Inclusive quarkonium production and the NRQCD factorization approach

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I discuss the current status of the comparison between quarkonium-production measurements and the predictions of the NRQCD factorization formalism.

I. NRQCD FACTORIZATION

In calculating rates for quarkonium production or decay, one would like to separate the short-distance heavy-quark-antiquark (Q̅Q) annihilation or production process, which has a typical momentum scale $p \gtrsim m_Q$, and can be treated perturbatively, from the long-distance quarkonium dynamics, which have a typical momentum scale $p \lesssim m_Q v$ and are nonperturbative in character. Here $m_Q$ is the heavy-quark mass, and $v$ is the relative velocity of the $Q$ and $Q̅$ in the quarkonium rest frame. Such a separation of long- and short-distance scales in heavy-quarkonium production and decay can be expressed elegantly in terms of the effective field theory Nonrelativistic QCD (NRQCD) [1].

In the cases of inclusive quarkonium production at large transverse momentum $p_T$ and at large CM-frame momentum $p^*$, the cross section can be written in a factorized form as a sum of products of NRQCD matrix elements and short-distance coefficients:

$$\sigma(H) = \sum_n \frac{F_n(\Lambda)}{m_Q^2} \langle 0 | O^H_n(\Lambda) | 0 \rangle.$$ (1)

The $F_n(\Lambda)$ are short-distance coefficients. They are, essentially, the partonic cross sections to make a $Q̅Q$ pair and can be calculated as expansions in the strong-coupling constant $\alpha_s$. The (vacuum) matrix elements involve four-fermion operators, which have the form

$$O^H_n = \chi^\dagger \kappa_n \psi \left( \sum_X \langle H + X | H + X \rangle \right) \psi^\dagger \kappa'_n \chi.$$ (2)

Here, $\psi$ is the Pauli spinor field that annihilates a heavy quark, $\chi$ is the Pauli spinor field that creates a heavy antiquark, and $\kappa$ contains Pauli matrices, color matrices, and covariant derivatives. The operator creates a $Q̅Q$ pair in a state with certain color, spin, and orbital-angular-momentum quantum numbers, projects it onto an intermediate state containing a heavy quarkonium $H$ plus anything, and annihilates a $Q̅Q$ pair with specific quantum numbers from that state. The operator matrix elements contain all of the long-distance (nonperturbative) physics. They are the probabilities for a $Q̅Q$ pair to evolve into a heavy quarkonium.

A similar factorization formula applies to inclusive quarkonium decays. The decay matrix elements are the crossed versions of quarkonium production matrix elements. Only the color-singlet production and decay matrix elements are simply related.

The NRQCD factorization formalism gains much of its predictive power from the fact that the nonperturbative operator matrix elements are universal, i.e., process independent. Although some decay matrix elements have been computed on the lattice [2, 3], in general, the matrix elements must be extracted phenomenologically. The consistency of the phenomenological matrix elements from process to process is a key test of the NRQCD factorization formalism.

NRQCD also predicts velocity-scaling rules [1], which give the leading power behavior of the matrix elements as functions of $v$. It follows that the sum over operator matrix elements in Eq. (1) is actually an expansion in powers of $v$. For charmonium, $v^2 \approx 0.3$; for bottomonium, $v^2 \approx 0.1$.

An important feature of the NRQCD factorization formalism is that quarkonium decay and production occur through color-octet, as well as color-singlet, $Q̅Q$ states. If one drops all of the color-octet contributions, then the result is the color-singlet model (CSM). In contrast, NRQCD factorization is not a model. It sometimes is called, erroneously, “the color-octet model,” but it is, rather, a consequence of QCD in the limit $m, p_T \gg \Lambda_{QCD}$.

A proof of NRQCD factorization would rely both on NRQCD itself and on the machinery that is used to prove factorization of hard-scattering processes in QCD. Although it is widely believed that the NRQCD factorization formula (1) can be established by standard methods, no detailed proof exists in the literature. It is expected that corrections to the factorization formula are of order $\Lambda_{QCD}^2/p_T^2$ for unpolarized cross sections and $\Lambda_{QCD}/p_T$ for polarized cross sections [4].

II. SOME SUCCESSES OF THE NRQCD FACTORIZATION FORMALISM

Predictions of the NRQCD factorization formalism have been confirmed in a number of decay and production processes. In this talk, I will focus on quarkonium
The latter uncertainties were estimated by varying the choices of renormalization and factorization scales. Theoretical uncertainties arise mainly from uncertainties in the color-octet matrix elements and from the model. Theoretical uncertainties are exacerbated when one uses the fitted values of the matrix elements to make predictions for other processes. The Tevatron data for J/ψ production are also fit well by the NRQCD factorization prediction, and the lower set of curves is the color-singlet-model prediction. The data points are from the Delphi measurement [9]. From Ref. [10].

The color-octet contributions offer a possible explanation for the undershoot in the cross section at LEP. The leading-order NRQCD factorization and CSM predictions of Ref. [11] for the J/ψ inclusive production cross section $d\sigma/dp_T^2$ in $pp$ deep-inelastic scattering (DIS) are shown in Fig. 3, along with the H1 data. As can be seen, the data clearly favor the NRQCD factorization prediction over the CSM prediction. A similar situation holds for $d\sigma/dQ^2$. Surprisingly, the cross section $d\sigma/dz$, which is differential in the energy fraction (inelasticity) $z$, is not fit well by either the NRQCD factorization or CSM predictions. The former undershoots the data at large $z$ and undershoots the data small $z$, while the latter undershoots the data at all $z$. It is worth noting that the calculation of Ref. [11] disagrees with a number of previous results [13–17], which themselves are not fully consistent. Those discrepancies have not yet been resolved completely.

III. SOME PROBLEMATIC COMPARISONS WITH EXPERIMENT

There are several notable processes for which the comparison of NRQCD factorization predictions with the data is less than satisfactory.
A. Polarization of quarkonium at the Tevatron

The polarization of quarkonium produced at large $p_T$ at the Tevatron provides a potentially definitive test of the color-octet mechanism. Quarkonium production at large-$p_T$ ($p_T > 4m_c$ for the $J/\psi$) is dominated by gluon fragmentation into the quarkonium through the $^3S_1$ color-octet matrix element. At large $p_T$, the fragmenting gluon is nearly on its mass shell, and, so, is nearly transversely polarized. According to the NRQCD velocity-scaling rules, spin-flip interactions are suppressed relative to non-spin-flip interactions. Therefore, it is expected that most of the gluon’s polarization is transferred to the $J/\psi$ [18]. Radiative corrections and color-singlet production dilute this polarization [19–21]. Nevertheless, substantial polarization is expected at large $p_T$. In the $J/\psi$ case, feeddown from the $\psi'$ and the $\chi_c$ states is important and has now been taken into account [22]. The NRQCD factorization prediction for the $J/\psi$ polarization as a function of $p_T$ is shown, along with the CDF data, in Fig. 4. The quantity $\alpha$ parametrizes the angular distribution of the decay leptons in the $J/\psi$ rest frame: $d\sigma/d(\cos \theta) \propto 1 + \alpha \cos^2 \theta$. Here, $\theta$ is the angle between the three-momentum of the positive lepton in the $J/\psi$ rest frame and the boost vector from the $J/\psi$ rest frame to the CM frame of the colliding hadrons. $\alpha = 1$ corresponds to 100% transverse polarization; $\alpha = -1$ corresponds to 100% longitudinal polarization. Polarization of produced $\psi'$ mesons is simpler theoretically, since feeddown is not important, but, unfortunately, the experimental statistics are much poorer.

As can be seen, the observed $J/\psi$ polarization is generally smaller than the prediction and seems to trend in the wrong direction, decreasing with increasing $p_T$. However, the experimental error bars are large, and only the last data point truly disagrees with the prediction. Furthermore, there are large uncertainties in the theoretical predictions. There are uncertainties in the NRQCD matrix elements, which are reflected in the prediction band in Fig. 4. There are uncertainties from uncalculated contributions of higher order in $\alpha_s$, including effects from multiple soft-gluon emissions and $k$-factors. Since the polarization depends on a ratio of matrix elements, it probably is not strongly affected by corrections to the matrix-elements fits from such higher-order effects. Next-to-leading-order corrections have been calculated for $^3S_1$ color-octet fragmentation [19, 24, 25], which gives the bulk of the polarization. Corrections to the non-fragmentation process could conceivably increase the unpolarized contribution by a factor of two. There are also large order-$v^2$ corrections to gluon fragmentation to quarkonium [26], but the principal effect of these is to change the size of the corresponding color-octet matrix element in fits to the Tevatron data, which does not affect the polarization prediction. The large order-$v^2$ corrections do, however, raise questions as to the convergence of the NRQCD $v$ expansion.

Existing calculations assume that 100% of the $Q\overline{Q}$ polarization is transferred to the quarkonium. Spin-flip corrections are suppressed only by $v^2$, not $v^4$, relative to the non-flip part [1]. It could happen that the spin-flip corrections are anomalously large and depolarize the produced quarkonium. It has also been suggested that the velocity-scaling rules may need to be modified for the charmonium system [27, 28]. These issues should be resolved by a lattice calculation that is in progress [29].

B. Inelastic photoproduction at HERA

Theoretical calculations of the cross section for inelastic photoproduction of quarkonium at HERA have been
carried out in the NRQCD factorization formalism by several groups [30–36]. The compilation of predictions from Ref. [6] and the H1 and Zeus data are shown in Fig. 5, plotted as function of the energy fraction $z$. As can be seen, the color-octet contribution is poorly determined, owing to large uncertainties in the color-octet matrix elements. Even so, there is little room for a color-octet contribution. Furthermore, as is shown in Fig. 6, corrections of next-to-leading order in $\alpha_s$ (NLO) [41, 42] increase the color-singlet piece by about a factor of two at large $z$ and are, by themselves, in good agreement with the data. The data shown in Figs. 5 and 6 are for the cut $p_T > 1$ GeV. One can question whether factorization is valid at such small values of $p_T$. However, the data differential in $p_T$ are compatible with NLO color-singlet production alone at large $p_T$ [6]. It should be noted, though, that there are large uncertainties in the NLO color-singlet contribution, which arise primarily from uncertainties in $m_c$ and $\alpha_s$. The true color-singlet contribution could be lower than the central value by about a factor of two, leaving more room for a color-octet contribution.

Near $z = 1$, the leading-order color-octet contribution grows rapidly, in apparent disagreement with the data. However, in this region soft-gluon emission leads to large logarithms of $1 - z$ and also to large corrections of higher order in $v$, both of which must be resummed. The resummation of the corrections of higher order in $v$ leads to a nonperturbative “shape function” [43]. Both the shape function and the resummed logarithmic corrections significantly smear out the color-octet contribution near $z = 1$. The effects of a model calculation of the shape function [44] on the leading-order NRQCD factorization predictions are shown in Fig. 7, along with the H1 data. The Zeus data [40] show a similar behavior. As can be seen, the inclusion of shape-function effects may lead to a considerable improvement in the agreement of the NRQCD factorization predictions with the data. The resummation of logarithms is expected to have a comparable effect. (Note that the higher-$p_T$ data shown in Fig. 7 are more compatible with a color-octet contribution than the data shown in Fig. 5). Effects from resummation of logarithms of $1 - z$ and model shape functions have been calculated for the process $e^+ e^- \rightarrow J/\psi + X$ [45]. For shape functions that satisfy the velocity-scaling rules, these effects are comparable in size. It may be possible
to use this resummed theoretical prediction to extract the shape function from the Belle and BaBar data for $e^+e^- \rightarrow J/\psi + X$ and then use it to make firm predictions for $J/\psi$ photoproduction near $z = 1$.

C. Double $c\bar{c}$ production at Belle

For the exclusive double charmonium process $e^+e^- \rightarrow J/\psi + \eta_c$, the Belle Collaboration measures a cross section times a branching ratio into at least four charged tracks of $46\pm6_{-5}^{+7}$ fb [46]. In contrast, leading-order calculations predict a cross section of $2.31 \pm 1.09$ fb [47–49]. There are some uncertainties from uncalculated corrections of higher-order in $\alpha_s$ and $\alpha$ and from NRQCD matrix elements. However, because this is an exclusive process, only color-singlet matrix elements enter, and these are fairly well determined from the decays $J/\psi \rightarrow e^+e^-$ and $\eta_c \rightarrow \gamma\gamma$.

Since the Belle mass resolution is 110 MeV but the $J/\psi$ mass difference is only 120 MeV, it has been suggested that some of the $J/\psi + \eta_c$ data sample may consist of $J/\psi + J/\psi$ events [50, 51]. The state $J/\psi + J/\psi$ has charge-parity $C = +1$, and consequently, is produced in a two-photon process, whose rate is suppressed by a factor $(\alpha/\alpha_s)^2$ relative to the rate for $J/\psi + \eta_c$. However, as was pointed out in Refs. [50, 51], the two-photon process contains photon-fragmentation contributions that are enhanced by factors $(E_{\text{beam}}/2m_c)^4$ from photon propagators and log(8$(E_{\text{beam}}/2m_c)^4$) from a would-be collinear divergence. As a result, the predicted cross section

\[ \sigma(e^+e^- \rightarrow J/\psi + J/\psi) = 8.70 \pm 2.94 \text{ fb} \]

is larger than the predicted cross section

\[ \sigma(e^+e^- \rightarrow J/\psi + \eta_c) = 2.31 \pm 1.09 \text{ fb} \]

although corrections of higher order in $\alpha$ and $\alpha/\alpha_s$ likely reduce the former prediction by about a factor of three. These predictions spurred a re-analysis of the Belle data [52], with the result that there is no significant $J/\psi + J/\psi$ signal observed $\sigma(e^+e^- \rightarrow J/\psi + J/\psi) < 7 \text{ fb}$.

There are also Belle results on inclusive double-charmonium production. For the ratio $R_{J/\psi} = \sigma(e^+e^- \rightarrow J/\psi + c\bar{c})/\sigma(e^+e^- \rightarrow J/\psi + X)$, the most recent Belle analysis yields $R_{J/\psi} = 0.82 \pm 0.15 \pm 0.14$, with $R_{J/\psi} > 0.48$ (90% confidence level) [46]. Predictions based on NRQCD factorization [53–55] give $R_{J/\psi} \approx 0.1$. The measured and predicted $J/\psi + c\bar{c}$ absolute cross sections also disagree by almost an order of magnitude, with the Belle result [56] being about 0.6–1.1 pb and the prediction [53–55] being about 0.10–0.15 pb. This prediction is based only on the color-singlet contribution. However, corrections of higher order in $\alpha_s$ and $\alpha/\alpha_s$ are not expected to be large. Neither are corrections of higher order in $\alpha_s$.

The discrepancies in the double $c\bar{c}$ inclusive and exclusive cross sections are among the largest in the standard model. Theory and experiment differ by about an order of magnitude—a discrepancy which is larger than any known QCD k-factor. It is important to recognize that these discrepancies are problems not just for NRQCD factorization, but for perturbative QCD (pQCD) in general. In the case of the cross section for $e^+e^- \rightarrow J/\psi + \eta_c$, one obtains exactly the same predictions in the NRQCD factorization [47, 48] and light-front-QCD [49] formalisms. With regard to the fraction of $J/\psi + X$ events that are $J/\psi + c\bar{c}$, it is difficult to see how any perturbative calculation could give a value as large as 80%. The color-evaporation model, for example, proceeds through the same Feynman diagrams as the NRQCD factorization calculation, differing only in the specific treatment of the evolution of the $c\bar{c}$ pair into quarkonium, and would, therefore, be expected to give a prediction that is not too different from that of NRQCD factorization.

Clearly, it is very important to have independent checks of the Belle double-charmonium results, such as could be provided by the BaBar collaboration. If the Belle results are confirmed, then we would be forced to entertain some unorthodox possibilities: there are new charmonium production mechanisms within the standard model that have not yet been recognized, pQCD is inapplicable to double-charmonium production, or physics beyond the standard model plays an important role. It would be very surprising, however, if the last possibility could manifest itself at such low energies.

IV. SUMMARY

The NRQCD factorization approach provides a systematic method for calculating quarkonium decay and production rates a double expansion in powers of $\alpha_s$ and $\alpha$. Calculation of production rates also relies upon hard-scattering factorization, for which corrections are suppressed by powers of $\Lambda_{QCD}/p_T$. NRQCD factorization has enjoyed a number of successes, for example, in quarkonium production at the Tevatron, $\gamma\gamma \rightarrow J/\psi + X$ at LEP, and quarkonium production in DIS at HERA. Other experimental tests are, so far, more problematic. These include quarkonium polarization at the Tevatron, inelastic quarkonium photoproduction at HERA, and double $c\bar{c}$ production at Belle. The Belle double $c\bar{c}$ production results present a severe challenge to pQCD. It would be very useful for the BaBar collaboration to check these results. In other cases, inclusion of corrections of higher order in $\alpha_s$ and $\alpha$ and resummation of soft-gluon effects and endpoint corrections of higher order in $\alpha$ should help to achieve agreement between theory and experiment. More precise theoretical predictions are hampered by uncertainties in the NRQCD matrix elements. Lattice calculations can help to pin down the decay matrix elements, but it is not yet known how to formulate the calculation of production matrix elements on the lattice. This is an exciting time for heavy-quarkonium physics, with a great deal of experimental and theoretical activity in quarkonium decay and spectroscopy, as well as production, and we can expect to see continuing progress on the many challenging problems that remain.
