Observational Constraints on the Spectral Index

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Abstract

We address the possibility of bounding the spectral index $n$ of primordial density fluctuations, using both the cosmic microwave background (cmb) anisotropy, which probes scales $10^3$ to $10^4$ Mpc, and data on galaxies and clusters which probes scales 1 to 100 Mpc. Given $n$, sufficiently accurate large scale data on the cmb anisotropy can determine the normalisation of the primordial spectrum. Then the small scale data are predicted within a given model of structure formation, which we here take as the MDM model determined by the Hubble parameter $H_0$ and the neutrino fraction $\Omega_\nu$. Each piece of small scale data is reduced to a value of $\sigma(R)$ (the linearly evolved rms density contrast with top hat smoothing on scale $R$) which allows data on different scales to be readily compared. As a preliminary application, we normalise the spectrum using the ten degree variance of the COBE data, and then compare the prediction with a limited sample of low energy data, for various values of $n$, $\Omega_\nu$ and $H_0$. With $H_0$ fixed at 50 km sec$^{-1}$ Mpc$^{-1}$, the data constrain the spectral index to the range $0.7 \lesssim n \lesssim 1.2$. If gravitational waves contribute to the cmb anisotropy with relative strength $R = 6(1 - n)$ (as in some models of inflation), the lower limit on $n$ is increased to about 0.85. The uncertainty in $H_0$ widens this band by about 0.1 at either end.
Introduction

A widely explored hypothesis about large scale structure is that it originates as an adiabatic density perturbation, which is generated during inflation as a vacuum fluctuation. If this hypothesis is correct, the spectral index of the perturbation provides a unique window on the nature of the fundamental interactions at the inflationary energy scale, which is almost certainly many orders of magnitude higher than any scale that is directly accessible to either accelerator physics or astrophysics. The reason is that it is determined by the shape of the inflaton potential and the mechanism that ends inflation (Davis et al. 1992; Liddle & Lyth 1992, 1993a, 1993b; Salopek 1992). According to some models $n$ is a few percent less than 1, but there exist models where it is tens of percent below 1, others where it is indistinguishable from 1, and at least one model where it can be ten to twenty percent bigger than 1 (Linde 1991; Liddle & Lyth 1993a; Copeland et al. 1994).

The large angle cmb anisotropy provides a constraint on $n$ which is almost independent of any hypothesis about the dark matter or the value of the Hubble parameter (eg. Wright et al., 1994; Gorski et al., 1994), but the constraint is weak because the data probe only the decade of scales $10^3$ to $10^4$ Mpc. We here address the possibility of a more accurate determination of $n$ by combining the cmb anisotropy with data on galaxies and clusters, which probe the comparatively small scales 1 to 100 Mpc. The idea is to exploit the long lever arm provided by the simultaneous use of the two types of data.

Given $n$, sufficiently accurate large scale data on the cmb anisotropy can determine the normalisation of the primordial spectrum through the Sachs-Wolfe effect. Then the small scale data are predicted within a given model of structure formation, which we here take as the MDM model (Bonometto & Valdarnini, 1984; Fang, Li & Xiang, 1984; Shafi & Stecker 1984; Klypin et al. 1993; Schaefer & Shafi 1993), determined by the Hubble parameter $h$ and the neutrino fraction $\Omega_\nu$.[2] Each piece of small scale data is reduced to a value of $\sigma(R)$ (the linearly evolved rms density contrast with top hat smoothing on scale $R$), allowing data on different scales to be readily compared.

As a preliminary application, we normalise the spectrum using the ten degree variance of the COBE data, and present a limited set of low energy data which then constrain $n$, $\Omega_\nu$, and $h$. We present results with $h = 0.5$, and estimate roughly the effect of the uncertainty in $h$. Finally we look at the effect of including a gravitational wave contribution to the cmb anisotropy, with relative strength $R = 6(1 - n)$ as predicted in in some models of inflation.

A more detailed study including a full investigation of the effect of varying $h$ is in progress (Liddle et al. 1994).

Though differing in significant respects, the present work is not unrelated to early studies of the MDM model. Pogosyan and Starobinsky (1993) have looked at the effect of varying $H_0$ and $\Omega_\nu$ with $n = 1$. In addition, Liddle and Lyth (1993b) and Schaefer and Shafi (1993) have varied $n$ and $\Omega_\nu$ with $h = 0.5$; a comparison with these latter results is made in the Conclusion.

The observational constraints

Several different types of observation constrain the theory, on scales ranging from about 1 Mpc to $10^4$ Mpc. As usually presented the data refer to different quantities, so that one cannot plot them on a single graph to exhibit their scale dependence. To avoid this problem, we here focus on a single quantity $\sigma(R)$, defined as the linearly evolved rms of the density contrast, after smoothing with a top hat filter of radius $R$. As we shall discuss, practically all of the data can be presented in terms of this quantity which makes it preferable to the

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1 As usual $h$ is the Hubble constant in units Mpc km$^{-1}$ sec$^{-1}$. The baryon density may be considered fixed through the nucleosynthesis prediction $\Omega_B h^2 = 0.013 \pm 0.002$ (Walker et al. 1991).
more widely used power spectrum $P(k)$.

In this section we present various observational determinations of $\sigma(R)$. They are compared in Table 1 and Figure 1 with a benchmark theoretical model, taken to be the pure CDM model ($\Omega_{\nu} = 0$), normalised to the $10^0$ COBE data described below and with the canonical parameter choices $n = 1$, $h = 0.5$ and $\Omega_B = 0.05$. Then in the next section we see what parameters are needed to actually fit the data. Throughout we use the transfer functions of Schaefer and Shafi (1993).

Except for the pairwise galaxy velocity, we focus exclusively on data in the linear regime, defined by $\sigma(R, z) \lesssim 1$ where $z$ is the redshift. On a given scale, linear theory is valid as long as $\sigma(R, z) \lesssim 1$, which means that it is valid up to the present epoch on scales $R \gtrsim 10h^{-1}$ Mpc. On these scales, the data themselves can be taken to refer to the linear quantity. Smaller scales require special treatment, as described in Section E below.

A: The large scale cmb anisotropy

To normalise the amplitude we use the rms of the anisotropy observed by COBE, smeared on the ten degree scale which corresponds to a linear scale $10^3$ to $10^4$ Mpc. Its observed value from two years of COBE data (Bennett et al. 1994) is $\Delta T/T = (1.1 \pm 0.1) \times 10^{-5}$, which with a Gaussian window function and a flat spectrum would correspond to an expected quadrupole $Q_2 = 15\mu K$. The formal error is in fact less than the cosmic variance (10% with a weak dependence on $n$), which must be added in quadrature to yield an estimate of the underlying power spectrum amplitude which has an uncertainty of about 13%. Corrections for the non-Gaussian beam profile and incomplete sky coverage raise the central value to $Q_2 = 17.4\mu K$ (Wright et al. 1994a), so we adopt as the equivalent ten degree anisotropy for a Gaussian window the figure

$$\Delta T/T = (1.3 \pm .1) \times 10^{-5}$$  (1)

We note here an important caveat that although the interpretation of the 10 degree variance both observationally and theoretically is rather simple, alternative means of analysing the data (Wright et al. 1994b, Gorski et al. 1994) have recently given higher values for the normalisation which. If a higher value is confirmed it would have an important impact on the constraints that can be obtained, shifting the allowed range of $n$ to be somewhat lower.

B: The distribution of galaxies and galaxy clusters

The number density contrast $\delta_N(x)$ is known fairly well for IRAS galaxies, optical galaxies, radio galaxies and Abell galaxy clusters, out to a distance of several hundred Mpc. Given the biasing hypothesis that we discuss in a moment, one can deduce the mass density contrast $\delta(x)$, and hence the dispersion $\sigma(R)$. Alternatively, one can deduce the correlation function $\xi(R)$. From a number of possibilities, we have chosen to use a recent analysis by Peacock and Dodds (1993), which combines a variety of data sets.

The dispersion and the correlation function are related to the underlying power spectrum $P(k)$ (per unit logarithmic interval of wavenumber $k$) by

$$\sigma^2(R) = \int_0^\infty W^2(kR)P(k)dk/k$$  (2)

$$\xi(R) = \int_0^\infty W(kR)P(k)dk/k$$  (3)

where $W$ is the ‘top hat’ window function, and $\xi(R)$ is taken to be the volume averaged quantity (Peacock & Dodds 1993). As with $\sigma(R)$, we take $\xi(R)$ and $P(k)$ to denote the present linearly evolved quantities.
The biasing hypothesis is that for each type of object

$$\delta_N(x) \simeq b_N\delta(x)$$  \hspace{1cm} (4)

where $b_N$ is a scale independent bias parameter. If linear evolution is valid, the spectrum $\mathcal{P}_N$ observed in redshift space is then related to the linearly evolved spectrum $\mathcal{P}$ of the density contrast in real space by

$$\mathcal{P}_N = b_N^2 \left[ 1 + \frac{2}{3} \frac{1}{b_N} + \frac{1}{5} \frac{1}{b_N^2} \right] \mathcal{P}$$  \hspace{1cm} (5)

Towards the lower end of the linear regime $R \gtrsim 10 h^{-1} \text{Mpc}$ there are significant corrections, both to this formula and to the linear evolution of $\mathcal{P}$. After estimating them, one can deduce the ratios $b_I : b_0 : b_R : b_A$, and (because the corrections are non-linear) one can also determine the overall normalisation, specified say by $b_I$.

Peacock and Dodds estimate $b_I : b_0 : b_R : b_A = 1 : 1.3 : 1.9 : 4.5$ for the ratios, and $b_I = 1.0 \pm 0.2$ for the normalisation. Assuming the central value $b_I = 1.0$ they give an estimate of the present value of the linearly evolved spectrum $\mathcal{P}(k)$ over the range $k/h = 0.01$ to 0.45 $\text{Mpc}^{-1}$. The estimate was obtained from measured values of $\sigma(R)$ or $\xi(R)$ using the prescriptions

$$\sigma(R) = \mathcal{P}^{1/2}(k_R)$$  \hspace{1cm} (6)
$$\xi(R) = \mathcal{P}^{1/2}(\sqrt{2} k_R)$$  \hspace{1cm} (7)

where

$$k_R = \left[ \frac{1}{2} \Gamma \left( \frac{m + 3}{2} \right) \right]^{1/(m+3)} \sqrt{\frac{5}{R}}$$  \hspace{1cm} (8)

and $m \equiv (k/\mathcal{P})(d\mathcal{P}/dk)$ is the effective spectral index evaluated in the benchmark CDM model. These formulae are obtained by taking $m$ constant, and using the approximation

$$W(kR) = \exp(-k^2R^2/10)$$  \hspace{1cm} (9)

which is exact for $kR \ll 1$.

We have used these prescriptions to convert the estimates of $\mathcal{P}(k)$ into estimates of $\sigma(R)$. Note that since the original data consisted of measurements of $\sigma(R)$ and of $\xi(R)$ any error in the prescription will tend to cancel (it would cancel exactly if all of the data consisted of $\sigma(R)$ as opposed to $\xi(R)$). The results are shown in Table 1 and Figure 1. For each point, the inside error bar shows the fractional uncertainty in $b_I \mathcal{P}^{1/2}$, and the full error bar combines this in quadrature with the estimated 20% uncertainty in $b_I$. One has the freedom to move the entire set of points up or down by the same amount (on our logarithmic scale) within the full error bars, or to move each point separately within its own inside error bar.

Although this prescription relating $\sigma(R)$ to $\mathcal{P}(k)$ is adequate on scales $\gtrsim 10 \text{Mpc}$, it is too dependent on the shape of the transfer function to be useful on smaller scales. As we shall see, data on such scales directly constrain $\sigma(R)$, which is our main reason for focusing on that quantity rather than on $\mathcal{P}(k)$ or $\xi(R)$.

### C: The peculiar velocity field

Since the peculiar velocity field $v(x)$ is the gradient of a potential in linear theory, it can be constructed in principle from the radial component observed through the ratio of Doppler
shift to distance (Bertschinger & Dekel 1989). Then in principle one can deduce the density contrast from the equation \( \mathbf{v} = t \mathbf{a} \), which is equivalent to

\[
\nabla \cdot \mathbf{v} = -4\pi G t \rho \delta
\]

(10)

Unlike the density contrast, the peculiar velocity can reasonably be assumed to be the same as that of the underlying matter at least on large scales, which in principle makes it a better probe than the galaxy correlation and dispenses with the biasing hypothesis.

In practice one still needs the hypothesis at present in order to obtain really powerful results, which are obtained by comparing the density field obtained via velocities with that obtained via galaxy surveys. A recent study (Dekel et al. 1993) concludes that at 95% confidence level \( 0.5 < b_I < 1.3 \). This is consistent with the above estimate \( b_I = 1.0 \pm 0.2 \), and suggests that the upper limit of that estimate cannot be increased much. As a result the lower limits on \( \sigma(R) \) provided by the galaxy correlation data should be rather reliable.

Although the peculiar velocity alone does not yet give very powerful results, it is not completely useless. The standard way of utilising it is to calculate the theoretical rms of \( \mathbf{v} \) for a random location, after smoothing over a sphere of radius \( R h^{-1} \) Mpc, and compare with what we observe in the sphere around us. (There are several variants of this procedure, such as averaging the radial component over a sphere.) This method can be used on the scale \( R \simeq 20 h^{-1} \) to \( 60 h^{-1} \) Mpc, and according to Schaefer and Shafi (1993) it gives the estimate shown in Figure 1 when compared with the benchmark value, the uncertainty being dominated by the cosmic variance. The conclusion, shared by many earlier studies, is that there is broad agreement with the galaxy correlation result, but that the uncertainty is much bigger.

Ultimately the aim will be to use Eq. (11) directly. A preliminary study been done by Seljak and Bertschinger (1993) reports \( \sigma(R) = 1.3 \pm 0.3 \) at \( R = 8 h^{-1} \) Mpc. On this scale \( \sigma_{cdm}(R) = 1.22 \) which leads to \( \sigma(R)/\sigma_{cdm}(R) = 1.06 \pm 0.24 \). This is too high to be consistent with the other estimates (Figure 1) and we shall not consider it further.

D: The galaxy cluster number density

The average number density \( n(> M) \) of clusters with mass bigger than \( M \sim 10^{15} M_\odot \) gives information on a scale of order \( 10h^{-1} \) Mpc. Within linear theory one can estimate \( n(> M) \) by considering the density contrast \( \delta_R(x) \), smeared over a sphere of radius \( R \) which encloses mass \( M \). (For a review of this procedure see Liddle and Lyth (1993a).) The well known Press-Schechter estimate starts with the assumption that the matter in regions of space where the linearly evolved quantity \( \delta(R, x) \) exceeds some critical value \( \delta_c \simeq 1.7 \) is bound into objects with mass \( > M \) (the value of \( \delta_c \) is motivated by a spherical collapse model, which gives \( \delta_c = 1.68 \)). The Gaussian distribution gives the fraction of space occupied by such regions, and multiplying it by a more or less unmotivated factor 2 leads to the Press-Schechter estimate for the mass fraction bound into objects with mass bigger than \( M \),

\[
\Omega(> M) = \text{erfc} \left( \frac{\delta_c}{\sqrt{2} \sigma(R)} \right).
\]

(11)

An alternative prescription (Bardeen et al. 1986) is to identify \( n(> M) \) with the number density of the peaks of \( \delta_R(x) \) whose height exceeds \( \delta_c \), which gives a roughly similar result. Yet another method is to run an N-body simulation of the collapse, which again gives roughly similar results, and suggests (Lacey & Cole 1994) that the appropriate value for \( \delta_c \) is within 20% or so of the theoretically motivated 1.7. The equivalent value with Gaussian smearing at a fixed value of \( M \) is \( \delta_c \simeq 1.3 \), as one finds both by direct calculation of \( \sigma(R) \) with the two filters (Liddle & Lyth 1993a), and by N-body simulation (Lacey & Cole 1994, Efstathiou & Rees 1988).
A recent study of the number density of Abell clusters using these methods (White, Efstathiou & Frenk 1993) gives \( \sigma(R) = 0.57 \pm 0.05 \), at \( R = 8/h \) Mpc, which is compared with the benchmark in Figure 1. This estimate is lower than that obtained from the galaxy correlation, but compatible with it in view of the uncertainties.

A different quantity that can be observed is \( n(>v) \) where \( v \) is the velocity dispersion of the constituents (virial velocity). It can be converted into \( n(>M) \) using a spherical collapse model as reviewed for example by Liddle and Lyth (1993a). Using the Press-Schechter estimate and ignoring uncertainties due to the spherical collapse model, several authors have estimated \( \sigma(R) \) by this method, most recently Carlberg et al. (1993) who find \( \sigma(R) = 0.75 \pm 0.15 \) at \( 8h^{-1} \) Mpc, in agreement with the galaxy correlation estimate mentioned earlier.

The estimate just mentioned is actually at redshift \( z \approx 0.3 \), which was allowed for by taking into account the linear evolution \( \sigma \propto (1+z)^{-1} \). In the future high redshift estimates of \( n(>M) \) and \( n(>v) \) for clusters will be very informative, but at present the uncertainties involved are too large to permit very definite conclusions.

**E: The density of high-redshift objects**

Going down in scale from clusters to galaxies, linear evolution is still valid at high redshift even though it fails before the present. As a result one can use the Press-Schechter estimate Eq. (11) or N-body simulations with linear initial conditions to estimate the mass fraction \( \Omega(>M,z) \) and compare it with observation.

One approach is to use the observed number density \( n(>M,z) \) of quasars, together with reasonably astrophysics, to establish a lower bound on \( \Omega(>M,z) \). One such estimate (Haehnelt 1993) is \( \Omega(>10^{13} M_\odot, 4.0) > 1 \times 10^{-7} \), which using the Press-Schechter formula gives \( \sigma(R,4.0) > 0.33(\delta_c/1.7) \) at \( R = 3.3h^{-1} \) Mpc. Taking \( \delta_c = 1.7 \) and the linear evolution \( \sigma \propto (1+z)^{-1} \) appropriate for pure CDM, this gives the bound on \( \sigma(R)/\sigma_{cdm}(R) \) plotted in Figure 1. Allowing some hot dark matter tightens this bound because it gives less growth at early epochs, but the effect is not very big for \( \Omega_{\nu} \leq 0.3 \).

Potentially more restrictive bounds (Subramanian & Padmanabhan 1994; Mo & Miralda-Escude 1994; Kauffmann & Charlot 1994) are provided by damped Lyman alpha systems, which at these redshifts seem to contain a mass fraction comparable to that of present day galaxies. For instance, data presented by Wolfe (1993) indicate that at \( z = 3 \) the baryon mass fraction is bigger than 0.0023. Dividing this by the average baryon fraction \( \approx 0.05 \) for the universe, this translates to \( \Omega(>M,3.0) > 0.046 \). To calculate the corresponding bound on \( \sigma(R)/\sigma_{cdm}(R) \) one needs the mass \( M \) of the systems, which is not well known. Using \( M = 3 \times 10^{11} M_\odot \) corresponding to \( R = 0.5h^{-1} \) Mpc, one finds the bound \( \sigma(R)/\sigma_{cdm}(R) > 0.60(\delta_c/1.7) \), which is shown in Figure 1 for \( \delta_c = 1.7 \). It is not terribly sensitive to \( M \) in the range \( 10^{10} \) to \( 10^{12} M_\odot \).

Although they are potentially of great significance, these small scale constraints should be viewed with caution at present. One problem is that they involve astrophysics as well as direct observation; for example in the case of damped Lyman alpha systems our assumption of a universal baryon fraction is clearly questionable. In addition there is the uncertainty attached to the use of the Press-Schechter formula with the canonical value \( \delta_c = 1.7 \), and also a possible inadequacy on small scales of our adopted transfer function.

**F: Pairwise galaxy velocity dispersion**

The observations presented are the most useful ones pertaining to the linear regime. Additional information can be obtained by going to the non-linear regime and comparing with numerical simulations. The most important quantity to consider is probably the pairwise galaxy velocity dispersion. According to a study of pure CDM by Gelb, Gradwohl and Frie-
Table 1: The data set, which is discussed in detail in the text. The galaxy and cluster correlation points assume a bias parameter $b_I = 1.0$.

Our data set is summarised in Figure 1. Before comparing it with theory, one has to ask whether the different data points are compatible. The only definite discrepancy is the low value coming from the pairwise galaxy velocity, but as already discussed it may be raised by introducing hot dark matter, and the value of $R$ at which it should be applied is also rather uncertain.

Although not actually discrepant, the lower limit on $\sigma(R)/\sigma_{\text{cdm}}(R)$ at $R \sim 1h^{-1}\text{Mpc}$ coming from damped Lyman alpha systems will be puzzling if it turns out to be as high as the one shown in Figure 1. Such a high limit would seem to imply a minimum (or at least an extremely sharp flattening) for $\sigma(R)/\sigma_{\text{cdm}}(R)$ somewhere in the range $1 \lesssim R \lesssim 10h^{-1}\text{Mpc}$, which neither MDM nor any similar fix of the CDM model can provide.

Constraining the spectral index

So far we have compared the data only with the benchmark CDM model. Now we ask what, if any, regime of parameter space provides a fit to the data, and in particular what range of $n$ is allowed. For the reasons stated we ignore the pairwise galaxy velocity point (the lowest point at $8h^{-1}\text{Mpc}$), and for the moment also the Lyman alpha bound (extreme left hand point).

To precisely delineate the allowed regime, we would need a precise prescription as to what constitutes a fit. In the two earlier investigations, different viewpoints were taken in this respect. The first (Liddle & Lyth 1993b) used a data set even more limited than the one that we have exhibited, and simply demanded that the curve pass within the error bars.
of every point. Such an approach cannot be used if there are incompatible error bars, and is dangerous if some error bars are only just compatible. The second (Schaefer & Shafi 1993) considered a relatively full data set, somewhat akin to the set presented in Figure 1 but with many more points coming from galaxy correlations, calculated the weighted mean square difference $\chi^2$ between theory and observations, and drew contours in the $n$-$\Omega_\nu$ plane. Although potentially useful, this procedure too is somewhat problematical given the present state of the data. In particular, it is not clear that values of $\sigma(R)$ or $\xi(R)$ deduced from the galaxy correlation on nearby scales are statistically independent as the $\chi^2$ analysis assumes, even in regard to that part of the error not arising from the uncertainty in the bias parameter (shown as inside error bars in our Figures).

Thus it is difficult, at the present time, to decide how best to formalise the notion of an acceptable fit. Instead of making such a decision, we offer in Figures 1 and 2 some representative curves. On the basis of these, we argue that $n$ must lie within the advertised bands, if the curve is not to pass far outside the error bars of at least one piece of data.

Recall that we are normalising the curves using the COBE $10^6$ variance, which with $n = 1$ is equivalent to an rms quadrupole $Q_2 = (17.4 \pm 2.3) \mu K$. Consider first the lower bound $n \gtrsim 0.7$. Figure 1 shows the prediction with $n = 0.7$, $\Omega_\nu = 0$ and the central value of the normalisation. The prediction is clearly too low, and adding hot dark matter ($\Omega_\nu > 0$) obviously makes things worse. In Figure 2 the effect of raising the normalisation by 1-$\sigma$ is shown. Now the value $n = 0.7$ is seen to be marginally acceptable (excluding the two points already mentioned), though a better fit would clearly ensue if one added some hot dark matter and increased $n$. In this context, note particularly that the galaxy correlation points can be varied randomly only within their inside error bars, implying a significant positive slope (this is the famous result that the canonical CDM model has too much power on small scales). It is clear that a value of $n$ significantly below 0.7 cannot be accommodated.

Next consider the upper bound $n \lesssim 1.2$. In Figure 2 the prediction with $n = 1.2$ is plotted, with the normalisation reduced by 1-$\sigma$. Even with this reduction, the theoretical curve is significantly higher than the data, for any reasonable value of $\Omega_\nu$. (If one takes seriously the cluster abundance point it is clear that even $n = 1.2$ is completely ruled out.)

So far we have discounted the possibility of a significant gravitational wave contribution to the cmb anisotropy. Such a contribution is predicted by some models of inflation (Davis et al. 1992; Liddle & Lyth 1992, 1993a, 1993b; Salopek 1992). Of the models which are well motivated from particle physics, those which give a significant contribution also have $n < 1$, and in them the normalisation of the density perturbation is reduced by a factor $[1 + 6(1 - n)]^{-1/2}$ when gravitational waves are taken into account. In Figure 2, the effect of including this factor is shown for $n = 0.85$ and $\Omega_\nu = 0$, with the COBE normalisation raised by 1-$\sigma$. Its slope is clearly too small, but if the slope is increased by reducing $n$ or by making $\Omega_\nu > 0$ the normalisation will be too low. Thus, $n$ cannot be significantly less than 0.85 with the gravitational waves.

These conclusions are for the case $h = 0.5$, which is the central value in the range $0.4 \lesssim h \lesssim 0.6$ allowed by Hubble's law and the age of the universe. Increasing $h$ by 0.1 is roughly equivalent to reducing $n$ by 0.1 and vice versa (see eg. Liddle and Lyth, 1993), so the uncertainty in $h$ widens the allowed band for $n$ by roughly 0.1.

**Conclusion**

Although it is hampered at the present time by somewhat inadequate data, the simultaneous use of the large angle cmb anisotropy and of data on galaxies and clusters is a potentially very powerful tool for constraining the spectral index of the primordial adiabatic density perturbation. The model dependence introduced by considering the galaxy and cluster data
is likely to be more than compensated by the extra range of scales explored (about four decades as opposed to one decade for the large scale cmb anisotropy alone). Provided that the density perturbation is capable of accounting for both types of data, it may be possible in the foreseeable future to determine $n$ to an accuracy of a few percent, which would allow a unique window on nature of the fundamental interactions responsible for inflation.

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**Figure Caption**

*Figure 1.*
Observation versus theory, described in detail in the text. The theoretical curves are all COBE normalised and given as the ratio with respect to the benchmark CDM model. The observational data points are as follows. Lower limits: damped Lyman alpha systems (left-most point) and quasars. Star: galaxy pairwise velocity dispersion. Open triangle: cluster number density (error bar angled only for visual clarity). Filled triangle: cluster velocity dispersion. Cross: bulk flow in spheres around us. Squares, galaxy and galaxy cluster correlation functions; the inner error bars on the squares are without the uncertainty in $b_I$ (see text for details).

*Figure 2.*
Similar to figure 1, showing some extreme choices of parameters, but with $h$ kept at 0.5. These models are not COBE normalised; instead the COBE normalisation is allowed to shift...
by 1-sigma (13% with the incorporation of cosmic variance) in either direction to improve agreement with the data. Note that the introduction of hot dark matter is insufficient to compensate for tilt to $n = 1.2$, indicating a strong upper limit. For $n$ as low as 0.70, adding hot dark matter only makes things worse. For models with significant gravitational waves, hot dark matter must be introduced to obtain the right shape for the galaxy correlation function, but the cost is excessively reduced short-scale power.
