Escape of superheavy and highly energetic particles produced by particle collisions near maximally charged black holes

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Abstract

For particle collision near rapidly rotating Kerr black holes, the center-of-mass energy can be arbitrarily high if the angular momentum of either of the colliding particles is fine-tuned. Recently, it has been shown that particles which are produced by such a particle collision and escape to infinity cannot be very massive nor very energetic. For electrically charged black holes there is a similar phenomenon, where the center-of-mass energy for the collision of charged particles near the horizon can be arbitrarily high. One might expect that there would exist a similar bound on the energy and mass of particles that are produced by such a particle collision and escape to infinity. In this paper, however, we see that this expectation is not the case. We explicitly show that superheavy and highly energetic charged particles produced by the collision near maximally charged black holes can escape to infinity at least within classical theory if the backreaction and self-force of the particle can be neglected.

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I. INTRODUCTION

Bañados, Silk, and West [1] have shown that the center-of-mass (CM) energy of a particle collision near the horizon of a maximally rotating black hole can be arbitrarily high if either of the two colliding particles has a fine-tuned value of the angular momentum, which we call the Bañados-Silk-West (BSW) collision. Based on this demonstration, they have suggested that a maximally rotating Kerr black hole acts as a particle accelerator, so that a maximally rotating Kerr black hole is expected to create quite a massive or highly energetic particle in principle. If such an exotic particle is to be observed at infinity, it must escape to infinity. Jacobson and Sotiriou [2] have suggested that such a massive or energetic particle cannot escape to infinity.

For this problem, the energy extraction mechanism from a black hole might be relevant. Penrose considered the disintegration of an incident particle in the ergoregion and showed that energy can be extracted from a rotating black hole with the production of a negative-energy particle in the ergoregion [3]. This process is called the Penrose process. The energy extraction from a rapidly rotating black hole is expected to be a possible engine of active galactic nuclei. On the other hand, the net energy-extraction efficiency of the Penrose process is bounded by the upper limit $\simeq 20.7\%$ [4–6]. Piran, Shaham, and Katz [7–9] proposed a collisional Penrose process. They first expected that it is much more efficient than the original Penrose process, whereas they subsequently found that its efficiency is as modest as the original one. Very recently, Bejger, Piran, Abramowicz, and Håkanson [10] have numerically shown that the energy of a photon produced by the pair annihilation of the BSW type cannot be high, but instead rather modest. Harada, Nemoto, and Miyamoto have analytically given upper bounds on both the energy of the emitted particle and the energy-extraction efficiency for more general physical reactions of the BSW type [11]. Both these studies have revealed that the collisional Penrose process through the BSW collision is a valid energy-extraction mechanism, while the emitted particle escaping to infinity cannot be very energetic nor very massive.

As for an electrically charged black hole, Denardo, Hively, and Ruffini showed that there exists a region of the spacetime where the energy of a charged particle can be negative and hence the energy can be extracted [12, 13]. So the Penrose process can occur around a charged black hole. In fact, the energy extraction from a black hole with an electromagnetic
field can be more efficient than that from a neutral rotating black hole and its efficiency is not bounded [14–18]. Recently, Zaslavskii [19] has shown that there is an electromagnetic counterpart of the BSW collision. In the case of a static maximally charged black hole, a collision of arbitrarily high CM energy is possible if either of the two colliding particles has a fine-tuned value of the charge. One can expect that a more efficient energy extraction through the collision of this type near a Reissner-Nordström black hole may be possible than that through the BSW collision near a Kerr black hole. We show that this is the case. Unlike in the Kerr case, we explicitly demonstrate that the mass and energy of the product particle which escapes to infinity can be arbitrarily large. Note that, in the same context, Zaslavskii [20] has firstly shown that there exists no upper bound on the energy extraction from the collision of this type and noticed the fundamental difference in this regard between the Kerr case and the Reissner-Nordström case. His approach and our present approach are totally consistent with and complementary to each other. This phenomenon, in principle, opens a possibility that a distant observer might observe an exotic superheavy particle produced by the particle collision of extremely high CM energy near a maximally charged black hole. We use the geometrized units, in which $c = G = 4\pi\varepsilon_0 = 1$.

II. CHARGED PARTICLE AROUND A REISSNER-NORDSTRÖM BLACK HOLE

We briefly review the motion of a test charged particle in a Reissner-Nordström black hole spacetime. The line element in the Reissner-Nordström spacetime is given by

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\theta^2 + \sin^2 \theta d\phi^2,$$

(1)

where

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2},$$

(2)

and $M$ and $Q$ ($|Q| \leq M$) are the mass and electric charge of the black hole, respectively. We assume $Q \geq 0$ hereafter without loss of generality. An event horizon is located at $r = r_h = M + \sqrt{M^2 - Q^2}$, where $f(r)$ vanishes. If $Q = M$, $r_h = M$ and the black hole is said to be extremal.

The Lagrangian of a test charged particle in an electromagnetic field is given by

$$\mathcal{L} = \frac{1}{2} g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} + q A_\mu \frac{dx^\mu}{d\lambda},$$

(3)
where \( q \) is the electric charge of the particle, \( \lambda \) is the parameter of the particle’s world line, and \( A_\mu = -Q/r(dt)_\mu \) is a vector potential. The parameter \( \lambda \) of the particle with mass \( m \) is related to the proper time \( \tau \) by \( \tau = m\lambda \). The local four-momentum \( p^\mu \) is given by \( p^\mu = dx^\mu/d\lambda \), where \( p^\mu \) is normalized as \( p^\mu p_\mu = -m^2 \). If we put \( m = 0 \), we obtain the motion of a massless particle. Since the Reissner-Nordström spacetime is spherically symmetric, we can assume the motion of a particle is restricted on the equatorial plane (\( \theta = \pi/2 \)). Since the metric (1) does not depend on \( t \) or \( \phi \), we obtain from the Euler-Lagrange equation

\[
\frac{dt}{d\lambda} = \frac{P(r)}{f(r)} \tag{4}
\]

and

\[
\frac{d\phi}{d\lambda} = \frac{L}{r^2}, \tag{5}
\]

where

\[
P(r) \equiv E - \frac{Q}{r}, \tag{6}
\]

and \( E \) and \( L \) denote the conserved energy and angular momentum of the particle, respectively. The equation of the radial motion is written in the following form:

\[
\left(\frac{dr}{d\lambda}\right)^2 + V(r) = 0, \tag{7}
\]

where the effective potential \( V(r) \) is given by

\[
V(r) = -\left( E - \frac{qQ}{r} \right)^2 + f(r) \left( m^2 + \frac{L^2}{r^2} \right). \tag{8}
\]

In the region where the motion of the particle is allowed, \( V(r) \) must be nonpositive. In the limit \( r \to \infty \), \( V(r) \) becomes

\[
\lim_{r \to \infty} V(r) = -E^2 + m^2,
\]

implying that the particle can be at infinity if \( E \geq m \).

Since we are considering a causal world line, we need to impose \( dt/d\lambda \geq 0 \) along it. This is called a forward-in-time condition. Since we are interested in the outside of the horizon, the following inequality must hold in the region in which the particle is allowed to exist:

\[
E - \frac{qQ}{r} \geq 0. \tag{9}
\]

In particular, to reach the horizon from the outside of the horizon, the particle must satisfy

\[
E \geq \frac{qQ}{r_H}. \tag{10}
\]
We call a particle which satisfies the equality in Eq. (10) a critical particle and the equality a critical condition.

In the following, we consider particles with vanishing angular momentum, i.e., $L = 0$, in the maximally charged black hole spacetime. There are two turning points, where $V(r)$ vanishes, given by

$$r = r_{\pm} \equiv M \left( 1 + \frac{q - E}{E + m} \right).$$

(11)

If the particle is unbounded or $E > m$, $r_+$ is an outer turning point. If an unbounded particle satisfies $q > E$, $M < r_{-} < r_{+}$ holds and the region between $r_{-}$ and $r_{+}$ is prohibited.

### III. COLLISION OF ARBITRARILY HIGH CM ENERGY

We consider two charged particles falling radially into the maximally charged black hole. The CM energy $E_{\text{cm}}$ of the two colliding particles in the limit where the collision point approaches the horizon is given by

$$\lim_{r \to r_{H}} E_{\text{cm}}^2 = m_1^2 + m_2^2 + \left[ \frac{E_2 - q_2}{E_1 - q_1} m_1^2 + \frac{E_1 - q_1}{E_2 - q_2} m_2^2 \right].$$

(12)

The constants $E_i$, $m_i$, and $q_i$ ($i = 1, 2$) denote the conserved energy, mass, and electric charge of particle $i$, respectively. If either of the two particles satisfies $E_i = q_i$ or the critical condition, the right-hand side of Eq. (12) diverges. If the critical particle satisfies $E_i > m_i$, the potential does not prevent the critical particle from approaching the horizon. If particle 1 is critical and particle 2 is subcritical, i.e., $q_1 = E_1$ and $q_2 < E_2$, the CM energy of the two particles colliding at $r = r_{c} \equiv M(1 + \epsilon)$ ($\epsilon \ll 1$) is given by

$$E_{\text{cm}}^2 \approx \frac{2A(E_2 - q_2)}{\epsilon},$$

(13)

where $A \equiv E_1 - \sqrt{E_1^2 - m_1^2}$. We can see from Eq. (13) that the CM energy can grow without bound in the limit where the collision point approaches the horizon.

Now, we consider the reaction of particles 1 and 2 into particles 3 and 4, all of which are assumed to move radially, i.e., $L_i = 0$ ($i = 1, 2, 3, 4$). We assume that particle 4 is moving inwardly at the collision point, i.e., $p^r_4 < 0$.

The conservation of charge and four-momentum before and after the collision at the collision point $r = r_{c} \equiv M(1 + \epsilon)$ yields

$$q_1 + q_2 = q_3 + q_4$$

(14)
and
\[ p_1'' + p_2'' = p_3'' + p_4'', \quad (15) \]
respectively. Since all particles move radially, the \( \theta \) and \( \phi \) components of Eq. (15) vanish. Using Eq. (14), we obtain the energy conservation from the \( t \) component of Eq. (15). Since the product particles after the collision are timelike or null, particles 3 and 4 must satisfy \( m_i^2 \geq 0 \) \((i = 3, 4)\) and Eq. (9) in the region where they are allowed to exist. For particle 3, the outer turning point must be located inside the collision point so that it can escape to infinity.

**IV. ESCAPE OF SUPERHEAVY AND HIGHLY ENERGETIC PARTICLES**

Here, we explicitly show that particle 3 can be very massive and escape to infinity, simultaneously. For this purpose, we assume that particle 3 is moving inwardly or \( p_3^r < 0 \) is satisfied at the collision point and satisfies the following equations:
\[ q_3 = E_3(1 + \delta \epsilon) \quad (16) \]
and
\[ E_3(1 - \delta) = \frac{1}{2}(A + \frac{m_3^2}{A}), \quad (17) \]
where \( \delta \) is a constant between 0 and 1. \( E_3 > m_3 \) is automatically satisfied, which is necessary for particle 3 to escape to infinity. In the following, we check that particle 3 can be very massive and energetic and really escape to infinity, simultaneously.

First, we check the location of the outer turning point for particle 3. Using Eqs. (11) and (16), we find that the outer turning point for particle 3 is given by
\[ r_{+3} = M \left( 1 + \frac{E_3}{E_3 - m_3^2} \delta \epsilon \right). \quad (18) \]
Since we assume \( p_3^r < 0 \), particle 3 must be bounced by the potential barrier to escape to infinity. The outer turning point \( r_{+3} \) is located outside the horizon. From Eq. (18), the requirement \( r_c \geq r_{+3} \) is equivalent to
\[ E_3(1 - \delta) \geq m_3. \quad (19) \]
We find that the above inequality is automatically satisfied from Eq. (17).
Then, we check the forward-in-time condition for particle 3. Using Eqs. (16) and (18), we can see that Eq. (9) is automatically satisfied at \( r = r_{+,3} \), and it is also automatically satisfied in the outer region.

Then, we check that all of \( m_2^2 \geq 0, m_3^2 \geq 0, \) and \( m_4^2 \geq 0 \) can be satisfied. The reaction must satisfy the local momentum conservation. The \( r \) component of the four-momentum of particle \( i \) is given by \( |p_r^i| = \sqrt{-V_i} \). Using Eqs. (16) and (17), we can solve the \( r \) component of Eq. (15) for \( m_3^2 \). The result is the following:

\[
m_3^2 = A \frac{m_2^2 - m_4^2}{P_2 \sqrt{-V_2}} \left( 1 - \frac{M}{r} \right) + m_1^2,
\]

where \( P_2 \) and \( V_2 \) are given by Eqs. (6) and (8) of particle 2, respectively. Therefore, if \( m_2^2 > m_4^2 \geq 0 \), the right-hand side of Eq. (20) is positive. Thus, both \( m_3^2 \) and \( m_4^2 \) can be positive. Especially, from Eq. (20), \( m_3^2 \) for the near-horizon collision (\( \epsilon \ll 1 \)) is given by

\[
m_3^2 \approx \frac{2(E_2 - q_2)A}{\epsilon} \left( 1 - \frac{m_4^2}{m_2^2} \right) \approx E_{cm}^2 \left( 1 - \frac{m_4^2}{m_2^2} \right),
\]

where Eq. (13) has been used. Therefore, if \( m_2 > m_4 \) is satisfied, \( m_3^2 \) is positive. The above equation also implies that \( m_3 \) is typically of the order of the CM energy.

Finally, we check the forward-in-time condition for particle 4. Since particle 4 moves inside the collision point, we study the left-hand side of Eq. (9) for particle 4 between the collision point and the horizon. From the energy and charge conservation, \( E_4 \) and \( q_4 \) are written in terms of \( E_1, E_2, E_3, q_1, q_2, \) and \( q_3 \). To estimate the left-hand side of Eq. (9) for particle 4 at the horizon, using the energy and charge conservation and Eq. (16), we find

\[
E_4 - q_4 = E_2 - q_2 + E_3 \delta \epsilon.
\]

Since \( E_3 > 0, E_2 > q_2, \) and \( \delta > 0 \), the right-hand side of Eq. (22) is positive. To estimate the left-hand side of Eq. (9) of particle 4 at the collision point, using the energy and charge conservation, Eqs. (16), (17), and (21), we find

\[
E_4 - q_4 \left|_{1+\epsilon} \right. = \frac{1}{2} \left[ P_2 - \sqrt{P_2^2 - m_2^2 f(r)} + \frac{m_4^2}{m_2^2} \left( P_2 + \sqrt{P_2^2 - m_2^2 f(r)} \right) \right]_{r=M(1+\epsilon)}.
\]

Thus, if \( m_1^2 \geq 0 \), the right-hand side of Eq. (23) is positive. Since the left-hand side of Eq. (9) is a monotonic function, particle 4 satisfies the forward-in-time condition in the relevant region.
In the above, we have shown that the assumptions (16) and (17) are consistent with the forward-in-time condition, \( r_c > r_{+3} \), the momentum conservation, \( m_3^2 \geq 0, m_4^2 \geq 0 \), and \( E_3 > m_3 \). Now, we examine how energetic and massive particle 3 can be if \( \epsilon \ll 1 \). For simplicity, we assume \( m_1 = E_2 = m_2 = m \) and \( q_2 = 0 \). Since particle 1 is assumed to be critical, \( E_1 = q_1 \) must be satisfied. For simplicity, we assume that \( E_1 \) is slightly larger than the rest mass energy \( m_1 \), while an exactly marginally bound critical particle, i.e., \( E_1 = q_1 = m_1 \), must be at rest. Since \( m_4 \) must be smaller than \( m_2 \), let us choose \( m_4 = m/\sqrt{2} \). If particles 1 and 2 collide at \( r = M(1 + \epsilon) \) (\( \epsilon = 10^{-4} \)), we can find from Eq. (21) that the mass of particle 3 is given by \( m_3 \simeq 100m \). Since the constant \( \delta \) can take any value between zero and unity, let us choose \( \delta = 1/2 \). Using Eqs. (16) and (17), we can find \( E_3 \simeq q_3 \simeq 10^4m \). In general, for \( \epsilon \ll 1 \), we find that \( m_3 \sim E_{\text{cm}} \sim m/\sqrt{\epsilon} \) and \( E_3 \sim q_3 \sim m/\epsilon \) up to numerical factors of order unity.

V. CONCLUSION AND DISCUSSION

We have shown that a superheavy and highly energetic particle, which can be produced by the collision near an extremal Reissner-Nordström black hole, can escape to infinity and be in principle observed by a distant observer. In other words, kinematics does not forbid such a massive particle from escaping to infinity in contrast to the Kerr case, where kinematics forbids such a superheavy particle from escaping to infinity. In this study, we have neglected gravitational and electromagnetic radiation and backreactions on the metric and the electromagnetic field. Thus, one may expect that there is an upper bound on the energy and mass of escaping product particles in realistic situations due to these effects.

Finally, we should comment on the present mechanism in the context of particle physics in the real world. It is well known that electric charge is quantized by an elementary charge \( e \simeq 1.6 \times 10^{-19} \text{C} \). The energy of the critical particle in this paper is therefore discretized by the mass corresponding to \( e \), which is given by \( \sqrt{\alpha}E_{\text{Pl}} \), where \( \alpha \simeq 1/137 \) is the fine-structure constant and \( E_{\text{Pl}} \simeq 10^{19} \text{GeV} \) is the Planck energy. This clearly makes it problematic to interpret the present mechanism as a factory of superheavy elementary particles in a usual sense. To circumvent this problem, one might introduce a macroscopic object which satisfies a critical condition. For example, an object as massive as \( 10^{-6} \text{g} \) can be critical if it is charged with only one elementary charge.

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