Gauge invariant Lagrangian formulation of higher spin massive bosonic field theory in AdS space

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Abstract

We develop the BRST approach to Lagrangian construction for the massive integer higher spin fields in an arbitrary dimensional AdS space. The theory is formulated in terms of auxiliary Fock space. Closed nonlinear symmetry algebra of higher spin bosonic theory in AdS space is found and method of deriving the BRST operator for such an algebra is proposed. General procedure of Lagrangian construction describing the dynamics of bosonic field with any spin is given on the base of the BRST operator. No off-shell constraints on the fields and the gauge parameters are used from the very beginning. As an example of general procedure, we derive the Lagrangians for massive bosonic fields with spin 0, 1 and 2 containing total set of auxiliary fields and gauge symmetries.

1 Introduction

The various aspects of higher spin field theory attract much attention in modern theoretical physics (see reviews [1]). At present, research in this area is devoted to development of general methods allowing to construct Lagrangian formulations, to study the specific properties, to
clarify the possible underlying structures of such a theory, to see the relations with superstring theory and to find the ways leading to description of interacting higher spin fields (see e.g. [2–9] for recent progress in massive and [10–18] for recent progress in massless higher spin theories).

The present paper is devoted to formulating the general approach for deriving the Lagrangians of massive higher spin fields in AdS space of arbitrary dimension. The approach is based on development of BRST construction in higher spin field theory which automatically allows to obtain gauge invariant Lagrangian describing a consistent dynamics of higher spin fields. To be more precise we use BRST-BFV construction or BFV construction [19] (see also the reviews [20]).

BFV construction was originally developed for quantization of gauge theories. The gauge theories are formulated in phase space and characterized by first class constraints $T_a$ satisfying the involution relations in terms of Poisson brackets $\{T_a, T_b\} = f^c_{ab}T_c$ with structure functions $f^c_{ab}$. Basic notion of BFV construction is nilpotent BFV charge $Q$ defined as follows

$$Q = \eta^aT_a + \frac{1}{2}\eta^a\eta^b f^c_{ab}P_c + \ldots,$$

where $\eta^a$ and $P_a$ are canonically conjugate ghost variables. The dots mean the extra terms which in principle should be added in order to get the nilpotent charge for the case when the structure functions depend on phase coordinates of the theory. After quantization the BFV charge $Q$ becomes nilpotent Hermitian operator in extended space of states $|\Psi\rangle$ containing the ghosts. Subspace of physical states is defined by the equation $Q|\Psi\rangle = 0$ up to the transformation $|\Psi'\rangle = |\Psi\rangle + Q|\Lambda\rangle$ which is gauge transformation in the approach under consideration. It is proved that in subspace of physical states an unitary $S$-matrix exists [19]. We emphasize that initial point of this construction is a classical Lagrangian formulation of the gauge theory.

One of the problems of the higher spin field theory is its Lagrangian formulation. As was pointed out in the pioneer paper [21] the Lagrangian dynamics of higher spin fields demands to use, except a basic filed with given spin $s$, the auxiliary fields with less spins. Application of BRST approach to higher spin filed theory in principle allows to describe its dynamics. The matter is that the dynamics of gauge theory is completely concentrated into its constraints. Since the BRST charge is constructed on the base of the constraints, it includes whole information about dynamics. Therefore if we are able to express the Lagrangian in terms of BRST charge, we automatically obtain the consistent gauge invariant Lagrangian formulation of the theory with all auxiliary fields. Namely such a situation is realized in free higher spin field models [23, 30–33] (see also the early application of BRST approach to interacting higher spin filed theory in [34]).

Use of BRST construction in higher spin filed theory differs from its use for quantization in the following points. Classical Lagrangian formulation of the theory is unknown from the very beginning, moreover the main problem of the theory is to find such a formulation. Hence, classical system of constraints is also unknown and classical BRST charge can not be written. The only we know are the conditions on the fields defining irreducible representations of Poincare or AdS group with given mass and spin. These conditions are realized as some operators in auxiliary Fock space and interpreted as first class operator constraints. Then ones

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1 Following a tradition accepted in string field theory and massless higher spin field theory we call BRST-BFV construction as BRST construction.

2 We do not concern here the unclosed gauge algebras.

3 Following the accepted tradition we call this operator as BRST operator.

4 Application of BRST construction in higher spin field theory is analogous in some aspects to application of BRST construction in string field theory [27] (see also the reviews [28]). Relations of higher spin field theory to string field theory are discussed in [29].
demand that an algebra of these operators in terms of their commutators should be closed. This condition allows us to find a complete system of operators forming the closed algebra. As a result, from the very beginning we have the operators in Fock space and their commutators, the formulation looks like quantum although it is still a classical theory. Moreover, there are no any guaranties that the commutator algebra is a Lie algebra. As we will see further, the algebra in the case under consideration is nonlinear and analogous to $W_3$ algebra (see e.g [26]). Besides, it turns out that some of the operators closing the algebra can not be interpreted as constraints. Thus, application of BRST construction in higher spin field theory faces many principal problems.

In this paper we develop the gauge invariant approach to deriving the Lagrangian for massive higher spin fields in AdS space of arbitrary dimension. This approach is based on BRST construction and automatically yields a gauge invariant Lagrangian for field with any spin containing a complete set of auxiliary fields. As it should be, the gauge invariance of massive theory means that the corresponding Lagrangian formulation includes the Stueckelberg fields. The approach under consideration is a generalization of works [30–33] on Lagrangian construction of higher spin fields in arbitrary dimensional Minkowski space. The BRST approach to higher spin fields in AdS space was first used in [23], where the bosonic massless fields were considered. It was proved [23] that in four dimensions, obtained Lagrangian theory is reduced, after elimination of all the auxiliary fields, to the Fronsdal Lagrangian [22]. In this paper we develop a general method of Lagrangian construction for massive higher spin bosonic fields in AdS space and show that in massless limit it yields results presented in [23]

The paper is organized as follows. In Section 2 we introduce an auxiliary Fock space, which is a base of BRST construction for higher spin fields, and describe the differential operators corresponding to constraints. These constraints define an irreducible massive integer spin representation of the AdS group. In Section 3 we construct a set of operators forming a closed algebra in terms of commutators. It is shown that this algebra is nonlinear (quadratic algebra analogous to $W_3$ algebra) and includes two operators which are not constraints. In Section 4 we discuss an extension of the algebra by means of generalization of the method proposed in [30–33]. This generalization demands a deformation of the algebra and construction of a representation for the deformed algebra. Such a representation is found so that the new expressions for the operators consist of two parts: the initial expression of the operator plus an additional part. The additional parts for the operators are explicitly derived in Section 5. Then in Section 6 we discuss the construction of BRST operator for the deformed nonlinear algebra. After this in Section 7 we derive the Lagrangian describing propagation of massive bosonic field of arbitrary fixed spin in AdS space. There it is also shown that this theory is a gauge one and the gauge transformations are written down. In Section 8 we illustrate the procedure of Lagrangian construction by finding the gauge invariant Lagrangians for massive spin-0, spin-1, and spin-2 fields and their gauge transformations in explicit form. Section 9 summarizes the obtained results. Finally, in Appendix A we give general differential calculus in auxiliary Fock space on a gravitational background. Appendix B is devoted to obtaining of a representation of the operator algebra given in Table 2 in terms of creation and annihilation operators and in Appendix C we prove that the constructed Lagrangian reproduces the correct conditions on the field defining the irreducible representation of the AdS group.
2 Auxiliary Fock space for higher spin fields in AdS space-time.

Massive integer spin-$s$ representation of the AdS group is realized in space of totally symmetric tensor fields $\Phi_{\mu_1...\mu_s}(x)$ satisfying the following conditions (see e.g. [4])

$$\left(\nabla^2 + r[s^2 + s(d-6) + 6 - 2d] + m^2\right)\Phi_{\mu_1...\mu_s}(x) = 0,$$

$$\nabla^{\mu_1}\Phi_{\mu_1\mu_2...\mu_s}(x) = 0,$$

$$g^{\mu_1\mu_2}\Phi_{\mu_1\mu_2...\mu_s}(x) = 0,$$

where $r = \frac{R}{d(d-1)}$ with $R$ being the scalar curvature and $d$ being the dimension of the space-time. Our purpose is to construct a Lagrangian which reproduces these constraints as the consequences of the equations of motion.

In order to avoid an explicit manipulation with a number of indices it is convenient to introduce auxiliary Fock space generated by creation and annihilation operators with tangent space indices $(a, b = 0, 1, \ldots, d - 1)$

$$[a, a^+_b] = -\eta_{ab}, \quad \eta_{ab} = \text{diag}(+, -, \ldots, -).$$

An arbitrary vector in this Fock space has the form

$$|\Phi\rangle = \sum_{s=0}^{\infty} \Phi_{a_1...a_s}(x) a^{+a_1} \ldots a^{+a_s}|0\rangle = \sum_{s=0}^{\infty} \Phi_{\mu_1...\mu_s}(x) a^{+\mu_1} \ldots a^{+\mu_s}|0\rangle,$$

where $a^{+\mu}(x) = e^\mu_a(x)a^{+a}$, $a^\mu(x) = e^\mu_a(x)a_a$, with $e^\mu_a(x)$ being the vielbein. It is evident that

$$[a_\mu, a^+_{\nu}] = -g_{\mu\nu}.$$

We call the vector $|0\rangle$ as basic vector. The fields $\Phi_{\mu_1...\mu_s}(x)$ are the coefficient functions of the vector $|\Phi\rangle$ and its symmetry properties are stipulated by the properties of the product of the creation operators. We also suppose the standard relation

$$\nabla_\mu e^a_\nu = 0.$$

We want to realize the constraints (2)–(4) as some constraints on the vectors $|\Phi\rangle$. To do that ones introduce a special derivative operator acting on the vectors $|\Phi\rangle$.

$$D_\mu = \partial_\mu - \omega^a_\mu a^+_a a_b, \quad D_\mu|0\rangle = 0.$$

Then one can show that

$$[D_\mu, \Phi_{a_1...a_s}] = (\partial_\mu \Phi_{a_1...a_s}),$$

$$[D_\mu, a^+_{\nu}] = \omega^a_\mu a^+_a b^\nu, \quad [D_\mu, a^a] = -\omega^a_\mu a_b,$$

$$[D_\mu, a^+_{\nu}] = -\Gamma^\nu_{\mu\lambda} a^{+\lambda}, \quad [D_\mu, a^\nu] = -\Gamma^\nu_{\mu\lambda} a^\lambda,$$

$$[D_\mu, a^a_{\nu}] = \Gamma^\lambda_{\mu\nu} a^\lambda_{\nu}, \quad [D_\mu, a^\nu_{\lambda}] = \Gamma^\lambda_{\mu\nu} a^\lambda_{\nu}.$$

\[5\] Another (equivalent) form of the derivative operators acting on vectors of Fock space and differential calculus based on such operators are discussed in Appendix A.
where we suppose that $\partial_\mu a_a = \partial_\mu a_a^+ = 0$. The operator $D_\mu$ acts on the vectors $|\Phi\rangle$ as

$$D_\mu|\Phi\rangle = \sum_{s=0}^{\infty} (\nabla_\mu \Phi_{\mu_1...\mu_s}(x)) a^{+\mu_1} \cdots a^{+\mu_s}|0\rangle = \sum_{s=0}^{\infty} (\nabla_\mu \Phi_{\mu_1...\mu_s}(x)) a^{+\mu_1} \cdots a^{+\mu_s}|0\rangle,$$  

where $\nabla_\mu$ is the covariant derivative acting on tensor fields with tangent and space-time indices by standard rule.

For latter purposes it is useful to state the relations

$$g^{\mu\nu}(D_\mu D_\nu - \Gamma^\sigma_{\mu\nu} D_\sigma)|\Phi\rangle = \sum_{n=0}^{\infty} (\nabla_\mu \nabla_\nu \Phi_{\mu_1...\mu_n}(x)) a^{\mu_1} \cdots a^{\mu_n}|0\rangle,$$  

$$[D_\nu, D_\mu] = R_{\mu\nu\rho\tau} a^{\mu+} a^\rho = R_{\mu\nu\rho\tau} a^{\rho+} a^\tau,$$  

with $R_{\mu\nu\rho\tau}$ being the curvatures $\Gamma^{s}_{\mu\nu\rho}$, $\Gamma^{s}_{\mu\nu\rho\tau}$. Here and further we use following convention for AdS curvature tensor $R_{\mu\nu\alpha\beta} = g_{\mu\lambda} R^\lambda_{\nu\alpha\beta} = r(g_{\mu\alpha} g_{\nu\beta} - g_{\mu\beta} g_{\nu\alpha}).$

The operator $D_\mu$ is used for realization of the constraints in space of vectors $|\Phi\rangle$. Let us define the following operators

$$l_0 = D^2 + m^2 + r[X - 4g_0 + 6 - \frac{d(d-4)}{4}],$$  

$$D^2 = g^{\mu\nu}(D_\mu D_\nu - \Gamma^\sigma_{\mu\nu} D_\sigma),$$  

$$X = g_0^2 - 2g_0 - 4l_2^+ l_2,$$  

$$l_1 = -ia^{\mu} D_\mu, \quad l_1^+ = -ia^{\mu+} D_\mu,$$  

$$l_2 = \frac{1}{2} a^{\mu} a_\mu, \quad l_2^+ = \frac{1}{2} a^{\mu+} a_\mu,$$  

$$g_0 = -a^{\mu+} a_\mu + \frac{d}{2}. $$

Using the above relations, one can see that if the following constraints

$$l_0|\Phi\rangle = l_1|\Phi\rangle = l_2|\Phi\rangle = 0$$  

are fulfilled then each component $\Phi_{\mu_1...\mu_s}(x)$ of $|\Phi\rangle$ obeys the conditions describing spin-$s$ field in AdS space $[21, 22]$. Therefore if the conditions $[23]$ are fulfilled then vector $|\Phi\rangle$ will describe the fields with arbitrary integer spin $s$ in AdS space.

Let us turn to the arbitrary algebra generated by operators $l_0, l_1, l_2$.

### 3 Algebra generated by the constraints

We develop an approach to Lagrangian formulation for massive higher spin fields based on the following idea. We treat the conditions $[21, 22]$ or the equivalent conditions $[23]$ as the constraints in some unknown yet Lagrangian gauge theory. Our purpose is to restore a Lagrangian leading to given set of first class constraints. The most efficient way to realize this idea is to use general BRST method.

Basic notion of BRST method is Hermitian BRST operator which is constructed on the base of a set of first class constraints. To get Hermitian BRST operator, the set of constraints should be invariant under Hermitian conjugation. In the case under consideration, the operator $l_0$ is Hermitian, but the operators $l_1, l_2$ are not Hermitian. Therefore, to get a set of operators invariant under hermitian conjugation we should add two more operators $l_1^+, l_2^+$ to the operators $l_0, l_1, l_2$. As a result, the set of operators $l_0, l_1, l_1^+, l_2, l_2^+$ is invariant under
Hermitian conjugation. However, it is clear that the operators $l^+_1$, $l^+_2$ can not be interpreted as constraints on the vectors (6) on equal footing with the operators $l_0$, $l_1$, $l_2$. Indeed, taking Hermitian conjugation of (23) we see that they (together with $l_0$) are constraints on the bra vector

$$\langle \Phi | l_0 = \langle \Phi | l^+_1 = \langle \Phi | l^+_2 = 0.$$  

(24)

Nevertheless we will use the operators $l^+_1$, $l^+_2$ for BRST construction and show that they do not contribute to equations of motion for basic vector (6) obtained from final Lagrangian. This occurs due to these operators are multiplied on the annihilation ghost operators in BRST operator (see relations (55) and (71), some details are discussed in [31]).

Algebra of the operators $l_0$, $l_1$, $l^+_1$, $l_2$, $l^+_2$ is open in terms of commutators of these operators. To close the algebra we add to it all the operators generated by the commutators of $l_0$, $l_1$, $l^+_1$, $l_2$, $l^+_2$.

Before proceeding in this way let us introduce a more general operator $\tilde{l}_0$ instead of operator $l_0$

$$\tilde{l}_0 = D^2 + m^2 + (\alpha - 1)r \frac{d(d-4)}{4} + (\beta - 1)rX + \gamma rg_0$$  

(25)

and use this operator $\tilde{l}_0$ as a constraint. The operator $\tilde{l}_0$ depends on real parameters $\alpha$, $\beta$, $\gamma$. It is evident that $l_0$ (17) is obtained from $\tilde{l}_0$ at $\alpha = \frac{24}{d(d-4)}$, $\beta = 2$, $\gamma = -4$. We shall show that the correct on-shell condition (2) can be reproduced at any values of $\alpha$, $\beta$, $\gamma$, besides the expressions of the Lagrangian (C.7) will be simpler for $\tilde{l}_0$ at some specific values $\beta$, $\gamma$, which do not coincide with $l_0$ (see Appendix C). Also we note that the case of massless bosonic higher spin fields in AdS space considered in [23] corresponds to the following choice of the coefficients $m = \alpha = \gamma = 0$, $\beta = 2$.

Now let us close the algebra generated by the operators $\tilde{l}_0$, $l_1$, $l^+_1$, $l_2$, $l^+_2$. As a result we obtain two more Hermitian operators: $g_0$ (22) and

$$l = m^2 + \alpha \frac{d(d-4)}{4} r.$$  

(26)

The algebra of operators (24)–(26) is closed and it is given in Table 1 where

$$[l_1, \tilde{l}_0] = \beta r (4l^+_1l_2 + 2g_0l_1 - l_1) + \gamma rl_1,$$  

(27)

$$[\tilde{l}_0, l^+_1] = \beta r (4l^+_2l_1 + 2l^+_1g_0 - l^+_1) + \gamma rl^+_1,$$  

(28)

$$[l_1, l^+_1] = \tilde{l}_0 - l + (2 - \beta)rX - \gamma rg_0.$$  

(29)

In this Table and in the subsequent ones the first arguments of the commutators are listed in the first column, the second arguments are listed in the upper row. The algebra corresponding to Table 1 is a base for massive integer higher spin field Lagrangian construction in AdS space.

We will call this algebra as massive integer higher spin symmetry algebra in AdS space. We want to emphasize here two points. First, the operator $l$ commutes with all other operators and therefore it plays a role of central charge. Second, due to (27)–(29), the given algebra is non-linear, the right hand sides of these commutators are quadratic forms in operators.

Since operators $g_0$ and $l$ are obtained as commutators of a constraint on the ket vector (23) with a constraint on the bra vector (24), these operators $g_0$ and $l$ can not be considered as constraints neither in the space of bra-vectors nor in the space of ket-vectors.

One can show that a straightforward use of BRST construction as if all the operators are the first class constraints doesn’t lead to the proper equations (23) for any spin (see e.g. [30–33] for
the cases of higher spin fields in flat space). This happens because among the above hermitian operators there are operators which are not constraints (g₀ and l in the case under consideration) and they bring two more equations (in addition to (23)) on the physical field (6). Thus we must somehow get rid of these supplementary equations.

Method of avoiding the supplementary equations consists in constructing new enlarged expressions for the operators of the algebra given in Table 1 so that the hermitian operators which are not constraints will be zero.

### 4 Constructing the deformations of the symmetry algebra

Our purpose here is to construct the new enlarged operators satisfying some deformed algebra so that the new operators g₀ and l become zero or (as was shown in [31]) they contain arbitrary parameters which then defined from the condition that the supplementary equations do not give more restrictions on the basic vectors (6) in addition to (23).

The method we apply in this paper for constructing the enlarged expressions for the operators is a generalization of the method used in our papers [30–33] where the operator algebras are Lie algebras.

In case of Lie algebras we acted as follows. We enlarged the representation space introducing new creation and annihilation operators and constructed a new representation of the corresponding operators so that the expression for any operator was a sum of two parts: the initial expression for the operator plus a specific part which depends on the new creation and annihilation operators only. As a result in this new representation the operators which are not constraints were zero or contained arbitrary parameters whose values would define later

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6Introducing the new creation and annihilation operators in BRST construction for higher spin field theory is analogous to a conversion procedure (see e.g. [35]) which is used for quantization of constraint systems with
One more requirement was that the vector in the enlarged space (including the ghost fields) should be independent of the ghost fields corresponding to the operators which are not constraints [30–33].

The generalization of the above method to the case of non-linear symmetry algebra given by Table I is based on deformation of algebra of the enlarged operators in comparison with the corresponding initial algebra.

Ones describe the method of deformation. Let us denote all the operators of the algebra given in Table I as $l_i$. Then the structure of the algebra looks like

$$[l_i, l_j] = f^k_{ij} l_k + f^{km}_{ij} l_k l_m,$$  \hspace{1cm} (30)

where $f^k_{ij}$, $f^{km}_{ij}$ are constants. In case of $f^{km}_{ij} = 0$ we have a Lie algebra and the described method will be reduced to the method used in the flat space case [30–33]. As in the case of a Lie algebra we enlarge the representation space by introducing the additional creation and annihilation operators and construct the new operators of the algebra $l_i \rightarrow L_i$

$$L_i = l_i + l'_i,$$  \hspace{1cm} (31)

where $l'_i$ are the part of the operator which depends on the new creation and annihilation operators only (and constants of the theory like the mass $m$ and the curvature). It is evident that if we fix the structure of the new operators (31) then we are not able to preserve the algebra (30). Therefore there are two possibilities. The first one is to reject (31) and the second one is to deform the algebra (30). We choose the second possibility and demand that the new operators $L_i$ (31) are in involution relations

$$[L_i, L_j] \sim L_k.$$  \hspace{1cm} (32)

Since $[l_i, l'_j] = 0$ we have

$$[L_i, L_j] = [l_i, l_j] + [l'_i, l'_j] = f^k_{ij} L_k - (f^{km}_{ij} + f^{mk}_{ij}) l'_m L_k + f^{km}_{ij} l_k L_m - f^k_{ij} l'_k + f^{km}_{ij} l'_m l'_k + [l'_i, l'_j].$$  \hspace{1cm} (33)

Then in order to provide (32) the last three terms must be canceled. Therefore we put

$$[l'_i, l'_j] = f^k_{ij} l'_k - f^{km}_{ij} l'_m l'_k$$  \hspace{1cm} (34)

and as a result we have the deformed algebra for the enlarged operators

$$[L_i, L_j] = f^k_{ij} L_k - (f^{km}_{ij} + f^{mk}_{ij}) l'_m L_k + f^{km}_{ij} L_k L_m.$$  \hspace{1cm} (35)

Thus we see that the algebra (35) of the enlarged operators $L_i$ is deformed in comparison with the algebra (30) of the initial operators $l_i$. The second term in this algebra shows that some of the structure functions are not the constants and depend on the new creation and annihilation operators by means of $l'_m$. In case of Lie algebra $f^{km}_{ij} = 0$ the algebras of the initial operators $l_i$, of the additional parts $l'_i$, and of the enlarged operators $L_i$ coincide. Namely this situation takes place in the flat space [30–33].

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7 The method can easily be generalized to the case when the algebra has the structure

$$[l_i, l_j] = f^k_{ij} l_k + f^{km}_{ij} l_k l_m + \cdots + f^{k_1 \cdots k_N}_{ij} l_{k_1} \cdots l_{k_N}$$

where all $f$’s are constants.

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second class constraints.
Before we go further, ones discuss some details concerning the construction of the algebra (35). First, we should find the operators (additional parts) $l'_i$ satisfying the algebra (34). Second, these additional parts corresponding to the operators which are not constraints ($g'_0$ and $l'_1$ in the case under consideration) should contain an arbitrary parameter or vanish in sum with their initial expression ($g_0$ and $l$ in the case under consideration). Third, the additional part corresponding to the Hermitian constraint $\tilde{l}'_0$ should contain an arbitrary parameter which value will be defined from the condition of reproducing the correct conditions (23). After the additional parts $l'_i$ will be calculated we proceed as follows: ones construct BRST operator as if all the operators $L_i$ are the first class constraints and the vector in the enlarged space (including the ghost fields) must be independent of the ghost fields corresponding to the operators which are not constraints.

In next Section we study the algebra (34) of the additional parts of the operators $l'_i$ and find their explicit expressions.

### Table 2: Algebra of the additional parts for the operators

|        | $\tilde{l}'_0$ | $l'_1$ | $l'^+_1$ | $l'_2$ | $l'^+_2$ | $g'_0$ | $l'$ |
|--------|---------------|--------|----------|--------|----------|--------|-----|
| $\tilde{l}'_0$ | 0             | $-\left(36\right)$ | $-2\gamma r l'_2$ | $2\gamma r l'^+_2$ | 0        | 0     |     |
| $l'_1$  | $\left(36\right)$ | 0     | $\left(38\right)$ | 0         | $-l'^+_1$ | $l'_2$ | 0   |
| $l'^+_1$ | $-\left(37\right)$ | $-\left(38\right)$ | 0     | $l'_1$  | 0         | $-l'^+_1$ | 0   |
| $l'_2$  | $2\gamma r l'_2$ | 0     | $-l'_1$ | 0         | $g'_0$    | $2l'_2$ | 0   |
| $l'^+_2$ | $-2\gamma r l'^+_2$ | $l'^+_1$ | 0     | $-g'_0$  | 0         | $-2l'^+_2$ | 0   |
| $g'_0$  | 0             | $-l'_1$ | $l'^+_1$ | $-2l'_2$ | $2l'^+_2$ | 0     | 0   |
| $l'$    | 0             | 0     | 0        | 0         | 0         | 0     | 0   |

5 Constructing the additional parts of the operators

The procedure described in Section 4 in the case under consideration leads to algebra (34) for the operators of the additional parts in the form given by Table 2 where

\[
[l'_1, l'_0] = -\beta r (4l'^+_2 l'_2 + 2g'_0 l'_1 - l'_1) + \gamma r l'_1, \quad (36)
\]
\[
[l'_0, l'^+_1] = -\beta r (4l'^+_2 l'_1 + 2l'^+_1 g'_0 - l'^+_1) + \gamma r l'^+_1, \quad (37)
\]
\[
[l'_1, l'^+_1] = \tilde{l}'_0 - l' - \gamma r g'_0 + (\beta - 2)r(g'_0^2 - 2g'_0 - 4l'^+_2 l'_2). \quad (38)
\]

Ones point out that in the case of massless fields considered in [23] $m = \alpha = \gamma = 0$, $\beta = 2$, and therefore there was possible to construct the additional parts so that $\tilde{l}'_0 = l' = l'^+_1 = l'^+_2 = 0$.

In the more general case considered here the explicit expressions for the additional parts
can be calculated with the help of the method described in \cite{24,30}. As a result we get

\[
\tilde{l}'_0 = m_0^2 + \gamma \rho (b_1^+ b_1 + 2b_2^+ b_2) - \beta r (2h - 1) b_1^+ \sum_{k=0}^{\infty} \left[ \frac{-8r}{m_1^2} \right]^k \frac{b_2^{2k+1}}{(2k+1)!} \\
- 2\beta r b_1^+ b_2^2 \sum_{k=0}^{\infty} \left[ \frac{-8r}{m_1^2} \right]^k \frac{b_2^{2k+2}}{(2k+2)!} + \frac{1}{2} \beta M^2 \sum_{k=1}^{\infty} \left[ \frac{-8r}{m_1^2} \right]^k \frac{b_2^{2k}}{(2k)!},
\]

(39)

\[
l'_1 = -m_1 b_1^+ b_2 + m_1 b_1^+ \frac{2h - 1}{4} \sum_{k=1}^{\infty} \left[ \frac{-8r}{m_1^2} \right]^k \frac{(b_2^+)^{k-1} b_1^{2k}}{(2k)!} \\
+ \frac{1}{2} m_1 b_1^+ \sum_{k=1}^{\infty} \left[ \frac{-8r}{m_1^2} \right]^k \frac{(b_2^+)^{k-1} b_1^{2k+1}}{(2k+1)!} + \frac{M^2}{m_1} \sum_{k=0}^{\infty} \left[ \frac{-8r}{m_1^2} \right]^k \frac{b_2^{2k+1}}{(2k+1)!},
\]

(40)

\[
l'_2 = (b_1^+ b_2 + b_1^+ b_1 + h) b_2 - \frac{2h - 1}{4} b_1^+ \sum_{k=1}^{\infty} \left[ \frac{-8r}{m_1^2} \right]^k \frac{(b_2^+)^{k-1} b_1^{2k+1}}{(2k+1)!} \\
- \frac{1}{2} b_1^+ \sum_{k=1}^{\infty} \left[ \frac{-8r}{m_1^2} \right]^k \frac{(b_2^+)^{k-1} b_1^{2k+2}}{(2k+2)!} - \frac{M^2}{m_1} \sum_{k=0}^{\infty} \left[ \frac{-8r}{m_1^2} \right]^k \frac{b_2^{2k+2}}{(2k+2)!},
\]

(41)

\[
l'_1^+ = m_1 b_1^+,
\]

(42)

\[
l'_2^+ = b_2^+,
\]

(43)

\[
g'_0 = b_1^+ b_1 + 2b_1^+ b_2 + h,
\]

(44)

\[
l' = -m^2 - \alpha \frac{d(d-4)}{4} r,
\]

(45)

where we have denoted

\[
M^2 = m^2 + m_0^2 + \alpha \frac{d(d-4)}{4} r + (\beta - 2) h(\beta - 2) r - \gamma hr.
\]

(46)

In the above expressions \(h\) is a dimensionless arbitrary constant, \(m_0\) and \(m_1\) are the arbitrary constants with dimension of mass. For constructing the additional parts \(39\)–\(45\) we have introduced, in accordance with method given in Section 4, two pairs of new bosonic creation and annihilation operators satisfying the standard commutation relations

\[
[b_1, b_1^+] = [b_2, b_2^+] = 1.
\]

(47)

The found additional parts of operators possess all the necessary properties described in Section 4. The operators which are not constraints give zero in sum with their initial expressions for the operator \((l + l' = 0)\) or contain an arbitrary parameter \((g'_0\) contains an arbitrary parameter \(h)\) which value will be defined later from the condition that they do not generate the extra restrictions on the physical states. The additional part for the Hermitian operator \(\tilde{l}'_0\) also contains an arbitrary parameter \(m_0^2\) which value will be defined from the condition of reproducing the correct conditions \(24\). The massive parameter \(m_1\) remains arbitrary, in particular it can be expressed through the other massive parameters of the theory

\[
m_1 = f(m, r) \neq 0.
\]

(48)

---

\(^8\) To be completed a detailed calculation of the additional parts is given in Appendix \[B\].
For example one can put \( m_1 = m \) or \( m_1 = \lambda \), where \( \lambda \) is the inverse radius of the AdS space \( \lambda^2 = r \). This arbitrariness does not affect on the equations for the basic vector (6).

We note that the operators of the additional parts do not satisfy the usual properties
\[
(l_0^+ + l_0^-) \neq l_0^+ l_0^-, \quad (l_1^+ + l_1^-) \neq l_1^+ l_1^-, \quad (l_2^+ + l_2^-) \neq l_2^+ l_2^-
\]
if we use the standard rules of Hermitian conjugation for the new creation and annihilation operators
\[
(b_1^+ = b_1^+, \quad b_2^+ = b_2^+).
\]

To restore the proper Hermitian conjugation properties for the additional parts we change the scalar product in the Fock space generated by the new creation and annihilation operators
\[
\langle \Psi_1 | \Psi_2 \rangle_{\text{new}} = \langle \Psi_1 | K \Psi_2 \rangle
\]
with some unknown yet operator \( K \). This operators is defined from the condition that all the operators must have the proper Hermitian properties with respect to the new scalar product
\[
\langle \Psi_1 | K l_0^- | \Psi_2 \rangle = \langle \Psi_1 | (l_0^+) K | \Psi_2 \rangle, \quad \langle \Psi_1 | K g_0^- | \Psi_2 \rangle = \langle \Psi_1 | (g_0^+) K | \Psi_2 \rangle,
\]
\[
\langle \Psi_1 | K l_1^- | \Psi_2 \rangle = \langle \Psi_1 | (l_1^+) K | \Psi_2 \rangle, \quad \langle \Psi_1 | K l_2^- | \Psi_2 \rangle = \langle \Psi_1 | (l_2^+) K | \Psi_2 \rangle, \quad \langle \Psi_1 | K l_1^+ | \Psi_2 \rangle = \langle \Psi_1 | (l_1^-) K | \Psi_2 \rangle, \quad \langle \Psi_1 | K l_2^+ | \Psi_2 \rangle = \langle \Psi_1 | (l_2^-) K | \Psi_2 \rangle.
\]
The explicit expression for the operator \( K \) can be found using the method given in [25,30,31].

The calculations of the operator \( K \) are described in Appendix B.

6 The deformed algebra and BRST operator

In this section we find BRST operator and discuss the aspects stipulated by nonlinearity of the operator algebra.

Construction of BRST operator is based on following general principles (see the details in reviews [20]):

1. BRST operator \( Q' \) is Hermitian, \( Q'^+ = Q' \), and nilpotent, \( Q'^2 = 0 \).

2. BRST operator \( Q' \) is built using a set of first class constraints. In the case under consideration the operators \( \mathcal{L}_0, L_1, L_1^+, L_2, L_2^+ \), \( g_0 \) are used as a set of such constraints.

3. BRST operator \( Q' \) satisfies the special initial condition
\[
Q' \bigg|_{\mathcal{P} = 0} = \eta_0 \mathcal{L}_0 + \eta_1^+ L_1 + \eta_1 L_1^+ + \eta_2^+ L_2 + \eta_2 L_2^+ + \eta_G g_0.
\]

Here \( \eta_0, \eta_1, \eta_1^+, \eta_2, \eta_G \) are ghost ‘coordinates’ with ghost number \( gh(\eta) = +1 \) and \( \mathcal{P}_0, \mathcal{P}_1, \mathcal{P}_1^+, \mathcal{P}_2, \mathcal{P}_2^+ \), \( g_0 \) are their canonically conjugate ghost ‘momenta’ with ghost number \( gh(\mathcal{P}) = -1 \) satisfying the anticommutation relations
\[
\{\eta_0, \mathcal{P}_0\} = \{\eta_G, \mathcal{P}_G\} = \{\eta_1, \mathcal{P}_1^+\} = \{\eta_1^+, \mathcal{P}_1\} = \{\eta_2, \mathcal{P}_2^+\} = \{\eta_2^+, \mathcal{P}_2\} = 1.
\]

Our purpose here is to construct such an operator \( Q' \) in an explicit form.

Let us turn to the algebra of the enlarged operators \( L_i \). Straightforward calculation of commutators (in accordance with Section 4) allows to to find this algebra in the form given in Table 3, where
\[\text{In what follows we forget about operator } l, \text{ since its enlarged expression is zero } l + l' = 0. \text{ See formulas (26) and (53).}\]
The relations (57)–(59) show that the symmetry algebra is nonlinear (quadratic). Using the commutation relations one can write the right hand sides of quadratic products of the operators (57)–(59) in various equivalent forms. Each such a form of writing the algebra can, in principle, lead to different ordering prescriptions. It means, to control of constructing the BRST operator we should fix an ordering for quadratic products of operators in the algebra. We consider most general ordering prescription. All possible ways to order the operators in right hand sides of (57)–(59) are described in terms of arbitrary real parameters $\xi_1, \xi_2, \xi_3, \xi_4, \xi_5$. The ordered commutation relations look like

$$
[L_1, \bar{L}_0] = (\gamma - \beta)rL_1 + 4\beta rL_1^+L_2 - 4\beta r l_1^+L_2 - 4\beta r l_2^+L_1^+ \\
= 2\beta r G_0L_1 - 2\beta r l'_0G_0 - 2\beta r g'_0L_1, 
$$

(57)

$$
[\bar{L}_0, L_1^+] = (\gamma - \beta)rL_1^+ + 4\beta rL_2^+L_1 - 4\beta r l_2^+L_1 - 4\beta r l_1^+L_2^+ \\
= 2\beta r L_1^+G_0 - 2\beta r l'_0G_0 - 2\beta r g'_0L_1^+, 
$$

(58)

$$
[L_1, L_1^+] = \bar{L}_0 - \gamma rG_0 + 4(2 - \beta)r(l'_2^+L_2 + l'_2L_2^+) \\
- 2(2 - \beta)r g'_0G_0 + (2 - \beta)r(G_0^2 - 2G_0 - 4L_2^+L_2). 
$$

(59)

Before we begin a construction of the BRST operator ones point out that the algebra of the enlarged operators in the case under consideration is very similar to one considered in [26]. The only difference of our case from [26] consists in the symmetry property of constants $f_{ij}^{km}$.

|      | $\bar{L}_0$ | $L_1$ | $L_1^+$ | $L_2$ | $L_2^+$ | $G_0$ |
|------|------------|------|--------|------|--------|------|
| $\bar{L}_0$ | 0          | $\gamma r$ | $\beta r L_1$ | $-2\gamma r L_2$ | $2\gamma r L_2^+$ | 0     |
| $L_1$     | (57)       | 0    | $\gamma r$ | $-\beta r L_1$ | 0     | $-\gamma r L_1$ |
| $L_1^+$   | (58)       | $\beta r L_1$ | 0    | $\gamma r$ | $\beta r L_1$ | 0     |
| $L_2$     | $2\gamma r L_2$ | 0    | $-\gamma r L_1$ | 0     | $-\gamma r L_1$ | $G_0$ |
| $L_2^+$   | $-2\gamma r L_2^+$ | $L_1^+$ | 0    | $-G_0$ | 0     | $-2L_2^+$ |
| $G_0$     | 0          | $-L_1$ | $L_1^+$ | $-2L_2$ | $2L_2^+$ | 0     |

Table 3: Algebra of the enlarged operators

$$
[L_1, \bar{L}_0] = \gamma rL_1 + (2 - \xi_1)\beta rL_1^+L_2 + (2 + \xi_1)\beta rL_2L_1^+ - 4\beta r l_1^+L_2 - 4\beta r l_2^+L_1^+ \\
+ (1 - \xi_2)\beta r G_0L_1 + (1 + \xi_2)\beta r L_1G_0 - 2\beta r l'_0G_0 - 2\beta r g'_0L_1 \\
+ (\xi_1 - \xi_2)\beta r L_1, 
$$

(60)

$$
[\bar{L}_0, L_1^+] = \gamma rL_1^+ + (2 + \xi_3)\beta rL_2^+L_1 + (2 - \xi_3)\beta rL_1L_2^+ - 4\beta r l_2^+L_1 - 4\beta r l_1^+L_2^+ \\
+ (1 + \xi_4)\beta r L_1^+G_0 + (1 - \xi_4)\beta r G_0L_1^+ - 2\beta r l'_0G_0 - 2\beta r g'_0L_1^+ \\
+ (\xi_4 - \xi_3)\beta r L_1^+, 
$$

(61)

$$
[L_1, L_1^+] = \bar{L}_0 - \gamma rG_0 + 4(2 - \beta)r(l'_2^+L_2 + l'_2L_2^+) - 2(2 - \beta)r g'_0G_0 \\
+ (2 - \beta)r(G_0^2 - \xi_5G_0 - (2 + \xi_5)2L_2^+L_2 - (2 - \xi_5)L_2L_2^+). 
$$

(62)
In [26] this quantity is symmetric in upper indices $f_{ij}^{km} = f_{ij}^{mk}$, but in the present paper we leave them having arbitrary symmetry by means of introducing the arbitrary parameters $\xi_i$. At $\xi_i = 0$ we have the symmetrized product of the operators in rhs of (60)–(62) and our algebra will be a partial case of the algebra considered in [26]. It was proved in [26] that the BRST operator includes the terms of first, thirds and seventh order in ghosts. Therefore one can expect in our case the analogous terms in BRST operator plus some extra terms stipulated by difference in symmetry of the constants $f_{ij}^{km}$.

Now let us turn to construction of BRST operator. Demanding that BRST operator be Hermitian (with respect to the new scalar product (51)) we find equations on $\xi_i$. These equations leave us only one arbitrary coefficient which we denote $\xi$, the others are expressed through it as follows

$$\xi = \xi_1 = \xi_2 = \xi_3 = \xi_4, \quad \xi_5 = 0.$$  \hspace{1cm} (63)

In this case the commutators (60)–(62) take the form

$$[L_1, \tilde{L}_0] = \gamma r L_1 + (2 - \xi) \beta r L_1^+ L_2 + (2 + \xi) \beta r L_2 L_1^+ - 4\beta r l_1^+ L_2 - 4\beta r l_2^+ L_1^+ + (1 - \xi) \beta r G_0 L_1 + (1 + \xi) \beta r L_1 G_0 - 2\beta r l_1^+ G_0 - 2\beta r g_0 L_1, \hspace{1cm} (64)$$

$$[\tilde{L}_0, L_1^+] = \gamma r L_1^+ + (2 + \xi) \beta r L_2^+ L_1 + (2 - \xi) \beta r L_1 L_2^+ - 4\beta r l_2^+ L_1 - 4\beta r l_1^+ L_2 + (1 + \xi) \beta r L_1 G_0 - 2\beta r g_0 L_1^+ - 2\beta r l_1^+ G_0 - 2\beta r g_0 L_1^+, \hspace{1cm} (65)$$

$$[L_1, L_1^+] = \tilde{L}_0 - \gamma r G_0 + 4(2 - \beta) r (l_2^+ L_2 + l_2^+ L_2^+) - 2(2 - \beta) r g_0 G_0 + (2 - \beta) r (l_1^+ L_1 + l_1^+ L_1^+) - 2(2 - \beta) r g_0 G_0.$$  \hspace{1cm} (66)

To find the BRST operator we write it as a Hermitian operator polynomial of seventh degree in ghosts and demand its nilpotency. Such a procedure leads to the following result for the BRST operator $Q'$

$$Q' = \eta_0 \tilde{L}_0 + \eta_1^+ L_1 + \eta_1 L_1^+ + \eta_2 L_2 + \eta_2 L_2^+ + \eta G_0 - \eta_1^+ \eta_1 P_0 - \eta_2^+ \eta_2 P_G + (\eta \eta_1^+ + \eta_2^+ \eta_0) P_1 + (\eta \eta_2 G + \eta_1^+ \eta_2) P_1^+ + 2\eta \eta_2^+ P_2 + 2\eta_2 \eta_2 P_G$$

$$+ \beta r \eta_0 \eta_1^+ \left[ 4l_1^+ P_2 + 4l_2^+ P_1 + 2g_0^+ P_1 + 2l_1^+ P_G \right] + \beta r \eta_0 \eta_1 \left[ 4l_1^+ P_2 + 4l_2^+ P_1 + 2g_0^+ P_1 + 2l_1^+ P_G \right] - \beta r \eta_1^+ \eta_0 \left[ (2 - \xi) L_1 P_2 + (2 + \xi) L_2 P_1 + (1 - \xi) G_0 P_1 + (1 + \xi) L_1 P_G \right] - \beta r \eta_0 \eta_1 \left[ (2 - \xi) L_1 P_2 + (2 + \xi) L_2 P_1 + (1 - \xi) G_0 P_1 + (1 + \xi) L_1 P_G \right] - (2 - \beta) \gamma r \eta_1^+ \eta_1 \left[ G_0 P_G - 2L_2 P_2 - 2L_2 P_2^+ - 2g_0^+ P_G + 4l_1^+ P_2 + 4l_2^+ P_2 \right]$$

$$+ \gamma \gamma r \eta_0 \eta_1 P_G + \gamma \gamma r \eta_0 \left[ \eta_1 P_1 - \eta_1 P_1^+ + 2\eta_2^+ P_2 - 2\eta_2 P_2^+ \right] + 3\beta \gamma r \eta_0 \left[ \eta_1 \eta_2 P_2^+ P_2 + \eta_2^+ \eta_2^+ P_1 P_2 + \eta_2 \eta_2^+ P_2 P_2 \right.$$ \hspace{1cm} (67)

It is interesting to compare this result for BRST operator with the result of [26]. We see that BRST operator (67) unlike to [26] has no terms of the seventh degree in the ghosts (three ghost momenta and four ghost coordinates). The matter is that, according to [26] the presence (or the absence) of such terms in BRST operator depends on the number of operators whose commutators have quadratic rhs. To get the seventh degree terms we need at least four such
operators. In our case we get only three operators whose commutators have quadratic rhs \((64)-(66)\). These commutators are of \(\tilde{L}_0, L_1, L_1^+\) operators, therefore we have only three ghost coordinates \(\eta_0, \eta_1^+, \eta_1\) whereas the terms of seventh degree in ghosts demand at least four ghost coordinates. New point in compare with \([26]\) is appearance in BRST operator the terms of fifth degree in ghosts. Their origin is another symmetry property of the constants \(f_{ij}^{km}\) in the case under consideration in comparison with \([26]\). If we put \(\xi = 0\) (what corresponds to the symmetric ordering of the operators in rhs of \((64)-(66)\)) the BRST operator \(Q'\) \((67)\) has no terms of the fifth degree in ghosts as it should be in accordance with \([26]\).

Further we will show that the arbitrariness in BRST operator stipulated by the parameter \(\xi\) is resulted in arbitrariness of introducing the auxiliary fields in the Lagrangians and hence does not affect on dynamics of the basic field \((6)\).

7 Construction of Lagrangians

To construct the Lagrangian we use the procedure developed in \([31]\) for massive bosonic higher spin fields in the flat space. According to this procedure we should extract dependence of \(Q'\) \((67)\) on the ghosts \(\eta_G, P_G\)

\[
Q' = Q + \eta_G (\sigma + h) - \eta_2 \eta_2 P_G + 2\beta r \eta_0 (\eta_1 l_1^+ - \eta_1^+ l_1) P_G
\]

\[
+ (1 + \xi) \beta r \eta_0 (\eta_1^+ l_1 - \eta_1 L_1^+) P_G - (2 - \beta) r \eta_1^+ \eta_1 (G_0 - 2g_0) P_G
\]

\[
+ \gamma \eta_1^+ \eta_1 P_G + 3\beta \xi r \eta_0 (\eta_1^+ \eta_2 P_1^+ - \eta_2^+ \eta_1 P_1) P_G,
\]

where

\[
\sigma + h = G_0 + \eta_1^+ P_1 - \eta_1 P_1^+ + 2\eta_2 P_2 - 2\eta_2^+ P_2^+,
\]

\[
[\sigma, Q] = 0.
\]

\[
Q = \eta_0 \tilde{L}_0 + \eta_1^+ L_1 + \eta_1 L_1^+ + \eta_1 L_1 + \eta_2 L_2 + \eta_2 L_2^+ - \eta_1^+ \eta_1 P_0 + \eta_2^+ \eta_1 P_1 + \eta_1^+ \eta_2 P_1^+
\]

\[
+ \beta r \eta_1^+ \eta_0 \left[4l_1^+ P_2 + 4l_2^+ P_1 + 2g_0 P_1\right] + \beta r \eta_0 \eta_1 \left[4l_1^+ P_2 + 4l_2^+ P_1 + 2g_0 P_1\right]
\]

\[
- \beta r \eta_1^+ \eta_0 \left[(2 - \xi) L_1^+ P_2 + (2 + \xi) L_2 P_1^+ + (1 - \xi) G_0 P_1\right]
\]

\[
- \beta r \eta_0 \eta_1 \left[(2 - \xi) L_1 P_2^+ + (2 + \xi) L_2^+ P_1 + (1 - \xi) G_0 P_1^+\right]
\]

\[
+ (2 - \beta) r \eta_1^+ \eta_1 \left[2 L_2^+ P_2 + 2 L_2 P_2^+ - 4 l_2^+ P_2 - 4 l_2^+ P_2^+\right]
\]

\[
+ \gamma \eta_0 \left[\eta_1^+ P_1 - \eta_1 P_1^+ + 2 \eta_2 P_2 - 2 \eta_2^+ P_2^+\right]
\]

\[
+ 3\beta \xi r \eta_0 \left[\eta_1^+ P_1 + \eta_2^+ P_2^+ + \eta_1^+ \eta_2^+ P_1 P_2 + \eta_2^+ \eta_1^+ P_1 P_2 + \eta_2^+ \eta_1^+ P_2^+ P_1\right].
\]

We consider that the ghost operators act on the vacuum state as follows

\[
P_0 |0\rangle = P_G |0\rangle = \eta_1 |0\rangle = P_1 |0\rangle = \eta_2 |0\rangle = P_2 |0\rangle = 0
\]

and suppose \([31]\) that the vectors and the gauge parameters do not depend on the ghost \(\eta_G\). As a result we have from the equation defining the physical vectors \(Q' |\Psi\rangle = 0\), where \(|\Psi\rangle\) is a vector in the extended space (including all ghosts except \(\eta_G\))

\[
Q |\Psi\rangle = 0, \quad (\sigma + h) |\Psi\rangle = 0, \quad gh(|\Psi\rangle) = 0,
\]

\[
\delta |\Psi\rangle = Q |\Lambda\rangle, \quad (\sigma + h) |\Lambda\rangle = 0, \quad gh(|\Lambda\rangle) = -1,
\]

\[
\delta |\Lambda\rangle = Q |\Omega\rangle, \quad (\sigma + h) |\Omega\rangle = 0, \quad gh(|\Omega\rangle) = -2.
\]
Since we can not write a gauge parameter with ghost number $-3$, the chain of gauge transformation is finite.

Let us discuss construction of Lagrangian for the field with a given value of the spin $s$. The middle equation of (72) is equation for defining the possible values of the arbitrary parameter $h$. Ones can see that it takes the values $-h = s + \frac{d}{2} - 3$ and these values are connected with the spin of the field. Having fixed a value of the spin we define the parameter $h$. This value of $h$ is substituted into the other equations (72)–(74) (including operator $Q$). Let us denote $Q_s$ the operator $Q$ (70) where we substituted $s + \frac{d}{2} - 3$ instead of $-h$

$$Q_s = Q \mid_{-h \rightarrow s + \frac{d}{2} - 3}$$

(75)

and let us denote the eigenvectors of the operator $\sigma$ (69) corresponding to the eigenvalue $s + \frac{d}{2} - 3$ as $\mid \chi \rangle$,

$$\sigma \mid \chi \rangle = (s + \frac{d}{2} - 3) \mid \chi \rangle, \quad Q_s^2 \mid \chi \rangle = 0. \quad (76)$$

Thus for a given spin-$s$ we have from (72)–(74) the equation of motion and the gauge transformation

$$Q_s \mid \Psi \rangle_s = 0, \quad (77)$$

$$\delta \mid \Psi \rangle_s = Q_s \mid \Lambda \rangle_s, \quad (78)$$

$$\delta \mid \Lambda \rangle_s = Q_s \mid \Omega \rangle_s \quad \text{(79)}$$

and the middle equations of (72)–(74) are satisfied by construction.

Next step [31] is to extract the Hermitian ghosts $\eta_0$, $P_0$ in equations (77)–(79). We have$^{10}$

$$Q_s = \Delta Q + \eta_0 \tilde{L}_0 - \eta_1^+ \eta_1 P_0, \quad (80)$$

$$\Delta Q = \eta_1^+ L_1 + \eta_1 L_1^+ + \eta_2^+ L_2 + \eta_2 L_2^+ + \eta_2^+ \eta_1 P_1 + \eta_1^+ \eta_2 P_1^+ + (2 - \beta)\eta_1^+ \eta_1 [2L_2^+ P_2 + 2L_2^+ P_2^+ - 4l_2^+ P_2 - 4l_2^+ P_2^+], \quad (81)$$

$$\tilde{L}_0 = \tilde{L}_0 - \beta r \eta_1^+[4l_1^+ P_2 + 4l_2^+ P_1^+ + 2g_0^+ P_1] + \beta r \eta_1[4l_1^+ P_2^+ + 4l_2^+ P_1 + 2g_0^+ P_1^+] + \beta r \eta_1[2 - \xi L_1^+ P_2 + (2 + \xi) L_2^+ P_1 + (1 - \xi) G_0 P_1] - \beta r \eta_1[2 - \xi L_1^+ P_2 + (2 + \xi) L_2^+ P_1 + (1 - \xi) G_0 P_1^+] + \gamma r [\eta_1^+ P_1 - \eta_1 P_1^+ + 2\eta_2^+ P_2 - 2\eta_2 P_2^+] + 3 \beta \xi [\eta_1 \eta_2^+ P_1 + \eta_1^+ \eta_2 P_1^+ + \eta_1^+ \eta_2 P_2 + \eta_1^+ \eta_2^+ P_2], \quad (82)$$

for the operator $Q_s$ and

$$\mid \Psi \rangle = \mid S \rangle + \eta_0 \mid A \rangle, \quad (83)$$

$$\mid \Lambda \rangle = \mid \Lambda_0 \rangle + \eta_0 \mid \Lambda_1 \rangle, \quad (84)$$

$$\mid \Omega \rangle = \mid \Omega_0 \rangle.$$

$^{10}$Here and further ones assume that we substituted $s + \frac{d}{2} - 3$ instead of $-h$ in all the operators $l_i'$.  

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for the gauge parameters. As a result we have the equations of motion and the gauge transformation

\[ \tilde{L}_0 |S\rangle - \Delta Q |A\rangle = 0, \quad \Delta Q |S\rangle - \eta^+_1 \eta_1 |A\rangle = 0, \quad (85) \]
\[ \delta |S\rangle = \Delta Q |\Lambda_0\rangle - \eta^+_1 \eta_1 |\Lambda_1\rangle, \quad \delta |A\rangle = \tilde{L}_0 |\Lambda_0\rangle - \Delta Q |\Lambda_1\rangle, \quad (86) \]
\[ \delta |\Lambda_0\rangle = \Delta Q |\Omega\rangle, \quad \delta |\Lambda_1\rangle = \tilde{L}_0 |\Omega\rangle. \quad (87)\]

It is easy to show that the equations of motion (85) can be derived from the following Lagrangian

\[ L_s = \langle S | K \{ \tilde{L}_0 |S\rangle - \Delta Q |A\rangle \} + \langle A | K \{ -\Delta Q |S\rangle + \eta^+_1 \eta_1 |A\rangle \} \quad (88) \]

which can also be written in a more compact form

\[ L_s = \int d\eta_0 s \langle \Psi | KQ_s |\Psi\rangle_s. \quad (89) \]

Now we fix the arbitrary parameter \( m^2_0 \). It is defined from the condition of reproducing the conditions (23) for the basic vector \(|\Phi\rangle\) (6). The general vector \(|\Psi\rangle\) includes the basic vector \(|\Phi\rangle\) as follows

\[ |\Psi\rangle = |\Phi\rangle + |\Phi_A\rangle, \quad (90) \]

where the vector \(|\Phi_A\rangle\) includes only the auxiliary fields as its components. One can show, using the part of equations of motion and gauge transformations, that the vector \(|\Phi_A\rangle\) can be completely removed. The details are given in Appendix [C]. As a result we obtain the equation of motion for the basic vector (77) in the form

\[ (l_0 - m^2 + M^2) |\Phi\rangle = l_1 |\Phi\rangle = l_2 |\Phi\rangle = 0, \quad (91) \]

where \( M^2 \) is defined in (46). We see that the Lagrangian reproduces the basic conditions (23) if \( M^2 = m^2 \). It leads to

\[ m^2_0 = -\alpha \frac{d(d-4)}{4} r - (\beta - 2) h(h - 2) r + \gamma hr. \quad (92) \]

It is interesting to note that in case of \( \alpha = \gamma = 0, \beta = 2 \) we will have \( m^2_0 = 0 \). Such a situation was considered in [23] where the higher spin massless bosonic fields in AdS space were studied.

One can prove that the Lagrangian (89) indeed reproduces the basic conditions (23). The details of the proof are given in Appendix [C]. Relation (89) where the parameter \( m^2_0 \) is fixed by (92) is our final result.

Construction of the Lagrangian describing propagation of all massive bosonic fields in AdS space simultaneously is analogous to that in the flat space [31] and we do not consider it here.

Let us turn to examples.

8 Examples

8.1 Spin-0 field

For a scalar field with spin \( s = 0 \) we have \( h = 3 - \frac{d}{2} \). Then the vector \(|\Psi\rangle_s\) which satisfies the condition (76) and has the proper ghost number (72) looks like

\[ |\Psi\rangle_0 = \varphi(x)|0\rangle. \quad (93) \]
Then using (92), (B.32) the relation for Lagrangian (88) gives

\[ L_0 = \varphi \left\{ \nabla^2 + r(6 - 2d) + m^2 \right\} \varphi \quad (94) \]

It is easy to see that this Lagrangian reproduces the equation (2) for \( s = 0 \).

### 8.2 Spin-1 field

For a vector field we have \( h = 2 - \frac{d}{2} \). Then the vector \( |\Psi\rangle_s \) and the gauge parameter \( |\Lambda\rangle_s \), which satisfy the condition (76) and have the proper ghost numbers (72), (73), look like

\[ |\Psi\rangle_1 = \left[ -ia^{\mu+} A_\mu(x) + b_1^+ A(x) \right]|0\rangle + \eta_0 P_1^+ \varphi(x)|0\rangle, \quad (95) \]

\[ |\Lambda\rangle_1 = P_1^+ \lambda(x)|0\rangle \quad (96) \]

Using (92), (B.32) we find the Lagrangian (88) for vector field

\[ L = -A^\mu \left\{ \left( \nabla^2 - r(d - 1) + m^2 \right) A_\mu - \nabla_\mu \varphi \right\} \]

\[ + \frac{m^2}{m_1^2} A \left\{ \left( \nabla^2 - 4r + m^2 \right) A - m_1 \varphi \right\} + \varphi \left\{ \varphi - \nabla^\mu A_\mu - \frac{m^2}{m_1} A \right\} \quad (97) \]

and the gauge transformations (86)

\[ \delta A_\mu(x) = \nabla_\mu \lambda, \quad \delta A(x) = m_1 \lambda, \quad \delta \varphi(x) = (\nabla^2 + m^2) \lambda. \quad (98) \]

The relations (97) and (98) are the final result for massive vector field. We obtain the gauge invariant theory in terms of vector filed \( A_\mu \) and two auxiliary scalar fields \( A \) and \( \varphi \).

Let us consider the massless limit of this theory. To do that we put \( m \to 0 \) in (97)\(^{11}\). Then the field \( A \) vanishes and we obtain the theory of the massless field \( s = 1 \) in AdS space.

Now we show how the Lagrangian (97) is transformed to the conventional form. Let us rescale the field \( A \)

\[ \frac{m}{m_1} A = A' \to A. \quad (99) \]

In this case Lagrangian (97) and the gauge transformation (98) are rewritten as

\[ L = -A^\mu \left\{ \left[ \nabla^2 + m^2 - r(d - 1) \right] A_\mu - \nabla_\mu \varphi \right\} \]

\[ + A \left\{ \left( \nabla^2 + m^2 \right) A - m \varphi \right\} + \varphi \left\{ \varphi - \nabla^\mu A_\mu - mA \right\}, \quad (100) \]

\[ \delta A_\mu(x) = \nabla_\mu \lambda, \quad \delta A(x) = m \lambda, \quad \delta \varphi(x) = (\nabla^2 + m^2) \lambda \quad (101) \]

and they become independent of \( m_1 \).

Ones exclude the field \( \varphi(x) \) from Lagrangian (100) with the help of its equation of motion. It leads to

\[ L = \frac{1}{2} F^{\mu\nu} F_{\mu\nu} - (mA_\mu - \nabla_\mu A)(mA^\mu - \nabla^\mu A), \quad (102) \]

where

\[ F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu. \quad (103) \]

Now removing the field \( A \) with the help of its gauge transformation we arrive at the conventional form of Lagrangian for spin-1 field (up to an overall factor)

\[ L = \frac{1}{2} F^{\mu\nu} F_{\mu\nu} - m^2 A_\mu A^\mu. \quad (104) \]

\(^{11}\)Ones remind that arbitrary parameter \( m_1 \) arose in Section 5 when we constructed the additional parts of the operators. In general it has no relation to physical mass \( m \) and should not tend to zero in massless limit.
8.3 Spin-2 field

For spin-2 field we have $h = 1 - \frac{d}{2}$. Then the vector $|\Psi\rangle_s$ and the gauge parameter $|\Lambda\rangle_s$, which satisfy the condition (76) and have the proper ghost numbers (72), (73), look like

$$|S_1\rangle = \left[(-1)^2 - a^\mu a^\nu + H \mu \nu (x) + b_1^\mu H_1 (x) - i a^\mu b_1^\nu A_\mu (x) + b_1^{+2} \varphi (x)\right]|0\rangle$$
$$|S_2\rangle = H(x)|0\rangle,$$
$$|A_1\rangle = \left[-i a^\mu + H_\mu (x) + b_1^\nu A(x)\right]|0\rangle, \quad |A_2\rangle = H_2(x)|0\rangle,$$
$$|\lambda_1\rangle = \left(-i a^\mu + \lambda_\mu (x) + b_1^\nu \lambda(x)\right)|0\rangle, \quad |\lambda_2\rangle = \lambda_2(x)|0\rangle.$$  

Here $|S_1\rangle$, $|A_1\rangle$, $|\lambda_1\rangle$ are defined in (C.1)–(C.4).

Using (32), (88), (B.32) we find Lagrangian for the massive spin-2 field in the form

$$\mathcal{L} = \frac{1}{2} H^{\mu \nu} \left\{ \nabla^2 + m^2 - 2r \right\} H_{\mu \nu} - 2(\beta - 1) r g_{\mu \nu} H^\sigma_{\sigma}$$
$$+ (2 + \xi) \beta r g_{\mu \nu} H + g_{\mu \nu} H_2 - 2 \nabla_\mu H_\nu \right\}$$
$$- \left(\frac{d - 2}{2} H_1 + \frac{m_1^2}{m_2^2} \varphi\right) \left\{ \nabla^2 + m^2 + 2r(d - 1) \right\} H_1$$
$$- 4 \beta r \left(\frac{d - 2}{2} H_1 + \frac{m_1^2}{m_2^2} \varphi\right) + \beta (2 - \xi) r H - H_2 \right\}$$
$$- \frac{m_1^2}{m_2^2} A^\mu \left\{ \nabla^2 + m^2 + r(d - 1) \right\} A_\mu - \nabla_\mu A - m_1 H_\mu \right\}$$
$$+ \frac{m_1^2}{m_2^2} \left[2m^2 + 2r(d - 1) \right] \varphi - H_1 \right\} \left\{ \nabla^2 + m^2 + 2r(d - 1) \right\} \varphi - m_1 A \right\}$$
$$- H \left\{ \nabla^2 + m^2 + 2r(d - 1) + 2 \xi \beta r \right\} H - \nabla^\mu H_\mu - \frac{m_2^2}{m_1} A + H_2$$
$$- \frac{1}{2} (2 + \xi) \beta r H^{\mu \nu} + \beta r(2 - \xi) \frac{d - 2}{2} H_1 + \beta r(2 - \xi) \frac{m_2^2}{m_1} \varphi \right\}$$
$$+ H^{\mu} \left\{ \nabla^\nu H_{\mu \nu} - H_\mu - \nabla_\mu H + \frac{m_2^2}{m_1} A_\mu \right\}$$
$$- \frac{m_1^2}{m_2^2} A \left\{ \nabla^\nu A_\mu - m_1 (H + H_1) - A + \frac{2m_2^2 + 2r(d - 1)}{m_1} \varphi \right\}$$
$$+ H_2 \left\{ \frac{1}{2} H^{\mu \nu} + \frac{d - 2}{2} H_1 + \frac{m_2^2}{m_1} \varphi - H \right\}.$$  

Using (88), (32) one finds the gauge transformations

$$\delta H_{\mu \nu} = \nabla_{\mu} \lambda_{\nu} + \nabla_{\nu} \lambda_{\mu} - g_{\mu \nu} \lambda_2, \quad \delta H_1 = \lambda_2,$$
$$\delta A_{\mu} = \nabla_{\mu} \lambda + m_1 \lambda_{\mu}, \quad \delta \varphi = m_1 \lambda,$$
$$\delta H = \nabla^\mu \lambda_{\mu} + \frac{m_2^2}{m_1} \lambda - \lambda_2,$$
$$\delta H_{\mu} = \left(\nabla^2 + m^2 + r(d - 1)\right) \lambda_{\mu},$$
$$\delta A = \left(\nabla^2 + m^2 + 2r(d - 1)\right) \lambda,$$
$$\delta H_2 = (2 - \xi) \beta r \nabla^\mu \lambda_{\mu} - (2 + \xi) \beta r \frac{m_2^2}{m_1} \lambda$$
$$+ (\nabla^2 + m^2 + 2r(d - 1) + \beta r(2 - 2d + \xi)) \lambda_2.$$  


The relation (109) and the above gauge transformations are our final result for massive spin-2 theory in AdS space. We get the gauge theory in terms of basic filed $H_{\mu\nu}$ and some number of auxiliary fields.

In massless limit $m \to 0^{12}$ in (109) we get Lagrangian for the massless spin-2 field in terms of the fields $H_{\mu\nu}$, $H_{\mu}$, $H_1$ and $H_2$.

Now we rewrite the Lagrangian (109) in more conventional form. Let us make the following transformation. First, we redefine the fields
\[ \frac{d}{2} - \frac{2}{2} \frac{H_1 + m^2}{m_1^2} \varphi = \frac{d}{2} - \frac{2}{2} \frac{H_1'}{H_1}, \] (116)
\[ \frac{m}{m_1} A = A' \to A, \quad \frac{m}{m_1} A_{\mu} = A'_{\mu} \to A_{\mu}, \quad \frac{m^2}{m_1^2} \varphi = \varphi' \to \varphi, \] (117)
and the gauge parameters
\[ \frac{m}{m_1} \lambda = \lambda' \to \lambda. \] (118)
Then we use the equations of motion for $H_{\mu}$, $A$ in (109) and remove field $H_1$ with the help of its gauge transformation. After that the gauge parameters $\lambda$ and $\lambda_2$ are not independent
\[ \lambda_2 + \frac{2m}{d-2} \lambda = 0. \] (119)
Next we use the equations of motion for the fields $H_2$ and $H$ in previously obtained Lagrangian. Finally, after one more rescaling the field $\varphi$
\[ \left[ 2(d-1) \left( \frac{1}{d-2} + \frac{r}{m^2} \right) \right]^{1/2} \varphi = \varphi' \to \varphi \] (120)
we arrive at Zinoviev’s Lagrangian [6]
\[ \mathcal{L} = \frac{1}{4} H^{\mu\nu} \left[ \nabla^2 + m^2 - 2r \right] H_{\mu\nu} - \frac{1}{2} H_{\mu}^{\mu} \left[ \nabla^2 + m^2 + r(d-3) \right] H_{\nu}^{\nu} \]
\[ + H^{\mu\nu} \left[ \nabla_\mu \nabla_\nu H_\sigma^\sigma - \nabla_\mu \nabla_\sigma H_\sigma^{\nu} \right] \]
\[ - A^{\mu} \left[ \left[ \nabla^2 + r(d-1) \right] A_\mu - \nabla_\mu \nabla^\nu A_\nu \right] + \varphi \left( \nabla^2 - \frac{d}{d-2} m^2 \right) \varphi \]
\[ + \left[ 2(d-1) \left( \frac{m^2}{d-2} + r \right) \right]^{1/2} \varphi \left[ m H_{\mu}^{\mu} - 2 \nabla^\mu A_\mu \right] \]
\[ + 2m \left[ H_\sigma^{\nu} \nabla_\mu A_\mu - H^{\mu\nu} \nabla_\mu A_\nu \right]. \] (121)
with the gauge transformations
\[ \delta H_{\mu\nu} = \nabla_\mu \lambda_\nu + \nabla_\nu \lambda_\mu + g_{\mu\nu} \frac{2m}{d-2} \lambda, \] (122)
\[ \delta A_{\mu} = \nabla_\mu \lambda + m \lambda_\mu, \] (123)
\[ \delta \varphi = \left[ 2(d-1) \left( \frac{m^2}{d-2} + r \right) \right]^{1/2} \lambda. \] (124)
It is easy to see that at
\[ r + \frac{m^2}{d-2} = 0 \] (125)

---

12 We assume as in Subsection 8.2 that the arbitrary parameter $m_1$ does not tend to zero in this limit.
Lagrangian (121) splits into a sum of two independent parts with helicities \( \pm 2, \pm 1 \) \((H_{\mu\nu}, A_\mu)\) and the scalar field \(\phi\). Thus the field \(H_{\mu\nu}\) becomes partial massless at

\[
m^2 = -(d-2)r
\]

in accordance with [3].

9 Summary

We have developed the gauge invariant approach to deriving the Lagrangians for bosonic massive higher spin fields in arbitrary dimensional AdS space. The Lagrangian includes the Stueckelberg fields, providing the gauge invariance in the massive theory, and the complete set of the auxiliary fields which should be introduced for Lagrangian formulation in higher spin theory. The approach is based on BRST construction for special nonlinear symmetry algebra formulated in the paper.

We begin with imbedding the massive higher spin fields into the vectors of auxiliary Fock space. All such fields are treated as the components of the vectors of the Fock space and the theory is completely formulated in terms of such vectors. The conditions defining the irreducible representation of the AdS group with given mass and spin are realized by differential operators acting in this Fock space. The above conditions are interpreted as the constraints on the vectors of the Fock space and generate the closed nonlinear symmetry algebra which is the main object of our analysis. As we proved, derivation of the correct Lagrangian demands an extension and deformation of the algebra. BRST construction for such nonlinear symmetry algebra is given. It is shown that the BRST operator corresponding to the extended deformed algebra defines the consistent Lagrangian dynamics for bosonic massive fields of any value of spin. We constructed the gauge invariant Lagrangians in terms of Fock space in the concise form for any higher spin fields propagating in AdS space of arbitrary dimension. It is interesting to point out that the gauge transformations in the theory under consideration are reducible, the corresponding order of reducibility is equal to unit. As an example of the general scheme we obtained the gauge invariant Lagrangians and the corresponding gauge transformations for the component spin-0, spin-1, and spin-2 massive fields in the explicit form. The Lagrangians for component massive fields with other spins can be analogously found using the simple manipulations with creation and annihilation operators in the Fock space.

The main results of the paper are given by the relations (88), (89) where Lagrangian for the massive field with arbitrary integer spin is constructed, and (86), (87) where the gauge transformations for the fields and the gauge transformations for the gauge parameters are written down.

The procedure for Lagrangian construction developed here for higher spin massive bosonic fields can also be applied to fermionic higher spin theories in AdS background and for fields with mixed symmetry tensor or tensor-spinor fields (see [25] where bosonic massless fields with mixed symmetry were considered in Minkowski space). The results obtained open principle possibility for derivation of the interacting vertices for massive higher spin fields in AdS space.

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Appendix

A Differential calculus in auxiliary Fock space.

We consider an arbitrary $d$-dimensional Riemann space with metric $g_{\mu\nu}(x)$ which can be expressed in terms of the vielbein $e^b_\mu(x)$

$$g_{\mu\nu}(x) = \eta_{bc} e^b_\mu(x) e^c_\nu(x), \quad g_{\mu\lambda}(x)g^{\lambda\nu}(x) = \delta^\nu_\mu,$$

$$e^a_\mu(x) e^b_\mu(x) = \delta^a_b, \quad e^a_\mu(x) = g^{\mu\nu}(x) \eta_{bc} e^c_\nu(x), \quad \eta_{bc} \eta^{cd} = \delta^d_b,$$

where $\eta_{bc}$ is the Minkowski metric with the signature $\eta_{bc} = (+, -, ..., -)$.

We introduce the creation and annihilation operators $a^+_b$ and $a^b$ carrying flat-space indices $b$

$$[a_b, a^+_c] = -\eta_{bc}, \quad [a_b, a_c] = 0, \quad [a^+_b, a^+_c] = 0,$$

$$a_b|0\rangle = 0, \quad \langle 0|0\rangle = 1, \quad \partial_\mu a^b = \partial_\mu a^{b+} = 0,$$

where $|0\rangle$ is the vacuum vector of the Fock space. Any $s$-particle vector $|\Phi^s\rangle$ can be presented in the form

$$|\Phi^s\rangle = \Phi_{b_1...b_s}(x) a^{b_1+}...a^{b_s+}|0\rangle, \quad N|\Phi^s\rangle = s|\Phi^s\rangle, \quad N = -a^{b+}a_b, \quad a^{b+} = \eta^{bc}a^+_b,$$

where $\Phi_{b_1...b_s}(x)$ is a symmetric tensor field of rank $s$. It is useful to introduce the creation and annihilation operators $a^+_\mu(x)$ and $a^\mu(x)$ by the relations

$$a^+_\mu(x) = e^b_\mu(x) a^+_b, \quad a^\mu(x) = e^b_\mu(x) a_b.$$

They obey the properties

$$[a^+_\mu(x), a^+_\nu(x)] = 0, \quad [a^\mu(x), a^\nu(x)] = 0, \quad [a^\mu(x), a^+_\nu(x)] = g_{\mu\nu}(x), \quad a^\mu(x)|0\rangle = 0$$

and create the $s$-particle vector

$$|\Phi^s\rangle = \Phi_{\mu_1...\mu_s}(x) a^{\mu_1+}...a^{\mu_s+}(x)|0\rangle, \quad N|\Phi^s\rangle = s|\Phi^s\rangle, \quad N = -a^{\mu+}(x)a^\mu(x), \quad a^{\mu+} = g^{\mu\nu}(x)a^+_\mu(x).$$

where $\Phi_{\mu_1...\mu_s}(x)$ is a symmetric tensor on the Riemann space.

Ones introduce the operator $D_\mu$ acting on any operator tensor with curved-space and with flat-space indices by the rule

$$D_\mu Q^\ldots = \left(\nabla_\mu Q^\ldots\right) - \omega^b_\mu c [d^{c+}a_b, Q^\ldots],$$

where the dots denote the curved-space and flat-space indices. The $\nabla_\mu$ is the proper covariant derivative. It is clear that this operator possesses the Leibniz rule

$$D_\mu \left(A \cdot B\right) = \left(D_\mu A\right) \cdot B + A \cdot \left(D_\mu B\right).$$
From definition of $D_\mu$ it follows that it acts on usual tensor fields (which do not depend on creation and annihilation operators) as the covariant derivative

$$D_\mu \Phi_{\mu_1 \mu_2 ... \mu_s}(x) = \nabla_\mu \Phi_{\mu_1 \mu_2 ... \mu_s}(x), \quad D_\mu \Phi_{b_1 b_2 ... b_s}(x) = \nabla_\mu \Phi_{b_1 b_2 ... b_s}(x). \quad (A.9)$$

In particular we have

$$D_\mu g_{\alpha \beta}(x) = \nabla_\mu g_{\alpha \beta}(x) = 0. \quad (A.10)$$

Consider the action of $D_\mu$ on the creation and annihilation operators with flat-space indices. One obtains

$$D_\mu a^b = \partial_\mu a^b + \omega^b_{\mu c} a^c - \omega^c_{\mu d} [a^{d+} a_c, a^b] = (\omega^b_{\mu c} + \omega^c_{\mu b}) a^c = 0, \quad D_\mu a^{b^+} = 0, \quad (A.11)$$

because of $\partial_\mu a^b = \partial_\mu a^{b^+} = 0$ and $\omega^b_{\mu c} = -\omega^c_{\mu b}$. Therefore

$$D_\mu a^\nu = \partial_\mu c_0 a^b + \Gamma^\nu_{\alpha \mu} c_0 a^b - \omega^d_{\mu} c^\nu [a^{d+} a_b, a^d] = (\partial_\mu c_0^\nu + \Gamma^\nu_{\alpha \mu} c_0^a + \omega^c_{\mu b} c_0^\nu) a^b =
\begin{equation}
= (\nabla_\mu c_0^\nu) a^b = 0, \quad D_\mu a^{\nu^+} = 0. \quad (A.12)
\end{equation}$$

The property of covariant constancy of the creation and annihilation operators (A.11) (A.12) allow us to consider the operator $D_\mu$ as representation of the covariant derivative on vectors $|\Phi^s\rangle$ if we assume that $D_\mu|0\rangle = 0$. Indeed, we have

$$D_\mu|\Phi^s\rangle = (\nabla_\mu \Phi_{\mu_1 \mu_2 ... \mu_s}(x)) a^{\mu_1+}(x)...a^{\mu_s+}(x)|0\rangle = \left(\nabla_\mu \Phi_{b_1 b_2 ... b_s}(x)\right) a^{b_1+}...a^{b_s+}|0\rangle \quad (A.13)$$

In a similar way we can find the action of the commutator of the covariant derivative on the vectors $|\Phi^s\rangle$:

$$[D_\mu, D_\nu]|\Phi^s\rangle = -R^a_{\beta \mu \nu} a^\beta a^\alpha |\Phi^s\rangle = -\mathcal{R}^b_{c \mu \nu} a^{c^+} a_b |\Phi^s\rangle, \quad (A.14)$$

where we used notations

$$R^a_{\beta \mu \nu} = \partial_\mu \Gamma^a_{\beta \nu} - \partial_\nu \Gamma^a_{\beta \mu} - \Gamma^\lambda_{\beta \mu} \Gamma^a_{\lambda \nu} + \Gamma^\lambda_{\beta \nu} \Gamma^a_{\lambda \mu}, \quad (A.15)$$

$$\mathcal{R}^b_{c \mu \nu} = \partial_\mu \omega^b_{\nu c} - \partial_\nu \omega^b_{\mu c} - \omega^b_{\mu d} \omega^d_{\nu c} + \omega^b_{\nu d} \omega^d_{\mu c}, \quad (A.16)$$

for the curvature tensors.

## B Calculation of the additional parts

In this Appendix we show how the representation of the algebra given in Table 2 can be constructed in terms of some creation and annihilation operators.

Let us consider a representation of this algebra with the vector $|0\rangle_V$ annihilated by the operators $l_1'$ and $l_2'$

$$l_1'|0\rangle_V = l_2'|0\rangle_V = 0 \quad (B.1)$$

and being the eigenvector of the operators $\tilde{g}_0$, $g_0'$ and $l'$

$$\tilde{g}_0|0\rangle_V = m_0^2|0\rangle_V, \quad g_0'|0\rangle_V = h|0\rangle_V, \quad l'|0\rangle_V = m_2^2|0\rangle_V, \quad (B.2)$$

$$l_1'|0\rangle_V = l_2'|0\rangle_V = 0 \quad (B.1)$$

and being the eigenvector of the operators $\tilde{g}_0$, $g_0'$ and $l'$

$$\tilde{g}_0|0\rangle_V = m_0^2|0\rangle_V, \quad g_0'|0\rangle_V = h|0\rangle_V, \quad l'|0\rangle_V = m_2^2|0\rangle_V, \quad (B.2)$$

$$l_1'|0\rangle_V = l_2'|0\rangle_V = 0 \quad (B.1)$$

and being the eigenvector of the operators $\tilde{g}_0$, $g_0'$ and $l'$

$$\tilde{g}_0|0\rangle_V = m_0^2|0\rangle_V, \quad g_0'|0\rangle_V = h|0\rangle_V, \quad l'|0\rangle_V = m_2^2|0\rangle_V, \quad (B.2)$$

$$l_1'|0\rangle_V = l_2'|0\rangle_V = 0 \quad (B.1)$$

and being the eigenvector of the operators $\tilde{g}_0$, $g_0'$ and $l'$

$$\tilde{g}_0|0\rangle_V = m_0^2|0\rangle_V, \quad g_0'|0\rangle_V = h|0\rangle_V, \quad l'|0\rangle_V = m_2^2|0\rangle_V, \quad (B.2)$$

$$l_1'|0\rangle_V = l_2'|0\rangle_V = 0 \quad (B.1)$$

and being the eigenvector of the operators $\tilde{g}_0$, $g_0'$ and $l'$

$$\tilde{g}_0|0\rangle_V = m_0^2|0\rangle_V, \quad g_0'|0\rangle_V = h|0\rangle_V, \quad l'|0\rangle_V = m_2^2|0\rangle_V, \quad (B.2)$$
where \( m_0, m_2 \) are arbitrary constants with dimension of mass and \( h \) is a dimensionless arbitrary constant\(^{13}\). They are the arbitrary constants which must be contained in the additional parts of Hermitian operators. Next we choose the basis vectors as follows

\[
|n_1, n_2\rangle_V = \left( \frac{m_1}{l_{n_2}^+} \right)^{n_1} (l_{n_2}^+)^{n_2} |0\rangle_V, \tag{B.3}
\]

where \( m_1 \) is an arbitrary nonzero constant with dimension of mass. It may be constructed from the parameters of the theory \( m_1 = f(m, r) \neq 0 \). It is easy to find how the operators \( l', l'^+_1, l'^+_2, g_0' \) act on these basis vectors

\[
l'|n_1, n_2\rangle_V = m_2^2 |n_1, n_2\rangle_V, \tag{B.4}
\]
\[
l'^+_1|n_1, n_2\rangle_V = m_1 |n_1 + 1, n_2\rangle_V, \tag{B.5}
\]
\[
l'^+_2|n_1, n_2\rangle_V = |n_1, n_2 + 1\rangle_V, \tag{B.6}
\]
\[
g_0'|n_1, n_2\rangle_V = (n_1 + 2n_2 + h) |n_1, n_2\rangle_V. \tag{B.7}
\]

Calculation of how the rest operators \( \tilde{l}_0', \tilde{l}_1', \tilde{l}_2' \) act on the basis vectors is much more difficult. First we find that

\[
\tilde{l}_0'|n_1, n_2\rangle_V = 2\gamma r n_2 |n_1, n_2\rangle_V + (l'^+_2)^{n_2} \tilde{t}_0'|n_1, 0\rangle_V, \tag{B.8}
\]
\[
\tilde{l}_1'|n_1, n_2\rangle_V = -m_1 n_2 |n_1 + 1, n_2 - 1\rangle_V + (l'^+_2)^{n_2} \tilde{t}_1'|n_1, 0\rangle_V, \tag{B.9}
\]
\[
\tilde{l}_2'|n_1, n_2\rangle_V = (n_2 - 1 + n_1 + h) n_2 |n_1, n_2 - 1\rangle_V + (l'^+_2)^{n_2} \tilde{t}_2'|n_1, 0\rangle_V. \tag{B.10}
\]

Thus it remains to calculate how the operators \( \tilde{l}_0', \tilde{l}_1', \tilde{l}_2' \) act on the basis vectors \( |0\rangle_V \).

To do this we define some auxiliary operators and their action on the vector \( |0\rangle_V \)

\[
K_0 \equiv g_0'^2 - 2g_0 - 4l'^+_2 l'^+_2, \quad K_0|0\rangle_V = h(h-2)|0\rangle_V, \tag{B.11}
\]
\[
K_1 \equiv \left[ K_0, l'^+_1 \right] = 4l'^+_2 l'^+_1 + 2l'^+_2 g_0' - l'^+_1, \quad K_1|0\rangle_V = m_1(2h - 1)|1, 0\rangle_V. \tag{B.12}
\]

\[
K_2 \equiv \left[ K_1, ll'^+_1 \right] = 2l'^+_2 + 4l'^+_2 K', \tag{B.13}
\]
\[
K'_2 = \tilde{l}_0' + m^2 + \alpha \frac{d(d-4)}{4} r + (\beta - 2)r K_0 - \gamma r g_0', \tag{B.14}
\]
\[
K_2|0\rangle_V = 4M^2 |0, 1\rangle_V + 2m_1^2 |2, 0\rangle_V, \quad K'_2|0\rangle_V = M^2 |0\rangle_V, \tag{B.15}
\]

where

\[
M^2 = m^2 + m_2^2 + \alpha \frac{d(d-4)}{4} r + (\beta - 2)r \hbar(h-2) - \gamma r h. \tag{B.16}
\]

In terms of these operators we have

\[
\left[ l'_1, l'^+_1 \right] = K'_2, \quad \left[ \tilde{l}_0', l'^+_1 \right] = -\beta r K_1 + \gamma r l'^+_1, \quad \left[ K_2, l'^+_1 \right] = -8l'^+_2 r K_1. \tag{B.17}
\]

Using the above formulas we obtain

\[
\tilde{l}_0'|n_1, 0\rangle_V = m_0 |n_1, 0\rangle_V + \gamma r n_1 |n_1, 0\rangle_V - \beta r(2h - 1) \sum_{k=0} \left( \frac{-8r}{m_1^2} \right)^k C_{2k+1}^{m_1} |n_1 - 2k, k\rangle_V
\]
\[
- 2\beta r \sum_{k=0} \left( \frac{-8r}{m_1^2} \right)^k C_{2k+2}^{m_1} |n_1 - 2k, k\rangle_V
\]
\[
- \frac{4\beta r M^2}{m_1^2} \sum_{k=0} \left( \frac{-8r}{m_1^2} \right)^k C_{2k+2}^{m_1} |n_1 - 2k - 2, k + 1\rangle_V \tag{B.18}
\]

\(^{13}\)The representation given by (B.5) and (B.6) is called in the mathematical literature the Verma module. It explains the subscript \( V \) at the vectors.
where \( C_k^m = \frac{n!}{k!(n-k)!} \). Substituting (B.18) into (B.8) ones get
\[
\tilde{l}_0|n_1, n_2\rangle_V = m_0|n_1, n_2\rangle_V + \gamma r(n_1 + 2n_2)|n_1, n_2\rangle_V
- \beta r(2h - 1) \sum_{k=0}^{\infty} \left[ \frac{-8r}{m_1^2} \right]^k C_{2k+1}^{m_1} |n_1 - 2k, n_2 + k\rangle_V
- 2\beta r \sum_{k=0}^{\infty} \left[ \frac{-8r}{m_1^2} \right]^k C_{2k+2}^{m_1} |n_1 - 2k, n_2 + k\rangle_V
- 4\beta r M^2 \sum_{k=0}^{\infty} \left[ \frac{-8r}{m_1^2} \right]^k C_{2k+2}^{m_1} |n_1 - 2k, n_2 + k + 1\rangle_V. \tag{B.19}
\]

Analogously one can first find
\[
l'_1|n_1, 0\rangle_V = m_1 \frac{2h - 1}{4} \sum_{k=1}^{\infty} \left[ \frac{-8r}{m_1^2} \right]^k C_{2k}^{m_1} |n_1 - 2k + 1, k - 1\rangle_V
+ \frac{m_1}{2} \sum_{k=1}^{\infty} \left[ \frac{-8r}{m_1^2} \right]^k C_{2k+1}^{m_1} |n_1 - 2k + 1, k - 1\rangle_V
+ \frac{M^2}{m_1} \sum_{k=0}^{\infty} \left[ \frac{-8r}{m_1^2} \right]^k C_{2k+1}^{m_1} |n_1 - 2k + 1, k - 1\rangle_V, \tag{B.20}
\]
\[
l'_2|n_1, 0\rangle_V = 1 - \frac{2h}{4} \sum_{k=1}^{\infty} \left[ \frac{-8r}{m_1^2} \right]^k C_{2k+1}^{m_1} |n_1 - 2k, k - 1\rangle_V
- \frac{1}{2} \sum_{k=1}^{\infty} \left[ \frac{-8r}{m_1^2} \right]^k C_{2k+2}^{m_1} |n_1 - 2k, k - 1\rangle_V
- \frac{M^2}{m_1} \sum_{k=0}^{\infty} \left[ \frac{-8r}{m_1^2} \right]^k C_{2k+2}^{m_1} |n_1 - 2k - 2, k\rangle_V, \tag{B.21}
\]
and then substituting (B.20) and (B.21) into (B.9) and (B.10) respectively we find how operators \( l'_1 \) and \( l'_2 \) act on the basis vectors \(|n_1, n_2\rangle_V\)
\[
l'_1|n_1, n_2\rangle_V = -m_1 n_2|n_1 + 1, n_2 - 1\rangle_V + \]
\[
+ m_1 \frac{2h - 1}{4} \sum_{k=1}^{\infty} \left[ \frac{-8r}{m_1^2} \right]^k C_{2k}^{m_1} |n_1 - 2k + 1, n_2 + k - 1\rangle_V
+ \frac{m_1}{2} \sum_{k=1}^{\infty} \left[ \frac{-8r}{m_1^2} \right]^k C_{2k+1}^{m_1} |n_1 - 2k + 1, n_2 + k - 1\rangle_V
+ \frac{M^2}{m_1} \sum_{k=0}^{\infty} \left[ \frac{-8r}{m_1^2} \right]^k C_{2k+1}^{m_1} |n_1 - 2k - 1, n_2 + k\rangle_V. \tag{B.22}
\]
\[
\ell'_2|n_1, n_2\rangle_V = (n_2 - 1 + n_1 + h)n_2|n_1, n_2 - 1\rangle_V = \frac{2h - 1}{4} \sum_{k=1}^{\infty} \left[ -\frac{8r}{m_1^2} \right]^k C_{2k+1}^m n_1 - 2k, n_2 + k - 1 \rangle_V \\
- \frac{1}{2} \sum_{k=1}^{\infty} \left[ -\frac{8r}{m_1^2} \right]^k C_{2k+2}^m n_1 - 2k, n_2 + k - 1 \rangle_V \\
- \frac{M^2}{m_1^2} \sum_{k=0}^{\infty} \left[ -\frac{8r}{m_1^2} \right]^k C_{2k+2}^m n_1 - 2k, n_2 + k \rangle_V.
\] (B.23)

Now let us turn to construction of a representation of the operator algebra given in Table 2 in terms of creation and annihilation operators. The number of pairs of these operators and their statistics is defined by the number of operators used in the definition of the basis vectors (B.3). Thus we introduce two pairs of the bosonic creation and annihilation operators with the standard commutation relations

\[
[b_1, b_1^+] = 1, \quad [b_2, b_2^+] = 1,
\] (B.24)

corresponding to \(\ell'_1\), \(\ell'^+_1\) and \(\ell'_2\), \(\ell'^+_2\) respectively. After this we map the basis vectors (B.3) and the basis vectors of the Fock space generated by \(b_1^+, b_2^+\)

\[
|n_1, n_2\rangle_V \leftrightarrow (b_1^+)^{n_1} (b_2^+)^{n_2} |0\rangle = |n_1, n_2\rangle
\] (B.25)

and find from (B.14), (B.17), (B.19), (B.22), (B.23) form of the operators in terms of the creation and annihilation operators

\[
\ell' = m_2^2, \quad \ell'^+_1 = m_1 b_1^+, \quad \ell'^+_2 = b_2^+, \quad g'_0 = b_1^+ b_1 + 2b_2^+ b_2 + h,
\] (B.26)

\[
\tilde{l}_0 = m_0^2 + \gamma r(b_1^+ b_1 + 2b_2^+ b_2) - \beta r(2h - 1) b_1^+ \sum_{k=0}^{\infty} \left[ -\frac{8r}{m_1^2} \right]^k \frac{b_2^+ b_1^{2k+1}}{(2k+1)!} \\
- 2\beta r b_1^{2} \sum_{k=0}^{\infty} \left[ -\frac{8r}{m_1^2} \right]^k \frac{b_2^+ b_1^{2k+2}}{(2k+2)!} + \frac{1}{2} \beta M^2 \sum_{k=1}^{\infty} \left[ -\frac{8r}{m_1^2} \right]^k \frac{b_2^+ b_1^{2k}}{(2k)!},
\] (B.27)

\[
\ell'_1 = -m_1 b_1^+ b_2 + m_1 b_1^+ \frac{2h - 1}{4} \sum_{k=1}^{\infty} \left[ -\frac{8r}{m_1^2} \right]^k \frac{(b_2^+)^{k-1} b_1^{2k}}{(2k)!} \\
+ \frac{1}{2} m_1 b_1^+ \sum_{k=1}^{\infty} \left[ -\frac{8r}{m_1^2} \right]^k \frac{(b_2^+)^{k-1} b_1^{2k+1}}{(2k+1)!} + \frac{M^2}{m_1} \sum_{k=0}^{\infty} \left[ -\frac{8r}{m_1^2} \right]^k \frac{b_2^+ b_1^{2k+1}}{(2k+1)!},
\] (B.28)

\[
\ell'_2 = (b_2^+ b_2 + b_1^+ b_1 + h)b_2 - \frac{2h - 1}{4} b_1^+ \sum_{k=1}^{\infty} \left[ -\frac{8r}{m_1^2} \right]^k \frac{(b_2^+)^{k-1} b_1^{2k+1}}{(2k+1)!} \\
- \frac{1}{2} b_1^+ \sum_{k=1}^{\infty} \left[ -\frac{8r}{m_1^2} \right]^k \frac{(b_2^+)^{k-1} b_1^{2k+2}}{(2k+2)!} + \frac{M^2}{m_1} \sum_{k=0}^{\infty} \left[ -\frac{8r}{m_1^2} \right]^k \frac{b_2^+ b_1^{2k+2}}{(2k+2)!}.
\] (B.29)

Finally we put the arbitrary constant \(m_2^2\) to be equal to \(m_2^2 = -m^2 - \alpha \frac{(d-4)}{4} r\). Thus we have obtained the expressions of the additional parts (439–445) for the operator algebra given in Table 2.
Let us find an explicit expression for the operator $K$ which used in the definition of the scalar product (51). The defining relations for this operator are given by (52)–(54). These relations shall be satisfied for any $|\Psi\rangle$ if they shall be satisfied for the basis vectors of the Fock space $|n_1, n_2\rangle$. Then using arguments of [25,30] one can check that the following operator satisfy (52)–(54)

$$K = Z^+ Z, \quad Z = \sum_{n_1, n_2=0}^{\infty} |n_1, n_2\rangle \langle n_1, n_2| \frac{1}{n_1! n_2!} \langle n_1, n_2|.$$  \hfill (B.31)

For practical calculations it is useful to note that $V \langle n'_1, n'_2|n_1, n_2\rangle V \sim \delta_{{n'_1+2n'_2}}^{n_1+2n_2}$. For low numbers $n_1 + 2n_2$ the operator $K$ is

$$K = |0\rangle \langle 0| + \frac{M^2}{m_1^2} b_1^+ |0\rangle \langle 0| b_1 + h b_2^+ |0\rangle \langle 0| b_2 - \frac{M^2}{2m_1^2} (b_1^+)^2 |0\rangle \langle 0| b_2 + b_2^+ |0\rangle \langle 0| b_1^2$$ 
$$+ \frac{M^2}{m_1^2} \frac{M^2 + r(1 - 2h)}{2m_1^2} b_1^+ b_2^+ |0\rangle \langle 0| b_1^2 + \ldots \hfill (B.32)$$

This expression for the operator $K$ is used when we give the examples.

### C Removing of the auxiliary fields

In this Appendix we explain how the conditions (23) on basic vector (6) can be obtained from equations of motion (85) following from Lagrangian (89).

First we explicitly extract the dependence of the fields and the gauge parameters on the ghost fields

$$|S\rangle = |S_1\rangle + \eta_1^+ P_1^+ |S_2\rangle + \eta_1^+ P_2^+ |S_3\rangle + \eta_2^+ P_1^+ |S_4\rangle + \eta_2^+ P_2^+ |S_5\rangle$$
$$+ \eta_0^+ \eta_2^+ P_1^+ P_2^+ |S_6\rangle, \hfill (C.1)$$

$$|A\rangle = P_1^+ |A_1\rangle + P_2^+ |A_2\rangle + \eta_1^+ P_1^+ P_2^+ |A_3\rangle + \eta_2^+ P_1^+ P_2^+ |A_4\rangle \hfill (C.2)$$

$$|\Lambda\rangle = |\Lambda_0\rangle + \eta_0 |\Lambda_1\rangle, \hfill (C.3)$$

$$|\Lambda_0\rangle = P_1^+ |\lambda_1\rangle + P_2^+ |\lambda_2\rangle + \eta_1^+ P_1^+ P_2^+ |\lambda_3\rangle + \eta_2^+ P_1^+ P_2^+ |\lambda_4\rangle \hfill (C.4)$$

$$|\Lambda_1\rangle = P_1^+ P_2^+ |\lambda_5\rangle, \hfill (C.5)$$

$$|\Omega\rangle = P_1^+ P_2^+ |\omega\rangle. \hfill (C.6)$$

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In this notation the Lagrangian \([S9]\) takes the form

\[
\mathcal{L} = \langle S_1 | K \{ \tilde{L}_0 | S_1 \} - L^+_1 | A_1 \rangle - L^+_2 | A_2 \rangle - \beta r \left[ (2 + \xi) L^+_2 - 4 l^+_2 \right] | S_2 \rangle \\
- \beta r \left[ (1 - \xi) G_0 - 2 g^0 \right] | S_1 \rangle - 3 r \beta | S_1 \rangle - 3 r \xi | S_1 \rangle \\
- \langle S_2 | K \{ \tilde{L}_0 | S_2 \} - L_1 | A_1 \rangle + | A_2 \rangle - L^+_2 | A_3 \rangle \\
+ \beta r \left[ (1 - \xi) G_0 - 2 g^0 \right] | S_2 \rangle + \beta r \left[ (2 + \xi) L_2 - 4 l^+_2 \right] | S_1 \rangle \\
+ \beta r \left[ (2 - \xi) L^+_1 - 4 l^+_1 \right] | S_4 \rangle - \gamma r | S_2 \rangle + 3 r \beta | S_2 \rangle \\
- \langle S_3 | K \{ \tilde{L}_0 | S_3 \} - L_1 | A_2 \rangle + L^+_1 | A_3 \rangle \\
+ \beta r \left[ (2 - \xi) L_1 - 4 l^+_1 \right] | S_2 \rangle + \beta r \left[ (2 - \xi) L^+_1 - 4 l^+_1 \right] | S_5 \rangle \\
+ (2 - \beta) r \left[ 2 L_2 - 4 l^+_2 \right] | A_1 \rangle - (2 - \beta) r \left[ 2 L^+_2 - 4 l^+_2 \right] | A_4 \rangle \\
- \langle S_5 | K \{ \tilde{L}_0 | S_5 \} - L_2 | A_2 \rangle - | A_3 \rangle + L^+_1 | A_4 \rangle \\
+ \beta r \left[ (2 - \xi) L_1 - 4 l^+_1 \right] | S_4 \rangle - \beta r \left[ (1 - \xi) G_0 - 2 g^0 \right] | S_5 \rangle \\
+ \beta r \left[ (2 + \xi) L^+_2 - 4 l^+_2 \right] | S_6 \rangle + \gamma r | S_5 \rangle + 3 r \beta | S_5 \rangle \\
+ \langle S_6 | K \{ \tilde{L}_0 | S_6 \} + L_2 | A_3 \rangle - L_1 | A_4 \rangle + \beta r \left[ (1 - \xi) G_0 - 2 g^0 \right] | S_6 \rangle \\
- \beta r \left[ (2 + \xi) L_2 - 4 l^+_2 \right] | S_5 \rangle + 3 \gamma r | S_6 \rangle - 3 r \beta | S_6 \rangle \\
- \langle A_1 | K \{ L_1 | S_1 \} - L^+_1 | S_2 \rangle - L^+_2 | S_3 \rangle - | A_1 \rangle \\
+ (2 - \beta) r \left[ 2 L^+_2 - 4 l^+_2 \right] | S_4 \rangle \\
- \langle A_2 | K \{ L_2 | S_1 \} + | S_2 \rangle - L^+_2 | S_3 \rangle - L^+_1 | S_4 \rangle \} \\
+ \langle A_3 | K \{ L_2 | S_2 \} + | S_5 \rangle - L_1 | S_4 \rangle + L^+_2 | S_6 \rangle \} \\
- \langle A_4 | K \{ L_1 | S_5 \} - L_2 | S_3 \rangle + L^+_1 | S_6 \rangle + | A_4 \rangle \\
- (2 - \beta) r \left[ 2 L_2 - 4 l^+_2 \right] | S_4 \rangle \}. \tag{C.7}
\]

Here we see at \( \beta = \gamma = 0 \) (these values don’t correspond to that which give \( l_0 \) \([17]\) many terms in \([C.7]\) disappear and the expression for the Lagrangian is simplified.
The equations of motion which follow from Lagrangian (C.7) are

\[
\begin{align*}
\tilde{L}_0|S_1 \rangle - L_1^+|A_1 \rangle - L_2^+|A_2 \rangle & - \beta r \left[ (2 + \xi) L_2^+ - 4l_2^+ \right]|S_2 \rangle \\
& - \beta r \left[ (1 - \xi) G_0 - 2g_0' + 3\xi \right]|S_1 \rangle - 3\gamma r|S_1 \rangle = 0, \\
\tilde{L}_0|S_2 \rangle - L_1|A_1 \rangle + |A_2 \rangle - L_2^+|A_3 \rangle & + \beta r \left[ (1 - \xi) G_0 - 2g_0' + 3\xi \right]|S_2 \rangle - \gamma r|S_2 \rangle \\
& + \beta r \left[ (2 + \xi) L_1 - 4l_1' \right]|S_1 \rangle + \beta r \left[ (2 - \xi) L_1^+ - 4l_1'^+ \right]|S_4 \rangle = 0,
\end{align*}
\]

\[
\begin{align*}
\tilde{L}_0|S_3 \rangle - L_1|A_2 \rangle + L_1^+|A_3 \rangle & + \beta r \left[ (2 - \xi) L_1 - 4l_1' \right]|S_2 \rangle + \beta r \left[ (2 - \xi) L_1^+ - 4l_1'^+ \right]|S_5 \rangle \\
& + (2 - \beta) r \left[ 2L_2 - 4l_2' \right]|A_1 \rangle - (2 - \beta) r \left[ 2L_2^+ - 4l_2'^+ \right]|A_4 \rangle = 0,
\end{align*}
\]

\[
\begin{align*}
\tilde{L}_0|S_4 \rangle - L_2|A_1 \rangle - L_2^+|A_4 \rangle & = 0, \\
\tilde{L}_0|S_5 \rangle - L_2|A_2 \rangle - |A_3 \rangle + L_1^+|A_4 \rangle & - \beta r \left[ (1 - \xi) G_0 - 2g_0' - 3\xi \right]|S_5 \rangle + \gamma r|S_5 \rangle \\
& + \beta r \left[ (2 - \xi) L_1 - 4l_1' \right]|S_4 \rangle + \beta r \left[ (2 + \xi) L_2^+ - 4l_2'^+ \right]|S_6 \rangle = 0,
\end{align*}
\]

\[
\begin{align*}
\tilde{L}_0|S_6 \rangle + L_2|A_3 \rangle - L_1|A_4 \rangle & + \beta r \left[ (1 - \xi) G_0 - 2g_0' - 3\xi \right]|S_6 \rangle + 3\gamma r|S_6 \rangle \\
& - \beta r \left[ (2 + \xi) L_2 - 4l_2' \right]|S_5 \rangle = 0,
\end{align*}
\]

\[
\begin{align*}
L_1|S_1 \rangle - L_1^+|S_2 \rangle - L_2^+|S_3 \rangle - |A_1 \rangle & + (2 - \beta) r \left[ 2L_2^+ - 4l_2'^+ \right]|S_4 \rangle = 0, \\
L_2|S_1 \rangle + |S_2 \rangle - L_2^+|S_5 \rangle - L_1^+|S_4 \rangle & = 0, \\
L_2|S_2 \rangle + |S_5 \rangle - L_1|S_4 \rangle + L_2^+|S_6 \rangle & = 0, \\
L_1|S_5 \rangle - L_2|S_3 \rangle + L_1^+|S_6 \rangle + |A_4 \rangle & - (2 - \beta) r \left[ 2L_2 - 4l_2' \right]|S_4 \rangle = 0.
\end{align*}
\]

These equations of motion and the Lagrangian (C.7) are invariant under the gauge transformations

\[
\begin{align*}
\delta|S_1 \rangle &= L_1^+|\lambda_1 \rangle + L_2^+|\lambda_2 \rangle, \\
\delta|S_2 \rangle &= L_1|\lambda_1 \rangle - |\lambda_2 \rangle + L_2^+|\lambda_3 \rangle, \\
\delta|S_3 \rangle &= L_1|\lambda_2 \rangle - L_1^+|\lambda_3 \rangle - |\lambda_5 \rangle - (2 - \beta) r \left[ 2L_2 - 4l_2' \right]|\lambda_1 \rangle \\
& + (2 - \beta) r \left[ 2L_2^+ - 4l_2'^+ \right]|\lambda_4 \rangle, \\
\delta|S_4 \rangle &= L_2|\lambda_1 \rangle + L_2^+|\lambda_4 \rangle, \\
\delta|S_5 \rangle &= L_2|\lambda_2 \rangle + |\lambda_3 \rangle - L_1^+|\lambda_4 \rangle, \\
\delta|S_6 \rangle &= -L_2|\lambda_3 \rangle + L_1|\lambda_4 \rangle, \\
\delta|A_1 \rangle &= (\tilde{L}_0 - 2\gamma r)|\lambda_1 \rangle + L_2^+|\lambda_5 \rangle, \\
\delta|A_2 \rangle &= (\tilde{L}_0 - \gamma r)|\lambda_2 \rangle - L_1^+|\lambda_5 \rangle + \beta r \left[ (2 - \xi) L_1 - 4l_1' \right]|\lambda_1 \rangle \\
& - \beta r \left[ (1 - \xi) G_0 - 2g_0' \right]|\lambda_2 \rangle - \beta r \left[ (2 + \xi) L_1^+ - 4l_1'^+ \right]|\lambda_3 \rangle, \\
\delta|A_3 \rangle &= (\tilde{L}_0 + \gamma r)|\lambda_3 \rangle - L_1|\lambda_5 \rangle + \beta r \left[ (2 + \xi) L_2 - 4l_2' \right]|\lambda_2 \rangle \\
& + \beta r \left[ (1 - \xi) G_0 - 2g_0' \right]|\lambda_3 \rangle + \beta r \left[ (2 - \xi) L_1^+ - 4l_1'^+ \right]|\lambda_4 \rangle, \\
\delta|A_4 \rangle &= (\tilde{L}_0 + 2\gamma r)|\lambda_4 \rangle - L_2|\lambda_5 \rangle,
\end{align*}
\]
in its turn the gauge parameters are not uniquely defined but are invariant under transformations
\[
\begin{align*}
\delta|\lambda_1\rangle &= -L_2^+|\omega\rangle, \\
\delta|\lambda_2\rangle &= L_1^+|\omega\rangle, \\
\delta|\lambda_3\rangle &= L_1|\omega\rangle, \\
\delta|\lambda_4\rangle &= L_2|\omega\rangle, \\
\delta|\lambda_5\rangle &= \tilde{L}_0|\omega\rangle.
\end{align*}
\]
(C.28) (C.29)

One can see again that the gauge transformations (C.18)–(C.27) are simplified at \(\beta = \gamma = 0\).

**C 1. The gauge fixing**

First we discard the gauge parameter \(|\lambda_5\rangle\) and consider \(|\lambda_1\rangle, |\lambda_2\rangle, |\lambda_3\rangle, |\lambda_4\rangle\) to be independent.

Next we remove dependence of \(|S_1\rangle\) on the operator \(b_1^+\) using the parameter \(|\lambda_1\rangle\) and using the gauge parameter \(|\lambda_2\rangle\) we can remove the field \(|S_3\rangle\) using some components of \(|\lambda_2\rangle\)

\[
\delta|S_3\rangle = L_1|\lambda_2\rangle + \ldots = \left(\frac{M^2}{m_1}b_1 + \ldots\right)|\lambda_2\rangle + \ldots
\]
(C.30)

After this we have used parameter \(|\lambda_1\rangle\) completely and we have the restricted parameter \(|\lambda_2\rangle\):
\(b_1|\lambda_2\rangle = 0\), i.e. \(|\lambda_2\rangle\) is independent of \(b_1^+\). Using this restricted gauge parameter \(|\lambda_2\rangle\) we can remove dependence of \(|S_1\rangle\) on \(b_2^+\) getting \(|S_1\rangle = |\Phi\rangle\). After this the parameter \(|\lambda_2\rangle\) is exhausted. Next we remove dependence of the vectors \(|S_2\rangle\) and \(|S_4\rangle\) on \(b_2^+\) with the help of the parameters \(|\lambda_3\rangle\) and \(|\lambda_4\rangle\) respectively. Now we have used all the gauge parameters. Let us show that the rest auxiliary fields are zero as consequence of the equations of motion in this gauge.

**C 2. Removing of the auxiliary fields with the equations of motion**

After the gauge fixing we have the gauge conditions
\[
|S_1\rangle = |\Phi\rangle, \quad b_2|S_2\rangle = 0, \quad |S_3\rangle = 0, \quad b_2|S_4\rangle = 0,
\]
(C.31)

where \(|\Phi\rangle\) is the basic vector \(\mathcal{6}\). Let us look at the equation of motion (C.15)
\[
l_2|\Phi\rangle + |S_2\rangle - L_2^+|S_5\rangle - L_1^+|S_4\rangle = 0.
\]
(C.32)

Acting on this equation by the operator \(b_2\) and using (C.31) we get
\[
b_2L_2^+|S_5\rangle = 0 \implies |S_5\rangle = 0.
\]
(C.33)

Acting by \(b_2\) on (C.14) we get
\[
b_2|A_1\rangle = 2r(\beta - 2)|S_4\rangle \implies b_2b_2|A_1\rangle = 0.
\]
(C.34)

Thus we can decompose \(|A_1\rangle\)
\[
|A_1\rangle = |A_1\rangle' + b_2^+|A_1''\rangle,
\]
(C.35)

and (C.34) takes the form
\[
|A_1''\rangle = 2r(\beta - 2)|S_4\rangle.
\]
(C.36)

Acting twice by \(b_2\) on (C.8) we get
\[
b_2b_2L_2^+|A_2\rangle = 0 \implies b_2|A_2\rangle = 0,
\]
(C.37)
thus $|A_2\rangle$ does not depend on $b_2^\pm$.

Let us expand the fields $|S_2\rangle$, $|S_4\rangle$, $|A_1\rangle$, $|A_1''\rangle$, $|A_2\rangle$, in powers of $b_1^\pm$

\[
|S_2\rangle = \sum_{k=0}^{s-2} (b_1^\pm)^k |S_{2k}\rangle, \quad |S_4\rangle = \sum_{k=0}^{s-3} (b_1^\pm)^k |S_{4k}\rangle, \quad (C.38)
\]

\[
|A_1\rangle = \sum_{k=0}^{s-1} (b_1^\pm)^k |A_{1k}\rangle, \quad |A_1''\rangle = \sum_{k=0}^{s-3} (b_1^\pm)^k |A_{1k}'\rangle, \quad |A_2\rangle = \sum_{k=0}^{s-2} (b_1^\pm)^k |A_{2k}\rangle. \quad (C.39)
\]

Then decomposing the equations of motion in powers of $b_1^\pm$ and $b_2^\pm$ ones obtain (here we assume $|\Phi\rangle$)

**Equations of motion** (C.38)

\[
(b_1^\pm)^s \quad m_1 |A_{1,s-1}\rangle = 0, \quad (C.40a)
\]

\[
(b_1^\pm)^{s-1} \quad m_1 |A_{1,s-2}\rangle = l_1^+ |A_{1,s-1}\rangle, \quad (C.40b)
\]

\[
(b_1^\pm)^k \quad m_1 |A_{1,k-1}\rangle = l_1^+ |A_{1k}\rangle + l_2^+ |A_{2k}\rangle + \beta r(2+\xi)l_2^+ |S_{2k}\rangle, \quad (C.40c)
\]

\[
(b_1^\pm)^0 \quad \left(l_0 + 4(2-\beta)l_2^+ l_2^\dagger\right) |\Phi\rangle = l_1^+ |A_{10}\rangle + l_2^+ |A_{20}\rangle + \beta r(2+\xi)l_2^+ |S_{20}\rangle, \quad (C.40d)
\]

**Equations of motion** (C.41)

\[
(b_1^\pm)^{s-1} \quad m_1 |S_{2,s-2}\rangle = |A_{1,s-1}\rangle, \quad (C.41a)
\]

\[
(b_1^\pm)^{s-2} \quad m_1 |S_{2,s-3}\rangle = |A_{1,s-2}\rangle + l_1^+ |S_{2s-2}\rangle, \quad (C.41b)
\]

\[
(b_1^\pm)^k \quad m_1 |S_{2,k-1}\rangle = |A_{1k}\rangle + l_1^+ |S_{2k}\rangle + 2r(\beta-2)l_2^+ |S_{4k}\rangle, \quad (C.41c)
\]

\[
(b_1^\pm)^0 \quad l_1 |\Phi\rangle = l_1^+ |S_{20}\rangle + |A_{10}\rangle + 2r(\beta-2)l_2^+ |S_{40}\rangle, \quad (C.41d)
\]

**Equations of motion** (C.42)

\[
(b_1^\pm)^{s-2} \quad m_1 |S_{4,s-3}\rangle = |S_{2,s-2}\rangle, \quad (C.42a)
\]

\[
(b_1^\pm)^{s-3} \quad m_1 |S_{4,s-4}\rangle = |S_{2,s-3}\rangle - l_1^+ |S_{4,s-3}\rangle, \quad (C.42b)
\]

\[
(b_1^\pm)^k \quad m_1 |S_{4,k-1}\rangle = |S_{2k}\rangle - l_1^+ |S_{4k}\rangle, \quad (C.42c)
\]

\[
(b_1^\pm)^0 \quad l_2 |\Phi\rangle = l_1^+ |S_{40}\rangle - |S_{20}\rangle, \quad (C.42d)
\]

**Equations of motion** (C.43)

\[
b_2^\pm (b_1^\pm)^k \quad |A_{1k}\rangle = 2r(\beta-2) |S_{4k}\rangle, \quad (C.43)
\]

**Equations of motion** (C.44)

\[
b_2^\pm (b_1^\pm)^{s-2} \quad |A_{2,s-2}\rangle = \beta r(2-\xi) |S_{2,s-2}\rangle - m_1 |A_{1,s-3}\rangle, \quad (C.44a)
\]

\[
b_2^\pm (b_1^\pm)^{s-3} \quad |A_{2,s-3}\rangle = \beta r(2-\xi) |S_{2,s-3}\rangle - m_1 |A_{1,s-4}\rangle - l_1^+ |A_{1,s-3}\rangle, \quad (C.44b)
\]

\[
b_2^\pm (b_1^\pm)^k \quad |A_{2k}\rangle = \beta r(2-\xi) |S_{2k}\rangle - m_1 |A_{1k}'\rangle - l_1^+ |A_{1k}\rangle, \quad (C.44c)
\]

\[
b_2^\pm (b_1^\pm)^0 \quad |A_{20}\rangle = \beta r(2-\xi) |S_{20}\rangle - l_1^+ |A_{10}\rangle, \quad (C.44d)
\]
One can show that the solution of equations of motion (C.40)–(C.44) is
\[ |S_2⟩ = |S_4⟩ = |A_1⟩ = |A_2⟩ = 0. \] (C.45)

To see this one should start from the two first equations of (C.40). They give
\[ |A'_{1,s-1}⟩ = |A'_{1,s-2}⟩ = 0. \]

Then we go down to next set of equations (C.41). The first two of them give us
\[ |S_{2,s-2}⟩ = |S_{2,s-3}⟩ = 0. \]

Going down to the subsequent sets of equations (C.42), (C.43), (C.44) we obtain one after another that
\[ |S_{4,s-3}⟩ = |S_{4,s-4}⟩ = 0, |A''_{1,s-3}⟩ = |A''_{1,s-4}⟩ = 0, |A_{2,s-2}⟩ = |A_{2,s-3}⟩ = 0. \]

After this we return to the first set of equations (C.40) and repeat the procedure until we obtain (C.45). After this we get from (C.40d), (C.41d), (C.42d) that the equations on the basic fields are (23).

Thus now we have
\[ |S_1⟩ = |Φ⟩, |S_2⟩ = |S_3⟩ = |S_4⟩ = |S_5⟩ = |A_1⟩ = |A_2⟩ = 0. \] (C.46)

Substituting these solutions into (C.16) we get
\[ L^+_2 |S_6⟩ = 0 \implies |S_6⟩ = 0, \] (C.47)

then substituting into (C.17) we obtain
\[ |A_4⟩ = 0, \] (C.48)

and finally substituting into (C.12) ones have
\[ |A_3⟩ = 0. \] (C.49)

Thus we remove all the auxiliary fields and the equations of motion for the basic fields |Φ⟩ are (23).

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