Beam Size Measurement with Pair Monitor and BeamCal

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At the International Linear Collider (ILC), measurement of the beam profile at the interaction point (IP) is a key issue to achieve high luminosity. We studied the beam size measurement by combination of Pair Monitor and BeamCal. We obtained measurement accuracies of 2.7%, and 6.6% for the horizontal ($\sigma_x$), and vertical ($\sigma_y$) beam sizes, respectively, for 50 bunch crossings.

1 Introduction

In the ILC, measurement of the beam size at IP is important to keep high luminosity because the luminosity critically depends on beam size. Since the vertical beam size is very small (5.7 nm), it must be measured with in 1 nm accuracy. We have studied measurement of the beam profile with Pair Monitor \(^1\)\(^2\) and BeamCal. Since the beam energy and the particle density is high at ILC, a large number of $e^+e^-$ pairs are created during the bunch crossing by three incoherent processes; Breit-Wheeler process, Bethe-Heitler process and Landau-Lifshitz process. The generated $e^\pm$ pairs are usually referred to as the pair background. The particles with the same charge as the oncoming beam are scattered with large angles and carry information on the beam profile at IP\(^3\).

Pair Monitor and BeamCal offer the independent information, and combining them results in better measurement accuracies. In this paper, we report a reconstruction of the beam sizes ($\sigma_x$, $\sigma_y$) using the Taylor matrixes and present the expected measurement accuracies. The beam profile can be reconstructed by measurement of the azimuthal hit distribution of scattered $e^+e^-$ pairs by Pair Monitor and the energy deposit in BeamCal.

2 Simulation

The performance of Pair Monitor and BeamCal was studied, using the geometry of the GLD \(^4\) scaled to match with 3.5 Tesla and a simulation package Jupiter \(^5\). The pair background was generated by CAIN \(^6\). BeamCal was located at 430 cm from IP, and Pair Monitor was located in front of BeamCal. Solenoid field (3.5T) with the anti-DID (reversed Detector Integrated Dipole) was used for the magnetic field. BeamCal is a cylinder of 18 cm radius, which consists of 33 layers of 3.5 mm thick tungsten for the absorber and 0.3 mm thick silicon for the sensor. Pair Monitor is a silicone disk of 10 cm radius and 200 µm thickness. There are two holes on Pair Monitor and BeamCal whose radius are 1.0 cm and 1.8 cm for the incoming and outgoing beams, respectively.

In this study, the nominal beam sizes are 639 nm and 5.7 nm for the horizontal and vertical beam sizes, respectively, as the nominal beam size. The center of mass energy is set to 500 GeV with the beam crossing angle of 14 mrad.

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3 The measurement variables

3.1 Measurement variables of Pair Monitor

The maximum radius of hit reflects the maximum transverse momentum of the pairs, which in turn is given by the electromagnetic fields of the oncoming beam. Since the vertical beam size is much smaller than the horizontal beam size, the maximum electromagnetic field is inversely proportional to the horizontal beam size. We defined the shoulder of the radial distribution ($R_{shl}$) as the radius to contain 97.5% of all the hits. Figure 1 shows $R_{shl}$ as a function of the horizontal beam size for several vertical beam sizes. As expected, $R_{shl}$ decreases for larger horizontal beam size.

The azimuthal scattering angle of the pairs at the bunch crossing would depend on the horizontal to vertical aspect ratio of the bunch, which would then affect the azimuthal distribution of the hit density on Pair Monitor. We thus studied the distribution of the hit density as a function of the radius from the center of the extraction beam pipe ($R$) and the angle around it ($\phi$). We define $N_0$ as the number of hits in $-\pi < \phi < -2.0$ radian and $2.8 < \phi < \pi$ radian for $0.6 \cdot R_{shl} < R < 0.8 \cdot R_{shl}$. In order to derive the beam information from the azimuthal distribution, we compared $N_0$ to the total number of hits ($N_{all}$). $N_0/N_{all}$ decreases for larger horizontal and vertical beam sizes.

The total number of the hits on Pair Monitor, $N_{all}$, reflects the luminosity which is inversely proportional to the vertical and horizontal beam size. Since the total number of the pair backgrounds are nearly proportional to luminosity, the number of all the hits on Pair Monitor is inversely proportional to the vertical and horizontal beam sizes.

3.2 Measurement variables of BeamCal

We also investigated the measurement variables related to BeamCal. We defined the average radius ($R_{ave}$) as follows:

$$R_{ave} = \frac{\sum R_i \times E_{dep_i}}{\sum E_{dep_i}},$$

where $E_{dep_i}$ means the energy deposit and $R_i$ is the radius from the center of the extraction beam pipe of the $i$-th cell. $R_{ave}$ decreases for larger horizontal beam size similar to $R_{shl}$ of Pair Monitor.

Figure 1: $R_{shl}$ vs. $\sigma_x$ fitted with second-order polynomials. $R_{shl}$ decreases for larger $\sigma_x$.

Figure 2: $1/E_{dep_{all}}$ vs. $\sigma_y$ fitted with second-order polynomials. $1/E_{dep_{all}}$ increases for larger $\sigma_x$ and $\sigma_y$.
The total energy deposit on BeamCal, $E_{\text{dep}}$, reflects the luminosity similar to $N_{\text{all}}$ in the case of Pair Monitor. Figure 2 shows $1/E_{\text{dep}}$ as functions of the vertical beam size for several horizontal beam sizes. $1/E_{\text{dep}}$ is proportional to the beam size.

4 Reconstruction of beam sizes

We reconstructed the beam sizes from the hit distribution of the pair backgrounds on Pair Monitor and the energy deposit in BeamCal. Five measurement variables ($R_{\text{shl}}$, $N_{0}/N_{\text{all}}$, $1/N_{\text{all}}$, $R_{\text{ave}}$, and $1/E_{\text{dep}}$) are used for the reconstruction. These measurement variables $(m_i, i=1, 2, \ldots, 5)$ should depend on the beam sizes $(\sigma_x, \sigma_y)$, they can be expanded around the nominal beam sizes $(\sigma_{0x}, \sigma_{0y})$ by the Taylor series. Since these measurement variables are fitted with second-order polynomials in this study, and $m_i$ are expanded up to the second-order as follows:

$$\Delta \vec{m} = \vec{m}(\sigma_x, \sigma_y) - \vec{m}(\sigma_{0x}, \sigma_{0y})$$

$$= \sum_{a=x,y} \frac{\partial \vec{m}}{\partial \sigma_a} \Delta \sigma_a + \sum_{a=x,y} \sum_{\beta=x,y} \frac{1}{2} \Delta \sigma_\beta \frac{\partial^2 \vec{m}}{\partial \sigma_a \partial \sigma_\beta} \Delta \sigma_a$$

$$= [A_1 + \Delta \vec{m}^T \cdot A_2] \cdot \Delta \vec{m},$$

(2)

where $\vec{m} = (m_1, \ldots, m_5)$, $\Delta \vec{m} = (\Delta \sigma_x, \Delta \sigma_y)$, $\Delta \sigma_\alpha = \sigma_\alpha - \sigma_{0\alpha}$, and $A_1$ is a $5 \times 3$ matrix of the first order coefficients of the Taylor expansion and $A_2$ is a tensor of the second derivative coefficients. The beam size is reconstructed by multiplying the inverted matrix of a coefficient of $\Delta \vec{m}$ in Equation (2) as follows:

$$\Delta \vec{m} = [A_1 + \Delta \vec{m}^T \cdot A_2]^+ \cdot \Delta \vec{m},$$

(3)

where the superscript “+” indicates the Moore Penrose inversion which gives the inverse matrix of a non-square matrix $A$ as $A^+ = (A^T A)^{-1} A^T$.

We obtain the numerical values of the matrix $(A_1)$ and the tensor $(A_2)$ of Equation (3) by fitting the data by second-order polynomials. The beam sizes at IP is then reconstructed by the inverse matrix method as shown Equation (3), where it is solved iteratively as follows (7):

$$\Delta \vec{m}_n = [A_1 + \Delta \vec{m}^T_{n-1} \cdot A_2]^+ \cdot \Delta \vec{m}$$

(4)

The iteration was repeated until consecutive iterations satisfied $(\Delta \vec{m}_n - \Delta \vec{m}_{n-1})/\Delta \vec{m}_n < 1\%$.

Figure 3 shows the relative deviations of the reconstructed vertical beam size from the true beam size for 50 bunch crossings which are measured by BeamCal, Pair Monitor, and combination with them. The 95% C.L. intervals for these distributions were 13.4%, 8.6%, and 6.6%, respectively. On the other hand, we obtained the 95% C.L. intervals of 4.4%, 3.3%, and 2.7% for the horizontal beam size. From these results, we conclude that the measurement accuracies for the vertical and horizontal beam size are 6.6% and 2.7% with Pair Monitor and BeamCal.

5 Conclusions
We studied a technique of beam size measurement with the Pair Monitor and BeamCal. The method utilizes the second-order inversion of the Taylor expansion. Five measurement variables were used to reconstruct the beam sizes ($\sigma_x, \sigma_y$), and the matrix elements of the expansion were obtained by fitting with second-order polynomials of the beam sizes. The combined measurement with the BeamCal and Pair Monitor produces better accuracies than that with only one of them. The measurement accuracies of the horizontal and vertical beam sizes for 50 bunch crossings were found to be 2.7% (17.3 nm) and 6.6% (0.38 nm), respectively.

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References

[1] Presentation: http://ilcagenda.linearcollider.org/contributionDisplay.py?contribId=311&sessionId=13&confId=2628
[2] K. Ito et al., Study of Beam Profile Measurement at Interaction Point in International Linear Collider, arXiv:0901.4151
[3] T. Tauchi and K. Yokoya, Nanometer Beam-Size Measurement during Collisions at Linear Colliders, KEK preprint 94-122.
[4] GLD Concept Study Group: K. Abe et al. GLD Detector Outline Document. physics/0607154, 2006.
[5] Jupiter web-page: http://acfahep.kek.jp/subg/sim/simtools/
[6] CAIN web-page: http://lcedev.kek.jp/~yokoya/CAIN/cai25/
[7] M. Ternick, Fast Beam Diagnostics through Beamstrahlung at TESLA.
[8] FCAL collaboration web-page: http://www-zeuthen.desy.de/ILC/fcal/

Figure 3: Relative deviations of the vertical beam size by, from the top, the BeamCal, Pair Monitor, and combination with the BeamCal and Pair Monitor.