Acoustic cloaking in two dimensions: a feasible approach

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Abstract. This work proposes an acoustic structure feasible to engineer that accomplishes the requirements of acoustic cloaking design recently introduced by Cummer and Schurig (2007 New J. Phys. 9 45). The structure, which consists of a multilayered composite made of two types of isotropic acoustic metamaterials, exactly matches the conditions for the acoustic cloaking. It is also shown that the isotropic metamaterials needed can be made of sonic crystals containing two types of material cylinders, whose elastic parameters should be properly chosen in order to satisfy (in the homogenization limit) the acoustic properties under request. In contrast to electromagnetic cloaking, the structure here proposed verifies the acoustic cloaking in a wide range of wavelengths; its performance is guaranteed for any wavelength above a certain cutoff defined by the homogenization limit of the sonic crystal employed in its fabrication.

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1. Introduction

In a recent work by Cummer and Schurig [1], it was predicted that acoustic cloaking is possible in a two-dimensional (2D) geometry by means of a cloak made of an acoustic material having a strong mass anisotropy not existing in nature. This result in acoustics follows an analogous result in electromagnetism where different groups demonstrated that strong anisotropy of the corresponding electromagnetic (EM) parameters was also required [2]–[5]. However, while the EM cloaking has been experimentally demonstrated by using a specially designed metamaterial [6], its acoustic counterpart has not yet been demonstrated. Moreover, the demonstration for the acoustic cloaking is still waiting for some proposal of engineered material (metamaterial) that accomplishes the requirements on mass anisotropy predicted in [1]. In this regard, the work by Milton et al [7] describes conceptually how the mass anisotropy could be possible by spring-loaded masses. Besides, in a recent advancement in the physical realization of metamaterial with mass density anisotropy these authors have demonstrated that such an uncommon property can be made possible by using non-symmetric lattices of solid cylinders [8].

In this work, we present an acoustic cloak that could be physically realizable. In brief, the proposed cloak is based on a multilayered structure consisting of two layers with the same thickness and made up of two different acoustic isotropic metamaterials. These metamaterials are built with sonic crystals (i.e. periodic arrays of sonic scatterers) based on two types of elastic cylinders that have to accomplish certain requirements on their mass density and effective sound speed. Numerical experiments based on multiple scattering method are presented to support the exact performance of the proposed cloak.

The paper is organized as follows. First, in section 2, we review the solution in [1] and report our approach to get the acoustic cloaking. Numerical experiments demonstrating the performance and properties of the proposed cloak are also presented and discussed. Section 3 describes the recipe to build the metamaterials needed to fabricate the multilayered cloak making it physically feasible. Finally, the work is summarized in section 4.

2. The acoustic cloak: a proposal

The solution reported by Cummer and Schurig for the acoustic cloaking in [1] requires a fluid material with an anisotropic density and a scalar bulk modulus. Moreover, these parameters must be dependent on the radial distance to the hidden object. The predicted functional form is

\[
\frac{\rho_r}{\rho_b} = \frac{r}{r - R_1}, \quad (1a)
\]

\[
\frac{\rho_0}{\rho_b} = \frac{r - R_1}{r}, \quad (1b)
\]

\[
\frac{B}{B_b} = \left( \frac{R_2 - R_1}{R_2} \right)^2 \frac{r}{r - R_1}, \quad (1c)
\]

where \(R_1\) and \(R_2\) are the inner and outer radii of the cloaking shell, \(B\) is the bulk modulus of the shell, \(\rho_r\) and \(\rho_0\) are the components of the diagonal mass density tensor, and the quantities with subscript \(b\) are those of the surrounding background that is a fluid or a gas.
Materials with such uncommon properties do not exist in nature and, therefore, some engineered material should be introduced to accomplish them. In a recent paper [8], these authors have shown that, in the low frequency limit, arrangements of cylinders in non-symmetric lattices lead to acoustic metamaterials with anisotropic mass density and scalar bulk modulus, as required by equations (1). The lattices considered in that work are single-cylinder lattices and, as a consequence, when the mass density of the cylinder is larger (smaller) than that of the background, the effective mass density tensor is always larger (smaller) than that of the background. This is an important drawback, because in conditions (1) one component of the mass density tensor is the reciprocal of the other and consequently the radial (angular) component of the mass tensor is always larger (smaller) than that of the background. Therefore, a material having certain mass density in the radial direction and its reciprocal along the tangential direction cannot be engineered by using the theory developed in [8]. However, we suggest below a path to get the actual realization of such a property by using periodic structures.

We arrive at the solution proposed here by exploring the possibility of building anisotropic materials based on sonic crystals with two types of material cylinders, following a combination of two approaches previously introduced by these authors [8, 9]. Unfortunately, the practical realization of conditions (1) was impossible to achieve because of the limitation imposed by the close packing condition of the lattice. Therefore, in a natural way, we conclude that a multilayered structure made of two materials overcomes such problem and give a solution to accomplish the required conditions. It is interesting to note that an approach similar to this was also proposed to get EM cloaking [10]. However, while the EM cloak only verifies a reduced set of conditions imposed for EM cloaking, the one reported here exactly matches the conditions for acoustic cloaking.

Let us consider a cloaking shell consisting of a multilayered structure that is made of alternating layers of materials of types 1 and 2. For any periodic system the bulk modulus (in the homogenization limit) does not depend on the type (isotropic or anisotropic) of lattice. In fact, the effective bulk modulus (at large wavelength), $B^*$, can be determined by simply doing a volume average of its reciprocal. Thus, for a 1D multilayered system of materials 1 and 2, it was shown that [11]

$$
\frac{1}{B^*} = \frac{1}{d_1 + d_2} \left[ \frac{d_1}{B_1} + \frac{d_2}{B_2} \right],
$$

(2)

where $B_1$ ($B_2$) is the bulk modulus of material 1 (2) and $d_1$ ($d_2$) is the length of layer 1 (2).

To obtain the tensor associated with the effective speed of sound, we need to calculate first the dispersion relation $K(\omega)$ of the system, i.e. the wavenumber as a function of the frequency. This calculation is very simple if the procedure explained in textbooks like [12] is followed,

$$
\cos K_i d = \cos k_1 d_1 \cos k_2 d_2 - \frac{1}{2} \left[ \frac{\rho_1 k_{2x}}{\rho_2 k_{1x}} + \frac{\rho_2 k_{1x}}{\rho_1 k_{2x}} \right] \sin k_1 d_1 \cos k_2 d_2,
$$

(3)

where

$$
K_{ix}^2 = \frac{\omega^2}{c_i^2} - K_y^2,
$$

(4)

for $i = 1, 2$.

The effective speed of sound is defined (in the low frequency limit) as the ratio between the angular frequency $\omega$ and the wavenumber $K$. This ratio can be obtained by making a power expansion of the trigonometric functions up to the second power of their arguments. It is easy
to show that, $c_\perp$ and $c_\parallel$, the diagonal components of the speed tensor for the propagation along
the perpendicular and parallel directions, are
\begin{equation}
  c_\perp^2 = \frac{B^*}{\rho_\perp}, \quad c_\parallel^2 = \frac{B^*}{\rho_\parallel},
\end{equation}
where $\rho_\perp$ and $\rho_\parallel$ are given by
\begin{equation}
  \rho_\perp = \frac{1}{d_1 + d_2} (d_1 \rho_1 + d_2 \rho_2), \quad \frac{1}{\rho_\parallel} = \frac{1}{d_1 + d_2} (d_1/\rho_1 + d_2/\rho_2)
\end{equation}
and represent the diagonal components of the effective dynamical mass density tensor.

Similar expressions were also reported by Schoenberg and Sen [11]. So the components of
the reciprocal mass density tensor for $d_1 = d_2 = d/2$ and $\rho_2/\rho_b = \rho_b/\rho_1$ are
\begin{equation}
  \frac{\rho^{-1}_\perp}{\rho^{-1}_b} = \frac{2}{\rho_1/\rho_b + \rho_b/\rho_1}, \quad \frac{\rho^{-1}_\parallel}{\rho^{-1}_b} = \frac{\rho_1/\rho_b + \rho_b/\rho_1}{2}.
\end{equation}

This structure satisfies the conditions (1a) and (1b); i.e. $\rho_\perp/\rho_b = \rho_b/\rho_1$. Moreover, the
component of the mass density tensor along the perpendicular direction is always larger than
one, as also required by condition (1a).

Now, we have to determine the material properties of media 1 and 2. If material 1 is selected
as the high density material, from (7), the dependence of $\rho_1$ as a function of $r$ is
\begin{equation}
  \rho_1(r) = \rho_0(r) + \sqrt{\rho_1^2(r) - \rho_b^2} = \frac{r}{r - R_1} + \sqrt{\frac{2R_1}{r - R_1}}.
\end{equation}

The bulk modulus of both materials must also depend on the radial distance. To accomplish
this condition, one possibility is assuming that both materials have the same speed of sound $c_1$. Therefore, the dependence of this quantity as a function of $r$ can be obtained by inserting
$B_1 = \rho_1 c_1^2$ and $B_2 = \rho_2 c_1^2$ into equations (1) and (2)
\begin{equation}
  c_1(r) = \sqrt{\frac{B^*\rho_1}{\rho_b^2}} = \frac{R_2 - R_1}{R_2} \frac{r}{r - R_1} c_b.
\end{equation}

Equations (8) and (9) define the properties of medium 1, whereas those for medium 2 are
derived from them as explained above
\begin{equation}
  \rho_2(r) = \frac{\rho_2^2/\rho_1}{r + \sqrt{2R_1(r - R_1)}}, \quad c_2(r) = \frac{R_2 - R_1}{R_2} \frac{r}{r - R_1} c_b.
\end{equation}

The proposed cloak is shown schematically in figure 1, where the 1D structure is
transformed into a circular-shaped shell that it is expected to cloak a rigid core placed in its interior.

To check the functionality of the multilayered cloak, we have performed multiple scattering
simulations by using the method developed in [14, 15]. Maps of the acoustic pressure at a time
instant are represented by the real part of the complex amplitude $p$ and are shown in figure 2
for the case of a rigid core of radius $R_1$ that is placed inside a multilayered shell of radius
$R_2 = 2R_1$. The full structure is submitted to an acoustic field of wavelength $\lambda = R_1/2$. The
performance of two different shells are depicted in figure 2, where the left panel corresponds to
Figure 1. Schematic view of the cloaking shell. It consists of a circular-shaped multilayered structure made up of two different materials of the same thicknesses.

Figure 2. Pressure map for a planar wave incident on a rigid cylindrical scatterer surrounded by a multilayered acoustic shell made up of 50 layers (left panel) and 200 layers (right panel). The radius of the shell is twice that of the core ($R_2 = 2R_1$). The wavelength of the incident field is $\lambda = R_1/2$.

A shell made of 50 layers and the right panel to one composed of 200 layers, where each layer of thickness $d$ is composed of two alternate layers of thickness of materials 1 and 2, respectively, i.e. $d = d_1 + d_2 = d/2 + d/2$. The cloaking effect is evident in both the representations, but that corresponding to 200 layers can be considered to be near perfect. These results can be compared with the case of the rigid cylinder with no cloak that is represented in the left panel of figure 3, where the incident wave is strongly scattered by the cylinder. On the other hand, in the right panel of figure 3 is depicted the acoustic cloaking by an extremely thin cloak, its thickness being two orders of magnitude smaller than the hidden cylinder, but it also is made of 200 layers. This result is very promising, because it indicates the possibility of building cloaks as thin as the available technology allows.
Figure 3. Left panel: pressure map for a planar wave incident on a rigid cylindrical scatterer of radius $R_1$. Right panel: map corresponding to the same scatterer surrounded by an extremely thin cloak shell ($R_2 - R_1 = 0.01 R_1$) made of 200 layers. The wavelength of the incident field is $\lambda = R_1/2$.

Now, it is also interesting to analyze the cloaking effect as a function of the number of layers employed in the fabrication of the cloak. The resulting behavior is important in order to simplify the fabrication of the cloaking shell as much as possible. We have studied the backscattered field as a typical parameter characterizing the cloak’s performance and it is represented (in a logarithmic scale) in figure 4 as a function of the frequency for different numbers of layers. It is remarkable that in figure 4 only 50 layers are able to reduce by more than one order of magnitude (for a wide range of frequencies) the backscattered field in comparison with that for the corresponding naked rigid cylinder. Other interesting cases like penetrable and void regions are not reported here, but we expect results analogous to those that have already been published [13].

3. Building the layers of the cloak: a feasible approach

It has been shown above that the acoustic cloak can be exactly realized by using a set of $N$ layers, each one made up of one isotropic fluidlike material of type 1 and another of type 2, their acoustic parameters being described by conditions (8), (9) and (10). This section is devoted to show that the requested dynamical properties of the fluidlike materials 1 and 2 can be actually realized by using lattices of solid cylinders embedded in the background where the sound propagates, i.e. by using sonic crystals. Particularly, we have shown that sonic crystals are a class of metamaterials that dynamically behave (in the homogenization limit) as true fluidlike materials whose properties can be tailored with practically no limitation. For example, the acoustic parameters of the homogenized sonic crystal made up of only one type of component basically depend on the filling fraction of the lattice [17, 18]. More recently, we have reported [9]
that sonic crystals made up of two material cylinders increase the possibilities of metamaterial design. Here, we have used the last approach to design the acoustic properties needed for the layers of the cloaking shell.

For instance, let us assume that metamaterials 1 and 2 are going to be made up of two types of cylinders with acoustic parameters: \((\rho_{1\alpha}, c_{1\alpha})\) and \((\rho_{1\beta}, c_{1\beta})\) for metamaterial 1, and \((\rho_{2\alpha}, c_{2\alpha})\) and \((\rho_{2\beta}, c_{2\beta})\) for metamaterial 2. The parameters of metamaterials 1 and 2 can be tailored with the filling fractions \(f_{1\alpha}, f_{1\beta}, f_{2\alpha}\) and \(f_{2\beta}\) of the components involved in their fabrication. Thus,

\[
\frac{1}{B_1} = \frac{1 - f_{1\alpha} - f_{1\beta}}{B_b} + \frac{f_{1\alpha}}{B_{1\alpha}} + \frac{f_{1\beta}}{B_{1\beta}},
\]

\[
\eta_1 = \eta_{1\alpha} f_{1\alpha} + \eta_{1\beta} f_{1\beta},
\]

\[
\frac{1}{B_2} = \frac{1 - f_{2\alpha} - f_{2\beta}}{B_b} + \frac{f_{2\alpha}}{B_{2\alpha}} + \frac{f_{2\beta}}{B_{2\beta}},
\]

\[
\eta_2 = \eta_{2\alpha} f_{2\alpha} + \eta_{2\beta} f_{2\beta},
\]

where \(\eta_i = (\rho_i - \rho_b)/(\rho_i + \rho_b)\).

It is important to point out that the expressions (12) and (14) are only valid for small and moderate filling fractions. For large filling fractions (near the close-packing condition), the multiple scattering interaction between cylinders must be included [16] and calculations should be performed by using the exact expressions already reported in previous works [9, 17, 18].

In figure 5, we have represented the so-called phase diagram [9] of the metamaterial that can be obtained by using composites made of two component sonic crystals. This diagram has been obtained by considering cylinders made of fully elastic materials characterized by their

Figure 4. Frequency response of the backscattered pressure for a rigid cylinder surrounded by a cloaking shell consisting of a multilayered structure as described in figure 1. \(N\) is the number of layers in the structure. The case of the bare rigid cylinder without the shell is also represented (black line).
Figure 5. Phase diagram showing that properties (8)–(9) for metamaterial 1 and (10a)–(10b) for metamaterial 2 can be satisfied by using sonic crystal composites made of only two different elastic materials (see table 1). The continuous lines represent the range of possible values for metamaterials 1 and 2. The dotted lines enclose the area of values that is possible to tailor with the two-component sonic crystal.

Table 1. Acoustic parameters of the materials forming the composites. The third column reports the effective velocity of the elastic cylinder [9]; \( c = \sqrt{c_\ell^2 - c_t^2} \), where \( c_\ell \) and \( c_t \) are the longitudinal and transverse velocities, respectively.

| Material | \( \rho/\rho_b \) | \( c/c_b \) |
|----------|-----------------|--------------|
| 1\( \alpha \) | 400 | 100 |
| 1\( \beta \) | 2 | 50 |
| 2\( \alpha \) | 0.1 | 0.5 |
| 2\( \beta \) | 0.001 | 200 |

parameters of density, \( \rho \), longitudinal velocity, \( c_\ell \), and transverse velocity, \( c_t \). Table 1 reports the parameters of the materials considered, where \( c = \sqrt{c_\ell^2 - c_t^2} \) represents the associated fluidlike velocity [9]. Lines of the same color enclose the area in which a metamaterial with parameters \((\rho^*, c^*)\) are available by just changing the filling fraction of the materials employed in the composite. For instance, the area enclosed by the blue lines defines the range of parameter that can be tailored using materials 1\( \alpha \) and 1\( \beta \) in the composition of metamaterial 1. Within this area, the blue straight line represents the variation in \((\rho^*, c^*)\) needed by material 1 in order to accomplish the cloaking by the multilayered structure. In other words, the conditions for material 1 given by (8) and (9) can be fully accomplished by using materials 1\( \alpha \) and 1\( \beta \) in table 1. For the case of material 2, the red straight line in figure 5 plots the range of variation requested for cloaking. Let us stress that the set of elastic materials in table 1, which were selected to get the condition of acoustic cloaking, is not unique and another combination of materials could achieve a similar performance. In fact, in order to make the cloak real, a more appropriate set should be chosen to match the availability of materials in nature.

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4. Summary

It has been shown that acoustic cloaking shells are possible by means of multilayered structures made with two types of acoustic isotropic metamaterials, whose acoustic parameters should change as a function of the distance from the layer to the center of the shell. The practical realization of the required radial dependence can be achieved by using homogenized 2D sonic crystals having two full elastic cylinders per unit cell. The parameters of the sonic crystal needed to get the cloaking effect can be obtained through a phase diagram analysis of the selected materials used in the cylinders. An important advantage of our proposed structures in comparison with its EM counterpart to get electromagnetic cloaking is their performance in a wide range of wavelengths. Besides, our proposed metamaterials are very robust against positional and/or structural disordering [17, 18].

We hope that this proposal motivates future experimental work demonstrating its performance. Finally, we should point out that the present proposal can be extended to the recently reported case of acoustic cloaking in three dimensions [19, 20], a result that was implicitly treated in [21]. By using an analogous development, we can foresee that the corresponding shell could be made of multilayers of two different isotropic metamaterials, which might consist of spheres of different solid materials.

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