Unsupervised Ensemble Classification with Dependent Data

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Abstract—Ensemble learning, the machine learning paradigm where multiple algorithms are combined, has exhibited promising performance in a variety of tasks. The present work focuses on unsupervised ensemble classification. The term unsupervised refers to the ensemble combiner who has no knowledge of the ground-truth labels that each classifier has been trained on. While most prior works on unsupervised ensemble classification are designed for independent and identically distributed (i.i.d.) data, the present work introduces an unsupervised scheme for learning from ensembles of classifiers in the presence of data dependencies. Two types of data dependencies are considered: sequential data and networked data whose dependencies are captured by a graph. Moment matching and Expectation Maximization algorithms are developed for the aforementioned cases, and their performance is evaluated on synthetic and real datasets.

Index Terms—Ensemble learning, unsupervised, sequential classification, crowdsourcing, dependent data.

1 INTRODUCTION

As social networks, connected “smart” devices and highly accurate scientific instruments have permeated society, multiple machine learning, signal processing and data mining algorithms have been developed to process the generated data and draw inferences from them. With most of these algorithms typically designed to operate under different assumptions, combining them can be beneficial because different algorithms can complement each other's strengths.

Ensemble learning refers to the task of designing an algorithm that can combine the results provided by multiple different learners or annotators [4], see Fig. 1. This algorithm, often referred to as meta-learner, should generally be able to outperform the individual learners. In particular, ensemble classification refers to fusing the results provided by different classifiers. Each classifier observes data, decides one class, out of $K$ possible, each of these data belong to, and provides the meta-learner with those decisions. Such a setup emerges in diverse disciplines including medicine [1], biology [2], team decision making, distributed detection, and economics [3], and has recently gained attention with the advent of crowdsourcing [4], as well as services such as Amazon’s Mechanical Turk [3], and Figure8, to name a few.

When training data are available, a meta-learner can learn how to combine the results from individual classifiers, based on these ground-truth labels [6]. In many cases however, labeled data are not available to train the combining meta-classifier, justifying the need for unsupervised (or blind) ensemble learning methods. One such manifestation of unsupervised ensemble learning is crowdsourcing, where people are tasked with providing classification labels. Multiple algorithms attempt to address the unsupervised ensemble classification problem, and a common assumption is that the data are independent and identically distributed (i.i.d.) from an unknown distribution [8], [9], [10], [11], [12], [13], [14]. In several cases however, additional domain knowledge may be available to the meta-learner. This domain knowledge provides information regarding the data distribution, as well as data dependencies. In this paper, two types of data dependence are considered: sequential and networked data. Classification of sequential data arises in many natural language processing tasks such as part-of-speech tagging, named-entity recognition, and information extraction, to name a few [15]. Examples of networked data, where data correlations or dependencies are captured in a known graph, include citation, social, communication and brain networks among others.

The present work puts forth a novel scheme for unsupervised ensemble classification with data dependencies, built upon simple concepts from probability, as well as recent advances in tensor decompositions [16] and optimization theory, that enable assessing the reliability of multiple annotators and combining their answers. Similarly to prior works for i.i.d. data, in our proposed model each annotator has a fixed probability of deciding that a datum belongs to class $k$, given that the true class of the datum is $k'$. These probabilities parametrize the annotators. Data dependencies are then encoded in the marginal probability mass function (pmf) of the data labels. For sequential data, the pmf of data labels is assumed to be a Markov chain, while for networked data the pmf of data labels is assumed to be a Markov Random Field (MRF). Assuming that annotators make decisions independent of each other, the proposed method extracts these probabilities from annotator responses. As an initial step, the moment-matching method we introduced in [12] for ensemble classification of i.i.d. data, is adopted to provide rough estimates for annotator parameters. These parameters are then refined using expectation maximization (EM) algorithms tailored for the dependencies present in the data, to
produce the final label estimates. To assess the performance of the proposed algorithms, extensive numerical tests, on synthetic as well as real data are presented.

The rest of the paper is organized as follows. Section 2 states the problem, and provides preliminaries along with a brief description of the prior art in unsupervised ensemble classification for i.i.d. data. Section 3 introduces the proposed approach to unsupervised ensemble classification for sequential data, while Section 4 deals with its counterpart for networked data. Section 5 presents numerical tests to evaluate our methods. Finally, concluding remarks and future research directions are given in Section 6.

**Notation:** Unless otherwise noted, lowercase bold letters, \( x \), denote vectors, uppercase bold letters, \( X \), represent matrices, and calligraphic uppercase letters, \( X \), stand for sets. The \((i,j)\)th entry of matrix \( X \) is denoted by \( [X]_{ij} \). \( X^\top \) denotes the transpose of matrix \( X \). \( \mathbb{E} \) stands for the \( D \)-dimensional real Euclidean space, \( \mathbb{R}_+ \) for the set of positive real numbers, \( \mathbb{E} [ \cdot ] \) for expectation, and \( || \cdot || \) for the \( \ell_2 \)-norm. Underlined capital letters \( \overline{X} \) denote tensors; while \([A, B, C]\) is used to denote compactly a \( K \) parallel factor (PARAFAC) analysis tensor \([16]\) with factor matrices \( A = [a_1, \ldots, a_K], B = [b_1, \ldots, b_K], C = [c_1, \ldots, c_K] \), that is \([A, B, C]\) = \( \sum_{k=1}^{K} a_k \circ b_k \circ c_k \). Finally, \( \mathcal{I}(A) \) denotes the indicator function of event \( A \), i.e. \( \mathcal{I}(A) \) = 1 if \( A \) occurs and is 0 otherwise.

## 2 Problem Statement and Preliminaries

Consider a dataset consisting of \( N \) data (possibly vectors) \( \{x_n\}_{n=1}^{N} \) each belonging to one of \( K \) possible classes with corresponding labels \( \{y_n\}_{n=1}^{N} \), e.g. \( y_n = k \) if \( x_n \) belongs to class \( k \). The pairs \( \{x_n, y_n\}_{n=1}^{N} \) are drawn from an unknown joint distribution \( D \), and \( X \) and \( Y \) denote random variables such that \( (X, Y) \sim D \). Consider now \( M \) annotators that observe \( \{x_n\}_{n=1}^{N} \), and provide estimates of labels. Let \( f_m(x_n) \in \{1, \ldots, K\} \) denote the label assigned to datum \( x_n \) by the \( m \)-th annotator. All annotator responses are then collected at a centralized meta-learner or fusion center. Collect all annotator responses in the \( M \times N \) matrix \( F \), that has entries \( [F]_{mn} = f_m(x_n) \), and all ground-truth labels in the \( N \times 1 \) vector \( y = [y_1, \ldots, y_N]^\top \). The task of unsupervised ensemble classification is: Given only the annotator responses \( \{f_m(x_n), m = 1, \ldots, M\}_{n=1}^{N} \), we wish to estimate the ground-truth labels of the data \( \{y_n\} \); see Fig. 1.

### 2.1 Prior work

Most prior works on blind or unsupervised ensemble classification focus on the i.i.d. data case. Possibly the simplest scheme is majority voting, where the estimated label of a datum is the one that most annotators agree upon. Majority voting presumes that all annotators are equally “reliable,” which is rather unrealistic, both in crowdsourcing as well as in ensemble learning setups. Other blind ensemble methods aim to estimate the parameters that characterize the annotators’ performance, namely the annotator confusion matrices.

A popular approach is joint maximum likelihood (ML) estimation of the unknown labels and the aforementioned performance parameters using the expectation-maximization (EM) algorithm [8]. As EM iterations do not guarantee convergence to the ML solution, recent works pursue alternative estimation methods. For binary classification, [9] assumes that annotators adhere to the “one-coin” model, meaning each annotator \( m \) provides the correct (incorrect) label with probability \( \delta_m \) \((1 - \delta_m)\). Recently, [11] advocated a spectral decomposition technique of the second-order statistics of annotator responses for binary classification, that yields the reliability parameters of annotators.

In the multiclass setting, spectral methods such as [10], [12] utilize third-order moments and tensor decompositions to estimate the unknown reliability parameters. These estimates can then initialize the EM algorithm of [8].

Recent works advocate blind ensemble approaches for sequential data. A method to aggregate annotator labels for sequential data relies on conditional random fields (CRFs) [17]. However, this method operates under strong and possibly less realistic assumptions requiring e.g., that only one annotator provides the correct label for each datum. To relax the assumptions of [17], extensions of the standard Hidden Markov Model (HMM) to incorporate annotator responses have been reported along with a variational EM [18] algorithm to aggregate them [19]. As both aforementioned methods require tuning of hyperparameters, a training step is necessary, which can be unrealistic in unsupervised settings.

When features \( \{x_n\}_{n=1}^{N} \) are available at the meta-learner, approaches based on Gaussian Processes can be used to classify the data based on annotator responses [20], [21], [22]. These approaches can take advantage of some data dependencies, however, in addition to requiring the data features at the meta-learner, these methods have only been developed for binary classification.

The present work puts forth a novel scheme for unsupervised multiclass ensemble classification in the presence of data dependencies. Our approach builds upon our previous work on blind ensemble classification of i.i.d. data [12], [13], and markedly extends its scope to handle sequential as well as networked data, without requiring training or access to data features. For sequentially dependent data case we present a moment matching algorithm that is able to estimate annotator confusion matrices as well as the parameters characterizing the Markov chain of the labels. These confusion matrices and parameters are then refined using EM iterations tailored for the sequential data classification task. For the networked dependencies, the moment matching method for i.i.d. data is used to initialize an EM algorithm designed for networked data. To our knowledge, no existing
work tackles the ensemble classification task under the networked data regime. Compared to our conference precursor in [23], here we have included extensive numerical tests as well as a new EM algorithm for sequential data along with algorithms that tackle networked data.

2.2 Unsupervised ensemble classification of i.i.d. data

Consider that each annotator has a fixed probability of deciding that a datum belongs to class $k'$, when presented with a datum of class $k$; and all annotators behavior is presumed constant throughout the dataset. Clearly, the performance of each annotator $m$ is then characterized by the so-called confusion matrix $\Gamma_m$, whose $(k', k)$-th entry is

$$ [\Gamma_m]_{k', k} := \Pr(f_m(X) = k'| Y = k). \tag{1} $$

The $K \times K$ matrix $\Gamma_m$ has non-negative entries that obey the simplex constraint, since $\sum_{k'=1}^K \Pr(f_m(X) = k'| Y = k) = 1$, for $k = 1, \ldots, K$; hence, entries of each $\Gamma_m$ column sum up to 1, that is, $\sum_{k'=1}^K \Gamma_m[k, k'] = 1$ and $\Gamma_m \succeq 0$. The confusion matrix showcases the statistical behavior of an annotator, as each column provides the annotator’s probability of deciding the correct class, when presented with a datum from each class. Collect the set of constraints per matrix to the convex set $\mathcal{C} := \{ \Gamma \in \mathbb{R}^{K \times K} : \Gamma \succeq 0, \sum_{k'=1}^K \Gamma[k, k'] = 1 \}$, where essentially each column lies on a probability simplex, and let $\mathcal{C}_p := \{ u \in \mathbb{R}^K : u \succeq 0, u^\top 1 = 1 \}$ denote the simplex constraint for a $K \times 1$ vector. Before proceeding, we adopt the following assumptions, which will hold throughout this paper.

As1. Responses of different annotators per datum are conditionally independent, given the ground-truth label $Y$ of the same datum $X$; that is,

$$ \Pr(f_1(X) = k_1, \ldots, f_M(X) = k_M | Y = k) = \prod_{m=1}^M \Pr(f_m(X) = k_m | Y = k) $$

As2. Most annotators are better than random.

As1 is satisfied by annotators making decisions independently, which is a rather standard assumption [8], [10], [11]. Graphically, this model is depicted in Fig. 2. Under As2, the largest entry per $\Gamma_m$ column is the one on the diagonal

$$ [\Gamma_m]_{kk} \geq [\Gamma_m]_{k', k}, \text{ for } k', k = 1, \ldots, K. $$

As2 is another standard assumption, used to alleviate the inherent permutation ambiguity in estimating $\Gamma_m$ blindly.

If all annotator confusion matrices were ideally known, the label of $x_n$ could be estimated using a maximum a posteriori (MAP) classifier. The latter finds the label $k$ that maximizes the joint probability of $y_n$ and observed annotator responses $\{f_m(x_n) = k_m\}_{m=1}^M$

$$ \hat{y}_n = \arg\max_{k \in \{1, \ldots, K\}} L(x_n | k) \Pr(y_n = k) \tag{2} $$

$$ = \arg\max_{k \in \{1, \ldots, K\}} \log \pi_k + \sum_{m=1}^M \log(\Gamma_m(k_m, k)) $$

where $\pi_k := \Pr(y_n = k) = \Pr(Y = k)$ and $L(x_n | k) := \Pr(f_1(x_n) = k_1, \ldots, f_M(x_n) = k_M | Y = k)$, and the second equality of (2) follows from As1 and properties of the logarithm. In addition, if all classes are assumed equiprobable, (2) reduces to the maximum likelihood (ML) classifier.

But even for non-equiprobable classes, unsupervised ensemble classification requires estimates of the class priors $\pi := [\pi_1, \ldots, \pi_K]^\top$ as well as all the annotator performance parameters $\{\Gamma_m\}_{m=1}^M$.

2.2.1 EM algorithm for i.i.d. data

Here we outline how the EM algorithm can be employed to estimate the wanted annotator performance parameters by iteratively maximizing the log-likelihood of the observed annotator responses; that is, $\log \Pr(F; \theta)$, where $\theta$ collects all the annotator confusion matrices (prior probabilities are assumed equal and are dropped for simplicity). Each EM iteration includes the expectation (E)-step and the maximization (M)-step.

At the E-step of the $(i + 1)$st iteration, the estimate $\theta^{(i)}$ is given, and the so-termed Q-function is obtained as

$$ Q(\theta; \theta^{(i)}) = \mathbb{E}_{y | F, \theta^{(i)}}[\log \Pr(y, F; \theta)] = \mathbb{E}_{y | F, \theta^{(i)}}[\log \Pr(F | y; \theta)] + \mathbb{E}_{y | F, \theta^{(i)}}[\log \Pr(y; \theta)]. \tag{3} $$

Since the data are i.i.d., we have under As1 that

$$ \mathbb{E}_{y | F, \theta^{(i)}}[\log \Pr(F | y; \theta)] = \frac{1}{M} \sum_{m=1}^M \sum_{k=1}^K \log \Gamma_m(f_m(x_n), k) q_{nk} $$

where $q_{nk} := \Pr(y_n = k | \{f_m(x_n)\}_{m=1}^M, \theta^{(i)})$ is the posterior of label $y_n$ given the observed data and current parameters, and

$$ \mathbb{E}_{y | F, \theta^{(i)}}[\log \Pr(y; \theta)] = \frac{1}{M} \sum_{m=1}^M \sum_{k=1}^K \log \Pr(y_n = k; \theta) q_{nk}. $$

Under this model, it can be shown that [8], [10]

$$ q_{nk}^{(i+1)} = \frac{1}{Z} \exp \left( \sum_{m=1}^M \sum_{k'=1}^K I(f_m(x_n) = k') \log(\Gamma_m^{(i)}(k', k)) \right) \tag{4} $$

where $Z$ is the normalization constant.

At the M-step, annotator confusion matrices are updated by maximizing the Q-function to obtain

$$ \theta^{(i+1)} = \arg\max_{\theta} Q(\theta; \theta^{(i)}). \tag{5} $$

It can be shown per annotator $m$ that (5) boils down to

$$ \Gamma_m^{(i+1)}_{k', k} = \frac{\sum_{n=1}^N q_{nk}^{(i+1)} I(f_m(x_n) = k')}{\sum_{k''=1}^K \sum_{n=1}^N q_{nk}^{(i+1)} I(f_m(x_n) = k'')} \tag{6} $$

Fig. 2. Graphical depiction of the Dawid-Skene model for i.i.d. data. Shaded ellipses are observed variables (here annotator responses).
The E- and M-steps are repeated until convergence, and ML estimates of the labels are subsequently found as

\[
y(x_n) = \arg \max_{k \in \{1, \ldots, K\}} \Pr \left( \{ f_m(x_n) \}_{m=1}^M, y_n = k \right)
\]

\[
= \arg \max_{k \in \{1, \ldots, K\}} q_{nk}.
\]

2.2.2 Moment-matching for i.i.d. data

As an alternative to EM, the annotator performance parameters can be estimated using the moment-matching method we introduced for i.i.d. data in \cite{12}, which we outline here before extending it to dependent data in the ensuing sections.

Consider representing label \( k \) using the canonical \( K \times 1 \) vector \( e_k \), namely the \( k \)-th column of the \( K \times K \) identity matrix \( I \). Let \( f_m(X) \) denote the \( m \)-th annotator’s response in vector format. Since \( f_m(X) \) is just a vector representation of \( f_m(x) \), it holds that \( \Pr(f_m(X) = k' | Y = k) = \Pr(f_m(X) = e_k | Y = k) \). With \( \gamma_{m,k} \) denoting the \( k \)-th column of \( \Gamma_m \), it thus holds that

\[
\gamma_{m,k} := \mathbb{E}[f_m(X) | Y = k] = \sum_{k' = 1}^K e_k \cdot \Pr(f_m(X) = k' | Y = k) \tag{7}
\]

where we used the definition of conditional expectation. Using \( \Box \) and the law of total probability, the mean vector of responses from annotator \( m \), is hence

\[
\mathbb{E}[f_m(X)] = \sum_{k=1}^K \mathbb{E}[f_m(X) | Y = k] \Pr(Y = k) = \Gamma_m \pi. \tag{8}
\]

Upon defining the diagonal matrix \( \Pi := \text{diag}(\pi) \), the \( K \times K \) cross-correlation matrix between the responses of annotators \( m \) and \( m' \neq m \), can be expressed as

\[
\mathbf{R}_{mm'} := \mathbb{E}[f_m(X) f_{m'}^\top(X)]
\]

\[
= \sum_{k=1}^K \mathbb{E}[f_m(X) | Y = k] \mathbb{E}[f_{m'}^\top(X) | Y = k] \Pr(Y = k)
\]

\[
= \Gamma_m \text{diag}(\pi) \Pi_{m'} = \Gamma_m \Pi \Pi_{m'}^\top. \tag{9}
\]

where we successively relied on the law of total probability, \( A \boxplus \) and \( \Box \). Consider now the \( K \times K \times K \) cross-correlation tensor between the responses of annotators \( m \), \( m' \neq m \), and \( m'' \neq m \), namely

\[
\Psi_{mm'm''} := \mathbb{E}[f_m(X) \circ f_{m'}(X) \circ f_{m''}(X)]. \tag{10}
\]

It can be shown that \( \Psi_{mm'm''} \) obeys a PARAFAC model with factor matrices \( \Gamma_m, \Gamma_{m'} \) and \( \Gamma_{m''} \) \cite{12}; that is,

\[
\Psi_{mm'm''} = \sum_{k=1}^K \tau_k \gamma_{m,k} \circ \gamma_{m',k} \circ \gamma_{m'',k} \tag{11}
\]

Note here that the diagonal matrix \( \Pi \) can multiply any of the factor matrices \( \Gamma_m, \Gamma_{m'} \), or \( \Gamma_{m''} \).

Having available the sample average counterparts of \( \Box \), \( \Box \) and \( \Box \), correspondingly \( \{ \mu_m \}_{m=1}^M \), \( \{ S_{mm'} \}_{m,m'=1}^M \), and \( \{ \hat{r}_{mm'm''} \}_{m,m',m''=1}^M \), estimates of \( \{ \Gamma_m \}_{m=1}^M \) and \( \pi \) can be readily obtained. This approach is an instantiation of the method of moments estimation method, see e.g. \cite{24}, and can be cast as the following constrained optimization task

\[
\min_{\pi \in \mathcal{C}, \{ \Gamma_m \}_{m=1}^M} \left\{ g(\{ \Gamma_m \}_{m=1}^M, \pi) \right\} \tag{12}
\]

where

\[
g(\{ \Gamma_m \}, \pi) := \sum_{m=1}^M \| \mu_m - \Gamma_m \pi \|_F^2
\]

\[
+ \sum_{m=1}^M \sum_{m' > m} |S_{mm'} - \Gamma_m \Gamma_{m'}^\top|_F^2
\]

\[
+ \sum_{m=1}^M \| \hat{r}_{mm'm''} - [\Gamma_m \Pi, \Gamma_{m'}, \Gamma_{m''}] \tau_k \|_F^2.
\]

The non-convex optimization in \cite{12} can be solved using the alternating optimization method described in \cite{12}, which is guaranteed to converge to a stationary point of \( g \) \cite{25}. A \( \hat{\mathbf{\Gamma}} \) is used here to address the permutation ambiguity that is induced by the tensor decomposition of \( \mathbf{\Gamma} \). Interested readers are referred to \cite{12} for implementation details, and theoretical guarantees.

Upon obtaining \( \{ \hat{\mathbf{\Gamma}}_m \}_{m=1}^M \) and \( \hat{\pi} \), a MAP classifier can be subsequently employed to estimate the label for each datum; that is, for \( n = 1, \ldots, N \), we obtain

\[
y(x_n) = \arg \max_{k \in \{1, \ldots, K\}} \log \hat{\pi}_k + \sum_{m=1}^M \log \hat{\Gamma}_m(f_m(x_n), k). \tag{13}
\]

where \( \hat{\Gamma}_m(k', k) = [\hat{\mathbf{\Gamma}}_m]_{k' k} \), and \( \hat{\pi}_k = [\hat{\pi}]_k \). The estimates \( \{ \hat{\mathbf{\Gamma}}_m \}_{m=1}^M, \hat{\pi}, \hat{y}_n \), can be improved using the EM iterations in Sec. 2.2.2. Such a refinement is desirable when \( N \) is relatively small, and thus moment estimates are not as reliable.

Next, we will introduce novel approaches for ensemble classification with sequential and networked data.

3 Sequentially Dependent Data

Suppose now that our data belong to a sequence, and the \( n \)-th datum depends on the \( (n-1) \)-st one. Pertinent settings emerge with speech and natural language processing tasks such as word identification, part-of-speech tagging, named-entity recognition, and information extraction \cite{15}.

In order to take advantage of this dependence, we will encode it in the marginal pmf \( \Pr(y) \) of the labels. Specifically, we postulate that the sequence of labels \( \{ y_n \}_{n=1}^N \) forms a one-step time-homogeneous Markov chain; that is, variable \( y_n \) depends only on its immediate predecessor \( y_{n-1} \). This Markov chain is characterized by a \( K \times K \) transition matrix \( \mathbf{T} \), whose \( (k, k') \)-th entry is given by

\[
[T]_{kk'} = T(k, k') = \Pr(y_k = k | y_{k-1} = k').
\]

Matrix \( \mathbf{T} \) has non-negative entries that satisfy the simplex constraint; hence, \( \mathbf{T} \in \mathcal{C} \). With \( y := [y_1, \ldots, y_N]^\top \) collecting the sequence of labels, the marginal probability of \( \{ y_n \}_{n=1}^N \) can be expressed using successive conditioning, as

\[
\Pr(y) = \Pr(y_1 = k_1) \prod_{n=2}^N T(k_n, k_{n-1}). \tag{14}
\]
where \( k := [k_1, \ldots, k_N]^\top \). Accordingly, the data \( \{x_n\}_{n=1}^N \) depend only on their corresponding \( y_n \), and are generated from an unknown conditional pdf as \( x_n \sim \Pr(x_n|y_n = k_n) \). The data pairs \( \{(x_n, y_n)\}_{n=1}^N \) form a hidden Markov model (HMM), where the labels \( \{y_n\}_{n=1}^N \) correspond to the hidden variables of the HMM, while \( \{x_n\}_{n=1}^N \) correspond to the observed variables of the HMM.

As with the i.i.d. case of Sec. 2.2, \( M \) annotators observe \( \{x_n\}_{n=1}^N \), and provide estimates of their labels \( f_m(x_n) \). Under As1, the responses of different annotators per datum are conditionally independent, given the ground-truth label \( y_n \) of the same datum \( x_n \); that is

\[
\Pr( f_1(x_n) = k_1, \ldots, f_M(x_n) = k_M | y_n = k) = \prod_{m=1}^M \Pr( f_m(x_n) = k_m | y_n = k) \quad \text{for } n = 1, \ldots, N. \tag{15}
\]

A graphical representation of this HMM is provided in Fig. 3. At this point, we require an additional assumption.

As3. The Markov chain formed by the labels \( \{y_n\} \) has a unique stationary distribution \( \pi := [\pi_1, \ldots, \pi_K]^\top = [\Pr(Y = 1), \ldots, \Pr(Y = K)]^\top \), and is also irreducible.

As3' here will lead to our two-step moment matching algorithm for sequentially dependent labels.

Building on the HMM, we will now present our novel moment-matching approach to blind ensemble learning for classifying sequential data. Similar to [26], our method decouples the problem of learning the parameters of interest in two steps. First, estimates of the confusion matrices \( \{\Gamma_m\}_{m=1}^M \) and stationary priors \( \hat{\pi} \) are obtained; and subsequently, the transition matrix is estimated as \( \hat{T} \) before obtaining an estimate of the labels \( \{\hat{y}_n\}_{n=1}^N \).

### 3.1 Label estimation for sequential data

Given only annotator responses for all data in a sequence, an approach to estimating the labels of each datum, meaning the hidden variables of the HMM, is to find the sequence \( \{\hat{y}_n\}_{n=1}^N \) that maximizes the joint probability of the labels \( y \) and the annotator responses \( F \), namely

\[
\Pr( y = k, F) = \Pr(y_1 = k_1) \prod_{n=2}^N T(k_n, k_{n-1}) \prod_{m=1}^M \Gamma_m(f_m(x_n), k_n) \tag{16}
\]

where the equality is due to [14] and [15]. This can be done efficiently using the Viterbi algorithm [27], [28]. In order to obtain estimates of the labels, \( \{\Gamma_m\}_{m=1}^M \) and \( \hat{T} \) must be available. The next subsection will show that \( \{\Gamma_m\}_{m=1}^M \) and \( \hat{T} \) can be recovered by the statistics of annotator responses, using the aforementioned two-step procedure.

### 3.2 Confusion and transition matrix estimation

Under As3 the HMM is mixing and assuming that \( y_0 \) is drawn from the stationary distribution \( \pi \), the responses of an annotator \( m \) can be considered to be generated from a mixture model, see e.g. [26]

\[
f_m(X) \sim \sum_{k=1}^K \pi_k \Pr(f_m(X)|Y = k). \tag{17}
\]

Based on the latter, the remainder of this subsection will treat labels \( \{y_n\}_{n=1}^N \), as if they had been drawn i.i.d. from the stationary distribution \( \pi \), that is \( y_n \sim \pi \) for \( n = 1, \ldots, N \). Then, the procedure outlined in Sec. 2.2 can be readily adopted to obtain estimates of the stationary distribution \( \hat{\pi} \) and the confusion matrices \( \{\Gamma_m\}_{m=1}^M \).

With estimates of annotator confusion matrices \( \{\Gamma_m\} \) and stationary probabilities \( \hat{\pi} \) at hand, we turn our attention to the estimation of the transition matrix \( \hat{T} \). To this end, consider the cross-correlation matrix of consecutive vectorized observations between annotators \( m \) and \( m' \), namely \( \hat{R}_{mm'} = \mathbb{E}[f_m(x_n)f_{m'}(x_{n-1})] \). Under the HMM of Sec. 3 the latter is given by

\[
\hat{R}_{mm'} = \Gamma_m T \text{diag}(\hat{\pi}) \Gamma_{m'}^\top = \Gamma_m A \Gamma_{m'}^\top \tag{18}
\]

where \( A := \text{diag}(\pi) \). Letting \( \hat{S}_{mm'} \) denote the sample counterpart of \( \hat{R}_{mm'} \), and with \( \{\Gamma_m\}_{m=1}^M \) available, we can recover \( T \) as follows. First, we solve the convex moment-matching optimization problem

\[
\min_{A \in \mathbb{C}_S} \sum_{m=m'}^{M} \| \hat{S}_{mm'} - \Gamma_m A \Gamma_{m'}^\top \|_F^2 \tag{19}
\]

where \( \mathbb{C}_S \) is the set of matrices whose entries are positive and sum to 1, namely \( \mathbb{C}_S := \{X \in \mathbb{R}^{K \times K}: X \geq 0, 1^\top X 1 = 1\} \). The constraint is due to the fact that \( 1^\top T = 1^\top \), \( \text{diag}(\pi)1 = \pi \), and \( \pi^\top 1 = 1 \). Note that (19) is a standard constrained convex optimization problem that can be solved with off-the-shelf tools, such as CVX [29]. Having obtained \( A \) from (19), we can then estimate the transition matrix as

\[
\hat{T} = A \text{diag}(\hat{\pi})^{-1}. \tag{20}
\]

Note here that explicit knowledge of \( \pi \) is not required, as its estimate can be recovered from \( A \) as

\[
\hat{\pi}^\top = 1^\top A = 1^\top \hat{T} \text{diag}(\hat{\pi}) = 1^\top \text{diag}(\hat{\pi}).
\]

The following proposition argues the consistency of the transition matrix estimates \( \hat{T} \).

**Proposition 1.** Given accurate estimates of \( \{\Gamma_m\} \) and \( \pi \), the estimate \( \hat{T} \) given by (19) and (20) approaches \( T \) as \( N \to \infty \).
Algorithm 1 EM algorithm for Sequential Data

Input: Annotator responses \( \{f_m(x_n)\}_{m=1,n=1}^{M,N} \), initial estimates \( T^{(0)}, \{\Gamma_m^{(0)}\}_{m=1}^M \).

Output: Estimates \( T, \{\Gamma_m\}_{m=1}^M \).

1: while not converged do
2: \( \hat{q}_{nk}^{(i+1)} \) and \( \xi_n^{(i+1)}(k,k') \) using the forward-backward algorithm (App. A).
3: \( \hat{\Gamma}_m^{(i+1)} \) via (24).
4: \( \hat{T}^{(i+1)} \) via (23).
5: \( i \leftarrow i + 1 \)
6: end while

Algorithm 2 Unsupervised Ensemble Classifier for Sequential Data

Input: Annotator responses \( \{f_m(x_n)\}_{m=1,n=1}^{M,N} \).

Output: Estimation of data labels \( \{\eta_n\}_{n=1}^N \).

1: Estimate \( \pi, \{\Gamma_m\}_{m=1}^M \) via (12).
2: Estimate \( \hat{T} \) via (19) and (20).
3: Estimate \( \hat{y}_n \) using the Viterbi algorithm [cf. Sec. 3.1].
4: If needed refine estimates of \( \hat{T}, \{\hat{\Gamma}_m\} \) and \( \{\hat{y}_n\} \) using Alg. 1.

Proof. By the law of large numbers, \( \hat{S}_{mn'} \to \bar{R}_{mn'} \) as \( N \to \infty \) for all \( m, m' \). Since the objective in (19) is convex, from [30] we have that \( \hat{A} \) will converge to \( A = T \text{diag}(\pi) \) as \( N \to \infty \). Finally, as \( T \) can be recovered from \( \hat{A} \) in closed form [cf. (20)], the proof is complete.

With estimates of \( \{\Gamma_m\}, \pi \) and \( \hat{T} \) at hand, estimates of the labels \( \{\eta_n\}_{n=1}^N \) can be obtained using the method described in Sec. 3.1. Furthermore, the estimates of \( \{\Gamma_m\}, \pi \) and \( \hat{T} \) can be used to initialize an EM algorithm (a.k.a. Baum-Welch), whose details are provided in the next subsection.

Remark 1. While here we employed the algorithm of [12] to estimate \( \{\Gamma_m\} \), any other blind ensemble classification algorithm, such as [8], [10], can be utilized too. In addition, methods that jointly estimate confusion matrices and Markov chain parameters such as [31], can also be appropriately modified for the ensemble classification task.

3.3 EM algorithm for sequential data

As with the i.i.d. case of Sec. 2.2. here we develop an EM algorithm to iteratively maximize the log-likelihood of the observed annotator responses. In order to update the parameters of interest \( \theta := \text{vec}([T, \Gamma_1, \ldots, \Gamma_M]) \) per iteration, the following two quantities have to be found

\[ q_{nk} = \Pr(y_n = k | \mathbf{F}, \theta) \]  

and

\[ \xi_n(k, k') = \Pr(y_n = k, y_{n+1} = k' | \mathbf{F}, \theta). \]

Luckily, due to the causal structure of \( \Pr(y) \), the aforementioned quantities can be estimated efficiently using the forward-backward algorithm [27], whose details can be found in Appendix A.

At iteration \( i \), after obtaining \( q_{nk}^{(i+1)}, \xi_n^{(i+1)}(k, k') \) for \( k, k' = 1, \ldots, K \) and \( n = 1, \ldots, N \), via the forward-backward algorithm, the transition and confusion matrix estimates can be updated as

\[ \hat{T}_{kk'}^{(i+1)} = \frac{\sum_{n=1}^{N-1} \xi_n^{(i+1)}(k, k')}{\sum_{n=1}^{N-1} q_{nk}^{(i+1)}} \]  

\[ \hat{\Gamma}_m^{(i+1)} = \frac{\sum_{n=1}^{N} \xi_n^{(i+1)}(k, k') \Pr(f_m(x_n) = k')}{\sum_{n=1}^{N} \sum_{k''=1}^{K} q_{nk}^{(i+1)} \Pr(f_m(x_n) = k'')} \]  

The EM iterations for sequential data with dependent labels is summarized in Alg. 1 while the overall ensemble classifier for sequential data is tabulated in Alg. 2.

4 NETWORK DEPENDENT DATA

In many cases, additional information pertaining to the data is available in the form of a directed graph \( G(V, E) \), where \( V \) and \( E \) denote the vertex (or node) and edge sets of \( G \), respectively. Examples of such networked data include social and citation networks [32], [33]. Each node of this graph corresponds to a data point, thus \( |V| = N \), while the edges capture pairwise relationships between the data.

As with Sec. 3, we will encode data dependence, meaning the pairwise relationships provided by the graph \( G \) in the marginal pmf of the labels \( \Pr(y) \). Specifically, we model the labels \( \{\eta_n\}_{n=1}^N \) as being drawn from an MRF, and in this setup we will need one more assumption.

As4. The conditional pmf of \( y_n \), for all \( n = 1, \ldots, N \), satisfies the local Markov property

\[ \Pr(y_n | y_{-n}) = \Pr(y_n | y_{N_n}) \]  

where \( y_{-n} \) is a vector containing all labels except \( y_n \) and \( y_{N_n} \) is a vector containing the labels of node \( n \) neighbors.

Under As4, the joint pmf of all labels is

\[ \Pr(y) = \frac{1}{Z} \exp(-U(y)) \]  

where \( Z := \sum_y \exp(-U(y)) \) is the normalization constant, and \( U(y) \) is the so-called energy function. Computing the normalization constant \( Z \) involves all possible configurations of \( y \); hence, it is intractable for datasets with moderate to large size \( N \). By the Hammersley-Clifford theorem the energy function can be written as

\[ U(y) = \frac{1}{2} \sum_{(n,n') \in E} V(y_n, y_{n'}) \]  

where \( V(y_n, y_{n'}) \) denotes the so-called clique potential of the \((n, n')\)-th edge. Here, we select the clique potential as

\[ V(y_n, y_{n'}) := \begin{cases} 0 & \text{if } y_n = y_{n'} \\ \delta_n & \text{if } y_n \neq y_{n'} \end{cases} \]  

where \( \delta_n > 0 \) is some predefined scalar, which controls how much we trust the given graph \( G \). The local energy at node (datum) \( n \) of the graph is then defined as

\[ U_n(y_n) = \frac{1}{2} \sum_{n' \in N_n} V(y_n, y_{n'}). \]
This particular choice of clique potentials forces neighboring nodes (data) of the graph to have similar labels, and has been successfully used in image segmentation [35, 36].

Similar to the i.i.d. and sequential cases, data vectors \( \{x_n\}_{n=1}^N \) are generated from an unknown conditional pdf \( x_n \sim \Pr(x_n|y_n = k) \) that depends only on their corresponding label \( y_n \). M annotators observe \( \{x_n\}_{n=1}^N \) and provide estimates of their labels \( f_m(x_n) \). With \( \text{Arg}_{2} \) still in effect, annotator responses are conditionally independent given the label \( Y \). Thus, we can express the joint pmf of label \( y_n \) and corresponding annotator responses \( f_m(x_n) \) given the neighborhood \( y_{N_n} \) of node \( n \), as

\[
Pr \left( \left\{ f_m(x_n) \right\}_{m=1}^M | y_n = k \right| y_{N_n} = k_{N_n} \right) = \prod_{m=1}^M \Gamma_m(f_m(x_n), k) Pr(y_n = k | y_{N_n} = k_{N_n})
\]  

and accordingly the posterior probability of label \( y_n \)

\[
Pr \left( y_n = k \right| \left\{ f_m(x_n) \right\}_{m=1}^M, y_{N_n} = k_{N_n} \right) \propto \prod_{m=1}^M \Gamma_m(f_m(x_n), k) Pr(y_n = k | y_{N_n} = k_{N_n})
\]  

\[
= \exp \left( -U_n(k) + \sum_{m=1}^M \log \Gamma_m(f_m(x_n), k) \right).
\]

4.1 Label estimation for networked data

Finding ML estimates of the labels \( \hat{y} \), under the aforementioned model, involves the following optimization problem

\[
\hat{y} = \arg \max_y \Pr(F, y) = \arg\max_y \Pr(F|y) \Pr(y)
\]

\[
= \arg\max_y \frac{1}{Z} \exp(-U(y)) \Pr(F|y).
\]

Unfortunately, (32) is intractable even for relatively small \( N \), due to the structure of (25). This motivates well approximation techniques to obtain estimates of the labels.

Popular approximation methods include Gibbs sampling [37] and mean-field approximations [35]. Here, we opted for an iterative method called iterated conditional modes (ICM), which has been successfully used in image segmentation [36]. Per ICM iteration, we are given estimates \( \{\hat{\Gamma}_m\}_{m=1}^M \), and update the label of datum \( n \) by finding the \( k \) maximizing its local posterior probability; that is,

\[
\hat{y}^{(i)}_n = \arg\max_{k \in \{1, \ldots, K\}} \Pr \left( y_n = k \right| \left\{ f_m(x_n) \right\}_{m=1}^M, \hat{y}_n^{(i-1)}, y_{N_n} = k_{N_n} \right)
\]

\[
= \arg\min_{k \in \{1, \ldots, K\}} U_n(k) - \sum_{m=1}^M \log \left( \hat{\Gamma}_m(f_m(x_n), k) \right)
\]

where the superscript denotes the iteration index, \( \hat{y}_N \) denotes the label estimates provided by the previous ICM iteration, and the second equality is due to (31). The optimization in (33) is carried out for \( n = 1, \ldots, N \) until the values of \( \hat{y} \) have converged or until a maximum number of iterations \( T_{\text{max}} \) has been reached.

The next subsection puts forth an EM algorithm for estimating \( \{y_n\}_{n=1}^N \) and \( \{\hat{\Gamma}_m\}_{m=1}^M \).

Algorithm 3 EM algorithm for networked data

\[\text{Input:} \text{Annotator responses } \{f_m(x_n)\}_{m=1}^M, \text{ initial } y_n^{(0)}, \left\{ \Gamma_m^{(0)} \right\}_{m=1}^M, \text{ Data graph } G(V, E). \]

\[\text{Output:} \text{Estimates of data labels } \{\hat{y}_n\}_{n=1}^N. \]

1: \text{while not converged do}
2: \hspace{1em} 2: \hspace{1em} \text{while not converged AND } t < T_{\text{max}} \text{ do}
3: \hspace{2em} \text{for } n = 1, \ldots, N \text{ do}
4: \hspace{3em} \text{Update } \hat{y}_n^{(t)} \text{ using (33).}
5: \hspace{2em} \text{end for}
6: \hspace{1em} \text{end while}
7: \text{end while}
8: \text{Compute } \hat{y}_n^{(i+1)} \text{ using (34).}
9: \text{Compute } \left\{ \hat{\Gamma}_m^{(i+1)} \right\}_{m=1}^M \text{ using (36).}
10: \text{end while}

4.2 EM algorithm for networked data

As with the i.i.d. case in Sec. 2.2.2 and the sequential case in Sec. 2.1 the EM algorithm of this section seeks to iteratively maximize the marginal log-likelihood of observed annotator responses. However, the Q-function [cf. Sec. 2.2.1] is now cumbersome to compute under the MRF constraint on \( y \).

For this reason, we resort to the approximation technique of the previous subsection to compute estimates of \( q_{nk} = \Pr(y_n = k | \left\{ f_m(x_n) \right\}_{m=1}^M; \theta) \). Specifically, per EM iteration \( i \), we let \( \hat{y}^{(i)} := [\hat{y}^{(i)}_1, \ldots, \hat{y}^{(i)}_N] \) denote the estimates obtained by the iterative procedure of Sec. 4.1. Then, estimates \( \hat{q}_{nk}^{(i+1)} \) are obtained as [cf. 31]

\[
\hat{q}_{nk}^{(i+1)} = \frac{1}{Z'} \exp \left( -U_n^{(i+1)}(k) + \sum_{m=1}^M \log \left( \hat{\Gamma}_m^{(i)}(f_m(x_n), k) \right) \right)
\]

where

\[
Z' = \sum_k \exp \left( -U_n^{(i+1)}(k) + \sum_{m=1}^M \log \left( \hat{\Gamma}_m^{(i)}(f_m(x_n), k) \right) \right)
\]

is the normalization constant, and \( U_n^{(i+1)}(k) \) is given by

\[
U_n^{(i+1)}(k) = \frac{1}{2} \sum_{n' \in N_n} V(k, \hat{y}_{n'}^{(i+1)}).
\]

Finally, the M-step that involves finding estimates of \( \left\{ \hat{\Gamma}_m \right\}_{m=1}^M \) is identical to the M-step of the EM algorithm of Sec. 2.2.1 for i.i.d. data; that is,

\[
\hat{\Gamma}_m^{(i+1)}(k) = \frac{\sum_{n=1}^N \hat{q}_{nk}^{(i+1)} \mathcal{I}(f_m(x_n) = k')}{{\sum}_{k''=1}^K \sum_{n=1}^N \hat{q}_{nk}^{(i+1)} \mathcal{I}(f_m(x_n) = k'')}.
\]

Similar to the i.i.d. case, the aforementioned EM solver deals with a non-convex problem. In addition, the ICM method outlined in Sec. 4.1 is a deterministic approach that performs greedy optimization. Therefore, proper initialization is crucial for obtaining accurate estimates of the labels and annotator confusion matrices.

As with the decoupling approach of Sec. 3 here we first obtain estimates of annotator confusion matrices \( \left\{ \hat{\Gamma}_m \right\}_{m=1}^M \) and labels \( \hat{y} \) using the moment-matching algorithm of Sec. 2.2.2. These values are then provided as initialization to
error is also evaluated.

For synthetic data, the average confusion matrix estimation and the per-class True Positive (TP), False Positive (FP), and False Negative (FN) performances are as follows:

\[
\text{Precision}_k = \frac{\text{TP}_k}{\text{TP}_k + \text{FP}_k}
\]

\[
\text{Recall}_k = \frac{\text{TP}_k}{\text{TP}_k + \text{FN}_k}
\]

and the per-class True Positive (TP), False Positive (FP), and False Negative (FN) performance metrics as

\[
\text{TP}_k = \sum_{n=1}^{N} I(y_n = k, \hat{y}_n = k)
\]

\[
\text{FP}_k = \sum_{n=1}^{N} I(y_n \neq k, \hat{y}_n = k)
\]

\[
\text{FN}_k = \sum_{n=1}^{N} I(y_n = k, \hat{y}_n \neq k).
\]

For synthetic data, the average confusion matrix estimation error is also evaluated as

\[
\mathcal{E}_{CM} := \frac{1}{M} \sum_{m=1}^{M} \frac{||\Gamma_m - \hat{\Gamma}_m||_1}{||\Gamma_m||_1} = \frac{1}{M} \sum_{m=1}^{M} ||\Gamma_m - \hat{\Gamma}_m||_1
\]

All results represent averages over 10 independent Monte Carlo runs, using MATLAB [33]. Vertical lines in some figures indicate standard deviation.

5 Numerical Tests

The performance of the proposed algorithms for both sequential and networked data is evaluated in this section using synthetic and real datasets. The metric utilized in all experiments is the F-score given by

\[
\text{F-score} = \frac{2}{K} \sum_{k=1}^{K} \frac{\text{Precision}_k \cdot \text{Recall}_k}{\text{Precision}_k + \text{Recall}_k}
\]  

where we have defined the per-class Precision and Recall as

\[
\text{Precision}_k = \frac{\text{TP}_k}{\text{TP}_k + \text{FP}_k}
\]

\[
\text{Recall}_k = \frac{\text{TP}_k}{\text{TP}_k + \text{FN}_k}
\]

and the per-class True Positive (TP), False Positive (FP) and False Negative (FN) performance metrics as

\[
\text{TP}_k = \sum_{n=1}^{N} I(y_n = k, \hat{y}_n = k)
\]

\[
\text{FP}_k = \sum_{n=1}^{N} I(y_n \neq k, \hat{y}_n = k)
\]

\[
\text{FN}_k = \sum_{n=1}^{N} I(y_n = k, \hat{y}_n \neq k).
\]

For sequential data, Alg. 2 with and without EM refinement (denoted as Alg. 2 + Alg. 1 and Alg. 2 respectively) is compared to majority voting (denoted as MV), the moment-matching method of [12] described in Sec. 3.2.2 (denoted as MM), Alg. 1 initialized with majority voting (denoted as MV + Alg. 1), and “oracle” classifiers, based on Viterbi’s algorithm that uses the ground-truth confusion and transition matrices. For synthetic data, the transition matrix estimation error \(||T - \hat{T}||_1\) is also evaluated. For real data tests, instead of MM the EM algorithm of [8] initialized with MM is evaluated (denoted as DS).

All datasets in this subsection are split into sequences. Here, we assume that per dataset these sequences are drawn from the same ensemble HMM [cf. 3]. The reported F-score represents the averaged F-score from all sequences.

5.1 Sequential data

For synthetic tests, S sequences of \(N_s, s = 1, \ldots, S\), ground-truth labels each, were generated from a Markov chain, whose transition matrix was drawn at random such that \(T \in C\). Each of the \(N = \sum_s N_s\) ground-truth labels \(\{y_n\}_{n=1}^{N}\) corresponds to one out of \(K\) possible classes. Afterwards, \(\{\Gamma_m\}_{m=1}^{M}\) were generated at random, such that \(\Gamma_m \in C\), for all \(m = 1, \ldots, M\), and \(M/2 + 1\) annotators are better than random, as per Alg. 1. Then annotators’ responses were generated as follows: if \(y_n = k\), then the response of annotator \(m\) will be generated randomly according to the \(k\)-th column of its confusion matrix, \(\gamma_{m,k}\) [cf. Sec. 2], that is \(f_m(x_n) \sim \gamma_{m,k}\).

Fig. 6 shows the average F-score for a synthetic dataset with \(K = 4, M = 10\) annotators and a variable number of data \(N\) and \(N_s = 40\) for all \(s = 1, \ldots, S\). Fig. 6 shows the average confusion and transition matrix estimation errors for varying \(N\). As the number of data \(N\) increases the performance of the proposed methods approaches the performance of the “oracle” one. Accordingly, the confusion and transition matrix estimates are approaching the true ones as \(N\) increases. This is to be expected, as noted in [12], since the estimated moments are more accurate for large \(N\). Interestingly, Alg. 1 performs well when initialized with majority voting, even though it reaches a performance plateau as \(N\) increases. For small \(N\) however, it outperforms the other proposed methods. This suggests that initializing Alg. 1 with majority voting is preferable when \(N\) is not large enough to obtain accurate moment estimation.

The next experiment evaluates the influence of the number of annotators \(M\) for the sequential classification task. Figs. 6 and 7 showcase results for an experiment with \(K = 4\), fixed number of data \(N = 10^4\), \(N_s = 40\) and a varying number of annotators \(M\). Clearly, the presence of multiple annotators is beneficial, as the F-score increases for all algorithms, while the confusion and transition matrix errors decrease. As with the previous experiment, the performance of Alg. 1 + Alg. 2 improves in terms of F-score, as \(M\) increases.

5.1.2 Real data

Further tests were conducted on two real datasets, the Part-of-Speech (POS) tagging dataset and the Biomedical Information Extraction (IE) [19] dataset.
For the POS dataset $M = 10$ classifiers were trained using NLTK [39] on subsets of the Brown corpus [40] to provide part-of-speech (POS) tags of text. The number of tags is $K = 12$. Then the classifiers provided POS tags for all words in the Penn Treebank corpus [41], which contains $N = 100,676$ words.

The Biomedical IE dataset consists of 5,000 medical paper Abstracts, on which $M = 91$ annotators were tasked with marking all text spans in a given Abstract that identify the population of a randomized controlled trial. The dataset consists of $N = 304,629$ words belonging into $K = 2$ classes: in a span identifying the population or outside. For this particular dataset we evaluate Precision and Recall per sequence in the following way, which was suggested in [19]

$$\text{Precision} = \frac{\# \text{ true positive words}}{\# \text{ words in a predicted span}}$$

$$\text{Recall} = \frac{\# \text{ words in a predicted span}}{\# \text{ words in ground-truth span}}.$$ 

Results for both datasets are listed in Tab. II. For the POS dataset, it can be seen that Alg. [1] + Alg. [2] performs best in all metrics. For the Biomedical IE dataset, while majority voting achieves the best precision of all algorithms, due to its low recall, the overall F-score is low. However, Alg. [1] + Alg. [2] outperforms competing alternatives with regards to recall and F-score.


5.2 Networked data

For networked data, Alg. 4 (denoted as Alg. 4) is compared to majority voting (denoted as MV), the moment-matching method of [12] described in Sec. 2.2.2 (denoted as MM) and Alg. 3 initialized with majority voting (denoted as MV + Alg. 3). For real data tests, instead of MM the EM algorithm of [8] initialized with MM is evaluated (denoted as DS).

5.2.1 Synthetic data

For the synthetic data tests, an $N$-node, $K$ community graph is generated using a stochastic block model [42]. Each community corresponds to a class, and the labels $y_n = k$ if node $n$ belongs to the $k$-th community. Afterwards, $\Gamma_{m}^{M}$ were generated at random, such that $\Gamma_{m} \in C$, for all $m = 1, \ldots, M$, and annotators are better than random, as per Alg. 2. Then annotators’ responses were generated as follows: if $y_n = k$, then the response of annotator $m$ will be generated randomly according to the $k$-th column of its confusion matrix, $\gamma_{m,k}$ (cf. Sec. 2), that is $f_{m}(x_n) \sim \gamma_{m,k}$. For the synthetic data tests, we set $\delta_n = M$. Fig. 8 shows the average F-score for a synthetic dataset with $K = 4$ and $M = 10$ annotators for varying number of data $N$. Fig. 9 shows the average confusion estimation error as $N$ increases. As with sequential data, the F-score of the proposed algorithms increases with $N$ growing, and confusion matrix estimation error decreases. MV+ Alg. 3 quickly reaches a plateau of performance as $M$ also does not improve with increasing $N$. At the same time Alg. 4 capitalizes on the initialization provided by MM.

5.2.2 Real data

Further tests were conducted on five real datasets. For the Cora, Citeseer [32] and Pubmed [33] datasets the graph $G$ and data features $\{x_n\}$ are provided with the dataset. In these cases, $M = 10$ classification algorithms from MATLAB’s machine learning toolbox were trained on different randomly selected subsets of the datasets. Afterwards, these algorithms provided labels for all data in the dataset. For these datasets, we set $\delta_n = M$. For the Music genre and Sentence Polarity datasets [22] the features $\{x_n\}$ and annotator responses $F$ are provided with the dataset. In these cases, the graphs were generated from the data features using $k$-nearest neighbors. Since the graphs are generated from the data features, here we set $\delta_n = M_n/2$, where $M_n$ is the number of annotators that have provided a response for the $n$-th datum.

The Cora, Citeseer and Pubmed datasets are citation networks and the versions used here are preprocessed by [43]. The Cora dataset consists of $N = 2,708$ scientific publications classified into $K = 7$ classes. The features $\{x_n\}$ of this dataset are sparse 1,433-dimensional vectors and for this dataset each classification algorithm was trained on a random subset of 150 instances. The Citeseer dataset consists of $N = 3,312$ scientific publications classified into one of $K = 6$ classes. The features $\{x_n\}$ of this dataset are sparse 3,703-dimensional vectors, and each classification algorithm was trained on a subset of 100 instances. The Pubmed dataset is a citation network that consists of $N = 19,717$ scientific publications from the Pubmed database pertaining to diabetes, classified into one of $K = 3$ classes. The features $\{x_n\}$ of this dataset are 500-dimensional vectors, and each classification algorithm was trained on a subset of 300 instances. The Music genre dataset contains $N = 700$ song samples (each of duration 30secs), belonging into $K = 10$ music categories, annotated
by $M = 44$ annotators. The sentence polarity dataset contains $N = 5,000$ sentences from movie reviews, classified into $K = 2$ categories (positive or negative), annotated by $M = 203$ annotators.

In most datasets Alg. 4 exhibits the best performance in terms of F-score followed closely by $MV + Alg. 3$. For the Music Genre dataset however, $MV + Alg. 3$ outperforms Alg. 4. This is to be expected, as $N$ is relatively small for this dataset and as such the estimated annotator moments are not very accurate. This can also be seen from the fact that $MV$ outperforms $DS$.

6 CONCLUSIONS AND FUTURE DIRECTIONS

This paper introduced two novel approaches to unsupervised ensemble and crowdsourced classification in the presence of data dependencies. Two types of data dependencies were investigated: i) Sequential data; and ii) Networked data, where the dependencies are captured by a known graph. The performance of our novel schemes was evaluated on real and synthetic data.

Several interesting research avenues open up: i) Distributed and online implementations of the proposed algorithms; ii) ensemble classification with dependent classifiers and dependent data; iii) development of more realistic annotator models for dependent data; and iv) extension of the proposed methods to semi-supervised ensemble learning.

APPENDIX A

THE FORWARD-BACKWARD ALGORITHM

Let $b_{n,k}$ denote the probability of observing $\{f_{m}(x_n)\}_{m=1}^{M}$ given that $y_n = k$, that is

$$b_{n,k} = \prod_{m=1}^{M} \Pr(f_{m}(x_n)|y_n = k) = \prod_{m=1}^{M} \Gamma_{m}(f_{m}(x_n), k).$$

(44)

The forward-backward algorithm [27] seeks to efficiently obtain the probability of the observed variable sequence $\{f_{m}(x_n)\}_{n=1,m=1}^{N,M}$ given current HMM parameter estimates $\theta$. It takes advantage of the fact that the past and future states of a Markov chain are independent given the current state. Tailored for our ensemble HMM, we have

$$\Pr(F|\theta) = \sum_{k=1}^{K} \alpha_{n,k} \beta_{n,k},$$

(52)

which holds for any $n \in \{1, \ldots, N\}$. Then the variables of interest, $q_{nk}$ and $\xi_{n}(k,k')$, can be obtained as

$$q_{nk} = \Pr(y_n = k|F, \theta) = \frac{\alpha_{n,k} \beta_{n,k}}{\sum_{k'=1}^{K} \alpha_{n,k'} \beta_{n,k'}},$$

(53)

$$\xi_{n}(k,k') = \Pr(y_n = k, y_{n+1} = k'|F, \theta) = \frac{\alpha_{n,k} T(k,k') \beta_{n+1,k} \beta_{n+1,k'}}{\sum_{k''=1}^{K} \alpha_{n,k''} T(k,k'') \beta_{n+1,k''} \beta_{n+1,k'}}.$$  

(54)

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| Dataset   | K  | M  | N  | MV  | DS | Alg | MV + Alg |
|-----------|----|----|----|-----|----|-----|----------|
| Cora      | 10 | 10 | 2  | 708 | 0.278 | 0.422 | 0.306 |
| CiteSeer  | 7  | 10 | 3  | 3,112 | 0.4257 | 0.4449 | 0.3524 |
| Pubmed    | 3  | 10 | 19 | 7,117 | 0.6908 | 0.7437 | 0.7067 |
| Music Genre | 10 | 44 | 700 | 0.7046 | 0.4746 | 0.7410 | 0.8029 |
| Sent. Polarity | 2 | 203 | 6,000 | 0.8895 | 0.9129 | 0.9153 | 0.9139 |

**TABLE 2**: F-score for Real data experiments with Networked data.

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