Synthesis of a time-optimal control system for an extremal object

G.A. Pikina¹, F.F. Pashchenko², A.F. Pashchenko³
¹ Moscow Power Engineering Institute, Moscow, Russia
² Institute of Control Sciences of RAS, Moscow, Russia

E-mail: paschenko_alex@mail.ru

Abstract. The paper presents the derivation of the synthesis method for the algorithm of the time-optimal controller for a third order dynamic system. A model with an extreme second-order transient response with delay was adopted as an object of research. The constant speed actuator is represented by an integrator. The synthesis is based on using the Pontryagin’s maximum principle and describing the dynamics of a system in the state space using canonical variables. The verification of the correctness of the result obtained by the theorem of Feldbaum A.A. on the number of switchings of the direction of movement of the regulating body during the control interval has been executed. To calculate the canonical state variables, it is proposed to use the position of the regulator, the controlled value and the derivative calculated from its values, measured on real objects.

1. Introduction

Among the optimal control algorithms, the most interesting are the time-optimal algorithms, which, under deterministic influences, provide a minimum transient time, and under random influences, most likely, the minimum value of the variance of the controlled variable. The widespread use of time-optimal algorithms is hindered, in particular, by the difficulties in obtaining an exact analytical solution for controlled systems of high order (higher than second order). Most of the works devoted to this issue deal with second-order systems. In [1], an exact solution is given for a third-order system with zero roots, in [2, 3], an algorithm is obtained for a speed-optimal control for a third-order system with two real and one zero roots, which reflects the control of an object with an s-shaped transient characteristic by an integral executive mechanism.

![Figure 1](image-url)  

Figure 1. Extreme transient response of the object
However, when managing objects of thermal and nuclear power plants, there are cases when the transient characteristic of the object has a pronounced extremum (Figure 1).

Such objects include the channels “reactivity - neutron flux density” and “reactivity - steam pressure in front of the turbine” of a nuclear power unit with a VVER reactor [4], “position of a three-way shunting valve - temperature of secondary steam superheating” of a boiler, “air flow - dust consumption” of a boiler hammer mill.

To reflect the extremum of the transient characteristic of the object, we take for its transfer function the simplest second-order model with a first-order polynomial in the numerator and pure delay

$$W_o(p) = \frac{K(T_3p + 1)}{(T_1p + 1)(T_2p + 1)}e^{-p\tau_o} = \frac{y(p)}{\mu(p)},$$

(1)

in which the value of the extreme is determined by the choice of the value of the time constant $T_3$. Here $K$ is the amplification factor; $T_1$, $T_2$, $T_3$ - time constants; $p$ is the Laplace variable; $y(t)$, $\mu(t)$ - controlled value and control action. Let us supplement the object model (1) with a model of an actuator of a regulating body of the MEO type, which uses an asynchronous electric motor of constant speed $S$. Considering that the time-optimal control has a relay character, the actuator can be represented by the transfer function of an integrating link

$$W_{im}(p) = \frac{S}{p} = \frac{\mu(p)}{u(p)},$$

(2)

and restriction on the control action module (direction of valve movement) $|u(t)| \leq 1$.

2. Obtaining the maximum performance algorithm

Let us formulate the synthesis problem:

for a linear third-order control system with delay, consisting of an object with a transfer function (1) and an actuator with a transfer function (2), find an optimal control algorithm $u^*(X)$ that minimizes the integral criterion

$$I = \int_0^T dt = T \rightarrow \min$$

the initial state of the system $X(0)$ is arbitrary, the final state of the system is given $X(T) = 0$, the control satisfies the constraint $|u(t)| \leq 1$.

We synthesize the optimal control algorithm $u^*(X)$ using Pontryagin’s maximum principle and representing the dynamics of the system in the state space $X$.

In accordance with the relay character of the optimal control, each point of the phase space corresponds to control $+U$ or $-U$, i.e. the entire phase space is divided into two areas: in one of them, $+U$ control is optimal, in the other $-U$ control is optimal. These areas are separated by the switching hypersurface $\Pi$, shown in Figure 2.
The problem of synthesizing a maximum performance system is to find the equation of the switching surface, which consists of two half-surfaces for positive and negative control, respectively.

It is known [1], that the synthesis of the maximum performance algorithm for a system with delay is carried out without taking into account the delay. However, in the obtained control algorithm, one should substitute not the current values of the state variables \( X(t) \), but their predicted values for the lag time of the object \( X(t + \tau_0) \).

To preserve the linear character of the system of equations after the forthcoming transformations, it is advisable to use the canonical state variables, therefore, the fractional-rational part of the transfer function of system (1), (2) is represented as a sum of simple fractions:

\[
W(p) = \frac{KS(T_3 p + 1)}{p(T_1 p + 1)(T_2 p + 1)} = \frac{b_1}{p} + \frac{b_2}{p + \alpha_1} + \frac{b_3}{p + \alpha_2} = \frac{x_1(p)}{u(p)} + \frac{x_2(p)}{u(p)} + \frac{x_3(p)}{u(p)}
\]

where

\[
\alpha_1 = \frac{1}{T_1}, \quad \alpha_2 = \frac{1}{T_2}, \quad b_1 = KS, \quad b_2 = KS \frac{T_3 - T_1}{T_1 - T_2}, \quad b_3 = -KS \frac{T_3 - T_2}{T_1 - T_2}.
\]

System of differential equations for state variables \( x_1, x_2, x_3 \) according to the decomposition of \( W(p) \) will look like:

\[
\begin{align*}
x_1' &= b_1 \cdot u, \\
x_2' &= b_2 \cdot u - \alpha_1 x_2, \\
x_3' &= b_3 \cdot u - \alpha_2 x_3, \\
y &= x_1 + x_2 + x_3.
\end{align*}
\]

We solve the system of differential equations for arbitrary initial conditions in reverse time by replacing \( dt = -d\tau \), taking into account the constancy of the optimal control \( u \) on separate control sections. Adding a condition for turning off the control in a small neighborhood of the origin, we obtain an algorithm with the maximum control speed:
3. Example

Let us check the reliability of the obtained algorithm (3) using the example of an inertial system object-executive mechanism with following parameters of transfer functions:

\[ K=1; \quad T_1=80 \text{ s}; \quad T_2=50 \text{ s}; \quad T_3=150 \text{ s}; \quad \tau_0=0; \quad S=1/30 \text{ s}^{-1} \]

The control processes in the optimal system with the initial value \( x_1(0) = 0.5 \), which with the accepted initial data is equivalent to applying a step action to the input of the object \( \Delta \mu(t) = 0.5 \cdot 1(t) \), are given in Figure 3.

![Figure 3. Regulation processes with internal step action: 1 – change of controlled variable; 2 – change of position of the regulatory body](image)

Can be seen that during the control interval the regulator changes the direction of movement of the regulating body twice, which is fully consistent with the Feldbaum’s theorem on the number of switchings. At the end of the interval, the controlled variable returns to its original state. The regulation time was approximately 130 s, i.e. the sum \( (T_1 + T_2) \) of the time constants of the transfer function of the object.

4. Conclusion

The paper proposes a method for synthesizing an algorithm for time-optimal controller of a third-order dynamic system. A model with an extreme second-order transient response with delay was adopted as an object of research. The constant speed actuator is represented by an integrator. The synthesis is based on using the Pontryagin’s maximum principle and description of the system dynamics in the state space using canonical variables.

The canonical state variables used for analytical synthesis are mathematical abstractions that can be calculated from the physical variables measured on a real object. The output value \( y(t) \) and the position of the regulator \( \mu(t) \) are almost always measurable. In addition to them, calculated derivatives \( y'(t) \) and \( \mu'(t) \) can be used.
Shall be noted that in the time optimal system the speed of movement of the actuator is always constant and equal to $|\mu'(t)| = \dot{S}$. Only the sign of the derivative changes depending on the sign of the control $u$.

A feature of real control objects is the need to add a delay link to the transfer function of the inertial object model. In contrast to the probabilistic approach of correcting the controller algorithm in the presence of a delay [5], one can use a simpler approach - predicting the vector of state variables $X(t + \tau)$ over time $\tau_0$. Even the simplest - linear forecast of the form

$$x_i(t + \tau_0) = x_i(t) + x_i'(t)\tau_0$$

provides a quite satisfactory result.

The predicted values of the state variables should be substituted into the equations of system (13) of the time-optimal algorithm.

5. References

[1] Feldbaum A.A. Foundations of the Theory of Optimal Automatic Systems), Moscow: Nauka, 1970.
[2] Pikina G.A., Kocharovsky D.N. Comparison of the efficiency of the time-optimal predictive algorithm and the PID algorithm in a closed-loop automatic control system // Teploenergetika. No. 1, 2007. pp. 62-68.
[3] G.A. Pikina. Predictive Time Optimal Algorithm for a Third-Order Dynamical System with Delay. Proceedings of PTPPE International Conference 9—11 October 2017. // «Journal of Physics: Conference Series». V. 891. No. 012278. 2017.
[4] Pikina G.A., Dinh L.V., Pashchenko A.F., Pashchenko F.F. The dynamic models of water-water nuclear reactor with temperature reactivity coefficients // Proceedings of the 2015 10th IEEE Conference on Industrial Electronics and Applications, ICIEA 2015. IEEE, 2015. pp. 1014-1019.
[5] Novoseltsev V.N. On optimal control in the presence of delay // Automation and Remote Control. 1964. Vol. 25, No. 11.