An improved adaptive weighted median filter algorithm

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Abstract. An improved adaptive weighted median filtering method is proposed to deal with the interference noise of ultrasonic RF signal. Firstly, edge pixel points are determined to be filtered by the method of extending edge points; secondly, mean value is used to replace the median value which considered to be noise points; finally, weighted smoothing processing is carried out. The final experimental results in this paper show that the proposed method has better effect on RF signal processing.

1. Introduction
The RF signal required by ultrasound elastography technology [1] will be affected by the instrument in the process of producing, causing noise interference, and even submerging the effective signal. Therefore, the elimination of noise interference has become an important point in ultrasound elastography technology. Filtering plays an important role in signal processing.

In the process of signal processing, the size and shape of the traditional median filter window are defined, and edge details will be lost while filtering. In order to solve the contradiction between denoising and retaining details at the same time, researchers propose many improved median filtering algorithms. Literature [2] is a standard adaptive median filter algorithm, which uses the median value in the window as the response for pixel points containing noise, but is not applicable for high-density noise images. In order to improve the adaptive median filtering algorithm, literature [3] solves the problem of repeated operation of pixel points in the process of window iteration, but it is easy to misjudge the extreme points as noise points for filtering. Literature [4] is a center-weighted median filtering algorithm. By weighting, the proportion of the center pixel in the window increases, but it is easily affected by noise points. Literature [5] is an image adaptive median filtering algorithm, combining one-dimensional and two-dimensional median filtering to propose adaptive median filtering, but there are certain limitations in window selection.

Based on the above, this paper proposes an improved adaptive weighted median filtering algorithm, which can alleviate the loss of edge details while reducing noise.

2. Another section of your paper

2.1. Median filtering
Median filter (MF) [6] is a nonlinear signal processing technique based on sorting statistics theory that can effectively suppress noise. It is a neighborhood operation that sorts pixels in the neighborhood according to gray level. And the intermediate value of the group is then selected as the output pixel.
value. In 1971, JWTukey proposed the concept of median filter on time series analysis. The advantage of this filter is that it is simple and fast, and it shows excellent performance in filtering out superimposed white noise and long tail superimposed noise.

The idea of median filtering is to compare the size of pixels in a certain domain, and take out the median of this field as the new value of the center pixel of this field. The standard median filtering algorithm relies on a fast sorting algorithm. It is a nonlinear filtering method with less edge blur. It can not only remove or reduce random noise and pulse interference, but also preserve the information at the edge of the image.

The standard median filter (MF) is defined as:

\[ f(i, j) = \text{Median}\{g(s, t), (s, t) \in S_y\} \] (1)

In equation (1), \( g(s, t) \) is noise, the median filtering method is to sort the pixels in the sliding filter window, and the output pixel value \( f(i, j) \) of the filtering result is the median value of the sequence.

### 2.2. Adaptive median filtering

According to the basic nature of median filtering, the size of the sliding window plays a crucial role in signal filtering performance. The classic median filter sliding window will remain unchanged, the smaller window can protect the signal well, but cannot effectively remove the noise; the larger sliding window can better suppress the noise, but at the same time the edge of the signal is blurred or even lost effective information, which is also an important drawback of median filtering.

In order to compensate for this defect, adaptive median filtering (AMF) \[^7\] is used to adaptively adjust the sliding window length, which can effectively denoise and protect the effective signal details.

Assuming that the image size is \( M \times N \), \( (x, y) \) is the coordinates of the image signal point \( p \), and \( S_{xy} \) is the domain window centered on \( (x, y) \). Which defines:

- \( Z_{\min} \) represents the smallest gray value in \( S_{xy} \), \( Z_{\max} \) is the largest gray value, \( Z_{\text{med}} \) is the median of all gray values in the representation, \( Z_{xy} \) is the gray value of the \( x \) row and the \( y \) column, \( S_{\text{max}} \) is the maximum window size allowed by \( S_{xy} \).

The adaptive median filtering algorithm can be represented as two processes, processes A and B:

**Processes A:**

\[
\begin{align*}
A_1 &= Z_{\text{med}} - Z_{\min} \\
A_2 &= Z_{\text{med}} - Z_{\max}
\end{align*}
\] (2)

For process A, if \( A_1 > 0 \) and then \( A_2 < 0 \), go to process B, otherwise judge the window suspected noise point, increase the window size; if the window size \( n \ll S_{\text{max}} \), repeat process A, otherwise output \( Z_{xy} = Z_{\text{med}} \), think that the window has noise points.

**Processes B:**

\[
\begin{align*}
B_1 &= Z_{xy} - Z_{\min} \\
B_2 &= Z_{xy} - Z_{\max}
\end{align*}
\] (3)

For process B, if \( B_1 > 0 \) and \( B_2 < 0 \), it is judged that there is no noise point in the window, output \( Z_{xy} = Z_{xy} \), otherwise output \( Z_{xy} = Z_{\text{med}} \).

### 2.3. Improved adaptive weighted median filter (IAWMF)

The traditional adaptive median filtering is based on the extreme point of the adaptive window as the basis for determining the noise point. There are three defects: 1) the adaptive window takes the point of the image as the center point and filtering step-by-step using a template of \( N \times N (N \geq 3) \), and the pixel points at the edge of the signal are ignored, which affects the overall filtering effect. 2) When the
detected suspected noise point is judged beyond the filter window size, the median value of the output may be a noise point. 3) For the window containing the noise point, the gray value of the pixel of the region in which it is located as the processing result of the filtering, and the magnitude of the median has a large effect on the filtering effect of the noise.

In response to the above defects, this paper proposes the following solutions:

**Extended edge**: the adaptive median filter takes (x, y) as its center point in the process of implementation, and ignores its edge value in the process of filtering, which affects the experimental results. For a matrix of size 5*5, in the process of taking the median, it is equivalent to convolving with a matrix of size 3*3. In this process, only the region of size 3*3 that is not the edge has an impact, and the edge remains the same. In order to make up for this defect, this paper adopts the extended adjacent edge method of ad = 1, so that all pixel points can be detected by noise in the filtering window. After expanding the image, the image becomes a 7*7 size matrix, and the convolution operation of the non-edge 5*5 matrix (corresponding to the original 5*5 matrix).

**Combined mean filtering**: Mean filtering \[^8\] is one of the classic algorithms for image denoising. The noise-free gray value is estimated by the average gray value in the neighborhood of each pixel of the noise image. Mean filtering is a typical linear filtering algorithm, which refers to giving a template which includes neighboring pixels around it to a target pixel on an image, and then replaces the original pixel value with the average of all the pixels in the template.

In the traditional median filtering algorithm, it is considered that the median point \(_{\text{med}}\) when \(_{\text{min}} < _{\text{med}} < _{\text{max}}\) is satisfied is not a noise point. And if \(_{\text{med}} = _{\text{min}}\) or \(_{\text{med}} = _{\text{max}}\) is satisfied, it is a noise point. If the window size is not satisfied, the median point of the output is a noise point. Therefore, a way to combine the mean filtering is proposed, and the mean value \(_{\text{mean}}\) of the domain is used instead of the median.

\[
_{\text{xy}} = _{\text{mean}}
\]

**Weighted filtering**: The weighted median filtering is to multiply each pixel in the window by a corresponding weight, and then statistically sort, taking the median instead of the noise value. The traditional median filtering can be seen as a weighted median filter with a weight of 1 for each pixel.

\[
W = \frac{1}{n} \cdot \text{one}(n, n)
\]

In this paper, the weight matrix of equation (5) is selected where \(n \geq 3\), then convolution operation on the signal after adaptive median filtering, to obtain a new weighted adaptive median filtered signal which replacing the original signal value.

3. **Experiment and result analysis**

The experimental data in this paper were derived from the University of Michigan's agar-graphite tissue mimicking phantoms and ex vivo kidneys. The size and dimensions of the experimental data cannot be easily estimated which is a 1536*128*225 three-dimensional matrix.

To further verify the filtering effect, the signal-to-noise ratio and the root mean square error of the signal are compared, and the definitions are respectively:
\[ SNR = 10 \log \left( \frac{\sum_{i=1}^{N} S(i)^2}{\sum_{i=1}^{N} (\hat{S}(i) - S(i))^2} \right) \]  

\[ RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (\hat{S}(i) - S(i))^2} \]  

Where \( S(i) \) represents the original signal, \( \hat{S}(i) \) representing the filtered signal. The results are shown in Table 1.

| Table 1 | Filtering vs. SNR and RESE of experimental data |
|---------|-----------------------------------------------|
|         | SNR      | RMSE  | Time(s) |
| MF      | 0.96     | 931.82| 0.096   |
| AMF     | 6.56     | 479.32| 1.31    |
| MAMF    | 6.85     | 472.75| 1.62    |
| AWMF    | 6.85     | 472.06| 1.93    |
| IAWMF   | 7.61     | 433.76| 3.46    |

It can be seen from Table 1 that for large-scale experimental data, Mean Adaptive Filtering (MAMF) and Adaptive Weighted Median Filtering (AWMF) have almost the same filtering effect, but the latter consumes longer running time than the former. The SNR of this method is the highest, and the RMSE is the smallest, indicating that the filtering result is the best, but the RMSE error value is very large for this experimental data.

In order to further verify the experimental method, another set of 500*200 data is selected for testing while a noise signal is added, and the experimental results are compared (the selected filtering window size \( S_{\text{max}} = 7 \), the added noise is uniform noise) as shown in Table 2.

| Table 2 | Filtered SNR and RMSE comparison |
|---------|----------------------------------|
|         | SNR      | RMSE  | Time(s) |
| MF      | 6.87     | 3.24  | 0.011   |
| AMF     | 13.86    | 0.145 | 0.048   |
| MAMF    | 15.0     | 0.127 | 0.081   |
| AWMF    | 23.68    | 0.0425| 0.1179  |
| IAWMF   | 41.36    | 0.0039| 0.1532  |

It is concluded from Table 2 that the improved method has much higher SNR than other methods and the RMSE is the smallest which indicating that the signal processing effect is better.

Trying to change the filter window during the experiment and finding that the filtering effect will change accordingly. In order to further verify the conjecture, the experiment chooses to change the window size \( S_{\text{max}} \) and noise types to experiment:

1. Select the added noise as uniform noise, and change the value of \( S_{\text{max}} \), \( S_{\text{max}} = 3i + 2(i = 0,1,2 \ldots) \).
2. Select \( S_{\text{max}} = 7 \) to change the type of noise added (uniform noise, Gaussian noise, Rayleigh noise, salt and pepper noise).
Figure 2. (a) and (b) respectively represent SNR and RMSE that change with window size; (c) and (d) respectively represent SNR and RMSE that change with different noise types.

Combining the above experimental data, the following conclusions are drawn:

1. The method presented in this paper has a good filtering effect. As shown in figure 2(a), its SNR is much higher than that of other methods, and RMSE of root mean square error is close to 0 in figure 2(b). For different filter window sizes, RMSE of IAWMF remains almost unchanged, indicating that IAWMF has a certain stability.

2. As the size of the filter window increases to a certain extent, the filtering effect is getting better and better. As shown in Figure 2(a), the larger the SNR value, the smaller the corresponding RMSE. For AWMF and IAWMF, when $S_{max} \leq 7$, the SNR increases with the increase of $S_{max}$. When $S_{max} \geq 7$, the value of SNR gradually decreases, indicating that the filter window size $S_{max} = 7$ of weighted filtering is the most suitable. The same result compares AMF and MAMF, the filter window size $S_{max} = 9$ is the best.

3. The type of noise will have a certain impact on the experimental results. The filtering effect on the addition of salt and pepper noise is the best, and the processing of Rayleigh noise is the worst. The method in this paper has a very close effect on the processing of various noises which indicating that this method is almost immune to the type of noise.

In summary, the method IAWMF has better processing effect on noise, and has better filtering performance than other methods.
4. Discussion
In this paper, on the basis of adaptive median filtering, an improved adaptive weighted median filter is proposed. The median value of the suspected noise point is replaced by means of mean value, and the weight matrix convolution operation of the filter window after adaptive median filtering is performed. The final results are better than other algorithms, and the feasibility of this method is illustrated. It laid the foundation for the subsequent ultrasonic elastography experiment.

References
[1] G. Cortela, L. Leija. Elastograms of the diabetic foot by ultrasonic impulse elastography [J]. IEEE Transactions on Image Processing, 2016.
[2] Wang D D, Wang F M. Research on digital image noise removal technology based on Matlab [J]. Mechanical engineering and automation, 2015(2): 98-99.
[3] Liu pengyu, ha rui, jia kebin. Improved adaptive median filtering algorithm and its application [J]. Journal of Beijing university of technology, 2017, 43(4):581-586.
[4] Ko S J, Lee Y H. Center weighted median filters and their applications to image enhancement [J]. IEEE Transactions on Circuits and Systems, 1991, 38(9):984-993.
[5] Liu hai. An image adaptive median filtering algorithm [J]. Software guide, 2018(5).
[6] Gong S R, Liu C P, Wang Q. Digital image processing and analysis [M]. Beijing: Tsinghua University press, 2006.
[7] Department of ECE, Shri JJT University, et al. A Novel Algorithm for Image Denoising using Modified Adaptive Median Filter [J]. Research Journal of Applied Sciences, Engineering and Technology, 2015, Vol.10 (4):373-375.
[8] Mehdi Mafi, Hoda Rajaei, Mercedes Cabrerizo. A Robust Edge Detection Approach in the Presence of High Impulse Noise Intensity through Switching Adaptive Median and Fixed Weighted Mean Filtering[J]. IEEE Transactions on Image Processing, 2018.