Neutrino mass and lepton number violation in the Little Higgs model

Ashok Goyal 1
Department of Physics and Astrophysics, University of Delhi, Delhi-110 007, India
Inter-University Centre for Astronomy and Astrophysics, Pune-410 007, India

Abstract

We study lepton number violation in Little Higgs model and find that the choice of putting triplet Higgs vev equal to zero so as not to have any tree level neutrino Majorana mass is not natural in the sense that such a term is generated at the one loop level. We investigate the contribution of exotic lepton number violating terms on neutrinoless double beta decay, K meson decay and on trilepton production in $\nu$-N scattering.

1 Introduction

There is compelling evidence for the existence of non zero small neutrino masses and large neutrino flavour mixing from the Solar, Atmospheric and Accelerator neutrino data [1]. The SK atmospheric neutrino and K2K data are best described by dominant $\nu_\mu \leftrightarrow \nu_\tau$ vacuum oscillations with best fit values $|\Delta M_A|^2 = 2.1 \times 10^{-3}$ eV$^2$ and $\sin^22\theta_A = 1.0$ at 99.73% CL. The Solar neutrino data is described by $\nu_e \leftrightarrow \nu_\mu$ oscillations with best fit value $|\Delta M_0|^2 = 7.9^{+0.6}_{-0.5} \times 10^{-5}$ eV$^2$ and $\tan^2\theta_0 = 0.40^{+0.09}_{-0.07}$. The Troitzk and Mainz tritium $\beta$-decay experiments [2] provide information on the absolute $\bar{\nu}_e$ mass measurement $m_{\bar{\nu}_e} < 2.2$ eV at 95% CL. From the study of anisotropy in the CMBR and large scale structure, the WMAP data [3] has severe constraints on the masses of all active neutrino species $\Sigma m_j < (0.7 - 1.8)$eV (95% CL). The Katrin experiment expected to start in 2007 is planned to reach sensitivity $\sim 0.20$ eV (95% CL) for $m_{\bar{\nu}_e}$ and the WMAP and future PLANCK experiments and the data on weak lensing of galaxies by large scale structure may allow one to determine $\Sigma m_j$ with an uncertainty of (0.04-0.10) eV.

In the minimal Standard Model(SM), the neutrinos are massless because of the existence of simple Higgs structure which leads to Global lepton number conservation and forbids the Majorana mass term $\bar{\nu}_L^C \nu_L$ and the absence of singlet $\nu_R$ forbids Dirac mass term as well. Thus

1E-mail :agoyal@iucaa.ernet.in
the masslessness of neutrinos in SM arise due to restricted particle content of the SM. There are several extensions of SM that can give non-zero neutrino mass viz. by i) extension of the lepton sector, ii) extension of the Higgs sector and iii) extension of both the sectors. The smallness of the neutrino mass in comparison to the lightest charged lepton mass requires fine tuning of parameters and is generally considered unnatural. An attractive option currently favoured is the celebrated See-Saw mechanism which naturally leads to a set of three light Majorana masses for the three neutrinos along with the existence of large Majorana mass right-handed electro-weak singlet neutrinos. It is thus imperative to go beyond the SM and look for new physics. Neutrino mass generation is not the only problem afflicting SM. The other problem is the so called Hierarchy problem, that is enormous difference between the electro-weak and GUT/Planck scale. The precision electro-weak data prefers the existence of light Higgs and thus SM with light Higgs can be considered as an effective theory valid to a high scale perhaps all the way to GUT/Planck scale whereas the Higgs mass is not protected and gets quadratically divergent contribution to its mass and requires fine tuning. One way out of this difficulty is to invoke Supersymmetry where quadratic divergences in Higgs mass are cancelled by the corresponding fermionic contribution. In fact there are several post SM scenarios devised to ameliorate this difficulty namely : SUSY, Left-Right symmetric gauge theories, GUTS, theories of extra dimensions etc. All these theories can naturally incorporate Majorana neutrinos.

Recently there has been a proposal to consider Higgs fields as Nambu-Goldstone bosons of a Global symmetry which is spontaneously broken at some high scale $f$ by acquiring vacuum expectation value (vev). The Higgs field gets a mass through electro-weak symmetry breaking triggered by radiative corrections leading to Coleman-Weinberg type of potential. Since the Higgs is protected by approximate Global symmetry, it remains light and the quadratic divergent contributions to its mass are cancelled between particles of the same statistics. The Littlest-Higgs (LH) model [4] is a minimal model of this genre which accomplishes this task of cancelling quadratic divergence to one loop order with a minimal matter content. The LH model consists of an SU(5) non-linear sigma model which is broken down to SO(5) by a vacuum expectation value $f$. The gauged subgroup $[SU(2) \times SU(1)]^2$ is broken at the same time to diagonal electro-weak SM subgroup $[SU(2) \times SU(1)]$. The new heavy states in this model consist of vector 'top quark' which cancels the quadratic divergence coming from the SM top quark along with the new heavy gauge bosons ($W_H, Z_H, A_H$) and a triplet Higgs $\Phi$, all of masses of order $f$ and in the TeV range. The effect of these new states on electro-weak precision parameters has been studied to put constraints on the parameters of the model [4]. In particular no stringent limit on $v'$ the vev of triplet Higgs exist apart from the bound $\frac{v'^2}{v^2} < \frac{1}{167^2}$ from demanding positive definite mass for the triplet Higgs. It should be noted that $v'$ can be made to vanish by tuning the parameters so that the triplet Higgs does not couple to standard doublet Higgs.
The existence of triplet Higgs allows the lepton number violating interaction
\[ L_{LV} = -g_{\phi ll} \bar{l}_L^C \tau_2 \bar{\tau} l_L + h.c \] (1)
which is invariant under the electro-weak gauge symmetry of the LH model. After electro-weak symmetry breaking, such an interaction will generate Majorana masses for the left handed neutrino of order \( g_{\phi ll} v' \) and will allow for lepton number violating processes like neutrinoless double \( \beta \)-decay. In order to achieve neutrino Majorana mass consistent with current bounds on neutrino masses from neutrino oscillation data which does not differentiate between the Majorana and Dirac masses and from costraints from neutrinoless double \( \beta \)-decay experiments, either the coupling \( g_{\phi ll} \) or the triplet Higgs vev \( v' \) should be unnaturally small i.e. \( g_{\phi \nu \nu} v' \sim 10^{-10} \) GeV.

The vacuum expectation value \( v' \) can be taken to be zero as discussed above, on the other hand putting the coupling \( g_{\phi ll} \) equal to zero by hand ammounts to an adhoc imposition of lepton number conservation. Vanishing of \( v' \) admittedly requires fine tuning but is perhaps a small price to be paid to achieve non-zero neutrino mass and lepton number violation in the attractive frame work of LH model without any further extension.

![Figure 1: Neutrino Majorana mass at one loop level](image1)

![Figure 2: Generic diagram for \( \Delta L = 2 \) Majorana neutrino mediated processes](image2)
2 Neutrino mass and lepton number violating processes

In the scenario in which the triplet Higgs does not develop a vev, the neutrinos will get masses through loop diagrams shown in Fig.1 in which one of the the black circles stand for the lepton number violating vertex [1] and the cross represents charged lepton mass insertion. The diagram is log divergent. The divergence can be absorbed in tree level mass term arising from nonzero vev $v'$ which in turn would mean that the choice $v'$ is not natural. In fact the above diagrams will generates nonzero vev through $g_{\Phi\nu\nu} \times \bar{\nu}_L \nu_L$ term in (1). Some of the important lepton number violating processes that we consider are:

1. The neutrinoless double $\beta$-decay process $(N, Z) \rightarrow (N - 2, Z + 2) + e^- + e^-$
2. K meson decay $K^+ \rightarrow \pi^- \mu^+ \mu^+$
3. $\tau$ decay $\tau^- \rightarrow e^+ + \pi^- + \pi^-$
4. Nuclear muon-positron conversion $^{32}S + \mu^- \rightarrow ^{32}S + e^-$
5. Trimuon production in neutrino factories and in the ultra high energy cosmic ray neutrino interaction processes $\nu N \rightarrow \mu^- \mu^+ \mu^+ X$

The above lepton number violating $\Delta L=2$ processes mediated by Majorana neutrinos are depicted in fig.2 and the matrix elements are proportional to neutrino flavor mixing matrix elements and lepton number violating Majorana neutrino mass insertion. In the presence of lepton number violating interaction in the LH-model, the dominant generic diagram is through the decay of doubly charged Higgs into lepton pair and is shown in fig.3. For the neutrinoless
double $\beta$ decay these diagrams are characterised by the following couplings

$$M_\nu \sim \left( \frac{g}{2\sqrt{2}} \right)^4 \frac{1}{M_W^4} < m >$$

$$M_\Phi \sim \left( \frac{g}{2\sqrt{2}} \right)^2 \frac{1}{M_W^2} \frac{8}{M_\Phi^4} m_f (\frac{v}{f} - 2s_+ ) g_{\Phi l_1 l_2}$$

In the case of $K^+ \rightarrow \pi^- \mu^+ \mu^+$ decay, the dominant s-channel diagrams yield the amplitude

$$M_\nu = 2G^2_F f_K f_\pi (V_{ud} V_{us})^* \frac{M_\nu}{q^2 - m_\nu^2} p_{K\mu} p_{\pi\nu} [L^{\mu\nu}(k_1, k_2) - L^{\mu\nu}(k_2, k_1)]$$

(2)

where

$$L^{\mu\nu}(k_1, k_2) = \bar{v}(k_1) \gamma^\mu \gamma^\nu P_R v(k_2)$$

Decay rates can be calculated and we get

$$\Gamma_\nu = G^4_F f_K f_\pi (V_{ud} V_{us})^* \frac{M_\nu}{q^2 - m_\nu^2} p_{K\mu} p_{\pi\nu} [L(k_1, k_2) - L(k_2, k_1)]$$

(4)

$$\Gamma_\Phi = G^2_F f_K f_\pi \frac{m_K^2}{f^2} \frac{m_\Phi^8}{m_\Phi^8} m_K |V_{ud} V_{us}|^2 g_{\Phi \mu \mu}^2 I_\Phi$$

(5)

where $I_\nu$ and $I_\Phi$ are dimensionless phase space integrals.

In the trimuon production process $\nu_\mu N \rightarrow \mu^- \mu^+ \mu^+ X$, the relative effective couplings in the amplitude from the two diagrams are

$$M_\nu \sim G^2_F \frac{m_\mu}{<q^2>}$$

and

$$M_\Phi \sim G_F \left( \frac{m_\mu m_\mu}{M_\Phi^2} \right)^{1/2} \frac{1}{f} \left( \frac{v}{f} - 2s_+ \right) g_{\Phi \mu \mu}$$

(3)

3 Summary and Conclusions

The above lepton number violating processes mediated by Majorana neutrino exchange have been studied in the literature [6] and limits on the Majorana neutrino mass placed. The best limits on Majorana neutrino mass come from neutrinoless double $\beta$-decay of $^{76}Ge$ which gives $< m >_{ee} < 0.3$ eV. Limits on $< m >_{e\mu}$ varies from 17 MeV to < 1.3 TeV from muon-positron conversion in sulphur. Limits on $< m >_{\mu\mu}$ and $< m >_{\tau\tau}$ are very weak and are obtained from K and $\tau$ decays and are < 0.48 and < 12 TeV respectively. In the presence of lepton number violating interactions in the LS model, the corresponding decay rates and branching ratios need not be supressed by the low neutrino masses and can indeed be large. They are proportional to the coupling $g_{\Phi l_l}$ and the most stringent limit on the coupling comes from the neutrino-less double $\beta$-decay. It should be emphasised that the experimental bounds on the lepton number violating processes discussed here except for neutrinoless double $\beta$-decay are so weak that any meaningful
bounds on either the light majorana neutrino masses or couplings $g_{\Phi l}$ do not exist. Alternatively, if lepton number violating processes are indeed observed, they would strongly point towards the existence of exotic lepton number violating interactions such as discussed above arising in the LS model.

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