Structural Properties of Planar Graphs of Urban Street Patterns

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Recent theoretical and empirical studies have focused on the structural properties of complex relational networks in social, biological and technological systems. Here we study the basic properties of twenty 1-square-mile samples of street patterns of different world cities. Samples are represented by spatial (planar) graphs, i.e. valued graphs defined by metric rather than topological distance and where street intersections are turned into nodes and streets into edges. We study the distribution of nodes in the 2-dimensional plane. We then evaluate the local properties of the graphs by measuring the meshedness coefficient and counting short cycles (of three, four and five edges), and the global properties by measuring global efficiency and cost. As normalization graphs, we consider both minimal spanning trees (MST) and greedy triangulations (GT) induced by the same spatial distribution of nodes. The results indicate that most of the cities have evolved into networks as efficient as GT, although their cost is closer to the one of a tree. An analysis based on relative efficiency and cost is able to characterize different classes of cities.

I. INTRODUCTION

During the last decade, the growing availability of large databases, the increasing computing powers, as well as the development of reliable data analysis tools, have constituted a better machinery to explore the topological properties of several complex networks from the real world \cite{1,2,3,4}. This has allowed to study a large variety of systems as diverse as social, biological and technological. The main outcome of this activity has been to reveal that, despite the inherent differences, most of the real networks are characterized by the same topological properties, as for instance relatively small characteristic path lengths and high clustering coefficients (the so-called small-world property) \cite{5}, scale-free degree distributions \cite{6}, degree correlations \cite{7}, and the presence of motifs \cite{8} and community structures \cite{9}. All such features make real networks radically different from regular lattices and random graphs, the standard topologies usually used in modeling and computer simulations. This has led to a large attention towards the comprehension of the evolution mechanisms that have shaped the topology of a real network, and to the design of new models retaining the most significant properties observed empirically.

Spatial networks are a special class of complex networks whose nodes are embedded in a two or three-dimensional Euclidean space and whose edges do not define relations in an abstract space (such as in networks of acquaintances or collaborations between individuals), but are real physical connections \cite{4}. Typical examples include neural networks \cite{10}, information/communication networks \cite{11,12}, electric power grids \cite{13} and transportation systems ranging from river \cite{14}, to airport \cite{15,16} and street \cite{17} networks. Most of the works in the literature, with a few relevant exceptions \cite{11,18,19}, have focused on the characterization of the topological properties of spatial networks, while the spatial aspect has received less attention, when not neglected at all. However, it is not surprising that the topology of such systems is strongly constrained by their spatial embedding. For instance, there is a cost to pay for the existence of long-range connections in a spatial network, this having important consequences on the possibility to observe a small-world behavior. Moreover, the number of edges that can be connected to a single node is often limited by the scarce availability of physical space, this imposing some constraints on the degree distributions. In few words, spatial networks are different from other complex networks and as such they need to be studied in a different way.

In this paper we focus on a particular class of spatial networks: networks of urban street patterns. We consider a database of 1-square mile samples of different world cities and for each city we construct a spatial graph by associating nodes to street intersections and edges to streets. In this way, each of the nodes of the graph is given a location in a 2-dimensional square, and a real number, representing the length of the corresponding street, is associated to each edge. By construction, the resulting graphs are planar graphs, i.e. graphs forming nodes whenever two edges cross. After a previous work on the distribution of centrality measures \cite{20}, here we present a comparative study of the basic properties of spatial networks of different city street patterns. In particular we evaluate the characteristics of the graphs both at a global and at a local scale. The main problem with spatial graphs is that, in most of the cases, the random graph or the complete graph are no more a good way to normalize the results. In fact, the common procedure in relational (non-spatial) complex networks is to compare the properties of the original graph derived from the real system with those of some randomized versions of the graph, i.e. of graphs with the same number of nodes and links as the original one, but where the links are distributed at random. This is, for instance, the standard
way proposed by Watts and Strogatz in Ref. 5 to assess whether a real system is a small world. One quantifies the structural properties of the original graph by computing its characteristic path length $L$ and clustering coefficient $C$, where $L$ measures the typical separation between two vertices in the graph (a global property), whereas $C$ measures the cliquishness of a typical neighbourhood (a local property). Then, the graph is a small-world if $L$ assume a value close to that obtained for the randomized version of the graph, $L_{\text{rand}}$, while the value of $C$ is much larger than $C_{\text{rand}}$. Similarly, in the efficiency-based formalism proposed in Refs. 21,22, a small-world network is defined as a system being extremely efficient in exchanging information both at a global and at a local scale. Again, the values of global and local efficiency are compared with those obtained for a randomized version of the graph. A similar method is used in the counting of short cycles or specific motifs in a graph representing a real system 5. The research of the motifs and cycles is based on matching algorithms counting the total number of occurrences of each motif and each cycle in the original graph and in the randomized ones. Then, a motif or a cycle is statistically significant if it appears in the original graph at a number much higher than in the randomized versions of the graph. In a planar graph, as those describing urban street patterns, the randomized version of the graph is not significative because it is almost surely a non-planar graph due to the edge crossings induced by the random rewiring of the edges. Moreover, because of the presence of long-range links, a random graph corresponds to an extremely costly street pattern configuration, where the cost is defined as the sum of street lengths 22. The alternative is to compare urban street patterns with grid-like structures. Following Ref. 18, we shall consider both minimum spanning trees and greedy triangulations induced by the real distribution of nodes in the square. Spanning trees are the planar graphs with the minimum number of links in order to assure connectedness, while greedy triangulations are graphs with the maximum number of links compatible with the planarity. Spanning trees and greedy triangulations will serve as the two extreme cases to normalize the structural measures we are going to compute.

II. NETWORKS OF URBAN STREET PATTERNS

The database we have studied consists of twenty 1-square mile samples of different world cities, selected from the book by Allan Jacobs 23. We have imported the twenty maps into a GIS (Geographic Information System) environment and constructed the correspondent spatial graphs of street networks by using a road-centerline-between-nodes format 22. Namely, each urban street pattern is transformed into a undirected, valued (weighted) graph $G = (\mathcal{V}, \mathcal{L})$, embedded in the 2-dimensional unit square. In Fig. 1 we show the case for the city of Savannah: in the upper-left panel we report the original map, and in upper-right panel the obtained graph. $\mathcal{V}$ is the set of $N$ nodes representing street intersections and characterized by their positions $(x_i, y_i)_{i=1,...,N}$ in the square. $\mathcal{L}$ is the set of $K$ links representing streets. The links follow the footprints of real streets and are associated a set of real positive numbers representing the street lengths, $(l_k)_{k=1,...,K}$. The graph is then described by the adjacency $N \times N$ matrix $A$, whose entry $a_{ij}$ is equal to 1 when there is an edge between $i$ and $j$ and 0 otherwise, and by a $N \times N$ matrix $L$, whose entry $l_{ij}$ is equal to the length of the street connecting node $i$ and node $j$. In this way both the topology and the geography (metric distances) of the system will be taken into account. A list of the considered cities is reported in Table I together with the basic properties of the derived graphs. The considered cases exhibit striking differences in terms of cultural, social, economic, religious and geographic context. In particular, they can be roughly divided into two large classes: 1) patterns grown throughout a largely self-organized, fine-grained historical process, out of the control of any central agency; 2) patterns realized over a short period of time as the result of a single plan, and usually exhibiting a regular grid-like structure. Ahmedabad, Cairo and Venice are the most representative examples of self-organized patterns, while Los Angeles, Richmond, and San Francisco are typical examples of mostly-planned patterns. We have selected two different parts of the city of Irvine, CA, (named Irvine 1 and Irvine 2) for two highly diverse kinds of urban fabrics: the first is a sample of an industrial area showing enormous blocks with few intersections while the second is a typical residential early sixties “lollipop” low density suburb based on a tree-like layout with a lot of dead-end streets. The differences between cities are already evident from the basic properties of the derived graphs. In fact, the number of nodes $N$, the number of links $K$, and the cost of the wiring, defined as the sum of street lengths:

$$Cost = \sum_{i,j} a_{ij} l_{ij}$$

and measured in meters, assume widely different values, notwithstanding the fact we have considered the same amount of land. Notice that Ahmedabad has 2870 street intersections and 4387 streets in a surface of 1-square mile, while Irvine has only 32 intersections and 37 streets. Ahmedabad and Cairo are the cities with the largest cost, while the cost is very small (less than 40000 meters) in Barcelona, Brasilia, Irvine, Los Angeles, New Delhi, New York, San Francisco, Washington and Walnut Creek. A large difference is also present in the average edge (street) length $\langle l \rangle$, that assumes the smallest values in cities as Ahmedabad, Cairo and Venice, and the largest value in San Francisco, Brasilia, Walnut Creek and Los Angeles. In Ref. 20 we have studied the edges length distribution $P(l)$ for the two different classes of cities, showing that self-organized cities show single peak distributions, while
mostly planned cities exhibit a multimodal distribution, due to their grid pattern. We now have gone deeper into the characterization of the distributions of nodes (street intersections) in the unit square: we have calculated the fractal dimension of the distributions, by using the box counting method [25]. In all the samples, except Irvine 1 that is too small to draw any conclusion, we have found that the nodes are distributed on a fractal support with a fractal dimension ranging from 1.7 to 2.0. This result is similar to that obtained by Yook et al. for the spatial distribution of the nodes of the Internet, considered both at the level of routers and at the level of autonomous systems [11].

### A. Minimum Spanning Tree (MST) and Greedy Triangulation (GT)

Planar graphs are those graphs forming vertices whenever two edges cross, whereas non-planar graphs can have edge crossings that do not form vertices [26]. The graphs representing urban street patterns are, by construction, planar, and we will then compare their structural properties with those of minimally connected and maximally connected planar graphs. In particular, following Buhl et al. [18], we consider the Minimum Spanning Tree (MST) and the Greedy Triangulation (GT) induced by the distribution of nodes (representing street intersections) in the square. The Minimum Spanning Tree (MST) is the shortest tree which connects every nodes into a single connected component. By definition the MST is an acyclic graph that contains $K_{\text{min}} = N - 1$ links. This is the minimum number of links in order to have all the nodes belonging to a single connected component [26]. At the other extreme, the maximum number of links, $K_{\text{max}}$, that can be accommodated in a planar graph with $N$ nodes (without breaking the planarity) is equal to $K_{\text{max}} = 3N - 6$ [27]. The natural reference graph should be then the Minimum Weight Triangulation (MWT), which is the planar graph with the highest number of edges $K_{\text{max}}$, and that minimize the total length. Since no polynomial time algorithm is known to compute the MWT, we thus consider the Greedy Triangulation (GT), that is based on connecting couples of nodes in ascending order of their distance provided that no edge crossing is introduced [28]. The GT is easily computable and leads to a maximal connected planar graph, while minimizing as far as possible the total length of edges considered.

To construct both the MST and the GT induced by the spatial distribution of points (nodes) $\{x_i, y_i\}_{i=1,\ldots,N}$ in the unit square, we have first sorted out all the couples of nodes, representing all the possible edges of a complete graph, by ascending order of their length. Notice that the length of the edge connecting node $i$ and node $j$ is here taken to be equal to the Euclidean distance $d_{\text{Euc}} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$. Then, to compute the MST we have used the Kruskal algorithm [29]. The algorithm consists in browsing the ordered list, starting from the shortest edge and progressing toward the longer ones. Each edge of the list is added if and only if the graph obtained after the edge insertion is still a forest or it is a tree. A forest is a disconnected graph in which any two elements are connected by at most one path, i.e., a disconnected ensemble of trees. (In practice, one checks whether the two end-nodes of the edge are belonging or not to the same component). With this procedure, the graph obtained after all the links of the ordered list are considered is the MST. In fact, when the last link is included in the graph, the forest reduces to a single tree. Since in the Kruskal algorithm an edge producing a crossing would also produce a cycle, following this procedure prevents for creating edge crossings. To compute the GT we have constructed a brute force algorithm based on some particular characteristics of planar GT [28]. The algorithm consists in browsing the ordered list of edges in ascending order of length, and checking for each edge whether adding it produces any intersections with any other edge already added.

For each of the twenty cities we have constructed the respective MST and GT. These two bounds make also sense as regards the possible evolution of a city: the most primitive forms are close to trees, while more complex forms involve the presence of cycles. We can then compare the structural properties of the origial graphs representing the city with those of the two limiting cases represented by MST and GT. As an example in Fig. 1 in the bottom-left and in the bottom-right panel we report respectively the MST and the GT obtained for the city.

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| CITY     | N  | K   | Cost | $\langle l \rangle$ | $D_{\text{box}}$ |
|----------|----|-----|------|---------------------|-----------------|
| Ahmedabad| 2870| 4387| 121037| 27.59               | 1.92            |
| Barcelona| 210 | 323 | 36179 | 112.01              | 1.99            |
| Bologna  | 541 | 773 | 51219 | 66.26               | 1.95            |
| Brasilia | 179 | 230 | 30910 | 134.39              | 1.83            |
| Cairo    | 1496| 2255| 84395 | 37.47               | 1.82            |
| Irvine 1 | 32  | 36  | 11234 | 312.07              |                |
| Irvine 2 | 217 | 227 | 28473 | 128.26              | 1.81            |
| Los Angeles | 240 | 340 | 38716 | 113.87              | 1.90            |
| London   | 488 | 730 | 52800 | 72.33               | 1.94            |
| New Delhi| 252 | 334 | 32281 | 96.56               | 1.85            |
| New York | 248 | 419 | 36172 | 86.33               | 1.72            |
| Paris    | 335 | 494 | 44109 | 89.29               | 1.88            |
| Richmond | 697 | 1086| 62608 | 57.65               | 1.78            |
| Savannah | 584 | 958 | 62050 | 64.77               | 1.85            |
| Seoul    | 869 | 1307| 68121 | 52.12               | 1.87            |
| San Francisco | 169 | 271 | 38187 | 140.91              | 1.90            |
| Venice   | 1840| 2407| 75219 | 31.25               | 1.81            |
| Vienna   | 467 | 692 | 49035 | 72.16               | 1.88            |
| Washington| 192 | 303 | 36342 | 119.94              | 1.93            |
| Walnut Creek | 169 | 197 | 25131 | 127.57              | 1.80            |

TABLE I: Basic properties of the planar graphs obtained from the twenty city samples considered. $N$ is the number of nodes, $K$ is the number of edges, Cost and $\langle l \rangle$ are respectively the total length of edges and the average edge length (both expressed in meters), $D_{\text{box}}$ is the box-counting fractal dimension.
FIG. 1: The urban pattern of Savannah as it appears in the original map (top-left), and reduced into a spatial graph (top-right). We also report the corresponding MST (bottom-left) and GT (bottom-right).

B. Graph local properties

The degree of a node is the number of its direct connections to other nodes. In terms of the adjacency matrix, the degree $k_i$ of node $i$ is defined as $k_i = \sum_{j=1, N} a_{ij}$. In many real networks, the degree distribution $P(k)$, defined as the probability that a node chosen uniformly at random has degree $k$, or, equivalently, as the fraction of nodes in the graph having degree $k$, significantly deviates from the Poisson distribution expected for a random graph and exhibits a power law (scale-free) tail with an exponent $\gamma$ taking a value between 2 and 3 \cite{1, 2, 4}. As already mentioned in the introduction, we do not expect to find scale-free degree distributions in planar networks because the node degree is limited by the spatial embedding. In particular, in the networks of urban street patterns considered, it is very improbable to find an intersection with more than 5 or 6 streets. In Fig. 2, we report the average degree $\langle k \rangle$, and the degree distribution $P(k)$ for $k$ going from 1 to 5. The cities are labeled with an index going from 1 to 20, the same index we used in Table I. In all the samples considered, the largest number of nodes have a degree equal to 2 or 3. Self-organized cities as Ahmedabad, Bologna, Cairo and Venice have $P(k = 3) > P(k = 4)$, while the inverse is true for most of the single-planned cities as New York, San Francisco and Washington, because of their square-grid structure. It is not the aim of this article to discuss the meaning of such differences in terms of their possible impacts on crucial factors for urban life, such as pedestrian movement, wayfinding, land-uses or other cognitive or behavioural matters. However, it is worth noting that, for instance, 3-arms and 4-arms street junctions are expected to perform very differently in human orienteering within an urban complex system due to the differences in the angle widths involved in each turn \cite{31, 32}. It is also interesting to notice the significative frequency of nodes with degree 1 in cities as Irvine and Walnut Creek. Such nodes correspond to the dead-end cul-de-sac streets typical of the suburban early Sixties “lollipop” layouts, which in turn leads to highly debated topics in the current discussion about safety and liveability of modern street patterns as opposite to more traditional ones \cite{33, 34}.

Many complex networks show the presence of a large number of short cycles or specific motifs \cite{1, 2, 4}. For instance, the so called local clustering, also known as transitivity, is a typical property of acquaintance networks, where two individuals with a common friend are likely to know each other \cite{15}. The degree of clustering is usually quantified by the calculation of the clustering coefficient $C$, introduced in Ref. \cite{5}, that is a measure of the fraction of triangles present in the network. Such a quantity is known each other \cite{9}.

The clustering coefficient $C$ is defined as

$$C = \frac{1}{N} \sum_i \frac{1}{k_i(k_i-1)} \sum_{j \neq i} a_{ij} a_{ji}$$

where $N$ is the number of nodes in the network, $k_i$ is the degree of node $i$, and $a_{ij}$ is 1 if there is an edge between nodes $i$ and $j$ and 0 otherwise. The clustering coefficient is a measure of the fraction of triangles present in the network, and it is a key characteristic of many real-world networks. A high clustering coefficient indicates that nodes tend to form tightly connected groups, while a low clustering coefficient suggests a more disordered network structure. In the context of urban street networks, the clustering coefficient can provide insights into the connectivity and organization of the network, helping to understand the spatial arrangement and accessibility of different areas. For instance, in cities with a high clustering coefficient, streets are more likely to form compact and tightly connected neighborhoods, which can affect various aspects of urban life, such as transportation, social interactions, and land-use planning. Understanding the clustering coefficient in urban networks is crucial for urban planning and design, as it can guide the creation of more efficient and liveable urban environments. The nature and distribution of clustering coefficients in urban street networks can be influenced by various factors, including historical development, cultural preferences, and planning policies. Therefore, studying the clustering coefficient in urban networks can contribute to a deeper understanding of urban dynamics and inform evidence-based urban design strategies.
TABLE II: Local properties of the graphs of urban street patterns. We report the meshedness coefficient $M$ [13], and the number $C_k$ of cycles of length $k = 3, 4, 5$ normalized to the number of cycles in the GT, $C_k^{GT}$.

| CITY               | M   | $C_3/C_3^{GT}$ | $C_4/C_4^{GT}$ | $C_5/C_5^{GT}$ |
|--------------------|-----|----------------|----------------|----------------|
| 1 Ahmedabad        | 0.262 | 0.023 | 0.042 | 0.020 |
| 2 Barcelona        | 0.275 | 0.019 | 0.101 | 0.019 |
| 3 Bologna          | 0.214 | 0.015 | 0.048 | 0.013 |
| 4 Brasilia         | 0.147 | 0.020 | 0.027 | 0.012 |
| 5 Cairo            | 0.253 | 0.020 | 0.043 | 0.019 |
| 6 Irvine 1         | 0.085 | 0.035 | 0.022 | 0.005 |
| 7 Irvine 2         | 0.014 | 0.007 | 0.004 | 0.000 |
| 8 Los Angeles      | 0.211 | 0.002 | 0.075 | 0.011 |
| 9 London           | 0.249 | 0.011 | 0.060 | 0.000 |
| 10 New Delhi       | 0.154 | 0.011 | 0.020 | 0.011 |
| 11 New York        | 0.348 | 0.024 | 0.136 | 0.028 |
| 12 Paris           | 0.241 | 0.028 | 0.063 | 0.016 |
| 13 Richmond        | 0.279 | 0.034 | 0.068 | 0.022 |
| 14 Savannah        | 0.322 | 0.002 | 0.111 | 0.026 |
| 15 Seoul           | 0.253 | 0.021 | 0.051 | 0.021 |
| 16 San Francisco   | 0.309 | 0.003 | 0.148 | 0.003 |
| 17 Venice          | 0.152 | 0.016 | 0.030 | 0.010 |
| 18 Vienna          | 0.242 | 0.007 | 0.063 | 0.018 |
| 19 Washington      | 0.293 | 0.026 | 0.132 | 0.022 |
| 20 Walnut Creek    | 0.084 | 0.000 | 0.011 | 0.003 |

One of the possible mechanisms ruling the growth of an urban system is the achievement of efficient pedestrian and vehicular movements on a global scale. This has important consequences on a number of relevant factors affecting the economic, environmental and social performances of cities, ranging from accessibility to micro-criminality and land-uses [37]. The global efficiency of an urban pattern in exchanging goods, people and ideas should be considered a reference when the capacity of that city to support its internal relational potential is questioned. It is especially important to develop a measure that allows the comparison between cases of different form and size, which poses a problem of normalization [38]. The global structural properties of a graph can be evaluated by the analysis of the shortest paths between all pairs of nodes. In a relational (unweighted) network the shortest path length between two nodes $i$ and $j$ is the minimum number of edges to traverse to go from $i$ to $j$. In a spatial (weighted) graph, instead we define the shortest path length $d_{ij}$ as the smallest sum of the edge lengths throughout all the possible paths in the graph from $i$ to $j$ [21, 22]. In this way, both the topology and the geography of the system is taken into account. As a measure of the efficiency in the communication between the nodes of a spatial graph, we use the so-called global efficiency $E$, a measure defined in Ref. [21] as:

$$E = \frac{1}{N(N-1)} \sum_{i,j,i \neq j} \frac{d_{ij}^{Euc}}{d_{ij}}$$  \hspace{1cm} (2)$$

Here, $d_{ij}^{Euc}$ is the distance between nodes $i$ and $j$ along a straight line, defined in Section 11A and we have adopted
TABLE III: The efficiency $E$ of each city is compared to the minimum and maximum values of the efficiency obtained respectively for the MST and the GT. The cities are labeled from 1 to 20 as in Table II.

| CITY          | $E$  | $E^{\text{MST}}$ | $E^{\text{GT}}$ |
|---------------|------|------------------|------------------|
| Ahmedabad     | 0.818| 0.351            | 0.944            |
| Barcelona     | 0.814| 0.452            | 0.930            |
| Bologna       | 0.799| 0.473            | 0.936            |
| Brasilia      | 0.695| 0.503            | 0.931            |
| Cairo         | 0.800| 0.385            | 0.943            |
| Irvine 1      | 0.755| 0.604            | 0.943            |
| Irvine 2      | 0.374| 0.533            | 0.932            |
| Los Angeles   | 0.780| 0.460            | 0.930            |
| London        | 0.800| 0.475            | 0.936            |
| New Delhi     | 0.766| 0.490            | 0.930            |
| New York      | 0.835| 0.433            | 0.931            |
| Paris         | 0.838| 0.473            | 0.938            |
| Richmond      | 0.800| 0.502            | 0.839            |
| Savannah      | 0.793| 0.341            | 0.922            |
| Seoul         | 0.814| 0.444            | 0.941            |
| San Francisco | 0.792| 0.448            | 0.893            |
| Venice        | 0.673| 0.386            | 0.943            |
| Vienna        | 0.811| 0.423            | 0.937            |
| Washington    | 0.837| 0.452            | 0.930            |
| Walnut Creek  | 0.688| 0.481            | 0.938            |

By definition the MST has a relative cost $C_{\text{rel}} = 0$, while GT has $C_{\text{rel}} = 1$. In Fig. 3 we plot for each city $E_{\text{rel}}$ as a function of $C_{\text{rel}}$. The cities can be a-priori divided into different classes: 1) medieval fabrics, including both arabic (Ahmedabad and Cairo) and european (Bologna, London, Venice, and Vienna); 2) grid-iron fabrics (Barcelona, Los Angeles, New York, Richmond, Savannah and San Francisco); 3) modernist fabrics (Brasilia and Irvine 1); 4) baroque fabrics (New Delhi and Washington); 5) mixed fabrics (Paris and Seoul); 6) “lollipop” layouts (Irvine 2 and Walnut Creek). We shall see that the plot $E_{\text{rel}}$ vs. $C_{\text{rel}}$ has a certain capacity to characterize the different classes of cities listed above. The plot indicates an overall increasing behavior of $E_{\text{rel}}$ as function of $C_{\text{rel}}$, with a saturation at $E_{\text{rel}} \sim 0.8$ for values of $C_{\text{rel}} > 0.3$. Grid-iron patterns exhibit a high value of relative efficiency, about 70–80% of the efficiency of the GT, with a relative cost which goes from 0.342 to 0.383. Medieval patterns have in general a lower cost and efficiency than grid-iron patterns although, in some cases as Ahmedabad and Cairo (the two medieval cities with the largest efficiency), they can also reach a value of $E_{\text{rel}} \sim 0.8$ with a smaller cost equal to 0.29. Modernist and “lollipop” layouts are those with the smallest value of Cost but also with the smallest value of efficiency.

Thus define a normalized cost measure, $C_{\text{rel}}$, as:

$$ C_{\text{rel}} = \frac{C - C_{\text{MST}}}{C_{\text{GT}} - C_{\text{MST}}} $$

Of course, the counterpart of an increase in efficiency is an increase in the cost of construction, i.e., an increase in the number and length of the edges. The cost of construction can be quantified by using the measure $Cost$ defined in formula (1). Given a set of $N$ nodes, the shortest (minimal cost) planar graph that connects all nodes corresponds to the MST, while a good approximation for the maximum cost planar graph is given by the GT. We
III. CONCLUSIONS

We have proposed a method to characterize both the local and the global properties of spatial graphs representing urban street patterns. Our results show that a comparative analysis on the structure of different world cities is possible by the introduction of two limiting auxiliary graphs, the MST and the GT. A certain level of structural similarities across cities as well as some difference are well captured by counting cycles and by measuring normalized efficiency and cost of the graphs. The method can be applied to other planar graphs of different nature, as highway or railway networks.

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