Acoustic resonance coupling for directional wave control: from angle-dependent absorption to asymmetric transmission

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Abstract
We investigate acoustic wave coupling between two resonators of very different intrinsic losses, e.g. lossy and lossless resonators as an extreme case. We find that the resonator pair placed on a reflector exhibits angle-dependent absorption, showing an order of magnitude difference for opposite angles of incidence. The results obtained from a harmonic oscillator model show excellent agreement with numerical results. Moreover, our analytical model explicitly describes the contribution of radiation leakage and coupling to the angle-dependent absorption, enabling one to design a simple resonant structure demonstrating asymmetrical transmission.

1. Introduction
Wave-structure interaction is of fundamental importance, enabling intriguing wave phenomena [1–5]. Particular examples include an optical analogy of electromagnetically induced transparency (EIT) [1–3], extraordinary optical transmission in a subwavelength aperture (denoted as bull’s eye) [4], and a single structure supporting for both EIT and super-scattering [5]. Combined with resonance, such optical coupling enables to design subwavelength metamaterials having desired dispersion [6]. Similarly, in acoustic metamaterials, resonance coupling has been used to realize perfect acoustic absorption in a two-port system [7, 8], EIT-like acoustic transmission [9], and extraordinary acoustic transmission [10]. Moreover, there have been theoretical considerations in the study of such coupling between two identical resonators [11], or frequency-detuned resonators (two closely-placed resonances) [8, 12]. These coupled resonator systems mostly have used resonators of similar characteristics. Recently, the coupling of resonators with asymmetric losses has been considered [13–15].

Interaction between resonators with a large loss contrast has drawn attention, because it is often unavoidable and, more importantly, functions as an essential building block of non-Hermitian metamaterials, such as a lossy acoustic metasurface [16, 17], a lossy acoustic topological insulator [18] and a passive PT symmetry structure [19]. To study coupled resonance in optics, couple-mode theory has proven useful for understanding the interaction between resonance modes [20, 21]. Similarly, in acoustics, formalisms based on a Lorentzian model have been used to capture physical characteristics as well as spectral behaviors of coupled resonators [7, 22]. However, such models were considered for simple scenarios, e.g. wave propagation in a waveguide [22]. In addition, these models often implemented a lumped loss parameter accounting for both internal loss and resonance coupling [7], partly because of complexity involved in characterizing radiation leakage and coupling between resonators [23].

Here, we investigate the interaction between a pair of lossy and lossless resonators placed on an acoustic reflector using an analytical formula based on a harmonic oscillator model. As an isolated system, we study the absorption cross section of the resonator pair for acoustic waves incident on the same side of the reflector. Our formalism includes closed-form expressions of radiation leakages from the individual resonators and radiation coupling between the two resonators, thereby enabling to characterize the radiatively-coupled resonators from an interference perspective. We observe angle-sensitive absorption in the resonator pair, and our simulations show an order of magnitude difference in absorption between two oblique incidence angles (tilted to the right...
To study coupled resonances, we consider an isolated pair of lossy and lossless harmonic oscillators on an acoustic reflector in a two-dimensional (2D) domain, as illustrated in Figure 1(a). These two resonators operate at the same resonance frequency ($f_0$), but only the left oscillator has an intrinsic loss (otherwise identical with the same mass $m$ and spring constant $k$). The harmonic oscillators, whose widths ($s$) are identical and subwavelength ($s < \lambda_0 = c/f_0$, $c$ the speed of sound), isotropically radiate acoustic waves to the half-space as a point scatterer. The resonators placed with a spacing of $d$ are coupled each other via the radiation. Note that these harmonic oscillators interacting with incident acoustic waves represent acoustic resonators such as Helmholtz resonators, quarter-wavelength resonators, and membrane-type resonators, enabling us to focus on coupling phenomena applicable to all these resonator types instead of limiting to a specific type of acoustic resonators. In addition, such a simple model system as the isolated resonator pair has relevance to practical systems consisting of acoustic sources (speaker) or detectors (microphone) in combination of adjacent lossless resonators, and in this system, the lossless resonators radiatively coupled with the acoustic sources and detectors can control the characteristics (e.g. directivity and sensitivity) of the sources and detectors.

The coupling between the resonators is characterized by radiation coupling rate ($\eta_2$), while the interaction between the resonators and free space is characterized as radiation leakage rate ($\eta_1$). Using the radiation leakage rate ($\eta_1$) and coupling rate ($\eta_2$), the coupled equation of motion is expressed by [24, 25]

$$\begin{cases}
    m \frac{d^2y_1(t)}{dt^2} + \eta_1y_1(t) + \eta_1y_1(t) \frac{dy_1(t)}{dt} + \eta_2y_1(t) \frac{dy_2(t)}{dt} + ky_1(t) = F_1(t) \\
    m \frac{d^2y_2(t)}{dt^2} + (\eta_2 + \eta_1)y_2(t) \frac{dy_2(t)}{dt} + \eta_2y_2(t) \frac{dy_2(t)}{dt} + ky_2(t) = F_2(t)
\end{cases}$$

where the subscripts 1 and 2 denote the individual resonators, respectively, $y$ is the vibration amplitude, $\eta_1$ is the internal loss rate having the same unit (kg s$^{-1}$) as the leakage and coupling rates, and $F$ is the force acting on the resonators due to incident acoustic waves. The radiation leakage rate serves as the damping term of the coupled equations (equation (1)), indicating that a larger radiation leakage leads to a decrease in the vibration amplitude. Assuming a plane wave excitation under normal incidence ($\theta = 0$), we have $F_1(t) = F_2(t)$. For oblique incidences ($\theta \neq 0$) and a spacing of $d$ $\sim \lambda_0$, there exists an excitation phase difference of $\Delta \varphi = 2\pi d \sin(\theta)/\lambda$, i.e. $F_{21} = F_2/F_1 = \exp \left[ -i\Delta \varphi \right]$.

To characterize the absorption of the resonator pair, we use absorption cross section ($\sigma_{abs}$), defined as the absorbed power per unit depth for 2D ($P_{abs}$ [W m$^{-1}$]) relative to the incident acoustic power per unit area
\( P_{\text{inc}} \) (W m\(^{-2}\)), i.e. \( \sigma_{\text{abs}} [\text{m}] = \frac{P_{\text{inc}} [\text{W m}^{-2}]}{P_{\text{inc}} [\text{W m}^{-2}]} \). Using equation (1) and the definition of \( Y_{21} = Y_2 / Y_1 \) (i.e. the ratio of the vibration amplitudes of the resonators), the absorption cross section spectrum of the lossy and lossless resonators \((\eta_1 = \eta_2 = 0)\) is derived \(^{26}\) and given by

\[
\sigma_{\text{abs}} (\omega) = 2\sigma_{\text{abs}} \left\{ -\frac{\omega}{\omega_0} + \frac{\omega_0}{\omega} - 2\text{Im} \left( \frac{\eta_1 + \eta_2 Y_{21}}{\eta_0} \right) \right\}^2 + 4 \left( \frac{\eta_1}{\eta_0} + \text{Re} \left( \frac{\eta_1 + \eta_2 Y_{21}}{\eta_0} \right) \right), \tag{2a}
\]

\[
Y_{21} = \left\{ \left( -\omega / \omega_0 + \omega_0 / \omega \right) \eta_0 + 2i \left( \eta_1 + \eta_2 - F_{12} \eta_1 \right) \right\} F_{21}, \tag{2b}
\]

where \( \omega \) is the frequency, \( \omega_0 \) is the resonance frequency \( \omega_0 = \sqrt{k / m} = 2\pi f_0 \), \( \eta_0 \) is the critical damping \( \eta_0 = 2 \sqrt{km} \), \( \eta' \) is \( \text{Re} \left( \eta_0 \right) = \frac{\pi Z s^2}{\lambda} \), \( Z \) is the acoustic impedance, \( \lambda \) is the wavelength, and \( \sigma_{\text{abs}} = \lambda / 2\pi \) (i.e. the upper limit of a single-resonance subwavelength resonator in 2D) \(^{27}\).

Our formulism is based on explicit descriptions of radiation leakage \((\eta_1)\) and radiation coupling \((\eta_2)\) rather than using a lumped loss \((\eta_1 + \eta_2)\) as a fitting parameter or considering \( \eta_2 \) as a real value (i.e. lack of phase information). To find closed-form expressions of \( \eta_1 \) and \( \eta_2 \), we consider the radiated pressure field induced by a harmonically-vibrating resonator. The radiation leakage rate \((\eta_1)\) and radiation coupling rate \((\eta_2)\) per unit depth \((\text{kg} \text{ m}^{-2} \text{s}^{-1})\), both derived from the 2D dipole Rayleigh integral, are expressed, respectively, by

\[
\eta_1 \approx \frac{\pi Z s^2}{\lambda} \left\{ 1 + i \frac{2}{\pi} \frac{2}{3} - \gamma - \ln \left( \frac{\pi s}{\lambda} \right) \right\}, \tag{3a}
\]

\[
\eta_2 \approx \frac{\pi Z s^2}{\lambda} H_0^{(2)} \left( \frac{2\pi d}{\lambda} \right), \tag{3b}
\]

where \( Z \) is the acoustic impedance, \( s \) is the width of the resonator, \( \gamma \) is the Euler’s constant of \( \gamma = 0.5332 \), and \( H_0^{(2)} \) is the zero-order Hankel function of second kind. The detailed derivation can be found in appendix A.

As plotted in figure 2, the leakage \((\eta_1)\) and coupling \((\eta_2)\) rates have complex values. The real part of each complex value corresponds to leakage or coupling resistance, whereas its imaginary part is related to acoustic reactance, which does not contribute to acoustic power transfer \(^{28}\). The radiation leakage rate of equation (3a) corresponds to acoustic radiation reactance derived in \(^{29}\). Note that the radiation leakage rate is proportional to the square of the resonator width \((s)\) relative to the wavelength \((\lambda)\), i.e. \( \eta_1 \propto s^2 / \lambda \). In figure 2(a), as decreasing the width \((s)\), the leakage rate (its magnitude) is reduced and its reactance becomes more dominant than its resistance, i.e. \( \text{Im}(\eta_1) > \text{Re}(\eta_1) \). Similarly, the coupling rate \((\eta_2)\) is gradually decreased with the distance \( d \), whereas its phase angle is drastically varied by \( d \) (see figure 2(b)). From equation (2a), the peak absorption occurs at \( \omega = \omega_p \), satisfying

\[
-\frac{\omega}{\omega_0} + \frac{\omega_0}{\omega} - 2\text{Im} \left( \frac{\eta_1 + \eta_2 Y_{21}}{\eta_0} \right) = 0. \tag{4}
\]

Equation (4) indicates that the peak absorption frequency \((\omega_p)\) depends on the imaginary parts of \( \eta_1 \) and \( \eta_2 \). When both the imaginary parts are zero, \( \omega_p \) is equal to the natural frequency \((\omega_0 = \sqrt{k / m})\), i.e. \( \omega_p = \omega_0 \).

Figure 2. (a) Normalized radiation leakage rate \((\eta_1 / \pi Z)\) in the complex domain as a function of the width \((s)\) relative to the wavelength \((\lambda)\). (b) Normalized radiation coupling rate \((\eta_2 / \pi Z)\) in the complex domain as a function of the distance \((d)\) relative to the wavelength \((\lambda)\). The blue symbols indicate the distances \((d = 0.15\lambda, 0.6\lambda, \text{and} 1.13\lambda)\) for \( \text{Im}(\eta_1) = 0 \).
figure 2(b), the spacings of \( d = 0.15 \lambda_0, 0.6 \lambda_0 \) and 1.13 \( \lambda_0 \) lead to zero-imaginary values of \( n_\eta \) [i.e. \( \text{Im}(n_\eta) = 0 \)], and thus coupling of the resonator pair influences the oscillation of the resonator pair without affecting \( \omega_p \). In contrast, for \( \text{Re}(n_\eta) = 0 \), the coupling changes \( \omega_p \) without increasing or decreasing the vibration amplitudes of the resonator pair.

### 3. Results

#### 3.1. Angle-dependent absorption and its mechanism

Based on the resonance coupling discussed above, we demonstrate angle-dependent acoustic absorption, applicable to acoustic sensing or acoustic antennas (directive responses are desired). Figure 3(a) shows the calculated absorption cross section (\( \sigma_{abs} \)) of a pair of lossy and lossless resonators for opposite angles of incidence (\( \theta = \pm 40^\circ \)) and parameters of \( \lambda_0/20, d = 0.3 \lambda_0, f_0 = 800 \text{ Hz (}= \omega_0/2\pi) \). Here, \( \sigma_{abs} \) is normalized to \( \sigma_{abs} = \lambda_0/2\pi \). For the positive angle of \( \theta = 40^\circ \), in which incident acoustic waves first excite the left lossy resonator, the resonator pair exhibits high absorption (\( \sigma_{abs} \geq 4\sigma_{abs} \) at resonance) with a Lorentzian spectral characteristic. The corresponding absorption coefficient, defined as \( A = \sigma_{abs}/d_{inc} \), has \( A = 1 \) for an incident width of \( d_{inc} < 4\sigma_{abs} \). Remarkably, there is a large contrast in absorption cross section \( \sigma_{abs} \) between these incidence angles (\( \theta = \pm 40^\circ \)), showing an order of magnitude difference at a frequency of \( f_p = \omega_0/2\pi = 799 \text{ Hz (}= \sigma_{abs} \theta) \). The numerical results (symbols) show excellent agreement with the analytical results (solid lines) obtained from the full analytical model of equation (2), validating the closed-form expressions of \( n_\eta \) and \( n_\xi \) (equation (3)). For other angles of incidence, absorption cross section spectra can be found in appendix B. The numerical results are obtained by using 2D finite-difference frequency-domain simulation (COMSOL Multiphysics 5.3). In the numerical calculation, the vibrating masses of the...
harmonic oscillators are modeled as rigid domains, to whose bottom surface boundary conditions are applied with input parameters of viscous damping constant ($\eta_i$) and spring constant ($k_i$).

Typical acoustic absorbers on a reflector are known to have a characteristic of quasi-omnidirectional sound absorption (rather insensitive to incident angles) [30], which is typically desirable for noise reduction. In contrast, the angle-dependent acoustic absorption, observed in our study, can be useful in applications requiring directional acoustic sensing. In such an example, the lossy harmonic oscillator may be replaced by an acoustic detector (transducer), and a lossless acoustic resonator (e.g., Helmholtz resonator) can be placed nearby so as to enable high-sensitivity detection of acoustic waves from a specific direction while preventing sensing of acoustic waves from the opposite angle.

To find conditions for high absorption, equation (2a) is simplified at resonance to

$$\sigma_{\text{abs}} = 2\sigma_{\text{abs}}^0 \frac{4\eta_{i}^{0} \eta_{i}^{0}}{\eta_{i}} + \text{Re}(\eta_{i} + \eta_{i} Y_{21})^2, \tag{5}$$

We first consider a case of a single-resonance system (no coupling with a neighboring resonator). For single resonance ($Y_{21} = 0$) and critical coupling ($\eta_i = \eta_i^0$) [31], it leads to $\sigma_{\text{abs}} = 2\sigma_{\text{abs}}^0 \frac{4\eta_{i}^{0} \eta_{i}^{0}}{[\eta_{i} + \eta_{i} Y_{21}]^2}$, which is the upper limit of $\sigma_{\text{abs}}$ for a single-resonance resonator on a reflector. For dual resonances ($Y_{21} = 0$, owing to resonance coupling the absorption cross section ($\sigma_{\text{abs}}$) can exceed the single resonance limit of $2\sigma_{\text{abs}}^0$. For $\sigma_{\text{abs}}$ to exceed this limit ($\sigma_{\text{abs}} > 2\sigma_{\text{abs}}^0$), the denominator of equation (5) needs to be minimized, thus requiring a negative sign of the coupling term [i.e. $\text{Re}(\eta_{i} Y_{21}) < 0$], due to the positive leakage term $\left[\text{Re}(\eta_{i}) = \frac{-2\pi^2}{h^2} > 0\right]$. Physically, such a condition indicates that the radiation leakage of the lossy resonator destructively interferes with the radiated wave from the lossless resonator ($\eta_{i} Y_{21}$). In other words, the effective radiation leakage (or damping) is reduced owing to the resonance coupling. For this destructive interference, the phase condition of the coupling term [$\phi = \text{Arg}(\eta_{i} Y_{21})$] is expressed by

$$\phi \approx \text{Arg}(F_{21}) + \text{Arg}(\eta_{i}) = -\frac{2\pi d}{\lambda} \sin(\theta) + \text{Arg}\left[H_{0}^{(2)}\left(\frac{2\pi d}{\lambda}\right)\right] = (2n - 1)\pi, \tag{6}$$

where $n$ is the integer number. The strongest coupling occurs by the smallest $d$ when $n = 0$, i.e. $\phi = -\pi$. Here, we have used the approximation of $\text{Arg}(\eta_{i} Y_{21})$ by $\text{Arg}(\eta_{i} F_{21})$ (appendix C). In figure 3(b), we confirm that the positive angle of $\theta = 40^\circ$ meets the condition of equation (6), leading to $\phi \approx -\frac{\pi}{2} - \frac{\pi}{2} = -\pi$ (destructive interference), whereas the negative angle ($\theta = -40^\circ$) results in $\phi \approx \frac{\pi}{2} + \frac{\pi}{2} = 0$ (constructive interference). For this constructive interference, the effective radiation leakage is increased, and thus the vibration of the lossy resonator is suppressed.

The angle-dependent absorption in this paper is based on the ideal system consisting of the lossy and lossless resonator. In a realistic system, losses may be unintentionally introduced. We observe the angle-dependent absorption even in a realistic system consisting of two Helmholtz resonators (see appendix D). The lossless Helmholtz resonator is realized by using a short neck length, and thus the loss in the neck is negligible. For losses comparable to that of the lossy resonator, the transmission contrast is compromised while the angle-dependent absorption is preserved.

### 3.2. Acoustic focusing

In the high absorption condition, the resonance coupling allows the lossless resonator to effectively transfer acoustic energy into the lossy resonator. In other word, acoustic energy is focused without using a focusing lens. From equation (6), we can notice that the resonance coupling is also dependent on the distance ($d$) of the two resonators. Thus, at a given angle of incidence, the absorption cross section as well as the resonance coupling can be controlled by $d$. Thus, effective acoustic focusing requires a proper selection of the distance ($d$). For the normal incidence ($\theta = 0^\circ$), with no excitation phase difference [$\text{Arg}(F_{21}) = 0$], the coupling phase angle is simply dictated by only the radiation coupling rate, i.e. $\phi \approx \text{Arg}\left[H_{0}^{(2)}\left(\frac{2\pi d}{\lambda}\right)\right]$. Figure 3(c) show the absorption cross section spectra for two different spacings of $d = 0.6\lambda_0$ and $0.15\lambda_0$. There is a large contrast in peak absorption between the two spacings. Interestingly, despite their weak coupling strength, the larger spacing of $d = 0.6\lambda_0$ leads to $\sigma_{\text{abs}} = 5\sigma_{\text{abs}}^0$ at 799 Hz, because $\phi \approx -\pi$ is satisfied (figure 3(d)). In contrast, the closely-placed resonators ($d = 0.15\lambda_0$) result in near zero of $\sigma_{\text{abs}}$ at 799 Hz due to $\phi \approx 0$.

As acoustic absorption in the lossy resonator is increased by the neighboring lossless resonator, one may consider increasing the number of lossless resonators surrounding the lossy resonator to further increase $\sigma_{\text{abs}}$. For a simple case, another lossless resonator is placed to the right of the lossy resonator with the same interval of $d = 0.6\lambda_0$ (see the inset of figure 4). Two of these lossless resonators increase the vibration amplitudes of the lossy resonator, showing an increase to $\sigma_{\text{abs}} \approx 7\sigma_{\text{abs}}^0$ in figure 4. Note that this enhancement is not as high as it is expected. In fact, we have expected $\sigma_{\text{abs}} = 8\sigma_{\text{abs}}^0(=2\sigma_{\text{abs}}^0 + 3\sigma_{\text{abs}}^0 + 3\sigma_{\text{abs}}^0)$ [contributions by the lossy resonator.
(2\sigma_{abs}^{\phi}) and by the two lossless resonators (3\sigma_{abs}^{\phi} + 3\sigma_{abs}^{\phi}), as the contribution of a single lossless resonator (i.e. 3\sigma_{abs}^{\phi}) is confirmed in figure 3(c). The discrepancy arises because the direct interaction between the lossless resonators negatively affects overall absorption by dampening each other’s vibration due to the distance between the lossless resonators (1.2\lambda_0) corresponding to \phi \approx 0 (i.e. increase of the effective radiation leakage).

3.3. Asymmetric acoustic transmission

Based on the resonance coupling, we propose a structure for unidirectional acoustic transmission, as illustrated in figure 5(a). Such a unidirectional acoustic transmission is desirable for practical application scenarios. For example, acoustic waves can be separated and moved from their noise sources such that formation of standing waves is inhibited. In other applications, acoustic energy harvesters enclosed by such panels enabling asymmetric transmission can collect acoustic energy without losing them once acoustic waves pass through the panels.

The proposed structure consists of a thin rigid panel and a dipolar lossless resonator (e.g., membrane-type resonators) in the middle of the panel surrounded by two pairs of lossless resonators, one pair on the bottom and the other on the top. Here, the dipolar resonator interacts with the top and bottom acoustic domains. For asymmetric transmission, the spacings within each pair are different, i.e. d_T = 0.6\lambda_0 and d_B = 1.2\lambda_0 for the top and bottom pairs, respectively. The distance of the top pair (d_T = 0.6\lambda_0) is chosen from the result in figure 4 to enhance the oscillation of the middle resonator for high transmission. Figure 5(b) shows simulated transmission power spectra for the forward and backward propagations. By considering an incidence width of 2d_B (i.e. the device width), the transmission (power) coefficient (T) is calculated by \( T = \frac{E_{\text{trans}}}{E_{\text{inc}}} \) with \( E_{\text{trans}} [\text{W m}^{-1}] \) being the transmitted power and \( E_{\text{inc}} [\text{W m}^{-1}] \) the incident power (i.e. \( E_{\text{inc}} = 2d_B E_{\text{inc}} \)). In the forward (F) direction (from top to bottom), we observe a peak acoustic transmission of 30% near 790 Hz. The top resonator pair (d_T = 0.6\lambda_0) surrounding the middle resonator works in a way that their coupling with the middle resonator effectively reduces resistance to the vibrating mass of the middle resonator because of \phi \approx -\pi. In contrast, for the backward (B) direction (from bottom to top), the acoustic transmission is significantly suppressed, because of a coupling phase angle of \phi \approx 0 for d_B = 1.2\lambda_0. Note that the transmission contrast of the forward to backward waves reaches approximately 15 at 795 Hz. In addition, we find that there is the distinct difference in the total pressure fields between the propagation directions, as observed in figures 5(c), (d). Similarly, [32] demonstrated a structure consisting of a channel (\sim 2\lambda width) for asymmetric acoustic transmission, which is based on wave-vector direction conversions. Although the peak transmission of our device is relatively low compared to that of [32], our proposed design shows the large transmission contrast and is enabled by the simpler structure using the resonance coupling. The forward transmission in our design can be further increased by engineering the middle resonator (e.g. replacing it by a monopole resonator such as a Helmholtz resonator). The substitution of the monopole resonator is feasible as long as the monopolar resonator interacts with both top and bottom acoustic domains (see appendix E).
4. Discussion

We have developed an analytical model that describes the interaction between two acoustic resonators on an acoustic reflector. Our model captures the physical characteristics of the radiatively-coupled resonators. Based on the understanding of the resonant coupling, we have shown angle-dependent absorption as well as asymmetric acoustic wave transmission. Although our formulism is based on the isolated resonator pair, it offers physical insights into periodic structures [33–35]. Our results, based on only one lossy resonator, is different from the previous studies on resonant coupling [5, 36, 37], where two lossy (lossless) resonators had a spectral overlap for a higher absorption (transmission). Moreover, our approach enabling the angle-sensitive absorption can be useful for applications, e.g. acoustic antennas capable of absorbing the sound wave in a specific angle while isolating unwanted acoustic waves (noise) from other angles [38].

Appendix A. Derivation of the radiation leakage rate ($\eta_1$) and radiation coupling rate ($\eta_2$)

We start by considering the radiated pressure field induced by one resonator harmonically vibrating with a velocity of $u_0$, which is given by the 2D dipole Rayleigh integral as

$$p(x, y) = \int_{S_0} \frac{\partial}{\partial y} p(x_0, y)|_{y=0} g(x, y|x_0, y_0) dS_0,$$  \hspace{1cm} (A.1)

where $S_0$ is the area of the resonator, and $g$ is the Green’s function. Here, the pre-factor in front of $g$ describes the boundary coupling condition applied to the point sources on the surface of the resonator, which is given by $\frac{\partial}{\partial y} p(x_0, y)|_{y=0} = -ikZu_0$ with $k$ being the wave number, and $Z$ being the acoustic impedance [29]. Thus, equation (A.1) gives pressure at any point $(x, y)$ due to all the point sources at $(x_0, y_0)$ on $S_0$. The analytical forms of $\eta_1$ and $\eta_2$ are then obtained by radiation force ($F_r$) induced by the pressure field $[p(x, y)]$ in the absence of the incident acoustic wave, which are expressed by

![Figure 5. (a) Asymmetric acoustic transmission by coupled resonators for the forward (F) and backward (B) incident waves. (b) Transmitted power spectra ($T_F$ and $T_B$) and transmission contrast ($T_F/T_B$). (c), (d) Simulated pressure field for the forward and backward incident waves at 795 Hz. The gray arrows indicate the locations of the surrounding resonators.](image-url)
The difference between the leakage and coupling is that the radiation coupling $c_{21}$ is determined by radiation from the neighboring resonator ($S_2 \rightarrow S_1$). Here, for the lossy/lossless pair, we use the equality of $rr_{12} = rr_h$ and $cc_{12} = cc_h$ because of the same size ($SS_s = SS_s$) and reciprocity. 2D cylindrical waves emitted from a point source are described with the 2D Green’s function (modified for an infinite reflector) given by [29]

$$g(x, y | x_0, y_0) = \frac{i}{2} H_0^{(2)}(kr),$$

where $H_0^{(2)}$ is the zero-order Hankel function of second kind, and $R = \sqrt{(x - x_0)^2 + (y - y_0)^2}$.

### Appendix B. Absorption cross section spectra for other angles of incidence

The large contrast in $\sigma_{abs}$ between $\pm \theta$ at resonance, seen in figure 2(a), occurs when the peak $\sigma_{abs}(+\theta)$ is exactly aligned with the dip $\sigma_{abs}(-\theta)$. Figure B.1 shows the calculated $\sigma_{abs}$ for different angles of incidence. With increasing $\theta$, the peak (marked with red arrows) and dip (blue arrows) are red-shifted and blue-shifted, respectively, getting closer to each other.

### Appendix C. Approximation of $\text{Arg}(\eta_e Y_{21})$ by $\text{Arg}(\eta_e F_{21})$

Figures C.1(a) and (b) confirm that the line for high $\sigma_{abs}$ is exactly matched with that for $|\text{Arg}(\eta_e Y_{21})| \approx \pi$, i.e. equivalent with $\text{Re}(\eta_e Y_{21}) < 0$. For visual guidance, the identical white dashed lines are overlaid on the surface plots. Moreover, we find that $\text{Arg}(\eta_e Y_{21})$ can be approximated to $\text{Arg}(\eta_e F_{21})$, as shown in figure C.1(c).
Appendix D. Realistic model of the lossy and lossless harmonic oscillators

The harmonic oscillator pair, considered in the main text, can be implemented with a realistic model composed of lossy and lossless Helmholtz resonators (HRs), as illustrated in figure D.1(a). The two HRs have the same neck width ($w_n$), but different neck lengths ($h_n$). Although our design is based on the lossless HR in combination of the lossy HR, the lossless HR inevitably have thermal-viscous losses. To minimize the loss in the neck, the lossless HR has a smaller neck length than the lossy HR. The thermal and viscous losses of the HRs occurring in the necks are considered by using the Thermoviscous Acoustic module in COMSOL Multiphysics. The angle-dependent absorption of the HR pair is shown in figure D.1(b). In addition, for normal incidence ($\theta = 0^\circ$), the distance-dependent absorption is observed, as shown in figure D.1(c). These results indicate that our design approach is valid for the realistic system, where the lossless resonator has losses.

When considering non-negligible losses in the lossless resonator, the coupled resonator exhibits a decrease in the absorption contrast while it preserves the angle dependence. Figure D.2 shows simulation results for different loss inclusions in the lossless resonator: $\eta_{l2} = 0$ (lossless), $\eta_{l2} = 0.1 \eta_{l1}$, and $\eta_{l2} = 0.2 \eta_{l1}$.

Figure C.1. (a) Analytically-calculated peak absorption cross section ($\sigma_{\text{del}}$) as functions of $d$ and $\theta$. (b) Analytically-calculated $|\text{Arg}(\eta_1 Y_{21})|$. (c) Analytically-calculated $|\text{Arg}(\eta_2 F_{21})|$. The white dashed lines in (a)–(c) are all identical and correspond to $d$ and $\theta$ for $|\text{Arg}(\eta_2 Y_{21})| = \pi$.

Figure D.1. (a) Schematic of a pair of lossy (left) and lossless (right) Helmholtz resonators. (b) Angle-dependent absorption cross section spectra (simulation) for $\theta = \pm 40^\circ$ and $d = 0.3 \lambda_0$. The geometrical parameters are given by $w_1 = 1.7$ mm, $h_{n1} = 4.3$ mm, $w_{c1} = 12.5$ mm, and $h_{c1} = 17.7$ mm for the lossy HR and $w_2 = 1.7$ mm, $h_{n2} = 1.1$ mm, $w_{c2} = 12.5$ mm, and $h_{c2} = 25.5$ mm for the lossless HR. (c) Distance-dependent absorption cross section spectra (simulation) for normal incidence ($\theta = 0^\circ$) and $d = 0.6 \lambda_0$ and $0.15 \lambda_0$. 

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Appendix E. Asymmetric transmission using a monopolar resonator in the middle

Figure E.1 shows the asymmetrical transmission by using a monopolar resonator as the middle resonator. For the forward incidence, the peak transmission reaches $\sim 50\%$ and the transmission contrast between the forward and backward incidences is larger than 10. Compared to the dipolar middle resonator (see figure 5(b)), the monopolar middle resonator has a decrease in the transmission bandwidth with the increased transmission due to a decreased leakage rate.

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References

[1] Boller K-J, Imamoglu A and Harris S E 1991 Observation of electromagnetically induced transparency Phys. Rev. Lett. 66 2593
[2] Papasimakis N, Fedotov V A, Zholudev N I and Prosvirnin S L 2008 Metamaterial analog of electromagnetically induced transparency Phys. Rev. Lett. 101 253903
[3] Liu N, Languth L, Weiss T, Kästel J, Fleischhauer M, Pfau T and Giessen H 2009 Plasmonic analogue of electromagnetically induced transparency at the Drude damping limit Nat. Mater. 8 758–62
[4] Lezec HJ, Degiron A, Devaux E, Linke RA, Martin-Moreno I, Garcia-Vidal FJ and Ebbesen TW 2002 Beaming light from subwavelength aperture Science 297 820
[5] Verselegers L, Yu Z, Ruan Z, Catrysse P B and Fan S 2012 From electromagnetically induced transparency to supercursing with a single structure: a coupled-mode theory for doubly resonant structures Phys. Rev. Lett. 108 083902
[6] Luk’yanchuk B, Zheludev N I, Maier S A, Halas N J, Nordlander P, Giessen H and Chong C T 2010 The Fano resonance in plasmonic nanostructures and metamaterials Nat. Mater. 9 707–15
[7] Yang M, Meng C, Fu C, Li Y, Yang Z and Sheng P 2015 Subwavelength total acoustic absorption with degenerate resonators Appl. Phys. Lett. 107 104104
[8] Santillán A and Bızhovertını S I 2011 Acoustic transparency and slow sound using detuned acoustic resonators Phys. Rev. B 84 064304
[9] Zhou Y, Lu M-H, Feng L, Ni X, Chen Y-F, Zhu Y-Y, Zhu S-N and Ming N-B 2010 Acoustic surface evanescent wave and its dominant contribution to extraordinary acoustic transmission and collimation of sound Phys. Rev. Lett. 104 164301
[10] Zhang J, Cheng Y and Liu X 2017 Extraordinary acoustic transmission at low frequency by a tunable acoustic impedance metasurface based on coupled Mie resonators Appl. Phys. Lett. 110 233502
[11] El Boudouti E H, Mrabtil T, Al-Wahsh H, Akjouj A and Dobrzynski L 2008 Transmission gaps and Fano resonances in an acoustic waveguide: analytical model J. Phys.: Condens. Matter 20 255212
[12] Ryoo H and Jeon W 2018 Dual-frequency sound-absorbing metasurface based on visco-thermal effects with frequency dependence J. Appl. Phys. 123 115110
[13] Ding K, Ma G, Zhang Z Q and Chan C T 2018 Experimental demonstration of an anisotropic exceptional point Phys. Rev. Lett. 121 085702
[14] Merkel A, Romero-Garcia V, Groby J-P, Li J and Christense J 2018 Unidirectional zero sonic reflection Phys. Rev. Lett. 98 201102
[15] Sheng C, Li J, Peng X and Cummer S A 2018 Synthetic exceptional points and unidirectional zero reflection in non-Hermitian acoustic systems Phys. Rev. Mater. 2 125203
[16] Li Y, Shen C, Xie Y, Li J, Wang W, A Cummer S A and Jing Y 2017 Tunable asymmetric transmission via lossy acoustic metasurfaces Phys. Rev. Lett. 119 035501
[17] Li Y and Assouar B M 2016 Acoustic metasurface-based perfect absorber with deep subwavelength thickness Appl. Phys. Lett. 108 063505
[18] Lee T and Iizuka H 2019 Bragg scattering based acoustic perfect absorber with deep subwavelength thickness Phys. Rev. B 99 064305
[19] Liu T, Zhu X, Chen F, Liang S and Zhu J 2018 Unidirectional wave vector manipulation in two-dimensional space with an all passive acoustic parity-time-symmetric metamaterials crystal Phys. Rev. Lett. 120 142402
[20] Yariv A 1973 Coupled-mode theory for guided-wave optics IEEE J. Quantum Electron. 9 919
[21] Fan S, Wonjoo Suh W and Joannopoulos J D 2003 Temporal coupled-mode theory for the Fano resonance in optical resonators J. Opt. Soc. Am. A 20 569
[22] Achilleos V, Theocharis G, Richoux O and Pagneux V 2017 Non–Hermitian acoustic metamaterials: role of exceptional points in sound absorption Phys. Rev. B 95 144303
[23] Haus H A 1984 Waves and Fields in Optoelectronics (Englewood Cliffs, NJ: Prentice-Hall)
[24] Rau S S 2011 Mechanical Vibrations 5th edn (Englewood Cliffs, NJ: Prentice Hall)
[25] Langley R S 2007 Numerical evaluation of the acoustic radiation from planar structures with general baffle conditions using wavelets J. Acoust. Soc. Am. 121 766
[26] Lee T, Nomura T, Dede E M and Iizuka H 2019 Ultra-sparse perfect acoustic absorbers enabling fluid flow and visible-light control Phys. Rev. Appl. 11 024022
[27] Ruan Z and Fan S 2012 Temporal coupled-mode theory for light scattering by an arbitrarily shaped object supporting a single resonance Phys. Rev. A 85 043828
[28] Fahy F J 1987 Sound and Structural Vibration: Radiation, Transmission, and Response (San Diego, CA: Academic)
[29] Mellow T and Kärkkäinen L 2011 On the sound fields of infinitely long strips J. Acoust. Soc. Am. 130 153
[30] Jiménez N, Huang W, Romero-Garcia V, Pagneux V and Groby J-P 2016 Ultra-thin metamaterial for perfect and quasi-unidirectional sound absorption Appl. Phys. Lett. 109 121902
[31] Lee T and Iizuka H 2018 Heavily overdamped resonance structurally engineered in a grating metasurface for ultra-broadband acoustic absorption Appl. Phys. Lett. 113 101903
[32] Zhu Y-F, Zou X-Y, Liang B and Cheng J-C 2015 Acoustic one-way open tunnel by using metasurface Appl. Phys. Lett. 107 113501
[33] Christensen J, Fernandez-Dominguez A L, De Leon-Perez F, Martin-Moreno L and Garcia-Vidal F J 2007 Collimation of sound assisted by acoustic surface waves Nat. Phys. 3 8351
[34] Mei J, Hou B, Ke M, Peng S, Jia H, Liu Z, Shi J, Wen W and Sheng P 2008 Acoustic wave transmission through a bull’s eye structure Appl. Phys. Lett. 92 124106
[35] Quan L, Qian F, Liu X, Gong X and Johnson P A 2015 Mimicking surface plasmons in acoustics at low frequency Phys. Rev. B 92 104105
[36] Li J, Wang W, Xie Y, Popa B-I and Cummer S A 2016 A sound absorbing metasurface with coupled resonators Appl. Phys. Lett. 109 091908
[37] Merkel A, Theocharis G, Richoux O, Romero-Garcia V and Pagneux V 2015 Control of acoustic absorption in one-dimensional scattering by resonant scatterers Appl. Phys. Lett. 107 244102
[38] Zhang Z, Tian Y, Wang Y, Gao S, Cheng Y, Liu X and Christensen J 2018 Directional acoustic antennas based on valley-hall topological insulators Adv. Mater. 30 1803229