Electroweak Vacuum metastability in a natural minimal Type-III Seesaw Model

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Abstract

We study the minimal type-III seesaw model in which we extend the SM by adding two $SU(2)_L$ triplet fermions with zero hypercharge to explain the origin of the non-zero neutrino masses and mixing. We show that the naturalness conditions and the limits from lepton flavor violating decays provide very stringent bounds on the model along with the constraints on the stability/metastability of the electroweak vacuum. We perform a detailed analysis of the model parameter space including all the constraints for both normal as well as inverted hierarchies of the light neutrino masses. We find that most of the region that are allowed by lepton flavor violating decay and naturalness fall in the metastable region.

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I. INTRODUCTION

The discovery of the Higgs boson \cite{1, 2} at the Large Hadron Collider (LHC) has confirmed the mode of generation of the masses of the fundamental particles via the mechanism of electroweak (EW) symmetry breaking and has put the Standard Model (SM) on a solid foundation. However, despite its success in explaining most of the experimental data, the SM can not address certain issues. One of the most important experimental observation that necessitates the extension of the SM is the phenomenon of neutrino oscillation. The solar, atmospheric, reactor and accelerator neutrino oscillation experiments have shown that the three neutrino flavors mix among themselves and they have very small but non-zero masses, unlike as predicted in the SM.

Among the various beyond standard model scenarios that are proposed in the literature to explain the small neutrino masses, the most popular one is the seesaw mechanism. This is based on the assumption that the lepton number is violated at a very high energy scale by some heavier particles. The tree level exchange of these heavy particles would give rise to the lepton number violating dimension-5 Weinberg operator $\kappa L L H H$ \cite{3}. This will give rise to small neutrino masses once the EW symmetry is broken. Here, $L$ and $H$ are the lepton and the Higgs doublets respectively and $\kappa$ is a proportionality constant with negative mass dimension and is inversely proportional to the energy scale at which the new physics enters. Depending on the nature of the heavy particles added for the ultraviolet completion, one can have three types of seesaw mechanism. If the seesaw is generated by adding extra neutral fermionic singlets, it is called a type-I seesaw mechanism \cite{4–7}. Similarly, type-II seesaw mechanism is generated by adding a triplet scalar \cite{8–11} to the SM whereas the addition of fermionic triplets give rise to type-III seesaw mechanism \cite{12}. It is known that in order to get a neutrino mass of the sub-eV scale, one has to take the new particles to be extremely heavy or else take the new couplings to be extremely small. This spoils the testability of the theory. However, there are various TeV scale extensions of the minimal scenarios proposed in the literature \cite{13–17} which can be probed at the experiments (For recent reviews see for instance \cite{18, 19}). In the case of type-I and type-III seesaw models, one can also have large Yukawa couplings and new fermions of masses in the TeV scale by choosing some particular textures of the neutrino Yukawa coupling matrix \cite{20, 22}.

An important aspect to be considered while studying seesaw models is the issue of naturalness.
It is well known that the Higgs mass gets large corrections from the higher order loop diagrams due to its self-interaction as well as the couplings with gauge bosons and fermions. The theory is perceived unnatural if a severe fine-tuning between the quadratic radiative corrections and the bare mass is needed to bring down Higgs mass to the observed scale. It is well known that although the dimensional regularization can throw away the quadratic divergences, the presence of other dangerous logarithmic and finite contributions cause similar naturalness problem. In the case of seesaw models in which the new particles couple to the SM Higgs, this naturalness problem is enhanced \cite{23,33}. Demanding the correction to the Higgs mass to be of the order of TeV sets an upper limit on the masses of the new heavy particles and can be deemed as another motivation for bringing down the seesaw scale.

Another aspect of low scale seesaw models which have received attention lately is the implications of such scenarios for the stability of the EW vacuum. It is to be noted that the observed value of the Higgs mass of $125.7 \pm 0.3$ GeV is quite intriguing from the viewpoint of the EW vacuum stability. The measured values of the SM parameters, especially the top mass $M_t$ and strong coupling constant $\alpha_s$ suggest that an extra deeper minima resides near the Planck scale, threatening the stability of the present EW vacuum \cite{34,35}, i.e., the EW vacuum might tunnel into that true (deeper) vacuum. The decay probability has been calculated using the state of the art NNLO corrections and it suggests that the present EW vacuum is metastable at $3\sigma$ which means that the decay time is greater than the age of the universe. It is well known that the scalar couplings pull the vacuum towards stability whereas the Yukawa couplings push it towards instability. Thus, in the case of seesaw models, the Yukawa couplings as well as the masses of the new fermions will also get bounded by the constraints from the stability/metastability of the EW vacuum \cite{32,36,47}. In particular, in reference \cite{48}, the authors have discussed the implications of vacuum stability and gauge-Higgs unification in the context of the type-III seesaw model and reference \cite{45} has discussed the EW vacuum metastability in the context of type-I as well as type-III seesaw models. In reference \cite{32}, the authors have studied the implications of naturalness and vacuum stability in a minimal type-I seesaw model. Similarly, the naturalness and vacuum stability in the case of the type-II seesaw model has been studied in reference \cite{31}.

In this paper, we study the consequences of naturalness in the minimal type-III seesaw model, in which we extend the SM by adding two $SU(2)_L$ triplet fermions with zero hypercharge to
explain the origin of the non-zero neutrino masses and mixing. To give mass to all the three light
active neutrinos, one needs three triplet fermions. Hence, in the minimal type-III seesaw model,
the lightest active neutrino will be massless. We use the Casas-Ibarra (CI) parametrization for
the neutrino Yukawa coupling matrix \[49, 50\] and by choosing the two triplets to be degenerate,
we have only three independent real parameters, namely the mass of the triplet fermions and a
complex angle in the CI parametrization. We study and constrain the bounds on these model
parameters by demanding the theory to be natural. In addition, we also study the bounds on the
model from the EW vacuum metastability as well as lepton flavor violating (LFV) decays.

The rest of the paper is organized as follows: In Sec. II, we review the minimal type-III seesaw
model and the parametrization used for our studies. In Sec. III, we discuss the implications of
naturalness in the minimal type-III seesaw model and in section IV, we have discussed the con-
straints from the LFV decays. After this, we discuss the effective Higgs potential in the presence of
the extra fermion triplets and the renormalization group (RG) evolution of the different couplings.
This is followed by the detailed discussion of the results and we summarize in Sec. VII.

II. THE MINIMAL TYPE-III SEESAW MODEL

We extend the standard model with two fermionic triplets $\Sigma_{R_i}$, $i = 1, 2$ with zero hypercharge,
which can be represented as,

$$
\Sigma_R = \begin{bmatrix}
\Sigma^0_R/\sqrt{2} & \Sigma^+_R \\
\Sigma^-_R & -\Sigma^0_R/\sqrt{2}
\end{bmatrix} \equiv \frac{\Sigma^i_R \sigma^i}{\sqrt{2}},
$$

(2.1)

where $\Sigma^\pm_R = (\Sigma^1_R \mp i\Sigma^2_R)/\sqrt{2}$. The parts of the Lagrangian that are relevant to neutrino mass
generation are,

$$
-\mathcal{L}_\Sigma = \tilde{\phi}^\dagger \Sigma_R \sqrt{2} Y_\Sigma L + \frac{1}{2} \text{Tr} [\Sigma_R M \Sigma_R^c] + \text{h.c.},
$$

(2.2)

where the generation indices have been suppressed. In the above equation, $L = (\nu_l \ l^-)^T$ is the
lepton doublet and $\tilde{\phi} = i\sigma_2 \phi^*$ ($\sigma_2$ is the second Pauli matrix). For simplicity, we consider the
scenario in which the Majorana mass matrix $M$ is proportional to the identity matrix so that the
heavy fermions have degenerate masses which we denote by $M_{\Sigma}$. Once the Higgs field $\phi$ acquires
a vacuum expectation value (VEV), the neutral fermion mass matrix could be written as,
\[
M_\nu = \begin{pmatrix} 0 & M_T^D \\ M_D & M \end{pmatrix}.
\]  
(2.3)

Here, \( M_D = Y_\Sigma v/\sqrt{2} \), where \( v = 246 \) GeV is the VEV of the SM Higgs. The above given mass matrix could be diagonalized by a unitary matrix \( U_0 \) as,
\[
U_0^T M_\nu U_0 = M_\nu^{\text{diag}} = \text{diag}(m_1, m_2, m_3, M_\Sigma, M_\Sigma),
\]  
(2.4)

where \( M \) is the mass of the heavy triplet fermions. Also, we will have the lightest neutrino mass as 0, that is, \( m_1 = 0 \) for the normal hierarchy (NH) and \( m_3 = 0 \) for the inverted hierarchy (IH). We can write the matrix \( U_0 \) as \( [51] \),
\[
U_0 = W U_\nu \simeq \begin{pmatrix} (1 - \frac{1}{2} \epsilon) U & M_D^\dagger (M^{-1})^* U_R \\ -M^{-1} M_D U & (1 - \frac{1}{2} \epsilon') U_R \end{pmatrix} \equiv \begin{pmatrix} U_L & T \\ S & U_H \end{pmatrix}.
\]  
(2.5)

Here, \( W \) brings the full 5 \( \times \) 5 mass matrix to the block diagonal form and \( U_\nu \) and \( U_R \) diagonalizes the light and heavy neutrinos mass matrices respectively. In our case, \( U_R \) is 2 \( \times \) 2 identity matrix. \( U_L \) is the Pontecorvo-Maki-Nakagava-Sakata (PMNS) mixing matrix with a small non-unitary correction. The non-unitarity is characterized by \( \epsilon \) and \( \epsilon' \) and are given by,
\[
\epsilon = T T^\dagger = M_D^\dagger (M^{-1})^* M_D, \quad \epsilon' = S S^\dagger = M^{-1} M_D M_D^\dagger (M^{-1})^*.
\]  
(2.6)

In the limit \( M \gg M_D \), the light neutrino mass matrix is given by,
\[
m_{\text{light}} = -M_D^T M^{-1} M_D.
\]  
(2.7)

We use the Casas-Ibarra parametrization \([49,50]\) for the Yukawa coupling matrix \( Y_\Sigma \), such that the constraints on the light neutrino mixing angles as well as the mass squared differences as predicted from the oscillation data are automatically satisfied. In this parametrization,
\[
Y_\Sigma = \frac{\sqrt{2}}{v} \sqrt{D_\Sigma} R \sqrt{D_\nu} U^\dagger,
\]  
(2.8)

where \( D_\Sigma = \text{diag}(M_\Sigma, M_\Sigma) \), \( D_\nu = \text{diag}(m_1, m_2, m_3) \), and \( R \) is an arbitrary complex 2 \( \times \) 3 orthogonal matrix which parametrizes the information that is lost in the decoupling of the triplet fermions. The light neutrino masses for the normal and inverted hierarchies are given by,
\[
m_1 = 0, \quad m_2 = \sqrt{\Delta m_{\text{sol}}^2}, \quad m_3 = \sqrt{\Delta m_{\text{atm}}^2} \quad \text{(NH)}
\]
\[ m_1 = \sqrt{\Delta m_{atm}^2}, \quad m_2 = \sqrt{\Delta m_{sol}^2 + \Delta m_{atm}^2}, \quad m_3 = 0 \quad \text{(IH)}. \]  

We use the following parametrization of the PMNS matrix \( U \) in which,

\[
U = \begin{pmatrix}
  c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\
 s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13}
\end{pmatrix} P, \tag{2.10}
\]

where \( c_{ij} = \cos \theta_{ij}, \ s_{ij} = \sin \theta_{ij} \) and the phase matrix \( P = \text{diag}(e^{-i\alpha}, e^{+i\alpha}, 1) \) contains the Majorana phases.

| Parameter | NH | IH |
|-----------|----|----|
| \( \Delta m_{21}^2/10^{-5}eV^2 \) | 6.80 \( \rightarrow \) 8.02 | 6.80 \( \rightarrow \) 8.02 |
| \( \Delta m_{31}^2/10^{-3}eV^2 \) | +2.399 \( \rightarrow \) +2.593 | −2.562 \( \rightarrow \) −2.369 |
| \( \sin^2 \theta_{12} \) | 0.272 \( \rightarrow \) 0.346 | 0.272 \( \rightarrow \) 0.346 |
| \( \sin^2 \theta_{23} \) | 0.418 \( \rightarrow \) 0.613 | 0.435 \( \rightarrow \) 0.616 |
| \( \sin^2 \theta_{13} \) | 0.01981 \( \rightarrow \) 0.02436 | 0.02006 \( \rightarrow \) 0.02452 |

**TABLE I:** The oscillation parameters in their 3\( \sigma \) range, for both NH and IH as given by the global analysis of neutrino oscillation measurements with three light active neutrinos \[52\].

In our numerical analysis, we have used the values of mass squared differences and mixing angles in the 3\( \sigma \) ranges as shown in table \[1\] \[52\] and vary the phases \( \delta \) and \( \alpha \) between \( -\pi \) to \( +\pi \). It has been shown in reference \[50\] that the matrix \( R \) could be parametrized as,

\[
R = \begin{pmatrix}
  0 & \zeta \sin z \\
 0 & \zeta \cos z \\
\cos z & \zeta \sin z \\
\sin z & \zeta \cos z
\end{pmatrix} \quad \text{(NH)}
\]

\[
R = \begin{pmatrix}
  0 & \zeta \sin z \\
 0 & \zeta \cos z \\
\cos z & \zeta \sin z \\
\sin z & \zeta \cos z
\end{pmatrix} \quad \text{(IH)}, \tag{2.11}
\]

where \( z \) is a complex parameter and \( \zeta = \pm 1 \). We fix the value of \( \zeta \) to be \( +1 \) for all our analysis and this doesn’t change any of our results. Thus the only free parameters in the model are the mass of the triplet fermions, \( M_\Sigma \) and the complex angle \( z \).
Note that in this model, the charged components of the triplet fermions mix with the SM charged leptons. This is governed by the Lagrangian \[53\],

\[
L = - \left( \bar{\ell}_R \, \bar{\Psi}_R \right) \begin{pmatrix} m_l & 0 \\ \sqrt{2}M_D & M \end{pmatrix} \begin{pmatrix} \ell_L \\ \Psi_L \end{pmatrix} + \text{h.c.}, \tag{2.12}
\]

where we have defined,

\[
\Psi = \Sigma^+_R + c + \Sigma^-_R. \tag{2.13}
\]

The charged fermion mass matrix given in the above eqn. can be diagonalized by a bi-unitary transformation.

Since the additional heavy triplet fermions have the \(SU(2)\) gauge interactions, it could be produced and detected in the collider experiments through the process(es) \(pp \rightarrow \Sigma^+\Sigma^- \rightarrow mj + nl + \not{E}_T\) \((m, n\) indicate the integer). The collider study of extra triplet fermions was first explored in reference \[54\] in the context of a \(SU(5)\) GUT model. Since then, a lot of works have been done on the phenomenology of type-III seesaw model in the context of LHC \[55-62\]. The experimental searches performed by the CMS and the ATLAS have put lower bounds on the triplet masses. CMS \[63\] has set a lower limit of 430 GeV on the triplet mass with the data from \(\sqrt{s} = 13\) TeV run whereas depending on the various scenarios studied, the ATLAS results rule out masses in the range below \(325 - 540\) GeV \[64\]. Recently, the authors of reference \[65\] have studied the phenomenology of type-III seesaw model in the context of high energy \(e^+e^-\) colliders.

### III. NATURALNESS

One of the problems associated with the high-scale seesaw models is that the associated heavy particles give a very large corrections to the Higgs mass making the theory unnatural. Here, we shall see the implications of naturalness in the context of the type-III seesaw. The tree level SM Higgs potential is given by,

\[
V = -\mu^2(\Phi^\dagger\Phi) + \lambda(\Phi^\dagger\Phi)^2, \tag{3.1}
\]

where,
\[ \Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} G^+ \\
 v + h + iG^0 \end{pmatrix} \]  

(3.2)

where the vev \( v = 246 \text{ GeV} \) and this will give us the physical Higgs particle with tree level mass as \( m_h^2 = 2\lambda v^2 \). For the naturalness of the Higgs mass, the heavy right handed neutrino loop corrections to the mass parameter \( \mu \) should be smaller than \( O(\text{TeV}^2) \). In the \( \overline{\text{MS}} \) scheme, the correction is given by,

\[ \delta \mu^2 \approx \frac{3}{4\pi^2} \text{Tr}[Y^\dagger \Sigma D^2 Y \Sigma]. \]  

(3.3)

Note that we have taken the quantity \( (\ln[\frac{M_{\Sigma}}{\mu_R}] - \frac{1}{2}) \) to be unity (where \( \mu_R \) is the renormalization scale). Now, using the parametrization in eqn. (2.8), we get,

\[ \delta \mu^2 \approx \frac{3}{4\pi^2} \frac{2}{v^2} \text{Tr}[D_\nu R^i D_\Sigma^3 R] = \frac{3M^3_{\Sigma}}{2\pi^2 v^2} \cosh(2\text{Im}[z]) \times \begin{cases} 
\sqrt{\Delta m^2_{\text{sol}}} + \sqrt{\Delta m^2_{\text{atm}}} & (\text{NH}) \\
\sqrt{\Delta m^2_{\text{atm}}} + \sqrt{\Delta m^2_{\text{sol}}} + \Delta m^2_{\text{atm}} & (\text{IH}). 
\end{cases} \]  

(3.4)

From the above expressions, we can see that the only unknown parameters are \( M_{\Sigma} \) and \( \text{Im}[z] \).

(a) Naturalness contour for NH  
(b) Naturalness contours for IH

FIG. 1: Naturalness contours in the \( \text{Im}[z]-M_{\Sigma} \) plane. The left (right) plot is for NH (IH). In the shaded regions, \( \delta \mu^2 \) is less than \( p\% \) of \( 1\text{TeV}^2 \) where \( p = 500, 100, 50, 20, 10, 5, 1 \) (from top to bottom). The unshaded regions are disfavored by naturalness.

In figure (1), we have presented the naturalness contours in the \( \text{Im}[z]-M_{\Sigma} \) plane for both NH and IH. In the shaded regions, \( \delta \mu^2 \) is demanded to be less than \( p\% \) of \( 1\text{TeV}^2 \) where \( p = \)
From these plots, we can see that higher the mass of the triplet, smaller the allowed values of the \( \text{Im}[z] \). For instance, demanding \( \delta \mu^2 < (1 \text{ TeV})^2 \) implies that \( M_\Sigma \leq 1.84 \times 10^7 \text{ GeV} \) for \( \text{Im}[z] = 0 \) and \( M_\Sigma \leq 3 \times 10^5 \text{ GeV} \) for \( \text{Im}[z] = 6 \). These bounds become even more stringent as we demand \( \delta \mu^2 \) to be smaller as could be seen from the plots. Also, from eqn (3.4), we can see that the \( \delta \mu^2 \) values for NH and IH differ roughly by a factor of half (\( \Delta m^2_{\text{atm}} >> \Delta m^2_{\text{sol}} \)). This effect could be seen from the fact that for a given value of \( \text{Im}(z) \), the maximum allowed value of \( M_\Sigma \) for NH is slightly higher than that for IH.

IV. CONSTRAINTS FROM THE LEPTON FLAVOUR VIOLATION

The decay widths and the branching ratios (BR) for the various lepton flavor violating decays in the context of type-III seesaw model have been worked out in the reference [53]. This model can have the decays \( \mu \to e\gamma \) and \( \tau \to l\gamma \) at the one loop level and \( \mu \to 3e \) as well as \( \tau \to 3l \) processes in the tree level due to the charged lepton mixing. However, among all the LFV decays, the most stringent bound is the one coming from \( \mu \) to \( e \) conversion in the nuclei. The \( \mu \to e \) conversion rate to the total nucleon muon capture rate ratio (\( R^{\mu\to e} \)) puts a bound on \( \epsilon_{e\mu} \). For the \(^{48}\text{Ti} \) nuclei, we have [66],

\[
R^{\mu\to e} < 4.3 \times 10^{-12},
\]  

(4.1)

and the bound from this is the most stringent among all the LFV bounds in the triplet fermion model and is given as [53],

\[
\epsilon_{e\mu} < 1.7 \times 10^{-7}.
\]  

(4.2)

We present the constraints on \( z \) and \( M_\Sigma \) from this bound in figure 2 for both NH and IH. The region above the blue dotted line are disallowed by the LFV bounds whereas the regions to the right of the purple, magenta and brown solid lines are disallowed by the naturalness bounds depending on the naturalness condition used. We can clearly see that the naturalness bounds restricts larger values of \( M_\Sigma \) whereas the LFV bound constrains the larger values of \( \text{Im}(z) \) corresponding to the smaller values of \( M_\Sigma \). The unshaded region is the one that is allowed by both LFV as well as the naturalness bounds. One can notice from these plots that for both NH and IH, the maximum
allowed value of $\text{Im}(z)$ is $\sim 10$ which corresponds to a triplet mass of $\sim 10^4$ GeV. In generating these plots, we have varied the light neutrino mass squared differences and mixing angles in their $3\sigma$ ranges and the Dirac and Majorana phases are varied in the range $0 - 2\pi$ and presented the most stringent bounds.

![Graphs](image)

**FIG. 2:** Bounds on $Z$ from lepton flavour violation (blue dotted line) and naturalness (purple, magenta and brown solid lines). The left (right) plot is for NH (IH). The unshaded region is allowed by both LFV as well as naturalness bounds.

V. VACUUM STABILITY

In this section, we discuss how the stability of the EW vacuum is modified in the presence of the extra fermionic triplets if we assume that there is no other new physics up to the Planck scale. It is well known that if we have extra fermions, they will destabilize the EW vacuum. We aim to quantify this effect and obtain constraints in the context of the model outlined. In the following, we discuss the theoretical background and tools needed in the stability analysis of the EW vacuum up to the Planck scale such as the Higgs effective potential which determines the instability, metastability, stability and perturbative-unitary scales, the proper matching conditions...
which give the initial values of the model parameters at the electroweak (EW) scale and the RGEs
delineating the running of the couplings and the other parameters from the EW scale up to the
Planck scale $M_{Pl}$.

The SM one-loop effective Higgs potential in the $\overline{MS}$ scheme and the Landau gauge can be
written as

$$V_{1}^{\text{SM}}(h) = \sum_{i=1}^{5} \frac{n_{i}}{64\pi^{2}} M_{i}^{4}(h) \left[ \ln \frac{M_{i}^{2}(h)}{\mu^{2}(t)} - c_{i} \right], \quad (5.1)$$

where the index $i$ is summed over all SM particles, $M_{i}^{2}(h) = \kappa_{i}(t) h^{2}(t) - \kappa'_{i}(t)$ and $c_{h,G,f} = 3/2$, $c_{W,Z} = 5/6$ [67–71]. $n_{i}$ is the number of degrees of freedom of the particle fields. The values of
$n_{i}$, $\kappa_{i}$ and $\kappa'_{i}$ are given in the eqn.(4) in [67]. The above contribution comes with a positive sign
for the gauge and scalar bosons, whereas it is negative for the fermion fields. The running energy
scale $\mu$ is related to a dimensionless parameter $t$ as $\mu(t) = M_{Z} \exp(t)$.

The additional contribution to the one-loop effective potential from the fermionic triplet is [38, 47, 72],

$$V_{1}^{\Sigma}(h) = 3 \left[ \frac{M_{D}(h) M_{D}(h)}{\mu^{2}(t)} \ln \frac{M_{D}(h) M_{D}(h)}{\mu^{2}(t)} - \frac{3}{2} \right] - \frac{3}{32\pi^{2}} \left[ \ln \frac{M_{D}(h) M_{D}(h)}{\mu^{2}(t)} \right]^{2} \left[ \ln \frac{M_{D}(h) M_{D}(h)}{\mu^{2}(t)} - \frac{3}{2} \right] \frac{3}{2}, \quad (5.2)$$

where $M_{D}(h) = \frac{Y_{\Sigma}}{\sqrt{2}} h$ and $i, j$ run over the three light neutrinos and the two triplet fermions
respectively. In this analysis, we use the two-loop contributions to the effective potential for the
SM particles whereas extra fermion triplet is considered up to one-loop only. For high field value
$h(t) >> v$, the effective potential could be approximated as, $V_{e\text{ff}}^{\text{SM}+\Sigma} = \lambda_{e\text{ff}}(h) \frac{h^{4}}{4}$. The one- and
two-loop SM expressions for $\lambda_{e\text{ff}}(h)$ can be found in reference [35]. The contributions due to the
extra fermionic triplet is obtained as,

$$\lambda_{e\text{ff}}^{\Sigma}(h) = -3 e^{\Gamma(h)} \left( \frac{Y_{\Sigma} Y_{\Sigma}^{\dagger}}{2} \ln \frac{Y_{\Sigma} Y_{\Sigma}^{\dagger}}{2} - \frac{3}{2} \right) + \left( Y_{\Sigma} Y_{\Sigma}^{\dagger} \right)_{jj} \left( \ln \frac{Y_{\Sigma} Y_{\Sigma}^{\dagger}}{2} - \frac{3}{2} \right) \frac{3}{2}, \quad (5.3)$$

where, the factor $\Gamma(h) = \int_{M_{t}}^{h} \gamma(\mu) d \ln \mu$ indicates the wave function renormalization of the Higgs
field. Here $\gamma(\mu)$ is the anomalous dimension of the Higgs [67–71], the contribution to which from
the fermion triplet at one loop is $\frac{3}{2} \text{Tr} \left( Y_{\Sigma} Y_{\Sigma}^{\dagger} \right)$. We also assume that $\mu = h$. In this choice, all the
running coupling constants ensure faster convergence of the perturbation series of the potential [73].
We compute the RG evolution of all the couplings to analyse the Higgs potential up to the Planck Scale. We first calculate all the SM couplings at the top mass scale $M_t$, taking care of the threshold corrections \cite{74,77}. We use one-loop RGEs to calculate $SU(2)$ and $U(1)$ gauge couplings $g_2(M_t)$ and $g_1(M_t)$ \cite{74}. For $g_3(M_t)$, the $SU(3)$ gauge coupling, we use three-loop RGEs considering contributions from the five quarks and the effect of the sixth, i.e., the top quark has been taken using an effective field theory approach. We also include the leading term in the four-loop RGE for $\alpha_s$. The mismatch between the top pole mass and the $\overline{MS}$ renormalized coupling has been taken care of using the threshold correction $y_t(M_t) = \sqrt{2} \frac{M_t}{v} (1 + \delta_t(M_t))$ where $\delta_t(M_t)$ is the matching correction for $y_t$ at the top pole mass. We use $\lambda(M_t) = \frac{M_t^2}{v^2} (1 + \delta_H(M_t))$ for the Higgs quartic coupling $\lambda$. To calculate this at the scale $M_t$, we have included the QCD corrections up to three loops \cite{78}, electroweak corrections up to one-loop \cite{79,80} and the $O(\alpha\alpha_s)$ corrections to the matching of top Yukawa and top pole mass \cite{75,81}. We have reproduced the SM couplings at $M_t$ as in references \cite{35,77} by using these threshold corrections. We evolve them up to the heavy fermionic mass scale using the SM RGEs \cite{82–85}. The extra contributions due to the femionic triplets are included after the threshold heavy fermionic mass scale. Then we evolve all the couplings up to the Planck scale to find the position and depth of the new minima at the high scale.

It is well known that if the EW vacuum of the Higgs potential is not the global minimum, then a quantum tunneling to the true vacuum may occur. This happens because the RG running can make the quartic coupling $\lambda$ negative at a high energy scale. However, this does not pose a threat to the theory if the decay time is greater than the lifetime of the Universe $\tau_U \sim 10^{17}$ secs \cite{86} and in such a case, we say that the EW vacuum is metastable. The decay probability of the EW vacuum to the true vacuum at the present epoch has been computed using the bounce solution of the euclidean equations of motion of the Higgs field \cite{35,87,88},

$$P_0 = 0.15 \frac{\Lambda_B^4}{H^4} e^{-S(\Lambda_B)}, \quad \text{where } S(\Lambda_B) = \frac{8\pi^2}{3|\lambda_{eff}(\Lambda_B)|}. \quad (5.4)$$

Here, $H$ is the Hubble constant and $S(\Lambda_B)$ is the minimum action of the Higgs potential at the bounce size $R = \Lambda_B^{-1}$ which gives the dominant contribution to the tunneling probability $P_0$. The

\footnote{Our result will not change significantly even if we use the two-loop RGEs for $g_1$ and $g_2$.}
metastable EW vacuum implies that the decay probability $P_0 < 1$. This can be translated into a bound on the Higgs effective quartic coupling $\lambda_{\text{eff}}$ which can be read as \[ \lambda_{\text{eff}} > \lambda_{\text{eff}}(\Lambda_B) = \frac{-0.06488}{1 - 0.00986 \ln \left(\frac{v}{\Lambda_B}\right)}. \] (5.5)

$\lambda_{\text{eff}}(\Lambda_B) < \lambda_{\text{eff}}(\Lambda_B)$ corresponds to the unstable region and the EW vacuum is absolutely stable at $\lambda_{\text{eff}}(\Lambda_B) > 0$. Also, the theory violates the perturbative unitarity at $\lambda_{\text{eff}}(\Lambda_B) > \frac{4\pi}{3}$. \[90\]

In figure 3, we show the running of the Higgs quartic coupling for four different sets of benchmark points for the type-III seesaw model. In the first figure, the purple and gray lines correspond to $M_t = 171.3$ and $174.9$ GeV respectively with the value of $\text{Tr}[Y_\Sigma^\dagger Y_\Sigma]^{\frac{1}{2}}$ fixed as 0.283 and $= M_\Sigma = 10^7$ GeV. For the first case, we can see that the Higgs quartic coupling $\lambda$ remains positive up to the Planck scale, i.e., the EW vacuum is absolutely stable up to the $M_{\text{Pl}}$. For $M_t = 174.9$ GeV, we can see that $\lambda \sim \lambda_{\text{eff}}$ becomes negative at the energy scale $\sim 10^{19}$ GeV, the so called instability scale $\Lambda_I$, and remains negative up to $M_{\text{Pl}}$. However, we find that the beta function of the Higgs quartic coupling $\beta_\lambda(\equiv dV(h)/dh)$ becomes zero around the energy scale $\sim 10^{17}$ GeV, which implies that there is an extra deeper minima at that scale and we have checked that the EW vacuum corresponding to this point is metastable. Similarly in the second figure, we have given the running of the quartic coupling for two different values of $\text{Tr}[Y_\Sigma^\dagger Y_\Sigma]^{\frac{1}{2}}$ with fixed $M_t$ and $M_\Sigma$. We notice that as the value of the $\text{Tr}[Y_\Sigma^\dagger Y_\Sigma]$ is increased from 0.283 to 0.636, the EW vacuum shifts from the metastable to the unstable region. Thus, the conditions of stability and metastability can put constraints on the allowed values of $\text{Tr}[Y_\Sigma^\dagger Y_\Sigma]^{\frac{1}{2}}$.

A. Phase diagram of Vacuum stability

As we have already discussed previously, the present values of the SM parameters, especially the top Yukawa coupling $y_t$ and strong coupling constant $\alpha_s$ with Higgs mass $M_h \approx 125.7$ GeV (within the $3\sigma$ range) imply that an extra deeper minima exists near the Planck scale. Hence, there is a tension that the EW vacuum might tunnel into that true (deeper) vacuum. In the type-III seesaw model, depending upon the new physics parameter space, the stability of the EW vacuum is modified compared to that in the SM and there are two effects contributing to this. The first one is the negative contribution to the running of $\lambda$ as well as to the effective Higgs potential due
FIG. 3: RG evolution of the Higgs quartic coupling. The figure in the left side shows the running of \( \lambda \) for different values of \( M_t \) with fixed \( M_\Sigma \) and \( \text{Tr}[Y^\dagger_\Sigma Y_\Sigma]^{1/2} \) whereas the figure in the right side shows the running of \( \lambda \) for different values of \( \text{Tr}[Y^\dagger_\Sigma Y_\Sigma]^{1/2} \) with \( M_\Sigma \) and \( M_t \) fixed. For both the plots, we have taken \( M_{\Sigma 1} = M_{\Sigma 2} = M_\Sigma = 10^7 \text{ GeV} \).

to the triplet fermion Yukawa coupling (see the eqns. 5.3 and [A1]). The second one is through the modified RGE for the SU(2) gauge coupling, \( g_2 \), which in turn gives a positive contribution to the running of \( \lambda \) (Since we are adding SU(2) triplets, this will modify the SU(2) gauge coupling RGE as given in [A3]). These effects have also been discussed in reference ([45]).

In figure 4, we have given the phase diagram in the \( \text{Tr}[Y^\dagger_\Sigma Y_\Sigma]^{1/2} - M_\Sigma \) plane. The figure in the left hand side is for NH whereas the one in the right hand side is for IH. We have generated these plots for the fixed values of the SM parameters \( M_t = 173.1, M_h = 125.7 \) and \( \alpha_s = 0.1184 \). Here, the red dashed line separating the unstable region (red) and the metastable (yellow) region is obtained when \( \beta_\lambda(\mu) = 0 \) along with \( \lambda(\mu) = \lambda_{\text{min}}(\Lambda_B) \). From this plot, we can see that the parameter space with \( \text{Tr}[Y^\dagger_\Sigma Y_\Sigma]^{1/2} \gtrsim 0.64 \) with the heavy fermion mass scale \( 200 - 10^{10} \text{ GeV} \) are excluded by instability of the EW vacuum. The gray dashed line corresponds to the points for which the beta function of the quartic coupling \( \lambda \) is zero at the Planck scale, i.e., the second minima is situated at that scale. Also, we can see a very small green region near small mass and small coupling values corresponding to which the EW vacuum is absolutely stable. This is due to the positive contribution from \( g_2 \) to the running of \( \lambda \), and this effect is not very important for the
large values of Yukawa couplings as well as for large masses of the fermionic triplets. However, this region is disfavored from the LFV constraints.

In addition, we have also given the bounds from LFV as well as naturalness in this figure. One can see that the area that are allowed both by naturalness as well as LFV falls in the metastability region if we demand $\delta \mu^2 \leq 5 \text{ TeV}^2$. So, there is no extra threat from the instability of the EW vacuum to this allowed parameter space.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{phase_diagram}
\caption{The phase diagram in the $\text{Tr} [Y_{\Sigma}^\dagger Y_{\Sigma}]^{1/2} - M_\Sigma$ plane. The one in the left side is for NH whereas the right one is for IH. The color coding of the lines (blue, purple, magenta and brown) are the same as in figure 2. The red dashed line separates the unstable and the metastable regions of the EW vacuum.}
\end{figure}

It is also important to look at the change in the confidence level at which the (meta)stability is excluded or allowed (one-sided) in the context of minimal type-III seesaw model. The confidence level plot(s) will provide a quantitative measurement of the (meta)stability for the new physics parameter space. In figure 5, we show how the confidence level at which EW vacuum is allowed(excluded) from the metastability(instability) depends on new Yukawa couplings of the heavy fermions for the type 3 seesaw model.

In figure 5a, we show the dependence of confidence level against the trace of the Yukawa coupling, $\text{Tr}[Y_{\Sigma}^\dagger Y_{\Sigma}]^{1/2}$ for the fixed values of the SM parameters for the triplet mass of $M_\Sigma = 10^4$ GeV. Similar plot with a higher value of heavy fermion masses $M_\Sigma = 10^{12}$ GeV is shown in figure 5b.
In both cases the EW vacuum is metastable for smaller values of the new Yukawa coupling. For $M_h = 125.7$ GeV, $M_t = 173.1$ GeV and $\alpha_s(M_Z) = 0.1184$, the confidence level (one-sided) at which the EW vacuum is metastable (yellow region) increases with the increase of $\text{Tr} \left [ Y^\dagger_\Sigma Y_\Sigma \right ]^{1/2}$ and the EW vacuum becomes unstable for $\text{Tr} \left [ Y^\dagger_\Sigma Y_\Sigma \right ]^{1/2} \sim 0.64$ in both the cases. Also, one can see that the confidence level at which the EW vacuum is metastable increases with the increase in the mass of the fermion triplets. This is because the effect of the extra fermions remain decoupled up-to $M_\Sigma$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig5}
\caption{Dependence of confidence level at which the EW vacuum stability is excluded/allowed on $\text{Tr} \left [ Y^\dagger_\Sigma Y_\Sigma \right ]^{1/2}$ for two different values of $M_\Sigma$.}
\end{figure}

VI. SUMMARY

In this paper we have analyzed the implications of naturalness and the stability of the electroweak vacuum in the context of a minimal type-III seesaw model. We have also studied the constraints from lepton flavor violating decays. We have found that the lighter masses of the fermionic triplets, $M_\Sigma \simeq 400$ GeV are disallowed by the constraints from the $\mu \rightarrow e$ conversion in
the nucleus. At the same time, the heavier triplet masses are disfavored by the naturalness. If we demand the correction to the Higgs mass to be less than 200 GeV, it will put an upper bound of \( \sim 10^5 \text{ GeV} \) on the masses of the triplets. Also, the maximum value of \( \text{Tr}[Y^\dagger \Sigma Y]\) that is allowed is 0.1, corresponding to \( M_\Sigma \sim 10^4 \text{ GeV} \). Also, another important result we have found is that in the parameter space that is allowed by both the LFV as well as naturalness constraints, the EW vacuum is metastable. Hence, One does not really have to worry about the instability of the vacuum in this model. The major part of the allowed parameter space lies in a region that could be tested in the future collider experiments.

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**Appendix A: Renormalization Group Equations**

The beta functions for the various couplings are defined as,

\[
\beta_{\chi_i} = \frac{\partial \chi_i}{\partial \ln \mu} = \frac{1}{16\pi^2} \beta^{(1)}_{\chi_i} + \frac{1}{(16\pi^2)^2} \beta^{(2)}_{\chi_i}.
\]

For the running scale \( \mu < M_\Sigma \),

\[
\beta_{\chi_i} = \beta^{SM}_{\chi_i}, \quad \beta^{(1)}_{g_2} = -\frac{19}{6} g_2^3 \quad \text{and} \quad \beta_{Y_\Sigma} = 0,
\]

and for \( \mu > M_\Sigma \), the one-loop RGEs for \( \lambda, y_t, g_2 \) and \( Y_\Sigma \) are as given below.

\[
\beta_{\lambda} = \frac{3}{8} g_1^4 + \frac{3}{4} g_1^2 g_2^2 + \frac{9}{8} g_2^4 - 3g_1^2 \lambda - 9g_2^2 \lambda + 24\lambda^2 + 12\lambda y_t^2 - 6y_t^4
\]

\[
+ 12\lambda \text{Tr}(Y_\Sigma Y^\dagger_\Sigma) - 10\text{Tr}(Y_\Sigma Y^\dagger_\Sigma Y^\dagger_\Sigma Y_\Sigma) \tag{A1}
\]

\[
\beta_{y_t} = y_t \left( \frac{9}{2} y_t^2 - 8g_3^2 - \frac{17}{12} g_1^2 - \frac{9}{4} g_2^2 + 3\text{Tr}(Y_\Sigma Y^\dagger_\Sigma) \right) \tag{A2}
\]
Two-loop RGEs used in this work have been generated using SARAH [92]. In our work, we have taken only the top-quark contributions. The other SM-Yukawa couplings are comparatively smaller and their inclusion does not alter our result.

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