Fayet-Iliopoulos terms in five-dimensional orbifold supergravity

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ABSTRACT

We derive an off-shell formulation for the boundary Fayet-Iliopoulos (FI) terms in locally supersymmetric $U(1)$ gauge theory on 5D $S^1/Z_2$ orbifold. Some physical consequences of such FI-terms, e.g., the generation of 5D kink mass for hypermultiplet are studied within the full supergravity framework.

1. Introduction

When we consider physics beyond the standard model (SM), the supersymmetry (SUSY) is one of the promising candidate which can stabilize the electroweak scale, and achieve better gauge coupling unification at the grand unification scale. One of the remarkable feature is that the SUSY breaking is parameterized by $F$ and $D$ order parameters which is the auxiliary component of the chiral and vector multiplet respectively. As is well known, for instance, the tadpoles of the $F$ and $D$ are SUSY invariant. These terms cause O’Raifeartaigh and Fayet-Iliopoulos (FI) mechanism of SUSY breaking, respectively.

Another perspective beyond the SM is models with extra dimensions. We extend not only the internal space but also the space-time itself. In this direction, orbifold models are particularly interesting which makes a correlation between the internal and the external space. In such model because of the nontrivial parity in extra dimension, we naturally obtain the quasi-localization of fields that results in some hierarchical structures in the 4D effective theory. Furthermore they provide some SUSY breaking and the sequestering mechanism by the spatiality of extra dimension, and the various mediation mechanisms from the hidden to the visible sector.

Here we will focus our attention on the Fayet-Iliopoulos term in orbifold models. Let us consider the 5D $S^1/Z_2$ orbifold model with global SUSY where the orbifold direction is chosen in the $\sigma_3$ direction of $SU(2)_R$. In this case, we have two 4D fixed plane with $N = 1$ SUSY. Then it was pointed out in Ref. [12] that the $U(1)$ vector multiplet has FI-term at the fixed plane in the form of $\left(\xi_0 \delta(y) + \xi_\pi \delta(y - \pi R)\right)D$ where $D$ is the $N = 1$ auxiliary field that consists of the gauge scalar $\phi$ and the third component of the $SU(2)$-triplet auxiliary field $Y^{(3)}$ in 5D $U(1)$ vector multiplet as $D = 2Y^{(3)} - \partial_y \phi$. If the FI-coefficients $\xi_0$ and $\xi_\pi$ take the integrable form $\xi_\pi = -\xi_0 \equiv \xi$, the $D$-flat condition gives the periodic sign-function $c(y)$ type vacuum expectation value (VEV) of the gauge scalar field $\langle \phi \rangle = \xi c(y)$. This VEV produces the kink mass for the charged hypermultiplet and results in the quasi-localization of the hyperino field $\xi Y^{(3)}$. In this manner, FI-term is
generically important in global orbifold models. Then our question is about the situation in local supersymmetry.

In 4D supergravity (SUGRA), the FI-term is forbidden by the local SUSY except the case of gauged $U(1)_R$ or so called pseudo-anomalous $U(1)$. However in 5D orbifold SUGRA, the on-shell analysis in Ref. [1] showed that the existence of some bulk $Z_2$-odd operator with the $\epsilon(y)$-type coefficient implies the existence of boundary FI-term for a (normal) $U(1)$ vector multiplet. But the exact off-shell formulation for such FI-term in 5D orbifold SUGRA was absent. In our work [4], we derive the off-shell formulation of such boundary FI-term by utilizing the off-shell 5D SUGRA [5] with four-form Lagrange multiplier [6] which can generate the $\epsilon(y)$-type coefficient in a consistent way with the local SUSY.

2. Formulation

In the off-shell formulation of 5D SUGRA [5], we introduce three $U(1)$ vector multiplets $V_A = (M^A, A^A, \Omega^{Ai}, Y^{Aij})$ where $A = Z, X, S$. We use the notation for the gauge scalar fields as $M^A = (\alpha, \beta, \gamma)$ with $Z_2$-parity $(+, -, -)$ respectively. The $V_Z$ is the graviphoton multiplet, $V_X$ is the multiplet which finally has boundary FI-term and $V_S$ will be eliminated by the constraint generated by the four-form Lagrange multiplier in order to get $\epsilon(y)$ type coefficient dynamically. We also introduce a compensator and a physical hypermultiplet whose scalar component is represented by the quaternion $A$ and $\Phi$ respectively. These are gauged by the vector multiplets with the charge assignment of $(T_Z, T_X, T_S)A = (0, \tilde{q}, -\frac{3}{2}k)i\sigma_3A$ and $(T_Z, T_X, T_S)\Phi = (0, q, c)i\sigma_3\Phi$. We set $\tilde{q} = 0$ in order not the $V_X$ to take part in the $R$-gauging. We remark that the charge $k$ determines the AdS curvature of the background geometry and the $c$ gives bare kink mass for the hypermultiplet. In addition, we introduce the linear (four-form) multiplet $L|_{V_Z}$ as a Lagrange multiplier constraining $V_S$, whose vector and real scalar component is dualized into the three- and four-form filed respectively under the background of $V_Z$

We start from the off-shell SUGRA Lagrangian with Weyl multiplet, the above three vector multiplets and two hypermultiplets, plus the Lagrange multiplier terms with the four-form multiplet which was derived in Ref. [5]. The norm function of our model is defined as

$$N = \alpha^3 - \frac{1}{2}\alpha^2\beta + \frac{1}{2}\xi_{FI}\alpha\beta\gamma,$$

which after integrating out four-field results in $N = \alpha^3 - \frac{1}{2}\alpha^2\beta + \frac{1}{2}\xi_{FI}\epsilon(y)\alpha^2\beta$. The action after integrating out four-form multiplet is given by the sum of bulk and boundary contributions. Besides the usual bulk action, the brane action (bosonic part) is induced as

$$e^{-1\pi R}_4 L_{\text{brane}} = \left[\frac{1}{2}\xi_{FI}\alpha^2(\text{tr}[i\sigma_3 Y' X] + \partial_4 \beta) - 2\alpha(3k + \frac{3}{2}\text{tr}[\Phi^i \Phi] + c\text{tr}[\Phi^i \sigma_3 \Phi \sigma_3])\right] (\delta(y) - \delta(y - \pi R)).$$

where we find the boundary FI-term for the $V_X$ multiplet in the first line with the coefficient $\xi_{FI}$ defined in the norm function. (The full result is shown in Ref. [4].) We note
that this brane action proportional to $\delta(y) - \delta(y - \pi R) = \partial_y \epsilon(y)/2$ is the consequence of the four-form mechanism [3].

On the 4D Poincaré invariant background geometry $ds^2 = e^{2K} \eta_{\mu \nu}dx^\mu dx^\nu - dy^2$, the 4D energy density is calculated as

$$E = \int dy \ e^{4K} \left( \frac{1}{2} a_{IJ} D^I D^J + \frac{2}{1 + v^2} |F|^2 - 6|\kappa|^2 \right),$$

where $I, J = (Z, X)$, $a_{IJ} = -\frac{1}{2} \frac{\partial^2 \ln N^r_{IJ}}{\partial y^i \partial y^j}$ and we assume $\langle \Phi(0) \rangle = v$ and $\langle \Phi(r) \rangle = 0$ for the quaternionic hyperscaler field $\Phi = \Phi(0) 1_2 + i \sum_{r=1}^{3} \Phi^{(r)} \sigma_r$. The superconformal gauge fixing $N = 1$ reduces the number of scalar field as

$$\alpha(\phi) = (1 + \frac{e^2}{4} \epsilon^2(y)/8)^{-1/3} \cosh^{2/3}(\phi),$$
$$\beta(\phi) = \alpha(\phi)[(2 + \frac{1}{4} \epsilon^2 \epsilon^2(y))]^{1/2} \tanh(\phi) + \frac{1}{2} \epsilon \epsilon \epsilon(\phi)],$$

where we find that $\beta$ almost carries the physical degree of freedom $\phi$ at the leading order. The $D$, $F$ and $\kappa$ are the gaugino, hyperino and gravitino ($N = 1$) Killing order parameters respectively,

$$\kappa = \partial_y K - \mathcal{P}/3, \quad F = \partial_y v - (q\beta + \epsilon(y)\alpha - \mathcal{P}/2)v,$$
$$D^I = N_{IJK} \left( \partial_y N + 2N J \mathcal{P}/3 + 6k \epsilon(y)\delta^2 + 4((3k/2 + c)\epsilon(y)\delta + q\delta X) v^2 \right),$$

where $\mathcal{P} = -2 \left[ \frac{3}{2} k \epsilon(y) \alpha + \left( \frac{3}{2} k + c \right) \epsilon(y) \alpha + q \beta \right] v^2$ and $N_{IJK} = \delta_{I}^{K}$. We have a ($\beta$-independent) boundary FI-contribution in the first term $N^{X} \frac{\partial}{\partial y} N_X$ in $D^X$. We should note that the stationary condition of the energy functional is just given by Killing conditions $\kappa = F = D^I = 0$.

3. Some physical consequences

Above we succeeded to formulate boundary FI-term off-shell. In this section we show some physical consequences of the FI-term. Combining $\kappa = 0$ and $D^I = 0$, we find

$$e^{-2K} \partial_y (e^{2K} N_I) = -6k \epsilon(y)\delta^2 - 4((3k/2 + c)\epsilon(y)\delta^2 + q\delta X) v^2. \quad (1)$$

Obviously $F = 0$ is satisfied by $v = 0$, and for such hyperscalar vacuum value, Eq. (1) for $I = X$ is satisfied by $N_X = -\alpha\beta + \epsilon e(y)\alpha^2/2 = 0$ resulting

$$\beta(\phi) = \frac{1}{2} \epsilon \epsilon \epsilon(\phi). \quad (2)$$

With this relation, $N = 1$ condition gives $\alpha(\phi) = (1 + \frac{e^2}{4} \epsilon^2(y)/8)^{-1/3}$ that means

$$\langle \phi \rangle = 0.$$

With these VEVs of $\phi$ and $v$, the $I = Z$ component of Eq. (1) yields

$$K \simeq -k|y|,$$
and we find from Eq. (2) that

$$\langle \beta \rangle = \frac{1}{2} \xi_{FI} \epsilon(y) \alpha(\phi = 0) = \frac{1}{2} \xi_{FI} \epsilon(y) + O(\xi_{FI}^2).$$

(3)

From this result, we conclude that there exists a supersymmetric vacuum with the FI-term $\xi_{FI} \neq 0$, and at $O(\xi_{FI})$ the only vacuum deformation induced by the FI-term is the nonvanishing VEV of $\beta$ given by Eq. (3) irrespective of the value of $k$, i.e., irrespective of whether the background geometry is flat or AdS [7].

4. Summary

We have shown an exact formulation of the boundary FI-term in the fully local context. We utilized off-shell formulation with four-form Lagrange multiplier to realize the bulk odd operator which is relevant to the boundary FI-term in the orbifold SUGRA. We analyzed the BPS stationary conditions of the scalar potential with the FI-term $\xi_{FI} \neq 0$, and found that there is a supersymmetric vacuum. At $O(\xi_{FI})$, the only vacuum deformation induced by the FI-term is the nonvanishing VEV of $\beta$ which generates the kink mass for the charged hypermultiplets. This result is irrespective of whether the background geometry is flat or AdS [7].

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6. References

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