Radiatively Generated Isospin Violations in the Nucleon

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Abstract

Isospin violating valence and sea distributions are evaluated due to QED leading $\mathcal{O}(\alpha)$ corrections to the standard QCD evolution equations. Unique perturbative predictions are obtained within the radiative parton model, and confirm earlier results. Nonperturbative contributions have been estimated and depend on a single free parameter chosen to be the current quark mass. The relevance of our predictions for extracting $\sin^2 \theta_W$ from DIS $\nu(\bar{\nu})N$ data (‘NuTeV anomaly’) is discussed as well.
The NuTeV collaboration recently reported [1] a measurement of the Weinberg angle $s_W^2 \equiv \sin^2 \theta_W$ which is approximately three standard deviations above those presented in [2] for the world average of other electroweak measurements. Possible sources for this discrepancy (see, for example, [3, 4, 5, 6, 7]) include, among other things, an isospin violating contribution $\delta R_I$ to the relation [8]

$$R_I^+ - R_I^- = \frac{1}{2} - s_W^2 + \delta R_I^{-}.$$  \hspace{1cm} (1)

This contribution is given, for $N = \frac{1}{2}(p + n)$, by [3, 5, 7]

$$\delta R_I^- = \left( \frac{1}{2} - \frac{7}{6} s_W^2 \right) \frac{\delta U_v - \delta D_v}{U_v + D_v}$$  \hspace{1cm} (2)

where the second moments $\delta Q_v(Q^2)$ and $Q_v(Q^2)$ of the valence distributions $\langle \delta q_v(x, Q^2) \rangle$ are $Q_v(Q^2) = \int_0^1 x q_v(x, Q^2) dx$, $q_v = q - \bar{q}$, and $\delta Q_v(Q^2) = \int_0^1 x \delta q_v(x, Q^2) dx$ with

$$\delta u_v(x, Q^2) = u_v^p(x, Q^2) - d_v^n(x, Q^2)$$

$$\delta d_v(x, Q^2) = d_v^p(x, Q^2) - u_v^n(x, Q^2).$$  \hspace{1cm} (3)

NLO QCD corrections do not significantly [4, 6, 7] contribute to (1). Nonperturbative calculations [9, 10, 11] of $\delta q_v$ in (3) were found [5] to possibly reduce the discrepancy between the neutrino and electroweak measurements of $s_W^2$ by up to 40%. A somewhat different calculation [12], based on QED contributions to $\delta q_v(x, Q^2)$, resulted in a comparable reduction of this discrepancy.

We shall approach the issue of the isospin violations within the framework of the radiative parton model [13] and obtain predictions for $\delta q_v$ which depend on a single free parameter required by the nonperturbative contribution and chosen to be, as in [12], the current quark mass. Although our method differs from those in [5], the results turn out to be similar. On the other hand, our starting position is only a little different to [12], the results, as anticipated in that work, turn out to be quite similar.

Following [12] we shall evaluate the modifications of the parton distributions due to QED radiative effects. In contrast to [12, 14] we shall calculate these effects only to
leading order in $\alpha$, which is sufficient for our purpose here and furthermore simplifies the calculations considerably. The inclusion of QED $\mathcal{O}(\alpha)$ corrections modifies the QCD evolution equation for charged parton distributions by an additional term which, in an obvious symbolic notation, reads \[15, 16\]
\[
\dot{q}(x, Q^2) = \frac{\alpha_s}{2\pi} [P_{qq} * q + P_{qg} * g] + \frac{\alpha}{2\pi} e_q^2 P_{\gamma q} * q
\] with $P_{\gamma q}(z) = \left(\frac{1+ze^2}{1-z}\right)^\gamma_\gamma$. Notice that the addition [12, 14] of a further term $(\alpha/2\pi)e_q^2 P_{\gamma q} * \gamma$ to the r.h.s. of eq. (4) would actually amount to a subleading $\mathcal{O}(\alpha^2)$ contribution since the photon distribution $\gamma(x, Q^2)$ of the nucleon is of $\mathcal{O}(\alpha)$ \[17, 18, 19, 20, 21, 22\]. The standard QCD evolution equation for the gluon distribution remains unaltered at leading $\mathcal{O}(\alpha)$. The perturbative $(p) \mathcal{O}(\alpha)$ modifications of the valence quark distributions in (3), as implied by (4), are now given by
\[
\delta_p u_v(x, Q^2) = \frac{\alpha}{2\pi} \int_{Q_0^2}^{Q^2} d\ln q^2 \int_x^1 dy P\left(x \frac{y}{x}\right) u_v(y, q^2)
\]
\[
\delta_p d_v(x, Q^2) = -\frac{\alpha}{2\pi} \int_{Q_0^2}^{Q^2} d\ln q^2 \int_x^1 dy P\left(x \frac{y}{x}\right) d_v(y, q^2)
\] with $P(z) = (e_u^2 - e_d^2) P_{qq}(z)$ and $Q_0^2 \equiv \mu_0^2 = 0.26 \text{ GeV}^2$, $u_v(x, q^2)$, $d_v(x, Q^2)$ are taken from the dynamical (radiative) parton model \[13\]. Notice that this treatment of the perturbative components differs from that in \[12\].

For the nonperturbative $(np)$ modifications we estimate, following eq. (12) in \[12\],
\[
\delta_{np} u_v(x) = \frac{\alpha}{2\pi} \ln \frac{Q_0^2}{\mu_0^2} \int_x^1 dy P\left(x \frac{y}{x}\right) u_v(y, Q_0^2)
\]
\[
\delta_{np} d_v(x) = -\frac{\alpha}{2\pi} \ln \frac{Q_0^2}{\mu_0^2} \int_x^1 dy P\left(x \frac{y}{x}\right) d_v(y, Q_0^2)
\] where we take $\mu_0 = m_q \approx 10 \text{ MeV}$, i.e. of the order of the current quark masses \[2\]. Here the parton distributions at $Q^2 \leq Q_0^2$ were taken to equal their values at the perturbative input scale $Q^2 = Q_0^2$, i.e. were ‘frozen’.

In fig. 1 we show the total $\delta_p q_v(x, Q^2) + \delta_{np} q_v(x)$ and the purely perturbative $\delta_p q_v(x, Q^2)$ isospin violating distributions at a typical scale of $Q^2 = 10 \text{ GeV}^2$. Our total
isospin violating distributions are quite similar to those in [12], as well as to those in [5] which were obtained by rather different methods. The fact that only our total results are compatible with nonperturbative (bag) model expectations [5, 9, 10, 11] is indicative for the necessity of our nonperturbative estimates in (6) and that the rather marginal perturbative contributions in (5) are not sufficient for a realistic estimate of isospin violating effects. It is, furthermore, particularly interesting to note that our results depend, as mentioned above, only on a single free parameter, i.e. $\mu_0$ in (6).

Encouraged by this agreement we present in fig. 2 and 3 the corresponding predictions for the isospin violating distribution $\delta \bar{u}$ and $\delta \bar{d}$ of sea quarks as defined in (3) and obtained from (5) and (6) by replacing $u_v$ and $d_v$ by $\bar{u}$ and $\bar{d}$, respectively. At $Q^2 = 10$ GeV$^2$ the perturbative contribution $\delta p \bar{q}(x, Q^2)$ in fig. 2 does not dominate the total result $\delta p \bar{q}(x, Q^2) + \delta np \bar{q}(x, Q^2)$ which is obviously in contrast to the predictions at higher scales as illustrated, for example, at $Q^2 = M_W^2$ in fig. 3. Similar results are obtained for the LO CTEQ4 parton distributions [23] where also valence–like sea distributions are employed at an input scale $Q^2_0 = 0.49$ GeV$^2$, i.e., $x\bar{q}(x, Q^2_0) \to 0$ as $x \to 0$. Such predictions may be tested by dedicated precision measurements of Drell–Yan and DIS processes employing neutron (deuteron) targets as well.

As a possible immediate application of our predictions for the isospin violating valence distributions let us finally turn to the relation in (1). Since the isospin violation generated by the QED $O(\alpha)$ correction is such as to remove more momentum from up–quarks than down–quarks, as is evident from fig. 1, it works in the right direction to reduce the NuTeV anomaly [1], i.e. $\sin^2 \theta_W = 0.2277 \pm 0.0013 \pm 0.0009$ as compared to the world average of other measurements [2] $\sin^2 \theta_W = 0.2228(4)$. This effect is also slightly $Q^2$ dependent because of the perturbative contribution in (5). At $Q^2 = 10$ GeV$^2$, appropriate for the NuTeV experiment, our total distributions in fig. 1 imply $\delta U_v = -0.002226$ and $\delta D_v = 0.000890$ which, together with $U_v + D_v = 0.3648$ in (2), leads to a change in the measured value of $\sin^2 \theta_W$ of $\delta \sin^2 \theta_W = \delta R_I^- = -0.0020$ according to (1). Since the value
of sin²θ_W from the NuTeV experiment, with δR_I⁻ ≡ 0 in (1), is 0.0049 larger than the world average of other measurements, our predicted isospin violation δR_I reduces this discrepancy by about 40%. This reduction may be slightly diminished if it is corrected for experimental acceptance cuts [3].

To summarize, we evaluated the modification of parton distributions due to QED leading O(α) corrections to the standard QCD evolution equations. For the isospin violating valence δq_v(x, Q^2) and sea δq(¯q, x, Q^2) distributions (q = u, d) unique perturbative predictions are obtained within the dynamical (radiative) parton model. The nonperturbative contributions to δq_v and δq have been estimated and depend on a single free parameter chosen to be the current quark mass. The results for the valence distributions bear some similarity to nonperturbative bag–model expectations. Our total predictions for δq_v reduce significantly the discrepancy between the large result for sin²θ_W as derived from deep inelastic ν(¯ν)N data (‘NuTeV anomaly’) and the world average of other measurements.

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Figure 1: The isospin violating ‘majority’ $\delta u_v$ and ‘minority’ $\delta d_v$ valence quark distributions at $Q^2 = 10$ GeV$^2$ as defined in (3). The perturbative GRV(pert.) predictions are calculated according to (5). With the non–perturbative contribution in (6) added, one obtains the total estimates GRV(total). The bag model estimates are taken from ref. [5].
Figure 2: The isospin violating sea distributions $\delta \bar{u}$ and $\delta \bar{d}$ at $Q^2 = 10$ GeV$^2$ as defined in (3) with $u_v, d_v$ replaced by $\bar{u}, \bar{d}$. The perturbative (pert.) predictions are calculated according to (5) with $u_v, d_v$ replaced by $\bar{u}, \bar{d}$. The same replacement holds for (6) when calculating the non-perturbative contributions which have to be added to the ‘pert.’ predictions in order to obtain the total results. The LO GRV sea distributions [13] are used throughout.
Figure 3: As in fig. 2 but at $Q^2 = M_w^2$. 