Sequential recommendation with metric models based on frequent sequences

Corentin Lonjarret1,2 · Roch Auburtin2 · Céline Robardet1 · Marc Plantevit3

Received: 31 January 2020 / Accepted: 17 February 2021 / Published online: 12 March 2021
© The Author(s), under exclusive licence to Springer Science+Business Media LLC, part of Springer Nature 2021

Abstract
Modeling user preferences (long-term history) and user dynamics (short-term history) is of greatest importance to build efficient sequential recommender systems. The challenge lies in the successful combination of the whole user’s history and his recent actions (sequential dynamics) to provide personalized recommendations. Existing methods capture the sequential dynamics of a user using fixed-order Markov chains (usually first order chains) regardless of the user, which limits both the impact of the past of the user on the recommendation and the ability to adapt its length to the user profile. In this article, we propose to use frequent sequences to identify the most relevant part of the user history for the recommendation. The most salient items are then used in a unified metric model that embeds items based on user preferences and sequential dynamics. Extensive experiments demonstrate that our method outperforms state-of-the-art, especially on sparse datasets. We show that considering sequences of varying lengths improves the recommendations and we also emphasize that these sequences provide explanations on the recommendation.

Keywords Recommender systems · Sequential recommendation · Collaborative filtering

1 Introduction
As society becomes ever more digital, the number of situations where users are faced with a huge number of choices increases. In this context, a selection process by exhaustive examination of all possibilities is not feasible. Hence, accessing a virtually infinite set of digital contents – or digital descriptions of real objects called hereafter items – is only possible if the search process is equipped with powerful recommendation tools. This is what recommender systems (Aggarwal 2016; Gomez-Uribe and Hunt
In this paper, we tackle the problem of sequential recommendation that aims to predict/recommend to a user the next item from collaborative data. This is a challenging problem whose importance has been gradually recognized by researchers. Indeed, user preferences (i.e., the long-term dynamics) and user sequential dynamics (i.e., the short-term dynamics) need to be fruitfully combined to account for both personalization and sequential transitions. The first methods, which paved this research field, aim at capturing either user preferences or user sequential dynamics. User preferences have been identified using methods such as Matrix Factorization (MF) (Koren 2009; Koren and Bell 2015) that uses inner products to decompose a compatibility matrix between users and items, or FISM (Kabbur et al. 2013) that splits a similarity matrix between items to better account for transitive relationships between items. Sequential dynamics has been caught by Markov Chains (MC) for the calculation of the conditional probability of appearance of an item according to a fixed number $L$ of passed items. The next generation of recommender systems is based on models that combine both sequential dynamics and user preferences, like FPMC (Rendle et al. 2010) or FOSSIL (He and McAuley 2016) which fuse Matrix Factorization and Markov Chains. PRME (Feng et al. 2015) improves FOSSIL and FPMC by replacing inner product with Euclidean distance. Such a metric embedding brings a better generalization, mostly due to the respect of the triangle inequality. However, it still suffers from a lack of personalization on the short term part, for example due to the fact that Markov chains have a fixed length regardless of the users and their considered items. Moreover, these models combine the user preferences and sequential dynamics using two embeddings – one for the user preferences and another one for the sequential dynamics – which can damage the quality of the model. More recently, and especially since the traces left by the users become longer, a growing research effort (Zhang et al. 2019; Tang and Wang 2018; Chen et al. 2018) has been dedicated to the investigation of the use of deep learning technique for sequential recommendation. Yet, this kind of methods need large amounts of data and can have troubles on sparse datasets. These statements are confirmed by our experiments.

To cope with these limitations, we propose a new model REBUS (Recommendation Embedding Based on freqUent Sequences) that uses (1) frequent sequences to identify the part of user history that is the most relevant for recommendation and using these sequences to estimate Markov Chains of variable orders and (2) a unified metric embedding model based on user preferences and user sequential dynamics.

Figure 1 illustrates how REBUS works. It consists of embedding items into latent vectors and using these vectors to represent user preferences and user sequential dynamics: In Fig. 1(a), user preferences are represented by the weighted sum of all the items of the user history and in Fig. 1(b) user sequential dynamics is represented by the weighted sum of the most representative recent items of the user history. These most representative items are those that match frequent sequences over the entire dataset. These frequent sequences are used as a proxy to know which items in the user history are important to capture sequential dynamics. In Fig. 1(c), REBUS recommend the item that is the nearest to the vector resulting from the weighted sum of user preferences and sequential dynamics.
Sequential recommendation with metric models...

Fig. 1 Overview of **REBUS** that accommodates user preferences and sequential dynamics through Euclidean distances.  

**a User preferences** are represented by the embedding of all the user’s history items.  
**b Sequential Dynamics** is represented by the embedding of the user’s items that match a frequent sequence. 
The temporal order of the items is taken into account with a temporal damping factor.  
**c REBUS** takes the nearest item of the weighted sum of user preferences and sequential dynamics for recommendation.

Our contributions are summarized as follows. We develop a new method, **REBUS**, that integrates and unifies metric embedding models to capture both user preferences and sequential dynamics. Especially, the use of personalized sequential patterns makes it possible to select the part of the recent user’s history that is of most interest for the recommendation. In an empirical study over 12 datasets, we show that **REBUS** outperforms 10 state-of-the-art algorithms (that cover the different types introduced above) on sparse dataset and have nearly the same performance on dense datasets. Furthermore, we show that the sequences we use to model user sequential dynamics can provide additional insights on the recommendations. Finally, for reproducibility purposes, data and code are made available here: https://bit.ly/3gwZAOF.

The remaining of the paper is organized as follows. Section 2 introduces the notations. We review the literature in Sect. 3. **REBUS** model is defined in Sect. 4. We report an extensive empirical study in Sect. 5 and conclude in Sect. 6.

2 Notations

In this section, we introduce the notations we use throughout this paper to both define our proposal and review the literature.

\( U \) and \( I \) are used to respectively denote the set of users and the set of items. The symbols \( u \) and \( i \) stand for individual users and items (i.e., \( u \in U \) and \( i \in I \)). The trace left by a user \( u \) is the sequence of items \( s_u = (i_1, \ldots, i_p) \), with \( i_\ell \in I, \ \ell = 1 \ldots p \), and \( i_p \) being the most recent item. Let \( s_u^{[1,t]} \) be the substring of \( s_u \) starting at the 1st and oldest item, and ending at the \( t^{th} \) and more recent one. We use \( I_{s_u} \) to denote items that appear in \( s_u \) and \( I_{s_u^{[1,t]}} \) for those that belong to \( s_u^{[1,t]} \). Notations used throughout this paper are summarised and exemplified in Table 1.
Table 1  Notations

| Symbol | Description |
|--------|-------------|
| U      | User set    |
| I      | Item set    |
| S      | Set of user sequences |
| F      | Set of frequent substrings of S. F = \{(1), (2), (4), (5), (4, 5)\} |
| su     | Sequence of items associated to user u. su = ⟨1, 2, 3, 2, 2, 4, 5⟩ |
| Isu    | Set of items that appear in su. Isu = {1, 2, 3, 4, 5} |
| msu    | Sequence of F that best matches su. msu = ⟨4, 5⟩ |
| su t   | Item that appears in su at position t. su_5 = 2 |
| msu t  | Item that appears in msu at position t. msu_5 = 5 |
| su[1,t] | Substring of su starting at position 1 and ending at position t. su[1,5] = ⟨1, 2, 3, 2⟩ |
| msu[1,t] | Sequence of F that best matches su[1,t]. msu[1,5] = ⟨2⟩ |
| \(\hat{p}_{u,i,t}\) | Prediction that measures the probability for user u to choose item i while only considering su[1,t] |
| \(>_{u,t}\) | Personalized total order of user u at position t |

3 Related work

In this paper, we consider sequential recommendation that aims to predict the next item that will interest a user based on a sequence of items that he previously purchased/consumed.

There are several variants to this problem which lie outside the scope of this study. For instance, temporal recommendation (Ding and Li 2005; Koren 2009; Xiong et al. 2010; Wu et al. 2017) takes into account the timestamps of users’ actions to make recommendations based on a specific time (e.g. recommending a glass of wine in the evening but not in the morning). Also, we exclude context-aware (Rendle 2012; Pasricha and McAuley 2018; Zhang et al. 2016; Huang et al. 2018) and time-aware (Sanchez and Bellogín 2020) recommendations, as these approaches require contextual information (timestamps, item categories, user features, etc...) which limits their use to specific data. Hence, we restrain our study to methods that only take as input users’ sequences to make recommendation, without considering additional information.

In the following, we detail the related works that consider recommender systems as an estimation problem of \(\hat{p}_{u,i}\), values that measure the interest of user u for item i using real observations D. Such methods rely on the optimization of a loss function \(L(D, \hat{P})\), which evaluates the proximity between D and \(\hat{P} = (\hat{p}_{u,i})\). We present below the recent approaches that are related to our proposal.

3.1 User preferences (long term dynamics)

Early recommendation methods aim at identifying user preferences using Collaborative Filtering techniques such as K-Nearest Neighbour algorithms (Sarwar et al. 2001) or Matrix Factorization (MF) (Koren and Bell 2015). In many state-of-the-art
recommender systems, MF is used to model the interactions between users and items. It consists in decomposing a matrix, $D_{\text{user} \times \text{item}} = (d_{u,i})$ with $d_{u,i} = 1 \iff i \in I_u$, in a product of $k$-ranked matrices: $D_{\text{user} \times \text{item}} \approx R \times Q$. The prediction $\hat{p}_{u,i}$, that the user $u$ chooses the item $i$, is estimated by the inner product:

$$\hat{p}_{u,i} \propto \langle R_u, Q_i \rangle,$$

with $R_u$ and $Q_i$ the latent vectors associated to user $u$ and item $i$. However, as usually users give implicit feedback on few items of $I$, the matrix $D$ is sparse and these approaches suffer from loss of precision when the numbers of users and items grow. In order to overcome these problems, other approaches such as FISM (Kabbur et al. 2013) – which is an improvement of SLIM (Ning and Karypis 2011) – decompose an implicit item-to-item similarity matrix in two $k$-ranked matrices $P$ and $Q$ so that:

$$\hat{p}_{u,i} \propto \beta_i + \beta_u + \frac{1}{|I_u \setminus \{i\}|} \sum_{j \in I_u \setminus \{i\}} \langle P_j, Q_i \rangle,$$

where $\beta_i$, $\beta_u$ are biases respectively associated to item $i$ and user $u$, $\frac{1}{|I_u \setminus \{i\}|}$ normalized long set of items and $\alpha$ is used to control the degree of agreement between items. Hence, the more $i$ is similar to items already chosen by user $u$, the more likely $i$ will be a good choice for $u$. Taking into account these transitive relations between items makes it possible to increase the quality of the recommendation.

Recently, deep learning techniques have been introduced in many recommender systems. For example, user preferences are modeled by Multi-Layer Perceptions (MLP) in (He et al. 2017b) or using Auto-encoder in (Sedhain et al. 2015).

### 3.2 Sequential dynamics (short term dynamics)

Another trend in recommender system design is to use sequential information (the order in which the items are chosen) to model the user sequential dynamics. Short term dynamics can be modeled using first order Markov Chains. It consists in using the probability $p(i \mid j)$ of predicting $i$ given the last item $j$ in the sequence of the user. The transition matrix $(p(i \mid j))_{i,j}$ is decomposed in a product of two $k$-ranked matrices $M_j$ and $N_i$ that represent the latent/embedding vectors of item $j$ and $i$, so that the probability of having $i$ after $j$ is estimated by the inner product:

$$\hat{p}_{i\mid j} \propto \langle M_j, N_i \rangle,$$

with $M_j$ and $M_i$ the latent vectors associated to items $j$ and $i$. It is also possible to consider high order Markov Chains with the same principle.

There are other approaches, called session-based model (Quadrana et al. 2018; Wang et al. 2019), that give more weight to recent events by construction: The user sequences are divided in sessions. Therefore, they focus on shorter user sequences (i.e., the sessions) fostering short term dynamics. Considering sessions makes it possible to focus the recommendation on similarities with recent items. Some of these methods,
like *Session-based recommendations with Recurrent Neural Networks* (GRU4Rec) (Hidasi et al. 2016) or Session-based KNN (Ludewig and Jannach 2018), are widely used and deserve to be mentioned and evaluated within the framework of sequential recommendation systems.

### 3.3 Unifying user preferences and sequential dynamics

Current recommendation methods accommodate user preferences and sequential dynamics as it has been observed that it increases their performances. FPMC (Rendle et al. 2010) is one of the first method that uses both Matrix Factorization and first-order factorized Markov Chains. The probability that user \( u \) chooses item \( i \) just after having taken item \( j \) is estimated by the sum of two inner products:

\[
\hat{p}_{u,i|j} \propto \langle R_u, Q_i \rangle + \langle M_j, N_i \rangle
\]

Fossil (He and McAuley 2016) improves FPMC by associating an implicit item-to-item similarity matrix decomposition method with a Markov Chains model of order \( L \). On top of that, PRME (Feng et al. 2015) enhances FPMC and Fossil by replacing the inner products with Euclidean distances. As argued in (Chen et al. 2012; Feng et al. 2015), metric embedding model brings better generalization ability than Matrix Factorization to represent Markov chains because of the triangle inequality assumption. The probability that user \( u \) takes the item \( i \) after item \( j \) is estimated by the sum of the following Euclidean distances:

\[
\hat{p}_{u,i|j} \propto -\left( \alpha \cdot ||R_u - Q_i||_2^2 + (1 - \alpha) \cdot ||M_j - M_i||_2^2 \right)
\]

with \( \alpha \) a weight that controls the long term and short term dynamics.

TransRec (He et al. 2017a) is based on a novel metric embedding model that unifies user preferences and sequential dynamics with translation. To achieve this, items are embedded as points \( P_i \) in a latent transition space and users are modeled as translation vectors \( T_u \) in the same space:

\[
\hat{p}_{u,i|j} \propto \beta_i - d(P_j + T_u, P_i)
\]

with \( d() \) a distance (\( L_1 \) or squared \( L_2 \)), \( T_u \) a translation vector representing \( u \), and \( P_i \), \( P_j \) points in the transition space related to items \( i \) and \( j \). The sequential dynamics is captured with first order Markov chains.

Recently, a growing research effort has been dedicated to the investigation of the use of deep learning models (Zhang et al. 2019) for sequential recommendation. In that context, Recurrent Neural Networks (RNN) have been widely used for session-based recommendation. Such networks use Long Short Term Memory (LSTM) or Gated Recurrent Units (GRU) and have shown good results to model user sequential Dynamics (Hidasi et al. 2016; Hidasi and Karatzoglou 2018; Jannach and Ludewig 2017; Devooght and Bersini 2017). Another line of work has investigated Convolutional
Neural Network based methods. Convolutional Sequence Embedding Recommendation Model (Caser) (Tang and Wang 2018) is the first CNN-based method that captures both user preferences and user sequential dynamics. Caser embeds $L$ previously considered items as an “image” and learns sequential patterns with convolution operations. Inspired by a new sequential model Transformer for machine translation tasks (Vaswani et al. 2017), Self-Attentive Sequential Recommendation (SASRec) (Kang and McAuley 2018) is a new sequential recommendation model that outperforms many advanced sequential models on sparse and dense datasets.

Finally, we can cite the recent work of (Sanchez and Bellogín 2020), which, alike to our approach, computes a similarity based on the longest common subsequence. However, this method requires temporal information, which makes it out of the scope of our study.

### 3.4 Summary and desiderata

Few approaches combine user preferences and sequential dynamics into a metric embedding model. The proposed approaches consider Markov chains of fixed small order $L$ or black box model based on deep neural networks. In this paper, we aim to personalize the sequential dynamics of users by identifying the most salient items from the user’s history, and use them with user preferences in order to have a unified embedding metric model. Our goal is also to have a model that is as interpretable as possible and gives some explanations on the recommendation made.

### 4 REBUS model

**REBUS** is a metric embedding model in which only items are projected. Their corresponding embedding vectors are influenced by both the preferences of the user and their sequential dynamics. The user preferences are wrapped in the model by constraining the latent vector $P_i$ of an item that should be recommended to a user to be as close as possible to the average vector of the embedding of the items contained in the user history. In order to identify the part of the sequence that is most characteristic of a user, we consider frequent sequential patterns. These frequent sequences are both present in the history of several users (whose minimum number is specified by a threshold) while allowing to ignore certain items which are not sufficiently characteristic of their general behavior. Once the most important items are identified for a user, their order is taken into account in the model thanks to a damping factor based on the rank of the item in the sequence. This allows sequences of different lengths to be used in a unified manner. The embedding vectors are then learned using the Bayesian personalized ranking optimization method (Rendle et al. 2009) on our model. These steps are detailed below.
4.1 Long-term metric-based model

To model user preferences, we follow the way paved by FISM (Kabbur et al. 2013) and FOSSIL (He and McAuley 2016) while replacing inner product with Euclidean distance. The objective is to compute a latent vector $P_i$ for each item $i$, so that the prediction for a user $u$ to choose $i$ varies in the opposite way to the distance between the sum of the items already chosen by $u$ and the item $i$. Moreover, in order to not overweight items that appear in long transactions – items selected with many others – we normalize the user preferences by the inverse of the number of items in the sequence of user $u$. As in FISM or FOSSIL, the hyperparameter $\alpha$ controls the degree of agreement between items: When $\alpha = 1$, the long-term part is equivalent to the barycenter of the latent vectors; The closer $\alpha$ is to 0, the more it is equivalent to the sum of latent vectors. In our experiments, we observed that the best values of $\alpha$ lie between 0.7 and 1.

\[
\hat{p}_{u,i} \propto -\frac{1}{|I_{su}\setminus\{i\}|^\alpha} \sum_{j \in I_{su}\setminus\{i\}} P_j - P_i^2.
\]

In the learning phase, we estimate the prediction associated to item $i$ and user $u$ at position $t$, where $i$ is the $t^{th}$ item taken by $u$. Note that, for a user $u$, the first item is the oldest one ($t = 1$) and the last taken item is the most recent one ($t = |su|$). It seems also rightful to restrict the set of items to be considered to those older than $t$ ($I_{su}[1..t]$) (even if it is not the choice made by FISM and FOSSIL). This choice was confirmed empirically. In addition, we limit the number of items to be considered to the recent ones with max_length hyperparameter. It allows REBUS to be more flexible by controlling the temporal window that influences the user’s preferences and to get rid of the old past. All in one, this leads to the following equation for estimating user long-term preferences:

\[
\hat{p}_{u,i,t} \propto -\frac{1}{|I_{su}[x..t]\setminus\{i\}|^\alpha} \sum_{j \in I_{su}[x..t]\setminus\{i\}} P_j - P_i^2,
\]  

with $x = \max(t - \text{max\_length}, 1)$.

4.2 Short-term dynamics modeled by frequent patterns

As explained in Sect. 3, it has been shown that taking into account short-term individual dynamics improves the recommendation. However, existing approaches only consider a fixed, short and consecutive part of the history to make the recommendation. In contrast, REBUS takes into account parts of the user history which may be of different lengths for each user and also not necessarily consecutive. Finding the most adapted sequential pattern for a user $u$ at a position $t$ is accomplished in two steps: (1) computing a set $F$ of representative sequences of users’ histories, and (2) identifying a sequence that personally represents a user $u$. 
However, taking into account inter-individual variability – by allowing items to be in the representative sequence of a user – can be done either in step 1 or in step 2. In the first scenario, we extract frequent subsequences from user’s histories, and identify the subsequence that perfectly matches the history of a given user when making the recommendation. In the second scenario, we extract frequent substrings from user’s histories and identify the substring that best characterizes a user using a string alignment algorithm that allows to skip some uninformative items.

We implemented both scenarios and found that the quality of the results are very similar. However, the model based on frequent subsequences has a higher cost due to the size of the collection of frequent subsequences, that is much greater than the one of frequent substrings. We thus detail below the second scenario.

4.2.1 Computing representative sequences from users’ histories

A recommendation made for a user is a generalization of behaviors observed for many other users of the system. To capture the short-term dynamics of users, we want to identify substrings of purchased items that characterize the possible short-term dynamics in the system. That is to say, we want to get sets of items ordered in time that the users of the system are likely to consider in the same order. Substrings that can account for user sequential dynamics are the ones that appear in many user’s histories. We identify them by extracting frequent substrings (Gusfield 1997) that appear in at least $\text{minCount}$ user sequences and are at most of size $L$. $\text{minCount}$ and $L$ are hyperparameters of our model. The obtained substrings constitute the set $F$. Three important points should be stressed here:

– The set $F$ is computed once at the beginning of REBUS and then is used during the learning phase;
– The computation time of $F$ takes at most ten seconds in our experiments and is therefore not a computational bottleneck;
– Each substring of a frequent substring is also frequent and thus belongs to $F$.

4.2.2 Deriving from $F$ a personalized sequence for $u$

Once $F$ is computed, the objective is to find which substring to use as personalized context for user $u$ at position $t$. This substring, denoted $m_{s_u^{[1..t]}}$, must be included in the user’s history while allowing the latter to contain additional items. To do that, we adapt the “Exact matching with wildcards” algorithm (Gusfield 1997) to compute the longest substring among the substrings in $F$ that end to the most recent item in $s_u^{[1..t]}$ that belongs to $F$. The function is presented in Algorithm 1. It consists to pick up the most recent item of $s_u^{[1..t]}$ that matches a substring of $F$. After that, it identifies the longest substring in $F$ that ends with the previously found item and which is contained in the user sequence. If this process ends without any substring matching, $s_u^{t-1}$ is taken as personalized context for $u$ at position $t$, where $s_u^{t-1}$ is the most recent item considered by $u$ excluding $s_u^t$, the ground truth item. It allows our model to always consider sequential dynamics. It is important to notice that the personalized context for a given user relies on its last actions and therefore varies according to position $t$. 

 Springer
As an example, let us consider the set of frequent substrings $F = \{\langle 0 \rangle, \langle 1 \rangle, \langle 2 \rangle, \langle 3 \rangle, \langle 4 \rangle, \langle 0, 1 \rangle, \langle 1, 3 \rangle, \langle 0, 1, 3 \rangle, \langle 1, 2 \rangle, \langle 2, 4 \rangle, \langle 1, 2, 4 \rangle\}$ and consider the user sequence $s_u^{[1,7]} = \langle 0, 1, 2, 3, 4, 5 \rangle$. The most recent item that can be exploited is 4 as item 5 does not appear in $F$. The longest sequence of $F$ that ends with 4 and that only contains items of $s_u^{[1,7]}$ in the same order is $m_{s_u^{[1,7]}} = \langle 1, 2, 4 \rangle$. Indeed, the substring $\langle 1, 2, \ast, 4 \rangle$ matches $s_u^{[2,5]}$ with $\ast$ a wildcard. If the user sequence is $s_u^{[1,4]} = \langle 7, 8, 9 \rangle$, none of the substrings of $F$ matches $s_u^{[1,4]}$, and $m_{s_u^{[1,4]}} = \langle 9 \rangle$.

Algorithm 1: ExactMatchWithWildCard($s_u^{[1,t]}$, $F$)

Input: A user sequence $s_u^{[1,t]} = \langle s_u^1, \cdots, s_u^{t-1} \rangle$ and the set of frequent sequences $F$

Output: $m_{s_u^{[1,t]}}$, the longest substring among the substrings in $F$ that end to the most recent item $s_u^t$ in $s_u^{[1,t]}$ that belongs to $F$, or $s_u^{t-1}$ if any.

1. $\text{sequence} \leftarrow \langle s_u^1, \cdots, s_u^{t-1} \rangle$
2. $\text{path} \leftarrow ()$
3. for $i = t - 1$ downto 1 do
   4.   item $\leftarrow s_u^i$
   5.   if $\text{path} = ()$ then
      6.       if item $\in F$ then
      7.          $\text{path} \leftarrow (\text{item})$
      8.       else
      9.          if $\langle \text{item} \rangle \cdot \text{path} \in F$ then
         10.             $\text{path} \leftarrow \langle \text{item} \rangle \cdot \text{path}$  \Comment{The concatenation of strings (item) and path}
   11. if $\text{path} = ()$ then
      12.       $\text{path} \leftarrow \langle s_u^{t-1} \rangle$
   13. return $\text{path}$  \Comment{Here $m_{s_u^{[1,t]}} \leftarrow \text{path}$}

4.2.3 Constraining $P_t$ with short-term dynamics

The personalized sequence $m_{s_u^{[1,t]}}$ is then used to constrain the latent vector $P_t$ to be as close as possible to the items contained in $m_{s_u^{[1,t]}}$. A damping factor $\eta_r$, depending on the item position $r$, is used to increase the importance of recent items of $m_{s_u^{[1,t]}}$. With $P_{m_{s_u^{[1,t]}}}^r$ be the vector corresponding to the item of $m_{s_u^{[1,t]}}$ at position $r$, we have:

$$\hat{p}_{u,i,t} \propto -\left\| \sum_{r=1}^R \eta_r P_{m_{s_u^{[1,t]}}}^r - P_t \right\|_2^2. \tag{2}$$

where $R = |m_{s_u^{[1,t]}}|$ is the number of items in $m_{s_u^{[1,t]}}$. The value of $\eta_r$ increases with $r$ to give more weight to recent items. To normalize the short-term part of REBUS, we make $\eta_r$ follow the softmax (normalized exponential) (Bishop 2006) of the following linear function:

\[ \eta_r = \frac{e^{\theta r}}{\sum_{r=1}^R e^{\theta r}} \]
Let us consider the same example as above where $m_{s_{1,2,4}^{1,7}} = \{1, 2, 4\}$ and $\{1, 2, *, 4\}$ matches $s_{u}^{[2,5]}$. Positions of items $\{1, 2, 4\}$ are respectively 1, 2 and 3 and $|m_{s_{1,2,4}^{1,7}}| = 3$ because we do not take into consideration wildcard item *. Thus $\eta_1 \approx 0.23$, $\eta_2 \approx 0.321$ and $\eta_3 \approx 0.448$. As expected the most recent item 4 has a greater importance compared to items 1 and 2.

4.3 The long-term and short-term metric embedding model

The embedding of items into a metric space has two main advantages. First, it brings better generalization as Euclidean distances preserve the triangle inequalities. Second, it makes it possible to fully unify the long and short-term dynamics (as expressed by equations 1 and 2) of each item into a single embedding vector resulting from the computation of one distance:

$$\hat{p}_{u,i,t} \propto -||| \text{Long-term} + \text{Short-term} | |^2.$$

It is also usual to add a bias term $\beta_i$ specific to each item, and a hyperparameter $\gamma$ to weigh the importance between long-term and short-term dynamics:

$$\hat{p}_{u,i,t} \propto -\left( \beta_i + \sum_{j \in I_s^{x,t} \setminus \{i\}} \gamma \sum_{j \in I_s^{x,t} \setminus \{i\}} P_j + (1 - \gamma) \sum_{r=1}^R \eta_r P_{m_{s_{1,2,4}^{1,7}}^{x,t} s_{u}^{1,t}} - P_i | |^2 \right)$$

4.4 Bayesian personalized ranking optimization criterion

The goal for a sequential recommender system is, for all users, to rank the ground-truth item higher than all other items. Bayesian Personalized Ranking (BPR) (Rendle et al. 2009) formalizes this problem as maximizing the posterior probability of the model parameters $\theta$, given the order relation $>_{u,t}$ on items: $p(\theta | >_{u,t})$. Using Bayes’ rule, the probability is proportional to $p(\theta | >_{u,t}) p(\theta)$. The goal is thus to identify the parameters that maximize the likelihood of correctly ordering items. It is formally expressed as having $i >_{u,t} j$, which means that $i$ is ranked higher than item $j$ for user $u$ at position $t$ with $i = s_{u}^{t}$ the ground truth item. Assuming independence of items, their orders and users, this leads to estimate the model parameters by the maximum a posteriori probability (MAP):

$$\arg \max_{\theta} = \ln \prod_{u \in U} \prod_{t=2} \prod_{j \neq s_{u}^{t}} p(s_{u}^{t} >_{u,t} j | \theta) p(\theta)$$
The parameters of REBUS model are the embedded vectors $P_i$ and the bias terms $\beta_i$ for $i \in I$. $p(s_u^t > u, t | j | \theta)$ is the probability that the ground truth item $s_u^t$ is correctly ranked with respect to $j$ by the model, that is:

$$p(s_u^t > u, t | j | \theta) = p(\hat{p}_{u, s_u^t, t} > u, t \mid \hat{p}_{u, j, t} | \theta) = p(\hat{p}_{u, s_u^t, t} - \hat{p}_{u, j, t} > u, t 0 | \theta)$$

This quantity is approximated by $\sigma(\hat{p}_{u, s_u^t, t} - \hat{p}_{u, j, t})$, where $\sigma(z) = \frac{1}{1 + e^{-z}}$ is the logistic sigmoid function. Taking as prior probability for $\theta$ a normal distribution with zero mean and $\lambda_{\theta} I$ as variance-covariance matrix, the criterion to optimize (equation 4) becomes:

$$\arg \max_{\theta} = \sum_{u \in U} \sum_{t=2}^{\lvert s_u \rvert} \sum_{j \neq s_u^t} \ln \sigma(\hat{p}_{u, s_u^t, t} - \hat{p}_{u, j, t}) - \lambda_{\theta} \lVert \theta \rVert^2,$$

where $\lambda_{\theta}$ is a regularization hyperparameter.

### 4.5 Model training

REBUS learns the embedded vectors $P_i$ and the bias terms $\beta_i$ by maximizing equation 5. Hyperparameters – that is to say $\alpha, \gamma, L, \text{minCount}, \text{max_length}$ and regularization hyperparameters $\lambda_{\theta}$ – are chosen using a grid search strategy. The learning rate as well as $k$, the length $k$ of embedded vectors, are fixed by the analyst. The hyperparameter $k$ must be chosen knowing that the larger it is, the more precise the item vectors and the more costly the computation of the model.

Item embedding $P_i$ are randomly initialized using Xavier initialization (Glorot and Bengio 2010) and the bias terms $\beta_i$ are initialized to zero.

The parameters are learned using a variant of Stochastic Gradient Descent (SGD) called Adam optimizer (Kingma and Ba 2015) with a batch size of 128 examples. For each batch, it consists in uniformly sampling a user $u$, a position $t$, that gives the positive item $i = s_u^t$, and a 'negative’ item $j$.

### 4.6 Recommendation

Once REBUS has been trained, it can be used to make recommendations. Considering the past actions of a user $s_u^{1:t}$, the most appropriate frequent sequence $m_u^{1:t}$ is found and used in equation 3 with $i$ a candidate item. The item with the highest $\hat{p}_{u,i,t}$ value (or equivalently the smallest Euclidean distance) is recommended. Note that REBUS can recommend more than one item, e.g. the Top-N items with the $N$ smallest Euclidean distances.
4.7 Discussion

As said before, one of the key characteristics of REBUS is to only embed items, like SASRec, GRU4Rec and FMC do. This characteristic brings two advantages compared to other models that embed items and users (like FPMC, PRME, TransRec and CASER). First, it has a smaller space complexity and second it suffers less from the cold start problem.

*Space complexity:* The learned parameters of REBUS are the item embeddings – each being of dimension $k$ – and the items bias terms. This results in a number of parameters in $O(|I| \times k + |I|)$. Thus, the complexity of our model does not grow with the number of users, unlike other models that embed users and items.

*Recommending items to new users:* We can say that REBUS suffer less from the cold-start problem, since it can make recommendations to new users who have interacted only with a single item of the system. We show in the next section that REBUS has good performance compared to other models when it comes to the problem of cold start users.

5 Experiments

In this section, we present a thorough empirical study. We first begin by describing the 12 real-world datasets we consider, as well as the questions these experiments aim to answer. Then, we report an extensive comparison of REBUS with 10 state-of-the-art algorithms for sequential recommendation. Results demonstrate that REBUS outperforms state-of-the-algorithms according to several metrics in most cases. Finally, we provide a deeper analysis of REBUS, especially how frequent substrings are actually used in sequential recommendation, the impact of the user preferences with regard to the sequential dynamics and how REBUS can be tuned to give better results. For reproducibility purposes, the source code and the data are made available. 1

5.1 Datasets and aims

To evaluate the performance of REBUS on both sparse and dense datasets from different domains, we consider 5 well-known benchmarks and introduce a new dataset:

Amazon was introduced by (McAuley et al. 2015). It contains Amazon product reviews from May 1996 to July 2014 from several product categories. We have chosen to use the 3 following diversified categories: Automotive, Office product and video games.

MovieLens 1M 2 (Harper and Konstan 2015) is a popular dataset including 1 million movie ratings from 6040 users between April 2000 and February 2003. We used the datasets pre-processed by selecting the most recent $x$ ratings for each user, $x \in \{5, 10, 20, 30, 50\}$. These datasets allow us to study the performance of REBUS on

1 https://bit.ly/3gwZAOF.
2 http://grouplens.org/datasets/movielens/1m/.
Table 2  Main characteristics of the datasets

| Datasets   | #Users | #Items | #Actions | #A/#U | #A/#I | Sparsity  |
|------------|--------|--------|----------|-------|-------|-----------|
| Others     |        |        |          |       |       |           |
| Epinions   | 5015   | 8335   | 26,932   | 5.37  | 3.23  | 99.94%    |
| Foursquare | 43,110 | 13,335 | 306,553  | 7.11  | 22.99 | 99.95%    |
| Adressa    | 141,933| 3257   | 1,861,901| 13.12 | 571.66| 99.60%    |
| Viisitiv   | 1398   | 590    | 16,417   | 11.74 | 27.83 | 98.01%    |
| Amazon     |        |        |          |       |       |           |
| Ama-Auto   | 34,316 | 40,287 | 183,573  | 5.35  | 4.56  | 99.99%    |
| Ama-Office | 16,716 | 22,357 | 128,070  | 7.66  | 5.73  | 99.97%    |
| Ama-Game   | 31,013 | 23,715 | 287,107  | 9.26  | 12.11 | 99.96%    |
| Movielens  |        |        |          |       |       |           |
| ML-5       | 6040   | 2848   | 30175    | 5.00  | 10.60 | 99.82%    |
| ML-10      | 6040   | 3114   | 59,610   | 9.87  | 19.14 | 99.68%    |
| ML-20      | 6040   | 3212   | 111,059  | 18.39 | 33.41 | 99.45%    |
| ML-30      | 6040   | 3391   | 152,160  | 25.19 | 44.87 | 99.26%    |
| ML-50      | 6040   | 3467   | 215,676  | 35.71 | 62.21 | 98.87%    |

the same dataset but with different levels on sparsity (i.e. MovieLens with the 5 most recent ratings will be more sparse than MovieLens with the 50 most recent ratings). **Epinions** describes consumer reviews for the website Epinions from January 2001 to November 2013. This dataset was collected by the authors of (Zhao et al. 2014).

**Foursquare** depicts a large number of user check-ins on the Foursquare website from December 2011 to April 2012.

**Adressa** (Gulla et al. 2017) includes news articles (in Norwegian). The dataset was offered by Adresseavisen, a local newspaper company in Trondheim, Norway.

**Visiativ** is a new dataset that gathers the downloads of Computer-Aided-Design resources by engineers and designers from November 2014 to August 2018. This dataset seems relevant to us to evaluate temporal recommendation since the downloaded documents are tutorials that users read to train themselves and improve their comprehension of some specific software.

For each dataset, ratings are converted into implicit feedback and we only consider users and items that have at least 5 interactions. The main characteristics of these datasets are reported in Table 2.

To evaluate the performance and the limits of **REBUS**, we propose to answer the following questions:

- What are the performances of **REBUS** compared to those of state-of-the-art algorithms for sequential recommendation for sparse and dense datasets? What about **REBUS**’s performance in presence of cold-start users?
- Does **REBUS** take benefit from sequential behaviors in a better way than Markov chains of fixed order do?
- What is the most important component? The user preferences? The sequential dynamics or both?
- What about the recommendations? Does **REBUS** provide diverse recommendations?

3 https://www.visiativ.com/en-us/.
5.2 Comparison methods

We compare REBUS to 10 state-of-the-art methods designed for both item and sequential recommendation:

- Popularity (POP), the naive baseline that ranks items according to their popularity (Aggarwal 2016);
- Bayesian Personalized Ranking (BPR) (Rendle et al. 2009), that recommends items by considering only user preferences with matrix factorization techniques;
- Factorized Markov Chains (FMC) (Rendle et al. 2010), that is based on the factorization of the item-to-item transition matrix;
- Factorized Personalized Markov Chains (FPMC) (Rendle et al. 2010), that considers both user preferences and user dynamics thanks to matrix factorization and first-order Markov chains;
- Personalized Ranking Metric Embedding (PRME) (Feng et al. 2015), that embeds user preferences and user dynamics into two Euclidean distances;
- Translation-based Recommendation (TransRec) (He et al. 2017a), that unifies user preferences and sequential dynamics with translations;
- Convolutional Sequence Embedding Recommendation (CASER) (Tang and Wang 2018), a CNN-based method that captures both user preferences and user sequential dynamics;
- Self-Attentive Sequential Recommendation (SASRec) (Kang and McAuley 2018), a self-attention based model that captures both user preferences and user sequential dynamics;
- Session-based KNN (S-KNN) (Jannach and Ludewig 2017), a nearest-neighbor-based approach designed for session-recommendation. S-KNN compares the current session with past sessions in training data. In our experiments, a user’s sequence is assimilated to a session.
- GRU4Rec (Hidasi et al. 2016), a RNN-based method tailored for session-based recommendation. More precisely, we use the improved version of GRU4Rec (Hidasi and Karatzoglou 2018) which adopts new loss functions and extends the sampling strategy.

Table 3 provides a comparison of the above methods according to several criteria: whether they are personalized, sequentially aware, metric-based, integrated into a unified model, based on personalized order Markov chains or on a model that only embed items. Indications about the time needed to train the models are also reported in the last column. For fair comparisons, we return the time required for 25 epochs on Amazon Office. As we can see, the time needed for REBUS is comparable to the one of other methods. It includes the time needed to extract frequent substrings.

5.3 Experimental settings

We apply the same partition as (Bayer et al. 2017; He et al. 2017b, a; Kang and McAuley 2018) for user sequences (also known as leave-one-out evaluation). Indeed, for each dataset, the user sequences are split into 3 parts:
Table 3  Overview of the different models. P: Personalized?, S: Sequentially-aware?, M: Metric-based?, U: Unified model?, O: personalized Order Markov chains?, I: embed only items?, T: Time (in seconds) for the Amazon Office dataset with 25 epochs

| Property                  | P | S | M | U | O | I | T       |
|---------------------------|---|---|---|---|---|---|---------|
| Pop                       | ✗ | ✗ | ✗ | ✗ | ✗ | ✗ | ~1s     |
| BPR                       | ✓ | ✗ | ✗ | ✗ | ✗ | ✗ | ~21     |
| FMC                       | ✗ | ✓ | ✗ | ✗ | ✗ | ✗ | ~19     |
| FPMC                      | ✓ | ✓ | ✗ | ✗ | ✗ | ✗ | ~26s    |
| PRME                      | ✓ | ✓ | ✓ | ✗ | ✗ | ✗ | ~26s    |
| TransRec                  | ✓ | ✓ | ✓ | ✗ | ✗ | ✗ | ~27s    |
| CASER                     | ✓ | ✓ | ✓ | ✗ | ✗ | ✗ | ~75s    |
| SASRec                    | ✓ | ✓ | ✓ | ✗ | ✗ | ✗ | ~20s    |
| S-KNN                     | ✓ | ✓ | ✓ | ✗ | ✗ | ✗ | ~1s     |
| GRU4Rec                   | ✓ | ✓ | ✓ | ✗ | ✗ | ✗ | ~19s    |
| REBUS                     | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ~30s    |

1. The most recent item is used for the test. It is named ground-truth item and denoted $g_u$;
2. The second most recent item is used for the validation when learning the model;
3. Other items of the user sequence are used to train the models.

The performances of the models are assessed by three widely used metrics for sequential recommendation (Rendle et al. 2009; He et al. 2017a; Kang and McAuley 2018):

**Area Under the ROC Curve (AUC).**

$$\text{AUC} = \frac{1}{|U|} \sum_{u \in U} \frac{1}{|I \setminus I_{su}|} \sum_{j \in I \setminus I_{su}} 1(\hat{p}_{u,g_u,t} > \hat{p}_{u,j,t}),$$

where the indicator function $1(b)$ returns 1 if its argument $b$ is True, 0 otherwise. This measure calculates how high the ground-truth item of each user has been ranked in average.

**Hit Rate at position X (HIT_X).**

$$\text{HIT}_X = \frac{1}{|U|} \sum_{u \in U} 1(R_{g_u} \leq R_X),$$

where $R_{g_u}$ is the ranking of the ground-truth item and $R_X$ is the Xth ranking. The HIT_X function returns the average number of times the ground-truth item is ranked in the top X items. We compute HIT_5, HIT_10, HIT_25 and HIT_50. Note that HIT_X is equal to Recall@X and proportional to Precision@X, two others common metrics in recommendation, as we only have one test item per user.

**Normalized Discounted Cumulative Gain at position X (NDCG_X).**

$$\text{NDCG}_X = \frac{1}{|U|} \sum_{u \in U} \frac{1(R_{g_u} \leq R_X)}{\log_2(R_{g_u} + 1)}.$$
The NDCG_X is a position-aware metric which assigns larger weights to higher positions. We compute NDCG_5, NDCG_10, NDCG_25 and NDCG_50.

We also consider two metrics to assess how diverse are the recommendations made by a model.

**Popularity rate X (Pop_X).** The POP_X is the proportion of predicted items that are similar to the X most popular items. We can note that the POP model (the most popular) should have a Pop_X value equal to 1. However, it is not always the case since already consumed items are not recommended to user.

**Diversity rate X (Div_X).** The DIV_X is the proportion of items that are predicted at least once to all users (the number of unique predicted items divided by the total number of items). Div_X is sometimes referred to as aggregate diversity (Adomavicius and Kwon 2012).

Contrary to (Kang and McAuley 2018; He et al. 2017b; Huang et al. 2018), which follow the strategy of taking a sample of $x$ negative items during the evaluation (i.e. items that a user has not interacted with), we compute the metrics with all possible negative items (also known as *TrainItems* (Said and Bellogín 2014)) to have an impartial assessment of all methods with exact measures instead of approximated ones.

**REBUS** is implemented using *TensorFlow* and *Adam* optimizer (Kingma and Ba 2015). In order to make a fair comparison, we implemented BPR, FMC, FPMC, PRME, TransRec with the same architecture as **REBUS**. For CASER, GRU4Rec and SASRec, we use the code provided by the authors. To limit the numbers of combinations to explore with grid search, we fix the following parameters for all models: The learning rate to 0.001, the batch size to 128, the dimension of the learned latent vectors $k$ is set to 10 and we stop the training if there is no improvement of the AUC validation for 250 epochs. All regularization hyperparameters are taken in \( \{0, 0.01, 0.01, 0.1, 1\} \). For models with other hyperparameters we have tried those given by the authors, except for CASER and GRU4Rec which require too many hyperparameters: There are about 50000 possible combinations for CASER and 100000 for GRU4Rec. Unlike the other models, we did not use early stopping for GRU4Rec because it degrades its performance, and therefore, for equity issues, the number of epochs and the learning rate were determined via a random search with the other hyperparameters. Regarding specific hyperparameters of **REBUS**, $\alpha$ is taken in \( \{0.3, 0.5, 0.7, 1.0\} \), $\gamma$ in \( \{0.3, 0.5, 0.7\} \), $\text{minCount}$ has for default value 2, and $L$ takes its value between 3 and 5 for small datasets (Visiativ, Epinions, ML-5) and between 8 and 15 for other datasets$^5$.

### 5.4 Performance study

The performances of the different methods on every dataset are reported in Table 4 and Table 5$^6$. We can observe that **REBUS** outperforms other models on most datasets,

---

$^4$ Only for Foursquare dataset, we observed that it is better not to have $\gamma$ (remove $\gamma$ and $(1-\gamma)$ in Equation 3). It is noteworthy that recommendations may be different to the case where $\gamma = 0.5$ due to the bias terms $\beta_i$.

$^5$ Best hyperparameters for each dataset are reported in supplementary material (Lonjarret et al. 2020a).

$^6$ We only show HIT_25, HIT_50, NDCG_25 and NDCG_50 in the tables. HIT_5, HIT_10, NDCG_5 and NDCG_10 are reported in supplementary material (Lonjarret et al. 2020a).
having the best AUC value. **REBUS** also performs well on all other metrics, with an average rank between 3.58 to 4.00, and an improvement between 1.35% to 6.94% over the best competitor, and the best HIT_25, HIT_50, NDCG_25 and NDCG_50 in overall. For HIT_5, HIT_10, NDCG_5 and NDCG_10, we respectively found improvements of 7.15%, 3.22%, 11.56% and 8.74% compared to the best competitor. However, these high percentages have to be nuanced because the obtained values are often low (i.e., in comparison to AUC) and the results vary a lot from one dataset to another.

In general, BPR-MF outperforms FMC. This confirms that user preferences have a more important role than the sequential dynamics in the recommendations of the next-item. As expected, SASRec performs very well on dense datasets (i.e., ML30, ML50 where \( \#A/\#U > 20 \)). However, this model is outperformed by several others on sparse datasets. We can observe that **REBUS** also outperforms PRME and FPMC on sparse datasets. It gives evidence that having independent latent vectors to model user preferences and sequential dynamics is not an advantage for sparse datasets. As SAS-rec, GRU4Rec performs very well on dense datasets especially on ML20, ML30 and ML50 with the best HIT_X and NDCG_X. Except for these three datasets, GRU4Rec is outperformed by several competitors on the remaining datasets. S-KNN exhibits bad AUC performance because of its architecture\(^7\): S-KNN cannot rank items that are not in the neighborhood of the user target. Besides, S-KNN reports fair performance on HIT_X and NDCG_X and outperforms all models on Amazon datasets – this confirms the observations made in (Ludewig and Jannach 2018) that despite the simplicity of S-KNN, it can perform as well or even better than complex models – but it is outperformed by **REBUS** for all other datasets.

The improvement of **REBUS** according to each model is reported in Table 6. Considering all datasets, **REBUS** outperforms all the studied models. However, some models exhibits better performances when only focusing on MovieLens datasets (e.g., SASRec, GRU4Rec, TransRec, FPMC and FMC for which **REBUS** has negative improvements).

In Fig. 2, we study the effect of the size of the latent vectors \( k \in \{10, 20, 30, 40, 50\} \)\(^8\) on AUC. In most of the cases, **REBUS** has the best performances for each \( k \) value. It is worth to mention that **REBUS** obtains better results when the size of the latent vectors increases, which is not the case of other methods. For instance, SASRec becomes worse when the number of latent dimensions increases for 3 of 4 datasets.

Eventually, we study the diversity of recommendations provided by each model. Figure 3 reports POP_X and DIV_X for each model. For POP_X, the higher the value, the more the model tends to recommend popular items. For DIV_X the higher the value, the more the model tends to recommend different items. These results give evidence that **REBUS**’ recommendations are rarely based on the most popular items. However, only a subset of items (between 20% and 50%) is recommended to the users, which remains comparable to most other models.

---

7 Performances of other nearest-neighbor-based approaches as Item-based KNN and Sequence-Aware Extensions (V-S-KNN, S-S-KNN and SF-S-KNN) are reported in supplementary material (Lonjarret et al. 2020a).

8 To avoid running again the grid search, we took the best combination of hyperparameters that we previously found for \( k = 10 \).
Table 4: AUC, HIT_25, HIT_50, NDCG_25 and NDCG_50 for the different models on Epinions, Foursquare, Adressa, Visiativ, Amazon-Automotive, Amazon-Office and Amazon-Video-Games datasets. The last row, called *Improv. vs Best*, shows the improvement in percentage of our method compared to the other best model (best obtained results are in bold and best obtained results for concurrent models are underlined). The last column is the average performance on these 7 datasets, the average of all datasets is included in Table 5.

| Metric | Epinions | Foursq | Adressa | Visiativ | Auto | Office | Games | Avg  |
|--------|----------|--------|---------|----------|------|--------|-------|------|
| POP    |          |        |         |          |      |        |       |      |
| AUC    | 0.4575   | 0.9169 | 0.9582  | 0.7864   | 0.5856| 0.6412 | 0.7484| 0.7277|
| HIT25  | 2.25%    | 46.66% | 19.39%  | 32.04%   | 2.54%| 0.62%  | 3.50% | 15.29%|
| HIT50  | 3.42%    | 55.65% | 30.89%  | 48.24%   | 3.75%| 1.67%  | 5.16% | 21.25%|
| NDGC25 | 0.80%    | 18.92% | 7.21%   | 12.87%   | 0.96%| 0.15%  | 1.37% | 6.04% |
| NDGC50 | 1.03%    | 20.68% | 9.40%   | 15.97%   | 1.19%| 0.36%  | 1.69% | 7.19% |
| FMC    |          |        |         |          |      |        |       |      |
| AUC    | 0.5421   | 0.9508 | 0.9842  | 0.8354   | 0.6251| 0.6771 | 0.8473| 0.7803|
| HIT25  | 1.67%    | 53.06% | 64.23%  | 45.66%   | 2.57%| 1.35%  | 10.64%| 25.60%|
| HIT50  | 2.63%    | 64.19% | 77.99%  | 58.78%   | 3.78%| 2.67%  | 15.71%| 32.25%|
| NDGC25 | 0.77%    | 26.13% | 29.19%  | 23.24%   | 1.05%| 0.52%  | 4.19% | 12.15%|
| NDGC50 | 0.95%    | 28.29% | 31.84%  | 25.75%   | 1.28%| 0.77%  | 5.16% | 13.44%|
| BPR    |          |        |         |          |      |        |       |      |
| AUC    | 0.5593   | 0.9474 | 0.9656  | 0.8270   | 0.6649| 0.7072 | 0.8590| 0.7901|
| HIT25  | 2.88%    | 46.95% | 25.83%  | 42.72%   | 2.90%| 1.58%  | 7.82% | 18.67%|
| HIT50  | 4.49%    | 58.38% | 40.16%  | 57.42%   | 4.59%| 2.59%  | 12.42%| 25.72%|
| NDGC25 | 1.20%    | 20.44% | 9.35%   | 17.13%   | 1.10%| 0.63%  | 2.85% | 7.53% |
| NDGC50 | 1.51%    | 22.66% | 12.10%  | 19.95%   | 1.43%| 0.82%  | 3.73% | 8.88% |
| FPMC   |          |        |         |          |      |        |       |      |
| AUC    | 0.5536   | 0.9492 | 0.9848  | 0.8433   | 0.6482| 0.6958 | 0.8777| 0.7932|
| HIT25  | 1.63%    | 56.46% | 65.36%  | 46.67%   | 2.36%| 1.60%  | 12.28%| 26.62%|
| HIT50  | 2.51%    | 65.38% | 79.39%  | 60.36%   | 3.64%| 2.53%  | 18.33%| 33.16%|
| NDGC25 | 0.70%    | 30.88% | 30.14%  | 22.68%   | 0.88%| 0.56%  | 4.71% | 12.94%|
| NDGC50 | 0.87%    | 32.61% | 32.85%  | 25.35%   | 1.13%| 0.74%  | 5.87% | 14.20%|
| Metric    | Epinions | Foursq | Adressa | Visiativ | Auto | Office | Games | Avg   |
|-----------|----------|--------|---------|----------|------|--------|-------|-------|
| PRME      |          |        |         |          |      |        |       |       |
| AUC       | 0.6071   | 0.9538 | 0.9849  | 0.8572   | 0.6749| 0.7154 | 0.8759| 0.8099|
| HIT25     | 1.88%    | 55.64% | 64.78%  | 45.09%   | 2.34%| 2.78%  | 12.63%| 26.45%|
| HIT50     | 2.72%    | 67.23% | 78.19%  | 59.43%   | 3.85%| 4.87%  | 18.74%| 33.58%|
| NDGC25    | 0.85%    | 30.71% | 30.33%  | 24.86%   | 0.85%| 0.95%  | 4.89% | 13.35%|
| NDGC50    | 1.01%    | 32.97% | 32.92%  | 27.61%   | 1.14%| 1.34%  | 6.07% | 14.72%|
| TransRec  |          |        |         |          |      |        |       |       |
| AUC       | 0.6138   | 0.9619 | 0.9852  | 0.8647   | 0.6991| 0.7320 | 0.8869| 0.8205|
| HIT25     | 1.98%    | 58.67% | 65.70%  | 51.25%   | 3.98%| 3.97%  | 10.76%| 28.04%|
| HIT50     | 3.14%    | 67.81% | 78.81%  | 64.73%   | 5.99%| 6.37%  | 16.51%| 34.77%|
| NDGC25    | 0.94%    | 33.13% | 31.99%  | 26.66%   | 1.56%| 1.38%  | 4.08% | 14.25%|
| NDGC50    | 1.16%    | 34.90% | 34.52%  | 29.26%   | 1.95%| 1.84%  | 5.18% | 15.55%|
| S-KNN     |          |        |         |          |      |        |       |       |
| AUC       | 0.0684   | 0.8330 | 0.9367  | 0.8369   | 0.1476| 0.2938 | 0.6322| 0.5355|
| HIT25     | 2.51%    | 63.90% | 27.77%  | 44.87%   | 5.15%| 5.36%  | 13.94%| 23.36%|
| HIT50     | 3.70%    | 70.75% | 40.33%  | 61.15%   | 6.71%| 7.32%  | 19.54%| 29.93%|
| NDGC25    | 1.32%    | 45.92% | 45.92%  | 21.07%   | 2.49%| 2.77%  | 5.82% | 13.30%|
| NDGC50    | 1.55%    | 47.25% | 15.36%  | 24.95%   | 2.79%| 3.15%  | 6.89% | 14.56%|
| GRU4Rec   |          |        |         |          |      |        |       |       |
| AUC       | 0.5813   | 0.9563 | 0.9829  | 0.8384   | 0.6568| 0.7228 | 0.8517| 0.7986|
| HIT25     | 2.09%    | 49.38% | 57.01%  | 45.30%   | 2.12%| 3.49%  | 8.92% | 24.04%|
| HIT50     | 3.74%    | 60.85% | 74.03%  | 57.85%   | 3.56%| 5.96%  | 13.59%| 31.37%|
| NDGC25    | 0.71%    | 28.96% | 23.33%  | 22.30%   | 0.78%| 1.28%  | 3.41% | 11.54%|
| NDGC50    | 1.03%    | 31.18% | 26.61%  | 24.72%   | 1.06%| 1.75%  | 4.31% | 12.95%|
| Metric | Epinions | Foursq | Adressa | Visiativ | Auto | Office | Games | Avg |
|--------|----------|--------|---------|----------|------|--------|-------|-----|
| CASER  | AUC      | 0.6238 | 0.9259  | 0.9750   | 0.8544 | 0.6872 | **0.7510** | 0.8282 | 0.8065 |
|         | HIT25    | 2.44%  | 46.45%  | 60.65%   | 46.74% | 2.63%  | 1.42%  | 6.45% | 23.83% |
|         | HIT50    | 3.88%  | 56.73%  | 74.86%   | 61.08% | 3.64%  | 2.50%  | 10.23% | 30.42% |
|         | NDGC25   | 0.96%  | 18.39%  | 26.29%   | 21.35% | 0.94%  | 0.45%  | 2.37%  | 10.11% |
|         | NDGC50   | 1.24%  | 20.40%  | 29.04%   | 24.12% | 1.14%  | 0.66%  | 3.10%  | 11.38% |
| SASRec  | AUC      | 0.6251 | 0.9617  | **0.9870** | 0.8731 | 0.6861 | 0.7392 | 0.8812 | 0.8219 |
|         | HIT25    | 2.60%  | 55.04%  | **67.30%** | 51.76% | 2.55%  | 3.65%  | 9.16% | 27.44% |
|         | HIT50    | 4.00%  | 64.97%  | **82.15%** | 65.38% | 4.07%  | 6.12%  | 14.54% | 34.46% |
|         | NDGC25   | 1.03%  | 30.70%  | 29.23%   | 25.00% | 0.95%  | 1.30%  | 3.56%  | 13.11% |
|         | NDGC50   | 1.30%  | 32.62%  | 32.10%   | 27.61% | 1.24%  | 1.77%  | 4.59%  | 14.46% |
| REBUS   | AUC      | **0.6524** | 0.9677  | 0.9854   | **0.8735** | 0.7184 | 0.7507 | **0.8934** | **0.8345** |
|         | HIT25    | **3.39%** | 62.77%  | 66.73%   | **53.33%** | 4.24%  | 4.12%  | 9.30% | **29.13%** |
|         | HIT50    | **5.32%** | 70.41%  | 79.77%   | **67.17%** | 6.20%  | 6.24%  | 14.61% | **35.68%** |
|         | NDGC25   | 1.31%  | **46.02%** | **32.00%** | 24.20% | 1.68%  | 1.52%  | 3.52%  | **15.75%** |
|         | NDGC50   | **1.68%** | **47.49%** | **34.52%** | 26.86% | 2.05%  | 1.92%  | 4.54%  | **17.01%** |
| Rank    | AUC      | 1      | 1       | 2       | 1      | 1      | 2      | 1     | 1.29    |
|         | HIT25    | 1      | 2       | 2       | 1      | 2      | 2      | 6     | 2.29    |
|         | HIT50    | 1      | 2       | 2       | 1      | 2      | 3      | 6     | 2.43    |
|         | NDGC25   | 2      | 1       | 1       | 4      | 2      | 2      | 7     | 2.71    |
|         | NDGC50   | 1      | 1       | 1       | 4      | 2      | 2      | 7     | 2.57    |
Table 4 continued

| Metric  | Epinions | Foursq | Adressa | Visiativ | Auto | Office | Games | Avg |
|---------|----------|--------|---------|----------|------|--------|-------|-----|
| Improv. vs Best |          |        |         |          |      |        |       |     |
| AUC     | 4.37%    | 0.60%  | −0.16%  | 0.05%    | 2.76%| −0.04% | 0.73% | 1.53%|
| HIT25   | 17.71%   | −1.77% | −0.85%  | 3.03%    | −17.67%| −23.13%| −33.29%| 3.89%|
| HIT50   | 18.49%   | −0.48% | −2.90%  | 2.74%    | −7.60%| −14.75%| −25.23%| 2.62%|
| NDGC25  | −0.76%   | 0.22%  | 0.03%   | −9.23%   | −32.53%| −45.13%| −39.52%| 10.53%|
| NDGC50  | 8.39%    | 0.51%  | 0.00%   | −8.20%   | −26.52%| −39.05%| −34.11%| 9.39%|
Table 5  AUC, HIT._25, HIT._50, NDCG._25 and NDCG._50 for the different models on ML-5, ML-10, ML-20, ML-30 and ML-50 datasets. The last row, called Improv. vs Best, shows the improvement in percentage of our method compared to the other best model (best obtained results are in bold and best obtained results for concurrent models are underlined). The last column is the average performance on the 12 datasets, including datasets of Table 4.

|       | POP   | FMC   | BPR   | FPMC  |
|-------|-------|-------|-------|-------|
|       | Metric | ML-5  | ML-10 | ML-20 | ML-30 | ML-50 | Avg(ML) | Avg(All) |
| AUC   | 0.7352 | 0.7722 | 0.7919 | 0.7981 | 0.8032 | 0.7801 | 0.7496 |
| HIT25 | 7.48%  | 7.60%  | 7.57%  | 7.57%  | 7.72%  | 7.59%  | 12.08%  |
| HIT50 | 12.65% | 12.93% | 12.82% | 12.72% | 13.13% | 12.85% | 17.75%  |
| NDGC25| 2.64%  | 2.75%  | 2.73%  | 2.66%  | 2.70%  | 2.70%  | 4.65%   |
| NDGC50| 3.63%  | 3.76%  | 3.74%  | 3.64%  | 3.74%  | 3.70%  | 5.74%   |
| AUC   | 0.7414 | 0.8054 | 0.8446 | 0.8560 | 0.8689 | 0.8233 | 0.7982  |
| HIT25 | 9.84%  | 13.84% | 16.44% | 16.74% | 18.63% | 15.10% | 21.22%  |
| HIT50 | 15.25% | 20.60% | 24.95% | 25.58% | 27.72% | 22.82% | 28.32%  |
| NDGC25| 3.57%  | 5.72%  | 6.62%  | 6.69%  | 7.36%  | 5.99%  | 9.59%   |
| NDGC50| 4.61%  | 7.01%  | 8.25%  | 8.39%  | 9.11%  | 7.47%  | 10.95%  |
| AUC   | 0.7623 | 0.8241 | 0.8517 | 0.8570 | 0.8629 | 0.8316 | 0.8074  |
| HIT25 | 9.65%  | 11.96% | 11.49% | 10.66% | 11.24% | 11.00% | 15.47%  |
| HIT50 | 15.98% | 19.74% | 19.56% | 18.74% | 18.56% | 18.52% | 22.72%  |
| NDGC25| 3.52%  | 4.16%  | 4.06%  | 3.68%  | 3.87%  | 3.86%  | 6.00%   |
| NDGC50| 4.73%  | 5.65%  | 5.61%  | 5.22%  | 5.21%  | 5.30%  | 7.39%   |
| AUC   | 0.7309 | 0.8150 | 0.8629 | 0.8766 | 0.8891 | 0.8349 | 0.8106  |
| HIT25 | 7.82%  | 14.65% | 17.42% | 18.65% | 18.96% | 15.50% | 21.99%  |
| HIT50 | 13.26% | 22.90% | 26.28% | 27.60% | 28.63% | 23.74% | 29.23%  |
| NDGC25| 2.87%  | 5.57%  | 6.63%  | 7.27%  | 7.41%  | 5.95%  | 10.02%  |
| NDGC50| 3.91%  | 7.15%  | 8.33%  | 8.99%  | 9.26%  | 7.53%  | 11.42%  |
| Metric | ML-5     | ML-10    | ML-20    | ML-30    | ML-50   | Avg(ML)  | Avg(All) |
|--------|----------|----------|----------|----------|---------|----------|----------|
| PRME   |          |          |          |          |         |          |          |
| AUC    | 0.7771   | 0.8302   | 0.8645   | 0.8765   | 0.8867  | 0.8470   | 0.8254   |
| HIT25  | 10.51%   | 15.58%   | 14.56%   | 16.19%   | 18.00%  | 14.97%   | 21.67%   |
| HIT50  | 16.94%   | 23.81%   | 24.39%   | 25.55%   | 27.14%  | 23.57%   | 29.41%   |
| NDGC25 | 3.78%    | 5.79%    | 5.16%    | 5.94%    | 6.62%   | 5.46%    | 10.06%   |
| NDGC50 | 5.01%    | 7.37%    | 7.04%    | 7.72%    | 8.37%   | 7.10%    | 11.55%   |
| TransRec |       |          |          |          |         |          |          |
| AUC    | 0.7900   | 0.8477   | 0.8736   | 0.8816   | 0.8883  | 0.8563   | 0.8354   |
| HIT25  | 14.36%   | **16.99%** | 16.49%   | 16.00%   | 17.09%  | 16.18%   | 23.10%   |
| HIT50  | 22.45%   | **25.60%** | 26.51%   | 26.30%   | 27.87%  | 25.75%   | 31.01%   |
| NDGC25 | **5.33%** | **6.30%** | 5.96%    | 5.95%    | 6.35%   | 5.98%    | 10.80%   |
| NDGC50 | 6.88%    | **7.95%** | 7.89%    | 7.93%    | 8.42%   | 7.81%    | 12.32%   |
| S-KNN  |          |          |          |          |         |          |          |
| AUC    | 0.2778   | 0.7164   | 0.8115   | 0.8297   | 0.8421  | 0.6955   | 0.6022   |
| HIT25  | 9.46%    | 14.99%   | 12.85%   | 12.62%   | 11.97%  | 12.38%   | 18.78%   |
| HIT50  | 13.08%   | 22.62%   | 21.13%   | 20.50%   | 19.27%  | 19.32%   | 25.51%   |
| NDGC25 | 3.87%    | 5.55%    | 4.54%    | 4.48%    | 4.26%   | 4.54%    | 9.65%    |
| NDGC50 | 4.57%    | 7.01%    | 6.13%    | 5.99%    | 5.66%   | 5.87%    | 10.94%   |
| Metric     | ML-5     | ML-10     | ML-20     | ML-30     | ML-50     | Avg(ML)   | Avg(All) |
|------------|----------|-----------|-----------|-----------|-----------|-----------|----------|
| GRU4Rec    |          |           |           |           |           |           |          |
| AUC        | 0.7523   | 0.8151    | 0.8569    | 0.8727    | 0.8833    | 0.8361    | 0.8142   |
| HIT25     | 7.67%    | 12.49%    | 19.09%    | **21.26%**| **22.50%**| **16.60%**| 20.94%   |
| HIT50     | 13.26%   | 20.30%    | **27.84%**| **30.77%**| **32.31%**| 24.89%    | 28.67%   |
| NDGC25    | 2.92%    | 5.14%     | **7.88%** | **8.76%** | **9.27%** | **6.79%** | 9.56%    |
| NDGC50    | 3.99%    | 6.64%     | **9.56%** | **10.59%**| **11.15%**| **8.38%** | 11.05%   |
| CASER     |          |           |           |           |           |           |          |
| AUC        | 0.7493   | 0.8118    | 0.8534    | 0.8650    | 0.8764    | 0.8312    | 0.8168   |
| HIT25     | 7.40%    | 11.36%    | 14.27%    | 15.95%    | 18.66%    | 13.53%    | 19.54%   |
| HIT50     | 13.58%   | 18.20%    | 22.62%    | 24.57%    | 28.18%    | 21.43%    | 26.67%   |
| NDGC25    | 2.63%    | 4.06%     | 5.35%     | 5.89%     | 7.07%     | 5.00%     | 7.98%    |
| NDGC50    | 3.82%    | 5.37%     | 6.95%     | 7.54%     | 8.90%     | 6.52%     | 9.36%    |
| SASRec    |          |           |           |           |           |           |          |
| AUC        | 0.8000   | 0.8537    | 0.8760    | **0.8860**| **0.8957**| **0.8623**| 0.8387   |
| HIT25     | 12.77%   | 15.91%    | 16.11%    | 18.26%    | 18.55%    | 16.32%    | 22.80%   |
| HIT50     | 20.83%   | 25.32%    | 25.96%    | 28.20%    | 29.29%    | **25.92%**| 30.90%   |
| NDGC25    | 4.53%    | 6.08%     | 5.81%     | 6.97%     | 7.09%     | 6.10%     | 10.19%   |
| NDGC50    | 6.08%    | 7.89%     | 7.70%     | 8.87%     | 9.15%     | 7.94%     | 11.74%   |
| REBUS     |          |           |           |           |           |           |          |
| AUC        | **0.8031**| **0.8563**| **0.8772**| 0.8826    | 0.8884    | 0.8615    | **0.8458**|
| HIT25     | **14.42%**| 15.27%    | 15.60%    | 15.80%    | 16.72%    | 15.56%    | **23.48%**|
| HIT50     | **22.79%**| 25.50%    | 26.01%    | 26.21%    | 26.91%    | 25.48%    | **31.43%**|
| NDGC25    | 5.32%    | 5.59%     | 5.60%     | 5.80%     | 6.03%     | 5.66%     | **11.55%**|
| NDGC50    | **6.92%**| 7.55%     | 7.59%     | 7.80%     | 7.98%     | 7.57%     | **13.07%**|
| Metric   | ML-5 | ML-10 | ML-20 | ML-30 | ML-50 | Avg(ML) | Avg(All) |
|----------|------|-------|-------|-------|-------|---------|----------|
| Rank     |      |       |       |       |       |         |          |
| AUC      | 1    | 1     | 1     | 2     | 3     | 1.60    | 1.42     |
| HIT25    | 1    | 4     | 6     | 8     | 8     | 5.40    | 3.58     |
| HIT50    | 1    | 2     | 4     | 5     | 8     | 4.00    | 3.08     |
| NDGC25   | 2    | 5     | 6     | 8     | 8     | 5.80    | 4.00     |
| NDGC50   | 1    | 3     | 6     | 8     | 8     | 4.80    | 3.50     |
| Improv. vs Best |    |       |       |       |       |         |          |
| AUC      | 0.39%| 0.30% | 0.14% | −0.38%| −0.82%| −0.09%  | 0.85%    |
| HIT25    | 0.42%| −10.12%| −18.28%| −25.68%| −25.69%| −6.27%  | 1.65%    |
| HIT50    | 1.51%| −0.39%| −6.57%| −14.82%| −16.71%| −1.70%  | 1.35%    |
| NDGC25   | −0.19%| −11.27%| −28.93%| −33.79%| −34.95%| −16.64%| 6.94%    |
| NDGC50   | 0.58%| −5.03%| −20.61%| −26.35%| −28.43%| −9.67%  | 6.09%    |
Table 6  Average improvement of **REBUS** compared to each model on others datasets (e.g., Epinions, Foursquare, Adressa, Visiativ, Amazon-Automotive, Amazon-Office-Product and Amazon-Video-Games datasets), ML datasets (e.g., ML-5, ML-10, ML-20, ML-30 and ML-50) datasets) and all datasets

| Metric       | SASRec | TransRec | PRME  | CASER  | GRU4Rec | FPMC   | BPR   | FMC   | POP   | S-KNN  |
|--------------|--------|----------|-------|--------|---------|--------|-------|-------|-------|--------|
| Avg Others   |        |          |       |        |         |        |       |       |       |        |
| AUC          | 1.53%  | 1.71%    | 3.04% | 3.47%  | 4.50%   | 5.21%  | 5.62% | 6.95% | 14.68%| 55.84% |
| HIT25        | 6.16%  | 3.89%    | 10.13%| 22.24% | 21.17%  | 9.43%  | 56.03%| 13.79%| 90.52%| 24.70% |
| HIT50        | 3.54%  | 2.62%    | 6.25% | 17.29% | 13.74%  | 7.60%  | 38.72%| 10.64%| 67.91%| 19.21% |
| NDGC25       | 20.14% | 10.53%   | 17.98%| 55.79% | 36.48%  | 21.72% | 109.16%| 29.63%| 160.76%| 18.42% |
| NDGC50       | 17.63% | 9.39%    | 15.56%| 49.47% | 31.35%  | 19.79% | 91.55%| 26.56%| 136.58%| 16.83% |
| Avg ML       |        |          |       |        |         |        |       |       |       |        |
| AUC          | −0.09% | 0.61%    | 1.71% | 3.65%  | 3.04%   | 3.19%  | 3.60% | 4.64% | 10.43%| 23.87% |
| HIT25        | −4.66% | −3.83%   | 3.94% | 15.00% | −6.27%  | 0.39%  | 41.45%| 3.05% | 105.01%| 25.69% |
| HIT50        | −1.70% | −1.05%   | 8.10% | 18.90% | 2.37%   | 7.33%  | 37.58%| 11.66%| 98.29%| 31.88% |
| NDGC25       | −7.21% | −5.35%   | 3.66% | 13.20% | −16.64% | −4.87% | 46.63%| −5.51%| 109.63%| 24.67% |
| NDGC50       | −4.66% | −3.07%   | 6.62% | 16.10% | −9.67%  | 0.53%  | 42.83%| 1.34% | 104.59%| 28.96% |
| Avg All      |        |          |       |        |         |        |       |       |       |        |
| AUC          | 0.85%  | 1.24%    | 2.47% | 3.55%  | 3.88%   | 4.34%  | 4.76% | 5.96% | 12.83%| 40.45% |
| HIT25        | 2.98%  | 1.65%    | 8.35% | 20.16% | 12.13%  | 6.78%  | 51.78%| 10.65%| 94.37%| 25.03% |
| HIT50        | 1.72%  | 1.35%    | 6.87% | 17.85% | 9.63%   | 7.53%  | 38.34%| 10.98%| 77.07%| 23.21% |
| NDGC25       | 13.35% | 6.94%    | 14.81%| 44.74% | 20.82%  | 15.27% | 92.50%| 20.44%| 148.39%| 19.69% |
| NDGC50       | 11.33% | 6.09%    | 13.16%| 39.64% | 18.28%  | 14.45% | 76.86%| 19.36%| 127.70%| 19.47% |
5.5 Cold-start user study

We now investigate how **REBUS** behaves when dealing with cold-start users, i.e., users whose sequence of actions is too short to take benefit from. We compare **REBUS** to solely 5 methods – POP, FMC, GRU4Rec, SASRec and S-KNN – since the other ones are not applicable for cold-start user recommendation. Only models that just embed items can make recommendations in this context. To evaluate the performance of **REBUS** and its competitors, we have chosen 7 datasets: Epinions, Foursquare, Adressa, Visiativ, Amazon-Automotive, Amazon-Office and Amazon-VideoGames. We then split each dataset into two parts:

1. One that contains users and items with at least 5 interactions. This is the same filter we used previously since we want to train the model with the same configuration and have the same hyperparameters;
2. A second one that contains cold-start users, that is to say users who do not appear in the first part but who interact with items that appear in the first part. Here, the users have a short history (between 1 and 4 items). This part is not used to train the models but only to evaluate their performance on the cold-start users.

For each dataset, the first parts are used as done in the previous studies. The second parts are split as follows:

1. The most recent item is used for the test;
2. The other items of the user sequence are used to predict the test item.

The performances of the different methods on every dataset are reported in Table 79. We can observe that **REBUS** outperforms other models except S-KNN on most datasets, having the best AUC and HIT_50 values. S-KNN returns the best score for HIT_25 and NDCG_X. This can be explained by the fact that user sequences in this testbed are similar to those used in session-based recommendations (short sequences). In this context, S-KNN is well adapted for cold-start users. Surprisingly, GRU4Rec does not get good results and is outperformed by REBUS, SASRec and S-KNN. We report in Fig. 4 some additional indicators (i.e., POP_X and DIV_X) of the recommendations made for cold-start users. Results are similar to those obtained in Fig. 3.

---

9 We only show HIT_25, HIT_50, NDCG_25 and NDCG_50 in the table. HIT_5, HIT_10, NDCG_5 and NDCG_10 are reported in supplementary material (Lonjarret et al. 2020a).
S-KNN provides more diverse recommendations than other models. This may explain it is the best model for HIT_{25} and NDCG_{X}. This study demonstrates that REBUS outperforms other models for the cold-start user problem for some performance metrics (i.e., AUC and HIT_{50}). For others metrics, REBUS is outperformed by only S-KNN. These good performances are mainly due to the embedding we use to model the data which is applicable on short user sequences.

5.6 Study of the impact of user preferences and sequential dynamics in the recommendation

We have demonstrated that REBUS outperforms the state-of-the-art algorithms in most configurations (i.e., datasets and metrics). We now study how our model effectively behave. Especially, we investigate the impact of both user preferences and sequential dynamics on recommendation. To this end, we consider independently these two components. Furthermore, we also examine short Markov chains in our model instead of frequent sequences to capture the sequential dynamics. REBUS_UP refers to the long term part, REBUS_SD refers to the short term part using personalized context via frequent sequences, XMC means that the model uses Markov chains with a fixed order X. The performances of the different configurations on each dataset are reported in Tables 8, 9.

As expected, we can observe that the configurations of our model that combine user preferences and sequential dynamics (i.e. REBUS and REBUS_{XMC}) outperform other configurations – that only model either user preferences or the sequential dynamics – (1) for all metrics on sparse datasets and (2) only for AUC on dense datasets. Indeed,
Table 7  AUC, HIT\_25, HIT\_50, NDCG\_25 and NDCG\_50 for the different models that do not suffer of the problem of cold-start users. The 3 last rows, called Improv. vs Best, Improv. vs SASRec and Improv. vs S-KNN, shows the improvement in percentage of our method compared to the best model, SASRec and S-KNN (best obtained results are in bold and best obtained results for concurrent models are underlined)

| Metric | Epinions | Foursq | Adressa | Visiativ | Auto | Office | Games | Avg |
|--------|----------|--------|---------|----------|------|--------|-------|-----|
| POP    |          |        |         |          |      |        |       |     |
| AUC    | 0.5741   | 0.8862 | 0.9689  | 0.7810   | 0.6400| 0.6933 | 0.7702| 0.7591|
| HIT25  | 3.62%    | 41.38% | 43.41%  | 31.21%   | 2.78% | 0.46%  | 3.98% | 18.12%|
| HIT50  | 5.41%    | 51.93% | 59.32%  | 46.08%   | 4.06% | 1.69%  | 5.89% | 24.91%|
| NDGC25 | 1.41%    | 15.82% | 18.55%  | 12.98%   | 1.09% | 0.12%  | 1.55% | 7.36% |
| NDGC50 | 1.76%    | 17.88% | 21.59%  | 15.81%   | 1.34% | 0.36%  | 1.91% | 8.66% |
| FMC    |          |        |         |          |      |        |       |     |
| AUC    | 0.5829   | 0.9326 | 0.9776  | 0.8316   | 0.6644| 0.7020 | 0.8529| 0.7920|
| HIT25  | 2.28%    | 48.89% | 59.37%  | 45.54%   | 3.32% | 1.32%  | 10.88%| 24.51%|
| HIT50  | 3.61%    | 60.15% | 72.33%  | 57.97%   | 4.84% | 2.72%  | 15.63%| 31.04%|
| NDGC25 | 0.91%    | 22.98% | 27.35%  | 23.84%   | 1.50% | 0.54%  | 4.27% | 11.63%|
| NDGC50 | 1.16%    | 25.16% | 29.84%  | 26.22%   | 1.79% | 0.81%  | 5.19% | 12.88%|
| GRU4Rec|          |        |         |          |      |        |       |     |
| AUC    | 0.5846   | 0.9330 | 0.9731  | 0.8335   | 0.6962| 0.7177 | 0.8257| 0.7948|
| HIT25  | 3.11%    | 42.60% | 52.94%  | 45.24%   | 2.63% | 2.19%  | 7.37% | 22.30%|
| HIT50  | 4.67%    | 54.81% | 65.75%  | 57.07%   | 4.01% | 4.37%  | 11.25%| 28.85%|
| NDGC25 | 1.27%    | 22.98% | 24.98%  | 23.05%   | 1.06% | 0.75%  | 2.81% | 10.98%|
| NDGC50 | 1.57%    | 25.33% | 27.44%  | 25.34%   | 1.33% | 1.16%  | 3.55% | 12.24%|
| S-KNN  |          |        |         |          |      |        |       |     |
| AUC    | 0.0495   | 0.7012 | 0.9515  | 0.8048   | 0.1138| 0.1297 | 0.4103| 0.4515|
| HIT25  | 3.09%    | 54.48% | 58.82%  | 51.01%   | \textbf{6.91%} | \textbf{7.20%} | 14.83%| 28.05%|
| HIT50  | 3.90%    | 61.65% | 70.61%  | 61.18%   | \textbf{8.21%} | \textbf{8.62%} | 18.93%| 33.30%|
| NDGC25 | \textbf{1.74%} | \textbf{34.67%} | \textbf{34.60%} | \textbf{27.89%} | \textbf{3.68%} | \textbf{4.15%} | \textbf{6.91%} | \textbf{16.24%}|
| NDGC50 | \textbf{1.90%} | \textbf{36.06%} | \textbf{36.87%} | \textbf{29.84%} | \textbf{3.93%} | \textbf{4.43%} | \textbf{7.70%} | \textbf{17.25%}|

116 C. Lonjaret et al.
Table 7 continued

| Metric  | Epinions | Foursq | Adressa | Vistativ | Auto | Office | Games | Avg |
|---------|----------|--------|---------|----------|------|--------|-------|-----|
| SASRec  |          |        |         |          |      |        |       |     |
| AUC     | 0.6504   | 0.9407 | 0.9806  | 0.8655   | 0.6893 | 0.7340 | 0.8698 | 0.8186 |
| HIT25   | 3.36%    | 47.57% | 61.29%  | 49.11%   | 2.85% | 2.96%  | 9.26%  | 25.20% |
| HIT50   | 5.07%    | 58.44% | 74.98%  | 61.59%   | 4.20% | 4.84%  | 14.12% | 31.89% |
| NDGC25  | 1.29%    | 23.12% | 29.14%  | 24.56%   | 1.12% | 1.05%  | 3.66%  | 11.99% |
| NDGC50  | 1.61%    | 25.22% | 31.77%  | 26.97%   | 1.38% | 1.41%  | 4.60%  | 13.28% |
| REBUS   |          |        |         |          |      |        |       |     |
| AUC     | 0.6253   | 0.9537 | 0.9815  | 0.8612   | 0.7267 | 0.7309 | 0.8747 | 0.8220 |
| HIT25   | 3.94%    | 53.75% | 63.42%  | 51.61%   | 4.57% | 5.09%  | 10.46% | 27.55% |
| HIT50   | 6.01%    | 64.47% | 75.65%  | 62.78%   | 6.76% | 7.65%  | 15.88% | 34.17% |
| NDGC25  | 1.58%    | 30.21% | 33.22%  | 26.23%   | 1.79% | 2.07%  | 4.04%  | 14.16% |
| NDGC50  | 1.97%    | 32.28% | 35.58%  | 28.39%   | 2.21% | 2.56%  | 5.08%  | 15.44% |
| Rank    |          |        |         |          |      |        |       |     |
| AUC     | 2        | 1      | 1       | 2        | 1    | 2      | 1     | 1.43 |
| HIT25   | 1        | 2      | 1       | 1        | 2    | 2      | 3     | 1.71 |
| HIT50   | 1        | 1      | 1       | 1        | 2    | 2      | 2     | 1.43 |
| NDGC25  | 2        | 2      | 2       | 2        | 2    | 3      | 2     | 2.14 |
| NDGC50  | 1        | 2      | 2       | 2        | 2    | 3      | 3     | 2.00 |
| Improv. vs Best |          |        |         |          |      |        |       |     |
| AUC     | -3.86%   | 1.38%  | 0.09%   | -0.50%   | 5.43% | -0.42% | 0.56%  | 0.42% |
| HIT25   | 8.84%    | -1.34% | 3.48%   | 1.18%    | -33.86% | -29.31% | -29.47% | -1.78% |
| HIT50   | 11.09%   | 4.57%  | 0.89%   | 1.93%    | -17.66% | -11.25% | -16.11% | 2.61% |
| NDGC25  | -9.20%   | -12.86%| -3.99%  | -5.95%   | -51.36% | -50.12% | -41.53% | -12.81% |
| NDGC50  | 3.68%    | -10.48%| -3.50%  | -4.86%   | -43.77% | -42.21% | -34.03% | -10.49% |
| Metric       | Epinions | Foursq | Adressa | Viisiativ | Auto | Office | Games | Avg   |
|--------------|----------|--------|---------|-----------|------|--------|-------|-------|
| Improv. vs SASRec | AUC      | −3.86% | 1.38%   | 0.09%     | −0.50%| 5.43%  | −0.42%| 0.56% | 0.42% |
|              | HIT25    | 17.26% | 12.99%  | 3.48%     | 5.09%| 60.35% | 71.96%| 12.96%| 9.33% |
|              | HIT50    | 18.54% | 10.32%  | 0.89%     | 1.93%| 60.95% | 58.06%| 12.46%| 7.15% |
|              | NDGC25   | 22.48% | 30.67%  | 14.00%    | 6.80%| 59.82% | 97.14%| 10.38%| 18.10%|
|              | NDGC50   | 22.36% | 27.99%  | 11.99%    | 5.27%| 60.14% | 81.56%| 10.43%| 16.27%|
| Improv. vs S-KNN  | AUC      | 1163%  | 36.01%  | 3.15%     | 7.01%| 538.58%| 463.53%| 113.19%| 82.06%|
|              | HIT25    | 27.51% | −1.34%  | 7.82%     | 1.18%| −33.86%| −29.31%| −29.47%| −1.78%|
|              | HIT50    | 54.10% | 4.57%   | 7.14%     | 2.62%| −17.66%| −11.25%| −16.11%| 2.61% |
|              | NDGC25   | −9.20% | −12.86% | −3.99%    | −5.95%| −51.36%| −50.12%| −41.53%| −12.81%|
|              | NDGC50   | 3.68%  | −10.48% | −3.50%    | −4.86%| −43.77%| −42.21%| −34.03%| −10.49%|
Fig. 4 Box plot of POP_X (first row) and DIV_X (second row) with \( X \in \{1, 5, 25\} \) for the different models that do not suffer of the problem of cold-start users. Values are averaged over all datasets.

Surprisingly, \( \text{REBUS}_{SD_{X}}\text{MC} \), for \( X > 1 \), performs very well on ML-30 and ML-50 and outperforms all models (GRU4Rec included) on ML20 dataset, for HIT_X and NDGC_X (see Table 5). Despite the fact that \( \text{REBUS}_{SD_{X}}\text{MC} \) (when \( X > 1 \)) outperforms all non-combining configurations (that is to say \( \text{REBUS}_{SD_{1}}\text{MC}, \text{REBUS}_{SD} \) and \( \text{REBUS}_{UP} \)), there is no improvement when \( XMC \) is used in \( \text{REBUS} \) (i.e. \( \text{REBUS}_{XMC} \) with \( X > 1 \)). In overall, \( \text{REBUS}_{SD} \) provides better performances than \( \text{REBUS}_{SD_{1}}\text{MC} \), which proves that there is an interest in using personalized sequences to model sequential dynamics. To conclude, the most consistent configurations are \( \text{REBUS}_{1MC} \) and \( \text{REBUS} \).
Table 8 AUC, HIT\_25, HIT\_50, NDCG\_25 and NDCG\_50 for REBUS on Epinions, Foursquare, Adressa, Visiativ, Amazon-Automotive, Amazon-Office-Product and Amazon-Video-Games datasets (best obtained results are in bold). The last column is the average performance on these 7 datasets, the average of all datasets is included in Table 9.

| Metric     | Epinions | Foursq | Adressa | Visiativ | Auto   | Office | Games | Avg     |
|------------|----------|--------|---------|----------|--------|--------|-------|---------|
| REBUS\_UP |          |        |         |          |        |        |       |         |
| AUC        | 0.6495   | 0.9661 | 0.9828  | 0.8521   | 0.7147 | 0.7484 | 0.8860| 0.8285  |
| HIT\_25    | 3.21%    | 61.16% | 63.48%  | 43.80%   | 4.08%  | 3.88%  | 8.83% | 26.92%  |
| HIT\_50    | 5.00%    | 69.20% | 76.93%  | 59.64%   | 6.02%  | 6.06%  | 14.14%| 33.86%  |
| NDCG\_25   | 1.27%    | 42.07% | 31.80%  | 18.29%   | 1.62%  | 1.53%  | 3.28% | 14.26%  |
| NDCG\_50   | 1.61%    | 43.62% | 34.40%  | 21.34%   | 1.99%  | 1.94%  | 4.30% | 15.60%  |
| REBUS\_SD\_1\_MC | | | | | | | | |
| AUC        | 0.6257   | 0.9624 | 0.9846  | 0.8591   | 0.6855 | 0.7184 | 0.8710| 0.8153  |
| HIT\_25    | 2.65%    | 55.12% | 64.26%  | 50.97%   | 3.80%  | 4.08%  | 9.16% | 27.15%  |
| HIT\_50    | 3.84%    | 65.09% | 77.89%  | 63.58%   | 5.55%  | 6.74%  | 13.74%| 33.78%  |
| NDCG\_25   | 1.19%    | 28.93% | 30.23%  | 27.11%   | 1.47%  | 1.52%  | 3.59% | 13.44%  |
| NDCG\_50   | 1.42%    | 30.85% | 32.87%  | 29.55%   | 1.81%  | 2.03%  | 4.47% | 14.71%  |
| REBUS\_SD\_2\_MC | | | | | | | | |
| AUC        | 0.6428   | 0.9660 | 0.9852  | 0.8720   | 0.7132 | 0.7440 | 0.8872| 0.8301  |
| HIT\_25    | 3.00%    | 55.26% | 64.75%  | 53.55%   | 4.11%  | 4.40%  | 9.01% | 27.73%  |
| HIT\_50    | 5.30%    | 65.07% | 78.66%  | 66.45%   | 5.96%  | 6.93%  | 13.95%| 34.62%  |
| NDCG\_25   | 1.29%    | 30.11% | 30.67%  | 25.82%   | 1.61%  | 1.59%  | 3.47% | 13.51%  |
| NDCG\_50   | 1.73%    | 32.01% | 33.63%  | 28.31%   | 1.96%  | 2.08%  | 4.42% | 14.84%  |
| REBUS\_SD\_3\_MC | | | | | | | | |
| AUC        | 0.6485   | 0.9663 | 0.9850  | 0.8725   | 0.7192 | 0.7536 | 0.8892| 0.8335  |
| HIT\_25    | 3.35%    | 55.21% | 64.56%  | 52.90%   | 4.15%  | 4.60%  | 8.68% | 27.64%  |
| HIT\_50    | 5.30%    | 64.92% | 78.17%  | 66.31%   | 6.15%  | 7.31%  | 13.57%| 34.53%  |
| NDCG\_25   | 1.34%    | 30.91% | 30.36%  | 24.23%   | 1.60%  | 1.66%  | 3.36% | 13.35%  |
| NDCG\_50   | 1.71%    | 32.79% | 32.99%  | 26.81%   | 1.99%  | 2.18%  | 4.30% | 14.68%  |
| Metric     | Epinions | Foursq | Adressa | Vissiativ | Auto | Office | Games | Avg   |
|------------|----------|--------|---------|-----------|------|--------|-------|-------|
| REBUS SD   | AUC      | 0.6114 | 0.9653  | 0.9849    | 0.8610 | 0.6904 | 0.7232 | 0.8796 | 0.8166 |
|            | HIT25    | 3.02%  | 55.37%  | 64.78%    | 51.25% | 3.89%  | 4.11%  | 9.09%  | 27.36% |
|            | HIT50    | 4.25%  | 65.20%  | 78.41%    | 63.80% | 5.84%  | 6.69%  | 13.78% | 34.00% |
|            | NDGC25   | 1.20%  | 30.72%  | 30.47%    | 26.35% | 1.51%  | 1.49%  | 3.53%  | 13.61% |
|            | NDGC50   | 1.43%  | 32.62%  | 33.10%    | 28.78% | 1.89%  | 1.99%  | 4.43%  | 14.89% |
| REBUS 1MC  | AUC      | 0.6584 | 0.9682  | 0.9854    | 0.8751 | 0.7190 | **0.7542** | 0.8935 | 0.8363 |
|            | HIT25    | 3.67%  | 63.21%  | 66.45%    | 53.48% | 4.11%  | 4.27%  | 9.28%  | **29.21**% |
|            | HIT50    | 5.18%  | 70.81%  | 79.71%    | **67.60**% | 5.99% | 6.44% | **14.73**% | **35.78**% |
|            | NDGC25   | **1.46**% | 46.16% | 31.77%    | 24.59% | 1.62%  | 1.55% | 3.53%  | 15.81% |
|            | NDGC50   | **1.75**% | **47.63**% | 34.33%    | 27.31% | 1.99%  | 1.97% | **4.57**% | **17.08**% |
| REBUS 2MC  | AUC      | 0.6660 | 0.9676  | **0.9855** | **0.8754** | 0.7189 | 0.7537 | **0.8936** | **0.8372** |
|            | HIT25    | 3.25%  | 62.76%  | 66.61%    | 52.40% | 4.16%  | 4.15% | 9.17%  | 28.93% |
|            | HIT50    | **5.56**% | 70.26% | 79.68%    | 66.16% | 6.07%  | 6.51% | 14.55% | 35.54% |
|            | NDGC25   | 1.31%  | 45.92%  | 31.92%    | 23.58% | 1.62%  | 1.60% | 3.46%  | 15.63% |
|            | NDGC50   | **1.75**% | 47.37% | 34.44%    | 26.21% | 1.99%  | 2.05% | 4.49%  | 16.90% |
| REBUS 3MC  | AUC      | 0.6624 | 0.9669  | 0.9849    | 0.8729 | 0.7179 | 0.7528 | 0.8923 | 0.8357 |
|            | HIT25    | 3.49%  | 62.92%  | 64.60%    | 50.39% | 4.13%  | 4.19% | 8.82%  | 28.36% |
|            | HIT50    | 5.44%  | 70.24%  | 78.53%    | 64.95% | 6.01%  | 6.32% | 14.06% | 35.08% |
|            | NDGC25   | 1.35%  | 46.14%  | 30.36%    | 22.57% | 1.62%  | 1.61% | 3.31%  | 15.28% |
|            | NDGC50   | 1.73%  | 47.55%  | 33.05%    | 25.36% | 1.98%  | 2.02% | 4.32%  | 16.57% |
### Table 8 continued

| Metric | Epinions | Foursq | Adressa | Visiativ | Auto | Office | Games | Avg |
|--------|----------|--------|---------|----------|------|--------|-------|-----|
| **REBUS** |          |        |         |          |      |        |       |     |
| AUC    | 0.6524   | 0.9677 | 0.9854  | 0.8735   | 0.7184 | 0.7507 | 0.8934 | 0.8345 |
| HIT25  | 3.39%    | 62.77% | **66.73%** | 53.33%   | **4.24%** | 4.12% | **9.30%** | 29.13% |
| HIT50  | 5.32%    | 70.40% | **79.77%** | 67.17%   | **6.20%** | 6.24% | 14.61% | 35.67% |
| NDGC25 | 1.31%    | 46.04% | **32.00%** | 24.20%   | **1.68%** | 1.52% | 3.52% | 15.75% |
| NDGC50 | 1.68%    | 47.52% | **34.52%** | 26.86%   | **2.05%** | 1.92% | 4.54% | 17.01% |
Table 9 AUC, HIT, NDCG for REBUS on ML-5, ML-10, ML-20, ML-30 and ML-50 datasets (best obtained results are in bold). The last column is the average performance on the 12 datasets, including datasets of Table 8.

| Metric | ML-5   | ML-10  | ML-20  | ML-30  | ML-50  | Avg(ML) | Avg(All) |
|--------|--------|--------|--------|--------|--------|---------|----------|
| REBUS_UP |        |        |        |        |        |         |          |
| AUC    | 0.7941 | 0.8428 | 0.8603 | 0.8631 | 0.8671 | 0.8455  | 0.8356   |
| HIT25  | 12.95% | 12.49% | 11.97% | 11.61% | 11.39% | 12.08%  | 20.74%   |
| HIT50  | 21.51% | 20.72% | 19.99% | 19.47% | 19.49% | 20.24%  | 28.18%   |
| NDGC25 | 4.80%  | 4.40%  | 4.10%  | 4.06%  | 3.88%  | 4.25%   | 10.09%   |
| NDGC50 | 6.43%  | 5.97%  | 5.64%  | 5.56%  | 5.43%  | 5.81%   | 11.52%   |
| REBUS_SD\(_1\)MC |        |        |        |        |        |         |          |
| AUC    | 0.7848 | 0.8207 | 0.8556 | 0.8657 | 0.8725 | 0.8399  | 0.8255   |
| HIT25  | 13.45% | 15.66% | 17.83% | 18.08% | 18.20% | 16.65%  | 22.77%   |
| HIT50  | 20.83% | 23.22% | 26.38% | 27.14% | 27.39% | 24.99%  | 30.12%   |
| NDGC25 | 5.10%  | 6.18%  | 6.74%  | 6.99%  | 7.03%  | 6.41%   | 10.51%   |
| NDGC50 | 6.52%  | 7.63%  | 8.39%  | 8.73%  | 8.80%  | 8.01%   | 11.92%   |
| REBUS_SD\(_2\)MC |        |        |        |        |        |         |          |
| AUC    | 0.7954 | 0.8468 | 0.8718 | 0.8773 | 0.8850 | 0.8553  | 0.8406   |
| HIT25  | 14.06% | 18.15% | 19.32% | 19.71% | 19.75% | 18.20%  | 23.76%   |
| HIT50  | 22.07% | 26.89% | 29.03% | 27.84% | 27.39% | 25.92%  | 31.72%   |
| NDGC25 | 5.17%  | 6.75%  | 7.28%  | 7.74%  | 7.49%  | 6.88%   | 10.75%   |
| NDGC50 | 6.71%  | 8.43%  | 9.14%  | 9.71%  | 9.50%  | 8.70%   | 12.28%   |
| REBUS_SD\(_3\)MC |        |        |        |        |        |         |          |
| AUC    | 0.7969 | 0.8531 | 0.8763 | 0.8804 | 0.8879 | 0.8589  | 0.8441   |
| HIT25  | 13.78% | 17.34% | 19.56% | 19.44% | 19.47% | 17.92%  | 23.59%   |
| HIT50  | 21.87% | 27.57% | 29.92% | 29.86% | 30.44% | 27.93%  | 31.78%   |
| NDGC25 | 5.03%  | 6.48%  | 7.35%  | 7.51%  | 7.28%  | 6.73%   | 10.59%   |
| NDGC50 | 6.58%  | 8.44%  | 9.34%  | 9.51%  | 9.39%  | 8.65%   | 12.17%   |
### Table 9 continued

| Metric  | ML-5  | ML-10 | ML-20 | ML-30 | ML-50 | Avg(ML) | Avg(All) |
|---------|-------|-------|-------|-------|-------|---------|----------|
| **REBUS_SD** |       |       |       |       |       |         |          |
| AUC     | 0.7758 | 0.8282 | 0.8572 | 0.8677 | 0.8756 | 0.8409  | 0.8267   |
| HIT25   | 13.96% | 16.05% | 17.50% | 18.26% | 18.43% | 16.84%  | 22.98%   |
| HIT50   | 21.29% | 23.88% | 26.73% | 27.77% | 27.79% | 25.49%  | 30.45%   |
| NDGC25  | 5.16%  | 6.13%  | 6.49%  | 7.03%  | 6.83%  | 6.33%   | 10.58%   |
| NDGC50  | 6.57%  | 7.64%  | 8.26%  | 8.86%  | 8.62%  | 7.99%   | 12.02%   |
| **REBUS1MC** |       |       |       |       |       |         |          |
| AUC     | 0.8023 | 0.8555 | 0.8770 | 0.8829 | 0.8883 | 0.8612  | 0.8466   |
| HIT25   | 14.82% | 15.17% | 15.80% | 16.58% | 16.87% | 15.85%  | 23.64%   |
| HIT50   | 22.67% | 24.66% | 26.61% | 27.21% | 27.34% | 25.70%  | 31.58%   |
| NDGC25  | 5.44%  | 5.48%  | 5.68%  | 6.08%  | 6.21%  | 5.78%   | 11.63%   |
| NDGC50  | 6.95%  | 7.30%  | 7.76%  | 8.12%  | 8.21%  | 7.67%   | 13.16%   |
| **REBUS2MC** |       |       |       |       |       |         |          |
| AUC     | 0.8012 | 0.8557 | 0.8770 | **0.8837** | **0.8905** | **0.8616** | **0.8474** |
| HIT25   | 14.46% | 14.85% | 15.35% | 16.48% | 15.91% | 15.41%  | 23.30%   |
| HIT50   | 22.31% | 24.82% | 25.53% | 27.17% | 26.78% | 25.32%  | 31.28%   |
| NDGC25  | 5.24%  | 5.38%  | 5.45%  | 6.04%  | 5.83%  | 5.59%   | 11.45%   |
| NDGC50  | 6.74%  | 7.29%  | 7.40%  | 8.09%  | 7.92%  | 7.49%   | 12.98%   |
| **REBUS3MC** |       |       |       |       |       |         |          |
| AUC     | 0.8000 | 0.8538 | 0.8764 | 0.8825 | 0.8894 | 0.8604  | 0.8460   |
| HIT25   | 14.03% | 14.41% | 14.82% | 15.78% | 15.63% | 14.93%  | 22.77%   |
| HIT50   | 21.96% | 24.06% | 25.30% | 26.31% | 25.62% | 24.65%  | 30.73%   |
| NDGC25  | 5.08%  | 5.25%  | 5.29%  | 5.67%  | 5.53%  | 5.36%   | 11.15%   |
| NDGC50  | 6.61%  | 7.10%  | 7.29%  | 7.69%  | 7.45%  | 7.23%   | 12.68%   |
### Table 9 continued

| Metric | ML-5  | ML-10 | ML-20 | ML-30 | ML-50 | Avg(ML) | Avg(All) |
|--------|-------|-------|-------|-------|-------|---------|----------|
| REBUS  |       |       |       |       |       |         |          |
| AUC    | 0.8031| 0.8563| 0.8772| 0.8826| 0.8884| 0.8615  | 0.8458   |
| HIT25  | 14.42%| 15.27%| 15.60%| 15.80%| 16.72%| 15.56%  | 23.48%   |
| HIT50  | 22.79%| 25.50%| 26.01%| 26.21%| 26.91%| 25.48%  | 31.43%   |
| NDGC25 | 5.32% | 5.59% | 5.60% | 5.80% | 6.03% | 5.66%   | 11.55%   |
| NDGC50 | 6.92% | 7.55% | 7.59% | 7.80% | 7.98% | 7.57%   | 13.08%   |
Table 10 The first five columns show the percentage of substrings \( m_{s_u}^{1:t_l} \) that are equivalent to: (A) no matched items, (B) a first order Markov chain, (C) a first order Markov chain but not with the most recent item, (D) a \( L \) order Markov chain \((L > 0)\). (E) shows the percentage of substrings \( m_{s_u}^{1:t_l} \) that cannot be modeled by Markov chains (substrings that are mapped in a non-consecutive way on the user history). The two last columns show the mean of \( m_{s_u}^{1:t_l} \) sizes (F) and the mean of their occupation length in the user sequence (G).

| Dataset         | (A) no match | (B) MC1 | (C) MC1_old | (D) MC_L | (E) Seq. | (F) Size | (G) Occup. |
|-----------------|-------------|--------|------------|---------|---------|---------|-----------|
| Epinions_2_3    | 3.39%       | 62.21% | 34.06%     | 0.16%   | 0.18%   | 1.01    | 1.01      |
| Foursquare_2_15 | 0.06%       | 26.15% | 1.39%      | 43.85%  | 28.54%  | 2.36    | 2.93      |
| Adressa_2_2     | 0.00%       | 1.66%  | 0.02%      | 93.05%  | 5.26%   | 1.98    | 2.07      |
| Vissiativ_2_10  | 0.00%       | 40.27% | 0.21%      | 28.97%  | 30.54%  | 1.71    | 2.91      |
| Ama-Auto_2_8    | 2.12%       | 72.82% | 22.48%     | 1.31%   | 1.27%   | 1.03    | 1.04      |
| Ama-Office_2_8  | 1.19%       | 72.86% | 20.79%     | 2.69%   | 2.48%   | 1.05    | 1.10      |
| Ama-Games_2_10  | 0.06%       | 80.74% | 5.57%      | 6.13%   | 7.50%   | 1.14    | 1.38      |
| ML-5_2_3        | 0.00%       | 87.05% | 7.50%      | 3.08%   | 2.37%   | 1.06    | 1.08      |
| ML-10_2_8       | 0.00%       | 79.35% | 1.94%      | 8.06%   | 10.65%  | 1.20    | 1.48      |
| ML-20_2_8       | 0.00%       | 59.11% | 0.81%      | 13.41%  | 26.67%  | 1.42    | 2.90      |
| ML-30_2_10      | 0.00%       | 47.37% | 0.51%      | 16.71%  | 35.41%  | 1.55    | 4.21      |
| ML-50_2_10      | 0.00%       | 35.33% | 0.17%      | 20.94%  | 43.56%  | 1.68    | 5.96      |

5.7 Study of the used personalized sequences

We study the characteristics of the sequences used by REBUS for recommendation. The first five columns (from (A) to (E)) of Table 10 report the percentage of sequences \( m_{s_u}^{1:t_l} \) that are equivalent to: (A) no matched items, (B) a first order Markov chain \(^{11}\), (C) a first order Markov chain but not with the most recent item, (D) a \( L \) order Markov chain \((L > 1)\). In column (E) we report the percentage of sequences \( m_{s_u}^{1:t_l} \) that cannot be modeled by Markov chains (sequences that are not strings). The column (A) confirms that it is very uncommon that none of the sequences of \( F \) match the sequence \( s_u^{1:t_l} \). REBUS often takes into account the most recent item within the user’s sequence, but there are some cases for which older items are used. This cannot be obtained by first order Markov chains. Furthermore, depending on the dataset, between 10% to 40% (Columns (C) + (E)) of the sequences are different from Markov Chains. The added value of our approach can be assessed by (C), (D) and (E). The two last columns\(^{12}\) of Table 10 reports the mean of \( m_{s_u}^{1:t_l} \) sizes (F) and the mean of their occupation in \( s_u \) (G), i.e., difference between the last position and the first position in \( m_{s_u}^{1:t_l} \). In most cases, short sequences are exploited to model the sequential dynamics. Nevertheless, we have observed that there are different sizes ranging from

\(^{10}\) The two numbers after the name of the dataset are respectively the value of \( \text{minCount} \) and \( L \).

\(^{11}\) Column (A) and (B) are both equivalent to a first order Markov chain but we split it into two different columns to point out the fact that it is very uncommon that no item from \( F \) matches \( s_u^{1:t_l} \).

\(^{12}\) Sequences that do not match any substring of \( F \) are omitted for columns (F) and (G).
1 to 15. The mean of the occupation of the sequences is rather low. **REBUS** often uses compact sequences. This can explain the good results of short Markov chains to capture sequential dynamics in **REBUS**. We can also observe that **REBUS** sometimes uses non-consecutive items with an arbitrary number of spaces between items. Such sequences cannot be obtained by fixed order Markov chains. This is why frequent sequences outperforms Markov chain when focusing only on the sequential dynamics (see Table 8).

### 5.8 Relative importance of user preferences on sequential dynamics

The user preferences modelling part is very important to provide accurate recommendations. Models that only focus on user preferences (e.g., BPRMF, **REBUS_UP**) outperform those that only consider the sequential dynamics (e.g., FMC, **REBUS_SD**, **REBUS_SD1_MC**) on the AUC metric. However, there are also models that only focus on sequential dynamics and outperform those that only consider the user preferences on HIT_X or NDGC_X. The trade off between user preferences and sequential dynamics is very important to make **REBUS** the most robust and accurate as possible for all kind of datasets and metrics. The $\gamma$ hyperparameter makes it possible to weight the importance given to each part of the model: $\gamma = 0.5$ means that the two parts have the same importance, whereas the greater the $\gamma$, the greater the importance of the user preferences in the recommendations. The best values of $\gamma$ determined by grid search ($\gamma \in \{0.0, 0.1, \ldots, 1.0\}$) are reported in supplementary material (Lonjarret et al. 2020a). For most datasets, $\gamma$ is between 0.3 and 0.7. This means that both user preferences and sequential dynamics are important, but one may dominate the other to get accurate recommendations. In Fig. 5, we analyze the impact of $\gamma$ on AUC, HIT_50, NDCG_50. We can observe different cases: (1) $\gamma$ does not really have an impact on performance of **REBUS** (e.g., Adressa dataset); (2) the best performance on AUC appears when **REBUS** focused on the sequential dynamics (e.g., ML-50 dataset, $\gamma = 0.3$); (3) best results are obtained when **REBUS** is focus on the user preferences (e.g., Office dataset, $\gamma = 0.7$); (4) the best performance on AUC is reached by **REBUS** when the user preferences and the sequential dynamics have the same importance (e.g., Visiativ). Notice that for Office dataset, HIT_50 and NDCG_50 do not follow the same behavior as AUC: a greater $\gamma$ increases the AUC performance but decreases the HIT_50 and NDCG_50 performances. Eventually, the green ellipse in Fig. 5 is the best $\gamma$ identified for **REBUS** in Sect. 5.4 (we have selected the hyperparameters that maximize the AUC performance) but we can see that if we change $\gamma$ for **REBUS_XMC** we can sometimes obtain better performance on AUC, HIT_50, NDCG_50 than those reported on Tables 4 and 5.

### 5.9 Examples of recommendations

Figure 6 shows some recommendations made by **REBUS** on Amazon-Video Games. As can be seen on these examples, **REBUS** captures the sequential dynamics and recommends items which are similar to the ground truth. For instance, for the first user, **REBUS** recommends *Final Fantasy X-2* because the user sequentially bought
the previous editions of the Final Fantasy games (the red squared items). The ground truth is a similar game: The Legend of Dragoon. On this example, the sequential dynamics is captured with a 4-item substring mapped with a discontinuity (a hole) on the user sequence. Some recommendations are based on smaller and more compact sequences, e.g., the two last recommendations use only the most recent item to capture the sequential dynamics. Based on the last user action – Pokemon Mystery Dungeon on Nintendo 3DS and Zumba Fitness on Nintendo Wii – REBUS respectively recommends The Legend of Zelda: Ocarina of Time on Nintendo 3DS (instead of Nintendo 3DS XL) and a Nintendo Wii.

5.10 Summary and discussion

This experimental study demonstrates that our sequential recommendation model REBUS provides the most accurate recommendations for most datasets. Indeed REBUS outperforms other methods in sparse data. When considering dense data, our model remains competitive and we have seen that the performances of REBUS can even be increased with the use of shorter users’ histories to model the sequential dynamics. For instance, REBUS_SD – that uses Markov chains of length 3 and the damping factor to model the sequential dynamics – outperforms the state-of-the-art models on dense data (e.g., ML-50 or ML-30 or ML-20) except GRU4Rec. This
Fig. 6 Recommendations for a random sample of users by REBUS on Amazon Video Games. The red squares are items in $m$ _s_ [1, t].

empirical study also gives evidence that REBUS shows good performance for the problem of cold-start users, outperforming most state-of-the-art models. Actually, the item embedding is powerful even if the history of the user is very short (e.g., 1 or 2 items).

We also have shown that REBUS can provide some insight to explain a recommendation based on the personalized sequences which capture the sequential dynamics. However, this is not enough to explain the whole recommendation in particular the user preferences part of the recommendation. In (Lonjarret et al. 2020b), we propose a method, based on subgroup discovery with different pattern languages (i.e., item-sets and sequences), that provides interpretable explanations of the recommendations made by REBUS, but also by other models. Such an approach is useful for comparing different models and explaining the rationale for recommendations.

6 Conclusion

REBUS is a unified metric model that only embeds items in a Euclidean space in order to learn a representation of the user preferences and sequential dynamics. While other existing models only use fixed-order Markov Chains, regardless of the users and their considered items, one of the strengths of REBUS is that it uses frequent sequences to identify the part of user history that is the most relevant for recommendation. These
sequences are then used to estimate Markov Chains of variable orders, adapted to the user’s profile. Despite its simple architecture, we have demonstrated in an extensive empirical study on numerous datasets that REBUS outperforms the state-of-the-art sequential recommendation models on sparse datasets – which is the most standard configuration in real cases – and has comparable performances on dense datasets. This empirical study also provides evidences that REBUS has good performance even if the user histories are very short (that corresponds to cold-start users). Finally, REBUS is easily customizable, as it is based on a trade-off between sequential dynamics and user preferences fixed by the end-user or by hyperparameter optimization techniques, and also as it relies on the use of a set of sequences that can be automatically learnt or replaced by any other Markov Chains (of fixed or variable order). These elements makes it possible to adapt REBUS to different contexts, such as sparse or dense datasets.

This work opens up several avenues for further research. A first improvement could be to learn $\gamma$ during the learning phase. We can also learn a new vector $\gamma_u$ associated to each user to have a more personalized trade off between user preferences and sequential dynamics. This can be done, for instance, by self attention model architecture (Vaswani et al. 2017).

A second research direction concerns the analysis of the sequential dynamics. The sequences used in REBUS provide additional insights about the recommendations. However, REBUS is a complex and efficient model assimilated to a black box model whose interpretation requires smarter approaches. This can be achieved with methods that analyze recommender systems independently of the model used, and provide enlightenment of recommendations (Lonjarret et al. 2020b). Explanations based on the study of the internal functioning of the model would be also insightful. The learning of interpretable metric between items based on their embedding is a promising research direction (Yoshida et al. 2019). Another interesting future direction is the investigation of more sophisticated pattern syntaxes to better model both the user preferences and the sequential dynamics and thus supporting a better embedding.

Acknowledgements This work was supported by the ACADEMICS grant of the IDEXLYON, project of the University of Lyon, PIA operated by ANR-16-IDEX-0005.

References

Adomavicius G, Kwon Y (2012) Improving aggregate recommendation diversity using ranking-based techniques. IEEE Transactions on Knowledge & Data Engineering 24(05):896–911. https://doi.org/10.1109/TKDE.2011.15

Aggarwal CC (2016) Recommender Systems: The Textbook, 1st edn. Springer Publishing Company, Incorporated

Bayer I, He X, Kanagal B, Rendle S (2017) A generic coordinate descent framework for learning from implicit feedback. In: Proceedings of the 26th International Conference on World Wide Web, International World Wide Web Conferences Steering Committee, Republic and Canton of Geneva, CHE, WWW ’17, p 1341–1350, 10.1145/3038912.3052694,

Bishop CM (2006) Pattern Recognition and Machine Learning (Information Science and Statistics). Springer-Verlag, Berlin, Heidelberg

Chen S, Moore JL, Turnbull D, Joachims T (2012) Playlist prediction via metric embedding. In: Yang Q, Agarwal D, Pei J (eds) The 18th ACM SIGKDD International Conference on Knowledge
Sequential recommendation with metric models... 1131

Discovery and Data Mining, KDD’12, Beijing, China, August 12–16, 2012, ACM, pp 714–722, 10.1145/2339530.2339643,

Chen X, Xu H, Zhang Y, Tang J, Cao Y, Qin Z, Zha H (2018) Sequential recommendation with user memory networks. In: Proceedings of the Eleventh ACM International Conference on Web Search and Data Mining, ACM, New York, NY, USA, WSDM ’18, pp 108–116, 10.1145/3159652.3159668,

Devooght R, Bersini H (2017) Long and short-term recommendations with recurrent neural networks. In: Proceedings of the 25th Conference on User Modeling, Adaptation and Personalization, ACM, New York, NY, USA, UMAP ’17, pp 13–21, 10.1145/3079628.3079670,

Ding Y, Li X (2005) Time weight collaborative filtering. In: Proceedings of the 14th ACM International Conference on Information and Knowledge Management, ACM, New York, NY, USA, CIKM ’05, pp 485–492, 10.1145/1099554.1099689,

Feng S, Li X, Zeng Y, Cong G, Chee YM, Yuan Q (2015) Personalized ranking metric embedding for next new poi recommendation. In: Proceedings of the 24th International Conference on Artificial Intelligence, AAAI Press, IJCAI’15, pp 2069–2075, http://dl.acm.org/citation.cfm?id=2832415.2832536

Glorot X, Bengio Y (2010) Understanding the difficulty of training deep feedforward neural networks. In: Teh YW, Titterington M (eds) Proceedings of the Thirteenth International Conference on Artificial Intelligence and Statistics, PMLR, Chia Laguna Resort, Sardinia, Italy, Proceedings of Machine Learning Research, vol 9, pp 249–256, http://proceedings.mlr.press/v9/glorot10a.html

Gomez-Uribe CA, Hunt N (2016) The netflix recommender system: Algorithms, business value, and innovation. ACM Trans Manag Inf Syst 6(4):13:1–13:19, 10.1145/2843948,

Gulla JA, Zhang L, Liu P, Özgöbek O, Su X (2017) The adressa dataset for news recommendation. In: Proceedings of the International Conference on Web Intelligence, Association for Computing Machinery, New York, NY, USA, WI ’17, p 1042–1048, 10.1145/3106426.3109436,

Gusfield D (1997) Algorithms on Strings, Trees, and Sequences - Computer Science and Computational Biology. Cambridge University Press. https://doi.org/10.1017/cbo9780511574931

Harper FM, Konstan JA (2015) The movielens datasets: History and context. ACM Trans Interact Intell Syst 5(4):19:1–19:19, 10.1145/2827872,

Huang J, Zhao WX, Dou H, Wen JR, Chang EY (2018) Improving sequential recommendation with knowledge-enhanced memory networks. In: The 41st International ACM SIGIR Conference on Research & Development in Information Retrieval, ACM, New York, NY, USA, SIGIR ’18, pp 505–514, 10.1145/3209978.3210017,

Jannach D, Ludewig M (2017) When recurrent neural networks meet the neighborhood for session-based recommendation. In: Proceedings of the Eleventh ACM Conference on Recommender Systems, ACM, New York, NY, USA, RecSys ’17, pp 306–310, 10.1145/3109859.3109872,

Kabbur S, Ning X, Karypis G (2013) Fism: Factored item similarity models for top-n recommender systems. In: Proceedings of the 19th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining, ACM, New York, NY, USA, KDD’13, pp 659–667, 10.1145/2487575.2487589,
Kang W, McAuley JJ (2018) Self-attentive sequential recommendation. In: IEEE International Conference on Data Mining, ICDM 2018, Singapore, November 17-20, 2018, IEEE Computer Society, pp 197–206, 10.1109/ICDM.2018.00035,

Kingma DP, Ba J (2015) Adam: A method for stochastic optimization. In: Bengio Y, LeCun Y (eds) 3rd International Conference on Learning Representations, ICLR 2015, San Diego, CA, USA, May 7-9, 2015, Conference Track Proceedings

Koren Y (2009) Collaborative filtering with temporal dynamics. In: Proceedings of the 15th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining, ACM, New York, NY, USA, KDD ’09, pp 447–456, 10.1145/1557019.1557072,

Koren Y, Bell RM (2015) Advances in collaborative filtering. In: Recommender Systems Handbook, Springer, pp 77–118

Lonjarret C, Auburtin R, Robardet C, Plantevit M (2020a) REBUS: Supplementary materials, source code and datasets. https://bit.ly/3gwZAOF

Lonjarret C, Robardet C, Plantevit M, Auburtin R, Atzmueller M (2020b) Why should i trust this item? explaining the recommendations of any model. In: 2020 IEEE International Conference on Data Science and Advanced Analytics (DSAA), IEEE, pp 526–535

Ludewig M, Jannach D (2018) Evaluation of session-based recommendation algorithms. User Modeling and User-Adapted Interaction 28(4–5):331–390. https://doi.org/10.1007/s11257-018-9209-6

McAuley J, Targett C, Shi Q, van den Hengel A (2015) Image-based recommendations on styles and substitutes. In: Proceedings of the 38th International ACM SIGIR Conference on Research and Development in Information Retrieval, Association for Computing Machinery, New York, NY, USA, SIGIR ’15, p 43–52, 10.1145/2766462.2767755,

Ning X, Karypis G (2011) Slim: Sparse linear methods for top-n recommender systems. In: Proceedings of the 2011 IEEE 11th International Conference on Data Mining, IEEE Computer Society, Washington, DC, USA, ICDM’11, pp 497–506, 10.1109/ICDM.2011.134,

Pasricha R, McAuley J (2018) Translation-based factorization machines for sequential recommendation. In: Proceedings of the 12th ACM Conference on Recommender Systems, ACM, New York, NY, USA, RecSys ’18, pp 63–71, 10.1145/3240323.3240356,

Quadrana M, Cremonesi P, Jannach D (2018) Sequence-aware recommender systems. ACM Comput Surv 51(4):66:1–66:36, 10.1145/3190616,

Rendle S (2012) Factorization machines with libfm. ACM Trans Intell Syst Technol 3(3):57:1–57:22, 10.1145/2168752.2168771,

Rendle S, Freudenthaler C, Gantner Z, Schmidt-Thieme L (2009) Bpr: Bayesian personalized ranking from implicit feedback. In: Proceedings of the Twenty-Fifth Conference on Uncertainty in Artificial Intelligence, AUAI Press, Arlington, Virginia, United States, UAI ’09, pp 452–461, http://dl.acm.org/citation.cfm?id=1795114.1795167

Rendle S, Freudenthaler C, Schmidt-Thieme L (2010) Factorizing personalized markov chains for next-basket recommendation. In: Proceedings of the 19th International Conference on World Wide Web, ACM, New York, NY, USA, WWW ’10, pp 811–820, 10.1145/1772690.1772773,

Resnick P, Varian HR (1997) Recommender systems - introduction to the special section. Commun ACM 40(3):56–58. https://doi.org/10.1145/245108.245121

Said A, Bellogín A (2014) Comparative recommender system evaluation: Benchmarking recommendation frameworks. In: Proceedings of the 8th ACM Conference on Recommender Systems, Association for Computing Machinery, New York, NY, USA, RecSys ’14, p 129–136, 10.1145/2645710.2645746,

Sanchez P, Bellogín A (2020) Time and sequence awareness in similarity metrics for recommendation. Information Processing & Management 57:102228. https://doi.org/10.1016/j.ipm.2020.102228

Sarwar B, Karypis G, Konstan J, Riedl J (2001) Item-based collaborative filtering recommendation algorithms. In: Proceedings of the 10th International Conference on World Wide Web, ACM, New York, NY, USA, WWW ’01, pp 285–295, 10.1145/371920.372071,

Sedhain S, Menon AK, Sanner S, Xie L (2015) Autorec: Autoencoders meet collaborative filtering. In: Proceedings of the 24th International Conference on World Wide Web, ACM, New York, NY, USA, WWW ’15 Companion, pp 111–112, 10.1145/2740908.2742726,

Tang J, Wang K (2018) Personalized top-n sequential recommendation via convolutional sequence embedding. In: Proceedings of the Eleventh ACM International Conference on Web Search and Data Mining, ACM, New York, NY, USA, WSDM ’18, pp 565–573, 10.1145/3159652.3159656,

Vaswani A, Shazeer N, Parmar N, Uszkoreit J, Jones L, Gomez AN, Kaiser L, Polosukhin I (2017) Attention is all you need. In: Guyon I, von Luxburg U, Bengio S, Wallach HM, Fergus R, Vishwanathan SVN,
Sequential recommendation with metric models...

Garnett R (eds) Advances in Neural Information Processing Systems 30: Annual Conference on Neural Information Processing Systems 2017, 4-9 December 2017, Long Beach, CA, USA, pp 5998–6008, http://papers.nips.cc/paper/7181-attention-is-all-you-need

Wang S, Cao L, Wang Y (2019) A survey on session-based recommender systems. CoRR arxiv:abs/1902.04864.

Wu CY, Ahmed A, Beutel A, Smola AJ, Jing H (2017) Recurrent recommender networks. In: Proceedings of the Tenth ACM International Conference on Web Search and Data Mining, ACM, New York, NY, USA, WSDM ’17, pp 495–503, 10.1145/3018661.3018689.

Xiong L, Chen X, Huang T, Schneider JG, Carbonell JG (2010) Temporal collaborative filtering with bayesian probabilistic tensor factorization. In: Proceedings of the SIAM International Conference on Data Mining, SDM 2010, April 29 - May 1, 2010, Columbus, Ohio, USA, SIAM, pp 211–222, 10.1137/1.9781611972801.19.

Yoshida T, Takeuchi I, Karasuyama M (2019) Learning interpretable metric between graphs: Convex formulation and computation with graph mining. In: Proceedings of the 25th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining, Association for Computing Machinery, New York, NY, USA, KDD ’19, p 1026–1036, 10.1145/3292500.3330845.

Zhang F, Yuan NJ, Liu D, Xie X, Ma WY (2016) Collaborative knowledge base embedding for recommender systems. In: Proceedings of the 22Nd ACM SIGKDD International Conference on Knowledge Discovery and Data Mining, ACM, New York, NY, USA, KDD ’16, pp 353–362, 10.1145/2939672.2939673.

Publisher’s Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Authors and Affiliations

Corentin Lonjarret1,2 · Roch Auburtin2 · Céline Robardet1 · Marc Plantevit3

1 Céline Robardet
celine.robardet@insa-lyon.fr
Corentin Lonjarret
corentin.lonjarret@insa-lyon.fr
Roch Auburtin
rauburtin@visiativ.com
Marc Plantevit
marc.plantevit@liris.cnrs.fr

1 Univ Lyon, INSA Lyon, CNRS, LIRIS UMR 5205, 69621 Villeurbanne, France
2 Visiativ, 26 r Benoit Bennier, 69260 Charbonnières-les-Bains, France
3 Univ Lyon, Université de Lyon 1, CNRS, LIRIS UMR 5205, 69622 Villeurbanne, France