Thermoelectric radiation detector based on a superconductor-ferromagnet junction: calorimetric regime

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We study the use of a thermoelectric junction as a thermal radiation detector in the calorimetric regime, where single radiation bursts can be separated in time domain. We focus especially on the case of a large thermoelectric figure of merit $ZT$ affecting significantly for example the relevant thermal time scales. This work is motivated by the use of hybrid superconductor/ferromagnet systems in creating an unprecedentedly high low-temperature $ZT$ even exceeding unity. Besides constructing a very general noise model which takes into account cross correlations between charge and heat noise, we show how the detector signal can be efficiently multiplexed by the use of resonant LC circuits giving a fingerprint to each pixel. We show that for realistic detectors operating at temperatures around 100 to 200 mK, the energy resolution can be as low as 1 meV. This allows for a broadband single-photon resolution at photon frequencies of the order or below 1 THz.

I. INTRODUCTION

Some of the most sensitive sensors of electromagnetic radiation are based on using superconducting films absorbing the radiation and measurement systems converting this process into detectable electronic signal. The best-studied example of such sensors is the superconducting transition edge sensor (TES) [1], which has already been used for many types of applications, such as in security imaging [2], materials analysis [3, 4] and cosmic microwave background radiation detection [5, 6]. In TES sensors the absorbed radiation heats the electrons above the critical temperature $T_c$ of superconducting films, and results into measurable changes in the film resistance. This resistance is often read out by utilizing an applied bias voltage or current [7], fixing the operating point close to $T_c$, and allowing for additional read-out features such as electrothermal feedback, and bias-based multiplexing strategies [8]. However, the presence of bias-induced dissipation also leads to an overall heating of the system, and increases the thermal noise, thus reducing the sensitivity. In addition, in multi-pixel systems fabricating bias lines for each pixel becomes a technological challenge. Another important sensor in this context is the kinetic inductance detector (KID) [8–12] based on the read-out of the kinetic inductance signal in superconducting microwave resonators. Also KIDs require probe signals for read-out, resulting into increased dissipation within the pixels. As the desire in many applications is to further increase the number of detector pixels [13], such probe-based sensors become increasingly difficult to operate.

In general, one would prefer to only have the effect of the coupling between radiation and the detector in the measured signal, and therefore to get rid of the probe signal. This desire can be achieved with a thermoelectric detector (TED) [14–18], where the absorption of radiation leads to a temperature difference, which creates a measurable thermoelectric current or voltage. In this work we consider such thermoelectric detectors. They have indeed the advantage of the lack of probe signals. However, for most systems the thermoelectric effects are very weak, and therefore sensitivity close to those of TES and KID cannot be expected. This changed with the discovery of the giant thermoelectric effect in superconductor-ferromagnet hybrids [19–21], which can in principle be utilized to create sub-Kelvin thermoelectric heat engines with figures of merit $ZT$ exceeding even those of the best thermoelectric devices. Hence, such systems may offer sensitivity rivaling those of TES and KID sensors, but without the need of probe signals [18].

Compared to probe-based sensors, there are also issues with the proper read-out of the signal, as many existing multiplexing strategies are based on modulating the probe signal. We address this here by analyzing in detail the use of the thermoelectric detector in the calorimetric regime, where radiation arrives at bursts separated by long times compared to the relevant time scales of the detector. This is opposite to the bolometric regime analyzed in the earlier work [18]. In contrast to many recent works [15, 19] utilizing an ad hoc noise model, we derive the energy resolution of such a thermoelectric calorimeter by taking into account all the relevant noise terms, including the cross-correlation of heat and current noises in the thermoelectric junction, as required by the linear response theory. As a result, we obtain the energy resolution and the relevant thermal time scales of the calorimeter modified by the large $ZT$. We also analyze the resulting time-dependent thermoelectrically generated current profile in various parameter regimes.

In this work we consider a single pixel of the thermoelectric detector based on a superconducting film and the ferromagnetic junction [18], as depicted in Fig. 1. The single pixel of the proposed detector is built with an element made of a thin film of a superconductor-ferromagnetic insulator (S-FI) bilayer coupled to superconducting antennas via a clean (Andreev) con-
II. THEORY

Here we study the thermoelectric detector in the calorimetric regime, where the process of relaxation of the detector is much faster than the arrival of the consecutive incident pulses of energies to the detector. In what follows we first consider the heat balance equation of a generalized thermoelectric detector within linear response assumption given by \[18\]

\[ C_h \frac{d\Delta T(t)}{dt} = P_\gamma(t) - G_{\text{th}}^{\text{tot}} \Delta T(t) - \alpha V_{\text{th}}(t), \]  

where \(P_\gamma(t)\) is the power of the incident radiation, \(V_{\text{th}}(t)\) is the time dependent voltage between the ferromagnetic electrode and the superconductor, \(C_h\) is the heat capacity of the absorber and \(\alpha\) is the response coefficient for the Peltier heat current. In Eq. \(1\) \(\Delta T(t) = T_S(t) - T\) is the change of temperature in the superconductor due to the incident power where \(T_S(t)\) is the time dependent temperature of the superconductor and \(T\) is the bath temperature. Here, we consider \(T\) equal to the temperature of ferromagnetic electrode \((T_F)\) and phonon temperature \((T_{ph})\). The quantity \(G_{\text{th}}^{\text{tot}} = G_{\gamma-\text{ph}} + G_{\text{th}}\) represents the total heat conductance of the superconducting film to the heat bath, and \(G_{\gamma-\text{ph}}\) and \(G_{\text{th}}\) stand for the heat conductance of the quasiparticles in the superconductor to the phonons and to the ferromagnetic electrode, respectively. Another possible (spurious) heat conduction channel could be due to quasiparticle-magnon scattering, but we disregard it below as it depends on the microscopic details of the magnets. Notably, there has various other possibility of spurious heat conduction processes such as via quasiparticle-magnon scattering. Equation \(1\) in the frequency \((\omega)\) space gives the following solution for the change of temperature, \(\Delta T(\omega) = (P_\gamma(\omega) - \alpha V_{\text{th}}(\omega)) / (G_{\text{th}}^{\text{tot}} + i\omega C_h) \) \[26\]. Next, within linear response assumption, we consider the thermoelectric current from the superconductor to the ferromagnetic electrode \[18\],

\[ I_{\text{th}}(t) = -\frac{\alpha}{T} \Delta T(t) - C V_{\text{th}}(t), \]  

where \(C\) is the conductance of the thermoelectric junction. This thermoelectric current through the thermoelectric junction ultimately reaches an amplifier. Disregarding the back-action noise from the amplifier and considering the amplifier as a capacitor or an inductor, the thermal current through the amplifier due to the thermoelectric voltage, \(V_{\text{th}}\), in frequency \((\omega)\) space is \(I_{\text{th}}(\omega) = V_{\text{th}}(\omega) [i\omega C + 1/(i\omega L)]\), where \(C\) and \(L\) are the capacitance and inductance of the detection circuit, respectively. Here \(L\) and \(C\) can represent elements that have been designed on purpose for identifying the pixel (see below). Next, using Eqs. \(1\) and \(2\), along with the expression of the current through detectors.

FIG. 1. Schematic of the thermoelectric detector based on superconductor (S) and ferromagnetic (F) electrode a spin-filter junction. S is also coupled with ferromagnetic insulator (FI) which provides a spin splitting exchange field to S. I is an insulating layer and \(P_\gamma(t)\) is the time dependent power of incident radiation which needs to be detected. \(T_S\), \(T_F\) and \(T_{ph}\) are the temperature of the superconducting film, the temperature of the ferromagnetic electrode and phonon temperature, respectively.
the thermoelectric junction $V_{th}(\omega) = \lambda_V(\omega)P_\gamma(\omega)$, where $\lambda_V(\omega) = \alpha/\sqrt{\omega^2 - \omega^2 G_{th}(\omega)}$, and $V_{tot}(\omega) = G_{th} + i\omega C_h$ and $Y_{tot}(\omega) = G_{th} + i\omega C_h$ and $Y_{tot}(\omega) = G_{th} + i\omega C_h$ and $Y_{tot}(\omega) = G_{th} + i\omega C_h$. The current through the inductor is $I_L(\omega) = V_{th}(\omega)/(i\omega L)$, which $\lambda_I(\omega) = \lambda_V(\omega)/(i\omega L)$. Finally, we obtain the expressions of $V_{th}(t)$ and $I_L(t)$.

$$V_{th}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \lambda_V(\omega)P_\gamma(\omega)e^{-i\omega t}$$  \hspace{1cm} (3)

$$I_L(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \lambda_I(\omega)P_\gamma(\omega)e^{-i\omega t}.$$  \hspace{1cm} (4)

The integrals in Eq. (3) and (4) can easily be solved through the Cauchy residue theorem, but the general expressions are too long to be presented here. Rather, below we present some limiting cases.

Next, for the analyses of the various fluctuation processes present in the system, relevant for predicting the energy resolution of detection, we consider a Langevin noise circuit model. We denote $\delta T$, $\delta V$ and $\delta I_L$ as the temperature fluctuation on the absorber, voltage noise across the capacitor and the current noise across the inductor. These noise are governed by the charge current noise $\delta Q_J$ and heat current noise $\delta Q_J$ through the thermoelectric junction, and the heat current noise $\delta Q_{ph}$ due to quasiparticle-phonon scattering. These noise terms satisfy the heat balance equation and the Kirchhoff law for the noise terms in $\omega$ space [27], as

$$Y_{th}(\omega) \delta T(\omega) = \delta Q_J(\omega) + \delta Q_{ph}(\omega) - \alpha \delta V(\omega)$$  \hspace{1cm} (5)

$$Y_{tot}(\omega) \delta V(\omega) = \delta I(\omega) - \frac{\alpha}{T} \delta T(\omega).$$  \hspace{1cm} (6)

Solving Eqs. (5) and (6), we can obtain the noise voltage $\delta V(\omega)$ through the capacitor and the current noise through the inductor as $\delta I_L(\omega) = \delta V(\omega)/(i\omega L)$. We obtain the expressions of the noise correlations $\langle \delta V(t)\delta V(t') \rangle$ and $\langle \delta I_L(t)\delta I_L(t') \rangle$, in time domain through $4\pi^2 \int_0^\infty d\omega \delta(\omega - \omega') \langle \delta V(\omega)\delta V(\omega') \rangle e^{-i\omega t}e^{-i\omega' t}$ and $4\pi^2 \int_0^\infty d\omega \delta(\omega - \omega') \langle \delta I_L(\omega)\delta I_L(\omega') \rangle e^{-i\omega t}e^{-i\omega' t}$. To obtain the second order noise correlations of these noise terms, we consider the intrinsic correlations of the detector as

$$\langle \delta I(\omega)\delta I(\omega') \rangle = 4\pi k_B T G_\theta(\omega + \omega')$$

$$\langle \delta Q_J(\omega)\delta Q_J(\omega') \rangle = 4\pi k_B T^2 G_{th}(\omega + \omega')$$

$$\langle \delta I_L(\omega)\delta Q_J(\omega') \rangle = -4\pi k_B T \alpha \delta(\omega + \omega')$$

$$\langle \delta Q_{ph}(\omega)\delta Q_{ph}(\omega') \rangle = 4\pi k_B T^2 G_{ph}(\omega + \omega')$$

$$\langle \delta Q_J(\omega)\delta Q_{ph}(\omega') \rangle = 0$$

$$\langle \delta I(\omega)\delta Q_{ph}(\omega) \rangle = 0.$$

Vanishing intrinsic correlations signify that the noises of the corresponding processes are independent. Finally, we obtain the following simplified expressions for the second order noise correlations as

$$\langle \delta V(t)\delta V(t') \rangle = \int_{-\infty}^{\infty} d\omega \lambda_V(\omega)^2 \left[ 1 + (1 + Z_T)\frac{\alpha^2}{G_{th}} \right] e^{i\omega(t-t')}$$  \hspace{1cm} (7)

$$\langle \delta I_L(t)\delta I_L(t') \rangle = \int_{-\infty}^{\infty} d\omega \lambda_I(\omega)^2 \left[ 1 + (1 + Z_T)\frac{\alpha^2}{G_{th}} \right] e^{i\omega(t-t')}.$$  \hspace{1cm} (8)

In Eqs. (7) and (8), $Z_T = \alpha^2/(G_{th} - \alpha^2)$ is the thermoelectric figure of merit and $\tau_{th} = C_h/G_{th}$ is the thermal relaxation time. Our theoretical formalism for the current through the inductor and the second order noise correlation help us to analyze the optimum energy resolution of the thermoelectric detector in the calorimetric regime. As a result we can find the condition for single photon detection in the far-infrared regime, as shown below.

### III. RESULTS

In this section we discuss the results obtained from the formalism in the previous section. First we evaluate $I_L(t)$. Next, we obtain the optimal energy resolution in the calorimetric regime, with the idea of optimal filtering. This approach helps us analyzing a scheme for multiplexing the read-out. Finally, we predict the energy resolution in a SF based TED.

#### A. Current through the inductor in the calorimetric regime

Here we analyze the behavior of the current through the inductor with respect to time at various circumstances in the calorimetric regime, that is when $P_\gamma(t) = E\delta(t)$. Instead of the current, one could measure the voltage across the capacitor. The results are qualitatively similar, and the intrinsic energy resolution is the same in both cases.

In what follows we denote the charge relaxation time $\tau_{RC} = C/G$ and LC time $\tau_{LC} = \sqrt{LC}$. For simplicity, we also define the corresponding frequencies by $\omega_h = 1/\tau_{th}$, $\omega_{RC} = 1/\tau_{RC}$ and $\omega_{LC} = 1/\tau_{LC}$. Using Eq. (4), first we obtain $I_L(t)$ for finite $t \gg \tau_{RC}$, when the charge relaxation process is fastest, that is for $\tau_{RC} \ll \tau_{th}$, as

$$I_L(t) = \frac{E\delta}{C_h T(1 + Z_T)\omega_{RC}^2 \omega_{RC} \omega_{th} \phi(t)}$$  \hspace{1cm} (9a)

$$\phi(t) = \exp \left[ -\omega_{th} t (1 + Z_T) \right] - \frac{\omega_{th}^2 (1 + Z_T) t}{\omega_{RC}}.$$  \hspace{1cm} (9b)

Thus in the case of fast charge relaxation $I_L(t)$ tends to zero for $t \gtrsim \omega_{RL}/(\omega_{RC}(1 + Z_T))^2 = Lg/(1 + Z_T)$, but
of $\omega$ termin regimes: fast charge relaxation $\tau_{RC} \ll \tau_{th}$, fast thermal relaxation $\tau_{th} \ll \tau_{RC}$, and high resonator frequency $\tau_{LC} \ll \tau_{th}$, $\tau_{RC}$ (green). In all curves $ZT = 1$. $M$ is a scaling factor which takes different values for blue, orange and green curves as $10^8$, $10^2$ and 1, respectively. The time scale $\tau_{th}$ has the values 20 $\tau_{th}$, 50 $\tau_{th}$ and 5 $\times 10^{-4}$ $\tau_{th}$ for blue, orange and green curves respectively.

The initial decay is governed by the time scale $(ZT + 1)\tau_{th}$. Next, we obtain the expression of $I_L(t)$ for finite non-zero $t \gg \tau_{th}$, when the thermal relaxation time is the fastest one, that is $\tau_{th} \ll \tau_{RC}, \tau_{LC}$. It is

$$I_L(t) = \frac{4E_0\omega_Y}{1 + ZT} \exp \left[-\frac{\omega_{RC}t}{2(1 + ZT)}\right] \sin \left(\frac{Yt}{1 + ZT}\right)$$

$2Y = \sqrt{4\omega_{LC}^2(1 + ZT)^2 - \omega_{RC}^2}$. (10)

In this case $I_L(t)$ is a decaying oscillatory function with a decaying time scale $2(ZT + 1)\tau_{RC}$ if $Y$ is real. On the other hand, $I_L(t)$ simply decays, if $Y$ is imaginary. Finally, we analyze $I_L(t)$ for $t \gg \tau_{LC}$ for a high resonator frequency $\omega_{LC}$, that is when $\tau_{LC} \ll \tau_{th}, \tau_{RC}$. In this case

$$I_L(t) = \left(\frac{E_0\omega_Y^2}{CC_hLT}\right) \left[4e^{-\omega_{RC}t/2} \cos(\omega_{LC}t) - e^{-\omega_{th}t}\right].$$

In this case $I_L(t)$ oscillates with the frequency $\omega_{LC}$. The oscillations decay within the charge relaxation time. These oscillations are visible especially when the first term dominates, i.e., $\tau_{RC} \gg \tau_{th}$. Example time-dependent currents corresponding to these three regimes are shown in Fig. 2.

**B. Energy resolution in frequency domain with optimal filtering**

In what follows we first optimize the energy resolution using the optimal filtering technique [28]. From Eqs. (3) and (4), we obtain expressions for the thermoelectric voltage and current through the inductor as

$$V_{th}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dw \ E\lambda_Y(\omega)e^{-i\omega t}p(\omega),$$

$$I_L(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dw \ E\lambda_I(\omega)e^{-i\omega t}p(\omega),$$

where we have considered $P_{th}(\omega) = EP(\omega)$ in Eqs. (3) and (4). Next, following Eqs. (7) and (8) we have the noise correlations for the thermoelectric voltage and current through the inductor as

$$\langle \delta V(t)\delta V(t') \rangle = \frac{1}{2\pi} \int_{-\infty}^{\infty} dw \ e^{2}\bar{\omega}(\omega)e^{-i\omega(t-t')}$$

$$\langle \delta I_L(t)\delta I_L(t') \rangle = \frac{1}{2\pi} \int_{-\infty}^{\infty} dw \ e^{2}\bar{\omega}(\omega)e^{-i\omega(t-t')}.$$ (16)

From Fig. 2 we can see that when $\tau_{RC}$ is not the shortest time scale, $I_L(t)$ decays with oscillation as $t$ increases. Here our aim is to find the best estimate of $E$ in the presence of signal noise terms as Eqs. (15) and (16). Equations (15) and (16) indicate that the noise terms are correlated at different times and correlated in frequency space, therefore it is easier to make an analysis in frequency domain. Now, let us choose a weight function $W(\omega)$ in the frequency domain, and therefore define the expected values for the signal and the corresponding noise terms as $\langle \bar{V}_{th} \rangle = \frac{1}{2\pi} \int_{-\infty}^{\infty} dw \ W(\omega)E\lambda_Y(\omega)p(\omega)$, $\langle \bar{I}_L \rangle = \frac{1}{2\pi} \int_{-\infty}^{\infty} dw \ W(\omega)E\lambda_I(\omega)p(\omega)$, $\langle \bar{\delta V}^2 \rangle = \frac{1}{2\pi} \int_{-\infty}^{\infty} dw \ \left|W(\omega)\right|^2 e^{2}\bar{\omega}(\omega)$, and $\langle \bar{\delta I}_{L}^2 \rangle = \frac{1}{2\pi} \int_{-\infty}^{\infty} dw \ \left|W(\omega)\right|^2 e^{2}\bar{\omega}(\omega)$. Next, we can define the energy resolution of a generalized thermoelectric detector in terms of noise fluctuations and expected signals as $\Delta E = E\sqrt{\bar{\delta V}^2}/\langle \bar{V}_{th} \rangle = E\sqrt{\bar{\delta I}_{L}^2}/\langle \bar{I}_L \rangle$ [28]. At this point, as we desire to have the maximum value of signal to noise ratio, and hence the minimum energy resolution, we need to search for an optimal filter, that is an optimal $W(\omega)$. The desired optimal filter can be found out by finding a zero of the functional derivative of $\Delta E$ with respect to $W(\omega)$ [28]. The optimal filter is $W(\omega) = E\lambda_Y(-\omega)p(\omega)/e^{2}\bar{\omega}(\omega) = E\lambda_I(-\omega)p(\omega)/e^{2}\bar{\omega}(\omega)$. Using this weight function we obtain an expression of the optimum energy resolution through such an optimal filter in the calorimetric regime, that is when $P_{th}(t) = E\delta(t)$ or equivalently $p(\omega) = 1$, given by

$$\Delta E_{opt}^{(Fil)} = NEP/\tau_{eff},$$

where $NEP^2 = 4k_B T^2G_{th}^{tot}/ZT$ and $\tau_{eff} = \tau_{th}\sqrt{1 + ZT}$. In Eq. (19) the effective time constant, $\tau_{eff}$ is affected by $ZT$. This is the generalization of the energy resolution obtained earlier for a thermoelectric detector in the calorimetric regime with a non-zero $ZT$ [19].
In the above analysis about energy resolution with the idea of optimal filtering, we have not considered the added noise in amplification, described by the low-frequency power spectral density $S_A$. Now, in the measured signal, we consider an effect due to noise term $\delta I_A(t)$ of a current amplifier, where amplifier noise term is uncorrelated with other intrinsic noise terms of the detector. We assume the amplifier current noise fluctuation in the calorimetric regime is 
$$\langle \delta I_A(t)\delta I_A(t') \rangle = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \ S_A \ e^{-i\omega(t-t')}$$
Therefore in order to obtain the optimal energy resolution, we design the optimal filter by including the effect due to amplifier current noise term. As a result, we get the optimal energy resolution $\Delta E_{\text{opt}} = NEP_{\text{tot}}\sqrt{\tau_{\text{eff}}}$, where $NEP_{\text{tot}}^2 = 4k_B T^2 G_{th}^2 / ZT_{\text{tot}}$ and $\tau_{\text{eff}} = \tau_{th}\sqrt{1+ZT_{\text{tot}}}$. Here $ZT_{\text{tot}}^{-1} = ZT^{-1} + ZA^{-1}$, where $1/ZTA = S_A G_{th}^2 / (2k_B T^2)$. The effect of the amplifier can hence be disregarded if $ZT/ZTA = S_A(1 + ZT)/(2k_B T) \ll 1$.

C. Comment about multiplexing

In this section we consider a practical multiplexing case by doing a numerical experiment. For this, we define the filtered signal in the calorimetric regime from the previous section as
$$I_L^{(\text{Fil})}(t) = \frac{E}{2\pi} \int_{-\infty}^{\infty} d\omega \ W(\omega) \lambda_{\omega}(\omega) e^{-i\omega t} \quad (20)$$
Now, in Eq. (20), if we consider $W(\omega)$ to be the optimal filter as obtained in the previous section, then we have
$$I_L^{(\text{Fil})}(t) = A \exp \left[-t/ \left(\tau_{th}\sqrt{1+ZT_{\text{tot}}}\right)\right]$$
$$A = \frac{E^2 ZT_{\text{tot}}}{8k_B T^2 G_{th}^2} (1 + ZT_{\text{tot}})^{-1/2}.$$ 
Therefore for the optimal filter, the plot of $-\ln I_L^{(\text{Fil})}(t)$ vs $t$ is simply a straight line with the slope $\tau_{th}\sqrt{1+ZT_{\text{tot}}}$. On the other hand for any random filter $I_L^{(\text{Fil})}(t)$ does not have simple decaying form but further oscillates in time. As a result, the time-averaged current becomes very small. From this feature of $I_L^{(\text{Fil})}(t)$ we can identify the pixel for which the designed filter is approximately optimal. In Fig. 3 we represent the $I_L^{(\text{Fil})}(t)$ when the filter is optimal (blue curve), and when the filter is not optimal (orange curve).

D. Energy resolution of the SF based TED

Above discussion is valid for a generic TED. In what follows we evaluate the energy resolution $\Delta E_{\text{opt}}$ of a SF based TED, disregarding the effect of amplifier noise in the measured signal. We can express the coefficients of the thermoelectric detector as $[18, 19]$

$$G = G_T \int_{-\infty}^{\infty} dE \ \frac{N_0(E)}{4k_B T \cosh^2 \left(\frac{E}{2k_B T}\right)} \quad (21)$$
$$G_{th} = \frac{G_T}{e^2} \int_{-\infty}^{\infty} dE \ \frac{E^2 N_0(E)}{4k_B T^2 \cosh^2 \left(\frac{E}{2k_B T}\right)} \quad (22)$$
$$\alpha = \frac{P G_T}{2e} \int_{-\infty}^{\infty} dE \ \frac{E N_z(E)}{4k_B T \cosh^2 \left(\frac{E}{2k_B T}\right)}. \quad (23)$$
Here $P = (G_T - G_L) / (G_T + G_L)$ is the spin polarization, $G_\sigma$ is the normal-state conductance for spin $\sigma$, $N_0(E) = (N^\uparrow + N^\downarrow)/2$ and $N_z(E) = N^\uparrow - N^\downarrow$ are the spin-averaged and spin-difference density of states of the superconductor, normalized to the normal-state density of states, $N_0$, at the Fermi level. Here $N_L = N_S(E \pm h)$ with $N_S(E) = \text{Re} \left[ \frac{E + i\Gamma}{\sqrt{(E + i\Gamma)^2 - h^2}} \right]$, $h$ is the spin splitting exchange field, and $\Gamma \ll \Delta$ describes pair-breaking inside the superconductor. The heat capacity of the absorber with the volume $\Omega$ of the superconductor is $C_h = \nu_F \Omega e^2 G_{th}/G_T$ $[18]$. Finally, the electron-phonon heat conductance is obtained from $[18, 21]$

$$G_{q-ph} = \frac{\Sigma}{96\zeta(5)k_B T^3} \int_{-\infty}^{\infty} dE \ E \int_{-\infty}^{\infty} d\omega \ \omega^2 |\omega| \times L_{E,E+\omega} F_{E,\omega} \quad (24a)$$
$$L_{E,E'} = \frac{1}{2} \sum_{\sigma=\uparrow,\downarrow} N_\sigma(E) N_\sigma(E') \times \left[ 1 - \Delta^2 / [(E + \sigma h)(E' + \sigma h)] \right] \quad (24b)$$

![Fig. 3. Filtered current $I_L^{(\text{Fil})}(t)$ through the inductor. In all curves $\tau_{th}/\tau_{th} = 10^{-3}$ and $ZT_{\text{tot}} = 1$. In the blue curve $W(\omega)$ the optimal filter. On the other hand for the orange curve, we consider the $W(\omega)$ is equal to the optimal filter of the blue curve.](image)
$$F_{E,\omega} = \frac{1}{2} \left[ \sinh \left( \frac{\omega}{2k_BT} \right) \cosh \left( \frac{E}{2k_BT} \right) \right] \times \cosh \left( \frac{E + \omega}{2k_BT} \right)^{-1}.$$  \hfill (24c)

In Eq. (24a) $\Sigma$ is the material dependent electron-phonon coupling constant and $\zeta(5)$ is the Riemann zeta function. For $k_BT \ll \Delta - h$, the thermoelectric coefficients have the analytical estimates [18, 19]

$$G \approx G_T \sqrt{2\pi\Delta} \cosh(h)e^{-\Delta}$$ \hfill (25)

$$G_{th} \approx \frac{k_B G_T \Delta}{e^2} \sqrt{\frac{\pi}{2\Delta}} e^{-\Delta} \left[ e^{\Delta}(\Delta - h) \right]^2$$

$$+ e^{-h}(\Delta + h)^2$$ \hfill (26)

$$\alpha \approx \frac{P G_T}{e} \sqrt{2\pi\Delta} e^{-\Delta} \left[ \Delta \sinh(h) - h \cosh(h) \right]$$ \hfill (27)

$$G_{q-ph} \approx \frac{\Sigma \Omega}{96\zeta(5)} T^4 \left[ \cosh(h)e^{-\Delta} f_1(\Delta) \right]$$

$$+ \pi \Delta^5 e^{-2\Delta} f_2(\Delta),$$ \hfill (28)

where $\tilde{h} = h/k_BT$ and $\tilde{\Delta} = \Delta/k_BT$. In Eq. (28) the terms $f_1$ and $f_2$ represent the scattering and recombination processes. The functions $f_1(x) = \sum_{n=0}^{3} C_n/x^n$ and $f_2(x) = \sum_{n=0}^{2} B_n/x^n$, where $C_0 = 440$, $C_1 = -500$, $C_2 = 1400$, $C_3 = -4700$, $B_0 = 64$, $B_1 = 144$, $B_2 = 258$.

The analytical estimate of the $\Delta E_{opt}^{(Fil)}$ can now be obtained by substituting Eqs. (25)- (28) in Eq. (19). As at low temperatures the scattering contribution dominates over recombination in the quasiparticle-phonon heat conductance, we neglect the recombination process and assume $f_1(x) \approx 400$ for $k_BT \lesssim 0.1\Delta$ to obtain a simplified analytic estimate of the energy resolution $\Delta E_{opt}^{(Fil)}$ of the SF based TED.

$$\Delta E_{opt}^{(Fil)} \approx \left[ \sqrt{4\nu_T \Omega k_B^3 T^3 / \chi} / ZT \right] (1 + ZT)^{1/4},$$ \hfill (29)

$$\chi = (2\pi \tilde{\Delta})^{1/2} e^{-\tilde{\Delta}} \left[ (\tilde{\Delta}^2 + \tilde{h}^2) \cosh(\tilde{h}) \right]$$

$$-2\tilde{\Delta} \tilde{h} \sinh(\tilde{h}),$$ \hfill (30)

$$ZT \approx \frac{P^2}{1 - P^2 + \frac{\Delta^2 + Z_{spur} \cosh^2(h)}{h \sinh(h) - \Delta \sinh(h)^2}},$$ \hfill (31)

$$Z_{spur} = \frac{e^2 \Sigma \Omega / k_B}{G_T k_B^3 \Delta} \left[ \frac{220}{96\zeta(5)} \right].$$ \hfill (32)

In the following, we use the formulas of Eqs. (19)-(24c) to find $\Delta E_{opt}^{(Fil)}$ as a function of the exchange field and the bath temperature of the SF based TED. We also compare the numerical results with the analytical estimate in Eq. (29).

We consider an Al absorber of volume $\Omega = 10^{-19}$ m$^3$ and the superconducting critical temperature at zero exchange field $T_c = 1.2$ K [29], $\nu_T = 10^{17}$ J$^{-1}$m$^{-3}$ and $G_T = 5 \times 10^{-4} e^2 \Sigma \Omega / k_B \approx 25$ $\mu$S [18]. With these choices, we get an overall scaling factor $\sqrt{4\nu_T \Omega / \Delta^3} = 20$ meV. This factor is used in Figs. 4 and 5 as the unit energy resolution. We thus find that the optimal energy resolution at $k_BT = 0.1\Delta$, corresponding to $T = 200$ mK for Al, can be below 1 meV. This corresponds to single-photon resolution at frequencies $f = 2\pi \Delta E_{opt}^{(Fil)}/h$ above 240 GHz. Figure 5 shows the corresponding temperature dependence of the optimal energy resolution (solid lines). The analytical estimate in Eq. (29) fits the numerics up to $k_BT \lesssim 0.1\Delta$. Above this the quasiparticle-phonon recombination process starts affecting the results.
Let us again discuss the added noise in current amplification. A good cryogenic SQUID amplifier can reach \( S_A \sim 0.3/(fa)^2/\text{Hz} \) \(^{[30]}\). With the above-chosen tunnel conductance and superconducting gap \( \Delta \), this would translate into \( ZT/ZT_A \approx 2 \times 10^{-4} \times (1 + ZT)/G_T \Delta/(Gk_BT) \). This becomes of the order of unity or larger for \( k_BT \lesssim 0.1\Delta \). Below that temperature it would be advantageous to either measure the voltage instead of the current, or use higher-conductance junctions. Contrary to the noise equivalent power in bolometers \(^{18}\), the current, or use higher-conductance junctions. Consequently, increasing the contact transparency may be challenging especially with Al/EuS based spin-filter junctions.

IV. CONCLUSIONS

In this work we present the first full noise analysis of a generic thermoelectric detector TED, especially including the possibility of a high thermoelectric figure of merit. In particular, we show that TEDS based on superconductor-ferromagnet systems may rival the best transition edge sensor (TES)-type calorimeters, reaching wide-band energy resolution below 1 meV (with unit quantum efficiency). Hence such SF TEDs present a viable alternative for TES devices especially in the case of large arrays where the lack of required probe power leads to reduced heating and simplified design of the detectors.

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