Product Portfolio Management in Competitive Environments

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Product diversity is one of the prominent factors for customers’ satisfaction, while from the firms’ perspective, the additional engineering costs required for product diversity should not exceed the acquired profits from the increase in their market share. Thus, one of the critical decision-making tasks for companies is the selection of an optimal mix of products, namely product portfolio management (PPM). Traditional studies on PPM problem have paid relatively less attention to the actions of other competitors. In this paper, we study PPM problem in a competitive environment where each firm’s objective is to maximize its expected shared surplus. We model the competition with an $n$-player game that optimal product portfolios are driven from its Nash equilibrium. Utility functions are determined by the expected value of the shared surplus. We analyze the strategic behavior of firms to determine their optimal product portfolios.

Key words: Product portfolio management, Shared surplus, Market segmentation, Nash equilibrium

1. Introduction

Portfolio management is strategic decision-making or strategic planning to forecast how firms should spend their scarce engineering, operation resources, and marketing resources to maximize their objective functions which is categorized as resource allocation problems. It focuses on having the right balance between number of projects and available resources or capabilities Cooper et al. (1999). However, an organization must optimize its product diversity to increase revenues Lancaster (1990). There are two methods for a business to succeed new products: doing projects right, and doing the right projects Cooper et al. (2000). Hence, an important decision-making for the firms is to offer the right product variety to the target market instead of creating various products in relation to anticipate all needs of customers Jiao et al. (2007).

Portfolio management has been extensively studied with different approaches. A linear programming method is applied for R&D project selection Jackson (1983). Financial
models and financial indexes, probabilistic financial models, options pricing theory, strategic approaches, scoring models and checklists, analytical hierarchy approaches, behavioral approaches, mapping approaches or bubble diagrams are different methods for selecting a new portfolio Cooper et al. (1999). While there are several approaches allowing companies to increase R&D productivity, their implementations impose new challenges for product portfolio management. In Cooper and Sommer (2020), new solutions are proposed to deal with these emerging challenges. The problem of product portfolio optimization is analyzed by population dynamic approach in which company’s product portfolio is considered as a product population Wang et al. (2021). The authors analyze the product population’s growth balance by the logistic model.

It also considers how the product population’s scale and structure can be continuously optimized in a way that balances enterprise output, product synergy, and resource allocation.

The conjoint-based approach to optimal product portfolio problem ends in integer optimization problems which are NP-hard problems Nair et al. (1995). For instance, the problem of designing a new product line in order to maximize its surplus which is determined based on customer preference is NP-hard Kohli and Krishnamurti (1989). In Belloni et al. (2008), efficient methods have been devised for the customer preference perspective. Furthermore, product portfolio decisions have been examined with a focus on the engineering implications, the cost and complexity of actions among multiple products Simpson (2004).

Furthermore, Jiao and Zhang Jiao and Zhang (2005) investigated product portfolio management with the view of customer-engineering interaction and established a maximizing shared-surplus model for PPM problem in the absent of competition view point where the detail of produce design has been included. They formulate the problem as an integer programming which is an NP-hard problem and also a heuristic genetic algorithm is applied to solve the relaxed version of the integer linear programming problem. However, diverse analyses have been applied to examine PPM problems. One of them is the concept of Nash equilibrium has been employed to model competitive reactions in produce design Choi and DeSarbo (1993) and product line design Kuzmanovic and Martic (2012).

The new entrant firm into a competitive market has been studied in Choi et al. (1990), Steiner (2010), Liu et al. (2015). The new entrant firm has more resources and pre-experience on the rivals’ behavior. Hence, it is can be expected to become a leader against
the other firms in the market (Choi et al., 1990). The Stackelberg (leader-followers) game has been applied to find out an optimal for a single product design (Steiner, 2010) and product portfolio (Liu et al., 2015). The objective of Liu et al. (2015) is to maximize the expected shared surplus of the new market entrant. Steiner formulates optimal product design problem based on the perspective of a profit-maximizing new entrant (the leader) who wants to launch a brand onto an existing product market and acts with foresight by anticipating price-design reactions of the incumbent firms (the Nash followers) (Steiner, 2010) while Liu et al.’s concern is to maximize the expected shared surplus of the new market entrant.

The competitive interactions of two firms to find an optimal product portfolio has been modeled by a non-cooperative complete information game (Sadeghi and Zandieh, 2011). In this game, utility functions for firms are determined based on the customer-engineering interaction model which is proposed in (Jiao and Zhang, 2005) and Nash equilibrium of this game is calculated for just one numerical example where two firms with four different products compete in a market with 3 segments. Product line design as an instance of PPM problem has been studied with the game theoretical approach. In Liu et al. (2017), the competitive interactions of firms to design product line has been formulated with the Stackelberg model and for an industrial case of cell phones and the analysis of finding an equilibrium of this model has been implemented. Moreover, the competition environment between two firms from the viewpoint of the product cost and customer satisfaction has been modeled by a Bayesian game (Yang et al., 2019).

John Nash introduced a solution concept for strategic-form games, called a Nash Equilibrium (NE) (Nash, 1951). He proved that every finite strategic-form game has a mixed strategy equilibrium. He presented two existence proofs: the first one was based on Kakutani’s Fixed Point Theorem (Nash et al., 1950), and the second one was based on Brouwer’s Fixed Point Theorem (Nash, 1951).

C. Papadimitriou (Papadimitriou, 1994) proposed the complexity class PPAD (Polynomial Parity Argument in a Directed graph) which is the class of all search problems that can be polynomially reduced to the END OF THE LINE problem: given two circuits $S$ and $P$, each with $n$ input bits and $n$ output bits, such that $P(0^n) = 0^n = S(0^n)$, find an input $x \in \{0, 1\}^n$ such that $P(S(x)) \neq x$ or $S(P(x)) \neq x \neq 0^n$. PPAD class contains several important problems that are suspected to be hard (Daskalakis et al., 2009). Moreover, it is shown
that finding a NE for a finite strategic-form game is PPAD-complete Daskalakis et al. (2009).

In this paper, we study an $n$-agent game which is a generalization of 2-agent game was developed in Sadeghi and Zandieh (2011) to model a competitive market with different segments for product portfolio management. The self-interest of each firm is to maximize its total expected shared surplus which is formulated in Jiao and Zhang (2005). Luxury brands in which the central point of such markets concentrates on the rich consumers is one example of a single market segmentation. In Sadeghi and Zandieh (2011), an example of two firm who compete in a single market segmentation has been examined. We analyze the process of finding a mixed Nash equilibrium for these game in a single market segmentation when all strategies in NE are inner points of strategy spaces which we call it an interior Nash equilibrium.

This paper is organized as follows. In Section 2 the PPM problem in competitive environment is formulated with an $n$-agent game. An analysis for finding an interior Nash equilibrium for a single market segmentation is presented in Section 3. Finally, the conclusion are presented in Section 4 with a number of areas for future works.

2. The Model

Suppose that $Z$ is a potential product set for a market which has multiple market segments, represented by $\{G_j : 1 \leq j \leq m\}$ such that each segment $G_j$ contains homogeneous customers with total demand $Q_j$. We enumerate the product set $Z$ with $\{1, \ldots, \rho\}$ where $\rho = |Z|$. Assume $n$ firms, $I = \{1, \ldots, n\}$, compete in the market. Let $Z_i \subseteq Z$ be the set of products which are technically possible for firm $i \in I$. Thus, a product portfolio for firm $i$ is a subset of $\Lambda_i \subseteq Z_i$ such that $\Lambda_i \neq \emptyset$. Hence, the pure strategy set for firm $i \in I$ which we denote it by $S_i$ is a set of non-empty subsets of $Z_i$. Consequently, a mixed strategy for this firm is a distribution on $S_i$.

Suppose that $\hat{\sigma}_i : S_i \rightarrow [0, 1]$ is a mixed strategy for firm $i$. We show that it induces a distribution $\sigma_i$ on $Z_i$. Let

$$\sigma_i(p) = \sum_{\substack{A \subseteq S_i \\ p \in A}} \frac{\hat{\sigma}_i(A)}{|A|}.$$
As each $\Lambda \in S_i$ appears as many times as its elements, it follows that $\sum_{p \in Z_i} \sigma_i(p) = 1$, which in turn shows that $\sigma_i$ is a distribution on $Z_i$. On the other hand, if $\sigma_i$ is a distribution on $Z_i$ and $\Lambda \in S_i$, we may define

$$\hat{\sigma}_i(\Lambda) = \sum_{p \in \Lambda} \frac{\sigma_i(p)}{|\{\Gamma \in S_i: p \in \Gamma\}|}$$

which gives a distribution on $S_i$.

Every product has certain engineering costs; so different firms may have different production costs due to different technologies. Thus, we denote the price of product $p \in Z$ produced by firm $i \in I$ in $G_j$, $1 \leq j \leq m$ by $\beta_{ijp}$. Assume, for firm $i$, customer preference for product $p \in Z_i$ in $G_j$ is represented by respective utility, $u_{ijp}$. Now, we model the competition among $n$ firms as a game $G$ and call it a PPM game.

**Definition 1.** A PPM game $G$ is a game in strategic form with the following structure

1. The set of firms, $I = \{1, \ldots, n\}$.
2. The set of $m$ market segments $G_j$ where $1 \leq j \leq m$.
3. Firm $i$’s strategy space

$$\Sigma_i = \{(\sigma_{i1}, \ldots, \sigma_{ip}) \in [0, 1]^{\rho} : \sum_{p=1}^{\rho} \sigma_{ip} = 1, \ p \notin Z_i \Rightarrow \sigma_{ip} = 0\}.$$  

4. Firm $i$’s payoff for the strategy profile $(\sigma_i, \sigma_{-i}) \in \Sigma_i \times \Sigma_{-i}$ is

$$u_i(\sigma_i, \sigma_{-i}) = \sum_{j=1}^{m} \sum_{p=1}^{\rho} \beta_{ijp} \cdot Q_j \cdot \frac{\exp(u_{ijp}) \cdot \sigma_{ip}}{\sum_{r=1}^{m} \sum_{q=1}^{\rho} \exp(u_{rjq}) \cdot \sigma_{rq}} \cdot \sigma_{ip}.$$  

For convenience, let $e_{ijp} = \exp(u_{ijp})$ for each $i \in I$, $1 \leq j \leq m$ and $1 \leq p \leq \rho$.

**Remark 1.** Let $(\sigma_i, \sigma_{-i}) \in \Sigma_i \times \Sigma_{-i}$. By the multinomial logit (MNL) model McFadden et al. (1977), the probability that a customer chooses product $p$ produced by firm $i$ in the segment market $G_j$ is equal to

$$P_{ijp}(\sigma_i, \sigma_{-i}) = \frac{e_{ijp} \cdot \sigma_{ip}}{\sum_{r=1}^{m} \sum_{q=1}^{\rho} e_{rjq} \cdot \sigma_{rq}}.$$  

for $i \in I$, $1 \leq j \leq m$ and $1 \leq p \leq \rho$.

### 3. Equilibrium Analysis

John Nash introduced a solution concept for strategic games which is called Nash Equilibrium Nash (1951). He proved that every finite strategic-form game has a mixed strategy
equilibrium. He presented two existence proofs: the first one was based on Kakutani’s Fixed
Point Theorem [Nash et al. (1950)], and the second one was based on Brouwer’s Fixed Point
Theorem [Nash (1951)]. Since a PPM game is a finite game, it has a mixed Nash equilibrium.
In this section, we explain how to find it when it is in \((0, 1)^n\).

Nash stated that a rational agent will only play the strategy which is a best response to
the strategies actually taken by its opponents. To formalize the statement, let

\[
\Delta(S_i) = \{\sigma : \sigma \text{ is a distribution on } S_i\},
\]

where \(S_i\) is a strategy set for agent \(i\). The best response of agent \(i\) in a strategic-form game
\(G\) is a correspondence \(BR_i : \Delta(S_{-i}) \to \Delta(S_i)\) given by

\[
BR_i(\sigma_{-i}) = \arg \max_{\sigma_i \in S_i} u_i(\sigma_i, \sigma_{-i})
\]

for each \(i\). Clearly, strategy profile \((\sigma_1, \ldots, \sigma_n) \in \prod_{i=1}^n \Delta(S_i)\) is a Nash equilibrium if and
only if \(\sigma_i \in BR_i(\sigma_{-i})\) for each agent \(i\).

Let \(G\) be a PPM game, and suppose that \(\sigma^*\) is a Nash equilibrium. Then \(\sigma_i^*\) is a
constrained (local-) maximum for \(u_i(\sigma_i, \sigma_{-i}^*)\) subject to \(\sum_{p=1}^\rho \sigma_{ip} = 1\) for each \(1 \leq i \leq n\); so if
\(\sigma^*\) is an interior point of \([0, 1]^n\) then we may use the Lagrange multipliers method to find
a subset of \(\mathbb{R}\) that must include \(\sigma^*\). To set things up, we first define

\[
g_i(\sigma_i) = \sum_{p=1}^\rho \sigma_{ip} - 1, \quad 1 \leq i \leq n.
\]

Then

\[
\frac{\partial u_i}{\partial \sigma_i}(\sigma_i^*, \sigma_{-i}^*) \parallel \frac{\partial g_i}{\partial \sigma_i}(\sigma_i^*, \sigma_{-i}^*), \quad 1 \leq i \leq n
\]

where

\[
\frac{\partial u_i}{\partial \sigma_i}(\sigma_i, \sigma_{-i}) = \left[ \frac{\partial u_i}{\partial \sigma_{i1}}(\sigma_i, \sigma_{-i}), \ldots, \frac{\partial u_i}{\partial \sigma_{i\rho}}(\sigma_i, \sigma_{-i}) \right],
\]

\[
\frac{\partial g_i}{\partial \sigma_i}(\sigma_i, \sigma_{-i}) = \left[ \frac{\partial g_i}{\partial \sigma_{i1}}(\sigma_i, \sigma_{-i}), \ldots, \frac{\partial g_i}{\partial \sigma_{i\rho}}(\sigma_i, \sigma_{-i}) \right].
\]

Hence, there are constants \(\lambda_1, \ldots, \lambda_n \in \mathbb{R}\) such that

\[
\frac{\partial u_i}{\partial \sigma_i}(\sigma_i^*, \sigma_{-i}^*) = \lambda_i \frac{\partial g_i}{\partial \sigma_i}(\sigma_i^*, \sigma_{-i}^*), \quad 1 \leq i \leq n.
\]
As \( \frac{\partial u_i}{\partial \sigma_i} = [1, \ldots, 1] \), it follows that

\[
\frac{\partial u_i}{\partial \sigma_{is}} (\sigma^*_i, \sigma^*_{-i}) = \frac{\partial u_i}{\partial \sigma_{it}} (\sigma^*_i, \sigma^*_{-i}), \quad 1 \leq i \leq n, \ 1 \leq s, t \leq \rho.
\]

Doing some calculations we get

\[
\frac{\partial u_i}{\partial \sigma_{is}} (\sigma^*_i, \sigma^*_{-i}) = \sum_{j=1}^{m} \frac{2 \beta_{ij}s Q_j e_{ij}s \sigma^*_{is} \left( \sum_{p=1}^{\rho} \sum_{r=1}^{n} e_{rjp} \sigma^*_r - \sum_{p=1}^{\rho} \beta_{ijp} Q_j e_{ijp} e_{ij}s \sigma^*_{ip}^2 \right)}{\left( \sum_{p=1}^{\rho} \sum_{r=1}^{n} e_{rjp} \sigma^*_r \right)^2}.
\]

From now on, we consider the case \( m = 1 \); so for all \( 1 \leq s, t \leq \rho \) we must have

\[
2 \beta_{il}s Q_1 e_{il}s \sigma^*_{is} \left( \sum_{p=1}^{\rho} \sum_{r=1}^{n} e_{rlp} \sigma^*_r \right) - \sum_{p=1}^{\rho} \beta_{ipl} Q_1 e_{ip} e_{il}s \sigma^*_{ip}^2
\]

\[
= 2 \beta_{il} Q_1 e_{il} \sigma^*_{it} \left( \sum_{p=1}^{\rho} \sum_{r=1}^{n} e_{rlp} \sigma^*_r \right) - \sum_{p=1}^{\rho} \beta_{ipl} Q_1 e_{ip} e_{il}t \sigma^*_{ip}^2.
\]

Doing some manipulations, we get

\[
(e_{il}s - e_{il}t) \sum_{p=1}^{\rho} \beta_{ipl} e_{ip} \sigma^*_{ip}^2 = 2(\beta_{il}s e_{il}s \sigma^*_{is} - \beta_{il}t e_{il}t \sigma^*_it) \sum_{p=1}^{\rho} \sum_{r=1}^{n} e_{rlp} \sigma^*_r
\]

or

\[
\frac{\beta_{il}s e_{il}s \sigma^*_{is} - \beta_{il}t e_{il}t \sigma^*_it}{e_{il}s - e_{il}t} = \frac{\sum_{p=1}^{\rho} \beta_{ipl} e_{ip} \sigma^*_{ip}^2}{2 \sum_{p=1}^{\rho} \sum_{r=1}^{n} e_{rlp} \sigma^*_r}.
\]

As the right hand side does not depend on \( s \) or \( t \), we denote it by \( k_i(\sigma^*) \) and rewrite the above equation as

\[
\sigma^*_i = \frac{e_{il}s - e_{il}t}{\beta_{il}s e_{il}s} k_i(\sigma^*) + \frac{\beta_{il}t e_{il}t}{\beta_{il}s e_{il}s} \sigma^*_it, \quad 1 \leq s, t \leq \rho.
\]

Summing on \( s \) from 1 to \( \rho \) gives

\[
1 = k_i(\sigma^*) \left( \sum_{p=1}^{\rho} \frac{e_{ilp} - e_{il}t}{\beta_{ipl} e_{ilp}} \right) + \sigma^*_it \left( \sum_{p=1}^{\rho} \frac{\beta_{il}t e_{il}t}{\beta_{il}s e_{il}s} \right).
\]

Setting

\[
E_{it} = \sum_{p=1}^{\rho} \frac{e_{ilp} - e_{il}t}{\beta_{ipl} e_{ilp}}, \quad B_{it} = \sum_{p=1}^{\rho} \frac{\beta_{il}t e_{il}t}{\beta_{il}s e_{il}s}
\]

we may write

\[
k_i(\sigma^*) = \frac{1 - \sigma^*_it B_{it}}{E_{it}}
\]
and hence
\[
\frac{1 - \sigma^*_is}{E_is} = \frac{1 - \sigma^*_it}{E_it}, \quad 1 \leq s, t \leq \rho.
\]

It follows that
\[
\sigma^*_is = \frac{E_{it} - E_is}{E_{it}B_is} + \frac{E_isB_{it}}{E_{it}B_is}\sigma^*_it.
\]

So we may write each $\sigma^*_is$ in terms of $\sigma^*_i1$ and appropriate constants.

To write the relations in a more compact form, let
\[
a_{is} = \frac{E_{i1} - E_is}{E_{i1}B_is}, \quad b_{is} = \frac{E_isB_{i1}}{E_{i1}B_is},
\]

hence we may write $\sigma^*_is = a_{is} + b_{is}\sigma^*_i1$ and
\[
\sum_{r=1}^{n} \sum_{p=1}^{\rho} e_{r1p}\sigma^*_rp = \sum_{r=1}^{n} \sum_{p=1}^{\rho} (a_{rp}e_{r1p} + b_{rp}e_{r1p}\sigma^*_r1)
\]
\[
= \left( \sum_{r=1}^{n} \sum_{p=1}^{\rho} a_{rp}e_{r1p} \right) + \sum_{r=1}^{\rho} \left( \sum_{p=1}^{\rho} b_{rp}e_{r1p} \right) \sigma^*_r1
\]
\[
= a + \sum_{r=1}^{n} b_r\sigma^*_r1,
\]

where
\[
a = \sum_{r=1}^{n} \sum_{p=1}^{\rho} a_{rp}e_{r1p}, \quad b_r = \sum_{p=1}^{\rho} b_{rp}e_{r1p}, \quad 1 \leq r \leq n,
\]

so
\[
u_i(\sigma^*_i1, \ldots, \sigma^*_in) = \frac{\sum_{p=1}^{\rho} \beta_{i1p}Q_1 e_{i1p}\sigma^*_ip^2}{a + \sum_{r=1}^{n} b_r\sigma^*_r1}
\]
\[
= \frac{\sum_{p=1}^{\rho} \beta_{i1p}Q_1 e_{i1p}(a_{ip} + b_{ip}\sigma^*_i1)^2}{a + \sum_{r=1}^{n} b_r\sigma^*_r1}.
\]

Now, for each $i \in I$, we define function $v_i$ from $[0,1]^n$ to $\mathbb{R}$ by
\[
v_i(\tau_1, \ldots, \tau_n) = \frac{\sum_{p=1}^{\rho} \beta_{i1p}Q_1 e_{i1p}(a_{ip} + b_{ip}\tau_i)^2}{a + \sum_{r=1}^{n} b_r\tau_r}.
\]

Since $(\sigma^*_i1, \ldots, \sigma^*_in)$ locally maximizes $u_i$, we may find it with usual techniques for maximizing $v_i$. As each $\sigma^*_is$ can be expressed in terms of $\sigma^*_i1$, this gives the whole point $\sigma^*$. 
4. Conclusion
This research captures the competition among firms in a market where firms have to
decide on which subset of products to produce with differentiated products. The object
for the firms is to maximize their expected shared surplus. The competition among firms
is modeled by a non-cooperative game, called PPM game, where the utilities of agents is
measured by their expected shared surplus. The Lagrange multipliers method is used to
compute an interior Nash equilibrium (if there exist any) for a single market segmentation.

Future studies can focus on analyzing Nash equilibrium to predict strategic behavior
of firms in more general markets with different markets. The solution concept of $\varepsilon$-Nash
equilibrium provides the permission of the unilateral deviation of $\varepsilon$ value which may be
useful for using Lagrange multipliers method for finding a mixed Nash equilibrium of a
PPM game in which supports of some mixed strategies in equilibrium are singletons.

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