Relationship of Rainfall and IOD Phenomena by using Spectral Analysis and SARIMA

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Abstract. Rainfall is very essential for life and it has complexity process. To obtain the relationship between rainfall in time-spatial of Aceh region and the Indian Ocean Dipole (IOD) phenomenon, we used spectral-analysis and SARIMA model. Identification results show that the rainfall data monthly in Aceh Barat (ABR), Aceh Besar (ABS), Sabang (S) and Aceh Utara (AU) has a similar pattern. However, due to the sensitivity of the occurrence of the IOD, the ABR region has higher rainfall compared to ABS, S and AU regions. Rainfall in all regions observation has a peak time, namely at the end of the year (October-November-December). It has the negative peak of the IOD event occurred in September-2016 while the positive peak point occurred in October-2019. Overall, the normal phase occurred from January to April-2019. In peak season in ABR and ABS, the rainfall intensity increased. The results showed that the rainfall model in ABR and S has normal distribution, whereas in ABS and AU has nonnormal distribution. Furthermore, the rainfall model in ABR with SARIMA(2,0,2)(0,1,1)₆ is the best model compared to model of three other regions with high accuracy forecasting.

1. Introduction

Aceh province has position in western Sumatra island that around in the Indian Ocean and Malacca strait. The climate phenomenon in the western region of Sumatra, Indonesia and the western part of the Indian Ocean is often referred to the Indian Ocean Dipole (IOD). The IOD phenomenon is characterized by the presence of a positive sea surface temperature (SST) anomaly in the western Indian Ocean while a negative SST anomaly occurs in the western region of Sumatra, Indonesia. When a positive IOD phenomenon occurred, a collection of convective clouds around the east coast of Africa causes the intensity of rainfall in the western part of Indonesia to be relatively low, while when the IOD is negative, the intensity of rainfall in the western part of Indonesia is relatively high [1].

Rainfall in Indonesia generally is influenced by well-scale atmospheric circulation phenomenon in global, regional, and local. Global phenomena such as MJO (Madden Julian Oscillation), ENSO (El Niño–Southern Oscillation), Australia Monsoon and Asia Monsoon [2]. Generally, rainfall variability in Indonesia is affected by Australia and Asia Monsoons, however, for seasonal and inter-seasonal rainfall is affected by ENSO and IOD phenomena [3]. Statistical analysis of rainfall is related time and spatial, so that rainfall forecasting in quantities is not easy due to complexity process of atmospheric, geographic, and oceanographic.
Rainfall analysis in the Indian Ocean has been investigated by using Markov chains in \cite{4} with focused for state transition. In this study, we investigate the relationship between rainfall and IOD phenomena in the Indian Ocean by using time series approach, in special is method spectral analysis and SARIMA (seasonal autoregressive integrated moving average) model.

2. Methods

Spatially, we use rainfall dataset in the four districts from Station Climatology and Geophysics Meteorological Agency in Aceh region with the Indian Ocean Dipole (IOD) phenomenon from the period of January 2010 to December 2019. The data used in this study is a secondary data, namely monthly IOD index data (°C) sourced from the Japan Agency for Marine-Earth Science Technology (JAMSTEC) \cite{5}. Furthermore, monthly rainfall data (mm / month) in the West Aceh Regency region for the period January 2010 to December 2019 are sourced from the Meteorology, Climatology and Geophysics Agency (BMKG) \cite{6} and can be accessed through rainfall satellite data (mm / month) for the Aceh Province sourced from the Center for Hydrometeorology and Remote Sensing (CHRS).

In time series analysis, if we viewed rainfall dataset from a domain perspective, it is divided into two, namely time series analysis in the time domain and time series analysis in the frequency domain. The analysis was carried out on the time domain series in the form of the use of autocorrelation, partial autocorrelation, and auto covariance functions. Meanwhile, models in the time domain series include Autoregressive (AR), Moving Average (MA), Autoregressive Moving Average (ARMA), and Autoregressive Integrated Moving Average (ARIMA). However, the frequency domain time series analysis is considered as a result of the existence of a cycle component at different frequencies and is difficult to obtain in the time domain \cite{7}.

Spectral analysis or also known as spectrum analysis was introduced by Schuster, a social worker in the 19th century who did research in the fields of marine, meteorology and astronomy with the aim of knowing hidden periodicity \cite{8}. Spectral analysis method is inferential statistical analysis based on the concept of frequency, visually represented by a spectrum. Spectral analysis is a method used for time series analysis in the frequency domain. The basic concept of spectral analysis is calculating the periodogram and drawing the power spectrum lines \cite{9}. The periodogram equation can be written as follows

\[
I(\omega_k) = \begin{cases} \frac{n}{2} (a_k^2 + b_k^2), & k = 1, \ldots, \left\lfloor \frac{n-1}{2} \right\rfloor \\ n \frac{\hat{a}_k^2}{\pi}, & k = 0 \end{cases} \quad (2.1)
\]

where \(a_k\) and \(b_k\) are Fourier coefficient written as follows:

\[
a_k = \begin{cases} \frac{1}{n} \sum_{t=1}^{n} Z_t \cos \omega_k t, & k = 0 \text{ dan } k = \frac{n}{2} \text{ if } n \text{ even}, \\ \frac{2}{n} \sum_{t=1}^{n} Z_t \cos \omega_k t, & k = 1, 2, \ldots, \left\lfloor \frac{n-1}{2} \right\rfloor \text{ if } n \text{ odd}, \end{cases} \quad (2.2)
\]

\[
b_k = \frac{2}{n} \sum_{t=1}^{n} Z_t \sin \omega_k t, \quad k = 1, 2, \ldots, \left\lfloor \frac{n-1}{2} \right\rfloor, \quad (2.3)
\]

with

\[
\omega_k = \frac{2 \pi k}{n}, \quad k = 0, 1, 2, \ldots, \left\lfloor \frac{n}{2} \right\rfloor \quad (2.4)
\]

where:

- \(I(\omega_k)\) : periodogram th-\(k\)
- \(a_k\) : parameter \(a\) of Fourier coefficient
- \(b_k\) : parameter \(b\) of Fourier coefficient
- \(\omega_k\) : Fourier frequency th-\(k\)
- \(\pi\) : radian \(180^\circ\)
- \(n\) : total observation
- \(Z_t\) : data observation at time th-\(t\)
- \(t\) : time
In this study, we used Spectral analysis and SARIMA models to obtain relationship of rainfall and IOD phenomena in several districts in Aceh region. The detail of spectral analysis and SARIMA models as follows:

1. Identification IOD index and rainfall dataset.
2. Periodogram analysis of IOD index and rainfall.
3. Stationary testing, if it is not stationary then using transformation or differencing and if yes then identification of SARIMA model.
4. Significance of parameters
5. Diagnostic testing of model by white noise testing and normality
6. Goodness of fit and accuracy model
7. IOD prediction model
8. Relationship IOD and rainfall in different districts
9. Analysis and conclusion.

3. Results and Discussion

3.1 Time Series and Boxplot of Rainfall

The rainfall pattern time series from Jan 2010 to Dec 2019 was illustrated as in Figure 1. Based on the figure, the time series pattern of rainfall in Aceh Barat, Aceh Besar, Sabang, and Aceh Utara has a pattern that tends to similar for each month. Then it can be seen that the West Aceh region has high rainfall (blue color) compared to the Aceh Besar, Sabang and North Aceh regions. This is because West Aceh is a sensitive area with close to the Indian Ocean to the occurrence of the IOD phenomenon so that it often gets the impact of high rainfall.

![Figure 1. Rainfall time series plot for 4 districts and IOD Index Jan 2010 to Dec 2019](image_url)

Based on Figure 2, it shows that the peak point of rainfall occurred at the end of the year, which is around October, November, and December. In fact, it shows that the rainfall data contains the outliers, for instance as appeared in Aceh Barat region which is in September and December, in Aceh Besar region, which is in April and August, in Sabang region which is in February, March, April, August, and November, and in Aceh Utara region which is in July and December. This indicates that the months with outliers, the regions involved have a high rainfall (negative IOD). Other things, rainfall in Aceh Barat shows fluctuate compared other three regions.

Furthermore, investigation regarding rainfall dataset from four districts (stations), specially to obtain rainfall variability monthly, we can see boxplot of rainfall in 4 districts as follows:
All results are summarized in the table of rainfall for four districts as seen in Table 1.

Table 1. Statistic Summary of Rainfall in 4 Stations

| Rainfall (mm) | Min | Mean   | Standard Deviation | Max  |
|---------------|-----|--------|--------------------|------|
| Aceh Barat    | 47.6| 327.4333 | 160.7194         | 775.0|
| Aceh Besar    | 0.00| 157.6783 | 126.1285         | 542.1|
| Sabang        | 0.00| 162.4700 | 134.0414         | 673.0|
| Aceh Utara    | 0.00| 111.0342 | 91.6564          | 571.3|

The total observations to obtain the results as in Table 1 were 120 samples. As can be seen in Table 1, the lowest rainfall was in Aceh Barat at 47.6 mm, whereas the rainfall in Aceh Besar, Sabang and Aceh Utara was 0 mm (no rain occurred). In contrast, the highest rainfall occurred in West Aceh at 775.0 mm, followed by 673.0 mm, 542.1 mm, and 571.3 mm occurred in respectively Sabang, Aceh Besar, and Aceh Utara. The average rainfall in Aceh Barat, Aceh Besar, Sabang and Aceh Utara was 327.4333 mm, 157.6783 mm, 162.4700 mm, and 111.0342 mm, respectively.

3.2 Impact of IOD phenomena on Rainfall
To show the relationship between the IOD phenomena and rainfall for 4 districts in Aceh, we can see Figure 3.
Figure 3. Plot of rainfall anomaly for 4 districts and IOD in period 2016 - 2019

Based on Figure 3, the rainfall anomaly plot for 4 districts and the IOD index in 2016 and 2019 has different phases. In 2016, when the IOD index was in a negative phase, rainfall in Aceh Barat, Aceh Besar, Sabang, and Aceh Utara areas increased. In contrast, in 2019, when the IOD index was in a positive phase, the regions of Aceh Barat, Aceh Besar, Sabang and Aceh Utara experienced a decrease in rainfall. From this figure also we can see that the negative peak of IOD events occurred in September 2016 while the positive peak of IOD events occurred in October 2019. Meanwhile, the normal phase occurred in January to April 2019.

Figure 4. Classification and prediction of IOD index (BMKG, 2019)

IOD is identified based on an index called the Dipole Mode Index (DMI). The DMI index is obtained from the difference in SST anomaly between the western Indian Ocean (West Box) and the eastern Indian Ocean (East Box). DMI can be used to determine the strength of the IOD, the greater
the DMI index value, the stronger the IOD that occurs [10]. According to BMKG [6] the DMI classification consists of 3 phases, namely a positive DMI phase at an index range of greater than 0.4, a normal DMI phase at an index range from -0.4 to 0.4, and a negative DMI phase at an index range of less than -0.4 as shown in Figure 4.

We can see that peak point IOD in October-November (Figure 3), shifted in September-October (Figure 4). In the peak season, Aceh Barat and Aceh Besar have the increase of rainfall intensity (Figure 3). The IOD phases of minimum (negative) and maximum (positive) means a condition that we should concern on observing an effort to risk reduction of disasters or the effects of these phenomena events. For more details, see a map of the distribution of rainfall during the negative IOD phase in September 2016 and positive IOD phase in October 2019 in the Aceh Province region as shown in Figure 5 below.

![Figure 5: Map of rainfall distribution in Aceh Province in the (a) negative IOD and (b) positive IOD phases](image)

Based on Figure 5, in September 2016 in the region of Aceh Province has experienced an increase in rainfall, it can be seen on the map of the western coast of Aceh Province which is blue, meaning that the rainfall in the western region of Aceh Province is above normal or has the increase of rainfall, particularly in Simeulue and Aceh Singkil areas with the highest rainfall of 1891 mm and the lowest of 91 mm. However, on the map of the distribution of positive IOD, the phase rainfall is in October 2019 in the west region of Aceh Province, it was seen that the overall map was red, meaning that the rainfall in the Aceh Province area was below normal or experienced a decrease in rainfall from normal with rainfall, the highest was 981 mm and the lowest was 47 mm. In addition, you can also see a map of the distribution of rainfall in the normal phase in the Aceh Province, namely in February and March 2019 as shown in Figure 5 below.

Based on Figure 6, on the map of the distribution of normal phase rainfall in the Aceh Province, it can be seen that in February and March 2019 the rainfall in Aceh Province is normal, namely in general on the map it is marked in red and yellow, but in the areas of Aceh Barat, Subulussalam, and Aceh Singkil has high rainfall on the map marked in blue. Overall, the highest rainfall in February
2019 was 576 mm and the lowest was 34 mm, while in March 2019 the highest rainfall was 415 mm and the lowest was 69 mm.

![Map of rainfall distribution in Aceh Province during the normal phase of the month (a) February and (b) March 2019](image)

**Figure 6.** Map of rainfall distribution in Aceh Province during the normal phase of the month (a) February and (b) March 2019

The following significance testing of parameter model for rainfall in four districts are

| Parameter | Estimate | P-value | Remark |
|-----------|----------|---------|--------|
| Rainfall Model in Aceh Barat of SARIMA (2,0,2)(0,1,1)$^6$ |
| AR(1) $\theta_1$ | 1.5154 | 2.2 x $10^{-16}$ | Significant |
| AR(2) $\theta_2$ | -0.9035 | 2.2 x $10^{-16}$ | Significant |
| MA(1) $\theta_1$ | -1.5140 | 2.2 x $10^{-16}$ | Significant |
| MA(2) $\theta_2$ | 0.9999 | $1.131 \times 10^{-13}$ | Significant |
| SMA(1) $\theta_1$ | -0.9498 | $2.876 \times 10^{-8}$ | Significant |
| Rainfall Model in Aceh Besar of SARIMA (2,0,2)(1,1,0)$^12$ |
| AR(1) $\theta_1$ | 0.8735 | 2.2 x $10^{-16}$ | Significant |
| AR(2) $\theta_2$ | -0.7816 | 2.2 x $10^{-16}$ | Significant |
| MA(1) $\theta_1$ | -0.9947 | 2.2 x $10^{-16}$ | Significant |
| MA(2) $\theta_2$ | 0.9999 | 2.2 x $10^{-16}$ | Significant |
| SAR(1) $\varphi_1$ | -0.4258 | 3.548 x $10^{-7}$ | Significant |
| Rainfall Model in Sabang of SARIMA (2,0,2)(1,1,0)$^12$ |
| AR(1) $\theta_1$ | -0.2353 | 4.11 x $10^{-9}$ | Significant |
| AR(2) $\theta_2$ | -0.9530 | 2.2 x $10^{-16}$ | Significant |
| MA(1) $\theta_1$ | 0.1453 | 0.004001 | Significant |
| MA(2) $\theta_2$ | 0.9999 | 2.2 x $10^{-16}$ | Significant |
| SAR(1) $\varphi_1$ | -0.4481 | 5.845 x $10^{-7}$ | Significant |
| Rainfall Model in Aceh Utara of SARIMA (2,0,2)(0,1,1)$^6$ |
| Parameter | Estimate | P-value | Remark |
|-----------|----------|---------|--------|
The results of the parameter significance test in Table 2 shows that p-value of all parameter for overall model less than α (0.05), it means can reject $H_0$ and parameter model is significant. So it can be concluded that all the parameters for the rainfall models for Aceh Barat, Aceh Besar, Sabang and Aceh Utara are significant at the error level 5%.

Further, a diagnostic test is performed for all models whose estimated parameter values are significant.

1. Testing of Diagnostic Model
   a. White Noise Test
   The residual white noise assumption test can be seen in Table 4 below:

   | Rainfall       | SARIMA model          | P-value   |
   |----------------|-----------------------|-----------|
   | Aceh Barat     | (2,0,2)(0,1,1)$^6$    | 0.3338    |
   | Aceh Besar     | (2,0,2)(1,1,0)$^{12}$ | 0.7856    |
   | Sabang         | (2,0,2)(1,1,0)$^{12}$ | 0.4859    |
   | Aceh Utara     | (2,0,2)(0,1,1)$^{12}$ | 0.8948    |

   The result of white noise test in Table 3 shows that p-value of all model of rainfall more than α (0.05), it means cannot reject $H_0$. So it can be concluded that all rainfall model in Aceh Barat SARIMA $(2,0,2)(0,1,1)^6$, Aceh Besar SARIMA $(2,0,2)(1,1,0)^{12}$, Sabang SARIMA $(2,0,2)(1,1,0)^{12}$ and rainfall model in Aceh Utara SARIMA $(2,0,2)(0,1,1)^{12}$ qualify white noise.

   b. Testing of Residual Normality
   The normality test using the Shapiro Wilk test can be seen in Table 5 below:

   | Rainfall       | SARIMA model          | P-value   |
   |----------------|-----------------------|-----------|
   | Aceh Barat     | (2,0,2)(0,1,1)$^6$    | 0.07995   |
   | Aceh Besar     | (2,0,2)(1,1,0)$^{12}$ | 0.00284   |
   | Sabang         | (2,0,2)(1,1,0)$^{12}$ | 0.06260   |
   | Aceh Utara     | (2,0,2)(0,1,1)$^{12}$ | 1.161 x 10$^7$ |

   Normality test results in Table 4 shows that p-value of rainfall model in Aceh Barat and Sabang more than α (0.05), it means cannot reject $H_0$. It can be concluded that rainfall model in Aceh Barat and Sabang has normal distribution. Meanwhile, p-value of rainfall model in Aceh Besar and Aceh Utara less than α(0.05), it means can reject $H_0$. So it can be concluded that rainfall model in Aceh Besar and Aceh Utara has non normal distribution.

2. Goodness of fit and accuracy model

   | Rainfall   | Model               | AIC   | MAPE          | RMSE          |
   |------------|---------------------|-------|---------------|---------------|
   | Aceh Barat | SARIMA $(2,0,2)(0,1,1)^6$ | 2.01  | 6.797608      | 0.2092061     |
   | Aceh Besar | SARIMA $(2,0,2)(1,1,0)^{12}$ | 136.25 | 16.72115      | 0.3943151     |
   | Sabang     | SARIMA $(2,0,2)(1,1,0)^{12}$ | 106.68 | 13.60940      | 0.3438413     |
   | Aceh Utara | SARIMA $(2,0,2)(0,1,1)^{12}$ | 126.76 | 26.87394      | 0.3496797     |
Table 5 shows that rainfall model in Aceh Barat with SARIMA \((2,0,2)(0,1,1)\) is the best model compared with rainfall model in Aceh Besar, Sabang, and Aceh Utara, due to it has AIC, MAPE, and RMSE value less than other. Rainfall model in Aceh Barat has high accuracy forecasting in MAPE category under 10% [11].

4. Conclusion
We applied the spectral analysis and SARIMA model to obtain the relationship between the rainfall in four districts (or stations) and the Indian Ocean Dipole (IOD) phenomenon from the period of January 2010 to December 2019. The results showed that the rainfall anomaly for four districts and the IOD index in 2016 and 2019 has different phases. In 2016, however, when the IOD index was in a negative phase, rainfall in the Aceh Barat, Aceh Besar, Sabang, and Aceh Utara areas increased. In contrast, in 2019, when the IOD index was in a positive phase, the regions of Aceh Barat, Aceh Besar, Sabang and Aceh Utara experienced a decrease in rainfall.

A peak point of IOD in October-November, shifts from September-October. In the peak season, Aceh Barat and Aceh Besar have an increase rainfall intensity. The rainfall model in Aceh Barat and Sabang has the normal distribution, whereas rainfall model in Aceh Besar and Aceh Utara has the nonnormal distribution. Furthermore, rainfall model in Aceh Barat with SARIMA \((2,0,2)(0,1,1)\) is the best model compared to rainfall model in Aceh Besar, Sabang, and Aceh Utara with high accuracy forecasting.

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