Probabilistic Harmonic Analysis on Distributed Photovoltaic Integration Considering Typical Weather Scenarios

Che Bin¹, Yu Ruoying², Dang Dongsheng¹ and Wang Xiangyan²

¹State Grid Ningxia Electric Power Company Economic Research Institute, Yinchuan 750002, Ningxia, China
²State Key Laboratory of Operation and Control of Renewable Energy & Storage Systems, China Electric Power Research Institute. Beijing 100192, China

Abstract. Distributed Generation (DG) integrating to the network would cause the harmonic pollution which would cause damages on electrical devices and affect the normal operation of power system. On the other hand, due to the randomness of the wind and solar irradiation, the output of DG is random, too, which leads to an uncertainty of the harmonic generated by the DG. Thus, probabilistic methods are needed to analyse the impacts of the DG integration. In this work we studied the harmonic voltage probabilistic distribution and the harmonic distortion in distributed network after the distributed photovoltaic (DPV) system integrating in different weather conditions, mainly the sunny day, cloudy day, rainy day and the snowy day. The probabilistic distribution function of the DPV output power in different typical weather conditions could be acquired via the parameter identification method of maximum likelihood estimation. The Monte-Carlo simulation method was adopted to calculate the probabilistic distribution of harmonic voltage content at different frequency orders as well as the harmonic distortion (THD) in typical weather conditions. The case study was based on the IEEE33 system and the results of harmonic voltage content probabilistic distribution as well as THD in typical weather conditions were compared.

1. Introduction
The traditional power grid has some disadvantages: long construction period, large investment and complex reliability guarantee technology which would easily cause the accidents like power shortage and large area blackout. Distributed power generation such as solar power and wind power, is environmentally efficient, flexible, cost-effective, short construction period and less risk, which supplements the central power generation. The coordinated development between the distributed generation and the central generation could help save the investment, reduce energy consumption, and improve the reliability and flexibility of power system. The distributed generation (DG)[1-2] thus becomes an important direction of the world's energy strategy.

The most common types of DG are wind power, solar power, small hydropower, micro-sized gas turbine, and fuel cell. Mostly, the frequency of the power produced by DG has to be converted into working frequency through electronic devices which would cause voltage fluctuations and flicker, unbalance of three-phase voltage and harmonic distortion, damaging on electrical devices and affecting the normal operation of power system[3-5].
On the other hand, due to the randomness of the wind and solar irradiation, the output of DG is random, too, which leads to an uncertainty of the harmonic generated by the DG. Thus, probabilistic methods are needed to analyse the impacts of the DG integration. Common methods of probability analysis are analytic methods[6-9] and simulation methods[10].

Since weather condition has great impacts on PV power output[11-13], this paper analysed the harmonic distribution and harmonic distortion in distribution network caused by distributed photovoltaic (PV) power under different weather conditions. Parameter identification method has been applied to obtain the probability distribution model which is the base of Monte-Carlo simulation to calculate the harmonic voltage probability distribution and the total harmonic distortion (THD).

2. Probability Model of PV Output

2.1. Model of PV Output

The principle of PV power generation mainly is transmitting the solar radiation energy into electrical energy by utilizing the photoelectric effect. The major components include the PV array, which is consisted of the solar cell panel, the grid inverter and controller, shown in Figure 1.

![Figure 1. Composition of the PV System](image)

The power output of the PV cells is primarily related with the solar radiation intensity, the area of PV array and the photoelectric conversion efficiency. Therefore, the total PV power output can be approximately described as:

\[ P_{pv} = r \cdot A \cdot \rho \]  

(1)

Where, \( r \) is the intensity of solar radiation, \( A \) is the area of PV array, \( \rho \) is the photoelectric conversion efficiency.

2.2. The solar radiation probability model in different weather conditions

The probability distribution of the solar radiation density is commonly described as Beta distribution, whose density function[14] is:

\[ f(r) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \left( \frac{r}{r_{\text{max}}} \right)^{\alpha-1} \left( 1 - \frac{r}{r_{\text{max}}} \right)^{\beta-1} \]

(2)

Where, \( r \) is the intensity of solar radiation, \( r_{\text{max}} \) is the maximum value of the intensity of solar radiation, \( \alpha, \beta \) are the shape parameters of the Beta distribution.

Within one certain area, there are many factors that could affect the solar radiation, including the weather conditions, which is the domain factor. The solar radiation probability distribution varies with the weather conditions, thus makes it necessary to acquire the typical solar radiation probability distribution of different weather. According to the weather data in Golmud, China, the typical weather types are summarized as: sunny day, cloudy day, rainy day and snowy day, corresponding to the PV power output curves shown in Figure 2.

![Figure 2. PV Power Output Curves](image)
2.3. The Probability Distribution Model of PV Power Output
Combine equation (1) with equation (2), the PV power output probability distribution can be described as:

\[ f(P_{PV}) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha) \Gamma(\beta)} \left( \frac{P_{PV}}{P_{\text{max}}} \right)^{\alpha-1} \left( 1 - \frac{P_{PV}}{P_{\text{max}}} \right)^{\beta-1} \]  

(3)

Where, \( P_{\text{max}} \) is the maximum value of PV, which could be calculated through:

\[ P_{\text{max}} = r_{\text{max}} \cdot A \cdot \rho \]  

(4)

Hence the PV power output also obeys the Beta distribution whose shape parameters of the typical weather conditions could be acquired through the PV power output data by using parameter identification methods.

3. Harmonic Analysis Basis

3.1. Harmonic Analysis model of Distribution system
The Harmonic model of the distribution network including the harmonic model of electrical components such as distributed PV, transformer, transmission line and load. In order to study the harmonic voltage distribution of the ideal distribution network in the condition of steady state, the model of the distribution network integrated with PV should be done, and the equivalent circuit diagram of distribution network of single harmonic is shown in Figure 3.

Figure 3. Equivalent Circuit Diagram of The Distribution Network of single harmonic

In Figure 3, the ideal distribution network does not have harmonic sources except the PV inverter, that is to say, there is no background harmonic at all. The distributed PV system acts as a harmonic current source injecting into the distribution network with the \( h \)th harmonic current \( I_{\text{sh}} \) after integrating. \( Z_{Lb}^h \) is the \( h \)th harmonic equivalent impedance behind the point of connection(PC), \( Z_{L}^h \) is the \( h \)th harmonic equivalent impedance before PC, \( Z_{\text{eq}}^h \) is the \( h \)th harmonic equivalent impedance between point of load(PL) and point of common coupling(PCC), which depends on the harmonic impedance of transformer and lines. \( I_{Lbh} \) is the \( h \)th harmonic current of \( I_{Lb} \), \( I_{Lh} \) is the \( h \)th harmonic current flowing from PC to PL, \( I_{Lh} \) is the \( h \)th harmonic current of \( I_{L} \), \( I_{\text{eqh}} \) is the \( h \)th harmonic current of \( I_{\text{eq}} \). According to the Kirchhoff’s Current Law, the following equations are founded:

\[ I_{\text{sh}} - I_{Lbh} - I_{Lh} = 0 \]  

(5)

\[ (Z_{eq}^h / Z_L^h) I_{Lh} = Z_{Lb}^h I_{Lbh} \]  

(6)

Thus, the current flowing to the PCC \( I_{\text{eqh}} \) could become:

\[ I_{\text{eqh}} = \frac{1}{Z_{eq}^h / Z_L^h + Z_{eq}^h / Z_{Lb}^h + 1} I_{\text{sh}} \]  

(7)

From the equation (7), it can be seen that the current flowing to the PCC \( I_{\text{eqh}} \) is determined by \( Z_{eq}^h \), \( Z_L^h \) and \( Z_{Lb}^h \). Since \( Z_{eq}^h \) is far less than \( Z_L^h \) and \( Z_{Lb}^h \), the harmonic current mostly flow into the PCC.

3.2. Voltage THD and Individual Harmonic Components
Harmonic voltages are assessed both individually and as total harmonic distortions. Individual harmonic component $U_h$ is defined as the percentage of voltage for order $h$ with respect to the fundamental voltage $U_1$, as given in (8).

$$U_h(\%) = \frac{U_h}{U_1} \times 100\%$$

$THD_U$, on the other hand, is the ratio of the RMS value of all the harmonic components of the signal, to the fundamental voltage $U_1$. Equation (9) shows the correlation between $THD_U$ and individual components.

$$THD_U = \sqrt{\frac{\sum_{h=2}^{\infty} (U_h)^2}{U_1}}$$

### 4. Framework of Harmonic Probabilistic Assessment

In this paper, the Monte Carlo method is adopted to analyse the harmonic probability distribution of the network with the distributed PV system integrated in.

Seen from the flowchart of the calculation in Figure 4, the shape parameters ($\alpha$, $\beta$) of the probability distribution of PV power output should be calculated firstly by using the parameter identification method. Thus the probability distribution model of PV output is obtained to sample the value of distributed PV power output, which is used to finish the harmonic power flow calculation. Then, the harmonic injection current can be calculated and thus the harmonic voltage of nodes is acquired. After accomplishment of all the sampling missions, the harmonic voltage probability distribution and the $THD_U$ value are obtained.

![Figure 4. Flowchart of the Calculation](image)

### 5. Case Study and Results

The study system shown in Figure 5 is a 33-bus 12.6 kV distributed network. Bus0 is the slack bus connected to the main grid. The slack bus voltage is set to 1.05 p.u. to keep all node voltages maintained at least at 0.95 p.u. The branch impedance parameters and the bus load is given in [15], in which the bus load is the total three-phase load of the bus. The system total active load is 3715.0 kW, reactive load is 2300.0 kvar.
The DPV source is integrated to the system at bus17, which is the end of the distributed network. The rated power of DPV is 10 MW and the penetration is around 23%. The DPV power output curves in typical weather conditions are shown in Figure 2.

![Figure 5. 33-Bus Test System](image)

5.1. Parameter Identification for Typical weather condition

Formula (4) shows that the PV power output also obeys the Beta distribution as the solar radiation does. Corresponding to the typical PV power output curves shown in the Figure 2, the Beta distribution parameters are estimated through maximum likelihood method. The results are in Table 1.

|        | Sunny | Cloudy | Snowy | Rainy |
|--------|-------|--------|-------|-------|
| $\alpha$ | 4.1160 | 1.1472 | 1.0656 | 1.9589 |
| $\beta$  | 3.1084 | 4.2373 | 5.9681 | 7.9845 |

According to the results in Table 1, the probabilistic distribution functions of DPV power output in typical weather conditions are acquired. Adopting the Monte-Carlo simulation, the harmonic voltage content of different orders at PCC, harmonic voltage probabilistic distribution at PCC and the THD$U$ are calculated, as follows:

5.2. Harmonic Voltage Content

Harmonic voltage content of different orders at PCC of sunny, cloudy, snowy and rainy weather conditions are shown in Figure 6.

![Figure 6. Harmonic Voltage Content of Typical Weather Conditions](image)

5.3. Harmonic Voltage Probability Distribution

5.3.1. Sunny condition. When the weather condition is sunny, the simulation result is shown in Figure 7.
5.3.2. **Cloudy condition.** When the weather condition is cloudy, the result is shown in Figure 8.

![Figure 7. Harmonic Voltage of sunny condition](image)

**Figure 7.** Harmonic Voltage of sunny condition

![Figure 8. Harmonic Voltage of Cloudy Condition](image)

**Figure 8.** Harmonic Voltage of Cloudy Condition

5.3.3. **Snowy condition.** When the weather condition is cloudy, the result is shown in Figure 9.

![Figure 9. Harmonic Voltage of Snowy Condition](image)

**Figure 9.** Harmonic Voltage of Snowy Condition

5.3.4. **Rainy condition.** When the weather condition is rainy, the result is shown in Figure 10.

![Figure 10.](image)

**Figure 10.**
5.4. THD<sub>U</sub> Results

| Weather Conditions | sunny  | cloudy | snowy  | rainy  |
|--------------------|-------|--------|--------|--------|
| THD<sub>U</sub> Value | 1.03×10<sup>-2</sup> | 1.07×10<sup>-2</sup> | 1.26×10<sup>-2</sup> | 1.01×10<sup>-2</sup> |

From Table 2 and figures above, we could infer that:

(1) The DPV power output beta distribution parameter α gets the maximum value when it is sunny and gets the minimum value when it is rainy; parameter β gets the maximum value when it is rainy and gets the minimum value when it is sunny.

(2) The harmonic voltage content varies as the weather changes, however, the trends are almost the same and the orders that have the highest content are constant: 7<sup>th</sup>, 13<sup>th</sup>, 16<sup>th</sup>, 17<sup>th</sup>.

(3) The probability distribution of the harmonic voltage varies as the weather changes, and in the same weather condition, the distribution of different voltage orders are similar.

(4) The THD<sub>U</sub> varies as the weather changes. The most serious distortion happens when it is snowy, and the second is cloudy. The smallest THD<sub>U</sub> happens when it is rainy.

6. Conclusion
This paper studied the effects of different weather conditions on the PV harmonic distribution variations. The maximum likelihood method was adopted to estimate the probabilistic distribution model of the PV power output in different weathers. And the harmonic probabilistic distribution as well as the THD of typical weathers could be calculated via Monte-Carlo simulation.

Based on the 33-bus case study, the probability distribution for several orders of harmonic was evaluated. Relatively, the odd order harmonics are dominant and have a wide variance, and the even order harmonics are smaller. The THD in snowy weather is more serious than other typical conditions. This study could help planning the PV integrating into distribution network and designing the filters by finding the boundaries for each harmonic.

Acknowledgments
This work is supported by project of State Grid Ningxia Electric Power Company

References
[1] Thomas, A. GOran, S. Lennart, Distribution generation: a definition, Electric PowerSyst. Res.57(3)(2001)195-204
[2] Dugan, R.C, McDermott, T.E, Distributed generation[J],IEEE industry Applications Magazine,2002,8(2):19-25
[3] D Ruoss, S Taiana, Grid Interconnection of Building Integrated and Other Dispersed Photovoltaic Power Systems Report IEA PVPS T5-01: 1998
[4] B Verhoeven UTILITY ASPECTS OF GRID CONNECTEDPHOTOVOLTAIC POWER SYSTEMS, 1998.
[5] AE Emanuuel, WF Horton, WT Jewel, DJ Phileggi IEEE Task Force on the Effects of Harmonics. Effects of harmonic on equipment[J]. IEEE Trans on Power Delivery, 1993,8(2).
[6] Allan R N, Leite de Silva A M, Burchett R C. Evaluation methods and accuracy in probabilistic load flow solutions. IEEE Trans on Power Apparatus and Systems[J].1981,100(5):2539-2546
[7] Zhang P, Lee S T. Probabilistic load flow computation using the method of combined cumulants and Gram-Charlier expansion IEEE Trans on Power Systems[J].2004,19(1):676-682
[8] Allan R N, Leitede SAM, Burchett R C. Evaluation methods and accuracy in probabilistic load flow solutions IEEE Trans on PAS[J].1981,100(5):2539-2546
[9] SH Shehadeh, HH Aly, ME El-Hawary. PV Harmonic Distribution Analysis for Various Conditions studied the effect of changing the blanking angle of the square-wave inverter on the harmonics [C]. IEEE Canadian Conference on Electrical & Computer Engineering, 2016
[10] Pallavi T. Sawant; C. L. Bhattar. Optimization of PV System Using Particle Swarm Algorithm under Dynamic Weather Conditions 2016 IEEE 6th International Conference on Advanced Computing (IACC).
[11] Hong-Tzer Yang; Chao-Ming Huang; Yann-Chang Huang; Yi-Shiang Pai. A Weather-Based Hybrid Method for 1-Day Ahead Hourly Forecasting of PV Power Output IEEE Transactions on Sustainable Energy Year: 2014, Volume: 5, Issue: 3 Pages: 917 – 926.
[12] N. Mutoh; M. Ohno; T. Inoue. A Method for MPPT Control While Searching for Parameters Corresponding to Weather Conditions for PV Generation Systems IEEE Transactions on Industrial Electronics Year: 2006, Volume: 53, Issue: 4 Pages: 1055 – 1065.
[13] Karaki SH, Chedid R B, Ramadan R. Probabilistic performance assessment of autonomous solar-wind energy conversion systems [J]. IEEE Trans on Energy Conversion, 1999, 14(3): 766-772.
[14] Electrical Installation Guide According to IEC International Standards, Schneider Electric S.A, 2016.
[15] MewE. Baran Felix F. Wu, Network reconfiguration in distribution systems for loss reduction and load balance. [J]. IEEE Transactions on Power Delivery, Vol. 4, No. 2, April 1989.