Nonminimal Lorentz-Violating Extensions of Gauge Field Theories

Zonghao Li
Physics Department, Indiana University, Bloomington, IN 47405, USA

A general method is presented to build all gauge-invariant terms in gauge field theories, including quantum electrodynamics and quantum chromodynamics. It is applied to two experiments, light-by-light scattering and deep inelastic scattering, to extract first bounds on certain nonminimal coefficients for Lorentz violation.

1. Introduction

Lorentz violation has been a popular topic in recent years in the search for new physics beyond the Standard Model (SM). The Standard-Model Extension (SME) developed by D. Colladay and V.A. Kostelecký studies Lorentz violation in the context of effective field theory.\(^1\) It includes all possible Lorentz-violating modifications to the SM coupled to General Relativity to describe all possible Lorentz-violating experimental signals. All minimal terms (mass dimensions \(d \leq 4\)) have been established;\(^1,2\) most nonminimal free-propagation terms have been established;\(^3\) and some low-dimension (\(d \leq 6\)) interaction terms in quantum electrodynamics (QED) have been established.\(^4\) However, general Lorentz-violating terms in gauge field theories are still unknown. Here, we present a general method to build all Lorentz-violating terms in gauge field theories and apply these to two experiments, light-by-light scattering and deep inelastic scattering (DIS), to get first bounds on certain SME coefficients. Related techniques can be applied in the gravity context.\(^5\) The present contribution to the CPT’19 proceedings is based on results in Ref. 6.

2. Theory

The SME preserves gauge invariance, so we need to find all gauge-invariant terms to build general Lorentz-violating extensions of gauge field theories.
The gauge-covariant operator is a powerful tool in building gauge-invariant terms. An operator $O$ is called gauge covariant if it transforms to $UO\mathbf{U}^\dagger$ under the gauge transformation, where $U$ is a unitary representation of the gauge group $G$, and the fermion field $\psi$ transforms to $U\psi$ under the gauge transformation. If $O_1, O_2$ are gauge-covariant operators, we can build gauge-invariant operators by taking traces of them, $\text{Tr}(O)$, or combining them with Dirac bispinors, $(O_1\psi)(O_2\psi)$. Therefore, we can first build gauge-covariant operators and then get gauge-invariant operators.

A direct calculation shows that the gauge-covariant derivative $D_\mu$ and the gauge field strength tensor $F_{\alpha\beta}$ are gauge-covariant operators. Moreover, we find that any operator formed as a mixture of $D$ and $F$ is gauge covariant. In principle, we can construct gauge-invariant operators from all those operators. However, this would introduce a lot of redundancies because $F$ is related to the commutator of $D$ with itself. Therefore, we need to characterize those gauge-covariant operators in terms of a set of standard bases with controlled or no redundancy. The key result is that any operator formed as a mixture of $D$ and $F$ can be expressed as a linear combination of operators of the form

$$D_{(n_1)} F_{\beta_1 \gamma_1} (D_{(n_2)} F_{\beta_2 \gamma_2}) \cdots (D_{(n_m)} F_{\beta_m \gamma_m}) D_{(n_{m+1})},$$

(1)

where $D_{(n)} = (1/n!) \sum_{\alpha_1, \alpha_2, \ldots, \alpha_n} D_{\alpha_1} D_{\alpha_2} \cdots D_{\alpha_n}$ is totally symmetrized with the summation performed over all permutations of $\alpha_1, \alpha_2, \ldots, \alpha_n$. The basic idea behind Eq. (1) is absorbing the symmetric parts in the totally symmetrized $D_{(n)}$ and the antisymmetric parts in $F$. The detailed proof uses Young tableaux and can be found in Ref. 6.

We proceed to build general gauge-invariant terms from Eq. (1). Both QED and quantum chromodynamics (QCD) are based on gauge field theories, so the general Lorentz-violating extensions of QED and QCD can be constructed. We remark in passing that the extensions include both Lorentz-invariant and Lorentz-violating terms, so we are actually building general gauge-invariant extensions of QED and QCD. The reader is referred to Ref. 6 for the details of the Lagrange densities. In the next two sections, we look at two experimental applications.

3. Light-by-light scattering

Light-by-light scattering is a nonlinear effect of the electromagnetic field, which is hidden in the classical linear Maxwell equations but can arise in QED via radiative loop corrections. Experimental measurements of light-by-light scattering can provide important tests of QED. Since the cross
section is tiny, light-by-light scattering was directly measured only recently at the LHC by the ATLAS collaboration. They measured ultraperipheral Pb+Pb collisions at \( \sqrt{s_{NN}} = 5.02 \text{ TeV} \). By the equivalent-photon approximation, the collision of high-energy ultraperipheral heavy ions can be treated as collisions of photons from the heavy ions.

The QED extension built in the last section can describe all possible deviations from the SM prediction in light-by-light scattering experiments. The dominant contribution comes from a \( d = 8 \) term:

\[
\mathcal{L}^{(8)}_g \supset -\frac{1}{38} k^{(8)}_{F} \kappa^{\lambda\mu\rho\sigma\tau\upsilon} F_{\alpha\lambda} F_{\mu\nu} F_{\rho\sigma} F_{\tau\upsilon},
\]

where \( k^{(8)}_{F} \kappa^{\lambda\mu\rho\sigma\tau\upsilon} \) are coefficients for Lorentz violation. This term creates a new interaction vertex with four photon lines and contributes to the light-by-light scattering at tree level. Many possible Lorentz-violating signals can arise from this. It produces new contributions to the total cross section of light-by-light scattering in addition to the SM ones. The SM coefficients are assumed to be approximately constant in the Sun-centered frame, so the experimental cross section can depend on the sidereal time with the Earth rotating about its axis and revolving around the Sun. The experimental results can also depend on the location and orientation of the laboratory. The Lorentz-violating term can produce a new energy dependence for the differential cross section as well.

Due to statistical limitations of the data, we compare here only the total cross sections to get bounds on the SME coefficients. Future improvements in the experiment can lead to more detailed investigations of possible Lorentz-violating signals. The LHC experiment measured the total cross section as \( 70 \pm 24 \text{ (stat.)} \pm 17 \text{ (syst.)} \text{ nb} \). The theoretical SM prediction is \( 49 \pm 10 \text{ nb} \). Comparing these two results gives bounds on 126 components of the coefficients \( k^{(8)}_{F} \). The bounds on the Lorentz-invariant and isotropic components of the coefficients \( k^{(8)}_{F} \) are also extracted. All these components are constrained to approximately \( 10^{-7} \text{ GeV}^{-4} \).

4. Deep inelastic scattering

DIS provides key experimental support for the existence of quarks and the predictions of QCD. It is also an essential tool in the search for new physics beyond the SM and can be used to test Lorentz symmetry. The QCD+QED extension built via the method presented in Sec. 2 can describe all Lorentz-violating signals in DIS experiments.

The contributions from the minimal SME to DIS have been considered before. Here, we focus on contributions from nonminimal terms.
Since most DIS experiments use unpolarized beams, we consider spin-independent terms. The leading-order spin-independent contribution from the nonminimal SME is

\[ L^{(5)} = -\frac{1}{2} \sum_{f} a_{f}^{(5)\mu\alpha\beta} \bar{\psi}_f \gamma_{\mu} iD_{(\alpha} D_{\beta)} \psi_f + \text{h.c.}, \]

where the parentheses around the lower indices mean symmetrization on \( \alpha \) and \( \beta \) with a factor of \( 1/2 \), \( f = u, d \) includes the dominant quark flavors, and \( a_{f}^{(5)\mu\alpha\beta} \) are \( d = 5 \) coefficients for Lorentz violation. This term modifies both the free propagation of fermions and interactions among photons and fermions. The cross section with the corrections can be found in Ref. 6.

Based on the simulations in Ref. 11 for \( c^{\mu\nu} \) coefficients, we can estimate the bounds on \( a_{f}^{(5)\mu\alpha\beta} \) to be around \( 10^{-7} - 10^{-4} \) GeV\(^{-1} \). We can also expect that the corrected DIS cross section depends on up to the third-order harmonics of the sidereal-time variables because the coefficients \( a_{f}^{(5)\mu\alpha\beta} \) contain three indices. The \( a_{f}^{(5)\mu\alpha\beta} \) coefficients also provide CPT-odd contributions to DIS for protons and antiprotons. Experimental measurements of those Lorentz-violating signals can provide fruitful insight into new physics beyond the SM.

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