Determination of Viscoelastic Properties of human Carotid Atherosclerotic Plaque by Inverse Boundary Value Analysis

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Abstract. In this study, we assessed the mechanical response of samples from human atherosclerotic diseased media and fibrous cap via uniaxial tensile testing. Results show a pronounced hysteresis phenomenon caused by viscoelasticity during the loading-unloading process. An inverse analysis method with finite element modeling was employed to identify the material parameter values for a viscoelastic anisotropic (VA) constitutive model through matching simulation predictions of load-displacement curves with experimental measurements. The identified material parameter values can be used in simulation studies of diseased human carotid arteries, including investigations of inflation processes associated with stenting or angioplasty.

1. Introduction
Atherosclerotic plaque rupture is the main cause of acute cardiovascular events such as myocardial infarction and stroke, which cause life-threatening consequences [1, 2]. Plaque rupture is a complicated process involving interactions among the arterial wall, blood pressure, flow-induced shear stress on the fibrous cap, and clinical interventions (stenting and angioplasty).

Finite element modeling has been used in the analysis of the fibrous cap failure phenomenon to evaluate stress, strain and damage and to assess the vulnerability of the plaque tissue under supraphysiological expansion [3]. Numerical predictions of the mechanical behavior of plaque tissue are dependent on the availability of material properties, including viscoelastic properties. The instantaneous maximum stresses inside the plaque tissue are governed by viscoelasticity, which tends to reduce the degree of damage under dramatic and large deformation conditions. The viscoelastic response of arterial tissues plays an important role in the interaction between the arterial wall and vasoactive drugs, hypertension, or vascular trauma [4, 5]. Given the complicated interaction modalities and the biomechanical behavior of diseased vascular tissues during clinical interventions, there is a pressing need to characterize the viscoelastic properties of atherosclerotic plaque tissues.
In this study, uniaxial tension experiments on human carotid plaque diseased media and fibrous caps were performed. A viscoelasticity formulation by Holzapfel [6] was incorporated into the Holzapfel-Gasser-Ogden (HGO) hyperelasticity model [7], leading to the development of a viscoelastic anisotropic (VA) model for artery materials including plaque tissues. The VA model parameter values for layer-specific plaque tissues were then identified through inverse modeling of the tensile experimental data. The VA model was implemented in a general-purpose finite element code [8] via user subroutines, enabling finite element simulations of the uniaxial experiments during the inverse analysis procedure.

2. Material and methods

2.1. Experimental process
In this study, specimens were made of diseased media (DM) and fibrous cap (FC) obtained from human carotid artery after endarterectomies. The experimental setup is similar to the uniaxial tensile tests of porcine abdominal aortas [9, 10]. The diseased media and fibrous cap were isolated, washed in PBS and dissected from the perivascular tissue. One end of the specimen was gripped by a pair of clamps connected to an actuator (Bose ELF 3200, Biodynamic Co, MN), which controlled the sequential loading-unloading cycle. The other end was fixed by a pair of clamps connected to load cell (Bose ELF 3200, Biodynamic Co, MN) for load data recording. The uniaxial tensile loading process was recorded by a microscopic computer vision system which was placed at a fixed distance in the front of the specimen [11], as shown in Figure 1.

Figure 1. Experimental process: (a) schematic of carotid artery media and fibrous cap specimens obtained from endarterectomies which were cut into strips transversely (A: axial direction; C: circumferential direction); (b) experimental setup of stress relaxation and uniaxial tensile tests; (c) specimen setup (zoomed-in view).

2.2. Theoretical framework

2.2.1. Viscoelastic Anisotropic Model for Soft Material. The constitutive model for describing the arterial tissue is acquired from a decomposition of the stress expressions into three different parts; the viscoelastic stress response (non-equilibrium state) [12], the volumetric part and the isochoric stress response (equilibrium condition).

2.2.2. Anisotropic Hyperelastic Model: The Holzapfel-Gasser-Ogden (HGO) Model. The free energy potential $\Psi$ per unit reference volume of arterial material in a decoupled form is expressed as:

$$\Psi = \frac{1}{D} \left( \frac{J^2 - 1}{2} - \ln J \right) + \frac{\mu}{2} (I_1 - 3) + \frac{k_1}{2k_2} \left[ e^{k_2[I_{1}+(1-3\kappa)I_{41}-1]} - 1 \right] + \frac{k_1}{2k_2} \left[ e^{k_2[I_{1}+(1-3\kappa)I_{42}-1]} - 1 \right]$$

(1)
where $\frac{1}{\bar{\rho}}$ is analogous to the bulk modulus of the material, $I_1 = \text{tr}(\bar{C})$ denotes the first invariant of $\bar{C}$, and $\mu$ is the neo-Hookean parameter, which characterizes the shear modulus of the material without fibers; $I_{41} = a_{01} \cdot \bar{C} a_{01}$ and $I_{42} = a_{02} \cdot \bar{C} a_{02}$ are tensor invariants equal to the square of the stretch in the direction of $[a_{01}] = [\cos \gamma, \sin \gamma, 0]$ and $[a_{02}] = [\cos \gamma, -\sin \gamma, 0]$, respectively [13]

The constitutive parameter $k_1$ is related to the relative stiffness of fibers, which is determined from mechanical tissue experiments; $k_2$ is a dimensionless stiffness parameter. The parameter $\kappa$ is the fiber dispersion parameter.

2.2.3. Viscoelastic Anisotropic (VA) Model. A viscoelastic anisotropic (VA) model was developed based on the HGO model [14, 15] by incorporating a viscoelasticity formulation [16]. The second Piola-Kirchhoff stress tensor of VA model $S_{m+1}$ at time $t_{m+1}$ is given by [17, 18]:

$$ S_{m+1} = \left( S_{\text{vol}}^\infty + S_{g}^\infty + S_{f}^\infty + \sum_{\alpha=1}^{n} Q_{\alpha} \right)_{m+1} $$  \hfill (2)

where “$\infty$” expresses the equilibrium condition when the time approaches infinity.

The non-equilibrium stress tensor $Q_{\alpha (m+1)}$ includes mechanical response from matrix material $Q_{g\alpha (m+1)}$ and fibers $Q_{f\alpha (m+1)}$ at time $t_{m+1}$ which are denoted as:

$$ Q_{g\alpha (m+1)} = \exp \left( - \frac{\Delta t}{T_{g\alpha}} \right) \left[ \exp \left( - \frac{\Delta t}{T_{g\alpha}} \right) Q_{g\alpha (m)} - \beta_{g\alpha} (S_{g}^\infty)_{m} \right] + \beta_{g\alpha} \exp \left( - \frac{\Delta t}{T_{g\alpha}} \right) (S_{g}^\infty)_{m+1} $$ \hfill (3)

$$ Q_{f\alpha (m+1)} = \exp \left( - \frac{\Delta t}{T_{f\alpha}} \right) \left[ \exp \left( - \frac{\Delta t}{T_{f\alpha}} \right) Q_{f\alpha (m)} - \beta_{g\alpha} (S_{f}^\infty)_{m} \right] + \beta_{f\alpha} \exp \left( - \frac{\Delta t}{T_{f\alpha}} \right) (S_{f}^\infty)_{m+1} $$ \hfill (4)

The Cauchy stress tensor $\sigma_{m+1}$ at time $t_{m+1}$:

$$ \sigma_{m+1} = \left( J^{-1} F^T S F \right)_{m+1} $$ \hfill (5)

In the above equations, $\Delta t$ is the time increment from time $t_{m}$ to $t_{m+1}$; $T_{g\alpha} (\alpha = 1-n)$ and $T_{f\alpha}$ represent the relaxation time for ground matrix material and collagen fibers, respectively. In addition, $\beta_{g\alpha}$ and $\beta_{f\alpha}$ denote the non-dimensional parameter for ground matrix material and fibers, respectively. For the finite element analyses, a single set of viscoelastic parameters such as $T_{g1}, \beta_{g1}, T_{f1}$ and $\beta_{f1} (\alpha = 1)$ was used.

3. Inverse analysis method

Inverse analysis utilizes the history of variables (e.g., force, displacement and stress) measured in an experiment (e.g., uniaxial tensile loading) to identify a set of model parameter values for the test. This set of parameter values can be used in a model of the experiment to predict the history of the variables that provide a best fit to the measured history according to an error minimization criterion [19, 20]. In the current study, the objective function in the error minimization criterion describes the difference between the predicted and measured force history and is given by

$$ f(\mu, k_1, k_2, \gamma, \kappa, T_{g1}, \beta_{g1}, T_{f1}, \beta_{f1}) = \sum_{i=1}^{n} \left[ F_{pi}(\mu, k_1, k_2, \gamma, \kappa, T_{g1}, \beta_{g1}, T_{f1}, \beta_{f1}) - F_{ei} \right]^2 $$ \hfill (6)

where $F_{pi}(\mu, k_1, k_2, \gamma, \kappa, T_{g1}, \beta_{g1}, T_{f1}, \beta_{f1})$ and $F_{ei}$ are predicted and measured force data, respectively, at the $i$th increment. A reasonable set of parameters values was obtained when the objective function is minimized to an acceptable value.

4. Numerical implementation

In order to identify the VA model material parameters for the human carotid plaque tissue, inverse modeling of the uniaxial experiment was used to automatically predict and compare the load-displacement data with the experimental measurements. A set of converged parameter values were acquired when the objective function diminished to a small value and the convergence criteria was met (Figure 2).
5. Results
The data of load vs. stretch ratio from uniaxial tensile tests were used for identification of material parameters of VA model. The material parameter values of VA model, Eq. (5), were obtained from the best fit between experimental and simulation results through inverse analysis method, which is shown in Figure 3. The parameter values for samples of diseased media and fibrous cap are shown in Table 1 and Table 2, respectively.

In our study, the viscoelastic response is well captured through a matrix-fibers linear viscoelastic anisotropic model, in which the parameters ($\mu, k_1, k_2, \gamma$ and $\kappa$) represent the anisotropic hyperelastic behavior of the arterial tissue, and the remaining parameters ($T_{g1}, B_{g1}, T_{f1}$ and $\beta_{f1}$) characterize the viscoelastic response from the matrix material and the collagen fibers. The material parameters obtained from the loading-unloading cycle of uniaxial experiments are diversely distributed which may be attributed to sample variability among individual patients.
Table 1. Best-fit parameter values for the viscoelastic anisotropic (VA) model of human diseased media samples under uniaxial tensile tests.

| Sample | $\mu$ (kPa) | $k_1$ (kPa) | $k_2$ | $\gamma$ (angle) | $\kappa$ | $T_{g1}$ | $\beta_{g1}$ | $T_{f1}$ | $\beta_{f1}$ | Residual |
|--------|-------------|-------------|-------|------------------|---------|---------|-------------|---------|-------------|----------|
| DM1    | 0.04        | 3010.00     | 36.80 | 50.08            | 0.31    | 0.18    | 0.02        | 1.07    | 0.43        | 0.00057  |
| DM2    | 0.12        | 3610.00     | 81.11 | 48.75            | 0.31    | 0.18    | 0.02        | 1.36    | 0.53        | 0.00011  |
| DM3    | 1.17        | 650.00      | 0.02  | 41.85            | 0.21    | 0.18    | 0.02        | 1.97    | 0.41        | 0.00001  |
| DM4    | 0.03        | 2470.00     | 33.69 | 61.22            | 0.30    | 0.18    | 0.02        | 1.36    | 0.53        | 0.00009  |
| DM5    | 0.01        | 6230.00     | 100.00| 57.03            | 0.32    | 0.18    | 0.02        | 0.75    | 1.11        | 0.00065  |
| Average|             |             |       |                  | 0.29    | 0.18    | 0.02        | 1.42    | 0.58        | 0.00029  |
| SD     | 0.45        | 1812.20     | 35.79 | 6.74             | 0.04    | 0.00    | 0.00        | 0.49    | 0.27        | 0.00027  |

Table 2. Best-fit parameter values for the viscoelastic anisotropic (VA) model of human fibrous cap samples under uniaxial tensile tests.

| Sample | $\mu$ (kPa) | $k_1$ (kPa) | $k_2$ | $\gamma$ (angle) | $\kappa$ | $T_{g1}$ | $\beta_{g1}$ | $T_{f1}$ | $\beta_{f1}$ | Residual |
|--------|-------------|-------------|-------|------------------|---------|---------|-------------|---------|-------------|----------|
| FC1    | 0.30        | 2333.00     | 72.43 | 30.89            | 0.33    | 0.18    | 0.02        | 1.08    | 0.54        | 0.000569 |
| FC2    | 1.04        | 5570.00     | 13.02 | 30.84            | 0.27    | 0.01    | 2.92        | 1.84    | 0.61        | 0.000114 |
| FC3    | 1.05        | 1370.70     | 21.37 | 43.57            | 0.21    | 5.17    | 1.39        | 1.50    | 0.45        | 0.00032  |
| FC4    | 1.15        | 479.00      | 13.96 | 36.33            | 0.25    | 0.34    | 1.20        | 0.69    | 1.09        | 0.000003 |
| FC5    | 0.60        | 512.00      | 17.39 | 41.17            | 0.20    | 1.57    | 2.73        | 0.54    | 1.23        | 0.00009  |
| FC6    | 0.80        | 1451.00     | 2.81  | 24.55            | 0.26    | 0.12    | 7.08        | 0.77    | 0.84        | 0.000138 |
| Average|             |             | 23.50 | 34.56            | 0.25    | 1.23    | 2.56        | 1.07    | 0.79        | 0.000144 |
| SD     | 0.30        | 1735.29     | 22.60 | 6.53             | 0.04    | 1.84    | 2.25        | 0.46    | 0.29        | 0.000197 |

6. Conclusion
The aim of this study was to characterize the viscoelastic properties of human carotid artery atherosclerotic plaque tissues. The passive viscoelastic responses were manifested in terms of the load-stretch curves for loading-unloading cycles in uniaxial tensile tests. The VA model parameter values were identified through an automated inverse analysis procedure by matching finite element simulation predictions with experimental measurements of the observed viscoelastic responses from the uniaxial tensile tests. The proposed VA model, along with the identified model parameter values, will enable solutions of boundary-value problems in terms of predicting arterial mechanical responses and arterial wall stress distributions under various loading scenarios.

Several sources of approximations were involved in the current study. The test sample geometry models were reconstructed from images of front and side views of the samples, assuming the sample thicknesses is constant along the sample width direction. Secondly, the samples under uniaxial tensile test did not undergo pure tensile deformation because shearing forces may occur inside the samples with large width-length ratio. Thirdly, the material parameter values identified from this study were identified from a single stretch ratio which may not fully characterize the mechanical behavior of the arterial tissue.

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