Optimal voltage regulator of power supply system for submersible processing equipment

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Abstract. The article considers the synthesis of the optimal voltage regulator for the electric power supply system of the electromechanical complex of the submersible apparatus with the power transmission through the cable line. Based on the reduced linearized model of the multidimensional nonlinear control object the regulator with adjustable parameters is synthesized, providing the DC voltage stabilization at the power supply system load. The variant of building proportional feedback on output state variables is used for the regulator in the system with parametric and external disturbances. Due to the technical impossibility of measuring the voltage on the load, a combined optimal regulator with negative feedback is organized, which makes it possible to stabilize the voltage at the payload within the specified limits when receiving data from the output filter of voltage source inverter.

1. Introduction
The efficiency of submersible processing equipment such as electric submersible pumps for extracting oil and electromechanical complex of the deep-diving submersibles is largely determined by the characteristics of power supply systems (PSS) with the power transmission through the cable line. The main problem in the construction of such special power supply systems is the problem of synthesis of optimal control, providing voltage stabilization at the remote payload of the submersible part [1–3]. At the same time, the impossibility of measuring the regulated voltage on electromechanical load makes it difficult to directly apply the classical structures of closed loop systems with negative feedback on the output variable [4, 5]. In addition, the limitations on the computing resources of the control device that implements the laws and control algorithms, predetermines the allowable simplification of the mathematical model of complex, multidimensional and nonlinear PSS with non-stationary parameters. Therefore, along with the task of synthesis of the optimal regulator, the task of reduction of the initial model of PSS becomes actual, i.e. construction of a lower order model, which adequately reflects the behavior of the power supply system. It is assumed that the basis for the construction of the law providing the required quality of the output voltage stabilization of PSS can be a linearized mathematical model that allows the use of methods of linear control theory. The obtained reduced model of PSS in the form of differential equations allows to further take into account the variable disturbance pattern and nonlinear properties of the control object.
2. Structure of the power part of the PSS and principles of formation of the reduced mathematical model

The power supply system (Figure 1) considered in this paper contains a three-phase voltage source inverter (VSI) with pulse width modulation (PWM), to the input of which a DC voltage $U_s$ is supplied through the L-shaped LC-filter (F1). To smooth out the higher harmonics between the VSI and the step-up transformer (T1), the LC-filter (F2) is included. The increased voltage from the transformer T1 is supplied to the cable line (CL). The reduced voltage from the output of transformer T2 through three-phase bridge rectifier (R) and smoothing LC-filter with parameters $L_d$, $C_d$ (‘filter 3’ in Figure 1) is supplied to the load of the considered power supply system, which may be active and shown in the circuit by an equivalent resistance $R_n$. The windings of the step-up transformer T1 are connected according to the Δ/Y connection, and the step-down transformer T2 according to the Δ/Δ connection. The transformers parameters are windings active resistance and leakage inductance, taking into account the leakage flux influence.

![Diagram of the power part of the PSS](image)

Figure 1. Structure of the power part of the PSS.

Control of VSI in PSS is organized on the basis of PWM with third harmonic injection. For the description of the output phase voltage VSI the switching functions [6] are usually used. In our previous papers the mathematical model of PSS is described in detail. It is presented by the equations in the form of Koshi, which allows to simulate both dynamic and steady-state modes of operation of PSS power part [7, 8]. The complexity of the analysis of processes in PSS lies in the high order of the system (eleven differential equations per phase), as well as the presence of switching functions that describe the operation of VSI and rectifier, which consequently brings the complexity to the synthesis of a voltage regulator. To correctly simplify the mathematical model of PSS the following assumptions were accepted: discrete switching functions on the basis of the analysis of an amplitude spectrum were represented by continuous functions; the part of the circuit including transformers and a cable line, was replaced by an equivalent second order $RLC$-circuit. Such replacement allowed reducing the number of differential equations from eleven to eight per one phase [9].

The parameters of the equivalent circuit were determined from the condition of conformity of parameters of the second order transfer function on voltage and parameters of the transfer function on the basis of the analysis of the magnitude Bode plot of the PSS. The verification of the adequacy of the reduced mathematical model with the initial circuit of replacement of PSS was carried out by comparison of dependences of flowing currents and voltages in power elements [9]. As researches have shown, the approximation error does not exceed 4% that indicates the adequacy of mathematical model of PSS on the basis of transfer functions.

3. Analytical synthesis of the optimal regulator for PSS

Assuming that the parameters of the power supply system are stationary at the $i$-th interval of its electromechanical load operation, it is possible to obtain a reduced model of an open loop system (an object under to be controlled):

$$x(t) = A_i x(t) + b_i u_i(t), \quad x(t = t_0) = x_0, \quad t_0 \neq 0,$$  

(1)
where $x(t)$ is $n$-dimensional vector of control object state variables; $A_i$ is $n \times n$ matrix of stationary parameters of PSS; $b_i$ is $n$-dimensional vector stationary control parameters; $u_i(t)$ is scalar control input vector, which is the output variable of the voltage regulator (in our case it is the pulse width factor for VSI); $t_0$ is arbitrary initial moment of time; $x_0$ is $n$-dimensional vector of initial values of state variables.

Without loss of commonality for the procedure of synthesis of the optimal regulator it is possible to consider a variant of measurement of the entire vector $x(t)$ of state variables, which takes place at a stage of mathematical modeling of the PSS adequate model. The optimized criterion of quality of regulated processes in PSS can be written in the form of a smooth quadratic function $J_i(x, u)$ of two components:

$$J_i(x, u) = \int_{t_0}^{t_*} \left( x^T(t)Qx(t) + r_iu_i^2(t) \right) dt,$$

where $(*)^T$ is transpose symbol; $t_*$ is moment of time of the final state of the object; $Q$ is positive semidefinite matrix $n \times n$, the components of which are penalty coefficients for the corresponding state variables $x(t)$; $r_i$ is positive coefficient, the value of which reflects the influence of scalar control actions $u_i(t)$ on the state of extreme quality criteria and indirectly reflects the requirements for energy efficiency management.

The most suitable form of weight (penalty) matrix $Q$ in the formation of requirements to the processes of many objects is a diagonal form. This form of matrix $Q$ does not require checking the necessary condition of its positive semidefinition and significantly simplifies the structure of the quality criterion. For optimal regulators, the structure of which is determined by the law of linear feedback on variables of the state of the controlled object of the type (1), the minimum functionality (2) will be provided for the control action:

$$u_i(t) = -r_i^{-1}b_i^T P_i(t)x(t),$$

where $P_i(t)$ is $n \times n$, the components of which are determined by solving the differential (algebraic) Riccati's equation or Lyapunov's equation [10].

In the process of synthesis of an optimum regulator for considered system of an electrical supply the transfer function was determined during its operation in a mode close to a no-load one with a pulse width factor of $k_m = 0.7$. According to the obtained transfer function, the system of differential equations in the form of Cauchy (1) is obtained, which allows to write the matrices $A$ and $b$. Regulator coefficients were calculated based on the expression $k^T = r^T b^T P$.

Figure 2 shows a block diagram of the PSS with an optimal regulator (OR), which allows to stabilize the load voltage. The equivalent resistance varies from $R_n = 10R_{nr}$ to $R_n = 3.4R_{nr}$, where $R_{nr}$ is the rated load resistance, $L$ is the load.

![Figure 2. Structure of the PSS model with optimal output regulator.](image-url)
Figure 3a shows the result of the regulator’s operation when operation in a no-load mode with the load resistance $R_n = 10R_{nr}$.

\[ u_{Cd}(t), \text{ V} \]

\[ R_n = 10R_{nr} \]

\[ R_n = 6R_{nr} \]

\[ u_{Cd}(t), \text{ V} \]

\[ R_n = 10R_{nr} \]

\[ R_n = 3.4R_{nr} \]

\[ t, \text{ s} \]

\[ t, \text{ s} \]

Figure 3. Load voltage in PSS with optimal regulator (switching at a time of 50 ms: a – $R_n = 6R_{nr}$; b – $R_n = 3.4R_{nr}$).

After reaching the steady-state mode, the load increases at the moment of time $t_p = 50$ ms (resistance changes up to $R_n = 6R_{nr}$). Figure 3b reflects that at the moment of time 50 ms the load resistance changes from $R_n = 10R_{nr}$ to $R_n = 3.4R_{nr}$, and the error of voltage stabilization increases from 2% (Figure 3a) to 10%. If the load continues to increase, the voltage stabilization error increases.

It is known that according to the technical requirements to the existing PSS, the error of voltage stabilization at the load in 20 ms after changing the operating mode should not exceed 10%. To extend the limits of voltage stabilization it was proposed to use a regulator with reconfigurable parameters, which includes three optimal regulators, the coefficients of which were calculated for a certain range of changes in the load of the PSS (close to a no-load mode, mode of reduced load, the nominal mode).

Figure 4a shows the results of the combined optimal regulator operation when the system is switched to a no-load mode (the regulator tuned to the mode close to a no-load one is activated) and then the nominal load is connected (the regulator tuned to nominal mode is switched on). Figure 4b shows the operation of the combined regulator when connecting a load with resistance $R_n = 2R_{nr}$, the voltage is stabilized by the regulator tuned on a reduced load mode. In all the Figures shown, the load is connected at 50 ms and the stabilization error is less than 10% of the specified voltage $U$. 
As in power supply system there is no technical possibility to measure voltage on load, it is expedient to use rectified voltage from the output VSI filter for construction of the optimal regulator with negative feedback. Figure 5 shows the combined optimal regulator scheme, which allows to stabilize the voltage at the power supply system load in case of direct measurement at the LC-filter output (F2). The voltage received from the filter capacitance F2 is rectified by rectifier R2 and smoothed by capacitive filter F4.

Assuming that the mathematical model of the investigated PSS is linear, we can suggest that in the steady state of operation the load voltage and the rectified voltage from the filter F4 will be linked by a linear ratio. The coefficient of linear coupling $k=15.9$ was determined experimentally, which was used in the synthesis of the optimal regulator to recalculate the change in load voltage reduced to the measured rectified voltage of the filter F4 (Figure 5). The proposed regulator also consists of three optimal regulators. Each of them is tuned to their own control range (mode close to a no-load one, a reduced load mode, the nominal mode). The regulators are switched according to the signal $S$ coming from the control system CS.
Figure 5. Combined optimal regulator structure for voltage measurement from the output VSI filter.

Figure 6 shows the result of the work of the proposed combined optimum regulator. At the moment of start the PSS, the control system CS acquires a control signal $S$, which causes the activation of the regulator configured to stabilize the voltage in a no-load mode, after 50 ms the nominal load is connected, and the control system sends a signal to disconnect the first regulator and connect the second regulator configured for the operating modes close to the nominal ones (Figure 6a).

$$u_{Cd}(t), \text{ V}$$

![Graph a) showing voltage $u_{Cd}$ over time](image)

$t, \text{s}$

$R_n = 10R_{nr}$

$R_n = R_{nr}$

Figure 6. Load voltage with combined optimum regulator and measuring the voltage from the filter VSI: a – change $R_n$ from $R_n = 10R_{nr}$ to $R_n = R_{nr}$; b – from $R_n = 10R_{nr}$ to $R_n = 2R_{nr}$.
In case of further load changes, CS switches regulators according to the current state of the system. So, in Figure 6b at the moment of time \( t=50 \) ms there is an abrupt change of system load to \( R_n = 2R_m \), thus the operating regulator is switched off and the regulator adjusted on reduced load mode is included into operation. In all operating modes of the PSS with the proposed regulator the voltage error at the equivalent load does not exceed 10%.

4. Summary
The research showed that the optimum regulator with a variable structure allows to stabilize voltage on PSS payload and to provide the quality indicators established for the voltage measured on the output VSI filter. This is due to the assumption that the characteristics of the PSS are linear. For a more accurate description of the PSS, it is necessary to take into account the non-linearity of the elements, caused in particular by the influence of the hysteresis in transformers. The load conversion factor for the regulator will be a non-linear relationship. Synthesis of the optimal regulator, taking into account nonlinearities of the PSS elements, is the subject for further research.

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