A cascade of e\(^{-}\)e\(^{+}\) pair production by a photon with subsequent annihilation to a single photon in a strong magnetic field

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Abstract
The process of electron–positron pair production by a photon with subsequent annihilation to a single photon in a strong magnetic field has been studied. The general amplitude has been calculated and the process rates have been found in a low Landau levels approximation (resonant and nonresonant cases). The comparison of resonant and nonresonant cases shows a significant excess of the resonant rate. The polarization of the final photon in a strong magnetic field has also been found. It has been shown that polarizations of the initial and final photons are independent except for the case of normal linear polarization of the initial photon.

Keywords: electron–positron pair, resonance, strong magnetic field

(Some figures may appear in colour only in the online journal)

1. Introduction
The process of photon propagation in a strong external electromagnetic field remains topical, despite an extensive literature on this subject [1–15].

The first theory of this process has been developed in [1–3] using polarization operator methods, and the refractive index of polarized vacuum has been found. In an external electromagnetic field, vacuum becomes an optically active medium, and an effect similar to light birefringence can be observed. In works [4–8] considerable attention was paid to vacuum polarization and the singularity of the polarization operator in a constant magnetic field. Also, the influence of vacuum polarization on the propagation and absorption of electromagnetic waves near pulsars was considered.

It should be emphasized that in the resonant case the intermediate electron–positron pair can still annihilate to the final photon instead of producing a real electron–positron pair. However, the divergence of the polarization operator is usually associated only with one-photon pair creation. The cascade process of pair production with subsequent annihilation requires more detailed study. A similar process of production of an electron–positron pair and successive annihilation has also been recently considered in a plane-wave field in [16]. Also, there is another resonance in the problem, which is the result of the formation of a bound state between the created electron and positron [14, 15].

It should be noted that the resonant divergences are typical for two vertex QED processes in an external electromagnetic field. In this case, the intermediate particle goes to the mass shell [17–27].

In this work the process of electron–positron pair production by a photon with subsequent annihilation to a single photon in a magnetic field is studied using diagram technique. Magnetic field is considered to be comparable with the critical Schwinger one, \( H_c = m^2 c^3 / e \hbar = 4.41 \cdot 10^{13} \text{G} \). Such fields are believed to exist in neutron stars [11–13]. Subcritical magnetic field can be generated in heavy ion collisions with impact parameter comparable to the Compton wavelength of an electron [28]. The process rate has been found in the low Landau levels (LLL-) approximation and the polarization degree of the final photon has been considered in resonant and nonresonant cases. It should be noted that the process rate contains the divergence (Dirac delta-function), since there is a
2. The process amplitude

Let us consider the propagation of an arbitrary polarized photon in a uniform magnetic field. The Landau gauge of electromagnetic 4-potential is chosen, which is (0,0, xH, 0).

The Feynman diagram of the process is shown in figure 1. The wavy line is the photon wave function and the internal double lines are the Green’s functions of the virtual electron and the positron in a magnetic field.

The wave function of a photon has a standard form (we will use natural units, where \( h = c = 1 \), throughout) [31]:

\[
A_\mu = \frac{2\pi}{\sqrt{\omega V}} e^{i k \cdot x} e^{-i \omega t},
\]

where \( V \) is the normalization volume and \( e_\mu = (0, e) \) is the polarisation 4-vector of the photon:

\[
\bar{e} = \begin{pmatrix}
\cos \phi \cos \theta \cos \alpha - e^{i \beta} \sin \phi \sin \alpha \\
\sin \phi \cos \theta \cos \alpha + e^{i \beta} \sin \phi \sin \alpha \\
- \sin \theta \cos \alpha
\end{pmatrix}.
\]

Here, \( \phi \) and \( \theta \) are the azimuthal and polar angles, \( \alpha \) and \( \beta \) are the parameters that determine photon polarization.

The Green’s function of an electron in a magnetic field has the following form [22–27]

\[
G(x,x') = \frac{-m\sqrt{h}}{(2\pi)^3} \int d^3k e^{-i k \cdot x} \sum_n \frac{G_{\text{He}}(x,x')}{\bar{g}_n - E_n},
\]

where \( h \) is magnetic field strength in the units of the quantum of critical magnetic field of \( H_\gamma \), \( \gamma \) are the Dirac gamma matrices, \( U_n \) is the Hermite function, \( \rho(x) = m\sqrt{h} x + g_\mu mH \) is the argument of \( U_n \) and the primed functions in the formula (4) depend on \( x' \),

\[
\Phi = g_0(t - t') - g_1(y - y') - g_2(z - z'),
\]

\[
\tau = \frac{1}{2}(1 + i\gamma_2\gamma),
\]

\[
P = (g_0,0,0,g_\mu).
\]

According to standard rules of quantum electrodynamics [31], the amplitude of the process of photon propagation in vacuum in the presence of a magnetic field can be written as

\[
S_{\text{ii}} = e^2 \int d^3x d^3x' \text{Tr}[\mathcal{A}_1^+ \gamma_\mu \mathcal{A}_2 \gamma_\mu \mathcal{A}_2^+ \times \mathcal{A}_1^+],
\]

where the primed wave function of the final photon depends on \( x' \).

Taking into account the expressions for the Green’s functions (3) and the photon wave functions (1) and carrying out corresponding mathematical transformations, it is easy to obtain the following expression for the general amplitude of the process,

\[
S_{\text{ii}} = -\frac{4\pi e^2\delta(k - k')}{V\omega} \sum_{n_p,n_q \to 0} \int d\theta d\phi d\phi' d\phi'' \frac{e^{i\theta' \phi'' - i\theta \phi} B_{j_p}}{(g_0 - E_\gamma)^2 (f_{j_p} - E_\gamma^2)}.
\]

where \( B_{j_p} \) have the form:

\[
B_1 = e_1 e_2 (J_{n_1,n_2} + J_{n_1,n_2}^*) (m^2 + P_f),
\]

\[
B_2 = (T_1^+ T_2^+ J_{n_1,n_2} + T_1^- T_2^- J_{n_1,n_2}^*) (m^2 + P_f),
\]

\[
B_3 = - 2n_1 \hbar m g_\mu J_{n_1,n_2,n_1,n_2} (T_{n_1}^+ e_2 + e_1 T_{n_2}^+)
\]

\[
\quad \quad \quad + J_{n_1,n_2,n_1,n_2} (e_1 T_{n_2}^+ + T_{n_1}^+ e_2),
\]

\[
B_4 = 2n_1 \hbar m J_{n_1,n_2,n_1,n_2} (e_1 T_{n_2}^+ + T_{n_1}^+ e_2)
\]

\[
\quad \quad \quad + J_{n_1,n_2,n_1,n_2} (e_1 T_{n_2}^+ + T_{n_1}^+ e_2),
\]

\[
B_5 = 2n_1 \hbar \mu J_{n_1,n_2,n_1,n_2} (T_{n_1}^+ e_2 + e_1 T_{n_2}^+),
\]

\[
B_6 = 4e_1 e_2 \hbar m^2 J_{n_1,n_2,n_1,n_2} (T_{n_1}^+ e_2 - T_{n_2}^+ e_2).
\]

Here,

\[
\Phi' = \frac{k_x' - k_x - g_\mu}{2hm^2} + \frac{g_\mu(k_y' - k_y)}{2hm^2} + (n_q - n_f)(\phi' - \phi),
\]

\( n_{\phi,f} \) are the Landau levels of the intermediate particles, \( e_{1,2} \) are the components of the polarization vectors of the photons parallel to the field, \( J_{n_1,n_2} \) are the known functions appearing in quantum electrodynamics in a magnetic field [22–30]. The following notation is introduced:

\[
P' = (g_0,0,0,-g_\mu),
\]

\[
T_{1,+}^\pm = \cos \theta \cos \alpha \pm ie^{i\beta} \sin \alpha,
\]

\[
T_{2,+}^\pm = \cos \theta' \cos \alpha' \pm ie^{i\beta'} \sin \alpha'.
\]

Note that the functions \( T_{1,2}^\pm \) determining the polarization properties of photons coincide with the similar functions in the process of two-photon electron–positron pair production in a magnetic field [26].

After carrying out the integration (9) over \( d\phi, d\phi' \), the amplitude of the process has the form

\[
P'(t) = (g_0,0,0,-g_\mu)(1 \pm ie^{i\beta} \sin \alpha),
\]

\[
T_{1,2}^\pm = \cos \theta \cos \alpha \pm ie^{i\beta} \sin \alpha.
\]
Note that the amplitude contains $\delta^4(k - k')$. In other words, in the process considered both energy and momentum are conserved, despite the presence of a magnetic field. It should be emphasized that this is not the case for most QED processes in a magnetic field, and the amplitude normally contains only 3-delta function ($\delta(k - k')$ is absent for Landau gauge of the potential).

In the process considered the dispersion law is satisfied for both the initial and final photons, $k^2 = 0$ and $k'^2 = 0$. Taking into account the 3-delta functions $\delta^4(k - k')$, it is easy to find that $k_z = \pm k'_z$. The case $k_z = -k'_z$ should be rejected as non-physical, since it corresponds to reflection of the photon regardless of the magnetic field strength and the photon frequency.

Hereinafter, without loss of generality, we will use a system of reference in which the photon momentum parallel to the field is absent,

$$k_z = 0,$$  

(14)

since the Lorentz transformation along $\mathbf{H}$ does not change the magnetic field.

It is known that the amplitude of this process diverges. Therefore, it is necessary to carry out the procedure of regularization or renormalization. We will use the Bogoliubov regularization method [32]. Specifically, the denominator in the expression for the Green’s function should be replaced as follows:

$$(g, j)^2_0 - E^2_{g,f} + i\epsilon)^{-1} \rightarrow (g, j)^2_0 - E^2_{g,f} + i\epsilon)^{-1} - (g, j)^2_0 - E^2_{g,f} - M^2 + m^2 + i\epsilon)^{-1},$$  

(15)

where $M$ is the additional mass. In the Bogoliubov representation [32] (which is similar to the Schwinger’s proper-time method [33]), the integrations in (9) can be performed using the relation

$$\frac{1}{s^2_0 - E^2_{g,f} + i\epsilon} = \frac{1}{i} \int_0^\infty d\zeta e^{i(s^2_0 - E^2_{g,f} + i\epsilon)\zeta}. $$  

(16)

After carrying out the procedure of regularization, the general amplitude of the process is given by the formula

$$S_{\text{fi}} = \frac{8\pi^2 e^2 \hbar m^2 \delta^4(k - k')}{V\omega} \times \sum_{n_p, n_f = 0}^{\infty} \int_0^1 d\zeta$$

$$\times \left( a - \frac{b}{2} \ln \frac{m^2 \zeta (1 - \zeta)}{\omega^2 (\zeta - \zeta_1) (\zeta - \zeta_2)} + \frac{m^2}{\omega^2 (\zeta - \zeta_1) (\zeta - \zeta_2) - i \frac{\epsilon}{\omega}} \right).$$  

(17)

Here, $\epsilon$ is a small positive quantity. The following notation is used:

$$a = e^2 v^2 (J^2_{n_f, n_f} + J^2_{n_f, n_f - 1, n_f - 1} + T_1 T_2^* (J^2_{n_f, n_f} + J^2_{n_f, n_f - 1})), $$  

(18)

$$b = 2T_1 T_2^* (J^2_{n_f, n_f} + J^2_{n_f, n_f - 1}), $$  

(19)

$$c(\zeta) = a + 4e^2 v^2 \hbar \sqrt{n_p n_f} J_{n_p, n_f} J_{n_f - 1, n_f - 1} + a (1 + 2n_f h) + 2haN_2 \zeta, $$  

(20)

$$\zeta_{1,2} = \frac{1}{2} - \frac{m^2}{\omega^2} hN_2 \pm \sqrt{\frac{\omega^2}{m^2} - 4(1 + N_2 h)} + 4h^2 N_2^2. $$  

(21)

3. The resonant conditions and the process rate

Let us consider the resonant case when virtual particles become real ones. In this case, both first-order poles in (17) coincide with each other:

$$\zeta_1 = \zeta_2 = \zeta_{\text{res}}. $$  

(22)

Equation (22) has two solutions, one of which is spurious. Indeed, $\zeta_{\text{res}}$ does not belong to the integration interval (17), hence the singularity in the integrand vanishes. Therefore, we will use the other solution, which has the form

$$\omega_{\text{res}} = m \left( \sqrt{1 + 2n_f h} + \sqrt{1 + 2n_f h} \right). $$  

(23)

Thus, the resonant frequency is equal to the sum of the electron and the positron energies with zero longitudinal momenta. It should be noted that the same condition was obtained in [2] when analyzing the polarization operator in the presence of a magnetic field.

In the resonant case (23) the amplitude of the process (17) takes on the form

$$S_{\text{fi}}^{\text{res}} = -\frac{i 8\pi^2 e^2 m^4}{V\omega^2 \Gamma^2} \delta^4(k - k') \sum_{n_p, n_f = 0}^{\infty} c(\zeta_{\text{res}}), $$  

(24)

where $\Gamma$ is the width of the resonance. Hereinafter, we will use LLL-approximation,

$$n_{p,f} \sim 1, \quad \hbar n_{p,f} \ll 1. $$  

(25)

These conditions are common in subcritical magnetic fields, when the field strength approaches the critical value of $H_c$. In this approximation in the first order in $h$ the resonant photon frequency can be rewritten as:

$$\omega_{\text{res}} = m (2 + N_2 h). $$  

(26)

In view of the above expressions, the resonant amplitude in the linear approximation on $h$ has the form

$$S_{\text{fi}}^{\text{res}} = \frac{-i 2\pi^2 e^2 m^2}{V\Gamma} \delta^4(k - k')\times \hat{J}^2 (v^2 [2 + h(N_2 - 2n_f h)] + hN_2 T_1 T_2^*), $$  

(27)
where
\[ J = \left( -1 \right)^{n_1} \frac{\eta}{\sqrt{m_e^2 n_1^2}} e^{-i \frac{n_1}{2} \tau^2}. \] (28)

Here, \( \eta = (k^2 + k^2) / 2 \hbar \). It should be noted that based on the formula for the amplitude (27) the optical theorem can be derived. In other words the rate of single-photon electron–positron pair production is determined by the imaginary part of the amplitude of the process studied. In the case of an unpolarized photon, the optical theorem looks like
\[ W_{\text{pair}} = -\frac{2\Gamma}{m_e^2 \delta \omega} \text{Im}(S^o_{\text{fi}}), \] (29)
where \( W_{\text{pair}} \) is the probability of the process of one-photon electron–positron pair creation (see for example [26, 29]), \( \delta \omega \) is the detuning from the threshold of the one-photon pair creation. It should be emphasized that in the formula (29) \( \delta \omega \) goes to zero if the pair is created on fixed Landau levels with zero longitudinal momenta.

It should be noted that the methods of quantum field theory (QFT) [34–36], in particular the optical theorem, can be applied to the problem of a heavy charged particle passing through a magnetized electron gas, which is related to the method of electron cooling. In this method the emittance of a heavy charged particle beam is reduced due to collisions with electrons which have a small velocity spread [37–39]. The energy loss of a charged particle in the first Born approximation is determined by the imaginary part of the polarization operator. It should be noted that despite the widespread use of the electron cooling method in acceleration technology, there is no complete theoretical description of the difference in the friction force for positive and negative particles, which is experimentally observed [40]. This problem will be important for electron cooling of antiproton beams at the high energy storage ring (HESR) [41, 42]. The difference in cooling of antiprotons and protons can be described using QFT methods in the second Born approximation.

Let us find the resonance rate of the process of photon propagation in a strong magnetic field. According to the well-known QED rules the differential rate is determined as
\[ dW = |S|^2 \frac{Y d^3 k}{(2\pi)^3}. \] (30)
The integration over \( d^3 k \) can be carried out using \( \delta \)-function properties. The rate accurate to the first corrections in \( \hbar \) can be obtained in the form
\[ W_{\text{res}} = \frac{\pi \hbar^2 m_e^4}{2 \Gamma^2} \delta(\omega - \omega') J^4 (1 + \xi_3)(1 + \xi'_3) \times [1 + (N_3 - 2n_3 n_3^*) \hbar] + (\xi_3 \xi'_3 + \xi_3 \xi_3^*) N_3 \hbar, \] (31)
where \( \xi_1,2,3 \) are the Stokes parameters of photons [43]:
\[ \xi_1 = \sin 2\alpha \cos \beta, \]
\[ \xi_2 = \sin 2\alpha \sin \beta, \]
\[ \xi_3 = \cos 2\alpha. \]

It follows from (23) that the rate is substantially dependent on the initial photon polarization and vanishes in the case of normal linear polarization (\( \xi_3 = -1 \)) within the accuracy of the approximation. Additionally, the suppression of process rate also occurs in the case \( \xi_3 = -1 \) in other processes in a magnetic field (for example, in the process of electron–positron pair by a single photon [24]). The account of the next correction in \( \hbar \) gives a small but non-zero rate,
\[ W_{\text{res}}^{(\xi_3 = -1)} = \frac{\pi \hbar^2 m_e^4}{2 \Gamma^2} \delta(\omega - \omega') J^4 N_3 (1 - \xi_3^2). \] (32)
As is known, the two vertex processes have resonant divergence. In this case, the intermediate particle goes to the mass shell and the second-order process can be represented by two subsequent first-order processes. The resonant process of photon propagation in a magnetic field can be represented as a sequence of processes of one-photon pair creation and annihilation of pair in a single photon. According to [24], in the case when the photon has normal linear polarization (\( \xi_3 = -1 \)), the rate of the process of pair production by a photon is suppressed. Thus, the rate of the process (figure 1) is also suppressed when the initial photon has normal linear polarization.

It can be concluded from expressions (23) and (32) that the rate is considerably less in the latter case. Figure (2) shows the dependence of the dimensionless process rate on the magnetic field in the units of the critical strength for various initial photon polarizations. It should be noted that the width of resonance has the form [26]
\[ \Gamma = \frac{2}{3} e^2 \hbar^2. \] (33)

4. Nonresonant case

Let us consider the process in the nonresonant case when the condition (22) is not fulfilled. In this case, the initial photon frequency looks like
where $\kappa$ belongs to the interval $(0, 1)$.

Taking into account the general amplitude and the condition (34) the nonresonant amplitude in LLL-approximation (25) takes on the form

$$S_{\text{nonres}} = \frac{\hbar \pi e^2 m^2}{V \omega \sqrt{\kappa \hbar}} \delta(k - k')J^2(s_1e_1e_2 + s_2 T_1 T_2^*),$$

(35)

where

$$s_1 = 1 + \frac{24i}{\pi} \frac{\sqrt{\kappa \hbar}}{\hbar} = -\frac{13}{8} \frac{\kappa}{\hbar} + \frac{5}{4} N_+ + n_{p_f},$$

$$s_2 = \frac{\hbar N_+}{2}.$$

Similarly to the previous section, one can find the rate of the nonresonant process:

$$W_{\text{nonres}} = \frac{\pi \hbar m^2 e^4}{2^{11} \kappa} \delta(\omega - \omega') J^3 (1 + \xi_3^2)(1 + \xi_3^2).$$

(36)

In the same way, when the initial photon has normal linear polarization the nonresonant process rate is

$$W_{\text{nonres}}^{(\xi_3=-1)} = \frac{\pi \hbar m^2 e^4}{2^{11} \kappa} \delta(\omega - \omega') J^3 N_+^2 (1 - \xi_3^2).$$

(37)

Formulas (36) and (37) show that the rate in the case of normal linear polarization of the initial photon is much less than in other cases.

According to the expressions (23) and (36), the ratio of the resonant and nonresonant rates can be found as

$$\frac{W_{\text{res}}}{W_{\text{nonres}}} = \frac{144 \kappa}{e^3 \hbar^3}.$$  

(38)

It follows from the above expressions that the process rate is substantially greater in the resonant case compared to the nonresonant one.

5. Polarization of the final photon

Let us find the polarization of the final photon when the resonance condition (22) is fulfilled. By definition, the degree of polarization of the final photon can be written as

$$P = \frac{W(\xi') - W(-\xi')}{W(\xi') + W(-\xi')}.$$  

(39)

After developing equation (39) into a power series and keeping terms linear in $\hbar$, the degree of polarization takes on the form

$$P = \xi_3 + \frac{\xi_2 \xi_3 + \xi_3 \xi_4}{1 + \xi_3} N_+ \hbar.$$  

(40)

Equality (40) shows that the degree of polarization is almost independent on the polarization of the initial photon. Taking into account relation (40), the Stokes parameters of the final photon can be written as

$$\xi_1' = 1, \quad \xi_2' = \frac{\xi_1 - \xi_2}{1 + \xi_3}, \quad \xi_3' = \frac{\xi_2 - \xi_3}{1 + \xi_3}.$$  

(41)

Thus, the final photon polarization is almost always linear anomalous and $P \approx 1$.

The case $\xi_3 = -1$ is the only exception. According to the expression (32), the polarization of the final photon is $\xi_3' = -1$. Therefore, the photon propagates in a magnetic field without changing polarization when the initial polarization is the normal linear one. It should be noted that the photon polarization shows similar behavior in the nonresonant case.

Thus, if the photon is polarized as an eigenmode (normal and anomalous mode polarizations are $\xi_3 = -1$ and $\xi_3 = 1$ respectively), its polarization does not change after traveling through the magnetic field, in accordance with known results [1–3].

6. Conclusion

In the present work the process of photon propagation in a subcritical magnetic field has been considered. The photon polarization is assumed to have arbitrary values. The expressions of the process rate obtained have been analyzed in the LLL-approximation.

Considerable attention has been paid to the resonant case (23) when the virtual electron–positron pair goes to the mass shell. It should be emphasised that resonant photon propagation with production and annihilation of electron–positron pair is possible when condition (23) is true. In previous works, however, the resonance was associated only to the single-photon production of a real pair. The expressions for the process rate in the resonant and nonresonant cases substantially depend on the initial photon polarization. In the case of normal linear polarization ($\xi_3 = -1$) the rate has the minimum value.

It was found from consideration of the polarization degree that the final photon has anomalous linear polarization ($\xi_3' = 1$) in most cases. The exception is the case of normal linear polarization $\xi_3 = -1$, when the photon propagates without change of the polarization ($\xi_3' = -1$). Thus, vacuum in a magnetic field does not show optical activity when the photon polarization $\xi_3 = \pm 1$.

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