Kinematical correlations: from RHIC to LHC

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Our recent works on correlations

- Jet-jet correlations (with A. Rybarska and G. Ślipek)
- Photon-jet correlations (with T. Pietrycki)
- Charm-anticharm correlations (with M. Łuszczak)
- Correlations of leptons from semileptonic decays of heavy mesons
- Drell-Yan pair production (with G. Ślipek)
- $J/\psi - gluon$ correlations (with S. Baranov)
Plan of the first part

- Introduction/Motivation
- Theoretical approach(es)
- Matrix elements
- Unintegrated gluon distributions
- Results
- Conclusions

based on:
A. Szczurek, A. Rybarska and G. Slipek,
Phys. Rev. D76 (2007) 034001.
Experimental motivation:
New RHIC data for hadron-hadron correlations – indication of jet structure down to small transverse momenta
(→ Jan Rak)
New PHENIX data

Theoretical motivation:
Dynamics of gluon/parton ladders – a theoretical challenge.

The QCD dynamics (collinear, $k_t$-factorization) is usually investigated for inclusive reactions:

- $\gamma^*$-proton total cross section (or $F_2$)
- Inclusive production of jets
- Inclusive production of mesons (pions)
- Inclusive production of open charm, bottom, top
- Inclusive production of direct photons
- Inclusive production of quarkonia
Very interesting are:

- Dijet correlations ([Leonidov-Ostrovsky, Bartels et al.])
- $Q\bar{Q}$ correlations (→ Marta Luszczak)
- $\gamma^*$ – jet correlations (→ Tomasz Pietrycki)
- jet – $J/\psi$ correlations ([Baranov-Szczurek])
- Exclusive reactions: $pp \rightarrow pXp$ where $X = J/\psi, \chi_c, \chi_b, \eta', \eta_c, \eta_b$
  ([Matrin-Khoze-Ryskin, Szczurek-Pasechnik-Teryaev])

They contain much more information about QCD ladders.
QCD motivation

HERA $\gamma^* p$ total cross section ($F_2(x, Q^2)$)
In LO:

\[ \frac{d\sigma}{d\phi} = f(W) \delta (\phi - \pi) \]  

(1)

In NLO:

\[ h_1 \]

\[ X_1 \]

\[ \begin{array}{c}
 x_1 \\
 \end{array} \]

\[ (y_1, p_{1t}) \]

\[ (y_3, p_{3t}) \]

\[ \begin{array}{c}
 h_2 \\
 x_2 \\
 \end{array} \]

\[ (y_2, p_{2t}) \]

\[ correlation \]

Figure 1: A typical diagram for 2 → 3 contributions.
Figure 2: Typical diagrams for $\kappa_t$-factorization approach.
Pair of partons in $\kappa_t$-factorization approach

\[
\frac{d\sigma(h_1 h_2 \rightarrow j j)}{d^2 p_{1,t} d^2 p_{2,t}} = \int dy_1 dy_2 \frac{d^2 \kappa_{1t}}{\pi} \frac{d^2 \kappa_{2t}}{\pi} \frac{1}{16\pi^2 (x_1 x_2 s)^2} |\mathcal{M}(gg \rightarrow j j)|^2 
\cdot \delta^2(\vec{\kappa}_{1,t} + \vec{\kappa}_{2,t} - \vec{p}_{1,t} - \vec{p}_{2,t}) f(x_1, \kappa_{1,t}^2) f(x_2, \kappa_{2,t}^2)
\] (2)

where

\[
x_1 = \frac{m_{1t}}{\sqrt{s}} e^{y_1} + \frac{m_{2t}}{\sqrt{s}} e^{y_2},
\] (3)

\[
x_2 = \frac{m_{1t}}{\sqrt{s}} e^{-y_1} + \frac{m_{2t}}{\sqrt{s}} e^{-y_2}.
\] (4)

The final partonic state is $jj = gg, q\bar{q}$.

There are other (quark/antiquark initiated) processes ($\rightarrow$ see soon)
Pair of partons in $\kappa_t$-factorization approach

\[ f_1(x_1, \kappa_{1,t}^2) \rightarrow x_1 g_1(x_1) \delta(\kappa_{1,t}^2) \]  \hspace{1cm} (5)

and

\[ f_2(x_2, \kappa_{2,t}^2) \rightarrow x_2 g_2(x_2) \delta(\kappa_{2,t}^2) \]  \hspace{1cm} (6)

then one recovers the standard collinear formula.

Inclusive cross sections:

\[
\frac{d\sigma(h_1 h_2 \rightarrow j)}{dy_1 d^2 p_{1,t}} = 2 \int dy_2 \frac{d^2 \kappa_{1,t}}{\pi} \frac{d^2 \kappa_{2,t}}{\pi} (\ldots) \left| \vec{p}_{1,t} = \vec{\kappa}_{1,t} + \vec{\kappa}_{2,t} - \vec{p}_{2,t} \right. \]  \hspace{1cm} (7)

or equivalently

\[
\frac{d\sigma(h_1 h_2 \rightarrow j)}{dy_2 d^2 p_{2,t}} = 2 \int dy_1 \frac{d^2 \kappa_{1,t}}{\pi} \frac{d^2 \kappa_{2,t}}{\pi} (\ldots) \left| \vec{p}_{2,t} = \vec{\kappa}_{1,t} + \vec{\kappa}_{2,t} - \vec{p}_{1,t} \right. \]  \hspace{1cm} (8)
The integration with the Dirac delta function in (2)

\[ \int dy_1 dy_2 \frac{d^2 \kappa_{1t}}{\pi} \frac{d^2 \kappa_{2t}}{\pi} \left( \ldots \right) \delta^2 \left( \ldots \right) . \]  

(9)

can be performed by introducing the following new auxiliary variables:

\[ \vec{Q}_t = \vec{\kappa}_{1t} + \vec{\kappa}_{2t} , \]

\[ \vec{q}_t = \vec{\kappa}_{1t} - \vec{\kappa}_{2t} . \]  

(10)

The jacobian of this transformation is:

\[ \frac{\partial (\vec{Q}_t, \vec{q}_t)}{\partial (\vec{\kappa}_{1t}, \vec{\kappa}_{2t})} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = 2 \cdot 2 = 4 . \]  

(11)
Then:

\[
\frac{d\sigma(h_1 h_2 \rightarrow Q\bar{Q})}{d^2 p_{1,t} d^2 p_{2,t}} = \frac{1}{4} \int dy_1 dy_2 d^2 q_t d^2 q_t (\ldots) \delta^2 (\vec{Q}_t - \vec{P}_{1,t} - \vec{P}_{2,t})
\]

(12)

\[
= \frac{1}{4} \int dy_1 dy_2 d^2 q_t (\ldots) |_{\vec{Q}_t = \vec{P}_t} =
\]

(13)

\[
= \frac{1}{4} \int dy_1 dy_2 \left\{ q_t dq_t d\varphi (\ldots) \right\} |_{\vec{Q}_t = \vec{P}_t} =
\]

(14)

\[
= \frac{1}{4} \int dy_1 dy_2 \left\{ \frac{1}{2} dq_t^2 d\varphi (\ldots) \right\} |_{\vec{Q}_t = \vec{P}_t}.
\]

(15)

Above \( \vec{P}_t = \vec{p}_{1,t} + \vec{p}_{2,t} \).
If one is interested in the distribution of the sum of transverse momenta of the outgoing quarks, then it is convenient to write

\[ d^2 p_{1,t} \, d^2 p_{2,t} = \frac{1}{4} d^2 P_t d^2 p_t = \frac{1}{4} d\varphi + P_t dP_t \, d\varphi - p_t dp_t \]

\[ = \frac{1}{4} 2\pi P_t dP_t \, d\varphi - p_t dp_t . \]  

(16)

If one is interested in studying a two-dimensional map \( p_{1,t} \times p_{2,t} \) then

\[ d^2 p_{1,t} \, d^2 p_{2,t} = d\varphi_1 \, p_{1,t} dP_{1,t} \, d\varphi_2 \, p_{2,t} dp_{2,t} . \]

(17)

Then

\[ \frac{d\sigma(p_{1,t}, p_{2,t})}{dp_{1,t} dp_{2,t}} = \int d\varphi_1 d\varphi_2 \, p_{1,t} p_{2,t} \int dy_1 dy_2 \, \frac{1}{4} q_t dq_t d\varphi_{q_t} \, (...) . \]

(18)
Pair of partons in $k_t$-factorization approach

It is convenient to make the following transformation of variables

\[
(\phi_1, \phi_2) \rightarrow (\phi_{\text{sum}} = \phi_1 + \phi_2, \phi_{\text{dif}} = \phi_1 - \phi_2), \quad (19)
\]

where $\phi_{\text{sum}} \in (0, 4\pi)$ and $\phi_{\text{dif}} \in (-2\pi, 2\pi)$. Now the new domain $(\phi_{\text{sum}}, \phi_{\text{dif}})$ is twice bigger than the original one $(\phi_1, \phi_2)$.

\[
d\phi_1 d\phi_2 = \left(\frac{\partial \phi_1 \partial \phi_2}{\partial \phi_{\text{sum}} \partial \phi_{\text{dif}}} \right) d\phi_{\text{sum}} d\phi_{\text{dif}}. \quad (20)
\]

The transformation jacobian is:

\[
\left(\frac{\partial \phi_1 \partial \phi_2}{\partial \phi_{\text{sum}} \partial \phi_{\text{dif}}} \right) = \frac{1}{2}. \quad (21)
\]
Pair of partons in $\kappa_t$-factorization approach

$$d^2 p_{1,t} \, d^2 p_{2,t} = p_{1,t} dp_{1,t} \, p_{2,t} dp_{2,t} \frac{d\phi_{\text{sum}} d\phi_{\text{dif}}}{2}$$

$$= p_{1,t} dp_{1,t} \, p_{2,t} dp_{2,t} \, 2\pi d\phi_{\text{dif}} . \quad (22)$$

The integrals in Eq.(18) can be written equivalently as

$$\frac{d\sigma(p_{1,t}, p_{2,t})}{dp_{1,t} dp_{2,t}} = \frac{1}{2} \cdot \frac{1}{2} \int d\phi_{\text{sum}} d\phi_{\text{dif}} \, p_{1,t} p_{2,t} \int dy_1 dy_2 \, \frac{1}{4} q_t dq_t d\phi_{q_t} \, (...) . \quad (23)$$

First $\frac{1}{2} -$ jacobian, second $\frac{1}{2} -$ extra extension of the domain.
By symmetry, there is no dependence on $\phi_{\text{sum}}$

$$\frac{d\sigma(p_{1,t}, p_{2,t})}{dp_{1,t} dp_{2,t}} = \frac{1}{2} \cdot \frac{1}{2} \cdot 4\pi \int d\phi_{\text{dif}} \, p_{1,t} p_{2,t} \int dy_1 dy_2 \, \frac{1}{4} q_t dq_t d\phi_{q_t} \, (...) . \quad (24)$$
Matrix elements for $2 \rightarrow 2$ processes

The matrix elements for on-shell initial gluons/partons

\[
|\mathcal{M}_{gg \rightarrow gg}|^2 = \frac{9}{2} g_s^4 \left( 3 - \frac{\hat{t}\hat{u}}{\hat{s}^2} - \frac{\hat{s}\hat{u}}{\hat{t}^2} - \frac{\hat{s}\hat{t}}{\hat{u}^2} \right),
\]

\[
|\mathcal{M}_{gg \rightarrow q\bar{q}}|^2 = \frac{1}{8} g_s^4 \left( 6 \frac{\hat{t}\hat{u}}{\hat{s}^2} + \frac{4}{3} \frac{\hat{u}}{\hat{t}} + \frac{4}{3} \frac{\hat{t}}{\hat{u}} + \frac{3}{\hat{s}} \frac{\hat{t}}{\hat{s}} + \frac{3}{\hat{s}} \frac{\hat{u}}{\hat{s}} \right),
\]

\[
|\mathcal{M}_{gq \rightarrow gq}|^2 = g_s^4 \left( -\frac{4}{9} \frac{\hat{s}^2 + \hat{u}^2}{\hat{s}\hat{u}} + \frac{\hat{u}^2 + \hat{s}^2}{\hat{t}^2} \right),
\]

\[
|\mathcal{M}_{qg \rightarrow qg}|^2 = g_s^4 \left( -\frac{4}{9} \frac{\hat{s}^2 + \hat{t}^2}{\hat{s}\hat{t}} + \frac{\hat{t}^2 + \hat{s}^2}{\hat{u}^2} \right).
\]

(25)

The matrix elements for off-shell initial gluons – the same formulae but with $\hat{s}, \hat{t}, \hat{u}$ from off-shell kinematics. In this case $\hat{s} + \hat{t} + \hat{u} = k_1^2 + k_2^2$, where $k_1^2, k_2^2 < 0$. Our prescription – a smooth analytic continuation of the on-shell formula off mass shell.
2 → 3 processes in collinear approach

Standard parton model formula:

\[ d\sigma(h_1 h_2 \rightarrow ggg) = \int dx_1 dx_2 \; g_1(x_1, \mu^2) g_2(x_2, \mu^2) \; d\hat{\sigma}(gg \rightarrow ggg) \]

The elementary cross section can be written as

\[ d\hat{\sigma}(gg \rightarrow ggg) = \frac{1}{2\hat{s}} |M_{gg \rightarrow ggg}|^2 dR_3 \]

The three-body phase space element is:

\[ dR_3 = \frac{d^3p_1}{2E_1(2\pi)^3} \frac{d^3p_2}{2E_2(2\pi)^3} \frac{d^3p_3}{2E_3(2\pi)^3} (2\pi)^4 \delta^4(p_a + p_b - p_1 - p_2 - p_3) \]

(26)

(27)

(28)
processes in collinear-factorization approach.

It can be written in an equivalent way as:

\[
dR_3 = \frac{dy_1 d^2 p_1}{(4\pi)(2\pi)^2} \frac{dy_2 d^2 p_2}{(4\pi)(2\pi)^2} \frac{dy_3 d^2 p_3}{(4\pi)(2\pi)^2} (2\pi)^4 \delta^4(p_a + p_b - p_1 - p_2 - p_3),
\]

(29)

The last formula is useful for practical purposes. Now

\[
d\sigma = dy_1 d^2 p_1 dy_2 d^2 p_2 dy_3 \left( \frac{1}{(4\pi)^3 (2\pi)^2} \frac{1}{\hat{s}^2} x_1 f_1(x_1, \mu_f^2) x_2 f_2(x_2, \mu_f^2) \right) |\mathcal{M}_{2 \rightarrow 3}|^2
\]

(30)

where

\[
x_1 = \frac{p_1}{\sqrt{s}} \exp(+y_1) + \frac{p_2}{\sqrt{s}} \exp(+y_2) + \frac{p_3}{\sqrt{s}} \exp(+y_3),
\]

(31)
Repeating similar steps as for $2 \rightarrow 2$: 

$$d\sigma = \frac{1}{64\pi^4 \hat{s}^2} x_1 f_1(x_1, \mu_f^2) x_2 f_2(x_2, \mu_f^2) \left| \mathcal{M}_{2\rightarrow 3} \right|^2$$

$$p_{1t} dp_{1t} p_{2t} dp_{2t} d\Phi_- dy_1 dy_2 dy_3,$$  \hspace{1cm} (32)

where $\Phi_-$ is restricted to the interval $(0, \pi)$. 

Matrix elements for $2 \rightarrow 3$ processes

For the $gg \rightarrow ggg$ process $(k_1 + k_2 \rightarrow k_3 + k_4 + k_5)$ the squared matrix element is

$$|\mathcal{M}|^2 = \frac{1}{2} g_s^6 \frac{N_c^3}{N_c^2 - 1} \left[ (12345) + (12354) + (12435) + (12453) + (12534) + (12543) + (13245) + (13254) + (13425) + (13524) + (12453) + (14325) \right]$$

$$\times \sum_{i<j} (k_i k_j) / \prod_{i<j} (k_i k_j) ,$$

(33)

where $(ijlmn) \equiv (k_i k_j)(k_j k_l)(k_l k_m)(k_m k_n)(k_n k_i)$. 
Matrix elements for $2 \rightarrow 3$ processes

It is useful to calculate matrix element for the process $q\bar{q} \rightarrow ggg$. The squared matrix elements for other processes can be obtained by crossing the squared matrix element for the process $q\bar{q} \rightarrow ggg \ (p_a + p_b \rightarrow k_1 + k_2 + k_3)$

$$|M|^2 = g_s^6 \frac{N_c^2 - 1}{4N_c^4}$$

$$\sum_{i}^{3} a_i b_i (a_i^2 + b_i^2)/(a_1 a_2 a_3 b_1 b_2 b_3)$$

$$\times \left[ \frac{\hat{s}}{2} + N_c^2 \left( \frac{\hat{s}}{2} - \frac{a_1 b_2 + a_2 b_1}{(k_1 k_2)} - \frac{a_2 b_3 + a_3 b_2}{(k_2 k_3)} - \frac{a_3 b_1 + a_1 b_3}{(k_3 k_1)} \right) \right]$$

$$+ \frac{2N_c^4}{\hat{s}} \left( \frac{a_3 b_3(a_1 b_2 + a_2 b_1)}{(k_2 k_3)(k_3 k_1)} + \frac{a_1 b_1(a_2 b_3 + a_3 b_2)}{(k_3 k_1)(k_1 k_2)} + \frac{a_2 b_2(a_3 b_1 + a_1 b_3)}{(k_1 k_2)(k_2 k_3)} \right)$$

(34)
Matrix elements for $2 \to 3$ processes

The matrix element for the process $gg \to q\bar{q}g$ is obtained from that of $q\bar{q} \to ggg$ by appropriate crossing:

$$|\mathcal{M}|^2_{gg \to q\bar{q}g}(k_1, k_2, k_3, k_4, k_5) = \frac{9}{64} |\mathcal{M}|^2_{q\bar{q} \to ggg}(-k_4, -k_3, -k_1, -k_2, k_5).$$  \hspace{1cm} (36)

We sum over 3 final flavours ($f = u, d, s$).

For the $qq \to qgg$ process

$$|\mathcal{M}|^2_{qq \to qgg}(k_1, k_2, k_3, k_4, k_5) = \left(-\frac{3}{8}\right) |\mathcal{M}|^2_{q\bar{q} \to ggg}(k_1, -k_3, -k_2, k_4, k_5)$$  \hspace{1cm} (37)

and finally for the process $g\bar{q} \to \bar{q}gg$

$$|\mathcal{M}|^2_{g\bar{q} \to \bar{q}gg}(k_1, k_2, k_3, k_4, k_5) = \left(-\frac{3}{8}\right) |\mathcal{M}|^2_{q\bar{q} \to ggg}(-k_3, k_2, -k_1, k_4, k_5).$$  \hspace{1cm} (38)
Gaussian smearing

\[ F_{\text{naive}}(x, \kappa^2, \mu_F^2) = x g_{\text{coll}}(x, \mu_F^2) \cdot f_{\text{Gauss}}(\kappa^2) , \quad (39) \]

\[ f_{\text{Gauss}}(\kappa^2) = \frac{1}{2\pi\sigma_0^2} \exp \left( -\frac{\kappa_t^2}{2\sigma_0^2} \right) / \pi . \quad (40) \]

BFKL UGDF

\[ -x \frac{\partial f(x, q_t^2)}{\partial x} = \frac{\alpha_s N_c}{\pi} q_t^2 \int_0^\infty dq_{1t}^2 \frac{dq_t^2}{q_{1t}^2} \left[ \frac{f(x, q_{1t}^2) - f(x, q_t^2)}{|q_t^2 - q_{1t}^2|} + \frac{f(x, q_t^2)}{\sqrt{q_t^4 + 4q_{1t}^4}} \right] . \quad (41) \]
Golec-Biernat-Wuesthoff saturation model
from dipole-nucleon cross section to UGDF

\[ \alpha_s \mathcal{F}(x, \kappa_t^2) = \frac{3\sigma_0}{4\pi^2} R_0^2(x) \kappa_t^2 \exp(-R_0^2(x) \kappa_t^2), \]  

(42)

\[ R_0(x) = \left( \frac{x}{x_0} \right)^{\lambda/2} \frac{1}{GeV}. \]  

(43)

Parameters adjusted to HERA data for \( F_2 \).

Kharzeev-Levin gluon saturation

\[ \mathcal{F}(x, \kappa^2) = \begin{cases} f_0 & \text{if } \kappa^2 < Q_s^2, \\ f_0 \cdot \frac{Q_s^2}{\kappa^2} & \text{if } \kappa^2 > Q_s^2. \end{cases} \]  

(44)

\( f_0 \) adjusted by Szczurek to HERA data for \( F_2 \).
Kwiecinski parton distributions

QCD-most-consistent approach – CCFM.

For LO ($2 \rightarrow 1$) processes convenient to use UPDFs in a space conjugated to transverse momentum (Kwieciński et al.)

$$\tilde{f}(x, b, \mu^2) = \frac{1}{2\pi} \int d^2\kappa \exp \left(-i\vec{\kappa} \cdot \vec{b}\right) \mathcal{F}(x, \kappa^2, \mu^2)$$

$$\mathcal{F}(x, \kappa^2, \mu^2) = \frac{1}{2\pi} \int d^2 b \exp \left(i\vec{\kappa} \cdot \vec{b}\right) \tilde{f}(x, b, \mu^2)$$

The relation between

Kwieciński UPDF and the collinear PDF:

$$xp_k(x, \mu^2) = \int_0^\infty d\kappa_t^2 f_k(x, \kappa_t^2, \mu^2)$$
At $b = 0$ the functions $f_j$ are related to the familiar integrated parton distributions, $p_j(x, Q)$, as follows:

$$f_j(x, 0, Q) = \frac{x}{2} p_j(x, Q).$$

$$p_{NS} = u - \bar{u}, \quad d - \bar{d},$$
$$p_S = \bar{u} + u + \bar{d} + d + \bar{s} + s + ...,$$
$$p_{sea} = 2\bar{d} + 2u + \bar{s} + s + ...,$$
$$p_G = g,$$

where ... stand for higher flavors.
for a given impact parameter:

\[
\frac{\partial f_{NS}(x, b, Q)}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi Q^2} \int_0^1 dz \, P_{qq}(z) \left[ \Theta(z - x) \, J_0((1 - z)Qb) \, f_{NS} \left(\frac{x}{z}, b, Q\right) \right.
\]
\[
- f_{NS}(x, b, Q) \right]
\]

\[
\frac{\partial f_{S}(x, b, Q)}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi Q^2} \int_0^1 dz \left\{ \Theta(z - x) \, J_0((1 - z)Qb) \left[ P_{qq}(z) \, f_{S} \left(\frac{x}{z}, b, Q\right) \right]
\right.
\]
\[
+ P_{qg}(z) \, f_{G} \left(\frac{x}{z}, b, Q\right) \right\} - [zP_{qq}(z) + zP_{qg}(z)] \, f_{S}(x, b, Q) \}
\]

\[
\frac{\partial f_{G}(x, b, Q)}{\partial Q^2} = \frac{\alpha_s(Q^2)}{2\pi Q^2} \int_0^1 dz \left\{ \Theta(z - x) \, J_0((1 - z)Qb) \left[ P_{qg}(z) \, f_{S} \left(\frac{x}{z}, b, Q\right) \right]
\right.
\]
\[
+ P_{gg}(z) \, f_{G} \left(\frac{x}{z}, b, Q\right) \right\} - [zP_{gg}(z) + zP_{qg}(z)] \, f_{G}(x, b, Q) \}
Nonperturbative effects

Transverse momenta of partons due to:

- **perturbative effects**
  (solution of the Kwieciński-CCFM equations),
- **nonperturbative effects**
  (intrinsic momentum distribution of partons)

Take factorized form in the b-space:

\[
\tilde{f}_q(x, b, \mu^2) = \tilde{f}_q^{CCFM}(x, b, \mu^2) \cdot F_q^{np}(b).
\]

We use a *flavour and x independent* form factor

\[
F_q^{np}(b) = \exp \left( \frac{-b^2}{4b_0^2} \right)
\]

May be too simplistic?
Unintegrated gluon distributions (comparison)

$x = 10^{-2}$

- derivative UGDF
  - $\mu^2 = 0.3, 0.5, 0.7 \text{ GeV}^2$

- Bluemlein UGDF
  - $\alpha_s = 0.2, 0.4, 0.6$

- Kwiecinski UGDF
  - $\mu^2 = 10, 100 \text{ GeV}^2$
  - $b_0 = 1.2 \text{ GeV}^{-1}$
Processes included in our $k_t$-factorization approach

There are 4 important contributions:

- $\text{gluon+gluon} \rightarrow \text{gluon+gluon}$ (Leonidov-Ostrovsky)
- $\text{gluon+gluon} \rightarrow \text{quark+antiquark}$ (Leonidov-Ostrovsky)
- $\text{gluon+(anti)quark} \rightarrow \text{gluon+(anti)quark}$ (new !!!)
- $\text{(anti)quark+gluon} \rightarrow \text{(anti)quark+gluon}$ (new !!!)

First two processes discussed also by:
Bartels-Sabio-Vera-Schwennsen
New contributions

Figure 3:
Processes included in $k_t$-factorization

$gg \rightarrow gg$ (left upper),
$gg \rightarrow q\bar{q}$ (right upper),
$gq \rightarrow gq$ (left lower),
$qg \rightarrow qg$ (right lower).

Kwiecinski UPDFs with $b_0 = 1$ GeV$^{-1}$, $\mu^2 = 100$ GeV$^2$.
Full range of parton rapidities.
Processes included in $k_t$-factorization

Fractional contributions of different subprocesses

- $gg \to gg$ (left upper),
- $gg \to q\bar{q}$ (right upper),
- $gq \to gq$ (left lower),
- $qg \to qg$ (right lower).

Kwieciński UPDFs with $b_0 = 1 \text{ GeV}^{-1}$, $\mu^2 = 100 \text{ GeV}^2$.
5 GeV $< p_{1t}, p_{2t} < 20$ GeV.
Azimuthal correlations

\[ W = 200 \text{ GeV} \]
\[ p_{1t}, p_{2t} = (5, 15) \text{ GeV} \]

\[ \frac{d\sigma}{d\phi} \text{ (mb)} \]

\[ \varphi \text{ (deg)} \]

- \( gg \rightarrow gg \)
- \( gg \rightarrow q\bar{q}q\bar{q} \)
- \( gq \rightarrow gq \)
- \( qg \rightarrow qg \)
$\mu^2 = 0.25$ (black), $10$ (blue), $100$ (red) GeV$^2$
Different UGDFs

\[ W = 200 \text{ GeV} \]
\[ p_{1t}, p_{2t} = (5, 15) \text{ GeV} \]
\[ y_1, y_2 = (-4, 4) \]

\[ \frac{d\sigma}{d\varphi} (\text{mb}) \]

- Kwiecinski
- Kharzeev-Levin
- BFKL
- Ivanov-Nikolaev

\[ \varphi (\text{deg}) \]

\[ 0 \quad 50 \quad 100 \quad 150 \]

\[ 10^{-5} \quad 10^{-4} \quad 10^{-3} \quad 10^{-2} \quad 10^{-1} \quad 1 \]
2 → 3 processes in collinear approach

Figure 7: $gg \rightarrow ggg$ component for $W = 200$ GeV.

Singularities when $\vec{p}_1 \rightarrow 0$, $\vec{p}_2 \rightarrow 0$ and $\vec{p}_3 \rightarrow 0$. 
How to remove NLO singularities?

$k_t$-factorization – no singularities, no delta functions !!!
$gg \to gg$, different UGDFs vs $gg \to ggg$

- $-4 < y_1, y_2 < 4$.

KL (left upper),
BFKL (right upper),
Ivanov-Nikolaev (left lower),
$gg \to ggg$ (right lower).
Dijet correlations for $gg \rightarrow ggg$, leading jets

$p_{1t}(\text{selected}) > p_{3t}$ and $p_{2t}(\text{selected}) > p_{3t}$
Dijet correlations for $gg \rightarrow ggg$, leading jets

$p_{1t}(selected) > p_{3t}$ and $p_{2t}(selected) > p_{3t}$

Figure 9:
Figure 10: Definition of windows in $p_{1t} \times p_{2t}$ plane.
Figure 11:
Extra scalar cuts

- to eliminate LO and NLO singularities (yes!)
- to enhance resummation with respect to NLO (no!)

Figure 12: $|p_{1t} - p_{2t}| > \Delta_s$. 
Extra vector cuts

- to eliminate LO and NLO singularities (yes!)
- to enhance resummation with respect to NLO (no!)

Figure 13: $|\vec{p}_{1t} + \vec{p}_{2t}| > \Delta_v$. 
Summary/Conclusions of the first part

- **Dijet correlations** at RHIC have been calculated in the $k_t$-factorization approach with different UGDFs (UPDFs) from the literature.

- Two new mechanisms have been included compared to the literature. They are dominant at larger rapidities (or rapidity gaps) i.e. constitute competition for Mueller-Navelet (BFKL) jets.

- Results have been compared with collinear NLO calculations.

- At $\phi < 120^0$ and/or asymmetric jet transverse momenta the $k_t$-factorization is superior over the collinear NLO.

- This calculation is a first step for hadron-hadron correlations measured at RHIC. Here internal structure of both jets enters in addition.

- The method can be used in semihard region (small $p_t$) at LHC.
Photon-jet correlations
Plan of the second part of the talk

- Introduction
- Inclusive spectra
- Photon-jet correlations
- Results
- Conclusions

based partially on:
1) Phys. Rev. D75, 014023 (2007)
2) arXiv:hep-ph/0704.2158, Phys. Rev. D76 034003
in collaboration with T. Pietrycki
Cascade mechanism 1

\[
\begin{align*}
&h_1 & \rightarrow & X_1 \\
&h_2 & \rightarrow & X_2 \\
&(x_1, k_{1t}) & \rightarrow & p_{1t} \\
&(x_2, k_{2t}) & \rightarrow & p_{2t} \\
\end{align*}
\]

\[
\begin{align*}
&h_1 & \rightarrow & X_1 \\
&h_2 & \rightarrow & X_2 \\
&(x_1, k_{1t}) & \rightarrow & p_{1t} \\
&(x_2, k_{2t}) & \rightarrow & p_{2t} \\
\end{align*}
\]

\[
\begin{align*}
\text{soft emissions} & \quad \text{soft emissions} \\
\text{hard } \gamma, q & \quad \text{hard } \gamma, q \\
\text{soft emissions} & \quad \text{soft emissions}
\end{align*}
\]
Cascade mechanism 2

\[ h_1 \to X_1 \to (x_1, k_{1t}) \to p_{1t} \to \text{soft emissions} \]
\[ (x_2, k_{2t}) \to p_{2t} \to \text{soft emissions} \]

\[ h_2 \to X_2 \to (x_1, k_{1t}) \to p_{1t} \to \text{hard } \gamma, g \]
\[ (x_2, k_{2t}) \to p_{2t} \to \text{hard } \gamma, g \]

\[ \text{soft emissions} \]

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Kimber-Martin-Ryskin for \( k_t^2 > k_{t,0}^2 \)

\[
 f_q(x, k_t^2, \mu^2) = T_q(k_t^2, \mu^2) \frac{\alpha_s(k_t^2)}{2\pi} \\
 \cdot \int_x^1 dz \left[ P_{qq}(z) \frac{x}{z} q\left(\frac{x}{z}, k_t^2\right) \Theta(\Delta - z) + P_{qg}(z) \frac{x}{z} g\left(\frac{x}{z}, k_t^2\right) \right]
\]

\[
 f_g(x, k_t^2, \mu^2) = T_g(k_t^2, \mu^2) \frac{\alpha_s(k_t^2)}{2\pi} \\
 \cdot \int_x^1 dz \left[ P_{gg}(z) \frac{x}{z} g\left(\frac{x}{z}, k_t^2\right) \Theta(\Delta - z) + \sum_q P_{gq}(z) \frac{x}{z} q\left(\frac{x}{z}, k_t^2\right) \right]
\]

saturation for \( k_t^2 < k_{t,0}^2 \)
\[
\frac{d\sigma(h_1 h_2 \rightarrow \gamma, \text{parton})}{d^2 p_{1,t} d^2 p_{2,t}} = \int dy_1 dy_2 \frac{d^2 k_{1,t}}{\pi} \frac{d^2 k_{2,t}}{\pi} \frac{1}{16\pi^2 (x_1 x_2 s)^2} \sum_{i,j,k} |M(i,j \rightarrow \gamma k)|^2 
\cdot \delta^2(\vec{k}_{1,t} + \vec{k}_{2,t} - \vec{p}_{1,t} - \vec{p}_{2,t}) f_i(x_1, k_{1,t}^2) f_j(x_2, k_{2,t}^2)
\]

\[(i, j, k) = (q, \bar{q}, g), (\bar{q}, q, g), (g, \bar{q}, q), (q, g, q)\]

standard collinear formula

\[
f_i(x_1, k_{1,t}^2) \rightarrow x_1 p_i(x_1) \delta(k_{1,t}^2)
\]

\[
f_j(x_2, k_{2,t}^2) \rightarrow x_2 p_j(x_2) \delta(k_{2,t}^2)
\]
Differential cross section

$2 \rightarrow 2$ in $k_t$-factorization approach

$$d\sigma_{h_1h_2\rightarrow\gamma,k} = dy_1dy_2d^2p_{1,t}d^2p_{2,t} \frac{d^2k_{1,t}}{\pi} \frac{d^2k_{2,t}}{\pi} \frac{1}{16\pi^2(x_1x_2s)^2} \sum_{i,j,k} |M_{ij\rightarrow\gamma k}|^2 \cdot f_i(x_1,k_{1,t}^2)f_j(x_2,k_{2,t}^2)\delta^2(\vec{k}_{1,t} + \vec{k}_{2,t} - \vec{p}_{1,t} - \vec{p}_{2,t})$$

$2 \rightarrow 3$ in collinear-factorization approach

$$d\sigma_{h_1h_2\rightarrow\gamma kl} = dy_1dy_2dy_3d^2p_{1,t}d^2p_{2,t} \frac{1}{(4\pi)^3(2\pi)^2} \frac{1}{\hat{s}^2} \sum_{i,j,k,l} |M_{ij\rightarrow\gamma kl}|^2 \cdot x_1p_i(x_1,\mu^2)x_2p_j(x_2,\mu^2)$$

see Aurenche et al., Nucl. Phys. B286 553 (87)
Photon-jet correlations \( \frac{d\sigma}{d\phi} \)

2 → 2 in \( k_t \)-factorization approach

\[
\frac{d\sigma_{h_1 h_2 \rightarrow \gamma k}}{d\phi_-} = \int \frac{2\pi}{16\pi^2(x_1 x_2 s)^2} \frac{f_i(x_1, k_{1,t}^2)}{\pi} \frac{f_j(x_2, k_{2,t}^2)}{\pi} \sum_{i,j,k} |M_{ij \rightarrow \gamma k}|^2 
\cdot p_{1,t} dp_{1,t} p_{2,t} dp_{2,t} dy_1 dy_2 q_t dq_t d\phi_{qt}
\]

2 → 3 in \text{collinear}-factorization approach

\[
\frac{d\sigma_{h_1 h_2 \rightarrow \gamma kl}}{d\phi_-} = \int \frac{1}{64\pi^4 S^2} x_1 p_i(x_1, \mu^2) x_2 p_j(x_2, \mu^2) \sum_{i,j,k,l} |M_{ij \rightarrow \gamma kl}|^2 
\cdot p_{1,t} dp_{1,t} p_{2,t} dp_{2,t} d y_1 d y_2 d y_3
\]
Decorrelations in $(p_{1,t}, p_{2,t})$ space
Scale dependence in Kwieciński UPDFs
Photon-jet correlations $d\sigma/d\phi_-$

NLO collinear vs $k_t$-factorization approach

$\sqrt{s} = 1960$ GeV

$p_{1,t}, p_{2,t} \in (5, 20)$ GeV

$y_1, y_2, y_3 \in (-4, 4)$

NLO collinear

- Gauss $\sigma_0 = 1$ GeV
- KMR $k_{t0}^2 = 1$ GeV$^2$

Kwieciński $b_0 = 1/\text{GeV}$
Scalar cuts

\[ |p_{1,t} - p_{2,t}| > \Delta S \]

\[ \sqrt{s} = 1960 \text{ GeV} \]

\[ p_{1,t}, p_{2,t} \in (5, 20) \text{ GeV} \]

\[ y_1, y_2, y_3 \in (-4, 4) \]

NLO collinear

Gauss

KMR

Kwieciński

\[ k^2_{t0} = 1 \text{ GeV}^2 \]

\[ b_0 = 1/ \text{ GeV} \]
Vector cuts

\[ |\vec{p}_{1,t} + \vec{p}_{2,t}| > \Delta V \]

\[ \sqrt{s} = 1960 \text{ GeV} \]

\[ p_{1,t}, p_{2,t} \in (5, 20) \text{ GeV} \]

\[ y_1, y_2, y_3 \in (-4, 4) \]

**NLO collinear Gauss**

\[ \sigma_0 = 1 \text{ GeV} \]

**KMR**

\[ k_{t0}^2 = 1 \text{ GeV}^2 \]

**Kwieciński**

\[ b_0 = 1/ \text{ GeV} \]
Leading photon/jet

NLO collinear

\( \sqrt{s} = 1960 \text{ GeV} \)
\( p_{1,t}, p_{2,t} \in (5, 20) \text{ GeV} \)
\( y_1, y_2, y_3 \in (-4, 4) \)

(dashed) no limits on \( p_{3,t} \)
(solid) \( p_{3,t} < p_{2,t} \)
(dotted) \( p_{3,t} < p_{1,t} \)
\( p_{3,t} < p_{2,t} \)

\( p_{1,t} \) - photon
\( p_{2,t} \) - observed parton
\( p_{3,t} \) - unobs. parton
Leading photon/jet

NLO collinear versus $k_t$-factorization

(solid) $p_{3,t} < p_{2,t}$

(dotted) $p_{3,t} < p_{1,t}$

$p_{3,t}$ - unobs. parton

$p_{1,t}$ - photon

$p_{2,t}$ - observed parton

$\sqrt{s} = 1960$ GeV

$p_{1,t}, p_{2,t} \in (5, 20)$ GeV

$y_{1,2,3} \in (-4, 4)$

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Leading photon/jet in \((p_{1,t}, p_{2,t})\) space

- No limits on \(p_{3,t}\)
- \(p_{3,t} < p_{2,t}\)
- \(p_{3,t} < p_{1,t}\)
- \(p_{3,t} < p_{2,t}\)
Windows in \((p_{1,t}, p_{2,t})\)
Windows in $(p_{1,t}, p_{2,t})$ - RHIC
Photon hadron correlations

\[ h_1 \rightarrow i \rightarrow \gamma \rightarrow j \rightarrow h \]

\[ h_2 \rightarrow k \rightarrow h \]

\[ h_1 \rightarrow i \rightarrow \gamma \rightarrow l \rightarrow h \]

\[ h_2 \rightarrow k \rightarrow h \]
Photon hadron correlations - results

![Graphs showing photon-hadron correlations](image)

The graphs depict the differential cross section $d\sigma/d\Delta\phi$ in nb for different ranges of $p_{T,\gamma}$ and $p_{T,assoc}$, with $\Delta\phi$ being the angle between the photon and associated hadron. The graphs are divided into three sections, each showing the cross section for different $p_{T,\gamma}$ ranges:

1. $5 < p_{T,\gamma} < 7$ GeV
2. $7 < p_{T,\gamma} < 9$ GeV
3. $9 < p_{T,\gamma} < 12$ GeV

Each section further subdivides into cases where $p_{T,assoc}$ is less than 2 GeV and 5 GeV. The graphs are labeled 'total', 'gq + qg', and 'q\bar{q} + g\bar{q}' for different combinations of flavor and mass.
Good agreement with exp. data using Kwiecinski UPDFs (carefull treatment of the evolution of the QCD ladder)

Predictions made for LHC based on several UPDFs

The $k_t$-factorization approach is also better tool
  - for $\phi_- < \pi/2$ if leading parton/photon condition is imposed
  - for $\phi_- = \pi$ (no singularities)

RHIC measures $\gamma$-hadron, next step inclusion of jet hadronization
Drell-Yan with $k_t$ smearing

Lowest order process:

Initial quarks and antiquarks

Kwieciński UPDFs a good tool to include initial transverse momenta

Nonzero transverse momenta of the lepton pair.
Examples of higher order subprocesses:

Initial $k_t$ included
No singularities
Drell-Yan versus semileptonic decays

Drell-Yan, $O(\alpha_s^1)$  
Semileptonic decays of $D$ and $\bar{D}$

$W=200$ GeV, Kwieciński UPDFs for both
Summary of SLD and DY

We have calculated \((p_{1t}, p_{2t})\) and azimuthal \(e^+e^-\) correlations including:

(a) \(gg \rightarrow c\bar{c} \rightarrow D\bar{D} \rightarrow e^+e^-\),
(b) \(gg \rightarrow b\bar{b} \rightarrow B\bar{B} \rightarrow e^+e^-\),
(c) \(O(\alpha_s^0)\) and \(O(\alpha_s^1)\) Drell-Yan within \(k_t\)-factorization approach.

For SLD decorrelation in azimuthal angle \(\phi_{ee}\) is due to:

(a) decay \(D \rightarrow e^+ (\bar{D} \rightarrow e^-)\)
(b) initial \(k_t\)-smearing of gluons (UGDFs)

For Drell-Yan decorrelation in \(\phi_{ee}\) is due to:

(a) initial \(k_t\)-smearing of quarks and antiquarks.

At RHIC (\(W=200\) GeV) dominance of SLD over DY in the large part of the phase-space. At LHC it may be even worse!
$J/\psi$ - gluon correlations
$J/\psi$ - gluon correlations

Kwieciński UGDF

$\mu^2 = 10 \text{ GeV}^2$ (left), $\mu^2 = 100 \text{ GeV}^2$ (right)
$J/\psi$ - gluon correlations, gluons from the ladder
From RHIC to LHC

- Our future: LHC
- Calculations must be done, more difficulties, small x, saturation effects?
- Large rapidities will be accessible (very small $x$).
- Small $p_t$ with ALICE (saturation effects).
- Really large $p_t$ will be available (domain of NLO, NNLO).
- Good lack for LHC and the correlation program for proton-proton collisions.
- Nuclear correlation program is very interesting at RHIC, It will be the same at LHC.
For our future with LHC