Analogy Based Valuation of Currency Options

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Abstract

What happens when the anchoring and adjustment heuristic of Tversky and Kahneman (1974) is incorporated in currency option models? Surprisingly, it generates the peculiar features of currency smiles within the Black-Scholes framework, while adding power to stochastic volatility and jump diffusion models. Anchoring versions converge to corresponding Black-Scholes, stochastic volatility, and jump diffusion models if adjustments to underlying currency risks to get to option risks are correct or if uncovered interest-rate parity holds. Anchoring predicts that the slope of the smile is positively related to recent spot trend, whereas curvature is positively related to diversity of sentiment. Empirical evidence supports these predictions.

Keywords: Anchoring, Currency Options, Black-Scholes, Stochastic Volatility, Jump Diffusion, Risk Reversals, Sentiment

JEL Classification: G13, G12, G11, G02
Analogy Based Valuation of Currency Options

What happens when the anchoring and adjustment heuristic of Tversky and Kahneman (1974) is incorporated in currency option models? Surprisingly, it generates the peculiar features of currency implied volatilities within the Black-Scholes framework\(^1\), while adding power to stochastic volatility and jump diffusion approaches.

The key peculiar feature of implied volatilities in currency markets is the behavior of risk-reversals (Out-of-the-money call implied volatility of given delta minus the out-of-the-money put implied volatility of the same delta). Risk-reversal is high in bullish (underlying currency expected to appreciate) and low in bearish (underlying currency expected to depreciate) regimes. Hence, the implied volatility smile is similar to a forward-skew when the underlying currency sentiment is predominantly bullish, it looks like a reverse-skew when the underlying currency sentiment is bearish, and the smile is symmetric when the sentiment is generally neutral (see Derman (2008)).

This is in sharp contrast with equity index option where only the reverse-skew is typically observed. Unsurprisingly, risk-reversals are closely watched and reported by market professionals as a measure of market sentiment.\(^2\)

Even though market professionals read sentiment from the shape of the smile, there is no direct role of investor sentiment in currency option models. I show that adjusting the currency option pricing models for the anchoring and adjustment heuristic of Tversky and Kahneman (1974) creates a direct role for investor sentiment.

Achilles heel of stochastic volatility models is that they require implausibly high parameter values (high volatility of volatility, and correlation parameters) to match the observed skew (Bakshi, Cao, and Chen (1997), Bates (1994a), Bates (1994b)). Similarly, a key weakness of jump diffusion models is that they require implausibly large jump intensity to generate the observed skew (Jackwerth (2000)). Anchoring generates all three types of smiles (symmetric, forward-skew, and reverse-skew) within the geometric Brownian motion framework. The role of increasingly complex distributional assumptions is that they amplify the smile seen in the

\(^1\) Garman and Kohlhagen (1983) extend the Black-Scholes model to currency options. For simplicity and ease of reference, I refer to Garman and Kohlhagen extension of the Black-Scholes model as the Black-Scholes model in this article.

\(^2\) [https://www.dukascopy.com/fxcomm/fx-article-contest/?Using-Implied-Volatility-As-An=&action=read&id=1788&language=en](https://www.dukascopy.com/fxcomm/fx-article-contest/?Using-Implied-Volatility-As-An=&action=read&id=1788&language=en)

http://www.gfmi.com/sites/default/files/2014%20Risk%20Reversals.pdf
simplest framework. Hence, with anchoring, the smile requires smaller parameter values. Furthermore, anchoring approach links the underlying currency sentiment to the slope and curvature of the smile. Empirical evidence strongly supports these predictions.

Starting from Tversky and Kahneman (1974), over 40 years of research has demonstrated that while forming estimates, people tend to start from what they know and then make adjustments to their starting points. However, adjustments from such self-generated starting points typically remain biased towards the starting values (see Furnham and Boo (2011) for a general review of the literature). Describing the anchoring heuristic, Epley and Gilovich write (2001), “People may spontaneously anchor on information that readily comes to mind and adjust their response in a direction that seems appropriate, using what Tversky and Kahneman (1974) called the anchoring and adjustment heuristic. Although this heuristic is often helpful, the adjustments tend to be insufficient, leaving people's final estimates biased towards the initial anchor value.” (Epley and Gilovich (2001) page. 1).

A few examples illustrate this heuristic quite well. When respondents were asked which year George Washington became the first US President, most would start from the year the US became a country (in 1776). They would reason that it might have taken a few years after that to elect the first president so they add a few years to 1776 to work it out, coming to an answer of 1778 or 1779. George Washington actually became president in 1789, implying that, starting from 1776, adjustments are typically insufficient. The adjustments from self-generated starting values are insufficient because of the tendency to stop adjusting once a plausible value is reached (Epley and Gilovich (2006)). Another example is the freezing temperature of vodka. People typically know that vodka freezes at a temperature below the freezing point of water (0 Celsius). So, when asked about the freezing point of vodka, they tend to start from the freezing point of water and then adjust downwards to form an estimate. However, their adjustments do not go far enough, and fall short of the right answer (which is -24 Celsius).

Where would you start if you need to form a risk judgment about a given call option? One expects the risk of a call option to be related to the risk of the underlying asset. In fact, a call option creates a leveraged position in the underlying asset. Hence, a natural starting point is the risk of the underlying asset, which needs to be scaled-up. Defining $\sigma(r_s)$ as the standard deviation of underlying currency returns, and $\sigma(r_c)$ as the standard deviation of call returns, one expects the following to hold:

$$\sigma(r_c) = \sigma(r_s)(1 + A)$$  \hspace{1cm} (0.1)

where $(1 + A)$ is the scaling-up factor.
The Black-Scholes model specifies a particular value for $A$, which is equal to $\Omega - 1$ where $\Omega$ is the call price elasticity with respect to the underlying currency exchange rate. In stochastic volatility and jump diffusion approaches, the correct value of $A$ is also equal to $\Omega - 1$, as long as diffusive risk is the only priced factor. Anchoring and adjustment heuristic implies that starting from the risk of the underlying asset, the adjustments to get to the risk of a call option are insufficient. That is, $A < \Omega - 1$. In other words, with the anchoring bias, $A = m(\Omega - 1)$, where $0 \leq m < 1$.

Under the Black-Scholes assumptions, any value of $A$ different from $\Omega - 1$ creates a risk-less arbitrage opportunity. Under the stochastic volatility and jump diffusion approaches, deviations from correct value of $A$ do not generally create risk-less arbitrage opportunities as perfect replication of the option is not possible.

If one allows for a little ‘sand in the gears’ in the form of transaction costs while keeping the other assumptions of the Black-Scholes model the same, a whole range of values of $A$ different from the Black-Scholes specified value become plausible. That is, it becomes possible to support incorrect beliefs in equilibrium because such beliefs cannot be arbitrated away. In fact, among options, currency options tend to have the largest transaction costs as they are largely OTC instruments created in the inter-bank market.  

As the volatility of call returns and the underlying currency returns are related in accordance with equation (0.1), their (net) returns must also be related as follows:

\[
E[r_c] = E[r_s] + r_F + A \cdot (E[r_s] + r_F - r_D) \tag{0.2}
\]

where $r_F$ and $r_D$ are the foreign and domestic risk-free returns respectively, $E[r_s]$ is the expected return from the underlying currency, and $E[r_c]$ is the expected return from the call option.

Anchoring implies that $A$ is lower than $\Omega - 1$, a prediction that can be tested in controlled laboratory experiments. In a series of experiments, Siddiqi (2012) finds:

1) The call average return is so much less than the Black Scholes prediction that the hypothesis that a call option is priced by equating its return to the return available on the underlying asset outperforms the Black-Scholes hypothesis by a large margin. The call average return is always

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3 See chapter 2 in DeRosa (2011)
4 The derivation is discussed in section 1.
larger than the underlying asset average return; however, it always remains far below the Black-Scholes prediction suggesting that anchoring might be taking place.

2) If the similarity between the payoffs of call and the underlying asset is reduced, let’s say by increasing the strike price, then the statistical performance of the hypothesis, \( E[r_c] = E[r_s] \), weakens. That is, with weakly similar payoffs, participant price a call option in such a way that a large distance is allowed between \( E[r_c] \) and \( E[r_s] \). With anti-similar payoffs (such as that of a put option and the underlying asset), the hypothesis, \( E[r_{option}] = E[r_s] \) performs poorly. This suggests that analogical anchoring gets weaker as the analogy between the corresponding call and underlying asset payoffs gets weaker, with the effect disappearing when the payoff similarity disappears.

The experimental evidence in Siddiqi (2012) suggests that anchoring bias directly influences the price of a call option. There is no evidence of anchoring directly influencing the price of a put option as put and underlying asset payoffs are anti-similar. Hence, the perception of similarity between a call option and its underlying asset appears to be the driving mechanism here. It is worth mentioning that similar types of situational similarities have been used in the psychology and cognitive science literature to test for the influence of self-generated anchors on judgment (see Epley and Gilovich (2006) (2001) and references therein).

The contribution of this article is to adjust the standard currency option pricing models for anchoring. Surprisingly, I find that anchoring-adjusted option pricing model generates the peculiar features of implied volatility smile in currency markets. Furthermore, key predictions of the anchoring-adjusted model have strong empirical support.

This article is organized as follows. Section 1 derives option pricing formulas when the anchoring bias is combined with Black-Scholes, stochastic volatility, and jump diffusion models. Section 2 shows that the anchoring framework explains the peculiar features of the smile in currency markets. Section 3 discusses key predictions of the anchoring model in the light of empirical evidence. Section 4 concludes.

\[ \textit{Siddiqi (2015a) adjusts equity option pricing models for anchoring and finds that anchoring provides a unified explanation for key option pricing puzzles in equity markets. Siddiqi (2015b) and Siddiqi (2015c) extend the anchoring approach to CAPM and CCAPM respectively. The ability of anchoring to explain a wide variety of phenomena across financial markets suggests that it may be the unifying theme in finance.} \]
1. Anchoring in Currency Markets: The Basic Framework

In this section, I present the basic framework without assuming a particular distribution for the underlying currency returns. Option pricing formulas in continuous time with geometric Brownian motion are derived in section 1.1. Section 1.2 derives the anchoring-adjusted Hull and While stochastic volatility formula. Section 1.3 derives the anchoring-adjusted Heston stochastic volatility formula. Section 1.4 presents the anchoring-adjusted Merton jump diffusion PDE. Section 1.5 adjusts the Bates model for anchoring.

As a start, I consider a one-period situation with only two points in time, now and the future. (Mathematically, the first-order conditions from a multi-period version must decompose anyway into an overlapping sequence of first-order conditions from the two-points-in-time model. However, such details are not needed for the main message of this article, so I avoid creating unnecessary clutter of notation which could be distracting.)

Consider an exchange economy with a representative agent who seeks to maximize utility from consumption over two points, \( t \) and \( t + 1 \). At time \( t \), the agent chooses to split his endowment between investments in 4 asset types and current consumption. The 4 asset types are: a foreign currency risk-free bond with the return of \( r_F \), a domestic currency risk-free bond with a return of \( r_D \), a foreign currency call option with random payoff of \( X_c \), and a foreign currency put option with a random payoff of \( X_p \) and the same strike as the call option. If \( \sigma(S_{t+1}) \) is the standard deviation of future exchange rate level, \( \sigma(\tilde{X}_c) \) is the standard deviation of call payoffs, and \( \sigma(\tilde{X}_p) \) is the standard deviation of put payoffs, then, for corresponding options, one may write: \( \sigma(S_{t+1}) = \sigma(\tilde{X}_c) + \sigma(\tilde{X}_p) \). Assume that, at time \( t + 1 \), the representative agent consumes all his wealth.

The decision problem facing the representative agent is:

\[
\max u(c_t) + \beta E[u(c_{t+1})]
\]

subject to:

\[
c_t = e_t - S_t \cdot n_s \cdot (1 + \theta) - c_t \cdot n_c \cdot (1 + \theta) - P_F \cdot n_F \cdot (1 + \theta) - P_t \cdot n_p \cdot (1 + \theta)
\]

\[
\tilde{c}_{t+1} = e_{t+1} + S_{t+1} \cdot n_s \cdot (1 + r_F) \cdot (1 - \theta) + \tilde{X}_c \cdot n_c \cdot (1 - \theta) + P_F \cdot n_F \cdot (1 + r_D) \cdot (1 - \theta) + \tilde{X}_p \cdot n_p \cdot (1 - \theta)
\]
where $c_t$ and $c_{t+1}$ are current and next period consumption, $e_t$ and $e_{t+1}$ are endowments, $S_t$ is the domestic currency price of one unit of foreign currency, $n_s$ is the number of units of foreign currency invested in the foreign risk-free bond, and $C_t$ is the domestic currency price of a call option with one unit of foreign currency as the underlying. The number of call option is denoted by $n_C$, whereas $n_p$ and $n_F$ denote the number of put options and the domestic risk-free bond respectively. $P_t$ and $P_F$ denote the put option price and the domestic risk-free asset price respectively, and $\theta$ is the percentage transaction cost which is assumed to be proportional and symmetric for simplicity.

Using $SDF = \frac{\beta E[u'(c_{t+1})]}{u'(c_t)}$ as the stochastic discount factor, in equilibrium, the following must hold:

$$\frac{1 + \theta}{1 - \theta} \cdot \frac{1}{1 + r_F} = E\left[SDF \cdot \frac{S_{t+1}}{S_t}\right]$$

$$\frac{1 + \theta}{1 - \theta} = E[SDF] \cdot (1 + r_d)$$

$$\frac{1 + \theta}{1 - \theta} \cdot C_t = E[SDF \cdot \tilde{X}_c]$$

$$\frac{1 + \theta}{1 - \theta} \cdot P_t = E[SDF \cdot \tilde{X}_p]$$

Using $\tilde{R}_e = \frac{s_{t+1}}{S_t}$, $\tilde{R}_c = \frac{\tilde{X}_c}{c_t}$ and $\tilde{R}_p = \frac{\tilde{X}_p}{P_t}$, one may write:

$$\frac{1 + \theta}{1 - \theta} \cdot \frac{1}{1 + r_F} = E[SDF] \cdot E[\tilde{R}_e] + \rho_e \cdot \sigma[SDF] \cdot \sigma[\tilde{R}_e]$$

$$\frac{1 + \theta}{1 - \theta} = E[SDF] \cdot E[\tilde{R}_c] + \rho_c \cdot \sigma[SDF] \cdot \sigma[\tilde{R}_c]$$

$$\frac{1 + \theta}{1 - \theta} = E[SDF] \cdot E[\tilde{R}_p] + \rho_p \cdot \sigma[SDF] \cdot \sigma[\tilde{R}_p]$$

where $\frac{1 + \theta}{1 - \theta} \cdot \frac{1}{(1 + r_d)} = E[SDF]$ and $\rho$ is the correlation of the asset with the SDF.

The above equations can be simplified further by writing $\rho_c = \rho_e$, $\rho_p = -\rho_e$, and

$$\sigma(\tilde{R}_p) = a \cdot \sigma(\tilde{R}_e) - b \cdot \sigma(\tilde{R}_c),$$

where $a = \frac{s}{P}$ and $b = \frac{c}{P}$. Also, there exists an $A$ such that
\[ \sigma(\tilde{R}_c) = \sigma(\tilde{R}_e)(1 + A). \] Hence, the equilibrium conditions for corresponding call and put options are:

\[ \frac{1 + \theta}{1 - \theta} = E[SDF] \cdot E[\tilde{R}_c] + \rho_e \cdot \sigma[SDF] \cdot \sigma(\tilde{R}_e) \cdot (1 + A) \]

\[ \frac{1 + \theta}{1 - \theta} = E[SDF] \cdot E[\tilde{R}_p] - \rho_e \cdot \sigma[SDF] \cdot \sigma(\tilde{R}_e)(a - b(1 + A)) \]

It follows that:

\[ E[\tilde{R}_c] = (1 + r_p) + (1 + A) \left\{ E[\tilde{R}_e] - \frac{(1 + r_D)}{(1 + r_F)} \right\} \quad (1.1) \]

\[ E[\tilde{R}_p] = (1 + r_p) - (a - b(1 + A)) \left\{ E[\tilde{R}_e] - \frac{(1 + r_D)}{(1 + r_F)} \right\} \quad (1.2) \]

Assuming that \( SDF \) exists, one can use equations (1.1) and (1.2) directly to price corresponding call and put options. Equivalently and more simply, one can discount call expected payoff by (1.1) to recover call option price and then use the put-call parity for currency options to calculate the corresponding put option price.

In order to arrive at the Black-Scholes formula for a currency call option, use a continuous time version of (1.1), set \( A = \Omega - 1 \) where \( \Omega \) is the call price elasticity w.r.t the underlying currency exchange rate, and assume geometric Brownian motion for the exchange rate dynamics. For stochastic volatility models, if diffusive risk is the only priced factor, set \( A = \Omega - 1 \) and assume a stochastic volatility process for the exchange rate dynamics. For jump diffusion option pricing model with diffusive risk as the only priced risk, once again set \( A = \Omega - 1 \), however, assume a mixed jump diffusion process for the underlying currency dynamics.\(^6\)

For anchoring-adjusted models, use \( A = m(\Omega - 1) \) with \( 0 \leq m \leq 1 \). Hence, the anchoring-adjusted counterparts of Black-Scholes, stochastic volatility, and jump diffusion models can be derived by using the continuous time version of 1.1. Starting from the risk of the underlying currency exchange rate, if the adjustment is correct, that is, if \( m = 1 \), then the

\(^6\) Of course, by using a different formulation for \( A \), one can allow for other sources of risk such as stochastic volatility or jump risk.
anchoring-adjusted models would converge to the corresponding Black-Scholes, stochastic volatility, or jump diffusion models.

It is straightforward to see that the anchoring-adjusted models are equivalent to their corresponding rational counterparts if $E[S_{t+1}] = S_t \cdot \frac{(1+r_D)}{(1+r_F)}$. This is because the term multiplying $(1 + A)$ in (1.1) goes to 0. In other words, regardless of the magnitude of anchoring, if uncovered interest-rate parity holds, anchoring-adjusted and corresponding standard models are equivalent.

### 1.1. Anchoring-Adjusted Black-Scholes Model

The continuous-time version of (1.1) in terms of net returns is:

$$\frac{1}{dt} E\left[\frac{dC}{C}\right] = r_D + (1 + A_K) \left\{ \frac{1}{dt} E\left[\frac{dS}{S}\right] - r_D + r_F \right\}$$

$$= \frac{1}{dt} E\left[\frac{dS}{S}\right] + r_F + A_K \left\{ \frac{1}{dt} E\left[\frac{dS}{S}\right] - r_D + r_F \right\} \quad (1.3)$$

where $C$, and $S$, denote the call price, and the domestic currency price of one unit of foreign currency respectively. $A_K = m(\Omega_K - 1)$, where $0 \leq m \leq 1$. The subscript $K$ is added to emphasize the dependence of elasticity on the strike price. If $m = 1$, there is no anchoring bias. The anchoring approach converges to the Black-Scholes model in this case. If $m < 1$, there is anchoring bias, and the anchoring and the Black-Scholes formulas differ.

Writing $\frac{1}{dt} E[ds] = \mu$, (1.3) can be written as:

$$\frac{1}{dt} E\left[\frac{dC}{C}\right] = \mu + r_F + A_K(\mu - r_D + r_F) \quad (1.4)$$

The underlying currency exchange rate follows geometric Brownian motion:

$$dS = \mu S dt + \sigma S dZ$$

where $dZ$ is the standard Brownian process.
From Ito’s lemma:

\[ E[dC] = \left( \mu S \frac{\partial C}{\partial S} + \frac{\partial C}{\partial t} + \frac{\sigma^2 S^2}{2} \frac{\partial^2 C}{\partial S^2} \right) dt \]

Substituting (1.5) in (1.4) leads to:

\[ (\mu + r_F + A_K (\mu - r_D + r_F))C = \frac{\partial C}{\partial t} + \frac{\partial C}{\partial S} \mu S + \frac{\partial^2 C}{\partial S^2} \frac{\sigma^2 S^2}{2} \]

(1.6) describes the partial differential equation (PDE) that must be satisfied if anchoring determines call option prices.

To appreciate the difference between the anchoring PDE and the Black-Scholes PDE, consider the expected return under the Black-Scholes approach, which is given below:

\[ \frac{1}{dt} \frac{E[dC]}{C} = \mu + r_F + (\Omega_K - 1) (\mu + r_F - r_D) \]

That is, substituting \( A_K = \Omega_K - 1 \) in (1.4) gives the Black-Scholes expected return.

Substituting (1.5) in (1.7) and realizing that \( \Omega_K = \frac{S \frac{\partial C}{\partial S}}{C} \) leads to the following:

\[ r_D C = \frac{\partial C}{\partial t} + (r_D - r_F) S \frac{\partial C}{\partial S} + \frac{\partial^2 C}{\partial S^2} \frac{\sigma^2 S^2}{2} \]

(1.8) is the Black-Scholes PDE adapted for currency call options. If \( A_K = \Omega_K - 1 \), then (1.8) and (1.6) are the same. If \( A_K < \Omega_K - 1 \) due to anchoring bias, then (1.6) and (1.8) are different from each other.

Constantinides and Perrakis (2002) derive a stochastic dominance based upper bound (CP upper bound) on a call option’s price in the presence of proportional transaction costs. Their bound is considered the tightest option pricing bound derived in the literature under general conditions. The CP upper bound is the call price at which the expected return from the call option is equal to the expected return from the underlying asset net of round-trip transaction cost:

\[ \tilde{C} = \frac{(1 + \theta)S \cdot E[C]}{(1 - \theta)E[S]} \]

\(^7\) See Proposition 1 in Constantinides and Perrakis (2002).
It is easy to see that the anchoring price is always less than the CP upper bound. The anchoring-prone investor expects a return from a call option which is at least as large as the expected return from the underlying currency. That is, with anchoring, \( \frac{E[C]}{C} \geq \frac{E[S]}{S} > \frac{(1-\theta)E[S]}{(1+\theta)S}. \) It follows that the maximum price under anchoring is: \( \bar{C}_A < \bar{C} = \frac{(1+\theta)S E[C]}{(1-\theta)E[S]}. \)

Re-writing the anchoring PDE with the boundary condition, we get:

\[
(\mu + r_F + A_K(\mu - r_D + r_F))C = \frac{\partial C}{\partial t} + \frac{\partial C}{\partial S} \mu S + \frac{\partial^2 C}{\partial S^2} \frac{\sigma^2 S^2}{2}
\]

(1.9)

where \( 0 \leq A_K \leq (\Omega_K - 1) \), and \( C_T = \max\{S - K, 0\} \)

There is a closed form solution to the anchoring PDE. Proposition 1 puts forward the resulting European option pricing formulas.

**Proposition 1** If anchoring influences option prices, then the price of a European currency call option is obtained by solving the anchoring PDE. The formula is \( C = e^{-(r_F + A_K \delta)(T-t)} \left\{ SN(d_1^A) - Ke^{-(\mu)(T-t)} N(d_2^A) \right\} \) where \( d_1^A = \frac{\ln(S/K) + (\mu + \frac{\sigma^2}{2})(T-t)}{\sigma \sqrt{T-t}} \), \( d_2^A = \frac{\ln(S/K) + (\mu - \frac{\sigma^2}{2})(T-t)}{\sigma \sqrt{T-t}} \), \( A_K = m(\Omega_K - 1) \) with \( 0 \leq m \leq 1 \), and \( \delta = \mu + r_F - r_D \)

**Proof.**

See Appendix A. ■

It is easy to see that if the uncovered interest rate parity holds, that is, if \( \mu = r_D - r_F \), then the anchoring price is the same as the Black-Scholes price. The price of a corresponding European put option is given in Proposition 2.

**Proposition 2** The price of a European put option is given by:

\[
P = e^{-r_D(T-t)} \cdot K \left\{ 1 - N(d_2^A) \cdot e^{-(\mu + r_F - r_D)(T-t)} \right\} - e^{-r_F(T-t)}
\]

\[
\cdot S \left\{ 1 - N(d_1^A) \cdot e^{-A_K(\mu + r_F - r_D)(T-t)} \right\}
\]
Proof.

The formula can be derived in two ways:

1) Use put-call parity for currency options, which is, \( P - C = e^{-r_D(T-t)} \cdot K - e^{-r_F(T-t)} \cdot S \)

2) Use the continuous time version of (1.2) as applicable to net returns, and the Ito’s lemma for put options to derive a PDE for currency put option price. Use the procedure in Appendix A to solve the PDE to obtain the desired formula.

Next, I extend the anchoring approach to 1) the stochastic volatility models of Hull and White (1987) and Heston (1993), 2) jump diffusion model of Merton (1976), and 3) Bates (1996) model which combines stochastic volatility with Poisson jumps.

1.2 Anchoring-Adjusted Stochastic Volatility Model (Hull-White)

Pioneering work on stochastic volatility models is Hull and White (1987). The idea is that variance is generated by a stochastic process of its own apart from the process that derives the spot exchange rates. The interaction between the two processes is captured by a correlation parameter.

\[
\begin{align*}
    dS &= \mu S dt + \sqrt{V} S dw \\
    dV &= \varphi V dt + \varepsilon V dz \\
    E[dwdz] &= \rho
\end{align*}
\]

Where \( V = \sigma^2 \) (Instantaneous variance of stock’s returns), and \( \varphi \) and \( \varepsilon \) are non-negative constants. \( dw \) and \( dz \) are standard Brownian processes. Time subscripts in \( S \) and \( V \) are suppressed for notational simplicity. If \( \varepsilon = 0 \), then the instantaneous variance is a constant, and we are back in the Black-Scholes world. Bigger the value of \( \varepsilon \), which can be interpreted as the volatility of volatility parameter, larger is the departure from the constant volatility assumption of the Black-Scholes model. Hull and White (1987) is among the first option pricing models that allowed for stochastic volatility. A variety of stochastic volatility models have been proposed including Stein and Stein (1991), and Heston (1993) among others.
In this section, I consider the case when $\rho = 0$, the case when $\rho \neq 0$ is considered in the next section when Heston stochastic volatility model is discussed.

If anchoring influences prices and the underlying exchange rate and its instantaneous volatility follow the stochastic processes described above, then by the application of Ito’s lemma:

$$
\frac{\partial C}{\partial t} + \mu S \frac{\partial C}{\partial S} + \sigma V \frac{\partial C}{\partial V} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + \frac{1}{2} \sigma^2 V^2 \frac{\partial^2 C}{\partial V^2} = \left( \mu + r_F + A_K(\mu - r_D + r_F) \right) C \tag{1.10}
$$

where $0 \leq A_K \leq (\Omega_K - 1)$, and $C_T = \max\{S - K, 0\}$

By definition, under anchoring, the price of a call option is the expected terminal value of the option discounted at the rate which the marginal investor in the option expects to get from investing in the option. The price of the option is then:

$$
C(S_T, \sigma_T^2, t) = e^{- (\mu + r_F + A_K \delta)(T-t)} \int C(S_T, \sigma_T^2, T) p(S_T|S_t, \sigma_t^2) dS_T \tag{1.11}
$$

Where the conditional distribution of $S_T$ as perceived by the marginal investor is such that $E[S_T|S_t, \sigma_t^2] = S_t e^{(\mu)(T-t)}$ and $C(S_T, \sigma_T^2, T)$ is $\max(S_T - K, 0)$.

By defining $\bar{V} = \frac{1}{T-t} \int_t^T \sigma_t^2 d\tau$ as the means variance over the life of the option, the distribution of $S_T$ can be expressed as:

$$
p(S_T|S_t, \sigma_t^2) = \int f(S_T|S_t, \bar{V}) g(\bar{V}|S_t, \sigma_t^2) d\bar{V} \tag{1.12}
$$

Substituting (1.12) in (1.11) and re-arranging leads to:

$$
C(S_t, \sigma_t^2, t) = \int \left[ e^{-(\mu + r_F + A_K \delta)(T-t)} \int C(S_T)f(S_T|S_t, \bar{V}) dS_T \right] g(\bar{V}|S_t, \sigma_t^2) d\bar{V} \tag{1.13}
$$

By using an argument that runs in parallel with the corresponding argument in Hull and White (1987), it is straightforward to show that the term inside the square brackets is the anchoring price of the call option with a constant variance $\bar{V}$. Denoting this price by $Call_{AM}(\bar{V})$, the price of the call option under anchoring when volatility is stochastic (as in Hull and White (1987)) is given by:
\[ C(S_t, \sigma_t^2, t) \]
\[ = \int \text{Call}_{AM}(\bar{V}) g(\bar{V} | S_t, \sigma_t^2) d\bar{V} \quad (1.14) \]
where \( \text{Call}_{AM} = e^{-(r_F + A_K \delta)(T-t)} \{ SN(d_1^A) - K e^{-\mu(T-t)} N(d_2^A) \} \)

\[ d_1^A = \frac{\ln(S/K) + (\mu + \frac{\sigma^2}{2})(T-t)}{\sigma \sqrt{T-t}}, \quad d_2^A = \frac{\ln(S/K) + (\mu - \frac{\sigma^2}{2})(T-t)}{\sigma \sqrt{T-t}}, \quad A_K = m(\Omega_K - 1) \text{ with } 0 \leq m \leq 1, \text{ and } \]
\[ \delta = \mu + r_F - r_D \]

Equation (1.14) shows that the anchoring adjusted call option price with stochastic volatility is the anchoring price with constant variance integrated with respect to the distribution of mean volatility.

### 1.3 Anchoring-Adjusted Stochastic Volatility Model (Heston)

In this section, I extend the anchoring approach to the Heston model. In Heston model, the spot exchange rate and its volatility follow the processes given by:

\[ dS = \mu S dt + \sqrt{V} S dw \]
\[ dV = k(\theta - V) dt + \sigma \sqrt{V} dz \]
\[ E[dwdz] = \rho \]

where \( V \) is the initial instantaneous variance, \( \theta \) is the long run variance, \( k \) is the rate at which \( V \) moves towards \( \theta \), and \( \sigma \) is the volatility of volatility parameter. The model reverts to the Black-Scholes model when \( \sigma \) and \( k \) are set to zero.

By using Ito’s lemma, the anchoring-adjusted partial differential equation for the European call option is given by:

\[ \frac{\partial C}{\partial t} + \mu S \frac{\partial C}{\partial S} + k(\theta - V) \frac{\partial C}{\partial V} + \frac{1}{2} V S^2 \frac{\partial^2 C}{\partial S^2} + \frac{1}{2} \sigma^2 V \frac{\partial^2 C}{\partial V^2} + \rho \sigma V \frac{\partial^2 C}{\partial S \partial V} \]
\[ = (\mu + r_F + A_K(\mu - r_D + r_F)) C \quad (1.15) \]

where \( C_T = \max\{S - K, 0\} \). (1.15) can be solved by using Fourier methods. Proposition 3 provides the solution.
Proposition 3-Anchorong-Adjusted Heston Model: The anchoring-adjusted price of a European call option when the exchange rate dynamics are as in the Heston model is given by:

\[ C = e^{-(r_F + A_K \delta)(T-t)} \left( S P_1 - K e^{-(\mu)(T-t)} P_2 \right) \]

where

\[ \delta = \mu + r_F - r_D \]

\[ P_1 = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \text{Re} \left\{ \frac{e^{-i\varphi \ln k} f(\varphi - i)}{i\varphi f(-i)} \right\} d\varphi \]

\[ P_2 = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \text{Re} \left\{ \frac{e^{-i\varphi \ln k} f(\varphi)}{i\varphi} \right\} d\varphi \]

\[ f_{AH}(\varphi) = e^{A+B+C} \]

\[ A = i\varphi \ln S_t + i\varphi(\mu)(T-t) \]

\[ B = \frac{\varphi k}{\sigma^2} \left( (k - \rho \sigma i\varphi - d)(T-t) - 2\ln \left( \frac{1 - ge^{-d(T-t)}}{1 - g} \right) \right) \]

\[ C = \frac{V}{\sigma^2} \frac{(k - \rho \sigma i\varphi - d)(1 - e^{-d(T-t)})}{1 - ge^{-d(T-t)}} \]

\[ d = \sqrt{(\rho \sigma i\varphi - k)^2 + \sigma^2 (i\varphi + \varphi^2)} \]

\[ g = \frac{k - \rho \sigma i\varphi - d}{k - \rho \sigma i\varphi + d} \]

Proof.

See Appendix B
It is straightforward to check that if uncovered interest-rate parity holds, then the anchoring-adjusted model converges to the Heston stochastic volatility model. In the next section, I extend the anchoring approach to Merton’s jump diffusion model.

### 1.4 Anchoring-Adjusted Jump Diffusion Model (Merton)

Merton (1976) assumes that the underlying spot exchange rate process is a mixture of geometric Brownian motion and Poisson-driven jumps:

\[ dS = (\mu - \gamma \beta)Sdt + \sigma Sdz + (Y - 1)Sdq \]

Where \( dz \) is a standard Guass-Weiner process, and \( dq \) is a Poisson process. \( dz \) and \( dq \) are assumed to be independent. \( \gamma \) is the mean number of jump arrivals per unit time, \( \beta = E[Y - 1] \) where \( Y - 1 \) is the random percentage change in the underlying exchange rate if the Poisson event occurs, and \( E \) is the expectations operator over the random variable \( Y \). If \( \gamma = 0 \) (hence, \( dq = 0 \)) then the spot exchange rate dynamics are identical to those assumed in the Black Scholes model. Merton assumes that logarithm of jump size is normally distributed. That is, 

\[ \ln Y \sim N(\theta, \sigma^2) \]

It follows that \( \beta = E[Y - 1] = e^{\theta + \frac{\sigma^2}{2}} - 1 \).

If anchoring determines the price of the call option when the underlying spot exchange rate dynamics are a mixture of a geometric Brownian motion and a Poisson process as described earlier, then by Ito’s lemma, the following partial differential equation must be satisfied:

\[
\frac{\partial C}{\partial t} + \mu S \frac{\partial C}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + \gamma E[C(SY, t) - C(S, t)] - \gamma \beta S \frac{\partial C}{\partial S} \\
= (\mu + r_F + A_K(\mu - r_D + r_F))C
\]

(1.16)

Merton assumes that the jump risk is diversifiable. This means that the correct adjustment to foreign currency bond return \((\mu + r_F)\) to arrive at call return in Merton’s model is: \( A_K \cdot \)

\[ \{\mu - r_D + r_F\} = (\Omega_K - 1)\{\mu - r_D + r_F\} = \left( \frac{\partial C}{\partial S} - 1 \right)(\mu - r_D + r_F). \]

Substituting the correct adjustment term in (1.16) results in the following PDE:

\[
\frac{\partial C}{\partial t} + (r_D - r_F)S \frac{\partial C}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + \gamma E[C(SY, t) - C(S, t)] - \gamma \beta S \frac{\partial C}{\partial S} \\
= r_D C
\]

(1.17)
(1.17) is Merton’s jump diffusion PDE adapted for currency call options. Hence, if the adjustment term is equal to the theoretically correct adjustment then anchoring PDE reduces to Merton’s jump diffusion PDE, as expected. However, if the anchoring bias causes the adjustment term to be smaller, then the anchoring and Merton PDEs are different from each other. It is easy to solve (1.16) by using an argument analogous to the argument in Merton (1976).

1.5 Anchoring-Adjusted Stochastic Volatility with Poisson Jumps (Bates Model)

Bates model is an extension of the Heston model. The dynamics under Bates model are:

\[ dS = (\mu S - \lambda \mu J) dt + \sqrt{V} S dw + J dN \]
\[ dV = k(\theta - V) dt + \sigma \sqrt{V} dz \]
\[ E[dwdz] = \rho \]

Time subscripts are suppressed for simplicity. Bates model adds a compound Poisson process with jump intensity \( \lambda \) to the Heston model. A compound Poisson process is a Poisson process where the jump sizes follow the following distribution:

\[ \log(1 + J) \in N \left( \log(1 + \mu_j) - \frac{\sigma_j^2}{2} \right) \]

Using Ito’s lemma for the continuous part and an analogous lemma for the jump part, the anchoring-adjusted PDE for the price of European call option is:

\[
\frac{\partial C}{\partial t} + \mu S \frac{\partial C}{\partial S} + k(\theta - V) \frac{\partial C}{\partial V} + \frac{1}{2} V S^2 \frac{\partial^2 C}{\partial S^2} + \frac{1}{2} \sigma^2 V \frac{\partial^2 C}{\partial V^2} + \rho \sigma V \frac{\partial^2 C}{\partial S \partial V} \\
+ \lambda E[C(SY, t) - C(S, t)] - \lambda \mu_j \frac{\partial C}{\partial S} \\
= (\mu + r_F + A_K(\mu - r_D + r_F))C \quad (1.18)
\]

where \( C_T = \max(S - K, 0) \).

(1.18) can be solved by using Fourier methods as in the case of anchoring-adjusted Heston model. Proposition 4 provides the solution.
Proposition 4-Anchor Adjusted Bates Model: The anchoring-adjusted price of a European call option when the exchange rate dynamics are as in the Bates model is given by:

\[
C = e^{-\left(r_F + A_K \delta\right)(T-t)} \left[SP_1 - Ke^{-\left(\mu\right)(T-t)} P_2\right]
\]

where

\[\delta = \mu + r_F - r_D\]

\[
P_1 = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \text{Re} \left\{ \frac{e^{-i\varphi \ln k f(\varphi - i)}}{i\varphi f(-i)} \right\} d\varphi
\]

\[
P_2 = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \text{Re} \left\{ \frac{e^{-i\varphi \ln k f(\varphi)}}{i\varphi} \right\} d\varphi
\]

\[
f(\varphi) = e^{A + B + C} \cdot e^{-\lambda \varphi (T-t) + \lambda (T-t)} \left( (1+i\mu)^{i\varphi} e^{\frac{1}{2} i\varphi (\varphi - 1)} - 1 \right)
\]

\[A = i\varphi \ln S_t + i\varphi (\mu) (T-t)\]

\[B = \frac{\varphi k}{\sigma^2} \left( (k - \rho \sigma i\varphi - d)(T-t) - 2\ln \left( \frac{1 - ge^{-d(T-t)}}{1 - g} \right) \right)
\]

\[C = \frac{V}{\sigma^2} \frac{(k - \rho \sigma i\varphi - d)(1 - e^{-d(T-t)})}{1 - ge^{-d(T-t)}}
\]

\[d = \sqrt{(\rho \sigma i\varphi - k)^2 + \sigma^2 (i\varphi + \varphi^2)}
\]

\[g = \frac{k - \rho \sigma i\varphi - d}{k - \rho \sigma i\varphi + d}
\]

Proof.

See Appendix C
The anchoring-adjusted Bates model is closely related to the anchoring-adjusted Heston model. This is because original (without anchoring) Heston and Bates model are closely related to each other. The difference when compared with the Heston model is that the characteristic function is multiplied by a term accounting for the jump. In the Bates model, innovations in log-return can come from two sources (log-return is formed as a sum of two independent random variables); one accounting for the stochastic volatility part and one accounting for the jump-part. The characteristic function for the sum of two independent random variables is the multiplication of the two characteristic functions.

2. The Implied Volatility Smile in Currency Options

The peculiar features of the currency smile are easily generated with anchoring-adjusted models, even within the simplest framework of geometric Brownian motion. The role of complex distributional assumptions is to amplify the smile seen with the geometric Brownian motion. Section 2.1 illustrates the smile in the anchoring-adjusted Black-Scholes model, followed by the discussion of anchoring-adjusted stochastic volatility and anchoring-adjusted jump diffusion models in sections 2.2 and 2.3 respectively.

2.1 The Smile in Anchoring-Adjusted Black-Scholes Model

The expected spot exchange-rate return, \( \mu \), is the driver of the smile in anchoring-adjusted models. It appears directly in anchoring-adjusted formulas. Hence, sentiment directly matters in anchoring-adjusted models in line with the behavior of market professionals.

Currency options are unique in the sense they have certain symmetries. In particular, a call option on foreign currency is a put option on domestic currency, and a put option on foreign currency is a call option on domestic currency. Common sense suggests that an investor who expects foreign currency to appreciate would be interested in out-of-the-money call (most traded call is 25-delta) on foreign currency, and an investor who expects the foreign currency to depreciate would be interested in out-of-the-money put (25-delta is most traded put) on foreign currency or equivalently in out-of-the-money call on domestic currency. As discussed in the introduction, professional traders read the sentiment from the skew with this expectation in mind. The skew is the difference between same delta out-of-the-money call and put implied volatilities (known as risk-reversal in market parlance). An increase in skew is interpreted as the
market sentiment turning more bullish, whereas a decrease in the skew is taken to mean that the sentiment is becoming bearish. A symmetric smile is interpreted as reflecting the bullish and bearish sentiments of equal magnitude.

Anchoring-adjusted models are in accord with the above observation of market professionals. In anchoring-adjusted models, $\mu$, which measures the exchange rate expectations of the marginal investor, is the driver of the smile. One expects the marginal investors in out-of-the-money call and out-of-the-money put options to assign opposite signs to $\mu$. Relative magnitudes then determine whether we see a symmetric smile, forward skew, or reverse-skew.

Next, I illustrate the smile in anchoring-adjusted Black-Scholes model. Table 1-Panel A illustrates the symmetric smile corresponding to bullish ($\mu = +10\%$) and bearish sentiments ($\mu = -10\%$) of equal magnitude. Panel B illustrates the forward skew with stronger bullish ($\mu = +20\%$) than bearish sentiment ($\mu = -10\%$). Panel C illustrates the reverse-skew with stronger bearish ($\mu = -20\%$) than bullish sentiment ($\mu = +10\%$). Figures 1a, 1b, and 1c graphically illustrate Table 1, and helps in visualizing the impact of sentiment on the implied volatility as smile shown in Table 1.

As discussed earlier, a closely watched measure of investor sentiment is the value of risk-reversals (out-of-the-money call implied volatility minus out-of-the-money put implied volatility). In the above illustration, the risk-reversal values are 0.05094, 2.4574, and -2.2972 corresponding to symmetric, forward, and reverse skews respectively. Hence, incorporating the anchoring and adjustment heuristic of Tversky and Kahneman (1974) into currency option pricing provides a theoretical basis for the market practice of reading sentiment from risk-reversals.

An advantage of the anchoring approach is that it does not build the smile into the distribution of the underlying exchange rate. In contrast, stochastic volatility and jump diffusion approaches effectively insert the smile into the underlying distribution. These models are generally considered to be miss-specified as they require implausibly large parameter values to match the observed skews (see Bakshi, Cao, and Chen (1997), Bates (2008)). As anchoring generates the skew on its own, incorporating anchoring into stochastic volatility and jump diffusion models helps in mitigating this concern.
### Table 1-Panel A: Symmetric Smile

| Strike | $\mu$    | Call  | Put   | Call  | Put   | Implied Volatility |
|--------|----------|-------|-------|-------|-------|--------------------|
| 95     | -10%     | 5.667 | 0.667 | 5.7283| 0.7283| 22.08302%          |
| 100    | +5% or -5% | 2.2872| 2.2872| 2.4934| 2.4934| 21.80391%          |
| 105    | +10%     | 0.6449| 5.6449| 0.8252| 5.8252| 22.13396%          |

Parameter Values: $r_D = r_F = A_K = 0, S_t = 100, T - t = 1\ month, \sigma_e = 20\%$

For out-of-the-money options, put trader is marginal when $\mu$ is negative, and call trader is marginal when $\mu$ is positive. For at-the-money options, call with $\mu = +5\%$, and put with $\mu = -5\%$ give the same implied volatility so either can be assumed marginal.

### Panel B: Forward Skew

| Strike | $\mu$    | Call  | Put   | Call  | Put   | Implied Volatility |
|--------|----------|-------|-------|-------|-------|--------------------|
| 95     | -10%     | 5.667 | 0.667 | 5.7283| 0.7283| 22.08302%          |
| 100    | +5% or -5% | 2.2872| 2.2872| 2.4934| 2.4934| 21.80391%          |
| 105    | +20%     | 0.6449| 5.6449| 1.0411| 6.0411| 24.5404%           |

### Panel C: Reverse-Skew

| Strike | $\mu$    | Call  | Put   | Call  | Put   | Implied Volatility |
|--------|----------|-------|-------|-------|-------|--------------------|
| 95     | -20%     | 5.667 | 0.667 | 5.9228| 0.9228| 24.4312%           |
| 100    | +5% or -5% | 2.2872| 2.2872| 2.4934| 2.4934| 21.80391%          |
| 105    | +10%     | 0.6449| 5.6449| 0.8252| 5.8252| 22.13396%          |
Figure 1a

Figure 1b
2.2. Anchoring-Adjusted Stochastic Volatility Model and the Smile

As discussed earlier, a crucial weakness of stochastic volatility models is that they require implausibly large parameter values of correlation and volatility of volatility parameters, when compared with their time-series averages (see Bakshi, Cao, and Chen (1997)). Furthermore, these parameters must fluctuate a lot to match the fluctuating smile. In particular, symmetric-skew requires $\rho = 0$, forward-skew needs $\rho > 0$, and reverse-skew requires $\rho < 0$. As the smile is sometimes symmetric, sometimes forward-skew, and sometimes reverse-skew, rationalizing it in a stochastic volatility framework requires a widely fluctuating $\rho$.

As anchoring generates the skew without effectively building it in the underlying distribution, incorporating anchoring in a stochastic volatility model increases the power of the stochastic volatility approach. In particular, anchoring-adjusted stochastic volatility model can generate all three types of smiles even when $\rho = 0$. In general, smaller parameter values are needed to match a given skew.

To see this, in the Hull-White stochastic volatility process defined in section 1.2, assume that $\varphi = a + bV$ with $b < 0$ and $a > 0$. Recall, that $\rho = 0$ has been assumed in section 1.2. (1.14) can be simplified further by using the same procedure as in Hull and White (1988) to yield:

$$ Call^{A}_{SV} = Call^{A}_{BS} + Q \cdot \epsilon^2 $$

(2.1)

where $Call^{A}_{BS}$ is the anchoring-adjusted Black-Scholes price given in Proposition 1 evaluated at mean volatility over the life of the option, $\epsilon$ is the earlier defined volatility of volatility parameter, and $Call^{A}_{SV}$ denotes the anchoring-adjusted stochastic volatility price of a European call option.
\[ Q = \frac{V}{4b\theta^2} \cdot (e^{2\theta} - 4e^\theta + 2\theta + 3) \cdot \frac{\partial^2 \text{Call}_{BS}}{\partial V^2} \] and \( \theta = b(T - t) \)

In (2.1) if \( \varepsilon = 0 \), then we are back to the anchoring-adjusted Black-Scholes that generates all three types of smiles. On the other hand, if in (2.1), the anchoring bias is zero, which happens when \( A_K = (\Omega_K - 1) \) or equivalently when \( m = 1 \) in \((1 - m)(\Omega_K - 1)\), then \( \text{Call}_{BS}^A \) converges to the Black-Scholes call price. In that case, (2.1) can only generate a symmetric looking smile. Hence, anchoring bias introduces the forward-skew and the reverse-skew into the model. Furthermore, incorporating anchoring into a stochastic volatility model may lead to steeper smiles. This is important due to the difficulty that stochastic volatility models face in matching steep smiles from short dated options (DeRosa (2011)), and the general misspecification of models with stochastic volatility.

Note that the sign of \( Q \) in (2.1) depends on the sign of \( \frac{\partial^2 \text{Call}_{BS}}{\partial V^2} \), which is proportional to \( (d_1^A d_2^A - 1) \) (defined in proposition 1). Hence, the sign of \( (d_1^A d_2^A - 1) \) determines whether the introduction of stochastic volatility into the picture increases or decreases the implied volatility obtained from anchoring-adjusted Black-Scholes with constant volatility. Proposition 5 provides the range within which the implied volatility is decreased by allowing for stochastic volatility. Outside this range, the implied volatility is increased.

**Proposition 5.** The implied volatility is decreased by allowing for stochastic volatility (when compared with anchoring-adjusted Black-Scholes implied volatility) if the following is true:

\[ e^{-z} < \frac{S e^\mu(T-t)}{K} < e^z \] where \( z = \sqrt{V(T - t) + \frac{V^2}{4}(T - t)^2} \), \( S \) is the spot rate, \( K \) is the strike price, and \( V \) is mean average volatility over the life of the option.

Outside the above range, the implied volatility is increased.

**Proof.**

See Appendix D.

\[ \blacksquare \]
From Proposition 5, one can see that the investor sentiment reflected in the option price is crucial in determining whether the implied volatility is increased or decreased. One expects, out-of-the-money options to reflect stronger bullish and stronger bearish sentiments than at-the-money options. Hence, the magnitude of $\mu$ is larger for out-of-the-money than at-the-money options. It follows that stochastic volatility tends to reduce the implied volatility from at-the-money options and increases the implied volatility from out-of-the-money options. This amplifies the pattern obtained from anchoring-adjusted Black-Scholes. In the context of the example presented in section 1.1, what happens when stochastic volatility is introduced? It leads to an amplified smile pattern as shown in Figure 2 (corresponding constant volatility graph is from Figure 1a).

![Implied Volatility from Anchoring-Adjusted Models](image)

**Figure 2**

Similarly, adding stochastic volatility amplifies the patterns in Figure 1b and Figure 1c as well. Hence, the role of adding more richness to the underlying process is that such additions may amplify the smile which is seen in the simplest framework.

### 2.3 The Anchoring-Adjusted Jump Diffusion Model and the Smile

As anchoring alone (with geometric Brownian motion) is capable of generating a skew, it is straightforward to see that combining it with a jump diffusion model would generate the skew at much lower levels of jump intensity (compare solutions of 1.16 and 1.17). This is important
because a major criticism of jump diffusion models is that they require a counterfactually large jump intensity to match the observed skew. Furthermore, generating the forward-skew in a jump diffusion model requires assuming asymmetric jumps in one direction, whereas generating the reverse-skew requires asymmetry to be in the opposite direction. It is easy to see that incorporating anchoring in a jump diffusion framework helps in mitigating this concern as all three types of smiles can be generated with anchoring even if symmetric jumps are assumed.

3. Empirical Evidence

In traditional currency option pricing models such as the Black-Scholes, stochastic volatility, and jump diffusion models, sentiment does not directly affect option prices. It matters only to the extent that it reflects the underlying fundamentals. In sharp contrast, sentiment directly matters for option prices in the presence of anchoring bias, and affects prices beyond what can be justified by fundamentals. So, if sentiment matters directly then fluctuations in currency option prices must be larger than what can be justified by fluctuations in the underlying spot exchange rate. Fluctuations in risk-reversals are typically much larger than fluctuations in the spot exchange rate suggesting that sentiment matters.\(^8\)

If sentiment matters, then one expects an out-of-the-money call to reflect bullish sentiment, and an out-of-the-money put to reflect bearish sentiment, whereas at-the-money options are expected to reflect average sentiment. With anchoring bias, if the sentiment turns bullish, one expects to see a rise in the implied volatility from out-of-the-money call and a fall in the implied volatility from out-of-the-money put. One expects to see a rise in the corresponding spot rate as well. It follows that there must be co-movement between the spot rate and the difference between the implied volatility from out-of-the-money call and put of similar delta. Hence, the anchoring model makes the following prediction:

**Prediction 1:** Risk Reversal (Out-of-the-money call implied volatility minus out-of-the-money put implied volatility) is positively correlated with recent trend in the underlying currency spot exchange rate.

Empirical evidence strongly supports this prediction. Gudhus (2003) use data on JPY/USD and USD/GBP currency pairs to test the relationship between risk-reversal and recent spot trend and find that they are positively related and quite strongly so (extremely

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\(^8\) As one example, see Figure 1 in OFR Market Monitors (August 2015).
significant t statistics are obtained). In fact, the more positive the recent trend in the underlying currency, higher is the call implied volatility compared to the put implied volatility.

If sentiment becomes more dispersed, that is, if bullish investors become more bullish, and bearish investors become more bearish relative to the average sentiment, then one expects to see the implied volatilities of out-of-the-money options rise relative to the implied volatility of at-the-money options. It follows that the dispersion of sentiment has a direct impact on the curvature of the smile. The following prediction follows:

**Prediction 2:** The strangle implied volatility (average implied volatility from 25-delta call and 25-delta put minus at-the-money implied volatility) is positively related to how disperse the sentiment is.

Empirical evidence supports this prediction as well. With data on the following currency pairs, JPY/USD, Euro/USD, and GBP/USD, Beber et al (2013) find that strangle implied volatility rises as the standard deviation of reported exchange rate beliefs increases.

4. Conclusions

The behavior of the implied volatility smile has been puzzling and option pricing models that have been put forward to explain it are generally considered miss-specified as they require implausibly large parameter values. I show that incorporating the anchoring and adjustment heuristic of Tversky and Kahneman (1974) in standard currency option pricing models increases their power. It generates the three types of smiles commonly observed in currency options within the framework of geometric Brownian motion. Increasing distributional complexity (by introducing stochastic volatility and/or jumps) amplifies the smile. Hence, to match a given skew, smaller parameter values suffice. In particular, one may even assume zero correlation between the stock price and volatility processes and generate the common types of smiles seen in currency options. In addition, this article makes a key methodological contribution by incorporating the anchoring heuristic into Black-Scholes, Hull-White, Merton, Heston, and Bates models. By incorporating anchoring in a wide range of models, this article effectively demonstrates that any arbitrary set of distributional assumptions for the underlying dynamics can be combined with the anchoring heuristic. Furthermore, two predictions of the anchoring-adjusted model have strong empirical support.

This article is part of a research program that studies the implications of incorporating the anchoring heuristic into asset pricing. In particular, (Siddiqi 2015a) adjusts equity option pricing models for anchoring and finds that anchoring provides a unified explanation for key option pricing puzzles in equity markets. Siddiqi (2015b) and Siddiqi (2016) extend the anchoring
approach to CAPM and CCAPM respectively. Anchoring-adjusted CAPM provides a unified explanation for size, value, and momentum effects, whereas anchoring-adjusted CCAPM provides an explanation for the equity premium puzzle. The ability of anchoring to explain a wide variety of phenomena across financial markets suggests that anchoring could be the unifying theme in finance.

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