Statistical analysis of the wind speed at mountain site Chopok, Slovakia, using Weibull distribution

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Abstract. The 2-parameter Weibull distribution was used to analyse the wind speed at high-altitude mountain site Chopok, situated in the central part of Nízke Tatry mountains in Slovakia. Analysed wind speed data has been collected over 11 year period (2005–2015) and measured under standard conditions – in the height of 10 m over above ground using calibrated anemometers. Suitability of Weibull distribution was assessed using the coefficient of determination and root mean square error. From the results it is concluded that the 2-parameter Weibull distribution adequately describe the observed wind speed and can be used with acceptable statistical accuracy for prediction the wind speed data at this location.

1. Introduction

Mountain site Chopok is the windiest place in Slovakia and a popular mountain resort at the same time. Wind speed influences conditions for snowmaking, lifts operation, skiing, paragliding, and it is also a common factor in the design of structures and buildings. Therefore it is important to analyse its statistical properties precisely. In literature the Weibull distribution is widely used and accepted distribution for modelling the wind speed data [1–22].

In the presented paper the 2-parameter Weibull distribution was used to analyse the wind speed at high-altitude mountain site Chopok, situated in the central part of Nizke Tatry mountains in Slovakia. For this purpose, the hourly wind speed data from meteorological station Chopok collected over the 11-year period (2005–2015) and measured at the standard 10 m height above ground were analysed. The suitability of Weibull distribution was assessed by the coefficient of determination and root mean square error.

The parameters of the Weibull distribution were estimated using the maximum likelihood method. The calculations were performed using statistical software STATISTICA and software MATLAB.

The rest of the paper is organized as follows. In chapter 2, the wind speed data and methods, including the Weibull distribution, maximum likelihood method and performance criteria, are briefly described. In chapter 3, the results and discussion are presented. Finally, in chapter 4, major conclusions are summarized.
2. Data and methods
In this section the wind speed data and methods are briefly described.

2.1. Description of location and wind speed data
Chopok is the second highest peak of the mountain range Nizke Tatry. It has the most inclement climate and at the same time it is the windiest place of Slovakia with its average annual wind speed about 10 m s\(^{-1}\). This place could be characterized as the place of practically constant wind. Generally in the mountains of Slovakia, depending on the altitude, the average annual wind speed gets at 4–8 m s\(^{-1}\) and maximum wind speed up to 60 m s\(^{-1}\).

**Figure 1.** Meteorological station Chopok [23].

**Figure 2.** Universal anemograph sensor [26].

The meteorological station (MS) Chopok is characterized by its parameters like latitude 48°56'38’', longitude 19°35'32’’ and the altitude 2005 meters above sea level. It is situated under the peak of Chopok. Windiness is more noticeable in hilly and mountainous terrain with higher altitude, mainly in open ridge and saddle areas [24–26]. The first measurement and observation by this MS was in 1954. The station Chopok was in 2017 included into the European network EMEP and also into the worldwide network GAW (Global Atmosphere Watch) WMO.

Measuring method for wind direction is anemograph and measuring of wind speed is being done by anemometers. Spatial orientation in the terrain is characterized like a spot measurement.

In accordance with the internal guidelines of the Slovak Hydrometeorological Institute (SHMI), the land of surrounding area for measurement must be flat, grassy and without recesses. The wind tower is included within the specific land. Coordinates refer to this specific land. The wind tower does not need separately measured coordinates, because the specific land has standard dimensions required by internal regulations – that means \((20 \times 20)\) m. Geographic data are focused on GPS coordinates and relate to a rain gauge in accordance with the WMO manual. The standard direction and wind speed at SHMI monitoring stations is 10 m above the ground surface. Currently, automatic devices Vaisala and ultrasonic devices GILL are used to measure wind characteristics. Anemometers have a 2-year calibration interval.

In meteorological practice the direction and the power of a wind vector are recorded separately as the wind direction and the wind speed. The direction of wind is determined by the direction from which the wind blows. It is measured by wind vanes at meteorological stations and recorded as the average direction of wind in the past 10 minutes [26].
Data of selected MS, namely Chopok were recorded from meteorological reports. A total of 96,409 wind speed measurements were collected, but approximately 14% of them were excluded from statistical analyses because of obviously wrong or missing value, or power blackout.

2.2. Weibull distribution

A random variable \( X \) is said to have the 2-parameter Weibull distribution with parameters \( k > 0, \ c > 0 \) if its probability density function (PDF) is given by

\[
f(x) = \frac{k}{c} x^{k-1} \exp\left(-\left(\frac{x}{c}\right)^k\right), \quad \text{for} \ x > 0
\]

and cumulative distribution function (CDF) is given by

\[
F(x) = 1 - \exp\left(-\left(\frac{x}{c}\right)^k\right), \quad \text{for} \ x > 0
\]

where \( x \) is the wind speed, \( k \) is the dimensionless shape parameter and \( c \) is the scale parameter in units of the wind speed (here in m s \(^{-1}\)).

The Weibull distribution is right skewed, reflecting the fact that the strong winds are rare while the moderate and fresh winds are more common. The Rayleigh distribution is a special case of the Weibull distribution for \( k = 2 \). It is known, that the Weibull shape parameter \( k \) generally ranges from 1.5 to 3 for most wind speed conditions in the world [10, 27]. The value of the shape parameter \( k \) can be regarded as an indicator of the wind speed stability. The higher value of \( k \) indicates more stable wind speed while the lower value indicates very variable wind speed. A higher value of \( k \) such as 2.5 or 4 indicates that the variation of mean wind speed is small (e. g. location with steady winds). A lower value of \( k \) such as 1.5 or 2 indicates a greater deviation away from mean wind speed (e. g. location with gusty winds) [28]. The scale parameter \( c \) is proportional to the mean wind speed. The higher value of this parameter \( c \) indicates that the wind speed is higher. For \( c = 3.5 \) the Weibull distribution is similar to the normal one.

Various methods have been developed and compared for estimating the parameters of the Weibull distribution. The method of moments, the least square method and the maximum likelihood method are commonly used [2, 3, 7, 27, 29, 30]. In this paper the parameters of the Weibull distribution were estimated using the maximum likelihood method (MLM). The MLM is preferable and more efficient than the other known estimation methods for large sample size and tends to lower root mean square error [3, 12, 31].

Let \( x_1, x_2, \ldots, x_n \) be a realization of a random sample \( X_1, X_2, \ldots, X_n \) of size \( n \) from the Weibull distribution. The MLM parameter estimates are obtained by solving nonlinear equations

\[
\frac{1}{k} - \frac{1}{n} \sum_{i=1}^{n} x_i^{k} \ln x_i + \frac{1}{n} \sum_{i=1}^{n} \ln x_i = 0,
\]

\[
c = \left(\frac{1}{n} \sum_{i=1}^{n} x_i^k\right)^{1/k}.
\]

These equations are solved iteratively with Newton method. The estimates of the parameters \( k \) and \( c \) are denoted \( \hat{k} \) and \( \hat{c} \), respectively.

2.3. Performance criteria

In order to assess whether the Weibull distribution is suitable for wind speed data modelling or not, several statistical tests can be used [32]. In this paper the coefficient of determination \( (R^2) \) and the root mean square error \( (RMSE) \) were used. The most accurate probability distribution corresponds to smaller \( RMSE \) values and larger values of \( R^2 \). The coefficient of determination can be calculated using
\[
R^2 = \frac{\sum_{i=1}^{n}(\hat{F}(x_i) - \bar{F})^2}{\sum_{i=1}^{n}(\hat{F}(x_i) - \bar{F})^2 + \sum_{i=1}^{n}(F_n(x_i) - \hat{F}(x_i))^2},
\]
and the root mean square error can be calculated using
\[
RMSE = \left[1/n \sum_{i=1}^{n}(F_n(x_i) - \hat{F}(x_i))^2\right]^{1/2},
\]
where \(\hat{F}(x)\) is the estimated cumulative distribution function, \(\bar{F} = 1/n \sum_{i=1}^{n} \hat{F}(x_i)\) and \(F_n(x) = \frac{1}{n} \sum_{i=1}^{n} I(x_{(i)} \leq x)\) is the empirical distribution function, where \(I(x_{(i)} \leq x) = 1\), if \(x_{(i)} \leq x\) and 0 otherwise, and \(x_{(1)}, x_{(2)}, \ldots\) are observations in ascending order, so that \(x_{(1)} \leq x_{(2)} \leq \ldots\).

\(3.3.\) Statistical analysis and results

In this section the results and discussion are presented.

3.1. Wind speed characteristics

The average wind speed and standard deviation are the key characteristics from practical point of view. Tables 1 and 2 present the descriptive statistics of seasonal, annual and monthly wind speed in Chopok. The average wind speed \(\bar{x}\), standard deviation \(s\), skewness \(g_1\) and kurtosis \(g_2\), respectively, are calculated by:
\[
\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i, \quad s = \left[\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2\right]^{1/2}, \quad g_1 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^3, \quad g_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^4 - 3.
\]

It has been shown, that the annual average wind speed is 8.26 m s\(^{-1}\) with standard deviation 5.31 m s\(^{-1}\). The lowest value of the mean wind speed is observed in the summer season 6.61 m s\(^{-1}\) with standard deviation 4.29 m s\(^{-1}\) and the highest value in the winter season 9.74 m s\(^{-1}\) with standard deviation 5.68 m s\(^{-1}\). The highest value of the standard deviation was calculated for the winter season 5.68 m s\(^{-1}\) and the lowest in the summer season 4.29 m s\(^{-1}\).

The monthly average wind speed varies between 6.31 m s\(^{-1}\) and 10.28 m s\(^{-1}\) with minimum in August with standard deviation 4.09 m s\(^{-1}\) and maximum in December with standard deviation.
5.79 m s\(^{-1}\). The highest value of the standard deviation was calculated in March 6.04 m s\(^{-1}\) and the lowest in August 4.09 m s\(^{-1}\).

The monthly variation of average and maximum wind speed is illustrated in figure 3. The monthly variation of standard deviation is illustrated in figure 4.

The skewness is positive. It means that the seasonal (annual, monthly) wind speed data are skewed to the right.

### Table 1. Descriptive statistics of seasonal and annual wind speed (m s\(^{-1}\)).

| Season  | Average | St. dev. | Min | Max | Skewness | Kurtosis |
|---------|---------|----------|-----|-----|----------|----------|
| Spring  | 8.21    | 5.35     | 0.3 | 34.6| 1.02     | 0.71     |
| Summer  | 6.61    | 4.29     | 0.3 | 24.7| 0.98     | 0.48     |
| Autumn  | 8.25    | 5.25     | 0.2 | 29.1| 0.77     | -0.07    |
| Winter  | 9.74    | 5.68     | 0.3 | 33.2| 0.63     | -0.32    |
| Annual  | 8.26    | 5.31     | 0.2 | 34.6| 0.88     | 0.25     |

### Table 2. Descriptive statistics of monthly wind speed (m s\(^{-1}\)).

| Month   | Average | St. dev. | Min | Max | Skewness | Kurtosis |
|---------|---------|----------|-----|-----|----------|----------|
| January | 9.93    | 5.67     | 0.3 | 30.7| 0.67     | -0.15    |
| February| 9.28    | 5.64     | 0.6 | 33.2| 0.66     | -0.25    |
| March   | 9.39    | 6.04     | 0.3 | 34.6| 0.85     | 0.12     |
| April   | 7.91    | 5.31     | 0.5 | 30.7| 1.01     | 0.46     |
| May     | 7.42    | 4.47     | 0.4 | 30.0| 1.05     | 1.40     |
| June    | 6.95    | 4.41     | 0.3 | 24.5| 0.89     | 0.21     |
| July    | 6.68    | 4.45     | 0.5 | 24.7| 1.07     | 0.67     |
| August  | 6.31    | 4.09     | 0.4 | 22.5| 1.01     | 0.67     |
| September| 7.26    | 4.95     | 0.5 | 29.1| 0.98     | 0.51     |
| October | 7.82    | 4.91     | 0.2 | 25.1| 0.75     | -0.13    |
| November| 9.53    | 5.51     | 0.3 | 28.4| 0.59     | -0.40    |
| December| 10.28   | 5.79     | 0.6 | 30.8| 0.53     | -0.57    |

3.2. Analysis of Weibull parameters

Tables 3 and 4 present the seasonal, annual and monthly estimates of the parameters of the Weibull distribution and values of the statistical tests. The annual shape parameter \(k\) is 1.6274, while the annual scale parameter \(c\) is 9.2640 m s\(^{-1}\). The annual value of \(R^2\) is 0.9965 and the annual value of \(RMSE\) is 0.0175. Figure 5 shows the histogram of annual wind speed data and the fitted Weibull PDF. From this figure it is evident that the observed wind speed data is modelled well by 2-parameter Weibull distribution.

The seasonal value of shape parameter \(k\) ranges from 1.6191 in spring to 1.7916 in winter, while the value of the seasonal parameter \(c\) ranges from 7.4202 m s\(^{-1}\) in summer to 10.9805 m s\(^{-1}\) in winter. In general, the value of the scale parameter \(c\) is the highest in the winter season and the lowest in the summer season. The seasonal value of \(R^2\) ranges from 0.9955 to 0.9962 and the value of \(RMSE\) ranges from 0.0183 to 0.0196. Figure 6 shows the histograms of the seasonal wind speed data and the fitted Weibull PDFs. From these figures it is evident that the observed wind speed data are characterized at the well acceptable levels of \(R^2\) and \(RMSE\) by 2-parameter Weibull distribution.

Analyses show that the monthly shape parameter \(k\) ranges from 1.5291 in September to 1.8563 in December and the monthly scale parameter \(c\) ranges from 7.0801 m s\(^{-1}\) in August to 11.5945 m s\(^{-1}\) in December. The monthly values of \(R^2\) are greater than 0.9926 and the values of \(RMSE\) are less than 0.0252. It means that the Weibull distribution describes the data very well.
The lower values of the seasonal, annual and monthly shape parameter \( k \) (less than 2) implies that Chopok is a location with gusty winds. The values of the scale parameter \( c \) are quite high, this implies this location to be windy.

**Figure 5.** The histogram and fitted Weibull probability density distribution for the annual wind speed data.

**Figure 6.** The histograms and fitted Weibull probability density distributions for the seasonal wind speed data.
Table 3. Seasonal and annual estimates of the parameters and statistical tests.

| Parameter estimates | Statistical test |
|---------------------|------------------|
| \( \hat{k} \)       | \( \hat{c} \)    | \( R^2 \) | RMSE  |
| Spring              | 1.6191           | 9.2120   | 0.9960 | 0.0183 |
| Summer              | 1.6238           | 7.4202   | 0.9955 | 0.0196 |
| Autumn              | 1.6318           | 9.2410   | 0.9962 | 0.0184 |
| Winter              | 1.7916           | 10.9805  | 0.9962 | 0.0185 |
| Annual              | 1.6274           | 9.2640   | 0.9965 | 0.0175 |

Table 4. Monthly estimates of the parameters and statistical tests.

| Parameter estimates | Statistical test |
|---------------------|------------------|
| \( \hat{k} \)       | \( \hat{c} \)    | \( R^2 \) | RMSE  |
| January             | 1.8339           | 11.1985  | 0.9979 | 0.0134 |
| February            | 1.7097           | 10.4351  | 0.9945 | 0.0223 |
| March               | 1.6245           | 10.5224  | 0.9947 | 0.0217 |
| April               | 1.5698           | 8.8482   | 0.9926 | 0.0252 |
| May                 | 1.7505           | 8.3643   | 0.9992 | 0.0083 |
| June                | 1.6604           | 7.8168   | 0.9949 | 0.0211 |
| July                | 1.5907           | 7.4899   | 0.9942 | 0.0221 |
| August              | 1.6265           | 7.0801   | 0.9965 | 0.0173 |
| September           | 1.5291           | 8.0881   | 0.9959 | 0.0190 |
| October             | 1.6537           | 8.7705   | 0.9968 | 0.0169 |
| November            | 1.8071           | 10.7437  | 0.9954 | 0.0204 |
| December            | 1.8563           | 11.5945  | 0.9953 | 0.0206 |

4. Conclusion

In this paper the wind speed data at Chopok, located in Nízke Tatry, Slovakia, were statistically analysed using the 2-parameter Weibull distribution. For this purpose, the hourly wind speed data from meteorological station Chopok collected over the 11-year period (2005–2015) were used. The suitability of the Weibull distributions were assessed using the coefficient of determination and root mean square error. The main results obtained from the presented study can be listed as follows:

- The annual shape parameter \( k \) is 1.6274, while the annual scale parameter \( c \) is 9.2640 m s\(^{-1}\).
- The seasonal value of shape parameter \( k \) ranges from 1.6191 to 1.7916, while the value of the seasonal parameter \( c \) ranges from 7.4202 m s\(^{-1}\) to 10.9805 m s\(^{-1}\).
- The monthly shape parameter \( k \) ranges from 1.5291 to 1.8563, while the monthly scale parameter \( c \) ranges from 7.0801 m s\(^{-1}\) to 11.5945 m s\(^{-1}\).
- The shape parameter \( k \) is less than 2, this implies that Chopok is a location with gusty winds.
- The values of the scale parameter \( c \) are quite high, this implies this location to be windy.
- The annual value of \( R^2 \) is 0.9965 and value of \( RMSE \) is 0.0175. The seasonal value of \( R^2 \) ranges from 0.9955 to 0.9962 and the value of \( RMSE \) ranges from 0.0183 to 0.0196 and the monthly values of \( R^2 \) are greater than 0.9926 and the values of \( RMSE \) are less than 0.0252.

These results show that the 2-parameter Weibull distribution adequately describes the wind speed at Chopok, located in Nízke Tatry, Slovakia, and can be used with the acceptable statistical accuracy for prediction of the wind speed data at this location.
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