Inductance Calculation of Multilayer Circular Printed Spiral Coils

Tao Liu, Zhiqiang Wei*, Haokun Chi, Bo Yin
College of Information Science and Engineering, Ocean University of China, Qingdao, China

*Corresponding author e-mail: weizhiqiangouc@126.com

Abstract. In the past few years, implanted biomedical devices have been widely used in healthy and medical applications and wireless power transfer (WPT) techniques used in these devices have become a research hot topic. The printed spiral coils (PSCs), which have characteristics of small volume, low cost, high precision and high-integration, are better suited for WPT system. It is noteworthy that mutual inductance value of PSCs is a significant parameter which can influence the transfer efficiency of WPT system. Considering the geometry of multilayer circular PSCs and the distance between two adjacent layers, the previous methods are not suitable for calculating the mutual inductance of multilayer circular PSCs. In this study, an approximate calculation formula of mutual inductance between two single-layer circular PSCs is presented based on the existing research. On this basis, the self-inductance values of double-layer circular PSCs are calculated by MATLAB, and the calculation results are verified by the ANSYS Q3D simulation and measurement results.

1. Introduction
In the past few years, implanted biomedical devices have been widely used in healthy and medical applications and wireless power transfer (WPT) techniques used in these devices have been become a research hot topic [1-3]. The printed spiral coils (PSCs), which have characteristics of small volume, low cost, high precision and high-integration, are better suited for WPT system [4]. It is noteworthy that the inductance value of PSCs is a significant parameter which can influence the transfer efficiency of WPT system. Tavakkoli proposed an approximate calculation of the mutual inductance for two different shapes of printed spiral coils. Compared with Neumann’s method, using this formula could reduce computational effort [5]. Cheng proposed the inductance calculation of rectangular spiral coil, which is simpler than the method using the greenhouse [6]. Su presented a formula to calculate mutual inductance generated by two flat circular coils [7]. Khan proposed a magnetically coupled four-coil resonators based on WPT system and presented a simplified formula to calculate the mutual inductance between two coils [8]. Considering the geometry of circular PSCs and the distance between two adjacent layers, the previous methods are not suitable for calculating mutual inductance of multilayer circular PSCs. In this study, an approximate calculation formula of the mutual inductance between two single-layer circular PSCs is presented based on the existing research. On this basis, the self-inductance values of double-layer circular PSCs are calculated by MATLAB, and the calculation results are verified by the ANSYS Q3D simulation and measurement results.
2. Theoretical inductance calculation of multilayer circular PSCs

2.1. Inductance calculation of single-layer circular PSC

The inductance of single-layer circular PSC depends on its own structure. Fig.1 shows the geometry of the single-layer circular PSC. In Fig.1, \( d_{\text{in}} \) and \( d_{\text{out}} \) are the inside and outside diameter of the coil, respectively. \( n \) represents the turn number of the coil, \( w \) denotes the width of the copper track. \( s \) is the gap between two adjacent copper tracks. Equation (1) expresses the simplified inductance of the single-layer circular PSC [9]:

\[
L = \frac{\mu_0 \cdot n^2 \cdot d_{\text{avg}} \cdot C_1}{2} \left( \frac{\ln \left( \frac{C_2}{\sigma} \right)}{\sigma} + C_3 \cdot \sigma + C_4 \cdot \sigma^2 \right)
\]

Where \( \mu_0 \) is the magnetic permeability of free space, \( d_{\text{avg}} \) denotes the average diameter of the coil and \( d_{\text{avg}} = (d_{\text{out}} + d_{\text{in}})/2 \). \( \sigma \) represents the fill ratio of the coil, \( \sigma = (d_{\text{out}} - d_{\text{in}})/(d_{\text{in}} + d_{\text{out}}) \). \( C_i \) are the layout dependent coefficient based on the geometry of the coil, for a circle coil, \( C_1 = 1.0 \), \( C_2 = 2.45 \), \( C_3 = 0 \), \( C_4 = 0.2 \).

![Figure 1. Geometry of single-layer circular PSC.](image)

2.2. Inductance calculation of double-layer circular PSC

When multiple inductors are connected in series, multiple series inductors can be equivalent to one inductor. Fig.2 shows the equivalent circuit model of two single-layer circular PSCs in series.

![Figure 2. Equivalent circuit model of two single-layer circular PSCs in series.](image)

According to the Kirchhoff Voltage Laws, there is:

\[
u = u_{L1} + u_{12} + u_{L2} + u_{21} \]

\[
u = L_1 \frac{di}{dt} + M \frac{di}{dt} + L_2 \frac{di}{dt} + M \frac{di}{dt} \]

\[
u = (L_1 + L_2 + 2M) \frac{di}{dt} \]

\[
L' = L_1 + L_2 + 2M
\]

(2)
$L_1, L_2$ represent the inductance of two single-layer circular PSCs, respectively. $L'$ denotes the equivalent inductance of the two coils in series. $u(t)$ represents the voltage across the coils. $u_{L_1}(t)$ and $u_{L_2}(t)$ are the induced voltage generated by $L_1$ and $L_2$, respectively. $u_{12}$ and $u_{21}$ denote the induced voltage produced by mutual induction between $L_1$ and $L_2$. $M$ is the mutual inductance generated by $L_1$ and $L_2$. Considering that the two single-layer circular PSCs are the same, then $L_1 = L_2 = L$, the inductance of double-layer circular PSC can be expressed as:

$$L' = 2L + 2M$$

The double-layer circular PSC is assumed to be composed of a group of coaxial filamentary coils that were connected in series, the tracks of the coil are assumed as constant current carrying filamentary coils [10]. Fig.3 shows the structure of the double-layer circular PSC. $n$ is the turn number of the double-layer circular PSC. $R_i$ and $R_j$ represent the average radius of every filamentary coil, respectively. $d$ denotes the distance between two single-layer circular PSCs. Fig.4 shows the configuration of two coaxial filamentary coils.

![Figure 3. Structure of double-layer circular PSC.](image1)

![Figure 4. Configuration of two coaxial filamentary coils.](image2)

For the double-layer circular PSC, the mutual inductance $M$ can be approximated as accumulation of mutual inductance generated by all coaxial filamentary coils. According to the Neumann’s equation, the total mutual inductance is given by:
\[ M = \sum_{j=1}^{n} \sum_{i=1}^{n} M_{ij} \text{and } M_{ij} = \frac{\mu_0}{4\pi} \int_{l_i} \int_{l_j} \frac{dl_i \cdot dl_j}{r_{ij}} \]  

(4)

Where \( M_{ij} \) is the mutual inductance generated by two coaxial filamentary coils. \( dl_i \) and \( dl_j \) are differential length elements of the two coils, and \( dl_i = R_i \left( -\sin \theta a_i + \cos \theta a_i \right) d\theta \), \( dl_j = R_j \left( -\sin \phi a_j + \cos \phi a_j \right) d\phi \). \( r_{ij} \) represents the distance between two arbitrary points on the two filamentary coils, \( r_{ij} = \sqrt{R_i^2 + R_j^2 + d^2 - 2R_iR_j \cos(\phi - \theta)} \). Thus, the expression of the mutual inductance is written:

\[
M_{ij} = \frac{\mu_0}{4\pi} \int_{\phi=0}^{2\pi} \left\{ \int_{\theta=0}^{2\pi} \frac{R_iR_j \left( \cos(\phi - \theta) \right) d\phi d\theta}{\sqrt{R_i^2 + R_j^2 + d^2 - 2R_iR_j \cos(\phi - \theta)}} \right\} d\phi
\]

\[
= \frac{\mu_0}{4\pi} \int_{\phi=0}^{2\pi} \left\{ \int_{\theta=0}^{2\pi} \frac{R_iR_j \left( \cos \theta - \phi \right) d\theta}{\sqrt{R_i^2 + R_j^2 + d^2 - 2R_iR_j \cos( \cos \theta - \phi )}} \right\} d\phi
\]

(5)

We introduce a variable \( \vartheta = \theta - \phi \), equation (5) can be simplified as:

\[
M_{ij} = \frac{\mu_0}{4\pi} \int_{\vartheta=0}^{2\pi} \left\{ \int_{\vartheta=0}^{2\pi} \frac{R_iR_j \cos \vartheta d\vartheta}{\sqrt{R_i^2 + R_j^2 + d^2 - 2R_iR_j \cos \vartheta}} \right\} d\vartheta
\]

\[
= \frac{\mu_0}{4\pi} \int_{\vartheta=0}^{2\pi} \left\{ \int_{\vartheta=0}^{2\pi} \frac{R_iR_j \cos \vartheta d\vartheta}{\sqrt{R_i^2 + R_j^2 + d^2 - 2R_iR_j \cos \vartheta}} \right\} d\vartheta
\]

\[
= \frac{\mu_0 R_iR_j}{2} \int_{\vartheta=0}^{2\pi} \frac{\cos \vartheta}{\sqrt{R_i^2 + R_j^2 + d^2 - 2R_iR_j \cos \vartheta}} d\vartheta
\]

(6)

Equation (6) shows that the mutual inductance \( M_{ij} \) is decided by the every average radius of the two filamentary coils and the distance between them. The closer the distance and the larger the average radius of the coil, the bigger the mutual inductance.

The coupling coefficient \( k \) represents the coupling strength between two coils. The value of the coupling coefficient \( k \) is determined by the structure of the two coils and the distance between them.

\[
k = \frac{M}{\sqrt{L_1 \cdot L_2}}
\]

(7)

Considering the geometric parameters of the two single-layer circular PSCs are the same, then \( L_1 = L_2 = L \), thus the coupling strength is expressed as:

\[
k = \frac{M}{L}
\]

(8)

It can be seen from equation (8), \( 0 \leq k \leq 1 \). The bigger the value of \( k \), which indicates the tighter the coupling between two coils. \( k = 0 \), \( k = 1 \) are called no-coupling and full coupling, respectively.

The inductance of the double-layer circular PSC depends on its structure. The geometrical parameters of double-layer circular PSCs are shown in Tab.1. Fig.5 and Fig.6 show the variation of the
mutual inductance, coupling coefficient and total inductance with the parameter changes of double-layer circular PSCs.

Table 1. Geometrical parameters of double-layer circular PSCs.

| Inside diameter (mm) | Turns n | Width of the copper track (mm) | Gap (mm) | Distance (mm) |
|---------------------|---------|--------------------------------|----------|---------------|
| 6.8                 | 1-20    | 0.25, 0.5                     | 0.25, 0.5| 1, 4, 10      |

Figure 5. The variation of mutual inductance, coupling coefficient and total inductance with the parameter changes of double-layer circular PSCs.

![Figure 5](image)

Figure 6. The variation of mutual inductance, coupling coefficient and total inductance with the parameter changes of double-layer circular PSCs.

![Figure 6](image)

Fig.5 and Fig.6 show that the mutual inductance, the coupling coefficient and total inductance are the maximum as the distance \( d = 1 \text{ mm} \). Comparing Fig.5 with Fig.6, when the distance \( d \) and the number of turns \( n \) are kept constant, the parameter \( w \) is set from 0.25 mm to 0.5 mm and parameter \( s \) is set from 0.25 mm to 1 mm, the mutual inductance \( M \) and the coupling coefficient \( k \) increase, the total inductance \( L_{\text{tot}} \) decreases first and increases afterwards. When the outside diameter \( d_{\text{out}} = 20 \text{ mm} \), although the mutual inductance \( M \) and the coupling coefficient \( k \) increase, the total inductance \( L_{\text{tot}} \) decrease dramatically. The analysis indicates that we can increase the inductance of the double-layer circular PSCs by reducing the inside diameter, the gap, the width of the copper track and the distance.
3. Simulation and experiment
We designed 11 circular PSCs with different structures by Altium designer and fabricated them on FR4 printed circuit boards (PCB). The detailed parameters of 11 circular PSCs are shown in Tab.2. Fig.7 shows the 11 circular PSCs prototypes. We get every the self-inductance value of the 11 coils by MATLAB calculation, ANSYS Q3D simulation and actual measurement, respectively. The results are shown in Fig.8.

![Figure 7. Circular PSCs prototypes](image)

| Coil number | Inside diameter (mm) | Outside diameter (mm) | Turn number | Width of the copper track (mm) | Gap (mm) | Distance (mm) | Layer |
|-------------|----------------------|-----------------------|-------------|-------------------------------|----------|---------------|-------|
| 1           | 6.8                  | 25                    | 18          | 0.25                          | 0.25     | 1             | 2     |
| 2           | 9.8                  | 25                    | 15          | 0.25                          | 0.25     | 1             | 2     |
| 3           | 6.8                  | 22                    | 15          | 0.25                          | 0.25     | 1             | 2     |
| 4           | 13                   | 25                    | 12          | 0.25                          | 0.25     | 1             | 2     |
| 5           | 6.8                  | 25                    | 12          | 0.25                          | 0.5      | 1             | 2     |
| 6           | 11.4                 | 25                    | 9           | 0.25                          | 0.5      | 1             | 2     |
| 7           | 6.8                  | 16                    | 9           | 0.25                          | 0.25     | 1             | 2     |
| 8           | 17                   | 34                    | 6           | 0.5                           | 1        | /             | 1     |
| 9           | 17                   | 34                    | 6           | 0.5                           | 1        | /             | 1     |
| 10          | 10                   | 34                    | 8           | 0.5                           | 1        | /             | 1     |
| 11          | 19                   | 34                    | 4           | 0.5                           | 1.5      | /             | 1     |

![Figure 8. The self-inductance values of calculation, simulation and measurement.](image)
Fig. 8 shows that the inductance of ANSYS Q3D simulation and measurement results are basically in accordance with MATLAB calculation. However, the measurement value is lightly greater than the theoretical value and the simulated value. This may be due to the fact that the via connected two layers PCB increases the inductance of the double-layer circular PSC.

Fig. 9 shows that a conventional WPT system was created to light a light emitting diode lamp at a distance of 12mm. Two double-layer circular PSCs (Coil 5 in Tab. 2, \( d_i = 6.8\text{mm} \), \( d_{out} = 22\text{mm} \), \( n = 22 \), \( w = 0.25\text{mm} \), \( s = 0.25\text{mm} \), \( d = 1\text{mm} \)) were used as transmitter and receiver of the WPT system. The resonant frequency of the system was set at 3 MHz, and the power was set at 2 W. Fig. 10 shows the magnetic field distribution of the WPT system, which indicates that most of the energy is effectively transferred.

4. Conclusion
An approximate calculation formula of the mutual inductance between two single-layer circular PSCs was proposed. On this basis, the self-inductance values of double-layer circular PSCs are calculated by MATLAB, and the calculation results are verified by the ANSYS Q3D simulation and measurement results. Our next work will to consider the effect of the via on the mutual inductance of double layer circular PSCs.

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References

[1] P. Si, A. P. Hu, S. Malpas, and D. Budgett, A frequency control method for regulating wireless power to implantable devices, IEEE Trans. Biomed. Circuits Syst., vol. 2, no. 1, Mar. 2008, pp. 22–29.

[2] Ghovanloo, Maysam, Najafi, Khalil, A wireless implantable multichannel microstimulating system-on-a-chip with modular architecture, IEEE Transactions on Neural Systems and Rehabilitation Engineering. Sept. 2007, vol. 15, pp. 449–457.

[3] R. Wu, S. Raju, M. Chan, J. K. O. Sin, and C. P. Yue, Silicon-embedded coil for high-efficiency wireless power transfer to implantable biomedical ICs, IEEE Electron. Device Lett., vol. 34, no. 1, Jan. 2013, pp. 09–11.

[4] Uei-Ming Jow, Maysam Ghovanloo, Design and Optimization of Printed Spiral Coils for Efficient Inductive Power Transmission, IEEE Transactions on Biomedical Circuits and Systems, vol. 1, 2007, pp. 70-73.

[5] Tavakkoli, Hadi, Abbaspour-Sani, Ebrahim, Khalilzadegan, Amin, Analytical study of mutual inductance of hexagonal and octagonal Spiral planer coils, Sensors and Actuators A: Physical, vol. 247, Aug. 2016, pp. 53-6.

[6] Yuhua Cheng and Yaming Shu, A New Analytical Calculation of the Mutual Inductance of the Coaxial Spiral Rectangular Coils, IEEE Transaction on Magnetics, vol. 50, no. 4, Apr. 2014

[7] Y. P. Su, Xun Liu, and S. Y. Ron Hui, Mutual Inductance Calculation of Movable Planar Coils on Parallel Surfaces, IEEE Transactions on Power Electronics, vol. 24, no. 4, Apr. 2009

[8] Sadeque Reza Khan and GoangSeog Choi, Analysis and Optimization of Four-Coil Planar Magnetically Coupled Printed Spiral Resonators, SENSORS, vo. 16, no.1219, AUG 2016

[9] S. S. Mohan, M.del Mar Hershenson, S. P. Boyd, and T. H. Lee, Simple Accurate Expressions for Planar Spiral Inductances, IEEE Journal of Solid-state Circuits, vol.34, no. 10, Oct.1999, pp. 1419-1424.

[10] S. I.Babic, F.Sirois, C.Akyel, and C.Girardi, Mutual inductance calculation between circular filaments arbitrarily positioned in space: Alternative Grover’s formulas, IEEE Transactions on Magnetics, vol. 46, no. 9, Sep. 2010, pp. 3591–3600.