Volume Ignition via Time-like Detonation in Pellet Fusion

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March 12, 2015

Abstract: Relativistic fluid dynamics and the theory of relativistic detonation fronts are used to estimate the space-time dynamics of the burning of the D-T fuel in Laser driven pellet fusion experiments. The initial “High foot” heating of the fuel makes the compressed target transparent to radiation, and then a rapid ignition pulse can penetrate and heat up the whole target to supercritical temperatures in a short time, so that most of the interior of the target ignites almost simultaneously and instabilities will have no time to develop. In these relativistic, radiation dominated processes both the interior, time-like burning front and the surrounding space-like part of the front will be stable against Rayleigh-Taylor instabilities. To achieve this rapid, volume ignition the pulse heating up the target to supercritical temperature should provide the required energy in less than 10 ps.

Keywords: Time-like Detonation, Radiation dominated ignition

Recent experimental and theoretical efforts to achieve Inertial Confinement Fusion (ICF) have focussed on the compression of the fuel in a 2.263 mm diameter fuel capsule including a CH ablator, a thin D-T ice layer, and a D-T gas fill in the middle [Hurricane \textit{et al.}, 2014; Park \textit{et al.}, 2014]. The ablator layer was placed on the external surface to achieve a larger compression of the pellet. The outcome of these experiments was that most of the pellet broke into pieces due to Rayleigh-Taylor surface instabilities, which are common in surfaces under extreme dynamical pressure.

Already in the classical literature, (Zel’dovich \textit{et al.}, 1969) mentioned that the way to avoid these instabilities is to make the compression and the detonation front a high temperature radiation dominated front, which works to smooth out the Rayleigh-Taylor (RT) instabilities propagating with the sound-speed, while the radiation is propagating with the speed of light.

From this well known reason it must be obvious that the radiation dominated, high temperature process must be described with relativistic fluid dynamics, wherein the pressure is not neglected compared with the energy density, and the propagation of radiative energy is described in a consistent way with all other dynamical processes.

Unfortunately the overwhelming majority of theoretical models that attempt to describe ICF experiments are (i) non-relativistic, i.e. neglecting pressure, and (ii) radiation is assumed to have infinite speed in principle. (Of course such an “infinite speed” becomes an undefined speed, which depends on the space-time grid resolution and on the numerical method.)

In the present work we concentrate on the “volume ignition” of the fuel by neglecting (for the 1st approximation) compression. We use a relativistic Rankine-Hugoniot description originally described by (Taub, 1948), which description was then 39 years later corrected by (Csernai, 1987), and used since then widely in the field of relativistic heavy ion collisions (Csernai, 1994). Just as in the Rankine-Hugoniot non-relativistic description of shock waves the relativistic relations are also based on the energy-momentum tensor, \( T^{\mu\nu} \), and baryon charge current, \( N^\mu \), conservation across a hypersurface with a normal 4-vector \( \Lambda^\mu \), where the change of a quantity \( a \) across the hyper-surface is denoted by \( [a] = a_2 - a_1 \):

\[
[R^\mu] = [T^{\mu\nu} \Lambda_\nu] = 0, \quad [j] = [N^\mu \Lambda_\mu] = 0. \quad (1)
\]

These conservation laws lead to the relativistic shock or detonation equations for the energy density, \( \epsilon \), pressure, \( p \), and generalized specific volume, \( X = (e + p)/\rho \)

\[
j^2 = \Lambda^\mu \Lambda_\mu [p]/[X], \quad [p](X_1 + X_2) = [(e + p)X]. \quad (2)
\]

This description treats detonations also across hypersurfaces with time-like normal vectors (\( \Lambda^\mu \Lambda_\mu = +1 \)), and therefore has the name time-like detonation, which actually means simultaneous volume ignition.

E.g. this description gives a correct, and rigorous description of the sudden and rapid hadronization of the Quark-Gluon Plasma (QGP) with the release of large latent heat of the QGP to hadronic matter phase transition (Csernai, 1987). Csernai, 1994. Taub’s description could be applied to “slow”, space-like fronts only.

This simplified model gives a quantitative estimate of the dynamics of volume ignition, which occurs in such a time-like detonation, thus completely avoiding the possibility of RT type of mechanical instability.

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In the work of [Csernai, 1987], the compression of the fuel in a sphere is neglected and the heating is described by isotropic radiation inwards from all sides, where $Q$ is the heat radiated in unit time per unit surface. The opacity of the target is assumed to be uniform inside the sphere, a fraction, $C'$, of the radiated heat is absorbed in the fuel and it is assumed that the incoming energy is sufficient that it is absorbed by the fuel by the point when the incoming beam reached the surface of the fuel on the back side of the pellet and not before. Under these assumptions the temperature increases and reaches a critical (ignition) temperature at some space-time point, $T(r, t) = T_c = 4\pi QC/C_v$, where $C_v$ is the specific heat.

Under these assumptions an analytic solution is given by [Csernai, 1987], providing the space-time points of a constant temperature hyper-surface, see FIG.1

$$T(r, t) \propto \begin{cases} 0, & t < 1 - r \\ \frac{1}{r} \left( \ln \frac{1}{t-r} - 1 \right) + \frac{1}{r^2}, & 1 - r < t < 1 + r \\ \frac{1}{r} \ln \left( \frac{1}{t-r} - 2 \right), & t > 1 + r \end{cases}$$

The isotherm hyper-surface is such that in an external spherical layer the detonation starts from the outside with sub-luminous velocity propagating inwards. This “propagation” velocity increases, reaches $c$, the velocity of light, at a radius $R_A$ (indicated by red dots on the $T = \text{const.}$ contour lines, FIG.1).

Then the points of the detonation hyper-surface inside this radius are not in causal connection with each other and the ignition completes with a semi-simultaneous volume ignition. This process is not acausal, it is the consequence of the initial inward radiation, so that all points of the time-like detonation hyper-surface are within the light-cones of the energy emission at the initial external surface of the radius $R_0$. The idea of volume ignition was presented earlier based on different arguments by [Kasotakis et al., 1989], with a short laser pulse.

This model became relevant and applicable to the recently published ICF experiments performed at the National Ignition Facility (NIF). To achieve a rapid volume ignition the needed total ignition energy should be radiated inward in a time interval, $t_{in} < 1$ or $t_{in} \ll 1$ (in units of $[R_0/c]$).

Of course the model is oversimplified. Even if we irradiate the fuel with light the radiation pressure is one third of the radiation energy density, $p_R = e_R/3$, and this presses the fuel before ignition. But this lasts for a short enough time to avoid the build-up of a RT instability. The mechanical RT instability propagates with a speed much smaller than the speed of light, $10^4 - 10^5$ times less in a solid, and about 10 times smaller at nuclear matter density.

The longer is $t_{in}$ the greater probability we have for RT instability. If $t_{in} > 3$, the RT instability can hardly be avoided, and the possible volume ignition domain size becomes negligible. Thus for an $R_0 = 3 \text{ mm}$ pellet the ideal irradiation time for volume ignition would be $t_{in} \leq 10$ ps (while for an $R_0 = 30 \text{ cm}$ target it would be $t_{in} = 1$ ns).

At the NIF “High foot” pre-compression of the fuel capsule in fortunate configuration will not lead to RT instability. As presented by [Hurricane et al., 2014] this shorter, $\sim 6$ ns. “High foot” pre-compression led to a more isotropic polar view (Figure 2b in ref. Hurricane et al. [2014]) giving rise to less RT instability as it is shown by [Casey et al., 2014], and higher total yield. As also mentioned in ref. [Atzeni et al., 2014], the pre-compression stage should be concluded by an ignition spike of 10-30 ps. This is not very far from our suggestion of 10 ps or less to achieve volume ignition or a time-like detonation. In ref. [Atzeni et al., 2014] this is not considered and only conventional shocks are discussed in the form of an imploding shock followed by a rebounding outgoing shock. These mechanical, pressure driven processes are still subject to RT instability, while the somewhat shorter and more energetic irradiation can prevent the possibility of all mechanical insta-
In the present ignition experiments a compression stage precedes the ignition process \cite{Hurricane2014, Park2014}. In the most successful experiments with an initial “High foot” shock of \(\sim 6\) ns the capsule is heated up and compressed to a smaller size of a radius of about an order of magnitude smaller, making the target hotter and more dense. The hotter target became less opaque, and the target more compressed. Then two final, more energetic shocks lasting for \(\sim 8\) ns, resulted in a hot spot of \(\sim 37\) \(\mu\text{m}\) radius, which is (partly) ignited and emitted almost the same energy that was invested to its compression and heating. The rapid ignition of an \(R_0 = 300\) (100) \(\mu\text{m}\) target to ignition at a mostly time-like surface would need a short energetic pulse of \(t_{\text{pre}} \approx 1(0.3)\) ps, which is about \(10^3 - 10^4\) times less than it was done in the NIF experiment.

Nevertheless the radiation dominated implosion leads to unstable section in the space-like, mechanically unsteady part of the detonation adiabat under the Chapman-Jouget point \(\text{point “O” in Figure 2}\), and see also the discussion in Chapt. 5 of the textbook by \cite{Csernai1994} also). Thus, the shortening of the pulse of the last two shocks, without decreasing the total deposited energy, leads to radiation dominance and may increase the stability considerably compared to the relatively “slow” final compression and heating of \(\sim 8\) ns.

Apart from the (i) short and energetic ignition pulse, the other critical condition is the (ii) sufficiently small opacity, such that the ignition pulse should be able to penetrate the whole target. Luckily the more energetic preheating pulses lead to a smaller target opacity and better thermal conductivity as shown by \cite{Hu2014}, so the appropriate tuning of the laser irradiation must be possible. The present pre-compression and heating with the starting “High foot” pulse led to higher temperature after the pre-compression, and this may be sufficient or adequate to achieve the optimal (low) opacity for the final short ignition pulse. The presented simple analytic model can be best applied to the stage of the process after the “High foot” pulse heated up the fluid sufficiently to achieve sufficiently small opacity.

Both these conditions may be more easily satisfied by heavy ion beams, where the beam bunch length can be regulated by beam optics and could be adjusted so that the length at the target becomes as short as \(10\) ps, (or 3 mm) or less. The suggestions of \cite{Lalousis2014} and \cite{Hora2014} emphasize also fast ignition as this work, however, this is considered to happen in a well focussed location with subsequent spreading the fusion flame through the whole (pre-compressed) target. The expansion of the target is counteracted by a strong magnetic field generated by a previous laser pulse. The primary aim is to generate as strong mechanical compression as possible. This is not the mechanism suggested here.

The detailed study of \cite{Fernandez2014} has basically the same goal, although instead of the magnetic field the possibilities of using proton and ion beams are discussed. It is even discussed that to reach maximal acceleration and relativistic speeds in the implosion, the plasma should be made opaque to the extent that it becomes sufficiently thick to isolate the rear target surface from the laser’.

The problem with these approaches is that this extreme compression leads to RT instabilities, which leads to a strong rebound of most of the target material. (In a collapse of a cold, \(\sim 20\) Solar mass Supernova progenitor only up to a 2 Solar mass Neutron or Hybrid star remains compressed.) To describe these instabilities a very high resolution, \(3+1\) dimensional relativistic PICR code is necessary, \cite{Csernai2012}, with very small numerical viscosity. Using such a relativistic approach one can see that the explosive hadronization of QGP takes place in a large part of the QGP volume with an almost simultaneous detonation, see the works of \cite{Floerchinger2014} and \cite{Csernai2009}. This is essentially the only secure way to prevent instabilities, and this requires a radiation dominated relativistic process, as shown also in Figure 1. To achieve
this almost simultaneous ignition in a large part of the pre-heated target of ~ 100μm, this target should not be further compressed, rather heated up by radiation, possibly simultaneously from all sides, so that radiation can reach also “the rear target surface”. Thus, in this aspect, the present suggestion is an alternative of the focused single point fast ignition discussed by (Fernandez et al., 2014), (Lalousis et al., 2014) and (Hora et al., 2014). On different grounds it was also found that picosecond pulses may lead to volume ignition. This in any case excludes the development of most instabilities as shown by (Hora et al., a, 2014), (Hora et al., b, 2014) and (Hora 2013).

The penetration length of heavy ion beams can also be regulated by the well-chosen beam energy. This operation is well studied in connection with radiation therapy where the position of the absorption Bragg peak is tuned to the necessary depth in the body to reach the tumor. One could also study if in this case one can achieve a sufficiently short and energetic final pulse so that the detonation becomes time-like (in the section of the detonation adiabat between points A and A’ in FIG. 2) in the majority of the compressed target.

Enlightening discussions with Dujuan Wang, Dieter H.H. Hoffmann and Heinrich Hora are gratefully acknowledged.

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