An Analytical-Numerical Approach to Model and Analyse Squirrel Cage Induction Motors

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To the memory of Alessandro Costabeber.

Abstract—Nowadays, finite element analysis represents the most accurate tool to analyse electrical machines. However, the time domain resolution of electromagnetic problem, in some cases, requires long simulation time due to the induced nature of the currents. The computational burden increases when the machine features a skewed layout on the stator or rotor structures, since this requires 2D multi-slices approximated analysis or even a full 3D model. In this paper, a general analytical method to model electromagnetic devices is applied to a squirrel cage induction motor featuring a skewed rotor structure. The modelling approach is wisely implemented and adapted to pursue a fair balance between accuracy of the analysis and computational burden, taking advantage of all the symmetries existing in the rotor cage of the machine, aiming to minimize the model complexity. A comparative analysis in term of the inductances between analytical and finite element is proposed. The results provided by the model developed are compared with respect to the corresponding values provided by both finite element and experimental test performed on the reference machine. Such comparisons show that the proposed model is actually able to achieve a pretty good balance between accuracy and computational efficiency.

Index Terms—Analytical model, Inductances, Induction Motor, Numeric resolution, Rotor cage, Slotting effects, Winding function.

I. INTRODUCTION

Squirrel cage induction machines (SCIMs) have been very popular for a long time in a wide range of sizes thanks to several interesting features such as manufacturing simplicity, relatively low cost, robustness, potential for direct line supply, possibility to operate at relatively high temperature and easy adaptation for high speed operation [1]–[3]. In fact, SCIMs are still widely used for a lot of different applications, ranging from line-fed and inverter-fed motors for industrial and civil applications to propulsion drives for transportation and even to power generation [4]. Nevertheless, in many application fields, stringent requirements, especially in term of efficiency, are emerging: this is pushing the manufacturers to refine their designs to meet the expectations by better exploiting the mechanical and electromagnetic properties of the materials and by paying more attention to minimize the secondary phenomena that may affect unfavourably the performances, aiming to keep SCIMs competitive with respect to other solutions. In particular, the high frequency currents induced in the rotor cage due to the high order spatial harmonics exhibited by the stator windings represent an undesirable secondary effect which deteriorates the efficiency. These currents flow in the solid conductive bars and, due to skin and proximity effects, increase the AC resistance and therefore the Joule losses of the machine, deteriorating the efficiency. Furthermore, the interaction between the magnetomotive forces (MMFs) generated by the cage and stator windings produces asynchronous, synchronous and torque ripple components [5]. To reduce such parasitic effects produced by the non-ideal structure of the windings, SCIMs are often equipped with a skewed rotor, yet at the cost of an average torque reduction. Therefore, accurate models able to consider the interactions between stator and rotor MMFs are required to quantify the impact on the performance and intervene on the design to mitigate those parasitic phenomena. The most accurate tool available for a detailed investigation of the SCIM operation is transient-with-motion FEA However a systematic use of such tool is usually considered unpractical since investigating any specific steady-state condition may take a long computation time due to the necessity to await for the settling of the periodic currents induced in the rotor cage. When the machine is equipped with a skewed stator or rotor structure, an approximated multi-slice 2D-FEA or eventually a full 3D-FEA have to be used, further drastically increasing the computational burden, thus making such solution even less attractive. In fact, although accuracy is an important aspect, in practice, any simulation model has to feature an adequate computational speed to turn out useful during any industrial design process.

On the other hand, the most widespread methods to model SCIMs is the single-phase equivalent circuit (EC), which allows to predict the steady state performance at different operating conditions. However the analytical computation of the lumped parameters featuring the EC might leads to inaccuracy due to non-linearity of iron materials and eddy current phenomena in solid conductors. The time harmonic FEA has been demonstrated to be an effective tool to improve the characterization of the EC’s parameters related to the fundamental spatial harmonic. In [6] and [7] a simplified FE models to compute the EC parameters is presented. This approach consists in an iterative procedure where the electromagnetic problem is separated in two domains; a single stator slot and a single rotor slot. This allows low computational effort at the cost of a not full geometrical...
representation of the electromagnetic problem. Identification of the EC parameters are also proposed in [8]–[10] and [11] where FE models are built considering the natural minimum common periodicity of stator and rotor structures. Although further validation of the EC approach is proposes in [6], [7] and [12], limitations are exhibited when high order harmonics are taken into account. In literature are also reported multi-harmonic equivalent circuits where the lumped parameters are analyti-
cally computed per each harmonic order [13]. However, the calculation or separation of such parameters into their harmonic components by means of FEA has not yet been demonstrated to be an effective approach. Although improvements in the estimation of the lumped parameters of the EC accounting for the bell harmonic effects are considered in [14], the representation of the complex phenomena taking place inside SCIMs by means of a compact single-phase EC lack of details about the actual electromagnetic condition of the machine.

Alternatively to the EC model, a mathematical derivation of the model for a squirrel cage winding in dq-reference frame is presented in [15] and [16], considering the discrete distribution of rotor bars. More recently, a modelling method for SCIMs based on stator and rotor windings functions has been developed in [17] and increased accuracy is achieved in [18] and [19], accounting for the slot openings effects by means of permeance function implemented as reported in [20].

According to the previous considerations on the modelling techniques for SCIMs, the development of a computationally efficient time-stepping simulation model able to predict the interactions between time and space harmonics, including slotting effects, as well as to analyze skewed structures, is deemed to be potentially very interesting. In this work, the general analytical model [21] for the electromagnetic analysis of electrical machines featuring long axial length structure with respect to its diameter, is applied and tailored for the case of SCIMs. Such modelling method permits to calculate the inductances matrix, develops further theoretical considerations with respect to [22] and better take into account the peculiar structure of the rotor cage winding, the effects of slotting and the skewing. The efficient implementation of the proposed modelling approach is then addressed, focusing on achieving a suited balance between model accuracy and computational burden. A discussion of the model limitations is then presented, comparing significant results against experimental data obtained for the considered machine in the same conditions.

II. BACKGROUND

In [21] and [23], the general framework covering theoretical aspects concerning the modelling of a generic electromagnetic device is presented and used in this work. A first application for SCIMs of the above general model was presented in [22] with promising results; therefore, a similar approach is adopted also in this paper, developing further considerations aimed to improve its implementation and to keep into account further features such as slot openings and skewing. The main aspects of the modelling approach adopted are briefly recalled in this section for the sake of clarity.

A compact form of the main equations governing the electromechanical energy transformation can be then represented in a matrix form obtained by piling in a vector the quantities related to stator and rotor structure. The system of equations describing the terminals voltage \( \bar{v} \) of each equivalent stator and rotor phases and their current \( i \), is expressed as

\[
\bar{v} = \bar{R} \bar{i} + \bar{M}(\alpha, \bar{i}) \frac{d\bar{x}}{dt} + \bar{L}(\alpha) \frac{d\bar{i}}{dt}
\]

(1)

where \( \bar{R} \) is the resistance matrix and \( \bar{L}(\alpha) \) is the inductance matrix, which depends only on the position \( \alpha \) under the assumption of linear magnetic behaviour of the device. This may be achieved either when all the materials operate in their linear region or approximately when the MMF drop inside the ferromagnetic regions composing the cores are negligible with respect to the MMF drop across the layers of linear materials. The motional coefficient vector \( \bar{M}(\alpha, \bar{i}) \) is obtained as reported in (2).

\[
\bar{M}(\alpha, \bar{i}) = \frac{\partial \bar{L}(\alpha)}{\partial \alpha} \bar{i}
\]

(2)

The expression of the inductance matrix related to the main flux tubes (i.e. those crossing the airgap) and linked with the phases is expressed as

\[
\bar{L}(\alpha) = \ell \int_0^1 \mu_e(\lambda, \alpha) \bar{N}_e(\lambda, \alpha) \bar{N}_e^T(\lambda, \alpha) d\lambda
\]

(3)

where the coordinate \( \lambda \) represents the normalized tangential position along the air gap mapping the interval \([0,1)\), \( \ell \) is the axial length of the motor, \( \mu_e(\lambda, \alpha) \) and \( \bar{N}_e(\lambda, \alpha) \) are the equivalent permeability and winding functions vector, whose expressions are reported in (4) and (5), respectively.

\[
\mu_e(\lambda, \alpha) = \mu_0 \frac{\tau_\theta(\lambda, \alpha)}{\epsilon_\theta(\lambda, \alpha)}
\]

(4)

\[
\bar{N}_e(\lambda, \alpha) = \bar{N}(\lambda, \alpha) - \int_0^1 \int_0^1 \mu_e(\lambda, \alpha) d\lambda \bar{N}(\lambda, \alpha) d\lambda
\]

(5)

The generic function that belong to the vector \( \bar{N}(\lambda, \alpha) \) describes the spatial distribution of the active sides of the stator and rotor windings while the equivalent permeability function \( \mu_e(\lambda, \alpha) \) allows to account for the anisotropic conformation of the air gap. In (4), the function \( \epsilon_\theta(\lambda, \alpha) \) represents the length of the flux lines between the stator and rotor peripheries while the function \( \tau_\theta(\lambda, \alpha) \) defines the profile of the medium magnetic-equipotential surface in the air gap for every position \( \alpha \) along the geometric tangential coordinate \( \lambda \). The electromagnetic torque is obtained through the derivative of the coenergy with respect to the Lagrangian position \( \alpha \) as reported below

\[
W_E(\alpha, \bar{i}) = \frac{\partial C_E(\alpha, \bar{i})}{\partial \alpha} = \frac{1}{2} \bar{i}^T \bar{M}(\alpha, \bar{i})
\]

(6)

Considering that the whole set of windings is divided in stator and rotor sets, the vectors of currents, fluxes and voltages can
be split in two sections as follows
\[ \bar{\psi}(t) = \begin{bmatrix} \bar{\psi}_s(t) \\ \bar{\psi}_r(t) \end{bmatrix}, \quad \bar{v}(t) = \begin{bmatrix} \bar{v}_s(t) \\ \bar{v}_r(t) \end{bmatrix}, \quad \bar{i}(t) = \begin{bmatrix} \bar{i}_s(t) \\ \bar{i}_r(t) \end{bmatrix} \] (7)
The inductance and resistance matrices may be then divided as follows:
\[ \mathbf{L}(\alpha) = \begin{bmatrix} L_{ss}(\alpha) & L_{sr}(\alpha) \\ L_{rs}(\alpha) & L_{rr}(\alpha) \end{bmatrix}, \quad \mathbf{R} = \begin{bmatrix} R_{ss} & 0 \\ 0 & R_{rr} \end{bmatrix} \] (8)

III. ROTOR CAGE MODEL

The structure of a squirrel cage winding consists in a number \( b \) of parallel conductors (bars) connected at both ends through shared low resistance paths (rings). Therefore, while the stator phases are eventually composed of sections each one consisting of simple conductive paths with well identified terminals, at a first view the rotor conductive paths are not easily recognised as proper phases due to their articulated structure. This means that a good SCIM model requires an appropriate representation of the rotor cage. To such purpose, a useful topological representation of the cage can be obtained as in Fig. 1 where the thick and solid rotor bars conductors are idealised with thin paths for the induced currents and the whole structure is sketched in planar form, having the two rings shown as the internal and external circles. The idealised conductor approximation leads to neglect the phenomena that characterise solid conductors, such as the non-uniform spatial distribution of current density across the section and the partial linking of the leakage fluxes. According to the graph theory applied to the above circuitual representation, the number of equivalent current elements needed to coherently described the problem is equal to the number of natural loops that can be easily identified. However, this results equal to the number \( b \) of external loops delimited by adjacent bars (bar loops) plus the inner loop related for example to the front ring (ring loop). Considering that each loop is a simple closed curve, from an electromagnetic point of view the squirrel cage can be then represented as a set of \( b+1 \) equivalent phases in which the above loop currents flow as they are short-circuited. However, unless very unusual operating condition occurs, the loop current associated to the ring can be considered negligible: therefore, \( b \) is the number of phases necessary to fully represent the cage.

A. Resistance Matrix

According to the chosen topological representation, the sub-matrix of the rotor resistances \( \mathbf{R}_R \) feature a peculiar non-diagonal structure, due to the fact that the equivalent phases share parts of their physical path. In fact, as show in Fig. 1, when the bars and bar loops are labelled with consecutive numbers, bar \( k \) will be shared between the two adjacent loops \( k, k+1 \) while each sector of the front ring would be shared between a bar loop and the ring loop. Assuming to neglect the later, the resistance sub-matrix \( \mathbf{R}_R \) turns then out to be a quasi three-diagonal structure as shown in (9) where \( R'_m = -R_m \), \( R_d = 2(R_m + R_{ring}) \), \( R_m \) is the resistance of each bar and \( R_{ring} \) is the resistance of each ring sector.

\[ \mathbf{R}_R = 2\rho \begin{bmatrix} R_d & R_m & R'_m \\ R_m & R_d & R_m \\ R'_m & R_m & R_d \end{bmatrix} \] (9)

In reality, the value of the above resistances would be actually affected by the operating conditions due to the bulk nature of the cage bars and rings. Nevertheless, the bars are located within the slots whereas the rings are only adjacent to the rotor core by one side. This means that the rings are affected by non-uniform current distribution much less than the bars: therefore, as a first approximation the value of the resistance of each ring sector can be estimated as in (10)
\[ R_{ring} = \rho \frac{\pi D_{ring}}{b S} \] (10)
where \( \rho \) is the electrical resistivity while \( D_{ring} \) and \( S \) are diameter and cross section of the ring, respectively.

B. Inductance Matrix

Interesting considerations about the inductance matrix components related to the rotor structure (8) can be drawn applying the Gauss Law in its integral form to a closed surface generated by joining the elementary surfaces single out by the bar loops (Main portion). This give rise to a quasi-cylindrical surface, and the two quasi-circular plain surfaces delimited by the rings (Caps portions)
\[ \nabla \cdot \mathbf{B} = 0 \rightarrow \int \int_{Main} \mathbf{B} \cdot \hat{n} dS + \int \int_{Caps} \mathbf{B} \cdot \hat{n} dS = 0 \] (11)
The flux crossing the Main surface can be expressed as in (12)
\[ \int \int_{Main} \mathbf{B} \cdot \hat{n} dS = \sum_k \psi_{rk} = \mathbf{1}^T \bar{\psi}_r \] (12)
As a first approximation, the fluxes crossing the Caps portions could be apparently neglected. Nevertheless, considering (11) and (12), such assumption would imply \( \mathbf{1}^T \bar{\psi}_r = 0 \) meaning that the fluxes linked to the considered equivalent rotor phases turn out to be not linearly independent. Therefore one should conclude that in this way the matrix \( \mathbf{L}(\alpha) \) would turn out to be...
singular for any $i$ and $\alpha$, as
\[
\mathbf{i}^T \left[ \mathbf{L}_{rr}(\alpha) \right] \mathbf{i} = 0 \quad \forall \ (i, \alpha) \tag{13}
\]

Such modelling approach would make impossible to analyse the dynamic behaviour of rotor equivalent phase currents when only stator supply is imposed, as usual. Therefore, the full circuitual representation of an electrical machine equipped with a cage, must account for the inductive effects of end connections (leakage inductances) in order to make the model useful in most of cases. As a second level of approximation, the related leakage fluxes can be considered as mainly due to the self-inductance effect of the currents flowing in each ring sector delimited by adjacent bars, i.e. the mutual inductance effect may be neglected. The self-inductance of each ring sector may be estimated in different ways (e.g. [24]); for the scope of this paper, it was determined through FE analysis [25], obtaining the value $L_{ring} = 2.7 \cdot 10^{-7} [H]$ which was then taken into account within the matrix $\mathbf{L}_{rr}(\alpha)$.

C. Rotor skewing

The stator or rotor skewing is an effective method to reduce high order harmonics induced in both cage and stator windings. The ferromagnetic structure is axially twisted along the motor axis without changing the shapes of the cross section. For this reason, the estimation of the performance of skewed motor by means of FEA is a heavy task from a computational point of view thus requiring 3D-model or alternative 2D-model featuring an axial discretisation of the problem in multi-slices. The latter is proved to be effective and less time consuming than full 3D problem, but it is affected by not negligible errors unless an high number of slices is chosen. Since the skewing rate is usually a fraction of the slot pitch, the magnetic conditions at each cross section of the machine along the axial direction will not be much different from the 2D scenario considered in the analytical model recalled in Section II: therefore, a quasi-2D field map is expected to depend on the axial position, whereas the currents will be the same in all of the sections of each bar or stator phase. The general results recalled in Section II may be adapted to approximately analyse also a skewed machine by making all the descriptive functions depending on a linear coordinate spanning along the machine axis in the interval: $z \in [0, \ell]$. This leads to modify (3) into the general expression (14) for the inductances related to the main flux tubes:
\[
\mathbf{L}(\alpha) = \int_0^\ell \int_0^1 \mu_0 \mathbf{N}(\lambda, z, \alpha) \mathbf{N}^T(\lambda, z, \alpha) d\lambda dz
\tag{14}
\]

The above expression can be approximated in discrete form by considering a finite number of slices $n_S$ each one intended to represent the average contribution of an axial portion of the machine having length equal to $l_i$. Assuming that only the rotor is skewed, as usual in squirrel cage machines, the global inductance can be estimated according to (15)
\[
\mathbf{L}(\alpha) = \sum_{i=1}^{n_S} \frac{l_i}{\ell} \mathbf{L}(\alpha - \theta_{sk, i})
\tag{15}
\]

where $\theta_{sk, i}$ is the skew angle of rotor in the direction of the coordinate $\alpha$ at the axial position of slice $i$ and $l_i$ is the length of the rotor portion attributed to such slice. In this paper such approach has been used considering
\[
\theta_{sk, i} = \frac{i - 1}{n_S - 1} \theta_{sk} \quad i = 1...n_S
\tag{16}
\]
\[
l_i = \frac{l}{n_S - 1} \quad i = 2...n_S - 1 \quad l_1 = l_n_S = \frac{l}{2(n_S - 1)}
\tag{17}
\]

where $\theta_{sk}$ is the total skew angle and the slices are assumed to be equally spaced with the first and the last one located respectively at the beginning and end of the machine active length. Thanks to the lower computational burden involved in the use of the analytical model with respect to the 2D FE analysis, the implementation of such approach permits to increase the number of slices aiming to improve the results accuracy.

D. Model Reduction in Case of Symmetries

The stator usually features a symmetrical structure according to the number of pole pairs $p$. Moreover, often the squirrel cage features a fully symmetrical structure with identical bars evenly spaced and connected by regularly shaped rings. When it happens that $b = pk$ with $k \in \mathbb{N}$, i.e. there is an integer number for each pole pair, one should expect that the currents flowing in the cage also feature a symmetrical pattern according to the pole pairs symmetry. Furthermore, sometimes it happens that the number of bars per pole pair is even: in such case, one should expect that the currents flowing in any pair of bars displaced by one pole pitch have opposite waveforms. In such condition, the equivalent circuital modelling of the cage can be simplified by enforcing the above constrains among currents. This may be achieved by connecting in series all of the equivalent loop phases located 2 pole pitches apart, and eventually in anti-series all of the loop phases displaced by 1 pole pitch. In fact, as shown in Fig. 1, the current flowing in loops located at a pole pitch (i.e. the one indicated in blue) will be crossed by the same current with opposite sign while the current flowing in loops located at a pole pair pitch will be equal. This property thus permits to virtually connect in series and anti-series loops with the same current module of amplitude as shown in Fig. 2. The number of variables can be therefore drastically reduced from $b$ to $b/(2p)$, thus simplifying the model complexity and the computational effort required. It is worth to note that, as already mentioned...
in [26], the new version of rotor winding functions shown in
Fig. 3 permits to nullify the whole set of mutual-inductances
between rotor phases leading to a rotor matrix inductances which
assumes a diagonal structure. Such simplification in addition to
reduce the model complexity, allows the matrix $$L_{rr}(\alpha)$$
to be
not singular due to the fact that only the main diagonal hosts
elements different from zero. Therefore, the calculation of the
ring inductance necessary to invert the matrix $$L(\alpha)$$ in case of
a full cage model as reported in Section III-B is deceived when
a simplified model exploiting symmetries and anti-symmetries
is adopted yet allowing a straightforward comparison against a
2D-FE simulation.

IV. NUMERICAL IMPLEMENTATION

The analytical model is implemented numerically in Matlab-
Simulink platform. The stator and rotor windings representation
and the modelling of the slotting effects necessary to accurately
predict the performances of the machine are reported in this
Section IV-A and IV-B. The enhancements made with respect
to [23] are kept general to be suitable for any combination of
stator and rotor slots number and different angles of skewing.

A. The Winding Functions

An accurate implementation of the winding functions (5)
permits to represent the multi harmonic nature of the MMFs. In
[23], the circuital modelling technique is restricted to electrical
machines featuring phase windings hosted in slots with small
openings. In case of stator or rotor layouts with not negligible
slot openings dimension, the conductors can be considered
uniformly distributed along the opening width as represented in
Fig. 4 where, the profile of the obtained updated WF plotted in
red differs from the ideal case standard WF, draw with blue line.
This modification directly impact on the harmonics components
introduced in the model improving the results accuracy.

B. Slotting Effects

The modelling of the slot openings is accounted in the
computation of the inductances matrix (3), the next task consists in
identifying the leakage inductance mainly due to those flux lines
that cross the slots and the corresponding slot openings. Such
computation is performed by means of the simplified and linear
FE models reported in Fig. 6a) and b), where a single rotor and
stator slot are respectively modelled.

D. Model Resolution

The induced nature of the rotor currents requires a numerical
resolution of the analytical model. Therefore, the electromagnetic
system of equations reported in (1), is implemented in
Matlab-Simulink platform. The block-diagram representation is shown in Fig. 7. The matrix of the inductances is calculated off-line and organized in a look-up table. The input variables are the time evolution of the rotor position \( \alpha(t) \) and the vector of the imposed voltages \( \bar{v}(t) \); the vector of the currents are the outputs. The Y-connection of stator windings of the real platform, constrain the currents to an empty sum. This is emulated by the red blocks shown in Fig. 7 where \( R_0 \) is selected to minimize the error.

V. MODEL VALIDATION AND LIMITATIONS

The analytical-numerical modelling approach proposed in this paper is validated for a medium size 3-phase, 50 [Hz], SCIM with a rated power of 11 [kW], featuring 2 pole pairs, distributed single layer stator windings and equipped with a rotor cage consisting of a set of 28 short circuited rotor bars skewed by \( \theta_{sk} \) electrical degrees. A peculiarity of the motor under investigation is that the bars are hosted in closed slots. However, to be consistent with the model assumptions [23], the rotor bridges are considered opened. This is justified by the fact that the electromagnetic saturation of the bridges makes the relative permeability of the material to drastically drop. The tuning of the analytical model is performed thorough selection of the slot opening width in order to match the leakage inductance value of the close bridges configuration at rated operating condition. This is done by means of the simplified FE model reported in Fig. 6a). Deviations in the predicted results in term of stator current waveform and steady state torque, respect to FE results and test are highlighted and commented in this section. The analytical results are firstly compared against the FEA for \( 0 < s < 1 \) with \( s \in \mathbb{R} \). To highlight potentialities and limits of the proposed model, the predicted performance analytically computed are compared against four different FE models each one featuring different peculiarities.

In Table I the characteristics featuring the FE models are summarized. Open (A-B-C) and closed (D) rotor bridges are considered in order to assess the impact of the slot openings functions. The model featuring idealised conductor (A) is compared with the solid bar ones (B-C-D) in order to account for secondary phenomena due to the skin effect. The impact of the material non-linearities (C-D) is assessed with respect to the model featuring magnetically linear material behaviour (A-B). The analytical model developed in the previous section results close to Model A which is used for validation. The discrepancies that arise comparing the numerical-analytical solutions against the models B, C, and D is important to quantify the impact of the effects which are not included in the analytical model.

As final validation test results of the motor with skewed rotor cage are compared against the analytical model.

A. Inductances Identification

The inductances variation with respect to the rotor position \( \alpha \) computed analytically are validated in the following. The Model A is considered for the comparison and solved as 2D Model in the time-stepping domain. Fig. 8 presents the results where a fairly good match is highlighted for the sample inductances components selected. The rotor skewing impacts over the whole set of inductances therefore, considering a constant skewing angle of \( \theta_{sk} = \tau_p = 1/36 \), a time-stepping multi-slice FEA, modelling the machine with 3 slices is performed. It is important to notice that the number of slices selected to represent the skewing might affect the harmonics components of the inductances. For this reason, the analytical expression (15) is implemented for \( n_s = 3 \) to match the slice number of the FE model. A fairly good match is exhibited in Fig. 8 where \( \theta_{sk,i} = -\theta_{sk}/2 \) for \( i = 1 \). An high number \( n_s \) is selected in the analytical computation in order to closely replicate the integral form (14). The results are shown with black line and compared in Fig. 8.

B. FE Validation

The analytical-numerical model is solved on a Intel Xeon CPU with clock frequency of 3.5 GHz and 128 GB of RAM and the results are compared against the correspondent FE models, namely Model A and Model B. The stator current waveforms and the steady state torque are the benchmark quantities considered.
for the validation. A very accurate discretisation of a complete rotational period is obtained by dividing the coordinate $\lambda$ in 25200 divisions. Due to the simplified cage model introduced, the machine description is based on a state variable vector $i$ whose dimension can be shrunk from $3 + 28 = 31$ to $3 + 7 = 10$. Stator current waveforms are compared for $s = 0.02$ and $s = 0.04$ and reported in Fig. 9. The current waveforms predicted by the analytical model show an excellent match with respect to the idealized Model A with stranded bars in both the operating conditions, thus confirming the good representation of the inductances reported in Fig. 8. The steady state electromagnetic torque obtained numerically by applying (6) at different slip values is reported against the FE computation of Model A and Model B, in Fig. 10. The solid rotor bars featuring the cage of Model B affect the output torque, mainly due to the rotor resistance matrix which becomes a frequency-dependant quantity. A correction factor $\kappa_{e,c}(f_{e,r})$ for the rotor resistance (9) is therefore introduced in order to account for the skin effect. This factor is computed by means of the simplified FE model reported in Fig. 6a). In this way the resistance is obtained as function of the fundamental frequency of the rotor current. The good agreement of the curves reported in Fig. 10, highlights how the proposed model allows to account for the stator and rotor harmonics interaction producing torque components together with the skin effects of the rotor bars for the operating points considered, yet remarkably reducing the computational burden with respect to the FE approach.

### C. Model Limitations

The limitations of the analytical method are emphasised when comparing it with respect to models featuring non-linear magnetization curves. The harmonic spectrum deviation of the stator current with respect to the Model C in Fig. 12 is limited to few harmonic orders as the magnetic cores of the SCIM are dimensioned to operate below the saturation knee point. The discrepancy is however enhanced considering the effect of closed rotor slots in combination with the non-linear material.
behaviour as in Model D. The severe saturation that occurs in the bridges is highlighted in Fig. 11 and reflects into the peaks of specific harmonics (i.e. $6h \pm 1$ with $h \in \mathbb{N}$) of the stator current. The saturation of the bridges plays an important role in the dependency of the leakage flux with respect to the current, in the containment of the current magnitude during the starting condition, in the reduction of the magnetizing current and high frequency losses at no-load, and consequently in the efficiency and power factor improvements achieved [27]. Moreover, the bridge saturations change location and magnitude with respect to the rotor position, slip value and magnetic load, thus leading to a dissimilar behaviour of the cage with respect to the models with opened slots. Considering the above, the functions (19) and (18) modelling the equivalent opening dimensions of the slots would require aperiodic waveforms that change according to the rotor position and bar currents. Such model would imply a further identification of the inductances with respect to positions and currents that it will definitely complicate the resolution and it will lead the modelling and analysis beyond the aim of this manuscript. Given the rated rotor current, the equivalent slot opening of the rotor cage is implemented analytically considering a width opening dimension such as the leakage inductance matches the one with closed bridges. The model tuned in this way, by means of the simplified model in Fig. 6a), guarantees good agreement of SCIM’s output at low slip values as shown in Fig. 13, when small deviations of the bridge saturation is contained with respect to the reference case at nominal slip. However, the analytical steady state torque in Fig. 13 exhibits a discrepancy for slip values higher than $s = 0.1$. This is mainly due to the non-linearity of the soft magnetic materials as proven by the very similar behaviour exhibited by Model C (open slot) and Model D (closed slots) in Fig. 13. Although, a mismatch of the analytical resolution is exhibited in the steady state torque at high slip value reported Fig. 13, the model presented is capable to fairly represent the trends of the asynchronous and synchronous torques due to the high order harmonics interactions at low rotor speeds.

D. Skewed Model and Experimental Validation

The experimental platform consists of a DC machine acting in driving mode coupled through a torque meter with the SCIM under test. The test rig set up is shown in Fig. 14.

Stator current waveforms are measured at different mechanical power and compared against the analytical results. The SCIM under test is equipped with a skewed rotor structure and therefore, both analytical and FE models are updated to account for skewing considering $n_S = 300$ and $n_S = 5$ number of slices, respectively. In Fig. 15 the analytical stator current spectra at three different mechanical operative points are compared against multi-slice FE model and experimental test results. Although the analytical model proposed is based on the assumption of linear behaviour of the iron materials, the overall harmonic amplitudes show a fairly good match. However, the harmonic amplitudes discrepancy for $6h \pm 1$ harmonic orders, due to the asymmetric rotor bridges saturation, confirm again the limit of the analytical approach in predicting particular harmonics as already mentioned in Section V-B.
The phenomena due to the MMF harmonics interaction are successfully predicted. At high slip values the synchronous and asynchronous torque occurred and the effects of a skewed rotor in mitigating them, can be studied with reduced computational time with respect to FE analysis. The flexibility guaranteed by the modelling approach allows to investigate different motor configurations. The model proposed is a good candidate to be employed for implementing control strategies that require accurate models, featuring multi-space harmonic orders.

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