Simulations of parameters of the mirror trap-based neutron source limited by the development of DCLC instability

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Abstract. The axially symmetric mirror trap with deuterium-tritium plasma is considered as a neutron source for the hybrid reactor. The goal of this study is to select the facility parameters, providing the maximum neutron yield at a fixed input power. The procedure is proposed for verifying the sufficient criterion for stability of the drift-cone ion-cyclotron mode. Within a limited set of configurations under consideration, the maximum fusion reaction power of 5.8 MW is achieved at an input power of 100 MW.

1. Introduction
Currently, the issue of creating a neutron source (NS) for the hybrid fusion-fusion reactors is relevant. In this work, the NS is considered based on the axially symmetric mirror trap confining deuterium–tritium plasma. The plasma parameters and neutron yield are simulated using the DOL code [1]. Similar calculations have been previously performed [2], in which only the quick test was used to estimate the plasma stability. In this paper, the more realistic criterion is used based on the analysis of drift-cone ion-cyclotron instability. The stability criterion is described in detail in Section 2. Section 3 presents the results of the NS simulations. Only two facility configurations are studied, which allows estimating the range of the achievable parameters.

2. Stability criterion
The NS under consideration is a mirror trap facility confining the two-component plasma. The first component is the rather cold “background” plasma. This plasma is heated by the injected atomic beams. Due to the charge-exchange processes between the background plasma and the beam atoms, the second plasma component forms, called the “fast ions”. The average fast ion energy is considerably higher than the background component temperature. The GDT facility (Budker Institute of Nuclear Physics) [3] is considered as an experimental prototype of the NS proposed.

The description of the two-component plasma stability is generally difficult. We specify the main types of instabilities. In the hydrodynamic approximation, the flute instability is the most dangerous mode for the open traps. In the review [3], several methods for suppressing this mode are discussed that do not require the considerable modification of the facility configuration. A number of instabilities occur at high plasma pressures, such as the ballooning and mirror modes. The experiments at the GDT facility [4] demonstrated that, at relative pressures of \( \beta_v \leq 0.6 \), these modes are absent.
At last, the distribution function deviations from the Maxwellian one can result in the development of the kinetic instabilities.

We consider two modes that are the most dangerous. The first one is the Alfvén ion cyclotron (AIC) instability. In the presence of the strong inhomogeneity in the angular distribution of ions, it leads to the resonant build-up of the Alfvén waves. The AIC development was studied theoretically and experimentally at the GDT facility [5, 6]. The main conclusions are as follows. The development of this mode results in a slight scattering of the fast ions with the maximum energies. In the phase space, the location of the fast ion source is determined by the parameters of the injected atomic beams. In the NS under consideration, it is located far from the drain region, which, in this case, is the so-called “loss cone”. For this reason, the development of the instability of this type does not lead to the considerable particle or energy losses.

Another kinetic mode that may be even more dangerous is the drift-cyclotron loss-cone (DCLC) instability. The reason for development of this instability is the fast depletion of the “loss cone” region in the phase space. Such conditions can be realized when the gas-dynamic regime of the warm ion confinement is changed for the kinetic confinement regime typical of the classical mirror trap. We note that, in the kinetic regime, the estimated fusion reaction rates are maximal. However, in such estimates, the development of instabilities is not taken into account. Apparently, in reality, such a regime cannot be realized. Thus, in terms of the NS efficiency, the main question is, how close the NS confinement regime can be to the classical mirror confinement regime, while the plasma still remains stable. In this work, the DCLC mode stability criterion is assumed to be the general stability criterion.

The stability criterion is based on the results of [7]. In this study, the problem is considered, which is axially symmetric and homogeneous along the facility axis. The magnetic field is also assumed to be uniform and parallel to the axis of symmetry. The dispersion relation is derived in the electrostatic limit. Under these conditions, the plasma stability is determined by the “transverse distribution function” obtained by summation over the longitudinal velocities: \( f_L(v_L) = \int f(v_L, v_\parallel)dv_\parallel \). The resulting dispersion relation describes two types of instabilities: the DCLC instability developing in the presence of the radial plasma density gradient and the double-humped (DH) instability, which can develop even in the homogeneous plasma. The distinctive feature of the studies presented in [7] is the fact that the distribution function is approximated by a sum of the Gaussian profiles, which considerably simplifies the mathematical expressions:

\[
\begin{align*}
  f_1(v_L) &= \sum \frac{n_i}{\pi w_j^2} \exp \left( -\frac{v_J^2}{w_j^2} \right). \\
  \end{align*}
\]

The plasma parameters calculated using the DOL code cannot be directly used in formula (1). In Section 3, we describe the method for selecting parameters of the Gaussian profiles for any arbitrary point inside the facility based on the results of the DOL code calculations. We note that, to reduce the calculation time, each plasma component was approximated only by two Gaussian profiles.

The DOL code used for the fast ions calculates the distribution function \( f(v_L, v_\parallel) \), which makes it possible to rather simply find the approximation parameters. The loss cone for the fast ions is assumed to be empty, so the transverse distribution function vanishes at the origin: \( f_L(0) = 0 \). Therefore, the function proposed by Herver [8] is used to approximate the distribution function of this component:

\[
\begin{align*}
  f_L &= \frac{n_i}{\pi w_\parallel^2} \left( \frac{w_\parallel^2}{K} \right)_{\frac{K+1}{K-1}} \left( \exp \left( -\frac{w_\parallel^2}{K} \right) - \exp \left( -(K+1) \frac{w_\parallel^2}{K} \right) \right). \\
  \end{align*}
\]

Here, \( n \) is the fast ion density and \( K > 1 \). To determine the \( K \) and \( w \) parameters, approximation (2) is introduced (using the least square technique) into the distribution function calculated using the DOL code. In terms of plasma stability, the most important region is the region of low velocities where the transverse distribution function increases. To emphasize this, the additional condition is introduced: the approximation maximum should coincide with the maximum of the distribution function calculated using the DOL code.

For the background plasma component, the DOL code calculates only the integrated characteristics, such as the density and temperature. Moreover, for this component, the degree of the loss cone fill can vary in a wide range, depending on the NS parameters and the point inside the facility under
consideration. Therefore, the distribution function approximation for the background plasma is given by the more general formula:

\[ f_L = C_1 \exp \left( \frac{-v_i^2}{w_i^2} \right) - C_2 \exp \left( -K \frac{v_i^2}{w_i^2} \right), \]  

where \( C_1 \geq C_2 \geq 0, K > 1. \) The first term determines the distribution function behaviour at high energies, and the second one determines the degree of the loss cone fill. Four conditions are required to calculate the \( C_1, C_2, w, \) and \( K \) approximation parameters. The first two conditions are set by the integrated characteristics:

\[ n = \int f_L \cdot 2\pi v_L dv_L, \quad n \cdot \frac{zT}{m} = \int v^2 \cdot f_L \cdot 2\pi v_L dv_L, \]  

where \( n \) and \( T \) are the background plasma density and temperature, and \( m \) is the ion mass. The loss cone size \( K \) is determined by the local mirror ratio \( R = B_m/B, \) where \( B \) is the field at the point under study, and \( B_m \) is the maximum field in the magnetic mirror:

\[ K = 1 + C_R \cdot R. \]  

Such dependence can be obtained by comparing the approximation (3) and the transverse distribution function (1), calculated for the case of the classical mirror trap [9] in the limit \( R >> 1. \) Good agreement is observed at \( 2 < C_R < 3; \) in this work, we used \( C_R = 2.5. \) The last condition determines the degree of the loss cone fill, namely, the value of the transverse distribution function at the origin: \( f_L(0) \). Unfortunately, this value cannot be obtained from the DOL code calculations; it was estimated as follows. Since, in the major part of the facility, the loss cone is narrow, its filling occurs mainly due to the angular diffusion. At low energies, the collisions are more frequent and the diffusion proceeds faster. The characteristic energy \( E_{th}, \) below which the loss cone is filled, is determined by the following condition:

\[ 1 \approx \int R (l^i) \gamma_i(v) \cdot \frac{dv}{v}, \quad m_i w_i^2 + e_i \phi(l^i) = E_{th,i} \gamma_i(v) = \sum \frac{8e^2}{3m_i^i w_i^2} A_{i,j}, \]  

where \( i \) and \( j \) are the indices of the ion species, \( l \) is the coordinate along the field line, \( e_i \) and \( m_i \) are the ion charge and mass, \( A_{i,j} \) is the Coulomb logarithm, \( \phi \) is the electrostatic potential, \( w_i \) is the thermal velocity, and \( \gamma_i \) is the effective angular scattering frequency derived in the low-energy limit. In fact, expression (6) describes the angular scattering frequency, normalized to the loss cone width, integrated over the time required for ions to move from the magnetic mirror section to the point under study. Assuming that the distribution function in the filled part of the loss cone is close to the Maxwellian one, we obtain the last condition:

\[ f(0) \approx \frac{n}{2\pi w_i^2} \left( \text{erf}(\sqrt{E_{th,i}/T}) + \text{erf}(\sqrt{E_{th,i}/T}) \right), \]  

where \( E_{th,i} \) and \( E_{th} \) are the characteristic energies calculated according to condition (7) for ions moving in the positive and negative directions.

### 3. Numerical simulations

The numerical simulations of the NS parameters were performed for two configurations close to those considered in [2]. The magnetic field profiles used in these configurations are presented in Fig. 1. The NS main cell length was 20 m and the mirror ratio was \( R = 10. \) The atomic beam injectors were located near the magnetic mirrors and were oriented perpendicular to the facility axis. The zone between the injectors can be used for the arrangement of the hybrid reactor blanket. The fixed power of atomic beams was \( P_{inj} = 100 \text{ MW}. \) The ratios of the magnetic field at the beam injection point to the minimum (central) magnetic field were \( R_{inj} = 1.0 \) and 1.8 for the first and the second configurations, respectively. The plasma radius in the central section of the facility was 30 cm for both configurations.

A series of calculations was performed for each configuration, in which the energy of the atomic beam particles \( E_{inj} \) and the amount of gas \( J_{gas} \) injected into the system for maintaining the material balance were varied. Firstly, the plasma parameters were calculated using the DOL code. Then, in accordance with the criterion described in Section 2, the plasma stability was verified. The stability verification was carried out at two points: in the facility centre and at the point with the maximum fast ion density. The NS efficiency was characterized by the ratio of the fusion reaction power \( P_{fus} \) to
the injection power: \( Q = P_{\text{gas}}/P_{\text{inj}} \). A typical calculation result is shown in figure 2. Two unstable regions are formed at the axis of the gas flow maintaining the material balance. At low gas puffing \( J_{\text{gas}} \), the background plasma confinement regime is close to the classical one. In this case, the oscillations developed in the energy range that is considerably lower than the background plasma temperature. That is, the instability starts to develop because the loss cone of the background ions is empty. An increase in the gas puffing \( J_{\text{gas}} \) results in the cooling of the background plasma, an increase in its collision frequency, and the stabilization of these oscillations. With a further increase in the gas puffing \( J_{\text{gas}} \), the gap between the background plasma temperature and the characteristic energy of the fast ions becomes larger, which leads to the development of the double-humped instability. The oscillations are excited in the energy range of the order of several background plasma temperatures. At the large gas puffing, the plasma becomes stable again, since the fast ion density becomes insufficient for the instability development.

![Figure 1. Variation of the magnetic field B along the facility z-axis inside the main cell of the NS. The locations of the injectors \( z_{\text{inj}} = \pm 8 \text{ m} \) are indicated by arrows.](image)

![Figure 2. The NS efficiency \( Q \) at different \( J_{\text{gas}} \) and \( E_{\text{inj}} \) values for the configuration with \( R = 10 \) and \( R_{\text{inj}} = 1.8 \): unstable and stable modes are indicated by empty and filled dots, respectively.](image)

The calculation parameters for both configurations corresponding to the maximum NS efficiency \( Q \) are presented in table 1. We note that the relatively small \( Q \) values can be due to the too strict stability criterion. In particular, it is known that the longitudinal inhomogeneity and the effects of the finite plasma pressure improve the DCLC mode stability. These effects were not taken into account in [7]. We also note that, in [2], the following restriction imposed on the ratio of confinement times was used as the stability criterion: \( \tau_{\text{kin}}/\tau_{\text{gd}} \leq 1 \), where \( \tau_{\text{kin}} \) and \( \tau_{\text{gd}} \) are the estimates for the classical confinement regime with a low collision frequency and the gas-dynamic regime with a high collision frequency, respectively. Table 1 shows that, in the stable regimes, this ratio can be \( \tau_{\text{kin}}/\tau_{\text{gd}} \approx 10 \).

**Table 1.** Calculation parameters for configurations with the maximum \( Q \) values. The densities of fast and background ions are separated by a slash.

| Configuration                           | \( R = 10, R_{\text{inj}} = 1.8 \) | \( R = 10, R_{\text{inj}} = 1.0 \) |
|----------------------------------------|-----------------------------------|-----------------------------------|
| Energy of atomic beam particles \( E_{\text{inj}} \), keV | 100                               | 120                               |
| Gas flow introduced into the facility \( J_{\text{gas}} \), eq. kA | 3.7                               | 5                                 |
| Electron temperature, keV              | 1.2                               | 1                                 |
| Ion temperature, keV                   | 2                                 | 1.1                               |
| Ion density at the centre, \( 10^{13} \) cm\(^{-3} \) | 2.9/3.0                           | 3.6/1.9                           |
Ion density at the injection points, $10^{13} \text{ cm}^{-3}$

| Density          | Value 1 | Value 2 |
|------------------|---------|---------|
| 5.1/2.6          | 2.2/2.1 |

Maximal value of relative plasma pressure $\beta$

| Value          | Value |
|----------------|-------|
| 0.14           | 0.33  |

Ratio of confinement times $\tau_{\text{kin}}/\tau_{\text{gd}}$

| Value          | Value |
|----------------|-------|
| 11             | 5     |

NS efficiency $Q$

| Value          | Value |
|----------------|-------|
| 0.058          | 0.045 |

### 4. Conclusions

The procedure for verifying the DCLC mode stability based on the model [7] is proposed, while the plasma parameters are calculated using the DOL code. Two NS configurations are considered as an example. The maximum NS efficiency was $Q = 5.8\%$. Such low efficiency is explained by the too strict stability criterion.

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