Calibration Method for Industrial Robots Based on the Principle of Perigon Error Close

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This research was supported by the National Natural Science Foundation of China under Grant 52175489.

ABSTRACT For the problem that the self-error of industrial robot calibration device has influence on calibration accuracy, a calibration method of industrial robots based on the principle of Perigon Error Close is proposed. In the method, the theory that the sum of circular indexing interval errors around a circle is zero is applied to robot calibration to improve robot calibration accuracy. The calibration principle of the proposed method is provided in detail and the calibration equation is derived in this paper. The calibration system based on the proposed method was constructed with one semiconductor laser and two position sensing detectors (PSDs) fixed on a rotary table. Based on the position error data obtained from laser spot position on the PSDs, the robot kinematics parameter errors were identified by using Levenberg Marquardt (LM) algorithm. The robot calibration experimental setup was constructed and the related verification experiments were carried out. The model parameter identification experiment validates the feasibility of the proposed method for industrial robot calibration. The calibration compensation experiment results of industrial robots show that the maximum position error of the robot is reduced by 71.9% and the average position error is reduced by 77.8%, which validates the effectiveness of the proposed method for industrial robot calibration.

INDEX TERMS Calibration, industrial robots, principle of Perigon Error Close, laser, PSD

I. INTRODUCTION Industrial robots are widely used in various fields for the advantages of high repeatable positioning accuracy, high reliability and strong adaptability [1], [2]. In practical application, the long-term and high-intensity use of the robot could cause great wear and tear on the robot joints, which would make the actual model parameters of the robot deviate from the theoretical values and lead to the decline of positioning accuracy. Therefore, the timely calibration of robot is quite necessary and is the key to guaranteeing its working accuracy [3]–[6].

The basic principle of robot calibration is to identify the model parameter errors by using the measured end-pose data of the robot, and then compensate the robot model to improve its positioning accuracy [7]–[11]. The high measurement accuracy of laser tracker makes it an important equipment for precision robot calibration applications, while the high cost makes it difficult to be widely used [12]–[15].

Therefore, researchers have carried out a lot of studies on robot calibration methods, among which the calibration methods of imposing constraints on end-effector can be roughly divided into two types: contact and non-contact calibration methods [16]–[18]. The contact calibration methods generally require external reference measurement component to complete the calibration [19].

For example, Shi et al. [20] constructed a geometric point constraint calibration system by using a sampling needle installed on the robot end-effector and a sampling box used for placing the crop seeds, which improved the positioning accuracy of robot. He et al. [21] constructed a multi-position constraint system by using two standard devices and a non-bar device to improve the accuracy of robot. Joubair et al. [22] constructed a distance and sphere constraint calibration system with a precision touch probe installed on the robot end-effector and a special triangular plate with three datum spheres, which improved the accuracy of the robot in a specific workspace.

The problem existed among the above calibration methods is that the contact error during calibration has influence on the accuracy of the measurement data, which would affect the calibration accuracy. Comparing with contact calibration methods, non-contact calibration methods do not require physical contact, which can effectively avoid the influence of contact errors on the calibration accuracy [23], [24]. Gao et al. [25] proposed a calibration method by using a laser pointer installed on the robot end-effector and a position sensing detector (PSD) arbitrarily placed in the work space, and established the optimization model for calibration by imposing...
virtual point constraint on the laser beam. However, this method only considers the joint offset errors of the robot, and does not consider other parameter errors. Du et al. [26] constructed a calibration system by using a laser pointer, a rotatable PSD and fixed cameras, and applied virtual sphere constraint to the laser beam for calibration to improve the positioning accuracy of robot. Guo et al. [27] used one single laser displacement sensor (LDS) and one master sphere installed on the robot end-effector to construct the calibration system, and calibrated the kinematic parameters by imposing spherical center point constraint on the laser beam, which improved the calibration accuracy and efficiency of the robot.

Summing the above calibration methods with constraint imposed on the robot end-effector, they can improve the accuracy of robot to varying degrees, however, the common problem among these methods is that the self-error of robot calibration device could have influence on calibration accuracy. Therefore, this paper proposes a calibration method of industrial robots based on the principle of Perigon Error Close. The definition of the Perigon Error Close [28] is that the sum of circular indexing interval errors around a circle is zero, and the Perigon Error Close is applied to robot calibration to improve robot calibration accuracy.

The rest of the paper is organized as follows: In section II, the calibration system of the proposed method is introduced. In section III, the calibration principle of the proposed method is provided in detail and the calibration equations are derived. In section IV, three verification experiments and the corresponding experimental results are illustrated. Finally, the conclusion is presented in Section V.

II. CALIBRATION SYSTEM

The calibration system for industrial robot based on the principle of Perigon Error Close is shown in Figure 1. The calibration system include laser, rotary table, two PSDs, data processing module and PC software. Firstly, the laser is installed on the end-effector of the robot, and the axis direction of the laser is required to be the same as the axis direction of the end-effector. Two PSDs are fixed on the rotary table of the calibration system, one PSD is fixed at the center of the rotary table and another PSD is fixed on the circle with a radius of R. Secondly, the laser beam need to vertically incident onto the coordinate system relative to the base coordinate system can be obtained by using the six joint transformation matrices

$$T = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ (2)

where, \((p_x, p_y, p_z)\) is the position coordinate of the end coordinate system relative to the base coordinate system. The matrix composed of \((n_x, n_y, n_z)\), \((o_x, o_y, o_z)\) and \((a_x, a_y, a_z)\) is the direction cosine of the three unit vectors.

Suppose that the joint angle data of the industrial robot is \((\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6)\), by substituting the joint angle data into formula (2), then the end pose data \((p_x, p_y, p_z, \alpha, \beta, \gamma)\) of the robot can be obtained. In the calibration system shown in Figure 1, the axis direction of the laser is the same as that of the end-effector, and according to the principle of Perigon Error Close constraint shown in Figure 2, the position coordinate conversion equation from the end position coordinate of the robot to the center of laser spot on the PSD can be derived by

$$\frac{x - p_x}{a_x} = \frac{y - p_y}{a_y} = \frac{z - p_z}{a_z}$$ (3)

where, \((x, y, z)\) is the position coordinate of the laser spot center on the PSD.
Suppose that the nominal position coordinate of laser spot center on the PSD obtained by equation (3) is \( P_k(X_k, Y_k, Z_k) \), where, \( k \) is the number of data, and \( k \) is taken as 1, 2, \( k \) is the position of the PSD, and \( k \) is taken as O, A, B, C and D. As is shown in Figure 2, the measurement steps are as follows: Firstly, when the PSD_2 is at position A, the laser beam is adjusted to be vertically incident onto the photosensitive surface of PSD_2, and the laser beam is positioned to the spot center of the photosensitive surface through the data processing module. At the same time, the robot is controlled to move to position 1, and a set of robot joint angle data is collected by using the robot controller, thus the end pose data is obtained through the forward kinematics of the robot.

Secondly, the robot is controlled to move from position 1 to position 2 along the axis direction, and when the robot is at position 2, the other end pose data can be obtained in the same way. Thirdly, the end pose data of the two positions are substituted into equation (3) to establish a system of equations, and the nominal value of \( P_k(X_k, Y_k, Z_k) \) can be obtained by solving the system of equations. Fourthly, by using the same measurement method as getting the nominal position \( P_k(X_k, Y_k, Z_k) \) of the laser spot center on the PSD_1 can be obtained. Fifthly, the rotary table is controlled by PC software to rotate to the positions of B, C and D in sequence, and the nominal position coordinate of laser spot center on the PSD_2 at each position are obtained through the same measurement method, respectively, thus the nominal values corresponding to the three positions of B, C and D are expressed as \( P_b, P_c, \) and \( P_d \), respectively. Finally, all the measurement data of the five position points are used for robot model parameter identification.

![FIGURE 2. Principle of Perigon Error Close constraint.](image)

Suppose that the actual position coordinate of laser spot center on PSD under the robot base coordinate system is \( p_k(x_k, y_k, z_k) \). Due to the errors of robot kinematics parameter, the nominal position of the laser spot center on the PSD is deviated from the actual position, and its position deviation \( \Delta e_i \) can be derived by

\[
\Delta e_i = p_i - P_i = J_i \Delta \delta_i
\]

where, \( P_i \) is the nominal value of laser spot center on the PSD_1 when PSD_1 is at position O. \( P_a, P_b, P_c, P_d \) are the nominal values of laser spot center on the PSD_2 when PSD_2 is at four positions A, B, C and D, respectively. \( J_i \) is the Jacobian matrix. \( \Delta \delta_i \) are the kinematic parameter errors.

![FIGURE 3. Circular indexing angle measurement principle.](image)

In the proposed calibration method, as is shown in Figure 2, the four target positions of PSD_2 on the rotary table are at the four equipartition points of a circle, respectively, and the actual position coordinate \( p_k(x_k, y_k, z_k) \) under the robot base coordinate system can be obtained by equation (4). As is shown in Figure 3, according to the principle of similar triangles, the distance \( X \) from the point O to the line segment AB can be derived by

\[
\frac{p_b - p_o}{p_o - p_a} = \frac{X}{p_b - p_a}
\]

(5)

The distance \( X \) can be obtained from equation (5), and then according to the properties of the right triangle, the angle \( \theta \) can be expressed by

\[
\theta = \arctan \left( \frac{p_b - p_a}{2} \right) = \arctan \left( \frac{(p_b - p_a)^2}{2(p_b - p_o)(p_a - p_o)} \right)
\]

(6)

According to the circular indexing angle measurement principle shown in Figure 3, the circular indexing interval error of PSD_2 between the two adjacent positions of A and B can be derived by

\[
\mu_{AOB} = \frac{\pi}{2} - 2\theta = \frac{\pi}{2} - 2 \arctan \left( \frac{(p_b - p_a)^2}{2(p_b - p_o)(p_a - p_o)} \right)
\]

(7)

Through the same circular indexing interval error obtaining principle, the other three circular indexing interval errors can be obtained. And based on the theory that the sum of circular indexing interval errors around a circle is zero, then the sum of the obtained four circular indexing interval errors can be expressed by

\[
\mu_{AOB} + \mu_{BOC} + \mu_{COD} + \mu_{DOA} = 0
\]

(8)

The kinematic calibration equation based on the principle of Perigon Error Close can be obtained by substituting equation (4) and equation (7) into equation (8)

\[ G: \Delta \delta = H \]

(9)

where, \( G \) and \( H \) are calibration equation parameters, and the \( G \) and \( H \) can be derived by
lateral effect position sensors, whose detecting wavelength range is 320 to 1100 nm and effective detection aperture is 9 mm, and the measurement accuracy of PSDs is 0.75 µm. The two PSDs are fixed on a round plate made of invar material, and the round plate can rotate with the rotary table.

The kinematic calibration equation (9) can be used to identify the parameters of the robot model. In order to improve the robustness and accuracy of parameter identification, the analytical method is used to analyze and eliminate the redundant parameters in the robot model, and the model parameter errors after processing can be expressed as

\[
\Delta \delta = (\Delta a_1, \Delta a_2, \Delta a_3, \Delta \alpha_1, \Delta \alpha_2, \Delta \theta_1, \Delta \theta_2, \Delta d_1, \Delta d_2, \Delta d_3, \Delta d_4)^T
\]

where, \( \Delta a_i, \Delta a_i, \Delta \alpha_i, \Delta \theta_i, \Delta d_i \) are the link length errors, link twist errors, joint angle errors, link offset errors, respectively.

In order to improve the accuracy of robot model parameter identification, the robot kinematics parameter errors identification is carried out by LM algorithm, and the robot kinematics parameter errors can be derived by

\[
\Delta \delta_{\text{n+1}} = (G_n^T G_n + \lambda I)^{-1} G_n^T H_n, \quad n = 0, 1, 2, \ldots, N
\]

where, \( \lambda \) is the damping coefficient. \( n \) is the number of iterations. \( G_n, H_n \) are the calibration equation parameters of iteration.

The identification of model parameters based on LM algorithm can be iterated by equation (13), in which the initial values of the iterative identification parameter are the theoretical kinematics model parameters \( \delta_0 \), and the iterative identification equation of robot kinematics model parameters \( \delta \) can be derived by

\[
\delta_{\text{n+1}} = \delta_n + \Delta \delta_{\text{n+1}}, \quad n = 0, 1, 2, \ldots, N
\]

Substituting the robot joint angle data and the theoretical position coordinate data of the laser spot center on PSDs into equation (14) for iteration, during the iteration process, when the \( \Delta \delta \) is less than the preset threshold, iterative process ends, then the model parameter errors and robot kinematics model parameters \( \delta_0 \) can be obtained.

**IV. EXPERIMENTAL RESULTS**

In order to verify the feasibility and effectiveness of the proposed robot calibration method, three experiments were performed, including simulation experiment of model parameter identification, calibration compensation experiment and comparison experiment of calibration methods.

An experimental setup for industrial robot calibration based on the principle of Perigon Error Close was constructed, as is shown in Figure 4. The industrial robot to be calibrated is ER3B-C30, of which the repeated positioning accuracy is 0.02 mm. The laser X650NS used in the setup is a semiconductor laser, whose spot diameter of the output laser is 0.5 mm, the output power is 5 mw, and the laser wavelength is 650 nm. The two PSDs in the experimental setup are Thorlabs PDP90A

![Experimental setup](image)

**TABLE 1. Simulation experiment results.**

| Joint number | Kinematic parameters | Initial values | Preset errors | Identified errors |
|--------------|----------------------|----------------|---------------|------------------|
| Joint 1      | \( a_1 \) (mm)       | 0              | 0.2           | 0.247            |
|              | \( d_1 \) (mm)       | 367.5          | 0.5           | 0.563            |
|              | \( \theta_1 \) (°)   | 0              | 0.04          | 0.042            |
| Joint 2      | \( a_1 \) (mm)       | 295            | 0.2           | 0.199            |
|              | \( d_1 \) (mm)*      | 0              | -0.05         | -0.051           |
|              | \( \theta_1 \) (°)   | 90             | 0.02          | 0.019            |
| Joint 3      | \( a_1 \) (mm)       | 37             | 0.1           | 0.142            |
|              | \( d_1 \) (mm)       | 0              | 0.5           | 0.565            |
|              | \( \theta_1 \) (°)   | 0              | 0.07          | 0.076            |
Figure 5. Simulation experiment results before and after calibration.

The experimental results show that the position errors of the robot after calibration are smaller than the position errors before calibration, which verifies the feasibility of the proposed method for robot model parameter identification.

B. CALIBRATION COMPENSATION EXPERIMENT

In order to verify the effectiveness of the proposed calibration method for industrial robot model parameter identification, the calibration compensation experiment is performed with the setup shown in Figure 4. In the experiment, AT960 laser tracker is used to measure the actual position data of the robot end so as to obtain the position errors of the robot. The specific steps of the experiment are as follows: Firstly, 50 target sampling points are planned in the robot work space, and the robot is controlled to move to the target sampling points, at the same time, the position data of each point is measured with the laser tracker. Secondly, the proposed calibration method is used to identify the model parameters of the robot, and then the identified new model parameters are used to replace the original parameters of the robot, so as to compensate the robot model parameters. Thirdly, the robot is controlled to move to the 50 target sampling points again, and the laser tracker is used to simultaneously measure the position data of each point. Finally, the position errors before and after calibration compensation are compared to verify the effectiveness of the proposed method.

The actual model parameters of the robot identified by the proposed calibration method are shown in Table 2, and the comparison experimental results of robot position errors before and after calibration are shown in Figure 6.

TABLE 2. Identified kinematics parameters.

| Joint | a (mm) | d (mm) | α (°) | θ (°) |
|-------|--------|--------|-------|-------|
| 1     | 0.196  | 367.500| 89.897| -0.304|
| 2     | 295.386| 0     | 90.614|       |
| 3     | 36.973 | 0     | 89.857|       |
| 4     | -0.027 | 90    | 295.698| 0     |
| 5     | 0.196  | 0     | 90    | 0.004 |
| 6     | 0.196  | 0     | 78.528| 0     |

FIGURE 5. Simulation experiment results before and after calibration.

FIGURE 6. Experimental results before and after calibration.

The experimental results indicate that the maximum position error of the robot is reduced from 1.164 mm to 0.326 mm after calibration, and the average position error is reduced from 0.953 mm to 0.211 mm after calibration compensation. The calibration compensation experiment results demonstrate that the proposed calibration method effectively reduces position errors of the robot, which verifies the effectiveness of the proposed method for robot calibration.

C. COMPARISON EXPERIMENT OF CALIBRATION METHODS

In this experiment, the robot calibration method based on geometric radius constraint and the proposed calibration method are used for robot calibration, respectively, so as to compare the calibration compensation performance between them. The kinematic parameters of the robot based two
methods are shown in Table 3. Firstly, 50 sets of experimental measurement data are re-collected by using the measurement method in Section III. Secondly, the robot calibration method based on geometric radius constraint is used to identify and compensate the model parameters of the robot by using the same experimental steps in Section B. Thirdly, the robot is controlled to move to the 50 target sampling points again, and the laser tracker is used to simultaneously measure the position data of each point, so as to obtain the position errors data before and after calibration compensation based on this method.

**TABLE 3.** Kinematics parameters.

| Joint | $a$ (mm) | $\alpha$ (°) | $d$ (mm) | $\theta$ (°) |
|-------|----------|--------------|----------|--------------|
| 1     | 0        | 90           | 367.5    | 0            |
| 2     | 295      | 0            | 0        | 90           |
| 3     | 37       | 90           | 0        | 0            |
| 4     | 0        | 90           | 295.5    | 0            |
| 5     | 0        | -90          | 0        | 90           |
| 6     | 0        | 0            | 78.5     | 0            |

The actual model parameters of the robot identified by the proposed calibration method and geometric radius constraint method are shown in Table 4 and Table 5, and the comparison experimental results are shown in Table 6 and Figure 7.

**TABLE 4.** Identified kinematics parameters based on the principle of Perigon Error Close.

| Joint | $a$ (mm) | $\alpha$ (°) | $d$ (mm) | $\theta$ (°) |
|-------|----------|--------------|----------|--------------|
| 1     | 0.310    | 90.481       | 367.500  | -0.675       |
| 2     | 295.676  | -0.206       | 0        | 90.759       |
| 3     | 36.957   | 90.381       | 0.001    | -0.279       |
| 4     | -0.043   | 90           | 295.814  | 0            |
| 5     | 0.311    | -90          | -0.001   | 90           |
| 6     | 0.311    | 0            | 78.501   | 0            |

**TABLE 5.** Identified kinematics parameters based on the geometric radius constraint.

| Joint | $a$ (mm) | $\alpha$ (°) | $d$ (mm) | $\theta$ (°) |
|-------|----------|--------------|----------|--------------|
| 1     | 0.554    | 90.293       | 367.500  | -0.430       |
| 2     | 296.206  | -0.416       | 0        | 90.957       |
| 3     | 36.924   | 90.155       | -0.002   | -0.712       |
| 4     | -0.075   | 90           | 296.073  | 0            |
| 5     | 0.555    | -90          | 0.001    | 90           |
| 6     | 0.555    | 0            | 78.511   | 0            |

The experimental results indicate that the average position error of the calibration method based on geometric radius constraint is 0.299 mm, and that of the proposed calibration method is 0.218 mm. The experimental results demonstrate that both calibration methods can effectively reduce the position errors of the robot. Comparing with the robot calibration method based on geometric radius constraint, the proposed calibration method has better performance in reducing the position errors of robot.

**TABLE 6.** Position errors before and after calibration.

|                  | Before calibration | Geometric radius constraint | Principle of Perigon Error Close |
|------------------|--------------------|-----------------------------|----------------------------------|
| Average          | 0.953              | 0.299                       | 0.218                            |
| Max              | 1.164              | 0.439                       | 0.337                            |

**FIGURE 7.** Comparison experimental results of calibration methods.

**V. CONCLUSION**

In this paper, a calibration method for industrial robots based on the principle of Perigon Error Close is proposed. The principle of Perigon Error Close used in robot calibration was described in detail, and the corresponding robot calibration experimental setup was designed and constructed. The simulation experiment was performed, which verifies the feasibility of the proposed method for robot model parameter identification. In the calibration compensation experiment for ER3B-C30 robot, after calibration compensation, the maximum position error of the robot was reduced from 1.164 mm to 0.326 mm, and the average position error was reduced from 0.953 mm to 0.211 mm. In the comparison experiment, compared with the calibration method based on geometric radius constraint, the robot average position error was reduced by 0.08 mm by using the proposed calibration method. All these experimental results verify the feasibility and effectiveness of the proposed calibration method used in robot model parameter identification and compensation.

Considering the harsh working circumstances with proposed calibration system in the future, some measures should be taken to solve the problems of installation error and laser drift and to further improve calibration accuracy, which are our important work in the future.

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