Invited Comment

Primacy and ranking of UEFA soccer teams from biasing organization rules

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Received 28 November 2013, revised 9 June 2014
Accepted for publication 12 June 2014
Published 28 August 2014

Abstract
A question is raised on whether some implied regularity or structure, as found in the soccer team ranking by the Union of European Football Associations (UEFA), is due to an implicit game result value or score competition conditions. The analysis is based on considerations of complex systems, i.e. finding whether power or other simple law fits are appropriate to describe some internal dynamics. It is observed that the ranking is specifically organized: a major class comprising a few teams emerges after each season. Other classes, which apparently have regular sizes, occur subsequently. Thus, the notion of the Sheppard primacy index is envisaged to describe the findings. Additional primacy indices are discussed for enhancing the features. These measures can be used to sort out peer classes in more general terms. A very simplified toy model containing components of the UEFA ranking rules suggests that such peer classes are an extrinsic property of the ranking, as obtained in many nonlinear systems under boundary condition constraints.

Keywords: ranking, soccer, primacy index, dissipative structures, self-organization

(Some figures may appear in colour only in the online journal)

1. Introduction

Nonlinearity and complexity [1–3] are common features of a large number of systems studied in modern science [4–6]. They are often investigated by nonlinear dynamics methods [7–9]. In the last decade or so, these methods have been applied to many social, economic and financial systems [10]. In many complex systems, researchers have detected the existence of power laws for a large variety of characteristic quantities. Such power laws have become very useful tools for studying complex systems because the functional relations can indicate that the system is controlled by a few rules that propagate across a wide range of scales [11–14].

For example, ranking analysis has received a great deal of attention since Zipf [15] observed that a large number of size distributions, $N_r$, can be approximated by a simple scaling (power) law $N_r = N_0 r^{-\alpha}$, where $r$ is the ranking parameter, with $N_r \geq N_{r+1}$ (and obviously $r < r + 1$). This idea has led to a flurry of log-log diagrams showing a straight line through the displayed data. More generally, one considers the so-called rank-size scaling law:

$$y_r = \frac{a}{r^\alpha}. \quad (1.1)$$

where the scaling exponent $\alpha$ is considered indicative of whether the size distribution $y_r$ is close to some optimum (=equilibrium) state [15], i.e. when $\alpha = 1$. The amplitude $a$ can be estimated from the normalization condition. For the
discrete distribution, equation (1.1), \( a \approx r_d/\zeta (\alpha) \sim r_d/2 \), where \( r_d \) is the largest value of \( r \) and \( \zeta (\alpha) \equiv \sum_{k=1}^{\infty} k^{-\alpha} \) is the Riemann zeta function [16].

This scaling hypothesis might be applied in sport competition ranking, although the number of scales would obviously be finite. Nevertheless, measurements or ranking in sport competitions, while reported frequently in the media, lack the necessary descriptive power that studies of complex systems usually present and require in physics investigations. An analysis of data from a specific nonlinear complex system, the Union of European Football Associations (UEFA) team ranking, is presented below as a specific and interesting modern society example. Deviations at low and high rank \( r \), from the empirical fits to a single power law, indicate the existence of different regimes. For completeness, the UEFA rules leading to its team ranking coefficient are briefly recalled in appendix A. They suggest extrinsic biases.

Introducing an indirect ‘primacy measure’ based on the Sheppard hierarchy index [17] (see appendix B for a review of Sheppard’s original index), it is found that UEFA teams can be organized in several well defined classes. Whether or not this is related to or could be used for weighting performances is speculation, but it cannot be \textit{a priori} disregarded. However, practical applications cannot be recommended from this report, because any recommendations would be outside the present scientific aims, i.e. a search for structural features in a social complex system—team ranking.

The paper is organized as follows. In section 2, a brief review of the literature on ranking, and in particular for soccer teams, is presented. The data analysis is performed in section 3. Simple empirical laws are briefly reviewed in order to introduce possible fit laws. The rank-size relationship, equation (1.1), is assessed for UEFA teams. Various fits point to features, emphasized in figures displaying various empirical laws. In section 4, the hierarchy inside the top classes is further analyzed, starting from the conventional primacy index of Sheppard [17], through additional measures of primacy. Several remarks serve as conclusions in section 6.

A very simplified ‘toy model’ is numerically discussed in appendix C in order to show that under UEFA team ranking rules, gaps necessarily emerge between classes of teams, thereby suggesting that the features mimic thermodynamic dissipative structures in finite size systems.

2. State of the art

The rank-size relationship, equation (1.1), has been identified frequently and discussed sufficiently in the literature, and allows much of the present investigation to be based on such a simple empirical law. This may be ‘simply’ because the rank-size relationship can be reached from a wide range of specific situations. Indeed, Zipf’s law, equation (1.1) with \( \alpha = 1 \), and its generalizations, can be obtained in different models: one example is tied to the maximization of the entropy concept [18]; another stems from the law of proportionate effect, the so-called Gibrat’s law [19]. Recall that Gibrat’s law describes an \textit{evolution} process, supposing that the growth rate of something is independent of its size and previous rank.

Since specialized literature on team ranking is not common in the physics literature, a brief ‘state of the art’ is presented below, only pointing to \textit{a few} publications:

- Stefani [20]—pioneering survey of the major world sports rating systems in 1997;
- Cassady et al [21]—discussion of a customizable quadratic assignment approach for ranking sports teams in 2005;
- Churilov and Flitman [22]—proposal in favor of the data envelopment analysis (DEA) model for producing a ranking of teams or countries, like in the olympics games, in 2006;
- Broadie and Rendleman [23]—audacious question of whether the official World golf rankings are biased.

In one highly conclusive paper, entitled Universal scaling in sports ranking [24], the authors studied the distributions of scores and prize money in various sports, showing that different sports share similar trends in scores and prize money distributions, while pointing to many implications. In a related study, Pilavci’s MSc thesis (see [25] and references therein) evaluated economic, demographic and traditional factors that affect soccer clubs’ on-pitch success in UEFA games.

Other ranking studies, for the National Collegiate Athletics Association (NCAA) and for National Football League (NFL) teams [26, 27], have been based on subjective considerations or indirect measures.

Specifically for soccer ranking themes, the following studies should be mentioned:

- Kern and Paulusma [28] discussed FIFA rules’ complexity for competition outcomes and team ranking in 2001;
- Macmillan and Smith [29] explained such a country ranking in 2007;
- Ausloos et al [30] compared the country FIFA ranking, based on games between national squads, and the country UEFA ranking, based on team game results;
- Constantinou and Fenton [31] determined the level of ‘ability’ of soccer teams (over five English Premier League seasons) by ratings based on the relative discrepancies in scores between adversaries.

Other papers with some soccer related content can be mentioned for completion: e.g., the flash-lag error effect in soccer games [32], the ‘best’ team win frequency [33], the upset frequency as a measure of competitiveness [34], the ‘evaluation’ of goals scored in Euro 2012 [35], the goal distributions [36, 37], the relationship between the time of the first goal in the game and the time of the second goal [38], the goal difference as a better measure for the overall fitness of a team [39], the general dynamics of soccer tournaments [40], and the structure, speed and play patterns of World Cup soccer final games between 1966 and 2010 [41].
These interesting papers are at the interfaces of various disciplines and are often tied to various technical questions or are limited to the analysis of distribution functions, as often found for complex systems, but without conveying questions tied to self-organizations [42] or external constraints [43].

3. UEFA team analyzed data sets

Usually, a ranking represents the overall performance over the period of a whole season. In particular, UEFA soccer teams are ranked according to results based on five previous ‘seasons’ for teams having participated in the UEFA Champions League and the UEFA Europa League6. The rules leading to such a ranking are reviewed in appendix A. They are more complicated than a ‘win–draw–loss’ rating. The ratings depend on the success at some competition level and differ according to the competition. One should be aware that a UEFA’s country coefficient is used to pre-determine the number of teams participating in each association either in the UEFA Champions League or in the UEFA Europa League.

A UEFA coefficient table is freely available and is updated regularly depending on the competition timing. The present data, and its subsequent analysis, are based on the September 2012 downloaded table. The team UEFA coefficients are calculated as described in the appendix A tables, and are derived from http://fr.uefa.com/memberassociations/uefarankings/club/index.html.

The number of concerned teams is 445; the UEFA coefficients range from ∼ 134.7 down to ∼ 0.383. For the sake of illustration, the first 60 teams and their UEFA coefficient (in September 2012) are listed in table 1. The statistical characteristics of these UEFA coefficients are given in table 2, second column. Note that the kurtosis (a measure of the fourth moment of the distribution, in fact equivalent to a specific heat in thermodynamics) changes sign (near r ≈ 35) according to the number of data points taken into account. By analogy with phase transitions, one should imagine the existence of a ‘critical rank’. In fact, the μ/σ function, which represents the order parameter in phase transition studies, evolves as an exponential (∼e−α/r) rather than as a power law, toward some ‘critical rank’. If the analogy is conserved, such an exponential behavior suggests considering the feature as one found at the Kosterlitz–Thouless transition [44] in the 2D XY model, where magnetic vortices are topologically stable configurations, as in spin glasses or thin disordered superconducting granular films.

For completeness, and noting that the fits are non-linear, the present study uses the Levenberg-Marquardt algorithm [45–48], except in figure 4 where a simple least square fit has been used for simplicity. The error characteristics from the fit regressions, i.e. χ2, d, the number of degrees of freedom, the p-value [24, 49, 50] and the R2 regression coefficient, are given in table 3. It can be observed that in all cases the p-value is lower than 10−6 (abbreviated by 0 in the table). Each χ2 value is rounded to the closest integer.

3.1. Empirical ranking laws

Beside the classical two-parameter power law, equation (1.1), other, often used, three-parameter statistical distributions, can be used:

- the Zipf–Mandelbrot-Pareto (ZMP) law [51]:
  \[ y(r) = b/(\nu + r)^\alpha. \]  

- the power law with exponential cut-off [52]:
  \[ y(r) = c \cdot r^{\alpha} \cdot e^{-\nu r}, \]  

- the mere exponential (two-parameter fit) case
  \[ y(r) = d \cdot e^{-\nu r}. \]

The ZMP law leads to a curvature at low r in a log–log plot and presents an asymptotic power law behavior at large r. Note that both \( \alpha \) and \( \zeta \) exponents, in equations (1.1), and (3.1) must be greater than 1 for the distributions to be well-defined (also greater than two for the mean to be finite and greater than three for the variance to be finite). On the other hand, since \( \nu \) in equation (3.1) is not necessarily found to be an integer in a fitting procedure, \( r \) can be considered as a continuous variable, for mathematical convenience, without any loss of mathematical rigor; the same holds true for the fit parameters \( a \), \( b \), and \( c \), and the ‘relaxation ranks’ \( \lambda \) and \( \eta \).

3.2. Data analysis

A few simple and possible various rank-size empirical distributions are shown figures 1–3.

From figure 1, the exponential law would appear to be more appropriate than the power law, in view of the regression coefficient \( R^2 \) values (∼0.97 vs. ∼0.80). However, the origin of the numerical value of the coefficient in the exponential can be hardly imagined from theoretical arguments. One can merely attribute it to some ‘relaxation rank’ ∼50. The power law exponent \( a \sim 0.54 \) on the other hand is rather far from 1, and low7. In fact the marked deviations at low and high rank \( r \), from the empirical fits, in particular from the single power law fit, indicates the existence of different regimes. Note the accumulation of data points below the (power law) fit occurring for teams with very low \( r \), i.e. \( r \ll 5 \). This suggests the existence of a so-called queen effect [53] for the top five teams.

This effect can be emphasized through the use of the ZMP law, equation (3.1), as shown in figure 2, using a log-log plot for emphasis of the goodness of fit in the low rank regime. Note the successive deviations of the data from the fits, which again suggest different regimes. In figure 2, a fit by a power law with exponential cut-off, equation (3.2), is also displayed. These three-parameter ZMP, equation (3.1), and the power law with cut-off, equation (3.2), necessarily lead to a better \( R^2 (∼0.99) \) than the two-parameter fits. A goodness-

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6 One should notice that it is not easy to compare teams in different divisions or leagues.

7 Note that the amplitude of the power law is about \( r\mu/2 \).
Table 1. Top 60 UEFA ranked teams according to the September 2012 coefficients.

| Rank | Team name          | Coeff. rank | Team name          | Coeff. rank |
|------|--------------------|-------------|--------------------|-------------|
| 1    | FC Barcelona       | 134.6906    | 2                  | FC Bayern München | 114.8653 |
| 3    | Manchester United  | 113.9074    | 4                  | Chelsea FC     | 111.9074 |
| 5    | Real Madrid CF     | 111.6906    | 6                  | Arsenal FC     | 96.9074  |
| 7    | FC Internazionale  | 91.3962     | 8                  | Atletico de Madrid | 88.6906 |
| 9    | FC Porto           | 87.0668     | 10                 | Valencia CF    | 84.6906  |
| 11   | Olympique Lyonnais | 84.0332     | 12                 | FC Shakhtar Donetsk | 80.6520 |
| 13   | SL Benfica         | 79.1068     | 14                 | Milan AC       | 78.3962  |
| 15   | CSKA Moskva        | 76.4330     | 16                 | Olympique de Marseille | 75.0332 |
| 17   | Liverpool FC       | 68.9074     | 18                 | FC Schalke 04  | 66.8653  |
| 19   | Sporting Club Portugal | 66.0668 | 20                 | Manchester City | 64.9074  |
| 21   | Villarreal CF      | 64.6906     | 22                 | Dynamo Kiev    | 62.6520  |
| 23   | PSV Eindhoven      | 61.5461     | 24                 | FC Zenit St.Petersburg | 60.4330 |
| 25   | Ajax Amsterdam     | 59.4653     | 26                 | Sporting Braga | 59.1068  |
| 27   | SV Werder Bremen   | 57.8653     | 28                 | FC Twente Enschede | 56.5461 |
| 29   | Metalist Kharkiv   | 53.1520     | 30                 | Tottenham Hotspurs FC | 52.9074 |
| 31   | Hamburger SV       | 52.8653     | 32                 | Sevilla CF     | 52.1906  |
| 33   | Olympiakos Piraeus FC | 51.4000 | 34                 | AS Roma        | 50.8962  |
| 35   | VfB Stuttgart      | 49.8653     | 36                 | Juventus FC    | 49.3962  |
| 37   | Paris Saint Germain | 49.0332   | 38                 | Girondins de Bordeaux | 48.0332 |
| 39   | Athletic Bilbao    | 47.6911     | 40                 | Standard de Liège | 45.2000 |
| 41   | FC Basel           | 44.5830     | 42                 | Bayer Leverkusen | 43.8657 |
| 43   | Fulham FC          | 42.9074     | 44                 | FC Kobenhavn   | 42.8600  |
| 45   | Lille OSC          | 42.0322     | 46                 | Rubin Kazan    | 40.9330  |
| 47   | Fiorentina AC      | 40.3962     | 48                 | SSC Napoli     | 40.3962  |
| 49   | RSC Anderlecht     | 40.2000     | 50                 | Panathinaikos FC | 39.9000 |
| 51   | Udinese Calcio     | 39.3962     | 52                 | AZ Alkmaar     | 39.0456  |
| 53   | VfL Wolfsburg      | 38.8653     | 54                 | Spartak Moskva FC | 37.4330 |
| 55   | APOEL Nicosia      | 35.2170     | 56                 | Club Brugge KV | 35.2000  |
| 57   | Galatasaray SK     | 35.1050     | 58                 | BATE Borisov   | 33.9250  |
| 59   | Besiktas JK        | 33.6050     | 60                 | Borussia Dortmund | 32.6853 |

Table 2. Summary of statistical characteristics for the September 2012 UEFA coefficient team ranking data.

| Team name          | Coeff. rank | Team name          | Coeff. rank |
|--------------------|-------------|--------------------|-------------|
| Olympique Lyonnais | 84.0332     | FC Shakhtar Donetsk | 80.6520     |
| SL Benfica         | 79.1068     | Milan AC           | 78.3962     |
| CSKA Moskva        | 76.4330     | Olympique de Marseille | 75.0332    |
| Liverpool FC       | 68.9074     | FC Schalke 04      | 66.8653     |
| Sporting Club Portugal | 66.0668 | Manchester City    | 64.9074     |
| Villarreal CF      | 64.6906     | Dynamo Kiev        | 62.6520     |
| PSV Eindhoven      | 61.5461     | FC Zenit St.Petersburg | 60.4330    |
| Ajax Amsterdam     | 59.4653     | Sporting Braga     | 59.1068     |
| SV Werder Bremen   | 57.8653     | FC Twente Enschede | 56.5461     |
| Metalist Kharkiv   | 53.1520     | Tottenham Hotspurs FC | 52.9074    |
| Hamburger SV       | 52.8653     | Sevilla CF         | 52.1906     |
| Olympiakos Piraeus FC | 51.4000 | AS Roma            | 50.8962     |
| VfB Stuttgart      | 49.8653     | Juventus FC        | 49.3962     |
| Paris Saint Germain | 49.0332   | Girondins de Bordeaux | 48.0332    |
| Athletic Bilbao    | 47.6911     | Standard de Liège  | 45.2000     |
| FC Basel           | 44.5830     | Bayer Leverkusen   | 43.8657     |
| Fulham FC          | 42.9074     | FC Kobenhavn       | 42.8600     |
| Lille OSC          | 42.0322     | Rubin Kazan        | 40.9330     |
| Fiorentina AC      | 40.3962     | SSC Napoli         | 40.3962     |
| RSC Anderlecht     | 40.2000     | Panathinaikos FC   | 39.9000     |

Table 3. Summary of regression fit characteristics: $\chi^2$, d: number of degrees of freedom; $p$: $p$-value and $R^2$: regression coefficient [50].

| Empirical law | Equation | Figure | $\chi^2$ | d | $p$ | $R^2$ |
|---------------|----------|--------|----------|---|-----|-------|
| pw            | (1.1)    | 1      | 40678    | 444 | 0.797 |       |
| exp           | (3.3)    | 1      | 5826     | 444 | 0.972 |       |
| pwco          | (3.2)    | 2.3    | 1477     | 444 | 0.99  |       |
| ZMP           | (3.1)    | 2      | 2659     | 444 | 0.99  |       |

| pw            | (1.1)    | 4      | 2195     | 49  | 0.92  |       |
| pw            | (1.1)    | 4      | 1474     | 39  | 0.93  |       |
| pw            | (1.1)    | 4      | 975      | 29  | 0.93  |       |
| exp           | (3.3)    | 4      | 1417     | 49  | 0.95  |       |
| exp           | (3.3)    | 4      | 1076     | 39  | 0.95  |       |
| exp           | (3.3)    | 4      | 677      | 29  | 0.95  |       |
| pw            | (1.1)    | 5      | 216      | 394 | 0.98  |       |
| exp           | (1.1)    | 5      | 821      | 394 | 0.94  |       |
of-fit test indicates that the latter two empirical laws can be further considered. The meaning of the $\nu$ value, in the ZMP law equation (3.1), has been discussed elsewhere [53–57]. On the other hand, the power law with exponential cut-off [52] has been discussed as occurring from the ‘random group formation’ in a sport research context [24].

Thus, it can be admitted that marked deviations also occur for $r \geq 100 \sim 120$. In fact, when examining the $r \leq 100$ range, several regular size regimes appear, after zooming on the vicinity of the marked fit deviations at ranks between 10 and 100 as shown for the power law with cut-off case, on a log–log plot, in figure 3. Successive arrows indicate ‘data steps’ at $r \approx 16$, 28, (39), 50, 62, 73, at least. This allows us to emphasize an intrinsic structure in such regimes with a ‘periodicity’ $\approx 11$ or 12.

Therefore, since different regimes can be seen emerging at various intervals, the team ranking behavior can be more precisely re-examined. This leads to much statistical analysis and many fit trials. Two types, either an exponential or a power law, are shown for three different sub-selections in figure 4, i.e. the top 50, 40 and 30 teams. The statistical characteristics of the ‘sample’ distributions are given in table 2, columns 3–5. The increase in the $R^2$ value with respect to the overall regime (in figure 1) is remarkable, for the power law fits, i.e. $R^2 \approx 0.797 \rightarrow 0.98$, suggesting that the 50 top teams or so ‘behave’ in a different way from the others $r > 50$. Observe the value of the power law exponent: $\alpha \sim 0.3$, in figure 4, in this regime, instead of 0.53 for the whole set, in figure 1. The evolution of the regression coefficient is very mild when changing the ‘sample size’, see table 3. Observe that the fits in figure 4 do not indicate any striking difference between the exponential and power law fits, from the $R^2$ or $\chi^2$ value criteria. Note from figure 4 that the numerical value of the exponential ‘relaxation rank’, i.e. the prefactor $(\approx 0.02)$ for $-x$, is still 50, like the value of the ‘relaxation rank’ in figure 1.

Next, the behavior of the teams above $r = 50$ can be quickly examined for completeness: see figure 5. Observe that the exponent $\alpha \sim 5/3$ for these high ranking teams differs significantly from the corresponding one for (the best) low ranked teams, $\sim 0.36$, as displayed in figure 4. Again, this emphasizes some difference in ‘behavior’ between the teams.
ranked below or above \( r \approx 50 \). The method of ‘primacy analysis’ seems thus of subsequent interest.

4. Analysis of primacy

It has been seen here above that the UEFA team ranking distribution can be close to a rank-size relationship. However, these distributions are primate distributions [17], i.e. one or very few teams predominate the distribution shape leading to a convex distribution that corresponds to the presence of a number of teams, 50 or so, with much larger coefficients than the mean coefficient \( \sim 14 \) (std. dev. \( \sim 21.25 \); see table 2). Therefore, concentrating on such ‘top teams’, it is of interest to raise the question whether the UEFA coefficient ranking ‘method’ implicitly induces some inner structure. In order to do so, the notion of primacy measure is developed here below.

4.1. Sheppard primacy measure

Measures of ‘primacy’ can be of the kind

\[
P_{i,j}^{(k)} = \frac{N_j}{\sum_{r=2}^{k+j} N_r}, \quad \text{with } k = 2, 3, ..., \tag{4.1}
\]

measuring the percentage of a contribution in the whole distribution. In particular, equation (4.1) gives a numerical value for the primacy of the best ranked entity with respect to the next \( k - 1 \) entities, since these are ordered by decreasing values.

One can go on and define

\[
P_{i,j}^{(k)} = \frac{N_j}{\sum_{r=M-j+1}^{M} N_r}, \quad \text{with } k_M = 2, 3, ..., \tag{4.2}
\]

in order to measure the primacy of any entity \( j \) over selected \( k_M - j + 1 \) lower entities. Obviously this number is reduced when \( j \) increases, due to the necessarily finite size \( k_M \leq M \) of the system.

If a power law of the kind \( N_r = N/\alpha^r \), see equation (1.1), is substituted into each of these measures, it is obvious that the corresponding index of primacy depends on \( \alpha \). Hence rank-size relationships with different \( \alpha \) will have different ‘levels of primacy’, which consequently will be hardly comparable to each other.

Sheppard [17] tried to avoid this puzzle by formulating a primacy index that is independent of \( \alpha \) (see appendix B for some further introduction). He defined

\[
P_{r} = \frac{1}{N - 2} \sum_{r=1}^{N-2} \frac{\ln (N_{r+2}) - \ln (N_{r+1})}{\ln (N_{r+2}) - \ln (N_{r+1})} \times \frac{\ln (r+2) - \ln (r+1)}{\ln (r+1) - \ln (r)}. \tag{4.3}
\]

4.2. Modified Sheppard-idea based primacy indices

However, Sheppard’s primacy index, equation (4.3), contains the difference between two logarithms in the denominator. Thus, when two consecutive UEFA coefficients have almost the same value, this difference can be very small, thus leading to a huge value of the Sheppard index. In order to avoid such problems, other ‘local primacy measures’ can be considered, forcing the difference between two closely related (logarithms of) \( N_r \) to be present only in the numerator. Keeping \( N_{r+1} \leq N_r \),
and \( r + 1 > r \), these measures are

\[
V_r = -\frac{\ln (N_{r+1}) - \ln (N_r)}{\ln (r+1) - \ln (r)} \quad (4.4)
\]

and

\[
W_r = \frac{\ln (N_r) - \ln (N_{r+1})}{\ln (r+1) - \ln (r)} - \frac{\ln (N_{r+1}) - \ln (N_{r+2})}{\ln (r+2) - \ln (r+1)} \\
\equiv V_r - V_{r+1} \quad (4.5)
\]

Observe that \( V_r \) and \( W_r \) are related to the next team(s) in the ranking, and measure a sort of distance. The results of the applications of such formulae for the problem at hand are shown in figure 6 for \( V_r \) and figure 7 for \( W_{r+1} \).

These enlightening figures indicate well-defined regime borders, corresponding to specific teams, as indicated in figure 6. For \( r \geq 5 \), the method confirms that the successive ‘regime sizes’ remain rather equal in value (≈11 or 12). There is also some ‘accumulation’ of teams near the borders, e.g. as seen in figure 7, reminiscent of a sharp transition behavior.

These features are also reminiscent of the occurrence of some sort of granular structure along a 1-dimensional space, or more generally dissipative structures [59]. Several analogies come in mind, e.g.: (i) hot spots in ballast resistors [60, 61] or superconductors [62]; (ii) invasion waves in combustion [63, 64]; (iii) Marangoni tears, and (iv) Benard cells, as in cloud band structures [65], or (Spitzberg) stone structures [66, 67]. In all such so-called intermittent processes in an open medium, the interplay between feedback mechanisms is known to induce temporary structures.

In order to illustrate such features through a simple model, consider the nonlinear open system known as the 1-dimensional ballast resistor. It describes the interplay between the input current leading to a Joule effect (\( RI^2 \)) and the energy dissipation due to the heat transfer with the surrounding environment wherein the transfer is controlled by the heat capacity and heat conductivity of the wire [60, 61]. In a first approximation, this leads to searching for the stability points of a system in a cubic potential. Whatever the boundary conditions, the system can become heterogeneous with hot and cold domains, which themselves can become heterogeneous through Hopf bifurcations [62]. The sizes of the domain and their motion (and life time) can be calculated [60–62]. The connection to an open social system, for which the internal ranking rules look like unstable conditions for self-organization, is not immediate. However, the analogy seems pertinent enough to suggest that models of intermittent processes can serve as bases for further generalization.

For illustrative purpose, a (very imperfect) toy model is presented in appendix C in order to show that UEFA-like rules, recalled in appendix A, i.e. attributing points to teams in a (necessarily) biased way over several seasons, imply the opening of gaps in the UEFA coefficient structure, thus regimes. The model is quite simplified, hence a search for properties similar to the mentioned Kosterlitz–Thouless transitions or characteristics of dissipative structures is quite outside the scope of the present paper.

**5. Discussion**

As seen in figure 5, the exponent \( \beta \sim 5/3 \) for the high rank team regime differs greatly from the corresponding one \( \sim 1/3 \)

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8 The connection between transitive continuous ordering of states by a ranking function and thermodynamic entropy is known in conventional thermodynamics under the name of adiabatic accessibility. This principle was introduced by Caratheodory [68] and used by Lieb and Yngvason [69]. See physical arguments in favor of connecting the entropy potential to the rank in [63, 64].

9 The number of UEFA ranked teams changes every year or so.
for (the best) low rank teams (figure 4). In a scientific field, like condensed matter physics, where exponents are frequently used for sorting out processes, a low value of the exponent stresses the narrow range of ‘activity’. The five-fold ratio between two exponents would indicate somewhat ‘long range’ or ‘diffuse properties’ for the system (=regime) with the largest exponent.

As in thermodynamics, drastically different values for the $\beta$ exponent may indicate a ‘state phase’ change, between a compressed phase and one (or more) disordered phases. In fact, one can further analyze the data from a statistical point of view, arguing that the ratio $\mu/\sigma$ between the mean and the standard deviation of rank-size distributions can be considered as the order parameter, as in studies of phase transitions. As shown in table 2, such a ratio steadily increases when reducing the considered number of ‘top of the top’ teams, forming the ordered state. On the other hand, the ratio $\mu/\sigma$ decreases below one for the ‘below the top 50’ teams, thereby indicating a sort of ordered-glass state phase transition. The pointing down arrow in figure 3 marks such a ‘critical rank’ between the medium rank teams (glass or vortex state) and the high rank teams (disordered state), in the present case.

Such findings on structures in organized competitions, like the UEFA Champions League and Europa League, invite some thought. In these apparently specific (team) competitions, the ‘self-organization scores’ determine that the predominant direction of evolution of the system is directed towards regular structures, thus less opened competitiveness. These competitions seem to be paradigmatic cases. However, the findings suggest ways for discussing many other, more or less abstract, competitions, as complex social systems are, when there are imposed competition rules (and scores) in their generic form [63, 64]. This induced structuralization was also seen in other (peer-to-peer) competitions between industrial companies [70]. Therefore, one can audaciously state that ranks in social competition might be unavoidable and structures can hardly be modified because of classical thermodynamics principles. It remains to be studied and discussed whether quantum-like (probabilistic) constraints would allow more competitiveness and subsequent ranking modification.

6. Concluding remarks

In concluding, it should be stressed that it has been investigated whether the classical rank-size relationships [13, 58] can be treated as indicators of a sport team class system, taken as a complex system and worthy of a scientific approach, as are many dynamical systems nowadays. A norm of performance for teams or team budget effects have not been considered. In this paper, we discuss the ranking in the particular case of UEFA teams, at first not debating the value of the points attributed to victory or loss at some competition level. Yet, their somewhat relative values arising from some arbitrariness underlines the present investigation and leads to harsh conclusions.

First, a conclusion is reached that the distribution of UEFA team ranking does not follow a single power law nor an exponential. Instead, it appears that the ranking should be grouped into classes. The rank-size distribution for these classes can be approximated by a mere scaling (power) law, with a quite different exponent, $\sim 1/3$ or $5/3$, suggesting a sort of order-disorder phase transition, in a thermodynamics-like sense, between the lowest and highest rank teams. Moreover, through this log–log search for an empirical law, it is found that structures exist in the overall classification. The medium rank regime is made of several ‘glassy’ states. The very high ($r \geq 100$) ranks corresponding to a disordered state. Along this analogy, the rules establishing the team UEFA coefficients act like a magnetic field inducing various structures in a magnetic or superconducting system.

In addition, in section 4, an index has been introduced in order to enhance regimes in data ranking. Through the ‘team primacy’ investigation for the top group of teams, on the basis of a modification of the conventional Sheppard index of primacy, the regimes are emphasized as extrinsically induced structures. It has been indicated that measures, called $V_i$ and $W_i$, independent of the empirical law functional form, lead to subsequent numerical results pointing to some inner structure of the team rank-value distribution, according to the present set of UEFA rules. It can be guessed, without more mathematical work than is necessary, that the inner structure is likely tied to the pool, in the first rounds, followed by the ‘direct elimination’ tournament-like process [40] in the UEFA competition and the ‘increase in value’ of the team when progressing further in the competitions through an accelerating effect with a set of nonlinear constraints (similar to boundary conditions) inducing an unstable organization, similar to that found in intermittent processes, when forced cycles drive an interplay between (two or several) feedback mechanisms and leading to visible physically characterized zones [59].

The (very imperfect) toy model presented in appendix C points to the opening of gaps when values of game results and tournament forms are imposed. Nevertheless, this very simplified model should suggest to the reader further research, through simulation or analytic work in order to study equivalent properties similar to those characterizing the above-mentioned Kosterlitz–Thouless transitions or characteristics of dissipative structures, hinted to be appearing in the UEFA coefficient rank-size structure, thus biased regimes.

In general, strongly intransitive competitions, in any society, dare we say, can display types of behavior associated with complexity, like competitive co-operation and leaping cycles. External influences induce structures in many physical processes. Through an example, we bring some evidence of the universality of the features. Note that the analogy with strict thermodynamics weakens as competitive systems become more intransitive [63, 64].

In future work, it would be of interest to examine whether the Sheppard generalized ideas and measures could be used to quantify how much a complex system rank-size distribution deviates from the usual power-law rank-size relationship, equation (1.1). Moreover, one could ask whether
this hyperbolic form has truly to be chosen as an optimum basis for discussing rational ranking rules. These measures seem to allow one to introduce a finer description of classes. In so doing, they might also serve for weighting performances, as in the case of NCAA College Football Rankings [26], by organizing more homogeneously (depending on team budgets and expectations [24]) based team competitions or regulating various sport conditions in team ranking. This may also imply some consideration of relaxing the rule about the number of teams from a country which may play in the UEFA competitions, if more competitiveness is of interest. An impossible dream?

Sports other than soccer are open to further investigations. Then, it will be interesting to see whether some universal features occur or whether there are marked differences.

Acknowledgments

This work has been performed in the framework of COST Action IS1104 ‘The EU in the new economic complex geography: models, tools and policy evaluation’. MA and NKV acknowledge some support through the project ‘Evolution spatiale et temporelle d’infrastructures régionales et économiques en Bulgarie et en Fédération Wallonie-Bruxelles’, within the intergovernmental agreement for cooperation between the Republic of Bulgaria and the Communauté Française de Belgique. Partial support by BS39/2014 is acknowledged by AG. Thanks to Peter Richmond and Wayne K Augé II for much improving the readability of the paper.

Appendix A. UEFA score rules

Some brief explanation, e.g. based on the 2013-14 UEFA competitions, can be useful in order to follow the competition organization, team elimination rule and subsequent point counting, as indicated in the tables here above. How many teams are selected, i.e. qualified, for the UEFA Champions League and the UEFA Europa League, is somewhat irrelevant for the present purpose. This is not discussed further. Details can be found in, e.g., http://en.wikipedia.org/wiki/2013-14_-_UEFA_-_Champions_-_League and in http://en.wikipedia.org/wiki/2013-14_-_UEFA_-_Europa_-_League

A total of 76 teams participated in the 2013/14 UEFA Champions League. Such a number of teams and the subsequent number of rounds depend on the number of available days for such a competition in a season. First, note that the qualifying round and play-off eliminations are direct confrontations between two teams, somewhat drawn at random. The process went as follows in 2013/14: four less well ranked teams had to go to the so called first qualifying round elimination. The two losers got each 0.5 points; the winners joined 32 teams at the second qualifying round elimination. The 17 losers got 1 point each. The 17 winners of the direct confrontations qualified for the third qualifying round elimination with 13 other teams. The 15 losers went to the Europa League play-off round (see below). The 15 winners of direct confrontations qualified for the play-off round with five other teams. The 10 losers went to the Europa League group stage round (see below). The 10 winners joined 22 ‘directly best qualified teams’ for the group stage. These 32 teams were drawn into eight groups of four. In each group, teams played against each other (home-and-away) in a round-robin format. The group winners and runners-up advanced to the round of 16, while the eight third-placed teams were directly entered in the 2013/14 UEFA Europa League round of 32 (see below). Thereafter, teams played against each other over two legs on a home-and-away basis, in a direct knockout phase, up to the one-match final. For the group stage and later rounds, the points were attributed as mentioned in table A1.

In contrast, 194 teams participated in the 2013/14 UEFA Europa League. Among the 194 teams, 33 (= 15+10+8; see above) were transferred from the Champions’ League, according to the following process. The 161 qualified teams were ranked (from bottom to top) and inserted into the qualifying rounds as follows. In the first qualifying round, the 76 (less well ranked) teams played at home and away after a mere drawing (with the restriction that two teams from the same association could not play against each other). The 38 winners were merged with 42 better ranked teams for the second qualifying round. The 40 winners went to the third qualifying round and merged with 18 better ranked teams. The 29 winners from the third qualifying round were merged with better 18 ranked teams and 15 losers (see above) from the Champions League third qualifying round to make a ‘play-off round’ of 62 teams. The 31 winners from the play-off round went to the group stage made of 48 teams, i.e. seven directly qualified and 10 qualified from the play-off round of the Champions League (see above). The 24 winners were merged with the eight ‘third placed teams from the Champions League Group stage’ (see above) to form the ‘round of 32’. A direct knockout phase goes on thereafter on a home-and-away basis legs, up to the single match final. The 38 losers in the first qualifying round got 0.25 pt each; see table A2. It is easy to note the team points thereafter.

In so doing, it is easily noted that the teams get points according to the level at which they were eliminated. Moreover, the number of obtained points increases much when a team participates in games at the group stage level.

Table A1. Champions League points system.

| Round of participation | Points |
|------------------------|--------|
| First qualifying round elimination | 0.5 points |
| Second qualifying round elimination | 1 point |
| Group stage participation | 4 points |
| Group stage game win | 2 points |
| Group stage game draw | 1 point |
| Round of 16 participation | 4 points |

N.B. Since the 2009/2010 season, clubs have been awarded an additional point if they reach the round of 16, quarter-finals, semi-finals or final. Points are not awarded for elimination in the third qualifying round or play-offs because the teams go to the Europa League.
Sheppard [17] proposed the following index of primacy:

\[
P_{N} = \frac{1}{N-2} \sum_{r=1}^{N-2} \left[ \ln \left( N_{r} \right) - \ln \left( N_{r+1} \right) \right] \times \frac{\ln \left( r+2 \right) - \ln \left( r+1 \right)}{\ln \left( r+1 \right) - \ln \left( r \right)}.
\]  

(B.1)

Note the following logics behind this index: substitute the power law rank-size relationship, \( N_{r} = N_{1}r^{-\alpha} \), into the previous equation. The result is

\[
P_{N} = \frac{1}{N-2} \sum_{r=1}^{N-2} 1 = 1
\]

(B.2)

whichever the value of \( \alpha \). Thus, for a rank-size relationship obeying a perfect power law, the index \( P_{N} \) has a value of 1, irrespective of the slope of the relationship on a log–log plot.

If \( P_{N} \) is less than one, the value suggests a locally convex form for the distribution. This suggests the occurrence of some ‘strong primacy’.

However, if (i) the size histogram fit deviates from the perfect power law, like in the figures in the main text, and in fact this is quite generally so\(^{10}\), and (ii) if the data is on the left of the fitting curve, then \( P_{N} \) will necessarily exceed one, as in the queen effect.

In conclusion, the \( V_{s} \) and \( W_{s} \) measures well quantify the possible features. The structural findings are henceforth much enhanced.

### Appendix C. Toy model

In this appendix, we want to indicate that the rule (see Appendix A) clubs have been awarded an additional point if they reach the round of 16, quarter-finals, semi-finals or final necessarily implies the opening of gaps in the ranking coefficient, hence the appearance of classes. In order to do so, we are greatly simplifying the UEFA competition conditions, and refer only to a virtual set of top teams for a tournament. We leave more complicated numerical simulations for further work.

\(^{10}\) No need to emphasize that this is always the case if \( R^{2} \) is strictly <1.
obtain a distribution of points for the eight top teams. The final number of points is rounded to the nearest integer. Under such a (very simplified) tournament process, the final distribution is found to be that displayed in figure 8.

Even though this is not exactly equivalent to the UEFA cases, the main emphasis on attributing rewarding points after winning some group and level stage is conserved. The equivalence between reality and the toy model resides in the ‘legal decision’ of these attributed points through a ranking based on a set of games (irrespective of the detailed results). It should be obvious that the more group stages a team is winning, the more so the ranking level will be emphasized, and accelerated depending on the level of the stage. Exactly as in the toy example (no draw) tournament, the difference in the winning some group and level stage is conserved. The cases, the main emphasis on attributing rewarding points after legal decision, the final numerical feature, the envelope of the (absolute value) of the maxima of the derivative of the UEFA coefficients can be precisely obtained through a hyperbolic fit, see figure 9, as $y = 56.22x + 0.83$, with $\chi^2 = 1.62$ for nine degrees of freedom, pointing to a 0.998 confidence level—somewhat demonstrating the rigidity of the gap structure.

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