The axial anomaly in QCD at finite temperature

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Abstract

We study flavor mixing and the $U(1)_A$ anomaly in QCD at zero and finite
temperature. Using the instanton liquid model, we show that the strength of
the anomaly is essentially unchanged near the critical temperature for chiral
symmetry restoration. We demonstrate that nevertheless chiral symmetry
restoration has important consequences for the $\eta$ and $\eta'$. In particular, the
strange and non-strange components of the $\eta$ unmix near $T_c$. The anomaly
does not affect the strange eta, so we expect a light purely strange pseudo-
scalar near the phase transition.

11.30.Rd, 12.38.Lg, 12.38.Mh
1. In connection with the ongoing heavy ion program at AGS and CERN it is of great interest to identify possible changes in hadronic properties as matter is heated up and reaches the critical temperature for chiral symmetry restoration. Recently, a number of authors have argued that such changes might be very dramatic in the $\eta - \eta'$ sector [1–4] (see [5–8] for earlier work on the subject). At zero temperature, the $\eta' - \pi$ mass splitting, which is (mostly) due to the anomaly, is larger than any other mesonic mass splitting. This means that any tendency towards (partial) $U(1)_A$ restoration might lead to physical effects that are more easily observed than changes in, for example, the $\rho$ meson or nucleon channels.

In this letter, we wish to study the axial anomaly and the $\eta - \eta'$ system at finite temperature in the instanton liquid model [9–11]. Since instantons provide the mechanism for the $U(1)_A$ anomaly in QCD, the applicability of the model appears obvious. Nevertheless, we would like to make two additional comments. First, the model not only accounts for the anomaly, it gives a very successful description of hadronic phenomenology in general. In addition to that, the model describes spontaneous chiral symmetry breaking as well as its restoration at a critical temperature $T_c \simeq 140$ MeV. Our second comment concerns the anomaly. A quantitative description of the $\eta - \eta'$ system in the instanton model is more subtle than one might expect. Indeed, the simplest, random, instanton model fails to give an acceptable description of the $\eta'$ mass and $\eta - \eta'$ mixing.

The plan of this letter is as follows. First, we discuss the $\eta - \eta'$ system at zero temperature. Next, we review general arguments concerning the anomaly at finite temperature. Finally, we study $\eta$ and $\eta'$ correlation functions at finite temperature.

2. Before we go into detail, we should remind the reader of ’t Hooft’s mechanism for $U(1)_A$ breaking in QCD [12]. The QCD partition function receives contributions from special field configurations, instantons, that carry topological charge. In the field of an instanton, the Dirac operator has a chiral zero mode, $i\bar{\gamma} \gamma_5 \phi = 0$. In the case of an instanton, the zero mode is left handed $\gamma_5 \phi = -\phi$, while in the case of an anti-instanton it is right handed. The presence of a zero mode implies that for massless fermions, the amplitude for an isolated instanton vanishes, because it contains a factor $\det(i\gamma)$. However, when we
calculate a $U(1)_A$ violating observable, like the expectation value of the 't Hooft operator $\mathcal{O}_{\text{det}} = \det_f(\bar{\psi}_L\psi_R) + (L \leftrightarrow R)$, the $N_f$ zero modes in the determinant cancel against $N_f$ zero modes in the quark propagators. As a result, the instanton can absorb $N_f$ left handed quarks and turn them into right handed quarks, violating axial charge by $2N_f$ units.

In order to study the effect of the anomaly on the spectrum of pseudoscalar mesons, we have to consider correlation functions of the $SU(3)$ singlet and octet pseudoscalar meson currents $j_{0,8} = \bar{q}\gamma_5\lambda_{0,8}q$. The diagonal singlet and octet, as well as the off-diagonal singlet-octet mixing correlators are given by

$$\Pi_{00} = \frac{1}{3} \left\{ 2\Pi_{\text{con}}^u + \Pi_{\text{con}}^s - 4\Pi_{\text{dis}}^u - \Pi_{\text{dis}}^s - 4\Pi_{\text{dis}}^{us} \right\}, \quad (1)$$

$$\Pi_{88} = \frac{1}{6} \left\{ 2\Pi_{\text{con}}^u + 4\Pi_{\text{con}}^s - 4\Pi_{\text{dis}}^u - 4\Pi_{\text{dis}}^s - 8\Pi_{\text{dis}}^{us} \right\}, \quad (2)$$

$$\Pi_{08} = \frac{1}{3\sqrt{2}} \left\{ 2\Pi_{\text{con}}^u - 2\Pi_{\text{con}}^s - 4\Pi_{\text{dis}}^u + 2\Pi_{\text{dis}}^s + 2\Pi_{\text{dis}}^{us} \right\}, \quad (3)$$

where $\Pi_{\text{con}}^f = \langle \text{Tr} \left[ S^f(x,y)\gamma_5S^f(y,x)\gamma_5 \right] \rangle$ is a connected correlation function and $\Pi_{\text{dis}}^{fg} = \langle \text{Tr} \left[ S^f(x,x)\gamma_5 \right] \text{Tr} \left[ S^g(y,y)\gamma_5 \right] \rangle$ is a disconnected correlator. Here, $S^f(x,y)$ is the quark propagator of a quark with flavor $f$ and $\langle \cdot \rangle$ denotes averaging over all gauge field configurations. In deriving (1-3) we have assumed exact isospin symmetry. For comparison, the pion correlator is given by $\Pi_{\pi} = \Pi_{\text{con}}^u$ and the kaon correlator by $\Pi_K = \Pi_{\text{con}}^{us}$. Also, the correlators of the strange and non-strange components of the $\eta$ are given by $\Pi_{NS} = 2\Pi_{\text{con}}^u - \Pi_{\text{dis}}^u$ and $\Pi_S = \Pi_{\text{con}}^s - \Pi_{\text{dis}}^s$, while their mixing is determined by $\Pi_{NS,S} = \sqrt{2}\Pi_{\text{dis}}^{us}$. At zero temperature, we will exclusively focus on euclidean space correlation functions. This means that the long time behavior of the correlators is given by $\Pi \sim \exp(-mx)$, where $m$ is the ground state mass in the given channel.

Due to flavor $SU(3)$ symmetry breaking, the singlet and octet correlation functions mix. The physical $\eta$ and $\eta'$ states couple to linear combinations of the singlet and octet currents,

$$\eta' = \cos \theta \eta_0 + \sin \theta \eta_8. \quad (4)$$

Experimentally, the value of the mixing angle is $\theta \simeq -(10 - 20)\degree$. The sign of the mixing angle corresponds to a reduction of the strange component of the $\eta$ and a strangeness
enhancement in the $\eta'$. The uncertainty in the mixing angle is hard to assess. In fact, the concept of a mixing angle may not be very well defined, since the two states are rather far apart in mass.

Qualitatively, instanton effects can be understood from the propagator in the field of a single instanton, $S^I(x,y) = \phi_0(x)\phi_0(y)/m_f + S_{NZM}(x,y) + S_m(x,y)$. Here, $\phi_0(x)$ is the zero mode wave function, $S_{NZM}(x,y)$ is the non-zero mode propagator and $S_m(x,y)$ includes mass corrections [13]. Let us first consider the case of exact $SU(3)$ symmetry. It is important to note that zero modes give identical contributions to the connected and disconnected correlators. We find $\Pi_{00} = -2 \cdot I$, $\Pi_{08} = I$ and $\Pi_{08} = 0$, where we have defined

$$I = \int d\rho \frac{n(\rho)}{m^2} \int d^4z |\phi(x-z)|^2 |\phi(y-z)|^2. \quad (5)$$

Here, $n(\rho)$ is the single instanton density [14] and $z$ is the instanton position. Note that $I$ is not singular in the chiral limit $m \to 0$, because the instanton density contains a factor $m^2$. Strictly speaking, $n(\rho)$ is proportional to $m^3$, but if chiral symmetry is broken, zero modes can be absorbed by the quark condensate. This means that we can replace one power of $m$ by the effective mass $m^* = \frac{4}{3} \pi \rho^2 \langle \bar{q}q \rangle$ [9], which is finite in the chiral limit.

If we include $SU(3)$ flavor breaking, the situation becomes more complicated. In effective models, it is usually assumed that the ’t Hooft interaction is $SU(3)$ symmetric and the mixing is caused by quark or meson mass terms. In that case it is clear that the mixing angle is negative, since the mass term drives the system toward ideal mixing, corresponding to $\theta = -54.7^\circ$. In general, the situation is more complicated and $SU(3)$ flavor breaking in the ’t Hooft interaction is substantial. In the single instanton approximation, flavor symmetry breaking can be taken into account by replacing the effective mass $m^*$ by $m^* + m_f$ [15]. The off-diagonal correlators are completely determined by disconnected contributions, since the zero mode contributions to the connected and disconnected correlators with the same flavor cancel each other. We find [13]

$$\begin{align*}
\Pi_{08} &= \frac{\sqrt{2}}{3} \frac{m_u - m_s}{m^* + m_s} I^*, \\
\Pi_{NS,S} &= \sqrt{2} \frac{m^* + m_d}{m^* + m_s} I^*,
\end{align*} \quad (6)$$

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where we have replaced the current quark masses by effective masses in \( I^* \). In the single instanton approximation \( \Pi_{08} \) is negative, which corresponds to a positive mixing angle\(^1\). On the other hand, mass corrections in the non-zero mode part of the propagator give a positive contribution to \( \Pi_{08} \). This means that the sign of the mixing angle is determined by the competition between the flavor breaking in direct mass insertions and the ’t Hooft interaction.

In the following, we study this problem in the instanton liquid model. We refer the reader to \([11,16]\) for details of the model. For our purposes here it is only important that there are three different instanton ensembles, the random (RILM), quenched (QILM) and unquenched (IILM) models. In the random and quenched ensembles, the topological susceptibility is finite \( \chi_{\text{top}} \simeq (N/V) \), where \( (N/V) \) is the density of instantons. In the unquenched ensemble, topological charge is screened and \( \chi_{\text{top}} \sim m \langle \bar{q}q \rangle \). Eta meson correlation functions in the random model\(^2\) and two different unquenched ensembles (the “stream line” and “ratio ansatz” ensembles, see \([14]\)) are shown in figure \( I \). We note that the \( U(1)_A \) anomaly is “over-explained” in the random ensemble. The \( \eta' \) channel is so repulsive that the correlation function becomes unphysical \( (\Pi(x) < 0) \). Also, flavor symmetry breaking is very strong and the singlet-octet correlator is large and negative (corresponding to a positive mixing angle).

This situation is improved in the unquenched ensembles. The flavor-singlet correlation function is physical and singlet-octet mixing is smaller. The prediction for the masses and mixing angles depends sensitively on details of the interaction. In the streamline ensemble, the \( \eta' \) is still too heavy, \( m_{\eta'} \simeq 2 \text{ GeV} \). The mass of the \( \eta \) is given by \( m_\eta = (0.66 \pm 0.12) \text{ GeV} \)

\(^1\) The off-diagonal correlator in the single instanton approximation was first calculated in \([13]\), but the conclusions concerning the mixing angle appear to be wrong.

\(^2\) The results in the quenched ensemble are very similar. Eta meson correlation functions in the random ensemble were first studied in \([17]\). Unfortunately, this work contains an error in the flavor octet and the off-diagonal singlet-octet correlation functions. In particular, \( \Pi_{08} \) has the wrong sign.
and the mixing angle is small, $\theta = (1 \pm 3)^\circ$. A small mixing angle was also found in the Nambu and Jona-Lasinio model \cite{18} (see also \cite{19,20}). In particular, these authors showed that a small mixing angle is not necessarily incompatible with the observed $\eta \rightarrow 2\gamma$ rate.

3. A global measure of the strength of the anomaly is provided by the expectation value of the ’t Hooft operator. In figure \ref{fig:2} we show the temperature dependence of $\langle O_{\text{det}} \rangle$ in the finite temperature instanton ensembles\cite{11} obtained in \cite{11}. For comparison, we also show the quark condensate and two different four fermion operators. At zero temperature both the $SU(2) \times SU(2)$ chiral and the axial $U(1)_A$ symmetry are broken. Chiral symmetry breaking is caused by interactions between the zero modes associated with individual instantons. As a result, some of the lowest states become collective and form a condensate. Near $T = 125$ MeV, chiral symmetry is restored and the quark condensate goes to zero. This transition is due to a rearrangement of the instanton liquid, going from a disordered, random, system to an ensemble of topologically neutral instanton-anti-instanton pairs. Clearly, there is no tendency towards $U(1)_A$ restoration as chiral symmetry is restored. In fact, $\langle O_{\text{det}} \rangle$ has a maximum near the phase transition (although the uncertainty is also largest near $T_c$). The reason why $\langle O_{\text{det}} \rangle$ survives the chiral phase transition should be clear from our discussion above: the ’t Hooft operator can induce a tunneling event all by itself\cite{12}. At temperatures significantly above $T_c$ the semi-classical tunneling amplitude contains the suppression factor $n(\rho) \sim \exp\left(- \frac{2N_c}{3} + \frac{N_f}{3} \left(\frac{\pi \rho T}{\phi}\right)^2\right)$ \cite{23} and $\langle O_{\text{det}} \rangle$ becomes small. This suppression factor is mostly due to Debye screening of the instanton field. Therefore, it does not affect the

\footnote{All of these ensembles have total topological charge $Q = 0$. This means that the topological susceptibility evaluated for the entire volume is not correct. This should not affect local observables. Indeed, we have checked that topological charge fluctuations in a sub-volume have the expected dependence on the quark mass and volume \cite{21}.}

\footnote{This can also be verified in the mean field approximation to the instanton liquid for two flavors, see \cite{22}.}
instanton density below the phase transition. This was checked explicitly by performing a
soft pion calculation of the instanton density at small temperature [24] and in (quenched)
lattice calculations of the instanton density at finite temperature [25–27].

Local four fermion operators like $O_{det}$ are hard to measure on the lattice. Therefore, a number of authors have studied $U_A(1)$ violating mesonic susceptibilities. Here, mesonic susceptibilities are defined as integrals of the corresponding correlation function,$\chi_\Gamma = \int d^4x \Pi_\Gamma(x)$. The most natural candidate for a $U(1)_A$ order parameter is the difference $\chi_\pi - \chi_\eta'$, but this quantity also involves disconnected quark loops. A better observable is $\chi_\pi - \chi_\delta$, originally suggested by the Columbia group [28]. For $N_f = 2$ and if chiral symmetry is restored, this quantity is a measure of $U(1)_A$ breaking. A nice feature of $\chi_\pi - \chi_\delta$ is that it can be expressed in terms of the spectral density $\rho(\lambda)$ of the Dirac operator

$$\chi_\pi - \chi_\delta = 4m^2 \int d\lambda \frac{\rho(\lambda)}{(\lambda^2 + m^2)^2}. \quad (7)$$

For comparison, the quark condensate is given by

$$\langle \bar{q}q \rangle = -2m \int d\lambda \frac{\rho(\lambda)}{\lambda^2 + m^2}. \quad (8)$$

These results allow us to constrain the low virtuality part of the spectrum. For $U(1)_A$ to be broken but chiral symmetry restored, we require $\chi_\pi - \chi_\delta$ to be finite in the limit $m \to 0$ while $\langle \bar{q}q \rangle$ goes to zero. This requirement is clearly satisfied by $\rho(\lambda) \sim m^{N_f} \delta(\lambda)$, corresponding to a dilute system of instantons. However, interactions among zero and non-zero modes might alter the shape of the spectrum. It is therefore interesting to note that the criterion given above is also satisfied by a non-analytic spectral density $\rho(\lambda) \sim \lambda^\alpha$ with $\alpha \leq 1$.

In order to study this question in more detail, we have determined the spectrum of the Dirac operator in the instanton liquid (for $N_f = 2$) for several different values of the temperature and the quark masses, see figure 3. Above $T_c$ we clearly observe a peak in the spectrum near $\lambda = 0$. The number of eigenvalues in the peak is nicely consistent with $N(\lambda \simeq 0) \sim m^2$. Below $T_c$, most of the small eigenvalues are related to chiral symmetry breaking. Their number is proportional to the effective mass $m^*$ and almost independent of
the current mass $m$. There is a dip in the spectrum for small quark masses. This is a finite volume effect. In a finite volume, the spectral density near zero will always go to zero as $m \to 0$. We have checked that the width of the dip in the spectrum decreases as the volume is increased.

A number of groups have measured $\chi_\pi - \chi_\delta$ (or the corresponding screening masses) on the lattice [28–30]. Most of the published results indicate that $U(1)_A$ remains broken, although recent results by the Columbia group have questioned that conclusion [31]. From the literature, it is not clear whether lattice simulations find the peak in the spectrum observed in the instanton liquid. The Columbia group has measured the valence mass dependence of the quark condensate, which is a folded version of the Dirac spectrum [28]. Their result looks very smooth, not indicative of a small virtuality peak. One should note, however, that instanton calculations focus exclusively on the small virtuality part of the spectrum while the number of eigenvalues in lattice simulations is much larger. Also, both instanton and lattice simulations suffer from certain artefacts if the quark mass is made too small. In our case, if the mass is too small, isolated instantons are rare and the constraint $Q = 0$ affects the results. On the lattice, for small quark masses one might run into problems with chiral fermions.

4. The $U(1)_A$ anomaly at finite temperature is usually discussed in terms of the effective lagrangian [3]

$$\mathcal{L} = \frac{1}{2} \text{Tr} \left( (\partial_\mu \Phi)(\partial^\mu \Phi^\dagger) \right) - \text{Tr} \left( \mathcal{M}(\Phi + \Phi^\dagger) \right) + V(\Phi\Phi^\dagger) + c \left( \text{det} \Phi + \text{det} \Phi^\dagger \right),$$

(9)

where $\Phi$ is a meson field in the $(3, 3)$ representation of $U(3) \times U(3)$, $V(\Phi\Phi^\dagger)$ is a $U(3) \times U(3)$ symmetric potential (usually taken be quartic), $\mathcal{M}$ is a mass matrix and $c$ controls the strength of the $U(1)_A$ breaking interaction. If the coupling is taken to be $c = \chi_{\text{top}}/(12 f_\pi^3)$, the effective lagrangian reproduces the Witten-Veneziano relation $f_\pi^2 m^2_{\eta'} = \chi_{\text{top}}$. In a quenched ensemble, we can further identify $\chi_{\text{top}} \simeq (N/V)$. The temperature dependence of $c$ is usually estimated from the semi-classical tunneling amplitude $n(\rho) \sim \exp\left(-\frac{8}{3}(\pi \rho T)^2\right)$. As a result, the strength of the anomaly is reduced by a factor $\sim 5$ near $T_c$. If the anomaly
becomes weaker, the eigenstates are determined by the mass matrix. In that case, the mixing angle is close to ideal $\theta = -54.7^\circ$ and the non-strange $\eta$ is almost degenerate with the pion.

There are several points in this line of argument that are not entirely correct. The strength of the 't Hooft term is not controlled by the topological susceptibility ($\chi_{\text{top}} = 0$ in full QCD!), $\chi_{\text{top}}$ is not proportional to the instanton density (for the same reason), and, at least below $T_c$, the semi-classical estimate for the instanton density is not applicable. As we saw above, at $T_c$ the instanton liquid is rearranged but the strength of the $U(1)_A$ anomaly does not change very much. However, chiral symmetry restoration affects the structure of flavor mixing in the $\eta - \eta'$ system (see figure 4). The mixing between the strange and non-strange eta is controlled by the light quark condensate (see equation (6)), so $\eta_{\text{NS}}$ and $\eta_{\text{S}}$ do not mix above $T_c$. As a result, the mixing angle is not close to zero, as it is at $T = 0$, but close to ideal. Furthermore, the anomaly can only affect the non-strange $\eta$, not the strange one. Therefore, if the anomaly is sufficiently strong, the $\eta_{\text{NS}}$ will be heavier than the $\eta_{\text{S}}$.

We should compare this scenario to other possibilities discussed in the recent literature. A number of authors have noticed that for three massless flavors, both the $\eta$ and $\eta'$ are unaffected by the anomaly above $T_c$ \cite{16, 32–34}. This is a related effect, but not that relevant for QCD, where the strange mass is not small. Also, two recent papers have argued that the $\eta_{\text{NS}}$ and $\eta_{\text{S}}$ decouple near $T_c$ \cite{2, 3}. However, in these works the effect is caused by the disappearance of the anomaly and as a result, the $\eta_{\text{S}}$ is always predicted to be heavier than the $\eta_{\text{NS}}$. The scenario proposed here is consistent the effective lagrangian \cite{5}. However, most authors employ a “first order” treatment of flavor symmetry breaking and neglect terms of order $(m_s - m_u)c$. These terms are precisely what causes the effect discussed here.

5. To explore this phenomenon in a more quantitative way, we study $\eta - \eta'$ correla-

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5 Our scenario cannot be described in terms of the non-linear effective lagrangian employed in \cite{3}. This should not be surprising; non-linear effective meson theories have to be used with care near the chiral phase transition.
tion functions in the instanton model at finite temperature. The results were obtained for
three flavors with masses \( m_u = m_d = 22 \text{ MeV} \) and \( m_s = 155 \text{ MeV} \). We consider temporal
correlation functions, rather than the spacelike screening correlators usually calculated on
the lattice. Temporal correlators have the advantage that they are directly related to the
spectrum of physical excitations.

The results are shown in figure 5. Correlation functions below \( T_c \) are shown by the open
squares (\( T = 55 \text{ MeV} \)), pentagons (\( T = 78 \text{ MeV} \)) and hexagons (\( T = 111 \text{ MeV} \)), while the
correlators near and above \( T_c \) are denoted by closed squares (\( T = 126 \text{ MeV} \)) and pentagons
(\( T = 145 \text{ MeV} \)). Below \( T_c \) the singlet correlation function is strongly repulsive, while the
octet correlator shows some attraction at larger distance. The off-diagonal correlator is small
and positive, corresponding to a negative mixing angle. The strange and non-strange eta
correlation functions are very similar, which is a sign for strong flavor mixing. This is also
seen directly from the off-diagonal correlator between \( \eta_S \) and \( \eta_{NS} \).

Above \( T_c \), the picture changes. The off-diagonal singlet-octet changes sign and its value
at intermediate distances \( \tau \simeq 0.5 \text{ fm} \) is significantly larger. The strange and non-strange
eta correlators are very different from each other. The non-strange correlation function
is very repulsive, while the strange one is significantly larger. This clearly supports the
scenario presented above. Near \( T_c \) the eigenstates are essentially the strange and non-
strange components of the \( \eta \), with the \( \eta_S \) being the lighter of the two states. This picture
is not realized completely, \( \Pi_{S,NS} \) does not vanish and the singlet eta is still somewhat more
repulsive than the octet eta correlation function. This is due to the fact that the light quark
mass does not vanish. In particular, in this simulation the ratio \( (m_u + m_d)/(2m_s) = 1/7 \),
which is about three times larger than the physical mass ratio.

It is difficult to provide a quantitative analysis of temporal correlation functions in the
vicinity of the phase transition. At high temperature the temporal direction in a euclidean
box becomes short and there is no unique way to separate out the contribution from ex-
cited states. Nevertheless, under some simplifying assumptions one can try to translate
the correlation functions shown in figure 5 into definite predictions concerning the masses
of the $\eta$ and $\eta'$. For definiteness, we will use ideal mixing above $T_c$ and fix the threshold for the perturbative continuum at 1 GeV. In this case, the masses of the strange and non-strange components of the $\eta$ at $T = 126$ MeV are given by $m_{\eta_S} = (0.420 \pm 0.120)$ GeV and $m_{\eta_S} = 1.250 \pm 0.400$ GeV.

6. In summary, we studied flavor mixing and the axial anomaly at zero and finite temperature. At zero temperature, we have emphasized two points. The first one is that the correlations among instantons that lead to topological charge screening are also important in reproducing the $\eta$ and $\eta'$ correlation functions. The second one is that flavor symmetry breaking in the instanton induced interaction is substantial. It acts against the symmetry breaking from direct mass insertions. As a result, the $\eta - \eta'$ mixing angle is small.

At finite temperature, we have shown that the strength of the anomaly as measured by the expectation value of the 't Hooft operator is essentially independent of temperature below $T_c$. In our model, $\langle \mathcal{O}_{det} \rangle$ even peaks near $T_c$ and then drops at larger temperatures. In terms of the spectral density of the dirac operator, the anomaly is due to a spike $\rho(\lambda) \sim m_N^N \delta(\lambda)$ in the spectrum at zero virtuality.

Our most important result concerns the role of flavor mixing at finite temperature. Although the strength of the anomaly is not reduced near $T_c$, chiral symmetry restoration affects the $\eta - \eta'$ system. If the light quark condensate vanishes, transitions between light and strange pseudoscalars are suppressed. As a result, the eigenstates are given by the strange and non-strange components of the $\eta$. The anomaly does not affect the strange $\eta$, so we predict a purely strange, light pseudoscalar near the transition. We have estimated the mass of this state to be around 400 MeV. A light strange meson is of interest in connection with strangeness production in relativistic heavy ion collisions. This suggests that the coupling of a light $\eta_S$ to kaons and etas should be studied in more detail.

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FIGURES

FIG. 1. Singlet and octet eta correlation functions in different instanton ensembles at zero temperature. All correlation functions are normalized to free quark propagation.

FIG. 2. Temperature dependence of the quark condensate, the ’t Hooft operator and two different four fermion operators in the instanton liquid. All condensates are normalized to their $T = 0$ values.

FIG. 3. Dirac spectra below and above the chiral phase transition for different dynamical quark masses. The spectral density is given in arbitrary units. The instanton density was held fixed at $(N/V)\Lambda^{-4} = 1$, where $\Lambda$ is the QCD scale parameter. Quark masses and inverse temperature $\beta = T^{-1}$ are given in units of $\Lambda$.

FIG. 4. Leading contributions to flavor mixing in the $\eta - \eta'$ system below and above the chiral phase transition.

FIG. 5. Correlation functions for singlet and octet, off-diagonal singlet-octet as well as strange and non-strange etas at different temperatures. Open squares, pentagons and hexagons are below $T_c$, closed squares and pentagons above $T_c$. 

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FIG. 1.

FIG. 2.
FIG. 3.

FIG. 4.
FIG. 5.