Effect of interphase heat transfer on bulk condensation in dust-laden vapor-gas flow

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Abstract. The model was proposed for homogeneous-heterogeneous bulk condensation in vapor-gas flow with dust particles. The feature of the model is taking into account temperature difference between gaseous phase and dust particles (with condensate on them). Calculations were carried out for mixture of D_2O (vapor) and N_2 (gas). The model was verified by comparison of results for the flow without dust with experimental data. Results were obtained for different values of mass fraction of dust, radius of dust particles and their initial subcooling. It was shown that there is minimal initial subcooling at which there is liquid on dust particles at nozzle outlet. This value depends on size of particles and mass fraction of dust.

1. Introduction

Bulk condensation usually takes place in the presence of various heterogeneous nucleation sites. Hence, in the general case, the condensation is of a homogeneous–heterogeneous nature. The relationship between the homogeneous and heterogeneous mechanisms varies in a wide range depending on the vapor supersaturation and the rate of the supersaturation state development, as well as the concentration, sizes, and nature of heterogeneous sites of nucleation. Three processes should be considered for description of bulk condensation kinetics, these processes are nucleation, growth of droplets and interphase heat transfer. Such approach was used in [1] for homogeneous condensation in vapor-gas flow without dust. In this paper we use this approach for bulk condensation in dust-laden flow, in previous papers (for example, [2]) we considered only nucleation and growth of droplets with assumption that temperatures of liquid and gaseous phase are the same, so there was no interphase heat transfer. As well as in [2], the description of bulk condensation kinetics is based on that there are two groups of droplets in a flow. The first group is microdroplets resulting from the homogeneous nucleation in the bulk of the vapor–gas mixture, and the second group is macrodroplets resulting from the condensation on dust particles. Two kinetic equations for droplet size distribution function [3] are used:

\[
\frac{df}{dx} + \frac{\partial r f}{\partial r} = \frac{1}{\rho} \delta (r - r_c)
\]

(1)

\[
\frac{d f_M}{dx} + \frac{\partial r_M f_M}{\partial r} = 0
\]

(2)
Here \( f \) and \( f_M \) are distribution function of micro- and macrodroplets, \( u \) is velocity of the flow, \( r \) is droplet radius, \( \dot{r} \) and \( \dot{r}_M \) are growth rates of micro- and macrodroplets, \( I \) is nucleation rate, \( \delta \) is delta-function, and \( r_{cr} \) is critical radius of microdroplet. Number of macrodroplets is constant and equal to number of dust particles, so right part of equation (2) is zero. Distribution function in the inlet is zero for microdroplets; and it is equal to distribution function of dust particles for macrodroplets.

In general, temperatures of gaseous phase, micro- and macrodroplets are different, so three energy equations should be used for calculation of these three temperatures, if finite rate of interphase heat transfer is considered. Interphase heat flux and growth rate of droplets should be calculated in wide range of Knudsen number of growing droplets. Approximate approach was proposed in [1] for calculation of heat flux and growth rate. This approach has no Knudsen number limitations, but it is complicated, so three models can be used for numerical simulation sequentially. First model is model of one temperature, in which temperatures of gaseous phase and both groups of droplets are the same, it was used in our previous papers (for example, [2]). Second model is model of two temperatures, it is considered in this paper. In this model temperatures of gaseous phase and microdroplets are the same, but temperature of macrodroplets is not equal to them. Third model is model of three temperatures.

2. Problem
We considered one-dimensional stationary flow of vapor (D\(_2\)O), gas (N\(_2\)) and dust particles in supersonic part of Laval nozzle. Equation of gas dynamics were used for description of vapor-gas flow, velocities of gaseous phase and both groups of droplets are assumed to be the same due to small size of droplets and dust particles. The continuity and momentum equations are written as follows:

\[
\frac{d}{dx}(\rho u A) = 0, \quad \rho u \frac{du}{dx} = -\frac{dp}{dx}
\]

(3)

Here \( \rho \) and \( p \) are density and pressure, \( A \) is cross-section area of nozzle. Walls of nozzle are adiabatic, so sum of enthalpy \( h \) and kinetic energy is constant:

\[
\frac{d}{dx}\left(h + \frac{u^2}{2}\right) = 0
\]

(4)

Enthalpy of mixture is equal to sum of enthalpies of its components multiplied by their mass fractions:

\[
h = g_v h_v + g_g h_g + g_p h_p + g_l^{\text{hom}} h_l^{\text{hom}} + g_l^{\text{het}} h_l^{\text{het}},
\]

(5)

Here \( g_v, g_g, g_p \) are mass fractions of vapor, gas and solid particles, \( g_l^{\text{hom}}, g_l^{\text{het}} \) are mass fractions of liquid in micro- and macrodroplets:

\[
g_l^{\text{hom}} = \frac{4}{3} \pi \rho_i \int r^3 f \, dr, \quad g_l^{\text{het}} = \frac{4}{3} \pi \rho_i \int r^3 (f_M - f_0) \, dr,
\]

(6)

where \( \rho_i \) is density of liquid, and \( f_0 \) is size distribution function of dust particles at the inlet. Correlations of mass balance for components of mixture are written as follows:

\[
-\frac{dg_v}{dx} = \frac{dg_l^{\text{hom}}}{dx} + \frac{dg_l^{\text{het}}}{dx}, \quad \frac{dg_g}{dx} = \frac{dg_p}{dx} = 0
\]

(7)

Substitution of equations (5) and (7) into equation (4) leads to following equation:

\[
\left(g_v C_{pv} + g_g C_{pg} + g_l^{\text{hom}} C_{pl}\right) \frac{dT}{dx} + \left(g_p C_{pp} + g_l^{\text{het}} C_{pl}\right) \frac{dT_M}{dx} + u \frac{du}{dx} = \frac{dg_l^{\text{hom}}}{dx} (h_v - h_l^{\text{hom}}) + \frac{dg_l^{\text{het}}}{dx} (h_v - h_l^{\text{het}})
\]

(8)
Here $T$ is temperature of gaseous phase and microdroplets, $T_M$ is temperature of macrodroplets, $C_p$ is specific heat capacity. It was taken into account that $dh = C_p dT$. Differences of enthalpies in (8) can be calculated as follows:

$$h_i - h_i^{hom} = L(T), \quad h_i - h_i^{het} = L(T_M) - C_{pv}(T_M - T)$$

(9)

Here $L(T)$ and $L(T_M)$ are values of evaporation heat at temperatures $T$ and $T_M$, respectively. So equation (8) can be written as follows:

$$
\left( g_p C_{pp} + g_g C_{pg} + g_i^{hom} C_{pl} \right) \frac{dT}{dx} + \left( g_p C_{pp} + g_i^{het} C_{pl} \right) \frac{dT}{dx} + u \frac{du}{dx} = L(T) \frac{dg_i^{het}}{dx} + Q_L,
$$

(10)

Two energy equations can be obtained from equation (10). First of them is written for gaseous phase and microdroplets, and second one is written for dust particles and macrodroplets:

$$
\left( g_p C_{pp} + g_g C_{pg} + g_i^{hom} C_{pl} \right) \frac{dT}{dx} + u \frac{du}{dx} = L(T) \frac{dg_i^{hom}}{dx} + Q_L
$$

(11)

$$
\left( g_p C_{pp} + g_i^{het} C_{pl} \right) \frac{dT}{dx} = (L(T_M) - C_{pv}(T_M - T)) \frac{dg_i^{het}}{dx} - Q_L
$$

(12)

Here $Q_L$ is total heat flux from macrodroplets to gaseous phase per mass unit divided by velocity:

$$Q_L = \frac{4\pi}{u} \int_{r_c}^r q r^2 f dr,$$

(13)

Here $q$ is specific heat flux (per unit of surface area of macrodroplets).

Equation for model of one temperature can be also obtained from equation (10) if $T = T_M$:

$$
C_{pm} \frac{dT}{dx} + u \frac{du}{dx} = L(T) \frac{dg_i^{hom} + g_i^{het}}{dx},
$$

(14)

Here $C_{pm}$ is specific heat capacity of mixture of vapor, gas, dust particles and liquid.

Equation of state of ideal gas was used for both vapor and non-condensable gas.

Set of two kinetic equations (1) and (2) was used for description of condensation kinetics. The Frenkel–Zeldovich formula [4] was used for nucleation rate. The Fuchs formula [5] was used for growth rate of microdroplets:

$$r = \frac{\alpha (p_v - p_l)}{\rho_i \sqrt{2\pi RT/\mu_i} \left( 1 + \frac{\alpha}{D} \sqrt{\frac{RT}{2\pi\mu_i}} r + <l> \right)^{-1}}$$

(15)

Here $\alpha$ is condensation coefficient, $R$ is universal gas constant, $\mu_i$ is molar mass of vapor, $D$ is diffusion coefficient, $p_l$ is density of liquid, $<l>$ is mean free path of molecules of vapor-gas mixture. The Fuchs formula cannot be used for macrodroplets, because it was assumed in this formula that temperatures of gaseous phase and droplets are the same. Approximate approach for calculation of growth rate and interphase heat flux was proposed in [1] with no limitations for Knudsen number of growing droplets. However, nonlinear equation should be solved numerically in this approach for each value of droplet radius, and it leads to significant increase of calculation time. In this paper we considered continual regime of macrodroplets growth, when size of dust particles is much more than mean free path of molecules of gaseous phase. In this case growth of droplets and interphase heat transfer are determined by diffusion and heat conduction. Following formulas were used for growth rate and heat flux:
\[ \dot{r}_m = \frac{D \mu_v}{\rho RT_r} \left( p_r - p_s \left( \frac{T_r}{T_m} \right) \right) \]  

\[ q = \frac{\dot{\lambda}}{r} \left( T_M - T \right) \]

Here \( \lambda \) is thermal conductivity of gaseous phase.

So, we proposed the model of bulk condensation in dust-laden flow of vapor-gas mixture for one-temperature and two-temperatures models.

3. Results and discussion

Simulation of the process was carried out by numerical solving of the system of equations with use of the program COND-KINET-1, which was developed in G.M. Krzhizhanovskii Power Engineering Institute. Calculations were carried out for mixture of D\(_2\)O (vapor) and N\(_2\) (gas) for conditions of experiments [6]. Therefore, we have opportunity to verify our calculations by comparison of our results with experimental data for the flow without dust.

3.1. Model of one temperature

Results are presented in figures 1 and 2 for effect of dust presence on kinetics of bulk condensation in the nozzle. These results are coordinate dependencies of temperature of the flow and mass fraction of liquid in micro- and macrodroplets for different values of mass fraction of dust. Partial pressure of vapor at inlet was 520 Pa, stagnation pressure was 30.2 kPa and stagnation temperature was 308 K. If there is no dust, the flow expands without condensation. It can be seen in figure 2 that mass fraction of liquid is zero for coordinates less than 20 mm. For \( x > 25 \) mm temperature increases due to heat release during condensation. Intensity of heterogeneous condensation increases at increase of mass fraction of dust particles (and increase of their total surface area). Comparison of figures 1 and 2 shows that increase of temperature is a result of intensive homogeneous condensation. Heterogeneous condensation leads only to that temperature decreases more slowly and region of homogeneous nucleation moves downstream. Results for the flow without dust are in good agreement with experimental data [5]. There is no qualitative difference between new calculations and results presented in [2]. So, more results for model of one temperature and their discussion can be found in [2].

![Figure 1. Variation of temperature along nozzle axis for different mass fractions of dust: 0 – no dust, 1 – 3%, 2 – 5%, 3 – 7%.](image1)

![Figure 2. Variation of mass fraction of liquid in microdroplets (solid lines) and macrodroplets (dashed lines) along nozzle axis for different mass fractions of dust: 0 – no dust, 1 – 3%, 2 – 5%, 3 – 7%, symbols – experimental data [6].](image2)
3.2. Model of two temperatures

For model of one temperature parameters of calculation were partial pressure of vapor, stagnation pressure and temperature of the flow, mass fraction and radius of dust particles. For model of two temperatures initial subcooling of dust $\Delta T$ (temperature difference between gaseous phase and dust particles) was an additional parameter. Coordinate dependence of temperature of gaseous phase is shown in figure 3 for different values of initial subcooling. Intensity of condensation on dust particles increases with increase of $\Delta T$ according to equation (16) for growth rate of macrodroplets, so results become closer to ones for model of one temperature. All results below were obtained for partial pressure of vapor at inlet 346 Pa, stagnation pressure 30.2 kPa and stagnation temperature 308 K.

![Figure 3](image.png)

**Figure 3.** Coordinate dependence of temperature of gaseous phase for different values of initial subcooling of dust particles: $1 - 0$ K; $2 - 10$ K; $0$ – model of one temperature. Mass fraction of dust is 1%, radius of dust particles is 1 $\mu$m. Dashed line is for flow without dust.

Coordinate dependencies of mass fraction of liquid in macrodroplets and temperature of dust particles are shown in figure 4. Temperature of dust particles can change due to heat release at condensation on their surface and heat transfer between dust particles and gaseous phase. Condensation leads to increase of temperature, and heat transfer can result in increase or decrease of temperature depending on direction of heat flux. At size of particles $r_p = 1 \mu$m and $\Delta T = 0$ K temperature of dust particles is always higher than temperature of gaseous phase, but it decreases along nozzle axis due to higher effect of interphase heat transfer in comparison with effect of heat release at condensation. At the same radius of particles and $\Delta T = 10$ K temperature increases before $x \approx 5$ mm due to heat release at condensation and heat transfer from gaseous phase because temperature of particles is less than temperature of gaseous phase at this stage of process. At certain distance from inlet temperature of particles becomes equal to temperature of gaseous phase, direction of heat flux changes, and temperature of particles begins to decrease. Increase of radius of particles to 10 $\mu$m leads to that variation of their temperature along nozzle axis becomes insignificant due to several reasons. First, rate of temperature variation of a particle is inversely proportional to its size according to thermal balance equation of a particle because variation of particle’s energy is proportional to its volume, and supplied heat is proportional to its surface area. Second, increase of particles’ size at constant mass fraction of dust results in decrease of their total surface area. Third, specific values of mass and heat fluxes according to equations (16) and (17) decrease with increase of particle’s radius.
It follows from formula for growth rate (16) that there is such temperature of particles $T_0$ at which sign of growth rate changes:

$$\frac{p_v}{T} = \frac{p_v(T_0)}{T_0}$$

(18)

Evaporation of liquid from particles begins at this temperature. Equation (21) means that density of vapor in gaseous phase is equal to density of saturated vapor at $T_0$, because vapor is considered as ideal gas. At $r_p = 1 \mu m$ the heat released at condensation is transferred to gaseous phase well. Temperature of particles decreases along nozzle axis from certain value of coordinate, which depends on initial subcooling of dust. There is no evaporation in considered nozzle with 70 mm length. Mass fraction of liquid on dust particles increases, and this increase accelerates with increase of initial subcooling because growth rate macrodroplets increases with decrease of their temperature. Mass fraction of liquid at $\Delta T = 10 K$ is more than for model of one temperature, because there is part of nozzle where temperature of macrodroplets is lower than temperature of gaseous phase. At $r_p = 10 \mu m$ temperature of particles at inlet is less than $T_0$, so mass fraction of liquid in macrodroplets increases in beginning of process. Density of vapor decreases due to expansion of flow; at certain distance it becomes equal to density of saturated vapor at temperature of particles. Liquid begins to evaporate from dust particles, its mass fraction begins to decrease. The coordinate of evaporation beginning increases with increase of initial subcooling. There is no liquid on dust particles at outlet for zero subcooling. With increase of $\Delta T$ mass fraction of liquid at outlet increases because intensity of condensation on particles increases with decrease of their temperature, and intensity of evaporation decreases. It should be noted that intensity of condensation on dust particles is very low, order of mass fraction of liquid in macrodroplets is 0.01% at $r_p = 1 \mu m$ and 0.0001% at $r_p = 10 \mu m$.

Results are shown in figure 5 for minimal initial subcooling of particles at which there is liquid on particles at outlet. If $\Delta T$ is less than minimal value then vapor condenses on particles in beginning of nozzle, but later all this liquid evaporates in the nozzle of considered length. It should be noted that at higher partial pressure of vapor (in particular, 520 Pa) there is liquid on dust particles at outlet even at zero subcooling. Temperature of particles should be higher than temperature of gaseous phase for absence of liquid on dust particles at outlet. The reason is that growth rate of macrodroplets increases with increase of vapor’s partial pressure, and evaporation rate (after that temperature of particles becomes equal to $T_0$) decreases.

![Figure 4](image_url)
Figure 5. Minimal initial subcooling as function of mass fraction of dust and size of particles: 1 – 3 µm, 2 – 4 µm, 3 – 7 µm, 4 – 10 µm.

Coordinate dependencies of integral parameters (number of microdroplets per mass unit and mass fraction of liquid in microdroplets) of aerosol are shown in figure 6 for different values of particles’ radius and their initial subcooling. Initial subcooling does not affect parameters of condensational aerosol if size of particles is 10 µm due to extremely low intensity of heterogeneous condensation. Mass fraction of liquid on dust particles does not exceed 5 ppm even at initial subcooling 30 K, so presence of dust particles and their parameters do not affect flow of vapor-gas mixture, and, as a result, the process of homogeneous condensation. At \( r_p = 1 \) µm number of microdroplets and mass fraction of liquid decrease with increase of initial subcooling due to increase of intensity of heterogeneous condensation. Temperature of gaseous phase decreases more slowly (see figure 3), increase of supersaturation decelerates, and it leads to decrease of nucleation rate and growth rate of microdroplets.

Figure 6. Coordinate dependence of (a) number of microdroplets per mass unit and (b) mass fraction of liquid in microdroplets for different values of initial subcooling of dust particles: 1 – 0 K; 2 – 10 K; 3 – 20 K, 4 – 30 K, 0 – model of one temperature. Mass fraction of dust is 1%, solid lines are for \( r_p = 1 \) µm, and dashed lines are for \( r_p = 10 \) µm.
4. Conclusion
Results of calculation with use of model of one temperature show that there is opportunity to control the process of bulk condensation by adding of dust particles to the vapor-gas flow. Mass fraction of dust and size of particles are controlling parameters in this case. There is one more controlling parameter for model of two temperatures, it is initial subcooling $\Delta T$. For each combination of particles’ size and mass fraction of dust there is minimal initial subcooling $\Delta T_{\text{min}}$ at which there is liquid on dust particles at nozzle outlet. At $\Delta T < \Delta T_{\text{min}}$ liquid condenses on dust particles in the beginning, but later all this liquid evaporates within the nozzle of considered length. This is qualitative difference between two considered models for temperature of particles, there is no evaporation in model of one temperature. Goal of further study is improvement of the model by considering temperature difference between gaseous phase and not only macrodroplets but microdroplets (model of three temperatures).

5. References
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