Finite Element Analysis of Cylinder Shell Resonator and Design of Intelligent Density Meter

Sui X W, Fan Y M, Zhang G X and Qiu Z R
State Key Laboratory of Precision Measuring Technology and Instruments, Tianjin University, Tianjin, 300072, CHINA
E-mail: allensui@eyou.com

Abstract. On the basis of the mathematical model and finite element analysis of the cylinder shell resonator, a novel resonant liquid density meter is designed. The meter consists of a cylinder shell resonator fixed on both ends, a measurement circuit with automatic gain control and automatic phase control, and a signal processing system with microcomputer unit C8051F021. The density meter is insensitive to the liquid pressure, and it can intelligently compensate for the temperature. The experiment results show the meter characteristic coefficients of $K_0$, $K_1$, and $K_2$ at 25 centigrade are $-129.5668$ kgm$^{-3}$, $-0.2535 \times 10^6$ kgm$^{-3}$s$^{-1}$ and $0.6239 \times 10^{10}$ kgm$^{-3}$s$^{-2}$, respectively. The accuracy of the sensor is $\pm 0.1\%$ in range of 700-900 kgm$^{-3}$

1. Introduction
The measuring technology of liquid density is widely used in many scopes, such as oil field, automobile industry, beverage industry and many others. Compared with the traditional methods, the resonant density meter is a new kind of sensor for its precision, online-use and remote transmission. There are many kinds of resonators, such as resonant spring, resonant tube, resonant beam, tuning forks, and others. The structure of cylinder shell resonant liquid density meter is shown in Figure 1.

2. Analysis and design of the cylinder shell resonator
Cylinder shell resonator is a key component of the sensor, which vibrates at its natural frequency and mode. When it is in liquid, the liquid inside vibrates with the cylinder shell together and the vibrating frequency varies with the liquid mass in the shell. Because the volume of the cylinder shell is known, the liquid density determines the frequency. The following equation can be derived from elastic mechanics:
\[ \rho = \rho_0 \left( \frac{\omega_0^2}{\omega^2} - 1 \right) = \rho_0 \left( \frac{T^2}{T_0^2} - 1 \right) \]  

(1) 

where \( \rho \) is the density of liquid inside the shell; \( \omega_0, \omega \) are angular frequencies of the shell in air and in liquid, respectively; \( T_0, T \) are vibration periods of the shell in air and in liquid respectively; \( \rho_0 \) is a constant resting on the size and material of the cylinder shell. In practice, the period is much more convenient to measure than the frequency. For a given cylinder shell, \( \rho_0 \) and \( T_0 \) are constants, so equation (1) takes the form

\[ \rho = K_0 + K_2 T^2 \]  

(2) 

Considering other factors influencing the vibration frequency, equation (2) changes into equation (3) to improve the calculation precision.

\[ \rho = K_0 + K_1 T + K_2 T^2 \]  

(3)

\( K_0, K_1 \) and \( K_2 \) are determined from calibration experiments and the liquid density can be worked out by measuring the vibration period of the shell in liquid.

As a resonator, the cylinder shell has different frequencies in different vibration modes. The vibration mode can be expressed by the number of half-waves in axial direction \( m \) and the number of waves along radial direction \( n \). Calculation and experiments show the frequency of mode \( m=1 \) is easy to excite, the frequency of mode \( n=2 \) is insensitive to the liquid pressure. The resonator and vibration mode of \( m=1, n=2 \) are shown in Figure 2.

![Figure 2. The resonator and vibration mode of \( m=1, n=2 \)](image)

The finite element method is used to analyze the cylinder shell model. The program written in MATLAB gives the FEM analysis results. For the cylinder shell fixed at both ends, its vibration equation can be written as

\[ (K - \omega_0^2 M)a = 0 \]  

(4)

where \( K \) is the stiffness matrix of the system; \( M \) is its mass matrix; \( a \) is its vibration mode vector. The effective length, outside radius and thickness of the shell are 45mm, 9mm and 0.2mm, respectively. The material used in China is 3J53, which is equivalent to Ni-Span C in UK and US. The elastic modulus is \( 1.95 \times 10^5 \) Pa, the density of the material is 7900kgm\(^{-3}\), and the Poisson ratio is 0.3. The natural frequency of the resonator in air is 5741Hz. A bubble filter net is installed at the bottom of the meter additionally to vent the air bubble in the liquid and the measuring accuracy has been much improved.

3. Measuring and controlling circuit

The measuring and controlling circuit is a self-sustaining oscillator, which consists of a measuring circuit, a band-pass filter, an auto phase control, an auto gain control, and an exciting circuit. Three sets of electro-magnetic coils, one exciting coil and two measuring coils that produce a voltage induced by mechanical deformation, are mounted around the cylinder shell as shown in Figure 3.
Figure 3. The structure of measuring and exciting coils

For mode $n=2$, the voltage signals of two measuring coils are of the same phase, but for other modes, they are different. After summing up, the voltage of mode $n=2$ is enhanced, but the voltages of other modes are weakened.

The band-pass filter consists of a two-stage low-pass filter and an eight-stage high-pass one. The eight-stage high-pass filter is a Butterworth filter LTC1064 from LINEAR company designed by FilterCAD software with cut-off frequency 2.1 kHz, which is used to suppress the severe electromagnetic interference and mechanical vibration below 2 kHz. The two-stage low-pass filter is a Butterworth filter with cut-off frequency 6.0 kHz. These two filters working together form a band-pass filter with bandwidth between 2.1 kHz and 6 kHz.

The auto gain control and auto phase control are needed to keep the shell vibrating continuously in different liquids. The auto gain amplifier is LMH6503 with wide bandwidth, low power consumption and linear gain. After it is regulated, the signal is amplified in power, and then drives the exciting coil. The exciting coil is specially designed, it is small, and has enough exciting power to keep the resonator vibrating continuously.

4. Signal processing system
The measuring signal is fed into the signal processing system based on a microcomputer unit. The internal timing counter of the processor measures the period of the vibration. The period, in conjunction with $K_0$, $K_1$ and $K_2$ is used to calculate the liquid density, and intelligent compensation for the temperature is realized. The microcomputer unit is a single chip computer of model C8051F021 from Cygnal Company, which is a fully integrated system and compatible with 8051 system CIP-51. The MCU is fast and accurate with system clock of 22.1184 MHz, the potential counter error is one clock pulse, that is approximately equivalent to 0.05 $\mu$s, and less than 0.01% of the measured period.

Temperature is the main error source in measurement. The calculation reveals that one-centigrade change in temperature brings about a relative frequency change of $5 \times 10^{-6}$. A thermometer of AD590 with accuracy of 0.5 centigrade from AD Company is used to measure the liquid temperature. The temperature is also fed into MCU C8051F021. A digital filter with sliding average is implemented to improve the signal processing system immunity. The MCU communicates with the host computer through a RS232 interface. The density meter is calibrated at different temperatures, and respective coefficients of the sensor are stored in the host computer. A linear interpolation is applied to calculate the liquid density when the density meter is used for in-line usage. The computer calculates with high accuracy and speed. In this way, the error caused by temperature shift is intelligently compensated.

5. Experiments and conclusions
In the experiments, different kinds of oil, fuel are mixed to prepare special liquids with different densities. Density meter of model DE51 with measuring range of 0-3000 $\text{kg/m}^3$ and accuracy of 0.01 $\text{kg/m}^3$ is served as the reference. A mercury sensor with resolution of 0.1 $\degree$C is used to measure the liquid temperature. The MCU of the resonant density meter gives the period of the cylinder shell vibration. The least square method is applied to obtain the functional relationship between the liquid
density and the period measured shown by equation (3). The experimental results show that the sensor coefficients $K_0$, $K_1$ and $K_2$ at 25°C are $-129.5668 \text{ kgm}^{-3}$, $-0.2535 \times 10^6 \text{ kgm}^{-3}\text{s}^{-1}$ and $0.6239 \times 10^{10} \text{ kgm}^{-3}\text{s}^{-2}$ respectively. The calculation error is less than 0.02%. The sensor coefficients under other temperatures can also be obtained in the same way. The experimental results after calibration in the range of 700kgm$^{-3}$-900kgm$^{-3}$ are listed in Table 1.

Table 1. The results of density measurement tests

| No. of test | DE51 reading (kgm$^{-3}$) | Resonator display (kgm$^{-3}$) | Absolute error (kgm$^{-3}$) | Relative error (%) |
|-------------|----------------------------|--------------------------------|-----------------------------|-------------------|
| 1           | 700.55                     | 701.1                          | 0.55                        | 0.08              |
| 2           | 725.28                     | 725.6                          | 0.32                        | 0.04              |
| 3           | 738.60                     | 738.2                          | -0.40                       | -0.05             |
| 4           | 770.84                     | 770.9                          | 0.06                        | 0.01              |
| 5           | 800.67                     | 801.1                          | 0.43                        | 0.05              |
| 6           | 820.85                     | 821.1                          | 0.25                        | 0.03              |
| 7           | 842.97                     | 843.7                          | 0.73                        | 0.09              |
| 8           | 865.43                     | 865.2                          | -0.23                       | -0.03             |
| 9           | 881.88                     | 881.4                          | -0.48                       | -0.05             |
| 10          | 903.54                     | 904.2                          | 0.66                        | 0.07              |

In the whole range of 700 kgm$^{-3}$-900 kgm$^{-3}$, the maximum absolute error of the cylinder shell density meter is 0.73 kgm$^{-3}$. Taking into account the error of DE51, the maximum relative error of the meter can be considered as less than 0.1%. The cylinder shell density meter will find wide applications in many fields.

Reference

[1] Lian Y Y, 1982, Density measure technology (China: Machine Press)
[2] Liu G Y, Chen M, Wu Z H, Fan S C, 1995, Novel sensor technology and application, (China: Press of Beijing University of Aeronautics and Astronautics)
[3] Raszillier, H, Durst F, 1991, Coriolis-effect in mass flow metering, Ingenieur-Archiv, Vol. 61 pp. 192-214
[4] Zhang G X, Jin Z Z, 2001, Measuring and controlling Circuits (China: Machine Press)
[5] Pan Z J, Shi G J, 2002, Principle and application of high speed MCU C8051FXXX (China: Press of Beijing University of Aeronautics and Astronautics)