MHD Stagnation Point Flow with Heat Transfer Past a Porous Sheet along with Viscous Dissipation and Thermal Radiation

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Abstract: In this article, we study the magnetohydrodynamics stagnation point flow for the upper-convected Maxwell fluid with the viscous dissipation and thermal radiation effects using the Cattaneo-Christov heat flux model. The flow equations are reconstructed and the obtained set of partial differential equations is then converted into an arrangement of nonlinear, coupled O.D.E. by utilising some reasonable similarity transformations. After this, the set of O.D.E. is solved by applying shooting method. Graphs and tables describe the behavior of physical parameters.

Index Terms: Maxwell fluid; Viscous dissipation; Thermal radiation; Magnetohydrodynamics; Shooting method.

I. INTRODUCTION

“The point in the flow field where the fluid's velocity is zero is called stagnation point”. The study of viscous, incompressible, fluid past a permeable plate or sheet has great importance in the field of fluid dynamics. During the past few decades, the work on stagnation point flow of an incompressible fluid past a permeable sheet has got significant importance because of its large number of applications in manufacturing industries. Some of the main applications are refrigeration of electrical gadgets by fan, solar receiver, etc. The study of two-dimensional (2D) stagnation point flow was first investigated by Hiemenz [1], whereas for getting the accurate solution, Eckert [2] extended this problem by adding the energy equation. In view of that Mahapatra and Gupta [3], Ishak et al. [4], and Hayat et al. [5] have studied the effects of heat transfer in stagnation point over a permeable plate.

“The study of magnetic properties of electrically conducting fluids is known as Magnetohydrodynamics (MHD)”. The study of MHD fluid flow was first introduced by Swedish Physicist, Alfven [6]. The effect of heat transfer in Magnetohydrodynamics flow of Jeffrey fluid model over a permeable plate is invested by Hayat et al. [7]. Mustafa et al. [8] inspected the Magnetohydrodynamics flow of Maxwell fluid with heat transfer.

The study of flow behaviour and heat transfer generated by means of stretching medium, has plenty of significance in numerous industrialized developments (e.g. in the process of rubber and plastic sheets manufacturing, upgrading the solid materials like crystal, turning fibers etc). The most widely used coolant liquid among them is water. In above cases, flow behaviour and heat transfer investigation is of major importance because final product quality be determined to bulk level on the basis of coefficient of skin friction and heat transfer surface rate. Numerous investigators talked over different traits of stretching flow problem. Some of them are Crane [9], Chaim [10], Liao and Pop [11], Khan and Sanjayanand [12], and Fang et al. [13].

In future, advancement in nano-technology is expected for making unbelievable changes in our lives. A very big number of researchers are working in this area due to its great use in the engineering and its linked areas. In the process of air cleaning, development of microelectronics, safety of nuclear reactors etc, thermophoretic magnetohydrodynamic flow of heat and mass transfer consumes prospective uses. Choi [14] was the first who introduced the idea of “nanofluids” and presented the report on the heat transfer properties of nanofluids. The thorough exposure on thermophoretic flow was examined by Derjaguin and Yalamov [15]. Heat and mass transfer of MHD thermophoretic stream above plane surface was also studied by Issac and Chamka [16]. Thermophoresis effect on aerosol particles was investigated by Tsai [17]. In fluid temperature, no doubt, viscous dissipation produces a considerable ascend. This would happen because of change in kinetic motion of fluid into thermal energy.

Viscous dissipation is unavoidable in case of flow field in high gravitational field. Viscous flow past a nonlinearly stretching sheet was deliberated by Vajravelu [18]. For external natural convention flow over a stretching medium, the impact of viscous dissipation was also studied by Mollendro and Gebhart [19], whereas the impact of viscous dissipation and Joule heating on the forced convection flow with thermal radiation was presented by Duwairi [20].
Our prime objective is, we providing a review study of Shah et al. [21] and extend the flow analysis with viscous dissipation parameters.

II. MATHEMATICAL MODELING

Consider the time independent, incompressible, two-dimensional MHD, laminar, and steady state flow of a fluid past a semi-infinite stretching surface. The geometry of the flow model is given in Figure 1.

![Figure 1. Geometry for the flow under consideration.](image)

Here Cattaneo-Christov heat flux model is under consideration. Along y-axis, a constant magnetic field of strength $B_0$ is applied perpendicular to x-axis. Further it is supposed that the induced magnetic field is negligible. It is supposed that boundary layer approximations are appropriate to the governing equations considered by Renardy for “Maxwell fluid models”. By making use of boundary layer approximations, the arrangement of representing PDEs like continuity, momentum and energy equations can be expressed as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$  \hspace{1cm} (1)

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \lambda_1 \left( u^2 \frac{\partial^2 u}{\partial x^2} + u^2 \frac{\partial^2 u}{\partial y^2} + 2uv \frac{\partial^2 u}{\partial x \partial y} \right) = \nu \nabla^2 u - \frac{\sigma B_0^2 u^2}{\rho}$$  \hspace{1cm} (2)

$$\rho C_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = -\nabla q + \sigma B_0^2 u^2 - \frac{\partial q_r}{\partial y} + \frac{\nu}{\rho C_p} \left( \frac{\partial u}{\partial y} \right)^2$$  \hspace{1cm} (3)

where $u$ and $v$ are the components of velocity along the $x$ and $y$ directions respectively. Moreover, $\lambda_1$ denotes the relaxation time, $\rho$ denotes the fluid’s density, $B_0$ is constant magnetic field, $\sigma$ be the electric conductivity constant, kinematic viscosity is denoted by $\nu$, $C_p$ is the specific heat, fluid temperature is $T$, $q_r$ is the radiative heat flux. According to Christov, we have

$$q + \lambda_2 \left( \frac{\partial q}{\partial t} + V \cdot \nabla q + (\nabla V) \cdot q \right) = -k \nabla T$$  \hspace{1cm} (4)

On abolishing $q$ from Eqs. (3) and (4), we have

$$\left( \frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} \right)$$

$$+ \lambda_2 \left( \frac{u}{\partial x} + \frac{v}{\partial y} \right) \frac{\partial T}{\partial x} + \left( \frac{u}{\partial x} + \frac{v}{\partial y} \right) \frac{\partial T}{\partial y} +$$

$$\left( u^2 \frac{\partial^2 u}{\partial x^2} + v^2 \frac{\partial^2 u}{\partial y^2} + 2uv \frac{\partial^2 u}{\partial x \partial y} \right)$$

$$= \alpha \frac{\partial^2 T}{\partial y^2} + \sigma \frac{B_0^2 u^2}{\rho C_p} - \frac{\partial q_r}{\partial y} + \frac{\nu}{\rho C_p} \left( \frac{\partial u}{\partial y} \right)^2$$  \hspace{1cm} (5)

where $V$ denotes the fluid velocity, $\lambda_2$ is the relaxation time and thermal diffusivity is denoted by $\alpha$. Also, the radiative heat flux $q_r$, by using the Rosseland approximation for radiation, can be written as

$$q_r = \frac{-4\sigma k^4}{3k^4}$$  \hspace{1cm} (6)

where $\sigma$ and $k$ stand for the Stefan-Boltzmann constant and coefficient of mean absorption.

“Expansion of $T^4$ about $T_\infty$ by making use of Taylor’s series is”:

$$T^4 = T_\infty^4 + \frac{472\alpha}{21}(T - T_\infty) + \frac{1278\alpha^2}{21}(T - T_\infty)^2 + \frac{24\alpha}{3!}(T - T_\infty)^3 + \frac{24}{4!}(T - T_\infty)^4$$  \hspace{1cm} (7)

Disregarding the higher order terms,

$$T^4 = T_\infty^4 + 4T_\infty^3(T - T_\infty)$$

$$\Rightarrow \frac{\partial T}{\partial y} = 4T_\infty^3 \frac{\partial T}{\partial y}$$  \hspace{1cm} (8)

Using (8) in (6) and the differentiate w.r.t. y, we get

$$\frac{\partial q_r}{\partial y} = \frac{16\sigma^2 \alpha}{3k^4} \frac{\partial^2 T}{\partial y^2}$$  \hspace{1cm} (9)

The boundary conditions for the above system of PDE are

$$u = U, \quad v = 0, \quad T = T_w(x), \text{ at } y = 0$$

$$u \to 0, \quad T \to T_\infty, \text{ as } y = \infty$$  \hspace{1cm} (10)

III. DIMENSIONLESS FORM OF THE MODEL

Now, we introduce similarity transformations or (dimensionless variables) Shah et al. [21] which are useful in transforming the PDEs Eqs. (1) - (3) into the ODEs along with the boundary conditions Eqs. (8).

$$\eta = \frac{u}{\sqrt{\nu}(y)}, \quad \theta(\eta) = \frac{T - T_\infty}{T_\infty - T_w}$$

$$u = U\eta^{\prime}(\eta), \quad v = -\frac{1}{2} \nu \eta^{\prime} \left( \eta^{\prime} - \eta^2 \right)$$  \hspace{1cm} (11)

where the prime represents derivative w.r.t $\eta$. $T_\infty$ and $T_w$ are the ambient and constant fluid temperature at wall respectively and $\theta$ is the dimensionless temperature. The set of corresponding ODEs is:
The system of first order ODEs along with the boundary conditions are subject to boundary conditions
\[ f'(\eta) = 0, \ f(\eta) = 0 \text{ at } \eta = 0, \ f'(\infty) \to 0, \text{ as } \eta \to \infty, \]
\[ \theta(\eta) = 1 \text{ at } \eta = 0; \ \theta(\eta) \to 0, \text{ as } \eta \to \infty. \]

Let us use the notations
\[ f = y_4, \ \theta = y_4. \]

Further denote
\[ f' = y_4', \ f'' = y_4'', \ \theta' = y_5', \ \theta'' = y_5''. \]

The system of first order ODEs along with the boundary conditions becomes
\[ y_4' = y_2, \quad y_4(0) = 0 \]
\[ y_5' = y_3, \quad y_5(0) = 1. \]

Let \( f'' = \frac{1}{2\beta f} \left( \eta \beta f'^2 f'' + 2 \beta 2ff' f'' - f f'^{11} + 2Mf' \right) \)
\[ \theta'' = \frac{3Pr}{6+8R-3Prf^2} \left( 3yf f' \theta' - f \theta^3 - 2MEn c^2 f'^2 - Ec f'^2 \right) \]

subject to boundary conditions
\[ f'(0) = 1, \ f(0) = 0 \text{ at } \eta = 0, \ f'(\infty) \to 0, \text{ as } \eta \to \infty, \]
\[ \theta(0) = 1 \text{ at } \eta = 0; \ \theta(\infty) \to 0, \text{ as } \eta \to \infty. \]

The system of first order ODEs along with the boundary conditions becomes
\[ y_1' = y_2, \quad y_1(0) = 0 \]
\[ y_2' = y_3, \quad y_2(0) = 1. \]

\[ y_3' = \frac{1}{2\beta f^2} (\eta \beta y_2^2 y_3 + 2 \beta y_1 y_2 y_3 - y_1 y_3 + 2My_2), \]
\[ y_3(0) = s \]
\[ y_4' = y_5, \quad y_4(0) = 1 \]
\[ y_5' = \frac{3Pr}{6+8R-3Prf^2} \left( 3y y_1 y_2 y_3 - y_1 y_5 - 2MEn c y_2^2 - Ec y_2^2 \right), \]
\[ y_5(0) = t \]

For solving above system numerically, we replace the domain \([0, \infty], \text{ by the bounded domain } [0, \eta_{\text{max}}]\) where \( \eta_{\text{max}} \) is some suitable real number. In the above system of equations we have \( y_3(\eta) \) and \( y_5(\eta) \) at \( \eta = 0 \) i.e., \( s \) and \( t \) are missing conditions and are to be chosen such that \( y_3(\eta_{\text{max}}, s, t) \approx 0 \) and \( y_5(\eta_{\text{max}}, s, t) \approx 0 \).

Finally, the choice of \( \eta_{\text{max}} = 16 \) was more than enough for end condition. The convergence criteria is chosen to be successive value agree up to 2 significant digits.

IV. NUMERICAL SOLUTION

As system of Eqs. (12) - (15) with the associated boundary conditions is coupled and nonlinear, so approximate solution cannot be found directly. For this we use the numerical technique i.e., the shooting method along with Adams-Moulton method. By making use of this technique, we convert the system of higher order ODEs into the system of first order ODEs.

The boundary conditions for the governing ODEs are
\[ f(\eta) = 0, \ f'(\eta) = 1, \ \theta(\eta) = 1, \text{ at } \eta = 0, \]
\[ f'(\eta) \to 0, \ \theta(\eta) \to 1, \text{ at } \eta = \infty. \]

In Eqs. (12) - (13), \( \beta \) is the Deborah number, \( Pr \) is the Prandtl number, \( M \) is the magnetic parameter, radiational parameter is \( R, Ec \) is the Eckert number and \( y \) is the non-dimensional thermal relaxation time parameter. Some important dimensionless parameters are formulated as
\[ \beta = \frac{\lambda_1}{2x}, \ Pr = \frac{v}{\alpha}, \ M = \frac{\sigma \beta z}{\rho b}, \ R = \frac{4\sigma \gamma \beta z}{kk^2}, \]
\[ Ec = \frac{u^2}{C_p(T_{\infty} - T_{\infty})} \text{ and } y = \frac{\lambda_1 d}{2x}. \]
parameter $R$ on dimensionless temperature $\theta(\eta)$ is represented in Figure 5. In this graph it is observed that on increasing the value of radiational parameter $R$, temperature profile $\theta(\eta)$ also increases. So, the rate of heat transfer decreases with increase in radiational parameter $R$, and because of which temperature profile increases. In Figure 6, the influence of non-dimensional thermal relaxation time parameter $\gamma$ on temperature profile $\theta(\eta)$ is shown. This graph represents that on increasing the non-dimensional thermal relaxation time parameter $\gamma$, value of temperature profile $\theta(\eta)$ decreases, because of this fact that when non dimensional thermal relaxation time parameter increases results decreases in time of deformation which causes the decrease in temperature of fluid. Figure 7 shows the influence of Deborah number $\beta$ on velocity profile $f'(\eta)$. For the increasing values of Deborah number $\beta$, velocity increases near the plate while in the rest portion of the boundary layer it diminishes for expanding $\beta$. From Figure 8, it can be seen that by the increase in Deborah number $\beta$, temperature profile $\theta(\eta)$ increases. Figure 9 illustrates the difference of temperature $\theta(\eta)$ for different values of the Prandtl number $Pr$. It is perceived that the temperature decreases, for the increasing values of Prandtl number. Decrease in thermal boundary layer comes across when $Pr$ is larger and decrease in the thermal diffusivity causes rise in the Prandtl number. In this way increment in $Pr$ diminishes diffusivity and the variety in thermal characteristics increments.

### Table I.

| $Pr$  | $\gamma$ | $\beta$ | $M$ | $Ec$ | $R$  | $-\theta'(0)$ |
|-------|-----------|----------|-----|------|------|----------------|
| 0.72  | 0.5       | 0.5      | 0.1 | 0.1  | 0.23 | 0.20963440     |
| 0.3   |           |          |     |      |      | 0.10691930     |
| 0.5   |           |          |     |      |      | 0.15225160     |
| 0.7   |           |          |     |      |      | 0.20440960     |
|       | 0.2       |          |     |      |      | 0.21996200     |
|       | 0.3       |          |     |      |      | 0.21646850     |
|       | 0.4       |          |     |      |      | 0.21303900     |
|       |           | 0.2      |     |      |      | 0.22867970     |
|       |           | 0.5      |     |      |      | 0.20963430     |
|       |           | 0.7      |     |      |      | 0.20664860     |
|       |           |          |     | 0.3  |      | 0.17987930     |
|       |           |          |     | 0.5  |      | 0.15613890     |
|       |           |          |     | 0.7  |      | 0.13709520     |
|       |           |          |     | 0.3  |      | 0.13016200     |
|       |           |          |     | 0.9  |      | 0.05068963     |
|       |           |          |     | 1.1  |      | 0.01095345     |
|       |           |          |     | 0.3  |      | 0.19704990     |
|       |           |          |     | 0.7  |      | 0.14891000     |
|       |           |          |     | 1.8  |      | 0.10241040     |

### Table II.

Comparison of $-f''(\alpha)$ when $Pr = 0.72$, $\gamma = 0.5$, $Ec = 0.1$ and $R = 0.1$.

| $Pr$  | $\gamma$ | $\beta$ | $M$ | $Ec$ | $R$  | $-f''(\alpha)$ |
|-------|-----------|----------|-----|------|------|----------------|
| 0.72  | 0.5       | 0.2      | 0.1 | 0.1  | 0.23 | Present Value  |
|       |           |          |     | 0.5  |      | 0.51593330    |
|       |           |          |     | 0.7  |      | 0.48199610    |
|       | 0.1       |          |     |      |      | 0.45181500    |
|       | 0.3       |          |     |      |      | 0.48199610    |
|       | 0.5       |          |     |      |      | 0.4822495     |
|       | 0.7       |          |     |      |      | 0.4822495     |
|       |           | 0.1      |     |      |      | 0.64494780    |
|       |           | 0.3      |     |      |      | 0.6450524     |
|       |           | 0.5      |     |      |      | 0.78028870    |
|       |           | 0.7      |     |      |      | 0.89726330    |
|       |           |          |     | 0.5169288 | | 0.5169288     |
|       |           |          |     | 0.4822495 | | 0.4822495     |
|       |           |          |     | 0.4822495 | | 0.4822495     |
|       |           |          |     | 0.6450524 | | 0.6450524     |
|       |           |          |     | 0.7803249 | | 0.7803249     |
|       |           |          |     | 0.8972758 | | 0.8972758     |
Figure 2. Dimensionless Velocity vs $M = 0.1, 0.3, 0.5, 0.7$

Figure 3. Dimensionless Temperature vs $M = 0.1, 0.3, 0.5, 0.7$

Figure 4. Dimensionless Temperature vs $Ec = 1, 3, 5, 7$

Figure 5. Dimensionless Temperature vs $R = 0, 0.3, 0.5, 0.7$
Figure 6. Dimensionless Temperature vs $\gamma$

Figure 7. Dimensionless Velocity vs $\beta$

Figure 8. Dimensionless Temperature vs $\beta$

Figure 9. Dimensionless Temperature vs $Pr$
VI. CONCLUSIONS

Conclusions which are obtained:

- Because of strong Magnetic parameter $M$ it causes diminish in velocity and increment in temperature.

- Increase in Deborah number $\beta$ temperature increases, while the velocity decreases in the horizontal direction.

- Temperature profile rises while extending the radiation parameter and a same effect of Eckert number is seen on the temperature field.

- On temperature profile Prandtl number has decreasing effects.

- Velocity field $f'$ decreases for increasing values of $\beta$.

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