Metastable supersymmetry breaking configurations of $D$-branes and $NS5$-branes in string theory often owe their existence to classical gravitational interactions between the branes. We show that in the effective theory of the light fields, these interactions give rise to a non-canonical Kähler potential and other D-terms. String theory provides a UV completion in which these non-renormalizable terms can be computed. We use these observations to clarify the relation between the phase structure of ISS-type models and their brane realizations.

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1. Introduction

In the last few years there has been some work on metastable supersymmetry breaking vacua in supersymmetric field theories, following the observation of ISS [1] that such vacua may be rather generic. The particular example studied in [1] – massive supersymmetric QCD (SQCD) in the free magnetic phase – reduces in the infrared to an effective Wess-Zumino (WZ) model for the light fields, which captures the phase structure of the theory. The parameters of this “macroscopic” WZ model are determined in terms of those of the underlying “microscopic” SQCD, which provides an ultraviolet (UV) completion.

Extending the discussion of [1] to string theory is interesting since it gives rise to a more general class of UV completions, which may exhibit new phenomena. This can be done [2-5] by using the well known realization of SQCD as the low energy theory on intersecting $D$-branes and $NS5$-branes (see [6] for a review). In the brane description, the light fields of [1] correspond to open strings stretched between various $D$-branes. Their low energy dynamics is described by the field theory studied in [1], with corrections that depend on the parameters of the brane configuration.

The WZ model of [1] contains pseudo-moduli (i.e. complex massless scalar fields $\Phi$ with a classically flat potential), whose expectation values parameterize a moduli space of SUSY breaking vacua. This moduli space is lifted by one loop effects, which give a mass to $\Phi$ and stabilize it at the origin of pseudo-moduli space. In the brane description, when all the distances between the branes are large, the one loop effects of [1] are in general small and the dominant contribution to the potential on pseudo-moduli space is due to the classical gravitational attraction between $D$-branes and $NS5$-branes [5]. It leads to the same qualitative outcome, stabilizing $\Phi$ at the origin of pseudo-moduli space, but the origin of this stabilization is different from that of [1].

A natural question, which was not addressed in [5], concerns the interpretation of the above gravitational attraction in the low energy WZ model. The purpose of this paper is to fill this gap. We will see that it gives rise to a non-trivial Kähler potential and other, higher order, D-terms. From the low energy point of view, these terms correspond to non-renormalizable interactions, and thus depend on the UV completion of the WZ model. The embedding in string theory provides such a completion, and determines all these terms.

One of the main motivations for this work is to better understand the generalized ISS model studied in [7-9]. In field theory this model is obtained by deforming the superpotential of $\Phi$. The metastable vacuum structure of the resulting WZ model was analyzed
in [4] and [5] (which corrected a mistake in [4]). The corresponding brane system, which is obtained from that of [2-5] by rotating some of the branes, was analyzed in [8]. While the qualitative phase structures in field and string theory agree, some important aspects are different. In particular, the brane construction gives rise to metastable vacua not seen in the field theory. Interestingly, these vacua are the more phenomenologically promising ones.

The interpretation of the vacuum structure of the brane system in terms of the low energy effective field theory, that we will describe below, helps clarify the situation. The additional vacua that are found in string theory, but not in the WZ model, occur when the effective coupling at the UV cutoff scale exceeds a certain critical value. The resulting non-renormalizable field theory is strongly coupled at high energies and requires a UV completion. Such a completion is provided by string theory.

This situation is reminiscent of the Nambu-Jona-Lasinio (NJL) model [10], where the vacuum spontaneously breaks chiral symmetry due to an attractive non-renormalizable four-Fermi interaction. Symmetry breaking occurs when the four Fermi coupling at the UV cutoff scale exceeds a certain critical value. A brane system in string theory provides a UV complete theory with closely related dynamics [11].

The plan of this paper is the following. In section 2 we discuss the ISS model and its brane realization. After briefly reviewing some of the results of [2-5], we calculate the leading terms in the low energy effective action of the model and show that the gravitational interaction of the $D$-branes with an $NS5$-brane gives rise to D-terms that play an important role in the existence of a metastable SUSY breaking vacuum. We discuss the region in parameter space in which these D-terms give the leading contribution to the potential of the pseudo-moduli, and the one in which this potential is dominated by the one loop contribution computed in [1].

In section 3 we turn to the generalized ISS model of [7-9]. We exhibit all the metastable vacua described in [8] in the effective field theory and discuss their fate as the parameters of the brane system are varied towards the renormalizable field theory regime. As expected, many of the metastable states disappear in the process. In section 4 we summarize our results and comment on them. Some technical details are described in the appendix.
2. D-term supersymmetry breaking from branes

In this section we discuss the string theory realization of the ISS model \[2-5\]. As mentioned in the introduction, in string theory the ISS pseudo-moduli are stabilized primarily by classical gravitational effects. We show that in the low energy theory these effects give rise to non-trivial D-terms, which together with the superpotential lead to a metastable SUSY breaking state.

2.1. ISS from branes

We start by decomposing the 9 + 1 dimensional spacetime of type IIA string theory as follows:

$$\mathbb{R}^{9,1} = \mathbb{R}^{3,1} \times \mathbb{C}_{v} \times \mathbb{C}_{w} \times \mathbb{R}_{y} \times \mathbb{R}_{x^7},$$  \hspace{1cm} (2.1)

with

$$v = x^4 + ix^5, \quad w = x^8 + ix^9, \quad y = x^6. \hspace{1cm} (2.2)$$

The brane configuration we consider is depicted in figure 1. All branes are extended in the $\mathbb{R}^{3,1}$ labeled by $(x^0, x^1, x^2, x^3)$. The NS5-branes denoted by $NS$ and $NS'$ are further extended in $v$ and $w$, respectively. The $D6$-branes are extended in $w, x^7$, while the $D4$-branes are stretched between other branes as indicated in the figure.

**Fig. 1:** The ISS brane configuration.
At low energies, the brane configuration of figure 1 reduces to an $N = 1$ supersymmetric gauge theory with gauge group $U(N_f - N_c)$, $N_f$ flavors of fundamentals $q^i, \tilde{q}_i$, and gauge singlets $\Phi^i_j, i, j = 1, \cdots, N_f$. The chiral superfields $q, \tilde{q}$ and $\Phi$ have canonical Kähler potential,

$$K = q^\dagger q + \tilde{q}^\dagger \tilde{q} + \Phi^\dagger \Phi,$$

and superpotential

$$W_{\text{mag}} = h q^j \Phi^i_j \tilde{q}_i - h \mu^2 \Phi^i_i.$$

This theory is the Seiberg dual of $N = 1$ SQCD with gauge group $U(N_c)$, and $N_f$ flavors $Q_i, \tilde{Q}^i_i$, whose mass is proportional to $\mu^2$. It has been used to study metastable SUSY breaking in [1].

The parameters of the low energy theory are given in terms of the underlying string theory ones by

$$g_{\text{mag}}^2 = \frac{4\pi^2 g_s l_s}{y_1 - y_{\text{NS}}}, \quad \hbar^2 = \frac{8\pi^2 g_s l_s}{y_2 - y_1}, \quad \mu^2 = \frac{v_2}{16\pi^3 g_s l_s^2}.$$

Here and below we make a choice of phase such that $h$ and $\mu$ are real and positive.

The geometric description is reliable when the distances between the various branes are large (relative to $l_s$), and the string coupling $g_s$ is small. In this regime, the magnetic gauge coupling $g_{\text{mag}}$ and Yukawa coupling $h$ are small. The mass parameter $\mu$ is typically above the string scale. If $\mu$ is sufficiently small, the low energy dynamics of the branes is well described by magnetic SQCD. In general, the low energy effective Lagrangian receives contributions from other sources.

The brane configuration of figure 1 is unstable to reconnection of the $N_f - N_c$ color $D4$-branes with flavor $D4$-branes, leading to that of figure 2. The resulting configuration is marginally stable. It contains an $N_c \times N_c$ matrix $X$ of massless fields describing the positions in the $w$-plane of the $D4$-branes stretched between the $NS'$-brane and the $D6$-branes. The potential for $X$ is flat since, as is clear from figure 2, the energy of the branes is independent of $w$. 
Fig. 2: The marginally stable brane configuration.

The above discussion has a simple analog in the effective field theory. The Kähler potential (2.3) and superpotential (2.4) give rise to the bosonic potential

\[ V_0 = h^2 (|\tilde{q}q - \mu^2 I_{N_f}|^2 + |q\Phi|^2 + |\Phi\tilde{q}|^2), \tag{2.6} \]

where \( I_{N_f} \) is a rank \( N_f \) identity matrix. The configuration of figure 1 describes the origin of field space (as is clear from the fact that both the \( U(N_f - N_c) \) gauge symmetry and the global \( U(N_f) \) symmetry are unbroken in it). Expanding (2.6) around this point we see that the magnetic quarks \( q, \tilde{q} \) have a tachyonic mass term. Thus, they get an expectation value, which is the field theory analog of the brane reconnection process described above.

As is familiar from O’Raifeartaigh-type models, the potential (2.6) cannot be set to zero due to the fact that the rank of \( \tilde{q}q, N_f - N_c \), is smaller than that of the identity matrix \( I_{N_f} \). Hence, supersymmetry is broken. The minimum of the potential corresponds (up to global symmetries) to

\[ \tilde{q}q = \begin{pmatrix} \mu^2 I_{N_f - N_c} & 0 \\ 0 & 0 \end{pmatrix}, \quad \Phi = \begin{pmatrix} 0 & 0 \\ 0 & X \end{pmatrix}. \tag{2.7} \]

\( X \) is the \( N_c \times N_c \) matrix field described in terms of the brane system above. It has a classically flat potential, and thus parameterizes a pseudo-moduli space of non-supersymmetric vacua.

\(^{2}\) Following standard notation, we denote the superfields and their bottom components by the same letter.
Since in the leading approximation the potential of $X$ is flat, we need to consider corrections. The nature of these corrections is different in different regions of the parameter space of brane configurations. For $|v_2| \ll g_s l_s$, the analysis of ISS is valid and the leading contribution to the potential on pseudo-moduli space comes from one loop effects in the WZ model \((2.3), (2.4)\). The resulting Coleman-Weinberg potential behaves near the origin like

\[ V_1 = \frac{\ln 4 - 1}{8\pi^2} (N_f - N_c)|h^2 \mu|^2 \text{Tr}X^\dagger X + \cdots \]  

(2.8)

and gives a mass of order $|h^2 \mu|$ to the pseudo-moduli.

For $v_2$ that remains finite in the limit $g_s \to 0$, the field theory potential \((2.8)\) is a subleading effect. The dominant contribution in this limit comes from the gravitational attraction of the $N_c$ D4-branes in figure 2 to the NS-brane \([5]\). Our task in the rest of this section is to understand this gravitational effect in the low energy effective theory of the light fields $q, \tilde{q}$ and $\Phi$.

### 2.2. D-terms from branes

It turns out to be useful to first consider the brane configuration of figures 1, 2 in the special case $v_2 = 0$, for which $\mu = 0$ (see \((2.3)\)) and supersymmetry, gauge and global symmetries are unbroken. The WZ model \((2.3), (2.4)\) has in this case a moduli space of supersymmetric vacua labeled by $\langle \Phi \rangle$, with $q = \tilde{q} = 0$, along which the F-term potential \((2.6)\) vanishes. In the brane construction, this moduli space corresponds to placing the flavor D4-branes at arbitrary points in the $w$-plane.\(^3\) It is easy to check that the brane configuration preserves $N = 1$ supersymmetry everywhere in moduli space, in agreement with the field theory analysis.

In the above discussion, the brane configuration of figures 1, 2 (with $v_2 = 0$) is thought of as living in flat spacetime. This is a good approximation when the distances between the branes are large, but for our purposes it is important to include the leading corrections to this picture. Those are due to the fact that the flavor D4-branes actually live in the geometry of the NS-brane.\(^4\)

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\(^3\) This gives $N_f$ of the $N^2_f$ moduli seen in the low energy field theory. See e.g. \([6]\) for a description of the full moduli space.

\(^4\) The NS$'$ and D6-branes do not contribute to the discussion below since they are extended in $w$. 

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To analyze the effects of this geometry on the moduli space, we consider the Dirac-Born-Infeld (DBI) action for a $D4$-brane stretched in $y$ between the $NS'$ and $D6$-branes

$$S = -T_4 \int d^4 x \int_{y_1}^{y_2} dy e^{-\phi} \sqrt{-\det P(G + B)_{ab}} ,$$

where $T_4/g_s = 1/(2\pi)^4 l_s^5 g_s$ is the tension of the $D4$-brane, $\varphi$ is the dilaton, and $P(G + B)$ is the pullback of the spacetime metric and $B$-field to the worldvolume of the brane. The $D4$-brane lives in the fivebrane geometry \[12\],

$$ds^2 = dx_{\mu} dx^\mu + dv d\tau + H(x^n)[dy^2 + (dx^7)^2 + dwd\bar{w}] ,$$

$$e^{2(\varphi - \varphi_0)} = H(x^n) ,$$

$$H_{mnp} = -\epsilon_{mnp}^q \partial_q \varphi .$$

(2.10)

Here $\mu = 0, 1, 2, 3$, while $m, n, p = 6, 7, 8, 9$ run over the directions transverse to the $NS$-brane. $H_{mnp}$ is the field strength of the Neveu-Schwarz $B$ field sourced by the fivebrane; $g_s = e^{\varphi_0}$ is the string coupling far from the fivebrane. The harmonic function $H$ is given by

$$H(r) = 1 + \frac{l_s^2}{r^2} ,$$

(2.11)

with $r^2 = x_m x^m = (y - y_{NS})^2 + (x^7)^2 + |w|^2$. The background (2.10) is reliable when $r \gg l_s$, and we will assume this throughout our discussion.

In order to study the effects of the fivebrane background on the dynamics of the moduli, we take the position of the $D4$-brane, $w$, to be a function of $x^\mu$, and plug it into the DBI action (2.9). The resulting four dimensional effective Lagrangian is

$$\mathcal{L} = -\frac{T_4}{g_s} \int_{y_1}^{y_2} dy \sqrt{1 + H(r)|\partial_\mu w|^2 - \frac{1}{8} H^2 (\partial_\mu w \partial_\nu \bar{w} - \partial_\mu \bar{w} \partial_\nu w)^2} .$$

(2.12)

Taking $w(x^\mu)$ to be constant gives

$$\mathcal{L} = -\frac{T_4}{g_s} \Delta y ,$$

(2.13)

the (negative of) the energy density of a BPS $D4$-brane of length $\Delta y = y_2 - y_1$ in flat space. As expected from supersymmetry, the fivebrane background does not lift the moduli space labeled by $w$, and does not modify the energy of the brane.

The next term in the expansion of the square root in (2.12) gives the kinetic term of $w$,

$$\mathcal{L}_2 = -\frac{T_4}{2g_s} \int_{y_1}^{y_2} dy H(r)|\partial_\mu w|^2 .$$

(2.14)
In general, the integral in (2.14) depends non-trivially on all the parameters. It simplifies in the limit where the length of the $D$-brane, $\Delta y$, is much smaller than its distance to the $NS$-brane, i.e. $l_s \ll \Delta y \ll |y_{NS} - y_i|$. One can further choose the origin of $y$ such that the $NS'$ and $D$-branes are located near the origin, with the $NS$-brane far away from them, i.e. $l_s \ll |y_1|, |y_2| \ll |y_{NS}|$. In that case, one has

$$\int_{y_1}^{y_2} \frac{dy}{(y - y_{NS})^2 + |w|^2} \simeq \frac{\Delta y}{y_{NS}^2 + |w|^2},$$

so

$$\mathcal{L}_2 = -\frac{T_4}{2g_s} \Delta y H(|w|)|\partial_\mu w|^2,$$

where

$$H(|w|) = 1 + \frac{l_s^2}{y_{NS}^2 + |w|^2}.$$  

(2.15) – (2.17) capture the full dependence of the kinetic term (2.16) on the parameter $w/y_{NS}$ but neglect corrections that vanish in the limit $y_i/y_{NS} \to 0$. As $y_{NS} \to -\infty$, $H \to 1$, and the Lagrangian (2.16) takes a canonical form in terms of the field

$$\Phi = w \sqrt{\frac{T_4 \Delta y}{2g_s}}.$$ 

(2.18)

This relation can be alternatively written as

$$h\Phi = \frac{w}{2\pi l_s^2},$$

(2.19)

which is obtained by comparing the mass of $q, \tilde{q}$ at a point $\langle \Phi \rangle$ in moduli space to the energy of a fundamental string of length $w$.

For finite $y_{NS}$, the effect of the fivebrane geometry is to generate a non-trivial Kähler potential on the moduli space labeled by $\Phi$,

$$\partial_\Phi \partial_\bar{\Phi} \mathcal{K} \equiv \mathcal{K}_{\Phi \bar{\Phi}} = H(|\Phi|) = 1 + \frac{l_s^2}{y_{NS}^2 + |w(\Phi)|^2} = 1 + \frac{l_s^2}{y_{NS}^2} \left(1 + \frac{|\Phi|^2}{\Lambda^2}\right)^{-1},$$

(2.20)

with

$$\Lambda^2 = \frac{T_4 y_{NS}^2 \Delta y}{2g_s} = \left(\frac{y_{NS}}{2\pi l_s^2 h}\right)^2.$$ 

(2.21)
Near the origin, the Kähler potential \((2.20)\) behaves as
\[
K_{\Phi\Phi} \simeq 1 - \frac{|\Phi|^2}{\Lambda^2} \left[ 1 + O \left( \frac{|\Phi|^2}{\Lambda^2} \right) \right],
\]  
(2.22)
where
\[
\tilde{\Lambda} = \frac{y_{NS}}{l_s} \Lambda = \frac{y_{NS}^2}{2\pi l_s^3 h},
\]
(2.23)
and we neglected subleading terms for \(l_s/y_{NS} \ll 1\).

In the low energy theory, the non-canonical Kähler potential \((2.22)\) corresponds to including in the effective Lagrangian an infinite series of higher dimension operators. Such operators are typically suppressed by a UV scale. In our case, two such scales appear, \(\Lambda\) \((2.21)\) and \(\tilde{\Lambda}\) \((2.23)\). For \(y_{NS} \gg l_s\), the regime in which the geometric description is valid, they are widely separated, \(\tilde{\Lambda} \gg \Lambda\). All other dimensionful parameters in the model can be expressed in terms of \(h, \mu, \Lambda, \tilde{\Lambda}\). E.g. the string scale is given by
\[
2\pi l_s = \frac{h\tilde{\Lambda}}{(h\Lambda)^2};
\]
(2.24)
y_{NS} can be computed by plugging \((2.24)\) into \((2.23)\).

So far we took \(\Phi\) to be a single chiral superfield, or in the brane language restricted to the case \(N_f = 1\). In general, \(\Phi\) is an \(N_f \times N_f\) matrix and the kinetic term \((2.14)\) can be written as (the bosonic part of)
\[
\mathcal{L}_2 = \int d^4\theta \text{Tr} \mathcal{K}(\Phi, \Phi^\dagger),
\]
(2.25)
with the Kähler potential given by \((2.20)\).

In the discussion above we focused on the bosonic terms of the Lagrangian \((2.24)\). Terms involving the fermions are related to them by supersymmetry, which is preserved by the DBI action. Therefore, they are in principle guaranteed to agree with \((2.25)\). In appendix A we verify that this is indeed the case.

The Kähler potential \((2.20)\) is not the only effect of the fivebrane on \(\Phi\). Expanding the square root \((2.12)\) one finds derivative interactions associated with higher order D-terms. To illustrate this, consider the next (quartic) term in the expansion of \((2.12)\),
\[
\mathcal{L}_4 = \frac{T_4}{16g_s} \int_{y_1}^{y_2} dy H^2(r) \left[ 2|\partial_\mu w|^4 + (\partial_\mu w \partial_\nu w - \partial_\mu w \partial_\nu w)^2 \right]
\]
\[
= \frac{T_4}{8g_s} \int_{y_1}^{y_2} dy H^2(r) |\partial_\mu w \partial_\mu w|^2 = \frac{g_s}{2T_4\Delta y} H^2(|\Phi|) |\partial_\mu \Phi \partial^\mu \Phi|^2.
\]
(2.26)
In the last equality we used the approximation discussed after eq. (2.14), and (2.18).

In the low energy theory of the field $\Phi$, (2.26) is due to the following D-term:

$$L_4 = \int d^4 \theta G(\Phi, \overline{\Phi}) e^{\alpha \beta} e^{\dot{\alpha} \dot{\beta}} \mathcal{D}_\alpha \Phi \mathcal{D}_\beta \overline{\Phi} \mathcal{D}_{\dot{\alpha}} \Phi \mathcal{D}_{\dot{\beta}} \overline{\Phi}. \tag{2.27}\label{2.27}$$

Using the standard conventions for the expansion of a chiral superfield (see e.g. [13]), and performing the $\theta$ integrals, the bosonic terms in (2.27) take the form

$$L_4 = 16 G(\Phi, \overline{\Phi}) \left( |F|^4 - 2 |F|^2 |\partial_\mu \Phi|^2 + |\partial_\mu \Phi \partial^\mu \Phi|^2 \right). \tag{2.28}\label{2.28}$$

Setting $F = 0$ by using its equation of motion and comparing the four-derivative term in (2.28) with (2.26) we see that

$$G(\Phi, \overline{\Phi}) = \frac{g_s}{32 T_4 \Delta y} H^2(|\Phi|). \tag{2.29}\label{2.29}$$

Higher order terms in the expansion of the square root (2.12) give an infinite series of higher order D-terms, which can be calculated in the same way.

2.3. Supersymmetry breaking at non-zero $\mu$

We now turn to the general situation in figure 2, with $\nu_2 \neq 0$, for which supersymmetry is broken. As is clear from the figure, the origin of the breaking is twofold. First, the $N_c$ $D_4$-branes which support the (pseudo-)moduli $X$ (2.7) are not mutually BPS with the $N_f - N_c$ remaining $D_4$-branes. As a consequence, the spectrum of open strings connecting the two stacks of branes is non-supersymmetric. These open strings are massive, and can be integrated out. Since they must appear in pairs in intermediate states, they influence the low energy dynamics of $X$ only via string loops. We are interested in effects that survive in the classical limit $g_s \to 0$; hence, we will ignore them.

The second source of supersymmetry breaking in figure 2 is the $NS$-brane, which is also not mutually BPS with the $N_c$ $D_4$-branes. This gives rise to supersymmetry breaking effects that survive in the classical limit, and are sensitive to the distance between the fourbranes and fivebrane, $|y_{NS}|$, as well as to the position of the $D6$-branes in the $v$-plane, $v_2$. Our goal in this subsection is to analyze these effects in the low energy theory.

The $N_c$ $D_4$-branes in figure 2 are stretched along the line

$$v = a(y - y_1). \tag{2.30}\label{2.30}$$
The slope $a$ is given by
\[ a = \frac{v_2}{\Delta y} = 2\pi (l_s h\mu)^2 , \tag{2.31} \]
where we used (2.5) to relate $a$ to the field theory parameters. Plugging (2.30) into the DBI action (2.12) leads to the following effective Lagrangian for $w$:
\[ \mathcal{L} = -\frac{T_4 \Delta y}{g_s} \sqrt{1 + \frac{a^2}{H}} \sqrt{1 + H |\partial_\mu w|^2} + O(|\partial w|^4) . \tag{2.32} \]
Setting $w$ to a constant, we see that the energy of the $D4$-brane is no longer independent of $w$,
\[ V = \frac{T_4 \Delta y}{g_s} \left( \sqrt{1 + \frac{a^2}{H}} - 1 \right) , \tag{2.33} \]
where we have subtracted from (2.32) the energy of the supersymmetric configuration (the one with $a = 0$), as is standard in supersymmetric field theory [13].

We will find it convenient to expand in the slope $a$ (2.31). To leading order, (2.33) takes the form
\[ V = \frac{T_4 \Delta y}{2g_s} \frac{a^2}{H} + O(a^4) . \tag{2.34} \]
In the low energy theory of $\Phi$, this must be due to a non-zero superpotential. In general one has [13]:
\[ V = |\partial_\Phi W|^2 K^{\Phi \bar{\Phi}} . \tag{2.35} \]
Plugging in the potential (2.34) and Kähler potential (2.20) one finds:
\[ |\partial_\Phi W|^2 = \frac{T_4 \Delta y}{2g_s} a^2 = |h\mu|^2 . \tag{2.36} \]
In the last equality we used the explicit forms of $h$, $\mu$, (2.5). We see that the DBI calculation is compatible with the superpotential $W = -h\mu^2 \Phi$ (2.4), and Kähler potential (2.20).

Expanding the potential (2.33) around the origin of pseudo-moduli space, we find
\[ V = |h\mu|^2 + M_X^2 |X|^2 + O(|X|^4) , \tag{2.37} \]
with
\[ M_X = a \frac{l_s}{y_{NS}} = \frac{h\mu^2}{\Lambda} . \tag{2.38} \]

\textsuperscript{5} As in the previous subsection, for the purpose of this discussion we can consider each of the $N_c$ $D4$-branes individually. Thus, we set $N_c = 1$ below; the generalization to larger $N_c$ is straightforward.
The first equality shows that the pseudo-modulus $X$ (2.4) develops a mass linear in $a$.

The second expresses this mass in terms of the parameters appearing in the low energy
Lagrangian, where it is a consequence of the non-zero superpotential (2.30) and non-
canonical Kähler potential (2.22). In particular, if we send $y_{NS} \to -\infty$, or in field theory
language $\bar{\Lambda} \to \infty$, the mass (2.38) goes to zero, and supersymmetry is restored, despite
the fact that the linear superpotential (2.4) remains non-trivial in this limit.

This is due to the familiar fact that a chiral superfield with a linear superpotential and
canonical Kähler potential describes a free massless boson and fermion and does not really
break supersymmetry. More formally, one can in that case shift $\Phi$ by its non-zero F-term
and recover a Lagrangian with vanishing superpotential. When the Kähler potential is
non-trivial, this cannot be done.

From the brane perspective this is natural as well, since when the $NS$-brane is absent
(and ignoring the $N_f - N_c$ D4-branes in figure 2, as discussed above), the brane system
is supersymmetric, and the only difference with respect to the system with $a = 0$ is that
the length of the $N_c$ D4-branes, and thus their energy, is $a$-dependent. This $a$ dependence
leads to the linear superpotential of the low energy effective field theory.

Note also that, as mentioned above, the potential on pseudo-moduli space due to the
non-trivial Kähler potential (2.37), (2.38), is much larger than that due to the ISS one loop
potential, (2.8). Indeed, the former remains finite as $g_s \to 0$ with finite $a$, while the latter
goes to zero like $g_s$. This is due to the fact that the Kähler potential is a consequence of
classical gravitational interactions, while (2.8) is a one loop effect. From the point of view
of the low energy effective field theory, the above discussion is valid when the UV scale $\bar{\Lambda}$
is much smaller than $\mu/h$ (neglecting numerical factors).

To recapitulate, the kinetic terms for the light bosons and fermions at order $a^0$, as well
as the potential for the scalars at order $a^2$ (2.34) are compatible with the Kähler potential
(2.20), and superpotential (2.4), which go like $a^0$ and $a$, respectively. All these terms have
been computed to leading order in $a$. We next comment on the structure of subleading
corrections.

One can think of the perturbative expansion in $a$ as follows. The Kähler potential and
superpotential discussed above give a mass proportional to $a$, (2.38), to the pseudo-moduli.
This mass defines a natural momentum scale in the low energy effective theory. Therefore,
when expanding in powers of $a$, derivatives $\partial_\mu$ scale like $a$, super-derivatives $D_\alpha$ like $\sqrt{a}$,
etc. The bottom component of the chiral superfield $\Phi$ scales like $a^0$, while the fermion $\psi$
scales like $\sqrt{a}$ due to the factor of $\theta$ in front of it in the component expansion of the chiral superfield.

According to these rules the Kähler potential and superpotential terms computed above scale like $a^2$. The D-term (2.27) computed in the previous subsection scales like $a^4$. Other possible terms that scale like $a^4$ include a correction of order $a^2$ to the Kähler potential (2.20), and a contribution proportional to $a^3$ to the superpotential (2.4), (2.36).

In order to calculate these terms one can proceed as follows. From (2.32) we see that the kinetic term for $\Phi$ receives a correction at order $a^2$:

$$\delta L_2 = -\frac{1}{2} a^2 |\partial_\mu \Phi|^2. \quad (2.39)$$

In the low energy effective action there are two possible contributions to (2.39). One is from the D-term (2.27). Plugging the (order $a$) F-term,

$$|F|^2 = \left| \frac{\partial_\Phi W}{\partial \Phi} \right|^2 = \frac{a^2 T_4 \Delta y}{2 g_s H^2}, \quad (2.40)$$

into (2.28), we find that the D-term (2.27) exactly accounts for the correction (2.39). Since the only other contribution to the kinetic term at this order is from the Kähler potential, we conclude that (2.20) is in fact not corrected at order $a^2$.

We next turn to the superpotential. To leading order in $a$, it goes like

$$W = -a \sqrt{\frac{T_4 \Delta y}{2 g_s}} \Phi. \quad (2.41)$$

Since $\Phi$ is charged under a $U(1)$ global symmetry that corresponds geometrically to rotations of the $w$-plane, any higher order corrections to (2.41) must be proportional to $\Phi$ as well. At the order under consideration, the only possible correction goes like $\delta W \sim a^3 \Phi$.

To calculate it, we expand (2.33) to order $a^4$:

$$V = \frac{T_4 \Delta y}{2 g_s} a^2 - \frac{T_4 \Delta y}{8 g_s} \frac{a^4}{H^2} + O(a^6). \quad (2.42)$$

Since the Kähler potential is not corrected at order $a^2$, and thus does not contribute to the $a^4$ term in (2.42), there are again two possible contributions to this term. One comes from the D-term (2.27). Plugging (2.40) into (2.28) gives

$$V_4 = -16G |F|^4 = -\frac{T_4 \Delta y}{8 g_s} \frac{a^4}{H^2}. \quad (2.43)$$
Comparing to (2.42) we see that this agrees with the DBI calculation. Therefore, we conclude that the superpotential (2.41) does not receive corrections at order $a^3$
.

To summarize, we find that to order $a^4$ in the expansion of the effective Lagrangian described above, the Kähler potential and superpotential are given by (2.20) and (2.41), respectively. Corrections to these potentials that are down by $a^2$ from the leading contributions vanish. The non-zero corrections to the DBI results at this order are due to the higher derivative D-term (2.27).

It is natural to expect that this pattern persists to higher orders in $a$. Indeed, the Kähler potential and higher D-terms should not depend on the orientation of the $D4$-brane (labeled by $a$ (2.31)) in the limit $\Delta y \ll y_{NS}$, in which the fourbrane can be considered as a local probe of the geometry. The form of the superpotential is determined by the $SO(2)_v \times SO(2)_w$ R-symmetry (corresponding to rotations in the $v$ and $w$ planes, respectively) to be linear in $v_2$ and $w$, or equivalently in $\mu^2$ and $\Phi$.

One can use the DBI action (2.32) to calculate the mass of the pseudo-moduli (2.38) to all orders in $a$. A short calculation gives

$$M_X = \frac{a}{\sqrt{1 + a^2 y_{NS}^2}} l_s. \quad (2.44)$$

As discussed above, from the point of view of the low energy effective action, this mass receives contributions from the whole infinite tower of D-terms.

For $a$ that remains finite in the limit $g_s \to 0$, the contribution to the mass of the pseudo-moduli (2.44) of the non-canonical Kähler potential, (2.38), and other D-terms, is much larger than that computed in the low energy field theory in [1], (2.8). On the other hand, if one fixes all the parameters other than $v_2$ and sends $v_2 \to 0$, the ISS mass (2.8) eventually becomes larger, since it goes like $\sqrt{v_2}$, while (2.38) goes like $v_2$. Formally, the two masses become comparable when

$$v_2 \sim g_s y_{NS}^4 l_s^3. \quad (2.45)$$

However, in this regime both calculations are unreliable. The DBI calculation fails because $v_2$ (2.45) and thus the mass (2.38) are of order $g_s$, and one has to include string loop effects in the discussion. The field theory result (2.8) is not valid since (2.45) corresponds to

$$\mu \sim m_s \left(\frac{y_{NS}}{l_s}\right)^2, \quad (2.46)$$

which is much larger than $m_s$. As mentioned above, the field theory analysis is reliable for $\mu \ll m_s$, which corresponds to $v_2 \ll g_s l_s$. 

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3. R-symmetry breaking metastable vacua

In this section we turn to the generalized ISS model of [7-9]. We use the results of the previous section to describe the R-symmetry breaking metastable vacua of [8] in the low energy effective field theory, and discuss their fate as the parameters of the brane system are varied.

3.1. Deformed ISS from branes

The brane system we consider is depicted in figure 3. It is obtained from that of figure 2 by a rotation of the $D6$-branes by an angle $\theta$ in $(v, w)$. In the low energy theory, the rotation of the sixbranes corresponds [8] to adding a $\Phi^2$ term to the superpotential (2.4):

$$W = h q \Phi \bar{q} - h^2 \mu^2 \text{Tr}\Phi + \frac{1}{2} h^2 \mu \phi \text{Tr}\Phi^2.$$  \hfill (3.1)

The parameters $h, \mu$ are again given by (2.5), while

$$\mu_\phi = \frac{\tan \theta}{8\pi^2 g_s l_s}.$$  \hfill (3.2)

Fig. 3: The deformed ISS brane configuration.

The orientation of the $D6$-branes leads to an important difference in the dynamics of the configuration of figure 3 compared to that of figure 2. While the latter has (to leading order) a pseudo-moduli space of supersymmetry breaking vacua labeled by the positions of
the $N_c$ D4-branes in $w$, the former has a supersymmetric vacuum, in which the D4-branes are located at the point $(v, w) = (0, w_0)$, with

$$|w_0| = |v_2| \cot \theta . \quad (3.3)$$

At that point, the projections of the $NS'$ and D6-branes on the $(v, w)$ plane intersect, and the D4-branes stretch between them along the $y$ direction. In fact, the brane system has many supersymmetric vacua, which are described and compared to field theory in [8]. From the perspective of the WZ model with superpotential (3.1), these vacua are obtained by analyzing the zeroes of the bosonic potential corresponding to (3.1),

$$V_0 = h^2 (|\tilde{q} q - \mu^2 I_{N_f} + h\mu_\phi \Phi|^2 + |q\Phi|^2 + |\Phi \tilde{q}|^2) . \quad (3.4)$$

To study metastable vacua in the brane system, one needs to add to the above discussion the effects of the NS-brane background discussed in the previous section. This leads to a force attracting the D4-branes in figure 3 towards $w = 0$, as before. This force counteracts the one coming from (3.4), which pushes the D4-branes away from the origin. The balance of the two effects generically leads to the appearance of metastable supersymmetry breaking vacua at $w < w_0$ [8]. Our main task in this section is to understand these vacua in terms of the low energy field theory of the fields $q, \tilde{q}, \Phi$.

3.2. ISS-type vacua

As in section 2, in studying the metastable vacua of the brane configuration of figure 3 we will focus on one of the $N_c$ D4-branes stretched between the $NS'$ and D6-branes, and neglect the remaining $N_f - N_c$ D4-branes, whose effects are down by $g_s$. We will also continue to work in the regime $|y_i| \ll |y_{NS}|$, and neglect the curving of the D4-brane in the fivebrane background (which is discussed in [8,8]).

The D4-brane under consideration is displaced from the supersymmetric vacuum at $(v, w) = (0, w_0)$, and stretches between the $NS'$ and D6-branes along a straight line in $(v, y, w)$ space. This line can be parameterized by a variable $\lambda \in [0, 1]$ as follows:

$$(v, y, w) = (\lambda(w_0 - w_1) \sin \theta \cos \theta, \lambda \Delta y + y_1, \lambda(w_0 - w_1) \sin^2 \theta + w_1) . \quad (3.5)$$

$\lambda = 0, 1$ correspond to the endpoints of the D4-brane lying on the $NS'$ and D6-branes, respectively.
To study the dynamics of the fourbrane one can e.g. take $w_1$ (the position of the endpoint of the $D4$-brane on the $NS'$-brane) to be a dynamical field $w_1(x^\mu)$, and plug (3.3) into the DBI action (2.9). This gives

$$S = -T_4 \int d^4x \int_0^1 d\lambda e^{-\varphi} \sqrt{g_{\lambda\lambda}} \sqrt{-\det(\eta_{\mu\nu} + H \partial_{\mu} w_1 \partial_{\nu \bar{w}_1})},$$

where

$$e^{-\left(\varphi - \varphi_0\right)} \sqrt{g_{\lambda\lambda}} = \left[(\Delta y)^2 + |w_0 - w_1|^2 \sin^2 \theta \left(\sin^2 \theta + \frac{\cos^2 \theta}{H}\right)\right]^{\frac{1}{2}},$$

and

$$H = 1 + \frac{l_s^2}{y_{NS}^2 + |\lambda \sin^2 \theta (w_0 - w_1) + w_1|^2}.$$

The bosonic potential is given by (after subtracting the energy of the supersymmetric configuration)

$$V = \frac{T_4}{g_s} \int_0^1 d\lambda \left[e^{-\left(\varphi - \varphi_0\right)} \sqrt{g_{\lambda\lambda}} - \Delta y\right].$$

In general, the $D4$-brane (3.3) has a finite extent in $w$, and the integrals in (3.4), (3.9) are non-trivial. The situation simplifies for small $\theta$, where the extent of the fourbrane in $w$, $|w_0 - w_1| \sin^2 \theta$ is small (of order $\theta$) and the integrands of the above integrals can be taken to be approximately independent of $\lambda$. In that limit, the $D4$-brane becomes a local probe of the geometry of the fivebranes, as in section 2. To leading order in $a$ (2.31) and $\theta$, one can describe its location in the $w$ plane by a complex scalar field, $w(x^\mu)$, whose kinetic term is given by (2.16), or by the corresponding canonically normalized field $\Phi$ defined in (2.18).

The bosonic potential (3.9) is given to leading order in $\theta$ by

$$V = \frac{T_4}{2g_s \Delta y H} \theta^2 |w_0 - w|^2 = \frac{1}{H} v_2 \sqrt{\frac{T_4}{2g_s \Delta y}} - \frac{\theta}{\Delta y} X^2.$$  

In the second equality, $X$ is the field defined in (2.7). Using (2.5) and (3.2), (3.10) may be written as

$$V_0 = \frac{1}{H} |h\mu^2 - h^2 \mu_\phi X|^2.$$

Comparing to (2.35) we see that the potential (3.11) agrees with the one obtained from the superpotential (3.1) and Kähler potential (2.20). The potential vanishes at

$$X_{\text{susy}} = \frac{\mu^2}{h \mu_\phi},$$

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the supersymmetric vacuum. For small \( \mu_\phi \) this vacuum is located far from the origin of field space.

Near the origin of field space, the potential (3.11) behaves as follows:

\[
V_0 \simeq h^2 \left( \mu^4 - 2 h \mu_\phi \mu^2 X + \mu^4 \frac{X^2}{\Lambda^2} + \cdots \right),
\]

(3.13)

where the \( \cdots \) stand for terms that are negligible for small \( \mu_\phi \). In (3.13) we neglected the quadratic term in the expansion of the classical potential (3.11) relative to the one that comes from the Kähler potential (2.22). This is justified when

\[
h\tilde{\Lambda} \ll \frac{\mu^2}{\mu_\phi},
\]

(3.14)
i.e. the UV scale \( \tilde{\Lambda} \) is well below the value of \( X \) at the supersymmetric vacuum, (3.12).

The potential (3.13) has a local minimum at

\[
hX_{\text{min}} = \frac{\mu_\phi}{\mu^2} (h\tilde{\Lambda})^2.
\]

(3.15)

In the regime (3.14) this minimum is located well below \( \tilde{\Lambda} \). Since in obtaining (3.15) we approximated the Kähler potential (2.21) by (2.22), it is valid when \( X_{\text{min}} \ll \Lambda \). This implies the following hierarchy of scales:

\[
X_{\text{min}} \ll \Lambda \ll \tilde{\Lambda} \ll \sqrt{\Lambda X_{\text{susy}}} \ll X_{\text{susy}},
\]

(3.16)

where \( X_{\text{susy}} \) is given by (3.12).

We see that the brane analysis of the R-symmetry breaking brane configuration of figure 3 performed in [8] has a simple interpretation in the low energy effective field theory. Combining the non-trivial Kähler potential found in section 2, (2.21), with the superpotential corresponding to the deformed brane system, (3.1), leads to metastable vacua at non-zero \( \Phi \) (3.15), in which the two balance each other. As in section 2, the corresponding WZ model [7,9] exhibits similar vacua, but the stabilization mechanism is different – the role of the Kähler potential is now played by the one loop potential. We will comment on the transition between the two regimes later.

\[\text{6} \] Recall that the parameters \( h, \mu, \mu_\phi \) are taken to be real and positive.
3.3. Tachyonic branches

The brane system of figure 3 has a rich phase structure that was explored in [8], where it was found that vacua are labeled by two integers \( k, n \) (see figure 4). The \( N_f - N_c - k \) D4-branes stretched between the \( NS \) and \( NS' \)-branes give rise to an unbroken \( U(N_f - N_c - k) \) subgroup of the magnetic gauge group. A \( U(k) \times U(n) \times U(N_f - k - n) \) subgroup of the \( U(N_f) \) flavor group is unbroken as well.

The magnetic meson field \( \Phi \) can be decomposed as

\[
h\Phi = \begin{pmatrix} 0_k & 0 & 0 \\ 0 & h\Phi_n & 0 \\ 0 & 0 & \frac{\mu^2}{\mu_\phi} I_{N_f - k - n} \end{pmatrix}.
\]

The \( n \times n \) matrix \( \Phi_n \) describes the position of the \( n \) fourbranes that break supersymmetry in figure 4. The system can relax to a supersymmetric state in one of two ways. The \( n \) D4-branes can move to \( w = w_0, (3.3) \), where \( h\langle \Phi_n \rangle = \frac{\mu^2}{\mu_\phi} I_n \); this leads to the configuration of figure 4 with \( n = 0 \). Alternatively, they can connect with some of the \( N_f - N_c - k \) color branes, effectively increasing the value of \( k \). Of course, in general the system can relax by a combination of the two processes.

![Fig. 4: The vacuum structure of the deformed brane configuration.](image-url)

In addition to the supersymmetric vacua, the system has some non-supersymmetric metastable ones. For \( k = N_f - N_c \) they can be constructed using the results of the previous subsection. Here, we would like to discuss the case \( k < N_f - N_c \), where some new issues appear due to the presence of the color branes.
The analysis of the effective potential for $\Phi_n$ that is due to the Kähler potential (2.20) and superpotential (3.1) is identical to the one for $X$ in the previous subsection. It leads to a local minimum of the potential at (see (3.15))

$$h\langle \Phi_n \rangle = \frac{h\mu_0}{\mu^2}(h\tilde{\Lambda})^2 I_n.$$  

(3.18)

However, in this case there is a further potential instability. In brane language it corresponds to the reconnection process of some or all of the $n$ flavor branes with color ones. It is described in string theory by the condensation of an open string tachyon stretched between the two stacks of branes. The mass of this tachyon is given by [14]

$$\alpha' m^2 = -\frac{a}{2\pi}.$$  

(3.19)

For it to be non-tachyonic, the endpoints of the color and flavor branes on the $NS'$-brane must be a distance $w$ apart, with

$$w > l_s\sqrt{2\pi a}.$$  

(3.20)

Using (2.19) and (2.31), (3.20) implies that

$$\langle \Phi_n \rangle = x\mu I_n, \quad \text{with } x > 1.$$  

(3.21)

Comparing to (3.18) we conclude that from the brane perspective, the vacua with $k < N_f - N_c$ are metastable when

$$\tilde{\Lambda} > \tilde{\Lambda}_c = \sqrt{\mu \cdot \frac{\mu^2}{h\mu_0}} = \sqrt{\mu X_{\text{susy}}}.$$  

(3.22)

Note that (3.22) is consistent with (3.16) when $\mu \ll \Lambda$, which is natural in the effective field theory, and is satisfied in the geometric brane regime as well.

From the field theory perspective, the constraint (3.21) is due to the presence of unHiggsed magnetic quarks $q, \bar{q}$ in vacua with $k < N_f - N_c$. In the regime of interest, the Kähler potential for these quarks can be taken to be canonical, and their potential is given by (3.4).

Diagonalizing the mass matrix for the bosonic and fermionic components of $q, \bar{q}$, we find

$$m_b^2 = |h\mu|^2(|x|^2 \pm |1 - \epsilon x|), \quad \text{(degeneracy } 2n(N_f - N_c)),$$

$$m_{\tilde{b}}^2 = |h\mu|^2(|x|^2 \pm |1 - \epsilon x|),$$

(3.23)

For simplicity we restrict to the case $k = 0$, in which the analysis simplifies somewhat. Other cases are qualitatively similar.
\[ m_f^2 = |h \mu|^2 |x|^2, \quad \text{(degeneracy } 4n(N_f - N_c)) \), \quad (3.24) \]

where \( x \) is defined in (3.21) and

\[ \epsilon = \frac{h \mu}{\mu} = \frac{l_s \theta}{v_2} \sqrt{2\pi a} . \quad (3.25) \]

Requiring that the scalar masses (3.23) are non-tachyonic leads to the constraint \( x > 1 - O(\epsilon) \), in agreement with (3.21).

Thus, we see that the non-trivial Kähler potential for \( \Phi \), (2.20), gives rise to a large set of metastable vacua, some of which exist only in the range (3.22). Note that our results are consistent with the recent analysis of [9]. These authors showed that taking the Kähler potential of \( \Phi \) to be canonical, the one loop potential in the WZ model is not sufficient for pushing the metastable vacua discussed in [9] to the region (3.21), in which the unHiggsed magnetic quarks are non-tachyonic. From the point of view of [9], our analysis takes place at a finite value of the UV cutoff in the WZ model. The Lagrangian includes an infinite set of non-renormalizable operators, which seem unmotivated from the low energy point of view, but are determined in string theory. As we saw in (3.16), the scales associated with these operators are in fact rather low.

### 3.4. From Kähler to one loop

The brane system we are studying reduces to a WZ model with superpotential (3.1) and canonical Kähler potential when the displacement \( v_2 \) in figure 4 is taken to be well below \( g_s l_s \). In that regime, the leading corrections to the dynamics of \( \Phi \) come from the one loop potential, which is not sufficient to produce locally stable minima in the tachyonic branches. On the other hand, for larger \( v_2 \) the dominant effect is the non-trivial Kähler potential and other D-terms, which in general lead to the appearance of locally stable vacua in tachyonic branches.

It would be interesting to study the transition between the two regimes in detail by following the dynamics as one changes the parameters of the brane model. Unfortunately, this is difficult for reasons that were described above. In this subsection we consider a toy model, which we believe captures the essence of the problem.

We take the potential to be a sum of classical and one loop contributions,

\[ V = V_0 + V_1 , \quad (3.26) \]
where $V_0$ is the tree level potential (3.13), and $V_1$ the Coleman-Weinberg potential

$$V_1 = \frac{1}{64\pi^2} \text{Str} \left( m^4 \log m^2 \right).$$

(3.27)

The supertrace runs over the spectrum (3.23), (3.24), relevant for vacua with $k = 0$ in figure 4. We fix the parameters $h$, $\mu$, $\mu_\phi$ that enter the classical Lagrangian of the WZ model, and vary the UV scale $\tilde{\Lambda}$ that parameterizes the deviation from canonical Kähler potential.

Consider first the case $\mu_\phi \ll h\mu$. When $\tilde{\Lambda}$ is small, the non-canonical Kähler potential (2.22) is dominant, and the potential (3.26) has a local minimum at (3.15). The location of this minimum increases as $\tilde{\Lambda}$ increases; at the same time the size of the quadratic term in the potential for $X$, (3.13), decreases. At $\tilde{\Lambda} \sim \mu/h$, the quadratic term in the one loop potential $V_1$, (2.8), becomes comparable to the Kähler one, and as $\tilde{\Lambda}$ is increased further it dominates over it. In this regime, the location of the local minimum (3.15), $\langle \Phi_n \rangle \sim \mu_\phi/h \ll \mu$, no longer increases with $\tilde{\Lambda}$. Since this value is not in the regime (3.21), the corresponding vacua are unstable to condensation of the tachyonic modes of $q$, $\tilde{q}$ discussed above.

As $\tilde{\Lambda}$ increases further, additional local minima of the potential (3.26) appear. In particular, when $\tilde{\Lambda} \gg \mu^2/\mu_\phi$, one can show that the local minimum of $V_0$ (3.13) becomes again a local minimum of the full potential (3.26); the one loop correction to $V_0$ at that point is negligible. Since the value of (3.15) in that case is parameterically larger than $\mu$, this vacuum is metastable.

Increasing $\tilde{\Lambda}$ further leads eventually to violation of the bound (3.14), after which other contributions to the potential become important. For $\tilde{\Lambda}$ well above the supersymmetric vacuum (3.12), the contribution of the non-canonical Kähler potential to the dynamics can be neglected, and the analysis reduces to that of [9]. In particular, the metastable vacua due to the Kähler potential disappear in this regime.

For $\mu_\phi \sim h\mu$ or larger, the one loop contribution to (3.26) is small for all $\tilde{\Lambda}$ in the range (3.14). Thus, the analysis of the classical potential with a non-trivial Kähler potential is valid. For $\tilde{\Lambda}$ outside the range (3.14), the metastable states disappear, as before.

Thus, we see that metastable vacua in the tachyonic branches exist when the UV cutoff in the WZ model is sufficiently small. When the cutoff exceeds a value comparable to the scale set by the supersymmetric vacuum, (3.12), they disappear. This picture is in agreement with the results of [9] who found no metastable states in the limit $\tilde{\Lambda} \to \infty$.

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8 Here and below we neglect numerical factors that depend on $N_f$, $N_c$. 

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4. Discussion

In this paper we studied non-supersymmetric vacua of systems of intersecting $D$-branes and $NS5$-branes in string theory. Such systems are very useful for embedding supersymmetric gauge theory dynamics into string theory [3], and it is natural to ask whether they shed light on supersymmetry breaking as well.

Previous work on these systems showed that they exhibit some qualitative similarities to the field theories discussed by ISS [1], but some important differences were noted as well. In particular, the detailed phase structure and mechanism for the stabilization of the pseudo-moduli $\Phi$ are in general different in (renormalizable) field theory and in string theory.

The purpose of this work was to provide a description of the string theory analysis in terms of the light degrees of freedom. We found that while in [1] the pseudo-moduli were stabilized by one loop effects in the corresponding Wess-Zumino model, string theory leads naturally to a different class of effective field theories. In addition to the superpotential (2.4), (3.1), one finds a non-trivial Kähler potential, (2.20), (2.22), and higher order $D$-terms such as (2.27). These $D$-terms play an important role in analyzing the vacuum structure of the theory, and give rise to a rich landscape of metastable supersymmetry breaking vacua.

In the low energy theory the $D$-terms correspond to non-renormalizable operators that require a UV completion. Such a completion is provided by string theory. It is in general different from the UV completion proposed by ISS, in terms of an asymptotically free Seiberg dual field theory, but since in the brane system one can continuously interpolate between the two, many features of the phase structure are qualitatively similar.

We calculated the Kähler potential of the pseudo-moduli in the approximation where the $D$-branes giving rise to them can be treated as local probes of the geometry of the extra dimensions, and found that in that approximation the Kähler potential is determined by the geometry, (2.20). The fact that the metric on a supersymmetric moduli space of $D$-branes moving in a non-trivial geometry depends simply on that geometry is well known. Here we found that this property persists to non-supersymmetric (pseudo) moduli spaces. This was established to leading order in the supersymmetry breaking parameter, but is likely to be true more generally.

There are a number of natural extensions of this work that are worth pursuing. In the supersymmetric case, the structures seen in type IIA brane systems of the sort considered
here have IIB counterparts in terms of $D$-branes wrapping small cycles on Calabi-Yau manifolds. It would be interesting to extend these results to non-supersymmetric systems, and in particular calculate the Kähler potential and higher D-terms in that case.

It would also be interesting to consider the time-dependent dynamics of systems of the sort considered here, that might be relevant for early universe cosmology. If the $NS$-brane that produces the non-canonical D-terms is in motion with respect to the $D$-branes on which the pseudo-moduli live, the potential changes with time. The system could be trapped for a while in a metastable minimum, and then decay when the fivebrane reaches a critical distance from the $D$-branes. This might give interesting models of inflation in string theory.

One can try to use models of the sort described here as hidden sector models for supersymmetry breaking in nature. In particular, the metastable vacua that we found in the tachyonic branches, which owe their existence to the non-trivial Kähler potential, appear to be particularly promising candidates for phenomenology, due to the large breaking of R-symmetry in them.

As an example, one can consider the brane configuration of figure 4 in the metastable state with $k = 0$, and arbitrary $n$. In this vacuum, the system has a global symmetry $U(n) \times U(N_f - n)$. It is natural to embed the gauge group of the MSSM in the $U(n)$ factor. The pseudo-modulus $\Phi_n$ is the SUSY breaking chiral spurion, and the magnetic quarks $q, \tilde{q}$ are the messengers. Standard gauge mediation (see e.g. [15] for a review) leads to comparable gaugino and sfermion masses of the order of $\alpha\mu^2/X_{\text{min}}$. One can choose the parameters of the model such that these masses are around the weak scale.

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Appendix A. Fermionic terms in the low energy effective Lagrangian

In section 2.2 we calculated the leading terms in the low energy effective Lagrangian of the bosonic (bottom) component of the chiral superfield $\Phi$. In this appendix we show that the fermionic terms are consistent with (2.25), with the Kähler potential (2.20).

Expanding (2.25) in components (and restricting again to a single $D_4$-brane, for simplicity) gives [13]:

$$L_2 = -K_{\Phi\Phi} \left(|\partial_\mu \Phi|^2 + i\bar{\psi} D_\mu \psi\right) + \cdots$$

(A.1)

where $D_\mu \psi = \partial_\mu \psi + \partial_\mu \Phi \Gamma_{\Phi\Phi} \psi$, with $\Gamma_{\Phi\Phi} = \partial_\Phi H(|\Phi|)$. The supersymmetry transformation is

$$\delta \zeta \Phi = \sqrt{2} \zeta \psi \ .$$

(A.2)

To compute the fermionic term in (A.1) for the brane system, we start with the DBI action for the $D_4$-brane, keeping terms quadratic in the fermions [14-18]:

$$L_{D4 \text{ fermi}} = \frac{i}{2} T_4 \int d^5 \sigma e^{-\phi} \sqrt{-\det P(G + B) \Theta(1 + \tilde{\Gamma}_{D4})(P(\Gamma^a D_a) - \Delta) \Theta} \ .$$

(A.3)

Here $\Theta$ is the thirty-two component Majorana spinor, $D_A = \nabla_A + \frac{1}{4} H_{ABC} \Gamma^{BC} \mathbf{T}$ is the torsional connection, and $\Delta = \frac{1}{2} \left(\Gamma^A \partial_\Phi \varphi + \frac{1}{6} H_{ABC} \Gamma^{ABC} \right)$. The matrix $\tilde{\Gamma}_{D4} = \frac{1}{3!} \varepsilon^{a_1 \ldots a_5} \Gamma_{a_1 \ldots a_5} \mathbf{T}$ is the kappa symmetry projector, and $\mathbf{T}$ is the usual chirality matrix in ten dimensions. $H_{ABA}$, $a = 0, \ldots, 4$ are worldvolume coordinates, while $A, B$ are ten-dimensional tangent space indices.

The first step is to extract the superpartner $\psi$ of $\Phi$ from the thirty two component spinor $\Theta$. The supersymmetry transformations of the transverse scalars $X^I (= v, w, x^7)$ take the form

$$\delta X^I = -\frac{i}{2} \tau^{I} \Theta \ ,$$

(A.4)

The configuration we study (figure 1 with $v_2 = 0$) preserves four supercharges. The corresponding supersymmetry parameter, $\epsilon$, is determined by the projection conditions

$$\Gamma_{NS} \epsilon = \Gamma^0 \ldots \Gamma^3 \Gamma^w v_1 \Gamma^w_2 \epsilon = \epsilon \ ,$$

$$\Gamma_{D6} \epsilon = \Gamma^0 \ldots \Gamma^3 \Gamma^w v_1 \Gamma^w_2 \Gamma^7 \epsilon = \epsilon \ ,$$

$$\Gamma_{D4} \epsilon = \Gamma^0 \ldots \Gamma^3 \Gamma^w \mathbf{T} \epsilon = \epsilon \ .$$

(A.5)
We pick a basis for the gamma matrices compatible with the symmetries of the brane configuration
\[
\Gamma^\mu = \sigma^2 \otimes \gamma^\mu \otimes 1_2 \otimes 1_2 ,
\]
\[
\Gamma^{v_1,v_2,y} = \sigma^1 \otimes 1_4 \otimes \sigma^{1,2,3} \otimes 1_2 ,
\]
\[
\Gamma^{w_1,w_2,7} = \sigma^3 \otimes 1_4 \otimes 1_2 \otimes \sigma^{1,2,3} .
\]
(A.6)

The projection conditions imply \( \epsilon \) is of the form
\[
\epsilon = \chi_+ \otimes \epsilon_R \otimes \chi_+ \otimes \chi_+ + c.c. ,
\]
(A.7)

where the complex conjugate is determined by the Majorana condition \( \epsilon^* = B \epsilon \) (\( B \) is the product of all the real gamma matrices). \( \chi_\pm \) is a constant two-dimensional spinor satisfying \( \sigma^3 \chi_\pm = \pm \chi_\pm \), and \( \epsilon_R \) is a right-handed Weyl spinor in the \( \mathbb{R}^{3,1} \) worldvolume of the brane configuration.

Recalling the preserved supersymmetries of the DBI action are given by (A.4), using (A.7) and setting \( I = w \) in (A.4), we see that the superpartner of \( \Phi \) has the following embedding in \( \Theta \):
\[
\Theta = \chi_- \otimes \psi_L \otimes \chi_+ \otimes \chi_- + c.c.
\]
(A.8)

To relate this to the conventional form for a Weyl spinor in four-dimensions, we choose a Weyl basis such that \( \epsilon_R = \frac{1}{\sqrt{2}} (0, \tilde{\zeta}_\alpha)^T \) and \( \psi_L = \frac{1}{\sqrt{2}} (\tilde{\psi}^\alpha, 0)^T \). Then, using (2.18), (A.4) gives us the supersymmetry variation for \( \Phi \)
\[
\delta \Phi = \sqrt{2T_4 \Delta y} \frac{H(|\Phi|)^{-\frac{1}{2}}}{g_s} \tilde{\zeta} \tilde{\psi} ,
\]
(A.9)

where the \( H^{-1/2} \) comes from the vielbein in \( \Gamma^w \). Comparing to (A.2) we identify the canonically normalized superpartner of \( \Phi \),
\[
\psi = \sqrt{\frac{T_4 \Delta y}{g_s}} H(|\Phi|)^{-\frac{1}{2}} \tilde{\psi} .
\]
(A.10)

Finally, plugging (A.10), (A.8) into (A.3), and evaluating on the CHS background (2.10), we find the expected result (A.1).
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