The next generation of $\mu \to e\gamma$ and $\mu \to 3e$
CLFV search experiments

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Abstract

We explore the possibilities for extending the sensitivity of current searches for the charged lepton flavor violating decays $\mu \to e\gamma$ and $\mu \to eee$. A future facility such as Project X at Fermilab could provide a much more intense stopping $\mu^+$ beam, facilitating more sensitive searches, but improved detectors will be required as well. Current searches are limited by accidental and physics backgrounds, as well as by the total number of stopped muons. One of the limiting factors in current detectors for $\mu \to e\gamma$ searches is the photon energy resolution of the calorimeter. We present a new fast Monte Carlo simulation of a conceptual design of a new experimental concept that detects converted $e^+e^-$ pairs from signal photons, taking advantage of the improved energy resolution of a pair spectrometer based on a silicon charged particle tracker. We also study a related detector design for a next generation $\mu \to eee$ search experiment.

1 Introduction

Charged lepton flavor violating (CLFV) processes, such as $\mu^+ \to e^+\gamma$ and $\mu^+ \to e^+e^-e^+$, are mediated by neutrino oscillations in loop diagrams in the Standard Model (SM). While allowed, these reactions are highly suppressed due to the extremely small neutrino masses. For example, the branching fraction for $\mu \to e\gamma$ is given by

$$BR(\mu \to e\gamma) = \frac{3\alpha}{32\pi}\left| \sum_i U_{\mu i}^* U_{ei} \frac{m^2_V}{m^4_W} \right|^2 \sim 10^{-52},$$

where $U_{ei}$ are the leptonic mixing matrix elements, assuming neutrinos are Dirac particles. This is clearly well below the reach of any conceivable experiment. However, in many extensions of the SM, such as supersymmetric grand unified theories or theories with extra dimensions, larger contributions to CLFV are allowed [1]. Observing CLFV is therefore a clear indication of physics beyond the Standard Model (henceforth BSM physics). Figure 1 shows an example of BSM processes mediated by SUSY particles.

The effective Lagrangian relevant for the $\mu^+ \to e^+\gamma$ and $\mu^+ \to e^+e^-e^+$ decays can be parametrized, regardless of the origin of CLFV, as a sum of dipole terms and a “contact term”. The $\mu^+ \to e^+\gamma$ process is only sensitive to the dipole terms, while both dipole and contact terms contribute to $\mu^+ \to e^+e^-e^+$ decays [2]. Improving upper limits of the
µ⁺ → e⁺γ and µ⁺ → e⁺e⁻e⁺ branching fractions down to 10⁻¹⁴ and 10⁻¹⁶, respectively, could probe scales of BSM physics up to several thousands of TeV. In addition, the Dalitz plot of the μ → eee decays offers the possibility to determine the chirality of BSM physics, should it be observed with sufficient statistics [3].

Recent MEG measurement at PSI [5] sets a limit of $B(\mu^+ \rightarrow e^+\gamma) < 5.7 \times 10^{-13}$ at 90% confidence level using $3.6 \times 10^{14}$ stopped muons on target. The MEG detector consists of a set of drift chambers and scintillation timing counters, located inside a superconducting solenoid, and a liquid Xenon calorimeter with UV-sensitive photomultiplier tubes, located outside the solenoid.

There are two main sources of background. Over 90% of the background in the signal region comes from accidental background, that is, a positron from a regular Michel muon decay combined with a photon from a radiative muon decay (RMD) $\mu^+ \rightarrow e^+\nu_e \bar{\nu}_\mu \gamma$. Most of the remaining background is due to RMD where the neutrinos carry away minimum energy. The accidental background rate depends on the instantaneous stopping muon rate $R_\mu$, total integrating data acquisition time $T$, and detector resolutions:

$$N_{\text{acc}} \propto R_\mu^2 \times \Delta E_\gamma^2 \times \Delta P_e \times \Delta \Theta_{e\gamma}^2 \times \Delta t_{e\gamma} \times T,$$

where $\Delta E_\gamma$ and $\Delta P_e$ are the resolutions of photon energy and positron momentum, respectively; $\Delta \Theta_{e\gamma}$ and $\Delta t_{e\gamma}$ are the resolutions of $e\gamma$ opening angle and timing.

The MEG Collaboration has proposed an upgrade [6] aiming to improve the sensitivity to $\mu \rightarrow e\gamma$ decay by one order of magnitude below the current limit, i.e., to set a limit at $\sim 6 \times 10^{-14}$ in the absence of signal. They will replace their tracker with a lower-mass, higher-granularity device, reduce target thickness, use a faster timing counter array, and increase the granularity of the liquid xenon detector by replacing the PMTs with a larger
number of smaller solid state photosensors. The sensitivity estimate is based on a muon stopping rate of $7 \times 10^7$ muons/s for a three year run, assuming 180 DAQ days per year.

To improve the experimental reach beyond that of the MEG upgrade, one needs to further improve the detector sensitivity. The photon energy resolution is a major limiting factor in this search. A pair spectrometer that measures $e^+e^-$ pair tracks from photon conversions in a thin dense material can greatly improve the photon energy resolution. This approach was discussed at 2012 Project X Summer Study \[7\]. The loss of efficiency due to the small photon conversion probability can be compensated for by improved fiducial solid angle coverage and by the higher beam power at Project X at Fermilab.

We have conducted an initial study of this concept using a fast simulation tool (FastSim) originally developed for the Super$B$ experiment \[8\] using the BABAR software framework and analysis tools. FastSim allows us to model detector components as two-dimensional shells of simple geometries such as cylinders, cones, disks, and planes. The effect of physical thickness is modeled parameterically. Coulomb scattering and ionization energy loss are modeled with the standard parameterization in terms of radiation length and particle momentum and velocity. Bremsstrahlung and pair production are modeled by simplified cross-sections. Tracking measurements are described in terms of the single-hit and two-hit resolution, and the efficiency. Silicon strip detectors are modeled as two independent orthogonal projections. FastSim reconstructs high-level detector objects from simulated hits and energy deposits using the simulation truth to associate detector objects, bypassing pattern recognition. Errors associated with pattern recognition are introduced by perturbing the truth-based association, using models based on BABAR pattern recognition algorithm performance. The final set of hits on associated with a track is passed to the BABAR Kalman filter track fitting algorithm to obtain reconstructed track parameters.

The FastSim model in this study consists of a thin aluminum stopping target and a six-layer cylindrical silicon detector. A 0.56 mm thick lead (10% $X_0$) half cylinder covering $0–\pi$ in azimuthal angle at $R = 80$ mm serves as the photon converter. The target consists of two cones connected at their base; each cone is 50 mm long, 5 mm in radius, and 50 $\mu$m thick. Two silicon detector cylinders are placed close the target for better vertexing resolution; two layers are placed just outside the Pb converter, and two layers a few cm away. The layout is shown in Fig.2; a signal event display is shown in Fig.3. The silicon detector is modeled after Super$B$ inner silicon striplet modules but thinner. Each layer is formed of 50 $\mu$m thick double-sided striplets silicon sensors mounted on 50 $\mu$m of kapton. The hit spatial resolution is modeled as a sum of two components with resolutions of 8 $\mu$m and 20 $\mu$m, and a hit efficiency of 90%. The entire detector is placed in a 1T solenoidal magnetic field.

We generate muons at rest and have them decay via $\mu^+ \rightarrow e^+\gamma$ to study the reconstruction efficiency and resolution. Approximately 1.3% of generated signal events are well-reconstructed, passing quality and fiducial selection criteria. The photon energy resolution is approximately 200 keV (Fig.4), similar to the positron momentum resolution, which corresponds to 0.37% for 52.8 MeV photons. This is a substantial improvement compared to the 1.7%–2.4% resolution of the current MEG and the 1.0%–1.1% resolution goal of the MEG upgrade.

The positron angular resolution is slightly below 10 mrad in both $\theta$ and $\phi$ views, better than current MEG performance but worse than MEG upgrade projection. The photon direction, determined solely from $e^+e^-$ momenta, has a resolution similar to that of the
Figure 2: Schematic drawing (in the plane transverse to the muon beam axis) of the \( \mu \to e\gamma \) detector.

The energies and directions of the \( e^+ \) and \( \gamma \) are smeared according to the FastSim study using double-Gaussian functions. We study the scenarios with timing resolutions of 50 ps and 100 ps. The MEG experiment uses 5 independent variables \( E_{\gamma}, p_e, \phi_{e\gamma}, \theta_{e\gamma}, \) and \( \Delta t_{e\gamma} \), to construct their likelihood function. In our detector, we can take advantage of the excellent direction resolution of the converted photon. If the photon is produced at a different point from positron production point, as is the case for accidental backgrounds, the direction of the \( \gamma \to e^+e^- \) momentum and that of the line connecting the \( e^+e^- \) vertex and the primary positron. It can be further improved by using vertex information. Both the \( \gamma \to e^+e^- \) vertex and positron production vertex (by extrapolating positron track back to the target) have a position resolution of the order of 100 \( \mu m \). Therefore, the photon direction, determined by connecting the two vertices, has a resolution of the order of 1 mrad (given the lever arm of 80 mm). As a result, the resolution of the angle between \( e^+ \) and \( \gamma \) is dominated by \( e^+ \) angular resolution.

We then use a toy Monte Carlo technique to determine the sensitivity of this apparatus. For accidental background, we generate \( e^+ \) and \( \gamma \) from the Michel spectrum and the RMD spectrum \[9\], respectively. Only those momenta near the end points of the spectra could contribute to the background. The directions, production points, and production times of \( e^+ \) and \( \gamma \) are generated randomly without correlation. We ignore the other positron originating from the RMD. For the RMD background, we generate \( e^+ \) and \( \gamma \) according to the theoretical partial branching fraction formula \[9\]. Their directions are correlated, and their production times and positions are identical. The number of accidental background events is a product of \( R^2_\mu \), the partial branching fractions of the Michel decay and RMD, the selection timing window, the total DAQ time, phase space factors, and the reconstruction and selection efficiencies. For the RMD background, the scaling factor is \( R_\mu \), instead of \( R^2_\mu \).
Figure 4: Photon energy and $e\gamma$ invariant mass distributions. Fitted curve is a double-Gaussian distribution.

$e^+$ production point on the target will be different. Two additional variables $\Delta \theta_{\gamma}$ and $\Delta \phi_{\gamma}$ are therefore used in our study. Comparisons between signal and accidental background are shown in Fig. 5.

To estimate the 90% C.L. upper limit sensitivity, we use a cut-and-count approach to estimate the background level and then a Feldman-Cousins method [10] to calculate the upper limit sensitivity assuming no signal events are present.

Figure 5: Discriminating variables used in the $\mu^+ \to e^+\gamma$ search.

Figure 6 shows the background levels, signal efficiency, and 90% C.L. sensitivity under various selection cuts for $R_\mu = 1 \times 10^9$ muons/s, and 50-ps resolution on $t_{e\gamma}$. A sensitivity of $B(\mu^+ \to e^+\gamma) < 1.6 \times 10^{-14}$ could be reached with an integrated DAQ time of 1.5 years. The sensitivity reach as a function of integrated DAQ time for both 50-ps and 100-ps timing resolutions is also shown.
Increasing the muon rate further could improve the sensitivity. However, the sensitivity quickly moves away from the $O(1)$ background regime, because the accidental background grows as $\sim R_{\mu}^2$. A better approach is to increase the efficiency and reduce the muon rate to keep the background level low. Figure 7 shows a scenario in which the signal efficiency is 5-times higher and the muon stopping rate is slightly reduced to $R_{\mu} = 7 \times 10^8$. In this scenario, one can reach a sensitivity of $B(\mu^+ \to e^+\gamma) < 6 \times 10^{-15}$. Such an approach can be realized with multiple layers of thin photon converters and associated silicon tracking layers. Studies of the sensitivity of a multi-converter design are underway.

An alternative version of the photon conversion approach to a $\mu \to e\gamma$ experiment has also been discussed [11]. In this version, consider a large volume solenoidal magnet, such as the KLOE coil, which has a radius of 2.9 m, run at a field of perhaps 0.25 T. A large volume, low mass cylindrical drift chamber provides many ($\geq 100$) layers of tracking, utilizing small cells and having a total number of sense wires approaching $10^5$. Interspersed every ten layers is a 0.5 mm W converter shell. There are a sufficient number of points on the $e^+$ and $e^-$ tracks from converted photons behind each converter to reach a total conversion efficiency of perhaps 80%, with excellent photon mass resolution.

Figure 6: Left: $B(\mu^+ \to e^+\gamma)$ sensitivity optimization for a given scenario (see text). Right: sensitivity as a function of integrated DAQ time for both 50-ps and 100-ps $t_{e\gamma}$ resolutions.

In summary, using a converted photon to increase the $\mu^+ \to e^+\gamma$ detection sensitivity by improving the photon energy resolution appears to be a promising approach. More detailed studies are needed to quantify the requirements in detail, with the goal of improving upon the MEG upgrade sensitivity by an about order of magnitude.

3 $\mu^+ \to e^+e^-e^+$

The current bound on the $\mu^+ \to e^+e^-e^+$ decay has been set by the SINDRUM experiment at PSI [12]. No signal was observed; a limit of $B(\mu^+ \to e^+e^-e^+) < 1 \times 10^{-12}$ was therefore derived, assuming a decay model with a constant matrix element. This measurement was limited by the number of stopped muons, the background from $\mu^+ \to e^+e^-e^+\nu_e\bar{\nu}_\mu$ decays.
remaining negligible. The Mu3e experiment [13] has been proposed to improve this bound by four orders of magnitude, reaching a single event sensitivity (SES) at the level of $7 \times 10^{-17}$. The experiment consists of a silicon pixel detector immersed in a 1 T magnetic field and surrounding a double-cone target, and two timing detector systems. The dominant backgrounds arise from $\mu^+ \rightarrow e^+ e^- e^+ \bar{\nu}_e \nu_e$ events, as well as accidental coincidences of tracks from $\mu^+ \rightarrow e^+ \bar{\nu}_e \nu_e$ and $\mu^+ \rightarrow e^+ e^- e^+ \bar{\nu}_e \nu_e$ decays. Excellent momentum resolution ($< 0.5$ MeV) and timing resolution (50-500 ps depending on the detector system) reduce these backgrounds at an acceptable level.

Our study aims to increase the expected Mu3e sensitivity by an order of magnitude. This requires an improved detector to further reduce the physics and accidental backgrounds. We employed the fast simulation tool discussed above, and explored the improvements needed to achieve a SES at the level of $5 \times 10^{-18}$. The FastSim model consists of a silicon tracker composed of 6 cylindrical layers, surrounding an active target. Each layer is formed of 50 $\mu$m thick double-sided striplet silicon sensors mounted on 50 $\mu$m of kapton. The hit spatial resolution is modeled as a sum of two components with resolutions of 8 $\mu$m and 20 $\mu$m, and a hit efficiency of 90%. The active target is made of two hollow cones of silicon pixel detectors connected at their base. Each cone is 5 cm long, 50 $\mu$m thick and has a radius of 1 cm, with a pixel size of 50 $\mu$m by 50 $\mu$m. Although not included in the simulation, a time-of-flight system should be installed as well. We assume a time resolution of 250 ps, averaging the values of the corresponding Mu3e detector systems. The apparatus layout is displayed in Fig. 8, together with a simulated $\mu^+ \rightarrow e^+ e^- e^+$ event.

We generate $\mu^+ \rightarrow e^+ e^- e^+$ events according to phase space to study the detector resolution and efficiency. The stopped muons are reconstructed by combining three electrons, constraining the tracks to originate from the same pixel in the active target. To further improve the resolution, we require the probability of the constrained fit to be greater than 1%, and a reconstructed muon momentum less than 1 MeV. The absolute value of the cosine of the polar angle of each electron must also be less than 0.9. The resulting $e^+ e^- e^+$ invariant mass distribution, shown in Fig. 9, peaks sharply at the muon mass. We extract

Figure 7: Left: $B(\mu^+ \rightarrow e^+ \gamma)$ sensitivity optimization with 5-times higher signal sensitivity and lower $R_\mu$ than that in Fig. [6]. Best sensitivity is $6 \times 10^{-15}$. 
the resolution by fitting this spectrum with a double-sided Crystal Ball function (a Gaussian with power-law tails on both sides). The Gaussian resolution is found to be 0.3 MeV. To investigate the contribution of the active target to the resolution, we performed alternative fits, removing the geometric constraints, or taking the vertex position by considering all points from tracks intercepting the target, and choosing the one minimizing the $\chi^2$ of the constrained fit. While we observe an improvement compared to the unconstrained fit, the second method yields a similar signal resolution. However, the active target provides a better estimate of impact parameters of the tracks, improving background rejection.

Figure 8: Display of the experimental setup, together with a simulated $\mu^+ \rightarrow e^+ e^- e^+$ event.

Figure 9: The $e^+ e^- e^+$ invariant mass distribution after all selection criteria are applied fitted by a double-sided Crystal Ball function.

The signal efficiency is found to be 27%. To achieve a SES at the level of $5 \times \sim 10^{-18}$ after a 3-year run with 100% DAQ efficiency, a stopped muon rate of the order of $8 \times 10^9$ is needed. For comparison, the Mu3e stopped muon rate at the HiMB beam at PSI is expected to be of the order of $2 \times 10^9$.

To estimate the background contributions under these running conditions, we define a
signal window as $104.9 < m_{\text{ee}e} < 106.5$ MeV, containing approximately 90% of the signal. The irreducible background arises from $\mu \rightarrow e^+e^-e^+\nu\bar{\nu}$ events where the two neutrinos carry almost no energy. We estimate its contribution to be about 8 events by convolving the branching fraction with the resolution function and integrating in the signal region, as shown in Figure 10. However, this background depends strongly on the tail of the mass distribution, and small improvements translate into large background reductions. For example, decreasing the thickness of the silicon sensors and the supporting kapton structure by 20% (40%) reduces the background down to $\sim 4$ ($\sim 1$) events. Additional improvements of the reconstruction algorithms might further improve the resolution and reduce this contamination as well.

![Figure 10](image)

Figure 10: The $\mu^+ \rightarrow e^+e^-e^+\nu\bar{\nu}$ branching fraction before and after convolution with the detector resolution overlaid with signal at different branching fractions. Results are shown for 50$\mu$m thick silicon sensors (left) and 30$\mu$m thick silicon sensors (right).

We consider accidental backgrounds produced by the combination of a Michel decay and a radiative Michel decay (2M$\gamma$ decays), or three simultaneous Michel decays (3M decays), where one of the the positrons is misreconstructed or produces an electron by interacting with the detector. In both cases, we assume the decays occurs within the same pixel in the active target, and during the same time window. This yields position and time suppression factors $\delta S = 7.8 \times 10^{-7}$ and $\delta t = 2.5 \times 10^{-10}$, respectively. The number of background events per second can be expressed as:

$$N_{2M\gamma} = R_\mu^2\delta S\delta t B(\mu^+ \rightarrow e^+\nu_e\bar{\nu}_\mu)B(\mu^+ \rightarrow e^+\nu_e\bar{\nu}_\mu)P(\gamma \rightarrow e^+e^-)P_\mu \simeq 0.33P_\mu$$

$$N_{3M} = R_\mu^3(\delta S)^2 B(\mu^+ \rightarrow e^+\nu_e\bar{\nu}_\mu)^3(\delta t)^2P_\mu \simeq 0.02P_\mu$$

where $P(\gamma \rightarrow e^+e^-) \sim 0.18\%$ is the probability of photon conversion in the target and $P_\mu$ denotes the probability to reconstruct a muon candidate after all selection criteria are applied. The factors $P_\mu$ are estimated by Monte Carlo simulation using the matrix element and differential decay width given in Ref. [9][14]. Values of $P_\mu$ of the order of $O(10^{-8})$ ($O(10^{-9})$) are found for 2M$\gamma$ (3M) decays. Both backgrounds are estimated to be less than one event. A similar background level is expected from combinations of $\mu^+ \rightarrow e^+\nu_\mu\nu_e$ and $\mu^+ \rightarrow e^+e^-\nu_\mu\nu_e$ decays.

In summary, we have outlined the requirements needed to improve the projected sensitivity of the Mu3e experiment by an order of magnitude using a compact silicon tracker.
surrounding an active target. We estimate that a stopped muon rate of $\mathcal{O}(8 \times 10^9)$ would be required to achieve a SES of $5 \times 10^{-18}$ for a 3-year run with 100% DAQ efficiency. Relatively modest improvements on the resolution are needed to maintain the irreducible background at an appropriate level, while an active target proves to be essentially in the reduction of accidental backgrounds.

4 Conclusions

With plausible improvements in photon energy resolution provided by measuring the photon in the decay $\mu \rightarrow e\gamma$, time resolution and vertex location, it appears feasible to substantially improve the sensitivity of searches for this decay, provided that a sufficiently intense surface muon beam, such as that being studied in the context of Project X can be provided. The use of an active target and silicon tracking can similarly improve the sensitivity of searches for the rare decay $\mu \rightarrow 3e$.

Improvements in the sensitivity of searches for both $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$ decays beyond those in proposed in the MEG upgrade and Mu3e experiments are are well-motivated and appear to be quite possible. To achieve this improvement, will be necessary to improve the experimental resolution in the directions explored herein, and to develop a more intense surface muon beam.

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