Electron pairing in conventional superconducting metals is mediated by phonons. Strong interaction of electrons with the lattice also manifests itself through self-energy effects yielding fingerprints of the electron-phonon spectral function \( \alpha^2 F(\omega) \) in the tunneling density of states. An issue of whether or not the magnetic mode is capable of having a serious impact on the electronic self-energy, in view of the small intensity of the resonance, has been contested on theoretical grounds. Moreover, a recent re-examination of ARPES results has suggested that the totality of data is better described in terms of coupling to phonons and not to magnetic excitations. However, this latter claim is not supported by IR studies of isotopically substituted \( \text{YBa}_2\text{Cu}_3\text{O}_y \) (YBCO) which show no isotope effect for the feature in question. Currently available data leave ambiguities regarding the roles of lattice and magnetic degrees of freedom in carrier dynamics as well as in the superconductivity of cuprates.

Signatures of strong coupling effects in cuprate high-\( T_c \) superconductors have been authenticated through a variety of spectroscopic probes. However, the microscopic nature of relevant excitations has not been agreed upon. Here we report on magneto-optical studies of the CuO planes in a prototype of high-\( T_c \) superconductor \( \text{YBa}_2\text{Cu}_3\text{O}_y \) (YBCO). Infrared data are directly compared with earlier inelastic neutron scattering results by Dai et al. [Nature (London) 406, 965 (2000)] revealing a characteristic depression of the magnetic resonance in a magnetic field less than 7 T. This analysis has allowed us to critically assess the role of magnetic degrees of freedom in producing strong coupling effects for YBCO system.

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FIG. 1: (color online) Reflectance spectra obtained at 5 K in magnetic field for (top) \( y = 6.50 \) (\( T_c \sim 31 \) K), (middle) \( y = 6.65 \) (\( T_c \sim 60 \) K) YBCO crystals, and (bottom) graphite. Polarized light along the \( a \) axis is used for detwinned YBCO crystals. The magnetic field is applied along the \( c \) axis. Red (thin solid) lines: \( H = 0 \); blue (thick dashed) lines: \( H = 7 \) T. Sharp spikes near 2,900 cm\(^{-1}\) in the high field spectra are due to absorption in the windows of our cryostat.

We proceed by briefly outlining the fundamentals of an IR probe of the electronic self-energy. Interaction of the mobile charges with bosonic excitations leads to a frequency dependence of the scattering rate \( 1/\tau(\omega) \) in accord with the Allen formula:

\[
\frac{1}{\tau(\omega)} = \frac{2\pi}{\omega} \int_0^\omega d\omega' (\omega - \omega') \alpha^2 F(\omega') + \frac{1}{\tau_{imp}},
\]

where \( 1/\tau_{imp} \) is the impurity scattering. Experimentally, the frequency dependence of \( 1/\tau(\omega) \) can be inferred from the analysis of the complex optical conductivity \( \sigma(\omega) \) within the Extended Drude Model:

\[
1/\tau(\omega) = \omega_p^2/4\pi \cdot \text{Re}[1/\sigma(\omega)],
\]

where a total plasma frequency \( \omega_p^2 \) is determined by integration of \( \sigma_c(\omega) \) up to the charge transfer gap. Eq. (1) is commonly applied to the analysis of the data for cuprates and provides support for an idea of QPs coupling to a magnetic resonance. Nevertheless, Eq. (1) is not entirely adequate for a superconductor since it completely ignores the effect of the superconducting energy gap \( 2\Delta \) on the form of the \( 1/\tau(\omega) \) spectra. In order to treat the impact of the gap and of strong coupling to bosonic modes on equal footing we used the following result also derived by Allen:

\[
\frac{1}{\tau_s(\omega)} = \frac{2\pi}{\omega} \int_0^{\omega-2\Delta} d\omega' (\omega - \omega') \alpha^2 F(\omega') \\
\times E \left[ \left( 1 - \frac{4\Delta^2}{(\omega - \omega')^2} \right)^{1/2} \right],
\]

where \( E \) is the complete elliptic integral of second kind. Although the utility of Eq. (2) is obvious it is non-trivial to employ this formula for the extraction of \( \alpha^2 F(\omega) \) from experimental data since simple inversion prescriptions do not apply in this case. To circumvent this limitation Dordevic et al. developed a numerical procedure based on the inverse theory that is described in details elsewhere. In the bottom panels [(e) and (f)] of Fig. 2 we show the \( \alpha^2 F(\omega) \) spectrum extracted in this fashion from the \( H = 0 \) spectrum. We wish to point out an excellent agreement with INS results for the spin susceptibility \( \chi(\omega) \) [open symbols in Fig. 2(e)] without introducing a frequency offset. Indeed, both a sharp resonance and a broad incoherent background of the spin susceptibility appear to be reproduced in the \( \alpha^2 F(\omega) \) spectrum.

An important feature of the strong coupling formalism [Eqs. (1) and (2)] is the integral relationship between \( 1/\tau(\omega) \) and \( \alpha^2 F(\omega) \). This relationship implies that a depression of the intensity in \( \alpha^2 F(\omega) \) necessarily reduces the magnitude of \( 1/\tau(\omega) \) and consequently enhances the reflectivity level at all frequencies above the resonance mode in the spectral function. In order to quantify the magnitude of possible \( H \)-induced changes associated with a depression of the INS resonance in magnetic field we adopted the following protocol. We first reduced the intensity of the sharp peak near \( \sim 270 \) cm\(^{-1}\) (\( \sim 34 \) meV) in the \( \alpha^2 F(\omega) \) spectrum by 20 %: a factor suggested by INS measurements. The intensity of broad background remained intact [blue (thick dashed) line in Fig. 2(f)]. Evidently, this modification will produce a conservative estimate of the impact of the INS resonance on IR data. Using the spectral function with the suppressed intensity...
we calculated $1/\tau(\omega, 7 \text{ T})$ from Eq. (2) and also $m^*(\omega, 7 \text{ T})$ with the help of Kramers-Kronig analysis. Finally, a combination of $1/\tau(\omega, 7 \text{ T})$ and $m^*(\omega, 7 \text{ T})$ allowed us to generate the reflectance spectrum $R(\omega, 7 \text{ T})$ [blue (thick dashed) line in Fig. 2(b)]. Comparing this final output of modeling with the experimental curve for $H = 0$ one finds that the effect of the applied magnetic field is rather small in the far-IR but exceeds 5% at frequencies above 800 cm$^{-1}$. This is further detailed in the inset of Fig. 2 where we present the ratio $\Delta R(\omega, H) = R(\omega, 7 \text{ T})/R(\omega, 0 \text{ T})$ calculated from the model spectra. These anticipated changes of reflectance exceed the uncertainty of $R(\omega, H)$ in our apparatus and therefore should be readily detectable.

Empowered by modeling of the data we will now discuss the implications of the lack of magnetic field dependence of IR spectra for underdoped YBCO documented in Figs. 1 and 2. One possible interpretation of the data is that the magnetic resonance is irrelevant to QP dynamics. Within this view self-energy effects in the data can be assigned to excitations inherently insensitive to the magnetic field such as phonons or the spin fluctuations continuum. However, single-phonon processes have a well defined high-energy cut-off in cuprates that does not exceed 800 cm$^{-1}$ for YBCO. For this reason phonons alone cannot account for a high frequency background in the $\alpha^2F(\omega)$ spectra in Fig. 2. On the contrary, magnetic excitations extend to significantly higher frequencies and therefore can naturally account for the form of $1/\tau(\omega)$ spectra in mid-IR energy range. Thus our results are consistent with the viewpoint that distinct phonon modes in concert with the broad spin fluctuations continuum are jointly responsible for strong coupling effects in cuprates.

An intriguing interpretation of the magnetic resonance seen in the INS experiments is offered by $SO(5)$ theory also providing a unified view on superconductivity and antiferromagnetism in cuprates. This interpretation is in accord with our data as we will elaborate below. $SO(5)$ theory predicts a $\pi$-resonance in the particle-particle channel that is present both above and below $T_c$. Coupling of the $\pi$-resonance to neutrons is facilitated by the formation of the pair condensate in a $d$-wave superconductor. This latter attribute of the $\pi$-mode is important. First, it allows one to understand the quasiparticles self-energy effects at $T > T_c$ in the absence of superconductivity. Second, within the framework of the $SO(5)$ theory a suppression of the neutron mode in the high magnetic field INS experiments is only an apparent effect. Indeed, this suppression is fully accounted for by a reduction of the superconducting order parameter in type-II $d$-wave system that is expected to occur in the regime of constant intrinsic intensity of the $\pi$-mode. In this fashion magnetic field is expected to have only a small effect on the quasiparticles self-energy probed in the IR data despite apparent depression of the mode seen in the INS experiment. Thus the $SO(5)$ interpretation of the INS peak allows one to reconcile dissimilarities in the magnetic field effects in IR and neutron measurements.

The results reported here call for an examination of the self-energy effects seen in cuprates by other spectroscopic methods in magnetic field. While it may be impossible to carry out such experiments in the case of photoemission studies, tunneling measurements appear to be well suited for this task. It is also worthwhile to re-evaluate the role of interband transitions and other excitations in providing a direct contribution to the optical conductivity in mid-IR region. The so-called multi-component analysis of the optical data offers a complementary interpretation of some of the effects discussed here within the self-energy formalism.

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The Eliashberg spectral function $\alpha^2 F(\omega)$ accounts for an effective (and generally energy dependent) electron-phonon coupling $\alpha^2$; $F(\omega)$ is the phonon density of states.

For details, see Ref.\(^\text{33}\) . One distinction of the spectral function obtained using the inversion theory from that generated using a simplified procedure $W(\omega) = 1/2\pi \cdot d^2/d\omega^2[\omega/\tau(\omega)]$ is worth of attention. The latter approach always yields regions of negative spectral function which is unphysical. On the contrary, the inversion theory offers a self-consistent treatment of both energy gap and strong coupling effects in the optical data and yields the physically meaningful spectral function throughout the entire frequency range. For details, see Ref.\(^\text{33}\).

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