Developing geogebra-assisted reciprocal teaching strategy to improve junior high school students’ abstraction ability, lateral thinking and mathematical persistence

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Abstract. The development of science and technology requires reform in the utilization of various resources for mathematics teaching and learning process. One of the efforts that can be made is the implementation of GeoGebra-assisted Reciprocal Teaching strategy in mathematics instruction as an effective strategy in improving students’ cognitive, affective, and psychomotor abilities. This research is intended to implement GeoGebra-assisted Reciprocal Teaching strategy in improving abstraction ability, lateral thinking, and mathematical persistence of junior high school students. It employed quasi-experimental method with non-random pre-test and post-test control design. More specifically, it used the 2x3 factorial design, namely the learning factors that included GeoGebra-assisted Reciprocal Teaching and conventional teaching learning, and levels of early mathematical ability (high, middle, and low). The subjects in this research were the eighth grade students of junior high school, taken with purposive sampling. The results of this research show: Abstraction and lateral abilities of students who were taught with GeoGebra-assisted Reciprocal Teaching strategy were significantly higher than those of students who received conventional learning. Mathematical persistence of students taught with GeoGebra-assisted Reciprocal Teaching strategy was also significantly higher than of those taught with conventional learning.

1. Introduction
The Regulation of Minister of Education and Culture No. 64 Year 2013 states that one of the competencies expected to be acquired by junior high school students in learning mathematics is the ability to show logical, critical, analytical, creative, careful and accurate, responsible, responsive, and persistent attitudes in solving problems. The expected outcome of mathematics education is that students have creative, critical, scientific thinking, honest, discipline, and diligent personalities [1].

Thinking ability is a major factor in living every aspect of life [2]. Intelligent thinking is one of the main reasons why humans can survive [3]. The thinking style associated with intelligence is lateral thinking. This is as stated by [4] that intelligence is positively correlated with lateral thinking. [5] said that the thinking style used in solving problems is closely related to lateral and vertical thinking styles.

Lateral thinking ability will hone an individual’s creativity to overcome anything [6]. The problem of lateral thinking is the rich diversity of thoughts [7]. Ideally, students are not only presented with
closed-ended problems but also problems that have non-singular solutions and answers. Providing open-ended problems can give students the opportunity to gain new insights into their knowledge. In addition, with open-ended questions teachers have the opportunity to help students understand and elaborate mathematical ideas with the greatest breadth and depth possible. Midwest Consortium for Mathematics and Science Education [8] argued that too much emphasis on mathematical mechanics and procedural mathematics will inhibit meaningful learning.

Abstraction ability must be of particular concern, and abstraction can be called a process and an outcome of the process [9]. A student proficient in mathematics must have abstraction ability [10]. Abstraction in mathematics developed very rapidly in the mid-twentieth century [11]. With abstraction, mathematical formalism becomes a kind of necessity (requirement) in every mathematical development [12]. According to [13], one of the greatest strengths and beauties in mathematics is abstraction. Abstraction is a fundamental process in mathematics [14]. Indeed, the essence of mathematics is abstraction and abstracting concept. The same argument is expressed by [15], stating that abstraction is an important issue in mathematics education.

The kind of teaching and learning that can give the opportunity of freedom of thinking to learners so that they can use strategies appropriate to their knowledge and take an active role, with the teacher only acting as a facilitator, and with the opportunity for classroom discussion is Reciprocal Teaching strategy. Reciprocal Teaching is a learning activity that includes reading the compiled instructional materials, followed by students summarizing the materials, making inquiries, clarifying, and making predictions [16]. Teaching and learning through Reciprocal Teaching is expected to motivate students’ active participation in the classroom with the target of improving their lateral thinking ability. In addition to improvement in the ability of mathematical lateral thinking, mathematical persistence is expected to increase. Persistence is the tenacity that gives more emphasis on the positive side, namely the mental attitude to cultivate and encourage enthusiasm, optimism, and belief and perseverance in dealing with a problem [17].

Computer technology also enables students to learn mathematics more easily and allows for more development in mathematics learning. Kusumah [18] emphasized that high-level concepts and skills that are linked to each other are difficult to teach through books alone, for books have limitations.

Teaching and learning with GeoGebra-assisted Reciprocal Teaching strategy is an instructional approach optimally involving students, enabling them to do investigation and improve their abstraction ability, lateral thinking, and persistence by integrating thinking skills and conceptual understanding. This is in line with Wees’ opinion [19] that GeoGebra allows students to be active in building an understanding of geometry. Collen and Steven [20] added that students use computers every day to improve their basic mathematical skills and to develop effective problem-solving strategies.

2. Methods
The research employed a quasi-experimental method with non-random pretest-posttest control group design. More specifically, it used the 2x3 factorial designs, that is, two learning factors consisting of GeoGebra-assisted Reciprocal Teaching and conventional learning, and three levels of early mathematical ability (high, middle, and low). The population of this study included all eighth grade students in one junior high school in Bandung, while the sample consisted of eight grade students in two classes in the school, taken by simple random sampling, assigned to one experimental class and one control class. The instruments of this research consisted of mathematical abstraction test, lateral thinking test, mathematical persistence questionnaire, and observation sheet.

3. Results and Discussion

3.1. Abstraction Ability
Data of abstraction ability were obtained through pretest and posttest. The pretest and posttest scores were then calculated to obtain the normalized gain (N-gain) of mathematical abstraction ability scores in the experimental class that was treated with GeoGebra-assisted Reciprocal Teaching strategy and the
control class that was taught with conventional learning. Result of difference test for N-gain average scores of students’ abstraction ability for the total students was finally obtained.

Table 1. Result of Difference Test of N-Gain Average Scores for Abstraction Ability of the Whole Students

| T       | Sig. (2-tailed) | H₀   |
|---------|-----------------|------|
| 3.366   | 0.000           | Rejected |

Table 1 shows that the probability value (sig.) of 0.000 is smaller than 0.05 or p (sig) < 0.05, so that H₀ is rejected. It indicates that the improvement in the abstraction ability of students who were taught with GeoGebra-assisted Reciprocal Teaching model was significantly greater compared to that of students who received conventional learning based on N-Gain average scores of the whole students’ abstraction ability.

Subsequently, a significant difference test of the N-Gain average scores of abstraction ability based on early mathematical ability (EMA) was carried out using two-way ANOVA test, whose results are presented in Table 2.

Table 2. Results of two-way Anova of the N-Gain Average Scores for Abstraction Ability based on EMA

|            | df | F      | Sig. | H₀     |
|------------|----|--------|------|--------|
| Between Groups | 2  | 10.029 | 0.000| Rejected |
| Within Groups   | 32 |        |      |        |
| Total         | 34 |        |      |        |

Table 2 indicates that the significance value is smaller than α, i.e. 0.000 < α = 0.05, so H₀ is rejected, meaning there was a significant difference in the improved abstraction ability based on EMA groups (high, medium, and low) among students who were taught with GeoGebra-assisted Reciprocal Teaching. Because the result of ANOVA test indicates a difference, in order to know which EMA group is significantly different, a follow-up test (Post Hoc) was conducted, using Tukey’s test. Recapitulation of further test of N-gain scores of students’ abstraction ability based on EMA can be seen in Table 3.

Table 3. Results of Post Hoc Test of the N-Gain Scores for Abstraction Ability based on EMA

| (I) EMA  | (J) EMA  | Mean Difference (I-J) | Sig.   | H₀    |
|----------|----------|----------------------|--------|-------|
| High     | Medium   | 0.12784              | 0.002  | Rejected |
| Low      | Medium   | 0.14783              | 0.001  | Rejected |
| Medium   | High     | -0.12784             | 0.002  | Rejected |
| Medium   | Low      | 0.01999              | 0.822  | Accepted |
| Low      | Medium   | -0.14783             | 0.001  | Rejected |
|          | Medium   | -0.01999             | 0.822  | Accepted |

Table 3 shows results of Post Hoc Test using Tukey’s test, where the significance value for the high and medium EMA groups is 0.02 < α, for the high and low EMA groups 0.01 < α, and for the medium and low EMA groups 0.822 > 0.05 = α. The results mean there was a difference in the improved abstraction ability between the high and low EMA groups. With regard to N-gain average, the highest improvement in abstraction ability was gained by the high EMA group students. Similarly, there was a difference in the improvement of the high and low EMA groups. In terms of N-gain average, the greatest improvement in abstraction ability was obtained by the high EMA students. On the other hand, there was no difference in abstraction ability improvement between the medium and low EMA groups.

3.2. Lateral Thinking Ability
Data of mathematical lateral thinking ability were obtained through pretest and posttest. The pretest and posttest scores were then calculated to obtain the normalized gain (N-gain) scores of mathematical lateral thinking ability of the experimental class that was treated with GeoGebra-assisted Reciprocal Teaching strategy and the control class that was taught with conventional learning. Then, the result of difference test of N-gain average score for Abstraction Ability for the total students was finally obtained.

Table 4. Result of Difference Test of N-Gain Average Scores for Lateral Thinking Ability of the Whole Students

| T     | Sig. (2-tailed) | H₀       |
|-------|-----------------|----------|
| 5.267 | 0.000           | Rejected |

Table 4 informs that the probability value (sig.) of 0.000 is smaller than 0.05 or p (sig) <0.05, so that H₀ is rejected. It means that the lateral thinking ability improvement of students who were treated with GeoGebra-assisted Reciprocal Teaching model was significantly higher than that of students who were taught with conventional learning based on the N-Gain average scores of overall students’ lateral thinking ability.

Subsequently, a significant difference test of N-Gain average scores of lateral thinking ability based on early mathematical ability (EMA) using two-way ANOVA test was conducted, the results of which are presented in table 5.

Table 5. Results of Two-Way Anova of N-Gain Scores for Lateral Thinking Ability Based on EMA

|                  | df | F     | Sig. | H₀       |
|------------------|----|-------|------|----------|
| Between Groups   | 2  | 6.057 | 0.006| Rejected |
| Within Groups    | 32 |       |      |          |
| Total            | 34 |       |      |          |

Table 5 shows that the sig. value is smaller than α, that is 0.006 <α = 0.05, which means that there was a significant difference in the improved lateral thinking among EMA groups (high, medium, and low) of students who received learning with the GeoGebra-assisted Reciprocal Teaching strategy. Because the result of ANOVA test shows there was a difference, so to know which EMA group was significantly different, a follow-up test (Post Hoc), using Tukey’s test, was conducted. Recapitulation of further test for N-gain scores of students’ lateral thinking ability based on EMA can be seen in table 6.

Table 6. Result of Post Hoc Test of N-Gain scores for Lateral Thinking Ability Based on EMA

| (I) EMA | (J) EMA | Mean Difference (I-J) | Sig. | H₀       |
|---------|---------|-----------------------|------|----------|
| High    | Medium  | -0.04933              | 0.273| Accepted |
|         | Low     | 0.11900               | 0.004| Rejected |
|         | High    | 0.04933               | 0.273| Accepted |
| Medium  | Medium  | -0.06967              | 0.083| Accepted |
|         | High    | 0.11900               | 0.004| Rejected |
|         | Medium  | 0.06967               | 0.083| Accepted |

Table 6 presents results of Post Hoc Test using Tukey’s test, with significance value for high and middle EMA groups = 0.273 > α, meaning that there was no difference in the improved lateral thinking ability between the two groups. For the high and low EMA groups, the significance value is 0.004 < α, which means that there was a difference in the improved lateral thinking ability between the high and low EMA groups. In terms of the N-gain average, the highest improvement in mathematical lateral thinking ability was obtained by the high EMA group students. For the middle and low EMA groups,
the significance value obtained $= 0.083 > \alpha$, meaning that there was no difference in the improvement of mathematical lateral thinking ability between medium and low EMA groups.

### 3.3. Mathematical Persistence

The analysis of students’ mathematical persistence is aimed to determine students’ attitudes about the relevance and appreciation of mathematics, i.e. the tendency to think and act positively. The attitude scale model used was Likert scale. The student mathematical persistence scale was given to students after posttest implementation. The scale used in this research consists of 4 alternative answers. The score for each statement is 1 if student chooses Strongly Disagree (SD), 2 if student chooses Disagree (D), 3 if student chooses Agree (A), and 4 if student selects Strongly Agree (SA) for positive statements; on the other hand, the score is 4 if student chooses Strongly Disagree (SD), 3 if student chooses Disagree (D), 2 if student chooses Agree (A), and 1 if student chooses Strongly Agree (SA) for negative statements.

Data of students’ mathematical persistence were ordinal, so first they were converted into interval data. The process of converting ordinal data into interval data is known as MSI (Method of Successive Interval). For the analysis of students’ scores on the mathematical persistence scale, a difference test was conducted.

The result of the difference test of the students’ mathematical persistence scores using the Mann-Whitney non-parametric test are presented in table 7.

**Table 7. Results of Non-Parametric Mann-Whitney Test of Students’ Mathematical Persistence Average Scores**

| Null Hypothesis | Test | Sig. | Decision |
|-----------------|------|------|----------|
| The distribution of VAR00001 is the same across categories of VAR00002 | Independent Samples Mann-Whitney U Test | 0.000 | Reject the null hypothesis |

Table 7 presents results of Mann-Whitney non-parametric test. Based on the test, a significance value of 0.000 was obtained, which is smaller than the level of significance of 0.05. Thus, there was a significant difference in the mathematical persistence between students who were taught with GeoGebra-assisted Reciprocal Teaching strategy and students who received conventional learning.

### 4. Conclusion

The improvement of abstraction ability and lateral thinking of students who were taught with GeoGebra-assisted Reciprocal Teaching strategy was significantly greater than that of students who received conventional learning. There was a difference in the abstraction ability among groups of different early mathematical ability levels (high, medium, and low) who were treated with the GeoGebra-assisted Reciprocal Teaching strategy. The abstraction ability of the high EMA group students was significantly greater than that of the medium and low EMA group students, while the ability in the middle and low EMA groups did not differ significantly. There was a difference in the lateral thinking ability among the three groups of early mathematical ability (high, moderate, and low) who were taught with the GeoGebra-assisted Reciprocal Teaching strategy. The lateral thinking ability of middle EMA group students was not significantly different from that of the high and low EMA groups, whereas the high EMA group’s lateral thinking ability was significantly higher than that of the low EMA group. Finally, the mathematical persistence of students who were treated with GeoGebra-assisted Reciprocal Teaching strategy was significantly higher than that of the students who received conventional learning.

### 5. References

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