Parity assignments in $^{172,174}\text{Yb}$ using polarized photons and the $K$ quantum number in rare earth nuclei

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The 100 % polarized photon beam at the High Intensity γ-ray Source (HIγS) at Duke University has been used to determine the parity of six dipole excitations between 2.9 and 3.6 MeV in the deformed nuclei $^{172,174}\text{Yb}$ in photon scattering $(\gamma,\gamma')$ experiments. The measured parities are compared with previous assignments based on the $K$ quantum number that had been assigned in Nuclear Resonance Fluorescence (NRF) experiments by using the Alaga rules. A systematic survey of the relation between $\gamma$-decay branching ratios and parity quantum numbers is given for the rare earth nuclei.

Low-lying dipole excitations in heavy nuclei have been studied extensively using the Nuclear Resonance Fluorescence (NRF) or photon scattering method, which provides a model-independent way to determine excitation energies, spins, decay widths, decay branchings, and transition probabilities. The parity of a nuclear state can be determined by either scattering unpolarized $\gamma$-rays and measuring polarization in the exit channel, or using linearly polarized $\gamma$-ray beam and measuring the azimuthal angular distribution of the scattered photons. For deformed even-even nuclei the $K$ quantum number of $J = 1$ states can be assigned within the validity of the Alaga rules from the electromagnetic decay branching ratio

\[ R = \frac{B(\Pi I_1; 1^+_K \rightarrow 2^+_1)}{B(\Pi I_1; 1^+_K \rightarrow 0^+_1)} = \frac{\Gamma_1}{\Gamma_0} \cdot \frac{E^3(1^+_K \rightarrow 0^+_1)}{E^3(1^+_K \rightarrow 2^+_1)} \]  

(1)

where $\Gamma_1$ and $\Gamma_0$ denote the decay widths to the $2^+_1$ and $0^+_1$ levels, respectively, and $E(1^+_K \rightarrow 2^+_1)$ and $E(1^+_K \rightarrow 0^+_1)$ correspond to the energies of these transitions.

In general, there is no relation between the $K$ quantum number and the parity of a $J = 1$ excitation. However, restricting oneself to dipole excitations that carry the largest part of the excitation strength one selects collective modes for which certain selection rules may exist. Within realistic calculations for deformed nuclei in the framework of the interacting boson model (IBM) with $s$- and $d$-proton and neutron bosons (sd-IBM-2), where negative parity states are not included, all $J^\pi = 1^+$ levels have a branching ratio corresponding to $K = 1$ (e.g. the bandheads of the $K = 0$ octupole vibrational band). This suggests that those states with $J = 1$ and branching ratios corresponding to $K = 0$ have negative parity. Positive parity has generally been assumed in previous works for all $K = 1$ excitations in the energy range of the $M1$ scissors mode for calculating the summed $B(M1)$ strength, if no direct parity assignments were available. This rule of thumb was supported by $\gamma$-ray polarization measurements analyzing Compton-scattering asymmetries of the NRF $\gamma$-ray lines in some deformed nuclei of the Nd to Er even-even isotopic sequences. It was concluded that at least the strong dipole excitations in sufficiently axially-symmetrically deformed nuclei decay according to the Alaga rules for $\Delta K = 1$ (0) for positive (negative) parity.
The $K$ quantum number is a good quantum number only in the case of axially symmetric deformation and the aforementioned correlation between the parity of a strong dipole excitation and its decay branching ratio to the ground band has been tested in these nuclei for which their axial symmetry was considered to be well established. Very recently, it has been proposed that heavy rare earth nuclei in the mass region $A \approx 170$ might be close to the critical point of an axially-symmetric–to–triaxial shape phase transition. For triaxial shapes the Alaga rules do not hold and in this case the assignment of positive parity from the branching ratio is lacking a basis. If the stable nuclei in the $A \approx 170$ mass region would indeed exhibit a more pronounced triaxiality than the lighter stable rare earth nuclei, then the previous compilations of the total scissors mode's $M1$ excitation strength using parity assignments on the basis of decay branching ratios might contain a systematic error for nuclei with mass numbers $A \approx 170$.

The interacting boson model in its proton-neutron version (sd-IBM-2) represents a simple and useful model for the description of the evolution of the quadrupole-collective structure of heavy nuclei and of proton-neutron mixed-symmetry states such as the scissors mode. The description of triaxial deformation in the framework of the IBM involves two alternate approaches that differ significantly for the description of mixed-symmetry states. Either one may include cubic terms of the quadrupole-quadrupole interaction in an $F$-spin symmetric Hamiltonian for generating triaxial eigenstates, or one might consider a situation close to the dynamical symmetry limit SU(3)** of the standard two-body IBM-2 with structural parameters of opposite sign, $\chi_{\nu} = -\chi_{\pi}$ [11]. The latter approach breaks $F$-spin symmetry and could cause a significant amount of $F$-spin mixing into the low-energy states if the Majorana interaction were not too strong [10].

The properties of mixed-symmetry states with $F$-spin quantum number $F = F_{\text{max}} - 1$ are very sensitive to the strength of the Majorana interaction and, thus, to the amount of $F$-spin mixing in the low-energy wave functions [12]. In fact, $F$-spin multiplets of states of neighboring nuclei, including Yb nuclei, have been observed for symmetric states with $F = F_{\text{max}}$ [11] and the scissors mode with $F = F_{\text{max}} - 1$ [13]. These observations suggest that $F$-spin is not severely broken in the corresponding nuclei which may rule out an SU(3)**-like description even for the nuclei in the mass region $A \approx 170$. More information, particularly on the goodness of the $K$ quantum number for mixed-symmetry states of nuclei in this mass region, would be highly desirable for estimating the relevance of $F$-spin breaking descriptions of a possible triaxiality at $A \approx 170$.

The measured summed dipole excitation strength in Yb isotopes attributed to the $1^+$ scissors mode on the basis of decay branching ratios in the energy region 2–4 MeV is in good agreement with the observations for neighboring nuclei if one assumes positive parity for all $K = 1$ states [12]. In order to confirm these parity assignments and to extend the systematics between branching ratios and parities of dipole states in rare earth nuclei to the mass region $A \approx 170$, we performed a series of experiments for determining parity quantum numbers for some of the strongest dipole excitations of $^{172,174}$Yb.

Parity quantum numbers of strongly dipole excited states can be assigned in NRF experiments by using a linearly polarized photon beam for excitation and by measuring the azimuthal angular distribution of the scattered photons about the polarization plane of the incident beam. For $0^+ \rightarrow 1^+ \rightarrow 0^+$ elastic resonant photon scattering on the ground state of even-even nuclei due to dipole excitation, the resulting angular distribution is given by [14]

$$W(\theta, \phi) = \frac{3}{2} + \frac{3}{4} \left(1 - \cos^2 \theta\right) \cdot [\pi_1 \cos(2\phi) - 1],$$

(2)

where $\theta$ is the polar scattering angle with respect to the incident photon beam, $\phi$ is the azimuthal angle of the reaction plane with respect to the polarization plane of the incident $\gamma$-beam and $\pi_1$ is the parity quantum number of the excited state (+1 or −1). As shown in Fig. 1, the distribution for $E1$ transitions has a minimum at $\phi = 90^\circ$ and a maximum at $\phi = 0^\circ$, while the situation is vice versa for $M1$ transitions. Therefore, it is sufficient to measure the angular distribution at $\phi = 0^\circ$ and $\phi = 90^\circ$ to determine the parity of a dipole state unambiguously. For $\theta = 90^\circ$ the minima of the distributions are zero and the analyzing power is maximal [10]:

$$\Sigma = \frac{W(90^\circ, 0^\circ) - W(90^\circ, 90^\circ)}{W(90^\circ, 0^\circ) + W(90^\circ, 90^\circ)} \approx \pi_1 = \begin{cases} +1 \text{ for } J = 1^+ \\ -1 \text{ for } J = 1^- \end{cases}.$$

(3)

The $\gamma$-beam at the High Intensity $\gamma$-ray Source (HI-$\gamma$S) at the Duke Free Electron Laser Laboratory combined with a set-up of four HPGe detectors has been...
proven to be useful for this type of experiment \cite{10}. The beam is generated by Laser Compton backscattering of photons from relativistic electrons and is quasi-monoenergetic. A free electron laser is used, which is driven by the same electron beams. The backscattered laser photons are boosted in energy by six orders of magnitude resulting in \(\gamma\) rays of several MeV with tunable energies. By adjusting the wavelength of the FEL and the energy of the electrons, one can select the desired energy of the \(\gamma\) rays. In the present work \(\gamma\) rays in the vicinity of 3 MeV had an energy spread, as defined mainly by the collimator opening angle, of about 2 \(\%\) (FWHM) and the degree of horizontal linear polarization was better than 99 \(\%\). We used a 45 g natural Yb\(_2\)O\(_3\) target in order to be able to observe excitations in different Yb isotopes simultaneously. The natural abundances of \(^{172}\)Yb and \(^{174}\)Yb are 21.9 \(\%\) and 31.8 \(\%\), respectively. We measured at three mean energies of 2930 keV, 3005 keV and 3550 keV. Fig. 2 shows the summed spectra of all detectors at a beam energy of 3550 keV. A coincidence with the electron pulses had been used to reduce the beam uncorrelated room background. Peaks corresponding to elastic and inelastic photon scattering off the Yb nuclei are visible in the vicinity of the energy of the \(\gamma\) ray beam. Below this energy, the spectrum consists only of continuous background mainly due to Compton scattering on various components of the experimental apparatus. The spectrum measured with a germanium detector positioned directly in the beam path is also included in the figure. The peak at 3550 keV exhibits the energy distribution of the beam. The average intensity on target was about \(5 \cdot 10^6\) \(\gamma\) rays per second. The four Ge(HP) detectors were positioned at \(\theta = 90^\circ\) and \(\phi = 0^\circ, 90^\circ, 180^\circ\) and \(270^\circ\) at a distance of 5.5 cm from the target.

Fig. 2 shows the energy spectra obtained with a beam energy of 3005 keV in the polarization plane of the beam and perpendicular to it. Three \(J = 1\) states were populated at this beam energy. The dipole states at 3009 keV and 3002 keV were known from previous experiments \cite{14}. The first peak only appears in the upper spectrum, yielding \(J^\pi = 1^+\), while the second is only observed in the lower spectrum, yielding \(J^\pi = 1^-\). In \(^{174}\)Yb, the inelastic decay of the 3009 keV level to the first \(2^+_1\) state at 76.5 keV is visible in both spectra, as one would expect from the nearly isotropic angular correlation for the \(J = 1 \rightarrow J = 2\) transition in a \(0 \rightarrow 1 \rightarrow 2\) cascade. A new \(J^\pi = 1^-\) state at 2983 keV was also observed. The existence of this state was confirmed by reanalyzing data from previous NRF experiments using non-polarized bremsstrahlung \cite{14}. The experimental asymmetry is given by

\[
\epsilon = \frac{N_\parallel - N_\perp}{N_\parallel + N_\perp} = q \cdot \Sigma ,
\]

where \(N_\parallel\) and \(N_\perp\) denote the efficiency-corrected peak areas in the spectra obtained by detectors located in the polarization plane of the incident photon beam or perpendicular to it. The experimental sensitivity \(q\) of the setup is somewhat smaller than 1.0 as a result of the finite size of the target and the finite solid angles of the detectors. The asymmetries measured for the six excitations covered in our experiment are summarized in Table I and shown in Fig. 3. We note that only states within the narrow energy range of the beam are excited. The calculated sensitivity for the present setup is \(q = 0.761(5)\),
TABLE I: Measured asymmetries, assigned parities and branching ratios for the dipole excitations in $^{172,174}$Yb that were observed in this experiment.

| $E_x$ (keV) | Isotope | $\epsilon$ | $J^a$ (h) | $R_{exp}$ $^a$ |
|------------|---------|------------|-----------|----------------|
| 2920       | $^{174}$Yb | 0.77(13)   | $1^+$     | 0.44(8)        |
| 2983       | $^{172}$Yb | 1.00(45)   | $1^+$     | 0.37(13)       |
| 3002       | $^{172}$Yb | 0.57(19)   | $1^+$     | 0.55(11)       |
| 3009       | $^{174}$Yb | -0.61(19)  | $1^-$     | 2.75(52)       |
| 3527       | $^{174}$Yb | -0.69(9)   | $1^-$     | 1.87(28)       |
| 3562       | $^{174}$Yb | 0.81(7)    | $1^+ b$   | 0.50(10)       |

$^a$ taken from [14]
$^b$ parity also measured in [15]

which is in agreement with the observed asymmetries. For all cases, a parity assignment with a confidence level of more than four standard deviations is possible. All $J_K = 1_0$ states have negative parity, and all $J_K = 1_1$ states have positive parity. I.e., for the strongest dipole excitations that carry the largest fraction of the respective total dipole strength the previous parity assignments done on the basis of decay branching ratios were correct and no discrepancy is found for the summed $B(M1)$ strength calculated in [14].

In the present work we are dealing with well-deformed rotational nuclei with $E(4^+_1)/E(2^+_1) \geq 3.0$, where $K$ is expected to be a good quantum number. The correlation between the branching ratio $R_{exp}$ and the parity in well deformed rare earth nuclei is shown in Fig. 4 for all $J = 1$ states where both values are known. The additional data points are taken from previous NRF experiments on $^{150}$Nd [10], $^{160}$Gd [21], $^{162,164}$Dy [21], $^{166,168,170}$Er [22], and $^{176}$Hf [7]. All positive parity states have a branching ratio of $R_{exp} < 1$, and most are consistent with a $K = 1$ assignment, which is in agreement with the predictions for the scissors mode. Therefore, the assignment of negative parity for $J_K = 1_0$ states seems to be justified. But there are also negative parity states with $R_{exp} < 1$, especially some with a branching ratio that is consistent with $K = 1$. The $K$-quantum number does not provide enough information to make a parity assignment of $J_K = 1_1$ states. However, most of the comparatively strong excitations with $R_{exp} \approx 0.5$, which corresponds to $K = 1$, are positive parity states. Fig. 4 shows the summed dipole strength over all states shown in Fig. 5 in bins of $\Delta R_{exp} = 0.1$ interval. For better comparison the $\Gamma/E^3$ values are shown, which differ only by a factor of $2.87 \cdot 10^{-3}$ and $2.59 \cdot 10^{-1}$ from the corresponding $B(E1)$ and $B(M1)$ values, respectively. The fraction of electric dipole strength in the region below $R_{exp} \leq 1$ is almost negligible compared to the magnetic dipole strength in this region. Therefore, the calculated summed $B(M1)$ strength reported in previous works [4, 7] seems to be an acceptable upper limit.

It should be emphasized, that due to the low sensitivity of Compton polarimetry, the parity is nearly exclusively known only for the strongest excitations. The correlation between branching ratio and parity presented above may not be observed in the case of weaker excitations.

Let us finally return to the Yb nuclides studied in this work and address the goodness of the $K$ quantum number of the scissors mode of $^{172,174}$Yb. Our polarity measurement has not only proven the validity of previous positive parity assignments to some strong $M1$ excitations around 3 MeV as discussed above, but it supported the data on Yb nuclides used for previous systematic investigations of the scissors mode’s excitation energy and strength. With respect to excitation energy and decay pattern we consider the identified strongest fragments of the scissors mode (see $1^+$ states in Tab.I) as representatives for the mode. All four strong $M1$ excitations show,
FIG. 6: Summed dipole strength in bins of $\Delta R_{\text{exp}} = 0.1$. In case of $M1$ transitions the given values can be converted in $B(M1)$ strength by $B(M1)/\mu_N^2 = 2.59 \cdot 10^{-3} \cdot \Gamma/E^3$. In case of $E1$ transitions by $B(E1)/e^2 fm^2 = 2.87 \cdot 10^{-3} \cdot \Gamma/E^3$.

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