Few-nucleon physics

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Few-nucleon physics

Winfried Leidemann
Dipartimento di Fisica, Università di Trento and INFN (Gruppo Collegato di Trento),
via Sommarive 14, I-38100 Trento, Italy
E-mail: leideman@science.unitn.it

Abstract. Recent progress in few-nucleon physics achieved by Italian research groups is described. Results for relativistic effects in elastic three-nucleon form factors (Rome group), hadronic and electromagnetic observables in three- and four-nucleon systems with special emphasis on three-nucleon force effects (Pisa and Trento groups), and spectra of nuclei with more than 4 nucleons (Padua group) are presented.

1. Introduction
The field of few-nucleon physics constitutes an essential part of nuclear physics. It consists of ab initio calculations of hadronic and electromagnetic observables with a given well-defined Hamiltonian of an \( A \)-nucleon system, where exact solutions of the quantum mechanical \( A \)-body system are requested for bound and/or scattering states. In particular one can consider bound-state spectra and observables from hadronic reactions like scattering lengths, phase shifts, total and differential cross sections. A polarization of beam, target, and/or final particles leads to many other hadronic polarization observables. In addition reactions with electroweak probes (photons, electrons, neutrinos) without and with polarization degrees of freedom give further information on the nuclear dynamics and electroweak structure of few-nucleon systems. For the latter one has to consider a further important aspect, namely the consistency of the electroweak currents with the nuclear Hamiltonian.

Few-nucleon physics has many objectives: development of nuclear force models (nucleon-nucleon (NN) and three-nucleon (3N) forces) and check whether even higher many-body forces are necessary, test of nuclear force models via two- and three-nucleon hadronic observables with possible feedback on force model, further test of nuclear dynamics in electromagnetic reactions with two- and three-nucleon systems; the study of the electroweak structure of few-nucleon systems; the investigation of the importance of relativistic effects; the extension of nuclear force models beyond pion threshold; the determination of neutron electroweak properties in two- and three-nucleon electroweak observables; the determination of observables with astrophysical relevance.

Here follows a very brief summary on the state of the art for some selected aspects of few-nucleon physics. NN potential models have a long history (see e.g. [1]). Today we have the so called modern realistic NN potentials like AV18 [2], various Nijmegen models [3], and CD Bonn [4] as well as chiral models [5, 6, 7]. The potentials [2, 3, 4, 5] all lead to a perfect or almost perfect description of various thousands of NN scattering data for energies up to pion threshold, while the chiral potential [6, 7] is constructed more in the original spirit of effective field theory where open parameters are fitted to low-energy data only. The history of 3N forces started with
the inclusion of the Fujita-Miyazawa term (consideration of \( \Delta \) degrees of freedom). Today, also for 3N forces there are various conventional and chiral models, for example [8, 9, 10, 11, 12].

In the last decade the calculation of bound states with realistic forces has made great progress. Nowadays, such calculations are standard for \( A < 4 \). For \( A = 4 \) there exists a benchmark calculation [13], where seven different theory groups have participated by applying seven different theoretical methods. Calculations for p-shell nuclei bound-states with Green’s function Monte Carlo (GFMC) and no core shell model (NCSM) techniques have reached \( A = 10 \) and even beyond (see e.g. [14, 15]). There is hope that with additional methods like coupled cluster [16] and auxiliary field diffusion Monte Carlo [17] even considerably larger systems can be treated.

The calculation of continuum state wave functions has also made quite an improvement, but rigorous solutions with all possible channels exist only up to \( A = 3 \) (see e.g. [18]). There also exist exact calculations for the \( A = 4 \) [19, 20, 21] and the \( A = 5 \) [22] cases, however, for energies below the three-body break-up threshold. On the other hand with the Lorentz integral transform (LIT) approach [23, 24] there exists an alternative method for the calculation of continuum reaction observables, where an explicit calculation of continuum state wave functions is avoided. In fact the LIT approach reduces a scattering state problem to a bound-state like problem, which then can be solved with typical bound-state techniques. Up to now the LIT method has been applied to the calculation of inclusive electromagnetic observables with realistic nuclear forces for three- [25, 26, 27, 28, 29] and four-body systems [30, 31] and with semi-realistic forces for \( A = 6, 7 \) [32, 33]; for \( A = 4 \) also exclusive reactions have been considered (see e.g. [34]).

Concerning the electromagnetic interaction with few-nucleon systems it should be pointed out that review articles have been published for the two- [35] and three-nucleon systems [36] quite recently.

Here we will not consider few-nucleon physics beyond pion threshold, but it has to be mentioned that there are two longer recent reports on this topic with authors from Padua [37] and Trento [38].

In the following the work of the Italian few-nucleon physics community of the last two years is reviewed. To this end the work of the Pisa, Padua, Rome and Trento few-nucleon groups is considered. It should be mentioned that the term group has to be interpreted in a wider sense, because there are also external collaborators. Since the author of this article is from Trento he would like to mention long-term external collaborators of the Trento group explicitly: Sonia Bacca (TRIUMF), Nir Barnea (Hebrew University), Victor Efros (Kurchatov Institute Moscow) and Edward Tomusiak (University of Victoria).

The paper is organized as follows. In section 2 the basic ingredients of the hyperspherical harmonic (HH) expansion technique, heavily used by Pisa and Trento groups, are briefly described. In section 3 bound and low-energy scattering states of three- and four-body nuclei, calculated with realistic nuclear forces, are discussed. Reactions with electroweak probes are illustrated in section 4, where (i) relativistic effects on the elastic \((e,e')\) form factors of the trinucleons and (ii) LIT calculations for inelastic inclusive reactions with three- and four-body systems are considered. Finally, in section 5, results for weakly bound nuclei with the multi channel scattering approach (MCAS) are presented for \( A > 4 \).

2. Bound and continuum states with the hyperspherical harmonics expansion

In this section the HH expansion approach is briefly reviewed. More details and further references can be found in two detailed recent papers [19, 24].

The first step of an HH expansion consists of rewriting the \( 3(A - 1) \) internal coordinates of an \( A \)-nucleon system, the Jacobi coordinates \( \xi_i = (\xi_i, \theta_i, \phi_i) \), as follows. One keeps the \( 2(A - 1) \) angles \( \theta_i \) and \( \phi_i \) and introduces the hyperradius \( \rho \left( \rho^2 = \sum_i \xi_i^2 \right) \) and additional \( (A - 2) \) angles \( \alpha_i \). This leads to a subdivision of the space into a hyperradial \( \rho \) and a hyperspherical \( \Omega \). As
an example the defining equations for the additional angle $\alpha_1$ in the $A = 3$ case are given: $\xi_1 = \rho \cos \alpha_1$, $\xi_2 = \rho \sin \alpha_1$. Next one considers the internal kinetic energy of the $A$-body system,

$$T = -\frac{1}{2m} \sum_{i=1}^{A-1} \nabla_{\xi_i}^2,$$

where $m$ is a reference mass. With hyperspherical coordinates one has

$$\sum_{i=1}^{A} \nabla_{\xi_i}^2 = \frac{\partial^2}{\partial \rho^2} + \frac{3A - 4}{\rho} \frac{\partial}{\partial \rho} + \hat{K}^2,$$

where $\hat{K}$ is the generalized angular momentum operator, which has no $\rho$ dependence and is given in terms of the hyperspherical coordinates $\theta_i$, $\phi_i$, and $\alpha_i$. The hyperspherical harmonic functions $Y_{\lambda K}$ are eigenfunctions of $\hat{K}^2$ with eigenvalues $K(K + 3A - 5)$ (note that $[K]$ symbolizes a set of quantum numbers). For $A = 2$ one has $Y_{\lambda K} \rightarrow Y_{lm}$ and $K \rightarrow l$, where $Y_{lm}$ are the usual spherical harmonics.

An HH expansion has the following form:

$$\sum_{[K]} Y_{[K]}(\Omega) R_{[K]}(\rho)$$

with the hyperradial part

$$R_{[K]}(\rho) = \sum_{j} c_{[K],j} L_j(\rho) \exp(-b\rho),$$

where $L_j$ are the Laguerre polynomials. The expansion is carried out up to a maximal $K$-value $K_{\text{max}}$. The HH functions need to have proper symmetries in order to form together with the nuclear spin and isospin functions an antisymmetric nuclear wave function.

A delicate problem is the convergence of the expansion. Due to the NN short range repulsion one has usually a very slow convergence. The Pisa and Trento groups use various methods to accelerate the convergence: (i) introduction of additional two-nucleon correlation functions $f_{ij}$ (HH → CHH) and multiplication of each HH basis state with $\omega = \prod_{i<j} f_{ij}$, the method is also used with a simplified correlation factor for one pair only (PHH, Pisa group [19]) and with spin-isospin dependent correlations $f_{ST}(r_{ij})$ (Trento group [24]); (ii) selection of specific channels (Pisa group); (iii) calculation of an effective interaction by unitary transformation to the $K_{\text{max}}$-dependent model space (EIHH [39, 40]), similar as is done in the NCSM approach.

In addition the Pisa group performs HH expansions in momentum space [41]. This facilitates the treatment of non-local NN potentials like [4]. In this case one has a hypersphere with hypermomentum $Q = \sum_i \vec{k}_i$, where the $\vec{k}_i$ are the conjugate momenta of the Jacobi vectors $\xi_i$.

As mentioned above the Trento group uses the LIT method and does not calculate continuum states explicitly. The Pisa group, however, also performs HH expansions of two-fragment scattering states relevant for $N+Y \rightarrow N+Y$ reactions, where $N$ is a nucleon and $Y$ a two- or three-body nucleus. The N-Y scattering state with quantum numbers $[J] = (L, S, J, J_z)$ is given by

$$\Psi_{[J]} = \Psi_{[J]}^C + \Psi_{[J]}^A,$$

where $\Psi_{[J]}^C$ describes the region where N-Y are close, whereas $\Psi_{[J]}^A$ takes care of the asymptotic region. For $\Psi_{[J]}^C$ an HH expansion is carried out, whereas $\Psi_{[J]}^A$ is written as

$$\Psi_{[LSJz]}^A = N \Psi_{A, \text{free}}^{[LSJz]} + \sum_{L'S'} R_{[LSJz]}^{L'S'} \Psi_{A, \text{irregular}}^{[L'SJz]}.$$
Figure 1. (Color online) Various observables for \(pd\) elastic scattering at \(E_{cm} = 2\) MeV. The curves for the various potential models are defined in the insert. Experimental data from [46].

The matrix \(R^J\) is related to the K-matrix which has eigenvalues \(\tan \delta_{LSJ}\) and where \(\delta_{LSJ}\) are the phase shifts of N-Y scattering. The matrix elements \(R^J_{LS,S',L'}\), the normalization constant \(N\) and the expansions coefficients of \(\Psi^J_C\) are determined applying the Kohn variational principle.

3. Three- and four-body bound and low-energy scattering states

In [19] the Pisa group applies the HH expansion technique in order to calculate high-precision results for three- and four-nucleon bound and zero-energy scattering states. There bound-state energies of \(^3\)H, \(^3\)He, and \(^4\)He for semi-realistic and realistic nuclear forces are listed. In case of the zero-energy scattering one finds results for \(nd/pd\) doublet and quartet scattering lengths and \(n^3\)H/\(p^3\)He singlet and triplet scattering lengths with various realistic potentials.

Concerning the bound-state energies three realistic NN potentials (AV18 [2], CDBonn [4], N3LO-Idaho [5]) and three 3N forces (UIX[8], TM [10, 19], N2LO [12]) in the combinations AV18+UIX, CDBonn+TM, and N3LO-Idaho+N2LO have been taken into account. Since the 3N forces are fitted to describe the three-body binding energy it is interesting to see what happens in the four-body case. The results show that for all the considered cases the \(^4\)He binding energy is slightly overestimated, namely by 0.06 MeV (N3LO-Idaho+N2LO), 0.16 MeV (AV18+UIX), and 0.6 MeV (CDBonn+TM).

In case of the zero-energy scattering (no results given for CDBonn and CDBonn+TM) the following conclusions are drawn in [19]: (i) \(nd\) and \(pd\) quartet scattering lengths are almost model independent, while there is a large effect of the 3N force on the doublet scattering lengths. Once
Figure 2. The nucleon to nucleon spin transfer coefficients in $N\bar{d}$ elastic scattering at $E_{\text{lab}} = 22.7$ MeV. The crosses are the $pd$ data from [44, 45]. HH expansion results with AV18 potential without (dashed) and with Coulomb force (solid).

Figure 3. (Color online) As in figure 2, but results from Faddeev calculations (no Coulomb force included) with various potential models: light band (AV18, CDBon, Nijm I, Nijm II), dark band (additional consideration of TM-3N force), solid line (AV18+UIX). In addition x-es for the pseudo-neutron data (see text).

The 3N force is included also the latter results are rather model independent. Comparison with experiment is made for $nd$ scattering. A very good agreement is found for $^4a_{nd}$, while there are small discrepancies in comparison to an experimental high-precision result for $^2a_{nd}$. The relatively best agreement with theory is obtained for the N3LO-Idaho+N2LO potential. In case of the 4-body zero-energy scattering the 3N force does not lead to important contributions for the considered isospin T=1 channel. The experimental situation is a little bit less settled than in the three-body case and therefore the comparison with experimental results is not yet very conclusive. Where available, comparisons to results of other calculations for bound-state energies and four-body scattering lengths are made. Quite good and fairly good agreements are obtained for bound-state energies and scattering lengths, respectively.

A more detailed description and interpretation of the results for binding energy and zero-energy scattering can be found in the contribution of A Kievsky in this volume.

In figure 1 results for $pd$ scattering observables at somewhat higher energies ($E_{\text{cm}} = 2$ MeV) [42] are shown using as NN and 3N forces AV18 or N3LO-Idaho and UIX or N2LO, respectively. It is evident that the theoretical total cross section shows only negligible potential model effects.
and that there is an excellent agreement with experiment. Very good agreements are also found for $T_{20}$ and $T_{22}$. $T_{20}$ is only rather weakly potential model dependent, whereas for $T_{22}$ stronger effects are visible and the 3N force is necessary in order to have a good description of the minimum. Similar potential model effects can be observed for $T_{11}$, $T_{21}$, and $A_y$, but the agreement with experiment is not very good. In particular $T_{11}$ and $A_y$ show rather large discrepancies for the comparison of theory and experiment (well known $A_y$-puzzle). A more detailed discussion of $Nd$ scattering at low energies is found in the contribution of L E Marcucci in this volume.

**Figure 4.** The nucleon to deuteron spin transfer coefficients in $Nd$ elastic scattering at $E_{lab}^{N} = 22.7$ MeV. Notation as in figure 2.

**Figure 5.** (Color online) The nucleon to deuteron spin transfer coefficients in $Nd$ elastic scattering at $E_{lab}^{N} = 22.7$ MeV. Notation as in figure 3.

In figures 2-5 results for spin transfer coefficients for the reactions $d(\vec{N},\vec{N})d$ and $d(\vec{N},\vec{d})N$ are shown [43] at $E_{lab}^{N} = 22.7$ MeV. In figures 2 and 4 results from the Pisa group are given for the AV18 potential without ($nd$ scattering) and with ($pd$ scattering) Coulomb force. The Coulomb force effect is quite large for the nucleon to nucleon spin transfer coefficient $K_{X}^{X'}$ (figure 2) and for the nucleon to deuteron spin transfer coefficient $K_{Z}^{Y'Z'}$ (figure 4). The agreement with the experimental data [46] for the $d(\vec{p},\vec{p})d$ and $d(\vec{p},\vec{d})p$ scattering reactions is not sufficiently satisfying even if the Coulomb force is included. This is not surprising since the 3N force has not been considered. In the results of a Faddeev calculation [43] shown in the figures 3 and 5 the 3N force is included, however the Coulomb force is not taken into account. Therefore the proton data of figures 2 and 4 have been corrected for the effect of the Coulomb force resulting in pseudo experimental data for the $d(\vec{n},\vec{n})d$ and $d(\vec{n},\vec{d})n$ reactions. In general this leads to a rather good agreement of theory and pseudo-experiment, however, with the nucleon to deuteron spin transfer coefficient $K_{Z}^{Y'Z'}$ one finds also one exception. In fact for this case one has already
a good agreement with an NN force only, which is then considerably spoiled by the inclusion of a 3N force.

Now a somewhat different argument is discussed, namely the neutron spin rotation in $\vec{n}d$ scattering. Such an effect can only be obtained by parity violating (PV) components in the NN potential. In [47] two models for the PV potential are considered. The so called DHH with $\pi$, $\rho$ and $\omega$ exchanges, where weak meson-nucleon couplings are used, and pionless EFT with four-nucleon contact terms. The calculation is performed perturbatively in first order in the PV potentials using the AV18+UIX potential for the $n$-$d$ wave function. The results show only a rather weak PV model dependence and lead to an effect which is about an order of magnitude larger than in $\vec{n}p$ scattering. Many more details of the calculation are given in the contribution of L Girlanda to this volume.

The references of additional articles on few-nucleon physics of the Pisa group from the last two years, not discussed in this overview, are [48, 49, 50, 51, 52].

4. Reactions with electroweak probes

Perturbation-induced reactions can be divided in inclusive and exclusive processes. In the former case the final state of the particle system undergoing the perturbation is not observed. In the latter case the final state is at least partially observed, like e.g. in an $(e, e'p)$ reaction, where besides the scattered electron also an outgoing proton with energy $E_p$ and scattering angle $\Omega_p = (\theta_p, \phi_p)$ is detected.

Inclusive cross sections of perturbation-induced reactions have the following general form

$$\frac{d^2\sigma}{d\omega d\Omega_{ext}} = \alpha_{ext} \sum_{i=1}^{M} f_i(\omega, q, \theta_{ext}) R_i(\omega, q),$$

where $\omega$ and $q$ are energy and momentum transfer of the external probe to the particle system, $\Omega_{ext} = (\theta_{ext}, \phi_{ext})$ denotes the scattering angle of the external probe, $\alpha_{ext}$ is a constant characteristic for the external probe, and $f_i$ are kinematic functions. The functions $R_i$ describe the various responses of the particle system to the external probe and thus contain information about the dynamics of the particle system. They are defined as follows

$$R_i(\omega, q) = \sum_f |\langle f|\Theta_i|0\rangle|^2 \delta(\omega - (E_f - E_0)).$$

Here $E_0$ and $|0\rangle$ are ground state energy and wave function of the particle system under consideration, $E_f$ and $|f\rangle$ denote final state energy and wave function of the final particle system, and $\Theta_i$ is the operator inducing the response function $R_i$.

In the following only inclusive reactions will be considered and thus no further information about exclusive reactions is given here.

4.1. Trinucleon electromagnetic form factors and the light-front Hamiltonian dynamics

We start the discussion of the electromagnetic probes by considering elastic inclusive responses of the three-nucleon systems, i.e. the trinucleon electromagnetic form factors. With the intention to study relativistic effects they are calculated by the Rome group in continuation of their work on deuteron electromagnetic observables (see e.g. [55]). Their calculation is a manifestly relativistic calculation in the so called light-front (LF) relativistic Hamiltonian dynamics. In this approach one constructs operators under the condition that they fulfill the commutation rules of the Poincaré generators. Two different cases appear: (i) operators not containing the interaction and (ii) operators depending on the mass of the interacting system. It is important to note that the interacting mass operator depends upon intrinsic variables only and has to satisfy
Figure 6. Preliminary results for $^3$H charge form factor with $^3$H wave function from Pisa group and with LF nucleon form factors from [53] and with one-body electromagnetic operator only. Thick lines: LF calculations with AV18 NN potential (solid) and additional UIX 3N force (dash-dotted). Thin lines: non relativistic calculation with AV18 (solid) and AV18+UIX (dash-dotted). Data from [54].

The same constraints given by the Galilean group, and thus has the same properties implemented in non relativistic quantum mechanics. Therefore one can exploit realistic wave functions for light nuclei from non relativistic calculations, that depend upon Jacobi coordinates, in order to evaluate matrix elements of a Poincaré covariant current operator [56].

In figures 6-9 preliminary results of the Rome group for the three-nucleon electromagnetic elastic form factors are shown. As had to be expected relativistic effects are small for momentum transfers below $3 \text{ fm}^{-1}$, but become more important beyond the form factor minima. Two-body electromagnetic operators have not yet been considered in the calculation and thus an agreement with experimental data is not yet obtained. Concerning the two-body electromagnetic operators see also G. Salmé’s contribution in this volume and [57].

Additional recent publications on theoretical few-nucleon physics of the Rome group are found in [58, 59].

4.2. Inelastic inclusive electromagnetic reactions

As already mentioned above, the Trento group uses the LIT [23] in order to calculate inelastic reactions. The LIT method is motivated by the fact that an explicit calculation of continuum wave functions is avoided. Nonetheless the LIT approach allows the ab initio calculation of a reaction cross section, where a many-body continuum is involved in initial and/or final states, but without requiring the knowledge of the generally complicated many-body continuum wave function. In fact, the scattering problem is reduced to a calculation of a localized function with an asymptotic boundary condition similar to a bound-state wave function. Such an approach was already proposed by Efros in 1985, but with the Stieltjes instead of the Lorentz integral transform [60]. However, it has been found that the application of the Stieltjes transform is problematic since it leads to serious inversion problems [61]. Though the LIT can also be applied
to exclusive reactions (see [24]) here only inclusive response functions as defined in Eq. (8) are considered.

For the LIT calculation of $R_i$ of Eq. (8) one proceeds in the following way. First the ground state wave function $\langle \Psi \big| \rangle$ of the nucleon system under consideration is calculated. Then one solves the equation

$$ (H - E_0 - \sigma_R - i\sigma_I) \langle \tilde{\Psi}_i \rangle = \Theta_i \langle \Psi \big| \rangle, $$

where $H$ is the Hamiltonian of the particle system and $\sigma_R/I$ are parameters, whose meaning is explained below. Since the eigenvalues of $H$ have to be real the homogeneous version of Eq. (9) has only the trivial solution $\tilde{\Psi}_i = 0$ and thus Eq. (9) has a unique solution. In addition, due to the asymptotically vanishing ground state wave function $\langle \Psi \big| \rangle$ also the right-hand-side of Eq. (9) vanishes asymptotically. Therefore, and because of the complex energy, $E_0 - \sigma_R - i\sigma_I$, $\tilde{\Psi}_i$ has a similar asymptotic boundary condition as a bound state. It means that $\tilde{\Psi}_i$ is a so-called localized function, i.e. square-integrable with a norm $\langle \tilde{\Psi}_i \big| \tilde{\Psi}_i \rangle$. This has very important
Figure 10. Effects of the various contributions on $R_T(\text{^3He})$ in the quasi-elastic region at $q=174$ (upper left), 250 (upper right), 400 (lower left), and 500 MeV/c (lower right): one-body (dotted), one-body + implicit MEC via Siegert operator (dashed), one-body + $\pi$-MEC + $\rho$-MEC (dashed dotted), one-body + $\pi$-MEC + $\rho$-MEC + additional MEC via Siegert operator (solid).

consequences: even if one aims at a calculation of a reaction cross section in the continuum, one is not confronted with a scattering state problem any more, in fact one needs to apply only bound-state techniques for the solution of Eq. (9).

The key point of the LIT method consists of the fact that the Lorentz integral transform $L_i$ of the response function $R_i,$

$$L_i(\sigma_R, \sigma_I, q) = \int R_i(\omega, q) \mathcal{L}(\omega, \sigma_R, \sigma_I) d\omega,$$  \hspace{1cm} (10)

is related to the norm $\langle \tilde{\psi}_i | \tilde{\psi}_i \rangle,$ which can be obtained from the solution of Eq. (9). In fact one has

$$L_i(\sigma_R, \sigma_I, q) = \langle \tilde{\psi}_i(\sigma_R, \sigma_I, q) | \tilde{\psi}_i(\sigma_R, \sigma_I, q) \rangle$$  \hspace{1cm} (11)

($q$ dependence of $L_i$ and $\tilde{\psi}_i$ will be dropped in the following). In Eq. (10) $\mathcal{L}$ is a Lorentzian centered at $\sigma_R$ with a width $\Gamma = 2\sigma_I$:

$$\mathcal{L}(\omega, \sigma_R, \sigma_I) = \frac{1}{(\omega - \sigma_R)^2 + \sigma_I^2}.$$  \hspace{1cm} (12)
Now also the meaning of the parameters $\sigma_{R/I}$ becomes evident: $\sigma_I$ represents a kind of energy resolution, while with $\sigma_R$ a given energy range can be scanned.

With the above equations the principle idea of the LIT method can be explained: one solves the LIT equation (9) for many values of $\sigma_R$ and a fixed $\sigma_I$ and calculates $L_i(\sigma_R, \sigma_I = \text{const})$, and then one inverts the transform in order to determine $R_i(\omega, q)$. The inversion of the LIT has to be made with some care, since, in principle, it constitutes an ill-posed problem. In fact the LIT method has to be understood as an approach with a controlled resolution. The resolution is governed by $\sigma_I$, i.e. structures with a width considerably smaller than $\sigma_I$ cannot be easily resolved. The LIT inversion is ill-posed, since one can always add an oscillating response with a wave length much smaller than $\sigma_I$, which will then lead to an almost identical result for the transform. Therefore the LIT inversion is made with a regularization scheme (for details see [62, 63]). In case that a single resonance with a width much smaller than $\sigma_I$ is present this will be realized in the inversion process even if it cannot be completely resolved (see discussion of Coulomb monopole resonance in [64, 65]). If one wants to resolve such structures precisely one has to further reduce $\sigma_I$ (see [67]). Note that one has to make more and more accurate calculations for a smaller and smaller $\sigma_I$ in order to obtain a sufficiently converged LIT result. Thus, in principle, one can enhance the LIT resolution continuously, but one has to be aware that there can always be structures with an even smaller width that cannot be resolved. This is the cause of the ill-posed problem and also the reason why the LIT approach is a method with a controlled resolution. That the LIT approach works in an excellent way
has been shown in various benchmark calculations for two- and three-nucleon electromagnetic reactions [23, 24, 26, 27, 66, 67].

The LIT results illustrated in the following are calculated using CHH and EIHH expansions (see section 2) for three- and four-nucleon systems, respectively.

Figure 12. $R_T(\alpha)$ at $q=250$, 400, 500 MeV/c in upper, middle, lower panel. Theoretical $R_T$ with one-body (dotted) and one-body + $\pi$-MEC + $\rho$-MEC + additional MEC via Siegert operator (solid). Data from [68] (triangles), [69] (circles), and [70] (squares).

Figure 13. $R_T(\alpha)$ at $q=0.882$, 1.64, 2.47 fm$^{-1}$ in upper, middle, lower panel. Notation as in figure 12, but additional curve in upper panel for total result without Coulomb force in final state interaction (dash-dotted). Experimental data from [71].

As first LIT calculation the transverse response function $R_T(\omega, q)$ of $^3$He is discussed [29]. As NN potential the BonnRA model has been used and in addition also the TM 3N force has been taken into account. As current operators non relativistic one-body as well as $\pi$ and $\rho$ meson exchange currents (MEC) are taken. The MEC are uniquely defined by the meson theoretical BonnRA potential. Besides the explicit MEC calculation also an implicit one has been carried out by using Siegert operators. The latter lead to the dominant part of the electric multipole contributions only, while there are no magnetic multipole contributions at all via the Siegert operator. Besides the isovector MEC in the BonnRA additional MEC of isoscalar nature are present. They are in general much less important than the isovector $\pi$ and $\rho$ MEC. It is important to note that a part of the isoscalar MEC is taken into account by the Siegert operator and thus it can be checked if they become important for a specific kinematics. To this end one has to compare the result of a direct calculation of one- and two-body current operators with a result obtained in a calculation where the Siegert operator is used and where in addition the
non-Siegert contributions of the explicitly used current operators are added. A difference in the two calculations is then due to isoscalar exchange currents (for a more detailed description see [29]).

In figures 10 and 11 results for the $R_T(\omega, q)$ of $^3\text{He}$ are depicted at various momentum transfers. It is readily seen that there are rather strong MEC effects: at 15-30 MeV above threshold MEC enhance $R_T$ by more than 30% for the two higher $q$-values (very close to threshold even by up to 200%, see figure 13); they increase the quasi-elastic peak height up to about 10%. Effects due to the Siegert operator remain quite small in and below the quasi-elastic peak. The picture changes at higher energies, where Siegert contributions become more important, particularly at lower $q$. In the high-energy tail one also finds a small increase of $R_T$ due to residual exchange currents, which are taken into account only in the Siegert and not in the direct MEC calculation. In figure 12 the $R_T$ results are shown in comparison to experimental data. For $q=250$ and 400 MeV/c one finds a good agreement. However, data are not precise enough to allow a definite conclusion about the MEC contribution. As opposed to the lower $q$ cases one finds at $q=500$ MeV/c a difference between the theoretical and experimental peak positions. The shift amounts to about 5-10 MeV. Relativistic effects, in particular those arising from corrections to the kinetic energy, might be responsible for this difference. In fact in [28] it was shown for the longitudinal response function $R_L(q, \omega)$ that such effects lead at $q=500$ MeV/c to a shift of the peak position by 6 MeV. In figure 13 various $R_T$ theoretical and experimental low-energy results are illustrated at $q=0.882, 1.64$ and 2.47 fm$^{-1}$ corresponding to about 174, 324 and 487 MeV/c, respectively. One sees that the MEC contribution can be very important, e.g. at $q=487$ MeV/c one finds an increase of about 200% close to threshold. Contrary to the cases shown in figure 12 one can make a definite conclusion about the MEC contribution. It is evident that they lead to a considerably improved agreement between theory and experiment. For the two higher $q$-values theoretical and experimental results agree very well, whereas for $q=174$ MeV/c the theoretical result underestimates experimental data somewhat below 10 MeV. In the upper panel of figure 13 the effect of the Coulomb force is shown in addition. Within 2 MeV above threshold there is effect of more than 10%, while at 5 MeV above threshold the Coulomb effect still amounts to 4%.

In figure 14 preliminary $R_T$ results are shown for a calculation with AV18+UIX, where $\pi$ and $\rho$ MEC have been taken into account consistently with the AV18 potential applying the Arenhövel-Schwamb technique [76]. One sees that the results are very similar to those of BonnRA+TM potential shown in the lower panel of figure 13.

Now we turn to the $^4\text{He}$ electromagnetic responses. Already a few years ago the total $^4\text{He}$ photoabsorption cross section has been calculated with the LIT method using realistic nuclear forces [30, 73]. In the meantime also the $^4\text{He}$ longitudinal and transverse $(e, e')$ response functions have been considered with the LIT approach. In figure 15 $R_L$ results are shown. They
are obtained with AV18+UIX [65]. One sees that the 3N force reduces the quasi-elastic peak strength by about 10%. Experimental data are described quite well by our full result, but they are not precise enough to resolve the 3N force effect. In figure 16 $R_L$ is illustrated at lower $q$. One readily notes a very strong reduction at lower energies due to the 3N force, which reaches up to about 40%. The reduction cannot be attributed to a simple binding effect as becomes evident from the also shown $R_L$ results with a semi-realistic NN force (MT potential [77]). In fact, $^4$He binding energies are 24.3, 28.4, and 30.6 MeV for AV18, AV18+UIX, and MT potentials, respectively. Even though the MT energy is closer to that of AV18+UIX, the MT $R_L$ is more similar to the $R_L$ of AV18 than to the AV18+UIX $R_L$. At lower $q$ there is only one data set at 200 MeV/c [78], which is not sufficiently precise to draw concrete conclusions.

Also for the transverse response function $R_T$ there is a LIT calculation, however, presently only for the semi-realistic MT potential [64]. On the other hand in the calculation a consistent current for the MT potential has been taken into account consisting of: one-body spin current, total one-body operator, i.e. spin + convection current (IA), and a consistent MEC for the MT potential, however, the center of mass dependence of the MEC is neglected. In figure 17 the results are shown at various $q$. At higher $q$ the spin current dominates, but for $q = 100$ MeV/c the convection current presents a stronger effect. By comparing the IA with the full calculation, one can conclude that quite a strong MEC effect is found at low $q$. For $q = 100$ MeV and $\omega = 100$ MeV MEC lead to an enhancement of the strength by a factor of four with respect to the IA. At this $q$, the introduced approximation for the MEC is safely reliable. As $q$ increases the MEC effect is reduced dramatically. Unfortunately, there are no experimental data in the quasi-elastic region for the two lower $q$. For $q = 300$ MeV/c where experimental data from Bates [74] and Saclay [75] exist, one observes a tiny effect of the MEC with respect to IA result. The

![Figure 15.](image-url) (Color online) $R_L(\omega, q)$ of $^4$He at various $q$. Dashed lines: AV18; solid lines: AV18+UIX. Data from Bates [74] (squares), Saclay [75] (circles) and world data-set from [70] (triangles).
addition of the MEC leads to an increase of 4% and almost 10% at $\omega = 100$ and 150 MeV, respectively. The full calculation agrees rather well with the data of Bates in the energy range $55 \text{ MeV} \leq \omega \leq 115 \text{ MeV}$, but is lower than the data of Saclay in the quasi-elastic peak region and above. Here it should be mentioned that the approximation introduced in the MEC is estimated in [64] to be of about 10% to 20% in the peak and negligible in the tail for $q = 200$ to 300 MeV/c. Thus it does not affect the conclusion, since the overall MEC effect is small.

There are other recent publications of the Trento group not discussed here, they are found in [67, 79, 80, 81].

5. Weakly bound light nuclei with the multi channel algebraic scattering approach

Now we turn to a somewhat different subject, namely to calculations with the multi channel algebraic scattering (MCAS) method. Different from all the topics discussed above no realistic NN and 3N forces are used. In fact the Hamiltonian is purely phenomenological. The aim is the treatment of bound and low-energy scattering states of stable nuclei as well as of weakly bound nuclear systems. The MCAS starting point consists of a finite matrix representation of Heisenberg’s scattering matrix using the Sturmian theory as expansion method (see e.g. [82, 83]). Sturmians represent an efficient method for determining $S$-matrix, bound states, resonances, and scattering wave functions. Non-local and Coulomb interactions as well as the scattering process in coupled channel (CC) dynamics can be treated consistently. This is important, for example, when strong couplings to low-lying excitations of the target nucleus play a role.

It follows a brief introduction to the MCAS theory, which is taken from [85]. In case of the Schrödinger equation,

$$(E - H_0)\Psi_E = V\Psi_E,$$  \hfill (13)

one has a spectral variable $E$ and an eigenstate $\Psi_E$. However, for Sturmians one considers the following equation

$$(E - H_0)\Phi_i(E) = \frac{V\Phi_i(E)}{\eta_i(E)},$$  \hfill (14)

where $E$ is a parameter and the eigenvalue $\eta_i(E)$ is the potential scale. As boundary conditions one has (i) bound-state like and normalizable for $E < 0$ and (ii) for $E > 0$ purely
outgoing/incoming waves, which are not normalizable. The Sturmian spectrum consist of all the potential rescalings that give solution to Eq. (14) for given energy $E$ and with well-defined boundary conditions.

The CC S-matrix can be written as

$$S_{cc'} = \delta_{cc'} - i\pi \sqrt{k_c k_{c'}} \sum_i \hat{\chi}_{ci}(E^+; k_c) \frac{1}{1 - \eta(E^+)} \hat{\chi}_{c'i}(E^+; k_{c'}) ,$$

where in momentum space one has $\hat{\chi}_{ci}(E^+; k_c) = \langle k_c | V | \Phi_I(E) \rangle$. This form stands for a scattering process starting in the asymptotic channel $c$, then expanded into Sturmians, which freely propagate in the interaction region, and finally decay into the outgoing channels.

The nucleon-nucleus interaction is given as a potential matrix, where different operators are considered:

$$V_{cc'} = \sum_{n=C,LS,LL,SI} V_n \langle (LS) j I; J^\pi | O f_n (r, R, \theta, \phi, \alpha) | (L'S) j' I'; J'^\pi \rangle .$$

In the calculations of the Padua group the scattering of a nucleon with a light nucleus with a $0^+$ ground state is considered. The operators are expanded up to second order in the core
Table 1. Experimental data and theoretical results for $^7\text{Li}$ and $^7\text{Be}$ states (energies are in MeV, widths are in keV). All energies are defined with thresholds, $p^+\text{He} = 9.975$ MeV with respect to $^7\text{Li}$ ground state and $n^+\text{Be} = 10.676$ MeV with respect to $^7\text{Be}$ ground state ($^a$ for these states spin and parity are unknown, $^b$ spin-parity of this state has been assigned as $^1_2^−$).

| $J^π$ | $^7\text{Li}$ Exp. | $^7\text{Li}$ Theory | $^7\text{Be}$ Exp. | $^7\text{Be}$ Theory |
|-------|-------------------|----------------------|-------------------|----------------------|
| $^3_2^−$ | -9.975           | -9.975               | -10.676           | -11.046              |
| $^4_2^−$ | -9.497           | -9.497               | -10.246           | -10.680              |
| $^4_4^−$ | -5.323 [69]     | -5.323               | -6.106 [175]     | -6.409               |
| $^4_4^−$ | -3.371 [918]    | -3.371               | -3.946 [1200]    | -4.497               |
| $^1_2^−$ | -2.251 [80]     | -0.321               | -3.466 [400]     | -1.597               |
| $^1_2^−$ | -1.225 [4712]   | -2.244               | --                | --                   |
| $^1_2^−$ | -0.885 [2752]   | -0.885               | --                | --                   |
| $^1_2^−$ | -0.405 [437]    | -0.405               | -1.406 [?]       | -1.704               |
| $^1_2^−$ | 1.265 (260)     | 0.704 (56)           | 0.334 (320)      | -0.539               |
| $^1_2^−$ | 1.796 (1570)    | 0.727 (699)          | 0.776 [1800]     | -3.346               |
| $^1_2^−$ | 3.7 (800) $^a$  | 2.981 (990)          | 1.995 (231)      |                      |
| $^3_4^−$ | 4.7 (700) $^a$  | 3.046 (750)          | 2.009 (203)      |                      |
| $^3_4^−$ | 5.964 (230)     | 4.904 (150)          |                   |                      |
| $^3_4^−$ | 6.76 (2240)     | 6.5 (6500) $^b$     | 5.78 (1650)      |                      |

The functions $f_n(r, R, θ)$ have a standard Woods-Saxon form $WS(r)$ for $n = C, LL, SI$ and for $n = LS$ an additional derivation: $\frac{1}{r} \frac{d}{dr} WS(r)$.

Calculations with the CC Hamiltonian above can easily lead to deeply bound spurious states in the bound spectrum. They originate form a violation of the Pauli principle. The phase space corresponding to target nucleons in fully occupied shells has to be forbidden to the incoming nucleon, however, in the original CC model such a condition is missing. As described in [86] one can use the technique of orthogonalizing pseudo potentials (OPP) [87], which eliminates the spurious states by the addition of a new term in the nuclear potential.

Before describing some selected results obtained by the Padua group another recent publication should be mentioned, [88], where nonlocalities in nucleon-nucleus potentials are considered in greater detail.

There are various calculations of the Padua group with the MCAS theory: $n^+\text{C}$ system [83, 84]; $n^+\text{C}$ and $p^+\text{O}$ [89]; $N^+\text{He}$ and $N^+\text{Be}$ [90]. Here we discuss only the lightest of these systems, i.e. the $A = 7$ systems. In tables 1 and 2 results for the spectra are shown. A single CC potential has been used for all four $A = 7$ systems, however with a different OPP term for Li/He and Be/B. For the former only the $0s_{1/2}$ shells are blocked, while for the latter...
Table 2. Experimental data and theoretical results for $^7$He and $^7$B states. All energies are in MeV and relate to thresholds of $-0.445$ MeV for $n+^6$He and of $-2.21$ MeV for $p+^6$Be ($^a$ observed very recently and interpreted as a $\frac{1}{2}^-$ state, $^b$ spin-parity of this state is unknown).

| $J^\pi$   | $^7$He          | $^7$B          |
|-----------|-----------------|----------------|
| $\frac{3}{2}^-$ | Exp. 0.445 (150) 0.43 (100) 2.21 (1400) 2.10 (190) |
| $\frac{5}{2}^-$ | -- 1.70 (30) 3.01 (110) |
| $\frac{1}{2}^-$ | Exp. 1.0 (750) 2.79 (4100) 3.55 (340) |
| $\frac{3}{2}^-$ | Exp. 3.35 (1900) 3.55 (200) 5.35 (340) |
| $\frac{1}{2}^-_a$ | Exp. 6.24 (4000) 6.24 (1900) |

A more complex OPP scenario is considered (for details see [90]). It should be mentioned that in [90] also $^3$H-$^4$He and $^3$He-$^4$He scattering has been calculated in terms of a di-cluster model and a rather good description of the scattering data has been obtained.

A very recent application of the MCAS approach is the calculation of the radiative capture $^3$He($\alpha, \gamma$)$^7$Be [91]. To this end the MCAS method for nucleon-nucleus scattering has been extended to a treatment of a two-cluster system, where finite sizes and internal excitations of the cluster are taken into account. The initial and final state wave functions were calculated using the nuclear potential of [90], which had been fitted to reproduce best the states and resonance widths of $^7$Li. In figures 18 and 19 the results of the radiative capture calculation are shown. One sees that the MCAS S-factor reproduces the energy dependence of the data quite well, but overestimates the experimental S-factor somewhat. A change of the initial-state scattering state wave function by use of a rather simple repulsive hard-core model leads to a decrease of the S-factor showing that the result is rather sensitive to the wave function. However,
as the authors point out, for a consistent calculation one would need to construct a better MCAS potential. For a discussion of results from other calculations we refer to [91].

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