Numerical simulation research of subcooled flow boiling based on MUSIG model under low pressure

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Abstract. Subcooled boiling heat transfer is widely used in modern high-power and high-load internal combustion engines (ICEs) because of its efficient heat transfer characteristics. In the present work, a numerical simulation of subcooled boiling under low-pressure conditions is performed on CFX with a user-defined program. Lee’s subcooled boiling experiment results are used for calculations. The bubble departure diameter is simulated by Du’s model. In order to apply the simulation method to engine cooling system with complex channel, the Favre Averaged Drag Model and Tomiyama Lift Force Model are used which makes up for shortcomings of the experience factor needed to be adjusted. The Multiple Size Group (MUSIG) model considering the effects of vapor bubbles coalescence and breakup is used to describe the different sizes of bubbles in the liquid phase. A qualitatively good agreement with the experimental data is achieved without changing model parameters, which lays a good foundation for the application of the simulation method in an engine cooling system.

1. Introduction

With the development of modern engines towards high detonation pressure and high power density, the thermal stress of the engine cylinder head is one order of magnitude higher than that of the mechanical stress [1, 2]. The traditional heat transfer method can no longer meet the cooling requirements of the key area of cylinder head under high load [3]. Due to its high efficiency of heat transfer capability, the subcooled boiling flow has been paid more and more attention due to its high heat transfer efficiency [4]. Thereby, a high-temperature cooling system based on boiling heat transfer has become the future direction of the engine's cooling system. However, the mechanism to calculate the complex channel on the boiling heat transfer model is not mature currently. It is very necessary to establish a numerical model of subcooled flow boiling of water jacket and accurately predict the boiling state of the subcooled flow of a water jacket. In recent years, a series of simulation models for boiling heat transfer in cylinder head have been developed, such as: The single-phase boiling model, homogenous flow model, and two-fluid boiling model [5]. Two fluid simulation method based on the RPI wall-boiling model has become a trend in the simulation of boiling heat transfer [6]. There are many complex sub-models in the RPI wall-boiling model. Owing to the complexity of the sub-models, many scholars have carried out a series of research on the sub-models [6-9]. In these sub-models, bubble departure diameter, bubble departure frequency, and nucleation site density have a great influence on the accuracy of boiling heat transfer calculation. Each model has its own limitations. The selection and establishment of sub-models have become a research direction. As the current simulation...
of heat transfer has basically met the requirements, the research on improving the accuracy of void fraction distribution is still in progress.

In the present study, some recently proposed sub-models have been implemented in CFX by a user-defined function to predict the two-phase flow characteristics of the subcooled boiling flow. In order to apply the simulation method to engine cooling system with complex channel, the Favre Averaged Drag Model and Tomiyama Lift Force Model are used which makes up for shortcomings of the experience factor needed to be adjusted. The MUSIG model is adopted to solve the problem of the bubble Sauter diameter distribution in different positions. The model works well with the classical experiment data. It lays a good foundation for the design and optimization of a cooling water jacket.

2. The basic mathematical theory

2.1. Partitioning of the wall heat flux

Wall heat flux distribution determines the heat and mass transfer between phases near the heating surface. The schematic diagram of the wall heat flux partition model is shown in Figure 1. Many investigations on subcooled flow boiling confirm that there are actually three components of the wall heat flux \[10, 11\]: (1) heat flux from evaporation, \(Q_e\); (2) heat flux from transient conduction, \(Q_q\); and (3) heat flux from forced convection, \(Q_c\).

\[
Q = Q_e + Q_q + Q_c = A_h(T_w - T_l) + A_h(T_w - T_l) + \frac{\pi}{6} \rho g H_l \frac{d_w^3}{6} + \frac{\pi}{6} \rho g H_l \frac{d_w^3}{6} + \pi \rho g H_l \frac{d_w^3}{6} + \pi \rho g H_l \frac{d_w^3}{6} + \pi \rho g H_l \frac{d_w^3}{6} + \pi \rho g H_l \frac{d_w^3}{6}
\]

where \(A_h\) is the fraction influenced by the vapor bubbles on the heating surface \(A_h = \min(N_p/K, \pi d_w^2/4)\), \(K\) is a constant \((K=4)\). Fraction \(A_c\) is the rest of the wall surface, with \(A_c = \max(1-A_h, 10^{-4})\). \(f\) is the departure frequency of the vapor bubble. \(t_w\) is the vapor bubble waiting time defined as \(t_w = 0.8/f\). \(d_w\) is the bubble departure diameter. \(N_p\) is the active nucleation site density of the heating surface. \(T_l\) is the liquid temperature at a given non-dimensional distance \(y_l = 250\) from the wall.

![Figure 1. Schematic diagram of the wall heat flux partition model.](image)

2.2. The thermal phase change model

In subcooled flow boiling, the heat and mass transfer between the liquid phase and vapor phase is obtained by the evaporation and condensation of vapor bubbles in the subcooled liquid. The evaporation rate per unit volume \(\Gamma_w\) \[11\] at the heating surface can be expressed as

\[
\Gamma_w = \frac{q_e}{h_{lg} V_i} \frac{A_i}{V_i} = \frac{\pi}{6} d_w \rho g f N_p A_i \frac{A_i}{V_i}
\]

where \(A_i\) and \(V_i\) are the area of the heating surface and volume of the first cell near the heating surface.

For condensation, the mass transfer rate from vapor to liquid can be expressed as

\[
\Gamma_{lg} = h_{lg} A_{lg} \left(T_g - T_l\right) / H_{lg}
\]
where, $A_l$ is the interfacial area density ($= 6\alpha/d_b$), and $h_l$ is the heat transfer coefficient between the liquid phase and vapor phase, defined as $h_l = Nu_l \lambda_l / d_b$. $Nu_b$ is the Nusselt number of vapor bubble, calculated by the Ranz-Marshall model [12]:

$$Nu_b = 2 + 0.6 Re_b^{0.5} Pr_l^{1/3}$$  \hspace{1cm} (4)

where $Re_b$ is the vapor bubble Reynolds and $Pr_l$ is the liquid Prandtl Number.

2.3. Inter-phase momentum transfer

In the momentum equation, momentum transfer between the phases plays an important part in the distribution of the void fraction in subcooled flow boiling. The total interfacial forces include drag force and non-drag forces. In this paper, the non-drag force mainly considers lift force, virtual mass force, wall lubrication force and turbulent dispersion force.

The drag force between phases is caused by the friction between the interfaces of relative motion between liquid and vapor bubbles, and the different pressures acting on bubbles. The direction of the drag force is opposite to the direction of the relative motion of bubbles, and the drag force is affected by the shape and size of the bubbles. Ishii-Zuber drag model [13] is adopted in the paper.

$$F_D = C_D \frac{\rho_l}{2 \pi} \rho \left( \frac{U_g - U_l}{d_b} \right)^2 \left( \nabla \times \left( \frac{U_g - U_l}{d_b} \right) \right)$$  \hspace{1cm} (5)

where $C_D$ is the drag coefficient, and $d_b$ is the vapor bubble diameter.

The lift force affects the radial distribution of the void fraction. It can push bubbles towards the wall. It is expressed as

$$F_L = C_L \rho \left( \frac{U_g - U_l}{d_b} \right) \left( \nabla \times \left( \frac{U_g - U_l}{d_b} \right) \right)$$  \hspace{1cm} (6)

where $C_L$ is the lift force coefficient which is experiential and also set to 0.1-0.5. To improve the applicability, it is based on Tomiyama model [14].

Generally, the wall lubrication force only acts on the bubbles in a thin layer adjacent to the heating surface. It prevents the bubbles from adhering to the heating surface. [12]. It is defined as:

$$F_{WL} = -\alpha \rho_l \left( \frac{U_g - U_l}{d_b} \right) \nabla \times \left( \frac{U_g - U_l}{d_b} \right)$$  \hspace{1cm} (7)

where $y_w$ is the distance to the nearest wall. The coefficients $C_1$ and $C_2$ are -0.01 and 0.05, respectively.

The turbulent dispersion force is caused by the combined action of turbulent eddies and interphase drag, which causes the bubble to move away from the heating surface and move towards the bulk flow region. It is calculated as follows:

$$F_{TD} = C_{TD} \frac{3 \rho_l \mu_l}{4 d_b \sigma_t} \left( \frac{V_g - V_l}{a_g} - \frac{V_l}{a_l} \right)$$  \hspace{1cm} (8)

where $\sigma_t$ is the turbulent Schmidt number. $C_{TD}$ is the turbulent dispersion coefficient. These two empirical parameters need to be adjusted according to the working conditions. Considering it will be applied to complex systems, Favre Averaged Drag Model [15] is used in the paper

$$F_{TD} = -\frac{3 \rho_l \mu_l}{4 d_b \sigma_t} \left( \frac{U_g - U_l}{d_b} \right) \frac{V_l}{1-a}$$  \hspace{1cm} (9)

Because the vapor density is less than the liquid density, the virtual mass force is taken into account [15]. It can be calculated as:

$$F_{VM} = \rho_l C_{VM} \left( \frac{d^2 \bar{V}_g}{dt^2} - \frac{d \bar{V}_l}{dt} \right)$$  \hspace{1cm} (10)

where $C_{VM}$ is the virtual mass coefficient, set to 0.5.

2.4. Boiling sub-models

The active nucleation site density $N_a$ of the heating surface is an important parameter in subcooled flow boiling, which directly affects the number of bubbles generated. The model for $N_a$ adopted in the RPI model adopts Hibiki and Ishii’s model [16].

$$N_a = \bar{N}_a \left( 1 - \exp \left( -\frac{\rho^2}{ho_{\mu}^2} \right) \right) \exp \left( f \left( \frac{\rho^2}{\rho_{\mu}^2} \right) \right)$$  \hspace{1cm} (11)
where $N_c$ is average cavity density. $r_c$ is a critical cavity radius. $\theta$ is the contact angle.

Because the model of bubble departure frequency in CFX is mainly for pool boiling. In this paper, we mainly focus on subcooled flow boiling. Thereby, the frequency of the bubble adopted Basu’s model, It is calculated as:

$$f = \frac{1}{139.1(\Delta T_w^{1.4})+0.045a_1a_w\exp(-0.02f_a_{wub})}$$

(12)

where $Ja_w$ and $Ja_{sub}$ are Jacob number corresponding to wall superheat and subcooling respectively.

In the study of subcooled boiling flow, the bubble departure diameter $d_w$ is a research focus. It is a very important parameter that directly affects the accuracy of the calculation results. The model used in the simulation method is the correlation of Du [17].

$$d_w = 10^{-0.433}L_\sigma P_{\alpha}^{0.261}P_{\alpha}^{3.381}R_{\alpha}^{-0.323}$$

(13)

where the bubble Reynolds number $Re_b$ has adopted 0.322 mm. $Ja_{N,w}$ is Jacob number ($= \epsilon_p(T_w-T_{sat})/h_f$). $L_\sigma$ is Laplace length ($=\sqrt{\sigma/(\rho_\ell - \rho_v)}$).

2.5. Bubble breakup and coalescence model

The diameter of the bubble determines the size of the interface and the size of the interface force, which affects momentum exchange, heat and mass transfer. Due to the lack of relevant models, many scholars assume that the bubble diameter is a constant, and use a constant to describe the bubble size.

In the present work, the MUSIG model is adopted to describe the bubble Sauter diameter distribution. In the MUSIG model, bubbles are divided into $N$ groups according to the diameters, and all discrete phases are assumed to have the same velocity to reduce the number of momentum equations. The bubble population balance equations are established by using the models of bubble breakup and coalescence, as shown in Figure 2. The population balance equation [18] is expressed as:

$$\frac{\partial n_i}{\partial t} + \nabla \cdot \left( \vec{U} n_i \right) = \left( \sum_{j=1}^{N} S_{ij} \right) + \left( S_{ph} \right)_i (i=1,2,\ldots,N)$$

(14)

where $\left( \sum_{j=1}^{N} S_{ij} \right) = B_B - D_B + B_C - D_C$, and $B_B, D_B, B_C, D_C$ are the smaller bubbles birth rate caused by breakup of larger bubbles, the larger bubbles death rate caused by the breakup of larger bubbles, the larger bubbles birth rate caused by the coalescence of smaller bubbles, and the smaller bubbles death rate caused by the coalescence of other smaller bubbles, respectively. $\left( S_{ph} \right)_i = \varnothing_{\text{COND}}$.

The breakup model is calculated using the Luo and Svendsen model [19], which is a theoretical model for the breakup of bubbles in turbulent suspensions. The coalescence model is calculated using Prince and Blanch model [20].

3. Experimental facility and boundary conditions

3.1. Experimental facility

The model of subcooled flow boiling under low-pressure in the paper is validated against experimental data from the literature of Lee et al [21, 22]. The experimental facility is constructed and builds up for
water at low system pressure. The experimental section is a vertical upward annular pipe heated by an inner tube. The outer diameter of the inner tube is 19 mm and the inner diameter of the outer tube is 37.5 mm. The length of the heating section is 1670 mm. The measuring plane was installed 1610 mm downstream of the beginning of the heated section. The experimental conditions for simulation calculation are shown in Table 1.

### Table 1. Experimental conditions.

| Case | $P_{in}$ (MP) | $q_{in}$ (kW/m²) | $G$ (kg/m².s) | $T_{in}(^\circ C)$ |
|------|---------------|------------------|---------------|-------------------|
| case 1 | 0.13          | 114.8            | 477.0         | 95.6             |
| case 2 | 0.142         | 152.3            | 474.0         | 96.6             |
| case 3 | 0.115         | 169.8            | 478.1         | 83.9             |
| case 4 | 0.137         | 197.2            | 714.4         | 94.9             |
| case 5 | 0.125         | 139.1            | 715.2         | 93.9             |
| case 6 | 0.134         | 133.6            | 478.9         | 95.7             |
| case 7 | 0.121         | 196.9            | 718.4         | 83.8             |
| case 8 | 0.143         | 251.5            | 1059.2        | 92.1             |

#### 3.2. Boundary conditions

In order to avoid the influence of inlet and outlet effects on the measurement parameters, both the length of the extension section of the inlet and outlet is 330 mm. The Euler two-phase flow model is adopted to simulate subcooled boiling. Due to subcooled boiling flow belonging to bubbly flow, an SST turbulence model is employed for the continuous phase while the zero equation is employed for the dispersed vapor phase. For the liquid phase, a no-slip is applied and for the vapor phase, a free slip boundary condition is applied. The boundary conditions are shown in Figure 3. Considering the influence of the bubble-induced turbulence viscosity, the Sato’s eddy viscosity model is adopted in the paper. In order to make the result more close to the experimental data, actual liquid properties will be adopted.

![Figure 3. Mesh and Boundary conditions.](image)

![Figure 4. Void fraction distribution under different mesh.](image)

#### 3.3. Grid independent

Because the vapor bubbles are generated near the heating surface and the wall function is adopted in the model, the thickness of the first layer grid has a great influence on the accuracy of the model prediction. In order to improve the calculation accuracy, it is necessary to adjust the grid size of the rectangular channel in the experimental section constantly to ensure that $y^+$ distribution is between 20 and 90. In order to verify the independence of the grid, the model is divided into six kinds of meshes: $15 \times 7 \times 230$, $25 \times 5 \times 230$, $20 \times 7 \times 160$, $20 \times 7 \times 300$, $20 \times 10 \times 230$, $25 \times 7 \times 230$ (radial mesh number $\times$ circumferential mesh number $\times$ axial mesh number) and the local void fraction is used as the verification basis. Figure 4 presents the distribution of local void fraction in the radial direction under case 7. It can be seen that the local void fraction distribution does not change significantly with the increase of mesh number. Considering the balance between the cost of calculation and the accuracy of calculation, the computational domain is divided into $20 \times 7 \times 230$. 


4. Results and analysis

4.1. Simulation results contours

As shown in Figure 5 (a), the figure shows the contours of the liquid temperature, local void fraction and the Sauter mean diameter under the condition of case 3, respectively. It can be seen from the figure that the bubbles are mainly concentrated in the thin region near the heating surface. There is a large temperature gradient from the inner heating surface to the outer wall. The vapor phase exists near the heating surface and forms the peak value of the void fraction near the wall of the heating surface near the outlet side. Due to the action of different forces, such a void fraction distribution appears. The Souter diameter distribution is the same as the distribution of void fraction. This is because when the bubbles depart from the heating wall, they will flow together with the liquid phase and accumulate a large number of bubbles near the outlet. At the outlet of the heating pipe, the phenomenon of bubble coalescence is more and the phenomenon of bubble breakup is less. The high temperature in bulk flow near the outlet leads to the smaller bubble condensation rate. Thereby, the Souter diameter near the outlet is larger.

The effect of various forces on the void fraction distribution is shown in Figure 5 (b). Under the influence of comprehensive forces, the distribution of void fraction is more consistent with the experimental data. In Figure 5 (b), TD represents turbulent dissipation force; LF represents lift force and WL represents wall slip force. Through the comparison between the NONE curve and the TD curve, it can be seen that the role of the turbulent dissipation force is to promote the bubble dispersion from the main bubble concentration area to the low concentration area. Through the comparison of TD curve and TD+LF curve, it can be seen that the effect of lift force has a great influence on the radial distribution of bubbles. Through the comparison of TD + LF curve and TD + LF + WL curve, it can be seen that the effect of wall lubrication force has a great influence on the near wall area. The force pushes the bubble away from the heating surface and prevents the bubble from adhering to the heating surface, but the action distance is very short.

4.2. Comparison of experiment and simulation

The experimental and calculated results of the subcooled boiling void fraction are compared as shown in Figure 6-7. From those figures, it can be seen that the distribution of void fraction obtained by simulation is consistent with the experimental results under different working conditions. The simulation results reflect the experimental phenomenon that the peak value of void fraction is formed in the region near the heating surface. Bubbles are generated on the heating surface. When bubbles are separated from the wall, they are also affected by wall slip force, turbulent dissipation force and lift force. The wall lubrication force only acts on the bubbles in a thin layer adjacent to the heating surface. It prevents the bubbles from adhering to the heating surface. The turbulent dispersion force causes the bubble to move away from the heating surface and move towards the bulk flow region. Therefore, the void fraction will have a peak near the heating surface.
Figure 6. Comparison of experimental values of void fraction with simulated values in case 1,2,3,4.

Figure 7. Comparison of experimental values of void fraction with simulated values in case 5,6,7,8.

Figure 8 shows the measured and simulated radial distribution of the Sauter diameter at the measured position. From those figures, it can be seen that the simulated Sauter diameter has a good agreement with the measured results. It can also be concluded that the Sauter diameter increases first and then decreases in the direction far from the heating surface. This is because the bubbles will merge and break up in the liquid phase when they depart from the heating surface. In the area near the heating surface, bubbles coalescence is more common than bubbles breakup. Therefore, the Sauter diameter will be larger, but there is no particularly large bubble size. Because the larger the size, the easier to break up. Because the liquid temperature far from the heating surface is low, the condensation of the bubble is obvious, so the diameter of Sauter will be smaller.

Figure 8. Comparison of experimental values of void fraction with simulated values in case 2,4,6,8.
5. Conclusions
In the present work, an RPI model based on the Euler-Euler two-fluid equations has been implemented in CFX to predict the subcooled boiling flow in an annular channel. Some sub-models such as the bubble departure diameter, bubble diameter, bubble departure frequency, momentum exchange coefficients, etc. are implemented in CFX using a user-defined program. The simulation method is mainly used for subcooled boiling heat transfer under low pressure and low heat flux. Some main conclusions are as follows:

(1) The new embedded sub-models can well predict subcooled flow boiling under low pressure.

(2) The MUSIG model is adopted to describe the bubble the phenomenon of vapor bubbles breakup and coalescence. It can well describe the distribution of bubble size in subcooled boiling and make up for the lack of accurate bubble size formula in the RPI model.

(3) The results show that the model can well predict the important parameters such as void fraction, bubble diameter and so on. This lays a good foundation for Euler multiphase model used in the engine cooling system and provides a good theoretical basis for the optimization and design of a high-temperature cooling system.

Nomenclature

| Symbol | Definition |
|--------|------------|
| \( A_c \) | fraction area influenced by cooling liquid ( - ) |
| \( A_i \) | interfacial area density (m\(^2\)) |
| \( c_p \) | specific heat ((J/(kg K)) |
| \( C_D \) | drag coefficient (-) |
| \( C_l \) | lift force coefficient (-) |
| \( d_n \) | bubble departure diameter (m) |
| \( d_d \) | bubble diameter (m) |
| \( E_o \) | Eotvos number (-) |
| \( F_{LF} \) | lift force (N) |
| \( F_D \) | drag force (N) |
| \( F_{wL} \) | wall lubrication force (N) |
| \( F_{TD} \) | turbulent dispersion force (N) |
| \( F_{VM} \) | virtual mass force (N) |
| \( f \) | bubble detachment frequency (1/s) |
| \( g \) | gravitational acceleration (m/s\(^2\)) |
| \( G \) | Mass flux (kg/(m\(^2\)s)) |
| \( h \) | heat transfer coefficient (W/(m\(^2\)K)) |
| \( H \) | latent heat (J/kg) |
| \( J_a \) | Jacob number (-) |
| \( N_a \) | nucleation site density (1/m\(^2\)) |
| \( N_u \) | Nusselt number |
| \( k \) | thermal conductivity (W/(m K)) |
| \( P \) | pressure (Pa) |
| \( Pr \) | Prandtl number (-) |

| Symbol | Definition |
|--------|------------|
| \( q \) | heat flux (W/m\(^2\)) |
| \( Re \) | Reynolds number (-) |
| \( T \) | temperature (K) |
| \( U \) | velocity (m/s) |
| \( y^+ \) | wall plus (-) |
| \( y_w \) | distance to the nearest wall (m) |

Greek symbols

| Symbol | Definition |
|--------|------------|
| \( \alpha \) | void fraction |
| \( \Gamma \) | mass transfer (kg/(m\(^3\))s)) |
| \( \rho \) | density (kg/m\(^3\)) |
| \( \sigma \) | surface tension (N/m) |
| \( \sigma_t \) | turbulent Schmidt number (-) |
| \( \theta \) | contact angle (° ) |
| \( \mu \) | kinetic viscosity (Pa s) |

Subscripts

| Symbol | Definition |
|--------|------------|
| \( w \) | wall |
| \( \text{sat} \) | saturated |
| \( l \) | liquid |
| \( g \) | vapor |
| \( \text{in} \) | inlet |
| \( d \) | departure |
| \( \text{sub} \) | subcooled |

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