Surface Acoustic Wave Single-Electron Interferometry

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We propose an experiment to observe interference of a single electron as it is transported along two parallel quasi-one-dimensional channels trapped in a single minimum of a travelling periodic electric field. The experimental device is a modification of the surface acoustic wave (SAW) based quantum processor. Interference is achieved by creating a superposition of spatial wavefunctions between the two channels and inducing a relative phase shift via either a transverse electric field or a magnetic field. The interference can be used to estimate the decoherence time of an electron in this type of solid-state device.

Constructing a solid-state single-electron interferometer poses many challenges, especially single-electron transport through the device. Recent experiments on electron interferometers1,2 and double quantum dots3,4,5,6,7,8 have demonstrated interference, but do not deal with single electrons. These experiments have to take into account many-particle effects, the behaviour of electrons as quasi-particles, and the validity of the application of theories such as Fermi liquid theory. Besides not showing true single particle interference, these factors obscure the fundamental electron coherence time, which is of crucial importance for many prospective solid state quantum information processing schemes4,5,6,7,8.

Electron quantization using surface acoustic waves (SAW), originally studied in the context of current standards3,4,10, has recently lead to a proposal for the implementation of a quantum processor in the solid-state that uses this mechanism4,11. Advantages of the proposed SAW devices include the unique feature of creating a completely polarised initial state and of making ensemble measurements over billions of identical computations. Additionally, these systems are similar to quantum dots, but have the advantage that manipulation of qubits can be done with static potentials on surface gates without the need for expensive high-frequency pulse generation8. Furthermore, the mechanism of SAW transport eliminates the problem of backscattering from discontinuities in the electron trajectory which also detracts from the ideal interferometry experiment22,23. This opens up the range of mechanisms for inducing relative phase shifts required to observe interference fringes.

The acoustoelectric devices we consider in this paper are fabricated on modulation doped GaAs-AlGaAs heterostructures. Because GaAs is a piezoelectric material, applying a radio-frequency potential difference between a pair of interdigitated transducers produces vibrations that propagate through the structure as longitudinal waves (SAWs), which in turn induce an electrostatic potential. The SAWs then travel across the 2-dimensional electron gas and through a mesa patterned with surface gates that define two parallel quasi-one-dimensional channels. By altering the static potential on the surface gates it is possible to trap a single electron in each SAW potential minimum in each of the two channels with an accuracy greater than 1 part in 101512. A two level quantum system (qubit) can be defined by the presence of a single electron in either the lower or the upper channel (|0⟩ and |1⟩ respectively). Single qubit rotations can be implemented by variations in the static potentials defined by surface gates. The probability of the presence of an electron in either channel can be measured directly from the current output of each channel via Ohmic contacts.

A Mach-Zender single particle interferometer can be constructed from a single qubit SAW processor by a combination of σx and σz gates. The size of the interference fringes gives an indication of the fidelity of device which is a combination of the individual gate fidelities and decoherence. By varying the effective length of the interferometer, the dephasing time of single electrons in this system can be estimated, which is expected to be the limiting factor for coherent manipulation of these systems.

Decoherence of qubit can be characterised by two timescales, the T1 and the T2 time, which are a measure of the rate at which the system experiences unwanted transitions and dephasing between quantum levels respectively. In the Bloch sphere picture13,14,15, the T1 (amplitude damping) time is associated with the contraction of the Bloch sphere along the z-axis, in conjunction with a symmetrical contraction along the x- and y-axes consonant with complete positivity16. This transforms a pure state to a completely mixed state. The T2 (phase relaxation) time is associated with the contraction of the x- and y-axes only, resulting in as shrinkage of the Bloch sphere to a line along the z-axis. In the Markovian regime, an initially pure state, |ψ⟩ = α|0⟩ + β|1⟩,
evolves under phase relaxation as
\[ \rho(t) = \begin{pmatrix} |\alpha|^2 & \alpha \beta^* e^{-t/T_2} \\ \alpha^* \beta e^{-t/T_2} & |\beta|^2 \end{pmatrix}. \] (1)

The off-diagonal terms (coherences), responsible for interference, decrease in magnitude exponentially, where \( T_2 \) is the \( 1/e \) time constant.

By varying \( \phi \), interference fringes can be observed (Fig. 2). Using the standard definition of visibility, \( v = \frac{P_{\text{max}} - P_{\text{min}}}{P_{\text{max}} + P_{\text{min}}} \), we find that
\[ v_0 = \frac{v \sin^2 2\theta}{(\cos^2 \theta + \sin^2 \theta)} \] (3a)
\[ v_1 = v, \quad \forall \theta. \] (3b)

Therefore \( v_1 \) only depends on the dephasing.

![FIG. 1: A Mach Zender interferometer. A single particle at a time is sent horizontally towards the first beamsplitter. We label the state of the particle in the upper and lower arms of the interferometer |0⟩ and |1⟩ respectively. A phase shift is introduced into the upper arm. The two paths are directed to interfere at a second beamsplitter. Particle detectors determine from which direction the particle exits the interferometer.](image)

A Mach-Zender interferometer is shown in Fig. 1. Initially, a particle is in the localised state |0⟩ travelling horizontally towards the first beamsplitter. The actions of the beamsplitters, having transmittances \( t = \cos^2 \theta \) and reflectances \( r = \sin^2 \theta \), and phase shifter can be expressed as unitary operations, \( U_{BS} = \begin{pmatrix} \cos \theta & -\sin \theta \\ e^{i\gamma} \sin \theta & e^{i\gamma} \cos \theta \end{pmatrix} \) and \( \varphi = \begin{pmatrix} 1 \\ 0 \\ 0 \\ e^{i\phi} \end{pmatrix} \) respectively. The state may experience dephasing for a period of \( \tau \), the transit time between the two beamsplitters. The final state after the second beamsplitter is
\[ \rho_{00} = \cos^4 \theta + \sin^4 \theta + \frac{1}{2} v \sin^2 2\theta \cos(\gamma + \phi) \]
\[ \rho_{01} = \rho_{10}^* = \frac{1}{2} e^{-i\gamma} \sin 2\theta (\cos 2\theta + ve^{i(\gamma+\phi)}) - 2v \cos^2 \theta \cos(\gamma + \phi) \]
\[ \rho_{11} = \frac{1}{2} \sin^2 2\theta (1 - v \cos(\gamma + \phi)), \]

where \( v = e^{-\gamma/T_2} \). The probabilities of each detector clicking therefore are
\[ P_0 = \cos^4 \theta + \sin^4 \theta + \frac{1}{2} v \sin^2 2\theta \cos(\gamma + \phi) \] (2a)
\[ P_1 = \frac{1}{2} \sin^2 2\theta (1 - v \cos(\gamma + \phi)). \] (2b)

If the beamsplitters have different splitting ratios, the interference pattern will depend on \( v \) and the two angles \( \theta_1 \) and \( \theta_2 \). The average of \( P_1 \) or \( P_2 \) (with respect to \( \phi \)) will be \( \frac{1}{2} \) if at least one of the beamsplitter ratios is 50:50. This allows the possibility of tuning the interferometer by adjusting the first beamsplitter until the average value of \( P_0 \) or \( P_1 \) is \( \frac{1}{2} \), and then adjusting the second beamsplitter to maximize the visibility.

A two-channel SAW device is shown in Fig. 3. One channel is blocked off so that only one electron is carried in the wavefront of each SAW potential minimum. Information is encoded on the position of the electron, so that localisation to the upper and lower channels corresponds to the qubit states |0⟩ and |1⟩ respectively. A superposition of the two states can be created by lowering the potential barrier between the two channels with the aid of a gap in the surface gates. While the electron
Introducing an asymmetry to the double well potential via a transverse electric field separates the eigenstates of the systems into localised single particle eigenfunctions, evolving with different energies:

$$|\psi\rangle = \cos \theta e^{-iE_0 t/\hbar} |0\rangle - i \sin \theta e^{-iE_1 t/\hbar} |1\rangle.$$  \hfill (6)

The relative phase difference between the two paths is therefore given by the energy difference $\epsilon = E_0 - E_1$ between the two localised states

$$\Delta \phi = \epsilon = \frac{e}{\hbar} \int V dt,$$  \hfill (7)

where $V$ is the voltage difference between the two channels and $e$ is the electronic charge. Since the electrons are transported by the SAW, $\int dt = \tau = l/v$ where $l$ is the length of the channel region experiencing the electric field and $v$ is the velocity of the SAW ($\sim 2700\text{m/s}$ in GaAs). We can then rewrite Eq. (7) as

$$\Delta \phi = \frac{e|\vec{E}|}{\hbar} \frac{l}{v},$$  \hfill (8)

since $V = \vec{E} \cdot \vec{d}$, where $\vec{E}$ is the electric field and $\vec{d}$ is the displacement between the two channels, and therefore explicitly calculate $\Delta \phi$.

The lowest electron temperature achievable in a $^3\text{He} - ^4\text{He}$ dilution refrigerator is realistically around $100\text{mK}$ ($\sim 10\mu\text{eV}$), assuming that microwave heating is minimized. We take this thermal energy as the resolution of the experiment. In order to obtain clearly defined oscillations, the minimum transverse potential change needed for each $2\pi$ phase change is $\sim 100\mu\text{eV}$, corresponding to a maximum phase gate length of $0.1\mu\text{m}$. We cannot have a longer gate without decreasing the number of readings per fringe, given the voltage resolution due to thermal noise. We also require observation of several periods in order to obtain a good estimate of the visibility.

If the relative phase shift is introduced via the Aharonov-Bohm effect \cite{Aharonov1988}, we have that

$$\Delta \phi = \frac{e}{\hbar} \int \vec{B} \cdot \vec{n} dS$$  \hfill (9)

where $S$ is the surface enclosed by the two paths of the interferometer. In our setup, in order to obtain a $2\pi$ phase shift, if the area enclosed by two paths is of the order of $\sim 0.2\mu\text{m}^2$, a $|\vec{B}|$ field change of the order of $\sim 20\text{mT}$ is required. Interference of electrons has already been observed in the presence of large magnetic fields in \cite{Cabrera1994}; we thus expect that this small magnetic field should not produce much additional decoherence.

To measure the dephasing rate, we need to subject the superposition of localised electron states to increasing lengths of time and measure for each length the reduction of the visibility. This can be achieved by lengthening the effective path length of the interferometer, as shown in Fig. 4. We require at least 5 different times to obtain a reasonable estimate of $T_2$. The longest interferometer transit time should be of the order of $2.3\times T_2$, if we require
the minimum visibility to be 10% of the initial visibility. Although absolute estimates the $T_2$ time do not exist, recent experiments place a lower bound on decoherence of $\sim 1\text{ns}$ \cite{16}. Using this value, we find that the longest channel setting needs to be of the order of $vt \sim 6\mu m$. Increments in channel distance between each setting thus need to be of $\sim 1.2\mu m$ or less. This is easily achievable using current electron-beam lithography technology.

The $T_1$ time, corresponding to unwanted tunnelling, can be made extremely long in between the two beamsplitter regions and may be ignored. In the tunnelling regions however, effects like scattering from fluctuating impurity potentials (random telegraph noise) do become important. Estimates of the decoherence time for similar tunnelling regions have been made for a double dot system and found to be at least $1\text{ns}$ \cite{16}. Since our tunnel regions are $\sim 300\mu m$ long, the electron traverses them in a less than $100\text{ps}$, so we expect these errors to be small. In any case, these gate errors are constant and thus one can factor out their effect to determine $T_2$. Since the electron transported by the SAW is shielded from many particle effects, our system may show higher coherence than multi-electron quantum dots \cite{16}.

Increasing the channel length to estimate the dephasing time will be a challenge. A main concern will be that the environment of the qubits will change. However, the increase in static impurities will be small (for an average impurity density of $\sim 1\mu m^{-1}$) and techniques exist to ‘delete’ their effects on the qubits, once their presence is located \cite{18}. Calibration of the beamsplitters is vital to eliminate the contribution of mismatched splitting ratios to the variation in interference visibility.

We do not include in our analysis decoherence arising from spin-orbit coupling. This, however, we expect to be negligible because of the much longer decoherence times supported by the spin degree of freedom \cite{16}.

Finally, this device can also be used as an electric field measuring device, since changes in the transverse electric field will result in changes in the interference pattern. By means of a feedback circuit, the absolute size of the field can be measured. This measurement will be subject to shot noise, $\sqrt{N/N} = 1\sqrt{N}$, where $N = f\Delta t$ is the total number of electrons collected in time $\Delta t$ with SAWs of frequency $f$. There is a trade-off between increased sensitivity, by using a longer $\Delta t$, and measurement bandwidth.

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[22] Numerical studies by \cite{21} suggest that SAW assisted transport increases quantum coherence over ballistic transport.
[23] As this paper was being completed we were made aware of the work of \cite{21} who have considered a similar situation.
[24] The squeezing of the Bloch sphere in orthogonal directions are constrained by the structure of quantum mechanics to obey $|\eta_x \pm \eta_y| \geq \sqrt{1 \pm \eta_z}$ \cite{17}. This constraint stems from linearity and the possibility of the system being entangled with other systems \cite{21}.