Could Fermion Masses Play a Role in the Stabilization of the Dilaton in Cosmology?

Alejandro Cabo*∗∗∗ and Robert Brandenberger∗

∗ Department of Physics, McGill University, 3600 rue Université, Montréal, QC, H3A 2T8, Canada
∗∗ Perimeter Institute for Theoretical Physics, 31 Caroline Street, Waterloo, ON, N2L 2Y5, Canada and
∗∗∗ Group of Theoretical Physics, Instituto de Cibernética, Matematica y Fisica, Habana, Cuba
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We study the possibility that the Dilaton is stabilized by the contribution of fermion masses to its effective potential. We consider the Dilaton gravity action in four dimensions to which we add a mass term for a Dirac fermion. Such an action describes the interaction of the Dilaton with the fermions in the Yang-Mills sector of the coupled supergravity/super-Yang-Mills action which emerges as the low energy effective action of superstring theory after the extra spatial dimensions have been fixed. The Dilaton couples to the Fermion mass term via the usual exponential factor of this field which multiples the non-kinetic terms of the matter Lagrangian, if we work in the Einstein frame. In the kinetic part of the Fermion action in the Einstein frame the Dilaton does not enter. Such masses can be generated in several ways: they can arise as a consequence of flux about internal spatial dimensions, they may arise as thermal fermion masses in a quasi-static phase in the early universe, and they will arise after the breaking of supersymmetry at late times. The vacuum contribution to the potential for the Dilaton is evaluated up to two loops. The result shows a minimum which could stabilize the Dilaton for reasonable ranges of parameter values.

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INTRODUCTION

Superstring theory predicts the existence of the Dilaton, a scalar field partner of the graviton. Dilaton excitations appear in the mass spectrum of a superstring at the same level as the graviton fluctuations (see e.g. [1] for a textbook discussion). Consistent backgrounds for superstring theory must include the Dilaton field in addition to the metric, and the action for such a background is that of Dilaton gravity (see e.g. [2, 3]).

At the tree level, the Dilaton is a free massless scalar field with a very specific coupling to the matter sector of the theory. Phenomenologically, the existence of the Dilaton leads to two sets of problems. Since the Dilaton is coupled to matter, then a rolling Dilaton leads to time-dependent coupling constants. To avoid this problem, the Dilaton should be constant at the current stage of cosmological evolution. This is not sufficient, however: unless the Dilaton has a potential yielding a large mass, then it would lead to a “Fifth force”. Fifth force constraints lead to a lower bound on the mass of the Dilaton today which is of the order $m < 10^{-12}$ GeV [4] (but see [5] for an attempt to make a running Dilaton consistent with late time cosmology).

The question of Dilaton stabilization at various stages of the evolution of the universe has therefore been a subject of considerable interest in recent years. In fact, the Dilaton is just one of the various scalar fields which follow from superstring theory in the low-energy limit. Other scalar fields describe the sizes and shapes of the extra spatial dimensions predicted by superstring theory. Collectively, these fields are called “moduli fields”. In the context of recent work on moduli stabilization in Type IIB superstring theory, it has been realized that certain moduli fields - including the Dilaton - can be stabilized by the addition of fluxes about the compact spatial dimensions [6, 7]. Non-perturbative effects such as gaugino condensation [8] have also been used to stabilize the Dilaton, in particular in the context of heterotic superstring theory [8] and string gas cosmology [10].

Since the issue of Dilaton stabilization is a problem for late time cosmology, it is of interest to explore possible mechanisms to stabilize the Dilaton which do not depend on special assumptions about extra dimensions.

Another motivation to look for a different Dilaton stabilization mechanism comes from String Gas Cosmology (SGC). String Gas Cosmology [11] (see e.g. [12] for recent reviews) is a model of early universe cosmology which makes use of new degrees of freedom and new symmetries of string theory, coupling these ingredients to a classical background model containing gravity plus the Dilaton. The universe is assumed to start as a compact space filled with a gas of strings. Since there is a maximal temperature for a gas of closed strings, the early stage of cosmological evolution in SGC will be described by a phase of almost constant temperature, the “Hagedorn phase”. SGC leads to the possibility of a non-singular cosmology. As has recently been observed, thermal fluctuations in a gas of closed strings in the Hagedorn phase can lead to a scale-invariant spectrum of cosmological fluctuations [13, 14], with a slight blue tilt for gravitational waves forming a distinctive prediction of the model [15]. However, in order that this scenario work, the Dilaton needs to be fixed during the Hagedorn phase. Thus, in SGC we would like the Dilaton to be fixed both at very early and at very late times (it need not be fixed in intermediate phases).
DILATON GRAVITY WITH A MASSIVE FERMION

In this paper we will consider an action of a Dilaton coupled to a massive fermion of the following simple form:

\[
S = \int dx \sqrt{-g(x)} \left( -\frac{1}{2}g^{\mu\nu}(x)\partial_\mu \varphi(x)\partial_\nu \varphi(x) + \Psi(x)(i\frac{g^{\mu\nu}\gamma_\mu}{2} \bar{\partial}_\nu - \exp(\alpha \varphi)m)\Psi(x) \right).
\]

This action describes the coupling between the Dilaton \( \varphi \) and the Fermionic part of super-Yang-Mills matter in the Einstein frame, as we will show in the following. The form of introducing the derivative \( \bar{\partial} = \partial - \partial \) assures the reality of the Lagrangian in curved space [10]. Note that the kinetic term contains no Dilaton-dependent exponential factor, whereas the mass term does. This is demanded if we start from the low-energy action for superstring theory, as we show below. The Dirac matrices satisfy the usual commutation relations \( \{\gamma_\mu, \gamma_\nu\} = 2g_\mu\nu(x) \), where \( g_\mu\nu \) is the space-time metric, \( g^{\mu\nu} \) is its inverse, and \( g \) is its determinant.

Our Lagrangian (1) describes the coupling between the Dilaton \( \varphi \) and matter fermions \( \Psi \) resulting from the low energy effective action of superstring theory. The starting point is the Lagrangian for supergravity in ten space-time dimensions written in the Einstein frame:

\[
L_{SG} = \frac{1}{2\kappa^2} R - \frac{3}{4} e^{-(3/2)\varphi} H_{MNP}^2 - \frac{9}{16\kappa^2}(\partial_\mu \varphi)^2 \]

In the above, \( R \) is the Ricci scalar, \( \kappa^2 \) is the gravitational constant in ten dimensions, \( \varphi \) is the Dilaton, \( H_{MNP} \) is the three form which is the curl of the fundamental string theoretical two form \( B_{MN} \), and the capital Latin indices run over all space-time dimensions (see Chapter 13 of [11]). We have neglected the contribution from the gravitini and the dilatini.

To this Lagrangian, we add the Lagrangian of the super-Yang-Mills matter sector which is

\[
L_{SYM} = -\frac{1}{4} e^{-(3/4)\varphi} F^a_{MN} F^{MNa} - \frac{1}{2} \bar{\chi}^a \Gamma^M(D_\mu \chi)^a + \frac{1}{16} \sqrt{2} e^{-(3/4)\varphi} \bar{\chi}^a \Gamma^{MNP} H_{MNP} \chi^a, \]

where \( F^a_{MN} \) is the field strength of the gauge field of matter (the index \( a \) denotes the index in group space), \( \chi \) is the matter sector fermion field and the \( \Gamma^M \) are constant Dirac algebra matrices out of which the \( \Gamma \) matrices with more
indices are constructed by taking totally antisymmetrized products. Once again, we have neglected the contribution from the gravitini and the dilatini. In the presence of matter, the three form field $H_{MNP}$ gets a contribution from the Chern-Simons three form. Since we will in the following be neglecting the contribution of the gauge fields, the Chern-Simons term will not appear, either.

From (3) it follows that a non-vanishing three form flux about internal spatial dimensions can generate a mass term for the fermions $\chi$. If we fix gravity, then the action for the remaining fields (the Dilaton and the fermion $\chi$) takes on the form of our toy model (1) with $\alpha = -3/4$.

**EFFECTIVE POTENTIAL**

In the following, we will study the one and two loop approximations of the above effective action at zero temperature. The background field method of computing the effective action is reviewed in the Appendix. In this paper, we are mainly interested in local quantities such as the effective potential, and hence it is justified to work in the flat space-time approximation. Note that working in curved space-time would entail very nontrivial complications.

We will thus be working with the Minkowski metric $g_{\mu \nu} = \eta_{\mu \nu}$ and can view this approximation as working in comoving coordinates and conformal time $\eta$, the latter being defined in terms of the cosmological time $t$ via

$$d\eta = a(t)^{-1}dt,$$

(where $a(t)$ is the cosmological scale factor), and neglecting the evolution of the scale factor. For conformally coupled matter, this procedure is rigorous. However, in our case we are interested in mass generation, and hence our procedure is only an approximation (albeit a good one).

Therefore, the generating functional can be written in the form

$$Z[j, \bar{\eta}, \eta] = \exp \left\{ i \int dx \sqrt{-g} \left( -\frac{1}{2} g^{\mu \nu}(x) \partial_\mu \phi(x) \partial_\nu \phi(x) \right) + \int dx j(x) \phi(x) \right\} \times \frac{\text{Det} \left[ (i \square + \frac{1}{2} \exp(\alpha \phi)m) \right]}{\text{Det} \left[ i \partial^2 \right]^4} \times \exp \left[ -i \int dx \exp(\alpha \phi) \left( \sum_{\delta} \delta \left( \frac{\delta}{i \delta j(x)} \right) \right) \right] \times \exp \left[ -i \int dx_1 dx_2 \eta(x_1) G(x_1, x_2, \phi(x_2)) \right] \times \exp \left[ -i \int dx_1 dx_2 j(x_1) \frac{D(x_1, x_2)}{2} j(x_2) \right],$$

where the metric has been kept in the general form in the tree Dilaton part of the effective action since replacing it by the Minkowski metric is not required in terms of possible technical simplifications.

The inverse propagators now have the form they take on in Minkowski space-time:

$$D^{-1}(x_1, x_2) = \partial^2 \delta(x_1 - x_2),$$

$$G^{-1}(x_1, x_2 | \phi) = (\gamma^\nu \partial^\nu - \exp(\alpha \phi)m) \delta(x_1 - x_2),$$

Therefore, the effective action for the Dilaton field with the Fermi fields set to zero can be written as follows:

$$\Gamma[\phi, 0, 0] \equiv \Gamma[\phi] = \frac{1}{i} \log(Z[j, 0, 0]) - \int dx j(x) \phi \quad \text{(8)}$$

$$= \int dx \sqrt{-g(x)} \left( -\frac{1}{2} g^{\mu \nu}(x) \partial_\mu \phi(x) \partial_\nu \phi(x) \right) + \frac{1}{i} \log \text{Det}[i \gamma^\mu \partial^\nu - \exp(\alpha \phi)m] - \frac{1}{2i} \log \text{Det}[i \partial^2] \times \exp \left[ -i \int dx \exp(\alpha \phi) \left( \sum_{\delta} \delta \left( \frac{\delta}{i \delta j(x)} \right) \right) \right] \times \exp \left[ -i \int dx_1 dx_2 \eta(x_1) G(x_1, x_2, \phi(x_2)) \right] \times \exp \left[ -i \int dx_1 dx_2 j(x_1) \frac{D(x_1, x_2)}{2} j(x_2) \right].$$

The first term on the right hand side of (8) is the tree level effective action. We now turn to the evaluation of the one loop contribution.
One Loop Effective Action

Using standard techniques we can write down the one loop correction to the Effective action associated with vacuum fluctuations. According to the background field method, the one loop effective action \( \Gamma^{(1)}(\phi) \) can be obtained by calculating the Gaussian approximation of the generating functional about the background field (see e.g. [17, 18] for reviews). In terms of the shifted fields, this corresponds to the Gaussian approximation of (5) about the point where the field fluctuations are zero.

Returning to (5), we see that the exponent of the first term on the right hand side of the equation is the tree level effective action, whereas the one loop contribution resulting from the Gaussian integral approximation is the logarithm of the two determinants appearing in the second line of (5). Thus, the sum of the tree and one loop vacuum effective actions takes the form

\[
\Gamma^{(0,1)}[\phi] = \Gamma^{(0)}[\phi] + \Gamma^{(1)}[\phi] = \int dx \sqrt{-g(x)} \left( -\frac{1}{2} g^{\mu \nu}(x) \partial_\mu \phi(x) \partial_\nu \phi(x) \right) + \frac{1}{i} \log \det [i \gamma^\mu \partial_\mu - \exp(\alpha \phi) m] - \frac{1}{2i} \log \det [\partial^2].
\]

The last line is the one loop correction \( \Gamma^{(1)}[\phi] \), the first determinant coming from the fermionic term, the second from the term involving Dilaton fluctuations.

The functional determinants appearing in \( \Gamma^{(1)}[\phi] \) can be evaluated by going to the momentum space representation. Making use of the identity \( \ln(\det) = \text{Tr}(\ln) \) we can write the fermionic term as

\[
\Gamma^{(1)}_f = \frac{1}{i} V^4 \int \frac{dp^D}{(2\pi)^D} \text{Tr}^4 \ln \left[ e^{2\alpha \phi} m^2 - P^2 \right]
\]

where the remaining trace runs over the Dirac indices, and the superscripts 4 over the volume, the trace and the integration measure indicates the dimension of space-time. A similar expression holds for the contribution to the one loop effective potential coming from Dilaton loops (whose contribution, however, will vanish in the Minimal Subtraction scheme used below). In order to evaluate this integral, we perform Wick rotation. The trace over the Dirac indices is trivial and results in a multiplicative factor of 4.

The momentum integral diverges. To regularize and renormalize the resulting expression for the one loop contribution to the effective action, we evaluate the Euclidean integrals using dimensional regularization. We consider the integrals in \( D \) space-time dimensions. In \( D \neq 4 \) the integral converges and can be evaluated exactly. The renormalization consists of subtracting the contribution which gives the pole at \( D = 4 \). As is known from the Coleman-Weinberg construction [19], we need to introduce a mass scale \( \mu \) to define the subtraction point (“dimensional transmutation”). Given this mass scale, we can write the D-dimensional volume as

\[
V^D = V^4 \mu^{4-D},
\]

and we can re-scale the coupling constant \( \alpha \) as

\[
\alpha^D = \alpha \mu^{2 - \frac{D}{2}}.
\]

After Wick rotation and going to a general dimension, the expression for the one-loop effective potential has the form

\[
\Gamma^{(1)}[\phi] = \frac{1}{i} \log \det [i \gamma^\mu \partial_\mu - \exp(\alpha^D \phi) m] - \frac{1}{2i} \log \det [\partial^2]
\]

\[
= 2 V^D \int \frac{dp^D}{(4\pi)^D} \log \left[ p^2 + \exp(2\alpha^D \phi) m^2 \right] - \frac{1}{2} V^D \int \frac{dp^D}{(4\pi)^D} \log [p^2]
\]

\[
= 2 V^D \frac{\Gamma(1 - \frac{D}{2}) \exp(\alpha^D D \phi) m^D}{(4\pi)^{\frac{D}{2}}}. \tag{13}
\]
Note that in $D$ space-time dimensions the fields and $\alpha$ have the following energy dimensions:

$$D[\phi] = \frac{D}{2} - 1,$$

$$D[\Psi] = \frac{D - 1}{2},$$

$$D[\alpha^D\phi] = 0 \rightarrow D[\alpha^D] = 1 - \frac{D}{2}. \tag{16}$$

After evaluation of the integrals in $D$ space-time dimensions and after eliminating the divergences at $D = 4$ employing the Minimal Subtraction (MS) scheme, that is, by omitting the pole part in $D - 4$ of the result in (13), the finite part of the potential becomes (after some algebra):

$$V^{(1)}[\phi] = -\Gamma^{(1)}_{\text{finite}}[\phi] = \left(\frac{m^2}{4\pi}\right)^2 \exp(4\alpha\phi)\left[3 - \gamma - \log\left(\frac{m^2}{4\pi\mu^2}\right) - 2\alpha\phi\right] + C \tag{17}$$

$\gamma = 0.57721.\,$

An undetermined contribution $C$ to the cosmological constant has been added in the above expression for possible future applications to phenomenological questions. Note that the general equation for the effective action always admits this addition.

As a function of $\phi$, the one-loop potential $V^{(1)}$ has a local maximum. Thus, we conclude that at one-loop level, there is no Dilaton stabilization.

**FIG. 1:** The sum of the one and two loop contributions to the effective potential plotted, for fixed values of the parameters $m = 1, C = 0$ and $\alpha = -1$, as a function of $\mu$ and the mean value of the Dilaton field $\phi$. Note the appearance of a minimum of the potential as a function of the Dilaton field which becomes more pronounced for smaller values of $\mu$.

Vacuum two loop correction for small $\varphi$

Let us consider now the highly non-linear interaction term between the Dilaton and the fermion field in the action (1). We will assume that the mean value of the Dilaton and of its the quantum fluctuations are small in order that the
functional integral [27] is not affected by only retaining the order one term in the Taylor expansion of the $\exp(\alpha \varphi)$ in powers of $\varphi$. Thus, the action of the system becomes equivalent to the corresponding one in the Yukawa field theory.

In this "small Dilaton" approximation the two loop correction to the effective action can be written as the 1PI diagram term

$$\Gamma^{(2)}[\varphi] = -\frac{1}{2} \alpha^2 \exp(2\alpha \varphi) m^2 \int dx_1 dx_2 D(x_1, x_2) G^{r_1 r_2} (x_2, x_1|\varphi) G^{r_1 r_2} (x_1, x_2|\varphi)$$

(18)

After expressing the above in terms of momentum integrations by Fourier transforming the propagators, we obtain:

$$\Gamma^{(2)}[\varphi] = -\frac{1}{2} \alpha^2 \mu^{4-D} V^D \exp(2\alpha \varphi) m^2 \int \frac{dp_1}{(2\pi)^D} \frac{dp_2}{(2\pi)^D} \frac{1}{(p_1 - p_2)^2} \times \left( \frac{1}{\gamma^\mu p_1 \mu - \exp(\alpha \varphi) m} \right)_{r_1 r_2} \left( \frac{1}{\gamma^\mu p_2 \mu - \exp(\alpha \varphi) m} \right)_{r_2 r_1}$$

$$= -\frac{1}{2} \alpha^2 \mu^{4-D} V^D \exp(2\alpha \varphi) m^2 \int \frac{dp_1}{(2\pi)^D} \frac{dp_2}{(2\pi)^D} \frac{1}{(p_1 - p_2)^2} \times \left[ 4(p_2 \cdot p_3 + \exp(2\alpha \varphi) m^2) \right] / \left( p_1^2 - \exp(2\alpha \varphi) m^2 \right) (p_2^2 - \exp(2\alpha \varphi) m^2)$$

(19)

in which the scalar product between the two integration momenta can be eliminated by employing the identity

$$p_2 \cdot p_3 + \exp(2\alpha \varphi) m^2 = \exp(2\alpha \varphi) m^2 - \frac{(p_1 - p_2)^2}{2} + \frac{p_1^2 + p_2^2}{2},$$

(20)

which allows $\Gamma^{(2)}[\varphi]$ to be expressed as follows

$$\Gamma^{(2)}[\varphi] = -\alpha^2 \mu^{4-D} V^D \exp(4\alpha \varphi) m^4 \int \frac{dp_1}{(2\pi)^D} \frac{dp_2}{(2\pi)^D} \frac{1}{(p_1 - p_2)^2} \times \left[ 4(p_2 \cdot p_3 + \exp(2\alpha \varphi) m^2) \right] / \left( p_1^2 - \exp(2\alpha \varphi) m^2 \right) (p_2^2 - \exp(2\alpha \varphi) m^2).$$

(21)

The first integral term in the last expression is the square of a one-loop integral. The second terms is a two loop integral which expression reduces to a particular class of a master integral which has been evaluated, by example in Ref. 24. After employing the formulae for these integrals given in Ref. 24, the two-loop correction can be written as follows

$$\Gamma^{(2)}[\varphi] = -\alpha^2 (m^*)^2 \mathcal{V}^4 \frac{i^{4\epsilon - 6} \pi^4 - 2\pi^2 \Gamma^2 (\epsilon - 1)}{(2\pi)^{8\epsilon - 4\epsilon}} \left( \frac{m^*}{\mu^2} \right)^{-2\epsilon}$$

$$- \frac{4\alpha^2 (m^*)^2 \pi^4 - 2\pi^2 \mathcal{V}^4}{(2\pi)^{4(2\epsilon - \epsilon)}} \epsilon \Gamma^2 (1 + \epsilon) \left( \frac{\pi m^*}{\mu^2} \right)^{-2\epsilon},$$

$$m^* = m \exp(\alpha \phi), \quad \epsilon = 2 - \frac{D}{2}.$$  

(22)

The negative of the finite part of $\Gamma^{(2)}[\varphi]$ in the limit $\epsilon \to 0$ gives the two loop contribution to the effective potential

$$V^{(2)}[\varphi], = -\frac{\Gamma^{(2)}_{finite}[\varphi]}{\mathcal{V}^4}.$$  

(23)
The explicit formula for \( V^{(2)} \) can be evaluated using Mathematica and is given below:

\[
V^{(2)}(\phi) = \frac{1}{512\pi^4}e^{6\alpha\phi}m^6 \left[ 4(\log \left( \frac{m^2}{\mu^2} \right) + 2\alpha \phi) \times \left( 3 \left( \log \left( \frac{m^2}{\mu^2} \right) + 2\alpha \phi \right) - 2(5 + \log(64) + 3\log(\pi)) + 6\gamma \right) \\
+ 2(25 + 8\log(2)(5 + \log(8)) + \log(\pi)(20 + 6\log(16\pi))) \\
- 8\gamma(5 + \log(64) + 3\log(\pi)) + \pi^2 + 12\gamma^2 \right] \]

(24)

We see that the two-loop contribution to the potential leads to a term which is proportional to

\[
(\alpha\phi)^2 e^{6\alpha\phi} \]

(25)

which changes the character of the potential at small values of \(|\phi|\) dramatically. There is now a local minimum of the potential at a small and negative value of \(\phi\) which becomes deeper the smaller \(\mu\) is.

The total effective potential up to two loops is given by

\[
V[\phi] = V^{(1)}[\phi] + V^{(2)}[\phi].
\]

(26)

A plot of the resulting potential as a function of the Dilaton field and of the dimensional renormalization parameter \(\mu\), is shown in Figure 1 where we have chosen the particular value of the mass \(m = 1\) and the negative parameter \(\alpha = -1\).

The plot focuses on a region of the regularization parameter \(\mu\) which is of special interest. It is clear that in this zone, the Dilaton field can be stabilized at a negative value thanks to the appearance of a minimum value of the potential at small values of \(\mu\) relative to \(m\), which for simplicity was taken equal to one for the plot.

**DISCUSSION AND CONCLUSIONS**

In this paper we have computed one and two-loops contributions to the effective potential of the Dilaton coupled to Dirac fermions in a way motivated by the low energy effective action of superstring theory. The divergences in the resulting integrals were regularized and renormalized using dimensional regularization and Minimal Subtraction. This introduces a new mass scale \(\mu\) into the result. Our result shows that if the ratio of the Fermion mass \(m\) to the new mass scale \(\mu\) is sufficiently large, a minimum in the Dilaton effective potential appears at two-loop order. This leads to a possible mechanism that could stabilize the Dilaton field.

Note that since the minimum of the potential is an absolute minimum, the “overshoot problem” [21] is not an issue. A more important worry, on the other hand, is the stability of our stabilization mechanism to higher loop corrections. It is difficult to address this worry in a perturbative framework.

An immediate application of our result is to the stabilization of the Dilaton at late times, after the time of supersymmetry breaking. At that time, we would be making use of the fermions of the Standard Model. Supersymmetry is broken and so the use of our model Lagrangian is justified.

The application of our results to early universe cosmology are more speculative. Our model Lagrangian is not supersymmetric: we have neglected the gravitini and dilatinis. In a fully supersymmetric background, we would expect loops of the fields we have neglected to result in terms cancelling the contributions to the effective potential which we have computed. However, at finite density and finite temperature in the early universe supersymmetry is broken. Thus, one might in fact hope that our one loop potential is also applicable in early universe cosmology. In this context, one would use the thermal masses of fermions to stabilize the Dilaton.

In most cosmological background the scale factor changes rapidly with time in the early universe. This leads to another concern about the applicability of our analysis to early universe cosmology. However, in String Gas Cosmology, it is postulated that the early dense phase is a static Hagedorn phase at constant temperature. In this context, our analysis is justified, hence providing a new way of stabilizing the Dilaton in this phase of String Gas Cosmology.

Concerning the possible application of our mechanism to Dilaton stabilization in superstring cosmology, there are further caveats: Not only the Dilaton, but also the shape and size moduli of the extra dimensions need to be stabilized. Fluxes which wrap cycles of the extra-dimensional manifold and non-perturbative techniques such as gaugino condensation are often used for this purpose (see e.g. [22] for an overview). These mechanism can also generate a potential for the Dilaton which can compete with the potential we have derived here.
The form of the potential suggests that after solving for the cosmological evolution of the model, the thermal energy of the Universe could be gradually transformed in energy of the Dilaton, which then could play the role of a quintessence field describing Dark Energy. The study of this possibility will be considered elsewhere.

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APPENDIX: GENERATING FUNCTIONAL AND EFFECTIVE ACTION

The generating functional in terms the auxiliary sources $j(x), \eta(x)$ and $\bar{\eta}(x)$ for the Dilaton and fermion fields is defined as the functional integral (see e.g. \cite{17,18})

$$Z[j, \eta, \bar{\eta}] = \int \mathcal{D}\phi \mathcal{D}\bar{\phi} \mathcal{D}\Psi \exp \left\{ i [S + \int dx(j(x)\phi(x) + \eta(x)\bar{\Psi}(x) + \bar{\eta}(x)\eta(x))] \right\}. \tag{27}$$

In order to compute the one loop effective action it is beneficial to make use of the background field method. Thus, we shift the scalar field

$$\phi(x) \rightarrow \phi + \phi^r(x), \tag{28}$$

where $\phi$ is a classical background Dilaton field which solves the equations of motion in the absence of fermions. Inserting this expansion into (27), $Z[j, \eta, \bar{\eta}]$ can be written in the following form

$$Z[j, \eta, \bar{\eta}] = \exp \left\{ i \left[ \int dx \sqrt{-g(x)} \left( -\frac{1}{2} g^{\mu\nu}(x) \partial_\mu \phi(x) \partial_\nu \phi(x) \right) + \int dxj(x) \phi(x) \right] \right\}$$

$$\times \exp \left[ -i \int dx \sqrt{-g(x)} \exp(\alpha \phi) \left( \exp\left(\frac{\delta^\alpha \delta}{i \delta j(x)} \right) - 1 \right) \frac{\delta}{-i\delta \eta(x)} \frac{\delta}{i \delta \bar{n}(x)} \right]$$

$$\times \mathcal{D}\phi \exp \left\{ i \left[ \int dx \sqrt{-g(x)} \left( -\frac{1}{2} g^{\mu\nu}(x) \partial_\mu \phi(x) \partial_\nu \phi(x) \right) + \int dxj(x) \phi(x) \right] \right\}$$

$$\times \mathcal{D}\bar{\phi} \mathcal{D}\Psi \exp \left\{ i \int dx \sqrt{-g(x)} \left[ \bar{\Psi}(x)\left( i g^{\mu\nu} \gamma_\mu \frac{\partial}{\partial \nu} - \exp(\alpha \phi) m \right) \Psi(x) + \bar{\eta}(x)\Psi(x) + \bar{\eta}(x)\eta(x) \right] \right\}. \tag{29}$$

Now, it is possible to evaluate the two Gaussian integrals appearing in the above expression to obtain

$$Z[j, \eta, \bar{\eta}] = \exp \left\{ i \left[ \int dx \sqrt{-g(x)} \left( -\frac{1}{2} g^{\mu\nu}(x) \partial_\mu \phi(x) \partial_\nu \phi(x) \right) + \int dxj(x) \phi(x) \right] \right\}$$

$$\times \frac{\text{Det} \left[ \sqrt{-g(x)} \gamma_\mu \frac{\partial}{\partial \nu} - \exp(\alpha \phi) m \right]}{\text{Det} \left[ \partial_\mu \left( \sqrt{-g(x)}(x) \partial_\nu \right) \right]^\frac{1}{2}} \tag{30}$$

$$\times \exp \left[ -i \int dx \sqrt{-g(x)} \exp(\alpha \phi) \left( \exp\left(\frac{\delta^\alpha \delta}{i \delta j(x)} \right) - 1 \right) \frac{\delta}{-i\delta \eta(x)} \frac{\delta}{i \delta \bar{n}(x)} \right]$$

$$\times \exp \left[ -i \int dx_1 dx_2 \eta(x_1)G(x_1, x_2|\phi)\eta(x_2) \right] \exp \left[ -i \int dx_1 dx_2 j(x_1) \frac{D(x_1,x_2)}{2} j(x_2) \right]$$

where the boson and fermion propagators $D(x_1,x_2)$ and $G(x_1,x_2|\phi)$ are the inverses of the kernels

$$D^{-1}(x_1,x_2) = \delta^{(1)}(\sqrt{-g(x_1)} g^{\mu\nu}(x_1) \partial_\nu^{(1)}) \delta(x_1 - x_2), \tag{31}$$

$$G^{-1}(x_1,x_2|\phi) = \sqrt{-g(x_1)} \left( i g^{\mu\nu} \gamma_\mu \frac{\partial}{\partial \nu} \right) - \exp(\alpha \phi) m \delta(x_1 - x_2). \tag{32}$$
The effective action is defined as the Legendre transform of \( \frac{1}{i} \ln Z \), with the canonically conjugate fields being the sources and the respective mean fields. Thus, we have

\[
\Gamma[\phi, \psi, \bar{\psi}] = \frac{1}{i} \log Z[j, \bar{\eta}, \eta] - \int dx (j(x)\phi(x) + \bar{\eta}(x)\psi(x) + \psi(x)\eta(x)),
\]

where \( \phi(x), \psi(x) \) and \( \bar{\psi}(x) \) are the functional derivatives of the logarithm of the partition function with respect to \( ij(x), \bar{\eta}(x) \) and \( -i\eta(x) \), respectively. Very similar expressions can be written down for the finite temperature partition function and effective action.

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