Effects of the coupling of quasiparticle and collective vibrations on the properties of $^{120}$Sn

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Abstract. Assuming quasiparticles and collective vibrations as fundamental modes of excitation and taking into account their interplay within the framework of Nuclear Field Theory, it is possible to give an accurate and comprehensive description of the low-energy spectrum of the superfluid nucleus $^{120}$Sn.

1. Introduction
A relevant issue in the field of nuclear structure is whether it is possible to obtain a quantitative and comprehensive description of low-energy nuclear spectroscopic properties assuming quasiparticles and collective vibrations as fundamental modes of excitation, and considering the many-body processes associated with their interweaving in order to renormalise the properties obtained in the mean field approximation. This question was addressed (see [1] and refs. therein) in the framework of Nuclear Field Theory for the case of the superfluid nucleus $^{120}$Sn. It was shown that theory can indeed account, within a 10% average error level, for a set of structure and reaction data which provide a rather complete characterization of $^{120}$Sn. In the present contribution more details will be given concerning the influence of the mean field on the final results and concerning the possibility of determining an optimised set of bare single-particle energies in the valence shell, instead of taking them from a Hartree-Fock field obtained with an effective interaction fitted on global nuclear properties. After an outline of the formalism, I will discuss the renormalization of the pairing gap, which is enhanced by the pairing interaction induced by the exchange of collective phonons, and the renormalization of quasiparticles, leading to the fragmentation of the quasiparticle strength. Theoretical results will be compared to data, and the stability of the results will be addressed, varying the deformation of the lowest quadrupole vibration - which is the main factor determining the strength of the particle-vibration coupling - around its empirical value.

2. Theoretical framework
In this section the theoretical framework will be briefly summarised. See refs. [1, 2] for more details. The calculation will be divided in two steps. Starting from a fixed set of single particle levels, the HFB equations in the pairing $^1S_0$ channel are solved using the Argonne $v_{14}$ bare interaction. In order to obtain convergence a large set of levels must be included in the calculation, up to a cutoff of about 1 GeV, due to the effect of the hard core in the interaction. In this way a set of quasiparticles energies and of quasiparticles amplitudes is produced. An
Figure 1. (left) Diagonal part of the pairing gaps, resulting from the HFB calculation performed with the Argonne interaction and the SLy4 mean field. The circle encloses the gaps associated with the five valence orbitals lying close to the Fermi energy. They are shown in detail in the right part of the figure, together with the gaps \( \Delta \) obtained after the inclusion of the renormalization processes. Also indicated are the Fermi energy and the experimental value of the pairing gap derived from the odd-even mass difference, \( \Delta_{\text{exp}} \approx 1.4 \) MeV.

Many-body effects are included in the second step of the calculation, in which only the five lowest valence quasiparticles are included, neglecting the effects of the particle-vibration coupling on the radial wave functions. Starting from the quasi particle energies \( E_a \) and quasi particle amplitudes \( u_a, v_a \) obtained with the bare pairing interaction, one solves the energy-dependent eigenvalue problem, fragmenting the original quasiparticle strength into a set of states \( a(n) \) and obtaining a set of renormalized quasi particle energies \( \tilde{E}_a(n) \) and associated amplitudes \( \tilde{u}_a(n), \tilde{v}_a(n) \):

\[
\begin{pmatrix}
(\epsilon_a - \epsilon_F) + \tilde{\Sigma}_{11}^{11} \\
(\epsilon_a - \epsilon_F) + \tilde{\Sigma}_{22}^{12}
\end{pmatrix}
\begin{pmatrix}
\tilde{u}_a(n) \\
\tilde{v}_a(n)
\end{pmatrix}
= \begin{pmatrix}
\tilde{E}_a(n) \\
\tilde{E}_a(n)
\end{pmatrix},
\]

where \( \epsilon_a \) denotes the HF single-particle energy, while the new normal self-energies are given by

\[
\begin{align*}
\tilde{\Sigma}_{11}^{11} &= u_a^2\Sigma_{11}^{11} + v_a^2\Sigma_{22}^{11} - 2u_av_a\Sigma_{12}^{11} \\
\tilde{\Sigma}_{22}^{12} &= u_a^2\Sigma_{22}^{12} + v_a^2\Sigma_{11}^{12} + 2u_av_a\Sigma_{12}^{12}.
\end{align*}
\]

For a given eigenvalue \( \tilde{E}_a(n) > 0 \) there exists a corresponding eigenvalue \( \tilde{E}_a(-n) = -\tilde{E}_a(n) \).
The quantity $\Sigma^{11\text{pho}}_{a(n)}$ is given by

$$
\Sigma^{11\text{pho}}_{a(n)} = \sum_{b,m,\lambda,\nu} \frac{V^2(ab(m)\lambda\nu)}{E_{a(n)} - E_{b(m)} - \hbar \omega_{\lambda\nu}} + \sum_{b,m,\lambda,\nu} \frac{W^2(ab(m)\lambda\nu)}{E_{a(n)} + E_{b(m)} + \hbar \omega_{\lambda\nu}},
$$

(3)

and $\Sigma^{22\text{pho}}_{a(n)} = -\Sigma^{11\text{pho}}_{a(n)}$.

The quasiparticle-phonon matrix elements connecting the quasiparticle $a$ with the quasiparticle-phonon configuration $b(m)\lambda\nu$ are given by

$$
V(ab(m)\lambda\nu) = h(ab\lambda\nu)(u_a\bar{u}_{b(m)} - v_a\bar{v}_{b(m)}) ; \quad W(ab(m)\lambda\nu) = h(ab\lambda\nu)(u_a\bar{v}_{b(m)} + v_a\bar{u}_{b(m)}),
$$

(4)

where

$$
h(ab\lambda\nu) = -(-1)^{j_a - j_b}\beta_{\lambda\nu} < a| r_1 \frac{\partial U}{\partial r_1} | b > < j_b || Y_\lambda || j_a > \left[ \frac{1}{(2j_a + 1)(2\lambda + 1)} \right]^{1/2}
$$

(5)

is the basic particle-phonon vertex [3]. This vertex contains a finite range form factor and is suitable for the coupling to collective surface modes. It avoids convergence problems related to the coupling with uncorrelated particle-hole excitations, which arise when the coupling is calculated selfconsistently using zero-range Skyrme forces.

One can separate the abnormal self-energy into two terms, writing

$$
\tilde{\Sigma}^{12}_{a(n)} = \Delta_a^{\text{BCS}} + \tilde{\Sigma}^{12\text{pho}}_{a(n)}.
$$

(6)

The first term, $\Delta_a^{\text{BCS}}$, is the pairing gap associated with the bare interaction which was obtained in the first step of the calculation. The second term is equal to

$$
\tilde{\Sigma}^{12\text{pho}}_{a(n)} = -\sum_{b,m,\lambda,\nu} \frac{(2j_b + 1)}{2} V_{\text{ind}}(a(n)b(m))\bar{u}_{b(m)}\bar{v}_{b(m)}
$$

(7)

and contains the induced pairing interaction:

$$
V_{\text{ind}}(a(n)b(m)) = \sum_{\lambda,\nu} \frac{2h^2(ab\lambda\nu)}{(2j_b + 1)} \times \left[ \frac{1}{E_{a(n)} - E_{b(m)} - \hbar \omega_{\lambda\nu}} - \frac{1}{E_{a(n)} + E_{b(m)} + \hbar \omega_{\lambda\nu}} \right].
$$

(8)

The renormalized pairing gap $\tilde{\Delta}_{a(n)}$ is obtained multiplying $\tilde{\Sigma}^{12}_{a(n)}$ by the $Z$–factor (which is close to the fraction of the quasiparticle strength remaining in the $n$–fragment, although it does not coincide with it)

$$
Z_{a(n)} = \left( 1 - \frac{\Sigma^{11}_{a(n)} - \Sigma^{11}_{a(-n)}}{2E_{a(n)}} \right)^{-1}.
$$

(9)

The renormalised pairing gaps obtained with the Argonne interaction and the SLy4 mean field are shown in the right panel of Fig. 1. The induced interaction increases the value of the gap, and prevails over the reduction caused by the $Z$–factor.

The properties of the phonons are not explicitly renormalized. They will instead be obtained from QRPA calculations with separable forces, using coupling constants reproducing the empirical energy $\hbar \omega_{\lambda_1}$ and deformation parameter $\beta_{\lambda_1}$ for $\lambda = 2, 3$. 

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Figure 2. Basic diagrams taken into account in the present study, which renormalize the normal and abnormal self-energies obtained in mean field calculations: polarization (a), correlation (b), induced pairing interaction (c) processes.

Figure 3. Energies of the five valence single-particle energies associated with the Skyrme forces discussed in the text (ordered according to increasing effective mass). The last column shows the optimised energies discussed in text.

3. Pairing gaps and effective masses
In this section, the dependence of the pairing gaps on the effective mass associated with the mean field will be discussed. First of all let us study the case of the bare interaction. The pairing gaps calculated with the Argonne interaction with the mean field generated by seven Skyrme forces characterized by different values of $m_k$, the effective mass in nuclear matter at saturation density, are shown in Fig. 4(a) by green diamonds. One observes an overall increase of the gap with increasing $m_k$ superposed to significant fluctuations.

The fluctuations in the gaps are associated with the details of the energy distribution of the five valence orbitals, shown in Fig. 3, while the overall dependence on $m_k$ is determined by the whole single-particle spectrum. The two effects can be disentangled by solving the pairing gap equations using always the same single-particle energies for the five valence orbitals. In this
0.6 0.7 0.8 0.9 1

Figure 4. (a) Bare pairing gap of the $h_{11/2}$ orbital, as a function of the effective mass associated with the mean fields generated with the effective forces shown in Fig. 3 (cf. text). (b) Dependence of the monopolar pairing coupling constant on the effective mass.

0.6 0.7 0.8 0.9 1

Figure 5. Absolute difference between the experimental gap and the theoretical value of the renormalised gap, calculated as a function of the effective mass associated with the mean field.

case the pairing gap dependence becomes much smoother (red dots in Fig. 4(a), obtained using the valence levels of the SGII interaction). The results of these calculations can be accurately reproduced by a BCS calculation with a monopole interaction acting only in the valence shell, using the values of the pairing constant $G$ reported in Fig. 4(b) as a function of $m_k$. It is remarkable that, once the function $G(m_k)$ has been determined, the gaps shown in Fig. 4(a) by open squares, obtained by simple BCS calculations using only the corresponding five valence levels, can reproduce in detail the pairing gaps obtained by the calculation with the Argonne interaction using different mean fields (green diamonds in Fig. 4(a)).

The values of the renormalised gaps $\tilde{\Delta}$ are shown Fig. 5. The comparison with the experimental value favours effective masses in the range $m_k \approx 0.6m - 0.8m$. It should be remarked that the above discussion is oversimplified concerning at least three effects neglected here, which act on the pairing gap in different way. First, the momentum dependence associated with the Skyrme interaction is not realistic, and tends to make the gaps too low [4]. On the
The experimental excitation energies of the lowest valence levels (referred to the energy of the $3/2^+$ level) are compared to the theoretical renormalised quasiparticle energies, computed as a function of the coupling strength, starting from SLy4 single-particle energies. (b) The mean square root deviation between the experimental and the theoretical spectrum shown in (a) (solid line) is compared to the deviations obtained starting from with other three Skyrme forces and with the optimised single-particle levels shown in Fig. 7 (a) and discussed in the text (red dash-dotted line).

other hand, renormalization effects due to spin modes have not been included, and as a result gaps are about too large [2, 5]. Finally, the inclusion of three-body force, rarely considered until now, would tend to make the gap smaller, according to the results of ref. [6]. The corrections associated with these effects are difficult to quantify, but are of the order of 300 keV.

4. Renormalization processes and optimised levels
The quasiparticle spectrum obtained with the SLy4 mean field and the Argonne interaction is shown in Fig. 6(a) as a function of the ratio between the quadrupole deformation $\beta_2$ used in the calculation and the experimental value $\beta_{2,\text{exp}} = 0.13$, in order to assess the stability of the results. The mean square root deviation $\sigma$ between theory and experiment is shown in Fig. 6(b), also for other forces. Consistent results are obtained for the SGII, SLy4 and Ska interactions, which lead to $\sigma \approx 200$ keV and to a minimum close to $\beta_{\text{exp}}$ (although larger value of $\beta_2$ are equally acceptable in the case of SGII). It has been shown in ref. [1], that, using the SLy4 mean field, good quantitative agreement (within 10-20%) between theory and experiment can be obtained for a variety of observables, including electromagnetic transitions, multiplet splittings, one- and two-nucleon transfer absolute cross sections. On the other hand, it was observed that the $5/2^+$ strength distribution originating from the strongly fragmented $d_{5/2}$ orbital was not well reproduced, and that this was related to the specific position of the orbitals in the SLy4 mean field. Better results were obtained shifting the energy of the $d_{5/2}$ by 600 keV towards the Fermi energy.

We have already noticed that the results obtained with the Argonne interaction and a given mean field associated with an effective mass $m_k$ can be reproduced using a monopole pairing interaction with the value of the pairing coupling strength appropriate to $m_k$. We note that in such calculations, the essential ingredients are the values of the single-particle energies of the five valence orbitals; the change in the mean field leads to relatively small differences in the matrix elements. In the following, we study the possibility to determine optimized values of the single-particle energies, and show results obtained with matrix elements calculated with
Figure 7. (a) Energies of the optimised five valence single-particle energies $\epsilon_{\text{opt}}^a$ as a function of the quadrupole deformation parameter. (b) The experimental excitation energies of the lowest valence levels (referred to the energy of the $3/2^+$ level) are compared to the theoretical spectrum obtained starting from the optimized single-particle levels, as a function of the quadrupole deformation parameter. (c) The experimental energies of the members of the $h_{11/2} \otimes 2^+$ multiplet are compared with the theoretical values, calculated as a function of the quadrupole parameter. The associated mean square deviation is shown in (d).

The optimised single-particle energies have been determined minimising the mean square root difference between the experimental quasi particle spectrum and the theoretical renormalized energies $\tilde{E}(\bar{n})$, where $\bar{n}$ indicates the fragment carrying the largest quasiparticle strength for given quantum number. The resulting optimised levels have been calculated as function of $\beta/\beta_{\text{exp}}$ and are shown in Fig. 7(a). The spectrum obtained for $\beta = \beta_{\text{exp}}$ is compared in Fig. 3 with those associated with the different Skyrme forces representative of different values of the effective mass. The fit does not determine the absolute position of the levels, because the quasiparticle spectrum is unchanged by shifting all the single-particle energies and the Fermi energy $\epsilon_F$ by the same amount. One can fix the absolute position of the energy levels by comparing the experimental value of the neutron separation energy in $^{120}$Sn ($S_n = 9.1$ MeV) and $^{121}$Sn ($6.2$ MeV) with the theoretical value, $S_n \approx -\epsilon_F \pm \epsilon_0$, where $\epsilon_0$ denotes the values of the lowest calculated quasiparticle. It can be seen that all the considered Skyrme forces lead to a more bound scheme. The distance between the $g_{7/2}$ and the $d_{5/2}$ optimised levels is smaller than in most Skyrme spectra, in particular than in the case of SLy4. This confirms the analysis of the $5/2^+$ strength function mentioned above.

The renormalised quasiparticle spectrum is shown in Fig. 7(b). The deviation from experiment is smaller than 50 keV and is, by construction, smaller than that obtained starting...
Figure 8. The experimental occupation factors associated with the 5/2\(^+\) levels extracted from pickup and stripping one-transfer reaction are convoluted with a gaussian with \(\sigma = 0.25\) MeV in order to obtain the experimental strength function shown by the black curve. The other curves show the theoretical strength functions calculated with the optimised energy levels for two values of the quadrupole deformation parameter.

from the HF orbitals (Fig. 6(b)). It is noticed that relatively good accuracy is obtained with the Ska and the SGII mean fields, which have a low effective mass and \(g_{7/2}\) and \(d_{5/2}\) orbitals which lie close to each other. Also shown are the calculated levels belonging to the \(h_{11/2} \otimes 2^+\) multiplet with the associated deviation between theory and experiment (Fig. 7(c),(d)). Finally, in Fig. 8 the experimental and the theoretical 5/2\(^+\) strength function are compared. In this case, the best results are obtained with a value of the deformation parameter somewhat smaller than the experimental one.

5. Conclusions
The possibility of using optimised energy levels constitutes an interesting alternative to the use of single-particle levels from Hartree Fock calculations with effective interactions fitted on empirical bulk properties, which contain correlations which may lead in principle to double counting with respect to those explicitly considered in the many-body renormalization processes. In this respect, it seems anyway reassuring that the results obtained in the present analysis are in most cases quantitatively similar to those obtained with effective interactions associated with reasonable effective masses, lying in the range \(m_k = 0.6m - 0.8m\).

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