Monte-Carlo Based QCD Sum Rules Analysis of \(X_0(2900)\) and \(X_1(2900)\)

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Motivated by the very recent discovery of fully open-flavor exotic states \(X_0(2900)\) and \(X_1(2900)\) by the LHCb Collaboration, we study possible interpretations of these exotic states by QCD Sum Rules method. \(X_0(2900)\) and \(X_1(2900)\) are studied in molecular and diquark-antidiquark tetraquark pictures, respectively. Obtained results for masses are in good agreement with the observed masses in the experiment. We also made a Monte-Carlo based analysis within QCD Sum Rules for the mass distributions to investigate the possible assignments for the structures of \(X_0(2900)\) and \(X_1(2900)\).

I. INTRODUCTION

The conventional quark model presents part of the properties of QCD in agreement with experimental data up to 2003 in which the first exotic hadron candidate \(X(3872)\) was observed by Belle Collaboration [1]. According to quark model, hadrons can be classified into two groups: mesons made of quark-antiquark pair and baryons made of three quarks. The spectrum of QCD is more rich than the spectrum of conventional quark model since any color neutral (colorless) configuration is possible in QCD.

Very recently the LHCb Collaboration reported an important discovery of exotic particles with open quark flavors in the invariant mass distribution of \(D^- K^+\) of the \(B^\pm \to D^\mp D^- K^\pm\) [2]. The structure have been parametrized in terms of two Breit-Wigner resonances as

\[
\begin{align*}
X_0(2900) & : \quad J^P = 0^+, \quad M = 2866 \pm 7 \text{ MeV}, \quad \Gamma_0 = 57 \pm 13 \text{ MeV}, \\
X_1(2900) & : \quad J^P = 1^-, \quad M = 2904 \pm 5 \text{ MeV}, \quad \Gamma_0 = 110 \pm 12 \text{ MeV},
\end{align*}
\]

with global significance of more than 5\(\sigma\). \(X_0(2900)\) is a narrow state whereas \(X_1(2900)\) is a broader one. These two states are 502 MeV and 540 MeV higher than the \(DK\) threshold, respectively. Both of these states decay into \(D^- K^+\) and as a result their quark content should be \([ud\bar{s}c]\). Since they have four different flavors, they could have an exotic nature.

Actually, this is not the first state consisting of four different flavor. D0 collaboration reported an exotic open quark flavor state \(X(5568)\) decaying into \(B^0_s \pi^0\) in 2016 [3] but not confirmed by LHCb [4], CMS [5], CDF [6] and ATLAS [7]. For recent studies on this exotic state see [8, 9]. Based on this phenomena, confirmation of discovery of recent open quark flavor states by other collaborators may help understanding of the QCD, especially low-energy region of it.

After the observation of the LHCb Collaboration, these two new states were studied by simple quark model [10]. In [11], the molecular nature of \(X_0(2900)\) state is investigated. They extracted the mass positions of its heavy quark spin partners. Possible partners of \(X_{0,1}(2900)\) were studied by two-body Coulomb and chromomagnetic interactions in [11]. In [12], triangle singularity of \(X_0(2900)\) and \(X_1(2900)\) was studied in the \(B^+ \to D^+ D^- K^+\) decay via the \(\chi_c K^+ D^-\) and \(D_{12}^0 D_{12}^0 K^0\) rescattering diagrams. QCD Sum Rules method was applied in [14] by using four possible interpolating currents with \(J^P = 0^+\). The mass spectra of open charm and bottom tetraquarks \(qq\bar{q}Q\) within an extended relativized quark model is calculated in [15]. \(D^{(*)} K^{(*)}\) system was studied within one-boson exchange model in molecular picture [16]. Molecular of compact tetraquark pictures of \(X_0(2900)\) and \(X_1(2900)\) were studied by QCD Sum Rules in [17]. They concluded \(X_0(2900)\) as a molecule state and \(X_1(2900)\) as compact diquark-antidiquark tetraquark state. In [18], axialvector-diquark-axialvector-diquark type and scalar-diquark-scalar-diquark type fully open flavor tetraquark states with \(J^P = 0^+\) were studied in QCD Sum Rules method. \(X_0(2900)\) and \(X_1(2900)\) states are studied whether two-body strong decays into \(D^- K^+\) via triangle diagrams and three-body decays into \(D^{(*)} K\) in [19].

In [20], \(X_0(2900)\) and \(X_1(2900)\) are studied in qBSE approach. Tetraquarks composed of \(ud\bar{s}c\) are investigated with meson-meson and diquark-antidiquark structures in the quark delocalization color screening mode [19]. Two states \((1^+\text{ and } 2^+)\) stemming from the \(D^{(*)} K^*\) interaction was studied in [20]. In [23], the LHCb vector \(ud\bar{s}c\) state \(X(2900)\) was studied whether it can be interpreted as a triangle cusp effect arising from \(D^{(*)} K^*\) and \(D_{1} K^{(*)}\) interactions. Mass and coupling of \(X_0(2900)\) are determined using the QCD two-point sum rule method in [24]. Branching ratios of \(B^- \to D^- X_{0,1}(2900)\) was studied in [25].

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An intriguing property of this experimental study is the widths of the resonances. The higher resonance \( X_1(2900) \) with \( J^P = 1^- \) has a significantly larger width than the lower resonance \( X_0(2900) \) with \( J^P = 0^+ \). Considering the \( X_1(2900) \rightarrow D^- K^+ \) decay is \( P^- \)-wave and \( X_0(2900) \rightarrow D^- K^+ \) is \( S^- \)-wave, this difference of widths may help to understand the possible interpretations of these resonances. For this purpose, we use QCD Sum Rules which is a powerful and successful nonperturbative method to handle these resonances.

The rest of the paper is organized as follows. In Section II, a brief introduction to QCD Sum Rules (QCDSR) is given. In Section III, Monte-Carlo based analysis is done and numerical results are presented for \( X_0(2900) \). Section IV is the same as previous section but for \( X_1(2900) \). Section V is a brief summary of this work.

II. QCD SUM RULES

QCD Sum Rules are formulated by Shifman, Vainshtein and Zakharov in 1979 [26] for mesons and generalized to baryons by Ioffe [27] in 1981. It is one of the celebrated method among non-perturbative methods such as lattice QCD, AdS/QCD, Chiral Perturbation Theory etc. The method is based on the study of a suitable chosen correlation function in two different kinematical regions.

On one side, it is calculated in the deep Euclidean region where the correlation function receives dominant contribution from short distances. In this case, the correlation function can be calculated using operator product expansion (OPE). On the other side, one calculates the correlation function for positive momentum squared. In this kinematical region, the correlation function can be expressed in terms of the properties of the hadrons (phenomenological side). The two expressions are matched using spectral representation of the correlation function, and hadronic properties are extracted by this matching.

The fundamental object of the QCD Sum Rule is the correlation function

\[
\Pi(q^2) = i \int d^4xe^{i x q} \langle 0 | T[j(x)j^+(0)] | 0 \rangle ,
\]

where \( j(x) \) is the interpolating current, \( q \) is the momentum of the state and \( T \) is the time ordering operator. Currents are suitably chosen operators made of quark and gluon fields that can create the studied hadron from vacuum. The underlying idea of QCDSR method is the assumption that there is an interval in momentum \( q \) which the correlation function may be equivalently represented at the both quark and hadron levels.

A. The OPE Side

The QCD degrees of freedom are quarks and gluon fields and we have to take care the populating of these fields into the QCD vacuum which is the complex structure of QCD. We can work analytically in the perturbative regime so that we can calculate perturbative part of \( \Pi(q) \) in Eq. (3). Due to the complex structure of QCD vacuum, expectation values of the operators of quarks fields and gluon fields are non-zero resulting what we call condensates. Condensate (vacuum field) contributions to the correlation function can be calculated via Wilson OPE.

Applying OPE to the Eq. (3) gives

\[
\Pi(q^2) = i \int d^4xe^{i x q} \langle 0 | T[j(x)j^+(0)] | 0 \rangle = \sum_d C_d(Q^2) \langle O_d \rangle ,
\]

where \( C_d(Q^2)(Q^2 = -q^2) \) is the Wilson’s coefficients, and \( \langle O_d \rangle \) is the expectation value of the composite local operators. The coefficients include only short-distance effects and can be calculated perturbatively. Long-distance effects which are non-perturbative are contained in the local operators. In the right hand side of Eq. (4), the operators are ordered by their dimensions \( d \).

In the OPE side, we evaluate the correlation function in terms of the OPE expansion and we have to take account the contributions from condensates to obtain good OPE convergence and reliable results. Using dispersion relation, the OPE side of Eq. (3) can be represented as

\[
\Pi^{OPE}(q^2) = \int_{s_{\text{min}}}^{\infty} \frac{\rho^{OPE}(s) ds}{s - q^2} ,
\]

where

\[
\rho^{OPE}(s) = \frac{1}{\pi} \text{Im}[\Pi^{OPE}(s)] ,
\]

is the spectral density function and \( s_{\text{min}} \) is a kinematical limit.
B. The Phenomenological Side

In the phenomenological side, the correlation function is evaluated considering the hadron itself as the degree of freedom. Calculation of the correlation function proceeds with inserting intermediate states for the hadron of interest. The correlation can be written as follows:

$$
\Pi^{\text{Phen}}(q^2) = i \int_0^\infty ds \frac{\rho(s)}{s - q^2} + \text{subtraction terms},
$$

where $\rho(s)$ is the spectral function. In order to QCDSR be useful, one must parametrize spectral function with a small number of parameters. The lowest resonance is often fairly narrow, whereas higher-mass resonances are broader. Hence, one can parametrize the spectral function as a single sharp pole decoding the lowest resonance of mass $m_H$, plus a smooth continuum representing higher mass resonances

$$
\rho(s) = \lambda^2 \delta(s - m_H^2) + \rho^{\text{cont}}(s),
$$

where $\lambda$ gives the coupling of the current to the lowest mass hadron $m_H$ as $\langle 0 | j | H \rangle = \lambda$. Putting this relation to the Eq. (7), one can get the following expression for the phenomenological side

$$
\Pi^{\text{Phen}}(q^2) = \frac{\lambda^2}{q^2 - m_H^2} + \lim_{s \to m_H^2} \int_{s_0}^\infty \frac{ds \rho^{\text{OPE}}(s)}{s - q^2} + \text{subtraction terms}.
$$

Subtraction terms are suppressed when a Borel transform is applied:

$$
B^2_M[\Pi(q^2)] = \left[ \lim_{-q^2, n \to \infty} \left( \frac{(-q^2)^n}{n!} \left( \frac{d}{dq^2} \right)^n \Pi(q^2) \right) \right],
$$

Borel transformation kills the subtraction terms and exponentially suppresses the contribution from excited resonances and continuum states in phenomenological side. Furthermore, in the OPE side, it suppresses factorially the contribution from higher dimension condensates which have inverse power of $q^2$. Then we can extract the $\lambda$ as well as the mass of the low lying state coupling to the interpolating current $j$. After transferring the continuum contribution to the OPE side, and performing a Borel transformation on both sides, the sum rule can be written as

$$
\lambda^2 e^{-m_H^2/\lambda^2} = \int_{s_{\text{min}}}^{s_0} ds \rho^{\text{OPE}}(s) e^{-s/\lambda^2}
$$

By taking the derivative of Eq. (11) with respect to $1/M^2$ and dividing the result by Eq. (11), one can obtain the mass

$$
m_H^2 = \int_{s_{\text{min}}}^{s_0} ds s \rho^{\text{OPE}}(s) e^{-s/M^2}.
$$

For more details of the QCDSR technique see [28, 29].

III. INTERPRETATION AND MONTE-CARLO ANALYSIS OF $X_0(2900)$ WITH $J^P = 0^+$

The observed peak of $X_0(2900)$ by LHCb is around 2.9 GeV. This is very close to the $D^{*-}K^{+}$ threshold at 2902 MeV. In this section, we assume $X_0(2900)$ has a molecular picture and created in the vacuum by the following interpolating current with $J^P = 0^+$:

$$
j(x) = [\bar{c}^a(x)\gamma_5 d^a(x)] [\bar{s}^b(x)\gamma_5 u^b(x)].
$$

Here the subscripts $a$ and $b$ are color indices and, $u$, $d$, $s$ and $c$ represent the up, down, strange and charm quark fields, respectively. The components $\bar{c}\gamma_5 d$ and $\bar{s}\gamma_5 u$ in the interpolating current are two generic meson operators. They couple to the $D^{*-}$ and $K^{*-}$ mesons, respectively. The current can couple to the $S$ wave $D^{*-}K^{*+}$ molecular state with $J^P = 0^+$. The coupling of current $j(x)$ to the $X_0$ state can be defined as

$$
\langle 0 | j | X_0 \rangle = f_{X_0}.
$$

The QCDSR calculation starts with obtaining correlation function in terms of the physical degrees of freedom. This is the first step and ends up with Borel transformed form of the function $\Pi^{\text{phys}}(q)$:

$$B_q \Pi^{\text{phys}}(q) = m_{X_0}^2 f_{X_0}^2 e^{-m_{X_0}^2/M^2} + \cdots.$$  

(15)

The next step is to find the theoretical expression for the same function, $\Pi^{\text{OPE}}(q)$. Contracting the quark fields yields

$$\Pi^{\text{OPE}}(q) = i \int d^4xe^{iqx} \text{Tr}[\gamma_5 S^{a\dagger}_c(x)S^{a\dagger}_d(-x)] \text{Tr}[\gamma_5 S^{a\dagger}_u(x)S^{a\dagger}_d(-x)],$$  

(16)

where $S^{ab}_q(x)$ with $q = u, d, s$ and $S^{ab}_c(x)$ are the light and heavy quark propagators, respectively. The light quark propagator can be written as

$$S^{ab}_q(x) = \frac{i\delta_{ab}}{2\pi^2} \frac{x^2}{12} - \frac{\delta_{ab}}{2\pi^2} \frac{\delta_m(q)}{48} - \frac{i\delta_{ab}}{2\pi^2} \frac{x^2}{192} \langle \bar{q}g_\sigma Gq \rangle + i\delta_{ab} \frac{x^2 m_4}{1152} \langle \bar{q}g_\sigma Gq \rangle$$

$$- \frac{\delta_{ab}}{2\pi^2} \frac{\delta_m(q)}{7776} \langle \bar{q}g_\sigma Gq \rangle - \frac{\delta_{ab}}{2\pi^2} \frac{x^2}{27648} \langle \bar{q}g_\sigma Gq \rangle + \cdots.$$  

(17)

For the heavy quark propagator $S^{ab}_c(x)$, we employ the following expression

$$S^{ab}_c(x) = i \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \left[ \frac{\delta_m(k)}{k^2 - m_c^2} - \frac{g_\sigma G^{ab}_c}{4} \frac{\sigma_{\alpha\beta}(k + m_c) + (k + m_c)\sigma_{\alpha\beta}}{(k^2 - m_c^2)^2} \right]$$

$$+ \frac{g_\sigma G^{ab}_c}{12} \delta_{ab} m_c \frac{k^2 + m_c^2}{(k^2 - m_c^2)^2} + \cdots.$$  

(18)

Here, $a, b = 1, 2, 3$ and $A, B, C = 1, 2, \cdots, 8$ are color indices. $t^A = \lambda^A/2$, and $\lambda^A$ are the Gell-Mann matrices. In the nonperturbative terms the gluon field strength tensor $G^{A}_{ab} = G^{A}_{ab}(0)$ is fixed at $x = 0$.

The correlation function $\Pi^{\text{OPE}}(q)$ can be written by a dispersion integral

$$\Pi^{\text{OPE}}(q) = \int_{s_{\text{min}}}^\infty \rho^{\text{OPE}}(s) \frac{ds}{s - q^2},$$  

(19)

where we parametrize $s_{\text{min}} = (m_c + m_\lambda)^2$. The sum rules for mass and decay constant (residue) can be obtained as mentioned before by equating $\Pi^{\text{phys}}(q)$ and $\Pi^{\text{OPE}}(q)$. The mass can be found as

$$m_{X_0}^2 = \int_{s_{\text{min}}}^{s_0} \frac{dss\rho^{\text{OPE}}(s)}{s} e^{-s/M^2}.$$  

(20)

The obtained sum rules depend on vacuum condensates and quark masses. We use the following values $[30]$:

$$m_c = (1.275^{+0.025}_{-0.035}) \text{ GeV},$$

$$m_d(2 \text{ GeV}) = 95^{+9}_{-3} \text{ MeV},$$

$$\langle \bar{q}q \rangle = (0.8 \pm 0.2) \text{ GeV}^2,$$

$$\langle \bar{s}s \rangle = m_0^2(\bar{s}s),$$

$$\langle \bar{q}q \rangle = (0.24 \pm 0.01)^3 \text{ GeV}^3,$$

$$\langle \bar{q}q \rangle = (0.8 \pm 0.1)\langle \bar{q}q \rangle,$$

$$\langle g^2 G^2 \rangle = (27648) \text{ GeV}^4.$$  

(21)

The resulting sum rules for mass is a function of Borel parameter $M^2$ and continuum threshold $s_0$. The correct choice of these parameters is an important task for sum rule calculations. One needs to find working regions for Borel and continuum threshold parameters where physical quantities do not extremely depend on these values or have a weak dependence on them. The QCDSR method suffers from the theoretical uncertainties, which are its unavoidable property. There exist some procedures to extract $M^2$ and $s_0$ values which are well defined in the context of the
QCDSR method itself. In order to maintain desired results, working regions for \( M^2 \) and \( s_0 \) should satisfy some constraints on the pole contribution (PC)

\[
PC = \frac{\Pi(M^2, s_0)}{\Pi(M^2, \infty)},
\]

and OPE convergence

\[
R(M^2) = \frac{\Pi_{\text{DimN}}(M^2, s_0)}{\Pi(M^2, s_0)}.
\]

Here, \( \Pi_{\text{DimN}}(M^2, s_0) \) is a last term in the correlation function. In the present work, we have employed QCD two-point sum rule method and take into account vacuum condensates only with dimension eight. The continuum threshold \( s_0 \) depends on the energy of the first excited state with the same quantum numbers and structure as the particle under consideration. In the case of the exotic states, it is difficult to determine unambiguously this energy level. In standard recipes, the continuum threshold \( s_0 \) is taken to be as \( s_0 = (m_{\text{ground}} + \delta)^2 \) where \( \delta \) varies between 0.3 and 0.8 GeV. Here we consider the LHCb Collaboration resonance as a ground state. The working region for Borel parameter \( M^2 \) is found by using PC and \( R(M^2) \). PC is used to fix upper bound of \( M^2 \) and \( R(M^2) \) is to find lower bound of \( M^2 \). Our analysis shows that for \( PC > 0.2 \), the working regions are

\[
M^2 \in [2, 3] \text{ GeV}^2, \quad s_0 \in [10, 12] \text{ GeV}^2.
\]

The predicted value for the mass of \( X_0 \) is

\[
M_{X_0} = 2792 \pm 124 \text{ MeV}
\]

where the uncertainty results from the Borel mass parameter \( M^2 \), continuum threshold value \( s_0 \), and various quark and gluon parameters. This value is in good agreement with the mass of experimental result for \( X_0(2900) \). In Fig. 1, we display prediction of sum rules for mass \( m \) as a function of Borel parameter \( M^2 \).

The next task is to study mass of \( X_0(2900) \) by using QCD sum rules with Leinweber’s Monte-Carlo based uncertainty analysis [31]. For some applications of this method see [32-34]. To do this, we first need to estimate the standard deviation \( \sigma_{\text{OPE}}(M^2) \) of \( \Pi^{\text{OPE}}(M^2) \) at any point in the sum rule interval. This estimation can be done by randomly generating 250 set of Gaussian distributed input parameters of QCD (condensates and \( \Lambda_{\text{QCD}} \)) with given uncertainties. Once the standard deviation is obtained the phenomenological output parameters \( (s_0, f, m) \) can be obtained by minimizing a weighted \( \chi^2 \):

\[
\chi^2 = \sum_{j=1}^{n\rho} \frac{(\Pi^{\text{OPE}}(M^2) - \Pi^{\text{Phys}}(M^2, s_0, f, m))^2}{\sigma_{\text{OPE}}(M^2)},
\]

FIG. 1: The mass of the \( X_0 \) as a function of the Borel parameter \( M^2 \) at fixed \( s_0 \).
where \( M^2 = M^2_{\text{min}} + (M_{\text{max}}^2 - M_{\text{min}}^2)(j-1)/(n_B-1) \) which means dividing the sum rule window into \((n_B-1)\) even parts. We set \( n_B = 50 \) and generated 3000 Gaussian distributed input parameters with given uncertainties (10\% uncertainties, which are typical uncertainties in QCDSR). The QCD input parameters are quark condensate, gluon condensate, mixed condensate values which are given in Eq. 21 and \( \Lambda_{\text{QCD}} = 0.353 \text{ GeV} \). We have selected the physical results since in the sum rules some constraints exist such as \( s_0 > m^2 \). We also plot the histogram for 3000 different \( X_0(2900) \) masses obtained in the least-squares fitting procedure. Fig. 2 shows the distribution of \( X_0 \) meson masses.

\[
\begin{align*}
\text{FIG. 2:} & \quad \text{The histogram of the } X_0 \text{ masses obtained from 3000 matches.} \\
\text{The distribution corresponds to} & \quad M_{X_0} = 2865^{+20}_{-18} \text{ MeV,} \\
\text{which is in good agreement with the experimental value. The uncertainty is lower than the commonly assumed } & \quad 10 \% \text{. It can be seen from Fig. 2 that the distribution of mass is very close to Gaussian curve.}
\end{align*}
\]

\[
\text{IV. INTERPRETATION AND MONTE-CARLO ANALYSIS OF } X_1(2900) \text{ WITH } J^P = 1^- 
\]

In this section, we investigate the diquark-antidiquark structure for the observed peak \( X_1(2900) \) with quantum number \( J^P = 1^- \). We use the following interpolating current

\[
j_\mu(x) = s_a^T(x)C_b(x)\left[\bar{u}_a(x)\gamma_\mu\gamma_5C\bar{d}_b(x) - \bar{u}_b(x)\gamma_\mu\gamma_5C\bar{d}_a(x)\right],
\]

where \( a \) and \( b \) are color indices and \( C \) is the charge conjugation matrix.

The correlation function can be written as

\[
\Pi_{\mu\nu}(q) = i \int d^4x e^{iqx} \langle 0|T[j_\mu(x)j_{\nu}^\dagger(0)]|0\rangle = \left(-g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{q^2}\right)\Pi_1(q) + \frac{q_{\mu}q_{\nu}}{q^2}\Pi_0(q),
\]

where \( \Pi_1(q) \) and \( \Pi_0(q) \) are the invariant functions related to the spin-1 and spin-0 intermediate states, respectively. Since \( \Pi_1(q) \) receives contributions only from spin-1 intermediate state, we use it to perform numerical analysis. In order to obtain QCDSR expression, we first have to calculate the correlation function in terms of the physical parameters of the studied hadron. Saturating the correlation function with a complete set of the \( X_1(2900) \) state, we find

\[
\Pi_{\mu\nu}^{\text{phys}}(q) = \frac{(0|j_\mu|X_1(q))\langle X_1(q)|j_{\nu}^\dagger|0\rangle}{m_{X_1}^2 - q^2} + \cdots,
\]

where the dots indicate contributions to the correlation function arising from the higher resonances and continuum states. We can define coupling \( f_{X_1} \) using the matrix element

\[
(0|j_\mu|X_1(q)) = f_{X_1}m_{X_1}\epsilon_\mu,
\]

\[
(0|j_\mu|X_1(q)) = f_{X_1}m_{X_1}\epsilon_\mu,
\]

\[
(0|j_\mu|X_1(q)) = f_{X_1}m_{X_1}\epsilon_\mu,
\]
where $\epsilon_{\mu}$ is the polarization vector of the $X_1(2900)$ state. With this definition correlation function can be written as

$$\Pi^{\text{Phys}}_{\mu\nu}(q) = \frac{f_{X_1}^2 m_{X_1}^2}{m_{X_1}^2 - q^2} \left( -g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{m_{X_1}^2} \right) + \cdots. \quad (32)$$

Applying Borel transformation to Eq. (32) yields

$$B_{q^2}\Pi^{\text{Phys}}_{\mu\nu}(q) = \frac{f_{X_1}^2 m_{X_1}^2}{m_{X_1}^2} e^{-\frac{m_{X_1}^2}{M^2}} \left( -g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{m_{X_1}^2} \right) + \cdots \quad (33)$$

At second step, the correlation function has to be calculated from the QCD (OPE) side. Contracting the heavy and light quark fields yields

$$\Pi^{\text{OPE}}_{\mu\nu}(q) = i \int d^4xe^{iqx} \text{Tr} \left[ \gamma_{\mu} \tilde{S}^{a'b'}_d(x) \gamma_{\nu} S^b_a(-x) \right] \{ \text{Tr} \left[ \gamma_{\mu} \tilde{S}^{a'b'}_d(-x) \gamma_{\nu} S^b_a(-x) \right] - \text{Tr} \left[ \gamma_{\mu} \tilde{S}^{a'b'}_d(-x) \gamma_{\nu} S^a_{u'}(-x) \right] \}
+ \text{Tr} \left[ \gamma_{\mu} \tilde{S}^{a'b'}_d(-x) \gamma_{\nu} S^{b'a}_u(-x) \right] - \text{Tr} \left[ \gamma_{\mu} \tilde{S}^{a'b'}_d(-x) \gamma_{\nu} S^{b'a}_u(-x) \right]. \quad (34)$$

Here we use the notation $\tilde{S}_q(x) = CST(x)C$. The $\Pi^{\text{OPE}}_{\mu\nu}(q)$ can be written by a dispersion integral

$$\Pi^{\text{OPE}}_{\mu\nu}(q) = \int_0^{\infty} \frac{ds\rho^{\text{OPE}}(s)}{s - q^2}. \quad (35)$$

After normal procedure defined previous section, the mass can be found as

$$m_{X_1}^2 = \int_{(m_c + m_s)^2}^{s_0} ds\rho^{\text{OPE}}(s)e^{-s/M^2} \int_{(m_c + m_s)^2}^{s_0} ds\rho^{\text{OPE}}(s)e^{-s/M^2}. \quad (36)$$

The working regions for continuum threshold $s_0$ and Borel parameter $M^2$ are the same as given in previous section. The predicted value for the mass of $X_1$ is

$$M_{X_1} = 2963 \pm 64 \text{ MeV} \quad (37)$$

which is in good agreement with the mass of experimental result for $X_1(2900)$. In Fig. 3, we display prediction of sum rules for mass $m$ as a function of Borel parameter $M^2$.

![FIG. 3: The mass of the $X_1$ as a function of the Borel parameter $M^2$ at fixed $s_0$.](image)

The histogram for masses of $X_1$ can be seen in Fig. [4].
FIG. 4: The histogram of the $X_1$ masses obtained from 3000 matches.

The distribution corresponds to

$$M_{X_1} = 2900 \pm 19 \text{ MeV},$$  \hspace{1cm} (38)

which is in good agreement with the experimental value. The uncertainty is lower than the commonly assumed 10 %. The distribution of mass is very close to Gaussian curve.

V. DISCUSSION, SUMMARY AND CONCLUDING REMARKS

In this present work, we have studied two new resonances $X_0$ and $X_1$ observed by the LHCb collaboration. This is the first time observing fully open-flavor exotic states. We studied their possible interpretations using QCD sum rules method.

We used a molecular current (meson-meson tetraquark) with $J^P = 0^+$ to study mass of $X_0(2900)$. The extracted value is

$$M_{X_0} = 2792 \pm 124 \text{ MeV}$$  \hspace{1cm} (39)

which is in good agreement with the experimental mass of $X_0(2900)$ suggesting a possible molecular picture as $J^P = 0^+ D^- K^+$. This molecular picture was supported in [35], in which the authors made a prediction for the $0^+$ state with mass 2848 MeV and $\Gamma = 59$ MeV before the experiment.

A diquark-antidiquark tetraquark interpolating current with $J^P = 1^-$ was used to study mass of $X_1(2900)$. The extracted value is

$$M_{X_1} = 2963 \pm 64 \text{ MeV}$$  \hspace{1cm} (40)

which is in good agreement with the experimental mass of $X_1(2900)$.

The mass values of traditional analysis and Monte-Carlo analysis are given in Table I. As can be seen from Table I, results for both of the traditional analysis and Monte-Carlo analysis are in good agreement with experimental values. The error limit is very close to experimental values in Monte-Carlo analysis.

|        | Traditional | Monte-Carlo | Experiment |
|--------|-------------|-------------|------------|
| $X_0$  | 2792 $\pm$ 124 | 2865$^{+20}_{-18}$ | 2866 $\pm$ 7 |
| $X_1$  | 2963 $\pm$ 64  | 2900 $\pm$ 19   | 2904 $\pm$ 5 |

An interesting feature of this experiment is that $X_1(2900)$ has larger width than the $X_0(2900)$. For this purpose, we made a Monte-Carlo based analysis for the masses of these states. It can be seen from Figs. 2 and 4, $X_0(2900)$ has a narrower shape than the $X_1(2900)$. In other words, $X_0(2900)$ has a small width than the $X_1(2900)$ has. This analysis support the molecular picture for $X_0(2900)$ and diquark-antidiquark tetraquark structure for $X_1(2900)$. 
It is clear that the computation of mass alone does not allow us to make a conclusion on the internal structure of an exotic state. In QCDSR formalism, one cannot deduce if a state have a tetraquark configuration or molecular configuration. The two interpretations made above are just possible assignments for these states. More theoretical and experimental studies are needed to investigate these exotic states.

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