Computerized generation and simulation of meshing of a novel crown gear coupling avoiding edge contact

Yabin GUAN*, Xiaohui YANG* and Zongde FANG*
*School of Mechanical Engineering, Northwestern Polytechnical University, Xi'an 710072, China
E-mail: guanyabinpu@163.com

Received: 7 May 2019; Revised: 23 June 2019; Accepted: 21 July 2019

Abstract
To avoid the occurrence of the hub tooth tip edge contact with the sleeve tooth root flank and the sleeve tooth tip edge contact with the hub tooth root flank, a novel crown gear coupling composed of a hub with profile crowning, longitude crowning, and tip sphere teeth, and a sleeve with straight internal teeth is presented in this paper. The generating procedure of the hub with profile parabolic modification for the novel crown gear coupling is proposed. And the meshing model with angular misalignment of the novel crown gear coupling is developed. Tooth contact analysis (TCA) based on the theory of gearing and loaded tooth contact analysis (LTCA) based on finite element method (FEM) are employed to simulate contact path and to calculate contact stress distributions of the novel crown gear coupling. An example crown gear coupling is presented. The effects of profile parabolic modification coefficient and misalignment angle on contact path are investigated both for the conventional and novel crown gear couplings. Also, contact stress distributions are analyzed both for them with the maximum misalignment angle. The performed research proves that an appropriate amount of profile modification for the hub of the crown gear coupling permits the avoidance of edge contact in presence of angular misalignment, and can significantly reduce the maximum contact stress, compared with the conventional one.

Keywords: Crown gear coupling, Edge contact, Profile parabolic modification, Tooth contact analysis, Loaded tooth contact analysis

1. Introduction

A crown gear coupling mainly consists of a hub and a sleeve of the same number of teeth that are rotating about the same axis. The hub is an external gear that has longitude crowning and tip sphere teeth, and the sleeve is an internal spur gear. When a crown gear coupling is loaded, multiple crown gear coupling tooth pairs are engaged simultaneously to carry the load, leading to significantly high load carrying capacity. In addition, hub tooth surface with certain lead crown modification can also effectively accommodate some angular misalignment between two mating shafts. Owning these characters, crown gear couplings have been widely applied in the power transmission of industrial systems (Yu and Wang, 2009; Locke et al., 2013; Guo et al., 2013; Peng et al., 2018). However, there is still little investment into research concerning them compared to other parts, such as gears and bearings.

Misalignment of a crown gear coupling has been recognized as harmful because excessive misalignment failures due to an increased significant load concentration towards the end of the hub or sleeve teeth, or end loading, and accelerates wear and fretting fatigue of the crown gear coupling. Misalignment failures account for approximately 20% of the known crown gear coupling failures. Therefore, it is important to study contact analysis between teeth when the crown gear coupling is working with misalignment. Several studies have been performed for this by TCA. Using the hub surface formed by a positive continuous modification along the hub tooth width direction, Nakashima (1988) and Alfares et al. (2006) studied the influence of misalignment angle on the minimum clearance distribution along the circumferential direction and contact path on the hub tooth surface. Hakozaki and Shimachi (2004) changed the curvature along the hub tooth width direction to modify the bearing area for angular misalignment. A larger bearing area and a smaller tooth gap were obtained by the method, which were related to the tooth backlash-hit noise. Guan et
al. (2018) presented a tooth contact analysis technique for this special device based on gear meshing theory and coordinate transformation, and contact path obtained by the proposed method for the crown gear coupling with 0.3° misalignment is shown in Fig. 1(a). And some other studies have been performed for this by LTCA. Renzo et al. (1968) derived an equation for the load sharing between the teeth of the crown gear coupling by assuming that the ratio of the load on any single tooth to the load on the most loaded tooth is equal to the ratio of the difference between the elastic deformation of the most loaded tooth and the separation distance on any single tooth, to the elastic deformation of the most loaded tooth. Based on the load sharing equation proposed by Renzo et al. (1968) and hertz contact theory, Guo et al. (2015, 2016) developed an analytic model to analyze the contact characteristics of the crown gear coupling which fully considered the bending and contact deformation of the teeth. Alfares and Elkholy (2001) calculated the load sharing between the teeth of crown gear coupling with angular misalignment based on the clearance distribution calculated above and tooth stiffness. Keum (2003) studied the effect of misalignment angle on the distributions of stresses and contact forces by FEM. The result showed that with the increase of misalignment angle, the location of the maximum contact stress moves from the center to the edge along the hub tooth width direction, as expected. Guan et al. (2019) compared three different geometric models for the crown gear coupling on unloaded and loaded contact performances by FEM. And, they found that all of the teeth are in line contact, and the torque is transmitted evenly across the teeth, no matter which model is used, when the hub is perfectly aligned with the sleeve. However, a serious load concentration can occur along the hub tooth profile direction when the hub is misaligned with the sleeve, as shown in Fig. 1(b). Some other studies also have been performed for this by conducting experiments. Cuffaro et al. (2013), Cura et al. (2013) and Qureshi et al. (2017) conducted an experimental test by means of a dedicated test rig to compare the contact performance of the straight and crown gear couplings with different misalignment angles. Results show that the contact pattern on the hub tooth surface of the crown gear coupling is completely different with that of the straight gear coupling. He (2013) did the loading test of crown gear coupling combining the actual working condition of concrete truck, and checked the contact pattern on the hub tooth surface, as shown in Fig. 1(c). From TCA, LTCA, and experiments shown in Fig.1, it can be seen that edge contact occurs between the tip of one member and the root flank of the other for the convention crown gear coupling no matter when the crown gear coupling is unloaded or loaded.

Fig. 1 Contact analysis of the conventional crown gear coupling.

The above-mentioned researchers have made great contributions to contact analysis of the crown gear coupling. However, no research has been found on how to avoid the occurrence of the hub tooth tip edge contact with the sleeve tooth root flank and the sleeve tooth tip edge contact with the hub tooth root flank. Edge contact results in unexpected early failure of a gear transmission because of highly concentrated contact stress generated between the tip of one member and the root flank of the other (Litvin et al., 2003; Gonzalez-Perez et al., 2015; Fuentes-Aznar et al., 2017; Guo et al., 2019). And, the same is true for the crown gear coupling. Localization of bearing contact by crowning the hub tooth surface in both profile and longitude directions should be required to avoid the edge contact.
Therefore, profile modification should be added to the hub tooth surface of the conventional crown gear coupling. A novel crown gear coupling is presented to avoid edge contact in this paper. The mathematic model of an imaginary rack cutter with parabolic modification to determine the axial section of the form grinding wheel is constructed. After determination of the axial section of the form grinding wheel, it becomes possible to determine the form grinding wheel surface. Based on the generating principle and equation of the form grinding wheel surface, the equation of the hub tooth surface with profile parabolic modification is derived. And then the theoretical mesh model for the novel crown gear coupling with angular misalignment is developed. Based on the developed mathematical model of the novel crown gear coupling, the TCA is performed. Besides, the finite element mesh model is established to conduct LTCA. The instantaneous contact points of the novel crown gear coupling are calculated to study the effects of profile parabolic modification and misalignment angle. Moreover, the contact performances of the conventional and novel crown gear couplings are compared.

2. Geometry of the novel crown gear coupling

A conventional crown gear coupling mainly consists of a hub with longitude crowning and tip sphere teeth and a sleeve with straight internal teeth. To prevent edge contact, profile modification is added to the hub tooth surface of the conventional crown gear coupling. And, the hub with profile crowning, longitude crowning, and tip sphere teeth, and the sleeve with straight internal teeth form a novel crown gear coupling. Comparison of the hub tooth surfaces of the conventional and novel crown gear couplings is shown in Fig. 2.

![Comparison of the hub tooth surfaces](image)

Fig. 2 Comparison of the hub tooth surfaces.

The sleeve surface of the novel crown gear coupling can be manufactured by a shaper cutter (Guan et al., 2018) and the process of the derivation is omitted here. The hub surface of the novel crown gear coupling can be determined by the following processes: (I) in the meshing of the hub workpiece with an imaginary rack cutter with parabolic modification, when the initial profile crowned tooth surface $\Sigma_i$ is generated and the form grinding wheel surface can be determined because the axial section of the form grinding wheel is the same as the cross-section of surface $\Sigma_{\text{w}}$, (II) in the process of generation by the form grinding wheel, when the intermediate profile and longitudinally crowned hub tooth surface $\Sigma_h$ is obtained, (III) in the process of generation by a mill cutter, when the final, profile and longitudinally crowned hub tooth surface with a spherical tip is obtained. The process of generation by a mill cutter to generate the spherical tip is omitted here, which has been stated in the reference (Guan et al., 2018).

The generation of the initial profile crowned tooth surface $\Sigma_i$ during the process is equivalent to pure rolling of the pitch line of the rack cutter against the pitch circle of the hub workpiece, as shown in Fig. 3. Movable coordinate systems $S_i(O_i, x_i, y_i, z_i)$ and $S_w(O_w, x_w, y_w, z_w)$ are rigidly connected to the rack cutter that performs translational motion and the hub workpiece that performs rotational motion, respectively. The fixed coordinate system $S(O_a, x_a, y_a, z_a)$ is rigidly connected to the cutting machine. The parabolic profile of the rack cutter for hub workpiece generation in plane $z_a = l_a$ is shown in Fig. 3(a). Parameter $u_i$ and $l_i$ are the parabolic rack cutter surface parameters, wherein $u_i$ is measured from point $Q$ to the vertex $O_a$, and $l_i$ is measured from $O_a$ along the tangent to the point of intersection of $y_a$-axis and $x_a$-axis. Parabolic rack cutter surface $\Sigma_i$ is represented in coordinate system $S_i(O_i, x_i, y_i, z_i)$ whose origin is the vertex $O_a$ of the parabola by the following vector function $\mathbf{r}_i(u_i, l_i)$:
where \( a_c \) is the parabola coefficient of the parabolic rack cutter profile.

Then, using the coordinate transformation in transition from \( S_a \) to \( S_c \), the rack cutter surface \( \Sigma_c \) can be represented in coordinate system \( S_c \) by the following matrix equation

\[
\mathbf{r}_c^{(Q)}(u_c, l_c) = \mathbf{M}_{ca} \mathbf{r}_a^{(Q)}(u_c, l_c)
\]

(2)

where \( \mathbf{M}_{ca} \) represents the coordinate transformation from \( S_a \) to \( S_c \).

The angle \( \phi_h \) that the hub workpiece rotates and the displacement \( s \) that the rack cutter moves along \( x_c \)-axis are related by the following equation

\[
\phi_h = \frac{s}{r_p}
\]

(3)

where \( r_p \) is the pitch radius of the hub workpiece.

After determination of the initial profile crowned tooth surface, it becomes possible to determine the form grinding wheel surface \( \Sigma_d \). The form grinding wheel is shown in Fig. 4.

The axial section (plane \( z_d = 0 \), as shown in Fig. 4(a)) of the form grinding wheel surface \( \Sigma_d \) coincides with the cross section of the initial profile crowned tooth surface. Surface \( \Sigma_d \) is a surface of revolution formed by rotation of the form grinding wheel axial profile about its axis \( x_d \).

The procedure of derivation of grinding disk surface \( \Sigma_d \) is as follows: Taking in Eq. (4) \( l_c = 0 \), the cross-section of \( \Sigma_d \) by plane \( z_d = 0 \) is obtained. \( s \) can be expressed with parameter \( u_c \) using the meshing equation in Eq. (4) and the cross
section profile may be determined by vector function \( \mathbf{R}_d(u_c) \). The following coordinate systems are applied for derivation of \( \Sigma_d \): \( S_h(O_h,x_h,y_h,z_h) \) that is rigidly connected to hub workpiece, \( S_d(O_d,x_d,y_d,z_d) \) that is a coordinate system related to the form grinding wheel.

\[ \mathbf{r}_d(u_c) = \begin{bmatrix} r_d(u_c,0)_1 \\ r_d(u_c,0)_2 \\ 0 \\ 1 \end{bmatrix} \]  

(5)

The coordinate of point \( M_{dh} \) in \( S_d \) is derived from the matrix \( \mathbf{M}_{dh} \) describing transformation from \( S_h \) to \( S_d \)

\[ \mathbf{r}_d(u_c) = \mathbf{M}_{dh} \mathbf{r}_h(u_c) \]  

(6)

The form grinding wheel tooth surface \( \Sigma_d \) generated by rotating its axial profile about its axis \( x_d \) and it can be obtained by the following equation

\[ \mathbf{r}_d(u_c,\theta_d) = \begin{bmatrix} r_d(u_c,\theta_d)_1 \\ r_d(u_c,\theta_d)_2 \cos \theta_d \\ r_d(u_c,\theta_d)_2 \sin \theta_d \\ 1 \end{bmatrix} \]  

(7)

The profile and longitudinally crowned hub tooth surface \( \Sigma_h \) can be processed by the above form grinding wheel. The coordinate systems are used to derive of \( \Sigma_h \): \( S_h \) and \( S_d \) that are rigidly connected to the hub and the generating tool, \( S_d(O_d,x_d,y_d,z_d) \) that is an auxiliary fixed coordinate system, and \( S_1 \) that is rigidly connected to the displacement circle radius as shown in Fig. 5(a). Besides, the generating wheel performs the following motions (shown in Fig. 5(b)) during the process of generation: translational motion along the \( z_1 \)-axis; plunging motion along the \( y_1 \)-axis; rotating motion about origin \( O_d \).

During the process of generation the form grinding wheel rotates about axis \( x_d \) to provide the required magnitude of velocity of cutting. However, the rotating cutting motion of the wheel does not affect the mathematical determination of the envelope (hub tooth surface) since the form grinding wheel is a surface of revolution. Therefore, the rotating cutting motion of the form grinding wheel is ignored while the envelope is derived.

A curved shape \( y_1 \)-movement dependent upon the \( z_1 \)-position is defined by a function \( y_1(z_1) \). In this case, a circular
function has been chosen for additional movements $y_i(z_i)$.

\[ y_i^{(O_i)} = z_i^{(O_i)} \cot \theta_k \]  

(8)

The hub tooth surface $\Sigma_h$ is represented in $S_h$ by the coordinate transformation in transition from $S_d$ to $S_h$ and the meshing equation between the form grinding wheel and the hub as follows

\[ r_k(u, \theta_d) = r_d(u, \theta_d, \theta_k) = M_{hk} M_{kd} r_d(u, \theta_d) \]

\[ f(u, \theta_d, \theta_k) = \left( \frac{\partial r_k(u, \theta_d, \theta_k)}{\partial u} \times \frac{\partial r_d(u, \theta_d, \theta_k)}{\partial \theta_d} \right) \cdot \frac{\partial r_d(u, \theta_d, \theta_k)}{\partial \theta_k} = 0 \]  

(9)

where $M_{hk}$ and $M_{kd}$ represent the coordinate transformation from $S_k$ to $S_h$ and $S_d$ to $S_k$, respectively.

Fig. 5 Coordinate systems applied for derivation of the hub tooth surface.

3. Meshing model and contact analysis

After derivation of the geometry model of the novel crown gear coupling, the analytical mesh model of the novel crown gear coupling transmission is developed as shown in Fig. 6.

Fig. 6 Meshing of the hub and sleeve with angular misalignment.
θ is the angular misalignment error between the hub axis and sleeve axis. TCA for the crown gear coupling (Guan et al., 2018) can be applied to simulate the meshing condition, and the main goal is to determine contact path. The first step is to represent the two mating tooth surfaces in the same coordinate system, whose coordinates and unit normal should be the same at the point of contact. Then, the kinematic characteristics of the novel crown gear coupling transmission can be analyzed by combining TCA with the mathematical model. Description of TCA for the novel crown gear coupling is omitted here.

Coordinate systems for meshing are shown in Fig. 6(a). From Fig. 6(a), it can be seen that coordinate system $S_f (O_f, x_f, y_f, z_f)$ is the global coordinate system that is fixed to the frame of the novel crown gear coupling drive; coordinate systems $S_h$ and $S_s (O_s, x_s, y_s, z_s)$ are rigidly connected to the hub and the sleeve, respectively; auxiliary coordinate system $S_\theta (O_\theta, x_\theta, y_\theta, z_\theta)$ is applied for simulation of errors of alignment; coordinate system $S_t (O_t, x_t, y_t, z_t)$ is an auxiliary fixed coordinate system. And, the misaligned hub gear performs rotation about $z_h$-axis.

To perform TCA, the homogeneous surface coordinates and the normal vector of the hub and sleeve should be transferred to the global fixed coordinate system $S_f$ and the position and normal vector can be represented by

$$ r_j^{(h)}(\phi, u, \theta) = M_{ft}M_{\theta h}M_{fs}r_j(u, \theta) $$

$$ n_j^{(h)}(\phi, u, \theta) = \frac{\partial r_j^{(h)}(\phi, u, \theta)}{\partial u} \times \frac{\partial r_j^{(h)}(\phi, u, \theta)}{\partial \theta} $$

$$ r_j^{(s)}(\phi, \theta, l_s) = M_{rs}r_j(\phi, \theta, l_s) $$

$$ n_j^{(s)}(\phi, \theta, l_s) = \frac{\partial r_j^{(s)}(\phi, \theta, l_s)}{\partial \phi_1} \times \frac{\partial r_j^{(s)}(\phi, \theta, l_s)}{\partial l_s} $$

where $M_{ft}$, $M_{\theta h}$, $M_{fs}$ and $M_{rs}$ represent the coordinate transformation from $S_t$ to $S_f$, $S_\theta$ to $S_r$, $S_h$ to $S_\theta$, and $S_s$ to $S_r$ respectively.

Fig. 7 Finite element model of crown gear coupling.

Besides, the finite element mesh model is established to conduct loaded tooth contact analysis. The full-teeth finite
element models for the hub and sleeve are shown in Fig. 7(a) and (b). The assembled finite element model for the novel crown gear coupling and the boundary conditions in ANSYS are shown in Fig. 7(c). Nodes of the rim portion (red area) of the sleeve are fully constrained. One end of the rigid elements is connected to a reference node located at the coordinate origin, and the nodes on the inner surface (yellow area) of the hub. The reference node is constrained except the rotation about the rotating axis. Besides, the torque is applied directly to the reference node.

4. Results and discussion

Using the proposed mathematical model of the hub tooth surface of the novel crown gear coupling and the analytical mesh model considering angular misalignment, the effects of the profile modification parabolic coefficient and misalignment angle on contact path obtained by TCA will be examined. Besides, contact stress distributions on the hub tooth surface obtained by LTCA of the conventional and novel crown gear couplings with the maximum misalignment angle will be compared. Table 1 presents the geometry design parameters of the novel crown gear coupling analyzed in this study.

There are so many teeth of the hub that we cannot show contact position and contact stress distributions on every hub tooth surface in the context. The hub teeth are numbered starting from the one on the top (pure pivoted area) and continuing in the clockwise direction, as shown in Fig. 6(b). Here, contact position and contact stress distributions on the hub tooth surface of every hub tooth with a 4 teeth interval, are mapped to a plane as depicted in Figs. 8, 9, 10, 11 and 12.

Table 1  Parameters of the example system.

| Parameters                              | Hub  | Sleeve |
|-----------------------------------------|------|--------|
| Number of teeth z                       | 92   | 92     |
| Module m [mm]                           | 8    | 8      |
| Tooth face width b [mm]                 | 150  | 180    |
| Normal pressure angle α [°]             | 20   | 20     |
| Addendum of tooth ha [mm]               | 6.4  | 6.4    |
| Dedendum of tooth hf [mm]               | 8.4  | 6.4    |
| Displacement circle radius Rd [mm]      | 3000 | —      |
| Tool tip radius rt [mm]                 | 3.04 | 1.2    |
| Maximum value of misalignment angle θmax [°] | 0.2  | —      |
| Parabola coefficient of profile crowned rack cutter ac | -0.0007 | — |
| Parabola vertex position of profile crowned rack cutter lc | 0 | — |

4.1 Effect of profile modification coefficient

The profile parabola coefficient ac plays an important part in determining the maximum amount of profile modification, and directly affect contact position of the crown gear coupling. Contact position travels along a path that traverses the face width of the hub tooth. Fig. 8 shows contact path on the hub tooth surface having a misalignment angle of θ = 0.2° along with different profile modification coefficients of ac = 0, -0.0001, -0.0007 and -0.0013. The other parameters are listed in table 1 above.

When there is no profile modification (i.e. ac = 0, as shown in Fig. 8(a)), contact positions of teeth #1, #45, #49 and #89 are at the tip edge of the hub tooth surface, and contact positions of teeth #5-#41 and #53-#85 are at the root edge of the hub tooth surface. It can be explained that edge contact on teeth #1, #45, #49 and #89 occur when tip edge curve of the hub meshes with the sleeve tooth flank surface, and edge contact on teeth #5-#41 and teeth #53-#85 occur when the hub tooth flank surface meshes with tip edge curve of the sleeve. In a word, edge contact happens at every contact position no matter where the tooth is, which can produce a serious load concentration resulting in high contact stress.

As the magnitude of profile crowning increases from ac = 0 to ac = -0.0013, it is observed that profile crowning modification moves contact path along the hub profile direction from the edge towards the center of the hub tooth flank effectively, but it doesn’t have a significant effect on contact path along the hub tooth width direction. When the magnitude of profile crowning reaches a certain threshold, there is no change in contact path. Therefore, increasing the
magnitude of profile crowning can avoid the edge contact occurring at the tip edge of the hub tooth surface or the sleeve tooth surface, and improve contact stress distributions of the crown gear coupling.

\[ \text{(a) } a_c = 0 \]

\[ \text{(b) } a_c = -0.0001 \]

\[ \text{(c) } a_c = -0.0007 \]

\[ \text{(d) } a_c = -0.0013 \]

Fig. 8 Contact path of the novel crown gear coupling with different profile modification coefficients.

4.2 Effect of misalignment angle

In this subsection, the effects of misalignment angle on contact path of the crown gear coupling are investigated. The introduction of misalignment into the contacting gears is to represent the assembling and manufacturing errors occurring in actual system. The angular misalignment between the hub and sleeve is denoted as \( \theta \), as shown in Fig. 5.

Figs. 9 and 10 show the effect of misalignment angle on contact path on the novel and conventional hub tooth flank at different misalignment angles of \( \theta = 0^\circ, 0.1^\circ \) and \( 0.2^\circ \), respectively. When the hub rotates without a misalignment (\( \theta = 0^\circ \) as shown in Fig. 9(a) and Fig. 10(a)), contact path on the novel hub tooth flank forms a point and contact path on the conventional hub tooth flank forms a line. Line contact on the hub tooth surface of the conventional crown gear coupling occurs because the profile shape of the sleeve is the same as the profile shape of the hub tooth in the middle plane. Neither contact path on the hub tooth surface of the novel crown gear coupling nor that of the conventional crown gear coupling is a curve because there is no relative movement between the hub and sleeve.

When the hub is misaligned (such as \( \theta = 0.1^\circ \) as shown in Fig. 9(b) and Fig. 10(b)), contact position on the hub tooth surface shifts from the middle section (tooth #1, pivoted area) to the tooth side edge (tooth #21, tilted area) and then went back to the tooth side edge (tooth #45, pivoted area) along the hub tooth width direction during the half cycle of the hub rotating. Contact positions on the hub tooth surface of the conventional crown gear coupling are all at the tip or root edge of the hub tooth surface while contact positions on the hub tooth surface of the novel crown gear coupling are all at the center of the hub tooth surface along the hub tooth profile direction. Therefore, when the hub rotates from the pure pivoted area to the pure tilted area, contact position shifts on the conventional hub tooth flank from the tip to...
the root along the hub tooth profile direction and shifts from the middle to the side edge along the hub tooth width direction simultaneously, and contact position on the hub tooth surface of the novel crown gear coupling just shifts from the side edge to the middle along the hub tooth width direction.

![Diagram](image1)

![Diagram](image2)

![Diagram](image3)

Fig. 9 Contact path of the novel crown gear coupling at different misalignment angles.

![Diagram](image4)

![Diagram](image5)

![Diagram](image6)

Fig. 10 Contact path of the conventional crown gear coupling with different angular misalignments.

It is also observed from Figs. 9 and 10 that when misalignment angle increases, contact position moves further
away from the middle both for the conventional and novel crown gear couplings. Contact position on the hub tooth surface of the conventional crown gear coupling near the pure titled area (such as teeth #21, #65) varies much more sensitivity than that of the novel crown gear coupling.

Contact stress distributions can also be used to verify the accuracy of contact path above obtained by TCA. Here a torque of $T = 1.75$ MN·m and a misalignment angle $\theta = 0^\circ$ are simultaneously applied to the novel crown gear coupling. Fig. 11 shows contact stress distributions on the hub tooth surface obtained by LTCA for the novel crown gear coupling. Two main observations can be made from Fig. 11: contact stress distributions are identical for all teeth because the relative positions of contact points on the hub tooth surface of the novel crown gear coupling for all teeth are completely coincidence, as shown in Fig. 9(a); every contact ellipse is at the center along the hub tooth profile direction, which is consistent with contact position shown in Fig. 9(a). Thus, it can be seen that contact positions obtained by TCA and LTCA are in good agreement with each other.

![Fig. 11 Contact stress distributions of the novel crown gear coupling without misalignment.](image)

That using the novel crown gear coupling can avoid edge contact was further validated by comparing contact stress distributions of the conventional crown gear coupling with those of the conventional crown gear coupling. The comparison is shown in Fig. 12. And here a torque of $T = 1.75$ MN·m and the maximum misalignment angle $\theta = 0.2^\circ$ are simultaneously applied to the crown gear couplings. From Fig. 12(a), significant edge loading is observed in the conventional crown gear coupling working with the maximum misalignment angle. It is also noted that the maximum contact stress can reach 359 MPa. After making an improvement about the conventional crown gear coupling, contact stress distributions of the novel crown gear coupling are shown in Fig. 12(b). From the figure, it can be observed that there is no edge contact and load concentration at the root of the hub tooth surface along the hub profile direction no longer exists for the novel crown gear coupling with angular misalignment. The maximum contact stress is decreased by 27.6% from 359 MPa to 260 MPa, compared with that obtained using the conventional crown gear coupling. However, from Fig. 12, it can be also seen that the contact pattern is symmetrically located at the opposite sides of the hub tooth surface and the maximum contact stress occurs at the pure titled area (teeth #21, #65) both for the novel and conventional crown gear couplings.

![Fig. 12 Contact stress distributions of the crown gear coupling with $\theta = 0.2^\circ$ misalignment.](image)

5. Conclusion

In this study, a novel crown gear coupling has been presented. Based on the analysis and results, some conclusions can be obtained as follows:
1. The meshing of the novel crown gear coupling is in point contact in the condition of perfect alignment. Relative positions of contact points on the hub tooth flank for all teeth are completely coincidence and located in the center region of tooth surfaces. Moreover, torque is shared equally across all of the teeth.

2. With the increase of the amount of profile modification, contact path of the novel crown gear coupling shifts from the tip and root edges to the middle along the hub tooth profile direction, with the existence of angular misalignment. It means that optimum profile modification helps avoid the edge contact of the hub surface and sleeve surface. When the amount of profile modification increases to a certain value, contact path stays the same and contact points remain in the middle along the hub tooth profile direction.

3. Misalignment angle has little influence on contact positions along the hub tooth profile direction both of the novel and conventional crown gear couplings. When misalignment angle increases, contact points moves further away from the middle along the hub tooth width direction both for the novel and conventional crown gear couplings. And, contact position of the conventional crown gear coupling near the pure titled area varies much more sensitivity than that of the novel one.

4. For a crown gear coupling with misalignment, loaded tooth contact analysis based on FEM confirms the reduction of maximum contact stress of the novel crown gear coupling comparison with the conventional one and edge contact can be avoided by profile modification. And, the contact pattern is symmetrically located at the opposite sides of the coupling and the maximum contact stress occurs at the pure titled area both for the novel and conventional crown gear couplings.

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