Analysis of the deployment of a three-mass tethered satellite formation

Shumin Chen1,2, Aijun Li1, and Changqing Wang1

1 School of Automation, Northwestern Polytechnical University, West Youyi Road 127, Xi’an, P.R. China, 710072
2 Institute of IT, Mathematics and Electronics, Samara National Research University, Moskovskoe Shosse 34, Samara, Russia, 443086

E-mail: chenshumin@mail.nwpu.edu.cn

Abstract. Tethered satellite formation has obtained widespread attention in recent years. The main causes of the increasing study interest around tethered formation lie in its promised applications in space, such as interferometric measurements. The deployment of a tethered formation system in low Earth orbit is investigated in this paper. The orbital tethered system consists of three nanosatellites connected via tethers end-to-end, and the desired arrangement for the end masses is an equilateral triangle. For the sake of brevity, the formation system is modeled as a particle-rigid rod system, in which the elasticity of the tethers is omitted. The deployment process is carried out using tension forces and active external forces generated by low-thrust engines. The numerical results confirm that it is possible to deploy a triangular tethered system, which rotates at a given initial angular velocity, to an expected arrangement using the proposed control law of tether tensions and active forces.

1. Introduction

Research on spacecraft formation flying [1, 2] has drawn much attention for decades of years because of its attractive advantages compared with traditional large spacecraft in terms of overall mission cost, flexibility, reconfigurability, and increased redundancy. The implementation of distributing the function of a monolithic spacecraft among several smaller and cooperative spacecrafts is seen as one of the future development trends of space exploration. Spacecraft formation technique has been practiced on several missions, for example, large space apertures arrangement for space interferometry [3], formation flying of spacecrafts in close proximity for the purpose of capturing and docking to complete spacecraft recovery and servicing missions [4].

With the increasing pursuit of much better image resolutions, it will inevitably lead to the need for a huge aperture in observation missions. In order to cope with the limited capacity of current launch vehicles, the concept of using multiple apertures in a large formation to achieve a better resolution has been presented [5]. Compared to the concept of distributing the sparse apertures over long, lightweight and deployable booms, a tethered system can provide the optimum low mass structure with which to take space-based interferometric measurements [3]. Consequently, the tethered satellite formation (TSF) has become one of the current hot spots in the area of space interferometry. Besides, as a kind of space tether system (STS), tethered formation flying demonstrates its superiority not only in terms of system mass, but also promises much more potential applications owing to the inherent properties of STS, which have been fully discussed based on a lot of technical papers and experimental studies [6, 7].
The TFS can have various configurations according to the geometrical shapes of the fully deployed system. One of the basic geometries of tethered formation is closed-string configuration [8], in which the spacecrafts are connected sequentially via tethers end to end. Especially, the research on the triangular tethered configuration, which is regarded as one of the basic arrangements of closed-string formation, has great significance. Tragesser [9] assessed the stability of a spinning triangular tethered formation. When the formation of three satellites is in the cylindrical equilibrium case, the behavior of the system can be quite similar to a rigid body system at a rapid spinning velocity. In order to complete Earth sensing and surveillance missions, the conical equilibrium is more suitable for that the projection of the formation on the Earth in this case is not a line, a roughly ellipsoidal. However, it was found that the triangular satellite formation designed on the classical rigid body equilibrium (including cylindrical and conical) is not stable in the presence of tether flexibility, which requires the introduction of active control. Similarly, to expand the range of Earth observation, the Likins-Pringle equilibrium configuration was also used in [10] to create the triangular tethered constellation model. Yu et al [11] considered the spinning stability of a triangular TSF in low Earth orbit. The tethers were modeled as massless straight rods. In order to verify the stability analysis, ground experiments were carried out to simulate the spinning motion of an equilateral triangle formation. Both the analytical and experimental results confirmed that a large spinning angular rate can ensure stable motion of the formation.

It is worth mentioning that successful deployment of TSF is a prerequisite for the following on-orbit operations. Essentially, the deployment control of TSF is challenging due to the strong nonlinear characteristics of system, like what exists for the common dual-body space tether system [12]. Cai et al [13] considered the deployment/retrieval of a rotating triangular TSF near libration points, and developed a coupled dynamical formulation for the attitude and orbital motions. The influences of parameters on system stability were unveiled, which lays the foundation for controller design. Williams et al [14] investigated the optimal deployment and retrieval of a spinning triangular TSF within the orbital plane in low Earth orbit. The deployment/retrieval was performed by deploying and reeling tethers. Numerical results illustrated that there is symmetry between tether deployment and retrieval.

Compared to pure tension control, the strategy that combined tether tension with small external forces could create a balance between the brevity and validity of the control law, which constitutes the motivation of the current work. In order to design the controller, we derive the equations of motion of a triangular TSF in low Earth orbits. The remainder of this paper is organized as follows. In the following section, the mathematical model is developed by using Lagrange’s equations. Then, the deployment strategy is proposed. Later, the effectiveness of the proposed controller is demonstrated by numerical simulations. Finally, conclusions are presented.

2. Dynamic model of TSF

2.1. Description of the formation system

When studying the deployment process of space tether system, the simplified tether model that regards tether as inextensible and massless rod is often employed [13]. Considering the tether during deployment is always tight and its mass is much lighter than the end masses, it is reasonable to utilize such simplification of tethers in modeling. Moreover, in this paper the satellites are modeled as point masses.

The considered triangular TSF is illustrated in figure 1. The orbital coordinate system $C_{xyz}$ (denoted by $S_c$) is used to describe the motion of system. The origin of $S_c$ is located at the system’s center of mass (CM). The $Cx$ axis points positively from the Earth to the CM of the formation system, the $Cy$ axis lies in the orbital plane perpendicular to the $Cx$ axis and points along the direction of orbital velocity of the CM, and the $Cz$ axis is normal to the orbital plane.
2.2. Derivation of the dynamical equations

The masses of three satellites are denoted by $m_1, m_2, m_3$, and the three tethers are represented as $l_1, l_2, l_3$, respectively. The position vector of the $i$th satellites in the orbital coordinate system is denoted by $r_i = (x_i, y_i)$, $i = 1, 2, 3$.

As the origin of the orbital frame is located at the CM of system, the constraint equation can be expressed as [15]

$$\sum_{i=1}^{3} m_i r_i = 0$$

(1)

Additionally, notice that the position of $m_2$ relative to $m_1$ can be represented by $l_1$ and the libration angle $\theta_1$, and similarly, the position of $m_3$ relative to $m_2$ is defined by $l_2$ and the libration angle $\theta_2$, the following geometric relations can be obtained

$$x_2 = x_1 + l_1 \cos \theta_1, \quad y_2 = y_1 + l_1 \sin \theta_1$$
$$x_3 = x_2 + l_2 \cos \theta_2, \quad y_3 = y_2 + l_2 \sin \theta_2$$

(2)

Combining equations (1) and (2), one can obtain

$$x_i = \sum_{j=1}^{2} A_{ij} l_j \cos \theta_j, \quad y_i = \sum_{j=1}^{2} A_{ij} l_j \sin \theta_j$$

(3)

where the coefficients $A_{ij}$ are constants determined by the masses of satellites and are given by

$$[A] = \begin{bmatrix}
- (\mu_2 + \mu_3) & -\mu_3 \\
\mu_1 & -\mu_3 \\
\mu_1 & \mu_1 + \mu_2
\end{bmatrix}$$

(4)
where $\mu = m_i/m$, $m = \sum_{i=1}^{3} m_i$.

In order to derive the equations of motion using Lagrange’s approach, the kinetic and potential energy of the system are required. The kinetic energy of the system has the following form

$$T = \frac{1}{2} \sum_{i=1}^{3} m_i (\dot{\mathbf{r}}_i + \dot{\mathbf{r}}_c) \cdot (\dot{\mathbf{r}}_i + \dot{\mathbf{r}}_c)$$

$$= \frac{1}{2} m \dot{\mathbf{r}}_c \cdot \dot{\mathbf{r}}_c + \dot{\mathbf{R}}_c \cdot \mathbf{r} + \frac{1}{2} \sum_{i=1}^{3} m \dot{r}_i \cdot \dot{r}_i$$

where $\dot{\mathbf{R}}_c$ represents the orbital velocity vector of the CM of the system, while $\dot{r}_i$ is the relative velocity of the $i$th satellites with respect to the CM.

The potential energy is given by

$$U = \sum_{i=1}^{3} \frac{\mu m_i}{|\mathbf{R}_c + \mathbf{r}_i|}$$

Notice the fact that $|\mathbf{r}_i|$ is usually much smaller than the orbital amplitude of the CM. Expanding equation (7) into a binomial series and keeping to $O(1/R_c^3)$, one can obtain

$$U \approx -\sum_{i=1}^{3} \frac{\mu m_i}{R_c} + \sum_{i=1}^{3} \frac{\mu m_i x_i}{R_c^2} + \sum_{i=1}^{3} \frac{\mu m_i}{2R_c^3} \left( x_i^2 + y_i^2 - 3x_i^2 \right)$$

$$= -\sum_{i=1}^{3} m_i \omega^2 R_c^2 + \sum_{i=1}^{3} m_i \omega^2 (y_i^2 - 2x_i^2)$$

Thus, the dynamical equations of TSF can be derived from the Lagrange’s equations

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}} \right) - \frac{\partial T}{\partial q} + \frac{\partial U}{\partial q} = \mathbf{Q}$$

where $q = (l_1, l_2, \theta_1, \theta_2)^T$ is vector of the generalized coordinates, and $\mathbf{Q} = (Q_1, Q_2, Q_3, Q_4)^T$ is the vector of the generalized forces.

### 2.3. Evaluation of the generalized forces
The generalized forces in this work are derived in the presence of tether tensions, and external active forces acting on the satellites, which are assumed to be generated by low-thrust engines. Figure 2 shows the directions of the tension forces $T_i$ and the external forces $F_i$, where $\phi$ represents the angle between the tethers and the external forces.

The generalized forces $Q$ can be evaluated using the principle of virtual work and are given by

$$Q = (T_1 - T_3 + F_3) \cdot \frac{\partial r_1}{\partial q} + (T_2 - T_1 + F_2) \cdot \frac{\partial r_2}{\partial q} + (T_3 - T_2 + F_3) \cdot \frac{\partial r_3}{\partial q} \tag{10}$$

where the tension forces $T_i$ can be written as

$$T_i = T_i \left( \cos \theta_i, \sin \theta_i \right), \quad T_2 = T_2 \left( \cos \theta_2, \sin \theta_2 \right),$$

$$T_3 = T_3 \left( \frac{T_3}{l_i}, (-l_1 \cos \theta_1 - l_2 \cos \theta_2, -l_1 \sin \theta_1 - l_2 \sin \theta_2) \right) \tag{11}$$

and the length of the third tether satisfies the geometric relation $l_3 = \sqrt{l_1^2 + l_2^2 + 2l_1l_2 \cos (\theta_1 - \theta_2)}$ under the assumption that the tethers are not bent.

Similarly, the external forces can be expressed by

$$F_1 = F_1 \left( \cos \Delta \theta_1, \sin \Delta \theta_1 \right), \quad F_2 = F_2 \left( \cos \Delta \theta_2, \sin \Delta \theta_2 \right)$$

$$F_3 = F_3 \left( \cos \Delta \theta_3, \sin \Delta \theta_3 \right) \tag{12}$$

where $\Delta \theta_1 = \theta_1 - \phi, \Delta \theta_2 = \theta_2 - \phi, \Delta \theta_3 = \theta_3 - \phi$, while $\theta_i$ is the angle between the $Cx$ axis and the direction of $F_3$.

Considering the relations $I_3 = r_3 - r_1 = \left( l_1 \cos \theta_1 + l_2 \cos \theta_2, l_1 \sin \theta_1 + l_2 \sin \theta_2 \right)$ and $I_3 = l_3 \left( \sin \theta_3, -\cos \theta_3 \right)$, one obtains

$$\sin \theta_1 = \frac{l_1 \cos \theta_1 + l_2 \cos \theta_2}{l_3}, \quad \cos \theta_1 = \frac{l_1 \sin \theta_1 + l_2 \sin \theta_2}{l_3} \tag{13}$$

Thus, $F_3$ can be calculated as
\[ F_3 = F_3 \left( \cos \theta_3 \cos \varphi + \sin \theta_3 \sin \varphi \right) \]
\[ = F_3 \left( \frac{-l_1 \sin \theta_1 + l_2 \sin \theta_2 \cos \varphi + l_1 \cos \theta_1 \cos \theta_3 \sin \varphi }{l_3} \right) \]
\[ = F_3 \left( \frac{l_2 \cos \theta_1 + l_2 \cos \theta_2 \cos \varphi + \sin \theta_1 + l_2 \sin \theta_2 \sin \varphi }{l_3} \right) \quad (14) \]

Combining equations (10-12) and (14), the generalized forces \( Q \) may be obtained as

\[ Q_i = -T_i - (T_i/l_i) \left[ l_i + l_2 \cos (\theta_i - \theta_2) \right] \]
\[ - (\mu_1 + \mu_2) F_i \cos \varphi + \mu_1 F_i \cos (\theta_i - \theta_1 - \varphi) \]
\[ + \mu_1 F_i \left[ l_1 \sin \varphi + l_2 \sin (\theta_i - \theta_2 + \varphi) \right] \]
\[ = -T_i - (T_i/l_i) \left[ l_i + l_2 \cos (\theta_i - \theta_2) \right] \]
\[ - \mu_1 F_i \cos (\theta_i - \theta_2 - \varphi) - \mu_1 F_i \cos \varphi \]
\[ + (\mu_1 + \mu_2) F_i \left[ l_1 \sin \varphi + l_2 \sin (\theta_i - \theta_2 + \varphi) \right] \]
\[ = (T_i/l_i) \left[ l_i \cdot l_2 \sin (\theta_i - \theta_2) \right] \]
\[ + (\mu_1 + \mu_2) F_i \sin \varphi + \mu_1 F_i \sin (\theta_i - \theta_1 - \varphi) \]
\[ + \mu_1 F_i \left[ l_1 \cos \varphi + l_2 \cos (\theta_i - \theta_2 + \varphi) \right] \]
\[ = (T_i/l_i) \left[ l_i \cdot l_2 \sin (\theta_i - \theta_2) \right] \]
\[ + \mu_1 F_i \sin \varphi + \mu_1 F_i \sin (\theta_i - \theta_1 + \varphi) \]
\[ + (\mu_1 + \mu_2) F_i \left[ l_2 \cos \varphi + l_1 \cos (\theta_i - \theta_1 + \varphi) \right] \]
\[ = -T_i - (T_i/l_i) \left[ l_i + l_2 \cos (\theta_i - \theta_2) \right] \]
\[ - \mu_1 F_i \cos (\theta_i - \theta_2 - \varphi) - \mu_1 F_i \cos \varphi \]
\[ + (\mu_1 + \mu_2) F_i \left[ l_1 \sin \varphi + l_2 \sin (\theta_i - \theta_2 + \varphi) \right] \quad (16) \]

\[ Q_6 = -(T_i/l_i) \left[ l_i \cdot l_2 \sin (\theta_i - \theta_2) \right] \]
\[ + \mu_1 F_i l_2 \sin (\theta_i - \theta_1 + \varphi) + \mu_1 F_i l_2 \sin \varphi \]
\[ + (\mu_1 + \mu_2) F_i l_2 \left[ l_1 \cos \varphi + l_2 \cos (\theta_i - \theta_1 + \varphi) \right] \]
\[ = -(T_i/l_i) \left[ l_i \cdot l_2 \sin (\theta_i - \theta_2) \right] \]
\[ + \mu_1 F_i l_2 \sin (\theta_i - \theta_1 + \varphi) + \mu_1 F_i l_2 \sin \varphi \]
\[ + (\mu_1 + \mu_2) F_i l_2 \left[ l_1 \cos \varphi + l_2 \cos (\theta_i - \theta_1 + \varphi) \right] \quad (18) \]

3. **Deployment control strategy**

The expectation for deployment of TSF is to deploy tethers to form a desired physical geometry. Considering that the rotation is a critical aspect for tethered formation flying, moreover, a triangular TSF is promised to maintain dynamic stability by rotation in proper conditions [16], it is reasonable to assume that the TSF studied in this work has a stable initial rotation rate. In this case, the tether tensions are presented in the following form

\[ T_i = T_i^0 + k_i \cdot (l_i - l_d) + w_i \cdot \dot{l}_i \quad (i = 1, 2, 3) \]

where \( T_i^0 \) represent the nominal tether tensions at the initial rotation rate, \( l_d \) is the desired length of deployment, \( k_i, w_i \) are control gains.

The tensions \( T_i^0 \) can be obtained by employing the conditions \( \dot{l}_i = \ddot{l}_i = 0 \) and \( F_i = 0 \) into the dynamic model, which have the following forms
where $l_i^0$ are the initial lengths of the three tethers, $\dot{\theta}_i^0$ are the initial libration angle rates. It is worth noting that the value of $T_i^0$ calculated by equations (20-22) could be quite small, since the initial lengths of tethers and the libration angle rates are usually set to small values. As a result, the control tensions defined by equation (19) in the initial stage could be negative when the expected final length $l_i$ is much larger than the deployed lengths $l_i$. This means the tethers at this stage should carry compression forces, which is obviously contrary to the physical nature of tethers. To cope with this problem, it is necessary to introduce some constraint conditions on the control forces, that is, when the minimum boundary is not reached: $T_i < T_{\text{min}}$, then set $T_i = T_{\text{min}}$. In this way, the tethers are initially deployed under the minimum allowed tension force, and then the tensions change according to the proposed control law.

The active control forces are designed with the performance characteristics of low-thrust engines taking into consideration. A relay law is proposed as follows

$$F_i = \begin{cases} F, & t < t_k \\ 0, & t \geq t_k \end{cases}, \quad i = 1, 2, 3$$ (23)

where $F$ is a given constant, $t_k$ is the acting time of the active forces.

4. Numerical results and analysis

Numerical simulations were carried out to verify the effectiveness of the proposed control law. It was assumed that the orbital altitude is 500km. The physical parameters of the triangular TSF and the parameters of the controller are given in Table 1 and Table 2, respectively.

**Table 1.** Physical parameters of the TSF.

| Parameters | Values |
|------------|--------|
| $m_1, m_2, m_3$ | 10kg |
| $l_1^0, l_2^0, l_3^0$ | 1m |
| $j_1^0, j_2^0$ | 0.5m/s |
| $l_d$ | 500m |
| $\theta_1^0, \theta_2^0$ | $\pi/6, 5\pi/6$ |
| $\dot{\theta}_1^0, \dot{\theta}_2^0$ | 0.1rad/s |
Table 2. Parameters of the proposed control law.

| Parameters | Values       |
|------------|--------------|
| $T_{\text{min}}$ | 0.1N         |
| $k_1, k_2, k_3$ | 0.2, 0.2, 0.4 |
| $w_1, w_2, w_3$ | 10, 12, 15   |
| $F$          | 2N           |
| $\phi$       | 100°         |
| $t_k$        | 120s         |

The simulation time is set as $t = 500s$. Numerical results are presented in the following figures. Figure 3 depicts the time histories of the lengths of the three tethers during deployment. The variation curves show that tether deployment goes smoothly and no tether rewind occurs during the process. Moreover, it can be seen that all the tethers are deployed from the initial value (1m) to the commanded length (500m) simultaneously, and the overall time needed is 325s approximately. It is noteworthy that the errors between the length of the deployed tethers and the expected value at the end of the deployment are quite small since no perturbation is taken into account: for $l_1$ and $l_2$, the errors are $0.001m$ and $0.015m$, respectively; while for $l_3$, the value is $0.465m$, which is slightly larger.

Figure 4 shows the change rates of tether lengths during deployment. As shown in figure 4, the tethers are deployed with the length rates increasing in the initial stage, and then the velocity of deployment slows down and converges to zero gradually due to the termination of the external forces and the action of braking mechanism. It should also be noted here that the length rates of the three tethers do not exactly coincide during deployment.

Figure 5 demonstrates the changes of the libration angles during deployment. It can be seen that the libration angles $\theta_1$ and $\theta_2$ gradually increase as the system rotates around the center of mass. As a reminder, the initial difference between $\theta_1$ and $\theta_2$ is $2\pi/3$, which guarantees that the initial configuration of the TSF is a regular triangle. When the tethers are completely deployed, the difference between the two angles is $2.096rad$, which is almost the same as the difference of the initial angles. Figure 6 depicts the change rates of $\dot{\theta}_1$ and $\dot{\theta}_2$. The minimum values of $\dot{\theta}_1$ and $\dot{\theta}_2$ are $6.8 \times 10^{-4}rad/s$.
and $2.6 \times 10^{-3} \text{rad/s}$, respectively. Furthermore, as shown in the figure, $\dot{\theta}_1$ and $\dot{\theta}_2$ tend to be the same (0.01 rad/s) at the end of the deployment.

Figure 5. Variation curves of the libration angles versus time.

Figure 6. Variation curves of the rate of libration angles versus time.

Figure 7 shows the tether tensions during deployment. As noted previously, the tether tensions at the initial stage is restricted to the permissible minimum, which is set as 0.1N. As a result, the performance of the TSF at this stage is characterized by deploying the tethers at an increasing speed, which echoes what is shown in figure 4. Then, after a relatively long braking phase, the tensions eventually stabilize at the given minimum again.

Figure 7. The tether tensions during deployment.

Figure 8 shows the trajectories of the satellites during tether deployment in orbital frame. The final configuration of the TSF remains as an equilateral triangle.
Figure 8. The trajectories of satellites during tether deployment.

5. Conclusions
The present work considered the deployment of a triangular tethered formation in low Earth orbit. The control strategy is proposed by regulating the tether tensions and switching low-thrust engines. The simulation results show that the system is stably deployed until the expected configuration is formed. In future studies, the more complex scenarios about the deployment of TSF, such as the coupled dynamics of tether deployment and the orbital motion in the presence of perturbations, deserve further investigation.

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