One of the most challenging problems in relativistic heavy-ion collisions is to explain the observed enhancement of singly strange and multistrange hadrons. The shorter equilibration times for strange quarks in a quark-gluon plasma (QGP) than the strange hadrons in hadronic matter had led to the conjecture [1] that enhanced production of (multi)strange hadrons might be an indication of QGP formation in the early stage of the collision. However, a firm conclusion of strangeness enhancement as a signal for the QGP can only be established if baseline hadronic processes fail to explain the strange particle abundances. For the singly-strange kaons and lambdas, it was found that the observed enhancement at both the AGS [2] and the SPS [3] energies could indeed be understood within a hadronic scenario [4]. This scenario, on the other hand, cannot explain the enhanced production of the multistrange Ξ and Ω (and their antiparticles) observed in Pb+Pb collisions at SPS energies [5,6]. The latter enhancement has thus been associated with production mechanisms which include possible QGP formation [7]. The doubly strange Ξ has also been measured in heavy ion collisions at the AGS [8]. Compared to that expected from primary nucleon-nucleon collisions, the observed enhancement is even more pronounced than in the SPS experiments. Although this is not surprising as experimental data have shown that the ratio of strange to nonstrange particles (such as $K^+/π^+$) in A+A relative to p+p collisions increases gradually from RHIC energy to lower beam energies at SIS [9], it is interesting to find out if the measured Ξ yield in this collision is consistent with the hadronic or the QGP scenario. In this paper, we shall study the production of multistrange Ξ from Au+Au collisions at the AGS energy of 6.4 GeV using the relativistic hadronic transport model ART [10]. We shall show that strangeness-exchange reactions between the antikaon and lambda(sigma) in hadronic matter alone can explain the measured Ξ yield [8], and exotic processes for multistrange particle production at the AGS energies are thus not required.

II. THE ART MODEL AND THE STRANGENESS-EXCHANGE REACTIONS

The ART model is a hadronic transport model that includes baryons such as $N, Δ(1232), N^*(1440), N^*(1535), Λ, Σ$, and mesons such as $π, ρ, ω, η, K, K^*$. Both elastic and inelastic collisions among most of these particles are included by using the experimental data from hadron-hadron collisions. The ART model has been quite successful in explaining many experimental observations, including the surprisingly large kaon antiflow [11,12] in heavy ion collisions at AGS energies. The ART model also allows us to understand whether or not the strongly interacting matter formed in these collisions reaches chemical and/or thermal equilibrium. In the present study, we extend the ART model to include perturbatively the Ξ particle as in the studies for other rare particles using the transport model [6,13,14].

In hadronic matter, the Ξ particle is mainly produced via the strangeness-exchange reactions $K(Δ, Σ) → πΞ$ because the contribution from the associated production reactions $NN → ΞNNK$ and $π(Δ, Σ) → KΞ$ are Okubo-Zweig-Iizuka suppressed. Since there is no empirical information on the strangeness-exchange reaction for Ξ production, we use the cross section obtained from the gauged flavor SU(3)-invariant Lagrangian in the coupled-channel approach [15]. Because the large threshold for final states with the η particle, only the reactions $K(Δ, Σ) → πΞ$ are considered here. Writing the spin- and isospin-averaged cross sections for these reactions as

\[
\sigma_{KΛ→πΞ} = \frac{1}{4} \frac{p_τ}{p_K} |M_{KΛ→πΞ}|^2; \\
\sigma_{KΣ→πΞ} = \frac{1}{12} \frac{p_τ}{p_K} |M_{KΣ→πΞ}|^2, 
\]

where $p_K$ and $p_τ$ are initial antikaon and final pion momenta in the center-of-mass system, the theoretical cross sections of Ref. [15] are found to be well fitted by the
following squared invariant matrix elements $|M_{\bar{K}\Lambda \rightarrow \pi\Xi}|^2$ and $|M_{\bar{K}\Sigma \rightarrow \pi\Xi}|^2$ [16]:

$$|M_{\bar{K}\Lambda \rightarrow \pi\Xi}|^2 = 34.7 \frac{s_0}{s} \text{ mb},$$

$$|M_{\bar{K}\Sigma \rightarrow \pi\Xi}|^2 = 318 \left(1 - \frac{s_0}{s}\right)^{0.6} \left(\frac{s_0}{s}\right)^{1.7} \text{ mb}. \quad (2)$$

In the above, the threshold energy $s_0^{1/2}$ in the center-of-mass system is 1.611 GeV and 1.688 GeV for the reactions $\bar{K}\Lambda \rightarrow \pi\Xi$ and $\bar{K}\Sigma \rightarrow \pi\Xi$, respectively. We note that except near threshold, these cross sections are of the order of 5-10 mb.

The cross sections for the inverse reactions $\pi\Xi \rightarrow \bar{K}\Lambda$ and $\pi\Xi \rightarrow \bar{K}\Sigma$, which are needed for treating $\Xi$ annihilation, are related to those for $\Xi$ production by the principle of detailed balance, i.e.,

$$\sigma_{\pi\Xi \rightarrow \bar{K}\Lambda} = \frac{1}{3} \frac{p_{\bar{K}}^2}{p_\pi^2} \sigma_{\bar{K}\Lambda \rightarrow \pi\Xi},$$

$$\sigma_{\pi\Xi \rightarrow \bar{K}\Sigma} = \frac{1}{p_{\bar{K}}^2} \sigma_{\bar{K}\Sigma \rightarrow \pi\Xi}.$$. \quad (3)

We have also included the production of the resonance $\Xi^* (1530)$ via the reaction $\pi\Xi \rightarrow \Xi^* (1530)$, and its cross section is given by a Breit-Wigner form with the empirical width of $\Gamma_\Xi = 9.5$ MeV. The cross section for the inverse reaction of $\Xi^* (1530)$ decay, i.e., $\Xi^* (1530) \rightarrow \pi\Xi$, is also obtained from the detailed balance relation.

### III. RESULTS

#### A. Time evolution of hadron abundances

![Graph showing time evolution of hadron abundances](image1)

**Fig. 1.** Time evolution of central density and hadrons at midrapidity $|y| < 0.5$ in 6A GeV Au+Au collisions at impact parameter $b \leq 3$ fm in the ART model.

The time and spatial distribution of midrapidity $|y| < 0.5$ hadrons at freeze-out in 6A GeV Au+Au collisions at impact parameter $b \leq 3$ fm from ART model.

The time and spatial distribution of midrapidity $|y| < 0.5$ hadrons at freeze-out are shown in Fig. 2. Most of the $\Xi$’s freeze-out between 3 and 7 fm/c, whereas the freeze-out distribution of protons and pions is centered at about 12 fm/c (left panel). Due to the fact that strange hadrons have much smaller scattering cross sections with nucleons and pions, they decouple quite early from the system. The dominantly soft collisions among the non-strange hadrons lead to a long tail in the nucleon and pion distributions. The transverse radius distribution (right panel) indicates that the $\Xi$ and other hyperons have source sizes similar to the initial source.

#### B. Impact parameter dependence

![Graph showing impact parameter dependence](image2)

**Fig. 2.** Time and transverse radius distributions of midrapidity $|y| < 0.5$ hadrons at freeze-out in 6A GeV Au+Au collisions at impact parameter $b \leq 3$ fm from ART model.

In Fig. 3, we show the impact parameter dependence of the total yield of different particle species obtained...
from the ART model for Au+Au collisions at 6 A GeV. Except for very peripheral collisions, a $K^-/K^+$ ratio of 0.09 is obtained and found to be independent of collision centrality. This trend is also observed experimentally from $\sim 11 A$ GeV up to RHIC energies [17,18]. Within a statistical approach, this implies that $K^+$ and $K^-$ have similar freeze-out volumes. Indeed, in the transport calculation kaons and antikaons exhibit similar mean freeze-out times ($\sim 8.3$ fm/c) and transverse radii ($\sim 4.5$ fm) for central collisions. The $\Lambda + \Sigma^0$ exhibits a similar impact parameter dependence because of its associated production with kaons via the reaction $(\pi, \rho, \omega)(N, \Delta, N^*) \rightarrow K(\Lambda, \Sigma)$. Except for the very central collisions the $\Lambda + \Sigma^0$ yield in the ART model is found to be consistent with the E895 data.

![Graph](image)

FIG. 3. Calculated impact parameter dependence for the total yield of $K^+$, $K^-$, $\Lambda + \Sigma^0$ (top panel) and $\Xi^-$ (bottom panel) in Au+Au collisions at 6 A GeV. The dependence of $\Xi^-$ production on the cross section $\sigma$ for $K(\Lambda, \Sigma) \rightarrow \pi \Xi$ is also shown. The E895 data for the $\Lambda + \Sigma^0$ (top panel) and $\Xi$ yield (bottom panel) are shown by solid circles.

The $\Xi^-$ yield obtained by strangeness-exchange reactions between antikaons and lambda/omega for different impact parameters is also shown in Fig. 3. Similar to the singly strange hadrons, the $\Xi^-$ yield exhibits nearly a linear dependence on impact parameter. This is due to the fact that $\Xi^-$'s are mostly produced from secondary collisions in the ART model. Fig. 3 further shows that while the $\Lambda + \Sigma^0$ is enhanced by a factor of 4, the $\Xi^-$ yield grows steadily by a factor of 6 from peripheral to central collisions. This specific hierarchy of strangeness enhancement of $\Xi > \Lambda + \Sigma^0$ has been reported for Pb+Pb collision at the SPS by the WA97 collaboration [19]. However, at SPS energies, the $\Xi^-$ yield tends to saturate for a large number of participants $N_{\text{part}} > 100$, and its enhancement (by $\sim 3$) in central collisions is smaller than our prediction for AGS energies. The steady growth of $\Xi^-$ with centrality is a clear indication that its yield is sensitive to the volume of the fireball formed in the collision. Furthermore, we find that the $\Xi^-$ yield from strangeness-exchange reactions in the ART model is in reasonable agreement with the E895 data [8]. This is in contrast to SPS energies, where the cascade from the transport model was found [6] to underpredict somewhat the WA97 data, especially for central collisions.

C. Comparisons with the statistical model

It may be interesting to find if strange hadrons also reach chemical equilibrium in heavy ion collisions at 6 A GeV. To this end we compare results from the transport model with those from the statistical model based on the grand canonical ensemble [20,21] with complete thermal, chemical, and strangeness equilibrium. In this model, the particle density in the Boltzmann approximation is given by

$$
\frac{N_i}{V} = \frac{g m_i^2 T}{2 \pi^2} K_2(m_i/T)e^{m_i/T},
$$

in the usual notation [20,21]. Assuming $\Xi$ to be also in chemical, and strangeness equilibrium, the cascade to nucleon ratio is

$$
\frac{N_{\Xi}}{N_N} = \left(\frac{m_{\Xi}}{m_N}\right)^2 \left(\frac{K_2(m_{\Xi}/T)}{K_2(m_N/T)}\right)^{2n_s}
= 4 \left(\frac{N_\Lambda}{N_N}\right)^2 \left(\frac{m_\Xi m_N}{m_\Lambda}\right)^2 \left(\frac{K_2(m_{\Xi}/T)}{K_2(m_{\Lambda}/T)}\right)^2.
$$

For central Au+Au collisions, we use the ART model result for the ratio $N_\Lambda/N_N = 0.025$. This gives a $\Xi^-$ yield of $N_{\Xi} = 0.35 - 0.38$ at a freeze-out temperature of $T = 120 - 170$ MeV. Note that the production of the strange particle $\Xi$ is less than 1 per event. Thus, strangeness conservation must be implemented locally, i.e., a canonical instead of grand canonical ensemble must be used [22]. Therefore the present estimate of $\Xi^-$ gives only an upper bound. The higher value for the estimate of $\Xi^-$ from the statistical model compared to that obtained in the present transport approach indicates that $\Xi^-$ does not reach chemical equilibrium. Furthermore, like other strange hadrons, $\Xi$ freezes out relatively earlier than nonstrange hadrons with a mean time and transverse radius of 8.4 fm/c and 4.6 fm when the central energy density is about $\epsilon \approx 1$ GeV/fm$^3$.

The $\Xi^-/(\Lambda + \Sigma^0)$ ratio of 0.026 for $b = 0 - 3$ fm in the ART model is smaller than the equilibrium model estimate of 0.042 but slightly larger than the E895 data [8]. At midrapidity, the $\Xi^-/(\Lambda + \Sigma^0)$ ratio of 0.12 has been observed by the E810 Collaboration for central Si+Au collisions at 14.6 A GeV [23]. Since $\Xi$ is generated from $\Lambda, \Sigma$ by strangeness-exchange reactions, its yield would naturally be more peaked at midrapidity than that for As. This is corroborated by a higher $\Xi^-/(\Lambda + \Sigma^0)$ ratio of 0.03 at midrapidity $|y| < 0.50$. 

3
Another way to see if the yield of $\Xi$ approaches that for chemical equilibrium is to study how it changes when the cross section is artificially increased for the reaction $\bar{K}(\Lambda, \Sigma) = \pi \Xi$. This method was introduced in Ref. [24] to study whether or not $K^-$ reaches chemical equilibrium at energies available at SIS/GSI. The latter energies are below the threshold for $K^-$ production from nucleon-nucleon collisions in free space. Figure 3 shows that the $\Xi^-$ yield increases rather dramatically with increasing cross section $\sigma$. Equally important is the fact that for a given $\sigma$, the enhancement is even more pronounced for central collisions. Nonetheless, it is only for very large cross sections ($\sim 6\sigma$), where the hydrodynamical limit may have been reached, that the $\Xi^-$ approaches chemical equilibrium with $N_{\Xi^-} = 0.55$.

Interestingly, this value of $\Xi^-$ is even larger than that predicted from the statistical model. Possible reasons for this result are: (i) The temperature at complete chemical equilibrium, if reached at all, is much higher than that in the statistical model; (ii) The increased cross section has altered the reaction dynamics especially for $\bar{K}$ and $\Lambda$, i.e., a large abundance of $\bar{K}s$ and $\Lambda + \Sigma^0$s so produced will increase the $\Xi$ at a higher temperature; or (iii) large numbers of $\Xi$s, produced at an early and dense stage do not have sufficient time to be annihilated by rescattering because of rapid expansion of the system during its subsequent evolution. The possibility of large $\Xi$ production from case (i) alone is remote as its yield in the present statistical model is found to be rather insensitive to temperature.

D. Excitation functions

In Fig. 4, we show the excitation function for the strange particle abundances (top panel) in central Au+Au collisions from the ART model. As expected, with increasing beam energy the multiplicities of the hadrons increases. The enhancement is more pronounced for antikaons and cascades as the energy available for these massive and rare particle production increases. This is clearly evident in the bottom panel of the Fig. 4 where the $K^-/K^+$ ratio is found to increase monotonically with beam energy. The $\Xi^-$ has the largest rate of increase with energy resulting in an increasing of the ratios $\Xi^-/(\Lambda + \Sigma^0)$ and $\Xi^-/K^-$ with beam energy.

IV. SUMMARY

In summary, we have used a relativistic hadronic transport model to investigate the doubly strange cascade production. We find that the strangeness-exchange reactions between antikaons and hyperons lead to substantial production of $\Xi^-$ in Au+Au collisions at 6.4 GeV that is in reasonable agreement with the data. This is in stark contrast to that at SPS energies, where the strangeness-exchange reactions underpredict the data for the cascade yield in central collisions. The success of the purely hadronic scenario eliminates the scope of an exotica that may be necessary to explain the $\Xi$ yield at the AGS energies. Among all the strange hadrons, the $\Xi$ particle abundances reveal the strongest enhancement with centrality of collision and also with increasing beam energy.

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