Noncommutative D-branes from Covariant AdS Superstring

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Abstract

We explore noncommutative D-branes in the AdS$_5 \times$S$^5$ background from the viewpoint of $\kappa$-invariance of a covariant open string action in the Green-Schwarz formulation. Boundary conditions to ensure the $\kappa$-invariance of the action lead to possible configurations of noncommutative D-branes. With this covariant method, we derive configurations of 1/4 BPS noncommutative D-branes. The resulting D-branes other than D-string are 1/4 BPS at any places, and the D-string is exceptional and it is 1/2 BPS at the origin and 1/4 BPS outside the origin. All of them are reduced to possible 1/4 BPS or 1/2 BPS AdS D-branes in the commutative limit or the strong magnetic flux limit. We also apply the same analysis to an open superstring in the pp-wave background and derive configurations of 1/4 BPS noncommutative D-branes in the pp-wave. These D-branes consistently related to AdS D-branes via the Penrose limit.
1 Introduction

Noncommutative geometry in string theory has been well studied from the novel work of Seiberg and Witten [1]. When D-branes are considered in the presence of a constant NS-NS two-form $B$, a low-energy effective theory on the world-volume is realized as a field theory on a noncommutative space. The constant two-form flux may be taken as a magnetic flux on the world-volume. D-branes with constant fluxes (i.e. gauge field condensates on the branes) are often referred to as noncommutative D-branes. Noncommutative D-branes in flat spacetime have been well studied (for example, see [2–4]), but it seems not to be the case for those on curved backgrounds. Here we are interested in AdS-branes with gauge field condensates, which are interesting objects to study in the context of the AdS/CFT correspondence [5].

An AdS-brane gives a defect in the field theory side, and another field theory [6], which is called defect conformal field theory (dCFT), is realized on the defect [7]. When magnetic or electric flux is turned on through the AdS-brane worldvolume, the flux should deform the dCFT and it is interesting to study the resulting theory. For example, in the recent study of the AdS/CFT duality at a far-from BPS region, an integrable open spin chain appears in the analysis of the dCFT [8]. When the flux is turned on, it may be interpreted as a boundary magnetic field of the open spin chain. For this purpose, as the first step, we will study noncommutative D-branes in the AdS$_5 \times $S$^5$ and pp-wave backgrounds.

In a series of our previous works we showed the power of $\kappa$-invariance of type IIB superstrings in a covariant classification of possible D-branes in AdS$_5 \times $S$^5$ [12, 13] (For a short review see [14]) and in the pp-wave [15]. The $\kappa$-invariance of an open string requires that surface terms under the $\kappa$-variation should be deleted by imposing some additional conditions [22], which lead to the classification of the possible D-branes. This procedure has some advantages in comparison to the brane probe analysis with Dirac-Born-Infeld (DBI) action on the AdS and the pp-wave [18]. First, it is obvious that we do not need to analyze each of DBI actions of D$p$-branes. All we have to consider is just a string action. Secondly, our procedure is covariant and does not depend on the light-cone gauge. Finally we can show that the classification is valid at “full order” of the fermionic variables [13]. This procedure is also applicable to Dirichlet branes for an

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1 For other open spin chains, see [9] for D3-D7-O7 and [10] for a giant graviton.

2 Noncommutative D-branes in the pp-wave background are also discussed in [11]

3 For a classification of possible 1/2 BPS D-branes on a pp-wave with the light-cone gauge, see [16–21]
open supermembrane on the $\text{AdS}_{4/7} \times S^{7/4}$ background [23, 24] and the pp-wave [25, 26], and D-branes [27] of type IIA pp-wave string [28]. These results are consistently related via the double dimensional reduction [29].

In this paper we explore noncommutative D-branes in the $\text{AdS}_5 \times S^5$ background by analyzing a covariant type IIB string action in the Green-Schwarz formulation [30]. We apply the procedure developed in our previous papers to the case with a constant two-form $F$ composed of $B$ and the field strength on the D-brane. The boundary conditions to ensure that surface terms under the $\kappa$-variation vanish are shown to lead to possible noncommutative D-branes.\footnote{In this paper, we consider surface terms up to the fourth order in $\theta$. We expect that the result is valid even in the full order as was proven in [13] for the commutative case.} As the result, we find 1/4 BPS noncommutative D-branes. The D-branes other than D-string are 1/4 BPS anywhere, but the D-string is exceptional and it is 1/2 BPS at the origin and 1/4 BPS outside the origin. All of them are reduced to possible 1/4 BPS or 1/2 BPS AdS D-branes in the commutative limit or the strong magnetic flux limit. We also apply the same analysis to an open superstring in the pp-wave background and derive configurations of 1/4 BPS noncommutative D-branes in the pp-wave. These are consistently related to noncommutative AdS D-branes via the Penrose limit [31].

This paper is organized as follows. In section 2, we shall introduce a covariant action of the type IIB open string in the $\text{AdS}_5 \times S^5$ background in the Green-Schwarz formulation, which includes a constant two-form $F$. Then we write down surface terms under the $\kappa$-variation of the action, which play an important role in determining possible configuration of noncommutative D-branes. In section 3, we discuss noncommutative D-branes in flat spacetime. The well-known results obtained in the different methods can be reproduced and this justifies the consistency of our method with the others. In section 4, we discuss 1/4 BPS noncommutative D-branes in $\text{AdS}_5 \times S^5$. In section 5, after introducing a covariant action of a pp-wave superstring with a constant two-form, we find 1/4 BPS noncommutative D-branes in the pp-wave. Section 6 is devoted to a summary and discussions.

2 Open AdS superstring with a constant two-form

In this section we shall introduce the covariant action of the type IIB string on the $\text{AdS}_5 \times S^5$ background [30], including a constant two-form. The action has the $\kappa$-symmetry
which ensures the consistency of the theory. When we consider an open string rather than a closed string, the $\kappa$-variation of the action gives surface terms and so we need to impose some boundary conditions in order to preserve the $\kappa$-symmetry [22]. These conditions restrict possible configurations of D-branes in $\text{AdS}_5 \times S^5$. We will derive the surface terms here, and the boundary conditions will be considered in section 3 and 4.

### 2.1 The action of AdS superstring

The action of type IIB superstring on a supergravity background is given by

$$ S = \int_{\Sigma} d^2 \sigma \left[ L_{\text{NG}} + L_{\text{WZ}} \right], \quad (2.1) $$

where the Nambu-Goto (NG) part is

$$ L_{\text{NG}} = -\sqrt{-g(X, \theta)}, \quad (2.2) $$

$$ g_{ij} = E_M^i E_N^j G_{MN}, \quad E_i^A = \partial_i Z^M \tilde{E}_M^A, \quad Z^M = (X^M, \theta^\alpha), \quad (2.3) $$

and the Wess-Zumino (WZ) part consists of the two parts:

$$ L_{\text{WZ}} = L_{\text{WZ}}^0 + L_{\text{WZ}}^1, \quad (2.4) $$

$$ L_{\text{WZ}}^0 = -2i \int_0^1 dt \tilde{E}^A \bar{\theta} \Gamma_A \bar{\sigma} \tilde{E}, \quad (2.5) $$

$$ L_{\text{WZ}}^1 = \frac{1}{2} e^{ij} e_i^A e_j^B F_{AB} = \frac{1}{2} e^{ij} \partial_i X^M e^A_M \partial_j X^N e^B_N F_{AB}, \quad (2.6) $$

where $\tilde{E} = E_{|\theta \to -\theta}$ and $\sigma = \sigma_3$ for an F-string while $\sigma = \sigma_1$ for a D-string. We will focus upon the F-string case hereafter. The two-from field strength $F$ is defined by $F = B - da$ where $a$ is a gauge field on the D-brane.

For the case of the $\text{AdS}_5 \times S^5$ we are interested in, the supervielbein is given by [12]

$$ E^A = e^A + i\bar{\theta} \Gamma^A \left( \frac{\sinh(M/2)}{M/2} \right)^2 D\theta, \quad E^\alpha = \left( \frac{\sinh M}{M} D\theta \right)^\alpha, \quad (2.7) $$

where we have introduced the following quantities:

$$ M^2 = i\lambda \left( \tilde{\Gamma}_A i\sigma_2 \bar{\theta} \Gamma^A - \frac{1}{2} \Gamma_{AB} \bar{\theta} \Gamma^{AB} i\sigma_2 \right), \quad (2.8) $$

$$ D\theta = d\theta + \frac{\lambda}{2} e^A \tilde{\Gamma}_A i\sigma_2 \bar{\theta} + \frac{1}{4} \omega^{AB} \Gamma_{AB} \bar{\theta}, \quad \tilde{\Gamma}_A \equiv (-\Gamma_{a} \mathcal{I}, \Gamma_{a'} \mathcal{J}), \quad \tilde{\Gamma}_{AB} \equiv (-\Gamma_{ab} \mathcal{I}, \Gamma_{a'b'} \mathcal{J}), \quad \mathcal{I} = \Gamma^{01234}, \quad \mathcal{J} = \Gamma^{56789}. \quad (2.8) $$

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5 We follow the notation used in [12].

6 For alternative superstring actions in $\text{AdS}_5 \times S^5$, see [32–34] (see also [35]). For classical integrability of AdS superstring [36] in [34], see [37].
Here $e$ and $\omega$ are the vielbein and the spin connection, respectively, on the $\text{AdS}_5 \times \text{S}^5$ background.

In the presence of a non-trivial constant $\mathcal{F}$ along the D-brane world-volume (i.e., $\mathcal{F}_{AB} = \text{const.}$ and others = 0), the boundary condition for Neumann directions is modified and becomes a mixed boundary condition. The resulting boundary condition is as follows:

$$
\partial_t X^N e_\mathcal{A}^N + \partial_N X^e B^{\mathcal{A}} = 0, \quad \mathcal{A}_i (i = 0, \cdots, p) \in \text{Neumann}
$$

$$
\partial_t X^N e^{\mathcal{A}}_N = 0, \quad \mathcal{A}_i (i = p + 1, \cdots, 9) \in \text{Dirichlet}
$$

where $\partial_t$ and $\partial_n$ are derivatives on $\partial \Sigma$ tangential and normal to the D-brane world-volume, respectively. In the limit for a magnetic flux, $\mathcal{F} \to \infty$, Neumann directions $(\bar{A}, \bar{B})$ are replaced by Dirichlet directions. On the other hand, for an electric flux $F$, no dimensional reduction of the D-brane occurs because the range of the electric flux is restricted as $0 \leq \mathcal{F} \leq 1$.

### 2.2 The $\kappa$-variation of the open string action

According to our procedure explained in the introduction, let us examine the surface terms under the $\kappa$-variation of the type IIB string action on $\text{AdS}_5 \times \text{S}^5$ up to fourth order in $\theta$. For the case with $\mathcal{F} = 0$, the vanishing conditions on the surface terms were examined in [12].

To extract the surface terms of the fourth order in $\theta$ from $S^B_{\text{WZ}} \equiv \int_\Sigma d^2 \sigma \mathcal{L}^0_{\text{WZ}}$, we need to know the expression of the $\kappa$-variation up to fourth order in $\theta$. The $\kappa$-variation $\delta_\kappa E^A = \delta_\kappa Z^M E^A_M = 0$ in this background is given as [13]

$$
\delta_\kappa X^M e^A_M = -(1 + H + H^2 + \cdots) B e^B_{\bar{\alpha}} \delta_\kappa \theta^\alpha,
$$

where

$$
H^A_B = -i \bar{\theta} \Gamma^A \left( \frac{\sinh \mathcal{M}/2}{\mathcal{M}/2} \right)^2 [D \theta]_N e^N_B, \quad E^A_{\bar{\alpha}} = i \bar{\theta} \Gamma^A \left( \frac{\sinh \mathcal{M}/2}{\mathcal{M}/2} \right)^2.
$$

It is easy to derive

$$
\delta_\kappa X^M e^A_M = -i \bar{\theta} \Gamma^A \delta_\kappa \theta - \frac{i}{12} \bar{\theta} \Gamma^A \mathcal{M}^2 \delta_\kappa \theta
$$

$$
- \bar{\theta} \Gamma^A \left( \frac{\lambda}{2} \bar{\Gamma}_B i \sigma_2 \theta + \frac{1}{4} \omega_B^{CD} \Gamma_{CD} \theta \right) \bar{\theta} \Gamma^B \delta_\kappa \theta + O(\theta^6).
$$
By using this expression, we obtain the $\kappa$-variation surface term of $S^B_{WZ}$

$$
\delta_\kappa S^B_{WZ} = \int_{\partial \Sigma} d\xi \partial_t X^M e^A_M F^A_{AB} \\
\times \left[ -i \bar{\theta} \Gamma^B \delta_\kappa \theta - \frac{i}{12} \bar{\theta} \Gamma^B M^2 \delta_\kappa \theta \\
- \frac{\lambda}{2} \bar{\theta} \Gamma^B \tilde{\Gamma}^C i \sigma_2 \theta \bar{\theta} \Gamma^C \delta_\kappa \theta - \frac{1}{4} \bar{\theta} \Gamma^B \Gamma^{CD} \theta \omega^C_E \bar{\theta} \Gamma^E \delta_\kappa \theta \right] + O(\theta^6). \quad (2.13)
$$

It is known that the $\kappa$-variation of the total action $S$ gives surface terms only. In particular the NG part produces no surface term, and so the $\kappa$-variation surface terms emerge only from the WZ part.

The surface terms under the $\kappa$-variation of the WZ term are given by

$$
\delta_\kappa S_{WZ} = \int_{\partial \Sigma} d\xi \left[ L^0 + L^\lambda + L^{\text{spin}} \right], \quad (2.14)
$$

with

$$
L^0 = -i \partial_t X^M e^A_M (\bar{\theta} \Gamma_A \sigma \delta_\kappa \theta + F^A_{AB} \bar{\theta} \Gamma^B \delta_\kappa \theta) \\
+ \frac{1}{2} (\bar{\theta} \Gamma^A \delta_\kappa \theta \bar{\theta} \Gamma_A \sigma \bar{\theta} \Gamma_A \delta_\kappa \theta \bar{\theta} \Gamma^A \delta_\kappa \theta), \quad (2.15)
$$

$$
L^\lambda = \partial_t X^M e^A_M \left[ - \frac{\lambda}{2} (\bar{\theta} \Gamma_A \sigma \Gamma^A \delta_\kappa \theta + \bar{\theta} \Gamma^B \Gamma^C i \sigma_2 \theta) \bar{\theta} \Gamma^C \delta_\kappa \theta \\
+ \frac{\lambda}{4} (\bar{\theta} \Gamma^B \delta_\kappa \theta \bar{\theta} \Gamma_B \delta_\kappa \theta + \bar{\theta} \Gamma^B \sigma \delta_\kappa \theta \bar{\theta} \Gamma_B \delta_\kappa \theta) \\
- \frac{i}{12} (\bar{\theta} \Gamma_A \sigma \Gamma^C \delta_\kappa \theta + F^A_{AB} \bar{\theta} \Gamma^B \delta_\kappa \theta) \right], \quad (2.16)
$$

$$
L^{\text{spin}} = -\frac{1}{4} \omega^{CD} \partial_t X^M e^A_M \bar{\theta} \Gamma^E \delta_\kappa \theta (\bar{\theta} \Gamma_A \sigma \Gamma^CD \theta + F^A_{AB} \bar{\theta} \Gamma^B \Gamma^C \delta_\kappa \theta) \\
+ \frac{1}{8} \omega^{BC} \partial_t X^M e^A_M (\bar{\theta} \Gamma^A \delta_\kappa \theta \bar{\theta} \Gamma_A \Gamma^B \sigma \theta \bar{\theta} \Gamma^C \delta_\kappa \theta \bar{\theta} \Gamma_A \Gamma^C \theta), \quad (2.17)
$$

where $L^{\text{spin}}$ includes $\omega$-dependent terms, $L^\lambda$ includes $\lambda$-dependent (but $\omega$-independent) terms and $L^0$ includes $\lambda$-independent terms. These surface terms should vanish under some conditions to ensure the $\kappa$-invariance. From the next section, we will examine the conditions under which the surface terms vanish.

### 3 Noncommutative D-branes in flat spacetime

In this section we will examine the vanishing condition of $L^0$. This condition leads to the classification of the possible noncommutative D-branes in flat spacetime, since the AdS metric is reduced to the flat spacetime metric when we consider the $\lambda = 0$ case.
Noncommutative D-branes in the Green-Schwarz formulation have been already studied before [2–4], but our analysis here is covariant and so more general.

First, let us examine the first line of \( \mathcal{L}^0 \), which is the second order in \( \theta \),

\[
-i \partial_t X^M e_M \bar{\epsilon}_M (\bar{\theta} \Gamma A \sigma_\kappa \theta + \mathcal{F}_{AB} \bar{\theta} \Gamma B \delta_\kappa \theta)
\]

\[
= -i \partial_t X^M e_M \bar{\epsilon}_M \left[ (1 + \mathcal{F})_A^B \bar{\theta} \Gamma B \delta_\kappa \theta^1 - (1 - \mathcal{F})_A^B \bar{\theta} \Gamma_B \delta_\kappa \theta^2 \right], \quad (3.1)
\]

where we have used the boundary condition (2.9). We should carefully consider the gluing conditions for the fermionic variable \( \theta \) at boundaries in order to see that the surface terms vanish. For this purpose we define the following matrix,

\[
M \equiv g_0 \Gamma^{\bar{A}_0 \cdots \bar{A}_p} \prod_{n=1}^{[(p+1)/2]} \left( g_n + h_n \Gamma^{\bar{a}_{2n-1} \bar{a}_{2n}} \right), \quad (3.2)
\]

and demand

\[
\theta^2 = M \theta^1. \quad (3.3)
\]

Because \( \theta^1 \) and \( \theta^2 \) are Majorana-Weyl spinors, \( p \) must be odd, \( p = 1, 3, 5, \ldots \).

Let \( C \) be the charge conjugation matrix satisfying

\[
(\Gamma^A)^T = -CT^AC^{-1}, \quad (3.4)
\]

then one finds the following relation,

\[
\bar{\theta}^2 = \bar{\theta}^1 C^{-1} M^T C = \bar{\theta}^1 M', \quad (3.5)
\]

\[
M' \equiv (-1)^{p+1+[\frac{p+1}{2}]} g_0 \prod_{n=1}^{[p+1]} (g_n - h_n \Gamma^{\bar{a}_{2n-1} \bar{a}_{2n}}) \Gamma^{\bar{A}_0 \cdots \bar{A}_p}. \quad (3.6)
\]

If \( \bar{B} \in \{ \bar{a}_{2\ell-1}, \bar{a}_{2\ell} \} \), then we find that

\[
M \Gamma_{\bar{B}} M = -sg_0^2 \prod_{n \neq \ell} \left( g_n^2 + h_n^2 s_n \right) (g_\ell - h_\ell s_\ell + 2g_\ell h_\ell \epsilon_\ell) \Gamma^\bar{C} \Gamma^\bar{C} \quad (3.7)
\]

where \( (\epsilon_\ell)_{\bar{a}_{2\ell-1}} \bar{a}_{2\ell} = \eta_{\bar{a}_{2\ell-1} \bar{a}_{2\ell}}, (\epsilon_\ell)_{\bar{a}_{2\ell}} \bar{a}_{2\ell-1} = -\eta_{\bar{a}_{2\ell-1} \bar{a}_{2\ell-1}} \) and others are zero. Hence the vanishing condition of the first line of \( \mathcal{L}^0 \) is

\[
(1 + \mathcal{F})_A^B = (1 - \mathcal{F})_A^C \left( -sg_0^2 \prod_{n \neq \ell} \left( g_n^2 + h_n^2 s_n \right) (g_\ell^2 - h_\ell^2 s_\ell + 2g_\ell h_\ell \epsilon_\ell) \right)^C_B \quad (3.8)
\]

For the case without an electric \( \mathcal{F} \), since

\[
\epsilon_\ell = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & \mathcal{F}_{\bar{a}_{2\ell-1} \bar{a}_{2\ell}} \\ \mathcal{F}_{\bar{a}_{2\ell-1} \bar{a}_{2\ell}} & 0 \end{pmatrix} = \epsilon_\ell \mathcal{F}_{\bar{a}_{2\ell-1} \bar{a}_{2\ell}} \quad (3.9)
\]
and $s_n = 1$, (3.8) implies the following two equations:

$$1 = -s g_0^2 \prod_{n \neq \ell} (g_n^2 + h_n^2)(g_n^2 - h_n^2 - 2g_\ell h_\ell F_{\epsilon \ell}) ,$$  

for $1 \leq \ell \leq [\frac{p+1}{2}]$. These are solved by

$$g_0 = \sqrt{-s} , \quad g_n = \cos \varphi_n , \quad h_n = \sin \varphi_n , \quad F_{\bar{a}_2 a_{2n}} = \tan \varphi_n$$  

where $0 \leq \varphi_n \leq \pi/2$.

For the case with an electric $F$, say $F_{\bar{a}_1 a_2}$ and $\eta_{\bar{a}_1 a_1} = -\eta_{\bar{a}_2 a_2} = -1$, since

$$\epsilon_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \quad \begin{pmatrix} 0 & F_{\bar{a}_1 a_2} \\ F_{\bar{a}_2 a_1} & 0 \end{pmatrix} = \epsilon_1 F_{\bar{a}_1 a_2} ,$$  

$s_1 = -1$ and $s_n = 1 \ (n \geq 2)$, (3.8) implies

$$1 = -s g_0^2 \prod_{n \neq \ell} (g_n^2 + h_n^2 s_n)(g_n^2 - h_n^2 s_\ell - 2g_\ell h_\ell F_{\epsilon \ell}) ,$$  

for $1 \leq \ell \leq [\frac{p+1}{2}]$. These are solved by

$$g_0 = \sqrt{-s} ,$$  

$$g_1 = \cosh \varphi_1 , \quad h_1 = \sinh \varphi_1 , \quad F_{\bar{a}_1 a_2} = \tanh \varphi_1$$  

$$g_n = \cos \varphi_n , \quad h_n = \sin \varphi_n , \quad F_{\bar{a}_2 a_{2n}} = \tan \varphi_n \ (n \geq 2) ,$$  

where $0 \leq \varphi_1 \leq \infty$ and $0 \leq \varphi_n \leq \pi/2 \ (n \geq 2)$.

In both cases, $M$ is written as

$$M = \sqrt{-s} \exp \left( \sum_{n=1}^{[\frac{p+1}{2}]} \varphi_n \Gamma_{\bar{a}_{2n-1} a_{2n}} \right) \Gamma_{A_0 \cdots A_p} .$$  

It is clear from this expression that a noncommutative Dp-brane is reduced to a commutative Dp-brane for $\forall \varphi_n \to 0$. On the other hand, it is reduced to a noncommutative

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7For the case with an electric $F_{\bar{a}_1 a_2}$, we have used $\cosh \varphi_1 + \Gamma_{\bar{a}_1 a_2} \sinh \varphi_1 = \exp(\varphi_1 \Gamma_{\bar{a}_1 a_2})$. Because $\varphi_1 = \text{arctanh} F_{\bar{a}_1 a_2} = \frac{1}{2} \log \frac{1 + F_{\bar{a}_1 a_2}}{1 - F_{\bar{a}_1 a_2}}$, we believe that the gluing matrix in the earlier literature is valid only for an electric $F$.  

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D\((p - 2)\)-brane for a magnetic \(\mathcal{F}_{a_{2n-1}a_{2n}} \rightarrow \infty \) \((\varphi_n \rightarrow \pi/2)\), while remains to be a non-commutative Dp-brane for an electric \(\mathcal{F}_{a_1a_2} \rightarrow 1 \) \((\varphi_1 \rightarrow \infty)\).

It is convenient to rewrite the boundary condition \(\theta^2 = M \theta^1\) in the 2 \times 2 matrix form as follows:

\[
\theta = M \theta = \begin{pmatrix} 0 & M^{-1} \\ M & 0 \end{pmatrix} \begin{pmatrix} \theta^1 \\ \theta^2 \end{pmatrix}, \quad M^2 = 1. \tag{3.20}
\]

Here the matrix \(M\) is defined as

\[
M = \sqrt{-s} \exp(-\sum_{n=1} \varphi_n \Gamma^{a_{2n-1}a_{2n}} \sigma_3) \Gamma \rho \tag{3.21}
\]

and then the matrix \(\rho\) is represented by 2 \times 2 Pauli matrices, according to the value of \(p\),

\[
\rho = \begin{cases} 
\sigma_1 & \text{when } p = 1 \mod 4 \\
-i\sigma_2 & \text{when } p = 3 \mod 4 .
\end{cases} \tag{3.22}
\]

The range of the parameter \(\varphi_n\) depends on the type of the flux, namely magnetic or electric. For the magnetic flux case \(\varphi_n\) takes the value in the range of \(0 \leq \varphi_n \leq \pi/2\) and for the electric flux case it takes in \(0 \leq \varphi_n \leq \infty\). As we have just seen above, the gluing matrix \(M\) satisfies

\[
M' \Gamma_A \sigma_3 + \Gamma_A \sigma_3 M + \mathcal{F}_{AB}(M' \Gamma^B + \Gamma^B M) = 0 , \tag{3.23}
\]

or equivalently

\[
M'(\Gamma_A \sigma_3 + \mathcal{F}_{AB} \Gamma^B) = -(\Gamma_A \sigma_3 + \mathcal{F}_{AB} \Gamma^B) M \tag{3.24}
\]

where \(M'\) is defined as

\[
M' = C^{-1} M^T C = -M . \tag{3.25}
\]

It is obvious from (3.24) that the first line of \(\mathcal{L}^0\) vanishes.

Then, by using (3.23), the second line of \(\mathcal{L}^0\), the fourth order term in \(\theta\) vanishes as follows:

\[
\frac{1}{2} (\bar{\theta} \Gamma^A \delta_n \theta \, \bar{\theta} \Gamma_A \sigma + \bar{\theta} \Gamma^A \sigma \delta_n \theta \, \bar{\theta} \Gamma_A) \partial_t \theta
= -\frac{1}{2} \mathcal{F}_{AB}(\bar{\theta} \Gamma^A \delta_n \theta \, \bar{\theta} \Gamma^B + \bar{\theta} \Gamma^B \delta_n \theta \, \bar{\theta} \Gamma^A) \partial_t \theta = 0 , \tag{3.26}
\]

where we have used

\[
\bar{\theta} \Gamma^A \delta_n \theta = -\bar{\theta} M \Gamma^A \delta_n \theta = -\bar{\theta} \Gamma^A \delta_n \theta = 0 . \tag{3.27}
\]

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\[ F \to \infty \]

\[
\begin{array}{ccccccc}
\text{NC D9} & \to & \text{NC D7} & \to & \text{NC D5} & \to & \text{NC D3} & \to & \text{NC D1} & \to & \text{C D(1)} \\
& \downarrow & & \downarrow & & \downarrow & & \downarrow & & \\
\text{C D9} & & \text{C D7} & & \text{C D5} & & \text{C D3} & & \text{C D1} & & \\
\end{array}
\]

Table 1: The sequence of noncommutative D-branes in flat spacetime.

In summary we have discussed boundary conditions and constructed the gluing matrix. As the result we have found the well-known result that noncommutative D\(p\)-branes with \(p = \text{odd}\) in flat spacetime. By taking a strong flux limit \(F \to \infty\) for a magnetic case, D\(p\)-branes are reduced to D\((p-2)\)-branes. The commutative limit \(F \to 0\) leads to the standard commutative D-branes. The sequence of them via the two limits is depicted in Table 1. All of these D-branes are 1/2 BPS. In the case of flat spacetime higher order terms than the fourth order in \(\theta\) do not appear and so our result is rigorous with respect to \(\theta\).

4 Noncommutative D-branes in AdS\(_5\times S^5\)

In this section, we examine \(\lambda\)-dependent terms \(L^\lambda\) and \(\omega\)-dependent terms \(L^{\text{spin}}\). These terms include the parameter \(\lambda\), which characterizes the AdS geometry. Hence the analysis here is intrinsic to the AdS case and the boundary conditions lead to a classification of possible noncommutative AdS-branes.

It is straightforward to see that by using (3.23), \(L^\lambda\) is rewritten as

\[
\left[ \frac{\lambda}{2} \bar{\theta} \Gamma^C \delta_{\kappa} \theta \ (\Gamma_A \sigma_3 + F_{AB} \Gamma^B) M \hat{\Gamma}_{C} i \sigma_2 \theta \\
+ \frac{\lambda}{4} \bar{\theta} \Gamma^B \delta_{\kappa} \theta \ (\Gamma_B \sigma_3 + F_{BC} \Gamma^C) M \hat{\Gamma}_{A} i \sigma_2 \theta \\
+ \frac{\lambda}{4} \bar{\theta} \Gamma^B \sigma_3 \delta_{\kappa} \theta \ \bar{\theta} \Gamma_B \hat{\Gamma}_{A} i \sigma_2 \theta \\
+ \frac{i}{12} \bar{\theta} (\Gamma_A \sigma_3 + F_{AB} \Gamma^B) M \mathcal{M}^2 \delta_{\kappa} \theta \right] \partial_i X^M e^A_M . \tag{4.1}
\]

The first and the second lines in (4.1) vanish when

\[ M \hat{\Gamma}_{C} i \sigma_2 \theta = \hat{\Gamma}_{C} i \sigma_2 \theta , \tag{4.2} \]

which ensures that the third line vanishes as

\[ \bar{\theta} \Gamma_B \hat{\Gamma}_{A} i \sigma_2 \theta = - \bar{\theta} \Gamma_B M \hat{\Gamma}_{A} i \sigma_2 \theta = - \bar{\theta} \Gamma_B \hat{\Gamma}_{A} i \sigma_2 \theta = 0 \]. \tag{4.3}
Among odd $p$, we find that this is satisfied for $p = 1$. Noting that
\[ [\varphi_1 \Gamma^{a_1 a_2} \sigma_3, \widehat{\Gamma}_C i \sigma_2] = 0 \] (4.4)
is satisfied by $(2,0)$- and $(0,2)$-branes, we find that (4.2) is satisfied by the D1-branes. It is straightforward to see that the fourth line vanishes for the D1-branes. As we will see soon, $L^{\text{spin}}$ vanishes for these branes sitting at the origin, and thus we find that noncommutative D1-branes, $(0,2)$ and $(2,0)$, sitting at the origin are 1/2 BPS.

To proceed further, we shall introduce an additional gluing matrix $M_0$ satisfying $M_0^2 = 1$,
\[
M_0 = \ell_0 \Gamma^{\Lambda_0 \cdots \Lambda_p} 1 , \quad \ell = \begin{cases} \sqrt{-s} & \text{for } p = 1 \text{ mod } 4 \\ \sqrt{s} & \text{for } p = 3 \text{ mod } 4 \end{cases} \] (4.5)
and demand $\theta = M_0 \theta$. It satisfies
\[
M_0^\prime = C^{-1} M_0^T C = (-1)^{p+1+[\frac{p+1}{4}]} M_0 . \] (4.6)

The gluing matrices, $M$ and $M_0$, commute each other i.e., $[M, M_0] = 0$, and hence the branes obtained below are 1/4 BPS. Here we should note that the most general 1/4 projections are not always written in terms of mutually commuting gluing matrices only. Hence 1/4 BPS noncommutative D-branes we are considering is a part of possible 1/4 BPS D-branes.

By using the gluing condition $\theta = M_0 \theta$, we can derive the following equations:
\[
\bar{\theta} \Gamma^C \delta_\kappa \theta = \bar{\theta} M_0^\prime \Gamma^C M_0 \delta_\kappa \theta = 0 , \] (4.7)
when $p = 3$ mod 4, and
\[
\bar{\theta} (\Gamma_A \sigma_3 + \mathcal{F}_{AB} \bar{\Gamma}_B) M \widehat{\Gamma}_C i \sigma_2 \theta = 0 , \] (4.8)
when $p = 1 (3)$ mod 4 and $d, d' =$ odd (even). Here $d (d')$ is the number of Dirichlet directions in AdS$_5$ (S$^5$), respectively. It follows from these equations that the first and the second lines in (4.1) vanish when one of the followings is satisfied
\begin{itemize}
\item $p = 3$ mod 4,
\item $p = 1$ mod 4 and $d =$ odd.
\end{itemize}

On the other hand, by noting that
\[
\bar{\theta} \Gamma^C \sigma_3 \delta_\kappa \theta = 0 \quad \text{when } p = 1 \text{ mod } 4 , \] (4.9)
\[
\bar{\theta} \Gamma_B \widehat{\Gamma}_A i \sigma_2 = 0 \quad \text{when } p = 1 (3) \text{ mod } 4 \quad \text{and } d, d' =$ even (odd) , (4.10)
\end{itemize}
we find the third line vanishes when one of the followings is satisfied
\[ p = 1 \text{ mod } 4, \]
\[ p = 3 \text{ mod } 4 \text{ and } d, d' = \text{odd}. \]

In summary we find that the first, the second and the third lines vanish when \( d = \text{odd} \) and \( p = 1, 3 \text{ mod } 4 \), namely we see that (even,even)-branes are possible.

Let us examine the fourth line. Noting that (4.7) and

\[ \bar{\theta} \Gamma_{A} \delta_{3} \theta = 0 \quad \text{when } p = 1, 3 \text{ mod } 4 \text{ and } d, d' = \text{odd}, \]

we find the fourth line vanishes for (even,even)-branes.

Next, we examine \( \omega \)-dependent terms, \( \mathcal{L}^{\text{spin}} \), which are rewritten as

\[ \frac{1}{4} \omega_{\hat{E}}^{C \hat{D}} \partial_{t} X^{M} e_{M}^{\hat{A}} \bar{\theta} \Gamma_{\hat{D}} \delta_{3} \theta \bar{\theta}(\Gamma_{\hat{A}} \Gamma_{\hat{B}} \Gamma_{\hat{C}} \Gamma_{\hat{D}}) M \Gamma_{C D} \theta \]
\[ - \frac{1}{8} \omega_{\hat{D}}^{B C} \partial_{t} X^{M} e_{M}^{\hat{A}} \bar{\theta} \Gamma_{\hat{A}} \delta_{3} \theta \bar{\theta}(\Gamma_{\hat{B}} \Gamma_{\hat{C}} \Gamma_{\hat{D}}) M \Gamma_{B C} \theta \]
\[ + \frac{1}{8} \omega_{\hat{D}}^{B C} \partial_{t} X^{M} e_{M}^{\hat{A}} \bar{\theta} \Gamma_{\hat{A}} \delta_{3} \theta \bar{\theta} \Gamma_{\hat{B}} \Gamma_{\hat{C}} \theta. \]

Let us consider 1/2 BPS D1-branes, (2,0)- and (0,2)-branes, first. The vanishing condition for the first and the second lines is

\[ \omega_{E}^{C \hat{D}} M \Gamma_{C D} \theta = \omega_{E}^{C \hat{D}} \Gamma_{C D} \theta \]

which ensures that the third line vanishes. Since \( \omega_{E}^{C \hat{D}} = 0 \) (see Appendix B in [12]) and \( M \Gamma_{C D} = \Gamma_{C D} M \) for these branes, the first and the second lines vanish if \( \omega_{E}^{C \hat{D}} = 0 \), i.e., if branes are sitting at the origin. As \( \bar{\theta} \Gamma \Gamma_{B C} \theta = 0 \) for these branes, the last line vanishes if \( \omega_{D}^{B C} = 0 \), i.e., if the branes are sitting at the origin. Thus (2,0)- and (0,2)-branes sitting at the origin are 1/2 BPS.

Next we consider 1/4 BPS D-branes. First we examine the first and the second lines. They vanish when \( p = 3 \text{ mod } 4 \) due to (4.7). For \( p = 1 \text{ mod } 4 \), the terms proportional to \( \omega_{E}^{C \hat{D}} \) vanish due to (4.12). Noting that \( \omega_{E}^{C \hat{D}} = 0 \), we are left with the terms including \( \omega_{E}^{C \hat{D}} \). It is natural to expect that \( \omega_{E}^{C \hat{D}} = 0 \) for \( F_{B C} \neq 0 \neq F_{D E} \). This implies that (4.17) is satisfied for \( p = 1 \text{ mod } 4 \). Thus the first and the second lines vanish under
the configuration even outside the origin. Next we examine the last line. It vanishes for \( p = 1 \mod 4 \) due to (4.9). Since

\[
\bar{\theta} \Gamma_{\overline{A}} \Gamma_{\overline{BC}} \theta = 0 \quad \text{when } p = 3 \mod 4
\]  

(4.18)

and \( \omega^{BC}_D = 0 \), we are left with terms proportional to \( \omega^{BC} \). By the same reasoning above, we derive

\[
\omega^{BC}_D \bar{\theta} \Gamma_{\overline{A}} \Gamma_{\overline{BC}} \theta = \omega^{BC}_D \bar{\theta} \Gamma_{\overline{A}} \Gamma_{\overline{BC}} M \theta = 0,
\]  

(4.19)

so that the last line vanishes even for \( p = 3 \mod 4 \). We note that \( L_{\text{spin}} \) vanishes under the configuration even outside the origin.

Summarizing, we find 1/4 BPS noncommutative D-branes depicted in Table 2. (2,0)- and (0,2)-branes are 1/2 BPS when they are sitting at the origin, while 1/4 BPS when they move away from the origin.

| D1      | D3      | D5      | D7      | D9      |
|---------|---------|---------|---------|---------|
| (2,0), (0,2) | (4,0), (0,4), (2,2) | (4,2), (2,4) | (4,4) | absent |

Table 2: 1/4 BPS noncommutative D-branes in \( \text{AdS}_5 \times S^5 \)

When \( \forall \mathcal{F} \rightarrow 0 \), 1/4 BPS noncommutative Dp-branes with \( p = 3 \mod 4 \) are reduced to 1/4 BPS commutative Dp-branes, while 1/4 BPS noncommutative Dp-branes with \( p = 1 \mod 4 \) become 1/2 BPS commutative Dp-branes because surface terms vanish without \( M_0 \) in this limit [12]. We summarize commutative D-branes sitting at the origin in Table 3.

| SUSY | D(−1) | D1      | D3      | D5      | D7      | D9      |
|------|-------|---------|---------|---------|---------|---------|
| 1/2  | (0,0) | (2,0), (0,2) | (3,1), (1,3) | (4,2), (2,4) | (3,5), (5,3) | absent |
| 1/4  |       |         | (4,0), (0,4), (2,2) |         |         |         |

Table 3: Commutative D-branes in \( \text{AdS}_5 \times S^5 \) sitting at the origin

**Penrose limit**

Now let us discuss the Penrose limit [31] of the above branes. The Penrose limit is taken as follows [38]. First let us introduce the light-cone coordinates as

\[
X^\pm = \frac{1}{\sqrt{2}}(X^9 \pm X^0)
\]
and $\theta_{\pm} = P_{\pm}\theta_{\pm}$ with $P_{\pm} = \frac{1}{2} \Gamma_{+} \Gamma_{-}$, and scale as

$$X^{+} \to \Omega^{2} X^{+}, \quad \theta_{+} \to \Omega \theta_{+}.$$  \hspace{1cm} (4.20)

Then the limit $\Omega \to 0$ is taken. We distinguish the cases depending on the boundary conditions of the light-cone coordinate $(X^{+}, X^{-})$ as $(N,N)$ for $X^{\pm} \in$ Neumann directions and $(D,D)$ for $X^{\pm} \in$ Dirichlet directions. We consider the Penrose limits of $(N,N)$- and $(D,D)$-cases. Then we see that the noncommutative AdS-branes obtained above are reduced to noncommutative D-branes in the pp-wave as shown in Table 4.

| (4, 0) $\frac{1}{4}$ | $\leftrightarrow$ | (4, 0) $\frac{1}{4}$ | $\rightarrow$ | (4, 0) $\frac{1}{4}$ | $\rightarrow$ | (0, 4) $\frac{1}{4}$ | $\rightarrow$ | (4, 4) $\frac{1}{4}$ | $\rightarrow$ | (4, 4) $\frac{1}{4}$ |
|---------------------|-----------------|---------------------|-----------------|---------------------|-----------------|---------------------|-----------------|---------------------|-----------------|---------------------|
| DD                  | $\leftrightarrow$ | DD                  | $\rightarrow$ | DD                  | $\rightarrow$ | DD                  | $\rightarrow$ | DD                  | $\rightarrow$ | DD                  |
| NN                  | $\rightarrow$ | NN                  | (+-; 1, 1) $\frac{1}{4}$ | NN                  | (+-; 3, 3) $\frac{1}{4}$ | NN                  | (+-; 3, 1) $\frac{1}{4}$ | NN                  | (+-; 1, 3) $\frac{1}{4}$ |

Table 4: Penrose limit of noncommutative AdS-branes.

In the subsequent sections, we will discuss noncommutative D-branes in a pp-wave by studying the $\kappa$-invariance of a covariant string action on the pp-wave. We will see that the possible noncommutative D-branes are (+-;odd,odd)- and (even,even)-type configurations and all of them are consistently obtained in the Penrose limit considered above.

## 5 Noncommutative D-branes in the pp-wave

In this section we shall consider an open superstring in the maximally supersymmetric pp-wave background. D-branes in the pp-wave have been well studied and noncommutative D-branes in the pp-wave are also discussed in [11]. However, most of the earlier works are done in the light-cone gauge. As far as we know, there are no covariant studies of noncommutative D-brane in the pp-wave. Hence, by repeating the analysis in the case of the AdS superstring, let us classify possible noncommutative D-branes in the pp-wave. This classification is also our new result.

### 5.1 Open pp-wave superstring with a constant two-form

For the pp-wave background supervielbeins are given by [12]

$$E^{A} = e^{A} + \frac{i}{2} \bar{\theta} \Gamma^{A} \left( \frac{\sinh(\mathcal{M}/2)}{\mathcal{M}/2} \right)^{2} D\theta, \quad E^{a} = \left( \frac{\sinh \mathcal{M}}{\mathcal{M}} D\theta \right).$$
Here we have introduced several quantities:

\[ \mathcal{M}^2 = i \frac{\mu}{2} (f + g) i \sigma_2 \cdot \partial \Gamma^\nu + i \frac{\mu}{2} \Gamma_m i \sigma_2 \cdot \partial \Gamma^m + i \frac{\mu}{2} \Gamma_r \Gamma_+ \partial \Gamma^a f i \sigma_2 \]

\[ - i \frac{\mu}{2} \Gamma_r \Gamma_+ \partial \Gamma^a g i \sigma_2 - i \frac{\mu}{2} \Gamma_{mn} \partial \Gamma^{mn} i \sigma_2 , \]

\[ D\theta = d\theta + e^{- \frac{\mu}{2} (f + g) i \sigma_2 \theta + e_m \frac{\mu}{2} \Gamma_m i \sigma_2 \theta + e_m \frac{\mu^2}{2} \Gamma_m \Gamma_+ \theta ,} \]

\[ \hat{\Gamma}_m = (- \Gamma_r \Gamma_+ f, \Gamma_r \Gamma_+ g) , \quad \hat{\Gamma}_{mn} = (- \Gamma_{rs} \Gamma_+ f, \Gamma_{rs} \Gamma_+ g) , \]

\[ e^+ = dX^+ - \frac{\mu^2}{2} (X^m)^2 dX^- \quad e^- = dX^- \quad e^m = dX^m \quad e^m = X^m dX^- . \]

In this parametrization, the pp-wave metric becomes the standard form

\[ ds^2 = 2 e^+ e^- + (e^m)^2 = 2 dX^+ dX^- - \mu^2 (X^m)^2 (dX^-)^2 + (dX^m)^2 . \]

The WZ action in the case of the pp-wave is given by

\[ S_{WZ} = \int_\Sigma d^2 \sigma \left[ \mathcal{L}_{WZ}^0 + \mathcal{L}_{WZ}^1 \right] \quad (5.1) \]

with

\[ \mathcal{L}_{WZ}^0 = \epsilon^{ij} \partial_i X^M \epsilon^A_M \partial_j X^N \epsilon^B_N \mathcal{F}_{AB} , \quad (5.2) \]

\[ \mathcal{L}_{WZ}^1 = \epsilon^{ij} \left[ - i \partial_i X^M \epsilon^A_M \partial \Gamma_A \epsilon^B_B \mathcal{F}_{AB} - i \frac{\mu^2}{2} \Gamma_m \Gamma_+ \partial \Gamma^m \right] O(\theta^6) . \quad (5.3) \]

The universal feature of \( \kappa \)-variation is

\[ \delta_\kappa E^A = \delta_\kappa X^M E^A_M + \delta_\kappa \theta^a E^A_\alpha = 0 \quad (5.4) \]

and for the pp-wave case it can be rewritten, at the fourth order of \( \theta \), as

\[ \delta_\kappa X^M \epsilon^A_M = - \frac{i}{2} \partial \Gamma^A \delta_\kappa \theta - i \frac{\mu^2}{24} \partial \Gamma^A \mathcal{M}^2 \delta_\kappa \theta - \frac{1}{4} \partial \Gamma^B \delta_\kappa \theta \partial \Gamma^A (\varrho_B + \varrho_B^{spin}) \theta + O(\theta^6) , \quad (5.5) \]

where \( \varrho_A \) and \( \varrho_A^{spin} \) are defined, respectively, by

\[ \varrho_A = \epsilon^A_M \left( e^{- \frac{\mu}{2} (f + g) i \sigma_2 + e_m \frac{\mu}{2} \Gamma_m i \sigma_2} \right) , \quad \varrho_A^{spin} = \epsilon^A_M \left( e^m_M \frac{\mu^2}{2} \Gamma_m \Gamma_+ \right) . \quad (5.6) \]

It is straightforward to see that

\[ \varrho_+ = \frac{\mu}{2} (f + g) i \sigma_2 , \quad \varrho_m = \frac{\mu}{2} \Gamma_m i \sigma_2 , \quad \varrho_m^{spin} = \frac{\mu^2}{2} X^m \Gamma_m \Gamma_+ , \quad (5.7) \]

and others vanish.\[14\]
Under the $\kappa$-variation, $S_{NG}$ does not produce a surface term. The surface terms under the $\kappa$-variation of $S_{WZ}$ are obtained as

$$\delta_\kappa S_{WZ} = \int_{\partial \Sigma} d\xi \left[ L^0 + L^\mu + L^{\text{spin}} \right]$$  \hfill (5.8)

with

$$L^0 = -i \left( \bar{\theta} \Gamma_A \sigma \delta_\kappa \theta + \mathcal{F}_{AB} \bar{\theta} \Gamma^B \delta_\kappa \theta \right) \dot{X}^M e^A_M$$

$$+ \frac{1}{4} \left( \bar{\theta} \Gamma^A \delta_\kappa \theta \bar{\theta} \Gamma_A \sigma \dot{\theta} + \bar{\theta} \Gamma_A \sigma \delta_\kappa \theta \bar{\theta} \Gamma^A \dot{\theta} \right),$$  \hfill (5.9)

$$L^\mu = \left[ -\frac{1}{2} \bar{\theta} \Gamma^C \delta_\kappa \theta \left( \bar{\theta} \Gamma_A \sigma \varrho_C \theta + \mathcal{F}_{AB} \bar{\theta} \Gamma^B \varrho_C \theta \right) + \frac{1}{4} \left( \bar{\theta} \Gamma^B \delta_\kappa \theta \bar{\theta} \Gamma_B \sigma \varrho_A \theta + \bar{\theta} \Gamma_B \sigma \delta_\kappa \theta \bar{\theta} \Gamma^B \varrho_A \theta \right) - \frac{i}{12} \left( \bar{\theta} \Gamma_A \sigma \mathcal{M}^2 \delta_\kappa \theta + \mathcal{F}_{AB} \bar{\theta} \Gamma^B \mathcal{M}^2 \delta_\kappa \theta \right) \right] \dot{X}^M e^A_M ,$$  \hfill (5.10)

$$L^{\text{spin}} = \left[ -\frac{1}{2} \bar{\theta} \Gamma^C \delta_\kappa \theta \left( \bar{\theta} \Gamma_A \sigma \varrho_C^{\text{spin}} \theta + \mathcal{F}_{AB} \bar{\theta} \Gamma^B \varrho_C^{\text{spin}} \theta \right) + \frac{1}{4} \left( \bar{\theta} \Gamma^B \delta_\kappa \theta \bar{\theta} \Gamma_B \sigma \varrho_A^{\text{spin}} \theta + \bar{\theta} \Gamma_B \sigma \delta_\kappa \theta \bar{\theta} \Gamma^B \varrho_A^{\text{spin}} \theta \right) \right] \dot{X}^M e^A_M .$$  \hfill (5.11)

In the next subsection we will examine these surface terms and determine boundary conditions under which these vanish.

### 5.2 1/4 BPS noncommutative D-branes in the pp-wave

In order to determine the boundary conditions for the fermionic variable $\theta$, we shall introduce the gluing matrix $M$ given in (3.21) and demand that $\theta = M \theta$ in the same way as in the AdS case. Since it satisfies (3.23), $L^0$ vanishes.

By using (3.23), $L^\mu$ can be rewritten as

$$\left[ \frac{1}{2} \bar{\theta} \Gamma^C \delta_\kappa \theta \bar{\theta} (\Gamma_{\bar{A}} + \mathcal{F}_{\bar{A}B} \Gamma^B) M \varrho_C \theta \
- \frac{1}{4} \bar{\theta} \Gamma^B \delta_\kappa \theta \bar{\theta} (\Gamma_B + \mathcal{F}_{\bar{B}C} \Gamma^C) M \varrho_{\bar{A}} \theta \
+ \frac{1}{4} \bar{\theta} \Gamma_B \sigma \varrho_3 \delta_\kappa \theta \bar{\theta} \Gamma^B \varrho_A \theta \
+ \frac{i}{12} \bar{\theta} (\Gamma_{\bar{A}} + \mathcal{F}_{\bar{A}B} \Gamma^B) M M^2 \delta_\kappa \theta \right] \dot{X}^M e^A_M$$  \hfill (5.12)

where we have used

$$\bar{\theta} \Gamma^B \delta_\kappa \theta = \frac{1}{2} \bar{\theta} (M^B + \Gamma^B M) \delta_\kappa \theta = 0 .$$  \hfill (5.13)
The vanishing condition for the first and the second lines is

\[ M \varphi C \theta = \varphi C M \theta , \]  

(5.14)

which ensures that the third line vanishes. (5.14) means

\[ M \varphi \bar{m} \theta = \varphi \bar{m} M \theta \]  

(5.15)

for \(- \in D\), while for \(- \in N\), (5.15) and

\[ M \varphi \bar{z} \theta = \varphi \bar{z} M \theta \]  

(5.16)

Among other \(p\), \(p = 1\) is the special case. For \(p = 1\) and \(- \in D\), (5.15) is satisfied when

\[ \varphi_n \Gamma \bar{a}_{2n-1} \bar{a}_{2n} f = f \varphi_n \Gamma \bar{a}_{2n-1} \bar{a}_{2n} \quad \text{and} \quad d = \text{even} , \]  

(5.17)

where \(d \quad (d')\) is the number of Dirichlet directions contained in \(\{1, 2, 3, 4\} \quad \{5, 6, 7, 8\}\), respectively. This means that the condition is satisfied by \((2, 0)\)- and \((0, 2)\)-branes. On the other hand, for \(- \in N\), (5.16) implies that

\[ \varphi_n \Gamma \bar{a}_{2n-1} \bar{a}_{2n} (f + g) = -(f + g) \varphi_n \Gamma \bar{a}_{2n-1} \bar{a}_{2n} \quad \text{and} \quad d = \text{even} , \]  

(5.18)

which is not satisfied by the \((+\text{-})\)-brane. Thus we find that for \((0, 2)\)- and \((2, 0)\)-branes the first and the second lines vanish and so the third line vanishes. It is straightforward to see that the last line vanish for these branes. As will be seen below, \(\mathcal{L}^{\text{spin}}\) vanishes for these branes sitting even outside the origin. Thus we find that \((0, 2)\)- and \((2, 0)\)-branes sitting anywhere are 1/2 BPS.

For 1/4 BPS branes\(^8\), we introduce an additional gluing matrix \(M_0\) given in (4.15) which commutes with \(M\), \([M, M_0] = 0\), and satisfies

\[ M_0' = C^{-1} M_0^T C = (-1)^{p+1+\lfloor \frac{p+1}{2} \rfloor} M_0 . \]  

(5.19)

First we examine the first and the the second lines in (5.12). By using the boundary condition \(\theta = M_0 \theta\),

\[ \bar{\theta} \Gamma B \delta_\kappa \theta = 0 \quad \text{for} \quad p = 3 \text{ mod } 4 \]  

(5.20)

is derived and so they vanish for \(p = 3 \text{ mod } 4\). For \(p = 1 \text{ mod } 4\), since

\[ \bar{\theta} \Gamma B M \varphi C \theta = -\bar{\theta} M_0 \Gamma B M \varphi C \theta = \bar{\theta} \Gamma B M M_0 \varphi C \theta \]  

(5.21)

\(^8\)We should note that 1/4 BPS branes we consider here is a part of general 1/4 BPS branes. The 1/4 projectors here are composed of two of mutually commuting gluing matrices only.
and

\[ M_0 \theta_\lambda = \frac{\mu}{2} ((-1)^d f + (-1)^g i) \sigma_2 M_0 \]

\[ M_0 \theta_m = \begin{cases} -(-1)^d \theta_\nu M_0, & \nu \in D, \\ 0, & \nu \in N \end{cases}, \quad \nu \in D, \quad \nu \in N, \]  

(5.22)

they vanish when one of the followings is satisfied

- \( p = 3 \mod 4, \)
- \( p = 1 \mod 4, \) \( - \in D \) and \( d = \text{even}, \)
- \( p = 1 \mod 4, \) \( - \in N \) and \( d = \text{odd}. \)

Next, we examine the third line. It vanishes when \( p = 1 \mod 4 \) because

\[ \bar{\theta} \Gamma^B \sigma_3 \delta_n \theta = 0 \quad \text{for} \quad p = 1 \mod 4 . \]  

(5.23)

For \( p = 3 \mod 4, \) since

\[ \bar{\theta} \Gamma^B \theta = \bar{\theta} \Gamma^B M_0 \theta \]  

and (5.22), the third line vanishes when one of the followings is satisfied

- \( p = 1 \mod 4, \)
- \( p = 3 \mod 4, \) \( - \in D \) and \( d = \text{even}, \)
- \( p = 3 \mod 4, \) \( - \in N \) and \( d = \text{odd}. \)

Summarizing we find that the first, second and third lines vanish for \((+-;\text{odd},\text{odd})\)-branes and \((\text{even,even})\)-branes.

It is straightforward to see that for these branes the fourth line vanishes.

Next we examine \( \mathcal{L}^{\text{spin}} \), which is rewritten by using (3.23) as

\[ \left[ \frac{1}{2} \bar{\theta} \Gamma^C \delta_n \theta \bar{\theta} (\Gamma_A \sigma_3 + \mathcal{F}_{AB} \Gamma^B) M_0 \theta \right] \bar{\theta} \Gamma^B \theta = \frac{1}{4} \bar{\theta} \Gamma^B \delta_n \theta \bar{\theta} (\Gamma_B \sigma_3 + \mathcal{F}_{BC} \Gamma^C) M_0 \theta \]

\[ + \frac{1}{4} \bar{\theta} \Gamma^B \delta_n \theta \bar{\theta} \Gamma^C \theta \]  

\[ \bar{X}^M e_M \]  

\[ + \frac{1}{2} \bar{\theta} \Gamma^C \delta_n \theta \bar{\theta} (\Gamma_A \sigma_3 + \mathcal{F}_{AB} \Gamma^B) M_0 \theta \]  

(5.25)

In the presence of \( \mathcal{F} \) on the D-brane world-volume, the spin connection \( \theta_\lambda^{\text{spin}} \bar{d}X^\lambda = \omega_\lambda^{\text{spin}} dX^\lambda \) may develop additional components \( \omega_\lambda^{\text{spin}} dX^\lambda \). Even in this case, it is natural to expect that

\[ [\varphi_n \Gamma^A \Gamma_{AB} \Gamma, \omega_\lambda^{\text{spin}} \bar{d}X^\lambda] = 0 \]  

(5.26)
as in the AdS case. For \(- \in D\), it vanishes as \(g^\text{spin}_C \neq 0\) only for \(C = -\), so we consider the case with \(- \in N\) below. The first and the second lines vanish for \(p = 3 \mod 4\) due to (5.20). Even for \(p = 1 \mod 4\), they vanish since we derive

\[
\bar{\theta}(\Gamma_A\sigma_3 + \mathcal{F}_{AB}\Gamma^B)\Gamma_{CD}\theta\omega^{CD} = -\bar{\theta}(\Gamma_A\sigma_3 + \mathcal{F}_{AB}\Gamma^B)\bar{\Gamma}_{CD}M\theta\omega^{CD} = 0\, ,
\]  

and

\[
\bar{\theta}(\Gamma_A\sigma_3 + \mathcal{F}_{AB}\Gamma^B)M\frac{\mu^2}{2}X^m\Gamma_{m}\Gamma_+\theta = -\bar{\theta}M_0(\Gamma_A\sigma_3 + \mathcal{F}_{AB}\Gamma^B)M\frac{\mu^2}{2}X^m\Gamma_{m}\Gamma_+\theta = 0\, .
\]  

Next we examine the last line, which vanishes when \(p = 1 \mod 4\) due to (5.23). For \(p = 3 \mod 4\), it vanishes since we derive assuming (5.26)

\[
\bar{\theta}\Gamma_B\Gamma_{CD}\theta\omega^{CD} = -\bar{\theta}\Gamma_B\Gamma_{CD}M\theta\omega^{CD} = 0\, ,
\]  

and

\[
\bar{\theta}\Gamma_B\mu^2X^m\Gamma_m\Gamma_+\theta = \bar{\theta}\Gamma_BM_0\mu^2X^m\Gamma_m\Gamma_+\theta = 0\, .
\]  

Summarizing \(\mathcal{L}^\text{spin}\) vanishes for \((+-;\text{odd, odd})\)-branes and \((\text{even, even})\)-branes even if they are sitting outside the origin. Finally, it is obvious that \((0,2)\)- and \((2,0)\)-branes are 1/2 BPS even off the origin, because \(- \in D\) for these branes. As a result, we have obtained noncommutative D-branes summarized in Table 5.

| SUSY | D1 | D3 | D5 | D7 | D9 |
|------|----|----|----|----|----|
| 1/2  | (0,2), (2,0) | (4,0), (2,2), (0,4) | (4,2), (2,4) | (4,4) | absent |
| 1/4  | (+-;1,1) | (+-;3,1), (+-;1,3) | (+-;3,3) |

Table 5: Noncommutative D-branes in the pp-wave

In the commutative limit \(\forall\mathcal{F} \to 0\), noncommutative D-branes above are reduced to commutative D-branes. Combining the result obtained in the flat limit with that obtained in [12], we summarize commutative D-branes sitting at the origin of the pp-wave in Table 6.
We have considered some possible configurations of 1/4 BPS noncommutative D-branes in the $\text{AdS}_5 \times S^5$ and the pp-wave. The 1/4 BPS noncommutative AdS-branes are allowed to exist at arbitrary position in the spacetime. The D-string case is exceptional and it is 1/2 BPS at the origin and 1/4 BPS outside the origin. We also have seen that all of them are reduced to 1/2 BPS and 1/4 BPS commutative AdS-branes in the commutative limit or the strong magnetic flux limit. The 1/4 BPS noncommutative D-branes in the pp-wave background have also been classified by applying the same analysis as in the AdS case to the pp-wave case. The resulting possible D-branes are consistently reproduced from the noncommutative AdS-branes via the Penrose limit.

It would be possible to see that the result shown in this paper is still valid at full order in $\theta$ by following our previous paper [13], though the possible noncommutative D-branes have been classified at fourth order in $\theta$. This issue is more complicated and so we leave it as a future problem. We hope that we could report on this issue in the near future.

It is also interesting to consider the interpretation of our results before taking a near-horizon limit. According to the work of Skenderis and Taylor [18], an AdS-brane is surely related to a supersymmetric intersection of D-brane with a stack of $N$ D3-branes. Hence our result may also be interpreted in terms of the intersecting D-branes. In particular, the T-dual picture would be related to an intersecting D-branes at angles [39]. For this direction, it would also be useful to generalize the work [18, 40] by including a constant two-form. There the relation to dCFTs is also discussed. As a generalization of our work, it would be interesting to consider an intersecting AdS D-branes (for an intersecting D-branes on a pp-wave, see [41]), though we have discussed a single AdS D-brane here.

As another generalization of our works, it would be interesting to consider oblique D-branes [42] with gauge field condensates in the pp-wave background as discussed in [43,44].

### Table 6: Commutative D-branes in the pp-wave (sitting at the origin)

| SUSY | D(-1) | D1          | D3          | D5          | D7          | D9          |
|------|-------|-------------|-------------|-------------|-------------|-------------|
| 1/2  | (0,0) | (0,2), (2,0)| (1,3), (3,1)| (2,4), (4,2)|             | absent      |
|      |       |             | (+;0,2)     | (+;1,3)     | (+;2,4)     |             |
|      |       |             | (+;2,0)     | (+;3,1)     | (+;4,2)     |             |
| 1/4  |       | (4,0), (2,2),(0,4)| (4,4)     |             | (+;3,3)     |             |
|      |       |             | (+;1,1)     |             |             |             |
It would also be nice to study the AdS origin of the oblique D-branes by following [45].

As an interesting application of our procedure, we can consider noncommutative M-branes. For noncommutative M-branes, the classification of the possible Dirichlet branes is drastically modified and one can see some interesting features. For this subject we will report in other papers soon [46, 47].

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