Relativistic Quantum Field Theory with a Physical State Vector

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Evolution of a physical quantum state vector is described as governed by two distinct physical laws: Continuous, unitary time evolution and a relativistically covariant reduction process. In previous literature, it was concluded that a relativistically satisfactory version of the collapse postulate is in contradiction with physical measurements of a non-local state history. Here it is shown that such measurements are excluded when reduction is formulated as a physical process and the measurement devices are included as part of the state vector.

I. INTRODUCTION

Relativistic Quantum Field Theory (QFT) relies on a number of interpretational rules which allow to make measurable predictions. In the non-relativistic limit of Quantum Theory (QT) a state vector can be defined which evolves according to two laws: (a) Unitary, continuous time evolution in agreement with the non-relativistic equations of motion and (b) the state vector reduction or collapse postulate, which dictates how the state vector changes as a result of measurement. Together, they allow to define a unique state vector history. In the following the notation Physical State Vector (PSV) is used for a state vector which, in its coordinate space representation, is well defined at all spacetime points.

For the relativistic theory the definition of a PSV faces difficulties due to the fact that spacelike measurements which are instantaneous in one inertial frame are no longer instantaneous in another. Collapse has to take place in a covariant fashion. This requirement restricts us to using light cone sections. It turns out that the Forward Light Cone (FLC) is insufficient, as follows most stringent from experiments (see and references therein) performed in connection with measuring violations of Bell’s inequalities. Indeed, a prescription for reduction on the Backward Light Cone (BLC) was already proposed in an early paper on the subject. But, subsequently Aharonov and Albert (AA) concluded that a relativistically satisfactory version of the collapse postulate cannot be found at all. Their analysis relies on constructing local devices which allow to monitor a non-local history. As stressed by them, the empirical meaning of their procedure is crucial. Otherwise it would be without physical content to assert that the system is in an eigenstate of some non-local operator and, being in such a state, excluding collapse on the BLC. In summary, AA state that the relativistic theory still has the capacity to predict probabilities, but lacks the capacity to define a consistent PSV.

It is one purpose of the present paper to show that measurements of the Aharonov-Albert type are ruled out, once the measurement device is considered to be part of the PSV and reduction is understood as a physical process, distinct from unitary and continuous time evolution. In section II.A the non-relativistic PSV evolution is reviewed, followed by introducing its relativistic generalization. Section II.B introduces an unconventional formulation of the reduction process. Namely, each reduction is attributed to the action of a detector. For the non-relativistic limit this is shown to agree with the conventional formulation. The full relativistic spacetime picture is developed in section II.C and our rules for reduction are distinct from those of ref. It is shown in section II.D that AA measurements become impossible within the developed framework.

To give some illustrations, section III deals first with spacelike measurement of charge for a particle which is in a superposition of two waves localized at spacelike positions. The introduction of AA-like devices (for a precise definition see there) does not lead to inconsistencies. Section II.B considers spacelike measurements on the entangled singlet state of two distinguishable spin particles. This example, also discussed by Aharonov and Albert, is for various features of interest the technically simplest illustration. In particular, it turns out that the remains of the AA idea are still sufficient to show inconsistency for the proposal by Hellwig and Kraus. In section III.C measurement is briefly discussed for the Greenberger-Horne-Zeilinger (GHZ) state, mainly to give an example where three detectors at spacelike positions are involved. Conclusion are drawn in the final section IV.
II. PHYSICAL STATE VECTOR AND REDUCTION

We deal with states in the Schrödinger picture. The question may arise, whether statistical mixtures ought better to be considered. Let us first have a look at classical physics. There, the existence of a single state is guaranteed by definition and statistics is an essential tool in view of the practical impossibility to control a large number initial conditions. Here I take a similar stance on QT, I assume that a PSV of the world $|\Psi_W\rangle$ exists, although a complete set of measurements to determine the initial state cannot be performed. The essential feature of QT is that even complete knowledge of the state does only allow stochastic predictions. Introducing a density matrix approach at this point, where we are dealing with the fundamental nature of QT, may more obscure than enlighten this central issue. Whether a state or a density matrix reflects the underlying fundamental physics correctly, either case is an assumption, and Newton’s First Rule for Reasoning in Philosophy decides the matter in favor of the state.

A. General state vector evolution

In the following a microscopic object (photon, electron, proton, etc.), as studied in many QT applications, is called Quantum Object (QO). Let us define

$$t_+ = \lim_{\epsilon \to 0, \epsilon > 0} (t + \epsilon) \quad \text{and} \quad t_- = \lim_{\epsilon \to 0, \epsilon > 0} (t - \epsilon)$$

in the sense that whenever these symbols are used the indicated limits have to be taken outside of all other operations. In the non-relativistic QT state vectors of QOs $|\psi(t)\rangle$ are defined on the instantaneous hyperplane of 4d spacetime and evolve according to two laws:

(a) Unitary, causal time evolution ($t_+ > t_{0+}$)

$$|\psi(t_-)\rangle = U(t_- - t_{0+}) |\psi(t_{0+})\rangle \quad \text{with} \quad U(t) = \exp(-iHt)$$

when no measurements are carried out.

(b) An instantaneous, stochastic transformation (state vector reduction or collapse)

$$|\psi(t_-)\rangle \rightarrow |\psi(t_{1+})\rangle$$

when a measurement is carried out at time $t_1$.

The initial state $|\psi(t_{0+})\rangle$ can, in principle, uniquely be specified by measuring a complete, commuting set of observables. Let $A$ represent operators corresponding to such a set of observables and assume that the possible results of their measurements are given by $a_n$, ($n = 0, 1, 2, ...$), where each $a_n$ represents an entire set of eigenvalues and, hence, specifies a unique state $|\psi_n\rangle$ for which

$$A|\psi_n\rangle = a_n |\psi_n\rangle$$

holds. Assume, the operators of $A$ are measured at time $t_A > t_0$, then QT predicts that $a_n$ is found with probability

$$P_n = |\langle \psi_n | \psi(t_{A-}) \rangle|^2 .$$

Given that the particular set of eigenvalues $a_n$ is observed at time $t_A$, the state $|\psi(t_{A+})\rangle$ of (2.3) is $|\psi(t_{A+})\rangle = |\psi_n\rangle$. In this way a series of complete measurements $A, B, C, ...$ at times $t_A < t_B < t_C < ...$ (where each capital letter represents a complete set of operators) defines a physical state history $|\psi(t)\rangle$. This state history can be monitored by performing non-demolition experiments, which measure the last complete set of operators again, before the next reduction. For instance, for time $t$ in the range $t_A < t < t_B$ we would measure the observables $U(t - t_A) A U^{-1}(t - t_A)$.

This reproduces the result $a_n$ with certainty and without disturbing the state $|\psi(t)\rangle = |\psi_n(t - t_A)\rangle = U(t - t_A) |\psi_n\rangle$.

The idea of a PSV has considerable intuitive appeal and should not be given up easily. Here I embark on a starting point which is kind of opposite to the one of ref. 3. Assuming the existence of $|\Psi_W\rangle$, a distinct understanding of measurements is developed. As explained in the next subsections, each reduction is attributed to a detector and new restrictions emerge, because every detector has to be part of the PSV. While the non-relativistic limit is found to agree with the scenario sketched above, AA measurements become ruled out, and a consistent PSV may exist.
First, we need some notation. In the following we call two spacetime points \( x_0 = (ct_0, \vec{x}_0) \) and \( x_1 = (ct_1, \vec{x}_1) \) spacelike when
\[
(x_0 - x_1)^2 = c^2 (t_0 - t_1)^2 - (\vec{x}_0 - \vec{x}_1)^2 \leq 0 \tag{2.6}
\]
holds. In case of the equal sign, we assume that \( x_0 \) and \( x_1 \) are defined as the limit of points
\[
x_0 = \lim_{\delta \to 0+, \delta > 0} x_{0,\delta} \quad \text{and} \quad x_1 = \lim_{\delta \to 0+, \delta > 0} x_{1,\delta} \quad \text{such that} \quad (x_{0,\delta} - x_{1,\delta})^2 < 0 \quad \text{holds for all} \quad \delta > 0.
\]
A Lorentz Covariant Spacelike Hypersurface (LCSH) is now defined to be a hypersurface \( S \) which fulfills two conditions: (1) Equation (2.6) holds for any two points \( x_0 \in S \) and \( x_1 \in S \). (2) To insure Lorentz covariance of the collapse process, \( S \) is build from light cone sections. Let \( S_0 \) and \( S_1 \) be LCSHs. The hypersurface \( S_1 \) is said to be in the future of \( S_0 \), when for any two points \( x_0 \in S \) and \( x_1 \in S \) with \( \vec{x}_0 = \vec{x}_1 \) we have \( t_0 \leq t_1 \), and there exist some such points with \( t_0 < t_1 \). Similarly single spacetime points or sets of such points are defined to be in the future (or past) of a LCSH. Finally, for each LCSH \( S \) hypersurfaces \( S_+ \) and \( S_- \) are defined by taking the time component of all their spacetime points in accordance with the limits defined by equation (2.7). Assume a state vector is initially defined on some LCSH \( S_{0+} \). Its relativistic evolution follows then rules similar to the laws (a) and (b):

(a’) Unitary and causal evolution to some LCSH \( S_{1-} \) in the future of \( S_{0+} \)
\[
|\Psi(S_{1-})\rangle = U(S_{1-},S_{0+}) |\Psi(S_{0+})\rangle \tag{2.7}
\]
when no reduction happens on any hypersurface in-between \( S_{1-} \) and \( S_{0+} \).

(b’) A stochastic transformation
\[
|\Psi(S_{1-})\rangle \rightarrow |\Psi(S_{1+})\rangle \tag{2.8}
\]
describes reduction on \( S_1 \). From \( S_{1+} \) the state vector evolves to a next LCSH \( S_{2-} \) by unitary and causal evolution. On \( S_2 \) reduction takes place, and so on.

The process is graphically depicted in figure 1. A light cone section becomes added to the initial LCSH \( S_0 \) such that the created surface \( S_1 \) is in the future of \( S_0 \). On \( S_1 \) reduction takes place from \( S_{1-} \) to \( S_{1+} \). The \( S_{0\pm} \) and \( S_{1\pm} \) surfaces are not explicitly indicated in figure 1, their locations are obvious. In the next subsection we consider the reduction (b’) in detail and come to a process where the addition of a single new light cone section (not spelled out in rule (a’)) is typical. By reasons explained then, the definition of reduction in (b’) is not entirely identical with the one of (b). Namely, some measurements will constitute reductions and others not. This is just a notational issue. The physical content of (a) and (b) is fully recovered in the non-relativistic limit of (a’) and (b’). In particular, for the speed of light \( c \to \infty \) the LCSH of figure 1 becomes the instantaneous hyperplane.

B. Measurement and reduction

From the fact that experimental measurements are made, it follows that it is possible to construct bound states \( |\Psi_D\rangle \), called detectors, which have the ability to perform them. Eventually the detector may project some quantum state on an eigenstate of certain operators, or onto a state out of the space of eigenstates in case that the eigenvalue in question is degenerate. I proceed with a description in terms of the PSV: A detector \( |\Psi_D\rangle \) can be constructed and a QO \( |\psi\rangle \) (for instance a single electron) can be prepared, such that over some time \( \mathbb{R} \) period the factorization
\[
|\Psi_W\rangle = |\psi\rangle |\Psi_D\rangle , \tag{2.9}
\]
where \( |\Psi_D\rangle \) describes the rest of the world, is meaningful in the following sense: Although, due to interactions, this factorization can never be exact, the evolution of
\[
|\Psi\rangle = |\psi\rangle |\Psi_D\rangle , \tag{2.10}
\]
considered in isolation, describes nevertheless the features we are interested in correctly and proceeds as follows. Initially there is very little overlap between the wave functions of the QO and of the detector, such that approximately
\[
|\Psi\rangle = |\psi\rangle |\Psi_D\rangle . \tag{2.11}
\]
Subsequently, $|\psi\rangle$ and $|\Psi_D\rangle$ interact and this is expressed by the notation $|\psi \Psi_D\rangle$, which is already used in equations (2.9) and (2.10). At some point in its spacetime the detector performs a transition

$$|\psi \Psi_D\rangle \rightarrow |\Psi_D'\rangle$$

(2.12)

such that the (macroscopic) state $|\Psi'_D\rangle$ constitutes a measurement of an observable or set of observables $O_n$, according to the purpose for which the detector was constructed. The measurement (2.12) is here interpreted as a physical ability of the detector. Besides avoiding the word reduction at the moment and emphasizing the role of the detector as part of a state vector this is not distinct from standard QT. In the non-relativistic limit complete measurements (2.4) can be performed to the extent that it is possible to design a detector which measures the complete set of operators $A$ such that the state $|\Psi'_D\rangle$ factorizes in the form

$$|\Psi'_D\rangle = |\psi_n\rangle |\Psi_D\rangle$$

(2.13)

where $|\psi_n\rangle$ is the eigenstate of (2.4) and the state $|\Psi_D\rangle$ allows us to access this information (through the neglected interactions with $|\Psi_R\rangle$).

It should be noted that for real detectors the factorization (2.13) is the exception and not the rule. To give one example, to measure the $L_z$ spin component of an electron ala Stern-Gerlach one does first split the electron wave function by employing a magnetic field. Subsequently one detects the electron in one of two alternative branches, for instance by using a Channel Electron Multiplier (CEM) which actually digests the electron. We may employ just one CEM, say in the branch corresponding to $L_z = +\frac{1}{2}$, and distinguish two states of the CEM: $|\Psi_D\rangle$ when nothing happens and $|\Psi_D'\rangle$ when a signal current is put out. The interpretation of the latter case is that a $L_z = +\frac{1}{2}$ electron has been detected. Further, if the detection efficiency would be 100% and we could make sure by some initial measurement that one electron enters the device, we would interpret the $|\Psi_D\rangle$ state as a measurement of $L_z = -\frac{1}{2}$. In practice, one would rather count the electron in the $L_z = -\frac{1}{2}$ branch with a second CEM. To stay close with what actually happens, we use in the following the symbol $|\Psi'_D\rangle$ to denote measurement results, with the understanding that factorization (2.13) is included as a special case.

We are now ready to define reduction. For the measurement process (2.12) we distinguish two cases:

1. $|\psi\rangle = |\psi_n\rangle$ holds for the initial QO. Then the transformation (2.12) appears to be consistent with the time evolution ($a'$) and we assume that this is indeed the case. For the just given practical example this would be when an electron is either initially prepared in the spin $L_z = +\frac{1}{2}$ state and the resulting detector state is $|\Psi_D'\rangle$, or when the electron is initially prepared in the $L_z = -\frac{1}{2}$ state and the resulting detector state is $|\Psi_D\rangle$.

2. $|\psi\rangle = c_n |\psi_n\rangle + c_n |\psi_n\rangle$ with $|c_n|^2 + |c_n|^2 = 1$, both $|c_n|^2 > 0$ and $|c_n|^2 > 0$, and $\langle \psi_n | \psi_n \rangle = 0$ holds for the initial QO. Then the transformation (2.12) appears to be inconsistent with the time evolution ($a'$). We assume that this is indeed the case and understand reduction ($b'$) as a physical law distinct from ($a'$). In the following we limit the definition of reduction strictly to the situation where it constitutes an interruption of the time evolution ($a'$). For our practical example reduction would, for instance, take place when the electron is initially prepared in an $L_z$ eigenstate, say $L_z = \frac{1}{2}$. The possible outcomes, each with 50% probability, are then the CEM states $|\Psi_D\rangle$ versus $|\Psi_D\rangle$, both somehow labelled by the superscript $n$ in equation (2.12).

Case (1) is measurement without reduction. When, in addition, also the factorization (2.13) holds, we have a non-demolition measurement. Although not every measurement constitutes now a reduction, every detector has to be attributed the ability to perform reductions. This follows from the fact that a detector is a device constructed for performing measurements of some observable(s) on certain QOs. We can then confront the detector with a QO which is prepared to have some component orthogonal to the measured eigenstate(s). According to the standard rules of QT, the detector has then to make a decision between the eigenstate(s) and the orthogonal complement, i.e. performs a reduction in the sense just defined.

To move, in a finite time, from one detector state to another requires a difference in energy distribution $\Delta E$. Assuming that the only driving property which constitute the ability to perform reductions is being a boundstate confronted with a (not necessarily macroscopic) difference in energy distribution $\Delta E$, a heuristic approach to quantum measurement was developed in ref. [9]. Subsequently, this motivated the present work, which may allow to express the ideas of [1] within a more appropriate framework. Relevant questions about the physical nature of detectors are postponed to future work. This paper is devoted to defining a PSV and the next subsection deals with the central issue of constructing a relativistically consistent spacetime picture for the reduction process.
C. The reduction process in spacetime

We consider non-overlapping detectors $A, B, C, \ldots$, each localized in a well-defined spacetime region. The basic idea is that these detectors perform reductions in a well-defined order, which in the following is called reduction order and indicated by the symbol $r$. The labelling of the detectors may be chosen such that the reduction order

$$r_A < r_B < r_C < \ldots$$  \hspace{1cm} (2.14)

holds. If detectors measure at timelike positions with respect to one another, their reduction order is requested to agree with the time order of the measurements: $r_A < r_B \iff t_A < t_B$. For detectors at spacelike positions the reduction order is still assumed to exist, but does no longer correspond to a well-defined time order. We use the reduction order to construct a PSV $|\Psi\rangle = |\Psi_{W}\rangle$ in spacetime. As spacelike operators commute, measurable effects are supposed to be independent of the chosen reduction order.

By assumption, the PSV is initially defined on a LCSH $S_{0+}$: $|\Psi\rangle = |\Psi(S_{0+})\rangle$ and all reductions happen in the future (as defined in II.A) of $S_{0+}$. Detector $A$ is first in reduction order. Some section of its BLC is in the future of $S_{0+}$ and denoted $C_{A}^{blc}$. The next LCSH $S_{1-}$ is obtained by replacing the part of $S_{0+}$ which is in the past of $C_{A}^{blc}$ by $C_{A}^{blc}$. By unitary and causal time evolution of law (a') the PSV becomes defined on $S_{1-}$: $|\Psi\rangle = |\Psi(S_{1-})\rangle$. This is always possible, because the definition of the reduction order makes sure that no reduction process can be in the way. For this causal evolution it is sufficient to know the PSV on the section of $S_{0+}$ which is within the BLC of the detector’s reduction position. On $S_{1}$ detector $A$ performs then the stochastic reduction of law (b') and the PSV becomes defined on $S_{1+}$: $|\Psi\rangle = |\Psi(S_{1+})\rangle$. When the initial hypersurface $S_{0+}$ is at $t = -\infty$ for all its space points, then $S_{1}$ is just the BLC of $A$.

Involving detector $B$, the promotion of the PSV from $S_{1+}$ to $S_{2-}$ and from there to $S_{2+}$ follows precisely the same scheme. By induction we proceed from detector to detector, always promoting the LCSH, with the PSV defined on it, forward in time: $S_{0} < S_{1} < S_{2} < \ldots$ in the sense of time-ordering of LCSHs. Should there be a last detector, we can promote the PSV by unitary and causal time evolution to $t = +\infty$ for all space points. Figure 2 illustrates a situation involving three detectors.

Our construction exploits the fact that spacelike operators commute to attribute each reduction to the action of a local detector. Despite the fact that the reduction order cannot be verified experimentally, this has some physical appeal. In contrast to this the procedure proposed by Hellwig and Kraus \cite{4} is entirely formal and remains inconsistent (see section III.B) even after our critical assessment of AA measurements.

D. Aharonov-Albert measurements

The statement of Aharonov and Albert (AA) is that it is possible to design procedures which combine several local interactions to measure some non-local property of the quantum system. For instance, they propose two arrays of devices, called AA devices henceforth, to measure the non-local variable $x_{1} - x_{2}$ of two distinguishable particles. These devices interact with the quantum system $|\psi_{1}\rangle |\psi_{2}\rangle$ through the Hamiltonian

$$H_{int} = \int di \left(h_{i1} + h_{i2}\right) \text{ with } h_{i2} = g(t) q_{i} x_{j} \delta(x_{j} - x_{i}) \text{, } (j = 1, 2)$$  \hspace{1cm} (2.15)

where $x_{j}$ is the position of particle $j$, $x_{i}^{j}$ is the position of the $i$th AA device corresponding to particle $j$, and $q_{i}^{j}$ is some internal variable of the device. The coupling $g(t)$ is non-zero only during a short interval $t_{1} < t < t_{2}$, when the devices are switched on. The position of particle $j$ becomes then coupled to the canonical conjugate momentum

$$\Pi_{j} = \int di \, \pi_{j}^{i}$$

of array $j$, where $\pi_{j}^{i}$ is the canonical conjugate momentum of the correspondingly labelled AA device. AA claim that it is possible to prepare all devices in initial states such that, when separated, they allow for measurement $x_{1} - x_{2}$, but not for measurement of $x_{1}$, $x_{2}$ or $x_{1} + x_{2}$, in accordance with

$$[\Pi_{1} - \Pi_{2}, Q_{1} + Q_{2}] = 0 \text{ but } [\Pi_{j}, Q_{1} + Q_{2}] \neq 0 \text{ and } [\Pi_{1} + \Pi_{2}, Q_{1} + Q_{2}] \neq 0.$$  \hspace{1cm} (2.16)

See equations (7) to (16) in their paper \cite{5} for more details.
Let us try to combine the AA approach with the concept of measurement developed in this paper. First, the interactions of nature cannot be turned on and off at will. They are all supposed to be described by the Lagrangian $\mathcal{L}$ of a fundamental QFT. However, this point is minor. It is perfectly reasonable to assume that we can prepare AA devices $|\Psi^{i,j}_{\text{AA}}\rangle$ such that, through preparation of their initial state, their interaction with the quantum system $|\psi_1\rangle|\psi_2\rangle$ is effectively described by Hamiltonians $H^{i,j}_{\text{int}} = (h^i_1 + h^j_2)$ with $h^i_j$ given by (2.13). A second minor point is that in practice we can only employ a finite number of such devices, say $i = 1, \ldots, n$. It makes sense to assume that such a finite discretization still gives, within certain limitations of accuracy and response efficiency, the desired result. Hence, the relevant state vector is

$$ |\Psi\rangle = |\psi_1\rangle |\psi_2\rangle \prod_{i=1}^{n} |\Psi^{1,i}_{\text{AA}}\rangle |\Psi^{2,i}_{\text{AA}}\rangle $$

where the factorization has to be understood in the sense discussed for equation (2.11). In particular, at times when interaction takes place the expression

$$ |\Psi\rangle = |\psi_1\rangle \prod_{i=1}^{n} |\Psi^{1,i}_{\text{AA}}\rangle |\psi_2\rangle \prod_{i=1}^{n} |\Psi^{2,i}_{\text{AA}}\rangle $$

is appropriate to indicate it, compare equation (2.10). This product structure of devices limits non-local measurement severely and cannot be avoided, because every degree of freedom in QFT enters as a factor into the state vector.

An immediate consequence is that the non-local operator $\Pi_{\text{AA}}$ of particle $j$ is appropriate to indicate it, compare equation (2.10). This product structure of devices limits non-local measurement severely and cannot be avoided, because every degree of freedom in QFT enters as a factor into the state vector.

One may want to address the question, to where the state vector (2.17) eventually leads us. Neither of the two AA arrays alone can be a detector. Namely, by its purpose array $j$ is not allowed to produce a definite result for its conjugate momentum $\Pi_{\text{AA}}$. The only way the AA devices could possibly function is through unitary and causal interaction with the quantum system $|\psi_1\rangle|\psi_2\rangle$, then feeding the gained information into a final device $|\Psi_{C}\rangle$ which is a detector. Each of the two AA arrays has to be positioned within the BLC of the spacetime point at which detector $C$ performs its measurements in accordance with the scheme developed in the previous two sections. In the rest of this paper we interpret AA devices in this sense (AA-like devices in the introduction and in the figure captions).

Not only our notion of measurement is distinct from AA, but also our notion of a PSV. As element in Fock space our PSV exist only on certain spacelike hyperplanes (never cross reduction lines!). However, where it does not exist it cannot be monitored. An example is given in section III.A.

### III. EXAMPLES

In the next subsection we deal with detection of a single, charged particle by detectors $A$ and $B$ at spacelike positions. In addition it is shown why non-local monitoring, involving two AA devices and a third detector $C$ fails.

In the other subsections we work out various examples of spacelike measurements on states made from distinguishable spin $\frac{1}{2}$ particles. Detectors are denoted by $A$, $B$ and $C$ and assumed to act at spacelike positions in reduction order $r_A < r_B < r_C$ (or in a different reduction order when explicitly stated). Each detector can measure the spin of an incoming particle with respect to one of $n$ different axis $\hat{r}_i$, $i = 1, \ldots, n$. As the particles are distinguishable, we can (and do) label them by the detectors which eventually measure them. The following equation defines then eigenstates with respect to the axis $\hat{r}_i$ as measured by detector $A$

$$ L^A_i |a^{i+}\rangle = \frac{1}{2} |a^{i+}\rangle \quad \text{and} \quad L^A_i |a^{i-}\rangle = -\frac{1}{2} |a^{i-}\rangle $$

where the superscripts + and − label the $+\frac{1}{2}$ and $-\frac{1}{2}$ eigenstates, respectively. The same notation is used for the other detectors, replacing $A$ by $B$ or $C$ (and $a$ by $b$ or $c$).
A. Split charged particle

We consider a single charged particle which has been split by a magnetic field into a superposition of two localized waves

$$|\psi\rangle = |\psi_a\rangle + |\psi_b\rangle$$

where $|\psi_a\rangle$ heads towards detector $A$ and $|\psi_b\rangle$ towards detector $B$ along the lines indicated in figure 3. Each detector will measure simply whether a charged particle comes in or not. The initial LCSH $S_0$ is chosen to be at $t = -\infty$, out of the picture. AA devices are positioned on spacetime worldlines AA$_1$ and AA$_2$ of this figure (as they are not measurement devices the same kind of lines as for QOs are used). In the present example we assume that the charged particle is scalar or an electron with given spin. The no-cloning theorem does then allow to assume that an AA device is able to make a copy of the state it encounters and to send this copy to the detector $C$. Without bothering about the experimental feasibility, this seems to be the most favorable assumption about the eventual performance of such devices. The copy will be denoted by $|\psi^i_i\rangle$ ($i = 1, 2$) when originating from AA device AA$_i$. Detector $C$ performs the same kind of measurement as detectors $A$ and $B$ do and allows to trace the direction from where the charged particle comes.

Detector $A$ reduces the state vector on the LCSH $S_1$. On the $S_{1-}$ side of this surface the PSV, obtained from $S_{0+}$ by unitary and causal time evolution, reads

$$|\Psi(S_{1-})\rangle = (|\psi_a\rangle |\psi_1^1\rangle + |\psi_b\rangle |\psi_1^2\rangle) |\Psi_{AA}'\rangle |\Psi_A'\rangle |\Psi_B'\rangle |\Psi_C'\rangle,$$

where the state vector includes detectors and AA devices. AA$_1$ has created the copy $|\psi_1^1\rangle$ of $|\psi_a\rangle$. The reduction (100% efficiency assumed) by detector $A$ transforms $|\Psi\rangle$ either into

$$|\Psi(S_{1+})\rangle = |\psi_1^1\rangle |\Psi_{AA}'\rangle |\Psi_A'\rangle |\Psi_B'\rangle |\Psi_C'\rangle,$$

where the prime on $|\Psi_A'\rangle$ indicates that a charged particle has been detected, or into

$$|\Psi(S_{1+})\rangle = |\psi_b\rangle |\Psi_{AA}'\rangle |\Psi_A'\rangle |\Psi_B'\rangle |\Psi_C'\rangle,$$

when no particle is detected by $A$. In either case $|\Psi\rangle$ is then defined on the LCSH $S_{1+}$. Continuing with (3.3), the final state, defined on the spacelike surface $S_4$ of figure 3, will be

$$|\Psi(S_4)\rangle = |\Psi_{AA}'\rangle |\Psi_{AA}'\rangle |\Psi_A'\rangle |\Psi_B'\rangle |\Psi_C'\rangle,$$

i.e. a charged particle has now also been detected by detector $C$ in a measurement which is not a reduction. As the picture for this case is rather obvious, it is not drawn in figure 3, instead the next one is depicted. Continuing with (3.4) the second AA device has an opportunity to act and on the spacelike surface $S_{2-}$ of figure 3 the state becomes

$$|\Psi(S_{2-})\rangle = |\psi_b\rangle |\psi_2^2\rangle |\Psi_{AA}'\rangle |\Psi_{AA}'\rangle |\Psi_A'\rangle |\Psi_B'\rangle |\Psi_C'\rangle.$$

After measurements by $B$ and $C$ which are both no reductions the final state becomes

$$|\Psi(S_4)\rangle = |\Psi_{AA}'\rangle |\Psi_{AA}'\rangle |\Psi_A'\rangle |\Psi_B'\rangle |\Psi_C'\rangle.$$

In (3.6) as well as in (3.7) the detector $C$ has measured a charge, but it cannot be interpreted as measuring a non-local charge.

Let us push this example a bit further and assume now the reduction order $r_C < r_B < r_A$. The BLC of detector $C$ defines now the reduction surface $S_1$ as indicated in figure 4. On the $S_{1-}$ side the PSV is

$$|\Psi\rangle = (|\psi_a\rangle |\psi_1^1\rangle + |\psi_b\rangle |\psi_2^2\rangle) |\Psi_{AA}'\rangle |\Psi_{AA}'\rangle |\Psi_A'\rangle |\Psi_B'\rangle |\Psi_C'\rangle$$

where both AA devices have now done their work. Detector $C$ reduces the state to either

$$|\Psi(S_{1+})\rangle = |\psi_a\rangle |\Psi_{AA}'\rangle |\Psi_{AA}'\rangle |\Psi_A'\rangle |\Psi_B'\rangle |\Psi_C'\rangle,$$

or

$$|\Psi(S_{1+})\rangle = |\psi_b\rangle |\Psi_{AA}'\rangle |\Psi_{AA}'\rangle |\Psi_A'\rangle |\Psi_B'\rangle |\Psi_C'\rangle.$$

The final PSVs are identical to those obtained before: Equation (3.8) leads to (3.7) and equation (3.9) to (3.6). The latter one is drawn in figure 4. As observed in (3.6) the charge is not conserved on all spacelike hyperplanes. As charge is a superselection rule, this means on such hyperplanes a PSV as vector in Fock space does not exist. Our example enlightens that these non-existing states cannot be monitored and our PSV evolution remains empirically satisfactory.
B. Singlet decay

As in the previous subsection the $S_0$ LCSH is moved to $t = -\infty$. Including the detectors $A$ and $B$, the singlet state reads

$$|\Psi(S_{1-})\rangle = 2^{-\frac{1}{2}} \{ |a^{k-}\rangle |b^{k+}\rangle - |a^{k+}\rangle |b^{k-}\rangle \} |\Psi_A\rangle |\Psi_B\rangle.\tag{3.10}$$

Assume that $A$ measures with respect to axis $\hat{r}_i$ and that the result is $i+$. The corresponding reduction reads

$$2^{-\frac{1}{2}} \{ |a^{k-}\rangle |\Psi_A\rangle |b^{k+}\rangle - |a^{k+}\rangle |\Psi_A\rangle |b^{k-}\rangle \} |\Psi_B\rangle \rightarrow \{ |a^{i+}\rangle |a^{k-}\rangle |b^{k+}\rangle - |a^{i+}\rangle |a^{k+}\rangle |b^{k-}\rangle \} |\Psi_A^i\rangle |\Psi_B\rangle.$$

It is easy to work out the matrix elements $\langle a^{i+}|a^{k-}\rangle = -\sin(\theta/2)$ and $\langle a^{i+}|a^{k+}\rangle = \cos(\theta/2)$, where $\theta$ is the angle between axis $i$ and axis $k$. Further, $|b^{i-}\rangle = -\sin(\theta/2)|b^{k+}\rangle + \cos(\theta/2)|b^{k-}\rangle$. Hence, detector $A$ reduces the state to

$$|\Psi(S_{1+})\rangle = -|b^{i-}\rangle |\Psi_A^i\rangle |\Psi_B\rangle,\tag{3.11}$$

see figure 5 for the location of $S_1$, which cuts through the worldline of particle $|b\rangle$ before it reaches detector $B$. Equation (3.11) still holds on $S_{2-}$. Assume that $B$ measures with respect to axis $\hat{r}_j$ and that the result is $j+$. The final reduction simply reads

$$-|\Psi_A^j\rangle |b^{i-}\rangle |\Psi_B\rangle \rightarrow |\Psi(S_{2+})\rangle = -|\Psi_A^j\rangle |\Psi_B^j\rangle.\tag{3.12}$$

Note that the reduction performed by $B$ is entirely local, affecting the PSV only in the immediate neighbourhood of $B$. Given axis $\hat{r}_i$ for $A$ and $\hat{r}_j$ for $B$, let us find the QT probability $P(a^{i+}, b^{j+})$ that $i+$ is measured by $A$ and $j+$ by $B$. For the reduction step from (3.10) to (3.11) the probability is

$$|\langle\Psi(S_{1+})\rangle|\langle\Psi(S_{1-})\rangle|^2 = 2^{-\frac{1}{2}} \left[ |\langle a^{i+}|a^{k-}\rangle|^2 + |\langle a^{i+}|a^{k+}\rangle|^2 \right] = \frac{1}{2} \tag{3.13}$$

and for the step (3.12) it is $|\langle b^{i+}|b^{j-}\rangle|^2$. Together this implies

$$P(a^{i+}, b^{j+}) = \frac{1}{2} |\langle b^{i+}|b^{j-}\rangle|^2.\tag{3.14}$$

Note that for $j+ = i-$ the matrix element becomes $|\langle b^{i-}|b^{j-}\rangle|^2 = 1$: Detector $B$ still measures, but no reduction takes place. Finally, the reader can easily verify that the same probability $P(a^{i+}, b^{i+})$, namely $\frac{1}{2} |\langle a^{i+}|a^{i-}\rangle|^2$, is obtained when $B$ measures $j+$ first in reduction order, and $A$ measures then $i+$. We compare now with the construction by Hellwig and Kraus [3]. As in figure 3 of their paper, regions 1 to 4 are defined in our figure 6. Positions of the detectors $A$ and $B$ are identical with those in figure 5, but now both BLCs are drawn. We ignore detector $C$ at the moment and construct a state on a spacelike hypersurface within one region of figure 6 by crossing all worldline available there, but never leaving the region. For the measurement process described above, we obtain the following results:

- Equation (3.10) in region 1.
- $+|a^{j-}\rangle |\Psi_A\rangle$ in region 2 due to measurement at $B$.
- $-|b^{i-}\rangle |\Psi_B\rangle$ in region 3 due to measurement at $A$, compare equation (3.11).
- $-|\Psi_A^j\rangle |\Psi_B^j\rangle$ in region 4, compare equation (3.12).

The major difference to the construction of this paper is that the assignment in the regions 2 and 3 is a formal one, defining the results as if both detectors $A$ and $B$ would have carried out their reductions. Serious consequences are implied. Let us introduce AA devices $AA_1$ and $AA_2$ as in the previous subsection, see figure 3. They create the state $+|a^{j-}\rangle |c_1^{j-}\rangle |\Psi_A\rangle$ in region 2 and $-|b^{i-}\rangle |c_2^{i-}\rangle |\Psi_B\rangle$ in region 3. This amounts to sending the copy state $|c_1^{j-}\rangle |c_2^{i-}\rangle$ to detector $C$, resulting in the conditional probability $P(c_1^{j-}, c_2^{i-}) = 1$ in case that detector $C$ measures the spin of particle $|c_1\rangle$ with respect to the $\hat{r}_j$ axis and the spin of particle $|c_2\rangle$ with respect to the $\hat{r}_i$ axis. This probability is in disagreement with QT (i.e. false).

In contrast, the reduction process of this paper stays correct by the simple fact that $C$ performs just another spacelike, local measurement (i.e commutes with $A$ and $B$). Straightforward but lengthy algebra allows to verify
explicitly that all reduction orders lead to the same result. For the remainder of this subsection, we follow our standard order $r_A < r_B < r_C$ and include AA devices.

At the positions indicated in figure 3 we imagine the AA devices $AA_1$ and $AA_2$ in figure 5. For notational convenience, we do no longer include detectors and AA devices explicitly. After passing $AA_1$ the state vector becomes then

$$|\psi(S_{1-})\rangle = 2^{-\frac{3}{2}} \{ |a^k\rangle |b^k\rangle |c_1^k\rangle - |a^k\rangle |b^k\rangle |c_1^k\rangle \},$$

(3.15)

When reduction by detector $A$ has produced the measurement result $i+$ we have

$$|\psi(S_{1+})\rangle = \langle a^i + |a^k\rangle |b^k\rangle |c_1^k\rangle - \langle a^i + |b^k\rangle |b^k\rangle |c_1^k\rangle +$$

(3.16)

and this happens with with probability (3.13). After passing $AA_2$ the state vector becomes

$$|\psi(S_{2-})\rangle = \langle a^i + |a^k\rangle |b^k\rangle |c_2^k\rangle - \langle a^i + |b^k\rangle |b^k\rangle |c_2^k\rangle +$$

(3.17)

Remarkable with this equation are two points: (1) The same result is obtained when passing $AA_2$ proceeds the reduction by detector $A$. (2) Due to the locality of their interaction, the AA devices are unable to copy (i.e. measure non-locally) the fully entangled state (3.10), which would be

$$\sim \{ |c_1^k\rangle |b^k\rangle - |c_1^k\rangle |b^k\rangle \}.$$

I.e. the expression would have to show up as a simple overall factor. Instead, the copying process has produced a new entanglement. The device has no ability to monitor non-locally the spin zero content of the initial state, as evidenced in $AA$. Still, this new entanglement is non-local enough to produce an inconsistency for the prescription proposed by Hellwig and Kraus. Namely, the reduction by $B$ for the measurement $j+$ yields

$$|\psi(S_{2+})\rangle = const \{ |a^i + |a^k\rangle |b^k\rangle |c_2^k\rangle - |a^i + |b^k\rangle |b^k\rangle |c_2^k\rangle +$$

(3.18)

where the constant is determined by the normalization condition $\langle \psi | \psi \rangle = 1$. The probability for this to happen is a bit lengthy and spared the reader, because it is not really of importance. Important is, once it happened the conditional probability that detector $C$ finds $j-$ for particle $|c_1\rangle$ and $i-$ for particle $|c_2\rangle$ is in general less than one, as follows from calculating $P(c_1^j, c_2^j) = |\langle c_1^j | c_2^j | \psi(S_{2+}) \rangle|^2$.

C. The Greenberger–Horne–Zeilinger state

A GHZ state is a state of three distinguishable spin $\frac{1}{2}$ particles which gained some popularity as a theoretical example for which local realistic theories and QT differ in 100% of the results for some measurements. Here our interest in it is limited to illustrating reduction for a case involving three detectors. For the following discussion we define $\hat{r}_1 = \hat{x}, \hat{r}_2 = \hat{y}$ and $\hat{r}_3 = \hat{z}$ and do not consider any other axis. Reduction for our standard order is depicted in figure 7. Including the detectors, one of the eight GHZ states reads $[7]$

$$|\Psi(S_{1-})\rangle = 2^{-\frac{3}{2}} \{ |a^{3+}\rangle |b^{3+}\rangle |c_1^{3+}\rangle - |a^{3-}\rangle |b^{3-}\rangle |c_1^{3-}\rangle \} |\Psi_A\rangle |\Psi_B\rangle |\Psi_C\rangle.$$

(3.19)

Assume that $A$ measures the $L_x$ spin component and that the result is $+\frac{1}{2}$. The corresponding reduction is

$$|\Psi(S_{1-})\rangle \rightarrow |\Psi(S_{1+})\rangle = 2^{-\frac{3}{2}} \{ |b^{3+}\rangle |c_3^{3+}\rangle - |b^{3-}\rangle |c_3^{3-}\rangle \} |\Psi_A^{1+}\rangle |\Psi_B\rangle |\Psi_C\rangle.$$

(3.20)

Next, let $B$ measures $L_y$ and the result is assumed to be $+\frac{1}{2}$ again. Then the reduction on $S_2$ of figure 7 is

$$|\Psi(S_{2-})\rangle \rightarrow |\Psi(S_{2+})\rangle = \{ |c^{3+}\rangle + i |c^{3-}\rangle \} |\Psi_A^{1+}\rangle |\Psi_B^{2+}\rangle |\Psi_C\rangle = 2^{-\frac{3}{2}} \{ |c^{3+}\rangle |\Psi_A^{1+}\rangle |\Psi_B^{2+}\rangle |\Psi_C\rangle.$$

(3.21)

Reductions (3.20) and (3.21) are both non-local. Note that the geometry of figure 7 is chosen such that the change affecting particle $|c\rangle$ happens in a region where $S_2$ falls, up to the proper definition of limits, on $S_1$. This is the reason why different line symbols are used to indicate the continuation of the $|b\rangle$ and $|c\rangle$ worldlines after reduction. The reduction for $|b\rangle$ is on $S_1$, whereas $|c\rangle$ undergoes two reductions: on $S_1$ and $S_2$ in a region where those surfaces fall on top of one another. The final reduction by detector $C$ is local and when detector $C$ measures $L_y$ the result is with certainty $+\frac{1}{2}$, i.e. a measurement without reduction.
IV. CONCLUSIONS

The central point of this paper is that it is possible to understand reduction in QFT as a fundamental physical process. Each reduction is performed locally by a detector, whereas the consequences can be global. Essential for such an interpretation of the reduction process is the existence of a consistent spacetime picture. This was shown by explicit construction of a PSV. In contrast to the non-relativistic limit the PSV can no longer be monitored, because the order in which detectors at spacelike separations perform their reductions cannot be verified experimentally. Therefore, the developed spacetime picture has not to be taken literally.

Potentially, our framework is a building block of a theory of reduction. It opens the door towards investigating questions like: Which conglomerates of matter have under which circumstances the ability to perform reductions (i.e. can act as detectors)? Are there frequency laws of reduction? These issues will be addressed in future work. Preliminary thoughts exist [9] and, actually, motivated the present investigation. The goal is to formulate reduction as a stochastic process, which goes on independently of whether measurements are performed or not, such that the measurement process becomes explained. Interestingly, there can then be experimental implications beyond standard QT which allow to verify such a stochastic process.

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Figure 4

Diagram showing points A, B, C, S_1, S_2, S_3, S_4, AA_1, and AA_2.
Figure 5

The diagram shows a trajectory in a parameter space with axes labeled as $t$ and $x$. Points $A$ and $B$ are connected by lines that intersect at point $S$. The trajectory is represented by the points $S_1$ and $S_2$ on the axes.
Figure 7

A

S_1

S_3

C

S

S_2

B

x

t