Coding of Stereo Signals by a Single Digital ΔΣ Modulator

Sergio Callegari
ARCES/DEI, University of Bologna, Italy
sergio.callegari@unibo.it

Abstract—The possibility of using a single digital ΔΣ modulator to simultaneously encode the two channels of a stereo signal is illustrated. From the modulated stream, the two channels can be recovered with minimal processing and no cross-talk. Notably, demultiplexing does not affect the sample-depth so that, after it, one still has a data stream suitable for directly driving a power bridge and convertible into analog by mere low-pass filtering. Furthermore, the approach is very flexible and if one channel is unused, it lets the other get improved dynamic range and SNR. The approach can take advantage of recent techniques for the design of ΔΣ modulators, including methods for psychoacoustically optimal distribution of quantization noise. Code is available to replicate the proposed examples and as a general computer aided design tool.

I. INTRODUCTION

Digital ΔΣ Modulator (ΔΣM) are the widest adopted form of ΔΣ modulators in commercial integrated circuits [1]. Their role is that of information re-coders that are fed with a stream of high-resolution samples and deliver an equivalent high-rate low-depth (low-resolution, e.g., binary, ternary) stream. By equivalent, it means that from the output stream it is possible to recover the input information with very good approximation by mere linear filtering. In other words, the coding process assures that the artifacts created by resolution reduction have negligible energy in the signal band. The process can be functional to D/A conversion, frequency synthesis [2], switched mode power control [3], and so on.

A particularly interesting application of ΔΣM is in audio systems, where D/A conversion is associated to amplification. In conventional setups, a Nyquist rate D/A converter is followed by a smoothing filter and an analog amplifier, in an arrangement typically characterized by poor power efficiency. Conversely, by ΔΣ modulation, a high-resolution digital stream (as coded in a conventional digital media) can be converted in commands for a switched mode power bridge. Power amplification can so be achieved by a switched-mode unit with much better efficiency. The ΔΣM ability to deliver a low-depth stream is the key to directly driving the bridge, since the latter can only assume a very limited number of configurations.

In this paper, the possibility of using a single modulator to simultaneously encode the two channels of a stereo signal is illustrated. The proposal introduces a multiplexing mechanism where: (i) multiplexing and demultiplexing are inexpensive; (ii) ΔΣM design techniques specific of the audio realm remain applicable; and (iii) after demultiplexing the properties of the ΔΣ streams are preserved. By (iii), one means that the data stream obtained after de-multiplexing is still suitable to directly drive a switched mode power bridge as if no multiplexing/demultiplexing was involved.

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The proposed setup saves a ΔΣM with respect to a conventional stereo arrangement. Yet, this is not a major advantage, since a modulator with more complex filters is required in exchange. More interesting is the ability to save connections, particularly when the sound delivery is remote from the signal source, as in Fig. 1. Even if the loudspeakers need to be themselves separated from each other, their wiring can be simplified by a ‘daisy chain’ topology. The advantage increases in environments where passing wires is difficult or visually unappealing (e.g., domestic) or when specific connection requirements exist, such as galvanic isolation or optical link, since in this case a coupling element can be saved too. Obviously, a similar advantage could also be obtained by conventional digital multiplexers/demultiplexers in a 2 ΔΣM setup. Yet, the proposed arrangement is simpler and requires demultiplexing hardware just on one channel, as it will be shown shortly. Most important, this proposal turns out to be more flexible. If one channel is not needed, it lets the dynamic range and SNR be improved on the other.

Even if this paper focuses on audio systems, the same approach can be used whenever two signals need to be treated at once. The audio application is currently targeted both for its economic importance and since it represents an interesting benchmark. In fact, it imposes strong specifications on cross-talk and SNR. Furthermore, ΔΣMs for the audio domain may include specific design flows (e.g., for minimizing perceived noise according to psychoacoustic models [4], [5]). An appealing feature of the proposed multiplexing mechanism it its full compatibility with them. In a near future, the results presented here will be extended to a larger number of multiplexed signals and to a wider range of applications (transmission of multiple sensed data, multiplexing of hand-pass signals, storage of multiple analog waveforms on digital memories, etc.).

II. BACKGROUND

A brief review of ΔΣ modulation is needed to define the notation. ΔΣ coders are feedback-based nonlinear systems [6] requiring a sample rate \( f_s = 1/\tau \) exceeding twice the width \( B \) of the band \( B \) of their input signal \( u(nT) \) by a large factor known as Oversampling Ratio (OSR). Their behavior is typically analyzed relying on a linearized model that replaces the quantization process responsible for the resolution reduction by the superposition of a noise signal \( \epsilon(nT) \). Following the classical model of quantization, \( \epsilon(nT) \) is assumed to be independent from \( u(nT) \), white and uniformly distributed in value. With this, the modulator behavior gets characterized through two items: a Signal Transfer Function (STF), from the input \( u(nT) \) to the output \( x(nT) \), and a Noise Transfer Function (NTF), from \( \epsilon(nT) \) to \( x(nT) \), so that

\[
X(z) = STF(z)U(z) + NTF(z)E(z).
\]

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used to indicate the Laplace transforms of the signals named by the corresponding lower case symbol.

Under common assumptions, a relationship exists between STF, NTF and the internal filtering structures inside the modulator such that, once \( STF(z) \) and \( NTF(z) \) are assigned, the modulator functionality is fully defined (even if the actual implementation and arrangement of the filters can vary) [6]. For this reason, the modulator functional design substantially reduces to a suitable choice of these two transfer functions [7]. In (re)coding, typically one wants \( u(nT) \) to pass through the modulator unaltered, so that \( STF(z) = 1 \) or, at most, \( STF(z) = z^{-d} \) where \( d \) is an integer. The NTF should then be arranged to strongly attenuate the quantization noise in \( B \), so that the original information can be recovered from \( x(nT) \) by merely filtering away all that is out of \( B \). To have \( NTF(z) \) highly attenuating in \( B \), it must be allowed to amplify elsewhere (so that the net result appears like moving noise from one frequency region to another) [6] . This is due to many reasons, not last the requirement that the modulator feedback loop is non-algebraic. Since the loop transfer function accounts to \((1-NTF(z))/NTF(z))=1 \text{−} NTF(z) \) must be delaying, and thus \( NTF(z) \) must be biproper and show a unitary gain when factored in zero-pole-gain form. Other common constraints include avoiding the magnitude response of \( NTF(z) \) to peak above a certain value \( \gamma \) depending on the quantizer resolution, following the Lee stability criterion [8]. For binary quantizers, \( \gamma < 2 \) and typically \( 1.5 \) is used.

III. TWO WAY MULTIPLEXING IN ΔΣ MODULATION

As described in the previous section, the modulator input-output behavior is approximately equivalent to that of a linear channel. Consequently, multiplexing Low Pass (LP) signals into a single ΔΣ stream should be practicable by exploiting frequency division and superposition as in Fig. 2. This requires up converting the input signals to occupy different bands and using an NTF with multiple high-attenuation regions corresponding to them. Then a complementary down conversion can be applied at the decoding end to restore the original bands, so enabling the separation of the different streams by filtering. Appealingly, the same filter that would in any case be used to do signal reconstruction can here double as a channel separation filter for de-multiplexing.

Notwithstanding apparent simplicity, this approach involves some challenges. In fact, it requires a modulator whose NTF has multiple high-attenuation zones while respecting the structural constraints summarized in Sec. II. Furthermore, it involves designing the up- and down-conversion operations in a way that is acceptable cost-wise and capable of ‘passing-through’ the low-resolution property of ΔΣ streams after down-conversion. Without this property, the possibility to directly feed the down-converted output to a bridge or to transform it into analog by mere filtering would be hindered. These requirements rule out generic single- and double-side-band mixers as up/down-converters since such blocks involve multiplication by sinuosoidal carriers. Not only full-fledged multipliers are too expensive. They also require high-resolution arithmetic, thus delivering high-depth outputs unsuitable at the receiving end.

In practice, the sole operation guaranteed to leave the data depth unaltered is multiplication by \( \pm 1 \) (since any set of discrete levels balanced around zero is invariant under this operation). Consequently, down-conversion can only be based on mixing with carriers \( r(nT) \in \{-1,1\} \forall n \in Z \). Interestingly, this mixing is an involutory operator, namely an operator that is its own inverse. In fact, for an arbitrary \( u(nT) \) and \( r(nT) \in \{-1,1\} \), one has \( u(nT)r(nT)\tilde{r}(nT) = u(nT)r\tilde{r}(nT) = u(nT)r(nT) = u(nT) \). This assures that up-conversion, that must necessarily be the inverse of down-conversion, can rely on the very same binary mixing.

Once the restriction to binary mixing is established, what remains to be evaluated is how it can be used to shift spectral occupations in order to support the multiplexing and to guarantee that the $\Delta \Sigma M$ has sufficient space for its quantization noise. Mixing a generic \( u(nT) \) by a binary periodic \( r(nT) \), one gets a signal \( y(nT) \) whose Power Spectral Density (PSD) is

\[
\Psi_y(f) = \int_{0}^{1} \Psi_u(x) \Psi_r(f-x) dx. \tag{1}
\]

where \( \Psi_u(x) \) and \( \Psi_r(f) \) are the PSD of the carrier and the input signal respectively. In fact, a product in the time domain converts into a convolution in the frequency domain. Being periodic, \( r(nT) \) can be decomposed in the superposition of multiple complex-exponentials signals as in

\[
r(nT) = \sum_{k=0}^{N-1} r_k e^{i2\pi \frac{k}{N} nT} \tag{2}
\]

where \( N \) is the period length, \( r_k \) is the \( k \)-th Fourier coefficient and \( r_{N-k} = r_k \), with the asterisk indicating complex conjugation. Thus,

\[
\Psi_r(f) = \sum_{n=-\infty}^{\infty} \sum_{k=0}^{N-1} r_k e^{i2\pi \frac{k}{N} nT} e^{-i2\pi f n} = \sum_{k=0}^{N-1} r_k \sum_{n=-\infty}^{\infty} \delta(f - k/N + i) \tag{3}
\]

where \( \delta(\cdot) \) is the Dirac delta. When expression (3) is substituted into (1), its argument is restricted within \([0,1]\), so that only \( i = 0 \) needs to be evaluated and the inner sum disappears. With this,

\[
\Psi_y(f) = \int_{0}^{1} \Psi_u(\tilde{f} - \xi) \sum_{k=0}^{N-1} r_k \delta(\tilde{f} - k/N + i) dx = \sum_{k=0}^{N-1} r_k \Psi_u(\tilde{f} - k/N). \tag{4}
\]

In other words, \( \Psi_u(f) \) is the superposition of \( N \) scaled replicas of the input signal spectrum, shifted by \( 1/N \) from each other.

If one needs to multiplex two LP signals with identical spectral occupation, the above derivation ensures the possibility of doing so passing-through one of them without any up-conversion (as in Fig. 2), provided that for the other the up-conversion involves a binary carrier such that: (i) it has no dc component (namely \( r_0 = 0 \)); and (ii) \( N < 1/b \), with \( B = b/f_0 \). These two properties are sufficient conditions to assure that the up-converted signal and the passed-through one do not overlap in the frequency domain.

Once this requirement is satisfied, one needs to consider the interactions between multiplexing and the $\Delta \Sigma M$ operation. To make the architecture comparable to one using two modulators and operating with some $\Delta \Sigma M$ as OSR, one needs $f_0 = 4 \times OSR.B$, so that the overall data rate is the same. However, multiplexing makes the modulator operate with an input waveform whose spectral occupation is made of multiple frequency intervals each as large as $B$ (due to the spectral replicas), for an overall bandwidth $B_{eq} = NB$, where $N$ is the overall number of replicas. It is obviously desirable to have $N$ as low as possible. For one, the modulator ends up working at an
effective OSR given by $\text{OSR}_{\text{eff}} = 16/(2\pi N_\Phi) = (2\text{OSR})/N$, so that its noise performance is necessarily reduced when $N$ is large. Secondly, its NTF magnitude response needs to have as many valleys as $N$, which may require increasing too much the modulator order when $N$ is large. However, it is known that $N$ is certainly bound by $N$, since it counts the passed-through signal and at most $N-1$ replicas of the up-converted signal. Thus, picking $N = 2$ automatically ensures the smallest possible $\tilde{N}$. With this, $r(nT)$ can be set to $(-1)^n$ as in Fig. 3a. The resulting up-conversion produces a single replica shifted by $f = 1/2$, so that it basically converts the LP signal to a High Pass (HP) one, as shown in Fig. 3b.

IV. MODULATOR DESIGN

In the proposed 2-way multiplexed arrangement, the signal entering the modulator has two components, one LP and one HP. Consequently, the NTF needs to be Band Pass (BP). A suitable NTF can be built from a conventional NTF for LP signals as follows: (i) design an NTF for the base-band signals, say $\tilde{\text{NTF}}_{\text{LP}}(z)$; (ii) obtain the desired BP NTF as $\tilde{\text{NTF}}(z) = \tilde{\text{NTF}}_{\text{LP}}(z) \cdot \tilde{\text{NTF}}_{\text{LP}}(-z)$.

Let this procedure be examined in reverse. If $\tilde{\text{NTF}}_{\text{LP}}(z)$ is an HP transfer function, then the elementary $z \rightarrow z^\gamma$ spectral transformation makes $\tilde{\text{NTF}}_{\text{LP}}(-z)$ its equivalent LP function. Multiplying $\tilde{\text{NTF}}_{\text{LP}}(z)$ and $\tilde{\text{NTF}}_{\text{LP}}(-z)$, gets a transfer function that has a high-attenuation spectral peak approximately given by the squared peak gain of $\tilde{\text{NTF}}_{\text{LP}}(z)$. From this, one finds how $\tilde{\text{NTF}}_{\text{LP}}(z)$ needs to be designed. One must impose a peak gain $\gamma_{\text{LP}} = \sqrt{\gamma}$ where $\gamma$ is the Lee coefficient that would be used in a conventional non-multiplexed design. This in addition to taking an OSR given by $\text{OSR}_{\text{LP}} = 2\text{OSR}$ (i.e., twice the value that would have been used in a non-multiplexed setup), following the $f_\Phi$ choice illustrated before.

This design procedure has a major asset in being fully based on the design of LP modulators. Thus, it lets any specific design strategy devised for LP modulators be automatically portable to the 2-way multiplexed arrangement. For instance, suppose that one wants to base the design on the synthesizENTF design strategy proposed by Schreier [6]. This is implemented into a function synthesizENTF(order, OSR, opt, $H_{\text{inf}}$), where order is the modulator order, OSR is the OSR, opt is a flag controlling some optimization modes, and $H_{\text{inf}}$ is $\gamma$. Assuming that one would invoke it as synthesizENTF(order, OSR, opt, $H_{\text{inf}}$) for a conventional modulator, then to design $\tilde{\text{NTF}}_{\text{LP}}(z)$ it is sufficient to call synthesizENTF(order, OSR/2, opt, sqrt($H_{\text{inf}}$)), leading to a final BP $\tilde{\text{NTF}}(z)$ with an order twice as large as order. All the optimizations provided by synthesizENTF will be automatically present. Similarly, suppose that one has a function capable of designing psychoacoustically optimal NTFs (as in [4] or [5]), based on a required order, an OSR or $f_\Phi$ specification, a Lee coefficient value, and possibly other parameters. Again, it can be used for $\tilde{\text{NTF}}_{\text{LP}}(z)$, just remembering to double the OSR (or $f_\Phi$) and to take the square root of the Lee coefficient with respect to an equivalent non-multiplexed design.

A. Expected performance

Being $\Delta\Sigma$ modulators strongly non-linear objects, an accurate performance evaluation can only be based on simulation in actual operating conditions. Yet, some estimation of the noise-floor and SNR, is still possible by relying on the linearized model to compute the in-band power level due to quantization noise as in

$$P_N = \sigma_q^2 \cdot 2 \int_0^{\pi f_s} |\tilde{\text{NTF}}(e^{j2\pi f})|^2 df$$

where $\sigma_q^2$ is the power of $e(nT)$ [6], [7]. In typical cases, $P_N$ scales with the OSR in a rather predictable way, as shown in Fig. 4a. Specifically, doubling the OSR improves the noise floor by approximately 3 dB + 6 dB · (modulator order), which is an expected result [6]. Furthermore, it can also be empirically found that $P_N$ scales quite regularly with $\gamma$, as shown in Fig. 4b. Whenever $\gamma$ is reduced by taking its square root, the noise floor is worsened by approximately $-1$ dB + 6 dB · (modulator order).

This means that applying the 2-way multiplexing technique, which implies starting from a modulator design with twice the OSR and the square root of $\gamma$, improves the noise floor by about 5 dB at any order. It is worth underlining that this advantage does not reflect on the SNR. In fact, since $\Delta\Sigma$ modulators have a strict limit on the input signal range that they can tolerate, superimposing two signals at the input of the modulator requires halving the maximum acceptable signal level. This lowers the maximum signal power by 6 dB. Hence, all together, one can expect an almost unchanged SNR (in fact, almost negligibly degraded by about 1 dB).

V. SIMULATION AND PERFORMANCE VALIDATION

For validation, the following test setup is considered. The two channels to be simultaneously encoded have a 20 kHz bandwidth and the overall OSR is set to 64. This sets the sample clock of the multiplexed system at 5.12 MHz. Binary modulators with $STF(z) = 1$ are considered. Test tones with frequencies set at ~1 and ~3.2 kHz and nominal amplitudes set at 0.2 and 0.44 are used for the 1st and 2nd channel respectively (amplitudes normalized with respect to the quantization levels ±1). Merit factors include the quantization noise floor, SNR, cross-talk, and maximum tolerable signal amplitude. The
latter is evaluated by forcing the same amplitude on the two signals and rising it to the point where the modulator starts misbehaving. SNR is evaluated at the nominal and maximum amplitudes, in band. A benchmark obtained using two individual \( \Delta \Sigma \) coders for the two signals is also provided. The benchmark coders have a 20 kHz bandwidth, half the order, and are designed with the same technique (Schreiers’ synthesizeNTF [6]) and OSR as those under test, getting a 2.56 MHz sample rate. All the tests can be replicated using the PyDSM toolbox, available for download at http://pydsm.googlecode.com.

The achieved behavior is illustrated in Fig. 5. Plots (a) and (b) show the NTF magnitude response with a log and linear frequency axis respectively. Plot (c) is a fragment of the modulator output \( x(nT) \). Plot (d) illustrates the PSD of \( x(nT) \), obtained from time domain simulations. Here, the overall shape of the noise PSD agrees with the curve in (b), while the peaks corresponding to the test tones are almost invisible, being quite close to the plot frame. Plots (e) and (f) show fragments of the reconstructed signals \( \hat{u}_1(t) \) and \( \hat{u}_2(t) \). This visually illustrates that the approach works and that there is no distortion, noise, or cross-talk perceivable “by the eye”. Eventually, plots (g) and (h) show the PSD of the quantization noise for the modulator output \( x(nT) \) (from which the first output is obtained) and for \( x(nT) \cdot (-1)^m \) (from which the second output is obtained). Quantitatively, performance is summarized in Tbl. I, that includes comparison to a reference system. For the multiplexed arrangement, he tabled maximum input amplitude is cumulative. Namely, if both channels are active, each must be limited to half the maximum cumulative value. Maximum SNR is reported with respect to this situation.

VI. CONCLUSIONS

A method for multiplexing stereo signals on a single \( \Delta \Sigma \)M has been proposed. Simulation data confirms the theoretical expectation that, with respect to a reference system using two \( \Delta \Sigma \)Ms, noise floor is improved (-4 dBm) while SNR is degraded in an almost inappreciable way (-1 to -2 dB). With respect to the reference system, the approach saves 1 modulator and 1 data link. Yet, the saving on the modulator is relative, since the order of its filters is doubled. Note that the data link saving could in principle be achieved also using two modulators and a conventional digital multiplexing system. However, in the proposed arrangement demultiplexing is simpler and only requires hardware on a single channel. Most important, the proposed approach is more flexible than a two-\( \Delta \Sigma \)M one. In fact, if one channel is unused, it lets the input range of the other be risen, so that for the used channel the noise floor improvement can be capitalized into an SNR improvement (up to +4 dB).

| 8th order dual channel modulator | Channel 1 | Channel 2 |
|----------------------------------|-----------|-----------|
| Noise floor (in band)            | -102 dBm  | -101 dBm  |
| SNR (in band)                    | 98 dB     | 105 dB    |
| Max SNR (in band)                | 103 dB    | 103 dB    |
| Crosstalk                        | Below noise floor | Below noise floor |
| Max input amplitude (cumulative) | 0.68      | 0.68      |

Table I Performance indicators for a modulator system for stereo channels designed by the proposed method and conventional (reference) techniques

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