Gauge coupling renormalization in orbifold field theories

Kiwoon Choi
Department of Physics, Korea Advanced Institute of Science and Technology Daejeon
305-701, Korea
E-mail: kchoi@hep.kaist.ac.kr

Hyung Do Kim
School of Physics, Korea Institute for Advanced Study Seoul 235-010, Korea
E-mail: hdkim@kias.re.kr

Ian-Woo Kim
Department of Physics, Korea Advanced Institute of Science and Technology Daejeon
305-701, Korea
E-mail: iwikim@hep.kaist.ac.kr

ABSTRACT: We investigate the gauge coupling renormalization in orbifold field theory preserving 4-dimensional $N = 1$ supersymmetry in the framework of 4-dimensional effective supergravity. As a concrete example, we consider the 5-dimensional Super-Yang-Mills theory on a slice of AdS$_5$. In our approach, one-loop gauge couplings can be determined by the loop-induced axion couplings and the tree level properties of 4-dimensional effective supergravity which are much easier to be computed.

KEYWORDS: Renormalization Group, Field Theories in Higher Dimension, Supersymmetric Effective Theories.
1. Introduction

Recently higher-dimensional field theories compactified on orbifold have been proposed as models providing an efficient mechanism for symmetry breaking, e.g. the supersymmetry (SUSY) breaking [1, 2] and/or the grand unified gauge symmetry breaking [3]. One can construct realistic grand unified models more efficiently in such framework [4, 5]. Higher-dimensional orbifold models may also lead to a geometric understanding of various hierarchical mass scales in particle physics [6, 7], the suppression of some Yukawa couplings [8], and the $b$-$t$ mass ratio [9].

In this paper, we wish to discuss the gauge coupling renormalization in orbifold field theories preserving the 4-dimensional (4D) $N = 1$ supersymmetry in the framework of 4D effective supergravity (SUGRA). We are particularly interested in theories with large scale hierarchies, for instance a model in which the Kaluza-Klein (KK) threshold scale is significantly lower than the cutoff scale of the theory so that the KK towers are important for the gauge coupling renormalization. In such cases, it is quite convenient to consider the gauge coupling renormalization in the framework of 4D effective SUGRA since the 1-loop gauge couplings can be determined by the loop-induced axion couplings and the tree level properties of 4D effective SUGRA which are much easier to be computed. As a concrete example, we will consider 5D SUGRA-coupled super-Yang-Mills (SYM) theory on a slice of 5D Anti-de Sitter space (AdS$_5$) [10, 11, 12, 13, 14] with four well-separated mass scales: the KK scale, the orbifold length, the AdS curvature, and finally the cutoff scale. However, much of the discussions here can be easily extended to generic higher dimensional orbifold field theories. Also the results for a flat supersymmetric 5D geometry can be obtained from our AdS results by taking the limit that the AdS curvature becomes zero.

The organization of this paper is as follows. In section II, we discuss some features of the supersymmetric gauge theory on a slice of AdS$_5$, including the gauged $U(1)_R$ symmetry and also the possible form of gauge coupling renormalization. In section III, we derive the
4D effective SUGRA of the 5D theory on AdS\(_5\) and match the renormalized low energy gauge couplings in 5D theory with the moduli-dependent gauge couplings in 4D effective SUGRA. With this matching, one can determine the 1-loop gauge couplings by the 1-loop induced axion couplings and the tree level properties of 4D effective SUGRA. Section IV is the conclusion.

2. Supersymmetric gauge theory on a slice of AdS\(_5\)

In this section, we discuss some features of the SUGRA coupled SYM theory on a slice of AdS\(_5\) which is orbifolded w.r.t \(y \to -y \) and \(y \to y + 2\pi\) where \(y\) is the coordinate of the 5-th dimension. The model under consideration contains 5D vector multiplets for YM fields, 5D hypermultiplets for charged matter fields, as well as the 5D SUGRA multiplet. To be general, we also assume that there are charged brane matter fields living on 3-branes at the orbifold fixed points \(y = 0\) and \(\pi\).

The action of the SUGRA multiplet is given by \cite{10, 11}

\[
S_{\text{sugra}} = -\frac{1}{2} \int d^4x dy \sqrt{-G} M_*^2 \left\{ \mathcal{R} + \bar{\Psi}_i \gamma^{MNP} D_N \Psi_i P + \frac{3}{2} C^{MN} C_{MN} \right. \\
- \frac{1}{4\sqrt{-G}} \epsilon^{MNPQR} B_M C_{NP} C_{QR} - \frac{3}{2} k \epsilon(y) \bar{\Psi}_i \gamma^{MN} (\sigma_3)_{ij} \Psi^j_N \\
- 12k^2 + \frac{\left( \delta(y) - \delta(y - \pi) \right)}{\sqrt{G_{55}}} 12k + \ldots \right\},
\]

(2.1)

where \(\mathcal{R}\) is the 5D Ricci scalar for the metric \(G_{MN}\), \(C_{MN} = \partial_M B_N - \partial_N B_M\) is the field strength of the graviphoton \(B_M\), \(\Psi_i^i M\) \((i = 1, 2)\) are the symplectic Majorana gravitinos, \(M_*\) is the 5-dimensional Planck scale, \(k\) is the AdS curvature, and the ellipsis stands for the higher dimensional terms which are not relevant for the present discussion. Here the gravitino kink mass and the brane cosmological constants are determined by supersymmetry, and the indices \(i, j\) label the fundamental representation of the \(SU(2)_R\) automorphism group of 5D SUGRA, which is raised or lowered by \(\epsilon^{ij} = \epsilon_{ij} = (i\sigma_2)^{ij}\) in the NW-SE convention. The SUGRA multiplet has the standard \(Z_2\) boundary condition:

\[
e^A_M(-y) = Z^A_B Z^B_M e^B_N(y), \\
\Psi^i_M(-y) = Z^N_M (\sigma_3)^i_j \gamma^5 \Psi^j_N(y), \\
B_M(-y) = -Z^N_M B_N(y),
\]

(2.2)

where \(e^A_M\) is the 5-bein and

\[
Z^N_M = Z^B_A = \text{diag}(1, 1, 1, 1, -1).
\]

The above orbifolding breaks the \(N = 2\) SUSY down to \(N = 1\). One may break the residual \(N = 1\) SUSY by imposing a nontrivial boundary condition of \(\Psi^i_M\) for \(y \to y + 2\pi\) (the Sherk-Schwarz SUSY breaking). In this paper, we consider only the orbifolding preserving \(N = 1\) SUSY, so all gravity multiplets are assumed to be periodic under \(y \to y + 2\pi\). The
above action for SUGRA multiplet leads to the following AdS$_5$ metric as a solution of the equations of motion:

$$ds^2 = e^{-2k|y|R}g_{\mu\nu}dx^\mu dx^\nu + R^2dy^2,$$

(2.3)

where $g_{\mu\nu}$ denotes the massless 4D graviton and $R$ is the orbifold radius.

The action of vector multiplets is given by \[1\]

\[
S_{\text{vector}} = -\frac{1}{2} \int d^4xdy\sqrt{-G} \left\{ \frac{1}{2g_5^2} F^a_{MN} F^a_{MN} + D_M \phi^a D^M \phi^a + \bar{\lambda}^i \gamma^M D_M \lambda^i + \frac{1}{4\sqrt{-Gg_{5a}}} \epsilon^{MNPQR} B_M F^a_{NP} F^a_{QR} + \frac{1}{2} \kappa \epsilon(y) \bar{\lambda}^i (\sigma_3)_{ij} \lambda^j - 4k^2 \phi^a \phi^a + \frac{\delta(y)}{\sqrt{G_{55}}} \left( \frac{1}{2g_5^2} F^{a\mu\nu} F^a_{\mu\nu} + 4k\phi^a \phi^a + \ldots \right) + \frac{\delta(y-\pi)}{\sqrt{G_{55}}} \left( \frac{1}{2g_5^2} F^{a\mu\nu} F^a_{\mu\nu} - 4k\phi^a \phi^a + \ldots \right) + \ldots \right\},
\]

(2.4)

where $F^a_{MN}$ are the YM field strengths, $\lambda^i$ are the symplectic Majorana gauginos, $\phi^a$ are real scalar fields spanning a very special target manifold, and the ellipses denote the higher dimensional terms. Again the gaugino kink mass and the scalar (bulk and brane) masses are determined by supersymmetry. Here we are interested in the vector multiplets containing zero mode gauge fields, so the $Z_2$ boundary conditions are given by

$$A^a_M (-y) = Z^N_M A^a_N (y), \quad \lambda^i (-y) = (\sigma_3)^{ij} \gamma_5 \lambda^j (y), \quad \phi^a (-y) = -\phi^a (y),$$

(2.5)

where all fields in the vector multiplet are assumed to be periodic under $y \to y + 2\pi$.

The action of hypermultiplets has the form \[1\]

\[
S_{\text{hyper}} = \int d^4xdy\sqrt{-G} \left\{ |D_M h^i_I|^2 + i \bar{\Psi}_I \gamma^M D_M \Psi_I + ic_I k \epsilon(y) \bar{\Psi}_I \Psi_I + \sum_I \sum_i \left( \left( c_i^2 + \epsilon_i c_I - \frac{15}{4} k^2 + k(3 - 2\epsilon_i c_I) \right) \frac{\delta(y) - \delta(y - \pi)}{\sqrt{G_{55}}} \right) |h^i_I|^2 + \ldots \right\},
\]

(2.6)

where $h^i_I$ are two complex scalar fields in the $I$-th hypermultiplet spanning a quaternionic target manifold with the tangent space group $SU(2)_R \times Sp(2n_H)$ ($I = 1, \ldots, n_H$), $\Psi_I$ are the Dirac fermions with kink mass $c_I k \epsilon(y)$, and $\epsilon_{1,2} = \pm 1$. Under the $[SU(2)]^{n_H}$ subgroup of $[SU(2)]^{n_H} \subset Sp(2n_H)$, the hypermultiplets transform as

$$h^i_I \to e^{i\alpha_I} h^i_I, \quad \Psi_I \to e^{i\alpha_I} \Psi_I,$$

while the $SU(2)_R$ transformation can be read off from the index $i$. Here the hypermultiplets are allowed to have nontrivial boundary conditions for both $y \to -y$ and $y \to y + 2\pi$:

$$h^i_I (-y) = \omega_I (\sigma_3)^{ij} h^j_I (y), \quad h^i_I (-y + \pi) = \eta_I (\sigma_3)^{ij} h^j_I (y + \pi),$$

$$\Psi_I (-y) = \omega_I \gamma_5 \Psi_I (y), \quad \Psi_I (-y + \pi) = \eta_I \gamma_5 \Psi_I (y + \pi),$$

(2.7)

\[1\] Here we choose the scalar field fluctuations not being the coordinates of the quaternionic target manifold, but being a fundamental representation of the target space group $SU(2)_R \times Sp(2n_H)$, which is always possible with the $4n_H$-beins on the quaternionic manifold.
where \( \omega_I = \pm 1 \) and \( \eta_I = \pm 1 \). Note that \( y + \pi \rightarrow -y + \pi \) corresponds to the successive transformation of \( y \rightarrow -y \) and \( y \rightarrow y + 2\pi \).

To obtain supersymmetric AdS background, we need to gauge \( U(1)_R \) symmetry by graviphoton. It has been noted in [15] that the gauge coupling of \( B_M \) is required to be \( \mathbb{Z}_2 \)-odd when the 5D SUGRA model contains matter fields and nontrivial brane actions. Then the \( U(1)_R \) gauge transformation and the corresponding covariant derivative are given by

\[
\Phi \rightarrow e^{-i\epsilon(y)\Omega(x,y)}g_B T_B \Phi,
\]

\[
B_M \rightarrow B_M + \partial_M \Omega,
\]

\[
D_M \Phi = \partial_M \Phi + ic(y)g_B T_B B_M \Phi,
\]

(2.8)

where \( g_B \) is a coupling constant, \( T_B \) is the gauged \( U(1)_R \) generator, and \( \Omega \) is a continuous gauge function obeying the orbifolding boundary condition

\[
\Omega(y) = -\Omega(-y) = \Omega(y + 2\pi).
\]

Note that \( \Omega(0) = \Omega(\pi) = 0 \) guarantees that (i) \( D_M \Phi \) has the same gauged \( U(1)_R \) transformation as \( \Phi \), (ii) the Chern-Simons terms in the action are invariant under the gauged \( U(1)_R \), (iii) there is no \( U(1)_R \)-anomaly. It also means that bulk and brane matter fields at \( y = 0, \pi \) are all invariant under the gauged \( U(1)_R \).

In order for the \( U(1)_R \) covariant derivative to be consistent with the SUSY transformation, \( T_B \) is required to commute with the \( \mathbb{Z}_2 \) transformation associated with \( y \rightarrow -y \).

Without loss of generality, such \( U(1)_R \) generator can be written as

\[
g_B T_B = g_RT_{3R} + g_IT_{3I},
\]

(2.9)

where \( T_{3R} \) is the \( U(1) \)-generator of \( SU(2)_R \) and \( T_{3I} \) is the \( U(1) \)-generator of the \( I \)-th \( SU(2) \) subgroup of \( Sp(2n_H) \). It is then straightforward to find (see Appendix)

\[
g_R = -3k, \quad g_I = c_I k,
\]

(2.10)

yielding the following form of covariant derivatives:

\[
D_M h_I^i = \nabla_M h_I^i - i \left( \frac{3}{2} (\sigma_3)^i_j - c_I \delta^i_j \right) k \epsilon(y) B_M h_J^j,
\]

\[
D_M \Psi_I = \nabla_M \Psi_I + ic_I k \epsilon(y) B_M \Psi_I,
\]

\[
D_M \lambda^{ia} = \nabla_M \lambda^{ia} - i \frac{3}{2} (\sigma_3)^i_j k \epsilon(y) B_M \lambda^{aj},
\]

(2.11)

where \( \nabla_M \) denotes the covariant derivative containing other gauge fields including the spin connection.

In addition to \( S_{sugra} \), \( S_{vector} \) and \( S_{hyper} \), there can be additional brane actions involving the brane fields as well as the bulk fields at the fixed points. Such brane actions are required

\[\text{Note that} \ T_B \ \text{can always be matched with an isometry generator in the quaternionic manifold of hypermultiplet scalar fields.}\]
to be invariant under the unbroken $N = 1$ SUSY generated by the Killing spinor on AdS$_5$ which will be discussed in Appendix. Also the gauged $U(1)_R$ enforces that $B_M$ can appear in the brane actions only through $C_{\mu5} = \partial_{\mu}B_5 - \partial_5 B_{\mu}$, not through the covariant derivative of matter field. As a result, the brane actions do not have any non-derivative coupling of $B_5$ in the field basis of $S_{\text{vector}}$ and $S_{\text{hyper}}$.

Let us now consider the mass scales involved in the model. Generically, orbifold field theories on a slice of AdS$_5$ have the KK scale, i.e. the scale where the massive KK modes start to appear and/or the KK level spacing, given by

$$M_{KK} \simeq \frac{\pi k}{e^{\pi kR} - 1}. \quad (2.12)$$

In the limit of large AdS curvature $\pi kR \gg 1$, the KK scale is exponentially suppressed, $M_{KK} \simeq \pi k e^{-\pi kR}$, so we have the mass scale hierarchies

$$\mu \ll M_{KK} \ll 1/R \ll k \ll M_*, \quad (2.13)$$

where $\mu$ denotes the low energy scale for currently available experiments. In the other limit with small AdS curvature $\pi kR \ll 1$, the geometry is (approximately) flat and the KK scale is given by

$$M_{KK} \simeq 1/R, \quad (2.14)$$

for which the scale hierarchies are given by

$$\mu \ll M_{KK} \simeq 1/R \ll M_* \quad (2.15)$$

The low energy couplings of gauge field zero modes in AdS$_5$ would appear as a dimensionless function of the involved mass scales $M_{KK}, R, k, M_*$ as well as of the bare couplings $g_5^2, g_0^2, g_{\pi}^2 \, [13, 14]$. (Here we assume that $M_*$ corresponds to the cutoff scale of the model.) At tree level, the 4D gauge couplings $g_5^2$ are simply given by

$$\left( \frac{1}{g_5^2} \right)_{\text{tree}} = \frac{\pi R}{g_{5a}^2} + \frac{1}{g_0^2} + \frac{1}{g_{\pi a}^2}. \quad (2.12)$$

At 1-loop order, there can be two types of quantum corrections: those which are power-law dependent on the involved energy scales and the others which are logarithmic in scales. As for the power-law dependent part, it is dominated by the contribution from the cutoff scale $M_*$, while the logarithmic part receives equally important contributions from all scales. When $\pi kR \gg 1$ so that we have the mass hierarchy (2.13) with $M_{KK} \approx \pi k e^{-\pi kR}$, writing the dimensionless 1-loop 4D gauge coupling at low energy scale $\mu \ll M_{KK}$ in terms of the involved mass scales in a manner having sensible limiting behavior at large $R$, we find that

---

3Our orbifold field theory has many scalar fields, e.g. scalar fields in 5D vector or hypermultiplets, which can have nonzero vacuum expectation values (VEV) $\langle \sigma \rangle$ in general. In case that $\langle \sigma \rangle \gg \mu$, we should take into account this additional mass scale in the analysis. Here we assume that there is no such mass scale, so no more mass scales other than $M_{KK}, 1/R$ and $k$ between $\mu$ and $M_*$. 

---
$g_a^2(\mu)$ can be generically written as\(^4\).

$$
\frac{1}{g_a^2(\mu)} = \left[ \frac{1}{g_{5a}(M_5)} + \frac{\gamma_a}{8\pi^3 M_5} \right] \pi R + \frac{1}{g_{5a}(M_5)} + \frac{1}{g_{\pi a}(M_5)} + \frac{C_a}{8\pi^2} + \frac{b_a'}{8\pi^2} \ln \left( \frac{M_5}{k} \right) + \frac{b_a''}{8\pi^2} \ln \left( \frac{1}{M_{KK} R} \right) + \frac{b_a'''}{8\pi^2} \ln \left( \frac{M_{KK}}{\mu} \right),
$$

(2.16)

where $C_a$ are some constants which do not depend on any of $\mu, M_{KK}, R$ and $k$ at one-loop approximation, and $b_a$ are the conventional one-loop beta function coefficients receiving the contribution only from the massless 4D modes at scales below $\mu$. The other coefficients $b'_a, b''_a, b'''_a$ and $\gamma_a$ receive the contribution from the KK towers. Among them, $b'_a, b''_a$ and $b'''_a$ can be unambiguously calculated within the orbifold field theory as they reflect the infrared property of the model below the cutoff scale $M_5$, while $\gamma_a$ are uncalculable as they reflect the unknown UV physics around $M_5$. We stress that the logarithms $\ln(M_I/M_J)$ in (2.16) for $\{M_I\} = \{M_5, k, 1/R, M_{KK}\}$ can not be interpreted solely as the coupling running between $M_I$ and $M_J$. There can be some contribution from the coupling running, particularly from the running localized at the orbifold fixed points. However they include also the finite KK threshold corrections. So (2.16) should be interpreted as one simple way to reorganize the whole quantum corrections including both the KK threshold effects and the running effects.

In case with large scale ratios given by (2.13), the large logs of $b_a, b'_a$ and $b''_a$ provide important corrections to the 4D gauge couplings. As for $C_a$ which are independent of mass scales, they are generically of order unity, so subleading compared to the large-logs. One can also make a strong coupling assumption\(^4\) on the uncalculable bare brane couplings

$$
\frac{1}{g_{5a}^2(M_5)} \sim \frac{1}{g_{\pi a}^2(M_5)} = \mathcal{O} \left( \frac{1}{8\pi^2} \right),
$$

(2.17)

and then they are also subleading compared to the large-logs.

As for the power-law running corrections, their coefficients $\gamma_a$ highly depend on the way of UV cutoff. Of course, if the bulk gauge group is a simple group, $\gamma_a$ will be $a$-independent. In other cases that the bulk gauge group is not unified, one may compute $\gamma_a$ in certain regularization scheme and argue that different 5D gauge couplings rapidly approach to each other (when the scale is increased) due to the power-law running governed by $\gamma_a$.\(^7\) However, power-law running can not be considered as a calculable property of orbifold field theory since it is highly sensitive to the unknown UV physics.\(^18\)\(^19\). This can be easily noticed by changing the cutoff $M_5 \rightarrow c_a M_5$ for $a$-dependent constants $c_a$ which are generically of order unity. This change of cutoff leads to $\gamma_a \rightarrow c_a \gamma_a$ and represents the effects of unknown threshold effects at $M_5$. As usual, the cutoff-scheme dependence of $\gamma_a$ should be cancelled by the cutoff scheme dependence of the corresponding Wilsonian couplings $g_{5a}^2(M_5)$. It is thus not meaningful to split the power-law running part from the

\(^4\)Here we consider the case with $M_5 \gg k$ to see the gauge coupling renormalization proportional to $\ln(M_5/k)$. However in most of the practical applications of the model, one assumes $k \sim M_5$. The gauge coupling renormalization in such case can be obtained from our result by simply ignoring $\ln(M_5/k)$.
bare coupling $1/g_5^2$ in orbifold field theory. Rather, one has to consider the cutoff-scheme
independent combination
\[ \kappa_a M_* \equiv \frac{1}{g_5^2(M_*)} + \frac{\gamma_a}{8\pi^3} M_*. \] (2.18)

Summarizing the above discussions, when we have the scale hierarchy (2.13), the 1-loop
low energy gauge couplings in AdS$_5$ can be written as
\[ \frac{1}{g_a^2(\mu)} = \kappa_a \pi M_* R + \frac{1}{8\pi^2} \Delta_a + O \left( \frac{1}{8\pi^2} \right), \] (2.19)
where
\[ \Delta_a = b_a''' \ln(M_*/k) + b_a'' \ln(kR) + b_a' \ln(1/M_{KK} R) + b_a \ln(M_{KK}/\mu) \] (2.20)
for $\mu \ll M_{KK} \approx \pi k e^{-\pi kR}$. Here the 4D momentum $\mu$ is measured in the metric frame of
massless 4D graviton, and $\kappa_a$ are uncalculable bare parameters associated with the bare
5D gauge couplings $g_5^2$. The other uncalculable bare brane couplings $g_0^2, g_{\pi a}$ are assumed
to give subleading corrections of order $1/8\pi^2$. The 1-loop correction $\Delta_a$ include both the
KK threshold effects and the running effects which can be unambiguously calculated within
the orbifold field theory. In the next section, we will calculate $\Delta_a$ using the 4D effective
SUGRA of 5D theory on AdS$_5$.

3. Matching with 4D effective SUGRA

In this section, we derive the 4D effective SUGRA of the 5D SYM theory on a slice of AdS$_5$,
and calculate the low energy gauge coupling (2.16) in the framework of 4D effective SUGRA.
Generic 4D SUGRA action is determined by the real Kähler potential $K$, the holomorphic
gauge kinetic function $f_a$ and the holomorphic superpotential $P$. In superspace, the action
takes the form
\[ \int d^4x d^4\theta \left\{ -3 \exp \left( \frac{-K}{3} \right) \right\} + \int d^4x d^2\theta \left( \frac{1}{4} f_a W_a^{\alpha a} W^a_\alpha + P \right) + h.c \] (3.1)
where the gravity multiplet fields are set to their Poincare invariant VEV. The gauge kinetic
function $f_a$ determines the 4D Wilsonian gauge coupling ($g_{Wa}$) and the vacuum angle ($\Theta_a$) as
\[ f_a = \frac{1}{g_{Wa}} + i \frac{\Theta_a}{8\pi^2}, \] (3.2)
and the Kähler potential can be expanded in powers of gauge-charged chiral superfields:
\[ K = K_0(T, T^*) + Z_\Phi(T, T^*)\Phi^* e^{-V} + ..., \] (3.3)
where $T$, $\Phi$, and $V$ denote the gauge-singlet moduli superfields, gauge-charged chiral matter
superfields, and vector gauge superfields, respectively.

It has been found in [20] a relation between the beta function and the anomalous
dimension in 4D $N = 1$ supersymmetric gauge theory, which is exact in perturbation
theory. Using this relation, one can express the low energy one-particle-irreducible (1PI)
gauge couplings in terms of $f_a$ and the Kähler metric $Z_\Phi$ of charged matter fields. This procedure can be generalized to find a relation between the moduli-dependent 1PI gauge coupling and the moduli-dependent Wilsonian couplings in 4D SUGRA \cite{21}. For moduli-independent UV regulator, the super-Weyl and Kähler invariance of 4D SUGRA determines the 1PI gauge coupling as

$$\frac{1}{g_a^2(\mu)} = \text{Re}(f_a) + \frac{1}{8\pi^2} \left[ b_a \ln \left( \frac{M_{Pl}}{e^{-K_0/6} \mu} \right) - \sum_\Phi T_a(\Phi) \ln \left( \frac{Z_\Phi}{e^{K_0/3}} \right) + T_a(\text{Adj}) \ln \left( \frac{1}{g_a^2(\mu)} \right) \right],$$

(3.4)

where $M_{Pl}$ is the 4D Planck scale of the metric $g_{\mu\nu}$ which is used to measure the external momentum $\mu^2 = -g^{\mu\nu} \partial_\mu \partial_\nu$, $T_a(\Phi) = \text{Tr}([T_a(\Phi)]^2)$ is the Dynkin index of the gauge group representation $\Phi$, and $b_a$ is the one-loop beta function coefficient:

$$b_a = -3T_a(\text{Adj}) + \sum_\Phi T_a(\Phi).$$

Note that the moduli-dependence of $M_{Pl}$ differs in different metric frame. For instance, in the 4D superconformal frame in which the action is given by (3.1), the moduli-dependence of $M_{Pl}$ is given by $e^{-K_0/6}$, while in the 4D Einstein frame which is obtained from the superconformal frame after the Weyl scaling $g_{\mu\nu} \rightarrow e^{K_0/3} g_{\mu\nu}$, $M_{Pl}$ is moduli-independent. When one matches the 4D SUGRA coupling (3.4) with the 1PI coupling computed in underlying orbifold field theory, one has to use the same metric frame as well as the same moduli-independent regulator \cite{22}. In the one-loop approximation, the above low energy gauge couplings can be written as

$$\frac{1}{g_a^2(\mu)} = \text{Re}(f_a) + \frac{1}{8\pi^2} \left[ b_a \ln \left( \frac{M_{Pl}}{e^{-K_0/6} \mu} \right) - \sum_\Phi T_a(\Phi) \ln \left( \frac{Z_\Phi^{(0)}}{e^{K_0/3}} \right) + T_a(\text{Adj}) \ln (\text{Re}(f_a)) \right],$$

(3.5)

where $Z_\Phi^{(0)}$ is the tree level Kähler metric of $\Phi$.

Using (3.5), one can find the 1-loop gauge coupling in 5D gauge theory on a slice of AdS$_5$ by computing $f_a, K_0$ and $Z_\Phi^{(0)}$ of the 4D effective SUGRA. As was noted in the previous section, all scale hierarchies in 5D theory are determined by the orbifold radius $R$, so the dominant 1-loop renormalization can be determined by computing the $R$-dependence of the 4D effective SUGRA. The only part of (3.5) which is undetermined by tree-level analysis is the 1-loop threshold correction to $f_a$. However due to the holomorphicity of $f_a$, in our case of 5D theory on AdS$_5$, the 1-loop piece of $f_a$ is determined by the loop-induced axion coupling which can be easily computed by using the chiral anomaly structure of the theory.

\footnote{This relation was confirmed by an explicit computation in \cite{21} at 1-loop order.}
In order to derive the 4D effective SUGRA action of the 5D theory on AdS$_5$, it is convenient to rewrite the 5D actions $S_{\text{vector}}$ and $S_{\text{hyper}}$ in terms of the $N = 1$ superfields of unbroken SUSY[23]. This procedure involves a $R$ and $B_5$-dependent field redefinition which would generate a new piece of brane action through the chiral anomaly. The Killing spinor generating the unbroken $N = 1$ global SUSY is given by (see Appendix)

$$e^{-\frac{1}{2}k|y|(R-3iB_5)}\eta,$$

where $\eta$ is a constant 4D Weyl spinor. It is then straightforward to see that the 5D vector multiplet $\{\phi^a, A^a_\mu, \lambda^a \}$ can be decomposed into an $N = 1$ vector superfield $V^a$ and a chiral superfield $\chi^a$ whose component fields are given by $\{A^a_\mu, \lambda^a \}$ and $\{\tilde{\phi}^a + iA_5, \tilde{\zeta}^a \}$, respectively, where

$$\tilde{\lambda}^a(x, y) = e^{-\frac{3}{2}k|y|(R+iB_5)}\lambda^a(x, y),$$

$$\tilde{\zeta}^a(x, y) = Re^{-\frac{1}{2}k|y|(R-3iB_5)}\zeta^a(x, y),$$

$$\tilde{\phi}^a(x, y) = R\phi^a(x, y).$$

where $\lambda^a = \frac{1}{2}(1 + \gamma_5)\lambda^1a$ and $\zeta^a = \frac{1}{2}(1 - \gamma_5)\lambda^2a$. The 5D hypermultiplet $\{h_I^I, \chi_I \}$ can be similarly decomposed into two $N = 1$ chiral superfields $H_I$ and $h_I^I$ whose component fields are given by $\{h^I_1, \tilde{\psi}_I \}$ and $\{\tilde{h}^2_I, \tilde{\phi}_I \}$, respectively, where

$$\tilde{h}^1_1(x, y) = e^{-(\frac{3}{2} - c_1)|y|(R+iB_5)}h^1_1(x, y),$$

$$\tilde{h}^2_1(x, y) = e^{(\frac{3}{2} + c_1)|y|(-R+iB_5)}h^2_1(x, y),$$

$$\tilde{\psi}_I(x, y) = e^{-k|y|(2 - c_1)R-ic_1B_5}\psi_I(x, y),$$

$$\tilde{\phi}_I(x, y) = e^{-k|y|(2 + c_1)R+ic_1B_5}\phi_I(x, y),$$

for $\psi_I = \frac{1}{2}(1 + \gamma_5)\psi_I$ and $(\phi_I^*)^* = \frac{1}{2}(1 - \gamma_5)\phi_I$.

On each brane, there can be a brane action containing $N = 1$ supersymmetric ($B_5$-dependent) gaugino-scalar-fermion interactions $\phi_{\text{br}}^a\psi_{\text{br}}\lambda^a$ and/or $\phi_{\text{br}}^a\psi_{\text{br}}\lambda_{\text{br}}$, where $\{\phi_{\text{br}}, \psi_{\text{br}}\}$ and $\{A^a_{\text{br}}, \lambda_{\text{br}}\}$ are the $N = 1$ matter and gauge multiplets, respectively, living only on the brane. In order to rewrite such $U(1)_R$-invariant brane actions in $N = 1$ superspace, one needs also to redefine the brane fermions as follows:

$$\tilde{\psi}_{\text{br}} = e^{-\frac{1}{2}k|y_{\text{br}}|(R-3iB_5)}\psi_{\text{br}},$$

$$\tilde{\lambda}_{\text{br}} = e^{-\frac{1}{2}k|y_{\text{br}}|(R+iB_5)}\lambda_{\text{br}},$$

where $y_{\text{br}} = 0$ or $\pi$ denotes the location of the brane where the field lives on. Then the $N = 1$ chiral brane superfield $Q_{\text{br}}$ is given by the redefined component fields $\{\phi_{\text{br}}, \tilde{\psi}_{\text{br}}\}$ and the $N = 1$ vector brane superfield $V_{\text{br}}$ by $\{A^a_{\text{br}}, \tilde{\lambda}_{\text{br}}\}$.

After the above field redefinitions, the bulk actions $S_{\text{vector}}$ and $S_{\text{hyper}}$ can be written in $N = 1$ superspace[12, 23] as follows

$$S_{\text{bulk}} = \int d^5x \left[ \int d^4\theta \left( \frac{1}{2}(T + T^\ast)e^{-(T+T^\ast)k|y|} \left( M_3 + M_4 e^{(\frac{3}{2} - c_1)(T+T^\ast)k|y|} H_I e^{-V} H_I \right) \right) \right].$$

- 9 -
\[
+ M_s e^{\frac{1}{2}(T + T^*) k |y|} H_I^e V H_I^e) + 2 \kappa_a M_s e^{-\left(\frac{1}{2}(T + T^*) k |y|\right)}(\partial_y V_a - \frac{1}{\sqrt{2}}(\chi^a + \chi^{a*}))^2 \right]
\]
\[
+ \int d^2 \theta \left\{ \frac{\kappa_a M_s}{4} T W^{a\alpha} W_a^\alpha + H^c(\partial_y - \frac{1}{\sqrt{2}}(\chi) H + h.c. \right\}, \tag{3.9}
\]

where \( T \) is the radion superfield given by
\[
T = R + iB_5 + \theta \psi_5 + \theta^2 F_T,
\]
for the fifth-component of the graviphoton \((B_5)\) and \(\psi_5 = \frac{1}{2}(1 + \gamma_5)\Psi^{i=2}_5\). Here we included the pure radion action coming from \(S_{\text{sugra}}\). The brane actions can be also written in \(N = 1\) superspace as
\[
S_{\text{brane}} = \int_{y=0}^{d^4 x} \left[ \int d^4 \theta Q^*_{UV} e^{-V} Q_{UV} + \int d^2 \theta \left( \frac{1}{4g^2_{UV}} W^\alpha_{UV} W_{UV\alpha} + P_{UV}(Q_{UV}, H_I) \right) \right]
\]
\[
+ \int_{y=\pi}^{d^4 x} \left[ \int d^4 \theta e^{-\kappa(T + T^*)} Q^*_{IR} e^{-V} Q_{IR} + \int d^2 \theta \left( \frac{1}{4g^2_{IR}} W^\alpha_{IR} W_{IR\alpha}
\right.
\]
\[
\left. + e^{-\kappa \pi T} P_{IR}(Q_{IR}, e^{\frac{(3-c)}{2}\kappa T} H_I) \right) \right] + \text{h.c.}, \tag{3.10}
\]

where \(Q_{UV}\) and \(Q_{IR}\) are the brane chiral superfields living on the UV \((y = 0)\) and IR \((y = \pi)\) brane, respectively, and \(W_{UV}\) and \(W_{IR}\) are the chiral spinor superfields for the brane vector superfields \(V_{UV}\) and \(V_{IR}\).

At classical level, the superfield action \(S_{\text{bulk}} + S_{\text{brane}}\) describes the same theory as the component field action before the field redefinitions \([3.1], \tag{3.7}\) and \([3.8]\). However at quantum level, we should include the anomaly terms induced by these field redefinitions at one-loop order. Since there is no chiral anomaly in 5D bulk, anomalies appear only at the orbifold fixed points \([24, 25]\), which can be easily calculated to be
\[
S_{\text{anomaly}} = \int d^5 x \int d^2 \theta \left[ \left\{ \frac{3}{4} T_a(\chi^a)(\delta(y) + \delta(y - \pi)) - \frac{1}{2} c_T T_a(\Psi_I)(\omega_I \delta(y) + \eta_I \delta(y - \pi)) \right. \right.
\]
\[
\left. \left. - \frac{3}{2} T_a(\psi_{IR}) \delta(y - \pi) \right\} \frac{k|y|T}{16\pi^2} W^{a\alpha} W_a^\alpha \right.
\]
\[
+ \frac{3}{4} \left( T_{IR}(\lambda_{IR}) - T_{IR}(\psi_{IR}) \right) \delta(y - \pi) \frac{k\pi T}{16\pi^2} W_{IR\alpha} W_{IR\alpha} \right]. \tag{3.11}
\]

It is now straightforward to derive the tree level 4D effective action from the above 5D actions in \(N = 1\) superspace. The bulk superfields \(T\) and \(W_a\) have \((y\text{-independent})\) zero modes as they have \((+, +)\) boundary condition under \(y \to -y\) and \(y + \pi \to -y + \pi\). On the other hand, \(H_I\) can have \((y\text{-independent})\) zero mode only when \((\omega_I, \eta_I) = (+, +),\) while \(H_I^e\) does only when \((\omega_I, \eta_I) = (-, -).\) By integrating over \(y\), one easily finds the following radion Kähler potential \(K_0\), the tree level Kähler metric \(Z^{(0)}_{H_I}\), and the tree level gauge kinetic function \(f_a^{(0)}\) for the massless 4D modes of \(T, H_I\) or \(H_I^e,\) and \(W_a:\)
\[
e^{-\frac{K_0}{3}} = (1 - e^{-\pi k(T + T^*)}) \frac{M_s^3}{k} = M_{Pl}^2,
\]
\[
e^{-\frac{K_0}{3}} Z^{(0)}_{H_I} = \frac{M_s}{(\frac{1}{2} - \omega_I c_I)k}(e^{(\frac{1}{2} - \omega_I c_I)\pi k(T + T^*)} - 1),
\]
\[
f_a^{(0)} = \kappa_a \pi M_s T. \tag{3.12}
\]
Here we ignored the contribution to $f_{a}^{(0)}$ from the fixed point gauge couplings $1/\hat{g}_{a}^{2}$ based on the strong coupling assumption (3.17). The contribution to $Z_{H_{I}}^{(0)}$ from the fixed point kinetic terms of bulk fields are ignored also by the similar reasoning. The tree-level Kähler metrics and gauge kinetic functions of brane superfields are also easily obtained to be

$$e^{-K_{0}/3}Z_{Q_{UV}}^{(0)} = 1, \quad e^{-K_{0}/3}Z_{Q_{IR}}^{(0)} = e^{-\pi k(T+T^{*})},$$

$$f_{IR}^{(0)} = \frac{1}{\hat{g}_{IR}^{2}}, \quad f_{UV}^{(0)} = \frac{1}{\hat{g}_{UV}^{2}}. \quad (3.13)$$

The $T$-dependent one-loop threshold corrections to gauge kinetic functions can be determined by the loop-induced axion ($B_{5}$) couplings to gauge fields: $B_{5}e^{\mu\nu\rho\sigma}F_{\mu\nu}^{a}F_{\rho\sigma}^{a}$. There can be two sources of such axion couplings. One is $S$-anomaly and the other is the one-loop threshold effects of massive KK modes. A nice feature our field basis is that $B_{5}$ does not have any non-derivative coupling in $S_{\text{bulk}}$ other than the Chern-Simons coupling. As a result, in our field basis, integrating out the massive KK modes does not generate any $B_{5}$-coupling to gauge fields. Then the $T$-dependent one-loop corrections to gauge kinetic functions are entirely given by $S_{\text{anomaly}}$, yielding:

$$\Delta f_{a} = -\frac{3}{8\pi^{2}} \left( T_{a}(Q_{IR}) - \frac{1}{2} T_{a}(\text{Adj}) + \frac{1}{3} \eta_{I} c_{I} T_{a}(H_{I}) \right) k\pi T,$$

$$\Delta f_{IR} = \frac{3}{8\pi^{2}} \left( T_{IR}(\text{Adj}) - T_{IR}(Q_{IR}) \right) k\pi T,$$

$$\Delta f_{UV} = 0. \quad (3.14)$$

With (3.3), (3.12), (3.13) and (3.14), the low energy bulk gauge couplings are given by

$$\kappa_{a}\pi M_{a} \text{Re}(T) + \frac{1}{8\pi^{2}} \left[ T_{a}(Q_{UV}) \ln \left( \frac{M_{a}}{\mu} \right) + T_{a}(Q_{IR}) \ln \left( \frac{M_{a} e^{-\frac{1}{2}\pi k(T+T^{*})}}{\mu} \right) \right. \left. - T_{a}(\text{Adj}) \left\{ 3 \ln \left( \frac{M_{a}}{\mu} \right) - \frac{3}{4} \pi k(T+T^{*}) - \ln(M_{a}(T+T^{*})) \right\} + \sum_{\omega_{I}=\eta_{I}} T_{a}(H_{I}) \left\{ \ln \left( \frac{k}{\mu} \right) - \ln \left( \frac{e^{(\frac{1}{2}-\omega_{I}c_{I})\pi k(T+T^{*})} - 1}{\pi(1-2\omega_{I}c_{I})} \right) \right\} - \sum_{I} \frac{1}{2} T_{a}(H_{I}) \eta_{I} c_{I} \pi k(T+T^{*}) \right], \quad (3.15)$$

where $\mu^{2} = -g^{\mu\nu}\partial_{\mu}\partial_{\nu}$ is the external 4D momentum below the KK threshold scale. Here $\sum_{I}$ denotes the summation over all hypermultiplets, while $\sum_{\omega_{I}=\eta_{I}}$ denotes the summation over the hypermultiplets having a zero mode. The brane gauge couplings at low energies can be similarly obtained as

$$\frac{1}{g_{UV}^{2}(\mu, k, R)} = \frac{1}{g_{UV}^{2}} + \frac{b_{UV}}{16\pi^{2}} \ln \left( \frac{M_{a}^{2}}{\mu^{2}} \right),$$

$$\frac{1}{g_{IR}^{2}(\mu, k, R)} = \frac{1}{g_{IR}^{2}} + \frac{b_{IR}}{16\pi^{2}} \ln \left( \frac{e^{-\pi k(T+T^{*})} M_{a}^{2}}{\mu^{2}} \right). \quad (3.16)$$
where \( b_A = -3T_A(\text{Adj}) + \sum \Phi T_A(\Phi) \) \( (A = \text{UV or IR}) \) are the one-loop beta function coefficients for the couplings of gauge fields living only on the branes.

The above expression of low energy brane couplings is what one would expect based on the notion of position-dependent effective cutoff \[14\]. The bare brane couplings \( \hat{g}_{2A}^2 (A = \text{UV or IR}) \) are the one-loop beta function coefficients for the couplings of gauge fields living only on the branes.

The above expression of low energy brane couplings is what one would expect based on the notion of position-dependent effective cutoff \[14\]. The bare brane couplings \( \hat{g}_{2A}^2 (A = \text{UV or IR}) \) are the one-loop beta function coefficients for the couplings of gauge fields living only on the branes. However in the metric frame of \( g_{\mu\nu} \) in which \( \mu^2 = -g_{\mu\nu} \partial_\mu \partial_\nu \) is measured (see Eq. \[2.3\]), \( \hat{g}_{IR}^2 \) can be identified as the Wilsonian coupling at the effective cutoff scale \( e^{-\pi k R} M_\ast \), while \( \hat{g}_{UV}^2 \) is still the Wilsonian coupling at \( M_\ast \).

As for the low energy bulk gauge couplings, our result \[3.15\] shows that the (calculable) quantum corrections are generically of order \( \frac{1}{8\pi^2} \ln \left( \frac{M_\ast^2}{\mu^2} \right) \sim \frac{1}{8\pi^2} \ln \left( \frac{M_{\text{Pl}}^2}{M_\text{weak}}^2 \right) \).

It reproduces correctly the known 1-loop gauge coupling in flat 5D orbifold in the limit \( kR \to 0 \). In this flat limit, \[3.15\] is reduced to

\[
\frac{1}{g_a^2(\mu)} = \kappa_a \pi M_\ast \text{Re}(T) + \frac{1}{8\pi^2} \left[ b_a \ln \left( \frac{1}{\mu R} \right) + b''_a \ln \left( M_\ast R \right) \right] + \ldots \tag{3.17}
\]

where the ellipsis stands for the subleading pieces of \( O(1/8\pi^2) \), and

\[
b''_a = -2T_a(\text{Adj}) + \sum T_a(Q_{UV}) + \sum T_a(Q_{IR}).
\]

This value of \( b''_a \) agrees with what one would obtain in the explicit KK loop computation \[19\]. In fact, \[3.15\] can be obtained also through a totally independent (but more general) 5D calculation using dimensional regularization \[22\], providing a nontrivial check of our result.

The result \[3.15\] is valid for arbitrary value of \( k \) and \( \mu \) as long as \( k \lesssim M_\ast \) and \( \mu \lesssim M_{\text{KK}} \).

When \( \pi k(T + T^*) \gg 1 \) so that we have the scale hierarchy \[2.13\], one can rewrite \[3.15\] in terms of the logarithms of four distinctive mass scales \( M_{\text{KK}} \approx \pi ke^{-\pi k(T + T^*)/2}, 1/R, k \) and \( M_\ast \). In this regard, for the hypermultiplets with \( \omega_I = \eta_I \) having a massless mode, it is convenient to consider the following three different classes:

\[
\text{Class 1:} \quad \frac{1}{2} - \omega_I c_I \gg \frac{1}{\pi k(T + T^*)},
\]

\[
\text{Class 2:} \quad \left| \frac{1}{2} - \omega_I c_I \right| \ll \frac{1}{\pi k(T + T^*)},
\]

\[
\text{Class 3:} \quad \frac{1}{2} - \omega_I c_I \ll -\frac{1}{\pi k(T + T^*)}. \tag{3.18}
\]

Then \[3.15\] can be written as

\[
\frac{1}{g_a^2(\mu)} = \kappa_a \pi M_\ast \text{Re}(T) + \frac{b_a}{8\pi^2} \ln \left( \frac{M_{\text{KK}}}{\mu} \right) + \frac{b''_a}{8\pi^2} \ln \left( \frac{1}{M_{\text{KK}} R} \right) + \frac{b''''_a}{8\pi^2} \ln (k R) + \frac{b''''_a}{8\pi^2} \ln \left( \frac{M_\ast}{k} \right) + \ldots, \tag{3.19}
\]

where

\[
b_a = -3T_a(\text{Adj}) + \sum T_a(Q_{IR}) + \sum T_a(Q_{UV}) + \sum \omega_I = \eta_I T_a(H_I),
\]
\[ b'_a = -\frac{3}{2} T_a(\text{Adj}) + \sum T_a(Q_{UV}) + \left[ -\sum_{I} \eta_{IcI} T_a(H_I) + \sum_{\omega_I = \eta_I} T_a(H_I) \right] \]

\[ b''_a = \frac{1}{2} T_a(\text{Adj}) + \sum T_a(Q_{UV}) + \left[ -\sum_{I} \eta_{IcI} T_a(H_I) + \sum_{\omega_I = \eta_I} T_a(H_I) \right] \]

\[ b'''_a = -2 T_a(\text{Adj}) + \sum T_a(Q_{UV}) + \sum T_a(Q_{IR}), \quad (3.20) \]

and the ellipsis stands for the subleading pieces of \( O(1/8\pi^2) \). Obviously \( b_a \) represents the conventional coupling running at scales between \( \mu \) and \( M_{KK} \). On the other hand, other coefficients contain the KK threshold effects, so their logarithms cannot be interpreted as a coupling running.

### 4. Conclusion

In this paper, we have examined the gauge coupling renormalization in orbifold field theory in the context of 4D effective SUGRA. The 4D effective SUGRA is a convenient framework to study the gauge coupling renormalization since the 1-loop gauge couplings can be determined by the 1-loop induced axion couplings and the tree level properties of 4D effective SUGRA which are much easier to be computed. To be explicit, we take an example of the 5D SUGRA-coupled SYM theory on a slice of AdS\(_5\) with four well-separated mass scales, the KK threshold scale, the orbifold length, the AdS curvature, and the cutoff scale. In this case, the relevant axion couplings are those of the graviphoton which can be determined simply by the chiral anomaly structure of the model. The calculable piece of gauge coupling renormalization in AdS\(_5\) is logarithmic, which is generically of order \( \frac{1}{8\pi^2} \ln(M_{Pl}^2/M_{\text{weak}}^2) \), so is numerically of order unity. We have calculated such logarithmic corrections in generic 5D SYM models defined on a slice of AdS\(_5\) preserving \( N = 1 \) SUSY.

**Note added:** After this paper is completed, there have appeared several papers discussing the gauge coupling renormalization in AdS\(_5\) [27, 28, 29, 30, 31, 22, 32, 33].

### Acknowledgments

HK thanks the particle theory group of University of Washington, Seattle for their warm hospitality and thanks D. B. Kaplan, A. Katz and A. Nelson for discussions. This work is supported in part by the BK21 program of Ministry of Education, KRF Grant No. 2000-015-DP0080, the KOSEF Sundo Grant, and the Center for High Energy Physics(CHEP), Kyungpook National University.
A. 5D SUSY transformation and Gauged $U(1)_R$

In this appendix, we briefly discuss how the $U(1)_R$-gauging (2.11) is related with the AdS curvature and the mass parameter of the hypermultiplets through the 5D SUGRA transformation. Let us first set up the notation for 5D spinor. Generic spinor field in 5D SUGRA can be represented by symplectic-Majorana spinor satisfying

$$\bar{\Psi}^i = \Psi^i \gamma^0 = (\Psi^i)^T C.$$  

where

$$C^T = -C, \quad (\gamma^A)^T = C \gamma^A C^{-1},$$

and the $SU(2)_R$ doublet index $i = 1, 2$ is raised or lowered by $\varepsilon^{ij} = \varepsilon_{ij} = (i\sigma_2)_{ij}$:

$$\Psi^i = \varepsilon^{ij} \Psi_j, \quad \Psi_i = \Psi^j \varepsilon_{ji}$$

A symplectic-Majorana spinor contains two independent left-handed 4D Weyl spinors $\psi$ and $\psi^c$:

$$\Psi^{i=1} = \begin{pmatrix} \psi \\ \psi^c \end{pmatrix}, \quad \Psi^{i=2} = \begin{pmatrix} \psi^c \\ -\psi \end{pmatrix},$$

so $\Psi^{i=1}$ or $\Psi^{i=2}$ can be regarded as a Dirac spinor.

The 5D SUGRA action of general vector multiplets and hypermultiplets and their SUGRA transformations are given in [26]. In our notation, the SUGRA transformation of the gravity multiplet $\{e^A_M, \Psi^i_M, B_M\}$ is given by

$$\delta \xi e^A_M = \frac{1}{2} \bar{\xi}^i \gamma^A \Psi_{iM}, \quad \delta \xi B_M = \frac{i}{2} \bar{\Psi}^i_M \xi_i,$$

$$\delta \xi \Psi^i_M = D_M \xi^i + \frac{1}{2} k e(y)(\sigma^3)^i_j e^A_M \gamma^A \xi^j$$

$$+ \frac{i}{8} e^A_M (\gamma_{ABC} \xi^i - 4 \eta_{AB} \gamma^A C \xi^i) C^{BC} + ..., \quad (A.1)$$

where $C_{MN} = \partial_M B_N - \partial_N B_M$ and the ellipsis denotes the piece which is bilinear in hyperino fields. Comparing $D_M \xi^i$ in $\delta \xi \Psi^i_M$ with the next term, one easily finds

$$D_M \xi^i = \nabla_M \xi^i - i \frac{3}{2} k e(y)(\sigma^3)^i_j B_M \xi^j,$$

and this determines the $U(1)_R$-gauging for the fields carrying $SU(2)_R$ index $i$, e.g

$$D_M \lambda^{ia} = \nabla_M \lambda^{ia} - i \frac{3}{2} k e(y)(\sigma^3)^i_j B_M \lambda^{ja}.$$

The SUGRA transformation of hypermultiplet $\{h^i_I, \Psi_I\}$ is given by [26]

$$\delta \xi h^i_I = -i \bar{\xi}^i \Psi_I,$$

$$\delta \xi \Psi_I = -i \left[ \gamma^A \xi e^A_M D_M h^j_I \varepsilon^{}_{ji} + k e(y) \xi^i \left( \frac{3}{2} (\sigma_3)^j_k - c_I \delta^j_k \right) h^k_I \varepsilon_{ji} \right]. \quad (A.2)$$
Again comparing the first term in $\delta \Psi_I$ with the second, one easily finds
\[ D_M h_I^1 = \partial_M h_I^1 - i k \epsilon(y) \left( \frac{3}{2} (\sigma_3)_{ij}^i - c_I \delta_j^i \right) k \epsilon(y) B_M h_I^1, \]
and so
\[ D_M \Psi_I = \nabla_M \Psi_I + i c_I k \epsilon(y) B_M \Psi_I. \]

Let us now make sure that the above $U(1)_R$ covariant derivatives correctly lead to the mass parameters in $S_{\text{hyper}}$ of (2.6). For the SUGRA transformation (A.1), the Killing spinor equation is given by
\[ D_M \xi^i + \frac{1}{2} k \epsilon(y) (\sigma^3)_i^j c_M A \xi^j = 0, \]
which has the solution
\[ \xi^{i=1} = \begin{pmatrix} \xi \\ 0 \end{pmatrix}, \quad \xi^{i=2} = \begin{pmatrix} 0 \\ \bar{\xi} \end{pmatrix}, \]
where
\[ \xi(y) = e^{-\frac{1}{2} k |y|(R-3iB_5)} \eta \]
for a constant 4D Weyl spinor $\eta$ generating the unbroken $N = 1$ global SUSY.

From (A.3) and (A.2), one can see that $\{ \hat{h}_1^I, \hat{\psi}_I \}$ and $\{ (\hat{\psi}_2^I)^*, \hat{\psi}_I \}$ form $N = 1$ chiral superfields $H_I$ and $H^c_I$ under the global SUSY generated by $\eta$, where $\hat{h}_1^I, \hat{\psi}_I$ and $\hat{\psi}_I^c$ are defined in (3.7). One can also find from (A.3) and (A.2) that the $F$-components of these chiral superfields are given by
\[ F_{H_I} = e^{-\frac{1}{2} (-2c_I) k |y|R} \partial_5 h_I^2, \]
\[ F_{H^c_I} = -e^{-\frac{1}{2} (2c_I) k |y|R} \partial_5 (\hat{h}_1^2)^*. \]

Using the kinetic terms, it is also straightforward to find that the D-term action of hypermultiplets is given by (23, 12)
\[ \int d^5 x \int d^4 \theta \frac{1}{2} (T + T^*) \left[ e^{\frac{1}{2} k |y|(R+T^*)} H_I H_I^* + e^{\frac{1}{2} k |y|(R-T^*)} H^c_I H^c_I^* \right]. \]

In order for the $F$-components of $H_I$ and $H^c_I$ to be given as (A.4), the $F$-term action should be given by
\[ \int d^5 x \left[ \int d^2 \theta H_I^c \partial_5 H_I + \text{h.c.} \right]. \]

It is then straightforward to see that the above D-term and $F$-term actions give the correct hypermultiplet masses:
\[ M_\Psi(y) = c_I k \epsilon(y), \]
\[ M^2(h_i^{i=1}) = \left( e_I^2 + c_I - \frac{15}{4} \right) k^2 + (3 - 2c_I) k (\delta(y) - \delta(y - \pi)), \]
\[ M^2(h_i^{i=2}) = \left( e_I^2 - c_I - \frac{15}{4} \right) k^2 + (3 + 2c_I) k (\delta(y) - \delta(y - \pi)). \]
References

[1] J. Scherk and J. H. Schwarz, *Spontaneous Breaking Of Supersymmetry Through Dimensional Reduction*, Phys. Lett. B 82 (1979) 60; How To Get Masses From Extra Dimensions, Nucl. Phys. B 153 (1979) 61.

[2] A. Pomarol and M. Quiros, *The standard model from extra dimensions*, Phys. Lett. B 438 (1998) 355; I. Antoniadis, S. Dimopoulos, A. Pomarol and M. Quiros, *Soft masses in theories with supersymmetry breaking by TeV-compactification*, Nucl. Phys. B 544 (1999) 533; A. Delgado, A. Pomarol and M. Quiros, *Supersymmetry and electroweak breaking from extra dimensions at the TeV-scale*, Phys. Rev. D 60 (1999) 095008.

R. Barbieri, L. J. Hall and Y. Nomura, *A constrained standard model from a compact extra dimension*, Phys. Rev. D 63 (2001) 105007.

[3] Y. Kawamura, *Gauge symmetry reduction from the extra space S(1)/Z(2)*, Prog. Theor. Phys. 103 (2000) 613; *Triplet-doublet splitting, proton stability and extra dimension*, Prog. Theor. Phys. 105 (2001) 999; *Split multiplets, coupling unification and extra dimension*, Prog. Theor. Phys. 105 (2001) 691.

[4] L. J. Hall and Y. Nomura, *Gauge coupling unification from unified theories in higher dimensions*, hep-ph/0111068.

L. J. Hall and Y. Nomura, *Gauge unification in higher dimensions*, Phys. Rev. D 64 (2001) 055003.

[5] G. Altarelli and F. Feruglio, *SU(5) grand unification in extra dimensions and proton decay*, Phys. Lett. B 511 (2001) 257; A. B. Kobakhidze, Phys. Lett. B 514 (2001) 131; A. Hebecker and J. March-Russell, *A minimal S**1/(Z(2) x Z'(2)) orbifold GUT*, Nucl. Phys. B 613 (2001) 3.

[6] I. Antoniadis, *A Possible New Dimension At A Few Tev*, Phys. Lett. B 246 (1990) 377; N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, *The hierarchy problem and new dimensions at a millimeter*, Phys. Lett. B 429 (1998) 263.

[7] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, Phys. Rev. Lett. 83 (1999) 3370.

[8] N. Arkani-Hamed and M. Schmaltz, *Hierarchies without symmetries from extra dimensions*, Phys. Rev. D 61 (2000) 033005.

[9] H. D. Kim, J. E. Kim and H. M. Lee, *Top-bottom mass hierarchy, s - mu puzzle and gauge coupling unification with split multiplets*, hep-ph/0112094.

[10] R. Altendorfer, J. Bagger and D. Nemeschansky, *Supersymmetric Randall-Sundrum scenario*, Phys. Rev. D 63 (2001) 125025; A. Falkowski, Z. Lalak and S. Pokorski, *Supersymmetrizing branes with bulk in five-dimensional supergravity*, Phys. Lett. B 491 (2000) 172; J. Bagger, D. Nemeschansky and R. J. Zhang, *Supersymmetric radion in the Randall-Sundrum scenario*, J. High Energy Phys. 08 (2001) 057.

A. Falkowski, Z. Lalak and S. Pokorski, *Four dimensional supergravities from five dimensional brane worlds*, Nucl. Phys. B 613 (2001) 189.

[11] T. Gherghetta and A. Pomarol, *Bulk fields and supersymmetry in a slice of AdS*, Nucl. Phys. B 586 (2000) 141; *A warped supersymmetric standard model*, Nucl. Phys. B 602 (2001) 3.
[12] D. Marti and A. Pomarol, *Supersymmetric theories with compact extra dimensions in N = 1 superfields*, Phys. Rev. D 64 (2001) 105025.

[13] A. Pomarol, *Grand unified theories without the desert*, Phys. Rev. Lett. 85 (2000) 4004.

[14] L. Randall and M. D. Schwartz, *Quantum field theory and unification in AdS5*, J. High Energy Phys. 11 (2001) 003; *Unification and the hierarchy from AdS5*, Phys. Rev. Lett. 88 (2002) 081801.

[15] E. Bergshoeff, R. Kallosh and A. Van Proeyen, *Supersymmetry in singular spaces*, J. High Energy Phys. 10 (2000) 033.

[16] Z. Chacko, M. A. Luty and E. Ponton, *Massive higher-dimensional gauge fields as messengers of supersymmetry breaking*, J. High Energy Phys. 07 (2000) 036.

Y. Nomura, *Strongly coupled grand unification in higher dimensions*, hep-ph/0108170.

Y. Nomura, D. R. Smith and N. Weiner, *GUT breaking on the brane*, Nucl. Phys. B 613 (2001) 147.

[17] K. R. Dienes, E. Dudas and T. Gherghetta, *Extra spacetime dimensions and unification*, Phys. Lett. B 436 (1998) 51; *Grand unification at intermediate mass scales through extra dimensions*, Nucl. Phys. B 537 (1999) 47.

This point was noted in N. Arkani-Hamed, A. G. Cohen and H. Georgi, *Accelerated unification*, hep-th/0108089 in the framework of deconstruction.

[18] R. Contino, L. Pilo, R. Rattazzi and E. Trincherini, *Running and matching from 5 to 4 dimensions*, Nucl. Phys. B 622 (2002) 227.

[19] V. A. Novikov, M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, *Exact Gell-Mann-Low Function Of Supersymmetric Yang-Mills Theories From Instanton Calculus*, Nucl. Phys. B 229 (1983) 351.

*Beta Function In Supersymmetric Gauge Theories: Instantons Versus Traditional Approach*, Phys. Lett. B 166 (1986) 329.

[20] V. Kaplunovsky and J. Louis, *Field dependent gauge couplings in locally supersymmetric effective quantum field theories*, Nucl. Phys. B 422 (1994) 57.

*On Gauge couplings in string theory*, Nucl. Phys. B 444 (1995) 191.

[21] K. Choi and I. W. Kim, *One loop gauge couplings in AdS(5)*, hep-th/0208071.

[22] N. Arkani-Hamed, T. Gregoire and J. Wacker, *Higher dimensional supersymmetry in 4D superspace*, J. High Energy Phys. 03 (2002) 053.

[23] N. Arkani-Hamed, A. G. Cohen and H. Georgi, *Anomalies on orbifolds*, Phys. Lett. B 516 (2001) 393.

[24] C. A. Scrucca, M. Serone, L. Silvestrini and F. Zwirner, *Anomalies in orbifold field theories*, Phys. Lett. B 525 (2002) 169.

[25] A. Ceresole and G. Dall’Agata, *General matter coupled N = 2, D = 5 gauged supergravity*, Nucl. Phys. B 585 (2000) 143.

[26] W. D. Goldberger and I. Z. Rothstein, *High energy field theory in truncated AdS backgrounds*, hep-th/0204166.

[27] K. Agashe, A. Delgado and R. Sundrum, *Gauge coupling renormalization in RS1*, hep-ph/0206095.
[29] K. Choi, H. D. Kim and I. W. Kim, *Radius dependent gauge unification in AdS(5)*, hep-ph/0207013.

[30] R. Contino, P. Creminelli and E. Trincherini, *Holographic evolution of gauge couplings*, hep-th/0208002.

[31] A. Falkowski, H. D. Kim, *Running of gauge couplings in AdS5 via deconstruction*, hep-ph/0208056.

[32] W. D. Goldberger and I. Z. Rothstein, *Effective Field Theory and Unification in AdS Backgrounds*, hep-th/0208060.

[33] L. Randall, Y. Shadmi and N. Weiner, *Deconstruction and gauge theories in AdS(5)*, hep-th/0208120.