxies, as selected optically by Abell[21] or by X-ray surveys[22]. These surveys typically probe out to redshift $z \sim 0.1$. Several studies[23, 24] suggest that on scales of $\sim 600 \, h^{-1} \text{Mpc}$ the distribution of Abell clusters is homogeneous.

A more controversial result on the distribution of galaxies suggests a ‘characteristic scale’ of clustering of $\sim 128 \, h^{-1} \text{Mpc}$[25, 26]. It is not clear yet if this feature is real, or just due to small number statistic or survey geometry[27]. Recently Einasto et al.[28] suggested that the distribution of Abell clusters is a quasiregular three-dimensional network of superclusters and voids, with regions of high density separated by about $120 \, h^{-1} \text{Mpc}$. The reality of such a ‘periodicity’ in galaxy clustering will soon be revisited by two new large redshift surveys. The American-Japanese Sloan Digital Sky Survey (SDSS) will yield redshifts for about 1 million galaxies and the Anglo-Australian ‘2 degree Field’ (2dF) survey, will produce redshifts for 250,000 galaxies (both with median redshift of $\bar{z} \sim 0.1$). These big galaxy surveys will give good statistics on scales larger than $\sim 100 \, h^{-1} \text{Mpc}$.

**Tracers at high redshift**

We need to observe beyond $z = 0.1$ in order to sample a big enough volume to probe clustering on scales above $300 \, h^{-1} \text{Mpc}$ and to fill the gap between scales probed by galaxy surveys and the scales probed by COBE. However this then introduces the extra complication that we cannot interpret the data without taking account of how the clustering evolves with time, and also possible cosmic evolutionary effects in the brightness of objects. Here we discuss the X-Ray Background (XRB) and radio sources as probes with median redshift $\bar{z} \sim 1$. Other possible high-redshift tracers are quasars and clusters of galaxies.

The XRB and radio sources are tracers of galaxies (or at least of that subset which is active). We see from Figure 3 that the limits they set to large scale inhomogeneities ($> 100 \, h^{-1} \text{Mpc}$) are less stringent than those implied by CMB measurements, but they provide independent constraints,
since they sample ‘luminous’ objects rather than the total mass.

Radio galaxies

Radio sources in surveys have typical median redshift $\bar{z} \sim 1$, and hence are useful probes of clustering at high redshift. Earlier studies[29] claimed that the distribution of radio sources supports the ‘Cosmological Principle’. However, the redshift distribution of radio sources is now better understood and it is clear that the wide range in intrinsic luminosities of radio sources would dilute any clustering when projected on the sky. Recent analyses of new deep radio surveys[30] suggest that radio sources are clustered at least as strongly as local optical galaxies[31, 32, 33, 34, 35, 36, 37]. Nevertheless, on very large scales the distribution of radio sources seems nearly isotropic. Comparison of the measured quadrupole in a radio sample to the theoretically predicted ones[17] offers a crude estimate of the fluctuations on scales $\lambda \sim 600 h^{-1}$ Mpc. The derived amplitudes are shown in Figure 3 for the two assumed CDM models. Given the problems of catalogue matching and shot-noise, these points should be interpreted as significant ‘upper limits’, not as detections.

The X-Ray Background

Although discovered in 1962, the origin of the X-ray Background (XRB) is still controversial, but its sources, whatever they turn out to be, are likely to be at high redshift[38, 39].

The XRB is a unique probe of fluctuations on intermediate scales between those of local galaxy surveys and COBE[40, 41, 12, 13, 43, 44, 45], although the interpretation of the results depends somewhat on the nature of the X-ray sources and their evolution. The rms dipole and higher moments of spherical harmonics can be predicted[18] in the framework of gravitational instability and assumptions on the distribution of the X-ray sources with redshift. By comparing the predicted multipoles to those observed by HEAO1[19] it is possible to estimate the amplitude of fluctuations for an assumed shape of the density fluctuations (e.g. CDM model). Figure 3 shows the amplitude of fluctuations derived at the effective scale $\lambda \sim 600 h^{-1}$ Mpc probed by the XRB. Assuming a specific epoch-dependent biasing scheme[46] (see Box 1) and a range of models of evolution of X-ray sources and clustering, the
present-epoch density fluctuations of of the X-ray sources is found to be no more than twice the amplitude of fluctuations in mass[19]. Below we shall use this estimate to derive a fractal dimension of the universe on large scales, which turns out to be indistinguishable from the one for a homogeneous universe.

Quasars, high redshift galaxies, and Lyman-α clouds

Until the mid-1990s, quasars – hyperactive galactic nuclei – were the only objects luminous enough to be identified in substantial numbers at redshifts $z > 2$. It is still unclear how their clustering evolves with redshift[47], but there is no evidence that they are any more clustered than extragalactic radio sources (which are a related population). The advent of 10-metre-class telescopes now allows the detection of galaxies out to equally large redshifts. Already, several hundred galaxies with $z = 3$ have been detected, and they display about the same level of clustering as nearby galaxies[48]. Since gravitational effects enhance density contrasts during cosmic expansion, one might at first sight have expected weaker clustering of galaxies at earlier times. However, the luminous galaxies that have already formed at the epoch corresponding to $z = 3$ belong to an exceptional subset associated with unusually high density peaks, which display enhanced clustering (see Box 1). When this is taken into account, the data are compatible with CDM models normalised to match the degree of clustering at low redshifts[48]. Larger samples of high-$z$ galaxies will soon provide direct evidence on the clustering of early galaxies on scales up to and beyond $100 \, h^{-1} \text{Mpc}$; comparison with the lower-redshift samples will then elucidate exactly how the clustering evolves. Such data will perhaps reveal larger-scale clustering at high redshifts, but the amplitude of this is already constrained by the radio sources and XRB data. The rich absorption-line spectra of high redshift quasars offer a probe for the distribution of intervening material. In particular, the ‘Lyman-α forest’ in quasar spectra reveals the distribution of gas clouds (probably intimately related to low-mass protogalaxies) along the line of sight. The absorption features in each spectrum are smoothly distributed in redshift. There are no large ‘clearings’ in the Lyman-α forest: indeed the relevant clouds seem even more smoothly distributed than galaxies[19]. Moreover, the spectra of different quasars indicate that the clouds have the same properties along all lines of sight. Relating the spacings of these clouds to the overall mass distri-
bution (or even to the galaxy distribution) is not straightforward. However, if these distributions manifested scale-free fractal-like structure it would be remarkable if this structure were not plainly apparent in quasar absorption spectra [8].

**Direct probes of mass distribution**

As already discussed, the luminous objects selected by surveys may not trace the total mass. There are however at least three independent probes of inhomogeneities in the gravitational field induced by the total mass fluctuations: lensing, the CMB, and peculiar velocities. Gravitational lensing – the distortion of distant galaxy images by intervening potential wells – can only constrain the mass fluctuations on scales smaller than $20 h^{-1} \text{Mpc}$ or so [50, 51]. Here we focus on the other two probes of mass fluctuations, which show good consistency with the picture that fluctuations are significant on small scales (as deduced from peculiar velocities) but are tiny on the very large scales (as inferred from the CMB).

**The CMB**

The CMB is well described by a black-body radiation spectrum at a temperature of $2.73^\circ K$, hence providing crucial evidence for the hot Big Bang model. This sea of radiation is highly isotropic, the major anisotropy being due to the motion of our Galaxy (the Milky Way) at 600 km/sec relative to the CMB. This motion is remarkably reconstructed in both amplitude and direction by summing up the forces due to masses represented by galaxies [52, 53] at distances nearer than $\sim 100 h^{-1} \text{Mpc}$. The dipole anisotropy in the distribution of nearby supernovae also indicates that most of the Galaxy’s motion arises from local inhomogeneities [54]. The agreement between the CMB dipole and the dipole anisotropy of relatively nearby galaxies is a good argument in favour of large scale homogeneity. In an arbitrarily lumpy universe this would be a coincidence. A given overdensity $\delta \rho$ on a scale $\lambda$ produces a peculiar velocity proportional to $\lambda$ (in the linear regime), so the absence of very large peculiar velocities is in itself evidence that $\delta \rho$ decreases as steeply as $\propto 1/\lambda$.

Apart from the dipole anisotropy, the other main anisotropies in the CMB
radiation are imprints of the potential wells at the last scattering surface and their influence on the plasma motions at this epoch. As the photons climbed out of different gravitational potential wells they experienced gravitational redshift. On angular scales larger than 1°, separate regions on the sky were not causally connected. In 1992 the COBE satellite detected fluctuations in the CMB, at the level of $10^{-5}$ on scales of $10^{6}$; which corresponds to a present-epoch length-scale of $\sim 1000 \, h^{-1} \text{Mpc}$ (see Figure 3). These tiny CMB fluctuations are attributed to ‘metric’ or ‘curvature’ fluctuations of this order in a universe which has a Friedmann-Robertson-Walker (FRW) metric of space-time (corresponding to a homogeneous and isotropic universe). Moreover, the concept of ‘inflation’ has suggested a physical reason why the universe may end up close to FRW metric but with fluctuations in the metric that are equal on all scales. Further support for this hypothesis comes from studies of how the present-day clustering of galaxies might have evolved via gravitational instability: the relevant fluctuations here are on much smaller scales than those probed by COBE but the required amplitude is again of order of $10^{-5}$. The metric fluctuations on larger scales would not have evolved into bound structures: the associated density contrast would still be small, but for constant metric fluctuations it would have an amplitude inversely proportional to the square of the length-scale.

On scales smaller than 1 deg causal physical processes took place. The plasma oscillated acoustically in response to the fluctuations in the potential wells of the dark matter. This interaction between plasma and gravity translates to peaks in the angular power-spectrum of the temperature fluctuations. The angular scales of these peaks correspond to few hundreds of $h^{-1} \text{Mpc}$ today, hence can be compared with other probes of the fluctuations such as galaxies, radio sources and the XRB (see Figure 3). The existing and new experiments (e.g. Tenerife, CAT, Saskatoon, Planck, MAP, VSA) will map the fluctuations on these scales. This is important because the position and height of the peaks can be used to determine the cosmological parameters with very high precision. Also, these smaller-scale fluctuations are the precursors of those which have developed into clusters and superclusters by the present time. Indeed, the imprint of fluctuations represented by the CMB angular ‘peaks’, even though at the level of few percent, might show up in the deep galaxy maps soon to be produced by the Sloan and 2dF surveys.

One may question if large amplitude inhomogeneities along the line of
sight can wash out large intrinsic fluctuations in the CMB and make it look very smooth. The general effect would however be merely to distort the the angular distribution of the fluctuations rather than to homogenize the temperature map\[61\].

**Peculiar velocities**

Peculiar velocities (like the 600 km/sec motion of the Local Group described above) are deviations from the recession velocity that would be expected due to the smooth expansion of the Universe. Their measurement in other galaxies requires redshift-independent distances, derived by observing the apparent brightness or size of a galaxy and relating this to some parameter, e.g. internal stellar velocities, that is known to be a measure of its intrinsic luminosity. The gross features of the local peculiar velocity field inferred in this way correlate well with overdensities in galaxy distribution, e.g. the Virgo cluster and the Great Attractor\[62, 63, 64\], although in some regions the agreement is not perfect, perhaps due to systematic measurement errors.

Unfortunately, the distance measurement errors increase with distance, so the observed peculiar velocity field\[65\] can only probe scales $\lambda < 20 h^{-1}$ Mpc. Lauer and Postman\[66\] claimed that a sample of Abell clusters out to 150 $h^{-1}$ Mpc is moving at $\sim 700$ km/sec with respect to the CMB, suggesting that the CMB dipole (caused by such relative motion) is generated largely by mass concentrations beyond $\sim 100 h^{-1}$ Mpc, but most other studies suggest bulk flows on smaller scales.

**Fractal vs. homogeneity on large scales**

A fractal is a distribution or shape that is not homogeneous, but possesses the property that each part of it is a version of the whole reduced in scale. In other words it 'looks' the same on all scales: from ‘afar’, or close up for an enlarged view of some portion. Fractals abound in nature and are fundamental in physics\[3, 4\], for they accompany the basic idea of the scaling of physical laws. For example, the coastline of a small peninsula drawn on paper could look equally valid for a large continent; and fractals have long been studied in solid state physics. The clustering of galaxies (see Fig. 1) lends itself to a fractal description since the clumpiness prevails over a wide
range of scales. In the language of fractals (see Box 2), fractal dimensions are used to characterise the degree of clustering. Fractal dimensions generalise our intuitive concepts of dimension and can take any positive value less than 3. On scales below $\sim 10 \, h^{-1} \, \text{Mpc}$ galaxies are distributed with the correlation dimension $D_2 = 1.2 - 2.2$ (see also Box 2 and Table 1). This is well below the homogeneous value of 3.

The other side of the coin is that the mass distribution has to approach homogeneity on large scales in order for the FRW metric, the standard model of space-time, to hold. To reconcile the two models one supposes that the fractal model crosses over gradually to homogeneity, i.e. $D_2 = 3$ (see Figure 4). Although luminous matter does not necessarily trace mass, the detection of this large-scale homogeneity is naturally a very important quest. If the galaxy distribution were a fractal on very large scales, it may imply seeding of galaxy formation by topological defects (e.g. strings) uncorrelated with the large scale mass distribution, or it may even have important implications for the application of the FRW metric, unlikely though this may be given the many successes of the ‘standard cosmology’.

In recent years Pietronero and coworkers have strongly advocated that the scale of homogeneity has not been detected even in the deepest redshift surveys. Most analyses of density fluctuations had assumed large-scale homogeneity but these authors applied methods that made no such assumptions and argued that the fractal behaviour extended to the largest scales probed ($\sim 1000 \, h^{-1} \, \text{Mpc}$), with $D_2 \approx 2$. However, Cold Dark Matter models of density fluctuations (which fit reasonably well the available observational data) predict that at scales above $\sim 10 \, h^{-1} \, \text{Mpc}$ one should begin to detect values of $D_2$ greater than 2, with $D_2 \approx 3$ on scales larger than $\sim 100 \, h^{-1} \, \text{Mpc}$ (see Box 2 and Figure 4), in conflict with Pietronero’s claim.

Several authors have therefore made further analyses of galaxy distributions using fractal algorithms. All of them obtained results which were consistent with standard models of density fluctuations and all appeared to detect an approach to homogeneity on the largest scales analysed by them. We list some results for $D_2$ in Table 1. Although the statistics are still poor, one can already see the steady increase towards $D_2 = 3$. In particular, the results do not support a constant $D_2$ for all scales, and the latest results by Scaramella et al. provide the closest fractal measurement yet to homogeneity. Scaramella et al. pointed out the importance of appropriate corrections to the observed flux from high-redshift galaxies to account for
redshifting of their spectrum across the observer’s waveband. These spectral band corrections are crucial to the interpretation of high-redshift photometry and should not be left out, approximate though they may be. On smaller scales, no other group has substantiated the value $D_2 \approx 2$ on all scales larger than $10 \, h^{-1}\text{Mpc}$. Lemson & Sanders\cite{69}, who did obtain this value on scales below $30 \, h^{-1}\text{Mpc}$ (Table 1), also detected a crossover to homogeneity for larger scales.

The debate has therefore brought up technical issues which have been useful. For example, it was correctly pointed out\cite{5} that if a survey is too small, then one cannot define the mean density (for the galaxies do form a fractal on small scales) and hence related tools such as correlation functions can be misleading. On the other hand, the fractal proponents have not helped their cause by using highly incomplete and inhomogeneous samples. Detailed technical arguments for\cite{7} and against\cite{8, 73, 74} fractals on large scales have been given. The continued application of fractal algorithms (as opposed to traditional methods which assume homogeneity) to larger and deeper surveys should definitively resolve the matter.

Alternative arguments against the large-scale fractal model have been given by Peebles\cite{2}. He pointed out that the variation of both the number counts and the angular correlation function of galaxies with apparent luminosity point strongly against a pure fractal universe. Furthermore, since properties of a pure fractal are independent of scale, the projected galaxy distribution in shells of increasing size should look the same. This is strongly in conflict with the observations\cite{8}, which show decreasing clumpiness in larger shells. However, we note that visual impression alone cannot indicate the closeness to isotropy.

Direct estimates of $D_2$ are not possible for much larger scales, but we can calculate values of $D_2$ at the scales probed by the XRB and CMB by using CDM models normalised with the XRB and CMB as described above. The resulting values are extremely close to 3 and are given in the lower part of Table 1. (They are even tighter than the constraint $3 - D_2 \leq 0.001$ obtained by Peebles\cite{2} from the XRB using a different argument.) We consider the agreement of XRB and CMB fluctuations with the popular CDM models within the framework of a homogeneous universe to argue strongly against a pure fractal galaxy or mass distribution. (We remind the reader that the XRB traces galaxies, while the CMB traces mass.) Can we go further? Isotropy does not imply homogeneity, but the near-isotropy of the CMB can
be combined with the Copernican principle that we are not in a preferred position. All observers would then measure the same near-isotropy, and an important result has been proven that the universe must then be very well approximated by the FRW metric\(^7\), \(^6\).

While we reject the pure fractal model in this review, the performance of CDM-like models of fluctuations on large scales have yet to be tested without assuming homogeneity \textit{a priori}. On scales below, say, \(30 h^{-1} \text{Mpc}\), the fractal nature of clustering implies that one has to exercise caution when using statistical methods which assume homogeneity. As a final note, we emphasize that we only considered one ‘alternative’ here, which is the pure fractal model where \(D_2\) is a constant on all scales.

**Discussion**

To study galaxy evolution and the validity of the FRW metric on large scales it is important to explore density fluctuations at higher redshift. The examples of the X-ray Background and radio sources are encouraging, but redshift information is required to constrain the growth of cosmic structure with time. Our main conclusion is that, while the galaxy distribution can be described as a fractal on scales smaller than \(20 h^{-1} \text{Mpc}\), there is strong evidence from present data on the XRB and the CMB that homogeneity and isotropy approximately prevails on scales larger than \(300 h^{-1} \text{Mpc}\). However, the scale of ‘cross-over’ to homogeneity is not well determined yet. We emphasize that it is difficult to ‘prove’ the Cosmological Principle. However, the recent observational tests described in this review, in combination, offer extremely strong support for this crucial hypothesis.

We have derived constraints on large-scale (\(> 100 \text{Mpc}\)) irregularities in the distribution of galaxies and other luminous objects. These could have turned out to be much less homogeneous than the overall mass distribution. On the largest scales (\(\sim 1000 h^{-1} \text{Mpc}\)) possible inhomogeneities in the matter distribution are strongly constrained by the smallness of the CMB fluctuations – it is this evidence which tells us, most convincingly, how remarkably accurately the Friedmann-Robertson-Walker models seem to fit our universe. Of course, we cannot formally exclude a pre-Copernican model universe, such that the isotropy around us is atypical of what would be measured by hypothetical observers elsewhere\(^7\). But, leaving such aside, there
is a well-defined sense in which our universe is homogeneous on the largest accessible scales; neither its mass distribution, nor that of the galaxies resembles a pure fractal. Cosmological parameters such as $\Omega$ therefore have a well-defined meaning — indeed these considerations tell us over what volume we need to average in order to determine them with any specified level of precision.

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**Box 1: Quantitative measures of galaxy clustering**

One popular measure of galaxy clustering is the two-point correlation function[^1] defined as the excess probability, relative to a random distribution, of finding a galaxy at a distance \( r \) from another galaxy. It is now well established that on scales smaller than \( \sim 10 \, h^{-1} \text{Mpc} \) it has roughly the form

\[
\xi(r) = \left( \frac{r}{r_0} \right)^{-\gamma}.
\]

For optically selected galaxies \( \gamma = 1.8 \), \( r_0 \approx 5 \, h^{-1} \text{Mpc} \), while for galaxies observed in the infrared with the IRAS satellite, which include spiral galaxies but under-represent ellipticals, \( r_0 \approx 4 \, h^{-1} \text{Mpc} \) with a somewhat shallower slope[^78]. The clustering of galaxy clusters (as selected by Abell[^21] or by X-ray surveys) obeys a similar law[^79] but with a much stronger clustering amplitude, \( r_0 \approx 15 - 20 \, h^{-1} \text{Mpc} \). The Fourier transform of the correlation function \( \xi(r) \) is the power-spectrum \( P(k) \) (where \( k \) is the wavenumber), which corresponds to the square of Fourier coefficients of the fluctuations. The rms fluctuations (see Figure 3) can be written as \( \langle (\frac{\delta \rho}{\rho})^2 \rangle \propto k^3 P(k) \).

It is not likely that the fluctuations in the density of particular galaxy types are exactly the same as the fluctuations in mass. The simplest assumption, which has been widely adopted, is that the galaxy and mass density fluctuations at any point \( \vec{x} \) are related by

\[
\delta_g(\vec{x}) = b \delta_m(\vec{x}),
\]
where \( b \) is the ‘bias parameter’. Usually \( b > 1 \) which implies that the galaxies are more clustered than the mass distribution. By modelling galaxies as peaks of the underlying mass distribution and using an argument analogous to that which explains why the highest ocean waves come in groups, Kaiser showed that in the linear approximation the correlation function of galaxies is related to the mass correlation function by

\[
\xi_{gg}(r) = b^2 \xi_{mm}(r),
\]

where \( r \) is the separation between galaxies or mass elements. Note that although eq. (3) does follow from eq. (2), it is more general and does not imply eq. (2). Various theoretical and observational considerations suggest that \( b \approx 1 - 2 \).

Biasing must certainly be more complicated than eqs. (2) and (3): indeed, clustering is not the same for galaxies of different galaxy morphologies. For example, elliptical galaxies are more strongly clustered than spiral galaxies on scales \( \lesssim 10h^{-1}\text{Mpc} \). The appropriate value of \( b \) may depend on scale, as well as on the local overdensity. Furthermore, it is not clear a priori that \( \delta_g \) is just a function of \( \delta_m \). The efficiency of galaxy formation could in principle be modulated by some large-scale environmental effects (e.g. heating by early quasars, or the proximity of a cosmic string) which are uncorrelated with \( \delta_m \). Biasing might therefore be non-local, non-linear, stochastic and epoch-dependent.

**Box 2: The fractal dimension**

If we count, for each galaxy, the number of galaxies within a distance \( R \) from it, and call the average number obtained \( N(< R) \), then the distribution is said to be a fractal of correlation dimension \( D_2 \) if \( N(< R) \propto R^{D_2} \). Of course \( D_2 \) may be 3, in which case the distribution is homogeneous rather than fractal. In the pure fractal model this power law holds for all scales of \( R \), whereas in a hybrid model it holds for \( R \) less than some scale, above which \( D_2 \) increases towards 3 to accommodate the Cosmological Principle. To allow for varying \( D_2 \) one often writes

\[
D_2 \equiv \frac{d \ln N(< R)}{d \ln R}.
\]
Using the above, the fractal proponents have estimated $D_2 \approx 2$ for all scales up to $\sim 1000 \, h^{-1} \text{Mpc}$, whereas other groups have obtained, in general, scale-dependent values as listed in Table 1.

These measurements can be directly compared with the popular Cold Dark Matter models of density fluctuations, which predict the increase of $D_2$ with $R$ for the hybrid fractal model. If we now assume homogeneity on large scales, then the mean density $\bar{n}$ and the correlation function $\xi(r)$ can be defined, and

$$N(< R) = \frac{4\pi}{3} R^3 \bar{n} + 4\pi \bar{n} \int_0^R dr r^2 \xi(r), \quad (5)$$

for a flat universe with $\Omega = 1$. Hence we have a direct mapping between $\xi$ and $D_2$. If we choose a power-law form for $\xi(r)$ (eq. 1), then it follows that $D_2 = 3 - \gamma$ if $\xi \gg 1$. If $\xi(r) = 0$ we obtain $D_2 = 3$. The CDM models give us the correlation function $\xi(r)$ on scales greater than $\sim 10 \, h^{-1} \text{Mpc}$, where we do not need to worry about non-linear gravitational effects. The function $N(< R)$ can then be calculated from these correlations. The predicted runs of $D_2(R)$ from three different CDM models are given in Fig. 4. They may differ somewhat but they all show the same qualitative behaviour: above $30 \, h^{-1} \text{Mpc}$ we should be able to measure dimensions close to 3, not 2. Above $100 \, h^{-1} \text{Mpc}$ they become indistinguishably close to 3. They also illustrate that it is inappropriate to quote a single crossover scale to homogeneity, for the crossover is gradual. Here we have described but one statistical fractal measure, $D_2$, out of a much larger set known as the ‘multifractal spectrum’, which is a useful tool for the statistical description of redshift surveys.

**Figures and Table**

**Figure 1:**

The distribution of 2 million galaxies with magnitude $17 \leq b_j \leq 20.5$ shown in an equal area projection centred on the South Galactic pole. The data from APM scans over a contiguous area of 4300 square degrees. The small empty patches in the map are regions excluded around bright stars, nearby dwarf galaxies, globular clusters and step wedges. Although in projection, the pattern of the distribution of galaxies is seen to be non-uniform, with clusters, filaments and voids.
Figure 2:
The redshift distribution of more than 10,000 galaxies, from the Las Campanas Redshift Survey [10]. The plot shows the superposition of 3 slices in the North Galactic cap, and likewise for the South Galactic cap, plotted in redshift versus angular (RA) coordinates. Clustering of galaxies is seen on scales smaller than $\sim 30 h^{-1} \text{Mpc}$, but on larger scales the distribution approaches homogeneity. Note that the diluted density of galaxies at higher redshifts is an artifact, due to the selection of galaxies by their apparent flux.

Figure 3:
A compilation of density fluctuations on different scales from various observations: a galaxy survey, deep radio surveys, the X-ray Background and Cosmic Microwave Background experiments. The measurements are compared with two popular Cold Dark Matter models. The Figure shows mean-square density fluctuations $\langle (\delta \rho / \rho)^2 \rangle$. The solid and dashed lines correspond to the standard Cold Dark Matter power-spectrum (with shape parameter $\Gamma = 0.5$) and a ‘low-density’ CDM power-spectrum (with $\Gamma = 0.2$), respectively. Both models are normalized such that the rms fluctuation within $8 h^{-1} \text{Mpc}$ spheres is $\sigma_{8,M} = 1$. The open squares at small scales are estimates of the power-spectrum from 3-dimensional inversion of the angular APM galaxy catalogue [11, 12]. The elongated ‘boxes’ at large scales represent the COBE 4-yr [13, 14, 16] (on the right) and Tenerife [15] (on the left) CMB measurements. The solid triangles represent constraints from the quadrupole moment of the distribution of radio sources [17]. This quadrupole measurement probes fluctuations on scale $\lambda_*^{-1} \sim 600 h^{-1} \text{Mpc}$. The top and bottom solid triangles are upper limits of the amplitude of the power-spectrum at $\lambda_*$, assuming CDM power-spectra with shape parameters $\Gamma = 0.2$ and 0.5 respectively, and an Einstein-de Sitter universe. The crosses represent constraints from the XRB HEAO1 quadrupole [18, 19]. Assuming evolution, clustering and epoch-dependent biasing prescriptions, this XRB quadrupole measurement probes fluctuations on scale $\lambda_*^{-1} \sim 600 h^{-1} \text{Mpc}$, very similar to the scale probed by the radio sources. The top and bottom crosses are estimates of the amplitude of the power-spectrum at $\lambda_*$, assuming CDM power-spectra with shape parameters $\Gamma = 0.2$ and 0.5 respectively, and an Einstein-de Sitter universe. The fractional error on the XRB amplitudes (due to the shot-noise...
of the X-ray sources) is about 30%.

**Figure 4:**

The fractal correlation dimension $D_2$ versus length scale $R$ assuming three Cold Dark Matter models of power-spectra with shape and normalization parameters ($\Gamma = 0.5; \sigma_8 = 0.6$), ($\Gamma = 0.5; \sigma_8 = 1.0$) and ($\Gamma = 0.2; \sigma_8 = 1.0$). Regardless of model they all exhibit the same qualitative behaviour of increasing $D_2$ with $R$, becoming vanishingly close to 3 for $R > 100 \, h^{-1} \text{Mpc}$. Pietronero’s pure fractal model\[5, 6, 7\] corresponds to the horizontal axis $D_2 = 2$, in conflict with CDM models and the data presented in Table 1.
| Sample                      | $R \ (h^{-1} \text{ Mpc})$ | $D_2$ |
|-----------------------------|-----------------------------|-------|
| Guzzo et al. [67]           | Perseus-Pisces              | 1.0–3.5 | 1.2 |
|                             |                             | 3.5–20  | 2.2 |
| Martinez & Coles [68]       | QDOT                        | 1.0–10  | 2.25|
|                             |                             | 10–30   | 2.77|
| Lemson & Sanders [69]       | CfA                         | 1.0–30  | 2.0 |
| Martínez et al. [72]        | Stromlo-APM                 | 30–60   | 2.7–2.9 |
| Scaramella et al. [24]      | ESP                         | 300–400 | 2.93|
| X-Ray Background            | $\sim 500$                 | $3 - D_2 = 10^{-4}$ |
|                             |                             | with $\sigma_8 = 2$, $\Gamma = 0.5$ |
| COBE 4 year normalisation   | $\sim 1000$                | $3 - D_2 = 2 \times 10^{-5}$ |
|                             |                             | with $\sigma_8 = 1.4$, $\Gamma = 0.5$ |

**Table 1**

Estimates of the fractal correlation dimension $D_2$ obtained from galaxy surveys, showing a general increase with scale. Scaramella et al. [24] analysed a number of subsamples with different methods, from which we chose one of their largest. All their results that include necessary ‘k-corrections’, which account for the effect of galaxy spectra being redshifted relative to the observer’s pass band, are consistent with $D_2 = 3$ within their errors. Also given are estimates of $D_2$ from the X-Ray Background and the Cosmic Microwave Background, obtained by normalising a standard CDM model to match measured anisotropy results (see box). Unlike the other measurements in this Table, the CMB probes directly the fluctuations in mass.
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