Heavy cosmic strings

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(Dated: 21 February 2006)

We argue that cosmic strings with high winding numbers generally form in first order gauge symmetry breaking phase transitions, and we demonstrate this using computer simulations. These strings are heavier than single-winding strings and therefore more easily observable. Their cosmological evolution may also be very different.

I. INTRODUCTION

Interest in cosmic strings [1, 2] has recently been resparked by new observational and theoretical findings. It now seems that cosmic strings are relatively generic prediction of superstring theory [3], which is currently thought to be the most promising candidate for a theory of quantum gravity. They also appear to be unavoidable in grand unification based on field theory, if the same theory is to explain cosmological inflation as well as unification of elementary particle interactions [4]. Discovery of cosmic strings would therefore open up an observational window to the very early universe, to extremely high energy physics and possibly also to quantum gravity. A pair of apparently identical galaxies at redshift \( z \approx 0.46 \) known as CSL-1 [5] caused a great deal of excitement because it seemed to bear the hallmarks of gravitational lensing by a string. Even though pictures taken by the Hubble Space Telescope showed that it is, in fact, nothing but two separate galaxies [6], it led to a large amount of theoretical work and significant advances in the field [7].

In 1980’s, cosmic strings were seen as a potential explanation for the origin of structure in the universe [2], and therefore their observational signatures have been studied in detail. The strength of these signatures depends on the dimensionless number \( G_\mu \), in which \( G \) is Newton’s gravitational constant and \( \mu \) is the tension of the string, defined as the energy per unit length. So far, cosmic string searches have not found anything, which sets an upper bound on the tension. The strongest bounds arise from the cosmic microwave background [8] and timing of pulsars [9, 10], both give \( G_\mu \lesssim \text{few} \times 10^{-9} \). The natural value of \( G_\mu \) in superstring theory models seems to be significantly lower, around \( 10^{-10} \ldots 10^{-9} \). While these strings would be observable with planned gravitational wave experiments such as LISA, [11] they are clearly out of reach of our current observations.

However, theories with cosmic strings predict rather generally a spectrum of different string tensions. This is the case in field theory models in the “Type-I” regime, where strings with any integer winding number are stable, as well as superstring models in which there are stable bound states of D and F-strings known as \((p, q)\)-strings [8]. As was mentioned in Ref. [12], cosmic string searches are generally constraining a somehow averaged value of the tension. To be specific, let us denote the tension of a single-winding string by \( \mu_0 \), and the ratio of the tension of a given string to this by \( x = \mu/\mu_0 \). Because the amplitude of gravitational waves behaves as \( \Omega_g \propto (G_\mu)^{2/3} \) in the relevant range [11], the pulsar bound actually constrains \( x^{2/3} G_\mu \) rather than the tension \( G_\mu \) of an elementary single-winding string. This means that if multiple-winding strings exist, these searches can be much more sensitive than they appear.

The cosmological evolution of a network of strings with multiple windings is an open problem, although some studies have recently been done. For instance, Ref. [12] treats the interactions of strings as local events and ignores their extended nature. Field theory simulations of multiple-winding cosmic string interactions [13, 14] have shown that interactions typically lead to lower winding numbers. For multiple-winding strings to exist, they must have been formed with high winding numbers in the first place. The aim of this paper is to show that this can indeed happen.

II. SETUP

We will focus on cosmic string formation in a finite-temperature phase transition in the Abelian Higgs model. Realistic grand unified theories should behave qualitatively in the same way, and experience with superstring theory models seems to indicate that the same general principles would apply also in that case [15]. The model consists of two fields, the complex Higgs scalar \( \phi \) and the gauge field \( A_\mu \) associated with an U(1) gauge symmetry. The Lagrangian is

\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \phi)^* D^\mu \phi - \lambda \left( \phi^* \phi - \eta^2 \right)^2 ,
\]

where we have used the field strength tensor \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \) and the covariant derivative \( D_\mu = \partial_\mu + ie A_\mu \). The model has two dimensionless parameters, the gauge coupling constant \( e \) and the Higgs self-coupling \( \lambda \), and the dimensionful scale \( \eta \), which corresponds to the vacuum expectation value of \( \phi \). The corresponding equa-
tions of motion in Minkowski space are
\[
\partial_\mu F^{\mu\nu} = -2\text{e}^{-\text{Im}\phi^*} D^\nu \phi,
\]
\[
D_\mu D^\mu \phi = -2\lambda \left( \phi^* \phi - \eta^2 \right) \phi. \tag{2}
\]

To model cosmic string formation in the early universe \cite{1,16}, we study our model at a non-zero temperature. It has two phases: the high-temperature symmetric (Coulomb) phase characterized by a massless photon field, and the low-temperature broken (Higgs) phase which is essentially a relativistic version of a superconducting phase.

As in superconductors, the “magnetic field” \( \vec{B} = \nabla \times \vec{A} \) cannot exist in the broken phase. Instead, it gets confined into Nielsen-Olesen vortex lines, which are our cosmic strings. The flux \( \Phi \) carried by a string is an integer multiple of the flux quantum \( \Phi_0 = 2\pi/e \), and this defines the winding number \( N_W = \Phi/\Phi_0 \) of the string. At zero temperature, strings with \( N_W > 1 \) are stable if \( e^2/\lambda > 1 \), and at high temperatures one requires \( e^2/\lambda \gtrsim 1.7 \) \cite{17}. In this so-called Type I regime, the transition is of first order.

We investigate the model in the Type I regime. The system is initially in thermal equilibrium in the symmetric phase, and the temperature is then gradually decreased. When the temperature falls below a certain value, the symmetric phase becomes metastable. Bubbles of broken phase nucleate, grow and eventually fill the whole space. In our analytical calculations, we assume that the transition is strongly first order so that by this time their typical size is much larger than the microscopic scales such as the thickness of bubble walls or their radius at nucleation.

### III. STRING FORMATION

It is often assumed that cosmic strings are formed by the Kibble mechanism \cite{1}, even though it is strictly speaking only valid in the global limit \( e = 0 \). According to this picture, the complex phase of \( \phi \) has a random but constant value in each bubble. When two bubbles collide, the phase angle takes the shortest way to interpolate between the two values. Whenever three bubbles meet, it is possible that a hole of symmetric phase is left between them. There is then a probability of 1/4 that, when going from bubble 1 to bubble 2 to bubble 3, the phase angle changes by \( 2\pi \). If this happens, a string is formed when the hole eventually closes. The number density of strings per unit cross-sectional area is therefore roughly \( 1/R^2 \), where \( R \) is the typical bubble size. Note that only strings with \( N_W = 1 \) can form. Even \( N_W = 2 \) would require a simultaneous collision of at least five different bubbles at the same point, and since we are assuming that the wall thickness is negligible, this never happens.

Away from the global limit, the Kibble mechanism does not fully describe string formation \cite{16,18}. Strings are also produced by thermal fluctuations of the magnetic field, which become trapped in regions of symmetric phase \cite{16,19}. As we will show, this mechanism can form strings with \( N_W > 1 \).

In thermal equilibrium the state of the system is given by an ensemble of configurations with a Boltzmannian probability distribution, \( p \propto \exp(-E/T) \), where \( E = \int d^3x \rho \) is the total energy of the system. In the temporal gauge \( (A_0 = 0) \), the energy density is
\[
\rho = \frac{1}{2} \left( \vec{E}^2 + \vec{B}^2 \right) + \phi^* \phi + \left| \partial \phi \right|^2 + \lambda \left( \phi^* \phi - \eta^2 \right)^2, \tag{3}
\]
where \( \vec{E} = -\partial_\mu \vec{A} \) and \( \vec{B} = \nabla \times \vec{A} \). Most importantly for us, \( \vec{B} \) has approximately Gaussian thermal fluctuations. Magnetic flux may get swatted between two colliding bubbles \cite{13}, but in this paper we will focus on three-bubble collisions, because they can produce higher winding numbers \cite{19}. When the three bubbles coalesce leaving a hole of symmetric phase between them, any flux that was there initially will remain trapped in the hole because flux lines cannot penetrate the broken phase. When the hole closes, the flux trapped in it forms a string.

The winding number of the string is determined by the trapped flux, with a possible contribution of \( \pm 1 \) from the Kibble mechanism, which we will ignore from now on. We must therefore estimate how much flux there was in the region before it became trapped. At that time, it was still in thermal equilibrium with its surroundings, and therefore the flux is determined by the thermal equilibrium distribution. We can calculate its typical value using the saddle point method. In the symmetric phase, where \( \phi = 0 \), the energy density is proportional to \( \vec{B}^2 \).

This means that the minimal energy \( E_{\text{min}}(\Phi) \) for a configuration with given magnetic flux \( \Phi \) through the region is proportional to \( \Phi^2 \). For dimensional reasons it has to be of the form \( E_{\text{min}}(\Phi) = C\Phi^2/R \), where \( R \) is the typical bubble size and \( C \) is a dimensionless number of order one.

The typical value \( \Phi_{\text{rms}} \) of the flux, given by the square root of the variance of the probability distribution \( p(\Phi) \propto \exp(-E_{\text{min}}(\Phi)/T) \), is \( \Phi_{\text{rms}} \approx \sqrt{RT} \). When the hole between the bubbles eventually closes, all this flux turns into a string with typical winding number
\[
N_W \approx \Phi_{\text{rms}}/\Phi_0 \approx (e/2\pi)\sqrt{RT} \approx (e/2\pi)(v/T)^{1/8}T^{1/2}. \tag{4}
\]

Here we have used dimensional analysis to express \( R \) in terms of the bubble wall velocity \( v \) and the bubble nucleation rate \( \Gamma \) as \( R \approx (v/T)^{1/4} \).

**TABLE I: Parameter values used in the simulations.**
FIG. 1: Time evolution of $\langle \phi^2 \rangle$ for different cooling rates. The time axis has been rescaled by the cooling rate, so that it is in one-to-one correspondence with the effective temperature.

Because $R$ is much greater than the thermal wavelength $1/T$, $N_W$ can be large. If we, for instance, assume that $R$ is comparable to the Hubble radius $1/H$, we find

$$N_W \approx \frac{e}{2\pi g_s^{1/4}} \sqrt{\frac{m_{Pl}}{T}} \approx \frac{e}{2\pi (g_s G \mu_0)^{1/4}}, \quad (5)$$

where $g_s$ is the effective number of thermal degrees of freedom. If the tension $\mu_0$ of a single-winding string is low, the typical winding number can therefore be high. In particular, this means that the typical observed tension $\mu \approx N_W \mu_0$ scales as $(G \mu_0)^{3/4}$ rather than $G \mu_0$, making the strings more easily detectable. For $e = 1$ and $g_s \approx 100$, this would strengthen the pulsar bound by about a factor of two.

IV. SIMULATIONS

To test this theoretical picture, we carried out numerical simulations of first-order phase transitions in the Abelian Higgs model. We discretized the theory on a $256^3$ lattice in the standard way using the non-compact formulation (see, for instance, Ref. [21]).

FIG. 2: Bubbles of broken phase at the time when $\langle \phi^2 \rangle = 0.33$ in the simulations with cooling rates (a) $\sigma = 2.5 \times 10^{-4}$ and (b) $\sigma = 2.5 \times 10^{-5}$. With lower cooling rate, the bubbles are larger and there are fewer of them, because the nucleation rate was lower at the time of their nucleation.

FIG. 3: String networks formed in simulations with two different cooling rates $\sigma$. In both cases, we have used a short period of gradient flow to remove short-distance thermal noise, and plotted isosurfaces of $|\phi|$. We have made movies of the full time evolution available at http://www.imperial.ac.uk/people/a.rajantie/research/heavystrings/.

The values of all parameters are given in Table I. We first generated thermal initial conditions by evolving the system in time using the classical equations of motion and generating random values at random intervals for those components of the time derivatives $\dot{\phi}$ and $\vec{E} = -\partial_0 \vec{A}$ that do not appear in Gauss’s law $\vec{\nabla} \cdot \vec{E} = 2e \text{Im} \phi^* \dot{\phi}$.

After 512 units of time and 64 randomization steps, we were satisfied that the state of the system was in equilibrium. After that, we added a small damping term $\sigma$ to the equations of motion, and evolved the system in time without any random noise. The damping term was implemented in such a way that it only affected the com-
ponents of the field that do not appear in Gauss’s law. It cools the system down as $T_{\text{eff}} \propto \exp(-\sigma t)$ and serves two purposes: It causes the phase transition to start earlier because the bubble nucleation rate depends on the temperature, and it removes the latent heat released by the growing bubbles. Without damping, the system would end up in a state of two coexisting phases. We ran the simulation with two different cooling rates $\sigma = 2.5 \times 10^{-4}$ and $2.5 \times 10^{-5}$.

The time evolution of the variance $\langle \phi^2 \rangle$ of the scalar field is shown in Fig. 1 and consists of three stages: First, the system is in the symmetric phase, with a small and almost constant $\langle \phi^2 \rangle$. Then, at around $t \approx 0.05/\sigma$, there is a rapid transition to the broken phase, after which the system equilibrates, apart from a small number of strings that were formed in the transition. When the cooling is slower, the transition takes places at a higher temperature. Because the nucleation rate is then lower, fewer bubbles are nucleated and they can become larger. To confirm this, we have plotted in Fig. 2 the regions that are in the broken phase at the instant when $\langle \phi^2 \rangle = 0.33$.

Pictures of the string networks produced in the two runs are shown in Fig. 3. As expected, slower cooling produced fewer strings. The ratio of the total string length between the runs is 2.3, indicating a ratio of typical bubble sizes of 1.5.

In both cases, one can see several junctions where two strings merge into an heavier winding-2 string. This would not happen in the Kibble mechanism. The length ratio of windings 2 and 1 is higher in the slower transition (0.18 vs 0.15), supporting our theory. However, because of the low statistics and finite-size effects, this cannot be directly compared with the prediction in Eq. (4), which furthermore assumes that $N_W > 1$. This condition is not satisfied in this simulation or indeed in any similar simulation unless one can use several orders of magnitude larger lattice sizes.

V. CONCLUSIONS

We have shown that thermal first-order phase transitions can produce strings with high winding numbers and predicted the typical winding in Eq. (1). Our numerical simulations confirm the formation of strings with $N_W > 1$. Quantitative tests are not possible with simulations of this type because of the computing power they would require, but other numerical tests will be reported in a future publication [18]. It is not known how a string network with different winding numbers evolves, but if the heavy strings survive until today, they will be much more easily observable than ordinary single-winding strings. The current observational upper bounds would therefore become much stronger. This will, however, require a large scale simulation of the later evolution of the string network and is beyond the scope of this paper. It would also be interesting to extend this work to brane collisions in superstring theory models to find out whether heavy strings are also produced in that case by a similar mechanism.

Acknowledgments

This research was conducted in cooperation with SGI/Intel utilising the Alritx 3700 supercomputer and was supported by the ESF Coslab programme. MD was supported by EPRSC and The Cambridge European Trust, and AR by Churchill College, Cambridge.

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