Scalar emission in a rotating Gödel black hole

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Abstract

We study the absorption probability and Hawking radiation of the scalar field in the rotating Gödel black hole in minimal five-dimensional gauged supergravity. We find that Gödel parameter \( j \) imprints in the greybody factor and Hawking radiation. It plays a different role from the angular momentum of the black hole in the Hawking radiation and super-radiance. These information can help us know more about rotating Gödel black holes in minimal five-dimensional gauged supergravity.

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I. INTRODUCTION

The standard Friedmann-Robertson-Walker (FRW) model is a popular phenomenological description of the rather idealized isotropic and homogeneous universe filled with perfect fluid. However, this model is too ideal to be used to describe the universe with global rotation. Considering that all compact objects in the universe are rotating, it would be natural to think that the rotation is a universal phenomenon which may also apply to the global universe. An exact solution for the rotating universe was found by Gödel [1], which satisfies Einstein’s field equations containing a cosmological constant and homogeneous pressureless matter. Gödel universe possesses several special features including, in particular, the presence of closed timelike curve (i.e. the time machine) through every point.

A great deal of effort [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14] has been spent in recent years in studying Gödel-type solutions in the context of five-dimensional minimal supergravity. These solutions are related by T duality to pp-waves [7, 8], which make the Gödel-type universes important since they might provide the possibility of quantizing strings in these backgrounds and building relations to the corresponding limits of super-Yang Mills theories.

The black hole solution must embed in the Gödel universe. The solutions of the neutral non-rotating and rotating black holes immersed in the rotating Gödel universe were found in the five-dimensional minimal supergravity [9], which are called Schwarzschild and Kerr Gödel black holes. These black hole solutions also satisfy the usual black hole thermodynamics [10, 11, 12, 13, 14]. In the study of the wave dynamical properties of Schwarzschild Gödel black hole [15], it was found that the Schwarzschild Gödel black hole is stable at least for the small Gödel parameter regime. Recently, there have been more solutions of black holes found in Gödel universe and various properties have been investigated accordingly [16, 17, 18, 19]. In this paper we are going to study the Hawking radiation in the rotating black hole embedded in the Gödel universe. We will calculate the greybody factor of scalar particles propagating in this Kerr Gödel black hole and show rich physics brought by the rotation parameter \( a \) of the black hole and the Gödel parameter \( j \) by using the matching technique, which has been extensively used in evaluating the absorption probabilities and Hawking radiations of various black holes [20, 21, 22, 23, 24, 25, 26, 27, 28, 29].

The paper is organized as follows: in the following section we will introduce the Kerr Gödel metric and derive the master equation of scalar field in the limit of small Gödel parameter \( j \). In Sec.III, we will obtain the approximate solution in the low energy and low angular momentum limit. In section IV, we will calculate
the absorption probability and the luminosity of Hawking radiation for the scalar field. Finally in the last section we will include our conclusions.

II. MASTER EQUATION IN THE KERR GÖDEL BLACK HOLES

The equations of the bosonic field in the minimal supergravity theory in five dimensions are written in the form

\[ R_{\mu \nu} = 2F_{\mu \alpha}F_{\nu}^{\alpha} - \frac{1}{6}g_{\mu \nu}F^2, \]

\[ D_{\mu}F^{\mu \nu} = \frac{1}{2\sqrt{3}} \epsilon^{\alpha \beta \gamma \mu \nu}F_{\alpha \beta}F_{\gamma \mu}, \]

(1)

where \( \epsilon_{\alpha \beta \gamma \mu \nu} = \sqrt{-g} \epsilon_{\alpha \beta \gamma \mu \nu} \) and \( F^2 = F_{\alpha \beta}F^{\alpha \beta} \). The solutions of equations (1) consist of the metric and a \( U(1) \) gauge field which has an additional Chern-Simons interaction.

The Gödel universe is a solution of the equation (1), whose metric and \( U(1) \) gauge field are given by [9]

\[ ds^2 = -(dt + j r^2 \sigma_3)^2 + dr^2 + \frac{r^2}{4}d\Omega_3^2, \quad \tilde{A} = \frac{\sqrt{3}}{2} j r^2 \sigma_3, \]

(2)

with

\[ d\Omega_3^2 = d\theta^2 + \sin^2 \theta d\phi^2 + \sigma_3^2, \quad \sigma_3 = d\psi + \cos \theta d\phi, \]

(3)

where \( \theta, \phi \) and \( \psi \) are Euler angles. \( j \) is the Gödel parameter and is responsible for the rotation of the Gödel universe. The angular velocity of the universe is \( \Omega_u = \frac{4j}{1 - 4j r^2} \). When \( r > 1/(2j) \), the closed timelike curves appear. The matter content of Gödel universe [2] consists of pressureless dust and the energy-momentum tensor for the field \( F_{\mu \nu} \) has vanishing pressure and constant energy density proportional to \( j^2 \). In addition, just like the original four dimensional Gödel universe [1], the solution [2] is homogeneous. When \( j = 0 \), the metric [2] reduces to the Minkowski spacetime.

The solution of the equation (1) which describes a five dimensional rotating black hole (with the two equal rotation parameters) embedded in the Gödel universe can be written as [9]

\[ ds^2 = -u(r)dt^2 - 2g(r)\sigma_3 dt + h(r)\sigma_3^2 + f(r)^{-1}dr^2 + \frac{r^2}{4}d\Omega_3^2, \]

(4)

with

\[ u(r) = 1 - \frac{2M}{r^2}, \quad h(r) = -j^2 r^2 (r^2 + 2m) + \frac{Ma^2}{2r^2}, \]

\[ g(r) = jr^2 + \frac{Ma}{r^2}, \quad f(r) = 1 - \frac{2M}{r^2} + \frac{8jM(a + 2jM)}{r^2} + \frac{2Ma^2}{r^4}. \]

(5)
Here $M$ and $a$ are related to the mass and angular momentum of the black hole, respectively. When $M = a = 0$, the metric reduces to that of the five-dimensional G"{o}del universe \(2\). When $j = 0$ it reduces to the five-dimensional Kerr black hole with two equal rotation parameters. When $a = 0$ the solution becomes the Schwarzschild-G"{o}del black hole.

The Kerr G"{o}del Black Hole \(4\) has an outer horizon at $r_+$ and an inner horizon at $r_-$,

$$r^2_\pm = M(1 - 4aj - 8j^2M) \pm \sqrt{M^2(1 - 4aj - 8j^2M)^2 - 2Ma^2}.$$  

(6)

which are determined by $f(r) = 0$. The metric is well behaved at the horizon but the gauge field becomes singular there. The function $u(r)$ is equal to zero when $r = \sqrt{2M}$, corresponding to an ergosphere \(13\). The Hawking temperature $T_H$ of the black hole and the angular velocity $\Omega_H$ at horizon are described as

$$T_H = \frac{r^2_+ - r^2_-}{r^2_+ \sqrt{4h(r_+) + r^2_+}},$$  

(7)

and

$$\Omega_H = \frac{4g(r_+)}{4h(r_+) + r^2_+},$$  

(8)

respectively.

In the following we will investigate Hawking radiation of a scalar field in the small $j$ case since the small rotation of the G"{o}del cosmological background seems the most reasonable in phenomenology \[15\]. In the limit of small $j$, the metric coefficients \(4\) can be rewritten as

$$u(r) = 1 - \frac{2M}{r^2}, \quad h(r) = \frac{Ma^2}{2r^2},$$

$$g(r) = \frac{jr^2}{r^2} + \frac{Ma}{r^2}, \quad f(r) = 1 - \frac{2M}{r^2} + \frac{8jMa}{r^2} + \frac{2Ma^2}{r^4}.$$  

(9)

The non-zero components of the inverse metric $g^{\mu\nu}$ and the determinant $g$ can be expressed as

$$g^{00} = -\frac{r^4 + 2Ma^2}{\Delta}, \quad g^{11} = \frac{\Delta}{r^4}, \quad g^{22} = \frac{4}{r^2},$$

$$g^{04} = -\frac{4(jr^4 + Ma)}{\Delta}, \quad g^{34} = -\frac{4 \cos \theta}{r^2 \sin^2 \theta}, \quad g^{33} = \frac{4}{r^2 \sin^2 \theta},$$

$$g^{44} = \frac{4 \cos^2 \theta}{r^2 \sin^2 \theta} + \frac{4(r^2 - 2M)}{\Delta}, \quad g = -\frac{g^{00}}{64 \sin^2 \theta},$$  

(10)

with

$$\Delta = r^4 - 2M(1 - 4aj)r^2 + 2Ma^2.$$  

(11)

The wave equation for the massless scalar field $\Phi(t, r, \theta, \phi, \psi)$ in the Kerr G"{o}del black hole spacetime obeys

$$\frac{1}{\sqrt{-g}} \partial_\mu \left( \sqrt{-g} g^{\mu\nu} \partial_\nu \Phi(t, r, \theta, \phi, \psi) \right) = 0.$$  

(12)
Taking the ansatz $\Phi(t, r, \theta, \phi, \psi) = e^{-i\omega t}R(r)e^{im\phi+i\lambda \psi}S(\theta)$, where $S(\theta)$ is the so-called spheroidal harmonics, we can obtain the equation

$$\frac{1}{\sin \theta} \frac{d}{d\theta} \left[ \sin \theta \frac{dS(\theta)}{d\theta} \right] - \left[ \frac{(m - \lambda \cos \theta)^2}{\sin^2 \theta} - E_{lm\lambda} \right] S(\theta) = 0,$$

(13)

for the angular part. Obviously, this angular equation is independent of the rotating parameters $a$ and $j$, and is exactly identical to that in the static five-dimensional black hole spacetime. This is not surprising and in [30] the same angular equation as that in the Schwarzschild spacetime was also obtained in the five-dimensional Kerr black hole with two equal rotational parameters. The eigenvalue of the angular equation (13) is $E_{lm\lambda} = l(l+1) - \lambda^2$. The radial part reads

$$\frac{1}{r^3} \frac{d}{dr} \left[ \frac{\Delta dR(r)}{r} \right] + \left[ \frac{K^2}{\Delta} - \frac{4E_{lm\lambda} + \Lambda}{r^2} \right] R(r) = 0,$$

(14)

with

$$K = \sqrt{r^4 + 2Ma^2} \left[ \omega - \frac{4\lambda (jr^4 + Ma)}{r^4 + 2Ma^2} \right], \quad \Lambda = \frac{4\lambda^2 r^4}{r^4 + 2Ma^2}.$$  

(15)

The solution of the radial function $R(r)$ will help us to obtain the absorption probability $|A_{lm\lambda}|^2$ and the luminosity of Hawking radiation for a massless scalar particle propagating in the black hole spacetime.

### III. GREYBODY FACTOR IN THE LOW-ENERGY REGIME

Now we try to obtain an analytic solution of the radial equation (14) by using the well-known approximation technique [27]: we first solve the equation in the near horizon regime ($r \sim r_+$) and then in the far field limit ($r \gg r_+$), finally we smoothly match these two solutions in an intermediate region. In this way we can get an analytic expression in the low energy and low angular momentum approximation for the radial part of the field valid throughout the whole spacetime.

Let us first focus on the near-horizon regime. In order to express equation (14) into the form of a known differential equation, we perform the following transformation of the radial variable [27]

$$r \rightarrow f = \frac{\Delta}{r} \Rightarrow \frac{df}{dr} = (1 - f) \frac{A}{r},$$

(16)

where

$$A = 2 \left[ 1 - \frac{a^2}{(1 - 4aj)r^2 - a^2} \right].$$

(17)

The equation (14) near the horizon ($r \sim r_+$) can be expressed as

$$f(1 - f) \frac{d^2R(f)}{df^2} + (1 - D_+ f) \frac{dR(f)}{df} + \left\{ \frac{K^2}{A(r_+)^2(1 - f)} - \frac{4E_{lm\lambda} + \Lambda(r_+)}{A(r_+)^2(1 - f)} \right\} R(f) = 0,$$

(18)
where
\[ K_* = \sqrt{1 + \frac{2Ma^2}{r_+^2}} \left( \omega - \frac{4\lambda (j r_+^2 + Ma)}{r_+^2 + 2Ma^2} \right), \]
\[ D_* = 1 - 2a^2 \frac{[(1 - 4aj)r_+^2 - a^2]}{[(1 - 4aj)r_+^2 - 2a^2]^2}. \]

Employing the transformation \( R(f) = f^\alpha (1 - f)^\beta F(f) \), we can write the equation (18) into the form of the hypergeometric equation
\[ f(1 - f) \frac{d^2 F(f)}{df^2} + [c - (1 + a_1 + b)f] \frac{dF(f)}{df} - a_1 b F(f) = 0, \tag{20} \]
with
\[ a_1 = \alpha + \beta + D_* - 1, \quad b = \alpha + \beta, \quad c = 1 + 2\alpha. \tag{21} \]

Due to the constraint from coefficient of \( F(f) \), the power coefficients \( \alpha \) and \( \beta \) must satisfy the second-order algebraic equations
\[ \alpha^2 + \frac{K_*^2}{A(r_+)^2} = 0, \tag{22} \]
and
\[ \beta^2 + \beta(D_* - 2) + \frac{K_*^2}{A(r_+)^2} \frac{4E_{lm\lambda} + \Lambda(r_+)}{A(r_+)^2} = 0, \tag{23} \]
respectively. Solving these two equations, we obtain the solutions for \( \alpha \) and \( \beta \)
\[ \alpha_\pm = \pm \frac{iK_*}{A(r_+)}, \tag{24} \]
\[ \beta_\pm = \frac{1}{2} \left( 2 - D_* \right) \pm \frac{1}{2} \sqrt{(D_* - 2)^2 - \frac{4K_*^2}{A(r_+)^2} \frac{4E_{lm\lambda} + \Lambda(r_+)}{A(r_+)^2}}. \tag{25} \]

The general solution of the master equation (14) near the horizon can be expressed as
\[ R_{NH}(f) = A_+ f^\alpha (1 - f)^\beta F(a_1, b, c; f) + A_- f^{-\alpha} (1 - f)^\beta F(a_1 - c + 1, b - c + 1, 2 - c; f), \tag{26} \]
where \( A_\pm \) are arbitrary constants. Considering the boundary condition that no outgoing mode exists near the horizon, we choose \( \alpha = \alpha_- \) and \( A_+ = 0 \), which brings the near horizon solution to the final form
\[ R_{NH}(f) = A_- f^\alpha (1 - f)^\beta F(a_1, b, c; f). \tag{27} \]

Moreover, the above boundary condition also demands that near the horizon the hypergeometric function \( F(a_1, b, c; f) \) must be convergent, i.e. \( Re(c - a_1 - b) > 0 \), which implies that we must choose \( \beta = \beta_- \).
In order to construct a full analytic solution valid for the whole radial regime, we need to smoothly match the near horizon and far field solutions in the intermediate zone. For the benefit of the following discussion, we change the expression of the hypergeometric function of the near horizon solution from $f$ to $1-f$ by using the relation

$$R_{NH}(f) = A_- f^\alpha (1-f)^\beta \left[ \frac{\Gamma(c)\Gamma(c-a_1-b)}{\Gamma(c-a_1)\Gamma(c-b)} F(a_1,b,a_1+b-c+1;1-f) \right. \right.$$  

$$+ \left(1-f)^{c-a_1-b}\frac{\Gamma(c)\Gamma(a_1+b-c)}{\Gamma(a_1)\Gamma(b)} F(c-a_1,c-b,a_1-b+1;1-f) \right], \quad (28)$$

and stretch it towards the intermediate regime. In the limit $r \gg r_+$, the function $(1-f)$ can be approximated as

$$1-f \simeq \frac{2M(1-4aj)}{r^2}, \quad (29)$$

and the near horizon solution (28) can be simplified further to

$$R_{NH}(r) \simeq A_1 r^{-2\beta} + A_2 r^{2(\beta+D_\ast-2)}, \quad (30)$$

and

$$A_1 = A_- [2M(1-4aj)]^\beta \frac{\Gamma(c)\Gamma(c-a_1-b)}{\Gamma(c-a_1)\Gamma(c-b)}, \quad (31)$$

$$A_2 = A_- [2M(1-4aj)]^{-(\beta+D_\ast-2)}\frac{\Gamma(c)\Gamma(a_1+b-c)}{\Gamma(a_1)\Gamma(b)}. \quad (32)$$

Now let us turn to the far field region. Assuming that $r \to \infty$ and keeping only the dominant terms, we can expend the wave equation (14) for the massless scalar field as a power series in $1/r$

$$\frac{d^2 R_{FF}(r)}{dr^2} + \frac{3}{r} \frac{dR_{FF}(r)}{dr} + \left[ \tilde{\omega}^2 - \frac{4(l(l+1))}{r^2} \right] R_{FF}(r) = 0, \quad (33)$$

with

$$\tilde{\omega}^2 = \omega^2 - 8\lambda j\omega. \quad (34)$$

This is a Bessel equation. Thus, the general solution of radial master equation (14) in the far field region can be expressed as

$$R_{FF}(r) = \frac{1}{r} \left[ B_1 J_\nu(\tilde{\omega} r) + B_2 Y_\nu(\tilde{\omega} r) \right], \quad (35)$$

where $J_\nu(\tilde{\omega} r)$ and $Y_\nu(\tilde{\omega} r)$ are the first and second kind Bessel functions, $\nu = 2l+1$. $B_1$ and $B_2$ are integration constants. We now extend the far-field asymptotic solution (35) towards small radial coordinate. Taking the
Comparing equations (30) and (36) in the low energy and low angular momentum limit, we can obtain two relations between $A_1$ and $B_1$, $B_2$ in the limit $\omega r_+ \ll 1$. Making use of equations (31) and (32) and removing $A_-$, we find the constraint for $B_1$, $B_2$
\begin{equation}
B = \frac{B_1}{B_2} = -\frac{1}{\pi} \left[ \frac{2}{M(1-4aj)\tilde{\omega}^2} \right]^{2l+1} \frac{(2l+1)\Gamma(2l+1)\Gamma(c-a_1-b)\Gamma(a_1)\Gamma(b)}{\Gamma(a_1+b-c)\Gamma(c-a_1)\Gamma(c-b)}.
\end{equation}

In the asymptotic region $r \to \infty$, the solution in the far field can be expressed as
\begin{equation}
R_{FF}(r) \approx \frac{B_1 + iB_2}{\sqrt{2\pi\omega r}} e^{-i\tilde{\omega}r} + \frac{B_1 - iB_2}{\sqrt{2\pi\omega r}} e^{i\tilde{\omega}r} = A^{(\infty)}_{\text{in}} e^{-i\tilde{\omega}r} + A^{(\infty)}_{\text{out}} e^{i\tilde{\omega}r}.
\end{equation}

We need to point out that only when the condition $\tilde{\omega} \geq 0$ (i.e., $\omega \geq 8\lambda j$) is satisfied, the solution (38) denotes an incoming and an outgoing spherical waves at large distance from the black hole.

The absorption probability can be calculated by
\begin{equation}
|A_{lm\lambda}|^2 = 1 - \left| \frac{A^{(\infty)}_{\text{out}}}{A^{(\infty)}_{\text{in}}} \right|^2 = 1 - \left| \frac{B - i}{B + i} \right|^2 = \frac{2i(B^* - B)}{BB^* + i(B^* - B) + 1}.
\end{equation}

Combining the above result and the expression $B$ given in equation (37), we can analyze the properties of absorption probability for the massless scalar field in a rotating Gödel black hole background in the low-energy and low-angular momentum limits.

**IV. THE ABSORPTION PROBABILITY AND HAWKING RADIATION IN THE ROTATING GÖDEL BLACK HOLE**

We are now in a position to compute the absorption probability and discuss Hawking radiation of a Kerr black hole embedded in the Gödel universe.

In Fig.(1), we examine the influence of the angular momentum $a$ of black hole and Gödel parameter $j$ on the absorption probability. In the left figure, we plot the absorption probability for the first partial waves ($l = 0$, $m = 0$, $\lambda = 0$) by fixing $j = 0.2$. One can easily see that the absorption probability decreases with the increase of the parameter $a$, which is similar to that in the general rotating black hole spacetime shown in [20, 21, 22, 23, 24, 25, 26, 27, 28, 29]. For the fixed $j$, we observed that for smaller $a$, the potential peak becomes lower, which allows more radiation to leak to the infinity. In the right figure, we fix $a$ (with constant angular momentum ) and exhibit the dependence of the absorption probability on the Gödel parameter $j$. It shows that with the increase of $j$, the absorption probability decreases. In the low-energy and low-angular
momentum limit the absorption probability $|A_{l m \lambda}|^2 \sim \omega^3 r_+^3 [1 + 2Ma^2/r_+^4]^{\frac{5}{2}}$. The radius of the black hole event horizon decreases with the increase of the parameters $j$ and $a$. Thus for the first partial waves the influence of the Gödel parameter $j$ on the absorption probability is similar to that of the angular momentum $a$ of the black hole.

In Fig. (2), we find that for positive $\lambda$, in some range of the frequency $\omega$, the absorption probability can be negative, which presents us the super-radiance. This is the property brought by the rotation as also disclosed.
Besides the effect of $a$, we observed that the Gödel parameter $j$ can also contribute to the super-radiation. For the fixed $a$, we find that with the increase of $j$, both the magnitude of the super-radiance and the range of $\omega$ for the super-radiance to happen decreases. It is interesting to see that the effect of the Gödel parameter $j$ on the super-radiance is different from that caused by the angular momentum $a$ of the black hole. This property of the Gödel parameter $j$ has not been observed elsewhere.

As in [27], in the low energy limit $BB^* \gg i(B^* - B) \gg 1$, we can simplify our (39) to the form

$$|A_{l\lambda\lambda}|^2 = 2i\left(\frac{1}{B} - \frac{1}{B^*}\right) = 4\pi \left[M(1 - 4aj)\omega^2\right]^{2l+1} \frac{K^*}{A(r_+)} \frac{\Gamma^2(2\beta + D_\omega - 2)\Gamma^2(1 - \beta)(2 - D_\omega - 2\beta)\sin^2\pi(2\beta + D_\omega)}{(2l + 1)^2(2l + 1)\Gamma^2(\beta + D_\omega - 1)\sin^2\pi(\beta + D_\omega)}.$$ \quad (40)

From (25) we learnt that the quantity $2 - D_\omega - 2\beta$ is always positive. Using (17) we have $A(r_+) = 1 - \frac{2Ma^2}{r_+^4}$, which is positive since $r_+^2 - \sqrt{2Ma} = M(1 - 4aj) - \sqrt{2Ma} + \sqrt{M^2(1 - 4aj)^2 - 2Ma^2} > 0$. $\tilde{\omega} \geq 0$ is required to describe the outgoing and incoming spherical waves in the large distance in (38). The possibility to make $|A_{l\lambda\lambda}|^2 < 0$ is $K_\ast < 0$. From (19) and (34), $K_\ast < 0$ and $\tilde{\omega} \geq 0$ lead to

$$0 \leq \omega \leq \omega_c = \frac{4\lambda(r_+^4 + Ma)}{r_+^4 + 2Ma^2},$$ \quad (41)

and

$$\omega \geq \omega_0 = 8\lambda j,$$ \quad (42)

respectively. The condition for the occurrence of the super-radiance in this black hole background is $\omega_0 \leq \omega_c$.

From (41) and (12), we obtain the ratio

$$\frac{\omega_0}{\omega_c} = \frac{2j(r_+^4 + 2Ma^2)}{jr_+^4 + Ma}.$$ \quad (43)

When $a = 0$, from the above formula we have $\omega_0 = 2\omega_c$. This means that although the Schwarzschild Gödel black hole contains the rotational parameter $j$ of the Gödel universe it is impossible to occur super-radiance.

Fig.(3) shows the changes of the ratio $\omega_0/\omega_c$ with $a$ and $j$, which tells us that for the fixed $j$, there exists a lower bound of $a$ for the super-radiance to occur. But for the fixed $a$, there is the upper bound of $j$ for the super-radiance to happen. This is also shown in Fig.(2).

Now let us turn to study the luminosity of the Hawking radiation for the mode $l = 0$, $\lambda = 0$ which plays a dominant role in the greybody factor. Performing an analysis similar to that in [27, 28, 29], we can rewrite the greybody factor (39) as

$$|A_{l\lambda\lambda}|^2 \simeq \frac{\pi \omega^3 r_+^3}{2} \left[1 + \frac{2Ma^2}{r_+^4}\right].$$ \quad (44)
FIG. 3: Ratio of the value $\omega_0/\omega_c$ with the change of $a$ and $j$.

FIG. 4: The luminosity of Hawking radiation $L$ of scalar particles propagating in the rotating Gödel Black Holes in Minimal Five-Dimensional Gauged Supergravity, for $l = 0$ and $\lambda = 0$ and different $j$ and $a$.

For the slowly rotating black hole, when $j$ increases, the greybody factor decreases. In the limit $a \to 0$, the form of the formula (44) reduces to that in the usual five dimensional Schwarzschild black hole spacetime [31]. Combining it with equation (7), the luminosity of the Hawking radiation is given by

$$L = \int_0^\infty \frac{d\omega}{2\pi} |A_{lm\lambda}|^2 \frac{\omega}{e^{(\omega-\Omega_H \lambda)/T_H} - 1} \approx \frac{3\zeta(5)}{16\pi^5} \frac{r_+^2}{r_+^4 + 2Ma^2} \left(1 - \frac{r_+^2}{r_+^4}\right)^5 = GT_H^5,$$

where $G = 6\zeta(5)r_+^4 \left[1 + \frac{2Ma^2}{r_+^4}\right]^{\frac{3}{2}}$. In figure (4), we show the dependence of the luminosity of Hawking radiation.
on the G"odel parameter $j$ for different angular momentum parameters. In the limit $j \to 0$, the luminosity of Hawking radiation $L$ decreases with the increase of $a$, which is consistent with that in usual five dimensional Kerr black hole spacetime. However, when $j \neq 0$, from (45) we find that there is a peak of the luminosity of the Hawking radiation when $dL/da = 0$. For fixed $M = 1$ but different $j$, peaks appear at different $a_p$ which are listed in table I. With the increase of $j$, the $a_p$ increases. When $a < a_p$, we observe in Fig.(4) that with the increase of $a$, $L$ increases. For $a > a_p$, we see in Fig.(4) that the luminosity of Hawking radiation decreases with the further increase of $a$. When $a \to 0$, the formula (45) reduces to that in the five dimensional Schwarzschild G"odel black hole spacetime. In the small $a$ case, we find that the luminosity of the Hawking radiation $L$ increases with $j$, which can be explained by Hawking temperature of the black hole. In the small $a$ regime the behavior of the Hawking temperature $T_H$ can be expressed as

$$T_H \sim \frac{1}{\sqrt{2M}} + \frac{\sqrt{2}j}{\sqrt{M}} a + \frac{12j^2M - 1}{\sqrt{2M}} a^2 + O(a^3).$$

Thus, as $j$ increases the Hawking temperature increases. This leads to that the luminosity of Hawking radiation $L$ increases with $j$ in the small $a$ case. Our result also implies that the G"odel parameter can enhance the Hawking radiation. The influence of the G"odel parameter on the luminosity of Hawking radiation is in agreement with that observed in the quasinormal modes results in [15]. The larger G"odel parameter $j$ enhances the power emission of the black hole so that it is more difficult for the perturbation outside the black hole to die out.

V. SUMMARY

In this paper, we have studied the greybody factor and Hawking radiation for a massless scalar field in the background of a five dimensional rotating G"odel black hole in the low energy and low angular momentum approximation. We have found that the absorption probability and Hawking radiation contain the imprint of the G"odel parameter. With the inclusion of the G"odel parameter $j$, we have observed richer physics which has not been shown in the usual rotating black hole background. The effects on the super-radiance and the luminosity of the Hawking radiation due to the G"odel parameter $j$ are different from that of the angular

| $j$  | 0  | 0.05 | 0.1  | 0.15 | 0.2 |
|------|----|------|------|------|-----|
| $a_p$ | 0  | 0.0392 | 0.0742 | 0.1025 | 0.1238 |

TABLE I: The change of $a_p$ with different $j$. Here $M = 1$. 
momentum of the black hole. The observation that the Gödel parameter can enhance the Hawking radiation is interesting and this might open a window to detect whether our universe is rotating or not. It would be of interest to generalize our study to other fields emission, such as the gravitational field etc. Work in this direction will be reported in the future.

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