Mathematical modeling of free convection problems in a gravity field in OpenFOAM

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Abstract. A mathematical model of natural convection in the gravity field, using Boussinesq approximation, has been presented. This model contains the continuity equation, where density variations are ignored, the Navier-Stokes equation and the equation for heat flow. Rayleigh-Benard-convection in a rectangular box, with different types of border conditions, has been investigated. The equations solved numerically by an original solver with the use of object-oriented programming language OpenFOAM. Solver is based on PISO (Pressure-Implicit with Splitting of Operators) algorithm and finite volume method.

The Prandtl number, Grashof number and Rayleigh number has been examined. Rayleigh-Benard-convection between parallel planes of different temperatures, steady convection in horizontal fluid layer and natural convection flow in a square box, enclosed by non-isothermal wall has been used for solver verification. Results have been visualized with the use of open source application ParaView.

1. Introduction
Experiments of Benard (1900) initiated the study of convection. He observed the appearance of hexagonal honeycomb structures (Benard cells). Later Relay (1916) theoretically investigated the occurrence of convection in a horizontal fluid layer for two free boundaries. He established a transition from the thermal conductivity to the convection (the critical Rayleigh number). Further development was severely limited due to significant computational difficulties.

Lately, the study of convection processes developed very rapidly. Advances in the study of convection are associated with the progress of numerical calculations and with surviving interest in convection.

2. Mathematical model
A mathematical model of natural convection in the gravity field is determined by the continuity equation, where density variations are ignored, the Navier-Stokes equation and the equation for heat flow. In the Boussinesq approximation, they have the following form [1]

\begin{equation}
\begin{aligned}
& \text{div} \vec{U} = 0, \\
& \rho \left( \frac{\partial \vec{U}}{\partial t} + (\vec{U} \nabla) \vec{U} \right) - \nu \Delta \vec{U} = -\nabla p + g \left[ \beta (T - T_0) \right], \\
& \frac{\partial T}{\partial t} + \vec{U} \nabla T - \alpha \Delta T = 0,
\end{aligned}
\end{equation}
where $\beta$ — coefficient of thermal expansion of fluid, $T_0$ — temperature, $\alpha$ — the thermal diffusivity, $\nu$ — effective kinematic viscosity, $\rho$ — the density of the fluid.

First boundary conditions:

$$
\begin{align*}
T(0, y, t) &= T_H, \quad T(l, y, t) = T_c, \quad 0 \leq y \leq h, \quad t > 0, \\
\frac{\partial T}{\partial y}|_{y=0} &= \frac{\partial T}{\partial y}|_{y=h} = 0, \quad 0 \leq x \leq l, \quad t > 0, \\
U|_{\partial \Omega} &= 0, \\
p &= f(T|_{\partial \Omega}, \bar{U}|_{\partial \Omega}).
\end{align*}
$$

(2)

Second boundary conditions:

$$
\begin{align*}
T(x, 0, t) &= T_c, \quad T(x, h, t) = T_H, \quad 0 \leq x \leq l, \quad t > 0, \\
\frac{\partial T}{\partial x}|_{x=0} &= \frac{\partial T}{\partial x}|_{x=l} = 0, \quad 0 \leq y \leq h, \quad t > 0, \\
U|_{\partial \Omega} &= 0, \\
p &= f(T|_{\partial \Omega}, \bar{U}|_{\partial \Omega}).
\end{align*}
$$

(3)

Third boundary conditions:

$$
\begin{align*}
T(x, 0, t) &= T_H, \quad T(x, h, t) = T_c, \quad 0 \leq x \leq l, \quad t > 0, \\
T(0, y, t) &= T_c + (T_H - T_c)(h - y), \quad 0 \leq y \leq h, \quad t > 0, \\
T(l, y, t) &= T_c, \quad 0.05 \leq y \leq h, \quad t > 0, \\
\bar{U}|_{\partial \Omega} &= 0, \\
p &= f(T|_{\partial \Omega}, \bar{U}|_{\partial \Omega}).
\end{align*}
$$

(4)

Where $f$ - Navier-Stokes equation solved with respect to pressure.

Initial condition:

$$
\begin{align*}
T(x, y, t)|_{t=0} &= T_0, \quad 0 \leq x \leq l, \quad 0 \leq y \leq h, \\
\bar{U}(x, y, t)|_{t=0} &= 0, \\
p(x, y, t)|_{t=0} &= \rho g (h - y).
\end{align*}
$$

(5)

These equations solved numerically by an original solver with the use of object-oriented programming language OpenFOAM. To solve Navier-Stokes equation, PISO(Pressure-Implicit with Splitting of Operators) algorithm has been used [2]. It splits the operators into an implicit predictor and multiple explicit corrector steps, without iterations.

3. Solver verification

3.1. The case of a vertical fluid layer, task (1),(2),(5)

Solution of steady free convection in an infinite vertical fluid layer between parallel planes of different temperatures, has already been investigated [3]. Test case considered a rectangular enclosure of height $h$ and length $l$, with different ratio ($\frac{h}{l} = 1; 2; 4; 8$). Results confirmed the validity of the solver as shown in Fig. 1.

3.2. The case of a horizontal fluid layer, task (1),(3),(5)

As for the horizontal fluid layer, the onset of steady convection has been calculated. It was determined by the critical value of the Rayleigh number. The Rayleigh number is defined as the product of the Grashof number and the Prandtl number. When the Rayleigh number is below
Figure 1. Comparison between numerical and exact solution, where 1 — numerical solution for $h/l = 1$, 2 — $h/l = 2$, 3 — $h/l = 4$, 4 — $h/l = 8$, 5 — exact solution and $l = 0.2$ m.

A critical value, heat transfer is primarily in the form of conduction, when it exceeds the critical value, heat transfer is primarily in the form of convection. Was established, that critical value of the Rayleigh number belongs to the range of values between 1949 and 1952.3 (theoretical value 1708, experimental value [5] 1820).

a) $Ra = 1949$, $\Delta T = 14.1$ K, $U_{max} = 10^{-6}$ m$^2$/s

b) $Ra = 1952$, $\Delta T = 14.12$ K, $U_{max} = 10^{-3}$ m$^2$/s

c) $Ra = 2756$, $\Delta T = 20$ K, $U_{max} = 10^{-2}$ m$^2$/s

Figure 2. Horizontal fluid layer, top wall cooler then the bottom one. Rayleigh number $Ra$, temperature difference $\Delta T$ and maximum speed $U_{max}$ specified under the pictures.

3.3. The case of natural convection in square encloser, task (1),(4),(5)
The content of the article [6] was used for verification. The Navier-Stokes equation in the Boussinesq approximation in porous medium has the following form

$$\rho\left(\frac{\partial \vec{U}}{\partial t} + (\vec{U} \nabla)\vec{U}\right) - \nu \Delta \vec{U} = -\nabla p - \frac{\nu}{K} \vec{U} + g[\beta(T - T_0)].$$

First, the value of the permeability coefficient was taken as infinity $K \approx \infty$. It was done to compare the content of the article [6] with results from the solver without adding modifications.
Figure 3. Variation of maximum velocity with Rayleigh number.

Figure 4. Dimensionless numbers $Pr = 0.7$, $Ra = 10^5$ Permability coefficient $K \approx \infty$, isotherms.

Figure 5. Dimensionless numbers $Pr = 0.7$, $Ra = 10^5$ Permability coefficient $K \approx \infty$, streamlines.

Second, the value of the permeability coefficient was taken as $K = 0.001$. And modified solver was used. Constants were taken with the values indicated in the Table 1.

In Fig. 4, 5 results of the task (1), (4), (5) with $K \approx \infty$. In Fig. 6, 7 results with $K = 0.001$. Two oppositely directed fluid flows are obtained, which converges with the simulation results taken for verification. In addition, the obtained isotherms have the form presented in the article [6], which allows to draw a conclusion about the correctness of the implemented solver.
Table 1. Values of constants and dimensionless numbers for the task (1),(4),(5)

| Constant | Value |
|----------|-------|
| $\nu$   | $7 \cdot 10^{-4}$ m$^2$/s |
| $a$     | $1 \cdot 10^{-3}$ m$^2$/s |
| $\beta$ | $5 \cdot 10^{-4}$ K$^{-1}$ |
| $g$     | $9.81$ m/s$^2$ |
| $Pr$    | $0.7$ |
| $Gr$    | $1.5 \cdot 10^5$ |
| $Ra$    | $1.05 \cdot 10^5$ |
| $l$     | $1$ m |
| $h$     | $1$ m |

4. Conclusion
The processes of natural convection in the field of gravity in the Boussinesq approximation has been examined. A mathematical model and numerical simulation has been made. Original solver and test cases were implemented using OpenFOAM. Simulation results for solver verification has been presented.

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