Markovian and Post-Markovian dynamics of nonequilibrium thermal entanglement

Ilya Sinayskiy (Sinaysky)∗
Quantum Research Group, School of Physics, University of KwaZulu-Natal, Durban, 4001, South Africa

Francesco Petruccione†
Quantum Research Group, School of Physics and National Institute for Theoretical Physics,
University of KwaZulu-Natal, Durban, 4001, South Africa

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The dynamics of two spins coupled to bosonic baths at different temperatures is studied. The analytical solution for the reduced density matrix of the system in the Markovian and Post-Markovian case with exponential memory kernel is found. The dynamics and temperature dependence of spin-spin entanglement is analyzed.

I. INTRODUCTION

The influence of the environment plays an essential role in the description of the realistic quantum system [1]. Usually, environment destroys entanglement in the subsystem of interest. However in some cases it can create quantum correlations in the system [2, 3, 4]. One of the ways to understand the role of the parameters of the system is to study exactly solvable models. Here, we study the dynamics of a model that was recently introduced by L. Quiroga [5]. It consists of two interacting spins in contact with two reservoirs at different temperatures. In such a non-equilibrium case most studies are restricted to the steady-state solutions [5, 6, 7, 8] or to the zero temperature limit [9].

This paper is organized as follows. In Sec. II we describe the model of a spin chain coupled to bosonic baths at different temperatures and derive a master equation in Born-Markov approximation. In Sec. III we present the analytical solution for the system dynamics in the Markovian case, details of the solution can be found in Ref. [10]. In Sec. IV we present the analytical solution of the Post-Markovian master equation recently introduced by Shabani and Lidar [11]. Finally, in Sec. V we discuss the results and conclude.

II. MODEL

We consider a system of two interacting spins, with each spin coupled to a separate bosonic bath. The total Hamiltonian is given by

\[ \hat{H} = \hat{H}_S + \hat{H}_{B1} + \hat{H}_{B2} + \hat{H}_{SB1} + \hat{H}_{SB2}, \]

where

\[ \hat{H}_S = \frac{\epsilon_1}{2} \hat{\sigma}_1^z + \frac{\epsilon_2}{2} \hat{\sigma}_2^z + K(\hat{\sigma}_1^- \hat{\sigma}_2^- + \hat{\sigma}_1^+ \hat{\sigma}_2^+) \]

is the Hamiltonian describing spin-to-spin interactions and \( \hat{\sigma}_i^\pm, \hat{\sigma}_i^z \) are the Pauli matrices. Note, that the units are chosen such that \( k_B = \hbar = 1 \). The constants \( \epsilon_1 \) and \( \epsilon_2 \) denote the energy of spins 1 and 2, respectively and \( K \) denotes the strength of the spin-spin interaction. The Hamiltonians of the bosonic "baths" for each spin \( j = 1, 2 \) are given by

\[ \hat{H}_{Bj} = \sum_n \omega_{n,j} \hat{b}^\dagger_{n,j} \hat{b}_{n,j}. \]

The interaction between the spin subsystem and the reservoir with creation operators \( \hat{b}_{n,j}^\dagger \) is described by

\[ \hat{H}_{SBj} = \hat{\sigma}_j^+ \sum_n g_n^{(j)} \hat{b}_{n,j} + \hat{\sigma}_j^- \sum_n g_n^{(j)^\dagger} \hat{b}_{n,j}^\dagger = \sum_{\mu} \hat{V}_{j,\mu} \hat{f}_{j,\mu}. \]

∗Electronic address: ilsinay@gmail.com
†Electronic address: petruccione@ukzn.ac.za
The operators of the transitions in dynamical subsystem $\hat{V}_{j,\mu}$ are chosen to satisfy $[\hat{H}_S, \hat{V}_{j,\mu}] = \omega_{j,\mu} \hat{V}_{j,\mu}$, and the $\hat{f}_{j,\mu}$ act on the reservoir degrees of freedom. The total system (two spins with reservoirs) is described by the Liouville equation

$$\frac{d}{dt} \hat{\alpha} = -i[\hat{H}, \hat{\alpha}].$$

We assume that the evolution of the dynamical subsystem (coupled spins) does not influence the state of the environment (bosonic reservoirs) so that the density operator of the whole system $\hat{\alpha}(t)$ can be written as:

$$\hat{\alpha}(t) = \hat{\rho}(t) \hat{B}_1(0) \hat{B}_2(0),$$

where each bosonic bath is described by a canonical density matrix $\hat{B}_j = e^{-\beta_j \hat{H}_B_j}/\text{tr}[e^{-\beta_j \hat{H}_B_j}]$ and $\hat{\rho}(t)$ denotes the reduced density matrix of the spin subsystem.

In Born-Markov approximation the equation for the evolution of the reduced density matrix [12] is:

$$\frac{d}{dt} \hat{\rho} = -i[\hat{H}_S, \hat{\rho}] + \mathcal{L}_1(\hat{\rho}) + \mathcal{L}_2(\hat{\rho})$$

with dissipators

$$\mathcal{L}_j(\hat{\rho}) = \sum_{\mu, \nu} J^{(j)}(\omega_{j,\nu}) \{ [\hat{V}_{j,\mu}, [\hat{V}_{j,\nu}^\dagger, \hat{\rho}]] - (1 - e^{-\beta_j \omega_{j,\nu}}) [\hat{V}_{j,\mu}, \hat{V}_{j,\nu}^\dagger \hat{\rho}] \}$$

and where the spectral density is given by

$$J^{(j)}(\omega_{j,\nu}) = \int_0^\infty ds e^{i\omega_{j,\nu}s} \langle e^{-is \hat{H}_B_j} \hat{f}_{j,\nu}^\dagger e^{is \hat{H}_B_j} \hat{f}_{j,\nu} \rangle_j.$$

To find a solution we go to the basis of the eigenvectors $|\lambda_i\rangle$ with eigenvalues $\lambda_i$ of the Hamiltonian $\hat{H}_S$,

$$|\lambda_1\rangle = |0, 0\rangle, \quad \lambda_1 = -\frac{\epsilon_1 + \epsilon_2}{2},$$

$$|\lambda_2\rangle = |1, 1\rangle, \quad \lambda_2 = \frac{\epsilon_1 + \epsilon_2}{2},$$

$$|\lambda_3\rangle = \cos(\theta/2) |1, 0\rangle + \sin(\theta/2) |0, 1\rangle, \quad \lambda_3 = \kappa,$$

$$|\lambda_4\rangle = -\sin(\theta/2) |1, 0\rangle + \cos(\theta/2) |0, 1\rangle, \quad \lambda_4 = -\kappa,$$

where $\kappa \equiv \sqrt{K^2 + (\Delta \epsilon)^2}$ and $\tan \theta \equiv 2K/(\Delta \epsilon)$. In this representation the dissipators $\mathcal{L}_i(\hat{\rho})$ becomes

$$\mathcal{L}_j(\hat{\rho}) = \sum_{\mu=1}^2 J^{(j)}(\omega_\mu)(2\hat{V}_{j,\mu}^\dagger \hat{\rho} \hat{V}_{j,\mu} - \{ \hat{\rho}, \hat{V}_{j,\mu} \} +) + J^{(j)}(\omega_\mu)(2\hat{V}_{j,\mu}^\dagger \hat{\rho} \hat{V}_{j,\mu} - \{ \hat{\rho}, \hat{V}_{j,\mu} \} +),$$

with transition frequencies

$$\omega_1 = \lambda_2 - \lambda_3,$$

$$\omega_2 = \lambda_2 + \lambda_3.$$
we choose $\gamma$s where for the diagonal elements of the reduced density matrix is given by the following form and transition operators $B$ where $(\gamma_j(\omega_i) = (e^{i\omega_i} - 1)^{-1}$ and $J^{(j)}(-\omega_i) = e^{i\omega_i} J^{(j)}(\omega_i)$. For simplicity we choose the coupling constant to be frequency independent $\gamma_1(\omega) = \gamma_1$ and $\gamma_2(\omega) = \gamma_2$. In the basis $|\lambda_i\rangle$ the equation for the diagonal elements of the reduced density matrix is given by

$$\frac{d}{dt} \begin{pmatrix} \rho_{11}(t) \\ \rho_{22}(t) \\ \rho_{33}(t) \\ \rho_{44}(t) \end{pmatrix} = B \begin{pmatrix} \rho_{11}(t) \\ \rho_{22}(t) \\ \rho_{33}(t) \\ \rho_{44}(t) \end{pmatrix},$$

where $B$ is a $4 \times 4$ matrix with constant coefficients. The time-dependence for the non-diagonal elements has the following form

$$\rho_{i,j}(t) = e^{t s_{i,j}} \rho_{i,j}(0),$$

where $s_{i,j}$ is a complex number. For the initial state of the system in the computational basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ we choose

$$\dot{\rho}(0) = p_0 |00\rangle\langle 00| + p_1 |01\rangle\langle 01| + p_2 |10\rangle\langle 10| + (1 - p_0 - p_1 - p_2) |11\rangle\langle 11| + c_{12} |01\rangle\langle 10| + c_{12}^* |10\rangle\langle 01|.$$ 

III. EXACT SOLUTION IN THE MARKOVIAN CASE

The analytical solution in the basis of eigenvectors $|\lambda_i\rangle$ is given by:

$$\rho_{ii}(t) = \frac{1}{X_1 Y_2} \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} \begin{pmatrix} \rho_{11}(0) \\ \rho_{22}(0) \\ \rho_{33}(0) \\ \rho_{44}(0) \end{pmatrix},$$

where the coefficients $a_{ij}$ are given by:

$$a_{11} = (X_1^+ + X_1^- e^{-tX_1})(Y_2^+ + Y_2^- e^{-tY_2}),$$

$$a_{12} = (1 - e^{-tX_1})(1 - e^{-tY_2})X_1^+ Y_2^+, $$

$$a_{13} = (1 - e^{-tX_1})X_1^+(Y_2^+ + Y_2^- e^{-tY_2}),$$

$$a_{14} = (X_1^+ + X_1^- e^{-tX_1})(1 - e^{-tY_2})Y_2^+, $$

$$a_{21} = (1 - e^{-tX_1})(1 - e^{-tY_2})X_1^- Y_2^-, $$

$$a_{22} = (1 - e^{-tX_1})(1 - e^{-tY_2})X_1^+ Y_2^-, $$

$$a_{23} = (1 - e^{-tX_1})X_1^+(Y_2^+ + Y_2^- e^{-tY_2}),$$

$$a_{24} = (X_1^+ + X_1^- e^{-tX_1})(1 - e^{-tY_2})Y_2^+, $$

$$a_{31} = (1 - e^{-tX_1})(1 - e^{-tY_2})X_1^- Y_2^-, $$

$$a_{32} = (1 - e^{-tX_1})(1 - e^{-tY_2})X_1^+ Y_2^-, $$

$$a_{33} = (1 - e^{-tX_1})X_1^+(Y_2^+ + Y_2^- e^{-tY_2}),$$

$$a_{34} = (X_1^+ + X_1^- e^{-tX_1})(1 - e^{-tY_2})Y_2^+, $$

$$a_{41} = (1 - e^{-tX_1})(1 - e^{-tY_2})X_1^- Y_2^-, $$

$$a_{42} = (1 - e^{-tX_1})(1 - e^{-tY_2})X_1^+ Y_2^-, $$

$$a_{43} = (1 - e^{-tX_1})X_1^+(Y_2^+ + Y_2^- e^{-tY_2}),$$

$$a_{44} = (X_1^+ + X_1^- e^{-tX_1})(1 - e^{-tY_2})Y_2^+.$$
\(a_{22} = (X_1^- + X_1^+ e^{-tX_1})(Y_2^- + Y_2^+ e^{-tY_2}),\)

\(a_{23} = (X_1^- + X_1^+ e^{-tX_1})(1 - e^{-tY_2}) Y_2^-,\)

\(a_{24} = (1 - e^{-tX_1}) X_1^-(Y_2^- + Y_2^+ e^{-tY_2}),\)

\(a_{31} = (1 - e^{-tX_1}) X_1^-(Y_2^+ + Y_2^- e^{-tY_2}),\)

\(a_{32} = (X_1^- + X_1^+ e^{-tX_1})(1 - e^{-tY_2}) Y_2^+,\)

\(a_{33} = (X_1^- + X_1^+ e^{-tX_1})(Y_2^+ + Y_2^- e^{-tY_2}),\)

\(a_{34} = (1 - e^{-tX_1})(1 - e^{-tY_2}) X_1^- Y_2^+,\)

\(a_{41} = (X_1^+ + X_1^- e^{-tX_1})(1 - e^{-tY_2}) Y_2^-,\)

\(a_{42} = (1 - e^{-tX_1}) X_1^+(Y_2^- + Y_2^+ e^{-tY_2}),\)

\(a_{43} = (1 - e^{-tX_1})(1 - e^{-tY_2}) X_1^+ Y_2^-,\)

\(a_{44} = (X_1^+ + X_1^- e^{-tX_1})(Y_2^- + Y_2^+ e^{-tY_2}).\)

Taking into account the initial conditions, the non-vanishing non-diagonal elements are:

\[\rho_{34}(t) = \exp\left(-\frac{i2t\lambda_3}{2} \cdot \frac{t}{2} \frac{(X_1^+ + Y_2^-)}{2}\right) \rho_{34}(0),\]

\[\rho_{43}(t) = \bar{\rho}_{34} = \exp\left(\frac{i2t\lambda_3}{2} \cdot \frac{t}{2} \frac{(X_1^- + Y_2^+)}{2}\right) \rho_{43}(0).\]

In the solution we have introduced some constants:

\[X_i = X_i^+ + X_i^-;\]

\[Y_i = Y_i^+ + Y_i^-;\]

\[X_i^\mp = 2\cos^2(\theta/2) J^{(1)}(\pm \omega_i) + 2\sin^2(\theta/2) J^{(2)}(\pm \omega_i);\]

\[Y_i^\mp = 2\sin^2(\theta/2) J^{(1)}(\pm \omega_i) + 2\cos^2(\theta/2) J^{(2)}(\pm \omega_i);\]

or

\[X_i^\mp = (J^{(1)}(\pm \omega_i) + J^{(2)}(\pm \omega_i)) + \frac{\Delta \epsilon}{\sqrt{4K^2 + (\Delta \epsilon)^2}} (J^{(1)}(\pm \omega_i) - J^{(2)}(\pm \omega_i));\]

\[Y_i^\mp = (J^{(1)}(\pm \omega_i) + J^{(2)}(\pm \omega_i)) - \frac{\Delta \epsilon}{\sqrt{4K^2 + (\Delta \epsilon)^2}} (J^{(1)}(\pm \omega_i) - J^{(2)}(\pm \omega_i)).\]
One can easily see that the only steady-state solution possible in this system corresponds to the time moment \( t = \infty \):

\[
\lim_{t \to \infty} \rho_{ii}(t) = \frac{1}{X_1Y_2} \begin{pmatrix}
X_1^+Y_2^- \\
X_1^-Y_2^+ \\
X_1^-Y_2^- \\
X_1^+Y_2^+
\end{pmatrix},
\]

\[
\lim_{t \to \infty} \rho_{34}(t) = 0.
\]

In the regular basis \( \rho_\infty \) is:

\[
\rho_\infty = \frac{1}{X_1Y_2} \times
\begin{pmatrix}
X_1^-Y_2^- & 0 & 0 & 0 \\
0 & c^2X_1^-Y_2^+ + s^2X_1^+Y_2^- & s(X_1^-Y_2^+ - X_1^+Y_2^-) & 0 \\
0 & s(X_1^-Y_2^+ - X_1^+Y_2^-) & s^2X_1^+Y_2^+ + c^2X_1^-Y_2^- & 0 \\
0 & 0 & 0 & X_1^+Y_2^+
\end{pmatrix},
\]

where \( c = \cos(\theta/2) \) and \( s = \sin(\theta/2) \).

In order to quantify the entanglement between the spins we consider the concurrence [13]. In the steady-state \((t \to \infty)\) it is given by

\[
C_\infty = \frac{2}{X_1Y_2} \max \left( 0, \frac{\sin \theta}{2} \sqrt{|X_1^+Y_2^- - X_1^-Y_2^+|} - \sqrt{|X_1^-Y_2^+ - X_1^+Y_2^-|} \right).
\]

**IV. EXACT SOLUTION IN THE POST-MARKOVIAN CASE**

It is a well known fact that positivity is guaranteed only in the case of the Markovian dynamics and in general even in the Born-approximation one can find that evolution in no longer positive. Recently Shabani and Lidar [11, 14] suggested and studied an equation which describes positive and non-Markovian dynamics of the reduced system, so called Post-Markovian dynamics

\[
\frac{d\rho}{dt} = \mathcal{L} \int_0^t dt' k(t') \exp(t'\mathcal{L})\rho(t - t'),
\]

or

\[
\frac{d\rho}{dt} = \mathcal{L}k(t) \exp(t\mathcal{L}) \ast \rho(t).
\]

Note, that the above dynamics contains a phenomenological memory kernel \( k(t) \).

Solution of the post-Markovian equation can be constructed with the help of the Laplace transform

\[
s\rho(s) - \rho(0) = [k(s) \ast \frac{\mathcal{L}}{s - \mathcal{L}}] \rho(s).
\]

Then, the eigenvector-problem for the Lindbladian

\[
\mathcal{L}\rho = \lambda \rho
\]

can be solved and one gets the solution

\[
\rho(t) = \sum_i Tr[\mathcal{L}_i \rho(t)] R_i = \sum_i \mu_i(t) R_i,
\]
where
\[ \mu_i(t) = \text{Lap}^{-1}[\frac{1}{s - \lambda_i k(s - \lambda_i)}] \mu_i(0) = \xi_i(t) \mu_i(0). \]

Particularly, in the case considered in this article the post-Markovian equation takes the following form:
\[
\frac{d\rho}{dt} = -i[H_s, \rho] + (L_1 + L_2) \int_0^t dt' k(t') \exp((t'(L_1 + L_2))\rho(t - t').
\]

In order to solve the eigenvector problem we find the Jordan decomposition for the Lindbladian
\[ \mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 = S J S^{-1}, \]

where
\[
S = \begin{pmatrix}
\frac{Y_2^+/Y_2^-}{Y_2^+/Y_2^-} & \frac{Y_2^+/Y_2^-}{Y_2^+/Y_2^-} & -1 & -1 \\
\frac{X_1^-/X_1^+}{X_1^-/X_1^+} & -1 & \frac{X_1^-/X_1^+}{X_1^-/X_1^+} & -1 \\
\frac{X_1 Y_2^-/X_1 Y_2^-}{1} & -\frac{Y_2^+/Y_2^-}{1} & -\frac{X_1^-/X_1^+}{1} & -1 \\
1 & 1 & 1 & 1
\end{pmatrix}
\]

and
\[ J = \text{diag}(0, -X_1, -Y_2, -X_1 - Y_2). \]

In this paper we consider the exponential memory kernel [15]
\[ k(t) = \gamma_0 e^{-\gamma_0 t}, \]

which implies that
\[ \xi(\lambda_i, t) = \frac{\gamma_0 e^{\lambda_i t} + \lambda_i e^{-\gamma_0 t}}{\gamma_0 + \lambda_i}. \]

The analytical solution for the diagonal elements is
\[ \rho_{ii}(t) = S \times \text{diag}(1, \xi(J_{22}, t), \xi(J_{33}, t), \xi(J_{44}, t)) \times S^{-1} \begin{pmatrix}
\rho_{11}(0) \\
\rho_{22}(0) \\
\rho_{33}(0) \\
\rho_{44}(0)
\end{pmatrix}. \]

Taking into account that for the non-diagonal elements the Lindbladian has Jordan form the dynamics of the corresponding elements are given by \( \xi(\lambda_{\text{non-diag}}, t) \) with corresponding value.

V. RESULTS AND DISCUSSION

The dynamics of entanglement is analyzed in Figures 1-3. In Figures 1 and 2 the dynamics of the concurrence between the two qubits is shown for different coefficients \( \gamma_0 \) in the memory kernel \( k(t) \) (for the Markovian case \( \gamma_0 = \infty \)). One can see that with decreasing \( \gamma_0 \) memory effects play a more essential role in the system dynamics and practically suppress the oscillations in the concurrences due to the Hamiltonian dynamics of the system. In Figure 3 one can see that increasing the temperature of the baths destroys quantum correlations in the system. From Figure 3 and curve (c) on Figure 2 one can see the phenomenon of “sudden death” of entanglement Refs. [16, 17].

The steady-state concurrence is analyzed in Figures 4 and 5. The detailed analysis of steady state concurrence for this model is given in Ref. [10]. In Figures 4 and 5 we plot the steady-state concurrence for the symmetric (\( \Delta \epsilon = 0 \)) and non-symmetric (\( \Delta \epsilon \neq 0 \)) cases as a function the mean temperature (\( T_M = (T_1 + T_2)/2 \)) and the temperature difference (\( \Delta T = T_1 - T_2 \)) of the baths. One can see that in the symmetric case (Fig. 4) the maximal value of the entanglement corresponds to the thermodynamically equilibrium case (\( T_1 = T_2 \)) and in the non-symmetric case (Fig. 5) the maximum of the quantum correlations reaches in the thermodynamically non-equilibrium case.

In conclusion, we have found an analytical solution for a simple spin system coupled to bosonic baths at different temperatures in Markovian and Post-Markovian cases. We studied the influence of memory effect on the dynamics of entanglement.
FIG. 1: Dynamics of the concurrence $C(t)$ for the initial reduced density matrix $\hat{\rho}_0 = |1, 0\rangle\langle 1, 0|$. The parameters of the model are chosen to be $\gamma_1 = \gamma_2 = 0.001$, $\epsilon_1 = 2$, $\epsilon_2 = 1.1$, $K = 1$, $T_1 = 0.2$, $T_2 = 0.5$ for different coefficients $\gamma_0$ in the memory kernel $k(t)$: curve (a) corresponds to the Markovian case $k(t) = \delta(t)$; curves (b)-(d) post-Markovian cases; curve (b) $\gamma_0 = 1$; curve (c) $\gamma_0 = 0.1$; curve (d) $\gamma_0 = 0.01$.

FIG. 2: Dynamics of the concurrence $C(t)$ for the initial reduced density matrix $\hat{\rho}_0 = |1, 0\rangle\langle 1, 0|$. The parameters of the model are chosen to be $\gamma_1 = \gamma_2 = 0.001$, $\epsilon_1 = 2$, $\epsilon_2 = 1.1$, $K = 1$, $T_1 = 1.2$, $T_2 = 1.5$ for different coefficients $\gamma_0$ in the memory kernel $k(t)$: curve (a) corresponds to Markovian case $k(t) = \delta(t)$; curves (b)-(d) post-Markovian cases; curve (b) $\gamma_0 = 1$; curve (c) $\gamma_0 = 0.1$; curve (d) $\gamma_0 = 0.01$. 
FIG. 3: Dynamics of the concurrence $C(t)$ for the initial reduced density matrix $\hat{\rho}_0 = |1, 0\rangle\langle 1, 0|$. The parameters of the model are chosen to be $\gamma_1 = \gamma_2 = 0.001$, $\epsilon_1 = 1.5$, $\epsilon_2 = 1.1$, $K = 1$, $\gamma_0 = 0.1$ for different temperatures of the "baths": curve (a) corresponds to $T_1 = 0.2$, $T_2 = 0.5$; (b) $T_1 = 1.2$, $T_2 = 1.5$; (c) $T_1 = 2.2$, $T_2 = 2.5$.

\[ C(T_M, \Delta T)_{(\epsilon_1 = 2, \epsilon_2 = 2, K = 1)} \]

FIG. 4: Steady-state concurrence $C(T_M, \Delta T)$ as a function of the mean bath temperature $T_M = (T_1 + T_2)/2$ and temperature difference $\Delta T = T_1 - T_2$ in the symmetric case $\epsilon_1 = \epsilon_2 = 2$ with $K = 1$. 
FIG. 5: Steady-state concurrence $C(T_M, \Delta T)$ as a function of the mean bath temperature $T_M = (T_1 + T_2)/2$ and the temperature difference $\Delta T = T_1 - T_2$ in the case $\epsilon_1 = 2, \epsilon_2 = 1, K = 1$.

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