Probing high momentum nucleons in nucleus by $\pi$ production in nucleus-nucleus collisions

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Unlike the general description in the textbook, about 20% nucleons in fact have momenta greater than the so-called fermi momentum. Kinetic energy from these high-momentum nucleons may greater than that from nuclear Fermi movement. Based on the transport model of nucleus-nucleus collisions at intermediate energies, high-momentum nucleons caused by 2-nucleon short-range correlations (SRC) in nucleus is first studied through $\pi$ meson production in nucleus-nucleus collisions. We find that $\pi^+$ meson production in nucleus-nucleus collisions is very sensitive to the high-momentum cutoff value of nucleon in nucleus. By comparing the ratio of $\pi^+$ meson productions in Au+Au collisions at two beam energies with the FOPI experimental data, the high-momentum cutoff value of about 1.8 times nuclear Fermi momentum is obtained. This study may have broad implications in both nuclear physics and astrophysics.

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I. INTRODUCTION

What is the maximum momentum of fermions in a fermion system and how the high momentum fermions are stably held in fermion system so that they do not escape from the system? This interesting issue would help us to understand not only the force among the fermions at very short distance, but also the stability of the fermion system with high momentum fermions. In this study, using a nuclear fermion system as an example, we try to explore the high-momentum of nucleons in nucleus.

The picture of nucleons have maximum momentum—so called the fermi momentum $p_f$—in a nuclear system and move independently in the average field created by their mutually attractive interactions has been established since the 1950s. However, later proton-removal experiments using electron beams with energies of several hundred MeV showed that only about 80% (including 10 ~ 20% long range correlations) nucleons participate in this type of independent particle motion [1–3]. Recent high-momentum transfer measurements have shown that nucleons in nuclear can form pairs with large relative momentum and small center-of-mass momentum [4,5]. This was understood based on the dominance of the nucleon-nucleon tensor interaction in the inter-nucleon distances of about 1 fm [4,6]. The nucleon-nucleon short-range correlations (SRC) in nucleus leads to a high-momentum tail in the single-nucleon momentum distribution around 300 ~ 600 MeV/c [3,6]. And interestingly, the high-momentum tail’s shape caused by two-nucleon SRC is almost identical for all nuclei from deuteron to very heavy nucleus [10–13], i.e., roughly exhibits a $C/k^4$ tail [14–17]. Nucleon momentum distributions at even higher momenta are due to three or many-nucleon correlations. This part of momentum distribution probability was deduced to be less than 1% [18]. We thus in this study neglect this kind of high-momentum nucleons due to many-nucleon short range correlations.

In the high-momentum tail of nucleon momentum distribution, nucleonic component is strongly isospin-dependent, i.e., the number of n-p SRC pairs are about 18 times that of the p-p or n-n SRC pairs [3], thus in neutron-rich heavy nucleus protons have a greater probability than neutrons to have momentum greater than the nuclear Fermi momentum [17]. In neutron stars, protons only have a small proportion. The above n-p SRC in neutron stars would cause proton’s average kinetic energy far greater than the neutron’s [19]. And the stronger the n-p SRC is, the larger the difference of the average kinetic energy of protons and neutrons is seen. Therefore, besides directly reflecting the nuclear force at short rang [6,7], the high-momentum cutoff parameter $\lambda (= p_{max}/p_f$, i.e., the ratio of nucleonic maximum momentum and the nuclear Fermi momentum) strongly affects the study of the nuclear kinetic symmetry energy [15, 16], the latter is known plays crucial role in both nuclear physics and astrophysics [20]. The SRC caused off-fermi shell nucleons in supra-dense asymmetric nuclear matter also have broad implications in the studies of cooling of a neutron star, superfluidity of protons and isospin locking in the neutron stars [19].

However, the high-momentum cutoff value of nucleons due to SRC in nucleus was seldom reported. One can deduce the high-momentum cutoff parameter $\lambda$ from the momentum distribution of deuteron [14, 15] or from the high-energy electron scattering measurements [17, 18]. However, the distribution of nucleons in very light nuclei may be different from the case of the center of heavy nucleus. And the high-energy electron scattering measurements mainly probe the nucleons at the surface of nucleus [21]. Since due to SRC protons in heavy nucleus have a greater probability than neutrons to have momenta greater than the nuclear Fermi momentum and also $\pi^+$ production in nucleus-nucleus collisions at intermediate energies is mainly from proton-proton collision [22], we in this study propose a direct probe, i.e., using $\pi^+$ production in nucleus-nucleus collisions as a probe of the high-momentum cutoff value in nucleus.
II. THE BUU TRANSPORT MODEL

To study the high-momentum cutoff value of nucleon in nucleus by nucleus-nucleus collisions, we use the Isospin-dependent Boltzmann-Uehling-Uhlenbeck (BUU) transport model \cite{22}, which has been very successful in studying heavy-ion collisions at intermediate energies. The BUU transport model describes time evolution of the single particle phase space distribution function \( f(\vec{r}, \vec{p}, t) \), which reads

\[
\frac{\partial f}{\partial t} + \nabla_{\vec{r}} E \cdot \nabla f - \nabla_{\vec{p}} E \cdot \nabla f = I_c. \tag{1}
\]

The phase space distribution function \( f(\vec{r}, \vec{p}, t) \) denotes the probability of finding a particle at time \( t \) with momentum \( \vec{p} \) at position \( \vec{r} \). The left-hand side of Eq. (1) denotes the time evolution of the particle phase space distribution function due to its transport and mean field, and the right-hand side accounts for binary particle-particle collisions. \( E \) is a particle’s total energy, which is equal to kinetic energy \( E_{\text{kin}} \) plus its average potential energy \( U \). While the mean-field potential \( U \) of the single particle depends on position and momentum of the particle and is given self-consistently by its phase space distribution functions. The collision item \( I_c \) between particles accounts for the modifications of phase space distribution function by elastic and inelastic two body collisions \cite{22, 25}.

In the present version of the BUU model, nucleon-density distribution in initial colliding nuclei is given by the Skyrme-Hartree-Fock with Skyrme \( M^* \) force parameters \cite{26}. According to Refs. \cite{3, 17, 27, 28}, the proton and neutron momentum distributions can be given by

\[
n_{\text{proton(neutron)}}(p) = \begin{cases} 
C_{1, \text{proton(neutron)}} \frac{4}{2\pi^2} \frac{2}{\xi^2}, & p \leq p_f; \\
\frac{A_{\text{proton(neutron)}}}{2\pi^2} \frac{2}{\xi^2}, & p_f < p \leq \lambda p_f; \\
0, & p > \lambda p_f.
\end{cases} \tag{2}
\]

Where \( n(p) \) is nucleon momentum distribution in initial colliding nucleus, \( p_f \) is the nuclear Fermi momentum. \( \lambda = p_{\text{max}}/p_f \), is the factor of maximum momentum of nucleon relative to the nuclear Fermi momentum, i.e., the so-called high-momentum cutoff parameter. \( A, Z, N \) are the nuclear mass number, proton number and neutron number, respectively \cite{17}. The \( C_1 \) and \( C_2 \) parameters are determined by the normalization condition

\[
4\pi \int_{p_f}^{\lambda p_f} n(p) p^2 dp = 1 \quad \text{and roughly 20\% percentage of nucleons above the nuclear Fermi momentum} \ [3], i.e.,
\]

\[
4\pi \int_{p_f}^{\lambda p_f} n(p) p^2 dp \times 100\% = 20\%. \tag{3}
\]

The 20\% percentage of nucleons with high momenta is randomly distributed in nucleus. Since high-momentum nucleon pairs are in fact bound together in nucleus, there should be a negative narrow potential well to balance their high momenta to \( p \sim p_f \) (something like quarks in nucleon or meson). Thus these nucleon pairs “seems” to have average momenta below the nuclear Fermi momentum. To simplify the question, we did not consider both the negative narrow potential well of SRC and the possible correction of the mean-field potential of nucleons. The stability of colliding nucleus is not affected much by above considerations.

The schematic illustration of the initial momentum distributions of neutron and proton in nucleus is shown in Fig. 1. We can see that proton has larger probability than neutron to be in the high-momentum tail. It is thus not surprising that proton has larger average kinetic energy than neutron. From this schematic illustration we can also see that the high-momentum cutoff parameter \( \lambda \) strongly affects the average kinetic energies of protons and neutrons.

In this model, an isospin- and momentum-dependent mean-field single nucleon potential was recently updated \cite{22, 25}, which reads

\[
U(\rho, \delta, \vec{p}, \tau) = A_u(x) \frac{\rho_\tau}{\rho_0} + A_\tau(x) \frac{\rho_\tau}{\rho_0} \\
+ B \left( \frac{\rho}{\rho_0} \right) \delta \left( 1 - x \delta \right) - 8x\tau \frac{B}{\sigma + 1} \frac{\rho^{\sigma - 1}}{\rho_0^\sigma} \delta \tau, \\
+ 2C_{\tau, \tau} \frac{x^2 \rho^{\sigma - 1}}{1 + (\vec{p} - \vec{p}')^2/\Lambda^2}, \tag{4}
\]

where \( \tau, \tau' = 1/2(-1/2) \) for neutrons (protons), \( \delta = (\rho_n - \rho_p)/(\rho_n + \rho_p) \) is the isospin asymmetry, and \( \rho_n, \rho_p \) denote neutron and proton densities, respectively. The parameter values \( A_u(x) = -31.96 \) - 125.34 \( x \) MeV, \( A_\tau(x) = -101.96 + 125.34 \frac{x}{1 + (\vec{p} - \vec{p}')^2/\Lambda^2} \), \( B = 141.96 \) MeV, \( C_{\tau, \tau} = -38.2 \) MeV, \( C_{\tau, \tau'} = -122 \) MeV \( \sigma = 1.265 \), and \( \Lambda = 630.24 \) MeV/c are obtained by fitting seven empirical constraints of the saturation density \( \rho_0 = 0.16 \) fm\(^{-3}\), the binding energy \( E_0 = -16 \) MeV, the incompressibility \( K_0 = 230 \) MeV, the isoscalar effective mass \( m^*_s = 0.7 \) m, the
single-particle potential \( U_{\infty}^0 = 75 \text{ MeV} \) at infinitely large nucleon momentum at saturation density in symmetric nuclear matter, the symmetry energy \( S(\rho) = 32 \text{ MeV} \) and the symmetry potential \( U_{\infty}^{\text{sym}} = -35 \text{ MeV} \) at infinitely large nucleon momentum at saturation density \( \rho_0 \). \( f_+(\vec{r}, \vec{p}) \) is the phase-space distribution function at coordinate \( \vec{r} \) and momentum \( \vec{p} \) and solved by using the test-particle method numerically. Different symmetry energy’s stiffness parameter \( x \) can be used in the above single nucleon potential to mimic different forms of the symmetry energy predicted by various many-body theories \([31]\) without changing any property of the symmetric nuclear matter and the symmetry energy at normal density. Note here that in this study, in contrast to \( \pi^- \), \( \pi^+ \) production in nucleus-nucleus collisions at intermediate energies in fact does not depend on the symmetry energy or symmetry potential parameters used \([32]\). This is the other reason why we use \( \pi^+ \), not \( \pi^- \) to probe the high-momentum cutoff parameter in nucleus.

According to baryon effective masses, the isospin-dependent baryon-baryon (BB) scattering cross section in medium \( \sigma_{\text{BB medium}}^{\text{elas}} \) is reduced compared with their free-space value \( \sigma_{\text{BB free}}^{\text{elas}} \) by a factor of \([22]\)

\[
R_{\text{medium}}(\rho, \delta, \vec{p}) = \frac{\sigma_{\text{BB elastic}}^{\text{medium}}}{\sigma_{\text{BB elastic}}^{\text{free}}} = \left(\frac{\mu_{\text{BB}}^*}{\mu_{\text{BB}}^0}\right)^2,
\]

where \( \mu_{\text{BB}}^* \) and \( \mu_{\text{BB}}^0 \) are the reduced masses of the colliding baryon-pair in free space and medium, respectively. Note here that this form of reduced elastic baryon-baryon scattering cross section in medium agrees well with our recent study \([33]\). Since the inelastic baryon-baryon scattering cross section in medium is less known but crucial for \( \pi \) production \([34]\), we in the present model extend the above reduced factor \( R_{\text{medium}}(\rho, \delta, \vec{p}) \) to inelastic baryon-baryon scattering. Other treatments related to \( \pi \) production and absorption are similar with that in Ref. \([34]\).

### III. RESULTS AND DISCUSSIONS

It is known that at relative lower incident beam energy of heavy-ion collisions at intermediate energies, different density-distributions of nucleons in nuclear initialization have evident effects in nucleus-nucleus collisions \([35]\). It is not clear if different momentum-distributions of nucleons in the colliding nuclei also have effects in nucleus-nucleus collisions. Fig. 2 shows \( \pi^+ \) production as a function of high-momentum cutoff parameter \( \lambda \) of colliding nucleus in the Au + Au collisions at 0.4 and 1 GeV/nucleon incident beam energies. One can clearly see that as the high-momentum cutoff parameter \( \lambda \) increases, more \( \pi^+ \)'s are produced. Larger high-momentum cutoff parameter \( \lambda \) causes larger nucleon average kinetic energy, especially proton average kinetic energy \([17]\), thus the average center-of-mass energy of proton-proton collision also becomes larger. As a consequence more \( \pi^+ \) are produced in nucleus-nucleus collision \([22]\). This is the reason why one sees in the upper panel of Fig. 2 more \( \pi^+ \)'s are produced with large high-momentum cutoff parameter \( \lambda \). As incident beam energy increases, the initial movement of nucleons in nucleus becomes less important in nucleus-nucleus collisions. We thus see in the lower panel of Fig. 2 at 1 GeV/nucleon incident beam energy, \( \pi^+ \) production is less sensitive to the high-momentum cutoff parameter \( \lambda \).

In order to cancel out some uncertainties and reduce systematic errors, one usually uses the ratio of different kinds (especially the same kind but different charge states or isospins \([36]\)) of particles as observables in heavy-ion collisions. Fig. 3 shows such ratio of pion production with different charge states as a function of high-momentum cutoff parameter \( \lambda \) in Au + Au collisions. Because in neutron-rich nuclei, with SRC, protons have a greater average kinetic energy than neutrons \([17]\), the average center-of-mass energy of proton-proton collision is larger.
than that of neutron-neutron collision, especially for large high-momentum cutoff parameter $\lambda$. Since proton-proton collision mainly accounts for $\pi^+$ production and neutron-neutron collision mainly accounts for $\pi^-$ production [22], one thus sees the value of $\pi^-/\pi^+$ ratio evidently decreases with the high-momentum cutoff parameter $\lambda$. Again, at 1 GeV/nucleon incident beam energy the value of $\pi^-/\pi^+$ ratio is not sensitive to the high-momentum cutoff parameter $\lambda$. However, it is very worth noting that $\pi^-/\pi^+$ ratio is also a symmetry-energy-sensitive observable [37, 38] and presently the high-density behavior of the nuclear symmetry energy is still not well constrained [39]. Therefore $\pi^-/\pi^+$ ratio is not suitable to probe the high-momentum cutoff parameter.

Since $\pi^+$ production is not sensitive to the nuclear symmetry energy [32], but sensitive to the high-momentum cutoff parameter $\lambda$ only at relative low beam energy, one can construct the ratio of $\pi^+$'s produced from low and high incident beam energies to probe the high-momentum cutoff parameter. Shown in Fig. 4 is the ratio of $\pi^+$ productions at 0.4 and 1 GeV/nucleon incident beam energies as a function of high-momentum cutoff parameter $\lambda$. As expected, the ratio of $\pi^+$ multiplicity produced respectively from low and high beam energies are still very sensitive to the high-momentum cutoff parameter $\lambda$. By comparison with the FOPI pion production data [40], a constraint of $\lambda = 1.8 \pm 0.4$ is obtained. This result is similar to that in Ref. [18].

In nucleus-nucleus collisions at different incident beam energies, the densities of nuclear matter formed in nucleus-nucleus collisions are different [41]. As density increases, SRC of nucleons may become stronger, thus more and more protons move to the high momentum tail [19]. However, the high excitation energy of formed nuclear matter in nucleus-nucleus collisions may cancel out the effect of nucleon-nucleon short-rang correlations [42]. We therefore did not consider 2-nucleon SRC in nuclear transport at beam energies above 400 MeV/nucleon.

IV. CONCLUSIONS

In conclusions, based on the transport model we studied how the high-momentum cutoff parameter $\lambda$ affects $\pi$ production in nucleus-nucleus collision at intermediate energies. It is found that $\pi^+$ production in nucleus-nucleus collision at lower beam energy is very sensitive to the value of the high-momentum cutoff parameter $\lambda$. By comparing the ratio of $\pi^+$ productions at respectively low and high incident beam energies with the FOPI data, a constraint on the high-momentum cutoff value is obtained. Constraints on the high-momentum cutoff parameter $\lambda$ in nucleus have impacts in the studies of nuclear force at short range, in the construction of the transport model of heavy-ion collisions at intermediate energies, in the studies of equation of state of dense nuclear matter and the nuclear symmetry energy at super-density, or in the physics of neutron stars, etc.

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