Numerical modeling of the strain of elastic rubber elements

E N Moskvichev¹, A V Porokhin² and I V Shcherbakov²

¹ Tomsk State University, 36 Lenin Ave., Tomsk 634050, Russia
² JSC «Scientific-Production Centre «Polyus», 56 Kirova St., Tomsk 634050, Russia

E-mail: porokhin93@mail.ru

Abstract. A comparative analysis of the results of experimental investigation of mechanical behavior of the rubber sample during biaxial compression testing and numerical simulation results obtained by the finite element method was carried out to determine the correctness of the model applied in the engineering calculations of elastic structural elements made of the rubber. The governing equation represents the five-parameter Mooney-Rivlin model with the constants determined from experimental data. The investigation results showed that these constants reliably describe the mechanical behavior of the material under consideration. The divergence of experimental and numerical results does not exceed 15%.

1. Introduction

An increase in the role of finite element modeling in the engineering calculations is provided by the rapid development of computing techniques and software products. Nowadays, the use of such technologies makes it possible to reduce considerably the time and efforts required to determine the operational parameters of the designed products and to identify design flaws, in particular, the stress-strain state of the products made of hyperelastic materials, and to make the necessary changes at the design stage [1 – 3]. The specific feature of hyperelastic materials is the ability to preserve elastic properties at high strain. Many polymers [4], in particular, the rubber, can serve as an example of such materials whose behavior model under mechanical impact cannot be described by Hooke’s law. The reliable description of their mechanical behavior at high strain is the main purpose of the engineering predictions of strength and durability of the important devices and their elements [5].

Currently, many software products are developed to solve such problems. User can achieve the required convergence of the calculation results using a set of various input parameters inherent in a particular material model. The examples of such problem solutions are described in many literature sources, for example, [6 – 9]. Since the hyperelastic models of polymers are very popular, their description and application instructions are well known [6]. However, there is lack of the studies devoted to the description of nonlinear hyperelastic behavior of the selected material class. Therefore, the investigation in this field is very urgent.

In this work, a nontrivial approach with the Mooney-Rivlin model applied as a governing equation is used and it is accompanied by experimental study of mechanical behavior of the rubber sample during biaxial compression testing. The procedure realized by the authors is similar to that in [8] where the stress-strain state of the elastic element of vibration isolator is examined in the NX NASTRAN software environment and the results are compared with the corresponding experimental data.
The purpose of this study is to develop the physical and mathematical model and to select the governing equation that reliably describes the behavior of the rubber, which is used in the production of elastic elements of vibration isolators, at strain conditions.

2. Experimental procedure
The standard samples of type \( B \) are used as an object of research (figure 1, where \( 1 \) – rubber cylinder, \( 2 \) – steel plates glued to the ends of cylinder). The cylindrical samples have been made by curing the rubber IRP-1378-1 TR 38005924-2002 whose operating temperature under the long exposure ranges from -60 to +50 °C and the hardness is of 40 - 50 Shore units (A). The vibration dampers made of such rubber have passed all of the tests being part of the shock absorbers of ARMOO \([10]\) and ARMOO-M series \([11]\). The plates made of the structural steel have been glued to the ends of cylindrical samples with the glue 88-SA TR 38 1051760–89. For all samples, the time between curing of the elastic element and its testing is not less than 24 hours and more than 1 month.

The tests were carried out using the method \( C \) according to GOST ISO 7743-2013, which established the methods for determining the mechanical properties of the rubbers and thermoplastic elastomers during compression. The stress-strain curves for the considered material were obtained as a result of biaxial compression of the five prepared samples using the INSTRON 4483 unit equipped with the devices for automatic recording of the force versus strain curve with an accuracy corresponding to 1 class. In the tests, the samples were compressed at a rate of 10 mm / min until the strain of 70% was achieved. The control recording of the force and strain values was implemented after the samples had passed three loading-unloading cycles with an interval of 1 minute between them. When calculating the stress caused by exerted force, the sample cross-sectional area was assumed to be constant and equal to the initial one.

3. Statement of the problem for hyperelastic material modeling
The study of the sample model was carried out using the standard formulation of the problem of continuum mechanics including the equilibrium equation, kinematic correlations, and constitutive equations with an application of the ANSYS software. The Mooney-Rivlin approach \([6]\), which describes the nonlinear elastic behavior of a material at high strain, was taken as a basis of the governing correlations. The strain energy density was assumed to be a function of linear combination of two invariants of the Cauchy-Green strain tensor whose general form can be written as:
where \( W \) is the strain energy density (strain energy per unit volume); \( D_m \) are the constants related to the volumetric response; \( J=\text{det}(F) \) is the determinant of the gradient \( F \) of elastic deformations; \( C_{pq} \) are the constants associated with the form change \((C_{00}=0)\); \( \bar{I}_1, \bar{I}_2 \) are the first and second invariants of the deviator deformations.

The Mooney-Rivlin model written in the five-parameter form is as follows:

\[
W = \frac{1}{d} (J-1)^2 + C_{10} (\bar{I}_1 - 3) + C_{01} (\bar{I}_2 - 3) + C_{11} (\bar{I}_1 - 3)(\bar{I}_1 - 3) + C_{20} (\bar{I}_1 - 3)^2 + C_{02} (\bar{I}_2 - 3)^2,
\]

(2)

\[
d = \frac{(1-2\nu)}{C_{10} + C_{01}},
\]

(3)

where \( d \) is the material incompressibility parameter; \( C_{10}, C_{01}, C_{11}, C_{20}, C_{02} \) are the Mooney-Rivlin constants; \( \nu \) is the Poisson’s ratio.

The system of equations was solved using the finite element method in Lagrangian coordinates. The biaxial compression of cylindrical sample corresponding to the implemented experiment was simulated. On the sample lower end surface, the boundary conditions prohibiting the linear displacements along the axes were imposed due to the rigid gluing of the end to the steel plate. As for the upper end, the axial displacements \( u_0 \) whose values were limited to a maximum strain of 70% were assigned. The linear displacements in the directions perpendicular to the cylinder axis were also forbidden. The general form of the boundary conditions according to the scheme in figure 2a can be presented as follows:

\[
u_{x,y,z} |_{s_2} = 0,
\]

(4)

\[
u_{x,z} |_{s_1} = 0; \nu_y |_{s_1} = u_0,
\]

(5)

\[\sigma |_{s_5, s_4, s_3} = 0,
\]

(6)

where \( \nu_{x,y,z} \) are the displacement vector components defined on the corresponding model surfaces, \( \sigma |_{s_5, s_4, s_3} \) are the forming surfaces of cylindrical rubber sample and steel plates.

The finite elements in the form of hexahedrons were used to create the finite-element model of the sample (figure 2b). The grid density was determined from the grid convergence condition.

**Figure 2.** Boundary conditions (a) and finite-element model of the sample (b).
4. Results and discussion
The experimental results of the loading of the five samples show that the strain up to 70% is elastic; the stress-strain curves have an evident nonlinear character (figure 3).

Figure 3. Stress-strain curves obtained using the INSTRON 4483 unit for the five samples.

Based on the values obtained in experiment, the constants for the five-parameter Mooney-Rivlin equation were calculated (Table).

Table. The values of the constants for the Mooney-Rivlin model.

| Constant | C_{10} | C_{01} | C_{11} | C_{20} | C_{02} |
|----------|--------|--------|--------|--------|--------|
| Value, MPa | 1.63   | -1.33  | -1.22  | 1.91   | 0.27   |

The compression testing of the rubber samples is accompanied by the essential changes in their cross-section area. Since the values of the parameters describing the stress-strain state are represented in true coordinates in the ANSYS software, the evaluation of simulation results when comparing with experimental data requires taking into account the change in the area of experimental sample. Therefore, the experimental curves were converted into true values.

Assuming that the flat cross-section hypothesis is true and the stress distribution along the cross-section of rubber sample under compression is uniform, the true stress values can be determined by the formula [12]

\[
\sigma_{\text{tru}} = \sigma_{\text{con}} \frac{1}{1 + 2\varepsilon_1},
\]

where \(\sigma_{\text{tru}}\) are the stress values in true coordinates, \(\sigma_{\text{con}}\) are the stress values in conditional coordinates, \(\varepsilon_1\) is the sample strain.

Based on the experimental data and numerical simulation results, the true stress-strain curves were plotted (figure 4). The confidence intervals denoted on the experimental curve correspond to the permissible statistical error. The maximum discrepancy in the values of confidence interval and simulation results is of 15%.
Figure 4. True stress-strain curves obtained experimentally (1) and numerically (2)

A comparison of the finite-element model (figure 5a) and real sample (figure 5b) under biaxial compression confirms that the mathematical model is correct because it accurately describes the experimental sample behavior, and the stress arising in the test sample is approximately similar to that obtained numerically.

Figure 5. Strain distribution in the sample under biaxial compression at the strain of 48%: mathematical model (a), experimental sample (b).

5. Conclusion
An experiment on compression of the samples made of the rubber IRP-1378-1 was carried out. It showed that the tested material could strain elastically up to 70%, and the strain exhibited a strong nonlinear character.

The physical and mathematical model developed in the ANSYS software environment describes the nonlinear hyperelastic behavior of the material when compressing the standard cylindrical samples. The five-parameter Mooney-Rivlin model was chosen as the governing equation whose constants were determined from experimental data. The comparison of experimental and numerical
results made it possible to determine the maximum discrepancy which is equal to 15%. Consequently, it was stated that the mathematical model accurately described the real strain behavior of nonlinear elastic sample at high strain degree.

The mathematical model presented in this paper can be used in the engineering calculations of elastic elements of vibration isolators to determine the operating parameters at the design stage.

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