Soft $b$-continuous and soft $b$-irresolute multifunctions

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Abstract

In this paper, $b$-continuity (and $b$-irresolute) of multifunctions, which defined between soft topological spaces, is introduced and studied. We also investigate some properties of these multifunctions. Then we discussed the relationships among pre continuity, semi continuity and $\beta$-continuity of these multifunctions.

Keywords: Soft $b$-open sets, soft continuity, soft multifunctions.

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1. Introduction

Researchers deal the complexity of uncertain data in many fields such as economics, engineering, environment science, sociology, medical science etc. In 1999, Molodtsov initiated the novel concept of soft set as a new mathematical tool for dealing with uncertainties. Molodtsov [1] initiated a novel concept of soft set theory, which is a new mathematical tool for dealing with such uncertainties. He successfully applied the soft set theory into several directions such as smoothness of functions, game theory, Riemann Integration, theory of measurement, and so on. In the recent years, in development the fields of soft set theory and its applications has been taking place in a rapid pace. Maji and et al. [2], Pei and et al. [3], Çağman and et al. [4], Shabir and Naz [5], I. Zorlu et al. [6], Aygunoglu et al. [11], Kharal et al. [9] and many researchers studied this theory and they got many important results. Soft $b$-open sets firstly defined by Akdağ and Ozkan [7].

In this paper, $b$-continuity (and $b$-irresolute) of multifunctions which defined between soft topological spaces is introduced and studied. We also investigate the properties of these multifunctions. Then, we discussed the relationships among pre continuity, semi continuity and $\beta$-continuity of these soft multifunctions.
2. Preliminaries

Definition 2.1. [1] Let $X$ be an initial universe and $E$ be a set of parameters. Let $P(X)$ denote the power set of $X$ and $A$ be a non-empty subset of $E$. A pair $(F, A)$ is called a soft set over $X$, where $F$ is a mapping given by $F : A \rightarrow P(X)$. In other words, a soft set over $X$ is a parameterized family of subsets of the universe $X$. For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of $\varepsilon$-approximate elements of the soft set $(F, A)$.

Definition 2.2. [2] A soft set $(F, A)$ over $X$ is called a null soft set, denoted by $\Phi$, if $e \in A$, $F(e) = \emptyset$.

Definition 2.3. [2] A soft set $(F, A)$ over $X$ is called an absolute soft set, denoted by $\tilde{A}$, if $e \in A$, $F(e) = X$.

Definition 2.4. [5] Let $Y$ be a non-empty subset of $X$, then $\tilde{Y}$ denotes the soft set $(Y, E)$ over $X$ for which $Y(e) = Y$, for all $e \in E$.

Definition 2.5. [2] The union of two soft sets of $(F, A)$ and $(G, B)$ over the common universe $X$ is the soft set $(H, C)$, where $C = A \cup B$ and for all $e \in C$,

\[ H(e) = \begin{cases} F(e), & \text{if } e \in A - B, \\ G(e), & \text{if } e \in B - A, \\ F(e) \cup G(e), & \text{if } e \in A \cap B. \end{cases} \]

We write $(F, A) \tilde{\cup} (G, B) = (H, C)$.

Definition 2.6. [2] The intersection $(H, C)$ of two soft sets $(F, A)$ and $(G, B)$ over a common universe $X$, denoted $(F, A) \tilde{\cap} (G, B)$, is defined as $C = A \cap B$, and $H(e) = F(e) \cap G(e)$ for all $e \in C$.

Definition 2.7. [2] Let $(F, A)$ and $(G, B)$ be two soft sets over a common universe $X$. $(F, A) \tilde{\cap} (G, B)$, if $A \subset B$, and $H(e) = F(e) \subset G(e)$ for all $e \in A$.

Definition 2.8. [5] Let $\tau$ be the collection of soft sets over $X$, then $\tau$ is said to be a soft topology on $X$ if satisfies the following axioms.

1. $\Phi, \tilde{X}$ belong to $\tau$,
2. the union of any number of soft sets in $\tau$ belongs to $\tau$,
3. the intersection of any two soft sets in $\tau$ belongs to $\tau$.

The triplet $(X, \tau, E)$ is called a soft topological space over $X$. Let $(X, \tau, E)$ be a soft topological space over $X$, then the members of $\tau$ are said to be soft open sets in $X$. A soft set $(F, A)$ over $X$ is said to be a soft closed set in $X$, if its relative complement $(F, A)^c$ belongs to $\tau$.

Definition 2.9. [8] For a soft set $(F, A)$ over $X$ the relative complement of $(F, A)$ is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$, where $F^c : A \rightarrow P(X)$ is a mapping given by $F^c(\alpha) = X - F(\alpha)$ for all $\alpha \in A$.

Definition 2.10. [7] A soft set $(F, A)$ in a soft topological space $X$ is called:

1. soft $b$-open set iff $(F, A) \tilde{\cap} \text{int}(\text{cl}((F, A))) \tilde{\cup} \text{cl}(\text{int}((F, A))))$.
2. soft $b$-closed set iff $(F, A) \tilde{\cap} \text{int}(\text{cl}((F, A))) \tilde{\cup} \text{cl}(\text{int}((F, A))))$.

Theorem 2.11. [7] For a soft set $(F, A)$ in a soft topological space $X$,

1. $(F, A)$ is a soft $b$-open set iff $(F, A)^c$ is a soft $b$-closed set.
2. $(F, A)$ is a soft $b$-closed set iff $(F, A)^c$ is a soft $b$-open set.

Definition 2.12. [7] Let $(X, \tau, E)$ be a soft topological space and $(F, A)$ be a soft set over $X$. 

Theorem 2.15. [7] In a soft topological space \((F, A)\) in \(X\) is denoted by
\[
\text{bcl}((F, A)) = \overline{\{((F, E) \supseteq (F, A) : (F, E) \text{ is a soft b-closed set of } X\}}.
\]
(2) Soft b-interior of a soft set \((F, A)\) in \(X\) is denoted by
\[
\text{bint}((F, A)) = \bigcup\{((O, A) \supseteq (F, A) : (O, A) \text{ is a soft b-open set of } X\}.
\]
Clearly \(\text{bcl}((F, A))\) is the smallest soft b-closed set over \(X\) which contains \((F, A)\) and \(\text{bint}((F, A))\) is the largest soft b-open set over \(X\) which is contained in \((F, A)\).

Theorem 2.13. [7] Let \((F, A)\) be any soft set a in soft topological space \(X\). Then:

1. \(\text{bcl}((F, A)^{c}) = \overline{X} - \text{bint}((F, A))\).
2. \(\text{bint}((F, A)^{c}) = \overline{X} - \text{bcl}((F, A))\).

Remark 2.14. [7]
1. If \((F, A)\) is a soft set of soft topological space \(X\), then \(\text{bcl}((F, A))\) is the smallest soft b-closed set containing \((F, A)\). Thus \(\text{bcl}((F, A)) = (F, A) \overline{\cup \{\text{int}(\text{cl}(\text{bint}(F, A)))\} \cup \text{cl}(\text{bint}(F, A))}\).
2. If \((F, A)\) is a soft set of soft topological space \(X\) then \(\text{bint}((F, A))\) is the largest soft b-open set contained in \((F, A)\). Thus \(\text{bint}(F, A) = (F, A) \overline{\cap \{\text{int}(\text{cl}(F, A))) \cup \text{cl}(\text{bint}(F, A))\}}\).

Theorem 2.15. [7] In a soft topological space \(X\) the following hold for sb-closure.

1. \(\text{bcl}(\Phi) = \Phi\).
2. \(\text{bint}(\Phi) = \Phi\).
3. \(\text{bcl}(F, A)\) is a sb-closed set in \(X\).
4. \(\text{bcl}(\text{bcl}(F, A)) = \text{bcl}((F, A))\).
5. \((F, A)\) is soft b-closed set if and only if \((F, A) = \text{bcl}(F, A)\).
6. \((F, A)\) is soft b-open set if and only if \((F, A) = \text{bint}(F, A)\).

Theorem 2.16. [7] Let \(X\) be a soft topological space. If \((F, B)\) is an open set and \((F, A)\) is a soft b-open set in \(X\). Then \((F, A) \overline{\cap (F, B)}\) is a soft b-open set in \(X\).

Definition 2.17. [10] Let \(S(X, E)\) and \(S(Y, K)\) be two soft classes. Let \(u : X \rightarrow Y\) be multifunction and \(p : E \rightarrow K\) be mapping. Then a multifunction \(F : S(X, E) \rightarrow S(Y, K)\) is defined as follows: For a soft set \((G, E)\) in \((X, E)\), \((F(G, E), K)\) is a soft set in \((Y, K)\) given by \(F(G, E)(k) = \bigcup_{u(x) = k} u(G(x))\) for \(k \in K\). \((F(G, E), K)\) is called a soft image of a soft set \((G, E)\). Moreover, \(F(G, E) = \bigcup\{F(E_{x}^{e}) : E_{x}^{e} \subseteq (G, E)\}\) for a soft subset \((G, E)\) of \(X\).

Definition 2.18. [10] Let \(F : (X, E) \rightarrow (Y, K)\) be a soft multifunction. The soft upper inverse image of \((H, K)\) denoted by \(F^{+}(H, K)\) and defined as \(F^{+}(H, K) = \{E_{x}^{e} \subseteq X : F(E_{x}^{e}) \supseteq (H, K), e \in E\}\). The soft lower inverse image of \((H, K)\) denoted by \(F^{-}(H, K)\) and defined as \(F^{-}(H, K) = \{E_{x}^{e} \subseteq X : F(E_{x}^{e}) \subseteq (H, K) \neq \Phi\}\).

Definition 2.19. [10] Let \(F : S(X, E) \rightarrow S(Y, K)\) and \(G : S(X, E) \rightarrow S(Y, K)\) be two soft multifunctions. Then, \(F\) equal to \(G\) if \(F(E_{x}^{e}) = G(E_{x}^{e})\), for each \(E_{x}^{e} \subseteq X\).

Definition 2.20. [10] The soft multifunction \(F : S(X, E) \rightarrow S(Y, K)\) is called surjective if \(p\) and \(u\) are surjective.

Theorem 2.21. [10] Let \(F : S(X, E) \rightarrow S(Y, K)\) be a soft multifunction Then, for soft sets \((F, E), (G, E)\) and for a family of soft sets \((G_{i}, E)_{i \in I}\) in the soft class \(S(X, E)\) the following are hold:

1. \(F(\Phi) = \Phi\).
2. \(F(\overline{X}) \subseteq Y\).
(3) \( F((G, A) \cup (H, B)) = F(G, A) \cup F(H, B) \) in general \( F(\bigcup_{i \in I}(G_i, E)) = \bigcup_{i \in I} F(G_i, E) \).

(4) \( F((G, A) \cap (H, B)) \subseteq F(G, A) \cap F(H, B) \) in general \( F(\bigcap_{i \in I}(G_i, E)) \subseteq \bigcap_{i \in I} F(G_i, E) \).

(5) If \((G, E) \subseteq (H, E)\), then \( F(G, E) \subseteq F(H, E) \).

**Theorem 2.22.** [10] Let \( F : S(X, E) \to S(Y, K) \) be a soft multifunction. Then, for soft sets \((G, K), (H, K)\) in the soft class \( S(X, E) \) the following are hold:

1. \( F^-(\Phi) = \Phi \) and \( F^+(\Phi) = \Phi \).
2. \( F^-(\overline{Y}) = \overline{X} \) and \( F^+(\overline{Y}) = \overline{X} \).
3. \( F^-(\overline{(G, K) \cup (H, K)}) = F^-((G, K) \cup F^-(H, K)) \).
4. \( F^+(G, K) \cap F^+(H, K) \subseteq F^+((G, K) \cap F^+(H, K)) \).
5. \( F^-(G, K) \cap F^+(H, K) \subseteq F^-(G, K) \cap F^+(H, K) \).
6. \( F^+(G, K) \cap F^+(H, K) = F^+((G, K) \cap F^+(H, K)) \).
7. If \((G, K) \subseteq (H, K)\), then \( F^-(G, K) \subseteq F^-(H, K) \) and \( F^+(G, K) \subseteq F^+(H, K) \).

**Proposition 2.23.** [10] Let \((G_i, K)\) be soft sets over \( Y \) for each \( i \in I \). Then the followings are true for a soft multifunction \( F : (X, \tau, E) \to (Y, \sigma, K) \):

1. \( F^-(\bigcup_{i \in I}(G_i, K)) = \bigcup_{i \in I} F^-(G_i, K) \).
2. \( \bigcap_{i \in I} F^+(G_i, K) = F^+(\bigcap_{i \in I} G_i, K) \).
3. \( \bigcup_{i \in I} F^+(G_i, K) \subseteq F^+(\bigcup_{i \in I} G_i, K) \).
4. \( F^-(\bigcap_{i \in I} G_i, K) \subseteq F^-(\bigcap_{i \in I} G_i, K) \).

**Proposition 2.24.** [10] Let \( F : (X, \tau, E) \to (Y, \sigma, K) \) be a soft multifunction. Then the followings are true:

1. \( (G, A) \subseteq F^+(F(G, A)) \subseteq F^-(F(G, A)) \) for a soft subset \((G, A)\) in \( X \). If \( F \) is surjective then \( (G, A) = F^+(F(G, A)) = F^-(F(G, A)) \).
2. \( F(F^+(H, B)) \subseteq (H, B) \subseteq F(F^-(H, B)) \) for a soft subset \((H, B)\) in \( Y \).
3. For two soft subsets \((H, B)\) and \((U, C)\) in \( Y \) such that \((H, B) \cap (U, C) = \Phi\), then \( F^+(H, B) \cap F^-(U, C) = \Phi \).

**Proposition 2.25.** [10] Let \( F : (X, \tau, E) \to (Y, \sigma, K) \) and \( G : (Y, \sigma, K) \to (Z, \eta, L) \) be soft multifunction. Then the followings are true:

1. \( (F^{-})^{-} = F \).
2. For a soft subset \((T, C)\) in \( Z \), \((G \circ F)^-(T, C) = F^-(G^-(T, C)) \) and \((G \circ F)^+(T, C) = F^+(G^+(T, C)) \).

**Proposition 2.26.** [10] Let \((G, E)\) be a soft set over \( Y \). Then the followings are true for a soft multifunction \( F : (X, \tau, E) \to (Y, \sigma, K) \):

1. \( F^+(\overline{Y} - (G, K)) = \overline{X} - F^-(G, K) \).
2. \( F^-(\overline{Y} - (G, K)) = \overline{X} - F^+(G, K) \).

### 3. \( b \)-continuity of soft multifunctions

**Definition 3.1.** Let \((X, \tau, E)\) and \((Y, \sigma, K)\) be two soft topological spaces. Then a soft multifunction \( F : (X, \tau, E) \to (Y, \sigma, K) \) is said to be:

1. soft upper \( b \)-continuous at a soft point \( E^+_\varepsilon \subseteq X \) if for every soft open set \((G, K)\) such that \( F(E^+_\varepsilon) \subseteq (G, K) \), there exists a soft \( b \)-open neighborhood \((P, E)\) of \( E^+_\varepsilon \) such that \( F(E^+_\varepsilon) \subseteq (G, K) \) for all \( E^+_\varepsilon \subseteq (P, E) \).
2. soft lower \( b \)-continuous at a point \( E^-_\varepsilon \subseteq X \) if for every soft open set \((G, K)\) such that \( F(E^-_\varepsilon) \subseteq (G, K) \), there exists a soft \( b \)-open neighborhood \((P, E)\) of \( E^-_\varepsilon \) such that \( F(E^-_\varepsilon) \subseteq (G, K) \) for all \( E^-_\varepsilon \subseteq (P, E) \).
3. soft upper(lower) \( b \)-continuous if \( F \) has this property at every soft point of \( X \).
Theorem 3.2. For a soft multifunction $F: (X, \tau, E) \rightarrow (Y, \sigma, K)$ the following are equivalent:

1. $F$ is soft upper $b$-continuous.
2. $F^+(G, K)$ is soft $b$-open set in $X$, for each soft open subset $(G, K)$ in $Y$.
3. $F^-(H, K)$ is soft $b$-closed set in $X$, for each soft closed subset $(H, K)$ in $Y$.
4. $bcl(F^-(B, K)) \subseteq F^-(cl(B, K))$, for any soft subset $(B, K)$ in $Y$.
5. $F^+(int(G, K)) \subseteq bint(F^+(G, K))$, for every soft subset $(G, K)$ in $Y$.

Proof. (1)$\Rightarrow$(2): Let $(G, K)$ be any soft open set in $Y$ and $E_x^e \subseteq F^+(G, K)$. Then $F(E_x^e) \subseteq (G, K)$ and by (1), there exists soft $b$-open set $(U, E)$ containing $E_x^e$ such that $F(U, E) \subseteq (G, K)$. Since

$$E_x^e \subseteq \text{int}(cl(U, E)) \cap \text{cl}(\text{int}(U, E)) \cap \text{cl}(\text{int}(F^+(G, K))) \cap \text{cl}(\text{int}(F^+(G, K)))$$

we have

$$F^+(G, K) \subseteq \text{int}(cl(F^+(G, K))) \cap \text{cl}(\text{int}(F^+(G, K)))$$

Thus $F^+(G, K)$ is soft $b$-open set in $X$.

(2)$\Rightarrow$(1): Obvious.

(2)$\Rightarrow$(3): Let $(H, K)$ be any soft closed set in $Y$. Then $\tilde{Y} - (H, K)$ is soft open set in $Y$. By (2), $F^+(\tilde{Y} - (H, K)) = \tilde{X} - F^+(H, K)$ is soft $b$-open set and thus $F^-(H, K)$ is soft $b$-closed set.

(3)$\Rightarrow$(4): For every soft set $(B, K)$ in $Y$, $cl(B, K)$ is soft closed set. By (3), $F^-(cl(B, K))$ is soft $b$-closed set. Then $bcl(F^-(B, K)) \subseteq bcl(F^-(cl(B, K))) = F^-(cl(B, K))$.

(4)$\Rightarrow$(5): Let $(G, K)$ be any soft subset in $Y$. Then by (4), $bcl(F^-(G, K)) \subseteq F^-(cl(G, K))$ and $\tilde{X} - F^-(cl(G, K)) \subseteq \tilde{X} - bcl(F^-(G, K))$. Hence

$$F^+(\tilde{Y} - (G, K)) \subseteq bint(F^+(\tilde{Y} - (G, K)))$$

and

$$F^+(\text{int}(\tilde{Y} - (G, K))) \subseteq bint(F^+(\tilde{Y} - (G, K)))$$

Thus we have $F^+(\text{int}(G, K)) \subseteq bint(F^+(G, K))$.

(5)$\Rightarrow$(2): Let $(B, K)$ be any soft open set in $Y$. By (5), $F^+(G, K) = F^+(\text{int}(G, K)) \subseteq bint(F^+(G, K))$. Then $F^+(G, K) = bint(F^+(G, K))$. Therefore, $F^+(G, K)$ is soft $b$-open set in $X$. Thus, $F$ is soft upper $b$-continuous.

Theorem 3.3. For a soft multifunction $F: (X, \tau, E) \rightarrow (Y, \sigma, K)$ the following are equivalent:

1. $F$ is soft lower $b$-continuous.
2. $F^+(G, K)$ is soft $b$-open set in $X$, for each soft open subset $(G, K)$ in $Y$.
3. $F^-(H, K)$ is soft $b$-closed set in $X$, for each soft closed subset $(H, K)$ in $Y$.
4. $bcl(F^+(B, K)) \subseteq F^+(cl(B, K))$, for any soft subset $(B, K)$ in $Y$.
5. $F^-(int(G, K)) \subseteq bint(F^-(G, K))$, for every soft subset $(G, K)$ in $Y$.
6. $F(bcl(A, E)) \subseteq cl(F(A, E))$, for every soft set $(A, E)$ in $X$.

Proof. It is shown similarly to the proof of previous theorem that the (1), (2), (3), (4), (5) are equivalent. So, we only prove that (6) is equivalent them.

(5)$\Rightarrow$(6): Let $(A, E)$ be any soft subset in $X$. Then

$$F(bcl(A, E)) = F((A, E) \cap \text{int}((A, E)) \cap \text{cl}((A, E)))) = F(A, E) \cup F(int(cl(A, E)) \cap \text{cl}(int(A, E)))) \subseteq F(A, E) \cup F(cl(A, E))$$

(6)$\Rightarrow$(5): Let $(H, K)$ be any soft subset in $Y$. Since $F(F^+(H, K)) \subseteq (H, K)$, for a soft set $F^+(H, K)$ by (6), we have that

$$F(bcl(F^+(H, K))) \subseteq cl(F(F^+(H, K))) \subseteq cl(H, K) = (H, K)$$

Hence $bcl(F^+(H, K)) \subseteq F^+(H, K)$ and thus $F^+(H, K)$ is soft $b$-closed set.
Example 3.4. Let $X = \{x_1, x_2, x_3, x_4\}$, $Y = \{y_1, y_2\}$, $E = \{e\}$, $K = \{k\}$, $\tau = \{\Phi, \tilde{X}, \{(e, \{x_4\})\}$, $\{(e, \{x_1, x_2\})\}$ and $\sigma = \{\Phi, \tilde{Y}, (H, K)\}$, where $(H, K) = \{(k, \{y_1\})\}$. Let $u : X \rightarrow Y$ be multifunction defined as: $u(x_1) = \{y_2\}$, $u(x_2) = \{y_2\}$, $u(x_3) = \{y_1\}$, $u(x_4) = \{y_1, y_2\}$ and $p : E \rightarrow K$ be mapping defined as: $p(e) = \{k\}$. Then the soft multifunction $F : (X, \tau, E) \rightarrow (Y, \sigma, K)$ is soft upper $b$-continuous. Because for a soft open set $(H, K)$ in $Y$, $F^+(H, K) = \{(e, \{x_1\})\}$ is soft $b$-open set in $X$.

Example 3.5. In Example 3.4, let $(H, K) = \{(k, \{y_2\})\}$. Then for a soft open set $(H, K)$ in $Y$, $F^-(H, K) = \{\tilde{X}\}$ is soft $b$-open set in $X$. Then the soft multifunction $F : (X, \tau, E) \rightarrow (Y, \sigma, K)$ is soft lower $b$-continuous.

Definition 3.6. Let $(F, A)$ be a soft set in a soft topological space $(X, \tau, E)$. Then $(F, A)$ is called:

1. [12] soft semi open set iff $(F, A) \subseteq cl(int((F, A)))$.
2. [13] soft pre open set iff $(F, A) \subseteq int(cl(F, A))$.
3. [14] soft $\beta$-open set iff $(F, A) \subseteq cl(int(cl(F, A)))$.

Remark 3.7. [7] In a soft topological space $(X, \tau, E)$, we have following implications:

![Soft semi-open sets diagram]

![Soft pre-open sets diagram]

![Soft $b$-open sets diagram]

![Soft $\beta$-open sets diagram]

Theorem 3.8. Let $F : (X, \tau, E) \rightarrow (Y, \sigma, K)$ be a soft multifunction. Then:

1. $F$ is soft lower (upper) semi continuous if and only if $F^-(G, K)$ ($F^+(G, K)$) is soft semi open (closed) set in $X$, for each soft open (closed) subset $(G, K)$ in $Y$.
2. $F$ is soft lower (upper) pre continuous if and only if $F^-(G, K)$ ($F^+(G, K)$) is soft pre open (closed) set in $X$, for each soft open (closed) subset $(G, K)$ in $Y$.
3. $F$ is soft lower (upper) $\beta$-continuous if and only if $F^-(G, K)$ ($F^+(G, K)$) is soft $\beta$-open (closed) set in $X$, for each soft open (closed) subset $(G, K)$ in $Y$.

Remark 3.9. In a soft topological space $(X, \tau, E)$, we have following implications:

![Soft upper (lower) semi-continuity diagram]

![Soft upper (lower) pre-continuity diagram]

![Soft upper (lower) $b$-continuity diagram]

4. Soft $b$-irresolute multifunctions

Definition 4.1. Let $(X, \tau, E)$ and $(Y, \sigma, K)$ be two soft topological spaces. Then a soft multifunction $F : (X, \tau, E) \rightarrow (Y, \sigma, K)$ is said to be:

1. soft upper $b$-irresolute at a soft point $E^+_e \subseteq \tilde{X}$ if for every soft $b$-open set $(G, K)$ such that $F(E^+_e) \subseteq (G, K)$, there exists a soft $b$-open $(P, E)$ containing $E^+_e$ such that $F(E^+_e) \subseteq (G, K)$ for all $E^+_e \subseteq (P, E)$.
(2) soft lower \( b \)-irresolute at a point \( E^+_b \in \tilde{X} \) if for every soft \( b \)-open set \((G, K)\) such that \( F(E^+_b) \cap \tilde{G}(G, K) \neq \emptyset\), there exists a soft \( b \)-open \((P, E)\) containing \( E^+_b \) such that \( F(E^+_b) \cap \tilde{G}(G, K) \neq \emptyset\) for all \( E^+_b \in (P, E)\).

(c) soft upper(lower) \( b \)-irresolute if \( F \) has this property at every soft point of \( X \).

**Theorem 4.2.** For a soft multifunction \( F : (X, \tau, E) \rightarrow (Y, \sigma, K) \) the following are equivalent:

1. \( F \) is soft lower \( b \)-irresolute.
2. \( F^{-}(G, K) \) is soft \( b \)-open set in \( X \), for each soft \( b \)-open subset \((G, K)\) in \( Y \).
3. \( F^{+}(H, K) \) is soft \( b \)-closed set in \( X \), for each soft \( b \)-closed subset \((H, K)\) in \( Y \).

**Proof.** (1)\( \Rightarrow \) (2): Let \((G, K)\) be any soft \( b \)-open set and let \( E^+_b \in F^{-}(G, K) \). Since \( F \) is soft lower \( b \)-irresolute, there exists a soft \( b \)-open set \((U, E)\) containing \( E^+_b \) such that \( (U, E) \cap F^{-}(G, K) \neq \emptyset \). Thus \( F^{-}(G, K) \cap bint(U, E) = \emptyset \) and \( E^+_b \cap bint(F^{-}(G, K)) \neq \emptyset \). Thus \( F^{-}(G, K) \cap \tilde{bint}(F^{-}(G, K)) \neq \emptyset \) and \( F^{-}(G, K) \) is soft \( b \)-open set.

(2)\( \Rightarrow \) (3): Let \((H, K)\) be any soft \( b \)-closed set, then \( \tilde{Y} - (H, K) \) is soft \( b \)-open set. By (2), \( F^{-}(\tilde{Y} - (H, K)) = \tilde{X} - F^{+}(H, K) \) is soft \( b \)-open set. Thus we have \( F^{+}(H, K) \) is soft \( b \)-closed set.

(3)\( \Rightarrow \) (1): Let \((G, K)\) be any soft \( b \)-open set and let \( E^+_b \in F^{-}(G, K) \). Then \( \tilde{X} - (G, K) \) is soft closed set and by (3), \( F^{+}(\tilde{Y} - (G, K)) = \tilde{X} - F^{-}(G, K) \) soft closed set and thus \( F^{-}(G, K) \) soft open set. Put \( F^{-}(G, K) = (U, E) \) is soft \( b \)-open set containing \( E^+_b \) such that \( (U, E) \cap F^{-}(G, K) \neq \emptyset \). Thus \( F \) is soft lower \( b \)-irresolute. \( \square \)

**Theorem 4.3.** For a soft multifunction \( F : (X, \tau, E) \rightarrow (Y, \sigma, K) \) the following are equivalent:

1. \( F \) is soft upper \( b \)-irresolute.
2. \( F^{+}(G, K) \) is soft \( b \)-open set in \( X \), for each soft \( b \)-open subset \((G, K)\) in \( Y \).
3. \( F^{-}(H, K) \) is soft \( b \)-closed set in \( X \), for each soft \( b \)-closed subset \((H, K)\) in \( Y \).

**Proof.** The proof is similarly to previous theorem. \( \square \)

**Theorem 4.4.** Let \( F : (X, \tau, E) \rightarrow (Y, \sigma, K) \) and \( G : (Y, v, K) \rightarrow (Z, \sigma, T) \) be two soft multifunctions. Then, for \( G \circ F : (X, \tau, E) \rightarrow (Z, \sigma, T) \) the following are hold:

1. if \( F \) is soft upper(lower) \( b \)-continuous and \( G \) is soft upper(lower) \( b \)-continuous, then \( G \circ F \) is soft upper(lower) \( b \)-continuous.
2. if \( F \) and \( G \) is soft upper(lower) \( b \)-irresolute, then \( G \circ F \) is soft upper(lower) \( b \)-irresolute.
3. if \( F \) is soft upper(lower) \( b \)-irresolute and \( G \) is soft upper(lower) \( b \)-continuous, then \( G \circ F \) is soft upper(lower) \( b \)-continuous.

**Proof.** (1) Let \((H, T)\) be any soft open set of \( Z \). Since \( G : Y \rightarrow Z \) is soft upper(lower) \( b \)-continuous, then \( G^{+}(H, T) \cap (G^{-}(H, T)) \) is soft open set of \( Y \). Since \( F \) is soft upper(lower) \( b \)-continuous, so \( F^{+}(G^{+}(H, T)) = (G \circ F)^{+}(H, T) \cap (F^{-}(G^{-}(H, T)) = (G \circ F)^{-}(H, T)) \) is soft \( b \)-open set in \( X \). Hence \( G \circ F \) is soft upper(lower) \( b \)-continuous.

(2) Let \((H, T)\) be any soft \( b \)-open set of \( Z \). Since \( G \) is soft upper \( b \)-irresolute, then \( G^{+}(H, T) \) is soft \( b \)-open set of \( Y \). Also \( F \) is soft upper \( b \)-irresolute, so \( F^{+}(G^{+}(H, T)) = (G \circ F)^{+}(H, T) \) is soft \( b \)-open set. Thus \( G \circ F \) is soft upper \( b \)-irresolute.

(3) Let \((H, T)\) be soft \( b \)-open set of \( Z \). Since \( G \) is soft upper \( b \)-continuous, \( G^{+}(H, T) \) is soft upper \( b \)-open set of \( Y \). Also \( F \) is soft upper \( b \)-irresolute, then \( F^{+}(G^{+}(H, T)) = (G \circ F)^{+}(H, T) \) is soft \( b \)-open set of \( X \). Thus \( G \circ F \) is soft upper \( b \)-continuous. \( \square \)
5. Conclusion

Recently, many researches had been done various works in the soft set theory and in practices. In this paper, we introduce $b$-continuous multifunctions which defined between soft topological spaces and some of their properties are studied. We also introduce $b$-irresolute multifunctions. We hope that this paper is useful to researchers which deals with soft sets (especially with uncertainty problems) and its applications. Because, soft sets represent a powerful tool for decision making about information systems.

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