Information-to-work conversion by Maxwell’s demon in a superconducting circuit quantum electrodynamical system

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Information thermodynamics bridges information theory and statistical physics by connecting information content and entropy production through measurement and feedback control. Maxwell’s demon is a hypothetical character that uses information about a system to reduce its entropy. Here we realize a Maxwell’s demon acting on a superconducting quantum circuit. We implement quantum non-demolition projective measurement and feedback operation of a qubit and verify the generalized integral fluctuation theorem. We also evaluate the conversion efficiency from information gain to work in the feedback protocol. Our experiment constitutes a step toward experimental studies of quantum information thermodynamics in artificially made quantum machines.

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he gedanken experiment of Maxwell’s demon has led to the studies concerning the foundations of thermodynamics and statistical mechanics. The demon measures fluctuations of a system’s observable and converts the information gain into work via feedback control. Recent developments in information thermodynamics have elucidated the relationship between the acquired information and the entropy production and generalized the second law of thermodynamics and the fluctuation theorems. Here we extend the scope to a system subject to quantum fluctuations by exploiting techniques in superconducting quantum circuit quantum electrodynamics. We implement Maxwell’s demon equipped with coherent control and quantum non-demolition (QND) projective measurements on a superconducting qubit, thereby verifying the generalized integral fluctuation theorems and the information-to-work conversion. This demonstrates the potential of superconducting circuits as a versatile platform for investigating quantum information thermodynamics under feedback control, which may be treated separately (see also Supplementary Note 1). Here, the absolute irreversibility is caused by the combination of the projective measurement that restricts possible forward events and the non-ideal property of the feedback operation that makes the backward events random. For example, in the process shown in Fig. 1a, the projective measurement and feedback operation, where $I_{sh}$ is the stochastic Shannon entropy the demon acquires in the projective measurement, $\sigma = -\beta (W + \Delta F)$ is the entropy production, $\beta$ is the inverse temperature $1/(k_B T)$ of the initial state of the qubit, $W$ is the work extracted from the qubit via the feedback operation $U$, and $\Delta F$ is the change in the equilibrium free energy of the system. The angle brackets $\langle \cdot \rangle_{PM}$ indicate the statistical average obtained with a protocol using a projective measurement for the feedback control. Below we focus on the case with $\Delta F = 0$, i.e., on the process with the same system Hamiltonian at the beginning and the end, for simplicity of discussions.

The constant $\lambda_B$ on the right-hand side of Eq. (1) gives the total probability of those events in the time-reversed process, whose counterparts in the original process do not exist. Such events, which we call absolutely irreversible events, involve a formal divergence of the entropy production and should therefore be treated separately (see also Supplementary Note 1). Here, the absolute irreversibility is caused by the combination of the projective measurement that restricts possible forward events and the non-ideal property of the feedback operation that makes the backward events random. For example, in the process shown in Fig. 1a, the projective measurement and feedback operation, where $I_{sh}$ is the stochastic Shannon entropy the demon acquires in the projective measurement, $\sigma = -\beta (W + \Delta F)$ is the entropy production, $\beta$ is the inverse temperature $1/(k_B T)$ of the initial state of the qubit, $W$ is the work extracted from the qubit via the feedback operation $U$, and $\Delta F$ is the change in the equilibrium free energy of the system. The angle brackets $\langle \cdot \rangle_{PM}$ indicate the statistical average obtained with a protocol using a projective measurement for the feedback control. Below we focus on the case with $\Delta F = 0$, i.e., on the process with the same system Hamiltonian at the beginning and the end, for simplicity of discussions.

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Fig. 1 Maxwell’s demon and absolute irreversibility. a Concept of the experiment. The system initially prepared in a canonical distribution $\tilde{\rho}_{ini}$ evolves in time. A projective measurement $\tilde{\sigma}$ with outcome $x = g$ or $e$] by the demon projects the system onto a quantum state. The demon gains the stochastic Shannon entropy $I_{sh}$ and converts it into work $W$ via a feedback operation $U$ ($= \hat{U}_b$ or $\hat{U}_e$). The forward process ends up in the final distribution $\tilde{\rho}_{fin}$. The time-reversed reference process starts from a reference state $\tilde{\rho}_r$, which we choose to be equal to $\tilde{\rho}_{fin}$. The absolute irreversibility is quantified with $\lambda_B$, the probability of those events in the time-reversed process whose counterparts in the original process do not exist (red dashed arrows). b Schematic of the feedback-controlled system in the experiment. The right panel shows a photograph of the qubit-resonator coupled system. The cavity resonator is disassembled to show its internal structure.
$\hat{U}_g$ or $\hat{U}_e$, always bring the system to the ground state. Therefore, the evolution of the excited state in the reverse process via the operation $\hat{U}_g^\dagger$ or $\hat{U}_e^\dagger$ does not have a counterpart in the forward process. The probability $\lambda_{th}$ of such events in the present protocol is given by $e^{-\beta\hbar\omega_0}/(1 + e^{-\beta\hbar\omega_0})$, i.e., the excited state occupation probability in $\rho_e$.

The absolute irreversibility makes a significant contribution to the generalized second law of thermodynamics including the effect of the feedback control. For achieving the ultimate bound on the extracted work $\langle W \rangle_{PM} = k_B T (I_{sh})_{PM}$, the final state distribution $\rho_{\text{fin}}$ of the system has to be the same as $\rho_{\text{ini}}$3, 8. However, the projective measurement together with the unoptimized feedback operation prevents it and limits the amount of the extractable work (see Eq. (4) below).

In our experiment, a superconducting transmon qubit (i.e., the system) is placed at the center of an aluminum-made superconducting cavity resonator (Fig. 1b). The qubit state is controlled with a resonant microwave pulse, which induces Rabi rotation. Owing to the interaction between the qubit and the detuned resonator, the resonance frequency of the resonator varies depending on the qubit state. We utilize this property for the QND readout of the qubit; the ground and excited states are distinguished in the phase shift of a readout microwave pulse reflected by the resonator7.

Protocol with projective measurements. Figure 2a shows the sequence of the experiment corresponding to Fig. 1a. The qubit is initialized with a projective measurement and postselection, followed by a resonant pulse excitation, which prepares as an input a superposition state $\alpha_g |g\rangle + \alpha_e |e\rangle$ (where $|g\rangle$ and $|e\rangle$ are the ground and excited states of the qubit) of the ground $|g\rangle$ and excited $|e\rangle$ states of the qubit. As the qubit is subject to the subsequent projective measurement, the coherence in the input state does not have any essential role here, and the coefficients of the superposition define the effective temperature of the system $T = (\hbar \omega_q / k_B) \ln (\alpha_g^2 / |\alpha_e|^2)$ after the projection, where $\hbar \omega_q$ is the qubit excitation energy.

We evaluate the work $W(x, z) = E(x) - E(z)$ extracted from the system by employing the two-point measurement protocol (TPM), in which QND projective measurements on the energy eigenbasis with outcomes $x = g$ or $e$ and $z = g$ or $e$ are applied respectively to the initial and final states of the system14. Here $E(g)$ and $E(e)$ denotes the energies of the qubit in the ground and excited states, respectively. Depending on the measurement outcome $x$ for the feedback control, the feedback operation does or does not flip the state of the qubit with a $\pi$-pulse. A positive amount of the work ($W > 0$) implies that the energy is extracted from the system via the stimulated emission of a single photon induced by a $\pi$-pulse, which flips the qubit state. The probability $p(x)$ of the state $x$ being found gives $I_{sh}(x) = -\ln p(x)$.

In Fig. 2b we compare the experimentally obtained statistical average $\langle e^{\delta W - I_{sh}} \rangle_{PM} = \sum_{x, z} p(x, z) e^W(x, z) - I_{sh}(x)$ (blue circles).
States, which are prepared at the effective temperature (Supplementary Note 2). Inset in information gain for the feedback control. To evaluate the state of the qubit is not completely projected. It also gives less imperfect projection in the readout. With a weak readout pulse, the absolute irreversibility.

Fluctuation theorem under feedback control. Furthermore, the joint probability of observing a particular outcome is the joint probability of the measured system is quantum and the measurement no longer gives restriction on forward events. Therefore, we obtain \( \langle W \rangle \), which is a function of the measurement strength. The subsequent readout with outcome \( e \) projects the qubit state before the feedback control. See Supplementary Note 2 for details. The outcome \( x \), \( z \), and \( w \) (red squares) vs. \( \epsilon_b \). The black dashed line represents the Shannon entropy (Supplementary Note 2). Inset in c information-to-work conversion efficiency \( \eta \) (green circles) and the simulated result (line-connected black dots). The gray dashed line indicates the value for the efficiency in the limit of the projective measurement due to the absolute irreversibility.

Effect of imperfect projection. Next, we investigate the effects of imperfect projection in the readout. With a weak readout pulse, the state of the qubit is not completely projected. It also gives less information gain for the feedback control. To evaluate the influence of the weak measurement, we add two more readout pulses to the pulse sequence (Fig. 3a). The TPM again starts with a projective readout with outcome \( x \), but now the feedback control is performed based on the subsequent variable-strength measurement with outcome \( k = g \) or \( e \). Then, to project the qubit state before the feedback control, we apply another strong measurement to obtain outcome \( x \) with \( \beta \). Using these measurement outcomes, we calculate the stochastic QC-mutual information \( I_{QC}(x, k, y) = \ln p(y | k) - \ln p(x) \). Here, QC indicates that the measured system is quantum and the measurement output is classical. And \( p(y | k) \) is the probability of outcome \( y \) being obtained conditioned on the preceding measurement outcome \( k \). The first term in \( I_{QC} \) quantifies the correction to \( I_{S_a} \) because of the imperfect projection. If the measurement for the feedback control is a QND projective measurement and there is no relaxation of the qubit, \( p(y | k) \) becomes unity and \( I_{QC} \) reduces to \( I_{S_a} \). On the other hand, for the measurement with imperfect projection, the absolute irreversibility disappears, because such measurement no longer gives restriction on forward events. Therefore, we obtain \( \lambda_b = 0 \). In this case, the generalized integral
fluctuation theorem is reformulated as $^9$ (see also Supplementary Note 1)

$$\langle e^{W-LQC} \rangle \leq 1.$$  \hfill (2)

Figure 3b plots the statistical averages $\langle e^{W-LQC} \rangle$ and $\langle e^{HW} \rangle$, evaluated from the measurement outcomes of the pulse sequence shown in Fig. 3a. For example, $\langle e^{W-LQC} \rangle$ is experimentally obtained as $\sum_{x,k,y,z} p(x,k,y,z) e^{W(x,y,z)-LQC(x,y,z)}$, where $p(x,k,y,z)$ is the joint probability of observing a combination of the outcomes. By changing the amplitude of the readout pulse, which measures $k$, it is possible to continuously vary the post-measurement state from the projected state to a weakly disturbed state. Accordingly, the feedback error probability $e_{fb}$ increases with decreasing the readout amplitude. (See the Supplementary Note 2 for details.) We see that $\langle e^{W-LQC} \rangle$ (blue circles), which involves the information gain due to the measurement, is almost unity regardless of the feedback error probability. The small deviation from unity is understood as the effect of the qubit relaxation during the measurement or study feedback schemes maintaining the coherence between the system and the memory to improve the energy efficiency of the feedback. Superconducting quantum circuits further allow us to extend the study of information thermodynamics to larger and more complex quantum systems. It will lead to an estimation of the lower bound of the thermodynamic cost for quantum information processing.

Methods

Sample. The transmon qubit has the resonance frequency $\omega_c/2\pi = 6.6296$ GHz, the energy relaxation time $T_1 = 24$ ms, and the phase relaxation time $T_2 = 16$ ms at the base temperature $-10$ mK of a dilution refrigerator. The cavity has the resonance frequency $\omega_{cav}/2\pi = 10.6180$ GHz, largely detuned from the qubit, and the relaxation time $1/\kappa = 0.076$ $\mu$s. The coupling strength between the qubit and the resonator is estimated to be $g/2\pi = 0.14$ GHz.

Pulse sequences. The pulse sequences for the experiments in Figs. 2 and 3 take about 2.5 and 4 $\mu$s, respectively. Each readout pulse has the width of 500 ns. The qubit excitation pulse and the feedback control pulse are both 20 ns wide. See the Supplementary Note 2 for details. We take the statistics of the outcomes by repeating the pulse sequence about $8 \times 10^4$ times, with a repetition interval 300 $\mu$s.

Conversion efficiency. The conversion efficiency from the QC-mutual information $\langle I_{QC} \rangle$ to the work $\langle W \rangle$ is defined for $T > 0$ as $^4, ^5$

$$\eta = \frac{\langle W \rangle}{k_B T \langle I_{QC} \rangle},$$  \hfill (3)

where we omit the contribution from the free-energy change by assuming $\Delta F = 0$. As shown in the inset of Fig. 3c, $\eta$ is 0.65 in the limit of $e_{fb} \to 0$ corresponding to the case with the projective measurement shown in Fig. 2.

The efficiency obtained with the projective measurement is to be compared with the following inequalities:

$$\langle W \rangle \leq \langle W \rangle_{PM} \leq k_B T \langle I_{sh(2)} \rangle_{PM} + k_B T \ln(1 - \lambda_b).$$  \hfill (4)

The first inequality describes the fact that for a given protocol the extracted work with a proper projective measurement is superior to that obtained with an imperfect projection, which is demonstrated in Fig. 3c. On the other hand, the second inequality derived for $T > 0$ from the fluctuation theorem Eq. (1) represents the generalized second law of information thermodynamics (Supplementary Note 1). We find that the contribution from the absolute irreversibility sets the limit of the efficiency, given by $\eta = 1 - \ln(1 - \lambda_b)/\langle I_{sh(2)} \rangle_{PM}$, which is indicated by the dashed line in the inset of Fig. 3c. The experimental result demonstrates that our feedback scheme achieves the equality condition in Eq. (4) and is optimal (though not ideal) in this sense.

Discussion

We have successfully implemented Maxwell’s demon in a setup based on superconducting circuit quantum electrodynamics and verified the generalized integral fluctuation theorem in a single qubit. In the present work, the measurement outcome obtained by the demon was analyzed in terms of the Shannon and the QC-mutual information. On the other hand, the effect of the coherence can be investigated in a similar setup$^{27}$, one can characterize the energy cost for the measurement$^{28}$ or study feedback schemes maintaining the coherence between the system and the memory to improve the energy efficiency of the feedback. Superconducting quantum circuits further allow us to extend the study of information thermodynamics to larger and more complex quantum systems. It will lead to an estimation of the lower bound of the thermodynamic cost for quantum information processing.

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Author contributions

Y. Masuyama, K.F., and Y. Murashita designed the experiments. Y. Masuyama conducted the experiments. S.K. and Y.T. assisted in setting up the measurement system. K.F., Y. Murashita, and M.U. provided theoretical supports. A.N., Y.T., and R.Y. participated in discussions on the analysis. Y. Masuyama and Y.N. wrote the manuscript with feedback from all authors. M.U. and Y.N. supervised the project.

Additional information

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