Stabilizing a purely dipolar quantum gas against collapse

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We report on the experimental observation of the dipolar collapse of a quantum gas which sets in when we reduce the contact interaction below some critical value using a Feshbach resonance. Due to the anisotropy of the dipole-dipole interaction, the stability of a dipolar Bose-Einstein condensate depends not only on the strength of the contact interaction, but also on the trapping geometry. We investigate the stability diagram and find good agreement with a universal stability threshold arising from a simple theoretical model. Using a pancake-shaped trap with the dipoles oriented along the short axis of the trap, we are able to tune the scattering length to zero, stabilizing a purely dipolar quantum gas.

Interactions between atoms dominate most of the properties of quantum degenerate gases [1]. In the ultracold regime these interactions are usually well described by an effective isotropic zero-range potential. The strength and sign of this contact interaction is determined by a single parameter, the scattering length $a$. The contact interaction is responsible for a variety of striking properties of quantum gases. Strongly influencing the excitation spectrum of the condensate it gives rise to e.g. the superfluidity of Bose-Einstein condensates (BEC) or the existence of vortex lattices. The contact interaction also plays a crucial role in the physics of strongly correlated systems like in the BEC-BCS crossover [2] or in quantum phase transitions like the Mott insulator transition [3].

Another fundamental topic is the question of the existence of a stable ground state depending on the modulus and sign of the contact interaction. In the homogeneous case repulsive contact interaction ($a > 0$) is necessary for the stability of the BEC. In contrast, if the contact interaction is attractive ($a < 0$), the BEC is unstable. This instability can be prevented by an external trapping potential. The tendency to shrink towards the center of the trap is in that case counteracted by the repulsive quantum pressure arising from the Heisenberg uncertainty relation. Detailed analysis [4] yields that a condensate is stable as long as the number of atoms $N$ in the condensate stays below a critical value $N_{\text{crit}}$ given by

$$N_{\text{crit}} = \frac{k a_{\text{ho}}}{|a|}$$

where $a_{\text{ho}}$ is the harmonic oscillator length and $k$ is a constant on the order of $1/2$. This scaling, as well as the collapse dynamics for $N > N_{\text{crit}}$, have been studied experimentally with condensates of $^7\text{Li}$ [5, 6] and $^{85}\text{Rb}$ [7, 8]. In [4, 10] the atom number dependance of the collapse of mixtures of bosonic $^{87}\text{Rb}$ and fermionic $^{40}\text{K}$ quantum gases has been investigated.

Being anisotropic and long-range, the dipole-dipole interaction (DDI) differs fundamentally from the contact interaction. Besides many other properties, the stability condition therefore changes in a system with a DDI present. Considering the case of a purely dipolar condensate with homogeneous density polarized by an external field, one finds that due to the anisotropy of the DDI, the BEC is unstable, independent of how small the dipole moment is [11]. As in the pure contact case a trap helps to stabilize the system. In the dipolar case, however, it is not only the quantum pressure that prevents the collapse but more importantly the anisotropic density distribution imprinted by the trap. Consider a cylindrically symmetric harmonic trap

$$V_{\text{trap}}(r, z) = \frac{1}{2} m (\omega_r^2 r^2 + \omega_z^2 z^2)$$

with the dipoles oriented along $z$ and $r$ being the distance from the symmetry axis. As can be seen in Fig. 1A, in a pancake-shaped trap (aspect ratio $\lambda = \omega_z/\omega_r > 1$) the dipoles predominantly repel each other and the BEC is stable. In contrast, a cigar-shaped trap ($\lambda < 1$, Fig. 1B) leads to mainly attractive forces and hence to a dipolar collapse. Following this simple argument one expects that in the prolate case a positive scattering length $a$ is needed to stabilize the BEC, whilst in the oblate case one can even afford a slightly negative $a$. The dependance of the stability of a dipolar BEC on the trap aspect ratio $\lambda$ and scattering length $a$ has been extensively studied theoretically [12, 13, 14], and is experimentally investigated in this paper.

Our measurements are performed with a BEC of $^{52}\text{Cr}$ [15] which is to date the only experimentally accessible quantum gas with observable dipole-dipole interaction [16, 17]. To compare contact and dipolar interactions we introduce the length scale of the magnetic DDI

$$a_{\text{dd}} = \frac{\mu_0 \mu^2 m}{12 \pi \hbar^2}.$$  

The numerical prefactors in $a_{\text{dd}}$ are chosen such that a homogeneous condensate becomes unstable to local den-
sity perturbations for $a \lesssim a_{dd}$ [18]. As Chromium has a magnetic dipole moment of $\mu = 6\mu_B$ ($\mu_B$ the Bohr magneton), $a_{dd} \approx 15a_0$, where $a_0$ is the Bohr radius. Far from Feshbach resonances, $a$ takes its background value $a_{bg} \approx 100a_0$ [19] and the BEC is stable for any $\lambda$. To explore the unstable regime we thus reduce the scattering length, which in the vicinity of a Feshbach resonance scales like

$$a = a_{bg} \left(1 - \frac{\Delta B}{B - B_0}\right)$$

with the applied magnetic field $B$. To be able to tune $a$ accurately we use the broadest of the resonances in $^{52}$Cr [19] which is located at $B_0 \approx 589$ G and has a width of $\Delta B \approx 1.5$ G [17].

The details of our experimental setup and procedure have already been described elsewhere [17, 20] and shall only be summarized here. We produce a BEC of approximately 25,000 atoms about 10 G above the resonance where the scattering length is still close to its background value. Once the BEC is obtained, we adiabatically shape the trapping potential to the desired aspect ratio $\lambda$ within 25 ms. In order to be able to vary $\lambda$ over a large range, we generate the trapping potential by a crossed optical dipole trap and a superimposed one-dimensional optical lattice along the $z$-direction (see Fig. 1C). The two lattice beams (wavelength $\lambda_{latt} = 1064$ nm, waist $w_{latt} = 110 \mu$m, maximum power per beam $P_{latt} = 5$ W) propagate in the $xz$-plane under a small angle of $\vartheta/2 = 4^\circ$ with respect to the $x$-axis. This configuration creates a standing wave along the $z$-axis with a spacing $d = \lambda_{latt}/[2\sin(\vartheta/2)] = 7.6 \mu$m. Due to the large spacing we load at most two sites when ramping up the optical lattice. Tunneling processes are completely negligible on the timescale of our experiments. By varying the powers in the beams we are able to provide nearly cylindrically symmetric traps, with aspect ratios $\lambda$ between $\sim 1/10$ and $\sim 10$, while keeping the average trap frequency $\tilde{\omega} = (\omega_x^2\omega_z)/3$ approximately constant [21].

We then ramp the magnetic field within 10 ms to adjust the value of the scattering length. The current providing the magnetic field is actively stabilized on the $10^{-5}$ level [22], which results in a resolution of $\Delta \lambda / a_0$ around the zero crossing of the scattering length. After an additional holding time of 2 ms we finally switch off the trap and take an absorption image along the $x$-axis, after a time of flight of 5 ms. The BEC atom number and radii are obtained by fitting the density profile using a bimodal distribution [20]. Knowing the atom number and radii we can calibrate the scattering length $a$ as a function of the magnetic field $B$ [17, 22].

We observe two effects when approaching the zero-crossing of the scattering length: The BEC shrinks in both directions due to the decreasing scattering length and the ellipticity of the cloud changes as a manifestation of the enhanced dipolar effects [17]. Finally, when we decrease the scattering length even further, the BEC atom number abruptly decreases. At this point the density distribution does not show a bimodal shape any more
but becomes thermal-like. The total atom number stays roughly constant during this collapse, excluding three-body loss processes causing the decrease in BEC atom number. The critical scattering length \( a_{\text{crit}} \) where the condensate collapses depends strongly on the trap aspect ratio \( \lambda \) (Fig. 2). For an isotropic trap (red) the collapse occurs at \( a \approx 15a_0 \), whereas the pancake-shaped trap (blue) can even stabilize a purely dipolar BEC (\( a \approx 0 \)).

We repeated this experiment for all the six traps listed in Table I, thereby covering a range of two orders of magnitude in the trap aspect ratio \( \lambda \). By fitting to the observed BEC atom numbers (Fig. 2) the threshold function

\[
N = \max \left[ 0, N_0(a - a_{\text{crit}})^\beta \right],
\]

where \( N_0, a_{\text{crit}} \) and \( \beta \) are fitting parameters, we find the critical scattering length \( a_{\text{crit}} \). The simple functional form (Eq. 5) was chosen because it accounts for the slowly decreasing BEC atom number when approaching the collapse point. The exponent \( \beta \) describing the steepness of the collapse was found to be \( \beta \approx 0.2 \) for all traps. The obtained values of \( a_{\text{crit}} \) versus the trap aspect ratio are plotted in Fig. 3 A. We observe a clear shift towards smaller \( a \) as \( \lambda \) increases. For the most oblate trap (\( \lambda = 10 \)) we can reduce the scattering length to zero and hence access the purely dipolar regime experimentally.

To get a more quantitative insight into the collapse threshold \( a_{\text{crit}}(\lambda) \) we numerically determine the critical scattering length (green curve in Fig. 3 A) as follows. The ground state wave function \( \Phi(\mathbf{r}) \) of a BEC can be found by the minimization of the Gross-Pitaevskii energy functional [1]

\[
E[\Phi] = \int \left[ \frac{\hbar^2}{2m} |\nabla \Phi|^2 + V_{\text{trap}}|\Phi|^2 + \frac{g}{2}|\Phi|^4 + \frac{1}{2}\mathbf{r}^2 \right] \Phi^2 \left( \Phi^* \right)^2 d\mathbf{r}.
\]  

Here the first term corresponds to the kinetic energy \( E_{\text{kin}} \), the second to the potential energy \( E_{\text{pot}} \) in the trap, while the third represents the contact interaction energy \( E_{\text{cont}} \), where \( g = 4\pi \hbar^2 a/m \) is the coupling constant. The last, non-local term \( E_{\text{dd}} \) arises from the magnetic DDI [24], where

\[
U_{\text{dd}}(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{1 - 3 \cos^2 \theta}{|\mathbf{r}|^3}
\]

is the interaction energy of two magnetic dipoles \( \mu \) aligned by an external field. Here \( \mathbf{r} \) is the relative position of the dipoles and \( \theta \) the angle between \( \mathbf{r} \) and the direction \( z \) of polarization. In order to obtain an estimate of \( a_{\text{crit}} \) we calculate the energy \( E(\sigma_r, \sigma_z) \) of a cylindrically symmetric Gaussian wave function

\[
\Phi(\mathbf{r}, z) = \left( \frac{N}{\pi^{3/4} \sigma_r \sigma_z a_0} \right)^{1/2} \exp \left( -\frac{1}{2a_0^2} \left( \frac{r^2}{\sigma_r^2} + \frac{z^2}{\sigma_z^2} \right) \right),
\]

with \( \sigma_r \) and \( \sigma_z \) as variational parameters. Using this ansatz, where \( a_0 = \sqrt{\hbar/(m\omega)} \), the contributions to the total energy are the zero point fluctuations

\[
E_{\text{kin}} \frac{\hbar}{N} \omega = \frac{1}{4} \left( \frac{\sigma_r^2}{\sigma_z^2} + \frac{\sigma_z^2}{\sigma_r^2} \right),
\]
to a metastable state) is determined by the trap aspect ratio \( \lambda \) (see Fig 3 C, where \( a_{\text{dd}} > a > a_{\text{crit}} \) and Fig D, where \( a = a_{\text{crit}} \)). Finally, below \( a \sim -2a_{\text{dd}} \) (Fig 3 E) the metastable state vanishes for any \( \lambda \) and the BEC is always unstable [13, 14].

Considering the limit \( Na_{\text{dd}}/\omega_0 \gg 1 \) where the terms [15] and [16] can be neglected [23] (grey curve in Fig. 3 A) we gain further insight into the nature of the dipolar collapse. In this case the stability is governed by the competition between the contact and dipole-dipole interaction only, that is by the sign of the last term in Eq. 11. Hence the critical scattering length is implicitly given by

\[
a_{\text{crit}}(\lambda) = a_{\text{dd}} f(\kappa(\lambda)).
\]

The asymptotic behavior of the theory curve \( a_{\text{crit}} = a_{\text{dd}} \) for \( \lambda \to 0 \) (respectively \( a_{\text{crit}} = -2a_{\text{dd}} \) for \( \lambda \to \infty \)) now becomes apparent as for extremely prolate (respectively oblate) traps the cloud shape follows the trap geometry and \( f \) takes on its asymptotic values. Another particular point is \( a_{\text{crit}} = 0 \) marking the aspect ratio \( \lambda \) needed to stabilize a purely dipolar BEC. More precisely, as \( f(1) = 0 \), we search for the trap in which the ground state of a purely dipolar BEC is isotropic. As the DDI tends to elongate the BEC along the \( z \)-direction and shrink it radially, it is clear that the desired trap is oblate. Using our model we obtain the criterion \( \lambda > \lambda_c \approx 5.2 \) for a purely dipolar BEC to be stable, a result that agrees well with the values found in [13, 14, 26].

The grey curve in Fig. 3 A that we obtain by numerically solving Eq. 12 shows a universal behavior in the sense that in the large \( N \) limit \( a_{\text{crit}}(\lambda) \) does not depend any more on the absolute values of the trap frequencies and \( N \). This fact clearly distinguishes the dipolar collapse from the pure contact case (red curve in Fig. 3 A). The former is ruled by the interplay between contact and dipolar interaction whereas in the latter the zero point energy and the contact interaction rival against each other. Due to the different \( N \)-scaling of the two competing terms in the pure contact case, the \( \lambda \)-dependance, which is already weak for finite \( N \) [27], completely vanishes in the limit of large \( N \) as the stability criterion reads \( a_{\text{crit}}(\lambda) = 0 \) (see Eq. 11 and red curve in Fig. 3 A). Furthermore the stability threshold obtained here applies for any dipolar system like e.g. hetero-nuclear molecules, where the only difference is the specific value of \( a_{\text{dd}} \).

In spite of the simplicity of our model we find good agreement between experiment and theory (Fig. 3 A). We checked that the different atom numbers and mean trap frequencies that we find for the six traps modify the green curve by much less than the error bars which arise mainly from the calibration of the scattering length. For the most oblate trap (\( \lambda = 10 \)) the 1/\( e \)-lifetime of the purely dipolar BEC (\( a = 0 \)) decreases to \( \sim 13 \) ms.

In conclusion, we experimentally mapped the stability diagram of a dipolar BEC. The dependance on scattering length and trap aspect ratio agrees well with a simple model based on the minimization of the energy of a Gaussian ansatz. By using a pancake-shaped trap we were able to enter the regime of purely dipolar quantum gases. This work opens up the route to new and exciting physics [26]. A clear subject for future studies is the dynamics of the dipolar collapse, which might show
anisotropic features. Another remarkable property of a dipolar BEC in a pancake-shaped trap is the existence of a roton minimum in its Bogoliubov spectrum [18]. Furthermore, close to the collapse threshold, the existence of structured ground states is predicted [28, 29], a precursor for the supersolid phase [30] that is expected to appear in dipolar BECs in three dimensional optical lattices. Finally, a field that has gained increasing interest in the recent past is the study of unusual vortex lattice patterns in rotating dipolar BECs [31].

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