Heligman-pollard modification by using the makeham death rate to predict the life table of the elderly

M Riyana¹, S M Belwawin¹, N Hasanah¹ and M Ahmad²
¹Department Early Childhood Education, Universitas Musamus, Merauke, Indonesia
²Midwifery Department, Graduated School, Hasanuddin University, Indonesia

E-mail: minuk_fkip@unmus.ac.id

Abstract. Increasing the number of elderly people in the world is increasing every year, indicating that the economic factors of the community are getting better and also improving services and public health sciences and other factors. The purpose of this research is to find out the number of elderly people in a country. This can be seen by using a complete life table. But in some developing countries in the world do not have a complete life table and only have an abridged life table with an age interval of 5 years or 10 years. Based on the increase in the elderly population in the world researchers want to make the life table of the elderly based on a complete life table by using the interpolation method. Interpolation methods, for example, are the Heligman-Pollard, Elandt-Johnson, Kostaki, and others. In this study, the researchers tried to modify the Heligman-Pollard interpolation method by flexing it with the death rate of the Gompertz distribution. Thus formed the Heligman-Pollard modification using the death rate of the Gompertz distribution. Based on the modification, the result is that the method can predict the complete life table by the original complete life table data from the United States of America in 2014. With the $R^2$ result of 0.985017

1. Introduction
Currently, throughout the world, there is a population increase in the number of elderly people, this is obtained based on the results of WHO. The increase in the elderly population throughout the world indicates that the standard of living of the people in the world is increasing. Where according to the Indonesian government someone is said to be elderly if it has reached the age of 60 years and over [1]. In line with that according to the United Nations in Albis and Collard mentioning someone is said to be an elderly person if they are 60 years or older [2]. Improving people's living standards are usually in line with improving the community's economy and improving the health sector in an area. Which indicates that community income is getting better and increasing and the better handling of an existing disease. So that the increase in the elderly population causes some countries to establish programs specifically for the elderly so that the elderly have activities that can support their productivity and not become a burden on the productive age [3]. To find out how many elderly people in a country need data that can show the data in detail.

Data on the elderly population can be obtained from life tables, where life tables also contain data on mortality [4], elderly life expectancy, the chance of death and much more [5]. Where life table contains data based on old age. It is important to know that the life table is divided into two: a complete life table and a short life table. The complete life table is a life table containing data on deaths based on 1 year age intervals [5]. Whereas a short life table is a table containing data on mortality with a 5-year or 10-year age interval [5]. The two life tables contain data from the age of 0
years to 110 years old. A complete life table is usually only owned by developed countries and some developing countries. This is because developed countries already have a very good population registration system, in contrast to some developing countries. For example, in developed countries, someone who dies will surely be registered by a representative of a family member to get a death certificate for a deceased family member. This is due to very strict regulations that are applied to developed countries and several other developing countries. Therefore in some developing countries usually only have a short life table with 5 yearly or 10 yearly age intervals. So that some countries that do not have a complete life table will interpolate a short life table data to get a complete life table.

Interpolation of concise life table data can be done with several methods, namely by using the Kostaki, Elandt-Johnson, Heligman-Pollard, Brass-logit, and others. Here the author tries to modify the Heligman-Pollard method. This was done because the formula of the method explained all ages of deaths [6]. Which were divided into three classes which were the first class for children, the second class for young adults and the third class for the elderly. So the author took the formula in the third class and modified it using the Gompertz distribution. Where the distribution is also a distribution intended for the elderly [7]. This research is expected to be able to predict the complete life table of the elderly based on a short life table that can approach the actual complete life table that is owned by a particular country.

2. Methods
The researcher modified the Heligman-Pollard interpolation method by using the death rate of the Makeham distribution. Where the modification will use a concise life table data and a complete life table for US data in 2014 as a test of a modified method. The short life table will be used to estimate the complete life table by using a modified method. Meanwhile, the complete life table of the United States in 2014 will be used as comparative data to determine errors. Where this data is obtained from the Human Mortality Database (www.mortality.org). Modifications were made based on the Heligman-Pollard interpolation method using the Gompertz distribution mortality rate.

2.1. Heligman-pollard method
The mathematical model for interpolation using the Heligman-Pollard method, namely

\[
\frac{q_x}{p_x} = A^{x+B}C + D\exp\left[-E \left(\ln\left(\frac{x}{F}\right)\right)^2\right] + GH^x
\]

with \(q_x\) is the opportunity for people exactly \(x\) years old to die before reaching the age of \(x + 1\) year, \(p_x = 1 - q_x\) and \(A, B, C, D, E, F, G, H\), which are positive parameters [5]. This method is divided into three parts which represent different components of death. The first part describes the mortality rate for infants and early age children less than 10 years, death has 3 component parameters, namely \(A, B,\) and \(C\). The second part of parameters \(D, E,\) and \(F\) describes the age of death between the ages of 10-40 years or young adults. The third part describes the mortality rate of adults and the elderly between the ages above 40 years [7] where death has 2 components, namely \(G\) and \(H\) [8]. So that the formula used is:

\[
\frac{q_x}{p_x} = GH^x
\]

2.2. Gompertz distribution
In traditional actuarial models such as Gompertz, the mortality rate increases exponentially as we age, and has no limited function [9]. It is known that the Survival distribution function of the Gompertz distribution has a formula that is

\[
S_x(x) = \exp\left(\frac{R}{a} (1 - e^{ax})\right); R > 0, a > 0, x > 0.
\]

Based on the survival distribution function, the cumulative distribution function was obtained
\[ F_X(x) = 1 - \exp\left(\frac{R}{a} (1 - e^{ax})\right) \quad (4) \]

and probability distribution function
\[
f_X(x) = \frac{d}{dx} \left( 1 - \exp\left(\frac{R}{a} (1 - e^{ax})\right) \right)
= -\exp\left(\frac{R}{a} (1 - e^{ax})\right) \frac{d}{dx} \left(\frac{R}{a} (1 - e^{ax})\right)
= -\exp\left(\frac{R}{a} (1 - e^{ax})\right) \left(-Re^{ax}\right)
= Re^{ax} \exp\left(\frac{R}{a} (1 - e^{ax})\right)
\]

Furthermore, the force of mortality of population age \( x \) (\( \mu_x \)) is obtained as follows:
\[
\mu_x = \frac{f_X(x)}{s_X(x)} = \frac{Re^{ax} \exp\left(\frac{R}{a} (1 - e^{ax})\right)}{\exp\left(\frac{R}{a} (1 - e^{ax})\right)} = Re^{ax}
= R \exp(ax); R > 0 \text{ dan } a > 0 \quad (5)
\]

with \( R \) being the general death rate and \( a \) is changing the value of \( \mu_x \) from year to year, so Gompertz is said to use two-parameter functions \([9]\).

### 2.3. Estimation of heligman-pollard modifications using the death rate of the gompertz distribution

Model Mathematical models for interpolation using the old Helligman-Pollard method from formula (2), while formula (5) for the death rate of Gompertz distribution is used. So that the \( q_x \) and \( s_q_x \) models of the Gompertz distribution death rate are obtained as follows:
\[
q_x = 1 - \exp\left(\frac{R}{a} \exp(ax)(1 - \exp(a))\right) \quad (6)
\]
and
\[
\tilde{s}_q_x = 1 - \exp\left(\frac{R}{a} \exp((5a) - 1)\exp(ax)\right). \quad (7)
\]

For the stages of compiling a complete table of life for the elderly using the Heligman-Pollard modification of the death rate of the Gompertz distribution, there were two stages, namely:

a. Estimating parameters is done by using a mathematical program, and using equations:
\[
\tilde{s}_q_x = 1 - \prod_{i=0}^{5-1} 1 - \left( \exp\left(\exp\left(\frac{R}{a} \exp(a(x+i))(1 - \exp(a))\right)\right) G H^{x+i} \right).
\quad (8)
\]

where \( \tilde{s}_q_x \) is the approximate value of the chance of someone just age \( x \) will die before reaching the age of \( x + 5 \).

b. After the parameters are obtained, the chance of death in the complete life table can be calculated using the following equation:
\[
q_x = \exp\left(\frac{R}{a} \exp(ax)(1 - \exp(a))\right) G H^x. \quad (9)
\]

### 2.4. Data conformity test

Test of data compatibility was done to see how much error was obtained from the Heligman-Pollard modification by using the Gompertz distribution mortality rate. This is the matching of data from the results of modification interpolation and the full the United States life table data in 2014. According to
Agresti and Barbara [10] can conduct data suitability tests using the coefficient of determination with the following equation:

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2} \quad (10)$$

With information $y_i$ is the actual value, $\hat{y}_i$ is the estimated value and $\bar{y}$ is the average value.

3. Result and discussion

Heligman-Pollard modification of the death rate of the Gompertz distribution is a Heligman-Pollard combination and the death rate of the Gompertz distribution which is converted into the $q_x$ equation. Decreasing this equation is based on the Heligman-Pollard elderly method equation (2) with $p_x = 1 - q_x$. The model of death opportunity for the complete life table of the Heligman-Pollard modification with the Gompertz death rate is:

$$q_x = \frac{G^x}{p_x}$$
$$q_x = \frac{G^x}{1 - q_x}$$
$$q_x = (1 - q_x)G^x \quad (11)$$

by substituting equation (11) in equation (6) obtained

$$q_x = \left(1 - 1 + \exp \left(\frac{R}{a} \exp (ax)(1 - \exp (a))\right)\right)G^x$$

$$= \left(\exp \left(\frac{R}{a} \exp (ax)(1 - \exp (a))\right)\right)G^x. \quad (12)$$

The relationship $q_x$ with $q_x$ in the Heligman-Pollard modification with the Gompertz distribution death rate obtained

$$q_x = 1 - \prod_{i=0}^{5-1} (1 - q_{x+i}) \quad (13)$$

evidence:

$$5p_x = 1 - 5q_x$$
$$= 1 - \frac{5d_x}{l_x}$$
$$= \frac{l_{x+5}}{l_x}$$
$$= \frac{l_{x+4}}{l_{x+3}} \cdots \frac{l_{x+1}}{l_x}$$
$$= p_x p_{x+1} \cdots p_{x+5-1}$$
$$= \prod_{i=0}^{5-1} p_{x+i}$$
$$5q_x = 1 - \prod_{i=0}^{5-1} p_{x+i}$$
$$5q_x = 1 - \prod_{i=0}^{5-1} (1 - q_{x+i}) \quad \blacksquare$$

by translating equation (12) to (13) the mortality opportunity model is obtained for a summary table of elderly life:

$$5\hat{q}_x = 1 - \prod_{i=0}^{5-1} \left(1 - \left(\exp \left(\frac{R}{a} \exp (a(x + i))(1 - \exp (a))\right)\right)G^{x+i}\right). \quad (14)$$

The parameter values of equation (14) are obtained using the mathematical program obtained by the values of the parameters $G, H, R$ and $a$ shown in Table 1.
Table 1. Heligman-Pollard Modification parameter values using the Gompertz mortality rate

| Parameter | Nilai         |
|-----------|--------------|
| \( G \)   | 7.93914E-06  |
| \( H \)   | 1.11674      |
| \( R \)   | 0.0000703186 |
| \( a \)   | 0.0862238    |

The parameter estimation values obtained in Table 1 are substituted in equation (12) to produce an estimated value of \( q_x \) in the complete life table of the elderly. The results of the comparison of the \( q_x \) curve between the interpolation values of the Heligman-Pollard modification and the death rate of the Gompertz distribution and the full United States 2014 data life table are shown in Figure 1.

![Figure 1. Estimated value curve \( q_x \) life table elderly](image.png)

Based on Figure 1, it can be seen that the results of Heligman-Pollard's fascination using the death rate of the Gompertz distribution have the same pattern as the United States \( q_x \) data in 2014. Which is a monotonically upward curve, which means that the elderly are getting older big. And it produces an \( R^2 \) value of 0.985017 which means that the method has been able to predict the complete life table of the elderly.

4. Conclusion

Heligman-Pollard modification by using the death rate of Gompertz distribution can be an alternative in estimating the \( q_x \) value for a complete elderly life table. Because it has the same curve pattern with the original data and has an \( R^2 \) value of 0.985017.

References

[1] R1 K K 2013 *Buletin Jendela Data dan Informasi Kesehatan* (Jakarta: Bakti Husada.)
[2] d’Albis H and Collard F 2013 Age groups and the measure of population aging *Demogr. Res.* 29 617–40
[3] Penduduk P and Usia L 2009 Profil Penduduk Lanjut Usia 2009 25
[4] Németh L and Missov T I 2018 Adequate life-expectancy reconstruction for adult human mortality data *PLoS One* 13 e0198485
[5] Ibrahim R I Estimating a complete life table using the heligman-pollard model, six-point lagrangian interpolation and king’s osculatory interpolation
[6] Sharrow D J, Clark S J, Collinson M A, Kahn K and Tollman S M 2013 The age pattern of increases in mortality affected by HIV: Bayesian fit of the Heligman-Pollard model to data
from the Agincourt HDSS field site in rural northeast South Africa Demogr. Res. 29 1039
[7] Mazzuco S, Scarpa B and Zanotto L 2018 A mortality model based on a mixture distribution function Popul. Stud. (NY). 72 191–200
[8] Kostaki A and Panousis V 2001 Expanding an abridged life table Demogr. Res. 5 1–22
[9] Doray L G 2008 Inference for Logistic-type Models for the Force of Mortality Living to 100 symposium
[10] Agresti A and Barbara F 1986 Statistical Methods for the Social Sciences (California: D. Ellen Publishing Company.)