Quantum Energy Teleportation without Limit of Distance

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Quantum energy teleportation (QET) is, from an operational viewpoint, a protocol whose users, the transportation of energy via local operations and classical communication. QET has various links to fundamental research fields, including black hole physics, the quantum theory of Maxwell’s demon, and condensed-matter entanglement. There are promising signs that QET will be experimentally verified using the chiral boson fields of quantum Hall edge currents. In this Letter, we prove that, using the vacuum state of a quantum field, the upper bound of the amount of energy teleported by QET is inversely proportional to the transfer distance. This distance bound can be overcome by using squeezed states with local-vacuum regions.

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The physics of zero-point fluctuations in quantum fields provides a profound understanding of Nature. In the usual sense, the vacuum state implies nothingness, possessing no particles and zero energy. However, vacuum fluctuations can still be induced by the uncertainty relation of field operators. This yields nontrivial physical phenomena, including the Casimir effect [1] and the Unruh effect [2]. In general, zero-point fluctuations carry a nonvanishing zero-point energy. Though this is true even for quantum fields in a flat spacetime, we usually discard it in quantum field theory by subtracting a divergent constant from the total Hamiltonian. This is mainly because the zero-point energy exhibits a fundamental property known as passivity [3]. Any local operation, which would extract zero-point energy out of the field, actually injects energy and excites the field. Thus, the zero-point energy can be negligible in ordinary situations. However, this energy can be glimpsed through a spatial region with negative energy density [4]. Though the total energy of a quantum field is always nonnegative, its energy density can take a negative value over a spatial region, induced by quantum interference between particle-number eigenstates. This implies that the zero value of the vacuum state is not the minimum energy density. The quantum field can afford to attain the smaller value because this actually saves the zero-point energy in the vacuum state. Of course, we have another region with enough positive energy to ensure the total energy is greater than zero.

Recently, quantum information theory has shed light on some exotic aspects of the zero-point fluctuation. It has been proven that quantum energy teleportation (QET) is possible using the entanglement of fluctuations [5,6]. However, as shown in this Letter, QET suffers from a distance bound on the transferred energy as long as vacuum-state QET protocols are adopted. In order to overcome this bound, we propose a QET protocol using a squeezed state in place of the vacuum state. The proposed protocol is in principle capable of achieving long-distance energy teleportation. For instance, let us consider a quantum field in 1+1 spacetime dimensions. Imagine that two separate experimenters (for example, Alice and Bob) are able to execute local operations and classical communication on this field in the vacuum state. First, Alice performs a local measurement of the zero-point fluctuation in her region. Because of passivity, her measurement device excites the fluctuation, and injects energy $E_A$ to the field. This generates a wave packet with energy $E_A$. At the expense of measurement energy consumption, she obtains information about the quantum fluctuation, and announces this to Bob via a classical channel. As seen later, Alice is able to perform the measurement such that the wave packet with energy $E_A$ moves away from Bob. Hence, Bob obtains the measurement result without the direct energy propagation of $E_A$ from Alice, except for the information-carrier energy. Note that this measurement result includes information about the zero-point fluctuation in Bob’s region via the vacuum-state entanglement. The information enables Bob to form a strategy to suppress his zero-point fluctuation. This results in the extraction of the excess energy from the field. Thus, Bob gains additional energy $E_B$, minus the information-carrier energy. This bonus energy $E_B$ comes from nothing, which is the field in a local vacuum state with zero energy density. Simultaneously, Bob’s operation generates a wave packet with negative energy $-E_B$, which compensates for his energy extraction, in accordance with the local energy conservation law. From the operational viewpoint of protocol users, the energy $E_A$ injected to the field in the vacuum state by Alice can be regarded as the input of the protocol, and the bonus energy $E_B$ extracted by Bob from the local vacuum state can be considered as output. Hence, in effect, we have an energy transportation from Alice to Bob, albeit a one-time transfer for each entangled vacuum state, just like the quantum information transfer by conventional quantum teleportation protocols [7]. Thus, this protocol is termed quantum energy teleportation. The teleported energy $E_B$ is not larger than $E_A$ because of the nonnegative property of the total Hamiltonian. QET has not yet been experimentally verified, but a re-
the general solution is obtained as a sum of a left-energy inequality, adopting the natural unit $c = \hbar = 1$.

Introducing the light-cone coordinates $x^\pm = t \pm x$, the general solution is obtained as a sum of a left-moving component $\hat{\varphi}_+ (t^+$) and a right-moving component $\hat{\varphi}_- (t^-)$. In our later discussion, we can focus solely on the left-mover. The same argument is, of course, possible for the right-mover. The left-mover is quantized as

$$\hat{\varphi}_+ (t^+) = \int_0^{\infty} \left( \hat{a}_\omega e^{-i \omega t^+} + \hat{a}^\dagger_\omega e^{i \omega t^+} \right) \frac{d\omega}{\sqrt{4\pi \omega}}$$

with $[\hat{a}_\omega, \hat{a}^\dagger_\omega] = \delta (\omega - \omega')$, $[\hat{a}_\omega, \hat{a}_\omega'] = 0$. The vacuum state $|0\rangle$ is defined by $\hat{a}_\omega |0\rangle = 0$. The energy flux operator is given by $\hat{T}_+ (x^+) = : \hat{\Pi}_+ (x^+) \hat{\Pi}_+ (x^+) \hat{\Pi}_+ (x^+) \hat{\Pi}_+ (x^+) :$, where $\hat{\Pi}_+ (x^+) = \partial_{x^+} \hat{\varphi}_+ (x^+)$ and the total energy operator is calculated as $\hat{H}_+ = \int_{-\infty}^{\infty} \hat{T}_+ (x^+) dx^+$. The total Hamiltonian is the sum of $\hat{H}_+$ and the right-mover contribution $\hat{H}_-$. By concentrating on the left-mover, we are able to treat $\hat{H}_+$ as the total Hamiltonian, assuming no excitation in the right-mover. Let us consider a vacuum-state QET using $\hat{\varphi} \equiv \hat{\varphi}_+$. Alice stays in the spatial region $[x_{1A}, x_{2A}]$ and Bob stays in $[x_{1B}, x_{2B}]$, with $x_{2A} < x_{1B}$. Bob’s region is located to the right-hand side of Alice’s region, and the distance between them is defined as $L = x_{1B} - x_{2A}$. Let us assume that the initial state is the vacuum state $|0\rangle \langle 0|$.

![FIG. 1. (Color online) Schematic diagram of the quantum energy teleportation (QET) protocol](image)

thereby, long-distance QET sends only a small amount of energy. However, it is possible to overcome this bound using squeezed states with local-vacuum-state regions. We adopt a quantum state in which the local quantum fluctuations around Alice and Bob when they operate QET are the same as the zero-point fluctuation in the vacuum state, but are squeezed in an intermediate region between them. By adopting this squeezed state, Bob, who is far away from Alice, can gain the same amount of energy $E_B$ as that of the short-distance vacuum-state QET, avoiding the limit imposed by the distance. This result may have a great advantage for future applications of QET in quantum technology, and provide an impact on fundamental physics.

Let us consider a quantum free massless scalar field $\hat{\varphi}$ in 1+1 dimensions that obeys the equation of motion:

$$\left( \partial^2_t - \partial^2_x \right) \hat{\varphi} = 0.$$ 

Introducing the light-cone coordinates $x^\pm = t \pm x$, the general solution is obtained as a sum of a left-moving component $\hat{\varphi}_+ (x^+) \hat{\varphi}_- (x^-)$. In our later discussion, we can focus solely on the left-mover. The same argument is, of course, possible for the right-mover. The left-mover is quantized as

$$\hat{\varphi}_+ (x^+) = \int_0^{\infty} \left( \hat{a}_\omega e^{-i \omega t^+} + \hat{a}^\dagger_\omega e^{i \omega t^+} \right) \frac{d\omega}{\sqrt{4\pi \omega}}$$

with $[\hat{a}_\omega, \hat{a}^\dagger_\omega] = \delta (\omega - \omega')$, $[\hat{a}_\omega, \hat{a}_\omega'] = 0$. The vacuum state $|0\rangle$ is defined by $\hat{a}_\omega |0\rangle = 0$. The energy flux operator is given by $\hat{T}_+ (x^+) = : \hat{\Pi}_+ (x^+) \hat{\Pi}_+ (x^+) \hat{\Pi}_+ (x^+) \hat{\Pi}_+ (x^+) :$, where $\hat{\Pi}_+ (x^+) = \partial_{x^+} \hat{\varphi}_+ (x^+)$ and the total energy operator is calculated as $\hat{H}_+ = \int_{-\infty}^{\infty} \hat{T}_+ (x^+) dx^+$. The total Hamiltonian is the sum of $\hat{H}_+$ and the right-mover contribution $\hat{H}_-$. By concentrating on the left-mover, we are able to treat $\hat{H}_+$ as the total Hamiltonian, assuming no excitation in the right-mover. Let us consider a vacuum-state QET using $\hat{\varphi} \equiv \hat{\varphi}_+$.

Alice stays in the spatial region $[x_{1A}, x_{2A}]$ and Bob stays in $[x_{1B}, x_{2B}]$, with $x_{2A} < x_{1B}$. Bob’s region is located to the right-hand side of Alice’s region, and the distance between them is defined as $L = x_{1B} - x_{2A}$. Let us assume that the initial state is the vacuum state $|0\rangle \langle 0|$. At time $t = 0$, Alice instantaneously conducts a general measurement $|1\rangle$ in $[x_{1A}, x_{2A}]$ with measurement operators

$$\hat{M}_{0A} = \cos \left( \hat{A} - \frac{\pi}{4} \right),$$

$$\hat{M}_{1A} = \sin \left( \hat{A} - \frac{\pi}{4} \right),$$

corresponding to the binary measurement result $\mu = 0, 1$. Here, $\hat{A}$ is a local Hermitian operator defined as $\hat{A} = \int_{-\infty}^{\infty} g_A (x) \hat{\Pi}_+ (x) dx$, with the real function $g_A (x)$ localized in $[x_{1A}, x_{2A}]$. The positive operator-valued measure (POVM) of this measurement is given by $\Xi_{\mu A} = \hat{M}_{\mu A}^\dagger \hat{M}_{\mu A}$. A straightforward computation shows that the emergence probability of each result $\mu$ is the same: $p_{\mu} = \langle 0 | \Xi_{\mu A} | 0 \rangle = 1/2$. The post-measurement state for each $\mu$ is calculated as

$$\hat{\rho}_\mu = \frac{1}{p_{\mu}} \hat{M}_{\mu A} |0\rangle \langle 0| \hat{M}_{\mu A}^\dagger.$$

As a result of the passivity, this measurement injects average energy of

$$E_A = \sum_{\mu} p_{\mu} \text{Tr} \left[ \hat{H}_+ \hat{\rho}_\mu \right] - \text{Tr} \left[ \hat{H}_+ |0\rangle \langle 0| \right]$$

to the field, and generates a positive energy wave packet. The energy $E_A$ is computed as

$$E_A = \int_0^{\infty} |g_A (\omega)|^2 \omega^2 d\omega / 4\pi,$$  \hspace{1cm} (4)
where \( \bar{g}_A(\omega) = \int_{-\infty}^{\infty} g_A(x) e^{-i\omega x} \, dx \). Because \( \hat{M}_{\mu A} \) includes only the left-mover operator \( \hat{\Pi}_+(x) \), the wave packet moves to the left, further away from Bob. Thus, Bob cannot receive its energy by direct propagation of the wave packet. Assume that Bob receives the information of \( \mu \) from Alice at time \( t = T \). It should be emphasized here that the field in Bob’s region is in a local vacuum state with zero energy, though we have the wave packet localized in \([x_1B, x_2B]\). The post-operation state is computed as

\[
\hat{U}_{\mu B} = \exp \left( -i\theta_{\mu} \hat{B} \right),
\]

where \( \theta_{\mu} \) is a \( \mu \)-dependent real parameter, and \( \hat{B} = \int_{-\infty}^{\infty} g_B(x) \hat{\Pi}_+(x) \, dx \) with the real function \( g_B(x) \) localized in \([x_1B, x_2B]\). The post-operation state is computed as

\[
\hat{\rho}_{\text{QET}} = \sum_{\mu} \hat{U}_{\mu B} \exp \left( -iT \hat{H}_+ \right) \hat{M}_{\mu A} \langle 0 | \hat{M}_{\mu A}^\dagger \times \exp \left( iT \hat{H}_+ \right) \hat{U}_{\mu B}^\dagger.
\]

Note that the two-point correlation function \( C_{\mu A B} = \langle 0 | \hat{M}_{\mu A B}^\dagger(T) | 0 \rangle / \rho_0 \) with \( \hat{B}^\dagger(T) = \int_{-\infty}^{\infty} g_B(x - T) \partial_x \hat{\Pi}_+(x) \, dx \) is real because of the operator locality: \( \langle \hat{\Xi}_{\mu A}, \hat{B}^\dagger(T) \rangle = 0 \). Let us fix the real parameter \( \theta_{\mu} \) as

\[
\theta_{\mu} = \frac{2C_{\mu A B}}{G_B},
\]

where \( G_B = \int_{-\infty}^{\infty} (\partial_x g_B(x))^2 \, dx \). It can then be verified that the total energy decreases during this local operation by Bob. This implies that positive energy of

\[
E_B = \text{Tr} \left[ \hat{H}_+ \sum_{\mu} p_{\mu} \exp \left( -iT \hat{H}_+ \right) \hat{\rho}_{\text{QET}} \exp \left( iT \hat{H}_+ \right) \right] - \text{Tr} \left[ \hat{H}_+ \hat{\rho}_{\text{QET}} \right] > 0
\]

is extracted from the field in the local vacuum state as a negative work by Bob’s operation. Simultaneously, a wave packet with negative energy \(-E_B\) is generated in Bob’s region and begins to move to the left. The teleported energy \( E_B \) can be evaluated as

\[
E_B = \frac{1}{G_B} \sum_{\mu} p_{\mu} C_{\mu A B}^2.
\]

Using the correlation function

\[
\langle 0 | \hat{\Pi}_+(x_B) \hat{\Pi}_+(x_A) | 0 \rangle = -\frac{1}{4\pi (x_B - x_A - i\epsilon)^2},
\]

\( E_B \) can be explicitly computed as

\[
E_B = \frac{1}{\pi^2} \int_{-\infty}^{\infty} \left( \int_{-\infty}^{\infty} \frac{\bar{g}_B(x_B) g_A(x_A)}{(x_B - x_A + T)^2} \, dx_B \, dx_A \right)^2 \, dx \exp \left( \frac{1}{\pi} \int_0^\infty |\bar{g}_A(\omega)|^2 \omega \, d\omega \right).
\]

Note that, as the distance \( L \) between Alice and Bob becomes large, \( E_B \) in Eq. (5) decreases as \( E_B \propto 1/L^6 \). This damping behavior can be slightly improved to \( E_B \propto 1/L^4 \) by replacing \( \hat{B} \) in Eq. (5) with \( \hat{B} = \int_{-\infty}^{\infty} \tilde{g}_B(x) \tilde{\phi}(x) \, dx \). A natural question then arises: To what extent does any other QET protocol improve this long-distance behavior of \( E_B \)? As mentioned above, a stringent bound on the long-distance damping of \( E_B \) is given by Eq. (1) for any vacuum-state QET. Let us outline the derivation of this bound below. Its essence comes from Franagan’s theorem [12], which asserts the following. Consider a nonnegative continuous function \( \xi(x) \) with \( \xi(x \to \pm \infty) = 0 \), and define a Hermitian operator as

\[
\hat{H}_\xi = \int_{-\infty}^{\infty} \xi(x) \hat{T}_+(x) \, dx.
\]

Then, the inequality

\[
\text{Tr} \left[ \hat{H}_\xi \hat{\rho} \right] \geq -\frac{1}{12\pi} \int_{-\infty}^{\infty} \left( \partial_x \sqrt{\xi(x)} \right)^2 \, dx
\]

holds for an arbitrary state \( \hat{\rho} \). Applying this theorem to the vacuum-state QET yields Eq. (1). Let \( \hat{\rho}_{\text{QET}} \) denote the post-operation state following an arbitrary vacuum-state QET. Assume a wave packet with negative energy \(-E_B\) is generated in \([x_1B, x_2B]\). In the intermediate region between Alice and Bob, \([x_2A, x_1B]\), the average energy density vanishes. In the region to the left of Alice, \((-\infty, x_2A]\), we have a wave packet with positive energy \( E_A \). Let us impose the values \( \xi(x) = 0 \) for \( x \in (-\infty, x_2A] \) and \( \xi(x) = 1 \) for \( x \in [x_1B, x_2B] \) on \( \xi(x) \) in Eq. (9). In the region \([x_2B, \infty)\), it is sufficient to assume that \( \xi(x) \) slowly decreases to 0. As a result, \( \text{Tr} \left[ \hat{H}_\xi \hat{\rho}_{\text{QET}} \right] = -E_B \) for an arbitrary \( \xi(x) \) satisfying the above conditions. Thus,

\[
E_B \leq \frac{1}{12\pi} \inf_{\xi(x)} \int_{-\infty}^{\infty} \left( \partial_x \sqrt{\xi(x)} \right)^2 \, dx
\]

must be satisfied. The infimum of the \( \xi(x) \) satisfying the above boundary conditions is then taken. Using a variation method, the infimum is obtained from a function \( \xi_{\text{opt}}(x) \) obeying \( \xi_{\text{opt}}(x) = (x/L)^2 \) for \( x \in [x_2A, x_1B] \). This derives the inequality of Eq. (1).

Next, let us describe how to overcome the distance bound for the vacuum-state QET. This can be attained using a squeezed state instead of the vacuum state. Consider a non-decreasing \( C^1 \) function \( f(x) \) such that \( f(x) = x \) for \( x \leq x_2A \) and \( f(x) = x - l \) for \( x \geq x_1B + T \). Here, \( l \) is a length parameter. Due to \( \partial_x f(x) \geq 0 \), \( l \) is upper-bounded by \( x_1B + T - x_2A = L + T \). Using \( f(x) \), let us define a mode function \( v_\omega(x) \) of \( \tilde{\phi}_+(x) \) as \( v_\omega(x) = \frac{1}{\sqrt{4\pi\omega}} \exp(-i\omega f(x)) \). Then, \( \tilde{\phi}_+(x) \) can be expanded in terms of this mode function as

\[
\tilde{\phi}_+(x) = \int_0^\infty \left( \int f_\omega v_\omega(x) + f_\omega^\dagger v_\omega^\dagger(x) \right) \, d\omega,
\]
where \( \hat{f}_\omega, \hat{f}_\omega \) are creation and annihilation operators obeying \( [\hat{f}_\omega, \hat{f}_\omega'] = \delta (\omega - \omega') \) and \( [\hat{f}_\omega, \hat{f}_\omega] = 0 \). A squeezed state \( |f\rangle \) is defined by \( \hat{f}_\omega |f\rangle = 0 \). For \( |f\rangle \), the following relation holds for \( x_A \leq x_{2A} \) and \( x'_A \leq x_{2A} \).

\[
\langle f | \hat{\Pi}_+(x_A) \hat{\Pi}_+(x'_A) | f \rangle = -\frac{1}{4\pi (x_A - x'_A - i\epsilon)^2}. \tag{10}
\]

Because \( |f\rangle \) is a Gaussian state completely specified by a two-point correlation function \( \langle f | \hat{\Pi}_+(x) \hat{\Pi}_+(x') | f \rangle \), Eq. (10) means that the quantum fluctuation of \( (-\infty, x_{2A}) \) is the same as the zero-point fluctuation of \( |0\rangle \). This implies that \( (-\infty, x_{2A}) \) is a local-vacuum-state region with zero energy. Similarly, for \( x_B \geq x_{1B} + T \) and \( x'_B \geq x_{1B} + T \),

\[
\langle f | \hat{\Pi}_+(x_B) \hat{\Pi}_+(x'_B) | f \rangle = -\frac{1}{4\pi (x_B - x'_B - i\epsilon)^2} \tag{11}
\]

holds and implies that \( [x_{1B} + T, \infty) \) is also a local-vacuum-state region with zero energy. After time \( T \) has elapsed, the local-vacuum-state region moves to \( [x_{1B}, \infty) \) due to the left-mover evolution. Let us consider the case of a large \( L \). At time \( t = 0 \), Alice (who stays in the zero-energy region of \( (-\infty, x_{2A}) \)) performs the measurement in Eqs. (2) and (3) to \( \hat{\phi} \) in the state \( |f\rangle \). The measurement result \( \mu \) is sent to Bob (who stays in the zero-energy region of \( [x_{1B}, \infty) \)) at \( t = T \). During the communication time \( T \), the information of \( \mu \) jumps across the long-distance region \( (x_{2A}, x_{1B}) \) with nonzero energy \( E_c \), which is evaluated as

\[
E_c = \frac{1}{48\pi} \int_{x_{2A}}^{x_{1B} + T} (\partial_x \ln (\partial_x f(x)))^2 dx.
\]

At \( t = T \), Bob is able to extract the teleported energy from \( \hat{\phi} \) by performing the same operation as in Eq. (5) on the local zero-point fluctuation. Simply replacing Eq. (7) by

\[
\langle f | \hat{\Pi}_+(x_B) \hat{\Pi}_+(x_A) | f \rangle = -\frac{1}{4\pi (x_B - x_A - l - i\epsilon)^2}, \tag{12}
\]

the amount of teleported energy \( E_{BF} \) can be evaluated as

\[
E_{BF} = \frac{\left( \int_{-\infty}^{\infty} \left( \int_{-\infty}^{x_{1B} + T} \hat{g}_B(x) g_A(x_A) dx_A \right)^2 dx_B \right)^2}{\pi^2 \int_{-\infty}^{\infty} (\partial_x g_B(x))^2 dx \exp \left( \frac{1}{2} \int_0^{\infty} \left| g_A(\omega) \right|^2 \omega d\omega \right)}. \tag{13}
\]

The difference between Eq. (8) and Eq. (13) is just the appearance of \( l \) in the correlation function between the two separate regions of the numerator integral. It is worth stressing that \( l \) can, in principle, take a large value satisfying \( l \leq L + T \). By taking an \( L \)-dependent squeezed state \( |f\rangle \) such that \( l \sim L + T \), the long-distance damping of \( E_{BF} \) behaves not as \( O(L^{-6}) \), but \( O(L^0) \) as the distance \( L \) becomes large. Therefore, this squeezed-state QET indeed overcomes the distance bound in Eq. (1).

Note that \( E_{BF} \) for an \( L \)-independent \( |f\rangle \) with a fixed \( l \) again exhibits the original damping behavior of \( O(L^{-6}) \) when \( L \gg l \), as it should be.

The essence of QET without limit of distance is as follows. As seen in Eq. (4), the distance dependence of \( E_B \) for the vacuum-state QET comes from \( C_{\mu AB} \). This correlation is generated by the vacuum-state entanglement between local zero-point fluctuations in the regions of Alice and Bob. The point is that entanglement itself is a distance-independent concept. For instance, one qubit of a Bell pair can be placed apart from the other qubit, preserving the maximum entanglement between them. Similarly, if we supply two local quantum fluctuations of \( \hat{\phi} \) that are far away from each other with the same entanglement and correlations as those of the local zero-point fluctuations of two close regions, the QET remains effective, independent of the distance, because \( C_{\mu AB} \) is the same. Thus, the above squeezed-state method is one strategy for realizing long-distance correlation.

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