Interference Alignment for MIMO Downlink Heterogeneous Networks

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ABSTRACT In this paper, we investigate the interference alignment schemes for the downlink of a heterogeneous network (HetNet), where the macro base station (BS) can serve arbitrary number of macro users. Furthermore, the number of data streams for macro users and pico users can be different. Firstly, we propose a basic grouping method aided interference alignment (IA) scheme and derive the transmit and receiver beamforming matrices in a closed-form expression. Furthermore, for the case where the number of antennas at pico BS is equal to the number of antennas at each user, we propose an advanced generalized eigenvalue decomposition (GEVD) based grouping method aided IA scheme which can reduce the number of transmit antennas required at the pico BS compared to the basic grouping method aided IA scheme. Moreover, we also derive the maximum total achievable degrees of freedom (DoF) for each scheme. In addition, it can be proved that the maximum total achievable DoF of the GEVD based grouping method aided IA scheme is higher than or equal to that of the basic grouping method aided IA scheme for the same configuration.

INDEX TERMS Interference alignment, heterogeneous network, basic grouping method, GEVD.

I. INTRODUCTION

Recent years, wireless networks have experienced exponential growing and the demand such as high data-rate transmission, high power/spectral efficiency and high quality of service (QoS) are considered in wireless communications. At same time, the heterogeneous network (HetNet) [1], where small cells, for example picocell and femtocell, are deployed within a macrocell, is an important technique to powerfully improve the total spectral efficiency of mobile cellular networks.

However, the application of HetNet will inevitably generate extra interference. Especially, users at the cell-edge of both the macrocell and small cells are under the influence of co-channel interference [2], [3]. Addressing the interference between a macrocell and small cells is critical to improve the performance of HetNet. Therefore, effective interference management is necessary to mitigate the interference [2].

As an advanced interference management technique, interference alignment (IA) was proposed to obtain maximal degrees of freedom (DoF), and the key idea is to maximize the overlapping among the subspaces of all interference signals at each receiver so that the interference-free dimensions can be maximized [4]. A specific linear IA scheme was presented for the MIMO X channel [5]. One IA scheme was proposed for cellular downlink networks, which aligned interferences into a multi-dimensional subspace [6]. Iterative algorithms were presented to jointly design receiver and transmit beamforming matrices of IA schemes of interference channels [7]. For a two-cell MIMO network, one grouping method aided IA scheme was proposed, where the inter-cell interference is aligned into the same subspace [8]. Then, it was extended to an uplink multi-cell scenario [9]. For a two-cell uplink MIMO network, one IA scheme was proposed where the interference signals from users on the other cell are aligned into a low dimensional interference subspace [10]. Furthermore, for the multiple cell uplink MIMO network, one IA scheme was proposed [11], where for each cell, the interference signals imposed by the users in the same cell are aligned into a low dimensional interference subspace [11]. Moreover, the generalized eigenvalue decomposition (GEVD) based grouping method aided IA scheme...
was proposed for downlink multi-cell networks [12]. Furthermore, for MIMO interfering broadcast channels (IBC), one IA scheme was investigated [13]. For MIMO IBC networks with common messages, the feasibility of IA schemes with finite channel uses was investigated [14] and for MIMO X networks, the iterative algorithm was proposed for realize the IA scheme [15].

However, the above IA schemes are only applicable for homogeneous network. For downlink HetNet scenarios, the hierarchical IA scheme was proposed, where two picocells were deployed within a macrocell and there only exists two macro users in a macrocell [16]. One IA scheme was proposed for the uplink MIMO HetNet, where the number of data streams for each user is the same [17]. One IA scheme was proposed for the uplink MIMO HetNet with the MMSE criterion, which cannot completely eliminate interference [18]. Furthermore, an IA scheme was proposed for the downlink of MIMO HetNet, where only one single data stream is transmitted for each user [19]. Moreover, the partial IA scheme was proposed in the downlink MIMO HetNet, in which there exist one macrocell and two picocells and only one macro user can be served [20]. Additionally, the feasibility of IA schemes for HetNet was investigated in [21].

Against this background, in this paper, we propose two new IA schemes for the downlink of HetNet with one macrocell and two picocells. Our main contributions are as follows:

1. We propose a basic grouping method aided IA scheme in the HetNet for the scenario where the number of antennas at pico BS can be different from that of each user, as well as an advanced GEVD based grouping method aided IA scheme for the scenario where the number of antennas at pico BS is equal to that of each user.

2. In our schemes, the macro BS serves arbitrary macro users instead of only two macro users in [16] or one macro user in [20]. Furthermore, our scheme can support different number of data streams for macro users and pico users instead of the same number of data streams [17].

3. We propose five theorems to characterize the maximum total achievable DoF of the basic grouping method aided IA scheme and the GEVD based grouping method aided IA scheme. The advantage of the GEVD based grouping method aided IA scheme is that the pico BS requires less number of antennas than that of the basic grouping method aided IA scheme. And also, for the same configuration, the GEVD based grouping method aided IA scheme can provide a higher maximum total achievable DoF than or equal to that of the basic grouping method aided IA scheme.

The remainder of this paper is organized as follows. We describe the system model in Section II. In Section III, we propose a basic grouping method aided IA scheme in HetNet to jointly design the transmit and receiver beamforming matrices. In Section IV, we propose an advanced GEVD based grouping method aided IA scheme. We make an analysis on the maximum achievable DoF for both the proposed schemes in Section V. Finally, we present several simulation results in Section VI and Section VII concludes this paper.

Notations: We use lower case for scalars, upper and lower boldface for matrix $A$ and vector $a$, respectively. $A^H$ represents the conjugate transpose operator of the matrix $A$. $I_M$ is used to denote the $M \times M$ identity matrix. The operator $\| \cdot \|$ denotes Euclidean norm operator and span($\cdot$) denotes the subspace spanned by the column vectors of a matrix, while null($\cdot$) denotes an orthonormal basis for the null space of a matrix. $| \cdot |$ denotes the modulus of a complex number. Finally, the operator eig($\cdot$) denotes a unit norm eigenvector.

**II. SYSTEM MODEL**

Fig. 1 shows the system model considered in this paper that consists of $L = 3$ cells, i.e., one macrocell and two picocells, where the macrocell is the second cell and two picocells are the first and third cells, respectively. Furthermore, in Fig. 1, BS2 denotes the macro BS, while BS1 and BS3 denote two pico BSs, respectively. Moreover, we assume that the macro BS serves $K_2$ macro users and each pico BS serves $K_1 = 1$ pico user. Then, we assume that each user is equipped with $N_r$ receive antennas, each pico BS has $N_t$ transmit antennas and the macro BS has $M$ transmit antennas. Moreover, we assume that each pico BS conveys $d_p$ independent data streams to its corresponding user and the macro BS conveys $d_m$ independent data streams to each of its corresponding users. In this case, the macro users receive two types of interferences which are inter-user interference (IUI) among macro users and inter-tier interference (ITI) from pico BSs. Similarly, the pico users also receive two types of interferences which are ITI from the macro BS and co-tier interference (CTI) from the other pico BS. It is supposed that a perfect knowledge of channel state information (CSI), which can be estimated by the sophisticated channel estimation method and limited feedback [22], is available at each transmitter and receiver.
The $k$th user in the $l$th ($l = 1, 2, 3$) cell is referred to as user $[k, l]$. The transmitted signal vector $\mathbf{x}^{[k,l]}$ intended for the user $[k, l]$ is written as

$$\mathbf{x}^{[k,l]} = \sum_{i=1}^{d[k,l]} \mathbf{v}_i^{[k,l]} s_i^{[k,l]} = \mathbf{V}^{[k,l]} \mathbf{s}^{[k,l]}.$$

(1)

where $d[k,l] = d_p$ for the pico user, and $d[k,l] = d_m$ for macro users. The $i$th transmitted symbol for the user $[k, l]$ is denoted as $s_i^{[k,l]}$ and $\mathbf{s}^{[k,l]} = [s_1^{[k,l]}, s_2^{[k,l]}, \cdots, s_{d[k,l]}^{[k,l]}]^T \in C^{d[k,l] \times 1}$ denotes the transmitted symbol vector for the user $[k, l]$. The $i$th linear transmit beamforming vector is denoted as $\mathbf{v}_i^{[k,l]}$ for the pico user, $\mathbf{v}_i^{[k,l]} \in C^{N_i \times 1}$ and for macro users, $\mathbf{v}_i^{[k,l]} \in C^{M_i \times 1}$, which corresponds to the $i$th transmitted symbol $s_i^{[k,l]}$, with a unity norm constraint $\|\mathbf{v}_i^{[k,l]}\| = 1$. The transmit beamforming matrix for the user $[k, l]$ is written as $\mathbf{V}^{[k,l]} = [\mathbf{v}_1^{[k,l]}, \mathbf{v}_2^{[k,l]}, \cdots, \mathbf{v}_{d[k,l]}^{[k,l]}]$, where $\mathbf{v}_1^{[k,l]}, \mathbf{v}_2^{[k,l]} \in C^{N_i \times d[k,l]}$, and $\mathbf{V}^{[k,2]} \in C^{M_i \times d[k,2]}$. Furthermore, the transmitted signal vector $\mathbf{x}^{[k,l]}$ satisfies an average power constraint, $E[\|\mathbf{x}^{[k,l]}\|^2] \leq P[k,l]$, where $P[k,l]$ denotes the transmit power from the $l$th BS to the user $[k, l]$. Moreover, we assume that $P[1,1] = P[1,3] = P_{\text{pico}}, \sum_{k=1}^{K_2} P[k,2] = P_{\text{macro}}$, where $P_{\text{pico}}$ and $P_{\text{macro}}$ denote the total transmit power at each pico BS and macro BS, respectively. The received signal vector $\mathbf{y}^{[k,l]}$ at the user $[k, l]$ is expressed as

$$\mathbf{y}^{[k,l]} = \sum_{j=1}^{L} \mathbf{H}^{[k,l]}_{j} \sum_{i=1}^{K_j} \mathbf{x}^{[j,i]} + \mathbf{n}^{[k,l]},$$

(2)

where $\mathbf{H}^{[k,l]}_{j}$ is the channel matrix from the $j$th BS to the user $[k, l]$ with each entry of it to be independent and identically distributed ($i.i.d.$) according to $CN(0, 1)$. Furthermore, for the pico cells, $\mathbf{H}^{[k,l]}_{j} \in C^{N_i \times N_j}$, $\mathbf{H}^{[k,2]}_{j} \in C^{N_i \times N_j}$ and for macro cells, $\mathbf{H}^{[k,2]}_{j} \in C^{N_i \times M_j}$. Moreover, the vector $\mathbf{n}^{[k,l]}$ is the AWGN vector with zero mean and variance matrix of $\sigma^2 I$ observed at the user $[k, l]$. The user $[k, l]$ decodes the desired signals sent from its corresponding BS by multiplying its receiver a beamforming matrix $\mathbf{U}^{[k,l]}$, which is denoted as $\mathbf{U}^{[k,l]} = [\mathbf{u}_1^{[k,l]}, \mathbf{u}_2^{[k,l]}, \cdots, \mathbf{u}_{d[k,l]}^{[k,l]}] \in C^{N_i \times d[k,l]}$, where $\mathbf{u}_i^{[k,l]} \in C^{N_i \times 1}$ denotes the $i$th linear receiver beamforming vector. The received signal vector $\mathbf{\hat{y}}^{[k,l]}$ at the user $[k, l]$ after processing by $\mathbf{U}^{[k,l]}$ is written as

$$\mathbf{\hat{y}}^{[k,l]} = \mathbf{U}^{[k,l]} H^H \mathbf{y}^{[k,l]} = \mathbf{U}^{[k,l]} H^H \sum_{j=1}^{L} \mathbf{H}^H_{j} \sum_{i=1}^{K_j} \mathbf{x}^{[j,i]} + \mathbf{n}^{[k,l]},$$

(3)

where $\mathbf{n}^{[k,l]} = \mathbf{U}^{[k,l]} H^H \mathbf{n}^{[k,l]}$ is the effective noise vector with a covariance matrix of $\sigma^2 \mathbf{U}^{[k,l]} H^H \mathbf{U}^{[k,l]}$. The capacity $C^{[k,l]}$ achieved by the user $[k, l]$ can be described as (4), as shown at the bottom of this page [9], where $\bar{k}$ and $\bar{l}$ denote $k \neq \bar{k}$ and $l \neq \bar{l}$, respectively. Furthermore, $p^{[k,l]}$ denotes the transmit power of the $l$th transmit symbol for the user $[k, l]$, which satisfies the constraint $\sum_{i=1}^{d[k,j]} p_i^{[k,l]} = P[k,l]$. The DoF is defined as a pre-log factor of the sum rate [4], which is a key metric to assess the performance of the multi-antenna based system in a high SNR regime, and may be expressed as [4],

$$d_{\Sigma} \triangleq \lim_{\text{SNR} \to \infty} \frac{C_{\text{SNR}}}{\log(\text{SNR})} = \sum_{l=1}^{L} \sum_{k=1}^{K_l} d[k,l] = 2d_p + K_2 d_m,$$

(5)

where $C_{\text{SNR}} = \sum_{l=1}^{L} \sum_{k=1}^{K_l} C^{[k,l]}$ denotes the sum capacity of all the users in HetNet that can be achieved for a given SNR.

For the macro user $[k, l]$, in order to decode the desired signal, both the ITI and IUI should also be taken into account. One possible solution is to invoke IA techniques to align all interference into a subspace, which is linearly independent of the desired signal vector. To this end, the equations below must be satisfied [9]

$$\mathbf{U}^{[k,2]} H^H \mathbf{H}^{[k,2]} \mathbf{v}^{[1,i]} = \mathbf{0}, i \neq l, i \in \{1, 2, \cdots, K_2\},$$

(6)

$$\mathbf{U}^{[k,2]} H^H \mathbf{H}^{[k,2]} \mathbf{v}^{[m,2]} = \mathbf{0}, \forall m \neq k, i \in \{1, 2, \cdots, K_2\},$$

(7)

$$\text{rank}[\mathbf{U}^{[k,l]} H^H \mathbf{H}^{[k,l]}] \mathbf{v}^{[k,l]}] = d[k,l] = d_m,$$

(8)

where (6) is used to remove the ITI, while equation (7) is for the IUI. Furthermore, equation (8) indicates the DoF that the macro user $[k, l]$ should obtain. It was demonstrated that the condition (8) can be automatically satisfied when the elements of channel matrices are $i.i.d.$ according to $CN(0, 1)$ [7].
Similarly, for pico user \([k, l]\), in order to decode the desired signal, both the ITI and CTI should also be aligned into the interference subspace at the receiver of the pico user \([k, l]\), which is linearly independent of the desired signal as well. Therefore, the equations below must be satisfied [9]

\[
U^{[1, l]}H^{[1, l]}V^{[1, 2]} = 0, \quad l \neq 2, j \in \{1, 2, \cdots, K_2\},
\]

(9)

\[
U^{[1, l]}H^{[1, l]}V^{[i, l]} = 0, \quad \forall i \neq l, i \neq 2, l \neq 2,
\]

(10)

\[
\text{rank}(U^{[k, l]}H^{[k, l]}V^{[k, l]}) = d^{[k, l]} = d_p,
\]

(11)

where (9) mainly targets the ITI, while (10) is for the CTI. Furthermore, (11) indicates the DoF that the pico user \([k, l]\) should obtain. Similarly (8), (11) could be automatically satisfied [7].

### III. BASIC GROUPING METHOD AIDED IA SCHEME

In this section, we propose a basic grouping method aided IA scheme for the system model described in Section II. Specifically, we divide the \(K_2\) macro users into two groups and the macro users in the same group are associated to one pico BS. Then, we design the receiver beamforming matrices for the macro users so that the macro users in the same group align the ITI from the associated pico BS into the same subspace [9]. After that, we formulate the transmit and receiver beamforming matrices for the pico users to remove the ITI imposed on the macro users and CTI between picocells, respectively. Finally, we construct the transmit beamforming matrices at the macro BS to remove the ITI imposed on pico users and IUI among macro users.

There are two different scenarios, i.e., the number of users served by the macro BS, \(K_2\), is even or odd.

#### A. THE NUMBER OF MACRO USERS \(K_2\) IS EVEN

As shown in Fig. 2.(a), we divide all the \(K_2\) macro users into two groups. Specifically, users \([k, 2]\), where \(k \in \{1, 2, \cdots, K_2\}\), are treated as Group 1, while users \([k, 2]\), where \(k \in \{K_2 + 1, K_2 + 2, \cdots, K_2\}\) are treated as Group 2. Furthermore, we associate Group 1 to BS1 and Group 2 to BS3, respectively. Now, we are ready to design the IA scheme in the following steps.

**Step 1**: designing the receiver beamforming matrices at macro users

We design the \((N_t \times d_m)\)-dimensional receiver beamforming matrix \(U^{[k, 2]}\) for the macro users \([k, 2]\), so that for each group, the ITI from the associated pico BS are aligned into the same subspace. By this way, each pico BS consider ITI on \(K_2 + 1\) effective macro users instead of \(K_2\) ones in the original scenario. To this end, the ITI from a pico BS to its associated group span a particular subspace described as follows [9],

\[
G_1 = \text{span}(H_1^{[1, 2]}U^{[1, 2]}),
\]

\[
= \text{span}(H_1^{[2, 2]}U^{[2, 2]}),
\]

\[
= \cdots = \text{span}(H_1^{[K_2, 2]}U^{[K_2, 2]}),
\]

(12)

\[
\begin{align*}
G_1 &= \text{span}(H_1^{[1, 2]}U^{[1, 2]}) \\
&= \text{span}(H_1^{[2, 2]}U^{[2, 2]}) \\
&= \cdots = \text{span}(H_1^{[K_2, 2]}U^{[K_2, 2]}),
\end{align*}
\]

where \(G_1 \in C^{N_t \times d_m}\) represents the subspace spanned by the aligned ITI from BS 1 to the macro users in Group 1 after multiplying the receiver beamforming matrices \(U^{[k, 2]}\).

Similarly, \(G_2 \in C^{N_t \times d_m}\) represents the subspace spanned by the aligned ITI from BS 3 to the macro users in Group 2 after multiplying the receiver beamforming matrices \(U^{[k, 2]}\).

The matrix \(G_1\) satisfying (12) can be obtained by solving the following equation [9],

\[
I_{N_t} - H_1^{[1, 2]}H_1^{[1, 2]} \cdots 0 \quad 0 \quad \cdots 0 \\
I_{N_t} 0 - H_1^{[2, 2]}H_1^{[2, 2]} \quad \cdots 0 \\
\vdots \quad \vdots \quad \ddots \quad \ddots \quad \vdots \\
I_{N_t} 0 0 \quad \cdots - H_1^{[K_2, 2]}H_1^{[K_2, 2]} \\
F_1 U^{[1, 2]} U^{[2, 2]} \cdots U^{[K_2, 2]} \\
X_1
\]

\[
= F_1 X_1 = 0,
\]

(14)
where $F_1$ is a $(K_2N_2 \times (N_r + K_2N_r)/2)$-dimensional matrix and $X_1$ is the $((N_r + K_2N_r)/2 \times d_m)$-dimensional matrix. In order to comply with the $((N_r \times d_m))$-dimensional requirement of the receiver beamforming matrices $U^{[k,2]}$, the condition below must be satisfied [9],

$$
\frac{K_2N_r}{2} - \left(\frac{K_2}{2} - 1\right)N_r \geq d_m. 
$$

(15)

Likewise, in order to satisfy (13), the receiver beamforming matrices $U^{[k,2]}$ for the macro users in Group 2 can be obtained [9],

$$
\begin{bmatrix}
I_{N_r} & -H_3^{[K_2+1,2]} & \cdots & 0 \\
I_{N_p} & 0 & -H_3^{[K_2+2,2]} & \cdots & 0 \\
& & \vdots & \ddots & \vdots \\
I_{N_r} & 0 & 0 & \cdots & -H_3^{[K_2,2]} \\
\end{bmatrix}
$$

$$
\begin{bmatrix}
G_2 \\
U^{[K_2+1,2]}_1 \\
U^{[K_2+2,2]}_1 \\
\vdots \\
U^{[K_2,2]}_1 \\
\end{bmatrix}
$$

\begin{bmatrix}
F_2 \\
X_2 \\
\end{bmatrix}
= 0. 

(16)

Similarly, $F_2$ is a $(K_2N_2 \times (N_r + K_2N_r)/2)$-dimensional matrix and $X_2$ is the $((N_r + K_2N_r)/2 \times d_p)$-dimensional matrix. Furthermore, (15) needs to be satisfied to make sure that $U^{[k,2]}$ is a $(N_r \times d_m)$-dimensional matrix.

Once $X_1$ and $X_2$ are obtained by solving (14) and (16), the receiver beamforming matrices $U^{[k,2]}$ can be determined correspondingly.

**Step 2:** designing the transmit beamforming matrices at pico BSs

Based on the $(N_r \times d_p)$-dimensional receiver beamforming matrix $U^{[k,2]}$ for macro user $[k, 2]$, we design the transmit beamforming matrices $V^{[1,1]}$ and $V^{[1,3]}$ at pico BSs, so that the signal vectors sent by BS 1 and BS 3 will not interfere with the macro users any more. To this end, $V^{[1,1]}$ and $V^{[1,3]}$ need to satisfy the following conditions,

$$
V^{[1,1]} \subset \text{null}((U^{[1,1]}H^{[1,2]}_1H^{[K_2+1,2]}_1)H
(U^{[K_2+2,2]}_1H^{[K_2+2,2]}_1)H \cdots (U^{[K_2,2]}_1H^{[K_2,2]}_1)H), 
$$

(17)

$$
V^{[1,3]} \subset \text{null}((G_2U^{[1,2]}_2H^{[1,2]}_3H^{[K_2+1,2]}_3)H
(U^{[2,2]}H^{[2,2]}_3H^{[2,2]}_3)H \cdots (U^{[K_2,2]}H^{[K_2,2]}_3H^{[K_2,2]}_3)H), 
$$

(18)

In order to make sure that the $(N_r \times d_p)$-dimensional transmit beamforming matrices $V^{[1,1]}$ and $V^{[1,3]}$ exist, the following condition must be satisfied,

$$
N_r - \left(\frac{K_2}{2} + 1\right)d_m \geq d_p. 
$$

(19)

**Step 3:** designing the receiver beamforming matrices at pico users

In order to remove the CTI between picocells, we construct the $(N_r \times d_p)$-dimensional receiver beamforming matrices $U^{[1,1]}$ and $U^{[1,3]}$ for the pico user $[1, 1]$ and user $[1, 3]$ respectively. To this end, we need to satisfy the following conditions,

$$
U^{[1,1]} \subset \text{null}((H^{[1,1]}_3V^{[1,3]}_1H), 
$$

(20)

$$
U^{[1,3]} \subset \text{null}((H^{[1,3]}_1V^{[1,1]}_{1})H). 
$$

(21)

In order to make sure the $(N_r \times d_p)$-dimensional receiver beamforming matrices $U^{[1,1]}$ and $U^{[1,3]}$ exist, the following condition must hold,

$$
N_r - d_p \geq d_p. 
$$

(22)

**Step 4:** designing the transmit beamforming matrices at the macro BS

We design the $(M_t \times d_m)$-dimensional transmit beamforming matrices $V^{[k,2]}(k \in \{1, 2, \cdots, K_2\})$ at macro BS, to remove the IUl among macro users and ITI at pico users. To this end, the transmit beamforming matrices $V^{[k,2]}$ need to satisfy the following conditions,

$$
V^{[1,2]} \subset \text{null}((U^{[1,1]}H^{[1,2]}_2H^{[1,1]}_2)H
(U^{[1,3]}H^{[1,2]}_2H^{[1,3]}_2)H \cdots (U^{[K_2,2]}H^{[K_2,2]}_2H^{[K_2,2]}_2)H), 
$$

(23)

$$
V^{[2,2]} \subset \text{null}((U^{[1,1]}H^{[1,2]}_2H^{[1,1]}_2)H
(U^{[1,3]}H^{[1,2]}_2H^{[1,3]}_2)H \cdots (U^{[K_2,2]}H^{[K_2,2]}_2H^{[K_2,2]}_2)H), 
$$

(24)

$$
V^{[K_2,2]} \subset \text{null}((U^{[1,1]}H^{[1,2]}_2H^{[1,1]}_2)H
(U^{[1,3]}H^{[1,2]}_2H^{[1,3]}_2)H \cdots (U^{[K_2-1,2]}H^{[K_2-1,2]}_2H^{[K_2-1,2]}_2)H). 
$$

(25)

In order to ensure the $(M_t \times d_m)$-dimensional transmit beamforming matrices $V^{[k,2]}$ exist, the condition below must hold,

$$
M_t - 2d_p - (K_2 - 1)d_m \geq d_m. 
$$

(26)

Following the grouping method described above, all the users in the HetNet can successfully receive the intended signals without interference.

**B. THE NUMBER OF MACRO USERS $K_2$ IS ODD**

For the case where $K_2$ is odd, we group the macro users $[k, 2]$, where $k \in \{1, 2, \cdots, K_2/2\}$, together as Group 1, while the macro users $[k, 2]$, where $k \in \{K_2/2 + 1, K_2/2 + 2, \cdots, K_2 - 1\}$, together as Group 2, as shown in Fig. 2(b). Furthermore, we associate Group 1 to BS1 and Group 2 to BS3, respectively. In this case, the macro user $[K_2, 2]$ is left which belongs to neither of groups. How to deal with user $[K_2, 2]$ will be discussed later on.

**Step 1:** designing the receiver beamforming matrices at macro users
We design the \((N_r \times d_m)\)-dimensional receiver beamforming matrix \(U^{[k, 2]}\) for the macro users, so that for each group, the ITI from the associated pico BS are aligned into the same subspace. Then, each pico BS will only consider ITI on \(K_2 - 1\) + 2 effective macro users instead of \(K_2\) ones in the original scenario. To this end, the ITI from a pico BS to its associated group span a particular subspace described as follows \([9]\),

\[
G_1 = \text{span}[H_1^{[1,2]} H] U^{[1,2]} \equiv \text{span}[H_2^{[1,2]} H] U^{[1,2]},
\]

\[
G_2 = \cdots = \text{span}[H_1^{[K_2-1,2]} H] U^{[K_2-1,2]},
\]

\[
G_1 = \text{span}[H_3^{[1,2]} H] U^{[1,2]} \equiv \text{span}[H_3^{[1,2]} H] U^{[1,2]},
\]

\[
G_2 = \cdots = \text{span}[H_3^{[K_2-1,2]} H] U^{[K_2-1,2]},
\]

where \(G_1\) and \(G_2\) are \((N_t \times d_m)\)-dimensional matrices. Furthermore, we also obtain the receiver beamforming matrices \(U^{[k, 2]}\) at the macro users in Group 1 and Group 2 by constructing the equations \((29)\) and \((30)\) [see \((30)\), as shown at the bottom of this page] \([9]\),

\[
\begin{bmatrix}
I_{N_t} & -H_1^{[1,2]} H & 0 & \cdots & 0 \\
I_{N_t} & 0 & -H_2^{[1,2]} H & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
I_{N_t} & 0 & 0 & \cdots & -H_1^{[K_2-1,2]} H \\
\end{bmatrix}
\begin{bmatrix}
G_1 \\
U^{[1,2]} \\
U^{[2,2]} \\
\vdots \\
U^{[K_2-1,2]} \\
\end{bmatrix}
= F_1 \begin{bmatrix} X_1 \end{bmatrix},
\]

where \(F_1\) and \(F_2\) is \((K_2-1)N_t \times (N_t + (K_2-1)N_t)\)-dimensional matrices and \(X_1\) and \(X_2\) are spanned by \((N_t + (K_2-1)N_t) \times d_m)\)-dimensional matrices, respectively. In order to comply with the \((N_t \times d_m)\)-dimensional requirement of \(U^{[k, 2]}\), the condition below must be satisfied \([9]\),

\[
(K_2 - 1)N_t - \left(\frac{K_2 - 1}{2} - 1\right) N_t \geq d_m.
\]

Once \(X_1\) and \(X_2\) are obtained by solving \((29)\) and \((30)\), the receiver beamforming matrices \(U^{[k, 2]}(k \in \{1, 2, \ldots, K_2 - 1\})\) can be determined correspondingly.

Furthermore, for the macro user \([K_2, 2]\), we could choose an arbitrary \((N_t \times d_m)\)-dimensional receiver beamforming matrix \(U^{[K_2,2]}\) which satisfies the constraint as follows,

\[
\text{rank} \{U^{[K_2,2]}\} = d_m.
\]

**Step 2**: designing the transmit beamforming matrices at pico BSs

We construct the \((N_t \times d_p)\)-dimensional transmit beamforming matrices \(V^{[1,1]}\) and \(V^{[1,3]}\) at pico BSs for the pico user \([1,3]\) and user \([1,1]\) to cancel out the ITI imposed on macro users. Then, \(V^{[1,1]}\) and \(V^{[1,3]}\) need to satisfy the following conditions,

\[
\begin{align*}
V^{[1,1]} & \subset \text{null}[G_1, (U^{[1,2]} H, U^{[K_2-1,2]} H H, U^{[K_2,2]} H H, U^{[K_2,2]} H H)], \\
V^{[1,3]} & \subset \text{null}[G_2, (U^{[1,2]} H, U^{[K_2-1,2]} H H, U^{[K_2,2]} H H, U^{[K_2,2]} H H)],
\end{align*}
\]

In order to make sure that the \((N_t \times d_p)\)-dimensional transmit beamforming matrices \(V^{[1,1]}\) and \(V^{[1,3]}\) exist, the condition below must hold,

\[
N_t - \left(\frac{K_2 - 1}{2} + 2\right)d_m \geq d_p.
\]

**Step 3**: designing the receiver beamforming matrices at pico users

Following the same step 3 described in Section III. A, we design the \((N_r \times d_m)\)-dimensional receiver beamforming matrices \(U^{[1,1]}\) and \(U^{[1,3]}\) at pico user \([1,1]\) and user \([1,3]\) to remove the CTI between picocells.

**Step 4**: designing the transmit beamforming matrices at the macro BS

We can follow the same step 4 described in Section III. A to construct the \((M_t \times d_m)\)-dimensional transmit beamforming matrix \(V^{[K,2]}\) at macro BS. Then, we can remove the IUI among macro users and ITI at pico users.

Similarly, as described in Section III. A, all users can successfully receive the intended signals without interference.
In this section, we propose an advanced GEVD based group-
ing method aided IA scheme for the system model described
in [12]. Now we are ready to develop IA scheme in the
following steps.

Step 1: designing the receiver beamforming matrices at macro
users

We construct the \((N_t \times d_m)\)-dimensional receiver beam-
forming matrices \(U^{[2k-1,2]}\) and \(U^{[2k,2]}\) for the macro user
\([2k - 1, 2]\) and user \([2k, 2]\), so that the ITI from the same
pico BS to the macro users in Group \(k\) are aligned into the
same subspace [12]. By this way, each pico BS just need to
consider ITI on one effective macro user in each group instead
to two macro users in the original scenario. To this end, \(U^{[k,2]}\)
can be written as follows [12], [16],

\[
\text{span}(H_1^{[2k-1,2]}H U^{[2k-1,2]}) = \text{span}(H_1^{[2k,2]}H U^{[2k,2]})
\]

(36)

\[
\text{span}(H_3^{[2k-1,2]}H U^{[2k-1,2]}) = \text{span}(H_3^{[2k,2]}H U^{[2k,2]})
\]

(37)

from (36) and (37), we can obtain

\[
\text{span}(U^{[2k-1,2]}) = \text{span}((H_1^{[2k-1,2]}H)^{-1}H_1^{[2k,2]}H U^{[2k,2]})
\]

= \text{span}((H_3^{[2k-1,2]}H)^{-1}H_3^{[2k,2]}H U^{[2k,2]}).
\]

(38)

Then, \(U^{[2k-1,2]}\) and \(U^{[2k,2]}\) for the macro user \([2k - 1, 2]\)
and user \([2k, 2]\) can be determined, respectively, as follows [12], [16],

\[
u_i^{[2k,2]} = \text{eig}((H_3^{[2k,2]}H)^{-1}(H_3^{[2k-1,2]}H)(H_1^{[2k-1,2]}H)^{-1}H_1^{[2k,2]}H),
\]

(39)

\[
u_i^{[2k-1,2]} = \frac{(H_1^{[2k-1,2]}H)^{-1}H_1^{[2k,2]}H u_i^{[2k,2]}}{||H_1^{[2k-1,2]}H u_i^{[2k,2]}||}
\]

(40)

where \(i \in \{1, 2, \cdots, d_m\}\), and \(u_i^{[2k,2]}\) is the eigenvector of the
matrix \((H_3^{[2k,2]}H)^{-1}(H_3^{[2k-1,2]}H)(H_1^{[2k-1,2]}H)^{-1}H_1^{[2k,2]}H\)
corresponding to its \(i\)th eigenvalue.

Step 2: designing the transmit beamforming matrices at
pico BSs

We construct the \((N_r \times d_p)\)-dimensional transmit beam-
forming matrices \(V^{[1,1]}\) and \(V^{[1,3]}\) at pico BSs to remove the
ITI imposed on macro users. Then, \(V^{[1,1]}\) and \(V^{[1,3]}\) need
to satisfy the following conditions,

\[
V^{[1,1]} \subset \text{null}((U^{[1,2]}H)^{[1,2]}(H^{[3,2]}H)^{[3,2]}H)
\]

\[
\cdots (U^{[K_2-1,2]}H^{[K_2-1,2]}H^{[3,2]}H)
\]

(41)

\[
V^{[1,3]} \subset \text{null}((U^{[1,2]}H)^{[1,2]}(H^{[3,2]}H)^{[3,2]}H)
\]

\[
\cdots (U^{[K_2-1,2]}H^{[K_2-1,2]}H^{[3,2]}H)
\]

(42)

In order to guarantee the existence of the transmit beam-
forming matrices \(V^{[1,1]}\) and \(V^{[1,3]}\) at pico BSs, the condition
below must hold,

\[
N - \frac{K_2 d_m}{2} \geq d_p.
\]

(43)
Step 3: designing the receiver beamforming matrices at pico users

We develop the \((N_r \times d_p)\)-dimensional receiver beamforming vectors \(\mathbf{U}^{[1,1]}\) and \(\mathbf{U}^{[1,3]}\) for the pico user [1, 1] and user [1, 3] to cancel out the CTI between picocells [16]. To this end, \(\mathbf{U}^{[1,1]}\) and \(\mathbf{U}^{[1,3]}\) need to satisfy the following conditions,

\[
\mathbf{U}^{[1,1]} \subseteq \text{null}(\mathbf{H}^{[1,1]}_3 \mathbf{V}^{[1,3]}), \quad (44)
\]

\[
\mathbf{U}^{[1,3]} \subseteq \text{null}(\mathbf{H}^{[1,3]}_1 \mathbf{V}^{[1,1]}), \quad (45)
\]

In order to make sure the existence of \(\mathbf{U}^{[1,1]}\) and \(\mathbf{U}^{[1,3]}\), the following condition must be satisfied,

\[
N - d_p \geq d_p. \quad (46)
\]

Step 4: designing the transmit beamforming matrices at the macro BS

We construct the \((M_t \times d_m)\)-dimensional transmit beamforming matrices \(\mathbf{V}^{[k,2]}\) at the macro BS to remove the IUI among macro users and ITI at pico users [16]. To this end, the transmit beamforming matrices \(\mathbf{V}^{[k,2]}\) need to satisfy the following conditions,

\[
\mathbf{V}^{[1,2]} \subseteq \text{null}(\{\mathbf{U}^{[1,1]}_1 \mathbf{H}^{[1,1]}_2\} \mathbf{H}^{[1,3]}_1 \mathbf{H}^{[1,3]}_3), \quad (47)
\]

\[
\mathbf{V}^{[2,2]} \subseteq \text{null}(\{\mathbf{U}^{[1,1]}_1 \mathbf{H}^{[1,1]}_2\} \mathbf{H}^{[1,3]}_1 \mathbf{H}^{[1,3]}_3), \quad (48)
\]

\[
\mathbf{V}^{[k,2]} \subseteq \text{null}(\{\mathbf{U}^{[1,1]}_1 \mathbf{H}^{[1,1]}_2\} \mathbf{H}^{[1,3]}_1 \mathbf{H}^{[1,3]}_3), \quad (49)
\]

In order to ensure the \((M_t \times d_m)\)-dimensional transmit beamforming matrices \(\mathbf{V}^{[k,2]}\) exist, condition (26) must hold.

Following the advanced GEVD based grouping method aided IA scheme described above, all the users in the HetNet can successfully receive the intended signals without interference.

B. THE NUMBER OF MACRO USERS \(K_2\) IS ODD

For the case where \(K_2\) is odd, we group the macro user [2k − 1, 2] and user [2k, 2] together as Group \(k\), where \(k \in \{1, 2, \cdots, \frac{K_2 - 1}{2}\}\), as proposed in [12], and the total number of groups is \(\frac{K_2 - 1}{2}\), as shown in Fig. 3(b). In this case, the macro user [K2, 2] is left which belongs to none of groups. How to deal with user [K2, 2] will be discussed later on.

Step 1: designing the receiver beamforming matrices at macro users

We design the \((N_r \times d_m)\)-dimensional receiver beamforming matrices \(\mathbf{U}^{[2k-1,2]}\) and \(\mathbf{U}^{[2k,2]}\) for the macro user [2k − 1, 2] and user [2k, 2], so that the ITI from the same BS to the macro user in Group \(k\) are aligned into the same subspace [12]. In this case, each pico BS just need to consider ITI on one effective macro user in each group instead of two macro users in the original scenario. To this end, we can follow equations from (36) to (40) to obtain \(\mathbf{U}^{[2k-1,2]}\) and \(\mathbf{U}^{[2k,2]}\) for the macro user [2k − 1, 2] and user [2k, 2].

Furthermore, for the macro user [K2, 2], we can choose an arbitrary \((N_r \times d_m)\)-dimensional transmit beamforming matrix \(\mathbf{U}^{[K2,2]}\) satisfying the condition (32) [12].

Step 2: designing the transmit beamforming matrices at pico BSs

We construct the \((N_r \times d_p)\)-dimensional transmit beamforming matrices \(\mathbf{V}^{[1,1]}\) and \(\mathbf{V}^{[1,3]}\) at pico BSs to eliminate the ITI imposed on macro users. Then \(\mathbf{V}^{[1,1]}\) and \(\mathbf{V}^{[1,3]}\) need to satisfy the constraints as follows,

\[
\mathbf{V}^{[1,1]} \subseteq \text{null}(\{\mathbf{U}^{[1,1]}_1 \mathbf{H}^{[1,1]}_3\} \mathbf{H}^{[1,3]}_1 \mathbf{H}^{[1,3]}_1) \mathbf{H}^{[1,3]}_3), \quad (50)
\]

\[
\mathbf{V}^{[1,3]} \subseteq \text{null}(\{\mathbf{U}^{[1,1]}_1 \mathbf{H}^{[1,1]}_3\} \mathbf{H}^{[1,3]}_1 \mathbf{H}^{[1,3]}_3), \quad (51)
\]

In order to guarantee the existence of \(\mathbf{V}^{[1,1]}\) and \(\mathbf{V}^{[1,3]}\) at pico BSs, the condition below must hold,

\[
N - (K_2 + 1)d_m \geq d_p. \quad (52)
\]

Step 3: designing the receiver beamforming matrices at pico users

Following the same step 3 described in Section IV. A, we design the \((N_r \times d_p)\)-dimensional receiver beamforming matrices \(\mathbf{U}^{[1,1]}\) and \(\mathbf{U}^{[1,3]}\) at the pico user [1, 1] and user [1, 3] to remove the CTI between picocells.

Step 4: designing the transmit beamforming matrices at the macro BS

We can follow the same step 4 described in Section IV. A to construct the \((M_t \times d_m)\)-dimensional transmit beamforming matrix \(\mathbf{V}^{[k,2]}\) at macro BS. Then, we can remove the IUI among macro users and ITI at pico users.

Similarly, as described in Section IV. A, all the users can successfully receive the intended signals without interference.

Lemma 1: For the same DoF, when \(N_r = N_r\), whether \(K_2\) is even or odd number, the number of antennas required at the pico BS by the GEVD based grouping method aided IA scheme reduces \(d_m\) compared to the basic grouping method aided IA scheme.

Proof of Lemma 1: When \(K_2\) is an even number, as seen by (19) and (43). When \(K_2\) is an odd number, as seen by (35) and (52).

V. MAXIMUM ACHIEVABLE DOF

In this section, we make an analysis on the performance of the proposed two IA schemes in terms of the total achievable DoF, respectively. We discuss the DoF of the basic grouping method aided IA scheme when the number of macro users \(K_2\) is even or odd. Similarly, we also discuss the DoF of the GEVD based grouping method aided IA scheme when the number of macro users \(K_2\) is even or odd.
A. THE DOF OF THE BASIC GROUPING METHOD AIDED IA SCHEME

1) THE NUMBER OF MACRO USERS \( K_2 \) IS EVEN AND \( K_2 > 2 \)

**Theorem 1:** For the HetNet that consists of one macrocell and two picocells, where the macro BS serves \( K_2 \) macro users and \( K_2 \) is an even number and \( K_2 > 2 \), the maximum total achievable DoF for the basic grouping method aided IA scheme is,

\[
\begin{align*}
\text{DoF} = & \begin{cases} 
N_r + \frac{(K_2 + 2)N_r}{K_2}, & \text{if } N_t \leq \frac{K_2 M_t}{2(K_2 + 1)} \text{ and } \frac{N_t}{N_r} \geq \frac{K_2^2 + 2K_2 + 2}{K_2^2} \\
N_r + \frac{2N_t - N_r}{K_2 + 2} K_2, & \text{if } N_t \leq \frac{M_t(K_2 + 2)}{2(K_2 + 1)} \text{ and } \frac{N_t}{N_r} < \frac{K_2^2 + 2K_2 + 2}{K_2^2} \\
2N_t, & \text{if } N_t \leq \frac{M_t}{2} \text{ and } \frac{N_t}{N_r} \leq \frac{1}{2} \\
M_t, & \text{otherwise.}
\end{cases}
\end{align*}
\]

**Proof of Theorem 1:** For the basic grouping method aided IA scheme, we modify (15) as follows,

\[
\frac{K_2 N_r}{2} - d_m \geq \frac{K_2}{2} - 1)N_t, \quad (54)
\]

where (54) means if \((\frac{K_2}{2} - 1)N_t\) exceeds \(K_2 N_r\), the basic grouping method aided IA scheme is not applicable. However, we can select \(N_t\) transmit antennas out of a total \(N_t\) antennas to be actively used to transmit signals at the pico BS [10]. \((15)\) and \((19)\) can be modified, respectively, as follows [10],

\[
\begin{align*}
\frac{K_2 N_r}{2} - d_m & \geq \frac{K_2}{2} - 1)N_t', \quad (55) \\
N_t' & \geq \frac{K_2}{2} + 1)d_m + d_p. \quad (56)
\end{align*}
\]

Combining (55) and (56) we have [15],

\[
K_2 N_r \geq \frac{K_2^2}{2}d_m + (K_2 - 2)d_p, \quad (57)
\]

In order to guarantee the feasibility of the basic grouping method aided IA scheme, \((15),\) \((19),\) \((22)\) and \((26)\) must be satisfied, which are repeated here for clarity,

\[
\begin{align*}
K_2 N_r & \geq \frac{K_2^2}{2}d_m + (K_2 - 2)d_p, \quad (1) \\
N_t - \left(\frac{K_2}{2} + 1\right)d_m & \geq d_p, \quad (2) \\
N_r - d_p & \geq d_p, \quad (2) \\
M_t - 2d_p - K_2 d_m & \geq 0, \quad (4)
\end{align*}
\]

where \((4)\) means the total achievable DoF is no more than \(M_t\).

From (58), we can calculate the intersection points of \((1)\) and \((2)\) at \(\left(\frac{N_t}{2} - \frac{K_2 N_r}{2}, \frac{2K_2 M_t + N_r}{2K_2}\right)\), \((1)\) and \((2)\) at \(\left(\frac{K_2 N_r}{2}, \frac{2K_2 M_t + N_r}{2K_2}\right)\), \((1)\) and \((3)\) at \(\left(\frac{N_t}{2}, \frac{(K_2 + 2)N_r}{K_2}\right)\), \((2)\) and \((3)\) at \(\left(\frac{N_t}{2}, 2N_t - N_r\right)\), \((2)\) and \((4)\) at \(\left(\frac{K_2 + 2N_t}{K_2}, \frac{2N_t - N_r}{K_2}\right)\). \((2)\) and \((3)\) at \(\left(\frac{N_t}{2}, \frac{K_2}{2} - \frac{M_t}{2}\right)\), \((3)\) and \((4)\) at \(\left(\frac{N_t}{2}, \frac{M_t - N_r}{K_2}\right)\). The intersection of \((3)\) and \((4)\) has no influence on the achievable DoF. Based on these five intersection points, there are six different cases to characterize the feasible region of DoF, which will be discussed below.

a) The intersection point of \((1)\) and \((2)\) is at the right hand side of \((3)\), while the intersection point of \((1)\) and \((4)\) is at the right hand side of \((3)\), as shown in Fig. 4.(a), i.e., the condition below is satisfied,

\[
\begin{align*}
\frac{K_2^2 N_r}{4} - \left(\frac{K_2}{2} + 2\right) N_r & \geq \frac{N_r}{2} \\
\frac{K_2^4 - 2K_2 N_r}{4} & \geq \frac{N_r}{2} \quad \Leftrightarrow \quad \frac{N_r}{2} \leq \frac{\frac{K_2^4}{4} + M_t}{2K_2 + 2}.
\end{align*}
\]

In this case, the feasible region of DoF is shown in Fig. 4.(a), where the maximum total achievable DoF is \(N_r + \frac{(K_2 + 2)N_r}{K_2}\), determined by (5), corresponding to the optimal intersection point of \((1)\) and \((3)\), i.e., \(\left(\frac{N_t}{2}, \frac{(K_2 + 2)N_r}{K_2}\right)\).

b) The intersection point of \((1)\) and \((2)\) is at the left hand side of \((3)\), while the intersection point of \((2)\) and \((3)\) is at the right hand side of \((3)\), as shown in Fig. 4.(b), i.e., the condition
below is satisfied,

\[
\begin{align*}
&\begin{cases}
N_t \leq \min(\frac{N_r}{2}, \frac{M_t}{2}), \\
N_t \leq \frac{M_t}{2}, \\
\frac{N_r}{2} \leq \frac{1}{2},
\end{cases} \\
\iff \begin{cases}
N_r > \frac{2K_2 M_t}{(K_2 + 1)}, \\
N_t > \frac{K_t^2 + 2K_2 + 2}{K_2^2}, \\
\frac{N_r}{N_t} \geq \frac{K_t^2 + 2K_2 + 2}{K_2^2}.
\end{cases}
\end{align*}
\]

In this case, the feasible region of DoF is shown in Fig. 4.(c), where the maximum total achievable DoF is \(M_t\), i.e., \(\frac{N_r}{2} - \frac{N_r}{2}\).

(60)

\(c\) The number of antennas for the pico BSs satisfies the following condition, as shown in Fig. 4.(c),

\[
N_t \leq \min(\frac{N_r}{2}, \frac{M_t}{2}) \iff \begin{cases}
N_t \leq \frac{M_t}{2}, \\
\frac{N_r}{2} \leq \frac{1}{2}.
\end{cases}
\]

(61)

In this case, the feasible region of DoF is shown in Fig. 4.(c), where the maximum total achievable DoF is \(2\), corresponding to the optimal point \((N_t, 0)\), at which there are no data streams from the macro BS to the macro users.

\(d\) The intersection point of \((1)\) and \((2)\) is at the right hand side of \((3)\), while the intersection point of \((1)\) and \((3)\) is at the left hand side of \((3)\), as shown in Fig. 4.(d), i.e., the condition below is satisfied,

\[
\begin{align*}
&\begin{cases}
N_r > \frac{2K_2 M_t}{(K_2 + 1)}, \\
N_t > \frac{K_t^2 + 2K_2 + 2}{K_2^2}, \\
\frac{N_r}{N_t} \geq \frac{K_t^2 + 2K_2 + 2}{K_2^2}.
\end{cases}
\end{align*}
\]

(62)

Then, the feasible region of DoF is shown in Fig. 4.(d), where the maximum total achievable DoF is \(M_t\).

\(e\) The intersection point of \((1)\) and \((2)\) is at the left hand side of \((3)\), while the intersection point of \((2)\) and \((3)\) is at the left hand side of \((3)\), as shown in Fig. 4.(e), i.e., the condition below is satisfied,

\[
\begin{align*}
&\begin{cases}
N_r > \frac{2K_2 M_t}{(K_2 + 1)}, \\
N_t > \frac{K_t^2 + 2K_2 + 2}{K_2^2}, \\
\frac{N_r}{N_t} \geq \frac{K_t^2 + 2K_2 + 2}{K_2^2}.
\end{cases}
\end{align*}
\]

(63)

Then, the feasible region of DoF is shown in Fig. 4.(e), where the maximum total achievable DoF is \(M_t\).

\(f\) The number of antennas for the pico BSs satisfies the following condition, as shown in Fig. 4.(f),

\[
\frac{M_t}{2} \leq \min(N_r, N_t) \iff \begin{cases}
N_t \geq \frac{M_t}{2}, \\
N_r \geq \frac{M_t}{2}.
\end{cases}
\]

(64)

Then, the feasible region of DoF is shown in Fig. 4.(f), where the maximum total achievable DoF is \(M_t\).

Remark 1: In the case \(K_2 = 2\), we also can use the same way from (58) to (64) to obtain the maximum total achievable DoF based on its corresponding feasible region.

2) \(\text{The number of macro users } K_2 \text{ is odd and } K_2 > 3\)

Theorem 2: For the HetNet that consists of one macrocell and two picocells, where the macro BS serves \(K_2\) macro users and \(K_2\) is an odd number and \(K_2 > 3\), the maximum total achievable DoF for the basic grouping method aided IA scheme is,

\[
\begin{align*}
\text{DoF} & = \begin{cases}
N_r + \frac{(K_2 + 1) K_2 N_r}{K_2^2 - 5}, & \text{if } N_r \leq \frac{(K_2^2 - 5) M_t}{2K_2^2 + 2K_2 - 5} \quad \text{and} \quad N_r \geq \frac{K_t^2 + 2K_2 - 1}{K_2^2 - 5} \\
N_t + \frac{2N_t - N_r}{K_2 + 3} K_2, & \text{if } N_r \leq \frac{M_t(K_2 + 3) - 2K_2 N_t}{3} \quad \text{and} \quad N_r < \frac{K_t^2 + 2K_2 - 1}{K_2^2 - 5} \\
2N_t, & \text{if } N_t \leq \frac{M_t}{2} \quad \text{and} \quad \frac{N_t}{N_r} \leq \frac{1}{2}
\end{cases}
\end{align*}
\]

(65)

Proof of Theorem 2: For the basic grouping method aided IA scheme, we modify condition (31) as follows,

\[
(\frac{K_2 - 1}{2}) N_r - d_m \geq (\frac{K_2 - 1}{2} - 1) N_t,
\]

(66)

where (66) means if \((\frac{K_2 - 1}{2}) N_r\) exceeds \((\frac{K_2 - 1}{2}) N_t\), the basic grouping method aided IA scheme is not applicable. However, it is not logically correct that the additional antennas decrease or preclude the achievable gain of the IA scheme [10]. Similar as [10], at the pico BS, we can select \(N_t\) transmit antennas out of a total \(N_r\) physical antennas to be actively used to transmit signals. The condition (31) and (35) can be modified, respectively, as follows [10],

\[
(\frac{K_2 - 1}{2}) N_r - d_m \geq (\frac{K_2 - 1}{2} - 1) N_t',
\]

(67)

\[
N_t' \geq (\frac{K_2 - 1}{2} + 2)d_m + d_p,
\]

(68)

Combining (67) and (68) we have [15],

\[
(K_2 - 1) N_r \geq \frac{K_2^2 - 5}{2} d_m + (K_2 - 3)d_p.
\]

(69)
In order to guarantee the feasibility of the basic grouping method aided IA scheme, (69), (35), (22) and (26) must be satisfied, which are repeated here for clarity,

\[
\begin{align*}
(K_2 - 1)N_t & \geq \frac{K_2^2 - 5}{2}d_m + (K_2 - 3)d_p, \tag{71} \\
N_r - \frac{K_2 - 1}{2} & + 2d_m \geq d_p, \tag{70} \\
N_r - d_p & \geq d_p, \tag{70} \\
M_r - 2d_p - K_r d_m & \geq 0. \tag{70}
\end{align*}
\]

From (70), we can calculate the intersection points of \(\circ\) and \(\square\) at \((M_r - \frac{2K_2(K_2 - 1)N_t - K_2 M_r(K_2 - 3)}{2(3K_2 - 5)}, \frac{2(3K_2 - 5)}{K_2 - 2})\), \(\circ\) and \(\triangle\) at \((\frac{K_2^2 - 5}{4}N_r - \frac{K_2^2}{4} - 2(3K_2 - 5)N_t, \frac{K_2^2}{4} - 2(3K_2 - 5)N_t)\), \(\circ\) and \(\heartsuit\) at \((\frac{N_r - (K_2 - 1)N_t}{2}, \frac{K_2 - 1}{2} - 1)N_t\), \(\diamondsuit\) and \(\heartsuit\) at \((\frac{K_2 + 3)M_r - 2K_r N_t}{6}, \frac{2K_r - 3 - 2K_2 N_r}{2(3K_2 - 5)}\). Based on these five intersection points, there are six different cases to characterize the feasible region of DoF, which will be discussed below.

1) The intersection point of \(\circ\) and \(\square\) is at the right hand side of \(\heartsuit\), while the intersection point of \(\circ\) and \(\triangle\) is at the right hand side of \(\diamondsuit\), as shown in Fig. 5.(a), i.e., the condition below is satisfied,

\[
\begin{align*}
M_r - 2K_2(K_2 - 1)N_t - K_2 M_r(K_2 - 3) \geq 2(3K_2 - 5)N_t \\
N_r & \geq \frac{K_2^2}{2} + K_2 - 1.
\end{align*}
\]

In this case, the feasible region of DoF is shown in Fig. 5.(a), where the maximum total achievable DoF is \(N_r + \frac{K_2(K_2 + 1)N_t}{K_2^2 - 5}\), corresponding to the optimal intersection point of \(\circ\) and \(\heartsuit\), i.e., \((\frac{N_r}{2}, \frac{K_2(K_2 + 1)N_t}{K_2^2 - 5})\).

b) The intersection point of \(\circ\) and \(\triangle\) is at the left hand side of \(\heartsuit\), while the intersection point of \(\circ\) and \(\triangle\) is at the right hand side of \(\diamondsuit\), as shown in Fig. 5.(b), i.e., the condition below is satisfied,

\[
\begin{align*}
\frac{(K_2^2 - 5)N_r}{4} & - \frac{(K_2^2 + 2K_2 - 3)N_r}{4} \geq \frac{N_r}{2} \\
M_r & - \frac{2K_2(K_2 - 1)N_t - K_2 M_r(K_2 - 3)}{2(3K_2 - 5)} \geq \frac{N_r}{2}
\end{align*}
\]

In this case, the feasible region of DoF is shown in Fig. 5.(b), where the maximum total achievable DoF is \(N_r + \frac{2N_r - N_t}{K_2 + 3}K_2\), corresponding to the optimal intersection point of \(\circ\) and \(\heartsuit\), i.e., \((\frac{N_r}{2}, \frac{2N_r - N_t}{K_2 + 3})\).

c) The number of antennas for the pico BSs satisfies the following condition, as shown in Fig. 5.(c),

\[
N_r \leq \min\left(\frac{N_r}{2}, \frac{M_r}{2}\right) \iff \begin{cases} N_r \leq \frac{M_r}{2}, \\ N_r \leq \frac{1}{2}, \\ N_r \leq \frac{N_r}{2} \end{cases}
\]

In this case, the feasible region of DoF is shown in Fig. 5.(c), where the maximum total achievable DoF is \(2N_r\), corresponding to the optimal point \((N_r, 0)\), at which there are no data streams from the macro BS to the macro users.

d) The intersection point of \(\circ\) and \(\heartsuit\) is at the right hand side of \(\heartsuit\), while the intersection point of \(\circ\) and \(\heartsuit\) is at the left hand side of \(\diamondsuit\), as shown in Fig. 5.(d), i.e., the condition below is satisfied,

\[
\begin{align*}
\frac{(K_2^2 - 5)N_r}{4} & - \frac{(K_2^2 + 2K_2 - 3)N_r}{4} \geq \frac{N_r}{2} \\
M_r & - \frac{2K_2(K_2 - 1)N_t - K_2 M_r(K_2 - 3)}{2(3K_2 - 5)} \geq \frac{N_r}{2}
\end{align*}
\]

Then, the feasible region of DoF is shown in Fig. 5.(d), where the maximum total achievable DoF is \(M_r\).

e) The intersection point of \(\circ\) and \(\heartsuit\) is at the left hand side of \(\heartsuit\), while the intersection point of \(\circ\) and \(\heartsuit\) is at the
left hand side of (3), as shown in Fig. 5.(e), i.e., the condition below is satisfied,
\[
\frac{(K_2^2 - 5)N_t - (K_2^2 + 2K_2 - 3)N_r}{2(K_2 + 3)M_t - 2K_2 N_t} \leq \frac{N_r}{2}.
\]
Then, the feasible region of DoF is shown in Fig. 5.(e), where the maximum total achievable DoF is \(M_t\).

Remark 2: In the case \(K_2 = 1, 3\), we also can use the same way from (70) to (76) to calculate the maximum total achievable DoF based on its corresponding feasible region.

B. THE DOF OF THE GEVD BASED GROUPING METHOD AIDED IA SCHEME

1) THE NUMBER OF MACRO USERS \(K_2\) IS EVEN

**Theorem 3:** For the HetNet that consists of one macrocell and two picocells, where the macro BS serves \(K_2\) macro users and \(K_2\) is an even number, the maximum total achievable DoF for the GEVD based grouping method aided IA scheme is independent of \(K_2\) and can be expressed as follows,
\[
\text{DoF} = \min[M_t, 2N].
\]

**Proof of Theorem 3:** In order to guarantee the feasibility of the GEVD based grouping method aided IA scheme, the conditions of (43), (46), and (26) must be satisfied, which are repeated here for clarity,
\[
\begin{align*}
2N &\geq K_2 d_m + 2d_p, \quad (1) \\
N - d_p &\geq d_p, \quad (2) \\
M_t - 2d_p - K_2 d_m &\geq 0. \quad (3)
\end{align*}
\]
From (78), we calculate the intersection points of (1) and (2) at point \(\left(\frac{N}{2}, \frac{N}{K_2}\right)\), (2) and (3) at \(\left(\frac{M_t - N}{2}, \frac{M_t - N}{K_2}\right)\). The slope of both (1) and (3) are \(\frac{2}{K_2}\), so that there is no intersection point between (1) and (3). Based on these two intersection points, there are two different cases to characterize the feasible region of DoF, which will be discussed below.

a) \(K_2\) and the numbers of antennas satisfy the condition below, as shown in Fig. 6.(a),
\[
\frac{M_t - N}{K_2} \geq \frac{N}{K_2} \iff N \leq \frac{M_t}{2}.
\]
Then, the feasible region of DoF is shown in Fig. 6.(a), where the maximum total achievable DoF is \(2N\) according to (5), corresponding to the optimal intersection point of (1) and (2), i.e., \(\left(\frac{N}{2}, \frac{N}{K_2}\right)\).

b) \(K_2\) and the numbers of antennas satisfy the condition below, as shown in Fig. 6.(b),
\[
\frac{M_t - N}{K_2} \leq \frac{N}{K_2} \iff N \geq \frac{M_t}{2}.
\]
Then, the feasible region of DoF is shown in Fig. 6.(b), where the maximum total achievable DoF is \(M_t\).

In conclusion, the maximum total achievable DoF is,
\[
\text{DoF} = \begin{cases} 
2N, & \text{if } N \leq \frac{M_t}{2} \\
M_t, & \text{otherwise.}
\end{cases} \quad (81)
\]

2) THE NUMBER OF MACRO USERS \(K_2\) IS ODD

**Theorem 4:** For the HetNet that consists of one macrocell and two picocells, where the macro BS serves \(K_2\) macro users and \(K_2\) is an odd number, the maximum total achievable DoF for the GEVD based grouping method aided IA scheme is,
\[
\text{DoF} = \begin{cases} 
N + K_2 N \frac{K_2}{K_2 + 1}, & \text{if } N \geq \frac{2K_2 + 1}{K_2 + 1} \\
M_t, & \text{otherwise.}
\end{cases} \quad (82)
\]

**Proof of Theorem 4:** In order to guarantee the feasibility of the GEVD based grouping method aided IA scheme, the condition of (52), (46), and (26) must be satisfied, which are repeated here for clarity,
\[
\begin{align*}
2N - d_p &\geq K_2 d_m + 2d_p, \quad (1) \\
N - d_p &\geq d_p, \quad (2) \\
M_t - 2d_p - K_2 d_m &\geq 0. \quad (3)
\end{align*}
\]
From (83), we calculate the intersection points of (1) and (2) at point \(\left(\frac{N}{2}, \frac{N}{K_2 + 1}\right)\), (2) and (3) at \(\left(\frac{M_t - N}{2}, \frac{M_t - N}{K_2}\right)\). The slope of (1) is \(\frac{2}{K_2 + 1}\) and the slope of (3) is \(\frac{2}{K_2}\), so that there exists the intersection points between (1) and (3) at point \(\left(\frac{K_2 + 1}{2} \frac{M_t - N}{2}, K_2 N, 2N - M_t\right)\). Based on these three intersection points, there are two different cases to characterize the feasible region of DoF, which will be discussed below.
a) The intersection point of (1) and (3) is at the right hand side of (2), as shown in Fig. 7.(a), i.e., the condition below is satisfied,
\[(K_2 + 1)M_t - K_2N \geq \frac{N}{2} \iff \frac{M_t}{N} \geq \frac{2K_2 + 1}{K_2 + 1}.\] (84)

Then, the feasible region of DoF is shown in Fig. 7.(a), where the maximum total achievable DoF is \(N + \frac{K_2N}{K_2+1}\), corresponding to the optimal intersection point of (1) and (2), i.e., \((\frac{N}{2}, \frac{K_2N}{K_2+1})\).

b) The intersection point of (1) and (3) is at the left hand side of (2), as shown in Fig. 7.(b), i.e., the condition below is satisfied,
\[(K_2 + 1)M_t - K_2N \leq \frac{N}{2} \iff \frac{M_t}{N} \leq \frac{2K_2 + 1}{K_2 + 1}.\] (85)

Then, the feasible region of DoF is shown in Fig. 7.(b), where the maximum total achievable DoF is \(M_t\).

**Theorem 5:** For the HetNet that consists of one macrocell and two picocells, where the macro BS serves \(K_2\) macro users, for the same configuration, the maximum total achievable DoF by the GEVD based grouping method aided IA scheme is higher than or equal to that of the basic grouping method aided IA scheme.

**Proof of Theorem 5:** See Appendix. ■

**VI. SIMULATION RESULTS**

In this section, we demonstrate the performance of the basic grouping method aided IA scheme and the GEVD based grouping method aided IA scheme in terms of the maximum achievable DoF.

Fig. 8.(a) shows the maximum total achievable DoF with different \(N_t\) for the configuration \(\{K_2, M_t\} = \{5, 12\}\) and \(N_r = N_t\), in the context of the basic grouping method aided IA scheme and the GEVD based grouping method aided IA scheme. In Fig. 8.(a), for both methods, the maximum total achievable DoF increases with \(N_t\) increasing and the GEVD one obtains a higher DoF than that of the basic one in the condition \(N_t < 8\). Because in the case \(N_t < 8\), for the basic one, the maximum total achievable DoF is \(N_r + \frac{2N_t - N_r}{K_2 + 1}K_2\) as depicted in Theorem 1, while for the GEVD one, the maximum total achievable DoF is \(\min\{M_t, 2N_t\}\) as shown in Theorem 3. For \(N_t \geq 8\), the maximum total achievable DoF of both methods is consistent with \(M_t\) as shown in Fig. 8.(b). This is because, in the case \(N_t < 8\), for the basic one, the maximum total achievable DoF is \(N_r + \frac{2N_t - N_r}{K_2 + 1}K_2\) as depicted in Theorem 1, while for the GEVD one, the maximum total achievable DoF is \(\frac{M_t}{N_t} \geq \frac{2K_2 + 1}{K_2 + 1}\) as shown in Fig. 8.(b). This is because, in the case \(N_t < 8\), for the basic one, the maximum total achievable DoF is \(N_r + \frac{2N_t - N_r}{K_2 + 1}K_2\) as depicted in Theorem 1, while for the GEVD one, the maximum total achievable DoF is \(\min\{M_t, 2N_t\}\) as shown in Theorem 3. For \(N_t \geq 8\), the maximum total achievable DoF of both methods is consistent with \(M_t\) as shown in Fig. 8.(b). This is because, in the case \(N_t < 8\), for the basic one, the maximum total achievable DoF is \(N_r + \frac{2N_t - N_r}{K_2 + 1}K_2\) as depicted in Theorem 1, while for the GEVD one, the maximum total achievable DoF is \(\min\{M_t, 2N_t\}\) as shown in Theorem 3.
achievable DoF of the GEVD one is consistent with 14, which is higher than that of the basic one. Because in this case, for the GEVD one, the maximum total achievable DoF is \( \min[M_i, 2N] \), which is independent of \( K_2 \) as shown in Theorem 3, while for the basic one, the maximum total achievable DoF is \( N_r + \frac{2N_r - N_d}{K_2 + 2} K_2 \) as depicted in Theorem 1, which is lower than that of the GEVD one. As we can see, the GEVD based grouping method aided IA scheme can improve up to 33.3% against the basic grouping method aided IA scheme.

Fig. 9(b) shows the maximum total achievable DoF with different even number of \( K_2 \) for the configuration \( \{M_i, N_r, N_d\} = \{15, 7, 7\} \) in the context of the basic grouping method aided IA scheme and the GEVD based grouping method aided IA scheme. In Fig. 9(b), for both methods, the maximum total achievable DoF increases with \( K_2 \) increasing and the GEVD one is higher than that of the basic one. Because the maximum total achievable DoF is \( N + \frac{K_2 N}{K_2 + 1} \) for the GEVD one, as depicted in Theorem 4, while for the basic one, the maximum total achievable DoF is \( N_r + \frac{2N_r - N_d}{K_2 + 2} K_2 \) as depicted in Theorem 2, which is lower than that of the GEVD one. As we can see, the GEVD based grouping method aided IA scheme can improve up to 16.7% against the basic grouping method aided IA scheme.

**VII. CONCLUSION**

In this paper, we investigate the IA scheme for the scenario of the HetNet, which consists of one macrocell and two picocells. Specifically, we propose two IA schemes, i.e., the basic grouping method aided IA scheme and the GEVD based grouping method aided IA scheme, where the macro BS can serve arbitrary macro users and the number of data streams for the macro user and pico user can be different. Furthermore, we make an analysis of the feasible region and characterize the corresponding maximum total achievable DoF by different schemes. The GEVD based grouping method aided IA scheme can reduce the number of antennas required at the pico BS compared to the basic grouping method aided IA scheme. Moreover, the GEVD based grouping method aided IA scheme can provide a higher maximum total achievable DoF than or equal to that of the basic grouping method aided IA scheme. The simulation results are provided to confirm our conclusions.

**APPENDIX**

*Proof of Theorem 5.* We consider two scenarios, i.e., \( K_2 \) is an even number or odd number.

**A. \( K_2 \) IS AN EVEN NUMBER**

In the case \( N_r = N_t \) for the basic grouping method aided IA scheme, we modify (88) as follows,

\[
\begin{align*}
K_2 N &\geq \frac{K_2^2}{2} d_m + (K_2 - 2)d_p, \quad \text{(1)} \\
2N - 2d_m &\geq K_2 d_m + 2d_p, \quad \text{(2)} \\
N - d_p &\geq d_p, \quad \text{(3)} \\
M_i - 2d_p - K_2 d_m &\geq 0, \quad \text{(4)}
\end{align*}
\]

Comparing (86) with (87), conditions (1) and (4) in (86) are the same with conditions (1) and (4) in (87). Then, combining all the constraints in (86) and (87), we can see that the GEVD based grouping method aided IA scheme can provide a higher maximum total achievable DoF than or equal to that of the basic grouping method aided IA scheme. Moreover, there is an additional constraint, i.e., (2) in (86). Then, from condition (2) in (86) and condition (2) in (87), we can see that the GEVD based grouping method aided IA scheme provide a higher maximum total achievable DoF.

**B. \( K_2 \) IS AN ODD NUMBER**

In the case \( N_r = N_t \) for the basic grouping method aided IA scheme, we modify the condition (70) as follows,

\[
\begin{align*}
(K_2 - 1)N &\geq \frac{K_2^2 - 5}{2} d_m + (K_2 - 3)d_p, \quad \text{(1)} \\
2N - 3d_m &\geq K_2 d_m + 2d_p, \quad \text{(2)} \\
N - d_p &\geq d_p, \quad \text{(3)} \\
M_i - 2d_p - K_2 d_m &\geq 0, \quad \text{(4)}
\end{align*}
\]

Comparing (87) with (83), conditions (3) and (4) in (87) are the same with conditions (2) and (3) in (83). Then, from condition (2) in (87) and condition (1) in (83), we can see that the GEVD based grouping method aided IA scheme can provide a higher maximum total achievable DoF than or equal to that of the basic grouping.
method aided IA scheme. Moreover, there is an additional constraint, i.e., $\mathbf{1}$ in (87). Then, combining all the constraints in (87) and (83), we can see that the GEVD based grouping method aided IA scheme provide a higher maximum total achievable.

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