Coordinated control of active front steering and active disturbance rejection sliding mode-based DYC for 4WID-EV

Tao Feng¹, Yunpeng Wang² and Qing Li²

Abstract
The four-wheel independent driven electric vehicle (4WID-EV) can easily realize the four-wheel independent drive, which is convenient for the development and design of the direct yaw moment control system (DYC). Based on the theory of ADRC and the sliding mode control, a new type of DYC controller coordinated with the AFS controller using PID method is designed for the 4WID-EV in this paper. The coordinated control work area is divided according to the tire lateral force linear area. An improved particle swarm optimization algorithm which introduces linear decreasing inertia weight and annealing strategy is adopted to obtain the coordination work weight. The effectiveness of the DYC controller is verified by the Simulink simulation. It also shows that the performance is further improved after the coordinated control.

Keywords
In-wheel motor drive, direct yaw moment control, coordinated control, active disturbance rejection control, sliding mode control

Introduction
Ecological deterioration, energy depletion and other issues are increasingly perplexing the survival of human beings, and the vehicles that people rely on for daily travel mainly rely on burning fossil fuels, which undoubtedly aggravates the ecological and energy problems. In addition to the good environmental protection effect, electric vehicles have the advantages of high-energy conversion rate, low noise, recyclable energy and so on.

The Four-wheel independent driven electric vehicle (4WID-EV) can drive each wheel independently by installing a hub motor on four wheels separately. The electric vehicle with this driving form is more compact and has higher transmission efficiency, which will make it an important direction for the development of new energy vehicles in the future. At present, the 4WID-EV has not been mass produced and only appears in the concept vehicle. As a result, the research on the handling stability and safety performance of the 4WID-EV has become one of the current research.

In the late 1990s, Toyota Motor Company of Japan began to develop 4WID-EV. Research focused on the key technologies of improving the design of traditional automobile to adapt to the installation of hub motor, including the control of anti-lock brake system (ABS), anti-skid drive system (TCS) and electronic stability control system (ESC) by hub motor.²

The laboratory led by Professor Yoichi Hori of Tokyo University transformed the traditional vehicle, and developed 4WID-EV “UOT Electric March I” and “UOT Electric March II” successively. Based on the advantages of accurate measurement and control of wheel hub motor and independent drive of four-wheel hub motor, the direct yaw moment control is studied.³

The PCG group of Tsinghua University led the development of the 4WID-EV Harry, and carried out research on permanent magnet synchronous wheel motor control, electric vehicle braking energy feedback, driving torque distribution, etc.⁴

¹Basic Experimental Center for Natural Science, University of Science and Technology Beijing, Beijing, PR China
²School of Automation and Electrical Engineering, University of Science and Technology Beijing, Beijing, PR China
Corresponding author:
Qing Li, School of Automation and Electrical Engineering, University of Science and Technology Beijing, Beijing, 100083, PR China.
Email: liqing@ies.ustb.edu.cn

Creative Commons CC BY: This article is distributed under the terms of the Creative Commons Attribution 4.0 License (https://creativecommons.org/licenses/by/4.0/) which permits any use, reproduction and distribution of the work without further permission provided the original work is attributed as specified on the SAGE and Open Access pages (https://us.sagepub.com/en-us/nam/open-access-at-sage).
With the popularization of 4WID-EV technology, vehicle stability has been paid more and more attention. During steering, the vehicle is continuously being disturbed by various kinds of interference, which may risk the stability or even lead to traffic accidents. Therefore, it is necessary to control the vehicle and keep the vehicle body running stably. Active front steering (AFS) system can add a front wheel angle to the front wheel of the vehicle by the driver’s turning of the steering wheel, and improves the stability of the vehicle by adjusting the lateral force of the tire. Direct yaw control (DYC) can generate the yaw moment by driving or braking the wheel, so as to produce the desired yaw motion. AFS and DYC are both used for vehicle lateral stability control, and there is likely to be interference and conflict between them, which will weaken the original functions of each system. Therefore the coordinated control of AFS and DYC can give full play to the advantages of both of them.\(^5,6\)

The working area of vehicle driving can be divided into linear area and nonlinear area. The nonlinear area can be divided into transition area and saturation area. The steering wheel input in the linear area has a simple linear relationship with the vehicle’s yaw rate response. Generally, the driver only has the experience of driving in the linear area. Once the vehicle deviates from the linear area, the driver often makes the wrong operation due to lack of experience, which often leads to traffic accidents.\(^7\) The lateral force characteristics of the tire can also be divided into linear area and nonlinear area (transition area, saturation area). The lateral force saturation of the tire is an important factor that causes the vehicle to deviate from the linear area.\(^8\) AFS, which controls the lateral force of tire, is suitable for the linear area of the tire, not for the nonlinear area, while DYC has no such limitation.

Karbalaei et al. used the fuzzy controller to coordinate AFS and DYC. The fuzzy coordinated controller, which adjusts the weight coefficients of AFS and DYC in real time according to different working conditions, only uses AFS in linear area, only uses DYC in saturated area, and uses AFS and DYC coordinated control in nonlinear area.\(^9\) Behrooz Mashadi designed the sliding mode controller of active steering and direct yaw moment. They use the sliding mode controller of the upper layer to get the total additional yaw moment, and then make AFS work in the linear area of the tire. If the maximum wheel angle of AFS’s output still cannot meet the additional yaw moment, DYC comes to work.\(^10\)

In China, Zhao Wei of Chang’an University designed the fuzzy controller of AFS and DYC respectively, and designed the coordinated controller to adjust the weight coefficient between the two fuzzy controllers.\(^11\) Tian Jie et al. Of Jiangsu University designed the threshold value according to the yaw rate deviation when turning. When the threshold value is exceeded, the coordinated controller is adopted. When the threshold value is not exceeded, only the AFS control is used to improve the response characteristics of the vehicle.\(^12\) Du Song of Jilin University uses the phase plane analysis method to divide the independent working area of active steering and active braking and the area where they work together. The particle swarm optimization algorithm is used to solve the corresponding parameters, and the distribution coefficient of active steering and active braking under the joint action is obtained.\(^13\)

According to the characteristics of AFS and DYC, this paper designs the AFS controller based on the PID method and the DYC controller based on the sliding mode active disturbance rejection method, and then divides the linear area and nonlinear area following up the tire lateral force curve so as to develop a coordinated control strategy: the AFS control is only used in linear area, the DYC control is only used in saturation area, and the weight coordinated control is used in transition area. An improved particle swarm optimization algorithm is adopted to obtain the coordination weight. The simulation proves the effectiveness of this control algorithm. The symbols and meanings used in this paper are as Table 1.

**Vehicle Model**

**Vehicle model**

(1) The three degrees of freedom of body movement

Ignore the influence of suspension, so as to ignore the car’s pitch, roll and other movements. The rolling resistance of the wheel and the aligning torque is ignored, too.\(^14\) It is considered that the center of mass is a constant, and the steering angle of left front wheel and right front wheel is the same. Then the dynamic equations considering three directions are obtained.

Vehicle longitudinal (along x-axis) dynamic model:

\[
m\dot{v}_x = mgy_y + \left( F_{yfr} \cos \delta + F_{yfr} \cos \delta - F_{yfr} \sin \delta - F_{yfr} \sin \delta + F_{xrl} + F_{xrr} \right) - F_v
\]

(1)

Vehicle lateral (along Y-axis) dynamic model:

\[
m\dot{v}_y = -mgy_x + \left( F_{yfl} \sin \delta + F_{yfr} \sin \delta + F_{yfr} \cos \delta + F_{yfr} \cos \delta + F_{yrr} + F_{yrr} \right)
\]

(2)

Vehicle yaw (around Z-axis) dynamic model:

\[
J_\gamma = a \left( F_{yfr} \sin \delta + F_{yfr} \sin \delta + F_{yfr} \cos \delta + F_{yfr} \cos \delta \right) - b \left( F_{xrl} + F_{xrr} \right) - \left( F_{yfr} \cos \delta - F_{yfr} \cos \delta - F_{yfr} \sin \delta + F_{yfr} \sin \delta \right) \frac{d}{2} + \left( F_{yrr} - F_{yrl} \right) \frac{d}{2}
\]

(3)
Table 1. Symbols and meanings.

| Symbol | Meaning | Symbol | Meaning |
|--------|---------|--------|---------|
| \(m\) | vehicle mass | \(\alpha_f\) | lateral slip angle |
| \(j\) | moment of inertia around z-axis | \(v_{s\delta}\) | longitudinal velocity at wheel center |
| \(b\) | distance from CoG to rear axle | \(F_{s\delta}\) | vertical force |
| \(a\) | distance from CoG to front axle | \(C_{\delta}\) | gravitational acceleration |
| \(d\) | track | \(C_{\delta f}\) | longitudinal stiffness of each wheel |
| \(v_e\) | longitudinal velocity | \(C_{\delta r}\) | front wheel curving stiffness |
| \(\gamma\) | yaw rate | \(\alpha_r\) | rear wheel curving stiffness |
| \(\beta\) | vehicle side-slip angle | \(\alpha_l\) | moment of inertia around \(z\)-axis |
| \(\delta\) | wheel angle | \(r\) | effective rolling radius |
| \(f_{\delta\ell}\) | the longitudinal tire forces | \(T_{\text{max}}\) | maximum peak torque of motor |
| \(f_{\delta r}\) | the lateral tire forces | \(P_{\text{max}}\) | maximum peak power of motor |
| \(F_w\) | wind resistance | \(n\) | rotation speed |
| \(\alpha_{\delta\ell}\) | front lateral slip angle | \(n_l\) | rated speed |
| \(\alpha_{\delta r}\) | rear lateral slip angle | \(f_l\) | left front wheel |
| \(I\) | equivalent moment of inertia | \(f_r\) | right front wheel |
| \(R\) | effective rolling radius | \(r_l\) | left rear wheel |
| \(\omega_{\delta\ell}\) | the wheel angular acceleration | \(r_r\) | right rear wheel |
| \(T_{\delta}\) | driving torque | \(\omega_{\delta r}\) | lateral slip angles of rear wheel \(\alpha_r\) can be approximated by the following two expressions: |

\[
\alpha_f = \beta + \frac{a\gamma}{V} - \delta \tag{12}
\]

\[
\alpha_r = \beta - \frac{b\gamma}{V} \tag{13}
\]

In this paper, in addition to the vehicle model used in the actual experiment using the (8) to (11) model, the other cases are using the simplified lateral slip angle model.

Auxiliary calculation model

(1) Lateral slip angle

When the wheel is rolling, the actual direction of the wheel speed is not parallel to the wheel plane, but presents a certain angle \(\alpha\) that is the lateral slip angle. During the driving process, the four-wheel lateral slip angles \(\alpha_{ij}\) are as follows:

\[
\alpha_{\delta\ell} = \arctan\left(\frac{v_y + a\gamma}{v_x - 0.5d\gamma}\right) - \delta \tag{8}
\]

\[
\alpha_{\delta r} = \arctan\left(\frac{v_y + a\gamma}{v_x + 0.5d\gamma}\right) - \delta \tag{9}
\]

\[
\alpha_{\delta l} = \arctan\left(\frac{v_y - b\gamma}{v_x - 0.5d\gamma}\right) \tag{10}
\]

\[
\alpha_{\delta r} = \arctan\left(\frac{v_y - b\gamma}{v_x + 0.5d\gamma}\right) \tag{11}
\]

In some cases, the following simplified treatment is often done. It is assumed that the lateral-angles of the two front wheels and the two rear wheels are equal. As a result, the lateral slip angles of front wheel \(\alpha_f\) and the lateral slip angles of rear wheel \(\alpha_r\) can be approximated by the following two expressions:

\[
\alpha_f = \beta + \frac{a\gamma}{V} - \delta \tag{12}
\]

\[
\alpha_r = \beta - \frac{b\gamma}{V} \tag{13}
\]
\[ F_{zrr} = \frac{m}{l} \left( \frac{gh}{2} + \frac{a_i h}{2} + \frac{b_i h}{d} \right) \]  \hspace{1cm} (21)

**Tire model**

The Dugoff model\(^1^7\) has the advantages of small amount of calculation, fast calculation speed, high accuracy and no need of experimental data fitting, which can basically reflect the tire force characteristics. Therefore, this paper selects the Dugoff mode as the tire model of the vehicle.

1. **Longitudinal slip**

   The longitudinal slip of the vehicle can be expressed as the following formula, while in case of driving condition: \( \lambda_{ij} = \frac{R_{aij} - V_{cij}}{\max(R_{aij}, V_{cij})} \) (22)

2. **Lateral slip**

   \[ S_{aij} = \tan(\alpha_{ij}) \]  \hspace{1cm} (23)

3. **Under combined conditions**

   \[ F_{xij} = C_{xij} \cdot \frac{\lambda_{ij}}{1 + \lambda_{ij}} \cdot f(S_{ij}) \]  \hspace{1cm} (24)

   \[ F_{yij} = C_{yij} \cdot \frac{\tan(\alpha_{ij})}{1 + \lambda_{ij}} \cdot f(S_{ij}) \]  \hspace{1cm} (25)

   \( S_{ij} \) can be determined by the following formula:

   \[ S_{ij} = \frac{\mu F_{zij} (1 - \frac{v_x}{\lambda_{ij}} + \tan^2(\alpha_{ij}))}{2 \sqrt{(C_{xij}\lambda_{ij})^2 + (C_{yij}(\tan(\alpha_{ij})))^2}} \cdot (1 + \lambda_{ij}) \]  \hspace{1cm} (26)

   \[ f(S_{ij}) = \begin{cases} 2 - S_{ij}, & \text{if } S_{ij} < 1; \\ 1, & \text{if } S_{ij} \geq 1. \end{cases} \]  \hspace{1cm} (27)

In this paper, we can think that there are \( C_{yff} = C_{yfr}, \quad C_{yrl} = C_{yrr} = C_{ylr}, \quad C_{xfl} = C_{xfr} = C_{xrl} = C_{xrr} = C_{x}, \) and \( \epsilon = 0.015 \).

**Wheel Hub Motor**

According to the research focus of this paper and for the sake of simplicity, this paper does not make a detailed modeling of the inner part of the hub motor and only considers the external characteristics of the motor. The PI controller is used to control the longitudinal speed of the vehicle. The motor demand torque is calculated according to the target speed, and then it is output by the four-wheel hub motors.

![Figure 1. The dynamic model of the linear two degrees of freedom.](image)

The external characteristic curve of the motor reflects the limit of rated speed to motor peak torque, which can be expressed as the following formula:\(^1^6\)

\[ T = \left\{ \begin{array}{ll}
cc T_{max} , & n \leq n_e; \\
\frac{9549 P_{max}}{n} , & n > n_e. \end{array} \right. \]  \hspace{1cm} (28)

According to the method of article\(^1^6-1^8\) the motor parameters are matched.

**Nominal model**

The dynamic model of the linear two degrees of freedom illustrated in Figure 1 is widely used in the vehicle control. It can better describe the driver’s intention in the linear area of the vehicle, and is often used to represent the ideal steering characteristics of the vehicle.\(^1^9,2^0\)

The model is as follows:

\[ m(\dot{v}_y + \gamma v_x) = F_{yf} + F_{yr} \]  \hspace{1cm} (29)

\[ J\dot{\gamma} = aF_{yf} - bF_{yr} \]  \hspace{1cm} (30)

Since additional yaw moment can be applied by the DYC system, an additional item \( M \) is added after formula (30), which becomes:

\[ J\dot{\gamma} = aF_{yf} - bF_{yr} + M \]  \hspace{1cm} (31)

Considering that when the car is in the linear area, the tire is also in the linear area, and the lateral force of the tire is directly proportional to the lateral slip angle of the tire, the simplified formula (12) and (13) of the lateral slip angle are as follows:

\[ F_{yf} = 2 C_{yf} \left( \frac{\beta}{v_x} + \frac{a\gamma}{v_x} - \delta \right) \]  \hspace{1cm} (32)

\[ F_{yr} = 2 C_{yf} \left( \frac{\beta}{v_x} - \frac{b\gamma}{v_x} \right) \]  \hspace{1cm} (33)

Substituting equations (32) and (33) into equations (29) and (31) into the form of state space:

\[ \dot{X} = AX + BM + H\delta \]  \hspace{1cm} (34)
where \( X = \begin{pmatrix} \beta \\ \gamma \end{pmatrix} \), \( A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \), \( B = \begin{pmatrix} b_{12} \\ b_{22} \end{pmatrix} \),
\[
H = \begin{pmatrix} b_{11} \\ b_{21} \end{pmatrix},
\]
\[
a_{11} = \frac{2C_{dr} + 2C_{r}}{m v_x}, \quad a_{12} = -1 + \frac{2C_{dr} - 2C_{r}}{m v_x},
\]
\[
a_{21} = \frac{-2C_{dr} - 2C_{r}}{J_v}, \quad a_{22} = \frac{2d^2 C_{dr} + 2bC_{r}}{J_v}, \quad b_{11} = \frac{-2C_{dr}}{m v_x},
\]
\[
b_{12} = 0, \quad b_{21} = \frac{-2C_{dr}}{m v_x}, \quad b_{22} = \frac{1}{J}.
\]

When the vehicle is under the uniform driving condition, \( \dot{X} = 0 \), which is to say, the vehicle side-slip angle and yaw rate are both fixed values, then nominal yaw rate can be calculated to get from the first term \( \dot{y} = 0 \):
\[
\gamma_{0} = \frac{v_x/l}{1 + \frac{K}{\gamma}} \delta \tag{35}
\]

where \( K \) is the stability factor, \( K = \frac{\gamma}{\gamma_{0}} \left( \frac{v}{v_x} - \frac{v}{v_x} \right) \). The selection of automobile parameters should make \( K \) greater than 0, otherwise, when the longitudinal speed reaches \( \sqrt{-1/K} \), there must be an infinite yaw rate in order to achieve the steady state, which is impossible and leads to instability. The ideal yaw rate is less than \( \gamma_{upper} \): \( \gamma_{upper} = 0.85 \frac{\mu g}{v_x} \tag{36} \)

The nominal yaw rate is taken as:
\[
\gamma_d = \begin{cases} 
\gamma_{0}, & \gamma_{0} \leq \gamma_{upper}; \\
\gamma_{upper} \text{sgn}(\gamma_{0}), & \gamma_{0} > \gamma_{upper}.
\end{cases} \tag{37}
\]

In this paper, the nominal vehicle side-slip angle is set to 0, which is conducive to the track keeping and helps to prevent the tire force saturation.

The controller design

The AFS controller design

The PID control is to generate the control quantity in the form of linear combination after the error between the value of controlled variable and the given value is passed through the proportion element(P), integral element(I) and differentiation element(D). It has the advantages of simple operation and strong applicability.\(^{22}\) The principle is shown in Figure 2.

The value of given signal is: \( y_d(t) \), the controlled variable: \( y(t) \), error is: \( e(t) = y(t) - y_d(t) \). The control quantity:

\[
u(i) = K_p e(t) + K_i \int_0^t e(t) dt + K_d \frac{de(t)}{dt} \tag{38}\]

Where \( K_p, K_i, K_d \) are respectively the proportional coefficient, integral coefficient and differential coefficient.

The PID control is usually based on the linear model of the linear control method, and the AFS system uses the front wheel lateral force, in the linear area of the car and therefore has a good control effect. Therefore, this paper does not develop a new control algorithm for AFS and adopts the simple and stable PID control to ensure a certain control accuracy. Considering that the D unit may amplify the noise of the system and have a serious adverse effect on the control effect, the PID control in this paper is actually a PI control. Because the vehicle side-slip angle is very small in the linear region, the stability of the vehicle is mainly determined by the yaw rate,\(^{23}\) therefore this paper only aims at the AFS system designed to control the yaw rate. The control quantity of the controller is: the error between the yaw rate and its nominal value \( \gamma_d(t) = \gamma(t) - \gamma_d \) is the additional front wheel angle:
\[
\Delta \delta(i) = K_p e(i) + K_i \int_0^t e(i) dt + K_d \frac{de(i)}{dt} \tag{39}\]

Considering the physical limitation, the amplitude of \( \Delta \delta(i) \) in this paper is limited to 1\(^{\circ}\).

The DYC controller design

There is a coupling relationship between the yaw rate and the vehicle side-slip angle. In order to solve the coupling between them, the following strategies are selected: when the vehicle side-slip angle is small, only the yaw rate is controlled; when the vehicle side-slip angle increases to a certain extent, the yaw rate and the vehicle side-slip angle are jointly controlled by the weighting coefficient. When the vehicle side-slip angle is too large, only the vehicle side-slip angle is controlled.\(^{23}\)
\[
M = \begin{cases} 
M_\gamma, & |\beta| < \beta_1; \\
GM_\gamma + (1 - G)Mb_b, & \beta_1 < |\beta| < \beta_2; \\
M_\beta, & |\beta| > \beta_2.
\end{cases} \tag{40}
\]
\[
G = 1 - \frac{|\beta| - \beta_1}{\beta_2 - \beta_1} \tag{41}
\]

**Decision layer of yaw moment.** The typical ADRC is designed as a structure composed of four parts:\(^{24} - 28\) fast tracking differentiator (TD), nonlinear state error feedback (NLSEF) extended state observer (ESO), and disturbance estimation compensation. The structure diagram is as Figure 3.

Based on the arrangement of the transition process, TD overcomes the limitation that the output variable can’t track the jump to quantitative because of the inertia. At the same time, it gives a proper method to extract the differential signal to avoid the noise amplification in the differential process. NLSEF gives the
nonlinear combination control quantity through the nonlinear function, which expands the application scope of the traditional PID proportional integral differential linear combination, and avoids the closed-loop sluggish, oscillation, integral saturation and other problems caused by integral feedback. ESO and disturbance estimation compensation can accurately estimate all kinds of unknown disturbances of the system. It is not necessary to measure the disturbance or know the disturbance model in advance, so it is convenient to realize simple error feedback control of the system.

In traditional ADRC, the sign function is often used in TD, which is not smooth or continuous. It is easy to produce jitter and slow convergence in the control process. In addition, the control function used is complex and requires too many parameters to be adjusted, which is not easy to adjust. In ESO, fal function is often used, which also has the problem of too many parameters to be set, and there is a boundary inflection point, which causes fluctuation near the boundary inflection point and reduces the dynamic performance.

In view of the above shortcomings, TD is designed based on hyperbolic tangent function in article, and ESO is designed by inverse hyperbolic sine function instead of fal function in article. The paper also extends the form of nonlinear combined control strategy by combining sliding mode control. Combined with the above results, this paper designs an active disturbance rejection controller combined with the sliding mode control to design the DYC system, control the yaw rate and the vehicle side-slip angle, and improve the vehicle yaw stability. The formula (34) can be converted into two second-order systems in the following forms, which are respectively used to design the yaw rate controller and the vehicle side-slip angle controller.

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -R_id[A_1 \tanh(B_1 x_1 - r) - R_0d]A_2 \tanh(B_2 x_2 / R_0d)]
\end{align*}
\]

\[A_1, A_2, B_1, B_2, R_{id} \text{ are all positive parameters greater than 0. Set } A_1 = A_2 = A_{id}, B_1 = B_2 = B_{id}, \text{ the setting parameters are reduced to three, making the parameter selection more convenient.}
\]

(2) ESO based on the inverse hyperbolic sine function

\[
\begin{align*}
\dot{y} &= f_y + b_y M_y \\
\gamma &= y \\
\dot{\beta} &= f_\beta + b_\beta M_\beta
\end{align*}
\]
\[
\begin{align*}
e &= (z_1 - x_1) \\
\dot{z}_1 &= z_2 - B_0e \\
\dot{z}_2 &= z_3 - B_0\arcsin(h(e)) + b_0u \\
\dot{z}_3 &= -B_0\arcsin(h(e))
\end{align*}
\]

In the formula, \( u \) is the output control quantity of the controller, \( B_0, B_02 \) and \( B_03 \) are all parameters greater than 0. Selecting appropriate parameters can make: \( z_1 \rightarrow x_1, z_2 \rightarrow x_2, z_3 \rightarrow x_3 \), where \( x_3 = f - b_0u \).

(3) NLSEF combined with the sliding mode control

Take sliding surface as \( S = \lambda e_1 + e_2 \), where \( e_1 = z_1 - v_1, e_2 = z_2 - v_2, v_1, v_2 \) tracking to the derivative of the regular signal \( \dot{r} \). Set \( \dot{s} = 0 \), adopting the law of exponential approach (\( k_1 > 0, k_2 > 0 \)), that

\[
\dot{s} = \lambda \dot{e}_1 + \dot{e}_2 = -k_1\text{sgn}(s) - k_2s
\]

Substitute \( e_1 = z_1 - v_1, e_2 = z_2 - v_2 \), that

\[
\lambda (\dot{z}_1 - \dot{v}_1) + (\dot{z}_2 - \dot{v}_2) = -k_1\text{sgn}(s) - k_2s
\]

Substitute the observer formula (47), that

\[
\lambda (z_2 - B_0e - \dot{v}_1) + z_3 - B_02\arcsinh(e) + b_0u - \dot{v}_2 = -k_1\text{sgn}(s) - k_2s
\]

\[
u = \frac{1}{b_0} [-k_1\text{sgn}(s) - k_2s - \lambda (z_2 - B_0e - \dot{v}_1) - (z_3 - B_02\arcsin(e) + b_0u - \dot{v}_2)]
\]

Lyapunov function \( V = \frac{1}{2}\dot{s}s \), that

\[
\dot{s}s = s(\lambda (\dot{z}_1 - \dot{v}_1) + (\dot{z}_2 - \dot{v}_2))
\]

\[
\quad = s(\lambda (z_2 - B_0e - \dot{v}_1) + (z_3 - B_02\arcsin(e) + b_0u - \dot{v}_2))
\]

Substitute \( u \),

\[
s\dot{s} = s \cdot (-k_1\text{sgn}(s) - k_2s) \leq -k_1|s| - k_2s^2 \leq -k_1|s| \leq 0
\]

So that the system is stable.

(4) Disturbance compensation

Considering that the direct output control quantity of the sliding mode control module is \( u_0 \), the final output control quantity \( u \) of the DYC controller can be obtained by subtracting the disturbance compensation:

\[
u_0 - z_3/b_0 = u
\]

Thus, the direct output control value of the sliding mode control module is obtained as:

\[
u_0 = z_3 - k_1\text{sgn}(s) - k_2s - \lambda (z_2 - B_0e - \dot{v}_1) - (z_3 - B_02\arcsin(e) + b_0u + \dot{v}_2)
\]

In order to avoid high frequency vibration caused by sign function, inverse hyperbolic sine function is used instead. The output control quantity \( u \) from DYC controller of the upper layer gives the desired yaw moment \( M_\beta \), which is then distributed by controller of the lower layer.

In this paper, the DYC controller based on the PID design is chosen as the comparison, which proves the superiority of the DYC controller designed in this paper. Using the strategy of (40)-(41), the yaw rate and the vehicle side-slip angle are controlled respectively, and the control quantity is output according to the weight. When the yaw rate is controlled, the input of the controller is: the yaw rate deviates from its nominal value \( e_\gamma(t) = \gamma(t) - \gamma_0(t) \), and the control quantity is the expected yaw moment of the motor:

\[
M_\gamma = K_p\gamma e_\gamma(t) + K_i\int_0^t e_\gamma(t)dt + K_d\frac{de_\gamma(t)}{dt}
\]

When the vehicle side-slip angle is controlled, the input of the controller is: the vehicle side-slip angle deviates from its nominal value \( e_\beta(t) = \beta(t) - \beta_0(t) \), and the control quantity is the expected yaw moment of the motor:

\[
M_\beta = K_p\beta e_\beta(t) + K_i\int_0^t e_\beta(t)dt + K_d\frac{de_\beta(t)}{dt}
\]

Driving force distribution layer. In order to simplify calculation and ensure real-time performance, the method of axle load ratio distribution\(^{14}\) is adopted:

\[
\begin{align*}
\frac{F_{sfl} + F_{sfr}}{F_{sf}} &= \frac{F_{sfl} + F_{srl}}{F_{sr}} \\
\frac{F_{sfr} - F_{sfl}}{F_{sf}} &= \frac{F_{sfr} - F_{srl}}{F_{sr}}
\end{align*}
\]

From the vertical load formula (18)-(21), the front and rear axle vertical forces are respectively:

\[
F_{sf} = F_{sf} + F_{fr} = \frac{mgh - a_i h}{l}
\]

\[
F_{sr} = F_{sf} + F_{fr} = \frac{mga + a_i h}{l}
\]

According to formula (58) and considering that the angular acceleration of formula (47) is small, it is considered that there is \( T_{ij} = F_{ij}R \) approximately, so the formulation torque of every four driving wheel is as follows:
\[ T_{\beta} = \frac{mgb - a_{\beta}h}{2mg} \left( T_{\beta 0} - \frac{M_{f} \cdot R}{d} \right) \]
\[ T_{\delta r} = \frac{mgb - a_{\delta r}h}{2mg} \left( T_{\delta r 0} + \frac{M_{r} \cdot R}{d} \right) \]
\[ T_{\delta l} = \frac{mg a + a_{l}h}{2mg} \left( T_{\delta l 0} - \frac{M_{l} \cdot R}{d} \right) \]
\[ T_{\delta r} = \frac{mg a + a_{r}h}{2mg} \left( T_{\delta r 0} + \frac{M_{r} \cdot R}{d} \right) \]  

(61)

Where \( T_{\beta 0} \) is the expected total output torque of the hub motor determined by the speed control. Input the above specified torque to the corresponding hub motor to generate the desired yaw moment. Considering the limit of peak torque at a certain wheel speed and the limit of vertical force on longitudinal force, the actual driving torque of each hub motor is

\[ |T_{ij}'| \leq \min\{ T_{\text{max}}, \mu F_{zij}R \} \]  

(62)

**Coordinate strategy**

The paper\(^{35}\) presents a method to divide the working area of AFS and DYC system according to the linear area of the curve of tire lateral force changing with the tire lateral slip angle, which will be used in this paper as well.

According to the vertical force formula (18) to (21), when the vehicle is turning, the vertical force is transferred to the outer side wheel, the vertical force of the four wheels is not equal, and each tire may be in different working areas. As a result, the influence of the vertical load must be considered.

In this paper, the critical values of the tire lateral slip angle in linear-transition area and transition-saturation area are measured on the road with the vehicle speed of 120 km/h, with front wheel cornering stiffness of \(-40,000 \text{n/rad}\), rear wheel cornering stiffness of \(-68,000 \text{n/rad}\), vertical force range of 1000 n to 70,000 n, and the adhesion coefficient of 0.4.

The measurement method is to take the tire lateral slip angle corresponding to the point where the slope of the tire lateral force curve starts to decline with the change of tire lateral slip angle as the critical value of the linear-transition area. According to the reference,\(^{35}\) the tire lateral slip angle corresponds to the point where the slope drops to 30% as the critical value of the transition-saturation area.

Then, the curves of the critical value of the lateral slip angle with the vertical force in the linear-transition area and the transition-saturation area were fitted, respectively. The fitting results are as Figures 4 to 7.

Based on the above results, the critical lateral slip angle (subscript 1 stands for the critical value of linear-transition area and 2 stands for critical value of transition-saturation area) obtained from the fitting curve under the corresponding adhesion and vertical force is compared with the front wheel lateral slip angle and rear wheel lateral slip angle calculated according to formula (12) to (13), if \( \alpha_{f} < \alpha_{f1} \) and \( \alpha_{r} < \alpha_{r1} \), it’s considered that at this time, the four wheels are in the linear area of each tire, only...
the AFS control is used, at this time, AFS output weight \( q \) is 1, DYC output weight \( p \) is 0. If \( \alpha_f > \alpha_{f2} \) or \( \alpha_r > \alpha_{r2} \) or \( \alpha_r > \alpha_{r2} \) or \( \alpha_r > \alpha_{r2} \). That is to say, at this time, the tire is working in the saturated area, at least one of the four wheels is saturated, and only the DYC control is used. At this time, DYC output weight \( q \) is 1, and AFS output weight \( p \) is 0. The control parameters are summarized as Table 2:

| Lateral force area | AFS weight | DYC weight |
|-------------------|------------|------------|
| Linear area       | \( p = 1 \) | \( q = 0 \) |
| Transition area   | \( p = p_t \) | \( q = q_t \) |
| Saturation area   | \( p = 0 \) | \( q = 1 \) |

Figure 6. The critical lateral slip angle of rear wheel in linear-transition area.

Figure 7. The critical lateral slip angle of rear wheel in transition-saturation area.

\[
\omega = \frac{\omega_{\text{start}} - \omega_{\text{end}}}{\text{iter}_{\text{max}}} \times \text{iter}
\]

where \( \omega \) is the maximum number of iterations, \( \omega_{\text{start}} \) is the initial inertia weight, and \( \omega_{\text{end}} \) is the final inertia weight. In addition, a heated metropolis annealing criterion is proposed to decide whether to accept the new solution or not, so that each particle will add a judgment process after each movement and the particle population can maintain a certain diversity, enhance the ability to jump out of the local optimal solution, and improve the global search ability of the algorithm. The formula of Metropolis criterion is as follows:

\[
P_T = \exp \left\{ \frac{f(x') - f(x)}{T} \right\}
\]

Among them, \( f(x) \) and \( f(x') \) are both the non-negative objective functions, \( T \) representing the annealing temperature, \( x \) and \( x' \) respectively representing the old solution and new solution. Each iteration will accept the new solution with probability. The initial value of annealing temperature \( T \) is very small in the early stage of iteration. If \( f(x') > f(x) \), then \( P_T \) is close to 0, and the deteriorating solution is not accepted. In the middle and later period, as the temperature \( T \) rises slowly, the deteriorating solution will be accepted with a higher probability. In the initial stage, each particle in the search space only accepts the optimal solution, instead of blindly converging near the global optimal solution at that time. It slowly searches the area near
each particle to find the nearby peak, so as to avoid “premature” of the algorithm and falling into the local optimal. In the middle and later period, as the particles are more likely to accept the deterioration solution, the particles with low fitness value will quickly approach the particles at the current peak, so as to increase the convergence speed of the particles. The particles with high fitness value will continue to search slowly near their own peaks according to the particle position direction of the current peak, so as to generate a kind of stage that particles search at the same time for multiple peaks on the search surface. These can enhance the ability of the algorithm to jump out of the local optimal solution and improve the convergence accuracy. The temperature update formula for the minimum value of the objective function is as follows:

$$T = -[f(g_{best})] \times (c_T)^{iter}$$

where $c_T$ is the temperature rise control parameter, which determines the speed of temperature rise, and is selected according to specific problems. In this paper, the objective function is selected as:

$$f = p_f \int_0^t |\gamma - \gamma_d| dt + q_f \int_0^t |\beta - \beta_d| dt$$

where $p_f$ and $q_f$ are weight coefficients respectively. The tracking errors of the yaw rate and vehicle side-slip angle are optimized.

The control flow of the improved particle swarm optimization algorithm is illustrated in Figure 8.

### Simulation

In this chapter, the low adhesion road with a coefficient of adhesion of 0.4 is selected to simulate under the front wheel angle with an amplitude of 3°. The DYC control based on PID, as a contrast test with the sliding mode ADRC controller designed in this paper, does not participate in the coordinated control experiment. In order to simulate the measurement noise in real environment and verify that the designed controller is not sensitive to noise interference, the input variables (including the yaw rate, the vehicle side-slip angle, the front wheel angle) of the controller designed in this paper are all added with white noise with an angle amplitude of about 0.05. Then DLC experimental conditions will be used for this test.

The simulation parameters are as Table 3.

The simulation input curve is as Figure 9 and the simulation results are as Figures 10 and 11.

### Table 3. Main body simulation parameters.

| Variable | Describe | Value (unit) |
|----------|----------|--------------|
| m | vehicle mass | 1760 kg |
| a | distance from CoG to front axle | 1.18 m |
| b | distance from CoG to real axle | 1.77 m |
| l | wheel base | 2.95 m |
| d | track | 1.575 m |
| J | moment of Inertia around z-axis | 2488 kgm² |
| $C_D$ | aerodynamic resistance coefficient | 0.398 |
| A | windward area of vehicle | 2.4 m² |
| R | effective rolling radius | 0.38 m |
| l | wheel’s equivalent moment of inertia | 2.1 kgm² |
| h | height of vehicle center of mass | 0.719 m |
| $C_x$ | longitudinal stiffness | 50000 N/rad |
| $C_{yf}$ | front wheel cornering stiffness | -40000 N/rad |
| $C_{yr}$ | rear wheel cornering stiffness | -68000 N/rad |
| $P_{max}$ | maximum peak power of motor | 157451 kw |
| $T_{max}$ | maximum peak torque of motor | 310 Nm |
| $n_e$ | rated speed | 485 rpm |
angle error are both very large and with poor stability. After control, the error of the yaw rate and the vehicle side-slip angle are obviously reduced. The control effect of the yaw rate of the DYC controller based on PID is obviously worse than that of the DYC controller proposed in this paper, and it is worse than that of the AFS controller in some time periods. The effect of the vehicle side-slip angle is only slightly better than that of the AFS controller, which is less than that of the DYC controller designed in this paper. The control effect of

Figure 9. Input angle of front wheel in DLC experiment.

Figure 10. Yaw rate in DLC experiment.

Figure 11. Vehicle side-slip angle in DLC experiment.
the yaw rate and vehicle side-slip angle of the DYC controller designed in this paper is better than that of AFS. After the coordinated control, the error of the vehicle side-slip angle is not significantly reduced, but the error of yaw rate is further improved.

Conclusion

The AFS controller based on PID and the DYC controller based on sliding mode active disturbance rejection designed in this paper can make the vehicle yaw rate and the vehicle side-slip angle better track the nominal value, which effectively improve the vehicle yaw stability. Meanwhile, the DYC controller designed in this paper is better than the DYC controller based on PID. After the coordinated control, both the tracking effect and stability are further improved. As the sensors to measure the lateral slip angle are very expensive, the algorithm to estimate the lateral slip angle rapidly and precisely will be considered in the future work.

Declarations of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) received no financial support for the research, authorship, and/or publication of this article.

ORCID ID

Feng Tao (https://orcid.org/0000-0002-5121-1989)

References

1. Guo L and Jianmin Y. Control of electric vehicle body stability system based on l2 disturbance attenuation theory. J Beijing Inform Sci Technol Univ 2019; 34(1): 4–9.
2. Satoshi M. Innovation by in-wheel-motor drive unit. Vehicle Syst Dyn 2012; 50(6): 807–830.
3. Geng C, Mostefai L, Denai M, et al. Direct yaw-moment control of an in-wheel-motorized electric vehicle based on body slip angle fuzzy observer. IEEE Trans Ind Elect 2009; 56(5): 1411–1419.
4. Liangfeng M. Research on the Torque Control for Distributed Drive Electric Vehicle. Master’s Thesis, Tsinghua University, 2016.
5. Chen T. Research on Integrated Control Strategy of Active Front Steering and Direct Yaw Control Based on the Kalman Filer. Master’s Thesis, Hunan University, 2017.
6. Song C, Changgao X, Xu S, et al. Simulation of automobile stability control based on sliding mode control. J Chongqing Jiaotong Univ 2013; (4): 701–704.
7. Xingxing W. Research on Linear Range Characteristics of Vehicle Handling Performance. Master’s Thesis, Jilin University, 2013.
8. Harty, D., “Brand-by-Wire - A Possibility?,” SAE Technical Paper 2003-01-0097, 2003, https://doi.org/10.4271/2003-01-0097.
9. Karbalaei R, Ghaffari A and Kazemi R. A new intelligent strategy to integrated control of afs/dyc based on fuzzy logic. Int J Math Phys Eng Sci 2007; 1(1): 47–52.
10. Mashadi B and Majidi M. Integrated AFS and DYC sliding mode controller design for hybrid electric vehicle. In: ASME 2010 10th Biennial Conference on Engineering Systems Design and Analysis.
11. Wei Z. Simulation Research on Combined Control Strategy of Yaw Moment and Active Steering for Vehicle Dynamic Stability. PhD Thesis, Chang’an University, 2008.
12. Jie T, Xiang G, Ning C. Study of vehicle stability based on coordinated control of AFS and DYC[J]. J Mach Design, 2010; 27(10): 18–21.
13. Song D. The Research on Phase Plane Vehicle Coordination Control of Active Front Steering and Active Braking. Master’s Thesis, Jilin University, 2015.
14. Li G, Feng Z and Wang X. The fuzzy robust cooperative control of the vehicle steering/anti-lock brake system. In: 2011 International Conference on Mechatronic Science, Electric Engineering and Computer (MEC), pp. 2054–2057.
15. Li G and Wang H. The cooperative control of the vehicle steering/anti-lock braking system using the coordination variables. In: Jin D and Lin S (eds) Advances in Mechanical and Electronic Engineering, Berlin, Heidelberg: Springer Berlin Heidelberg. pp. 467–472.
16. Shen Y. Research on Yaw Stability Integrated Control Method of In-wheel Motor Drive Electric Vehicles. Master’s Thesis, Chongqing University of Technology, 2017.
17. Lishuang Z and Fei L. A control method study for vehicle direct yaw-moment. Mach Design Manuf 2010; (2): 126–128.
18. Zhiwei L. Design and Research on Four-side-wheel-Motor-Driving System of Electric Vehicle. Master’s Thesis, Zhejiang University, 2018.
19. Wei L. Research on Algorithm of Vehicle Stability Control System under Accelerating Conditions. Master’s Thesis, Jilin University, 2004.
20. Tseng HE, Ashrafi B, Madau D, et al. The development of vehicle stability control at ford. IEEE/ASME Trans Mechatron 1999; 4(3): 223–234.
21. Chao L. Research on Coupling Dynamics of in-wheel Motor Driving Electric Vehicles. Master’s Thesis, Shan- dong University of Technology, 2016.
22. Yang J. Research on Active Safety Control Strategy of Electric Vehicle Based on Active Front Steering (AFS) and Direct Yaw Moment Control (DYC). Master’s Thesis, Jiang Su University, 2018.
23. Wang Yue LC and Lei X. Research on yaw moment of distributed electric vehicle based on optimal allocation. Vehicle Power Technol 2019; (1): 16–22.
24. Jingqing H. From PID technique to active disturbances rejection control technique. Basic Autom 2002; (3): 13–18.
25. Mao Haijie L and Xiaolin F. Design of nonlinear tracking differentiator based on hyperbolic tangent function. J Comp Appl 2016; 36(S1): 305–309.
26. Wenchao X and Yi H. Performance analysis of 2-DOF tracking control for a class of nonlinear uncertain systems.
with discontinuous disturbances. *Int J Robust Nonlinear Cont* 2018; 28: 1456–1473.

27. Xue W, Madonski R, Lakomy K, et al. Add-on module of active disturbance rejection for set-point tracking of motion control systems. *IEEE Trans Ind Appl* 2017; 53(4): 4028–4040.

28. Yang F, Tan S, Xue W, et al. Extended state filtering with saturation-constrained observations and active disturbance rejection control of position and attitude for drag-free satellites. *Acta Automatica Sinica* 2020; 45: 1–14.

29. Wei Z. Design of improved ADRC for drum water level regulation of ship boiler. *Navig China* 2018; 41(4): 35–37.

30. Wand S, Sun G and Liu S. Active disturbance rejection sliding mode control for time-delay systems. *J Syst Simul* 2019; 31(1): 102–109.

31. Chen R, Song X, Sun H, et al. Active disturbance rejection control for yaw rate of four in-wheel driven electric vehicle. *Comput Meas Control* 2016; 24(9): 95–98.

32. Chen R, Sun H, Song X, et al. Active disturbance rejection control for side-slip angle of four in-wheel driven electric vehicle. *Appl Elect Tech* 2016; 42(10): 92–95.

33. Wang QD, Huang H and Chen WW. A new DYC control strategy based on feed forward control in the linear region. *Adv Mater Res* 2012; 442: 482–487.

34. Zhihua H. *Study on Integrated Control of AFS and DYC for In-Wheel-Motor Electric Vehicle*. Master’s Thesis, Chongqing University, 2018.

35. Farazandeh A, Ahmed A and Rakheja S. An independently controllable active steering system for maximizing the handling performance limits of road vehicles. *Proc Inst Mech Eng Part D J Autom Eng* 2014; 229(10): 1291–1309.

36. Qun L. *Design and Application of Fuzzy Controller Based on Particle Swarm Optimization*. Master’s Thesis, Shenyang University of Technology, 2017.