Penrose limits in massive type-IIA AdS$_3$ background

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ABSTRACT

In this paper we consider the non-Abelian T-dual geometry of the type IIB supergravity theory on $AdS_3 \times S^3 \times T^4$ background along a convenient $SU(2)$ subgroup of the $SO(4)$ R-symmetry. We examine various null geodesics of the resulting massive type IIA supergravity theory and investigate the Penrose limits along these geodesics. We find that one of the resulting backgrounds admits pp-wave geometry in the neighbourhood of a suitable null geodesic. We carry out the supersymmetry analysis of the resulting pp-wave geometry and observe that it preserves sixteen supercharges. Further we comment on the possible gauge theory dual of the resulting pp-wave background.
1 Introduction

The gauge-gravity correspondence \cite{1,2} plays an extremely important role in relating seemingly different theories each other. The BMN correspondence \cite{3} provides one such example where the string theory on a pp-wave background is related to a large $R$ charge sector of the $\mathcal{N} = 4$ superconformal theory in four dimensions. The BMN correspondence is used to compute interacting string states from perturbative gauge theory \cite{3,4}. The pp-wave background results upon taking the Penrose limit of supergravity theory in ten and eleven dimensions \cite{5–8}. They provide an exact string theory background to all orders of $\alpha'$ and $g_s$ \cite{9,10}. The exact nature of the pp-wave background enables one to compute the interacting string states using the BMN correspondence \cite{3}. In view of this, exploring pp-wave geometries as the Penrose limits of various supergravity theories in ten and eleven dimensions have become an important arena of research during the past several decades.

A particularly interesting set of backgrounds which are of recent interest in this context are the ones obtained from the conventional supergravity theories by applying non-Abelian T-duality \cite{11}. Non-Abelian T-duality is a generalization of the standard Abelian T-duality, where a non-Abelian isometry group of a supergravity solution is used for dualization. However, unlike the Abelian case, it is not a symmetry of the full string theory. It is mainly used as a solution generating technique to obtain new supergravity backgrounds from known ones. Initially non-Abelian T-duality was formulated for supergravity backgrounds with NS-NS fields. However, this duality has received a wide range of applicability after it has been generalized to incorporate the RR sector \cite{12}. Non-Abelian T-dual backgrounds has been constructed for a large class of theories and their relevance in the gauge-gravity correspondence has been envisaged \cite{13–25}. The Penrose limits for some of these dual geometries has been analysed \cite{26–29}. It has been shown that a number of these geometries indeed admit pp-wave solutions upon taking the Penrose limit. Of particular interest among these backgrounds are the non-Abelian duals of $AdS_5 \times S^5$ background \cite{26}, the Klebanov-Witten background \cite{27} and the Klebanov-Tseytlin background \cite{28}. Some of these dual background admit pp-wave geometries even when the original geometry before dualization provide no such solution \cite{28,30}.

Most of the above results focus on pp-waves geometries in non-Abelian T-dual geometries containing an $AdS_5$ factors. However there has not been much progress in analysing the Penrose limits of backgrounds containing an $AdS_3$ factor. In the present work we plan to consider one such background and inspect the Penrose limits in it. We focus on the non-Abelian T-dual of type $IIB$ supergravity theory on $AdS_3 \times S^3 \times CY_2$. This background has been constructed recently and its gauge theory dual has also been studied \cite{24}. Here we show that this geometry indeed admits pp-wave solution upon taking the Penrose limit in the vicinity of appropriate null geodesics. The plan of the paper is as follows. In the following section, we first briefly review the background. Subsequently we consider the Penrose limits along appropriate null geodesics and obtain the pp-wave geometry. In §3 we analyse the spinor conditions and show that the resulting pp-wave solution preserves sixteen supercharges. Finally in §4 we comment upon the possible gauge theory dual before concluding our results in §5. Some technical details are outlined in the appendix.
2 The non-Abelian T-dual of AdS$_3 \times$ CY$_2 \times$ S$^3$ and its Penrose limits

In this section we will first focus on the non-Abelian T-dual of the type IIB supergravity on the background of AdS$_3 \times$ CY$_2 \times$ S$^3$. This geometry arises as the near horizon limit of a stack of intersecting D1 – D5 configuration. This is one of the simplest examples used to apply non-Abelian T-duality in the presence of background RR-fields. The metric is given by

$$ds^2 = 4L^2 ds^2(AdS_3) + L^2 ds^2(CY_2) + 4L^2 ds^2(S^3) .$$

(2.1)

where

$$ds^2(AdS_3) = - \cosh^2 r dt^2 + dr^2 + \sinh^2 r d\chi^2 ,$$

$$ds^2(S^3) = d\alpha^2 + d\beta^2 + d\gamma^2 + 2 \cos \alpha d\beta d\gamma .$$

(2.2)

and $ds^2(CY_2)$ represents the metric of a Calabi-Yau twofold. Here the parameter $L$ represents the size of the internal manifold CY$_2$. In the present work we will entirely focus on the internal manifold $T^4$. We denote $z_i, i = 1, 2, 3, 4$ to parameterize the $T^4$. Thus,

$$ds^2(T^4) = \sum_{i=1}^{4} dz^2_i .$$

(2.3)

In addition to the above, the supergravity background is supported by the dilaton field $e^{2\Phi} = 1$ and the RR three-form flux

$$F_3 = 8L^2 \text{ Vol}(S^3) .$$

(2.4)

We will now consider the non-Abelian T-dual of the above background [12]. The background (2.1)-(2.4) has an SO(4) R-symmetry realised by the presence of $S^3$ factor in the background metric. The authors of [12] considered an SU(2) subgroup of this R-symmetry group to perform the duality transformation. The T-dual background is specified by the metric:

$$d\hat{s}^2_{\text{NATD}} = 4L^2 ds^2(AdS_3) + L^2 ds^2(T^4) + \frac{L^2}{4} dp^2 + \frac{L^2}{4 + \rho^2} d\Omega^2_2(\theta, \phi) ,$$

(2.5)

where $d\Omega^2_2(\theta, \phi) = d\theta^2 + \sin^2 \theta d\phi^2$ is the round metric on the 2-sphere. Here we have set $\alpha' = 1$ for convenience. Also we have rescaled the $\rho$-coordinate as $\rho \rightarrow L^2 \rho$ in order to get an overall $L^2$ factor in the metric.

The remaining fields in the NS – NS sector of the dual background are given by the expressions

$$e^{-2\hat{\Phi}} = \frac{L^6}{4g_s^2}(4 + \rho^2) , \quad \hat{B}_2 = -\frac{L^2 \rho^3}{2(4 + \rho^2)} \sin \theta d\theta \wedge d\phi ,$$

(2.6)
whereas the RR sector is described by the field strengths

\[ \hat{F}_0 = \frac{L^2}{g_s}, \quad \hat{F}_2 = -\frac{L^4 \rho^3}{2g_s (4 + \rho^2)} \sin \theta d\theta \wedge d\phi, \quad \hat{F}_4 = -\frac{L^6}{g_s} \text{Vol}(T^4). \]  

(2.7)

This geometry corresponds to a massive type IIA supergravity on \( AdS_3 \times T^4 \times M_3 \). It has a \( SU(2) \) global symmetry and it preserves only half the supersymmetry of the original background. The manifold \( M_3 \) consists of a \( S^2 \) fibration over a half line. For small \( \rho \) it resembles \( \mathbb{R}^3 \) where as for large \( \rho \) it has the product form \( S^2 \times \mathbb{R} \). It calls for recent interest in the context of two dimensional defect CFTs where this geometry appears as the leading order in an expansion of the number of gauge groups of the dual CFT [24].

We will now turn our discussion on the Penrose limits. For the \( AdS_3 \times S^3 \times T^4 \) background the Penrose limits have been extensively studied in [31]. The authors showed that the Penrose limits indeed give rise to pp-wave geometry. They have also considered Abelian T-duality of the background geometry along a Hopf-fibre of \( S^3 \) and examined the Penrose limits in the resulting Abelian T-dual background [31].

Here we will examine the Penrose limits for the non-Abelian T-dual background. We will consider appropriate null geodesics and take the Penrose limits along these geodesics. Consider first the geodesic equation

\[ \frac{d^2 x^\mu}{du^2} + \Gamma^\mu_{\nu\rho} \frac{dx^\nu}{du} \frac{dx^\rho}{du} = 0. \]  

(2.8)

Here \( \{x^\mu\} \) denote the spacetime coordinates and \( u \) is the affine parameter along the geodesic. Let \( x^\lambda \) denotes an isometry direction. Then, for motion along this isometry direction, the geodesic equation reduces to the following simple form

\[ \partial^\mu g_{\lambda\lambda} = 0. \]  

(2.9)

This is due to the fact that for motion along such an isometry direction the velocity as well as the acceleration along any \( x^\mu, \mu \neq \lambda \) will be zero:

\[ \frac{dx^\mu}{du} = 0 = \frac{d^2 x^\mu}{du^2}, \text{ for } \mu \neq \lambda. \]  

(2.10)

In order to obtain the Penrose limit we need in addition to focus in the vicinity of null geodesics. Thus, we impose \( ds^2 = 0 \) in addition to the geodesic condition (2.9).

We will now focus on various isometry directions of the background of interest (2.5). Let us first consider the \( z_i \)'s along the \( T^4 \). Though, these form isometries of the metric, the geodesic condition for them is trivially satisfied. Moreover we can write the \( T^4 \) as \( \mathbb{R} \times S^3 \) and consider isometries of this \( S^3 \). However, the geodesic condition for any of these isometries do not give any non-trivial constraint. Thus we will no longer consider these directions any further. This leaves behind the motion along the \( \phi \)-direction which we will now focus. The geodesic equation for this case becomes

\[ \partial_\mu g_{\phi\phi} = 0. \]  

(2.11)
This on the other hand gives nontrivial solutions as we can notice from the relevant metric component $g_{\phi\phi}$ in (2.5):

$$g_{\phi\phi} = \frac{L^2 \rho^2}{4 + \rho^2} \sin^2 \theta .$$

(2.12)

This has a nontrivial dependence on $\rho$ and $\theta$. Now for $\mu = \rho$, the geodesic condition (2.11) gives $\rho = 0$ and $\theta = \{0, \pi\}$. Similarly setting $\mu = \theta$, we obtain $\rho = 0$ and $\theta = \{0, \pi, \frac{\pi}{2}\}$. However, $\rho = 0$ and $\theta = \{0, \pi\}$ lead to singular geometries as $g_{\phi\phi}$ component vanishes for all these geodesics. Thus, we will not consider the Penrose limits for such singular geometries.

This leaves us the only possibility of considering the motion of a particle carrying nonzero angular momentum in the $(\rho, \phi)$ plane. We will consider null geodesics for such motion in the neighbourhood of $r = 0 = z_i$ and $\theta = \frac{\pi}{2}$. The Lagrangian for a massless particle with background metric $g_{\mu\nu}$ is

$$\mathcal{L} = \frac{1}{2} g_{\mu\nu} \dot{X}^\mu \dot{X}^\nu .$$

(2.13)

In the above we consider the affine parameter $u$ and the dots denote derivative with respect to it. We consider the background (2.5). Substituting the explicit expression for the metric we find

$$\mathcal{L} = \frac{L^2}{2} \left( -4 \dot{t}^2 + \frac{1}{4} \dot{\rho}^2 + \frac{\rho^2}{4 + \rho^2} \dot{\phi}^2 \right) .$$

(2.14)

The above system is completely integrable and admits a one parameter family of solutions as we can see in the following. Let us first analyse the symmetries of the above Lagrangian. Clearly, the coordinates $t$ and $\phi$ are cyclic. Hence the generalized momenta conjugate the to coordinates $t$ and $\phi$ are conserved. Consider the momentum conjugate to $t$ first:

$$\frac{\partial \mathcal{L}}{\partial \dot{t}} = -4L^2 \dot{t} .$$

(2.15)

Thus, we find $\dot{t} = \text{const}$. Choosing the affine parameter $u$ suitably we set $\dot{t} = 1$. Now consider the equation of motion pertaining to the $\phi$ coordinate:

$$\frac{\partial \mathcal{L}}{\partial \dot{\phi}} = L^2 \frac{\rho^2}{4 + \rho^2} \dot{\phi} = \text{const} .$$

(2.16)

We introduce the constant $J$ to denote this conserved quantity:

$$J = -\frac{\rho^2}{4 + \rho^2} \dot{\phi} .$$

(2.17)
Finally, we need to consider the equation for $\rho$-coordinate. Requiring the geodesics to be null i.e. $\mathcal{L} = 0$, we obtain

$$\dot{\rho}^2 = 4 \left( 4 - \frac{4 + \rho^2}{\rho^2} J^2 \right).$$

(2.18)

This equation admits exact analytical solution which has a simple form:

$$\rho^2 = \frac{4J^2 + 4(J^2 - 4)(u + c_{\rho})^2}{4 - J^2},$$

(2.19)

with $c_{\rho}$ being the constant of integration. We can set it to zero by redefining the affine parameter $u$ by a constant shift. Substituting the above in (2.17) and integrating we find

$$\phi = \frac{1}{J} \left[ u - \tan^{-1} \left( \frac{(J^2 - 4)u}{J} \right) \right],$$

(2.20)

We will now obtain the Penrose limit for the above null geodesic carrying an angular momentum $J$ around $r = 0 = z_i$ and $\theta = \frac{\pi}{2}$. First, we redefine the coordinates as follows

$$r = \frac{\bar{r}}{L}, \ z_i = \frac{y_i}{L}; \ i = 1, 2, 3, 4, \ \theta = \frac{\pi}{2} + \frac{x}{L}.$$

(2.21)

In addition to the above, we rescale the string coupling as $g_s = L^3 \tilde{g}_s$, in order to keep the dilaton finite upon taking the Penrose limit. Finally, we will consider the following large $L$ expansion:

$$dt = c_1 du, \ d\phi = c_2 du + c_3 \frac{dw}{L} + c_4 \frac{dv}{L^2}, \ d\rho = c_5 du + c_6 \frac{dw}{L}.$$

(2.22)

The expansion includes the coefficients $c_i$ which we need to determine. Notice that the geodesic to be null determines three of the coefficients as follows:

$$c_1 = 1, \ c_2 = -\frac{4 + \rho^2}{\rho^2}J, \ c_5 = 2 \left[ 4 - \frac{(4 + \rho^2)}{\rho^2} J^2 \right]^\frac{1}{2}.$$

(2.23)

We will now substitute the expansion (2.22) in the T-dual metric (2.5) and take the limit $L \to \infty$. It may be noted that the expansion contains terms of order $\mathcal{O}(L^2)$. However, they cancel each other upon imposing the null geodesic condition. In addition, it also contains terms of order $\mathcal{O}(L)$. Requiring the metric to be finite imposes the following condition on the coefficients $c_i$:

$$c_5 c_6 + \frac{4\rho^2}{4 + \rho^2} c_2 c_3 = 0.$$

(2.24)
We can substitute the values of $c_2$ and $c_5$ from (2.23) in the above equation. This leads to the following relation between the coefficients $c_3$ and $c_6$:

$$\frac{c_3}{c_6} = \frac{1}{2J} \left[ 4 - \frac{(4 + \rho^2)}{\rho^2} J^2 \right]^{\frac{1}{2}}.$$  

(2.25)

Subsequently we will show that upon requiring the background to satisfy the Einstein’s equations determines the coefficient $c_3$. This leaves behind the only undetermined coefficient $c_4$. Choice of an appropriate normalization for the cross term $dudv$ in the metric determines the value of this coefficient to be $c_4 = -\frac{1}{2}$.

We will now substitute the above results in the background metric (2.5) and take the limit $L \to \infty$. The leading term of the resulting metric takes the form

$$ds^2 = 2dudv + 4dr^2 + 4r^2d\chi^2 + dy_1^2 + dy_2^2 + dy_3^2 + dy_4^2 + \left( \frac{c_6}{4} + \frac{\rho^2}{4 + \rho^2} c_3^2 \right) dw^2$$

$$+ \frac{\rho^2}{4 + \rho^2} dx^2 - \left( 4r^2 + \frac{4 + \rho^2}{\rho^2} J^2 x^2 \right) du^2.$$  

(2.26)

This solution indeed corresponds to a pp-wave geometry. However, it is not in the standard Brinkmann form. Subsequently we will rewrite it in the Brinkmann form upon suitable redefinition of the coordinates. Before that, we will consider the Penrose limit for the remaining background fields. It is straightforward to show that the NS-NS two-form $\hat{B}_2$ and the dilaton $\Phi$ takes the form

$$\hat{B}_2 = \frac{c_3\rho^3}{2(4 + \rho^2)} dw \wedge dx ,$$

$$e^{-2\Phi} = \frac{1}{4\tilde{g}_s^2} (4 + \rho^2) .$$  

(2.27)

In obtaining the expression for $\hat{B}_2$ we have ignored exact terms which can be gauged away. The field strength corresponding to the NS-NS two-form $\hat{B}_2$ is given by

$$\hat{H}_3 = \rho \sqrt{\frac{4\rho^2 - (4 + \rho^2) J^2}{(4 + \rho^2)^2}} \left[ (12 + \rho^2)c_3 + \rho(4 + \rho^2)c_3' \right] du \wedge dw \wedge dx .$$  

(2.28)

In the above we have used the result $d\rho = c_5 du$ where the expression for the coefficient $c_5$ is given by (2.23). We can similarly carry out the Penrose limit for the fields corresponding to the RR sector. We find that the field strengths $\hat{F}_0$ and $\hat{F}_4$ vanish whereas the RR two-form field strength $\hat{F}_2$ takes the simple form

$$\hat{F}_2 = \frac{J}{2\tilde{g}_s} dx \wedge du .$$  

(2.29)
We will now transform the metric (2.26) into the standard Brinkmann form [5] using the formalism developed in [26]. We will first consider a metric of the form

\[ ds^2 = 2dudv + \sum_i A_i(u) \, dx_i^2 . \]  

(2.30)

Notice that the resulting pp-wave background (2.26) is indeed of the above form for suitable choice of the functions \( A_i(u) \). We will now redefine the transverse coordinates \( x_i \) as

\[ x_i \rightarrow \frac{x_i}{\sqrt{A_i}} . \]  

(2.31)

Further we make the following redefinition of one of the lightcone coordinates

\[ v \rightarrow v + \frac{1}{4} \sum_i \frac{\dot{A}_i}{A_i} x_i^2 . \]  

(2.32)

The resulting metric is in the Brinkmann form

\[ ds^2 = 2dudv + \sum_i dx_i^2 + \left( \sum_i F_i(u) \, x_i^2 \right) du^2 , \]  

(2.33)

with

\[ F_i = \frac{1}{4} \frac{\dot{A}_i^2}{A_i^2} + \frac{1}{2} \frac{d}{du} \left( \frac{\dot{A}_i}{A_i} \right) . \]  

(2.34)

For the present case we can read the functions \( A_i(u) \) from our pp-wave metric (2.26) as

\[ A_\bar{r} = 4 , \quad A_w = \frac{c_6^2}{4} + \frac{\rho^2}{4 + \rho^2} c_3^2 , \quad A_x = \frac{\rho^2}{4 + \rho^2} . \]  

(2.35)

Now we need to make the redefinition

\[ \bar{r} \rightarrow \frac{\bar{r}}{\sqrt{A_\bar{r}}} , \quad w \rightarrow \frac{w}{\sqrt{A_w}} , \quad x \rightarrow \frac{x}{\sqrt{A_x}} \quad \text{and} \]

\[ v \rightarrow v + \frac{1}{4} \left[ \frac{\dot{A}_w}{A_w} w^2 + \frac{\dot{A}_x}{A_x} x^2 \right] , \]  

(2.36)

to obtain

\[ ds^2 = 2dudv + d\bar{r}^2 + \bar{r}^2 d\chi^2 + dy_1^2 + dy_2^2 + dy_3^2 + dy_4^2 + dw^2 + dx^2 - \left[ \bar{r}^2 + \left( \frac{(4 + \rho^2)}{\rho^4} J^2 - F_x \right) x^2 - F_w w^2 \right] du^2 . \]  

(2.37)

The functions \( F_i \) are determined using the expression (2.34).
In the Brinkmann coordinates the NS-NS fields take the form

\[
e^{-2\Phi} = \frac{1}{4g_s^2}(4 + \rho^2) ,
\]

\[
\hat{H}_3 = \frac{\sqrt{4\rho^2 - (4 + \rho^2)J^2}}{(4 + \rho^2)^{\frac{3}{2}}}
\left[
\left(12 + \rho^2\right)c_3 + \rho\left(4 + \rho^2\right)c_3'
\right]
\left[
\frac{c_6^2}{4} + \frac{\rho^2 c_3^2}{4 + \rho^2 c_3^2}
\right]^{-\frac{1}{2}}
\]

\[du \wedge dw \wedge dx . \tag{2.38}\]

Similarly, for the RR sector, we find

\[
\hat{F}_2 = \frac{J}{2g_s} \sqrt{4 + \rho^2} \, dx \wedge du ,
\]

\[
\hat{F}_0 = 0 = \hat{F}_4 . \tag{2.39}\]

We will now verify and show that the above background indeed satisfies the equations of motion. We will first consider the Bianchi identities and the gauge field equations. Subsequently we will turn our attention to the Einstein’s equations. A quick inspection of the background fields (2.38)-(2.39) shows that the field strengths \(\hat{H}_3\) and \(\hat{F}_2\) are indeed closed. In addition we have \(\hat{F}_0 = 0 = \hat{F}_4\). Thus the Bianchi identities

\[
d\hat{H}_3 = 0 , \quad d\hat{F}_2 = \hat{F}_0 \hat{H}_3 , \quad d\hat{F}_4 = \hat{H}_3 \wedge \hat{F}_2 \tag{2.40}\]

are trivially satisfied.

We will now turn our attention to the type-IIA supergravity equations:

\[
d\left(e^{-2\Phi} \wedge \hat{H}_3\right) = \frac{J}{2g_s} \sqrt{4 + \rho^2} \, dx \wedge du ,
\]

\[
d\hat{F}_2 = \frac{J}{2g_s} \sqrt{4 + \rho^2} \, dx \wedge du ,
\]

\[
d\hat{F}_0 = 0 = \hat{F}_4 . \tag{2.41}\]

For the background fields (2.38)-(2.39), the above set of equations reduces to

\[
d\left(e^{-2\Phi} \wedge \hat{H}_3\right) = 0 \tag{2.42}\]

To verify that they hold, consider the Hodge duals of the field strengths \(\hat{H}_3\) and \(\hat{F}_2\):

\[
\star \hat{H}_3 = \frac{\sqrt{4\rho^2 - (4 + \rho^2)J^2}}{(4 + \rho^2)^{\frac{3}{2}}}
\left[
\left(12 + \rho^2\right)c_3 + \rho\left(4 + \rho^2\right)c_3'
\right]
\left[
\frac{c_6^2}{4} + \frac{\rho^2 c_3^2}{4 + \rho^2 c_3^2}
\right]^{-\frac{1}{2}}
\]

\[du \wedge d\Omega_2 \wedge dy_1 \wedge dy_2 \wedge dy_3 \wedge dy_4 . \tag{2.43}\]

\[\star \hat{F}_2 = \frac{J}{2g_s} \sqrt{4 + \rho^2} \, du \wedge d\Omega_2 \wedge dy_1 \wedge dy_2 \wedge dy_3 \wedge dy_4 \wedge dw . \tag{2.43}\]
Using the above expression it can be shown $\star \hat{H}_3, (e^{-2\phi} \star \hat{H}_3)$ as well as $\star \hat{F}_2$ are all closed. This is due to the fact that all the nonvanishing components of the Hodge dual are functions of the $\rho$-coordinate only. Because of the presence of $du$ factor both the Hodge duals are all closed forms. It is interesting to see that the gauge field equations are satisfied irrespective of the value of the undetermined coefficient $c_3$. However this is not the case for the Einstein’s equations as we will see in the following.

Consider the Einstein’s equations for type IIA supergravity:

$$
\hat{R}_{\mu
u} + 2D_\mu D_\nu \hat{\Phi} = \frac{1}{4} \hat{H}^2_{\mu\nu} + e^{2\Phi} \left[ \frac{1}{2} (\hat{F}_2^2)_{\mu\nu} + \frac{1}{12} (\hat{F}_4^2)_{\mu\nu} - \frac{1}{4} g_{\mu\nu} \left( \hat{F}_0^2 + \frac{1}{2} \hat{F}_2^2 + \frac{1}{4!} \hat{F}_4^2 \right) \right],
$$

(2.44)

along with the equation of motion for the dilaton

$$
\hat{R} + 4D^2 \hat{\Phi} - 4(\partial \hat{\Phi})^2 - \frac{1}{12} \hat{H}^2 = 0.
$$

(2.45)

A careful analysis of these equations show that the equation of motion for the dilaton is automatically satisfied for our pp-wave background. In addition the Einstein’s equations automatically hold for all the values of $\mu, \nu$ except for the case $\mu = \nu = u$, where the equation takes form

$$
\hat{R}_{uu} + 2D_u D_u \hat{\Phi} = \frac{1}{4} \hat{H}^2_{uu} + \frac{1}{2} e^{2\phi} \left( \hat{F}_2^2 \right)_{uu}.
$$

(2.46)

This indeed provides a nontrivial constraint involving the coefficient $c_3$. The coefficient $c_3$ can be determined upon solving this equation. Thus, the Einstein’s equation determines the coefficient and the pp-wave background satisfies the equations of motion. In the appendix we outline the computation involving the Einstein’s equation leading to the determination of the coefficient $c_3$.

### 3 Supersymmetry of pp-wave

In this section we will consider the supersymmetry preserved by our pp-wave background. In the case of backgrounds with an $AdS_5$ factor such as $AdS_5 \times S^5$, non-Abelian T-duality breaks the supersymmetry from $\mathcal{N} = 4$ to $\mathcal{N} = 2$ [21]. Whereas, for the case of Klebanov-Witten as well as the Klebanov-Tseytlin background, the dual background has $\mathcal{N} = 1$ supersymmetry [14, 15]. Because, for the Klebanov-Witten and Klebanov-Tseytlin case the Killing spinor of the original background does not carry any $SU(2)$-charge that used for dualization [32, 33]. However, in some of these examples it has been shown that the resulting pp-wave background restores some of the supersymmetries. For the present case the non-Abelian T-dual of $AdS_3 \times S^3 \times T^4$ itself preserves half the supersymmetries of the original background [12]. Our goal here is to analyse the spinor conditions in order to obtain the number of supersymmetries preserved by the pp-wave background as given in (2.37)–(2.39).
To analyse the supersymmetry conditions, let us introduce the Brinkmann coordinates \( X^i \) such that

\[
d\bar{r}^2 + r^2d\chi^2 = (dX^i)^2 ; \quad i = 1, 2 \ , \ y_i = X^i \ ; \ i = 3, 4, 5, 6 \ , \ w = X^7 , \ x = X^8 .
\] (3.1)

With this notation, the pp-wave background (2.37)-(2.39) takes the form

\[
ds^2 = 2dudv + \sum_{i=1}^{8} dX_i^2 + \mathcal{H} \ du^2 ,
\]

\[
\hat{\Phi} = \hat{\Phi}(u) ,
\]

\[
\hat{H}_3 = f_1(u) \ du \wedge dX^7 \wedge dX^8 ,
\]

\[
\hat{F}_2 = f_2(u) \ dX^8 \wedge du .
\] (3.2)

where, for easy reading, we have introduced the notations

\[
\hat{\Phi}(u) = \frac{1}{2} \ln \left[ \frac{4\hat{g}_s^2}{4 + \rho^2} \right] ,
\]

\[
\mathcal{H} = \sum_{i,j=1}^{8} F_{ij}X^iX^j ; \quad F_{ij} = F_{ji} ,
\]

\[
f_1(u) = \sqrt{4\rho^2 - (4 + \rho^2)J^2} \left\{ (12 + \rho^2)c_3 + \rho(4 + \rho^2)c_3' \right\} \left[ \frac{c_6^2}{4} + \frac{\rho^2}{4 + \rho^2}c_3' \right]^{-\frac{1}{2}} ,
\]

\[
f_2(u) = \frac{J}{2\hat{g}_s} \sqrt{4 + \rho^2} .
\] (3.3)

In the above, the functions \( F_{ij} \) are given by the expressions

\[
F_{11} = F_{22} = -1 ,
\]

\[
F_{77} = F_w , \quad F_{88} = F_x - \frac{(4 + \rho^2)^2}{\rho^4}J^2 .
\] (3.4)

Further, we introduce the frame \( \{ e^a \} \) such that

\[
e^- = du , \quad e^+ = dv + \frac{1}{2} \mathcal{H} du , \quad e^i = dX^i .
\] (3.5)

The pp-wave metric (3.2) in this frame is given by

\[
ds^2 = 2e^+e^- + \sum_{i=1}^{8} (e^i)^2 = \eta_{ab}e^ae^b ,
\] (3.6)
with \( \eta_{+-} = \eta_{-+} = 1 \) and \( \eta_{ij} = \delta_{ij} \). The remaining background fields in the frame takes the form

\[
\begin{align*}
\dot{\Phi} &= \dot{\Phi}(u), \\
\dot{H}_3 &= f_1(u) \, e^- \wedge e^7 \wedge e^8, \\
\dot{F}_2 &= f_2(u) \, e^8 \wedge e^-.
\end{align*}
\]  

(3.7)

We also need to use the spin connections in the supersymmetry conditions. From (3.5)-(3.6) it is easy to compute the spin connections. The non-vanishing components are given by

\[
\omega_{-i} = \omega^{+i} = \frac{1}{2} \partial_i \mathcal{H} \, du .
\]  

(3.8)

Let us now focus on the spinor conditions in detail. The supersymmetric variations for the dilatino and gravitino are given by

\[
\begin{align*}
\delta \hat{\lambda} &= \frac{1}{2} \left[ \dot{\Phi} \hat{\epsilon} - \frac{1}{24} \dot{H} \sigma_3 \hat{\epsilon} + \frac{3}{16} e^\Phi \hat{F}_2 (i\sigma_2) \hat{\epsilon} , \\
\delta \hat{\psi}_\mu &= D_\mu \hat{\epsilon} - \frac{1}{8} \dot{H}_{\mu\nu\rho} \Gamma^\nu \rho \sigma_3 \hat{\epsilon} + \frac{1}{16} e^\Phi \hat{F}_2 (i\sigma_2) \Gamma_\mu \hat{\epsilon} .
\end{align*}
\]

(3.9)

In the above we use the conventions of [13,26]. In particular, we note that the action of the covariant derivative \( D_\mu \) on the Killing spinors is given as \( D_\mu \hat{\epsilon} = \partial_\mu \hat{\epsilon} + \frac{1}{4} \omega^a_{\mu b} \Gamma_{ab} \hat{\epsilon} \). In addition we use the notation \( \hat{F}_n \equiv \hat{F}_{i_1 \ldots i_n} \Gamma^{i_1 \ldots i_n} \) and follow the standard convention for \( \sigma_i \) to denote the Pauli matrices. The Killing spinor \( \hat{\epsilon} \) is given as

\[
\hat{\epsilon} = \left( \begin{array}{c} \hat{\epsilon}_+ \\ \hat{\epsilon}_- \end{array} \right) ,
\]

(3.10)

in terms of two Majorana-Weyl spinors \( \hat{\epsilon}_± \). It satisfies the relation \( \Gamma_{11} \hat{\epsilon} = -\sigma_3 \hat{\epsilon} \). where we denote

\[
\Gamma^\pm = \frac{1}{\sqrt{2}} \left( \Gamma^9 \pm \Gamma^0 \right) .
\]  

(3.11)

We will now focus on the dilatino variation. Substituting the pp-wave metric and the background fields as given by (3.7) and simplifying we find

\[
\Gamma^- \left[ \dot{\Phi} - \frac{1}{2} f_1(u) \Gamma^{78} \sigma_3 - \frac{3 e^\Phi}{4} f_2(u) \Gamma^8 \left( i\sigma_2 \right) \right] \hat{\epsilon} = 0 .
\]  

(3.12)

The above equation is satisfied for \( \Gamma^- \hat{\epsilon} = 0 \). We will now proceed to solve the spinor conditions pertaining to the gravitino variation. We need to set \( \delta \hat{\psi}_\mu = 0 \) for \( \mu = +, -, i \). First consider the variation \( \delta \hat{\psi}_+ \). From the expression for the NS-NS three form flux, we
find $\hat{H}_{+\mu\nu} = 0$ for all $\mu, \nu$. Using it along with the condition $\Gamma_+ \hat{\epsilon} = \Gamma^- \hat{\epsilon} = 0$ in the variation $\delta \hat{\psi}_+ = 0$ we find that the Killing spinor $\hat{\epsilon}$ is independent of the light cone coordinate $v$, i.e., $\partial_+ \hat{\epsilon} = 0$. Thus, we have $\hat{\epsilon} = \hat{\epsilon}(u, X^i)$.

We will now turn our attention to the variations $\delta \hat{\psi}_i$, $i = 1, \ldots, 8$. The condition $\delta \hat{\psi}_i = 0$ implies

$$\partial_i \hat{\epsilon} = \Gamma^\prime \hat{\epsilon},$$

where we have introduced the notation

$$\mathcal{R} = \frac{1}{4} f_1(u) \left( \delta_{88} \Gamma^7 - \delta_{77} \Gamma^8 \right) \sigma_3 + \frac{e\hat{\Phi}}{8} f_2(u) \Gamma^8 (i\sigma_2) \Gamma^i.$$  \(3.13\)

A quick inspection of the above expression shows that $\Gamma^\prime$ anticommutes with $\mathcal{R}$. Thus $\delta \hat{\psi}_i$ becomes

$$\partial_i \hat{\epsilon} = \mathcal{R} \Gamma^\prime \hat{\epsilon}.$$  \(3.15\)

Since $\Gamma^- \hat{\epsilon} = 0$, we find $\partial_i \hat{\epsilon} = 0$. Thus, $\hat{\epsilon} = \chi(u)$ for some $\chi(u)$ such that $\Gamma^- \chi(u) = 0$. This leaves behind us to verify the only remaining condition $\delta \hat{\psi}_- = 0$. After some simplification this condition gives rise to

$$\partial_u \chi(u) - \frac{1}{4} f_1(u) \Gamma^7 \Gamma^8 \sigma_3 \chi(u) + \frac{e\hat{\Phi}}{4} f_2(u) \Gamma^8 (i\sigma_2) \chi(u) = 0.$$  \(3.16\)

Introducing the matrix

$$\mathcal{M}(u) = \frac{1}{4} \left( f_1(u) \Gamma^7 \Gamma^8 \sigma_3 - e\hat{\Phi} f_2(u) \Gamma^8 (i\sigma_2) \right)$$  \(3.17\)

the above condition can be rewritten as

$$\partial_u \chi(u) - \mathcal{M}(u) \chi(u) = 0.$$  \(3.18\)

This equation can be integrated to give rise $\chi(u) = e^{\int du \mathcal{M}(u)} \chi_0$. Thus the gravitino and dilatino variations are compatible with each other for the above choice of $\chi(u)$ along with $\Gamma^- \chi_0 = 0$. Since the later condition retains sixteen components of the Killing spinors, we find that the pp-wave background (3.2)-(3.4) preserves 16 supercharges. It is interesting to note that there is an enhancement of the number of supersymmetries preserved by the pp-wave solution originated from the non-Abelian T-dual background. This was the case even for the non-Abelian T-dual of the Klebanov-Tseytlin background [28]. Here the pp-wave solution preserves the supersymmetry of the original type $IIB$ supergravity theory on $AdS_3 \times S^3 \times T^4$ before applying T-duality.
4 Field theory dual

The field theory dual corresponding to the non-Abelian T-dual background (2.5)-(2.7) has been studied in detail in [24]. The construction of the dual theory is based upon the analysis of quantized brane charges. The T-dual geometry includes a nontrivial $S^2$ in the transverse space which interpolates between $\mathbb{R}^3$ and $\mathbb{R} \times S^2$ as we move from $\rho \to 0$ to $\rho \to \infty$. Presence of this nontrivial two cycle allows one to construct an intersecting brane configurations involving $D2$ and $D6$ branes stretched between $NS5$ branes. The dual gauge theory consists of a two dimensional $(0,4)$ quiver theory consisting of two infinite family of nodes with gauge groups of increasing rank with no flavor [24]. In support of this construction, the authors of [24] computed the central charge of the quiver theory and shown its agreement with the holographic central charge computed from the gravity dual.

The pp-wave background obtained upon taking the Penrose limit will correspond to a class of operator of the above quiver gauge theory. To understand it better we will consider the brane charges for the pp-wave background (2.37)-(2.39). The Page charges of various D-branes in type IIA supergravity are given by [20]

\[
\hat{Q}_{\text{page},D6} = \frac{1}{2\kappa_{10}^2 T_{D6}} \int_{C_2} \hat{F}_2 - \hat{F}_0 \hat{B}_2 ,
\]

\[
\hat{Q}_{\text{page},D4} = \frac{1}{2\kappa_{10}^2 T_{D4}} \int_{C_4} \hat{F}_4 - \hat{B}_2 \wedge \hat{F}_2 + \frac{1}{2} \hat{F}_0 \hat{B}_2 \wedge \hat{B}_2 ,
\]

\[
\hat{Q}_{\text{page},D2} = \frac{1}{2\kappa_{10}^2 T_{D2}} \int_{C_6} \hat{F}_6 - \hat{B}_2 \wedge \hat{F}_4 + \frac{1}{2} \hat{B}_2 \wedge \hat{F}_2 - \frac{1}{6} \hat{F}_0 \hat{B}_2 \wedge \hat{B}_2 \wedge \hat{B}_2 ,
\]

\[
\hat{Q}_{\text{page},D8} = \frac{1}{2\kappa_{10}^2 T_{D8}} \int \hat{F}_0 .
\]

Similarly, the Maxwell charges are given as [15]

\[
\hat{Q}_{\text{Max},D6} = \frac{1}{\sqrt{2\pi}} \int_{C_2} \hat{F}_2 ,
\]

\[
\hat{Q}_{\text{Max},D4} = \frac{1}{\sqrt{3\pi}} \int_{C_4} \hat{F}_4 ,
\]

\[
\hat{Q}_{\text{Max},D2} = \frac{1}{\sqrt{4\pi}} \int_{C_6} \hat{F}_6 ,
\]

\[
\hat{Q}_{\text{Max},D8} = \sqrt{2} \int \hat{F}_0 .
\]

Here $C_n$ corresponds to appropriate $n$-cycle in the associated geometry. In addition, we have the $NS5$ branes with charge

\[
\hat{Q}_{NS5} = \frac{1}{4\pi \alpha'} \int_{C_3} \hat{H}_3 .
\]
Substituting the values of the background fields for the pp-wave solution (2.37)-(2.39), we find the non-vanishing charges

\[ \hat{Q}_{\text{page},D6} = \hat{Q}_{\text{Max},D6} = \frac{1}{2\kappa_{10}^2 T_{D6}} \int_{C_2} \hat{F}_2 , \]
\[ \hat{Q}_{\text{NS}5} = \frac{1}{4\pi\alpha'} \int_{C_3} \hat{H}_3 , \]

where the expression of \( \hat{H}_3 \) and \( \hat{F}_2 \) are given in (2.38) and (2.39) respectively. This indicates that at the Penrose limit we are left with only \( D6 \) and \( NS5 \) branes. Thus the dual gauge theory will be described by an intersecting configuration of \( D6 \) and \( NS5 \) branes. The BMN operators corresponding to the holographic dual of the pp-wave geometry will reside in this quiver theory.

5 Conclusion

In this paper we have studied pp-wave geometries in non-Abelian T-dual backgrounds with an \( AdS_3 \) factor. We focused on the non-Abelian T-dual of type IIB supergravity theory on \( AdS_3 \times S^3 \times T^4 \) background. The resulting T-dual background consists of \( AdS_3 \times M_3 \times T^4 \), where the three dimensional space \( M_3 \) consists of a \( S^2 \) fibration over a half line. Denoting \( \phi \) to be the azimuthal coordinate on \( S^2 \) and \( \rho \) to be the coordinate parameterizing the half line, we considered null geodesics carrying non-zero angular momentum along the \((\rho, \phi)\) plane. We considered the Penrose limit in the vicinity of such null geodesics and showed that the resulting geometry gives rise to a pp-wave solution. We solved the spinor conditions for this pp-wave background and showed that it preserves sixteen supercharges. Finally we commented on the possible field theory dual for our pp-wave background. It would be interesting to explore the possibility of exploring pp-wave backgrounds in other non-Abelian T-dual geometries admitting \( AdS_3 \) factors. We hope to report on this in future.

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A Einstein’s Equations

Here we will analyse the Einstein’s equations for the pp-wave background (3.2)-(3.4). For type IIA supergravity the Einstein’s equations are given by

\[ \hat{R}_{\mu\nu} + 2D_\mu D_\nu \hat{\Phi} = \frac{1}{4} \hat{H}_{\mu\nu}^2 + e^{2\hat{\Phi}} \left[ \frac{1}{2} (\hat{F}_2^3)_{\mu\nu} + \frac{1}{12} (\hat{F}_4^3)_{\mu\nu} - \frac{1}{4} g_{\mu\nu} \left( \hat{F}_0^3 + \frac{1}{2} \hat{F}_2^2 + \frac{1}{4!} \hat{F}_4^2 \right) \right] . \]
The equation of motion corresponding to the dilaton is

$$\hat{R} + 4D^2 \hat{\Phi} - 4(\partial \hat{\Phi})^2 - \frac{1}{12} \hat{H}^2 = 0 .$$  \hfill (A.2)

In the above equations we have used the conventions of \[13\]. In particular, we denote \(\hat{H}_{\mu\nu} = H_{\mu\alpha\beta} g^{\alpha\rho} g^{\beta\sigma} \hat{H}_{\nu\rho\sigma}\) and similar expressions for \((\hat{F}^2)_{\mu\nu}\) and \((\hat{F}^4)_{\mu\nu}\).

Let us first consider the equation of motion corresponding to the dilaton \(\hat{A}.2\). We will show that each of the terms in this equation vanish identically. For the pp-wave background \(\hat{R} = 0\). To compute the second term, consider

$$D^2 \hat{\Phi} = g^{\mu
u} D_\mu \hat{D}_\nu \hat{\Phi} = g^{uv} D_u \hat{D}_v \hat{\Phi} + g^{vu} D_v \hat{D}_u \hat{\Phi} + g^{ij} D_i \hat{D}_j \hat{\Phi} .$$ \hfill (A.3)

Using \(\partial \hat{\Phi} = 0 = \partial \hat{\phi}\) for the partial derivatives and \(\Gamma^u_{uv} = \Gamma^v_{vu} = \Gamma^u_{ij} = 0\) we find

$$D^2 \hat{\Phi} = 0 .$$ \hfill (A.4)

Similarly, we can show that \((\partial \hat{\Phi})^2\) also vanishes identically:

$$\left( \partial \hat{\Phi} \right)^2 = g^{\mu
u} \partial_\mu \hat{\Phi} \partial_\nu \hat{\Phi} = 2g^{uv} \partial_u \hat{\Phi} \partial_v \hat{\Phi} + g^{ij} \partial_i \hat{\Phi} \partial_j \hat{\Phi} = 0 .$$ \hfill (A.4)

Finally, from the expression for \(\hat{H}_3\) \(\hbox{(2.38)},\) it is straightforward to see that \(\hat{H}_3 = 0\). This shows that the dilaton equation \(\hbox{(A.2)}\) is satisfied trivially.

We will now consider the Einstein’s equations \(\hbox{(A.1)}\). For our background \(\hat{F}_0 = 0 = \hat{F}_4\) and though \(\hat{F}_2\) is nonzero, from \(\hbox{(2.39)}\) we can see that \(\hat{F}_2 = 0\). Further, a straightforward calculation shows that the only the \(\mu\nu\)-components of \(\hat{H}^2_{\mu\nu}, (\hat{F}^2)_{\mu\nu}\) together with \(D_u D_v \hat{\Phi}\) are non-vanishing. Similar result holds for the Ricci tensor in in Brinkmann coordinates \(\hbox{[5]},\) i.e., \(\hat{R}_{uu}\) is the only nonvanishing component of the Ricci tensor. Thus, for the pp-wave background \((\hbox{3.2)-(3.4)},\) the Einstein’s equation \(\hbox{(A.1)}\) reduces to

$$\hat{R}_{uu} + 2D_u D_v \hat{\Phi} = \frac{1}{4} \hat{H}^2_{uu} + \frac{1}{2} e^{2\hat{\phi}} (\hat{F}^2)_{uu} .$$ \hfill (A.5)

Each of the terms in the above equation can be evaluated in a straightforward manner. We find

$$\hat{R}_{uu} = \left[ 2 + \frac{(4 + \rho^2)^2}{\rho^4} J^2 - F_x - F_w \right] ,$$

$$e^{-2\hat{\phi}} = \frac{1}{4\tilde{g}^2} \left( 4 + \rho^2 \right) ,$$

$$D_u D_v \hat{\Phi} = \frac{4 (\rho^2 - 4) - (4 + \rho^2) J^2}{(4 + \rho^2)^2} ,$$

$$\hat{H}^2_{uu} = 2 \frac{4\rho^2 - (4 + \rho^2) J^2}{(4 + \rho^2)^3} \left[ (12 + \rho^2)c_3 + \rho(4 + \rho^2)c'_3 \right] \left[ \frac{c_0^2}{4} + \frac{\rho^2}{4 + \rho^2} c_3^2 \right]^{-1} ,$$

$$(\hat{F}^2)_{uu} = \frac{J^2}{4\tilde{g}^2} \left( 4 + \rho^2 \right) .$$ \hfill (A.6)
Clearly, substituting the above expressions in (A.1) we can see that unlike the remaining equations, it is not satisfied identically. However, notice that this equation does involve the only undetermined coefficient $c_3$ in the expansion (2.22), and its derivative. Thus, it gives rise to a first order differential equation involving $c_3$. This equation can be integrated to write an exact expression for $c_3$ in closed form.

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