CORRELATORS IN $\mathcal{N} = 1$ SYM(2+1)

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We present a calculation of the correlator $\langle 0 | T^{++}(r) T^{++}(0) | 0 \rangle$ in $\mathcal{N} = 1$ SYM theory in 2+1 dimensions. In the calculation, we use supersymmetric discrete light-cone quantization (SDLCQ), which preserves the supersymmetry at every step of the calculation. For small and intermediate $r$ the correlator converges rapidly for all couplings. At small $r$ the correlator behaves like $1/r^6$, as expected. At large $r$ the correlator is dominated by the BPS states of the theory. We find a critical coupling where the large-$r$ correlator goes to zero; it grows like the square root of the transverse cutoff.

1 SDLCQ of $\mathcal{N} = 1$ SYM(2+1)

In this note we display the results of numerical calculations of the stress-energy correlator in $\mathcal{N} = 1$ SYM theory in three dimensions. The motivation for this calculation is twofold. Firstly, correlators of gauge-invariant operators, such as the stress-energy tensor, are of particular interest for string/field theory correspondence scenarios because they can be evaluated in both of the (conjectured dual) theories. Secondly, the correlator uses all spectral information, and therefore tests also the wave functions of the states, not only the masses. Additionally, it is technically non-trivial to go in a numerical approach to higher dimensional problems.

The method of SDLCQ has been described elsewhere, so we can be brief here. Following the DLCQ approach, we use the light-cone coordinates

$$x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^1), \quad x^\perp = x^\perp,$$

where $x^+$ plays the role of a time and $x^-$ is a spatial coordinate. In the sequel, the total longitudinal momentum is denoted by $P^+$, $P^-$ is light-cone energy or the Hamiltonian, and $P^\perp$ the total transverse momentum. It was shown in Ref. that DLCQ preserves supersymmetry, if the supercharge $Q^-$ rather than the Hamiltonian $P^-$ is discretized. Supersymmetry is the most important symmetry in this calculation, because it keeps the theory finite even at finite discretization parameter, and we shall follow this approach.

Let us now formulate $\mathcal{N} = 1$ SYM(2+1) within this framework. The action is

$$S = \int d^2 x \int_0^L dx_\perp \text{tr} (-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + i \bar{\Psi} \gamma^\mu D_\mu \Psi),$$

where the spinor has two components $(\psi, \chi)$. We use the light-cone gauge, $A^+ = 0$. We express everything in terms of the physical degrees of freedom, namely $\psi$ and $\phi \equiv A^\perp$, and can derive the light-cone supercharges

$$Q^+ = 2^{1/4} \int dx^- \int_0^L dx_\perp \text{tr} \left[ \psi \partial_- \phi - \phi \partial_- \psi \right],$$

$$Q^- = 2^{3/4} \int dx^- \int_0^L dx_\perp \text{tr} \left[ 2 \partial_- \phi \psi + g_{YM} \left( i[\phi, \partial_- \phi] + 2 \psi \bar{\psi} \right) \frac{1}{\partial_-} \right].$$

*WORK WITH S. PINSKY AND J. HILLER.
which fulfill the supersymmetry algebra
\[ \{ Q^+, Q^- \} = 2\sqrt{2} P^+, \quad \{ Q^-, Q^- \} = 2\sqrt{2} P^-, \quad \{ Q^+, Q^- \} = -4P_\perp. \]

To evaluate the theory on a computer, we need a finite-dimensional Hamiltonian operator. This is achieved by discretizing the theory in the following way. We compactify \( x^- \) on a circle of period \( 2L = 2\pi K/P^+ \), where the harmonic resolution \( K \) is effectively a cutoff in particle number. The longitudinal momenta can therefore take integer values \( n_i = 1, 2, \ldots, K \), in proper units. The transverse direction \( x^\perp \) is compactified on circle of period \( l \), with the transverse cutoff being \( 2\pi T/l \). This cutoff is symmetric, and allows the transverse momenta to take values \( n_\perp = 0, \pm 1, \pm 2, \ldots, \pm T \). It is a fundamentally different cutoff in the sense that it is only a momentum, and not a particle-number cutoff.

Close inspection of the resulting supercharges reveals two exact symmetries, namely transverse parity, which accounts for a 'parity doubling', and a non-degenerate \( S \)-Symmetry, associated with a flip of the color indices.

2 Correlation functions

We want to compute an expression of the form
\[
F(x^+, x^-, x^\perp) = \langle 0 | T^{++}(x^+, x^-, x^\perp) T^{++}(0, 0, 0) | 0 \rangle, \tag{1}
\]
where \( T^{++}(x) \) is a component of the stress-energy tensor, namely the momentum operator
\[
T^{++}(x) = \text{tr} \left( (\partial_\phi)^2 + \frac{1}{2} (i\psi \partial_\phi \psi - i(\partial_\phi \psi)\psi) \right) = T_B^{++}(x) + T_F^{++}(x).
\]

Both the boson and the fermion contributions are two-body operators, so that only two-particle states will contribute to the correlation function.

To evaluate the correlator, Eq. (1), we take the collinear limit, \( x^\perp = 0 \), and insert complete set of states \( |\alpha\rangle \) with energy eigenvalues \( P_\alpha \) into the r.h.s. of Eq. (1). It is straightforward to calculate the correlator, Eq. (1), in the free case and it then shows the expected \( 1/r^6 \) behavior for small distances. The real challenge is the evaluation of the correlator when the complete set of bound states is inserted in Eq. (1). We now have to write
\[
F(x^+, x^-, 0) = \sum_{n,m,s,t} \left( \frac{\pi}{2L^2} \right)^2 \frac{L}{\pi} T^{++}(n, m) e^{-iP^+_\perp x^\perp - iP^+ x^+} \frac{L}{\pi} T^{++}(s, t) | 0 \rangle, \tag{2}
\]
where \( P_\perp \) is Hamiltonian operator. We introduce the convenient notation
\[
\frac{1}{N_\perp} | u \rangle \propto \frac{L}{\pi} T^{++}(n, m) | 0 \rangle.
\]

Inserting a complete set of bound states \( |\alpha\rangle \) with masses \( M_\alpha \), and evaluating the sums over \( K \) and \( N_\perp \), as integrals yields
\[
\frac{1}{\sqrt{-i}} \left( \frac{x^-}{x^+} \right)^2 F(x^+, x^-, 0) = \sum_\alpha \frac{1}{2(2\pi)^{3/2}} \frac{M_\alpha^{3/2}}{\sqrt{r}} K_{9/2}(M_\alpha r) \left[ \frac{|\langle u | \alpha \rangle|^2}{K^3 N_\perp^2} \right]. \tag{3}
\]

The term in square brackets is constructed in such a way that it is free of any unphysical constants. It is calculated numerically, and multiplied by a function including the modified Bessel function \( K_{9/2} \). Collecting powers of \( r \) reveals that the individual terms in the sum of Eq. (3) behave like \( 1/r^3 \) in the small \( r \) limit.
3 Numerical Results

Let us now look at the numerical results, Fig. 1. First we focus on the small distance behavior of the correlator. Here, the correlator converges from below with increasing $T$. We expect a $1/r^6$ behavior from the free particle case, but as we saw in the last section, each individual bound state behaves like $1/r^5$. Therefore a coherent behavior of all the states has to take place. This makes the reproduction of the $1/r^6$ behavior a non-trivial check. It turns out that the theory passes this test, see Fig. 1(a); for $-0.5 \leq \log r \leq 0.5$ the plot of $r^5$ times the correlator falls like $1/r$. At larger $K$, Fig. 1(b), we see the same behavior at smaller $r$ as expected.

Now let us look at the correlation function at large distances. This region is totally determined by massless states. There are actually two types of massless states in the theory. The massless states at vanishing coupling $g = g_{\text{YM}} \sqrt{N_c} / \sqrt{2 \pi^{3/2}} = 0$ are mere reflections of the states of the dimensionally reduced theory. They behave as $g^2 M_{2+1}^2$, where $M_{2+1}^2$ are the masses of the two-dimensional theory. We therefore expect for $g \approx 0$ no dependence of the correlator on the transverse momentum cutoff $T$ at large $r$. This is exactly what we see in Fig. 1(a). The other kind of massless states are exactly massless for all couplings and are BPS states. Since they are guaranteed to be massless by the BPS symmetry, they have to have a complicated dependence on the coupling $g$ through their wavefunction, since we see that the large $r$ behavior of the correlator changes with $g$. In other words, the correlator provides us with information on wave functions of the BPS states.

It turns out that the coupling dependence of the large-$r$ limit of the correlator is very interesting. In Fig. 2 we show the large $r$ limit of the correlator as a function of the coupling for two different longitudinal cutoffs. Surprisingly, the logarithm of the correlator does not change monotonically with $g$, but has singularity at a 'critical' coupling which is a function of $K$ and $T$. If we plot the 'critical' couplings vs. $\sqrt{T}$ we find that coupling is a linear function of $\sqrt{T}$ and the coefficient is largely independent of the cutoff $K$. 
4 Conclusions

In this note we showed the first calculation of the stress-energy correlator from first principles in three-dimensional $\mathcal{N} = 1$ SYM theory. We recover the $1/r^6$ behavior of the free particle correlator, because the individual $1/r^5$ behaviors of the bound states add up coherently. The BPS states show an interesting behavior: although their masses are fixed by the BPS symmetry, their wavefunctions have to depend on the coupling, as can be inferred by the correlator data. Surprisingly, we find a vanishing of the correlator at large distances at a critical coupling $g_{\text{crit}} \propto \sqrt{T}$.

A word on the computer code may be in order. The latest version of the code handles two million states, and uses all the present symmetries, which in turn reduces the size of the matrix to be diagonalized by a factor of eight. We are therefore optimistic that calculations in full 3+1 dimensions are within reach. Another interesting theory in three dimensions is $\mathcal{N} = (8, 8)$ SYM(2+1), which is conjectured to correspond to a string theory of D2-branes, and we are working to generalize our code to tackle the problems associated with a large number of 'flavors'. Our current work should result soon in a complete (low-lying) spectrum of $\mathcal{N} = 1$ SYM(2+1), including all the information in the wavefunctions. These results could then be compared to future lattice results, namely the supersymmetric glueball spectrum.

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