A correspondence between standard model fermions and degrees of freedom of polycrystalline materials

Ilja Schmelzer

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Abstract

We identify natural degrees of freedom of polycrystalline materials – affine transformations of grains – with those of a three-dimensional lattice theory for \((T \otimes \Omega)(\mathbb{R}^3)\). We define a lattice Dirac operator on this space and identify its continuous limit with the free field limit of the whole fermionic sector of the standard model. Fermion doubling is used here as a tool to obtain the necessary number of steps of freedom. The correspondence extends to important structural properties (families, colors, flavor pairs, electromagnetic charge). We find a lattice version of chiral symmetry similar to the Ginsparg-Wilson approach.

This correspondence suggests to propose a “polycrystalline ether”. Combined with GLET, a general Lorentz ether theory of gravity with GR limit, this becomes a concept for a theory of everything. The extension to gauge fields is the major open problem and requires new concepts.

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1 Introduction

The main idea of this paper may be formulated like the prototype of crank physics: As a proposal of a *the theory of everything* which may described in a single sentence: *We have absolute time and an Euclidean space, which is filled with a polycrystalline ether.* To add further evidence in this direction, the author almost fits into the “engineer” pattern typical for cranks: He has no PhD and works in the domain of mesh generation for scientific computing.

For those who nonetheless have not yet stopped reading, let’s start. The key of this paper is a lattice version of the Dirac operator on \((T \otimes \Omega)(\mathbb{R}^3)\). Here \(\Omega(M^n)\) denotes the exterior bundle (de Rham complex) of a manifold \(M^n\). For a given metric \(g_{\mu\nu}\) on \(M^n\) Hodge theory (see app. [A.1]) defines not only a Dirichlet operator \(\Delta\) on \(\Omega(M^n)\), but also a natural square root \(D, D^2 = \Delta\) called “Dirac operator”. There exists also a square root for the operator \(\Box = \partial^2_t - \Delta\) which has the form \(\hat{D} = \gamma^0\partial_t - D\). We define also a complex structure on \((T \otimes \Omega)(\mathbb{R}^3)\) which corresponds to a quaternionic structure on \(\Omega(\mathbb{R}^3)\).

We consider a simple lattice discretization of this three-dimensional Hodge theory Dirac operator for Euclidean metric, leaving time continuous. The key observation is the identification of the large distance limit of this lattice equation with the free field limit of the whole fermionic sector of the standard model. The effect which allows to reach this identification is known as “fermion doubling” in lattice theory. If we try a naive (central differences) discretization of the Dirac equation and then consider the large distance limit we observe that we obtain not only the fermion which we have tried to put on the lattice, but also highly oscillating solutions which fulfill the same equation – so called doublers or spurious modes. This doubling effect happens in each direction. Once we discretize only in spatial direction, we obtain a factor \(2^3 = 8\). Now, the space \((T \otimes \Omega)(\mathbb{R}^3)\) has dimension 24. Together with the doubling factor 8 this gives exactly what we need to model the 24 standard model fermions, with their 8 real (4 complex) degrees of freedom for each fermion.

The identification does not reduce to 192 = 192, but gives all the structural properties of the standard model we would like to have: A representation of the standard Dirac matrices \(\gamma^\mu\) which allows to identify fermions, three fermion families, flavor pairs, three colors for quarks. We have also a natural candidate for electromagnetic charge in terms of the number of oscillating directions. Moreover, we have found a version of chiral symmetry on the lattice so that the lattice Dirac operator has chiral symmetry. It is defined by a pair of lattice operators \(\gamma^5, \tilde{\gamma}^5\) so that chiral symmetry for an operator \(O\) means \(\gamma^5O + O\tilde{\gamma}^5 = 0\), \(\tilde{\gamma}^5O + O\gamma^5 = 0\). Such a notion of chiral
symmetry on the lattice is similar (but not identical) to the generalization of chiral symmetry used in the Ginsparg-Wilson approach to chiral gauge theory.

On the other hand, we can identify on the kinematical level our discretization of \((T \otimes \Omega)(\mathbb{R}^3)\) with the degrees of freedom of a classical polycrystalline material. All what we need for this identification is the granular structure, which defines a spatial lattice, and the idea that the deformation of the granular structure may be described by an affine transformation. \(q_\mu^i(n) = (q_0^i(n), q_j^i(n)) \in A(3)\) for each grain \(n\). Together with the related momentum variables \(p_\mu^i(n) = (p_0^i(n), p_j^i(n)) \in A(3)\) we obtain the same 24 degrees of freedom per lattice node, and, again, with preservation of the most important structural properties:

\[
\psi^i(n) = q_0^i(n) + p_j^i(n)dx^j + q_j^i(n) * dx^j + p_0^i(n)dxdydz
\]

(with \(*dx^i = \frac{1}{2} \varepsilon_{ijk}dx^jdx^k\).

The extension of this kinematical identification to dynamics faces an old problem of ether theory. The Dirac equation is almost equivalent to the d’Alembert equation, which is already a classical wave equation. But in classical condensed matter we usually have different speeds for longitudinal and transversal waves, while this d’Alembert equation gives the same speed for all waves. We propose an “atomic ether” variant which allows to motivate the d’Alembert equation as a free field limit. But this variant has less predictive power in comparison with the “polycrystalline ether”: The number of different types of atoms – four – we have to put in by hand.

Then we consider the question of extension of the whole correspondence to gauge fields. Unfortunately, the standard Wilson approach to gauge fields on the lattice does not work. The reason is that multiplication with \(i\), which is assumed to be a pointwise operation in this approach, does not have these properties in our case. Real and imaginary components of a fermion are located on different lattice nodes. Therefore the lattice versions of the operators \(i\) as well as \(\gamma^5\) cannot be pointwise operators. This may be a feature, not a bug: It prevents the application of standard standard no-go theorems like the famous Nielson-Ninomiya theorem. But this is an open question for future research.

Nonetheless we can derive some general principles how gauge fields have to be handled in an ether-theoretical approach. These general principles differ from those in the standard relativistic approach in essential points. The major difference is that we have no factorization (BRST cohomology)

\[\text{Here and in the following, if not mentioned otherwise, Latin indices } i, j, k, \ldots \text{ vary over } x, y, z, \text{ Greek indices } \kappa, \lambda, \mu, \nu, \ldots \text{ over } t, x, y, z.\]
but the gauge degrees of freedom have to be physical degrees of freedom. Thus, we have to go back before the Gupta-Bleuler [11], [3] proposal to use an indefinite Hilbert space structure in the “big” space to obtain manifest Lorentz symmetry.

The “polycrystalline ether” proposal presented here nicely fits into a general Lorentz ether theory (GLET) for gravity proposed by the author in [23]. We introduce this theory shortly in app. A.3. In this theory, the gravitational field $g_{\mu\nu}$ is identified with density $\rho$, velocity $v^i$ and pressure tensor $p^{ij}$ of an ether in a classical Newtonian framework. The identification is a variant of the ADM decomposition:

$$
g^{00}\sqrt{-g} = t^{00} = \rho$$

$$
g^{i0}\sqrt{-g} = t^{i0} = \rho v^i$$

$$
g^{ij}\sqrt{-g} = t^{ij} = \rho v^i v^j + p^{ij}
$$

GLET describes only a general framework. It allows, but does not specify further “inner steps of freedom” – fields $\varphi^m(x)$ which describe the material properties of the ether. These fields $\varphi^m(x)$ are identified with matter fields. The Lagrangian which follows from this general theory is

$$L = L_{GR}(g^{\mu\nu}) + L_{matter}(g^{\mu\nu}, \varphi^m) - (8\pi G)^{-1}(\Upsilon g^{00} - \Xi(g^{11} + g^{22} + g^{33})) \sqrt{-g} \quad (2)$$

The only restriction which follows for the matter Lagrangian $L_{matter}$ is independence from the preferred coordinates – the Einstein equivalence principle. Thus, GLET defines an ideal framework for an ether-based theory of everything. The final ether theory of everything has to specify the material properties of the ether and to derive from this the Lagrangian of the standard model or some generalization. Our “polycrystalline ether” nicely fits into this scheme: The specification “polycrystalline” gives the standard model fermionic particle content.

We consider some steps into the direction of unification of our flat space Dirac operator with GLET: We remember shortly standard Hodge theory which defines the continuous Dirac operator for a general three-dimensional metric. We also consider the compatibility of this approach with ADM decomposition, which allows to extend this continuous operator to the arbitrary four-dimensional metrics which appear in GLET. We have not yet found a nice lattice variant of this operator for a general lattice.

The paper is organized as follows: First we consider continuous theory for the Dirac operator on $(T \otimes \Omega)(\mathbb{R}^3)$, then the lattice Dirac operator and
the identification with standard model fermions. Then we consider ether models. Last not least we discuss gauge fields. Parts which seem essential for the understanding of the whole picture but have been considered in earlier papers, especially GLET [23], Hodge theory and ADM decomposition [25] and general remarks about quantization [24], we have shortly introduced in appendices.

2 Continuous theory for the Dirac operator

on $(T \otimes \Omega)(\mathbb{R}^3)$

Let’s at first consider the lattice theory of the Dirac operator on $(T \otimes \Omega)(\mathbb{R}^3)$ and its connection to standard model fermions. The Dirac operator on $\Omega(\mathbb{R}^3)$ is the Dirac operator on the exterior bundle $\Omega(M^n)$ defined in Hodge theory on arbitrary manifolds $M^n$ (we introduce this operator for this general case in sec. A.1) for the case $M^n = \mathbb{R}^3$.

The Dirac operator used in QFT differs from the Hodge theory Dirac operator in several questions: First, the Dirac operator in QFT has a well-defined, fixed complex structure. For the Dirac operator on $\Omega(M^n)$ no such structure is specified. We will see below (sec. 2.2) that several complex structures exist on $\Omega(\mathbb{R}^3)$, which form a quaternionic structure. This may be used to define a natural complex structure on $(T \otimes \Omega)(\mathbb{R}^3)$. The complex structure is not important as long as we consider the Dirac equation alone. It becomes important later, when we want to define interactions with gauge fields.

Then, the QFT Dirac operator has eight steps of freedom on $(3 + 1)$-dimensional spacetime, while the Dirac operator on $\Omega(M^n)$ has dimension $2^n$. Thus, the necessary number of steps of freedom for the QFT Dirac operator we obtain for $\Omega(\mathbb{R}^3)$, not for $\Omega(\mathbb{R}^3 \otimes \mathbb{R})$. Therefore the consideration of the space $\Omega(\mathbb{R}^3)$ to describe fermions does not fit into the relativistic spacetime paradigm. It becomes much more natural in a theory which handles space and time differently. We consider such proposals in sec. 4. In this context, we can extend the three-dimensional operator using an essentially three-and-one-half dimensional approach to a QFT Dirac operator.
2.1 A Matrix representation of the Dirac operator on $\Omega(\mathbb{R}^3)$

The matrix representation of the Dirac matrices $\gamma^\mu$ in $\Omega(\mathbb{R}^3)$ may be defined in the following way:

$$ (\gamma^0 \partial_t - \gamma^i \partial_i) \phi = \text{def} \begin{pmatrix} \partial_t & \partial_z & \partial_y & \partial_x \\ -\partial_z & -\partial_t & \partial_y & \partial_x \\ -\partial_y & -\partial_t & -\partial_z & \partial_x \\ -\partial_x & -\partial_y & -\partial_z & -\partial_t \end{pmatrix} \begin{pmatrix} \varphi_{000} \\ \varphi_{001} \\ \varphi_{010} \\ \varphi_{011} \end{pmatrix} $$

where $\varphi = \sum \varphi_{ijk}(dx)^i(dy)^j(dz)^k$ is the decomposition of an element $\varphi \in \Omega(\mathbb{R}^3)$. In the context of this representation, it seems also natural to define the following operators $\beta^i$ by their combination with coefficients $m_i$:

$$ m_i \beta^i = \text{def} \begin{pmatrix} m_z & m_y & m_y & m_x \\ m_y & -m_z & -m_z & m_x \\ -m_z & -m_y & -m_y & -m_y \\ -m_z & -m_y & m_z & m_z \end{pmatrix} $$

The following operator equation holds:

$$ (\gamma^0 \partial_t - \gamma^i \partial_i + m_i \beta^i)^2 = -\Box + \delta^{ij} m_i m_j $$

This can be easily seen – this operator iterates three times, in each coordinate direction, the same trick:

$$ \left( \begin{pmatrix} A & (m_i + \partial_i)I \\ (m_i - \partial_i)I & -A \end{pmatrix} \right)^2 = (A^2 + (m_i + \partial_i)(m_i - \partial_i)I) \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} $$

This observation also suggests how to iterate this construction to arbitrary dimension.

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2This observation also suggests how to iterate this construction to arbitrary dimension.
It follows, as it should, that the $\gamma^\mu$ define a representation of the Dirac matrices. It also follows immediately that the matrices $\beta^i$ fulfill the following anti-commutation relations:

$$\beta^i \beta^j + \beta^j \beta^i = \delta^{ij} \tag{7}$$

and anti-commute with all $\gamma^\mu$:

$$\beta^i \gamma^\mu + \gamma^\mu \beta^i = 0 \tag{8}$$

It is also easy to see that

$$\gamma^0 (\gamma^1 \beta^1)(\gamma^2 \beta^2)(\gamma^3 \beta^3) = 1. \tag{9}$$

## 2.2 Complex structures on $\Omega(\mathbb{R}^3)$

This representation of the Dirac matrices does not have a complex structure. But such a structure seems necessary to define the interaction with gauge fields, at least if we want to do it in the standard way.

In a real representation, a complex structure is defined by a linear operator $i$. The properties required for $i$ to define a complex structure are are $i^{-1} = i^* = -i$ (where $i^*$ denotes the Euclidean adjoint) and $[\gamma^\mu, i] = 0$. Now, an interesting point is that there are several candidates for such a structure:

$$i = \beta^y \beta^z = i \beta^x \tag{10}$$

$$j = \beta^z \beta^x = i \beta^y \tag{11}$$

$$k = \beta^x \beta^y = i \beta^z \tag{12}$$

which together define a quaternionic structure:

$$ij = -ji = -k; \quad jk = -kj = -i; \quad ki = -ik = -j; \quad i^2 = j^2 = k^2 = -1 \tag{13}$$

We see that every complex structure is connected in a natural way with a preferred direction. This allows to define a natural complex structure on $(T \otimes \Omega)(\mathbb{R}^3)$, where in each of the three components $\Omega(\mathbb{R}^3)$ we use the preferred complex structure of this direction.

This complex structure on $(T \otimes \Omega)(\mathbb{R}^3)$ no longer has a preferred direction. Nonetheless, it has a preferred orientation (chirality).

\footnote{The classical representation $ij = k$ can be obtained using reverse signs for $i, j, k$, but we prefer this sign convention because it gives $\gamma^5 = \beta^x$.}
2.3 Getting rid of unnecessary dependencies on the complex structure

As we see, the complex structure is a subtle issue. As we will see below, it becomes even more subtle on the lattice. In this context, it seems useful to clarify what really depends and what does not depend on the choice of the complex structure, and to get rid of unnecessary dependencies on the complex structure.

2.3.1 The $\gamma^5$ operator

The “classical” operator $\gamma^5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$ depends on the complex structure. A natural replacement which does not depend on it – the expression $\gamma^0\gamma^1\gamma^2\gamma^3$ – we denote with $\iota$:

$$\iota \overset{\text{def}}{=} \gamma^0\gamma^1\gamma^2\gamma^3 = \beta_x\beta_y\beta_z\iota \gamma^\mu + \gamma^\mu \iota = 0 \quad \iota\beta_i = \beta_i \iota \quad (\iota)^2 = -1 \quad (14)$$

For each candidate $i$ for a complex structure, we obtain an own operator $\gamma^5 =_{\text{def}} -i\iota$. Especially for $i = \iota\beta^x$ we obtain $\gamma^5 = -i\beta^x = \beta^x$.

2.3.2 Hermitian and Euclidean structure

In the standard approach a Hermitian scalar product $\langle .,. \rangle$ is widely used. In our real representation we have only a standard Euclidean scalar product $\langle .,. \rangle$ yet. Now, for a complex structure $i$ these notions are closely related in a simple way: The Hermitian scalar product defines an Euclidean scalar product by

$$\langle \psi, \phi \rangle \overset{\text{def}}{=} \frac{1}{2}(\langle \psi, \phi \rangle + \langle \phi, \psi \rangle) \quad (15)$$

so that $(i\psi, i\phi) = (\psi, \phi)$. For a complex structure $i$, $i^2 = -1$, with this property this Hermitian scalar product is defined by the Euclidean scalar product as

$$\langle \psi, \phi \rangle = (\psi, \phi) - i(\psi, i\phi). \quad (16)$$

Thus, we should not care about the Hermitian scalar product, the Euclidean scalar product is all we need.
2.3.3 Adjoint operators

Another notion we can get rid of are Hermitian adjoint operators. A complex linear operator $A$ is simply a real linear operator with $[A, i] = 0$. For such operators, the Hermitian adjoint operator $A^+$ and the Euclidean adjoint operator $A^*$ coincide: $\langle A^* \psi, \phi \rangle = \langle \psi, A \phi \rangle$. As a consequence, the classical properties of the $\gamma$-matrices

$$
(\gamma^\mu)^+ = \gamma^0 \gamma^\mu \gamma^0
$$

are equivalent to

$$
(\gamma^\mu)^* = \gamma^0 \gamma^\mu \gamma^0.
$$

These properties are fulfilled in our representation for the standard Euclidean scalar product $(.,.)$ in $\mathbb{R}^8$.

3 Lattice theory of the Dirac operator on $(T \otimes \Omega)(\mathbb{R}^3)$ and standard model fermions

This representation is appropriate for a discretization of the Dirac equation on a regular hyper-cubic lattice. It can be obtained in a quite simple way: We start with a naive central difference approximation

$$
\partial_i \varphi(n) \rightarrow \frac{1}{2h_i} (\varphi(n + h_i) - \varphi(n - h_i)).
$$

In [26] we have seen that this naive discretization gives, as it should be expected, $2^4 = 16$ fermions instead of one (fermion doubling). We have also found that these doublers decompose in a simple way. First, there is a decomposition into eight pairs:

$$
\psi_{\kappa\lambda\mu} = (\varphi_{\kappa\lambda\mu} \text{ on spatial node } [2k + \kappa + \kappa', 2l + \lambda + \lambda', 2m + \mu + \mu'])
$$

with $\kappa, \lambda, \mu, \kappa', \lambda', \mu' \in 0, 1, k, l, m \in \mathbb{Z}$. Then, each pair decomposes into the part defined on even and odd nodes of the spatial-temporal lattice. Here, a node $[k, l, m, t], k, l, m, t \in \mathbb{Z}$ is odd/even if $k + l + m + t$ is odd resp. even.

In [26] we have thrown away seven of the eight pairs, leaving on the spatial node $(2k + \kappa, 2l + \lambda, 2m + \mu)$ only $\varphi_{\kappa\lambda\mu}$. On the other hand, we have suggested not to throw away the last doubler. Instead, we have proposed to interpret the remaining pair of doublers as a physical flavor doublet, as formed by the
quarks of each color and generation or, if the neutrino appears to be a Dirac particle, by leptons of each generation.

In this paper, we consider the reverse approach: On one hand, we use an “accurate” method, without doublers, for time. For example, we can do this in discrete time by hand, throwing away all odd nodes of the space-time grid. In the context of an ether theory (see sec. 4) this happens automatically: The natural discrete structure (granular structure of polycrystalline ether, atomic structure of atomic ether) gives only a spatial discretization but leaves time continuous.

On the other hand, we do not remove the remaining eight doublers and suggest to interpret them as the eight fermions of a whole family (two leptons, two quarks, each quark in three colors).

3.1 Identification of the particles

Now, the representation of the eight doublers in terms of $\psi_{\kappa\lambda\mu}$ seems useful to understand some of their symmetry properties. In the free field limit all of them have identical physical properties. Indeed, they may be transformed into each other by simple operators:

\begin{align*}
  s_x \psi_{\kappa\lambda\mu}(n) &= \psi_{(1-\kappa)\lambda\mu}(n) \\
  s_y \psi_{\kappa\lambda\mu}(n) &= \psi_{\kappa(1-\lambda)\mu}(n) \\
  s_z \psi_{\kappa\lambda\mu}(n) &= \psi_{\kappa\lambda(1-\mu)}(n)
\end{align*}

(21)

(22)

(23)

Let’s consider now the eigenvectors of the commuting set of operators $s_x, s_y, s_z$. Because $s_x^2 = s_y^2 = s_z^2 = 1$, they have only eigenvalues $\pm 1$. The resulting eight eigenspaces have dimension one and suggest a simple identification scheme with the fermions of one standard model family$^4$.

$^4$We use here symbols for the first family only for the purpose of illustration.
\[ \nu = \sum_{\kappa \lambda \mu} \psi_{\kappa \lambda \mu} \]  
(24)

\[ d_r = \sum_{\kappa \lambda \mu} (-1)^\kappa \psi_{\kappa \lambda \mu} \]  
(25)

\[ d_g = \sum_{\kappa \lambda \mu} (-1)^\lambda \psi_{\kappa \lambda \mu} \]  
(26)

\[ d_b = \sum_{\kappa \lambda \mu} (-1)^\mu \psi_{\kappa \lambda \mu} \]  
(27)

\[ u_r = \sum_{\kappa \lambda \mu} (-1)^{\lambda+\mu} \psi_{\kappa \lambda \mu} \]  
(28)

\[ u_g = \sum_{\kappa \lambda \mu} (-1)^{\mu+\kappa} \psi_{\kappa \lambda \mu} \]  
(29)

\[ u_b = \sum_{\kappa \lambda \mu} (-1)^{\kappa+\lambda} \psi_{\kappa \lambda \mu} \]  
(30)

\[ e = \sum_{\kappa \lambda \mu} (-1)^{\kappa+\lambda+\mu} \psi_{\kappa \lambda \mu} \]  
(31)

It seems remarkable that in this identification a color symmetry appears almost automatically as rotational symmetry. This identification also gives a natural suggestion for the electromagnetic charge of the particles: It may be identified with the number of “sign factors” of type \((-1)^\kappa\) in this definition. Moreover, there is a natural duality operation which connects the flavor pairs connected by weak interactions.

### 3.2 Some operators on fermions

Let’s define now some operators which seem to be useful for the understanding of the lattice theory. First, the following operators seem to be interesting:

\[ \epsilon_x \psi_{\kappa \lambda \mu} = (-1)^\kappa \psi_{\kappa \lambda \mu} \]  
(32)

\[ \epsilon_y \psi_{\kappa \lambda \mu} = (-1)^\lambda \psi_{\kappa \lambda \mu} \]  
(33)

\[ \epsilon_z \psi_{\kappa \lambda \mu} = (-1)^\mu \psi_{\kappa \lambda \mu} \]  
(34)

and \( \epsilon = \epsilon_x \epsilon_y \epsilon_z \). These operators transform the fermions into each other. Following the previous particle identifications, the operators \( \epsilon_i \) change electromagnetic charge by 1/3, while \( \epsilon \) changes the flavor.
3.3 Chiral symmetry on the lattice

The problem with “naive” Dirac fermions as well as with standard staggered fermions [13] is not only that they have the wrong number of doublers (sixteen resp. four) to allow a natural physical interpretation in the standard model. The problem is also that there is exact chiral $\gamma^5$ symmetry on the lattice. As a consequence, the doublers appear in pairs with reverse chiral charge. This does not fit the situation in the standard model (cf. [12]). Now, in our approach we do not have exact chiral $\gamma^5$ symmetry. Instead, we have a replacement for this symmetry. This replacement fulfills properties which define a generalization of the famous Ginsparg-Wilson (GW) relation [9].

Let’s consider one fermion with family index $x$, thus, with complex structure $i = i\beta^x$, $\gamma^5 = \beta^x$. To understand chiral symmetry we have to define $\gamma^5 = \beta^x$ on the lattice. It cannot be a pointwise operator as for Wilson fermions and staggered fermions – it connects components which are located in different points. Now, we propose to consider the following operator as a candidate for $\gamma^5$ on the lattice:

\[
\begin{align*}
(\gamma^5 \phi)(n_{\text{even}}) & = \phi(n_{\text{even}} - h_x) \\
(\gamma^5 \phi)(n_{\text{odd}}) & = \phi(n_{\text{odd}} + h_x)
\end{align*}
\]

It is easy to see that it approximates the continuous $\gamma^5$. More interesting is that some exact properties remain valid:

\[
(\gamma^5)^* = \gamma^5; \quad (\gamma^5)^2 = 1
\]

We can also define, as an alternative, the operator $\tilde{\gamma}^5$ by

\[
\begin{align*}
(\tilde{\gamma}^5 \phi)(n_{\text{even}}) & = \phi(n_{\text{even}} + h_x) \\
(\tilde{\gamma}^5 \phi)(n_{\text{odd}}) & = \phi(n_{\text{odd}} - h_x)
\end{align*}
\]

Similarly, we obtain

\[
(\tilde{\gamma}^5)^* = \tilde{\gamma}^5; \quad (\tilde{\gamma}^5)^2 = 1
\]

If we define the operators $V, O$ by $\tilde{\gamma}^5 = \gamma^5 V = \gamma^5 (1 - h_x O)$ we obtain the Ginsparg-Wilson (GW) relation for $O$:

\[
O \gamma^5 + \gamma^5 O = h_x O \gamma^5 O
\]

Moreover, we have also the following important commutation properties with $D$
\[ \gamma^5 D + D \gamma^5 = 0 \]  
\[ \gamma^5 D + D \gamma^5 = 0 \]  
\[ VD - DV = 0 \]  
\[ OD - DO = 0 \]

This allows to define two sets of chiral projector operators

\[ \tilde{P}_\pm = \frac{1}{2}(1 \pm \gamma^5), \]  
\[ P_\pm = \frac{1}{2}(1 \pm \tilde{\gamma}^5). \]

Similar pairs of projectors play a central role in approaches to chiral gauge theory based on the GW relation (10, 17) and its generalizations (14) as domain wall fermions [28], Neuberger’s overlap operator [19], and proposals by Fujikawa [8] and Chiu [5].

On the other hand, there are some differences: The operators \( V, O \) do not have the spectral properties of the similar operators considered, for example, by [10], [14]. At least partially this difference may be understood as caused by different aims. The aim of the standard GW approach is to obtain a single Weyl fermion on the lattice, without any doublers. In our approach we do not want to get rid of doublers at all. Instead, we want nontrivial chiral symmetry only to obtain nontrivial chiral interactions between the doublers.

The more important difference is that in our approach the complex structure is also not defined as a pointwise operator on the fermions. This prevents the use of the standard Wilson approach to lattice gauge theory and therefore also of the standard GW approach. On the other hand, we should not forget that this non-trivial character also prevent the application of standard no-go theorems like the famous Nielson-Ninomiya [20] theorem.

### 4 Ether models for \((T \otimes \Omega)(\mathbb{R}^3)\) lattice theory

In the previous sections we have found a way to describe the fermionic content of the standard model starting with the Dirac operator on \((T \otimes \Omega)(\mathbb{R}^3)\) combined with “naive” spatial discretization in space but not in time. We find here several occurrences of the number three – the dimension of space, not of space-time. This suggests physical interpretation in terms of theories which handle space and time differently.
We consider below two candidates for such theories: A polycrystalline ether where the discrete structure of space is obtained by crystal grains, and atomic ether theory where it is obtained by the atomic structure. In above cases, time is classical continuous absolute time, without any discrete structure.

4.1 Polycrystalline ether theory

In this section, we consider the derivation of \((T \otimes \Omega)(\mathbb{R}^3)\) from a simple “polycrystalline ether” hypothesis.

The derivation itself is quite simple. A polycrystalline material consists of small crystalline grains. If we want to do elasticity theory for such a material, we have to describe distortions of such a material. If the grains are more rigid than the material between them, the state of a grain \(n\) may be described, in good approximation, by the position of its center \(q^0_i(n)\) and a linear transformation \(q^i_j(n)\). Together with these state variables, we also need related momentum variables \(p^0_i(n), p^i_j(n)\). Now, we can identify these steps of freedom with \((T \otimes \Omega)(\mathbb{R}^3)\) by \(q^0_i + p^i_j dx^j + q^i_j * dx^i + p^0_i dx dy dz\).

This identification preserves not only the number of steps of freedom, but preserves also important structural properties. Duality between configuration and momentum variables gives flavor doublets, rotational symmetry gives color symmetry, three spatial directions give three families.

4.1.1 The special role of time

An important point for the identification is that space and time appear in the ether approach in a non-symmetric way: The polycrystalline structure leads to a natural lattice structure in space, but not in time. Therefore, the lattice-related doubling effect appears only in the three spatial directions. Therefore we obtain only \(2^3 = 8\) doublers, instead of \(2^4 = 16\) doublers as in a naive space-time lattice.

4.1.2 Empirical content of the polycrystalline ether hypothesis

Starting from a single phrase – “polycrystalline ether” – we have obtained not only the correct number of fermionic steps of freedom of the standard model (192 real fields), but also the most important structural properties of these steps of freedom \((192 = 3 \cdot 2 \cdot (1 + 3) \cdot 8)\). This identification does not look like something made up. The polycrystalline ether proposal seems very restrictive at least in some parts. For example, there would be no possibility for a fourth fermion family, the neutrino should be a standard Dirac particle.
This gives the theory sufficient empirical content. On the other hand, there will be large freedom in the choice of various material parameters. Therefore we should not expect predictions of all SM parameters from this theory.

4.1.3 Explanatory power

The polycrystalline ether proposal is quite satisfactory also from another point of view. It is well understood how polycrystalline materials may appear. Especially they appear in a quite general situation: Near second order phase transitions. The concept of second order phase transitions is a beautiful, attractive concept. The parameters which will be left unexplained seem to be almost as unimportant from metaphysical point of view, comparable with the parameters of planetary orbits in Newtonian gravity. Whatever the underlying microscopic theory, it will give some set of material parameters which depends on the set of parameters of the underlying microscopic theory.

4.1.4 Open problems

Of course, this state has not yet been reached. What we have found is, until now, only a partial success: the explanation of the fermionic part of the standard model.

Already in the free field limit of dynamics – the Dirac equation without any gauge fields considered here – we are faced with a classical problem known already from classical ether theory: In usual solid materials, including usual polycrystalline materials, we have different speeds of sound for longitudinal and transversal waves. This is a quite general property, it follows in standard elasticity theory from quite simple symmetry considerations. Instead, the Dirac equation gives the same maximal speed for all types of excitations.

4.2 Connection between Dirac and d’Alembert equation

Leaving this problem open, let’s assume we have a polycrystalline material which in some limit has the same d’Alembert equation equation for all of its steps of freedom

\[ \Box q_0(n, t) = 0 \quad \Box q_i(n, t) = 0 \] (48)

where \( \Box = \partial_t^2 - \Delta \) is the d’Alembert equation for the discrete Laplace operator \( \Delta \) on the lattice. This second order wave equation is already more close to equations for classical polycrystalline materials than the first order Dirac equation. Therefore it is worth to consider shortly their connection.
The three-dimensional lattice Dirac operator $D$ has the property $D^2 = \Delta$. Each solution of the Dirac equation $(\gamma^0 \partial_t \pm D)\psi = 0$ defines also a solution of the d'Alembert equation. In the other direction, the situation is less trivial. Assume we have a solution of the d'Alembert equation for initial values $Q = \{q_0(n), q_j^i(n)\}$ and their first derivatives $\dot{Q} = \{\dot{q}_0(n), \dot{q}_j^i(n)\}$. This does not define in general a unique solution $\{Q, P\}$ of the Dirac equation. The problematic part are homogeneous solutions $D\psi(n) = 0$, especially constants $\psi(n) = \psi_0 = \{Q_0, P_0\}$. This leads to non-uniqueness because $P \rightarrow P + P_0$ does not change $Q$, as well as non-existence of $P$ for solutions of type $Q_0 t$.

But these differences for constant solutions will not lead to physical effects which are observable for internal observers. Moreover we can suppress them using appropriate boundary conditions. In this sense, the formulation using the Dirac equation can be considered as equivalent to the formulation in terms of the d'Alembert equation.

Note that the shift from Dirac equation to d'Alembert equation may be important for the understanding of symmetry breaking. Indeed, the Laplace operator $\Delta$ on the lattice has no preferred orientation. Instead, the lattice Dirac operator depends on a choice of orientation. This choice of orientation is equivalent to the choice of the sign of the square root in $D^2 = \Delta$.

### 4.3 Atomic ether theory

Most essential points of polycrystalline ether theory will be present also in an alternative approach: Atomic ether theory. An atomic ether theory proposes some set of atoms which may be of different type. They form a lattice. Each type of atoms is described by a sub-lattice. This gives, for $k$ types of atoms, $3k$ real steps of freedom. Moreover, we have the same structure of space-time: The classical equations are continuous in time but discrete in space. Therefore we obtain a doubling with factor $2^3 = 8$.

Comparison with the standard model gives now $k = 4$. Thus, we need four types of atoms to describe the SM fermions. This is a special example of a general property: In comparison with the polycrystalline theory, we have more freedom in the construction of the atomic theory. This gives atomic ether theory less empirical content: It is much easier to modify atomic theory to fit observation. Nonetheless, it may be that an atomic ether theory appears to be very simple, comparable in simplicity with a polycrystalline ether theory.

The reason why we have introduced it here is that it allows to solve the problem with the different speed of longitudinal and transversal waves. Indeed, if we have four atoms, we can assume strong forces between atoms of the same type, and by Ockham’s razor we would prefer a theory where
these forces do not depend on the type. The free field limit would be, in this case, the limit where we have no interaction between atoms of different type, which leads to the same wave equation for all four atoms. Thus, in this atomic theory the d’Alembert equation as a free field limit seems to be much more natural than in a polycrystalline ether theory.

Let’s mention another difference between these two variants of ether theory. There is a quite natural process in polycrystalline materials: crystal growth. This process changes the average distance between the grains. Now, such a change of the critical distance leads to a renormalization which, from point of view of internal observers, seems equivalent to an expansion of their universe. Atomic ether theory does not give such a natural mechanism for renormalization.

5 Gauge Fields

Gauge fields are not yet described in our ether proposal. The main reason for this is the unorthodox realization of the complex structure on the lattice. Indeed, while we have found a nice complex structure in the continuous case, it does not define a pointwise complex structure on the lattice. Instead, the real and imaginary parts of a “complex number” of a fermion are located on different nodes. Once multiplication with $i$ is no longer a pointwise operation, the standard Wilson approach to lattice gauge theory fails.

On the other hand, we have found some nice prerequisites which seem to be useful to build such a theory. We have already described operators which seem to be related with electromagnetic charge, flavor and color of the particles. We have also found a nice nontrivial realization of chiral symmetry on the lattice, similar to the lattice version of chiral symmetry used in the Ginsparg-Wilson approach to chiral gauge theory.

Moreover, there are some general principles about the nature of gauge fields which follow from the ether approach: The gauge steps of freedom should be handled like real steps of freedom. Thus, there should be some physical evolution equation for these steps of freedom. The natural candidate is the Lorenz gauge. Gribov copies should be interpreted as really different field configurations.

5.1 Some general principles for gauge fields in ether theory

Let’s consider here some general principles for the realization of gauge fields in the context of an ether theory. It appears that the ether theory concept
is already quite restrictive about these general principles.

5.1.1 Gauge degrees of freedom as physical steps of freedom

There is the famous Bohm-Aharonov experiment which shows effects of gauge fields in a region where the field strength $F_{\mu\nu} = 0$. This seems to exclude the possibility of description of gauge fields using only the $F_{\mu\nu}$. Thus, it seems necessary to use the gauge potential $A_{\mu}$ to describe gauge fields. Thus, we can assume that the gauge potential $A_{\mu}$ or some equivalent on the lattice (like integrals $\int A_{\mu} dx^\mu$ over edges) is used to describe gauge steps of freedom.

Once non-gauge-invariant objects have to be used to describe steps of freedom of the ether, they have to be physical steps of freedom. This has consequences:

- Quantization has to be done in the “big” space, without factorization.
- We have a definite Hilbert space structure on this space.
- We need an evolution equation for the gauge degrees of freedom. Theories with different evolution equations are different as ether theories, even if they appear to be indistinguishable by observation.
- For this purpose, the Lorenz gauge condition is a natural candidate, without any reasonable competitor.
- Gribov copies are physically different states.

5.1.2 Definite Hilbert space structure

In this context, it seems worth to note that the use of an indefinite Hilbert space structure in the standard quantization approach has been developed only 1950 by Gupta and Bleuer [11], [3] and is not the only possibility. Instead, quantization is possible also based on a standard Hilbert space. In this way the electromagnetic field has been initially quantized by Fermi [7] and Dirac [6].

The advantage of the Gupta-Bleuer approach is manifest Lorentz symmetry. But this advantage does not have much value in our ether-theoretical approach, in comparison with a standard, physical Hilbert space structure which gives manifest unitarity. Indeed, if something goes wrong with gauge symmetry, the use of an indefinite Hilbert space leads to non-unitarity, and the resulting theory is obviously physically meaningless. If we, instead, start with a definite Hilbert space, something may go wrong with Lorentz symmetry, but the theory certainly remains to be physically meaningful.
Note that it is the ether approach which forces us to use such a definite Hilbert space. We have no choice here. But the choice we are forced to accept here seems to be a reasonable one.

5.1.3 Lorenz gauge and Gribov copies

The Lorenz gauge
\[ \partial_\mu A_\mu = 0 \]  \hspace{1cm} (49)

is not really a gauge, because it does not fix the gauge degrees of freedom. Instead, it is an evolution equation for them. Indeed, for a gauge transformation \( A_\nu \rightarrow A_\nu + \partial_\nu \omega \) we obtain
\[ \partial_\mu g^{\mu\nu} \sqrt{-g} \partial_\nu \omega = 0 \]  \hspace{1cm} (50)

thus, the classical harmonic equation. Thus, we have a whole field, defined by arbitrary initial values \( \omega(x, t_0) \), \( \partial_t \omega(x, t_0) \), which is not fixed by the Lorenz condition.

Now, this gauge degree of freedom should be interpreted as a physical field. Especially, Gribov copies define different physical states.

The choice of the Lorenz condition is also in good correspondence with the approach used for gravity (see app. A.3). Indeed, in gravity we use a similar coordinate condition – the harmonic condition – which is interpreted as a physical equation. Moreover, the harmonic condition also has the form of a conservation law, and this form has been used there to identify these equations with classical conservation laws. This analogy suggests not only the choice of the Lorenz condition. It also suggests to search for a physical interpretation of the Lorenz condition in terms of conservation of something.

6 Summary

What we have proposed here is a new paradigm for unification of all fundamental forces of nature, based on completely different metaphysics. We revive the old ether idea in its full beauty: With absolute space, absolute time, and an ether described by classical condensed matter equations. Relativistic symmetry as well as gauge symmetry are not fundamental, but should be derived. They are secondary symmetries for observable effects, caused by the restricted possibilities of internal observers, not true symmetries of reality itself.

The ether paradigm essentially simplifies quantization (see app. B) and supports also a revival of classical realism in quantum theory. It is compatible
with Bohmian mechanics \cite{4} and Nelson’s stochastics \cite{18} and defines a way to extend them to theories of everything.

The general ether theory of gravity (app. \ref{A.3}) sufficiently explains relativistic gravity, with interesting modifications: Frozen stars instead of black holes, no big bang singularity, a dark matter term. The polycrystalline ether proposal made here derives the whole fermionic content of the standard model, starting from almost nothing. We obtain not only the correct number of degrees of freedom (192 real fields), but all the basic structural properties: Eight components of fermions, three families, flavor pairs, color symmetry between quarks. We have considered the free Dirac operator and found natural operators which change flavor, color, electromagnetic charge and chirality.

A lot of interesting questions remain open. The gauge sector has not yet been understood. We have found only some general principles for handling gauge fields which differ from the standard relativistic approach: Going back before Gupta-Bleuer, unitarity should be made manifest, problems with unitarity in the relativistic approach should be transformed into violations of Lorentz symmetry. Gauge degrees of freedom are physical, the Lorenz gauge is proposed as a physical equation, Gribov copies understood as physically different configurations. We have no Faddejev-Popov ghost fields.

Moreover, nothing has been done yet in the Higgs sector or for the understanding of the fermion masses. The modifications in the general principles for gauge fields are too large to tell if or how the Higgs mechanism has to be modified.

Despite these open problems, the initial success of the polycrystalline ether hypothesis seems much too large to be accidental.

\section*{A Gravity}

A theory of everything should be able to describe gravity too. We consider here two questions: First, the way how to generalize a theory based on the “flat” Dirac operator for \((T \otimes \Omega)(\mathbb{R}^3)\) to a general four-dimensional metric background. Second, we introduce a general Lorentz ether theory which gives a metric theory of gravity and seems compatible with the ether proposals made here for the explanation of the standard model fermions.

The first question may be subdivided into two parts. First, the generalization of the three-dimensional Dirac operator on \((T \otimes \Omega)(\mathbb{R}^3)\) to the case of a general three-dimensional metric. Here, the continuous theory is well-known standard Hodge theory, we shortly remember the main results. Unfortunately we have not found yet a nice generalization of our “naive” lat-
tice discretization for a general lattice. The second part is the use of the ADM decomposition to extend the three-dimensional Dirac operator to an essentially three-and-one-half-dimensional Dirac operator on a four-dimensional space-time metric background.

The ether theory we propose can be understood as a modification of GR which breaks covariance and fixes harmonic coordinates, combined with an ether interpretation, which is based on the ADM decomposition and the interpretation of the harmonic condition in terms of classical conservation laws. The theory identifies the observable gravitational field with the classical energy-momentum tensor of the ether by $g^\mu_\nu \sqrt{-g} = t^\mu_\nu$. The material properties of the ether, which are not specified in this theory, have to be identified with the observable matter fields.

The most surprising observation is that this theory may be derived from a few axioms which may be motivated from simple classical principles. Essentially, we need a Lagrange formalism and the identification of the conservation laws given by Noether’s theorem with the conservation laws of classical condensed matter theory. This identification fixes four general equations, which are closely related to the preferred coordinates. They do not depend on the special material properties of the ether. As a consequence of the “action equals reaction” symmetry of the Lagrange formalism, the equations for these material properties of the ether do not depend on the preferred coordinates. But this is already the Einstein equivalence principle.

A.1 The Dirac operator on the de Rham complex

Until now we have used only a special case of the Dirac operator – the Dirac operator on $\Omega(\mathbb{R}^3)$ with a standard Euclidean metric. We extend now this operator to a general metric background. We need it only for a general Riemann metric on $\mathbb{R}^3$, but this generalization is well-known from Hodge theory for a general metric $g_{\mu\nu}(x)$ on a general manifold $M^n$ (see, for example, [21]). Let’s remember here the basic formulas:

The exterior bundle or de Rham complex $\Omega = \sum_{k=0}^{n} \Omega^k$ consists skew-symmetric tensor fields of type $(0,k), 0 \leq k \leq n$ which are usually written as differential forms

$$\psi = \psi_{i_1...i_k} dx^{i_1} \wedge \cdots \wedge dx^{i_k} \in \Omega^k$$

(51)

The exterior bundle $\Omega$ has dimension $2^n$ in the n-dimensional space. The most important operation on $\Omega$ is the external derivative $d : \Omega^k \to \Omega^{k+1}$
defined by

\[ (d\psi)_{i_1...i_{k+1}} = \sum_{q=1}^{k+1} \frac{\partial}{\partial x^{i_q}} (-1)^q \psi_{\hat{i}_q i_1...i_{k+1}} \]  

(52)

where \( \hat{i}_q \) denotes that the index \( i_q \) has been omitted. Its main property is \( d^2 = 0 \). In the presence of a metric, we have also the important \(*\)-operator \([\Omega^k] \rightarrow [\Omega^{n-k}]\):

\[ (*\psi)_{i_{k+1}...i_n} = \frac{1}{k!} \varepsilon_{i_1...i_n} g^{i_1 j_1} \cdots g^{i_k j_k} \psi_{j_1...j_k} \]  

(53)

with \(*^2 = (-1)^{k(n-k)} \text{sgn}(g)\). This allows to define a global inner product by

\[ (\phi, \psi) = \int \phi \wedge (*\psi) = \int \psi \wedge (*\phi) \]  

(54)

It turns out that the adjoint operator of \( d^* \) of \( d \) is

\[ d^* = (-1)^{rn+n+1} * d \]  

(55)

In this general context we can define the Laplace operator as

\[ \Delta = dd^* + d^* d \]  

(56)

Then, the Dirac operator (as its square root) can be defined as

\[ D = d + d^*. \]  

(57)

Indeed, we have \( d^2 = 0 \) as well as \((d^*)^2 = 0\).

The \( \mathbb{Z}_2 \) graduation is also useful: \( \varepsilon \psi = (-1)^k \psi \) if \( \psi \in \Omega^k \). The subspaces \( \varepsilon = 1 \) and \( \varepsilon = -1 \) have equal dimension \( 2^{n-1} \).

The operator \( \gamma^0 \) in our representation can be understood as a specialization of the graduation operator. Indeed, the operator \( \varepsilon \) anti-commutes with the Dirac operator \( D \).

Now, it would be nice to have a similar natural generalization of the “naive” discrete Dirac operator for a general lattice. Unfortunately, the author has not found a nice generalization. At the current moment, the author favors the idea to get rid of the Dirac equation in the regular situation, following sec. 4.2. Then what we have to generalize and to discretize on a general lattice is only the d’Alembert equation, which is less problematic.
A.2 Compatibility with ADM decomposition

Our approach is in essential points three-dimensional. Especially we have used the Dirac operator on $\Omega(\mathbb{R}^3)$, combined with an operator $\gamma^0$ defined by the graduation $\varepsilon$.

For the compatibility of the approach described here with a general metric background we propose to use the ADM decomposition. While this consideration is independent of an ether interpretation, we nonetheless use the denotations which we use later in our ether theory. Here they are simply nonstandard denotations for the standard ADM decomposition:

\[
\begin{align*}
g^{00}\sqrt{-g} &= \rho \\
g^{i0}\sqrt{-g} &= \rho v^i \\
g^{ij}\sqrt{-g} &= \rho v^i v^j + p^{ij}
\end{align*}
\]

The ADM spatial coordinates are simply “comoving” spatial coordinates which remain constant along the “velocity field” $v^i = g^{0i}/g^{00}$. The harmonic operator of the metric $g_{\mu\nu}$ in these coordinates reduces to

\[
\Box \psi = -(\rho \partial_t^2 - \Delta)\psi
\]

where $\Delta$ is already the standard three-dimensional (harmonic) Laplace operator of the spatial metric. This is already sufficient to generalize the concept. We can define now the four-dimensional Dirac equation using the three-dimensional Dirac operator $D$ as

\[
\sqrt{\rho \varepsilon} \partial_t \psi = \pm D\psi
\]

Moreover, in this decomposition we can also introduce a spatial lattice which (following the comoving coordinates) moves continuously in time. Such a discretization gives, as required for our identification, a doubling effect only in spatial directions.

A.3 Definition of General Lorentz Ether Theory

Let’s introduce now an ether theory of gravity proposed in \cite{27} and described in more detail in \cite{23, 14}. The theory has been named “General Lorentz Ether Theory” (GLET) for three reasons: It generalizes the Lorentz ether to gravity, competes with general relativity in a similar way as the Lorentz ether with special relativity, and is also “general” as an ether theory because it specifies only a few general properties of the ether, not its material properties (which define the matter content).
GLET preserves the essential features of classical ether theories: We have a classical Newtonian framework of absolute Euclidean space and absolute time. The space is filled with an ether. This ether is described using some general condensed matter variables: positive density $\rho(x)$, velocity $v^i(x)$, and a pressure tensor $p^{ij}(x)$, as well as some other material properties $\varphi^m(x)$. These other properties are not specified by this general theory. They have to be specified in special ether models. The pressure tensor is proposed to be negative definite. The gravitational field is defined by the classical energy-momentum tensor of the ether in a variant of ADM decomposition:

$$
g_{00} \sqrt{-g} = t_{00} = \rho \\
g_{i0} \sqrt{-g} = t_{i0} = \rho v^i \\
g_{ij} \sqrt{-g} = t_{ij} = \rho v^i v^j + p^{ij}
$$

The equations of the theory are Euler-Lagrange equations for the Lagrangian

$$L = L_{GR}(g_{\mu\nu}) + L_{\text{matter}}(g_{\mu\nu}, \varphi^m) - \left(8\pi G\right)^{-1}(\Upsilon g^{00} - \Xi(g^{11} + g^{22} + g^{33}))\sqrt{-g}$$

(60)

where $L_{GR}, L_{\text{matter}}$ are the most general covariant Lagrangians known from classical general relativity. The additional background-dependent terms break general covariance and fix a coordinate condition. The preferred Newtonian coordinates $X^\mu$ are the harmonic coordinates:

$$\Box X^\nu = \partial_\mu(g^{\mu\nu}\sqrt{-g}) = 0$$

(61)

Rewritten in the original ether variables, these conditions become the classical conservation laws – continuity equation

$$\partial_t \rho + \partial_i(\rho v^i) = 0$$

(62)

and Euler equation

$$\partial_t(\rho v^j) + \partial_i(\rho v^i v^j + p^{ij}) = 0$$

(63)

From this point of view, the theory looks like a minor modification of GR. An additional term breaks covariance and fixes harmonic coordinates. This is combined with an ether interpretation which is surprisingly close to classical condensed matter theory.
A.4 Axioms of General Lorentz Ether Theory

Much more interesting is that the Lagrangian of the theory, and especially the covariance of the matter Lagrangian (that means, the Einstein equivalence principle) may be derived from simple ether theory axioms. We will shortly introduce them, for a more detailed consideration see [23].

The formulation of the main axiom as well as the derivation is based on an unorthodox variant of the Lagrange formalism and Noether’s theorem as well as a slightly unorthodox choice of variables for the ether.

- We have a subdivision into the energy-momentum variables \( t^{\mu\nu} \) and other material steps of freedom \( \phi^m \), with the \( t^{\mu\nu} \) as independent variables. Instead, in classical condensed matter theory, pressure \( p_{ij} \) is usually defined as a given function of the other material steps of freedom, therefore, the \( t^{\mu\nu} \) are not independent from the \( \phi^m \).

- We require that dependence on the preferred coordinates of the Newtonian background \( X^\mu \) is made explicit.

This point needs explanation. The preferred coordinates \( X^\mu \) are also, as all physical fields, functions \( X^\mu(x) \) on space-time. Expressions can depend on them in explicit and implicit ways. An example is the expression \( u^\mu T_{\mu\nu} \), which explicitly depends on \( T \). It is equivalent to \( u^0 \), which therefore also depends on \( T \) – but this dependence is no longer explicit.

In general, we name the coordinate-dependence of an expression \( F(\phi, \phi^\mu, \ldots, X^\alpha, X^\alpha_{\mu}, \ldots) \) explicit if after replacement of the occurrences of \( X^\alpha(x) \) by four scalar fields \( U^\alpha(x) \) no coordinate-dependence is left. Thus, the expression \( F(\phi, \phi^\mu, \ldots, U^\alpha, U^\alpha_{\mu}, \ldots) \) should be covariant.

Once the coordinate-dependence in the Lagrangian is explicit, we can vary over the coordinates \( X^\mu \) and obtain Euler-Lagrange equations for them by the standard rules\[5\]

If we have translational symmetry \( X^\mu(x) \to X^\mu + c^\mu \) of the Lagrangian, Noether’s theorem tells that we obtain conservation laws. But in our formalism we don’t need Noether’s theorem to find them. Instead, the Euler-Lagrange equations for \( X^\mu \) are conservation laws: Indeed, \( \partial L/\partial X^\mu = 0 \), and all other terms appear in the Euler-Lagrange equations under some partial derivative\[6\].

\[5\] Note that only variations \( \delta X^\mu \) so that \( X^\mu + \delta X^\mu \) define valid coordinates are allowed. But for any \( \delta X^\mu(x) \) with finite support this holds for \( \varepsilon \delta X^\mu(x) \) for small enough \( \varepsilon \). Therefore this does not restrict the variation.

\[6\] In this formalism it is also easy to understand why these conservation laws disappear if the Lagrangian is covariant. In this case, the Euler-Lagrange equation for the \( X^\mu \) disappears.
Now we can formulate our main axiom: We require that the Euler-Lagrange equation for the preferred coordinates are proportional to the classical energy-momentum conservation laws. In the preferred coordinates we obtain:

\[
\frac{\delta S}{\delta X^\mu} = \gamma_{\mu\nu} \partial_\kappa t^{\kappa\nu}
\]

for some constant matrix \( \gamma_{\mu\nu} = 4\pi G \text{diag}(-\Upsilon, \Xi, \Xi, \Xi) \).

A.5 Derivation of the Lagrangian

The expression on the right side of equation 64 contains an implicit dependence on the coordinates – the index \( \nu \) in \( t^{\kappa\nu} \). Making it explicit, and using the metric variables \( g^{\mu\nu} \sqrt{-g} = t^{\mu\nu} \) gives

\[
\frac{\delta S}{\delta X^\mu} = \gamma_{\mu\nu} \partial_\kappa (g^{\kappa\lambda} \sqrt{-g} X^\nu_{\kappa\lambda}) \equiv \gamma_{\mu\nu} \Box X^\nu
\]

for the harmonic operator \( \Box \) of the metric \( g^{\mu\nu} \). There is a simple particular solution for this:

\[
L_{\text{part}} = \gamma_{\mu\nu} g^{\kappa\lambda} \sqrt{-g} X^\mu_{\kappa\lambda} X^\nu_{\kappa\lambda}
\]

For the general solution follows

\[
\frac{\delta S - S_{\text{part}}}{\delta X^\mu} = 0,
\]

which defines the Lagrangian of general relativity in its most general form:

\[
L - L_{\text{part}} = L_{\text{GR}}(g_{\mu\nu}) + L_{\text{matter}}(g_{\mu\nu}, \varphi^m)
\]

Especially, because \( L_{\text{part}} \) does not depend on the variables \( \varphi^m \), the whole “matter Lagrangian” should be covariant.

A.6 Physical predictions

Some differences between this ether theory and general relativity are considered in [23]. The most interesting one seems that for \( \Upsilon > 0 \) the gravitational collapse stops immediately before horizon formation. Thus, there are no black holes, but frozen stars. For sufficiently small \( \Upsilon > 0 \) this does not seem to lead to really observable differences. But the differences are at least theoretically important. Especially there will be no Hawking radiation.

Ether theory predicts a flat universe, in good agreement with observation. The \( \Xi \)-term gives an interesting cosmological term for dark energy. \( \Upsilon > 0 \)
prevents the big bang singularity. The solution for a flat Robertson-Walker universe will be a “big bounce”, but it remains open if this big bounce survives if we take into account inhomogeneities and viscosity. For details, see [24], [23].

B Quantization

The quantization of gravity is considered to be one of the greatest problems of modern science. From point of view of ether theory, it seems hard to understand what is problematic here. Most hard problems disappear. If an atomic ether theory, which defines a natural regularization, is given, quantization itself seems possible and quite unproblematic following the classical canonical quantization scheme. Thus, the problem seems to be more a problem of definition of an appropriate classical atomic ether theory which gives the classical continuous ether theory in the large distance limit than a quantization problem.

B.1 Relativistic quantization problems which disappear in ether theory

Quantization of an ether theory in a classical Newtonian framework is certainly a much simpler job than quantization of general relativity. The list of quantization problems of GR quantization which disappear in ether theory includes:

• The problem of time [13]: Once we have absolute time, we have no such problem.

• The information loss problem [22]: We have no black holes but stable frozen stars, without Hawking radiation.

• Quantum uncertainty of the light-cone and, therefore, of relativistic causality: Causality in ether theory is classical causality, connected with absolute time, and does not become uncertain.

• Closed causal loops: They do not appear in ether theory because they violate the condition $\rho > 0$. If $\rho \to 0$ the field theory limit fails, but not the fundamental atomic ether theory which is quantized.

• Physical meaning of the Hamiltonian constraint: We have a classical Hamilton formalism because we have classical absolute time.
B.2 Quantization of first order equations

In quantum field theory, the quantization of first order equations like the Lorenz gauge condition or the harmonic coordinate condition is considered to be especially problematic. Because we propose to use these conditions in our ether theory, let’s consider how they may be quantized.

Assume we have a classical atomic model of the ether. For the quantization of this model we do not have to use any field theory, multi-particle Schrödinger theory is sufficient. If we have defined this multi-particle theory, we have obtained a well-defined quantum theory of this ether. Now, conservation of particle number leads, in the classical large distance limit, to a continuity equation:

$$\partial_t \rho + \partial_i \rho v^i = 0$$ (69)

Note, especially, that this conservation law holds exactly, there are no “quantum fluctuations” of the particle number in multi-particle Schrödinger theory. Thus, quantization of such first order equations is not only possible, but a standard feature of atomic theories. What we need to quantize such an equation is, therefore, an atomic model so that the equation in question is interpreted as the conservation of some number of atoms.

This does not mean that the situation in the quantum field theory limit is nice. It is not. Nontrivial problems appear also in quantum field theory for usual condensed matter. For example, the definition of the operator $\hat{\rho}$ is not unproblematic. Especially, the canonical commutation relations proposed by Landau lead to a continuous, unbounded spectrum for $\hat{\rho}$ which is incompatible with its positivity. Such observations suggests that there may be a lot of problems of the field theory limit. But these are not problems of the fundamental atomic ether theory, nor are they quantization problems.

B.3 EPR realism and hidden variables

If we quantize some atomic ether theory using classical multi-particle Schrödinger theory, we can easily extend this to obtain hidden variable theories like Bohmian mechanics or Nelson’s stochastics which have been defined in this classical framework. Thus, ether theory, if successful, allows to generalize these hidden variable theories to a theory of everything.
On the other hand, these hidden variable theories, in combination with Bell’s theorem and the observed violation of Bell’s inequality, give independent strong support for the existence of a preferred frame. Indeed, in Bell’s theorem Einstein causality and EPR realism are the only ingredients we need. It’s violation proves that one of these two principles is wrong. Thus, we have a conflict between Einstein causality and EPR-realism.

In this conflict, EPR-realism is the more fundamental principle: It is easy to imagine a world without Einstein causality, but a world which is not EPR-realistic contradict common sense. While we agree that disagreement with common sense is not a decisive argument, it is certainly a very strong argument and cannot be simply dismissed.

Moreover, the existence of EPR-realistic hidden variable theories for quantum theory proves that EPR-realism is compatible with quantum theory. Therefore, quantum theory cannot give an independent argument for the rejection of EPR-realism. On the other hand, the existing problems of GR quantization give such independent evidence against Einstein causality. Thus, the question “which principle is compatible with quantum principles” clearly favors EPR-realism.

Therefore, it seems much more reasonable to reject Einstein causality than EPR-realism.

The situation can be reformulated in another way: If we start with EPR-realism as an axiom, Einstein causality is falsified, and we can derive that there exists a preferred frame. These considerations can be found, in more detail, in [24].

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