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Mathematical model to assess the imposition of lockdown during COVID-19 pandemic

Isa Abdullahi Baba\textsuperscript{a}, Abdullahi Yusuf\textsuperscript{b,c}, Kottakkaran Sooppy Nisar\textsuperscript{d,\textsuperscript{*}}, Abdel-Haleem Abdel-Aty\textsuperscript{e,f}, Taher A. Nofal\textsuperscript{g}

\textsuperscript{a} Department of Mathematical Sciences, Bayero University Kano, Kano, Nigeria
\textsuperscript{b} Department of Computer Engineering, Biruni University, Istanbul 34010, Turkey
\textsuperscript{c} Department of Mathematics, Federal University Dutse, Jigawa 7156, Nigeria
\textsuperscript{d} Department of Mathematics, College of Arts and Sciences, Wadi Aldawaser 11991, Prince Sattam bin Abdulaziz University, Saudi Arabia
\textsuperscript{e} Department of Physics, College of Sciences, University of Bisha, P.O. Box 344, Bisha 61922, Saudi Arabia
\textsuperscript{f} Physics Department, Faculty of Science, Al-Ashar University, Assiut 71524, Egypt
\textsuperscript{g} Department of Mathematics and Statistics, College of Science, Taif University, P.O. Box 11099, Taif 21944, Saudi Arabia

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\textbf{ABSTRACT}

Nigeria, like most other countries in the world, imposes lockdown as a measure to curtail the spread of COVID-19. But, it is known fact that in some countries the lockdown strategy could bring the desired results while in some the situation could worsen the spread of the virus due to poor management and lack of facilities, palliatives and incentives. To this regard, we feel motivated to develop a new mathematical model that assesses the imposition of the lockdown in Nigeria. The model comprises of a system of five ODE. Mathematical analysis of the model were carried out, where boundedness, computation of equilibria, calculation of the basic reproduction ratio and stability analysis of the equilibria were carried out. We finally study the numerical outcomes of the governing model in respect of the approximate solutions. To this aim, we employed the effective ODE45, Euler, RK-2 and RK-4 schemes and compare the results.

1. Introduction

Mathematical models for the propagation mechanisms of infectious disease exist in literature. These models play a major role in determining the methods of quantifying and analyzing effective control and preventive measures of infectious diseases [1–3]. There are several infectious diseases; as for compartmental diseases models, beginning with the very classic SIR model and making more complex proposals [4].

It is well known that the COVID-19 pandemic has caused a quagmire worldwide, with more than eight millions people infected worldwide, the mortality-recovery ratio appears to be in a positive proportion. Nevertheless, due to the sensitivity of Polymerase Chain Reaction, the absence or presence of the previously infected host is observed and the recovery rate appears to be promising in the absence of any curative vaccine. The challenge facing health care professionals, the World Health Organization and the Center for Disease Control in each quarter was whether reinfection could occur after a COVID-19 patient had been clinically treated. The subtle nature of the disease has brought to the attention of several scientists and medical practitioners to embark massively on multiple research to fully counterattack and stop the spread of the disease [13–22].

Of the seven known human coronaviruses, 4 are common human influenza pathogens. SARS-CoV, MERS-CoV and 2019-nCoV are responsible for severe respiratory diseases [5]. Although the COVID-19 has long been established and studied by researchers and medical practitioners, but many people lack awareness towards the disease and still vaccines and antiviral drugs to specifically prevent or treat the infection is not available. SARS-CoV in 2003 was the last major outbreak in China. It was an acute respiratory infectious disease with high fatality rate. The outbreak of SARS was controlled by China through multiple controls and good preventative steps. The incubation period for COVID-19 is significant and rather long when compared to SARS. Many studies calculated different incubation periods for the disease, for example; 5.2 days [6], 3.0 days [7] and 4.75 days [8]. In [7], an incubation period of

\textsuperscript{*} Corresponding author.

E-mail addresses: iababa.mth@buk.edu.ng (I.A. Baba), yusufabdullahi@fud.edu.ng (A. Yusuf), n.sooppy@psau.edu.sa (K.S. Nisar), amabdelaty@ub.edu.sa (A.-H. Abdel-Aty), t.a.hameed@tu.edu.sa (T.A. Nofal).

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up to 24 days is reported, this extends to 38 days as reported in Enshi Tujia and Miao autonomous prefecture in Hubei province of China. Note that asymptotically infected people are quite numerous [11] and compared to SARS-CoV and MERS-CoV the fatality is far inferior [7]. Genetic virus studies show that SARS-CoV and 2019-nCoV are 85% homologous [9]. But 2019-nCoV binds ACE2 to an affinity higher than SARS-CoVs [10]. At the end of 29 January 2020, SARS will be exceeded in the optimistic confirmed cases induced at COVID-19. The uncertainty of the asymptomatic cases, incubation period and the virus’ super-transmissibility bring great difficulties in the epidemics control.

The subtle characteristics of COVID-19 pandemic necessitated leaders from each country to impose some rules to curb the massive spread of the virus. Both developed, under-developed and developing countries agree to apply lock-down measure in order to restrict the movement of the people. In some countries the lock-down strategy could bring the desired results while in some the situation could worsen the spread of the virus due to poor management and lack of facilities, palliative and incentives. To this regard, we feel motivated to develop a new mathematical model that consider the lock-down strategy with the case study of Nigeria.

Nigeria recorded its first case of COVID-19 on February 27, 2020, in Lagos city [12]. From the mid of March 2020, the Federal Ministry of Health started to take necessary measures to prevent the spread of the virus. Travelers started to be screened and schools were short down. It is crystal clear that unlike the western countries, either the epidemic in Nigeria progressed through a slow phase or the number of asymptomatic cases in Nigeria is high. Since effective vaccine for the diseases is not available, the only available options are contact tracing and prevention of migration into the country. While China succeeded in preventing the spread of the disease by completely shutting down the country, Italy recorded worst case scenario by negligence of communities towards simple public health strategies. Observing these and many more cases, made the government of Nigeria to impose 14 - day lock down in some of its states which starts on March 30, 2020 [12].

Many recent researches in literature studied mathematical modeling of COVID-19 [23,24]. Some of these researches were concerned with fractional order models [25–31].

The aim of this research is to study the dynamics of COVID-19 in Nigeria with and without the lock down. This will enable us assess the significance or otherwise of the imposition of lock down in the country. We use real data from Nigeria Centre for Disease Control for our numerical simulations.

This paper is organized as follows. In Section 1, Introduction is given. In Section 2, the COVID-19 model with lock down is formulated. In Section 3, analysis of the model is carried out. In chapter 4, local stability analysis of the solutions of the models are conducted. Chapter 5, compares three different numerical schemes; Euler, Runge - Kutta of order 2 and Runge - Kutta of order 4 for the numerical simulations to illustrate the theoretical results. Finally, conclusion is given in chapter 6.

2. Model formulation

Let $S(t)$ represents the population of Susceptible individuals that aren’t yet under lock down, $S_L(t)$ represents Susceptible population that are under lock down, $I(t)$ represents Infective population that aren’t under lock down (isolation is referred to as lock down for convenience) and $I_L(t)$ refers Infective population that are under lock down and then cumulative density of the lock down program is $I_L(t)$. Then the total population is denoted by $N(t)$. System of ODE is used to represent the dynamics of this population. The parameter meanings as used in the model are given in Table 1.

### Table 1

| Parameter | Description |
|-----------|-------------|
| $\lambda$ | recruitment rate |
| $\beta$ | infection contact rate |
| $\lambda_1, \lambda_2$ | imposition of lock down on susceptible and infective respectively |
| $\gamma_1, \gamma_2$ | recovery rate in $I$ and $I_L$ respectively |
| $\alpha_1, \alpha_2$ | death rate due to infection in $I$ and $I_L$ respectively |
| $d$ | natural death rate |
| $\theta_1$ | rate of transfer of susceptible lock down individuals to susceptible class |
| $\theta_2$ | rate of transfer of infective lock down individuals to infective class |
| $\mu$ | rate of implementation of the lock down program |
| $\phi$ | rate of depletion of the lock down program |

3. Analysis of the model

In this section, some important properties of the proposed model such as boundedness, existence of equilibrium solutions and basic reproduction number will be analyzed.

3.1. Boundedness

The system trajectories are confined within a compact set. Then, the total population $N(t) = S(t) + S_L(t) + I(t) + I_L(t)$. Thus taking the derivative leads to

$$\frac{dN(t)}{dt} = \frac{dS(t)}{dt} + \frac{dS_L(t)}{dt} + \frac{dI(t)}{dt} + \frac{dI_L(t)}{dt}$$

$$= \lambda - (S + S_L + I + I_L)d - \alpha_1 I - \alpha_2 I_L$$

Therefore

$$\frac{dN(t)}{dt} \leq \lambda - dN.$$  \hspace{1cm} (2)

Consequently,

$$N(t) \leq \frac{\lambda}{d} + Ce^{-dt}.$$  \hspace{1cm} (3)

where $C$ is constant. The initial value condition at $t = 0$ gives

$$N(0) \leq \frac{\lambda}{d} + C.$$  \hspace{1cm} (4)

This implies that

$$C = N(0) - \frac{\lambda}{d}.$$  \hspace{1cm} (5)

We get
\[ \lim_{t \to \infty} N(t) = \lim_{t \to \infty} \left( \frac{\Lambda}{d} + N(0) \frac{\Lambda}{d} e^{-\lambda t} \right) = \frac{\Lambda}{d} \]  
and this gives
\[ \lim_{t \to \infty} N(t) = \frac{\Lambda}{d}. \]  
Hence the population is bounded above.

3.2. Existence of equilibrium

In order to obtain the equilibrium solution, we equate system (1) to zero and solve simultaneously. We obtain three different equilibrium solutions, viz; Disease free equilibrium, Endemic equilibrium in the absence of lock down, Endemic equilibrium in the presence of lock down.

Disease free equilibrium (DFE): \[ E_0 = [S_0, 0, 0, 0, 0, \frac{\gamma}{\mu}, \frac{\lambda}{\mu}, \frac{\phi}{\mu}, 0, 0, 0]. \]

Endemic equilibrium in the absence of lock down: \[ E_1 = [\frac{\Lambda}{d}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]. \]

Where \[ S_1 = \frac{\Lambda}{d} \] and \[ I_1 = \frac{\Lambda}{d}. \]

The endemic equilibrium in the presence of lock down: \[ E_2 = [S_{12}, S_{13}, I_{12}, I_{13}, I_{22}, I_{23}, L_2]. \]

Where
\[ S_2 = \frac{d\gamma + a + d + (\lambda_2 - \theta_2)\mu L_2}{\beta_\phi} S_{21}, \]
\[ I_2 = \frac{d\gamma + a + d + (\lambda_2 - \theta_2)\mu L_2}{\beta_\phi} I_{21}, \]
and \[ L_2 \] can be obtained by solving the quadratic equation \[ A_1 I_2^2 + A_2 I_2 + A_3 = 0. \]  
Where
\[ A_1 = -\frac{\mu(\lambda_2 - \theta_2)}{\beta_\phi} \left[ 1 + \frac{\gamma_2}{\alpha_2 + \gamma_2 + d} \right], \]
\[ A_2 = -\left( \gamma_2 + a + d \right) \left( \frac{\beta_\phi}{\beta_\phi + (\alpha_2 + \gamma_2 + d)(d + \theta_2)} \right), \]
\[ A_3 = -\frac{\mu(\lambda_2 - \theta_2)}{\beta_\phi} \left( \frac{\gamma_2}{\alpha_2 + \gamma_2 + d} \right). \]

Consider the following Jacobian matrix from (1):
\[
J = \begin{bmatrix}
-\beta I - \lambda L - d & \theta_1 & -\beta S + \gamma_1 & 0 & 0 & -\lambda S \\
\beta L & 0 & -\beta S - (\gamma_1 + a_1 + \lambda L - d) & \theta_2 & -\lambda I \\
0 & 0 & \lambda I & -d + \theta_2 + a_2 & \lambda I \\
0 & 0 & \mu & 0 & -\phi \\
\end{bmatrix}
\]

Theorem 5. The disease free equilibrium, \( E_0 \) is locally asymptotically stable if \( R_0 < 1 \).

Proof:
Consider the Jacobian matrix at \( E_0 \) then,
\[
J(E_0) = \begin{bmatrix}
-\lambda_{A} & -\beta \frac{\Lambda}{d} & 0 & 0 & -\frac{\Lambda}{d} \\
0 & \lambda_{A} & -\beta \frac{\Lambda}{d} & 0 & 0 \\
0 & 0 & \frac{\beta L}{d} - (\gamma_1 + a_1 + d) & \theta_2 & 0 \\
0 & 0 & 0 & -d + \theta_2 + a_2 & 0 \\
0 & 0 & 0 & \mu & -\phi \\
\end{bmatrix}
\]

The eigenvalues are \( -d, -(d + \theta_1), -(d + \theta_2 + a_2 + \beta_\phi), -\phi_\phi, -\phi \).

Define \( R_0 = \frac{\beta_L}{\beta_\phi + (\alpha_2 + \gamma_2 + d)} < 1 \). That is, \( E_0 \) is locally stable if \( R_0 < 1 \).

Theorem 6. The endemic equilibrium without lock down \( E_1 \) is locally asymptotically stable if \( R_0 > 1 \).

Proof:
Consider the Jacobian matrix at \( E_1 \) then,
\[
A_1 = \frac{\mu(\lambda_2 - \theta_2)}{\beta_\phi} \left[ 1 - \frac{\gamma_2}{\alpha_2 + \gamma_2 + d} \right], \quad A_2 = -\left( \gamma_2 + a + d \right), \quad A_3 = \frac{\mu(\lambda_2 - \theta_2)}{\beta_\phi}. \]

The eigenvalues are \( -(d + \theta_1), -(d + \theta_2 + a_2 + a_2 + \beta_\phi), -\phi_\phi, -\phi \).

Define \( R_0 = \frac{\beta L}{\beta_\phi + (\alpha_2 + \gamma_2 + d)} > 1 \). That is, \( E_1 \) is locally stable if \( R_0 > 1 \).

Theorem 7. The endemic equilibrium with lock down \( E_2 \) is locally asymptotically stable if \( \mu_2 I_2 < \mu_2 I_2 + a_2 \mu_2 + \phi_2 \).

Proof:
Consider the Jacobian matrix at \( E_2 \) then,
where

\[ A_1 = -\lambda_1 \frac{\phi'(y_1 + \alpha_1 + d) + (\lambda_2 - \theta_2)\mu I_2}{\beta \phi} \]

\[ A_2 \lambda_2 \frac{\phi'(y_1 + \alpha_1 + d) + (\lambda_2 - \theta_2)\mu I_2}{\beta \phi} \]

The eigenvalues are \(-(d + \theta_1), -(d + \theta_2 + \gamma_2 + \alpha_2), -\frac{\beta \phi_{i+1} - \phi_i}{\Delta t}, \frac{\lambda_2 \mu I_2}{\mu} -(d + \theta_2 + \gamma_2 + \alpha_2) \lambda_2 I_2\). Where \( K = \mu \lambda_2 I_2 - \mu \theta_2 I_2 - \alpha_2 \phi I_2 - \phi_2 \). This equilibrium point is stable if \( \mu \lambda_2 I_2 < \mu \theta_2 I_2 + \alpha_2 \phi I_2 + \phi_2 \).

8. Numerical simulations

In this portion we implement three effective numerical schemes: Euler, second-order Runge–Kutta, and fourth-order Runge–Kutta. The obtained solutions is compared with that of the ode4s GNU Octave function.

8.1. Euler’s technique

For the Euler, assume that a well-posed initial-value condition is given by

\[ \frac{dy}{dt} = f(t, y) \quad \text{and} \quad y(a) = \chi \quad a \leq t \leq b. \]  

A sequence of approximation point \((t, y) \approx (t, y(t))\) is established by EM to the exact solutions of ODE by \( t_{i+1} = t_i + h \) and \( w_{i+1} = w_i + hf(t_i, w_i) \), \( i = 0, 1, \ldots, N-1 \), and \( t_0 = a, w_0 = \alpha, h = \frac{b-a}{N} \).

8.2. Second order Runge–Kutta method

Suppose that a well-posed IVP is given, the RK technique of order two establishes a sequence of approximation points \((t, y) \approx (t, y(t))\) to the exact solution of the ODE by \( t_{i+1} = t_i + h, K_1 = f(t_i, w_i), K_2 = f(t_i + h, w_i + h K_1) \), and \( w_{i+1} = w_i + h \frac{K_1 + K_2}{2} \), for each \( i = 0, 1, \ldots, N-1 \), where \( t_0 = a, w_0 = \alpha \), and \( h = \frac{b-a}{N} \).

Fig. 1. ODE45 versus Euler for the governing model.
8.3. Fourth order Runge-Kutta method

Suppose that a well-posed IVP is given, the RK technique of order two establishes a sequence of approximation points \((t, w) \approx (t, y(t))\) to the exact solution of the ODE by \(w_{i+1} = t_i + h, K_1 = f(t_i, w_i), K_2 = f(t_i + \frac{h}{2}, w_i + \frac{h}{2}K_1), K_3 = f(t_i + \frac{h}{2}, w_i + \frac{h}{2}K_2), K_4 = f(t_i + h, w_i + hK_3),\) and \(w_{i+1} = w_i + \frac{h}{6}(K_1 + 2K_2 + 2K_3 + K_4),\) for each \(i = 0, 1, \ldots, N-1,\) where \(t_0 = a, w_0 = \alpha,\) and \(h = \frac{b-a}{N}.

Now we discuss the obtained numerical outcomes of the governing model in respect of the approximate solutions. To this aim, we employed the effective ODE45, Euler, RK-2 and RK-4 schemes and compare the results. The initial conditions are assumed as \(S(0) = 900, S_L(0) = 300.\)
\[ I(0) = 300, I_L(0) = 497, I_L(0) = 200 \]

and the parameters values are taken as in the table below:

| Parameters | Values  | References |
|------------|---------|------------|
| \( \Lambda \) | 400     | [10]       |
| \( \beta \)  | 0.000017 | [11]       |
| \( \gamma_1 \) | 0.0002  | [10]       |
| \( \gamma_2 \) | 0.002   | Assumed   |
| \( r_1 \)   | 0.16979 | [5]        |
| \( r_2 \)   | 0.03275 | [5]        |
| \( \alpha_2 \) | 0.03275 | [5]        |
| \( d \)     | 0.0096  | [12]       |
| \( \sigma_1 \) | 0.2     | [10]       |
| \( \sigma_2 \) | 0.02    | Assumed   |
| \( \mu \)   | 0.0005  | [10]       |
| \( \phi \)  | 0.06    | [10]       |

Fig. 1 depicts the solution of the system Eq. (1) with the stated initial conditions, established by the ode45 versus the Euler method. It is well-known that, EM is one of the simplest scheme that gives a captivating approximation on the behaviour of each of the system variables. The same discretization knots is used for both implementations within the interval \([0, T]\) with a step size given by \( h = T/100 \).

Fig. 2 depicts the solution of the system of Eq. (1) with the stated initial value conditions establish by the ode45 versus the RK-2 method of order two. It can be observe that, RK’s-2 method gives a better approximation than EM. This is due to the indistinguishable in this case.

Fig. 3 depicts the solution of Eq. (1) with the stated initial values computed by the ode45 versus the RK-4 method. The results of the RK-4 method give extremely good outcome. In addition, RK-4 requires four evaluations per step and its global truncation error is \( O(h^4) \). Fig. 4 depicts the overall comparison of all the schemes that have been used for the approximation of the system Eq. (1).

9. Conclusion

In conclusion, this paper consists of a system of five non–linear ordinary differential equations. The aim of the model is to study and assess the imposition of lock-down on the dynamics of COVID-19 in Nigeria. Mathematical analysis of the model were carried out, where boundedness, computation of equilibria, calculation of the basic reproduction ratio and stability analysis of the equilibria were carried out. We finally study the numerical outcomes of the governing model in respect of the approximate solutions. To this aim, we employed the effective ODE45, Euler, RK-2 and RK-4 schemes and compare the results. It will be important if the fractional order analogue to this model will be studied. Also the rest of the COVID-19 control measures are to be studied and their significance will be analysed.

Data availability statement

We don’t have data attached to this research.

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Isa Abdullahi Baba: Conceptualization, Writing - original draft. Abdullahi Yusuf: Writing - original draft, Software. Kottakkaran Sooppy Nisar: Writing - original draft, Writing - review & editing, Supervision. Abdel-Haleem Abdel-Aty: Formal analysis, Writing - review & editing. Taher A. Nofal: Writing - review & editing, Funding acquisition.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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