Linear stability studies including resistive wall effects with the CASTOR/STARWALL code

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Abstract. CASTOR/STARWALL is a code package for linear stability studies consisting of the CASTOR3DW code, the STARWALL code, and a hybrid version of both codes named CASTOR3D that is currently under development. The latter solves an extended eigenvalue problem consisting of the plasma part defined by the weak form of the perturbed single-fluid MagnetoHydroDynamic (MHD) equations, and a vacuum part which is derived from an energy functional. This new code can describe wall resistivity and plasma inertia simultaneously. Furthermore, besides the so far used straight field line coordinates, additionally, general flux coordinates are implemented which are more appropriate for the description of instabilities located close to the separatrix (e.g. Edge Localized Modes (ELMs)). The formulation of the eigenvalue problem in the new coordinates is not limited to axisymmetric equilibria, so that, finally, stability studies of resistive and rotating, 3D equilibria should be possible. In this paper we will sketch the theory, report on the progress of the code development, and present first results.

1. Introduction
The interaction of the plasma with currents induced in external conducting structures plays an important role for many tokamak instabilities, such as Vertical Displacement Events (VDEs), Resistive Wall Modes (RWMs), etc. Linear stability studies yield the growth rates and mode structures of instabilities in the initial linear phase. They are numerically less expensive, more robust and flexible than non-linear calculations. This allows quick parameter studies and is very useful for benchmarking the linear phase of non-linear simulations, e.g. made with the non-linear JOREK-STARWALL code [1, 2].

There exist several linear stability codes, e.g. CASTOR [3, 4, 5], CASTOR3DW [6], MARS-F [7], CAS3D [8], STARWALL [9], and CarMa [10], which use various methods to take external conducting structures into account. The response of ideal conducting structures to unit perturbations of the plasma can be computed independently of the eigenvalue problem of the plasma interior. The resulting response matrix is the only information needed to compute the boundary terms of the eigenvalue problem. This method is, for example, used in the CASTOR and CASTOR3DW codes. While the CASTOR code is limited to axisymmetric, closed walls, the CASTOR3DW code has been extended to 3D wall geometry for which the STARWALL code provides the vacuum response matrix. However, the response of resistive conducting structures depends on the growth rate. For slowly growing RWMs the plasma inertia can be neglected. In that case the plasma response to unit perturbations is computed, and the resulting plasma response matrix is used for the solution of the vacuum problem. This method is, for example,
used in the CarMa code [10], and in the STARWALL code [9]. The latter makes use of the potential energy of the plasma, which is computed with the CAS3D stability code for given unit perturbations. In contrast to the CarMa code, which is able to treat volumetric 3D conducting structures, but is restricted to axisymmetric equilibria, the STARWALL code can also be applied to 3D equilibria. However, it uses the ‘thin wall approximation’ and is restricted to ideal plasmas without flow. In order to study stability properties of realistic configurations, the separation of the problems as described above is usually not possible. That is, resistive walls with complex 3D geometry and plasma inertia (e.g. in case of a toroidally rotating plasma) have to be considered simultaneously. In the CarMa code this problem is solved by the so-called ‘backward’ coupling strategy [11]. There, the dynamic response of the conducting structures is integrated in the MHD equations.

The CASTOR3D, which is currently under development, follows another strategy. It is a hybrid of the CASTOR3DW code and the STARWALL code. This code solves an extended eigenvalue problem consisting of the plasma part defined by the weak form of the perturbed single-fluid MHD equations (CASTOR3DW code), and a vacuum part which is derived from an energy functional (STARWALL code). Furthermore, besides the so far used straight field line coordinates, additionally, general flux coordinates are implemented, which are more appropriate for the description of instabilities located close to the separatrix (e.g. ELMs). The matrix elements derived for the new coordinates are not limited to axisymmetric equilibria, so that, finally, stability studies of resistive and rotating, 3D equilibria should be possible.

The paper is organized as follows. The theory is briefly described in section 2. The present status of the code and first results are subject of section 3, and, finally, an outlook is given in section 4.

2. Theory

2.1. Coordinate systems

Because of the reasons mentioned in the introduction, general flux coordinates describing also 3D equilibria are currently implemented in the CASTOR3D code. These coordinates, named \((s, v, u)\), describe a right-handed coordinate system with \(s\) being the radial coordinate, and \(v, u\) being toroidal and poloidal angle-like coordinates, respectively. While the original CASTOR code [3, 4] and its subsequent versions use straight field line coordinates, being a special choice of flux coordinates, the CASTOR3D code supports any flux coordinate system. Furthermore, either the square root of the normalized poloidal flux, \(\rho_{\text{pol}}\), or the square root of the normalized toroidal flux, \(\rho_{\text{tor}}\), can be chosen as radial coordinate \(s\). Using an equilibrium input provided by the NEMEC code [12], \(s = \rho_{\text{tor}}\), and \(v, u\) being the angle-like coordinates of the NEMEC code, are the logical choice for these coordinates.

2.2. The basic equations

The linearized, ideal, single fluid MHD equations for the perturbations read

\[
\lambda \rho = -\vec{v} \cdot \nabla \rho_0 - \rho_0 \nabla \cdot \vec{v},
\]

\[
\lambda \rho_0 \vec{v} = -\nabla (\rho_0 T + \rho_0 T_0)/m + (\nabla \times \vec{B}_0 \times \vec{B} + \nabla \times \vec{B} \times \vec{B}_0)/\mu_0,
\]

\[
\lambda T = -\vec{v} \cdot \nabla T_0 - (\Gamma - 1)T_0 \nabla \cdot \vec{v},
\]

\[
\lambda \vec{B} = \nabla \times (\vec{v} \times \vec{B}_0),
\]

(1)
with the time dependence of the perturbed quantities taken to be \( \sim e^{\lambda t} \), and \( \lambda = \gamma + i\omega \) being the complex eigenvalue. The quantities \( \rho_0, T_0 \), and \( \vec{B}_0 \) denote density, temperature, and magnetic field of the unperturbed equilibrium. Here the fluid velocity \( \vec{v}_0 \) is assumed to be zero. The compressibility of the plasma is described by the coefficient \( \Gamma \), with \( \Gamma = 5/3 \) set during the following computations. The quantities \( m \) and \( \mu_0 \) denote ion mass and vacuum permeability, respectively. For the corresponding perturbed quantities \( \rho, T, \vec{v} \), and \( \vec{B} = \nabla \times \vec{A} \) an analogue ansatz [3, 4] is made as in the original CASTOR code. That is,

\[
\rho = \frac{1}{s} \bar{\rho}, \quad T = \frac{1}{s} \bar{T}, \quad \vec{A} = -i \bar{A}_s \nabla s + \bar{A}_v \nabla v + \bar{A}_u \nabla u, \quad \vec{v} = v^s \vec{r}_s + v^v \vec{r}_v + v^u \vec{r}_u = \frac{R^2}{\sqrt{g}} \bar{v}_s \vec{r}_s + \frac{R^2}{i \mu \bar{f}} \bar{v}_v \vec{B}_0 + \frac{R^2}{i \sqrt{g}} \bar{v}_u \vec{r}_u. \tag{2}
\]

Here \( \sqrt{g} \) is the Jacobian, \( \bar{f} = \partial \Phi / \partial s \) the derivative of the toroidal flux \( \Phi \), and \( \mu \) the rotational transform. The variables \( \bar{\rho}, \bar{v}_s, \bar{v}_v, \bar{v}_u, \bar{T}, \bar{A}_s, \bar{A}_v, \) and \( \bar{A}_u \), are collected in the symbolic vector \( \bar{w} = (\bar{\rho}, \bar{v}_s, \bar{v}_v, \bar{v}_u, \bar{T}, \bar{A}_s, \bar{A}_v, \bar{A}_u) \). Each of its components is expressed by a complex Fourier series in poloidal and toroidal direction, and a finite element interpolation, \( h^j_{k,p}(s) \), in radial direction:

\[
w_k(s, u, v) = \sum_{m,n,j,\bar{\beta}} \bar{m},\bar{n},\bar{j} \hat{g}^{k,\bar{\beta}}_{m,n,j,\bar{\beta}} \hat{h}_{k,p}^j(s) e^{2\pi i (\bar{m} u + \bar{n} v)} + \alpha^{k,\bar{\beta}}_{m,n,j,\bar{\beta}} \hat{h}_{k,p}^j(s) e^{-2\pi i (\bar{m} u + \bar{n} v)}. \tag{3}
\]

In (3) \( k \) labels the eight variables, \( m, n \) the poloidal and toroidal Fourier harmonics, and \( j \) the radial grid points. The interpolating functions are either quadratic \( (k=1,3,4,5,6) \) or cubic \( (k=2,7,8) \) Hermite polynomials. \( \bar{\beta} = 1, 2 \) labels the two different quadratic, \( h^j_{1,p} \), and cubic Hermite polynomials, \( H^j_{1,p} \), at each grid point. In 3D geometry the conjugate complex term in (3) becomes necessary, because the two orthogonal solutions have different eigenvalues in non-axisymmetric configurations.

In case of the equation for the radial velocity component \( v^r \) the formulation of the weak form leads to surface integrals of the form

\[
W_s = -\int_0^1 dv \int_0^1 du \frac{H^j_{1,p} e^{\pm 2\pi i (mu+nv)}}{R^2} |\vec{B} - \bar{B}|_{s=1}, \tag{4}
\]

where \( \vec{B} \) has to fulfill the boundary condition, \( \bar{n}_p \vec{B} = \bar{n}_v \vec{B} \), with \( \bar{n}_p = \nabla s / |\vec{r}_v \times \vec{r}_u| \) being the normal vector on the plasma boundary in outward direction, and \( \vec{B} \) being the perturbation of the vacuum field.

In order to determine the vacuum field perturbation in the presence of an ideal or resistive wall a variational method is used with the Lagrangian [9]

\[
\mathcal{L} = \frac{\mu_0}{8\pi} \int_{S_p} df_p \int_{S_p} df'_{p'} \frac{\vec{K}_p \cdot \vec{K}'_{p'}}{|\vec{r}_p - \vec{r}'_{p'}|} + \frac{\mu_0}{4\pi} \int_{S_p} df_p \int_{S_w} dw \frac{\vec{K}_p \cdot \vec{K}_w}{|\vec{r}_p - \vec{r}_w|} \tag{5}
\]

\[
+ \frac{\mu_0}{8\pi} \int_{S_w} dw \int_{S_w} dw' \frac{\vec{K}_w \cdot \vec{K}'_{w'}}{|\vec{r}_w - \vec{r}'_{w'}|} + \frac{1}{2\lambda \sigma w} \int_{S_p} dw \vec{K}_w \cdot \vec{K}_w + \int_{S_p} df_p (\bar{n}_p \cdot \hat{\xi}) \bar{n}_p \cdot (\vec{K}_p \times \vec{B}_0).
\]

The divergence-free surface currents, \( \vec{K}_{p,w} = \bar{n}_{p,w} \times \nabla \Phi_{p,w} \), generate the magnetic field perturbation in the vacuum domain, where \( \Phi_p \) and \( \Phi_w \) are the current potentials. The indices
p and w label plasma boundary and wall, respectively. The wall is characterized by its specific resistivity $\sigma_w$, and its width $d_w$. The first three terms of the Lagrangian (5) are the energies produced by surface currents in the plasma boundary and the wall. The fourth term is the energy dissipated in the resistive wall, while the boundary condition at the plasma boundary, $\vec{n}_p \times \vec{A} = - (\vec{n}_p \cdot \vec{\xi}) \vec{B}_0$, leads to the last term. Furthermore, the displacement vector $\vec{\xi}$ is related with the velocity perturbation $\vec{v}$ and the eigenvalue $\lambda$

$$\vec{\xi} = \frac{\vec{v}}{\lambda}$$  \hspace{1cm} (6)

In case of the plasma boundary or a closed wall with smooth geometry a Fourier ansatz

$$\Phi(u, v) = \sum_{m,n} \hat{\phi}_{mn} \sin(2\pi (mu + nv)) + \hat{\phi}_{nn} \cos(2\pi (nu + nv))$$  \hspace{1cm} (7)

is made for the corresponding current potentials, while a wall with complex, multiply-connected structure is discretized into triangles. Then $\vec{K}$ is assumed to be constant on a triangle and is defined by the values of the current potential at the vertices of the triangle. That is,

$$\vec{K}_\Delta = \frac{\hat{\phi}_1 \vec{r}_{23} + \hat{\phi}_2 \vec{r}_{41} + \hat{\phi}_3 \vec{r}_{12}}{\vec{r}_{21} \times \vec{r}_{32}}.$$  \hspace{1cm} (8)

Fig. 1: Triangle wall element.

The variation of the Lagrangian (5) with respect to the variables $\hat{\phi}_{mn}^{sc}$ and/or $\hat{\phi}_i$, and the use of (6) yields a coupled set of linear equations

$$\lambda (M_{pp} \vec{x}_p + M_{pw} \vec{x}_w) = -R_{ps} \vec{x}_s, \quad \lambda (M_{wp} \vec{x}_p + M_{ww} \vec{x}_w) = -R_{wu} \vec{x}_w.$$  \hspace{1cm} (9)

with the matrices $M_{pp}, M_{pw},$ and $M_{ww}$ resulting from its first three terms, $R_{ww}$ from its fourth, and $R_{ps}$ from its last term. The symbolic vector, $\vec{x}_p = (\{\hat{\phi}_{mn}^{sc}\}, \{\hat{\phi}_{mn}\})$, represents the Fourier coefficients of $\Phi^p$, while $\vec{x}_w = (\{\hat{\phi}_{mn}^{sc}\}, \{\hat{\phi}_{mn}\})$ or $\vec{x}_w = (\{\hat{\phi}_{mn}\})$ ($i = 1, \ldots, N_\Delta$, with $N_\Delta$ being the number of vertices), represent $\Phi^w$. Finally, $\vec{x}_s = (\{a_{k,\bar{n},\bar{j}}^{w}\}, \{a_{k,\bar{n},\bar{j}}^{w}\})$ is related to the components of the symbolic vector $\vec{w}$ at the plasma boundary.

2.3. The extended eigenvalue problem

The combination of the weak form of the perturbed MHD equations and equations (9), the latter resulting from the variation of the Lagrangian (5), yields the extended eigenvalue problem

$$\lambda \begin{pmatrix} B_{ll} & B_{ls} & 0_{lp} & 0_{lw} \\ B_{sl} & B_{ss} & 0_{sp} & 0_{sw} \\ 0_{pl} & 0_{ps} & M_{pp} & M_{pw} \\ 0_{wl} & 0_{ws} & M_{wp} & M_{ww} \end{pmatrix} \begin{pmatrix} \vec{x}_l \\ \vec{x}_s \\ \vec{x}_p \\ \vec{x}_w \end{pmatrix} = \begin{pmatrix} A_{ll} & A_{ls} & 0_{lp} & 0_{lw} \\ A_{sl} & A_{ss} & R_{sp} & R_{sw} \\ 0_{pl} & -R_{ps} & 0_{pp} & 0_{pw} \\ 0_{wl} & 0_{ws} & 0_{wp} & -R_{ww} \end{pmatrix} \begin{pmatrix} \vec{x}_l \\ \vec{x}_s \\ \vec{x}_p \\ \vec{x}_w \end{pmatrix}.$$  \hspace{1cm} (10)

Here the sub-matrices $A_{ll}, A_{ls}, A_{sl}, A_{ss}$ and $B_{ll}, B_{ls}, B_{sl}, B_{ss}$, and the sub-vectors $\vec{x}_l$ and $\vec{x}_s$ are related to the weak form of the MHD equations describing the interior (index $l$) and/or
the plasma boundary (index s), respectively. The magnetic field perturbation $\vec{B}$ in the surface integral (4) is obtained from the surface currents in the plasma boundary and the wall

$$\vec{B} = \vec{B}_p + \vec{B}_w = \frac{\mu_0}{4\pi} \int_{S_p} df' \vec{K}_{p'} \times \frac{(\vec{r}_p - \vec{r}_{p'})}{|\vec{r}_p - \vec{r}_{p'}|^3} + \frac{\mu_0}{4\pi} \int_{S_p} df' \vec{K}_{w'} \times \frac{(\vec{r}_w - \vec{r}_{w'})}{|\vec{r}_w - \vec{r}_{w'}|^3}. \tag{11}$$

Inserting (11) and the corresponding expressions for the surface currents $\vec{K}_{p'}$ and $\vec{K}_{w'}$ into (4) yields the matrices $R_{sp}$ and $R_{sw}$ in (10). The remaining elements of (10) have already been explained in the previous section.

### 3. Results

In the following we present first results obtained for an ideal external kink mode in the presence of an ideal, and a resistive wall by solving the extendend eigenvalue problem. For these computations the straight field line coordinates are used. Furthermore, the growth rates of internal kink modes are calculated using the new coordinates.

#### 3.1. Solution of the extended eigenvalue problem

The results of the CASTOR3D code are compared with analytical results calculated for a large aspect-ratio, low-$\beta$, circular cross-section tokamak plasma surrounded by a thin, uniform, resistive shell. In Ref. [13] plasma and shell are approximated by a cylinder of radius $a$, and a thin, concentric cylindrical shell of radius $r_w > a$, respectively. Then, the wall time $\tau_w$ is defined by $\tau_w = \mu_0 \sigma_w d_w r_w$ with $\sigma_w$ and $d_w$ being the wall conductivity and thickness, respectively. In this analytical model the dispersion relation for a $m,n$ resistive wall mode is $\gamma \tau_w = \Delta_w^m$, with

$$\Delta_w^m = \frac{2m}{(r_c/r_w)^{2m} - 1}. \tag{12}$$

Here $r_c$ is the critical radius which corresponds to the ideal stability boundary. That is, an ideal wall located at $r_w < r_c$ stabilizes the mode. Then, the normalized growth rate $\bar{\gamma}$ results to

$$\bar{\gamma} = \mu_0 \sigma_w d_w \gamma = \frac{2m}{r_w (r_c/r_w)^{2m} - 1}. \tag{13}$$

For the numerical calculations we assume an aspect ratio of $A = 10$, and a $q$-profile analogue to the one defined in Ref. [13] with $q_0 = 1.3$ and $q_a = 2.9$ being the values of the safety factor at magnetic axis and plasma boundary, respectively. This equilibrium is unstable with respect to a $m=3,n=1$ ideal external kink mode, and the critical radius numerically results to $r_c = 1.28073$ m.

**Fig. 2:** Comparison of the analytically computed normalized growth rate (black line) with the CASTOR3D results (red dots) as function of the wall position. The plasma boundary is located at $r_w = 1$ m. The dashed blue vertical line indicates the critical radius $r_c$. 
The analytical and the numerical results agree very well as long as the resistive wall is located well inside $r_c$. Very close, but still within this radius the analytical growth rate rapidly increases, while the increase of the numerical growth rate is limited by the plasma inertia. The solution of the extended eigenvalue problem is valid for every distance of the wall, as shown in Fig 3. There, the growth rates as function of the wall position are plotted for an ideal, and a resistive wall.

![Image](image_url)

**Fig. 3:** (a) Growth rates of an external kink mode as function of the position of an ideal wall (blue solid line), and a resistive wall (red dashed line), both computed with the CASTOR3D code. The green stars show the results obtained with the STARWALL code in case of the resistive wall. (b) Enlargement of the region around the critical radius $r_c$.

In the case of the ideal wall the growth rate converges towards the no wall limit for large wall distances, and it shows an almost singular decrease, when the wall approaches the ideal stability boundary. However, in the case of a resistive wall, the growth rate increases continuously from very small values typically for a RWM to the high values of the no wall limit. The results of the STARWALL code agree with the solution of the extended eigenvalue problem for small growth rates, but due to the neglect of the kinetic energy the code fails when the wall position approaches or exceeds the critical radius.

### 3.2. Use of general flux coordinates

The CASTOR3DW code has been extended to the computation of growth rates in the presence of 3D wall configurations. Although the toroidal mode coupling has been implemented, its use is still restricted to axisymmetric equilibria. Furthermore, the used straight field line coordinates are not a suitable choice if the modes are located close to the separatrix. In Fig. 4 flux surfaces up to the plasma boundary are plotted for a simple test equilibrium with circular cross-section (Fig. 4a), and an ASDEX Upgrade (AUG)-like equilibrium (Fig. 4b). For the latter a flux surface close but inside the separatrix has been chosen as plasma boundary. For both equilibria poloidal coordinate lines are plotted for the straight field line coordinates, and for the ‘NEMEC’ coordinates. While the latter show only a slight bending close to the separatrix, the poloidal coordinate lines of the straight field line coordinates become almost parallel to the radial ones. For these reasons, the matrix elements in the weak form of the ideal MHD equations have been derived in general 3D form and implemented in the CASTOR3D code.

The two test equilibria considered are unstable with respect to a $n=1$ (circular equilibrium) and a $n=3$ (AUG-like equilibrium) internal kink mode, respectively. The growth rates of these modes are determined for both, straight field line and ‘NEMEC’ coordinates, as shown in Figs 5 and 6. There, eigenfunctions and growth rates of the modes are compared. Because of the
different poloidal coordinates the Fourier spectra of the eigenfunctions are different, but the leading harmonics are the same for either coordinate system. Furthermore, the eigenvalues agree very well within 0.24% for the $n = 1$, and 1% for the $n = 3$ mode.

Fig. 4: Flux surfaces (black), poloidal coordinate lines for 'NEMEC' coordinates (blue), and straight field line coordinates (red) for a plasma with circular cross-section (a), and an AUG-like equilibrium (b).

Fig. 5: Circular equilibrium: Real parts of the harmonics, $\hat{v}^{mn}$, of the eigenfunctions in dependence of $\rho_{tor}$ for straight field line coordinates (a), and 'NEMEC' coordinates (b).

Fig. 6: AUG-like equilibrium: Real parts of the harmonics, $\hat{v}^{mn}$, of the eigenfunctions in dependence of $\rho_{tor}$ for straight field line coordinates (a), and 'NEMEC' coordinates (b).
Since, in case of these internal modes, the CASTOR3D code works very well, it will be tested for external and edge localized modes in a further step.

4. Outlook
The CASTOR3D code is a hybrid of the CASTOR3DW and the STARWALL codes. The new implementations, namely the solution of the extended eigenvalue problem and the use of general flux coordinates, yield very promising first results. However, there is still a plenty of analytical, programming and computational work to do. That is, further matrix elements describing plasma resistivity, plasma flow, etc., have to be derived in 3D, general form. The STARWALL part has to be adapted to the new coordinates, and further conducting structures, e.g. coils, have to be implemented in the CASTOR3D code. Although the 3D form of the matrix elements was used for the computations presented in the previous section, these computations were performed for axisymmetric equilibria. That is, terms that only play a role in 3D geometry still have to be tested. In order to perform stability studies for realistic axisymmetric and 3D equilibria, and complex, multiply-connected conductor structures, an efficient organization and parallelization of the CASTOR3D code is necessary. Since a non-hermitian eigenvalue problem has to be solved, a parallel eigenvalue solver of the Scalable Library for Eigenvalue Problem Computations (SLEPc) [14] is currently implemented. All new implementations will be carefully tested and benchmarked with other codes, e.g. the CAS3D code (ideal, 3D stability studies) and the non-linear JOREK/STARWALL code. The goal is a fully 3D, linear stability code that includes physical effects, such as plasma resistivity, plasma rotation, viscosity, etc., and which simultaneously takes 3D, resistive wall structures into account.

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