Modified Gravity Tomography

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Collaboration with A.C. Davis, B. Li and H. Winther.
arXiv:1203.4812,1111.6613
See talks by A.C. Davis, G. Zhao.

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Unique Lorentz invariant spin 2 effective theory = General Relativity (Weinberg 1965)

GR + ordinary matter does not lead to acceleration

Dark energy and modified gravity require extra degrees of freedom: scalars

Scalars acting on cosmological scales have a low mass and mediate a long range force

If coupled to baryons, then need to screen the scalar force locally (solar system, earth, laboratory)
Two types of screening:

Vainshtein mechanism (Galileon...)

Chameleon-like mechanism (chameleon-f(R), dilaton, symmetron...)

\[ S = \int d^4x \sqrt{-g} \left( \frac{1}{16\pi G_N} R - F(\phi, \partial \phi) - V(\phi) \right) + S_m(\psi_m, A^2(\phi) g_{\mu\nu}) \]

Non-linear effects
\[ S = \int d^4x \sqrt{-g} \left( \frac{1}{16\pi G_N} R - \frac{1}{2} (\partial \phi)^2 - V(\phi) \right) + S_m(\psi_m, A^2(\phi) g_{\mu\nu}) \]

matter dependent effective potential

\[
V_{eff}(\phi) = V(\phi) + \rho_m A(\phi)
\]

Khoury-Weltman

Environment dependent minimum.

The field generated from deep inside is Yukawa suppressed. Only a thin shell radiates outside the body. Hence suppressed scalar contribution to the fifth force.
For all models with a density dependent vacuum in a dense environment, the scalar force is screened provided:

\[ |\phi_{\text{in}} - \phi_{\text{out}}| \leq 2\beta_{\text{out}} m_{\text{Pl}} |\Phi_N| \]

This generalises the thin-shell condition of chameleons to dilatons, symmetrons ...
The non-linear potential of the model and the values of the field can be evaluated using:

\[
\phi(a) = \phi_i + \frac{3}{m_{Pl}} \int_{a_i}^{a} da \frac{\beta(a) \rho(a)}{am^2(a)}
\]

\[
V(a) = V_i - \frac{3}{m_{Pl}} \int_{a_i}^{a} da \frac{\beta^2(a) \rho^2(a)}{am^2(a)}
\]

The full non-linear dynamics is reconstructed parametrically using the mass and the coupling function as a function of redshift!

As the Universe evolves from pre-BBN to now, the density of matter goes from the density of ordinary matter (10g/cm3) to cosmological densities. The minimum of the effective potential experiences all the possible minima from sparse densities (now) to high density (pre-BBN).
This implies a reformulation of the screening condition:

\[
\frac{3}{m_P^2} \int_{a_{\text{in}}}^{a_{\text{out}}} da \frac{\beta(a) \rho(a)}{a m^2(a)} \leq 2 \beta_{\text{out}} \Phi_N
\]

where the bounds of the integral are such that the densities inside and outside the body correspond to the matter density at these respective scale factors.

Inverse power law chameleons (3/2 < r < 3) and large curvature f(R) models (r > 3) are described by:

\[
m(a) \sim m_0 a^{-r}
\]

Given \(m(a)\) and \(\beta(a)\), one can construct a model defined by \(V(\phi)\) and , and then check explicitly if screening occurs.
The loosest screening conditions requires that the Milky way is marginally screened:

\[
\frac{9 \Omega m_0 H_0^2}{m_0^2} \left( \int_{a_G}^{1} \frac{da}{a^4} \frac{\beta(a)}{\beta_0} \frac{m_0^2}{m^2(a)} \right) \leq 2 \Phi_G
\]

This implies the crucial bound:

\[
\frac{m_0}{H_0} \geq 10^3
\]

Effects of modified gravity can appear at most on the Mpc scale.
Inverse power law chameleons | Large curvature f(R)
At the background level, these models are cosmologically extremely simple:

BBN constraints imply that the field must follow the minimum of the effective potential since well before BBN. This is a stable configuration as soon as $m \gg H$.

In the late time Universe, the equation of state of the scalar fluid is such that:

$$\omega_\phi + 1 = \mathcal{O}\left(\frac{H^2}{m^2}\right)$$

No deviation from Lambda-CDM since BBN in practice.

Only astrophysical effects on large scale structure at the perturbation level.
Outlook:

**m(a)-β(a) parameterisation**

- **Linear perturbations:**
  \[
  \delta'' + \mathcal{H}\delta' - \frac{3}{2}\mathcal{H}^2(1 + \frac{2\beta(a)^2}{1 + \frac{m(a)^2a^2}{k^2}})\delta = 0
  \]

- **Non-linear reconstruction of V(φ) and β(φ)**

- **Screening condition**

- **N-body simulations**