Particle-Production Mechanism in Relativistic Heavy-Ion Collisions

Brian W. Bush and J. Rayford Nix
Los Alamos National Laboratory
Los Alamos, New Mexico 87545, USA

Abstract

We discuss the production of particles in relativistic heavy-ion collisions through the mechanism of massive bremsstrahlung, in which massive mesons are emitted during rapid nucleon acceleration. This mechanism is described within the framework of classical hadrodynamics for extended nucleons, corresponding to nucleons of finite size interacting with massive meson fields. This new theory provides a natural covariant microscopic approach to relativistic heavy-ion collisions that includes automatically spacetime nonlocality and retardation, nonequilibrium phenomena, interactions among all nucleons, and particle production. Inclusion of the finite nucleon size cures the difficulties with preacceleration and runaway solutions that have plagued the classical theory of self-interacting point particles. For the soft reactions that dominate nucleon-nucleon collisions, a significant fraction of the incident center-of-mass energy is radiated through massive bremsstrahlung. In the present version of the theory, this radiated energy is in the form of neutral scalar ($\sigma$) and neutral vector ($\omega$) mesons, which subsequently decay primarily into pions with some photons also. Additional meson fields that are known to be important from nucleon-nucleon scattering experiments should be incorporated in the future, in which case the radiated energy would also contain isovector pseudoscalar ($\pi^+, \pi^-, \pi^0$), isovector scalar ($\delta^+, \delta^-, \delta^0$), isovector vector ($\rho^+, \rho^-, \rho^0$), and neutral pseudoscalar ($\eta$) mesons.

1. Introduction

Many particles are produced in a typical relativistic heavy-ion collision. For the production of these particles, we would like to discuss the mechanism of massive bremsstrahlung, in which massive mesons are emitted during rapid nucleon acceleration. This mechanism is described within the framework of classical hadrodynamics for extended nucleons, corresponding to nucleons of finite size interacting with massive meson fields. This approach, which satisfies a priori the physical conditions that exist...
Fig. 1: Slice through the center of a nucleon. The circle indicates the location of the root-mean-square radius, where the exponentially decreasing mass density is only 3% of its central value.

at relativistic energies, is manifestly Lorentz-covariant and allows for nonequilibrium phenomena, interactions among all nucleons, and particle production.

Although the nucleon is a composite particle made up of three valence quarks plus additional sea quarks and gluons, when nucleons collide at very high energies, only a few rare events correspond to the head-on or hard collisions between the individual quarks and/or gluons. Whereas the underlying quark-gluon structure of the nucleon is of crucial importance for describing particle production in hard collisions, such collisions are nevertheless extremely rare, typically one in a billion. The vast majority of events correspond to soft collisions not involving individual quarks or gluons. For describing particle production in such events, an appealing idea is to regard the nucleon as a single extended object interacting with other nucleons through the conventional exchange of mesons (whose underlying quark-antiquark composition is ignored).

Experiments involving elastic electron scattering off protons have determined that the proton charge density is approximately exponential in shape, with a root-mean-square radius of $0.862 \pm 0.012$ fm. Although many questions remain concerning the relationship between the proton charge density and the nucleon mass density, it should be a fairly accurate approximation to regard them as equal. We therefore take the nucleon mass density to be

$$\rho(r) = \frac{\mu^3}{8\pi} \exp(-\mu r),$$  \hspace{1cm} (1)
with \( \mu = \sqrt{12}/R_{\text{rms}} \) and \( R_{\text{rms}} = 0.862 \text{ fm} \). We show in fig. 1 a gray-scale plot of the mass density through the center of a nucleon calculated according to this exponential, with the root-mean-square radius indicated by a circle.

The physical input underlying our new approach consists of Lorentz invariance (which includes energy and momentum conservation), nucleons of finite size interacting with massive meson fields, and the classical approximation applied in domains where it should be reasonably valid. At bombarding energies of many GeV per nucleon, the de Broglie wavelength of projectile nucleons is extremely small compared to all other length scales in the problem. In addition, the Compton wavelength of the nucleon is small compared to its radius, so that effects due to the intrinsic size of the nucleon dominate those due to quantum uncertainty. Finally, the angular momentum is typically several hundred \( \hbar \), and the radiated energy corresponds to several meson masses. The classical approximation for nucleon trajectories should therefore be valid, provided that the effect of the finite nucleon size on the equations of motion is taken into account.

We describe in sect. 2 the present version of our theory, which includes the neutral scalar (\( \sigma \)) and neutral vector (\( \omega \)) meson fields. This permits a qualitative discussion of not only particle production through massive bremsstrahlung, but also such other physically relevant points as the effect of the finite nucleon size on the equations of motion and an inherent spacetime nonlocality that may be responsible for significant collective effects. The \( \sigma \) and \( \omega \) mesons that are produced will subsequently decay primarily into pions with some photons also. The resulting classical relativistic equations of motion are solved in sect. 3 for soft nucleon-nucleon collisions at \( p_{\text{lab}} = 14.6, 30, 60, 100, \) and \( 200 \text{ GeV}/c \). Section 4 discusses the future incorporation of additional meson fields that are known to be important from nucleon-nucleon scattering experiments, including the isovector pseudoscalar (\( \pi^+, \pi^-, \pi^0 \)), isovector scalar (\( \delta^+, \delta^-, \delta^0 \)), isovector vector (\( \rho^+, \rho^-, \rho^0 \)), and neutral pseudoscalar (\( \eta \)). Further details are given in a series of papers, although not all of the equations appearing in some of the earlier publications are in their final form.

2. Equations of motion

Our action for \( N \) extended, unexcited nucleons interacting with massive scalar and vector meson fields is

\[
I = -M_0 \sum_{i=1}^{N} \int d\tau_i \sqrt{\dot{q}_i^2} + \frac{1}{8\pi} \int d^4x \left( \frac{1}{2} G^2 - m_{\pi}^2 \phi^2 \right) \\
- \frac{1}{8\pi} \int d^4x \left( \frac{1}{2} G^2 - m_{\pi}^2 V^2 \right) - \int d^4x \left( j\phi + K \cdot V \right),
\]

where \( M_0 \) is the bare nucleon mass and \( q_i \) is the four-position of the \( i \)th nucleon, whose trajectory is given by \( q_i = q_i(\tau_i) \). A dot represents the derivative with respect
to $\tau_i$. In the action the four-velocities are not constrained so that $\dot{q}_i^2 = 1$ and $\tau_i$ is not yet identified as the proper time; it is only in the equations of motion, which are derived as a result of the variation of $I$, that this is true. We use the metric $g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$, write four-vectors as $q^\mu = (q^0, \mathbf{q}) = (q^t, q^x, q^y, q^z)$, and use units in which $\hbar = c = 1$. The scalar potential is denoted by $\phi$, the four-vector potential by $V$, and the meson masses by $m_{s,v}$. The vector field strength tensor is

\[ G^{\mu\nu} = \partial [V^\nu] \equiv \partial^{\mu} V^\nu - \partial^{\nu} V^\mu , \tag{3} \]

the scalar source density is

\[ j(x) = g_s \sum_{i=1}^{N} d\tau_i \rho(x - q_i, \dot{q}_i) \sqrt{\dot{q}_i^2} , \tag{4} \]

and the vector source density is

\[ K^{\mu}(x) = g_v \sum_{i=1}^{N} d\tau_i \rho(x - q_i, \dot{q}_i) \dot{q}_i^\mu , \tag{5} \]

where $\rho$ is the four-dimensional mass density of the nucleon, the spatial part of which we assume to be exponential in the nucleon’s rest frame. The values of the six physical constants appearing in our theory are nucleon mass $M = 938.91897$ MeV, scalar ($\sigma$) meson mass $m_s = 550$ MeV, vector ($\omega$) meson mass $m_v = 781.95$ MeV, scalar interaction strength $g_{s2} = 7.29$, vector interaction strength $g_{v2} = 10.81$, and nucleon r.m.s. radius $R_{\text{rms}} = 0.862$ fm.

In ref. [12] we have derived exact equations of motion for the above action in two limits: (1) relativistic point nucleons and (2) nonrelativistic extended nucleons. We then generalize covariantly to obtain relativistic equations of motion for extended nucleons, which can be written as

\[ M_i^* a_i^\mu = f_{s,i}^{\mu} + f_{v,i}^{\mu} + f_{s,\text{ext},i}^{\mu} + f_{v,\text{ext},i}^{\mu} . \tag{6} \]

The effective mass is given by

\[ M_i^* = \tilde{M}^* + \Delta M_{\text{self},i} + g_s \tilde{\phi}_{\text{ext},i} , \tag{7} \]

\[ \tilde{M}^* = M + \frac{2}{3} M_s + \frac{1}{3} M_v' - \frac{4}{3} M_v + \frac{1}{3} M'_v , \tag{8} \]

\[ \Delta M_{\text{self},i} = -g_{s2} \int_{0}^{\infty} d\sigma \left[ \tilde{h} \left( \frac{\sigma}{2} \right) - m_s \int_{0}^{s_i} d\zeta \tilde{h} \left( \frac{\sqrt{s_i^2 - \zeta^2}}{2} \right) J_1(m_s \zeta) \right] - 2 M_s , \tag{9} \]

\[ g_s \tilde{\phi}_{\text{ext},i} = -g_{s2} \sum_{j \neq i} \int_{0}^{\infty} d\sigma \left[ w \left( \frac{k_j'}{2}, \sqrt{k_j^2 - s_j^2} \right) - m_s \int_{0}^{k_j'} d\zeta w \left( \frac{\sqrt{k_j^2 - \zeta^2}}{2}, \sqrt{k_j^2 - s_j^2} \right) \right] \times J_1(m_s \zeta) \tag{10} \]
The hadrostatic self-energies of the scalar and vector fields are denoted by \( M_{s,v} \) and their logarithmic derivatives with respect to meson mass by \( M'_{s,v} \). The self-forces are given by

\[
\begin{align*}
\chi_{s,i} &= \frac{g_s^2}{12} \mathcal{P}_{i}^{\mu
u} \int_{0}^{\infty} d\sigma \left[ h' \left( \frac{\sigma}{2} \right) - m_s \int_{0}^{\sigma} d\zeta \ h' \left( \frac{\sqrt{s_i^2 - \zeta^2}^2}{2} \right) J_1 (m_s \zeta) \right] s_{i\nu}^{'} , \quad (11) \\
\chi_{v,i} &= \frac{g_v^2}{6} \mathcal{P}_{i}^{\mu
u} \int_{0}^{\infty} d\sigma \left\{ \frac{(v_i^s \cdot v_i^s)(s_i^2 \cdot v_j^s)}{s_i^2} \left\{ \frac{h' \left( \frac{\sigma}{2} \right) - m_v \int_{0}^{\sigma} d\zeta \ h' \left( \frac{\sqrt{s_i^2 - \zeta^2}^2}{2} \right)}{s_i^2} \right\} J_1 (m_v \zeta) \right\} s_{i\nu}^{'} , \quad (12)
\end{align*}
\]

and the external forces by

\[
\begin{align*}
\chi_{s,ext,i} &= \frac{g_s^2}{2} \mathcal{P}_{i}^{\mu
u} \sum_{j \neq i} \int_{0}^{\infty} d\sigma \left[ w' \left( \frac{k_j^s}{2} \sqrt{k_j^s - s_j^2} \right) \right] - m_s \int_{0}^{\infty} d\zeta \ w' \left( \frac{\sqrt{k_j^s - \zeta^2}^2}{2} \right) \\
\times J_1 (m_s \zeta) \right] s_{j\mu}^{'} , \quad (13) \\
\chi_{v,ext,i} &= -\frac{g_v^2}{2} \mathcal{P}_{i}^{\mu
u} \sum_{j \neq i} \int_{0}^{\infty} d\sigma \left[ w' \left( \frac{k_j^s}{2} \sqrt{k_j^s - s_j^2} \right) \right] - m_v \int_{0}^{\infty} d\zeta \ w' \left( \frac{\sqrt{k_j^s - \zeta^2}^2}{2} \right) \\
\times J_1 (m_v \zeta) \right] s_{j\nu}^{'} \right] . \quad (14)
\end{align*}
\]

In the above \( \mathcal{P}_{i}^{\mu
u} = g_{i\mu}^{\mu\nu} - v_i^{\mu\nu} \), \( s_j^s = q_j (\tau_i) - q_j (\tau_j - \sigma) \), \( v_j = \frac{\partial q_j}{\partial \tau_j} \), \( \tilde{q}_j (\tau_j) \), and the retarded proper time \( \tau_j \) is determined implicitly from the condition \( k_j^s = 0 \), with \( \sigma = 0 \).

These equations are written in terms of the nucleon structure functions \( h \) and \( w \) and quantities derived from them. We define the interaction energy function to be

\[
W(m, r) \equiv \int d^3x_1 \int d^3x_2 \rho(x_1) e^{-mR} \rho(x_2) , \quad (15)
\]

where \( R \equiv |x_1 - x_2| \) and \( r \) is the distance between the centers of the two particles. Here \( \rho(r) \) is the nucleon mass density normalized so that \( 4\pi \int_{0}^{\infty} r^2 dr \rho(r) = 1 \). A Laplace transform relates the structure function \( w \) and its derivative \( w'(\sigma, r) \equiv r^{-1} \partial w / \partial r \) to the interaction energy \( W \):

\[
w(\sigma, r) \equiv 2L^{(-1)} [ W(m, r); 2\sigma ] . \quad (16)
\]

The self-interaction structure function is

\[
h(\sigma) \equiv 32\pi^2 \int_{0}^{\infty} d\sigma' \left( \sigma'^2 - \sigma^2 \right) \rho(\sigma + \sigma') \rho(|\sigma - \sigma'|) , \quad (17)
\]

5
with \( h'(\sigma) \equiv dh(\sigma)/d\sigma, \quad \tilde{h}(\sigma) \equiv \int_0^\sigma d\sigma' h(\sigma'), \) and \( \tilde{h}(\sigma) \equiv h'(\sigma) - 6m_e^2 \bar{h}(\sigma). \) The first-order Bessel function of the first kind is denoted by \( J_1. \)

These equations of motion, which are second-order, nonlinear, integrodifferential equations with four dimensions per particle, can be solved numerically without further approximation. In particular, we do not need to make either a mean-field approximation, a perturbative expansion in coupling strength, or a superposition of two-body collisions. To solve them we use a fourth-order Adams-Moulton predictor-corrector algorithm with adaptive step sizes. The integrations over proper time are done with a special error-minimizing application of Lagrange’s four-point (cubic) interpolation formulas.

3. Radiated energy and other results for soft nucleon-nucleon collisions

We now present some results obtained by solving our equations of motion for the soft collision of two nucleons at laboratory momentum \( p_{\text{lab}} = 14.6, 30, 60, 100, \) and \( 200 \) GeV/c. At three of these momenta substantial experimental data exist for heavy-ion collisions,\( 1, 2 \) and at the remaining two momenta experimental data exist for proton-proton collisions.\( 3 \) We will concentrate our discussion here on such physically observable quantities as scattering angle, transverse momentum, and radiated energy in the center-of-mass system, in which frame the computations are performed.

As shown in fig. 2, for a given incident momentum, the center-of-mass scattering angle for the dominating soft reactions described by our theory has a maximum value.
at a certain impact parameter and decreases to zero for both head-on and distant collisions. With increasing incident momentum in this range both the maximum angle and the impact parameter at which this maximum occurs decrease. For ultrarelativistic collisions this impact parameter is approximately the distance at which the transversely dominating static vector force for extended nucleons has its maximum. At the other extreme of low incident momentum, the opposing scalar and vector forces are of similar magnitude and give rise for small impact parameter to the more complicated behavior of the double-dot–dashed curve in fig. 2.

The transverse momentum has a related behavior, as shown in fig. 3. For a given incident momentum, the transverse momentum for soft reactions also has a maximum value at a certain impact parameter and decreases to zero for both head-on and distant collisions. The maximum transverse momentum increases slowly with increasing incident momentum in this range, and the impact parameter at which this maximum occurs decreases.

The center-of-mass radiated energy per nucleon for soft reactions shown in fig. 4 also has a maximum value at a certain impact parameter. However, this quantity decreases to a finite value for head-on collisions and to zero for distant collisions. The maximum center-of-mass radiated energy per nucleon increases strongly with increasing incident momentum. In the present version of the theory, this radiated energy will be in the form of $\sigma$ and $\omega$ mesons, which will subsequently decay primarily into pions with some photons also. The classical approximation is expected to be valid only when
the amount of radiated energy in the center-of-mass system exceeds the mass of the lightest meson, which is 550 MeV. As seen in fig. 4, this condition is well satisfied for impact parameters of physical interest at the three highest incident momenta, but not at the two lowest incident momenta.

The qualitative behavior of these results can be understood in terms of the nature of the external forces. The repulsive vector force scales as the Lorentz factor $\gamma$ in both the longitudinal and transverse directions, whereas the attractive scalar force scales as $\gamma^2$ in the longitudinal direction and as unity in the transverse direction. This implies that the vector force will dominate the transverse acceleration and the scalar force will dominate the longitudinal acceleration. For a given impact parameter the scattering angle and transverse momentum will be essentially proportional to the vector interaction strength $g_v^2$, and the radiated energy will be essentially proportional to $\gamma$ times the scalar interaction strength $g_s^2$.

4. Future directions

From nucleon-nucleon scattering experiments we know that several additional meson fields are important and must be included for a quantitative description.

- Isovector pseudoscalar ($\pi^+, \pi^-, \pi^0$)
- Isovector scalar ($\delta^+, \delta^-, \delta^0$)
- Isovector vector ($\rho^+, \rho^-, \rho^0$)
Fig. 5: Effective mass density for an extended nucleon. The intrinsic mass density is exponential with a root-mean-square radius of 0.862 fm.

- Neutral pseudoscalar (η)

The next step in the systematic development of the theory should be the inclusion of these additional meson fields. Once this is done, the massive bremsstrahlung in our theory would include pions, deltas, rhos, and etas in addition to the sigmas and omegas that are produced in the current version.

The effects of quantum uncertainty on the equations of motion should also be studied and included if they are important. This should be possible by use of techniques analogous to those used by Moniz and Sharp for nonrelativistic quantum electrodynamics. There appear in the classical nonrelativistic equations of motion for an extended electron terms of the form \( \int_\infty \int_\infty \rho (r) \mathcal{O} \rho (r') \, d^3r \, d^3r' \), where the operator \( \mathcal{O} \) is a function of \( r \) and \( r' \). Moniz and Sharp have shown in nonrelativistic quantum electrodynamics that the effect of quantum mechanics on the equations of motion is to replace such terms by terms of the form \( \int_\infty \int_\infty \rho (r) \mathcal{O}_{\text{eff}} \rho_{\text{eff}} (r') \, d^3r \, d^3r' \) and derivatives with respect to \( \lambda \) of these terms, where \( \lambda = 1/(\hbar M_0) \) is the Compton wavelength associated with the particle’s bare mass and the effective operator \( \mathcal{O}_{\text{eff}} \) is a function of \( r, r' \), and \( \lambda^2 \nabla r'^2 \). As illustrated in fig. 5, the effective mass density for an extended nucleon oscillates around the intrinsic mass density as a function of radial distance from the origin.

In conclusion, we have shown that classical hadrodynamics requires minimal physical input, leads to equations of motion that can be solved numerically without further approximation, and provides a suitable framework for describing particle production.
in relativistic heavy-ion collisions through the mechanism of massive bremsstrahlung. This work was supported by the U. S. Department of Energy.
5. References

[1] M. J. Tannenbaum, Int. J. Mod. Phys. A4 (1989), 3377.

[2] Quark Matter '91, Proc. Ninth Int. Conf. on Ultra-Relativistic Nucleus-Nucleus Collisions, Gatlinburg, Tennessee, 1991, Nucl. Phys. A544 (1992), 1c.

[3] D. H. Perkins, Introduction to High Energy Physics, Third Edition (Addison-Wesley, Menlo Park, 1987), p. 142.

[4] G. G. Simon, F. Borkowski, C. Schmitt, and V. H. Walther, Z. Naturforsch. A35 (1980), 1.

[5] R. K. Bhaduri, Models of the Nucleon (Addison-Wesley, New York, 1988).

[6] A. J. Sierk, R. J. Hughes, and J. R. Nix, in Proc. 6th Winter Workshop on Nuclear Dynamics, Jackson Hole, Wyoming, 1990, Lawrence Berkeley Laboratory Report LBL-28709 (1990), p. 119.

[7] A. J. Sierk, R. J. Hughes, and J. R. Nix, in Contributed Papers, Symp. in Honor of Akito Arima: Nuclear Physics in the 1990’s, Santa Fe, New Mexico, 1990, Report (1990), p. 118.

[8] B. W. Bush, J. R. Nix, and A. J. Sierk, in Advances in Nuclear Dynamics, Proc. 7th Winter Workshop on Nuclear Dynamics, Key West, Florida, 1991 (World Scientific, Singapore, 1991), p. 282.

[9] B. W. Bush, J. R. Nix, and A. J. Sierk, in Proc. 4th Conf. on the Intersections between Particle and Nuclear Physics, Tucson, Arizona, 1991, AIP Conference Proceedings 243 (American Institute of Physics, New York, 1992), p. 835.

[10] B. W. Bush and J. R. Nix, in Contributed Papers and Abstracts, Quark Matter '91, Ninth Int. Conf. on Ultra-Relativistic Nucleus-Nucleus Collisions, Gatlinburg, Tennessee, 1991, Report (1991), p. T98.

[11] B. W. Bush and J. R. Nix, in Advances in Nuclear Dynamics, Proc. 8th Winter Workshop on Nuclear Dynamics, Jackson Hole, Wyoming, 1992 (World Scientific, Singapore, 1992), p. 311.

[12] B. W. Bush and J. R. Nix, Ann. Phys. (N. Y.) 227 (1993), 97.

[13] B. W. Bush and J. R. Nix, Nucl. Phys. A560 (1993), 586.

[14] J. R. Nix, in Advances in Nuclear Dynamics, Proc. 10th Winter Workshop on Nuclear Dynamics, Snowbird, Utah, 1994 (World Scientific, Singapore, 1994), to be published.

[15] B. W. Bush and J. R. Nix, in Proc. Fifth Int. Conf. on Nucleus-Nucleus Collisions, Taormina, Italy, 1994, to be published.

[16] B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. 16 (1986), 1.

[17] K. Hikasa et al., Phys. Rev. D45 (1992), S1.

[18] B. D. Serot, Phys. Lett. 68B (1979), 146.

[19] G. P. Yost et al., Lawrence Berkeley Laboratory Report LBL-90 revised (1986).

[20] R. Machleidt, Adv. Nucl. Phys. 19 (1989), 189.

[21] E. J. Moniz and D. H. Sharp, Phys. Rev. D15 (1977), 2850.