Drinfeld Basis And a Nonclassical Free Boson Representation of
Twisted Quantum Affine Superalgebra \( U_q[osp(2|2)^{(2)}] \)

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Abstract

We derive from the super RS algebra the Drinfeld basis of the twisted quantum
affine superalgebra \( U_q[osp(2|2)^{(2)}] \) by means of the Gauss decomposition technique.
We explicitly construct a nonclassical level-one representation of \( U_q[osp(2|2)^{(2)}] \) in
terms of two \( q \)-deformed free boson fields.

Keywords: Drinfeld basis, quantum affine superalgebras, free boson representation.

1 Introduction

The algebraic analysis method based on infinite dimensional non-abelian symmetries has
proved extremely successful in both formulating and solving lower dimensional integrable
systems \([1,2] \). The key elements of this method are infinite-dimensional highest weight
representations of quantum (super) algebras and vertex operators. Drinfeld bases of quantu-
mm (super) algebras are of great importance in constructing the infinite-dimensional rep-
resentations and vertex operators. The Drinfeld bases of quantum affine bosonic algebras
were given by Drinfeld \([3] \). For the case of quantum affine superalgebras, the Drinfeld
bases have been known only for \( U_q[gl(m|n)^{(1)}] \) \([4,5,6] \) and \( U_q[osp(1|2)^{(1)}] \) \([7,8] \). In this pa-
per, we derive the Drinfeld basis of the twisted quantum affine superalgebra \( U_q[osp(2|2)^{(2)}] \)
by using the super version of the Reshetikhin-Semenov-Tian-Shansky (RS) algebra \[ \] the Gauss decomposition technique of Ding-Frenkel \[ \]. Moreover, we explicitly construct a nonclassical level-one representation of \( U_q[osp(2|2)^{(2)}] \) by means of two \( q \)-deformed free boson fields.

2 Drinfeld Basis of \( U_q[osp(2|2)^{(2)}] \)

The symmetric Cartan matrix of the twisted affine superalgebra \( osp(2|2)^{(2)} \) is \( (a_{ij}) \) with \( a_{11} = a_{22} = 1, \ a_{12} = a_{21} = -1 \). Twisted quantum affine superalgebra \( U_q[osp(2|2)^{(2)}] \) is a \( q \)-analogue of the universal enveloping algebra of \( osp(2|2)^{(2)} \) and is generated by the Chevalley generators \( \{E_i, F_i, K_i^{\pm 1} | i = 0, 1 \} \). The \( \mathbb{Z}_2 \)-grading of the Chevalley generators is \( [E_i] = [F_i] = 1, \ [K_i] = 0, \ i = 0, 1 \). The defining relations are

\[
\begin{align*}
K_i K_j &= K_j K_i, \\
K_j E_i K_j^{-1} &= q^{a_{ij}} E_i, & K_j F_i K_j^{-1} &= q^{-a_{ij}} F_i, \\
\{E_i, F_j\} &= \delta_{ij} \frac{K_i - K_i^{-1}}{q - q^{-1}} \hspace{2cm} (2.1)
\end{align*}
\]

plus \( q \)-Serre relations which we omit. Here \( \{X, Y\} \equiv XY + YX \).

\( U_q[osp(2|2)^{(2)}] \) is a \( \mathbb{Z}_2 \)-graded quasi-triangular Hopf algebra endowed with the coproduct \( \Delta \), counit \( \epsilon \) and antipode \( S \) given by

\[
\begin{align*}
\Delta(E_i) &= E_i \otimes 1 + K_i \otimes E_i, & \Delta(F_i) &= F_i \otimes K_i^{-1} + 1 \otimes F_i, \\
\Delta(K_i) &= K_i \otimes K_i, & \epsilon(E_i) &= \epsilon(F_i) = 0, & \epsilon(K_i) &= 1, \\
S(E_i) &= -K_i^{-1} E_i, & S(F_i) &= -F_i K_i, & S(K_i) &= K_i^{-1}, & i &= 0, 1. \hspace{2cm} (2.2)
\end{align*}
\]

The antipode \( S \) is a \( \mathbb{Z}_2 \)-graded algebra anti-homomorphism, i.e. for homogeneous elements \( a, b \in U_q[osp(2|2)^{(2)}] \), we have \( S(ab) = (-1)^{|a||b|}S(b)S(a) \).

We now give our definition of \( U_q[osp(2|2)^{(2)}] \) in terms of Drinfeld generators.

**Definition 1** \( U_q[osp(2|2)^{(2)}] \) is an associative algebra with unit 1 and the Drinfeld generators: \( X^\pm(z) \) and \( \psi^\pm(z) \), a central element \( c \) and a nonzero complex parameter \( q \). \( \psi^\pm(z) \) are invertible. The gradings of the generators are: \( [X^\pm(z)] = 1 \) and \( [\psi^\pm(z)] = [c] = 0 \). The relations are given by

\[
\begin{align*}
\psi^\pm(z) \psi^\pm(w) &= \psi^\pm(w) \psi^\pm(z), \\
\psi^+(z) \psi^-(w) &= \frac{(z_+ q + w_-)(z_- + w_+ q)}{(z_+ + w_- q)(z_- q + w_+)} \psi^+(w) \psi^+(z), \\
\psi^\pm(z^{-1}) X^+(w) \psi^\pm(z) &= \frac{z_+ + w q}{z_+ q + w} X^+(w),
\end{align*}
\]
\[ \psi^\pm(z)X^-(w)\psi^\pm(z)^{-1} = \frac{z^\pm + wz}{z^\pm q + w}X^-(w), \]
\[ (z + wq^{\pm 1})X^\pm(z)X^\pm(w) + (zq^{\pm 1} + w)X^\pm(w)X^\pm(z) = 0, \]
\[ \{X^+(z), X^-(w)\} = \frac{1}{(q - q^{-1})zw} \left[ \delta\left(\frac{w}{z}q^r\right)\psi^+(w+) - \delta\left(\frac{w}{z}q^{-r}\right)\psi^-(z+) \right]. \quad (2.3) \]

Expand the currents in the form
\[ X^\pm(z) = \sum_{n \in \mathbb{Z}} X^\pm_n z^{-n-1}, \]
\[ \psi^\pm(z) = \sum_{n \in \mathbb{Z}} \psi^\pm_n z^{-n} = K^{\pm 1} \exp \left( \pm (q - q^{-1}) \sum_{n > 0} H_{\pm n} z^n \right). \quad (2.4) \]

In terms of modes \( \{H_n | n \in \mathbb{Z} - \{0\}, X^\pm_n | n \in \mathbb{Z} \} \) and \( K \), the defining relations of \( U_q[osp(2|2)^{(2)}] \) are given by

\[ [K, H_n] = 0, \quad [KX^\pm_n, K^{-1}] = q^{\pm 1}X^\pm_n, \]
\[ [H_n, H_m] = \delta_{n+m,0}(-1)^n \frac{[n]_q [nc]_q}{n}, \quad n \neq 0, \]
\[ [H_n, X^\pm_m] = \frac{1}{n}(-1)^n \frac{[n]_q [\mp n]_q}{n} X^\pm_{n+m}, \quad n \neq 0, \]
\[ X^\pm_{n+1} X^\pm_m + q^{\pm 1} X^\pm_{m+1} X^\pm_n + X^\pm_{n+1} X^\pm_{m+1} X^\pm_n + X^\pm_{m+1} X^\pm_n = 0, \]
\[ \{X^+_n, X^-_m\} = \frac{1}{q - q^{-1}} \left( q^{\pm(n-m)} \psi^+_n \psi^-_{n+m} - q^{\mp(m-n)} \psi^-_n \psi^+_m \right). \quad (2.5) \]

Here and throughout \( [i]_q = (q^i - q^{-i})/(q - q^{-1}) \).

The Chevalley generators are obtained by the formulae:
\[ K_1 = K, \quad E_1 = X^+_0, \quad F_1 = X^-_0, \]
\[ K_0 = q^c K^{-1}, \quad E_0 = X^-_1 K^{-1}, \quad F_0 = -K X^+_1. \quad (2.6) \]

In terms of the Chevalley generators, the Drinfeld generators can be built up recursively by
\[ H_1 = q^c K^{-1} (X^+_0 X^-_1 + X^-_0 X^+_1), \]
\[ H_{-1} = q^{-c} K (X^+_1 X^-_0 + X^-_1 X^+_0), \]
\[ X^\pm_{n+1} = \mp q^{\pm c} [H_1, X^\pm_n], \quad X^\pm_{n-1} = \mp q^{\pm c} [H_{-1}, X^\pm_n], \quad n \geq 0 \quad (2.7) \]

plus the formulae for \( H_n, \ H_{-n} \ (n > 0) \) given as follows
\[ H_{\pm n} = \pm \frac{1}{q - q^{-1}} \sum_{p_1 + 2p_2 + \cdots + np_n = n} \frac{(-1)^{\sum p_i - 1} \prod (\sum p_i - 1)!}{p_1! \cdots p_n!} (K^{\mp 1} \psi^\pm_{\pm})^{p_1} \cdots (K^{\mp 1} \psi^\pm_{\pm})^{p_n}, \quad (2.8) \]

where
\[ \psi^\pm_{\pm n} = \pm (q - q^{-1}) q^{\mp c(n-2)} \{X^\pm_{\mp n+1}, X^\pm_{\pm 1}\}, \quad n > 0. \quad (2.9) \]
3 Derivation of Drinfeld Basis from Super RS Algebra

Let us recall the definition of the super RS algebra. Let \( R(z) \in \text{End}(V \otimes V) \), where \( V \) is a \( \mathbb{Z}_2 \)-graded vector space, be a R-matrix which satisfies the graded Yang-Baxter equation

\[
R_{12}(z)R_{13}(zw)R_{23}(w) = R_{23}(w)R_{13}(zw)R_{12}(z). \tag{3.10}
\]

We introduce \([5, 7]\)

**Definition 2**: The super RS algebra \( U(R) \) is generated by invertible L-operators \( L^\pm(z) \), which obey the relations

\[
R\left(\frac{z}{w}\right)L^\pm_1(z)L^\pm_2(w) = L^\pm_2(w)L^\pm_1(z)R\left(\frac{z}{w}\right),
\]

\[
R\left(\frac{z^\pm}{w^\mp}\right)L^\pm_1(z)L^-_2(w) = L^-_2(w)L^\pm_1(z)R\left(\frac{z^\mp}{w^\pm}\right), \tag{3.11}
\]

where \( L^\pm_1(z) = L^\pm(z) \otimes 1 \), \( L^\pm_2(z) = 1 \otimes L^\pm(z) \) and \( z^\pm = zq^\pm \). For the first formula of (3.11), the expansion direction of \( R(z/w) \) can be chosen in \( z/w \) or \( w/z \), but for the second formula, the expansion direction must only be in \( z/w \).

The multiplication rule for the tensor product is defined by

\[
(a \otimes b)(a' \otimes b') = (-1)^{[b][a']} (aa' \otimes bb'), \tag{3.12}
\]

for homogeneous elements \( a, b, a', b' \) of \( U_q[osp(2|2)\otimes 2] \).

In the following we apply the super RS algebra to derive the Drinfeld basis of \( U_q[osp(2|2)\otimes 2] \).

We take \( R(z/w) \) to be the R-matrix associated to the 3-dimensional representation \( V \) of \( U_q[osp(2|2)\otimes 2] \). Let \( v_1, v_2, v_3 \) be the basis vectors of \( V \) with the \( \mathbb{Z}_2 \)-grading \( [v_1] = [v_3] = 0 \) and \( [v_2] = 1 \). It can be shown that the R-matrix has the following form:

\[
R\left(\frac{z}{w}\right) = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & a & 0 & b & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & d & 0 & c & 0 & r & 0 & 0 & 0 \\
0 & f & 0 & a & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & g & 0 & e & 0 & c & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & a & 0 & b & 0 \\
0 & 0 & s & 0 & g & 0 & d & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & f & 0 & a & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}. \tag{3.13}
\]
where
\[
\begin{align*}
    a &= \frac{q(z - w)}{zq^2 - w}, & b &= \frac{w(q^2 - 1)}{zq^2 - w}, & c &= -\frac{q^{1/2}w(q^2 - 1)(z - w)}{(zq^2 - w)(zq + w)}, \\
    d &= \frac{q(z - w)(z + qw)}{(zq^2 - w)(zq + w)}, & e &= a - \frac{zw(q^2 - 1)(q + 1)}{(zq^2 - w)(zq + w)}, \\
    f &= \frac{z(q^2 - 1)}{zq^2 - w}, & g &= -\frac{q^{1/2}z(q^2 - 1)(z - w)}{(zq^2 - w)(zq + w)}, \\
    r &= \frac{w^2(q - 1)(q + 1)^2}{(zq^2 - w)(zq + w)}, & s &= \frac{z^2(q - 1)(q + 1)^2}{(zq^2 - w)(zq + w)}.
\end{align*}
\]

As in the non-super case \([10]\), \(L^\pm(z)\) allow the unique Gauss decomposition
\[
L^\pm(z) = \begin{pmatrix}
1 & 0 & 0 \\
0 & k_1^\pm(z) & 0 \\
0 & 0 & k_3^\pm(z)
\end{pmatrix}
\begin{pmatrix}
1 & f_{1,3}^\pm(z) \\
f_1^\pm(z) & f_2^\pm(z) \\
0 & 0 & 1
\end{pmatrix},
\]
where \(e_{i,j}^\pm(z), f_{i,j}^\pm(z)\), and \(k_i^\pm(z)\) \((i > j)\) are elements in the super RS algebra and \(k_i^\pm(z)\) are invertible; and \(e_i^\pm(z) \equiv e_{i,i+1}^\pm(z), f_i^\pm(z) \equiv f_{i,i+1}^\pm(z)\). Let us define
\[
X_i^+(z) = f_i^+(z) - f_i^-(z), \\
X_i^-(z) = e_i^+(z) - e_i^-(z).
\]

By the definition of the super RS algebra and the Gauss decomposition formula \((3.15)\), and after tedious calculations parallel to those of the \(U_q[osp(1|2)^{(1)}]\) case, we arrive at
\[
\begin{align*}
    k_i^\pm(z)k_j^\pm(w) &= k_j^\pm(w)k_i^\pm(z), & i, j &= 1, 2, 3, \\
    k_i^\pm(z)k_j^\mp(w) &= k_j^\mp(w)k_i^\pm(z), \\
    k_i^\pm(z)k_j^\pm(w) &= k_j^\pm(w)k_i^\pm(z), \\
    \frac{z_\mp - w_\mp}{z_\mp q^2 - w_\mp}k_i^\pm(z)k_j^\mp(w) &= \frac{z_\mp - w_\mp}{z_\mp q^2 - w_\mp}k_j^\pm(w)k_i^\pm(z), \\
    \frac{(z_\mp - w_\mp)(z_\mp + w_\mp)}{(z_\mp q^2 - w_\mp)(z_\mp q + w_\mp)}k_i^\pm(z)k_j^\mp(w)^{-1} &= \frac{(z_\mp - w_\mp)(z_\mp + w_\mp)}{(z_\mp q^2 - w_\mp)(z_\mp q + w_\mp)}k_j^\pm(w)^{-1}k_i^\pm(z), \\
    \frac{(z_\mp - w_\mp q^2)(z_\mp q + w_\mp)}{(z_\mp q^2 - w_\mp)(z_\mp + w_\mp)}k_i^\pm(z)k_j^\mp(w) &= \frac{(z_\mp - w_\mp q^2)(z_\mp q + w_\mp)}{(z_\mp q^2 - w_\mp)(z_\mp + w_\mp)}k_j^\pm(w)k_i^\pm(z), \\
    \frac{z_\mp - w_\mp}{z_\mp q^2 - w_\mp}k_i^\pm(z)^{-1}k_j^\mp(w)^{-1} &= \frac{z_\mp - w_\mp}{z_\mp q^2 - w_\mp}k_j^\pm(w)^{-1}k_i^\pm(z)^{-1}, \\
    k_i^\pm(z)X_j^-\mp(w) &= \frac{z_\mp q^2 - w}{q(z_\pm - w)}X_j^-\mp(w), \\
    k_i^\pm(z)^{-1}X_j^+\mp(w) &= \frac{z_\mp q^2 - w}{q(z_\pm - w)}X_j^+\mp(w),
\end{align*}
\]
\[ k_2^\pm(z)X_1^-(w)k_2^\pm(z)^{-1} = \frac{(z_\pm q^2 - w)(z_\pm + wq)}{q(z_\pm - w)(z_\pm q + w)} X_1^-(w), \]
\[ k_2^\pm(z)^{-1}X_1^+(w)k_2^\pm(z) = \frac{(z_\pm q^2 - w)(z_\pm + wq)}{q(z_\pm - w)(z_\pm q + w)} X_1^+(w), \]
\[ k_3^\pm(z)X_1^-(w)k_3^\pm(z)^{-1} = \frac{z_\pm + wq}{z_\pm q + w} X_1^-(w), \]
\[ k_3^\pm(z)^{-1}X_1^+(w)k_3^\pm(z) = \frac{z_\pm q + w}{z_\pm q + w} X_1^+(w), \]
\[ k_4^\pm(z)X_2^-(w)k_4^\pm(z)^{-1} = \frac{z_\pm q + w}{z_\pm q + w} X_2^-(w), \]
\[ k_4^\pm(z)^{-1}X_2^+(w)k_4^\pm(z) = \frac{z_\pm q + w}{z_\pm - w} X_2^+(w), \]
\[ k_5^\pm(z)X_2^-(w)k_5^\pm(z)^{-1} = \frac{z_\pm q + w}{z_\pm q + w} X_2^-(w), \]
\[ k_5^\pm(z)^{-1}X_2^+(w)k_5^\pm(z) = \frac{z_\pm q + w}{z_\pm q + w} X_2^+(w), \]
\[ (z - wq^2)X_1^+(z)X_2^+(w) + q(z - w)X_2^+(w)X_1^+(z) = 0, \]
\[ q(z - w)X_1^-(z)X_2^-(w) + (z - wq^2)X_2^-(w)X_1^-(z) = 0, \]
\[ (z + wq^{\pm 1})X_1^+(z)X_2^+(w) + (zq^{\pm 1} + w)X_1^+(w)X_1^+(z) = 0, \]
\[ (z + wq^{\pm 1})X_2^+(z)X_2^+(w) + (zq^{\pm 1} + w)X_2^+(w)X_2^+(z) = 0, \]

\[
\{X_1^-(w), X_1^+(z)\} = (q - q^{-1}) \left[ -\delta \left( \frac{z}{w} q^{-1} \right) k_2^+(z_+) k_1^+(z_+)^{-1} \right. \\
+ \delta \left( \frac{z}{w} q^{-1} \right) k_2^-(w_+) k_1^-(w_+)^{-1} \right],
\\
\{X_2^-(w), X_2^+(z)\} = (q - q^{-1}) \left[ \delta \left( \frac{z}{w} q^{-1} \right) k_3^+(z_+) k_2^+(z_+)^{-1} \right. \\
- \delta \left( \frac{z}{w} q^{-1} \right) k_3^-(w_+) k_2^-(w_+)^{-1} \right],
\\
\{X_2^-(w), X_1^+(z)\} = (q - q^{-1}) q^{1/2} \left[ -\delta \left( \frac{z}{w} q^{-1} \right) k_3^+(z_+) k_1^+(z_+)^{-1} \right. \\
+ \delta \left( \frac{z}{w} q^{-1} \right) k_3^-(w_+) k_2^-(w_+)^{-1} \right],
\\
\{X_1^-(w), X_2^+(z)\} = (q - q^{-1}) q^{1/2} \left[ \delta \left( \frac{z}{w} q^{-1} \right) k_3^-(w_+) k_1^+(z_+)^{-1} \right. \\
- \delta \left( \frac{z}{w} q^{-1} \right) k_3^-(w_+) k_2^-(w_+)^{-1} \right],
\]

where
\[
\delta(z) = \sum_{l \in \mathbb{Z}} z^l
\]  
(3.17)
is a formal delta function which enjoys the following properties:

\[
\delta \left( \frac{z}{w} \right) = \delta \left( \frac{w}{z} \right), \quad \delta \left( \frac{z}{w} \right) f(z) = \delta \left( \frac{z}{w} \right) f(w). \tag{3.19}
\]

Defining the algebraic homomorphism

\[
X^\pm(z) = z(q - q^{-1}) \left[ X_1^\pm(z) + X_2^\pm(-zq^{-1}) \right],
\]

\[
\psi^-(z) = (1 + q^{-\frac{1}{2}} - q^{\frac{1}{2}}) \phi_1(z) - \phi_2(-zq^{-1}),
\]

\[
\psi^+(z) = \psi_1(z) - (1 + q^{\frac{1}{2}} - q^{-\frac{1}{2}}) \psi_2(-zq^{-1}), \tag{3.20}
\]

where \(\phi_i(z) = k_{i+1}(z)k_i(z)^{-1}\), \(\psi_i(z) = k_{i+1}(z)k_i(z)^{-1}\), \(i = 1, 2\), then we obtain from (3.17), the current commutation relations (2.3).

\[4\] Level Zero Representation

We consider the evaluation representation \(V_x\) of \(U_q[osp(2|2)^{(2)}]\), where \(V\) is the 3-dimensional graded vector space with basis vectors \(v_1, v_2\) and \(v_3\). Let \(e_{ij}\) be the \(3 \times 3\) matrix satisfying \(e_{ij} v_k = \delta_{jk} v_i\). In the homogeneous gradation, the Chevalley generators of \(U_q[osp(2|2)^{(2)}]\) are represented on \(V_x\) by

\[
E_1 = e_{12} - e_{23}, \quad F_1 = e_{21} + e_{32}, \quad K_1 = q^{e_{11}-e_{33}},
\]

\[
E_0 = xq^{-1}(-e_{21} + e_{32}), \quad F_0 = x^{-1}q(e_{12} + e_{23}), \quad K_0 = q^{-e_{11}+e_{33}}. \tag{4.21}
\]

Let \(V_x^{*S}\) denote the dual module of \(V_x\), defined by \(\pi_V^*(a) = \pi_V(S(a))^st\). On \(V_x^{*S}\), the Chevalley generators are given by

\[
E_1 = -(q^{-1}e_{21} + e_{32}), \quad F_1 = qe_{12} - e_{23}, \quad K_1 = q^{-e_{11}+e_{33}},
\]

\[
E_0 = -xq^{-1}(e_{12} + q^{-1}e_{23}), \quad F_0 = x^{-1}q(-e_{21} + qe_{32}), \quad K_0 = q^{e_{11}-e_{33}}. \tag{4.22}
\]

The following proposition can be proved by induction.

**Proposition 1**: The Drinfeld generators are represented on \(V_x\) by

\[
H_m = -x^m \left[ \frac{m}{m} q^m \left( -(-1)^m q^m e_{11} + (1 - (-1)^m q^m) e_{22} + e_{33} \right) \right],
\]

\[
X_m^+ = -x^m \left( -(-1)^m q^m e_{12} + q^{-m} e_{23} \right), \quad X_m^- = x^m \left( -(-1)^m q^m e_{21} + q^{-m} e_{32} \right), \quad K = q^{e_{11}-e_{33}}, \tag{4.23}
\]

and on \(V_x^{*S}\) by

\[
H_m = -(1)^m x^m \left[ \frac{m}{m} q^{-m} \left( e_{11} + (1 - (-1)^m q^{-m}) e_{22} - (1)^m q^{-m} e_{33} \right) \right],
\]

\[
X_m^+ = -(1)^m x^m q^{-m} \left( q^{-m-1} e_{21} + (-1)^m e_{32} \right), \quad X_m^- = (1)^m x^m q^{-m} \left( q^{-m+1} e_{12} - (-1)^m e_{23} \right), \quad K = q^{-e_{11}+e_{33}}. \tag{4.24}
\]
5 Nonclassical Free Boson Realization at Level One

We use the notation similar to that of \cite{11, 12, 13}. Let us introduce two sets of bosonic oscillators \( \{a_n, c_n, Q_a, Q_c | n \in \mathbb{Z} \} \) which satisfy the commutation relations

\[
[a_n, a_m] = \delta_{n+m,0} (-1)^n \frac{[n]_q^2}{n}, \quad [a_0, Q_a] = 1,
\]

\[
[c_n, c_m] = \delta_{n+m,0} \frac{[n]_q^2}{n}, \quad [c_0, Q_c] = 1.
\]

The remaining commutation relations are zero. Introduce the \( q \)-deformed free boson fields

\[
a(z; \kappa) = Q_a + a_0 \ln z - \sum_{n \neq 0} \frac{a_n}{[n]_q} q^{[n]_q} z^{-n},
\]

\[
c(z) = Q_c + c_0 \ln z - \sum_{n \neq 0} \frac{c_n}{[n]_q} z^{-n}
\]

and set

\[
a_\pm(z) = \pm (q - q^{-1}) \sum_{n > 0} a_\pm n z^{\mp n} \pm a_0 \ln q.
\]

Then

**Theorem 1**: The Drinfeld generators of \( U_q[osp(2|2)^{(2)}] \) at level one have the following nonclassical realization by the free boson fields

\[
\psi^\pm(z) = e^{a_\pm(z)}:
\]

\[
X^\pm(z) = e^{a(z; \mp \frac{i}{2})} Y^\pm(z): F^\pm,
\]

where \( F^+ = q^{1/2} + q^{-1/2}, \quad F^- = 1/(q - q^{-1}) \) and

\[
Y^+(z) = e^{c(q^{1/2}z)} + e^{-c(-q^{-1/2}z)},
\]

\[
Y^-(z) = e^{c(-q^{1/2}z)} + e^{-c(q^{-1/2}z)}.
\]

This free boson realization is nonclassical in the sense that \( X^-(z) \) defined by (5.28) does not have classical or \( q \to 1 \) limit.

**Proof.** We prove this theorem by checking that the bosonized currents (5.28) satisfy the defining relations (2.3) of \( U_q[osp(2|2)^{(2)}] \) with \( c = 1 \). It is easily seen that the first two relations in (2.3) are true by construction. The third and fourth ones follow from the definition of \( X^\pm(z) \) and the commutativity between \( a_n \) and \( c_n \). So we only need to check the last two relations in (2.3).

We write

\[
Z^\pm(z) = e^{\pm a(z; \mp \frac{i}{2})}:
\]
We obtain the operator products

\[ Z^\pm (z) Z^\pm (w) = (z + q^\mp w) : Z^\pm (z) Z^\pm (w) : , \]
\[ Z^\pm (z) Z^-(w) = (z + w)^{-1} : Z^+(z) Z^-(w) : . \]
\[ Y^\pm (z) Y^\pm (w) = \pm q^{1/2}(z - w) : e^{c(\pm q^{1/2} z)} e^{c(\pm q^{1/2} w)} : \mp q^{-1/2}(z - w) : e^{-c(\mp q^{-1/2} z)} e^{-c(\mp q^{-1/2} w)} : , \]
\[ Y^+(z) Y^-(w) = q^{1/2}(z + w) : e^{c(q^{1/2} z)} e^{c(q^{-1/2} w)} : -q^{-1/2}(z + w) : e^{-c(q^{-1/2} z)} e^{c(q^{1/2} w)} : + \frac{q^{-1/2}}{z - q^{-1} w} : e^{c(q^{1/2} z)} e^{c(q^{-1/2} w)} : - \frac{q^{1/2}}{z - q w} : e^{-c(q^{-1/2} z)} e^{c(q^{1/2} w)} : . \]

Then the second last relation in (2.3) is easily seen to be true, and as to the last relation we have

\[ \{ X^+(z), X^-(w) \} = \frac{q^{1/2} - q^{-1/2}}{q - q^{-1}} : Z^+(z) Z^-(w) : \]
\[ = \left[ \frac{q^{-1/2}}{(z + w)(z - q^{-1} w)} + \frac{q^{1/2}}{(w + z)(w - q z)} \right] : e^{c(q^{1/2} z)} e^{-c(q^{-1/2} w)} : \]
\[ = \frac{1}{(q - q^{-1}) z w} : Z^+(z) Z^-(w) : \]
\[ \left[ \delta \left( - \frac{w}{z} \right) - \delta \left( \frac{w}{z} q^{-1} \right) \right] : e^{c(q^{1/2} z)} e^{-c(q^{-1/2} w)} : \]
\[ = \frac{1}{(q - q^{-1}) z w} \left[ \delta \left( \frac{w}{z} q \right) - \delta \left( \frac{w}{z} q^{-1} \right) \right] : Z^+(z) Z^-(w) : \]
\[ = \frac{1}{(q - q^{-1}) z w} \left[ \delta \left( \frac{w}{z} q \right) \psi^+(w q^{1/2}) - \delta \left( \frac{w}{z} q^{-1} \right) \psi^-(w q^{-1/2}) \right] . \] \hspace{1cm} (5.31)

This completes the proof.

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**References**
[1] C. Itzykson, H. Saleur, J.-B. Zuber, eds: *Conformal invariance and applications to statistical mechanics*, World Scientific, Singapore, 1988.

[2] M. Jimbo, T. Miwa, *Alegbraic Analysis of Solvable Lattice Models*, CBMS Regional Conference Series in Mathematics, **Vol.85**, AMS, 1994.

[3] V.G. Drinfeld, Sov. Math. Dokl. **36** (1988) 212.

[4] H.Yamane, e-print [q-alg/9603015](http://arxiv.org/abs/q-alg/9603015).

[5] Y.-Z. Zhang, J. Phys. **A30** (1997) 8325; Phys. Lett. **A234** (1997) 20.

[6] J.-F. Cai, S.K. Wang, K. Wu, W.-Z. Zhao, J. Phys. **A31** (1998) 1989.

[7] M.D. Gould, Y.-Z. Zhang, Lett. Math. Phys. **44** (1998) 291.

[8] J. Ding, e-print [math.QA/9905086](http://arxiv.org/abs/math.QA/9905086).

[9] N.Yu. Reshetikhin, M.A. Semenov-Tian-Shansky, Lett. Math. Phys. **19** (1990) 133.

[10] J. Ding, I.B. Frenkel, Commun. Math. Phys. **155** (1993) 277.

[11] H. Awata, S. Odake, J. Shiraishi, Commun. Math. Phys. **162** (1994) 61.

[12] K. Kimura, J. Shiraishi, J. Uchiyama, Commun. Math. Phys. **188** (1997) 367.

[13] Y.-Z. Zhang, e-print [math.QA/9812084](http://arxiv.org/abs/math.QA/9812084).