Change of mechanical structures of spacecraft with variable quantity of degrees of freedom in purposes of reaction/momentum wheels unloading

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Abstract. The using of the released (“defreezed”) additional degrees of freedom is considered to control by the relative angular momentum of reaction/momentum wheels and to unload this relative angular momentum. The change of the relative angular momentum of wheels is fulfilled with the help of its translation to additional internal rotors and main body of spacecraft.

1. Introduction
It is well known that spacecraft can be controlled and stabilized in its angular motion by internal rotors (reaction/momentum wheels) rotating relative the main body of spacecraft [1]. These rotors collect the angular momentum during control processes (Fig.1-a); and as the result the rotors become saturated in the sense of reaching the limiting angular velocities of relative rotation. In this case, it becomes necessary to unload the accumulated angular momentum of the rotors, which is realized through various schemes of interaction of the spacecraft with external forces and fields (magnetic, gravitational, aerodynamic). In the traditional scheme, the accumulated kinetic energy of the relative rotation of the rotors is directly or indirectly spent on interaction with external forces without any recuperation and the possibility of its reuse.

![Figure 1. The rotors unloading scheme with additional degrees of freedom release.](image)
As an alternative, rotors unloading scheme can be proposed, which based on the additional degrees of freedom that are being released (fig. 1). This scheme allows the recuperation of the rotational energy.

Let in the saturation mode the main rotors have a total angular momentum $K$ (Fig. 1-a). When implementing the proposed unloading scheme, the connections between the body and additional rotors are removed (Fig. 1-b). After releasing additional rotors they are closed on the main rotors (Fig. 1-c), joining them directly and receiving an equivalent in value and reverse in the sign the relative angular momentum $K'$. In this case, the total relative angular momentum of the new system of rotors instantly becomes equal to zero; and the angular momentum $K$ will be instantly added to the spacecraft main body. Further, in the dynamic sense, the spacecraft will fulfill the angular motion like an ordinary rigid body (“mono body”) that does not contain internal rotating masses. In this “mono-body mode”, the angular momentum of the spacecraft is damped by interaction with external forces, that is implemented by the special equipment.

At the same time, in the process of damping the angular momentum of the spacecraft, the electric motors of the connected (closed) rotors are switched to the mode of generating electricity, slowing down rotors relative rotation, and it allows recuperating a part of the initial energy of the relative rotation of the main rotors.

Moreover, there are possible schemes for connecting the main rotors with released additional rotors in different directions, for example, according to the schemes in Figs. 2 and 3. Here different values and directions of the relative angular momentum will be before and after the closure of the rotors. These schemes allow instantaneous translations of the angular momentum to the main body, and the spacecraft will switch to a different dynamic mode of the angular motion.

![Figure 2](image1.png)  
**Figure 2.** The planar connection of the main rotor with an additional released rotor.

![Figure 3](image2.png)  
**Figure 3.** The orthogonal connection of the main rotor with an additional released rotor.

Summarizing the above indicated possibilities for the release of new degrees of freedom and for the connection of rotors, it is possible to represent the mechanical scheme of the spacecraft in the form of a multi-rotor system with instantly superimposed/removed (closed/opened) in various sequences complexes of nonholonomic couplings of rotors, the motion of which will correspond to
nonholonomic dynamics [2-10] system (Fig. 4) [1]. So, in the presented paper the nonholonomic dynamics of such multi-rotor systems is considered at the activation/deactivation of nonholonomic constraints between different rotors.

2. The mathematical model of the nonholonomic multi-rotor system angular motion around center of mass

2.1. The general approach to the nonholonomic system dynamics investigation

Let us shortly indicate the general methodology of the nonholonomic systems description [2-10]. Assume that we have $s$ nonholonomic constraints in the following form:

$$
\sum_{j=1}^{m} b_{\beta j} (q_{1}, \ldots, q_{m}, t) \dot{q}_{j} + b_{\beta} (q_{1}, \ldots, q_{m}, t) = 0, \quad \beta = 1, 2, \ldots, s
$$

(1)

Here $q_{j}$ – are coordinates corresponded to the system’s degrees of freedom ($m$ – is the number of degrees of freedom). The general equations of the system dynamics are ($T$ is the kinetic energy):

$$
\sum_{j=1}^{m} \left( \frac{d}{dt} \frac{\partial T}{\partial \dot{q}_{j}} - \frac{\partial T}{\partial q_{j}} - Q_{j} \right) \delta q_{j} = 0
$$

(2)

where variations $\delta q_{j}$ now are not independent; they are connected (due to (1)) by the $s$ independent expressions:

$$
\sum_{j=1}^{m} b_{\beta j} (q_{1}, \ldots, q_{m}, t) \delta q_{j} = 0
$$

(3)

To build the dynamical equations for nonholonomic system, it is possible to subtract from equations (2) expressions (3) multiplied by indefinite Lagrangian multipliers $\lambda_{\beta}$:

$$
\sum_{j=1}^{m} \left( \frac{d}{dt} \frac{\partial T}{\partial \dot{q}_{j}} - \frac{\partial T}{\partial q_{j}} - Q_{j} - \sum_{\beta=1}^{s} \lambda_{\beta} b_{\beta j} (q_{1}, \ldots, q_{m}, t) \right) \delta q_{j} = 0
$$

(4)

Taking in mind that from (3) we can obtain $n$ (where $n=m-s$) dependences for variations $\delta q_{n+1}, \ldots, \delta q_{m}$ (where $k=1, \ldots, s$) expressed through variations $\delta q_{1}, \ldots, \delta q_{n}$; and then variations $\delta q_{m+1}, \ldots, \delta q_{n}$ can be considered as independent main variations.

Let us now choice the multipliers $\lambda_{\beta}$ such way that variations $\delta q_{n+1}, \ldots, \delta q_{m}$ will be equal to zeros. Therefore, in (4) we will see only independent variations $\delta q_{1}, \ldots, \delta q_{n}$, and consequently their
corresponded coefficients in brackets will equal to zeros, that allows to write the following dynamical equations:

\[
\frac{d}{dt} \frac{\partial T}{\partial \dot{q}_{ij}} - \frac{\partial T}{\partial q_{ij}} - \sum_{\beta=1}^{s} \lambda_{\beta} \beta_{\beta}(q_{i}, \ldots, q_{m}, t) = 0 \quad (j = 1, \ldots, m)
\]

And we must add to equations (5) the constraints (1). From (5) and (1) it is possible to completely find all of the coordinates \( q_j \) and multipliers \( \lambda_{\beta} \). So, the nonholonomic system will be fully described in terms of its coordinates.

2.2. The mathematical model of nonhomoclinic multi-rotor system angular motion

The mathematical model of the multi-rotor system with the structure presented at the figure (Fig.5)

\[
\begin{align*}
Aq + \sum_{j=1}^{N} I_{ij} \{ \dot{\sigma}_{uj} + \dot{\sigma}_{uj} \} + (C - B) qr + \left[ q \sum_{i=1}^{N} I_{ij} \{ \sigma_{uj} + \sigma_{ui} \} - r \sum_{i=1}^{N} I_{ij} \{ \sigma_{uj} + \sigma_{ui} \} \right] &= M_{x}^* \\
Bq + \sum_{j=1}^{N} I_{ij} \{ \dot{\sigma}_{uj} + \dot{\sigma}_{ui} \} + (A - C) pr + \left[ r \sum_{i=1}^{N} I_{ij} \{ \sigma_{uj} + \sigma_{ui} \} - p \sum_{i=1}^{N} I_{ij} \{ \sigma_{uj} + \sigma_{ui} \} \right] &= M_{y}^* \\
Cr + \sum_{j=1}^{N} I_{ij} \{ \dot{\sigma}_{uj} + \dot{\sigma}_{ui} \} + (A - C) qr + \left[ q \sum_{i=1}^{N} I_{ij} \{ \sigma_{uj} + \sigma_{ui} \} - r \sum_{i=1}^{N} I_{ij} \{ \sigma_{uj} + \sigma_{ui} \} \right] &= M_{z}^* \\
I_{i} \{ \dot{\phi} + \ddot{\phi} \} &= M_{\phi}^*; \\
I_{i} \{ \dot{\gamma} + \ddot{\gamma} \} &= M_{\gamma}^*; \\
I_{i} \{ \dot{\alpha} + \ddot{\alpha} \} &= M_{\alpha}^*; \\
I_{i} \{ \dot{\psi} + \ddot{\psi} \} &= M_{\psi}^*; \\
I_{i} \{ \dot{\omega} + \ddot{\omega} \} &= M_{\omega}^*.
\end{align*}
\]

Here \( \sigma_{ui} \) is the relative angular velocity of the \( kl \)-th rotor (relatively the main body); \( \{ A, B, C \} \) - are the main inertia moments of the main body with the “frozen” rotors; \( I_{i} \) are the longitudinal inertia moments of the \( l \)-layer-rotor. \( M_{x,y,z}^* \) - are the external torques acting on the main body and the internal torques acting on the rotors from the side of the main body. The system (6) is the main body of spacecraft equations. The system (7) corresponds to equations of the relative angular motion of rotors.

Figure 5. The mechanical model of the nonholonomic multi-layer multi-rotor system.

This system was described in [1]. It contains \( N \) layers with rotors on the six general directions coinciding with the principle axes of the main body. The motion equations of this system can be written in the following form [1]:

\[
\begin{align*}
Aq + \sum_{j=1}^{N} I_{ij} \{ \dot{\sigma}_{uj} + \dot{\sigma}_{uj} \} + (C - B) qr + \left[ q \sum_{i=1}^{N} I_{ij} \{ \sigma_{uj} + \sigma_{ui} \} - r \sum_{i=1}^{N} I_{ij} \{ \sigma_{uj} + \sigma_{ui} \} \right] &= M_{x}^* \\
Bq + \sum_{j=1}^{N} I_{ij} \{ \dot{\sigma}_{uj} + \dot{\sigma}_{ui} \} + (A - C) pr + \left[ r \sum_{i=1}^{N} I_{ij} \{ \sigma_{uj} + \sigma_{ui} \} - p \sum_{i=1}^{N} I_{ij} \{ \sigma_{uj} + \sigma_{ui} \} \right] &= M_{y}^* \\
Cr + \sum_{j=1}^{N} I_{ij} \{ \dot{\sigma}_{uj} + \dot{\sigma}_{ui} \} + (A - C) qr + \left[ q \sum_{i=1}^{N} I_{ij} \{ \sigma_{uj} + \sigma_{ui} \} - r \sum_{i=1}^{N} I_{ij} \{ \sigma_{uj} + \sigma_{ui} \} \right] &= M_{z}^* \\
I_{i} \{ \dot{\phi} + \ddot{\phi} \} &= M_{\phi}^*; \\
I_{i} \{ \dot{\gamma} + \ddot{\gamma} \} &= M_{\gamma}^*; \\
I_{i} \{ \dot{\alpha} + \ddot{\alpha} \} &= M_{\alpha}^*; \\
I_{i} \{ \dot{\psi} + \ddot{\psi} \} &= M_{\psi}^*; \\
I_{i} \{ \dot{\omega} + \ddot{\omega} \} &= M_{\omega}^*.
\end{align*}
\]
Let us consider in this paper possibility of the superimposition of the orthogonal connection of some two rotors in the same layer. It implies the following nonholonomic constrain, which includes the relative rotors angular velocity:

\[ \sigma_{w} - n_{w} \sigma_{i} = 0 \]  \hspace{1cm} (8)

In (8) coefficient \( n_{w} \) (it is constant) means the gear ratio between the rotor \#Wl and the rotor \#Wi.

2.3. The concrete example of nonholonomic multi-rotor system angular motion

For example, in this paper we consider the case of the single superimposition of the orthogonal connection of the rotor \#3l and \#5l, when then the single constrain takes place. Also we assume that we have only one layer of rotors (and we will not indicate the index \( l \)). The constrain (8) takes the form:

\[ \sigma_{5} - n_{s} \sigma_{3} = 0 \]  \hspace{1cm} (9)

The external torque-free motion equations then are:

\[
\begin{align*}
A \dot{p} + I \{ \dot{\sigma}_{i} + \dot{\sigma}_{j} \} + (C - B) q r + \left[ q l \{ \sigma_{s} + \sigma_{e} \} - rl \{ \sigma_{s} + \sigma_{e} \} \right] &= 0, \\
B \dot{q} + I \{ \dot{\sigma}_{i} + \dot{\sigma}_{j} \} + (A - C) p r + \left[ rl \{ \sigma_{s} + \sigma_{e} \} - pl \{ \sigma_{s} + \sigma_{e} \} \right] &= 0, \\
C \dot{r} + I \{ \dot{\sigma}_{i} + \dot{\sigma}_{j} \} + (A - C) q p + \left[ pl \{ \sigma_{s} + \sigma_{e} \} - q l \{ \sigma_{s} + \sigma_{e} \} \right] &= 0, \\
I \left( \dot{p} + \dot{\sigma}_{i} \right) &= M_{i}; \\
I \left( \dot{q} + \dot{\sigma}_{j} \right) &= M_{j}; \\
I \left( \dot{r} + \dot{\sigma}_{k} \right) &= M_{k}.
\end{align*}
\]

If we differentiate the constrain (9), we obtain

\[ \dot{\sigma}_{s} - n_{s} \dot{\sigma}_{3} = 0 \]  \hspace{1cm} (12)

And from (12) and (11) we can find the Lagrangian multiplier:

\[ \lambda = -\frac{1}{n_{s} + 1} \left( n_{s} \left[ I \dot{q} - M_{i} \right] - \left[ I \dot{r} - M_{j} \right] \right) \]  \hspace{1cm} (13)

which represents in substance the reactive torque of the nonholonomic constrain.

So, to investigate the nonholonomic dynamics we should now integrate the differential equations (10), (11) and (12) jointly.

The considered above example completely show the application of the general methodology of the nonholonomic multi-rotor motion research. In the next section the modelling results for this case and corresponded comments will be presented.

3. Numerical modelling of the nonholonomic multi-rotor angular motion

Let us to integrate numerically the equations of motion in case of indicated above example (9)-(12). Assume that the constrain (9) (or its differential form (12)) is fulfilled during the time-interval \( t \in [0, T] \), and outside this interval the system has its natural holonomic dynamics.

The internal torques for rotors spin-up/down control are selected to modelling in following forms:

\[
\begin{align*}
M_{i} &= v_{i} \sigma_{i} H(t - T); \\
M_{j} &= v_{j} \sigma_{j} H(t - T); \\
M_{k} &= v_{k} \sigma_{k} H(t - T); \\
M_{i}' &= I \dot{p}; \\
M_{j}' &= I \dot{q}; \\
M_{k}' &= I \dot{r}.
\end{align*}
\]  \hspace{1cm} (14)

where \( H(*) \) – is the Heaviside function. Internal torques (14) mean that the after the action of the nonholonomic constrain, the damping \( (v_{i}\sigma_{i}) \) are fulfilled for rotors \#1, 3, 5. The rotors ##2, 4, 6 we consider as fixed relative the main body \( \sigma_{2} = \sigma_{4} = \sigma_{5} = 0 \), and to provide relative immovability of “frozen” rotors in (14) we formally selected such torques. The system parameters and initial conditions are: \( A=5, B=7, C=10, I=1 \; [\text{kg}\cdot\text{m}^{2}]; \; T=150 \; [\text{s}]; \; n_{s}=1.5; \; p(0)=0.0, q(0)=0.1, r(0)=0.1, \sigma_{1}(0)=0.15, \sigma_{3}(0)=-0.1, \sigma_{5}(0)=-0.15 \; [1/\text{s}]; \; v_{1}=-0.1, v_{3}=-0.6, v_{5}=-0.6 \; [\text{kg}\cdot\text{m}^{2}/\text{s}]. \)
Figure 6. The modelling results for angular velocities (a) of the main body ($p$-red, $q$-blue, $r$-green) and for relative angular velocities of rotors (b).

Figure 7. The modelling results for constrain (9) fulfilment at $t \in [0, T]$ (a) and the corresponded Lagrangian multiplier (b).

Figure 8. The modelling results for the conservation of the system angular momentum – it is constant.
4. Conclusion

The possibility of the wheels unloading and reorientation of spacecraft with the help of additional degrees of freedom releasing is considered. The proposed scheme of saturated wheels unloading has the following advantages:

- It can instantly bring the spacecraft into the “mono-body” dynamics at connecting but rotating rotors. This mode of the “mono-body” dynamics allows to unload the angular momentum from the main body of spacecraft by usual ways (thrusters, magnet coils, etc.), without influence of energy recuperating processes into rotors.
- It is possible to divide the value of the relative angular momentum between the rotors in different layers; and it is possible to fulfil a reset of the accumulated angular momentum step by step (layer by layer), and here sizes and masses of rotors can be as small as we wish.
- The spacecraft can instantly reorient its attitude using complete value of the accumulated angular momentum – this allows to fulfil “large amplitude” manoeuvres.
- Different variants of energy recuperating schemes are possible inside connected rotors, which allows to fully or partially collection of the relative rotation energy.
- Different variants of rotors connection are available: planar, orthogonal, mixed schemes of rotors connecting in multilayer rotors system.

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