DIPOLE POLE POLARIZABILITIES OF $\pi^\pm$–MESONS

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The main experimental works, where dipole polarizabilities of charged pions have been determined, are considered. Possible reasons for the differences between the experimental data are discussed. In particular, it is shown that the account of the $\sigma$-meson gives a significant correction to the value of the polarizability obtained in the latest experiment of the COMPASS collaboration. We present also new fit results for the $(\gamma\gamma \to \pi\pi)$ reaction.

Keywords: polarizability, pion, sigma meson, dispersion relations, chiral perturbation theory

I. INTRODUCTION

Pion polarizabilities are fundamental structure parameters values of which are very sensitive to predictions of different theoretical models. Therefore, an accurate experimental determination of these parameters is very important for testing the validity of such models.

The most of experimental data obtained for the difference of the electric ($\alpha_1$) and ($\beta_1$) dipole polarizabilities of the charged pions are presented in Table I.

The polarizabilities were determined by analyzing the processes of the high energy pions scattering in the Coulomb field of heavy nuclei ($\pi^- A \to \gamma\pi^- A'$) via the Primakoff effect, radiative pion photoproduction from proton ($\gamma p \to \gamma\pi^+ n$), and two-photon production of pion pairs ($\gamma\gamma \to \pi\pi$). As seen from Table I, the data vary from 4 up to 40 and are in conflict even for experiments performed with the same method. In this paper we will consider possible reasons for such disagreements.

II. SCATTERING OF PIIONS IN THE COULOMB FIELD OF HEAVY NUCLEI

The charged pion polarizability was obtained at the first time in the work from the scattering of $\pi^-$ mesons off the Coulomb field of heavy nuclei.

| Experiments | $(\alpha_1 - \beta_1)_{\pi^\pm}$ |
|-------------|---------------------------------|
| $\gamma p \to \gamma\pi^+ n$ MAMI (2005) [1] | $11.6 \pm 1.5_{\text{stat}} \pm 3.0_{\text{syst}} \pm 0.5_{\text{mod}}$ |
| $\gamma p \to \gamma\pi^+ n$ Lebedev Phys. Inst. (1984) [2] | $40 \pm 20$ |
| $\pi^- A \to \gamma\pi^- A'$ Serpukhov (1983) [3] | $13.6 \pm 2.8 \pm 2.4$ |
| $\pi^- A \to \gamma\pi^- A'$ COMPASS (2007) [4] | $4.0 \pm 1.2 \pm 1.4$ |
| $\gamma\gamma \to \pi^+\pi^-$, D. Babusci et al. (1992) [5] | $38.2 \pm 9.6 \pm 11.4$ |
| PLUTO [6] | $34.4 \pm 9.2$ |
| DM I [7] | $4.4 \pm 3.2$ |
| MARK II [8] | $5.4 \pm 0.95$ |
| J.F. Donoghue, B.R. Holstein (1993) [9], Mark II [8] | $5.4$ |
| A.E. Kaloshin, V.V. Serebryakov (1994) [10], Mark II [8] | $5.25 \pm 0.95$ |
| L.V. Fil’kov, V.L. Kashevarov (2006) [11] | $13^{+2.6}_{-1.9}$ |
| the fit to the data [8, 12–16] up to 2.5 GeV | |
| R. Garcia-Martin, B. Moussallam (2010) [17] | $4.7$ |

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The cross section of the radiative pion scattering $\pi A \rightarrow \pi\gamma A'$ via the Primakoff effect can be written as

$$\frac{d\sigma_{\pi A}}{d\sigma_{\pi A} dQ^2 d\cos \theta_{\gamma\gamma}} = \frac{Z^2\alpha}{\pi(s-\mu^2)} F(f)^2(Q^2) \frac{Q^2-Q_{\min}^2}{Q^4} \frac{d\sigma_{\pi A}}{d\cos \theta_{\gamma\gamma}}$$

(1)

where $F_{eff}$ is the electromagnetic form-factor of nucleus, $\alpha$ is the fine-structure constant, $Z$ is the charge number of the nucleus, and $Q^2$ is the negative 4-momenta transfer squared, $Q^2 = -(p_A - p'_A)^2/(4E^2_{beam})$, where $s$ is the square of the total energy of the process $\gamma + \pi^\pm \rightarrow \gamma + \pi^\pm$, $E_{beam}$ is the pion beam energy. This cross section has a Coulomb peak at $Q^2 = 2Q_{\min}^2$ with a width equal to $\approx 6.8Q_{\min}^2$.

The experiment was carried out at a beam energy equal to 40 GeV. In this case if the energy of the incident photon in the pion rest frame $\omega_1 = 600$ MeV, then $Q_{\min}^2$ is equal to $4.4 \times 10^{-6}$ (GeV/c)^2. It was shown that the Coulomb amplitude dominates in this case for $Q^2 \leq 2 \times 10^{-4}$ (GeV/c)^2. The experiment was carried out at $Q_{cut}^2 < 6 \times 10^{-4}$ (GeV/c)^2. Events in the region of $Q^2$ of $(2 - 8) \times 10^{-3}$ (GeV/c)^2 were used to estimate the strong interaction background. This background was assumed to behave either as $\approx Q^2$ in the region $Q^2 \leq 6 \times 10^{-4}$ (GeV/c)^2 or as a constant. The polarizability was determined from the ratio (assuming $(\alpha_1 - \beta_1)_{x\pi} = 0$)

$$R_\pi = \frac{(d\sigma_{\pi A})}{d\Omega}(\frac{d\sigma_{\pi A}^0}{d\Omega}) = 1 - \frac{3\mu^3 x^2}{2\alpha - x} \alpha_\pi,$$

(2)

where $d\sigma_{\pi A}/d\Omega$ refers to the measured cross section and $d\sigma_{\pi A}^0/d\Omega$ to simulated cross section expected for $\alpha_\pi = 0$, $x = E_\gamma/E_{beam}$ in the laboratory system of the process $\pi A \rightarrow \pi\gamma A'$. As a result they have obtained: $(\alpha_1 - \beta_1)_{x\pi} = 13.6 \pm 2.8 \pm 2.4$.

The new result of the COMPASS collaboration [4] ($\alpha_\pi = 2.0 \pm 0.6_{stat.} \pm 0.7_{syst.}$) has been found also by studying the $\pi^-$-meson scattering off the Coulomb field of heavy nuclei. This value is in agreement with the result obtained in a very similar experiment in Serpukhov [3], but with $H_\pi$.

This experiment was performed with $E_{beam} = 190$ GeV. For such values of $E_{beam}$ the quantity of $Q_{\min}^2(COMPASS)$ must be smaller by 22.5 times than $Q_{\min}^2$ (Serpukhov). However, the authors of the experiment considered $Q_{\min}^2 \leq 0.0015$ (GeV/c)^2, which are essentially greater than $Q_{cut}^2$ in work [3].

As shown in Ref. [18] the basic ratio $R_\pi$ is applicable for the Coulomb peak only. On the other hand, in Ref. [19] it is elaborated that the Coulomb amplitude interference with the coherent nuclear amplitude is important for $0.0005 \leq Q^2 \leq 0.0015$ (GeV/c)^2. This means that the Serpukhov analysis could safely apply the ratio $R_\pi$ in [2], whereas COMPASS has to consider the interference of the Coulomb and strong amplitudes. The phase determined with the simple considerations in Ref. [20] for the Serpukhov experiment is close to $\pi/2$ meaning that the subtraction of a nuclear background assumed to be incoherent is justified.

In Refs. [18, 19], the strong amplitude is described by the Glauber model (elastic multiple scattering of hadrons in nuclei). The conditions and limitations of the Glauber approximation are discussed in the classical article about diffraction by U. Amaldi, M. Jacob, and G. Matthiae [21]. Göran Fälld and Ulla Tengblad [18, 19] assume that the hadron-nucleon potential of the nucleons in the nucleus is local and also real, then the phases between the incoming hadron and the nucleons add up linearly. However, at high energies - and the COMPASS energy of the incoming $\pi$ with 180 (GeV/c)^2 is high - the strong phases become complex and the summed amplitude acquires an additional energy dependent phase. The associated profile function must take into account multiple scattering and will be complex, i.e. an unknown phase appears.

Moreover, the simulation of the distribution in Fig. 3(c) in the work [4] does not reproduce the diffraction bumps at $Q > 0.04$ (GeV/c). With a more realistic "absorbing disc" for the profile function [21] all bumps in Fig. 3(c) could be reproduced well and again a phase would be close to $\pi/2$ [22]. Without a real fit to the data it is impossible to estimate the effect of the model dependence of the diffractive background, but that it will have an influence is clear from Ref. [19].

Comparison of data with different targets provides the possibility to check the $Z^2$ dependence for the Primakoff cross section and estimate a possible contribution of the nuclear background. Such an investigation was performed by the Serpukhov collaboration and they have obtained $Z^2$ dependence with good enough accuracy. The COMPASS collaboration really have gotten their main result using only Ni target but they wrote that they also considered other targets on small statistic and obtained approximate $\sim Z^2$ dependence.

It should be noted that in order to get an information about the pion polarizabilities, the authors considered the cross section of the process $\gamma \pi^- \rightarrow \gamma \pi^-$ equal to the Born cross section and the interference of the Born amplitude with the pion polarizabilities only. The COMPASS collaboration analyzed this process up to the total energy $W = 490$ MeV in the angular range $0.15 > \cos \theta_{\gamma\pi} > -1$. However, the contribution of the $\sigma$-meson to the cross section of the Compton scattering on the pion could be very substantial in this region of the energy and angles [23]. Therefore, we consider this contribution.
III. $\sigma$-MESON CONTRIBUTION

According the dispersion relation (DR) from Ref. [23] the contribution of the $\sigma$-meson can be determined as

$$ReM_{\sigma}^{\pm +} = \frac{t}{\pi} \int_{4\mu^2}^{\infty} \frac{ImM_{\sigma}^{\pm +}(t', s = \mu^2)}{t'(t' - t)} dt'.$$

(3)

The imaginary amplitude $ImM_{\sigma}^{\pm +}(t, s = \mu^2)$ has to be evaluated taking into account that the $\sigma$-meson is a pole on the second Riemann sheet. The relation between amplitudes on the first and the second sheets can be written [24] as

$$F_0^{I}(t + i\epsilon) = F_0^{I}(t + i\epsilon)(1 + 2\epsilon T_0^{II}(t + i\epsilon)),$$

(4)

where

$$T_0^{II} = -\frac{g_{\sigma\pi\pi}}{t_\sigma - t}, \quad F_0^{II} = \sqrt{2} \frac{g_{\sigma\gamma\gamma}g_{\sigma\pi\pi}}{t_\sigma - t},$$

(5)

$$t_\sigma = (M_\sigma - i\Gamma_{\sigma 0}/2)^2, \quad \rho = \frac{\sqrt{1 - 4\mu^2/t}}{16\pi}, \quad \Gamma_{\sigma 0} = \Gamma_{\sigma} \left( \frac{t - 4\mu^2}{M_\sigma^2 - 4\mu^2} \right)^{1/2}.$$

(6)

Using the relation (11), we have

$$ImM_{\sigma}^{\pm +}(t, s = \mu^2) = \frac{1}{\pi t} \sqrt{\frac{2}{3}} \frac{2g_{\sigma\gamma\gamma}g_{\sigma\pi\pi} R}{D^2 + R^2}, \quad D = (M_\sigma^2 - t - \frac{1}{4}\Gamma_{\sigma 0}^2), \quad R = M_\sigma \Gamma_{\sigma 0} + 2\rho g_{\sigma\pi\pi}^2.$$

(7)

We can get influence of the $\sigma$-meson on the extracted value of $(\alpha_1 - \beta_1)_{\pi \pm}$ by equating the cross section without $\sigma$-meson contribution to the cross section when $\sigma$-meson is taken into account [25]:

$$d\sigma_{\gamma\pi\rightarrow\pi\gamma}(B, (\alpha_1 - \beta_1)_{\pi \pm})/d\Omega = d\sigma_{\gamma\pi\rightarrow\pi\gamma}(B, M_{\sigma}^{\pm +}(\alpha_1 - \beta_1)_{\pi \pm})/d\Omega,$$

(8)

where $(\alpha_1 - \beta_1)_{\pi \pm}$ is the value of $(\alpha_1 - \beta_1)_{\pi \pm}$ without the $\sigma$ contribution obtained in [4] and $B$ is the Born term. For backward scattering $(z = -1)$, we have the following expression:

$$(\alpha_1 - \beta_1)_{\pi \pm} = \frac{1}{4\pi\mu} \left\{ -(B + ReM_{\sigma}^{\pm +}) + \frac{B + 4\pi\mu B(\alpha_1 - \beta_1)_{\pi \pm}}{B + ReM_{\sigma}^{\pm +}} \right\}, \quad B = \frac{2e^2\mu^2}{(s - \mu^2)(u - \mu^2)}.$$

(9)

In the case of integration over the region $-1 \leq z \leq 0.15$ we have

$$(\alpha_1 - \beta_1)_{\pi \pm} = F_0/F_1,$$

(10)

where

$$F_0 = \frac{1}{4\pi\mu} \left\{ \int_{-1}^{0.15} \left[-ReM_{\sigma}^{\pm +}(ReM_{\sigma}^{\pm +} + 2B) + 4\pi\mu B(\alpha_1 - \beta_1)_{\pi \pm} \right] (1 - z)^2 dz \right\},$$

(11)

$$F_1 = \left\{ \int_{-1}^{0.15} (B + ReM_{\sigma}^{\pm +})(1 - z)^2 dz \right\}.$$

(12)

In the calculation we used the parameters of the $\sigma$-meson from Ref. [24]: $M_\sigma = 441$MeV, $\Gamma_\sigma = 544$MeV, $\Gamma_{\sigma\gamma} = 1.98$keV, $g_{\sigma\pi\pi} = 3.31$GeV, $g_{\sigma\gamma\gamma} = 16\pi\Gamma_{\sigma\gamma\gamma} M_\sigma$. The results of the calculations using Eq. (9) (line (1)) and Eq. (10) (line (2)) are shown in Fig. 1. Line (3) is the result of Ref. [4]. As a result we have obtain $(\alpha_1 - \beta_1)_{\pi \pm} \sim 10$. However the magnitude of $(\alpha_1 - \beta_1)_{\pi \pm}$ is very sensitive to parameters of the $\sigma$-meson and can reach a value of $\sim 11$ for the parameters from [26].

So, the contribution of the $\sigma$-meson can essentially change the COMPASS result. It should be noted that the contribution of the $\sigma$-meson was not considered in Serpukhov as well. However, in this case, the contribution of the $\sigma$-meson for the Serpukhov kinematics is $\Delta(\alpha - \beta)_\sigma \gtrsim 2.7$ within the experimental error of the Serpukhov result.
IV. TWO-PHOTON PRODUCTION OF PION PAIRS

Investigation of the $\gamma\gamma \rightarrow \pi^+\pi^-$ process was carried out in the frameworks of different theoretical models and, in particular, within dispersion relations (DR) \[5\, 10\, 17\, 27\, 28\]. Authors of most dispersion approaches restricted the partial wave content by $s$ and $d$ waves only. Moreover, they often used additional assumptions, for example, to determine subtraction constants. The pion polarizabilities in a number of the works were obtained from the analysis of the experimental data in the region of the low energy ($W < 700$ MeV). The most of results for the charged pion polarizabilities obtained in these works are close to the ChPT prediction $[29\, 30]$. On the other hand, the values of the experimental cross section of the process $\gamma\gamma \rightarrow \pi^+\pi^-$ in this region are very ambiguous. Therefore, as it has been shown in Refs. $[9\, 11]$, even changes of these values of the polarizabilities by more than 100% are still compatible with the present error bars in the energy region considered. More realistic values of the polarizabilities could be obtained analyzing the experimental data on $\gamma\gamma \rightarrow \pi^+\pi^-$ in a wider energy region.

The processes $\gamma\gamma \rightarrow \pi^0\pi^0$ and $\gamma\gamma \rightarrow \pi^+\pi^-$ were analyzed in Refs. $[11, 23, 31]$ using DR with subtractions for the invariant amplitudes $M_{++}$ and $M_{+-}$ without an expansion over partial waves. The subtraction constants are uniquely determined in these works through the pion polarizabilities. The values of polarizabilities have been found from the fit to the experimental data of the processes $\gamma\gamma \rightarrow \pi^+\pi^-$ and $\gamma\gamma \rightarrow \pi^0\pi^0$ up to 2500 MeV and 2250 MeV, correspondently. As a result the following values of $(\alpha_1 - \beta_1)_{\pi^0} = 13.0^{+2.6}_{-1.9}$ and $(\alpha_1 - \beta_1)_{\pi^\pm} = -1.6 \pm 2.2$ have been found in these works. In addition, for the first time there were obtained quadrupole polarizabilities for both charged and neutral pions.

The new fit to the total cross section of the process $\gamma\gamma \rightarrow \pi^+\pi^-$ at $|\cos \theta_{\pi^+\pi^-}| < 0.6$ in the frame of the DR $[11]$ has been performed with the $\sigma$-meson considered as a pole on the second Riemann sheet. The DR for the charged pions were saturated by the contributions of the $\rho(770)$, $b_1(1235)$, $a_1(1270)$, and $a_2(1320)$ mesons in the $s$ channel and $\sigma$, $f_0(980)$, $f_0'(1500)$, $f_0(1710)$, $f_0(2020)$, $f_2(1270)$, and $f_2(1565)$ in the $t$ channel.

As the two $K$ mesons give a big contribution to the decay width of the $f_0(980)$ meson and the threshold of the reaction $\gamma\gamma \rightarrow KK$ is very close to the mass of the $f_0(980)$ meson, the Flatté approximation $[32]$ for $f_0(980)$ meson contribution was used. Besides we took into account a nonresonance contribution of the $s$ waves with the isospin $I = 0$ and 2 using $\pi^+\pi^-$ loop diagrams. The fit result using Eq. (7) for $ImM_{++}(t, s = s^2)$ with the following parameters of the $\sigma$-meson: $M_\sigma = 441$ MeV, $\Gamma_\sigma = 544$ MeV, $\Gamma_{\sigma\gamma} = 1.298$ keV, $g_{\sigma\pi\pi} = 3.31$ GeV, is shown in Fig. 2. As a result of the fit we have obtained: $(\alpha_1 - \beta_1)_{\pi^+} = 10^{+2.9}_{-1.6}$ and $(\alpha_1 + \beta_1)_{\pi^\pm} = 0.11^{+0.09}_{-0.02}$, that agrees well within the errors with our previous fit $[11]$ and predictions of the Dispersion Sum Rules (DSR), see Table I in Ref. $[31]$.

It should be noted that in the region of $W \leq 500$ MeV the main contribution is given by the Born term, the dipole and quadrupole polarizabilities, and the $\sigma$-meson. It would be very important to have new more accurate data in this energy region.

![Figure 1](image.png)

**Figure 1:** Dependence of $(\alpha_1 - \beta_1)_{\pi^\pm}$ on $W$. Lines (1) and (2) correspond to the calculation of Eqs. (9) and (10), respectively. Line (3) is the COMPASS result $[4]$. 
FIG. 2: The cross section of the process $\gamma\gamma \rightarrow \pi^+\pi^-$ (with $|\cos\theta_{cm}| < 0.6$). Experimental data of TPC/2γ [12], Mark II [8], and CELLO [13] Collaborations are shown with statistical uncertainties only. Statistical uncertainties for the most of the Belle Collaboration data [16] are smaller than corresponding blue circles. Vertical light-blue error bars are systematic uncertainties for these data. The red line is our global fit result.

V. DSR AND CHPT

Here we discuss possible reasons of the disagreement between DSR and ChPT predictions for the charge pion polarizabilities. DSR for the difference and sum of electric and magnetic pion polarizabilities have been constructed in Refs. [11, 23, 31, 33]. The main contribution to DSR for $(\alpha_1 - \beta_1)_{\pi^\pm}$ is given by $\sigma$-meson. However, this meson is taken into account only partially in the ChPT calculations [29].

In the case of the $\pi^0$-meson, the big contribution of the $\sigma$-meson to DSR is cancelled by the big contribution of the $\omega$-meson. The contribution of vector mesons in DSR can be written in the narrow width approximation as

$$ReM^{++}(s = \mu^2, t = 0) = \frac{-4g^2\mu^2}{(M^2_V - \mu^2)}.$$  \hspace{1cm} (13)

In the case of ChPT, the authors of Ref. [29] used

$$ReM^{++}(s = \mu^2, t = 0) = \frac{-4g^2\mu^2}{(M^2_V - \mu^2)}.$$  \hspace{1cm} (14)

The absolute value of the amplitude (14) is smaller than (13) by a factor $M^2_V/\mu^2$. So, $\sigma$-meson is included in the ChPT calculations only partially and, according Eq. (14), the $\omega$-meson also gives a very small contribution. As a result, the predictions of DSR and ChPT for $(\alpha_1 - \beta_1)_{\pi^0}$ are very close, see Table II in Ref. [31].

VI. SUMMARY AND CONCLUSIONS

We have considered the main experimental works concerning charge pion polarizabilities. The values of $(\alpha_1 - \beta_1)_{\pi^\pm}$ obtained in the Serpukhov [3], Mainz [1], and LPI [2] experiments are at variance with the ChPT predictions [29]. The result of the COMPASS Collaboration [4] is in agreement with the ChPT calculations. However, this result is very
model dependent. It is necessary to correctly investigate the interference between Coulomb and nuclear amplitudes and take into account the contribution of the $\sigma$-meson.

It should be noted that the most model independent result was obtained in the Serpukhov experiment [3].

We have presented our new fit results for the $\gamma\gamma \rightarrow \pi^+\pi^-$ reaction. The obtained value of $(\alpha_1 - \beta_1)_{\pi^\pm}$ agrees well with the DSR predictions and contradicts to ChPT where the $\sigma$-meson contribution is taken into account only partially.

In conclusion, further experimental and theoretical investigations are needed to determine the true values of the pion polarizabilities.

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