Thermal photon dispersion law and modified black-body spectra

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Abstract

Based on the postulate that photon propagation is governed by an SU(2) gauge principle we numerically compute the one-loop dispersion for thermalized photon propagation on the radiatively induced mass shell. Formerly, the dispersion was addressed by assuming $p^2 = 0$. While this approximation turns out to be excellent for temperatures $\leq 2T_{\text{CMB}}$ the exact result exhibits a much faster (power-like) shrinking of the gap in the black-body spectral intensity with rising temperature. Our previous statements on anomalous large-angle CMB temperature-temperature correlations, obtained in the approximation $p^2 = 0$, remain valid.
1 Introduction

The possibility that photon propagation is governed by an SU(2) rather than a U(1) gauge principle would have exciting astrophysical, cosmological, and conceptual consequences. To make them quantitative a precise understanding of SU(2) gauge dynamics is required. For the physics of photon propagation an analytical grasp of the deconfining phase is sufficient. Thermodynamically, this is feasible \[2, 3, 4, 5, 6, 7\].

In \[8\] we have computed the one-loop polarization tensor $\Pi$ of the massless mode in the deconfining phase of thermalized SU(2) Yang-Mills theory. By numerically finding estimates on the moduli of the one-particle irreducible contributions to the pressure at three-loop level \[7\] we conclude that the associated two-loop contributions to $\Pi$, obtained by cutting a massless line in the corresponding diagram for the pressure, are extremely suppressed with respect to the one-loop result (by a factor of the order $10^{-3}$). This entails that a modification of the massless mode’s dispersion law by radiative effects is in the effective theory for all practical purposes exhaustively described at the (resummed) one-loop level.

To be able to explicitly solve the kinematic constraints for the integration variables in the one-loop expression for $\Pi$, which are imposed by a maximal, temperature-dependent resolution scale $|\phi|$ owing to a thermal ground state $^2$, we have so far resorted to the approximation that the square of the four momentum $p$ of the massless mode be zero: $p^2 = 0$. In this approximation, there is no imaginary contribution to the screening function $G$. For temperatures not far above the critical temperature $T_c$ for the deconfining-preconfining phase transition, $G$ exhibits a sizable small-momentum regime where $G > p^2$. This regime is associated with a strong screening of the massless mode. On the other hand, there is a momentum regime where $G$ is sizably negative (antiscreening) thus describing the average loss of the massless mode’s energy to the ground-state dynamics. Microscopically, this effect is related to the generation of stable and nonrelativistic monopole-antimonopole pairs out of small-holonomy (anti)calorons. The latter’s holonomy is shifted to large values by inelastic scattering of the massless mode off the originally short-lived monopole-antimonopole systems (caloron or anticaloron dissociation). The rare generation of stable magnetic matter by scattering off short-lived ground-state constituents, subject to euclidean thermodynamics after spatial coarse-graining thus is described by a particular radiative correction in the effective theory which can be computed in the real-time formulation of thermalized quantum field theory and interpreted in purely energetic grounds \[9\].

The purpose of the present paper is to go beyond the approximation $p^2 = 0$ by numerically computing the function $G$ selfconsistently on the radiatively induced

\[ \text{References:} 2, 3, 4, 5, 6, 7, 8, 9. \]

\footnote{For a feasibility discussion of this postulate see \[1\].}
\footnote{The notion of a thermal ground state emerges upon the execution of a sufficiently local, self-consistent spatial coarse-graining over interacting calorons and anticalorons of topological charge modulus unity \[3, 4\].}
mass shell. Qualitatively, all the above described, essential features are reproduced by the full calculation: \( \text{Im} G = 0 \), screening at low and antiscreening at large spatial momentum modulus. At \( T \sim 2 T_c \) there is practically no difference between the full result and its approximation, and thus all results in applications, which rely on the low-temperature behavior of \( G \), remain valid \([10, 11, 12]\). The high-temperature behavior of \( G \) is profoundly different in the full calculation: While the onset of the gap \( Y^* = \frac{\omega^*}{T} \) towards small frequencies in black-body spectra falls off at high temperatures as \( \lambda^{-2/3} \) in the approximate result the full calculation yields a high-temperature behavior as \( Y^* \sim \lambda^{-3/2} \) where \( \lambda \equiv \frac{2\pi T}{\Lambda} \) is temperature scaled dimensionless by the Yang-Mills scale \( \Lambda \). Thus the cutoff frequency \( \omega^* \) absolutely increases as \( \lambda^{-1/2} \) with increasing temperature and not only relatively to the maximum of the black-body spectrum as was suggested by the approximate result.

This article is organized as follows. In Sec. 2 we first discuss the essential steps leading to the effective theory for the deconfining phase of SU(2) Yang-Mills thermodynamics and subsequently remind the reader of how the part of \( \Pi \) associated with the screening function \( G \) for propagating massless modes is computed. We detail some technical aspects in computing \( G \) selfconsistently, provide our results, and compare them to those of the approximate calculation (assuming \( p^2 = 0 \) for the external four-momentum \( p \)). This includes a discussion of the high-temperature behavior of \( G \). We also stress that the function \( G \) now saturates to finite values as \( |p| \to 0 \). In Sec. 3 we employ the results of Sec. 2 to derive a modification of black-body spectra once it is assumed that photon propagation is subject to an SU(2) gauge principle. In particular, we point out that in the high-temperature regime the approximation \( p^2 = 0 \) yields differing results compared to the full calculation: The Planck spectrum is approached much faster now. A summary of our work is given in Sec. 4.

2 Calculation of the screening function \( G \) on the radiatively induced mass shell

2.1 Essential steps in deriving the effective theory

Before we address the actual topic of the present work we would like to briefly discuss essential steps taken in \([2, 3, 4]\) to arrive at the effective theory for the deconfining phase.

The first step is to derive a thermal ground state composed of topologically nontrivial field configurations. The key observation is that BPS saturated, (anti)seldual field configurations in the Euclidean formulation exhibit vanishing energy-momentum and thus do not propagate in real time. Thus, if a selfconsistent spatial coarse-graining over these configurations can be executed then the resulting ground-state describing, adjoint scalar field \( \phi \) cannot propagate and fluctuate either. In a thermodynamical situation, the spatial and temporal homogeneity of one-point functions
then implies the spatial and temporal homogeneity of \( \phi \)’s modulus \( |\phi| \) (inertness). By selfconsistent it is meant that the spatial resolution scale of the effective theory emerges in the process of performing the coarse-graining in terms of temperature \( T \) and an integration constant \( \Lambda \). Because of \( \phi \)’s inertness its dynamics is carried by its dimensionless phase \( \hat{\phi} \). It turns out that on dimensional grounds only one single concrete (nonlocal) definition is possible in terms of BPS saturated field configurations describing the kernel of a differential operator which contains \( \hat{\phi} \). Moreover, because of their sufficiently low-dimensional moduli spaces only those configurations possessing topological charge modulus \( |Q| = 1 \) may contribute. Finally, only configurations of trivial holonomy may contribute since a non-dynamical holonomy parameter, which is required for BPS saturation, gives rise to total suppression in the infinite-volume limit \( [13] \). This leaves one with Harrington-Shepard calorons and anticalorons.

Executing the above-mentioned, nonlocal definition, which is an average over the two-point function of the field strength, points out that only magnetic-magnetic correlations do actually invoke a nontrivial result. Moreover, all ambiguities arising in the course of the calculations are one-to-one related to the undetermined parameters spanning the kernel of the operator \( D = \partial_t^2 + \left( \frac{2\pi}{\beta} \right)^2 \), where \( \beta \equiv 1/T \), which is linear. Thus \( D\phi = 0 \) is the sought after equation of motion. BPS saturation and this second-order equation are consistent if and only if \( \phi \)’s potential \( V(|\phi|^2) \) satisfies \( \partial_\phi^2 V(|\phi|^2) = -V(|\phi|^2)/|\phi|^2 \) \( [5] \) with solution \( V(|\phi|^2) = \Lambda^6/|\phi|^2 \) (\( \Lambda \) an integration constant of dimension mass). Notice that in this way the Yang-Mills scale \( \Lambda \) sneaks into the game purely nonperturbatively. Now, disregarding any interactions between the topologically trivial and the nontrivial sector of the fundamental theory, perturbative renormalizability \( [14, 15] \) states that a coarse-graining over the former sector doesn’t change the form of its effective action as compared to the fundamental action. The gauge invariance plus the invariance under the allowed spacetime symmetries does then only allow for a minimal coupling between \( \phi \) and the coarse-grained topologically trivial gauge field \( a_\mu \), see also \( [9] \), leading to the following effective action

\[
S_{\text{dec}} = \text{tr} \int d^4x \left\{ \frac{1}{2} G_{\mu\nu} G^{\mu\nu} + D_\mu \phi D_\mu \phi + \frac{\Lambda^6}{|\phi|^2} \right\} . \tag{1}
\]

Here \( D_\mu \) is an adjoint covariant derivative involving the yet unknown effective gauge coupling \( e \) and \( G_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu + ie[a_\mu, a_\nu] \). On tree-level, the action in Eq. (1) describes a situation where the thermal ground state only knows about the generation of monopoles and antimonopoles that collapse onto each other before they get re-separated through the absorption by (small-holonomy) (anti)calorons of propagating fundamental gauge modes that are, compared to the scale \( |\phi| \), far off their mass shell. The processes associated with a large (temporary) holonomy, leading to (anti)caloron dissociation into isolated and screened magnetic monopoles and antimonopoles \( [18] \), collectively are described by radiative corrections in the effective
Mesoscopically, the field $\phi$ gets domainized by these stable magnetic charges [16] (departure from the exponentially fast saturated infinite-volume limit considered in deriving $\phi$) but the thermal average over these domainizations, which is described by the loop expansion in the effective theory, assures the spatial homogeneity of one-point functions. Notice that this loop expansion, which converges rapidly due to kinematical constraints imposed by the nontrivial thermal ground state in physical unitary-Coulomb gauge, does not count powers of the effective coupling $e$. The latter’s running with temperature follows from the invariance of Legendre transformations under the applied spatial coarse-graining. For $T \gg T_c$ one has $e \equiv \sqrt{8\pi}$ where $T_c$ is the critical temperature for the deconfining-preconfining transition (more precisely: coexistence of these phases).

2.2 Set-up and computation of real part of $\Pi$

In Sec. 3 of [8] we have given the Feynman rules in unitary-Coulomb gauge for the computation of radiative corrections in the effective theory for the deconfining phase of SU(2) Yang-Mills thermodynamics [2]. For the sake brevity we do not repeat them here. On the one-loop level the two diagrams contributing to the polarization tensor $\Pi^{\mu\nu}(p,T)$ of the massless mode of four-momentum $p$ are shown in Fig.1. Thus $\Pi^{\mu\nu} = \Pi_A^{\mu\nu} + \Pi_B^{\mu\nu}$ where

![Feynman diagrams](image-url)
The Higgs mechanism in the effective theory and

\[ \Pi^\mu_\nu(p) = \frac{1}{2i} \int \frac{d^4k}{(2\pi)^4} e^{2\epsilon_{ace}} [g^\mu\rho(-p - k)^\lambda + g^\rho\lambda(k - p + k)^\mu + g^\lambda\mu(p - k + p)^\rho] \times \\
\epsilon_{dbf} [g^\sigma\nu(-k + p)^\kappa + g^\nu\kappa(p + k)^\sigma + g^\kappa\sigma(-p + k + k)^\nu] \times \\
(-\delta_{cd}) \left( g_{\rho\sigma} - \frac{k_\rho k_\sigma}{m^2} \right) \left[ \frac{i}{k^2 - m^2} + 2\pi \delta(k^2 - m^2) n_B(|k_0|/T) \right] \times \\
(-\delta_{ef}) \left( g_{\kappa\lambda} - \frac{(p - k)\lambda(p - k)\kappa}{(p - k)^2} \right) \times \\
\left[ \frac{i}{(p - k)^2 - m^2} + 2\pi \delta((p - k)^2 - m^2) n_B(|p_0 - k_0|/T) \right] \right] \]

(2)

and

\[ \Pi^\mu_\nu(p) = \frac{1}{i} \int \frac{d^4k}{(2\pi)^4} (-\delta_{ab}) \left( g_{\rho\sigma} - \frac{k_\rho k_\sigma}{m^2} \right) \left[ \frac{i}{k^2 - m^2} + 2\pi \delta(k^2 - m^2) n_B(|k_0|/T) \right] \times \\
(-i\epsilon^2) [\epsilon_{abc} \epsilon_{cd} (g^{\mu\rho} g^{\sigma\nu} - g^{\mu\sigma} g^{\rho\nu}) + \epsilon_{ace} \epsilon_{bde} (g^{\mu\rho} g^{\sigma\nu} - g^{\mu\sigma} g^{\rho\nu}) + \\
\epsilon_{ade} \epsilon_{bce} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma})] \right], \]

(3)

where \( m \equiv 2\epsilon|\phi| \) is the common mass of the two vector modes induced by the adjoint Higgs mechanism in the effective theory and \(|\phi| \equiv \sqrt{\Lambda^3/2\pi T}\). The relation between Yang-Mills scale \( \Lambda \) and critical temperature \( T_c \) for the deconfining-preconfining phase transition is \( \Lambda = \frac{2\pi}{13.87} T_c \) [2]. It was shown in [3] that \( \Pi^\mu_\nu \equiv 0 \) for \( p^2 = 0 \). We will see below that this continues to hold on the radiatively induced mass shell so that \( \Pi^\mu_\nu \) is always real.

Without constraining generality we assume \( p \) to be parallel to the z-axis. In this case, the screening function \( G(p_0(p), p) \), which enters the radiatively induced dispersion law as

\[ p^2(p) = p^2 + G(p_0(p), p), \]

(4)
is given as

\[ \Pi^B_{11} = \Pi^B_{22} = G(p_0, p). \]

(5)

The integration in Eq. (3) is subject to constraints imposed by the existence of the thermal ground state in the effective theory. Namely, the momentum transfer in the four-vertex of diagram B in Fig. 1 satisfies the following condition

\[ |(p + k)^2| = |G + 2k \cdot p + 4\epsilon^2|\phi|^2| \leq |\phi|^2, \]

(6)

where \( |\phi| = \sqrt{\Lambda^3/2\pi T} \). Notice that in a selfconsistent calculation of \( G \) on the radiatively induced mass shell in performing the square on the left-hand side of inequality (6) the following relations hold

\[ p^2 = G, \quad k^2 = 4\epsilon|\phi|^2, \]

(7)
where $e$ is the effective gauge coupling whose high-temperature value is given as $e = \sqrt{8\pi}$ \cite{17}.

In going over to dimensionless variables

$$y \equiv \frac{k}{|\phi|} \quad (8)$$

and by transforming to cylindrical coordinates \cite{8}

$$y_1 = \rho \cos \varphi , \quad y_2 = \rho \sin \varphi , \quad y_3 = \xi . \quad (9)$$

we have\textsuperscript{3}

$$\frac{G}{T^2} = \int d\xi \int d\rho \, e^2 \lambda^{-3} \left( -4 + \frac{\rho^2}{4e^2} \right) \rho \frac{n_B \left( 2\pi \lambda^{-3/2} \sqrt{\rho^2 + \xi^2 + 4e^2} \right)}{\sqrt{\rho^2 + \xi^2 + 4e^2}} , \quad (10)$$

where $n_B(x) = 1/(e^x - 1)$. In addition, we define $X \equiv |p|$. The limits of integration in Eq. (10) need yet to be determined in accord with Eq. (6). In cylindrical coordinates $\rho, \xi$ the support of the integration in Eq. (10) is the region where $\rho$ and $\xi$ satisfy one of the two following conditions

$$\left| \frac{G}{T^2} \lambda^3 \pm \frac{\lambda^{3/2}}{\pi} \left( \sqrt{X^2 + \frac{G}{T^2} \sqrt{\rho^2 + \xi^2 + 4e^2}} - X \xi \right) + 4e^2 \right| \leq 1 . \quad (11)$$

In Fig. 2 the integrand in Eq. (10) subject to the constraints in Eq. (11) is plotted for $\lambda = 2 \lambda_c$ and $\lambda = 4 \lambda_c$ where $\lambda_c = 13.87$ is the dimensionless critical temperature for the deconfining-preconfining phase transition signalled by a logarithmic pole in $\epsilon(\lambda)$ \cite{2, 4}. Through the conditions (11) the right-hand side of Eq. (10) becomes a function of $G$ in contrast to the approximate calculation where $p^2 = 0$. The strategy to determine $G$ at a given value of $\lambda$ in the full calculation thus is to prescribe a value for $G$ in (11), subsequently to compute the integral by Monte-Carlo methods, and to list this integral as a function of $G$. The final step is to determine the zero of left-hand side minus right-hand side of Eq. (10) to find $G$ selfconsistently. Numerically, we use Newton’s method for this task.

In Fig. 3 we compare plots of $\log \left| \frac{G}{T^2} \right|$ obtained in the full calculation with those of the approximation $p^2 = 0$, both as a function of $X$ and $Y \equiv \sqrt{X^2 + \frac{G}{T^2}}$ at $\lambda = 2 \lambda_c$, $\lambda = 3 \lambda_c$, and $\lambda = 4 \lambda_c$. Notice the saturation of $\frac{G}{T^2}$ in the full calculation to finite values as $X \to 0$. This is in contrast to the result obtained in the approximation $p^2 = 0$ and an important observation since it is likely to explain the quadratic rise of the spatial string tension with high temperature in terms of a radiative effect in the effective theory for deconfining SU(2) Yang-Mills thermodynamics \cite{9}. Notice

\textsuperscript{3}The contribution of the vacuum parts in the propagators of the two tree-level massive modes is excluded by the constraints on the maximal resolution in the effective theory, see \cite{2}.
Figure 2: The integrand in Eq. (10) subject to the constraints in Eq. (11) for $\lambda = 50$, $X = 0.8$, $\frac{G}{T^2} = -0.00146$, and $0 \leq \rho \leq 35$, $30 \leq \xi \leq 150$. This example illustrates the considerable constraining power of (11), here in the regime of antiscreening.

also the more rapid approach to zero of the critical value $Y^*$ for total screening (intersection point of $2 \log Y$ and $\log \frac{G}{2T}$) in the full calculation.$^4$ The regimes of screening (left of the zero of $\frac{G}{T^2}$) and antiscreening (right of the zero of $\frac{G}{T^2}$) are the same for the full and approximate calculation. Finally, let us investigate the high-temperature behavior of $Y^*(\lambda) = \frac{G}{T^2}(X = 0, \lambda)$ which cuts off the spectrum towards low frequencies. We fit the high-temperature behavior of $Y^*(\lambda)$ to a power-law model

$$Y^*(\lambda) = C\lambda^\nu, \quad (\lambda \gg \lambda_c),$$

(12)

where $C$ and $\nu$ are real constants. For the full (approximate) result we obtain $C \sim 20$ and $\nu \sim -3/2$ ($C \sim 2$ and $\nu \sim -2/3$), see also Fig.4. Thus the approach to abelian behavior is much faster in the full calculation as compared to the approximate result.

2.3 Check that $\text{Im}\, G = 0$

So far we have considered diagram B in Fig.1. Keeping in mind that only the thermal parts of the massive vector propagators contribute in diagram A $^2$ and by inspecting the analytic expression in Eq. (2), it is clear that if this diagram is finite then it necessarily is purely imaginary. Taking into account that $p^2 = G$, we define

$^4$Modes of frequency less than $Y^*$ do no longer propagate.
Figure 3: Plots of $\log |\frac{G}{T^2}|$ in the full calculation (solid grey curves) and for the approximation $p^2 = 0$ (dashed grey curves). The cusps in $\log |\frac{G}{T^2}|$ correspond to zeros separating the regime of screening ($G > 0$) from the regime of antiscreening ($G < 0$). The left panel depicts $\log |\frac{G}{T^2}|$ as a function of $X$. The right panel shows $\log |\frac{G}{T^2}|$ as a function of $Y \equiv \sqrt{X^2 + \frac{G}{T^2}}$. Here the dashed black curve is the function $2 \log Y$. In order of increasing lightness the curves correspond to $\lambda = 2 \lambda_c$, $\lambda = 3 \lambda_c$, and $\lambda = 4 \lambda_c$.

Figure 4: Plots of $Y^*(\lambda) = \frac{G}{T^2}(X = 0, \lambda)$ in the full calculation (solid line) and in the approximation $p^2 = 0$ (dashed line).
in dimensionless cartesian coordinates the following functions

\[ y_3(j) \equiv -\frac{\lambda^{3/2} T^2}{4\pi G} \left( -\frac{X}{T^2} + \frac{1}{2} \right) \left( \frac{X G}{T^2} \right)^2 + 4 \frac{G^2}{T^2} \left[ \frac{G^2}{4T^4} - 2\pi \lambda^{-3/2}(X^2 + \frac{G}{T^2})(y_1^2 + y_2^2 + 4e^2) \right] \],

\[ s(j) \equiv \sqrt{y_1^2 + y_2^2 + y_3(j)^2 + 4e^2} \],

\[ r(i, j) \equiv 2\pi \lambda^{-3/2} \left( -1 \right)^i s(j) \sqrt{X^2 + \frac{G}{T^2} - 2\pi \lambda^{-3/2} y_3(j)X} \],

where \( i, j = 0, 1 \). Then we have

\[
\frac{\Pi_{11}^A}{T^2} = \frac{\Pi_{22}^A}{T^2} = \frac{i e^2}{(2\pi)^2} \sum_{i,j=0}^1 \int_{-\infty}^{\infty} dy_1 \int_{-\infty}^{\infty} dy_2 \times \left\{ \begin{array}{l}
-2r(i,j) - \left( \frac{\lambda^{3/2}}{2\pi} \right)^2 \frac{r(i,j)^2}{4e^2} + 7 \frac{G}{T^2} - \left( \frac{\lambda^{3/2}}{2\pi} \right)^2 \left( \frac{G}{2eT^2} \right)^2 \\
48 \pi^2 \lambda^{-3} - 2\frac{r(i,j)}{4e^2} - 3 \frac{G}{4e^2 T^2} + \left( \frac{\lambda^{3/2}}{2\pi} \right)^2 \left( \frac{G}{4e^2 T^2} \right)^2 \end{array} \right\} \times \pi \frac{\pi}{2\lambda^{3/2} y_3(j) \sqrt{X^2 + \frac{G}{T^2}}} \right\} \right\} \times \left\{ \begin{array}{l}
\frac{\pi}{2\lambda^{3/2} y_3(j) \sqrt{X^2 + \frac{G}{T^2}}} \right\} \right\} \times \left\{ \begin{array}{l}
\frac{\pi}{2\lambda^{3/2} y_3(j) \sqrt{X^2 + \frac{G}{T^2}}} \right\} \right\} \times \left\{ \begin{array}{l}
\frac{\pi}{2\lambda^{3/2} y_3(j) \sqrt{X^2 + \frac{G}{T^2}}} \right\} \right\}
\]

and no further constraints need to be imposed. We have computed the integral in Eq. (14) employing Monte-Carlo methods and by using our result for \( G \) (determined by \( \Pi_{11}^B \)) for various temperatures \( \lambda \). We constantly obtain a vanishing result. Thus there is no finite photon width, and the problem of computing \( G \) is selfconsistently solved by computing \( \Pi_{11}^B \) only.

3 Modified black-body spectra

Here we would like to compare the implication of the full one-loop result for \( G \) for black-body spectra with that of the approximation \( p^2 = 0 \) under the postulate that photon propagation is described by the propagation of the massless mode in the deconfining phase of an SU(2) Yang-Mills theory (SU(2)\(_{\text{CMB}}\)) of scale \( \Lambda \sim 10^{-4} \text{eV} \) which corresponds to \( T_c = 2.73 \text{K} \) [2].
Figure 5: Plots of the dimensionless spectral intensity $I/T^3$ as a result of the full calculation (solid grey curves) and for the approximation $p^2$ (dashed grey curves). Solid black curves depict the conventional Planck spectrum for $T = 2T_c \sim 5.45 K$. Notice the excellent agreement with the result of the approximate calculation.

As was explained in [10, 19] the modified black-body spectral intensity $I_{SU(2)}(\omega)$ of the SU(2) theory is obtained from the conventional counterpart $I_{U(1)}(\omega)$ of the U(1) theory as follows

$$I_{U(1)}(\omega) \rightarrow I_{SU(2)}(\omega) = I_{U(1)}(\omega) \times \left( \omega - \frac{1}{2} \frac{d}{d\omega}(G) \right) \frac{\sqrt{\omega^2 - G}}{\omega^2} \theta(\omega - \omega^*)$$

where $\omega^* \equiv \sqrt{G(p = 0, T)}$, 

$$I_{U(1)}(\omega) \equiv \frac{1}{\pi^2} \frac{\omega^3}{\exp]\frac{\omega}{T}] - 1$$

and $\theta(x)$ is the Heaviside step function. In Fig.5 a comparison of the modification of low-frequency black-body intensities obtained in the full calculation and in the approximation $p^2 = 0$ is depicted for the temperature $T = 2T_c \sim 5.45 K$. Notice the excellent agreement of the approximate and the exact result. In Fig.6 this comparison is made for temperatures $T = 3T_c \sim 8.2 K$ and $T = 4T_c \sim 10.9 K$. Notice the faster approach to the Planck spectrum with rising temperature in the full as compared to the approximate calculation.

4 Summary and Conclusions

Let us summarize our results. In [8] the components of the polarization tensor of the massless mode (photon) were computed. These are relevant for the radiatively induced dispersion law in the effective theory for the deconfining phase of SU(2)
Figure 6: Plots of the dimensionless spectral intensity $\frac{1}{T^3}$ in the full calculation (solid grey curves) and for the approximation $p^2$ (dashed grey curves). Solid black curves depict the conventional Planck spectrum for (a) $T = 3 T_c \sim 8.2 K$ and (b) $T = 4 T_c \sim 10.9 K$. Notice the much faster approach with rising temperature to the Planck spectrum in the full as compared to the approximate calculation.

Yang-Mills thermodynamics. To make the calculation as analytically transparent as possible (resolution of kinematic constraints as imposed by the nontrivial thermal ground state) and also out of convenience, reference [8] has resorted to the approximation that the photon’s four momentum be on its free mass shell ($p^2 = 0$). Under the assumption that physical photon propagation is described by an SU(2) Yang-Mills theory of scale $\Lambda \sim 10^{-4}$ eV in [10] implications were investigated for the low-frequency part of low-temperature black-body spectra obtaining a sizable gap and a regime of sizable antiscreening.

The calculation of the photon’s polarization tensor as performed in the present paper no longer resorts to the above approximation. Rather, the incoming photon is now placed selfconsistently on its radiatively induced mass shell. For black-body spectra we obtain excellent agreement with the approximate calculation for temperatures up to $\sim 6$ Kelvin. At higher temperatures the one-loop exact shows a less dramatic effect than the approximate calculations suggests. Namely, the black-body spectrum in the low-frequency part of the antiscreening regime is less bumpy, and the spectral gap decreases faster with temperature.

Finally, notice that since results obtained in the approximation $p^2 = 0$ yield agreement with the full calculation up to $T \sim 6$ Kelvin and since in applications [11, 12] we see an effect only for temperatures up to this size the conclusions spelled out in these papers remain quantitatively valid.
Acknowledgments

The authors gratefully acknowledge fruitful discussions with Francesco Giacosa, Jochen Keller, and Markus Schwarz. We also would like to thank Markus Schwarz for his helpful comments on the manuscript.

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