Equation of State in a Generalized Relativistic Density Functional Approach

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1 Introduction

In many simulations of astrophysical objects and phenomena, the equation of state (EoS) of dense matter is an essential ingredient. It determines, e.g., the dynamical evolution of core-collapse supernovae [1, 2] and neutron star mergers [3], and the structure of compact stars [4]. The application of an EoS is reasonable if the timescales of reactions are much smaller than those of the system evolution and thermodynamic equilibrium can be assumed to hold. In general, a global EoS is required that covers a wide range in temperature, density and isospin asymmetry. These conditions affect the chemical composition of matter and the nucleosynthesis.

A critical examination of existing global EoS models [5] suggests that the development of an improved EoS is worthwhile. The set of constituent particles should be enlarged considerably including not only nucleons, charged leptons and photons but also a “complete” table of nuclei, mesons, hyperons or even quarks as degrees of freedom at high densities and temperatures. The model parameters have to be constrained better taking, e.g., properties of nuclei, results of heavy-ion collisions or compact star observations into account. Correlations should be considered more seriously, e.g., at low-densities where the virial equation of state (VEoS), which is determined by nucleon-nucleon correlations, is a model-independent benchmark [6, 7]. For composite particles such as nuclei the dissolution in the medium (Mott effect) has to be described properly [8, 9]. Electromagnetic correlations are essential in order to model the solidification/melting at low temperatures. Phase transitions and the appearance of ‘non-congruent’ features have to be treated correctly [10] with non-negligible differences between nuclear matter and stellar matter. Obviously, it is a tremendous challenge to cover the full range of thermodynamic variables in a single unified model.

Information on correlations are encoded in spectral functions, which have a complicated structure in general. Often, a quasiparticles (QP) approach is employed as an approximation. The QP properties change inside the medium and the size of residual correlations is reduced. The QP concept is very successful in nuclear physics, e.g., in phenomenological mean-field models (Skyrme, Gogny, relativistic) or the treatment of pairing correlations using a Bogoliubov transformation [11]. In the ultimate limit, an
exact diagonalisation of the Hamiltonian of the interacting many-body system leads to a system of independent QP that can be many-body states. At low densities, clusters appear as new degrees of freedom as described in the VEoS. In order to consider these features, a generalized relativistic density functional (gRDF) was developed. It takes the correct limits and explicit cluster degrees of freedom into account.

2 Generalized relativistic density functional

The gRDF model [12, 13, 14, 15] is based on a grand canonical approach. It is an extension of a conventional relativistic mean-field model with density dependent couplings [16]. All thermodynamic quantities are derived from a grand canonical potential density \( \omega(T, \{\mu_i\}) \), which depends on the temperature \( T \) and the set of chemical potentials \( \mu_i \) of all particles. The present set of particle species comprises baryons (nucleons and hyperons), nuclei, charged leptons and photons. Besides light nuclei (\(^2\)H, \(^3\)H, \(^3\)He, \(^4\)He) a full table of heavy nuclei (\(^A\)Z with \( A > 4, N, Z \leq 184 \)) is included, too. Experimental binding energies are used or, if not available, predictions from the DZ model [17]. Internal excitations of heavy nuclei are considered with temperature dependent degeneracy factors obtained with appropriate level densities. Effective continuum resonances represent nucleon-nucleon scattering correlations and ensure the correct low density limit in accordance with the VEoS [13].

All massive particles are treated as QP with scalar (\( S_i \)) and vector (\( V_i \)) potentials. The effective interaction is modeled by an exchange of mesons (\( \sigma, \omega, \rho \)) with density-dependent couplings to the nucleons, both free and bound in nuclei, using the well constrained DD2 parametrization [12]. It gives very reasonable nuclear matter parameters at a saturation density of \( n_{\text{sat}} = 0.149 \text{ fm}^{-3} \), such as a binding energy per nucleon \( E/A = 16.02 \text{ MeV} \), a compressibility \( K = 242.7 \text{ MeV} \), a symmetry energy \( J = 31.67 \text{ MeV} \) and a slope parameter \( L = 55.04 \text{ MeV} \). The neutron matter EoS lies within the error bounds of recent chiral effective field theoretical calculations [18, 19]. Both potentials \( S_i \) and \( V_i \) receive contributions from the meson fields. For composite particles, the scalar potential contains an additional mass shift \( \Delta m_i \) that depends on all particle densities and temperature. It mainly takes the blocking of states by the Pauli exclusion principle into account and serves to describe the dissolution of clusters by reducing the particle binding energy. This microscopically motivated approach replaces the traditional, purely geometric concept of the excluded-volume mechanism [20]. The vector potential \( V_i \) includes a “rearrangement” contribution due to the density dependence of the meson-nucleon couplings, which is required for the thermodynamic consistency of the model, and an electromagnetic correction to account for electron screening effects in stellar matter.
3 Symmetry energy and neutron skins of nuclei

The isospin dependence of the effective interaction in the gRDF model determines the density dependence of the symmetry energy. It is crucial for a proper description of the structure of neutron stars, see, e.g., the topical issue on the symmetry energy [21]. A strong correlation of the neutron skin thickness $\Delta r_{np}$ of heavy nuclei with the slope of the neutron matter equation of state [22, 23] or the slope parameter $L$ of the symmetry energy is observed when the predictions of a large number of mean-field calculations, both relativistic and non-relativistic, are compared, see, e.g., [24]. In recent years, many attempts were made to determine the symmetry energy at saturation $J$ and the parameter $L$ from experiments, e.g., by measuring the neutron skin thickness of $^{208}$Pb and using the $\Delta r_{np}$ vs. $L$ correlation. Since the calculations of neutron skin thicknesses are based on mean-field models, the question arises whether few-nucleon correlations can effect the results.

The gRDF approach can be employed to describe the formation of nuclei inside matter at finite temperatures by using an extended Thomas-Fermi approximation in spherical Wigner-Seitz cells [14]. In a calculation with nucleons and light clusters as degrees of freedom, it is observed that the probability of finding light clusters is enhanced at the surface of the heavy nucleus as compared to the surrounding low-density gas. The gRDF model can be extended to the description of heavy nuclei in vacuum at zero temperature to study cluster correlations. In this case, only the $\alpha$-particle remains as the relevant light cluster. Its density distribution is obtained from the $\alpha$-particle ground state wave function that is calculated self-consistently in the WKB approximation. For the chain of Sn nuclei, a distinct reduction of the neutron skin thickness is observed when $\alpha$-particle correlations are considered [25]. However, the effect vanishes for very neutron-rich nuclei or for nuclei with roughly the same neutron and proton numbers without a neutron skin. A variation of the isovector dependent part of the effective interaction allows to study the $\Delta r_{np}$ vs. $L$ correlation, e.g., for a $^{208}$Pb nucleus. A systematic shift is observed that might affect the determination of the slope parameter $L$ from measurements of the neutron skin thickness, at least as a systematic error. It is envisaged to investigate experimentally the predicted formation of $\alpha$-particles at the surface of Sn nuclei in quasi-elastic ($p,p\alpha$) reactions at RCNP, Osaka [26].

4 Outlook

The present version of the gRDF model includes only hadronic and leptonic degrees of freedom where nuclei are described as clusters composed of nucleons. At high densities or temperatures a phase transition to quark matter is expected. Hence, quark degrees of freedom should be incorporated into the approach. On the other hand, at
low densities and temperatures, quarks should be confined in nucleons. In a preliminary extension of the gRDF model with quarks, a phenomenological description of confinement will be implemented. The idea is to apply an “inverse” excluded-volume approach that permits the quarks to propagate freely only above a certain (scalar) density of the system. For this purpose, the classical excluded-volume mechanism is generalized by allowing more general dependencies of the “available volume fraction”. The correct quantum statistics and a relativistic description are considered, too. The relevant theoretical formulation to guarantee the thermodynamic consistency of the approach has been developed and exploratory calculations have been preformed.

Another extension of the gRDF model concerns the introduction of more general meson-nucleon couplings in the Lagrangian density. In conventional RMF approaches with density-dependent couplings, the nucleon self-energies only depend on densities. As known from Dirac-Brueckner calculations of nuclear matter, they should also depend on the nucleon momentum or energy. This dependence can be mapped to modified effective density dependent meson-nucleon couplings [27], but the full dependence should be kept in order to comply with the optical potential constraint at high nucleon energies. This can be achieved in a RMF model with density-dependent and non-linear derivative meson-nucleon couplings of general functional form [28]. Preliminary studies indicate a softening of the EoS at high densities, however, for a reliable fit of the model parameters, the approach has to be applied to the description of finite nuclei. Work in this direction is in progress.

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