Breaking the degeneracy between polarization efficiency and cosmological parameters in CMB experiments

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(Dated: February 9, 2021)

Accurate cosmological parameter estimates using polarization data of the cosmic microwave background (CMB) put stringent requirements on map calibration, as highlighted in the recent results from the Planck satellite. In this paper, we point out that a model-dependent determination of polarization calibration can be achieved by the joint fit of the TE and EE CMB power spectra. This provides a valuable cross-check to band-averaged polarization efficiency measurements determined using other approaches. We demonstrate that, in ΛCDM, the combination of the TE and EE constrain polarization calibration with sub-percent uncertainty with Planck data and 2% uncertainty with SPTrot data. We arrive at similar conclusions when extending ΛCDM to include the amplitude of lensing A_s, the number of relativistic species N_\text{eff}, or the sum of the neutrino masses \( \sum m_\nu \). The uncertainties on cosmological parameters are minimally impacted when marginalizing over polarization calibration, except, as can be expected, for the uncertainty on the amplitude of the primordial scalar power spectrum \( \ln(10^{10} A_s) \), which increases by 20 – 50%. However, this information can be fully recovered by adding TT data. For current and future ground-based experiments, SPT-3G and CMB-S4, we forecast the cosmological parameter uncertainties to be minimally degraded when marginalizing over polarization calibration parameters. In addition, CMB-S4 could constrain its polarization calibration at the level of \( \sim 0.2\% \) by combining TE and EE, and reach \( \sim 0.06\% \) by also including TT. We therefore conclude that relying on calibrating against Planck polarization maps, whose statistical uncertainty is limited to \( \sim 0.5\% \), would be insufficient for upcoming experiments.

I. INTRODUCTION

The Λ cold dark matter (ΛCDM) model has emerged to be the leading model in describing our universe since the advent of precision measurements of the anisotropies in the cosmic microwave background (CMB). On the largest angular scales, we have satellite measurements from WMAP and Planck that reach cosmic-variance limits in the temperature anisotropy spectrum up to multipoles \( \ell \sim 500 \) and \( \ell \sim 1600 \) respectively [1–3]. On small angular scales, large aperture ground-based experiments like the Atacama Cosmology Telescope (ACT) and the South Pole Telescope (SPT) provide high signal-to-noise measurements of the CMB damping tail [4, 5], in both temperature and polarization.

As elucidated and forecasted in [6] and demonstrated by Planck and recent results from ground-based telescopes, polarization measurements of the CMB are increasingly dominating over the temperature measurements in terms of their statistical constraining power on cosmological parameters. However, in order to fully take advantage of these upcoming data sets, systematic errors that could bias the polarization measurements must be sufficiently mitigated and controlled. Specifically, recent Planck results show that cosmological parameters can be biased by one of the main polarization systematics—errors in the estimates of the polarization efficiencies of the detectors [7, 8]. For Planck, the polarization efficiencies of its detectors as measured in-flight were discrepant from what were expected from laboratory measurements by up to 5 times the statistical uncertainties of the laboratory measurements [9]. To account for this discrepancy, the Planck polarization calibrations at different frequencies were then re-evaluated by requiring the polarization spectra to recover the ΛCDM cosmology inferred by the temperature spectrum measurements, effectively modeling the detector polarization efficiencies as overall calibration of the polarization maps per frequency, \( P_{\text{cal}} \).

In this work, we propose an alternative method to extract polarization calibration as a potential cross-check for direct approaches. Typically, polarization calibration parameters are included in cosmological parameter estimation as nuisance parameters with priors informed by external calibration steps [e.g. 10–12]. Here, we jointly fit the ΛCDM and extension models to the CMB TE and EE spectra allowing the polarization calibration parameters to float, i.e., we let the data to self-calibrate \( P_{\text{cal}} \) given a model. We show that the combination of just TE and EE is sufficient in providing a tight \( P_{\text{cal}} \) constraint, and the \( P_{\text{cal}} \) uncertainty can be further improved by including the temperature power spectrum TT. Atmospheric noise degrades the ground-based TT measurement more than satellite TT or ground-based TE and EE measurements. For this reason, the ability to self-calibrate \( P_{\text{cal}} \) with only TE and EE as demonstrated by this work is of particular interest to current and upcoming ground-based experiments [e.g. 13–17].

The inferred polarization calibration from our proposed method can produce tight constraints because of the different dependence on \( P_{\text{cal}} \) of TE and EE, which breaks parameter degeneracies with other cosmological parameters. While this inferred polarization calibration is admittedly model-dependent,
it is nevertheless useful as a consistency check against polarization calibration estimated through other methods. Furthermore, we show that most $\Lambda$CDM parameter constraints are only mildly to negligibly degraded when marginalizing over $P_{\text{cal}}$, and common extensions to $\Lambda$CDM are insensitive to marginalizing over this extra parameter.

In the following, we apply this method to SPTroI and $\textit{Planck}$ data and show that $P_{\text{cal}}$ are constrained to percent level precision for these experiments across $\Lambda$CDM and its extensions, including the lensing amplitude $A_s$, the effective number of relativistic species $N_{\text{eff}}$, and the sum of neutrino masses $\sum m_\nu$. We take inputs from a recent SPTroI power spectrum analysis [18, hereafter H18] and $\textit{Planck}$’s latest data release [19] and sample parameter spaces without imposing priors on their respective polarization calibration parameters.

With the recent release of the ACTpol DR4 data, we apply this method to the publicly available $\textit{ACTPollite}$ likelihood [20, 27] to demonstrate the ease of application of this approach. We use CosmoMC [21] for sampling the posterior distributions of SPTroI and $\textit{Planck}$, and COBAYA [22] for ACTpol. To check the relevance of this method for upcoming and future data sets, we forecast the $P_{\text{cal}}$ uncertainty and the changes in cosmological parameter uncertainties when marginalizing over $P_{\text{cal}}$ for SPT-3G and CMB-S4. While this paper was in its final stages of preparation, the results from the first season of the SPT-3G experiment were released [23]. We leave the application of our method to this data set to future work.

This paper is organized as follows. In Sec. II, we summarize polarization calibration as defined in SPTroI and $\textit{Planck}$. We present results for SPTroI, ACTpol, and $\textit{Planck}$ in Sections III to V. Our forecasts for SPT-3G and CMB-S4 are detailed in Sec. VI. We conclude in Sec. VII.

II. POLARIZATION EFFICIENCY AND EFFECTIVE CALIBRATION

The power absorbed by a polarized detector in an experiment such as $\textit{Planck}$ or SPTroI at time $t$ can be modeled as:

\[ P(t) = G [I + \rho (Q \cos(\psi(t)) + U \sin(\psi(t)))] + n(t), \]

where $I$, $Q$, and $U$ are the Stokes parameters that characterize the intensity and polarization fields, $G$ is the effective gain (setting the absolute calibration), $\rho$ is the detector polarization efficiency, $\psi(t)$ is the angle of the detector with respect to the sky and $n(t)$ is the detector noise. Here we have omitted effects from beams and bandpasses without loss of generality.

Intensity and polarization $I$, $Q$, and $U$ maps per frequency are then produced via map-making [e.g., 9] by co-adding observations at different times and from different detectors. Relative calibration corrections are applied across detectors and the co-addition is weighted given the noise of the time-ordered data over some observing period. In the following, we focus on the impact of errors in the estimate of detector polarization efficiency at the coadded map level, which can be effectively captured at each frequency by a polarization calibration correction parameter $P_{\text{cal}}$.\(^1\)

For the SPTroI, TE and EE analysis in H18, polarization maps are first made incorporating detector polarization efficiencies and angles measured on ground. Then, before forming data power spectra, the temperature and polarization maps are calibrated against $\textit{Planck}$ maps. The calibration factors $\epsilon$ are formed by first taking the ratio of the cross-spectrum between two halves of SPTroI maps and the cross-spectrum between $\textit{Planck}$ maps and SPTroI maps. The $\textit{Planck}$ maps are masked and filtered identically as the SPTroI maps and thus have the same filter transfer function and mode-coupling. The remaining differences from the beams $B_b$ and the pixel-window function $\sqrt{F_b}$ of the input $\textit{Planck}$ maps are accounted for as follows:

\[ \epsilon_b = \frac{P_{\text{Planck}}^{B_{\text{Planck}}}}{P_{\text{SPT}}^{B_{\text{SPT}}}} \frac{Q_{\text{SPT},x\text{SPT}}}{Q_{\text{Planck},x\text{Planck}}}, \]

where subscript $b$ denotes binned multipole, and $i,j$ denote different halves of the SPTroI data. The calibration factors are extracted by averaging across the multipole ranges $600 < \ell < 1000$ for temperature and $500 < \ell < 1500$ for polarization. The $\textit{Planck}$ DR2 Commander polarization maps are used to obtain the polarization calibration factor, and provide a $\sim 6\%$ correction to the $Q$ and $U$ maps (see sections 4.5.2 and 7.3 in H18 for further details). The uncertainties of the calibration factors are incorporated when sampling cosmological and nuisance parameters. Specifically, the theoretical spectra to which the data are compared are scaled by $1/(T_{\text{cal}}^2 P_{\text{cal}})$ for TE and $1/(T_{\text{cal}} P_{\text{cal}})^2$ for EE, where $T_{\text{cal}}$ denotes the overall residual calibration of the maps and $P_{\text{cal}}$ denotes the polarization calibration correction. Gaussian priors with mean of unity and uncertainties of $0.34\%$ and $1\%$ are applied to $T_{\text{cal}}$ and $P_{\text{cal}}$ respectively, based on the uncertainties of the ratio estimates in Eq. 2. It is the prior on $P_{\text{cal}}$ that we remove in this work.

For $\textit{Planck}$, the modeling of polarization calibration is different from the one used in H18 in two ways. First, the $\textit{Planck}$ likelihood at high-$\ell$ \(^2\) includes maps from 3 frequencies, 100, 143, and 217 GHz, in contrast to the single-frequency analysis done in H18 at 150 GHz. Second, while the SPTroI $P_{\text{cal}}$ is defined at the map level, the $\textit{Planck}$ effective polarization calibration parameters $c_{\ell}^{E} E$ are defined at the power spectrum level for each frequency spectrum $\nu \times \nu$ used in the high-$\ell$ likelihood.\(^3\) Thus, $P_{\text{cal}} = \sqrt{c_{\ell}^{E} E}$ for each frequency.

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\(^1\)The polarization calibration correction parameter, $P_{\text{cal}}$, are sometimes called polarization efficiency corrections in $\textit{Planck}$ papers. Unless specifically referring to detector polarization efficiencies, we use polarization calibration $P_{\text{cal}}$ as applied at the map level to refer to this correction. In this paper, we would often shorten “polarization calibration correction parameter” to polarization calibration.

\(^2\)The high-$\ell$ likelihood covers $\ell > 30$. We assume here that polarization efficiency corrections have a negligible impact on the low-$\ell$ polarization likelihood due to the large uncertainties in this regime due a combination of cosmic variance, noise, and systematic uncertainties.

\(^3\)Thus, the polarization efficiency for a cross-frequency spectrum $\nu \times \nu$ in, e.g., EE is $\sqrt{c_{\ell}^{E} E} \times \epsilon_{\rho}$. |
Specifically, the theory power spectra to which the data is compared are multiplied by a calibration factor $g$ defined as

$$g^{XY}_{\nu \nu'} = \frac{1}{2 \nu_P} \left( \frac{1}{\sqrt{c_{XX}^{XY} \nu \nu'}}, \frac{1}{\sqrt{c_{YY}^{XY} \nu \nu'}} \right). \tag{3}$$

Here, $\nu \times \nu'$ indicate the frequency spectra with $\nu, \nu' = 100, 143, 217 \text{ GHz}$; the spectra are then either for $XY = TE$ or $XY = EE$. $c^{XY}_{TT}$ denotes temperature calibration parameters, which are separately determined and on which priors are set. $c_{143}^{TT}$ is set to unity so that the 143 GHz temperature map is taken as a reference. Finally, $\nu_P$ is the overall Planck calibration parameter defined at the map level, on which a Gaussian prior is set (see Section 3.3.4 of [7] for further details). As detailed in Sec. V, in the baseline Planck analysis, $c^{EE}_{\nu \nu}$ are fixed to the values obtained by comparing the EE data spectra to the theory spectra computed given the best-fit cosmology to the TT spectra. In this work, $c^{EE}_{\nu \nu}$ are nuisance parameters to be constrained by the data themselves. Given the different definitions of the polarization calibration in these SPTroo and Planck works, in the rest of this paper we will always specify whether the quoted uncertainties refer to the map-level ($P_{\text{cal}}$) or power-spectrum level ($\nu^{EE}_{\nu \nu}$) corrections. In Sec. V, we will provide results for the Planck data using both definitions.

### III. SPTPOL

#### A. Data and model description

We use the SPTroo, TE and EE power spectrum measurements from H18. The generation of these measurements is described in detail in H18 and here we highlight relevant aspects of that work. Data in H18 came from the 150 GHz band observations made by the SPTroo camera on the South Pole Telescope over an effective area of 490 deg$^2$. The power spectra cover angular multipoles $\ell$ between 50 and 8000. The polarization noise level measured in the range $1000 < \ell < 3000$ of this data set is 9.4 $\mu$K arcmin.

For the $\Lambda$CDM baseline case, we sample the identical model space as in H18 using the same covariance matrix with CosmoMC [21]. The model parameter space is composed of $\Lambda$CDM, foreground, and nuisance parameters. The $\Lambda$CDM parameters are the cold dark matter density $\Omega_c h^2$; the baryon density $\Omega_b h^2$; the amplitude and tilt of the primordial scalar power spectrum $\ln(10^{10} A_s)$ and $n_s$; the optical depth to reionization $\tau$; CosmoMC’s internal proxy to the angular scale of the sound horizon at decoupling, $\theta_{\text{MC}}$. A Gaussian prior is set on $\tau : (0.0544, 0.0073^2)$ given the Planck results [24]. The sum of neutrino mass $\sum m_\nu$, when not sampled, is fixed to 0.06 eV. On the other cosmological parameters, we set large uniform priors.

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We consider Galactic dust foregrounds and the extragalactic foregrounds from polarized point sources. We model and set priors for them identically as in H18. The priors on the amplitudes of dust at $\ell$ of $80$, $A^{EE}_{80}$ and $A^{TE}_{80}$, are set to be uniform with $[0, 2\mu\text{K}^2]$; the priors on the spatial spectral indices, $\alpha_{TE}$ and $\alpha_{EE}$, are set to $(-2.42, 0.02^2)$. Finally, the prior on the amplitude of polarized sources $P_{3000}$ is set to $[0, 2.5\mu\text{K}^2]$.

As in H18, the nuisance parameters are beam uncertainties, super-sample lensing [25], and temperature and polarization calibrations. We include effects from super-sample lensing with the prior on $\kappa$ to be $(0, 0, 0.001^2)$. We model beam uncertainties using two eigenmodes with prior $(0, 0.01^2)$ on each mode. The overall residual calibration parameter $P_{\text{cal}}$ has prior $(1.0, 0.0034^2)$. Finally, as for the focus of this paper $P_{\text{cal}}$, we either set a prior of $(1.0, 0.01^2)$, which is the baseline of H18, or no prior, which is the method we propose to let $P_{\text{cal}}$ be determined by the data.

In the following, we will report results obtained either from TE and EE separately, or from the combination of the two, which we will refer to as TE,EE.

#### B. Main results

To illustrate the idea, in Fig. 1, we show the 2D posterior of $\ln(10^{10} A_s)$ and $P_{\text{cal}}$ from TE, EE, and TE,EE without imposing a $P_{\text{cal}}$ prior. We see that without a $P_{\text{cal}}$ prior, the constraints on $A_s$ from TE alone and EE alone are very degenerate with $P_{\text{cal}}$. However, since the $P_{\text{cal}}$ dependence from TE and EE are different (linear versus quadratic in $P_{\text{cal}}$ respectively), the combined TE,EE constraint on $A_s$ and $P_{\text{cal}}$ without a prior are significantly reduced. This illustrates the potential of combining the TE and EE spectra in constraining $P_{\text{cal}}$ without significantly degrading constraints on $\Lambda$CDM parameters. Furthermore, we find that the $P_{\text{cal}}$ parameter as sampled is consistent with unity. This serves as cross-check to the polarization calibration determined by the comparison to the Planck Commander polarization maps. In the following, we first show that the constraints on $P_{\text{cal}}$ are sufficiently precise and stable across different models to be used as a cross-check for other sources of measurements. We then discuss effects on cosmological parameter uncertainties when marginalizing over $P_{\text{cal}}$.

For this SPTroo data set, we obtain a $\sim 2\%$ constraint on $P_{\text{cal}}$ in $\Lambda$CDM and three extensions—$A_1$, $N_{\text{eff}}$, and $\sum m_\nu$, as listed in Tab. I and shown in Fig. 2. This level of precision is sufficient to cross-check the baseline approach used in H18 in which the SPTroo polarization maps are calibrated against the Planck Commander maps. In other words, without applying the polarization calibration correction from comparing against Planck, one would arrive at a similar conclusion that a $6\%$ correction should be applied to the calibration of the polarization maps if one lets $P_{\text{cal}}$ float while sampling the $\Lambda$CDM and extension model spaces with the TE,EE data set. We note that in all three extension scenarios, the $P_{\text{cal}}$ constraint does not degrade significantly, which shows that this approach is useful as cross-checks beyond just the $\Lambda$CDM model.

The stable uncertainties on $P_{\text{cal}}$ across $\Lambda$CDM and the few

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4We denote Gaussian priors with mean $\mu$ and standard deviation $\sigma$ as $(\mu, \sigma^2)$, and uniform priors between $\nu_{\text{min}}$ and $\nu_{\text{max}}$ as $[\nu_{\text{min}}, \nu_{\text{max}}]$. 
extensions suggest that $P_{\text{cal}}$ has little degeneracy with other parameters. Indeed, most cosmological parameter constraints are only negligibly to mildly degraded when we relax the $P_{\text{cal}}$ prior for the SPTpol TE, EE data set. We show in Fig. 3 the ratios of cosmological parameter uncertainties between the no $P_{\text{cal}}$ prior and the baseline $P_{\text{cal}}$ prior case for the models considered. The constraints on $A_s$ degrade most, by 40–60% depending on the model. This is expected given the correlation between $\ln(10^{10} A_s)$ and $P_{\text{cal}}$. The correlation is 84% for the $\Lambda$CDM case, as suggested in Fig. 1. All of the rest of the parameter uncertainties increase by $\lesssim 10\%$ when marginalizing over the broadened $P_{\text{cal}}$ posterior space. We show in Sec. VI that the degradation in $A_s$ disappears if we include the temperature spectrum measurement $TT$ as part of the input. This is because $TT$ tightly constrains $A_s$ independent of $P_{\text{cal}}$. For data sets similar to SPTpol, not only are the constraints on $P_{\text{cal}}$ precise enough for cross-checks with other approaches, most cosmological parameter constraints are also minimally degraded when no $P_{\text{cal}}$ priors are imposed.

As an alternative way of demonstrating consistency, we compare the inferred $P_{\text{cal}}$ values from the TE-only and EE-only data sets when the rest of the parameters are fixed to the best-fit from the TE, EE joint fit in $\Lambda$CDM with the baseline $P_{\text{cal}}$ prior. The marginalized $P_{\text{cal}}$ are $P_{\text{cal}}=0.997 \pm 0.020$ and $P_{\text{cal}}=0.991 \pm 0.005$ for the TE and the EE data set respectively. This shows that the individual data set does not prefer a statistically different $P_{\text{cal}}$; there is no significant systematic residuals that project onto $P_{\text{cal}}$.

We define the correlation between two parameters $x, y$ as $\rho_{xy} = \frac{\text{cov}(x, y)}{\sqrt{\text{cov}(x, x)\text{cov}(y, y)}}$, with $\text{cov}(x, y)$ the elements of the parameter covariance matrix.
We apply this method to the recent ACTpol DR4 data set on the frequency-combined CMB-only spectra, using the ACTPollite likelihood [20, 27]. We note that the flat prior applied on $y^p$, the ACTpol polarization calibration parameter, is sufficiently broad ([0.9, 1.1]) that it is already allowing $y^p$ to float to that extent. Here we estimate how well this ACTpol data set can constrain polarization calibration using just the TE and EE spectra, with the prior on $y^p$ further widened. We also check if the TE,EE $y^p$ result is consistent with the T,TE,EE $y^p$ result.

We use only the TE and EE frequency-combined spectra without TT on both the wide and the deep patch. We then transform the $y^p$ samples by applying an inverse to match the $P_{\text{cal}}$ definition, $P_{\text{cal}} = 1/y^p$. With this setup, we find $P_{\text{cal}} = 1.0113 \pm 0.0150$ in the ΛCDM model. It is consistent with the $y^p$ result from [20], which includes the TT spectra, of $y^p = 1.0008 \pm 0.0047$.

IV. ACTPOL

A. Data and model description

In this section, we test whether jointly fitting the Planck TE and EE spectra with no prior on $P_{\text{cal}}$ would produce sufficiently precise $P_{\text{cal}}$ measurements to serve as useful cross-checks for other approaches. We also test the level of impact of this approach on the uncertainties on cosmological parameters.

In Planck, polarization efficiencies, as well as polarization angles, were measured on the ground in [28] and taken into account in the map-making algorithm SRoll [9]. At the frequencies used in the high-multipole likelihood (100, 143, 217 GHz), polarization efficiencies per detector were found to be between 83% and 96%, with estimated uncertainties between 0.1 and 0.3% at the map level. However, tests performed on the maps, which compared strongly emitting polarized galactic dust regions as observed by different detectors, suggested that residual polarization efficiency errors are several times larger than the expected uncertainties reported in [28], as shown in [9]. Left uncorrected, these residuals in the polarization efficiencies can impact cosmological parameters up to fractions of a sigma by biasing the overall amplitude of the TE and EE spectra used in the high-multipole likelihood.

In order to correct for this effect, effective polarization calibrations were estimated by the Planck collaboration by comparing the TE and EE power spectra at 100, 143, and 217 GHz to fiducial TE and EE spectra computed from the ΛCDM best-fit to the TT data. Polarized galactic contamination was cleaned using information from the 353 GHz channel [7]. The fits were performed on a limited range of multipoles ($\ell = 200 - 1000$) to discard regions affected by foreground cleaning or noise uncertainties and over about $\sim 60\%$ of the sky (see [7] for details). The advantage of this method is that it provides an absolute reference with small uncertainties.
The disadvantage is that the polarization efficiency corrections found in this way depend on the cosmological model fitted to the temperature data (although this was tested to have a small impact). This method enabled determinations of the polarization calibration for EE with uncertainties below \(\leq 0.5\%\) at the map level (\(\leq 1\%\) at power spectrum level) and for TE with uncertainties below \(\leq 1\%\) (\(\leq 2\%\)) in each of the three frequencies used in the high-multipole Planck likelihood. Up to a global polarization calibration, the derived \(c_\ell^{EE}\)s were found to be consistent with the results of the component separation algorithm SMICA [9], which measures relative interference frequency calibration ratios between foreground-cleaned polarization maps. Furthermore, in [7], it was noted that the estimates obtained separately from EE and TE should agree given the same polarization maps. However, the two measurements were found to differ by up to \(1.7\%\) at the map level at 143 GHz (see Section 3.3.4 of [7]). As we will show below, this difference cannot be reconciled by the approach we propose in this work—leaving polarization efficiencies to freely vary. Since the difference in polarization calibration from TE and EE is small enough that it could be caused by statistical fluctuations, we leave the investigation of potential biases to parameters to future work and focus on the constraints on \(\sigma_{\text{cal}}\) given the Planck data set and impact on cosmological parameters.

We consider the 2018 final release of the Planck data [7]. We use the low-multipole likelihood in polarization SimAll \((\ell = 2 – 29\) in EE only), which we will refer to as “lowE.” For high multipoles, we use the Plik likelihood \((\ell = 30 – 1997\) in EE and TE), which we will refer to as TE and EE separately or TE,EE when used in combination. For cross-checks, we use the TT Commander likelihood at low-\(\ell\) \((\ell = 2 – 29)\) and Plik at high-\(\ell\) \((\ell = 30 – 2508)\) and we refer to the combination of the two as TT. We model polarization calibration only for the high-\(\ell\) likelihoods, because their impact on low-\(\ell\) spectra are negligible compared to cosmic variance, noise, and systematic uncertainties in this multipole range. In the baseline Planck results using the Plik likelihood, the polarization calibration to the TE and EE spectra are fixed to the ones obtained from comparing the EE spectra at different frequencies to the ΛCDM best-fit of the TT+lowE data combination. These baseline parameters are listed in Table II.

### 2. Main results and robustness assessment

We first discuss the uncertainties on \(\sigma_{\text{cal}}\) for the Planck data set when it is free to vary. Using TE,EE+lowE, we find one can determine the polarization calibrations with uncertainties smaller than \(\sim 1\%\) at the map level. More specifically we find uncertainties of 0.65\%, 0.6\% and 0.8\% at the map level for \(\nu = 100, 143, 217\) GHz respectively (corresponding to 1.3\%, 1.2\% and 1.7\% at the power spectrum level). Furthermore, we compare these uncertainties to the ones obtained with the TT power spectra included. We find that the error bars shrink by almost a factor of 2 to 0.35\%, 0.31\% and 0.51\% at the map level for the three frequencies and similarly at the power spectrum level. The measurements and uncertainties are reported in Tab. II and shown in Fig. 5. With and without TT, the uncertainties on the \(\sigma_{\text{cal}}\) factors are comparable to ones used in the Plik likelihood. This demonstrates that this approach yields relevant constraints on \(\sigma_{\text{cal}}\) for cross-checks of other approaches.

In Tab. II, we observe shifts in the mean values of the polarization calibrations when TT are added to TE and EE. To check that the shifts are consistent with statistical fluctuations, we employ the formalism described in [29], which is applicable for comparing two data sets in which one is a subset of the other. We find that the observed shifts are consistent with statistical fluctuations at better than the \(2\sigma_{\text{exp}}\) level, with

\[
\sigma_{\text{exp}} = \sqrt{\sigma_{\text{TE,EE}}^2 - \sigma_{\text{TT,TE,EE}}^2}.
\]

Finally, we note that the mean values recovered from the TT,TE,EE combination are slightly different from the ones used in the baseline because of statistical fluctuations due to the different multipole range and sky mask used in the two cases (see also the discussion in section 3.7 of [7]).

We further check how much the constraints degrade when we exclude the cross-frequency spectra and only use the combination of the TE and EE frequency auto-spectra \(100 \times 100, 143 \times 143,\) and \(217 \times 217\) GHz. We find in this case comparable constraints on polarization calibrations to our baseline results. Furthermore, if we include TE and EE from only one frequency instead of all three as in our previous cases, i.e., we use only the \(100 \times 100, 143 \times 143,\) or \(217 \times 217\) GHz.

### Table II. Polarization calibrations at power spectrum level obtained from Planck data assuming different cosmological models. We also report the corresponding polarization calibrations at map level (\(\sigma_{\text{cal}} = \sqrt{\sigma_{\text{cal}}^2}, \sigma_{\text{cal}} \sim (c_\ell^{EE} + c_\ell^{TT})/2\)), to ease the comparison with those obtained for SPT in Section III. The column "baseline" lists the fixed values used in the baseline Planck likelihood, which were determined with an uncertainty of \(\sim 1\%\) at the power-spectrum level (~0.5\% at the map level).

| Parameter | Planck|TE, EE+lowE | Planck|TT, TE, EE+lowE | baseline |
|-----------|------|-------------|------|----------------|---------|
| σ_{\text{cal}}\(^{\ell=100}\) | 0.985 ± 0.013 | 1.007 ± 0.007 | 1.021 |
| σ_{\text{cal}}\(^{\ell=143}\) | 0.954 ± 0.012 | 0.973 ± 0.006 | 0.966 |
| σ_{\text{cal}}\(^{\ell=217}\) | 1.036 ± 0.017 | 1.056 ± 0.011 | 1.04 |
| P_{\text{cal}}\(^{\ell=100}\) | 0.9925 ± 0.0066 | 1.0035 ± 0.0035 | |
| P_{\text{cal}}\(^{\ell=143}\) | 0.9767 ± 0.0064 | 0.9864 ± 0.0031 | |
| P_{\text{cal}}\(^{\ell=217}\) | 1.0178 ± 0.0081 | 1.0276 ± 0.0051 | |
logical parameters and polarization calibrations. For the other frequencies, the degradation of the constraint is smaller than a factor of 2.

Fig. 6 shows the degeneracies between the $P_{\text{cal}}$ parameters at different frequencies and the most degenerate cosmological parameter, $\ln(10^{10}A_s)$. When using TE,EE+lowE, $\ln(10^{10}A_s)$ has a $\sim 40\%$ correlation with each of the three $P_{\text{cal}}$ parameters. The second most degenerate parameter is $\Omega_b h^2$ ($\sim 30\%$ correlation), while all other parameters have smaller correlations. As can be expected, we also find the degeneracies amongst the $P_{\text{cal}}$ parameters to be large: $\rho_{\ln(A_{s},A_{s})} = 81\%$, $\rho_{\ln(A_{s},c_{s})} = 60\%$ and $\rho_{\ln(A_{s},\nu_{s})} = 66\%$. These correlations are then lifted when adding information from TT.

In terms of the impact on cosmological parameter constraints when allowing $P_{\text{cal}}$ parameters to float, we show the fractional difference in $\Lambda$CDM parameter uncertainties in Fig. 7 for TE,EE and Fig. 8 for TT,TE,EE. Similar to what we see in SPT, we observe negligible to mild degradation in $\Lambda$CDM parameter uncertainties besides those for $\ln(10^{10}A_s)$, given the correlations between the $P_{\text{cal}}$ parameters and $\ln(10^{10}A_s)$. For the TE,EE data set, the uncertainty of $\ln(10^{10}A_s)$ increases by $\sim 20\%$ when the $P_{\text{cal}}$ parameters are allowed to float. Once TT is included, which independently constrains $\ln(10^{10}A_s)$, we see that floating $P_{\text{cal}}$ has negligible impact on all $\Lambda$CDM parameters. We will see similar trends in our forecasts in Sec. VI.

C. Extended models

We now turn to extensions to the $\Lambda$CDM model. Similar to Sec. III, we check the constraints on $P_{\text{cal}}$ for three extensions, $A_s$, $\sum m_r$, and $N_{\text{eff}}$. The $P_{\text{cal}}$ uncertainties are shown in Fig. 5 for TE,EE and TT,TE,EE. We see that in all cases, the uncertainties of the $P_{\text{cal}}$ parameters are similar to those in $\Lambda$CDM. As for the cosmological parameter uncertainties, Figures 7 and 8 show the increase in their error bars when marginalizing over polarization calibration parameters for Planck TE,EE and TT,TE,EE respectively.

The parameter uncertainties in $\Lambda$CDM+$N_{\text{eff}}$ are little affected, with increases in the error bars by less than 15%. On the contrary, we find a somewhat larger effect on parameter uncertainties in the $\Lambda$CDM+$\sum m_r$ model for the TE,EE data. In this case, marginalizing over $P_{\text{cal}}$ increases the upper limit on $\sum m_r$ by almost 40%, while degrading the uncertainties on $H_0$ and $\sigma_8$ by almost 30%. We note that the main source causing the degradation in $\sum m_r$ does not come from a drastic increase in posterior uncertainty given the degeneracy between $\sum m_r$ and $P_{\text{cal}}$. The main effect rather comes from a shift in the best-fit values of correlated parameters $\sum m_r$, $\ln(10^{10}A_s)$, and $P_{\text{cal}}$. For this data set, TE dominates the fit and causes $\sum m_r$ and $\ln(10^{10}A_s)$ to be anti-correlated. With $P_{\text{cal}}$ free, the best fit for $\ln(10^{10}A_s)$ shifts to lower values by about 0.7 $\sigma$. Thus, a lower value of $\ln(10^{10}A_s)$ induces a shift of the $\sum m_r$ posterior distribution to higher values. Since this distribution is single-tailed with $\sum m_r > 0$, this shift is perceived as a change in the upper bounds. These degradations disappear once the TT data is included, because TT strongly constrains $\ln(10^{10}A_s)$. While

![Fig. 5. Marginal mean and 68% confidence level error bars on the three Planck $P_{\text{cal}}$ frequency parameters when they are let free to vary assuming different cosmological models. The top plot shows the results for Planck TE,EE, while the bottom one shows Planck TT,TE,EE. Estimates on the $P_{\text{cal}}$ parameters do not change significantly when varying the cosmological model.](image-url)
For the $\Lambda$CDM+$A_L$ model, it was noted in Planck that the $A_L$ parameter is high compared to the $\Lambda$CDM expectation—at the 2.8σ or 2.1σ levels\(^6\) for polarization calibrations estimated using Planck’s baseline or estimated using separate fits of TE and EE respectively, as already described above in Sec. VA. Here we show that leaving the polarization calibrations free to vary cannot alleviate the difference between these two results. This is due to the fact that the difference between the two Planck estimates of polarization efficiency from TE alone or from EE alone ($\Delta P_{\text{cal}} \sim 0.017$ at 143 GHz at map level) is larger than the $P_{\text{cal}}$ posterior width when $P_{\text{cal}}$ is free to vary when fitting the TE,EE or TT,TE,EE data ($\sigma(P_{\text{cal}}) \lesssim 0.01$). Furthermore, the $P_{\text{cal}}$ mean values measured from these fits are in good agreement with those of the baseline estimates.\(^7\) Therefore, leaving $P_{\text{cal}}$ free to vary provides results which are similar to the baseline case. Specifically, using the TE,EE+lowE data set, the $A_L$ parameter best fit is $A_L = 1.09 \pm 0.13$, which is within $0.8\sigma_{\exp}$ of the value obtained when fixing $P_{\text{cal}}$ in the baseline case, $A_L = 1.13 \pm 0.12$, with negligible impact on the uncertainties. Similarly when also including TT, varying the polarization calibrations leads to $A_L = 1.19 \pm 0.069$, in agreement with the baseline result obtained with $P_{\text{cal}}$ fixed $A_L = 1.18 \pm 0.068$ (see also the discussion in section 3.7 of \([7]\)). Thus, leaving the polarization calibrations free to vary has a very small impact on the value and error bar of the $A_L$ parameter, which remains higher than unity at the 2.8σ level, due to the tight constraint provided by the TE,EE or TT,TE,EE data combinations which agree with the baseline estimate. For the same reason, the other cosmological parameters are little affected as well.

VI. FORECASTS

In this section, we forecast how well $P_{\text{cal}}$ could be measured with our method and the impact on cosmological parameter uncertainties when marginalizing over $P_{\text{cal}}$ for ongoing and future experiments. We consider two experiment configurations: SPT-3G, the third-generation camera currently installed on the South Pole Telescope \([13, 30]\), and CMB-S4, a next-generation ground-based CMB experiment \([17]\).

A. SPT-3G

The SPT-3G receiver observes in three frequency bands 95, 150, and 220 GHz in both intensity and polarization with $\sim 16000$ detectors over $\sim 1500$ deg$^2$ of the sky in its main survey field. The full-width half-maximum of the beams are approximately 1.7, 1.2, and 1.1 arcminutes at 95, 150, and 220 GHz respectively. The first science results from SPT-
3G using TE and EE spectra measured using data collected in 2018 have recently been released [23]. However, the data were only collected for half of the observing season with part of the focal plane operable. Therefore, for this forecast, we use noise level projections starting from 2019 when the active detector count nearly doubled. With five seasons of observations on the main survey field (2019–2023 inclusive), the noise levels in the final coadded temperature maps are projected to be 3.0, 2.2, and 8.8 μK arcmin in the three frequency bands, and those in the polarization maps are a factor of \( \sqrt{2} \) higher [13, 30].

We forecast the \( P_{\text{cal}} \) constraints along with constraints on \( \Lambda \)CDM and extension parameters for SPT-3G for two scenarios. First, we use data from only one of the three frequency bands, 150GHz, for more direct comparison with SPTpol, described in Sec. III, and to verify the impact of using only one frequency channel. Second, we report the constraints when combining maps from all three bands.

We use the Fisher Matrix formalism and code described in [6] for extracting the 1-\( \sigma \) parameter uncertainties. As inputs, we use lensed power spectra of TT, TE, and EE; we do not include the lensing reconstruction spectrum \( C_{\ell}^{\phi \phi} \). We present constraints from the combination of TE and EE as a baseline and also those including all three spectra to study the effect of including TT. We restrict the power spectrum angular multipole range to \( \ell = 100 - 3500 \), and we adopt a Gaussian prior on the optical depth to reionization of \( \sigma(\tau) = 0.007 \), based on the Planck constraint [24]. We check that including 1/\( f \) noise or marginalizing over foregrounds do not change these results substantially.

Table III shows results for the \( \Lambda \)CDM case. The SPT-3G TE and EE combination is projected to constrain \( P_{\text{cal}} \) at the level of \( \sim 0.8\% \), either using only one frequency or combining the information from all three frequencies. When freeing \( P_{\text{cal}} \), the constraint on \( \ln(10^{10} A_s) \) is degraded by about 50\% while the rest of the \( \Lambda \)CDM parameters are mildly affected (below the 15\% level). Similar to what is seen in the Planck case, the degraded constraints can be recovered by adding the TT data. In this case, marginalizing over \( P_{\text{cal}} \) has negligible impact on cosmological parameters and the constraint on \( P_{\text{cal}} \) tightens to 0.2\%.

We verify that similar constraints on \( P_{\text{cal}} \) are obtained in extensions of the \( \Lambda \)CDM model, such as \( \Lambda \)CDM+\( N_{\text{eff}} \), \( \Lambda \)CDM+\( A_s \), or \( \Lambda \)CDM+\( \Sigma m_\nu \) for both the TE+EE and the TT+TE+EE data combination. As for the cosmological parameters, we highlight here the ones with constraints degraded when marginalizing over \( P_{\text{cal}} \). In \( \Lambda \)CDM+\( \Sigma m_\nu \), the \( \ln(10^{10} A_s) \) uncertainty increases by 40\% for the TE+EE data combination. In \( \Lambda \)CDM+\( N_{\text{eff}} \), the \( \ln(10^{10} A_s) \) uncertainty increases by 70\% and the uncertainties on \( \Omega_b h^2 \) and \( H_0 \) increase by \( \sim 30\% \). However, similar to the \( \Lambda \)CDM case, when including the TT data, the marginalization over \( P_{\text{cal}} \) has minimal impact on the constraints on cosmological parameters.
TABLE III. Fisher matrix forecast on cosmological parameters and $P_{\text{cal}}$ for SPT-3G, using the 150 GHz channel alone or all of the three channels. As a comparison, we also show constraints when fixing $P_{\text{cal}}$.

|                   | $\Omega_b h^2$ $[\times 10^{-3}]$ | $\Omega_c h^2$ $[\times 10^{-3}]$ | $H_0$ $[\text{[km/s/Mpc]}]$ | $\tau$ $[\times 10^{-3}]$ | $n_s$ | $\ln[10^{10}A_s]$ | $P_{\text{cal}}$ $[\times 10^{-3}]$ |
|-------------------|----------------------------------|----------------------------------|-------------------------------|-----------------|-------|-------------------|-------------------------------------|
| $\Lambda$CDM      |                                  |                                  |                               |                 |       |                   |                                      |
| SPT-3G TE+EE 150GHz | 1.4                              | 2.0                              | 7.5                           | 6.6             | 8.0   | 1.3               |                                      |
| SPT-3G TE+EE      | 1.3                              | 1.9                              | 7.1                           | 6.6             | 7.7   | 1.3               |                                      |
| SPT-3G TT+TE+EE   | 1.4                              | 1.7                              | 6.5                           | 6.4             | 7.4   | 1.2               |                                      |
| $\Lambda$CDM+P_{\text{cal}} |                      |                                  |                               |                 |       |                   |                                      |
| SPT-3G TE+EE 150GHz | 1.6                              | 2.1                              | 8.0                           | 6.6             | 8.2   | 2.0               | 7.6                                  |
| SPT-3G TE+EE      | 1.5                              | 2.0                              | 7.7                           | 6.6             | 7.9   | 1.9               | 7.4                                  |
| SPT-3G TT+TE+EE   | 1.4                              | 1.8                              | 6.8                           | 6.4             | 7.4   | 1.2               | 2.1                                  |

B. CMB-S4

CMB-S4 is a next-generation ground-based CMB experiment aiming to observe ~70% of the sky. It is planned to have a frequency coverage from 20 to 270 GHz and the full-width half-maximum of its beam at 150 GHz is ≤1.5 arcminutes [17]. There will be telescopes observing from both the South Pole and from the Atacama desert in Chile, for a deep and a wide area survey respectively.

In this work, we forecast the constraints on $P_{\text{cal}}$ given the wide survey from Chile. We use noise curves from [31], which combine information from all frequencies using an internal linear combination method. The per-frequency noise input includes atmospheric noise; the output noise curves include residuals from component separation. We assume $f_{\text{sky}} = 0.42$, which excludes the area covering the galaxy in the wide survey. As in the forecast for SPT-3G, we use lensed power spectra in the multipole range of $\ell = 100-3500$ and we do not include information from lensing reconstruction $C_{\ell}^{\text{pol}}$.

Table IV shows results for the $\Lambda$CDM case. We find that with just TE and EE, CMB-S4 data could constrain $P_{\text{cal}}$ at the level of ~0.2%, which further tightens to 0.056% when we add TT. When freeing $P_{\text{cal}}$, constraints on cosmological parameters are mildly degraded without TT, and negligibly degraded with TT. As in the previous sections, we verify that extending the $\Lambda$CDM model with $\sum m_\nu$, $N_{\text{eff}}$, and $A_s$ does not significantly change the constraints on $P_{\text{cal}}$. Conversely, leaving the $P_{\text{cal}}$ parameter free has the largest impact on the constraints on $\Omega_b h^2$, $H_0$ and $\ln(10^{10}A_s)$ in the $\Lambda$CDM+$N_{\text{eff}}$ model for TE+EE, at the level of 30%. Similarly to previous cases, including the TT data allows us to marginalize over $P_{\text{cal}}$ with no loss of precision on cosmological parameters.

VII. CONCLUSIONS

In this paper, we demonstrate that effective polarization calibrations $P_{\text{cal}}$ could be precisely determined by fitting CMB TE and EE spectra to the $\Lambda$CDM model and its common extensions with $P_{\text{cal}}$ as a free parameter. This is possible thanks to the different dependence of the TE and EE spectra on $P_{\text{cal}}$. While allowing $P_{\text{cal}}$ to float does increase the posterior volume and therefore degrades some constraints on cosmological parameters, we show that the degradation becomes negligible once TT is included.

We apply the method to SPTpol. and Planck. For the SPTpol. 150 GHz TE and EE data set presented in H18, we extract $P_{\text{cal}}$ with an uncertainty of ~2% at the map level, independent of the considered models. For the data set from the Planck 2018 data release, combining TE and EE allows us to measure $P_{\text{cal}}$ at 100, 143, and 217 GHz with uncertainties of 0.7%, 0.6% and 0.8% at the map level. We highlight how this method can be useful for detecting inconsistencies in the data. In particular, $P_{\text{cal}}$ determined using TE and EE should agree with the ones determined with TT included or the ones measured from external data sets. If not, this could suggest the existence of unaccounted for systematics which project into these multiplicative factors.

Finally, we forecast the capabilities of current and future experiments to constrain $P_{\text{cal}}$. We find that using its 3 frequency channels, SPT-3G will be able to measure $P_{\text{cal}}$ with an uncertainty of 0.7% from TE and EE, and the uncertainty can be improved to 0.2% when including TT. We find that leaving $P_{\text{cal}}$ free to vary will degrade the constraints on $A_s$ from TE and EE by 30%, while constraints from TT,TE,EE are not affected. Furthermore, we find that CMB-S4 could further tighten the uncertainty of $P_{\text{cal}}$ to 0.2% with its TE and EE measurements and to 0.06% with TT,TE,EE. Similarly to SPT-3G, while constraints on $A_s$ are affected by the variation of $P_{\text{cal}}$ by about 20% when using TE,EE, the constraints from TT,TE,EE are unaffected.

We highlight that these uncertainties on $P_{\text{cal}}$ are comparable to or tighter than those derived for the Planck baseline (~ 0.5%). As a consequence, relying on Planck to calibrate polarization maps will ultimately limit the accuracy of these experiments, provided that the Planck uncertainty is folded in the power spectrum covariance matrix. Furthermore, if the external $P_{\text{cal}}$ determination is biased and the systematic uncertainties are not properly included, cosmological parameters constraints would be biased. We observe a possible hint of this in the Planck TE,EE $\sum m_\nu$ upper limits between the (baseline) fixed $P_{\text{cal}}$ case and the free $P_{\text{cal}}$ case. For Planck however, the difference of $\sum m_\nu$ upper limits due to a shift in the $P_{\text{cal}}$ values is still compatible with a statistical fluctuation.

For future experiments, we emphasize that stringent control of $P_{\text{cal}}$ is important for accurate and precise inference on cosmological parameters, such as the sum of neutrino masses.
TABLE IV. Fisher matrix forecast on cosmological parameters and $P_{\text{cal}}$ for CMB-S4. As a comparison, we also show constraints when not varying the $P_{\text{cal}}$.

| $\Omega_b h^2$ [x10^{-4}] | $\Omega_c h^2$ [x10^{-3}] | $H_0$ [x10^{-1}] | $\tau$ [x10^{-3}] | $n_s$ [x10^{-3}] | $\ln[10^{10}A_s]$ [x10^{-2}] | $P_{\text{cal}}$ [x10^{-3}] |
|---------------------------|---------------------------|-----------------|-----------------|-----------------|--------------------------|----------------|
| **ACDM**                  |                           |                 |                 |                 |                          |               |
| CMB-S4 TE+EE              | 0.36                      | 0.71            | 2.7             | 5.1             | 2.5                      | 0.88          |
| CMB-S4 TT+TE+EE           | 0.36                      | 0.67            | 2.5             | 4.9             | 2.3                      | 0.85          |
| **ACDM+$P_{\text{cal}}$** |                           |                 |                 |                 |                          |               |
| CMB-S4 TE+EE              | 0.42                      | 0.75            | 2.9             | 5.1             | 2.5                      | 1.0           | 2.0          |
| CMB-S4 TT+TE+EE           | 0.37                      | 0.70            | 2.6             | 4.9             | 2.3                      | 0.86          | 0.56         |

Beyond using the primary CMB spectra TT, TE, and EE, we acknowledge the possibility of adding lensing potential power spectrum measurements to further tighten constraints on $P_{\text{cal}}$ and reduce degradations in cosmological parameters. We leave this for future work. In conclusion, this paper demonstrates that a significant source of systematic error for future CMB polarization experiments can be self-calibrated without major consequences on the constraints on cosmological parameters.

**ACKNOWLEDGMENTS**

We thank the participants of the CMB systematics and calibration focus workshop hosted virtually by Kavli IPMU for helpful comments and feedback. This work was completed in part with resources provided by the University of Chicago’s Research Computing Center. This work has received funding from the French Centre National d’Etudes Spatiales (CNES). This research used resources of the IN2P3 Computer Center (http://cc.in2p3.fr). WLKW is supported in part by the Kavli Institute for Cosmological Physics at the University of Chicago through grant NSF PHY-1125897, an endowment from the Kavli Foundation and its founder Fred Kavli, and by the Department of Energy, Laboratory Directed Research and Development program and as part of the Panofsky Fellowship program at SLAC National Accelerator Laboratory, under contract DE-AC02-76SF00515. TC is supported by the National Science Foundation through South Pole Telescope grant OPP-1852617.

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