Photon Inhibited Topological Transport in Quantum Well Heterostructures

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Here we provide a picture of transport in quantum well heterostructures with a periodic driving field in terms of a probabilistic occupation of the topologically protected edge states in the system. This is done by generalizing methods from the field of photon assisted tunneling. We show that the time dependent field dresses the underlying Hamiltonian of the heterostructure and splits the system into side-bands. Each of these sidebands is occupied with a certain probability which depends on the drive frequency and strength. This leads to a reduction in the topological transport signatures of the system because of the probability to absorb/emit a photon. Therefore when the voltage is tuned to the bulk gap the conductance is smaller then the expected $2e^2/h$. We refer to this as photon inhibited topological transport. Nevertheless, the edge modes reveal their topological origin in the robustness of the edge conductance to disorder and changes in model parameters. In this work the analogy with photon assisted tunneling allows us to interpret the calculated conductivity and explain the sum rule observed by previous authors\cite{1}.

Introduction.—Topological states of matter are currently at the forefront of research in condensed matter physics. From the quantum hall effect to topological superconductors, these states are of interest for a variety of reasons. In topological insulators the in-gap edge states are of primary interest. These states are topologically protected, meaning they are insensitive to deformations of the Hamiltonian’s parameters that leave the topological gap intact and the effects of disorder. The existence of such states provides a physical signature of the topology in the charge and spin conductance.

Recently, there has been a growing amount of attention paid to the generation and/or manipulation of topological states of matter through the application of a time-periodic perturbation\cite{1,20}. Experimental progress in this direction has been made in both photonic crystals\cite{21} and in a solid-state context in Bi$_2$Se$_3$\cite{22}. In the Letter we study how a time-periodic perturbation can be used to manipulate the transport properties of a quantum spin Hall insulator. For example: such a system is expected to manipulate the transport properties of a quantum spin Hall effect. We apply a time dependent field and allow for on-site disorder. Our Hamiltonian is as follows

$$H_S = H_{QW} + H_{\text{disorder}} + H_{\text{ext}}(t)$$

where $H_{QW} = \sum_k \psi_k^\dagger \begin{pmatrix} \hat{H}(k) & 0 \\ 0 & \hat{H}^*(-k) \end{pmatrix} \psi_k$ where $\psi_k^\dagger$ is a four component creation operator for electrons at momenta $k$ in state $m_J = (1/2,3/2,-1/2,-3/2)$ of the clean heterostructure and $\hat{H}(k) = \epsilon_k \sigma_0 + \mathbf{d}(k) \cdot \sigma$, with $\sigma$ being a vector of Pauli matrices. In the typical language of these structures\cite{23,25}, we take $\mathbf{d}(k) = (A \sin k_x, A \sin k_y, M - 4B + 2B(\cos k_x + \cos k_y))$ and $\epsilon_k = C - 2D(2 - \cos k_x - \cos k_y)$. In order to focus on transport without additional complications we follow Lindner and coworkers\cite{2} and set $C = D = 0$, $A = B = 0.2|M|$. All energies are in units of $M$. As we are interested in a “topological” system we take $M = 1$ so that $\text{sgn}(M/B) = 1\frac{2}{2}\frac{2}{2}$. Next, $H_{\text{ext}}(t) = 2(\mathbf{V} \cdot \sigma) \cos \Omega t$ is an electromagnetic field polarized in the direction $\mathbf{V}$\cite{2,27,28}. For concreteness, we will take $\mathbf{V} = \sqrt{\text{ext}} \hat{z}$; although this is not necessary for what follows. Note $H_{\text{ext}}(t)$ obeys the periodic generalization of time-reversal invariance\cite{2}.
FIG. 1: The two device geometries considered in this work. Left is a two-terminal device labeled with leads left ('L'), and right ('R'). On the right is a six terminal device labeled with leads 1through 6. The sites coupled to leads have a solid rectangle around them.

\[ T H_{\text{ext}}(t) T^{-1} = H_{\text{ext}}(-t + \tau) \text{ for some } \tau. \]

Finally, \( H_{\text{disorder}} = -\sum_{i,\alpha} w_i \psi_{i,\alpha}^\dagger \psi_{i,\alpha} \) (with \( \psi_i^\dagger \) the Fourier transform of \( \psi_i \)). This corresponds to charge impurities (disorder) changing the chemical potential on each site by \( w_i \). We draw the \( \{ w_i \} \) randomly from an evenly distributed sample between \(-W/2\) and \( W/2\). We call \( W \) the disorder strength.

Our numerical study employs the Floquet-Landauer formalism\cite{1, 7, 29, 30}. Similar to Ref. \cite{2}, we consider two different device geometries (see Fig. 1). First, we consider a two-terminal device with the left and right end of the system attached to leads whose Fermi level lies at the "lead energy" \( E \) with a slight offset bias between the two leads \cite{31}. In this set-up the quantity of interest is \( \sigma(E) \), the differential conductivity given that the chemical potential of the leads is at energy \( E \).

For a spin Hall insulator (e.g. our model above) in equilibrium when the lead energy \( E \) in a two-terminal device is tuned to lie in the gap (i.e. on the edge states), a value \( \sigma(E) = 2e^2/h \) is expected\cite{11, 34}. This is the first signature in which we are interested. For convenience, we define \( \sigma_{\text{TT}} = \sigma(V \approx 0)/h/e^2 \). Secondly, we consider a six-terminal device. This device allows us to probe whether the current is carried by bulk or edge modes\cite{7, 35, 36}. In equilibrium, it is found that the only non-zero values of the transmission elements between leads \( \lambda \) and \( \lambda' \), \( T_{\lambda,\lambda'}(\epsilon) \) (with \( \epsilon \) in the gap), come from tunneling between adjacent leads in the device. Thus \( T_{\lambda,\lambda'}(\epsilon_F) = 0 \), unless \( \lambda = \lambda' \pm 1 \) (where \( 6 + 1 \rightarrow 1 \)). Moreover, it is argued that \( T_{\lambda,\lambda+1}(\epsilon_F) = 1 \) as, because of the helical edge states, a quasiparticle originating at lead \( \lambda \) must tunnel to one of the neighbouring leads. Later in this Letter we look for similar properties in the non-equilibrium system.

Before proceeding we comment on recent criticisms of Floquet states in periodically driven systems\cite{27, 30}. Floquet states are often thought of as the steady-states of a time-periodic system\cite{11}. Refs. \cite{27, 39, 40} argue that the long time evolution of an isolated, periodically driven system leads to an effectively infinite temperature state for some driving periods. Our formalism for calculating transport properties attaches leads to the system (i.e. it is not isolated anymore) and only makes assumptions about the state of the leads in the distant past, namely that the leads are in a thermal equilibrium and no assumptions on the state of the leads in the distant past, namely that the leads are in a thermal equilibrium and no assumptions on the state of the system\cite{31}. This assumption provides the state of the system at the present time and does not rely on "evolving" any particular Floquet state.

Transport Results.— We begin with a clean system (\( \Omega = 0 \)) in a two terminal geometry. We fix \( \Omega = 2.3|M| \), and tune \( V_{\text{ext}} \). We plot \( \sigma_{\text{TT}} \) for \( V_{\text{ext}} = 0/|M|/|M| \) in Fig. 2. As \( V_{\text{ext}} \) is increased from zero, the quantization of \( \sigma_{\text{TT}} \) is lost. For moderately strong \( V_{\text{ext}} \), we see that it reaches \( \sigma_{\text{TT}} \approx 1.5 \). This shows that for a quantum spin Hall insulator, the (bare) conductivity is not, in general, quantized to the traditional equilibrium value under the application of a periodic perturbation.

Looking again at Fig. 2, we see that these values are robust to the strength of the disorder potential. The deviation from the clean limit is insignificant, even up to disorder strengths of \( M/2 \). Additionally, these values are insensitive to the coupling strength of the system to the leads\cite{34}, \( \Gamma \), and the system size. This robustness leaves the impression that despite the deviation of the conductivity from \( \sigma_{\text{TT}} = 2 \), the values it takes appear to be topologically protected. Our six-terminal calculations provide additional evidence of topological, edge conductance. With the lead energy set in the gap of the system, we find that \( T_{\lambda,\lambda'} = 0 \), except the off-diagonal elements \( T_{\lambda,\lambda+1} \) and \( T_{\lambda+1,\lambda} \). In contrast to equilibrium, we find that \( T_{\lambda,\lambda+1} + T_{\lambda+1,\lambda} < 1 \). In spite of this, we observe that the conduction takes place only between adjacent leads suggesting that the current is only flowing on the edges.

To explain the above behavior, we borrow insight from the field of photon assisted tunneling (PAT). PAT, as first proposed by Tien and Gordon\cite{23}, was originally used to describe a superconducting-insulator-superconductor tunnel junction. When a periodic AC voltage \( V_{\text{ac}} \) is applied to one of the leads, the energy eigenstates of these leads split into sidebands at energy \( E + n\hbar\Omega \) for integer \( n \) and driving frequency \( \Omega = 2\pi/T \). The probability that each one of these side bands is occupied is given
by $J_2^a(\alpha)$, where $\alpha = eV_{ac}/\hbar \Omega$, and $J_n$ is the $n^{th}$ Bessel function of the first kind. The consequence of this side-band splitting is that when a lead energy $E$, is applied across the tunnel junction, the electrons can tunnel into the system not just at energy $E$, but at $E + n\hbar \Omega$ with a probability of $J_n^2(\alpha)$. One interprets this as the electrons absorbing $(n > 0)$ or emitting $(n < 0)$ $|n|$ photons. As a result the conductivity in the driven system is given by $\sigma_{\text{PAT}}(E) = \sum_n J_n^2(\alpha)\sigma_0(E + n\hbar \Omega)$ \[23, 24\]. Here $\sigma_0(E)$ is the conductivity of the junction in the absence of the AC voltage.

Here we do not have a simple periodic modulation of the sample system, rather the modulation itself has some internal structure given by $\mathbf{V} \cdot \sigma$. The result of this is that the system is not simply split into side-bands. The fact that $H_{\text{ext}}(t)$ does not commute with the static Hamiltonian, leads to interesting effects. In the case of off-resonant light (light where $\hbar \Omega$ is not commensurate with the static spectrum), we can make some simplifying assumptions to obtain an effective description in line with PAT. We describe this simpler case here and leave the discussion of on-resonant light, where more care must be taken, for later \[12\].

In the field of Floquet topological insulators \[11, 13–15, 20\] with off-resonant light, it is known that one can think of the periodic perturbation as “dressing” the static system by modifying its underlying physical parameters to produce a new, effective static Hamiltonian. However, this approach is incomplete from a transport point of view. One must take into account the splitting of states of this effective Hamiltonian into side-bands. Thus off-resonant light has a two-sided effect: First, it dresses the static Hamiltonian to produce a new effective static Hamiltonian. Second, the eigenstates of this effective Hamiltonian are split into side-bands in a process analogous to PAT. This picture is not specific to the illustrative system we have chosen here, it is more general. One must take into account the splitting of the bands near the Fermi level are separated in energy from the other bands by a sufficient amount so that they can be neglected. Experimental validation of this comes from Ref. \[22\] where the experimental results can be understood by using only the bands near the Fermi level. As a result we have $\sigma_F(E + m\hbar \Omega) = \sigma_F(V)\delta_{m,0}$ for $E \approx 0$; no states exist at $m\hbar \Omega$. Therefore, we have

$$\sigma(E) = J_0^2 \left(\frac{2V_{\text{ext}}}{\hbar \Omega}\right) \sigma_F(E) \quad (E \approx 0)$$

Thus with an off resonant driving frequency, we describe the underlying system with an effective static Hamiltonian which may give rise to the signature transport properties. In the present case, we are interested in a Hamiltonian showcasing the quantum spin Hall effect. This state should have a two-terminal conductance of $2e^2/h$, and six-terminal transmission elements as described above. In the presence of a driving field, the in-gap edge states are only occupied with a certain probability due to the prospect of absorption/emission of photons. Thus, the transport property we are interested in only shows up with a certain probability. In the present case we expect $\sigma_F(V) = 2e^2/h$, and so the actual conductivity we measure will be $\sigma(E) = 2J_0^2 \left(\frac{2V_{\text{ext}}}{\hbar \Omega}\right)$. Plotting this against our numerical data produces excellent agreement (see Fig. \[2\]).

One may look at this expression as a correction to the quantized value of $2e^2/h$. One can show for in-gap energies $E$ that $\sigma(E) \approx 2 \left(1 - \left(\frac{V_{\text{ext}}}{\hbar \Omega}\right)^2\right)$, i.e. this correction is second order in $V_{\text{ext}}/\hbar \Omega$.

This explains our observation in the opening of this section. Despite the fact that we do not obtain the values $\sigma = 2e^2/h$, or $T_{\lambda, \lambda \pm 1} = 1$, , the values that we do see are robust in the same way as the equilibrium values. The underlying system is topological in nature, with helical edge states that give rise to $2e^2/h$ conductance and $T_{\lambda, \lambda \pm 1} = 1$. However there is only a certain probability that the electrons tunneling from the leads are at the correct energy to take advantage of these channels. Thus, the presence of these photons in the system inhibits the ability of these edge channels to transport charge.

Our discussion so far has not relied on the fact that the original Hamiltonian is topological in nature, rather
it is enough that the effective Hamiltonian be topologi-
ical. In other systems, it is possible to drive topological
states in otherwise trivial systems with off-resonant light.
The most prevalent example of this is graphene, where
the light produces an effective Hamiltonian with a topo-
logical mass. Thus the suppression described above may
also apply to these other systems. 

Connection to Floquet Sum Rule.—We now connect
our work to a sum rule proposed recently by Kundu and
Seradjeh in the context of a system with Floquet Majo-
ranana modes \cite{Kundu2020}. Similar to the current work,
these authors find that in the presence of a periodic perturbation, a sys-
tem with Majorana modes will not showcase the expected
zero-bias quantized conductance of $2e^2/h$. Instead, the
quantized conductivity is found in the sum
\[
\bar{\sigma}(E) = \sum_n \sigma(E + n\hbar\Omega). \tag{4}
\]
Physically, the above corresponds to performing mea-
surements of $\sigma(E)$ not just at an in gap energy $E$, but
for lead energies placed any number of $\hbar\Omega$’s above or
below this. The results of these measurements are then
summed up. Let us apply this sum rule to our system.
Using Eq. \[(4)\] we have
\[
\bar{\sigma}(E) = \sum_{n,m} J^2_m \left( \frac{2V_{ext}}{\hbar\Omega} \right) \sigma_F(E + (m + n)\hbar\Omega). \tag{5}
\]
Shifting $n \rightarrow n - m$, using the off resonance light conduc-
tivity $\sigma_F(E + m\hbar\Omega) = \sigma_F(E)\delta_{m,0}$ and the Bessel func-
tions property $\sum_n J^2_n(x) = 1$ leads to $\bar{\sigma}(E) = \sigma_F(E)$.

Therefore, if $\sigma_F(E)$ is quantized to $2e^2/h$, then $\bar{\sigma}(E)$
should be as well.

The above result is intuitive from a PAT point of view.
At a two-terminal lead energy $E \approx n\hbar\Omega$ the electrons
must emit $n$ photons to enter the quantized conduc-
tance channel and thus enter it with probability $P_{-n}$,
the probability to emit $n$ photons. This gives a conduc-
tance of $\sigma_F(0)P_{-n}$. Summing over all the lead energies is
then effectively summing over all of the probabilities as
$\bar{\sigma}(0) = \sigma_F(0) \sum_n P_n = \sigma_F(0)$, i.e. the sum rule recovers
the underlying conductance.

The above derivation can be generalized to on-resonant
driving under certain conditions \cite{Kundu2020}. In particular, one
expects the sum rule to hold when edge states are visible
in the so-called ”quasi-energy” spectrum. Nonetheless,
the derivation presented here contains all of the intuition
required to understand the sum rule.

In Fig. \[3\] we show $\bar{\sigma}(E)$ at $E = 0$ for various different
disorder strengths as well as $\sigma_F(E)$. Firstly, our data for
the clean system is in excellent agreement with $\sigma_F(E)$.

Secondly, the system shows noticeable deviations from
$\bar{\sigma}(E \approx 0) = 2e^2/h$ in two regimes of $V_{ext}$ and occur in
both the clean and disordered systems. Here the bulk
gap in the effective Hamiltonian closes, and the topologi-

cal edge states becoming washed out by bulk conduction
states. This is most obvious when looking at the disorder
averaged data where the regions with $\bar{\sigma}(E) = 2e^2/h$ are
insensitive to disorder, while the peaks are sensitive to
disorder, as bulk conduction states should be. This re-

tult is interesting from a PAT perspective. Not only has
the periodic field split the system into side bands, but it
has modified the underlying system in a non-trivial way.
In a traditional PAT context only the sideband splitting
would take place.

Conclusions.—We have developed an analogue of PAT
to describe the transport signatures of topologically pro-
tected edge states in the quantum well heterostructures.
Our picture entails electrons only accessing the topologi-
cal edge states of the system probabilistically. The prob-
ability of the electrons to absorb/emit a photon reduces
the traditional values associated with transport measure-
ments in these systems. These reduced values are, how-
ever, still insensitive to disorder and other deformations.
We refer to this phenomenon as “photon inhibited topo-
logical transport”.

By using this picture we related our system to a Flo-
quet sum rule proposed before \cite{Kundu2020}. Our picture of PAT
is able to offer a physical description of why one would
expect such a rule to hold. Namely, the sum rule is
adding up all of the probabilities of accessing the edge
state which, by itself, should have the traditional trans-
port signatures. This sum then reveals the underlying
transport properties.

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