What is the speed of quantum information?

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We study the apparent nonlocality of quantum mechanics as a transport problem. If space is a physical entity through which quantum information (QI) must be transported, then one can define its speed. If not, QI exists apart from space, making space in some sense ‘nonphysical’. But we can still assign a ‘speed’ of QI to such models based on their properties. In both cases, classical information must still travel at $c$, though in the latter case the origin of local spacetime itself is a puzzle. We consider the properties of different regimes for this speed of QI, and relevant quantum interpretations. For example, we show that the Many Worlds Interpretation (MWI) is nonlocal because it is what we call spatially complete.

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Since the seminal work of Einstein, Podolsky and Rosen [1], it has been clear that quantum mechanics appears to be nonlocal. Of course, classical observables respect relativistic causality, but this was imposed upon something else. Although QI could include ‘information’ about all dynamical entities in the formalism of a complete model with local hidden variables [2], and experiments agree with quantum predictions [3]. So there is something beyond classical information in quantum theory, and if we were to try to model it in terms of hidden variables, it would behave nonlocally. We believe that this apparent nonlocality must either be reified or explained away in any complete theory.

By quantum information (QI), we mean the set of all dynamical entities in the formalism of a complete quantum theory. QI could include ‘information’ about the wave function, phases, entanglement, histories, or something else. Although QI could specify unique outcomes for observables, like the nonlocal hidden variables of Bohm’s model, and our results should apply to such models, we will stress more the possibility that QI does not involve hidden variables. Instead, QI would allow a deeper understanding of the theory without adding classical determinism. In any case, our goal is simply to categorize the spacetime behavior this QI can have, and ask questions such as “Is it local?” “Is it causal?” and even more simply, “Where is it?”

In classical mechanics, information resides in physical space. To affect another location, it must be transported across space, with limiting speed $c$—i.e., it can affect things only in its forward lightcone. For QI, one possibility is that space is ‘physical’ in this way, and QI exists in it. Then QI needs transport through space at some speed $v_{QI}$, and the theory should explain the dynamics of this new kind of transport. Alternatively, space could be ‘nonphysical’ in that QI does not exist in it and thus does not need transport through it. As we will see later, it is still useful to characterize this kind of world by a ‘speed’. So either way, we believe that any complete theory should give a definitive answer to the title question.

For the cases where QI resides in physical space, let us define $v_{QI}$ by the extent of spacetime through which one quantum entity can affect another. Consider the smallest cone which can bound this region. Let $\theta_{QI}$ be the angle of this cone from the vertical. Then $v_{QI} = 0$ corresponds to $\theta = 0$, $v_{QI} = c$ to $\theta_{QI} = \pi/4$ (coincides with the forward lightcone), and $v_{QI} = \infty$ to $\theta_{QI} = \pi/2$. But these cases cover only half of spacetime, and one can in principle have speeds “faster than infinity” for $\theta_{QI} > \pi/2$. These correspond to quantum entities which can interact with others backwards through time. For example, $\theta_{QI} = 3\pi/4$ means that QI can travel along the backward lightcone, which we define as having ‘speed’ $v_{QI} = -c$. We can then conveniently define an inverse speed,

$$w_{QI} = \cot \theta_{QI} = (v_{QI}/c)^{-1},$$

so that the cases of $v_{QI} = c$, $\infty$, and $-c$ correspond to $w_{QI} = 1$, 0, and -1. For the rest of the paper, we will be asking what it means for a quantum model to correspond to different values of $w_{QI}$ (coupled with a characteristic spacetime). We summarize these results in Table I. Categories are written such that “Y” is theoretically desirable (‘spatial completeness’ possibly excepted), and we list “N” when there is no reason the answer has to be “N”. The final column is just opinion.

Let us dispense with the first line of Table I, which we will call a ’scripted universe’. Each point in space has all the QI it will ever need from the start, and there is no transport of any (including classical) information. This universe of disjoint points then appears to have dynamics in the same way that players appear to have an impromptu conversation in a carefully written play. This is anathema to physics because dynamics are an illusion. There are no real causes and effects, just the script.

The second possibility is $v_{QI} = c$ ($w_{QI} = 1$). Such a model would be local and causal. The question of Lorentz invariance has two aspects, for one-way and two-way QI transport. Recall that distance and time measurements are ultimately based upon two-way communication of light, from the origin to a reference point and back in a given frame. We can then use such points to measure one-way communication. Thus let us define a normalized
TABLE I: Regimes for the speed of QI. The Y/N’s answer “Can there be a model in this regime with QI which is/has...” E is for “effectively Y”. Strong causality is violated in nonphysical space models without a preferred frame, though weak causality is still preserved. In $w_{QI} = -1$ models, strong and weak causality are violated, though not beyond any $t$ cutoff. The Y/N in the latter models is for finite/infinite space.

| $w_{QI}$ | $\theta_{QI}$ | $v_{QI}$ | spacetime | example models | local? | causal? | $\tilde{w}$ Lorentz invariant? | preferred frame? | one time? | spatially complete? | nice? |
|--------|--------------|----------|-----------|----------------|------|-------|--------------------|-----------------|---------|------------------|-----|
| $\infty$ | 0 | 0 | Disjoint | Scripted | Y | n/a | Y | Y | Y | Y | N |
| 1 | $\pi$ | c | Local | Not allowed? | Y | Y | Y | Y | Y | N | N! |
| $(0,1)$ | $(\pi, 0)$ | $(c, \infty)$ | Not Local | Not allowed? | N | Y | N | N | N | N | N |
| $\epsilon$ | $\pi - \epsilon$ | $c/\epsilon$ | Discrete | MWI, Bohm, Collapse | N | Y | E | N | Y | E | Y |
| 0 | $\pi$ | $\infty$ | Continuous | MWI, Bohm, Collapse | N | Y | Y | N | N | Y | Y |
| 0 | $\pi$ | $\infty$ | Nonphysical | MWI+space source? | N | Y/N | N | N | Y | Y | Y |
| $-1$ | $\pi$ | $'-c'$ | Cutoff | Unknown | N | Y | Y | N | Y/N | Y | N |
| $-1$ | $\pi$ | $'-c'$ | Acausal | Transactional | N | $w_{\epsilon, s}$ | Y | Y | N | N | Y |

The inverse speed for the round trip to a reference point,

$$\tilde{w} = \frac{QI \text{ round trip time}}{\text{light round trip time}},$$

which can in principle be frame-dependent. In whatever frame we use to define $w_{QI}$ (which is the preferred frame if there is one), we have $\tilde{w} = w_{QI}$ (for $w_{QI} \geq 0$). For $w_{QI} = c$, we have $\tilde{w} = w_{QI} = 1$ in all frames, and thus two-way transport of QI in this regime is Lorentz invariant. But we still want to know if the one-way transport picks out a preferred frame (apart from the one from non-relativistic quantum mechanics). For the $v_{QI} = c$ case, there is nothing a priori which does.

But is such a $v_{QI} = c$ model allowed? As we said, experiments rule out only $v_{QI} = c$ hidden variable models. However, it also cannot be a collapse model, because such collapses are inherently nonlocal. Even the measurement of a single particle on a spacelike screen shows this, since all the other points on the screen instantly know they can no longer be the one to fire.

What about a noncollapse model such as the Many Worlds Interpretation (MWI)? By the MWI, we mean a quantum model based upon unitary evolution of a universal wave function. It would not entail new ‘worlds’ coming into existence with each quantum fork in the road (which would be a horribly nonlocal phenomenon) but rather mundane Schrödinger evolution of a single wave function of the Universe allowing arbitrary macroscopic superpositions. QI in such a model could consist of that wave function, but possibly also include information about a preferred basis for reality. Can such a model be local?

No. Consider spatial completeness, which means that some or all dynamical ‘information’ describing a state of the Universe is present (‘stored’ or otherwise accessible to dynamics there) at every point in space $x$. Spatially complete theories are necessarily nonlocal. Classical mechanics is spatially incomplete because it is local: dynamical information at $(x_1, t)$ differs from that at $(x_2, t)$.

For a quantum model to be local, it would need to be spatially incomplete, so that its QI differed from point to point. But dynamics in the MWI are encoded in a universal wave function, which is spatially complete. Thus the MWI is nonlocal. There is only a single wave function at time $t_0$, $|\Psi(t_0)\rangle$, with no provision for different points having different $|\Psi(t_0)\rangle$. A spin singlet with particles at points $x_1$ and $x_2$ is represented by the wave function $|0; x_1\rangle|0; x_2\rangle + |1; x_1\rangle|1; x_2\rangle|\sqrt{2}$, whether one speaks about it at $x_1$ or $x_2$, or any other point. It is a function of multiple spacetime points and does not “belong” to any one of them. It is this property which makes the wave function nonseparable (i.e., $\psi(x_1, x_2)$ cannot be written as $\phi(x_1)|\chi(x_2)\rangle$), and thus lies at the core of quantum nonlocality. An augmented MWI could have other QI such as preferred basis information or histories, which could differ from point to point, but since the theory relies on a spatially complete wave function, it is nonlocal.

An argument is sometimes made that the MWI is local because one cannot measure EPR-like correlations until the measured subsystems are brought together, e.g. that if Alice and Bob each measure half of an EPR pair, they can see violations of Bell’s inequalities only after pooling their information in a relativistically causal way. But the wave function in the MWI, which contains QI about both of them. It is thus this property which makes the wave function nonseparable (i.e., $\psi(x_1, x_2)$ cannot be written as $\phi(x_1)|\chi(x_2)\rangle$), and thus lies at the core of quantum nonlocality. An augmented MWI could have other QI such as preferred basis information or histories, which could differ from point to point, but since the theory relies on a spatially complete wave function, it is nonlocal.

There are other arguments that noncollapse models are local. Deutsch and Hayden argue that the Heisenberg representation of the MWI is local, but their formulation does not include position operators. When one does, position is again quantal and the theory is again nonlocal. Griffiths argues that there is no evidence for nonlocal influences in the Consistent Histories Interpretation if one enforces a ‘one-framework rule’. However, the formalism of the approach still contains a spacelike wave function, and thus has nonlocal QI. Quantum field theory does not seem to evade nonlocal-
ity either. The Lagrangian and canonical commutation relations respect locality, but this ensures only that inter-
actions and classical observables are local. The formalism
of field theory still relies on a spatially complete qua-
tern state. Unless quantum theory can be written using a
formalism which is manifestly spatially incomplete, with
QI at each point confined to its forward lightcone, one
cannot claim that the theory is truly local.

The third regime is \( c < v_{QI} < \infty \) (distinguishably less
than \( \infty \)). Such models would be spatially incomplete
and thus could not include the MWI. They would also
be nonlocal and would require a preferred reference frame
in which \( v_{QI} \) is isotropic. Further, even their two-way
communication breaks Lorentz invariance. Viewed in a
frame traveling at \( \beta \) with respect to the preferred frame,
\( \tilde{w}' = \tilde{w}(1 - \beta^2)/(1 - \beta^2 \tilde{w}^2) \approx \tilde{w}(1 - \beta^2) \) for \( \tilde{w} = w_{QI} \)
close to zero. Since \( w_{QI} \) is distinguishable from zero, we
can tell that \( \tilde{w}' \) differs from \( \tilde{w} \).

Next come the discrete and continuous physical space-
times with infinite \( v_{QI} \) (zero \( w_{QI} \)). They can accom-
modate collapse models, or noncollapse models such as
the Bohm model and the MWI. They can be spatially
complete, are nonlocal, and require a preferred reference frame
with a corresponding preferred time. We need to be
cautious about defining the term ‘causal’, though. A
weak criterion we would want any causal theory to meet
is that QI at each spacetime point cannot affect QI in its
past lightcone. A stronger criterion would be that there
is at least one local frame in which QI cannot affect QI
at prior times. ‘Relativistic causality’ is even stronger,
but identical to locality.) Models with zero \( w_{QI} \) satisfy
weak causality, since QI cannot propagate into the past
lightcone, but strong causality is satisfied as well, since
in the preferred frame, no QI can affect an earlier time.
One might argue that an infinite velocity, when viewed
in another frame, seems to go backward in time. But
we can simply insist that the theory be defined in the
preferred frame. And as we showed above, when \( w_{QI} \)
is immeasurably close to zero, \( \tilde{w} \) appears the same in
all frames. Thus a model in these regimes can in prin-
ciple be written in a causal and Lorentz invariant way,
except for the imposition of a preferred frame. We note
that one can define a set of preferred frames in a gener-
gally covariant way by specifying a timelike unit vector \( u^a \)
at each point. The \( u^a \) define spacelike hypersurfaces
which one might say specify a ‘preferred simultaneity’
with each hypersurface having a unique temporal index,
\( \tau \) (this requires that the hypersurfaces do not cross each
other, which is met if \( u^a [\nabla_b u_c] = 0 \)). At each point
on such a hypersurface, the tangent vectors correspond
to infinite velocity in the local preferred frame.

In the discrete infinite case, \( v_{QI} = c/\epsilon \) with \( \epsilon = w_{QI} \)
fixed but indistinguishable from zero. This follows if QI
can travel the length of the Universe in a single unit
of time, i.e., \( \epsilon < c \Delta \tau / L_{Universe} \). For example, if
\( \Delta \tau \sim 1_{\text{Planck}} \sim 10^{-43} \) sec, and \( L_{Universe} \sim 10^{10} \) lyr,
then if \( v_{QI} > 10^{61} \), one can never distinguish it from
\( \infty \). Such models can be effectively spatially complete,
since QI at each location is at most \( \Delta \tau \) out of date. All
QI dynamics could happen in sub-\( \Delta \tau \) time, according
to some new parameter \( \mathcal{T} \), which cycles through those
dynamics each \( \Delta \tau \). In some sense this is a second ‘time’,
which governs QI dynamics. But in this regime we can
also think of \( \tau \) and \( \mathcal{T} \) as different aspects of a single
underlying time \( T \) via a mapping like

\[
\tau = \Delta \tau \int (T/\Delta \tau), \quad \mathcal{T} = T \mod \Delta \tau.
\]

In the continuous case, \( \Delta \tau \to 0 \) and \( v_{QI} = \infty \). Many
physicists, if pressed, would probably pick this regime
since it seems that the wave function is updated every-
where “instantly”, i.e., it is spatially complete. A prob-
lem with this regime is that, since all QI dynamics and
transport take place in zero \( \tau \), one really needs a sec-
ond ‘time’ because the above mapping becomes singular.
One cannot base these dynamics (whatever they are) on
\( \tau \), which along each preferred hypersurface is constant.
But what does it mean to have two times? Picture a
curve \( \gamma(T) \) tracing a path through a cylindrical \( \tau \sim \mathcal{T} \)
space. The path is a helix going through all values of \( \mathcal{T} \)
before advancing infinitesimally in \( \tau \). Thus the helix
has vanishing pitch. In that limit, the curve \( \gamma(T) \)
becomes two-dimensional, because any point belonging
to the two-dimensional \((\tau, \mathcal{T})\) space (e.g., \( S^1 \times \mathbb{R} \)) also
belongs to the curve \( \gamma(T) \) (this is analogous to a singular
mapping of a line to a plane, which are equivalent infinite
sets). Thus in this regime we need two different temporal
parameters: one for the observed temporal evolution to
each new hypersurface, and one for QI transport dynam-
ics along each hypersurface. But unless one can probe
QI transport dynamics, one cannot detect the preferred
frame or the \( \mathcal{T} \) dimension, and all observations will cor-
respond to the usual SO(3,1) Lorentz invariance with one
apparent time.

All the above regimes are specified by QI and the
spacetime on which it exists. Now suppose that QI
does not exist on spacetime, but rather spacetime in-
formation, such as the metric, is part of QI. Then space
would be ‘nonphysical’ in the sense that QI would nei-
ther be stored in nor transported through space. QI
would exist elsewhere, such as a reified Hilbert or Fock
space. If QI encodes all spacetime behavior, space has
at most a secondary role. This is appealing from the
quantum point of view because the spatial wave func-
tion \( \langle \psi(x_1, x_2, t) \rangle = \langle x_1 | x_2 | \psi(t) \rangle \) is only one possible
representation of \( \psi(t) \). In such a model, all QI could
be instantly associated with any spatial point (though
without actually needing transport there), which is like
the infinite speed cases above. So we assign this case
\( w_{QI} = \tilde{w} = 0 \). (If local quantum models were allowed,
one could construct a nonphysical space version, which
would have the same properties as \( w_{QI} = 1 \) physical
space models.) Like those cases, this case is nonlocal
and can be spatially complete (now even spacetime
information could be spatially complete). For causality,
there are two possibilities. If the theory has a preferred
frame, then, just as in physical space, weak and strong causality are satisfied. But it is possible that the theory is completely Lorentz-invariant. In that case, the theory would be only weakly causal because it is the preferred frame which allows us to specify a unique past and future for spacelike interactions. In either case, we do not need the second time $T$ since there is no need to transport QI.

What is the role of ‘space’ here? First, it could be just an abstract entity, a convenient way of expressing certain properties of QI. Then it should be possible to write the theory without reference to space at all. It is not clear how to do this, especially when one brings in gravity. Second, space could be a construct arising from QI. Then space would still exist in some sense and could have a dynamical role, but it would ultimately be explained solely in terms of QI. We note that recently there have been attempts to construct discrete (extra) spatial dimensions out of gauge degrees of freedom [12], but these do not yet include a complete description of gravity.

The fundamental question for nonphysical space models is not “why is there nonlocality”, since nonlocality can arise trivially, but “why is there locality?” Why do classical observables have a limiting speed $c$ if QI (which must account for all classical information) is unfettered by space at all? How would such a theory meld with general relativity where space itself has a dynamical role? Some argue [13] that general relativity prefers space to classical observables having a limiting speed $c$. How would such a theory meld with general relativity where space itself has a dynamical role? Some argue [13] that general relativity prefers space to be nonphysical, with all (quantum) dynamics occurring via the relations between entities. But this might make things more complicated. For example, to specify $N$ points purely relationally requires of order $N^2$ numbers instead of order $N$ (where $N$ is huge). Finally, we have the regimes with $\nu_Q > \infty$, where QI can travel into the past. If any QI can travel arbitrarily far backwards in time, then $\nu_Q$ is really $-\infty$. However, we refer to this case as $\nu_Q = -1$ because the only Lorentz-invariant choice is to restrict propagation to the forward and backward lightcones [13]. Here the definition of “round trip” is ambiguous. It can mean return as close as possible to the starting spacetime point, in which case $\tilde{w} = 0$ (zero round trip time), or as “fast” a return as possible, in which case we could have $\tilde{w} = -1$ (negative round trip time). Both of these possibilities are Lorentz invariant. While classical observables seem causal, the underlying QI dynamics would depend upon QI from the distant past and future. We again need a second time to parametrize this, but it is less clear what this means since causality for QI is, in a real sense, lost.

Instead, one could have $\nu_Q = -1$ with a hard cutoff on how far QI can propagate backwards in time. Such a model would be causal in macroscopic time. In fact, at scales above the cutoff, this case is identical to the infinite $\nu_Q$ cases with the cutoff picking out the preferred frame. Although we again need both $\tau$ and $T$, if classical evolution happens only on scales above the cutoff, and space is finite, then backwards and forwards evolution of $\tau$ in $\tau-T$ space can in principle be represented with a nonsingular mapping $\gamma(T)$, with one underlying time $T$. But if space is infinite, the mapping becomes singular.

So it is useful to characterize quantum models by $\nu_Q$. Since the MWI is spatially complete, it cannot have $\nu_Q = c$ and thus is nonlocal. It is probably not possible to have any local dynamical quantum model, in which case the appealing regimes are those with infinite effective speed. Continuous physical space is appealing from the classical point of view, but such models require a second time for QI transport. Discrete models with $\nu_Q = c/\epsilon$ avoid this, but only if the Universe is finite [14]. Nonphysical space, where spacetime information is part of or arises from QI, is appealing from the quantum point of view, but no complete model exists yet. This tension between classical local spacetime and quantum nonlocality will need to be resolved in any quantum theory of gravity.

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