Modal parameter identification of subsynchronous oscillation based on stochastic subspace algorithm

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Abstract. At present, under the general trend of interconnection of power grid systems, the problem of sub-synchronous oscillation of power systems has gradually become the focus of attention. Stochastic subspace modal identification (SSI) algorithm plays an important role in vibration detection and damage identification of large motors. Finally, experiments are performed on the data collected in the field, and the results show that the SSI algorithm can better identify the mode of the signal. Compared with the traditional Prony algorithm and ARMA algorithm, the recognition accuracy of the stochastic subspace algorithm constructed in this paper is higher than that of Prony algorithm and ARMA algorithm, which has the advantage of strong anti-noise.

1. Introduction

The identification of modal parameters of power system sub-synchronous oscillation is the core of power system safety and stability. With the expansion of the modern power system in China and the complexity of the power grid structure, the problem of sub-synchronous oscillations has become increasingly prominent. In the current engineering projects, some traditional modal parameter recognition methods[1-4] are still commonly used, such as Prony method[5-6], ARMA method[7-9], and so on. After a lot of experiments and test analysis, these algorithms have their own characteristics and application scope. For example, the existence of Gaussian noise will cause estimation bias, which will affect the accuracy and stability of modal identification. Therefore, how to judge and eliminate noise modes has always been the focus of research in the field of modal identification[10].

Among them, the Prony algorithm represents the identification signal as a series of linear combination of complex exponential signals with arbitrary amplitude, phase, frequency and attenuation factor, and has a nonlinear multi-bit filtering characteristic. But the Prony algorithm can only be used for oscillating signal analysis[11], and the recognition accuracy is poor in the case of noise interference. The ARMA algorithm takes white noise as input, which solves the problem of unrecognizable input when the input is unknown in system recognition, and broadens the application field of system recognition. However, the ARMA algorithm can only compare and identify ideal signals. When the signal noise is relatively large, many modal parameters cannot be identified[12]. The stochastic subspace method is a system identification method developed in recent years and is applicable to many fields. In the work of this paper, a random subspace algorithm is used and it is used for the torsional vibration identification of the sub-synchronous turbine shafting torsional vibration[13-14], which also provides a basis for the safe and stable operation of the power system generator set.

This paper chooses to use SSI algorithm for modal identification. Firstly, the feasibility of this method was confirmed by modal identification of ideal signals. Then, a comparison and analysis is
performed on the noise added to the ideal oscillating signal, which proves the anti-noise performance of the SSI algorithm. Finally, compared with the traditional ARMA algorithm and Prony algorithm in the actual acquisition signal, it proves that the method has high recognition accuracy and good anti-noise performance.

2. Stochastic subspace modal identification algorithm

The stochastic process refers to the change form of the physical object without a certain form of change, which cannot be described by a deterministic function, that is different function values can be taken for each time. The stochastic subspace method [15] allows to refer to the time-domain method of reconstructing the response data into a matrix and obtaining linear modal parameters from it. It uses the output response data to identify the modal parameters of the structure based on the state space model. Its core technology is to project the "future" row space of the response data onto the "past" row space, retain all the information of the "past" data in the projection result, and use the projection result to predict the "future" data features, avoid the traditional manual identification and iterative process in the calculation process.

2.1. Basic principles of stochastic subspace

For a multi-degree-of-freedom linear vibration system, its state space model can be described as:

\[
\begin{align*}
x_{k+1} &= Ax_k + \omega_k \\
y_k &= Cx_k + v_k
\end{align*}
\]

(1)

Among them, the state vector \( x_k \in \mathbb{R}^n \); the output vector \( y_k \in \mathbb{R}^m \); System matrix \( A \in \mathbb{R}^{m \times n} \); output matrix \( C \in \mathbb{R}^{m \times n} \); Process white noise disturbance vector \( \omega_k \in \mathbb{R}^n \); Measuring white noise \( v_k \in \mathbb{R}^m \). Where \( \omega_k \) and \( v_k \) satisfy mutually unrelated conditions. According to the output matrix \( y_k \), Hankel matrix \( Y_{02i-1} \) can be constructed as follows:

\[
Y_{02i-1} = \begin{pmatrix} Y_{0i-1} \\ Y_{2i-1} \end{pmatrix} = \begin{pmatrix} Y_p \\ Y_f \end{pmatrix}
\]

(2)

The subscripts 0/2i-1 represent the subscripts of the first and last rows of the first column of the Hankel matrix, and the subscripts p and f represent "past" and "future" respectively. Then QR decomposition the Hankle matrix:

\[
\begin{pmatrix} Y_p \\ Y_f \end{pmatrix} = \begin{pmatrix} R_{11} & 0 & 0 \\ R_{21} & R_{22} & 0 \end{pmatrix} \begin{pmatrix} Q_1^T \\ Q_2^T \end{pmatrix}
\]

(3)

From the nature of the spatial projection, we can conclude that the orthogonal projection of the row space is defined as:

\[
R_i = \begin{pmatrix} Y_f \\ Y_p \end{pmatrix} = R_{2i}Q_i^T
\]

(4)

According to the subspace system detection theory [8], the projection can be decomposed into the product of the observable matrix \( O_i \) and the Kalman filter state sequence \( \hat{X}_i \).

\[
R_i = O_i \hat{X}_i
\]

(5)

Singular value decomposition of \( R_i \):

\[
R_i = (U_1 \quad U_2) \begin{pmatrix} S_1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} v_1^T \\ v_2^T \end{pmatrix} = U_1 S_1 V_1^T
\]

(6)
State space model equations can be composed of the following linear equations:

\[
\begin{pmatrix}
X_{i+1} \\
Y_{j}
\end{pmatrix} = \begin{pmatrix} A & X \end{pmatrix} \begin{pmatrix} \rho_w \\
\rho_e \end{pmatrix}
\]

(7)

Where: \(\rho_w\) and \(\rho_e\) are residuals; \(A\) and \(C\) can be obtained by least squares:

\[
\begin{pmatrix} A \\
C \end{pmatrix} = \begin{pmatrix} X_{i+1} \\
Y_{j} \end{pmatrix} \hat{X}_{i}
\]

(8)

2.2. Modal parameter identification process

After the state matrix \(A\) and the output matrix \(C\) are determined, the modal parameters of the structure need to be obtained by eigen decomposition. First, eigenvalue decomposition is performed on the state matrix \(A\), as follows:

\[
A = \Psi \Lambda \Psi^{-1}
\]

(9)

\[
C = [C_1 \ C_2 \ ... \ C_n]^T
\]

(10)

\(\Psi\) is the feature vector of the state space model, \(\Lambda = \text{diag} (\lambda_i) \in C^{2n \times 2n}, i = 1, 2, \ldots, L, n\). \(\lambda_i\) is the eigenvalue of the discrete-time system, and \(C^{2n \times 2n}\) is the eigenvector matrix of the system. The relationship between model characteristic \(\lambda_i^*, \lambda_i^{\epsilon*}\) and system natural vibration frequency \(\omega\), amplitude \(A\) and modal damping \(\xi\) is as follows:

\[
\omega_i = \sqrt{a_i^2 + b_i^2}
\]

(11)

\[
\lambda_i^*, \lambda_i^{\epsilon*} = -\xi \omega_i \pm j \omega_i \sqrt{1 - \xi_i^2}
\]

(12)

\[A = 2|C_i|
\]

(13)

3. Experimental verification

3.1. Ideal signal verification

Assume that the test set \(T\) contains a three-mode simulation signal, which has a frequency component close to the generator frequency, and the simulation signal is:

\[
X(t) = \sum_{k=1}^{N} B_k e^{-\xi_k \omega_k t} \cos(\omega_k t + \theta_{0k})
\]

(14)

Where \(N\) is the order, \(\omega_{nk} \cdot \xi_k \), and \(\omega_{dk}\) are the undamped natural frequency, damping ratio, and damped frequency of the \(k\)-th order of the system \((k = 1, 2, \ldots, N)\), and the relationship between them is:

\[
\omega_{dk} = \omega_{nk} \sqrt{1 - \xi_k^2}
\]

(15)

In order to verify the correctness and accuracy of SSI modal parameter identification, a three-degree-of-freedom viscous damping is taken as an example, and the frequency of the analog signal is set to \(f_1 = 10 Hz\), \(f_2 = 15 Hz\), \(f_3 = 25 Hz\), with amplitudes of 1, 0.5 and 1.5. The formula for the attenuation factor is as follows:

\[
\alpha = \xi_k \times \omega_{dk}
\]

(16)
The attenuation factors are set to 0.05, 0.15 and 0.03, the sampling frequency is \( f_s = 1000 \text{Hz} \). Then the spectrum diagram is shown in Figure 1. From the first 50Hz amplitude-frequency diagram of the analog signal, it can be seen that the third-order frequency is \( f_1 = 10 \text{Hz}, \ f_2 = 15 \text{Hz}, \ f_3 = 25 \text{Hz} \). It is exactly the same as the frequency set in the analog signal.

![Figure 1. The first 50Hz amplitude-frequency diagram of an analog signal.](image)

The SSI algorithm is used for modal identification of the ideal signal, and traditional algorithms such as Prony and ARMA are used to identify the ideal signal at the same time. The comparison results are shown in Table 1 below:

| Order | Raw data | SSI algorithm | Prony algorithm | ARMA algorithm |
|-------|----------|---------------|----------------|----------------|
|       | Frequency (Hz) | Attenuation factor | Frequency (Hz) | Attenuation factor | Frequency (Hz) | Attenuation factor | Frequency (Hz) | Attenuation factor |
| 1     | 10       | 0.05          | 10             | 0.05           | 10             | 0.05           | 9.98           | 0.05 |
| 2     | 15       | 0.15          | 15             | 0.15           | 15             | 0.15           | 14.83          | 0.15 |
| 3     | 25       | 0.03          | 25             | 0.03           | 25             | 0.03           | 24.98          | 0.03 |

3.2. Effect of noise intensity on modal identification

In order to study the influence of noise on the accuracy of SSI identification, adding white noise to the analog signal generates random noise with normal distribution with a mean value of 0 and a standard deviation of \( \sigma = 0.1 \). A comparative experiment was performed on the three algorithms, and the comparison results are shown in Table 2 below:

| Order | SSI algorithm | Prony algorithm | ARMA algorithm |
|-------|---------------|----------------|----------------|
|       | Frequency (Hz) | Attenuation factor | Frequency (Hz) | Attenuation factor | Frequency (Hz) | Attenuation factor |
| 1     | 9.99          | 0.0467         | \   \   \   \   |  \   \   \   \   |  \   \   \   \   |  \   \   \   \   |
| 2     | 14.99         | 0.1595         | 13.5306        | \   \   \   \   |  \   \   \   \   |  \   \   \   \   |
| 3     | 25            | 0.0301         | 24.7651        | \   \   \   \   |  \   \   \   \   |  \   \   \   \   |

The SSI algorithm was used to calculate the frequencies of 9.99, 15 and 25, the corresponding attenuation factors were 0.0467, 0.1595 and 0.0301. Through experiments, it can be found that the SSI algorithm can identify structural modal parameters of different frequency noises, and the identification results are good. The Prony algorithm can only identify two valid modal modes, while the ARMA algorithm cannot effectively identify the modal parameters under noise interference. The time domain
waveform of the ideal signal is compared with the time domain waveform of the signal restored by the modal parameters obtained by the SSI algorithm:

Figure 2. Time domain waveform of the noisy signal.

Figure 3. First-order waveform of the restored signal.

Figure 4. Second-order waveform of the restored signal.

Figure 5. Third-order waveform of the restored signal.

3.3. Case Analysis
In order to study the algorithm situation under real signals, the modal identification and comparison of the signals collected in the field is performed in this paper. The data sample time is 60s, the sampling rate is 1000Hz, and the sampling points are 60,000.

The difference between the two ends is used as the input signal for Fourier transform processing. The three frequencies obtained are 13.38 Hz, 23.68 Hz and 26.52 Hz. These three frequencies are preliminarily judged to be the modal frequencies of the first three orders of the shafting. The amplitude-frequency diagram is as follows:

Figure 6. The first 50Hz amplitude-frequency diagram of the real signal.
The following three methods are used to perform modal identification on real data. The identification results are shown in Table 3 below:

| Order | Fourier transform | SSI algorithm | Prony algorithm | ARMA algorithm |
|-------|------------------|---------------|-----------------|----------------|
|       | Frequency (Hz)   | Frequency (Hz)| Frequency (Hz)  | Frequency (Hz) |
| 1     | 13.38            | 13.31         | 13.32           | 13.63          |
| 2     | 23.68            | 24.2          | 25.47           | 37.74          |
| 3     | 26.52            | 26.67         | 49.67           | 52.25          |

Through the calculation of real data, it can be found that the SSI algorithm is more accurate and the frequency results are better. Due to the strong noise of the field signal, in order to obtain better identification accuracy, the filtering preprocessing is performed first, and then the attenuation factors of the three modes identified by the SSI algorithm are 0.0215, 0.0878, and 0.0289. The amplitudes calculated by the SSI algorithm are 0.1404, 0.0133 and 0.0181. In the traditional algorithm, the Prony algorithm cannot accurately identify the closer modal frequencies. Among the three modal frequencies, the ARMA algorithm is relatively good at identifying low-frequency frequencies, and the other two frequencies have poor results. The time-domain waveform of the real signal and the time-domain waveform of the signal restored by the modal parameters obtained by the SSI algorithm are compared as follows:

4. Conclusions
This paper first discusses the principle of modal identification based on the SSI algorithm, and verifies the correctness and accuracy of the modal identification of the SSI algorithm with analog signals. Then the noise immunity of the SSI algorithm is verified by adding noise to the analog signal. Finally,
by comparing the data collected on the actual site, it is determined that compared with other traditional algorithms Prony and ARMA, the SSI algorithm has better noise immunity and identification accuracy.

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