Spin light of neutrino in matter and electromagnetic fields

A.Lobanov, A.Studenikin*

Abstract

A new type of electromagnetic radiation by a neutrino with non-zero magnetic (and/or electric) moment moving in background matter and electromagnetic field is considered. This radiation originates from the quantum spin flip transitions and we have named it as "spin light of neutrino" (SLν). The neutrino initially unpolarized beam (equal mixture of $\nu_L$ and $\nu_R$) can be converted to the totally polarized beam composed of only $\nu_R$ by the neutrino spin light in matter and electromagnetic fields. The quasi-classical theory of this radiation is developed on the basis of the generalized Bargmann-Michel-Telegdi equation. The considered radiation is important for environments with high effective densities, $n$, because the total radiation power is proportional to $n^4$. The spin light of neutrino, in contrast to the Cherenkov or transition radiation of neutrino in matter, does not vanish in the case of the refractive index of matter is equal to unit. The specific features of this new radiation are: (i) the total power of the radiation is proportional to $\gamma^4$, and (ii) the radiation is beamed within a small angle $\delta\theta \sim \gamma^{-1}$, where $\gamma$ is the neutrino Lorentz factor. Applications of this new type of neutrino radiation to astrophysics, in particular to gamma-ray bursts, and the early universe should be important.

There exist at present convincing evidences in favour of neutrino non-zero mass and mixing, obtained in the solar and atmospheric experiments (see [1] for a review on the status of neutrino oscillations). Apart from masses and mixing non-trivial neutrino

*E-mail: studenik@srd.sinp.msu.ru
electromagnetic properties such as non-vanishing magnetic, \( \mu \), and electric, \( \epsilon \), dipole moments are carrying features of new physics. It is believed that non-zero neutrino magnetic moment could have an important impact on astrophysics and cosmology.

It is well known \([2]\) that in the minimally extended Standard Model with \( SU(2) \)-singlet right-handed neutrino the one-loop radiative correction generates neutrino magnetic moment which is proportional to neutrino mass

\[
\mu_\nu = \frac{3}{8\sqrt{2}\pi^2} e G_F m_\nu = 3 \times 10^{-19} \mu_0 \left( \frac{m_\nu}{1\text{eV}} \right),
\]

where \( \mu_0 = e/2m \) is the Bohr magneton, \( m_\nu \) and \( m \) are the neutrino and electron masses. There are also models \([3]\) in which much large values for magnetic moments of neutrinos are predicted. So far, the most stringent laboratory constraints on the electron neutrino magnetic moment come from elastic neutrino-electron scattering experiments: \( \mu_{\nu e} \leq 1.5 \times 10^{-10} \mu_0 \) \([4]\). More stringent constraints are obtained from astrophysical considerations \([5]\).

In this paper we study a new mechanism for emission of photon by the massive neutrino in presence of matter, assuming that the neutrino has an intrinsic magnetic (and/or electric) dipole moment. This phenomenon can be expressed as a process

\[
\nu \rightarrow \nu + \gamma
\]

that is the transition from a flavour neutrino in the initial state to the same flavour neutrino plus a photon in the final state. The mechanism under consideration can be also effective in the case of neutrino transitions with change of flavour if neutrino transition moment is not zero.

Different other processes characterized by the same signature of Eq.\((2)\) have been considered previously:

i) the photon radiation by massless neutrino \((\nu_i \rightarrow \nu_j + \gamma, \ i = j)\) due to the vacuum polarization loop diagram in presence of an external magnetic field \([6,7]\);

ii) the photon radiation by massive neutrino with non-vanishing magnetic moment in constant magnetic and electromagnetic wave fields \([8,9]\);

iii) the Cherenkov radiation due to the non-vanishing neutrino magnetic moment in homogeneous and infinitely extended medium which is only possible if the speed of neutrino is larger than the speed of light in medium \([10,11]\);

iv) the transition radiation due to non-vanishing neutrino magnetic moment which would be produced when the neutrino crosses the interface of two media with different refractive indices \([12,13]\).
v) the Cherenkov radiation by massless neutrino due to its induced charge in medium [14];

vi) the Cherenkov radiation by massive and massless neutrino in magnetized medium [15, 16];

vii) the neutrino radiative decay ($\nu_i \rightarrow \nu_j + \gamma$, $i \neq j$) in external fields and media or in vacuum [17, 18, 19, 20, 21].

The process we are studying in this paper has never been considered before. We discover a mechanism for electromagnetic radiation generated by the neutrino magnetic (and/or electric) moment rotation which occurs due to electroweak interaction with the background environment. It should be noted that generalization to the case of a photon emission by neutrino due to the neutrino transition magnetic moment is straightforward. If neutrino is moving in matter and an external electromagnetic field is also superimposed, the total power of this radiation contains three terms which originate from (i) neutrino interaction with particles of matter, (ii) neutrino interactions with electromagnetic field, (iii) interference of the mentioned above two types of interactions. This radiation can be named as "spin light of neutrino" ($SL\nu$) in matter and electromagnetic field to manifest the correspondence with the magnetic moment dependent term in the radiation of an electron moving in a magnetic field. A review on the spin light of electron can be found in [22]. Whereas the radiation of a neutral particle moving in external electromagnetic field in the absence of matter has been considered previously starting from [23], the mechanism of radiation produced by interaction with matter is considered in this our paper for the first time. It should be emphasize that the neutrino spin light can not be described as the Cherenkov radiation.

The $SL\nu$ in the background matter (similar to the radiation by neutrino moving in the magnetic field [8]) originates from the quantum spin flip transitions $\nu_L \rightarrow \nu_R$. Within the quantum approach the corresponding Feynman diagram of the proposed new process is the standard one-photon emission diagram with the initial and final neutrino states described by the "broad lines" that account for the neutrino interaction with matter (given, for instance, by the effective Lagrangian of Eq. (9) below). In this paper we develop the quasi-classical approach to the radiation process when the neutrino recoil can be neglected. This approach is valid for a wide range of neutrino and photon energies that could be of particular interest for different astrophysical and cosmological applications. For example, if the initial neutrino energy is about 10 $MeV$ the allowed range of the radiated photon energies span up to gamma-rays.
We should like also to emphasize here that the initially unpolarized neutrino beam (equal mixture of active left-handed and sterile right-handed neutrinos) can be converted to the totally polarized beam composed of only $\nu_R$ due to the spin light in contrast to the Cherenkov radiation which can not produce the neutrino spin self-polarization effect.

Our approach is based on the quasi-classical Bargmann-Michel-Telegdi (BMT) equation \[24\]

\[
\frac{dS^\mu}{d\tau} = 2\mu \left\{ F^{\mu \nu} S_\nu - u^\mu (u_\nu F^{\nu \lambda} S_\lambda) \right\} + 2\epsilon \left\{ \tilde{F}^{\mu \nu} S_\nu - u^\mu (u_\nu \tilde{F}^{\nu \lambda} S_\lambda) \right\},
\]

that describes evolution of the spin $S_\mu$ of a neutral particle with non-vanishing magnetic, $\mu$, and electric, $\epsilon$, dipole moments in electromagnetic field, given by its tensor $F_{\mu \nu}$. This form of the BMT equation corresponds to the case of the particle moving with constant speed, $\vec{\beta} = \text{const}$, $u^\mu = (\gamma, \gamma \vec{\beta})$, in presence of an electromagnetic field $F_{\mu \nu}$. The spin vector satisfies the usual conditions, $S^2 = -1$ and $S^\mu u_\mu = 0$. Note that the term proportional to $\epsilon$ violates $T$ invariance.

In our previous studies \[26,27\] (see also \[29\]) we have shown that the Lorentz invariant generalization of Eq.(3) for the case when effects of neutrino weak interactions are taken into account can be obtained by the following substitution of the electromagnetic field tensor $F_{\mu \nu} = (\vec{E}, \vec{B})$:

\[
F_{\mu \nu} \rightarrow E_{\mu \nu} = F_{\mu \nu} + G_{\mu \nu},
\]

where the tensor $G_{\mu \nu}$ accounts for the neutrino interactions with particles of the environment. The derivation of the quasi-classical Lorentz invariant neutrino spin evolution equation taking into account general types of neutrino non-derivative interactions with external fields is given in \[28\]. Within the quantum approach the neutrino spin flip under the influence of different types of interactions was also considered in \[30\].

In evaluation of the tensor $G_{\mu \nu}$ we demand that the neutrino evolution equation must be linear over the neutrino spin, electromagnetic field and such characteristics of matter (which is composed of different fermions, $f = e, n, p...$) as fermions currents

\[
j^\mu_f = (n_f, n_f \vec{v}_f),
\]

\[1\]Within this approach neutrino spin relaxation in stochastic electromagnetic fields without account for matter effects was considered in \[25\].
and fermions polarizations

\[ \lambda_f^\mu = \left( n_f(\vec{\zeta}_f \vec{v}_f), n_f \vec{\zeta}_f \sqrt{1 - v_f^2} + \frac{n_f \vec{v}_f(\vec{\zeta}_f \vec{v}_f)}{1 + \sqrt{1 - v_f^2}} \right). \] (6)

Here \( n_f, \vec{v}_f, \) and \( \vec{\zeta}_f (0 \leq |\vec{\zeta}_f|^2 \leq 1) \) denote, respectively, the number densities of the background fermions \( f \), the speeds of the reference frames in which the mean momenta of fermions \( f \) are zero, and the mean values of the polarization vectors of the background fermions \( f \) in the above mentioned reference frames. The mean value of the background fermion \( f \) polarization vector, \( \vec{\zeta}_f \), is determined by the two-step averaging of the fermion relativistic spin operator over the fermion quantum state in a given electromagnetic field and over the fermion statistical distribution density function.

Thus, in general case of neutrino interaction with different background fermions \( f \) we introduce for description of matter effects antisymmetric tensor

\[ G^{\mu\nu} = \varepsilon^{\mu\nu\rho\lambda} g^{(1)}_{\rho\lambda} u_\lambda - (g^{(2)}_{\mu\nu} u_\nu - u_\mu g^{(2)}_{\nu}), \] (7)

where

\[ g^{(1)}_{\mu} = \sum_f \rho_f^{(1)} j^\mu_f + \rho_f^{(2)} \lambda^\mu_f, \quad g^{(2)}_{\mu} = \sum_f \xi_f^{(1)} j^\mu_f + \xi_f^{(2)} \lambda^\mu_f, \] (8)

(summation is performed over the fermions \( f \) of the background). The explicit expressions for the coefficients \( \rho_f^{(1),(2)} \) and \( \xi_f^{(1),(2)} \) could be found if the particular model of neutrino interaction is chosen. For example, if one consider the electron neutrino propagation in moving and polarized gas of electrons within the extended standard model supplied with \( SU(2) \)-singlet right-handed neutrino \( \nu_R \), then the neutrino effective interaction Lagrangian reads

\[ L_{\text{eff}} = -f^\mu \left( \bar{\nu} \gamma^\mu \frac{1 + \gamma^5}{2} \nu \right), \] (9)

where

\[ f^\mu = \frac{G_F}{\sqrt{2}} \left( (1 + 4 \sin^2 \theta_W) j^\mu_e - \lambda^\mu_e \right). \] (10)

In this case the coefficients \( \rho_e^{(1),(2)} \) are

\[ \rho_e^{(1)} = \frac{\tilde{G}_F}{2\sqrt{2} \mu}, \quad \rho_e^{(2)} = -\frac{G_F}{2\sqrt{2} \mu}, \] (11)

where \( \tilde{G}_F = G_F(1 + 4 \sin^2 \theta_W) \).

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In the usual notations the antisymmetric tensor $G_{\mu\nu}$ can be written in the form,

$$G_{\mu\nu} = (-\vec{P}, \vec{M}),$$

where

$$\vec{M} = \gamma \{ (g_0^{(1)} \vec{\beta} - \vec{g}^{(1)}) - [\vec{\beta} \times \vec{g}^{(2)}] \}, \quad \vec{P} = -\gamma \{ (g_0^{(2)} \vec{\beta} - \vec{g}^{(2)}) + [\vec{\beta} \times \vec{g}^{(1)}] \}. \quad (13)$$

It worth to note that the substitution (14) implies that the magnetic $\vec{B}$ and electric $\vec{E}$ fields are shifted by the vectors $\vec{M}$ and $\vec{P}$, respectively:

$$\vec{B} \rightarrow \vec{B} + \vec{M}, \quad \vec{E} \rightarrow \vec{E} - \vec{P}. \quad (14)$$

We should like to emphasize here that precession of the neutrino spin can originate not only due to neutrino magnetic moment interaction with external electromagnetic fields but also due to the neutrino weak interaction with particles of the background matter. This is a very important point for understanding of the nature of the neutrino spin light in non-magnetized matter. In order to demonstrate how the neutrino spin procession appears in the background matter we consider below the neutrino spin evolution in matter in the absence of electromagnetic fields. We start with Eq.(3) and for simplicity neglect the neutrino electric dipole moment, $\epsilon = 0$. Then, in the absence of external electromagnetic field, we get the neutrino spin evolution equation in non-moving matter:

$$\frac{dS^\mu}{d\tau} = 2\mu \{ G^{\mu\nu} S_\nu - u^\mu (u_\nu G^{\nu\lambda} S_\lambda) \}. \quad (15)$$

The tensor $G_{\mu\nu}$ is given by Eqs.(7),(8) and (11). For the further simplifications we consider unpolarized ($\lambda_f = 0$) matter composed of only one type of fermions, so that there is now summation over $f$ in definition of $g^{(1)}_f$ and we shall omit the index $f$. Then we get

$$G^{\mu\nu} = \gamma \rho^{(1)} n \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\beta_3 & \beta_2 \\ 0 & \beta_3 & 0 & -\beta_1 \\ 0 & -\beta_2 & \beta_1 & 0 \end{pmatrix}, \quad (16)$$

where $\vec{\beta} = (\beta_1, \beta_2, \beta_3)$ is the neutrino three-dimensional speed. It is easy to show that

$$u_\nu G^{\nu\mu} = 0. \quad (17)$$
and from (15) we get the equation for the neutrino spin evolution in unpolarized and non-moving matter:

$$\frac{dS^\mu}{d\tau} = 2\mu G^{\mu\nu} S_\nu.$$  \hspace{1cm} (18)

In the laboratory reference frame the corresponding equation for the three-dimensional neutrino spin is

$$\frac{d\vec{S}}{dt} = 2\mu_\rho^{(1)} n [\vec{S} \times \vec{\beta}].$$  \hspace{1cm} (19)

If neutrino is propagating along the OZ axis, \(\vec{\beta} = (0, 0, \beta)\), then solutions of these equations for the neutrino spin components are given by

$$S^1 = S_0^\perp \cos \omega t, \quad S^2 = S_0^\perp \sin \omega t, \quad S^3 = S_0^3, \quad S^0 = S_0^0,$$

where

$$\omega = 2\mu_\rho^{(1)} n \beta,$$  \hspace{1cm} (21)

\(S_0^\perp\) and \(S_0^{3,0}\) are constants determined by the initial conditions.

From the above consideration it follows that if the initial neutrino state is not polarized longitudinally in respect to the neutrino momentum then the neutrino spin precession in the background matter always occurs. In its turn, a neutrino with processing magnetic momentum have to emit electromagnetic radiation. This is just the radiation which we have called the spin light of neutrino.

Recently we have derived the total radiation power of a neutral unpolarized fermion with anomalous magnetic moment \cite{31}. In that derivation we have supposed that the spin dynamics of a neutral particle is governed by the Bargmann-Michel-Telegdi equation and that the energy of the radiated photons is much less than the particle energy. Now in order to treat the electromagnetic radiation by the neutrino moving in background matter and electromagnetic field we use the substitution prescription of Eq.(4) and for the total radiation power get (for simplicity the case of \(T\)-invariant model of neutrino interaction is considered hereafter)

$$I = \frac{16}{3} \mu^2 \left[ 4(\mu^2 u_\rho E^\rho\lambda E_{\lambda\sigma} u^\sigma)^2 + \mu^2 u_\rho \mathcal{E}^{\rho\lambda} \mathcal{E}_{\lambda\sigma} u^\sigma \right].$$  \hspace{1cm} (22)

\footnote{The generalization to the case of neutrino with non-zero electric dipole moment in electromagnetic field can be found in \cite{32}}
We also get for the solid angle distribution of the radiation power:

\[
\frac{dI}{d\Omega} = \frac{\mu^2}{\pi \gamma (u\ell)^3} \left\{ \left[ 4(\mu^2 u_\mu \tilde{E}^{\rho\lambda} \tilde{E}_{\lambda\sigma} u_\sigma) + (\mu^2 u_\mu \dot{\tilde{E}}^{\rho\lambda} \dot{\tilde{E}}_{\lambda\sigma} u_\sigma) \right] (u\ell)^2 + 
\right.
\]

\[
+ 4(\mu^2 u_\mu \tilde{E}^{\rho\lambda} \tilde{E}_{\lambda\sigma} u_\sigma)(\mu u_\mu \tilde{E}^{\rho\lambda} l_\lambda)^2 + (\mu u_\mu \tilde{E}^{\rho\lambda} l_\lambda)^2 + 
\]

\[
+ 4\mu^2 (\mu u_\mu \tilde{E}^{\rho\lambda} l_\lambda)\epsilon^{\mu\nu\rho\lambda} u_\mu \tilde{E}_{\nu\sigma} u_\sigma \tilde{E}_{\rho\delta} l_\delta \right\},
\]

where \( \tilde{E}_{\mu\nu} = -\frac{1}{2} \epsilon_{\mu\nu\alpha\beta} E^{\alpha\beta} \) is the dual tensor \( E_{\mu\nu}, \) \( l^\mu = (1, \vec{l}) \) and \( \vec{l} \) is the unit vector pointing the direction of radiation. The derivatives in the right-hand side of Eqs. (22) and (23) are taken with respect of proper time \( \tau \) in the rest frame of the neutrino.

Using Eqs. (14) and (22) we find the total radiation power as a function of the magnetic field strength in the rest frame of the neutrino, \( \vec{B}_0, \) and the vector \( \vec{M}_0 \) which accounts for effects of the neutrino interaction with moving and polarized matter:

\[
I = \frac{16}{3} \mu^4 \left[ (2\mu (\vec{B}_0 + \vec{M}_0))^2 + (\vec{B}_0 + \vec{M}_0)^2 \right],
\]

where

\[
\vec{B}_0 = \gamma \left( \vec{B}_{\perp} + \frac{1}{\gamma} \vec{B}_{\parallel} + \sqrt{1 - \gamma^{-2}} \left[ \vec{E}_{\perp} \times \vec{n} \right] \right),
\]

\[
\vec{M}_0 = \vec{M}_{0\parallel} + \vec{M}_{0\perp},
\]

\[
\vec{M}_{0\parallel} = \gamma \beta \frac{n_0}{\sqrt{1 - v_e^2}} \left\{ \rho^{(1)} \left( 1 - \frac{\vec{v}_e \vec{\beta}}{1 - \gamma^{-2}} \right) - 
\right.
\]

\[
- \rho^{(2)} \left( \vec{\zeta}_e \vec{\beta} \sqrt{1 - v_e^2} + \frac{(\vec{\zeta}_e \vec{v}_e)(\vec{\zeta}_e \vec{v}_e)}{1 + \sqrt{1 - v_e^2}} \right) \frac{1}{1 - \gamma^{-2}} \right\},
\]

\[
\vec{M}_{0\perp} = -\frac{n_0}{\sqrt{1 - v_e^2}} \left\{ \vec{v}_e \left( \rho^{(1)} + \rho^{(2)} \left( \frac{(\vec{\zeta}_e \vec{v}_e)}{1 + \sqrt{1 - v_e^2}} \right) + \vec{\zeta}_e \rho^{(2)} \sqrt{1 - v_e^2} \right) \right\},
\]

and \( n_0 \) is the invariant number density of matter given in the reference frame for which the total speed of matter is zero.

It is evident that the total radiation power, Eq. (24), is composed of the three contributions,

\[
I = I_F + I_G + I_{FG},
\]

where \( I_F \) is the radiation power due to the neutrino magnetic moment interaction with the external electromagnetic field, \( I_G \) is the radiation power due to the neutrino weak
interaction with particles of the background matter, and $I_{FG}$ stands for the interference effect of electromagnetic and weak interactions. It should be pointed out that the electromagnetic contribution $I_F$ to the radiation of a neutrino (or a neutral fermion) in different field configurations has been considered in literature (see, for example, (23, 28, 29)), whereas the contribution to radiation by neutrino moving in matter, $I_G$, and also the interference term $I_{FG}$ have never been considered before.

These three types of neutrino radiation have common nature: they originate as effects of the neutrino interactions with the external electromagnetic field and background matter under which the neutrino spin is rotating. As it has been pointed out above, the discussed here radiation cannot be treated as the neutrino Cherenkov (10, 11, 14, 15, 16) and transition (12, 13) radiations. Contrary to the Cherenkov and transition radiations the considered new type of neutrino radiation is not forbidden when the refractive index of photons in the background environment is equal to $n_{\gamma}^{\text{ref}} = 1$. In order to highlight this distinction we consider here the particular case of $n_{\gamma}^{\text{ref}} = 1$, however generalization to the case $n_{\gamma}^{\text{ref}} > 1$ is straightforward.

One of the most important properties of the $SL\nu$ in the background matter is the strong dependence of the total radiation power on the density of matter,

$$I_G \sim n_0^4, \quad (30)$$

The total radiation power, Eq. (24), contains different terms in respect to dependence on the neutrino magnetic moment $\mu$. If we consider non-derivative terms and compare contributions to the neutrino spin light radiation power from the interaction with matter and electromagnetic field we come to the conclusion that, as it follows from Eq. (11), the ratio $I_G/I_F$ is proportional to $\mu^{-4}$. From Eq. (24) it is also obvious that the ratios of the radiation power in matter with varying density (in varying electromagnetic fields) to the radiation power in matter with constant density (in constant electromagnetic fields) are proportional to $\mu^2$. Accounting for smallness of the neutrino magnetic moment, it follows that the proposed new mechanism of neutrino spin light radiation could be efficient in the environments with varying densities of matter (with varying electromagnetic fields).

We consider at first the $SL\nu$ in presence of constant density matter and constant magnetic field. For definiteness we assume that the spin light radiation is produced by the electron neutrino $\nu_e$ moving in unpolarized ($\zeta_e = 0$) matter composed of only
electrons. Then for the total spin light radiation power we get

\[ I = \frac{64}{3} \mu^6 \gamma^4 \left[ \left( \vec{B}_{\perp} + \frac{1}{\gamma} \vec{B}_{\parallel} + n^{(1)} \vec{\beta} (1 - \vec{\beta} \vec{v}_e) - \frac{1}{\gamma} n \rho^{(1)} \vec{v}_e \right)^2 \right], \]  

(31)

where the terms proportional to \( \gamma^{-2} \) in the brackets are neglected. Here we use the notation,

\[ n = \frac{n_0}{\sqrt{1 - \nu_e^2}}, \]  

(32)

\( \vec{B}_{\perp,\parallel} \) are the transversal and longitudinal magnetic field components in respect to the neutrino motion, \( \vec{v}_e \) is the transversal component of the matter speed in the laboratory frame of reference. From Eq. (31) it follows that the correlation term \( I_{FG} \) is suppressed in respect to the terms \( I_F \) and \( I_G \) by the presence of additional neutrino Lorentz factor. That is why there is a reason to compare contributions to the radiation power produced by the neutrino interaction with matter which is moving longitudinally with non-relativistic speed, \( \nu \ll 1 \),

\[ I_G = \frac{64}{3} \mu^6 \gamma^4 \left( n \rho^{(1)} \vec{\beta} \right)^4, \]  

(33)

and the contribution to the radiation power produced by the interaction with the transversal magnetic field\(^3\),

\[ I_F = \frac{64}{3} \mu^6 \gamma^4 B_{\perp}^4. \]  

(34)

Note that from Eq. (33) it follows that in the case of the standard model, when the neutrino magnetic moment is proportional to its mass, and constant neutrino speed the radiation power goes with the neutrino mass squared, as one might expect. If we take \( B_{\perp} \sim 3 \times 10^5 \) G and \( n \sim 10^{23} \) cm\(^{-3} \) that corresponds to the case of the solar convective zone and \( \mu_{\nu_e} \sim 10^{-18} \mu_0 \), then we get that the matter term in the spin light exceeds the transversal magnetic field term by a factor of \( \sim 2 \times 10^{26} \). With increasing of the neutrino magnetic moment due to the inverse proportionality \( \rho^{(1)} \sim \mu_{\nu_e}^{-1} \), the ratio \( I_G/I_F \) decreases and becomes equal to unit only for \( \mu_{\nu_e} \sim 3 \times 10^{-12} \mu_0 \).

Let us turn to consideration of the contribution to the \( SL\nu \) that is generated by the neutrino interaction with matter of varying density. We again assume that matter is composed of electrons and neutrino interaction with matter is given by Lagrangian of Eq. (9). Since \( \dot{\nu}_e = \gamma \vec{\beta} \vec{\nabla} n_e \), we get for the total power of the neutrino spin light in this case

\[ I_G = \frac{2}{3} \mu^2 \gamma^4 \left\{ \frac{1}{2} \vec{G}_F n_e^4 + \vec{G}_F^2 \left[ (\vec{\beta} \vec{\nabla}) n_e \right]^2 \right\}. \]  

(35)

\(^3\)This our result is smaller by a factor 1/2 relative to the result of \(^2\) for the radiation power of the polarized neutral particle.
If we consider the case of moving and polarized matter then the effective number density depends on the value of the total matter speed $\vec{v}_e$ and polarization $\vec{\zeta}_e$, as well as on neutrino speed $\vec{\beta}$ and correlations between these three vectors (see [27, 33]):

$$n_e = \frac{n_0}{\sqrt{1 - v_e^2}} \left\{ (1 - (1 + 4 \sin^2 \theta_W)^{-1} \vec{\zeta}_e \vec{v}_e) (1 - \vec{\beta} \vec{v}_e) - (1 + 4 \sin^2 \theta_W)^{-1} \sqrt{1 - v_e^2} \left[ (\vec{\zeta}_e \vec{v}_e)(\vec{\beta} \vec{v}_e) - \vec{\zeta}_e \vec{\beta} \right] + O \left( \frac{1}{\gamma} \right) \right\}.$$ (36)

Using Eqs. (23) and (35) for the solid angle distribution we get

$$\frac{dI_G}{d\Omega} = \frac{3}{8\pi} I_G \left\{ \gamma^{-4}(1 - \beta \cos \vartheta)^{-3} - \gamma^{-6}(1 - \beta \cos \vartheta)^{-4} + \frac{1}{2} \gamma^{-8}(1 - \beta \cos \vartheta)^{-5} \right\}. \quad (37)$$

From the last formula it follows that the $SL\nu$ is strongly beamed and is confined within the cone given by $\delta \theta \sim \gamma^{-1}$. It should be noted here that this is a common feature of different contributions to the neutrino spin light radiation which is the inherent property of radiation by ultra-relativistic particles.

In conclusion, we have discovered a new mechanism of the electromagnetic radiation emitted by a neutrino with non-vanishing magnetic moment moving in the background matter and external electromagnetic field ("spin light of neutrino"). The generalization to the case of a neutrino with non-zero electric dipole moment is just straightforward. Our general new result is the prediction of a new type of electromagnetic radiation that is emitted by a massive neutral particle with non-vanishing magnetic (and/or electric) moment moving in the background matter. The $SL\nu$ originates from the quantum spin flip transitions. Therefore, the initially unpolarized neutrino beam can be converted to the totally polarized beam composed of only right-handed neutrinos $\nu_R$ in the considered radiation process if the right-handed neutrinos are sterile states, i.e. do not interact with the background environment.

We have also developed the quasi-classical theory of the $SL\nu$ that is valid in the case when the neutrino recoil can be neglected or when the energy of the radiated photon is less than the neutrino energy.

The considered radiation must be important for environments with high effective densities, $n$, because the total radiation power is proportional to $n^4$. It is also shown that the $SL\nu$ is strongly beamed in the direction of neutrino propagation and is confined within a small cone given by $\delta \theta \sim \gamma^{-1}$. The total power of the $SL\nu$ is increasing with the neutrino energy increase and is proportional to the fourth power of the neutrino Lorentz factor, $I \sim \gamma^4$. It is also possible to show that the average energy of photons
of the spin light in matter is

$$\omega_{\gamma} \sim G_F n_e \gamma^2.$$  \hfill (38)

Thus, the spin light emission rate in this case is proportional to $\gamma^2$,

$$\Gamma_{SL} = \frac{\sqrt{2}}{3} \gamma^2 \mu^2 G_F^2 n^3.$$  \hfill (39)

From these estimations we predict that the $SL\nu$ is effectively produced if the neutrino energy and matter density are large. Such a situation can be realized in the dense matter of the early Universe.

It is interesting to compare the rate of the $SL\nu$ in matter with the rate of the Cherenkov radiation in magnetic field that is widely discussed in literature (see, for example, [16]). First of all, in contrast to the Cherenkov and transition radiations by neutrino, the spin light is produced by neutrino even in the case when the refractive index of photons in the background matter is equal to $n_{\gamma}^{\text{ref}} = 1$. Let us compare the rate of the neutrino spin light, Eq.(39), with the rate of the Cherenkov radiation in magnetic field given by formula (25) of Ref. [16]. If the magnetic field is less than the electron critical field, $B < B_0 = 4.41 \times 10^{13} G$, the ratio of the two rates is

$$\frac{\Gamma_{SL}}{\Gamma_{Ch}} = 3.4 \times 10^8 \frac{\gamma^2 \mu^2 G_F^2 n^3}{p_0^5} \left(\frac{B}{B_0}\right)^6,$$  \hfill (40)

where $p_0$ is the neutrino energy. This ratio is equal to

$$\frac{\Gamma_{SL}}{\Gamma_{Ch}} = 13$$  \hfill (41)

for the two sets of neutrino energy, strength of magnetic field and density of matter:

1) $p_0 = 10 \text{ MeV}, B = 10^{-4} B_0, n = 10^{30} \text{ cm}^{-3}$,

2) $p_0 = 1 \text{ MeV}, B = 10^{-3} B_0, n = 10^{31} \text{ cm}^{-3}$.

In both cases we take the neutrino mass equal to $m_\nu = 1 \text{ eV}$ and magnetic moment equal to $\mu = 0.3 \times 10^{-10} \mu_0$ that is near the present experimental limit for the electron neutrino. The neutrino spin light rate also dominates over the Cherenkov radiation rate in strong magnetic field ($B \geq B_0$) if the matter density is increased to the level of $n \geq 10^{33} \text{ cm}^{-3}$. However, for such densities, neutrino magnetic moment and energies in the range span from $1 \text{ MeV}$ to $10 \text{ MeV}$ the quantum approach [34] to the $SL\nu$ has to be used. If we consider the case of much higher density, $n = 10^{37} \text{ cm}^{-3}$, and smaller magnetic moment, $\mu = 0.3 \times 10^{-16} \mu_0$, and neutrino energy $p_0 = 1 \text{ MeV}$ then
the quasi-classical approach to the spin light is valid and the ratio of the two rates is given again by Eq. (41). Thus, we predict that the spin light of neutrino can be more effective than the neutrino Cherenkov radiation if the density of matter is rather large.

The spin light of neutrino together with the synchrotron mechanism of radiation by charged particles, should be important for understanding of astrophysical phenomena where powerful beams of gamma-rays are produced. One of the possible examples could be the gamma-ray bursts.

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