Testing a hadronic rescattering model for RHIC collisions using the subdivision method

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Abstract

Recently it has been shown that calculations based on a hadronic rescattering model agree rather well with experimental results from the first RHIC run. Because of the large particle densities intrinsically present at the early time steps of Monte Carlo calculations attempting to model RHIC collisions undesirable artifacts resulting in non-causality may be present. These effects may compromise the results of such calculations. The subdivision method, which can remove such artifacts, has been used to test the present rescattering model calculations. It is shown that no appreciable changes are seen in the present calculations in using the subdivision method, thus strengthening the confidence in the results of this rescattering model for RHIC.

Key words: rescattering model, subdivision, RHIC
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1 Introduction

Recent hadronic rescattering model calculations have been shown to agree reasonably well with experimental results obtained in the first physics run for the Relativistic Heavy Ion Collider (RHIC) with $\sqrt{s} = 130$ GeV Au + Au collisions [1][2]. More specifically, these calculations provide good representations of the data for 1) the particle mass dependence of the $p_T$ distribution slope parameters (i.e. radial flow), 2) the $p_T$, particle mass, and pseudorapidity dependences of the elliptic flow, and 3) the $p_T$ and centrality dependences.

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of the pion Hanbury-Brown-Twiss (HBT) measurements. This is a remarkable accomplishment for a single model.

Many hadrons are initially produced in a relatively small volume in RHIC-type collisions, particularly at early times in the interaction. It is a challenging task for a Monte Carlo calculation of this type to deal accurately with binary collisions between particles in such a large particle density environment without introducing numerical artifacts which may affect the results. One such undesirable artifact that can occur when the interaction range between two particles is much greater than the scattering mean-free-path is non-causality of the collisions [3][4]. The method of subdivision can be used to minimize these artifacts associated with high particle density[5].

The goal of this work is to apply the method of subdivision to the present RHIC-energy rescattering calculations to test whether the results from these calculations without subdivision are valid. Calculations will be compared with and without subdivision for the pion observables 1) $(1/p_T)dN/dp_T$, 2) elliptic flow vs. $p_T$ and $\eta$, and 3) HBT vs. $p_T$, at a collision impact parameter of 8 fm. Section II will describe the calculational methods used and Section III with give the results of the study.

2 Calculational Methods

2.1 Hadronic rescattering calculation

A brief description of the rescattering model calculational method is given below. The method used is similar to that used in previous calculations for lower CERN Super Proton Synchrotron (SPS) energies [6]. Rescattering is simulated with a semi-classical Monte Carlo calculation which assumes strong binary collisions between hadrons. The Monte Carlo calculation is carried out in three stages: 1) initialization and hadronization, 2) rescattering and freeze out, and 3) calculation of experimental observables. Relativistic kinematics is used throughout. All calculations are made to simulate RHIC-energy Au+Au collisions in order to compare with the results of the $\sqrt{s} = 130$ GeV RHIC data.

The hadronization model employs simple parameterizations to describe the initial momenta and space-time of the hadrons similar to that used by Herrmann and Bertsch [7]. The initial momenta are assumed to follow a thermal transverse (perpendicular to the beam direction) momentum distribution for
all particles,
\[
\frac{1}{m_T} \frac{dN}{dm_T} = C m_T / \left[ \exp \left( \frac{m_T}{T} \right) \pm 1 \right]
\]

(1)

where \( m_T = \sqrt{p_T^2 + m_0^2} \) is the transverse mass, \( p_T \) is the transverse momentum, \( m_0 \) is the particle rest mass, \( C \) is a normalization constant, and \( T \) is the initial “temperature parameter” of the system, and a gaussian rapidity distribution for mesons,
\[
\frac{dN}{dy} = D \exp \left[ -\left( y - y_0 \right)^2 / (2\sigma_y^2) \right]
\]

(2)

where \( y = 0.5 \ln \left[ (E + p_z) / (E - p_z) \right] \) is the rapidity, \( E \) is the particle energy, \( p_z \) is the longitudinal (along the beam direction) momentum, \( D \) is a normalization constant, \( y_0 \) is the central rapidity value (mid-rapidity), and \( \sigma_y \) is the rapidity width. Two rapidity distributions for baryons have been tried: 1) flat and then falling off near beam rapidity and 2) peaked at central rapidity and falling off until beam rapidity. Both baryon distributions give about the same results. The initial space-time of the hadrons for \( b = 0 \) fm (i.e. zero impact parameter or central collisions) is parameterized as having cylindrical symmetry with respect to the beam axis. The transverse particle density dependence is assumed to be that of a projected uniform sphere of radius equal to the projectile radius, \( R \) (\( R = r_0 A^{1/3} \), where \( r_0 = 1.12 \) fm and \( A \) is the atomic mass number of the projectile). For \( b > 0 \) (non-central collisions) the transverse particle density is that of overlapping projected spheres whose centers are separated by a distance \( b \). The particle multiplicities for \( b > 0 \) are scaled from the \( b = 0 \) values by the ratio of the overlap volume to the volume of the projectile. The longitudinal particle hadronization position (\( z_{had} \)) and time (\( t_{had} \)) are determined by the relativistic equations [8],
\[
z_{had} = \tau_{had} \sinh y; t_{had} = \tau_{had} \cosh y
\]

(3)

where \( y \) is the particle rapidity and \( \tau_{had} \) is the hadronization proper time. Thus, apart from particle multiplicities, the hadronization model has three free parameters to extract from experiment: \( \sigma_y \), \( T \) and \( \tau_{had} \). The hadrons included in the calculation are pions, kaons, nucleons and lambdas (\( \pi, K, N, \) and \( \Lambda \)), and the \( \rho, \omega, \eta, \eta', \phi, \Delta, \) and \( K^* \) resonances. For simplicity, the calculation is isospin averaged (e.g. no distinction is made among a \( \pi^+, \pi^0, \) and \( \pi^- \)). Resonances are present at hadronization and also can be produced as a result of rescattering. Initial resonance multiplicity fractions are taken from Herrmann and Bertsch [7], who extracted results from the HELIOS experiment [9]. The initial resonance fractions used in the present calculations are: \( \eta/\pi = 0.05, \rho/\pi = 0.1, \rho/\omega = 3, \phi/(\rho + \omega) = 0.12, \eta'/\eta = K^*/\omega = 1 \) and, for simplicity, \( \Delta/N = 0 \).
The second stage in the calculation is rescattering which finishes with the freeze out and decay of all particles. Starting from the initial stage \((t = 0 \text{ fm/c})\), the positions of all particles are allowed to evolve in time in small time steps \((dt = 0.1 \text{ fm/c})\) according to their initial momenta. At each time step each particle is checked to see a) if it decays, and b) if it is sufficiently close to another particle to scatter with it. Isospin-averaged s-wave and p-wave cross sections for meson scattering are obtained from Prakash et al. \[10\]. The calculation is carried out to 100 fm/c, although most of the rescattering finishes by about 30 fm/c. The rescattering calculation is described in more detail elsewhere \[6\].

Calculations are carried out assuming initial parameter values and particle multiplicities for each type of particle. In the last stage of the calculation, the freeze-out and decay momenta and space-times are used to produce observables such as pion, kaon, and nucleon multiplicities and transverse momentum and rapidity distributions. The values of the initial parameters of the calculation and multiplicities are constrained to give observables which agree with available measured hadronic observables. As a cross-check on this, the total kinetic energy from the calculation is determined and compared with the RHIC center of mass energy of \(\sqrt{s} = 130 \text{ GeV}\) to see that they are in reasonable agreement. Particle multiplicities were estimated from the charged hadron multiplicity measurements of the RHIC PHOBOS experiment \[11\]. Calculations were carried out using isospin-summed events containing at freezeout for central collisions \((b = 0 \text{ fm})\) about 5000 pions, 500 kaons, and 650 nucleons \((\Lambda's \text{ were decayed})\). The hadronization model parameters used were \(T = 300 \text{ MeV}, \sigma_y=2.4, \text{ and } \tau_{had}=1 \text{ fm/c}\). It is interesting to note that the same value of \(\tau_{had}\) was required in a previous rescattering calculation to successfully describe results from SPS Pb+Pb collisions \[6\].

2.2 Subdivision method

The method of subdivision is based on the invariance of Monte Carlo particle-scattering calculations for a simultaneous decrease of the scattering cross sections by some factor, \(l\), and increase of the particle density by \(l^2\)\[3\]. As \(l\) becomes sufficiently large, non-causal artifacts become insignificant. The present rescattering calculation will be tested comparing pion observables for no-subdivision, i.e. \(l = 1\), with subdivision of \(l = 5\). A subdivision of \(l = 5\) was chosen since when applied to other Monte Carlo scattering calculations this value has been shown to produce large changes in observables when non-causal artifacts are present\[3\][4]. Since the particle density increase is accomplished by increasing the particle number by a factor \(l\), the computer CPU time taken per event increases by a factor \(l^2\), effectively “slowing down” a subdivision study by a factor \(l\) compared with not using subdivision. For the present study, 1640
events and 124 events were generated for the $l = 1$ and $l = 5$ samples, respectively (the statistical value of the $l = 5$ events being 5 times greater per event). The CPU time taken to generate the $l = 5$ sample of events was 420 CPU-hours on a 1 GHz PC processor.

3 Results

Figures 1-4 show comparisons of $l = 1$ with $l = 5$ hadron rescattering model calculations for pion observables. Descriptions of how the observables are extracted from the rescattering calculation are given elsewhere[2]. All calculations are carried out for an impact parameter of 8 fm to simulate a medium non-central collision which should result in significant elliptic flow for the purposes of the present test. Statistical errors are shown either as error bars or are of the order of the marker size when error bars are not shown.

Figure 1 compares the $p_T$ distributions for a rapidity cut of $-2 < y < 2$. As seen, the $l = 1$ and $l = 5$ calculations agree within statistical errors, although there is a slight decreasing trend for $l = 5$ compared with $l = 1$ for $p_T > 3$ GeV/c.

Figures 2 and 3 compare elliptic flow ($V_2$). Figure 2 shows $V_2$ vs. $p_T$ for a rapidity cut of $-2 < y < 2$ and Figure 3 shows $V_2$ vs. $\eta$ for a $p_T$ cut of $0 < p_T < 3$ GeV/c. For both cases, the $l = 1$ and $l = 5$ calculations are seen to agree within statistical errors, reproducing the “flattening” in $V_2$ at large $p_T$ and the “peaked” dependence of $V_2$ on $\eta$.

Finally, Figure 4 compares HBT results vs. $p_T$ for a rapidity cut of $-2 < y < 2$. Each result is calculated in a $p_T$ bin of 0.2 GeV/c. A three-dimensional fit is made to extract the pion source parameters $R_{T\text{side}}$, $R_{T\text{out}}$, $R_{T\text{long}}$, and $\lambda$. As seen in this figure, the $l = 1$ and $l = 5$ calculations agree within statistical errors for all parameters at all $p_T$ bins, reproducing the decreasing trend of the $R$-parameters and increasing trend of the $\lambda$-parameter with increasing $p_T$.

It can be concluded from Figures 1-4 that non-causal artifacts of the type which can be removed by the subdivision method are not present in the hadronic rescattering code used in this study. It is possible to speculate on why this is the case. There are three main features of the code which might tend to reduce these artifacts: 1) individual particles are allowed to scatter only once per time step, 2) a “scattering time” of two time steps is defined during which particles that have scattered are not allowed to rescatter, and 3) once two particles have scattered with each other, they are not allowed to scatter with each other again in the calculation. The present study was performed for non-central collisions ($b = 8$ fm), but it is expected that the same
Fig. 1. Pion $p_T$ distributions for $l = 1$ and $l = 5$.

conclusion, i.e. that subdivision does not significantly effect the results of the calculation, would also be obtained for central collisions. This is because the particle density is not very different for central collisions in the model than for mid-peripheral collisions since the particle multiplicities are scaled by the overlap volume for $b > 0$, as mentioned earlier.

In summary, the subdivision method has been used to test the validity of the present rescattering model calculations. It is found that no appreciable changes are seen comparing $l = 1$ and $l = 5$ calculations, thus strengthening the confidence in the results presented previously from this rescattering model for RHIC.

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Rescattering model: RHIC, pions
\( b = 8 \text{ fm}, \; 0 < p_T < 3 \text{ GeV/c} \)

Fig. 3. Pion Elliptic flow vs. pseudorapidity (\( \eta \)) for \( l = 1 \) and \( l = 5 \).
Fig. 4. Pion HBT vs. $p_T$ for $l = 1$ and $l = 5$. The ordinate scale for $\lambda$ is shown to the right.