Load Level and Target COF of Frames with Given Reliability Indices

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Abstract

The objectives of the present paper are to evaluate probabilistically the load level of column over-designed frame structures under a specific reliability level and to present target values for the column over-design factor (COF) by which undesirable story mechanisms can be avoided probabilistically. After the linear relationship between the load level and COF is derived, the likely story mechanisms are analyzed. The basic and optimum COFs probabilistically to avoid story mechanisms are proposed and evaluated for multi-story frames. It is found that the basic and optimum COFs increase with the increase in the number of stories and decrease with the increase in target reliability level of the frame. An evaluation example of a frame with non-uniform column strength over the height is also presented.

Keywords: frame structure; failure mode; story mechanism; column over-design factor; probabilistic analysis; target reliability level

1. Introduction

In earthquake-resistant design, it is necessary to ensure that frame structures have adequate capacity to absorb earthquake energy before failure, and the entire beam-hinging pattern mode (simplified as beam-hinging mode in the following text) in which all the beams in flexure yield before the columns yield is generally considered to be preferable (Clough and Penzien 1982; Lee 1996). To ensure that frame structures collapse according to the beam-hinging mode, structures are generally designed with a column over-design factor larger than one to make the columns relatively stronger than the beams.

However, practical observations have shown that structures may not collapse according to the designed failure modes, and in contrast, considerable number of structural failures were caused by the story failure mechanism, even though the structures were designed and expected to collapse according to the entire beam-hinging mode. An important factor incurring such a phenomenon is the uncertainties included in the member strengths and earthquake loads. These uncertainties may change the designed COF and lead to a possible reversal of balance between the strengths of the beams and columns, and ultimately the structure may collapse according to an undesirable failure mode, such as a partial column failure pattern, or a story mechanism. Due to uncertainties in member strengths and external loads, it is difficult to absolutely ensure that structures will collapse according to the preferable failure modes, a better strategy may be to ensure the structures collapse according to the preferable mode probabilistically, namely, to ensure an occurrence probability of the preferable failure mode larger than the probabilities of undesirable modes.

Many studies have been performed on various aspects of COF requirements of structures. In a study by Kuwamura et al. (1989), the variation of the yield strength was taken into account in COF evaluation. Kawano et al. (1998) investigated COF by a finite element method and some basic properties of the COF were elucidated. Nakashima and Sawaizumi (1999) simplified a frame structure into a fishbone shaped model to perform dynamic analysis with earthquake motions as input, and indicated that the necessary COF value to ensure no plastic hinges formed in columns increased approximately linearly with the increase of peak velocity of the earthquake wave. Dooley and Bracci (2001) stated that the minimum COF requirements should be about 2.0 to have a significant probability of satisfying the building performance objectives of a reinforced concrete frame during a design-basis seismic event. This value is much greater than the 1.2 that is recommended by the American Concrete Institute for concrete building design (ACI 1999). Almost all the aforementioned investigations were conducted on specific structures using specific earthquake inputs, but it is also necessary to develop a general approach that can be easily implemented to evaluate the target COFs for structures with arbitrary
numbers of stories and bays.

Ono et al. (2000) proposed a probabilistic evaluation method to evaluate the target COF that limits the failure probabilities of undesirable failure modes to a specific tolerance. However, since the criterion used in that investigation is to ensure the frame structures collapse strictly according to the beam failure pattern, in which any hinging in columns except the first-story column bases is not allowable, a much larger value of COF is needed in cases where both the load and the member strengths are uncertain than in cases with deterministic values of those parameters.

In design application, some hinges in columns are allowable, particularly in severe earthquakes, only if the structures still tend to fail in a preferable entire failure pattern, and basically only the story mechanisms should be avoided. Based on this point, the present paper presents an evaluation of the load level of column over-designed frame structures designed with a specific reliability level and estimation of the target values of COF that avoids the undesirable story mechanisms probabilistically.

2. Definitions and Assumptions

2.1 Definition of COF

The COF used in the investigation is defined for each beam-column node as the ratio of the sum of the moment capacity of columns to the sum of moment capacity of beams as:

\[
C_{of}(k) = \frac{\sum_{i} \mu_{mci}}{\sum_{i} \mu_{mbi}}
\]  

where \(k\) is the node number and \(\mu_{mci}\), \(\mu_{mbi}\) are the mean values of the ultimate moment strength of the column and beam connected to the \(k\)th node, respectively.

2.2 Computational assumptions

The following basic assumptions are used in this study:

(1) The strengths of beams and columns are designed to give the structure an identical COF at all beam-column nodes.

(2) The external loads consist only of the lateral earthquake loads triangularly distributed over the structure. This assumption is often used in simple seismically design.

(3) The plastic moment capacities of sections are statistically independent of one another, and independent of the earthquake load.

(4) All random variables are assumed to follow a lognormal distribution (Torrent 1978). Because the coefficient of variation (COV) of an earthquake load is generally taken to be 0.6-0.7 (Kanda 1993; AIJ 1993) and considering other uncertainties included in the load model, the COV for the lateral forces is assumed to be 0.8.

3. Determination of Load Level

In the entire beam-hinging mode, plastification occurs at all beam edges and bases of the first-story columns as shown in Fig.1. For an \(n\)-story, \(m\)-span frame structure, the performance function of the entire beam-hinging mode \(G(X)\), of which \(X\) is the vector of random variables of structural system, is generally given by:

\[
G(X) = 2\sum_{i=1}^{m} M_{bi} + 2 \sum_{j=1}^{n-1} M_{bij} + \sum_{i=1}^{m} M_{ci} - \sum_{j=1}^{n} jP_j
\]

where \(M_{bi}\) is the moment strength of the beam of the \(i\)th span and \(M_{bij}\) is the moment strength of the beam of the \(i\)th and \(j\)th story, \(M_{ci}\) is the moment strength of an interior column, and \(M_{cij}\) is the moment strength of an exterior column, \(P_j\) is the load acting on the \(j\)th story of the structure, \(n\) and \(m\) are the number of stories and spans, respectively, and \(h\) is the story height. The member strength and load are considered as random variables, while \(n\), \(m\), and \(h\) are assumed to be deterministic.

The mean strengths of structural members are determined through Eq. (3) to make the structure designed with the same COF in all beam-column nodes. Equation (4) gives the coefficient of variation of each random variable.

\[
\mu_{bai} = \mu_{bij} \mu_{cij} = 2 \mu_{bij} \mu_{cij} = 2 \mu_{cij} \frac{V_1}{V_2}
\]

where \(\mu_{bai}\) is the mean yield strength of the top beam, \(\mu_{bij}\) is the mean value of the load applied to the first floor of the structure, \(V_1\) and \(V_2\) in Eq. (4) are coefficients of variation of member strength and load, respectively. The mean value and COV defined for
From Eq. (5) the mean value of load applied to the first top beam is assumed to be 106.2 kN·m; the mean strengths of other members can be obtained by applying Eq. (3). For example, given \( C_{of} = 1.1 \), the mean strength of 212.4 kN·m can be obtained for beams B2 and B1, 116.8 kN·m for exterior columns C1, C2, and C3, and 233.6 kN·m for columns C4, C5, and C6. For other cases of COF, the member strengths can be similarly designed.

Using the first-order reliability method (FORM) (Ang and Tang 1984), the failure probability of the entire beam-hinging mode is obtained and shown in Fig.3., from which one can see that the load level has a great effect on the failure probability of the entire beam-hinging mode. In the present study, the target COF investigation was conducted under a specific reliability level, which means that for a given reliability index of the entire beam-hinging failure mode, the load levels are adjusted to ensure that the first order reliability index \( \beta_f \) remains the same as the target reliability index \( \beta_1 \) for frame structures designed with various COFs. The target reliability index \( \beta_1 \) reflects the reliability of the structure with respect to the entire beam-hinging mode, and it should be determined according to the significance and safety requirements of the structure.

In order to understand the relationships between \( \mu_p, C_{of}, \mu_m, \) and \( \beta_f, \) consider the reliability index of the entire beam-hinging mode with the performance function described in Eq. (2). When \( V_i \) is so much smaller than \( V_t \) that it can be ignored, the second moment reliability index can be approximately given as (Zhao et al. 2002):

\[
\beta_{SM} \approx [2m \mu_b (2n-1+C_{of}) - \mu_p] \frac{\sum_{j=1}^{n} j^2}{V_2 \mu_p} \frac{\sum_{j=1}^{n} j^4}{V_2 \mu_p}
\]

From Eq. (5) the mean value of load applied to the first story of structure is obtained as:

\[
\mu_p \approx 2 m \mu_b (2n-1+C_{of}) / (h \beta_{SM} V_2) \frac{\sum_{j=1}^{n} j^4}{\sum_{j=1}^{n} j^2}
\]

Because \( \beta_{SM} \) is unknown, Eq. (6) cannot be directly used to determine \( \mu_p \) under a specific reliability level of FORM. However, since the higher order moments of the performance function Eq. (2) are not affected by COF when \( V_i \) is far smaller than \( V_t \) (Zhao et al. 2002), one can easily understand that \( \mu_b \) is basically a linear function of COF. Meanwhile, \( \beta_{SM} \) is also not affected by the mean value of the member strength or the number of bays, so \( \mu_p \) also changes linearly with respect to \( \mu_b \) and \( n \).

Therefore, the mean value of load can be given by the following formula as a linear function of \( m, \mu_b \), and \( C_{of} \):

\[
\mu_p = m \mu_b (a + b C_{of})
\]

where \( a \) and \( b \) are constants independent of \( \mu_b, m, \) and \( C_{of} \) but related to \( \beta_f \) and \( n \). Comparing Eq. (6) with Eq. (7), one can see that:

\[
b = a/(2n-1)
\]

then the mean value of load can be given as:

\[
\mu_p = a m \mu_b (1 + \frac{1}{2n-1}) C_{of}
\]

Table 1. Values of \( a \) for Different \( V_i \) (\( \times 10^{-4} \))

| n | \( V_i = 0.1 \) | \( V_i = 0.2 \) | \( V_i = 0.3 \) | \( V_i = 0.4 \) |
|---|---|---|---|---|
| \( \beta_f = 2 \) | \( \beta_f = 3 \) | \( \beta_f = 4 \) | \( \beta_f = 2 \) | \( \beta_f = 3 \) | \( \beta_f = 4 \) | \( \beta_f = 2 \) | \( \beta_f = 3 \) | \( \beta_f = 4 \) |
| \( n=2 \) | 10.96 | 5.59 | 2.84 | 10.72 | 5.46 | 2.77 | 10.27 | 5.18 | 2.62 |
| \( n=3 \) | 7.54 | 3.99 | 2.06 | 7.41 | 3.97 | 2.00 | 7.11 | 3.79 | 1.93 |
| \( n=4 \) | 5.52 | 3.03 | 1.59 | 5.42 | 3.01 | 1.57 | 5.22 | 2.88 | 1.50 |
| \( n=5 \) | 4.25 | 2.40 | 1.29 | 4.20 | 2.39 | 1.27 | 4.05 | 2.29 | 1.22 |
| \( n=6 \) | 3.38 | 1.96 | 1.07 | 3.34 | 1.96 | 1.06 | 3.22 | 1.88 | 1.02 |
| \( n=7 \) | 2.74 | 1.65 | 0.91 | 2.70 | 1.64 | 0.91 | 2.60 | 1.51 | 0.88 |

Table 1. gives the values of \( a \) varying with \( \beta_f \) and \( n \) for \( V_i = 0.1, 0.2, 0.3, \) and 0.4.

Assuming \( V_t = 0.1 \), the mean values of load corresponding to different COFs are shown in Fig.4.(a) in terms of a solid line, from which a linear relationship between \( \mu_b \) and \( C_{of} \) can be recognized. The \( \mu_b-m \) curves and \( \mu_b-C_{of} \) curves in Fig.4.(b) and Fig.4.(c) are obtained in the same way, and the linear relationships between \( \mu_b \) and \( C_{of} \) are also observed. Figs.5.-7. show the results for \( V_i = 0.2, 0.3, \) and 0.4, which follow a similar trend. The mean values of load computed through Eq. (9) are illustrated in Figs.4.-7. as dotted lines. It can be seen that the formula presented for the mean load value gives almost the same results as those obtained by FORM. From Figs.4.-7., one can see that although Eq. (9) is derived under the condition that \( V_i \) is far smaller than \( V_t \), it can be also applied to cases where \( V_t \) is larger. Fig.8. compares the results of \( V_i = 0.5, 0.6, \) and 0.7, one can see that the discrepancy between the results obtained by FORM and the empirical formula Eq. (9) is very small. That is to say, the application of Eq. (9) is not limited to cases of small \( V_i \).

4. Evaluation Method for Target COF

4.1 Evaluation method

Because the structure is deterministically designed
Fig. 4. Relationship Curves for $V_1=0.1$: (a) $\mu_p$-$C_{of}$ Curves, (b) $\mu_p$-$m$ Curves ($\beta=2$), (c) $\mu_p$-$m$ Curves ($\beta=2$)

Fig. 5. Relationship Curves for $V_1=0.2$: (a) $\mu_p$-$C_{of}$ Curves, (b) $\mu_p$-$m$ Curves ($\beta=2$), (c) $\mu_p$-$m$ Curves ($\beta=2$)

Fig. 6. Relationship Curves for $V_1=0.3$: (a) $\mu_p$-$C_{of}$ Curves, (b) $\mu_p$-$m$ Curves ($\beta=2$), (c) $\mu_p$-$m$ Curves ($\beta=2$)

Fig. 7. Relationship Curves for $V_1=0.4$: (a) $\mu_p$-$C_{of}$ Curves, (b) $\mu_p$-$m$ Curves ($\beta=2$), (c) $\mu_p$-$m$ Curves ($\beta=2$)
to collapse according to the entire beam failure mode, the most likely failure mode is generally the preferable beam-hinging mode and all the likely story failure modes are undesirable. To probabilistically avoid the story mechanisms, the relative occurrence ratio of the most likely story mechanism to the entire beam-hinging mode \( \gamma \) should be limited to a specific allowable level of \( \gamma_0 \) lower than or equal to 1 as follows:

\[
\gamma = \frac{P_{f_2}}{P_{f_1}} \leq \gamma_0 \leq 1
\]  

(10)

where \( P_{f_1} \) is the occurrence probability of the desirable beam-hinging mode, and \( P_{f_2} \) is the occurrence probability of the most likely story mechanism. It should be noted that what is defined by \( P_{f_2} \) is not the joint probability of all possible story mechanisms but the probability of an individual story mechanism. The joint probability, which means the probability of appearance of any story mechanism, may be somewhat different from that of an individual failure mechanism. An evaluation with consideration of the other story mechanisms needs a separate study.

4.2 The most likely story mechanisms

The number of potential story mechanisms for a multi-story frame is large, and it increases rapidly with the increase in the number of stories. In the target COF evaluation, however, only the most likely story mechanisms are required. An earlier study on the story failure modes of frame structures (Zhao et al. 2007) showed that an upper story mechanism with the most failure stories and all the lower story mechanisms would be the most likely.

4.3 Evaluation example

The target COF evaluation of the two-bay, three-story structure shown in Fig.2. is presented to demonstrate the method. As the results of failure mode analysis, the failure modes that are necessary for COF evaluation are shown in Fig.9., where mode 1 is the preferable entire beam-hinging failure mode; mode 2 is the most likely upper story mechanism; and modes 3 and 4 are the most likely lower story mechanisms. In order to conduct the COF evaluation at the same reliability level, the external load level should be adjusted to ensure that the reliability index corresponding to the entire beam-hinging pattern mode of the structure remains the same even though the value of COF is changed. From Table 1. one obtains \( a = 7.54 \times 10^{-4} \) for \( \beta_\tau = 2 \). Then the load level can be expressed as:

\[
\mu_p = 7.54 \times 10^{-4} m \mu_\tau (1 + \frac{1}{5} C_{of})
\]  

(11)

Using the load level obtained, probabilistic analyses for the story mechanisms are conducted. Fig.10.(a) shows the \( P_r C_{of} \) curves. Table 2. gives the occurrence order of failure modes shown in Fig.9. In Table 2., because mode 1 is the preferable mode, the occurrence probability corresponding to mode 1 is \( P_{f_1} \). \( P_{f_2} \) will be the maximum occurrence probability corresponding to the likely story mechanisms, i.e., modes 2, 3, and 4. When the COF ranges from 1.0 to 1.09, the occurrence order is mode 3-4-1-2, and mode 3 is the most likely story mechanism; therefore, \( P_{f_3} \) is the occurrence probability corresponding to mode 3. Similarly, when COF ranges from 1.09 to 1.547, \( P_{f_4} \) corresponds to mode 4 and if COF is larger than 1.547,
Table 2. The Occurrence Order of Collapse Modes

| COF Range     | Mode 3 | Mode 4 | Mode 4 | Mode 4 | Mode 1 | Mode 1 |
|---------------|--------|--------|--------|--------|--------|--------|
| 1.0–1.09      | 3      | 4      | 4      | 4      | 1      | 1      |
| 1.09–1.155    | 4      | 3      | 1      | 1      | 2      | 2      |
| 1.155–1.24    | 4      | 3      | 1      | 1      | 2      | 2      |
| 1.24–1.245    | 4      | 3      | 1      | 1      | 2      | 2      |
| 1.245–1.547   | 4      | 3      | 1      | 1      | 2      | 2      |
| 1.547–5.0     | 4      | 3      | 1      | 1      | 2      | 2      |

Mode for \( P_f \)

Table 3. Basic COF and Optimum COF

| \(\beta\) | 2story | 3story | 4story | 5story | 6story | 7story |
|----------|--------|--------|--------|--------|--------|--------|
| Basic    | 1.098  | 1.234  | 1.377  | 1.568  | 1.792  | 2.011  |
| Optimum  | 1.279  | 1.543  | 1.784  | 1.992  | 2.174  | 2.464  |
| Basic    | 1.055  | 1.108  | 1.187  | 1.284  | 1.399  | 1.522  |
| Optimum  | 1.146  | 1.253  | 1.377  | 1.489  | 1.596  | 1.758  |
| Basic    | 1.039  | 1.072  | 1.092  | 1.142  | 1.198  | 1.270  |
| Optimum  | 1.046  | 1.122  | 1.177  | 1.235  | 1.301  | 1.388  |

As described before, \(\gamma\) is the ratio of the failure probability of the most likely story mechanism to the failure probability of the preferable entire beam failure mode. The allowable level \(\gamma_0\) equal to 1 implies the same likelihood of story mechanism as that of preferable collapse mode. Since the failure probability of the story mechanism should be restricted to a relatively low level, obviously the COF value corresponding to \(\gamma=1\) should be the lowest limit. For this reason, the lowest COF is defined as the basic COF \(C_{\text{min}}\), as illustrated in Fig.10.(c). When the COF of a frame is larger than \(C_{\text{min}}\), the occurrence probability of...

5. Basic COF and Optimum COF

The analysis above indicates that the allowable level \(\gamma_0\) denotes the safety requirement and that the target COF is sensitive to \(\gamma_0\). The smaller the value of \(\gamma_0\), the larger the target COF and the lower the occurrence probability of undesirable story failure modes will be. However, if COF increases, construction costs more. The target COF in application must satisfy the requirements of both safety and economy. At present, what level of evaluation index \(\gamma_0\) should be used is still unclear. In order to provide COF values applicable for design, basic and optimum COFs are proposed in this section.
any story mechanism will be less than that of the entire beam-hinging failure mode. The basic COF ensures the occurrence probability of the story mechanism is at least not larger than that of the preferable beam-hinging pattern.

From Fig. 10.(c), an obvious salient point can be observed in the COF evaluation curve. On the left of the salient point, the evaluation index changes rapidly with COF; on the right side of this point, the variation of the evaluation index with COF becomes slow. That is because the shape of the evaluation curve is determined by the maximum failure probabilities of the undesirable collapse modes, and this salient point is the point where the controlling failure mode changes from the lower story failure mode to the upper story failure mode. Obviously, the evaluation index is very sensitive to the variation of COF to the left of the salient point. In the region right of this salient point, increasing the COF becomes less efficient in reducing the occurrence probabilities of story mechanisms. In general, the evaluation curve of every specific structure has this salient point. Herein the COF value at this salient point is defined as the optimum COF $C_{opt}$, as shown in Fig.10.(c). The COF range between the basic and optimum COFs is recommended for use in COF design. In this recommended range, increasing the target COF is very efficient for avoiding story mechanisms.

Fig. 11. shows the evaluation curves of two- to seven-story frames under a reliability level of $\beta_F = 2, 3, \text{and} 4$, respectively. The basic and optimum COF values obtained from the evaluation curves are listed in Table 3., from which one can see that the basic and optimum COFs increase with the increase in the number of stories and decrease with the increase of the target reliability level of the frame. Because the effect of variation of member strength is very small, the reliability indices of upper story mechanism and lower story mechanism are almost independent of $m$ (Zhao et al. 2007); therefore, the basic and optimum COFs are not influenced by $m$.

6. Evaluation of Target COF of Frame with Nonuniform Column Strength

The investigations performed above were based on the results of story mechanism analyses that assume uniform column strengths in each story. In application, the columns in lower stories of most structures are often designed to be stronger than those in upper stories because they carry much larger loads. The target COF evaluations associated with these structures can be realized similarly by comparing the failure probabilities between entire beam-hinging mode and story mechanisms, whereas all the probable story mechanisms have to be taken into account. In this section, a four-story frame with column strengths irregularly distributed over the height of the frame is investigated to show the evaluation process.

Assume that the strength of columns in the first and second floor is 1.2 times the strength of columns in the third and top floor; consequently, the COF of the lower two stories will be in proportion to the COF of the upper stories. Once the target COF of the top floor is obtained, the target COF for lower stories can
be determined. The COF of top floor is taken as the investigation subject representing the COF level of the structure. The other parameters are assumed to be the same as those used in previous investigations.

Fig. 12. shows the nine story mechanisms that will probably occur in a four-story two-span frame. The obtained failure probabilities for these modes are illustrated in Fig. 13., from which one can see that mode 3 and mode 6 are the most likely to occur among the COF range in consideration. Dividing the largest failure probabilities of story mechanisms by the probability of entire beam-hinging mode produces the $\gamma$-COF curve, as shown in Fig. 14. Given an allowable level, the target COF can be determined from the $\gamma$-COF curve. Assuming that $\gamma_0 = 0.9$, a target COF of 1.48 is obtained for the top story of the frame considered. Consequently, the moment strength requirements to the column strengths are determined to be 157.1 ($=1.48 \times 106.2$) kN·m for the upper two stories and 188.6 ($=1.2 \times 157.1$) kN·m for the lower two stories to ensure a probability of story mechanism as low as 90% of that of entire beam-hinging mode.

7. Conclusions

The load levels for target COF evaluation were analyzed, and an empirical formula describing the relationship between the load and COF was proposed. Investigations of the COF requirements for probabilistically avoiding story mechanisms of frame structures were based on the comparison of occurrence probabilities between entire beam-hinging failure mode and the most likely story mechanisms. The investigation presented in this paper can be summarized by the following points.

1) Under a specific reliability level, the load level is approximately a linear function of the column over-design factor, the mean strength value, and the number of bays. The empirical load function presented provides results that are in good agreement with those obtained by FORM.

2) Basic and optimum COFs are proposed. The basic COF is the lowest limit that ensures the occurrence probability of the story mechanism is at least not larger than that of the preferable beam-hinging pattern, and the optimum COF is the one that avoids story mechanisms most efficiently. Use of a COF between basic and optimum COFs is recommended in design applications.

3) Both the basic and optimum COFs increase with the increase in the number of stories and decrease with the increase in target reliability level of the frame.

4) An example frame with column strength varying for different stories was evaluated. It is suggested that all the potential story mechanisms be taken into consideration before the most likely modes are known.

It should be noted that the investigation was conducted under some restrictive assumptions described in this paper. Other factors that were not considered in the paper, such as second-order effects and axial deformation, type of ground motion and dynamic response, distribution type of random variable, definition of the beam-hinging pattern, and correlations among member strengths, may be important. The consideration of all of these factors requires further research.

Acknowledgement
Prof. Y. Mori, Nagoya University, and Prof. H. Idota, Nagoya Institute of Technology, are acknowledged for their valuable discussions concerning this study.

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