Program Synthesis from Visual Specification

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Abstract

Program synthesis is the process of automatically translating a specification into computer code. Traditional synthesis settings require a formal, precise specification. Motivated by computer education applications where a student learns to code simple turtle-style drawing programs, we study a novel synthesis setting where only a noisy user-intention drawing is specified. This allows students to sketch their intended output, optionally together with their own incomplete program, to automatically produce a completed program. We formulate this synthesis problem as search in the space of programs, with the score of a state being the Hausdorff distance between the program output and the user drawing. We compare several search algorithms on a corpus consisting of real user drawings and the corresponding programs, and demonstrate that our algorithms can synthesize programs optimally satisfying the specification.

1 Introduction

The problem of program synthesis is an important one in AI. Synthesis has many settings, from the fully automated settings, where code is synthesized at the level of machine code, to the interactive, where synthesis assists a professional developer in an IDE. We focus on a novel synthesis setting where synthesis may be used to facilitate computer science education.

Consider a student in an educational programming task, drawing an image with a turtle-style program. They may have an intention expressible as a trajectory they would like to draw, as shown in Figure 1(a). This trajectory can be considered a complete but noisy specification of the intended program. And they may have some program built up in their workspace, but this program may be “incomplete” or “buggy”. For example, Figure 1(b) shows a program the user might have composed with the intention of drawing Figure 1(a), along with the trajectory it actually creates. The user might be uncertain how to proceed in order to correct this. It is in this setting where we seek to formulate a synthesis task that can provide the solution shown in Figure 1(c) where the arguments to both the repeat and turn blocks are corrected and a move block is added to yield the program nearest to the intended trajectory.
In this paper, we describe how to synthesize code from such user-provided visual specification. We ultimately formulate the synthesis problem as an optimization problem suitable for combinatorial search.

2 Setting

2.1 Programming Language

We will consider the space $\mathcal{P}$ of turtle programs generated by the following grammar:

\begin{align*}
    \text{Program} & \rightarrow \text{Statement}* \\
    \text{Statement} & \rightarrow \text{MOVE} \mid \text{TURN Angle} \mid \text{REPEAT Int Program}
\end{align*}

Here $\text{Angle}$ takes on values at increments of 30 degrees and $\text{Int}$ takes on values from 2 to 5. Throughout the paper we will speak of elements of this space equivalently as either programs or blocks. As can be seen in Figure 1 and elsewhere, blocks may be connected by being nested horizontally within repeat statements or vertically. If a block is connected vertically beneath another, we refer to it as a child block of its ancestor. If a block has no ancestors, we refer to it as a root block.

Figure 1

(a) An intended trajectory.  (b) A partial program.  (c) The synthesized solution.

Figure 2: Examples of workspaces.
We implement this turtle language using the Blockly\footnote{https://developers.google.com/blockly} visual block programming language and its editor. The semantics of this language are as follows:

- A MOVE statement translates the turtle in its current direction by some fixed magnitude.
- A TURN statement rotates the turtle by the specified number of degrees.
- A REPEAT statement executes a subprogram some number of times.

A user may position several such elements of $\mathcal{P}$ on their workspace as shown in Figure 2(a). Taking a more abstract perspective, we can define a workspace as a list of elements in $\mathcal{P}$ and we denote the space of workspaces by $\mathcal{P}$. We call a set of points on the two-dimensional plane $t \in T$ a trajectory. And we write $I(p) = t$ for the interpretation function which maps a workspace $p$ to its trajectory $t$. This interpretation function can be thought of as executing each block in the workspace on the canvas in the order that it appears in workspace list.

### 2.2 Editing Environment

We can describe the user interface of an editor for our programming language through editing commands. These commands represent discrete mouse manipulations performable through Blockly. Note that for these commands to make sense we must label each block on the workspace with an identifying number.

The families of commands are:

1. Get a \{Type\} block.
2. Remove block \{BlockId\}.
3. Connect block \{BlockId\} under block \{BlockId\}.
4. Connect block \{BlockId\} inside block \{BlockId\}.
5. Disconnect block \{BlockId\}.
6. Change \{Val\} in block \{BlockId\} to \{Val\}.

Figure 3: Family of available editing commands.

(1) adds a new block to the workspace, as a root block not connected to any other block on the workspace. The type parameter can be one of Move, Turn, or Repeat.

(2) removes the block and all its child blocks. This command matches the Blockly semantics of dragging a block to the trash bin.

(3) and (4) move a block and all its children to a new location. (3) moves a source block under a target block and connects them. If the target block has children, they are appended under the source block’s children. (4) is distinguished from (3) in that
the target must be a repeat block, and it places the source block at the top of the repeat body.

(5) disconnects a block from its parent, making it into a root block on the workspace.
(6) modifies the parameter values of blocks, such as the angle in the turn block or the integer in the repeat block.

The user writes a program by applying a sequence of editing commands beginning from an empty workspace. That is, a command, when specialized by a choice of feasible values for its parameters, can be thought of as mapping a workspace \( p \) to a successor workspace \( p' \). For example, beginning at an empty workspace, the following sequence of commands produces the program in Figure 1(b):

1. Get a repeat block.
2. Get a move block.
3. Connect block 2 inside block 1.
4. Get a turn block.
5. Connect block 3 under block 2.
6. Change 30 in block 3 to 120.

This family of commands represents an abstraction of the editor’s capabilities, as the user would typically be manipulating the editing environment with keyboard and mouse.

3 Problem Formulation

Having described the programming language and editing environment, we are now in a position to formulate our synthesis problem as search. The user in our a programming environment intends to produce a trajectory \( t^* \in T \) by means of a turtle program. Consider this as a search on a graph \( G \) whose vertices are the set of workspaces \( P \). There is an edge from \( p \) to \( p' \) in \( G \) if there is an editing command which produces workspace \( p' \) when applied to workspace \( p \). All edges have unit cost. We let \( \text{cost}(p, p') \) designate the weight of the shortest path from \( p \) to \( p' \) in \( G \). We will designate the initial state of their workspace by \( p_0 \).

We measure similarity of trajectories with Hausdorff distance. The Hausdorff distance is a commonly used metric for tasks in object matching and image analysis [3] and serves as a natural metric for the quality of the fit of a candidate program to a trajectory. We denote the Hausdorff distance between sets of points \( X \) and \( Y \) by

\[
d_H(X,Y) := \max \left( \max_{x \in X} \min_{y \in Y} d(x,y), \max_{y \in Y} \min_{x \in X} d(x,y) \right)
\]

where \( d(\cdot, \cdot) \) is the ordinary Euclidean distance.

Synthesizing a good solution \( \hat{p} \) from \( p_0 \) for trajectory \( t^* \) involves a tradeoff between two types of distance or error. On the one hand, a candidate solution \( p \) has some
distance from the target trajectory $d_H(I(p), t^*)$, reflecting the quality of its fit. On the other hand, it may depart significantly from $p_0$, reflected in a large value of cost($p_0, p$), the minimum number of editing steps required to create $p$ if beginning from $p_0$. We trade these off in a constraint formulation. Given intended trajectory $t^*$ and current program $p_0$, we define our synthesis problem as:

$$\arg\min_{p \in P} d_H(I(p), t^*) \quad \text{st} \quad \text{cost}(p, p_0) \leq C.$$ 

In all experiments below, we choose $C = 6$. This choice was made to ensure practical run times.

4 Algorithms

4.1 IDPS

Our first algorithm is a variant of iterative deepening, which we call iterative deepening program search (IDPS). We use a path-checking depth-limited search as a subroutine to minimize space requirements. Unlike traditional AI search procedures, IDPS returns not one but a sequence of programs $\hat{p}_1, \hat{p}_2, \ldots$ whose Hausdorff distance from the target trajectory $t^*$ strictly decreases. At iteration $d$, we initialize $\epsilon$ to $d_H(I(p_0), t^*)$ and expand the search graph up to depth $d$. Then, we iterate through all depth-$d$ programs $p$, checking if $d_H(I(p), t^*) \leq \epsilon$. When such a program is found, we emit it to our output sequence, and update $\epsilon$ to $d_H(I(p), t^*)$ so that future goal states must be strictly better than $p$.

Figure 4 displays an example of such a sequence, where $t^*$ is a noisy square and the initial program draws only a line. By convention, we include the initial program $p_0$ in the sequence.
4.2 Sampling search

Our second algorithm is a sample-based search shown in Algorithm 1. Here, we use a corpus of user programs and trajectories to guide our search. Algorithm 1 can be described as searching from a graph rooted at $p_0$. The initial program $p_0$ is represented by a sequence of editing commands. We use the notation $\bar{p} = p : c$ to represent the program $\bar{p}$ resulting in appending command $c$ to the end of command list $p$. The algorithm samples commands $c$ from a distribution $P(c | p)$ parameterized by command sequence $p$. A budget of $b$ candidates are sampled in total. Each of the $b$ sampling rounds begins at $p_0$ and samples $C$ commands, sequentially appending and evaluating them. The candidate minimizing $d_H(I(\cdot), t^*)$ among all samples candidates is returned.

**Algorithm 1 Sampling Search**

input: Budget $b$, cost $C$, initial program $p_0$, visual specification $t^*$

output: Program best

procedure SAMPLINGSEARCH($b, C, p_0, t^*$)

$best \leftarrow p_0$

for each $i$ in $1 \ldots b$

for each $j$ in $1 \ldots C$

$c_j \sim P(c | p_{j-1})$

$p_j \leftarrow p_{j-1} : c_j$

if $d_H(I(p_j), t^*) < d_H(I(best), t^*)$ then

$best \leftarrow p_j$

return $best$

What distinguishes variants of this algorithm is how the distribution $P(c | p)$ is modeled. We factor $P(c | p)$ this into a bigram model over command types and a distribution over command arguments. That is, we map the command sequence $p = \{c_i\}_{i=1}^{|p|}$ to a coarsened sequence $\{\tilde{c}_i\}_{i=1}^{|p|}$ by discarding arguments so that $\tilde{c}_i \in \{\text{Get}, \text{Remove}, \text{Connect}, \text{Change}, \text{Separate}\}$. We then draw $\tilde{c}_{|p|+1}$ from $P(\cdot | \tilde{c}_{|p|})$, a Markov chain over the coarsened tag sequence.

To sample $c_{|p|+1}$ from $\tilde{c}_{|p|+1}$, we have two models: a uniform model and a non-uniform model. On the uniform model, we uniformly sample the $\{\text{Type}\}$, $\{\text{BlockId}\}$, $\{\text{Val}\}$ arguments from the command language from all available types, values, and – in the case $\{\text{BlockId}\}$ – all available positions in the current program.

For the non-uniform model, we make the following observation about the process of editing programs: locations to modify the current program are chosen with a particular focus in mind. Specifically, when a user is connecting a block to another, the source block is more often the last block added to the workspace, while the destination block is often the next-to-last block added to the workspace. We construct a simple model to accommodate this observation. In choosing $\{\text{BlockId}\}$ arguments, we designate a probability $\lambda_{-1}$ that the source block is the last block added to the workspace. We assign probability mass $(1 - \lambda_{-1})$ uniformly over other feasible blocks. Then we sample a destination block as the next-to-last block with probability $\lambda_{-2}$, reserving probability $(1 - \lambda_{-2})$ to be uniformly distributed over all remaining feasible choices.
• Get a repeat block
• Get a turn block
• Connect block 2 inside block 1
• Change 30 in block 2 to 270
• Get a move block
• Connect block 3 under block 2
• Get a repeat block

Figure 5: Fragment of a program from the corpus.

All of these probabilities in the above models can be estimated from our corpus. We estimate the transition probabilities \( P(\tilde{c}_j|\tilde{c}_{j-1}) \) over the coarsened command sequences smoothing with pseudo-counts of 1. We estimate \( \lambda_{-1} \) and \( \lambda_{-2} \) by taking the empirical proportion of such decisions over the corpus. The product of the distribution over command types and the distribution over command arguments defines the distribution \( P(c|p_{j-1}) \) shown in Algorithm [4].

4.3 A Computational Speedup

While the Hausdorff distance \( d_H \), used in both IDPS and sampling search algorithms, is a natural choice for scoring the quality of a fit, the computation of \( d_H(X,Y) \) becomes prohibitive, as it is quadratic in the number of points of the two point sets \( x \) and \( y \) to be compared. Let us say we have some threshold \( \alpha > 0 \) and we wish to determine if \( d_H(X,Y) < \alpha \). In many cases where \( d_H(X,Y) \geq \alpha \), we may avoid some of this computation. If there exists a point \( x \in X \) such that \( \forall y \in Y, d(x,y) \geq \alpha \), then we may terminate our computation, as \( d_H(X,Y) \geq \alpha \). Furthermore, if for any \( x \in X \), we can find some \( y \in Y \) such that \( d(x,y) < \alpha \), we may omit computation of any further \( d(x,y') \) for \( y' \in Y \), because \( \min_{y \in Y} d(x,y) < \alpha \). These speed ups, however, avoid computation only if \( d_H \geq \alpha \). They do not compute \( d_H \). Therefore, these speed ups are employed in the algorithms above by replacing any inequality condition involving \( d_H \). If the \( d_H < \alpha \), the full quadratic computation of \( d_H \) is performed to compute its value.

5 Evaluation

We solicit a corpus of \( n = 23 \) programs and their visual specifications from 11 volunteer study participants, who range in programming experience from novice to professional. Figure 5 and Figure 6 give an examples of a participant program and some participant specifications, respectively. To construct this corpus, we employ the following data collection procedure:

Step 1 Each participant is educated in the capabilities of our turtle language and the editor environment by completing an introductory set of exercises.
Step 2 The participant is instructed to draw a trajectory on a standard canvas. Let $t^{(i)}$ represent this visual specification of the intended of the program, as drawn by participant $i$.

Step 3 The participant is instructed to compose a program in the turtle language which follows the drawn trajectory as closely as possible. Let $p^{(i)}$ represent the matching program from participant $i$.

We record each step in the participant’s programming process as a formal editing command, c.f. Section 2.2. We represent the complete program $p^{(i)}$ as the sequence of editing commands $\{c^{(i)}_1, \ldots, c^{(i)}_{|p^{(i)}|}\}$ which produce $p^{(i)}$ if performed in the editor beginning at an empty workspace.

Now consider each of these programs with the final $k$ commands removed, that is $p^{(i)}_{-k} = \{c^{(i)}_1, \ldots, c^{(i)}_{|p^{(i)}|-k}\}$. Letting $A$ denote the search algorithm at hand, our interest is in evaluating the performance of $\hat{p}^{(i)} = A(p^{(i)}_{-k}, t^{(i)})$, the $k$-ahead performance of our search algorithm.

As our search procedure does not discriminate between syntactically differing programs which produce the same trajectory (share the same semantics), we consider programs up to semantic equivalence. We define the semantic equivalence class of $p \in \mathcal{P}$ as $\text{SemEq}(p) = \{p' \in \mathcal{P} : I(p) = I(p')\}$. We then define our $k$-ahead accuracy in terms of our procedure achieving any semantically equivalent program to the target.

| Metric              | Notation                  | Equation                                      |
|---------------------|---------------------------|-----------------------------------------------|
| Accuracy            | $\text{Acc}_k(p^{(i)}_{-k}, t^{(i)})$ | $\{p^{(i)} \in \text{SemEq}(p^{(i)}_{-k})\}$ |
| Hausdorff Error     | $\text{Err}_k(p^{(i)}_{-k}, t^{(i)})$ | $d_H(t^{(i)}, I(\hat{p}^{(i)}))$              |
| Relative Error      | $\Delta_k(p^{(i)}_{-k}, t^{(i)})$ | $\frac{\text{Err}_k(p^{(i)}_{-k}, t^{(i)}) - \text{Err}_k(\hat{p}^{(i)}_{-k}, t^{(i)})}{\text{Err}_k(p^{(i)}_{-k}, t^{(i)})}$ |

Table 1: $k$-ahead metrics
Figure 7: Performance of each algorithm against $k$. (a) mean accuracy (b) mean Hausdorff distance (c) the relative error reduction.

Unfortunately, if we only consider $\hat{p}^{(i)}$ correct when $\hat{p}^{(i)} \in {SemEq}(p)$, we overlook the important case where $I(\hat{p}^{(i)})$ is a better fit for $t^{(i)}$ than $I(p^{(i)})$. This may happen, for example, if the participant has little programming experience and writes $p_i$ incorrectly. A softer measure of accuracy is simply the Hausdorff distance between $I(\hat{p}^{(i)})$ and the user’s trajectory, though this does not consider scaling. Still softer is the relative reduction in Hausdorff distance from $p_i - k$ to $\hat{p}^{(i)}$. Table 1 summarizes these $k$-ahead metrics.

We evaluate our method against all metrics. We also report runtime.

6 Related Work

The problem of generating a computer program from some specification has been studied since the beginnings of AI. Relevant work here falls into the two broad camps of synthesis where an explicit program is generated and induction where a latent representation may be used to generate input output pairs [1].

In the inductive setting, there is a large literature of work relating to the search for algorithms which correct a sketch. [6] uses recurrent neural networks to model conditional sketch generation, an image-to-image transformation problem. [4] uses probabilistic program induction to perform one-shot modeling. In the programming synthesis community, there is a large body of work synthesizing program from logical specifications [8].

Synthesis settings can further be distinguished by whether a given specification is partial or total. Abstractly, we may think of synthesis as attempting to infer some $f \in \mathcal{F}$ where $\mathcal{F}$ is a family of programs taking inputs from $\mathcal{X}$ and outputting $\mathcal{Y}$. A partial specification comes in the form of a set of input-output pairs $\{(x_1, y_1), \ldots, (x_n, y_n)\}$ whose input items are a proper subset of $\mathcal{X}$. This induces a problem of inference as the synthesis algorithm must settle on some choice of output for unobserved input. By way of contrast, a total specification fully specifies $f$ as a function. But even if we know the specification, we may have difficulty finding a matching program. The total specification synthesis problem is a problem of search rather than inference.

A further distinction can be made between settings where the specification is noisy
or noiseless. More recent work such as [1] and [5] naturally handle noisy specifications owing to their use of neural models.

[2] and [7] allow the user to sketch a partial program in addition to a specification through inputs/output pairs. Our method is distinct in that the user provides not a sketch but a partial program which may contain errors. That is, our synthesis is not constrained by the given partial program.

7 Results

We compute $k$-ahead completions for $k = 1, \ldots, 6$. To ensure practical run times, we allot a state budget of $b = 50,000$ programs for each algorithm and $k$. Likewise, we enforce a static horizon of $C = 6$ so that cost$(p(i), p(i)) \leq 6$ for all completions.

Figure 7(a) plots mean $\text{Acc}_k$ for each algorithm against $k$. For small $k$ such as $k = 1$, IDPS will always recover $p(i)$ unless a better fit is nearby, while sampling search manages a less reliable 63% recovery rate. Under our state budget constraint, IDPS’s performance decreases almost monotonically because it exhausts its budget before exploring deep states. By contrast, sampling search distributes its exploration equally across all depths, and consequently scales well to large $k$. Viewed another way, the best IDPS can do for small $k$ is recover the original trajectory. Figure 7(b) and Figure 7(c), which plot mean $\text{Err}_k$ and mean $\Delta_k$ respectively against $k$, reflect that sampling search has greater opportunity to explore deeper and more structured programs.

The dashed line of Figure 7(b) represents the mean Hausdorff distances of the user’s true completion $p(i)$, that is, the mean of $d_H(I(p(i)), t(i))$ across our corpus. As we regard the trajectory $t(i)$ as the true label, we can see that for smaller values of $k$ our algorithms improved upon the user’s completed program, $p(i)$. For small lookaheads, this suggests that users may benefit from viewing programs returned by our synthesis method.

Nonuniform sampling search dominates the sampling search regime, outperforming uniform sampling in $\text{Err}_k$ and $\Delta_k$ for all $k$. By localizing block targets in a manner statistically more consistent with observed human behavior, the algorithm restricts its search to programs produced by high-level groupings of commands, such as adding a turn block, then immediately connecting it to the penultimate block, and then changing its angle parameter. This appears to produce more reliable completion candidates.

It is worth noting that both uniform and nonuniform sampling search converge on better-fit trajectories when faced with idiosyncratic programming techniques by the programmer. For example, one participant wrote a large loop body and only added the loop itself as his last step, while most other programmers added the loop first. Under this setup, the block being connected inside the loop body is not local (in fact it is as distant as possible), so the nonuniform algorithm is unlikely to sample an essential command. Nevertheless, the algorithm produced a better Hausdorff fit through an altogether different program, an indication of robustness in our method.

Figure 8 shows that sampling search is consistently faster than IDPS, the nonuniform (resp. uniform) variant requiring an average of 5.97 minutes (resp. 4.82 minutes) for convergence as opposed to 9.83 minutes. That the former is nearly twice as fast
as the latter is not surprising. For both the uniform and nonuniform variant, sampling search is bottlenecked by the quadratic Hausdorff computation, while IDPS, on top of Hausdorff, must expand many useless states.

A qualitative look at some specific results can further insight. For example, consider item 18 from our corpus as presented in Figure 9. Here we show in the top row the solution obtained by the uniform algorithm for lookaheads $k = 3$ and $k = 4$ and in the bottom row the user-drawn trajectory and the nonuniform solution, which was the same in both cases of $k$. As we can see the uniform algorithm returns a solution which “cheats” with respect to our metric, constructing a trajectory which covers the region, without really approximating it. The Hausdorff distance of the $k = 3$ and $k = 4$ solutions in the top row is 89 and 90, respectively. By way of contrast, the nonuniform algorithm found the user’s solution, which had a distance to the specification trajectory of 9. We conjecture that two factors contributed to the poor performance of the uniform algorithm. First, intuitively, there are a small number of ways to correctly complete the program, but a vast number of ways to construct spirals of the form found by the uniform algorithm. Second, the user’s partial program, the starting point of search, was characterized by a large number of repeat blocks, from which spiral solutions of the form found were plentiful in the search space, as attachments which tended to nest repeats tend to draw such trajectories. The nonuniform solution, modeling as it does the attention or focus of the programmer, overcame these shortcomings.

Nevertheless, the nonuniform algorithm did not perform strictly better. For example, item 3 in our corpus is presented in Figure 10. Here, the uniform algorithm
returned a reasonable solution, while the nonuniform seemed to lose its way. This example is notable for another reason in that it demonstrates how the Hausdorff distance may not encode all features of the user’s intention. We see here that the program returned by the uniform algorithm adds a bend to the line inside the square, while the user’s trajectory suggests a straight curve. This raises the following question: is the angle of the line within the square the more salient feature of the user’s intention or its straightness? The Hausdorff distance “fudges” by adding a curve, while it could be argued the trajectory suggests that the particular angle is less important than the fact that the line is straight.

![Figure 10: Trajectories drawn by participants and uniform, nonuniform solutions](image)

As the quantitative results show, however, the nonuniform algorithm fared significantly better. We wish to emphasize the conceptual significance of this better performance. From a statistical point of view, we would argue that our corpus is not drawn from the “true distribution” for our task, namely some kind of distribution arising from expert performance. So we may wonder if such a distribution would even be helpful in guiding a search algorithm. This can be framed as a tradeoff: if we hew too closely to the observed behavior of users struggling to complete a task, we could overfit their idiosyncrasies and blind spots, and our algorithm could inherit their limitations. If we ignore their focus entirely and do not guide our search by some knowledge of how programs are written, our search would be too uninformed to perform well. What our succession of models suggests is that there is still enough statistical signal even in the work of novice programmers to guide and improve search.

8 Conclusion

We have formulated program synthesis with visual specification in the frame of classical AI search and have proposed two algorithms. Sampling methods produce improved solutions and scale more readily to larger problem instances. A sampling method informed by the attention and distribution of behaviors observed from novice programmers leads to further improvement. We demonstrated that these algorithms can outperform humans at their own intended tasks for smaller lookahead values, suggesting a practical benefit for a synthesis method that can complete a user’s programs.
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