RADIATIVE CORRECTION SCHEMES IN DEEP INELASTIC MUON SCATTERING

B. Badelek\textsuperscript{1}, D. Bardin\textsuperscript{2}, K. Kurek\textsuperscript{3} and C. Scholz\textsuperscript{4}

\textsuperscript{1} Department of Physics, Uppsala University, P.O.Box 530, 751 21 Uppsala, Sweden and Institute of Experimental Physics, Warsaw University, Hoża 69, 00-681 Warsaw, Poland
\textsuperscript{2} Theory Division, CERN, 1211 Geneva 23, Switzerland and Joint Institute for Nuclear Research, 141 980 Dubna, Moscow Region, Russia
\textsuperscript{3} Institute of Physics, Warsaw University Branch, Lipowa 41, 15-424 Białystok, Poland
\textsuperscript{4} Max Planck Institute of Nuclear Physics, P.O.Box 103980, 69 029 Heidelberg, Germany

Abstract

A description and a detailed comparison of the Mo and Tsai and the Dubna radiative correction schemes is presented. Numerical comparisons made in the kinematical region of the NMC high energy deep inelastic electroproduction experiment are discussed. An overall agreement between the two approaches in the region of low $x$ and high $y$, where the radiative corrections are largest, is better than 2%.

1 Introduction

It is well known that information on the nucleon internal structure is contained in the electromagnetic structure functions or in the one photon exchange cross section, $\sigma_{1\gamma}$, for the deep inelastic lepton scattering off nucleons. However, a determination of the one photon cross section from the data, which is a goal of electroproduction (i.e. electron– and muon scattering) experiments, demands excluding contributions from other electroweak processes. These processes account for a large fraction of the measured cross section, especially in the low $x$ and high $y$ region. They cannot be discarded from the measured cross section on the event–by–event basis; the measured differential cross section can instead be multiplied by a correction factor calculated theoretically. This is called a radiative correction procedure.

The first radiative correction scheme was created in the sixties by L.W. Mo and Y.S. Tsai\textsuperscript{1,2,3} (MT scheme) in connection with the early SLAC electron scattering experiments. Another approach, originally formulated for the CERN muon scattering experiment planned...
by the BCDMS Collaboration, was suggested by the Dubna group in the seventies \cite{4} and upgraded later \cite{5} (D scheme). In the analysis of the data from the deep inelastic electroproduction experiments, both schemes were extensively used (cf. e.g. \cite{6}), the MT scheme often in the exact (i.e. no ‘peaking approximation’) and upgraded version. However, the precision of the recent experiments is so high, see e.g. \cite{7}, that also the radiative correction procedure has to be based on more precise theoretical calculations. Therefore understanding similarities and differences in the two radiative correction schemes is of ultimate importance for concluding about consistency of results coming from different experiments.

The goal of this paper is to compare analytically as well as numerically the most upgraded versions of the two approaches. Our experience with the radiative correction procedure in the deep inelastic experiments carried out at CERN by the EMC, NMC and BCDMS Collaborations allows to point out the problems encountered during the application of this procedure in the data analysis. The previous comparisons between the two considered methods were limited to analysing the numerical results of the early versions of the schemes \cite{8} or only dealt with a subset of the radiative processes \cite{9}.

The paper is organized as follows. In Section 2 the deep inelastic kinematics and cross sections are defined. Sections 3 and 4, together with Appendices A and B contain a description of the MT and D schemes respectively. A complete set of formulae is given for each scheme using the original notation. Only minor simplifications and changes are introduced for the sake of clarity. Useful relations between the respective notations are given in the Appendix C. The schemes are then compared in Sections 5 (theoretical ideas) and 6 (numerical results) and finally a summary is given in Section 7.

\section{Basic Definitions and Kinematics of Deep Inelastic Scattering}

As mentioned above, a goal of electroproduction experiments is to extract the differential cross section in the one photon exchange approximation (fig. 1a) from the data. This cross section can be expressed in the following way by the structure functions \(F_1(x, Q^2)\) and \(F_2(x, Q^2)\) of the target:

\[
\frac{d^2\sigma_{1\gamma}(x, Q^2)}{dQ^2dx} = \frac{4\pi\alpha^2}{Q^4} \left[ \left(1 - y - \frac{Mxy}{2E}\right) \frac{F_2(x, Q^2)}{x} + \left(1 - \frac{2m^2}{Q^2}\right)y^2F_1(x, Q^2) \right]. \tag{1}
\]

In this equation \(\alpha\) is the fine structure constant, \(m\) is the electron (muon) mass, \(Q^2 = -q^2\) where \(q^2\) is the square of the four–momentum transfer between the incoming and outgoing lepton, \(x = Q^2/(2M\nu)\) the Bjorken scaling variable, \(M\) is taken as the proton mass, \(E\) and \(\nu\) are the lepton’s incident energy and energy transfer in the proton rest frame and \(y = \nu/E\).

The one photon exchange process described by the eq. (1), is a part of the lowest order (or Born) electroproduction cross section, \(\sigma^B\). The other part of \(\sigma^B\) proceeds from the \(Z^0\) boson exchange. The two contributions cannot be separated experimentally. However in
the present fixed target experiments the involved virtualities (i.e. the $Q^2$ values) are small comparing to the $Z^0$ mass squared and therefore eq.(11) is a good approximation of the Born cross section. The $Z^0$ contribution will be discussed later in more detail.

The differential cross section (1) can also be expressed in terms of structure functions $F_2(x,Q^2)$ and $R(x,Q^2)$:

$$\frac{d^2\sigma_{1\gamma}(x,Q^2)}{dQ^2dx} = \frac{4\pi\alpha^2}{Q^4} F_2 \left[ 1 - y - \frac{Mxy}{2E} + \left( 1 - \frac{2m^2}{Q^2} \right) \frac{y^2(1 + 4M^2x^2/Q^2)}{2(1 + R)} \right],$$

where $R$ is defined as:

$$R(x,Q^2) = \frac{\sigma_L}{\sigma_T} = \frac{(1 + 4M^2x^2/Q^2)F_2}{2xF_1} - 1;$$

$\sigma_L$ and $\sigma_T$ denote the cross sections for the longitudinally and transversally polarised virtual photon respectively.

As mentioned earlier the radiative events account for a large fraction of the measured cross section, $\sigma_{\text{meas}}$. These effects may lead to a wrong interpretation of the measured event kinematics. For example an elastic scattering from the target can be mistaken for a deep inelastic event if it is accompanied by an energetic bremsstrahlung photon not measured in the experiment. The magnitude of the radiative effects in the measured cross section will be characterized by the so called radiative correction factor, $\eta(x,y)$, defined as follows:

$$\eta(x,y) = \frac{\sigma_{1\gamma}}{\sigma_{\text{meas}}}.\quad(4)$$

This factor is used in the data analysis and will also be employed in comparing results of calculations between different radiative correction schemes.

Finally we have to stress that in the most of the deep inelastic experiments only the inclusive measurements are performed. This means that only incident and scattered leptons are measured and kinematics of the reaction is defined by the leptonic observables. However the kinematics of radiative events cannot be defined by leptonic variables only, e.g. a bremsstrahlung photon emission by the lepton results in a measured $Q^2$ different from the actual one. Therefore, when appropriate, we shall make a clear distinction between the hadron– and lepton–defined variables.

3 Mo and Tsai Scheme

In the original MT scheme the following processes were considered to contribute to the measured electron–proton inelastic cross section$^1$: real photon bremsstrahlung from

$^1$The original MT scheme was formulated for the electron scattering; therefore a large effort was put to quantify the effects of the energy loss in the electron passage through the target. We shall neglect them in this paper since in the examples of practical applications we shall deal with muon scattering only.
an initial and final electron (fig. 1b), vertex correction (fig. 1c) and vacuum polarisation
correction (fig. 1d). In the latter one only the electron and muon loops were originally
considered. The proton structure was accounted for through the two unspecified structure
functions. The MT scheme formulation is thus model independent. The mathematical
formulation of the MT is non-covariant.

Corrections depicted by the diagrams in fig. 1 can be divided into two groups: emission
of real photons (fig. 1b) with energy larger than ∆ and those with energy smaller than ∆
(fig. 1b) together with virtual corrections (fig. 1c,d). Contributions from the soft photon
emission and from the vertex correction separately are infrared divergent but the divergences
cancel when the contributions are considered jointly [10]. The parameter ∆ may have a
meaning of the energy resolution or in another words a maximal energy of the emitted
photon which is still not detectable in the experiment. It is often called the ‘infrared cut–off’
parameter. This interpretation implies that ∆ may not be too large. Too small ∆ must also
be avoided since it may cause numerical instabilities in the computation of the soft photon
contribution. The numerical results of the calculations should not depend on ∆.

In the MT approach the measured cross section can be expressed as follows (see Appendix
A for the definitions of variables used in MT publications):

\[
d^2σ_{\text{meas}} / dνdΩ = e^{-δ_R(∆)} F(Q^2) d^2σ_γ / dνdΩ \quad + \quad d^2σ_{\text{tails}} / dνdΩ ,
\]

where

\[
δ_R(∆) = \frac{α}{π} \left( \ln \frac{E_s}{∆} + \ln \frac{E_µ}{∆} \right) \left( \ln \frac{Q^2}{m^2} - 1 \right)
\]

is a residuum of the cancellation of the infrared divergent terms and takes into account
all soft photon emissions in the lowest order of α. It is a well known fact that the infrared
divergencies cancel in each order in α [10] and therefore it is possible to sum up contributions
from all soft photon emissions [11]. The exponential factor, \(e^{-δ_R(∆)}\), in eq.(6) is a result of
this summation. The function \(F(Q^2)\) contains all \(Δ−\)independent terms:

\[
F(Q^2) = 1 + δ_vac^e + δ_vac^µ + δ_{vtx} + δ_s,
\]

where\[2\]

\[
δ_vac^e,µ = \frac{2α}{π} \left[ -\frac{5}{9} + \frac{4m_{e,µ}^2}{3Q^2} + \frac{1}{3} \left( 1 + \frac{4m_{e,µ}^2}{Q^2} \right) \ln \left( \frac{\sqrt{1 + 4m_{e,µ}^2/Q^2} + 1}{\sqrt{1 + 4m_{e,µ}^2/Q^2} - 1} \right) \right]
\]

\[
δ_{vtx} = \frac{2α}{π} \left( -1 + \frac{3}{4} \ln \frac{Q^2}{m^2} \right)
\]

\[
δ_s = \frac{α}{π} \left[ \frac{1}{6} \pi^2 - \Phi \left( \cos^2 \frac{θ}{2} \right) + \Phi \left( \frac{E_µ - E_s}{E_p} \right) + \Phi \left( \frac{E_s - E_µ}{E_s} \right) \right]
\]

\[2\] \(δ_vac^e,µ\) given here is the full formula. The approximation \(2α/π(-5/9+1/3\ln Q^2/m_{e,µ}^2)\) holds for \(Q^2 \gg m_{e,µ}^2\)
(see e.g. eqs 5 and A1 from ref. [1]). For extremely low \(Q^2\) formula (8) converges to \(Q^2/15m_{e,µ}^2\).
and $\Phi$ the Spence function\(^{3}\). The second term in eq. (3), $\sigma_{\text{tails}}$, accounts for contribution from processes where the real photons of energy larger than $\Delta$ are emitted:

$$
\frac{d^2\sigma_{\text{tails}}}{d\nu d\Omega} = \frac{d^2\sigma(\omega > \Delta)}{d\nu d\Omega} = \int_{M_j=M}^{M_j^\text{max}} dM_j \frac{d^2\sigma_{j,r}}{d\nu d\Omega},
$$

where $M$ denotes a target mass, $M_j^\text{max} = \sqrt{M^2 - Q^2 + 2M(\nu - \Delta)}$ and $\sigma_{j,r}$ is given by the formula (A.24) in \(\text{[3]}\) and quoted in Appendix A; $M_j$ is an effective mass of the hadronic final state. The integration over $M_j$ in eq. (11) means that all the final hadronic states contribute to the cross section measured in a kinematical point ($Q^2, \nu$), fig. 2, so called ‘radiative tails’: elastic ($M_j = M$), resonance production ($M_j = M_{\text{res}}$) ... and deep inelastic tails, i.e. tails from the continuum.

Equation (3) does not take into account double– (multi–) photon exchange reactions (fig. 1e) nor any radiative correction from the hadron current (fig. 1f,g). These effects were estimated only for the elastic $e^- p$ interaction \(\text{[2]}\). Application of these calculations to the inelastic interaction is incorrect. Thus these results were discarded in practical applications of the MT scheme. Evaluation of these effects in the inelastic $e^- p$ interactions was not done in this scheme. To do this a model of the proton internal structure is necessary. The best framework presently is the quark parton model (QPM).

In the upgraded version of the MT scheme, used in the analysis of the NMC results \(\text{[12]}\), the $\tau^+\tau^-$ and $q\bar{q}$ loops in the vacuum polarisation \(\text{[10, 13]}\) and the virtual photon $-Z^0$ boson interference \(\text{[6]}\) were also included.

## 4 Dubna Scheme

In the Dubna scheme the calculations of the deep inelastic processes are based on a mixed approach which uses both a model independent and the quark parton model treatment of the radiative corrections. The radiative corrections to the leptonic current (fig. 1b,c) are calculated within the same model independent approach as in the Mo and Tsai scheme except the way of treating the soft bremsstrahlung photons. All other corrections are calculated using the QPM approach.

In the inclusive electroproduction experiments the radiative and non–radiative events cannot be distinguished and therefore the $\sigma_{\text{meas}}$ should not depend on any parameter $\Delta$, a property, which implicitly holds in Mo and Tsai scheme. The D scheme is explicitly $\Delta$ independent as a result of an integration over the whole bremsstrahlung photon phase space. The relevant procedure is described in detail in \(\text{[14]}\).

Applying the QPM and the fact that quarks are point–like objects permit in principle to calculate all QED processes to all orders of $\alpha$. This means that in addition to the diagrams

\(^{3}\)In the original MT formulation \(\text{[8]}\) the $q^2$ symbol was used in eqs \(\text{[5]} - \text{[8]}\). This variable, however, was then redefined to become the measured four momentum transfer, $q^2 = (s - p)^2$, and thus coincides with our definition of $-Q^2$. The latter was thus used here for clarity.
in fig. 1.a-d also the hadron current corrections (fig. 1.e-g) can be evaluated. The diagrams of fig. 1.e-g thus become as in fig. 3a-c. All these processes were taken into account as well as the lowest order weak corrections, not shown here, which can also be calculated in the framework of the quark-parton model. The outline of the radiative corrections calculations in the D scheme is given below; the details are given in refs [14, 15].

The measured cross section is now expressed as follows:

$$\frac{d^2\sigma_{\text{meas}}}{dQ^2dx} = \frac{d^2\sigma^B}{dQ^2dx} \left\{ e^{-\delta_R(x,Q^2)} + \delta^{VR}(x,Q^2) \right\} + \frac{d^2\sigma_{\text{in.tail}}}{dQ^2dx} - \frac{d^2\sigma^{IR}}{dQ^2dx} + \frac{2\pi \alpha^2}{Q^4} \sum_{B=\gamma,t,Z} \sum_{b=t,q} \sum_{Q,Q'} c_b K(B, p) \left[ V(B, p) R^V_B(B) + p A(B, p) R^A_B(B) \right] + \frac{d^2\sigma_{\text{el.tails}}}{dQ^2dx}.$$ (12)

The first two rows of this formula represent the results of the model independent calculations of the radiative corrections to the leptonic current. The third row represents the quark parton model description of the lepton–hadron interactions (fig. 3a) and of the radiative corrections to the hadronic current (fig. 3b,c) as well as certain interference terms. Finally the last term in eq. (12), \(d^2\sigma_{\text{el.tails}}/dQ^2dx\), describes the elastic and resonance radiative ‘tails’.

The \(d^2\sigma^B/dQ^2dx\) cross section in eq. (12) denotes the full Born cross section for the deep inelastic scattering, i.e. the cross section containing both the one photon– and one \(Z^0\) boson exchange contributions:

$$\frac{d^2\sigma^B}{dQ^2dx} = \frac{2\pi \alpha^2 y}{S_x} \sum_{i=1}^{3} A_i(x, Q^2) \frac{1}{Q^4} S^B_i(y, Q^2),$$ (13)

where the functions \(S^B_i(y, Q^2)\) are:

\[
S^B_1(y, Q^2) = Q^2 - 2m^2, \quad S^B_2(y, Q^2) = 2[(1-y)S^2 - M^2Q^2], \quad S^B_3(y, Q^2) = 2Q^2S(2-y). \quad (14)
\]

with \(Q^2 = S_{xy}\) and \(S = 2ME\). The functions \(A_i\) are given in Appendix B.

The \(\delta_R\) in eq. (12) is responsible for those parts of the soft and hard collinear photon emissions which could be resummed to all orders using the covariant exponentiation procedure, [11, 16]. It reads:

$$\delta_R = -\frac{\alpha}{\pi} \left( \ln \frac{Q^2}{m^2} - 1 \right) \ln \frac{y^2(1-x)^2}{(1-yx)(1-y(1-x))}. \quad (15)$$

\(^4\)Additional factor \(\alpha/\pi\) in the third row of eq. (12) is hidden in the definitions of the \(R\) functions.
The $\delta^{VR}$ correction factor in eq. (12) is a remnant of the exponentiation and of the subtraction procedure used to disentangle the infrared divergent terms from the $d^2\sigma_{\text{in.tail}}/dQ^2dx$ cross section [14], see below. It thus contains the vertex correction, fig. 1.c and is given by

$$\delta^{VR} = \delta_{\text{vtx}} - \frac{\alpha}{2\pi} \ln^2 \left( \frac{1 - yx}{1 - y(1-x)} \right) + \Phi \left[ \frac{1 - y}{(1 - yx)(1 - y(1-x))} \right] - \Phi(1).$$

(16)

The $d^2\sigma_{\text{in.tail}}/dQ^2dx$ in eq. (12) describes the inelastic radiative tail for the lepton current correction only:

$$d^2\sigma_{\text{in.tail}}/dQ^2dx = \frac{2\alpha^3y}{S_x} \int \int dQ_h^2 dM_h^2 \sum_{i=1}^{3} A_i(x_h, Q_h^2) \frac{1}{Q_h^2} S_i(y, Q^2, y_h, Q_h^2).$$

(17)

In this formula $M_h$ is the invariant mass of the final hadronic system and the variables bearing a subscript ‘h’ refer to virtual photon–target vertex in contrast to the variables measured in the inclusive electroproduction experiment. For their definitions see Appendix B where also the explicit expressions for the radiator functions $S_i$ are given.

It is a well known fact that the $d^2\sigma_{\text{in.tail}}/dQ^2dx$ cross section is infrared divergent. To regularize it a simple trick (‘fixation procedure’) of adding and subtracting an extra term

$$d^2\sigma_{\text{in.tail}}/dQ^2dx = d^2\sigma^B/dQ^2dx \int \int dQ_h^2 dM_h^2 \mathcal{F}^{IR}(y, Q^2, y_h, Q_h^2),$$

(18)

to $d^2\sigma_{\text{in.tail}}/dQ^2dx$ was employed [11, 14]. In the added term an integration over a full photon phase space was carried out, resulting in the above given expressions for $\delta_R$ and $\delta^{VR}$. The subtracted term appears explicitly in the second row of eq. (12), so that the difference $d^2\sigma_{\text{in.tail}}/dQ^2dx - d^2\sigma^{IR}/dQ^2dx$ in eq. (12) is finite over the full kinematic domain of $Q_h^2$ and $M_h^2$. This method is a key point of the D scheme, making it explicitly $\Delta$ independent. The function $\mathcal{F}^{IR}$ is given in Appendix B.

The third row in the eq. (12) represents the quark parton model calculations and contains clearly visible vector ($\gamma$ and $Z^0$) and axial (only $Z^0$) contributions. Index $B$ runs over the photon exchange ($\gamma$), the $Z^0$ boson exchange ($Z$) and their interference ($I$). Index $b$ stands for the scattering with the single photon emission by the quark ($q$), fig. 3c, and its interference with the photon emission by the lepton ($i$). The double photon exchange (fig. 3a) as well as the vertex corrections on the quark line (fig. 3b) are also hidden there. Both scattering off quarks ($Q$) and antiquarks ($\bar{Q}$) were considered. Coefficients $c_b$ are equal to $Q^2_Q$ or $Q_{\mu}Q_Q$ for the process of bremsstrahlung photon emission from the quark or for the interference term between the photon emission from the lepton and from the quark respectively; $Q_{\mu}$ and $Q_Q$ are the charges of the lepton and of the quark given in the electron charge units. The sign factor $p$ is defined as: $p = p_{\mu}p_Q$ where $p_{\mu,Q} = \pm 1$ for particle (antiparticle). For a detailed form of the coupling strength factors $K(B,p)$, modified vector, $V(B,p)$ and axial, $A(B,p)$ couplings as well as of the functions $R^{VA}_b(B)$ we refer the reader to ref. [13].

---

Footnote: In the original formulation [15] the index $b$ assumes also values 0 and 1 which correspond to the contributions from processes in fig. 1a,b,c. Here they are included in the model independent parts of eq. (12).
The problem of the soft photons’ emission from the quarks can be uniquely solved since the quarks are not observed and thus the quark states are all summed up in the cross section. The infrared divergencies for initial– and final state quarks as well as the quark mass singularities for the final state quarks cancel in each order in \( \alpha^1 \). Mass singularities associated with initial quark are included in the definition of the proton structure function.

The last row in eq. (12), \( d^2\sigma_{\text{el,tails}}/dQ^2 dx \), is in the D scheme treated essentially in the same way as in the MT except that it is formulated in a covariant way. Finally the vacuum polarisation, fig. 1d, was taken into account via the ‘running’ \( \alpha(Q^2) \) which in the \( Q^2 \gg m_f^2 \) approximation (\( m_f \) stands for the lepton and quark effective masses), \( 13 \) is defined as follows:

\[
\alpha(Q^2) = \frac{\alpha}{1 - \frac{1}{2} \sum f c_f Q_f^2 \delta_{\text{vac}}}.
\]

where \( c_f \) and \( Q_f \) are the colour factor and the electric charge of fermions \( f \) \( (c_f = Q_f = 1 \) for leptons); ‘\( f \)’ runs over all leptons and quarks.

We shall close this section with the following remarks: first, although not shown explicitly in eq. (12), the weak loop correction contribution is also present in the D formulation. It is calculated within the QPM framework; details are given in \( 15 \). Second, the \( \mathcal{O}(\alpha^2) \) corrections (\( \alpha^4 \) contributions to the cross section) not shown explicitly, were also implemented. For the elastic radiative tail they were calculated completely in the first paper of ref. \( 4 \), while for the inelastic continuum they were implemented in an approximate way, described in the second paper of ref. \( 4 \).

5 Comparison of the MT and D Schemes

Results of deep inelastic experiments were analysed using either MT or D radiative correction schemes. Therefore it is of ultimate importance to understand the differences and similarities between the two approaches. In this section we shall make a brief summary of theoretical ideas in the two schemes; numerical comparison will be presented in the next section. To facilitate the comparison, the relations between the variables used in the MT and D formulae are given in Appendix C.

The D scheme is formulated in a covariant way and its model dependent part is based on the quark parton model. The covariant formulation means that all formulae are expressed in terms of the Lorentz invariants and are thus independent of the choice of the reference frame. This also means that the D scheme is explicitly independent of the infrared cut–off parameter \( \Delta \), cf. eq. (12). The \( \Delta \)– independence was obtained through a special mathematical procedure.

The MT scheme is not covariantly formulated. It should also be \( \Delta \)–independent which formally means that the derivative over \( \Delta \) of the right hand side of eq. (13) should be equal to zero. The calculations show that this does not hold and that a certain dependence of the
results on $\Delta$ should be observed. For not too small $\Delta$ this dependence is very weak. For very small $\Delta$ (apart of numerical instabilities), eq.(3) depends much stronger on the value of $\Delta$ which in this case should be calculated separately for every kinematical point (i.e. for every $(x,y)$ value) in order to minimise the dependence.

The elastic and resonance tails (i.e. the contribution of the reactions with the elastic and resonance final states) are calculated in the same way in both schemes. The inelastic tails (the contribution of the reactions with the inelastic final states) originating from the leptonic bremsstrahlung are treated differently but the differences are purely mathematical. The inelastic tail in the MT is calculated with the same formula as the elastic one (eq. (11)) where the inelasticity of the process is taken into account through integration over kinematically allowed final state masses. The corresponding structure functions, $W_{1,2}^j$ (see Appendix A), are not specified; in practical applications the electromagnetic structure functions are used. The leptonic inelastic tail in D is also calculated with arbitrary structure functions while tails originating from the hadronic bremsstrahlung were calculated within the quark parton model framework.

The usage of the parton model in the D scheme allows to calculate the hadron current corrections in the inelastic case as well as a double photon exchange process; they are calculated up to $\alpha^3$. The hadron current corrections cannot be calculated in the MT approach and therefore such corrections were not taken into account. The $q\bar{q}$ loop is also naturally present in the vacuum polarisation process in the D scheme while in the MT it was only added later. The same is true for the weak interactions contributions ($Z^0$ exchange and the $\gamma-Z^0$ interference). The weak contributions are however very small as compared to the present experimental resolution.

Finally the $\alpha^4$ lepton current corrections were partially taken into account in the D scheme but not in the MT.

6 Numerical Calculations in the MT and D schemes

A tremendous increase of the accuracy of deep inelastic electroproduction experiments demands a similar increase of the accuracy of the radiative corrections calculations. Therefore the early versions of these calculations have constantly been improved. At the same time comparisons between the two considered schemes were made, [6, 8, 9]. In the analysis of the recent very precise measurements by the NMC [7, 12] both schemes were used in their most upgraded versions and applied for nucleon and nuclear targets. The MT code, named FERRAD35, apart of the processes contained in the eqs (5)–(11) included also the tau lepton– and quark loops and the photon–$Z^0$-boson interference. The tails were treated in an exact way in contrast to the peaking approximation [2, 3] applicable for the electron deep inelastic scattering. A detailed input information (structure functions, form factors, nuclear structure models, etc) was introduced [13]. The D code, named TERAD86, was in principle used in the same version as sketched by the formulae (12)–(19) and employed in the dedicated BCDMS experiment [4], except that the FERRAD35 input information was supplied. In
this section we shall compare the FERRAD35 and TERAD86 results on the radiative correction factors, \( \eta \), as well as on the elastic radiative tails. Comparisons will be done in the kinematical region of the NMC positive muon–proton deep inelastic scattering experiment, i.e. for \( 0.003 < x < 0.9 \) and \( 0.1 < y < 0.9 \) at the incident muon energy 280 GeV. The input information in the calculations was as follows: Gari and Krümpelman proton form factor parametrisation, \[13\], structure function \( F_2 \) as measured and parametrised by the NMC, \[7\], and finally for the \( R(x,Q^2) \) the parametrisation of SLAC was taken for all \( x \) and \( Q^2 > 0.35 \) GeV\(^2\). For smaller \( Q^2 \) the value of \( R \) was assumed to be constant and equal to the value at \( Q^2 = 0.35 \) GeV\(^2\). The proton resonances were neglected. For further details see refs \[12\] and \[18\].

The main numerical problem in the radiative correction programs is an integration of the radiative tails. For example, the integrand of eq. (11) changes by 27 orders of magnitude within the integration interval. It is obvious that this function should be integrated either by a high accuracy routines (e.g. the CERNLIB GAUSS routine demands an accuracy parameter \( \varepsilon < 10^{-12} \)) or by dividing the integration interval into many small sections. It seems however that the best method would be to change the integration variable from \( \cos \theta_k \) to \( \log (-q^2) \) but also in this case high accuracy of the integrating routines is needed. This method was used in TERAD86 while in FERRAD35 dividing the \( \cos \theta_k \) integration interval into many sections was normally employed (logarithmic integration in FERRAD35 was also tried and gave the same results). Due to the subtraction procedure in D the integrands in eqs (17) and (18) are fairly smooth and do not demand any extreme precision of integration.

The dependence of the FERRAD35 results on the parameter \( \Delta \) was carefully studied. The results are presented in fig. 4. In the region of low \( x \) and large \( y \) where the radiative correction factor \( \eta \) is largest, the results are only weakly dependent on \( \Delta \) for \( \Delta > 200 \) MeV. Results given below were obtained with \( \Delta = 280 \) MeV.

Comparison of the FERRAD35 and TERAD86 results is presented in figs 5–6. Radiative corrections are very large, exceeding 50% at low \( x \) and high \( y \), cf. fig. 5. In that region the agreement between the results of the two schemes is better than 2%, cf. fig.6 (closed symbols). However the \( \tau \bar{\tau} \) and \( q\bar{q} \) loop contributions in the vacuum polarisation process (fig.1.d), absent in the original version of the MT scheme and included in FERRAD35, give up to 2% contribution to the radiative correction factor in most of the kinematic region (fig.6, open symbols denote results of calculations \textit{without} those contributions). Fluctuations visible in the high \( x \) part of the curves in fig.6 come from numerical instabilities of FERRAD35 in that region.

7 Summary

The two existing schemes of radiative correction procedure, the Mo and Tsai and the Dubna ones are differently formulated and are (partially) based on a different physics approaches. Both were extensively used in analysing the high energy experimental data. In this paper we presented the two schemes in detail and compared them analytically and numerically from the point of view of their effect on the results of the deep inelastic positive muon scattering
from a proton target at 280 GeV. To this aim we used the latest version of the D and the upgraded version of the MT programs. The latter included the $\tau^+\tau^-$ and $q\bar{q}$ loops in the vacuum polarisation and the virtual photon–$Z^0$ boson interference terms, all absent in the original formulation. In contrast to the $\gamma–Z^0$ interference the quark loop contribution turned out to be quite substantial, changing the total radiative correction by about 2% in the measured region.

The MT scheme contains the ‘infrared cut–off’ parameter, $\Delta$. The results should not depend on its value (provided it is not too large and not too low) and indeed it is approximately so when $\Delta$ is equal to about 0.1% of the beam energy value. The covariant formulation of the D scheme excludes the existence of such parameter.

The overall radiative correction reaches 50% at low $x$ and high $y$. Calculated from the two schemes the corrections agree to better than 2% in this region. Differences are thus insignificant over the most of the phase space covered in the fixed target DIS experiments. They are of the order of other systematic errors in the data analysis [7].

Neither of the two radiative correction schemes contain a contribution from the multiphoton exchange process to the elastic radiative tail which may be important for heavy nuclear targets. Results of quantitative estimates of those processes, relevant for the heavy target data currently analysed by the NMC, are discussed in a separate paper [20].

Acknowledgements

We thank our colleagues from the EMC and NMC for never–ending discussions of radiative corrections and for the enjoyable research collaboration. We are indebted to A. Akhundov for critical reading of the manuscript and important comments. DB is very much obliged to L. Kalinovskaya and T. Riemann for valuable discussions. This research was supported in part by the Polish Committee for Scientific Research, grant number 2 P302 069 04 and by Bundesministerium für Forschung und Technologie.

8 Appendix A

Below we summarize variables used in the MT formulation. The metric used is such that $ps = E_pE_s - p\bar{s}$ and four vector components are in the laboratory system. Notation is explained in fig. 7 and the coordinate system is that of fig. 8.

\[
\begin{align*}
  s &= (E_s, \vec{s}) & \text{four momentum of the incident lepton} \\
  p &= (E_p, \vec{p}) & \text{four momentum of the scattered lepton} \\
  \theta(\Omega) & & \text{lepton scattering angle (solid angle),} \\
  \cos \theta &= \frac{\vec{s} \cdot \vec{p}}{||s|| \cdot ||p||} \\
  \theta_k, \phi_k & & \text{bremsstrahlung photon emission angles} \\
  \theta_v, \phi_v & & \text{virtual photon emission angles} \\
  t &= (M, \vec{0}) & \text{four momentum of the target proton}
\end{align*}
\]
\[ k = (\omega, \mathbf{k}) \]
\[ p_f = s + t - p - k \]
\[ q^2 = (s - p - k)^2 = (p_f - t)^2 \]
\[ -Q^2 = (s - p)^2 \]

four momentum of the bremsstrahlung photon
four momentum of the final hadronic system
four momentum transfer
measured four momentum transfer

In the one photon exchange approximation and assuming one photon emission, the radiative tail from the j’th mass level can be written as (formula (A.24) in [3]):

\[
\frac{d^2 \sigma_{j,r}}{d\Omega dE_p} = \frac{\alpha^3}{2\pi} \left( \frac{E_p}{E_s} \right) \int_{-1}^{1} \frac{2M\omega \delta(\cos \theta_k)}{q^4(u_0 - |\mathbf{\pi}| \cos \theta_k)} \\
\left( W_2^j(q^2) \left\{ \frac{-am^2}{x^3} \left[ 2E_s(E_p + \omega + \frac{q^2}{2} - \frac{m^2}{E_s} \right] - \frac{a'm^2}{y^3} \left[ 2E_p(E_s - \omega) + \frac{q^2}{2} \right] \right\} \right. \\
-2 + 2\nu (x^{-1} - y^{-1}) \left\{ m^2 (s \cdot p - \omega^2) + (s \cdot p) [2E_sE_p - (s \cdot p) + \omega(E_s - E_p)] \right\} \\
+ x^{-1} \left[ 2 \left( E_sE_p + E_s \omega + E_p^2 \right) + \frac{q^2}{2} \right] - (s \cdot p) - m^2 \right] \\
- y^{-1} \left[ 2 \left( E_sE_p - E_p \omega + E_s^2 \right) + \frac{q^2}{2} \right] - (s \cdot p) - m^2 \right] \\
+ W_1^j(q^2) \left[ \left( \frac{a}{x^3} + \frac{a'}{y^3} \right) m^2 (2m^2 + q^2) + 4 \right. \\
\left. + 4\nu \left( x^{-1} - y^{-1} \right) (s \cdot p) \left( (s \cdot p - 2m^2) + (x^{-1} - y^{-1}) \left( 2s \cdot p + 2m^2 - q^2 \right) \right) \right] \\
\right)
\]

where

\[
\omega = \frac{1}{2} \left( u^2 - M_j^2 \right) / (u_0 - |\pi| \cos \theta_k) \\
u = s + t - p = p_f + k \\
u_0 = E_s + M - E_p \\
|\pi| = \left( u_0^2 - u^2 \right)^{1/2} \\
\omega = 2m^2 + M^2 - 2(s \cdot p) + 2M (E_s - E_p) \\
q^2 = 2m^2 - 2(s \cdot p) - 2\omega (E_s - E_p) + 2\omega |\pi| \cos \theta_k \\
a = \omega (E_p - |\pi| \cos \theta_p \cos \theta_k) \\
a' = \omega (E_s - |\pi| \cos \theta_s \cos \theta_k) \\
b = -\omega |\pi| \sin \theta_p \sin \theta_k \\
\nu = (a' - a)^{-1} \\
\cos \theta_p = \left( |\pi| \cos \theta - |\pi| \right)/|\pi| \\
\cos \theta_s = \left( |\pi| - |\pi| \cos \theta \right)/|\pi| \\
x = (a^2 - b^2)^{1/2}
\[ y = \left( a'^2 - b^2 \right)^{1/2} \]

\( W_1, W_2 \) denote structure functions; in particular \( W_{1,2}^j(q^2) \) are the structure functions at four momentum transfer \( q^2 \) and invariant mass of the hadronic final state \( M_j \). \( W_{1,2} \) are connected with the functions \( F_{1,2} \) of eq. (1) in the following way: \( F_2 = \nu W_2 \) and \( F_1 = MW_1 \). Observe that the meaning of the \( \nu, x \) and \( y \) variables used in the MT formulation is different from their generally accepted meaning as the DIS variables.

9 Appendix B

Below the exact expressions for certain functions in the D scheme will be given. The ‘generalized’ structure functions \( A_i(x_h, Q_h^2) \), in eq. (13) are:

\[
\begin{align*}
A_1(x, Q^2) &= 2 F_{1NC}(x, Q^2), \\
A_2(x, Q^2) &= \frac{1}{yS} F_{2NC}(x, Q^2), \\
A_3(x, Q^2) &= \frac{1}{2yS} F_{3NC}(x, Q^2),
\end{align*}
\]

with

\[
\begin{align*}
F_{1,2}^{NC}(x, Q^2) &= F_{1,2}(x, Q^2) + 2 |Q_e| (v_e + \lambda a_e) \chi(Q^2) G_{1,2}(x, Q^2) + \left( v_e^2 + a_e^2 + 2\lambda v_e a_e \right) \chi^2(Q^2) H_{1,2}(x, Q^2), \\
F_{3}^{NC}(x, Q^2) &= -2 \text{sign}(Q_e) \left\{ |Q_e| (a_e + \lambda v_e) \chi(Q^2) G_3(x, Q^2) + 2v_\epsilon a_\epsilon + \lambda \left( v_\epsilon^2 + a_\epsilon^2 \right) \right\} \chi^2(Q^2) H_3(x, Q^2),
\end{align*}
\]

Here structure functions \( F_i, G_i \) and \( H_i \) describe the hadronic tensor respectively for the \( \gamma \), \( \gamma - Z \) and \( Z \) exchange, \( \lambda = \xi Q_e / |Q_e| \), \( \xi \) is the lepton beam polarisation, \( v_e \) and \( a_e \) are the vector and axial-vector couplings of the lepton to the \( Z \) boson:

\[ v_e = 1 - 4 |Q_e| \sin^2 \theta_W, \quad a_e = 1, \]

\( \theta_W \) is the weak mixing angle, \( Q_e \) is the lepton charge, \( Q_e = -1 \), and

\[ \chi = \chi(Q^2) = \frac{G_\mu}{\sqrt{2}} \frac{M_Z^2}{8\pi \alpha} \frac{Q^2}{Q^2 + M_Z^2}, \]

with the Fermi constant, \( G_\mu = 1.16639 \cdot 10^{-5} \text{GeV}^{-2} \).
The 'radiator' functions \( S_i \) and the function \( F^{IR} \) are:

\[
S_1(y, Q^2, y_h, Q^2_h) = \left\{ \frac{1}{\sqrt{C_2}} \left[ \frac{Q_h^2 - Q^2}{2} + \frac{(Q^2 + 2m^2)(Q_h^2 - 2m^2)}{Q_h^2 - Q^2} \right] \right. \\
- m^2(Q_h^2 - 2m^2) \frac{B_2}{C_2^{3/2}} \left. \right\} - \left\{ S \leftrightarrow -X \right\} + \frac{1}{\sqrt{A_2}},
\]

\[
S_2(y, Q^2, y_h, Q^2_h) = \left\{ \frac{1}{\sqrt{C_2}} [M^2(Q_h^2 + Q^2) - XS_h] \right. \\
+ \frac{1}{(Q_h^2 - Q^2) \sqrt{C_2}} \left[ Q_h^2[S(S - S_h) + X(X + S_h) - 2M^2(Q_h^2 + 2m^2)] \right. \\
+ 2m^2[(S - S_h)(X + S_h) + SX] \left. \right\} - \left\{ S \leftrightarrow -X \right\} - \frac{2M^2}{\sqrt{A_2}},
\]

\[
S_3(y, Q^2, y_h, Q^2_h) = \left\{ \frac{1}{\sqrt{C_2}} \left[ \frac{2Q_h^2(Q_h^2 + 2m^2)(S + X)}{Q_h^2 - Q^2} - 2XQ_h^2 - S_h(Q_h^2 + Q^2) \right] \right. \\
- 2m^2Q_h^2 \frac{B_2}{C_2^{3/2}}(2S - S_h) \left. \right\} + \left\{ S \leftrightarrow -X \right\},
\]

\[
F^{IR}(y, Q^2, y_h, Q^2_h) = \frac{Q^2 + 2m^2}{Q^2 - Q_h^2} \left( \frac{1}{\sqrt{C_1}} - \frac{1}{\sqrt{C_2}} \right) - m^2 \left( \frac{B_1}{C_1^{3/2}} + \frac{B_2}{C_2^{3/2}} \right),
\]

where

\[
A_2 = \lambda_1 \equiv A_1, \\
B_2 = 2M^2Q_h^2(Q^2 - Q_h^2) + X(S_hQ_h^2 - S_hQ^2) \\
+ SQ^2(S_l - S_h) \equiv -B_1(S \leftrightarrow -X), \\
C_2 = [XQ_h^2 - Q^2(S - S_h)]^2 + 4m^2 \left[ (S_l - S_h)(S_lQ_h^2 - S_hQ^2) \right. \\
- M^2(Q_h^2 - Q^2)^2 \left. \right] \equiv C_1[S \leftrightarrow -X],
\]

with

\[
X = S(1 - y) = 2ME' \\
S_h = Sy_h
\]

and the hadron defined variables, \( x_h, y_h \) are given by the following equations

\[
M_h^2 = M^2 + S_yh(1 - x_h) \\
Q_h^2 = Sx_hy_h
\]
10 Appendix C

Below we list the relations between the variables used in the MT (Appendix A) and in the D schemes (Appendix B):

\[
\begin{align*}
E & \quad E_s \\
E' & \quad E_p \\
S_l = S_y & \quad 2M\nu \\
Q_h^2 & \quad -q^2 \\
\lambda_l = S_l^2 + 4M^2Q_h^2 & \quad (2M|\bar{\nu}|)^2 \\
\frac{B_2}{2\lambda_l} & \quad a \\
\frac{C_2}{4\lambda_l} & \quad -x^2 \\
\frac{M_h}{4\lambda_l} & \quad M_j \\
\frac{2M}{(\lambda_l)^{1/2}} \left( E - E' + M \left( \frac{Q_h^2 - Q_h^2}{Q_h^2 - S_l} \right) \right) \cos \theta_k & \\
\frac{dQ_h^2}{Q_h^4\lambda_l^{1/2}} & \quad \frac{d(\cos \theta_k)}{q^4(u_0 - |\bar{\nu}| \cos \theta_k)}
\end{align*}
\]

References

[1] Y.S. Tsai, *Phys.Rev.* **120** (1960) 269.

[2] L.W. Mo and Y.S. Tsai, *Rev. Mod. Phys.* **41** (1969) 205.

[3] Y.S. Tsai, SLAC–PUB–848 (1971).

[4] A.A. Akhundov et al., *Sov. J. Nucl. Phys.* **26** (1977) 660; *JINR–Dubna preprints* E2–10147 (1976), E2–10205 (1976); D. Bardin and N. Shumeiko, *Sov. J. Nucl. Phys.* **29** (1979) 499.

[5] A.A. Akhundov et al., *Sov. J. Nucl. Phys.* **44** (1986) 988; *JINR–Dubna preprint* E2–86–104 (1986).

[6] T. Sloan, G. Smadja and R. Voss, *Phys. Rep.* **162** (1988) 45.

[7] NMC; P. Amaudruz et al., *Phys. Lett.* **B295** (1992) 159 and CERN–PPE/92–124 (with Errata: Oct 26th, 1992 and April 19th, 1993).
[8] T. Sloan, *EMC internal report* EMC/87/4; C. Lietzke and S.J. Wimpenny, *Univ. of California (Riverside) preprint* UCR/DIS–89–06.

[9] P.P. Kuzhir and N.M. Shumeiko, *Sov. J. Nucl. Phys.* **55** (1992) 1086.

[10] J.D. Bjorken and S.D. Drell, *Relativistic Quantum Fields*, McGraw–Hill (1965); C. Itzykson and J.B. Zuber, *Quantum Field Theory*, 3rd ed., McGraw–Hill (1987).

[11] D.R. Yennie, S. Frautchi and H. Suura, *Ann. Phys. (N.Y.)* **13** (1961) 379.

[12] NMC; P. Amaudruz et al., *Nucl. Phys. B* **273** (1992) 3.

[13] H. Burkhardt et al., *Z. Phys.* **C43** (1989) 497.

[14] D. Bardin et al., Proceedings of the Zeuthen Workshop on Elementary Particle Theory – Deep Inelastic Scattering, Teupitz, Germany, April 1992, *Nucl. Phys. B* (Proc. Suppl.) **29A** (1992) 209.

[15] D. Bardin et al., *Z. Phys.* **C42** (1989) 679.

[16] N. Shumeiko, *Sov. J. Nucl. Phys.* **29** (1979) 807.

[17] J. Drees, *Univ. of Wuppertal preprint* WU–B 78–16.

[18] see e.g. M. Arneodo (NMC), *Ph. D. thesis, Princeton University*, 1992.

[19] M. Gari and W. Krümpelman, *Z. Phys.* **A322** (1985) 689.

[20] K. Kurek, *to be submitted to Z. Phys. C*.

**Figure Captions**

1. Feynman diagrams for the deep inelastic scattering in the one photon exchange approximation (a) and the lowest order radiative processes: real photon bremsstrahlung from the charged lepton (b), vertex correction (c), vacuum polarisation (d), double photon exchange (e), hadron current corrections (f,g). In the MT scheme evaluated were diagrams b–d.

2. Range of kinematical variables from which the radiative tails contribute to the cross section measured at the point A($Q^2$, $\nu$).

3. Double photon exchange (a) and hadron current corrections (b,c) in the Dubna scheme.

4. Infrared cut–off parameter $\Delta$ dependence of the FERRAD35 results obtained for the 280 GeV muon – proton scattering. Radiative correction factor $\eta$ is defined in eq. (4).

5. Radiative correction factor $\eta$ calculated in FERRAD35 (open symbols) and TERAD86 (closed symbols) for the muon – proton scattering at 280 GeV.
6. Ratio of the radiative correction factors $\eta$ calculated in FERRAD35 ($\eta_F$) and TERAD86 ($\eta_T$) for the muon – proton scattering at 280 GeV (closed symbols). The open symbols give the $\eta_F/\eta_T$ ratio when only $e^+e^-$ and $\mu^+\mu^-$ contribute to the vacuum polarisation (fig.1d) in FERRAD35.

7. Definition of kinematic variables describing the hard photon emission in MT, cf. Appendix A (from [17]).

8. The coordinate system used in the integration over the solid angle of the photon in the formula A.24 in [3], quoted in Appendix A (from [3]).
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9403238v1
This figure "fig2-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9403238v1
This figure "fig1-2.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9403238v1
This figure "fig2-2.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9403238v1
This figure "fig1-3.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9403238v1
This figure "fig2-3.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9403238v1