Weak Signal Detection Based on Pseudo Wigner Ville Distribution

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Abstract. This paper proposes a weak signal detection technique based on Pseudo Wigner Ville Distribution (PWVD). The one-dimensional N-period sliding window is used to perform Pseudo Wigner transformation on the power signal data in the time domain direction, and analyze the PWVD regular on the frequency domain axis. Corresponding to extract PWVD on time domain, then analyze and accurately locates the weak signal changes, then performs the switching event detection. Finally, MATLAB is used to simulate the experiment, and the effectiveness of the proposed method is verified.

1. Introduction
At present, China's economy is still growing rapidly. It is estimated that by 2020, China's GDP will increase by more than 120% compared with 2010, and the demand for electricity will also double. The electricity consumption of the whole society will reach 8 trillion kWh, which put forward higher requirements for the power industry's support capabilities. In most cases, power users can only get the monthly power consumption provided by the power company, and can not know the detailed power consumption information of the internal electrical equipment of the user, but the research [1] shows that if the user can get real-time equipment-level energy consumption information rather than the monthly total electricity consumption, it can help users to maximize energy conservation work. For energy saving purposes, through the continuous efforts of researchers, the appliance load monitoring technology has been rapidly developed. As the basis of load monitoring, the load switch detection algorithm can provide more accurate data for the power sector to analyze, and can find a more energy-saving solution for individual users.

The main contents and structure of the remaining chapters of this paper are as follows. The second section mainly describes the method of switching event detection. The third section mainly discusses the principle of weak signal detection of PWVD. The fourth section carries out the experiment in this paper. Finally, the article is summarized in section 5.

2. Switch event detection methods
In general, the active power and reactive power in the time series are used for switching event detection[2]. Since 1980, a variety of switching event detection methods have also been proposed. Reviewing the previous literature, Hart first proposed the segmentation detection method[3] in 1992. According to the current and voltage data obtained in the bus, the active power is calculated and
displayed as a ladder signal for event detection. The wavelet algorithm is used in the literature [4] to decompose the acquired signal into edges, processes and other signals for event detection. In literature [5], a window-based event detection method is used. A scan window is used to scan the active power curve waveform, and the average value between the samples is calculated. Finally, the difference of the average value is compared to the threshold specified by the user for event detection.

The above results are basically based on changes in power or changes in current to study how to improve the accuracy of single-load event detection. But in practical application scenario, there is a switch between power similar appliances. At this time, it is difficult to correctly judge the switching event by means of power or current change. In addition, for small power appliances, the current change is not obvious, and the background noise in the circuit has a great influence on it. The traditional switch detection based on power or current mode has a larger probability of leakage. At the same time, the electrical devices that generate weak signals are usually non-pure resistive appliances, which have higher harmonics that are easily generated by internal inductive or capacitive devices, and have different time-frequency PWVD for power signals of different electrical appliances. Therefore, this paper proposes a weak signal detection technique based on PWVD, which detects the change of power signal by time-frequency analysis method, and then performs switch judgment.

3. Weak signal detection algorithm based on PWVD

Wigner Ville Distribution (WVD) was introduced by Eugene Paul Wigner, winner of the 1963 Nobel Prize in Physics. In 1932, a new equation was first introduced, which is different from the STFT and wavelet transform of linear transformation. It is a common time-frequency analysis method of bilinear transform, which is defined as the Fourier transform of the central covariance function of the signal. It can fully reflect the instantaneous power spectral density characteristics of the signal. Due to the operation of not adding window in the calculation, the mutual restriction between time domain resolution and frequency domain resolution is avoided. However, because the window function is not involved, it will be interfered by the cross term when analyzing the multi-component signal. In order to suppress the influence of cross terms, a method of PWVD is proposed, which is to smooth the WVD window.

The self-Wigner Ville distribution of continuous-time signals[6]\( s(t) \) is defined as shown in equation (1).

\[
W_s(t, f) = \int_{-\infty}^{\infty} x(t + \frac{\tau}{2})x^*(t - \frac{\tau}{2})e^{-j2\pi ft\tau} d\tau
\]

(1)

The WVD of the signal \( s(t) \) spectrum is defined as shown in equation (2).

\[
W_x(f, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(f + \frac{\xi}{2})X^*(f - \frac{\xi}{2})e^{j\xi t} d\xi
\]

(2)

Where \( x(t) \) is the analytical signal of \( s(t) \), * is the complex conjugate, \( \tau \) is the time difference variable, \( \xi \) is the frequency difference variable, \( \omega \) is the frequency, and \( X(f) \) is the Fourier transform of \( x(t) \). Define \( \gamma_x(t, \tau) = x(t + \tau / 2)x^*(t - \tau / 2) \), Then \( \gamma_x(t, \tau) \) is the Fourier transform of \( \tau \), which is the representation of signal \( x(t) \) at time \( t \) and frequency \( f \).

In the actual calculation process, the WVD defined by the above formula requires integration over the entire time axis or frequency axis, which is not conducive to real-time processing. Therefore, when the WVD distribution is obtained for the time \( t_0 \), the data length involved is limited. That is equivalent to adding a window function at the current moment, let

\[
x_{t_0}(t) = x(t)w(t - t_0)
\]

(3)

Usually the window function is symmetric about \( t_0 \) and is a real function of \( W(0) = 1 \). The WVD for intercepting signal \( x_{t_0}(t) \) is expressed as:
\[ W_{w0}(t, f) = \frac{1}{2\pi} \int_{-\infty}^{\infty} W_w(t, \xi)W_w(t-t_0, f-\xi)d\xi \]  

(4)

Where \( W_w(t, f) \) is the WVD of the window function, which is

\[ W_w(t, f) = \int_{-\infty}^{\infty} w(t+\tau/2)w^*(t-\tau/2)e^{\pi if\tau}d\tau \]  

(5)

The WVD of the windowed signal does not require all the data on the time axis. It only needs \( W_{w0}(t_0, f) \) at the midpoint of the window function. For the Wigner distribution of the above formula, take \( t = t_0 \) and replace \( t_0 \) with \( t \). You can get PWVD as follows:

\[ PW_x(t, f) = \frac{1}{2\pi} \int_{-\infty}^{\infty} W_x(t, \xi)W_w(0, f-\xi)d\xi \]  

(6)

It can be seen from the above formula that PWVD is the convolution of the signal WVD and the window function WVD with respect to the frequency variable \( f \). For a certain moment, the influence of the window function on the WVD can be understood as the filtering operation. The frequency response of the filter is \( W_w(0, f) \), so the performance of PWVD depends on the selection of the window function. If it is a real function, there is

\[ W_w(0, f) = \int_{-\infty}^{\infty} e^{-\pi if\tau}w^2(t/2)d\tau \]  

(7)

The above formula is a low-pass filter. The PWVD at this time is the smoothing result of the original WVD distribution frequency axis. The PWVD solution is a three-dimensional amplitude spectrum, so it is more difficult to apply the computer for efficient processing. Therefore, the result needs to be further simplified. That is, the feature with a certain amplitude is extracted from the multiplier signal to the greatest extent. The calculation amount should be reduced as much as possible to ensure the real-time performance.

Since the circuit environment mainly includes the power frequency signal and the frequency multiplication signal, the frequency should be basically concentrated under the condition of 50 Hz and its integral multiple. First, along the frequency axis direction, Select the maximum value of the pseudo Wigner transform on each frequency value to form the vector \( fm \). Select the frequency maximum value \( pk \) in \( fm \). Extract the corresponding frequency of \( pk \), and finally extract the pseudo-Wigner amplitude information of the frequency on the time axis to normalize and find the standard deviation. The amplitude information extracted by the method basically reflects the amplitude variation graph of the processed signal with time at different frequencies. And the final processing information is small but the information content is comprehensive, and is convenient for the program to implement. The process is as follows:

**Input:** PWVD result \( M_{t,f} \), where \( t \) is the time axis and \( f \) is the frequency axis.

**Output:** The extracted frequency information \( f_i \) and the corresponding normalized amplitude standard deviation \( std_{f_i} \).

1. Calculate the maximum value \( f_{\text{max}} = \max(M_{f}) \) at different frequencies along the frequency axis \( f \).
2. Extract the extreme value data index \( fpk_{id} = \text{findpeak}(f_{\text{max}}) \) in \( f_{\text{max}} \).
3. Extract the pseudo-Wigner amplitude of the extreme value index on the time-axis data \( y(f_i), i = 1, 2, 3 \), normalize and find the standard deviation.

\[ ypk_i = \text{std}(\frac{y(f_i) - \min(y(f_i))}{\max(y(f_i)) - \min(y(f_i))}) \]  

(8)
In the above step (2), due to the influence of interference, a plurality of extremum data indexes $f_{pk}$ may be generated in the vicinity of the actual extremum data, thereby generating a larger amount of calculation for the step (3) and outputting some unnecessary results. The process needs to further process the extracted extremum data as follows:

1. Perform differential processing on $f_{pk}$ and return a data index of less than 40 in the difference result.
   \[ \beta = \text{FindIndex}(\text{diff}(f_{pk}) < 40) + 1 \]

2. Traversing $\beta$, if the index of the adjacent element is not continuous, look for the maximum in the peak values of the current index and the one-digit index after the current segment, and the maximum value is defined as the peak data actually needed to achieve further optimization of extreme value data.

4. Experiment

In order to verify the effectiveness of the above process, the method proposed in this paper is simulated and verified, and the sampling rate is set to 7100. The following waveform signal is constructed and 8dB Gaussian white noise is added. The formula is as shown in equation (9), The electrical signal current time waveform is shown in figure 1.

\[ y = \sum_{n=1}^{4} x_{n,m} \sin(100(2m-1)\pi t_n) \quad (m = 1, 2, 3, 4) \]

![Current-time change diagram](image):

**Figure 1.** Electric signal current time waveform.

The above figure contains four types of waveform signals, each of which lasts for 6 cycles. The midpoint of the initial window function is set as $n_0 = 0.12$, the step length of each sliding of the window function is $n_{0,\text{step}} = 0.12$, and the distance between the endpoint of the window function and the midpoint of the window function is $k = 0.12$. Therefore, the signals in Figure 1 are all intercepted by moving the window function three times. First, the first segment of the power signal $s(n_0)$ is intercepted, the current time waveform, the contour diagram of pseudo Wigner transform and pseudo Wigner transform of the electrical signal is shown in figure 2 (left). The extreme frequency of $s(n_0)$ segment signals and the amplitude information of corresponding time axis are shown in figure 2(right).
Figure 2. Information about the first signal

It is easy to see that the amplitude of pseudo-Wigner transform changes greatly before and after the time \( t = 0.12 \) when the frequency of \( s(n_0) \) signal is about 100Hz. By normalizing the extremum data and solving the standard deviation and comparing with the set empirical threshold, it is possible to achieve high sensitivity discrimination of the waveform signal changes in the segment waveform. The threshold value \( \alpha = 0.3 \) is defined, that is, when the standard deviation is greater than \( \alpha \), switch events are considered to have occurred, and when the standard deviation is less than \( \alpha \), no switch events are considered to have occurred. The \( s(n_0) \) segment signal pseudo Wigner transform extreme frequency and corresponding normalized data standard deviation are shown in table 1.

Table 1. \( s(n_0) \) pseudo-Wigner transform extreme frequency and corresponding normalized data standard deviation

| Serial number | Frequency | Standard deviation |
|---------------|-----------|--------------------|
| 1             | 50        | 0.17564            |
| 2             | 102       | 0.34992            |
| 3             | 143.75    | 0.28889            |

Similarly, moving this window function intercepts the second signal \( s(n_1) \), the current time waveform, the contour diagram of pseudo Wigner transform and pseudo Wigner transform of the electrical signal is shown in Figure 3 (left). The extreme frequency of \( s(n_1) \) segment signals and the amplitude information of corresponding time axis are shown in figure 3 (right). The \( s(n_1) \) segment signal pseudo Wigner transform extreme frequency and corresponding normalized data standard deviation are shown in table 2.

Table 2. \( s(n_1) \) pseudo-Wigner transform extreme frequency and corresponding normalized data standard deviation

| Serial number | Frequency | Standard deviation |
|---------------|-----------|--------------------|
| 1             | 50        | 0.20958            |
| 2             | 100       | 0.38079            |
| 3             | 150       | 0.37011            |
| 4             | 200       | 0.26282            |
Moving this window function intercepts the third signal $s(n_3)$, the current time waveform, the contour diagram of pseudo Wigner transform and pseudo Wigner transform of the electrical signal is shown in Figure 4 (left). The extreme frequency of $s(n_3)$ segment signals and the amplitude information of corresponding time axis are shown in figure 4 (right). The $s(n_3)$ segment signal pseudo Wigner transform extreme frequency and corresponding normalized data standard deviation are shown in table 3.

| Serial number | Frequency | Standard deviation |
|---------------|-----------|--------------------|
| 1             | 50        | 0.14512            |
| 2             | 100       | 0.38396            |
| 3             | 150       | 0.34242            |
| 4             | 197.92    | 0.27629            |

5. Conclusions
In summary, the signal is transformed by PWVD to obtain the analysis results, and the weak signal is detected according to the method in this paper. The waveform changes are determined by comparing
the standard deviation of normalized data and threshold, so as to determine whether there is a switch event and state switching. The waveform state change is the first step in the determination of the switching event and the change of the operating state in the power signal. The accurate judgment of the process determines the accuracy and quality of the subsequent extracted waveform data. The PWVD method is used to fully preserve the pseudo-Wigner distribution information of the signal, and the method proposed in this paper can simplify the calculation steps and improve the computational efficiency. The simulation experiment proves that the method has good judgment ability for state change under the condition that the effective value of the waveform is close and has certain noise, and it has certain reference value and practical application ability.

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