Asymmetric cluster structure and isoscalar monopole/dipole transitions of $^{28}$Si

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Abstract. The asymmetric cluster states in $^{28}$Si are discussed on the basis of antisymmetrized molecular dynamics. It is found that the inversion doublet bands having the $^{24}$Mg+$\alpha$, $^{20}$Ne+$^8$Be and $^{16}$O+$^{12}$C cluster configurations appear in the energy range of $E_x = 10 \sim 15$ MeV. It is demonstrated that the $^{24}$Mg+$\alpha$, $^{20}$Ne+$^8$Be cluster states have pronounced isoscalar monopole (IS0) and dipole (IS1) transition strengths from the ground state. This result suggests that the IS0 and IS1 strength are good probe for these cluster states.

1. Introduction
Clustering is one of the important aspects in atomic nuclei and its role in light $p$-shell nuclei is established well. On the other hand, the clustering of mid $sd$-shell nuclei is still unknown because of the experimental and theoretical difficulties. One of such examples is $^{24}$Mg+$\alpha$ cluster states in $^{28}$Si that is of astrophysical interest. The antisymmetrized molecular dynamics (AMD) calculation suggested the positive-parity $^{24}$Mg+$\alpha$ cluster bands around $E_x = 10 \sim 15$ MeV [1]. Experimentally, two candidates of $^{24}$Mg+$\alpha$ cluster bands were proposed by the resonance scattering [2] and the $\alpha$-transfer reaction [3]. However, it is unclear if these observed states really corresponds to the AMD result due to the lack of the convincing evidence of the clustering.

An evidence of the parity-asymmetric clustering is the inversion doublet bands [4]. Recently, it was proved that the pair of the $0^+$ and $1^-$ states that constitute the doublet are strongly excited by the isoscalar monopole (IS0) and dipole (IS1) transitions [5, 6]. Indeed, it was observed that the $0^+$ and $1^-$ resonances around $E_x = 10 \sim 15$ MeV are populated by the IS0 and IS1 transitions [7]. Therefore, in this work, we investigate the cluster states and their IS0 and IS1 strengths in $^{28}$Si on the basis of AMD [8, 9]. Our aims are (1) the identification of the inversion-doublet of $^{24}$Mg+$\alpha$ cluster bands and (2) the prediction of their IS0 and IS1 transition strengths.

2. Theoretical Framework
In this work, we employ the Gogny D1S interaction as an effective interaction. The variational wave function of AMD is given as

$$\Phi_{AMD} = A \{ \varphi_1 \varphi_2 \cdots \varphi_A \}, \quad (1)$$
The variational parameters are the width \( \nu \) and the centroids \( Z_i \) of Gaussian wave packet, and the spin \( \alpha_i \) and \( \beta_i \). These parameters are determined by the constrained energy minimization after the projection to the positive- or negative-parity states. The constraint on the quadrupole deformation parameters \( \beta \) and \( \alpha \) \cite{10} are imposed. In addition to this, the \( d \)-constraint method \cite{11} is introduced to describe the various kinds of cluster configurations. After the energy-minimization, we performed angular-momentum-projection and generator coordinate method (GCM).

3. Results and Discussions

By adopting \( d \)-constraint method, we obtained the various kind of cluster configurations as shown in Fig. 1. The obtained cluster basis wave functions can be classified to four groups, \( ^{24}\text{Mg}+\alpha(T) \), \( ^{24}\text{Mg}+\alpha(A) \), \( ^{16}\text{O}+^{12}\text{C} \) and \( ^{20}\text{Ne}+^{8}\text{Be} \) cluster configurations. Here, (T) and (A) denote the triaxial and axial deformation of \( ^{24}\text{Mg}+\alpha \) cluster configurations, respectively.

We superposed these cluster configurations as well as the mean-field configurations, and performed the GCM calculation. Thus-obtained energy spectrum is shown in the Fig. 2, and compared with the observed energy spectra \cite{2, 3}. Although the excitation energies are slightly higher, AMD reproduces the existence of several bands in the energy range of \( E_x = 10 \sim 15 \) MeV. It is found that each of cluster configurations, \( ^{24}\text{Mg}+\alpha(T) \), \( ^{24}\text{Mg}+\alpha(A) \), \( ^{16}\text{O}+^{12}\text{C} \) and \( ^{20}\text{Ne}+^{8}\text{Be} \), generates the inversion-doublet band owing to their parity-asymmetric structure.
Then, the IS0 and IS1 transition strengths \( B(IS0) \) and \( B(IS1) \) of those band-head states are calculated, which is defined as,

\[
B(IS0) = |\langle \Phi(0^+_n) | \sum_{i=1}^{A} r_i^2 | \Phi(0^+_i) \rangle|^2, \quad B(IS1) = |\langle \Phi(1^+_n) | \sum_{i=1}^{A} r_i^2 Y_1(r_i^2) | \Phi(0^+_i) \rangle|^2.
\]  

(3)

The calculated \( B(IS0) \) and \( B(IS1) \) values are shown in the Fig. 2. As clearly seen, the IS0 and IS1 transitions are very sensitive to clustering. Their strengths to the \(^{24}\text{Mg}+\alpha(T)\) and \(^{20}\text{Ne}+^8\text{Be}\) cluster states are comparable to the single particle estimates \( B(IS0)_{s.p.} = 63 \text{ fm}^4 \) and \( B(IS1)_{s.p.} = 140 \text{ fm}^4 \). On the other hand, the transitions to the \(^{24}\text{Mg}+\alpha(A)\) and \(^{16}\text{O}+^{12}\text{C}\) cluster states are negligible. This sensitivity can be understood in terms of the dual nature of the ground state. In the Elliot’s SU(3) shell model, the ground state of \(^{28}\text{Si}\) belongs to the \((\lambda, \mu) = (0, 12)\) irreducible representation. It is easy to show that the \(^{24}\text{Mg}+\alpha(T)\) and \(^{20}\text{Ne}+^8\text{Be}\) cluster structure become identical to this \((0, 12)\) representation in the zero limit of the inter-cluster distance. This means that \(^{24}\text{Mg}+\alpha(T)\) and \(^{20}\text{Ne}+^8\text{Be}\) cluster wave functions are equivalent to the ground state wave function. In the other words, the degree-of-freedom of \(^{24}\text{Mg}+\alpha(T)\) and \(^{20}\text{Ne}+^8\text{Be}\) cluster excitations are embedded in the ground state. The IS0 and IS1 transitions activate the clustering degree-of-freedom embedded in the ground state as discussed in [5, 6] and then developed \(^{24}\text{Mg}+\alpha(T)\) and \(^{20}\text{Ne}+^8\text{Be}\) cluster states are populated. On the other hand, \(^{24}\text{Mg}+\alpha(A)\) and \(^{16}\text{O}+^{12}\text{C}\) cluster states belong to the \((12,8)\) and \((12,0)\) representations which are different from the ground state. Therefore, the IS0 and IS1 transitions from the ground state cannot yield these cluster states. Thus, the IS0 and IS1 transitions are very sensitive and selective probe for clustering.

4. Summary

We investigated the inversion doublets in \(^{28}\text{Si}\) using AMD with Gogny D1S interaction. It is found that \(^{24}\text{Mg}+\alpha(T)\), \(^{24}\text{Mg}+\alpha(A)\), \(^{16}\text{O}+^{12}\text{C}\) and \(^{20}\text{Ne}+^8\text{Be}\) configurations generates the doublet bands in the energy range of \( E_x = 10 \sim 15 \text{ MeV} \), which are an important evidence of those asymmetric cluster structure. The calculated IS0 and IS1 strengths to \(0^+\) and \(1^-\) states show that they are very sensitive and selective to cluster structure. The strengths to \(^{24}\text{Mg}+\alpha(T)\) and \(^{20}\text{Ne}+^8\text{Be}\) cluster states are enhanced but those to \(^{24}\text{Mg}+\alpha(A)\) and \(^{16}\text{O}+^{12}\text{C}\) cluster states are negligible \(0\). This sensitivity and selectivity owes to the duality of shell and cluster in the ground state. Our results remarks that IS0 and IS1 transitions are a good probe of asymmetric cluster structure of \(^{28}\text{Si}\).

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