Quark spin content of the proton, hyperon semileptonic decays, and the decay width of the Θ⁺ pentaquark

Ghil-Seok Yang,¹,∗ Hyun-Chul Kim,² † and Klaus Goeke¹ ‡

¹Institut für Theoretische Physik II, Ruhr-Universität Bochum, D–44780 Bochum, Germany
²Department of Physics, and Nuclear Physics & Radiation Technology Institute (NuRI), Pusan National University, 609-735 Busan, Republic of Korea

(Dated: March 2007)

Abstract

Using the existing experimental data for hyperon semileptonic decays and the flavor-singlet axial-vector charge $g_A^{(0)}$ from polarized deep inelastic scattering of the proton, we derive the decay width of the Θ⁺ pentaquark baryon. We take into account the effects of flavor SU(3) symmetry breaking within the framework of the chiral quark-soliton model. All dynamical parameters of the model are fixed by using the five experimental hyperon semileptonic decay constants and flavor singlet axial-vector charge. We obtain the numerical results of the decay width of the Θ⁺ pentaquark baryon as a function of the pion-nucleon sigma term $\Sigma_{\pi N}$ and investigate the dependence of the decay width of the Θ⁺ on the $g_A^{(0)}$, varying the $g_A^{(0)}$ within the range of the experimental uncertainty. We demonstrate that the combined values of all known semileptonic decays with the generally accepted value of $g_A^{(0)} \approx 0.3$ for the proton are compatible with a small decay width $\Gamma_{\Theta KN}$ of the $\Theta⁺$ pentaquark, i.e. $\Gamma_{\Theta KN} \leq 1$ MeV.

PACS numbers: 13.30.Ce, 13.30.Eg, 14.20.Dh, 14.20.Jn

Keywords: Pentaquark baryons, Quark spin content of the nucleon, semileptonic decays, decay widths of the pentaquark baryons, chiral quark-soliton model

∗Electronic address: yangg@tp2.rub.de
†Electronic address: hchkim@pusan.ac.kr
‡Electronic address: Klaus.Goeke@rub.de
I. INTRODUCTION

Since Diakonov, Petrov and Polyakov [1] predicted in the chiral quark-soliton model (χQSM) the mass and narrow decay width of the pentaquark baryon Θ⁺ with strange quantum number S = +1 and leading quark Fock structure uudd̄s, there has been an enormous amount of theoretical and experimental works (see, for example, recent reviews for the experimental results [2] and for the theoretical investigations [3, 4, 5]). Note that there is an earlier prediction by Praszalowicz of the mass in the soliton approach of the Skyrme model [6]. Many experiments have announced the existence of the Θ⁺ after the first independent observations by the LEPS [7] and DIANA [8] collaborations, while the Θ⁺ has not been seen in almost all high-energy experiments. Moreover, an exotic Ξ¹⁰ state was observed by the NA49 experiment at CERN [9], though its existence is still under debate.

A very recent CLAS experiment dedicated to search for the Θ⁺ has announced null results of finding the Θ⁺ in the reaction γp → K⁰Θ⁺ [10]. The subsequent experiment has also not found any evidence for the Θ⁺ in γd → pK⁻Θ⁺ [11]. Though these experiments are the measurements with high statistics, it is too early to conclude the absence of the Θ⁺. Note e.g. that the DIANA collaboration has continued to search for the Θ⁺ and found the formation of a narrow pK⁰ peak with mass of 1537 ± 2 MeV/c² and width of Γ = 0.36 ± 0.11 MeV in the K⁺n → K⁰p reaction [12]. Moreover, several new experiments searching for the Θ⁺ are in progress [13, 14]. In this obscure status for the Θ⁺, more efforts are required for understanding the Θ⁺ theoretically as well as experimentally. In addition, a recent GRAAL experiment [15] announced the evidence of a new nucleon-like resonance with a seemingly narrow decay width ~ 10 MeV and a mass ~ 1675 MeV in the η photoproduction from the neutron target. This new nucleon-like resonance, N*(1675), may be regarded as a non-strange pentaquark because of its narrow decay width and dominant excitation on the neutron target which are known to be characteristic for typical pentaquark baryons [16], though one should not exclude a possibility that it might be one of the already known πN resonances (possibly, D₁₅) [17]. This GRAAL data is consistent with the results for the transition magnetic moments in the chiral quark-soliton model (χQSM) [18] as well as the partial-wave analysis for the non-strange pentaquark baryons [19]. Moreover, a recent theoretical calculation of the γN → ηN reaction [20] describes qualitatively well the GRAAL data, based on the values of the transition magnetic moments in Refs. [18, 19], which implies that the N* seen in the GRAAL experiment could be favorably identified as one of the pentaquark baryons.

However, there is general opinion that, if the Θ⁺ exists, its width should be extremely small. Its value may even possibly lie below 1 MeV [21, 23]. As far as theory is concerned, the decay width of the Θ⁺ has been investigated in many different approaches [22, 23, 24, 25, 26, 27, 28, 29] and is mostly estimated to be very small. In the present work, we want to study the decay width of the Θ⁺ baryon within the framework of the chiral quark-soliton model (χQSM), including the effects of flavor SU(3) symmetry breaking and using the “model-independent approach” [30]. Recently, this approach has been applied to evaluate the magnetic moments of the baryon decuplet and antidecuplet, with parameters fixed by

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1 The approach is “model-independent” insofar that it does not perform self-consistent calculations leading to some solitonic profile but uses only the semiclassical rotational picture of the χQSM and determines the dynamical coefficients by fitting them to experimental data. In fact the pioneering paper on Θ⁺ [1] used this method.
experimental magnetic moments of the baryon octet \([31, 32]\) and the baryon octet, decuplet, and antidecuplet mass-splittings and the mass of the \(\Theta^+\). The same method was employed to get various transition magnetic moments \([18]\) and the results are in a good agreement with the SELEX and GRAAL data. Thus, in the present work, we want to analyze in the same way the axial-vector coupling constant of the \(\Theta^+\), based on the experimental data for hyperon semileptonic decay (HSD) constants \((g_1/f_1)^{B_1\rightarrow B_2}\) and the flavor-singlet axial-vector constant of the proton \(g_A^{(0)}\). It is in particular interesting to use the \(g_A^{(0)}\) as an input, since it carries information on the quark spin content of the proton. It is extracted from deep inelastic polarized electron-proton scattering and hence its information is independent of HSD. In fact, the \(g_A^{(0)}\) is related to the pseudoscalar coupling \(G_2\) in Ref. \([1]\) by the Goldberger-Treiman relation but there it has been neglected, its effect being assumed to be rather small. In Ref. \([1]\) the \(\Gamma_{\Theta KN}\) was determined by the empirical value of the pion-nucleon coupling constant \(g_{\pi NN}\) \([34]\). Moreover, the effects of flavor SU(3) symmetry breaking were neglected.

In the present work, we will perform a more general analysis of the \(\Gamma_{\Theta KN}\), emphasizing its dependence on \(g_A^{(0)}\) of the proton and the effects of SU(3) symmetry breaking. While the decay constants of HSDs are relatively well determined \([36]\), the value of the \(g_A^{(0)}\) is experimentally only known to be in the range of 0.15 – 0.35 \([37]\). Thus, we need to examine explicitly the dependence of the \(\Gamma_{\Theta KN}\) on the \(g_A^{(0)}\). We will later show that the \(\Gamma_{\Theta KN}\) is rather sensitive to \(g_A^{(0)}\) which is in contrast to the assumptions made in Ref. \([1]\). Moreover, we will see that the \(\Gamma_{\Theta KN}\) is constrained by the value of the \(\Sigma_{\pi N}\). In the end, we will see that the known data on HSDs and the experimental value of the \(g_A^{(0)}\) are compatible with a small width of \(\Gamma_{\Theta KN} \leq 1\) MeV.

II. FORMALISM

Using a formalism similar to the one of ref. \([18]\) the form factors of HSDs are defined by the following transition matrix elements of the vector and axial-vector currents:

\[
\begin{align*}
\langle B_2 | V_{\mu}^X | B_1 \rangle & = \bar{u}_{B_2}(p_2) \left[ f_1(q^2)\gamma_{\mu} - \frac{i f_2(q^2)}{M_1} \sigma_{\mu\nu} q^\nu + \frac{f_3(q^2)}{M_1} q_{\mu} \right] u_{B_1}(p_1), \\
\langle B_2 | A_{\mu}^X | B_1 \rangle & = \bar{u}_{B_2}(p_2) \left[ g_1(q^2)\gamma_{\mu} - \frac{i g_2(q^2)}{M_1} \sigma_{\mu\nu} q^\nu + \frac{g_3(q^2)}{M_1} q_{\mu} \right] \gamma_5 u_{B_1}(p_1),
\end{align*}
\]

where the vector and axial-vector currents are defined as

\[
V_{\mu}^X = \bar{\psi}(x)\gamma_{\mu} \lambda_X \psi(x), \quad A_{\mu}^X = \bar{\psi}(x)\gamma_{\mu} \gamma_5 \lambda_X \psi(x)
\]

with \(X = \frac{1}{2}(1 \pm i2)\) for strangeness conserving \(\Delta S = 0\) currents and \(X = \frac{1}{2}(4 \pm i5)\) for \(|\Delta S| = 1\). The \(q^2 = -Q^2\) stands for the square of the momentum transfer \(q = p_2 - p_1\). The form factors \(g_i\) and \(f_i\) are real quantities due to \(CP\)-invariance, depending only on the square of the momentum transfer. We can safely neglect \(g_3\) for the reason that its contribution to the decay rate is proportional to the ratio \(\frac{m^2_{\ell}}{M_1} \ll 1\), where \(m_{\ell}\) represents the mass of the lepton (\(e\) or \(\mu\)) in the final state and \(M_1\) that of the baryon in the initial state. Taking into account the \(1/N_c\) rotational and \(m_n\) corrections, we can write the resulting axial-vector
constants $g_1^{(B_1 \rightarrow B_2)}(0)$ as follows:

$$g_1^{(B_1 \rightarrow B_2)}(0) = a_1 \langle B_2 | D^{(8)}_{X^3} | B_1 \rangle + a_2 d_{pq3}(B_2) D^{(8)}_{Xp} \hat{J}_q | B_1 \rangle + \frac{a_3}{\sqrt{3}} (B_2 | D^{(8)}_{X^3} \hat{J}_3 | B_1 \rangle)
+ m_s \left[ a_1 d_{pq3}(B_2) D^{(8)}_{Xp} \hat{J}_q | B_1 \rangle + a_5 (B_2) \left( D^{(8)}_{X^3} D^{(8)}_{ss} + D^{(8)}_{X^3} D^{(8)}_{ss} \right) | B_1 \rangle
+ a_6 (B_2) \left( D^{(8)}_{X^3} D^{(8)}_{ss} - D^{(8)}_{X^3} D^{(8)}_{ss} \right) | B_1 \rangle \right],$$

(3)

where $a_i$ denote parameters encoding the specific dynamics of the chiral soliton model. $\hat{J}_q$ ($\hat{J}_3$) stand for the $q$-th (third) components of the collective spin operator of the baryons, respectively. The $D^{(R)}_{ab}$ denote the SU(3) Wigner matrices in representation $R$.

The collective Hamiltonian describing baryons in the SU(3) χQSM takes the following form [33]:

$$\hat{H} = \mathcal{M}_{sol} + \frac{J(J+1)}{2I_1} + \frac{C_2(SU(3)) - J(J+1) - \frac{N_s^2}{12}}{2I_2} + \hat{H}'$$

(4)

with the symmetry breaking piece given by:

$$\hat{H}' = \alpha D^{(8)}_{ss} + \beta Y + \frac{\gamma}{\sqrt{3}} D^{(8)}_{ss} \hat{J}_i,$$

(5)

where parameters $\alpha$, $\beta$, and $\gamma$ are proportional to the strange current quark mass $m_s$.

Taking into account the recent experimental observation of the mass of $\Theta^+$, the parameters in Eq.(5) can be conveniently parameterized in terms of the pion-nucleon $\Sigma_{\pi N}$ term (assuming $m_s/(m_u + m_d) = 12.9$) as [28]:

$$\alpha = 336.4 - 12.9 \Sigma_{\pi N}, \quad \beta = -336.4 + 4.3 \Sigma_{\pi N}, \quad \gamma = -475.94 + 8.6 \Sigma_{\pi N}$$

(6)

(in units of MeV). Moreover, the inertia parameters which describe the splittings of SU(3) baryon mass representations take the following values (in MeV)

$$\frac{1}{I_1} = 152.4, \quad \frac{1}{I_2} = 608.7 - 2.9 \Sigma_{\pi N}.$$  

(7)

Equations (6) and (7) follow from the fit to the masses of octet and decuplet baryons and of $\Theta^+$ as well. If one imposes the additional constraint that $M_{\Xi^{++}} = 1860$ MeV, then $\Sigma_{\pi N} = 73$ MeV [28] (see also [40]) in agreement with recent experimental estimates [41, 42]. However, since the measurement of $M_{\Xi^{++}}$ is still under debate, we will not fix $\Sigma_{\pi N}$ but vary it within a certain range, i.e. $\Sigma_{\pi N} = 45 - 75$ MeV.

Because the Hamiltonian of Eq.(5) mixes different SU(3) representations, the collective wave functions are given as linear combinations [39]:

$$| B_8 \rangle = | 8^{1/2}, B \rangle + c_B^{10} | 10^{1/2}, B \rangle + c_B^{27} | 27^{1/2}, B \rangle,$$

$$| B_{10} \rangle = | 10^{1/2}, B \rangle + d_B^{10} | 8^{1/2}, B \rangle + d_B^{27} | 27^{1/2}, B \rangle + d_B^{35} | 35^{1/2}, B \rangle,$$

(8)

where $| B_R \rangle$ denotes the state which reduces to the SU(3) representation $R$ in the formal limit $m_s \rightarrow 0$. The spin index $J_3$ has been suppressed. The $m_s$-dependent (through the linear $m_s$ dependence of $\alpha$, $\beta$, and $\gamma$) coefficients in Eq.(8) can be found in Ref.[31].
where

\[
M[\Sigma_{\pi N}] = \begin{bmatrix}
-7 & 4c_{27} & -2c_{10} & 7 & -8c_{27} & -2c_{10} & 1 & +2c_{27} & -c_{10} & -11 & -2 & -2 \\
-15 & 45 & 5 & 15 & 30 & 45 & 3 & 30 & 15 & -135 & 9 & 15 \\
-4 & 2c_{27} & 4c_{27} & 2 & 2c_{27} & c_{10} & 2 & -c_{27} & 1 & -2 & 1 \\
15 & 45 & 3 & 15 & 3 & 15 & 10 & 6 & 45 & 0 & 15 \\
1 & -c_{27} & 1 & 2c_{27} & 2c_{27} & 1 & 135 & 15 & 15 & 15 \\
15 & 15 & 15 & 30 & 15 & 30 & 10 & 10 & 15 & 15 & 15 \\
-7 & 4c_{27} & 4c_{27} & 1 & 4c_{27} & 2c_{27} & c_{10} & 15 & 15 & 15 & 15 \\
15 & 45 & 3 & 45 & 3 & 30 & 15 & 6 & 270 & 9 & 15 \\
0 & 0 & 0 & 1 & 0 & 0 & -1/5 & 1 & 1/5 
\end{bmatrix}
\]

Inverting Eq. (9), we finally obtain the values of dynamical parameters \(a_i\) as functions of \(\Sigma_{\pi N}\) and \(g_A^{(0)}\).
III. RESULTS AND DISCUSSION

Table II lists as example the results of the dynamical parameters $a_i$ for $g_A^{(0)} = 0.3$. The $a_i$ depend in general on $g_A^{(0)}$ and $\Sigma_{\pi N}$ nonlinearly, except for $a_6$ which is independent of $g_A^{(0)}$ as well as of $\Sigma_{\pi N}$. As a matter of fact, the $a_i$ are generally weakly dependent on $\Sigma_{\pi N}$ and the $a_2$, $a_4$, $a_5$ are rather sensitive to $g_A^{(0)}$.

| $\Sigma_{\pi N}$[MeV] | $a_1$  | $a_2$  | $a_3$  | $a_4$  | $a_5$  | $a_6$  |
|------------------------|--------|--------|--------|--------|--------|--------|
| 45                     | -2.4811 | 0.8933 | 0.3190 | 1.3580 | 0.1399 | 0.0450 |
| 50                     | -2.4113 | 1.0303 | 0.3196 | 1.3464 | 0.1432 | 0.0450 |
| 55                     | -2.3608 | 1.1255 | 0.3210 | 1.3226 | 0.1499 | 0.0450 |
| 60                     | -2.3221 | 1.1952 | 0.3228 | 1.2904 | 0.1591 | 0.0450 |
| 65                     | -2.2909 | 1.2481 | 0.3250 | 1.2517 | 0.1700 | 0.0450 |
| 70                     | -2.2650 | 1.2895 | 0.3275 | 1.2076 | 0.1825 | 0.0450 |
| 75                     | -2.2426 | 1.3224 | 0.3303 | 1.1587 | 0.1964 | 0.0450 |

**TABLE II:** The dynamical parameters $a_i$ determined with $g_A^{(0)} = 0.3$. The $\Sigma_{\pi N}$ is varied from 45 to 75 MeV.

We now insert the values of $a_i$ listed in Table II and similar ones with different $g_A^{(0)}$ into Eq.(3) together with the matrix elements of the $D_{ab}^{(R)}$ functions so that we can determine the transition axial-vector constant for the decay $\Theta^+ \rightarrow K^+ n$. Fig. 1 draws for various values of $g_A^{(0)}$.

![Fig. 1](image)

**FIG. 1:** The transition axial-vector coupling constant for $\Theta^+ \rightarrow K^+ n$ as a function of $\Sigma_{\pi N}$. The solid curve denotes that with $g_A^{(0)} = 0.3$, while the dashed and dot-dashed ones represent that with $g_A^{(0)} = 0.2, 0.4$, respectively.

For a given $g_A^{(0)}$ the dependence of the $g_A^{* (\Theta \rightarrow n)}$ on the $\Sigma_{\pi N}$. The larger $g_A^{(0)}$ we use, the smaller $g_A^{* (\Theta \rightarrow n)}$ we obtain. Moreover, the $g_A^{* (\Theta \rightarrow n)}$ turns out to be negative when $g_A^{(0)}$ is larger than around 0.37. For a given $g_A^{(0)}$ the $g_A^{* (\Theta \rightarrow n)}$ decreases as the $\Sigma_{\pi N}$ increases. The $g_A^{* (\Theta \rightarrow n)}$ at $\Sigma_{\pi N} = 70$ MeV is 70% smaller than that at $\Sigma_{\pi N} = 45$ MeV.
Using the effective Lagrangian for the $\Theta^+ \to K^+ n$ decay:

$$\mathcal{L} = -\frac{g^A_{\Theta n}}{2f_K} \overline{\Theta} \gamma_\mu \gamma_5 (\partial^\mu K^+) n + \text{h.c.},$$ (11)

where h.c. stands for the hermitian conjugate. We obtain from the effective Lagrangian the invariant amplitude for the decay as follows:

$$i\mathcal{M} = i\frac{g^A_{\Theta n}}{2f_K} \overline{\Theta}(p)n \gamma_\mu \gamma_5 u(p),$$ (12)

where $\overline{\Theta}$, $K^+$, and $n$ denote the fields of the $\Theta^+$, of the positively charged kaon, and of the neutron, respectively. The $f_K = 112$ MeV represents the kaon decay constant. The $\overline{\Theta}(p)n$ and $u(p)$ are the Dirac spinors for the neutron and $\Theta^+$ with the corresponding momenta, respectively and the $p$ denotes the kaon momentum. The decay width of the $\Theta^+ \to KN$ is proportional to the square of the transition axial-vector constant:

$$\Gamma_{\Theta KN} = 2\Gamma_{\Theta K+n} = \left(\frac{g^A_{\Theta n}}{16\pi f_K M_{\Theta}^2}\right)^2 \frac{|\vec{p}|}{(M_{\Theta} - M_N)^2 - m_K^2} (M_{\Theta} + M_N)^2,$$ (13)

where $|\vec{p}| = \sqrt{(M_{\Theta}^2 - (M_N + m_K)^2)(M_{\Theta}^2 - (M_N - m_K)^2)/2M_{\Theta}}$ is the kaon momentum and $M_{\Theta} = 1540$ MeV, $M_N = 939$ MeV, and $m_K = 494$ MeV stand for the masses of the $\Theta^+$, the nucleon, and the kaon, respectively. The factor 2 in front of the decay width $2\Gamma_{\Theta K+n}$ takes care of the fact that $\Theta^+$ has two distinct decay channels, $K^+n$ and $K^0p$, which are equally populated due to isospin-symmetry. The $\Gamma_{\Theta KN}$ is sensitive to the value of the $g^A_{\Theta n}$. Moreover, since the $\Gamma_{\Theta KN}$ is given as a function of the square of the $g^A_{\Theta n}$, it is independent of the sign of the $g^A_{\Theta n}$. Thus, the decay width $\Gamma_{\Theta KN}$ decreases until the sign of $g^A_{\Theta n}$ changes and then increases again.

| $\Sigma_{\pi N}$ [MeV] | $\Gamma_{\Theta KN}^{\text{total}}$ | Input $g^A_{\Theta n}^{(0)}$ |
|-----------------|-------------------------------|-----------------|
| 45              | 33.41                        | 0.28            |
| 50              | 22.25                        | 0.32            |
| 55              | 15.22                        | 0.36            |
| 60              | 10.45                        | 0.40            |
| 65              | 7.04                         | 0.26            |
| 70              | 4.54                         | 0.10            |
| 75              | 2.70                         | 0.01            |

TABLE III: The decay width of $\Theta^+ \to KN$ determined with $g^A_{\Theta n}^{(0)}$ varied from 0.28 to 0.40. The $\Sigma_{\pi N}$ is varied from 45 to 75 MeV.

In Table III we list the total decay width of the $\Theta^+ \to KN$ as a function of $\Sigma_{\pi N}$ and $g^A_{\Theta n}^{(0)}$. Actually, the decay width decreases until $g^A_{\Theta n}^{(0)} = 0.4$, and then starts to increase, while it gets smaller almost monotonically as the larger value of the $\Sigma_{\pi N}$ is used. The region where proper combinations of $g^A_{\Theta n}^{(0)}$ and $\Sigma_{\pi N}$ yield a small width $\Gamma_{\Theta KN} \leq 1$ MeV of the $\Theta^+$ pentaquark can easily be identified.
In Fig. 2 we draw the results of the total decay width of the $\Theta^+ \to KN$ as a function of $\Sigma_{\pi N}$ and $g_A^{(0)}$. The smaller the $\Gamma_{\Theta KN}$ the more restricted are the values $\Sigma_{\pi N}$ and $g_A^{(0)}$. The shaded rectangle indicates the area where one has generally accepted experimental values of $g_A^{(0)}$ and $\Sigma_{\pi N}$, i.e. $0.3 - 0.4$ and $65 - 75$ MeV, respectively, and simultaneously a $\Gamma_{\Theta KN} \leq 1$ MeV. It is of great interest to see that the range of $g_A^{(0)}$ is compatible with a theoretical investigation [40], based on the $\chi$QSM, on the COMPASS and HERMES measurements of the deuteron spin-dependent structure function [47,48,49]. It is worthwhile to mention that the values of $g_A^{(0)}$ in the present analysis is almost the same as theoretical results within the $\chi$QSM [50,51]. The range of $\Sigma_{\pi N}$ given above is consistent with a recent analysis [40]. If one interprets the result of the DIANA collaboration [12] as identification of the $\Theta^+$, namely the formation of a narrow $pK^0$ peak with mass of $1537 \pm 2$ MeV/$c^2$ and width of $\Gamma = 0.36 \pm 0.11$ MeV in the $K^+n \to K^0p$ transition, then that result is inside the shaded area of Fig. 2.

**FIG. 2:** The total decay width of $\Theta^+ \to KN$ as a function of $g_A^{(0)}$ and $\Sigma_{\pi N}$. The shaded square denotes the ranges of $g_A^{(0)}$: $0.3 - 0.4$ and of $\Sigma_{\pi N}$: $65 - 75$ MeV.

**IV. SUMMARY AND CONCLUSION**

In the present work, we analyzed within the framework of the chiral quark-soliton model the total decay width of the $\Theta^+ \to KN$, based on the experimental data of hyperon semileptonic decays and the flavor-singlet axial-vector constant $g_A^{(0)}$. The parameters in the collective Hamiltonian were fixed by the splittings of the SU(3) baryon mass representation [28]. The dynamical parameters in the collective axial-vector operators were fitted (“model-independent approach”) to the existing data of hyperon semileptonic decays and of $g_A^{(0)}$, where the value of the $g_A^{(0)}$ was varied within the range of $0.2 - 0.5$. Since all these parameters depend on the value of the $\Sigma_{\pi N}$, we took its value to be $45 - 75$ MeV.
We first computed the transition axial-vector coupling constant for the $\Theta^+ \to K^+ n$, $g_A^{*(\Theta \to n)}$. We showed that the $g_A^{*(\Theta \to n)}$ decreases as $g_A^{(0)}$ increases. Furthermore, the $g_A^{*(\Theta \to n)}$ depends on the $\pi N$ sigma term, $\Sigma_{\pi N}$: it is getting smaller as the $\Sigma_{\pi N}$ increases. Thus, the $g_A^{*(\Theta \to n)}$ turns out to be smaller with $\Sigma_{\pi N} = 70$ MeV by 70%, compared to that with $\Sigma_{\pi N} = 45$ MeV. It was also found that the $g_A^{*(\Theta \to n)}$ becomes negative around $g_A^{(0)} \approx 0.37$.

The total width $\Gamma_{\Theta KN}$ of the $\Theta^+ \to K N$ decay was finally investigated. Since it is proportional to the square of the transition axial-vector constant $g_A^{*(\Theta \to n)}$, it is rather sensitive to the $g_A^{*(\Theta \to n)}$. The $\Gamma_{\Theta KN}$ is getting suppressed as the singlet axial-vector constant $g_A^{(0)}$ increases. However, since the $g_A^{*(\Theta \to n)}$ turns out to be negative around 0.37, the $\Gamma_{\Theta KN}$ starts to increase around 0.37. As a result, the total decay width $\Gamma_{\Theta KN}$ turns out to be smaller than 1 MeV for values of the $g_A^{(0)}$ and $\Sigma_{\pi N}$ larger than 0.31 and 65 MeV, respectively.

As conclusion of present analysis, which uses the “model-independent approach to the chiral quark soliton, one can state: The known data of semileptonic decays combined with $0.3 \leq g_A^{(0)} \leq 0.4$ and $\Sigma_{\pi N} \geq 65$ MeV is compatible with the existence of a $\Theta^+$ pentaquark having a small width of the total decay $\Theta^+ \to K N$: $\Gamma_{\Theta KN} \leq 1$ MeV. Since all dynamical parameters in the present approach are fitted the existing experimental data, it is difficult to understand the origin of the strong correlation between the singlet axial-vector constant and the $\Theta^+$ decay width. The corresponding investigation is under way, in order to give a theoretical explanation of this strong correlation [52].

Acknowledgments

The authors are grateful to J.K. Ahn, D. Diakonov, J.H. Lee, S.i. Nam, M.V. Polyakov, and M. Praszalowicz for helpful discussion and comments. The present work is supported by the Korea Research Foundation Grant funded by the Korean Government(MOEHRD) (KRF-2006-312-C00507). The work is also supported by the Transregio-Sonderforschungsbereich Bonn-Bochum-Giessen, the Verbundforschung (Hadrons and Nuclei) of the Federal Ministry for Education and Research (BMBF) of Germany, the Graduiertenschaffen Bonn-Bochum-Dortmund, the COSY-project Jülich as well as the EU Integrated Infrastructure Initiative Hadron Physics Project under contract number RI3-CT-2004-506078.

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