I. INTRODUCTION

Interest in traversable wormholes, initiated by Morris and Thorne \cite{1}, has been greatly renewed in part because of the discovery that our Universe is undergoing an accelerated expansion \cite{2,3}. In other words, \( \dot{a} > 0 \) in the Friedmann equation \( \dot{a}/a = -\frac{4\pi}{3}(\rho + 3p) \), using units in which \( G = c = 1 \). The cause of this acceleration is taken to be a negative pressure \textit{dark energy} with equation of state \( p = -K\rho \), \( K > \frac{1}{3} \), and \( \rho > 0 \), where \( K \) is a constant, \( \rho \) the spatially homogeneous pressure, and \( p \) the energy density. Of particular interest is the case \( K > 1 \), referred to as \textit{phantom energy}, since it leads to a violation of the null energy condition, an essential requirement for maintaining a wormhole \cite{1}. Matter violating the null energy condition is usually called \textit{exotic}. Since the notion of dark or phantom energy ordinarily applies only to a homogeneous distribution of matter, phantom energy is not automatically a candidate for exotic matter. Fortunately, the extension to spherically symmetric homogeneous spacetimes has been carried out \cite{4}.

An alternative model is based on Chaplygin gas, whose equation of state is given by \( p = -\frac{A}{\rho} \). Another possibility is generalized Chaplygin gas (GCG), whose equation of state is \( p = -\frac{A}{\rho^\alpha} \), \( 0 < \alpha \leq 1 \) \cite{5,6}. Cosmologists became interested in this form of matter when it turned out to be a candidate for unifying dark matter and dark energy. To see this, consider the energy conservation equation \( \dot{\rho} = -3\dot{a}(\rho + p)/a \) in a flat FRW spacetime and substitute the equation of state \( p = -\frac{A}{\rho^\alpha} \). The result is

\[
\rho = \left( A + \frac{B}{\rho^{3(1+\alpha)}} \right)^{1/(1+\alpha)},
\]

where \( B \) is a constant of integration. It is now seen that \( \rho \sim a^{-3} \) at early times, that is, \( \rho \) behaves like matter, while in later times like a cosmological constant (\( \rho = \text{constant} \)). Previous models required two distinct fields, one to describe dark matter and the other dark energy, but one can argue, as in Ref. \cite{7}, that these ought to be different manifestations of the same entity. One possible motivation for this model came from the field theory points of view \cite{8}. Another interesting possibility is discussed in Ref. \cite{9}: starting with the Nambu-Goto action of string theory, the Chaplygin gas appears after considering \( d \)-branes in a \( (d + 2) \)-dimensional spacetime. Another attraction is that it admits a supersymmetric extension.

Observationally, the GCG model has not been without its problems. As noted in Ref. \cite{10}, while the model has successfully withstood various phenomenological tests over several years, there is some concern that it produces unphysical oscillations in the matter power spectrum. It is shown in Ref. \cite{10}, however, that these problems can be circumvented. Furthermore, very recent studies \cite{11,12} have concluded that the earlier criticisms were based on the oversimplifying assumption of an adiabatic cosmic medium.

On the other hand, according to another recent study (Ref. \cite{13}), the special case \( \alpha = 1 \), corresponding to the original Chaplygin gas, may very well have to be excluded. One should therefore concentrate on the GCG case by keeping \( \alpha < 1 \).

In a recent paper \cite{14} the author made a systematic study of exact solutions of wormhole spacetimes supported by phantom energy by starting with the general line element and equation of state. It is shown that there are only two ways to insert the redshift function “by hand.” Doing so leads to the exact solutions in Refs. \cite{15} and \cite{16}. Assigning a specific function to \( \rho \) leads to the exact solution in Ref. \cite{17}. Additional exact solutions that simultaneously avoid an event horizon are extremely rare. Nevertheless two new solutions were found. Included in the discussion are the junction conditions for matching each solution to an exterior Schwarzschild solution, as well as traversability criteria.

Ref. \cite{14} makes use of specific redshift and shape functions in the manner of the original Morris and Thorne paper. In this paper we continue in the spirit of Ref. \cite{14},...
that is, we start with the general line element and the Einstein tensor, together with the equation of state, and continue the analysis while avoiding, as much as possible, assigning specific functions to the metric coefficients.

II. THE PROBLEM

Consider the general line element for describing a wormhole:

\[ ds^2 = -e^{2\Phi(r)} dt^2 + e^{2\Lambda(r)} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \]  

(2)

In this context \( \Phi(r) \) is called the redshift function; for this function, \( e^{2\Phi(r)} \) must never vanish to avoid an event horizon. \( \Lambda(r) \) is related to the shape function \( b(r) = r(1 - e^{-2\Lambda(r)}) \), i.e., \( e^{2\Lambda(r)} = 1/[1 - b(r)/r] \). The shape function determines the spatial shape of the wormhole as viewed, for example, in an embedding diagram. By the very definition of wormhole, if the throat is at \( r = r_0 \), then \( b(r_0) = r_0 \). As a consequence, \( \Lambda(r) \) has a vertical asymptote at \( r = r_0 \); \( \lim_{r \to r_0^-} \Lambda(r) = +\infty \). To obtain a traversable wormhole, the shape function must obey the usual flare-out conditions at the throat \[ b'(r_0) < 1 \] and \( b(r) < r \); also required is asymptotic flatness, i.e., \( b(r)/r \to 0 \) as \( r \to \infty \).

The next step is to list the components of the Einstein tensor in the orthonormal frame \[ G_{\hat{t}\hat{t}} = \frac{2}{r} e^{-2\Lambda(r)} \Lambda'(r) + \frac{1}{r^2} \left( 1 - e^{-2\Lambda(r)} \right), \]  

(3)

\[ G_{\hat{r}\hat{r}} = \frac{2}{r} e^{-2\Lambda(r)} \Phi'(r) - \frac{1}{r^2} \left( 1 - e^{-2\Lambda(r)} \right), \]  

(4)

\[ G_{\hat{\theta}\hat{\theta}} = G_{\hat{\phi}\hat{\phi}} = e^{-2\Lambda(r)} \left( \Phi''(r) - \Phi'(r)\Lambda'(r) + [\Phi'(r)]^2 \right) + \frac{1}{r} \Phi'(r) - \frac{1}{r} \Lambda'(r). \]  

(5)

Since the Einstein field equations \( G_{\alpha\beta} = 8\pi T_{\alpha\beta} \) imply that the stress-energy tensor \( T_{\alpha\beta} \) is proportional to the Einstein tensor, the only nonzero components are \( T_{\hat{t}\hat{t}} = \rho(r) \), \( T_{\hat{r}\hat{r}} = p(r) \), and \( T_{\hat{\theta}\hat{\phi}} = T_{\hat{\phi}\hat{\theta}} = p_t(r) \), the transverse pressure. From the Einstein field equations and the equation of state \( p = -\frac{\rho}{\alpha} \), we obtain \( G_{\hat{t}\hat{t}} = 8\pi \rho \) and \( G_{\hat{r}\hat{r}} = 8\pi (\alpha \rho - \alpha) \), yielding the following equation:

\[ \frac{1}{8\pi} \left[ \frac{2}{r} e^{-2\Lambda(r)} \Phi'(r) - \frac{1}{r^2} \left( 1 - e^{-2\Lambda(r)} \right) \right] A = - \left\{ \frac{1}{8\pi} \left[ \frac{2}{r} e^{-2\Lambda(r)} \Lambda'(r) + \frac{1}{r^2} \left( 1 - e^{-2\Lambda(r)} \right) \right] \right\}^\alpha. \]  

(6)

Unlike the phantom energy case \( p = -K\rho \), the equation of state has the form of a quotient. The resulting differential equation (6) does not have an obvious exact solution, so that the analysis depends on numerical/graphical techniques.

Recall next the null energy condition, which requires the stress-energy tensor \( T_{\alpha\beta} \) to obey \( T_{\alpha\beta} \epsilon^\alpha \mu^\beta \geq 0 \) for all null vectors. In our orthonormal frame, for \( \left( p^\hat{t}, p^\hat{r} \right) = \left( 1, 0, 0 \right) \), a radial outgoing null vector, the condition becomes \( T_{\hat{t}\hat{t}} + T_{\hat{r}\hat{r}} = \rho + p \geq 0 \). Wormholes must necessarily violate this condition at the throat \[ 1 \]. Since \( \lim_{r \to r_0^+} \Lambda(r) = +\infty \), Eq. (6) now implies that

\[ \frac{1}{8\pi} \left( \frac{1}{r_0^2} - \frac{1}{r_0^2} \right) + \frac{A}{\left( \frac{1}{r_0^2} \right) \left( \frac{1}{r_0^2} \right)} = 0. \]

Solving for \( A \), we have \( A = 1/(8\pi r_0^2)^{\alpha + 1} \). We will see below that we actually need

\[ A < \frac{1}{(8\pi r_0^2)^{\alpha + 1}}. \]  

(7)

Using the above equation of state,

\[ p = -\frac{A}{\rho^2}, \quad 0 < \alpha \leq 1, \]

(8)

in conjunction with \( \rho + p < 0 \), yields the constraint

\[ \rho < A^{1/(\alpha + 1)}, \]

(9)

which, in turn, implies that \( B < 0 \) as \( B \) is discussed in Eq. [10]. The constraints (7) and (9) are also discussed in Ref. [10].

III. THEORETICAL ANALYSIS

As already noted, Eq. (6) does not lend itself to finding a simple exact solution, so that some numerical techniques will be needed. That is the topic of Sec. [11]. In this section we determine some general characteristics of GCG wormholes. The analysis is based on the assumption that a solution \( \Lambda(r) \) of Eq. (6) exists for a wide range of choices for \( \Phi'(r) \) and such that \( \lim_{r \to r_0^+} \Lambda(r) = +\infty \). We also assume that \( \Phi'(r) \) is continuous.

Suppose we rewrite Eq. (6) as follows:

\[ \frac{(8\pi)^{1/\alpha} r^2 (r^2)^{1/\alpha} A^{1/\alpha}}{1 - e^{-2\Lambda(r)}} \left[ 1 - e^{-2\Lambda(r)} - 2r e^{-2\Lambda(r)} \Phi'(r) \right]^{1/\alpha} = \frac{1}{r} + \frac{2e^{-2\Lambda(r)} \Lambda'(r)}{1 - e^{-2\Lambda(r)}}. \]  

(10)

Now define the dimensionless function

\[ F(r) = \int_{r_0}^{r} \frac{(8\pi)^{1/\alpha} r^2 (r')^2 (r')^{1/\alpha} A^{1/\alpha} dr'}{\left[ 1 - e^{-2\Lambda(r')} - 2r' e^{-2\Lambda(r')} \Phi'(r') \right]^{1/\alpha}}. \]

(11)

Observe that since \( e^{-2\Lambda(r)} \to 0 \) as \( r \to r_0^+ \), the integrand is sectionally continuous. This implies that \( F(r) \)
is defined and continuous on any interval \((r_0, r]\). Moreover, \(F(r_0) = 0\).

Returning to Eq. (10), integration yields

\[
F(r) = \ln \left(1 - e^{-2\Lambda(r)}\right) + ln r + ln c.
\]

So

\[
e^{F(r)} = cr \left(1 - e^{-2\Lambda(r)}\right).
\]

Once again, \(\lim_{r \to r_0^+} \Lambda(r) = +\infty\) and \(F(r_0) = 0\). So by Eq. (12), \(c = 1/r_0\) and

\[
e^{2\Lambda(r)} = \frac{1}{1 - \frac{r}{r_0} e^{F(r)}}.
\]

The line element then takes on the form

\[
ds^2 = -e^{2\Phi(r)} dt^2 + \frac{dr^2}{1 - \frac{r}{r_0} e^{F(r)}} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2).
\]

The next step is to show that the flare-out conditions are satisfied at the throat. Given that \(b(r) = r \left(1 - e^{-2\Lambda(r)}\right)\), it follows that \(b(r) < r\) for \(r > r_0\). Also, Eq. (13) implies that

\[
b(r) = r_0 e^{F(r)}.
\]

By Eq. (11),

\[
b'(r) = r_0 e^{F(r)} \times \frac{(8\pi)^{-1 + 1/\alpha} r^{2 - 1/\alpha} \Lambda^{1/\alpha}}{\left(1 - e^{-2\Lambda(r)}\right) \left[1 - e^{-2\Lambda(r)} - 2r e^{-2\Lambda(r)} \Phi'(r)\right]^{1/\alpha}}.
\]

If we assume that

\[
A < \frac{1}{(8\pi r_0^2)^{\alpha + 1}},
\]

then

\[
\Lambda^{1/\alpha} < \frac{1}{(8\pi)^{1 + 1/\alpha} (r_0^2)^{1 + 1/\alpha}}
\]

whence \(b'(r_0) < 1\).

In summary, the analysis in this section depends primarily on the asymptotic behavior of \(\Lambda(r)\) at the throat. It has not been established that the existence of \(\Lambda(r)\) yields a traversable wormhole. In addition, its dependence on \(\Phi(r)\) has to be closely examined, just as it is for phantom energy supported wormholes [14]. These ideas are discussed in the next section.

IV. NUMERICAL ANALYSIS

In this section we turn to the solution of Eq. (10), rewritten as follows:

\[
\Lambda'(r) = \frac{\frac{1}{2} (8\pi)^{1 + 1/\alpha} r^{2 - 1/\alpha} A^{1/\alpha} e^{2\Lambda(r)}}{\left[1 - e^{-2\Lambda(r)} - 2r e^{-2\Lambda(r)} \Phi'(r)\right]^{1/\alpha}} - \frac{1}{2r} \left(e^{2\Lambda(r)} - 1\right).
\]

Here \(A\) is taken to be less than \(1/(8\pi r_0^2)^{\alpha + 1}\) and \(\alpha\) to be close to unity. (In fact, for convenience we let \(\alpha = 1\), at least for now.) In order to generate numerical/graphical output, it is necessary to choose some specific value for the size of the throat (such as \(r_0 = 2\)) and to pick an initial value [such as \((2.000001, 5)\)] to simulate the asymptotic behavior. As one might expect, qualitatively, the form of \(\Lambda(r)\) remains roughly the same for a large range of \(\Phi\)'s.

A. Regression and traversability

In this section we turn to the question of traversability by humanoid travelers. One concern is the calculation of the proper distance from the throat of the wormhole to a point outside:

\[
\ell(r) = \int_{r_0}^{r} e^{\Lambda(r')} dr'.
\]

For this calculation a specific \(\Lambda(r)\) would be desirable. Regression equations from the numerical output are easy to obtain but tend to produce a poor fit—with one notable exception: \(\Lambda(r) = a + b \ln(r - r_0)\). This form yields an excellent fit for any \(\Phi(r)\) with small \(|\Phi'(r)|\) and an almost perfect fit for \(\Phi'(r) \equiv 0\). (\(\Phi'(r)\) can be either positive or negative; as \(\Phi'(r)\) increases, \(\Lambda(r) = a + b \ln(r - r_0)\) yields an ever poorer fit.) Some of the regression equations using \(r_0 = 2\) with the corresponding \(\Phi'\) are given next:

\[
\Phi'(r) \equiv 0:\quad \Lambda(r) = 1.20 - 0.23 \ln(r - 2)
\]

\[
\Phi'(r) = \frac{10}{r} : \quad \Lambda(r) = 1.25 - 0.22 \ln(r - 2)
\]

\[
\Phi'(r) = 10 e^{-r} : \quad \Lambda(r) = 1.32 - 0.20 \ln(r - 2)
\]

\[
\Phi'(r) = \frac{10}{r^2} : \quad \Lambda(r) = 1.45 - 0.18 \ln(r - 2)
\]

\[
\Phi'(r) = \frac{10}{r^{1.5}} : \quad \Lambda(r) = 1.57 - 0.16 \ln(r - 2)
\]

Observe that each \(\Lambda(r)\) has a vertical asymptote at \(r = 2\) and that \(\Lambda'(r) < 0\) for \(r > 2\). The numerical solutions were obtained only in the vicinity of the throat. The reason for this is discussed in Subsec. C.

Remark 1. We may assume that the equations are fairly typical for GCG wormholes since there is nothing
special about the choice of $r_0$ or the initial conditions. Also implied is that the wormholes are macroscopic.

From the standpoint of traversability, a relatively small $|\Phi'(r)|$ results in relatively small tidal forces [1]. In fact, for $\Phi'(r) \equiv 0$, the radial tidal force is zero.

The proper distance from the throat $r = r_0$ to a point outside, say $r = kr_0$ for some $k > 0$, is also going to be relatively small thanks to the resulting ln-form. As an example, for $\Lambda(r) = 1.20 - 0.23 \ln(r - r_0)$, we have from Eq. (15)

$$
\int_{r_0}^{kr_0} e^{1.20 - 0.23 \ln(r - r_0)} dr = e^{1.20} \int_{r_0}^{kr_0} (r - r_0)^{-0.23} dr = e^{1.20} r_0^{0.77} (k - 1)^{0.77} \frac{0.77}{77},
$$

(16)
corresponding to the coordinate distance $[r_0, kr_0]$.

We conclude that a wormhole with small $|\Phi'(r)|$ in the vicinity of the throat is likely to be traversable in the sense of having low tidal forces and short proper distances resulting from the ln-form. A wormhole is therefore traversable in this sense if, and only if, the numerical output for $\Lambda(r)$ closely fits the form $\Lambda(r) = a + b \ln(r - r_0)$.

Remark 2. Although of primary importance, $|\Phi'(r)|$ is not the only concern involving the redshift function. Returning to line element (2), even if $\lim_{r \to +\infty} \Phi(r) = 0$, a large $|\Phi'(r)|$ near the throat is also undesirable: depending on the sign of $\Phi(r)$, clocks fixed at $r = r_0$ will either run much faster or much slower than clocks outside the wormhole. So our definition of traversability does not include a study of proper traversal times.

Suppose we consider the expansion

$$
\ln(r - r_0) = \ln r - \frac{r_0^2}{2r^2} - \frac{r_0^3}{3r^3} - \cdots
$$

(17)
obtained by expanding $(d/dr)\ln(r - r_0) = 1/(r - r_0)$ in a geometric series and then integrating. Returning to Eq. (16),

$$
\ell(r) = \int_{r_0}^{kr_0} e^{1.20} e^{0.23[\ln r - \sum_{i} \frac{1}{i} \Phi(\frac{r}{r_0})^i]} dr,
$$

(18)
which is equal to the value of $\ell(r)$ in Eq. (16). To measure the deviation from this form, let us call $\Lambda(r)$ “log-like” if its expansion is

$$
\ln r - \frac{r_0}{r} - \frac{1}{2} \frac{r_0^2}{r^2} - \frac{1}{3} \frac{r_0^3}{r^3} - \cdots.
$$

(The condition $p < 1$ makes the function “less favorable” than the ln-function.) The series itself converges for all $r > r_0$ by the ratio test. The proper distance now becomes

$$
\ell(r) = e^{a} \int_{r_0}^{kr_0} e^{b[\ln r - \sum_{i} \frac{1}{i} \Phi(\frac{r}{r_0})^i]} dr.
$$

(19)

To study the behavior of $\ell(r)$ as a function of $p$, we let $r_0 = 4$, $k = 2$, $a = 1$, and $b = -0.20$. To show how $\ell(r)$ drifts away from the original values (corresponding to $p = 1$), $1/\ell(r)$ is plotted against $p$, as shown in Fig. 1. The graph resembles the right half of a normal curve. So

**B. Other solutions**

One of the main conclusions in Ref. [14] is the extreme difficulty in obtaining exactly solvable wormhole models without getting an unwanted event horizon. While the question of exact solutions does not arise in this study, based on Eq. (13) and numerical trials, choosing an “arbitrary” $\Phi'(r)$ would not ordinarily yield an event horizon. The choices for $\Phi$ are therefore enormous.

Another fortunate circumstance is the occurrence of $\Lambda(r) = a + b \ln(r - r_0)$ as a best-fitting curve. This function causes $e^{\Lambda(r)}$ to go to infinity relatively slowly, producing a small proper distance, as exemplified by Eq. (17). So for small $|\Phi'(r)|$ we are dealing with a situation that can hardly be improved. As $|\Phi'(r)|$ increases, however, the ln-functions used so far produce an ever poorer fit. The question is whether this deviation from the ln-form can be quantified. A related question might be: given the abundance of solutions and the resulting possibility of naturally occurring wormholes, can one estimate the likelihood of finding one that is actually traversable by humanoid travelers?

Suppose we consider the expansion

$$
\ln(r - r_0) = \ln r - \frac{r_0^2}{2r^2} - \frac{r_0^3}{3r^3} - \cdots
$$

(17)
obtained by expanding $(d/dr)\ln(r - r_0) = 1/(r - r_0)$ in a geometric series and then integrating. Returning to Eq. (16),

$$
\ell(r) = \int_{r_0}^{kr_0} e^{1.20} e^{0.23[\ln r - \sum_{i} \frac{1}{i} \Phi(\frac{r}{r_0})^i]} dr,
$$

(18)
which is equal to the value of $\ell(r)$ in Eq. (16). To measure the deviation from this form, let us call $\Lambda(r)$ “log-like” if its expansion is

$$
\ln r - \frac{r_0}{r} - \frac{1}{2} \frac{r_0^2}{r^2} - \frac{1}{3} \frac{r_0^3}{r^3} - \cdots.
$$

(The condition $p < 1$ makes the function “less favorable” than the ln-function.) The series itself converges for all $r > r_0$ by the ratio test. The proper distance now becomes

$$
\ell(r) = e^{a} \int_{r_0}^{kr_0} e^{b[\ln r - \sum_{i} \frac{1}{i} \Phi(\frac{r}{r_0})^i]} dr.
$$

(19)

To study the behavior of $\ell(r)$ as a function of $p$, we let $r_0 = 4$, $k = 2$, $a = 1$, and $b = -0.20$. To show how $\ell(r)$ drifts away from the original values (corresponding to $p = 1$), $1/\ell(r)$ is plotted against $p$, as shown in Fig. 1. The graph resembles the right half of a normal curve. So

**FIG. 1: A measure of traversability.**

if we regard $1/\ell(r)$ as a measure of traversability, then, according to Fig. 1, the likelihood of encountering a naturally occurring GCG wormhole that is also traversable
may be higher than one should expect, perhaps even as high as 25% (corresponding roughly to $1 \geq p \geq 0.6$).

This outcome is fairly typical: increasing the throat size $r_0$ has little effect on the shape of the graph, while the values of $k$ and $\alpha$ have no effect at all. As $b$ decreases toward $-0.5$, the graph falls off more rapidly, but the percentage of traversable wormholes appears to remain between 10% and 20%. Fortunately, even the worst-fitting cases have led to values between $-0.35$ and 0.

So far we have assumed that $\alpha$ in the equation of state $p = -A/\rho^\alpha$ is close to unity, thereby producing the best-fitting curves. As $\alpha$ decreases, one sees the same kind of falling off in the measure of traversability that appears in Fig. 1. So the best chance for obtaining a traversable wormhole is a GCG that is close to the original Chaplygin gas. However, this falling-off behavior can be partially compensated for by decreasing the constant $A$ in the equation of state. This can also be seen from Eq. (13): an increasing $1/\alpha$ is compensated for by decreasing $A$. So a traversable wormhole could in principle exist for small $\alpha$, provided that the size of $A$ can be controlled (for example, by an advanced civilization). In that case, however, the inequality $\rho < A^{1/(\alpha+1)}$, becomes a constraint on the wormhole material.

C. Junction to an external spacetime

Observe that any curve of the form $\Lambda(r) = a + b \ln(r - r_0)$ with $a > 0$ and $b < 0$ will eventually cross the horizontal axis. So our spacetime is not asymptotically flat, the same problem that occurs in Refs. [14] and [19]. We would like to join $\Lambda(r)$ smoothly to a positive curve that goes to zero, as in Ref. [18].

Assume that the extended curve has the form $g(r) = K/r^n$, starting at some $r = r_1$. We require that $g(r_1) = \Lambda(r_1)$ and $g'(r_1) = \Lambda'(r_1)$. Eliminating $K$ and $n$, we obtain

$$g(r) = \Lambda(r_1) \left( r/r_1 \right)^{n / (n - 1)}.$$

Since we are concerned mainly with the region around the throat, let us opt for an early cut-off, say at $r = 2.2$. (Besides, according to Ref. [19], $r$ lies in a fairly narrow range, which is also apparent from the numerical/graphical output.) As an example, for the case $\Phi' = 0$, we have $\Lambda(r) = 1.20 - 0.23\ln(r - 2)$. Then for $r = 2.2$,

$$g(r) = 1.57 \left( r/2.2 \right)^{-1.61}.$$

Joining $\Lambda(r)$ to a positive curve going to zero makes the spacetime asymptotically flat.

V. COMPARISON TO LOBO’S SOLUTION

The case $\Phi' = 0$ allows a comparison to Lobo’s exact solution [19]. From $\Lambda(r) = 1.20 - 0.23\ln(r - 2)$, we calculate

$$b_1(r) = r \left[ 1 - e^{-2.4(r - 2)^{0.46}} \right].$$

Lobo obtains

$$b_2(r) = r_0 \left[ \frac{64 A \pi^2}{3} r_0^2 (r^6 - r_0^6) + 1 \right]^{1/2}.$$

On the above interval $[2, 2.2]$, using the same $A$ and $r_0$, $b_1(r) = b_2(r)$ after rounding off to two significant figures.

VI. SUMMARY

This paper extends the general analysis in Ref. [14] to wormholes supported by generalized Chaplygin gas (GCG). The function $\Lambda(r)$ in line element (2) is obtained numerically from the Einstein field equations and the equation of state. The conclusions are:

1. The wormhole spacetime meets the flare-out conditions at the throat.

2. Qualitatively, $\Lambda(r)$ remains the same for a wide range of redshift functions $\Phi(r)$. Compared to the phantom energy supported wormholes in Ref. [14], event horizons are much less likely to occur.

3. Redshift functions with relatively small $|\Phi'(r)|$ yield regression equations closely fitting the form $\Lambda(r) = a + b \ln(r - r_0)$.

4. Item 3 implies the existence of macroscopic wormholes with low tidal forces and relatively short proper distances near the throat, making the wormholes traversable by humanoid travelers. (This criterion does not include proper traversal times.)

5. The junction to an external solution produces a spacetime that is asymptotically flat.

6. It is assumed above that $\alpha$ is close to unity in the equation of state $p = -A/\rho^\alpha$. A smaller $\alpha$ produces less favorable results but can be compensated for by a smaller $A$. However, a smaller $A$ would tighten the constraint $\rho < A^{1/(\alpha+1)}$ on the wormhole.

7. The abundance of solutions suggests that GCG wormholes may occur naturally. A rough measure of traversability implies that the chances of finding one that is traversable by humanoid travelers may be fairly good.
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