The New Numerical Galaxy Catalogue ($\nu^2$GC): Properties of Active Galactic Nuclei and Their Host Galaxies

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ABSTRACT
We present the latest results of a semi-analytic galaxy formation model, “New Numerical Galaxy Catalogue”, which utilises large cosmological $N$-body simulations. This model can reproduce many statistical properties of galaxies at $z \lesssim 6$. We focus on the properties of active galactic nuclei (AGNs) and supermassive black holes, especially on the accretion timescale onto black holes. We find that the number density of AGNs at $z < 1.5$ and at hard X-ray luminosity $< 10^{44}$ erg/s is underestimated compared with recent observational estimates when we assume the exponentially decreasing accretion rate and the accretion timescale which is proportional to the dynamical time of the host halo or the bulge, as is often assumed in semi-analytic models. We show that to solve this discrepancy, the accretion timescale of such less luminous AGNs should be a function of the black hole mass and the accreted gas mass. This timescale can be obtained from a phenomenological modelling of the gas angular momentum loss in the circumnuclear torus and/or the accretion disc. Such models predict a longer accretion timescale for less luminous AGNs at $z < 1.0$ than luminous QSOs whose accretion timescale would be $10^7$–$10^8$ yr. With this newly introduced accretion timescale, our model can explain the observed luminosity functions of AGNs at $z < 6.0$.

Key words: methods: analytical – galaxies: active – galaxies: evolution – galaxies: nuclei – (galaxies:) quasars: supermassive black holes – galaxies: statistics

1 INTRODUCTION

Galaxies are one of the main components of the Universe. Understanding galaxy formation and evolution is thus one of the main goals of astrophysics. Almost all galaxies have a super massive black hole (SMBH) at their centre and the mass of SMBHs correlates with properties of their host galaxies, such as the mass and velocity dispersion of the bulges (e.g. Magorrian et al. 1998; Ferrarese & Merritt 2000; Haring & Rix 2004; McConnell & Ma 2013). SMBHs and their host galaxies would thus have co-evolved with each other. The gas can lose its momentum via the growing processes of the bulge and/or galactic bars, and a part of the gas would get accreted onto the SMBH (e.g. Milos & Hernquist 1994; Wada & Habe 1995), which could be observed as active galactic nuclei (AGNs). After that, AGN radiation, jets, and outflow inject the energy and/or angular momentum to the surrounding gas, which would cause the increase/decrease of the star formation rate (SFR) of their host galaxies (e.g. Wagner et al. 2016, for a review). This “co-evolution” is a

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standing question in astrophysics, and has been the subject of theoretical and observational studies over three decades. Such work has focused on the mechanism of black hole (BH) feeding and the energetic feedback related with BH growth in the context of galaxy formation (see, however, Jahnke & Macciò 2011).

Understanding the growth mechanisms and evolution of SMBHs is challenging because they cannot be directly observed. AGNs are the main sources to obtain information on SMBHs observationally, which emit light when material is accreted onto the SMBHs. To overcome the difficulty in investigating growth mechanisms and evolution of SMBHs, we need close comparisons between model predictions and observations of both galaxies and AGNs.

Semi-analytic models of galaxy formation (hereafter SA models) are powerful tools for making theoretical predictions that can be directly compared with observations. In SA models, merging histories of dark matter (DM) haloes are obtained from N-body simulations (e.g. Roukema et al. 1997; Okamoto & Nagashima 2003; Nagashima et al. 2005; De Lucia et al. 2010; Makiya et al. 2016; Guo et al. 2016) or analytic algorithms based on the extended Press-Schechter formalism (e.g. Press & Schechter 1974; Lacey & Cole 1993; Nagashima & Yoshii 2004; Menci et al. 2005; Valiante et al. 2011). The evolution of baryonic components such as gaseous haloes, galaxies, and SMBHs is followed by phenomenological modellings to diminish the computational cost and to enlarge the sample size. Therefore, SA models are an excellent approach for statistical studies of galaxies and SMBHs and particularly useful for theoretical studies of rare objects, such as AGNs.

There are a large number of previous studies using SA models aimed at revealing the evolution of SMBHs within their host galaxies. The evolution of the AGN luminosity function (LF), which has been well known to imply the “anti-hierarchical trend” of SMBH growth, is regarded as one of the main observational constraints on the models. In the earlier studies, they tested the “merger-driven AGN scenario” by comparing their QSO LFs with observational ones in optical bands (e.g. Kauffmann & Haehnelt 2000; Enoki et al. 2003). Other triggering mechanisms of AGN activities such as disc instabilities (e.g. Lagos et al. 2008; Fanidakis et al. 2011; Hirschmann et al. 2012), direct accretion from its hot gas halo (e.g. Fanidakis et al. 2012; Griffin, Lacey, Gonzalez-Perez, del P. Lagos, Baugh & Fanidakis Griffin et al.), the effect of BH spins (e.g. Lagos et al. 2009; Fanidakis et al. 2011; Griffin, Lacey, Gonzalez-Perez, del P. Lagos, Baugh & Fanidakis Griffin et al.), the effect of the seed BH mass (e.g. Volonteri & Natarajan 2009; Shirakata et al. 2016), and AGN clustering (e.g. Fanidakis et al. 2013; Oguri et al. 2016, 2017).

There are, however, uncertainties related with phenomenological modellings of the SMBH evolution, e.g. the triggers and the duration of gas accretion, the relation between the accretion rate and the AGN luminosity, the dust attenuation, Compton absorption, BH seeding, and AGN feedback. Unfortunately, several physical processes degenerate. Different combinations of phenomenological modellings and free parameters in a model could equally well explain observational properties of AGNs. Therefore, it is important to understand the effect of each phenomenological modelling on properties of SMBHs and AGNs.

In this paper, we focus on the accretion timescale onto SMBHs. Estimation of this timescale is important as it reveals the co-evolution between SMBHs and their host galaxies. If all galaxies have undergone the AGN phase, the duration of this phase should be short to explain the observed AGN LFs. In contrast, AGNs should be long-lived if a small fraction of galaxies have experienced this phase (e.g. Soltan 1982).

There are some constraints on the accretion timescales obtained from previous studies (see Martini 2004, for more details). Yu & Tremaine (2002) estimate the timescale by comparing present-day mass density of BHs with the integrated accreted mass density in luminous AGN phases obtained from optical AGN LFs at various redshifts. They suggest that the average “AGN lifetime” is $3 - 13 \times 10^7$ years for $10^{45 - 47} M_{\odot}$ BHs if the radiation efficiency, $\epsilon$, is 0.1 - 0.3. On the theoretical side, Kauffmann & Haehnelt (2000, hereafter KH00) estimate the AGN lifetime by using an SA model. They assume a constant radiation efficiency for AGNs, which are triggered only by major mergers of galaxies. They derive the average AGN lifetime to explain observed AGN LFs with $M_B \lesssim -23$ (where $M_B$ is the $B$-band absolute magnitude). They suggest that the lifetime is $\sim 3 \times 10^8$ yr at $z = 0$ and that the timescale would scale with the dynamical time of the halo; $\propto (1 + z)^{-1.5}$.

In these studies, the AGN lifetime is assumed to be the timescale within which SMBHs are observed as optical AGNs. This timescale is not necessarily equal to the accretion timescale onto SMBHs. Hopkins et al. (2005) estimate not the AGN lifetime but the “total” accretion timescale considering the obscured accretion phases by using hydro-dynamic simulations. They suggest that the accretion onto an SMBH is not visible at first because gas and dust components are surrounding the nuclear region. After blowing out these components by AGN winds, AGNs can be observed as optical sources. The AGN lifetime is then $\sim 20$ Myr and the total accretion timescale is $\sim 100$ Myr for AGNs with $M_B < -22$.

There are still two uncertainties about the accretion timescale. One is the physical processes that govern the timescale. Several authors have proposed different mechanisms that determine the accretion timescale. KH00 suggest it is proportional to the dynamical time of the host halo. Norman & Scoville (1988) propose that the gas accretion continues during a starburst in its host galaxy, because they assume that the gas fueling to an SMBH is promoted by the mass loss from large star clusters. Granato et al. (2004) and Fontanot et al. (2006) assume the accretion rate to be determined by the viscosity of the accretion disc. The effect of these different assumptions on statistical properties of AGNs and SMBHs remains unclear.

It is also unclear whether the timescale of less luminous AGNs is the same order as that of luminous ones. Previous work has focused on the timescale of optical AGNs with $M_B < -22$ (hard X-ray (2-10 keV) luminosity, $L_X$, corresponds to $\sim 5 \times 10^{43}$ erg/s) whose SMBH mass is larger than $\sim 10^5 M_{\odot}$. Less luminous AGNs with $L_X \lesssim 10^{44}$ erg/s would have wide range of SMBH masses. The accretion timescale
of such less luminous AGNs is not necessarily in the same order as luminous AGNs.

There is a well-known problem of SMBH growth scenario. Assuming that AGN activities are triggered only by mergers of galaxies and that the accretion timescale is \( \sim 10^{7-8} \) yr, the number density of less luminous AGNs are underestimated in SA models. This implies that to explain the observed “anti-hierarchical trend” of SMBH growth, we need to consider other triggering mechanisms of SMBHs and/or to reconsider the accretion timescale. ¹ As an example, Hirschmann et al. (2012) assume that AGNs are triggered solely by galaxy mergers, and set the accretion timescale is proportional to the Salpeter timescale,

\[
e_{\text{acc}} \frac{L_{\text{Edd}}}{4\pi G m_p} L \sim 4.5 \times 10^7 \left( \frac{\epsilon}{0.1} \right) \left( \frac{L_{\text{Edd}}}{L} \right) \text{yr},
\]

for all AGNs, where \( \epsilon = L/\dot{M} c^2 \) is the radiation efficiency, \( L \) is the bolometric luminosity of AGNs, \( L_{\text{Edd}} = 4\pi G m_p c M_{\text{BH}} / \sigma_T \) is the Eddington luminosity, \( M_{\text{BH}} \) is the SMBH mass, \( \sigma_T, G, m_p, c \), are cross section of Tompson scattering, gravitational constant, proton mass, and the speed of light, respectively. Their model also underestimates the number density of less luminous AGNs at \( z < 1.5 \). They solve this problem by introducing a disc instability as a triggering mechanism of AGNs. Other SA models also try to reproduce AGN LFs by introducing additional triggering mechanisms of SMBH growth without reconsidering the accretion timescale.

In this paper, we test a new phenomenological and physically-motivated model of the accretion timescale and investigate whether the effect of the accretion timescale can explain the cosmic evolution of the AGN number density. We employ a revised version of an SA model, “New Numerical Galaxy Catalogue” (hereafter “New Numerical Galaxy Catalogue” (hereafter \( \nu^2 GC \); Makiya et al. 2016, hereafter M16). The model more accurately explains statistical properties of galaxies and AGNs at various redshifts than the model of M16. We show the statistical properties of AGNs and SMBHs obtained by the model. This paper is organized as follows. In Sec. 2 we present the details of the modellings which is relevant to the growth of SMBHs and their host bulges. In Sec. 3, we present the statistical properties of model SMBHs and AGNs. We mainly focus on the effect of the accretion timescale on AGN properties. Finally, in Sec 4, we discuss and summarise the results.

## 2 MODEL DESCRIPTIONS

We create merging histories of DM haloes from large cosmological \( N \)-body simulations (Ishiyama et al. 2015) ², which have higher mass resolution and larger volume compared with previous simulations (e.g. 4 times better mass resolution compared with Millennium simulations, Springel et al. 2005). Table 1 summarises basic properties of the simulations. The \( \nu^2 GC - \nu \) and -SS simulations have the same mass resolution with different box sizes (\( L = 560 \) and 70 \( h^{-1} \)Mpc, respectively). The \( \nu^2 GC - \nu \) simulation is one of the high-resolution simulations for our SA model which has \( ~ 64 \) times higher mass resolution than the \( \nu^2 GC - SS \) simulation with the same box size. Throughout this paper, we assume a \( \Lambda \)CDM universe with the following parameters: \( \Omega_0 = 0.31, \lambda_0 = 0.69, \Omega_b = 0.048, \sigma_8 = 0.83, n_s = 0.96, \) and a Hubble constant of \( H_0 = 100 \) \( h \) km \( s^{-1} \) Mpc\(^{-1} \), where \( h = 0.68 \) (Planck Collaboration et al. 2014, 2016).

The model used in this work is based upon the original \( \nu^2 GC \) model of M16, although it has undergone numerous improvements. This model originates from Nagashima & Yoshii (2004) and Nagashima et al. (2005) (\( \nu^2 GC \) model). Both \( \nu^2 GC \) and \( \nu^2 GC \) models have been used for a variety of astrophysical studies including gravitational waves, Ly\( \alpha \) emitters, star formation, and AGN clustering (Enoki et al. 2003; Enoki & Nagashima 2007; Kobayashi et al. 2007; Makiya et al. 2014; Oogi et al. 2016, 2017). The SMBH growth and AGN properties in M16 are based on Enoki et al. (2003), Enoki et al. (2014), and Shirakata et al. (2015). In this section, we describe the processes which relate to the growth of SMBHs (and their host bulges). Other modellings are shown in Appendix A. Schematics of the model are shown in Fig. 1. In our model, the values of adjustable parameters are determined by a Markov Chain Monte Carlo (MCMC) method. The details of fitting procedures, its result, and the resulting statistical properties of galaxies with the fiducial model are shown in Appendix B.

### 2.1 Bulge growth by mergers and disc instability

We assume that the bulge (spheroid) component within a galaxy grows via starbursts and the migration of disc stars, both of which are triggered by mergers of galaxies and disc instabilities. Our model for these processes is based on Shirakata et al. (2016).
2.1.1 Mergers of galaxies

When DM haloes merge with each other, the newly formed halo should contain several galaxies which are classified as satellite galaxies and a single central galaxy. All members of this galaxy group would eventually merge under the gravitational attraction of the resultant halo. Mergers of galaxies occur via dynamical friction (central-satellite merger) and random collision (satellite-satellite merger). We estimate the timescales of dynamical friction and random collision in the same manner as M16. For the dynamical friction, we set the merger timescale, $t_{\text{merg}}$, as $t_{\text{merg}} = f_{\text{merg}} t_{\text{nic}}$, where $f_{\text{merg}}$ is an adjustable parameter (in this paper, $f_{\text{merg}} = 0.81$) and $t_{\text{nic}}$ is the timescale of dynamical friction, for which we adopt the formula by Jiang et al. (2008) and Jiang et al. (2010). \(^3\)

These types of mergers induce bulge formation and growth within a galaxy. We introduce the model of the merger-driven bulge growth proposed by Hopkins et al. (2009a) based on hydrodynamic simulations. When galaxies merge, stars and gas lose their angular momentum through bar instabilities induced by the merger.

We define a primary galaxy as the galaxy with a larger baryon mass, $M_1$ (cold gas + stars + a central BH), between the merging pair, and secondary galaxy as the one with smaller baryon mass, $M_2$. We assume that the secondary is absorbed in the bulge of the primary. The bulge also obtains the cold gas and stars from the primary’s disc. The migrated stellar mass, $\Delta M_{\text{idg}}$, is determined as $M_{\text{IN}}(f_s, M_2, M_{\text{idg}})$, where $f_s = G(\mu) = 2\mu/(1 + \mu)$ is the mass fraction of the disc that is destroyed as a function of $\mu = M_2/M_1$ (Hopkins et al. 2009a). This results in the bulge of the primary gaining the stellar mass of $M_2 + \Delta M_{\text{idg}} \lesssim 2M_2$ per a merger.

The gas mass which migrates in from the primary’s disc is assumed to depend on the disc fraction of the primary, $f_{\text{id}} = (M_{\text{id}} + M_{\text{idg}})/M_1$ (where $M_{\text{idg}}$ is the cold gas mass in a primary’s disc before the merger), the gas mass fraction in the primary’s disc, $f_{\text{idg}}$, and a pair of orbital parameters, $b$ and $\theta$. The parameter, $b$, is the peri-galacticon distance before coalescence and $\theta$ is the inclination of the orbit of the secondary relative to the primary’s disc. Assuming the disc has an exponential surface density profile, we obtain the radius in which the gas migrates to the bulge, $R_{\text{gas}}$, following the Eq. 7 of Hopkins et al. (2009a):

$$R_{\text{gas}} = r_{\text{ds}}^b \left(1 - f_{\text{id}} f_{\text{idg}} F(\theta, b) G(\mu)\right),$$  \hspace{1cm} (2)

where $r_{\text{ds}}$ is the scale radius of the disc and $F(\theta, b)$ is a function of $b$ and $\theta$. \(^4\) Since we cannot obtain $b$ and $\theta$ from merger trees of the DM haloes, we employ the average value of $F(\theta, b)$ suggested by Hopkins et al. (2009a), $F(\theta, b) = 1.2$. The mass of the cold gas inside $R_{\text{gas}}$, $\Delta M_{\text{idg}} < R_{\text{gas}}$, migrates to the bulge and is exhausted by a starburst. The mass is described as follows:

$$\Delta M_{\text{idg}} = M_{\text{idg}} \times \left(1 - \left(1 + \frac{R_{\text{gas}}}{r_{\text{ds}}}\right) \exp\left(-\frac{R_{\text{gas}}}{r_{\text{ds}}}\right)\right).$$  \hspace{1cm} (3)

As seen in Eq. 2, $R_{\text{gas}}$ is larger for smaller $f_{\text{idg}}$ because gas can lose its angular momentum by the torques induced by stars (Hopkins et al. 2009a).

As shown in Eqs. 2 and 3, $\Delta M_{\text{idg}}$ is smaller than $M_{\text{idg}}$ even when $\mu = 1$ (i.e., an equal-mass merger). In this case, we cannot form pure bulge galaxies. We thus assume that the disc of the primary galaxy is completely destroyed when $\mu > f_{\text{major}}$, where $f_{\text{major}}$ is a free parameter ($f_{\text{major}} = 0.89$). We then set $\Delta M_{\text{idg}} = M_{\text{ids}}$ and $\Delta M_{\text{idg}} = M_{\text{idg}}$.

The cold gas in the bulge is consumed by a starburst even when only a minor merger occurs. The time evolution of the mass of stars, gas, metals (hot and cold phases), and BHs are calculated by Eqs. A11, A12, A13, A14, A15, and A16 with $t_{\text{bar}} \to 0$. The mass of newly formed stars by a starburst, $\Delta M_{\text{star,burst}}$ is described as:

$$\Delta M_{\text{star,burst}} = \frac{\alpha}{\alpha + \beta + f_{\text{BH}}} M^0_{\text{cold}}$$  \hspace{1cm} (4)

where $M^0_{\text{cold}}$ is the cold gas mass in the bulge immediately after a merger, $\alpha$ is the locked-up mass fraction, $f_{\text{BH}}$ is the fraction of the gas which gets accreted onto the SMBH, and $\beta$ is defined in Eq A10 in Appendix A2. Most of the cold gas in the bulge is turned into stars by the starburst and the remaining small fraction of the gas is accreted onto the central BH as described in Sec. 2.2.

2.1.2 Disc instability

We also consider bulge growths via disc instabilities. When a galactic disc becomes gravitationally unstable, a small fraction of the gas is accreted onto the bulge.

Following Efstathiou et al. (1982), a galactic disc becomes bar unstable when

$$V_{\text{max}} \left(\frac{GM_{\text{disc}}}{r_{\text{bar}}^2}\right)^{1/2} < \kappa_{\text{DLcrit}},$$  \hspace{1cm} (5)

where $V_{\text{max}}$ is the maximum rotation velocity. The scale length, $r_{\text{bar}}$, is estimated as $r_{\text{bar}} = (1/\sqrt{2}) \langle 4\pi \rangle R_{\text{init}}$, where $R_{\text{init}}$ is the initial radius of the hot gas sphere and $\langle 4\pi \rangle$ is the mean value of the dimensionless spin parameter. We employ

\(^3\) M16 set the orbital circularity as 0.5 for determining $t_{\text{nic}}$, which is the average value obtained from Wetzel (2011). In this paper, we consider the halo mass dependence on the circularity obtained from the previous work (Wetzel 2011).

\(^4\) We assume that gas and stars in the disc have the same scale radius (see, however, Mitchell et al. 2018).

\(\text{Table 1. Properties of the } v^2GC\text{ simulations we have employed in this paper. } N\text{ is the number of simulated particles, } L\text{ is the comoving box size, } m\text{ is the individual mass of a dark matter particle, } M_{\text{min}}\text{ is the mass of the smallest haloes } (= 40 \times m)\text{ which corresponds to the mass resolution, and } M_{\text{max}}\text{ is the mass of the largest halo in each simulation.}\)
\[ V_{\text{max}} = \sqrt{V_{\text{max, NFW}}^2 + V_{\text{max, bulge}}^2} \]

\[ V_{\text{max, NFW}} \sim 0.465 \frac{c}{\ln(1+c) - c/(1+c)} V_{\text{circ}}. \]

\[ V_{\text{max, bulge}} = \left\{ \begin{array}{ll} \sigma_{1D} & \left( r_{ds} < r_b \right) \\ \frac{M_{\text{bulge}} G}{r_b} & \left( r_{ds} > r_b \right). \end{array} \right. \]

where \( c \) is the concentration parameter of a DM halo, \( \sigma_{1D} \), and \( r_b \) are the 1D velocity dispersion and the size of the bulge, respectively. We assume that a bulge has the isothermal density profile (see Sec. A3.2).

The critical value for disc stabilities, \( \epsilon_{\text{DL,crit}} \) (Eq. 5), depends on the gas fraction and density profile of a galactic disc (e.g. Efstathiou et al. 1982; Christodoulou et al. 1995). If the velocity dispersion of galactic discs is neglected, the value of \( \epsilon_{\text{DL,crit}} \) is \( \sim 1.1 \) for the exponential stellar disc (Efstathiou et al. 1982) and \( \sim 0.9 \) for the gaseous disc (Christodoulou et al. 1995). We, however, treat \( \epsilon_{\text{DL,crit}} \) as an adjustable parameter, whose value should be \( \leq 1.1 \) since the disc actually has the velocity dispersion and becomes more stable. We set \( \epsilon_{\text{DL,crit}} = 0.75 \) to explain the observed cosmic SFR density. If we set \( \epsilon_{\text{DL,crit}} = 1.1 \), the cosmic SFR density becomes constant at \( 4 < z < 6 \), which is inconsistent with the previous suggestions and such model cannot explain the observed stellar mass – SFR relation.

We note that some other SA models (e.g. Cole et al. 2000; Lacey et al. 2016) use the circular velocity and the half-mass radius of the disc instead of \( V_{\text{max}} \) and \( r_{ds} \). The circular velocity would change by the effect of the supernovae (SNe) explosions. We thus use \( V_{\text{max}} \) following original prescription by Efstathiou et al. (1982). If we assume an exponential disc, the effective radius is only \( \sim 1.67 \) times larger than the scale length.

When a galactic disc becomes gravitationally unstable, a fraction of the cold gas and stars in the disc is added to the bulge component. The migrated stellar mass from the disc to bulge, \( \Delta M_{\text{ds,bulge}} \), is determined as:

\[ \Delta M_{\text{ds,bulge}} = f_{\text{bar}} M_{\text{ds}}. \]

where \( f_{\text{bar}} \) is a free parameter and \( M_{\text{ds}} \) is the stellar mass of the disc. The gas mass which migrates in from the disc, \( \Delta M_{\text{ds,gas}} \), is determined as:

\[ \Delta M_{\text{ds,gas}} = M_{\text{ds}} \times \left[ 1 - \left( 1 + \frac{R_{\text{gas}}}{r_{\text{ds}}} \right) \exp(-R_{\text{gas}}/r_{\text{ds}}) \right]. \]

\[ \frac{R_{\text{gas}}}{r_{\text{ds}}} = (1 - f_{\text{gas}}) f_{\text{bar}}. \]

\[ r_{\text{ds}} = \frac{1}{f_{\text{bar}}} \]

where \( M_{\text{ds}} \) is the gas mass of the disc. Eqs. 10, 11 and 11 are analogous to our galaxy merger case with \( G(\mu) = f_{\text{bar}} \) and \( F(\theta, b) = 1.0 \). The value of the free parameter, \( f_{\text{bar}} \), is set to 0.63.

The spheroids formed through this process might be so-called ‘pseudo-bulges’, although we do not differentiate between bulges formed by these instabilities and those formed by mergers. Starbursts triggered by these instabilities are also treated in the same way as those by mergers.

### 2.2 Growth of SMBHs and properties of AGNs

#### 2.2.1 BH seeding

A seed BH is immediately placed within a newly formed galaxy. We use a mass of the seed BHs, \( M_{\text{BH,seed}} = 10^2 M_\odot \), for all galaxies independent from the redshift. The minimum mass of the halo in which the gas cools and possibly forms a galaxy depends on redshift and the mass resolution of N-body simulations (see Fig. 2 in M16). A seed BH is, therefore, placed a halo with different mass with different mass resolution and/or at different redshift. The seed BH mass, however, does not affect the main results of this paper, focusing mainly on AGNs at \( z \lesssim 6 \), since the seed mass is negligible compared with the total amount of the accreted gas onto a BH (see Shirakata et al. 2016). Shirakata et al. (2016) suggest that the mass of the seed BHs should be dominated by \( \sim 10^2 M_\odot \) to reproduce the \( M_{\text{BH}} - M_{\text{bulge}} \) relation at \( z \sim 0 \), including galaxies with \( M_{\text{bulge}} < 10^6 M_\odot \).

#### 2.2.2 Mass accreted by SMBHs

When a starburst is triggered by a galaxy merger or disc instability (Sec. 2.1), a small fraction of the gas is supplied to the central SMBH. The accreted gas mass per starburst, \( \Delta M_{\text{acc}} \), is given by:

\[ \Delta M_{\text{acc}} = f_{\text{BH}} \Delta M_{\text{starburst}}. \]

where \( f_{\text{BH}} = 0.02 \), in this paper. We calculate the time evolution of the mass accretion rate, \( \dot{M}_{\text{BH}} \), from \( \Delta M_{\text{acc}} \) and the accretion timescale, \( \tau_{\text{acc}} \), as

\[ \dot{M}_{\text{BH}} = \frac{\Delta M_{\text{acc}}}{\tau_{\text{acc}}} \exp \left( \frac{1 - t_{\text{start}}}{\tau_{\text{acc}}} \right). \]

where \( t_{\text{start}} \) is the starting time of accretion, which is the same as that of the starburst. The prescription for \( \tau_{\text{acc}} \) is the main topic of this paper and will be described in Sec. 2.2.3 in detail. The starting time of the starburst, \( t_{\text{start}} \), is assigned randomly within the time step. Shirakata et al. (2015) suggest that \( t_{\text{start}} \) must be delayed from the starting time of the starburst so that the dust extinction of a galaxy becomes negligible for AGNs. In this paper, we do not include this delay to show clearly the effect of varying the modelling of the accretion timescale.

We note that Eqs. 13 and 14 are valid for SMBH growth via both galaxy mergers and disc instabilities. Practically, the value of \( f_{\text{BH}} \) is not necessarily the same for both galaxy mergers and disc instabilities. There are, however, almost no suggestions about the difference of the fraction of the cold gas mass which gets accreted onto an SMBH with different triggering mechanisms. We, thus, employ the common \( f_{\text{BH}} \) for diminishing the degree of freedom.

SMBHs also increase their mass via SMBH-SMBH coalescence following mergers of galaxies. As in M16, we simply
assume that SMBHs merge instantaneously after the merger of their host galaxies.

### 2.2.3 The accretion timescale for SMBHs

In this paper, we test three types of the accretion timescale summarised in Table 2. The $\text{KH00model}$, $t_{\text{acc}} = 3 \times 10^7 (1 + z)^{-1.5}$ yr, means that the accretion timescale is proportional to the dynamical time of the host halo (originally introduced by KH00).

Some SA models (e.g. Fanidakis et al. 2012; Pezzulli et al. 2017) instead use the $\text{GalModel}$, $t_{\text{acc}} = \alpha_{\text{bulge}} t_{\text{dyn, bulge}}$ by assuming the accretion continues until the gas supply from the host galaxy continues. The accretion timescale is proportional to the dynamical time of the host bulge, $t_{\text{dyn, bulge}} = r_b / V_b$ (where $r_b$ and $V_b$ are the size and 3D velocity dispersion of the bulge, respectively), and the coefficient, $\alpha_{\text{bulge}}$, is a free parameter. We choose the value of $\alpha_{\text{bulge}}$ so that the bright-end of the model AGN LFs are consistent with observed AGN LFs. In this paper, we set $\alpha_{\text{bulge}} = 0.58$.

We newly introduce the $\text{GalADModel}$ considering that the accretion would continue when gas is left in the circumnuclear torus or the accretion disc even when there is no gas supply from the host galaxy. We assume that $t_{\text{acc}}$ is the sum of the gas supply timescale from its host galaxy, which is assumed to relate with the dynamical time of the bulge, and the timescale for the angular momentum loss of the accreted gas at $\lesssim 100$ pc, $t_{\text{loss}}$:

$$t_{\text{acc}} = \alpha_{\text{bulge}} t_{\text{dyn, bulge}} + t_{\text{loss}}.$$  \hfill (15)

The second term of Eq. 15 includes the angular momentum loss timescale in a circumnuclear torus and/or in the accretion disc. We construct a simplified and phenomenological model for the angular momentum loss in the central region. The gas accretion should continue beyond the starburst phase of the host galaxies if the accreted gas requires a longer timescale to lose its angular momentum in the circumnuclear torus and the accretion disc. In this region, the gravitational potential is dominated by the SMBH. The timescale thus should depend on the mass of the SMBH. Considering a circumnuclear torus in which the mass accretion rate depends on the gravitational stability (e.g. Kawakatu & Wada 2008), the accretion timescale would become longer for the more massive SMBH. This timescale would also depend on the mass ratio between the accreted gas and the SMBH. When this ratio becomes higher, the self-gravity of the accreted gas works more effectively and thus the outer edge of the accretion disc becomes smaller. The dynamical timescale then becomes shorter. We hence describe $t_{\text{loss}}$ as a function of $M_{\text{BH}}$ and $\Delta M_{\text{acc}}$:

$$t_{\text{loss}} = \frac{t_{\text{loss},0}}{\text{Gyr}} \left( \frac{M_{\text{BH}}}{M_\odot} \right)^{\gamma_{\text{BH}}} \left( \frac{\Delta M_{\text{acc}}}{M_\odot} \right)^{\gamma_{\text{gas}}},$$  \hfill (16)

where $t_{\text{loss},0}$, $\gamma_{\text{BH}}$, and $\gamma_{\text{gas}}$ are free parameters which are tailored to match the observed AGN LFs from $z \sim 0$ to 5. We set values of $t_{\text{loss},0}$, $\gamma_{\text{BH}}$, and $\gamma_{\text{gas}}$ to be 1 Gyr, 3.5, and $-$4.0, respectively. We show that $\gamma_{\text{BH}}$ would be $> 0$ and $\gamma_{\text{gas}}$ would be $\lesssim 0$, considering the $v_z$-viscosity in the accretion disc, and these signs would be the same by considering CNDs (Appendix C).

When using this model, we find that there are SMBHs whose accretion timescale exceeds the age of the universe. In this case, we set $M_{\text{BH}} = 0$ implicitly assuming that accreted gas becomes gravitationally stable in a circumnuclear torus and/or an accretion disc, which cannot be accreted onto an SMBH. This treatment does not affect the shape of the AGN LFs since the accretion rates of such SMBHs are negligibly small.

There are some analytical estimates for the timescale of the angular momentum loss in a circumnuclear torus (e.g. Kawakatu & Umemura 2002; Kawakatu & Wada 2008), which have been employed by some SA models (e.g., Antonini et al. 2015; Bromley et al. 2004; Granato et al. 2004). We note that there are large uncertainties as to whether a circumnuclear torus with some common properties exists for all types of AGNs.

We do not consider an obscured phase (e.g. Hopkins et al. 2005), in which SMBHs do not appear as luminous AGNs at optical bands despite sufficiently large accretion rates onto SMBHs. To avoid this uncertainty, we compare the model results with observations by using AGN LFs in hard X-ray (2-10 keV) (see also Sec. 2.2.5).

### 2.2.4 AGN luminosity

We calculate the AGN bolometric luminosity, $L_{\text{bol}}$, from the accretion rate (Eq. 14). Hereafter we define the bolometric luminosity normalised by the Eddington luminosity ($L_{\text{Edd}}$) as $\lambda_{\text{Edd}} \equiv L_{\text{bol}} / L_{\text{Edd}}$ and the accretion rate normalised by Eddington rate ($M_{\text{Edd}} = L_{\text{Edd}} / c^2$) as $\dot{m}$. We employ the following relation between $\lambda_{\text{Edd}}$ and $\dot{m}$ (based on Kawaguchi 2003):

$$\lambda_{\text{Edd}} = \frac{1}{1 + 3.5 (1 + \text{tanh} (\log(\dot{m}/m_{\text{crit}})))} \left( \frac{m_{\text{crit}}}{\dot{m}} \right)^{-1},$$  \hfill (17)

where $m_{\text{crit}}$ is an adjustable parameter, whose value should be $2.5 \lesssim m_{\text{crit}} \lesssim 16.0$. We set $m_{\text{crit}} = 10.0$ and in this case, $\lambda_{\text{Edd}}$ has similar dependence on $\dot{m}$ to that obtained by Watarai et al. (2000) and Mineshige et al. (2000).

Although the gas accretion rate (Eq. 14) decreases monotonically with time, $L_{\text{bol}}$ does not necessarily decrease with time due to the difference of the change rate between $\lambda_{\text{Edd}}$ and $L_{\text{Edd}}$. When the following condition is satisfied, $L_{\text{bol}}(t)$ becomes larger than $L_{\text{bol}}(t_{\text{star}})$:

$$\frac{\lambda_{\text{Edd}}(t)}{\lambda_{\text{Edd}}(t_{\text{star}})} > \frac{L_{\text{Edd}}(t_{\text{star}})}{L_{\text{Edd}}(t)}.$$  \hfill (18)

A part of AGNs with $\lambda_{\text{Edd}} > 1.0$ satisfies this condition. We show the evolution of two SMBHs with $M_{\text{BH}} = 10^6 M_\odot$ in Fig. 2. We assume $t_{\text{acc}} = 10^7$ yr and $\Delta M_{\text{acc}} = 10^6$ and $10^7 M_\odot$ (top and bottom panels, respectively).

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Table 2. Summary of the accretion timescale model (Sec. 2.2.3).

| Model Name  | $t_{\text{acc}}$ | free parameters |
|-------------|------------------|-----------------|
| $\text{KH00model}$ | $3 \times 10^7 (1 + z)^{-1.5}$ yr | None |
| $\text{GalModel}$ | $\alpha_{\text{bulge}} t_{\text{dyn, bulge}}$ | $\alpha_{\text{bulge}}$ |
| $\text{GalADModel}$ | $\alpha_{\text{bulge}} t_{\text{dyn, bulge}} + t_{\text{loss}}$ | $\alpha_{\text{bulge}}$, $t_{\text{loss},0}$, $\gamma_{\text{BH}}$, $\gamma_{\text{gas}}$ |

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5 This also corresponds to the star formation timescale for a starburst (Nagashima et al. 2005).
bolometric correction. In this paper, we do not introduce the change of the radiative efficiency to keep the consistency and to diminish the degree of freedom of the model.

2.2.5 “Observable fraction” of AGNs

To compare the calculated AGN LFs with observed UV AGN LFs, we need to define “observable fraction” in UV-band, $f_{\text{obs, UV}}$, because we can only obtain the intrinsic luminosity of AGNs from our model. Since AGN obscuration and absorption processes are very complicated, we derive an empirical formula by the following procedures. Recent work (e.g. Aird et al. 2015; Ueda et al. 2014) has estimated the hydrogen column density distribution around AGNs by a compilation of available samples obtained by Swift/BAT, MAXI, ASCA, XMM-Newton, Chandra and ROSAT. Therefore, one can estimate the “intrinsic” luminosity in hard X-ray of observed AGNs by utilizing the hydrogen column density distribution. We thus use the observed hard X-ray LFs (Aird et al. 2015, Table 9) to obtain the “observable fraction”. The procedures are as follows.

First, we convert hard X-ray luminosities to UV luminosities with Eqs 19 and 20 and we obtain “intrinsic” UV LFs. Second, we assume the shape of the observable fraction as

$$f_{\text{obs, UV}} = A(z) \left( \frac{L_{\text{bol}}}{10^{46} \text{erg/s}} \right)^{\beta(z)} ,$$

(22)

where $L_{\text{bol}}$ is the bolometric luminosity. We assume that $A$ and $\beta$ are a function of redshift, $A(z) = A_0 (1 + z)^{A_1}$ and $\beta(z) = \beta_0 (1 + z)^{\beta_1}$, considering that the dust-to-gas ratio evolves with redshift. The value of $\beta_0$ should be positive, considering the luminosity dependence of AGN obscuration (e.g. Lawrence 1991). Third, we fit parameters, $A_0, A_1, \beta_0$, and $\beta_1$ by a Markov Chain Monte Carlo (MCMC) method to fit observed UV LFs (see the caption of Fig. 11). After $10^5$ iterations of the MCMC fitting, we obtain the best fit values ($A_0, A_1, \beta_0, \beta_1 = (0.16, -0.05, 0.07, 0.00)$) with which the observable fraction does not exceed 1.

Hopkins et al. (2007) propose an alternative formula for the “observable fraction”. They employ an observed distribution of hydrogen column density and assume a dust attenuation curve, then they derive intrinsic AGN LFs in hard X-ray (2-10 keV), soft X-ray (0.5-2 keV), optical $B$, and mid-IR (15 $\mu$m). We show the difference between observable fractions obtained from Hopkins et al. (2007) and this paper in Appendix E.

Ricci et al. (2017) suggest that observed UV LFs of AGNs are well explained by their hard X-ray LFs, whose hydrogen column densities are less than $10^{21-22} \text{cm}^{-2}$. Since the modelling of the distribution of gas around an SMBH is difficult for SA models, we estimate the observable fraction by an empirical formulation.

2.3 “Radio mode” AGN feedback

We introduce the so-called radio-mode AGN feedback process to prevent gas in massive haloes from cooling and forming stars. Following Bower et al. (2006), gas cooling in a halo is quenched when the following two conditions are satisfied:
log(M) 

\[ \frac{dN}{d\log M} \propto M^{-3} \]

where \( L_{\text{cool}} \) is the cooling luminosity of the gas, \( t_{\text{dyn}} \) is the dynamical time of the halo, \( \alpha_{\text{cool}} \) and \( \epsilon_{\text{SMBH}} \) are free parameters which are determined to reproduce the bright-end of the LFs of galaxies at \( z \approx 0 \). We set \( (\alpha_{\text{cool}}, \epsilon_{\text{SMBH}}) = (1.14, 2.19 \times 10^{-3}) \).

3 STATISTICAL PROPERTIES OF AGNS AND SMBHs

We present statistical properties of model AGNs and SMBHs, and show their dependence on the models of the accretion timescale onto SMBHs. We first present the local SMBH MF in Fig. 3 and the \( M_{\text{BH}} - M_{\text{bulge}} \) relation (including both AGNs and quiescent BHs) in Fig. 4. We show the results with the \( \nu^2 \text{GC-SS} \) and \( \nu^2 \text{GC-H2} \) simulations in both figures for checking the effect of the mass resolution. The model SMBH MF at \( z \approx 0 \) are shown as the grey dashed and black solid lines in Fig. 3. The SMBH MF is roughly consistent with the observational estimates (Shankar et al. 2004) (grey shaded region). The \( M_{\text{BH}} - M_{\text{bulge}} \) relation at \( z \approx 0 \) is consistent with observations at \( M_{\text{BH}} > 10^{8.5} M_{\odot} \) (Fig. 4) since we adjust the parameter, \( \epsilon_{\text{BH}} \), to reproduce this relation. We, however, find that the median value of the \( M_{\text{BH}} - M_{\text{bulge}} \) relation obtained by the fiducial model deviates from the observational estimates for \( M_{\text{bulge}} < 10^{9.5} M_{\odot} \). We do not use such low mass galaxies for the model calibration since the observed sample is too small. Most observational data for less massive galaxies with \( M_{\text{bulge}} < 10^{9.5} M_{\odot} \) have the same relation as the AGNs. In addition, the bulge mass of less massive galaxies is difficult to estimate by observations since the bulge is more rotational-support.

3.1 The effect of the accretion timescale on AGN LFs

We show the AGN properties obtained with \( \nu^2 \text{GC} \). We present the luminosity of AGNs in the hard X-ray band because the effect of obscuration and absorption is small. We show how AGN LFs change when we use three different models of the accretion timescale in Fig. 5. Black lines show the model hard X-ray LFs with different accretion timescales. We also show the fitting function of the LFs from Aird et al. (2015) with grey dotted lines and observed data from Aird et al. (2015), Ueda et al. (2014), and La Franca et al. (2005). We have confirmed that the results have no statistical differences when we employ the high resolution N-body simulations.

Black dashed lines show the hard X-ray (2-10 keV) AGN LFs with the KH00model, which is the timescale proportional to the dynamical time of the host halo. The model is consistent with observational results at \( \log L_X / \text{erg s}^{-1} > 43.5 \) within the dispersion of the observed data. We, however, find that the model underestimates the number density of AGNs at \( z < 1.0 \) with \( \log(L_X / \text{erg s}^{-1}) < 43.5 \) (i.e., nuclei of Seyfert galaxies), whose UV (1450Å) magnitude, \( M_{\text{UV}} \), corresponds to \( \sim -20.6 \). Such less luminous AGNs are not considered in the estimation of the AGN lifetimes in KH00 and their lifetimes could significantly differ for luminous AGNs.

Black dot-dashed lines show hard X-ray AGN LFs by the model in which the Galmodel. This modelling is similar to previous SA models (e.g. Fanidakis et al. 2012; Shirakata et al. 2016; Pezzulli et al. 2017). The accretion timescale does not cause a big difference in the faint-end slope of AGN LFs. We show how AGN LFs change when we use three different models of the accretion timescale in Fig. 5. Black lines show the model hard X-ray LFs with different accretion timescales. We also show the fitting function of the LFs from Aird et al. (2015) with grey dotted lines and observed data from Aird et al. (2015), Ueda et al. (2014), and La Franca et al. (2005). We have confirmed that the results have no statistical differences when we employ the high resolution N-body simulations.

Black dashed lines show the hard X-ray (2-10 keV) AGN LFs with the KH00model, which is the timescale proportional to the dynamical time of the host halo. The model is consistent with observational results at \( \log L_X / \text{erg s}^{-1} > 43.5 \) within the dispersion of the observed data. We, however, find that the model underestimates the number density of AGNs at \( z < 1.0 \) with \( \log(L_X / \text{erg s}^{-1}) < 43.5 \) (i.e., nuclei of Seyfert galaxies), whose UV (1450Å) magnitude, \( M_{\text{UV}} \), corresponds to \( \sim -20.6 \). Such less luminous AGNs are not considered in the estimation of the AGN lifetimes in KH00 and their lifetimes could significantly differ for luminous AGNs.

Black dot-dashed lines show hard X-ray AGN LFs by the model in which the Galmodel. This modelling is similar to previous SA models (e.g. Fanidakis et al. 2012; Shirakata et al. 2016; Pezzulli et al. 2017). The accretion timescale does not cause a big difference in the faint-end slope of AGN LFs.
AGN LFs compared with that with the KH00model, since the Galmodel has the accretion timescale with the same order as the KH00model as shown later in Fig. 6. Black solid lines show the hard X-ray AGN LFs with GalADmodel, implicitly considering the timescale of angular momentum loss in the circumnuclear torus and the accretion disc. The model enables us to reproduce not only bright-ends of the LFs but also the faint-ends, especially at $z < 1.5$. When this model of the accretion timescale is employed, a significant fraction of low-luminosity AGNs sustain their activity for a long time as we will show later. The model thus reproduces both the bright and faint-ends of AGN LFs much better than the other models.

Next, Fig. 6 shows the redshift evolution of the accretion timescale of KH00model and Galmodel, and $t_{\alpha_{\text{bulge/dyn.bulge}}}$. We select AGNs with $\log(L_X/\text{erg s}^{-1}) > 41.0$. The red circles and blue squares with error bars show the median value of $\alpha_{\text{bulge/dyn.bulge}}$ and $t_{\alpha_{\text{bulge/dyn.bulge}}}$ with 25th and 75th percentiles. The redshift evolution of the dynamical time of the bulge (red circles) and the halo (black solid line) are similar although the difference becomes larger at higher redshift. This explains why the AGN LFs with the KH00model and Galmodel are similar. While $t_{\alpha_{\text{bulge/dyn.bulge}}}$ distributes broadly, it is longer especially at lower redshift. This results in the increase of the number density of AGNs at $\log(L_X/\text{erg s}^{-1}) > 43.5$ and $z < 1.5$. We also plot $t_{\alpha_{\text{bulge/dyn.bulge}}}$ only for luminous AGNs with $\log(L_X/\text{erg s}^{-1}) > 43.5$ as green triangles. The timescale is more than 1 order of magnitude shorter than that of AGNs with $\log(L_X/\text{erg s}^{-1}) > 41.0$ at all redshifts.

The GalADmodel predicts the longer accretion timescales for the less luminous AGNs due to the effect of $t_{\alpha_{\text{bulge/dyn.bulge}}}$ as shown in Fig. 7. This figure shows the relation between hard X-ray luminosity and timescales ($t_{\alpha_{\text{bulge/dyn.bulge}}}$ and $t_{\alpha_{\text{bulge/dyn.bulge}}}$) at $z \sim 0.2$, and 4. We find that the timescale is almost constant ($\sim 2 \times 10^7$ yr) for AGNs with $\log(L_X/\text{erg s}^{-1}) > 44.0$ (corresponds to $M_{\text{UV}} < -22.3$), which is consistent with the constraints obtained by previous studies (Yu & Tremaine 2002; Kniazev & Haehnelt 2000; Hopkins et al. 2005). Less luminous AGNs, in contrast, have negative correlations between the timescale and $L_X$. We also find that the total accretion timescale becomes longer at lower redshift for all AGNs.

Figure 5. AGN LFs in hard X-ray (2-10 keV) at $z < 0.5$, $z \sim 0.7$, $z \sim 1.3$, $z \sim 2.0$, $z \sim 3.25$, and $z \sim 4.25$. The model LFs are obtained with the v2GC-M simulation. Black dashed, dot-dashed, and solid lines are the model LFs with different models of accretion timescale; the KH00model, Galmodel, and GalADmodel, respectively. Observational results are obtained from Red circles, blue triangles, and green squares are the data taken from Ueda et al. (2014), Aird et al. (2015), and La Franca et al. (2005), respectively. Grey dotted lines show the fitting LFs of observed data (Aird et al. 2015).
Figure 6. The redshift evolution of the accretion timescale with KH00model, Galmodel, and tloss. The black solid line shows the KH00model, which corresponds to the dynamical time of haloes. The red circles and blue squares with error bars show the median value of t_{dyn,bulge} and t_{loss} of AGNs with log(L_X/erg s^{-1}) > 41.0 obtained by the GalADmodel. The errorbars are 25th and 75th percentiles. We also show the value of t_{loss} of AGNs with log(L_X/erg s^{-1}) > 44.0 by green triangles.

The results obtained with the GalADmodel naturally explains the evolution of the AGN number density, which is sometimes called as “anti-hierarchical trend” of SMBH growth. The AGN number density. Figure 8 shows the number density of AGNs obtained with the GalADmodel, and those obtained from observations (Ueda et al. 2014; Aird et al. 2015). The model result shows mild anti-hierarchical trends would be partially because we consider the obscured fraction in hard X-ray (2-10 keV) is 0 at all redshift. We will show a more detailed analysis in future.

3.2 The effect of the timescale on other properties of AGNs

To see dependencies of the accretion timescale on M_{BH} and ΔM_{acc}, we show the relation between AGN bolometric luminosity and BH mass, M_{BH} (top panels), and accreted gas mass onto an SMBH, ΔM_{acc} (bottom panels) at z ~ 0, in Figs. 9 and 10. In Fig. 9, x-axes are the AGN bolometric luminosity at t = t_{start}, L_{bol}(t_{start}), while these are AGN bolometric luminosity at the output time, L_{bol}(out), in Fig. 10. The left panels show the result with the Galmodel and the right panels show that obtained by the GalADmodel. We note that the model AGNs have a weak correlation between M_{BH} and ΔM_{acc}, of the form M_{BH} ∝ ΔM_{acc}^{1/2}, with a large dispersion. This positive correlation comes from the fact that the host galaxy of the heavier SMBH is more massive and has large amount of the cold gas.

Fig. 9 shows the clear correlation between L_{bol}(t_{start}) and ΔM_{acc} with the Galmodel (bottom left panel). Since t_{dyn,bulge} is similar for galaxies at the same redshift (see Fig. 6), the peak accretion rate, M_{peak} ∝ ΔM_{acc}/t_{acc}, is mainly determined by ΔM_{acc}. The higher peak bolometric luminosity therefore implies a larger amount of the accreted gas. The relation between L_{bol}(t_{start}) and M_{BH} with the same model (top left panel) comes from the correlation, M_{BH} ∝ ΔM_{acc}^{1.1}.

The correlations obtained by the GalADmodel (right panels) show bimodal distributions, which are quite different from the model with the Galmodel. The peak accretion rate is proportional to M_{BH}^{γ_{BH}} ΔM_{acc}^{1−γ_{gas}} if α_{bulge/dyn,bulge} is smaller than t_{loss}. Since γ_{BH} = 3.5 and γ_{gas} = 4.0, M_{peak} ∝ M_{BH}^{1.5} ΔM_{acc}. The peak accretion rate, thus, can be written as M_{peak} ∝ ΔM_{acc}^{1.15} (or ∝ M_{BH}^{0.05}). These positive correlations appear as contour peaks at log(L_{bol}(t_{start})/erg s^{-1}) < 44.0.

Fig 10 shows the same relations as shown in Fig. 9, but instead plotting bolometric luminosity estimated at an out-

\[
\log(1 + z)
\]

\[
10^{0.2} 10^{0.4} 10^{0.6}
\]

\[
\log(1 + z)
\]

\[
10^{4} 10^{5} 10^{6}
\]

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10^{4} 10^{5} 10^{6}
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10^{4} 10^{5} 10^{6}
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put time. Since $L_{\text{bol}}(t_{\text{start}})$ has positive correlations with $M_{\text{BH}}$ and $\Delta M_{\text{acc}}$ when the Galmodel is employed, the dispersions of the correlation between $L_{\text{bol}}$ and $M_{\text{BH}}$ and $\Delta M_{\text{acc}}$ (left panels) reflect the elapsed time from their AGN activity.

The relation between AGN luminosity and SMBH mass allows us to compare theoretical models with observations and to potentially place a stronger constraint on the accretion timescale. There are numerous previous studies which present the relation between AGN luminosities and the SMBH mass at various redshifts (e.g., Schulze & Wisotzki 2010; Nobuta et al. 2012; Ikeda et al. 2017). Schulze & Wisotzki (2010) and Steinhardt & Elvis (2010) show the relation between the bolometric luminosity and the SMBH mass for broad line AGNs at $z < 0.3$ and $0.2 < z < 2.0$, respectively. Since their sample are limited at $L_{\text{bol}} > 10^{41.5}$ erg/s, we cannot distinguish the two models of the accretion timescale. If complete AGN sample with $L_{\text{bol}} > 10^{41.5}$ are obtained, we could put a stronger constraint on the accretion timescale.

In Fig. 11, we present AGN LFs in UV–band (1450 Å) from $z \sim 0$ to $z \sim 6$. The results are roughly consistent with observed UV AGN LFs (Croom et al. 2001, 2009; Fan et al. 2001; Richards et al. 2005, 2006; Fontanot et al. 2007; Siana et al. 2008; Glikman et al. 2011; Fiore et al. 2012; Ikeda et al. 2012; Palanque-Delabrouille et al. 2013; Ricci et al. 2017; Akiyama et al. 2018), especially at $z > 1.5$. We, however, overproduce UV LFs at lower redshift. In such redshift range, we also overproduce hard X-ray LFs (see Fig. 5) compared with the fitting LFs of Aird et al. (2015) although the model LFs are consistent with observed data points within the range of a dispersion. We need to take the dispersion of observed hard X-ray LFs into account for estimating the observable fraction although we leave it for future studies. The UV LFs do not place a strong constraint on the accretion timescale since the observed UV LFs are well determined only at $L_{\text{bol}} < 10^{42}$ (corresponds to $\log (L_{\text{bol}}/\text{erg s}^{-1}) > 44.6$) because of the contamination of galaxies’ emission (Parsa et al. 2016). The hard X-ray LFs obtained from models with the different assumption of the accretion timescale show little difference at $\log (L_{\text{bol}}/\text{erg s}^{-1}) > 44.6$.

We show Fig. 12 to show the effect of the timescale on the Eddington ratio distribution function. The black solid and dashed lines are results obtained with the Galmodel and GalADmodel, respectively. We select all AGNs with $M_{\text{BH}} > 10^6 M_\odot$ and $L_{\text{bol}} > 10^{43.5}$ erg/s at $z \sim 0$. The results at $L_{\text{bol}} > 10^{41.5}$ are roughly consistent with that obtained by the observation (Schulze & Wisotzki 2010) at $z \sim 0.3$. We, however, note that it is difficult to compare model Eddington ratio distribution functions with observations since (1) optical observational sample is limited in type-1 AGNs with well-estimated SMBH mass, (2) SMBH masses of X-ray AGNs are simply estimated from e.g., the BH mass – stellar mass relation, (3) observational sample seems to be incomplete for less massive SMBHs, and (4) the obscured fraction of AGNs would depend on both their luminosity and Eddington ratio (e.g., Oh et al. 2015; Khim & Yi 2017). Also, if there is an obscured growing phase before visible AGN phase suggested by, e.g., Hopkins et al. (2005), then the super-Eddington accreting phase should be preferentially missed.

Fig. 12 clearly show the difference caused by the implementation of the accretion timescale. The GalADmodel increases the number of AGNs with $\log (L_{\text{bol}}/\text{erg s}^{-1}) < 1.5$ and the difference between the two models becomes larger at smaller Eddington ratio. We find that the GalADmodel and Galmodel have no difference for active BHMF with AGNs $M_{\text{BH}} > 10^6 M_\odot$, $L_{\text{bol}} > 10^{43.5}$ erg/s, and $L_{\text{bol}} > 0.03$ (roughly similar selection as that of Schulze & Wisotzki 2010). As we can expected from AGN LFs (Fig. 5), the Eddington ratio distribution functions at $z > 1.0$ also have little difference between the Galmodel and Galmodel. The evolution of the Eddington ratio will appear in a future paper.
3.3 Triggers of the gas supply from host galaxies

Fig. 13 shows the fraction of AGN host galaxies at $0.0 < z < 7.0$ in each luminosity bin, divided by triggering situations. We classify the galaxies by the mass ratio of the merging galaxies: major (mass ratio $> 0.7 = f_{\text{major}}$; blue dash dotted line), intermediate ($0.4 - 0.7$; green dotted line), and minor ($< 0.4$; red solid line). The grey dashed line shows the fraction of AGNs triggered only by a disc instability. For merger-driven AGN activities, the typical merging mass ratio becomes larger for more luminous AGNs. Interestingly, we find that the primary trigger of AGNs at $z < 4.0$ is mergers of galaxies, although, at higher redshift, disc instabilities become essential for less luminous AGNs. This result is inconsistent with Fanidakis et al. (2012) and Griffin, Lacey, Gonzalez-Perez, del P. Lagos, Baugh & Fanidakis (Griffin et al.), who suggest that disc instabilities and “hot halo mode accretion” are dominant triggering mechanisms of AGNs even at $z < 4.0$. As we described in Sec. 2.1, we employ the smaller $\epsilon_{\text{DL crit}}$ for reproducing the properties of star formation galaxies at $z > 4.0$. Also, we consider the effect of the bulge potential on the stability of galactic discs. With this effect, the number of disc-unstable galaxies becomes 60% smaller at $z \sim 1$ with $\epsilon_{\text{DL crit}} = 0.75$. These are why our model suggests such low efficiency of disc instabilities as a triggering mechanism. The critical point is that the observed number density of AGNs can be sufficiently reproduced at $z < 4$ only by mergers of galaxies, and the importance of disc instabilities and other processes should be investigated in more detail. Our model predicts disc instabilities drive only less than 20% of AGNs at $z \sim 0$. We will come back this topic in Sec. 4.

4 DISCUSSION AND CONCLUSIONS

We have presented the latest results of an updated version of an SA model, $v^3GC$. The most important changes are related to the bulge and SMBH growth model. We assume that the gas accretion onto the SMBH and the bulge growth are triggered by mergers of galaxies and disc instabilities. For bulge and SMBH growths by mergers of galaxies, we em-
Fraction of AGNs and SMBHs in $v^2GC$

**Figure 12.** The Eddington ratio distribution functions at $z \sim 0$ obtained with GalADmodel and Galmodel (black solid and dashed lines, respectively). In both models, AGNs with $M_{BH} > 10^8 M_\odot$ and $L_x > 10^{43}$ erg/s are selected. Also, we compare the results with that obtained by Schulze & Wisotzki (2010) at $z \sim 0.3$ (blue filled circles with error bars).

**Figure 13.** Fraction of the AGN host galaxies whose AGN activity is triggered by mergers of galaxies or disc instabilities. We pick out AGNs (in $v^2GC$-M box) with $log(L_x/\text{ergs}) = [41.5, 42.5]$, $[42.5, 43.5]$, $[43.5, 44.5]$, and $> 44.5$. Mergers are classified according to the mass ratio of merging galaxies: $> 0.70$ (major, blue dash dotted), between $0.4$ and $0.7$ (middle, green dotted) and $< 0.4$ (minor, red solid). We also show the fraction of AGNs triggered only by the disc instability (grey dashed).

We employ a phenomenological model proposed by Hopkins et al. (2009a), whose model is based on results of hydrodynamic simulations. Along with this revision, we have also updated the way of calculating the velocity dispersion and size of bulges when bulges grow via minor mergers. For bulge and SMBH growths by disc instabilities, we employ a classical model originally proposed by Efstathiou et al. (1982). We consider the effect of the bulge potential on the gravitational stability of the disc.

We have investigated the effect of the accretion timescale on statistical properties of AGNs, such as their luminosity functions. We stress that the impact of the accretion timescale especially for low luminosity ($L_x < 10^{44}$ erg/s) AGNs has been almost neglected in previous SA models. When we assume that the accretion timescale is proportional to the dynamical time of the host halo or the host bulge, as in the previous SA models, the number density of the low luminosity AGNs is one order of magnitude smaller than observational estimates. We have found that the number density of such less luminous AGNs becomes consistent with the observational data when we take a phenomenological and physically-motivated model for the timescale of the AGN triggering mechanisms. As another point, some SA models (e.g. Fanidakis et al. 2012; Griffin, Lacey, Gonzalez-Perez, del P. Lagos, Baugh & Fanidakis Griffin et al.) assume that a disc instability destroys a galactic disc entirely and all the gas is exhausted by a starburst forming a spheroidal galaxy just as major mergers. By these two effects (ignoring bulge potential and the complete destruction of a disc), some SA models are likely to overproduce the number density of AGNs induced by disc instabilities. Further updates are necessary, and we leave it for future studies. Menci et al. (2014) suggest that fly-by interactions are important instead of disc instabilities. Although we do not introduce fly-by interactions, the random collision of galaxies may have similar effects. The “hot halo mode” (Fanidakis et al. 2012; Griffin, Lacey, Gonzalez-Perez, del P. Lagos, Baugh & Fanidakis Griffin et al.) is the same as our “radio mode” AGN feedback model, both of which are based on Bower et al. (2006). In our fiducial models, we do not calculate the AGN luminosity with this mode because the bolometric correction and the radiative efficiency are unclear. When we assume the same bolometric correction as that of QSOs, and the radiative efficiency is 0.1, the contribution of the radio mode AGN to the AGN LFs becomes the same order as that of AGNs induced by mergers of galaxies and disc instabilities at $L_x \sim 10^{41}$ erg/s at $z \sim 0$. The contribution becomes smaller at more luminous regime and at higher redshift. Our results based on the timescales show that observed AGN LF can be reproduced without “radio mode” or “hot halo mode” accretions. Even without the “radio mode” AGN feedback, GalADmodel produces a large number of AGNs with low Eddington ratios, which would be AGN jet and outflow sources. Considering the injected energy and momentum from the low Eddington
ratio AGNs, they may have non-negligible impact on the star formation quenching of massive galaxies. We will examine which explanation is more plausible in a future study.

Marulli et al. (2008) suggest the importance of AGN light curve for determining the shape of AGN LFs. They assume three types of the Eddington ratio evolution models based on observations and hydrodynamical simulations. The faint end slope of AGN LFs at $z < 1.0$ are well fitted when they assume the constant Eddington ratio, namely, $= 0.3(1+z)/d^2$ at $z < 3$, and $= 1$ at $z > 3$. By using this Eddington ratio, the accretion timescale should be $\sim 0.17$ Gyr at $z = 0$, which is larger than the dynamical time of bulges (Fig. 6) and is qualitatively consistent with our suggestion. However, the model with this assumption of the constant Eddington ratio understimates the number density of luminous AGNs at $z > 1$. They also introduce introduce AGN light curve with two stages; rapid, Eddington-limited growth phase, and longer quiescent phase with lower Eddington ratios. By using this light curve, the accretion timescale should be longer when the SMBH mass is smaller or the accreted gas mass is larger, which is the opposite to that suggested in the GalADmodel. The resulting faint end slope of AGN LFs at $z < 1$ is shallower than observations. They cannot explain the shape of the AGN LFs by changing just the Eddington ratio distribution. Finally they introduce SMBH mass dependency to the $f_{\text{BH}}$ and successfully reproduce AGN LFs at $z < 5$.

Hydrodynamic simulations (e.g. Sijacki et al. 2015; Khandai et al. 2015; Hirschmann et al. 2014) do explain AGN LFs well, assuming Bondi-Hoyle-Littleton (BHL) accretion for all SMBH growths. Generally, hydrodynamic simulations assume that the "effective" accretion rate onto SMBHs is roughly 200 times larger than the BHL accretion rate, which is too small compared to that of observed AGNs (e.g. Ho 2009). The assumption of the accretion rate with ~ 200 times larger than the BHL accretion, independent of any properties of galaxies and SMBHs, might be a too simplified assumption. Besides, we must care about another uncertainty; different AGN feedback models are employed in different cosmological simulations, which reproduce AGN LFs at the same extent.

As we have shown, there are several prescriptions to explain the faint end slopes of AGN LFs at $z < 1$. For discriminating the models, comparisons of model results with observed properties of AGNs and their host galaxies are necessary. We have shown the relation between $M_{\text{BH}}$ and $L_X$ (Figs. 9 and 10) the Eddington ratio distribution function at $z \sim 0.3$ (Fig. 12), and the fraction of AGNs with different triggering mechanisms (Fig. 13). Since the difference between the Galmodel and GalADmodel is clear for low luminosities AGNs with the smaller SMBH masses, the comparisons with observations are challenging. The other possible way would be comparing the clustering properties with observations. Fanidakis et al. (2013) suggest that thehost halo mass of luminous AGNs like QSOs and low luminosity ones is different. In their model, luminous AGNs are triggered by starbursts induced by mainly disc instabilities (and mergers of galaxies) and their typical host halo mass is $\sim 10^{12} M_{\odot}$. Low luminosity AGNs, on the other hand, are triggered mainly "hot halo mode" and their halo mass is larger than those of luminous AGNs, namely $\sim 10^{13} M_{\odot}$. The "hot halo mode" is efficient for cluster galaxies whose host halo is cooling inefficient. On the other hand, Oogi et al. (2016) suggest that when they assume AGNs are mainly triggered by mergers of galaxies, the host halo mass weakly depends on the AGN luminosities at $1 < z < 4$. The GalADmodel also shows the same trend as Oogi et al. (2016) at $1 < z < 4$. We, thus, can discriminate effects of the accretion timescale and AGN triggering mechanisms by detailed comparisons with observational results.

One might think that the underproduction of less luminous AGNs results from the underestimation of the velocity dispersion of the bulge and/or the underestimation of the cold gas mass in galaxies. As shown in Fig. B5, the velocity dispersion of the bulge tends to be smaller than those obtained from observations, although the bulge size is broadly consistent with the observational data at $z \sim 0$ (Fig. B6). The dynamical time of the bulge evaluated in the fiducial model is statistically longer than the value estimated from the observed velocity dispersion and bulge size. We thus underestimate the gas accretion rate onto SMBHs since the peak accretion rate is proportional to $r^2_{\text{dyn,bulge}}$. In addition, low mass galaxies in the model seem to have smaller gas masses than observed galaxies (Fig. B2) due to the insufficient resolution, which could also cause the underestimation of the gas accretion rate. In Fig. 14, we check these effects and find that both are insufficient to compensate the underproduction of the less luminous AGNs. We compare hard X-ray LFs at $z \sim 0$ obtained by the following three models: (1) the Galmodel with the $\nu^2 GC$-SS simulation (black solid line), (2) the Galmodel with the $\nu^2 GC$-H2 simulation (black dotted line), and (3) the model with $f_{\text{acc}} = 0.2 \times \frac{t_{\text{bulge, dyn,bulge}}}{t_{\text{bulge, dyn,bulge}}}$ (black dashed line). The number density of AGNs obtained by the model (3) becomes smaller than that obtained by the model (1) since $t_{\text{dyn,bulge}}$ is set to be smaller, and the AGN activity shut off sooner. Also, we find no effect of the gas deficiency by comparing (1) and (2), while the number of galaxies with $M_{\text{HII}} < 10^8 M_{\odot}$ increases when we employ the $\nu^2 GC$-H2 simulation. The comparison (1) and (2), therefore, suggests the gas deficiency is not the cause of the underestimation of the abundance of the less luminous AGNs. Even at $z \sim 1$, the model (3) does not solve the inconsistency of the faint-end slope since the shorter accretion timescale causes the shallower slope. We have confirmed that the faint-end slope of the AGN LF at $z \sim 1$ also does not change with model (3). We conclude the underestimation of the gas mass of galaxies is not a primary cause of the underestimation of the number density of faint AGNs.

Another problem of the AGN LFs obtained with the $\nu^2 GC$ is that there are no AGNs with $\log(L_X/\text{erg s}^{-1}) > 45.3$ at $z > 2.6$. Such luminous AGNs do not appear even when we employ $N$-body simulations with larger volumes. The modelling of the radio-mode AGN feedback is likely to be responsible for this, which was originally proposed by Bower et al. (2006) and is similar to other SA models. Host halo masses of AGNs with $\log(L_X/\text{erg s}^{-1}) < 45.0$ at $z \sim 4$ in the fiducial AGN model are $10^{12-13} M_{\odot}$. Such massive haloes could satisfy conditions of Eqs. 23 and 24 and the gas cooling is quenched even at high redshifts. This is shown in Fig. 15, which shows the fraction of galaxies whose gas cooling is quenched by the radio-mode AGN feedback. We find that about half of galaxies are quenched when $M_{\text{halo}} > 10^{12.5} M_{\odot}$ at $z \sim 4$. We will address this problem in future studies.
The fraction of central galaxies whose gas cooling is shut off by radio-mode AGN feedback at \( z \) is given by three models: (1) the \textit{Galmodel} with the \( v^2GC-SS \) simulation (black solid line), (2) the \textit{Galmodel} with the \( v^2GC-H2 \) simulation (black dotted line), and (3) the model in which \( t_{\text{acc}} = 0.2 \times \alpha_{\text{bulge}}/M_{\text{halo}} \) (black dashed line). Observational results are the same as the top left panel of Fig. 5.

Figure 14. AGNLFs at \( z \sim 0 \). To check the effect of the determination of \( f_{\text{dyn,bulge}} \) and the accreted gas mass, we compare three models: (1) the \textit{Galmodel} with the \( v^2GC-SS \) simulation (black solid line), (2) the \textit{Galmodel} with the \( v^2GC-H2 \) simulation (black dotted line), and (3) the model in which \( t_{\text{acc}} = 0.2 \times \alpha_{\text{bulge}}/M_{\text{halo}} \) (black dashed line). Observational results is the same as the top left panel of Fig. 5.

Figure 15. The fraction of central galaxies whose gas cooling is shut off by the radio-mode AGN feedback at \( z \sim 4 \). The x axis is the host halo mass of the galaxies. The number means [Quenchedhalo]/[Totalhalo].

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APPENDIX A: GALAXY MODELLINGS

A1 Gas cooling

Here we describe the calculation of the amount of the cold gas, which is accreted onto a central galaxy. In the model, we define a central galaxy of a new common halo as the central galaxy of the most massive progenitor halo.

The mass fraction of the baryonic matter in a DM halo has been calculated with the following procedures, identical to that of M16. Before reionization of the universe, the mass fraction is given as \( f_b = \Omega_b/\Omega_D \). The mass fraction, however, deviates from \( f_b \) after cosmic reionization because of the photoionization heating due to the UV radiation from galaxies and quasars. Small haloes with shallow gravitational potential wells cannot hold the gas heated by photoionization. We treat this effect following Okamoto et al. (2008) who performed high-resolution cosmological hydrodynamical simulations with a time-dependent UV background radiation field. They proposed the fitting formulae as the mass fraction of the baryonic matter as a function of the halo mass, \( M_h \), and redshift, \( z \), which was originally proposed by Gnedin (2000):

\[
 f_b(M_h, z) = \langle f_b \rangle \times \left[ 1 + (2^{2/3} - 1) \frac{M_h}{M(C)} \right]^{-3/2}, \quad (A1)
\]

where \( \Omega_{UV} = 2 \) controls the rate of decrease of \( f_b \) in low mass haloes. The characteristic mass as a function of \( z \), \( M_c(z) \), is described by using the fitting formula to the simulation results of Okamoto et al. (2008):

\[
 M_c(z) = 6.5 \times 10^9 \exp(-0.604z) \exp(-z/8.37) \Omega_M^{-1.6} \Omega_{UV}^{-1} M_{\odot}. \quad (A2)
\]

We assume reionization occurs at \( z = 9.0 \). See Sec. 2.3 of M16 for a more in-depth description.

All baryonic matter in a halo is diffuse hot gas soon after halo formation. To calculate the cold gas mass, we firstly calculate cooling radius, \( r_{cool}(t) \). We assume the Navarro-Frenk-White (NFW) density profile (Navarro et al. 1997) for DM haloes and the isothermal density profile with a finite core radius, \( r_c \), for hot gas haloes:

\[
 \rho_{NFW}(r) = \frac{\rho_{DM}}{\left( r/r_s \right) \left( 1 + r/r_s \right)^2}, \quad (A3)
\]

\[
 \rho_{hot}(r) = \frac{\rho_{hot,0}}{1 + (r/r_c)^2}, \quad (A4)
\]

where \( r_s \) is the scale radius of the DM halo, which is described by using the concentration parameter, \( c \), and virial radius, \( R_{vir} \), as \( R_{vir}/r_s \equiv c \). We assume \( r_c = 0.22 r_s \) (Makino et al. 1998), and use an analytical formulation of \( c \) obtained from Okamoto et al. (2008).
by fitting to the results of cosmological $N$-body simulations (Prada et al. 2012). After the collapse of a DM halo, the hot gas gradually cools via radiative cooling. The cooling time at a radius, $r$, is defined as

$$t_{\text{cool}}(r) = \frac{3}{2} \frac{t_{\text{cool}}(r)}{\mu m_p k_B n_e N(r)} \frac{k_B T_{\text{vir}}}{n_e^2(r) N(T_{\text{vir}}, \mu_{\text{vir}})}.$$  

(A5)

where $\mu$, $m_p$, $k_B$, and $n_e$ are the mean molecular weight, proton mass, Boltzmann constant, and electron number density, respectively. We employ a cooling function, $\Psi$, provided by Sutherland & Dopita (1993), which is a function of hot gas metallicity, $Z_{\text{hot}}$, and virial temperature, $T_{\text{vir}}$. Virial temperature is calculated from the circular velocity of the host DM halo, $V_{\text{circ}}$, as

$$T_{\text{vir}} = \frac{1}{2} \frac{\mu m_p}{k_B} V_{\text{circ}}^2.$$  

(A6)

The cooling radius, $r_{\text{cool}}(t)$, is defined as the radius at which $t_{\text{cool}}$ (Eq. A5) is equal to the time elapsed since the halo formation epoch. We can calculate the mass which cools in a given time step from Eqs. A4 and A5.

We evaluate the accretion radius, $r_{\text{acc}}(t)$, in which gas can cool and be accreted onto the central galaxy. We set $r_{\text{acc}} = M16(r_{\text{cool}}(t) = t_{\text{cool}}, R_{\text{vir}})$, similar to Lacey et al. (2016). Free-fall time, $t_f$, and free-fall radius, $r_f$, have the following relationship:

$$t_f(r_f) = \sqrt{2GM(r < r_f)}.$$  

(A7)

where $G$ is the gravitational constant and $M(r < r_f)$ is obtained by the volume integration of Eq. A3 from $r = 0$ to $r = r_f$.

We note that we assume the existence of a "cooling hole" in the same way as M16. Since we assume that the radial profile of the remaining hot gas is unchanged until the DM halo mass doubles, there is no hot gas at $r < r_{\text{cool}}$ once the gas cools and is accreted onto the central galaxy.

A2 Star formation

Our model includes star formation in cold gas discs and reheating of the gas by SNe. The implementation is similar to that of M16.

When the diffuse hot gas cools, it forms a cold gas disc and triggers star formation. The SFR, $\Psi$, is given by $\Psi = M_{\text{cold}}/\tau_{\text{star}}$, where $M_{\text{cold}}$ is the cold gas mass in a disc and $\tau_{\text{star}}$ is the star formation timescale. We assume that $\tau_{\text{star}}$ can be described with the dynamical timescale of the disc, $\tau_{d} = r_d/V_d$ (where $r_d$ and $V_d$ are the half-mass radius and the circular velocity of the disc, respectively):

$$\tau_{\text{star}} = \tau_d^{-1} \left[ 1 + \left( \frac{V_d}{V_{\text{star}}} \right)^{-\alpha_{\text{star}}} \right].$$

(A8)

where $\alpha_{\text{star}}, V_{\text{star}}$, and $\tau_{\text{star}}$ are free parameters, whose values are $0.46, 197$ km/s, and $-2.14$, respectively. The cold gas is reheated by SNe explosions at a rate of $M_{\text{cold}}/\tau_{\text{reheat}}$. The timescale for the reheating is given as follows:

$$\tau_{\text{reheat}} = \frac{\tau_d}{\beta(V_d)}. $$

(A9)

6 In M16, $V_{\text{star}}$ is assumed to be identical to $V_{\text{hot}}$, defined in Eq. A10.

and

$$\beta(V_d) = \left( \frac{V_d}{V_{\text{hot}}} \right)^{-\alpha_{\text{hot}}}. $$

(A10)

We calculate the chemical enrichment associated with the star formation and SNe explosions following Maeder (1992). We assume instantaneous recycling for SNe II and neglect any effects by SNe Ia.

The gas reheated by SNe would not be available for gas cooling immediately. We do not severely differentiate the ejected and reheated gas by SNe. In our model, the gas with mass $M_{\text{reheat}}$ cannot cool immediately and is stored in a reservoir due to the reheating and ejection by SNe. A fraction of this gas might return to the hot gas halo and cool with some timescale. Lacey et al. (2016) assume the returned gas mass as $\alpha_{\text{return}} M_d^7$, where $\alpha_{\text{return}}$ is a free parameter.

We, however, simply assume that $\alpha_{\text{return}} = 0$ and that all of the reheated gas falls back to the halo as hot gas when the halo mass doubles without escaping from the halo. If we set $\alpha_{\text{return}} = 1.0$, the cosmic star formation density at $z < 1.0$ becomes only $\sim 1.3$ times larger.

We obtain the time evolution of the masses of stars, hot gas, cold gas, and metals in cold and hot gas for a given SFR, $\Psi(t)$, as follows:

$$M_{\text{star}}(t) = \Phi_{\text{star}}(t).$$

(A11)

$$M_{\text{BH}}(t) = \Phi_{\text{BH}}(t).$$

(A12)

$$M_{\text{reheat}}(t) = \Phi_{\text{reheat}}(t).$$

(A13)

$$M_{\text{cold}}(t) = \Phi_{\text{cold}}(t).$$

(A14)

$$M_{\text{cold},Z}(t) = \Phi_{\text{cold,Z}}(t).$$

(A15)

$$M_{\text{gas}}(t) = \Phi_{\text{gas}}(t).$$

(A16)

where $M_{\text{star}}, M_{\text{BH}},$ and $M_{\text{reheat}}$ are the masses of stars, central BHs, and reheated gas mass by SNe in a galaxy, respectively, and $\Phi_{\text{BH}}$ is a free parameter tuned to match observational estimates of the relation between masses of bulges and SMBHs at $z = 0$. The metallicities of the cold and hot gas are denoted by $Z_{\text{cold}}$ and $Z_{\text{hot}}$, respectively. The value of the locked-up mass fraction, $\alpha$, and chemical yield, $\beta$, depend on the initial mass function (IMF). We adopt the Chabrier IMF (Chabrier 2003) with which the corresponding values are $(\alpha, \beta) = (0.52, 1.68Z_\odot)$. In this paper, we assume $Z_{\odot} = 0.019$. From Eq. A11 to A16, we analytically derive increments/decrements of the mass and metallicity of each component during a time step (see Eq. 15 - 19 of M16).

A3 Size of galaxies

Here we describe how to estimate galaxy size, the circular velocity of galactic discs, and the velocity dispersion of bulges.

A3.1 Disc size and circular velocity

We assume that DM and hot gas haloes have the same specific angular momentum and that the angular momentum is $\sim 3 \times 10^7$ km s$^{-1}$ kpc.

$M_d$ in Lacey et al. (2016) is the same as $M_{\text{reheat}}$ in $\nu^2 G \mathcal{C}$ although both are calculated with the same procedure.

$M_{\text{reheat}}$ is given as $M_{\text{hot}}$ in M16.
conserved during the formation of a cold gas disc. We adopt the log-normal distribution for the dimensionless spin parameter, \( \eta \equiv L/E^{1/2}/GM^{3/2} \), where \( L, E, \) and \( M \) are the angular momentum, binding energy, and DM halo mass, respectively, the same prescription as M16. The mean value of \( \eta \) is 0.042 and the logarithmic variance is 0.26, which are obtained from \( N \)-body simulations of Bert et al. (2007).

The effective radius of a cold gas disc, \( R_d \), is given by the following relation:

\[
R_d = (1.68/\sqrt{2}) \eta R_{\text{init}}.
\]

where the initial radius of the hot gas sphere, \( R_{\text{init}} \), is set to the accretion radius, \( r_{\text{acc}} \), introduced in Sec. A1. Disc rotation velocity, \( V_d \), is given as the circular velocity of its host halo. In the model, \( R_{\text{init}} \) and \( V_d \) are renewed when the disc mass increases from the previous time step and when the new \( R_{\text{init}} \) is larger than the previous time step.

We note that \( R_d \) becomes smaller than that at the previous time step when a merger of galaxies or disc instability occurs, by which the disc mass of the primary galaxy decreases. We then consider the conservation of the angular momentum and set the new effective radius \( R_{\text{d, new}} \), as

\[
R_{\text{d, new}} = (M_{d, i} + M_{b, i}) \times R_d,
\]

where \( M_{d, i} \) and \( M_{b, i} \) are the disc mass (stellar + cold gas) of the primary galaxy after and before the merger or disc instability, respectively.

### A3.2 Bulge size and velocity dispersion

We describe how to estimate bulge size and velocity dispersion when a merger of galaxies or a disc instability occurs. There have been several previous studies (e.g. Hopkins et al. 2009b; Covington et al. 2011; Shankar et al. 2013) which investigate how to calculate the size and velocity dispersion of the bulge from the Virial theorem and energy conservation. They, however, only study the major merger case. Applying their result to galaxies experiencing minor mergers or a disc instability, by which a galactic disc is not completely destroyed, is not straightforward. In this paper, we apply the similar formula to M16 9 to obtain size and velocity dispersion of bulges formed not only by major mergers but also by minor mergers and disc instability.

We first consider merging galaxies. The total energy of each galaxy which contributes to the bulge formation is given by the Virial theorem:

\[
E_i = -\frac{1}{2}(M_{b, i} + M_{BH, i}) V_{b, i}^2 + (M_{d, i} + M_{\text{cold, i}}) V_{d, i}^2.
\]

where \( M_{b, i}, M_{d, i} \), and \( M_{\text{cold, i}} \) are the masses of the bulge stars, disc stars, and cold gas, respectively, and \( V_b \) and \( V_d \) denote the velocity dispersion of the bulge and the rotation velocity of the disc, respectively. The subscripts, \( i = \{0, 1, 2\} \), indicate the merger remnant, the primary progenitor, and the secondary progenitor, respectively.

We consider the effect of the gravitational potential of the DM halo which hosts the primary galaxy on the bulge dynamics. The method is similar but slightly different from Lacey et al. (2016). Assuming that a fraction of the DM halo mass, \( M_{\text{DM, 1}} \), affects the bulge dynamics, we simply replace \( M_{b, 1} \) to \( M_{b, 1} + M_{\text{DM, 1}} \) in Eq. 18. The mass, \( M_{\text{DM, 1}} \), is given by:

\[
M_{\text{DM, 1}} = \frac{\Omega_0}{\Omega_b} \left( \frac{M_h}{M_{\text{vir}}} \right)^{\alpha_1}.
\]

where \( M_h \) and \( \alpha_1 \) are free parameters and the values are determined to reproduce the observed relation between the bulge size and K-band magnitude of galaxies at \( z \sim 0 \). In this paper, the values of \( M_h \) and \( \alpha_1 \) are \( 10^{11} M_\odot \) and \( 1.82 \), respectively. Since we do not utilize sub-halo merger trees, we ignore the effect of the DM potential for the secondary galaxies. We will update the model in the near future by including this effect.

As described in Sec. 2.1.1, a fraction of the disc mass in the primary galaxy, \( \Delta M_{d 1} + \Delta M_{d 2} \), migrates to the bulge. The remaining energy in the disc, \( E_{0,d} \), is then:

\[
E_{0,d} = -\frac{1}{2} (M_{d, 1} + M_{\text{cold, 1}} - (\Delta M_{d 1} + \Delta M_{d 2})) V_{d, 1}^2.
\]

The total energy of the bulge of the merger remnant, \( E_{0,b} \), can be described as follows:

\[
E_{0,b} = E_0 - E_{0,d}.
\]

Considering the energy dissipation, we obtain the energy conservation relation as follows:

\[
f_{\text{dis}} (E_1 + E_2 + E_{\text{orb}}) = E_{0,b}.
\]

where \( f_{\text{dis}} \), is the fraction of energy dissipated from the merging system. We simply parameterize \( f_{\text{dis}} \) by following M16:

\[
f_{\text{dis}} = 1 + k_{\text{dis}}, f_{\text{gas}}.
\]

where

\[
f_{\text{gas}} = \frac{\Delta M_{b 1} + \Delta M_{b 2}}{M_1 + M_2}.
\]

The orbital energy, \( E_{\text{orb}} \), is given as follows:

\[
E_{\text{orb}} = \frac{E_1 E_2}{(M_1/(M_1 + M_{\text{DM, 1}})) E_1 + (M_1 + M_{\text{DM, 1}})/M_2) E_2}.
\]

where \( M_1 \) and \( M_2 \) are the total mass of each galaxy (cold gas + stars + a BH).

We calculate the velocity dispersion and the size of a bulge, \( r_b \), as

\[
v_{b, 0}^2 = -\frac{2E_{0,b}}{M_{\text{lock, 0}}},
\]

\[
r_{b, 0} = \frac{G M_{\text{lock, 0}}}{2v_{b, 0}^2}.
\]

where \( M_{\text{lock, 0}} \) is the total mass of the merger remnant (including \( M_{\text{DM, 1}} \)). To obtain the 1D velocity dispersions, \( \sigma_{1D} \), we assume the bulge structure can be described by an isothermal sphere. The 1D velocity dispersion is simply given by

\[
\sigma_{1D} = v_{b, 0}/\sqrt{3}.
\]

For the disc instability, we employ the same formulae as those for the merger of galaxies while subscripts, \( i = \{1, 2\} \), indicate the bulge and disc, respectively and the orbital energy, \( E_{\text{orb}} \), is set to be 0.

---

9 M16 assume that only major mergers are induced starbursts in bulges and a galactic disc is completely destroyed by a major merger while it does not change by a minor merger.
A3.3 Dynamic response caused by SNe feedback

We consider the change of the size and velocity caused by SN feedback. The SN feedback continuously expels gas from a galaxy. As a result, the gravitational potential well becomes shallower and the gravitationally bound system expands and its rotation speed slows down (Yoshii & Arimoto 1987). We refer to this effect as dynamical response, which is taken into account the same way as M16. This affects the size of galactic discs and bulges, the rotation velocity of galactic discs and their host haloes, and the velocity dispersion of galactic bulges. See Sec. 2.8 of M16 for farther details.

A4 Photometric properties and morphological identification

In order to compare our results with observations, we have to convert the mass of galaxies to observed luminosities. We employ a stellar population synthesis model of Bruzual & Charlot (2003) and obtain the spectral energy distribution (SED) of model galaxies. To estimate the extinction effect for galaxies, we make the same assumptions as M16; first, the dust-to-cold gas mass ratio is proportional to the metallicity of the cold gas; second, the dust optical depth is proportional to the dust column density. The dust optical depth, \( \tau_{\text{dust}} \), is then calculated from the following relation:

\[
\tau_{\text{dust}} = \frac{M_{\text{cold}}}{M_\odot} \left( \frac{Z_{\text{cold}}}{Z_\odot} \right) \left( \frac{R_e}{kpc} \right)^{-2},
\]

where \( R_e \) is the effective radius of the galaxy, and \( \tau_0 \) is a tunable parameter determined to reproduce the local galactic properties, such as LFs. We set \( \tau_0 = 2.5 \times 10^{-3} \) following Nagashima et al. (2005), which is the dust attenuation coefficient in V-band. We calculate the optical depth of the disc and bulge separately. The effective radius, \( R_e \) is \( R_d \) for the disc, and \( R_b = 0.744 R_d \) for the bulge (Nagashima & Yoshii 2003). We employ the Calzetti extinction law (Calzetti et al. 2000), and assume a slab model for the dust distribution in the disc and the bulge.

The morphological types of model galaxies are determined in the same manner as M16; using bulge-to-total (B/T) luminosity ratio in B-band, galaxies with \( 0.6 < B/T < 0.4 \) are classified as elliptical, lenticular, and spiral galaxies, respectively (Simien & de Vaucouleurs 1986).

APPENDIX B: GENERAL RESULTS OF GALAXIES

In this section, we present properties of galaxies obtained from the fiducial model and compare them with those obtained from observations. Firstly, we run the MCMC fitting with the \( \nu\circ\text{cc-SS} \) simulation to tune parameters. For the model calibration, we use observed K- and r-band LFs at \( z = 0 \) obtained from the Galaxy and Mass Assembly (GAMA) survey, \( H_\text{I} \) mass function at \( z = 0 \) extracted from the data of the Arecibo Legacy Fast ALFA (ALFALFA) survey, \( M_{\text{BH}} - M_{\text{MBH}} \) relation at \( z = 0 \) (Eq. 11 Kormendy & Ho 2013), scaling relations of galactic discs and bulges at \( z = 0 \) (Courteau et al. 2007; Forbes et al. 2008, respectively) cosmic SFR density obtained from observations (UV- and IR-bands, and radio 1.4 GHz), K-band LFs at \( z = 1, 2, 3 \) obtained with the UKIDSS Deep Survey (Cirasuolo et al. 2010), and AGN hard X-ray LFs at \( z = 0.4, 1.2 \) (Ueda et al. 2014).

We summarised the fiducial values of our free parameters and related equations in Table B1. We run the calculation with 50000 realisations, excluding the initial 10000 steps of the “burn-in” phase (for more details, see Sec. 3.2 in Makiya et al. 2016). The reduced \( \chi^2 \) decreases at 3.4 % of the initial value after the first 10000 iterations, and at 1.5 % after 20000 iterations. After 20000 iterations, \( \chi^2 \) becomes a little larger (2.2 %/2.3 % of the initial value after 40000/50000 iterations). The dispersion of values of MCMC-fitted parameters after 50000 iterations is 1.69 / 1.29 times larger than that after 20000 / 40000 iterations. The averaged values of parameters, on the other hand, seems to be converged. The change of the averaged values of parameters is 4.7 % from 20000 to 50000 iterations and 1.4 % from 400000 to 50000 iterations. The increase of the iterations would thus cause the increase of the dispersion values.

We have checked the correlations between values of two different parameters by using the Pearson’s r (Table B2). The correlation is weak for most combinations of two parameters although some \( (\alpha_{\text{star}} - \nu_{\text{star}}, \kappa_{\text{gas}} - \epsilon_{\text{MBH}}, M_{\text{BH}} - \alpha_{\text{bulge}}, M_{\text{BH}} - \nu_{\text{gas0}}, \alpha_{\text{bulge}} - \nu_{\text{gas0}}, \text{and } \gamma_{\text{gas}} - \gamma_{\text{BH}}) \) have strong correlations, \( \rho \gtrsim 0.8 \).

The MCMC fitting has two crucial problems. First, since the \( \nu\circ\text{cc-SS} \) simulation has only \( 70^3 h^{-3} \text{Mpc}^3 \), we cannot fit the bright end slope of AGN LFs. The larger box simulations are not realistic considering the computational cost. Second, we have to fit parameter values so that all observational results are equally well reproduced. In other words, we cannot prioritise observational properties to fit. We, therefore, use the \( \nu\circ\text{cc-SS} \) simulation and refit some ill-fitted parameters by hand so that they are in 1σ in the MCMC-fitted values. The parameters which are refitted by hand are shown in Table B1. We cannot determine the values of \( \nu_{\text{mag}}, \nu_{\text{crit}}, \epsilon_{\text{BH}}, \gamma_{\text{gas}}, \) and \( \gamma_{\text{BH}} \) because of the degeneracy and the small box size.

The main results of this paper on the statistical properties of SMBHs and AGNs appear in Sec 3. Additional properties of galaxies such as size/velocity – magnitude relations of galactic discs, stellar mass – SFR relations appear in Appendix. B2.

B1 Properties of galaxies at \( z \sim 0 \)

Fig. B1 shows the K- and r-band LFs at \( z \sim 0 \). The results of the fiducial model with the \( \nu\circ\text{cc-SS} \) and -H2 simulations shown to test the resolution effect. We overplot the results obtained by M16 in grey dash-dotted lines. Red points with errorbars are the observational estimates by the GAMA survey (Driver et al. 2012). Fig. B2 shows the \( H_\text{I} \) mass function (MF) at \( z \sim 0 \). We assume the relation between the cold gas mass and the atomic hydrogen gas mass, \( M_{\text{HI}} \), as \( M_{\text{HI}} = 0.54 M_{\text{cold}} \), which is the same relation used in M16.

The bright-end slopes of the LFs and the massive-end slope of the \( H_\text{I} \) MF is sensitive to the values of \( \alpha_{\text{cool}} \) and \( \epsilon_{\text{MBH}} \) which are both related to the radio-mode AGN feedback. The faint-end slopes are determined by the energy of the SN feedback determined by \( \epsilon_{\text{SN}} \) and \( \nu_{\text{gas}} \). The low mass end slope of the \( H_\text{I} \) MF is also sensitive to the values of \( \alpha_{\text{star}} \).
Galaxies:

| parameter | related equation | value range | MCMC best | MCMC dispersion | adopted value |
|-----------|------------------|-------------|-----------|----------------|--------------|
| \(\sigma_{\text{star}}\) | Eq. A8 | \([-3.0,0.0]\) | -2.14 | 0.10 | -2.14 |
| \(V_{\text{star}}\) | Eq. A8 | \([100.0,400.0]\) | 211.30 | 14.37 | 197.00 |
| \(\epsilon_{\text{star}}\) | Eq. A8 | \([0.05,0.50]\) | 0.48 | 0.02 | 0.46 |
| \(V_{\text{gas}}\) | Eq. A10 | \([50.0,400.0]\) | 121.64 | 2.74 | 121.64 |
| \(\sigma_{\text{gas}}\) | Eq. A10 | \([0.0,4.0]\) | 3.92 | 0.07 | 3.92 |
| \(\alpha_{\text{return}}\) | Sec. A2 | | | | 0.00 |
| \(f_{\text{fe}}\) | Sec. 2.1.1 | \([0.8,1.0]\) | 0.98 | 0.01 | 0.81 |
| \(f_{\text{major}}\) | Sec. 2.1.1 | \([0.3,1.0]\) | 0.89 | 0.08 | 0.89 |
| \(\kappa_{\text{gas}}\) | Eq. A23 | \([1.0,3.0]\) | 2.70 | 0.20 | 2.75 |
| \(M_{\text{BH}}[10^{14}M_{\odot}]\) | Eq. A19 | \([0.1,10.0]\) | 2.10 | 1.43 | 1.00 |
| \(\epsilon_{\text{gas}}\) | Eq. A19 | \([0.5,2.0]\) | 1.82 | 0.13 | 1.82 |
| \(\tau_{\text{cool,star}}\) | Eq. 5 | \([0.7,1.1]\) | 1.05 | 0.01 | 0.75 |
| \(\beta_{\star}\) | Sec. 2.1.2 | \([1e^{-3},1.0]\) | 0.63 | 0.10 | 0.63 |
| \(\tau_{\text{BBH}}\) | Sec. A4 | | 2.5 \times 10^{-9} | | |

SMBHs and AGNs:

| parameter | related equation | value range | MCMC best | MCMC dispersion | adopted value |
|-----------|------------------|-------------|-----------|----------------|--------------|
| \(\alpha_{\text{cool}}\) | Eq.23 | \([0.8,1.2]\) | 1.14 | 0.04 | 1.14 |
| \(\log(M_{\text{MBH}})\) | Eq.24 | \([-3.0,0.0]\) | -2.66 | 0.53 | -2.66 |
| \(\beta_{\text{in}}\) | Eq.12 | \([1e^{-3},3e^{-2}]\) | 0.06 | 0.01 | 0.02 |
| \(M_{\text{BH}}[10^{14}M_{\odot}]\) | Sec. 2.2.1 | | \(10^{3}\) | | |
| \(\tau_{\text{cool}}\) | Eq.15 | \([0.1,1.2]\) | 0.77 | 0.24 | 0.58 |
| \(\tau_{\text{cool,cool}}\) | Eq.16 | \([0.1,5.0]\) | 1.56 | 0.71 | 1.00 |
| \(\gamma_{\text{gas}}\) | Eq.16 | \([-5.0,0.0]\) | -3.28 | 0.41 | -4.0 |
| \(\gamma_{\text{BBH}}\) | Eq.16 | \([0.0,5.0]\) | 4.40 | 0.42 | 3.5 |
| \(h_{\text{init}}\) | Eq.17 | | 10.0 | | |

Table B1. Summary of free parameters in the fiducial model. Almost all parameters are fitted with the MCMC method (iteration = 50000). We show the (1) parameter name, (2) related equation or section, (3-5) parameter range, best fit value, and dispersion (if MCMC fitted parameter), and (6) adopted value.

Table B2. List of the Pearson’s \(r\).
Next, we compare the predicted effective radius and velocity dispersion of elliptical and S0 galaxies at $z \sim 0$ with observations since these values are used for calculating the dynamical time of bulges. Here we also employ the $v^2 G$-SS and $v^2 G$-H2 simulations, although the effect of the mass resolution of the simulations is negligible. We use the data obtained from Forbes et al. (2008) who calculate the half-light radii are from 2MASS $K$-band 20th isophotal by using an empirical relation based on Sérsic light profiles (Forbes et al. 2008). Figs. B5 and B6 are the scaling relations between the bulge velocity dispersion and the $K$-band magnitude (the so-called Faber-Jackson relation; Faber & Jackson 1976) and the effective radius and the $K$-band magnitude, respectively. The data obtained from Forbes et al. (2008) are shown in red points. The results of the fiducial model with $v^2 G$-SS/H2 are described as grey squares/black diamonds with error bars indicating 10th and 90th percentiles. For comparison, we overplot the model results with $M_{DM,1} = 0$ as grey diamonds with error bars. We find that the effective radius of bulges with $M_K - 5 \log h < -23$ becomes smaller and $v^2 G$-H2 simulations, although the effect of the mass resolution of the simulations is negligible. We use the data obtained from Forbes et al. (2008) who calculate the half-light radii are from 2MASS $K$-band 20th isophotal by using an empirical relation based on Sérsic light profiles (Forbes et al. 2008). Figs. B5 and B6 are the scaling relations between the bulge velocity dispersion and the $K$-band magnitude (the so-called Faber-Jackson relation; Faber & Jackson 1976) and the effective radius and the $K$-band magnitude, respectively. The data obtained from Forbes et al. (2008) are shown in red points. The results of the fiducial model with $v^2 G$-SS/H2 are described as grey squares/black diamonds with error bars indicating 10th and 90th percentiles. For comparison, we overplot the model results with $M_{DM,1} = 0$ as grey diamonds with error bars. We find that the effective radius of bulges with $M_K - 5 \log h < -23$ becomes smaller and $v^2 G$-H2 simulations, although the effect of the mass resolution of the simulations is negligible. We use the data obtained from Forbes et al. (2008) who calculate the half-light radii are from 2MASS $K$-band 20th isophotal by using an empirical relation based on Sérsic light profiles (Forbes et al. 2008). Figs. B5 and B6 are the scaling relations between the bulge velocity dispersion and the $K$-band magnitude (the so-called Faber-Jackson relation; Faber & Jackson 1976) and the effective radius and the $K$-band magnitude, respectively. The data obtained from Forbes et al. (2008) are shown in red points. The results of the fiducial model with $v^2 G$-SS/H2 are described as grey squares/black diamonds with error bars indicating 10th and 90th percentiles. For comparison, we overplot the model results with $M_{DM,1} = 0$ as grey diamonds with error bars. We find that the effective radius of bulges with $M_K - 5 \log h < -23$ becomes smaller
when we set \( M_{\text{DM,1}} = 0 \). The results obtained from the fiducial model have some discrepancies with the observational results, especially for the velocity dispersion while the bulge MF at \( z \sim 0 \) is consistent with observed bulge MF obtained from Moffett et al. (2016), and Thanjavur et al. (2016), as shown in Fig. B7.

The velocity dispersion obtained from the fiducial model becomes smaller with massive galaxies than those obtained from observations. There might be two possible reason for the inconsistency. First, due to the underestimate of gas mass especially in the small galaxies. We find that the model overproduces gas-poor galaxies, whose \( r \)-band magnitude are less than \( \sim -18.5 \). The dissipation process plays important roles for calculation of the velocity dispersion (Sec. A3.2). Since the dissipated energy becomes larger with mergers of more gas-rich galaxies, underestimation of the cold gas mass would cause the underestimation of the velocity dispersion. Another possibility to reproduce Faber-Jackson relation might be related with the estimation of the gravitational potential of galactic discs. Galaxies which experience bulge growths should contain a galactic disc. The potential energy of the remained disc is estimated assuming that the rotation velocity of the disc remain unchanged (Eq. A20). When the discs have a shallower potential, the bulge should display a larger velocity dispersion.

To check these two effects, we test arbitrary models with the gas fraction \( f_{\text{gas, test}} \) of the galaxy and that with 0.3 times smaller \( R_{d, \text{disc}} \) value. The new gas fraction, \( f_{\text{gas, test}} \) is described as:

\[
 f_{\text{gas, test}} = f_{\text{gas}} \left( \frac{M_d}{10^{11} M_\odot} \right)^{-0.2}, \tag{B1}
\]

where \( f_{\text{gas}} \) and \( M_d \) are the same definition in Sec. 2.1.1 and 2.3.2. As an example, we consider a galaxy with \( M_K - 5 \log h \sim -20 \). The re-estimated gas fraction, \( f_{\text{gas, test}} \) is \( \sim 1.3 \) times larger than the fiducial value. We use \( f_{\text{gas, test}} \) instead of \( f_{\text{gas}} \) in Eq. A24, and re-estimate velocity dispersion. Fig. B8 shows Faber-Jackson relation obtained from these simple tests. The model result is roughly consistent with observational one. We conclude that the discrepancy of bulge velocity dispersion with observational estimates would become smaller when we can reproduce observed colour-magnitude relation and HI MF of less massive galaxies.
Figure B7. Bulge mass function at $z=0$ obtained with $v^2$GC-SS and -H2 simulations. The black solid line denotes the result obtained from the model. Red filled circles and blue filled triangles present observed MFs obtained from Moffett et al. (2016) and Thanjavur et al. (2016), respectively.

Figure B8. Velocity dispersions of elliptical and S0 galaxies as a function of $K$-band magnitude (Faber-Jackson relation). The black solid, red dashed, and blue dashed lines show the median value obtained by the fiducial model ($v^2$GC-SS), that by the artificially fixed gas fraction (Eq. B1), and that by the artificially fixed energy which remains in the galactic disc, respectively. Grey points show the observational data obtained from Forbes et al. (2008).

Figure B9. Cosmic SFR density as a function of redshift. The black solid line is the model results obtained with the $v^2$GC-SS and $v^2$GC-H2 simulations. Red filled triangles and stars and cyan filled squares are obtained from dust continuum emission (Pascale et al. 2009; Rodighiero et al. 2010; Karim et al. 2011, respectively). Blue filled circles, filled diamonds, and stars are from UV continuum emission (Cucciati et al. 2012; Bouwens et al. 2014; Ouchi et al. 2004, respectively). Black crosses are obtained from Hopkins (2004), which is a compilation of various other observational results.

Next, we present the evolution of $K$- and $B$-band LF of galaxies obtained by the fiducial model with the $v^2$GC-M and -H2 simulations to show the result of bright and rare populations of galaxies. The LFs and MFs presented here are volume-weighted. The details of the calculation of LFs and MFs from the simulation are described in Appendix D.

Fig. B10 shows the model $K$-band LFs (black solid lines) compared with observational results (Bell et al. 2003; Huang et al. 2003; Pozzetti et al. 2003; Drory et al. 2003; Caputi et al. 2006; Saracco et al. 2006; Devereux et al. 2009; Cirasolo et al. 2010; Driver et al. 2012). Model LFs reproduce observational results well for $z<3.5$ including faint-end slopes. The model of M16 also explains observed $K$-band LFs for $z<2.0$ well (Fig. 21 of M16), although it over estimates number density of less luminous galaxies ($M_K>-22$).

Fig. B11 compares the model $B$-band LFs (black lines) with observational results (Norberg et al. 2002; Gabasch et al. 2004; Ilbert et al. 2005; Giallongo et al. 2005; Jones et al. 2006). The dust-attenuated model LFs are shown by the solid lines (for dust correction, see Sec. A4) and LFs without dust attenuation are shown by the dashed lines. We note that the data obtained from Norberg et al. (2002) and Jones et al. (2006) at $z<0.25$ are not dust attenuation-corrected. Therefore, their results allow a fair comparison with the LF of the dust-attenuated model. The dust attenuation-corrected model LFs at $z>0.8$ seem to be inconsistent with observational estimates. The observational data of Giallongo et al. (2005) are dust attenuation-corrected by assuming SMC and Calzetti extinction curves. Considering the correction for the dust attenuation, the model reproduces observed $B$-band LFs at $z<3.5$ reasonably well. The data of Ilbert et al. (2005) and Gabasch et al. (2004) are consistent with observational data.
not dust attenuation corrected. Since the bright-end of LF's of Galliano et al. (2005), Ilbert et al. (2005), and Gabasch et al. (2004) are similar and the dust attenuation in B-band should have less impact than those suggested from the fiducial model, we conclude that some modifications of the dust attenuation are needed, which we leave for future studies.

Fig. B12 shows the stellar MFs from \( z \sim 0 \) to \( z \sim 4.5 \). We adopt Chabrier IMF (Chabrier 2003) as described in Sec. A2. We compare our results (black lines) with observational estimates by Li & White (2009), Baldry et al. (2012), Santini et al. (2012), Muzzin et al. (2013), Moustakas et al. (2013), and Tomczak et al. (2014), who employ either a Chabrier IMF (Chabrier 2003) or Kroupa IMF (Kroupa 2001). While the model can reproduce the massive end of the stellar MFs at \( z < 3.5 \), we find that the model underestimates the number of massive galaxies at \( z > 3.5 \) (bottom right panel). A similar feature is seen in other SA models (e.g. Hirschmann et al. 2012; Lacey et al. 2016). The derivation of stellar masses from observations is commonly performed by the broad-band SED fitting with galaxy templates assuming a single dust attenuation law. Alternatively, Mitchell et al. (2013) suggest that the discrepancy between SA models and observations in the stellar MFs at high redshifts stems from the uncertainties in the dust attenuation curve. For less massive galaxies, we also find that we overproduce their number density at \( 0.4 < z < 2.5 \), which is similar to other well studied SA models (e.g. Weinmann et al. 2012). Some previous studies with SA models investigate this problem. Henriques et al. (2013) show that the ejected gas should be reincorporated into the system on a timescale which depends on the halo mass; the smaller halo should have the larger timescale, and the gas returns to the system more slowly. The importance of the timescale to reproduce SMFs are also proposed by White et al. (2015). They also suggest the mass-loading factor which strongly depends on the redshift also plays a role in reproducing SMFs. White et al. (2015) imply a detailed comparison with observations are required to discriminate these two effects. Hirschmann et al. (2016) consider the decrease of the gas inflow rate by “pre-heating” and find that their model can reproduce not only the low mass end of SMFs but also the metal enrichment of galaxies. We need to consider such effects in the \( \nuGC \), which we leave it for future studies to decrease of the degree of freedom. As White et al. (2015) suggest, the values of parameters which are required for reproducing SMFs strongly depends on the treatment of the reservoir of reheated and/or ejected gas in each SA models. The value of these parameters, therefore, have almost no constraints now.

For checking the mass resolution effect, we overplot the results with the \( \nuGC-H2 \) simulation as grey dashed lines in Figs. B10 to B12, although the \( \nuGC-H2 \) simulation has \( 8^3 \) times smaller box size than the \( \nuGC-M \) simulation. We find the effect of the resolution is negligible.

We also present the relation between total stellar mass and SFR at \( z < 6.0 \) obtained from the fiducial model with the \( \nuGC-M \) simulation and compare it with that obtained from observations (Elbaz et al. 2007; Daddi et al. 2007; Salmon et al. 2015) in Fig. B13. We select all galaxies (central + satellite) without any luminosity or surface density limitations. The result is shown as the orange density map. In addition, blue points with errorbars show the relation for luminous galaxies with \( M_{\nuFUV} < -19.0 \) (where \( M_{\nuFUV} \) is the magnitude of the GALEX FUV band) obtained by the fiducial model, which are consistent with that of Salmon et al. (2015) at \( z > 4.0 \). The galaxies obtained by the fiducial model have larger SFRs than those obtained by observations when we take the selection effect into account at \( z > 4 \). Since the \( M_{*}-\text{SFR} \) relation obtained by Salmon et al. (2015) with \( \log(M_{*} / M_\odot) > 10.3 \) has a large dispersion, the slope of the \( M_{*}-\text{SFR} \) relation would not be strictly constrained. We note that the number of luminous galaxies obtained by the fiducial model with the \( \nuGC-M \) simulation is 135.1, 180.9, and 108.1 times larger than that of Salmon et al. (2015) at \( z \sim 4.5, 6 \), and 2, respectively. The galaxies with \( \log(M_{*} / M_\odot) > 10.5 \) and \( M_{\nuFUV} < -19.0 \) at \( z < 2 \) have smaller SFR than those predicted by the observational fitting. This could be a result of the AGN feedback effect (see also Sec. 4). At \( z < 2 \), gas cooling of most of such massive galaxies are quenched by the AGN feedback. The cold gas mass, thus, becomes smaller, resulting in lower SFRs.

Izumi et al. (2018) compare this relation obtained from the fiducial model employing the \( \nuGC-L \) simulation with the data of four observed AGN host galaxies at \( z \sim 6 \). These four AGNs, which are optically low-luminosity quasars (\( M_{\nuFUV} < -25 \)), are originally detected with Subaru Hyper Sprime Cam (HSC) (Matsuoka et al. 2017) and are observed with Atacama Large Millimeter/Submillimeter Array (ALMA) to investigate their host galaxies’ properties. They find that the sample galaxies are on or below the so-called “main sequence” at \( z \sim 6 \), which are very rare population in the fiducial model of \( \nuGC \). Luminous quasars (\( M_{\nuFUV} < -25 \)) at \( z \sim 6 \), on the other hand, have host galaxies with higher SFR than the “main sequence”. The fiducial model of \( \nuGC \) can reproduce such a bursty population. As shown in Fig. 8 in Izumi et al. (2018) and Fig. B13, the distribution of the SFR seems to have several sub-sequences. These sub-sequences should be artificial which result from time and mass resolution of the simulations and/or the discrete treatment of the time evolution of the hot gas density profiles and cooled gas mass. As we show in Sec. A1, the radial profiles of hot gas haloes remain unchanged until the DM halo mass doubles. It means that no hot gas distributes in \( r < r_{\text{cool}} \) until the DM halo mass doubles. Since the minimum halo mass of \( \nuGC-M \) and -SS simulations is \( 7.89 \times 10^5 M_\odot \), the radial profile of the hot gas halo of galaxies with \( M_\star < 10^9 M_\odot \) is not updated from the formation time. A part of such galaxies, therefore, would contain an unphysically smaller amount of the cold gas.

**APPENDIX C: THE TIMESCALE DEPENDENCY ON BH AND ACCRETED GAS MASS**

We firstly show that the accretion timescale from the accretion disc to the SMBH has a negative (positive) dependency on the mass of the accreted gas (SMBH), following the viscous timescale in the accretion discs. We classify the accretion discs by their accretion rate following Kato et al. (2008). Then, we analytically calculate the radial velocity of the gas.

---

10 Since the stellar mass difference between Chabrier and Kroupa IMF is only ~ 0.04 dex (Muzzin et al. 2013), we assume a negligible difference in our results.
$|v_t|$, and the outer radius of the accretion disc which is
determined as the boundary between self gravitating and
non-self gravitating disc, $r_{sg}$. The details appear in Kawaguchi
et al. (2004). Here we define the Schwarzschild radius, $r_{Sch}$,
as $2GM_{BH}/c^2$, the distance from the BH normalised by $r_{Sch}$.
$f = 1 - \sqrt{3r_{Sch}/r}$. The accretion rate is simply described
as $\Delta M_{acc}/\tau_{vis}$ for this calculation, where $\tau_{vis}$ is the viscous
timescale determined as $\tau_{vis} = r_{sg}/|v_t|$. The accretion rate
normalised by the Eddington mass accretion rate, $\dot{m}$ (the
Eddington mass accretion rate: $L_{Edd}/c^2$), is employed. The disc is classified according to the dominant opacity and pressure
sources as follows.

(i) The outer region in which the main opacity source is
electron scattering and the gas is the dominant pressure
source. Then

$$|v_t| \propto a^{4/5} M_{BH}^{-1/5} \dot{m}^{3/10} r^{-1/4} f^{-7/10}$$
and

$$r_{sg}/r_{Sch} \propto a^{28/45} M_{BH}^{-52/45} \dot{m}^{-22/45}.$$  

We obtain $\tau_{vis} \propto M_{BH}^{15/2} \Delta M_{acc}^{-41/4}$.

(ii) The middle region in which the main opacity source is
electron scattering and the gas is the dominant pressure
source. Then

$$|v_t| \propto a^{4/5} M_{BH}^{-1/5} \dot{m}^{3/5} r^{-2/5} f^{-3/5}$$
and

$$r_{sg}/r_{Sch} \propto a^{14/27} M_{BH}^{-26/27} \dot{m}^{-8/27}.$$  

We obtain $\tau_{vis} \propto M_{BH}^{18/5} \Delta M_{acc}^{-22/5}$.

(iii) The inner region in which the main opacity source is
electron scattering and the radiation is the dominant pressure
source. Then

$$|v_t| \propto a M_{BH}^{0} \dot{m}^{2} r^{-5/2} f$$
and

$$r_{sg}/r_{Sch} \propto a^{2/9} M_{BH}^{-2/9} \dot{m}^{4/9}.$$  

We obtain $\tau_{vis} \propto M_{BH}^{6/5} \Delta M_{acc}^{-6/5}$.

\begin{figure}[h]
\centering
\includegraphics[width=\columnwidth]{figureB10.png}
\caption{K - band LFs of galaxies at $z < 0.13$, $z = 0.2 - 0.8$, $z = 0.75 - 1.3$, and $z = 2.0 - 3.5$. The model LFs (volume-weighted) by the $v^2$GC-M simulation appear as black solid lines. Observational estimates are taken from Bell et al. (2003), Huang et al. (2003), Pozzetti et al. (2003), Drory et al. (2003), Caputi et al. (2006), Saracco et al. (2006), Devereux et al. (2009), Cirasuolo et al. (2010), and Driver et al. (2012).}
\end{figure}
AGNs and SMBHs in $\nu^2$GC

Considering these conditions, we conclude that the viscous timescale has a positive correlation to the BH mass and negative correlation to the accreted gas mass at all radii.

Next, we consider the Circumnuclear disc (CND). We consider the CND model of Kawakatu & Wada (2008), as an example, although the physical mechanisms of how the CND maintains its structure is still under discussion. In Kawakatu & Wada (2008), SNe occurred in the CND induces the tidal torque which enhances the gas accretion rate from the CND to the SMBH. When the CND becomes unstable considering from the Toomre criterion (Toomre 1964), then the star formation occurs and the accretion rate increases. Since the CND becomes stable for the massive SMBH, $\gamma_{\text{BH}}$ should be positive. On the other hand, since the SFR becomes more significant for the more gas-rich galaxies, $\gamma_{\text{gas}}$ should be negative. We cannot obtain constraints on the values of $\gamma_{\text{BH}}$ and $\gamma_{\text{gas}}$ since the model of CND is too complicated to construct a single phenomenological model of the accretion timescale (i.e. the outer radius of the CND depends on the SMBH mass, mass density of CND itself and their host galaxy; see Sec. 2.3 in Kawakatu & Wada 2008). With the simple assumptions (based on Kawakatu & Wada 2008), we estimate $\gamma_{\text{BH}} \sim -0.5$ and $\gamma_{\text{gas}} \sim 1.0$, assuming a constant star formation efficiency, constant surface densities of the host galaxy and CND, the outer radius of the CND which is proportional to $M_{\text{BH}}^{0.5}$.

APPENDIX D: THE CALCULATION OF LUMINOSITY AND MASS FUNCTION

We describe the calculation of the volume-weighted LFs and MFs from the model output. We obtain LFs and MFs from the model at discrete output redshifts. On the other hand, LFs and MFs are estimated from observations in continuous redshift ranges. We thus should estimate model LFs and MFs in the same redshift ranges as observations by averaging model LFs and MFs. We will now describe the derivation of the model LFs. The calculation of MFs is the same as that of LFs, with the magnitude replaced by the logarithmic stellar mass.

The average model LFs have the constant co-moving
volume ($dV$), while the solid angle ($d\Omega$) is constant for observations. The luminosity function, $\phi(z, M)$, in which $z$ and $M$ are the redshift and magnitude, respectively, is described as follows:

$$
\phi(z, M) = \frac{dN(z, M)}{dV},
$$

where $N(z, M)$ is the number density of objects over the whole sky at $z$ with a magnitude, $M$. The differential volume (co-moving), $dV$, is written with the differential solid angle, $d\Omega$, as

$$
dV = \frac{c r^2(z)}{H(z)} dz d\Omega.
$$

We calculate the model LF at a magnitude ($M$) which is averaged in a redshift range ($z_0 < z < z_n$), $\bar{\phi}(M)$, as follows:

$$
\bar{\phi}(M) = \frac{\sum_{i=0}^{n} W_i \phi_i(z_i)}{\sum_{i=0}^{n} W_i},
$$

$$
W_i = \frac{r^2(z_i) dz_i}{H(z_i)},
$$

$$
dz_i = \frac{(z_{i+1} - z_{i-1})}{2},
$$

where $i$ means the corresponding output number, $r(z)$ and $H(z)$ are the line-of-sight distance and Hubble parameter, respectively. At the larger redshift, the weight becomes larger. Then we can obtain averaged LFs/MFs at a constant solid angle.

**APPENDIX E: THE DIFFERENCE OF OBSERVABLE FRACTION WITH HOPKINS ET AL. 2007**

Here we show the difference of observable fractions obtained from Hopkins et al. (2007) and this paper (Eq. 22). Hopkins et al. (2007) derives an observable fraction as follows. They obtain intrinsic bolometric correction which is a similar shape to that of Marconi et al. (2004). By employing the observed hydrogen column density distribution (Ueda et al. 2003), they calculate the photoelectric absorption in X-ray. For optical and mid-IR bands, they adopt a canonical gas-to-dust ratio and SMC-like dust attenuation curve (Pei 1992) to obtain the probability of observing AGNs in optical/mid-IR bands. By the bolometric correction and the correction of the photoelectric absorption and the dust attenuation, they obtain intrinsic bolometric AGN LFs. Using this bolometric
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Figure B13. The relation between total stellar mass and SFR at $z < 6.0$. The model results (obtained with the $v^2GC$-M simulation) including all galaxies and those including only luminous galaxies ($M_{FUV} < -19.0$) are shown by the orange colour map and the blue points with errorbars (10th and 90th percentiles), respectively. For comparison, we overplot the results obtained from observations at $z \sim 0.0$ (Elbaz et al. 2007), $z \sim 1.0$ (Daddi et al. 2007), and $z \sim 2.0, 3.0, 4.0, 5.0, 6.0$ (Salmon et al. 2015).

AGN LF, they estimate the probability of observing AGNs with an intrinsic luminosity of hard-/soft- X-ray and optical $B$-band. They fit the probability as a function of the bolometric luminosity, $L_{bol}$, which is the observable fraction of AGNs:

$$f(L_{bol}) = f_{bol} \left( \frac{L_{bol}}{10^{46} \text{ erg s}^{-1}} \right)^{\beta},$$

where $(f_{bol}, \beta)$ is $(1.243, 0.066)$ in hard X-ray (2-10 keV), $(0.260, 0.082)$ in $B$-band (4400 Å).

The method for the estimation of the observable fraction in this paper is slightly different from that of Hopkins et al. (2007). We convert hard X-ray (2-10 keV) LFs obtained from Aird et al. (2015) to UV (1450 Å) LFs by using a bolometric correction (Marconi et al. 2004) and $M_{UV} = M_B + 0.85$ (Kawaguchi et al. 2001). The LFs obtained from these processes are regarded as the intrinsic UV LFs since hard X-ray (2-10 keV) LFs of Aird et al. (2015) are absorption-corrected. By comparing these intrinsic UV LFs with LFs obtained from observations, we obtain the parameters of observable fractions as $(A_0, A_1, \beta_0, \beta_1) = (0.16, 0.07, -0.05, 0.00)$ (Eq. 22).

We show the differences of observable fractions obtained by Hopkins et al. (2007) and by our new method in Fig. E1. The grey dotted line indicates intrinsic UV LFs and blue dashed and black solid lines show LFs considering observable fraction obtained from Hopkins et al. (2007) and this paper, respectively. We assume that the observable fraction obtained by Hopkins et al. (2007) is the same in $B$ and $UV$ bands. We find that in such a simple assumption, the observable fraction obtained in this paper is roughly consistent with those obtained by Hopkins et al. (2007), although they have a small ($\sim 20 \%$, at most) difference.

We note that UV LFs with observable fractions obtained from both Hopkins et al. (2007) and our calculation are inconsistent with observations at $z > 5.0$ since the fitting function of hard X-ray LFs obtained from Aird et al. (2015) can explain the observational results only at $z < 5.0$. We also note that the scatter of the conversion from the hard X-ray to UV luminosity are not considered for deriving the observable fraction. Akiyama et al. (2018) suggest that this
scatter has significant effect on the shape of the LFs (see Fig. 21 in Akiyama et al. (2018)). We need to consider the effect although we leave it for future studies.

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Figure E1. AGN LFs in UV-band (1450 Å) in $0.0 < z < 6.5$. Grey dashed line is the intrinsic UV LFs. Blue dashed and black solid lines are UV LFs considering observable fractions obtained from Hopkins et al. (2007) and this paper (Sec. 2.2.5), respectively. Observational results are obtained from Croom et al. (2001), Croom et al. (2009), Fan et al. (2001), Richards et al. (2005), Richards et al. (2006), Fontanot et al. (2007), Siana et al. (2008), Glikman et al. (2011), Fiore et al. (2012), Ikeda et al. (2012), Palanque-Delabrouille et al. (2013), Ricci et al. (2017), and Akiyama et al. (2018).