Covariant propagator and chiral power counting
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Abstract
Some one-loop diagrams with one and two external baryons/nucleons are revisited using covariant baryon propagators in chiral effective theory. We showed that it is enough to separate and subtract all the local terms that violate chiral power counting to recover chiral power counting, no need to introduce extra operations. The structures of leading chiral corrections and IR enhancement or threshold effects are ’stable’ or persist as long as covariant propagators are employed for all particles.

Keywords: chiral effective theories, covariant propagator, power counting

1. Introduction
Chiral perturbation theory is a beautiful effective theory of QCD at low energies that works perfectly in pure pion sector. In the one baryon sector, it is convenient to adopt the heavy baryon (HB) formalism[1, 2] for many topics[3]. Since Weinberg’s seminal paper in 1990[4] and the pioneering works[5], chiral perturbation theory has been extended to nuclear systems with significant successes[6, 7, 8, 9, 10]. Nevertheless, things become rather complicated when baryons are involved in loops[11]: If one works with covariant baryon propagators, baryon masses contribute extra large scales that upset the standard chiral power counting[12, 13]. In spite that it is enough to employ the HB formalism in one baryon sector to preserve chiral power counting, there do be occasions where it is flawed (mainly due to the loss of correct kinematics controlled by relativistic propagators) and not trustworthy[14]. This complication persists in the chiral effective theories for nucleonic systems or nuclear forces and has been intensively studied, see, e.g., Refs.[15, 16, 17, 18, 19, 20, 21]. Of course, the nonperturbative nature of nuclear system due to the large scattering lengths, etc., renders things even more difficult.

In recent years, there appear some evidences in favor of using covariant formulation of baryonic sectors[22, 23] where the key observation is that the large items that violate the standard chiral power counting are local and hence could be subtracted away by adopting an extended on-mass-shell (EOMS) prescription[24]. The recent efforts in constructing nuclear forces from a covariant approach show encouraging prospects[22, 25]. In our view, one could work in a very convenient and unified manner for all the cases involving heavy hadron propagators in chiral loops: merely isolating the local large (non-chiral) terms and remove them via local counter terms. The retained chiral terms are what one would have in HB formulation. For multi-baryonic processes, there may be nonlocal large terms due to ’infrared enhancement’ and/or threshold effects that become singular in HB formalism[14] and incur extra power like divergences due to non-relativistic treatment, which could only be definite within covariant formulation[23].

In this report, we wish to revisit some one-loop diagrams involving one and two baryons/nucleons to examine the chiral structures in a unified perspective. These diagrams will be recalculated using covariant baryonic propagators throughout the whole work, first with covariant vertices and then with HB vertices and composite operators. The standard chiral power counting is indeed revived after removing some local pieces. Moreover, provided that covariant propagators are employed, different version of pion-baryon interactions do not affect the leading chiral contributions (and the possible IR enhanced terms) after the large local pieces are separated and removed. This ’stability’ of the chiral components suggests a extended subtraction scheme: First separate and remove all the local large terms due

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to baryon/nucleon masses, then subtract the remaining chiral divergences. Generically, one may anticipate that in covariant formulation for heavy hadrons (including bosons), all the local non-chiral (large) terms warrant an extended subtraction according to the spirit of decoupling [26, 27, 28], a prescription that naturally preserves chiral power counting, while dispensing extra manipulations and automatically covering the sector with purely chiral degrees. In this sense, we arrived at a unified scheme for chiral effective theories for hadronic and nuclear physics. The nonperturbative treatment or resummation of certain definite items, according to our view, should also be pursued in covariant or relativistic formulation.

This report is organized as follows: In Sec. 2 we present the set up for our investigation. In Sec. 3 we present our main analysis of the one-loop diagrams calculated in covariant and mixed formulation, demonstrating the detailed chiral properties of these amplitudes. The summary is given in Sec. 4.

2. Theoretical setup

Here we present the main tools to be used and the objects to be examined. We will consider diagrams with two and four external lines of nucleons for simplicity. As a low-energy effective theory of QCD, chiral perturbation theory has been first given in covariant formulation in history [11]. In this work, we focus on the SU(2) chiral effective field theory. The covariant Lagrangian we work with reads (see Ref. [10])

\[ \mathcal{L}_{\chi ET} = \mathcal{L}_{\chi} + \mathcal{L}_{\pi N} + \cdots, \]

where \( \mathcal{L}_{\chi} \) is the effective theory of QCD, chiral perturbation theory

\[ \mathcal{L}_{\pi N} = \left[ - \frac{g_A}{2 f_\pi} \gamma^5 \tau \cdot \partial \phi \right] \Psi_N \left[ - \frac{1}{4 f_\pi^2} \tau \cdot (\pi \times \partial \phi) + O(\pi^3) \right] \Psi_N. \]

The HB form of the \( \pi - N \) interaction reads

\[ \mathcal{L}_{HB, int} = \bar{\Psi}_\nu \left[ - \frac{g_A}{2 f_\pi} \gamma^5 \tau \cdot \partial \phi \right] \Psi_\nu \left[ - \frac{1}{4 f_\pi^2} \tau \cdot (\pi \times \partial \phi) + O(\pi^3) \right] \Psi_\nu, \]

where \( S^\mu \equiv \frac{1}{2} \gamma^5 \sigma^{\mu\nu} \gamma_\nu = -\frac{i}{2} \gamma^5 (\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu), \gamma^2 = 1, \Psi_\nu(x) = \epsilon^{\mu
u\rho\sigma} P_\nu \psi(x), P_\nu \equiv \frac{1}{2} (1 \pm \gamma^5). \]

In Refs. [29, 30], it is shown that in low momentum transfer the chiral properties or non-analytical quark mass dependence of the matrix elements of traceless twist-2 operators between hadronic states could be computed through chiral perturbation theory in HB formulation. In this work we will illustrate our points with the simple case \( n = 1 \), namely, the diagrams with following composite operators:

\[ \mathcal{O}^{(1)}_{\mu}(n) = 2 \epsilon^{\mu\nu\rho\sigma} \partial_\rho \psi^\sigma + O(\pi^3), \]

\[ \mathcal{O}^{(1)}_{\mu, \text{cov}}(N) = A^{(n)} \left[ \gamma_\mu \left( \frac{1}{2 f_\pi^2} \tau \cdot \partial \phi \right) + \frac{g_A}{f_\pi} \gamma^5 (\tau \times \pi)^n \right] \Psi_N + O(\pi^3), \]

\[ \mathcal{O}^{(1)}_{\mu, \text{Hb}}(N) = A^{(n)} \left[ \gamma_\mu \left( \frac{1}{2 f_\pi^2} \tau \cdot \partial \phi \right) + \frac{g_A}{f_\pi} \gamma^5 (\tau \times \pi)^n \right] \Psi_N + O(\pi^3). \]

Here, we note that the third term with single pion is not listed out in Ref. [30] as it does not contribute in the HB formulation. However, it does contribute in covariant formulation, which is necessary to match with the HB results, as will be shown below.

The specific diagrams to be recalculated are listed in Fig. 1, while the last two diagrams (i.e., (f) and (g)) actually do not contribute in the HB formulation. The matrix elements of such operators \( (n = 1) \) between nucleon states will be denoted as

\[ M^{(1)}_{\mu}(p, p) \equiv \langle N(p) | \mathcal{O}^{(1)}_{\mu}(N) | N(p) \rangle, \quad p^2 = M_N^2. \]
We will also recalculate the one-loop diagrams for \( NN \) scattering listed in Fig. 2, namely the triangle and box diagrams for our purpose, the foot-ball diagram will not be considered as it does not contain baryon lines. To focus on our main points, we put that \( \rho = \bar{\rho} = p' = \bar{p}' = (M_N, 0) \):

\[
\mathcal{M}_{NN} (p, \bar{p}; p', \bar{p}') \equiv \langle N(p), N(\bar{p}) | T_{\chi ET} | N(p'), N'(\bar{p}') \rangle.
\]

(9)

3. Calculation and Analysis

3.1. Covariant and mixed calculations

The detailed results of completely covariant calculation are listed in the Appendix A. Here, we make some remarks about the results: First, it is straightforward to see that by requiring that \( A^{(1)} = a^{(1)} = 1 \), the items from Fig. 1 constitute the one-loop corrections to the vector current operator sandwiched between nucleon state, hence sum up to zero, i.e., the current is not renormalized as is the usual case for a conserved current:

\[
\sum_{i=a}^{g} \mathcal{M}^{(1)}_{\mu(i)} = 0.
\]

(10)

Like Fig. 2(a), Fig. 1(c) and Fig. 1(d) are completely chiral and will not be considered from now on. Second, there is a nonlocal and hence definite 'large piece' \( \sim -\frac{1}{2\rho} \arctan \sigma \) in Fig. 2(a) due to IR enhancement, which is also the dominant contribution to the contact \( NN \) interaction \( C_0(\bar{N}N)^2 \). This term remains definite in the mixed calculation, see Appendix B.

In the mixed calculation using covariant propagators and HB vertices and composite operators (Appendix B), vector current conservation is no longer valid

\[
\sum_{i=a}^{g} \mathcal{M}^{\mu(i)}_{\text{subtracted}} = -\bar{u} r^\mu v_{ij} \frac{5g_3^2 m_N^2}{16(4\pi f_\pi)^2} \left\{ \frac{3}{2} \left( \Gamma_\epsilon + \ell_N + \frac{13}{30} \right) + \left( \Gamma_\epsilon + \ell_N + \frac{3}{5} \right) + \frac{4}{5} (\mathcal{R}_{1a} - \mathcal{R}_{1b}) \right\} \neq 0.
\]

(11)

Obviously, the main sources of violation in the mixed calculation are the local large or non-chiral pieces. In other words, the chiral terms still preserve vector current conservation! This observation suggests that we should subtract all the local non-chiral terms, then canonical relations like current conservation would be preserved in chiral limit \( (\rho \to \infty) \):

\[
\sum_{i=a}^{g} \mathcal{M}^{\mu(i)}_{\text{subtracted}} = -\bar{u} r^\mu v_{ij} \frac{5g_3^2 m_N^2}{16(4\pi f_\pi)^2} \cdot \frac{4}{5} (\mathcal{R}_{1a} - \mathcal{R}_{1b}) = o(\rho^{-1/2}).
\]

(12)

For a more transparent analysis, it is instructive to decompose the amplitudes into terms of different chiral status, as will be done in next subsection.
3.2. Decomposition and separation

To explore the properties of these one-loop diagrams, we decompose each of them into four categories: local items that are non-chiral, local item that is chiral, nonlocal relativistic corrections (i.e., the $R$ terms) that vanish as $\rho \rightarrow \infty$, and possible nonlocal ’large’ items that arise from IR enhancement [4] or threshold effects [14] and could not be subtracted by local counterterms, hence termed ’anomalous’ here. To this end, we need the following crucial identity

$$
\ln \rho = \Gamma_\epsilon + \ell_\sigma - (\Gamma_\epsilon + \ell_N)
$$

(13)

for terms tied with powers $\rho^k$, $k = 0, 1$.

The results from covariant calculation (Appendix A) are then decomposed as below:

$$
M_{\mu\gamma(1)}^{(1)} = -A^{(1)\mu} \gamma^\mu a \frac{g_5^2 m_2^2}{4(4\pi f_2)^2} \left\{ 2 \left( \Gamma_\epsilon + \ell_N + \frac{3}{2} \right) - 3 \left( \Gamma_\epsilon + \ell_\sigma + R_{1a} \right) \right\} = \frac{1}{3} M_{\mu\gamma(1)}^{(1)},
$$

(14)

$$
M_{\mu\gamma(1)b}^{(1)} = a^{(1)\mu} \gamma^\mu a \frac{g_5^2 m_2^2}{4(4\pi f_2)^2} \left\{ 2 \rho \left( \Gamma_\epsilon + \ell_N + 1 \right) + 4 \left( \Gamma_\epsilon + \ell_N + \frac{7}{4} \right) - 3 \left( \Gamma_\epsilon + \ell_\sigma + R_{1b} \right) \right\},
$$

(15)

$$
M_{\mu\gamma(1)c}^{(1)} = -A^{(1)\mu} \gamma^\mu a \frac{g_5^2 m_2^2}{4(4\pi f_2)^2} \left\{ \rho \left( \Gamma_\epsilon + \ell_N + 1 \right) + 4 \left( \Gamma_\epsilon + \ell_N + \frac{7}{4} \right) - 3 \left( \Gamma_\epsilon + \ell_\sigma + R_{1f} \right) \right\} = M_{\mu\gamma(1)}^{(1)},
$$

(16)

$$
M_{NN:2(b)}^{(1)} = (\tau_1 \cdot \tau_2) \frac{g_5^2 m_2^2}{128\pi^2 f_2^2} \left\{ 2 \rho \left( \Gamma_\epsilon + \ell_N + 1 \right) + 4 \left( \Gamma_\epsilon + \ell_N + \frac{7}{4} \right) - 3 \left( \Gamma_\epsilon + \ell_\sigma + R_{2b} \right) \right\} = M_{NN:2(b)},
$$

(17)

$$
M_{NN:2(c)}^{(1)} = - (3 - 2 (\tau_1 \cdot \tau_2)) \frac{g_5^2 m_2^2}{128\pi^2 f_2^2} \left\{ \rho \left( \Gamma_\epsilon + \ell_N + 3 \right) - 4 \left( \Gamma_\epsilon + \ell_N + \frac{13}{16} \right) + \frac{15}{4} \left( \Gamma_\epsilon + \ell_\sigma \right) \right\},
$$

(18)

$$
M_{NN:2(d)}^{(1)} = - (3 + 2 (\tau_1 \cdot \tau_2)) \frac{g_5^2 m_2^2}{128\pi^2 f_2^2} \left\{ 3 \rho \left( \Gamma_\epsilon + \ell_N + \frac{1}{3} \right) + 4 \left( \Gamma_\epsilon + \ell_N + \frac{17}{16} \right) - \frac{15}{4} \left( \Gamma_\epsilon + \ell_\sigma \right) + R_{2d} \right\}.
$$

(19)

Similarly, for those from mixed calculation (Appendix B) we have

$$
M_{\mu\gamma(1)\text{ext}}^{(1)}_{\mu\gamma(1)a} = -A^{(1)\mu} \gamma^\mu a \frac{g_5^2 m_2^2}{4(4\pi f_2)^2} \left\{ \frac{5}{2} \left( \Gamma_\epsilon + \ell_N + \frac{3}{5} \right) + \frac{15}{4} \left( \Gamma_\epsilon + \ell_N + \frac{5}{3} \right) - 3 \left( \Gamma_\epsilon + \ell_\sigma + R_{1a}^{\text{ext}} \right) \right\},
$$

(20)

$$
M_{\mu\gamma(1)\text{ext}}^{(1)\text{ext}}_{\mu\gamma(1)\text{ext}} = a^{(1)\mu} \gamma^\mu a \frac{g_5^2 m_2^2}{4(4\pi f_2)^2} \left\{ \frac{5}{2} \left( \Gamma_\epsilon + \ell_N + \frac{11}{10} \right) + \frac{5}{2} \left( \Gamma_\epsilon + \ell_N + \frac{11}{5} \right) - 3 \left( \Gamma_\epsilon + \ell_\sigma + R_{1b}^{\text{ext}} \right) \right\} = M_{\mu\gamma(1)\text{ext}}^{(1)\text{ext}},
$$

(21)

$$
M_{\mu\gamma(1)\text{ext}}^{(1)\text{ext}} = M_{\mu\gamma(1)\text{ext}}^{(1)\text{ext}} = 0;
$$

(22)

$$
M_{NN:2(b)}^{(1)\text{ext}} = (\tau_1 \cdot \tau_2) \frac{g_5^2 m_2^2}{128\pi^2 f_2^2} \left\{ \frac{5}{8} \rho \left( \Gamma_\epsilon + \ell_N + \frac{11}{10} \right) + \frac{5}{2} \left( \Gamma_\epsilon + \ell_N + \frac{11}{5} \right) - 3 \left( \Gamma_\epsilon + \ell_\sigma + R_{2b}^{\text{ext}} \right) \right\} = M_{NN:2(b)}^{(1)\text{ext}},
$$

(23)

$$
M_{NN:2(c)}^{(1)\text{ext}} = - (3 - 2 (\tau_1 \cdot \tau_2)) \frac{g_5^2 m_2^2}{128\pi^2 f_2^2} \left\{ \frac{45}{64} \rho \left( \Gamma_\epsilon + \ell_N + \frac{881}{270} \right) - \frac{53}{64} \left( \Gamma_\epsilon + \ell_N + \frac{3745}{2238} \right) \right\},
$$

(24)
In contrast to most literature, we do not perform any additional manipulation (truncations and/or additional regulators) in our work. Instead, we could find support from the idea of 'decoupling'\cite{26,27,28}, which is employed to protect chiral power counting\cite{23}. Retained terms except those due to IR enhancement or threshold effects that must be kept anyway. This prescription could find support from the idea of 'decoupling'\cite{26,27,28}. Here it is employed to protect chiral power counting\cite{23}.

To make things more transparent, we have underlined all the local non-chiral terms, above-braced all the local chiral terms, and under-braced the 'anomalous' term. The residual terms are just those denoted by $R$'s that vanish as $\rho \to \infty$, close examination of these terms showed that they are suppressed by $\rho^{-1/2}$. The above decomposition and classification are also summarized in Table 1 for covariant and mixed calculations.

Table 1: Various components in covariant/mixed calculation

| Diagram | local, non-chiral | local, chiral | anomalous | suppressed |
|---------|------------------|--------------|-----------|------------|
| Fig.1(a,b,e) | yes/yes | yes/yes | no/no | yes/yes |
| Fig.1(f,g) | yes/no | no/no | no/no | yes/no |
| Fig.2(b,d) | yes/yes | yes/yes | no/no | yes/yes |
| Fig.2(c) | yes/yes | yes/yes | yes/yes | yes/yes |

Evidently, the large or non-chiral local terms and the suppressed terms $R$'s in each diagram are variant with different versions of vertices, the rest are 'stable'. The anomalous term in the box diagram for $NN$ scattering (Fig. 2(c)) becomes IR singular using HB propagator for baryons\cite{4}. If the nonrelativistic propagator is used, this term becomes $\sim -3\pi$ in dimensional scheme and contaminated by linear divergence in cut-off scheme\cite{23}. It becomes definite only when covariant baryonic propagator is employed.

3.3. Chiral 'stability' from covariant propagators

Removing all the underlined terms that are local and non-chiral, we arrive at the following 'stable' or 'preserved' components plus suppressed ones for each diagram:

$$\tilde{M}^{c(1)}_{\rho:3(1)} = -A^1(1)\bar{\tau}^a\gamma_a u \frac{g_s^2 m_{\rho}^2}{(4\pi f_{\pi})^2} \left\{ -3(\Gamma_\rho + \ell_\rho) + o(\rho^{-1/2}) \right\} = \frac{1}{3} \tilde{M}^{c(1)}_{\rho:3(1)},$$

$$\tilde{M}^{c(1)}_{\rho:1(1)} = d^1(1)\bar{\tau}^a\gamma_a u \frac{g_s^2 m_{\rho}^2}{(4\pi f_{\pi})^2} \left\{ -3(\Gamma_\rho + \ell_\rho) + o(\rho^{-1/2}) \right\},$$

$$\tilde{M}^{c(1)}_{\rho:1(1)} = -A^1(1)\bar{\tau}^a\gamma_a u \frac{g_s^2 m_{\rho}^2}{(4\pi f_{\pi})^2} \left\{ 0 + o(\rho^{-1/2}) \right\} = \tilde{M}^{c(1)}_{\rho:1(1)},$$

$$\tilde{M}_{NN:2(1)} = \langle \tau_1 \cdot \tau_2 \rangle \frac{g_s^2 m_{\rho}^2}{128\pi^2 f_{\pi}^3} \left\{ -3(\Gamma_\rho + \ell_\rho) + o(\rho^{-1/2}) \right\} = \tilde{M}_{NN:2(1)},$$

$$\tilde{M}_{NN:2(1)} = -(3 - 2(\tau_1 \cdot \tau_2)) \frac{g_s^2 m_{\rho}^2}{128\pi^2 f_{\pi}^3} \left\{ \frac{15}{4}(\Gamma_\rho + \ell_\rho) + \frac{12\rho}{\sigma} \arctan \sigma + o(\rho^{-1/2}) \right\},$$

$$\tilde{M}_{NN:2(1)} = -(3 - 2(\tau_1 \cdot \tau_2)) \frac{g_s^2 m_{\rho}^2}{128\pi^2 f_{\pi}^3} \left\{ -\frac{15}{4}(\Gamma_\rho + \ell_\rho) + o(\rho^{-1/2}) \right\}.$$
in each 'supposed' chiral loop, which could be smoothly extended from the standard subtraction algorithm that has been widely used in particle physics (say, \( MS \) or \( \overline{MS} \)) and automatically recovers the standard one for Goldstone sector. In this sense, we may call it extended \( MS \) (\( \overline{MS} \)), or EMS (\( \overline{EMS} \)) for short.

Before closing this section, we would like to note that the prescription suggested here has already been employed in effect in some literature addressing hadronic physics using chiral effective field theories. For example, in Ref.\([31]\), the chiral components are extracted in the computation of the pion momentum distributions in nucleons with the rest parts simply discarded.

**4. Summary and prospectives**

In summary, we have shown at one-loop level that the chiral components of the one-loop diagrams with one or two external baryonic/nucleonic lines are well ‘preserved’ as long as covariant propagators are employed for baryons/nucleons. These results naturally suggest us to advance an extended and unified prescription for chiral effective theories that dispenses extra operations and/or regularizations as long as covariant formulation is used.

As baryons still participate the dynamical processes in a ‘mild or modest’ sense, unlike in the standard decoupling theories where heavy particles are completely removed from the formulation of effective theories, the strategy suggested above may be seen as a modified or mitigated implementation of ‘decoupling’. This, in our view, may be somehow underlying the soft cut-off or lattice regularization methods advocated in Refs.\([32,33]\) for removing the large pieces that violate the standard chiral power counting in SU(3) chiral effective theory for baryonic decuplet. As we have only considered the SU(2) effective theories here, it is natural to investigate if the extended prescription or strategy still works well in SU(3) cases. It is also interesting to test this extended prescription beyond one-loop level, which might provide some clues for the issue of covariant resummation of the ‘anomalous’ components.

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**Appendix A. Covariant calculation of one-loop diagrams**

The explicit expressions of the one-loop diagrams listed in Fig. 1 and Fig. 2 will be given below. First, let us introduce the following shorthand notations

\[
\rho \equiv \frac{M^2}{m^2}, \quad \sigma \equiv \sqrt{4\rho - 1}, \quad \ell_N \equiv \ln \frac{4\pi \sigma^2}{M^2_N}, \quad \ell_\pi \equiv \ln \frac{4\pi \mu^2}{m^2_\pi}. \quad (A.1)
\]

Then, the completely covariant calculation yields the following

\[
\mathcal{M}^{(1)}_{\mu; (a)} = -A^{(1)}\mu \tau \eta_{\mu} u \frac{g_2^2 m^2_\pi}{4(4\pi f^2)}\left\{ -\Gamma_e - \ell_\pi - 2\ln \rho + 3 + \mathcal{R}_1 \right\} = \frac{1}{3} \mathcal{M}^{(1)}_{\mu; (a),} \quad (A.2)
\]

\[
\mathcal{M}^{(1)}_{\mu; (b)} = A^{(1)}\mu \tau \eta_{\mu} u \frac{g_2^2 m^2_\pi}{4(4\pi f^2)} \left\{ 2\rho (\Gamma_e + \ell_N + 1) + \Gamma_e + \ell_N - 3\ln \rho + 7 + \mathcal{R}_1 \right\}, \quad (A.3)
\]

\[
\mathcal{M}^{(1)}_{\mu; (c)} = A^{(1)}\mu \tau \eta_{\mu} u \frac{m^2_e}{4(4\pi f^2)} (\Gamma_e + \ell_\pi + 1), \quad \mathcal{M}^{(1)}_{\mu; (d)} = -\frac{m^2_e}{4(4\pi f^2)} A^{(1)}\mu \tau \eta_{\mu} u (\Gamma_e + \ell_\pi + 1), \quad (A.4)
\]

\[
\mathcal{M}^{(1)}_{\mu; (e)} = -A^{(1)}\mu \tau \eta_{\mu} u \frac{g_2^2 m^2_\pi}{4(4\pi f^2)}\left\{ \rho (\Gamma_e + \ell_N + 1) + (\Gamma_e + \ell_N + 2) + \mathcal{R}_f \right\} = \mathcal{M}^{(1)}_{\mu; (e)}, \quad (A.5)
\]

\[
\mathcal{M}_{NN; (1)} = \langle \tau_1 \cdot \tau_2 \rangle \frac{g_2^2 m^2_\pi}{128\pi^2 f^2} \left\{ 2\rho (\Gamma_e + \ell_N + 1) + \Gamma_e + \ell_N - 3\ln \rho + 7 + \mathcal{R}_2 \right\} = \mathcal{M}_{NN; (1)}, \quad (A.6)
\]
\[
M_{NN:2(c)} = - (3 - 2(\tau_1 \cdot \tau_2)) \frac{g_\mu^2 m_\pi^2}{128\pi^2 f_\pi^2} \left\{ \rho (\Gamma_e + \ell_N + 3) - \frac{1}{4} (\Gamma_e + \ell_x + 1) + 4 \ln \rho - 3 \right. \\
- \frac{12\rho}{\sigma} \arctan \sigma + R_{2c} \right\}, \\
M_{NN:2(d)} = - (3 + 2(\tau_1 \cdot \tau_2)) \frac{g_\mu^2 m_\pi^2}{128\pi^2 f_\pi^2} \left\{ 3\rho (\Gamma_e + \ell_N + 1) + \frac{1}{4} (\Gamma_e + \ell_x + 1) - 4 \ln \rho + 4 + R_{2d} \right\},
\]

where \((3 + 2(\tau_1 \cdot \tau_2)) \equiv \bar{u}_1 u_1 \bar{u}_2 u_2 \pm 2 \bar{u}_1 \tau_1 u_1 \bar{u}_2 \tau_2 u_2\). The detailed expressions of the \(R\)'s terms here and below vanish in the limit \(\rho \to \infty\) and will be given in Appendix C.

### Appendix B. Mixed calculation of one-loop diagrams

The calculation with HB vertices and composite operators yields the following

\[
M_{\mu1:2(a)}^{(1)UV} = -A^{(1)\bar{u}r\gamma\nu} (\frac{g_\mu^2 m_\pi^2}{4(4\pi f_\pi^2)} \left\{ \frac{5}{2} \rho (\Gamma_e + \ell_N + \frac{3}{2}) + \frac{3}{4} (\Gamma_e + \ell_x + \frac{1}{2}) \right\} - 3 \ln \rho + 6 + R_{1a}^{\mu1} \right\}, \tag{B.1}
\]

\[
M_{\mu1:2(b)}^{(1)UV} = A^{(1)\bar{u}r\gamma\nu} (\frac{g_\mu^2 m_\pi^2}{4(4\pi f_\pi^2)} \left\{ \frac{5}{8} \rho (\Gamma_e + \ell_N + \frac{11}{10}) + \frac{1}{2} (\Gamma_e + \ell_x + 1) - \frac{5}{2} \ln \rho + 6 + R_{1b}^{\mu1} \right\} \\
= \frac{4}{3} M_{\mu1:1}^{(1)UV} \text{ (provided } A^{(1)} = a^{(1)} = 1), \tag{B.2}
\]

\[
M_{\mu1:2(c)}^{(1)UV} = A^{(1)\bar{u}r\gamma\nu} (\frac{m_\pi^2}{(4\pi f_\pi^2)} (\Gamma_e + \ell_x + 1)), \quad M_{\mu1:1(d)}^{(1)UV} = -A^{(1)\bar{u}r\gamma\nu} (\frac{m_\pi^2}{(4\pi f_\pi^2)} (\Gamma_e + \ell_x + 1)), \tag{B.3}
\]

\[
M_{\mu1:2(d)}^{(1)UV} = M_{\mu1:1(f)}^{(1)UV} = 0, \tag{B.4}
\]

\[
M_{NN:2(b1)}^{m} = (\tau_1 \cdot \tau_2) \frac{g_\mu^2 m_\pi^2}{128\pi^2 f_\pi^2} \left\{ \frac{5}{8} \rho (\Gamma_e + \ell_N + \frac{11}{10}) + \frac{1}{2} (\Gamma_e + \ell_x + 1) - \frac{5}{2} \ln \rho + 6 + R_{2b}^{m} \right\} \\
= M_{NN:2(b2)}^{m}, \tag{B.5}
\]

\[
M_{NN:2(c)}^{m} = - (3 - 2(\tau_1 \cdot \tau_2)) \frac{g_\mu^2 m_\pi^2}{128\pi^2 f_\pi^2} \left\{ \frac{45}{64} (\Gamma_e + \ell_N + \frac{881}{270}) - \frac{133}{64} (\Gamma_e + \ell_N) + \frac{15}{4} \ln \rho - \frac{3745}{384} \right. \\
- \frac{12\rho}{\sigma} \arctan \sigma + R_{2c}^{m} \right\}, \tag{B.6}
\]

\[
M_{NN:2(d)}^{m} = - (3 + 2(\tau_1 \cdot \tau_2)) \frac{g_\mu^2 m_\pi^2}{128\pi^2 f_\pi^2} \left\{ \frac{417}{64} (\Gamma_e + \ell_N + \frac{587}{834}) + \frac{197}{64} (\Gamma_e + \ell_N) - \frac{15}{4} \ln \rho + \frac{1399}{128} \right. \\
+ R_{2d}^{m} \right\}. \tag{B.7}
\]

### Appendix C. Definitions of \(R\)'s

\[
R_{1a}^{\mu} \equiv \frac{1}{\rho} \left( \frac{4 - 12\rho}{\sigma} \arctan \sigma + 2 \ln \rho \right), \tag{C.1}
\]

\[
R_{1b}^{\mu} \equiv \frac{1}{\rho} \left( \frac{6 - 20\rho}{\sigma} \arctan \sigma + 3 \ln \rho \right) = R_{2b}, \tag{C.2}
\]

\[
R_{1f}^{\mu} \equiv \frac{1}{\rho} \left( -\sigma \arctan \sigma + \frac{\ln \rho}{2} \right), \tag{C.3}
\]

\[
R_{2c}^{\mu} \equiv \frac{1}{\rho} \left( \frac{14\rho - 3}{\sigma} \arctan \sigma - \frac{3}{2} \ln \rho \right), \tag{C.4}
\]


\[ \mathcal{R}_{2d}^{c} \equiv \left( -15 + \frac{3}{\rho} - \frac{1}{\sigma^2} \right) \frac{\arctan \sigma}{\sigma} + \frac{3 \ln \rho}{2 \rho} + \frac{1}{\sigma^2}, \]  
(C.5)

\[ \mathcal{R}_{2a}^{\rho} \equiv \frac{5}{4 \rho} \left( \frac{(1 - 4\rho) \sigma}{\rho} \arctan \sigma + \frac{6 \rho - 1}{2 \rho} \ln \rho - 1 \right), \]  
(C.6)

\[ \mathcal{R}_{1b}^{\rho} \equiv \frac{5}{16 \rho} \left( (-2 + 12 \rho - 16 \rho^2) \sigma \arctan \sigma + 1 - 8 \rho + 18 \rho^2 \ln \rho - 13 \rho^2 - 2 \right) = \mathcal{R}_{2d}^{c}, \]  
(C.7)

\[ \mathcal{R}_{2c}^{\rho} \equiv \left( 30 - \frac{19}{\rho} + \frac{31}{8 \rho^2} \right) \frac{\arctan \sigma}{\sigma} + \left( \frac{393}{64} + \frac{57}{32 \rho} - \frac{7}{128 \rho^2} - \frac{3}{128 \rho^3} \right) \ln \rho \]  
\[ + \frac{128 \rho}{128 \rho - 128 \rho^2 - \frac{3}{64 \rho^2}}, \]  
(C.8)

\[ \mathcal{R}_{2d}^{\rho} \equiv \left( -35 + \frac{133}{4 \rho} - \frac{5}{16 \rho^2} - \frac{121}{64 \rho^3} - \frac{23}{64 \rho^4} - \frac{1}{32 \rho^2 \sigma^2} \right) \frac{\arctan \sigma}{\sigma} + \left( \frac{591}{64} + \frac{85}{32 \rho} - \frac{155}{128 \rho^2} - \frac{21}{128 \rho^3} \right) \ln \rho \]  
\[ + \frac{1029}{128 \rho - 128 \rho^2 - 64 \rho^3} + \frac{13}{8 \rho^2 \sigma^2}. \]  
(C.9)

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