About One Task of Managing a Hierarchical System

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Abstract. The metrological support system for special equipment and special objects considered in the paper is modeled by a three-level hierarchical system. The metrological support system is considered as park of measuring equipment: high-precision instruments installed, working standards and working measuring instruments in a hierarchical sequence. The statement and solution of the problem of program - target planning for the development of the measuring equipment park is given. The goal of the development of the park is to provide the required number of verifications with working measuring instruments. The verified devices are measuring devices installed at special techniques and special objects. As the control actions on the park, the procurement of high-precision installations, working standards and working measuring instruments is strictly used in a certain proportion, determined in accordance with the hierarchy of the system. The mathematical model of the relationship between hierarchies is proposed. The model is based on the equation of the temporal balance of the park’s potential power and the required amount of time for verification, taking into account the frequency of verifications. In the general case, in a hierarchical model, correspondences between levels can be of the type: one to one, one to several and several to one. This circumstance leads to the necessity of solving NP – complete problem of discrete optimization. The article gives an algorithm for setting this NP – complete problem for different cases of violation of the uniqueness of correspondence between elements of different levels. The effective algorithm for estimating the dimension of this problem has been developed. Assessment of the dimension of the problem allows you to choose the appropriate algorithm for solving the discrete optimization problem (branch and bound method, genetic algorithm, ant algorithm, exhaustive search method). The results of mathematical modeling are presented.

1. Introduction
The characteristic features of hierarchical managed systems (HMS) are: vertical decomposition of the system into subsystems, the priority of subsystems of the upper level with respect to the underlying ones, and the presence of directs and feedbacks between levels [1-9].

The widespread use and versatility of the HMS is due to a number of advantages compared to radial (centralized) control systems:

· Freedom of local actions within the framework of imposed restrictions;
· The possibility of an appropriate combination of local criteria for the functioning of individual subsystems and the global optimality criterion for the system as a whole;
· The possibility of a compressed, aggregated presentation of relevant information on the results of control coming through feedback channels;
- Increased reliability of the control system, the presence of the properties of controllability, adaptability, organization and a number of other properties specific to specific systems;
- Universality of the management concept and approaches to solving management problems in hierarchical systems;
- Economic feasibility compared to control systems of a different structure. The latter quality requires justification in each case.

The management theory of HMS includes the following main sections: structural analysis and synthesis of the HMS, coordination problem in HMS, optimization of the functioning of HMS.

The principle of hierarchical management is an expression of the integrity of systems; by predetermining organization, it allows you to find ways to manage complex systems.

The problem of managing by Park of Measuring Equipment (PME) has been studied by many authors in various settings [10–13]. However, in practice, tasks periodically arise, for the solution of which it is necessary to develop new methods, techniques and technologies, since the known methods do not allow to fully take into account the specifics of considered in the paper a three-level hierarchical system.

This article is devoted to the development of a model of metrological support for measuring equipment (ME) installed on Special Techniques and Special Objects (STSO). The basis of the model is the equation of the time balance: the required working time fund and the working time fund, which can potentially be ensured with the full use of production capacities, equipment and production areas (for each types of measurements). The model allows you to create rational options for retrofitting the existing PME, providing the necessary (required) number of verifications of measuring equipment (ME) of various physical quantities. The results of calculations are presented.

2. Materials and methods

2.1. General formulation of the problem

The main costs for the metrological support of ME at STSO (expenses for ensuring the verification of ME installed on STSO) are the costs of the purchase, modernization and operation of samples of ME of Metrological Service Organizations (MSO). These MSO are provide verifications ME at STSO. These organizations also have a hierarchical structure. Three levels are MSO-1, MSO-2, MSO-3.

The basis of the metrological support tools are High-Precision Instruments (HPI), working standards (WS) and Working Measuring Instruments (WMI) [10-13].

Ensuring the correctness of the transfer of the size of units of physical quantities in all links of the metrological chain is carried out by means of verification schemes - regulatory documents establishing the subordination of HPI, WS, WMI involved in the transfer of unit size from HPI to WS and WMI with indication of measurement methods and error [14-16]. The MSO-3 provide verification of ME at STSO by means of using WMI.

The paper proposes a scientific and methodological approach to assessing the needs of MSO-1, MSO-2, MSO-3 in ME of various accuracy classes for different types of measurements in order to ensure the possibility of carrying out the required number of verifications of ME at STSO on different types of measurements.

It is assumed that the verification scheme [14-17] is a hierarchical structure (pyramid), at the top of which there are HPI located on the balance of MSO-1.
Under them in the pyramid are WS that are on the balance of MSO-2. Below there are located WMI, served by MSO-3.

In this paper, we consider PME, that managed by MSO-1, MSO-2, MSO-3. It is assumed that at the initial time, the PME consists of a certain amount of HPI, WS and WMI to conduct a certain number of verifications per year. Retrofitting of PME in order to ensure the required number of verifications of ME at STSO is carried out by procuring the required number of WMI, WS and HPI. Fig. 2 shows the part of scheme of metrological subordination of means of metrological support of different levels.

In the first level there are HPI, which is used for verification of the corresponding in appearance, range and accuracy of measurements of WS located at the second level. Each WS, in you turn, can be used for verification of the corresponding in appearance, range and accuracy of measurements of WMI samples located at the third level.

Note that in most cases each WMI sample can be verified by only one type of WS and, at the same time, this type of WS can be verified by only one type of HPI (see the chains from WSI-1,3,5,6). However, there are cases when one WS can be verified by many different HPI (see the chains across WS-3). We also note cases where one WMI can be verified by many different WS (see the chains across WSE-2). Such ambiguous verification capabilities are indicated by dashed arcs.

If you do not take into account the dashed arcs, then the subordination scheme (Fig. 2) will be a forest graph - a union of three trees. It is known that for a tree graph the condition for the existence of a single path from each vertex to any other vertexes is always satisfied. Including - the only way from the vertex of the first level to the vertex of the third level or vice versa:

2.2. Algorithm for calculating the amount of funding required for the procurement of WME, WS and HPI in order to ensure the required number of verifications of ME on ST SO

The PME is presented as a combination of two parts of the park: with an unambiguous chain of verifications and an ambiguous chain. The algorithm includes three stages. In the first two stages, the necessary calculations are made for the noticed parts of the PME. At the third stage - calculations for the PME as a whole.
I. Calculation the number of copies of HPI, purchases of WS and WME for that part of the PME for which each WMI can be verified by only one type of WS, and this type of WS can be verified by only one HPI (see WME-1,3,5, 6).

II. Preliminary calculation of the number of HPI copies, purchases of WS and WME for a variety of WME samples that can be verified by several types of WS and / or these types of WS can be verified by several HPI (see WME-2,4).

In the future, we will say that for this part of the Park the condition of uniqueness is violated.

III. Calculation the total number of copies of HPI, purchases of WS and WMI, as well as the total funding based on the use of the results of stages 1 and 2 and methods for solving the discrete optimization problem.

Let us describe these stages sequentially. At the first stage, we will carry out the calculations in the sequence: a) WMI (MSO-3), b) WS (MSO-2), c) HPI (MSO-1). We take the billing period equal to 1 year.

Ia). First, we describe the method of calculating the required number of WMI for the required number of verifications of ME at STSO.

Let \( n \) - number of verified WME installed on STS, and which must be metrologically provided (verified by a certain number of times a year). We introduce the following notation:

- \( \nu \) - frequency of verification of WMI [verifications per year];
- \( N = n \cdot \nu \) - the required number of verifications per year [verifications per year];
- \( K \) - the number of available WS samples [units];
- \( T \) - planned operating time of one WMI sample [year];
- \( p \) - productivity of one WME sample [verifications per year];
- \( C \) - the average cost of the purchased WMI sample; \( \Phi_{REQ} = N / p \) - the required fund of working time necessary for verification;

\[
\Phi_{O} = K \cdot T \quad \text{a fund of working time, which could potentially be ensured with the full use of production equipment and production areas;}
\]

\[
\Phi_{REQ} - \Phi_{O} = 0 \quad \text{equation of time balance;}
\]

\[
z = (\Phi_{REQ} - \Phi_{O}) / T \quad \text{the number of WMI samples that must be purchased in order to ensure that all verifications are carried out (provided);}
\]

\[
S = Z \cdot C \quad \text{the necessary amount of funding to ensure MSO-3;}
\]

\[
n + Z \quad \text{total number of WME, taking into account purchases.}
\]

Ib). We now describe a method for calculating the amount of WS required by MSO-2 to conduct a given number of tests of WMI. This method is completely identical to that described above in 1a).

Only a predetermined number of WMI verifications should be accepted \( Z \cdot \nu \), where \( n \) - is the number of WMI before the start of procurement, \( Z \) - is the number of WMI purchases, and \( \nu \) is the frequency of WS verifications by means of HPI.

Ic). For MSO-1, the algorithm for calculating the number of HPI copies is identical to that described in 1a), only the number of WS verifications should be taken equal to \( (n + Z) \cdot \nu \), where \( n \) - is the number of WS before the start of purchases, \( Z \) - is the number of WS purchases, where \( \nu \) is the frequency of WS verifications by means of HPI.

Note that when calculating the number of purchases of each type of ME, the result should be rounded up to the nearest whole number.

Note that if \( \Phi_{REQ} \leq \Phi_{O} \), then the number of purchases is taken equal to zero.

As a result of stage I, the required number of WMI, WS and HPI for the above-mentioned part of the PME with an unambiguous chain of verifications will be calculated. So we will provide the required number of verifications of ME for considered part of PME. In practice, the amount of financing for the relevant purchases is more than 80% of the total amount of financing. Because there are no so many “ambiguous verification chains” for the PME in practice.
We now turn to the description of the algorithm of stage II - a method for calculating the required amount of HPI, WS and WMI for “ambiguous verification chains”. In Fig. 3 the relevant vertexes are two (WMI-2 and WMI-4). The exact calculation of the minimum required number of purchases is NP-complete problem of discrete optimization [18-22]. At present, approximate multistep methods for solving such NP-complete problems, including the branch and bound method, which is a multistep method, have found wide application. The article proposes an approximate method for solving the problem. The proposed method is, in essence, the first step in solving NP-complete problem by means of the method of branch and bound.

We describe the essence of the method of stage II. For each vertexes of the third level with ambiguous verification chains. For definiteness, we will take the vertex of WMI-2. There exist three different paths lead to it. For each path, we calculate the required number of WMI, WS, HPI and the amount of funding in accordance with the algorithm described in stage I. Of all the paths found, “select” the one for which the amount of funding is minimal.

Now take the vertex WMI-4. There exist two different ways (paths) are brought into it. For these paths, we also calculate the required number of WMI, WS, HPI and the amount of financing in accordance with the algorithm described in stage I. Of these two ways (paths), we “single out” the one for which the amount of funding is minimal.

At the third stage, the discrete optimization problem is solved. As Boolean unknowns, various paths are used that lead to the vertices calculated in stage 2 for which the uniqueness condition is violated. As a result of solving the discrete optimization problem from many different paths leading to a vertex for which uniqueness is violated, the optimal path is chosen. Based on the results of solving the optimization problem, the total required number of WMI, WS and copies of HPI, as well as the corresponding amount of funding, will be found.

Note that as a first approximation for the branch and bound method, for each ambiguous vertex, from the paths calculated in step 2, we can select those “highlighted” paths that provide the minimum amount of funding. For the example considered in the article, when choosing “allocated” paths, we immediately obtain the optimal solution.

The method of calculating purchases, consisting in the implementation of stages I and II with the subsequent selection for ambiguous vertices of the “selected” paths, we will call the method of locally optimal paths.

3. Results

We present the results of mathematical modeling of the retrofitting of a part of the PME, for which each WMI sample can be verified in more than one way. For the above example, the number of different paths leading to the second vertex is 3, and the number of paths leading to fourth vertex is 2. Calculations showed high sensitivity of optimal financing options to the required number of ME verifications. For example, calculations with the required number of verifications of 10,000, 12,000 and 15,000 showed that the minimum amount of funding can be achieved on each of the three paths leading to the second vertex of the graph. Minimum amounts of funding are highlighted in gray in the indicated table. However, for the two paths leading to the fourth vertex, the minimum amount of funding is always achieved on the same first path (Table 1).

| Verifications | Parameter of task | WMI-2-1 | WMI-2-2 | WMI-2-3 | WMI-4-1 | WMI-4-2 |
|---------------|------------------|---------|---------|---------|---------|---------|
| ME            |                  |         |         |         |         |         |

Table 1. Dependence of the volumes of purchases and financing on the required number of verifications.
| Amount of financing | Number of purchases of WME | Number of purchases of WS | Number of purchases of HPI |
|---------------------|----------------------------|---------------------------|---------------------------|
| 10 000              | 240                        | 100                       | 615                       | 5190                     |
|                     | 0                          | 25                        | 125                       | 750                      |
|                     | 12                         | 20                        | 29                        | 48                       |
|                     | 0                          | 0                         | 0                         | 0                        |
|                     | 240                        | 600                       | 890                       | 6740                     |
| 12 000              | 240                        | 600                       | 890                       | 6740                     |
|                     | 0                          | 150                       | 250                       | 1000                     |
|                     | 12                         | 27                        | 44                        | 58                       |
|                     | 0                          | 0                         | 0                         | 2                        | 0                        |
|                     | 650                        | 1352                      | 1418                      | 9195                     |
| 15 000              | 125                        | 338                       | 438                       | 1375                     |
|                     | 20                         | 39                        | 68                        | 74                       |
|                     | 0                          | 0                         | 0                         | 3                        | 1                        |

It can be seen that the procurement structure (proportions between the purchased quantity of WMI and WS) also significantly depends on the required number of verifications. We also note that with an increase in the number of verifications (more than 15 000), the second variant of the path on the graph remains optimal. And when decreasing - the third version of the path on the graph. Moreover, with the
number of verifications not exceeding 9 600, “financing of the path” leading to WMI-4 is not required at all. All verification can be performed through the use of existing redundancy PME.

The described facts may be explained by the discrete properties of the investigated model of retrofitting the PME and the currently existing redundancy of the metrological support facilities. In the case when it is possible to carry out a certain number of verifications using the already available free capacities of the MP, then to minimize the amount of financing during retrofitting, you should first use the free capacities. At the same time, if there is enough free capacity to provide the required number of verifications, then additional financing for purchases and copies when retrofitting PME will not be required at all.

Fig. 3 presents the dependence of the volume of funding on the required number of verifications.

![Figure 3](image)

**Figure 3.** The dependence of the volume of funding on the required number of verifications.

Note that the dependencies “in general” have a relatively smooth appearance. With the increase in the number of required verifications, the necessary amount of financing increases.

Due to the fact that the number of different paths leading to the same vertexes of the third level is small and the number of “ambiguous vertices” is small, it turns out to be effective to solve the corresponding discrete optimization problem by enumerating all possible paths passing through the vertexes of level 2 for which the condition of uniqueness is violated.

Let us describe the method for constructing an estimate of the dimension of a discrete optimization problem.

Consider the part of the PME (part of the graph), for which the condition of uniqueness of the path is violated. Let there be vertexes of the second level in which the condition of uniqueness of the path passing through these vertexes is violated. Let each such vertexes, denoted as WS-r (r = 1, 2, ..., R), contain directed arcs and directed arcs (arrows) come out of each such vertex. Then the total number of ambiguous paths in this part of the graph will be equal \( \sum_{r=1}^{R} n_r \cdot m_r \) and the dimension of the discrete optimization problem can be calculated by the formula: \( L = \sum_{r=1}^{R} n_r \cdot m_r - M \), where \( M \) is the number of unique paths in the original graph.

For the considered example \( M = 4 \), \( m_1 = 1 \), \( m_2 = 1 \), \( m_3 = 2 \), \( n_1 = 1 \), \( n_2 = 2 \), \( n_3 = 2 \), \( n_4 = 2 \). Then \( L = 1 \cdot 1 + 1 \cdot 2 + 2 \cdot 2 + 1 \cdot 2 - 4 = 5 \).

For the simplest model, considered in the article, the results obtained by the exhaustive search method and the method of locally optimal paths coincide.

4. Discussion
The article has developed a three-level hierarchical model of metrological support. An algorithm is proposed for establishing sequential relationships between levels, starting from the lowest level to the
highest level. The algorithm is applicable to establish sequential relationships between levels for any number of levels. The algorithm is also applicable for establishing relationships with an incomplete hierarchy. For example, connections between level 1 and 3 or level 2 and 4, bypassing intermediate levels. With such connections, NP-complete problems also arise. Dimensions of such problems can also be estimated using the method described in the article.

Arising in case of violation of the strict hierarchy NP-complete problems can be presented in a concise form of the standard integer (boolean) programming problem, or set cover problem. However, the level of complexity of the solution does not depend on the representation of the problem in different form.

The method developed in the article is used in the formation of plans for the long-term development of PME. Note the redundancy inherent in the current metrological support system.

A method has been developed for calculating the number of purchased samples of WMI, WS and copies of HPI necessary to carry out the required number of verifications of ME on STSO. The model is based on the calculation of the fund of working time and the potential capacity of MSO. The model allows you to calculate rational procurement options for WMI and WS in order to ensure the required number of verifications of ME at STSO.

The method essentially uses the features of the structure and composition of the hierarchical verification scheme. It is shown that the calculation of the required number of ME purchases generally leads to the need to solve the discrete optimization problem (NP-complete problem).

The paper proposes a method for estimating the dimension of a discrete optimization problem. When solving practical problems with various hierarchical structures of ME subordination, after assessing the dimension of the corresponding discrete problem, a decision can be made on the use of one or another method to solve it.

5. Conclusions
The method for investigation a hierarchical system with three levels of hierarchy is developed.

The method has been applied for calculating the number of purchased WMI, WS and copies of HPI necessary to carry out the required number of verifications of ME at STSO.

The model is used in the formation of proposals in the program of medium-term and long-term development of PME. The formed plans turn out to be well balanced at all levels of the hierarchical system. Balanced purchases can be used to reduce redundancy.

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