Non-classical method of modelling of vibrating mechatronic systems

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Abstract. This work presents non-classical method of modelling of mechatronic systems by using polar graphs. The use of such a method enables the analysis and synthesis of mechatronic systems irrespective of the type and number of the elements of such a system. The method is connected with algebra of structural numbers. The purpose of this paper is also to introduce synthesis of mechatronic systems which is the reverse task of dynamics. The result of synthesis is obtaining system meeting the defined requirements. This approach is understood as design of mechatronic systems. The synthesis may also be applied to modify the already existing systems in order to achieve a desired result. The system was consisted from mechanical and electrical elements. Electrical elements were used as subsystem reducing unwanted vibration of mechanical system. The majority of vibration occurring in devices and machines is harmful and has a disadvantageous effect on their condition. Harmful impact of vibration is caused by the occurrence of increased stresses and the loss of energy, which results in faster wear machinery. Vibration, particularly low-frequency vibration, also has a negative influence on the human organism. For this reason many scientists in various research centres conduct research aimed at the reduction or total elimination of vibration.

1. Introduction

Vibration is a phenomenon which can be often observed in everyday life. Some types of vibration are used in the operation of machines and devices. However, most of them are of a harmful nature. This fact is related to the influence they exert on real objects causing their malfunction. Another important issue is the negative impact vibration has on the human body. For this reason the issue of the reduction of unrequired vibration is of great significance. That is why many research institutes are investigating methods of preventing the improper application or exploitation/operation of newly built machinery or methods of adjusting already existing and operating machines to specified requirements [1-3].

Methods of vibration prevention are many and varied. They have an effect on the machinery elements and components. One of the classifications is the division into passive, semi-active and active methods of vibration reduction [5, 6].

2. Non-classical method of modelling

In order to present the modelling of a vibratory mechanical system with active elements by means of the polar method, one should consider a system presented in Figure 1.

The system under consideration (Fig. 1) is composed of the following elements:
- inert elements $m_1, m_2, \ldots, m_n$,
- elastic elements $c_1, c_2, \ldots, c_m$,
- excitations acting on the system $F_1(t), F_2(t), \ldots, F_n(t)$,
- active excitations $G_1(t), G_2(t), \ldots, G_n(t)$.

*Figure 1.* Model of mechanical system of cascade structure

The description of the adopted discrete model utilises two basic sets of quantities $1S$ and $2S$ as well a set of elements $Z$,

where:

$1S$ – set of polar quantities – generalised linear coordinates,

$2S$ – set of flow quantities – generalised forces,

$Z$ – set of the coefficients of the polar equations of vibratory mechanical system elements, also referred to as dynamic rigidities.

A polar relation refers to relations $3S \subseteq 1S \times 2S \times 1S$, specified as follows [3]:

$$\left\{ i_1 s_i, 2 s_k, 1 s_j \right\} \Leftrightarrow \exists_{z_k} \left( \left( i_1 s_j - 1 s_i \right) z_k = 2 s_k \right) \land \left( 1 s_i, 1 s_j \in 1 S \land 2 s_k \in 2 S \land z_k \in Z \right)$$

where:

- $i, j = 0, 1, \ldots, n$,
- $i \neq j$;
- $\left( i_1 s_j - 1 s_i \right) z_k = 2 s_k$ – polar equation of the element $k$ in the system;
- $k = 1, \ldots, n, (n+1), \ldots, (n+m), ((n+m)+1), \ldots, ((n+m)+w), ((n+m)+w+1), \ldots$, 
- $\left( ((n+m)+w)+g \right)$

where:

- from 1 to $n$ inert elements,
- from $(n+1)$ to $(n+m)$ elastic elements,
- from $(n+m+1)$ to $(n+m+w)$ excitations acting on the system,
- from $(n+m+w+1)$ to $(n+m+w+g)$ excitations generated by active elements.

The polar relations is also designated as:

$$3 S_k = \left\{ i_1 s_i, 2 s_k, 1 s_j \right\} \in 3 S.$$  \hfill (2)

The dynamic structure is the representation of the mechanical model of a discrete system in the form:

$$S = \left\{ 1 S, 2 S, 3 S \right\}.$$  \hfill (3)

The graph $X$ of the structure $S$ of a discrete mechanical system is the triplet of sets in the form:

$$X = \left[ 1 X, 2 X, F \right] = \left[ X, F \right],$$  \hfill (4)

where:
\[ F = \{ f_i \}, (i=1,2) \] representation specified as follows:
\[ F : \{ s_1, s_2, s_3 \} \rightarrow \{ x_1, x_2 \}, \]
such that:
\[ f_i : s_i \rightarrow x_i \wedge f_2 : s_2 \subset s \rightarrow x_2, \]
in the following manner:
\[ f_i(s_i) = x_i (\text{Fig.2}), \]
where:
\[ s_i \in S, x_i \in X, (i=0, 1, 2, \ldots, n), \]

\[ \text{Figure 2. Representation of generalized coordinates in graph vertices} \]

\[ 2 f = f' \cup f'' \cup f''' \cup f''', \]
where:
\[ f'(s_k) = x_k \wedge f'(s_k) = x_k (\text{Fig.3}), \]
\[ s_k \in S, x_k \in X, k=1,2, \ldots, n \]
\[ s_k = \{ s_i, s_k \} \in S \text{ in the case of the inert elements } k \text{ of the system}, \]
\[ x_k = \{ x_i, x_0 \} \in X \text{ in the case of the edges } k \text{ of the graph}, \]
\[ x_i, x_0 \in X, (i=1, 2, \ldots, n). \]

\[ \text{Figure 3. Representation of inertial elements into graph edges} \]

For the sake of the clarity and the legibility of the representations, the grey-coloured lines in Figure 4 and subsequent figures designate the individual elements of the system relations (Fig. 1) in the graph edges.
where:

where:

where:

where:

where:

where:

\[ f''(2s_k) = 2x_k \quad (\text{Fig. } 4), \]

\[ f'''(2s_k) = 2x_k \quad (\text{Fig. } 5), \]

\[ f'''(2s_k) = 2x_k \quad (\text{Fig. } 6), \]

Fig. 4. Representation of elastic elements into graph edges

Fig. 5. Representation of excitations into graph edges

Fig. 6. Representation of active excitations into graph edges

Excitations acting on the system

Active excitations
$2\cdot x_k = \{x_0, x_1, x_i, x_i\} \in \mathbb{X}$ in the case of the edges $k$ of the graph,

$1\cdot x_0, x_1, x_i \in \mathbb{X}, \ (i=1, 2, \ldots, n)$. 

\[ F = \{i, f\}, \ i=1, 2, \ldots, n. \]

**Figure 6.** Representation of active excitations into graph edges

By way of mutual representations one obtains a graph (Fig. 2.7):

\[ X = [\mathbb{X}, F, F] = [X, F]. \]  \hspace{1cm} (13)

**Figure 7.** Polar graph as a model of system from figure 1
In order to modelling a system including an active subsystem in the form of electric elements reducing vibration (Fig. 8) by means of a non-classical method of designing, it is possible to use theory of graphs. Between vibratory mechanical systems and vibratory electric circuits (Fig. 9) there are analogies in the mathematical description [6,7]. The analogy between longitudinally vibrating models and electric models is presented in Table 1. Modelling of electric systems using method of polar graph is analogous like modelling mechanical systems, which are presented above. Relation between electrical and mechanical systems can be present by block graph (Fig. 10).

**Figure 8.** A model of the system with electric elements

**Figure 9.** Model of electric system

**Table 1.** Analogies between longitudinally vibrating models and electric models

| mechanical system longitudinal vibration | electric system |
|------------------------------------------|-----------------|
| \( m \) [kg]                             | \( L \) [H]     |
| \( c \) \[ \frac{N}{m} \]               | \( C \) [F]     |
| \( b \) \[ \frac{Ns}{m} \]             | \( R \) [\Omega]|
| \( F(t) \) [N]                          | \( U(t) \) [V]  |
| \( x(t) \) [m]                          | \( q(t) \) [C]  |
3. Conclusions

The presented manner of modelling mechatronic systems by means of polar graphs enables full automation and algorithmisation of calculations during the determination of the dynamic characteristics of the system as well as makes it possible to directly track implemented structural changes. In the case of systems composed of a great number of subsystems the determination of dynamic characteristics in a classical manner requires numerous labour-consuming activities. In situations when it becomes necessary to modify the structure of the system, each time it is necessary to formulate and solve the system of differential equations of motion, which is not required in the case of the system represented in the form of a polar graph.

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