Nonzero Mean Squared Momentum of Quarks in the Non-Perturbative QCD Vacuum

Li-Juan Zhou1,2, Leonard S. Kisslinger3, Wei-xing Ma4
1 Department of Information and Computing Science, Guangxi University of Technology, Liuzhou, 545006, P. R. China
2 Institute of Particle Physics, Hua-zhong Normal University, Wuhan, 430079, P. R. China
3 Department of Physics, Carnegie-Mellon University, Pittsburgh, PA. 15213, USA
4 Institute of High Energy Physics, Chinese Academy of Sciences, P. O. Box 918 Beijing, 100049, P. R. China
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The non-local vacuum condensates of QCD describe the distributions of quarks and gluons in the non-perturbative QCD vacuum. Physically, this means that vacuum quarks and gluons have nonzero mean-squared momentum, called virtuality. In this paper we study the quark virtuality which is given by the ratio of the local quark-gluon mixed vacuum condensate to the quark local vacuum condensate. The two vacuum condensates are obtained by solving Dyson-Schwinger Equations of a fully dressed quark propagator with an effective gluon propagator. Using our calculated condensates, we obtain the virtuality of quarks in the QCD vacuum state. Our numerical predictions are consistent with other theoretical model calculations such as QCD sum rules, Lattice QCD and instanton models.

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I. INTRODUCTION

Quantum mechanics dictates that even "empty" space is not empty, but rather filled with quantum fluctuations of all possible kinds. In many contexts, such as in atomic physics, these vacuum fluctuations are subtle effects which can only be observed by precision experiments. In other situations, especially when interactions of sufficient strength are involved, the vacuum fluctuations can be of substantial magnitude and even "condense" into a non-vanishing vacuum expectation value of some quantum fields, called vacuum condensates. These vacuum condensates can act as a medium[1], which influences the properties of particles propagating through it.

An important example of such a vacuum condensate is the Higgs vacuum expectation value, which is introduced in the Standard Model of particle physics to generate the masses of quarks, leptons, and the gauge bosons (W±, Z0) of the weak interaction. The vacuum expectation value of the Higgs field, ⟨φ⟩ = 246GeV, is uniquely determined in the Standard Model. The quark and lepton masses differ from one another only due to the different strength of the coupling of each fermion to the Higgs field. At the same time, the quark masses also receive additional contributions from the quark and gluon condensates in the QCD vacuum. In fact, the contribution of the QCD vacuum condensates to the masses for the three light quarks (u,d,s) considerably exceed the mass believed to be generated by the Higgs field[2].

The non-vanishing value of chiral quark vacuum condensates signals the spontaneous breaking of chiral symmetry in QCD, and quantitatively it is related to the pseudo-Goldstone bosons mass spectrum[3]. Due to non-perturbative effects of QCD, the vacuum of QCD has a nontrivial structure. The vacuum condensates are very important in the elucidation of the QCD structure and in description of hadron properties. If the vacuum acts as a medium and influences the properties of fundamental particles and their interactions, its properties can conceivably change. This idea has important implications in many aspects of physics.

The non-perturbative vacuum of QCD is densely populated by long-wave fluctuations of quark and gluon fields. The order parameters of this complicated state are characterized by the vacuum matrix elements of various singlet combinations of quark and gluon fields, such as

\[ \langle 0 | : \bar{q} q : | 0 \rangle, \quad \langle 0 | : \bar{q} i\gamma_{\mu}\sigma_{\mu\nu}G^{a}_{\mu\nu}\frac{\lambda^{a}}{2}| q : | 0 \rangle, \]

\[ \langle 0 | : \bar{q} \gamma_{\mu}\frac{\lambda^{a}}{2}\gamma_{\mu}\frac{\lambda^{a}}{2} q : | 0 \rangle, \]

where q is a quark field, and λa are the Gell-Mann matrices.
which are called vacuum condensates of QCD, where
\[ q(x) \] is the quark field, \( G_{\mu \nu}^a \) represents the gluon field strength tensor with \( a \) being color index (\( a = 1, \cdot \cdot \cdot , 8 \)), and can be expressed as
\[
G_{\mu \nu}^a(x) = \partial_\mu A_\nu^a(x) - \partial_\nu A_\mu^a(x) + g_s f^{abc} A_\mu^b(x) A_\nu^c(x),
\]
where \( \lambda^a \) in expression (1) is the \( SU(3) \) Gell-Mann matrix, \( f^{abc} \) represent the \( SU(3) \) structure constants, and \( g_s \) in Eq.(2) is the coupling constant related to the so-called QCD running coupling constant \( \alpha_s \) by \( \alpha_s(Q) = \frac{\alpha_s^2(Q)}{4\pi} \). \( A_\mu^a \) is the gluon field, \( \sigma_{\mu \nu} = \frac{i}{2} (\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu) \) in Euclidean space with \( \gamma_\mu \) being a Dirac Matrix.

In QCD by condensates we mean the vacuum mean values \( \langle 0 | O_1 | 0 \rangle \) of the local operators \( O_1(x) \), which arise due to non-perturbative effects. The latter point is very important and needs clarification. When determining vacuum condensates one implies the averaging only over non-perturbative fluctuations. If for some operators \( O \) the non-zero vacuum mean value appears also in the perturbative theory, it should not be taken into account in determination of the condensate. In other words, when determining condensates the perturbative vacuum mean values should be subtracted in calculation of the vacuum averages.

Separation of perturbative and non-perturbative contributions to the quark propagator, \( S_q(x) \), has some arbitrariness. For the nonperturbative propagator, defined in Sect. II, one makes an expansion of vacuum expectation values involving antiquark-quark fields that would vanish in a perturbative vacuum, the local vacuum condensates. The nonzero local quark vacuum condensate \( \langle 0 | : \bar{q}(0)q(0) : | 0 \rangle \) is responsible for the spontaneous breakdown of chiral symmetry. The nonzero local gluon vacuum condensate \( \langle 0 | : G_{\mu \nu}^a G_{\mu \nu}^a : | 0 \rangle \) defines the mass scale of hadrons through the trace anomaly[4].

The non-local vacuum condensates \( \langle 0 | : \hat{q}(x)q(0) : | 0 \rangle \) describe the distribution of quarks in the nonperturbative vacuum[5]. Physically, this means that vacuum quarks have a nonzero mean-squared momentum called virtuality. Indeed, the quark average virtuality is connected with the vacuum expectation values[6,7,8] which will be discussed in the Sect. III.

Studying the quark virtuality is of paramount importance for present day particle and nuclear physics, since it is not only related to the property of QCD vacuum states but also to quark vacuum condensates. In this work, we study the quark virtuality in the QCD vacuum by calculating quark and gluon vacuum condensates. The quark vacuum condensates are obtained by solving Dyson-Schwinger Equations (DSEs)[9] of fully dressed quark propagators with an effective gluon propagator under the constraints of an Operator Product Expansion (OPE)[10]. In Sect. II, we briefly introduce the DSEs for a fully dressed quark propagator and formulæ of various quarks vacuum condensates. In Sect. III the quark virtuality is defined; and the corresponding formulism of the virtuality is also derived in this section. In Sect. IV, our numerical results on the quark virtuality are presented. Our concluding remarks of this study are given in Sect. V.

\section{DSES for Quark Propagator}

To study quark virtuality, we need to know two vacuum condensates, and quark gluon mixed vacuum condensates. Therefore, we begin with a study of the quark propagators, which determine various quark condensates and quark gluon mixed vacuum condensates under the OPE constraints. The quark propagator is defined by
\[
S_q(x) = \langle 0 | T[q^a(x)\bar{q}^b(0)] | 0 \rangle
\]
where \( q^a(x) \) \( (\bar{q}^b(x)) \) is a quark field with color \( a \) \( (b) \), and \( T \) is the time-ordering operator. The fully dressed quark propagator in Eq.(3) can be decomposed into a perturbative part and a nonperturbative part. In other words, one can write the quark propagator[11,12] as
\[
S_q(x) = S_q^{PT}(x) + S_q^{NP}(x),
\]
where, expanding in the quark mass \( m_f \),
\[
S_q^{PT}(x) = \left( \frac{1}{2\pi^2} \frac{\gamma \cdot x}{x^4} - \frac{m_f}{2\pi^2 x^2} \right) \delta^{ab} + \cdots,
\]
and
\[
S_q^{NP}(x) = -\frac{1}{12} \langle [0 | : \bar{q}(x)q(0) : | 0 \rangle \\
+ \gamma_\mu(0 | : \bar{q}(x)\gamma^\mu q(0) : | 0 \rangle \rangle + \cdots,
\]
in configuration space, with a sum over color. For short distances, the Taylor expansion of the scalar part of \( S_q^{NP}(x) \), \( \langle 0 | : \bar{q}(x)q(0) : | 0 \rangle \), reads
\[
\langle 0 | : \bar{q}(x)q(0) : | 0 \rangle = \langle 0 | : \bar{q}(0)q(0) : | 0 \rangle \\
- \frac{x^2}{4} \langle 0 | : \bar{q}(0)[ig_\sigma G(0)]q(0) : | 0 \rangle + \cdots.
\]
In Eq.(7) the local operators of the expansion are the local quark vacuum condensates, the quark-gluon mixed condensate, and so forth.

An important observation is that the inverse quark propagator in momentum space can also be written in Euclidean space as

\[ S_f^{-1}(p) = i\slashed{p} \cdot A_f(p^2) + B_f(p^2), \]

which is renormalized at space - like point \( \mu^2 \) according to \( A_f(\mu^2) = 1 \) and \( B_f(\mu^2) = m_f(\mu^2) \), with \( m_f(\mu^2) \) being the current quark mass at renormalization point \( \mu^2 \). The subscript \( f \) in \( A_f \) and \( B_f \) stands for quark flavor \( u,d \) and \( s \).

Except for the current quark mass and perturbative corrections, the functions \( A_f(p^2) \) and \( B_f(p^2) \) are non-perturbative quantities which we refer to as the vector and scalar propagator condensates, respectively. The DSEs (in the Feynman gauge) satisfied by \( A_f \) and \( B_f \) can then be written as the set of coupled equations\(^{[6,13]}\),

\[ [A_f(s) - 1]s = \frac{1}{3\pi^3} \int_0^\infty s' ds' \int_0^\pi \sin^2 x D(s, s') \sqrt{s's'} A_f(s') \cos x \frac{2}{s' A_f^2(s') + B_f^2(s')} dx, \]

\[ B_f(s) = \frac{2}{3\pi^3} \int_0^\infty s' ds' \int_0^\pi \sin^2 x D(s, s') B_f(s') \frac{B_f(s)}{s' A_f^2(s') + B_f^2(s')} dx, \]

where \( s = p^2 \) and \( g_s^2 D(s, s') = g_s^2 D(s + s' - 2\sqrt{s's' \cos x}) \) is the dressed gluon propagator. Now, the task is to solve this set of coupled equations, Eqs. (9,10), and get the solutions \( A_f(s) \) and \( B_f(s) \).

One can solve the two coupling integral equations, Eqs. (9,10), using an effective gluon propagator such as

\[ g_s^2 D_{a\mu}^{ab}(q) = \delta^{ab} \delta_{\mu
u} g_s^2 D(q) = \delta^{ab} \delta_{\mu
u} \frac{4\pi\alpha(s)}{s}, \]

where \( \alpha(s) \) stands for quark-quark interaction which can be, for example, well approximated\(^{[10]}\) by

\[ \alpha(s) = 3\pi s \frac{\chi^2}{4s^2} \left[ e^{-s/\Delta} + \frac{\pi d}{\ln(s/A^2 + \epsilon)} \right]. \]

\( \chi \) in Eq. (12) is the strength of the interaction, and \( \Delta \) is its range parameter. The first term of Eq.(12) simulates the infrared enhancement and confinement, and the second term matches to the leading log renormalization group results. The parameter \( \epsilon \) can be varied in the range 1.0 ~ 2.5. We take \( \epsilon = 2.0 \) in the present calculations. The strength parameter \( \chi \) and the parameter \( \Delta \) are determined by fitting the solutions of DSEs to the pion decay constant\(^{[12]}\), and they are listed in table 1.

Table 1. Values of the strength parameter \( \chi \) and range parameters \( \Delta \) of the quark-quark interaction used in our present calculations.

| Set no. | Range \( \Delta \) | Strength \( \chi \) |
|---------|-------------------|-------------------|
| Set 1   | 0.40 GeV\(^2\)   | 1.84 GeV          |
| Set 2   | 0.20 GeV\(^2\)   | 1.65 GeV          |
| Set 3   | 0.02 GeV\(^2\)   | 1.50 GeV          |

The QCD scale parameter \( \Lambda \) and the value of \( d \) with the flavor number \( N_f = 3 \) are given by

\[ \Lambda = 0.2 \text{GeV}, \quad d = 12/(33 - 2N_f) = 12/27 \quad (13) \]

The non-local quark vacuum condensate \( \langle 0 | : \bar{q}(x)q(0) : | 0 \rangle \) is then given by the scalar part of Fourier transformed inverse quark propagator\(^{[12,13]}\),

\[ \langle 0 | : \bar{q}(x)q(0) : | 0 \rangle = (-4N_c) \int \frac{d^4p}{(2\pi)^4} \frac{B_f(p^2) e^{-ipx}}{p^2 A_f^2(p^2) + B_f^2(p^2)} \]

\[ = -\frac{3}{4\pi^2} \int_0^{\infty} ds s A_f^2(s) + B_f^2(s) \frac{2J_1(\sqrt{s x^2})}{\sqrt{s x^2}} \quad (14) \]

where the color number \( N_c = 3 \). Using the expansion of the \( J_1 \) Bessel function, \( 2J_1(\sqrt{s x^2})/\sqrt{s x^2} = 1 - s x^2/8 + \ldots \), and the definitions of the condensates given in Eq(7), one finds that the quark condensate is

\[ \langle 0 | : \bar{q}(0)q(0) : | 0 \rangle = -\frac{3}{4\pi^2} \int_0^{\infty} ds s B_f(s) A_f^2(s) + B_f^2(s) \quad (15) \]

while the local quark - gluon mixed vacuum condensate, \( \langle 0 | : \bar{q}(0)[ig_s \sigma G(0)]q(0) : | 0 \rangle \) is

\[ \langle 0 | : \bar{q}(0)[ig_s \sigma G(0)]q(0) : | 0 \rangle = -\frac{3}{8\pi^2} \int_0^{\infty} ds s B_f(s) A_f^2(s) + B_f^2(s) \quad (16) \]

Note that the upper limit of the integrals over \( s \) for the DSEs, Eqs(9,10), is infinity, while for the condensates, Eqs(15,16), there is a finite limit, \( s_o \). That is because the effective gluon propagator, Eqs(11,12),
provides a natural cutoff for the DSEs integrals, while the range for the condensates is given by the renormalization point, \( \mu \). For light quarks perturbative QCD begins to dominate nonperturbative QCD at about 3-4 GeV, so a renormalization point of \( \mu^2 = 10 \text{ GeV}^2 \) is expected. Therefore, we use \( s_o = 10 \text{ GeV}^2 \). This is explained in detail in Sect. IV.

A different derivation of quark-gluon mixed vacuum condensate explicitly used a form for the nonlocal quark condensate \( \langle 0 | \bar{q}(x)q(0) : 0 \rangle = g(x^2)(0) = g(0)q(0) : 0 \) (see Ref.[12]), and used a model for \( g(x) \) to derive an expression similar to that obtained in the model used in Ref.[14], but with very different results for the mixed quark-gluon condensate. In Sect. IV, we use the result for \( g(x) \) from Ref.[12] to estimate errors in our estimate for \( \lambda_q^2 \), defined in the following section.

Eqs.(15) and (16) will produce our numerical predictions of local quark and quark gluon mixed vacuum condensates, which will be then used to estimate the nonzero mean squared momentum of quarks in the non-perturbative QCD vacuum.

### III. NONZERO MEAN SQUARED MOMENTUM OF QUARK IN QCD VACUUM

The quantities \( f(\nu) \) were introduced to represent nonlocal condensates[15,16]. Their explicit form completely determines the coordinate dependence of the condensates, and describes the virtuality distribution of quarks in the non-perturbative vacuum. Its \( n \)-th moment is proportional to the vacuum expectation value of the local operator with the covariant derivative squared \( D^2 \) to the \( n \)-th power[17]:

\[
\int_0^\infty \nu^n f_q(\nu) d\nu = \frac{1}{\Gamma(n + 2)} \frac{\langle 0 | \bar{q}(D^2)^n q : 0 \rangle}{\langle 0 | \bar{q}q : 0 \rangle},
\]

where the covariant derivative \( D_\mu = \partial_\mu - ig_s A_\mu \), with \( A_\mu = A_\mu^a \lambda^a / 2 \) and \( \lambda^a \) is a SU(3) Gell-Mann matrix. It is natural to suggest that the vacuum expectation values in the right-hand side of Eq. (17) should exist for every \( n \). The two lowest moments \( (n = 0, n = 1) \) give the normalization condition \( (n = 0) \) and the average vacuum virtuality of quarks \( (n = 1) \), \( \lambda_q^2 \), respectively

\[
\int_0^\infty f_q(\nu) d\nu = 1,
\]

and

\[
\int_0^\infty \nu f_q(\nu) d\nu = \frac{1}{2} \frac{\langle 0 | \bar{q}D^2 q : 0 \rangle}{\langle 0 | \bar{q}q : 0 \rangle} = \frac{\lambda_q^2}{2}.
\]

To illustrate the definition of quark virtuality in the non-perturbative vacuum, \( \lambda_q^2 \), let us now consider the Taylor expansion of the simplest gauge invariant condensate

\[
\langle 0 | \bar{q}(0) E(0, x; A)q(x) : 0 \rangle \equiv \langle 0 | \bar{q}(0)q(x) : 0 \rangle
\]

\[
= \sum_{n=0}^\infty \frac{1}{n!} x_{\mu_1} \cdots x_{\mu_n} \langle 0 | \bar{q}D^{\mu_1} \cdots D^{\mu_n} q : 0 \rangle
\]

\[
= \langle 0 | \bar{q}q : 0 \rangle + \frac{x^2}{8} \langle 0 | \bar{q}D^2 q : 0 \rangle + \cdots ,
\]

where \( E = P \exp[i \int_x^y A_\mu(z) dz^\mu] \) is the path-ordered Schwinger phase factor (the integration is performed along the straight line) required for gauge invariance and \( A_\mu(z) = A_\mu^a(z) \lambda^a / 2 \).

The quantity \( \lambda_q^2 \), defined in Eq.[19],

\[
\lambda_q^2 = \frac{\langle 0 | \bar{q}D^2 q : 0 \rangle}{\langle 0 | \bar{q}q : 0 \rangle},
\]

was introduced for an expansion of the nonlocal quark condensate[5], which can be interpreted as the average virtuality of the vacuum quarks. Note that the operator \( \langle 0 | \bar{q}D^2 q : 0 \rangle \) can be presented in a different form:

\[
\langle 0 | \bar{q}D^2 q : 0 \rangle \equiv \langle 0 | \bar{q}D^2 D_\mu q : 0 \rangle
\]

\[
= \langle 0 | \bar{q}D^2 g_{\mu\nu} D^\nu q : 0 \rangle
\]

\[
= \langle 0 | \bar{q}D D q : 0 \rangle - \langle 0 | \bar{q}D^2 q : 0 \rangle
\]

\[
= -m_q^2 \langle 0 | \bar{q}q : 0 \rangle - \frac{1}{2} \langle 0 | \bar{q}D^2 q : 0 \rangle
\]

\[
= -m_q^2 \langle 0 | \bar{q}q : 0 \rangle + \frac{1}{2} \langle 0 | \bar{q}[i g_s G^{\mu\nu} \sigma_{\mu\nu}] q : 0 \rangle ,
\]

where \( D = \gamma_\mu D_\mu \), and we have used the identity

\[
g_{\mu\nu} = \gamma_\mu \gamma_\nu - \frac{\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu}{2} = \gamma_\mu \gamma_\nu - \sigma_{\mu\nu},
\]

the equation of motion

\[
D q(x) = -i m_q q(x),
\]

and the definition of the field strength tensor

\[
[D_\mu, D_\nu] = -i g G_{\mu\nu}.
\]

Thus, the "average virtuality" of the vacuum quarks \( \langle 0 | \bar{q}D^2 q : 0 \rangle \) is directly related to the "average vacuum gluon field strength" \( \langle 0 | \bar{q}[i g_s G^{\mu\nu} \sigma_{\mu\nu}] q : 0 \rangle \). In many papers[18] one can find the notation

\[
\langle 0 | \bar{q}[i g_s G^{\mu\nu} \sigma_{\mu\nu}] q : 0 \rangle \equiv m_q^2 \langle 0 | \bar{q}q : 0 \rangle.
\]
Using Eqs. (21,22,26), one can write the vacuum quark virtuality \( \lambda_q^2 \) as

\[
\lambda_q^2 = \frac{m_q^2}{2} - m_q^2.
\]

Namely,

\[
\lambda_q^2 = \frac{1}{2} \langle 0 | \hat{q} \hat{g} G^{\mu \nu} \sigma_{\mu \nu} q | 0 \rangle - \frac{m_q^2}{2}.
\]

For light quarks, the mass \( m_q^2 \) term is so small that it can be neglected. Finally, we arrive at

\[
\lambda_q^2 \sim \frac{1}{2} \langle 0 | \hat{q} \hat{g} G^{\mu \nu} \sigma_{\mu \nu} q | 0 \rangle.
\]

Eq. (29) is starting point of our calculations on quark virtuality. As we see from Eq. (29), in order to get \( \lambda_q^2 \) we have to calculate quark-gluon mixed vacuum condensate \( \langle 0 | \hat{q} \hat{g} G^{\mu \nu} \sigma_{\mu \nu} q | 0 \rangle \) and two quark vacuum condensate \( \langle 0 | \hat{q} q | 0 \rangle \) by the use of Eqs. (15) and (16).

IV. NUMERICAL RESULTS AND ERROR ESTIMATE

Using the solutions of DSEs of Eqs. (9,10), \( A_f \) and \( B_f \), with three different sets of the quark-quark interaction parameters given in Table 1, leads to our following theoretical predictions for the local two quark vacuum condensates, local quark-gluon mixed vacuum condensates via Eqs. (15,16). The corresponding theoretical results are listed in Table 2-3.

Table 2. The local two quark vacuum condensates of QCD, \( \langle 0 | \hat{q} q | 0 \rangle \rangle \). \( f \) stands for quark flavor and \( \mu \) denotes renormalization point, \( \pi^2=10 \) GeV^2.

| Set No. | \( \langle 0 | \hat{q} q | 0 \rangle \rangle \) | \( \langle 0 | \hat{q} q | 0 \rangle \rangle \) |
|---------|-----------------|-----------------|
| Set 1   | -0.013(Gr)^3    | -0.071(Gr)^3    |
| Set 2   | -0.0078(Gr)^3   | -0.068(Gr)^3    |
| Set 3   | -0.0027(Gr)^3   | -0.065(Gr)^3    |

Table 3. The local quark-gluon mixed vacuum condensates, \( \langle 0 | \hat{q} \hat{g} G q | 0 \rangle \). \( f \) stands for quark flavor and \( \mu \) denotes renormalization point, \( \pi^2=10 \) GeV^2.

| Set No. | \( \langle 0 | \hat{q} \hat{g} G q | 0 \rangle \rangle \) | \( \langle 0 | \hat{q} \hat{g} G q | 0 \rangle \rangle \) |
|---------|-----------------|-----------------|
| Set 1   | -0.015(Gr)^5    | -0.186(Gr)^5    |
| Set 2   | -0.010(Gr)^5    | -0.189(Gr)^5    |
| Set 3   | -0.0078(Gr)^5   | -0.193(Gr)^5    |

Our results for two quark local vacuum condensates are consistent with the predictions by Gall-Mann-Oakes-Renner relation (GMOR)\(^{19,20}\), \( (m_u + m_d)(0 | \hat{q} q | 0) = -\frac{1}{2} m_\pi f_\pi^2 \), where \( m_u \) and \( m_d \) are current quark masses with value of \( m_u = 9.7 \) MeV\(^{20}\), and \( m_\pi = 140 \) MeV, \( f_\pi = 93 \) MeV are the mass and decay constant of a pion, respectively. Substituting these values into the GMOR relation produces \( (0 | \hat{q} q | 0) = -0.0087 \) GeV\(^3\), which is reasonably consistent with the u,d quark condensates shown in Table 2, within errors.

Our theoretical results in Table 2 and 3 are also consistent with the empirical values used widely in QCD sum rules\(^{21}\), with \( (0 | \hat{q} q | 0) \rangle \rangle_u,d \sim -0.013 \) GeV\(^3\), as we obtained with Set 1. We also obtained a result consistent with the predictions of Lattice calculations\(^{22}\).

Using Eq.(29) and our numerical predictions of the two quark local vacuum condensates, and the local quark-gluon mixed vacuum condensates given respectively in tables 2 and 3, the quark virtuality, the nonzero mean squared momentum of quarks in non-perturbative QCD vacuum state, is given by

\[
\lambda_u^2 = \frac{1}{2} \langle 0 | \hat{q}(0) \hat{g} \sigma_{\mu \nu} G^\mu_\nu \sigma_{\mu \nu} \hat{q}(0) | 0 \rangle_{u,d} \]

\[
= 0.57 \text{ GeV}^2,
\]

for u, d quark, Set 1.

For s quark, we obtained

\[
\lambda_s^2 = \frac{1}{2} \langle 0 | \hat{q}(0) \hat{g} \sigma_{\mu \nu} G^\mu_\nu \sigma_{\mu \nu} \hat{q}(0) | 0 \rangle_s
\]

\[
= 1.31 \text{ GeV}^2,
\]

for Set 1.

In the summary of this section, our predictions of \( \lambda_{u,d}^2 \) and \( \lambda_s^2 \) with three different sets of quark-quark interaction parameters given in table 1 are listed in Table 4.

Table 4. The virtualities of three light quarks u, d and s in the QCD vacuum state.

| Set No. | \( \lambda_{u,d}^2 \) (GeV)^2 | \( \lambda_s^2 \) (GeV)^2 |
|---------|-----------------|-----------------|
| Set 1   | 0.57            | 1.31            |
| Set 2   | 0.68            | 1.38            |
| Set 3   | 1.43            | 1.48            |

All our theoretical results are in an acceptable range\(^{23}\) of \( \lambda_s^2 \) between 0.4 ~ 2.50 GeV^2. For example, for u and d quarks the standard QCD sum rule
estimation\textsuperscript{[24]} gives $\lambda_{u,d}^2 = 0.4 \pm 0.1 \text{GeV}^2$, the QCD sum rule analysis of pion form factor\textsuperscript{[25]} produces $\lambda_{u,d}^2 = 0.70 \text{GeV}^2$, and Lattice QCD calculations\textsuperscript{[26]} predicts $\lambda_{u,d}^2 = 0.55 \text{GeV}^2$. For $s$ quark, Lattice QCD \textsuperscript{[26]} gives $\lambda_s^2 = 2.50 \text{GeV}^2$, and the instanton model prediction\textsuperscript{[27]}, $\lambda_s^2 = 1.40 \text{GeV}^2$. The $\lambda_{u,d}^2$ is 1.43 for set 3 is larger than that for set 1 (0.57) and set 2 (0.68). The reason is that the range parameter $\Delta$ is an order of magnitude smaller. Therefore we observe that all our predictions are in a good agreement with the calculations cited by Refs. \textsuperscript{[23-27]}, but our method of calculation is quite different from others. However, it should be also pointed out that both the condensates and virtualities depend on renormalization point $\mu^2$. We discuss this dependence and estimated error of $\lambda_s^2$ in the following.

A. Dependence of $\lambda_s^2$ on $\mu^2$

As it has been mentioned in Sect. I, in QCD by condensates we mean the vacuum mean values $\langle 0 | O_i | 0 \rangle$ of the local operators $O_i$, which arise due to non-perturbative effects. When determining vacuum condensates one implies the averaging only over non-perturbative fluctuations. If for some operators $O_i$ the non-zero vacuum mean value appears also in perturbation theory, it should not be taken into account in the determination of the condensates. In other words, when determining condensates the perturbation vacuum mean values should be subtracted in calculation of the vacuum averages.

Separation of perturbation and non-perturbation contribution into vacuum mean values has some arbitrariness. Usually, this arbitrariness is avoided by introducing some renormalization point $\mu^2$. Integration over momenta of virtual quarks and gluons in the region below $\mu^2$ is referred to condensates, above $\mu^2$ is referred to perturbation theory. In such a formulation condensates depend on the renormalization point $\mu^2$, $\langle 0 | O_i | 0 \rangle_{\mu^2}$. Therefore, $\lambda_{s}^{2}$ depends on the renormalization point $\mu^2$, which we choose as 10 GeV$^2$ as explained in the paragraph following Eqs(15,16). This is consistent with our conclusions regarding our current calculations.

B. Estimate of Errors in $\lambda_s^2$

Our values of $\lambda_s^2$ for the u,d and s quarks are given by the quark condensate, $\langle 0 | : \bar{q}(0)q(0) :) | 0 \rangle$, and the quark-gluon mixed vacuum condensate, $\langle 0 | : \bar{q}(0)G_{\mu\nu}\sigma_{\mu\nu}q(0) :) | 0 \rangle$. The quark condensate is known to about 10%, but as discussed in Sect. II, approximations and models are needed to estimate the quark-gluon mixed condensate, and there can be some errors, depending on the model used.

Although it is difficult to give an accurate estimate of the errors, from the results for the condensates and virtualities given in Tables 2, 3, and 4 one sees that the virtuality is estimated to about a factor of two. Since the parameters for Sets 1, and 2 given in Table 1 are more reasonable than Set 3, one may estimate from Table 4 that our final results for the virtuality are within about 20%.

V. CONCLUDING REMARKS

We study the quark virtuality in the QCD vacuum based on the fully dressed confined quark propagator described by DSEs. The quark virtuality is determined by the ratio of the local quark-gluon mixed vacuum condensates $\langle 0 | : \bar{q}ig_{\mu\nu}\sigma_{\mu\nu}q :) | 0 \rangle$ to local quark vacuum condensates $\langle 0 | : \bar{q}q :) | 0 \rangle$. The local quark vacuum condensate and local quark gluon mixed vacuum condensate are obtained by solving the Dyson-Schwinger Equations in the “rainbow” approximation with an effective gluon propagator in Euclidean space and the Feynman gauge. The effective gluon propagator consists of two terms with two parameters: the strength of interaction $\chi$ and its range $\Delta$. The first term of the gluon propagator simulates the infrared enhancement and confinement, and the second term matches to the leading log renormalization group results. Our calculated results of local quark vacuum condensates and quark gluon mixed vacuum condensate are in good agreement with other theoretical model predictions such as QCD sum rules\textsuperscript{[24,25]}, Lattice QCD\textsuperscript{[26]} and instanton model\textsuperscript{[27]}.

Using the numerical results of our present calculations of local quark and quark-gluon mixed vacuum condensates, the virtualities $\lambda_q$ for light quarks (u, d and s) are obtained for three different sets of parameters $\chi$ and $\Delta$. The results are given in table 4. We find numerically that the contribution from the second term of gluon effective propagator in $g_{\mu\nu}^{2}D_{\mu\nu}$, Eq. (12), can be neglected. The dominant contribution to quark virtuality is from the first term of Eq. (12).

In conclusion, we predict the quark virtuality using a different method: solving DSEs, and using its numerical solutions $A_f$ and $B_f$. Our theoret-
ical results are consistent with all calculations of QCD sum rules, Lattice QCD and instanton models. However, it should be noticed that the predictions depend on renormalization point $\mu^2$, the separation between perturbative and non-perturbative part of QCD, since the vacuum condensate average only over non-perturbative vacuum fluctuations, and the perturbative contribution must be subtracted from any calculations. The detailed discussion on $\mu^2$ dependence will be published in our forthcoming paper. We believe this study is very important for investigation of QCD vacuum properties, and has many important applications both in particle physics and in nuclear physics.

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