Indeterministic Quantum Gravity and Cosmology

X. Probability-Theoretic Aspect:
A Hidden Selector for Quantum Jumps,
or How the Universe Plays the Game of Chance

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Abstract

This paper is a sequel to the series of papers [1-9]. The problem of the meaning of objective a priori probability for individual random trials without repetition is considered. A sequence of such trials, namely quantum jumps, is realized in indeterministic dynamics of the universe. A hidden selector for the quantum jumps is constructed.

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It is as natural as natural selection...

William Golding

Introduction

Indeterminism implies objective probability, so that indeterministic quantum gravity and cosmology (IQGC)—the theory being developed in this series of papers—relies on probability theory. In this regard, IQGC is notably specific. Generally the experimental verification of probability-theoretic results is considered in the context of identical conditions repeated many times. In IQGC (and not only in it), the repetitions are, in general, unavailable. Here we are faced with the problem of the meaning of objective a priori probability for individual situations. Thus the probability-theoretic aspect of IQGC should be specially studied.

The most solid way for handling the problem of individual situations is to get around it, namely, to reduce the choice in a sequence of the individual situations to the employment of the results of the sequence of one and the same random trial. In the case of IQGC this is done as follows.

The sequence of individual situations is that of quantum jumps numbered by $i \in \mathbb{Z} = \{0, \pm 1, \pm 2, \ldots\}$. A result of the $i$-th jump is one of the two states [5]: $\omega^+_i$ with energies $\epsilon^+_i$, $\epsilon^+_i > \epsilon^-_i$, and a priori probabilities $w^+_i$.

We take the interval $[0, 1]$ and divide it into two parts: $\Delta^-_i = [0, w^-_i]$, $\Delta^+_i = (w^-_i, 1]$. The random trial is the choice of a point from $[0, 1]$, the distribution being uniform. Denote the results $\omega^+_i$ of the $i$-th jump by $e_i = \pm 1$. Let the result of the $i$-th trial be $x_i$. Then $e_i = \chi_{\Delta^+_i}(x_i) - \chi_{\Delta^-_i}(x_i)$ where $\chi$ is the characteristic function.

Here we have the Bernoulli trials [10], the sequence $S = \{x_i : i \in \mathbb{Z}\}$ being its choice function, or selector. The latter is a hidden selector for quantum jumps in IQGC.

Now the universe $U$ as a dynamical system is $U = (M, S, \omega)$, where $M$ is a fixed spacetime manifold [6] and $\omega$ is a dynamical state: $\omega = (g, \dot{g}, \Psi)$, $g$ is a metric and $\Psi$ is a state vector of matter.

1 Indeterministic dynamics of the universe as a choice function of a stochastic process

In IQGC we have a denumerable set of quantum jumps,

$$J = \{e_i : i \in \mathbb{Z}\}, \quad Z = 0, \pm 1, \pm 2, \ldots,$$

where $e_i$ stands for the $i$-th event, i.e., the jump. A result of the $i$-th jump is one of the two admissible states $\omega^+_i$ of matter [5] with energies

$$\epsilon^+_i, \quad \epsilon^+_i > \epsilon^-_i$$

and a priori probabilities

$$w^+_i, \quad w^+_i + w^-_i = 1.$$
It is convenient to put
\[ e_i = \begin{cases} 
+1 & \text{for } \omega_i^+ \\
-1 & \text{for } \omega_i^- 
\end{cases} \tag{1.4} \]
Then we have
\[ J = \{ e_i \in \{+1, -1\} : i \in \mathbb{Z} \}. \tag{1.5} \]
We may regard \( J \) as a choice function of a stochastic process. In fact, \( J \) is the choice function: it is the only one realized, there are no repetitions for the process. Thus we are faced with the first problem—that of the meaning of objective a priori probabilities (1.3) in the case when only one choice function is realizable.

2 Probabilistic interpretation of a family of random trials without repetition

A natural general solution to the first problem was proposed in [11]. Let
\[ \{(X_\alpha, A_\alpha, P_\alpha) : \alpha = 1, 2, ..., N\} \tag{2.1} \]
be a family of probability spaces. Then
\[ f(x_1, x_2, ..., x_N) = \frac{1}{N} \sum_{\alpha=1}^{N} f_\alpha(x_\alpha), \quad x_\alpha \in X_\alpha, \tag{2.2} \]
is a random variable. A single measurement of \( f \) is performed, i.e., a single value for each \( x_\alpha \) (\( \alpha = 1, 2, ..., N \)) is obtained. Then for \( N \gg 1 \), the value of \( f \) obtained experimentally is close to the mean value:
\[ f \approx \langle f \rangle = \frac{1}{N} \sum_{\alpha=1}^{N} \langle f_\alpha \rangle. \tag{2.3} \]
Eq.(2.3) defines the meaning of a priori probability for individual situations. The usual approach corresponds to the case where
\[ (X_\alpha, A_\alpha, P_\alpha) = (X, A, P), \quad \alpha = 1, 2, ..., N, \quad f_1 = f_2 = ... = f_N. \tag{2.4} \]

3 The problem of the selection of a choice function

Although the approach given in the previous section elucidates the meaning of objective a priori probability for individual situations, we remain faced with the second problem: Is there a mechanism of the selection of an actual choice function \( J \) (1.5), which would grant that eq.(2.3) be fulfilled?
4 The Bernoulli trials and a hidden selector for quantum jumps

The most solid solution to the second problem consists in the employment of the results of the sequence of one and the same random trial. In other words, we employ the Bernoulli trials.

The random trial is the choice of a point \( x \) from the interval \([0, 1]\), the distribution being uniform:

\[
\text{for } [a, b] \subset [0, 1] \quad P([a, b]) = b - a.
\] (4.1)

Let

\[
S = \{x_i : i \in \mathbb{Z}\}
\] (4.2)

be a given choice function of the Bernoulli trials. For each \( i \) we divide the interval \([0, 1]\) into two parts:

\[
\Delta_i^- = [0, w_i^-], \quad \Delta_i^+ = (w_i^-, 1].
\] (4.3)

Now we put

\[
e_i = \chi_{\Delta_i^+}(x_i) - \chi_{\Delta_i^-}(x_i),
\] (4.4)

where \( \chi \) is the characteristic function, so that the result of the \( i \)-th quantum jump is determined by the following prescription:

\[
\omega_i = \begin{cases} 
\omega_i^+ & \text{for } x_i \in \Delta_i^+ \\
\omega_i^- & \text{for } x_i \in \Delta_i^-.
\end{cases}
\] (4.5)

Thus the choice function \( S \) (4.2) of the Bernoulli trials serves as a hidden selector for quantum jumps.

5 The universe as a dynamical system

The description of the universe \( U \) as a dynamical system may be given as follows. \( U \) is the triple,

\[
U = (M, S, \omega);
\] (5.1)

\( M \) is a fixed spacetime manifold described in [6]; \( S \) is a hidden selector; \( \omega \) is a dynamical state,

\[
\omega = (g, \dot{g}, \Psi)
\] (5.2)

where \( g \) is a metric and \( \Psi \) is a state vector of matter; the metric is of the form [5,6]

\[
g = dt \otimes dt - h_t,
\] (5.3)

so that

\[
\omega = (h, \dot{h}, \Psi).
\] (5.4)

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