Destabilization of the 2D conducting phase by an in-plane magnetic field

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We propose a mechanism for the recently reported destabilization by an in-plane magnetic field of the conducting phase of low density electrons in 2D. We apply our self-consistent approach based on the memory function formalism to the fully spin polarized electron system. This takes into account both disorder and exchange-correlation effects. We show that spin polarization significantly favors localization because of the enhancement of the exchange-correlations. A key outcome is that the conducting phase for the fully spin polarized system is significantly suppressed. The in-plane magnetic field needed to stabilize the fully spin polarized state lies in the range $0.1 < H < 1$ T, depending on the carrier density. We determine the metal-insulator phase diagram for the unpolarized and fully polarized systems, and we estimate the dependence of the critical magnetic field on carrier density.

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Even though the existence of a metal-insulator transition for two-dimensional electron systems has been known for several years, the nature of the insulating and metallic states near the transition is still a puzzle. In the presence of a magnetic field which is perpendicular to the electron plane, the familiar quantum Hall states are recovered. This is due to the dominant contribution of orbital effects in the magneto-conductance. If the magnetic field is parallel to the electron plane it can only couple directly to the electron spin. Recent experiments have reported that a weak parallel magnetic field is sufficient to destroy the conducting phase making the system insulating. The critical magnetic field needed varies with the carrier density but is less than or of the order of 1 T for both Si and GaAs.

Numerical simulations of the interacting electron system in the absence of imperfections in the substrate have shown that for electron densities $r_s \lesssim 20$, the ground state of the system is the unpolarized electron liquid. As the electron density is lowered and the electron correlations become stronger, the free energy per electron for the fully spin polarized state approaches the free energy of the unpolarized system. For $r_s \gg 10$ the free energies are very close and the Zeeman energy gain from a quite small parallel magnetic field could be sufficient to produce a fully polarized ground state. The critical magnetic field needed to induce this transition will become weaker for increasing $r_s$.

We have previously proposed that strong correlations in the presence of weak disorder in the substrate can localize the electrons into a glassy state. We have obtained reasonable agreement with the position of the metal-insulator transition in zero magnetic field for unpolarized electrons. We know from numerical simulations that spin polarized electrons are more strongly correlated than unpolarized electrons at the same density. This is due to the additional exchange contribution when all the electrons have parallel spin. This suggests that at the same electron density and the same level of disorder in the substrate, the polarized state is more likely to be in an insulating state than the unpolarized state.

In our approach we determine the metal-glass transition using the Kubo-relaxation function \( \Phi_{\nu}(q,t) \equiv \langle N_{\nu}(q,t) N_{\nu}(q,0) \rangle \). This is defined for the normalized dynamical density variable \( N_{\nu}(q,t) = \rho(q,t)/\sqrt{\chi_{\nu}(q)} \), where \( \rho(q) = \sum_q a_q^\dagger a_{q+q} \) is the density fluctuation operator. When the polarization index \( \nu = \mu \) the system is fully polarized with all the carrier spins aligned, while \( \nu = \mu \) for the unpolarized system. \( \chi_{\nu}(q) \) is the static susceptibility for the corresponding system. We are interested in the dynamics of relaxation processes as \( t \to \infty \). The order parameters for the glassy states are given by the relaxation function in this limit, \( f_{\nu}(q) = \lim_{t \to \infty} \Phi_{\nu}(q,t) \). When \( f_{\nu}(q) \) is non-zero, spontaneous fluctuations do not decay even at infinite time.

Within the Mori-Zwanzig formalism we calculate \( \Phi_{\nu}(q,z) \) in terms of the Memory Function \( M_{\nu}(q,z) \),

\[
\Phi_{\nu}(q,z) = \frac{1}{z - \Omega_{\nu}(q)/(1 + M_{\nu}(q,z))},
\]

where \( \Omega_{\nu}(q) = q^2/(m^* \chi_{\nu}(q)) \). In the limit \( z \to 0 \) the relaxation function is

\[
\lim_{z \to 0} -z \Phi_{\nu}(q,z) = f_{\nu}(q) = \frac{1}{1 + \Omega_{\nu}(q)/M_{\nu}(q)},
\]

where \( M_{\nu}(q) = \lim_{z \to 0} -z M_{\nu}(q,z) \). This is evaluated using mode-coupling theory. We obtain

\[
M_{\nu}(q) = M_{\nu}^{cc}(q) + M_{\nu}^{ic}(q).
\]

\( M_{\nu}^{cc}(q) \) is the contribution from interactions between the carriers. The effect of scattering off disorder in the substrate is contained in \( M_{\nu}^{ic}(q) \). Taking for the interaction between carriers \( V(q) = 2\pi e^2/eq \), where \( \epsilon \) is the substrate dielectric constant, we finally get for \( M_{\nu}^{ic}(q) \) the expression.
The correlations between carriers are taken into account through the static susceptibilities \( \chi_\nu(q) = \chi_\nu(q, \omega = 0) \). The \( \chi_\nu(q) \) are known for the disorder free system from the ground state properties of the system determined by numerical simulations \([9]\). We write

\[
\chi_\nu(q, \omega) = \frac{\chi_\nu^{(0)}(q, \omega)}{1 + V(q)[1 - G_\nu(q)]\chi_\nu^{(0)}(q, \omega)}.
\]

\( \chi_\nu^{(0)}(q, \omega) \) is the Lindhard function. The static local field factors \( G_\nu(q) \) contain the correlations for the polarization state \( \nu \). We use data from Ref. \([9]\) to determine the \( G_\nu(q) \) using Eq. \([9]\) and the fluctuation-dissipation theorem \([12]\).

The level of disorder can be measured in terms of the scattering rate \( \gamma_\nu \) calculated for carriers scattering from both impurities of density \( n_i \) and surface roughness. To evaluate \( \gamma_\nu \) we use the memory function formalism \([13]\) to obtain,

\[
M_\nu^{ic}(q) = \frac{1}{2m^*q^2} \sum_{q'} [V(q')(q\cdot q') + V(|q - q'|)]
\times (q\cdot (q - q'))^2 \chi_\nu(q') \chi_\nu(|q - q'|) f_\nu(|q - q'|), \quad (4)
\]

where \( n_i \) is the impurity density. Equations \([2]\) to \([4]\) form a self-consistent set which we can solve to determine the order parameters \( f_\nu(q) \).

For the disorder Memory Function \( M_\nu^{ic}(q) \) we consider scattering off monovalent Coulomb impurities randomly distributed within the plane of the carriers, \( U_{\text{imp}}(q) = [(2\pi e^2)/(\epsilon q)] F_i(q) \), and scattering off the surface roughness at the interface \( W_{\text{surf}}(q) \). \( F_i(q) \) is the impurity form factor. We take \( W_{\text{surf}}(q) = \sqrt{2\Delta \Gamma(q)} \exp(-q\Lambda)^2/8 \) appropriate for Si MOSFETs. Details of the parameters used are given in \([7]\). The final expression for \( M_\nu^{ic}(q) \) is

\[
M_\nu^{ic}(q) = \frac{1}{m^* q^2} \sum_q \left[n_i \langle |U_{\text{imp}}(q)|^2 \rangle + \langle |W_{\text{surf}}(q)|^2 \rangle \right]
\times (q\cdot (q - q'))^2 \chi_\nu(|q - q'|) f_\nu(|q - q'|), \quad (5)
\]
\[ i\gamma_{\nu} = -\frac{1}{2m^*n_i} \sum_q q^2 \left[ n_i \langle |U_{\text{imp}}(q)|^2 \rangle + \langle |W_{\text{surf}}(q)|^2 \rangle \right] \]
\[ \times \left( \frac{\chi_{\nu}(q)}{\chi_{\nu}^{(0)}(q)} \right)^2 \frac{\phi_{\nu}^{(0)}(q, i\gamma_{\nu})}{1 + i\gamma_{\nu} \phi_{\nu}^{(0)}(q, i\gamma_{\nu})/\chi_{\nu}^{(0)}(q)} \right) , \quad (7) \]

where \( \phi_{\nu}^{(0)}(q, i\gamma_{\nu}) = (1/i\gamma_{\nu}) \left[ \chi_{\nu}^{(0)}(q, i\gamma) - \chi_{\nu}^{(0)}(q) \right] \) is the relaxation spectrum for non-interacting carriers scattering off the disorder. \( h\gamma_{\nu} \) is in units of twice the Fermi energy. From the scattering rate we calculate the conductivity \( \sigma \) at the transition using the Drude relation.

In Fig. 1 we show the order parameters \( f_{\nu}(q) \) determined from Eqs. (4) to (5) for the polarized and unpolarized states for a range of impurity densities \( n_i \). When \( n_i \) is less than a critical density the \( f_{\nu}(q) \) is zero, indicating a conducting phase. At the critical \( n_i \) the \( f_{\nu}(q) \) jumps discontinuously, indicating a transition to an insulator [7]. The key point here is that for fixed \( r_s \) the critical impurity density is much smaller for the fully polarized system than it is for the unpolarized system.

We determined the critical \( n_i \) for both the polarized and unpolarized cases as a function of the carrier density. Figure 2 shows the resulting phase boundaries between conductor and insulator. The conducting phase for the fully polarized system which is represented by the shaded region is restricted to a small range of \( r_s \) below \( r_s \approx 10 \). In the absence of surface roughness scattering the conducting phase extends to \( r_s \approx 11 \). The conducting phase exists only for small levels of disorder. For the unpolarized system at the same \( r_s \) the critical level of disorder is significantly greater, and the conducting phase extends to much larger values of \( r_s \). The hatched region represents the reduction in the conducting phase region when going from the unpolarized to fully polarized system.

Fig. 2 shows that fully spin polarizing the system destabilizes the conducting phase except within a small range of carrier densities on the higher density side. The stable conducting phase is restricted to very small levels of disorder. This significant shrinkage of the conducting region is associated with the enhancement of exchange-correlations for the fully polarized system. This enhancement favors localization.

We propose that the disappearance of the conducting phase in the presence of an in-plane magnetic field is associated with polarization of the carrier spins. At these low carrier densities the energy cost for spin aligned states becomes very small and a weak magnetic field is sufficient to fully polarize the electrons. Using numerical simulations Rapisarda and Senatore [6] have calculated the ground state energies \( E_p \) and \( E_u \) for the fully polarized and unpolarized systems. They report for \( r_s \gtrsim 10 \) the energies of the two states differ by only a very small amount.

![FIG. 2. Phase boundaries between conducting and insulating states for unpolarized (solid line) and fully polarized systems (dashed line). Axes are impurity density \( n_i \) and \( r_s \). For fully polarized system shaded region is conducting, and remaining area is insulating. For unpolarized system the conducting phase is the shaded plus hatched regions, and remainder is insulating.](image1)

![FIG. 3. Magnetic field at which Zeeman energy equals the energy difference between the polarized and unpolarized states.](image2)

From these calculated \( E_p \) and \( E_u \) we can estimate the critical magnetic field needed to drive the system into the fully spin polarized ground state. We equate the Zeeman energy splitting at the critical field \( H_c \) with the energy difference

\[ \frac{g\mu_B}{h} H_c = E_p - E_u . \quad (8) \]

In Fig. 3 we plot \( H_c \) as a function of \( r_s \) for holes in GaAs. We use \( (g\sigma_z) = 1.1 \) taken from Ref. [4]. The energies \( E_p \) and \( E_u \) are calculated for the appropriate system parameters.

Hamilton et al. [5] reported for a GaAs sample with hole density \( p_s \) corresponding to \( r_s = 9 \) that a magnetic field \( \gtrsim 0.7 \) T drives the conducting state to an insulator. From Fig. 3 we find that at \( r_s = 9 \) the critical magnetic field needed to fully polarize the system is \( H_c = 0.6 \) T,
which is very close to this value. For electrons in Si MOSFETs the values of effective mass and \((g\sigma_s)\) are not too different from those for holes in GaAs. The measured value of \(H_c = 0.5\) T in Si by Simonian \textit{et al} at \(r_s = 9\) is also in good agreement with our value.

We find at \(r_s = 9\) that the critical disorder level needed to drive the fully polarized system to the insulating state corresponds to a conductivity of \(\sigma \simeq 4.5e^2/h\). This is consistent with the measured value at the transition of \(\sigma \simeq 5e^2/h\) for \(r_s = 9\).

Hamilton \textit{et al} give a phase diagram showing the metal-insulator phase boundary as a function of \(p_s\) and the magnetic field. For a conductivity \(\sigma = 5e^2/h\) we obtain an impurity density for their sample of \(n_i = 2.4 \times 10^9\) cm\(^{-2}\). Fig. 2 shows at this \(n_i\) that the \(r_s\) at the phase boundary drops from \(r_s = 11.5\) in the unpolarized system \((H = 0)\) down to \(r_s = 9.2\) for the fully polarized system \((H = 0.6\) T). To compare with the experimental points taken from Ref. \([5]\), we used a linear interpolation between \(p_s\) and \(H\) and determined the corresponding critical magnetic field as a function of \(p_s\) (solid line). Fig. 4 compares the experimental points with our calculated \(H_c\). We find reasonable agreement. If we increase \(n_i\) the solid line shifts to the right, and correspondingly there is an increase in the critical magnetic field (see Fig. 3).

In conclusion we have demonstrated that magnetic fields \(\lesssim 1\) T should be sufficient to fully spin polarize the carriers for \(r_s > 8\). We have shown that the enhanced exchange-correlations for the fully polarized system significantly favors the insulating phase. Our mechanism leads to results which are in reasonable quantitative agreement with experiment. We predict a re-emergence of a conducting phase for the fully polarized system at very small levels of disorder, but only for carrier densities \(r_s < 11\).

FIG. 4. Dependence of critical magnetic field for the metal-insulator transition in GaAs on hole density \(p_s\) (solid line) for impurity density \(n_i = 2.4 \times 10^9\) cm\(^{-2}\). Points are experimental data taken from Hamilton \textit{et al} [3].

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