Hadron tomography and its application to gravitational radii of hadrons

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Abstract Hadron tomography has been investigated by three-dimensional structure functions, such as generalized parton distributions (GPDs) and generalized distribution amplitudes (GDAs). The GDAs are s-t crossed quantities of the GPDs, and both functions probe gravitational form factors for hadrons. We determined the pion GDAs by analyzing Belle data on the differential cross section for the two-photon process $\gamma^*\gamma \rightarrow \pi^0\pi^0$. From the determined GDAs, we calculated timelike gravitational form factors of the pion and they were converted to the spacelike form factors by using the dispersion relation. These gravitational form factors $\Theta_1$ and $\Theta_2$ indicate mechanical (pressure, shear force) and gravitational-mass (or energy) distributions, respectively. Then, gravitational radii are calculated for the pion from the form factors, and they are compared with the pion charge radius. We explain that the new field of gravitational physics can be developed in the microscopic level of quarks and gluons.

Keywords Hadron tomography · QCD · Quark · Gravitational form factor

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1 Introduction

Three dimensional (3D) tomography has been investigated recently for the nucleon by using generalized parton distributions (GPDs) and transverse-momentum-dependent parton distributions (TMDs) from experimental measurements on the deeply virtual Compton scattering (DVCS) and semi-inclusive deep inelastic scattering, respectively [1]. Generalized distribution amplitudes (GDAs) are \( s \)-\( t \) crossed quantities of the GPDs and they can be investigated by the two-photon process \( \gamma^* \gamma \to hh \) to produce a hadron (\( h \)) pair. The major reasons for studying the 3D tomography is (1) to find the origin of nucleon spin including orbital angular momentum contributions [1], (2) to find internal structure of exotic hadron candidates [2], (3) to investigate gravitational properties of hadrons [3].

In 2016, the Belle collaboration published the cross-section data on \( \gamma^* \gamma \to \pi^0 \pi^0 \) [4], so that it became possible to discuss the GDAs in comparison with actual experimental measurements. We determined the pion GDAs by analyzing the Belle data, and then gravitational form factors were evaluated for the pion from the obtained GDAs [3]. We discuss these results in this report.

First, the definitions of the GPDs and GDAs are introduced in Sec. 2.1, gravitational form factors of the pion are explained in Sec. 2.2, and the differential cross section of \( \gamma^* \gamma \to \pi^0 \pi^0 \) is expressed by the GDAs in Sec. 2.3. Our analysis results are discussed for the GDAs and gravitational form factors in Sec. 3.

2 Theoretical formalism

2.1 Three-dimensional structure functions

The 3D structure functions, the GPDs and GDAs, for the hadron \( h \) are measured by the deeply virtual Compton scattering \( \gamma^* h \to \gamma h \) and two-photon process \( \gamma^* \gamma \to hh \), respectively, as shown in Fig. 1. In this article, we explain the GPDs and GDAs for the pion. The quark GPDs \( H_q^{\pi^0} \) for \( \pi^0 \) are defined by off-forward matrix elements of quark operators with a lightcone separation, and the quark GDAs \( \Phi_q^{\pi^0 \pi^0} \) are defined in the same way by the matrix element between the vacuum and the hadron pair [1]:

\[
H_q^{\pi^0}(x, \xi, t) = \int \frac{dy}{4\pi} e^{ixP^+y} \langle \pi^0(p') |\gamma(-y/2)\gamma^+q(y/2) | \pi^0(p) \rangle \mid_{y^+ = y_\perp = 0}, \tag{1}
\]

\[
\Phi_q^{\pi^0 \pi^0}(z, \zeta, W^2) = \int \frac{dy}{2\pi} e^{iz(2z-1)P^+y}/2 \times \langle \pi^0(p) \pi^0(p') |\gamma(-y/2)\gamma^+q(y/2) | 0 \rangle \mid_{y^+ = y_\perp = 0}. \tag{2}
\]

Here, the link variables to satisfy the color gauge invariance are not explicitly written.
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Fig. 1  (a) Virtual Compton process for GPDs.  (b) Two-photon process for GDAs.

Using the initial and final pion (photon) momenta \( p \) and \( p' \) (\( q \) and \( q' \)) in Fig. 1(a), we define average momenta \( \bar{P} = (p + p')/2 \), \( \bar{q} = (q + q')/2 \), and momentum transfer \( \Delta = p' - p = q - q' \). The GPDs are expressed by three kinematical variables, the Bjorken variable \( x \), the skewness parameter \( \xi \), and the momentum-transfer squared \( t \) as

\[
x = \frac{Q^2}{2p \cdot q}, \quad \xi = \frac{Q^2}{2P \cdot q}, \quad t = \Delta^2,
\]

where \( Q^2 = -q^2 \) and \( \bar{Q}^2 = -\bar{q}^2 \). The GDAs are expressed by three different variables, the momentum fractions \( z \) and \( \zeta \) in Fig. 1(b) and the invariant-mass squared \( W^2 \) as

\[
z = \frac{k \cdot q'}{P \cdot q'}, \quad \zeta = \frac{p \cdot q'}{P \cdot q'} = \frac{1 + \beta \cos \theta}{2}, \quad W^2 = (p + p')^2 = (q + q')^2 = s,
\]

where \( P \) is given by \( P = p + p' \), \( a^+ \) indicates the lightcone quantity \( a^+ = (a^0 + a^3)/\sqrt{2} \), \( \beta \) is defined by \( \beta = |p|/p^0 = \sqrt{1 - 4m^2/\pi^2} \), and \( \theta \) is the scattering angle in the center-of-mass frame of final pions.

The DVCS process is factorized into the hard perturbative QCD part and the GPDs as shown in Fig. 1(a), if the kinematical condition \( Q^2 \gg |t|, \Lambda^2_{QCD} \) where \( \Lambda_{QCD} \) is the QCD scale parameter, is satisfied. In the same way, the two-photon process is factorized with the GDAs as shown in Fig. 1(b) if the condition \( Q^2 \gg W^2, \Lambda^2_{QCD} \) is met. Then, the \( \gamma^* \gamma \rightarrow \pi^0 \pi^0 \) cross section is expressed by the GDAs, which can be determined by analyzing the Belle data.

2.2 Gravitational form factors

The GPDs and GDAs contain information on gravitational sources in the quark and gluon level. In order to show it, we take the \( n \)-th moment of the bilocal operator defining the GDAs in Eq. (2) as

\[
2(P^+/2)^n \int_0^1 dz (2z - 1)^{n-1} \int \frac{dy^-}{2\pi} e^{i(2z-1)p^+y^-/2q^-(-y/2)} \gamma^+ q(y/2) \bigg|_{y^+ = y^- = 0} = \mathcal{F}(0) \gamma^+ \left( i \gamma^+ \right)^{n-1} q(0), \tag{5}
\]
This equation indicates that the operator is the usual vector type \( \bar{q}\gamma^\mu q \) for \( n = 1 \), so that the electromagnetic form factor is probed for the pion as shown in Fig. 2(a). However, the \( n = 2 \) term indicates the energy-momentum tensor for quarks as shown in Fig. 2(b), and its form factors can be obtained by studying the GPDs and GDAs. In fact, the second moment of the GDAs is given by the matrix element of the quark energy-momentum tensor \( T_{\mu\nu}^q \) as

\[
\int_0^1 dz \left( 2z - 1 \right) \Phi_q^{0-0}(z, \zeta, W^2) = \frac{2}{(P^+)^2} (\pi^0(p) \pi^0(p') | T_{++}^q(0) | 0). \tag{6}
\]

The quark energy-momentum tensor is generally defined by \( T_{\mu\nu}^q(x) = q(x) \gamma^{(\mu} \bar{D}^{\nu)} q(x) \) with the covariant derivative \( D^\mu = \partial^\mu - ig\lambda^a A^a_{\mu} / 2 \). Here \( g \) is the QCD coupling constant and \( \lambda^a \) is the SU(3) Gell-Mann matrix. The gluon energy-momentum tensor is also defined in the same way. By the matrix element of the energy-momentum tensor, the timelike gravitational form factors \( \Theta_1^{+,q}(s) \) and \( \Theta_2^{+,q}(s) \) of the pion are defined as

\[
\langle \pi^0(p) \pi^0(p') | T_{\mu\nu}^q(0) | 0 \rangle = \frac{1}{2} \left[ (s g_{\mu\nu} - P^\mu P^\nu) \Theta_1^{+,q}(s) + \Delta^\mu \Delta^\nu \Theta_2^{+,q}(s) \right], \tag{7}
\]

where \( P \) and \( \Delta \) are \( P = p + p' \) and \( \Delta = p' - p \). Therefore, once the GDAs are determined, we can calculate these form factors by using Eqs. (6) and (7). We know that the energy-momentum tensors of quarks and gluons are the sources of gravity, so that they are called gravitational form factors. The gravitational interactions are generally too weak to be investigated in particle scattering experiments; however, the GPDs and GDAs provide a way to access them from the microscopic quark and gluon level.

2.3 Cross section for \( \gamma^* \gamma \rightarrow \pi^0 \pi^0 \) and GDAs

The cross section for the two-photon process \( \gamma^* \gamma \rightarrow \pi^0 \pi^0 \) is given by the matrix element of the form

\[
d\sigma = \frac{1}{4q \cdot q'} \sum_{\lambda, \lambda'} |M(\gamma^* \gamma \rightarrow \pi^0 \pi^0)|^2 \frac{d^3p}{(2\pi)^3} \frac{d^3p'}{(2\pi)^3} \frac{2E_p}{2E_p'} \frac{2E_{p'}}{2E_{p'}} (2\pi)^4 \delta^4(q + q' - p - p'). \tag{8}
\]

The matrix element is expressed by the hadron tensor \( T_{\mu\nu} \) and the photon polarization vector \( e^\mu \) as \( iM(\gamma^* \gamma \rightarrow \pi^0 \pi^0) = e^\mu(\lambda) e^{\nu}(\lambda') T_{\mu\nu} \) with

\[
T_{\mu\nu} = i \int d^4y e^{-iqy} \langle \pi^0(0) \pi^0(0') | T_{\mu\nu}(y) J_{\pi^0}^{\text{em}}(0) | 0 \rangle
= -g_{\mu\nu} e^2 \sum_q \frac{e_q^2}{2} \int_0^1 dz \frac{2z - 1}{z(1 - z)} \Phi_q^{0-0}(z, \zeta, W^2), \tag{9}
\]

\( g_{\mu\nu} \) is the metric tensor.
where $g^\mu\nu = -1$ for $\mu = \nu = 1, 2$ and $g^\mu\nu = 0$ for $\mu, \nu =$ others. Then, defining the helicity amplitude $A_{ij}$ as $A_{ij} = \varepsilon_{\mu}^{(i)}(q)\varepsilon_{\nu}^{(j)}(q') T^{\mu\nu}/s$ ($i = -, 0, +; j = -, +$), we obtain the cross section expressed by the GDAs as

$$
\frac{d\sigma}{d(cos\theta)} = \frac{\pi\alpha^2}{4(Q^2 + s)} \sqrt{1 - \frac{4m_p^2}{s}} |A_{++}|^2,
$$

$$
A_{++} = \sum \frac{\epsilon_2^2}{2} \int_0^1 dz \frac{2z - 1}{z(1-z)} \Phi_{q}^{\pi_+\pi^0}(z, \xi, W^2).
$$

In our analysis, higher-order effects of $\alpha_s$ and higher-twist terms are neglected. The gluon GDA contributes to the cross section as a higher-order term, so that they are not studied in this work.

### 3 Results for GDAs and gravitational form factors for pion

The GDAs are expressed by a number of parameters, which are determined by a $\chi^2$ analysis of the Belle cross-section data on $\gamma^*\gamma \rightarrow \pi^0\pi^0$. The possible isospin and angular momentum states for the final two pions are $I = 0$ and $L =$ even numbers ($0, 2, \cdots$). We only consider the lowest Gegenbauer polynomial $n = 1$ in setting up the $z$-dependent functional form of the asymptotic GDAs \[3\]. Then, the possible states are $L = 0$ (S wave) and $2$ (D wave). The GDAs are expressed by the addition of $S$- and $D$-wave contributions $\tilde{B}_{10}(W^2)$ and $\tilde{B}_{12}(W^2)$ as

$$
\Phi_q^{\pi^+\pi^0}(z, \xi, W^2) = N_\alpha z^\alpha (1-z)^\alpha (2z-1) [\tilde{B}_{10}(W^2) + \tilde{B}_{12}(W^2) P_2(cos\theta)],
$$

where $P_2(cos\theta)$ is the Legendre polynomial. In the scaling limit $Q^2 \rightarrow \infty$, the $z$ dependence is given $z(1-z)(2z-1)$ for the $\pi^0$ GDAs. In the above function, the parameter $\alpha$ is assigned for its functional variation.

There are contributions from the GDA continuum and resonances to the functions $\tilde{B}_{0i}(W^2)$: $\tilde{B}_{10}(W^2) =$ continuum + resonance $(f_0)$, $\tilde{B}_{12}(W^2) =$ continuum + resonance $(f_2)$. We consider the resonances $f_0(500)$ and $f_2(1270)$ in our analysis. Another scalar meson $f_0(980)$ is neglected because there is no clear signal of $f_0(980)$ in the differential cross-section data of Belle, although its effects are seen in the total cross section. Furthermore, its theoretical decay-constant estimate is not available by considering that it is a tetra-quark state, which is likely to be the $f_0(980)$ configuration \[5\]. The resonance terms contain resonance masses, total decay widths, two-pion coupling constants, and decay constants, which are taken from the particle-data-group tables and theoretical articles. The continuum terms are expressed by the momentum fraction carried by quarks and antiquarks in the pion $M_{2i(q)}^\pi$ and the overall timelike form factor $F_{q}(W^2) = 1/[1 + (W^2 - 4m_p^2)/A^2]^{n-1}$ with the constituent-counting factor $n = 2$ \[6\]. The cutoff parameter $A$ is one of the parameters in the $\chi^2$ analysis. Because of the page limitation, the details of $\tilde{B}_{ni}(W^2)$ and our parametrization are not explained here, and they should be found in the original paper \[5\].
We analyzed the Belle experimental data on the differential cross section for \( \gamma^* \gamma \rightarrow \pi^0 \pi^0 \), and the optimum GDAs are determined by the \( \chi^2 \) analysis. The comparison with some Belle data are shown in Fig. 3 where the \( Q^2 \) values are \( Q^2 = 8.92, 13.37, 17.23, \) and \( 24.25 \) GeV\(^2\) and the scattering angles are \( \cos \theta = 0.1 \) and 0.5. The smaller \( Q^2 \) data are not included in our analysis by considering the factorization condition \( Q^2 \gg W^2 \). The curves indicate our theoretical results, and they explain the data reasonably well. There are peaks in the \( W = 1.3 \) GeV region and it comes from the \( f_2(1270) \) resonance, and \( f_0(500) \) affects the cross section in the small-\( W \) region. There is an overall contribution from the GDA continuum in the whole-\( W \) range of Fig. 3.

Using the form-factor definition and the GDAs in Eqs. (7) and (11), we obtain the form factors expressed by the \( W \)-dependent functions of the GDAs as

\[
\Theta_{1,2,q}(s) = \frac{3}{5} \tilde{B}_{10}(W^2) + \frac{3}{10} \tilde{B}_{12}(W^2), \quad \Theta_{2,q}(s) = \frac{9}{10 \beta^2} \tilde{B}_{12}(W^2). \tag{12}
\]

Then, the total timelike gravitational form factors of the pion are obtained by adding them as \( \Theta_n(s) = \sum_{i=q} \Theta_{n,i}(s) \) where \( n = 1 \) or 2, and they are shown in Fig. 4. The D-wave term contributes to the form factor \( \Theta_2 \), which shows the resonance behavior at the \( f_2 \) mass. The function \( \Theta_1 \) has more complicated \( W \)-dependence due to the additional S-wave term. Since the imaginary parts of the form factors are determined in our analysis, they are used for calculating the spacelike gravitational form factors by using the dispersion relation. The
obtained spacelike form factors are shown in Fig. [3] There are significant differences in the $t$ dependence between the two form factors due to the additional S-wave term.

For understanding the meaning of the form factors $\Theta_1$ and $\Theta_2$, we may define the static energy-momentum tensor as $[7] T_{\mu\nu}^{\text{st}}(r) = \int d^3q/[[2\pi]^3 2E] e^{iqr} \langle \pi^0(q') T_{\mu\nu}(0) |\pi^0(p) \rangle$, where $E$ is the pion energy $E = \sqrt{m_\pi^2 + q^2/4}$. The $\mu\nu = ij \ (i, j = 1, 2, 3)$ components are expressed by the pressure $p(r)$ and shear force $s(r)$ as $T_{ij}^{\text{st}}(r) = p_\delta(r) \delta_{ij} + s_\delta(r) (r_i r_j - \delta_{ij}/3)$. The term $T_{ii}^{\text{st}}(r)$ is expressed only by $\Theta_1$, so that $\Theta_1$ indicates pressure and shear-force distributions. We may call it as mechanical (pressure, shear-force) form factor. On the other hand, the $\mu\nu = 00$ component satisfies relation $\int d^3r T^{00}_{\mu\nu}(r) = m_\pi \Theta_{2,0}(0)$. It means that $\Theta_2$ shows the gravitational mass (or energy) distribution in the pion. At finite $t$, $\Theta_1$ also contributes to this distribution.

By taking the Fourier transforms of the spacelike form factors, the space-coordinate densities are obtained as shown in Fig. [5] The mechanical distribution $\rho_1(r)$ extends to the larger-$r$ region than the gravitational-mass distribution $\rho_2(r)$. From the form factors or densities, the root-mean-square radii are calculated as: $\sqrt{\langle r^2 \rangle_{\text{mass}}} = 0.69$ fm and $\sqrt{\langle r^2 \rangle_{\text{mech}}} = 1.45$ fm, which are gravitational-mass and mechanical radii, respectively. There are some ambiguities in our results due to phase-factor assignment. If we consider such ambiguities, we obtain the radius ranges as [3]:

$$\sqrt{\langle r^2 \rangle_{\text{mass}}} = 0.56 \sim 0.69 \text{ fm}, \quad \sqrt{\langle r^2 \rangle_{\text{mech}}} = 1.45 \sim 1.56 \text{ fm}. \quad (13)$$

This should be the first result on the gravitational radii from the actual analysis of experimental measurements. The charge radius of the pion has been already measured as $\sqrt{\langle r^2 \rangle_{\text{charge}}} = 0.672 \pm 0.008$ fm [5]. It is our interesting finding that the gravitational-mass radius is similar or slightly smaller than the charge radius and that the mechanical radius is larger.

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