Oscillations simulation of the vibration protection suspended load with a movable base

M S Korytov¹, V S Shcherbakov¹, V V Titenko² and I E Pochekueva¹
¹Siberian State Automobile and Highway University, 5 Mira Pr., Omsk 644080 Russia
²Omsk State Technical University, 11 Mira Pr., Omsk, 644050, Russia

E-mail: kms142@mail.ru

Abstract. Reducing the vibration effects on a human operator of a construction, road or handling machine is an urgent task, since the vibration loads produced by the internal combustion engine, vehicle engine interacting with the support surface microrelief, as well as the machine working attachment, adversely affect the machine operator and increase the machine parts and assemblies wear. The operator may develop occupational diseases caused by vibrations. In this regard, the research objective is to develop the mathematical apparatus simulating the dynamic processes of the passive vibration protection systems oscillations at the design stage. Therefore, the problem developing a mathematical model that makes it possible to study the oscillations of a single degree of freedom passive vibration protection systems with any given static characteristic, was solved. On the basis of the dynamic equations of a single translational degree of freedom vibration system describing the vibration protection suspended operator’s seat, the mathematical model solving the differential equation of the system oscillations taking into account the spring loaded mass displacements damping and three-segment static characteristics of the vibration isolation mechanism with limiters was simulated using the package Simulink of MATLAB system. An example of using the developed mathematical model for modelling vibrations of a spring-loaded mass under the sinusoidal external influences is given. The preset vertical displacements of the machine base, i.e. its base chassis act as the external influences. The examples of the obtained simulation results, namely the vertical displacements and accelerations of the spring-loaded mass are presented. The maximum accelerations of the spring-loaded mass were found out to significantly increase, when the machine base forced displacements amplitude exceeds by half of the horizontal section of the quasi-zero static characteristic of the vibration protection suspension.

Key-words: vibrations, vibration protection, oscillations, acceleration, quasi-zero stiffness

1. Introduction
The speed and power of construction, road and handling machines tend to constantly increase, which leads to an increase in dynamic loads on the chassis and transmission assemblies of machines [1]. This causes an increase in the level of vibrations produced by these assemblies and parts. Vibration loads negatively affect the machine assemblies and parts, the operator and surrounding objects [2]. They cause the operator fatigue and increase the number of control errors [3], which reduce the performance of construction, road and handling vehicles [4]. Therefore, when designing and manufacturing modern
construction, road or handling machines, particularly much attention is given to protecting the operator from vibrations produced by the engine, chassis and various working attachments of the machine. Quite frequently, during machines operation the operator may develop occupational diseases caused by vibrations [5]. The problem is relevant for the operators of brush cutters [6], hand-held jackhammers [7], forklifts and pallet trucks [8], tractors [9] and various transport and construction machines [10] including motor graders [11]. Vibration protection systems are also used for processing equipment [12] and even for spacecraft [13]. Anti-vibration gloves can be used to protect hand tool operators from vibrations [14]. However, this approach cannot be applied for protecting a human operator of road construction and handling vehicles, whose workplace being a cab seat [15]. In this case, anti-vibration suspensions of seats and cabs are used [9]. Thus, reducing the vibrations eventually transmitted to the human operator of construction, road and handling vehicles through the workplace, i.e. through the seat, is a relevant task. The simplest and simultaneously most effective way to protect the operator from vibrations can be applying the anti-vibration suspended seats [2]. There are active and passive vibration protection systems [16]. In the active systems, electric and pneumatic drives are most often used for the additional power supply. In the passive systems, elastic springs, dampers and their combinations, as well as pneumatic shells are usually applied [17]. Some types of passive systems, such as traditional ones with metal springs and dampers have the advantages of simplicity, reliability and durability. It is advisable to use the passive vibration isolation systems, provided they successfully perform their functions. The key stage of designing new and improving existing vibration isolation systems is to study their properties using physical (full-scale) or mathematical modelling. Besides the diversity of designs, any single translational degree of freedom passive vibration protection systems possess a certain static force characteristic to be expressed in the “displacement and force” coordinates. A mathematical model that allows studying the dynamic vibrations of the passive vibration protection systems with any given static stiffness characteristic [18] including the advanced systems with a quasi-zero stiffness section in the operation area of displacements [19] should be developed. The model must also take into account the dynamic properties of the vibration protection system with the load. The dynamic parameters are the total reduced weight of the load with moving parts and the damping coefficient [20].

2. Problem statement

It is necessary to develop a single-mass dynamic model of the vibration protection system of the construction, road and handling vehicles operator. A single-mass vibrating system with one translational degree of freedom is characterized by the following set of parameters (figure 1): \( m \) is the total reduced mass of the load with moving parts of the machine (in this case - a human operator’s seat), kg; \( z \) is the vertical coordinate of the useful load mass center in a fixed coordinate system (CS), m; \( z_{op} \) is the vertical coordinate of the machine movable chassis (displacements of the chassis are hard-coded in a fixed CS), m; \( z_1 = z - z_{op} \) is the vertical coordinate of the load mass center relative to the movable chassis, m; \( \ddot{a} = \ddot{z} \) is the acceleration of the load mass center in a fixed CS, m/s\(^2\); \( b \) is the damping coefficient of the load vertical displacements, N/(m/s); \( g \) is the gravitational acceleration, m/s\(^2\); \( v_1 = \dot{z}_1 \) is the load mass center velocity relative to the movable chassis, m/s; \( z_{qz} \) is the width of the quasi-zero stiffness area of the anti-vibration mechanism with limiters, m; \( F \) is the vertical force produced by the system vibration isolation mechanism without considering a damper (constant-force mechanism with limiters), N; \( F_{plus} \) is the force value \( F \) at \( z_1 = -z_{qz} \) (the additional mechanism power upon reaching a conditional limit stroke of the mechanism), N; \( c_{lab} \) is the coefficient characterizing the limiters (bump stops) elasticity degree of the anti-vibration mechanism.
The force $F$ is set according to a composite, generally nonlinear static dependence, as a function of the coordinate $z_1$, i.e. the spring-loaded mass displacement inside the limits of the anti-vibration mechanism movement (figure 2, left axis).

The constant gravity of the useful load $mg$ is balanced by the constant static force of the vibration protection mechanism with limiters (the middle section of the power characteristic in figure 2, left axis), which allows to proceed to the equivalent simplified calculation model of the spring-loaded
mass vertical vibrations without taking into consideration the gravity $mg$. In this case, the static power characteristic of the vibration protection mechanism with limiters will shift down without changing the shape, and the force generated by the conditional equivalent mechanism will take zero values in the middle section (figure 2, right axis).

3. Theory
For the mechanical system which calculation model is shown in figure 1 without taking into account the gravity, the following dynamics equation can be derived based on the Lagrange-d'Alembert principle:

$$m \cdot a + b \cdot v_1 + F = 0$$

(1)

In the Cauchy's form, for simulating the mathematical model and solving a differential equation (DE) by the numerical methods, DE (1) can be solved with respect to the highest derivative and written as follows:

$$a = \frac{-F - b \cdot v_1}{m}$$

(2)

Under the sinusoidal vertical displacements of the machine chassis:

$$z_{eq} = A \cdot \sin(\omega \cdot t)$$

(3)

where $A$ is the oscillation amplitude of the base, $m$; $\omega = 2\pi/T$ is the angular (circular) frequency, $s^{-1}$; $T$ is the oscillation period, $s$, equation (2) has the following form:

$$a = \frac{-b \cdot \frac{d}{dt} (z - A \cdot \sin(\omega \cdot t)) - F}{m} = \frac{-b \cdot (v - A \cdot \omega \cdot \cos(\omega \cdot t)) - F}{m}$$

(4)

The force $F$ is a three-segment spline of the argument $z_1$, its analytical expression is as follows:

$$F = \begin{cases} 
  c_{ab} \left( -z_1 - \frac{z_{eq}}{2} \right)^s & \text{at } (-z_1) > \frac{z_{eq}}{2}; \\
  0 & \text{at } -\frac{z_{eq}}{2} \leq (-z_1) \leq \frac{z_{eq}}{2}; \\
  -c_{ab} \left( -z_1 + \frac{z_{eq}}{2} \right)^s & \text{at } (-z_1) < -\frac{z_{eq}}{2}, 
\end{cases}$$

(5)

where $s \geq 1$ is the power exponent (1 is the straight line, 2 is the parabola, etc.) of the mechanism bump stops elastic force curve (i.e., the end sections of a three-segment spline).

The values of the coefficient $c_{ab}$ are proposed to be calculated based on the dependence:

$$c_{ab} = \frac{F_{plus}}{\left( \frac{z_{eq}}{2} \right)^s}$$

(6)
Figure 3. Mathematical simulation model of vertical vibrations of the vibration protection suspended load on a movable base in MATLAB/Simulink symbols.

Figure 3 shows a mathematical model simulating a physical system, which design scheme is shown in figure 1 in MATLAB/Simulink symbols.

4. Experimental results
The results example of the separate simulation modeling is represented in figure 4. In this computational experiment, the model parameters took the following values: the vertical displacements amplitude of the machine chassis is $A=0.1\text{ m}$; the machine chassis oscillation period is $T=4\text{ s}$; the operator's seat mass is $m=200\text{ kg}$; the damping coefficient is $b=200\text{ N/(m/s)}$.

The machine chassis oscillations were set according to the sine law (3), the force $F$ was calculated according to the dependence (5) for the values of $s=2$ and $F_{plus}=2000\text{ N}$. 

---

**Figure 3.** Mathematical simulation model of vertical vibrations of the vibration protection suspended load on a movable base in MATLAB/Simulink symbols.
5. Results discussion

The developed simulation mathematical model is universal and makes it possible to vary all the above parameters of the system, as well as to investigate their influence on the parameters of the mass protected from vibrations by constantly restarting the simulation model. Figure 5 shows the functional dependencies of the sprung mass maximum acceleration achieved for 100 s. The total oscillation time was 200 s. The maximum acceleration of the sprung mass was defined in the second half of the transition process, i.e. in the period from 100 to 200 s for excluding the initial conditions influence.

Vertical displacements of the machine chassis have been sinusoidal in nature (3). The weight of the operator's seat assumed a constant value of \( m = 200 \text{ kg} \). The force \( F \) was calculated by the dependence (5), for the values of \( s = 2 \) and \( F_{\text{plus}} = 5000 \text{ N} \), \( z_{\text{op}} = 0.01 \text{ m} \).

The vertical displacement amplitude of the machine chassis \( A \), the chassis oscillation period \( T \) and the damping coefficient of the vibration protection suspension \( b \) varied. They took discrete values from...
the following series: $A=[0.02; 0.04; 0.06; 0.08]$ m; $T=[2; 3; 4; 5]$ s; $b=[2; 4; 8; 16; 32; 64; 128; 256; 512; 1024]$ N/(m/s).

Figure 5. Functional dependences of the spring-loaded mass maximum acceleration on the damping coefficient of the vibration protection suspension at the different chassis vertical displacements amplitude and oscillation period: (a) is $A=0.02$ m; (b) is $A=0.04$ m; (c) is $A=0.06$ m; (d) is $A=0.08$ m, $z_q=0.01$ m.

Figure 5 shows that increasing the values of the vibroprotective suspension damping coefficient at the different amplitudes of impact has an ambiguous effect on the maximum acceleration of the spring-loaded mass.

6. Conclusions

Based on the research conducted, a simulation mathematical model of the vertical vibrations of the vibration protection suspended load on a movable base was developed. MATLAB based software package Simulink was used.

The dynamic system and its model are characterized by the mass of the useful load protected from vibrations, the damping coefficient (viscous friction) and the static characteristic. The latter is represented as a dependence of the force produced by the anti-vibration suspension on the amount of deformation (the load displacement relative to the movable base). A static characteristic in a model can have any specified character. A promising static characteristic consisting of three sections was considered as an example. The central horizontal section reflects the quasi-zero stiffness, i.e. a constant force in a certain range of deformations. Two inclined end sections withstand the impact of bump stops preventing the displacement of the spring-loaded mass relative to the movable base and are characterized by a smooth change in the force relative to the horizontal section.

The developed mathematical model makes it possible to study the forced vibrations dynamic processes of the spring-loaded useful mass on a movable base. Forced displacements of the movable base can have an undefined character. Sinusoidal vibrations of a mobile base at the fixed amplitude and frequency were considered as an example.

The developed mathematical model makes it possible to obtain the time dependences of the accelerations, velocities and displacements of the spring-loaded mass in a fixed coordinate system, the
force acting on the load weight from the vibration protection suspension, the spring-loaded mass displacements relative to the vibration protection suspension (movable base). Using a mathematical simulation model developed in the MATLAB Simulink environment, a computational experiment to implement the machine mobile base displacements specified by the sinusoidal function, was performed. The amplitude and oscillation period of the machine movable base, as well as the damping coefficient of the vibration protection suspension, varied. Different combinations of the mentioned factors were considered.

The analysis of the computational experiment results showed that for the vertical displacements amplitude values of the machine chassis being less than half of the static characteristic horizontal section value (less than 0.05 m), an increase in the damping coefficient of the vibration protection suspension always increases the maximum acceleration of the spring-loaded mass. This is due to the displacement of the spring-loaded mass inside the horizontal, quasi-zero section and the lack of connection to the inclined sections of the static characteristic. At the small amplitudes of the forced oscillations, the maximum acceleration of the spring-loaded mass is generally low and tends to zero at zero values of the damping coefficient.

If the amplitude of the chassis vertical displacements is more than half of the horizontal quasi-zero section value of the static characteristic (more than 0.05 m), this leads to a regular entering the inclined sections of the static characteristic. With an increase in the damping coefficient, the maximum acceleration of the spring-loaded mass generally decreases, but having local minima and maxima at the short period oscillations. When activated the inclined sections of the static characteristic give the spring-loaded mass maximum acceleration values by one or two orders higher compared to the position only inside the horizontal section of the quasi-zero static characteristic.

It is appropriate to use the vibration protection suspensions with the quasi-zero stiffness length of more than double amplitude of the machine base forced displacements. This will significantly reduce the maximum vibration acceleration of the protected useful load mass.

7. References

[1] Pobedin A V, Dolotov A A and Shekhovtsov V V 2016 Procedia Eng. 150 1252–1257
[2] Korchagin P A, Teterina I A and Rahuba L F 2018 J. Phys. Conf. Ser. 944 012059
[3] Baranovskiy A M and Vikulov S V 2019 Marine Intellect. Technol. 3(1) 35–38
[4] Teterina I A, Korchagin P A and Letopolsky A B 2019 Lect. Notes Mech. Eng. 19 177–184
[5] Nehaev V A, Nikolaev V A and Zakernichnaya N V 2018 J. Phys. Conf. Ser. 1050 012057
[6] Sorica E, Vladut V, Cadre P and Sorica C 2018 Springer Proc. Phys. 198 165–172
[7] Ling X, Zhang L and Feng X 2019 Mech. Syst. Signal Pr. 118 317–339
[8] Rokosch F, Schick R and Schaefer K 2017 Zentralblatt Fur Arbeitsmedizin Arbeitsschutz Und Ergonomie 67(1) 15–21
[9] Lyashenko M V, Pobedin A V and Potapov P V 2016 Procedia Eng. 150 1245–1251
[10] Balakin P D, Dyundik O S and Zgonnik I P 2020 J. Phys. Conf. Ser. 1546 012125
[11] Korytov M S Shcherbakov V S, Titenko V V and Ots D A 2019 J. Phys. Conf. Ser. 1260 112015
[12] Burian Y A, Silkov M M and Trifonova E N 2019 AIP Conf. Proc. (Omsk) vol 2141(1) (USA: AIP Publishing) pp 030067
[13] Sayapin S N, Artemenko Yu N and Panteleev S V 2017 Studies in Systems, Decision and Control 95 213–230
[14] Hermann T and Dobry M W 2017 Int. J Occup. Saf. Ergo 23 (3) 415–423
[15] Teterina I A, Korchagin P A and Aleshkov D S 2018 IOP Conf. Series AMSD (Omsk) vol 6(3) (UK: Bristol, IOP Publishing) pp 1–5
[16] Ning D et al. 2019 Mech. Syst. Signal Pr. 133 106259
[17] Sorokin V N and Zakharhenkov N V 2018 J. Phys. Conf. Ser. 1050 012081
[18] Kim J et al. 2019 Int. J. Prec. Eng. Manuf. 20(9) 1573–1580
[19] Balakin P D et al. 2020 J. Phys. Conf. Ser. 1441 012086
[20] Liu Y et al. 2019 Arch. Appl. Mech. 89(9) 1743–1759