Research on Rebound Jamming Against Multi-Input Multi-Output Synthetic Aperture Radar

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ABSTRACT

Multi-Input Multi-Output Synthetic Aperture Radar (MIMO-SAR) emits orthogonal waveform, which reduces the probability of signal interception. Using multiple antennas to receive echoes, spatial filtering can be performed to avoid interference. MIMO-SAR waveform and structure characteristics determine that it has a certain anti-jamming ability with good prospect of application. For effective jamming MIMO-SAR, a theoretical model of rebound jamming against MIMO-SAR is established by taking the space-borne MIMO-SAR as object, the similarities and differences of rebound jamming against MIMO-SAR and conventional SAR are analyzed. Then, the effects of different signal systems on rebound jamming are discussed. Finally, the interference parameter selection for rebound jamming against the MIMO-SAR is analyzed. Simulation result verifies the validity and correctness of the proposed jamming method.

INDEX TERMS

Multi-input multi-output synthetic aperture radar (MIMO-SAR), deceptive jamming, rebound jamming, multi-frequency.

I. INTRODUCTION

MIMO-SAR system uses multiple antennas to radiate orthogonal signals to target region and multiple antennas to receive ground objects scattering echoes, which can obtain more ground information and form more equivalent phase centers than conventional Synthetic Aperture Radar (SAR), so that spatial sampling can replace time sampling and high-resolution imaging can be achieved by combining sub-band synthesis technology [1], [2]. MIMO-SAR emits orthogonal waveform, which reduces the probability of signal interception. Using multiple antennas to receive echoes, spatial filtering can be performed to avoid interference. MIMO-SAR structure and waveform characteristics determine that it has a certain anti-jamming ability with good prospect of application.

The research on SAR jamming mainly includes two aspects: suppression and deception. The former is easy to realize but requires high jamming power, and it is easy to be suppressed by Space-Time Adaptive Processing (STAP), Digital Beam Forming (DBF) and other technologies. The latter carries out interference by means of simulation or retransmission radar echo, which has the advantages of small power demand, good interference effect, simple equipment and strong operability, thus it is an effective means for jamming SAR. Rebound jamming is one type of scattered wave jamming. The intercepted SAR signal is retransmitted to an area and scattered by the scatterers of the ground to form the jamming signal. The scattered wave jamming aims to illuminate an area as large as possible by utilizing the wide beam antenna, and the realistic scene which covers the protected area is formed [3], [4]. The technology about active jamming space-borne SAR is discussed in [5], including noise jamming and deception jamming. Noise jamming includes blocking jamming and random pulse jamming and deception jamming includes transmitting responding and scattering jamming. The jamming effects on scatter wave jamming to SAR and analyzes jamming effects to SAR in the range direction is studied. Scatter wave jamming factor is put forward, and using the stationary phase principle, scatter wave jamming factor affected the imaging characteristics in the azimuth direction are analyzed in detail [6]. The principles of
scatter-wave jamming against spaceborne SAR is discussed, and the key problems of position control, direction guidance and transceiver isolation for engineering implementation are analyzed [7]. A scatter wave jamming to inverse synthetic aperture radar (ISAR) based on towed jammer is presented in [8]. Regarding the jammer and ISAR as the transmitter and the receiver of the equivalent bistatic ISAR, the model of the ISAR scatter wave jamming is discussed, and the jamming effects are analyzed in detail. To generate a verisimilar moving target based on a moving jammer, the scattered wave deception jamming method is proposed against displaced phase center antenna (DPCA) process. The jammer is set at unmanned aerial vehicle (UAV) and illuminates the calculated area [9]. At present, there is no research on the rebound jamming of MIMO-SAR and the analysis of jamming effect.

Meanwhile, the anti-rebound jamming methods to SAR are researched. Rosenberg studied the anti-jamming technology of SAR based on multi-channel and provided three algorithms to suppress fixed and non-fixed interference [10]–[14]. The first is an optimized STAP algorithm in the slow time domain, which can suppress direct wave interference and scattered wave interference to a certain extent. The second is the fast time domain STAP algorithm, which is much more effective than the first two algorithms in the presence of strong scattering wave interference, but this algorithm has some shortcomings in the processing of real data. References [15]–[17] used equivalent phase center cancellation and frequency agile methods to suppress MIMO-SAR deception jamming. Dual-path cancellation method to suppress rebound jamming is introduced in [18], which theoretically analyzes the working principle of dual-path cancellation and carries out simulation experiments on dual-path cancellation with simulation data. Sun etc. discussed cross spectrum measurement method of multi-channel SAR in the application of scattering wave interference suppression, where cross spectrum of covariance matrix with interference and noise was analyzed [19]. However, the proposed interference suppression methods are mainly carried out by means of cancellation. Although it has certain effect, it will cause the loss of ground feature information.

Based on the operation principle of MIMO-SAR system, this paper simulates and verifies the effectiveness of rebound jamming to MIMO-SAR, and further analyzes the influence of rebound jamming. In addition, the specific parameter requirements for accurate jamming are analyzed.

II. PRINCIPLE OF REBOUND JAMMING TO MIMO-SAR

Firstly, the conventional jamming mode is analyzed. Assuming that the jammer is stationary in the area of imaging scene, the jamming is implemented within the main beam width of the radar. The geometric relationship of spaceborne MIMO-SAR under jamming conditions is shown in Fig. 1. MIMO-SAR system receives echo in DBF mode in range direction. In imaging processing, the transmission path change caused by azimuth aperture should be considered. The azimuth coordinates of the point target P is \( x_p \). Then, at the moment \( t_m \), its distance from radar’s azimuth aperture is:

\[
\begin{align*}
R_p & = \sqrt{H_s^2 + Y_s^2 + \left( (k - K + \frac{1}{2} \right) d_{ant} + V_a t_m - x_p \right)^2} \\
& \approx R_p + \frac{\left( (k - K + \frac{1}{2}) d_{ant} + V_a t_m - x_p \right)^2}{2R_p} \\
& \approx \frac{R_p}{c} \left( (a - K + \frac{1}{2}) d_{ant} + V_a t_m - x_p \right)^2 + \frac{R_p}{c} \\
& + \frac{2R_p}{c} \left( (V_a t_m)^2 - \frac{2x_p V_a t_m}{c R_p} + \frac{(k + a - K - 1) d_{ant} V_a t_m}{c R_p} \right) \\
& = \tau_p (t_m) + \Delta \tau_{kap} (t_m)
\end{align*}
\]

where, \( \tau_p (t_m) \) is the signal transmission time delay of the target, when the azimuth center aperture emits and receives signals; \( \Delta \tau_{kap} (t_m) \) is the delay caused by the position difference between the transceiver aperture and the target. Equation (3) can be obtained:

\[
\begin{align*}
\tau_p (t_m) &= \frac{2R_p}{c} \left( (V_a t_m)^2 - \frac{2x_p V_a t_m}{c R_p} \right) \\
\Delta \tau_{kap} (t_m) &= \frac{(k + a - K - 1) d_{ant} V_a t_m}{c R_p}
\end{align*}
\]
According to equation (3), transmission delay of signal intercepted by jammer is:

\[
\tau_{ka}(t_m) = \frac{r_{kz}(t_m) + r_{aj}(t_m)}{c} = \frac{2R_i}{c} + \left(\frac{V_{atm}}{cr_i}\right)^2 - \frac{2x_iV_{atm}}{cr_i} + \frac{(k + a - K - 1) d_{ant} V_{atm}}{c r_i} \tag{4}
\]

Correspondingly, echo transmission time of the false point target is:

\[
\tau_{kj}(t_m) = \frac{r_{kj}(t_m) + r_{aj}(t_m)}{c} = \frac{2R_i}{c} + \left(\frac{V_{atm}}{cr_i}\right)^2 - \frac{2x_jV_{atm}}{cr_j} + \frac{(k + a - K - 1) d_{ant} V_{atm}}{c r_j} \tag{5}
\]

It can be seen from equation (5) that the jammer accurately simulates the MIMO-SAR echo at the false target, which also requires additional delay, doppler frequency shift and doppler rate of the signals intercepted by the jammer. Compared with the rebound jamming to conventional SAR, modulation function of rebound jamming to MIMO-SAR is more complex.

The rebound jammer is equipped on an airborne platform. After intercepting the radar signal, the jammer is processed by correlation and then radiates to the imaging area. The jamming signal is scattered by the ground targets and then received by the radar receiver. This section analyzes the influence on received echoes of MIMO-SAR with single static rebound jammer. The configuration of MIMO-SAR system under the condition of rebound jamming (double transmitter and multi-receiver) is shown in Fig. 2.

As shown in Fig. 2, MIMO-SAR system will transmit orthogonal signals from azimuth aperture, and all apertures receive echoes at the same time. The height of satellite platform is \(H_s\) with speed \(v_s\). The radar antenna array elements are uniformly distributed; the spacing between the adjacent array elements in azimuth is \(d_a\), and the spacing between the pitching array elements is \(d_r\). For a point target \(P\) in the scene, its coordinates can be expressed as \((x_p, y_p, 0)\), its distance from the jammer can be expressed as \(R_{jp}\), and its distance from MIMO-SAR antenna array element is \(R_{arp}\), the subscript \(a\) represents the azimuth aperture serial number, and the subscript \(r\) represents the range aperture serial number. The distance between jammer and radar transmitting array element is \(R_{aj}\).

The transmitting signal of MIMO-SAR system is \(q_k(\tau)\), and the baseband echo received by aperture \((a, r)\) is:

\[
r_{ar}(\tau, t_m) = \sum_{k=1}^{K} \sum_{p=1}^{P} rect\left(\frac{\tau - (R_{kp} + R_{arp})}{T_p} \right) q_k(\tau - \frac{R_{kp} + R_{arp}}{c}) \cdot \exp\left(-j2\pi f_0 \frac{R_{kp} + R_{arp}}{c}\right) \tag{6}
\]

where, \(R_{kp}\) represents the distance from transmit element \(k\) to scattering point \(P\). The jamming signal received by aperture \((a, r)\) is:

\[
J_{ar}(\tau, t_m) = \sum_{k=1}^{K} \sum_{p=1}^{P} \sigma_{arp} rect\left(\frac{\tau - (R_{kp} + R_{arp})}{T_p} \right) q_k(\tau - \frac{R_{kp} + R_{arp} + R_{arp}}{c}) \cdot \exp\left(-j2\pi f_0 \frac{R_{kp} + R_{arp} + R_{arp}}{c}\right) \tag{7}
\]

Therefore, the total echo received by the aperture \((a, r)\) is the summation of target echo and jamming signal:

\[
S_{ar}(\tau, t_m) = r_{ar}(\tau, t_m) + J_{ar}(\tau, t_m) = \sum_{k=1}^{K} \sum_{p=1}^{P} rect\left(\frac{\tau - (R_{kp} + R_{arp})}{T_p} \right) q_k(\tau - \frac{R_{kp} + R_{arp}}{c}) \times \exp\left(-j2\pi f_0 \frac{R_{kp} + R_{arp}}{c}\right) + \sum_{k=1}^{K} \sum_{p=1}^{P} \sigma_{arp} rect\left(\frac{\tau - (R_{kp} + R_{arp} + R_{arp})}{T_p} \right) q_k(\tau - \frac{R_{kp} + R_{arp}}{c}) \times \exp\left(-j2\pi f_0 \frac{R_{kp} + R_{arp} + R_{arp}}{c}\right) \tag{8}
\]
It can be known from the geometric relationship shown in Fig. 2:

\[
\begin{align*}
R_{kp} &= \sqrt{(k - 1) d_a + v_a l_m - x_p}^2 + y_p^2 + H_s^2 \\
&\approx R_p + \frac{(k - 1) d_a + v_a l_m - x_p}{2} \\
R_{arp} &= \sqrt{(a - 1) d_a + v_a l_m - x_p}^2 + y_p^2 + (r - 1) d_r - H_s^2 \\
&\approx R_p + \frac{(a - 1) d_a + v_a l_m - x_p}{2} \\
&\quad + (r - 1) d_r - 2H_s (r - 1) d_r \\
R_{kJ} &= \sqrt{(k - 1) d_a + v_a l_m - x_j}^2 + y_j^2 + (H_s - h_j)^2 \\
&\approx R_j + \frac{(k - 1) d_a + v_a l_m - x_j}{2} \\
R_{jp} &= \sqrt{(x_j - x_p)^2 + (y_j - y_p)^2 + H_j^2} = \text{constant}
\end{align*}
\]

where, \( R_p = \sqrt{y_p^2 + H_s^2} \) is the slant distance from the target point to the radar platform, and \( R_j = \sqrt{y_j^2 + (H_s - h_j)^2} \) is the slant distance from the jammer to the radar platform, and \( h_j \) is the platform height of rebound jammer.

A. JAMMING EFFECT IN RANGE

It can be seen from the above analysis that, compared with the signal echo transmitted at the same time, the main peak of the jamming echo appears later. This is because the propagation path of the jamming echo is larger than that of the signal echo, so the “false” distance produced by the jamming echo in the range compression imaging will be larger than the actual distance from the SAR to the target, and its distance offset will be

\[
\Delta R = (R_j + R_{jp} - R_p)/2
\]

where, \( R_j, R_{jp} \) and \( R_p \) are the slant distance from jammer to platform, jammer to target and target to platform.

In addition, the total variation of the \( R_J(l_m) \) curve of the distance function is called the range migration. Considering the length of a synthetic aperture, the possible maximum range migration is about

\[
\Delta R' = \frac{(a - 1) L^2}{8R_0}
\]

where, \( L \) is the synthetic aperture length and \( a \) is the interference factor of rebound jamming.

\[
a = \frac{\cos \theta_j}{(1 - h/H) \cos \theta_s}
\]

where, \( \theta_j \) and \( \theta_s \) are the angles between the slant range \( R_0 \) and \( R_j \) with the zero Doppler plane; \( h \) and \( H \) are the heights of the jammer and the radar platform respectively. Generally, jammer is located near the area of SAR beam irradiation, and the change of jammer height is the main factor affecting the change.

In general, \( \Delta R' \ll \Delta R \), the effect of the range migration is less than that of the “false” distance produced by the jamming echo, which can be ignored basically.

B. JAMMING EFFECT IN AZIMUTH

After the jamming echo is processed by azimuth pulse compression, the false target will produce an azimuth offset in the azimuth, and its value is

\[
\Delta \alpha = \frac{R_0 (\sin \theta_j - \sin \theta_s)}{2}
\]

It can be seen from the above formula that the relative position of the false target and the real target depends on the positive and negative of \( \Delta \alpha \) and takes the SAR flight direction as the positive reference direction, if \( \Delta \alpha > 0 \), the false target is in the “front” of the real target and vice versa.

At the same time, due to the influence of additional Doppler, the azimuth direction of the false target will be widened by

\[
|\delta \alpha| = \left| \frac{a - 1}{2} \cdot \frac{L}{2} \right|
\]

It can be seen that the absolute value of the azimuth broadening depends on the size of the rebound jamming factor \( a \), the larger the absolute value, the worse the azimuth compression effect, or even defocus. At the edge of the Doppler frequency band, if the Doppler modulation frequency mismatch can be ignored, the phase error caused by the Doppler modulation frequency mismatch is limited to:

\[
|\delta \phi_D| = \pi T_L^2 |\delta f_r| / 4 < \pi / 4
\]

where, \( T_L \) is synthetic aperture time. It can be seen from the change of Doppler modulation frequency that:

\[
a < 1 + \frac{\lambda}{R_0 \theta_H^2}
\]

where, \( \theta_H \) is the horizontal beam width. It can be seen from the formula of azimuth broadening that:

\[
|\delta \alpha| < \frac{\lambda}{2 \theta_H} = \frac{D}{2}
\]

where, \( D \) is the length of antenna azimuth aperture. It can be seen from the above formula that when the absolute value of the target azimuth broadening does not exceed the azimuth resolution of the SAR system, there will be no defocusing phenomenon in the azimuth, which is consistent with the actual situation. Therefore, according to the criterion of focusing depth [6], the jamming conditions needed by the rebound jamming method to generate false image deception jamming and defocused image suppression jamming can be obtained: when the position of the jammer meets equation (16), the rebound jamming to SAR can basically be regarded as false image deception jamming. For spaceborne SAR, equation (16) is generally satisfied, so its jamming effect is generally false image deception jamming.
TABLE 1. System parameters of multi-carrier FREQUENCY MIMO-SAR.

| Parameters                        | Symbol | Values  |
|-----------------------------------|--------|---------|
| Satellite altitude                | $H_s$  | 514.8 km|
| Transmitting signal carrier freq. | $f_c$  | 9.65 GHz|
| Transmitting signal bandwidth     | $B_t$  | 30 MHz  |
| Transmitting signal time width    | $t_t$  | 5 us    |
| Antenna elevation height          | $h_{ae}$ | 0.886 m |
| Subaperture length in azimuth     | $d_{ae}$ | 4 m     |
| Antenna aperture number in azimuth| $N_{ae}$| 3       |
| Doppler bandwidth                 | $B_d$  | 1774 Hz |
| Jamming to signal ratio           | JSR    | -10 dB  |
| Rebound jammer height             | $h_J$  | 1 km    |
| False image scene                 |        | Island, Pyramid |

III. SIMULATION OF REBOUND JAMMING TO MIMO-SAR

The rebound jamming method is verified by simulation data of three channels multi-carrier frequency MIMO-SAR. The system parameters used in simulation are shown in TABLE 1.

The jammer is considered to be equipped on the tethered balloon platform with fixed cable triangle configuration, and its height is 1km. The platform will vibrate under the action of wind force, which is characterized by large initial and fast stabilization. In the case of level four wind, the variation range of vibration is generally in meters. Through the analysis of jamming effect, it can be seen that for the rebound jamming to spaceborne MIMO-SAR, the influencing factor mainly depends on jamming factor, which is affected by the platform height. Compared with the spaceborne platform, the height change of meter level has little influence on jamming factor, which will make the jamming images slightly defocused in azimuth.

Under the ideal condition that the system parameters of MIMO-SAR are known and the signal characteristics can be accurately measured, the jamming simulation of multi-carrier frequency spaceborne MIMO-SAR system is carried out by the rebound jamming. Fig. 3 (a) and Fig. 3 (b) are false island and small pyramid image templates respectively. When JSR is $-10$ dB, the jamming effect of multi-carrier frequency spaceborne MIMO-SAR imaging is shown in Fig. 4 and Fig. 5, which shows that under the ideal condition of accurate measurement of MIMO-SAR system information and signal parameters, it is feasible to implement jamming on MIMO-SAR. Due to the different time delay, the jamming image shows leading or lagging in the range, at the same time, due to different phase modulation, there is a bending phenomenon in azimuth.

The quantitative evaluation of rebound jamming effect is carried out by mean square error, mean absolute error and structural similarity criteria, the results are showed in TABLE 2.

The evaluation results based on image quality show that the MIMO-SAR image quality becomes worse when there is rebound jamming, and the jamming is effective.

![MIMO-SAR imaging of coastal area without rebound jamming.](image1)

![MIMO-SAR imaging of coastal area with rebound jamming.](image2)

![False image template.](image3)

![Rebound jamming effect on coastal area (jammer located at the scene center, $H=1$ km).](image4)
The following is analysis of distance history $R_{\text{tag}}$ and $R_{\text{jam}}$ corresponding to target and jamming signal under different receiving array elements, so as to obtain the difference between sub-echoes.

$$
R_{\text{tag}} = R_{kp} + R_{arp} = R_p + \left( (k-1)d_a + v_a t_m - x_p \right)^2 / 2R_p + \frac{(a-1)d_a + v_a t_m - x_p}{2R_p} + \frac{(r-1)d_r)^2 - 2H_s (r-1)d_r}{2R_p}
$$

$$
R_{\text{jam}} = R_{kj} + R_{jp} + R_{arp} = R_j + \frac{(k-1)d_a + v_a t_m - x_j}{2R_j} + \text{constant}
$$

(19)

$$
\Delta R_{\text{tag}} = R_{\text{tag}} (k+1) - R_{\text{tag}} (k)
= \frac{(k \cdot d_a + v_a t_m - x_p)^2}{2R_p} - \frac{((k-1)d_a + v_a t_m - x_p)^2}{2R_p}
= \frac{d_a ((2k-1)d_a + 2v_a t_m - 2x_p)}{2R_p}
$$

(21)

$$
\Delta R_{\text{jam}} = R_{\text{jam}} (k+1) - R_{\text{jam}} (k)
= \frac{(k \cdot d_a + v_a t_m - x_j)^2}{2R_j} - \frac{((k-1)d_a + v_a t_m - x_j)^2}{2R_j}
= \frac{d_a ((2k-1)d_a + 2v_a t_m - 2x_j)}{2R_j}
$$

(22)

IV. INFLUENCE OF MIMO-SAR TRANSMITTING SIGNAL

A. ORTHOGONAL CODING SIGNAL

MIMO-SAR has the same carrier frequency transmitted from azimuth aperture, and the signal waveform is orthogonal. After the echo is received by each array element, the sub-echo separation and processing are completed by a matched filter bank, the sub-echo expression of aperture $(a, r)$ is:

$$
S_{ar,k}(\tau, t_m) = \sum_{p=1}^{P} \sigma_p \sin c \left( \tau - \frac{R_{kp} + R_{arp}}{c} \right) \exp \left( -j2\pi f_0 \frac{R_{kp} + R_{arp}}{c} \right) + \sum_{p=1}^{P} \sigma_p \sin c \left( \tau - \frac{R_{kj} + R_{jp} + R_{arp}}{c} \right) \times \exp \left( -j2\pi f_0 \frac{R_{kj} + R_{jp} + R_{arp}}{c} \right)
$$

(18)

where, $S_{ar,k}(\tau, t_m)$ represents the sub-echo generated by transmitting signal of array element $k$. From equation (18), it can be seen that the difference of sub-echoes after signal separation is mainly caused by the difference in transmission path.

(b) MIMO-SAR imaging of pyramid area with rebound jamming.

FIGURE 5. Rebound jamming effect on pyramid area (jammer located at the scene center, $H=1km$).
For different pitch receiving elements, equation (25) and (26) can be expressed as:

\[
\Delta R_{\text{tag},r} = R_{\text{tag}} (r+1) - R_{\text{tag}} (r) = \frac{2R_p (r + 1)}{2R_p} - \frac{2R_p (r)}{2R_p}
\]

\[
= \frac{(2r+1)d_x^2 - 2d_xH_s}{2R_p}
\]

\[
\Delta R_{\text{jam},r} = \frac{(2r+1)d_x^2 - 2d_xH_s}{2R_p}
\]

(25)

It can be known: (1) For the orthogonal coding MIMO-SAR system with rebound jamming, after the echo is separated, due to the different transmitting elements and the same receiving elements, the distance history difference of each sub-echo is related to the distances from the transmitting array element to the target or jammer. (2) For orthogonal coding MIMO-SAR system with rebound jamming, after the echo is separated, the jamming distance difference of sub-echoes received by different array elements is caused by the different receiving array elements. In particular, for receiving elements in same azimuth but different pitch, the distance history of target and jamming signal is same.

**B. MULTI-CARRIER FREQUENCY SIGNAL**

MIMO-SAR’s azimuth elements transmit linear frequency modulation (LFM) signals with different carrier frequencies. After receiving the echoes, each array element completes sub-echo separation through related demodulation and low-pass filtering. The sub-echo of element (\(a, r\)) is:

\[
S_{\text{ar},k} (\tau, t_m) = \sum_{p=1}^{P} a_p \exp \left\{ j\pi K_p \left( \tau - \frac{R_{kp} + R_{\text{arp}}}{c} \right)^2 \right\} \times \exp \left( -j2\pi f_k \frac{R_{kp} + R_{\text{arp}}}{c} \right) + \sum_{p=1}^{P} a_p \exp \left\{ j\pi K_p \left( \tau - \frac{R_{kp} + R_{\text{arp}}}{c} \right)^2 \right\} \times \exp \left( -j2\pi f_k \frac{R_{kp} + R_{\text{arp}}}{c} \right)
\]

(27)

After echo separation, the difference of sub-echo is not only determined by distance history, but also related to signal carrier frequency. The difference of the distance history between the echoes is the same as the orthogonal coding system, where the difference is the phase change caused by different carrier frequency.

\[
\theta_{\text{tag}} = \frac{2\pi R_{kp} + R_{\text{arp}}}{\lambda_k} = \frac{2\pi}{\lambda_k} \left[ \frac{(k-1)d_a + v_a t_m - x_p}{R_p} \right] + \frac{(a-1)d_a + v_a t_m - x_p}{2R_p} + \frac{(r-1)d_r}{2R_p}
\]

(28)

\[
\theta_{\text{jam}} = \frac{2\pi}{\lambda_k} \left[ \frac{R_{kp} + R_{\text{arp}}}{2R_p} \right] = \frac{2\pi}{\lambda_k} \left[ \frac{R_{kp} + R_{\text{arp}}}{2R_p} \right] + \frac{(a-1)d_a + v_a t_m - x_p}{2R_p} + \frac{(r-1)d_r}{2R_p}
\]

(29)

1) DIFFERENT TRANSMITTING ELEMENTS WITH THE SAME RECEIVING ELEMENT

Considering the case of adjacent array elements, the receiving aperture (\(a, r\)) is fixed but the transmitting element \(k\) is changed. Equation (30) and (31) can be obtained:

\[
\Delta \theta_{\text{tag}} = \theta_{\text{tag}} (k+1) - \theta_{\text{tag}} (k) = \frac{2\pi}{\lambda_{k+1}} \left[ \frac{(k-1)d_a + v_a t_m - x_p}{2R_p} \right] + \frac{(a-1)d_a + v_a t_m - x_p}{2R_p} + \frac{(r-1)d_r}{2R_p}
\]

(30)

\[
\Delta \theta_{\text{jam}} = \theta_{\text{jam}} (k+1) - \theta_{\text{jam}} (k) = \frac{2\pi}{\lambda_{k+1}} \left[ \frac{(k-1)d_a + v_a t_m - x_p}{2R_p} \right] + \frac{(a-1)d_a + v_a t_m - x_p}{2R_p} + \frac{(r-1)d_r}{2R_p}
\]

(31)

2) DIFFERENT RECEIVING ELEMENTS WITH THE SAME TRANSMITTING ELEMENT

When transmitting aperture is fixed, equation (32) and (33) can be obtained for the different receiving element in azimuth.

\[
\Delta \theta_{\text{tag},a} = \theta_{\text{tag}} (a+1) - \theta_{\text{tag}} (a) = \frac{2\pi}{\lambda_k} \left[ \frac{(a-1)d_a + v_a t_m - x_p}{2R_p} \right] + \frac{(a-1)d_a + v_a t_m - x_p}{2R_p}
\]

(32)

\[
\Delta \theta_{\text{jam},a} = \theta_{\text{jam}} (a+1) - \theta_{\text{jam}} (a) = \frac{2\pi}{\lambda_k} \left[ \frac{(a-1)d_a + v_a t_m - x_p}{2R_p} \right] + \frac{(a-1)d_a + v_a t_m - x_p}{2R_p}
\]

(33)

For different pitch receiving elements, it can be obtained:

\[
\Delta \theta_{\text{tag},r} = \theta_{\text{tag}} (r+1) - \theta_{\text{tag}} (r) = \frac{2\pi}{\lambda_k} \left[ \frac{(r-1)d_r^2 - 2r \cdot d_rH_s}{2R_p} \right] + \frac{((r-1)d_r)^2 - 2(r-1)d_rH_s}{2R_p}
\]

(34)
words, if the main lobe broadens to the same extent, the signal bandwidth error. At the same time, when the time width is large, the signal bandwidth allows greater absolute chirp rate accuracy increases with the increase of signal bandwidth. The value of the additional doppler shift is smaller when the position of false target and jammer is behind, and vice versa. In addition, the variation of peak position is inversely proportional to signal bandwidth. The larger the band width, the smaller the peak position variation under the same error.

For multi-carrier frequency MIMO-SAR system, its structural characteristics and signal characteristics determine its particularity. The following analysis is made from the two aspects: additional doppler frequency shift and carrier frequency error.

### A. INFLUENCE OF ADDITIONAL DOPPLER FREQUENCY SHIFT

As can be known from TABLE 3, when rebound jamming is implemented on MIMO-SAR, additional doppler frequency shift should be added to the jammer’s transmission function, and its expression is:

\[
\Delta f_{d,j} = \frac{V_a}{\lambda} \left( k + a - K - 1 \right) \frac{d_{ant}}{R_i} \frac{\Delta R_{ji}}{R_i}
\]

\[
\approx \frac{V_a}{\lambda} \left( k + a - K - 1 \right) \frac{2H_s \left( y_j^2 - y_i^2 \right)}{\left( 2H_s^2 + y_j^2 \right) \left( 2H_s^2 + y_i^2 \right)}
\]

(36)

where, \( k \) represents the serial number of transmitting array element; \( a \) represents the serial number of the receiving array element; \( K \) is the total number of transmitting array elements; \( d_{ant} \) is the spacing of array elements (the transmitting array is a uniform linear array); \( y_j \) is the distance coordinate of the jammer; \( y_i \) is the distance coordinates of the false target; \( H_s \) represents the height of radar platform.

The relation between the additional doppler frequency shift error and the change of transmitting array element is shown in Fig.6. The selected parameters are as follows: the height of the spaceborne radar platform is 514.8km; the speed is 7.6km/s; the carrier frequency of the transmitting signal is 9.65 G Hz; the number of antenna array elements is set as 10; the spacing of array elements is 2m; the radar incidence angle is 40 degrees; the jammer is deployed in the center of the imaging scene. In Fig. 6 (a), the additional doppler frequency shift error caused by different receiving array elements is compared. Fig. 6 (b) shows the additional doppler shift when the false target position changes.

It can also be known: (1) For the multi-carrier frequency MIMO-SAR system with rebound jamming, after the echoes are separated, due to the difference of transmitting array elements and carrier frequencies and the same receiving array element, the distance history and azimuth phase difference of each sub-echo is related to the different distances and carrier frequencies from the transmitting array elements to the target or jammer. (2) For multi-carrier frequency MIMO-SAR system with the same transmitting elements, the distance history difference of sub-echoes is caused by the different receiving array elements.

### VI. REQUIREMENTS FOR PARAMETER ACCURACY OF REBOUND JAMMING

According to the above analysis, MIMO-SAR system structure determines the implementation of rebound jamming is different from conventional SAR. At the same time, jamming MIMO-SAR requires corresponding guidance parameters, and there must be errors in the parameters estimation of the reconnaissance signals, so it is necessary to analyze the influence of the guidance parameter errors, so as to provide a reference for the implementation of rebound jamming. The related modulation terms of the rebound jammer transmission function for MIMO-SAR and conventional SAR are shown in TABLE 3.

Similar with conventional SAR deception jamming, measuring accuracy of chirp rate will affect peak value of output signal envelope and main lobe width, and center frequency error will lead to the change of the peak position: (1) In the condition of peak reduction with same value, the requirement of chirp rate accuracy increases with the increase of signal bandwidth; large signal bandwidth allows greater absolute bandwidth error. At the same time, when the time width is fixed and the chirp rate error is the same, the main lobe will broaden more with the increase of signal bandwidth. In other words, if the main lobe broadens to the same extent, the signal with larger bandwidth requires higher accuracy of chirp rate. (2) The change of theoretical peak position is proportional to the error of center frequency. When the measured center frequency is lower than the real value, the peak position lags behind, and vice versa. In addition, the variation of peak position is inversely proportional to signal bandwidth. The larger the band width, the smaller the peak position variation under the same error.

| Comparison items | MIMO-SAR | Conventional SAR |
|------------------|----------|------------------|
| Delay item       | \( \Delta \tau_s = \frac{2AR_{\text{ant}}}{c} \approx \frac{y_j^2 - y_i^2}{c \cdot H_s} \) | \( \Delta \tau_s = \frac{2AR_{\text{ant}}}{c} \approx \frac{y_j^2 - y_i^2}{c \cdot H_s} \) |
| Doppler shift    | \( \Delta \nu_{\text{d,j}} = \frac{2\nu}{\lambda} \left[ y_j + \frac{y_j + y_i}{R_i + R_i} \right] \) | \( \Delta \nu_{\text{d,j}} = \frac{2\nu}{\lambda} \left[ y_j + \frac{y_j + y_i}{R_i + R_i} \right] \) |
| Chirp rate       | \( \Delta K_{\text{ch}} = \frac{V_a^2}{\lambda} \left( y_j - y_i \right) \) | \( \Delta K_{\text{ch}} = \frac{V_a^2}{\lambda} \left( y_j - y_i \right) \) |
| Additional item with a longer delay | \( \Delta \tau_{\text{d,j}} = \frac{V_a^2}{\lambda} \left( y_j - y_i \right) \) | \( \Delta \tau_{\text{d,j}} = \frac{V_a^2}{\lambda} \left( y_j - y_i \right) \) |
the additional doppler frequency shift error has little impact
on the final jamming result for spaceborne MIMO-SAR.

B. INFLUENCE OF SUBCARRIER FREQUENCY ERROR

For the multi-carrier frequency MIMO-SAR system, the
azimuth elements simultaneously transmit the stepped LFM
signals. When rebound jamming is carried out, the transmis-
sion function of the jammer is rewritten as follows:

\[
H_k(f_r, t_m) = \sum_{p=1}^{P} \sigma_p \exp \left\{ -j 2 \pi \left( f_k t_m + \frac{\Delta f_{dpi}}{f_k} t_m + \frac{\Delta K_{api}}{f_k} t_m^2 \right) \right\}
\]

(37)

where,

\[
\Delta \tau_{pi} = \frac{2 \Delta R_{pi}}{c} \approx \frac{\gamma_p^2 - \gamma_i^2}{c \cdot H_s}
\]

\[
\Delta f_{dpi} = \frac{2 V_a}{\lambda_k} \left[ \frac{x_p R_i - x_i R_p}{R_p R_i} - (k + a - K - 1) d_{ant} \frac{\Delta R_{pi}}{2 R_p R_i} \right]
\]

\[
\Delta K_{api} = - \frac{V_a^2 \Delta R_{pi}}{\lambda_k R_p R_i}
\]

where, \( \lambda_k \) is the wavelength of transmitted signal of the \( k^{th} \) ekelelement.
If the reconnaissance equipment cannot accurately measure and estimate the signal parameters, and the jammer still modulates the intercepted signal according to the measured radar center frequency, then the modulation term changes to:

$$\frac{d f_{\text{mod}}}{\Delta f_{\text{mod}}} = 2V \left[ \frac{R_1 - R_2}{R_1 R_2} - (k + a - K - 1) \frac{\Delta R_{\text{pp}}}{R_1 R_2} \right]$$

$$\frac{d K_{\text{mod}}}{\Delta K_{\text{mod}}} = - \frac{V^2 \Delta R_{\text{pp}}}{R_1 R_2} \left( \frac{\lambda_k - \lambda_0}{\lambda_k \lambda_0} \right)$$

(38)

where, $\lambda_0$ is the wavelength of center carry frequency.

It can be seen that the measurement error of subcarrier frequency will have an impact on doppler frequency shift and chirp rate term, and the magnitude of the change is determined by $(\lambda_k - \lambda_0)/\lambda_k \lambda_0$, Fig. 7 shows the effect of measurement error of subcarrier frequency on false target imaging.

It can be obtained from Fig. 7: When reconnaissance system can not accurately measure each carrier frequency, jammer still modulates intercepted signal to radar signal center frequency, the coordinate offset of false target will occur, which is caused by doppler frequency shift error; the azimuth main lobe will be broaden, which is the result of incomplete chirp rate matching.

VI. CONCLUSION

The rebound jamming is applied in MIMO-SAR deception jamming in this paper, the orthogonal coding and multi-carrier frequency MIMO-SAR rebound jamming are analyzed, where the mathematical model of rebound jamming is generated, and the differences between MIMO-SAR and conventional SAR rebound jamming are compared. The following conclusions can be obtained: (1) Compare with the single channel SAR, jamming MIMO-SAR system by rebound method is more difficulty, but the rebound jamming is still an effective method. (2) The difference in the position of the transceiver elements will lead to the need for additional doppler frequency shift in the modulation function of the rebound jammer, but due to the limitation of the antenna size, the effect is small. (3) For the multi-carrier MIMO-SAR system, measurement error of sub-carrier frequency will lead to the position shift of the false target in range, and the main lobe broaden in azimuth.

This paper has carried out a preliminary analysis on the rebound jamming against MIMO-SAR, and will carry out further research on the rebound jamming enhancement technology, influence analysis of clutter suppression interference and signal polarization in the future work.

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