A Linear Fractional Transformation Based Approach to Robust Model Predictive Control Design in Uncertain Systems

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ABSTRACT

A novel robust model predictive control (RMPC) scheme is developed for uncertain nonlinear systems. To the RMPC design, firstly, the uncertain system would be described using a linear fractional transformation (LFT). Then, regarding the system’s uncertainties and control limitations, a linear matrix inequality (LMI) based control strategy is addressed to translate the RMPC synthesis into a minimization problem. Thus the controller’s gains are automatically updated at some time-instants by the solution of such optimization problem. Finally, the outcomes are numerically applied in some control examples. The simulation results show the effectiveness of the suggested robust predictive controller in comparison to similar RMPC techniques.

INDEX TERMS

Linear fractional transformation, linear matrix inequality, robust model predictive control, uncertain systems.

I. INTRODUCTION

Usually, the control issue of dynamical systems is increasingly affected by uncertain and unknown terms. There have been several standard frameworks for modeling the system’s uncertainties. The additive, multiplicative, polytopic, and linear fractional transformation (LFT) would be the most discussed ones [1]. In comparison to the other traditional forms, the LFT models support the following benefits:

a) Multiple types of uncertainties can be considered simultaneously. Nonetheless, the uncertainties may be raised from several sources.

b) All unknown expressions would be treated as a single uncertain term.

c) The conventional description formula can be counted as a particular form of the LFT model.

As a result, wide variations of the uncertain systems would be covered using the LFT tool. Until now, this methodology has been applied to the linear time-invariant (LTI) systems described via a transfer function [2]. Some robust discrete-time approaches have been developed for conic-type uncertain nonlinear systems [3], as well as the hidden Markov jump systems [4]. An iterative $H_\infty$ optimal control law is designed for nonlinear systems [5].

Commonly, the non-LFT forms (i.e., additive, multiplicative, polytopic types, and so on) may not entirely describe the system’s uncertainties. Thus, the characteristics of the closed-loop response, as well as the transient performance, and asymptotic stability, would be significantly degraded by applying the conventional control strategies. Consequently, current approaches to the robust controller design may be unsatisfactory in the uncertain systems. To date, stability analysis and controller synthesis play a crucial role in studying uncertain systems. Some mathematical tools have been developed to manage such difficulties. However, linear matrix inequality (LMI) has been attracted widespread interest among these techniques [6]. Some LFT based control methods have been addressed to uncertain linear systems. These briefly includes robust $H_\infty$ output regulation [7], output feedback control of the LFT systems under actuator saturation [8], and robust $H_2$ and $H_\infty$ filters [9].

In the last two decades, there has been a rapid rise in model predictive control (MPC) for industrial control applications [10]. To achieve this goal, firstly, a mathematical model is used to predict the plant’s future in the discrete-time domain. Then the limitations are appropriately formulated as
a constraint set. Finally, the optimization problem would be solved in an on-line way to determine the optimum control sequence. Hence, the MPC may be recognized as a real-time optimal control method [11]. The classical MPCs are mainly developed for the discrete-time representation [12]. They may be used in closed-form or require to solve an on-line optimization problem. Despite this, such an MPC suffers from a series of pitfalls due to the presence of unknown parameters. Thus robust model predictive control (RMPC) would still be an open problem in the uncertain control systems [13]. Similar to the standard MPCs, the suggested methodology has the following features:

a) A nominal model of the plant is used to predict the future response.

b) An optimization problem is solved in a real-time way to determine the control signals.

c) The control limitations can be handled as an extra constraint.

Thus, the proposed set-up can be categorized in the MPC family. Many attempts to the RMPC design, based on the LMI, have been reported in the literature. The works in this field focused primarily on the non-LFT models, as well as the tubes [14], [15], norm bounded uncertainty [16], and the constrained ones [17], [18]. A computationally efficient algorithm is suggested for the RMPC design in uncertain nonlinear systems [19]. Furthermore, the RMPC has been investigated in the switched linear systems [20], linear systems with disturbances [21], positive systems [22], linear parameter-varying systems [23]–[25], output tracking issues [26], and Markovian jump systems [27], [28]. Also, the RMPC is developed using two-stages neural network modeling [29], considering state-dependent uncertainties [30], under partial actuator faults [31], guaranteeing stability and satisfying constraints [32], assuming saturated inputs and randomly occurring uncertainties [33], involving finite-time convergence result [34], and employing collective neuro-dynamic optimization [35]. Although the RMPC synthesis is primarily discussed in the discrete-time system, it is extended to continuous-time representations [36], [37].

Despite the afore-mentioned LFT’s capabilities, to the author’s best knowledge, very few publications are available that discuss the RMPC design for the continuous-time LFT models. Thus, the above control methods may not be satisfactory. An LFT based MPC is designed for an autonomous vehicle [38]. The estimation of the nonlinear tunnel diode circuits [39] and the output feedback control of nonlinear systems [40] are studied using linear fractional parametric uncertainties. The finding motivates the authors to develop an RMPC for a class of the constrained uncertain systems. Thus, the uncertain terms and constrained control inputs would be the main complexities of the problem. Compared to the other control approaches, the key contribution and novelties of the method may be summarized as follows:

a) All uncertain terms of the dynamical system are formulated via the LFT model.

b) The control constraints are translated into another LMI condition.

c) A minimization problem, subjected to some LMIs, would be solved numerically at some known time-instants. Then the controller parameters are updated shortly.

These features would be remarkable points of the proposed approach. The LMI based control methods usually lead to some sufficient conditions. Thus these methods would be inherently conservative. However, the proposed RMPC contains some extra and free parameters that reduce the conservativeness. Besides, the RMPC method is numerically simulated in some control examples. The obtained results would also be compared with similar RMPC methods. The outcomes indicate that the proposed control method is robust regarding all the uncertainties and input constraints. Thus, the effectiveness of the addressed robust control method would be shown in the simulation.

The paper is organized as follows: The mathematical preliminaries and required inequalities are presented in Section 2. The control problem is formulated for uncertain systems in Section 3. The main results of the paper are addressed in Section 4. The proposed idea is simulated in some control examples in Section 5. Some concluding remarks are drawn in the last section.

II. MATHEMATICAL PRELIMINARIES

In this paper, the operator \( \| \| \) denotes the two-norm, \( I_n \) is \( n \times n \) identity matrix, the symbol * describes the symmetric property. Define a partitioned matrix \( N \in \mathbb{R}^{(p+r)\times(q+s)} \) as follows:

\[
N = \begin{bmatrix}
N_{11} & N_{12} \\
N_{21} & N_{22}
\end{bmatrix},
\]

where \( N_{11} \in \mathbb{R}^{p\times q} \), \( N_{12} \in \mathbb{R}^{p\times s} \), \( N_{21} \in \mathbb{R}^{r\times q} \), and \( N_{22} \in \mathbb{R}^{r\times s} \) are some compatible matrices. Then the lower LFT is defined as the following [1]:

\[
F_L(N, \Delta) \overset{\text{def}}{=} N_{11} + N_{12} \Delta (I - N_{22} \Delta)^{-1} N_{21},
\]

where \( \Delta \in \mathbb{R}^{r\times r} \) is an uncertain matrix. The LFT term (1) would be well-posed (or well-defined) if \( \det (I - N_{22} \Delta) \neq 0 \). Hence the boundedness of \( F_L(N, \Delta) \) is insured with respect to \( \Delta \). Applying the small gain theorem [41], a sufficient well-posedness condition may be found as follows:

\[
\| N_{22} \| \cdot \| \Delta \| < 1.
\]

Similarly, the upper LFT could be defined as [1]:

\[
F_u(N, \Delta) \overset{\text{def}}{=} N_{22} + N_{21} \Delta (I - N_{11} \Delta)^{-1} N_{12}.
\]

It can be seen that any upper LFT can be rewritten as an upper LFT. In this study, the uncertain terms would only be described through the lower LFT.

Schur Complement Lemma [6]: Let \( X \in \mathbb{R}^{q\times q} \), \( Y \in \mathbb{R}^{r\times r} \) be some symmetric matrices, and \( Z \in \mathbb{R}^{q\times r} \) be a rectangular matrix. Then the following inequalities would be
equivalent:
\[
\begin{bmatrix}
0 & -X - Z \gamma^T \\
0 & Y - Z^T 
\end{bmatrix} > 0, \quad (4)
\]

**Fact 1:** Let \( U \in \mathbb{R}^{r \times q}, V \in \mathbb{R}^{q \times p}, \) and \( \rho \) be a positive constant. Then the following inequality holds:
\[
U V + V^T U^T \leq \rho U U^T + \frac{1}{\rho} V V^T. \quad (5)
\]

**Fact 2:** The inequality \( X \leq \|X\| I_q \) holds for any matrix \( X \in \mathbb{R}^{q \times q}. \)

Next, an LFT based control law design would be investigated in a class of uncertain systems.

**III. PROBLEM FORMULATION**

Consider the following uncertain continuous-time system:

\[
\begin{bmatrix}
\frac{dx}{dt} \\
\phi
\end{bmatrix} = \begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix} \begin{bmatrix}
x \\
\omega
\end{bmatrix} + \begin{bmatrix}
B_1 \\
B_2
\end{bmatrix} u + \begin{bmatrix}
f_1(x) \\
f_2(x)
\end{bmatrix}, \quad t \geq t_0. \quad (6)
\]

In the uncertain system (6), \( x(t) \in \mathbb{R}^n \) is the state vector, \( x_0 = x(t_0) \) is the initial conditions, and \( u(t) \in \mathbb{R}^m \) is the control effort. The system’s uncertainties are limited to the matrix \( \Omega \in \mathbb{R}^{rs \times r} \) (i.e., \( \omega(t) = \Omega(t) \)). Many control systems, like the mass-spring-damper equation, can be described by Eq. (6). A simplified schematic of the uncertain system (6) may be depicted as Fig. 1. Additionally, the terms \( \omega(t) \in \mathbb{R}^r \) and \( \phi(t) \in \mathbb{R}^r \) denote the output and input of the uncertain block \( \Omega \), respectively.

**FIGURE 1. Schematic of the uncertain system (6).**

**Assumption 1:** The uncertain term \( \Omega \) is supposed to be unknown and satisfied with the following norm condition:

\[
\|\Omega\| \leq \delta, \quad (7)
\]

where \( \delta \) is a known constant while \( \delta \|A_{22}\| < 1 \). Moreover, the uncertain matrix \( \Omega \) may be a time-invariant or time-varying one.

**Assumption 2:** The states of the uncertain system (6) are available and measurable for the control purpose. Additionally, the uncertain system (6) is stabilizable in the presence of uncertainty \( \Omega \). To put it another way, there exists a control signal \( u(t) \) such that the closed-loop system is asymptotically stable.

**Assumption 3:** The nonlinear functions \( f_1(x) \) and \( f_2(x) \) are some unknown terms, but satisfy the following condition:

\[
\|f_1(x)\| \leq \|M_1 x\|, \quad (8)
\]

for some given matrices \( M_1 \) and \( M_2 \).

Substituting \( \omega = \Delta \phi \), Eq. (6) may be modified as follows:

\[
\frac{dx}{dt} = \left( A_{11} + A_{12} \Delta (I_r - A_{22} \Delta)^{-1} A_{21} \right) x + f_1(x) + A_{12} \left( B_1 + A_{12} \Delta (I_r - A_{22} \Delta)^{-1} B_2 \right) u + \Delta (I_r - A_{22} \Delta)^{-1} f_2(x). \quad (9)
\]

Therefore, the uncertain system (6) can be written as follows:

\[
\frac{dx}{dt} = A(\Delta) x + B(\Delta) u + f_\Delta(x), \quad (10)
\]

where \( A(\Delta), B(\Delta), \) and \( f_\Delta(x) \) are computed as:

\[
A(\Delta) = F_L \left[ \begin{bmatrix} A_{11} & A_{12} \end{bmatrix}, \Delta \right], \quad (11)
\]

\[
B(\Delta) = F_L \left[ \begin{bmatrix} B_1 & A_{12} \end{bmatrix}, \Delta \right], \quad (12)
\]

\[
f_\Delta(x) = f_1(x) + A_{12} \Delta (I_r - A_{22} \Delta)^{-1} f_2(x). \quad (13)
\]

The condition (7) guarantees that the LFT terms \( A(\Delta), B(\Delta), \) and \( f_\Delta(x) \) would be well-posed in the uncertain system (10). A straightforward form of Eq. (10) may be interesting by excluding the nonlinear terms and choosing \( A_{22} = 0 \) [42].

Utilizing the condition (8), there exists a matrix \( M \) such that the nonlinear term \( f_\Delta(x) \) satisfies the following inequality:

\[
\|f_\Delta(x)\| \leq \|M x\|. \quad (14)
\]

Ignoring the nonlinear term \( f_\Delta(x) \), Eq. (10) characterizes an LTI uncertain system. However, Eq. (10) would be found by applying the LFT approach. Hence the continuous-time uncertain system (10) is used for designing RMPC in this study. In order to realize the control system, a state feedback control law may be used as follows:

\[
u(t) = F(k) x(t), \quad t_k \leq t < t_{k+1}, \quad k = 0, 1, 2, \ldots \quad (15)
\]

The controller gain \( F(k) \) would be computed and updated at time-instant \( t_k \). At time-instant \( t_k \), the proposed algorithm is solved to find the gain \( F(k) \). Then the controller gains are updated. This process is repeated at next time-instant \( t_{k+1} \). Thus in the interval \([t_k, t_{k+1})\) the gain \( F(k) \) would be constant. The time-instants \( t_k \) may be either predefined or generated by defining a threshold. In the simulation section, the time-instants \( t_k \) are taken periodically.

In the MPC, the plant behavior would be anticipated with a nominal model at time-instant \( t_k \). The uncertain plant (10) is described by an LFT form. The matrices \( A_{11}, A_{12}, A_{21}, A_{11}, B_1, \) and \( B_2 \) are known terms in the LFT model. The system
uncertainties are limited to the matrix $\Delta$. Thus, the matrices mentioned above can predict the future behavior of the uncertain system (10) implicitly. The uncertain system (10) coupled with the state feedback control (12) may be written as follows:

$$\begin{bmatrix} \frac{dx}{dt} \\ \phi \end{bmatrix} = \begin{bmatrix} A_{11} + B_1F(\kappa) & A_{12} \\ A_{21} + B_2F(\kappa) & A_{22} \end{bmatrix} x + \begin{bmatrix} f_1(x) \\ f_2(x) \end{bmatrix}.$$ (13)

The closed-loop system (13) can be written as the following:

$$\frac{dx}{dt} = A_c x + f_\Delta(x),$$ (14)

where $A_c = A(\Delta) + B(\Delta) F$.

At time-instant $t_k$, the control law (12) is selected by minimizing the following cost function:

$$J(t_k) = \int_{t_k}^{+\infty} \left( x^T(\tau|t_k) Q x(\tau|t_k) + u^T(\tau) R u(\tau) \right) d\tau,$$ (15)

where the weight matrices $Q$ and $R$ are some positive definite or semi-definite ones. The matrices $Q$ and $R$ could be decomposed as follows:

$$Q = \left( Q^1 \right)^T Q^1, \quad R = \left( R^1 \right)^T R^1$$ (16)

Such weights determine the effects of the states and control signals on the performance index. The appropriate selection of the weight matrices $Q$ and $R$ are referred to as the MPC tuning. Many tuning algorithms have been suggested for the MPC, as well as reference model matching, optimization techniques, try and error.

The control law shall be designed to meet a control objective while some restrictions are satisfied. Nevertheless, many control goals may be met in a typical control application, as well as following a reference model, specifications of the desired response, and so on.

Although the MPC may seem quite classical, it has various abilities and many advantages. Some of the important ones are listed as follows:

1. The system’s uncertainties and disturbances can be handled in the control problem.
2. The control constraint, as well as the actuator saturations, can be considered in the formulation.
3. A cost function is minimized to compute the control signal. Thus it can be an optimal controller.

Accordingly, in many practical applications, the MPC would be the best control technique for the uncertain constrained system. Hence, the MPC approach is interested in this paper.

Assumption 4: The control input $u(t)$ is supposed to be bounded as follows:

$$\|u(t)\| \leq u_{\text{max}}.$$ (17)

The principal challenges and complexities, should be handled here, can be itemized as follows:

1. The nonlinear function $f_\Delta(x)$ is unknown.
2. The matrix $\Delta$ is an uncertain term.
3. The control signal is limited by $u_{\text{max}}$.
4. The nominal system may be stable or unstable.

Next, considering the system’s uncertainty $\Delta$, unknown function $f_\Delta(x)$, and the control limitations, an LMI based RMPC would be applied to the dynamical system (10).

IV. RMPC SYNTHESIS

In the uncertain system (10), the expected goals can be achieved via the control law (12). Hence, in some known and predefined time-instants $t_k$, a minimization problem may be solved numerically. Then, the gains of the controller (12) would be updated immediately. The main result is presented in the next proposition.

Proposition 1: Consider the uncertain system (10) with Assumptions 1-4. At time-instant $t_k$, if there exists a positive definite matrix $X \in \mathbb{R}^{n \times n}$, a rectangular matrix $Y \in \mathbb{R}^{m \times n}$ and positive constants $\rho_1, \rho_2, \gamma$ such that the following minimization problem admits a feasible solution:

$$\min \gamma$$ (18)

subject to

$$\begin{bmatrix} X & x(t_k) \\ * & 1 \end{bmatrix} \succeq 0,$$ (19)

$$\begin{bmatrix} X & \ast \\ Y & u_{\text{max}}^2 I_m \end{bmatrix} \succeq 0,$$ (20)

$$\begin{bmatrix} \Pi & \ast & \ast & \ast & \ast \\ \Omega - \rho_1 I_r & \ast & \ast & \ast & \ast \\ \rho_1 \alpha A^T_{12} & 0 & -\rho_1 I_s & \ast & \ast \\ M X & 0 & 0 & -\rho_2 I_m & \ast \\ Q^{1/2} X & 0 & 0 & 0 & -\gamma I_n \\ R^{1/2} Y & 0 & 0 & 0 & 0 & -\gamma I_m \end{bmatrix} \leq 0,$$ (21)

where $\Pi = A_{11} X + B_1 Y + X A_{11}^T + Y^T B_1^T + \rho_2 I_m$, $\Omega = A_{21} X + B_2 Y$, $\alpha = \frac{1}{4} A_{12}^T A_{12}$, then the closed-loop system would be asymptotically stabilized by the state feedback control law (12) with the gain $F(\kappa) = Y X^{-1}$. Furthermore, the constant $\gamma$ would be an upper-bound of the cost function (15) at time-instant $t_k$.

Proof Consider a quadratic Lyapunov function as follows:

$$V(x) = x^T Px,$$ (22)

where $P$ is a positive definite matrix.

Firstly, it is shown that the LMI (21) can be a sufficient condition for the following inequality:

$$\frac{d}{dt} V(x) \leq -(x^T Q x + u^T R u).$$ (23)

Secondly, the LMI (19) guarantees the minimum cost for the uncertain system (10). Finally, the control limitations are satisfied by the LMI (20).

Let us integrate both sides of the inequality (23) from $t_k$ to $+\infty$. Then

$$\lim_{t \to +\infty} V(x(t)) - V(x(t_k)) \leq -J(t_k).$$ (24)
The Barbalat lemma [41] implies \( \lim_{t \to \infty} V(x(t)) = 0 \). Thus, a reasonable upper-bound of the cost function \( J(t) \) can be found as follows:

\[
J(t_k) \leq V(x(t_k)) = x^T(t_k) P x(t_k). 
\]  

(25)

Hence \( V(x(t_k)) \) admits an upper-bound like \( \gamma \). Then

\[
x^T(t_k) P x(t_k) \leq \gamma. \tag{26}
\]

The inequality (23) can be written as follow:

\[
x^T P A_c x + x^T A_c^T P x + x^T P f(x) + f^T(x) P x \leq - \left( x^T Q x + u^T R u \right). \tag{27}
\]

Utilizing Fact 1, the nonlinear term of (27) may have an upper-bound as follows:

\[
x^T P f(x) + f^T(x) P x \leq \bar{\rho}_2 x^T P^2 x + \frac{1}{\bar{\rho}_2} x^T M^T M x. \tag{28}
\]

for any constant \( \bar{\rho}_2 > 0 \). Then, using the condition (11), inequality (28) is written as:

\[
x^T P f(x) + f^T(x) P x \leq \bar{\rho}_2 x^T P^2 x + \frac{1}{\bar{\rho}_2} x^T M^T M x. \tag{29}
\]

Thus inequality (27) would be satisfied if the following condition holds:

\[
A_c^T P + PA_c + \bar{\rho}_2 P^2 + \frac{1}{\bar{\rho}_2} M^T M \leq -Q - F^T R F. \tag{30}
\]

Let us pre and post multiply the inequality (30) by the matrix \( \gamma^2 P^{-1} \). Then

\[
\gamma P^{-1} A_c^T + \gamma A_c P^{-1} + \gamma \bar{\rho}_2 I_n + \gamma \frac{1}{\bar{\rho}_2} P^{-1} M^T M P^{-1} \leq - \gamma P^{-1} Q P^{-1} - \gamma P^{-1} F^T R F P^{-1}. \tag{31}
\]

Now, define the matrices \( X \overset{d}{=} \gamma P^{-1}, Y \overset{d}{=} \gamma F P^{-1}, \) and \( \rho_2 \overset{d}{=} \gamma \bar{\rho}_2 \). It is clear that the gain \( F(k) \) would be found as \( F(k) = Y X^{-1} \). Then

\[
XA_c^T + A_c X + \rho_2 I_n + \frac{1}{\rho_2} X M^T M X + \frac{1}{\gamma} X Q X + \frac{1}{\gamma} Y R Y \leq 0. \tag{32}
\]

The inequality (32) may be written as follows:

\[
\begin{align*}
A_{11} X + B_1 Y + X A_{11}^T + Y T B_1^T + \rho_2 I_n + \frac{1}{\rho_2} X M^T M X \\
+ \frac{1}{\gamma} X Q X + \frac{1}{\gamma} Y R Y + A_{12} \Delta (I_r - A_{22} \Delta)^{-1} (A_{21} X + B_2 Y) \\
+ (A_2 X + B_2 Y)^T \left( I_r - \Delta^T A_{22}^T \right)^{-1} \Delta^T A_{12}^T \leq 0. \tag{33}
\end{align*}
\]

Using Fact 1, the uncertain terms of the inequality (33) may be bounded as:

\[
\begin{align*}
A_{12} \Delta (I_r - A_{22} \Delta)^{-1} (A_{21} X + B_2 Y) \\
+ \left( X A_{21}^T + Y T B_1^T \right) (I_r - A_{22} \Delta)^{-T} \Delta^T A_{12}^T \\
\leq \rho_1 A_{12} \Delta (I_r - A_{22} \Delta)^{-1} (I_r - A_{22} \Delta)^{-T} \Delta^T A_{12}^T \\
+ \frac{1}{\rho_1} (A_2 X + B_2 Y)^T (A_{21} X + B_2 Y). \tag{34}
\end{align*}
\]

Hence, the inequality (33) can be modified as follows:

\[
\begin{align*}
A_{11} X + B_1 Y + X A_{11}^T + Y T B_1^T + \rho_2 I_n \\
+ \frac{1}{\rho_2} X M^T M X + \frac{1}{\gamma} X Q X + \frac{1}{\gamma} Y R Y \\
+ \rho_1 A_{12} \Delta (I_r - A_{22} \Delta)^{-1} (I_r - A_{22} \Delta)^{-T} \Delta^T A_{12}^T \\
+ \frac{1}{\rho_1} (A_2 X + B_2 Y)^T (A_{21} X + B_2 Y) \leq 0. \tag{35}
\end{align*}
\]

Employing Assumption 1 and Fact 2, we have:

\[
\begin{align*}
A_{12} \Delta \leq \delta \|A_{22}\| I_r. \tag{36}
\end{align*}
\]

Then the following inequalities would be deduced:

\[
\begin{align*}
(I_r - A_{22} \Delta)^{-1} \leq \frac{1}{\delta \|A_{22}\|} & I_r, \tag{37} \\
(I_r - A_{22} \Delta)^{-T} \leq \frac{1}{\delta \|A_{22}\|} & I_r.
\end{align*}
\]

Then the uncertain term of the inequality (35) may be bounded as:

\[
\begin{align*}
A_{12} \Delta (I_r - A_{22} \Delta)^{-1} (I_r - A_{22} \Delta)^{-T} \Delta^T A_{12}^T \\
\leq \frac{1}{\delta^2 \|A_{22}\|^2} A_{12} \Delta \Delta^T A_{12}^T. \tag{38}
\end{align*}
\]

Utilizing Fact 2, \( A_{12} \Delta \Delta^T A_{12}^T \) may have an upper-bound as follows:

\[
\begin{align*}
A_{12} \Delta \Delta^T A_{12}^T \leq \frac{\delta}{\delta - 1} A_{12} A_{12}^T. \tag{39}
\end{align*}
\]

Then inequality (38) would have an upper-bound as the following:

\[
\begin{align*}
A_{12} \Delta (I_r - A_{22} \Delta)^{-1} (I_r - A_{22} \Delta)^{-T} \Delta^T A_{12}^T \leq \alpha^2 A_{12} A_{12}^T, \tag{40}
\end{align*}
\]

where \( \alpha = \frac{\delta}{\delta - 1} \overset{\text{def}}{=} \frac{\delta}{\delta - 1} \|A_{22}\|. \) Hence, the inequality (35) is written as follows:

\[
\begin{align*}
A_{11} X + B_1 Y + X A_{11}^T + Y T B_1^T + \rho_2 I_n \\
+ \frac{1}{\rho_2} X M^T M X + \frac{1}{\gamma} X Q X + \frac{1}{\gamma} Y R Y + \rho_1 \alpha^2 A_{12} A_{12}^T \\
+ \frac{1}{\rho_1} (A_2 X + B_2 Y)^T (A_{21} X + B_2 Y) \leq 0. \tag{41}
\end{align*}
\]

Therefore, inequality (21) would be concluded using the Schur complement lemma. As a consequence, the inequality (23) is guaranteed through inequality (21).

The inequality (26) may be modified as the following form:

\[
x^T(t_k) X^{-1} x(t_k) \leq 1. \tag{42}
\]

As a result, the inequality (19) is deduced using the Schur complement lemma. The 2-norm of the control input is written as:

\[
\|u(t)\|^2 = x^T(t) F^T F x(t) = x^T(t) X^{-1} Y^T Y X^{-1} x(t). \tag{43}
\]
The LMI (20) is equivalent to the following inequality:
\[ Y^T Y \leq u_{max}^2 X. \] (44)

Let us multiply both sides of the inequality (44) by the matrix $X^{-1}$. Then
\[ X^{-1} Y^T Y X^{-1} \leq u_{max}^2 X^{-1}. \] (45)

The inequality (42) would lead to the following condition:
\[ x^T (t) X^{-1} x (t) \leq 1. \] (46)

Therefore, the control input $u (t)$ would be bounded above as follows:
\[ \|u (t)\| \leq u_{max}. \] (47)

It completes the proof.

The proposed control technique can be easily performed in practical applications. Hence, an implementable algorithm may be implemented as follows:

**Algorithm 1 RMPC Design for Uncertain Systems**

**Step 1.** Get the data $A_{11}, A_{12}, A_{21}, A_{22}, B_1, B_2, \delta, M, Q, R$, and $u_{max}$.

**Step 2.** Checking the MPC execution times $t_k$, $k = 0, 1, 2, \ldots$

**Step 3.** Get the values of the states $x (t_k)$.

**Step 4.** Solve the optimization problem suggested by Proposition 1.

**Step 5.** Update the controller’s gains $F (k) = YX^{-1}$.

**Step 6.** Return to Step 2.

In Fig. 2, a simple flowchart is illustrated for the proposed RMPC method.

**Corollary 1:** Proposition 1 can be used to design the exponentially stabilizing controller. Thus, the uncertain system (10) would be considered with Assumptions 1-3. If there exist some matrices $X > 0$, $Y$ and positive constants $\rho_1$, $\rho_2$ such that the following inequality admits a feasible solution:
\[
\begin{bmatrix}
\Pi + 2\mu X \\
A_{21} X + B_2 Y \\
\rho_1 \alpha A_{12}^T \\
MX
\end{bmatrix}
\begin{bmatrix}
* \\
-\rho_1 I_r \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
* \\
* \\
* \\
-\rho_2 I_n
\end{bmatrix} \leq 0, \tag{48}
\]

then, the control law (12) with $F = YX^{-1}$ would be an exponentially stabilizing controller for the uncertain system (10). The constant $\mu$ denotes the decay ratio of the closed-loop system. Moreover, asymptotic stability can also be verified by neglecting the term $\mu$.

**Proof:** Take $V (x) = x^T P x$ as the Lyapunov function. By a similar matrix manipulation, as seen in Proposition 1, the exponential stability condition $\frac{dV}{dt} \leq -2\mu V$ can be concluded from the inequality (48).

**Remark 1:** Proposition 1 may be used to design a robust optimal controller for the uncertain system (10). Thus, an off-line version of Proposition 1 would be preferred by setting $k = 0$ and $t_k = t_0$.

**Remark 2:** Proposition 1 is essentially suggested to the RMPC design in real-time applications. Thus, computation time may be a challenging issue. But it is revealed that the optimization problem would be solved in less than 1 second in the numerical simulations. Therefore, the method can be applied to practical applications.

**Remark 3:** In Proposition 1, a sufficient condition has been addressed to the RMPC design. Thus, the proposed control method would be implementable if the minimization problem admits a feasible solution.

**Remark 4:** In the proposed RMPC approach, all uncertain terms of the dynamical system are considered in the LFT model. Moreover, the control constraints are translated into another LMI. Hence, these would be the main advantages of the method over the existing ones. To RMPC synthesis, a minimization problem would be solved numerically at some known time-instants. Then the controller parameters are updated shortly.

**V. NUMERICAL SIMULATIONS**

In this section, the benefits of the proposed RMPC scheme is consistently validated in two uncertain numerical systems. Additionally, the results are also compared with the similar RMPC technique.

**Example 1:** Consider an uncertain system with the following parameters:

\[
A_{11} = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 \\
-2 & -1 & 0 & -5
\end{bmatrix},
\]

\[
A_{12} = \begin{bmatrix}
0.1 & 0 & 0.1 & 0 \\
0 & 0.1 & 0 & 0.1 \\
-0.1 & 0 & 0.1 & 0 \\
0 & 0 & 0 & 0.1
\end{bmatrix},
\]
The eigenvalues of the matrix $A_{11}$ are computed as $-5.0238, -0.6742, 0.3490 \pm 0.6846j$. Thus, the open-loop model of the nominal system would be unstable.

Moreover, the nominal system $(A_{11}, B_1)$ is stabilizable. The weights of the cost function (15) are selected as follows:

$$Q = \begin{bmatrix} 0.001 & 0 & 0 & 0 \\ 0 & 0.001 & 0 & 0 \\ 0 & 0 & 0.001 & 0 \\ 0 & 0 & 0 & 0.001 \end{bmatrix}, \quad R = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

The uncertain matrix $\Delta$ is specified as:

$$\Delta = \begin{bmatrix} 0.18 & -0.18 & 0.12 & 0.06 \\ 0.06 & 0.12 & 0.06 & -0.12 \\ -0.06 & 0.06 & -0.12 & 0.06 \\ -0.12 & -0.06 & 0.12 & 0.12 \end{bmatrix}.$$

It is seen that $\|\Delta\| = 0.3186$ and $\|A_{22}\| = 1.4782$. Thus, the well-posedness condition (Assumption 1) would be confirmed as $\|\Delta\| \cdot \|A_{22}\| = 0.4710 < 1$. The control constraint is taken as $u_{\text{max}} = 3$; the system’s state starts from $x_0 = [5 \ -5 \ 10 \ 5]^T$, the integration time is 1 millisecond. The inequality (11) can be matched for the nonlinear function $f_\Delta(x)$ with the following matrix $M$:

$$M = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.5 \end{bmatrix}.$$

In Example 1, Proposition 1 is executed with a period of 3 seconds. The solutions of the minimization problem (i.e., $X$, $\rho_1$, $\rho_2$, $\gamma$), as well as the controller gain $F$, are computed as below:

At initial time $t_0 = 0$, we have:

$$X = \begin{bmatrix} 397.4003 & -326.8478 & 239.0432 & 19.3651 \\ -326.8478 & 313.3216 & -257.3208 & -10.0746 \\ 239.0432 & -257.3208 & 304.8725 & -83.6640 \\ 19.3651 & -10.0746 & -83.6640 & 359.8634 \end{bmatrix},$$

$$\rho_1 = 1.3660 \times 10^{-3}, \quad \rho_2 = 7.2937, \quad \gamma = 25.6146,$$

$$F = \begin{bmatrix} -0.0329 & -0.2699 & -0.2322 & -0.1608 \\ -0.2294 & -0.4659 & -0.3169 & -0.0205 \end{bmatrix}.$$

Then, at time-instant $t_1 = 3$, they are found as:

$$X = \begin{bmatrix} 3.3841 & -2.8801 & 1.9893 & 0.3491 \\ -2.8801 & 2.7768 & -2.2201 & -0.1822 \\ 1.9893 & -2.2201 & 2.7462 & -1.3898 \\ 0.3491 & -0.1822 & -1.3898 & 5.3434 \end{bmatrix},$$

$$\rho_1 = 0.0148 \times 10^{-3}, \quad \rho_2 = 0.0916, \quad \gamma = 0.0713,$$

$$F = \begin{bmatrix} -0.3121 & -0.7725 & -0.5286 & -0.2778 \\ -0.7626 & -1.5260 & -0.9300 & 0.0139 \end{bmatrix}.$$

Similarly, at time-instant $t_2 = 6$, the minimum solutions are obtained as:

$$X = \begin{bmatrix} 0.6072 & -0.0062 & 0.0043 & 0.0012 \\ -0.0062 & 0.0057 & -0.0046 & -0.0007 \\ 0.0043 & -0.0046 & 0.0054 & -0.0032 \\ 0.0012 & -0.0007 & -0.0032 & 0.0147 \end{bmatrix},$$

$$\rho_1 = 0.2721 \times 10^{-7}, \quad \rho_2 = 0.3173 \times 10^{-3},$$

$$F = \begin{bmatrix} -2.2206 & -4.1136 & -2.0416 & -0.5111 \\ -6.4905 & -11.6842 & -5.6432 & -1.0434 \end{bmatrix}.$$

Next, at time-instant $t_3 = 9$, we have:

$$X = 10^{-5} \times \begin{bmatrix} 0.2262 & -0.1938 & 0.1323 & 0.0377 \\ -0.1938 & 0.1793 & -0.1443 & -0.0192 \\ 0.1323 & -0.1443 & 0.1705 & -0.1053 \\ 0.0377 & -0.0192 & -0.1053 & 0.4849 \end{bmatrix},$$

$$\rho_1 = 0.8895 \times 10^{-10}, \quad \rho_2 = 0.1071 \times 10^{-5},$$

$$\gamma = 0.3434 \times 10^{-6},$$

$$F = \begin{bmatrix} -10.2733 & -18.2633 & -8.6381 & -1.8738 \\ -29.5376 & -52.2021 & -24.5398 & -4.9652 \end{bmatrix}.$$

Finally, at time-instant $t_4 = 12$, the optimum solutions are calculated as:

$$X = 10^{-7} \times \begin{bmatrix} 0.6836 & -0.5700 & 0.3899 & 0.0870 \\ -0.5700 & 0.5122 & -0.4004 & -0.0481 \\ 0.3899 & -0.4004 & 0.4301 & -0.1928 \\ 0.0870 & -0.0481 & -0.1928 & 0.8794 \end{bmatrix},$$

$$\rho_1 = 0.2974 \times 10^{-12}, \quad \rho_2 = 0.1867 \times 10^{-8},$$

$$\gamma = 0.8583 \times 10^{-9},$$

$$F = \begin{bmatrix} -1.6221 & -3.2106 & -1.7333 & -0.4937 \\ -4.2746 & -8.1398 & -4.2438 & -0.7603 \end{bmatrix}.$$
The simulation results are compared with similar RMPC [37]. The outcomes are illustrated in Figs. 3-9. The states of the uncertain system are plotted in Figs. 3-6. The applied control inputs and the control constraints are drawn in Figs. 7-8. The required computation times are depicted in Fig. 9. It is seen that the computation time is less than 0.31 seconds. Thus the proposed RMPC can be implementable in real-time and practical control applications. The findings, as well as the transient response, verify the efficiency of the proposed method in comparison to the similar RMPC. So, it provides significant benefits in terms of transient response. The most striking result to emerge from the data is that the control restrictions and the uncertainties are successfully handled by employing the proposed control approach.

**Example 2:** Consider the following second-order uncertain system:

\[
m \frac{d^2 y}{dt^2} + b \frac{dy}{dt} + ky = u.
\]  

(49)

Many physical systems can be modeled by Eq. (49). Here, Eq. (49) describes an uncertain mass-spring-damper system. Hence, \(y\) denotes the position (meter), and \(u\) is the applied...
control force (Newton). The parameters $m$, $b$, and $k$ may be subjected to uncertainties as follows [1]:

$$
\begin{align*}
    m &= \bar{m} (1 + w_m \delta_m), \\
    b &= \bar{b} (1 + w_b \delta_b), \\
    k &= \bar{k} (1 + w_k \delta_k),
\end{align*}
\tag{50}
$$

In Eq. (50), the constants $\bar{m}$, $\bar{b}$, and $\bar{k}$ correspond to the nominal values. Moreover, the weights $w_m$, $w_b$, and $w_k$ would be some known terms.

Although the uncertain system (49) is always stable, the uncertainties can consistently degrade the transient performances. The differential equation (49) can be represented in the LFT form. Let define $x_1 \stackrel{\text{def}}{=} y$ and $x_2 \stackrel{\text{def}}{=} \frac{dy}{dt}$. Then an LFT representation of Eq. (49) would be found as follows:

$$
\begin{bmatrix}
    \frac{dx_1}{dt} \\
    \frac{dx_2}{dt} \\
    y_k \\
    y_b \\
    y_m
\end{bmatrix}
= \begin{bmatrix}
    0 & 1 & 0 & 0 & 0 \\
    -\frac{k}{\bar{m}} & -\bar{b} & \frac{1}{m} & -\frac{1}{m} & -w_m \\
    \frac{w_k \bar{b}}{0} & 0 & 0 & 0 & 0 \\
    -\frac{\bar{k}}{\bar{m}} & -\bar{b} & -1 & 1 & -w_m
\end{bmatrix}
\begin{bmatrix}
    x_1 \\
    x_2 \\
    \rho_k \\
    \rho_b \\
    \rho_m
\end{bmatrix} + \begin{bmatrix}
    0 \\
    0 \\
    0 \\
    0 \\
    0
\end{bmatrix} u. \tag{51}
$$

In comparison to Eq. (6), the matrix $\Delta$, the vectors $x$, $\omega$, and $\phi$ are taken as follows:

$$
\Delta = \begin{bmatrix}
    \delta_k & 0 & 0 \\
    0 & \delta_b & 0 \\
    0 & 0 & \delta_m
\end{bmatrix}, \quad x = \begin{bmatrix}
    x_1 \\
    x_2
\end{bmatrix},
$$

$$
\omega = \begin{bmatrix}
    \omega_k \\
    \omega_b \\
    \omega_m
\end{bmatrix}, \quad \phi = \begin{bmatrix}
    \phi_k \\
    \phi_b \\
    \phi_m
\end{bmatrix}.
$$

The system’s uncertainties would be limited to the matrix $\Delta$ as $\omega = \Delta \phi$. Therefore, the matrices $A_{11}, A_{12}, A_{21}, A_{11}, B_1, B_2$ are constructed as:

$$
A_{11} = \begin{bmatrix}
    0 & 1 & 0 \\
    -\frac{k}{\bar{m}} & -\bar{b} & \frac{1}{m} \\
    \frac{w_k \bar{b}}{0} & 0 & 0
\end{bmatrix}, \quad A_{12} = \begin{bmatrix}
    0 & 0 & 0 \\
    -\frac{1}{m} & -\frac{1}{m} & -w_m \\
    0 & 0 & 0
\end{bmatrix},
$$

$$
A_{21} = \begin{bmatrix}
    0 & 0 & 0 \\
    \frac{w_k \bar{b}}{0} & 0 & 0
\end{bmatrix}, \quad A_{22} = \begin{bmatrix}
    0 & 0 & 0 \\
    0 & 0 & 0 \\
    -1 & 1 & -w_m
\end{bmatrix},
$$

$$
B_1 = \begin{bmatrix}
    0 \\
    -1 \\
    \frac{1}{m}
\end{bmatrix}, \quad B_2 = \begin{bmatrix}
    0 \\
    0 \\
    0
\end{bmatrix}.
$$

The nominal values of Eq. (50) are selected as:

$$
\bar{m} = 2, \quad \bar{b} = 0.5, \text{ and } \bar{k} = 10.
$$

To examine the impact of the uncertainties on the transient performance, the following parameters are taken:

$$
w_m = 0.4, \quad w_b = 0.5, \text{ and } w_k = 0.5.
$$

Thus, the well-posedness condition would be validated as:

$$
\begin{align*}
    \delta_k &= 0.3, \quad \delta_b = -0.3, \quad \delta_m = 0.3.
\end{align*}
$$

It can be seen that $\|\Delta\| = 0.3$ and $\|A_{22}\| = 1.4697$. Thus, the well-posedness condition would be validated as $\|\Delta\| \cdot \|A_{22}\| = 0.4409 < 1$. Hence, all assumptions of the paper would be satisfied.

In Example 2, Proposition 1 is executed with a period of 3 seconds. The optimum solutions $(X, Y, \rho_1, \rho_2, \gamma)$ of the minimization problem, as well as the controller gain $F$, are computed as the following:

At initial time $t_0 = 0$, they are determined as below:

$$
X = \begin{bmatrix}
    1.0503 & -0.7881 \\
    -0.7881 & 1.8940
\end{bmatrix},
$$

$$
Y = \begin{bmatrix}
    5.9104 & -13.7589 \\
    -22.9249 & 0.4996 \times 10^{-4}
\end{bmatrix},
$$

$$
\rho_1 = 29.2249, \quad \rho_2 = 0.4996 \times 10^{-4}, \quad \gamma = 8.3295,
$$

$$
F = \begin{bmatrix}
    0.2566 & -7.1576 \\
    1.2269 & -11.3429
\end{bmatrix}.
$$

At time-instant $t_1 = 3$, we have:

$$
X = 10^{-3} \times \begin{bmatrix}
    0.1319 & -0.0698 \\
    -0.0698 & 0.2352
\end{bmatrix},
$$

$$
Y = \begin{bmatrix}
    0.0010 & -0.0028 \\
    0.0063 & 0.0027 \times 10^{-4}
\end{bmatrix},
$$

$$
\rho_1 = 0.0063, \quad \rho_2 = 0.0027 \times 10^{-4}, \quad \gamma = 1.9167,
$$

$$
F = \begin{bmatrix}
    1.2269 & -11.3429
\end{bmatrix}.
$$

At time-instant $t_2 = 6$, the solutions are

$$
X = \begin{bmatrix}
    0.2671 & -0.1569 \\
    -0.1569 & 0.4681
\end{bmatrix},
$$

$$
Y = \begin{bmatrix}
    0.2008 & -0.5159 \\
    0.1065 \times 10^{-6} & 0.5137 \times 10^{-11}
\end{bmatrix},
$$

$$
\rho_1 = 0.1065 \times 10^{-6}, \quad \rho_2 = 0.5137 \times 10^{-11}, \quad \gamma = 0.4147,
$$

$$
F = \begin{bmatrix}
    1.2994 & -10.5849
\end{bmatrix}.
$$

At time-instant $t_3 = 9$, we have:

$$
X = 10^{-11} \times \begin{bmatrix}
    0.1934 & -0.1103 \\
    -0.1103 & 0.3392
\end{bmatrix},
$$

$$
Y = 10^{-10} \times \begin{bmatrix}
    0.1416 & -0.4080 \\
    0.8861 \times 10^{-10} & 0.1238 \times 10^{-12}
\end{bmatrix},
$$

$$
\rho_1 = 0.8861 \times 10^{-10}, \quad \rho_2 = 0.1238 \times 10^{-12}, \quad \gamma = 0.0860,
$$

$$
F = \begin{bmatrix}
    0.5666 & -11.8458
\end{bmatrix}.
$$

At time-instant $t_4 = 12$, the solutions are calculated as:

$$
X = 10^{-13} \times \begin{bmatrix}
    0.1656 & -0.0870 \\
    -0.0870 & 0.2897
\end{bmatrix},
$$

$$
Y = 10^{-12} \times \begin{bmatrix}
    0.1146 & -0.6197 \\
    0.1597 \times 10^{-11} & 0.4649 \times 10^{-14}
\end{bmatrix},
$$

$$
\rho_1 = 0.1597 \times 10^{-11}, \quad \rho_2 = 0.4649 \times 10^{-14}, \quad \gamma = 0.0183,
$$

$$
F = \begin{bmatrix}
    -5.1278 & -22.9300
\end{bmatrix}.
$$

The control constraint is specified as $u_{\text{max}} = 10$. In the cost function (15), the weight matrices are selected as:

$$
Q = \begin{bmatrix}
    1 & 0 \\
    0 & 1
\end{bmatrix}, \quad R = 1.
$$
Similarly, the above computations are periodically repeated at $t_5 = 15$ and $t_6 = 18$. The simulation results are compared with the existing RMPC [37] and recent RMPC [42]. The outcomes are illustrated in Figs. 10-13. The position and velocity of the mechanical system (49) are plotted in Figs. 10-11. A similar control approach shows oscillating behaviors. But the oscillations of the states are well-damped by the suggested RMPC. The applied control force is displayed in Fig. 12. The control constraint $|u(t)| \leq 10$ is fulfilled, as depicted in Fig. 12. The computation times are seen in Fig. 13. Thus the proposed controller can be implementable in practical applications. The controller gains are drawn in Fig. 14. As expected, the proposed control law would have time-varying properties. The gains are updated via Proposition 1.

Consequently, compared to the same controller, the superiorities of the proposed control method are shown in terms of oscillation, settling-time, and overshoot. Thus, considerable progress has been made in this example. Therefore, the limitations, as well as the control constraint and the system uncertainties, are well-managed with the proposed control method.

VI. CONCLUSION AND FUTURE WORKS

An RMPC designing scheme is suggested for a class of uncertain systems described by the LFT. For achieving this goal, considering the control limitations and the LFT uncertainty, an LMI based minimization problem is addressed to the RMPC synthesis. Thus, the controller parameters are instantly updated by the proposed optimization method. From the transient responses, the simulation results demonstrate the effectiveness of the suggested RMPC thoroughly in comparing to similar control laws. Hence, the findings of the research are quite convincing. Therefore, the problem limitations, as well as the control constraints and the system’s uncertainties, are effectively handled by applying the proposed control approach.

This paper has underlined the importance of the LFT uncertainty on transient performance. For this purpose, an innovative solution is found for such an issue. In the future, the same problem can be studied in time-delay, singular, and under-actuated control systems. Moreover, the practical limitations of the actuators and sensors may also be included in the controller design. The computation aspect of the method can be improved as well. Hence, the feasibility regions of the LMI could be studied in depth. Accordingly, more efficient
control policy would be derived for uncertain dynamical systems.

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