I. INTRODUCTION

The inflationary scenario [1] has become the current paradigm of early universe cosmology. A period of exponential expansion of space solves a number of fine-tuning problems of Standard Big Bang cosmology, in particular the horizon and flatness problems. Without inflation, finely tuned initial conditions are required to explain the observed degree of spatial flatness, and acausal initial condition correlations are required in order to explain the observed near isotropy of the cosmic microwave background (CMB).

In order for these achievements of cosmological inflation to be counted as real successes, inflation should arise without having to impose very special initial conditions. At the present time, there are rather conflicting statements on the naturalness of initial conditions for inflation, from articles which claim that there is a very serious problem [2] to those which claim that inflation arises very naturally [3, 4].

The purpose of this article is to review the status of research on the nature of initial conditions required to obtain a period of accelerated expansion of space. In the context of Einstein gravity, matter with an equation of state $p < \frac{1}{3} \rho$ is required in order to obtain such expansion.

The inflationary scenario provides a theory for the origin of structure in the universe which can explain CMB anisotropies and the distribution of galaxies on large scales [5] (see also [6, 7]). Comparison between the predictions of this theory and the latest CMB observations [8] tells us that the expansion rate of space was nearly exponential during inflation.

Since exponential expansion corresponds to a de Sitter phase of expansion, the initial conditions question for inflation was first discussed in the context of de Sitter space. De Sitter space can be obtained by adding a cosmological constant $\Lambda$ to the gravitational action. At a classical level, one can then prove “no-hair” theorems for de Sitter space which show that classical fluctuations about de Sitter space redshift and space-time approaches pure de Sitter [9]. Assuming matter which satisfies the “strong energy condition”, the no-hair theorem was proved in [10] for an isotropic background metric, and by Starobinsky [58] (see also [12]) for anisotropic backgrounds. Extensions to the case of matter not satisfying the strong energy condition were recently studied in [13]. The stability of de Sitter space-time to inhomogeneities with vanishing Weyl curvature tensor [14], to the addition of linear gravitational waves [15] and to linear cosmological fluctuations [16] was also studied. The no-hair conjecture has also been proved at nonlinear level for a model containing dust matter plus a bare cosmological constant [17]. Further extensions and general considerations were made e.g. in [18]. All of these analyses assume that matter obeys the strong energy condition. If this assumption is dropped, then there is the possibility of unstable modes [19].

Quantum mechanically, the question of stability of de Sitter space-time is open. It is possible that infrared instabilities can destabilize de Sitter (see e.g. [20]), although this issue is hotly debated (for an opposing point of view see e.g. [21]). Studies of tensor fluctuations [22] also indicate the de Sitter space-time is unstable, and the onset of an instability due to scalar entropic fluctuations can also be shown [23], although these studies are only perturbative and hence cannot truly differentiate between an instability of de Sitter and a finite renormalization of the cosmological constant due to semi-classical
The initial condition problem for small field models of inflation was studied in detail in [28] at the level of a classical phase space analysis. It was shown that in order to obtain a sufficiently long period of inflationary expansion, the initial scalar field velocity has to be finely tuned (i.e., tuned to be much smaller than the characteristic value $|\dot{\varphi}| = H^2$) even if the initial value of the field itself is set by hand to be very close to zero.

Whereas this fine tuning of the initial scalar field velocity seems unnatural for simple potentials, there is one context in which it is natural, namely if the initial conditions for the small field inflationary phase are set by tunneling from a false vacuum [29]. This scenario cannot be realized with renormalizable scalar field potentials, but it may well arise in effective field theories obtained from string theory. What is required is a potential for which $\varphi = 0$ is a metastable minimum, and which has a sufficient degree of flatness for field values $|\varphi|$ beyond the nucleation value to obtain enough slow-roll inflation.

Another context in which the initial scalar field velocity might vanish is if the initial conditions for Minkowski space-time evolution are set via an analytic continuation from a Euclidean quantum gravity region. This is the quantum cosmology approach to the initial condition problem of small field inflation. The problem is that different ansätze for the wave function of the universe (specifically the Hartle-Hawking [30] wavefunction or the “tunneling” wavefunctions [31]) give very different results for the probability of inflation when applied to the same Lagrangian system (see e.g. [32] for a computation of the probability given the wave function of [30]).

The situation is very different in large field models of inflation. As early work in [33] already hinted at, and as was studied in mode detail in [34], the inflationary slow-roll trajectory is a local attractor (see [35] for a more mathematical discussion of the meaning of the term “attractor” in this context) in initial condition space, at least at the level of homogeneous and isotropic cosmology (see also [36]). In large field inflation models there is enough time for the excess kinetic energy compared to the energy required during slow-rolling to have time to redshift, whereas this is not the case for small field inflation.

The difference in the dynamics between small field and large field inflation can be illustrated with phase space diagrams, as shown in Figs. 1 - 4. In all four figures, the horizontal axis represents the value of the scalar field. Fig. 1 is a sketch of the potential energy density function (vertical axis) assumed in small field models, Fig. 3 is the analog for large field models of inflation. Figs. 2 and 4 are sketches of the phase space dynamics for a small field inflation model (Fig. 2) and a large field model (Fig. 4). The vertical axes represent the field momentum. The arrows on the trajectories indicate the evolution in time. It is clear from Figs. 2 and 4 that the slow-roll trajectory labelled with “SR” is a local attractor in initial condition space in the case of large field inflation, but not in the case of small field models. The difference in the likelihood of inflation (in the context of homogeneous and
isotropic cosmology) between large and small field models of inflation has also been more recently confirmed in [37].

The above considerations have been at the level of classical dynamical systems. Implicit in the analysis is that all initial conditions which have a fixed energy density are equally likely, and as initial density we take a density when the classical dynamical description becomes justified (e.g. the Planck density). However, in a quantum universe the wave function of the universe may not give equal probability to different initial conditions with the same energy density, but may prefer special initial conditions. On these grounds it was argued a long time ago that the initial conditions for inflation (at the mini superspace level) are very likely [38]. On the other hand, Penrose [39] and others [40] have argued that inflation is extremely unlikely. The different results are due to different measures chosen. Specifically, the analysis of [39, 40] uses a measure on phase space configurations today, and asks what set of configurations similar to the ones we observe now have come from inflationary initial conditions, whereas in the analysis of [38] one is studying measures on the set of initial conditions, a procedure which is more appropriate if one has in mind standard Cauchy evolution. In fact, it has been shown [41] (see also [42]) that the measure gets squeezed by time evolution into regions of phase space which yield inflation. However, general difficulties in making arguments based on a measure of initial configurations were discussed in [43].

III. INITIAL CONDITIONS FOR INHOMOGENEOUS DYNAMICS

A discussion of the probability to obtain inflation based on a mini-superspace analysis is only a very limited aspect of the initial conditions problem for inflation. If we assume homogeneity from the outset, then there is no horizon problem and inflation is not needed to explain the near isotropy of the CMB (it is still needed to explain the spatial flatness and to provide causal initial conditions for fluctuations). Thus, we must now consider how likely it is to obtain inflation from general inhomogeneous initial conditions. This issue is not yet completely settled. There are authors who argue that exponential fine tuning of initial conditions are required [44, 45], whereas others argue that even in the presence of matter and metric fluctuations the slow-roll trajectory of large field inflation is a local attractor in initial condition space [46, 47, 48]. In the following we will only discuss large field inflation models.

Why are such different conclusions reached? The analysis of [44, 45] starts from the assumption that "homogeneity" over a length scale of more than $H^{-1}$, where $H$ is the Hubble expansion rate at the beginning of inflation, is required in order to obtain inflation. By "homogeneity" it is here meant that the energy density has to be dominated by the almost constant mode of the scalar field. It is then argued that it is very unlikely to have such initial conditions. This argument, however, clearly will give only a lower bound on the probability of inflation. Since inhomogeneities redshift, but a homogeneous field only slowly rolls and hence decays much more slowly as a function of time than the fluctuating modes, Hence, homogeneity over a few Hubble patches is a sufficient condition to obtain inflation, but clearly it is not a necessary one.

In fact, it is argued in [34, 46, 48, 49] that even if the initial energy density was dominated by inhomogeneous modes (inhomogeneity scale smaller than $H^{-1}$) the universe is likely to eventually enter a period of inflation provided that there is some power in the quasi-homogeneous mode (homogeneous on scale $H^{-1}$) 1. The argument is as follows. Whatever the initial conditions are, the universe will expand. During this expansion phase the energy in the inhomogeneous modes will redshift whereas the zero mode of $\varphi$ will only slowly roll. Hence, as long as there is sufficient power in the zero mode, it will eventually come to dominate and the universe will start to inflate. Thus, the inflationary slow-roll trajectory is a local attractor in initial condition space, even in the presence of inhomogeneities. This argument, however, misses potential back-reaction effects. To second order in the amplitude of the fluctuating modes, these modes have an effect on the background. In particular, it is possible that they will effectively destroy the background and hence prevent the onset of inflation.

It has in fact been shown numerically [46] that in the absence of metric fluctuations the slow-roll trajectory for large field inflation is a local attractor. The attractor basin is in fact very large - the initial density in the inhomogeneous fluctuations may be orders of magnitude larger than that in the zero mode. A much improved numerical analysis of this problem has recently been published in [50].

However, it is not consistent to neglect metric fluctuations. One may worry that the presence of initial metric fluctuations may lead space-time to collapse into a gas of black holes rather than lead to an expanding cosmology. On the other hand, the work of [49] showed that even in the presence of metric fluctuations (linearized joint fluctuations of metric and matter) the large field slow-roll trajectory remains an attractor in initial condition space. To go beyond linear theory, methods of numerical relativity are required. The initial numerical studies of the onset of inflation with inhomogeneous initial conditions were performed in [28, 51] in the case of small field inflation models, and in [47] in the case of large field inflation. Both works used one space-dimensional codes. It

1 Note that having power on super-Hubble scales is expected. In fact, the absence of such fluctuations would require acausal initial conditions.
was shown that inflation was very unlikely in the case of small field inflation, but that it is an attractor in initial condition space for large field inflation.

With the improvements in numerical relativity codes, and the availability of more computational power, it has become possible to perform simulations in three spatial dimensions. Early work in three space dimensions is due to [52], and a more recent study was recently published in [53]. These studies all come to the conclusion that the slow-roll trajectory for large field inflation is a local attractor given the set of initial conditions which were studied. Specifically, in the recent work of [53] it was found that a Hubble patch with initial conditions in which the inhomogeneities (modelled as a set of Fourier modes with sub-Hubble wavelength) have an energy density which exceeds that of the background by a factor of $10^3$ will eventually undergo inflation, as long as the average spatial curvature is not positive, and as long as the field range remains in the slow-roll regime. If the field values entered the non-slow-roll region, it was found that sometimes the patch will not inflate. Thus, whereas the inflationary slow-roll trajectory is a local attractor, it is not a global attractor. Another new code is in development [54] and will be used to study the onset of inflation.

IV. CONCLUSIONS

As this review has shown, there has been a lot of work concerning initial conditions for inflation. The author feels that there is now a body of analytical and numerical work supporting the claim that in the case of large field models the slow-roll trajectory is a local attractor in initial condition space.

It is important to distinguish the issue of initial conditions for inflation from that of the initial conditions for the cosmological fluctuations and gravitational waves in an inflationary background. Concerning the latter, one usually argues that the Bunch-Davies [55] vacuum state is an attractor in the space of states (as far as the evolution of correlation functions is concerned) (see e.g. [56] for some early work and [57] for a recent review). The question of initial conditions for fluctuations is related to the “trans-Planckian problem” for cosmological fluctuations which will be touched on below.

Some researchers discount the entire discussion of initial conditions for inflation by saying that as long as there will be one patch which starts to inflate, its physical volume will rapidly come to dominate the Universe. In particular, if the field is allowed to explore the region in which stochastic forces driving the field up the poten-
FIG. 2: Sketch of phase space trajectories in small field inflation models. The horizontal axis is the field, the vertical axis is the momentum (equivalently the time derivative of the field). Typical initial conditions with nonvanishing $\pi$ do not lead to inflation.

Inflation becomes larger in magnitude than the classical force driving the field down the potential [58], then the physical volume of space will be dominated by inflationary patches. However, what “dominate” means depends on the measure one considers, and it is claimed in [45] that the probability of inflation remains low even if one takes the exponential growth of inflating regions into account. This debate may be academic, however, since small modifications of the potential at super-Planckian field values can easily eliminate the region of stochastic inflation [36]. As a bottom line, it is reassuring that a local dynamical systems analysis seems to show that the initial conditions for inflation do not have to be finely tuned.

Even if the inflationary slow-roll trajectory is a local attractor in the initial condition space, it is not a global attractor. If the inhomogeneities are too large, then we would expect space to collapse into a gas of black holes, leading to a picture of the early universe discussed in [59] or [60]. The author feels, however, that the inflationary slow-roll trajectory being a local attractor is already a promising achievement.

Having concluded that the slow-roll trajectory in large field inflation is a local attractor in initial condition space, one must keep in mind that there are problems in embedding large field inflation into a ultra-violet finite quantum theory such as string theory. Axion monodromy inflation [61] appears to be the most promising route, but there are concerns whether such models are safe against back-reaction effects [62] and considerations based on the “Weak Gravity Conjecture”, considerations which rule out large field inflation in other axion inflation models [63]. If these problems hold up and one is forced into small field inflation, then the initial condition problem for inflation would become rather severe.

It is important to keep in mind that the inflationary scenario is at best an incomplete picture of the very early universe. Inflation, at least in the context of scalar field matter coupled to Einstein gravity, is known to be past incomplete [64]. This implies that we need to go beyond inflationary cosmology if we really want to understand the very earliest moments of the universe.

There are other conceptual problems of inflation (see e.g. [65]). For one, if inflation last more than 70 e-folding times, then the wavelengths of all scales which are being observed today are smaller than the Planck length at the beginning of the period of inflation. As shown in [66] and many followup papers, this gives rise to the “trans-Planckian” problem for cosmological fluctuations: effects
of physics in the “zone of ignorance” of length scales smaller than the Planck length may well effect the nature of the observed fluctuations. Another issue concerns the sensitivity of the inflationary mechanism - almost constant potential energy density driving accelerated expansion, to our ignorance of what renders quantum vacuum energy gravitationally inert.

These and other conceptual issues have led to the development of alternative early universe scenarios such as the Pre-Big-Bang \cite{67}, the Ekpyrotic scenario \cite{68}, the “Matter Bounce” \cite{69} and String Gas Cosmology \cite{70}, to mention only a few. Whereas these scenarios do not suffer from the trans-Planckian problem for fluctuations, they all have their own conceptual problems. In particular, the initial conditions required for several of them (e.g. the Pre-Big-Bang and the Matter Bounce scenarios) are not attractors in initial condition space \cite{71} (on the other hand, the trajectory in the original Ekpyrotic scenario is an attractor \cite{72}).

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\[\text{FIG. 3: Sketch of the potential energy function (vertical axis) as a function of the scalar field value (horizontal axis) for large field inflation models.}\]

\[\text{[1] R. Brout, F. Englert and E. Gunzig, “The Causal Universe,” Gen. Rel. Grav. 10, 1 (1979); R. Brout, F. Englert and E. Gunzig, “The Creation Of The Universe As A Quantum Phenomenon,” Annals Phys. 115, 78 (1978); A. A. Starobinsky, “A New Type Of Isotropic Cosmological Models Without Singularity,” Phys. Lett. B 91, 99 (1980);}\]
D. Kazanas, “Dynamics of the Universe and Spontaneous Symmetry Breaking,” Astrophys. J. 241, L59 (1980); Guth AH, “The Inflationary Universe: A Possible Solution To The Horizon And Flatness Problems,” Phys. Rev. D 23, 347 (1981); K. Sato, “First Order Phase Transition Of A Vacuum And Expansion Of The Universe,” Mon. Not. Roy. Astron. Soc. 195, 467 (1981); L. Z. Fang, “Entropy Generation in the Early Universe by Dissipative Processes Near the Higgs’ Phase Transitions,” Phys. Lett. B 95, 154 (1980).

[2] A. Ijjas, P. J. Steinhardt and A. Loeb, “Inflationary paradigm in trouble after Planck2013,” Phys. Lett. B 723, 261 (2013) doi:10.1016/j.physletb.2013.05.023 [arXiv:1304.2785 [astro-ph.CO]]; A. Ijjas, P. J. Steinhardt and A. Loeb, “Inflationary schism after Planck2013,” Phys. Lett. B 736, 142 (2014) doi:10.1016/j.physletb.2014.07.012 [arXiv:1402.6980 [astro-ph.CO]].

[3] A. H. Guth, D. I. Kaiser and Y. Nomura, “Inflationary paradigm after Planck 2013,” Phys. Lett. B 733, 112 (2014) doi:10.1016/j.physletb.2014.03.020 [arXiv:1312.7619 [astro-ph.CO]].

[4] A. Linde, “Inflationary Cosmology after Planck 2013,” doi:10.1093/acprof:oso/9780198728856.003.0006 arXiv:1402.0526 [hep-th].

[5] V. Mukhanov and G. Chibisov, “Quantum Fluctuation And Nonsingular Universe. (In Russian),” JETP Lett. 33, 532 (1981) [Pisma Zh. Eksp. Teor. Fiz. 33, 549 (1981)].

[6] K. Sato, “First Order Phase Transition of a Vacuum and Expansion of the Universe,” Mon. Not. Roy. Astron. Soc. 195, 467 (1981).

[7] W. H. Press, “Spontaneous Production of the Zel’dovich Spectrum of Cosmological Fluctuations,” Phys. Scripta 21, 702 (1980). doi:10.1088/0031-8949/21/5/021

[8] P. A. R. Ade et al. [Planck Collaboration], “Planck 2015 results. XIII. Cosmological parameters,” arXiv:1502.01589 [astro-ph.CO].

[9] G. W. Gibbons and S. W. Hawking, “Cosmological Event Horizons, Thermodynamics, and Particle Creation,” Phys. Rev. D 15, 2738 (1977). doi:10.1103/PhysRevD.15.2738; S. W. Hawking and I. G. Moss, “Supercooled Phase Transitions in the Very Early Universe,” Phys. Lett. B 110, 35 (1982). doi:10.1016/0370-2693(82)90946-7

[10] R. M. Wald, “Asymptotic behavior of homogeneous cosmological models in the presence of a positive cosmological constant,” Phys. Rev. D 28, 2118 (1983). doi:10.1103/PhysRevD.28.2118

[11] A. A. Starobinsky, “Isotropization of arbitrary cosmological expansion given an effective cosmological constant,” JETP Lett. 37, 66 (1983).

[12] J. D. Barrow and J. Stein-Schabes, “Inhomogeneous cos-

FIG. 4: Sketch of phase space trajectories in large field inflation models. The axes are as in Figure 2. The slow-roll trajectories (the horizontal trajectories at $|\phi| > m_{pl}$ are now local attractors in initial condition space.
mologies with cosmological constant," Phys. Lett. A 103, 315 (1984). doi:10.1016/0375-9601(84)90467-5 ;
L. G. Jensen and J. A. Stein-Schabes, “Is Inflation Natural?,” Phys. Rev. D 35, 1146 (1987). doi:10.1103/PhysRevD.35.1146 ;
Y. Kitada and K. i. Maeda, “Cosmic no hair theorem in homogeneous space-times. 1. Bianchi models,” Class. Quant. Grav. 10, 703 (1993). doi:10.1088/0264-9381/10/4/008;
Y. Kitada and K. i. Maeda, “Cosmic no hair theorem in power law inflation,” Phys. Rev. D 45, 1416 (1992). doi:10.1103/PhysRevD.45.1416

[13] A. Maleknejad and M. M. Sheikh-Jabbari, “Revisiting Cosmoc No-Hair Theorem for Inflationary Settings,” Phys. Rev. D 85, 123508 (2012) doi:10.1103/PhysRevD.85.123508 [arXiv:1203.0219 [hep-th]].

[14] M. Bruni, S. Materrese and O. Pantano, “A Local view of the observable universe,” Phys. Rev. Lett. 74, 1916 (1995) doi:10.1103/PhysRevLett.74.1916 [astro-ph/9407054].

[15] A. A. Starobinsky, “Spectrum of relict gravitational radiation and the early state of the universe.” JETP Lett. 30, 682 (1979) [Pisma Zh. Eksp. Teor. Fiz. 30, 719 (1979)].

[16] J. D. Barrow, “Perturbations Of A De Sitter Universe,” In *Cambridge 1982, Proceedings, The Very Early Universe*, 273-278 and Preprint - BOUCHER, W. In *Cambridge 1982, Proceedings, The Very Early Universe*, 273-278 and Preprint - BOUCHER, W. [arXiv:1109.3535 [astro-ph.CO]].

[17] M. Bruni, F. C. Mena and R. K. Tavakol, “Cosmic no hair: Nonlinear asymptotic stability of de Sitter universe,” Class. Quant. Grav. 19, L23 (2002) doi:10.1088/0264-9381/19/5/101 [gr-qc/0107069].

[18] V. Muller, H. J. Schmidt and A. A. Starobinsky, “The Stability of the De Sitter Space-time in Fourth Order Gravity,” Phys. Lett. B 202, 198 (1988). doi:10.1016/0370-2693(88)90007-X;
J. D. Barrow and G. Goetz, “The Asymptotic Approach to De Sitter Space-time,” Phys. Lett. B 231, 228 (1989). doi:10.1016/0370-2693(89)90204-9;
J. Bicak and J. Podolsky, “Global structure of Robinson-Trautman radiative space-times with a cosmological constant,” Phys. Rev. D 55, 1985 (1997) doi:10.1103/PhysRevD.55.1985 [gr-qc/9901018];
S. Capozziello, R. de Ritis and A. A. Marino, “Recovering the effective cosmological constant in extended gravity theories,” Gen. Rel. Grav. 30, 1247 (1998) doi:10.1023/A:1026651129626 [gr-qc/9804053].

[19] J. D. Barrow, “Cosmic No Hair Theorems and Inflation,” Phys. Lett. B 187, 12 (1987). doi:10.1016/0370-2693(87)90063-3;
J. D. Barrow, “The Deflationary Universe: An Instability of the De Sitter Universe,” Phys. Lett. B 180, 335 (1986). doi:10.1016/0370-2693(86)91198-6

[20] A. M. Polyakov, “Decay of Vacuum Energy,” Nucl. Phys. B 834, 316 (2010) doi:10.1016/j.nuclphysb.2010.03.021 [arXiv:0912.5503 [hep-th]].

[21] D. Marolf and I. A. Morrison, “The IR stability of de Sitter QFT: results at all orders,” Phys. Rev. D 84, 044040 (2011) doi:10.1103/PhysRevD.84.044040 [arXiv:1010.5327 [gr-qc]].

[22] N. C. Tsamis and R. P. Woodard, “Relaxing the cosmological constant,” Phys. Lett. B 301, 351 (1993). doi:10.1016/0370-2693(93)91162-G

[23] R. H. Brandenberger, “Back reaction of cosmological perturbations and the cosmological constant problem,” hep-th/0210165;
G. Geshnizjani and R. Brandenberger, “Back reaction of perturbations in two scalar field inflationary models,” JCAP 0504, 006 (2005) doi:10.1088/1475-7516/2005/04/006 [hep-th/0310265];
G. Marozzi, G. P. Vacca and R. H. Brandenberger, “Cosmological Backreaction for a Test Field Observer in a Chaotic Inflationary Model,” JCAP 1302, 027 (2013) doi:10.1088/1475-7516/2013/02/027 [arXiv:1212.6029 [hep-th]].

[24] A. D. Linde, “A New Inflationary Universe Scenario: A Possible Solution of the Horizon, Flatness, Homogeneity, Isotropy and Primordial Monopole Problems,” Phys. Lett. B 108, 389 (1982). doi:10.1016/0370-2693(82)91219-9
A. Albrecht and P. J. Steinhardt, “Cosmology for Grand Unified Theories with Radiatively Induced Symmetry Breaking,” Phys. Rev. Lett. 48, 1220 (1982). doi:10.1103/PhysRevLett.48.1220

[25] S. R. Coleman and E. J. Weinberg, “Radiative Corrections as the Origin of Spontaneous Symmetry Breaking,” Phys. Rev. D 7, 1888 (1973). doi:10.1103/PhysRevD.7.1888

[26] A. D. Linde, “Chaotic Inflation,” Phys. Lett. B 129, 177 (1983). doi:10.1016/0370-2693(83)90837-7
G. F. Mazenko, R. M. Wald and W. G. Unruh, “Does a Phase Transition in the Early Universe Produce the Conditions Needed for Inflation?,” Phys. Rev. D 31, 273 (1985). doi:10.1103/PhysRevD.31.273

[27] D. S. Goldwirth and T. Piran, “Initial conditions for inflation,” Phys. Rept. 214, 223 (1992). doi:10.1016/0370-1573(92)90073-9

[28] J. Garriga, X. Montes, M. Sasaki and T. Tanaka, “Spectrum of cosmological perturbations in the one bubble open universe.” Nucl. Phys. B 551, 317 (1999) doi:10.1016/S0550-3213(99)00181-9 [astro-ph/9811257];
B. Freivogel, M. Kleban, M. Rodriguez Martinez and L. Susskind, “Observational consequences of a landscape,” JHEP 0603, 039 (2006) doi:10.1088/1126-6708/2006/03/039 [hep-th/0505232];
D. Yamauchi, A. Linde, A. Naruko, M. Sasaki and T. Tanaka, “Open inflation in the landscape,” Phys. Rev. D 84, 043513 (2011) doi:10.1103/PhysRevD.84.043513 [arXiv:1105.2674 [hep-th]].

[29] J. B. Hartle and S. W. Hawking, “Wave Function of the Universe,” Phys. Rev. D 28, 2960 (1983). doi:10.1103/PhysRevD.28.2960

[30] A. D. Linde, “Quantum Creation of the Inflationary Universe,” Lett. Nuovo Cim. 39, 401 (1984). doi:10.1007/BF02790571;
A. Vilenkin, “Creation of Universes from Nothing,” Phys. Lett. B 117, 25 (1982). doi:10.1016/0370-2693(82)90866-8;
A. Vilenkin, “Quantum Cosmology and the Initial State of the Universe,” Phys. Rev. D 37, 888 (1988). doi:10.1103/PhysRevD.37.888

[31] J. B. Hartle and T. Hertog, “The No-Boundary Measure in the Regime of Eternal Inflation,” Phys. Rev. D 82, 063510 (2010) doi:10.1103/PhysRevD.82.063510 [arXiv:1001.0262 [hep-th]]
A. Corichi and D. Sloan, “The Classical Universes of the No-Boundary Quantum State,” Phys. Rev. D 77, 123537 (2008) doi:10.1103/PhysRevD.77.123537 [arXiv:0803.1663 [hep-th]].

J. B. Hartle, S. W. Hawking and T. Hertog, “The Classical Universes of the No-Boundary Quantum State,” Phys. Rev. D 77, 123537 (2008) doi:10.1103/PhysRevD.77.123537 [arXiv:0803.1663 [hep-th]].

J. B. Hartle, S. W. Hawking and T. Hertog, “No-Boundary Measure of the Universe,” Phys. Rev. Lett. 100, 201301 (2008) doi:10.1103/PhysRevLett.100.201301 [arXiv:0711.4630 [hep-th]].

A. Albrecht and R. H. Brandenberger, “On the Realization of New Inflation,” Phys. Rev. D 31, 1225 (1985).

doi:10.1103/PhysRevD.31.1225

R. H. Brandenberger and J. H. Kung, “Chaotic Inflation as an Attractor in Initial Condition Space,” Phys. Rev. D 42, 1008 (1990). doi:10.1103/PhysRevD.42.1008

G. N. Remmen and S. M. Carroll, “Attractor Solutions in Scalar-Field Cosmology,” Phys. Rev. D 88, 083518 (2013) doi:10.1103/PhysRevD.88.083518 [arXiv:1309.2611 [gr-qc]].

V. Mukhanov, “Inflation without Selfreproduction,” Fortsch. Phys. 63, 36 (2015) doi:10.1002/prop.201400074 [arXiv:1409.2335 [astro-ph.CO]].

G. N. Remmen and S. M. Carroll, “How Many e-Folds Should We Expect from High-Scale Inflation?” Phys. Rev. D 90, no. 6, 063517 (2014) doi:10.1103/PhysRevD.90.063517 [arXiv:1405.5538 [hep-th]].

G. W. Gibbons, S. W. Hawking and J. M. Stewart, “A Natural Measure on the Set of All Universes,” Nucl. Phys. B 281, 736 (1987). doi:10.1016/0550-3213(87)90425-1; S. W. Hawking and D. N. Page, “How probable is inflation?” Nucl. Phys. B 298, 789 (1988). doi:10.1016/0550-3213(89)90008-9.

R. Penrose, “Difficulties with inflationary cosmology,” Annals N. Y. Acad. Sci. 571, 249 (1989). doi:10.1111/j.1749-6632.1989.tb50513.x

G. W. Gibbons and N. Turok, “The Measure Problem in Cosmology,” Phys. Rev. D 77, 063516 (2008) doi:10.1103/PhysRevD.77.063516 [hep-th/0609095]; S. M. Carroll and H. Tam, “Unitary Evolution and Cosmological Fine-Tuning,” arXiv:1007.1417 [hep-th].

A. Corichi and D. Sloan, “Inflationary Attractors and their Measures,” Class. Quant. Grav. 31, 062001 (2014) doi:10.1088/0264-9381/31/6/062001 [arXiv:1310.6399 [gr-qc]].

A. Corichi and A. Karami, “On the measure problem in slow roll inflation and loop quantum cosmology,” Phys. Rev. D 83, 104006 (2011) doi:10.1103/PhysRevD.83.104006 [arXiv:1011.4249 [gr-qc]].

J. S. Schiffrin and R. M. Wald, “Measure and Probability in Cosmology,” Phys. Rev. D 86, 023521 (2012) doi:10.1103/PhysRevD.86.023521 [arXiv:1202.1818 [gr-qc]].

T. Vachaspati and M. Trodden, “Causality and cosmic inflation,” Phys. Rev. D 61, 023502 (1999) doi:10.1103/PhysRevD.61.023502 [gr-qc/9811037].

L. Berezhiani and M. Trodden, “How Likely are Constituent Quanta to Initiate Inflation?,” Phys. Lett. B 749, 425 (2015) doi:10.1016/j.physletb.2015.08.007 [arXiv:1504.01730 [hep-th]].

A. Albrecht, R. H. Brandenberger and R. Matzner, “Numerical Analysis of Inflation,” Phys. Rev. D 32, 1280 (1985). doi:10.1103/PhysRevD.32.1280;

A. Albrecht, R. H. Brandenberger and R. Matzner, “Inflation With Generalized Initial Conditions,” Phys. Rev. D 35, 429 (1987). doi:10.1103/PhysRevD.35.429

H. Kurki-Suonio, R. A. Matzner, J. Centrella and J. R. Wilson, “Inflation From Inhomogeneous Initial Data in a One-dimensional Back Reacting Cosmology,” Phys. Rev. D 35, 435 (1987). doi:10.1103/PhysRevD.35.435

V. Muller, H. J. Schmidt and A. A. Starobinsky, “Power law inflation as an attractor solution for inhomogeneous cosmological models,” Class. Quant. Grav. 7, 1163 (1990). doi:10.1088/0264-9381/7/7/012

H. A. Feldman and R. H. Brandenberger, “Chaotic Inflation With Metric and Matter Perturbations,” Phys. Lett. B 227, 359 (1989). doi:10.1016/0370-2693(89)90441-4; R. H. Brandenberger and H. A. Feldman, “Effects of Gravitational Perturbations on the Evolution of Scalar Fields in the Early Universe,” Phys. Lett. B 220, 361 (1989). doi:10.1016/0370-2693(89)90888-5;

R. H. Brandenberger, H. Feldman and J. Kung, “Initial conditions for chaotic inflation,” Phys. Scripta T 36, 64 (1991). doi:10.1088/0031-8949/1991/T36/007

R. Easther, L. C. Price and J. Rasero, “Inflating an Inhomogeneous Universe,” JCAP 1408, 041 (2014) doi:10.1088/1475-7516/2014/08/041 [arXiv:1406.2869 [astro-ph.CO]].

D. S. Goldwirth and T. Piran, “Inhomogeneity and the Onset of Inflation,” Phys. Rev. Lett. 64, 2582 (1990). doi:10.1103/PhysRevLett.64.2582.

P. Laguna, H. Kurki-Suonio and R. A. Matzner, “Inhomogeneous inflation: The Initial value problem,” Phys. Rev. D 44, 3077 (1991). doi:10.1103/PhysRevD.44.3077; H. Kurki-Suonio, P. Laguna and R. A. Matzner, “Inhomogeneous inflation: Numerical evolution,” Phys. Rev. D 48, 3611 (1993) doi:10.1103/PhysRevD.48.3611 [astro-ph/9306009].

W. E. East, M. Kleban, A. Linde and L. Senatore, “Beginning inflation in an inhomogeneous universe,” arXiv:1511.05143 [hep-th].

K. Clough, P. Figueras, H. Finkel, M. Kunesch, E. A. Lim and S. Tunyasuvunakool, “GRChombo : Numerical Relativity with Adaptive Mesh Refinement,” Class. Quant. Grav. 32, no. 24, 245011 (2015) doi:10.1088/0264-9381/32/24/245011 [arXiv:1503.03436 [gr-qc]].

T. S. Bunch and P. C. W. Davies, “Quantum Field Theory in de Sitter Space: Renormalization by Point Splitting,” Proc. Roy. Soc. Lond. A 360, 117 (1978). doi:10.1098/rspa.1978.0060

R. H. Brandenberger and C. T. Hill, “Energy Density Fluctuations in de Sitter Space,” Phys. Lett. B 179, 30 (1986). doi:10.1016/0370-2693(86)90430-2

H. Jiang, Y. Wang and S. Zhou, “On the initial condition of inflationary fluctuations,” arXiv:1601.01179 [hep-th].

A. A. Starobinsky, “Stochastic De Sitter (inflationary) Stage In The Early Universe,” Lect. Notes Phys. 246, 107 (1986).
tion spectra,” Phys. Rev. D 60, 023507 (1999) [arXiv:gr-qc/9809062].

[61] E. Silverstein and A. Westphal, “Monodromy in the CMB: Gravity Waves and String Inflation,” Phys. Rev. D 78, 106003 (2008) [arXiv:0803.3085 [hep-th]]; L. McAllister, E. Silverstein and A. Westphal, “Gravity Waves and Linear Inflation from Axion Monodromy,” Phys. Rev. D 82, 046003 (2010) doi:10.1103/PhysRevD.82.046003 [arXiv:0808.0706 [hep-th]].

[62] A. Hebecker, P. Mangat, F. Rompineve and L. T. Witkowski, “Tuning and Backreaction in F-term Axion Monodromy Inflation,” Nucl. Phys. B 894, 456 (2015) doi:10.1016/j.nuclphysb.2015.03.015 [arXiv:1411.2032 [hep-th]]; J. P. Conlon, “Brane-Antibrane Backreaction in Axion Monodromy Inflation,” JCAP 1201, 033 (2012) doi:10.1088/1475-7516/2012/01/033 [arXiv:1110.6454 [hep-th]].

[63] A. Hebecker, F. Rompineve and A. Westphal, “Axion Monodromy and the Weak Gravity Conjecture,” arXiv:1512.03768 [hep-th]; L. E. Ibanez, M. Montero, A. Uranga and I. Valenzuela, “Relaxion Monodromy and the Weak Gravity Conjecture,” arXiv:1512.00025 [hep-th]; B. Heidenreich, M. Reece and T. Rudelius, “Weak Gravity Strongly Constrains Large-Field Axion Inflation,” arXiv:1506.03447 [hep-th]; J. Brown, W. Cottrell, G. Shiu and P. Soler, “On Axionic Field Ranges, Loopholes and the Weak Gravity Conjecture,” arXiv:1504.00659 [hep-th]; J. Brown, W. Cottrell, G. Shiu and P. Soler, “Fencing in the Swampland: Quantum Gravity Constraints on Large Field Inflation,” JHEP 1510, 023 (2015) doi:10.1007/JHEP10(2015)023 [arXiv:1503.04783 [hep-th]]; T. Rudelius, “Constraints on Axion Inflation from the Weak Gravity Conjecture,” arXiv:1503.0795 [hep-th]; T. Rudelius, “On the Possibility of Large Axion Moduli Spaces,” JCAP 1504, no. 04, 049 (2015) [arXiv:1409.5793 [hep-th]].

[64] A. Borde and A. Vilenkin, “Eternal inflation and the initial singularity,” Phys. Rev. Lett. 72, 3305 (1994) doi:10.1103/PhysRevLett.72.3305 [gr-qc/9312022].

[65] R. H. Brandenberger, “Inflationary cosmology: Progress and problems,” hep-ph/9910410.

[66] J. Martin and R. H. Brandenberger, “The TransPlanckian problem of inflationary cosmology,” Phys. Rev. D 63, 123501 (2001) doi:10.1103/PhysRevD.63.123501 [hep-th/0005209].

[67] M. Gasperini and G. Veneziano, “Pre - big bang in string cosmology,” Astropart. Phys. 1, 317 (1993) doi:10.1016/0927-6505(93)90017-8 [hep-th/9211021].

[68] J. Khoury, B. A. Ovrut, P. J. Steinhardt and N. Turok, “The Ekpyrotic universe: Colliding branes and the origin of the hot big bang,” Phys. Rev. D 64, 123522 (2001) [hep-th/0103239].

[69] F. Finelli and R. Brandenberger, “On the generation of a scale-invariant spectrum of adiabatic fluctuations in cosmological models with a contracting phase,” Phys. Rev. D 65, 103522 (2002) [arXiv:hep-th/0112249].

[70] R. H. Brandenberger and C. Vafa, “Superstrings In The Early Universe,” Nucl. Phys. B 316, 391 (1989); A. Nayeri, R. H. Brandenberger and C. Vafa, “Producing a scale-invariant spectrum of perturbations in a Hagedorn phase of string cosmology,” Phys. Rev. Lett. 97, 021302 (2006) [arXiv:hep-th/0511140]; R. H. Brandenberger, A. Nayeri, S. P. Patil and C. Vafa, “String gas cosmology and structure formation,” Int. J. Mod. Phys. A 22, 3621 (2007) [hep-th/0608121].

[71] Y. F. Cai, R. Brandenberger and P. Peter, “Anisotropy in a Nonsingular Bounce,” Class. Quant. Grav. 30, 075019 (2013) doi:10.1088/0264-9381/30/7/075019 [arXiv:1301.4703 [gr-qc]].

[72] S. Gratton, J. Khoury, P. J. Steinhardt and N. Turok, “Conditions for generating scale-invariant density perturbations,” Phys. Rev. D 69, 103505 (2004) doi:10.1103/PhysRevD.69.103505 [astro-ph/0301395].