Analytical Model for Predicting Productivity of Radial-Lateral Wells

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Abstract: An analytical model for predicting the productivity of Radial-lateral wells (RLW) drilled using radial jet drilling technology was developed in this work. The model assumes uniformly distributed equal-geometry laterals draining oil or gas under pseudo-steady state flow conditions within the lateral-reached drainage area. A numerical simulation and production data from three field cases of RLW were used to compare and validate the model. The result indicates that the model overestimates the well production rates for wells by 7.7%, 3.25%, and 8.8%, respectively. The error is attributed to several sources including, lack of data for well skin factor, uncertainty of horizontal permeability ($k_H$) in the well area, uncertainty of permeability anisotropy ($I_{ani}$), and uncertainty in bottom hole pressure ($p_w$). Error analysis of uncertainties in $k_H$, $I_{ani}$, and $p_w$ showed that the model could predict productivity well with an acceptable error (10%) over practical ranges of these parameter values. Parameter sensitivity analyses showed that an increasing number of laterals, lateral length, and horizontal permeability would almost proportionally increase productivity. Well productivity is sensitive to well skin factor and oil viscosity, but not sensitive to the radius of the lateral.

Keywords: radial lateral wells; radial jet drilling; unconventional reservoirs; analytical modeling; well completion

1. Introduction

Radial-lateral wells (RLW) are drilled and completed using coiled tubing with radial jet drilling (RJD) tools in both conventional and unconventional reservoirs. Just like most brachhanced wells, RLW’s are used to improve well productivity and recovery factor of oil and gas reserves [1,2]. Horizontal drilling and completions have long been used to increase production from low producing fields with traditional completions. The traditional horizontal techniques still incur high capital costs and may not be efficient for producing marginal oil/gas reservoirs. Radial jet drilling is a cost-efficient drilling method used to bypass damaged zones in the vicinity of the borehole, re-complete old wells where production has declined, and reach the untapped sweet spots, etc. [3]. The process generally involves running a milling tool downhole to a target depth to mill away the casing, followed by a jetting tool to make small diameter horizontal boreholes of approximately 0.066 ft. to 0.164 ft., and lengths up to 300 ft. using high-pressure water jets [3–6]. The early versions of RLW’s were short-radius multi-lateral wells [1,7]. A number of benefits from field application of RWLs have been reported in the literature [8–12]. In the specific case of Lu et al. [9] a 300% increase in production was reported when RJD was applied to recomplete an old field in Tarim in China. The results presented by Ashena et al. [12] showed limited success—only a 50% increase in production was realized.

In efforts to improve the unsatisfactory performance of RLW’s, people have considered improvements in the RJD design and techniques. As a result of the complex casing milling of the original RJD technology, a new proposed RJD method requires a bendable high-pressure pipe to complete the
milling of the casing, and carry highly Pressured fluid to break the rock [13]. Hydraulic fracturing was also investigated to integrate into the RJD procedure [14-17]. The creation of extended reach RLW’s was explored as a viable option to achieve satisfactory RJD performance [18]. Although all these improvements show added benefits, what constitutes satisfactory performance is not clear. This is because of the lack of a standard base of well conductivity for comparison. Simply considering the productivity of an RLW as the sum of productivity of its individual laterals will over-estimate well productivity because of the interactions between laterals.

A Rigorous mathematical model for productivity of RLW’s has not been found in the literature. Liu et al. [19] presented an inflow performance relationship (IPR) model for multi-branched configurations of horizontal wells. However, the performances of RLW’s are found to be highly inconsistent with model predictions. Raghavan and Joshi [20] produced a model to estimate the productivity of well with root-like laterals. The model utilizes effective wellbore radius (horizontal radial flow) to imitate flow of fluid into the horizontal boreholes. A mathematical model for productivity of multi-lateral wells under pseudo-steady-state flow conditions was provided by Retnanto and Economides [21]. Their model was developed by combining a one-dimensional linear, and a two-dimensional radial flow model. Larsen [22] provided a model related to that of Raghavan and Joshi [20]. They both consider horizontal drain holes modeled by vertical wellbores located at the midpoints of the well elements. These models are not rigorous because the effect of the distribution of radial laterals on well productivity is not considered in these models. Guo [23] illustrated a rigorous approach to integrate productivity of laterals to obtain productivity of a multi-lateral well. However, the resultant model is valid only for fishbone wells where the laterals are parallel, not radial.

Based on Furui et al. [24] model for horizontal wells, a mathematical model was derived in this study for predicting the productivity of RLW’s with consideration of interactions between radial laterals. The accuracy of the model was verified by comparison to the production performance of three wells. This model can be used for the optimization of RJD operations for maximizing RLW productivity.

2. Mathematical Model

Consider an RLW depicted in Figure 1. The following assumptions are made for mathematical modeling of well productivity:

1. Radial laterals are identical and distributed evenly in the drainage area.
2. Pseudo-steady state flow conditions are reached within the lateral-reached drainage area.
3. Single gas phase flow, and single oil phase flow, in dry gas reservoirs or in undersaturated oil reservoirs, respectively.

![Figure 1. Sketch of a Radial-lateral well (RLW) with four equal-geometry radial laterals.](image-url)
All of the currently available horizontal well productivity models, including those presented by Joshi [25], Economides et al. [26], Babu and Odeh [27], Butler [28], Borisov [29], Renard and Dupuy [30], Giger et al. [31], and Furui et al. [24] are for single lateral drain holes, not for multiple lateral drain holes. A single-lateral model describing the pseudo-linear flow regime between the virtual no-flow boundaries and the laterals is needed to integrate all laterals with interferences in the formulation of productivity of well structure with multiple horizontal drain holes in the radial directions. Among the above-mentioned models, only the models presented by Butler [28] and Furui et al. [24] can describe pseudo-linear flow. The Furui et al. [24] model was chosen because it is much newer than Butler’s model.

Starting from Furui et al. [24] model for a single lateral drain hole under steady-state flow conditions, the new model integrates multiple drain holes in a drilled area under pseudo-steady state flow conditions. Derivation of productivity models for radial lateral oil and gas wells is shown in Appendix A. The resultant equations are summarized as follows. The productivity equation for radial lateral oil wells is expressed as

$$ Q_o = \frac{7.08 \times 10^{-3} n k_H h (\bar{p} - p_w)}{\pi \mu_o B_o \sin(\frac{\theta}{2})} \ln \left\{ \frac{I_{ani} \ln \left[ \frac{bl_{ani}}{r_w (i_{ani} + 1)} \right] - I_{ani} (1.224 - s) + \frac{\pi}{2} \sin \left( \frac{\pi}{2} \right) L}{I_{ani} \ln \left[ \frac{bl_{ani}}{r_w (i_{ani} + 1)} \right] - I_{ani} (1.224 - s) + \frac{\pi}{2} \sin \left( \frac{\pi}{2} \right) R_w} \right\} \tag{1} $$

where $Q_o$ is well production rate in stb/day, $n$ is the number of radial laterals, $k_H$ is horizontal permeability in md, $\bar{p}$ is average reservoir pressure in psi, $p_w$ is wellbore pressure in psi, $h$ is pay zone thickness in ft, $\mu_o$ is oil viscosity in cp, $L$ is the length of lateral in ft, $B_o$ is oil formation volume factor in rb/stb, $r_w$ is radius of the lateral hole in ft, $s$ is skin factor, $R_w$ is the radius of the main wellbore in ft, and

$$ I_{ani} = \sqrt{\frac{k_H}{k_V}} \tag{2} $$

where $k_V$ is vertical permeability in md.

The productivity equation for radial lateral gas wells takes the form of

$$ Q_g = \frac{n k_H h (\bar{p}^2 - \bar{p}_g^2)}{1424 \pi \bar{p}^2 \pi^2 \sin(\frac{\pi}{2})} \ln \left\{ \frac{I_{ani} \ln \left[ \frac{bl_{ani}}{r_w (i_{ani} + 1)} \right] - I_{ani} (1.224 - (s + D Q_g)) + \frac{\pi}{2} \sin \left( \frac{\pi}{2} \right) L}{I_{ani} \ln \left[ \frac{bl_{ani}}{r_w (i_{ani} + 1)} \right] - I_{ani} (1.224 - (s + D Q_g)) + \frac{\pi}{2} \sin \left( \frac{\pi}{2} \right) R_w} \right\} \tag{3} $$

where $Q_g$ is well production rate in Mscf/day, $D$ is non-Darcy coefficient in day/Mscf, $\bar{p}_g$ is average gas viscosity in cp, $\bar{z}$ is average $z$-factor, and $T$ is reservoir temperature in °R.

### 3. Model Validation

#### 3.1. Numerical Simulation

To validate the model, production was simulated for two cases of RLWs with COMSOL Multiphysics and the results compared to model-calculated production rates. Fluid flow within the porous medium was modeled with COMSOL’s inbuilt Darcy’s law in the subsurface flow module. Parameters for the numerical simulation are based on Wells 1 and 3 in Table 1 for cases 1 and 2, respectively. The following assumptions were made in the simulation: reservoir pressure, $p_r = 2000$ psi; total compressibility, $C_t = 1 \times 10^{-5}$ psi$^{-1}$; and oil density, $\rho_o = 56.186$ lb/ft$^3$. The phase angle for case 1 was 60°, and that of case 2 was 90°. The simulation time was ten days. Figures 2 and 3 show the geometry and pressure profile for cases 1 and 2, respectively. The laterals were modeled to be uniformly distributed in a cylindrical medium and separated by the phase angles. The average reservoir pressure is obtained using the average function defined in COMSOL.
Table 1. Data for three wells in Belayim Land Oil Field, Egypt [8].

| Parameter                        | Well 1 | Well 2 | Well 3 | Units  |
|----------------------------------|--------|--------|--------|--------|
| Number of radial laterals, $n$   | 6      | 6      | 4      |        |
| Length of lateral, $L$           | 164.02 | 186    | 164.02 | ft     |
| Radius of lateral hole, $r_w$    | 0.083  | 0.083  | 0.083  | ft     |
| Horizontal permeability, $k_H$   | 15     | 26     | 20     | md     |
| Average reservoir pressure, $P$  | 900    | 1990   | 970    | psia   |
| Bottom-hole pressure, $p_{w}$    | 500    | 1500   | 570    | psia   |
| Pay-zone thickness, $h$          | 82     | 133    | 87     | ft     |
| Wellbore radius, $R_{wb}$        | 0.328  | 0.328  | 0.328  | ft     |
| Permeability ratio, $k_H/k_V$    | 10     | 10     | 10     |        |
| Permeability anisotropy, $I_{ani}$| 3.162  | 3.162  | 3.162  |        |
| Oil viscosity, $\mu_o$           | 7      | 7      | 7      | cp     |
| Oil formation volume factor, $B_o$| 1.03   | 1.03   | 1.03   | rb/stb |
| Skin factor, $s$                 | 0      | 0      | 0      |        |

Figure 2. (Left): Geometry of case 1 showing one lateral (Phase angle $= 60^\circ$). (Right): pressure profile after 10 days of simulation.

Figure 3. (Left): Geometry of case 2 showing one lateral (Phase angle $= 90^\circ$). (Right): pressure profile after 10 days of simulation.

Two statistical quantities, the Mean Absolute Percentage Error (MAPE) and the coefficient of determination ($R^2$), were used as evaluation criteria to assess the accuracy of prediction of the simulation results by the analytical model. MAPE was computed using Equation (4).

$$MAPE = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{Q_{si} - Q_{mi}}{Q_{si}} \right| \times 100$$

where $n$ is the number of data points, $Q_{si}$ is the production rate from simulation, and $Q_{mi}$ is the model predicted production rate. Smaller MAPE values are an indication of lower percentage errors [32].
Figures 4 and 5 compare the oil production rates computed by the analytical model, and the COMSOL Multiphysics software for cases 1 and 2, respectively. The MAPE value for case 1 is 2.98% whilst that of case 2 is 5.98%. According to Lewis [33], as reported by San Cristóbal [32], MAPE of less than 10% is a highly accurate prediction. Values in the range of 10% and 20%, are considered high for prediction accuracy, 20% and 50%, are average for prediction accuracy, and values greater than 50% are low for prediction accuracy. The values obtained in this work, therefore, demonstrate high accuracy of the model. Moreover, the $R^2$ for both cases is 0.999, which indicates high accuracy.

**Figure 4.** Comparison between the model-predicted production rate and the simulation results (Case 1).

**Figure 5.** Comparison between the model-predicted production rate and the simulation results (Case 2).

### 3.2. Field Case Example

In this section, the model is applied to estimate the production rate from real field examples of RLW drilled by RJD.

Ragab [8] presented the performance of three RLW’s drilled by RJD in the Belayim Land Oil Field in Egypt. These three wells were used to verify the model. The main reason for trying this new technique was to solve problems of well productivity, and field production decline. The field under study can be found in the central part of the Gulf of Suez. It is multi-layer reservoir of sand with interbedded shale and anhydrite. The three wells are from two zones in the layered reservoir, zones II-A and IV. Zone II-A is composed of 134.51 ft shales with some interbedded sandstone at the upper part of the member and followed downward by 160.76 ft of anhydrite and 45.93 ft of sandstone [34]. Zone IV is the thinnest (160.76 ft) of the Belayim formation and comprised of partly dolomitized sandstones with...
interspersed shale streaks [34]. A total of 23% of the Belayim original oil in place (OOIP) is contained in Zone IV and provides more than 27% of production [8].

Well #1 has a net pay thickness of 82 ft and was drilled to a total depth of about 8856 ft. The well is produced from zones II-A and zone IV with a mean daily production rate of 251 stb/day. Seven lateral holes with diameters ranging from 0.13 ft to 0.2 ft, and 164 ft long, were drilled from this well with 7000 psi jetting pressure. Each lateral was at a different depth but they were quite close to each other (3 ft between each lateral) and oriented like a coil by rotation of the bottomhole assembly (BHA) at the surface, by 1 1/2 turn. Drilling of lateral number 6 however failed, leaving 6 effective laterals for this well. The well-produced at a rate of 289 stb/day when it was opened to flow.

Well #2 has 6 laterals drilled at two levels with 7000 psi jetting pressure. The well produced from Zones II-A and IV. Laterals 1 to 3 were drilled at a depth of 7462 ft, with azimuth angles of 20°, 150°, and 225°, respectively. Laterals 4 to 6 were drilled at 7449 ft with azimuth angles of 20°, 150°, and 240°, respectively. The length of each lateral was 164 ft, except for the third lateral, which was drilled to 295 ft. The well was drilled to a depth of 8134 ft. The net pay is 133 ft. Well #2 produced at a rate of 245 stb/day when completed.

Well #3 was drilled successfully with four laterals at two levels and produced from Zone IV. The first two laterals were drilled at a depth of 8088 ft and the next two at 8059 ft. The azimuth angle at each depth was 115° and 295° from the north. The length of the laterals is 164 ft with diameters ranging from 0.13 ft to 0.2 ft. Two more laterals failed after several trials due to the loss of the flexible shaft. The well depth is about 8994 ft, with net pay thickness of 87 ft. Well #3 produced at a rate of 686 stb/day when completed.

Ragab [8] reported the permeability of the rock formation of the Belayim Land Oil Field as being heterogeneous, but the average porosity to be 20%. Abdel-Ghany et al. [7], in a separate study of RJD technology on the same field, reported permeability anisotropy ($I_{ani}$) to be 10. Table 1 summarizes well data for the three wells. Equation (1) was used to estimate the oil production rate for all three well cases, and the results are shown in Figure 6. The analytical model overestimated the production rate for well #1, well #2, and well #3 by 7.7%, 3.25%, and 8.8%, respectively.

![Figure 6](image-url)  
**Figure 6.** Comparison of oil production rate from three radial-lateral wells in Belayim Land Oil Field with model estimated production rate.

The overestimation could arise from uncertainties in horizontal permeability $k_H$, permeability anisotropy $I_{ani}$ and/or bottom-hole pressure, pw. These parameters are sensitized in Figures 7–9.
The shaded region on the figures shows a 10% error bound for over- and under-estimation. The percentage error in estimating $Q_o$ was calculated using Equation (5)

$$\text{% Error} = \frac{Q_{o(m)} - Q_{o(f)}}{Q_{o(f)}} \times 100$$  \hfill (5)$$

where $Q_{o(m)}$ is the model calculated production rate and $Q_{o(f)}$ is the field production rate. The absolute value of Equation (5) was not taken in order to differentiate between overestimation and underestimation by positive and negative values, respectively.

**Figure 7.** Errors in estimated well production rate; the coupled effect of uncertainties in bottom-hole pressure and horizontal permeability (Well #1).

**Figure 8.** Errors in estimated well production rate; the coupled effect of uncertainties in bottom-hole pressure and permeability anisotropy (Well #1).
Generally, from all three figures, the analytical model accuracy is demonstrated—it predicts the production rate quite well over the practical range of uncertainties, and thus values of the parameters most likely to occur in the field, as highlighted in the shaded region.

4. Parametric Analysis

Data for well #1 was used as a base case to study the sensitivity of various parameters on the oil production rate. Figure 10 shows the effect of the number of laterals on productivity. It can be observed that an increasing number of laterals increase production rate proportionally. Therefore, whenever reservoir conditions and wellbore integrity permits, more laterals can be drilled. However, although this study does not consider Net Present Value analysis—it is imperative to economically optimize the number of laterals, which will yield the most returns.

Traditionally, increasing the length of the radial increases productivity, and the model rightly captures this observation in Figure 11. The choice of radial length is a function of the reservoir properties [2]. A longer lateral contacts more of the reservoir formation and communicates better with the unswept areas of the reservoir. This results in higher production rates. The increase in production
rate, as shown in Figure 11 is proportional. This, however, might not be truly representative of what will be observed in the field, because of uneven pressure distribution along the lateral during production. There is likely an optimum point, after which further increase in the length of the lateral will not have a proportional increase in productivity. The length of the laterals depends on the jetting pressure as well as the formation strength [10]. The true length and direction of the lateral are difficult to predict because of limited access to the jetted holes—logging tools might not enter the hole [4]. Reinsch et al. [1] used acoustic measurements to monitor the direction of the jetting nozzle and, in principle, the direction and length of the jetted hole within the rock mass.

Figure 11. Modeled effect of lateral length on well production rate (Well #1).

Figure 12 illustrates the sensitivity of the radius of the lateral hole. It implies that an increasing radius of the lateral will improve production rate but not proportionally—the positive effect diminishes for higher radii. Maintaining the current lateral radius (0.08 ft), as was used in field cases presented above, will, therefore, be satisfactory for field practices. The main limitations associated with this small diameter hole size, as explained by Kamel [4] have to do with (1) completing the lateral holes with a liner. This has never been part of the design of RLW’s, and although the advantage of reduced cost due to the exclusion of a liner is there, it suffers from wellbore stability issues. This, however, has not been widely reported as a problem. (2) Reentering the lateral after it has been drilled, (3) no monitoring inside the lateral, (4) logging inside the lateral.

Figure 12. Modeled effect of lateral radius on well production rate (Well #1).
Removing the mechanical skin improves drain efficiency and subsequently oil production rate more than proportionally (Figure 13). Aside from the increase in productivity, skin in general, from operational practices, is never desirable and should be removed if possible. One of the benefits of RLW is to actually bypass the damaged zone in the near wellbore [3].

![Figure 13. Modeled effect of skin on well production rate (Well #1).](image1)

The effect of $k_H$ follows a similar trend as the length of radial; thus, an increase in $k_H$ increases productivity proportionally (Figure 14). The horizontal permeability, $k_H$ has high uncertainty due to heterogeneity, but with a good estimate of its value, one can determine candidate wells or formations for RJD. Figure 15 shows that less viscous oils increase productivity. As viscosity is reduced, the production rate increases markedly. This figure also depicts that highly viscous hydrocarbons might not be good candidates for RJD. However, if economics and technology permits, it could be possible to combine with steam flooding to increase productivity.

![Figure 14. Modeled effect of horizontal permeability on well production rate (Well #1).](image2)
not be good candidates for RJD. However, if economics and technology permits, it could be possible to combine with steam flooding to increase productivity.

From the foregoing, the model presented has the least effect. The optimizing radius of lateral for productivity improvement might not yield a significant result. However, many results. However, for mechanical reasons such as wellbore and/or lateral stability, the radius could be significant for optimization.

Figure 15. Modeled effect of oil viscosity on production rate (Well #1).

Figure 16 compares the sensitivities of the parameters discussed above. All parameters were increased by 20% and decreased by 20% for upper and lower cases, respectively. The skin has been excluded from this comparison because the base value was zero, and one cannot take a percentage increase or decrease from zero—the result will not be a real number. From the figure, it can be noticed that comparatively, increasing viscosity has the greatest effect on productivity. The radius of the lateral has the least effect. The optimizing radius of lateral for productivity improvement might not yield many results. However, for mechanical reasons such as wellbore and/or lateral stability, the radius could be significant for optimization.

Figure 16. A Tornado chart comparing parameter sensitivities.
5. Discussion

A data-driven mathematical model was developed for estimating the productivity of RLW’s. These wells are drilled with small diameter (1–2 inches) laterals in a radial fashion around a vertical wellbore. The presented model assumes identical and uniformly distributed radials in the drainage area. This assumption is valid for most applications where radial laterals are designed and drilled with today’s coiled tubing and jet drilling technologies. From the field examples presented, it is noticed that in some cases, the radials have different lengths. These radials may be placed in the same or different layers [2], depending on the thickness of the pay zone, and vertical communication. This is a limitation of the model. However, this should not present major errors as the depth difference for radials which do not occur at the same depth was about 3 ft. (for well #1). In this case, for the example in case 1 as described under model verification, it was assumed that all the radials occur at the same depth. In addition, the average length of radials was used when the length of radials was different.

The second assumption of pseudo-steady state flow becomes realistic after transient flow and the interference of pressure propagation as a result of fluid production through laterals. This assumption is suitable for describing well productivity during pressure depletion within the drilled drainage area.

The third assumption, single phase oil or gas flow, is not always valid, but it is acceptable when the effective permeabilities to oil and gas are used for under-saturated oil reservoirs and dry gas reservoirs, respectively. The model does not consider non-Darcy flow and does not capture the huge heterogeneity and complexities encountered in some formations, especially shale formations. These present some limitations to the model’s application.

One key advantage of this model is the ease of use. It is a data-driven model, and most of the required data do not cost much to acquire.

From the foregoing, the model presented has been shown to estimate productivity from three field cases with errors of less than 10%. The errors could be because of wellbore skin that was not considered in the model calculation due to a lack of data. The error can also be from the data uncertainties in horizontal permeability ($k_H$), permeability anisotropy ($I_{ani}$), and bottom hole pressure ($p_w$).

The spacing/angle between the lateral holes may not be equal. This was not factored into the model, resulting in over-prediction by the model. This is key in ensuring drain efficiency, for two laterals could drain the same reservoir area that is assumed to be drained by one lateral hole in the model. Field cases reported by Maut et al. [10] indicated a 90° phasing of laterals at each depth of RJD.

Further work to incorporate non-Darcy flow and multiphase flow into the model will be a great addition to ensure greater accuracy and wide application of the model.

6. Conclusions

An analytical model for predicting the productivity of RLWs completed with radial jet drilling technology was developed in this work. The model assumes single phase fluid flow and pseudo-steady state flow within the lateral-reached drainage area. Numerical simulations with COMSOL Multiphysics were used to validate the model. MAPE, which was used as an error measure to assess the accuracy of the model prediction showed values of 2.98% and 5.92% for the two cases analyzed. The $R^2$ statistic also used and showed good accuracy of the model with a value of 0.999. Three field cases of RLWs were used to verify the model. From the analysis, the following conclusions can be made.

1. The model overestimated the well production rate for wells 1, 2, and 3 by 7.7%, 3.25%, and 8.8%, respectively. The error is attributed to several sources, including (1) lack of data for well skin factor, (2) uncertainty of horizontal permeability ($k_H$) in the well area, (3) uncertainty of permeability anisotropy ($I_{ani}$), and (4) uncertainty in bottom hole pressure ($p_w$).

2. Error analysis of uncertainties in $k_H$, $p_w$, and $I_{ani}$ showed that the model could predict productivity well with an acceptable error (10%) over practical ranges of these parameter values.

3. Parameter sensitivity analyses showed that an increasing number of laterals, lateral length, and horizontal permeability would almost proportionally increase productivity. Well productivity
is very sensitive to well skin factor and oil viscosity, but less sensitive to the radius of lateral. The sensitivity to the lateral radius decreases as the radius increases.

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**Nomenclature**

- **BHA** Bottom hole assembly
- **Bo** oil formation volume factor, rb/stb
- **Ct** total compressibility, psi-1
- **D** non-Darcy coefficient, day/Mscf
- **h** pay-zone thickness, ft
- **Iani** permeability anisotropy
- **kH** horizontal permeability, md
- **kV** vertical permeability, md
- **L** length of lateral, ft
- **MAPE** Mean absolute percentage error, %
- **OOIP** Original oil in place
- **pet** reservoir pressure, psi
- **pw** wellbore pressure, psi
- **p** average reservoir pressure, psi
- **Qg** gas production rate, Mscf/day
- **Qo** oil production rate, stb/day
- **Rw** radius of wellbore, ft
- **rw** radius of lateral hole, ft
- **RJD** radial jet drilling
- **RLW** radial lateral well
- **s** skin factor
- **SD** drainage distance, ft
- **T** temperature, oR
- **z** average z-factor
- **pg** average gas viscosity, cp
- **µo** oil viscosity, cp
- **ρo** oil density, lb/ft$^3$
- **θ** angle, °

**Appendix A. Derivation of Analytical Model for Productivity of Radial-Lateral Wells**

Figure A1 depicts a well with $n$ radial laterals of equal-geometry. Consider an infinitesimal segment of a lateral $dx$ at a distance $x$ from the center of the main wellbore.

According to Furui et al. [24] work, the oil production rate from this segment is expressed as:

$$dq_o = \frac{7.08 \times 10^{-3} k_H (p_{avg} - p_w)}{\mu_o B_o \left\{ I_{ani} \ln \left[ \frac{k_{H}}{r_w (I_{ani} + 1)} \right] + \frac{SSD}{h} - I_{ani} (1.224 - s) \right\} } dx$$  \hspace{1cm} (A1)

where $q_o$ is oil production rate in stb/day, $p_{avg}$ is reservoir pressure in psi, $p_w$ is wellbore pressure in psi, $\mu_o$ is oil viscosity in cp, $k_H$ is horizontal permeability in md, $h$ is pay zone thickness in ft, $r_w$ is radius...
of the lateral wellbore in ft, $B_o$ oil formation volume factor in rb/stb, $s$ is skin factor, $S_D$ is drainage distance in ft, and $I_{ani}$ is permeability anisotropy.

$$I_{ani} = \sqrt{\frac{k_H}{k_V}}$$

(A2)

where $k_V$ is vertical permeability in md. The drainage distance is expressed as

$$S_D = x \sin(\theta) = x \sin\left(\frac{\pi}{n}\right)$$

(A3)

Figure A1. Sketch of a well with n radial laterals of equal-geometry.

Substituting Equation (A3) into Equation (A1) yields

$$dq = \frac{7.08 \times 10^{-3}k_H(\bar{p} - p_w)}{\mu_oB_o\left(I_{ani}\ln\frac{h_{ani}}{r_w(l_{ani}+1)} + \frac{n}{\pi} \sin\left(\frac{\pi}{n}\right)x - I_{ani}(1.224 - s)\right)}dx$$

(A4)

Which is integrated from the main wellbore to the lateral length $L$

$$\int_0^{q_o} dq = \int_{R_{w}}^{L} \frac{7.08 \times 10^{-3}k_H(\bar{p} - p_w)}{\mu_oB_o\left(I_{ani}\ln\frac{h_{ani}}{r_w(l_{ani}+1)} + \frac{n}{\pi} \sin\left(\frac{\pi}{n}\right)x - I_{ani}(1.224 - s)\right)}dx$$

(A5)

to yield

$$q_o = \frac{7.08 \times 10^{-3}k_Hh(\bar{p} - p_w)}{\pi \mu_oB_o \sin\left(\frac{\pi}{n}\right)} \ln\left\{ \frac{I_{ani}\ln\left[\frac{h_{ani}}{r_w(l_{ani}+1)}\right] - I_{ani}(1.224 - s) + \frac{n}{\pi} \sin\left(\frac{\pi}{n}\right)L}{I_{ani}\ln\left[\frac{h_{ani}}{r_w(l_{ani}+1)}\right] - I_{ani}(1.224 - s) + \frac{n}{\pi} \sin\left(\frac{\pi}{n}\right)R_w} \right\}$$

(A6)

For an oil well with $n$ radial laterals, the well production rate is expressed as

$$Q_o = \frac{7.08 \times 10^{-3}nk_Hh(\bar{p} - p_w)}{\pi \mu_oB_o \sin\left(\frac{\pi}{n}\right)} \ln\left\{ \frac{I_{ani}\ln\left[\frac{h_{ani}}{r_w(l_{ani}+1)}\right] - I_{ani}(1.224 - s) + \frac{n}{\pi} \sin\left(\frac{\pi}{n}\right)L}{I_{ani}\ln\left[\frac{h_{ani}}{r_w(l_{ani}+1)}\right] - I_{ani}(1.224 - s) + \frac{n}{\pi} \sin\left(\frac{\pi}{n}\right)R_w} \right\}$$

(A7)
where $Q_0$ is well production rate in stb/day. Similarly, for a gas well with $n$ radial laterals, the well production rate is expressed as

$$
Q_g = \frac{nk_Hh(p_2^g - p_s^g)}{1424\pi^2 \bar{z} T \sin(\frac{\pi}{n})} \ln \left( \frac{I_{an1} \left[ \frac{h_{an1}}{r_w(t_{an1} + 1)} \right] - I_{an2} \left[ 1.224 - (s + DQ_g) \right]}{I_{an1} \left[ \frac{h_{an1}}{r_w(t_{an1} + 1)} \right] - I_{an2} \left[ 1.224 - (s + DQ_g) \right] + \frac{n}{\pi} \sin(\frac{\pi}{n}) L} \right)
$$

(A8)

where $Q_g$ is well production rate in Mscf/day, $D$ is non-Darcy coefficient in day/Mscf, $p_s^g$ is gas viscosity in cp, $\bar{z}$ is the average $z$-factor, and $T$ is $r$.

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