Determination of main parameters of clay grinder

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Abstract. The article presents the main technological and power parameters of the clay grinder which depend on a number of geometric parameters, and various rheological characteristics of the processed material made it possible to derive pressure dependencies based on the flow of ductile masses which will allow designing such machines with rational geometric and kinematic parameters.

1. Introduction
The clay grinder is a machine with a complex effect of the working body on the clay mass. Plastic clay in the working channel between the blade and the body of the grinder is mixed and dispersed and simultaneously pressed through holes in the gratings of the body.

The performance, quality of processing and energy consumption are affected by a large number of different parameters, including bowl size, diameter and number of holes in the body, the number of revolutions of the impeller, the number of blades and their geometry, and the minimum gap between the blade and the bowl.

The main parameter on which the performance of the clay grinder depends is the pressure arising in the working channel between the blade and the wall of the body. The pressure should depend not only on the geometry of the working channel, but also to a large extent on the rheological properties of the processed plastic clays.

The processing of clay in the clay grinder is poorly understood and there is no method for calculating this machine.

The objective of this article is to determine the law of pressure change in the “working channel” of clay grinder, average pressure on the blade, friction force on the surface of the blade, torque, velocity gradient and some other parameters.

2. The nature of the movement of the plastic mass in the chamber of variable cross section
Due to the large size of the machine body and the small gap between the blade and the body the task can be simplified by considering the body deployed on the plane relative to which the blade moves (Figure 1).

The boundary conditions of the problem:
1. On the "lower" plane of the body
   \[ y = 0, u_x = 0, u_y = 0. \] (1)
2. On the surface of the blade

\[ y = h, u_x = V, u_y = 0. \]

3. \[ P = p_0 \text{ for } x = a. \]

The movement of the clay mass in the working channel formed by the wiper blade and the perforated clay grinder body occurs in the direction indicated in Figure 2. As a consequence, in the initial equations of motion of an incompressible viscous fluid [1–4] it is possible to discard terms containing \( u_z \) and derivatives with respect to \( z \). Neglecting the action of mass forces with an insignificant height of the channel for the deformation of the clay mass (\( F = 0 \)) and also considering the process of movement steady, i.e.

\[ \frac{\partial u_x}{\partial t} = 0 \]

we obtain the Navier Stokes system of equations describing the plane-parallel motion of plastic material

\[ \frac{\partial^2 u_x}{\partial x^2} = \frac{1}{\mu} \frac{dP}{dy} \]

\[ \frac{\partial u_x}{\partial x} + \frac{\partial u_{xy}}{\partial y} = 0 \]

Integrating equation (2) twice, we get

\[ u_x = \frac{1}{2\mu} \frac{dP}{dx} y^2 + Ay + B \]

\[ V_x = \frac{1}{2\mu} \frac{dP}{dx} yh^2 + Ah \]

From where

\[ A = -\frac{1}{2\mu} \frac{dP}{dx} h + \frac{V_x}{h} \]

Then

\[ u_x = \frac{1}{2\mu} \frac{dP}{dx} y^2 + \left( \frac{V_x}{h} - \frac{1}{2\mu} \frac{dP}{dx} h \right) y \]

and finally

\[ u_x = \frac{1}{2\mu} \left( \frac{dP}{dx} \left( y^2 - h_y \right) + \frac{V_{xy}}{h} \right) \]

\[ \frac{dP}{dx} \] in equation (6) it can be found using the fact that the “flow” of material \( Q \) through any cross-section of the working channel is the same:

\[ Q = \int_0^h u_x dy = -\frac{1}{2\mu} \frac{dP}{dx} \frac{h^3}{6} + \frac{V_x h}{2} \]
and then

$$\frac{dP}{dx} = -\frac{12}{h^3} \mu \left( Q - \frac{V_x h}{2} \right) = \frac{6\mu V_x}{h^3} \left( h - \frac{2Q}{V_x} \right)$$  \hspace{1cm} (7)$$

We introduce:

$$h_1 = \frac{2Q}{V_x}$$ \hspace{1cm} (8)

then the final equation for the pressure gradient takes the form

$$\frac{dP}{dx} = \frac{6\mu V_x}{h^3} (h - h_1)$$  \hspace{1cm} (9)

Substituting equation (9) into equation (6), we obtain

$$u_x = \frac{3V_x}{h^3} (h - h_1) (y^2 - h_1) + \frac{V_{xy}}{h}$$  \hspace{1cm} (10)

Because \( h = h_0 + \frac{x^2}{2R} \) (Figure 2).

\[ Figure 2. \] Graphs of pressure, velocity gradient, velocity field vsus length of the “working” part of the blade for \( V=0.59 \text{ m/s}; p=0.006 \text{ m}\)

From here, after differentiation, we get

$$\frac{dh}{dx} = \frac{x}{2R}$$  \hspace{1cm} (12)

and therefore

$$\frac{dP}{dx} = \frac{dP}{dh} \cdot \frac{dh}{dx}$$  \hspace{1cm} (13)

and

$$\frac{dP}{dx} = \frac{6\mu V_x}{h^3} (h - h_1)$$
Then
\[ \frac{dP}{dx} = \frac{6\mu v_x}{h^3} (h - h_1) \cdot \frac{R}{x} \]  
(14)

Using the expression \( x \) from (11) you obtain
\[ \frac{dP}{dh} = \frac{6\mu v_x (h - h_1)}{h^3 \sqrt{2R}} \]  
(15)

Integrating equation (15):
\[ \int \frac{dP}{dh} = \frac{6\mu v_x R}{\sqrt{2R}} \left( \int \frac{dh}{h^3 \sqrt{h - \frac{h_0}{h_1}}} - \int \frac{h_1 dh}{h^3 \sqrt{h - \frac{h_0}{h_1}}} \right) \]

As a result of integration, we obtain
\[ P = \frac{6\mu v_x R}{\sqrt{2R}} \left( \sqrt{\frac{h - h_0}{h_0 h}} + \frac{2h_0}{b h_0} \arctan \left( \frac{h - h_0}{h_0} \right) \right) \times \left( 1 - \frac{3h_1}{4h_0} - \frac{h_1 \sqrt{h - h_0}}{2h_0 h^2} \right) + b \]  
(16)

In order to find the integration constants \( b \) and \( h_1 \), we use the boundary conditions (1):
- for \( X = 0 \) \( P = P_0, h = h_0 \) \( (17) \)
- for \( X = a \) \( P = P_0, h = h_0 + \frac{a^2}{2R} \)

We find that
\[ b = P_0 \]  
(18)

Taking into account (18) from equation (16), we obtain after transformations
\[ h_1 = \frac{a}{\sqrt{2R \left( h_0 + \frac{a^2}{2R} \right)}} + \frac{1}{h_0} \arctan \frac{a}{\sqrt{2R h_0}} \]

Given the constants, the integration of equation (16) takes the form
\[ P = \frac{6\mu v_x R}{\sqrt{2R}} \left( \sqrt{\frac{h - h_0}{h_0 h}} + \frac{2h_0}{b h_0} \arctan \left( \frac{h - h_0}{h_0} \right) \right) \times \left( 1 - \frac{3h_1}{4h_0} - \frac{h_1 \sqrt{h - h_0}}{2h_0 h^2} \right) + P_0 \]  
(20)

Operation of clay grinders with clearances established by manufacturers is recommended within
\[ h_1 \approx 1.33 h_0 \]  
(21)

which allows for engineering calculations to write equation (20) in a simple form:
\[ P = \frac{2\mu v_x \sqrt{2R (h - h_0)}}{h^2} \]  
(22)

where \( h_1 \) for exact calculations is determined by the equation (19), and \( h \) by the formula (11). The pressure on the surface of the blade in any section allows calculating the formula (22).

To determine the average pressure on the curved surface of the working part of the blade, we integrate equation (22) along the length of the working blade in the range from 0 to \( a \).
\[ \int_0^a p \cdot dx = \int_0^a \frac{3\mu v_x h_1 b \cdot x \cdot dx}{2h_0 \left( h_0 + \frac{x^2}{2R} \right)^2} = \frac{3\mu v_x h_1 b \cdot x \cdot dx}{2h_0} \times \int_0^a \frac{4R^2 \cdot x \cdot dx}{\left( \left( \sqrt{2R h_0} \right)^2 + x^2 \right)^2} \]

after transformations we get
\[ P_{cp} = \frac{3\mu v_x h_1 R a^3 b}{2h_0^2 \left( 2R h_0 + a^2 \right)^2} \]  
(23)

To determine the velocity gradient on the surface of the blade we differentiate by the height of the channel the equation (16)
\[ \frac{dU}{dy} = \frac{h}{2\mu} \frac{dP}{dx} + \frac{U}{h} \]  
(24)
Substituting the value of \( dP/dx \) from equation (19) into equation (25) and performing the transformations, we obtain
\[
\left(\frac{du}{dy}\right)_{y=h} = \frac{h}{2\mu} \frac{6V_x}{h^3} (h - h_1) + \frac{V_x}{h} = \frac{4V_x}{h} - \frac{3V_x h_1}{h^2} \tag{25}
\]

Substituting the values of \( h \) into equation (25), we obtain the equation for determining the velocity gradient at any point on the surface of the blade:
\[
\left(\frac{du}{dy}\right)_{y=h} = \frac{4V_x}{h_0 + \frac{x^2}{2R}} - \frac{3V_x h_1}{h_0 + \frac{x^2}{2R}^2} \tag{26}
\]

Determining the average value of the velocity gradient on the surface of the blade allows you to perform the optimal geometric shape of the blade associated with the design of clay grinder body. To determine the average velocity gradient, we integrate equation (26) along the length of the working zone of the blade [5–7]:
\[
\left(\frac{du}{dy}\right)_{y=h} = \frac{4V}{a} \int_0^a \frac{dx}{h_0 + \frac{x^2}{2R}} - \frac{3V h_1}{a} \int_0^a \frac{dx}{h_0 + \frac{x^2}{2R}^2} = \frac{V R}{2h_0 a} \left[ (16 h_0 - 6 h_1) \arctg \frac{a}{\sqrt{2h_0 R}} - \frac{6 h_1 a}{2h_0 R + a^2} \right] \tag{27}
\]

The design of the drive of the clay grinder and the strength calculations of the units and parts of the machine require the determination of the friction force during the processing of the material. The shear stress acting on the blade in the zone of material capture is determined by the dependence
\[
\tau = \mu \left(\frac{du}{dy}\right)_{y=h} \cdot \cos a \tag{28}
\]
where \( \left(\frac{du}{dy}\right)_{y=h} \cdot \cos a \) — projection of the velocity gradient on the normal of the blade surface.

The total value of the friction force acting on the unit width of the blade is equal to
\[
F_{tr} = \int_0^l \mu \left(\frac{du}{dy}\right)_{y=h} \cdot \cos a \cdot dl \tag{29}
\]
where \( l \) — blade length in the area of material capture.

But \( l \approx a \) and \( dl \cdot \cos a = dx \) in range \( 0 \leq x \leq a \); therefore,
\[
F_{tr} = \mu \int_0^a \frac{du}{dy} \left(\frac{du}{dy}\right)_{y=h} dx \tag{30}
\]

Integration of the expression \( \left(\frac{du}{dy}\right)_{y=h} \) was made earlier in (27), finally the formula for calculating the friction force of the material on the blade can be written as
\[
F_{tr} = \frac{\mu V_x R b}{2h_0} \left[ (16 h_0 - 6 h_1) \arctg \frac{a}{\sqrt{2h_0 R}} - \frac{6 h_1 a}{2h_0 R + a^2} \right] \tag{31}
\]
where \( b \) — blade width.

3. Conclusion
The calculated dependencies given in the article can be used in the design of clay processing machines such as clay grinders. In addition to finding the strength characteristics of the working bodies of the machines the dependences take into account the rheological properties of plastic clays which sometimes differ by an order of magnitude and allow designing machines with optimal parameters.

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