RADIATIVE EFFICIENCIES OF CONTINUOUSLY POWERED BLAST WAVES

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ABSTRACT

We use general arguments to show that a continuously powered radiative blast wave can behave self-similarly if the energy injection and radiation mechanisms are self-similar. In that case, the power-law indices of the blast wave evolution are set by only one of the two constituent physical mechanisms. If the luminosity of the energy source drops fast enough, the radiation mechanisms set the power-law indices; otherwise, they are set by the behavior of the energy source itself. We obtain self-similar solutions for the Newtonian and the ultrarelativistic limits. Both limits behave self-similarly if we assume that the central source supplies energy in the form of a hot wind and that the radiative mechanism is the semiradiative mechanism of Cohen, Piran, and Sari. We calculate the instantaneous radiative efficiencies for both limits and find that a relativistic blast wave has a higher efficiency than a Newtonian one. The instantaneous radiative efficiency depends strongly on the hydrodynamics and cannot be approximated by an estimate of local microscopic radiative efficiencies, since a fraction of the injected energy is deposited in shocked matter. These solutions can be used to calculate gamma-ray-burst afterglows for cases in which the energy is not supplied instantaneously.

Subject headings: gamma rays: bursts — hydrodynamics — relativity — shock waves

1. INTRODUCTION

Afterglows from gamma-ray bursts (GRBs) have been discovered for 13 GRBs since the first detection of the afterglow of GRB 970228 (e.g., in 't Zand et al. 1998). The simplest cosmological fireball afterglow model (Paczynski & Rhoads 1993; Katz 1994; Mészáros & Rees 1997; Waxman 1997; Mészáros, Rees, & Wijers 1997; Piran 1998) seems to be in a good general agreement with the observed behavior (Wijers, Rees, & Mészáros 1997) or at least with the power-law decay. However, several works have tried to investigate more subtle effects that can change the afterglow characteristics. In particular, Rees & Mészáros (1998) and Panaitescu, Mészáros, & Rees (1998) have tried to explain the deviations from an ideal power law and the variety of light curves using a model in which a source emits shells with different Lorentz factors, which results in a gradual energy supply to the shell of shocked matter. In their work they neglected the thickness of the shell of shocked matter, and they did not calculate its structure. However, as evident from the classical treatment of adiabatic blast waves by Blandford & McKee (1976), the evolution of a blast wave depends strongly on the detailed structure of the matter inside it. Therefore, it remains to investigate the complete hydrodynamics and energy budget of a slow power-law decay blast wave.

The dynamics of blast waves with gradual energy supply is also relevant for the study of compact steep-spectrum objects (CSS), Gigahertz peak spectrum objects (GPS), and active galactic nuclei (AGN) radio lobes. The generally accepted model for these objects (see, e.g., Scheuer 1974; Begelman & Cioffi 1989; Bicknell, Dopita, & O'Dea 1997) describes a blast wave continuously powered by a jet from an AGN. Similar models have also been used for interstellar bubbles (Castor, McCray, & Weaver 1975; Weaver et al. 1977) for plerionic supernova remnants (see, e.g., Weiler 1983) and for galactic supershells (McCray 1987).

In this paper we consider blast waves with a gradual energy supply by a hot wind. Specifically, we obtain a new self-similar solution for a continuously powered radiative blast waves. Even if this situation may not be directly applicable to astrophysical objects, it is still important. Self-similar solutions are simple enough that they may be solved analytically (sometimes), and they are easy to grasp. Furthermore, it is likely that a generic blast wave will have tendency to approach self-similarity.

Adiabatic self-similar blast waves with energy injection have been treated previously by Blandford & McKee (1976) in the relativistic regime. In the Newtonian regime Castor et al. (1975) and Ostriker & McKee (1988) have investigated both radiative and adiabatic blast waves with energy injection, but they have not treated the case of fast decaying sources, which did not fit into their self-similar framework. In this paper we show that the inclusion of radiative losses has a qualitative influence on the solutions. Using our new solutions we obtain an accurate result for the fully radiative, steady injected blast waves, which were treated approximately by Blandford & McKee (1976). We add new solutions for a region of the parameter space that has not been treated yet either in the Newtonian or in the ultrarelativistic limits.

Self-similarity appears only if the physical situation can be characterized by a minimal number of dimensional parameters in such a way that physical scales can be constructed only from a single combination of those parameters. Therefore, by assuming that a blast wave behaves self-similarly, we can deduce the functional form of different physical processes. In fact, adiabatic blast waves already exhibit self-similar behavior without additional parameters. This requires that energy injection and radiation processes in self-similar blast waves will not have intrinsic scales. Specifically, the injection rate must be a power law.

The radiation process should also be self-similar. In order to satisfy that demand, we assume that the radiation process results with a semiradiative scenario where the cooling mechanisms are fast comparable to hydrodynamic timescales ("fast cooling"), but only a fraction of the energy produced by the shock is radiated away. This would take place, naturally, in any collisionless shock acceleration (see
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Fig. 1.—Schematic drawing of the regions in an energy injected blast wave. The regions are injected hot wind (a), shocked wind (b), shocked ISM (c), and ambient ISM (d). The boundaries between the regions are a reverse shock at $R_{rs}$, a contact discontinuity at $R_{cd}$, and a forward shock at $R_{sh}$. The central source ejects massless hot wind with a luminosity of $L_{in}$. Energy is transferred at a rate $L_{in}$ to the shocked matter and at $L_{cd}$ to the shocked ISM. The blast wave emits radiation with a luminosity from the shock front. In the Newtonian solution $E$ denotes the energy stored in shocked ISM, where in the ultrarelativistic limit $E$ includes the energy stored in shocked wind as well. Region (a) is negligible in the Newtonian regime due to the infinite sound velocity of the wind and $L_{rs} = L_{in}$.

Cohen, Piran & Sari 1998, hereafter CPS98). For example, in a GRB afterglow the cooling timescales are shorter than the hydrodynamic timescale (see, e.g., Waxman 1997; Mészáros et al. 1997; Sari, Piran, & Narayan 1998), and a fraction of the energy stays with shocked protons, which do not cool.

We describe our model in § 2. It is composed of a central source emitting a wind that interacts with a surrounding medium and creates a blast wave. We proceed in § 3 by calculating the blast wave energy and the radiated luminosity assuming self-similarity alone. In §§ 4 and 5 we split the discussion to the Newtonian and ultrarelativistic limits, assuming the self-similar radiation mechanism of CPS98. For each case we obtain the hydrodynamic solution and the radiative efficiencies (analytic solution for the ultrarelativistic limit and numerical ones for the Newtonian limit). Finally, we summarize our results in § 6.

2. THE MODEL

We consider a spherical semiradiative blast wave that appears when energy is released continuously into an ambient medium. This results in a strong shock wave that expands supersonically. We consider the regime where the influence of the injected and initial mass is negligible and the pressure of the surrounding medium is small compared to the energy density of the flow. These assumptions are necessary in order to obtain a self-similar solution (see, e.g., Landau & Lifshitz, their § 99).

We assume that the source supplies energy in the form of a hot wind with a negligible mass. This wind pushes away the surrounding medium and creates a cavity. This simple model leads to different qualitative behaviors, depending on the expansion velocity of the blast wave. If this velocity is much lower than the speed of light (Newtonian limit), the sound velocity of the ejected wind can be much higher than the expansion velocity. In this case the pressure inside the cavity settles fast and becomes isobaric. This isobaric bubble pushes away the ambient matter and creates a shock wave. (This model, where the injected wind is treated as an isobaric interior, has been widely used in the study of radio lobes in AGNs; Scheuer 1974).

In the ultrarelativistic case the hot wind has different dynamics. The sound velocity of the wind is of the order of the speed of light, but it is always comparable to the expansion velocity. In this case we assume that the wind is so sparse that its particles inside the cavity do not interact and simply move in straight lines, close to the speed of light, until they reach the edge of the cavity. Near this edge they undergo a strong shock, presumably a collisionless one, which compresses them. Consequently, this compressed
fluid produces a shock that advances into the interstellar medium (ISM).

Both in the Newtonian and in the ultrarelativistic limits, the blast wave is composed of four regions: (1) The injected wind, which contains wind that did not yet interact with the blast wave. In the Newtonian limit the reverse shock reaches the center, and this region disappears. (2) The shocked wind, which contains wind that was compressed because of the interaction with shocked ISM. It is separated from region 1 by a reverse shock. (3) The shocked ISM, which contains ISM that has been compressed by the forward shock at the blast wave front. A contact discontinuity separates this region from the shocked wind region. (4) The ISM at rest. A schematic view of the four regions appears in Figure 1.

3. SELF-SIMILARITY IN ENERGY INJECTION CASES

We look for self-similar solutions for semiradiative blast waves with central energy supply. If a blast wave contains only one of these two mechanisms (energy injection and radiation), each of them leads to a different temporal behavior. Because self-similarity does not allow the existence of several timescales in the solution, it is not known a priori whether such solutions exist.

Self-similarity requires a dimensionless energy injection mechanism. We therefore deduce that the energy is supplied to any interval along the self-similar solution at a rate \( L \propto t^\sigma \). The energy transfer rates along the self-similar hydrodynamic profile (especially \( L_{\text{rs}} \) and \( L_{\text{cd}} \); see Fig. 1) scale the same and differ only by a constant factor, which depends on the exact solution. However, this scaling can differ from \( L_{\text{int}} \) the energy supplied by the central source, as a fraction of the injected energy resides in the interior regions. Region (a) contains a freely expanding wind. If it exists, it does not depend on the blast wave parameters. For self-similarity the energy transfer rate from region (a) to region (b)–(c) must satisfy the self-similarity scaling laws. However, it is possible that the energy in region (a) will evolve differently.

Assuming that a self-similar solution exists, we can write the energy loss rate as

\[
L_{\text{rad}} = -\kappa(E(t)), \quad \kappa > 0,
\]

where \( E \) is the energy stored in a fixed interval of the self-similar profile. In the Newtonian limit we choose \( E \) to denote the energy stored in shocked ISM alone (and the corresponding injection rate is \( L_{\text{cd}} \)), and in the ultrarelativistic case \( E \) denotes the energy stored in the entire shocked matter (and the corresponding injection rate is \( L_{\text{rs}} \)). The value of the constant \( \kappa \) depends on this definition, but it is always dimensionless. This proportionality is evident from self-similarity, because there exists only one way to construct the dimensionality of \( L_{\text{rad}} \) (energy/time) using the characteristic parameters. Note that this argument holds for any self-similar radiation mechanism.

The energy conservation equation includes losses as well as injection from the blast wave center,

\[
\frac{dE}{dt} = L(t/t_0)^\sigma - \kappa(E/t).
\]

(2)

For \( \sigma \neq -1 - \kappa \) equation (2) has an analytic solution

\[
E = \frac{L}{\kappa + \sigma + 1} \left( \frac{t}{t_L} \right)^\sigma t + At^{-\kappa},
\]

(3)

where \( A \) is set by the initial conditions.

We proceed by investigating the asymptotic behavior of this equation for different values of \( \sigma \) and \( \kappa \). In cases where the equations behaves self-similarly, we find the power-law index of the energy temporal behavior \( E \propto t^\sigma \) and obtain the relations between the various power-law indices. A summary of the different limits appears in Table 1.

For \( L = 0 \), we reproduce the instantaneous energy injection case, and equation (3) trivially becomes a power law with \( \lambda = -\kappa \). Even in this simple case, an extrapolation toward \( t \to 0 \) results in an infinite energy. From equation (1) it is obvious that the radiated energy also diverges in this limit. These two infinities cancel each other, and at finite times the energy is always finite. A specific blast wave with an arbitrary initial energy evolves to this self-similar behavior at late times. Clearly the limit \( t \to 0 \) has no physical relevance, and only the late time behavior is interesting. It is therefore better to treat the solution from \( t = \infty \) backward in time, since this is the region where we expect the blast wave to evolve according to the self-similar solution. At any finite time all the energies are finite, and we encounter no infinities using this view. We discuss \( E_{\text{rad}}(t) \), which is the energy that will be radiated from time \( t \) to infinity. It is evident that \( E_{\text{rad}}(t) = E(t) \), because the energy stored in the shell decreases to zero when \( t \to \infty \).

Sources that scale with \( \sigma < -1 \) have not been treated so far. In this scenario the injected and radiated energies from \( t = 0 \) are infinite, similarly to the \( L = 0 \) case. This means that the solution is not physical near \( t = 0 \). As in the \( L = 0 \) case, we overcome the infinities by discussing \( E_{\text{rad}}(t) \) (the energy the will be injected from time \( t \) to infinity) instead of

| Condition | \( E_{\text{stored}} \) | \( E_{\text{rad}}/E_{\text{stored}} \) | \( E_{\text{rad}}/E_{\text{stored}} \) | \( E_{\text{rad}}/E_{\text{stored}} \) | \( E_{\text{rad}}/E_{\text{stored}} \) | \( E_{\text{rad}}/E_{\text{stored}} \) |
|-----------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| \( \sigma > -1 \) | \( \frac{L}{\kappa + \sigma + 1} \left( \frac{t}{t_L} \right)^\sigma t \) | \( \kappa \) | \( \infty \) | \( \frac{\kappa}{\sigma + 1} + 1 \) | \( \infty \) |
| \( \sigma = -1 \) | \( \frac{L}{\kappa + \sigma + 1} t \) | \( \infty \) | \( \infty \) | \( \infty \) | \( \infty \) |
| \( -1 > \sigma > -1 - \kappa \) | \( \frac{L}{\kappa + \sigma + 1} \left( \frac{t}{t_L} \right)^\sigma t \) | \( \kappa \) | \( \frac{1}{\sigma + 1} \) | \( \infty \) | \( \frac{\kappa}{|\sigma + 1|} - 1 \) |
| \( \sigma = -1 - \kappa \) | \( \cdots \) | \( \cdots \) | \( \cdots \) | \( \cdots \) | \( \cdots \) |
| \( -1 - \kappa > \sigma \) | \( t \) | \( \infty \) | \( 1 \) | \( \infty \) | \( 0 \) |
| \( L = 0 \) | \( E(t) \) | \( \infty \) | \( 1 \) | \( 0 \) | \( 0 \) |
$E_i^0(t)$ (the energy injected from $t = 0$ to $t$). The solutions are divided into three subclasses, depending on the relation between $\sigma$ and $\kappa$:

1. If $\sigma > -1 - \kappa$ the injected energy decreases slower than the rate of an uninjected blast wave, and the power laws are set by the injection law. The energies injected and radiated from $t = 0$ are infinite. However, the energy that will be radiated until infinity is always larger than the stored energy, and the energy that will be injected is larger (smaller) than the stored energy if $\sigma$ is larger (smaller) than $-1 - \kappa/2$. Note that even if the stored energy is larger than the energy injected until infinity, the evolution is set by the injected energy power law. In both cases $\lambda = \sigma + 1$.

2. If $\sigma = -1 - \kappa$, then equation (3) is not valid. The solution of equation (2) for this case is

$$E_{shell} = \left(\frac{t}{t_i}\right)^{-\kappa} \left[ A + Lt_t \log \left(\frac{t}{t_i}\right) \right].$$

This equation has no power-law asymptotics, and there is no self-similar solution in this case.

3. In cases where $\sigma < -1 - \kappa$, the late time behavior is identical to the instantaneous injection case, and the asymptotic of equation (3) is a power law with $\lambda = -\kappa$. Note that in all the subcases with $\sigma < -1$ the stored energy drops to zero with time.

Sources with $\sigma > -1$ were treated by Ostriker & McKee (1988). In this case $E_i^0(t)$, the energy injected from $t = 0$, is finite for every $t$, but it diverges as $t \to \infty$. This divergence allows us to neglect any initial energy in the blast wave. Equation (3) again has a power-law asymptotic, now with $\lambda = \sigma + 1$. The energy stored in the shocked matter is a constant fraction of the energy injected so far, and the same holds for the radiated energy. The energy injected is always larger than the energy stored in the shocked matter, and the radiated energy from $t = 0$ is larger (smaller) than the stored energy depending if $\kappa$ is larger (smaller) than $\sigma + 1$.

If the energy is supplied with $\sigma = -1$, the integral of injected energy is logarithmic, and it diverges in both limits of $t$. However, the asymptotic of equation (3) is again a power law, now with $\lambda = 0$; i.e., the shell holds a constant energy, which is set by initial conditions.

The study of the energy conservation equations revealed the temporal behavior of the energy if the solution is self-similar. However, $\kappa$ cannot be determined by self-similarity arguments alone. It is generally determined only by solving the complete hydrodynamic equations. We split the discussion of this solution to the Newtonian and the ultrarelativistic limits.

4. THE NEWTONIAN SOLUTION

To solve the hydrodynamic equations we use our model (§ 2), where the energy from the central source is supplied by an internal fluid with a high temperature and a low density, in the limit of infinite sound velocity. The large sound speed of the injected matter prevents the existence of an interior shock, and the flow contains only three regions (see Fig. 1): (b) an isobaric interior composed of shocked wind, (c) the shocked ISM, and (d) the ambient ISM. The hot internal fluid pushes the shocked matter at the contact discontinuity and accelerates it. The shocked matter accretes mass from the ambient medium and heats it at the shock front. The heated matter emits a fraction of its internal energy near the shock front.

If the evolution is governed by the injection mechanism, the blast wave is characterized by its instantaneous radiative efficiency, which is the ratio between emitted energy (at the shock front) and the energy released at the center, at the same time. Because of the structure of the blast wave (the three regions), it is helpful to divide the energy transfer into two stages: (1) energy transfer from the wind to the shocked matter and (2) energy transfer from the shocked matter to radiation.

Both stages show instantaneous efficiencies that are larger than unity if the source luminosity drops fast enough. This is caused by the delay between the time energy is supplied to the shocked matter and the time this energy is radiated. For example, if the source shuts down spontaneously it emits no energy, but energy is still radiated from the hot shocked matter. This results in an infinite instantaneous efficiency.

During the expansion, the hot internal fluid [region (b)] performs work on the shocked matter [region (c)], $L_{cd} = 4\pi r_{cd}^2 p_{cd} v_{cd}$, where $r_{cd}$, $p_{cd}$, and $v_{cd}$ are, respectively, the radius, pressure, and velocity of the contact discontinuity. Meanwhile, the internal fluid absorbs energy from the internal source which emits energy at a rate

$$L_{in} = \frac{4\pi}{3} \left[ \frac{d[R_{cd}^3 (\tilde{\gamma} - 1)]}{dt} + p_{cd} \frac{dr_{cd}^3}{dt} \right] .$$

(5)

Prior to solving the self-similar hydrodynamics, we use the self-similar relations to calculate the efficiency $L_{cd}/L_{in}$. A Newtonian blast wave must evolve according to the self-similar variable,

$$\tilde{\xi} = \frac{r}{[t^2 + 4(E_0/\tilde{\sigma})]^{1/2}} \frac{\rho_1}{R_{sh}} .$$

(6)

(see, e.g., Landau & Lifshitz, their § 106), where $\rho_1$ is the density of the surrounding medium, and $R_{sh}$ is the radius of the shell. The self-similar quantities, which turn the hydrodynamic equations into coupled ordinary differential equations (ODEs; see, e.g., Landau & Lifshitz, their § 106) are

$$\rho(r, t) = \frac{\tilde{\gamma} + 1}{\tilde{\gamma} - 1} \rho_1 \tilde{\rho}(\tilde{\xi}) ,$$

$$u(r, t) = \frac{2}{\tilde{\gamma} + 1} U_{sh} \frac{r}{R_{sh}(t)} \nu(\tilde{\xi}) ,$$

$$p(r, t) = \frac{2}{\tilde{\gamma} + 1} \rho_1 U_{sh}^2 \frac{r}{R_{cd}(t)} \nu(\tilde{\xi})^2 .$$

(7)

Keeping in mind that the ratio $r_{cd}/R_{sh}$ is constant (again because of self-similarity), we substitute $\rho(r, t)$ into equation (5) and use equation (6) to obtain

$$\eta_{in} = \frac{L_{cd}}{L_{in}} = \frac{3(\tilde{\gamma} - 1)(\tilde{\lambda} + 2)}{2(\tilde{\lambda} + 3) + 3(\tilde{\lambda} + 2))} .$$

(8)

For $\lambda > 0$ (which corresponds to energy injection cases with $\sigma > -1$), $\eta_{in} < 1$, as expected. However, for $\lambda < 0$, $\eta_{in} > 1$, because the radiated energy is mainly supported by energy that is already in the shocked matter. For $\lambda = -6(\tilde{\gamma} - 1)/(2 + 3\tilde{\gamma})$, $\eta_{in}$ is infinite. This corresponds to the Ostriker & McKee (1988) solution of an adiabatic interior with no internal energy supply that pushes a fully radiative shell.
This efficiency reveals the ratio between the injected energy $L_{in}$ and $L_{cd}$, the rate at which energy is supplied to the shocked ISM. To obtain the instantaneous global efficiency, we use equation (3) and find that $\eta_{rad} = L_{rad}/L_{cd} = \kappa/(\kappa + \sigma + 1)$.

To obtain $\kappa$ we need the full hydrodynamic solution. We use the energy loss rate of the semiradiative model

$$L_{rad} = -2\pi \rho_1 U_{sh}^3 R_{sh}^2 \epsilon$$

(CPS98), substitute the self-similar relations of equation (6), and obtain

$$\kappa = -2\pi \left(\frac{2 + \lambda}{5}\right)^3 \xi_0^3 \epsilon.$$  

(10)

The instantaneous efficiency is then

$$\eta_{tot} = \frac{L_{rad}}{L_{in}} = \frac{3(\hat{\gamma} - 1)(\lambda + 2)/[2(\lambda - 3) + 3\hat{\gamma}(\lambda + 2)]}{1 + 125(1 + \sigma)/[(3 + \sigma)^2 p \xi_0^3 \epsilon]}$$

(11)

where $\xi_0$ is the self-similar position of the shock.

To obtain $\xi_0$ and the instantaneous efficiency, we solve the self-similar equations. The solution is obtained by integrating numerically the self-similar hydrodynamic ODEs from the shock down to the contact discontinuity (where the fluid does not move on the self-similar profile). The boundary conditions for this integration are the semiradiative shock conditions

$$\alpha(\xi_0) = \frac{1}{1 - \delta}, \quad \nu(\xi_0) = 1 + \frac{\hat{\gamma} - 1}{\delta}, \quad \rho(\xi_0) = 1 + \frac{\hat{\gamma} - 1}{2 \delta}$$

(12)

(CPS98), where $\delta$ is related to the radiative efficiency by

$$\epsilon(\delta) = \frac{\delta}{1 + \hat{\gamma}} [2 + (\hat{\gamma} - 1)\delta].$$

(13)

We find $\xi_0$ using the normalization condition, which equates the energy defined in the self-similar parameter (eq. [6]) to the energy stored in the shocked matter:

$$E(t) = \rho_1 R_{sh}^3 U_{sh}^2 \frac{8\pi}{\xi_0^2 (\hat{\gamma}^2 - 1)} \int_{\xi_{cd}}^{\xi_0} [p(\xi) + \alpha(\xi)\nu(\xi)^2] \xi^4 d\xi.$$  

(14)

If the evolution is governed by the radiation mechanism, then $\lambda$ is not known prior to solving the hydrodynamics. In that case, in addition to equation (14) we use $E = -L_{rad}$ and obtain both $\lambda$ and $\xi_0$ simultaneously.

We present the hydrodynamic solutions for several cases in Figures 2–4 and in Table 2. In all cases where the evolution is set by the energy injection mechanism, a contact discontinuity appears in the solution. If the injected energy decreases fast enough with time, the shocked ISM profile is similar to the one found in instantaneously injected blast waves. The temperature rises monotonously toward the center, and it diverges at the contact discontinuity. The velocity is monotonous, and the fluid near the shock front has the maximal velocity. At the contact discontinuity the fluid is, at rest, relative to the profile. The density is also monotonous, and it reaches the contact discontinuity with a zero gradient. This results in a smooth transition between the shocked wind (which composes the internal isobaric

**TABLE 2**

| $\lambda$ | $\sigma$ | $\epsilon$ | $1 - \xi_{cd}/\xi_0$ | $\eta_{in}$ | $\eta_{tot}$ |
|----------|---------|-------------|---------------------|------------|-------------|
| 4        | 3       | 0           | 0.08                | ...        | ...         |
| 1        | 0       | 0.5         | 0.05                | 6/11       | 0.34        |
| $\frac{2}{3}$ | $\frac{1}{2}$ | 0 | 0.20 | ... | ... |
| $\frac{7}{2}$ | $\frac{2}{3}$ | 0.10 | $\frac{1}{2}$ | 0.23 |
| $\frac{2}{5}$ | $\frac{2}{3}$ | 0.62 | 14/9 | 8.35 |
| $\frac{1}{7}$ | $\frac{1}{2}$ | 0 | 1 | $\infty$ |
bubble) and the shocked ISM. Effectively, there is no discontinuity at the contact discontinuity.

The boundary between the shocked ISM and the internal hot isobaric bubble depends on the energy injection rate. As the injected energy becomes more dominant, i.e., it is injected with higher \( \sigma \), the matter tends to concentrate in a narrower shell near the front shock. If \( \sigma < (8 - 9\dot{\gamma})/(4 + 3\dot{\gamma}) \), the density gradient still vanishes at the contact discontinuity, which contains no jumps. For \( \sigma \) larger than this value, the density gradient is infinite, and the shocked matter region ends abruptly at the contact discontinuity. If \( \sigma > 2 \), the density itself also diverges to infinity. Despite these infinities the total mass and energy stored in the self-similar solution are finite, and the solution is a proper one. Increasing \( \sigma \) also changes the velocity profile considerably, since for large enough \( \sigma \) the maximal fluid velocity is not at the shock front but at the contact discontinuity.

Inclusion of a radiative mechanism decreases the shell width, with the limit of zero width if \( \epsilon \rightarrow 1 \). The qualitative effect of radiation is mainly observed in cases where \( \sigma < -1 \). There the solution is identical to the evolution of a blast wave with instantaneous energy release, unless the radiative efficiency is high enough. If \( \kappa > 1 + \sigma \), the evolution is set by the injection mechanism, and the blast wave has the same structure as ones with \( \sigma > -1 \).

The instantaneous efficiency as a function of \( \sigma \) is depicted in Figure 5. The efficiency is larger than unity if \( \sigma < -1 \) as the main source for radiation is the hot shocked matter that was heated at earlier times and not the energy that is currently being supplied. If \( \sigma > -1 \), the main contribution to the shocked matter energy is the recently released wind. The radiated energy is therefore directly connected to the wind released at that time, and the efficiency is lower than unity. It appears that the hydrodynamics are an important sink for injected energy. For example, a steady source with a semiradiative mechanism of \( \epsilon = 0.5 \) results in an overall efficiency of 0.34 if \( \dot{\gamma} = 5/3 \) and 0.19 is \( \dot{\gamma} = 4/3 \).

The limit of \( \epsilon = 1 \) cannot be calculated numerically because of the divergence in the semiradiative boundary conditions. However, a complete analytic solution exists for this case (see Appendix), assuming that the whole ISM is concentrated in a narrow shell. The limit of the numeric solution with \( \epsilon \rightarrow 1 \) reaches this solution.

### 5. The Ultrarelativistic Solution

If the blast wave expansion velocity is ultrarelativistic, the sound speed of the wind cannot be considered infinite, an internal shock exists, and the blast wave exhibits the four regions discussed in § 2. The inner region of the blast wave contains wind that flows unaltered inside the cavity until it reaches the blast wave edge. This wind has no dependence upon the blast wave parameters, and it can therefore scale differently in time.

In the derivation of the hydrodynamic solution we follow in the footsteps of Blandford & McKee (1976). We assume that the inner source varies with time as \( L_{\text{in}} = L_0 t^{m} \), and we look for a self-similar solution for the shocked matter (wind and ISM) only. According to our model, the shocked matter flows adiabatically in the entire self-similar region of the blast wave. The boundary conditions for the solution are found using the internal wind, because the energy supplied to the shell by the wind is equal to the sum of the radiated energy and the energy stored in the shell. Boundary conditions for the forward shock are found using the semiradiative jump conditions (CPS98).

Self-similar solutions for relativistic blast waves were found (Blandford & McKee 1976) to scale according to the self-similar parameter

\[
\chi = [1 + 2(m + 1)\Gamma^2(t)](1 - r/t), \quad m > -1 ,
\]

where \( \Gamma \) is the Lorentz factor of the shock, which scales as \( \Gamma^2 \propto t^{-m} \). A blast wave that evolves with \( m < -1 \) has no timelike relation between the central source and the blast wave edge and thus cannot be reached by injecting energy from a central source. However, in other cases, especially with a steep ambient density gradient, a faster acceleration is possible. This type of solutions will be discussed in CPS98.

We start by identifying the different regions in the self-similar solution. The reverse shock, which separates the self-similar portion of the blast wave from the inner wind must evolve self-similarly, and occur at a fixed self-similar location \( \chi = \chi_{rs} \). The velocity of the reverse shock is therefore
\( v = dr(r, t)/dt \), and its Lorentz factor is \( \Gamma_{rs} = \Gamma/(\chi_{rs})^{1/2} \). Moreover, because of the high velocity of the wind, the reverse shock is strong, and the shocked fluid must leave the shock with a velocity of \( c/3 \) (Landau & Lifshitz, their § 135), which translates into a Lorentz factor of \( \gamma = 2^{1/2} \Gamma_{rs} \) in the unshocked fluid frame.

We can therefore use the self-similar substitutions for the pressure \( p \), Lorentz factor \( \gamma \), and density \( \rho \),

\[
\begin{align*}
  p(r, t) &= \frac{1}{2} \rho_1 \Gamma(t)^2 f(\chi), \\
  \gamma(r, t) &= \frac{1}{2} \Gamma(t)^2 g(\chi), \\
  \rho(r, t) &= 2 \rho_1 \Gamma(t)^2 h(\chi)
\end{align*}
\]

(BM76), to obtain a condition for the reverse shock, \( \chi_{rs} \gamma_{rs} = 4 \). Similarly, for the contact discontinuity, where the fluid is at rest relative to the self-similar profile, we obtain \( \chi_{cd} \gamma_{cd} = 2 \). These relations enable us to identify the composition of a certain self-similar interval (shocked ISM or shocked wind) simply by the term \( \gamma(\chi) \).

The wind, which advances with a Lorentz factor that is much larger than \( \Gamma_{rs} \), overtakes the shocked shell with a velocity difference of \( 1/\Gamma_{rs}^2 \). The rate in which energy is supplied to the shell is therefore \( dE/dt = L_0(t_c)/2\Gamma_{rs}^2 \), where \( t_c \) accounts for the delay between the emission of the energy and the impact at the reverse shock.

We start by obtaining an equation of motion using the energy conservation equation. The energy stored in shocked matter is

\[
E = \int_0^{r(t)} \frac{4\pi}{3} r^2 [4\gamma^2(r, t) - 1] dr = \frac{8\pi \rho_1}{3(m+1)} \Gamma^2 \gamma^3,
\]

where we define the dimensionless energy integral

\[
\chi_s = \int_1^{t_c} fg d\chi.
\]

Using this energy and the semiradiative model, we write the energy conservation equation as

\[
dE/dt = L(t_c)/2\Gamma_s^2 - 8\pi \rho_1 R_{sh}^2 \Gamma^2 \gamma^3/3,
\]

and obtain

\[
\Gamma^2 = K \left( \frac{3L_0}{16\pi \rho} \right)^{1/(q+2)} t^{(q-2)(q+2)},
\]

where

\[
K = \left( 2^{3q} \chi_s(q+1) + \epsilon \right) / (q+2)^q \chi_s^{1/(q+2)} + \epsilon.
\]

To find \( \chi_s \), we solve the hydrodynamic self-similar ODEs (The analytic solution is given in Appendix B). The boundary conditions at the shock front \( (\chi = 1) \) are the semiradiative boundary conditions

\[
\begin{align*}
  f(1) &= 1, \quad g(1) = 1 + \epsilon, \quad h(1) = \frac{1 + \epsilon}{1 - \epsilon}
\end{align*}
\]

(CPS98).

The hydrodynamic profile begins at the shock with these boundary conditions, and \( g(\chi) = 1 + \epsilon \). Near the shock the fluid flows slower than the self-similar profile. Toward the blast wave center the fluid decelerates relative to that profile, where at \( g(\chi) = 2 \) the fluid is at rest. The contact discontinuity resides in this location. Closer to the center, the fluid accelerates (the fluid here moves faster than the self-similar profile) until it reaches the reverse shock, where \( \chi_{rs} \gamma_{rs} = 4 \).

We present the hydrodynamic solutions for several cases in Figures 6–8 and in Table 3. The solutions show the same qualitative behavior as the Newtonian blast waves. If the evolution is set by the injection mechanism, the contact discontinuity and the reverse shock exist. However, if the injected energy decreases fast enough with time, the shocked ISM profile is similar to the one found in blast waves without a continuous power supply. The density, pressure, and velocity are monotonous and reach the maximum at the shock front. The density vanishes at the contact discontinuity, with a gradient that diverges to \( -\infty \). The same structure appears also for \( q < -1 \), as long as the evolution is set by the injection mechanism. If \( q > 2 \), the density itself diverges to infinity at the contact discontinuity. A high injection rate also results in a velocity profile with a maximum at the contact discontinuity.

An immediate consequence of the solution applies for the steady injection fully radiative limit. Blandford & McKee (1976; their eq. [82]) have treated this case assuming that the entire radiation is emitted from some average location.
within this layer. We take into account the momentum losses due to radiation through the entire cooling layer and obtain

$$\Gamma^4 = 0.79 L_{\text{rad}} / 8\pi \rho_1 R^2.$$ \hspace{1cm} (21)

Similarly to the Newtonian case, we would like to characterize a blast wave by its radiative efficiency. Using equations (17) and (18), we calculate the ratio between the radiated energy rate and the energy supply rate to the shocked matter (which includes shocked ISM and shocked internal fluid) and obtain

$$\eta_{\text{rad}} = \frac{L_{\text{rad}}}{L_{\text{in}}} = \frac{1}{1 + \alpha_s(1 + q)\varepsilon}.$$ \hspace{1cm} (22)

Note that prior to the substitution of $$\alpha_s$$ from the full hydrodynamic solution, we can use a zero-order approximation and assume that $$\alpha_s$$ is constant. We can then immediately deduce that, similarly to the Newtonian case, the efficiency rises with increasing $$\varepsilon$$ and decreasing $$q$$.

The instantaneous efficiency is intrinsically ill-defined in the ultrarelativistic case. The luminosity scales with time as $$t^{(2 + 3q)/(2 + q)}$$ (using eq. [19]), which is generally different from the source behavior $$t^4$$. Therefore the overall radiation efficiency is not constant in time and cannot be used to characterize the blast wave. However, because of the relativistic motion of the shell, the radiation it emits is measured by the observer at a different time, $$t_{\text{obs}} = t/2/(m + 1)\Gamma^2$$. Still, the central source does not move. If it had been measured directly by an observer, it would evolve with $$t$$, and not $$t_{\text{obs}}$$. We define the luminosity in a more observable fashion by comparing the luminosity measured by an observer to the luminosity that would be measured if the blast wave shell had not existed and the source energy could escape freely to the observer. Transforming the radiated luminosity to the observer frame using equation (19) and dividing by the source energy injection rate, we obtain

$$\eta_{\text{tot}} = \frac{L_{\text{rad}}}{L_{\text{in}}} = \frac{\lambda_s^{1 + q}}{1 + \alpha_s(1 + q)\varepsilon},$$ \hspace{1cm} (23)

which is constant in time. Efficiencies for different cases appear in Figure 9. Note that the instantaneous efficiencies are higher than those of equation (22), which means that energy is supplied to the shocked matter faster than it is emitted. This apparent discrepancy is due to time contraction, which causes energy emitted over a long duration at the source to concentrate into a short observed time.

6. DISCUSSION AND CONCLUSIONS

Radiative blast waves with energy injection exhibit self-similar relations both in the Newtonian and in the ultrarelativistic limit. We have found that if a self-similar solution exists, the power-law index can be one of two: (1) the power-law index of a radiative blast wave with the same radiative mechanism but without energy injection or (2) the power-law index of the central source, disregarding radiative mechanism. The chosen power law is the one that

| TABLE 3 | Parameters of Similarity Solutions for Radiative Continuously Powered Ultrarelativistic Blast Waves |
|---------|---------------------------------------------|
| $$m$$   | $$q$$ | $$\varepsilon$$ | $$\chi_{cd}$$ | $$\chi_s$$ | $$\alpha_{cd}$$ | $$\alpha_s$$ | $$K$$ | $$\eta_{\text{rad}}$$ | $$\eta_{\text{tot}}$$ |
| -1 1 | 0 | 1 | 1 | 0 | 0 | $$\infty$$ | 0 | 0 |
| 2 | -1 | 3.37 | 6.28 | 0.89 | 1.24 | 7.50 | 0 | 0 |
| 2 | -1 | 1 | 1 | 1.86 | 0 | 1.20 | 2.22 | 0.71 | 0.88 |
| 3 | 1 | 1 | 1 | 2.38 | 0 | 1.41 | 8 | 1 | 1 |
| 4 | -1 | 1 | 1 | 2.59 | 8.1 | 0.93 | 1.42 | 127.46 | 2.32 | 1.53 |
| 7 | -1 | 1 | 1 | 9.04 | 1.68 | 17417.80 | 6.20 | 2.06 |
corresponds to an evolution with a faster energy increase or a slower energy decrease. Sources that emit energy with a steep decreasing power law ($\sigma < -1$) behave according to the same rule. This corrects a common belief, which states that such sources can be considered as instantaneous energy release sources. We have shown that this view is correct for adiabatic blast waves but that it does not hold for radiative blast waves.

The radiative efficiencies in the ultrarelativistic case are higher than those of Newtonian blast waves. Therefore the possibility that GRB afterglows are powered by continuously emitting sources (Katz & Piran 1997) cannot by ruled out on efficiency basis. Moreover, the instantaneous efficiency of a blast wave with a tuned power-law behavior can significantly exceed the efficiency of the microscopic radiative mechanism.

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APPENDIX A

ANALYTIC SOLUTION FOR THE NEWTONIAN FULLY RADIATIVE LIMIT

In the fully radiative case, the shocked matter is concentrated on a shell of zero thickness, where the interior contains hot isobaric matter. In this case the solution can be calculated analytically.

The interior pressure is

$$ P = \frac{(\gamma - 1)}{V(t)} \left[ E_0 \left( \frac{t}{t_0} \right)^\lambda - \frac{1}{2} \rho_1 V(t) U_{sh}^2 \right], \quad (A1) $$

where $V(t) = 4\pi R_{sh}^3/3$ is the blast waves volume, and $E_0(t/t_0)^\lambda$ is the energy of the whole blast wave. Using the equation of motion

$$ \frac{d(\rho_1 VU_{sh})}{dt} = 4\pi R_{sh}^2 P \quad (A2) $$

and equation (6), we obtain the radiative efficiency

$$ \eta = \frac{L_{rad}}{L_{rad} + \dot{E}} = \frac{9(\gamma - 1)(2 + \lambda)^2}{2(3 + 4\lambda)[2(\lambda - 3) + 3\gamma(\lambda + 2)]}. \quad (A3) $$

APPENDIX B

ANALYTIC SOLUTIONS FOR A SEMIRADIATIVE ULTERRARELATIVISTIC BLAST WAVE

The self-similar ODEs for the adiabatic interior of a blast wave are

$$ \mathcal{G}(y) = \frac{1}{g} \frac{d}{d\chi} g = \frac{(7m + 3k - 4) - (m + 2)g\chi}{(m + 1)(4 - 8g\chi + g^2\chi^2)}, $$

$$ \mathcal{F}(y) = \frac{1}{g} \frac{d}{d\chi} f = \frac{8(m - 1) + 4k - (m + k - 4)g\chi}{(m + 1)(4 - 8g\chi + g^2\chi^2)}, $$

$$ \mathcal{H}(y) = \frac{1}{g} \frac{d}{d\chi} h = \frac{2(9m + 5k - 8) - 2(5m + 4k - 6)g\chi + (m + k - 2)g^2\chi^2}{(m + 1)(4 - 8g\chi + g^2\chi^2)(2 - g\chi)}. \quad (B1) $$

(Blandford & McKee 1976), where $y = g(\chi)\chi$. These equations are applicable for external density gradient of $\rho \propto r^{-k}$. Following Blandford & McKee (1976), we use the relation

$$ g(\chi) \frac{d\chi}{dy} = \frac{1}{y\mathcal{G}(y) + 1}. \quad (B2) $$

We obtain

$$ \frac{d\ln f}{dy} = \frac{\mathcal{F}(y)}{y\mathcal{G}(y) + 1}, \quad (B3) $$

$$ \frac{d\ln h}{dy} = \frac{\mathcal{H}(y)}{y\mathcal{G}(y) + 1}, \quad (B4) $$

$$ \frac{d\ln \chi}{dy} = \frac{1}{y(y\mathcal{G}(y) + 1)}. \quad (B5) $$
Using equation (B5), which does not appear in Blandford & McKee (1976), we obtain $\chi(y)$ and a full solution, parameterized by $g(y)$. Because $g(y)$ has physical meaning (§ 5), this parameterization is even easier to use than the solution as a function of the self-similar parameter.

We integrate equations (B3)–(B5) from $y = 1 + \epsilon$, which corresponds to the blast wave edge toward the contact discontinuity ($y = 2$) and the reverse shock ($y = 4$). We obtain

$$\mathcal{I}_1(y) = e^{(\ln \gamma)dy} = y^{\gamma(16 + 28m + m^2 - 3km)/2}(b - m)/2 - 1,$$

$$\mathcal{I}_2(y) = e^{(\ln f)dy} = y^{(164 - 24m - m^2 + k(2m - 32 + 3k))/2}(b - 4 - 3m + 3m^2 + k(m + 1))/2(m + 1),$$

$$\mathcal{I}_3(y) = e^{(\ln h)dy} = y^{(-k_1h_2/2)(y - 2)^2(k - m)/h_2}(b - 24 - 13m + m^2 + k(-19 + 4m + 3k))/h_2,$$  \hspace{1cm} (B6)

using the following definitions:

$$\alpha \equiv [160 + 40m + m^2 - 6(12 + m) + 9k^2]^{1/2},$$  \hspace{1cm} (B7)

$$\beta \equiv -4 - 4m + y(12 + m - 3k) + y^2,$$  \hspace{1cm} (B8)

$$\gamma \equiv \alpha + 3k - m - 2y - 12,$$  \hspace{1cm} (B9)

$$h_1 \equiv 2\{480 - 368m + 13m^2 + m^3 + k[-464 + 136m + m^2 - 3k(-41 + 3m + 3k)]\},$$  \hspace{1cm} (B10)

and

$$h_2 \equiv 2(-12 + 3k + m).$$  \hspace{1cm} (B11)

Finally, we obtain the hydrodynamic profiles

$$\chi(y) = \frac{\mathcal{I}_1(y)}{\mathcal{I}_1(1 + \epsilon)},$$

$$f(y) = \frac{\mathcal{I}_2(y)}{\mathcal{I}_2(1 + \epsilon)},$$

$$h(y) = \frac{\mathcal{I}_3(y)}{\mathcal{I}_3(1 + \epsilon)} \frac{1 + \epsilon}{1 - \epsilon},$$  \hspace{1cm} (B12)

$$g(y) = \frac{y}{\chi(y)}.$$

We have used these relations to calculate the profiles in Figures 6–8. As discussed in § 5, the parameter $y$ distinguishes between the different regions within the blast wave. For example, the dimensionless energy (eq. [17]) held within shocked matter (between $1 + \epsilon < y < 4$) is simply

$$\alpha_s = \int_1^y f(y)g(\chi)dy = \int_{1 + \epsilon}^4 \frac{f(y)}{1 + yg(\chi)} dy,$$  \hspace{1cm} (B13)

without the explicit usage of the self-similar location of the reverse shock.

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