Linear quadratic regulator control with genetic algorithm convert applied to wind energy conversion

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Abstract. This document proposes a solution to the problem of physical sciences, such as control
systems, this happens since in the industry almost all systems with applications necessarily need
to guarantee stability and controllability. For this case we will implement a discrete linear
quadratic regulator controller for a back-to-back converter, with a genetic algorithm, for the
selection of the design matrices Q and R, for a doubly fed induction generator connected to a
wind power system. The optimization technique that is presented is an alternative to techniques
based on the reduction of errors through a measure, and the application problem is very current
due to renewable energies that are becoming increasingly important in the energy sector. The
description of the mathematical model of the converter in the state space is also relevant as a
spearhead for other types of application, such a technique is shown in detail.

1. Introduction
At present, power electronics intends to modify, through solid state devices, the way in which electrical
energy has been presented during the last century. Electronic converters allow the modification of
electrical energy for wide applications that are currently necessary due to the existing technology. one
among many advantages is its massification through the economy of scale which allows each day to be
more efficient improving the performance of the systems that use it, all this implies low costs and a
massive use that is amalgamated with high efficiency, high reliability, and low energy losses [1].

In this article, the back-to-back converter is implemented on wind turbines, which are in charge of
generating energy and subsequently delivering it to the grid, in which the main conversion structures
are: conversion with a double-fed induction generator (DFIG) [2], and conversion with a Full-Converter
synchronous machine.

The implementation of doubly fed generators has the advantage of the converter design which is
dimensioned for the rotor power and not for the entire system (turbine). and its disadvantage lies in the
use of slip rings, and its fragility to network disturbances.

2. Converter model
In Figure 1 shows the topology of the wind energy conversion system through the back-to-back
converter, this structure is one of the most used today.

The converter is made up of two elements, whatever the voltage levels, they are constituted in
insulated gate bipolar transistors (IGBT), the first is located next to the rotor and the second next to the
network, connected to each other through a direct current bus. Simply put, the rotor-side converter can
control the generator in terms of active and reactive power, while the grid-side converter can control the
voltage on the direct current (DC) bus and ensure power factor management during operation. performance [3].

Figure 2 represents a simplified model of the voltage-source converter (VSC) circuit, in which there are the inductors, the resistances, the DC side capacitors, two converters and inductance and resistors in series in the three lines that are connected to the alternating current systems. Inductance and capacitors are energy exchange elements and resistors are energy dissipative elements through heat, which translate into losses. Typically, \( R_1 = R_2 = R \) and \( L_1 = L_2 = L \) [4].

\[ \begin{align*}
\frac{d}{dt} \begin{bmatrix} i_{d1} \\ i_{q1} \end{bmatrix} &= \begin{bmatrix} -\frac{R}{L} & -\frac{\omega_1}{L} \\ -\frac{R}{L} & -\frac{\omega_1}{L} \end{bmatrix} \begin{bmatrix} i_{d1} \\ i_{q1} \end{bmatrix} - \begin{bmatrix} \frac{u_{dc}m_{d1} - E_{sd}}{L} \\ \frac{u_{dc}m_{q1}}{L} \end{bmatrix}, \end{align*} \]  

(1)
\[
\frac{d}{dt} [i_d, i_q] = \begin{bmatrix}
\frac{-R}{L} & \omega_2 \\
-\omega_2 & \frac{-R}{L}
\end{bmatrix} [i_d, i_q] - \begin{bmatrix}
\frac{V_{sd} - u_{dc}m_{d2}}{L} \\
-\frac{u_{dc}m_{q2}}{L}
\end{bmatrix},
\]

where \(\omega_1\) and \(\omega_2\) are the frequency of the phase voltage of system 1 and system 2, respectively; \(m_{d1}\) (\(m_{d2}\)) and \(m_{q1}\) (\(m_{q2}\)) are the components of the direct axis and the quadrature axis of the alternation ratio for the converter, which refers to the voltage from the direct current side to the peak of the phase to neutral voltage, in the terminals on the alternating current (AC) side of the converter; \(i_{d1}\) (\(i_{d2}\)) and \(i_{q1}\) (\(i_{q2}\)) are the components of the direct axis and the quadrature axis of the linear current vector, respectively; \(E_{sd}\) (\(V_{sd}\)) is the direct axis component of the line voltage vector; \(u_{dc}\) is the voltage of the intermediate circuit [5].

Through Equation (1) to Equation (6) that the back-to-back VSC is a coupled system of order 5 nonlinear. Where \(i_1, i_{q1}, i_{d2}, i_{q2}, u_{dc}\) are the state variables, \(m_{d1}, m_{q1}, m_{d2}, m_{q2}\) are the control variables.

Through the pivot directions of the currents in Figure 2. and Equation (1) to Equation (6) regardless of the loss of resistors, active power and reactive power can be delivered by VSC1 and delivered by VSC2. By Equation (3) to Equation (6).

\[
\begin{align*}
p_{c1} &= E_{sd}i_{d1} = u_{dc}i_{d1}, \\
q_{c1} &= -E_{sd}i_{d1}, \\
p_{c2} &= V_{sd}i_{d2} = u_{dc}i_{q2}, \\
q_{c2} &= -V_{sd}i_{d2},
\end{align*}
\]

For this work, the parameters used of the VSC back-to-back converter can be seen in Table 1.

| Table 1. Inerter parameters. |
|-----------------------------|
| Voltage source AC (L-L)     | 380 V          |
| Resistance \(R = R_1 = R_2\) | 0.04815 \(\Omega\) |
| Inductance \(L = L_1 = L_2\)  | 2.3 \(mH\)    |
| Capacitance DC               | 1,762 \(\mu F\) |
| Switching frequency          | 4 \(KHz\)     |

### 3. Optimal multivariable control

Figure 3 defines the general structure of a controller, where the control law is defined considering the accumulated error of the control signal [6]. With which the extended system will be defined by Equation (7) and Equation (8).

\[
[x(k + 1)] = [A \ 0] [x(k)] + [B \ 0] u(k) + [0 \ 1] r(k),
\]

Figure 3. Block diagram of the structure.
\[ y(k) = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x(k) \\ e(k) \end{bmatrix}, \]  
\[ \text{where the control signal is calculated as Equation (9),} \]
\[ u(k) = -[K \ K_1] \begin{bmatrix} x(k) \\ e(k) \end{bmatrix}, \]
\[ \text{where } K \text{ and } K_1 \text{ are constant matrices, which vary according to the control algorithm applied to the studied system.} \]
The way in which the discrete linear quadratic regulator (DLQR) problem is formulated in the case of discrete time is similar to the LQR problem of continuous time. Consider the time-invariant linear system described in Equation (10) and Equation (11).
\[ x(k+1) = Ax(k) + Bu(k), \]  
\[ y(k) = Cx(k) + Du(k), \]  
\[ \text{such that the vector } x(k) \text{ implies the variables to control [7]. The DLQR problem is to determine a control sequence } \{u^*(k)\}, k \geq 0, \text{ that minimizes the cost function Equation (12).} \]
\[ J(u) = \sum_{k=0}^{\infty} x^T(k)Qx(k) + u^T(k)Ru(k), \]  
\[ \text{such that the weighting matrices } Q \text{ and } R \text{ are positive definite. Where } (A, B, Q^{1/2} \text{ and } x^T) \text{ is accessible and observable; therefore, the solution to the DLQR problem is provided by the linear state feedback control law Equation (13).} \]
\[ u^*(k) = K^*x(k) = -[R + B^TP_c^*B]^{-1}B^TP_c^*Ax(k), \]  
\[ \text{such that } P_c^* \text{ unique, symmetric and the solution is positive defined by the Riccati algebraic equation (in discrete time), it is given by Equation (14).} \]
\[ P_c = A^T[P_c - P_cB[R + B^TP_c^*B]^{-1}B^TP_c]A + C^TQC. \]  
\[ \text{Likewise, in the continuous type, it is shown that the solution } P_c^* \text{ can be found through the eigenvectors of the Hamiltonian matrix, as seen in Equation (15).} \]
\[ H = \begin{bmatrix} A + BR^{-1}B^TA^TC^TQC & -BR^{-1}B^TA^T \\ -A^T C^TQC & A^T \end{bmatrix}. \]  
The linear state feedback control law can be extended for baseline tracking efficiency as follows Equation (16). The state control law can be expanded for monitoring effectiveness as follows.
\[ u(k) = -Kx(k) + K_1e(k), \]  
\[ K_1 \text{ steady-state gain matrix defined as Equation (13).} \]
The genetic algorithm is implemented as a heuristic optimization technique that allows efficiently finding through the cost function, which relates the restriction matrices Q and P which are penalized in the optimization problem according to the design conditions defined as [8,9] Equation (18).

\[ P_l = J(u) = \sum_{k=0}^{K_{\text{final}}} [x^T(k)Qx(k) + u^T(k)Ru(k)], \]  

(18)

where \( K_{\text{final}} \) is the total number of samples. The selection of \( Q \in \mathbb{R}^{n \times n} \), \( R \in \mathbb{R}^{p \times p} \) (number of states and \( p \) number of inputs) it is a difficult job as it involves specific conditions [10-12], Equation (19).

\[
Q = \begin{bmatrix}
q_{11} & q_{12} & \cdots & q_{1n} \\
q_{21} & q_{22} & \cdots & q_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
q_{n1} & q_{n2} & \cdots & q_{nn}
\end{bmatrix}, \quad R = \begin{bmatrix}
r_{11} & r_{12} & \cdots & r_{1p} \\
r_{21} & r_{22} & \cdots & r_{2p} \\
\vdots & \vdots & \ddots & \vdots \\
r_{p1} & r_{p2} & \cdots & r_{pp}
\end{bmatrix}
\]  

(19)

4. Results

For the model presented in section 2 and described in Equation (1) to Equation (6), the outputs are the currents in the \( q \) axis and the \( d \) axis of VSC1 and VSC2, that is, there is a system of four inputs and four outputs. Figure 4(a) and Figure 4(b) show the behavior of the open-loop system.

A DLQR controller was designed for the multivariable system to find a quick response. The selection of the Q and R matrices is carried out by implementing the algorithm described by the heuristic technique based on the genetic theory called genetic algorithms with an initial population of 50 individuals. Figure 5 and Figure 6 show the responses of the currents when applying the DLQR controller.

![Figure 4](image)

**Figure 4.** Behavior of the open-loop system. (a) outputs; (b) inputs.
5. Conclusions

In this work, a study of the back-to-back a converter was presented for the manipulation of the flow of both active and reactive power in the doubly connected induction generating machine to a wind turbine, excellent results were obtained when applying the discrete linear quadratic regulator controller using genetic algorithm to determine the matrices of Q and R. The control signals do not have variations or peaks as they are presented with some other classical control techniques such as proportional–integral–derivative controller.

The implementation of the optimal controllers in multivariable systems, provide an adequate selection of the controller parameters, starting from an objective function established for specific design conditions.

References
[1] Blaabjerg F, Liserre M, Ma K 2012 Power electronics converters for wind turbine systems *IEEE Transactions on Industry Applications* **48**(2) 708
[2] Poitiers F, Machmoum M, Le Doeuff R, Zaim M E 2001 Control of a doubly fed induction generator for wind energy conversion systems *International Journal of Renewable Energy Engineering* **3** 373
[3] Evangelista C, Valenciaga F, Puleston P 2012 Multivariable 2-sliding mode control for a wind energy system based on a double fed induction generator *International Journal of Hydrogen Energy* **37**(13) 10070
[4] Gan-gui Y, et al. 2008 Dynamic modeling and nonlinear decoupled control of back-to-back voltage source converter *International Conference on Electrical Machines and Systems* (China: Institute of Electrical and Electronics Engineers)
[5] Zhang K, Li G, Lian H 2005 Steady-state control strategy and simulation of VSC-HVDC *Electric Power Automation Equipment* **2005**(3) 79
[6] McKelvy T 2009 Least squares and instrumental variable methods *Control Systems, Robotics and Automation* (Sweden: Encyclopedia of Life Support Systems Publishers Company Limited) pp 206-229
[7] Antsaklis P, Michel A 2006 *Linear Systems* (Boston: Birkhäuser)
[8] Shen P 2011 LQR Control of double inverted-pendulum based on genetic algorithm *World Congress on Intelligent Control and Automation* (Taiwan: Institute of Electrical and Electronics Engineers)
[9] Michalewicz Z, Janikow C, Krawczyk J 1992 A modified genetic algorithm for optimal control problems *Computers and Mathematics with Applications* **23**(12) 83
[10] Montagner V, Maccari L, Dupont F, Pinheiro H 2011 A DLQR designed by means of a genetic algorithm for DC-DC boost converters *XI Brazilian Power Electronics Conference* (Brazil: Institute of Electrical and Electronics Engineers)
[11] Gessing R 2001 Discrete-time linear-quadratic output regulator for multivariable systems *IFAC Proceedings Volumes* **34**(8) 551
[12] Khajeh A, Ghazi R 2013 GA-based optimal LQR controller to improve LVRT capability of DFIG wind turbines *Iranian Journal of Electrical and Electronic Engineering* **9**(3) 167