Theoretical Analysis of Supersonic Low Reynolds Number Flow Control Technique Based on Flow Mixing

Xu Dachuan *, Ren Zebin, Chen Zhiqiang, Wang Haifeng and Guo Longde
Facility Design and Instrument Institute, China Aerodynamics Research and Development Center, Mianyang 621000, China

*E-mail: xude_ustc@hotmail.com

Abstract. The thickness of boundary layer increases rapidly in supersonic low Reynolds number flows. As a result, there will be shock wave-boundary layer interference. The disturbances of downstream can spread upstream through in boundary layer because the flow in boundary layer is subsonic, which may decrease flow field uniformity and stability. The pressure of this type of flow is always very low. For the sake of flow continuity, the increase of flow pressure is necessary. A kind of supersonic low Reynolds number flow control technique was proposed. The high pressure supersonic flow jetted from plates which located in the flow channel blew away boundary layer, cut off the disturbances spread from downstream to upstream, and eliminated the shock wave-boundary layer interference. The pressure of primary flow was also increased by mixing with the high pressure jetting flow. The calculation model of this flow control technique was established. A group of standard inputs were selected to calculate the parameters of flow and the preliminary law of flow was also obtained.

1. Introduction
In some supersonic low Reynolds number flows, such as flow with chemical reaction or combustion, the thickness of boundary layer increases rapidly, which will cause Shock wave/Boundary Layer Interation. The major feature of Shock wave/Boundary Layer Interation is strong adverse pressure gradient created near surface. Separation will be prompt by strong adverse pressure gradient. The separation shock wave in leading edge and reattachment shock wave in trailing edge will make the flow more complex. Unsteady fluctuation will be generated as unsteady characteristics of separation. Disturbances created downstream could spread upstream through boundary layer, as flow in boundary layer is subsonic, which will destroy flow field structure, decline the quality of chemical reaction or combustion. Besides, the exhaust needs to be pressurized for continuity of chemical reaction or combustion, as the pressure of flow is usually very low [1-2].

The flow control technique proposed in this paper is a type of active control. High pressure supersonic air flow streamed through pylons which are arranged in flow passage can blow away boundary layer, eliminate the shock waves interference, and improve the character of flow field, meanwhile increase pressure of exhaust.

2. Theoretical analysis model

2.1. Technique principle
As shown in figure 1, pylons, through which high pressure supersonic air flow jets out, are arranged in reaction cavity flow passage. As the name implies, central pylons are arranged in the central area of flow passage, periphery pylons are arranged around flow passage. Major flow and jetting flow enter into mixing section after preliminary mixing. It is assumed that after interaction and interblending, the major flow and jetting flow become a uniform flow in mixing section. The passage area of mixing section is not constant. The sectional area of front part is contractive, and constant in the middle part. It becomes a diffuser in the latter part. The purpose of this type of mixing section is decreasing pressure loss. The contractive area can lower flow Mach number, which will decrease the loss of shock loss. The uniform flow becomes subsonic from supersonic through several oblique shock wave near the inlet of diffuser. The pressure of subsonic flow in diffuser is increasing with area diffusing, and deceleration of velocity. In the calculation proposed in this paper, subscript 1 presents the parameters of major flow, subscript 2 presents the parameters of jetting flow, subscript 3 presents the parameters of mixing flow.

![Figure 1. Technique sketch diagram](image)

The main functions of periphery pylons are blowing away the boundary layer of major flow in reaction cavity, weakening or even eliminating the Shock wave/Boundary Layer Interaction by jetting high pressure supersonic air flow. The main functions of central pylons are eliminating the wave structures, improving the flow field in reaction cavity. Meanwhile, as the pressure of jetting flow is higher than major flow, the pressure of major flow can be increased by the mixing with jetting flow. The increase of flow pressure is beneficial for the design of downstream system.

Before deriving the theoretical analysis model, several hypotheses are listed as follows.
1) The flow is steady.
2) Both major flow and jetting flow are uniform, and the mixing flow is also uniform.
3) The fluid of major flow, jetting flow and mixing flow is perfect gas.
4) The shear stress of wall is ignored.
5) The flow is adiabatic.

2.2. Area ratio and flux ratio
Area ratio is the ratio of total area of jetting flow passage and total area of flow passage in cross section of pylons outlet, and can be written as:

\[ F = \frac{A_2}{A_1 + A_2} \]  

(1)

F is area ratio, \( A_2 \) is the area of jetting flow passage, \( A_1 \) is the area of major flow passage, and the position is in the cross section of pylons outlet.

Flux ratio is the ratio of major flow rate and jetting flow rate, and can be written as:

\[ n = \frac{\dot{m}_1}{\dot{m}_2} \]  

(2)

Cavity
Periphery pylons
Mixing section
Central pylons
Diffuser
n is flux ratio, \( \dot{m}_1 \) is major flow rate, \( \dot{m}_2 \) is jetting flow rate.

Both \( \dot{m}_1 \) and \( \dot{m}_2 \) are expanded by flow formula, and the equation of area ratio and flux ratio can be obtained.

\[
n = \frac{C_1 q_1 p_{01}(1-F)}{C_2 q_2 p_{02}F} \frac{T_{02}}{T_{01}}
\]

(3)

\( C_1 \) is aerodynamics parameter which is related to the gas physical property, \( q \) is the mass flow coefficient, \( P_0 \) is total pressure of flow, \( T_0 \) is total temperature of flow.

In the theoretical analysis model, the flux ratio is confirmed firstly, then the area ratio can be obtained by calculation. Once the geometrical parameters of reaction cavity are confirmed, both the major flow passage area and the jetting flow passage area can be obtained.

2.3. Momentum equation in mixing section

The flow process prediction and control of major flow and jetting flow in mixing section is one of the key problems in the theoretical analysis model establishment [3]. It is assumed that the major flow and jetting flow become a mixing uniform flow in mixing section. The variation rule of main flow parameters in mixing section can be described by momentum equation, which can be written as:

\[
\dot{m}_3 V_3 - (\dot{m}_1 V_1 + \dot{m}_2 V_2) = (p_1 A_1 + p_2 A_2) - p_3 A_3 + \int_{wall} p(l) dl
\]

(4)

\( \dot{m}_3, V_3, p_3 \) presents the mass flow rate, velocity and pressure of uniform flow at the outlet of contractive part in mixing section respectively. \( A_3 \) is the flow passage area at the outlet of contractive part. \( p(l) \) is the pressure distribution in mixing section, which reflects the interaction and interblending of major flow and jetting flow in mixing section. The pressure distribution is also related to the geometrical parameters of mixing section. The rule of pressure distribution is the key to solve the momentum equation. If there is significant difference between the pressure distribution adopted in solving momentum equation and real pressure distribution, the correct theoretical analysis model cannot be obtained [4-5].

Some special pressure distribution in mixing section can be adopted in order to simplify the calculation, for example, the linear pressure distribution as follows:

\[
\int_{wall} p(l) dl = -\frac{p_2 + p_3}{2} (A_1 + A_2 - A_3)
\]

(5)

Equation (4) and (5) give:

\[
E_3 \lambda_3 = E_2 \lambda_2 + n \sqrt{\theta} E_1 \lambda_1 + \frac{\pi c}{c_2 n_2} (\frac{\gamma+1}{2} k \Phi - \frac{k-1}{k}) - D \frac{n_3 (1+\Phi)}{2 c_3 q_3} \Phi
\]

(6)

\( \Phi \) is contraction ratio, \( c \) is ratio of specific heat at constant pressure of major flow and jetting flow, \( k \) is ratio of static pressure. D, E, \( \pi \) is the aerodynamics parameter related to gas physical property, \( \lambda \) is velocity coefficient.

\[
\Phi = \frac{A_3}{A_1 + A_2}, k = \frac{p_2}{p_1}, c = \frac{C_p_1}{C_p_2}, E = \frac{2 \sqrt{\gamma}}{\gamma + 1}, D = (n + 1) \sqrt{\frac{n c \theta + 1}{n c + 1}}, \pi = \left(1 - \frac{\gamma - 1}{\gamma + 1} \frac{\lambda^2}{\gamma - 1}\right)^{\frac{\gamma}{\gamma - 1}}
\]

The relationship of velocity coefficient \( \lambda_3 \) at mixing section outlet and contraction ratio \( \Phi \) can be obtained once the parameters of major flow and jetting flow are confirmed.

2.4. Calculation of parameters at outlet

Each velocity coefficient \( \lambda_3 \) at mixing section outlet corresponding a contraction ratio \( \Phi \). Through \( \lambda_3 \) and parameters of diffuser, the parameters at outlet can be obtained by calculation. In real calculation, a reasonable range of \( \lambda_3 \) can be selected firstly, then each corresponding contraction ratio and diffuser outlet parameters can be obtained by calculation. If the main objective is high pressure at diffuser...
outlet, the contraction ratio which corresponding the highest outlet pressure is selected as optimal contraction ratio.

The total pressure ratio at mixing outlet is defined as:

\[ P_m = \frac{P_{03}}{P_{01}} \]  

(7)

After derivation, \( P_m \) can be rewritten as:

\[ P_m = \frac{P_{02}C_2q_2DF}{P_{01}C_1q_1\sigma} \]  

(8)

The uniform flow becomes subsonic from supersonic through several oblique shock wave near the inlet of diffuser. Experiment results indicate: the pressure of flow which becomes subsonic from supersonic through several oblique shock wave approximately equals to the flow which through a normal shock wave. Therefore, the normal shock wave relation can be used to estimate the static pressure recovery of flow.

The relationship of normal shock wave pressure recovery coefficient \( \sigma \) and wavefront Mach number \( Ma \) can be written as follows:

\[ \sigma = \left( \frac{2\gamma}{\gamma+1}Ma^2 - \frac{\gamma-1}{\gamma+1} \right)^{\frac{1}{\gamma-1}} \left[ \frac{(\gamma+1)Ma^2}{(\gamma-1)Ma^2+2} \right]^\frac{\gamma}{\gamma-1} \]  

(9)

The total pressure recovery coefficient is defined as:

\[ P_t = P_m \sigma \]  

(10)

The contraction ratio which corresponding the highest total pressure coefficient is the optimal contraction ratio. Therefore, the calculation of theoretical analysis model can be accomplished once the parameters of major flow and jetting flow are confirmed.

3. Theoretical analysis

3.1. Standard input parameters

In order to summarize the effects of key parameters on flow performance, a group of standard input parameters is selected in theoretical analysis, which are listed in table 1 and table 2.

| parameter | value | unit |
|-----------|-------|------|
| \( Ma_1 \) | 0.3 |  |
| \( T_{01} \) | 500 | K |
| \( \gamma_1 \) | 1.4 |  |
| \( P_{01} \) | 592 | Pa |
| \( P_1 \) | 400 | Pa |
| \( R_1 \) | 287 | J/(kg*K) |
| \( \dot{m}_1 \) | 200 | g/s |
Table 2 Parameters of jetting flow

| parameter | value | unit |
|-----------|-------|------|
| Ma₂       | 4.0   |      |
| T₀₂       | 300   | K    |
| γ₂        | 1.4   |      |
| P₀₂       | 84.4  | kPa  |
| P₂        | 400   | Pa   |
| R₂        | 287   | J/(kg•K) |
| m₂        | 1.33  | kg/s |

3.2. Optimal contraction ratio
The variation curves of mixing section contraction ratio Φ, pressure recovery coefficient σ, total pressure ratio Pₘ with velocity coefficient λ₃ are shown in figure 2. The curves indicate that in the case of confirmed inlet flow parameters of mixing, the optimal contraction ratio exist which the corresponding total pressure recovery coefficient Pₘ is maximum. As shown in figure 2, with increase of velocity coefficient λ₃, the pressure recovery coefficient σ decreases gradually and total pressure ratio Pₘ increase. The product of σ and Pₘ is maximum at optimal contraction ratio, which means the total pressure recovery coefficient Pₘ is maximum.

Figure 2. Variation curves of key design parameters with λ₃

3.3. Flux ratio
The variation curve of total pressure recovery coefficient Pᵢ with flux ratio n is shown in figure 3. The diagram indicates that the total pressure recovery coefficient Pᵢ decrease gradually with increase of flux ratio n. The reason is that with increase of flux ratio n, the proportion of low energy major flow is increase, and the energy of mixing uniform flow is also decrease. In the case of constant major flow Mach number Ma₁ and jetting flow Mach number Ma₂, it result in the decreasing of total pressure
recovery coefficient $P_t$. Meanwhile, in order to keep the static pressure ratio of major flow and jetting flow constant, the area ratio decrease as the static pressure of major flow decrease.

![Figure 3. Variation curve of $P_t$ with $n$](image)

### 3.4. Jetting flow Mach number
The variation curve of total pressure recovery coefficient $P_t$ with jetting flow Mach number $Ma_2$ is shown in figure 4. The result indicates that the total pressure recovery coefficient $P_t$ increase with the increase of jetting flow Mach number $Ma_2$. The reason is that in the case of constant flux ratio $n$, increase of jetting Mach number $Ma_2$ means the pressure of jetting flow is higher, the area of pylons outlet decrease but kinetic energy increase. As a result, the total pressure recovery coefficient $P_t$ increases with the increase of jetting flow Mach number $Ma_2$.

![Figure 4. Variation curve of $P_t$ with $Ma_2$](image)

### 4. Conclusion
The technique principle of supersonic low Reynolds number flow control technique based on flow mixing is proposed in this paper. The basic flow process is analysed, and the theoretical analysis model is obtained. The effects of key parameters on flow performance are summarized by calculation through a group of standard input parameters. There is an optimal contraction ratio to achieve the
highest total pressure recovery coefficient. The lower flux ratio is beneficial for improving total
pressure recovery coefficient. Increase jetting flow Mach number is also beneficial for improving total
pressure recovery coefficient.

References
[1] Xu H 1987 Fundamentals of aerodynamics (Beijing: Beijing Aeronautics College Press)
[2] Liu Z 2003 Aerodynamic and structure design of high speed and low speed wind tunnel
(Beijing: National Defence Industry Press)
[3] Liao D and Ren Z 2006 High Power Laser and Particle Beams, 18 728
[4] Liao D 2003 An investigation of optimum performance and mixing enhancement for gaseous
ejectors (Xi’an: Northwestern Polytechnical University)
[5] Emanuel G 1974 AIAA Journal, 14 1292