An integrated methodology for fatigue life prediction of carbon fiber-reinforced polymer matrix composites under constant and variable $R$-ratio loadings

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Abstract. The progress in the anisomorphic constant fatigue life (CFL) diagram approach for efficiently identifying the fatigue of carbon fiber-reinforced composites under constant waveform loading over the whole range of mean stress is reviewed. The asset of this approach is the concept of critical stress ratio. The anisomorphic formulation of CFL diagram is flexible enough not only to increase the number of segments to accommodate the transient change in sensitivity to mean stress, but also to take a generalized form that considers the effect of temperature as well as the effect of variability in fatigue life. The anisomorphic approach is shown to succeed in describing the asymmetric and nonlinear CFL diagrams for carbon fiber composites of different kinds. It is suggested that the classic procedure for prediction of spectrum loading fatigue life of composites using the Miner rule with the rain flow cycle counting method is powered by integrating with the anisomorphic CFL diagram approach.

1. Introduction

Prediction of variable loading fatigue life of a given material raises three important issues to be taken into account [1-6]. A spectrum load that a material is required to withstand during service is a series of waveforms of changing stress level and stress ratio. It is complicated. To simplify the problem of fatigue life prediction, the spectrum loading is replaced with a set of constant waveform loading. The values of fatigue life for the reduced constant waveform loadings not only of different levels of peak/valley stress but also of different levels of mean stress are required to estimate their damaging effects. The ultimate state of damage accumulation has to be defined to judge fatigue failure under the spectrum loading. Typically, the rain flow cycle counting method [7, 8], a constant fatigue life diagram [9, 10] and the Miner rule [11, 12] are employed in the classic engineering scenario.

The understanding of the effects of constant stress level and stress ratio loading on fatigue life of composites has successfully been developed [13, 14]. The comprehensive investigation into the effect of stress ratio on constant waveform loading fatigue life of CFRPs over the whole range of mean stress has lead to significant progress in modeling of a CFL diagram that is suitable for composites.

In regard to the effect of variable loading on fatigue life of composites, however, we are still limited to an incomplete understanding the phenomenon and thus demand to establish an appropriate methodology for it [1-11]. To develop our understanding of the effect of load sequence on variable waveform loading fatigue life of CFRPs and to establish a method for predicting it as efficiently as possible, we need to study not only the effect of variation in alternating stress amplitude, but also the effect of variation in mean stress, i.e. the variation in $R$-ratio, on fatigue failure in CFRPs.
This article reviews the progress of the anisomorphic CFL diagram approach for carbon fiber-reinforced composites that has been made in the last decade [15-18], and demonstrates not only that it allows fully identifying the fatigue of composites under constant waveform loading, but also that it is flexible enough to be customized to take into account the effect of temperature [19, 20] as well as the effect of variability in fatigue life [21, 22]. It is further demonstrated that the classic fatigue life prediction methodology inviting the anisomorphic CFL diagram as its member scenario is a promising tool that allows consistently and moderately conservative prediction of fatigue life of CFRPs under variable waveform loading.

2. Anisomorphic CFL diagram approach

For metals of equal strength in tension and compression, as shown in figure 1(a), the particular value of stress ratio \( R = -1 \) is usually chosen to identify their fundamental S-N curves that are used to construct the Goodman diagrams for them. In contrast to metals, most fiber-reinforced composites exhibit different strengths in tension and compression. For those composites, therefore, it is suitable to replace...
If the tensile strength is larger than the compressive strength of a composite, the margin to static failure is larger in tension. Then, the value of mean stress of equal distance from the tensile and compressive strengths deviates from zero to a positive number as illustrated in figure 1(b). This suggests dividing T-C loading into tension-dominated T-C loading and compression-dominated T-C loading by means of the particular waveform whose mean stress takes a value of equal distance from the tensile and compressive strengths. The particular waveform corresponds to the critical stress ratio that is given by the ratio of compressive strength to tensile strength, and the use of it as the basis for construction of a CFL diagram is the distinguishing feature of the anisomorphic formulation [15-22].

2.1. Two-segment anisomorphic CFL diagram

In the two-segment anisomorphic formulation [15, 16], the CFL envelope for a given constant value of life is composed of two smooth curves associated with tension-dominated fatigue failure and compression-dominated fatigue failure, respectively. They are smoothly connected with each other at a point on the radial straight line with a constant amplitude ratio

$$\frac{\sigma_a}{\sigma_m} = \frac{1 - \chi}{1 + \chi}$$  \hspace{1cm} (1)

where $\chi$ is the critical stress ratio equal to the ratio of compressive strength $\sigma_c$ ($< 0$) to tensile strength $\sigma_t$ ($> 0$); i.e., $\chi = \sigma_c / \sigma_t \in (-\infty, 0)$. The two member curves of each anisomorphic CFL envelope on the two segments can be described, respectively, by means of the following piecewise defined functions:

[I] Tension-dominated segment ($\sigma_m^{(x)} \leq \sigma_m \leq \sigma_t^{(x)}$):

$$-\frac{\sigma_a - \sigma_a^{(x)}}{\sigma_a^{(x)}} = \left( \frac{\sigma_m - \sigma_m^{(x)}}{\sigma_t^{(x)} - \sigma_m^{(x)}} \right)^{\frac{1}{k_t^{(x)}}}$$  \hspace{1cm} (2)

[II] Compression-dominated segment ($\sigma_c \leq \sigma_m < \sigma_c^{(y)}$):

$$-\frac{\sigma_a - \sigma_a^{(x)}}{\sigma_a^{(y)}} = \left( \frac{\sigma_m - \sigma_m^{(y)}}{\sigma_c^{(y)} - \sigma_m^{(y)}} \right)^{\frac{1}{k_c^{(y)}}}$$  \hspace{1cm} (3)

where $\sigma_a^{(x)}$ and $\sigma_a^{(y)}$ represent the alternating stress and mean stress components of the maximum fatigue stress $\sigma_f^{(y)}$ for fatigue loading at the critical stress ratio $\chi$. The exponents $k_t^{(x)}$ and $k_c^{(y)}$ play the role to adjust the rate of change in the shape of a CFL curve from a straight line to a parabola on the tension-dominated and compression-dominated segments, respectively. The exponent $\psi^{(x)}$ represents the fatigue strength ratio associated with loading at the critical stress ratio $\chi$, and it is defined as

$$\psi^{(x)} = \frac{\sigma_f^{(x)}}{\sigma_c}$$  \hspace{1cm} (4)

The exponent $\psi^{(x)}$ is treated as a continuous function of the number of cycles to failure $N_f$; i.e.,

$$2N_f = f_f^{(x)}(\psi^{(x)})$$  \hspace{1cm} (5)

Figure 2 shows the two-segment anisomorphic CFL diagram identified for a woven CFRP laminate at RT [19]; dashed lines and symbols indicate the predicted and observed results, respectively. It was assumed that $k_t^{(x)} = k_c^{(y)} = 1$. From figure 2, it is seen that the two-segment anisomorphic CFL diagram approach succeeds in adequately describing the experimental CFL diagram over the whole range of mean stress in the tested range of fatigue life. The experimental results show that the CFL diagram for the woven CFRP laminate at RT is asymmetric about the alternating stress axis, and the peaks of the CFL envelopes for different constant values of life lie almost on the radial line of the constant...
amplitude ratio associated with fatigue loading at the critical stress ratio. The latter observation justifies the concept of critical stress ratio in the anisomorphic formulation of a CFL diagram. The two-segment anisomorphic CFL diagram approach has successfully been applied to non-woven CFRP laminates as well [15, 16].

2.2. Three-segment anisomorphic CFL diagram
A class of composite laminates exhibits a significant transition in mean stress sensitivity in fatigue between T-T and C-C loadings [17, 18]. In that case, it may be an oversimplification to rely on only the S-N curve for the critical stress ratio to construct the CFL diagram over the whole range of mean stress.

To cope with a significant distortion of a CFL diagram due to a large change in mean stress sensitivity, a transitional segment may be inserted between the two segments that are associated with T-T and C-C dominated fatigue failures, respectively [17]. It has a role of connecting the two neighboring segments, for simplicity, with the aid of linear interpolation. To define the transitional segment, another reference stress ratio is introduced which is called the sub-critical stress ratio and designated by \( \gamma \). The critical and sub-critical stress ratios divide the CFL diagram into three segments. The CFL curves in the three segments are described by means of the following piecewise-defined function over the domain of mean stress:

[I] Tension dominated zone \( (\sigma_m^{(1)} \leq \sigma_m \leq \sigma_m^{(l)}) \):

\[
\frac{\sigma_m - \sigma_m^{(1)}}{\sigma_m^{(l)} - \sigma_m} = \left( \frac{\sigma_m - \sigma_m^{(1)}}{\sigma_m^{(l)} - \sigma_m} \right)^{\gamma} \]

(6)

[II] Tension-compression transitional zone \( (\sigma_m^{(2)} \leq \sigma_m < \sigma_m^{(3)}) \):

\[
\frac{\sigma_m - \sigma_m^{(2)}}{\sigma_m^{(3)} - \sigma_m} = \frac{\sigma_m^{(2)} - \sigma_m^{(1)}}{\sigma_m^{(3)} - \sigma_m^{(1)}} \]

(7)

[III] Compression dominated zone \( (\sigma_c < \sigma_m \leq \sigma_m^{(2)}) \):
Figure 3. The three-segment anisomorphic CFL diagram for a carbon/epoxy [±60]_{3S} laminate [Kawai and Murata (2010)].

\[ \frac{\sigma_i^{(x_i)}}{\sigma_a^{(x_a)}} = \left( \frac{\sigma_{c}^{(x_c)}}{\sigma_{m}^{(x_m)}} \right)^2 \psi_{\chi}^{(x_i)} \]

(8)

where \( \sigma_i^{(x_i)} \), \( \sigma_a^{(x_a)} \), \( \sigma_c^{(x_c)} \), and \( \sigma_m^{(x_m)} \) represent the alternating and mean stress components of the maximum fatigue stresses \( \sigma_{max}^{(x)} \) and \( \sigma_{mix}^{(x)} \) that are associated with fatigue loadings at the critical and sub-critical stress ratios \( \chi \) and \( \chi' \), respectively.

The two variable exponents \( \psi_{z}^{(x)} \) and \( \psi_{\chi}^{(x)} \) involved by the piecewise-defined function for the three-segment anisomorphic CFL diagram approach play the roles which are similar to the role of \( \psi_{z}^{(x)} \) in the two-segment anisomorphic CFL diagram approach. The critical fatigue strength ratio \( \psi_{z}^{(x)} \) for the three-segment anisomorphic CFL diagram is defined by equation (4). The sub-critical stress ratio is defined as

\[ \psi_{\chi}^{(x)} = \begin{cases} \frac{\sigma_{max}^{(x_m)}}{\sigma_{c}^{(x_c)}}, & \chi \leq \chi' \leq 0 \\ \frac{\sigma_{mix}^{(x)}}{\sigma_{c}^{(x_c)}}, & -\infty < \chi < \chi' \end{cases} \]

(9)

As in the two-segment anisomorphic CFL diagram approach mentioned above, the critical and sub-critical fatigue strength ratios \( \psi_{z}^{(x)} \) and \( \psi_{\chi}^{(x)} \) are treated as continuous monotonic functions of \( N_f \); i.e.,

\( \psi_{z}^{(x)} = f^{-1}(2N_f) \) and \( \psi_{\chi}^{(x)} = f^{-1}(2N_f) \).

Figure 3 shows the three-segment anisomorphic CFL diagram for a non-woven carbon/epoxy [±60]_{3S} laminate, along with the experimental CFL data [17]. A transitional segment was inserted between the T-T and C-C dominated segments. The modification is found effective to improve the accuracy of prediction of the distorted CFL curves for the [±60]_{3S} laminate. The value of the shape transition exponent \( k_\chi = 0.2 \) retards the change in shape from a straight line to a parabola in the T-T dominated range \( \chi < \chi' < 1 \). The adjustment of the rate of shape change successfully improves the accuracy of prediction of the CFL curves for T-T loading.
2.3. Four-segment anisomorphic CFL diagram

Two-side modification to the two-segment anisomorphic CFL diagram results in a four-segment version that removes the step of judging which segment of the two-segment anisomorphic CFL diagram should be modified [18].

The right and left transitional segments are defined as \([L, \chi_L] = \{R: -\infty < \chi_L \leq R \leq \chi_R \leq 0\}\) and \([R, \chi] = \{R: -\infty \leq \chi_L \leq R \leq \chi < 0\}\), respectively, using two auxiliary stress ratios \(\chi_L\) and \(\chi_R\) in conjunction with the critical stress ratio \(\chi\). The right and left auxiliary stress ratios \(\chi_R\) and \(\chi_L\) are determined so as to satisfy the condition \(-\infty \leq \chi_L \leq \chi \leq \chi_R \leq 0\). The total interval of mean stress \([\sigma_{c}, \sigma_{n}]\) is thus separated into four subintervals: I. \([\sigma_{c}^{(L)}, \sigma_{c}^{(R)}]\); II. \([\sigma_{n}^{(L)}, \sigma_{n}^{(R)}]\); III. \([\sigma_{c}^{(L)}, \sigma_{c}^{(R)}]\); IV. \([\sigma_{c}^{(L)}, \sigma_{c}^{(R)}]\). The CFL envelopes on these subintervals are described by means of the following piecwise-defined functions:

[I] Tension-dominated zone \((\sigma_{m}^{(T)} \leq \sigma_{m} \leq \sigma_{c})\):

\[
\frac{\sigma_{a} - \sigma_{a}^{(T)}}{\sigma_{a}^{(T)}} = \left(\frac{\sigma_{m} - \sigma_{m}^{(T)}}{\sigma_{c}^{(T)}}\right)^{2} \psi_{T}^{(T)}
\]

(10)

[II] Right transitional zone \((\sigma_{m}^{(T)} \leq \sigma_{m} \leq \sigma_{c}^{(T)})\):

\[
\frac{\sigma_{a} - \sigma_{a}^{(T)}}{\sigma_{a}^{(T)}} = \frac{\sigma_{m}^{(T)} - \sigma_{m}}{\sigma_{c}^{(T)} - \sigma_{m}^{(T)}}
\]

(11)

[III] Left transitional zone \((\sigma_{m}^{(T)} \leq \sigma_{m} \leq \sigma_{c}^{(T)})\):

\[
\frac{\sigma_{a} - \sigma_{a}^{(T)}}{\sigma_{a}^{(T)}} = \frac{\sigma_{m}^{(T)} - \sigma_{m}}{\sigma_{c}^{(T)} - \sigma_{m}^{(T)}}
\]

(12)

[IV] Compression-dominated zone \((\sigma_{c} \leq \sigma_{m} \leq \sigma_{c}^{(C)})\):

\[
\frac{\sigma_{a} - \sigma_{a}^{(C)}}{\sigma_{a}^{(C)}} = \left(\frac{\sigma_{m} - \sigma_{m}^{(C)}}{\sigma_{c}^{(C)}}\right)^{2} \psi_{C}^{(C)}
\]

(13)

where \(\psi_{T}^{(T)}\) and \(\psi_{C}^{(T)}\) designate the fatigue strength ratios associated with the right and left auxiliary stress ratios, \(\chi_R\) and \(\chi_L\), respectively, and they are defined as

\[
\psi_{T}^{(T)} = \frac{\sigma_{m}^{(T)}}{\sigma_{c}^{(T)}}
\]

(14)

\[
\psi_{C}^{(T)} = \frac{\sigma_{m}^{(C)}}{\sigma_{c}^{(C)}}
\]

(15)

The variables \(\psi_{a}^{(T)}, \psi_{m}^{(T)}, \psi_{a}^{(C)}, \psi_{m}^{(C)}\) represent the alternating stress and mean stress components of the maximum fatigue stresses for fatigue loadings at the characteristic stress ratios.

Figures 4(a) and 4(b) show the four-segment anisomorphic CFL diagrams for on-axis and off-axis loadings of a unidirectional carbon/epoxy laminate, respectively [18]. For construction of these four-segment anisomorphic CFL diagrams, the standard auxiliary stress ratios \(\chi_R = 0\) and \(\chi_L = -\infty\) were used. As expected, the right and left transitional segments accommodate the anisomorphic CFL diagram to a large change in mean stress sensitivity from a higher level in tension-dominated fatigue loading to a lower level in compression-dominated fatigue loading, and thus the addition of these transitional segments to the anisomorphic CFL diagram successfully improves the accuracy of description.
3. Temperature-dependent anisomorphic CFL diagram approach

Consideration of the temperature dependence of the static strengths in tension and compression and of the reference S-N relationship for the critical stress ratio allows generalization of the anisomorphic CFL diagram approach for a given composite into a temperature-dependent version [19, 20]. In this attempt, the temperature dependences of tensile and compressive strengths are first modeled on the basis of the static test data at different temperatures. They determine the temperature dependence of the critical stress ratio as

\[ \chi(T) = \frac{\sigma_c(T)}{\sigma_s(T)} \]  

(16)

To predict the reference S-N relationship for the critical stress ratio at any temperature in a given range, we assume a grand master S-N curve for the critical stress that is described by means of a function

\[ p_{lm}^{(cr)} = g(\Psi_{\chi}) \]  

(17)

Figure 4. The four-segment anisomorphic CFL diagrams identified for unidirectional carbon/epoxy laminates; (a) [0]_{16}, (b) [30]_{16} [Kawai and Ito (2014)].
where $\Psi_z$ is the modified fatigue strength ratio [23] for the critical stress ratio

$$\Psi_z = \frac{\sigma_{s}^{(x)}}{\sigma_{T}^{(z)}}$$

and $P_{LM}^{(F)}$ is the life-temperature parameter of the Larson-Miller type [24]

$$P_{LM}^{(F)} = T(F + \log_{10} N_f)$$

Figure 5 shows the grand master S-N curve for the critical stress for a woven CFRP laminate [20]. The fatigue data for the critical stress ratios at different temperatures are doubly scaled in terms of stress and life using the modified fatigue strength ratio $\Psi_z(T)$ and the LM parameter for fatigue $P_{LM}^{(F)}$. The normalized grand master S-N relationship $\Psi_z(T)$ and $P_{LM}^{(F)}$ can successfully be identified.

Figures 6(a) and 6(b) show the predicted anisomorphic CFL diagrams for the woven CFRP laminate at RT and 100°C, respectively [20]. It is seen that the predicted CFL diagram agrees with the experimental CFL diagram, regardless of temperature. These comparisons demonstrate that the proposed temperature-dependent anisomorphic CFL diagram approach is useful for predicting the CFL diagrams for the woven CFRP laminate at different temperatures with reasonable accuracy over the whole range of mean stress.

4. Probabilistic anisomorphic CFL diagram approach

Using the statistical distributions for the static strength data in tension and compression and for the fatigue life data at the critical stress ratio, we can construct the anisomorphic CFL diagram for any constant value of probability of failure [21, 22]. The probabilistic anisomorphic CFL diagram allows predicting the P-S-N curves for any values of stress ratio.

The percentiles of the static strengths in tension and compression can be calculated using the distribution functions fitted to the observed static strength data. By contrast, however, it is not straightforward to deal with the P-S-N curve for the critical stress ratio since the critical stress ratio is
Figure 6. The predicted anisomorphic CFL diagrams for a woven carbon fabric composite laminate; (a) RT, (b) 100°C [Kawai, Matsuda and Yoshimura (2012)].

not a constant but a random variable. The modified fatigue strength ratio and fatigue life are random variables as well. To cope with the stress-ratio and probability dependence of the P-S-N curve for the critical stress ratio, we take advantage of the modified fatigue strength ratio that is useful for horizontal scaling to define a normalized master S-N curve. The modified fatigue strength ratio for any percentile of the critical stress ratio $\chi(P)$ is defined as follows:

$$\psi_f^{(\chi(P))} = \begin{cases} \frac{\sigma_{m}^{(\chi(P))}}{\sigma_f(P) \cdot \sigma_{m}^{(\chi(P))}}, & -1 < \chi(P) < 0 \\ \frac{\sigma_{m}^{(\chi(P))}}{\sigma_c(P) \cdot \sigma_{m}^{(\chi(P))}}, & -\infty < \chi(P) < -1 \end{cases}$$

The non-dimensional S-N curve for the critical stress ratio for any probability of failure is described by means of a function

$$2N_f^{(\chi(P))}(P) = g_f^{(\chi(P))}$$
Using the median S-N curve as the basis for transformation and assuming a small change in $\chi(P)$ with $P$, we can reduce equation (21) to the following form:

$$2N_f^{(x_p)}(P) = \Psi_p^{(x_p)}(P) = g_p(\Psi_p^{(x_p)}) \Lambda_f(P)$$

(22)

where the function $\Lambda_f(P)$ represents the ratio of $P^{th}$ percentile to median that can uniquely be determined using the cumulative distribution of fatigue life.

Figure 7 shows the normalized P-S-N curves for the critical stress ratio that are based on a lognormal distribution fitted to the fatigue life data for the experimental critical stress ratio [21]. The symbols indicate the median of the experimental fatigue life data, and the solid curve indicates the fitting to them. Figures 8(a) and 8(b) show the anisomorphic CFL diagrams for the different constant values of failure probability $P = 10\%$ and $90\%$, respectively. It is seen that the anisomorphic CFL envelopes predicted for different constant values of failure probability agree well with the experimental CFL data for $R = 0.1$ and 10, regardless of the probability of failure. Note that the experimental results for $R = 0.1$ and 10 were not used for the verification observed above.

5. Integration into a classic spectrum loading fatigue life prediction methodology

Kawai and Ishizuka [26] have performed variable $R$-ratio loading tests and evaluated the classical fatigue life prediction methodology. One of the variable $R$-ratio loading tests they performed is illustrated in figure 9. Two waveforms of different $R$-ratios that are equal in life are alternately repeated; i.e., $R_i \neq R_j$, $N_i^{(R_i)} = N_j^{(R_j)} = N^*$, $\sigma_{\max}^{(R_i)} \neq \sigma_{\max}^{(R_j)}$ and $\sigma_{\max}^{(R_i)} \neq \sigma_{\max}^{(R_j)}$. Since the alternation is along a vertical line as shown in figure 9(b), it was called a vertical alternating $R$-ratio (ARR) loading. It is a simple block loading. Nevertheless, this sort of variable loading tests allows us to examine the effect of simultaneous alternation in stress level and $R$-ratio on fatigue life.

Figure 10 shows the correlation between the predicted and observed fatigue life values for different vertical ARR loadings [26]. Note that the fatigue life predicted for vertical ARR loading that is shown in figure 10 was made by applying the Miner rule to the reduced waveforms counted by the rain flow method. The values of fatigue life for the counted constant waveforms were predicted using the anisomorphic CFL diagram that was constructed for the composite used in the ARR loading tests.
From figure 10, it is seen that all the data points are distributed in the left border of the range of a factor of two. The horizontal axis indicates the predicted fatigue life. From figure 10, therefore, we can see that the variable loading fatigue life prediction methodology allows predicting moderately conservative values of fatigue life for the vertical ARR loadings, regardless of the pair of $R$-ratios. The same feature was observed in the range of probability of failure lower than 90%. A moderately conservative prediction of fatigue life is favorable from an engineering point of view. The correlation results in figure 10 thus prove the potential usefulness of the integration of the Miner rule, the rain flow cycle counting method and the anisomorphic CFL diagram approach in the context of the variable waveform loading fatigue life prediction of composites.

6. Conclusions
A series of attempts at systematic modeling of a full shape of CFL diagram suitable for carbon fiber-reinforced composites that have been made by the author in the last decade was reviewed with validation at each stage of sophistication. The asset of the anisomorphic modeling of CFL diagram for composites, which distinguishes it from the attempts by other researchers, is to take into account the...
fatigue loading at the critical stress ratio under which damage accumulation occurs most significantly. The two-, three- and four-segment anisomorphic CFL diagrams have been formulated, and they can be used properly taking into account the significances of the asymmetry and nonlinearity in CFL curves for the composites to consider. The more the number of segments increases, the more the fatigue data are required for identification. Nevertheless, it is unique in its general formulation not only that allows identification by means of a limited amount of test data but also that reduces to a linear model of the Goodman type on one extreme and to a parabolic model of the Gerber type on the other, and thus allows of any in between. Another feature of the anisomorphic formulation is the ease of generalization that makes it possible for us to consider not only the effect of temperature but also the probability of failure.
A classical fatigue life prediction methodology consists of the three elements; typically 1) the rain flow cycle counting method to reduce a given loading spectrum into the effective constant \( R \)-ratio waveforms, 2) a CFL diagram approach to estimate the values of life for the counted effective waveforms of different constant values of \( R \)-ratio, and 3) the Miner rule to judge fatigue failure in composites under the given spectrum loading. This methodology is favored, because of its simplicity, in engineering fatigue analysis of composites. The join of the anisomorphic CFL diagram approach in the classic three-element methodology is expected to establish a powerful tool for engineering fatigue and reliability analyses of composites that undergo complicated cyclic loading.

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