More varieties of hybrid inflation

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Abstract

It is pointed out that hybrid inflation can be implemented with the inflaton field rolling away from the origin instead of towards it. This ‘inverted’ hybrid inflation has a spectral index \( n < 1 \), in contrast with ordinary hybrid inflation which has \( n \gtrsim 1 \), so a measured value of \( n \) substantially different from 1 would distinguish the two. Other generalisations of hybrid inflation are also considered.

I. INTRODUCTION

The most attractive models of inflation at present are the ‘hybrid’ inflation models [1-3], in which inflation ends due to the interaction of the inflaton field with other fields. They share with the very first models [4] the virtue that the inflaton field variation is small on the Planck scale [5], without requiring their fine-tuning. They have been widely studied, both in the original versions [1-3,6,7,5,8] and in modified ones [7,9-15].

Inflation generates an adiabatic density perturbation and gravitational waves. The former is supposed to be the origin of large scale structure and, together with a possible contribution from the gravitational waves, of the cosmic microwave background (cmb) anisotropy. The spectrum of the density perturbation is conveniently specified by a quantity \( \delta^2_H \), whose scale dependence is \( \delta^2_H \propto k^{n-1} \) where \( n \) is the spectral index. The spectrum of the gravitational waves is conveniently specified by their relative contribution \( R \) to the mean-square low multipoles of the cmb anisotropy seen by a randomly placed observer.

Within the usual paradigm of a single slow-rolling field\(^1\) (which includes most versions of hybrid inflation considered up to now, and we shall also restrict ourselves to this simple

\(^1\)A more general slow-roll paradigm [16] leaves the gravitational waves unchanged but increases \( \delta_H \), so that Eq. (1) becomes a lower bound, Eq. (3) an upper bound and Eq. (2) is no longer valid.
case) the predictions for $\delta_H^2$ \cite{17,18}, $n$ \cite{19,20} and $R$ \cite{21} are

$$
\delta_H^2(k) = \frac{1}{75\pi^2V^2}V^3
$$

(1)

$$
n - 1 = -3 \left(\frac{V'}{V}\right)^2 + 2 \frac{V''}{V}
$$

(2)

$$
R = 6 \left(\frac{V'}{V}\right)^2
$$

(3)

The units are $\hbar = c = M_{Pl} \equiv (8\pi G)^{-1/2} = 1$, and the potential $V$ and its derivatives are to be evaluated when cosmological scales leave the horizon $N$ e-folds before the end of inflation, where $N = 50$ to 25 and is given by $N = \int (V/V')d\phi$. In models with a small field variation, including in particular hybrid inflation models, $R$ is unobservably small \cite{22} and $n$ is either very close to 1 or is given by $n - 1 = 2V''/V$.

At present observation is consistent with $R = 0$, and with this value the COBE measurement of the low multipoles of the cmb anisotropy gives \cite{23} $\delta_H = 1.94 \times 10^{-5}$. The present constraint on $n$ is only \cite{24} $0.7 < n < 1.4$, though eventually one can expect a measurement with an accuracy $\Delta n \sim .01 \cite{25}$.

In most versions of hybrid inflation proposed so far, the inflaton field is near a minimum of the potential. For reasonable choices of the parameters the model can give practically any value of $n$ in the observed range bigger than 1. Two versions \cite{11,12} have a mildly concave potential leading to $n = 0.93$ to 0.97. We are going to point out the existence of ‘inverted’ hybrid inflation, which works near a maximum of the potential. As a result it can give practically any value below 1.

The outline of the paper is as follows. In the next section we study inverted hybrid inflation, and in the following one mention some generalizations of it. In Section IV we consider a whole class of models that generalize the mutated hybrid inflation of Ref. \cite{11}. In Section V we give our conclusions.

**II. INVERTED HYBRID INFLATION**

In the usual models of hybrid inflation $\phi$ is rolling towards zero. The potential is typically of the form

$$
V = V_0 + \frac{1}{2}m^2\phi^2 + \ldots
$$

(4)

and is dominated by the term $V_0$. This term arises because some other field $\psi$ is held at the origin by its interaction with $\phi$. When $\phi$ falls below some critical value $\phi_c$, the other field rolls to its vacuum value so that $V_0$ disappears and inflation ends. This model gives negligible $R$ and $n - 1 = 2m^2/V_0$.

We consider instead the opposite case of ‘inverted’ hybrid inflation, where $\phi$ rolls away from the origin and

$$
V = V_0 - \frac{1}{2}m^2\phi^2 + \ldots
$$

(5)
\[ V_0 = 7 \times 10^{-8} (1 - n)^2 \phi_c^2 e^{-(1-n)N} \]
\[ V = \Lambda^4 - \frac{1}{2} m^2 \phi^2 + \frac{1}{2} m^2 \psi^2 - \frac{1}{2} \lambda \phi^2 \psi^2 + \ldots \] (14)

An alternative way of implementing inverted hybrid inflation is given by

\[ V = V_0 - \frac{1}{2} m^2 \phi^2 - \frac{1}{2} m^2 \psi^2 - \frac{1}{2} m^2 \chi^2 + \frac{1}{2} \lambda^2 \phi^2 \chi^2 + \frac{1}{2} \lambda^2 \psi^2 \chi^2 + \frac{1}{4} \lambda^2 \chi^4 \] (15)

with \( m^2 \ll V_0 \lesssim m^2, m^2 \). Here \( \psi \) will be constrained to zero if \( \chi > m^2 / \lambda \). The minimum of \( \chi \)'s potential is at

\[ \chi = \sqrt{m^2 - \lambda^2 \phi^2} \] (16)

assuming \( \psi = 0 \) and \( \phi < m^2 / \lambda \). Therefore for

\[ \phi < \phi_c = \sqrt{\frac{\lambda^2 m^2 - \lambda^2 m^2}{\lambda^2 m^2}} \] (17)

\( \psi \) will be constrained to zero. Clearly we require \( \lambda \phi m^2 > \lambda \phi m^2 \). Also, to ensure that the terms involving \( \chi \) in Eq. (13) make a negligible contribution to the effective potential during inflation, we require \( \lambda \phi m^2 \ll \lambda \phi m^2 \). The contribution of the \( \phi \) dependence of \( \chi \) to the effective kinetic terms during inflation can be neglected if \( \lambda \phi \psi m^2 \ll \lambda^2 m^2 \).

Previous authors [26–30] considered the potential (6) in the context of a single-field model, and assumed that it holds until \( \phi \simeq f \), after which \( \phi \) settles down to the minimum located at a value somewhat bigger than \( f \). Since observation requires \( f \gg 1 \) (the Planck scale in our units), this places the model outside the regime of ordinary particle theory so that one can hardly justify the form of the potential. It has been suggested that \( \phi \) might be identified with one of the superstring moduli [27,29,30], but attempts [27,29,31,32] to construct a specific model using this idea have not been successful.

We have avoided these difficulties by ending inflation earlier using the hybrid inflation mechanism. An alternative would be to keep the single field \( \phi \), but to suppose that its potential steepens soon after cosmological scales leave the horizon so that inflation again ends at some value \( \phi_c \ll f \). Such a proposal is not unreasonable, though it does postulate a lot of structure in the single potential \( V(\phi) \).

III. GENERALIZATIONS OF INVERTED HYBRID INFLATION

Instead of assuming that the inflationary potential is quadratic, one can consider the possibility that it is of higher order. This might be because the quadratic term is absent, or it might be because one is not close to the origin though in that case there is no reason to suppose that a single term dominates. If the quadratic term is absent one might have

\[ V(\phi) \simeq -4.3 - 0.01 \cos(12\phi) \] for the canonically normalised field proportional to \( \text{Im} S \) after minimizing with respect to \( \text{Re} S \).
\[ V = V_0 - \frac{1}{4}\lambda \phi^4 + \ldots \] (18)

This form could arise from one of the generalizations of mutated hybrid inflation considered in the next section, and we use the results derived there. For \( \phi_c^2 \gg V_0/(2N\lambda) \) the predictions are independent of \( \phi_c \) and the same as for the non-hybrid case; \( 1 - n = 3/N \) with negligible gravitational waves and the normalization of \( \delta_H \) requiring \( \lambda \sim 10^{-12} \) independently of \( V_0 \). In the opposite case, \( 1 - n \) is reduced by a factor \( \phi_c^2/[V_0/(2N\lambda)] \) and the value of \( \lambda \) is reduced by this factor cubed.

Including both a quadratic and a quartic term one could have

\[ V = V_0 - \frac{1}{2}m^2 \phi^2 + \frac{1}{4}\lambda \phi^4 \] (19)

If inflation (after the observable Universe leaves the horizon) takes place near the maximum it reduces to the quadratic inverted hybrid inflation that we started out with. If it takes place near the minimum it reduces to the usual version of hybrid inflation, and in the intermediate case one gets something different.

Finally, we should point out that a related model of inflation, was proposed a long time ago \[33\]. In it, the inflaton is rolling away from the origin and it triggers the GUT Higgs transition at or before the end of inflation. The potential for this model is very complicated and it is not clear whether the Higgs potential is supposed to dominate the energy density. If there is a regime of parameter space in which it does, then the model would represent the first version of inverted hybrid inflation (and in fact the first version of hybrid inflation of any kind).

**IV. GENERALIZATIONS OF MUTATED HYBRID INFLATION**

Here we consider potentials of the form

\[ V = V_0 - \frac{\sigma}{p} \psi_p + \frac{\lambda}{q} \psi^q \phi^r \] (20)

with \( p \neq q \). The opposite case of \( p = q \) simply gives an ordinary or inverted hybrid inflation model where \( \psi \) is held at zero during inflation. In mutated hybrid inflation \[11\], and its generalisations \[12\] considered here, \( \psi \) is held close to zero, but not at zero, during inflation. An effective potential for the inflaton \( \simeq \phi \) is then generated from the couplings without requiring any additional term and, as we shall see, it can take unusual forms. In the case that this contribution to the effective potential is not the dominant one, it will not determine the spectrum but may still determine when inflation ends.

We assume \( \psi > 0 \) and \( \phi > 0 \). Then to get a model which, for fixed \( \phi \), has a minimum at \( \psi = \psi_* \) with \( \psi_* > 0 \), we require \( \sigma \lambda > 0 \) and \((q-p)\sigma > 0 \). See Eqs. (21) and (23) respectively below. \( \sigma > 0 \) corresponds to a generalisation of mutated hybrid inflation with the inflaton \( \phi \) rolling towards zero, while \( \sigma < 0 \) corresponds to a general inverted mutated hybrid inflation model with \( \phi \) rolling away from zero.

Now

\[ V_{\psi} = -\sigma \psi^{p-1} + \lambda \psi^{q-1} \phi^r \] (21)

5
and so \( V_\psi = 0 \) when \( \psi = 0 \) (for \( p \geq 2 \) and \( q \geq 2 \)) or
\[
\psi = \psi_* \equiv \left( \frac{\sigma}{\lambda} \right)^{\frac{1}{q-p}} \phi^{-\frac{q}{q-p}}
\] (22)

Now
\[
V_{\psi\psi}|_{\psi=\psi_*} = (q-p) \sigma \psi_*^{p-2}
\] (23)

For simplicity we will assume that \( V_{\psi\psi}|_{\psi=\psi_*} \gg V_0 \) so that \( \psi \) is held firmly at \( \psi = \psi_* \) during inflation. Then
\[
V|_{\psi=\psi_*} = V_0 - \left( \frac{q-p}{pq} \right) \sigma \psi_*^p
\] (24)

and the kinetic terms evaluated along \( \psi = \psi_* \) are
\[
\frac{1}{2} \left[ 1 + \left( \frac{r}{q-p} \right)^2 \frac{\psi_*^2}{\phi^2} \right] (\partial \phi)^2
\] (25)

Assuming \( \sigma \psi_*^p \ll V_0 \) so that \( V_0 \) dominates the energy density and \( \psi_* \ll \phi \) so that the kinetic terms are approximately canonical, we get the effective potential during inflation
\[
V(\phi) = V_0 \left( 1 - \mu \phi^{-\alpha} \right)
\] (26)

where
\[
\mu = \left( \frac{q-p}{pq} \right) \frac{\sigma^{\frac{q}{q-p}} \lambda^{-\frac{p}{q-p}}}{V_0} > 0
\] (27)

and
\[
\alpha = \frac{pr}{q-p} \neq 0
\] (28)

Now
\[
\frac{V'}{V} = \alpha \mu \phi^{-\alpha-1}
\] (29)

and
\[
\frac{V''}{V} = -\alpha(\alpha + 1) \mu \phi^{-\alpha-2}
\] (30)

The number of e-folds to the end of inflation is given by
\[
N = \int \frac{V}{V'} \, d\phi = \frac{\phi^{\alpha+2}}{\alpha(\alpha + 2) \mu} \quad \alpha > 0 \text{ or } \alpha < -2
\] (31)

\[
= \frac{1}{|\alpha| (2 - |\alpha|) \mu} \left( \phi_c^{2-|\alpha|} - \phi^{2-|\alpha|} \right) \quad -2 < \alpha < 0
\] (32)

\[
= -\frac{1}{2\mu} \ln \frac{\phi}{\phi_c} \quad \alpha = -2
\] (33)
The COBE normalisation \[24\] gives

\[5.3 \times 10^{-4} = \frac{V^{3/2}}{Vr} = \frac{V_0^{1/2} \phi^{\alpha+1}}{\alpha \mu} = \frac{2^{\alpha+1}}{\alpha+2} N^{\frac{\alpha+1}{2}} \mu^{-\frac{\alpha}{2}} V_0^{1/2} \quad \alpha > 0 \text{ or } \alpha < -2 \quad (34)\]

\[= \frac{V_0^{1/2}}{|\alpha| \mu} \left[ \phi_c^{-|\alpha|} - |\alpha| (2 - |\alpha|) \mu N \right] - 2 < \alpha < 0 \quad (35)\]

\[= \frac{e^{2\mu N} V_0^{1/2}}{2\mu \phi_c} \quad \alpha = -2 \quad (36)\]

The spectral index is given by

\[n \simeq 1 + 2 \frac{V''}{V} = 1 - 2\left( \frac{\alpha + 1}{\alpha + 2} \right) \frac{1}{N} \quad \alpha > 0 \text{ or } \alpha < -2 \quad (37)\]

\[= 1 - \frac{2|\alpha| (|\alpha| - 1) \mu}{\phi_c^{2-|\alpha|} - |\alpha| (2 - |\alpha|) \mu N} - 2 < \alpha < 0 \quad (38)\]

\[= 1 - 4\mu \quad \alpha = -2 \quad (39)\]

Note that \(n < 1\) in all cases except \(-1 \leq \alpha < 0\). However, to get \(\alpha\) in this range would require either \(r < 1\) or at least one of \(p, q\) or \(r\) to be negative. The case \(\alpha = -2\) is the one considered already in Section II, and for this as well as other \(\alpha\) in the range \(-2 \leq \alpha < 0\) we needed to specify that inflation ends at \(\phi = \phi_c\). If inflation ends because \(V_0\) stops dominating the energy density, we will have \(\phi_c \sim \mu^{1/\alpha}\), but higher order terms neglected in Eq. \[24\] may end inflation before this which would permit the desirable \(\phi_c \lesssim 1\).

Supersymmetric implementations of generalized mutated hybrid inflation

Potentials of the form \[24\] can be straightforwardly derived from supersymmetry along the lines of Ref. \[11\] for the case \(\sigma > 0\) (corresponding to \(\alpha > 0\)) and as in Section II for the opposite case. In both cases, the superpotentials involved will be compatible with the method of Ref. \[9\] for avoiding fatal supergravity corrections.

In supersymmetry one would prefer \(q\) and \(r\) to be even if \(\sigma > 0\), and \(p\) to be even if \(\sigma < 0\). Particularly natural possibilities are \(\sigma \sim V_0\) and \(p = 1\) or 2, and \(\sigma \sim -V_0\) and \(p = 2\), with the \(p = 1\) and \(p = 2\) cases corresponding respectively to a generic soft supersymmetry breaking term for a singlet and a non-singlet.

The three simplest possibilities compatible with these ideas are

1. \(p = 1, q = 2, r = 2\), leading to \(\alpha = 2\). This is the original mutated hybrid inflation model \[11\].

2. \(p = 2, q = 1, r = 1\), leading to \(\alpha = -2\) which is the inverted hybrid inflation model of Section II.

3. \(p = 2, q = 1, r = 2\), leading to \(\alpha = -4\) which is the quartic inverted hybrid inflation model of Section III.
V. CONCLUSION

The most important new model that we have discussed is inverted hybrid inflation. In contrast with all other known hybrid inflation models it can give a spectral index *significantly* below 1. The other models we have discussed either reproduce already known possibilities (though with a different prescription for the field value at which inflation ends), or else add to the list of models which give a spectral index *slightly* below 1. A future measurement of $n$ will be a powerful discriminator between hybrid inflation models.

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REFERENCES

[1] A. D. Linde, Phys. Lett. B249, 18 (1990).
[2] F. C. Adams and K. Freese, Phys. Rev. D43, 353 (1991).
[3] A. D. Linde, Phys. Lett. B259, 38 (1991).
[4] A. D. Linde, Phys. Lett. 108B (1982); A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. 48, 1220 (1982); Q. Shafi and A. Vilenkin, Phys. Rev. Lett. 52, 691 (1984).
[5] E. J. Copeland, A. R. Liddle, D. H. Lyth, E. D. Stewart and D. Wands, Phys. Rev. D49, 6410 (1994).
[6] A. R. Liddle and D. H. Lyth, Phys. Rep. 231, 1 (1993).
[7] A. D. Linde, Phys. Rev. D49, 748 (1994).
[8] S. Mollerach, S. Matarrese and F. Lucchin, Phys. Rev. D50, 4835 (1994).
[9] E. D. Stewart, Phys. Rev. D51, 6847 (1995).
[10] G. Dvali, Q. Shafi and R. Schaefer, Phys. Rev. Lett. 73, 1886 (1994).
[11] E. D. Stewart, Phys. Lett. B345, 414 (1995).
[12] G. Lazarides and C. Panagiotakopoulos, Phys. Rev. D52, 559 (1995); hep-ph/9606297.
[13] A. R. Liddle, D. H. Lyth and D. Roberts, Phys. Rev. D51, 4122 (1995).
[14] L. Randall, M. Soljacic and A. H. Guth, hep-ph/9512439.
[15] J. Garcia-Bellido, A. D. Linde and D. Wands, astro-ph/9605094.
[16] M. Sasaki and E. D. Stewart, Prog. Theor. Phys. 95, 71 (1996); T. T. Nakamura and E. D. Stewart, astro-ph/9604103.
[17] A. A. Starobinsky, Phys. Lett. 117B, 175 (1982); S. W. Hawking, Phys. Lett. 115B, 295 (1982); A. H. Guth and S.-Y. Pi, Phys. Rev. Lett. 49, 1110 (1982); J. M. Bardeen, P. S. Steinhardt and M. S. Turner, Phys. Rev. D28, 679 (1983).
[18] D. H. Lyth, Phys. Lett. 147B, 403 (1984); Phys. Lett. 150B, 465 (1985); Phys. Rev. D31, 1792 (1985).
[19] A. R. Liddle and D. H. Lyth, Phys. Lett. B291, 391 (1992).
[20] D. S. Salopek, Phys. Rev. Lett. 69, 3602 (1992).
[21] A. A. Starobinsky, Sov. Astron. Lett. 11, 133 (1985).
[22] D. H. Lyth, Lancaster preprint hep-ph/9606387.
[23] E. F. Bunn, D. Scott and M. White, Ap. J. 441, L9 (1995).
[24] A. R. Liddle et al., astro-ph/9511037; M. White et al., astro-ph/9605057.
[25] D. H. Lyth, in preparation.
[26] A. D. Linde, Phys. Lett. 132B, 317 (1983).
[27] P. Binetruy and M. K. Gaillard, Phys. Rev. D34, 3069 (1986).
[28] K. Freese, J. Frieman and A. V. Olinto, Phys. Rev. Lett. 65, 3233 (1990).
[29] F. C. Adams et al., Phys. Rev. D47, 426 (1993).
[30] T. Banks et al., Phys. Rev. D52, 3548 (1995).
[31] A. De la Macorra and S. Lola, Phys. Lett. B373, 299 (1996).
[32] M. C. Bento and O. Bertolami, gr-qc/9605070.
[33] K. Enqvist and D. V. Nanopoulos, Phys. Lett. 142B, 349 (1984); K. Enqvist and D. V. Nanopoulos, Nucl. Phys. B252, 508 (1985); C. Kounnas and M. Quiros, Phys. Lett. 151B, 189 (1985).