Generalized BRST symmetry for arbitrary spin conformal field theory

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ABSTRACT

We develop the finite field-dependent BRST (FFBRST) transformation for arbitrary spin-s conformal field theories. We discuss the novel features of the FFBRST transformation in these systems. To illustrate the results we consider the spin-1 and spin-2 conformal field theories in two examples. Within the formalism we found that FFBRST transformation connects the generating functionals of spin-1 and spin-2 conformal field theories in linear and non-linear gauges. Further, the conformal field theories in the framework of FFBRST transformation are also analyzed in Batalin–Vilkovisky (BV) formulation to establish the results.

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1. Introduction

Conformal field theories (CFT) [1] have been at the center of much attention during the last seventeen years mainly because they provide models for genuinely interacting quantum field theories, they describe two-dimensional critical phenomena, and they play a central role in string theory, at present the most promising candidate for a unifying theory of all forces. Much attention has been given to conformal field theories in higher dimensions due to their role in the AdS/CFT correspondence [2,3]. AdS3/CFT2 is one of the most hot topics nowadays as it may be amenable to the integrability approach that proved very successful especially in the case of AdS4/CFT3 [4]. The AdS/CFT correspondence has also been investigated for scalar fields [5–7], gauge fields [7], spinors [8], classical gravity [9] and type IIB string theory [10,11]. The AdS/CFT correspondence is used to calculate CFT correlators from the classical AdS theories of vector and Dirac fields and the connection between the AdS and boundary fields is properly treated via a Dirichlet boundary value problem [6].

Recently, in the framework of gauge invariant approach involving Stueckelberg fields the totally symmetric arbitrary spin-s anomalous conformal current and shadow field are studied and gauge invariant two-point vertex of the arbitrary spin anomalous shadow field is also obtained [12]. In Stueckelberg gauge frame, the two-point gauge invariant vertex becomes the standard two-point vertex of CFT. The logarithmic divergence of the BRST invariant action of arbitrary spin-s canonical shadow field turns out to be BRST invariant action of arbitrary spin-s conformal field [13]. The BRST invariant action of conformal field interprets geometrically the boundary values of massless AdS fields [13]. The study of BRST quantization which helps in proving the renormalizability of gauge theories is extremely important in the context of CFT.

Although BRST symmetry has been discussed for conformal field theory [13], the generalization of it by making the parameter field-dependent, so-called FFBRST transformation, has not yet been investigated. The FFBRST formulation, which was introduced for the first time by Joglekar and Mandal [14], has been studied considerably in various contexts [15–28]. For example, such formulation helps in calculating a correct prescription for poles in the gauge field propagators in noncovariant gauges by connecting the covariant gauges and noncovariant gauges of the theory [15,18]. The celebrated Gribov problem [29,30] of QCD has also been addressed through FFBRST transformation in Euclidean space [20]. Further, such formulation has been investigated for YM theory explaining low-energy dynamics via Cho–Faddeev–Niemi (CFN) decomposition. So, it is worth analyzing such formulation at both classical and quantum levels for conformal field theories. This provides a motivation for the analysis of FFBRST transformation in conformal field theory in present investigation.

We further like to extend our FFBRST formulation for CFT in the framework of Batalin–Vilkovisky (BV) formalism [31–35] which is one of the most powerful techniques to study gauge field theories and allows us to deal with very general gauge theories, including those with open or reducible gauge symmetry algebras. The BV method provides a convenient way of analyzing the possible violations of symmetries by quantum effects [32]. It is usually used to perform the gauge-fixing in quantum field theory, but was also
applied to other problems like analyzing possible deformations of the action and anomalies. The BRST-BV approach is successful for studying the manifestly Lorentz invariant formulation of string theory [36].

In this paper we generalize the FFBRST transformation for arbitrary spin-s conformal field theory by making the parameter finite and field dependent. Within the formulation, we find that the functional measure leads to a non-trivial Jacobian. This Jacobian can be exponentiated if it satisfies a certain condition. As a result the effective action gets modified. We compute the Jacobians for spin-1 and spin-2 conformal fields for particular choices of finite field-dependent parameters. We render that these calculated Jacobians play an important role in mapping of linear and nonlinear gauges. The analyzed BV formulation validates the results at quantum level. For BV formulation we extend the configuration space by introducing antifield corresponding to each field with opposite statistics. With such introduction of antifield the consequent extended action satisfies the mathematically rich quantum master equation.

The paper is presented in the following manner. In Section 2, we generalize the BRST transformation for arbitrary spin-s conformal field theory. We illustrate this generalization by two examples of spin-1 and spin-2 conformal fields in Section 3. We extend this formulation in the BV framework in Section 4. At the end we summarize the results.

2. Constructing FFBRST transformation for arbitrary spin-s conformal field theory

In this section we construct the FFBRST transformation for conformal field theory following the method advocated in [14]. Let us begin with the effective action for arbitrary spin-s conformal field theory defined by [13],

$$S_{\text{tot}} = \int d^dx \left[ \frac{1}{2s!} \sum_{\ell = 0}^{s} \mathcal{L}^{\ell} + \sum_{s' = 0}^{s-1} \mathcal{L}^{s'} \right],$$

where

$$\mathcal{L}^{\ell} = \frac{1}{2s!} \left( \phi^{a_1 ... a_{\ell'}} \phi^{\ell}_a \phi^{a_1 ... a_{\ell'}} \right) - \frac{a'(s'-1)}{4} \phi^{a_0 a_1 ... a_{s'}} \left( \phi^{a_{s'} a_1} \phi^{a_0 a_{s'+1} a_{s'}} \right),$$

$$\mathcal{L}^{s'} = \frac{1}{s!} \left( \phi^{a_1 ... a_{s'}} \right)^{s'} \phi^{a_1 ... a_{s'+1}} \phi^{a_1 ... a_{s'}}, \quad \nu_{s'} = s' + \frac{d-4}{2}.$$ (2)

This effective action is invariant under the usual BRST transformation for the collective fields $\phi^{a_1 ... a_{s'}} (x)$ as follows for the conformal field theory compactly as follows [13]

$$\delta_{\lambda} \phi^{a_1 ... a_{s'}} = \frac{d}{dk} \mathcal{R}[\phi^{a_1 ... a_{s'}} (x, k)] + \mathcal{S}_{\lambda}[\phi^{a_1 ... a_{s'}} (x, k)],$$

where $\mathcal{R}[\phi^{a_1 ... a_{s'}} (x)] = \phi^{a_1 ... a_{s'}} (x)$ is the generic Slavnov variation of the fields $\phi^{a_1 ... a_{s'}} (x)$ written collectively and $\delta_{\lambda}$ is the infinitesimal anticommuting global parameter of transformation.

Now we make the parameter $\delta_{\lambda}$ finite and field-dependent by interpolating a continuous parameter $k$ through fields which is bounded between 0 and 1. The infinitesimal-field dependent BRST transformation is constructed as follows [14]

$$\frac{d}{dk} = \mathcal{R}[\phi^{a_1 ... a_{s'}} (x, k)] = \mathcal{R}[\phi^{a_1 ... a_{s'}} (x, k)] + \mathcal{S}_{\lambda}[\phi^{a_1 ... a_{s'}} (x, k)],$$

where $\mathcal{R}[\phi^{a_1 ... a_{s'}} (x, k)]$ is an infinitesimal but the field-dependent parameter. The FFBRST transformation (denoted by $\delta_{\lambda}$) then can be obtained by integrating the above transformation from $k = 0$ to $k = 1$, as follows:

$$\delta_{\lambda} \phi^{a_1 ... a_{s'}} (x, k) = \mathcal{R}[\phi^{a_1 ... a_{s'}} (x, k)] + \mathcal{S}_{\lambda}[\phi^{a_1 ... a_{s'}} (x, k)].$$ (3)

where

$$\mathcal{S}_{\lambda}[\phi^{a_1 ... a_{s'}} (x, k)] = \mathcal{R}[\phi^{a_1 ... a_{s'}} (x, k)] - \mathcal{R}[\phi^{a_1 ... a_{s'}} (x, k)] = \mathcal{S}_{\lambda}[\phi^{a_1 ... a_{s'}} (x, k)],$$

is the finite field-dependent parameter and $\mathcal{R}[\phi^{a_1 ... a_{s'}} (x, k)]$ is given by

$$\mathcal{R}[\phi^{a_1 ... a_{s'}} (x, k)] = \sum_{i} \int d^4x \frac{\delta \mathcal{S}_{\lambda}[\phi^{a_1 ... a_{s'}} (x) \mathcal{S}]}{\delta \phi^{a_1 ... a_{s'}} (x, k)} S_{\lambda}[\phi^{a_1 ... a_{s'}} (x, k)].$$ (7)

This FFBRST transformation leaves effective action of a conformal field theories invariant. However, the functional measure changes non-trivially under such finite transformation.

Now we compute the Jacobian of the path integral measure defined generically by $D\phi^{a_1 ... a_{s'}} (x)$ for an arbitrary finite field-dependent parameter, $\mathcal{S}_{\lambda}[\phi^{a_1 ... a_{s'}} (x, k)]$, as follows

$$D\phi^{a_1 ... a_{s'}} (x) = J(k) D\phi^{a_1 ... a_{s'}} (x).$$ (8)

The Jacobian $J(k)$ of the path integral measure is thus obtained as a functional of fields. We exponentiate it by defining a local functional $S_{1}[\phi^{a_1 ... a_{s'}} (x, k)]$ in the following manner:

$$J(k) \mapsto e^{S_{1}[\phi^{a_1 ... a_{s'}} (x, k)]}.$$ (9)

Preserving the quantitative (physical) changes of the functional integral in conformal field theory leads to the following condition [14]

$$\int D\phi^{a_1 ... a_{s'}} (x) \left[ \frac{d}{dk} \ln J(k) \right] = -i dS_{1}[\phi^{a_1 ... a_{s'}} (x, k)] \exp[(S_{\text{tot}} + S_{1})] = 0.$$ (10)

The local functional $S_{1}[\phi^{a_1 ... a_{s'}} (x, k)]$ satisfies the following initial boundary condition $S_{1}[\phi^{a_1 ... a_{s'}} (x, k)] = 0$ to ensure $J = 1$, when fields do not change.

The infinitesimal change in Jacobian, $J(k)$, given in (10), has the explicit expression in terms of $\mathcal{S}_{\lambda}$ as follows

$$\frac{d}{dk} \ln J(k) = - \int d^4y \left[ \partial_{y_1} \frac{\partial \mathcal{S}_{\lambda}[\phi^{a_1 ... a_{s'}} (y, k)]}{\partial \phi^{a_1 ... a_{s'}} (y, k)} \right].$$ (11)

where, for bosonic fields, $+$ sign is used and $-$ for fermionic fields.

Therefore, performing FFBRST transformation changes the exponential action of the generating functional given in conformal field theory as following:

$$\int D\phi^{a_1 ... a_{s'}} (x) e^{S_{\text{tot}}} \mapsto \int D\phi^{a_1 ... a_{s'}} (x) e^{(S_{\text{tot}} + S_{1})},$$ (12)

where $S_{\text{tot}}$ is the most general effective action for CFT given in (1). To illustrate these results we would like to consider specific examples in the next sections.
3. BRST invariant conformal fields

In this section, we consider the two examples of BRST symmetric conformal field theory. We study the construction and implementation of FFBRST transformation on these theories explicitly.

3.1. Spin-1 conformal field

The BRST invariant action for spin-1 conformal field (a particular form of (11) in linear gauge is given by

\[
S_{\text{tot}} = \int d^4x \left[ -\frac{1}{4} F^{ab} (\partial^d \partial^e) (\partial^d \partial^e) - b (\partial^d \partial^e) (\partial^d \partial^e) + \frac{1}{2} b (\partial^d \partial^e) b + c (\partial^d \partial^e) b_{k+1} \right],
\]

where field-strength \( F^{ab} = \partial^a \phi^b - \partial^b \phi^a \). Here \( \phi^a, b, c \) and \( \partial \) are spin-1 conformal field, Nakanishi–Lautrup field, ghost field and antighost field, respectively. In terms of gauge-fixing fermion the above action can be described by

\[
S_{\text{tot}} = \int d^4x \left[ -\frac{1}{4} F^{ab} (\partial^d \partial^e) (\partial^d \partial^e) - b (\partial^d \partial^e) (\partial^d \partial^e) + \frac{1}{2} b (\partial^d \partial^e) b + c (\partial^d \partial^e) b_{k+1} S_0 \psi^L \right],
\]

where \( \psi^L = c (\partial^d \partial^e) (\partial^d \partial^e) + \frac{1}{2} b (\partial^d \partial^e) b \). The fermionic rigid BRST transformations of the fields are

\[
S_0 \psi^a = -\bar{\psi}^c \psi^a, \quad S_0 b = 0, \quad S_0 c = 0, \quad S_0 \bar{\psi} = b.
\]

The generating functional for spin-1 conformal field theory corresponding to (13) is defined by

\[
Z^L[0] = \int D\phi^a D\psi^L Dc D\bar{c} \exp(iS_{\text{tot}}).
\]

However, the BRST invariant action for spin-1 conformal field in non-linear (quadratic) gauge is given by

\[
S_{\text{tot}}^{\text{quad}} = \int d^4x \left[ -\frac{1}{4} F^{ab} (\partial^d \partial^e) (\partial^d \partial^e) + b (\partial^d \partial^e) (\partial^d \partial^e) - b (\partial^d \partial^e) b + \frac{1}{2} b (\partial^d \partial^e) b_{k+1} + c (\partial^d \partial^e) b + 2c (\partial^d \partial^e) b_{k+1} \right],
\]

which remains invariant under the same set of BRST transformations given in (15). Following the method given in Section 2, we construct the FFBRST transformation as follows:

\[
\delta \phi^a = -\bar{c} \Theta[\psi^a], \quad \delta b = 0, \quad \delta c = 0, \quad \delta \bar{\psi} = b \Theta[\psi^a],
\]

where \( \Theta[\psi^a] \) is an arbitrary finite field-dependent BRST parameter.

Now, we construct a particular \( \Theta[\psi^a] \) to calculate the Jacobian for path integral measure whose infinitesimal version is evaluated as follows

\[
\Theta[\psi^a] = -i \int d^4x \left[ \bar{c} (\partial^d \partial^e) (\partial^d \partial^e) \phi^a \right].
\]

Now we calculate the change in Jacobian with respect to continuous parameter \( \kappa \) as follows

\[
\frac{1}{f(\kappa)} \frac{d f(\kappa)}{d \kappa} = i \int d^4x \left[ -b (\partial^d \partial^e) (\partial^d \partial^e) \phi^a + 2c (\partial^d \partial^e) (\partial^d \partial^e) \phi^a \right],
\]

where we have utilized the relation (11).

To exponentiate the Jacobian we propose the following local functional

\[
S_{1}[\psi^a] = \int d^4x \left[ \left( \bar{c} (\partial^d \partial^e) (\partial^d \partial^e) \phi^a + 2c (\partial^d \partial^e) (\partial^d \partial^e) \phi^a \right) \right],
\]

where \( \xi_1 \) and \( \xi_2 \) are \( k \)-dependent arbitrary constant parameters. Eqs. (20) and (21) together with (10) yield the following linear differential equations:

\[
\xi_1 + 1 = 0, \quad \xi_2 - 2 = 0.
\]

The exact solutions of the above equations satisfying the boundary condition \( (\xi_1, \xi_0) = 0 \) are

\[
\xi_1 = -\kappa, \quad \xi_2 = 2 \kappa.
\]

With these identifications the expression of local functional becomes

\[
S_{1}[\psi^a] = \int d^4x \left[ -\kappa b (\partial^d \partial^e) (\partial^d \partial^e) \phi^a + 2\kappa c (\partial^d \partial^e) (\partial^d \partial^e) \phi^a \right].
\]

This is evident from above expression that at \( \kappa = 0 \) the functional \( S_1 \) vanishes. However, at \( \kappa = 1 \) this takes the following form:

\[
S_{1}[\psi^a]_{k=1} = \int d^4x \left[ -b (\partial^d \partial^e) (\partial^d \partial^e) \phi^a + 2c (\partial^d \partial^e) (\partial^d \partial^e) \phi^a \right].
\]

So, according to (12), after performing the FFBRST transformation on generating functional the effective action (13) modifies by

\[
S_{\text{tot}} + S_{1}[\psi^a]_{k=1} = S_{\text{tot}}^{\text{quad}}.
\]

Therefore, we observe that the FFBRST transformation on generating functional of spin-1 conformal theory in linear gauge changes the effective action from linear gauge to quadratic gauge within functional integral. Here we note that the FFBRST transformation amounts the precise change on the BRST exact part of the effective action. We construct the finite parameter in such a manner that Jacobian of the path integral measure amounts change in the BRST-exact part of the effective action.

3.2. Spin-2 conformal field

The classical action for spin-2 conformal field theory (a particular form of (11)) is given by

\[
S_{\text{inv}} = \int d^4x \left[ R_{\text{lin}}^{ab}(\partial^d \partial^e)_{k-1} R_{\text{lin}}^{ab} \right] - \frac{d}{4(d-1)} R_{\text{lin}}^{ab}(\partial^d \partial^e)_{k-1} R_{\text{lin}}^{ab}, \quad k = d - 2.
\]

where \( R^{ab} \) is expressed by

\[
R_{\text{lin}}^{ab} = \frac{1}{2} \left( (\partial^d \partial^e) (\partial^d \partial^e) + b \phi^a \phi^b - a \phi^a \phi^b \right).
\]

The gauge-fixing and ghost action is given together by

\[
S_{gf} = \int d^4x \left[ -b (\partial^d \partial^e) (\partial^d \partial^e) \phi^a - \frac{1}{2} b \phi^a \phi^b \right] + \frac{1}{u^2} (b - a b) (\partial^d \partial^e)_{k-1} + (a \phi^a \phi^e - (\partial^d \partial^e) (\partial^d \partial^e) \phi^a) + b (\partial^d \partial^e) (\partial^d \partial^e) + \frac{1}{2u^2} (b - a b)_{k-1} (b - a b) + b (\partial^d \partial^e)_{k-1} (b - a b) + \frac{1}{2} (\partial^d \partial^e) (\partial^d \partial^e) \phi^a \phi^b.
\]
So, the complete action is given by
\[ S_{\text{tot}} = S_{\text{inv}} + S_{\text{gf}}, \]
which is invariant under following BRST transformation:
\[
\delta_0 \phi^{ab} = - \left( \partial^a c^b + \partial^b c^a + \frac{2}{d-2} \eta^{ab} c \right) \delta \lambda, \\
\delta_0 \phi^a = - (\partial^a c - \partial^b c^b) \delta \lambda, \\
\delta_0 \phi = u \partial^a c \delta \lambda, \\
\delta_0 \phi^a = 0, \quad \delta_0 = 0, \\
\delta_0 c^a = b^a \delta \lambda, \quad \delta_0 c = b \delta \lambda, \\
\delta_0 b^a = 0, \quad \delta_0 b = 0, \
\]
where \( \delta \lambda \) is infinitesimal, anticommuting parameter. The FFBRST transformation is constructed by
\[
\delta_f \phi^{ab} = - \left( \partial^a c^b + \partial^b c^a + \frac{2}{d-2} \eta^{ab} c \right) \Theta [\phi^{a_1 a_2}], \\
\delta_f \phi^a = - (\partial^a c - \partial^b c^b) \Theta [\phi^{a_1 a_2}], \\
\delta_f \phi = u \partial^a c \Theta [\phi^{a_1 a_2}], \\
\delta_f \phi^a = 0, \quad \delta_f c^a = 0, \quad \delta_f c = 0, \\
\delta_f b^a = b^a \Theta [\phi^{a_1 a_2}], \quad \delta_f b = b \Theta [\phi^{a_1 a_2}], \\
\delta_f b^a = 0, \quad \delta_f b = 0. \
\]
To construct the finite field-dependent parameter \( \Theta [\phi^{a_1 a_2}] \) we choose the following infinitesimal parameter:
\[
\Theta [\phi^{a_1 a_2}] = - \int d^d x \left[ \sum_{l=1}^{\infty} \left( \phi^{a_1 a_2} \left( \phi^{b_1 b_2} - \frac{1}{2} \phi^a \phi^b \right) \right) \right]. \
\]
The change in Jacobian under FFBRST transformation is calculated by
\[
\frac{1}{J_f} \frac{d J_f}{d \kappa} = i \int d^d x \left[ - b^a (\partial^a a^1) \left( \phi^{b_1 b_2} - \frac{1}{2} \phi^a \phi^b \right) \right] \\
+ \phi^{a_1 a_2} \phi^{b_1 b_2} d \phi^{a_1 a_2}. \
\]
Keeping the forms of effective action in linear and quadratic gauges in mind we make an ansatz for \( S_1 \) in this case as follows
\[
S_1[\phi^{a_1 a_2}] = \int d^d x \left[ \xi_1 (\phi^{a_1 a_2})^k (\phi^{b_1 b_2} - \frac{1}{2} \phi^a \phi^b) \right] \\
+ \xi_2 (\phi^{a_1 a_2})^k \phi^{b_1 b_2} d \phi^{a_1 a_2} \\
+ \xi_3 (\phi^{a_1 a_2})^k \phi^{b_1 b_2} d \phi^{a_1 a_2} \\
+ \xi_4 (\phi^{a_1 a_2})^k \phi^{b_1 b_2} d \phi^{a_1 a_2} \\
+ \xi_5 (\phi^{a_1 a_2})^k \phi^{b_1 b_2} d \phi^{a_1 a_2} \\
+ \xi_6 (\phi^{a_1 a_2})^k \phi^{b_1 b_2} d \phi^{a_1 a_2} \\
+ \xi_7 (\phi^{a_1 a_2})^k \phi^{b_1 b_2} d \phi^{a_1 a_2} \\
+ \xi_8 (\phi^{a_1 a_2})^k \phi^{b_1 b_2} d \phi^{a_1 a_2} \\
+ \xi_9 (\phi^{a_1 a_2})^k \phi^{b_1 b_2} d \phi^{a_1 a_2} \\
+ \xi_{10} (\phi^{a_1 a_2})^k \phi^{b_1 b_2} d \phi^{a_1 a_2}. \
\]
The essential condition (10) together with (34) and (35) yields the following differential equations for \( \xi_i \):
\[
\xi_1 + 1 = 0, \quad \xi_2 - 1 = 0, \quad \xi_3 + 1 = 0, \quad \xi_4 - 1 = 0, \\
\xi_5 + 1 = 0, \quad \xi_6 + \frac{2}{d-2} = 0, \quad \xi_7 + \frac{1}{2} = 0, \quad \xi_8 - \frac{1}{2} = 0, \\
\xi_9 + 1 = 0, \quad \xi_{10} + \frac{1}{d-2} = 0. \
\]
The exact solutions of these differential equations satisfying boundary condition \( (\xi_i(\kappa = 0) = 0) \) are given by
\[
\xi_1 = -\kappa, \quad \xi_2 = \kappa, \quad \xi_3 = -\kappa, \quad \xi_4 = \kappa, \\
\xi_5 = \kappa, \quad \xi_6 = \frac{2}{d-2} \kappa, \quad \xi_7 = -\frac{1}{2} \kappa, \quad \xi_8 = \frac{1}{2} \kappa, \\
\xi_9 = -\kappa, \quad \xi_{10} = -\frac{1}{d-2} \kappa. \
\]
With these solutions the expression of \( S_1 \) reduces to
\[
S_1[\phi^{a_1 a_2}] = \int d^d x \left[ - b^a (\partial^a a^1)^k (\phi^{b_1 b_2} - \frac{1}{2} \phi^a \phi^b) \right] \\
+ \kappa \phi^{a_1 a_2} (\phi^{b_1 b_2} - \frac{1}{2} \phi^a \phi^b) \right] \\
+ \kappa \phi^{a_1 a_2} (\phi^{b_1 b_2} - \frac{1}{2} \phi^a \phi^b) \right] \\
+ \frac{2}{d-2} \kappa \phi^{a_1 a_2} \phi^{b_1 b_2} - \frac{1}{2} \kappa \phi^{a_1 a_2} \phi^{b_1 b_2} \\
+ \frac{1}{2} \kappa \phi^{a_1 a_2} \phi^{b_1 b_2} - \frac{1}{d-2} \kappa \phi^{a_1 a_2} \phi^{b_1 b_2} \\
+ \frac{1}{2} \kappa \phi^{a_1 a_2} \phi^{b_1 b_2} - \frac{1}{d-2} \kappa \phi^{a_1 a_2} \phi^{b_1 b_2}. \
\]
which vanishes for \( \kappa = 0 \). However, for \( \kappa = 1 \) this reduces to
\[
S_1[\phi^{a_1 a_2}] = \int d^d x \left[ - b^a (\partial^a a^1)^k (\phi^{b_1 b_2} - \frac{1}{2} \phi^a \phi^b) \right] \\
+ \frac{2}{d-2} \phi^{a_1 a_2} \phi^{b_1 b_2} - \frac{1}{2} \phi^{a_1 a_2} \phi^{b_1 b_2} \\
+ \frac{1}{2} \phi^{a_1 a_2} \phi^{b_1 b_2} - \frac{1}{d-2} \phi^{a_1 a_2} \phi^{b_1 b_2}. \
\]
Now, after performing the FFBRST transformation the extended action for spin-2 conformal field, as mentioned in (12), is calculated by
\[
S_{\text{tot}} + S_1[\phi^{a_1 a_2}] = \int d^d x \left[ R^{ab}_{\text{lin}} (\partial^a a^1)^k - R^{ab}_{\text{lin}} \frac{d}{4(d-1)} R^{ab}_{\text{lin}} (\partial^a a^1)^k - R^{ab}_{\text{lin}} \right] \\
- b^a (\partial^a a^1)^k (\phi^{b_1 b_2} - \frac{1}{2} \phi^a \phi^b) \right] \\
+ \frac{1}{2} \phi^{a_1 a_2} \phi^{b_1 b_2} - \frac{1}{d-2} \phi^{a_1 a_2} \phi^{b_1 b_2} \\
+ \frac{2}{d-2} \phi^{a_1 a_2} \phi^{b_1 b_2} - \frac{1}{d-2} \phi^{a_1 a_2} \phi^{b_1 b_2} \\
+ \frac{2}{d-2} \phi^{a_1 a_2} \phi^{b_1 b_2} - \frac{1}{d-2} \phi^{a_1 a_2} \phi^{b_1 b_2}. \
\]
+ \frac{1}{2} \bar{z}^2 (\partial' \partial'^k)^k \partial'^l \partial'^m \phi^m - \bar{z}^2 (\partial' \partial'^k)^k \partial^l \partial^m \phi^m - \frac{1}{d - 2} \bar{z} (\partial' \partial'^k)^k \eta^{ab} \partial^b \partial^a \phi^a \right]. \tag{40}

Here we observe that the final action obtained in (40) has non-linear gauge. Therefore, we observed that the FFRST transformation relates the generating functionals corresponding to linear and non-linear gauges for spin-2 conformal field also.

4. Conformal field theory in BV formulation

In this section, we extend the formulation using BV technique. For this purpose, we need to introduce the antifields (ψ^{a_2 \ldots a_r} \ast) corresponding to fields having opposite statistics in the configuration space. With the introduction of such antifields, the arbitrary extended quantum action, \( W_{\Psi}[\psi^{a_2 \ldots a_r}, \psi^{a_1 \ldots a_r} \ast] \), satisfies a certain rich mathematical relation, the so-called quantum master equation [32], which is given by

\[\Delta e^{iW_{\Psi}[\psi^{a_1 \ldots a_r} \ast, \psi^{a_1 \ldots a_r}]} = 0,\]

\[\Delta \equiv (-1)^{K} \frac{\partial^j}{\partial \phi^{a_1 \ldots a_r}} \frac{\partial^j}{\partial \partial^{a_1 \ldots a_r} \ast}, \tag{41}\]

where \( A \equiv (a_1, a_2, \ldots, a_r) \). Therefore, the extended quantum action \( W_{\Psi} \) with different gauge-fixing fermion \( \Psi \) are solutions of the quantum master equation. We would like to show that FFRST transformation with appropriate choice of finite field-dependent parameter relates different solutions of quantum master equation.

4.1. Spin-1 conformal field

In terms of field and antifields, the generating functional for the spin-1 conformal field theory in linear gauge is defined by

\[Z^L[0] = \int D\phi^a Db Dc D\epsilon \ e^{i} \int d^4x \left[ \frac{1}{4} F^a_{bc} (\partial' \partial'^k)^k \partial^b \phi^c + \frac{1}{2} (\partial' \partial'^k)^k b \right], \tag{42}\]

where \( \phi^a \) and \( \bar{c} \) are antifields corresponding to the \( \phi^a \) and \( c \) fields with opposite statistics. The above generating functional can further be recast compactly as

\[Z^L[0] = \int D\psi^a D\phi^a D\bar{c} D\epsilon \ e^{iW_{\Psi L}[\psi^a, \phi^a, \bar{c}, \epsilon]}, \tag{43}\]

where \( W_{\Psi L}[\psi^a, \phi^a, \bar{c}, \epsilon] \) is an extended quantum action (a solution of the quantum master equation defined later) for the conformal theory in linear gauge and \( \phi^a \) refers to the antifield generically corresponding to the collective field \( \phi^a (\equiv \phi^a, b, c, \bar{c}) \).

It is well-known that the antifields for a gauge theory can explicitly be computed from the gauge-fixed fermion. For the conformal theory in linear gauge the antifields are computed for the gauge-fixed fermion \( \Psi^L \) as follows:

\[\phi^a_L = \frac{\delta \Psi^L}{\delta \phi^a} = (\partial' \partial'^k)^k \partial^a \bar{c}, \]

\[b^a_L = \frac{\delta \Psi^L}{\delta b} = \frac{1}{2} (\partial' \partial'^k)^k \bar{c}, \]

\[\bar{c}^a_L = \frac{\delta \Psi^L}{\delta \bar{c}} = - (\partial' \partial'^k)^k \partial^a \phi + \frac{1}{2} (\partial' \partial'^k)^k b, \]

\[c^a_L = \frac{\delta \Psi^L}{\delta c} = 0. \tag{44}\]

With these identifications of antifields the extended quantum action in (42) coincides with the total effective action (13). However, for the non-linear gauge the gauge-fixing fermion is given by

\[\Psi^{NL} = \bar{c} \left[ -(\partial' \partial'^k)^k \partial^a \phi + (\partial' \partial'^k)^k \partial^a \phi^a + \frac{1}{2} (\partial' \partial'^k)^k b \right]. \tag{45}\]

The antifields for the above gauge-fixing fermion are estimated by:

\[\phi^a_L^{NL} = \frac{\delta \Psi^{NL}}{\delta \phi^a} = (\partial' \partial'^k)^k \partial^a \bar{c} - 2 (\partial' \partial'^k)^k \partial^a \bar{c}, \]

\[b^a_L^{NL} = \frac{\delta \Psi^{NL}}{\delta b} = \frac{1}{2} (\partial' \partial'^k)^k \bar{c}, \]

\[\bar{c}^a_L^{NL} = \frac{\delta \Psi^{NL}}{\delta \bar{c}} = - (\partial' \partial'^k)^k \partial^a \phi + (\partial' \partial'^k)^k \partial^a \phi^a + \frac{1}{2} (\partial' \partial'^k)^k b, \]

\[c^a_L^{NL} = \frac{\delta \Psi^{NL}}{\delta c} = 0. \tag{46}\]

Likewise the linear gauge case, the generating functional for the spin-1 conformal theory in non-linear gauge can be written in compact form as

\[Z^{NL}[0] = \int D\psi^a D\phi^a D\bar{c} D\epsilon \ e^{iW_{\Psi NL}[\psi^a, \phi^a, \bar{c}, \epsilon]}, \tag{47}\]

where \( W_{\Psi NL}[\psi^a, \phi^a \ast] \) is an extended quantum action (another solution of the quantum master equation) corresponding to non-linear gauge.

Now we construct the infinitesimal field/antifield dependent parameter as follows

\[\Theta[\phi^a, \phi^a \ast] = - i \int d^4 x \bar{c} \left( \bar{c}_L - \bar{c}_L^{NL} \right). \tag{48}\]

From this infinitesimal parameter the finite field/antifield dependent parameter can be calculated using the relation (6). The FFRST transformation with such field/antifield dependent parameter leads to the following Jacobian in the integrand of functional integral

\[J[\psi^{a_1 \ldots a_r} \ast (x)] = e^i \int d^4 x \left[ - \phi^a \partial^a \int_{\lim} \partial' \partial'^k \bar{c} + \phi^a \partial^a \int_{lim} \partial' \partial'^k \bar{c} \right], \tag{49}\]

which switches the generating functional of spin-1 conformal theory from one gauge to another.

Therefore, we establish the connection of the different solutions (\( W_{\Psi L}\) and \( W_{\Psi NL}\)) of the quantum master equation at quantum level through FFRST transformation with appropriately constructed finite field-dependent parameter.

4.2. Spin-2 conformal field

Introducing the antifields corresponding to fields, the generating functional for the spin-2 conformal field theory in linear gauge is defined by

\[Z^L[0] = \int D\phi^a D\psi^a D\phi^a Db D\bar{c} D\epsilon \ e^{i} \int d^4x \left[ R_{\lim} \left( \partial' \partial'^k \right)^{k-1} R_{ab} \right. \times \left. \exp \left[ i \int d^4x \left( R_{\lim} (\partial' \partial'^k)^k b + \phi^{a_2 \ast} \partial^a \partial'^k \partial^m \phi^m \right) \right] \right], \tag{50}\]

---

Footnote: We note in this case that the antifields depend on fields as these are expressed in terms of gauge-fixing fermion. Therefore this field/antifield dependent parameter actually depends on field only [37].
where \( \varphi^{a_1 a_2 \ast} \) are antifields corresponding to the \( \varphi^{a_1 a_2} (= \phi, \psi, \phi, \theta, b^i, \epsilon^a, c, \bar{c}, \bar{\epsilon}) \) fields generically with opposite statistics. This can further be written in compact notation as

\[
Z^L[0] = \int D\varphi^{a_1 a_2} e^{W_{\varphi^L}[\varphi^{a_1 a_2}, \varphi_0^{a_2 a_1 \ast}]},
\]

(51)

where \( W_{\varphi^L} [\varphi^{a_1 a_2}, \varphi_0^{a_2 a_1 \ast}] \) is the extended quantum action for spin-2 conformal theory in linear gauge.

In the same fashion, we define the generating functional for the spin-2 conformal theory for non-linear gauge in compact form as

\[
Z^{NL}[0] = \int D\varphi^{a_1 a_2} e^{W_{\varphi^{NL}}[\varphi^{a_1 a_2}, \varphi_0^{a_2 a_1 \ast}]},
\]

(52)

where \( W_{\varphi^{NL}} [\varphi^{a_1 a_2}, \varphi_0^{a_2 a_1 \ast}] \) is the extended quantum action corresponding to non-linear gauge.

We construct the infinitesimal field/antifield dependent parameter for this case as follows:

\[
G_{\varphi^L} [\varphi^{a_1 a_2}, \varphi_0^{a_2 a_1 \ast}] = -i \int d^d x \left[ \epsilon (\varphi_0^{a_2 a_1 \ast} - \varphi_0^{a_1 a_2}) + \varphi - \varphi_0^{a_1 a_2} \right].
\]

(53)

The finite field/antifield dependent parameter can be evaluated from relation (5). The FFRST formulation with such field/antifield dependent parameter leads to the following Jacobian in the integrand of functional integral

\[
J[\varphi^{a_1 a_2 \ldots a_r} (x)] = e^{i \int dx \left[ \varphi^{a_1 a_2 \ldots a_r} (\delta \varphi^{a_1 a_2}) - \varphi^{a_1 a_2 \ldots a_r} (\delta \varphi^{a_1 a_2}) \right]},
\]

(54)

which transforms the generating functional of spin-2 conformal theory from linear gauge to non-linear. Hence, the connection of the different solutions \( (W_{\varphi^L} \) and \( W_{\varphi^{NL}} \) of the quantum master equation for spin-2 is established through FFRST transformation with properly constructed parameter. In fact, any two solutions of quantum master equation are connected through FFRST transformation with different finite parameter.

5. Conclusions

In this paper, we have developed the FFRST transformation for arbitrary spin-s conformal field theory. We construct the FFRST transformation by making the transformation parameter finite and field-dependent. The parameter is made finite and field-dependent by making all the fields first (a continuous constant parameter) \( \kappa \)-dependent and then define an infinitesimal field-dependent BRST transformation. After that we integrate the parameter of infinitesimal field-dependent BRST transformation in the limiting values of \( \kappa \) which yields the finite field-dependent BRST parameter. The novelty of the FFRST transformation is that it leads to a local Jacobian for path integral measure and this Jacobian amounts a change in the BRST exact part of the effective action. Here we note that analogous to ordinary (non-conformal) quantum field theories the resulting Jacobians in the case of conformal field theories are still local in nature. This assures the consistency of generalized BRST formulation for CFTs also. For illustration purpose, we have considered the spin-1 and spin-2 conformal theories. For such theories we have explicitly constructed the specific finite field-dependent parameters. Furthermore, we have found that the Jacobians corresponding to different parameters switch the theories from one gauge to another (namely, linear to non-linear gauges). Furthermore, we have established the theory at quantum level by analyzing it through BV formulation. In BV formulation we have demonstrated that the finite field dependent BRST transformation connects the different solutions of quantum master equation for both spin-1 and spin-2 conformal theories. Thus our formulation will be helpful in estimating the observables of the conformal theory in different gauges.

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