Deep Spectral Descriptors: Learning the point-wise correspondence metric via Siamese deep neural networks

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Figure 1: In this paper, we introduce a novel Siamese deep learning framework to find an optimal embedding of spectral descriptors to improve point-wise matching performance between non-isometric models. A comparison with the state-of-the-art methods including heat kernel signature [SOG09], optimized spectral descriptor [LB14], and geodesic convolutions neural network [MBBV15] for correspondence matching between highly non-isometric models is shown in this figure. As shown in the figure, our approach outperforms the other state-of-the-art methods in matching point-wise correspondence between models with a significant shape difference.

Abstract

A robust and informative local shape descriptor plays an important role in mesh registration. In this regard, spectral descriptors that are based on the spectrum of the Laplace-Beltrami operator have gained a spotlight among the researchers for the last decade due to their desirable properties, such as isometry invariance. Despite such, however, spectral descriptors often fail to give a correct similarity measure for non-isometric cases where the metric distortion between the models is large. Hence, they are in general not suitable for the registration problems, except for the special cases when the models are near-isometry. In this paper, we investigate a way to develop shape descriptors for non-isometric registration tasks by embedding the spectral shape descriptors into a different metric space where the Euclidean distance between the elements directly indicates the geometric dissimilarity. We design and train a Siamese deep neural network to find such an embedding, where the embedded descriptors are promoted to rearrange based on the geometric similarity. We found our approach can significantly enhance the performance of the conventional spectral descriptors for the non-isometric registration tasks, and outperforms recent state-of-the-art method reported in literature.

Keywords: Deep Siamese networks, Spectral descriptor, Point-wise correspondence, Surface registration

1 Introduction

Deformable registration of surfaces is a fundamental problem in computer-aided design and computer graphics. The problem is critical for a wide range of applications such as shape interpolation [BLLL15; KMP07], statistical shape analysis and modeling [ACP03; BL12], geometry transfer [SP04], and such. Among others, finding the point-wise correspondence between surfaces is the main challenge for a successful registration, which, however, is known to be a NP-hard problem. To this end, a well defined local shape descriptor and an accurate similarity metric can increase the success rate of a registration algorithm significantly, as they provide a tool for quantifying the similarity between two points computationally.

For the last decade, a family of spectral descriptors (e.g., [RWP06; Rus07; SOG09; ASC11b; BK10]) that utilizes the eigendecomposition of the Laplace-Beltrami operator [Ros97] has drawn lots of attention among the researchers in relevant fields. This is because the spectrum of the Laplace-Beltrami operator has many desirable properties, such as isometry invariance, multiscaleness, parametrization independence, and such. Among others, the spectral descriptors in general show superior performances compared to other types of descriptors [LGB\textsuperscript{*}13], especially for the tasks such as shape retrieval [LH14; BK10], mesh segmentation [ASC11a; FSKR11] and isometric matching [OMMG10]. However, despite of all the desirable properties, they often struggle in most of the surface registration tasks, especially when the disparity between the surfaces is relatively large. This is because the spectral descriptors, by definition, are highly sensitive to the metric distortion. Therefore, even though they work generally well with the isometric registration tasks (e.g., surface scans of the same individual with different postures), but not so well with the deformable registration tasks (e.g., registering a skinny person to an obese person).

In order to mitigate such limitations of the spectral descriptors, we introduce a novel approach to embed the spectral descriptors to a new metric space using deep neural networks (DNNs) in such a way that the Euclidean distance between the descriptors in the new
embedding directly provides a desirable similarity metric for deformable registration tasks. To achieve so, we design a Siamese architecture of DNNs, in which two parallel sets of neural networks sharing the same coefficients are used to train the suitable embedding. The Siamese DNN is designed to find the optimal embedding of spectral descriptors by rearranging them in accordance of their correspondence among each other. In our experiment of detecting correspondence between human bodies, we found the newly embedded shape descriptors through our Siamese DNN have a superior performance in both aspects of the absolute number of correct matches and the distribution of the matches over entire shape.

2 Related Works

2.1 Spectral Descriptors

Spectral shape analysis is a branch of computational geometry that analyzes digital geometry using the spectrum of a linear operator defined on a surface. Among a variety of choices, the Laplace-Beltrami operator [Ros97] that generalizes the Laplacian on Riemannian manifolds has gained significant highlights because of several desirable properties. One such property is the isometry invariance of its eigenvalues [Lv06]. Since a lot of deformations in real-world can be characterized as an isometry or a near-isometry, the isometry invariance property of the eigenvalues of the Laplace-Beltrami operator gives advantage for many applications, including shape retrieval [RWP05; JZ07; LH14; BK10], correspondence matching [DK10; OMMG10], segmentation [RGB*09; ASC11a], and etc.

Based upon this idea, Reuter et al. [RWP06] defined an ordered set of numerical signatures of a shape that captures the unique fingerprints of a given geometry. The signatures, which they named ShapeDNA, essentially are an ascending sequence of eigenvalues of the Laplace-Beltrami operator. They proved such an encoding of a geometry data provides an effective analytic tool for quantifying the geometric dissimilarity between different shapes in such a way that is proportional to the metric distortion.

Similarly, Rustamov [Rus07] defined the Global Point Signature (GPS), also by utilizing the eigendecomposition of the Laplace-Beltrami operator. He defined the GPS at each point on the surface as a vector value whose elements are the eigenfunctions of different modes scaled by the corresponding eigenvalues.

More notably, Sun et al. [SOG09] proposed the Heat Kernel Signature (HKS) physically analogous to the different heat diffusion characteristics depending on the geometric shape of surfaces. They introduced the heat kernel equation that formulates the isotropic heat diffusion process on a manifold surface by using the Laplace-Beltrami spectrum, and defined a shape signature as a collection of function values of the heat kernel. For each point on the surface, the heat kernel function is sampled at \( n \) different time scales, forming an \( n \)-dimensional feature vector, to define the HKS. Due to the effectiveness of the HKS, a lot of variations of HKS have been introduced later on, including the works such as scale-invariant HKS (SIHKS) [BK10].

In the similar spirit of HKS, Aubry et al. [ASC11b] proposed the wave kernel signature (WKS) that is based on the quantum mechanical characterization of the wave propagation on manifolds. The WKS represents the average probability of measuring a quantum mechanical particle at a specific location. This is achieved by solving the Shrödinger’s equation, whose solution is represented also by the Laplace-Beltrami spectrum in a similar form to the HKS.

Many relevant literature (see e.g., [LH14; LGB*13]) reports that spectral descriptors outperform other types of shape representation methods for shape retrieval tasks in general, since they are invariant to the isometry and are proportional to a deformation, or a metric change. Especially, signatures such as HKS and WKS are also known to be multiscale in a sense that they inherently capture both the local and global shape characteristics through different time scales.

However, the spectral descriptors are not quite suitable for the dense correspondence problems or deformable registration problems that involve large, non-isometric deformations. This is because, for non-isometric registration cases, the spectral descriptors tend to fail in recognizing the corresponding points between two models due to a large difference in local metrics.

To mitigate such limitations, Litman et al. [LB14] introduced a machine learning approach for constructing a shape descriptor for the correspondence problems with large deformations. They generalized the well-known HKS and WKS equations into a generic form, and parametrized the generic form with a number of coefficients. They then proved that the Euclidean distance between the generic form of spectral descriptors are essentially the weighted distance with a metric tensor comprised of those coefficients. Therefore, by learning the optimal set of coefficients that generates the optimal metric maximizing the gap between true matching pairs and non-matching pairs, they derive an optimized spectral descriptor (OSD).

The OSD made a significant contribution to the field by overcoming the limitations of conventional spectral descriptors to some extent. However, we discovered that the performance of OSD is still unsatisfactory when the shape distortion between the individuals is relatively large (see Section 4).

2.2 Deep Neural Networks in Matching

Deep neural networks (DNNs) are a type of artificial neural networks that has multiple hidden layers as opposed to shallow neural networks where there is only one hidden layer. As like many other neural network approaches, DNNs try to find and generalize underlying rules and features of data on its own, from the observations on a training dataset, instead of using hard-coded or hand-crafted rules that are in general vulnerable to exceptions. This in fact is particularly useful for the tasks involving geometry data, such as computer vision, digital signal processing, and digital geometry processing, where features hold a highly sophisticated nature, and hence, manually coding sets of rules is extremely difficult or not necessarily available.

Indeed, in recent several years, there has been a phenomenal progress in the relevant fields of DNNs. Especially for the matching tasks, a number of works that utilize DNNs for deriving geometric correspondence metrics have been reported. For instance, [SSTF*15; KCR*16; BJTM16] employed the convolutional neural networks (CNNs) to define shape descriptors on two-dimensional (2D) images. Instead of being analytically defined, shape descriptors in these works are derived from the CNNs through a number of observations on training images. The descriptors developed in such a way are shown to outperform the conventional image descriptors that are hand-crafted.

Quite recently, the trend of CNN-based matching frameworks started to be transferred to 3D domains. Wei et al. [WHC*16] proposed using deep convolutional neural network to train a feature descriptor for correspondence of non-rigid shapes. Yi et al. [YSGG16] developed SyncSpecCNN by adapting the idea of graph convolutions on the spectral domain in order to achieve semantic segmentations of 3D shapes. Masci et al. [MBBV15] proposed a geodesic convolutional neural networks (GCNN) as a generalization of CNN to Riemannian manifolds based on geodesic-lo-
Global Points Signature [Rus07]: Given a point on a 2-manifold, the GPS at the point is defined as:

$$\text{GPS}(x) = \left[ \frac{\phi_1(x)}{\sqrt{\lambda_1}}, \frac{\phi_2(x)}{\sqrt{\lambda_2}}, \ldots, \frac{\phi_n(x)}{\sqrt{\lambda_n}} \right]^T,$$

where $\lambda_k$ and $\phi_k$ are the $k$-th eigenvalue and eigenfunction of the Laplace-Beltrami operator defined on the manifold respectively. An adequate number of eigenvalues suggested in [Rus07] is $n = 25$.

Heat Kernel Signature [SOG09]: Given a 2-manifold, the heat flow on the manifold can be approximated by the heat kernel function:

$$H_t(x,y) = \sum_{k=0}^{\infty} e^{-\lambda_k t} \phi_k(x) \phi_k(y).$$

Physically, the function returns the amount of heat diffused from a point $x$ to a point $y$ on the manifold during a certain time $t$. From this, the HKS is defined as a series of heat kernel values $H_t(x,x)$ measured at discrete samples of time $t_1, t_2, \ldots, t_n$:

$$\text{HKS}(x) = [H_{t_1}(x,x), H_{t_2}(x,x), \ldots, H_{t_n}(x,x)]^T.$$
two points indicates the difference between the spectral descriptors, and thus, the geometric dissimilarity between the corresponding surface points.

Essentially, our goal of training the neural network is to find a better, and hopefully the optimal, embedding of the spectral descriptors in a different metric space than the canonical Euclidean n-space. Hence, as a first step, it is critical to determine the dimensionality of the new embedding space in such a way that it preserves most of the geometric information encoded in the original spectral descriptors while keeping the dimensionality as concise as possible. In fact, although the original spectral descriptors are highly informative, they in general contain a great deal of redundancy as well. We prove this hypothesis by conducting an analysis on the intrinsic dimensionality of the spectral descriptors.

Simplest way of achieving such is to perform principal component analysis (PCA) on a set of spectral descriptors [Kir00; WEG87]. First, we collect 10,000 random samples from our database. Then, we randomly pick a point from the samples and find k-nearest neighbors to the selected point. Next, we conduct PCA on the set of k-nearest neighbors to estimate the local tangent space around the selected point. Finally, we analyze the residual variances with respect to the number of principal components, and estimate the intrinsic dimension of the local tangent space from it. Technically, we find d number of principal components that covers larger than 99% of the total variance, that is \( \sum_{i=1}^{d} \lambda_i \geq 0.99 \sum_{i=1}^{n} \lambda_i \) where \( \lambda_i \) is the eigenvalue associated with the i-th principal component. We repeat this process multiple times to statistically determine the intrinsic dimension of the spectral descriptors. More sophisticated methods (see e.g., [MM79; TDSL00; RS00]) may give better estimation on the intrinsic dimensionality of the data, we found them not critical from our experiments.

Figure 2 shows graphs of the average residual variances for GPS, HKS, and WKS. As can be noticed from the figure, the residual variances drop significantly up to the first five or so components, and the later components are close to zero. Such a trend was consistent when we varied the number of nearest neighbors k from 6 to 25. Interestingly, the same tendency was commonly observed across the different types of the spectral descriptors. We therefore conclude that the intrinsic dimension of the spectral descriptors is 5 and define the dimensionality of the embedding space to be 15, which is three times larger than the actual dimensionality to give enough degrees of freedom for distortion of the data manifold.

### 3.4 Siamese Neural Network

A “Siamese” architecture [BBB93] is an effective way of designing neural networks for comparative analysis. In Siamese network, two identical neural networks are placed in parallel, and the output layers of these networks are merged and are fed into another layers of neural network. The Siamese pairs share the same coefficients such that the weight values of the neurons and the biases are all identical between the pairs. The Siamese neural network is more advantageous than the other similar architectures. They can be trained with fewer parameters since the weights are shared across the pair of neural networks. In addition, it is rational to use similar neural networks to process similar inputs (e.g., 3D models). Features with the same semantics extracted by deep learning techniques can be compared with each other easily [KZS15].

In order to learn the optimal embedding of the spectral descriptors, we design a Siamese architecture as shown in Figure 3. First, each of the Siamese pairs takes a spectral descriptor defined at a given point as an input. Hence, the number of neurons on the input layer equals to the dimension of the spectral descriptor to be used. On the other hand, the output layer for each of the pairs is the final embedding of the spectral descriptors, and has a dimensionality of 15 as we discussed in Section 3.3. In-between the input and the output layers, there is a repetition of a fully-connected hidden layer and the rectified linear unit (ReLU) layer (Figure 4). Finally, the two output layers from the both side of the Siamese pair are then merged and are fed into the main body, which is purposed to find the distance metric in the embedding space. It is a common practice to place another layers of neural network as a main body; however, in our case, we just set a simple unit for computing the Euclidean distance between the two embedded descriptors and transforming it to a similarity metric.

The most critical part of our design of the Siamese network is the hidden layers at each of the Siamese pairs. There are two main design parameters in concern for the design of the hidden layers: how many layers we require and; how many neurons we should have for each of the layers. Unfortunately, there are no known guideline that works well for most of the cases. In this reason, we conducted several sessions of experiments by varying the number of hidden layers and the number of neurons associated to each of it.

Empirically, we found that more than two fully-connected hidden layers tend to overfit the training data provided, and unnecessarily complicates the training process of the neural network. On the other hand, shallow networks with a single hidden layer showed reasonable performance in general, but not quite good as the two fully-connected hidden layer cases.

Given these, we also conducted a series of analysis to determine the adequate number of neurons for each of the fully-connected layers. Considering that HKS and WKS are 100-dimensional vectors and the output embedding is 15-dimension, we varied the number of neurons from 15 to 100 and trained the neural network for each of the cases. In the case of GPS, whose initial dimensionality is 25, we varied the values from 15 to 25. To evaluate the performance for different cases, we compared the loss (LSS), true-negative ratio (TNR), false-negative ratio (FNR), and the misclassification error rate (ERR), whose details will be discussed in Section 3.6.

Figure 5 shows the result of our study. The horizontal axis represents the number of neurons in the first fully-connected hidden layer, and the vertical axis represents the second fully-connected hidden layer. Note that the upper triangle of the plot is empty because we excluded the cases where the next layers have the larger number of neurons than the previous layers. From such an analysis, we selected 78 and 32 as the adequate number of neurons for HKS.
Figure 3: A schematic diagram of the Siamese architecture used in this paper. Spectral descriptors computed at different points are fed into each branch of the Siamese network. The pair of the networks is identical, and shares the same coefficients (i.e., weights $W$, and bias $b$). The outputs of the Siamese pair, which mathematically are the spectral descriptors embedded into a different metric space, are then compared with Euclidean metric (or $l_2$ distance), which then gives a measure of similarity.

Figure 4: The structure of 5-layer DNN used in our method.

and WKS, and 20 and 18 for GPS.

### 3.5 Training

For the training of the neural network, we used the Dyna dataset [PMRMB15] to generate our training samples. The Dyna dataset is composed of over 40,000 human body meshes of ten subjects spanning a range of body shapes under various body postures. Each mesh contains 6,980 vertices. The meshes are obtained by aligning the body scans of the ten subjects. The body scans are collected by using a custom-built multi-camera active stereo system, which captures 14 assigned motions (e.g., running, jumping, shaking, etc.) at 60 frames per second. The system outputs 3D meshes of a size around 150,000 vertices in average. During the scanning, each subject wore tight pants and a bathing cap, and a sports bra for female subjects.

Such dense sets of vertices are then registered by conforming a template mesh to each of the scanned meshes. In this way, the topology of each mesh becomes compatible to each other, and the vertices with the same index get to correspond to each other geometrically. We exploit such by using them as the ground truth for the correspondence of a pair of vertices. On each of the meshes in Dyna database, we computed the spectral descriptors in a way that is described in Section 3.2, and stored their values as a data matrix.

For each batch at the training stage, we randomly select two models from the entire database, and pick 512 pairs of vertices at random. For each batch, the first half of the pairs are chosen from among the correct matching pairs, and the second half of the pairs are from the non-matching pairs.

The neural network then takes the pairs of spectral descriptors and updates its coefficients as a result of training. For the optimization, we found no significant difference between different types of optimization methods, but we found the Adam optimizer [KB14] performs well in general. For the learning rate we used initially 0.015 and let it exponentially decay by a factor of 0.9999 for each of the training steps. The objective of the optimization is to maximize the margin between non-matching pairs. This can be achieved by minimizing the sum of the following error terms over all training samples $k$.

\[
E(f_k, g_k) = y_k \|D(f_k) - D(g_k)\|^2 + (1 - y_k) \max (0, C - \|D(f_k) - D(g_k)\|^2),
\]

where $(f_k, g_k)$ are the $k$-th pair, $y_k$ is a Boolean value indicating the correspondence, $D(\cdot)$ is an embedding derived from the Siamese branch, and $\max(\cdot, \cdot)$ is a function comparing two values and returning the larger number. The constant $C$ is the minimum margin value between the non-matching pairs, and we set $C = 5$.

Here, one practical consideration that needs to be done for the selection of the Boolean value $y_k$ is that even the non-matching pairs might have a highly similar geometry. For instance, a point on the left thumb would probably be highly similar in geometry with a point on the right thumb. However, they are technically non-matching pair. If such cases happen in the training dataset, it may confuse the neural network since it is a sort of conflicting example. In this reason, instead of setting $y_k$ strictly either 0 or 1, we set some random value between 0 to 0.2 for the non matching pairs. The correct matching pairs are still kept to $y_k = 1$. 

\[
E(f_k, g_k) = y_k \|D(f_k) - D(g_k)\|^2 + (1 - y_k) \max (0, C - \|D(f_k) - D(g_k)\|^2),
\]
To produce the result presented in this paper, we trained the neural network with the total of 10,000 iterations, and hence the total number of training samples were 5,120,000 (batch size × number of iterations).

### 3.6 Evaluation criteria

For the quantitative measurement of the trained performance of our method, we use following criteria:

**Loss** is the sum of error terms in Equation 7 over test dataset. The loss is an indicator of how well the non-matching pairs are separated from each other in the embedding space.

**True-Negative Rate (TNR)** is the percentage of error that the correct matching pairs are classified as non-matching pairs. For the threshold of classification, we use half of the margin $C$ in Equation 7. Therefore, TNR shows what percentage of matching pairs are separated wider than $0.5C$.

**False-Positive Rate (FPR)** is the percentage of error that the non-matching pairs are classified as correct matching pairs. We use the same threshold with the above case. FPR shows what percentage of non-matching pairs are within the margin of $0.5C$.

**Misclassification Rate (ERR)** is the percentage of error that the pairs are classified to other than their ground truth. Conceptually, it is the union of TNR and FPR, and it is calculated as the sum of these two quantities.

### 4 Result

#### 4.1 Matching Performance

From the Dyna dataset, we randomly selected a pair of models from two different individuals that were not used for training of the Siamese DNN. We computed the spectral descriptors as well as the proposed descriptors on the models and matched their vertices one to the other through the nearest neighbor search using the simple Euclidean distance. We have rejected non-matching pairs, i.e., the pairs with the Euclidean distance over the threshold value $0.5C$. In addition, we also rejected the pairs with the geodesic distortion more than 5% of the shape diameter. We then counted the number of remaining matches, which could be considered as the correct matching pairs. We did not use any sophisticated algorithm for the calculation of the matching pairs, such as the ones presented in [LB14; LH05]. Instead, we directly compared the Euclidean distances among the descriptors such that a more straightforward comparison could be done on the quality of the metric.

Figure 6 and Table 1 show the result of such analysis. Overall, it is observed a significant improvement with the proposed method in terms of the absolute numbers of correct matching pair. Among them, GPS showed the largest improvement in average, followed by HKS and WKS. In overall matching performance, DeepHKS showed the most desirable result, followed by DeepWKS and DeepGPS.

Note even for the cases where the improvement in absolute number was not so significant or even negative, the quality of matching still has been improved in a sense that the pairs in deep signature results cover larger area than the original descriptors. For instance, the second row in Table 1 shows a relatively small improvement in HKS.
(2,209 → 2,300), and even a decrease of numbers in WKS (2,124 → 1,325) in terms of the absolute number of correct matching pairs. However, as can be observed from the corresponding images in Figure 6, the quality of matching is much better in DeepHKS and DeepWKS, such that the matching pairs are spread well over the larger area (e.g., areas around breast and belly). This is, in fact, because some of the body parts such as hands, feet, and faces, are near-isometry across the models. In this reason, the matching results of the original spectral descriptors are mostly concentrated around those body parts, whereas the matching results of the deep spectral descriptors are distributed widely over the entire body area, even including highly non-isometric areas.

4.2 Comparison with the OSD and GCNN

In addition to the aforementioned performance evaluation, we further verify the performance of our method by comparing with the OSD [LB14] and the GCNN[MBBV15]. We used 6 of the 10 individuals in the Dyna dataset for training the both algorithms. For the calculation of OSD, we used the MATLAB code came with the original OSD paper [LB14]. For each individual, we randomly picked 10 meshes over all assigned motions. In total, the training set contained 60 different models from 6 individuals. The training set for GCNN was the same as that for the OSD. The code to implement GCNN was written in Python and came with the original GCNN paper [MBBV15]. In order to make the comparison fair, we re-trained DeepHKS and DeepWKS using the same dataset. The rest of 4 individuals in the Dyna dataset were kept for testing.

To drill-down to details, we conducted seven separate analyses on each different group. The first four groups contained two models of the same individual but in different postures, which could be considered as isometric cases. The fifth group consisted of models of two different individuals but with similar body shapes, which was categorized as a near-isometric case. The last two groups contained two different individuals with significantly different body shapes, which were non-isometric cases. Among these two, the sixth group had a significant difference in height but a similar body shape, and the seventh group had significant differences both in height and the body shape. The comparison between the groups are also available in Table 2.

For better visualization, we randomly sampled 10% of all vertices uniformly distributed over the entire mesh. In the Dyna dataset, some of the body segments such as head, hands, and feet were rigid and thus remain unchanged across individuals due to the nature of how the dataset had been generated (see [PMRMB15] for the detail). Thus, these areas were excluded from the evaluation for more precise comparison between different methods.

Figure 7 and Table 3 show the result of such analyses. For the first four groups (i.e., isometric cases) the HKS showed the best performance in correspondence matching, followed by the OSD. Our method showed a comparable performance with OSD. The GCNN showed relatively high performance for the fourth group, but recorded bottom for all the other cases. For the fifth group (i.e., near-isometric case), the GCNN and our method showed similar performance, but the performance of the HKS and the OSD dropped to half. Lastly, for the last two groups (i.e., non-isometric case), ours performed the best, followed by GCNN and OSD.

It should be noted that our method showed quite consistent and reliable performance across different level of deformations (isometric to non-isometric) while maintaining acceptable level of accuracy, as opposed to other methods whose performances varied significantly.

It is also noteworthy that as the amount of metric distortion increases, the performance gap between our method and the other descriptors increases noticeably. Especially, for the last group, where the metric distortion was the largest, GCNN, HKS, and OSD were not quite successful in identifying correct matching pairs, whereas our method still maintained consistent level of accuracy.

5 Conclusion

In this paper, we proposed an approach to learn the similarity metric between the spectral descriptors for deformable registration tasks. We designed a Siamese neural network architecture carefully, based on the analysis of its design parameters. The neural network designed in this paper was able to find an optimal embedding of the spectral descriptors in a new metric space, in which a direct comparison based on the Euclidean distance already gives an excellent point-wise matching result between non-isometric models. The method was verified on a database containing a variety of body shapes and postures, where it showed significant improvement of the original spectral descriptors in non-isometric cases. The deep spectral descriptor developed through our approach outperformed the other state-of-the-art methods including the OSD and the GCNN, while the efforts required to implement and train the algorithm were much less.

A question might be raised if concatenating different spectral descriptors all together and apply the same Siamese learning procedure would produce any better result. We found that concatenating GPS, HKS, and WKS did not necessarily lead to a noticeable improvement of the performance immediately, possibly because of the increased dimensionality of the input layer, complicating the training procedure and the convergence of the neural network coefficients. We have not conducted a detailed study to find the optimal network parameters in such cases, as it would be out of scope of this paper. However, it is reasonable to assume having more descriptor components would provide richer information on geometric characteristics, and a further investigation on this would be valuable.

As another future work, it would be interesting to explore possible ways of improving our method to achieve scale-invariance. Current formulation is dependent upon scaling of the model. Furthermore, transferring the learned embedding from one training set to another (i.e., transfer learning [LC17]) would also be a possible direction for a future research work.

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Figure 6: Visualization of the matches within the geodesic distortion of 5% of the shape diameter. Each row shows a comparison between the GPS, HKS, and WKS, and their embeddings, namely DeepGPS, DeepHKS, and DeepWKS, respectively. Different models in different rows are randomly picked from the test dataset for the visualization. For a fair comparison, no sophisticated algorithm such as [LH05] is used, but a simple nearest neighbor search based on the Euclidean distances between the shape descriptors.
Table 1: Statistics of correct matching pairs for the results presented in Figure 6. Each of the rows in the table corresponds to each of the rows in Figure 6 in the same order. The left two entries of each row show the id number of an individual and the name and frame number of a motion, following the same naming convention in Dyna database. Different id numbers indicate different body shapes, and different motion names and frame numbers indicate different body postures. The rest of the entries show the number of correct matching pairs for each of the methods.

Table 2: Description of the selected correspondence tasks. Each of the rows in the table corresponds to each of the rows in Figure 7 in the same order. Model 1 and Model 2 correspond to the yellow and blue models in Figure 7 respectively.

Table 3: Percentage of correct matching pairs for the results presented in Figure 7. Each row in the table describes the result of the corresponding row in the figure in their respective order.
Figure 7: Comparison among HKS, OSD, GCNN, and DeepHKS. Each row corresponds to each group presented in Section 4.2 respectively. For a detailed drilled-down comparative analysis, the seven groups of models represent the gradual change from completely isometric cases to completely non-isometric cases. Our proposed DeepHKS outperforms the other algorithms in the non-isometric cases. Performances of all tested methods in each case have been presented in Table 3

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