Story Stability of a Structure- A Literature Review

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Abstract. Column is a slender beam, which carries load. Failure pattern of a column varies with different parameters such as buckling, compression, shear and tension. The initial imperfections in a column increases deflection and reduction in load carrying capacity. To accomplish stability, the key engineering elements such as connection and rigidity governs the effective length and width of the members. The researchers, covering the key engineering elements with different loading patterns, established numerous comprehensive studies. Further, advancement in the research were carried out to determine lateral stiffness, inter-story displacement and deflected beam shape under various loading patterns. The present study focuses on various literatures on effective length and governing factors, which determine the stability of the structure.

Keywords: Effective length, rigidity, displacement, lateral stiffness, inter-story displacement.

1. Introduction

In the design of steel structures, column and frame stability was described in different ways by different researchers. To determine the stability criteria, the key engineering elements such as system of loads, type of loads, end conditions, effective length, rigidity factors and point applications should be taken into consideration to attain the equilibrium. For decenniums, researchers have developed numerous perspectives for the assessment of frame stability in steel structures design. K factor length method is an important engineering element and to evaluate the frame stability and many codes rely on it. Effective length method interprets the effect of the frame behaviour of an individual column.[1] In order to attain this, alignment charts were used to evaluate the K factor.

2. Discussion on Effective Length

At the initial stage of research, the author noted that effective length could give birth to anomalies, which could influence the design approach, as effective length deals with effective length factor and the buckling strength of individual compressed components. In conditions such as buckling analysis, the value of the effective length can be higher or lower than 1.0 for an unbraced frame leaning column. However, the designers considered K value as 1.0 which created peculiarities as it is necessary to be accurate to attain to solve the moment magnifier equation. An interaction formula was proposed, which does not involve K-factor, which can restrict the peculiarities caused by the effective length in the aspects of leaning column only. [2] The proposed formula is represented in the equation (1).

$$\frac{P_u}{\phi_cP_{n1}} + \frac{\beta B_1 (M_G + B_2 M_F)}{\phi_p M_m} \leq 1.0$$  (1)
An approach was introduced on effective length factor that can be applied to any type of construction. The author has presented formulas to evaluate slender K-factor for prismatic columns considering three parameters such as side-sway parameter 1 (uninhibited); parameter 2 (partially inhibited) and parameter 3 (totally inhibited). The proposed methods were also applied to the stability of the frame with conditions such as condition 1 (unbraced); condition 2 (partially braced) and condition 3 (totally braced) with rigid, semi rigid and simple connections. It was concluded that effective length value of columns maybe less than 1.0 for condition 2 frames but not less than K factor in corresponding condition 3 having similar boundary conditions. Diagonal bracing is an effective way for determining bracing a frame of required size in relation to the column cross-area for simple or partially restrained and semi-rigid framing. The author had concluded that the fixity factors of the beam-column connections have minute effect on the size of required bracing and the fixity factors amongst them and girder has a substantial influence on effective length factor. [3]Improvements were made to the approach, which can evaluate slenderness K factor in cases such as a leaning column with side-sway parameters 1 & 2. The author has taken two famous arguments into confidence such as slenderness effective length factor method along with classical alignment method, which is the most commonly, used approach in the structure of columns with side-sway parameter 1. This approach was questioned in terms of K factor method and its impact on leaning column. Furthermore, scholars working on the effective length technique have established a strong case for the use of classical alignment charts, arguing that both variables may be used to extreme examples of leaning columns in frames with sway parameter 1. The author described a leaning column as a column with both ends hinged in the early stages of the study. In addition, side sway is partially unrestricted by a spring at one of the extremes or somewhere along its length. According to the author, the energy approach or the equilibrium method can be used to analyse the stability of a single column with a spring at the top, as described by Timoshenko (1961). The use of conventional alignment charts in the computation of the slenderness K factor for leaning columns in frames with unconstrained lateral sway produced erroneous results. To eliminate these inaccuracies two formulas were recommended [4]Which are represented in equation (2) and equation (3).

\[
P_{Cr} = \frac{S_\delta h^2}{L} \quad (2)
\]

\[
P_{iCr} = \frac{h_i^2}{L_i} \frac{S_\delta}{\sum_{i=1}^{n} \frac{P_j}{L_j} \times \frac{L_i}{L_j} \times \frac{h_i^2}{h_j^2}} \quad (3)
\]

An improved K-factor formula, which is suitable to all types of structures with the three conditions. The proposed formula is dependent on non-dimensional stiffness variable \(Sh^3/(EI)\). [5]

\[
\frac{Sh^3}{EI_c} = \frac{(\frac{\pi}{K})^4(c-d)}{(fC+FD)} \quad (4)
\]

For braced frames, equation (4) was reduced to the equation (5).

\[
\frac{Sh^3}{EI_c} = \left(\frac{\pi}{K}\right)^2 \left[1 - \frac{1}{1 - \frac{\pi}{K} \cot \left(\frac{\pi}{K}\right) + \left(\frac{\pi}{K}\right)^2} \right] \quad (5)
\]

And for the unbraced frames and partially braced frames with end conditions is represented in equation (6) and equation (7).
Further, Timoshenko stability functions were also used to quickly assess the elastic stability of 3D multiple column structures under gravity loading. Characteristic equations were deduced with respect to multiple column systems with the three side-sway parameters. In addition, the suggested method was to 3- dimensional framed structures with rigid, semi-rigid and simple connections. The drawbacks of the proposed equation were the floor diaphragms including the ground floor were to be assumed rigid in their own plan. With the primary axis aligned to the global axes, the columns were considered doubly symmetrical. Over all story flexural-torsional buckling takes place along XY-plane about Z-axis. Individual flexural column buckling occurring prior to over-all story buckling is included as a possible buckling mode. All members were not subjected to relative axial deformations. If the floor diaphragm is considered planar and horizontal buckling is assumed, axial constraints may be produced among the columns if there are more than three columns of varying heights. In terms of stability equation for 3D multiple-column systems with the first side-sway parameters indicated that, entire columns in a Storey system are combined. Whereas, behaviour of a three-dimensional framed structure totally rely on 14 factors in case of elastic stability. It is a misconception that each column in a multi-column system has its own essential load that is unaffected by the others. In doubly symmetrical frames with doubly symmetrical axial load distribution, instabilities are induced by a combination of flexural and torsional buckling. When this sort of coupled buckling is overlooked, the interaction between columns can lead to design flaws; the columns having the highest $EI/h$ and end fixity factors are responsible for over-all stability of framed buildings and framed structures with respect to buckling mode and inter-storey lateral bracing [6]. A simplified method for determining the critical buckling load of multiple-story frames with semi-rigid connections was proposed, as well as the capacity of the frames to sway. A column in a multi-story frame is treated as an individual in this approach. Equivalent springs were used to account for the contribution of members converging at the top, bottom, and top ends of the column in design aspects for assessment. Members of the frame to the rotations of the bottom and top nodes designed by rotational springs with constants were considered in terms of constraints. A constant translational spring is used to design the resistance provided by the braking system to the relative transverse movement of the end nodes. The impact of the connection non-linearity on the rotational stiffness of the springs was investigated. The author oriented his research in terms of interdependence of rotational stiffness of the members converging on the column, boundary conditions at their extremes and axial loads. [7]Study on sway and non-sway behaviour for assessing buckling length was done. In conventional usage, 2nd order analysis is traditionally associated with the study of a structure’s equilibrium in its deformed configuration, whereas first order analysis is associated with the study of a structure in its un-deformed configuration. Sway and non-sway behaviour are evaluated using common assumptions for beams and columns that connect one node with the same rotation, and the author has used this assumption to employ a new formulation that considers stiffness while giving lateral restrictions. [8]The effects of non-conservative factors on buckling loads, as well as the associated effective length for single and multiple-story framed buildings, was highlighted using the buckling approach method based on FEM. The effect of the non-conservativeness variable and the girder to column stiffness ratio on buckling loads and effective length was investigated. The FE-model was developed using Hamilton’s principles. For each element, general displacements and lateral displacements were expressed as a linear combination of a unidimensional Lagrange interpolation function for the axial displacement and the cubic Hermite interpolation function for the lateral displacement, respectively. The drawn conclusions were that the degree of instability increases when $\alpha = 0$ and is directly related to critical buckling loads of single
column and multiple storeys framed structures. When single storey frames are subjected to non-conservative stresses, the effective length remains constant if the stiffness ratio is greater than 10.0. When the stiffness ratio $\alpha$ is 1.0, the effective length is unaffected. Non-conservative forces have a bigger influence on buckling loads and effective length factors for a frame under the same axial force in the overall narrative than the sum axial forces along the bottom of the frames. The effective length is constant when the stiffness ratio at the loading points increases irrespective of $\alpha$ value [9]. Numerous codes of practice depend on effective length method to assess the stability of a structure were present. The study focused on the discrepancies in the approach presented by non-contradictory complementary information (NCCI) method. The errors identified by the author which fail to acknowledge contribution made by adjoining columns, to the rotational stiffness in relation to translational stiffness of end restraints by other columns in the same story. Simple improvements were made to calculate the distribution coefficients that can assess rotational stiffness of adjoining columns by considering the axial loads accurately. [10] While, in terms of effective length estimation, a procedure to determine buckling load and effective length and the global effects of the frame-compressed member in multiple-story braced and semi rigid jointed frames was proposed. The impact of the type of transfer elements between frame members, in terms of fixity factors were investigated and concluded that fixity factors variation significantly affects the buckling critical load of columns in frames of rigid members, unlike flexible structures. [11]

3. Discussion on Effective Length of Columns in Multi-storey Frames

As per the discussion above, [10] paradoxes created by the existing codes namely non-contradictory complementary information document (SN008a) to BS EN 1993-1 (BSI, 2005) and recommended additional changes to the approach to solve these paradoxes. In SN008a, the following equation (8) calculates the distribution coefficients from top and bottom column nodes.

$$\eta_X = \frac{\sum K_{c,X}}{\sum K_{c,X} + \sum K_{b,X}} \quad (8)$$

The rotational stiffness is assumed linear elastic in this operation. Furthermore, the following equation (9), can be used to compute the rotational stiffness of a member with end circumstances such as a fixed far end and no axial force.

$$K_{i,j} = \frac{I_{ij}}{K_{ij}} \quad (9)$$

The recommendations from NCCI SN008a was restricted to fully rigid connections. As per BS 5950 Annex E, it has stated that any restraining member required carrying more than 90% of its moment capacity and the $K$ value should be assigned as zero. The distribution coefficient $\eta$ is to be considered 1, when the end of the column is designed to carry more than 90% of its moment carrying capacity. When AISC and LFRD manuals are considered, it gives same recommendations as given by NCCI [12]. Both the manuals assume adjoining columns buckle simultaneously which lead to providing negative rotational stiffness at the restraints. In case of unbraced frames, the AISC [13] manual allows the modification of the column effective length, which is shown in the equation (10).

$$\psi_i = \sqrt{\frac{\sum P_u I_i}{P_u \sum I \psi_0^2}} \geq \sqrt{\frac{5}{8}} \psi_{i0} \quad (10)$$

The above equation (10) allows translational stiffness between zero to infinity, corresponding to weaker column support condition, that is being braced by the stronger one. The modified NCCI approach, in which the adjacent columns’ rotational stiffness is taken into account. The recommended distribution
coefficients are shown in equation (11) and equation (12) which are consistent with the NCCT’s design charts [10].

\[
\eta_i = \frac{K_{ij}}{K_{ij} + K_{XY,j} + \sum K_{b,j}} \\
\eta_j = \frac{K_{ji}}{K_{ji} + K_{XY,j} + \sum K_{b,j}}
\]

The author has rearranged the distribution coefficient and inducted the governing design chart equation. The governing design chart equation is represented in equation (13).

\[
\frac{S}{4} \left(1 - C^2 \left(\frac{S}{\frac{S}{4} + \left(\frac{1}{\eta_j} - 1\right)}\right) + \left(\frac{1}{\eta_j} - 1\right)\right) = 0
\]

\[
\frac{K_{ij}S}{4} \left(1 - C^2 \left(\frac{K_{ij}S}{\frac{K_{ij}S}{4} + K_{XY,j} + K_{b,j}}\right)\right) + K_{XY,j} + \sum K_{b,j} = 0
\]

From the equation (14), the criterion for buckling is represented in equation (15).

\[
K_{ij}^* + K_{XY,j}^* + \sum K_{b,j}^* = 0
\]

For sway frame, equation (16) approximated the rotational stiffness of an adjoining column with a fixed roller support at its far.

\[
K_{XY}^* = 0.25 \frac{I_{XY}}{L_{XY}} \left(1 - 0.82 \frac{P_{d,XY}}{P_{d,ij}} \left(\frac{L_{XY}}{L_{ij}}\right)^2\right)
\]

Equation (17) can be used to approximate the adjoining column with a fixed far end in a non-sway frame:

\[
K_{XY}^* = \frac{I}{L} \left(1 - 0.33 \frac{P_{d,XY}}{P_{d,ij}} \left(\frac{L_{XY}}{0.7L_{ij}}\right)^2\right)
\]

4. Discussion on Storey Stability
It was discovered that, for column stability under various loading situations, not only the effects of connection rigidity affect the semi rigid frames’ stability factors, but also the design of slender columns affects the median axial loads along the height, such as column weight. The author has proposed formulas to examine the K-factor of columns with semi-rigid connections considering parameters like semi-rigid connections with sideway parameter 1 or 2, which can affect uniformly distributed axial load. It was concluded that, a frame of around 6 percent of the column cross-area is relatively small for unrestrained or simple framing. In condition 1, the fixity factors have a significant effect on the final column size selection. The degree of fixity at beam-column connection and variation of the column weight appeared parabolic. When considered the effects of uniformly distribution of compressive axial loading which is designated as q on the total stability of a column becomes focal if loading ratio, \(\gamma = \frac{qh}{P}\) is greater than 0.1 and becomes tensile force if q is negative. In terms of end fixity factors related to fully restrained, the influence of q becomes more prominent than in case of partially restrained or semi rigid framing. Further, slenderness K factor equation based on the lateral sway at the ends subjected
to concentrated load and uniformly distributed axial load was developed. [14] This is represented in equation (18).

\[
K^2 = \frac{[40 + 8(\rho_a^2 + \rho_b^2) + \rho_a \rho_b (\rho_a + \rho_b + 3 \rho_a \rho_b - 34) + \gamma (20 + 10(\rho_a - \rho_b) + 3 \rho_a^2 + 5 \rho_b^2) - \rho_a \rho_b (3 \rho_a - 4 \rho_b - 1.5 \rho_a \rho_b + 17)]}{[3(4 - \rho_a \rho_b) (\rho_a + \rho_b + \rho_a \rho_b) + \frac{2 \pi^2}{\pi^2 EI} S_\Delta]} \tag{18}
\]

For a column subjected to heavily uniform distributed axial loading on one side and a light end load on other side, which is side-sway partially, inhibited [4] has proposed equation (19).

\[
K^2 = \frac{[20 + 10(\rho_a - \rho_b) + 3 \rho_a^2 + 5 \rho_b^2 - \rho_a \rho_b (3 \rho_a - 4 \rho_b - 1.5 \rho_a \rho_b + 17)]}{[3(4 - \rho_a \rho_b) (\rho_a + \rho_b + \rho_a \rho_b) + \frac{2 \pi^2}{\pi^2 EI} S_\Delta]} \tag{19}
\]

In due course research to evaluate conditions 1, 2 & 3 of story stability a classical approach was proposed. The author evaluated frames with prismatic columns as buckling modes for all three parameters. In addition, the author had proposed a method to evaluate the effective length factor, the critical axial load, and the sway magnification factor for a total story. The three-way stability equation proposed by the author overcomes the contradictions on two-way classification of first two conditions and their corresponding alignment charts. The author's proposed stability equations apply to multiple-story multiple-bay frames with rigid, semi-rigid, and simple connections. Moreover, the method put forth can provide analysis of elastic stability for real plane framed structures. This method uses the axial loads in the columns obtained from a first-order linear analysis to determine the critical axial loads, effective length factor for each column and the sway amplification factor. The author concluded that for the multiple systems with side-sway parameters 1 & 2, the proposed stability equations indicate that columns in the system are combined together. The story buckling of plane frames in-case of side-sway condition 2, stability of the frames depends on a set of variables such as \( \alpha_i, \beta_i, \gamma_i, \rho_{ai}, \rho_{bi}, \) and \( S_\Delta. \)

Additionally, the author has concluded that the proposed equations for stability and frame classification in which a column is subjected to minimal axial loading, the effective length factor becomes very large and when the applied axial load is zero effective length factor is infinity. In case of vertical members with minimal or no axial load and some degree of flexural end restraints serves as lateral bracing at that particular story level of framed structure. The authors had established major improvements in the design approach method and new formulas were proposed and new improvements have been made taking real time conditions into confidence. The proposed stability equations for story with side-sway inhibited, side-sway partially inhibited and side-sway uninhibited and the following equation (20), was adopted from (Salmon and Johnson 1980)[15]

\[
\frac{\phi_i^2}{R_{ai} R_{bi}} + \left( \frac{1}{R_{ai}} + \frac{1}{R_{bi}} \right) \left[ 1 - \frac{\phi_i}{\tan \phi_i} + \frac{\tan(\phi_i/2)}{\phi_i/2} \right] = 1 = 0 \tag{20}
\]

Which was further developed after adopting fixity factors as shown in equation (21)

\[
(1 - \rho_{ai})(1 - \rho_{bi}) \phi_i^2 + 3(\rho_{ai} + \rho_{bi} - 2 \rho_{ai} \rho_{bi}) \left[ 1 - \frac{\phi_i}{\tan \phi_i} + 9 \rho_{ai} \rho_{bi} \frac{\tan(\phi_i/2)}{\phi_i/2} \right] = 0 \tag{21}
\]

The proposed stability equation for story with side-sway parameter 2 is represented below in equations (22) and (23) represents for story with side-sway inhibited.

\[
\sum_{\gamma_i=1}^{\gamma_i=1} \left[ 1 - 3(\rho_{ai} + \rho_{bi} - 2 \rho_{ai} \rho_{bi}) + 9 \rho_{ai} \rho_{bi} \tan((\phi_i/2)/(\phi_i/2)) / \phi_i^2 (1 - \rho_{ai})(1 - \rho_{bi}) + 3(\rho_{ai} + \rho_{bi} - 2 \rho_{ai} \rho_{bi}) \left[ 1 - \frac{\phi_i}{\tan \phi_i} + 9 \rho_{ai} \rho_{bi} \frac{\tan(\phi_i/2)}{\phi_i/2} - 1 \right] \right] = \frac{S_\Delta}{\phi_i^2 (EI) / h_i^2} \tag{22}
\]
\[ \sum_{i=1}^{n} \frac{a_{ri}}{y_i} \left( 1 - 3(\rho_{ai} + \rho_{bi} - 2\rho_{ai}\rho_{bi}) + 9\rho_{ai}\rho_{bi}\tan((\phi_i/2)/(\phi_{i}/2))/(\phi_i^2(1 - \rho_{ai})(1 - \rho_{bi}) + \\
3(\rho_{ai} + \rho_{bi} - 2\rho_{ai}\rho_{bi}) \left( 1 - \frac{\phi_i}{\tan \phi_i} \right) + 9\rho_{ai}\rho_{bi}\left[ \frac{\tan(\phi_i/2)}{\phi_i^2} - 1 \right] \right) = 0 \] 

[16] extended his research on effects of column orientations in terms of column elastic stability and second order analysis of three-dimensional frames and presented a classical formulation. It can be utilized for ascertaining critical load and second order response of a three-dimensional multiple columns system that can be used instead of complex matrix models or eigenvalue numerical procedures. The author also worked on to indicate the significance of over-all frame twist and its catastrophic impact on lateral stability of three-dimensional framed structures. Additionally, work on merits of column orientation to minimize the twist effects had been established. The author had chosen semi-rigid end connections and with the principal cross-sectional axis oriented in any direction was analysed and categorized based on story side-sway twist buckling modes as frames with storey side-sway and twist inhibited, partially inhibited, and totally uninhibited. The author said that the floor diaphragms, including the ground level, are presumed rigid in their own plate, and the columns are assumed to be doubly symmetrical. The author did not focus on the impact of extensional and shear deformations. It was concluded that the proposed method allows studying the effects of topology of columns, load patterns, semi-rigid connections, flexural hinges, inter-storey bracings, and the properties of the members. Stability equations proposed for three-dimensional multiple column structures with side-sway twist uninhibited and partially inhibited show that columns in a storey are combined in terms of elastic stability behaviour of three-dimensional frames. It is a misconception to presume that each column in a multiple column structure has its own critical load independent from the rest of the storey columns. Stability is affected by the combination of storey side-sway in exceptional cases. Faulty designs may result due to negligence of combined buckling and coupling amongst columns in flexibility analysis; overall stability of the structure depends on orientation of the columns with EI/h and end fixity factors, column orientation, layout, load distribution among columns and stability of three-dimensional frames are inter-related. The author concluded that when the columns and their major axis form a regular polygon is oriented along its radius subjected to the combination critical and lateral loads offers a maximum over all twist stiffness.[17] Worked on the evaluation of elastic buckling loads of multiple-story unbraced steel frames under the effects of variable loading. The author performed stability assessment and design of framed structure depending on the assumptions of proportional loading. The author stated that the when unbraced steel frames are subjected to changeable loading resulting in pragmatic constrains which effects the life span of a structures. The effects of connection behaviour on the critical frame-buckling loads in a proportional case with distinct rigidity of beam to column connections were contemplated. The author concluded that the modification factor \( \beta_i \) which gives a significant measurement of the stiffness interactions amongst columns which counters the lateral instability. The author conception to determine critical frame buckling loads by a pair of constrained minimization and maximization problems, are defined as applied loads and total applied loads in a column. The capacity of the frame under maximal loading circumstances is characterized by the lower and upper boundaries of frame buckling loads, which represent minimization and maximization problems. \( N-\Delta \) type method is an approximate 2nd order analysis of frames with side-sway had been extensively in use. The primary concept behind this method was to calculate the drift and sway moments caused by gravity loads, which could then be applied to any hypothetical lateral stresses acting at the floor levels. The second order approach of sway analysis has little effect on the transition of discrete columns in the frame from sway to partially braced, nearly complete braced behaviour. The 2nd order approximation analysis for frames with sway-braced column interaction was extended by [18] The author discovered a general sway magnifier and critical load articulation that produces global and local second order effects, and suggested formulations in terms of lateral 1st order storey stiffness and the summation of both critical and free sway column loads. In terms of 1st order lateral storey stiffness and critical, free-sway column loads, as well as higher order shear relationship, the general sway magnifier, critical load, and effective length formulations were expanded. A new method was proposed for simplified bifurcation instability analysis considering vertical interaction effects of columns in different
storeys. The author had considered idealizing column model in the existing $G$-factor method and extended it by including all of story columns and their restraining beams. The author stated that the proposed stability matrix and system stability matrix can be used for sway-permitted and sway-prevented frames and the proposed method is computationally simple and mathematical software MATLAB can be used for easy implementation [19].

Geometrical imperfections in steel members and frames created during fabrication and construction might alter the extent of steel frames under differential loading. The focus was on column initial imperfections consisting of out-of-plumbness and out-of-straightness. These shortcomings can cause second-order effects and increase in deflection, which can result in catastrophic damage of the structure if not addressed properly. The author proposed a method for calculating lateral deflections, inter-story displacement, and evaluating lateral stability of a semi-braced storey frame under gravity loading. The author had represented the effect of the imperfections on deflections via notational load concept and also proposed a unique notational load to account for the effect of out-of-straightness imperfections on inter-story displacement. Minimization concept were adopted to account for variable loading. Instability, excessive inter-story displacement, excessive deflections, or the commencement of column yielding are all factors that have an impact on capacity. The author had stated that the frame is unstable when the lateral stiffness value diminishes. Even if the frames are built within the acceptable tolerance for initial imperfections, the author determined that buckling loads of frames are unaffected by the initial imperfections of the column, but it increases deflection, resulting in a reduction in capacity. [20]

The [20] defined capacity criterion, which is as follows in equation (24)

$$\Delta = \frac{Q + \sum_{i=1}^{n+1} \left[ \frac{P_i \Delta_{0,i}}{L_{c,i}} + \frac{P_i \delta_{0,i}}{L_{c,i}} \right]}{\sum_{i=1}^{n+1} S_{T,i} + K_b} = \Delta^*$$

Further, lateral stiffness equation of a semi-rigid connected frame in terms of shear deformation was derived. The author obtained the proposed equation by solving the Timoshenko system of governing differential equations for a semi-rigid connected column. The author stated that shear deformation can consequently lower the buckling load of unbraced steel frames and neglecting this statement leads to compromise in storey-based stability analysis also in terms of compressively loaded column, the effects of shear deformations impacted lateral stiffness and, also, the critical loads of a frame subjected to gravity loading were reduced. The author had employed shear deformation coefficient $\eta$ in lateral stiffness equation, assumed that columns in an unbraced frame experience equilateral displacement, and presented lateral stiffness equation of an unbraced story frame. It was concluded that variable loading approach should not exceed the minimum rotational buckling load of columns in a frame to minimize total axial load. In addition, in terms of the critical load of a lean-on frame containing a low slenderness supporting column ($L/r$ < 40) can be lowered by up to 41 percent even when the supporting column is not loaded. Analysis of multiple-story structure is commonly finished in design by assessing the capacity of the individual member but in reality, this is only possible when the members often interact especially in case of semi-rigid or rigid connections.[21]

Lateral stiffness and proposed empirical expression for semi-rigid connected frame with variable loading is as follows in equation (25) [21]

$$\sum S = \sum_{i=1}^{n+1} S_i = \sum_{i=1}^{n+1} \left[ \frac{12E_{c,i}L_{c,i}}{L_{c,i}^3} \beta_i \left( \frac{1}{1 + \zeta_i} \right) \right]$$

Storey based evaluation method for stability of multiple-storey frames with semi-rigid connections was introduced. The author considered global stability in relation to interaction of rotational stiffness among storeys. The proposed frame decomposition procedure is susceptible to the shape of buckling mode that
may be assumed, as it is difficult to solve. The buckling shape parameters developed by the author can be used to estimate critical loads. In the proposed decomposition technique, the assumption of asymmetric buckling, which was generally consistent with sway buckling mode in semi braced frames, yielded credible findings. Shape coefficients were introduced which is as follows in equation (26) \[22\]

\[v_{FN,b} = \frac{\theta_{u,F}}{\theta_{u,N}} = \frac{Q_{F}/S_{\theta,u,F}}{Q_{N}/S_{\theta,u,N}} = \frac{\Delta S_{\Delta,F}/S_{\theta,u,F}}{\Delta S_{\Delta,N}/S_{\theta,u,N}} \]

(26)

5. Conclusion
From the above literature review, it was understood that effective length of the column, shape functions, lateral stiffness, bracings and capacity would determine the stability of the structure. Several empirical and semi empirical formulas were proposed for better understanding of the stability. Indian codal provisions such as IS 800:2007 provide procedure to evaluate the effective length and determine the sway and non-sway ability but in terms of effective length, value with respect to increase in storey height and its implication towards stability needs to be studied. Stability of the structure is diverse in nature and suitable interpretations needs to be implemented with respect to bracing system and the effect of adjoining columns and code of practice provided were not sufficient to estimate the stability. This necessitates studying the effects of adjoining beam column connections and suitable modification factors need to be evolved.

6. Nomenclature

| Symbol | Description |
|--------|-------------|
| \(P\) | Column axial load |
| \(\sum P_E\) | story buckling load based upon \(K\)-Factor |
| \(\alpha_i\) | Load distribution |
| \(\beta\) | Function of amplification factor \(P-\Delta\) effect |
| \(\beta_i\) | Flexural stiffness ratio |
| \(\gamma_i\) | Height ratio |
| \(B_1\) | Amplification factor moment column |
| \(B_2\) | Amplification factor for \(P-\Delta\) effect |
| \(M_G\) | Moment caused by gravity load |
| \(M_F\) | first-order bending moment |
| \(\phi_c\) | resistance factor in compression |
| \(\phi_b\) | resistance factor in bending |
| \(P_{n1}\) | nominal axial capacity for \(K = 1.0\) |
| \(M_m\) | nominal moment capacity of beam |
\[ \Delta \]  story deflection

\[ S_\lambda \]  Story bracing stiffness

\[ \sum Q \]  total story shear

\[ L \]  Height of the column

\[ \rho_a, \rho_b \]  fixity factors at A and B

\[ S_\delta \]  external lateral stiffness

\[ E \]  Young's modulus

\[ I, I_c, i_j \]  Moment of inertia

\[ H \]  column height

\[ R_a \]  stiffness index at A

\[ R_b \]  stiffness index at B

\[ S \]  lateral restraint provided to a column

\[ R \]  Rotational restraints

\[ \theta_{XY} \]  Inter-story angle of twist of story floor

\[ \lambda_i \]  Angle made by local coordinates with global coordinates

\[ K \]  effective buckling length coefficient

\[ \phi_{ij} \]  Applied load ratio

\[ \beta_{ij} \]  modification factor

\[ Z \]  Function for maximization and minimization problems,

\[ L_{c, i_j} \]  length of column

\[ P_{ij} \]  applied axial load of individual column

\[ K_{nc} \]  load correction matrix with respect to non-conservative forces;

\[ \xi_{cr} \]  proportionality parameter;

\[ \eta_i, \eta_j \]  distribution coefficient;

\[ K_{ij} \]  nominal rotational stiffness;
\( K_{XY} \) rotational stiffness of an adjoining column;

\[
\sum K_{b,l}
\]
sum of rotational stiffness of the beams converging at nodes;

\( M_{lb} \) moment at the bottom end of the column;

\([S_i]\) Story stability matrix;

\( \Delta_{0,i} \) Initial out-of-plumb-ness;

\( \delta_{0,i} \) Initial out-of-straightness;

\( \zeta_i' \) Load due to gravity;

\( Q_F, Q_N \) Applied lateral load at far and near end

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