In this work we propose a new gravitational setup formulated in terms of two interacting vierbein fields. The theory is the fully diffeomorphism and local Lorentz invariant extension of a previous construction which involved a fixed reference vierbein. Certain vierbein components can be shifted by local Lorentz transformations and do not enter the associated metric tensors. We parameterize these components by an antisymmetric tensor field and give them a kinetic term in the action, thereby promoting them to dynamical variables. In addition, the action contains two Einstein-Hilbert terms and an interaction potential whose form is inspired by ghost-free massive gravity and bimetric theory. The resulting theory describes the interactions of a massless spin-2, a massive spin-2 and an antisymmetric tensor field. It can be generalized to the case of multiple massive spin-2 fields and multiple antisymmetric tensor fields. The absence of additional and potentially pathological degrees of freedom is verified in an ADM analysis. However, the antisymmetric tensor fluctuation around the maximally symmetric background solution has a tachyonic mass pole.

1. Introduction

The construction of classically consistent field theories is an on-going challenge which becomes disproportionately more complicated with increasing spin of the involved fields. The set of consistent interactions for fields up to spin-1 in flat space is reasonably well-understood but, when including gravity, the non-linearities of gravitational interactions and the spin-2 nature of the gravitational field always introduce further complexity. For a recent review on the programme of building new field theories in the presence of gravity, see [1].

A well-known example for a general class of consistent field theories including gravity is the Horndeski action, which contains the most general scalar (i.e. spin-0) interactions with second-order equations of motion [2]. The action has been generalized to the ‘beyond Horndeski’ class, which, despite its higher-order equations, does not give rise to Ostrogradski instabilities that would threaten the theory’s consistency [3, 4]. A recent further generalization of the consistent setup is the so-called ‘DHOST’ (degenerate higher-order scalar-tensor) theory [5, 6]. Examples for nonlinear vector (i.e. spin-1) interactions whose particular structure is chosen to avoid instabilities are standard Yang-Mills actions for massless fields [7], and the more recently constructed generalized Proca actions for a self-interacting massive field [8, 9].

In the spin-2 case, the Einstein–Hilbert action for general relativity (GR) delivers the nonlinear self-interactions for the massless field. The mass term that can be added to this theory has a very particular structure which is fixed by the absence of the Boulware-Deser ghost instability [10]. The nonlinear theory for massive gravity was constructed and shown to be ghost-free only a few years ago [11–13]. Formulating the action requires the introduction of a reference or fiducial metric tensor. This second metric can be promoted to a dynamical field, resulting in a classically consistent bimetric theory, which describes the nonlinear interactions of a massless and a massive spin-2 field [14]. The setup of consistent spin-2 interactions can also be generalized to the case of multiple massive fields [15]. For reviews on massive gravity and bimetric theory, see [16, 17] and [18], respectively.
The structure of the allowed massive spin-2 interactions assumes a remarkably simple form when written in terms of the vierbein fields related to the two metrics [15]. Moreover, the most general set of consistent interactions among multiple massive spin-2 fields actually requires the vierbein formulation [19]. This is not particularly astonishing since there is an example for this situation already in GR, where couplings to fermionic fields can only be expressed using the vierbein field.

The vierbein \( e^a_{\mu} \), related to the corresponding metric via \( g_{\mu
u} = e^a_{\mu} \eta_{ab} e^b_{\nu} \), has 16 independent components. Only 10 of these show up in the metric tensor since the latter is invariant under local Lorentz transformations, \( e^a_{\mu} \rightarrow N^a_{\mu} e^a_{\nu} \) with \( N^a_{\mu} \eta_{ab} N^b_{\nu} = \eta_{ab} \). In GR, the remaining 6 components are pure gauge and hence unphysical. Massive gravity and bimetric theory in vierbein formulation a priori contain these Lorentz components as dynamical fields, but they are required to vanish by the equations of motion. This property ensures the existence of an equivalent formulation in terms of metric tensors [20]. Moreover, as was shown in [21], the vanishing of the 6 additional Lorentz components is crucial for the consistency of the theory. It is also related to the possibility of having causal propagation [22].

Interestingly, the consistency problem of non-vanishing Lorentz components can be overcome by giving them a kinetic term. This was demonstrated explicitly in [28] for the case of a fixed reference frame field \( \tilde{e}^a_{\mu} \) (i.e., for the massive gravity case). The resulting theory describes the nonlinear interactions of a massive spin-2 with a massive antisymmetric tensor field. No additional degrees of freedom, which could potentially give rise to instabilities, enter at the nonlinear level. The structure of the resulting action is interesting because it involves the antisymmetric tensor combination \( B_{\mu\nu} = \tilde{e}^a_{\mu} \eta_{ab} \tilde{e}^b_{\nu} - \tilde{e}^a_{\nu} \eta_{ab} \tilde{e}^b_{\mu} \). The mass pole of its fluctuation around maximally symmetric backgrounds is tachyonic, indicating the instability of this vacuum solution.

Antisymmetric tensor fields, first considered in [31, 32], are objects of interests in supergravity theories, and thus in low-energy effective descriptions of string theory. Together with the graviton and the dilaton, they make up the massless bosonic excitations of the string. In \( D = 4 \) spacetime dimensions, antisymmetric tensors are dual to a scalar in the massless and to a vector in the massive case (see e.g., [33]), but in higher dimensions the duality does not necessarily relate them to lower-spin fields.

In this work we will build on the results of [28] and extend the setup by giving dynamics to the frame field \( \tilde{e}^a_{\mu} \).

1.1. Summary of results

We demonstrate that the action proposed in [28] can be generalized to a fully dynamical theory for two interacting vierbein fields \( e^a_{\mu} \) and \( \tilde{e}^a_{\mu} \). The result is a ghost-free bimetric action in vierbein formulation with dynamical Lorentz components which we parameterize in terms of the antisymmetric components \( B_{\mu\nu} = e^a_{\mu} \eta_{ab} \tilde{e}^b_{\nu} - \tilde{e}^a_{\nu} \eta_{ab} e^b_{\mu} \). The action is manifestly invariant under local Lorentz transformations and diffeomorphisms. The number of propagating degrees of freedom in this new setup is \( 2 + 5 + 3 \), corresponding to a massless spin-2, a massive spin-2 and a massive antisymmetric tensor. This is verified both at the linear and at the fully nonlinear level. The mass of the antisymmetric fluctuation around the maximally symmetric background is again tachyonic, implying that the bimetric vacuum of the extended theory is unstable. We then show how to further generalize the setup to the case of \( N \) interacting vierbeine and their independent antisymmetric components which are packaged into \( (N - 1) \) antisymmetric tensor fields.

1.2. Conventions

We work with metric signature \((- - + + +)\) in 4 spacetime dimensions for definiteness, but all of our results generalize to arbitrary dimension. Spacetime indices are denoted by Greek letters \( \mu, \nu \), Lorentz indices by Latin letters \( a, b \). Indices are raised and lowered by \( g_{\mu\nu} \) and the inverse \( g^{\mu\nu} \) on its curvatures and on objects related to the antisymmetric tensor. Indices on curvatures of \( f_{\mu\nu} \) are raised and lowered with \( f_{\mu\nu} \) and its inverse \( f^{\mu\nu} \). Lorentz indices are raised and lowered with \( \eta_{ab} \) and its inverse \( \eta^{ab} \). Brackets denoting symmetrization and antisymmetrization of indices are defined as \( T_{\mu\nu} = T_{(\mu\nu)} + T_{(\nu\mu)} \) with \( T_{(\mu\nu)} = \frac{1}{2}(T_{\mu\nu} - T_{\nu\mu}) \) and \( T_{[\mu\nu]} = \frac{1}{2}(T_{\mu\nu} + T_{\nu\mu}) \).

2. Review of gravity with antisymmetric components

Here we briefly review the results of [28], discussing first the massless case before adding the mass term.

---

1. In this paper we will not address the issue of causality. The term ‘consistent’ refers to the absence of ghost instabilities. For work on causal propagation in massive spin-2 theories in the framework of scattering amplitudes see [23, 24, 25–27].

2. The idea of making the Lorentz components dynamical in massive gravity was first mentioned in [29]. At the linearized level, their dynamics are discussed in the context of teleparallel theories in section 4.6 of [30].
2.1. Massless fields

The action for a massless antisymmetric tensor field $B_{\mu \nu}$, minimally coupled to a massless metric $g_{\mu \nu}$, is,

$$ S_{eB} = m_g^2 \int d^4x \sqrt{|g|} \left[ R(g) - 2\Lambda \right] - \frac{m_B^2}{2 \cdot 3!} \int d^4x \sqrt{|g|} \ H_{\mu \nu \rho} H^{\mu \nu \rho}, $$

(2.1)

where $H_{\mu \nu \rho} = 3\nabla_{(\mu} B_{\nu \rho)}$ is the 2-form field strength. We have included a Planck mass $m_g$ for $g_{\mu \nu}$ and also a mass scale $m_B$ for $B_{\mu \nu}$, such that both tensor fields are dimensionless. In four spacetime dimensions the massless antisymmetric tensor propagates one physical mode and the massless metric propagates two physical modes.

In [28], we repackaged the $10 + 6$ components contained in the tensor fields into the vierbein $e^a_{\mu}$. This was achieved by making the following identifications,

$$ g_{\mu \nu} \equiv e^a_{\mu} \eta_{ab} e^b_{\nu}, $$

(2.2a)

$$ B_{\mu \nu} \equiv e^a_{\mu} \eta_{ab} \tilde{e}^b_{\nu} - e^a_{\nu} \eta_{ab} e^b_{\mu}, $$

(2.2b)

where the auxiliary vierbein $\tilde{e}^a_{\mu}$ defines a fixed reference frame. For instance, one could take $\tilde{e}^a_{\mu} = \delta^a_{\mu}$. The symmetric field $g_{\mu \nu}$ is the spacetime metric with ordinary reference to the dynamical tetrad. It is invariant under local Lorentz transformations $e^a_{\mu} \rightarrow N^a_b e^b_{\mu}$ with $N^a_b N^d_e = \eta_{bd}$ and therefore depends on only 10 of the 16 components in $e^a_{\mu}$. The remaining 6 components enter the antisymmetric tensor $B_{\mu \nu}$.

The equations of motion for the vierbein following from the above action read,

$$ \mathcal{E}_a^{\mu} \equiv \delta S_{eB}^{\mu} / \delta e^a_{\mu} = 2\eta_{ab} e^b_{\nu} \mathcal{G}^{\nu \mu} + 2\eta_{ab} \tilde{e}^b_{\nu} \mathcal{B}^{\nu \mu} = 0. $$

(2.3)

Here we have defined,

$$ \mathcal{G}^{\nu \mu} = \mathcal{G}^+^{\nu \mu} \equiv R^{\nu \mu} - \frac{1}{2} (R - 2\Lambda) g^{\nu \mu} - m_g^2 4m_g^2 (H^{\nu \rho \sigma} H_{\rho \sigma} - \frac{1}{6} H^2 g^{\nu \mu}), $$

(2.4a)

$$ \mathcal{B}^{\nu \mu} = \mathcal{B}^+^{\nu \mu} \equiv -m_B^2 \nabla_\rho H^{\rho \nu \mu}, $$

(2.4b)

which correspond to the variations of the action with respect to the tensor fields. In the tensor formulation, $\mathcal{G}^{\nu \mu}$ and $\mathcal{B}^{\nu \mu}$ vanish separately. In fact, this is also the case in the vierbein formulation, as can be seen by looking at the antisymmetric combination of equations, $2\eta^{ab} e^a_{\mu} \mathcal{E}_c^{\mu} = 0$, which implies $\mathcal{B}^{\nu \mu} = 0$. Hence the vierbein and tensor formulations of the massless theory are equivalent.

2.2. Massive fields

Inspired by ghost-free massive gravity [11–13, 15, 28] added the following interaction term $S_V$ for the vierbein $e^a_{\mu}$ to the massless action,

$$ -m_g^2 m_B^2 \int e_{abcd} (b_1 e^a \wedge e^b \wedge e^c \wedge \tilde{e}^d) \equiv \nabla_a e^b \wedge e^c \wedge \tilde{e}^d + b_2 e^a \wedge e^b \wedge \tilde{e}^c \wedge \tilde{e}^d + b_3 e^a \wedge \tilde{e}^b \wedge \tilde{e}^c \wedge \tilde{e}^d. $$

(2.5)

It was shown in a nonlinear ADM analysis that these interactions make both the fields $g_{\mu \nu}$ and $B_{\mu \nu}$ massive. The action with this mass term propagates 5 + 3 degrees of freedom, corresponding to a massive spin-2 and a massive antisymmetric tensor (which is dual to a massive vector in $D = 4$). The latter is a tachyon at the linearized level. No additional degrees of freedom leading to ghost instabilities appear.

Denoting the variation of the mass term by $\mathcal{V}_a^{\mu} \equiv -\frac{1}{m_g^2 m_B^2} \nabla_c e^d \delta g_{cd}^{\mu} / \delta e^a_{\mu}$, the vierbein equations of motion now assume the form,

$$ \mathcal{E}_a^{\mu} = 2\eta_{ab} e^b_{\nu} \mathcal{G}^{\nu \mu} + 2\eta_{ab} \tilde{e}^b_{\nu} \mathcal{B}^{\nu \mu} + \mathcal{V}_a^{\mu} = 0. $$

(2.6)

These equations can still be separated into a symmetric and an antisymmetric part which read,

$$ \mathcal{B}^{\nu \mu} = \left( \tilde{P}^{-1} \right)^{a b} e^a_{\mu} e^b_{\nu} \mathcal{G}_{\rho \sigma}^{\rho \sigma} + \mathcal{V}_a^{\mu} = 0, $$

(2.7a)

$$ \mathcal{G}^{\nu \mu} = \left( \tilde{P}^{-1} \right)^{a b} e^a_{\mu} e^b_{\nu} \mathcal{G}_{\rho \sigma}^{\rho \sigma} + \frac{1}{2} e^a_{\mu} \eta_{ab} \mathcal{V}_a^{\nu} = 0. $$

(2.7b)

Here, $\tilde{P}$ denotes the inverse of the operator $P_{ab}^{\mu \nu} \equiv 2e^a_{[\mu} e^b_{\nu]}$ which is invertible on the space of antisymmetric matrices.

Note that our convention here slightly differs from the one in [28], where $B_{\mu \nu}$ had mass dimension 1.
3. Dynamical reference frame

The vierbein action with potential (2.5) explicitly breaks diffeomorphism and local Lorentz invariance, due to the presence of the fixed reference frame $\tilde{e}_{\mu}^a$. For various reasons it is desirable to restore these symmetries, which can be achieved by introducing dynamics for the reference vierbein $e_\mu^a$, as we shall do in the following.

3.1. Action and equations

The reference vierbein defines a second metric tensor, $f_{\mu\nu} = g^{ab} e_\mu^a e_\nu^b$. We can make it dynamical by augmenting the massive action by an Einstein-Hilbert term for $f_{\mu\nu}$. The full theory thus reads,

$$S_m = m_f^2 \int d^4x \sqrt{g} (R(g) - 2\Lambda) + m_f^2 \int d^4x \sqrt{f} (R(f) - 2\tilde{\Lambda}) - \frac{m_f^2}{2} \int d^4x \sqrt{g} H_{\mu\nu\rho} H^{\mu\nu\rho}$$

$$- m_f^2 m^2 \int d^4x \epsilon_{abcd} (b_1 e^a \wedge e^b \wedge e^c \wedge \tilde{e}^d + b_2 e^a \wedge e^b \wedge \tilde{e}^c \wedge \tilde{e}^d + b_3 e^a \wedge \tilde{e}^b \wedge \tilde{e}^c \wedge \tilde{e}^d).$$

For $B_{\mu\nu} = 0$, it reduces to ghost-free bimetric theory in vierbein formulation [14, 15]. For $B_{\mu\nu} \neq 0$ it is not obvious that the kinetic term for the antisymmetric components does not re-introduce the Boulware-Deser ghost. It is not obvious either that the dynamics for $e_\mu^a$ do not destroy the consistency of the model with fixed reference frame. In appendix we perform a $3 + 1$ split of the fields and explicitly show that the number of propagating degrees of freedom is $2 \alpha^2 + 3$, corresponding to a massless spin-2, a massive spin-2 and a massive antisymmetric field. The Boulware-Deser ghost is removed by a constraint, just like in ghost-free bimetric theory.

The action could in principle contain other ghosts, hidden in the kinetic terms for $g_{\mu\nu}$, $f_{\mu\nu}$ and $B_{\mu\nu}$. In the following we show that the equations of motion can again be separated in a way that preserves the kinetic structures with respect to the massless theory. This implies the absence of kinetic mixing introduced by the mass term, which is promising for the consistency of the theory.

Defining $\tilde{\gamma}_\mu^a \equiv -\frac{1}{m_f^2 \det \tilde{e}} \delta_{\mu}^a$ and using again $\gamma_\mu^a = -\frac{1}{m_f^2 \det \epsilon} \delta_{\mu}^a$, the equations of motions for $e_\mu^a$ and $\tilde{e}_\mu^a$, respectively, read,

$$E_\mu^a = 2\eta_{ab} \bar{e}_b \mathcal{G}^{\mu\nu} + 2\eta_{ab} \bar{e}_b \mathcal{B}^{\mu\nu} + \gamma_\mu^a = 0,$n

$$\tilde{E}_\mu^a = 2\eta_{ab} \bar{e}_b \mathcal{F}^{\mu\nu} - \frac{1}{\alpha^2} \frac{1}{\det \tilde{e}} \epsilon_{abcd} \bar{e}_b \mathcal{B}^{\mu\nu} + \tilde{\gamma}_\mu^a = 0.$$ (3.3a)

(3.3b)

Here, in addition to (2.4) we have used the definitions

$$\mathcal{F}^{\mu\nu} = \mathcal{F}^{\nu\mu} \equiv R^{\mu\nu}(f) - \frac{1}{2} (R(f) - 2\tilde{\Lambda}) f^{\mu\nu},$$

and,

$$\alpha \equiv \frac{m_f}{m_g}.$$ (3.5)

Using exactly the same arguments as in the case with non-dynamical $\bar{e}_\mu^a$, it is easy to show that either of the antisymmetric combinations of equations,

$$2\eta^{ab} e_\mu^a E_\mu^b = 0, \quad 2\eta^{ab} [e_\mu^a, \tilde{e}_\mu^b] = 0,$$

implies,

$$\mathcal{B}^{\mu\nu} - (\tilde{P}^{-1})_{\mu b}^{\alpha b} \bar{e}_a \mathcal{B}^{\nu d} \mathcal{Y}_d^\alpha = 0.$$ (3.6)

The fact that the two equations in (3.6) are equivalent is a direct consequence of the invariance of the action under diagonal local Lorentz transformations, which we shall discuss below. Plugging the expressions for $\mathcal{B}^{\mu\nu}$ back into the full equations, we obtain,

$$\mathcal{G}^{\mu\nu} + (\tilde{P}^{-1})_{\mu b}^{\alpha b} \bar{e}_a \mathcal{B}^{\nu d} \mathcal{Y}_d^\alpha = 0,$$ (3.8a)

$$\mathcal{F}^{\mu\nu} + (\tilde{P}^{-1})_{\mu b}^{\alpha b} \bar{e}_a \mathcal{B}^{\nu d} \mathcal{Y}_d^\alpha = 0.$$ (3.8b)

where $\tilde{P}^{\mu\nu}_{ab} \equiv 2 \epsilon^{\mu\nu}_{[a} \tilde{e}^{b]}$ is the same invertible operator as before.

4 On curvatures of the metric $f_{\mu\nu}$, we raise indices with the inverse metric $f^{\mu\nu}$. 

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The vierbein equations in (3.3) thus separate into one set of antisymmetric components, corresponding to either of the two equivalent expressions in (3.6), and two sets of symmetric components in (3.8). The kinetic structures are exactly those of the massless theory.

### 3.2. Local symmetries

The action in (3.2) is invariant under the following symmetry transformations.

- **Local Lorentz transformations** which infinitesimally transform the vierbeine as,
  \[ \Delta \omega^\alpha_{\mu} = \eta^{ab}_{\mu} \omega^c_{\nu} \epsilon^\nu_{\rho} \]  \[ \Delta \omega^c_{\rho} = \eta^{ab}_{\rho} \omega^c_{\nu} \epsilon^\nu_{\mu}, \]  \[ (3.9) \]
  with \( \omega^c_{\nu} = -\omega^c_{\nu} \). The transformation is diagonal since the gauge parameters \( \omega^c_{\nu} \) are the same for both fields. This is an obvious symmetry: In both metrics as well as in the antisymmetric tensor all Lorentz indices are contracted with the invariant tensor \( \eta^{ab}_{\mu} \) while in the interaction potential they are contracted with the invariant tensor \( \epsilon_{abcd} \).

- **Diffeomorphisms** under which the vierbeine infinitesimally transform as one-forms. These are generated by the Lie derivative,
  \[ \Delta \xi e^a_{\mu} = \xi^\nu \partial_{\nu} e^a_{\mu} + e^a_{\rho} \nabla_{\rho} \xi^\mu, \]
  \[ \Delta \xi e^a_{\nu} = \xi^\nu \partial_{\nu} e^a_{\mu} + e^a_{\rho} \nabla_{\rho} \xi^\mu, \]
  \[ (3.10) \]
  These transformations correspond to the diagonal subgroup of the \( \text{diff} \times \text{diff} \) symmetry which is broken by the mass term and the kinetic term for \( B_{\mu\nu} \). In fact the covariant derivatives in the transformations can be taken to be with respect to either metric since the Christoffel symbols of the two terms cancel each other out,
  \[ \Delta \xi e^a_{\mu} = \xi^\nu \partial_{\nu} e^a_{\mu} + e^a_{\rho} \nabla_{\rho} \xi^\mu, \]
  \[ \Delta \xi e^a_{\nu} = \xi^\nu \partial_{\nu} e^a_{\mu} + e^a_{\rho} \nabla_{\rho} \xi^\mu, \]
  \[ (3.11) \]
  where we have used \( \tilde{\Gamma}_{\mu\nu}^\sigma = \Gamma_{\mu\nu}^\sigma \). Hence we can also write the transformation of \( e^a_{\mu} \) as,
  \[ \Delta \xi e^a_{\mu} = \xi^\nu \partial_{\nu} e^a_{\mu} + e^a_{\rho} \nabla_{\rho} \xi^\mu, \]
  \[ (3.12) \]
  which is the proper transformation of a vector under diffeomorphisms of the metric \( g_{\mu\nu} \) compatible with \( \nabla^\nabla \). It then follows that the combination \( e^a_{\mu} \eta^c_{\nu} \tilde{e}^b_{\nu} \) as well as its symmetric and antisymmetric parts transform as tensors under the diagonal diffeomorphisms. Thus we have the desired transformation property of \( B_{\mu\nu} \).

- **Linear theory**

  We will now derive the spectrum of linear perturbations around maximally symmetric backgrounds. These solutions are obtained by making the ansatz \( e^a_{\mu} = c e^a_{\mu} \), for which the equations reduce to,
  \[ B_{\mu\nu} = 0, \quad R_{\mu\nu}(g) = \Lambda g_{\mu\nu}, \quad R_{\mu\nu}(\tilde{c}^2 g) = \Lambda f g_{\mu\nu}. \]
  \[ (3.14) \]
  Here we have defined the background curvatures,
  \[ \Lambda_g = \Lambda + 3m^2(3b_1 c + 2b_2 c^2 + b_3 c^3), \]
  \[ \Lambda_f = c^2 \Lambda + \frac{3m^2}{\alpha} (b_1 c + 2b_2 c^2 + 3b_3 c^3). \]
  \[ (3.15) \]
  Since \( R_{\mu\nu}(g) = R_{\mu\nu}(\tilde{c}^2 g) \), we obtain the background condition,
  \[ \Lambda_g = \Lambda_f, \]
  \[ (3.16) \]
  which is a polynomial equation in \( c \) whose roots fully determine the background solution. Next, we consider linear perturbations around the proportional backgrounds,
  \[ e^a_{\mu} = \tilde{e}^a_{\mu} + \delta e^a_{\mu}, \quad \tilde{e}^a_{\mu} = c e^a_{\mu} + \delta \tilde{e}^a_{\mu}. \]
  \[ (3.17) \]
  These can be combined into the three linear fluctuations of the tensor fields,
  \[ \delta g_{\mu\nu} \equiv g_{\mu\nu} - \bar{g}_{\mu\nu} = 2 \delta e^a_{(\mu} \tilde{e}^b_{\nu)} \eta_{ab}, \quad \delta f_{\mu\nu} \equiv f_{\mu\nu} - \bar{f}_{\mu\nu} = 2 \delta \tilde{e}^a_{(\mu} \tilde{e}^b_{\nu)} \eta_{ab}, \]
  \[ \delta B_{\mu\nu} \equiv 2 (c \delta e^a_{(\mu} \tilde{e}^b_{\nu)} - \delta \tilde{e}^a_{(\mu} \tilde{e}^b_{\nu)}) \eta_{ab}. \]
  \[ (3.18) \]
  It is then straightforward to show that the linearized equations of motions can be diagonalized into the following three equations,
  \[ \mathcal{E}_{\mu\nu} \epsilon^{\rho\sigma} m_{\rho\nu} - \Lambda_g \left( m_{\mu\nu} - \frac{1}{2} m_{\rho\sigma} g^{\rho\sigma} g_{\mu\nu} \right) - \frac{m_{\mu\nu}^2}{2} \left( m_{\mu\nu} - m_{\rho\sigma} g^{\rho\sigma} g_{\mu\nu} \right) = 0, \]
  \[ (3.19a) \]
\[
E_{\mu\nu} \, \gamma^\lambda \gamma^\sigma - \Lambda_g \left( \nu_{\mu\nu} - \frac{1}{2} \nu_{\mu\nu} g_{\rho\sigma} \right) = 0, \tag{3.19b}
\]
\[
\nabla^{\rho} \nabla^{\nu} b_{\mu\nu} - m_{\nu}^{\nu} b_{\mu\nu} = 0, \tag{3.19c}
\]
where we have defined,
\[
m_{\nu} \equiv \delta_{\mu\nu} - \frac{1}{c^2} \delta f_{\mu\nu}, \quad l_{\nu} \equiv \delta_{\mu\nu} + \alpha^2 \delta f_{\mu\nu}, \quad b_{\mu\nu} \equiv 6 B_{\mu\nu}.
\]
The linearized Einstein tensor in terms of the covariant derivative \( \nabla_{\mu} \) compatible with the background metric \( g_{\mu\nu} \) is given by,
\[
E_{\mu\nu} \, \gamma^{\rho} \gamma^{\sigma} = - \frac{1}{2} \left[ \delta_{\mu}^{\rho} \delta_{\nu}^{\sigma} \nabla_{\gamma}^2 + \bar{g}_{\mu\sigma} \nabla_\gamma \nabla_\gamma - \delta_{\mu}^{\rho} \delta_{\nu}^{\sigma} \nabla_\gamma - \delta_{\mu}^{\rho} \nabla_\gamma \nabla_{\sigma} - \delta_{\nu}^{\rho} \nabla_\gamma \nabla_{\sigma} \right] m_{\rho\sigma}.
\]
(3.21)

The masses for the spin-2 fluctuation \( m_{\nu} \) and the antisymmetric fluctuation \( b_{\mu\nu} \) are,
\[
m_{\nu}^2 = m^2 (1 + \alpha^{-2} c^{-2}) (3 b_1 c + 4 b_2 c^2 + 3 b_3 c^3), \tag{3.22a}
\]
\[
m_b^2 = - \frac{m^2}{3 c^2} (3 b_1 c + 4 b_2 c^2 + 3 b_3 c^3). \tag{3.22b}
\]

We note that these two masses are related by \( m_b^2 = - \frac{\alpha^2 m^2}{3 (1 + \alpha^2 c^2)} m_{\nu}^2 \). The linearized spectrum described by (3.19) consists of one massless spin-2, one massive spin-2 and one massive antisymmetric field with a tachyonic mass pole (at least for \( c^2 > 0 \)). The number of propagating degrees of freedom is therefore \( 2 + 5 + 3 = 10 \). In appendix we confirm that the number of degrees of freedom is the same in the nonlinear theory.

### 4. Generalization to multiple vierbeine

In this section we further generalize the bigravity theory with antisymmetric components to the case of \( N \) dynamical vierbein fields \( (e_I)_r^a \) with \( I = 1, \ldots, N \). We define the respective metric tensors as \( (g_I)_\mu^\nu = (e_I)_r^a (e_I)_r^b \delta_{ab} \).

#### 4.1. General structure

Ghost-free multi-vierbein theories contain the \( N \) Einstein-Hilbert kinetic terms,
\[
S_g = \sum_{I=1}^{N} m_I^2 \int d^4x \sqrt{g_I} \left( R(g_I) - 2 \Lambda_I \right). \tag{4.1}
\]

For \( N \) vierbein fields there exist \( \frac{1}{2} N(N-1) \) antisymmetric tensor combinations of the form \( \eta_{ab} (e_I)_r^a (e_I)_r^b \) with \( I = 1 \). Since the \( N \) vierbeine contain \( 6N \) Lorentz components, only \( N \) of the antisymmetric tensors can be taken to be independent. Furthermore, the overall Lorentz invariance of the multi-vierbein actions will render one combination unphysical. We can thus choose \( (N-1) \) independent combinations to define \( (N-1) \) antisymmetric tensor fields. The most convenient choice of these combinations depends on the types of couplings present in the multi-vierbein action. We will discuss several explicit examples below. The kinetic terms for the antisymmetric components in the action read,
\[
S_B = \frac{1}{2 \cdot 3!} \sum_{I=1}^{N} \int d^4 x \sqrt{g_I} \left( (H_I)_{\mu\nu\rho} (H_I)^{\mu\nu\rho} \right), \tag{4.2}
\]
with \( (H_I)_{\mu\nu\rho} = 3 \nabla_\mu (B_I)_{\nu\rho} \) and where \( g_{\mu\nu} \) is one metric which has picked out of the \( N \) symmetric fields \( (g_I)_\mu^\nu \). Moreover, the action will contain a potential,
\[
S_{\text{int}} = \int d^4 x \ V (e_I), \tag{4.3}
\]
and thus have the total form \( S = S_g + S_B + S_{\text{int}} \).

The interactions among the vierbein fields can now have two distinct forms: They can be pairwise couplings [15], corresponding to multiple copies of the bigravity case, or they can consist of determinant vertices [19], which are genuine multi spin-2 interactions involving more than just two vierbeine in one vertex. The pairwise couplings further split up into two categories: The center coupling, where one vierbein in the center interacts with all other vierbeine, and the chain coupling, in which each vierbein (except for the two at the ends of the chain) couples to the antisymmetric field with a tachyonic mass pole (at least for \( c^2 > 0 \)). The number of propagating degrees of freedom is therefore \( 2 + 5 + 3 = 10 \). In appendix we confirm that the number of degrees of freedom is the same in the nonlinear theory.

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5 We thank James Bonifacio for pointing this out.

6 In general, different choices for the metric \( g_{\mu\nu} \) which couples to the antisymmetric field lead to different theories. Only in cases where the interactions are symmetric under interchanges of the vierbein fields, the choice can be made without loss of generality.
chain) interacts with exactly two neighbours. The two distinct types of pairwise interaction graphs are displayed in figure 1; the left panel of figure 2 shows the determinant vertex.

The most general vierbein theory contains all these couplings. The only two restrictions are that the graph of vierbein interactions can never be closed into a loop and that no two vierbeine can share more than one determinant vertex. An example for such a graph is displayed in the right panel of figure 2. In the following we discuss giving dynamics to the antisymmetric components for the different types of couplings one by one.

4.2. Pairwise couplings

4.2.1. Center coupling

Denoting the three interaction terms in (2.5) by $V(e, \tilde{e}; b_0)$, the center coupling is a sum of pairwise vierbein interactions of the form,

$$V_{\text{center}} = \sum_{i=2}^{N} V(e_i, e_j; b_0^i).$$

(4.4)

The vierbein $(e_i)_a^m$ is in the center of the interaction graph and couples directly to all other vierbeine, which do not have any direct interactions among themselves, cf the left panel of figure 1. In this case, we define the set of $(N - 1)$ antisymmetric tensors as,

$$(B_1)_a^m \equiv (e_1)_a^m \eta_{ab} (e_1)_b^m - (e_1)_a^m \eta_{ab} (e_1)_b^m,$$

(4.5)

for $I = 2, \ldots, N$. These fields are given $(N - 1)$ kinetic terms which can be covariantized independently using any of the metrics $(\tilde{q}_I)_{ab}$. A straightforward generalization of the ADM analysis in appendix then verifies that the number of propagating degrees of freedom in this theory is $2 + 5(N - 1) + 3(N - 1)$, corresponding to one massless spin-2, $(N - 1)$ massive spin-2 and $(N - 1)$ massive antisymmetric tensor fields.
4.2.2. Chain coupling

The chain coupling is also a sum of pairwise vierbein interactions,

\[ V_{\text{chain}} = \sum_{i=2}^{N_i} V(e_i, e_{i-1}, b_i^e). \] (4.6)

The vierbeine \((e_i)_m^a\) and \((e_N)_m^a\) sit on the ends of the chain, in which any other vierbein interacts with its two neighbours only. In this case, we define,

\[ (B_{I})_{\mu\nu} \equiv (e_{I})_m^a \eta_{ab} (e_{I-1})_a^b - (e_{I-1})_m^a \eta_{ab} (e_{I})_b^a, \] (4.7)

for \(I = 2, \ldots, N_i\), and give kinetic terms to these \( (N_i - 1) \) antisymmetric tensors. Again each of these kinetic terms can be covariantized with any of the metrics \((g_I)_{\mu\nu}\). As in the case of the center coupling, the propagating degrees of freedom are one massless spin-2, \((N_i - 1)\) massive spin-2 and \((N_i - 1)\) massive antisymmetric tensor fields.

4.3. Determinant vertex

The genuine multiple vierbein interactions are of the following form,

\[ V_{\text{det}} = \det \left( \sum_{i=1}^{N_i} e_i \right). \] (4.8)

In contract to the pairwise interactions, in this coupling each vierbein interacts with all other \((N_i - 1)\) fields. We can now pick any vierbein, for instance \((e_I)_m^a\) and define the \((N_i - 1)\) independent antisymmetric tensors as,

\[ (B_{I})_{\mu\nu} \equiv (e_{I})_m^a \eta_{ab} (e_{I-1})_a^b - (e_{I-1})_m^a \eta_{ab} (e_{I})_b^a. \] (4.9)

for \(I = 2, \ldots, N_i\). Their covariant kinetic terms can again be written using any of the metrics \((g_I)_{\mu\nu}\). Since the determinant vertex is also of a totally antisymmetric structure, its \(3 + 1\) form will be a generalization of equation (A.9). The ADM analysis therefore generalizes exactly as in the case of pairwise interactions and the degrees of freedom are again one massless spin-2, \((N_i - 1)\) massive spin-2 and \((N_i - 1)\) massive antisymmetric tensor fields.

5. Discussion

We have generalized the massive gravity theory with dynamical antisymmetric components proposed in [28] to the case with a dynamical reference frame \(\tilde{e}^a_{\mu}\). The difference of the model with fixed reference frame and the fully dynamical setup is similar to the difference of massive gravity with a fixed fiducial metric and bimetric theory with two dynamical tensor fields. In particular, the theory proposed in this work is both local Lorentz and diffeomorphism invariant. The setup with fixed reference vierbein can be obtained from the fully dynamical theory by taking the limit \(m_{\nu} \to \infty\), while keeping all other parameters fixed.

Maximally symmetric background solutions (i.e. solutions that are invariant under the isometry groups ISO \((3, 1), \text{SO}(4, 1)\) or \(\text{SO}(4, 2)\)) require the vanishing of the antisymmetric components, \(B_{\mu\nu} = 0\). As we saw, their fluctuations are tachyonic and hence the corresponding vacua are unstable.

This issue can be resolved in Anti-de-Sitter spacetime with \(\Lambda_g < 0\) where a mass pole is unitary above the Breitenlohner–Freedman bound [34]. For spin-2 fields, this bound is \(m_{\nu} \geq \frac{2}{\sqrt{3}} \Lambda_g\). If the overall sign of the mass potential is swapped, the mass pole for the \(b_{\mu\nu}\) fluctuation becomes unitary. Then we can restrict the parameters in the action to satisfy the bound for \(m_{\nu} \geq \frac{2}{\sqrt{3}} \Lambda_g\), such that the spin-2 mass pole is unitary as well. It would be interesting to see whether the tachyonic instability also occurs for other physically relevant solutions, such as spherically symmetric or homogeneous and isotropic backgrounds. In any case, the fluctuations of the massive antisymmetric tensor will introduce nontrivial effects into the perturbation theory around such backgrounds. Whether the nonlinear Hamiltonian is bounded from below is an open question.

Matter can be coupled to the theory in at least three different ways without exciting additional degrees of freedom in the gravitational sector:

(i) through a minimal coupling to the vierbein \(e^a_{\mu}\),

(ii) through a minimal coupling to the vierbein \(\tilde{e}^a_{\mu}\),

(iii) through a minimal coupling to a linear combination \(e^a_{\mu} + a \tilde{e}^a_{\mu}\) with an arbitrary coefficient \(a\).

All these couplings will be linear in the lapse and shifts functions of the two vierbeine and therefore not destroy the constraint structure discussed in appendix.
Interestingly, option (iii) for the matter coupling (which was suggested for bimetric theory in [35, 36]) also opens up the possibility of defining the gravitational theory in a more symmetric way. Instead of coupling the kinetic term for the $B_{\mu\nu}$ field to the metric $g_{\mu\nu}$ of the vierbein $e^a_\mu$, we can couple it to the metric built from the linear combination of vierbeine, $G_{\mu\nu} = (e^a_\mu + ae^a_\mu)\eta_{ab}(e^b_\nu + ae^b_\nu)$. The analysis in appendix can be applied to this case with only minor modifications, which implies that the number of propagating degrees of freedom is again the same. Hence, we obtain another ghost-free action by replacing the metric $g_{\mu\nu}$ in the kinetic term for $B_{\mu\nu}$ in (3.2) by the metric $G_{\mu\nu}$.

Since the massive $B_{\mu\nu}$ field is dual to a massive vector in $D = 4$, it would be interesting to see whether there exists a dual formulation of our setup. This would deliver an equivalent action, possibly formulated in terms of the Lorentz invariant components of the vierbeine (i.e. the corresponding metric tensors $g_{\mu\nu}$ and $f_{\mu\nu}$) and a massive vector field $A_\mu$. The dualization of the vierbein potential may thus produce new types of interactions for massless and massive spin-2 with massive vector fields, possibly relating our work to generalized Proca theories [8, 9].

General relativity (GR) and its interpretation in terms of Riemannian geometry are a prime example of the interplay between geometric structures and fundamental physics. Understanding the underlying geometry of any theory which includes gravity is thus crucial. For example, the geometry of string theory and its web of dualities gives rise to interesting new mathematical structures such as the extended space of Double Field Theory (DFT). This opens up the possibility of de...

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Appendix. ADM analysis

This appendix contains an ADM constraint analysis and a degree of freedom counting in ADM variables for the vierbeine [47].

A.1. The $3+1$ parametrization

We parameterize the general vierbein $e^a_\mu$ as a Lorentz transformation of a gauge-fixed vierbein $E^a_\mu$,

$$e^a_\mu = N^a_\mu E^b_\mu = \left(\Gamma^\alpha_\nu \gamma^{\nu\beta} \begin{pmatrix} N & 0 \\ E^3 & E^1 \end{pmatrix} \right)$$

where

$$\Gamma \equiv \frac{1}{\sqrt{1 - v^a v^a}}, \quad \gamma^{\nu\beta} \equiv \delta^{\nu\beta} + \frac{\Gamma^2}{1 + \Gamma} v^\alpha v^\beta.$$  

(A.2)

Here $\alpha, \beta = 1, 2, 3$ are spatial Lorentz and $i, j = 1, 2, 3$ are spatial coordinate indices. Moreover, the Lorentz rotations sit entirely in $E^3_i$, such that we can write

$$E^3_i = R^3_i, E^0_i,$$

(A.3)

for some gauge-fixed $E^0_i$, with 6 independent components and $R^T = R^{-1}$. The second vierbein is parameterized as,

$$\bar{e}^a_\mu = \begin{pmatrix} L & 0 \\ \phi^a j L \end{pmatrix}.$$  

(A.4)

which does not require its own set of Lorentz parameters. They can be shifted into $e^a_\mu$ because the action is invariant under diagonal local Lorentz transformations. Thus $\phi^a_\mu$ has only 6 independent components.

The ADM expressions for the vierbeine induces the $3+1$ split for the antisymmetric tensor defined by $B_{\mu\nu} = 2\eta_{ab}\bar{e}^a_\mu e^b_\nu$. The result is,

$$B_{\mu\nu} = \Gamma \gamma_{\nu} (LE^\alpha_\mu + N\phi^\alpha_\mu) + \phi^\alpha_\nu \gamma_{\nu} (L_k^j E^3_i + \delta^k_3 N^j),$$  

(A.5a)
\[ B_{ij} = \gamma_{ij} \varphi^\alpha \varphi^\beta E^{\alpha \beta} = \gamma_{ij} R^\gamma \varphi^\alpha \dot{E}^{\gamma \beta} \]  

(A.5b)

In particular, \( B_{ij} \) is linear in the lapses and shifts while \( \dot{B}_{ij} \) is independent of them.

### A.2. Kinetic terms

The 3 + 1 splits for the Einstein–Hilbert terms written in terms of the vierbeine are of the form [48] (see also [49]),

\[
\mathcal{L}_E = \Pi_{\alpha} \dot{E}_i^{\alpha} - N \mathcal{C}^{(e)} - N^i \mathcal{C}_i^{(e)}, \quad (A.6a)
\]

\[
\mathcal{L}_d = \Pi_{\alpha} \dot{\varphi}_i^{\alpha} - L \mathcal{C}^{(d)} - L^i \mathcal{C}_i^{(d)}, \quad (A.6b)
\]

where \( \Pi_{\alpha} \) and \( \Pi_{\alpha}^i \) are the canonical momenta conjugate to the spatial vierbein components \( \dot{E}_i^{\alpha} \) and \( \dot{\varphi}_i^{\alpha} \), respectively. The kinetic terms for \( g_{\mu \nu} \) and \( f_{\mu \nu} \) are linear in \( N, N^i, L \) and \( L^i \) since the constraint contributions \( \mathcal{C}^{(e)}, \mathcal{C}_i^{(e)}, \mathcal{C}^{(d)} \) and \( \mathcal{C}_i^{(d)} \) do not depend on the lapse and shift functions. The precise form of the constraints will not be needed in the following.

The antisymmetric tensor is split into its components \( B_{ij} \) and \( \dot{B}_{ij} \) for which we insert the expressions in (A.5a). As was shown in [28], the kinetic term for \( B_{ij} \) then possesses the following form,

\[
\mathcal{L}_B = \Pi^{mn} \dot{B}_{mn} \frac{N}{\sqrt{g}} \Pi^{mn} \Pi_{mn} - 3N \Pi^{[i} \partial_k (\dot{\varphi}_i \dot{V}_{;ik}) + \partial_l \Pi^{[i} [\Gamma_{\alpha \beta \gamma} \dot{\varphi}_i + \varphi^\alpha \dot{V}_{;\beta} + \delta_{ij} \delta^{kl} \frac{\partial}{\partial \varphi} \dot{\varphi}_i] - \frac{1}{4} N \sqrt{g} \partial_i (\dot{\varphi}_i \dot{V}_{;ik} \dot{E}_i^{\gamma \beta}) \Xi^{(\delta)} \Pi_{ij} - \dot{\varphi}_i \dot{V}_{;ik}, \]

(A.7)

where \( \Pi^{mn} \) is the canonical momentum conjugate to \( B_{mn} \) and \( \Xi^{(\delta)} \Pi_{ij} \equiv \Xi^{(\delta)} \Pi_{ij} \equiv \gamma_{ij} \gamma^{\alpha \beta} \dot{\varphi}_i \dot{\varphi}_j + \gamma_{ij} \dot{\varphi}_i \dot{\varphi}_k + \gamma_{ij} \dot{\varphi}_k \dot{\varphi}_k \). This can be rewritten as,

\[
\mathcal{L}_B = \Pi^{mn} \dot{B}_{mn} - N \mathcal{C}^{(B)} - L \dot{\mathcal{C}}^{(B)} - N^i \mathcal{C}_i^{(B)} - L^i \dot{\mathcal{C}}_i^{(B)}, \quad (A.8)
\]

where none of the constraint contributions \( \mathcal{C}^{(B)}, \mathcal{C}_i^{(B)}, \mathcal{C}^{(d)} \) and \( \mathcal{C}_i^{(d)} \) depend on the lapses and shifts. As we argued in [28], a field redefinition can relate the 3 dynamical components \( B_{ij} \) to the 3 Lorentz rotation parameters in \( R^{\alpha \beta} \).

### A.3. Interaction potential

As was first shown in [15] and reviewed in detail in [28], the antisymmetric structure of the potential \( V \) ensures its linearity in the \( e^\alpha_0 \) and \( \varphi^\alpha_j \) components of the vierbeine. This in turn implies that the potential is linear in the lapse and shift functions of both \( e^\alpha_0 \) and \( \varphi^\alpha_0 \). Hence we can write,

\[
V = N \mathcal{C}^{(V)} + N^i \mathcal{C}_i^{(V)} + L \dot{\mathcal{C}}^{(V)} + L^i \dot{\mathcal{C}}_i^{(V)}, \quad (A.9)
\]

where \( \mathcal{C}^{(V)}, \mathcal{C}_i^{(V)}, \dot{\mathcal{C}}^{(V)} \) and \( \dot{\mathcal{C}}_i^{(V)} \) are functions of the remaining ADM variables alone.

### A.4. Full action

Putting together the results for the two kinetic terms and the mass potential, the whole action assumes the form,

\[
S = \int d^4x (\Pi_{\alpha} \dot{E}_i^{\alpha} + \Pi_{\alpha} \dot{\varphi}_i^{\alpha} + \Pi^{[i} \dot{B}_{ij} - N \mathcal{C} - N^i \mathcal{C}_i - L \dot{\mathcal{C}} - L^i \dot{\mathcal{C}}_i), \quad (A.10)
\]

where the \( \mathcal{C} \) and \( \mathcal{C}_i \), \( \dot{\mathcal{C}} \) and \( \dot{\mathcal{C}}_i \), contain the contributions from (A.6), (A.9) and (A.8) which do not depend on \( N, N^i, L \) and \( L^i \).

The equations for \( \nu^\alpha \) can now be solved for the components of one of the shift vectors, say \( L_i \). This gives a solution for \( L_i \) which is linear in \( N_i, L \) and \( N^i \). The \( L_i \) equations can be solved for \( \nu^\alpha \) and imply \( \dot{\mathcal{C}}_i = 0 \). Thus, after solving the constraints for \( L_i \) and \( \nu^\alpha \), the action is of the form

\[
S = \int d^4x (\Pi_{\alpha} \dot{E}_i^{\alpha} + \Pi_{\alpha} \dot{\varphi}_i^{\alpha} + \Pi^{[i} \dot{B}_{ij} - N \mathcal{C} - N^i \mathcal{C}_i - L \dot{\mathcal{C}}), \quad (A.11)
\]

where the remaining constraints \( \mathcal{C}, \mathcal{C}_i \) and \( \dot{\mathcal{C}} \) are functions of \( \dot{E}_i^{\alpha}, \dot{\varphi}_i^{\alpha}, \dot{B}_{ij} \) and their canonical momenta.

The equations for \( N, N_i \) and \( L \) impose 5 constraints on the dynamical variables \( \dot{E}_i^{\alpha}, \dot{\varphi}_i^{\alpha}, \dot{B}_{ij} \) (or \( R^{\alpha \beta} \)). The number of propagating degrees of freedom is thus expected to be \( 6 + 6 + 3 - 5 = 10 \), corresponding to a massless spin-\( 2 \) (2), a massive spin-\( 2 \) (5) and a massive \( B_{ij} \) field (3).

The shift constraint \( \dot{\mathcal{C}}_i \) together with a combination of the lapse constraints \( C \) and \( \dot{\mathcal{C}} \) will generate the diagonal diffeomorphism symmetry of the action. The remaining combination of lapse constraints only removes a full degree of freedom if it gives rise to a secondary constraint. We do not explicitly verify these expected features here. For pure bimetric theory, they were confirmed in [50, 51].
References

[1] Heisenberg L. 2019 Phys. Rept. 796 1–113 arXiv:1807.01725 [gr-qc]
[2] Horndeski G W 1974 Int. J. Theor. Phys. 10 363
[3] Gleyzes J, Langlois D, Piazza F and Vernizzi F 2015 Phys. Rev. Lett. 114 211101
[4] Gleyzes J, Langlois D, Piazza F and Vernizzi F 2015 J. Cosmol. Astropart. Phys. JCAP02(2015)018
[5] Langlois D and Noui K 2016 J. Cosmol. Astropart. Phys. JCAP02(2016)034
[6] Ben Acheur J, Crisostomi M, Koyama K, Langlois D, Noui K and Tasinato G 2016 J. High Energy Phys. JHEP12(2016)100
[7] Yang C N and Mills R L 1954 Phys. Rev. 96 191
[8] Tasinato G 2014 J. High Energy Phys. JHEP04(2014)067
[9] Heisenberg L 2014 J. Cosmol. Astropart. Phys. JCAP05(2014)015
[10] Boudware D G and Deser S 1972 Phys. Rev. D 6 3368
[11] de Rham C, Gabadadze G and Tolley A J 2011 Phys. Rev. Lett. 106 231101
[12] Hassan S F and Rosen R A 2012 Phys. Rev. Lett. 108 041101
[13] Hassan S F, Rosen R A and Schmidt-May A 2012 J. High Energy Phys. JHEP02(2012)026
[14] Hassan S F and Rosen R A 2012 J. High Energy Phys. JHEP02(2012)126
[15] Hinterbichler K and Rosen R A 2012 J. High Energy Phys. JHEP07(2012)047
[16] Hinterbichler K 2012 Rev. Mod. Phys. 84 671
[17] de Rham C 2012 Phys. Rev. D 85 024024
[18] Schmidt-May A and von Strauss M 2016 J. Phys. A 49 183001
[19] Hassan S F and Schmidt-May A 2019 Phys. Rev. Lett. 122 (no.25) 251101 arXiv:1804.09723 [hep-th]
[20] Deffayet C, Mourad J and Zahariade G 2013 J. High Energy Phys. JHEP03(2013)086
[21] de Rham C and Tolley A J 2015 Phys. Rev. D 92 024024
[22] Hassan S F and Kocic M 2018 J. High Energy Phys. JHEP05(2018)099
[23] Hinterbichler K, Joyce A and Rosen R A 2018 J. High Energy Phys. JHEP03(2018)051
[24] Bonifacio J, Hinterbichler K, Joyce A and Rosen R A 2018 J. High Energy Phys. JHEP06(2018)075
[25] de Rham C, Melville S, Tolley A J and Zhou S Y 2018 J. High Energy Phys. JHEP03(2018)011
[26] de Rham C, Melville S and Tolley A J 2018 J. High Energy Phys. JHEP04(2018)083
[27] de Rham C, Melville S, Tolley A J and Zhou S Y 2019 JHEP 03 182 arXiv:1804.10624 [hep-th]
[28] Markou C, Rudolph F I and Schmidt-May A 2019 Class. Quant. Grav. 36 095014
[29] Gabadadze G, Hinterbichler K, Pirskhalava D and Shang Y 2013 Phys. Rev. D 88 084030
[30] Ortin T 2004 Gravity and Strings (Cambridge: Cambridge Univ. Press) (https://doi.org/10.1017/CBO9780511616563)
[31] Ogievetsky V I and Polubarinov I V 1967 Sov. J. Nucl. Phys. 4 156 https://inspirehep.net/record/51411
[32] Ogievetsky V I and Polubarinov I V 1966 Yad. Fiz 4 216
[33] Kalb M and Ramond P 1974 Phys. Rev. D 9 2273
[34] Smalagic A and Spallucci E 2001 J. Phys. A 34 L435
[35] Breitenlohner F and Freedman D Z 1982 Phys. Lett. 115B 197
[36] Noller J and Melville S 2015 J. Cosmol. Astropart. Phys. JCAP01(2015)003
[37] Hinterbichler K and Rosen R A 2015 Phys. Rev. D 92 024030
[38] Siegel W 1993 Phys. Rev. D 47 5453
[39] Siegel W 1993 Phys. Rev. D 48 2826
[40] Hull C and Zweibach B 2009 J. High Energy Phys. JHEP09(2009)099
[41] Gualtieri M 2003 arXiv:math/0401221 [math-dg] inspirehep.net/record/640221
[42] Hitchin N 2003 Quart. J. Math. 54 281
[43] Freidel L, Rudolph F I and Svoboda D 2017 J. High Energy Phys. JHEP11(2017)175
[44] Freidel L, Rudolph F I and Svoboda D 2019 Commun. Math. Phys. : 1–32 arXiv:1806.05992 [hep-th]
[45] Chamsemdine A H 2001 Commun. Math. Phys. 218 283
[46] Chamsemdine A H and Mukhanov V 2010 J. High Energy Phys. JHEP03(2010)033
[47] Chamsemdine A H and Mukhanov V 2012 J. High Energy Phys. JHEP08(2012)036
[48] Arnowitt R L, Deser S and Misner C W 2008 Gen. Rel. Grav. 40 1997
[49] De S and Isham C J 1976 Phys. Rev. D 14 2505
[50] FeldP 1994 Class. Quant. Grav. 11 1087
[51] Hassan S F and Rosen R A 2012 J. High Energy Phys. JHEP04(2012)123
[52] Hassan S F and Lundkvist A 2018 J. High Energy Phys. JHEP08(2018)182