Trihedral lattice towers with optimal cross-sectional shape

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Abstract. The article discusses a new design of triangular lattice supports and suggests a technique for optimizing their cross-sectional shape. Optimization is carried out from the condition of the cross-section being equally stable in two planes. Such relationships are found between the geometric parameters of the section, at which the moments of inertia take the maximum value.

1. Introduction

This article will consider a new rational type of trihedral lattice structures used in the construction of tower structures for various purposes. The structure under study (patent for invention RU2584337 [1]) (Figure 1) contains belts 1 of a polyhedral closed section, lattice bars 2, attached to sheet gussets 3.

Figure 1. Trihedral lattice tower

Polyhedral belts of a closed section are made of sheet steel by bending it, while for the formation of gussets, the edges of the sheet steel are bent symmetrically in the opposite direction at an angle of 60 °,
and the walls of the lattice 2 of each face are attached to the gussets by welding. The formation of the closed polyhedral section 1 is carried out by performing the longitudinal weld 4 at the point of bending and the contact of the bent edges.

The purpose of this work is to determine the optimal parameters $b$, $h$, $l$ of the pentagonal section of the belts (Figure 2). The $l_1$ size is assigned for design reasons to ensure the required length of the weld, and we will consider it as given. An axial moment of inertia relative to the main central axis $y$ is selected as the objective function. A limitation is introduced in order to ensure the equal stability of the belt in two planes. Thus, optimization is performed from the stability condition. In the process of optimization, the perimeter of the section without taking into account the gussets is taken to be constant, which corresponds to the constancy of material consumption.

2. Methods

For simplicity, we will assume that the section is thin-walled and $\delta \ll L$. We represent the cross-section shown at fig. 2 as the set of seven rectangles (Figure 3).

![Figure 2. Optimized cross section](image)

The areas of each of the rectangles are defined as:

$$A_1 = b\delta; A_2 = A_3 = h\delta; A_4 = A_5 = l\delta; A_6 = A_7 = l_1\delta.$$  \hspace{1cm} (1)

Let us choose the coordinate system $zC_1y_1$ as the auxiliary one. The coordinates of the figures 1-7 centers of gravity are determined by the formulas:

$$z_{c1} = 0; z_{c2} = z_{c3} = \frac{h}{2}; z_{c4} = z_{c5} = h + \frac{l}{2} \sin \alpha = h + \frac{l}{2} \sqrt{1 - \frac{b^2}{4l^2}}. $$

$$z_{c6} = z_{c7} = h + l \sin \alpha + \frac{l_1}{2} \cos 30^\circ = h + l \sqrt{1 - \frac{b^2}{4l^2}} + \frac{l_1\sqrt{3}}{4}. $$  \hspace{1cm} (2)
The position of the entire section center of gravity is determined by the formula:

\[
z_c = \frac{2(A_2z_{c2} + A_4z_{c4} + A_6z_{c6})}{A},
\]

where \(A\) is the total cross-sectional area.

The moments of inertia of the entire section relative to the main central axes \(y\) and \(z\) are determined by the formulas:

\[
J_{y1}^I = \frac{b_1\delta_1^3}{12}; \quad J_{z1}^I = \frac{b_1^3\delta_1}{12}; \quad J_{y2}^I = J_{y3}^I = \frac{\delta h^3}{12}; \quad J_{z2}^I = J_{z3}^I = \frac{h_0\delta^3}{12};
\]

\[
J_{y4}^{IV} = J_{y5}^{IV} = J_{u4}^{IV} \cos^2 \alpha + J_{v4}^{IV} \sin^2 \alpha = \frac{l_{d3}^3 b^2}{12} + \frac{l_3^3 \delta}{12} \left(1 - \frac{b^2}{4l^2}\right);
\]

\[
J_{z4}^{IV} = J_{z5}^{IV} = J_{u4}^{IV} \sin^2 \alpha + J_{v4}^{IV} \cos^2 \alpha = \frac{l_{d3}^3}{12} \left(1 - \frac{b^2}{4l^2}\right) + \frac{l_3^3 \delta}{12} \left(1 - \frac{b^2}{4l^2}\right);
\]

\[
J_{y6}^{VI} = J_{y7}^{VI} = J_{u6}^{VI} \sin^2 30^\circ + J_{v6}^{VI} \cos^2 30^\circ = \frac{3l_1\delta^3}{12} + \frac{l_3^3 \delta}{12};
\]

\[
J_{z6}^{VI} = J_{z7}^{VI} = J_{u6}^{VI} \cos^2 30^\circ + J_{v6}^{VI} \sin^2 30^\circ = \frac{l_1\delta^3}{12} + \frac{3l_3^3 \delta}{12}.
\]
Finally, the expression for the moment of inertia $J_z$ takes the form:

$$J_z = J_{z1} + 2 \left( J_{z2} + A_2 \frac{b^2}{4} \right) + 2 \left( J_{z4} + A_4 \frac{b^2}{16} \right) + 2 \left( J_{z6} + A_6 \frac{l^2}{16} \right).$$

(5)

The expression for the moment of inertia $J_y$ is not presented here due to its cumbersomeness. Formula (6) can be presented in a simplified form if we neglect the terms that include the quantities $\delta^3$ and $\delta^2$.

In the problem under consideration, the objective function and constraints are nonlinear, therefore, to solve it, it is necessary to apply nonlinear optimization methods.

We carried out the solution in the Matlab environment using the Optimization Toolbox package. The function *fmincon* was used, which determines the minimum of the nonlinear objective function $J_y^{-1}$ with nonlinear constraints. The interior point method is selected as the nonlinear optimization method.

### 3. Results and Discussion

Table 1 shows the optimal values of the ratios $b / L$, $h / L$ and $l / L$ depending on the ratio $l_1 / L$. No solution was found for $l_1 / L > 0.125$. It probably does not exist for such relations between $l_1$ and $L$.

| $l_1/L$ | $b/L$   | $h/L$   | $l/L$   | $\alpha, {}^\circ$ |
|---------|---------|---------|---------|-------------------|
| 0       | 0.2677  | 0.2093  | 0.1569  | 31.5              |
| 0.025   | 0.2851  | 0.1826  | 0.1749  | 35.4              |
| 0.05    | 0.3022  | 0.1604  | 0.1885  | 36.7              |
| 0.075   | 0.3188  | 0.142   | 0.1986  | 36.6              |
| 0.1     | 0.3345  | 0.1265  | 0.2063  | 35.8              |
| 0.125   | 0.3495  | 0.1131  | 0.2122  | 34.6              |

Note that the pentagonal section without gussets at $l_1 = 0$ can act as a replacement for square tubes in truss chords of the Molodechno type [2-5]. Compared to a square tube, a pentagonal tube with optimal parameters $b$, $h$, $l$ for the same cross-sectional area has $5.2\%$ higher moments of inertia. The optimum angle $\alpha$ is $31.46^\circ$. When designing real structures, for convenience, you can take $\alpha = 30^\circ$.

The equality of the moments of inertia $J_y$ and $J_z$ is necessary to ensure equi-stability in the case of the same fixation of the rod in the xOz and xOy planes. At different reduced lengths in two planes, it becomes necessary to find the optimal section parameters for which the ratio $J_y / J_z$ is specified. The technique proposed by the authors and the developed program in the Matlab environment allows to do this. Table 2 shows the optimal values of $b / L$, $h / L$ and $l / L$ for various ratios $J_y / J_z$ at $l_1 = 0$.

| $J_y / J_z$ | $b/L$   | $h/L$   | $l/L$   |
|-------------|---------|---------|---------|
| 0.6         | 0.3135  | 0.1683  | 0.175   |
| 0.7         | 0.3     | 0.1803  | 0.1698  |
| 0.8         | 0.288   | 0.191   | 0.165   |
| 0.9         | 0.2773  | 0.2006  | 0.1608  |
| 1           | 0.2677  | 0.2093  | 0.1569  |
In a similar way, you can optimize the cross-section of the trihedral lattice tower with additional bends of the edges (Fig. 4). The values $b/L$, $h/L$ and $l/L$ obtained from the condition of equi-stability and the maximum moment of inertia, depending on the ratio $l_1/L$, are presented in Table 3.

| $l_1/L$ | $b/L$ | $h/L$ | $l/L$ | $\alpha, ^\circ$ |
|--------|-------|-------|-------|-----------------|
| 0      | 0.2677| 0.2093| 0.1569| 31.5            |
| 0.025  | 0.2933| 0.1714| 0.1819| 36.3            |
| 0.05   | 0.3175| 0.1432| 0.1981| 36.7            |
| 0.075  | 0.3398| 0.1214| 0.2087| 35.5            |
| 0.1    | 0.3604| 0.1039| 0.2159| 33.4            |

Table 3. Optimum parameters of the trihedral lattice tower section with additional bends of the edges, depending on the ratio $l_1/L$

Figure 4. Triangular lattice tower cross-section with additional bends of the edges

4. Summary
The optimal ratios between the pentagonal section side sizes of the trihedral lattice supports are determined, providing their maximum rigidity and uniform stability at a given material consumption. The optimal angle at the top was close to 120 degrees, and this value can be used in the design for simplification. In the future, it is advisable to consider the issues of local stability of the support elements with the proposed cross section.
References

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