INTUITIONISTIC FUZZY WEAKLY $g^\prime$-CLOSED SETS

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Abstract

In this paper, the concepts of intuitionistic fuzzy weakly $g^\prime$-closed sets and intuitionistic fuzzy weakly $g^\prime$-open sets and its properties in intuitionistic fuzzy topological space is introduced.

Keywords: Intuitionistic fuzzy topology, intuitionistic fuzzy weakly $g^\prime$ closed sets and intuitionistic fuzzy weakly $g^\prime$-open sets.

I. Introduction

In 1965, fuzzy sets was introduced by Zadeh and in 1968, fuzzy topology was introduced by Chang. After the introduction many discussions were performed on the generalization of this concepts. The intuitionistic fuzzy set was introduced by Atanassov. In 1997, Coker introduced the concept of intuitionistic fuzzy topological spaces. Here the concepts of intuitionistic fuzzy weakly $g^\prime$-closed sets and intuitionistic fuzzy weakly $g^\prime$-open sets and its properties are introduced.

II. Preliminaries

Here $(K, \tau)$ or K denoted as the IF topological space. The closure, the interior and the complement of a subset $F \subseteq K$ are denoted by $c(F), i(F)$ and $F^c$ respectively.

Definition 2.1: [I] Consider K is a non empty set. An IF set F in K is of the form $F=\{ \langle k, \mu_F(k), \nu_F(k) \rangle / k \in K \}$ where the functions $\mu_F(k): K \rightarrow [0,1]$ and $\nu_F(k): K \rightarrow [0,1]$ denotes the degree of membership and the degree of non membership of the element $k \in K$ to the set $F$, respectively and $0 \leq \mu_F(k) + \nu_F(k) \leq 1, k \in K$.

Definition 2.2: Consider F and E are the IFSets of the form $F=\{ \langle k, \mu_F(k), \nu_F(k) \rangle / k \in K \}$ and $E=\{ \langle k, \mu_E(k), \nu_E(k) \rangle / k \in K \}$. Then

- $F \subseteq E$ if and only if $\mu_F(k) \leq \mu_E(k)$ and $\nu_F(k) \geq \nu_E(k)$ for all $k \in K$,
- $F=E$ if and only if $F \subseteq E$ and $E \subseteq F$,
- $F=\{ \langle k, \nu_F(k), \mu_F(k) \rangle / k \in K \},$
- $F \cap E = \{ \langle k, \mu_F(k) \wedge \mu_E(k), \nu_F(k) \vee \nu_E(k) \rangle / k \in K \}$,

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Definition 2.3: An IF topology on $K$ is a family of IF Sets in $K$ satisfying:

- $0,1 \in \tau$,
- $H_1 \cap H_2 \in \tau$, for any $H_1, H_2 \in \tau$,
- $\cup H_i \in \tau$, the family $\{H_i|i \in J\} \subseteq \tau$.

The pair $(K, \tau)$ is called an IF topological space and any IFSet in $\tau$ is known as an IF open set in $K$.

The $F^c$ of an IF open set $F$ in an IF topological space $(K, \tau)$ is called an IF closed set (IFCS) in $K$.

Definition 2.4: Consider $(K, \tau)$ is an IF topological space and $F=\langle k, \mu^F(k), \nu^F(k) \rangle$ be an IFS in $K$. Then

\[
i(A) = \cup \{H/H \text{ is an IFOS in } K \text{ and } H \subseteq F\},
\]

\[
c(A) = \cap \{B/B \text{ is an IFCS in } K \text{ and } F \subseteq B\}.
\]

Note that for any IFS $F$ in $(K, \tau)$, $c(F^c) = (i(F))^c$ and $i(F^c) = (c(F))^c$.

Definition 2.5: An IFS $F=\langle k, \mu^F(k), \nu^F(k) \rangle$ in an IFTS $(K, \tau)$ is an

- intuitionistic fuzzy $\alpha$-open set if $F \subseteq i(c(i(F)))$,
- intuitionistic fuzzy regular open set if $F = i(c(F))$,
- intuitionistic fuzzy semi open set if $F \subseteq c(i(F))$.

Definition 2.6: An IFS $F=\langle k, \mu^F(k), \nu^F(k) \rangle$ in an IFTS $(K, \tau)$ is said to be an

- IFG"CS in [9] if $c(F) \subseteq Y$ whenever $F \subseteq Y$ and $Y$ is an IFGSOS in $K$,
- IFGCS [7] if $sc(F) \subseteq Y$ whenever $F \subseteq Y$ and $Y$ is an IFOS in $K$,
- IFRWGCS [8] if $c(i(F)) \subseteq Y$ whenever $F \subseteq Y$ and $Y$ is an IFROS in $K$,
- IFWGCS [10] if $c(i(F)) \subseteq Y$ whenever $F \subseteq Y$ and $Y$ is an IFOS in $K$.

Note: Every IFCS and IF$\alpha$CS in $(K, \tau)$ is an IFGSCS in $K$.

Result 2.1: Consider $F$ is IFSet in $(K, \tau)$, then

- $sc(F) = F \cup i(c(F))$,
- $si(F) = F \cap c(i(F))$.

If $F$ is an IFS of $K$ then $sc(F^c) = (si(F))^c$.

Result 2.2: Let $F$ be an IFS in $(K, \tau)$, then

- $ac(F) = F \cup c(i(c(F)))$,
- $ai(F) = F \cap i(c(F))$. 

III. Intuitionistic Fuzzy Weakly $g^\prime$-closed Sets

In this chapter intuitionistic fuzzy weakly $g^\prime$-closed sets and its properties are introduced.

**Definition 3.1:** An IFS $F \in (K, \tau)$ is said to be an IFWG$^\prime$CS if $c(i(F)) \subseteq Y$ whenever $F \subseteq Y$ and $Y$ is an IFGSOS in $(K, \tau)$.

**Ex.3.2:** Consider $K = \{e, f\}$ and $\tau = \{0, H, 1\}$ on $K$. $H = \langle k, (0.2, 0.2), (0.4, 0.5) \rangle$. Therefore $F = \langle k, (0.1, 0.1), (0.5, 0.6) \rangle$ is IFWG$^\prime$CS in $K$.

**Theorem 3.3:** Every IFCS in $K$ is IFWG$^\prime$CS, but the converse is not true.

**Proof:** Consider $F \subseteq Y$ and $Y$ is an IFGSOS in $K$. Since $F$ is an IFCS, $c(F) \subseteq Y$ and $c(i(F)) \subseteq Y$. Therefore $F$ is an IFWG$^\prime$CS in $K$.

**Ex.3.4:** Consider $K = \{e, f\}$ and $\tau = \{0, H, 1\}$ on $K$. $H = \langle k, (0.2, 0.2), (0.4, 0.5) \rangle$. Therefore $F = \langle k, (0.1, 0.1), (0.5, 0.6) \rangle$ is IFWG$^\prime$CS, which is not an IFCS in $K$. Since $c(F) = H \neq F$.

**Theorem 3.5:** Every IFRCS in $(K, \tau)$ is IFWG$^\prime$CS, but the converse is not true.

**Proof:** We know that IFRCS is IFCS and by Theorem 3.3, $F$ is IFWG$^\prime$CS in $K$.

**Ex.3.6:** Consider $K = \{e, f\}$ and $\tau = \{0, H, 1\}$ on $K$. $H = \langle k, (0.2, 0.2), (0.4, 0.5) \rangle$. Therefore $F = \langle k, (0.1, 0.1), (0.5, 0.6) \rangle$ is IFWG$^\prime$CS, which is not an IFRCS in $K$. Since $c(i(F)) = 0 \neq F$.

**Theorem 3.7:** Every IF$\alpha$CS in $K$ is IFWG$^\prime$CS, but the converse is not true.

**Proof:** Consider $F \subseteq Y$ and $Y$ is an IFGSOS in $(K, \tau)$. Since $c(F) \subseteq Y$, we have $c(i(c(F))) \subseteq F \subseteq Y$, which implies $c(i(F)) \subseteq c(i(c(F))) \subseteq F \subseteq Y$. Hence $F$ is an IFWG$^\prime$CS in $K$.

**Ex.3.8:** Consider $K = \{e, f\}$ and $\tau = \{0, H, 1\}$ on $K$. $H = \langle k, (0.2, 0.2), (0.4, 0.5) \rangle$. Therefore $F = \langle k, (0.1, 0.1), (0.5, 0.6) \rangle$ is IFWG$^\prime$CS, which is not an IF$\alpha$CS in $K$ since $c(i(F)) = 0 \neq F$.

**Theorem 3.9:** Every IFG$^\prime$CS in $K$ is IFWG$^\prime$CS, but the converse is not true.

**Proof:** Consider $F \subseteq Y$ and $Y$ is an IFGSOS in $(K, \tau)$. Since $c(F) \subseteq Y$, therefore $i(F) \subseteq c(F) \subseteq Y$. Hence $F$ is an IFWG$^\prime$CS in $K$.

**Ex.3.10:** Consider $K = \{e, f\}$ and $\tau = \{0, H, 1\}$ on $K$. $H = \langle k, (0.2, 0.2), (0.4, 0.5) \rangle$. Therefore $F = \langle k, (0.1, 0.4), (0.9, 0.3) \rangle$ is an IFWG$^\prime$CS, which is not an IFG$^\prime$CS in $K$.

**Theorem 3.11:** Every IFWG$^\prime$CS in $K$ is an IFWGCS, but the converse is not true.

**Proof:** Consider $F \subseteq Y$ and $Y$ is an IFOS in $K$. Since every IFOS is IFGSOS and by hypothesis, $c(i(F)) \subseteq Y$. Hence $F$ is an IFWGCS in $K$.

**Ex.3.12:** Consider $K = \{e, f\}$ and $\tau = \{0, H, 1\}$ on $K$. $H = \langle k, (0.6, 0.5), (0.4, 0.5) \rangle$. Therefore $F = \langle k, (0.7, 0.6), (0.3, 0.4) \rangle$ is IFWGCS, which is not an IFWG$^\prime$CS in $K$. Since $c(i(F)) = 1 \not\subseteq Y$, $Y$ is an IFGSOS.

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**Remark 3.13:** None of the implications are reversible.

$$
\begin{array}{c}
\text{IFG}^\ast \text{CS} \\
\text{IFRCS} \\
\text{IFCG}^\ast \text{CS} \\
\text{IFWG}^\ast \text{CS} \\
\text{IF\alpha CS} \\
\text{IF\gamma CS}
\end{array}
$$

**Theorem 3.14:** If $F$ is an IFWG$^\ast$CS and $F \subseteq E \subseteq c(i(F))$ then $E$ is IFWG$^\ast$CS in $K$.

**Proof:** Consider $Y$ is IFGSOS such that $E \subseteq U$. Since $F$ is an IFWG$^\ast$CS, we have $c(i(F)) \subseteq Y$. Since, $E \subseteq c(i(F))$ then $c(i(E)) \subseteq c(i(F))$. This implies $c(i(E)) \subseteq Y$. Hence $E$ is an IFWG$^\ast$CS in $K$.

**Theorem 3.15:** If $F$ is an IFG$^\ast$CS such that $F \subseteq E \subseteq c(F)$, where $E$ is IFS in an IFTS $K$, then $E$ is IFWG$^\ast$CS in $K$.

**Proof:** Consider $Y$ is IFGSOS such that $E \subseteq Y$. Since $F$ is an IFG$^\ast$CS and $c(i(F)) \subseteq c(F) \subseteq Y$. Now $c(i(E)) \subseteq c(E) \subseteq Y$. Therefore $E$ is an IFWG$^\ast$CS in $K$.

**Remark 3.16:** IFSCS and IFWG$^\ast$CS are independent to each other.

**Ex. 3.17:** Consider $K=\{e, f\}$ and $\tau=\{0, H, 1\}$ on $K$, $H=\{k, (.4, .4), (.6, .5)\}$. Therefore $F=\{k, (.5, .5), (.4, .5)\}$ is IFSCS, which is not an IFWG$^\ast$CS in $K$.

**Ex. 3.18:** Consider $K=\{e, f\}$ and $\tau=\{0, H, 1\}$ on $K$, $H=\{k, (.5, .5), (.4, .5)\}$. Therefore $F=\{k, (.3, .1), (.4, .6)\}$ is IFWG$^\ast$CS, which is not IFSCS in $K$.

**Remark 3.19:** The union of any two IFWG$^\ast$CSs need not be an IFWG$^\ast$CS.

**Ex. 3.20:** Consider $K=\{e, f\}$ and $\tau=\{0, H, 1\}$ on $K$, $H=\{k, (.4, .3), (.6, .7)\}$. Therefore $F=\{k, (.4, .2), (.6, .8)\}$ and the IFS $E=\{k, (.3, .3), (.6, .7)\}$ are IFWG$^\ast$CSs but $F \cup E$ is not an IFWG$^\ast$CS in $K$.

**Remark 3.21:** The intersection of two IFWG$^\ast$CSs need not be an IFWG$^\ast$CS as shown in the following example.

**Ex. 3.22:** Let $K=\{e, f\}$ and let $\tau=\{0, H_1, H_2, H_1 \cup H_2, H_1 \cap H_2, 1\}$ be an IFT on $K$, where $H_1=\{k, (.4, .3), (.6, .7)\}$ and $H_2=\{k, (.2, .5), (.8, .5)\}$. Then the IFS $F=\{k, (.6, .8), (.4, .2)\}$ and the IFS $E=\{k, (1, .4), (0, 0.6)\}$ are IFWG$^\ast$CSs but $F \cap E$ is not an IFWG$^\ast$CS in $K$.

**IV. Intuitionistic Fuzzy Weakly $g^\ast$-open Sets**

In this section intuitionistic fuzzy weakly $g^\ast$-open sets its properties are introduced.

**Definition 4.1:** An IFSet $F$ is said to be an IFWG$^\ast$OS if the complement $F^c$ is an IFWG$^\ast$CS in $K$.
Theorem 4.2: For any IFTS (K, τ), we have the following:

- all the IFOS is an IFWG″OS,
- all the IFROS is an IFWG″OS,
- all the IFαOS is an IFWG″OS,
- all the IFG″OS is an IFWG″OS. But the converses are not true in general.

Proof: Obvious.

Remark 4.3: The converse of the above theorem need not be true as shown in the following examples.

Ex. 4.4: Consider K={e,f} and τ={0,H,1} on K,H=〈k,(.2,.2),(.4,.5)〉. Therefore F=〈k,(.5,.6),(.1,.1)〉 is IFWG″OS, which is not IFOS in K.

Ex. 4.5: Consider K={e,f} and τ={0,H,1} on K,H=〈k,(.2,.2),(.4,.5)〉. Therefore F=〈k,(.5,.6),(.1,.1)〉 is IFWG″OS, which is not IFROS in K.

Ex. 4.6: Consider K= {e, f} and τ={0, H, 1} on K, H=〈 k, (.2, .2), (.4, .5) 〉. Therefore F=〈 k, (, .5, .6), (1, 1) 〉 is IFWG″OS, which is not IFαOS in K.

Ex. 4.7: Consider K={e, f} and τ={0, H, 1} on K, H=〈 k, (.8, .8), (.2, .1) 〉. Therefore F=〈 k, (, .1, .3), (.9, .7) 〉 is IFWG″OS, which is not IFG″OS in K.

Theorem 4.8: Every IFWG″OS in K is an IFWGOS in K.

Proof: Obvious.

Remark 4.9: The converse of the above theorem need not be true as shown in the following example.

Ex. 4.10: Consider K={e,f} and τ={0,H,1} on K,H=〈k,(.6,.5),(.4,.5)〉. Therefore F=〈k,(.3,.4),(.7,.6)〉 is IFWGOS, which is not IFWG″OS in K.

Ex. 4.11: Consider K={e,f} and τ={0,H,1} on K,H=〈k,(.8,.8),(.2,1)〉. Therefore F=〈k,(.1,.3),(.9,.7)〉 is IFGαOS, which is not IFG″OS in K.

Ex. 4.12: Consider K={e,f} and τ={0,H,1} on K,H=〈k,(.4,.3),(.6,.6)〉. Therefore F=〈k,(.2,.2),(.7,.8)〉 is IFGαOS, which is not IFG″OS in K.

Ex. 4.13: Consider K={e,f} and τ={0,H,1} on K,H=〈k,(.8,.8),(.2,1)〉. Therefore F=〈k,(.1,.3),(.9,.7)〉 is IFGα**OS, which is not an IFG″CS in K.
IV. Conclusion

The basic aim of this paper is to introduce a new set called Intuitionistic fuzzy weakly g"'-closed sets and some of its properties.

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