A Combinatorial BIT Bang Leading to Quaternions

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Abstract. This paper describes in detail how (discrete) quaternions - ie. the abstract structure of 3-D space - emerge from, first, the Void, and thence from primitive combinatorial structures, using only the exclusion and co-occurrence of otherwise unspecified events. We show how this computational view supplements, and provides an interpretation for, the mathematical structures. The build-up is emergently hierarchical, compatible with both quantum mechanics and relativity, and can be extended upwards to the macroscopic. The mathematics is that of Clifford algebras emplaced in the homology-cohomology structure pioneered by Kron. Interestingly, the ideas presented here were originally developed by the author to resolve fundamental limitations of existing artificial intelligence paradigms.

1 Introduction

We find ourselves in a universe of myriad, mystifying, and very nearly incomprehensible, complexity. At the same time, contemporary Big Bang cosmogenesis tells us that this complexity has apparently emerged from ‘nothing’ - from Void - via a poorly understood process. In this paper, I will attempt to describe a discrete, combinatorial, and computational framework for this process. I gladly acknowledge the inspiration of The Combinatorial Hierarchy of [Bastin&Kilmister, Parker-Rhodes], although the material presented here differs herefrom in many ways.

How can one get something from nothing? This question is currently being framed in terms of the concept of emergence - that novel properties can emerge from simpler constituents, while simultaneously these properties cannot be reduced to isolated actions of said constituents. From our point of view, the concept of emergence cannot be separated from that of hierarchy, in that emergent properties by definition inhabit a ‘higher’ - that is, more complex - level of organization than their constituents. The fact that the concept of emergence is controversial is, in our view, a result of the residual, but all-pervasive, influence of Newton’s physics, which is entirely reductionistic.

Nevertheless, a great advantage of the Newtonian view is that it provides an intuitive mechanism for how material entities influence each other: momentum exchange, as in

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the universal image of billiard balls colliding and rebounding. The felicity of mechanism is that it provides a blow-by-blow description of what is going on, and in so doing fertilizes our imagination while simultaneously guiding our modelling choices. The quantum-mechanical revolution put an end to this, evaporating ‘materia’ into a cloud of probability amplitudes, uncertainty, and non-determinism. Lacking the compass of a trustworthy mechanism, we have been collectively doomed to purblindly wander the jungles of mathematics, as Einstein well appreciated. For mathematics, in spite of appearances, does not really describe how things happen, but rather only their essential what.

Although we do not argue the case particularly here (see [Manthey94] for some initial conjectures), our model implicitly supplies an informational mechanism for quantum mechanics, and at that, one that is compatible with relativity theory. The mechanism we present is computational, in the sense that it is discrete, combinatorial in character, and described in terms of discrete computational operations. However, these operations are not the usual arithmetic “number crunching” most people associate with computing. Nor is the computation in question describable in terms of a Turing machine, which is - as [Penrose] essentially argued - equivalent to Newtonian mechanics. Rather, the concept of an evolving and expanding universe demands a distributed multi-process view. Thus the critical mechanisms are those that express the synchronization between the events constituting the various processes. The model that is built up on this basis, and explained in the following, we have dubbed the phase web paradigm; a corresponding program, called Topsy, has been implemented and is available to interested persons [www].

A major contribution of this paper is therefore that it shows how to unite computational, informational mechanisms - with the aforementioned advantages hereof - with ‘classical’ vector algebra, with surprising and intriguing results. In addition, the hierarchical aspect of the basic mechanisms shows how it is possible, at least in principle, to tell a detailed and rigorous story about the ascent from the microscopic to the macroscopic world.

The outline of the paper is as follows: the next section sketches our conceptual, and decidedly computational, framework, its mapping to Clifford algebras and a novel hierarchical structure that naturally captures emergent phenomena. The following section connects this with our implementation, revealing an important ambiguity in the mathematical description versus concrete informational mechanism, which ambiguity is then resolved. We then present a combinatorial “bit bang” based on the mechanisms introduced, and show how quaternions appear.
2 Mechanism, Clifford algebra, and Hierarchy

Initially, it is crucial to establish the validity of the concept of emergence in a mechanistic context. We will see that the presence of multiple processes is critical for this purpose.

The coin demonstration - Act I. A man stands in front of you with both hands behind his back, whilst you have one hand extended in front of you, palm up. You see the man move one hand from behind his back and place a coin on your palm. He then removes the coin with his hand and moves it back behind his back. After a brief pause, he again moves his hand from behind his back, places what appears to be an identical coin in your palm, and removes it again in the same way. He then asks you, “How many coins do I have?”.

It is important at the outset to understand that the coins are formally identical: indistinguishable in every respect. If you are unhappy with this, replace them with electrons or geometric points. Also, there are no ‘tricks’ in the prose formulation. What is at issue is the fact of indistinguishability, and we are simply trying to pose a very simple situation where it is indistinguishability, and nothing else, that is in focus.

The indistinguishability of the coins now agreed, the most inclusive answer to the question is “One or more than one”, an answer that exhausts the universe of possibilities given what you have seen, namely at least one coin. There being exactly two possibilities, the outcome can be encoded in one bit of information. Put slightly differently, when you learn the answer to the question, you will per force have received one bit of information.

The coin demonstration - Act II. The man now extends his hand and you see that there are two coins in it. [The coins are of course identical.]

You now know that there are two coins, that is, you have received one bit of information. We have now arrived at the final act in our little drama.

The coin demonstration - Act III. The man now asks, “Where did that bit of information come from??”

Indeed, where did it come from?! Since the coins are indistinguishable, seeing them one at a time will never yield an answer to the question. Rather, the bit originates in the simultaneous presence of the two coins. We call such a confluence a co-occurrence.

Penrose [Penrose] has argued that computational systems, not least parallel ditto, in principle cannot model quantum mechanics. However, his argument is based on Tur-

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2 Since we are dealing with informational mechanism, we prefer the term neo-mechanistic.
3 At this juncture, we hasten to mention that we are dealing here with local simultaneity, so there is no collision with relativity theory. Indeed, Feynman [Feynman65 p.63] argues from the basic principle of relativity of motion, and thence Einstein locality, that if anything is conserved, it must be conserved locally.
ing’s model, which in turn cannot capture co-occurrence.

Notice by the way how the matrix-based formulations of QM neatly get around the inherent sequentiality of \( y = f(x) \)-style (ie. algorithmic) thinking, namely by the literal co-occurrence of values in its vectors’ and matrices’ very layouts; and thereafter by how these values are composed *simultaneously* (conceptually speaking) by matrix operations. Instead of the matrix route, we have taken the conceptually compatible one of Clifford algebras, which are much more compact, elegant, and general, cf. [Hestenes].

We see from the Coin demonstration that there is information, *computational information*, available in the universe which *in principle* cannot be obtained sequentially. One can say that the information received from observing a co-occurrence is indicative of the fact that two states do not mutually exclude each other.

Co-occurrence and mutual-exclusion are in fact conceptual *opposites*, in that (say) two events cannot simultaneously both co-occur and mutually exclude. The following shows how this insight can be promoted to a concept of ‘action’.

**The block demonstration.** Imagine two ‘places’, \( p \) and \( q \), each of which can contain a single ‘block’. Each of the places is equipped with a sensor, \( s_p \) respectively \( s_q \), which can indicate the presence or absence of a block.

The sensors are the *only* source of information about the state of their respective places and are assumed *a priori* to be independent of each other, though they may well be correlated. The two states of a given sensor \( s \) are mutually exclusive, so a place is always either ‘full’, denoted (arbitrarily) by \( s \), or ‘empty’, denoted by \( \tilde{s} \); clearly, \( \tilde{\tilde{s}} = s \).

*Suppose there is a block on \( p \) and none on \( q \). This will allow us to observe the co-occurrence \( s_p + \tilde{s}_q \). From this we learn that having a block on \( p \) does not exclude not having a block on \( q \). Suppose at some other instant (either before or after the preceding) we observe the opposite, namely \( \tilde{s}_p + s_q \). We now learn that not having a block on \( p \) does not exclude having a block on \( q \). What can we conclude?*

First, it is important to realize that although the story is built around the co-occurrences \( s_p + \tilde{s}_q \) and \( \tilde{s}_p + s_q \), everything we say below applies equally to the ‘dual’ pair of co-occurrences \( s_p + s_q \) and \( \tilde{s}_p + \tilde{s}_q \). After all, the designation of one of a sensor’s two values as ‘\( \sim \)’ is entirely arbitrary. It is also important to realize that the places and blocks are story props: all we really have is two two-valued sensors reflecting otherwise unknown activities in the surrounding environment. Such sensors constitute the *boundary* between an entity and its environment in the phase web paradigm.

Returning to the question posed, we know that \( s_p \) excludes \( \tilde{s}_p \) and similarly \( s_q \) excludes...

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\(^4\)We are working in \( \mathbb{Z}_3 = \{0, 1, -1 = \tilde{1}\} \) rather than the traditional \( \mathbb{Z}_2 = \{0, 1\} \). We use the visual convention that a sensor written without a tilde is taken to be bound to the value 1, and vice versa; clearly, \( 0 = 0 \).
Furthermore, we have observed the co-occurrence of $s_p$ and $\bar{s}_q$ and vice versa. Since the respective parts of one co-occurrence exclude their counterparts in the other co-occurrence (cf. first sentence), we can conclude that the co-occurrences as wholes exclude each other.

Take this now a step further. The transition $s_p \rightarrow \bar{s}_p$ is indicative of some action in the environment, as is the reverse, $\bar{s}_p \rightarrow s_p$. The same applies to $s_q$. Perceive the transitions $s_p \leftrightarrow \bar{s}_p$ and $s_q \leftrightarrow \bar{s}_q$ as two sequential computations, each of whose states consists of a single value-alternating bit. By the independence of sensors, these two computations are completely independent of each other. At the same time, the logic of the preceding paragraph allows us to infer the existence of a third computation, a compound action, with the state transition $s_p + \bar{s}_q \leftrightarrow \bar{s}_p + s_q$, denoted $s_p s_q$. In effect, by combining in this way two single-bit computations to yield one two-bit computation, we have lifted our conception of the actions performable by the environment to a new, higher, level of abstraction. This inference we call co-exclusion, and can be applied to co-occurrence pairs of any arity $> 1$ where at least two corresponding components have changed.

Notice that the same reasoning applies to the action $s_p + s_q \leftrightarrow \bar{s}_p + \bar{s}_q$, also denoted $s_p s_q$. The two actions are, not surprisingly, dual to each other, so co-exclusion on two sensors can generate two distinct actions. Like co-occurrence, an action defined by co-exclusion also possesses an emergent property, generally comparable to spin $\frac{1}{2}$ [Manthey94].

Co-exclusion provides a very general mechanism for self-organization: simply observe co-excluding co-occurrences, since these then will represent an abstraction of the environment. However, the mechanism for actually discovering co-exclusions is as yet unspecified. Speaking mechanistically, how exactly does one (eg. the universe) discover the existence of a co-exclusive relationship between two co-occurrences?

Define a co-occurrence in terms of an “event buffer” with time-window-size $\Delta t$, where true simultaneity requires that $\Delta t = 0$, and larger values recognize the factual granularity with one can resolve events and/or the time-scale at which an environment varies. Suppose further that event identifiers are put into the event buffer as they occur, i.e. the new state engendered (and labelled) by the action associated with the event is inserted into the buffer. Finally, suppose that events in the buffer are successively discarded as their residence exceeds $\Delta t$ or the same event-state changes again. Clearly, this arrangement guarantees that the state changes contained in the buffer all took place within $\Delta t$, and thus occurred ‘simultaneously’ (modulo $\Delta t$). The reader is at this point encouraged to ponder the fact that this mechanism in fact solves the problem of discovering co-exclusions, and at that, in linear time and space and without pre-specification! [Reader pause, for a lovely aha! experience.]

To see why this claim is true, consider the fact that a sensor’s states are mutually exclusive, that is, if a sensor is currently in state $s$ then before it changed it was in the state $\bar{s}$. Furthermore, in $\mathbb{Z}_3$ at least, the opposite is also true: $\bar{\bar{s}} = s$. Hence, since the
buffer contains the co-occurrence (say) \( s_1 + s_2 \), and they both just changed, then before they entered the buffer, \( \tilde{s}_1 + \tilde{s}_2 \) obtained. But these two co-occurrences are exactly those necessary to define the co-exclusion \( s_1 + s_2 \leftrightarrow \tilde{s}_1 + \tilde{s}_2 \). The computation time and space are fundamentally linear because they are proportional to the buffer size. If we specify that all events are to pass through our event buffer, then the only pre-specification is the arity of the co-exclusion. Even this pre-specification can be avoided if all possible co-exclusions over the current buffer contents are instantiated as each event is entered into the buffer.

### 2.1 Co-occurrence and Co-exclusion via Clifford Algebras

This section presents, very informally, the mathematical foundation of the phase web paradigm. The point of departure is to view sensor states as vectors instead of scalars, as is conventionally done.

Let sensor state \( s = 1 \) indicate that sensor \( s \) is currently being stimulated, i.e. a synchronization token (an informational marker for a state’s existence) for that state is present, and \( s = \tilde{1} \) that \( s \) is currently not being stimulated, and hence a token for state \( \tilde{s} \) is present. Thus the two states of \( s \) are represented by their respective synchronization tokens, whose respective presences by definition exclude each other.

That a set of sensors qua vectors are orthogonal derives from the fact that, in principle, a given sensor says nothing about the state of any other sensor. A state of a multi-sensor system is then naturally expressed as the sum of the individual sensor vectors, and the state \( (s_a, \tilde{s}_b) = (1, \tilde{1}) \) is written as the vector sum \( s_a + \tilde{s}_b \). Since such states represent co-occurrences, it follows that co-occurrences are vector sums, usually denoting partial (local) states. Note how the commutativity of ‘+’ reflects the lack of ordering of the components of a co-occurrence; and as well that the co-occurrence \( 1 + \tilde{1} = 0 \) indicates that the interpretation of ‘zero’ is that the components of the sum exclude each other. Because \( \mathbb{Z}_2 \) does not distinguish state-value and exclusion, we take our algebra to be over \( \mathbb{Z}_3 = \{0, 1, 2\} = \{0, 1, \tilde{1}\} \).

The next step is to represent actions. [Manthey94] presents a detailed analysis of the group properties of co-occurrences and actions, concluding that the appropriate algebraic formalism is a (discrete) Clifford algebra [Hestenes], and that the state transformation effected by an action is naturally expressed using this algebra’s vector product. A prime characteristic of this product is that it is anti-commutative, that is, for \( (s_1)^2 = (s_2)^2 = 1 \), \( s_1 s_2 = -s_2 s_1 \). The magnitude of any such product is the area of the parallelogram its two components span, and the orientation of the product is

\[ ab = a \cdot b + a \wedge b, \]

where \( a \wedge b = -b \wedge a \) is the oriented area spanned by vectors \( a, b \). The basis vectors \( s_i \) of a Clifford algebra may have \( (s_i)^2 = \pm 1 \), and while here we choose \(+1\), reasons are appearing for choosing \(-1\). As long as they all have the same square, it doesn’t matter for what is said here. Note that \( (s_1 s_2)^2 = -1 \), so \( s_1 s_2 \cong \sqrt{-1} \).
perpendicular to the plane of the parallelogram and determined by the “right hand rule”. Applying the Clifford product to a state, one finds - using the square-rule and the anti-commutativity of the product given above - that

\[(s_1 + s_2)s_1s_2 = s_1s_1s_2 + s_2s_1s_2 = s_2 + \tilde{s}_1s_2s_2 = \tilde{s}_1 + s_2\]  

that is, that the result of the action \(s_1s_2\) is to rotate the original state by 90°, for which reason things like \(s_1s_2\) are called spinors. Thus state change in the phase web is modelled by rotation (and reflection) of the state space, and the effect of an ‘entire’ action can be expressed by the inner automorphism \(s_1s_2(s_1 + s_2)s_2s_1 = \tilde{s}_1 + \tilde{s}_2\), which corresponds to a rotation through 180°.

One of the felicities of Clifford algebras is that one needn’t designate one of the axes as ‘imaginary’ and the others as ‘real’. Rather, the \(i\)-business is implicit and the algebra’s anti-commutative product neatly bookkeeps the desired orthogonality and inversion relationships, no matter how many dimensions [ie. sensors (roughly)] are present.

The above 2-spinors are just one example of the vector products available in a Clifford algebra - any product of the basis vectors \(s_i\) is well-defined, and just as \(s_1s_2\) defines an area, \(s_1s_2s_3\) defines a volume, etc. Being by nature mutually orthogonal, the terms of a Clifford algebra

\[s_i + s_is_j + s_is_js_k + \ldots + s_is_j\ldots s_n\]  

themselves also define a vector space, which is the space in which we will be working (actually, hierarchies of such spaces). [The term (eg.) \(s_is_j\) above, for \(n = 3\), denotes \(s_1s_2 + s_2s_3 + s_3s_1\), that is, all possible non-redundant combinations.] It is perhaps worth stressing that this vector space is the space of the distinctions expressed by sensors, and as such has no direct relationship with ordinary 3+1 dimensional space.

A Clifford product like \(s_1s_2\) reflects both (1) the emergent aspect of a phase web action (via its perpendicularity to its components) and (2) its ability to act as a meta-sensor (since its orientation is \(\pm 1\)). Regarding (1), the emergence is rooted in the information gleaned from the co-occurrences underlying the co-exclusion inference that yields \(s_1s_2\), cf. the Coin demonstration. Regarding (2), the co-exclusion inference is an abstraction that produces a single action with two bits of state from two lower level actions each possessing a single bit of state. Since this abstraction has the same external behavior as its constituent sensors, namely \(\pm 1\), we can legitimately view it too as a sensor, a meta-sensor. By co-excluding meta-sensors, we can build a new set of abstractions - meta-meta-sensors - etc., and thus construct a hierarchy of interwoven co-occurrences and exclusions that directly reflects the observed activity of the surrounding environment. This hierarchy is the topic of the following.

### 2.2 From Clifford Algebra to Hierarchy

In analogy to \(s_1, s_2\) co-excluding to yield \(s_1s_2\), one might expect that the co-exclusion of two meta-sensors, say \(s_is_j\) and \(s_ps_q\), would be modelled by simply multiplying
them, to get the 4-action $s_1 s_2 s_3 s_4$. This turns out however to be inadequate, since although by the same logic the co-exclusion of (say) $s_i$ and $s_j$ in a phase web expresses explicitly a useful relationship (e.g. part-whole), the algebra’s rules reduce it from $s_i s_j$ to $s_j$, which is simply redundant.

Instead, we take as a clue the fact that change in a phase web occurs via trickling down through the layers of hierarchy, and draw an analogy with differentiation. In the present decidedly geometric and discrete context, differentiation corresponds to the boundary operator $\partial$. Define $\partial s = 1$ and let

$$\partial(s_1 s_2 \ldots s_m) = s_2 s_3 \ldots s_m - s_1 s_3 \ldots s_m + s_1 s_2 s_4 \ldots s_m - \ldots (-1)^{m+1} s_1 s_2 \ldots s_{m-1}$$

that is, drop one component at a time, in order, and alternate the sign. Using the algebra’s rules as before, one can show that $\partial(s_1 s_2 \ldots s_m) = (s_1 + s_2 + \ldots + s_m)s_1 s_2 \ldots s_m$ which is exactly the form of equation (1) for what an action does!

Take now equation (2) expressing the vector space of distinctions, segregate terms with the same arity, and arrange them as a decreasing series:

$$s_i \leftarrow s_i s_j \leftarrow s_i s_j s_k \leftarrow \ldots \leftarrow s_i s_j \ldots s_{n-1} \leftarrow s_i s_j \ldots s_n \quad (3)$$

Here as before, $s_i s_j$ is to be understood as expressing all the possible 2-ary forms (etc.), and hence the co-occurrence of pieces of similar structure. Each of the individuals is a simplicial complex, and the whole sequence is called a chain complex, expressing a sequence of structures of graded geometrical complexity in which the transition from a higher to a lower grade is defined by $\partial$. Furthermore, the entities at adjacent levels are related via their group properties - their homology, which we here assume is trivial.

The basic mechanism for expressing change or action in our hierarchical context is that of goal-driven computation. A goal is a local state whose presence causes an action to attempt to change its orientation, and a goal will typically be decomposed recursively into subgoals on that action’s constituents as it trickles down through the $\partial$-hierarchy. [Goals differ from the ‘imperatives’ traditionally used in computing - e.g. add x,2 or sine(x) - by not guaranteeing that the indicated computation will be achieved, but rather only a ‘best effort’, and success is contingent on the state of the environment and the rest of the phase web. There is no teleological baggage per se in this concept - ‘potential’ is a closer idea.]

It turns out that there is a second structure - a cohomology - that is isomorphic to the homology, but with the difference that arity increases via the $\delta$ (or co-boundary) operator, precisely opposite to $\partial$, cf. eqn. (3):

$$s_i \rightarrow s_i s_j \rightarrow s_i s_j s_k \rightarrow \ldots \rightarrow s_i s_j \ldots s_{n-1} \rightarrow s_i s_j \ldots s_n \quad (4)$$

6If one takes two components at a time, as we will do on occasion later on, then the sign-alternation disappears.

7More precisely, $(\sigma_p, \delta d^{p-1}) = (\sigma_p \partial, d^{p-1})$, where $\sigma_p$ is a simplicial complex with arity $p$, and $d^p$ the corresponding co-complex.
Building such increasing complexity is exactly what co-exclusion does. [We note that a Clifford algebra satisfies the formal requirements for the existence of the associated homology and cohomology.]

It is easily proven that $\partial \partial = 0$, and by isomorphism, so also $\delta \delta = 0$. For example, $\partial \partial (s_1 s_2) = \partial (\bar{s}_1 + s_2) = 1 + 1 = 0$, and similarly, $\partial \partial (s_2 s_1) = \partial (s_1 + \bar{s}_2) = 1 + 1 = 0$. Combining these now as the exclusion $\partial \partial (s_1 s_2 + s_2 s_1)$, we get $(1 + 1) + (1 + 1) = (1 + 1) + (1 + 1) = 0$, which are the two forms of the input to the determination of a co-exclusion relationship. Recalling the event-buffer mechanism for discovering co-exclusions, we see, especially if $\Delta t = 0$, that this mechanism is a realization of the isomorphic $\delta \delta = 0$!

Viewing $\delta$’s abstraction operation informationally, we see that two bits $(s_1, s_2)$ are being encoded in a single bit (the orientation of $s_1 s_2$), that is, information is being ‘abstracted away’. The missing bit indicates the phase of the action, ie. whether the state rotation/transformation is $s_1 + s_2 \leftrightarrow \bar{s}_1 + \bar{s}_2$ or $s_1 + \bar{s}_2 \leftrightarrow \bar{s}_1 + s_2$. What will actually occur is however well-defined by the other connections $s_1, s_2$ partake in, ie. the boundary conditions of the action. Note however that ‘well-defined’ does not necessarily imply ‘deterministic’. Isomorphically, the corresponding $\partial$ operation destroys the emergent information in the current state and replaces it by non-deterministic outcome.

Refer now to Figure [Bowden82], which we call a ladder diagram. The shaded shape points out a unique property of the homology-cohomology ladder, one that even many topologists seem unaware of, namely that the isomorphisms $\mu, \mu^{-1}$ are twisted, that is, the kernel of the group at one end of a rung is mapped by $\mu$ (respectively, $\mu^{-1}$) into the non-kernel elements of the group at the other end. [The isomorphisms $\mu, \mu^{-1}$ are matrices containing the terms’ $\mathbb{Z}_3$ coefficients.] This property was discovered by [Roth] in his proof of the correctness of Gabriel Kron’s then controversial methods for analyzing electrical circuits [Bowden82], and turns out to have profound implications: the entirety of Maxwell’s equations and their interrelationships can be expressed by a ladder with two rungs plus four terminating end-nodes [Bowden], and [Tonti] has - independently - shown similar relationships for electromagnetism and relativistic gravitational theory. Roth’s twisted isomorphism (his term) thus reveals the deep structure of the concept of boundary, and shows that the complete story requires both homology and cohomology.

### 2.3 Generalizing the Twisted Isomorphism Hierarchy

Each level of a ladder hierarchy, as presented so far, is built entirely from entities (ie. sensors) from the level immediately underneath, leading to what we call a ‘pancake’ hierarchy. But this is an unnecessary limitation, from which we now generalize.

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8Strictly speaking, $\partial, \delta, \mu/\mu^{-1}$ should all be indexed by level: $\partial_\ell, \delta_\ell, \mu_\ell/\mu_\ell^{-1}$. 
The left side of the ladder is the homology sequence generated by $\delta$ over the representation of actions as Clifford products cf. eqn. (3). The downward flow of decomposition of the structure into simpler pieces (ie. the crossing of successive boundaries) corresponds to the trickling down of goals to sub-goals described earlier.

The right side of the ladder is similarly the cohomology sequence generated by $\delta$ from sensory impressions, cf. eqn (4). The upward flow of composition of structure to form more complex structure corresponds to the effect of co-exclusion, up through which increasingly complex structure sensory impressions bubble.

The circles represent all the entities (Clifford algebra terms) at the particular level of complexity. The larger of the two circle halves holds those entities which will map to zero with the next hierarchical transition ($\delta$ or $\delta$) - the kernel of the group - as indicated by the pointed ‘beak’.

The rungs of the ladder, besides denoting the location and content of hierarchy levels, also express the existence of isomorphisms ($\mu$, $\mu^{-1}$) between the structures at either end of a given rung. The shaded portion, which can be seen to repeat in both directions, expresses the commutation relations that obtain.

**Figure 1:** Ladder diagram, illustrating homology-cohomology relationships.
Let $S_i$ be the set of sensors at $\delta$-level $i$. Similarly, let $G_i$ be the set of (sensors expressing the presence of) goals at $\partial$-level $i$. A pancake meta hierarchy of 2-actions can now be characterized by $S_i = S_{i-1} \times S_{i-1}$, where $\times$ is the cartesian product mediated by $\delta$.

Other, more general, hierarchical forms are now easily seen:

- $S_i = S_j \times S_k$, $j, k < i$, yielding non-pancake meta hierarchies; and of course the product may be over $>2$ levels. Aside from this, however, the semantics is roughly as before;
- $G \times G$, yielding a purely goal-based icarian hierarchy, roughly similar to a function-composition hierarchy;
- $S \times G$, yielding a combined abstraction over the underlying ladder level(s) that we call a morphic hierarchy.

Figure 2 illustrates the latter two, and we note that the morphic level in (a) may in principle ‘cross’ levels more radically, eg. as (b) does. We call these generalized forms ortho-hierarchies.

Icarian actions provide a means for a computation to express, self-reflectively, the way it carries out its goals. Morphic actions provide a means for a computation to express, self-reflectively, the relationship between $S$ and $G$ that is otherwise buried in $\mu, \mu^{-1}$. In the following, we will use the term ‘meta’ to denote all three types of abstraction.

3 Meta-sensor ambiguity and its resolution

At this point, we have seen two informational mechanisms for emergence - co-occurrence and co-exclusion, a mechanism for constructing the latter from the former, and a math-
emathematical framework that describes these and their composition into a very general hierarchical form. Still missing, though, is the mechanism that propagates information up and down this hierarchy.

The key issue here is that the algebraic meta-sensor symbol $s_1s_2$ codes two bits of information - the orientations of $s_1$ and $s_2$ - into one bit - the orientation of $s_1s_2$. Another way to put this is that the algebraic symbol does not distinguish between the duals: $s_1 + s_2 \leftrightarrow \tilde{s}_1 + \tilde{s}_2$ versus $\tilde{s}_1 + s_2 \leftrightarrow s_1 + \tilde{s}_2$.

Looking more closely at this, there are the following possibilities for a state-propagation mechanism. I.e. assuming that we begin in the state $s_1 + s_2$, flip the orientation of $s_1s_2$:

1. When both $s_1$ and $s_2$ flip.

   Here, we encode the two states $s_1 + s_2$ and $\tilde{s}_1 + \tilde{s}_2$ as the two possible orientations of $s_1s_2$. Unfortunately, there is a problem: the two dual states $\tilde{s}_1 + s_2$ and $s_1 + \tilde{s}_2$ are not represented at all in the meta-sensor state, and if one of these latter states obtains, then the meta-sensor’s state is undefined.

   If we therefore insist on a distinct meta-sensor for each of the duals, we give up any attempt at state compression (abstraction). We also get the problem of the very existence of the dual meta-sensor/meta-action...it may not even exist yet, and the changing of a single sensor is not enough to trigger its discovery (cf. the event window mechanism). This issue revolves around the fact that mathematically, all components of the space (here, Clifford algebra terms) are always implicitly present when needed, but this is not necessarily the case when, as we intend, they directly represent physical entities.

2. When one of $s_1$ or $s_2$ flips.

   In this encoding, one orientation of $s_1s_2$ indicates one dual, and the other orientation the other dual. However, this does not distinguish between the two states within a given dual (Exactly which state are we in? In which direction did we rotate?!), and $s_1s_2$ doesn’t flip at all if $s_1$ and $s_2$ flip simultaneously. In effect, we double the rate at which the meta-sensor flips to (partially) compensate for the fact that two bits simply cannot be encoded in one bit.

   This alternative (dubbed ‘symmetric’) succeeds at compressing state only because there are exactly two duals (which by definition exclude each other).

3. According to the orientation of $s_1$.

   Like the preceding alternative, this accepts ambiguity in the phase of the current and resulting states, in this case distinguishing one half of the $s_1 \times s_2$ plane, but not which quadrant. Mapping $s_2$ has a conjugal effect. Allowing both maps (to distinct meta-sensors) removes the ambiguity at the price of losing abstraction.
Note that we have only treated 2-actions. An action of arity $n$ possesses $\frac{n^2}{2}$ duals, so the above issues compound as arity increases. We will encounter this in the next section.

Note also that the existence of duals creates a naming problem, in that, on the one hand, all the duals of a given action should have a common name to reflect this familial relationship, while on the other they are all distinct from each other. Thus a given dual’s name must be a 2-tuple of the form $(\text{actionId},\text{dualId})$, where actionId is a (commutative) hash of (the names of) all the action’s constituent sensors (both polarities), whereas dualId need only be locally unique.

The above list is couched in terms of the bubbling of state change up the $\delta$-hierarchy. A similar analysis can be carried out from the point of view of goals trickling down the $\partial$-hierarchy, in which case the question is: given the goal $s_1 s_2 \rightarrow \tilde{s}_1 s_2$, which of the possible subgoals $s_1 \rightarrow \tilde{s}_1$, $s_2 \rightarrow \tilde{s}_2$, or both, should be issued, and when should they be retracted?

From either point of view, the second of the above alternatives seems preferable, for the following reasons:

- The phase ambiguity can be seen as a nifty way to model non-deterministic outcomes (true also of the third alternative, but not as symmetric).
- The second alternative satisfies the identity $s_1 s_2 (s_1 s_2 + s_1 + s_2) s_2 s_1 = s_1 s_2 + \tilde{s}_1 + \tilde{s}_2$, and hence preserves the semantics we arrived at in the preceding section.
- The third alternative has the effect that higher-level abstractions continue to ape primitive-level sensors ad infinitum, rather than the new exclusions they purport to reflect.
- The third alternative introduces a new problem: how to choose which of the two constituent sensors is to be mapped to the meta-sensor?

Unfortunately, the second alternative only works for arity 2, but Nature’s well-known affinity for symmetry should perhaps not be denied. Therefore we accept this hint and will try to solve our problems with arity 2 co-exclusions only. Nevertheless, since the choice of propagation model should have predictive consequences, this is an issue that can be resolved empirically (and we claim a certain amount of empirical support for our choice, as will become apparent).
4 The Bit Bang and quaternions

In this section we present a Big Bang scenario, but where, in contrast to the usual version, the expansion is in terms of information. This information is the result of making distinctions, and the maker is ‘the universe’ in the guise of an initial **Void**. In using the latter term, we intend no particular a priori interpretation, whether physical, logical, or metaphysical. This said, interpreting it in the present context as the vacuum is natural.

Regarding the making of distinctions, and in line with our development thus far, we will apply the distinction ‘co-occur vs. exclude’, which pair has the distinction-defining property that the two aspects necessarily exclude each other. In that the very utterance of one half of a distinction implies the other, they become conceptually co-occurrant in yin-yang fashion. This fact is in turn captured by the co-exclusion inference, which simultaneously introduces the hierarchical moment analyzed in §2.3.

Before presenting our *Bit Bang*, however, it is appropriate to motivate its relevance to our present endeavor. There are two aspects, the first being to demonstrate the emergence of space in the form of quaternions. Equally important, however, is the more theoretical problem of grounding the endeavor as whole. The point here is that in a hierarchical theory, such as the one we are presenting, there are two components: the entities that populate the various levels (eg. quaternions) and the mechanism of the hierarchy itself, ie. the mechanism by which the hierarchy is constructed.

The latter is responsible for the basic properties of the entities, and *these properties are by definition the same at every level*. We call this property of a hierarchical theory **level independence**, and it is both the blessing and the burden of any hierarchical theory. In the case at hand, the basic properties are those of co-occurrences and (co-)exclusions, and derivations and implications hereof. Level independence thus implies that the phase web’s hierarchy should be able to give a reasonable account of its creation ab initio, thereby **grounding** the entire construction. We interpret the ab initio construction as as an informational Big Bang, which information is the product of the distinctions afforded by co-occurrence (cf. the Coin demonstration) and co-exclusion (cf. the Block demonstration).

On the next page, then, is our Bit Bang. We divide it into a series of steps, where each step is intended to follow inevitably from the preceding one. One alternative branch is (as noted) from Step 0 to Step 1, where one could use $0 = \tilde{0}$ instead of $0 = 0 + 0$, yielding $\tilde{1}$ at Step 1 and thereafter reversing the logic of Step 2 to yield $1$; this branch thus rejoins the one given at the end of Step 2. In Step 1 one could also ask $1 + 0$, but this also yields $1$. Finally, Steps 2, 3, and 4 each ‘close’ a logical level.

Regarding the meta-physical language: what we are trying to convey cries out for verbal interpretation, and without such language the mathematical expression is more arcane than expressive, especially in the beginning.
Step | Symbolically | Commentary
--- | --- | ---
0 | Void = 0 | That about which nothing can be said. Even naming it implies the existence of something that is not Void. But Void is everything and nothing, paradoxically simultaneously thinkable and unthinkable. Physically, Void is (presumably) the vacuum.

Mathematically, we might attempt $0 = 0 + 0 = \bar{0} = \bar{0} + \bar{0} = 0 + \bar{0}$ but even this reifies distinctions we are forbidden: ‘+’ implies ‘parts’, and ‘∼’ implies ‘non-Void’.

But the universe undeniably exists! So from Void there must be a step. Suppose it was $0 = 0 + 0$, ie. the parts are as the whole (starting with $0 = \bar{0}$ yields $\bar{1}$, thence 1). Denote this distinction by the symbol...

1 | $I_0$ | which means ‘the same as’. But, having now admitted ‘parts’, we must ask, What is $I_0 + I_0$? [We now invoke $Z_3$ because: (1) $Z_0$ is not open to extension, (2) $Z_2$ doesn’t distinguish Void from ‘opposite’ so (3) $Z_3$ is the first possibility. Since co-occurrence together with exclusion exhaust/fulfill Void (see next step), it appears that $Z_3$ is sufficient to all future expansion.] Presuming then $Z_3$, the answer is $\bar{1}$.

2 | $\bar{I}_0$ | that is, $I_0$ is not the same as its parts ... $\bar{I}_0$ means ‘the opposite of’, and

$I_0 + \bar{I}_0$ means that the parts ‘the same as’ and ‘the opposite of’ $\equiv$ Void.

Ie. the marriage of sameness and oppositeness exhausts/fulfills Void. Denote this latter distinction, which is new, by...

3 | $I_1$ | $I_0 + \bar{I}_0 \rightarrow I_1$ is an “arity 1” co-exclusion, $\delta_0$. [Symbols with subscript $= 1$ are in effect ‘discrete variables in $Z_3$’ .]

Now that we have both sameness (co-occurrence) and oppositeness (exclusion), we can ask, What is $I_0 + \bar{I}_0 + I_0 + \bar{I}_0 = I_1 + \bar{1}$? This distinction is a true (arity $\geq 2$) co-exclusion.

[This step, and similar ones later, assumes that the Void can/will continue to produce new step-1 instances as needed. Logically, this is unproblematic; physically, it assumes the same vacuum activity as produced the first instance.]

The result of the co-exclusion of $I_1$ and $\bar{I}_1$ is

4 | $I_2$ | and, via step 2, $\bar{I}_2$ follows. Note that $(I_2)^2 = (\bar{I}_1)^2 = -1$, cf. §2.1.
Clearly, we could continue this listing of distinctions ad infinitum, but we choose to end it here, since step 4 has yielded $s_1s_2$, which is the basic quaternion building block.

Via the mappings
\[
\begin{align*}
1_0 & \mapsto 1, \quad \bar{1}_0 \mapsto -1 \\
1_1 & \mapsto s, \quad \bar{1}_1 \mapsto \bar{s} \\
1_2 & \mapsto s_1s_2 \mapsto e_1 \\
1_2 & \mapsto s_2s_3 \mapsto e_2 \\
1_2 & \mapsto s_3s_1 \mapsto e_3
\end{align*}
\]
it is easy to verify the defining quaternion relations
\[
\begin{align*}
e_i^2 &= -1 \\
e_i e_j &= -e_j e_i, \quad i \neq j \\
e_1e_2 &= e_3, \quad e_2e_3 = e_1, \quad e_3e_1 = e_2
\end{align*}
\]
whence we have redeemed the promissory note contained in the title of this paper.

Notice however that we have only witnessed the emergence of local “3-D-ness”. We are not claiming (nor do we wish to claim) that this 3-D-ness is a global Newtonian space with unique origin, nor even relativized multiple ditto. Rather, 3-D-ness is a property of co-exclusion-derived objects with sufficient information-carrying capacity (“complexity”). The globality of 3-D-ness can only be achieved by the need for consistency between objects sharing a given distinction (sensor), and a change in the state of a given distinction must therefore propagate through the structure. Thus it appears that our construction is entirely consistent with, although conceptually ‘under’ or ‘prior to’, general relativity in these respects. Moreover, since some distinctions lie below the level at which global 3-D-ness emerges, changes in these can appear to propagate more rapidly, since they are not constrained by the higher level structures (which will nevertheless always behave consistently vis-à-vis such changes). This is the phase web’s way of reconciling the locality conflict between relativity theory and quantum mechanics.

Given that we now have the three quaternion operators and the (local) 3-D spatial properties they define, it is natural to ask if the hierarchical buildup also can produce the 3-D objects we expect to find in such a space. We now address this question.

Our everyday experience tells us that three spatial dimensions can hold three-dimensional objects, so we should expect the extension to be straightforward. And it is, since $\delta(s_is_j + s_js_k + s_ks_i) = s_is_j s_k$ and $s_is_j s_k$ is an oriented volume (although it will develop that this is not quite right). [We postpone the issue of finding some mass to fill this volume.] Notice by the way that this formulation requires an arity-3 co-exclusion.

The next issue is how to propagate state up to this new, volumetric, entity. The problem is the same as before, except worse: instead of needing to encode two bits into one,
we now must encode three into one, that is, reflect eight possible states in two. Although we will eventually arrive at a similar solution as before (ie. arity 2), the details are instructive.

The table below lists the eight possibilities (viewing the three rightmost columns as binary numbers, the first column’s numbering is the decimal equivalent):

| s₁s₂ | s₂s₃ | s₃s₁ |
|------|------|------|
| 7    | 1    | 1    |
| 6    | 1    | 1    |
| 5    | 1    | 1    |
| 4    | 1    | 1    |
| 3    | 1    | 1    |
| 2    | 1    | 1    |
| 1    | 1    | 1    |
| 0    | 1    | 1    |

The pairs 7,0, 6,1, 5,2, 4,3 are co-exclusions, and are distributed symmetrically about the horizontal line. If we simply co-exclude these amongst themselves, we will get even more (six, to be exact) so this approach diverges. Rather, if we are to use an encoding similar to that of the earlier 2-coex case, we must look at little more closely at the dynamics of these entities. One could say that for these four 3-co-exclusions, the dynamics is that all three bits flip. What, then, if only one or two flip?

One change at a time yields so-called Grey-coded sequences, and the connectivity of the transitions is captured by a unit cube, each of whose vertices is labelled by one of the above states. That is, no compression of states occurs. We conclude therefore that this kind of distinction is not useful (nor is it a co-exclusion, so it’s not really a valid distinction anyway).

In the case where two meta-sensors flip (and one thus remains constant), it turns out that there are two disjoint families, 0, 3, 5, 6 and 1, 2, 4, 7, each defining a tetrahedron, ie. a plane plus a point outside of that plane, and hence 3-D orientation. See Figure 3.

Observe now the following:

- Half of each family is above/below the line, and the members of the halves pair complementarily, so above/below the line each contains all four rotation states around s₁s₂;

- Each family as well expresses the four possible rotations around s₁s₂, so the same information regarding the two bits that are flipping is available both to each family and above/below the line.
A transition across the line (i.e., between two 3-co-exclusive states) is a reflection (via a factor of $-1$, cf. $ab(a + b)ba = -(a + b)$);

There are three different possible meta-sensors, depending on which of $s_1s_2, s_2s_3, s_3s_1$ we choose to be the first column. To make use of our symmetric arity-2 meta-sensor construction/restriction, we choose these three meta-sensors to be the three co-exclusions (circularly) of $s_is_j$ with $s_k$, i.e. $\delta(s_is_j + s_k)$, written $s_is_j|s_k$. That such a meta-sensor form in fact describes $s_is_js_k$ is guaranteed by the fact that

$$\partial(s_1s_2s_3) = -[\partial(s_1s_2)s_3 + \partial(s_2s_3)s_1 + \partial(s_3s_1)s_2]$$

where the occurrences of $\partial$ on the righthand side can be interpreted as the corresponding events (=sensor changes).

Recalling (cf. §3) that a symmetric 2-meta-sensor flips when one, but not both, of its two constituents flips, the following table shows what happens to the three $s_is_j|s_k$-meta-sensors when, respectively, one, two, or all three constituents flip. Notation: $\times$ means a flip, a $-$ means no flip.

| What flips | $s_is_j$ | $s_k$ | $s_js_k$ | $s_i$ | $s_k|s_i$ | $s_j$ | Total |
|------------|----------|-------|----------|-------|-----------|-------|-------|
| $s_i$      | $\times$ | $-$    | $-$       | $\times$ | $\times$ | $-$    | 3     |
| $s_i, s_j$ | $-$      | $-$    | $\times$  | $\times$ | $\times$ | $\times$ | 0     |
| $s_i, s_j, s_k$ | $-$ | $\times$ | $-$ | $\times$ | $\times$ | $\times$ | 3     |

P C I
Since a symmetric meta-sensor only flips when one of its constituents flips, the only changes that count are those that pair a – with a ×, as reflected in the rightmost column. Thus a flip of one of the three sensors causes all three meta-sensors to flip, whereas a flip of two causes none, and when all three base-level constituents flip, so do all three meta-sensors. We now examine each case more closely; the bottom row assigns the names P, C, I respectively to the three mixed-level meta-sensors. (It is useful in thinking about this to have a picture of a little 3-D coordinate system in mind.)

Only \( s_i \) flips. This causes a reflection of both the \( s_i, s_j, s_k \) and PCI coordinate systems.

Both \( s_i \) and \( s_j \) flip. This causes a rotation in the \( s_i, s_j, s_k \) coordinate system, but no change in the PCI coordinate system (although the changes within P, C, and I are real enough).

All of \( s_i, s_j, s_k \) flip. This causes a reflection in the both the \( s_i, s_j, s_k \) and PCI coordinate systems.

Reminding ourselves of the CPT symmetry, this is exactly what should happen had we denoted the P meta-sensor as parity, the C meta-sensor as charge, and the I meta-sensor as isospin (which denotes the projection of the charge in one of three spatial directions).

In independent support of this identification, we tentatively offer the following. There are six quarks, occurring in families of two each, these two differing most critically in their charge: \( +\frac{2}{3} \) vs. \( -\frac{1}{3} \). In the mathematical formulation, \( s_1 s_2 \) and \( s_2 s_3 | s_3 s_1 \) are operationally equivalent, but in the actual realization, the latter is a distinct co-exclusion whose result just happens to have the same effect as \( s_1 s_2 \) but 180° out of phase. Because \( s_1 s_2 \) is half the ‘size’ of \( s_2 s_3 | s_3 s_1 \), we assign it charge \( -\frac{1}{3} \) and the latter \( +\frac{2}{3} \). The nicely logical way this works out together with the way changes in the three basis sensors \( \{s_1, s_2, s_3\} \) are coupled across the PCI meta-sensors in CPT-like fashion as just described, argue for identifying these three meta-sensors with parity, charge, and isospin.

We have been tempted to speculate in such matters in order to argue for our quaternion construction, and hasten to add that there are many details of the above quark structure that must be checked. In any event, it is important for the reader to understand that the only degree of freedom in this little game lies in how to arrange the pieces, i.e. the number and arity of possible co-exclusions and their mappings to corresponding meta-

\[^{9}\text{Having opened Pandora’s box here, we speculate that mass is proportional to the number of bits (distinctions) enclosed by a given co-exclusion envelope, but a glance at the quarks' empirical values shows that there is more to the story.}\]
sensor states. All co-exclusions denote distinctions (ie. bits) that the universe can and will make, which distinctions people denote by various quantum numbers (generally elements of $\mathbb{Z}_3$) eg. spin, parity, and charge. Hence, the combinatorial structure must provide every particle (known or otherwise) with a unique and consistent placement in that structure - one misfit means that the whole idea dies. In all cases, spin, quaternions and local 3-D-ness, parity, charge, and isospin are all clearly seen to be both distributed and emergent, and all are properties of the object $s_1s_2s_3$.

There is one final categorization of distinctions we must mention, namely that described by The Combinatorial Hierarchy (CH) [Bastin&Kilmister, Parker-Rhodes]. This hierarchy is traditionally constructed in $\mathbb{Z}_2$, but there is general agreement that it and the $\mathbb{Z}_3$ Bit Bang presented here are in some sense isomorphic. Howsoever, the key point is to examine the number of discriminately closed subsets (dcs’s), that is, subsets that close under the discrimination operation (in our case, exclusion; in the CH’s, exclusive-or). These are (cf. Figure 4)

$$\{s\}$$

$$\{s_1, s_2, \{s_1, s_2, s_1s_2\}\}$$,

$$\{s_1, s_2, s_3, \{s_1, s_2, s_1s_2\}, \{s_2, s_3, s_2s_3\}, \{s_3, s_1, s_3s_1\}, s_1s_2s_3\}$$

$$\ldots$$

Figure 4: Combinatorial Hierarchy categories (dcs’s) in $\mathbb{Z}_3$.

The key numbers are the successive sums of the dcs cardinalities: $1, 3, 7, 127, 2^{127} - 1 \rightarrow 3, 10, 137, 1.7 \times 10^{38}$, ie. column $c$ in the table below.
5 SUMMARY AND CONCLUSIONS

| (a) level | (b) # symbols per level | (c) cumulative sum $(b)$ | (d) map dim. | (e) # of map elements | (f) comment |
|-----------|-------------------------|-------------------------|-------------|-----------------------|-------------|
| 0         | 1                       | (1)                     | (1)         | $(1 \times 1)$       |             |
| 1         | 3                       | 3                       | 4           | $4 \times 4 = 16$    | 16 > 7      |
| 2         | 7                       | 10                      | 16          | $16 \times 16 = 256$ | 256 > 127   |
| 3         | 127                     | 137                     | 256         | $256 \times 256 = 65536$ | 65536 < $2^{127} - 1$ |
| 4         | $2^{127} - 1$           | $2^{127} + 136$         | $(256)^2$   |                       | cut-off reached |

Column (b) is simply the full number of ways a number of entities (symbols) can be combined - 1, 2, 3... at a time, which is $\sum_{p=1}^{n} \binom{n}{p} = 2^n - 1$. This sequence thus counts the number of symbols that can be formed from some given set of symbols by aggregation. A second sequence, column (d), is related to the number of symbols from column (b) which can via discrimination produce the remaining ones at the next level.

Especially the last two numbers in column (c) are thought provoking: more detailed combinatorial calculations yield a corrected value of the inverse fine structure constant very near the experimental value (137.0359 674 vs. observed 137.0359 895(61)), and similarly for the ratio of the electromagnetic and gravitational forces ($2^{127} + 136 = 1.69331 \times 10^{38}$ vs. observed $1.69358(21) \times 10^{38}$), respectively. Interestingly, the sequence in column (b) cuts off after the fourth step, since a symbol-basis of 65536 cannot span a space with $2^{127} + 136$ elements. See [Noyes] for these and a number of other physical constants calculated on this purely combinatorial basis. Note also that the dcs’s correspond to meta/morphic constructions (cf. §2.3) restricted to closure.

5 Summary and conclusions

We have described a truly distributed model of computation - the phase web - based on the distinction between co-occurrence and mutual exclusion of both states and events. This model, by virtue of its acceptance of true concurrency, exceeds Turing’s model of computation (which conclusion, while not widely appreciated, is not controversial in the computer science community). The importance of a computational model, in contrast to so many other kinds, is that it provides explicit mechanism, and we argued for the utility of mechanism as a tool for reasoning, not least in the context of 20th century physical theory. More concretely, without §3’s search for a mechanism for propagating state through the hierarchy, §4’s quaternion result would have been elusive, and perhaps impossible.

The fundamental hierarchy-building operation of co-exclusion - which expresses emergent phenomena naturally - turns out to be nicely modelled by Clifford algebra’s product, which algebra can thereafter be emplaced in the topological context of the twisted isomorphism between homology and cohomology. The coboundary operator $\delta$ was
seen to correspond to, precisely, co-exclusion; and the boundary operator $\partial$ to action. Thus the hierarchical moment implicit in the co-exclusion operation became mathematically explicit.

The fact that each level of the hierarchy is built via the same operation leads to the concept of the level independence of phenomena. Level independence is what gives power and scope to hierarchical theories, but also carries with it the complementary burden of showing that it truly does apply to any level of description, or, if you will, empirical fact. Turning this around, if we are to theorize meaningfully about (say) consciousness - which we believe our model can accommodate - we should have some reason to believe that our theoretical framework is grounded in reality.

To establish this, we modelled the cosmological Big Bang as a process of informational expansion deriving from the progressive compounding of distinctions, each distinction (co-exclusion) expressing one bit of information. Our demonstration in this paper of the emergence from this process of local 3-D-ness in the form of quaternions, besides its intrinsic interest, thus also allows us to discuss more complex phenomena and systems with rather greater confidence.

Relative to the quaternions themselves, we saw that the local 3-D space they define requires prior structure possessing sufficient information-carrying capacity to express the distinctions associated with 3-D-ness, of which the crucial one is parity. We saw that two other distinctions, intertwined with parity, appeared at the same time, which, inspired by the CPT theorem, we tentatively identified with charge and isospin. This broaching of the topic of particle structure gives a another way to test the validity of the points of view being advanced in this paper.

The extension of local 3-D-ness to 3+1 spacetime remains, and the path to be followed seems clear, although undoubtedly rocky.

Acknowledgements.

The basic structure of the Bit Bang was inspired by The Combinatorial Hierarchy of [Parker-Rhodes] and [Bastin&Kilmister], which in turn is based on a construction of the integers originally due to Conway. The Coin and Block demonstrations are reproduced with IEEE’s permission from [Manthey94]. Special thanks to Rainer Zimmermann and Achim Müller for their gracious hosting of the Natura Naturans ‘97 workshop in Bielefeldt, where a preliminary version of this work was presented.
References

Bastin, T. and Kilmister, C.W. *Combinatorial Physics*. World Scientific, 1995. ISBN 981-02-2212-2.

Bastin, T. and Kilmister, C.W. “The Combinatorial Hierarchy and Discrete Physics”. Int'l J. of General Systems, special issue on physical theories from information (in press).

Bowden, K. “Physical Computation and Parallelism (Constructive Post-Modern Physics)”. Int'l J. of General Systems, special issue on physical theories from information (in press).

Bowden, K. *Homological Structure of Optimal Systems*. PhD Thesis, Department of Control Engineering, Sheffield University UK. 1982.

Feynman, R. *The Character of Physical Law*. British Broadcasting Corp. 1965.

Hestenes, D. and Sobczyk, G. *From Clifford Algebra to Geometric Calculus*. Reidel, 1989.

Hestenes, D. *New Foundations for Classical Mechanics*. Reidel, 1986. The first 40 pages contain a very nice, historical introduction to the vector concept and Clifford algebras.

Manthey, M. “Synchronization: The Mechanism of Conservation Laws”. Physics Essays (5)2, 1992.

Manthey, M. “Toward an Information Mechanics”. Proceedings of the 3rd IEEE Workshop on Physics and Computation; D. Matzke, Ed. Dallas, November 1994. ISBN 0-8186-6715X.

Noyes, H.P. SLAC-PUB-95-7017 and SLAC-PUB-7205, pp-36-38.

Parker-Rhodes, F. *Theory of Indistinguishables - A Search for Explanatory Principles Below the Level of Physics*. Reidel. 1981.

Penrose, R. *The Emperor's New Mind*. Oxford University Press, 1989. ISBN 0-19-851973-7.

Roth, J.P. “An Application of Algebraic Topology to Numerical Analysis: On the Existence of a Solution to the Network Problem”. Proc. US National Academy of Science, v.45, 1955.

Tonti, E. “On the formal structure of the relativistic gravitational theory”. Accademia Nazionale Dei Lincei, Rendiconti della classe di Scienze fisiche, matematiche e naturali. Serie VIII, vol. LVI, fasc. 2 - Feb. 1974. (In english.)

www. Various phase web and Topsy publications, including (soon) code distribution, are available via www.cs.auc.dk/topsy.