Numerical simulation of high-speed A\textsuperscript{III}B\textsuperscript{V} photodetectors within drift-diffusion approximation

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Abstract. This paper is focused on a fundamental scientific and technical problem of research and development of high-performance on-chip interconnections for next-generation integrated devices of micro-, nano- and microwave electronics. Previously we proposed an injection laser with functionally integrated optical modulator and double heterostructure, which can be used as an efficient source of light for on-chip optical interconnections. To detect short laser pulses generated by the laser-modulator, a high-speed and technologically compatible photodetector is required. In this paper we develop the non-stationary drift-diffusion model of transient processes in high-speed photodetectors, algorithms of its numerical implementation and applied software intended for one- and two-dimensional simulation of photosensitive optoelectronic devices with various structures. According to the obtained results of drift-diffusion numerical simulation, it is reasonable to research the methods of carrier lifetime reduction in the active regions of photodetectors for on-chip optical interconnections.

1. Introduction

Nowadays the improvement of interconnections’ characteristics in integrated circuits (ICs) is one of the most pressing problems in the field of integrated electronics. State-of-the-art ICs use metal conductors for the inter-element commutation. However, traditional methods of interconnecting are gradually ceasing to meet the ever-increasing requirements of developers for several important characteristics such as response time, bandwidth capacity, reliability, energy efficiency, adaptability to streamlined manufacture and others. It is expected that in the immediate future the unsatisfactory performance of on-chip metal interconnections will limit the further scaling of transistors in ultra-large-scale multi-core ICs and will stop the sustainable growth of their integration density [1].

The problem of interconnections’ improvement in ICs can be solved in different ways [2]–[4]. Some approaches are aimed at the optimization and modernization of traditional commutation technologies and constructions (e.g. the application of new conductive alloys or circuit methods, chip cooling or special design concepts). But the most promising methods assumes the implementation of fundamentally new types of on-chip interconnections based on advanced physical principles (e.g. radiofrequency and optical lines, carbon nanotubes, graphene or nanoconductors).

Optoelectronic approach considers the possibility of constructive and technological integration of optical interconnections with electronic elements of ICs. High performance of integrated optoelectronic systems provides multiple advantages of this approach over other solutions of the
problem [5], [6]. Nevertheless, a lot of theoretical and practical problems in the field of on-chip optical interconnecting remain unsolved. Basic optoelectronic part of optical interconnection includes a laser, a high-speed optical modulator, an integrated waveguide and a photodetector. These devices should be parametrically and technologically matched with each other and with various electronic components of IC. Silicon is the most feasible material for on-chip optical interconnections due to its manufacturability and small losses, but the creation of effective silicon laser seems to be very problematic. That is why the development of novel $\text{A}^{\text{III}}\text{B}^{\text{V}}$ (A and B refer to group III and V semiconductors, respectively) on-chip optoelectronic devices and elaboration of techniques for $\text{A}^{\text{III}}\text{B}^{\text{V}}$ and silicon integration are the relevant research directions [7]–[9].

Previously we proposed an advanced injection laser with double $\text{A}^{\text{III}}\text{B}^{\text{V}}$ nanoheterostructure and functionally integrated modulator of optical radiation [10]–[12]. The laser-modulator applies the principle of controlled spatial relocation of carrier density peaks within quantum regions of conduction and valence bands for the generation of amplitude- or frequency-modulated optical signals. Numerical simulation showed that the device performance is not limited by relatively slow transients in power circuit. According to the simulation results, the maximum modulation frequency of the optical radiation generated by the laser-modulator is determined by subpicosecond processes of carrier relocation between quantum wells in the control heterostructure.

In paper [13] we introduced the concept of high-speed on-chip optical interconnections based on the lasers-modulators. However, the realization of such integrated optoelectronic systems requires a technologically compatible photodetector with subpicosecond response time. The aim of this paper is to develop the model of charge carrier transport and accumulation in high-speed $\text{A}^{\text{III}}\text{B}^{\text{V}}$ photosensitive devices and the appropriate simulation methods and tools. To implement the model, we chose the drift-diffusion approximation of the semiclassical approach to simulation of semiconductor devices [14].

2. Drift-diffusion model of photodetectors

The high-speed performance of semiconductor photodetectors depends on the following key factors [15]:

- the time of charge carriers’ transit through the device structure;
- the resistor-capacitor (RC) time constant.

The transit time of photogenerated charge carriers includes the fast component caused by their drift in a depletion region and the slow one determined by the carrier diffusion in neutral regions. The RC constant of photodetectors is defined by the equivalent capacity of its p-n junctions, internal and load resistances. The ways of the transit time reduction is of our main interest in this paper. The RC time shortening, by contrast, represents a technical problem which can be solved by circuit methods.

The processes of charge carrier transport and accumulation in semiconductor devices are described by physical and topological models [16]. The electrophysical properties of semiconductor materials and topological dimensions of the device regions are the initial parameters of these models. Physical and topological simulation can be performed within the quantum-mechanical or semiclassical approach [14], [16]. The fundamental semiclassical models describe electron-hole plasma as elements and in terms of the distribution functions. However, this approach is too detailed and too difficult for the practical simulation of carrier transport in semiconductor devices [17]. In the vast majority of cases, it is sufficient to use the simplified transport equations expressed in terms of the macroscopic physical parameters (e.g. charge carrier concentration, current density and others).

In this research we chose the drift-diffusion approximation of the semiclassical approach for the simulation of transients in high-speed photodetectors. The choice is caused by the following reasons [14], [16], [17]:

- drift-diffusion models take into account two- and three-dimensional character of charge carrier transport, various phenomena of electron and hole generation, recombination and scattering and provide extensions to high-field and quantum-mechanical cases;
- the model has a good balance between the adequacy of simulation results and the implementation complexity, especially in comparison with the hydrodynamic one;
the problem of the A\textsuperscript{III}B\textsuperscript{V} photodetector simulation does not require a thorough consideration of quantum-mechanical effects.

In spite of several limitations, the drift-diffusion model remains one of the most effective tools for the qualitative semiclassical analysis of modern semiconductor devices.

The basic drift-diffusion model consists of the following differential equations [16]–[18]:

\[
\frac{\partial n}{\partial t} = \frac{1}{q} \nabla j_n + G - R_n; \tag{1}
\]

\[
\frac{\partial p}{\partial t} = -\frac{1}{q} \nabla j_p + G - R_p; \tag{2}
\]

\[
j_n = -q \mu_n \left[ n \cdot \nabla (\varphi + V_n) - \varphi_T \cdot \nabla n \right]; \tag{3}
\]

\[
j_p = -q \mu_p \left[ p \cdot \nabla (\varphi - V_p) + \varphi_T \cdot \nabla p \right]; \tag{4}
\]

\[
\nabla (\varepsilon \cdot \nabla \varphi) = -\frac{q}{\varepsilon_0} \cdot (p - n + N_d - N_a), \tag{5}
\]

where \(n, p\) are the electron and hole densities; \(t\) is time; \(q\) is the elementary charge; \(G\) is the generation rate of electron-hole pairs; \(R_n, R_p\) are the recombination rates of electrons and holes; \(j_n, j_p\) are the electron and hole components of current density; \(\mu_n, \mu_p\) are the electron and hole mobilities; \(\varphi\) is the electrostatic potential; \(V_n, V_p\) are the bandgap offsets in the conduction and valence bands of a semiconductor, which are caused by heterojunctions; \(\varphi_T\) is the temperature potential; \(\varepsilon\) is the dielectric permittivity of a semiconductor material; \(\varepsilon_0\) is the vacuum permittivity; \(N_d, N_a\) are the concentrations of ionized donors and acceptors; \(\nabla\) is the del differential operator.

The specification of proper initial conditions for non-stationary drift-diffusion equations (1)–(5) necessitates the numerical solution of the full equation system in the stationary form. In the case of the photodetector simulation, it is reasonable to calculate initial conditions for the instant after the end of supply transient and before the start of illumination.

The operation mode with fixed bias voltage and changeable current density is typical for high-speed semiconductor photodetectors. In this mode the boundary conditions for equations (1)–(5) have the following form [16], [17], [19]:

- at the Schottky and ohmic contacts:

\[
n = \frac{N_{\text{eff}}}{2} + \left[ \left( \frac{N_{\text{eff}}}{2} + n_i^2 \right) \right]^{1/2} \cdot \exp \left( -\frac{\varphi_{\text{Sch}}}{\varphi_T} \right);
\]

\[
p = -\frac{N_{\text{eff}}}{2} + \left[ \left( \frac{N_{\text{eff}}}{2} + n_i^2 \right) \right]^{1/2} \cdot \exp \left( \frac{\varphi_{\text{Sch}}}{\varphi_T} \right);
\]

\[
\varphi = \varphi_T \cdot \ln \left[ \frac{n}{n_i} \right] + U(t) - \varphi_{\text{Sch}} = -\varphi_T \cdot \ln \left[ \frac{p}{n_i} \right] + U(t) - \varphi_{\text{Sch}},
\]

where \(N_{\text{eff}} = N_d - N_a; n_i\) is the intrinsic carrier concentration; \(\varphi_{\text{Sch}}\) is the Schottky barrier height \(\varphi_{\text{Sch}} = 0\) at ohmic contacts; \(U(t)\) is the bias voltage applied to a contact at the moment \(t\);

- at the contact-free boundaries:

\[
\frac{\partial n}{\partial \xi} = 0; \quad \frac{\partial p}{\partial \xi} = 0; \quad \frac{\partial \varphi}{\partial \xi} = 0,
\]

where \(\xi\) is the normal to the surface of a contact.
Equations (6)–(8) were derived under the assumptions of thermodynamic equilibrium and infinite recombination velocity at a contact.

Charge carrier mobilities $\mu_n$ and $\mu_p$ are the essential electrophysical parameters involved in drift-diffusion current density equations (3) and (4). In general, carrier mobility depends on various mechanisms of scattering in semiconductors, and accurate simulation of AlBiB\textsubscript{V} devices requires the experimental measurement of this parameter or the application of a complicated hydrodynamic approach. However, some phenomenological models are suitable for the adequate estimation of electron and hole mobilities within the drift-diffusion approximation. In this research we propose to apply the following mobility equations for the simulation of AlBiB\textsubscript{V} photodetectors:

- the low-field Caughey-Thomas model, which takes into account the non-linear dependences of electron and hole mobilities on the lattice temperature $T$ and the total impurity concentration $N_{sum}$ [14, 20]:

$$
\mu_{c,LF} = \mu_{c,min} + \frac{\mu_{c,min,300} \cdot (T / 300 \text{ K})^\gamma - \mu_{c,min}}{1 + \left[ N_{sum} / N_{c,ref} \right]^{\alpha_c}};
$$

where $c$ is the type of conductivity ($n$ or $p$); $\mu_{c,min}$, $\mu_{c,min,300}$, $\gamma$, $N_{c,ref}$, $\alpha_c$ are the material-dependent fitting parameters;

- the high-field model of hole mobility, which describes the effect of hole drift velocity saturation and considers the effective field $E_p$ incorporating electrostatic field and the gradient of hole density [16, 20]:

$$
\mu_p = \mu_{p,LF} \left[ 1 + \left( \frac{\mu_{p,LF} \cdot E_p^{\beta_p}}{\nu_{p,sat}} \right) \right]^{-1/\beta_p};
$$

$$
E_p = \nabla \phi_p = \nabla \left( \phi - V_p \right) + \frac{1}{p} \cdot \nabla \left( \phi \cdot n \right),
$$

where $\mu_{p,LF}$ is the low-field hole mobility; $\nu_{p,sat}$ is the saturation drift velocity of holes; $\beta_p$ is the fitting parameter; $\phi_p$ is the quasi-Fermi level for holes;

- the high-field model of electron mobility, which addresses the overshoot and saturation of field-velocity curve for AlBiB\textsubscript{V} semiconductors caused by the effect of electron inter-valley transition [20]:

$$
\mu_n = \left( \mu_{n,LF} + \nu_{n,sat} \cdot \frac{E_n^{\beta_n-1}}{E_0^{\beta_n}} \right) \cdot \left( 1 + \frac{E_n^{\beta_n}}{E_0^{\beta_n}} \right)^{-1};
$$

$$
E_n = \nabla \phi_n = \nabla \left( \phi + V_n \right) - \frac{1}{n} \cdot \nabla \left( \phi \cdot n \right),
$$

where $\mu_{n,LF}$ is the low-field electron mobility; $\nu_{n,sat}$ is the saturation drift velocity of electrons; $\beta_n$ is the fitting parameter; $E_n$ is the effective electric field for electrons; $E_0$ is the critical field intensity, at which the electron drift velocity reaches the maximum; $\phi_n$ is the quasi-Fermi level for electrons.

The values of fitting parameters entered into mobility equations (10)–(14) are given in [20] for the binary semiconductor compounds. In the ternary AlBiB\textsubscript{V} materials the resulting mobility can be evaluated as a harmonic mean of the appropriate parameters for binary semiconductors.

The influence of illumination on physical processes in photosensitive devices is taken into account by the setting of generation rate $G$ that appears in continuity equations (1) and (2) of the drift-diffusion model. To improve the quantum efficiency and, as a result, the sensitivity of photodetectors, the integrated resonant cavities are used [21]. If the light propagates along the $\gamma$ axis and transversely...
to the direction of built-in electric field (see figure 1), the rate of bipolar optical generation in resonant-cavity-enhanced (RCE) photodetector is described by the following equations [19], [22]:

\[
n_{ph}(y,t) = \sum_{m=1}^{M} \left[ \frac{P_{in}}{c / n_i} \left( \frac{-2L \left\lfloor \frac{m}{2} \right\rfloor + (-1)^{m+1} \frac{y}{y}}{cE_{ph}HW} \right) \cdot \frac{1 - R_1}{cE_{ph}2} \cdot \left( \frac{m-1}{2} \right)^\alpha \cdot \frac{m}{2} \right] \cdot \ldots \\
\text{...exp}\left( -2 \frac{m}{2} \alpha L + (-1)^m \alpha y \right),
\]

\[
P_{abs}(t) = \frac{E_{ph}HW}{\Delta t} \cdot \left[ 1 - \exp\left( -\alpha \frac{c}{n_i} \frac{\Delta t}{y} \right) \right] \cdot n_{ph}(y,t) \cdot \Delta y; \tag{16}
\]

\[
G(t) = \frac{P_{abs}(t)}{\Delta x \cdot W \cdot \Delta y \cdot E_{ph}}, \tag{17}
\]

where \( n_{ph}(y,t) \) is the photon density at the distance \( y \) and at the moment \( t \); \( M = \left\lceil \frac{e \cdot y}{n_i \cdot L} \right\rceil \) is the multiplicity of photon beam reflection; \( \left\lceil \right\rceil, \left\lfloor \right\rfloor \) are the floor and ceiling functions; \( P_{in}(t) \) is the time-dependent power of incident optical radiation; \( c \) is the speed of light in vacuum; \( n_i \) is the refractive index of the absorption region; \( L, H, W \) are the length (along the \( x \) axis), height (along the \( y \) axis) and width of the resonator; \( E_{ph} \) is the photon energy equal to the bandgap energy of the absorption region; \( \alpha \) is the absorption coefficient of a semiconductor at the wavelength of incident light; \( \Delta x, \Delta y \) are the coordinate grid steps along the \( x \) and \( y \) axes; \( \Delta t \) is the time grid step; \( P_{abs}(t) \) is the optical power absorbed by the whole resonant cavity at the moment \( t \). Two-dimensional model (15)–(17) considers the dynamics of photon propagation in the resonant cavity of a photodetector and the variation of generation rate \( G \) along the \( y \) axis due to the light absorption. After some simplifications this model of optical generation can be used together with the one-dimensional drift-diffusion equation system.

![Figure 1. The RCE p-i-n photodiode: 1 – totally reflecting optical mirror; 2 – half reflecting optical mirror; 3 – photon flux.](image)

The utilization of the following well-known model [23] is the feasible approach to the evaluation of the electron and hole recombination rates in semiconductor photodetectors:

\[
R_n = \frac{n - n_0}{\tau_n}, \tag{18}
\]

\[
R_p = \frac{p - p_0}{\tau_p}, \tag{19}
\]

where \( n_0, p_0 \) are the equilibrium concentrations of electrons and holes; \( \tau_n, \tau_p \) are the electron and hole lifetimes, which include several components connected with the different mechanisms of charge carrier recombination (e.g. the Shockley-Read-Hall, Auger, direct and others). In this research we specify the constant values of the electron and hole lifetimes according to the results of experimental measurements given in [20], [24].
Thus, the non-stationary drift-diffusion model of high-speed $A^{III}B^{V}$ photodetectors for on-chip optical interconnections involves basic equations (1)–(5), initial conditions, boundary conditions (6)–(9) and additional models of charge carrier mobilities (10)–(14) and of generation and recombination rates (15)–(19).

3. Simulation methods and software

To implement the full non-stationary drift-diffusion model in equations (1)–(19), the application of numerical methods is needed. A stable and convergent numerical solution of a system of differential equations requires an efficient discretization scheme and a high-performance computational algorithm. However, the universal approach to the efficient numerical implementation of the drift-diffusion models has not been developed yet. The non-stationary simulation of heterostructure devices is connected with some challenges such as the instability of discretization schemes and algorithms, significant time and resource consumption and inadequacy of simulation results caused by calculation errors. It is quite difficult to deal with the aforementioned challenges due to the features of the drift-diffusion model, variety of parameter values and lack of the appropriate mathematical investigations. Therefore, the development of drift-diffusion simulation techniques is an urgent problem in the field of computational electronics.

In this paper we propose two techniques designed for the numerical simulation of high-speed $A^{III}B^{V}$ photodetectors within the drift-diffusion approximation. Both techniques are based on the finite difference method [25] and use standard factors [14] for the scaling of drift-diffusion equations. The key distinctions of the techniques are considered in table 1.

Table 1. The comparison of finite-difference numerical simulation techniques for the drift-diffusion model of photodetectors.

| Feature                                      | Explicit technique                          | Semi-implicit technique                     |
|----------------------------------------------|---------------------------------------------|---------------------------------------------|
| Variable base                                | Combined\(^a\) variable base                | Slotboom variable base                      |
| Discretization scheme for time derivatives   | Explicit difference scheme [25]             | Semi-implicit difference scheme [25]        |
| Discretization scheme for coordinate derivatives | First-order upwind scheme [27]             | Traditional central differences             |
| Solution method for the system of finite-difference equations | Modified Gummel’s iterative method [16], [17] | Standard Newton’s iterative method [14], [16] |

\(^a\) $\{\varphi, F_n, F_p\}$ for equation (3), (4) and $\{n, p, \varphi\}$ for equations (1), (2) and (5)

\(^b\) $F_n = \exp(-\varphi_n)$, $F_p = \exp(\varphi_p)$, where $\varphi_n$, $\varphi_p$ are the quasi-Fermi levels for electrons and holes

The explicit technique allows for the reduction of computational resources, but requires a shorter time step than the other approach. The semi-implicit technique is characterized by fast quadratic convergence, but requires a lot of computational resources for the processing of large matrices. As a result, we recommend the explicit technique only for the non-stationary two-dimensional simulation. The semi-explicit one is suitable for the calculation of initial conditions and one-dimensional non-stationary simulation.

We developed the specialized software package designed for the one- and two-dimensional numerical simulation of carrier transport and accumulation in high-speed on-chip photosensitive devices based on $A^{III}B^{V}$ heterostructures. The package implements the drift-diffusion model and finite difference simulation techniques discussed above. We wrote the applied program in the Octave programming language and executed it using the GNU Octave software [28]. Octave is a high-level programming tool intended for the numerical solution of computational problems. In comparison with
the well-known MATLAB system, Octave has the similar syntax, capabilities and performance, but allows for free changing, copying and using. Scilab, Spyder and other non-proprietary environments for scientific computing have more specific syntax and worse benchmarking results than Octave [29].

The proposed modelling aids provide the simulation of semiconductor photodetectors with various electrophysical, constructive and technological parameters operating at different control actions.

4. Simulation results and discussion

Figures 2 and 3 demonstrate the results of the drift-diffusion numerical simulation of p-i-n photodiode with InP/In$_{0.53}$Ga$_{0.47}$As heterojunctions, 200-nm active region and two 100-nm heavily-doped regions. The background concentration of impurities is $10^{14}$ cm$^{-3}$, the donor density in n-region and acceptor one in p-region amount to $10^{18}$ cm$^{-3}$. The p-i-n photodetector has the integrated resonant cavity with the length and width of 1000 nm and height of 200 nm. The reflection coefficients of the cavity mirrors are 0.45 and 0.99. Initial conditions are calculated for the biased photodiode at the reverse voltage of 0.5 V. The illumination by 0.2-mW rectangular laser pulse starts at the reference time and continues until the instant of 2 ps, and the full simulation time is equal to 4 ps.

According to the simulation results, the bipolar optical generation caused by the illumination of the active region by the short laser pulse leads to the growth of electron and hole concentrations (figure 3(a)) and change in quasi-Fermi levels (figure 2(a) and (b)). In the case being considered the input power of incident optical radiation is quite small, and the distribution of electrostatic field intensity (figure 3(b)) remains approximately unchanged during the device operation.

![Figure 2](image1.png)

**Figure 2.** The results of the drift-diffusion numerical simulation of p-i-n photodetector with InP/InGaAs heterojunctions: a, b – the energy band diagrams (solid lines) and quasi-Fermi levels (dashed lines) before (a) and during (b) the structure illumination by 0.2-mW rectangular laser pulse.

![Figure 3](image2.png)

**Figure 3.** The results of the drift-diffusion numerical simulation of p-i-n photodetector with InP/InGaAs heterojunctions: a – the spatial distributions of electron and hole densities in the equilibrium state ($n_0, p_0$) and during the illumination ($n_{opt}, p_{opt}$); b – the spatial distribution of the electrostatic field intensity.
We researched multiple structures of p-i-n, uni-travelling-carrier (UTC) and Schottky-barrier photodiodes using the developed drift-diffusion model and simulation tools. According to the obtained results of numerical simulation, conventional photosensitive devices are characterized by the quite durable edge times in the order of several picoseconds. This outcome is caused by the following key factors:

- the negative influence of high-field transport effects on electron and hole mobilities;
- the slow recombination rate of photogenerated charge carriers.

The considerable lessening of the first factor seems to be very problematic in AIII BV semiconductors. By contrast, the reduction of carrier lifetime in photodetectors can be achieved with the help of advanced design methods.

Figure 4 shows the shapes of photocurrent pulses in the p-i-n photodetector with AIII BV heterojunctions at various lifetimes of electrons and holes. It was supposed that the specified parameters for electrons and holes are equal, but recombination rates were calculated independently through the corresponding carrier densities. According to the curves, the lifetime of charge carriers has significant effect on response time and sensitivity of the photodetector. On the one hand, smaller lifetime provide the fast return of carrier concentration to equilibrium value after the disappearance of illumination. On the other hand, short lifetime of electrons and holes affects the maximum value of photocurrent and, as a result, the device sensitivity.

![Figure 4. The comparable photoresponse characteristics of p-i-n photodiode for different lifetimes of charge carriers: in terms of A/cm² (a) and in the per-unit basis (b).](image)

It is worth noting that the lifetime of charge carriers in structures of typical photodetectors exceeds the values shown in figure 4 and amounts to 1 ns and more [15]. Therefore, the development of new methods aimed at the reduction of carrier lifetime in photosensitive AIII BV heterostructures is an urgent research direction in the field of optoelectronics engineering.

5. Conclusion
In this paper we developed the non-stationary drift-diffusion model of high-speed AIII BV photodetectors for on-chip optical interconnections in ICs, the finite difference techniques of its numerical implementation and applied software in the GNU Octave program. According to the results of numerical simulation, the reduction of charge carrier lifetime in photodetectors’ active regions allows for the improvement of the high-speed performance of integrated photosensitive devices.

6. References
[1] Ceyhan A and Naemi A 2013 IEEE Trans. Electron Devices 60 374-382
[2] Belkin M and Sigov A 2012 Nanoindustry 31 8-14
[3] Das D and Rahaman H 2014 Carbon Nanotube and Graphene Nanoribbon Interconnects (Boca Raton: CRC Press)
[4] Rakheja S and Naemi A 2011 IEEE Trans. Electron Devices 58 1319-1328
[5] Miller D A B 2010 Appl. Opt. 49 59-70
[6] Stucchi M, Cosemans S, Campenhout J V and Beyer G 2013 Microelectron. Eng. 112 84-91
[7] Li N, Liu K, Sgorr V J and Sadana D K 2015 Sci. Rep. 5 14067
[8] Spuesens T, Bauwelinck J, Regreny P and van Thourhout D 2013 IEEE Photonic Tech. L. 25 1332-1335
[9] Ohira K, Kobayashi K, Iizuka N, Yoshida H, Ezaki M, Uemura H, Kojima A, Nakamura K, Furuyama H and Shibata H 2010 Opt. Express 18 1544-1547
[10] Konoplev B G, Ryndin E A and Denisenko M A 2015 Russian Microelectronics 44 190-196
[11] Konoplev B G, Ryndin E A and Denisenko M A 2014 Proc. of SPIE 9440 944014
[12] Konoplev B G, Ryndin E A and Denisenko M A 2015 Tech. Phys. Lett. 41 587-590
[13] Konoplev B G, Ryndin E A and Denisenko M A 2011 Izvestiya SFedU. Engineering Sciences 117 21-27
[14] Vasileska D, Goodnick S M and Klimeck G 2010 Computational Electronics: Semiclassical and Quantum Device Modeling and Simulation (Boca Raton: CRC Press)
[15] Filachev A M, Taubkin I I and Trishenkov M A 2011 Solid-state Photoelectronics. Photodiodes (Moscow: Fizmatkniga)
[16] Abramov I I 1999 Simulation of Physical Processes in Silicon Elements of Integrated Circuits (Minsk: BGU Press)
[17] Abramov I I 2016 Fundamentals of Simulation of Micro- and Nanoelectronic Elements (Saarbrucken: LAP LAMBERT Academic Publishing)
[18] Mawby P A 1991 Simulation of semiconductor heterojunction devices Physics and Technology of Heterojunction Devices 3 53-110
[19] Pisarenko I and Ryndin E 2019 Electronics 8 106
[20] Palankovski V and Quay R 2004 Analysis and Simulation of Heterostructure Devices (Wien: Springer-Verlag)
[21] Unlu M S and Strite S 1995 J. Appl. Phys. 78 607-639
[22] Ryndin E A and Pisarenko I V 2016 Proc. 26th Int. Conf. "Microwave & Telecommunication Technology" (CriMiCo) 10 2405
[23] Shur M 1990 Physics of Semiconductor Devices (Prentice Hall)
[24] Adachi S 2009 Properties of Semiconductor Alloys: Group-IV, III-V and II-VI Semiconductors (Wiley)
[25] Samarskii A A 1987 Introduction to Numerical Methods (Moscow: Nauka)
[26] Slotboom J W 1973 IEEE Trans. Electron Devices 20 669-679
[27] Kulikova I V, Lysenko I E, Pristupchik N K and Lysenko A S 2014 Izvestiya SFedU. Engineering Sciences 158 106-111
[28] URL: https://www.gnu.org/software/octave/
[29] URL: http://roland65.free.fr/benchmarks/benchmarks-0.2.pdf

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