V_{ud} from Nuclear Mirror Transitions

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Abstract
We review the determination of V_{ud} from nuclear mirror transitions and describe current experimental and theoretical efforts aimed at improving its precision.

1 Introduction
The test of the Conservation of the Vector Current (CVC) in nuclear β decay and its relation to the determination of the Cabibbo angle has been the focus of much experimental and theoretical activity during more than 50 years [1]. Precision measurements in super-allowed pure Fermi transitions and refined theoretical corrections have enabled to extract the value of V_{ud} with a precision of $2 \times 10^{-4}$ [1].

In addition to Fermi transitions, super-allowed transitions between isospin doublets provide another set of nuclear transitions to test the CVC hypothesis and to extract V_{ud} [2]. Such transitions occur between mirror nuclei and are driven by the vector (V) and axial-vector (A) interactions.

This contribution reviews the determination of V_{ud} from nuclear mirror transitions and describes current experimental and theoretical efforts aimed at improving its precision.

2 Mirror transitions
β-decay transitions between isobaric analogue states within an isospin doublet are called “nuclear mirror transitions”. The initial and final states have therefore the same spins and parities (Fig.1). The extent to which the V and A components contribute to the transition is described by the mixing ratio, $\rho = C_A M_{GT}/(C_V M_F)$, where $M_{GT}$ and $M_F$ are respectively the Gamow-Teller (GT) and Fermi nuclear matrix elements and $C_A$ and $C_V$ are the axial and vector effective couplings respectively.
Figure 1: Scheme of a mirror transition indicating the spectroscopic quantities that are required to determine $V_{ud}$ (see text for details).

Except for the neutron and $^3$H, all mirror transitions proceed by $\beta^+$ emission. The number of protons and neutrons in the parent nucleus are related by $Z = N + 1$ so that the nuclei of interest lie on the proton rich side of the nuclear chart.

The master formula that relates $V_{ud}$ to the corrected decay rate in a nuclear transition can be casted in the form \[ F_t = \frac{K}{G_F^2 g_V^2 V_{ud}^2 (1 + \Delta_R^V)} \] (1)

where $K$ is a constant, $G_F$ is the Fermi constant, and $\Delta_R^V$ is a transition-independent radiative correction and whose values can be found elsewhere \[2\]. The interest in writing the relation between $F_t$ and $V_{ud}$ following Eq.(1) is that, if CVC holds, then $g_V = 1$ and the right-hand side is independent of the decay. This is valid for pure Fermi transitions, for nuclear mirror transitions and also for neutron decay. The left-hand side of Eq.(1) depends, in turn, on the decaying system. For a nuclear mirror transition it takes the form \[ F_t^m = f_V t (1 + \delta_R^V) (1 + \delta_{NS} - \delta_C) \left[ 1 + (f_A/f_V) \rho^2 \right] \] (2)

where $f_V$ is the statistical rate function for the vector part \[4\], $t$ is the partial half-life of the transition, $\delta_R^V$ and $\delta_{NS}$ are transition-dependent contributions to the radiative corrections, $\delta_C$ is the isospin symmetry breaking (ISB) correction, and $f_A/f_V$ is the ratio between the statistical rate functions for the axial and the vector parts \[3\].

The spectroscopic quantities required to determine $F_t^m$ are shown in Fig.1. The $Q_{EC}$-values determine the statistical rate functions $f_V$ and $f_A$. The branching ratio, $BR$, and half-life, $t_{1/2}$ determine the partial half-life, $t$.

3 Experimental status

The possibility offered by mirror transitions to determine $V_{ud}$ has motivated new measurements of the spectroscopic quantities. An example of recent progress, which
confirms the potential offered by nuclei for high precision experiments, is provided by measurements of the $^{19}$Ne half-life. Figure 2 shows the values of the half-life adopted in the survey of Ref. [3] as a function of the year of publication. The zoom for year 2012 shows new results from measurements carried out at KVI [5], TRIUMF [6] and GANIL [7], with relative uncertainties smaller than $5 \times 10^{-4}$. The technique used at GANIL, based on the implantation of nuclei in a superconductor, offers additional room for further improvements [7].

![Figure 2: Evolution of the values of the $^{19}$Ne half-life (see text for details).](image)

Preliminary measurements of the $^{21}$Na and $^{37}$K lifetimes have been attempted at KVI but no detailed analysis has yet been completed [5]. A new measurement of the $^{39}$Ca lifetime has been carried out [8]; the result did not significantly improve the precision on the world average value. Measurements of the mass [9] and of the lifetime [10] of $^{31}$S have been performed at Jyväskylä providing new data for the determinations of the $Q_{EC}$-value and the partial half-life.

Although the error on the current value of $V_{ud}$ obtained from mirror transitions is dominated by the error of the mixing ratio [11], it would be extremely useful to improve previous mass values deduced from reaction experiments, by using Penning trap mass spectrometry, in order to dispose of a robust data set.

The mixing ratio can be extracted from measurements of correlation coefficients like e.g. the $\beta-\nu$ angular correlation, $a$, the $\beta$ asymmetry parameter, $A$, or the $\nu$ asymmetry parameter.

The group from LPC in Caen has initiated a dedicated program at GANIL to measure $a$ in several nuclei. The setup uses a transparent Paul trap surrounded by detectors to record the recoil ions and the $\beta$ particles [12]. A measurement of $a$ in $^{35}$Ar decay has recently been completed. The collected data corresponds to a statistical precision of about 0.4-0.5%. The group is currently preparing a measurement of $a$ in $^{19}$Ne decay with the same setup.

Experiments with polarized nuclei can access the parameter $A$. For several mirror
candidates (e.g. $^{21}\text{Na}$, $^{23}\text{Mg}$, $^{29}\text{P}$, $^{35}\text{Ar}$, $^{37}\text{K}$), the mixed transition between the isobaric analogue states is accompanied by a pure GT transition to an excited state in the daughter nucleus, with a branching ratio of few %. Some of these nuclei can be polarized by optical pumping and be implanted in suitable targets. The measurement of $A$ for $\beta$ particles detected in coincidence with the de-exciting $\gamma$ ray provides a measurement of the initial nuclear polarization. Such relative measurements of $A$ will become feasible at the new BECOLA (BEam COoler and LAser spectroscopy) end-station at NSCL [13] and will provide new and improved determinations of $\rho$.

4 Theoretical corrections

The corrected decay rates, Eq. (2), require the inclusion of small nuclear structure and ISB corrections. The first systematic determination of such corrections for mirror nuclei used shell-model Wood-Saxon wave functions [3]. The ISB corrections have recently been calculated for both pure Fermi and mirror transitions, using density functional theory with independent Skyrme interaction [14]. The results of the two calculations are shown in Fig. 3 (left panel) along with the difference between the two calculations (right panel).

Figure 3: Calculated ISB corrections (see text for details).

Among the five parent nuclei ($^{19}\text{Ne}$, $^{21}\text{Na}$, $^{29}\text{P}$, $^{35}\text{Ar}$, $^{37}\text{K}$) considered so far to determine $V_{ud}$ from mirror transitions [2], the largest difference between the two calculations arises for $^{37}\text{K}$ and mounts to 0.7%. This has however a small impact on the average value, $\langle F \rangle^m$, and hence on $V_{ud}$ since the data from this transition has currently a smaller weight due to the precision of its mixing ratio.
5 Summary and Outlook

The value of $V_{ud}$ deduced from mirror transitions is [2]

$$V_{ud} = 0.9719(17)$$

where the precision is dominated by the experimental error on the mixing ratios.

Two new results of the $^{19}$Ne lifetime have recently been published [6, 7] and new systematic calculations of ISB corrections have been performed [14]. Measurements of spectroscopic quantities are required to build up a robust data set while theoretical ISB and nuclear structure corrections are crucial to estimate the associated theoretical uncertainties.

Progress towards a more accurate determination of $V_{ud}$ from mirror transitions definitely relies on improved correlation measurements for the extraction of $\rho$. Current measurements of the $\beta\nu$ angular correlation in a Paul trap and future relative $\beta$-asymmetries measurements with optically pumped low energy beams will provide significant contributions to this end.

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