RESPONSE IN THE CONTINUUM FOR LIGHT DEFORMED NEUTRON-RICH NUCLEI *

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The time-dependent Hartree-Fock calculation with a full Skyrme energy functional has been carried out on the three-dimensional Cartesian lattice space to study E1 giant dipole resonances (GDR) in light nuclei. The outgoing boundary condition for the continuum states is treated by the absorbing complex potential. The calculation for GDR in $^{16}$O suggests a significant influence of the residual interaction associated with time-odd densities in the Skyrme functional. We also predict a large damping for superdeformed $^{14}$Be at the neutron drip line.

1. Time-dependent approach to nuclear response in the continuum

The quantum-mechanical problems are usually solved in the energy (time-independent) representation. Namely, we either solve an energy eigenvalue problem for bound states or, for scattering states, we calculate a wave function with a proper boundary condition at a given energy. However, if one wishes to calculate physical quantities in a wide energy region, the time-dependent approach is very useful because a single time propagation provides information for a certain range of energy. Another advantage may be an intuitive picture provided by the time evolution of the wave function.

In Ref. 1, we have calculated molecular photoabsorption cross sections in the electronic continuum by using the time-dependent and time-independent approaches. The results indicate the capability and efficiency

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of the time-dependent method. In the present paper, we will show nuclear response calculations using the same technique, the time-dependent Hartree-Fock (TDHF) method combined with the absorbing boundary condition (ABC).

The TDHF state consists of $A$-occupied single-particle wave functions, $\{\phi_i(r, t)\}_{i=1,\ldots,A}$, each of which is complex and has two components (spinor). The three-dimensional (3D) Cartesian coordinate, $(x, y, z)$, is discretized in rectangular lattice and derivatives are estimated with the nine-point formula. The time evolution is determined by

$$\phi_i(r, t) = \exp \left(-i \int_0^t dt' h_{\text{HF}}[\rho(t')]\right) \phi_i(r, 0),$$

(1)

where the HF Hamiltonian, $h_{\text{HF}}[\rho(t)]$, depends on $\phi_i(r, t)$ and their derivatives. The initial state, $\{\phi_i(r, 0)\}_{i=1,\ldots,A}$, is chosen as the HF ground state wave function perturbed by an instantaneous external field, $\phi_i(t = 0) = e^{i k \hat{F}} \phi_i^0$. Here, the coupling parameter, $k$, can be arbitrary but should be small to validate the Fourier analysis (linear response approximation). The time variable is also discretized in a small step $\Delta t$ and the exponential operator in Eq. (1) is approximated by the fourth-order expansion,

$$\exp \left(-i \int_0^{\Delta t} dt' h_{\text{HF}}[\rho(t')]\right) \approx \sum_{n=0}^{4} \frac{1}{n!} (h_{\text{HF}}[\rho(\Delta t/2)])^n.$$

(2)

The time evolution of the expectation value of the external field, $\langle \Psi(t)|\hat{F}^\dagger\Psi(t)\rangle$, is computed, then we utilize the Fourier transform to obtain the strength function, $S(E) \propto E \sum_n \delta(E - E_n)||n|\hat{F}|0||^2$.

Next, let us discuss the treatment of the continuum. During the time evolution of the TDHF state, a part of the wave function may escape from the nuclear binding and become an outgoing wave. In the time-dependent approach, it is very difficult to impose the outgoing boundary condition (OBC) explicitly, because the outgoing particles have different energies. Furthermore, the boundary condition for non-spherical systems requires involved computation. Thus, instead of exactly treating the OBC, we use the absorbing boundary condition (ABC) which can be practically identical to the OBC. In the ABC method, the Cartesian coordinate space ($r < R_0$) is divided into two parts: an interacting region ($r < R_c$) and a non-interacting region ($R_c < r < R_0$). We introduce a complex absorbing potential, $-i \eta(r)$, active only in the non-interacting region. Then, we solve the TDHF equation of motion, Eq. (1), with the box boundary condition (BBC) at $r = R_0$, $\phi_i(r, t)|_{r=R_0} = 0$ for all occupied orbitals. This means
that the treatment of the continuum simply requires an addition of the absorbing potential to the HF Hamiltonian.

$$\left. h_{HF}[\rho(t)] \right|_{OBC} \mid_{r=R_0} \approx \left. \left( h_{HF}[\rho(t)] - i\tilde{\eta}(r) \right) \right|_{BBC} \mid_{r=R_0}. \quad (3)$$

We have demonstrated accuracy and practicality of the ABC method both in the time-dependent\textsuperscript{1,2,3,4} and the time-independent approaches.\textsuperscript{5,6}

2. Applications

2.1. Giant dipole resonance in $^{16}$O

Giant dipole resonance (GDR) in $^{16}$O has been studied extensively.\textsuperscript{7} Since $^{16}$O is a doubly-closed spherical nucleus, the continuum RPA\textsuperscript{8} is applicable. A microscopic calculation with the Skyrme energy functional was done\textsuperscript{9} and showed a two-peak structure for GDR in $^{16}$O. However, the calculation is not fully self-consistent because the residual Coulomb, spin-orbit, and $\sigma \cdot \sigma$ part of the interaction are neglected. According to the best of our knowledge, the continuum RPA calculation with the Skyrme functional has never been carried out fully self-consistently. Therefore, it is worthwhile to perform a fully self-consistent TDHF+ABC calculation and to investigate effects of the neglected part of the residual interaction.

We use the Skyrme energy functional of Ref.\textsuperscript{10} with the SGII parameter set. The 3D sphere of radius $R_0 = 22$ fm is adopted as a model space. The mesh spacing is $\Delta x \approx \Delta y \approx \Delta z \approx 1$ fm inside the interacting region but gradually increase up to $\Delta \approx 3$ fm in the non-interacting region. After obtaining the HF ground state of $^{16}$O, the isovector dipole field, $\vec{F} = z_n - z_p$, is activated instantaneously at $t = 0$. The time evolution is computed up to $T = 30$ $\hbar$/MeV. The result of the Fourier transform is shown in Fig. 1 (b). Apparently, we observe only one peak at $E \approx 20$ MeV, except for small peaks and a shoulder. This is inconsistent with results of the continuum RPA. The disagreement turns out to be due to the fact that the continuum RPA neglects part of the residual interaction. In the real-time calculation, we must use the same density functional as the one to define the HF ground state. Therefore, it is impossible to neglect the residual Coulomb and spin-orbit interaction. However, it is possible to neglect time-odd densities in the functional, because this does not affect the ground state. The calculation without the time-odd densities is shown in Fig. 1 (a). Now, the result becomes very similar to that of the continuum RPA and there are two peaks in the continuum. There are still some discrepancies related to the residual Coulomb and spin-orbit force. Our result indicates an important
role of the time-odd density part of the residual interaction. Since the
time-odd part of the Skyrme energy functional is not well determined by
the ground-state properties, this might provide a useful constraint.

![Graph](image-url)

Figure 1. Photoabsorption cross section for $^{16}$O calculated with different Skyrme energy
functionals. (a) Functional with SGII parameter set but neglecting all the time-odd
densities. (b) Full Skyrme functional with the SGII parameter set.

2.2. Giant dipole resonance in $^{8,14}$Be

Utilizing the same parameters and model space as $^{16}$O, we discuss $E1$ resonances in light deformed nuclei, $^{8}$Be and $^{14}$Be. The quadrupole deformation of the HF ground states of these nuclei are as large as $\beta \sim 1$ (prolate). Calculated $E1$ oscillator strengths are shown in Fig. 2. We can see a large deformation splitting of the GDR for $^{8}$Be. The low-energy peak is sharper and located around 7 MeV of excitation energy. This peak is associated with the isovector dipole oscillation along the symmetry axis ($z$-axis). The higher peak is rather broad and located around 22 MeV. The difference in the broadening leads to a greater peak height for the low-energy resonance, although the integrated oscillator strength is twice larger for the high-energy resonance.

For $^{14}$Be at the neutron drip line, the average peak positions for low- and high-energy resonances are similar to those in $^{8}$Be. However, the excess neutrons significantly increase the peak width of both resonances. The large deformation splitting almost vanishes in the total strength (thick solid line). It is worth noting that the present calculation takes account of the escape and the Landau damping width but does not include the spreading width. Some additional broadening might make the total strength look like a single peak with a large damping.
Figure 2. $E1$ Oscillator strength distribution for $^8$Be and $^{14}$Be calculated with the full Skyrme energy functional with the SGII parameters. The thin solid (dashed) line indicates the $E1$ oscillator strength associated with the oscillation parallel (perpendicular) to the symmetry axis.

3. Conclusion

We have carried out the Skyrme TDHF calculation with the ABC in real time for $E1$ resonances in the continuum. The time-odd density components in the Skyrme energy functional may influence considerably the isovector GDR strength distribution. The GDR in Be isotopes have been studied and the large deformation splitting and large broadening are predicted for a drip-line nucleus, $^{14}$Be.

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