The proton structure via double parton scattering

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Abstract. In this talk we present the results of the investigation on the so called double parton distribution functions (dPDFs), accessible quantities in high energy proton-proton and proton nucleus collisions, in double parton scattering processes (DPS). These new and almost unknown distributions encode information on how partons inside a proton are correlated among each other and represent a new tool to explore the three dimensional partonic structure of hadrons. In the present contribution, results of the calculations of dPDFs are presented also including phenomenological investigations on the impact of double correlations in experimental observables, showing how the latter could be observed in the next LHC run. In addition we discuss how present information on experimental observables could be related to the transverse proton structure.

1. Introduction
Thanks to the high luminosity reached in collider experiments, such as at the LHC, the investigation on multiple parton interactions (MPI) became relevant in the study of hadron-hadron collisions. In these kinds of events, more than one parton of a hadron interacts with partons of the other colliding hadron. However, the MPI contribution to the total cross section is suppressed with respect to the single partonic interaction. Nevertheless, MPI represent a background for the search of new Physics at the LHC and the measurement of their cross sections is an important experimental challenge. From a theoretical point of view, one of the main interests on MPI is the possibility of accessing new fundamental information on the partonic proton structure [1, 2]. To this aim, we focused our attention on the double parton scattering (DPS), the most simple case of MPI, which can be observed, in principle, in several processes, e.g., WW with dilepton productions and double Drell-Yan processes (see, Refs. [3, 4, 5, 6, 7] for recent reviews). At the LHC, DPS has been observed some years ago [8] and represents also a background for the Higgs production in several channels. Formally, the DPS cross section depends on the so called double parton distribution functions (dPDFs), \( F_{ij}(x_1, x_2, \vec{z}_\perp, \mu) \), which describe the joint probability of finding two partons of flavors \( i, j = q, \bar{q}, g \) with longitudinal momentum fractions \( x_1, x_2 \) and distance \( \vec{z}_\perp \) in the transverse plane inside the hadron, see Ref. [9]. Here \( \mu \) is the renormalization scale. Since for the moment being no data are available for dPDFs, they are usually approximated in experimental analyses through a fully factorized ansatz:

\[
F_{ij}(x_1, x_2, \vec{z}_\perp, \mu) = F_i(x_1, \mu)F_j(x_2, \mu)T(\vec{z}_\perp), \quad (1)
\]
where $F_i(x, \mu)$ are the usual one-body parton distribution functions (PDFs) and $T(z_{\perp})$, a distribution encoding the probability of finding two partons at distance $z_{\perp}$. In assumption (1) all possible unknown double parton correlations (DPC) between the two interacting partons (e.g. Ref. [10]) have been neglected. However, dPDFs are non perturbative quantities in QCD and they cannot be easily evaluated, thus constituent quark model (CQM) calculations could help to grasp their basic features. In particular we studied to which extent the factorized (1) all possible unknown double parton correlations (DPC) between the two interacting partons could help to grasp their basic features. In particular we studied to which extent the factorized (1) all possible unknown double parton correlations (DPC) between the two interacting partons have been neglected. However, dPDFs are non perturbative quantities in QCD and they cannot be easily evaluated, thus constituent quark model (CQM) calculations could help to grasp their basic features. In particular we studied to which extent the factorized approximation (1) is a suitable ansatz for dPDFs [2, 11, 12, 13, 14, 15]. Double PDFs are first of all calculated at the low hadronic scale of the model, $\mu_0$, then, in order to compare the outcome with future data taken at high momentum transfer, $Q > \mu_0$, it is necessary to perform a perturbative QCD (pQCD) evolution by using dPDF evolution equations, see Refs. [16, 17, 18]. In particular, this step is fundamental to understand to what extent DPC survive at the kinematic conditions of experiments. Thanks to this procedure, future data analyses of the DPS processes could be guided, in principle, by model calculations, see Ref. [19]. To this aim in Refs. [13, 20], DPC in dPDFs have been studied at the energy scale of the experiments and the so called $\sigma_{eff}$, evaluated in Refs. [2, 15, 19, 21, 22]. In fact, DPS cross section, in processes with final state $A + B$, is written through the following ratio (see e.g. Ref. [23]):

$$\sigma_{DPS}^{A+B} = \frac{m \sigma_{DPS}}{2 \sigma_{eff}},$$

where $m$ is a combinatorial factor depending on the final states $A$ and $B$ ($m = 1$ for $A = B$ or $m = 2$ for $A \neq B$) and $\sigma_{DPS}^{A(B)}$ is the single parton scattering cross section with final state $A(B)$. The present knowledge on DPS cross sections has been condensed in the experimental extraction of $\sigma_{eff}$ [23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36]. A constant value, $\sigma_{eff} \approx 15$ mb, is compatible, within errors, with data: a result obtained within the fully uncorrelated ansatz for dPDFs (all DPC are neglected). In the next sections the results of the calculations $\sigma_{eff}$ within CQM will be described in order to characterize “signals” of DPC.

2. Constituent Quark Models Calculations of dPDFs

In our first studies, dPDFs have been calculated within CQM, see Refs. [12, 13], and it has been shown that at the hadronic scale, DPC cannot be neglected. For example, as one can see in the left panel of Fig. 1, the ratio $uu(x_1, 0.4, k_{\perp})/uu(0.4, 0.4, k_{\perp})$ smoothly depends on $k_{\perp}$ reflecting the presence of correlations between $x_1, x_2$ and $k_{\perp}$. In fact, such a ratio would be constant if the dPDF factorizes as in Eq. (1). Here we denote $\tilde{F}_{uu}(x_1, x_2, k_{\perp}) = uu(x_1, x_2, k_{\perp})$, $k_{\perp}$ being the conjugate variable to $z_{\perp}$ and $F$ the Fourier Transform of $\tilde{F}$. Furthermore, in Ref. [2, 15], it has been demonstrated that relativistic effects, encoded in the Light-Front (LF) approach [37, 38], used here to implement the relativity in the calculations, introduce these kind of correlations, independently of the chosen model for the proton wave function or of the energy scales at which the dPDFs are evaluated.

In addition in Refs. [13, 20], we discuss the role of perturbative and non perturbative $x_1, x_2$ correlations in dPDFs at high energy scales. Also in this case we found that in principle they
cannot be neglected. In a recent work, we also show the impact of correlations in the pion dPDFs, see Ref. [39, 40]. As one can see in the right panel of Fig. 1, correlations are fundamental also for the pion case. Here we remark that within a model where the pion is described with two constituent quarks, the dPDF depends only on $x$ and $k_{\perp}$ since $x_2 = 1 - x$. All these analyses show the relevance of double correlations in dPDFs. However, since these distributions are not directly accessible in experiments, in the next section we discuss the impact of DPC in experimental observables.

3. The calculation of the effective cross section

A fundamental quantity relevant for the experimental analyses of DPS is the so called effective cross section, $\sigma_{\text{eff}}$. As previously discussed, this quantity represents the ratio between the product of one body quantities to a two body function, see in Eq. (2). Thus, $\sigma_{\text{eff}}$ represents a valid tool to explore the impact of double correlations in measurements of DPS. In Ref. [21] we considered a direct evaluation of $\sigma_{\text{eff}}$ in terms of PDFs and dPDFs calculated within the same CQM. A suitable expression of $\sigma_{\text{eff}}$ has been found:

$$ \sigma_{\text{eff}}(x_1, x'_1, x_2, x'_2) = \frac{\sum_{i,k,j,l} F_i(x_1) F_k(x'_1) F_j(x_2) F_l(x'_2) C_{ik} C_{jl}}{\sum_{i,j,k,l} C_{ij} C_{kl}} \int \tilde{F}_{ij}(x_1, x_2; k_{\perp}) \tilde{F}_{kl}(x'_1, x'_2; -k_{\perp}) \frac{dk_{\perp}}{(2\pi)^2}. \quad (3) $$

where here $C_{ij}$ are colour factors depending upon flavor indexes $i, j, k, l$, see Ref. [21] for details.

Eq. (3) reduces to a constant number if only one flavor and the full factorization ansatz (1) would be considered. In Refs. [2, 21, 22] the above quantity has been evaluated within different CQMs at different energy scales without any approximation.

As one can observe in the left panel of Fig. 2, once $\sigma_{\text{eff}}$ is evaluated within CQM, where correlations are not neglected, this function strongly depends on the rapidity, i.e. on $x_1$ and $x_2$. This feature has been addressed in further analyses in Refs. [2, 22, 39]. In Ref. [22], $\sigma_{\text{eff}}$ has been studied through dPDFs calculated within the AdS/QCD model. The mean value of $\sigma_{\text{eff}}$ and its $x$ dependence, obtained within this different model, is comparable with those found within the LF approach in Ref. [21]. Let us remark that the mean value of $\sigma_{\text{eff}}$ evaluated in Refs. [21, 22] is close to experimental analyses. Let us mention that even if the DPS involving
mesons is still unobserved, in Ref. [39] a model prediction of $\sigma_{\text{eff}}$ is provided. In this case the mean value is $\bar{\sigma}_{\text{eff}} \sim 40$ mb. Such a quantity is bigger than that evaluated for the proton. The interpretation of this result will be provided in the next section. Let us mention that $\bar{\sigma}_{\text{eff}}$ has been used in experimental analyses, see Ref. [41]. In order to establish whether these correlations could be observed in future experiments, in the next section we discuss the evaluation of $\sigma_{\text{eff}}$ for a specific DPS process.

### 3.1. The same-sign WW production at LHC

In order to establish to which extent double correlations could be accessed at the LHC, in Ref. [19] we have studied the same sign $W$ pair production process, a golden channel for the observation of DPS [42, 43, 44, 45]. See Ref. [46] for spin effects affecting this channel. The differential DPS cross section can be written as follows [9]:

$$d\sigma_{\text{DPS}}^{AB} = \frac{m}{2} \sum_{i,j,k,l} dz_{\perp} F_{ij}(x_1, x_2, \vec{z}_{\perp}, \mu) F_{ij}(x_3, x_4, \vec{z}_{\perp}, \mu) d\sigma_{\text{eff}}^A d\sigma_{\text{eff}}^B,$$

(4)

where $\vec{z}_{\perp}$ represents the elementary cross section. As non perturbative input of the calculations, use has been made of dPDFs evaluated in Ref. [13] without any approximations. Details of the fiducial DPS phase space adopted in the analysis are discussed in Ref. [19]. In particular we found that the total $W$-charge summed DPS cross section (considering both $W$ decays into same sign muons), calculated by means of dPDFs [13], is found to be $\sigma^{++} + \sigma^{--} \sim 0.69$. This result is consistent with those obtained by using for dPDFs the ansatz Eq. (1) with PDFs evaluated with the MSTW parametrization [47] and those obtained with dPDFs of the model described in Ref. [3]. The effects of DPC have been investigated by observing the $\bar{\sigma}_{\text{eff}}$ dependence on $\eta_1 \cdot \eta_2 \simeq 1/4 \ln(x_1/x_3)\ln(x_2/x_4)$. For this process we found a mean value $\langle \bar{\sigma}_{\text{eff}} \rangle \sim 21.04$ mb, consistent with calculations discussed in previous section. See Ref. [19] for details on the theoretical errors. A clear signature of the presence of double correlations is found by observing the departure of $\bar{\sigma}_{\text{eff}}$ from a constant, see right panel of Fig. 2. We have estimated that, with a luminosity of $L \sim 1000$ fb$^{-1}$, at 68% confidence level, such a departure of $\sigma_{\text{eff}}$ from a constant value can be measured in the next run of the LHC.

### 4. The link between $\sigma_{\text{eff}}$ and the transverse proton structure

While in the previous section it has been shown whether correlations could be observed in future experiments at the LHC, in this last part we show how collected experimental outcomes of $\sigma_{\text{eff}}$ could shed some light on the transverse proton structure. To this aim here we discuss the strategy adopted in Ref. [1]. In order to interpret many of the experimental estimates of $\sigma_{\text{eff}}$, the factorized ansatz (1) adopted in that study has been considered. In this scenario:

$$\sigma_{\text{eff}}^{-1} = \int d^2 z_{\perp} T(z_{\perp})^2 = \int \frac{d^2 k_{\perp}}{(2\pi)^2} \tilde{T}(\vec{k}_{\perp}) \tilde{T}(-\vec{k}_{\perp}),$$

(5)

where $\tilde{T}(k_{\perp})$ is the Fourier Transform of $T(z_{\perp})$ and called effective form factor (eff). In Refs. [1, 2], it has been shown how the probabilistic interpretation of $T(z_{\perp})$ and the asymptotic behaviour of $\tilde{T}(k_{\perp})$ allow to relate $\sigma_{\text{eff}}$ to the mean transverse distance $\langle z_{\perp}^2 \rangle$ between two active partons in a DPS process. In fact, $\tilde{T}(0) = 1$, $\tilde{T}(k_{\perp} \to \infty) \to 0$ and $\langle z_{\perp}^2 \rangle = -4 d\tilde{T}(k_{\perp})/dk_{\perp}^2 |_{k_{\perp}=0}$. In particular one can show that in the non relativistic limit one gets:

$$\tilde{T}(k_{\perp}) = \int dk_1 \int dk_2 \psi(\vec{k}_1 + \vec{k}_{\perp}, \vec{k}_2) \psi(\vec{k}_1, \vec{k}_2 + \vec{k}_{\perp}),$$

(6)
where $\vec{k}_i$ is the momentum of a parton $i$ and $\Psi$ is the proton wave function. As one can see the above expression is similar to the standard proton form factor (ff). However, since in this case there is a double momentum imbalance, i.e. $k_\perp$, the eff can be considered a double form factor. As deeply discussed in Refs. [1, 2], one should expect that in the extremely high $k_\perp$ region, the eff should fall to zero at least as the standard ff. Of course since the eff is a double ff it is reasonable that it goes to zero faster then the one body one, as discussed in Ref. [1, 2, 18].

Thanks to these almost general and model independent conditions, we found:

$$\frac{\sigma_{\text{eff}}}{3\pi} \leq \langle z^2 \rangle \leq \frac{\sigma_{\text{eff}}}{\pi}.$$  

(7)

The above expression has been validated by all models of dPDFs at our disposal, even for the pion target. In Fig. 3 the experimental values of $\sigma_{\text{eff}}$, related to different processes and different final states obtained by many collaborations, have been used in Eq. (7) to get the mean transverse distance of the active partons in a DPS process.

![Figure 3: The application or Eq. (7) using data of Refs. [31, 32, 33, 34, 35, 36]. The green vertical line stands for the transverse charge proton radius.](image)

The above relation has been properly generalised in Ref. [2] in order to include correlations between $x_1, x_2$ and $k_\perp$ going beyond the approximation Eq. (1). In addition, Eq. (7) has been extended to also include the splitting effects, i.e. the DPS occur between two partons produced by the splitting of a parent one. Within these results information on the transverse structure of the proton can be obtained from detailed experimental analyses of $\sigma_{\text{eff}}$.

5. Conclusions

In this contribution we have shown and discussed results of the calculations of dPDFs remarking on the impact of double parton correlations and the violation of the factorized ansatz usually assumed in experimental analyses. In particular we focus our attention on the role of correlations in experimental observables such as $\sigma_{\text{eff}}$. In this case we found that the dependence of $\sigma_{\text{eff}}$ upon the longitudinal momentum fraction of the partons represents clear sign ofn correlations. In particular we estimated that their effects could be observed in the next LHC run. From a different perspective, our analysis also shows how to unveil new information on the mean transverse distance of partons inside the proton from the present status of experimental investigations on DPS. All these studies point to the crucial role of dPDFs as fundamental tools to obtain new details on the non perturbative structure of the proton.

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