Neutrino mixings as a source of lepton flavor violations

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Abstract

Within the left-right symmetric model (LRM) the $Z$ boson decay into the channel $Z \rightarrow \tau \mu$ are investigated. The branching ratios of this decay is found in the third order of the perturbation theory. The obtained expression does not equal to zero only at the existence of the neutrino mixings. It means that from the point of view of the LRM nonconservation both of neutral and of charged lepton flavors has the same nature. As a result, elucidation of the decays $Z \rightarrow l_i \bar{l}_k$ ($i \neq k$) could provide data concerned the neutrino sector structure of the LRM. The neutrino sector parameters which could be measured in that case are as follows: (i) difference of the heavy neutrino masses; (ii) heavy-heavy neutrino mixing; (iii) heavy-light neutrino mixing.

Keywords: $Z$ boson decays, charged lepton flavor violation, left-right symmetric model, heavy and light neutrinos, mixing in the neutrino sector, Large Hadron Collider.

PACS numbers: 12.15.Ji, 12.15.Lk, 13.40.Ks, 12.60.Cn.

1 Introduction

The standard model (SM) of particle physics has been very successfully predicting or explaining most experimental results and phenomena. However it still has a few outstanding problems with empirical observations. One of them is connected with neutrinos. In the SM the lepton flavors $L_{e,\mu,\tau}$ are the conserved quantities. However, neutrino oscillation experiments demonstrated that the neutrinos have the masses and the neutral lepton flavors (NLF’s) is not conserved. It should be stressed that this nonconservation is caused by the mixing in the neutrino sector. Of course, the minimally extended SM (SM with the massive neutrinos) may be invoked for description of neutrino oscillation experiments but
processes involving violation of charged lepton flavors (CLF’s) are extremely suppressed in it because of the small neutrino masses. Owing to a positive signal in any of the experimental looking for CLF violation (CLFV) processes would automatically imply the existence of physics beyond the SM. Although no such processes have been detected to date, this is a very active field that is being explored by many experiments which have adjusted upper limits to this kind of CLFV processes.

The CLFV processes can be classified into high energy ones that are detected at colliders, such as the CLFV decays of the $Z$ and Higgs bosons, and low energy ones such as $\mu - e$ conversion in nuclei, rare radiative and pure leptonic decays of the $\mu$ and $\tau$ leptons. By now the strongest limits on the CLFV processes have been set in the $\mu - e$ conversions. For example, the branching ratios of the radiative $\mu \to e\gamma$ decay and $\mu - e$ transition in heavy nuclei have been bounded to be below $4.2 \times 10^{-13}$ and $7.0 \times 10^{-13}$ by the MEG [1] and SINDRUM II [2] collaborations, respectively. Next generation of experiments are expected to enhance in several orders of magnitude the sensitivities for $\mu - e$ transitions, reaching the impressive range of $10^{-18}$ for $\mu - e$ transition in nuclei by the PRISM experiment in J-PARC [3].

LEP, as $Z$ factory, looked for the CLFV decays $Z \to l_il_k$ ($i \neq k$) with no luck. In such a manner it established upper limits to these processes, which are relatively weak compared with the low-energy processes

$$\text{BR}(Z \to e\mu) < 1.7 \times 10^{-6} \quad [4],$$

$$\text{BR}(Z \to e\tau) < 9.8 \times 10^{-6}, \quad [4],$$

$$\text{BR}(Z \to \mu\tau) < 1.2 \times 10^{-5} \quad [5].$$

The currently running LHC could also throw light on CLFV processes. The LHC has been searching the $Z$ boson decays into two leptons of different flavor as well [7, 6]. ATLAS is already at the level of LEP results for the LFV $Z$ decay rates, and even better for $Z \to \mu e$ channel

$$\text{BR}(Z \to e\mu) < 7.50 \times 10^{-7} \quad [6],$$

$$\text{BR}(Z \to \mu\tau) < 1.3 \times 10^{-5} \quad [7],$$

$$\text{BR}(Z \to e\tau) < 5.8 \times 10^{-5} \quad [7].$$

The CLFV is also investigated in the Higgs boson decays $H \to l_il_m$, which are searched by the CMS [8, 9] and ATLAS [10] collaborations. There are a lot of models predicting the CLFV in the decays both of the Higgs [11, 12, 13] and $Z$ bosons [14, 15, 16]. It is clear that amongst them the models having common mechanism both for NLF violation and for CLFV are most attractive. The left-right model (LRM) [17] belongs among such models. The neutrino sector of the LRM, apart from light left-handed neutrinos $\nu_{LL}$, also includes heavy right-handed neutrinos $N_{IR}$ which are partners on the see-saw mechanism for $\nu_{LL}$. As this takes place, mixings in the neutrino sector become a principal source of the CLFV.
Within the LRM the CLFV has already been examined in Refs. [18, 19, 20]. The goal of our work is to consider the CLFV decays of the Z boson and establish what parameters of the LRM neutrino sector therewith could be determined. In the next chapter we give a short summary of the LRM (the detail description of the model could be found in the book [21]). In sections 3 we calculate the branching ratio of the decay $Z \rightarrow \tau^-\mu^+$ in the third order of the perturbation theory. Our results are discussed in section 4.

2 The left-right-symmetry and neutrino mixing

The LRM is built upon the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. It has three gauge coupling constants $g_L$, $g_R$ and $g'$ for the $SU(2)_L$, $SU(2)_R$ and $U(1)_{B-L}$ groups, respectively. In the LRM quarks and leptons appears in the left- and right-handed doublets

$$Q^a_L(\frac{1}{2}, 0, \frac{1}{3}) = \begin{pmatrix} u^a_L \\ d^a_L \end{pmatrix}, \quad Q^a_R(0, \frac{1}{2}, \frac{1}{3}) = \begin{pmatrix} u^a_R \\ d^a_R \end{pmatrix},$$

$$\Psi^a_L(\frac{1}{2}, 0, -1) = \begin{pmatrix} \nu_{aL} \\ l_{aL} \end{pmatrix}, \quad \Psi^a_R(0, \frac{1}{2}, -1) = \begin{pmatrix} N_{aR} \\ l_{aR} \end{pmatrix},$$

(7)

where in brackets the values of $S_L^W, S_R^W$ and $B - L$ are given, $S_L^W$ ($S_R^W$) is the weak left (right) isospin, $\alpha = e, \mu, \tau$. The scalar sector of the LRM, as a rule, contains the bi-doublet $\Phi(1/2, 1/2, 0)$ and two triplets $\Delta_L(1, 0, 2), \Delta_R(0, 1, 2)$ and, as a result, neutrinos are Majorana particles.

After spontaneous symmetry breaking which is realized by the following choice of the vacuum expectation values (VEV’s)

$$< \Delta_{L,R}^0 > = v_{L,R}/\sqrt{2}, \quad < \Phi^0 > = k_1, \quad < \Phi_2^0 > = k_2,$$

$$v_L < \text{max}(k_1, k_2) << v_R,$$

(8)

the gauge boson sector include two neutral $(Z_{1,2})$ and two charged $(W_{1,2})$ gauge bosons, where $Z_1$ and $W_1$ bosons are analogs of the $Z$ and $W$ bosons of the SM, respectively.

The Lagrangian describing interaction of the charged gauge bosons with the $Z_{1,2}$ bosons is conveniently expressed by the following form [21]

$$L_{WWV} = i \rho_{kl}^{(V)} B_{\mu\nu,\lambda\sigma} \{ [\partial^\mu W^\lambda_{k}(x)] W^\nu_{l}(x) V^\sigma(x) + W^\sigma_{k}(x) [\partial^\mu W^\lambda_{l}(x)] V^\nu(x) +$$

$$+ W^\nu_{k}(x) W^\sigma_{l}(x) [\partial^\mu V^\lambda(x)] \},$$

(9)

where $k, l = 1, 2, V = Z_1, Z_2$,

$$\rho_{il}^{(Z_1)} = \cos^2 \left( \xi + \frac{\pi}{2} \delta_{l2} \right) g_L M_{11} + \sin^2 \left( \xi + \frac{\pi}{2} \delta_{l2} \right) g_R M_{12},$$

$$\rho_{il}^{(Z_1)} = \rho_{lk}^{(Z_1)} = \frac{1}{2} \sin 2\xi (g_L M_{11} - g_R M_{12}), \quad k \neq l,$$

(10)

(11)
\[ W_1 = W_L \cos \xi + W_R \sin \xi, \quad W_2 = -W_L \sin \xi + W_R \cos \xi, \quad B^{\mu \nu, \lambda \sigma} = g^{\mu \nu} g^{\lambda \sigma} - g^{\mu \sigma} g^{\nu \lambda}, \]
\[ c_W = \cos \theta_W, \quad s_W = \sin \theta_W, \quad \theta_W \text{ is the Weinberg angle}, \quad \text{and } M_{ik} \text{ are elements of the matrix} \]
\[ M = \begin{pmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} e (g'^{-2} + g^{-2})^{1/2} & -e g^{-1} g_{R}^{-1} (g'^{-2} + g^{-2})^{-1/2} \\ 0 & g^{-1} (g'^{-2} + g^{-2})^{-1/2} \end{pmatrix}, \quad (12) \]
\( \phi \) is the mixing angle in the neutral gauge bosons sector). In its turn the expressions for \( \rho_{t}^{(Z_2)}, \rho_{kl}^{(Z_2)} \) follow from (10) and (11) under the substitutions
\[ g_L M_{11} \rightarrow g_R M_{22}, \quad g_R M_{12} \rightarrow g_L M_{21}, \quad \xi \rightarrow \xi + \frac{\pi}{2}. \quad (13) \]
The LRM made the following predictions about the values of the mixings in the gauge bosons sector (see, for example, the book [21] and references therein)
\[ \tan 2\phi \simeq \frac{k^2 (\cos 2\theta_W)^{3/2}}{2v_R^2 \cos^4 \theta_W} \simeq \frac{2m_Z^2}{m_Z^2} \sqrt{\cos 2\theta_W}, \quad (14) \]
and
\[ \tan 2\xi \simeq \frac{4g_L g_R k_1 k_2}{g_R^2 (2v_R^2 + k_+^2) - g_L^2 (2v_L^2 + k_+^2)}, \quad (15) \]
where \( k_+ = \sqrt{k_1^2 + k_2^2} = 174 \text{ GeV}. \) Further on we shall assume \( g_L = g_R \) and use the designation \( g \) for them.

Using the lower bounds on the masses of additional gauge bosons
\[ m_{W_2} \geq 3.7 \text{ TeV} \quad [22], \quad m_{Z'} > 4.4 \text{ TeV} \quad [23], \quad (16) \]
and definitions of the gauge boson masses one could obtain the limits on these mixing angles. For example, taking into account
\[ m_{W_2}^2 = \frac{1}{2} \left[ M_L^2 + M_R^2 - \sqrt{(M_L^2 - M_R^2)^2 + 4M_{LR}^4} \right], \]
where
\[ M_L^2 = \frac{g^2}{2} (k_1^2 + 2v_L^2), \quad M_R^2 = \frac{g^2}{2} (k_1^2 + 2v_R^2), \quad M_{LR}^2 = g^2 k_1 k_2, \]
we lead to the inequality \( v_R \geq 5.7 \text{ TeV} \) to give
\[ \sin 2\xi \leq 5 \times 10^{-4}. \quad (17) \]
Acting in an analogous way (see, for example, [21]), we get
\[ \tan 2\phi < 6 \times 10^{-4}. \quad (18) \]
In our calculation we also need the Lagrangian which governs the interaction between charged gauge bosons and fermions

\[ \mathcal{L}^{CC} = \frac{1}{2\sqrt{2}} \sum_i [g \bar{l}(x) \gamma^\mu (1 - \gamma_5) \nu_{iL}(x) W_{\mu L}(x) + g \bar{t}(x) \gamma^\mu (1 + \gamma_5) N_{iR}(x) W_{\mu R}(x)]. \]  

(19)

The neutrino states entering into the Lagrangian (19) have been specified in flavor basis. They do not represent physical states (mass eigenstates), but they are mixing of these states. For the sake of simplicity, in what follows, we shall be constrained by two flavor approximation. Then the connection between flavor and physical bases are determined by the following way

\[
\begin{pmatrix}
\nu_{aL} \\
N_{aR} \\
\nu_{bL} \\
N_{bR}
\end{pmatrix} =
\begin{pmatrix}
c_{\varphi_a} c_{\theta_{\nu}} & s_{\varphi_a} c_{\theta_{\nu}} & c_{\varphi_a} s_{\theta_{\nu}} & s_{\varphi_a} s_{\theta_{\nu}} \\
-s_{\varphi_a} c_{\theta_{\nu}} & c_{\varphi_a} c_{\theta_{\nu}} & -s_{\varphi_a} s_{\theta_{\nu}} & c_{\varphi_a} s_{\theta_{\nu}} \\
-c_{\varphi_b} s_{\theta_{\nu}} & -s_{\varphi_b} s_{\theta_{\nu}} & c_{\varphi_b} c_{\theta_{\nu}} & s_{\varphi_b} c_{\theta_{\nu}} \\
s_{\varphi_b} s_{\theta_{\nu}} & -c_{\varphi_b} s_{\theta_{\nu}} & -s_{\varphi_b} c_{\theta_{\nu}} & c_{\varphi_b} c_{\theta_{\nu}}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
N_1 \\
\nu_2 \\
N_2
\end{pmatrix},
\]

(20)

where \( \varphi_a \) and \( \varphi_b \) are the mixing angles inside \( a \) and \( b \) generations respectively, \( \theta_{\nu}(\theta_N) \) is the mixing angle between the light (heavy) neutrinos belonging to the \( a \)- and \( b \)-generations, \( c_{\varphi_a} = \cos \varphi_a \), \( s_{\varphi_a} = \sin \varphi_a \) and so on.

Within the LRM one could obtain the exact formula for the heavy-light neutrino mixing angle \( \varphi_{a,b} \) [19]

\[
\sin 2\varphi_a = 2 \frac{f_{aa}^2 v_R v_L - \left[ f_{aa}(v_R + v_L) - m_{\nu_1} c_{\theta_{\nu}}^2 - m_{\nu_2} s_{\theta_{\nu}}^2 \right] \left( m_{\nu_1} c_{\theta_{\nu}}^2 + m_{\nu_2} s_{\theta_{\nu}}^2 \right)}{f_{aa}(v_R + v_L) - 2 \left( m_{\nu_1} c_{\theta_{\nu}}^2 + m_{\nu_2} s_{\theta_{\nu}}^2 \right)},
\]

(21)

\[
\sin 2\varphi_b = \sin 2\varphi_a \left( f_{aa} \rightarrow f_{bb}, \theta_{\nu} \rightarrow \theta_{\nu} + \frac{\pi}{2} \right),
\]

(22)

where \( f_{aa} \) and \( f_{ab} \) are the triplet Yukawa coupling constants. We see that the heavy-light mixing angles belonging to different generations are practically equal in value

\[
\sin 2\varphi_a \simeq \sin 2\varphi_b \simeq 2 \frac{v_R v_L}{v_R + v_L} \equiv \sin 2\varphi.
\]

(23)

There are a lot of papers devoted to determination of experimental bounds on the value of the heavy-light neutrino mixing angle \( \varphi \) (see, for example [24] and references therein). One way to find such bounds is connected with searches for the neutrinoless double beta decay and disentangle the heavy neutrino effect. From the results of Ref. [25] considering the case of \( ^{76}\text{Ge} \), it follows that the upper limit on \( \sin \varphi \) is about \( \text{few} \times 10^{-3} \) for \( m_N = 100 \text{ GeV} \).

The other way is to directly look for the presence of the heavy-light neutrino mixing via their signatures, for example, in collider experiments. By way of illustration, we point Ref. [26] in which the final states with same-sign dileptons plus two jets without missing energy \( (l^\pm l^\pm jj) \), resulting from \( pp \) collisions were considered. Analysis of the channel

\[
p + p \rightarrow N^*_l l^\pm \rightarrow l^\pm + l^\pm + 2j
\]

(24)
led to the upper limit on \( \sin \varphi \) equal to \( 3.3 \times 10^{-2} \) for \( m_{W_R} = 4 \text{ TeV} \) and \( m_{N_1} = 100 \text{ GeV} \). So we see that the heavy-light neutrino mixing angle may not be so small.

In the next chapter, we will show that information about the value of this angle can also be obtained under investigation of the decay processes of the \( Z \) boson going with the lepton flavor violation.

### 3 CLFV decays of the \( Z \) boson

Let us investigate the \( Z_1 \) boson decay into the channel

\[
Z_1 \to \mu^+ + \tau^-.
\]  

Due to the mixing into the neutrino sector this decay could proceed in the third order of the perturbation theory. The corresponding diagrams are shown in Fig.1. For simplicity sake consider the individual contributions of each diagram to the total width of the decay (25). First we examine the diagrams shown in Fig.1a. It is clear that the main contribution to the decay width comes from the diagrams with the \( W_1^+ W_1^- \nu_L \) in the virtual state. In

\begin{align*}
\text{(a)} \quad & \begin{array}{c}
\begin{array}{c}
\tau^-
\end{array}
\end{array} \\
& \begin{array}{c}
\begin{array}{c}
\nu_L, N_R
\end{array}
\end{array} \\
& \begin{array}{c}
\begin{array}{c}
W_{1,2}
\end{array}
\end{array} \\
& \begin{array}{c}
\begin{array}{c}
\mu^+
\end{array}
\end{array} \\
& \begin{array}{c}
\begin{array}{c}
\tau^-
\end{array}
\end{array} \\
& \begin{array}{c}
\begin{array}{c}
\nu_L, N_R
\end{array}
\end{array} \\
& \begin{array}{c}
\begin{array}{c}
W_{1,2}
\end{array}
\end{array} \\
& \begin{array}{c}
\begin{array}{c}
\mu^+
\end{array}
\end{array}
\end{align*}

\begin{align*}
\text{(b)} \quad & \begin{array}{c}
\tau^-
\end{array} \\
& \begin{array}{c}
\nu_L, N_R
\end{array} \\
& \begin{array}{c}
W_{1,2}
\end{array} \\
& \begin{array}{c}
\mu^+
\end{array} \\
& \begin{array}{c}
\tau^-
\end{array} \\
& \begin{array}{c}
\nu_L, N_R
\end{array} \\
& \begin{array}{c}
W_{1,2}
\end{array} \\
& \begin{array}{c}
\mu^+
\end{array}
\end{align*}

\begin{align*}
\text{(c)} \quad & \begin{array}{c}
\tau^-
\end{array} \\
& \begin{array}{c}
\nu_L, N_R
\end{array} \\
& \begin{array}{c}
W_{1,2}
\end{array} \\
& \begin{array}{c}
\mu^+
\end{array}
\end{align*}

Figure 1: The Feynman diagrams contributing to the decay \( Z_1 \to \mu^+ + \tau^- \).
this case the internal neutrino line corresponds to convolution of the operators $\nu_\tau L(x)$ and $\bar{\nu}_\mu L(y)$. Then with the help of Eq. (20) we get

$$
\nu^*_\tau L(x)\bar{\nu}^*_\mu L(y) = \left\{ -\cos \varphi_\tau \sin \theta_\nu \nu_1(x) - \sin \varphi_\tau \sin \theta_\nu N_1(x) + \cos \varphi_\tau \cos \theta_\nu \nu_2(x) + \\
+ \sin \varphi_\tau \cos \theta_\nu N_2(x) \right\}^s \left\{ \cos \varphi_\mu \cos \theta_\nu \bar{\nu}_1(y) + \sin \varphi_\mu \cos \theta_\nu \bar{N}_1(y) + \cos \varphi_\mu \sin \theta_\nu \bar{\nu}_2(y) + \\
+ \sin \varphi_\mu \sin \theta_\nu \bar{N}_2(y) \right\}^s \simeq \sin^2 \varphi \sin \theta_\nu [N^s_2(x)\bar{N}^s_2(y) - N^s_1(x)\bar{N}^s_1(y)],
$$

(26)

where convolution of the operators is symbolized by $s$ and we have taken into account

$$
\nu^*_1(x)\bar{\nu}^*_1(y) \simeq \nu^*_2(x)\bar{\nu}^*_2(y), \quad \varphi_\mu = \varphi_\tau = \varphi.
$$

(27)

The matrix element corresponding to the diagram under consideration has the form

$$
M^{(a)} = \frac{g^3 c_W \sin 2\theta_\mu \sin^2 \varphi \cos \phi \cos^2 \xi}{8} \sqrt{\frac{m_\tau m_\mu}{2m_{Z1}E_\tau E_\mu}} \pi(p_1)\gamma^m(1-\gamma_5) \int \Omega \left\{ \delta \left( \hat{k} - \hat{p}_2 + m_{N_2} \right) - \delta \left( \hat{k} - \hat{p}_2 + m_{N_1} \right) \right\} \gamma^n(1-\gamma_5)v(p_2) \left[ g_{\sigma \lambda} \Lambda_{\mu \nu}(k - p) \Lambda_{\rho \sigma}(k) k_\rho - \\
- g_{\nu \lambda} \Lambda_{\mu \sigma}(k) \Lambda_{\mu \lambda}(k - p)(k - p)_\mu - g_{\nu \lambda} \Lambda_{\mu \sigma}(k - p) \Lambda_{\mu \nu}(k) p_\mu \right] B^{\mu \nu, \beta \sigma} Z^\lambda(p) d^4 k,
$$

(28)

where

$$
\Lambda_{\mu \nu}(k) = \frac{g_{\mu \nu} - k_\mu k_\nu / m_{W_1}^2}{k^2 - m_{W_1}^2},
$$

$m_{N_j}$ ($j = 1, 2$) is the mass of the heavy neutrino, $p_1$ ($p_2$) is the momentum of $\tau$-lepton ($\mu$-meson), $\theta_{\mu \tau}$ is the mixing angle between the heavy tau-lepton and muon neutrinos. Thanks to the current upper limits on the mixing angles in the gauge boson sector we may set $\cos \phi \cos^2 \xi$ equal 1.

The scheme of further calculations is as follows. Using the procedure of dimensional regularization and considering the motion equations we rewrite the expression (28) in the form

$$
M^{(a)} = \frac{i\pi^2 g^3 c_W \sin 2\theta_\mu \sin^2 \varphi}{4} \sqrt{\frac{m_\tau m_\mu}{2m_{Z1}E_\tau E_\mu}} \pi(p_1) \left[ (1 + \gamma_5)(A_{\gamma \lambda} + B p_{1 \lambda}) + (1 - \gamma_5)(C_{\gamma \lambda} + \\
+ D p_{1 \lambda}) \right] v(p_2) Z^\lambda(p),
$$

(29)

where the quantities $A, B, C$ and $D$ represent the two-dimensional integrals.

Let us find the part of the partial decay width connected with the diagram of Fig.1a. Substituting (29) into the formula

$$
d\Gamma = (2\pi)^4 \delta^{(4)}(p - p_1 - p_2) |M^{(a)}|^2 \frac{d^3 p_1 d^3 p_2}{(2\pi)^8},
$$
and integrating the obtained expression over $p_1$, $p_2$, we lead to the result
\[
\Gamma(Z_1 \to W_1^{-*}W_1^{+*}\nu_L^* \to \tau^- \mu^+) = \frac{g^6 c_W^2 \pi^3 \sin^4 \varphi \sin^2 2\theta_{\mu\tau} 1}{384 m_{Z_1}^2} \left\{ (m_{Z_1}^2 - m_{\mu}^2 - m_{\tau}^2) \left[ 3f_{A,A}(m_{N_1}, m_{N_2}) + \right. \right. \\
+ 3f_{C,C}(m_{N_1}, m_{N_2}) + \beta_\tau f_{B,B}(m_{N_1}, m_{N_2}) + f_{D,D}(m_{N_1}, m_{N_2}) \right\} + 4f_{A,C}(3m_{\mu}m_{\tau} - 2\beta_\tau) - \beta_\tau \left[ 4(m_{\mu} + m_{\tau}) \right] f_{C,B}(m_{N_1}, m_{N_2}) + f_{A,D}(m_{N_1}, m_{N_2}) \right\} \times \\
\times \sqrt{(m_{Z_1}^2 - m_{\mu}^2 - m_{\tau}^2)^2 - 4m_{\mu}^2 m_{\tau}^2},
\]
where
\[
\beta_\tau = \frac{(m_{Z_1}^2 + m_{\mu}^2 - m_{\tau}^2)^2}{m_{Z_1}^2} - m_{\tau}^2,
\]
\[
f_{A,A}(m_{N_1}, m_{N_2}) = [A(m_{N_1}) - A(m_{N_2})]^2,
\]
\[
f_{A,B}(m_{N_1}, m_{N_2}) = [A(m_{N_1}) - A(m_{N_2})] \times \\
\times [B(m_{N_1}) - B(m_{N_2})],
\]
and so on. Calculations demonstrate that the term $3(m_{Z_1}^2 - m_{\mu}^2 - m_{\tau}^2)f_{A,A}(m_{N_1}, m_{N_2})$
exceeds all remaining terms in the curly brackets on the several orders of magnitude.
Then, taking into account
\[
m_{Z_1}^2 \gg m_{\tau}^2, m_{\mu}^2, m_{\tau} m_{\mu}
\]
we get
\[
\Gamma(Z_1 \to W_1^{-*}W_1^{+*}\nu_L^* \to \tau^- \mu^+) \simeq \frac{g^6 c_W^2 \pi^3 \sin^4 \varphi \sin^2 2\theta_{\mu\tau} m_{Z_1}^2 f_{A,A}(m_{N_1}, m_{N_2})}{128},
\]
where
\[
A(m_j) = \int_0^1 dy \int_0^1 x dx \left\{ \left[ 8 + m_{W_1}^{-2} \left( \frac{21}{4} l_{xy}^2 - 22 p_x^2 + (p_1 p_2)(23x - 17xy - 2) \right) \right] \ln \left| \frac{l_{xy}^2}{l_{xy}^2 - p_x^2} \right| + \\
+ \frac{1}{l_{xy}^2 - p_x^2} \left[ 3p_x^2 - (p_1 p_2)(6x - 2xy - 4) + m_{W_1}^{-2} \left( -2p_x^4 + p_x^2 (p_1 p_2)(8x - 6xy) \right) - \\
- 4(p_1 p_2)^2(x - xy)(2x - xy) \right] \right\},
\]
\[
p_x = p_1(x - xy) + p_2 x, \quad l_{xy}^2 = \left( m_{\mu}^2 - m_{N_j}^2 - m_{Z_1}^2 + m_{W_1}^2 \right) xy + m_{Z_1}^2 x - m_{W_1}^2,
\]
\[
(p_1 p_2) = \frac{1}{2}(m_{Z_1}^2 - m_{\mu}^2 - m_{\tau}^2).
\]

Now we embarked on a consideration of contributions of the diagrams of Fig.1b and Fig.1c. It is clear that these diagrams transfer to each other under replacement
\[
m_{\mu} \longleftrightarrow m_{\tau},
\]
Therefore, we suffice to examine one of them. After the procedure of dimensional regularization the matrix element corresponding to the diagram shown in Fig. 1b could be represented in the form

\[
M^{(b)} = \frac{i\pi^2 g^3 \sin 2\theta_{\mu\tau}}{16c_W} \sqrt{\frac{m_{W_1}^2}{2m_{Z_1} E_\tau E_\mu}} \pi(p_1)\gamma^m(1 - \gamma_5)\left[F(m_{N_2}) - F(m_{N_1})\right]\gamma^n \times \frac{\hat{p}_1}{m_\tau^2 - m_\mu^2} \gamma^\nu(\gamma_5 - 1 + 4s_W^2)\nu(p_2)Z_\nu(p),
\]

where

\[
F(m_{N_1}) = \int_0^1 \left\{ g_{mn}\hat{p}_1(1-x)\ln \left| \frac{l_j}{l_j^2 - p_x^2} \right| - \hat{p}_1(1-x) \left[ (2p_xp_{xn} + g_{mn}(p_x^2 - l_j^2)) \ln \left| \frac{l_j}{l_j^2 - p_x^2} \right| + g_{mn}p_x^2 \right] + \frac{1}{2m_{W_1}^2} \left[ (p_x^2 - l_j^2)(\gamma_mp_{xn} + \gamma_np_{xn}) \ln \left| \frac{l_j}{l_j^2 - p_x^2} \right| + p_x^2(\gamma_mp_{xn} + \gamma_np_{xn}) \right] \right\} dx,
\]

\[
p_x = p_1 x, \quad l_j = (m_\tau^2 + m_{N_1}^2 - m_{N_1}^2)x - m_{W_1}^2.
\]

Using (34) we could find \(\Gamma(Z_1 \rightarrow \tau^-W_1^+\nu_L \rightarrow \tau^-\mu^+)\). In so doing the following approximate relation takes place

\[
\frac{\Gamma(Z_1 \rightarrow \tau^-W_1^+\nu_L \rightarrow \tau^-\mu^+)}{\Gamma(Z_1 \rightarrow W_1^-W_1^+\nu_L \rightarrow \tau^-\mu^+)} \approx 10^{-6} \div 10^{-7}.
\]

Therefore, the basic contribution to the decay (25) is caused by the diagram pictured in Fig. 1a.

Let us provide estimation of the branching ratio of the decay \(Z_1 \rightarrow \tau^-\mu^+\). Before we proceed further, we note that the function \(f_{A,A}(m_{N_1}, m_{N_2})\) depends on the difference of the heavy neutrino masses. For example, when \(m_{N_2}\) is varied from 100 up to 200 Gev and \(m_{N_1} = 100\) GeV (\(m_{N_1} = 150\) GeV) we have

\[
f_{A,A}(m_{N_1}, m_{N_2}) \in [0, 0.958], \quad (f_{A,A}(m_{N_1}, m_{N_2}) \in [0, 0.125]).
\]

Then setting

\[
\theta_{\mu\tau} = \frac{\pi}{4},
\]

we get

\[
\begin{align*}
\text{BR}(Z_1 \rightarrow \tau\mu) & \leq \begin{cases} 
9.7 \times 10^{-8}, \text{ at } \varphi = 3.2 \times 10^{-2}, & m_{N_1} = 100\text{ GeV}, \ m_{N_2} = 150\text{ GeV}, \\
1.4 \times 10^{-8}, \text{ at } \varphi = 5 \times 10^{-3}, & m_{N_1} = 100\text{ GeV}, \ m_{N_2} = 200\text{ GeV}, \\
7.6 \times 10^{-11}, \text{ at } \varphi = 10^{-3}, & m_{N_1} = 150\text{ GeV}, \ m_{N_2} = 200\text{ GeV},
\end{cases}
\end{align*}
\]

where

\[
\text{BR}(Z_1 \rightarrow \tau\mu) = \text{BR}(Z_1 \rightarrow \tau^-\mu^+) + \text{BR}(Z_1 \rightarrow \tau^+\mu^-).
\]

Notice that the expression (31) does not practically depend on the lepton masses. Therefore, all discrepancy between the branching ratios of the decays \(Z_1 \rightarrow \tau\mu, Z_1 \rightarrow \tau e\) and \(Z_1 \rightarrow e\mu\) is determined exclusively by the values of the mixing angles in the heavy neutrinos sector.
4 Conclusion

In the framework of the LRM the decay \( Z \rightarrow \tau \mu \) has been investigated in two flavor approximation. This decay is prohibited in the SM by virtue of the fact that it goes with the charged lepton flavor violation (CLFV). The obtained branching ratio of this decay does not equal to zero only at the existence of the neutrino mixings and at the absence of masses degeneracy in the heavy neutrino sector. From it follows that within the LRM nonconservation both of neutral and of charged lepton flavors has the same nature, namely, it is caused by the neutrinos mixing. As a result, elucidation of the decays \( Z \rightarrow l_i l_k \) \( (i \neq k) \) could provide data concerned the neutrino sector structure of the LRM. The neutrino sector parameters which could be measured in that case are as follows: (i) difference of the heavy neutrino masses; (ii) heavy-heavy neutrino mixing; (iii) heavy-light neutrino mixing. Note, that information about these parameters may be also obtained under investigation of the CLFV Higgs boson decays \[20\].

Using the maximal value of the heavy-light neutrino mixing angle \( \varphi \), which was found in collider experiments \[26\], we have get the upper bound on the branching ratio of the decay \( Z \rightarrow \tau \mu \). The obtained expression appears to be less on two order of magnitude than the upper bound \( \text{BR}(Z \rightarrow \tau \mu)_{\exp} < 1.2 \times 10^{-5} \) arrived by the experiments at ATLAS and CMS. However, it is well to bear in mind that this quantity is not the measured value of the branching ratio. It is nothing but the precision limit of the current experiments. In actual truth, the observed value of \( \text{BR}(Z \rightarrow \tau \mu) \) may prove to be less than \( 10^{-5} \). As a consequence the experiments on looking for the CLFV Z boson decays with higher precision than at present will certainly be continued during the new LHC runs and at future leptonic colliders where the more high statistics of \( Z \) boson events will be achieved. For example, the future LHC runs with \( \sqrt{s} = 14 \text{ TeV} \) and total integrated luminosity of first 300 \( \text{fb}^{-1} \) and later 3000 \( \text{fb}^{-1} \) expect the production of about \( 10^{10} \) and \( 10^{11} \) of the \( Z \) boson events, respectively. These large numbers provide an upgrading of sensitivities to \( \text{BR}(Z \rightarrow l_i l_m) \) of at least one order of magnitude with respect to the present sensitivity. However, the best sensitivities for these CLFV decays are expected from next generation of lepton colliders such as the International linear collider \[27\], Future Circular \( e^+e^- \) Collider (FCC-ee — TLEP) \[28\], Circular Electron-Positron Collider (CEPC) \[29\], in so far as they can work as \( Z \) factory with a very clean environment. For example, at TLEP \[30\], where up to \( 10^{13} \) \( Z \) bosons would be produced, the sensitivities to CLFV \( Z \) decay rates could be improved up to \( 10^{-13} \).

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