Vector Quark Model and $B \to X_s \gamma$ Decay

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Abstract

We study the $B$ meson radiative decay $B \to X_s \gamma$ in the vector quark model. Deviation from the Standard Model arises from the non-unitarity of the charged current KM matrix and related new FCNC interactions. We establish the relation between the non-unitarity of charged current mixing matrix and the mixing among the vector quark and the ordinary quarks. We also make explicitly the close connection between this non-unitarity and the flavor changing neutral currents. The complete calculation including leading logarithmic QCD correction is carefully carried out. Using the most updated data and the NLO theoretical calculation, the branching fraction of the observed $B$ meson radiative decay places a limit on the mixing angles as stringent as that from the process $B \to X \mu \bar{\mu}$.

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1 Introduction

A simple extension of Standard Model (SM) is to enlarge the particle content by adding vector quarks, whose right-handed and left-handed components transform in the same way under the weak $SU(2) \times U(1)$ gauge group. This extension is acceptable because the anomalies generated by the vector quarks cancel automatically and vector quarks can be heavy naturally. Vector quarks also arise in some Grand Unification Theory (GUT). For example, in some superstring theories, the $E_6$ GUT gauge group occurs in four dimensions when we start with $E_8 \times E_8$ in ten dimensions. The fermions are placed in a 27-dimensional representation of $E_6$. In such model, for each generation one would have new fermions including an isosinglet charge $-\frac{1}{3}$ vector quark.

In this article, we discuss the $B$ meson radiative decay in the context of a generic vector quark model and show that the experimental data can be used to constrain the mixing angles. In vector quark models, due to the mixing of vector quarks with ordinary quarks, the Kobayashi-Maskawa (KM) matrix of the charged current interaction is not unitary. The internal flavor independent contributions in the $W$ exchange penguin diagrams no longer cancel among the various internal up-type quarks. In addition, the mixing also generates non-zero tree level FCNC in the currents of $Z$ boson and that of Higgs boson, which in turn gives rise to new penguin diagrams due to neutral boson exchanges. All these contributions will be carefully analyzed in this paper. Leading logarithmic (LL) QCD corrections are also included by using the effective Hamiltonian formalism. The paper is organized as follows: In section 2, we review the charged current interaction and the FCNC interactions in a generic vector quark model. Through the diagonalization of mass matrix, the non-unitarity of KM matrix and the magnitude of the FCNC can both be related to the mixing angles between vector and ordinary quarks. In section 3, various contributions to $B$ meson radiative decays are discussed in the vector quark model. In section 4, we discuss constraints on the mixing angles from the new data on $B$ radiative decays and from other FCNC effects. There are many previous analyses on the same issue. We shall make detailed comparison at appropriate points (mostly in section 3.) of our discussion. Most vector quark models in the literature are more complicated than the one we considered here.

2 Vector Quark Model

We consider the model in which the gauge structure of SM remains while one charge $-\frac{1}{3}$ and one charge $\frac{2}{3}$ isosinglet vector quarks are introduced. Denote the charge $-\frac{1}{3}$ vector quark as
$D$ and the charge $\frac{2}{3}$ vector quark as $U$. Large Dirac masses of vector quarks, invariant under $SU(2)_L$, naturally arise:

$$M_U(U_L U_R + U_R U_L) + M_D(D_L D_R + D_R D_L) \tag{1}$$

All the other Dirac masses can only arise from $SU(2)_L$ symmetry breaking effects. Assume that the weak $SU(2)$ gauge symmetry breaking sector is an isodoublet scalar Higgs field $\phi$, denoted as

$$\phi \equiv \left( \begin{array}{c} \phi^+ \\ \phi^0 \end{array} \right) = \left( \begin{array}{c} \frac{1}{\sqrt{2}}(v + h^0) \end{array} \right) \tag{2}$$

We can express the neutral field $h$ in terms of real components:

$$h^0 = H + i\chi. \tag{3}$$

The conjugate of $\phi$ is defined as

$$\overline{\phi} \equiv \left( \begin{array}{c} \phi^{0*} \\ -\phi^- \end{array} \right) = \left( \begin{array}{c} \frac{1}{\sqrt{2}}(v + h^{0*}) \\ -\phi^- \end{array} \right) \tag{4}$$

Masses for ordinary quarks arise from gauge invariant Yukawa couplings:

$$-f_{ij}^d \bar{\psi}_i^d d_R^j \phi - f_{ij}^u \bar{\psi}_i^u u_R^j \tilde{\phi} - f_{ij}^{i*} \phi^i \bar{d}_R^i \psi_L^i - f_{ij}^{i*} \tilde{\phi}^i \bar{u}_R^i \psi_L^i \tag{5}$$

In addition, gauge invariant Yukawa couplings between vector quarks and ordinary quarks are possible, which give rise to mixing between quarks of the same charge. For the model we are considering, these are:

$$-f_{ij}^d \bar{\psi}_L^i D_R^j \phi - f_{ij}^u \bar{\psi}_L^i U_R^j \tilde{\phi} - f_{ij}^{i*} \phi^i \bar{D}_R^i \psi_L^i - f_{ij}^{i*} \tilde{\phi}^i \bar{U}_R^i \psi_L^i \tag{6}$$

In general, $U$ will mix with the up-type quarks and $D$ with down-type quarks. It is thus convenient to put mixing quarks into a four component column matrix:

$$(u_{L,R})_\alpha = \left[ \begin{array}{c} u_{L,R} \\ c_{L,R} \\ t_{L,R} \\ U_{L,R} \end{array} \right], \quad (d_{L,R})_\alpha = \left[ \begin{array}{c} d_{L,R} \\ s_{L,R} \\ b_{L,R} \\ D_{L,R} \end{array} \right] \tag{7}$$

where $\alpha = 1, 2, 3, 4$. All the Dirac mass terms can then be collected into a matrix form:

$$\bar{d}_L^o \mathcal{M}_d d_R' + \bar{d}_R^o \mathcal{M}_d^d d_L' \quad \text{and} \quad \bar{u}_L^o \mathcal{M}_u u_R' + \bar{u}_R^o \mathcal{M}_u^u u_L'.$$
In this article, we use fields with prime to denote the weak eigenstates and those without prime to denote mass eigenstates. $\mathcal{M}_{u,d}$ are $4 \times 4$ mass matrices. Since all the right-handed quarks, including vector quark, are isosinglet, we can use the right-handed chiral transformation to choose the right handed quark basis so that $U_L, D_L$ do not have Yukawa coupling to the ordinary right-handed quarks. In this basis, $\mathcal{M}_d$ and $\mathcal{M}_u$ can be written as

$$\mathcal{M}_d = \begin{pmatrix} \hat{M}_d & \hat{J}_d \\ 0 & M_D \end{pmatrix}, \quad \mathcal{M}_u = \begin{pmatrix} \hat{M}_u & \hat{J}_u \\ 0 & M_U \end{pmatrix}. \quad (9)$$

with

$$\hat{M}_{u,d} = \frac{v}{\sqrt{2}} f_{u,d}, \quad \hat{J}_{u,d} = \frac{v}{\sqrt{2}} f_{i4} \quad (10)$$

$\hat{M}_{u,d}$ (with hats) are the standard $3 \times 3$ mass matrices for ordinary quarks. $\hat{J}_{u,d}$ is the three component column matrix which determines the mixings between ordinary and vector quarks. We assume that the bare masses $M_{U,D}$ are much larger $M_W$. With $M_{U,D}$ factored out, $\mathcal{M}_{d,u}$ can be expressed in terms of small dimensionless parameters $a, b$:

$$\mathcal{M}_d = M_D \begin{pmatrix} \hat{a}_d & \hat{b}_d \\ 0 & 1 \end{pmatrix}, \quad \mathcal{M}_u = M_U \begin{pmatrix} \hat{a}_u & \hat{b}_u \\ 0 & 1 \end{pmatrix}. \quad (11)$$

The mixing matrix $U_{L,R}^{u,d}$ of the left-handed quarks and the corresponding one $U_{L,R}^{u,d}$ for right-handed quarks, defined as,

$$u'_{L,R} = U_{L,R}^{u} u_{L,R}, \quad d'_{L,R} = U_{L,R}^{d} d_{L,R}, \quad (12)$$

are the matrices that diagonalize $\mathcal{M}_{u,d} \mathcal{M}_{u,d}^\dagger$ and $\mathcal{M}_{u,d}^\dagger \mathcal{M}_{u,d}$ respectively. Hence the mass matrices can be expressed as

$$\mathcal{M}_u = U_L^{u,m_u} U_R^{u\dagger} \quad \mathcal{M}_d = U_L^{d,m_d} U_R^{d\dagger} \quad (13)$$

with $m_{u,d}$ the diagonalized mass matrices. The diagonalization can be carried out order by order in perturbation expansion with respect to small numbers $\hat{a}$ and $\hat{b}$. For isosinglet vector quark model, the right-handed quark mixings are significantly smaller. The reason is that $M_D^\dagger M_d$ is composed of elements suppressed by two powers of $a$ or $b$ except for the $(4,4)$ element. As a result, the mixings of $D_R$ with $d_R, s_R, b_R$ are also suppressed by two powers of $a$ or $b$. On the other hand, it can be shown that the mixings between $D_L$ and $b_L, s_L, d_L$ are only of first order in $a$ or $b$. To get leading order results in the perturbation, one can assume that $U_R = I$. For convenience, write $U_L$ as

$$U_L = \begin{pmatrix} \hat{K} & \hat{R} \\ \hat{S}^T & T \end{pmatrix}. \quad (14)$$
where \( \hat{K} \) is a \( 3 \times 3 \) matrix and \( \vec{R}, \vec{S} \) are three component column matrices. To leading order in \( a \) and \( b \), \( T \) is equal to 1. \( K \) equals the unitary matrix that diagonalizes \( \hat{a}\hat{a}^\dagger \). The columns \( \vec{R} \) and \( \vec{S} \), characterizing the mixing, are given by

\[
\vec{R} = \vec{b}, \quad \vec{S} = -\hat{K}\vec{b}.
\]  

(15)

Now we can write down the various electroweak interactions in terms of mass eigenstates. The \( Z \) coupling to the left-handed mass eigenstates are given by

\[
\mathcal{L}_Z = \frac{g}{\cos \theta_W} Z_\mu (J_3^\mu - \sin^2 \theta_W J_\text{em}^\mu),
\]

(16)

\[
J_3^\mu = \bar{u}'_L T^w_3 \gamma^\mu u'_L + \bar{d}'_L T^w_3 \gamma^\mu d'_L = \frac{1}{2} \bar{u}_L (z^u) \gamma^\mu u_L - \frac{1}{2} \bar{d}_L (z^d) \gamma^\mu d_L
\]

(17)

The 4 \( \times \) 4 matrices \( z \) are related to the mixing matrices by

\[
\begin{align*}
(z^u) &= U^u_L a_z U^u_L^\dagger \\
(z^d) &= U^d_L a_z U^d_L^\dagger.
\end{align*}
\]

(18)

with \( a_z \equiv \text{Diag}(1, 1, 1, 0) \). Note that the matrix \( z \) is not diagonal. Flavor Changing Neutral Current (FCNC) is generated by the mixings between ordinary and vector quarks\cite{2,3,4}.

The charged current interaction is given by

\[
\mathcal{L}_W = \frac{g}{\sqrt{2}} (W^-_\mu J^{\mu+} + W^+_\mu J^{\mu-}),
\]

(19)

\[
J^{\mu-} = \bar{u}'_L a_w \gamma^\mu d'_L = \bar{u}_L V \gamma^\mu d_L
\]

(20)

where \( a_w \equiv \text{Diag}(1, 1, 1, a) \) is composed of the Clebsch-Gordon coefficients of the corresponding quarks. For an isosinglet vector quark, \( a = 0 \). The 4 \( \times \) 4 generalized KM matrix \( V \) is given by:

\[
V = U^u_L a_w U^d_L.
\]

(21)

The standard 3 \( \times \) 3 KM matrix \( V_{\text{KM}} \) is the the upper-left submatrix of \( V \). Neither \( V \) nor \( V_{\text{KM}} \) is unitary. Note that the non-unitarity of \( V \) is captured by two matrices

\[
\begin{align*}
(V^\dagger V) &= U^d_L a_w^2 U^d_L \\
(VV^\dagger) &= U^u_L a_w^2 U^u_L.
\end{align*}
\]

(22)

In the model we are considering, these two matrices are identical to \( z^{u,d} \) of the FCNC effects in Eq. 18 since \( a_w^2 \) is equal to \( a_z \). Indeed

\[
V^\dagger V = (z^d), \quad VV^\dagger = (z^u)
\]

(23)
This intimate relation between the non-unitarity of $W$ charge current and the FCNC of $Z$ boson is important for maintaining the gauge invariance of their combined contributions to any physical process.

The off-diagonal elements of these matrices, characterizing the non-unitarity, is closely related to the mixing of ordinary and vector quarks. The off-diagonal elements are of order $a^2$ or $b^2$. To calculate it, in principle, the next-to-leading order expansion of $\hat{K}$, denoted as $\hat{K}_2$, is needed. In fact

$$(V^\dagger V)_{ij} = (\hat{K}_2^d + \hat{K}_2^{d\dagger})_{ij} + a^2(\vec{b}_d)_i(\vec{b}_d)_j^*$$

(24)

Fortunately, by the unitarity of the mixing matrix $U^d$, the combination $\hat{K}_2^d + \hat{K}_2^{d\dagger}$ is equal to $-(\vec{b}_d)(\vec{b}_d)^\dagger$.

$$\hat{K}_2^d + \hat{K}_2^{d\dagger} = -(\vec{b}_d)(\vec{b}_d)^\dagger$$

(25)

Thus the off-diagonal elements can be simplified

$$(V^\dagger V)_{ij} = (-1 + a^2)(\vec{b}_d)_i(\vec{b}_d)_j^*$$

(26)

For isosinglet vector quark, $a = 0$.

The Yukawa couplings between Higgs fields and quarks in weak eigenstate can be written in a matrix form as

$$-\frac{v}{\sqrt{2}} \left( \bar{\psi}'_L a_z d' R \phi + \bar{d}'_R M_d d'_L \psi'_L \phi^\dagger + \bar{\psi}'_L a_z M_u u'_ R \phi + \bar{u}'_R M_u d'_L \psi'_L \phi^\dagger \right)$$

(27)

Note that $\hat{a}_z$ is added to ensure that the left handed isosinglet vector quarks do not participate in the Yukawa couplings. The Yukawa interactions of quark mass eigenstates and unphysical charged Higgs fields $\phi^\pm$ are given by

$$\mathcal{L}_{\phi^\pm} = -\frac{g}{\sqrt{2} M_W} \left[ \bar{u}(m_u V L - V m_d R)d \right] \phi^+ + \frac{g}{\sqrt{2} M_W} \left[ \bar{d}(-m_d V^\dagger L + V^\dagger m_u R)u \right] \phi^-$$

(28)

while those of Higgs boson $H$ and unphysical neutral Higgs field $\chi$ by

$$\mathcal{L}_H = -\frac{g}{2 M_W} \left[ \bar{d}(m_d z^d L + z^d m_d R)d + \bar{u}(m_u z^u L + z^u m_u R)u \right] H$$

(29)

$$\mathcal{L}_\chi = -\frac{ig}{2 M_W} \left[ \bar{d}(-m_d z^d L + z^d m_d R)d + \bar{u}(m_u z^u L - z^u m_u R)u \right] \chi^0$$

(30)

3 \hspace{1em} B Meson Radiative Decay

The $B \rightarrow X_s \gamma$ decay, which already exists via one-loop $W$-exchange diagram in SM, is known to be extremely sensitive to the structure of fundamental interactions at the electroweak scale.
and serve as a good probe of new physics beyond SM because new interaction generically can also give rise to significant contribution at the one-loop level.

The inclusive $B \to X_s \gamma$ decay is especially interesting. In contrast to exclusive decay modes, it is theoretically clean in the sense that no specific low energy hadronic model is needed to describe the decays. As a result of the Heavy Quark Effective Theory (HQET), the inclusive $B$ meson decay width $\Gamma(B \to X_s \gamma)$ can be well approximated by the corresponding $b$ quark decay width $\Gamma(b \to s \gamma)$. The corrections to this approximation are suppressed by $1/m_b^2$ \cite{5} and is estimated to contribute well below 10\% \cite{6, 7}. This numerical limit is supposed to hold even for the recently discovered non-perturbative contributions which are suppressed by $1/m_c^2$ instead of $1/m_b^2$ \cite{9}. In the following, we focus on the dominant quark decay $b \to s \gamma$.

In SM, $b \to s \gamma$ arises at the one loop level from the various $W$ mediated penguin diagrams. The number of diagrams needed to be considered can be reduced by choosing the non-linear $R_\xi$ gauge as in \cite{10}. In this gauge, the tri-linear coupling involving photon, $W$ boson and the unphysical Higgs field $\phi^+$ vanishes. Therefore only four diagrams , as in Fig. 1. contribute.

![Figure 1: Charged boson mediated penguin.](image-url)
The on-shell Feynman amplitude can be written as

$$i\mathcal{M}(b \rightarrow s\gamma) = \frac{\sqrt{2} G_F}{\pi} \frac{e}{4\pi} \sum_i V_{ib} V_{is}^* F_2(x_i) q^\mu q^\nu \bar{s}\sigma_{\mu\nu}(m_R + m_L) b$$

with $x_i \equiv m_i^2/M_W^2$. The sum is over the quarks $u, c$ and $t$. The contributions to $F_2$ from the four diagrams are denoted as $f_1^{W,\phi}$, with the subscript 1 used to denote the diagrams with photon emitted from internal quark and 2 those with photon emitted from $W$ boson. The functions $f$’s are given by

$$f_1^W(x) = e_i \left[ \xi_0(x) - \frac{3}{2} \xi_1(x) + \frac{1}{2} \xi_2(x) \right]$$
$$f_2^W(x) = \xi_{-1}(x) - \frac{3}{2} \xi_0(x) + 2 \xi_1(x) - \frac{1}{2} \xi_2(x)$$
$$f_1^\phi(x) = \frac{1}{4} e_i x [\xi_1(x) + \xi_2(x)]$$
$$f_2^\phi(x) = \frac{1}{4} x [\xi_0(x) - \xi_2(x)]$$

Here the functions $\xi(x)$ are defined as

$$\xi_n(x) \equiv \int_0^1 \frac{z^{n+1}dz}{1 + (x-1)z} = - \ln x + (1-x) + \cdots + \frac{(1-x)^{n+1} - 1}{n+1}$$

and

$$\xi_{-1}(x) \equiv \int_0^1 \frac{dz}{1 + (x-1)z} = - \frac{\ln x}{1-x}$$

$F_2(x)$ is the sum of these functions and is given by

$$F_2(x) = f_1^W(x) + f_2^W(x) + f_1^\phi(x) + f_2^\phi(x) = \frac{8x^3 + 5x^2 - 7x}{24(1-x)^3} - \frac{x^2(2 - 3x)}{4(1-x)^4} \ln x + \frac{23}{36}$$

For light quarks such as $u$ and $c$, with $x_i \rightarrow 0$, the first two terms on the right hand side are small and can be ignored. $F_2(x_{u,c})$ is dominated by the $x$ independent term $\frac{23}{36}$. However these mass-independent terms get canceled among the up-type quarks due to the unitarity of KM matrix in SM

$$\sum_i V_{ib} V_{is}^* = 0$$

After the cancelation, the remaining contributions are essentially from penguins with internal $t$ quark.

It is convenient to discuss weak decays using the effective Hamiltonian formalism [11, 12], which is crucial for incorporating the QCD corrections to be discussed later. In this formalism, the $W$ and $Z$ bosons are integrated out at matching boundary $M_W$. Their effects
are compensated by non-renormalizable effective Hamiltonian operators. The important dim-6 operators relevant for \( b \to s\gamma \) include 12 operators

\[
H_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{12} C_i(\mu) O_i ,
\]

\[
\begin{align*}
Q_1 &= (\bar{c}_\alpha b_\beta)_{V-A} (\bar{s}_\beta c_\alpha)_{V-A} , \\
Q_2 &= (\bar{c}_\alpha b_\alpha)_{V-A} (\bar{s}_\beta c_\beta)_{V-A} \\
Q_3 &= (\bar{s}_\alpha b_\alpha)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V-A} \\
Q_4 &= (\bar{s}_\alpha b_\beta)_{V-A} \sum_\alpha (\bar{q}_\beta q_\alpha)_{V-A} \\
Q_5 &= (\bar{s}_\alpha b_\alpha)_{V-A} \sum_q (\bar{q}_\beta q_\beta)_{V+A} \\
Q_6 &= (\bar{s}_\alpha b_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V+A} \\
Q_7 &= \frac{3}{2} (\bar{s}_\alpha b_\alpha)_{V-A} \sum_q e_q (\bar{q}_\beta q_\beta)_{V+A} \\
Q_8 &= \frac{3}{2} (\bar{s}_\alpha b_\beta)_{V-A} \sum_q e_q (\bar{q}_\beta q_\alpha)_{V+A} \\
Q_9 &= \frac{3}{2} (\bar{s}_\alpha b_\alpha)_{V-A} \sum_q e_q (\bar{q}_\beta q_\beta)_{V-A} \\
Q_{10} &= \frac{3}{2} (\bar{s}_\alpha b_\beta)_{V-A} \sum_q e_q (\bar{q}_\beta q_\alpha)_{V-A} \\
Q_\gamma &= \frac{e}{8\pi^2} \bar{s}_\alpha \sigma^{\mu\nu} [m_b(1 + \gamma_5) + m_s(1 - \gamma_5)] b_\alpha F_{\mu\nu} \\
Q_G &= \frac{g_s}{8\pi^2} \bar{s}_\alpha \sigma^{\mu\nu} [m_b(1 + \gamma_5) + m_s(1 - \gamma_5)] (T^A_{\alpha\beta}) b_\alpha G^A_{\mu\nu}.
\end{align*}
\]

In Standard Model, the electroweak penguin operators \( Q_7, \ldots, Q_{10} \) are not necessary for a leading order calculation in \( O(\alpha) \). However, we will show later that in the vector quark model, FCNC effects exist as a linear combination of \( Q_7, \ldots, Q_{10} \). This effect will mix with \( Q_\gamma \) through RG evolution.

The Wilson coefficients \( C_i \) at \( \mu = M_W \) are determined by the matching conditions when \( W, Z \) bosons and \( t \) quark are integrated out. To the zeroth order of \( \alpha_s \) and \( \alpha \), the only non-vanishing Wilson coefficients at \( \mu = M_W \) for the above set are \( C_2, C_\gamma, C_G \). \( C_2 \) is given by

\[
C_2(M_W) = -V_{cs}^* V_{cb}/V_{ts}^* V_{tb}.
\]

It is equal to one if the KM matrix is unitary and \( V_{us}^* V_{ub} \) is ignored. \( C_\gamma \) at the scale \( M_W \) is
given by the earlier penguin calculations
\[ C_{\gamma}^{\text{SM}}(M_W) = \frac{1}{V_{ts}^*V_{tb}} \sum_i V_{tb}^*V_{ts}^*F_2(x_i) = -\frac{1}{2}D'_0(x_t) \simeq -0.193 \quad . \quad (43) \]
The numerical value is given when \( m_t = 170 \) GeV. Here the function \( D'_0 \) is defined as \[ D'_0(x) \equiv -\frac{8x^3 + 5x^2 - 7x}{12(1-x)^3} + \frac{x^2(2-3x)}{2(1-x)^4} \ln x. \quad (44) \]

\( C_{\gamma} \) retains only the mass-dependent contribution from the penguin diagram with internal t quark. The mass-independent terms get cancelled, due to unitarity, among the three internal up type quarks. The mass-dependent contributions from the penguin diagrams with internal \( u \) and \( c \) quarks (they are small anyway) appear both in the high energy and low energy theories and get canceled in the matching procedure. Similarly, in SM the \( b \to s\gamma \) transition arises from \( W \) exchange penguin diagrams which induce \( Q_G \). Since the gluons do not couple to \( W \) bosons, the gluonic \( W \) boson penguin consists only of two diagrams, which are given by \( f_1^W,\phi \) with \( Q \) replaced by one. With the mass-independent contribution canceled, the Wilson coefficient \( C_G \) can be written as
\[ C_G^{\text{SM}}(M_W) = -\frac{1}{2}E'_0(x_t) \simeq 0.096 \quad , \quad (45) \]
The function \( E'_0 \) is defined as \[ E'_0(x) \equiv -\frac{x(x^2 - 5x - 2)}{4(1-x)^3} + \frac{3}{2} \frac{x^2}{(1-x)^4} \ln x. \quad (46) \]
It is well known that short distance QCD correction is important for \( b \to s\gamma \) decay and actually enhances the decay rate by more than a factor of two. These QCD corrections can be attributed to logarithms of the form \( \alpha_s^n(m_b) \log^m(m_b/M_W) \). The Leading Logarithmic Approximation (LLA) resums the LL series \((m = n)\). Working to next-to-leading-log (NLL) means that we also resum all the terms of the form \( \alpha_s(m_b)\alpha_s^n(m_b) \log^m(m_b/M_W) \). The QCD corrections can be incorporated simply by running the renormalization scale from the matching scale \( \mu = M_W \) down to \( m_b \) and then calculate the Feynman amplitude at the scale \( m_b \).

The anomalous dimensions for the RG running have been found to be scheme dependent even to LL order, depending on how \( \gamma_5 \) is defined in the dimensional regularization scheme. It has also been noticed \[ \leftarrow \text{Oct}13 \] that the one-loop matrix elements of \( Q_5, Q_6 \) for \( b \to s\gamma \) are also regularization scheme dependent. The matrix elements of \( Q_{5,6} \) vanish in any four dimensional regularization scheme and in the \( \text{t Hooft-Veltman} \) (HV) scheme but are non-zero in the Naive dimension regularization (NDR) scheme. This dependence will exactly cancel.
the scheme-dependence in the anomalous dimensions and render a scheme-independent prediction. We refer to [11, 15] for a review and details. In following, we choose to use the HV scheme.

In the HV scheme, only $Q_\gamma$ has a non-vanishing matrix element between $b$ and $s\gamma$, to leading order in $\alpha_s(m_b)$. Thus we only need $C_\gamma(m_b)$ to calculate the LLA of $b \rightarrow s\gamma$ decay width. For $m_t = 170$ GeV, $m_b = 5$ GeV, $\alpha_s(5) = 0.117$ and in the HV scheme, $C_\gamma(m_b)$ is related to the non-zero Wilson coefficients at $M_W$ by [11, 14, 15]

$$C_\gamma(m_b) = 0.698 C_\gamma(M_W) + 0.086 C_G(M_W) - 0.156 C_2(M_W).$$

The $b \rightarrow s\gamma$ amplitude is given by

$$\mathcal{M}(b \rightarrow s\gamma) = -V_{tb}V_{ts}^* G_F \sqrt{2} C_\gamma(m_b) \langle Q_\gamma \rangle_{\text{tree}}$$

To avoid the uncertainty in $m_b$, it is customary to calculate the ratio $R$ between the radiative decay and the dominant semileptonic decay. The ratio $R$ is given, to LLA, by [14]

$$R \equiv \frac{\Gamma(b \rightarrow s\gamma)}{\Gamma(b \rightarrow ce\bar{\nu}_e)} = \frac{1}{|V_{cb}|^2} \frac{6\alpha}{\pi g(z)} |V_{ts}^* V_{tb} C_\gamma(m_b)|^2.$$  (48)

Here the function $g(z)$ of $z = m_c/m_b$ is defined as

$$g(z) = 1 - 8z^2 + 8z^6 - z^8 - 24z^4 \ln z.$$  (49)

In the vector quark model, deviations from SM result come from various sources: (1) charged current KM matrix non-unitarity, (2) Flavor Changing Neutral Current (FCNC) effects in neutral boson mediated penguin diagrams, and (3) the $W$ penguin with internal heavy $U$ vector quark. Since the last one can be incorporated quite straight-forwardly, we do not elaborate on this contribution which will not be relevant for models without the $U$ quark.

We concentrate on the first two contributions, which have been discussed in Refs.[16, 17, 18]. Here we make a more careful and complete analysis which supplements or corrects these earlier analyses. Refs.[16] have calculated effects due to non-unitarity of the KM matrix and effects due to the $Z$ mediated penguin in the Feynman gauge, however, their analysis did not include the FCNC contribution from the unphysical neutral Higgs boson, which is necessary for gauge invariance. The Higgs boson mediated penguins were also ignored. On the other

\footnote{It is customary in the literature to introduce the scheme independent "effective coefficients" for $Q_\gamma, Q_G$. These coefficients are defined so that $Q_\gamma, Q_G$ are the only operators with non-zero one loop matrix element for the process $b \rightarrow s\gamma(g)$. The "effective coefficients" are hence identical to the original Wilson coefficients in HV scheme.}
hand, Ref. [17], while taking the unphysical Higgs boson into account, did not consider effects
due to non-unitarity of the KM matrix, which gives the most important contribution. None
of the above treatments, except Ref. [18], included QCD corrections.

Our strategy is first to integrate out the vector quark together with $W$ and $Z$ bosons at
the scale $M_W$. As shown above, the KM matrix is not unitary in the presence of an isosinglet
vector quark. Therefore the mass-independent contributions in Eq. (38) from the magnetic
penguin diagrams with various up-type quarks no longer cancel. This contribution is related
to the short distance part of loop integration, i.e. when the loop momenta are large so that
the quark mass which appears in the propagator can be ignored. In the formalism of the
low energy effective Hamiltonian, it can be shown that these mass independent contributions
never arise if the theory is renormalized using DRMS or similar schemes. By dimensional
analysis, it is clear that the corresponding diagrams calculated using effective Hamiltonians
are always proportional to the square of the loop quark mass. When we match the two
calculations at the $M_W$ scale, the mass-independent contributions must be compensated by
new terms in Wilson coefficients. This is consistent with the notion that the effective field
theory formalism separates the short distance physics encoded in Wilson coefficients from
the long distance physics parameterized by matrix elements of the effective Hamiltonian.
Such separation enables us to calculate effects of new physics by simply calculating Wilson
coefficients perturbatively at the matching boundary. The matching results serve as initial
conditions when Wilson coefficients run to a lower scale by renormalization group. Since the
vector quarks have been integrated out, the anomalous dimensions are not affected by the
new physics.

Following this procedure, we calculate the extra contributions to the Wilson coefficient
$C_\gamma$ from non-unitarity:

$$\frac{(V^\dagger V)_{23}}{V_{ts}^*V_{tb}^*} \frac{23}{36} = \frac{\delta}{V_{ts}^*V_{tb}^*} \frac{23}{36}. \hspace{1cm} (50)$$

The parameter $\delta$, one of the off-diagonal elements of the matrix $V^\dagger V$, characterizes the
non-unitarity:

$$\delta = (V^\dagger V)_{23} = z_{sb} \hspace{1cm} (51)$$

The $b \to s\gamma$ transitions also arise from FCNC $Z$ boson and Higgs boson mediated penguin
diagrams as in Fig. 2. The FCNC contribution to $C_\gamma(M_W)$ can be denoted as follows:

$$\frac{z_{sb}}{V_{tb}^*V_{ts}^*} (f_{s,b}^Z + f_{s,b}^H) + \frac{z_{tb}^*z_{sb}^*}{V_{tb}^*V_{ts}^*} (f_{D}^{Z} + f_{D}^{H}) \hspace{1cm} (52)$$

For the sake of gauge invariance, $f_{s,b}^Z$ needs to be considered together with $f_X$. The $Z$ boson
penguins consist of internal charge $-\frac{1}{3}$ quarks. The contribution from internal $i = b, s$ quark,
$f_{s,b}^Z$, is given by $(y_i \equiv m_i^2/M_Z^2)$:

$$
\begin{align*}
    f_b^Z &= -\frac{1}{2} e_d \left\{ \left(-\frac{1}{2} - e_d \sin^2 \theta_W \right) \left[ 2\xi_0(y_b) - 3\xi_1(y_b) + \xi_2(y_b) \right] \\
    &\quad + e_d \sin^2 \theta_W \left[ 4\xi_0(y_b) - 4\xi_1(y_b) \right] \right\} \\
    f_s^Z &= -\frac{1}{2} e_d \left\{ \left(-\frac{1}{2} - e_d \sin^2 \theta_W \right) \left[ 2\xi_0(y_s) - 3\xi_1(y_s) + \xi_2(y_s) \right] \\
    &\quad + \frac{m_s}{m_b} e_d \sin^2 \theta_W \left[ 4\xi_0(y_b) - 4\xi_1(y_b) \right] \right\}
\end{align*}
$$

(53)

(54)

The last term in $f_s^Z$ has a mass insertion in the $s$ quark line. It is suppressed by $m_s/m_b$ and will be ignored. The calculation is similar to that of $f^W$. For a consistent approximation,

the two variables $y_b$ and $y_s$, which are the ratios of $m_b^2$, $m_s^2$ to $M_Z^2$, are also set to zero. Hence

$$
\begin{align*}
    f_b^Z + f_s^Z &\approx -\frac{1}{9} e_d \left\{ \left(-\frac{1}{2} - e_d \sin^2 \theta_W \right) \left[ 4\xi_0(0) - 6\xi_1(0) + 2\xi_2(0) \right] \\
    &\quad + e_d \sin^2 \theta_W \left[ 4\xi_0(0) - 4\xi_1(0) \right] \right\} \\
    &= -\frac{1}{9} - \frac{1}{27} \sin^2 \theta_W \approx -0.12
\end{align*}
$$

(55)

(56)

The $Z$-mediated penguin diagram with internal $D$ quark can also be calculated.

$$
\begin{align*}
    f_D^Z &= \frac{1}{4} e_d \left[ 2\xi_0(y_D) - 3\xi_1(y_D) + \xi_2(y_D) \right] \rightarrow -\frac{5}{72y_D} + O\left(\frac{1}{y_D^4}\right). \\
\end{align*}
$$

(57)

It approaches zero when $y_D \rightarrow \infty$ and thus $f_D^Z$ is negligible in large $y_D$ limit. For a gauge invariant result, the unphysical neutral Higgs $\chi$ mediated penguin needs to be considered together with the $Z$ boson penguin. In the non-linear Feynman gauge we have chosen, the mass of $\chi$ is equal to $M_Z$. The calculation is very similar to the $\phi^\pm$ penguin. For internal $s$, $b$, $D$ quarks, the contributions $f_{s,b,D}^\chi$ are given by

$$
\begin{align*}
    f_i^\chi &= \frac{e_d}{8} y_i \left[ \xi_1(y_i) + \xi_2(y_i) \right] \\
    &= -\frac{e_d}{8} y_i \left[ (2 - y_i) \ln y_i - \frac{5}{6} y_i^2 + 4 y_i^2 - \frac{13}{2} y_i + \frac{10}{3} \right] \frac{1}{(1 - y_i)^4}
\end{align*}
$$

(58)
It is obvious that the light quark contributions are suppressed by the light quark masses and thus negligible. The situation is quite different for the heavy $D$ quark. As an approximation, for $y_D \to \infty$, $f_D^X \to -\frac{5}{144} \sim -0.035$. This contribution, comparable to the $Z$ mediated penguin $f^Z$ from light quarks, has been overlooked in previous calculations\[16\]. Since the quark $D$ may not be much heavier than $Z$ boson, we expand $f_D^X$ in powers of $1/y_D$ and keep also the next leading term.

$$f_D^X \approx -\frac{5}{144} + \frac{1}{36} \frac{1}{y_D} + O\left(\frac{1}{y_D^2}\right). \quad (59)$$

The Higgs boson $H$ mediated penguin is similar to that of unphysical Higgs $\chi$:

$$f_i^H = -\frac{e_d}{8} w_i \left[3 \xi_1(w_i) - \xi_2(w_i)\right]$$

$$= -\frac{e_d}{8} w_i \left[(-2 + 3w_i) \ln w_i + \frac{7}{6} w_i^3 - 6w_i^2 + \frac{15}{2} w_i - \frac{8}{3}\right] \frac{1}{1-w_i} \frac{1}{y_D} \quad (60)$$

where $w_i \equiv m_i^2/M_H^2$. Similar to the $\chi$ penguin, $f_{s,b}^H$ can be ignored since $m_s, m_b \ll m_H$. For $f_D^H$, we again expand it in powers of $1/w_D$ and keep up to the next leading term:

$$f_D^H \approx +\frac{7}{144} - \frac{1}{18} \frac{1}{w_D} + O\left(\frac{1}{w_D^2}\right) \quad (61)$$

The leading term is $+0.048$, again comparable to the $Z$ penguin.

Put together, the Wilson coefficient $C_\gamma(M_W)$ in the vector quark model is given by

$$C_\gamma(M_W) = C_\gamma^{SM}(M_W) + \frac{\delta}{V_{tb}V_{ts}^*} \frac{23}{36} + \frac{z_{sb}}{V_{tb}V_{ts}^*} (f_s^Z + f_s^\chi + f_s^H + f_b^Z + f_b^\chi + f_b^H)$$

$$+ \frac{z_{sb}}{V_{tb}V_{ts}^*} (f_D^Z + f_D^\chi + f_D^H)$$

$$= C_\gamma^{SM}(M_W) + \frac{z_{sb}}{V_{tb}V_{ts}^*} \left(\frac{23}{36} - \frac{9}{27} \sin^2 \theta_W + \frac{5}{72y_D} + \frac{5}{144} - \frac{1}{36} \frac{1}{y_D} - \frac{7}{144} + \frac{1}{18} \frac{1}{w_D}\right)$$

$$\to -0.193 + \frac{z_{sb}}{V_{tb}V_{ts}^*} \times 0.506. \quad (62)$$

Here we have used the unitarity relations $z_{sb}z_{sb}^* = -|U_{44}|^2 z_{sb} \approx -z_{sb}$ to leading order in FCNC due to the unitarity of $U_L^d$ and $\delta = z_{sb}$ from Eq. (51). In the above numerical estimate we took $y_D, w_D$ to infinity.

Similarly the Wilson coefficient of the gluonic magnetic-penguin operator $Q_G$ is modified by the vector quark. In the vector quark model, the mass-independent term will give an extra contribution $\frac{1}{3} \delta$ if the KM matrix is non-unitary[14]. The FCNC neutral boson mediated gluonic magnetic penguin diagrams are identical to those of the photonic magnetic penguin,
except for a trivial replacement of $Q_d$ by color factors, since photon and gluons do not couple to neutral bosons. $C_G(M_W)$ in the vector quark model is given by

$$C_G(M_W) = C_G^{SM}(M_W) + \frac{\delta}{V_{tb}V_{ts}^*} \frac{1}{6} - 3 \frac{z_{sb}}{V_{tb}V_{ts}^*} (f_s^Z + f_s^x + f_s^H + f_b^Z + f_b^x + f_b^H)$$

$$-3 \frac{z_{sb}}{V_{tb}V_{ts}^*} (f_D^Z + f_D^x + f_D^H)$$

$$= C_G^{SM}(M_W) + \frac{z_{sb}}{V_{tb}V_{ts}^*} \left( \frac{1+1+\sin^2\theta_W}{3} - \frac{5}{24y_D} - \frac{5}{48} + \frac{1}{12y_D} + \frac{7}{48} - \frac{1}{6}w_D \right)$$

$$\rightarrow -0.096 + \frac{z_{sb}}{V_{tb}V_{ts}^*} \times 0.733 \ .$$

The above deviation from SM does not include QCD evolution. We can incorporate LL QCD corrections to these deviations in the framework of effective Hamiltonian. The key is that the deviation from Standard Model is a short distance effect at the scale of $M_W$ and $M_Q$. It can be separated into the Wilson coefficients at the matching scale, as we just did. The evolution of Wilson coefficients, which incorporates the LL QCD corrections, is not affected by the short distance physics of vector quark model and all the anomalous dimensions used in SM calculation still valid here. One only needs to use the corrected Wilson coefficients at $\mu = M_W$ and in so doing we resum all the terms of the form $z_{sb}\alpha_s^a(m_b)\log^n(m_b/M_W)$.

One subtlety in the vector quark model is that the quark mixing will generate Flavor Changing Neutral Current that couple to $Z$ boson, which in turn gives rise to $Z$ boson exchange interaction. This interaction is represented by an Effective Hamiltonian which is a linear combination of strong penguin operator $Q_3$ and electroweak penguin operator $Q_7,9$:

$$H_{NC} = 2G_F \sqrt{2} z_{sb} \left( -\frac{1}{2} (\bar{s}_\alpha b_\alpha)_{V - A} \sum_q (t^3 - e_q \sin^2\theta_W) (\bar{q}_\beta q_\beta)_{V \pm A} \right)$$

$$= -G_F \sqrt{2} z_{sb} \left[ -\frac{1}{6} Q_3 - \frac{2}{3} \sin^2\theta_W Q_7 + \frac{2}{3} (1 - \sin^2\theta_W) Q_9 \right]$$

which gives additional non-zero Wilson coefficients:

$$C_3(M_W) = -\frac{z_{sb}}{V_{tb}V_{ts}^*} \frac{1}{6} ,$$

$$C_7(M_W) = -\frac{z_{sb}}{V_{tb}V_{ts}^*} \frac{2}{3} \sin^2\theta_W ,$$

$$C_9(M_W) = \frac{z_{sb}}{V_{tb}V_{ts}^*} \frac{1}{3} (1 - \sin^2\theta_W ) .$$

To LL, the strong penguin and electroweak penguin operators could mixing among themselves and also with $Q_\gamma$ and $Q_G$. This will generate an additional LL QCD correction to
$b \to s\gamma$ decay in the vector quark model. The crucial Wilson coefficient $C_\gamma(m_b)$ obtain additional contributions:

$$C_\gamma(m_b) = 0.698 C_\gamma(M_W) + 0.086 C_G(M_W) - 0.156 C_2(M_W) + 0.143 C_3(M_W) + 0.101 C_7(M_W) - 0.036 C_9(M_W).$$  \hfill (66)

This FCNC LL QCD corrections is about one fifth the FCNC contribution of Z boson mediated penguin. The detail of the RG running calculation is given in the appendix.

The correction to ratio $R$ in the vector quark model, including its LL QCD corrections, is given by

$$\Delta R = \frac{6\alpha}{\pi g(z)} |V_{cb}|^2 \times 0.307 \times \text{Re} [V_{ts}^* V_{tb} z_{sb}] = 0.23 \text{Re} z_{sb}$$

to leading order in $\delta$. In this result, the difference between $V_{ts}^* V_{tb}$ and $-V_{cb}^* V_{cb}$, i.e.

$$V_{cs}^* V_{cb} = z_{sb} - V_{ts}^* V_{tb},$$  \hfill (67)

has been taken into account. We use the value $|V_{ts}^* V_{tb}|^2/|V_{cb}|^2 = 0.95$.

### 4 Constraints

The inclusive $B \to X_s\gamma$ branching ratio has been measured by CLEO with the branching ratio \cite{19, 20}

$$B(B \to X_s\gamma)_{\text{EXP}} = (3.15 \pm 0.54) \times 10^{-4}$$  \hfill (68)

This branching ratio could be used to constrain the mixing in the vector quark model. We calculate the vector quark model deviation to leading logarithmic order, i.e. all the terms of the form $z_{sb}\alpha_n(m_b)\log^n(m_b/M_W)$. The Standard Model prediction to leading logarithmic order is \cite{21}

$$B(B \to X_s\gamma)_{\text{LO}} = (2.93 \pm 0.67) \times 10^{-4}$$  \hfill (69)

The difference between the experimental data and the Standard Model LO prediction,

$$B(B \to X_s\gamma)_{\text{EXP}} - B(B \to X_s\gamma)_{\text{NLO}} = (0.22 \pm 0.86) \times 10^{-4}$$  \hfill (70)

It gives a range of possible vector quark model deviation and hence on $z_{sb}$ (with the input $B(B \to X_c e \nu) = 0.105$):

$$-0.0027 < z_{sb} < 0.0045$$  \hfill (71)

$$\Rightarrow$$ The SM prediction up to next-to-leading order has been calculated in Ref.\cite{7}, with $\alpha_{\text{LO}} = 0.13$. 

16
the result

\[ B(B \to X_s \gamma)_{\text{NLO}} = (3.28 \pm 0.33) \times 10^{-4} \] (72)

Ref. [8] later did a new analysis, which discards all corrections beyond NLO by expanding formulas like Eq. (48) in powers of \( \alpha_s \), and reported a slightly higher result:

\[ B(B \to X_s \gamma)_{\text{NLO}} = (3.60 \pm 0.33) \times 10^{-4} \] (73)

Here we also use these new next-to-leading order SM calculations and the leading order vector quark model correction to constraint \( Z_{sb} \). To be consistent in the estimate of the theoretical errors, however, a full next-to-leading order calculation of the vector quark model matching correction is required. The difference between the experimental data and the Standard Model NLO prediction, with the errors added up directly, is

\[
B(B \to X_s \gamma)_{\text{EXP}} - B(B \to X_s \gamma)_{\text{NLO}} = (-0.13 \pm 0.63) \times 10^{-4} \] [7]
\[
(-0.45 \pm 0.63) \times 10^{-4} \] [8] (74)

It gives a constraint on \( z_{sb} \):

\[-0.0032 < z_{sb} < 0.0021 \] [7]
\[-0.0045 < z_{sb} < 0.0007 \] [8] (75)

The previously strongest bound on \( z_{sb} \) is from \( Z \)-mediated FCNC effect in the mode \( B \to X \mu^+ \mu^- \) [3]:

\[-0.0012 < z_{sb} < 0.0012 \] (76)

Our new bound is as strong as that from FCNC. It shows that even though the vector quarks contribute to the radiative decay rate through one loop, as in SM, the data could still put strong bound.

On the other hand, in models like Ref. [1], operators of different chiralities such as

\[ O'_\gamma = \frac{e}{8\pi^2} \bar{s}_s \sigma^{\mu\nu}[m_b(1 - \gamma_5) + m_s(1 + \gamma_5)]b_\alpha F_{\mu\nu} \]
\[ O'_G = \frac{g_s}{8\pi^2} \bar{s}_s \sigma^{\mu\nu}[m_b(1 - \gamma_5) + m_s(1 + \gamma_5)](T_A^{\alpha\beta})b_\alpha G_{\mu\nu}^{A} \] (77)

occurs via the new interaction. Our study can be extended to these models too. However, the new amplitude for \( b \to s\gamma \) belongs to a different helicity configuration in the final state and it will not interfere with the SM contribution. Consequently, the constraint obtained from \( b \to s\gamma \) in these models is less stringent than that from \( B \to X \mu^+ \mu^- \).
In the upcoming years, much more precise measurements are expected from the upgraded CLEO detector, as well as from the $B$-factories presently under construction at SLAC and KEK. The new experimental result will certainly give us clearer evidence whether the vector quark model is viable.

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**Appendix**

The RG equations for the Wilson coefficients $C_r \equiv (C_1(\mu), \ldots, C_{10}(\mu))$ and $C_\gamma, C_G$ can be written as [15]

\[
\frac{d}{d \ln \mu} C_r(\mu) = \frac{\alpha_s}{4\pi} \hat{\gamma}_r^T C_r(\mu) \\
\frac{d}{d \ln \mu} C_\gamma(\mu) = \frac{\alpha_s}{4\pi} (\beta_\gamma \cdot C_\gamma(\mu) + \gamma_\gamma C_\gamma(\mu) + \gamma_G C_G(\mu)) \\
\frac{d}{d \ln \mu} C_G(\mu) = \frac{\alpha_s}{4\pi} (\beta_G \cdot C_r(\mu) + \gamma_G C_G(\mu))
\]

(78)

The 10 by 10 submatrix $\hat{\gamma}_r$ can be found in [15]. The anomalous dimension matrix entries $\beta_{\gamma,G}^{3-10}$ are extracted from the results of $\beta_{\gamma,G}^{3-6}$ in Ref. [15]. ⇒ In the HV scheme, $\beta_{\gamma,G}$ are \(\Leftarrow_{\text{Oct13}}\) given by:

\[
\beta_\gamma = \begin{pmatrix}
0 \\
-\frac{4}{27} N^2 - \frac{1}{2N} - \frac{2e_d N^2}{2N} \\
-\frac{2f}{27} N^2 - \frac{1}{2N} + \frac{2f N^2}{2N} \\
-\frac{4f}{27} N^2 - \frac{1}{2N} + \frac{2f N^2}{2N} \\
-\frac{2f}{9} e_d N^2 - \frac{1}{2N} + \frac{3f N^2}{2N} \\
-\frac{2f}{9} e_d N^2 - \frac{1}{2N} - \frac{3f N^2}{2N}
\end{pmatrix}, \quad \beta_G = \begin{pmatrix}
3 \\
\frac{11N}{9} - \frac{29}{9N} \\
\frac{22N}{9} - \frac{58}{9N} + 3f \\
6 + \frac{11N f}{9} - \frac{29f}{9N} \\
-2N + \frac{4}{N} - 3f \\
-4 - \frac{16N f}{9} + \frac{25f}{9N} \\
-3N e_d + \frac{6}{N} e_d - \frac{9f}{2} e_d \\
-6e_d - \frac{8N f}{3} e_d + \frac{25f}{6N} e_d \\
\frac{11N}{3} e_d + \frac{29}{3N} e_d + \frac{9f}{2} e_d \\
9e_d + \frac{11N f}{6} e_d - \frac{29f}{6N} e_d
\end{pmatrix}
\]

(79)

Here $u$ and $d$ are the numbers of active up-type quarks and down type quarks respectively, $f$ is the total number of active quark flavor. Between the scales $m_t$ and $M_W$, $u = 2$, $d = 3$, $f = 5$, $\bar{f} \equiv (e_d + e_u u)/e_d = -1$, $\bar{f} \equiv (e_d + e_u u)/e_d = -11/3$. For $SU(3)$ color, $N = 3$.  

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