Fault Detection for Timed FSM with Timeouts by Constraint Solving

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Abstract. Recently, an efficient constraint solving-based approach has been developed to detect logical faults in systems specified with classical finite state machines (FSMs). The approach is unsuitable to detect violations of time constraints. In this paper, we lift the approach to generated tests detecting both logical faults and violations of time constraints in systems specified with timed FSMs with timeouts (TFSMs-T). We propose a method to verify whether a given test suite is complete, i.e., it detects all the faulty implementations in a fault-domain and a method to generate a complete test suite. We conduct experiments to evaluate the scalability of the proposed methods.

Keywords: Finite State Machine; Timed Extended Finite State Machine; Conformance testing; Fault model-based test generation; Complete test suite

1 Introduction

The fault domain coverage criterion can be adopted to generate tests revealing faults in safety/security critical systems under test (SUT) [2]. The domain can be built from referenced databases \(^1\) or expert knowledge. Efficient test generation methods are needed especially for the fault domains of important sizes, which has motivated the development of an approach [18,19] leveraging on recent advances in the field of (Boolean) constraint solving. The approach has been elaborated to detect logical faults in reactive systems specified with finite state machines (FSMs). We plan to lift the approach to detect both logical faults and violations of time constraints in reactive systems; especially, we focus on reactive systems specified with timed FSMs with timeouts (TFSMs-T).

TFSM-T [15,4] is an extension of the classical FSM with timeout transitions for expressing time constraints. They define timeouts and the next states to be reached if no input is applied before the timeouts expire; otherwise outputs defined by input/output transitions are produced. Although they express limited types of time constraints as compared to other timed FSMs [4], TFSMs-T have been used to specify reactive systems such as web applications [27] and protocols [26,22,13]. Logical faults in TFSMs-T correspond to unexpected outputs or unexpected state changes. Reducing and increasing waiting time are violations of time constraints. An implementation under test for a

\(^1\) E.g.: \url{https://nvd.nist.gov/}
given specification TFSM-T can be represented with a mutated version of the specification TFSM-T also called a mutant. A mutant can be obtained by seeding the specification with an arbitrary number of faults. A fault domain for a specification is then a finite set of possible mutants; it can be built from a list of identified faults to be detected in systems under test. A mutant is nonconforming if its timed output sequence differs from that of the specification for some timed input sequences (tests). A complete test suite for a fault domain detects all nonconforming mutants in the domain.

Model-based testing with guaranteed fault coverage has been investigated for untimed and timed models. Finite state machines can be preferred over label transition systems for representing systems. This is probably because FSMs have been used early in testing digital circuits [25] and protocols [3], and they do not permit nondeterministic choices between the application of inputs and the production of outputs. Several approaches have been investigated for FSM-based test generation with guaranteed fault coverage [3][25]. FSMs have been extended to express time constraints, which has resulted in a variety of timed FSMs [16][4][9][27][10]. Timed FSMs are not compared to the well-known timed automata [1] for which testing approaches have been developed [14][8][6]. Testing approaches [9][27][10][24] for timed FSMs integrate the reasoning on time constraints in well-known FSM-based testing approaches [25][5]. The work [12] evaluates the application of different meta-heuristic algorithms to detect mutants of Simulink models. Meta-heuristic algorithms do not guarantee the detection of all predefined nonconforming mutants. The methods in [18][19] to verify and generate a complete test suite for FSM specifications are based on solving constraints or Boolean expressions, which allows to take advantage of the efficiency of constraint/SAT solvers [7][23]; this is a novelty as compared to the work in [21] and the well-known approaches such as the W-method. The high efficiency of using constraint solving in testing software code was demonstrated in [11]. The constraints specify the mutants surviving given tests; they are defined over the transitions in executions of the mutants. The executions are selected with a so-called distinguishing automaton of the specification and the fault domain that is compactly modeled with a nondeterministic FSM called a mutation machine. A solution of the constraints is a mutant which, if it is nonconforming, allows to generate a test detecting the mutant and many others; then the constraints are upgraded to generate new tests.

Our contribution is to lift the methods in [18][19] for verifying and generating complete test suites for fault domains for TFSM-T specifications. In our work, specifications and mutants are deterministic and input-complete TFSMs-T. We define a new distinguishing automaton with timeouts for a TFSM-T specification and a fault domain. The automaton serves to extract transitions in detected mutants and build constraints for specifying test-surviving mutants. Extracting the transitions, we pair input/output transitions with timeout-unexpired transitions allowing to pass the input/output transitions; this is formalized with a notion of "comb". We have implemented the methods in a prototype tool which we use to evaluate the efficiency of the methods and compare our results with those of the related work.

Organization of the paper. The next section introduces a fault model for TFSMs-T and the coverage of fault models with complete test suites. In Section 3 we build constraints for the analysis of timed input sequences and the generation of complete test suites.
The analysis and generation methods are presented in Section 4. Section 5 presents an empirical evaluation of the efficiency of the methods with the prototype tool. We conclude the paper in Section 6.

2 Preliminaries

Let \( \mathbb{R}_{\geq 0} \) and \( \mathbb{N}_{\geq 1} \) denote the sets of non-negative real numbers and non-null natural numbers, respectively.

2.1 TFSM with timeouts

Definition 1 (TFSM with Timeouts). A timed finite state machine with Timeouts (shortly, TFSM-T) is a 6-tuple \( S = (S, s_0, I, O, \lambda_S, \Delta_S) \) where \( S \) and \( O \) are finite non-empty set of states, inputs and outputs, respectively, \( s_0 \) is the initial state, \( \lambda_S \subseteq S \times I \times O \times S \) is an input/output transition relation and \( \Delta_S \subseteq S \times (\mathbb{N}_{\geq 1} \cup \{\infty\}) \times S \) is a timeout transition relation defining at least one timeout transition in every state.

Our definition of TFSM-T extends the definition in [4] by allowing multiple timeout transitions in the same state, which we use later to compactly represent sets of TFSMs-T. An input/output transition \((s, i, o, s') \in \lambda_S\) defines the output \(o\) produced in its source state \(s\) when input \(i\) is applied. A timeout transition \((s, \delta, s') \in \Delta_S\) defines the timeout \(\delta\) in state \(s\). A timeout transition can be taken if no input is applied at the current state before the timeout of the transition expires. It is not possible to wait for an input beyond the maximal timeout defined in the current state.

A TFSM-T uses a single clock for recording the time elapsing in the states and determining when timeouts expire. The clock is reset when the transitions are passed. A timed state of TFSM-T \(S\) is a pair \((s, x) \in S \times \mathbb{R}_{\geq 0}\) where \(s\) is a state of \(S\) and \(x \in \mathbb{R}_{\geq 0}\) is the current value of the clock and \(x < \delta\) for some \(\delta \in \mathbb{N}_{\geq 1} \cup \{\infty\}\) such that \((s, \delta, s') \in \Delta_S\). An execution step of \(S\) in timed state \((s, x)\) corresponds either to the time elapsing or the passing of an input/output or timeout transition; it is permitted by a transition of \(S\). Formally, \(stp = (s, x) a(s', x') \in (S \times \mathbb{R}_{\geq 0}) \times ((I \times O) \cup \mathbb{R}_{\geq 0}) \times (S \times \mathbb{R}_{\geq 0})\) is an execution step if it satisfies one of the following conditions:

- (timeout step) \(a \in \mathbb{R}_{\geq 0}, x' = 0\) and \(x + a = \delta\) for some \(\delta\) such that \((s, \delta, s') \in \Delta_S\); then \((s, \delta, s')\) is said to permit the step.
- (time-elapsing step) \(a \in \mathbb{R}_{\geq 0}, x' = x + a, s' = s\) and \(x + a < \delta\) for some \(\delta\) and \(s'' \in S\) such that there exists \((s, \delta, s'') \in \Delta_S\); then \((s, \delta, s'')\) is said to permit the step.
- (input/output step) \(a = (i, o)\) with \((i, o) \in I \times O\), \(x' = 0\) and there exists \((s, i, o, s') \in \lambda_S\); then \((s, i, o, s')\) is said to permit the step.

Time-elapsing steps satisfy the following time-continuity property w.r.t the same timeout transition:

if \((s_1, x_1) d_1(s_2, x_2) d_2(s_3, x_3) \ldots d_{k-1}(s_k, x_k)\) is a sequence of time-elapsing steps permitted by the same timeout transition \(t\), then \((s_1, x_1) d_1 + d_2 + \ldots + d_{k-1}(s_k, x_k)\) is a time-elapsing step permitted by \(t\). In the sequel, any time-elapsing step permitted by a timeout transition can be represented with a sequence of time-elapsing steps permitted by the same timeout transitions.
An execution of $S$ in timed state $(s, x)$ is a sequence of steps $e = stp_1 stp_2 \ldots stp_n$ with $stp_k = (s_{k-1}, x_{k-1}) a_k (s_k, x_k)$, $k \in [1, n]$ such that the following conditions hold:

- $(s_0, x_0) = (s, x)$,
- $stp_1$ is not an input/output step,
- $stp_k$ is a input/output step implies that $stp_{k-1}$ is a time-elapsing step for every $k \in [1, n]$.

If needed, the elapsing of zero time units can be inserted between a timeout step and an input/output step. Let $d_1 d_2 \ldots d_l \in \mathbb{R}_{\geq 0}^l$ be a non-decreasing sequence of real numbers, i.e., $d_k \leq d_{k+1}$ for every $k = 1, l-1$. The sequence $\sigma_e = ((i_1, o_1), d_1)((i_2, o_2), d_2) \ldots ((i_l, o_l), d_l)$ in $((I \times O) \times \mathbb{R}_{\geq 0})^*$ with $l < n$ is a timed input/output sequence of execution $e$ if $(i_1, o_1)(i_2, o_2) \ldots (i_l, o_l)$ is the maximal sequence of input/output pairs occurring in $e$. The delay $d_k$ for each input/output pair $(i_k, o_k)$, with $k = 1, l$, is the amount of the time elapsed from the beginning of $e$ to the occurrence of $(i_k, o_k)$. The timed...
input sequence and the timed output sequence of e are \((i_1, d_1)(i_2, d_2)...(i_t, d_t)\) and \((o_1, d_1)(o_2, d_2)...(o_t, d_t)\), respectively. We let \(\text{inp}(e)\) and \(\text{out}(e)\) denote the timed input and output sequences of execution e. Given a timed input sequence \(\alpha\), let \(\text{out}_S((s, x), \alpha)\) denote the set of all timed output sequences which can be produced by \(S\) when \(\alpha\) is applied in \(s\), i.e., \(\text{out}_S((s, x), \alpha) = \{\text{out}(e) \mid e\text{ is an execution of }S\text{ in } (s, x)\text{ and }\text{inp}(e) = \alpha\}\).

A TFSM-T \(S\) is deterministic (DTFSM-T) if it defines at most one input-output transition for each tuple \((s, i) \in S \times I\) and exactly one timeout transition in each state; otherwise, it is nondeterministic. \(S\) is initially connected if it has an execution from its initial state to each of its states. \(S\) is complete if for each tuple \((s, i) \in S \times I\) it defines at least one input-output transition. Note that the set of timed input sequences defined in each state of a complete machine \(S\) is \((I \times \mathbb{R}_{\geq 0})^*\).

We define distinguishability and equivalence relations between states of complete TFSMs-T. Similar notions were introduced in [27]. Intuitively, states producing different timed output sequences in response to the same timed input sequence are distinguishable. Let \(p\) and \(s\) be the states of two complete TFSMs-T over the same inputs and outputs. Given a timed input sequence \(\alpha\), \(p\) and \(s\) are distinguishable (with distinguishing input sequence \(\alpha\)), denoted \(p \not\simeq \alpha s\), if the sets of timed output sequences in \(\text{out}_S((p, 0), \alpha)\) and \(\text{out}_S((s, 0), \alpha)\) differ; otherwise they are equivalent and we write \(s \simeq p\), i.e., if the sets of timed output sequences coincide for all timed input sequence \(\alpha\).

**Definition 2 (Submachine).** TFSM-T \(S = (S, s_0, I, O, \lambda_S, \Delta_S)\) is a submachine of TFSM-T \(P = (P, p_0, I, O, \lambda_P, \Delta_P)\) if \(S \subseteq P\), \(s_0 = p_0\), \(\lambda_S \subseteq \lambda_P\) and \(\Delta_S \subseteq \Delta_P\).

**Example 1.** Figure 1 presents two initially connected TFSMs-T \(S_1\) and \(M_1\). \(M_1\) is nondeterministic; it defines two timeout transitions in states \(s_1\) and \(s_3\), which is not allowed in [4]. \(S_1\) is a complete deterministic submachine of \(M_1\).

Here are four executions of the TFSM-T \(M_1\) in Figure 1b where the transitions defining the steps appear below the arrows:

1. \((s_1, 0) \xrightarrow{\frac{2}{t_3} t_{16}} (s_1, 2) \xrightarrow{b_x t_2} (s_2, 0) \xrightarrow{\frac{1}{t_6}} (s_2, 1) \xrightarrow{a_x t_5} (s_3, 0) \xrightarrow{\frac{8}{t_{17}} t_s} (s_1, 0) \xrightarrow{\frac{4}{t_3}} (s_4, 0) \xrightarrow{\frac{0.5}{t_{12}}} (s_4, 0.5) \xrightarrow{a_y t_0} (s_1, 0)\)

2. \((s_1, 0) \xrightarrow{\frac{4}{t_3}} (s_4, 0) \xrightarrow{\frac{0.5}{t_{12}}} (s_4, 0.5) \xrightarrow{a_y t_0} (s_1, 0) \xrightarrow{\frac{3}{t_{16}}} (s_4, 0) \xrightarrow{\frac{0.7}{t_{12}}} (s_4, 0.7) \xrightarrow{a_y t_{13}} (s_2, 0)\)

3. \((s_1, 0) \xrightarrow{\frac{3.5}{t_3}} (s_1, 3.5) \xrightarrow{b_x t_2} (s_2, 0) \xrightarrow{\frac{1}{t_6}} (s_2, 1) \xrightarrow{a_x t_5} (s_3, 0) \xrightarrow{\frac{8}{t_{17}}} (s_1, 0) \xrightarrow{\frac{4}{t_3}} (s_4, 0) \xrightarrow{\frac{0.5}{t_{12}}} (s_4, 0.5) \xrightarrow{a_y t_{10}} (s_1, 0)\)

4. \((s_1, 0) \xrightarrow{\frac{3}{t_{16}}} (s_4, 0) \xrightarrow{\frac{0.5}{t_{12}}} (s_4, 0.5) \xrightarrow{b_x t_{11}} (s_4, 0) \xrightarrow{\frac{1}{t_{12}}} (s_4, 1) \xrightarrow{a_y t_{13}} (s_2, 0) \xrightarrow{\frac{12.5}{t_6}} (s_2, 12.5) \xrightarrow{a_x t_5} (s_3, 0)\)

Let us explain the first execution. It has 8 steps represented with arrows between timed states. The label above an arrow is either a delay in \(\mathbb{R}_{\geq 0}\) or an input-output pair. The
label below an arrow indicates the transitions permitting the step. The first, third and seventh steps of the first execution are time-elapsing. The second, fourth and last steps are input-output. The fifth and the sixth steps are timeout. The first step is permitted by either transition $t_3$ or $t_{16}$ because their timeouts are not expired 2 units after the machine has entered state $(s_1, 0)$. The timeout of transition $t_{12}$ permitting the seventh step has not expired before input-output transition $t_{10}$ is performed at the last step. The difference between the first and the third execution is that input $b$ is applied lately in the third execution, i.e., $3.5$ time units after the third execution has started. The timed input/output sequences for the four executions are $((b, x), 2)((a, x), 3)((a, y), 15.5)$, $((a, y), 4.5)((a, y), 8.2)$, $((b, x), 3.5)((a, x), 4.5)((a, y), 17)$ and $((b, x), 3.5)((a, y), 4.5)((a, x), 17)$, respectively. The timed input sequence and the timed output sequence for the first execution are $(b, 2)(a, 3)(a, 15.5)$ and $(x, 2)(x, 3)$ $(y, 15.5)$. Similarly, we can determine the timed input and output sequences for the three other executions. The third and the fourth executions have the same timed input sequence but different timed output sequences.

Henceforth the TFSMs-T are complete and initially connected.

### 2.2 Complete test suite for fault models

Let $S = (S, s_0, I, O, \lambda_S, \Delta_S)$ be a DTFSM-T, called the specification machine.

**Definition 3 (Mutation machine for a specification machine).** A nondeterministic TFSM-T $M = (M, m_0, I, O, \lambda_M, \Delta_M)$ is a mutation machine of $S$ if $S$ is a sub-machine of $M$. Transitions in $\lambda_M$ but not in $\lambda_S$ or in $\Delta_M$ but not in $\Delta_S$ are called mutated.

A mutant is a deterministic submachine of $M$ different from the specification. We let $\text{Mut}(M)$ denote the set of mutants in $M$. A mutant represents an implementation of the specification seeded with faults. Faults are represented with mutated transitions and every mutant defines a subset of them. Mutated transitions can represent transfer faults, output faults, changes of timeouts and adding of extra-states.

A transition $t$ is suspicious in $M$ if $M$ defines another transition $t'$ from the source state of $t$ and either both $t$ and $t'$ have the same input or they are timeout transitions. A transition of the specification is called untrusted if it is suspicious in the mutation machine; otherwise, it is trusted. The set of suspicious transitions of $M$ is partitioned into a set of untrusted transitions all defined in the specification and the set of mutated transitions undefined in the specification.

Let $P$ be a mutant with an initial state $p_0$ of the mutation machine $M$ of $S$. We use the state equivalence relation $\simeq$ to define conforming mutants.

**Definition 4 (Conforming mutants and detected mutants).** Mutant $P$ is conforming to $S$, if $p_0 \simeq s_0$; otherwise, it is nonconforming and a timed input sequence $\alpha$ such that $\text{out}_P((p_0, 0), \alpha) \neq \text{out}_S((s_0, 0), \alpha)$ is said to detect $P$. 
We say that mutant $P$ survives input sequence $\alpha$ if $\alpha$ does not detect $P$.

The set $\text{Mut}(\mathcal{M})$ of all mutants in mutation machine $\mathcal{M}$ is called a fault domain for $\mathcal{S}$. If $\mathcal{M}$ is deterministic and complete then $\text{Mut}(\mathcal{M})$ is empty. A general fault model is the tuple $\langle \mathcal{S}, \approx, \text{Mut}(\mathcal{M}) \rangle$ following [20,17]. Let $\lambda_{\mathcal{M}}(s, i)$ denote the set of input/output transitions defined in state $s$ with input $i$ and $\Delta_{\mathcal{M}}(s)$ denote the set of timeout transitions defined in state $s$. The number of mutants in $\text{Mut}(\mathcal{M})$ is given by the formula $|\text{Mut}(\mathcal{M})| = \Pi_{(s, i) \in \mathcal{S} \times I}|\lambda_{\mathcal{M}}(s, i)| \times \Pi_{s \in \mathcal{S}}|\Delta_{\mathcal{M}}(s)| - 1$, where $\lambda_{\mathcal{M}}(s, i)$ denotes the set of input-output transitions with input $i$ defined in $s$ and $\Delta_{\mathcal{M}}(s)$ denotes the set of timeout transitions in $s$. The conformance relation partitions the set $\text{Mut}(\mathcal{M})$ into conforming mutants and nonconforming ones which we need to detect.

**Definition 5 (Complete test suite).** A test for $\langle \mathcal{S}, \approx, \text{Mut}(\mathcal{M}) \rangle$ is a timed input sequence. A complete test suite for $\langle \mathcal{S}, \approx, \text{Mut}(\mathcal{M}) \rangle$ is a set of test detecting all nonconforming mutants in $\text{Mut}(\mathcal{M})$.

**Example 2.** In Figure 1, $M_1$ is a mutation machine for the specification machine $S_1$. $M_1$ and $S_1$ has the same number of states, meaning that the faults represented with mutated transitions in $M_1$ do not introduce extra-states. The mutated transitions are represented by dashed lines. The transitions $t_3$, $t_{14}$ and $t_7$ are suspicious; however $t_1$ is not. $t_1$ is trusted and $t_7$ is untrusted. $t_{14}$ is neither trusted nor untrusted because it does not belong to the specification. $t_{14}$ defines an output fault on input $\alpha$ since the expected output is defined with transition $t_7$. In state $s_3$, $t_{15}$ defines a transfer fault for input $b$ and $t_{17}$ increases the expected timeout for $s_3$ and defines a transfer fault. The transition $t_{16}$ implements a fault created by reducing the timeout of $t_3$; it is defined in the mutant $P_1$ in Figure 1c. For the timed input sequence $\langle b, 3.5 \rangle(a, 4.5)(a, 17)$, the specification $S_1$ and mutant performs the third and fourth executions in Example 1 respectively. $P_1$ is nonconforming because the produced timed input sequence $\langle x, 3.5 \rangle(y, 4.5)(x, 17)$ differ from $\langle x, 3.5 \rangle(x, 4.5)(y, 17)$, the timed output sequence produced by $S_1$. $M_1$ defines 31 mutants; some of them are conforming and we would like to generate a complete test suite detecting all the nonconforming mutants.

To generate a complete test suite, a test can be computed for each nonconforming mutant by enumerating the mutants one-by-one, which would be inefficient for huge fault domains. We avoid the one-by-one enumeration of the mutants with constraints specifying only test-surviving mutants.

### 3 Specifying test-surviving mutants

The mutants surviving a test cannot produce any execution with an unexpected timed output sequence for the test. We encode them with Boolean formulas over Boolean transition variables of which the values indicate the presence or absence of transitions of the mutation machine in mutants.

#### 3.1 Revealing combs and involved mutants

The mutants detected by a test $\alpha$ exhibit a revealing execution which produces an unexpected timed output sequence and has $\alpha$ as the test. Such an execution is permitted...
by transitions forming a comb-subgraph in the state transition diagram of the mutation
machine. Intuitively, a comb for an execution is nothing else but a path augmented with
timeout-unexpired transitions, i.e., transitions of which the timeouts have not expired
prior to performing an input-output step. These additional timeout transitions are also
needed to specify detected mutants and eliminate them from the fault domain. To sim-
plify the notation, we represent comb-subgraphs with sequences of transitions.

A comb of an execution \( e = stp_1stp_2 \cdots stp_n \) is the sequence of transitions \( t_1t_2 \cdots t_n \)
such that \( t_i \) permits \( stp_i \) for every \( i = 1 \ldots n \). We say that comb \( t_1t_2 \cdots t_n \) is enabled
by the input sequence of \( e \). Each timeout or input/output step in \( e \) is permitted with
a unique transition. However, each time-elapsing step is permitted by a timeout trans-
ition with an unexpired timeout, i.e., the timeout is not greater than the clock value
in the source timed state of the time-elapsing step. So, several combs can permit the
same execution since several timeout transitions permit the same time-elapsing step.
Note that timeout transitions with finite or infinite timeouts appear in combs when they
permit time-elapsing steps preceding input/output steps; later such timeout transitions
participate in Boolean encodings of combs involving detected mutants.

Example 3. There are two combs for the first execution in Example [1] this is because
the first step of the execution is permitted either by \( t_3 \) or \( t_{16} \). The first comb \( t_3t_6t_5t_{17}t_3t_{12}t_{10} \)
is represented in Figure [2]. The timeouts of the transitions represented with vertical ar-
rows have not expired in the execution when the input-output transition is performed.
The timeouts of the transitions represented with horizontal arrows have expired. The
first comb is deterministic whereas the second comb \( t_{16}t_6t_5t_{17}t_3t_{12}t_{10} \) is nondeter-
ministic because \( t_{16} \) and \( t_3 \) are two suspicious timeout transitions defined in \( s_1 \).
The first comb for the first execution is also a comb for the third execution in Example [1]
but the second comb is not a comb for the third execution. This is because the application
of \( b \) after 3.5 time units in \( s_1 \) is possible if the timeout transition \( t_{16} \) is not passed.
The second execution corresponds to a single nondeterministic comb \( t_3t_{12}t_{10}t_{16}t_{12}t_{13} \).
It is nondeterministic because \( t_{10} \) and \( t_{13} \) are two input/output transitions defined in \( s_4 \)
with the same input \( a \). Each occurrence of the timeout transition \( t_{12} \) before an input/output
transition indicates that the timeout of \( t_{12} \) has not expired before the input/output
transition is passed.

Combs for executions with unexpected timed output sequences reveal nonconform-
ing mutants unless they belong only to nondeterministic submachines. The combs be-
longing only to nondeterministic submachines have two transitions which are not de-
defined in the same mutant; such combs are called nondeterministic.
if it has two suspicious input-output transitions or two timeout transitions defined in an identical state of the mutation machine; otherwise, it is a deterministic comb.

Clearly, combs in a mutant or the specification are deterministic because two suspicious transitions cannot be defined in an identical state in a mutant or the specification. A nondeterministic submachine of a mutation machine can contain both deterministic and nondeterministic combs.

Definition 6 (Deterministic and nondeterministic combs). A comb is nondeterministic if it has two suspicious input-output transitions or two timeout transitions defined in an identical state of the mutation machine; otherwise, it is a deterministic comb.

Definition 7 (Revealing comb). Let \( \pi \) be a comb of an execution \( e_1 \) from \( (s_0, 0) \). We say that \( \pi \) is \( \alpha \)-revealing if there exists an execution \( e_2 \) of \( S \) such that \( \alpha = \text{inp}(e_1) = \text{inp}(e_2) \), \( \text{out}(e_1) \neq \text{out}(e_2) \) while this does not hold for any prefix of \( \alpha \). Comb \( \pi \) is revealing if it is \( \alpha \)-revealing for some input sequence \( \alpha \).

The specification contains no revealing comb because its executions always produce expected timed output sequences. Only mutants, nondeterministic or incomplete submachines of a mutation machine can contain revealing combs; but mutants contain only deterministic revealing combs.

Let \( \text{Rev}_\alpha(\mathcal{P}) \) denote the set of deterministic \( \alpha \)-revealing combs of machine \( \mathcal{P} \).

Lemma 1. \( \text{Rev}_\alpha(\mathcal{M}) = \bigcup_{\mathcal{P} \in \text{Mut}(\mathcal{M})} \text{Rev}_\alpha(\mathcal{P}) \), for any test \( \alpha \).

Proof. Mutants are deterministic submachines of the mutation machine. Each deterministic revealing comb in \( \mathcal{M} \) is a comb in a mutant, meaning that \( \text{Rev}_\alpha(\mathcal{M}) \subseteq \bigcup_{\mathcal{P} \in \text{Mut}(\mathcal{M})} \text{Rev}_\alpha(\mathcal{P}) \). Combs in mutants are necessarily deterministic and revealing combs in mutants are deterministic revealing comb in the mutation machine because mutants are deterministic submachines of the mutation machine. Thus \( \bigcup_{\mathcal{P} \in \text{Mut}(\mathcal{M})} \text{Rev}_\alpha(\mathcal{P}) \subseteq \text{Rev}_\alpha(\mathcal{M}) \).

Lemma 2. Let \( \pi \in \text{Rev}_\alpha(\mathcal{M}) \) for a test \( \alpha \) and \( \mathcal{P} \in \text{Mut}(\mathcal{M}) \). \( \mathcal{P} \) is not detected by \( \alpha \) if and only if \( \pi \notin \text{Rev}_\alpha(\mathcal{P}) \).

Proof. Assume that \( \mathcal{P} \in \text{Mut}(\mathcal{M}) \) is not detected by \( \alpha \). Then \( \mathcal{P} \) is either conforming or nonconforming. \( \mathcal{P} \) contains no revealing comb if it is conforming; then \( \pi \notin \text{Rev}_\alpha(\mathcal{P}) \).
\(Rev_\alpha(\mathcal{P})\) for every \(\pi \in Rev_\alpha(\mathcal{M})\). If \(\mathcal{P}\) is nonconforming and there is \(\pi \in Rev_\alpha(\mathcal{P}) \cap Rev_\alpha(\mathcal{M})\), we get a contradiction with the fact that \(\mathcal{P}\) is not detected by \(\alpha\) because \(\pi\) is the comb for an execution in \(\mathcal{P}\) with timed input sequence \(\alpha\) and an unexpected timed output sequence. Conversely, by Definition 4 and Definition 7 if \(Rev_\alpha(\mathcal{P}) \cap Rev_\alpha(\mathcal{M}) = \emptyset\) then \(\alpha\) does not detect \(\mathcal{P}\).

**Example 4.** The comb for the fourth execution in Example 1 namely \(t_{16}t_{12}t_{11}t_{12}t_{13}t_{14}\) is revealing and contained in the mutant and mutation machine in Figure 1. To prevent the fourth execution, we must prevent one the transitions in the comb. For example, if we prevent \(t_{16}\), the other timeout transition \(t_3\) defined in \(s_1\) will be performed, yielding to the third execution which cannot be performed in the mutant \(\mathcal{P}_1\), but rather in \(\mathcal{S}_1\). Clearly \(\mathcal{S}_1\) is not detected by \((b, 3.5)(a, 4.5)(a, 17)\), in the contrary of \(\mathcal{P}_1\).

We define a distinguishing automaton with timeouts for the specification and the mutation machine; the automaton define all revealing combs from which we will serve to extract deterministic combs which reveal nonconforming mutants.

**Definition 8 (Distinguishing automaton with timeouts).** Given a specification machine \(\mathcal{S} = (S, s_0, I, O, \Delta_S, \Delta)\) and a mutation machine \(\mathcal{M} = (M, m_0, I, O, \lambda_M, \Delta_M)\), a finite automaton \(\Delta = (C \cup \{\nabla\}, c_0, I, \Delta_D, \Delta_D, \nabla)\), where \(C \subseteq S \times S \times (\mathbb{N}_{\geq 0} \cup \{\infty\}) \times (\mathbb{N}_{\geq 0} \cup \{\infty\})\), \(\Delta_D \subseteq C \times I \times C\) is the input transition relation, \(\Delta_D \subseteq C \times (\mathbb{N}_{\geq 1} \cup \{\infty\}) \times C\) is the timeout transition relation and \(\nabla\) is the accepting (sink) state, is the distinguishing automaton with timeouts for \(\mathcal{S}\) and \(\mathcal{M}\), if it holds that:

- \(c_0 = (s_0, m_0, 0, 0)\)

- For each \((s, m, x_s, x_m) \in C\) and \(i \in I\)
  
- \((\mathcal{R}_1)\) : \(\{(s, m, x_s, x_m), i, (s', m', 0, 0)\} \in \lambda_D\) if there exists \((s, i, o, s') \in \lambda_S, (m, i, o', m') \in \lambda_M\) s.t. \(o = o'\)

- \((\mathcal{R}_2)\) : \(\{(s, m, x_s, x_m), i, \nabla\} \in \lambda_D\) if there exists \((s, i, o, s') \in \lambda_S, (m, i, o', m') \in \lambda_M\) s.t. \(o \neq o'\)

- For each \((s, m, x_s, x_m) \in C\) and the only timeout transition \((s, \delta_s, s') \in \Delta_S\)
  
- \((\mathcal{R}_3)\) : \(\{(s, m, x_s, x_m), \delta_s - x_m = \delta_m - x_m\} \in \Delta_D\) if there exists \((s, m, m') \in \Delta_M\) s.t. \(\delta_s - x_m < \delta_m - x_m\) and \(\delta_s = \infty\) and \(\delta_m - x_m > 0\)

- \((\mathcal{R}_4)\) : \(\{(s, m, x_s, x_m), \delta_s - x_m = \delta_m - x_m, (s, s', m, x_s + \delta_m - x_m, 0)\} \in \Delta_D\) if there exists \((s, m, m') \in \Delta_M\) s.t. \(\delta_m - x_m < \delta_s - x_m\) and \(\delta_m = \infty\) and \(\delta_m - x_m > 0\)

- \((\mathcal{R}_5)\) : \(\{(s, m, x_s, x_m), \delta_m - x_m, (s, m', 0, 0)\} \in \Delta_D\) if there exists \((s, m, m') \in \Delta_M\) s.t. \(\delta_m = \delta_m - x_m < \delta_s - x_m\) and \(\delta_s = \infty\) and \(\delta_m - x_m > 0\)

- \((\mathcal{R}_6)\) : \(\{(s, m, x_s, x_m), \delta_s - x_m, (s', m, 0, x_m + \delta_s - x_s)\} \in \Delta_D\) if there exists \((s, m, m') \in \Delta_M\) s.t. \(\delta_s - x_m < \delta_m - x_m\) and \(\delta_m = \infty\) and \(\delta_m - x_m > 0\)

- \((\mathcal{R}_7)\) : \(\{(s, m, x_s, x_m), \delta_s - x_m, (s', m, 0, 0, \infty)\} \in \Delta_D\) if there exists \((s, m, m') \in \Delta_M\) s.t. \(\delta_s - x_m < \delta_m - x_m\) and \(\delta_m = \infty\) and \(\delta_m - x_m > 0\), where \(\infty - x = \infty\) if \(x\) is finite or infinite and \(\infty + \infty = \infty\).

- \((\nabla, x, \nabla) \in \lambda_D\) for all \(x \in I\) and \((\nabla, \infty, \nabla) \in \Delta_D\)
The seven rules \( \{ \mathcal{R}_i \}_{i=1..7} \) introduce input transitions and timeout transitions in \( \mathcal{D} \). Each state \((s, m, x_s, x_m)\) of \( \mathcal{D} \) is composed of a state \( s \) of the specification, a state \( m \) of the mutation machine, the value \( x_s \) of the clock of the specification and the value \( x_m \) of the clock of the mutation machine. The clock values are needed for selecting timeout transitions. Input transitions are introduced with \( \mathcal{R}_1 \) and \( \mathcal{R}_2 \). According to \( \mathcal{R}_2 \), there is a transition to accepting state \( \nabla \) if different outputs are produced in \( s \) and \( m \) for the same input; otherwise the specification and the mutation machine move to next states, as described with rule \( \mathcal{R}_1 \).

Timeout transitions of the form \( ((s, m, x_s, x_m), \delta, (s', m', x'_s, x'_m)) \) are introduced by \( \{ \mathcal{R}_i \}_{i=3..7} \). The value of \( \delta \) is the delay for reaching the only timeout \( \delta_s \) defined in \( s \) from \((s, x_s)\) or a timeout defined in \( m \) from \((m, x_m)\), since multiple timeouts can be defined in states of nondeterministic mutation machines. \( \delta \) can be greater than the delays for reaching some timeouts defined in \( m \); however, \( \delta \) is never greater than \( \delta_s - x_s \), the delay for reaching the only timeout \( \delta_s \) in \( s \). So, \( x'_s = 0 \) if \( \delta = \delta_s - x_s \); a similar statement holds for the clock and a selected timeout transition of the mutation machine.

In \( \mathcal{R}_3 \), both the timeout in \( s \) and a timeout in \( m \) expire after \( \delta \) time units. In \( \mathcal{R}_4 \) only a timeout defined in \( m \) expires after \( \delta \) time units and the only finite timeout defined in \( s \) does not expire after \( \delta \) time units. A similar phenomenon is described with \( \mathcal{R}_5 \); but contrarily to \( \mathcal{R}_4 \), the only timeout in \( s \) is \( \infty \) and we set the clock of the specification to \( \infty \). Setting the clock to \( \infty \) expresses the fact that we do not care any more about finite values of \( x'_s \) because only the infinite timeout in \( s \) must be reached. Without this latter abstraction on the values of \( x'_s \), the size of \( C \) could be infinite because we could have to apply \( \mathcal{R}_4 \) infinitely. The rules \( \mathcal{R}_6 \) and \( \mathcal{R}_7 \) are similar to \( \mathcal{R}_4 \) and \( \mathcal{R}_5 \), except that the only timeout in \( s \) expires before a timeout in \( m \).

Each transition introduced in \( \mathcal{D} \) with \( \mathcal{R}_1 \), \( \mathcal{R}_2 \), and \( \mathcal{R}_3 \), is defined by a transition of the specification and a transition of the mutation machine. A transition introduced with \( \mathcal{R}_4 \) is defined by a timeout transition of the mutation machine and the only timeout transition of the specification in \( s \). Every comb of \( \mathcal{D} \) is defined by a comb of the specification and a comb of the mutation machine, i.e., it has been obtained by composing the transitions in a comb of the specification with the transitions in a comb of the mutation machine.

An execution of \( \mathcal{D} \) from a timed state \((c, x)\) is a sequence of steps between timed states of \( \mathcal{D} \); it can be defined similarly to that for a TFSM-T. An execution starting from \((c_0, 0)\) and ending at \( \nabla \) is called accepted. As for TFSMs-T, we can associate every execution of \( \mathcal{D} \) with a comb and a timed input sequence. A comb of \( \mathcal{D} \) is accepted if it corresponds to an accepted execution.

**Lemma 3.** A comb \( \pi \) of \( \mathcal{M} \) is revealing if it defines an accepted comb of \( \mathcal{D} \).

**Proof.** Let \( \pi' \) be an accepted comb of \( \mathcal{D} \) defined by a deterministic comb \( \pi \) of \( \mathcal{M} \). \( \pi' \) includes the sink state and it was obtained by composing the transitions in \( \pi \) with transitions in the specification with the rules in Definition 8. \( \pi \) corresponds to an execution of the mutation machine producing an unexpected timed output sequence, because otherwise \( \pi' \) would not have been accepted. Consequently \( \pi \) is revealing.

A revealing comb can be common to many mutants, in which case those mutants are said to be involved in the comb. Mutants involved in a revealing comb are detected.
by the tests enabling the comb. We let $\text{Sus}_X$ denote the set of suspicious transitions in $X$.

**Lemma 4.** A mutant $P$ is involved in a revealing comb $\pi$ of $M$ if and only if $\text{Sus}_\pi \subseteq \text{Sus}_P$.

**Proof.** Any mutant which does not contain a transition in a comb is not able to perform any execution defining the comb. Mutants contain all trusted transitions but a selected subset of suspicious transitions specified in the mutation machine. Consequently a mutant is involved in a revealing comb if and only if the suspicious transitions in the comb are included in the suspicious transitions contained in the mutant.

Lemma 4 assumes that mutants are known and indicates how to check if a mutant is involved in a given (deterministic or nondeterministic) revealing comb. However, we want to avoid the enumeration of the mutants in eliminating the nonconforming mutants detected by a test; we also want to generate tests corresponding to deterministic revealing combs because they detect nonconforming mutants as stated in Lemma 2. So we will focus on extracting only deterministic revealing combs of $M$ from $\mathcal{D}$. We can obtain deterministic revealing combs of $M$ by performing a Breadth-first search of sink state $\nabla$ in $\mathcal{D}$ while passing through transitions of $\mathcal{D}$ defined with transitions of $M$ which cannot be defined in an identical mutant. To compute $\text{Rev}_\alpha(M)$ the set of deterministic $\alpha$-revealing combs, we also apply Breath-first search of the sink state in the distinguishing automaton $\mathcal{D}$. This time the search is step-wise and guided by timed inputs in $\alpha$; it consists to pass a timeout transition in $\mathcal{D}$ whenever the delay between the current and the previous input in $\alpha$ is greater than the timeout of the transition or to pass an input transition in $\mathcal{D}$ when the current state in $\mathcal{D}$ defines a timeout smaller than delay between the current and the previous input. The result of the Breath-first search is $\text{Rev}_\alpha(M)$.

**Example 5.** Figure 3 presents an excerpt of the distinguishing automaton with timeouts for the $S_1$ and $M_1$ in Figure 1. It is the relevant fragment for extracting the revealing combs for the test $\alpha = (b, 0, 5)(a, 1)(b, 6.7)(a, 7.2)$. The whole distinguishing automaton is too big to fit in this paper. $[t]([R])$ indicates an input/output transition or a timeout transition $t$ of the mutation machine defining the transition of the automaton introduced with rule $R$; e.g., the timeout transition $((s_3, s_3, 0, 0), 5, (s_2, s_3, 0, 5))$ is defined by $t_{17}$ and the timeout of $t_{17}$ has not expired when the rule $R_6$ is applied. There are six deterministic $\alpha$-revealing combs: $t_{3}t_{2}t_{6}t_{5}t_{17}t_{8}t_{12}t_{10}$, $t_{3}t_{2}t_{6}t_{5}t_{17}t_{8}t_{12}t_{13}$, $t_{3}t_{2}t_{6}t_{5}t_{17}t_{15}t_{17}t_{14}$, $t_{16}t_{2}t_{16}t_{5}t_{17}t_{8}t_{12}t_{10}$, $t_{16}t_{2}t_{16}t_{5}t_{17}t_{8}t_{12}t_{13}$ and $t_{16}t_{2}t_{16}t_{5}t_{17}t_{15}t_{17}t_{14}$, where the transitions in bold are suspicious.

### 3.2 Encoding submachines involved in revealing combs

We introduce a Boolean variable for each suspicious transition in mutation machine $M$; Based on Lemma 4 we build Boolean formulas over these variables to encode the mutants involved in revealing comb. A solution of such a formula assigns a truth value to every transition variable. We say that a solution of a formula determine a submachine
\( \mathcal{P} \) of \( \mathcal{M} \) if \( \mathcal{P} \) is composed of the trusted transitions and the suspicious transitions of which the values of the corresponding transition variable is True in the solution. In general, the submachine for the solution of a formula can be noninitially-connected, nondeterministic or incomplete. Later we encode mutants (deterministic and complete submachines) with additional formulas. For now, let us encode the submachines involved in revealing combs of \( \mathcal{M} \) with Boolean formulas.

Let \( \alpha \) be a test and \( Rev_\alpha(\mathcal{M}) = \{ \pi_1, \pi_2, \ldots, \pi_n \} \) be the set of deterministic revealing combs of \( \mathcal{M} \) enabled by \( \alpha \). We encode a comb \( \pi = t_1t_2\ldots t_m \) of \( \mathcal{M} \) with the Boolean formula \( \phi_{\pi} = \bigwedge_{t_i \in Susp_{\pi}} \neg t_i \), the conjunction of all the suspicious transitions in \( \pi \). Clearly, any solution of \( \phi_{\pi} \) determines a submachine (of the mutation machine) containing the comb \( \pi \): The executions associated with \( \pi \) are defined in such a submachine which is detected by \( \alpha \). Conversely, each submachine determined by a solution of the negation of \( \phi_{\pi} \) does not contains \( \pi \); it cannot define any execution associated with \( \pi \) and is not detected by \( \alpha \). Such a submachine is not necessarily a mutant because it can be nondeterministic or incomplete. For the set of deterministic revealing combs in \( Rev_\alpha(\mathcal{M}) \), let us define the formula \( \phi_{\alpha} = \bigvee_{\pi \in Rev_\alpha(\mathcal{M})} \phi_{\pi} \). The set of (possibly nondeterministic or incomplete) submachines of \( \mathcal{M} \) detected by \( \alpha \) is determined by a solution of the negation of \( \phi_{\alpha} \), as stated in Lemma 5.

Lemma 5. A submachine of \( \mathcal{M} \) survives a test \( \alpha \) if and only if it can be determined by a solution of \( \neg \phi_{\alpha} \).

To obtain the mutants surviving \( \alpha \), we remove from the solutions of \( \neg \phi_{\alpha} \) those determining nondeterministic or incomplete submachines. This is possible with a Boolean formula encoding only the mutants in \( \mathcal{M} \).

3.3 Encoding the mutants in a mutation machine

Let \( \mathcal{T} = t_1, t_2, \ldots, t_n \) be a set of Boolean variables for all the transitions \( t_i \) of \( \mathcal{M} \), \( i = 1..n \). Let us define the Boolean formula \( \xi_\mathcal{T} \) as follows

\[
\xi_\mathcal{T} = \bigwedge_{k=1..n} \bigwedge_{l=k+1..n} (-t_k \lor -t_l) \land \bigvee_{k=1..n} t_i
\]

A solution of \( \xi_\mathcal{T} \) assigns True to exactly one selected variable and assigns False to all other variables. Note that \( \xi_\mathcal{T} \) is a CNF-SAT \([7]\) formula and it can be solved using an existing SAT solver \([23]\).

Let \( \mathcal{M} \) be a mutation machine for the specification machine \( \mathcal{S} \). Clearly, \( \lambda_S \subseteq \lambda_M \) and \( \Delta_S \subseteq \Delta_M \). A deterministic and complete submachine of \( \mathcal{M} \) selects one transition in \( \lambda_M(s,i) \) and one transition in \( \Delta_M(s) \) for every state \( s \) and input \( i \); it is therefore determined by a solution of \( \phi_M \) defined as follows.

\[
\phi_M = \bigwedge_{(s,i) \in S \times I} \xi_{\lambda_M(s,i)} \land \bigwedge_{s \in S} \xi_{\Delta_M(s)} \land \bigvee_{t \in \lambda_S \cup \Delta_S} \neg t
\]

The specification cannot be determined by a solution of \( \phi_M \) because its subformula \( \bigvee_{t \in \lambda_S \cup \Delta_S} \neg t \) encodes the rejection of transitions of the specification. The graph composed of the transitions selected by a solution can be disconnected, in which case it
does not represent any mutant; a mutant can be obtained by extracting all the selected transitions connected to the initial state. We can prove Lemma 6 based on the previous discussion.

**Lemma 6.** A submachine of $\mathcal{M}$ is complete and deterministic if and only if it is determined by a solution of $\varphi_M$.

We can prove the following theorem thanks to Lemma 6 and Lemma 5.

**Theorem 1.** A mutant survives the test $\alpha$ if it is determined by a solution of $\neg \varphi_\alpha \land \varphi_M$.

**Example 6.** Considering the revealing combs for $\alpha = (b, 0)(a, 0)$ $(b, 5)(a, 5)$, we use the suspicious transitions in the six revealing combs in Example 5 at Page 12 to compute $\neg \varphi_\alpha = (\neg t_3 \lor \neg t_{17} \lor \neg t_8 \lor \neg t_{10}) \land (\neg t_3 \lor \neg t_{17} \lor \neg t_8 \lor \neg t_{13}) \land (\neg t_3 \lor \neg t_{17} \lor \neg t_{15} \lor \neg t_{14}) \land (\neg t_{16} \lor \neg t_{17} \lor \neg t_8 \lor \neg t_{10}) \land (\neg t_{16} \lor \neg t_{17} \lor \neg t_8 \lor \neg t_{13}) \land (\neg t_{16} \lor \neg t_{17} \lor \neg t_{15} \lor \neg t_{14})$. The mutant composed with the transitions $t_1, t_2, t_3, t_4, t_5, t_7, t_9, t_{15}, t_{13}, t_{12}, t_{11}$ and $t_{16}$ is determined by a solution of $\neg \varphi_\alpha$ and it survives $\alpha$. The submachine with the transitions $t_1, t_2, t_3, t_4$ and $t_5$ is determined by another solution of $\neg \varphi_\alpha$; however, it is neither a mutant nor a solution of $\varphi_M$, defined as follows:

\[
\varphi_{M_\alpha} = (t_1) \land (t_2) \land (t_5) \land (t_4) \land (\neg t_7 \lor \neg t_{14}) \land (t_7 \lor t_{14}) \land (\neg t_8 \lor \neg t_{15}) \land (t_8 \lor t_{15}) \land (\neg t_{10} \lor \neg t_{13}) \land (t_{10} \lor t_{13}) \land (t_{11}) \land (t_3 \lor \neg t_{16}) \land (t_3 \lor t_{16}) \land (t_6) \land (\neg t_9 \lor \neg t_{17}) \land (t_9 \lor t_{17}) \land (t_{12}) \land (\neg t_{1} \lor \neg t_{2} \lor \neg t_4 \lor \neg t_5 \lor t_7 \lor \neg t_8 \lor \neg t_{11} \lor \neg t_{10} \lor \neg t_3 \lor \neg t_6 \lor \neg t_{12}).
\]

The mutants surviving a test $\alpha$ can be partitioned into conforming mutants and non-conforming mutants which can only be detected with a test different from $\alpha$. Non-conforming mutants can be used to generate additional tests and upgrade the constraints. The generated test suite is complete if the solutions of the constraints determine only conforming mutants. This is the intuition of the test verification and generation methods below. The methods avoid a one-by-one enumeration of the mutants because a single test eliminates many of them.

### 4 Verifying and Generating a Complete Test Suite

Let $E = \{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ be a test suite and $\langle S, \succeq, \text{Mut}(\mathcal{M}) \rangle$ be a fault model. Our method for verifying whether $E$ is complete works in three steps. First we build the Boolean expression $\bigwedge_{\alpha \in E} \neg \varphi_\alpha \land \varphi_M$ encoding the mutants surviving $E$; this is based on Theorem 1. Secondly, we use a solver to determine a mutant surviving the Boolean expression. Thirdly, we decide that $E$ is a complete test suite if there is no mutant surviving $E$ or all the mutants surviving the tests in $E$ are conforming. Procedure $\text{Verify_completeness}$ in Algorithm 1 implements the method. It makes a call to $\text{Determine_a_submachine}$ for obtaining a mutant in a fault domain specified with $\varphi_{fd}$. $\text{Determine_a_submachine}$ can use an efficient SAT-solver to solve $\varphi_{fd}$ and build mutants from solutions. $\text{Determine_a_submachine}$ returns $\text{null}$ when the $\varphi_{fd}$ is unsatisfiable, i.e., the fault domain is empty. $\text{Verify_completeness}$ always terminates; this is because the size of the fault domain and the number of revealing combs for a test are finite, and the SAT problem is decidable.
Procedure Verify_completeness \((\phi_{fd}, E, D)\):

**Input**: \(\phi_{fd}\), a Boolean expression specifying a fault domain

**Input**: \(E\), a (possibly empty) set of tests

**Input**: \(D\), the distinguishing automaton of \(M\) and \(S\)

**Output**: \(\alpha \neq \varepsilon\), a test detecting a nonconforming mutant surviving \(E\); \(\alpha = \varepsilon\), if \(E\) is a complete test suite

**Initialization**: \(\phi_E := \bigwedge_{\alpha \in E} \neg \phi_{\alpha}\)  \(\phi_{fd} := \phi_{fd} \land \phi_E\)  \(\phi_P := False\)  \(\alpha := \varepsilon\)

**repeat**

\(\phi_{fd} := \phi_{fd} \land \neg \phi_P\)

**end repeat**

**if** \(P \neq null\) **then**

**Build** \(D_P\), the distinguishing automaton of \(S\) and \(P\);

**if** \(D_P\) has no sink state **then**

\(\phi_P := \bigwedge_{t \in \lambda_P \cup \Delta_P} t\)

**else**

Set \(\alpha\) to the timed input sequence of an accepted comb of the distinguishing automaton \(D_P\);

**end if**

**end if**

**return** \((\phi_{fd}, \alpha)\);

**Algorithm 1**: Verifying the completeness of a given test suite

**Procedure** Generate_complete_test_suite in Algorithm 2 implements the iterative generation of a complete test suite. In each iteration step, a new test is generated to detect a surviving mutant returned by Verify_completeness if the mutant is nonconforming; otherwise the mutant is discarded from the set of surviving mutants. Generate_complete_test_suite always terminates because there are finitely many mutants in the fault domain, Verify_completeness always terminates and the number of surviving mutants is reduced at every iteration step.

**Example 7**. The result of an execution of Verify_completeness with input \(E_{init} = \{(b, 0.5)(a, 1)(b, 6.7)(a, 7.2)\}\) is the nonempty test \((a, 3)\), which indicates that \(E_{init}\) is not complete. We can generate additional tests to be added to \(E\) and obtain a complete test suite. An execution of Generate_complete_test_suite with \(E_{init}\) produces five tests detecting all the 31 mutants in the fault domain. The tests are the following: \((b, 0.5)(a, 1)(b, 6.7)(a, 7.2), (a, 3), (a, 4)(a, 8), (b, 0)(a, 0)(b, 0)(a, 0)\) and \((b, 0)(a, 0)\). The generated test suite includes identical untimed sequences applied after different delays, i.e., the delays are needed for the fault detection.

**5 Experimental results**

We implemented in the C++ language a prototype tool for an empirical evaluation of the efficiency of the proposed methods. The experiment was realised with a computer equipped with the processor Intel(R) Core(TM) i5-7500 CPU @ 3.40 GHz and 32 GB
Procedure Generate_complete_test_suite \( (E_{\text{init}}, (S, \simeq, \text{Mut}(M))) \);
Input : \( E_{\text{init}} \), an initial (possibly empty) set of timed input sequences
Input : \( (S, \simeq, \text{Mut}(M)) \), a fault model
Output : \( E \), a complete test suite for \( (S, \simeq, \text{Mut}(M)) \)

Compute \( \varphi_M \), the boolean formula encoding all the mutants in \( \text{Mut}(M) \);
Build \( D \), the distinguishing automaton of \( S \) and \( M \);
\( \varphi_{\text{fd}} := \varphi_M \);
\( E := \emptyset \);
\( E_{\text{curr}} := E_{\text{init}} \);
repeat
\( E := E \cup E_{\text{curr}} \);
(\( \varphi_{\text{fd}}, \alpha \)) := Verify_completeness(\( \varphi_{\text{fd}}, E_{\text{curr}}, D \));
\( E_{\text{curr}} := \{\alpha\} \);
until \( \alpha = \epsilon \);
return \( E \);

Algorithm 2: Generating a complete test suite \( E \) from \( E_{\text{init}} \)

RAM. The tool uses the solver cryptoSAT [23]. We present the results of the evaluation of the proposed methods with randomly generated specifications and a specification of the trivial file transfer protocol.

5.1 Case of the trivial file transfer protocol

We consider a TFSM-T specification of the Trivial File Transfer Protocol (TFTP) [22]. TFTP is timeouts-dependent and it has already been tested in [26]. Figure 4 shows our TFSM-T model for TFTP. The model was designed according to the specification in [22] and the modeling purposes in [26]. The modeling purposes focus on the behavior of reading files. No more than three packages are transferred and the timeout for waiting for a packet equals three seconds. Moreover, we assume the file exists. Unlike the model in [26] our model in Figure 4 is complete and deterministic. For the sake of clarity, multiple transitions from one state to another are represented with a single arrow.

The tool generated within 0.31s a complete test suite of size 23 for a mutation machine defining 1404928 mutants of the TFSM-T in Figure 4. The maximal length of the tests is 5. The mutation machine has 198 mutated transitions. The mutated transitions were introduced as follows. Firstly, we added 2 finite timeouts and one mutated infinite timeout in all but the initial state; they are 1 and 5. For each state and each input, we added a transition to every state and for each output, if the specified output for the input is not "Not defined". We noticed that the size of the generated complete test suite could be reduced to 16 by removing the seven tests which are prefixes of the others. Test suite optimization is a challenge to be addressed in further work.

We also generated a complete chaos machine (the maximal timeout is 5) for the specification in Figure 4. The chaos machine defines an input/output transition from any state to any other for each pair of input-output; it also defines a timeout transition from any state to any other for each timeout between 1 and 5, and \( \infty \). The resulting chaos mutation machine has 626 more transitions than the specification, and have approxi-
Fig. 4: A TFSM-T modeling the TFTP; Init is the initial state; multiple transitions from one state to another are represented with a single arrow.

| #mutants in the fault domain | #states | $\simeq 10^4$ | $\simeq 10^5$ | $\simeq 10^6$ | $\simeq 10^8$ |
|-----------------------------|---------|-------------|-------------|-------------|-------------|
| 4 states                    | (9, 0.04) | (26, 9.17) | (30, 319.19) | N/A         |
| 8 states                    | (9, 0.5)  | (19, 0.65) | (32, 5.31)  | (68, 864.06) |            |
| 10 states                   | (9, 4.73) | (20, 21.44) | (30, 682.97) | (58, 250.72) |            |
| 12 states                   | (8, 56.36) | (20, 1.32) | (25, 66.1)  | (47, 593.07) |            |
| 15 states                   | (5, 168.24) | (17, 227.56) | (33, 418.72) | (58, 64.55) |            |

Table 1: Size of the generated complete test suites and generating time; for an entry $(x, y)$, $x$ is the size of the test suite and $y$ is the generating time in seconds

We generated 50 tests within 1970s; they can be reduced to 32 tests by removing the tests’ prefixes. The maximal length of the tests is 5.

We relaxed the modeling purpose by allowing 15 packets instead of 3. The corresponding TFSM-T specification has 16 states and we built a mutation machine with 9438 mutated transitions defining $1.9 \times 10^{46}$ mutants. The tool generated a complete test suite of size 98 within 555.14s. The test suite can be amputated from 23 tests’ prefixes. The maximal length of the tests is 17.

We have generated complete test suites for fault domains of important sizes. Generates a complete test suite for a chaos TFSM-T mutation machine defining less than $10^9$ mutants. The efficiency of our approach depends on the complexity of the mutation machine. In the average, the higher the size of the fault domain, the longer is the time for generating complete test suites.

5.2 Case of randomly generated TFSMs-T

We randomly generated specification machines for given numbers of states and mutation machines for the specification machines. The generated specification and mutation
machines have 2 inputs and 2 outputs. The maximal timeout in the specification machines is 3 and the one in the mutation machines is 5. The result of the evaluation is presented in Table 1. We have measured the generating time for each test suite and the size of each test suite. An entry \((x, y)\) of Table 1 indicates the size of the test suite \(x\) and the corresponding generating time \(y\) in seconds (s).

6 Conclusion

We lifted a constraint solving-based test generation approach to generate complete test suite for fault models for TFSMs-T. We defined the distinguishing automaton with timeouts which is used to build SAT constraints, verify the completeness of test suites and generate complete test suites. We implemented a prototype tool for the proposed test verification and generation methods. The empirical evaluation of the methods indicates that they apply on industrial-size TFSMs-T specifying real systems.

Further work is in progress to reduce the size of the test suites and lift the proposed methods to TFSMs expressing time constraints beyond the timeouts.

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