

Optimized robust cruise control system for an electric vehicle

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Besides environmental advantages and fuel efficiency, electric motors are a suitable choice for powertrain of vehicles due to appropriate similarity between their torque-speed characteristics and the required vehicle torque for longitudinal performance. An applicable robust control system for the cruise controller of a DC motor-driven electric vehicle is proposed. The robust controller is tuned using a numerical optimization design method to compensate the disturbances from the road grade and changes in the vehicle weight. In this study, the vehicle’s powertrain model is developed and details of the controller design process are described. The robustness and performance of the designed controller is evaluated by performing computer simulations. The results demonstrate adequate robustness and disturbance rejection of the designed control system.

Keywords: robust controller; PID controller; PI controller; electric vehicle; cruise controller

1. Introduction

Electric vehicles (EVs) are becoming more popular these days and automobile manufacturers are introducing various types of EVs in the market. The main advantages of EVs are the emission elimination, low operating cost, high efficiency, simplicity and superior controllability over the powertrain. The EV powertrain consists of an electric motor, single or double speed transmission and the final drive unit. According to this relative simplicity of the powertrain and precise controllability of the electric motors, the EV powertrain are more controllable and reliable. In addition, the EV powertrain provides the regenerative braking capability.

Generally, the driver requires to appropriately pushing or releasing the acceleration pedal in order to maintain the vehicle’s speed. However, driving without the need to frequently control the speed provides safety, comfort and easiness for the driver. Cruise control system is developed for driving with constant speed on long stretched roads. This system performs as a speed-tracking controller and autonomously follows a pre-set vehicle speed. For instance, a well-tuned and robust cruise control system is an essential component of adaptive cruise control systems (Kim, Moon, Park, Kim, & Yi, 2009; Mayr, 1998).

The control logic of the cruise controller can be designed by employing different types of controllers, such as a proportional-integral-derivative (PID) controller (Arora, Lu, Diba, & Esmailzadeh, 2013) and fuzzy logic approach (Osman, Rahmat, & Ahmad, 2009). PID controllers are widely implemented in industrial applications due to their effectiveness and feasibility. The gains of this controller can be tuned using different control theories, such as the robust control theory. The robust controller minimizes the effect of uncertainties being encountered in the control system. These uncertainties can occur, due to the simplification of plant’s model or the surroundings effects, such as the temperature fluctuation, pressure fluctuation, noise, etc. Several methods have been developed to manage the uncertainties present in processes and improve the robustness and disturbance rejection of PID controller (Hu, He, & Li, 2008; Toscano, 2005). Examples of these methods are the pole-zero cancellation, internal model control (IMC) (Chien, 1990; Shamsuzzoha & Lee, 2006), tuning of the gain and phase margins (Åström & Hägglund, 1984; Ho, Hang, & Cao, 1995), interpolation of Bode plots (Bucz, Kozáková, & Vesely, 2013) and the particle swarm optimization (Xu, 2010). The $H_\infty$ theory is a well-known technique to design robust controller (Başar & Bernhard, 2008; Liu, Gu, & Zhang, 2003). System performance evaluation methods, such as frequency methods, easily describe the finding of constant loci of the stability margins and crossover frequencies, which can be used for the purpose of analysis and tuning the robust control system. This method is capable of examining the effects of any uncertainties present in the system and hence controlling the system robustness. Furthermore, the system performance evaluation methods can be used when system specifications are given in terms of the gain and phase margins (Krajewski, Lepschy, & Viaro, 2004; Krajewski, Lepschy, Miani, & Viaro, 2005). This technique presents an effective performance while it has its own complexity in the structure and implementation.

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The robust controller can be designed by using numerical optimization techniques, which involve fewer calculations and yet ensure good stability margins of the system (Toscano, 2005). In this paper, the robust proportional-integral (PI) torque and PID speed controllers are designed using the numerical optimization technique for the cruise controller of the EV. Simulation results show that the controller has sufficient robustness to compensate any disturbances, for the case of road grades, and presents a superior speed-tracking behaviour.

The major contribution of this research is to model a cruise controller and develop its complex design procedure with multi-layer controllers. The model performance and simulation results were verified with those reported in the literature, and showed an overall improvement of the cruise controller performance. Applicability of the developed control design procedure is carried out for an automotive system consisting of two-layer control loops with both PI and PID controllers.

2. System modelling

DC motors are an appropriate choice for industrial applications, which involve frequent starting–stopping and also require adjustable speed. This type of electric motor is reliable and easily controllable (Crowder, 1998). Furthermore, one of the widely used types of electric motors for the EV powertrains is the DC motor. The schematic model of the powertrain for a small-sized EV, including the cruise control system, is shown in Figure 1.

The DC motor is connected to the front wheel of the vehicle through a single-speed transmission. The resistant load of the vehicle and the frictional force of the motor’s bearings are considered as the resistant torques. Hence, the torque balance equation of the DC motor can be written as

\[ J_\theta \ddot{\theta}_m = M_t - M_f - M_v \]  

in which

\[ J_\theta = J_m + i_{tot} J_w + J_v, \]  

where \( J_\theta \) is the total moment of inertia, \( \theta_m \) is the angular position of the rotor of the DC motor, \( M_t \) is the torque of the motor, \( M_f \) is the bearing frictional torque, \( M_v \) is the load torque from the vehicle, \( J_m \) is the moment of inertia of the rotor and \( J_v \) is the moment of inertia of the front wheels, respectively. The parameter \( J_v \) is the equivalent vehicle moment of inertia, which could be determined as

\[ J_v = m \left( \frac{r_w}{i_{tot}} \right)^2, \]

where \( m \) is the mass of the vehicle, \( r_w \) is the wheel radius and \( i_{tot} \) is the total powertrain gear ratio. The required torque to maintain the vehicle’s longitudinal speed is considered as the load torque on the DC motor. According to the longitudinal dynamic of the vehicle, the load torque of the vehicle which consists of the inertia force, aerodynamic force, rolling resistance and the road grade force can be
formulated as (Preitl & Bauer, 2007):

$$M_v(t) = \frac{F_w}{I_{tot}} \left( m\dot{v}(t) + \frac{1}{2} \rho A_d C_d v^2(t) + mgC_r + mg\sin\alpha \right)$$

(4)

where $A_d$ is the projected frontal area, $C_d$ is the air drag coefficient, $\rho$ is the air density, $C_r$ is the rolling resistance coefficient, $g$ is the gravitational acceleration and $\alpha$ is the road grade.

According to the developed equations, the block diagram of the vehicle powertrain, including the cruise control system, can be constructed as shown in Figure 2. In this diagram, the sensitivity of the armature current sensor and the speedometer are modelled by gains $K_{cs}$ and $K_{ss}$, respectively. In addition, the transfer function in the blocks $G_{sc}$ and $G_{tc}$ represents the vehicle’s speed controller and DC motor torque controller, respectively.

3. Control system design

The main objective of a cruise control system is to momentarily track the desired speed of the vehicle. In an EV, speed can be actively controlled by continuously adjusting the torque of the electric motor. In this research, a robust optimized control system, consisting of both PID and PI controllers, has been utilized using the extended methodology presented in reference (Toscano, 2005) to constantly track the desired speed. Figure 3 depicts a feedback system, including the disturbance signal, that $G(s)$ represents the open-loop transfer function of the process model, i.e. the cruise controller system, and $G_c(s)$ is the transfer function of the controller. The sensitivity function $S^M_G(s)$ for the considered control system can then be defined as

$$S^M_G(s) = \frac{1}{1 + G_c(s) \cdot G(s)} = \frac{1}{1 + L(s)},$$

(9)

where $L(s)$ is the open-loop transfer function. Moreover, the closed-loop transfer function $M(s)$ of the system, can be written as

$$M(s) = 1 - S^M_G(s) = \frac{L(s)}{1 + L(s)}.$$ 

(10)

In order to achieve superior transient response, the following conditions must be satisfied:

$$\max_{\omega} |M(j\omega)| \leq M_{\text{max}},$$

(11)

$$M_P \leq M_{P\text{max}},$$

(12)
where $|M(j\omega)|$ is the magnitude of the closed-loop transfer function, $M_{\text{max}}$ is the upper bound of the amplitude of the closed-loop transfer function. The parameter $M_p$ is the first overshoot of the step-input response and $M_{\text{p_{max}}}$ is the upper bound of the overshoot for the system response. The magnitude of the closed-loop transfer function is equal to one in the low-frequency range, and therefore $M_{\text{p_{max}}}$ is greater than one. The minimum damping ratio, $\zeta_{\text{min}}$, of the control system, which corresponds to $M_{\text{p_{max}}}$ is

$$\zeta_{\text{min}} = \frac{|\ln(M_{\text{p_{max}}})|}{\sqrt{\pi^2 + \ln(M_{\text{p_{max}}})^2}}. \quad (13)$$

Moreover, the relationship between the upper bound of the closed-loop transfer function and the minimum damping ratio can be expressed as

$$M_{\text{max}} = \frac{1}{2\zeta_{\text{min}} \sqrt{1 - \zeta_{\text{min}}^2}}. \quad (14)$$

The gain margin, which is a measure of the stability of the system as shown in Figure 4, is defined as

$$K_g = \frac{1}{M(j\omega)}. \quad (15)$$

In order to increase the stability of the closed-loop system, the gain margin has to be maximized, which indicates that the distance between the real axis interception of the polar plot of the system and the critical point has to be increased. As a result, and according to Equation (10), the sensitivity of that function to the external disturbances needs to be decreased. Finally, in order to have a good disturbance rejection and reliable robustness, as well as suitable stability performance, the gains of the PI/PID controllers should satisfy the following condition:

$$\left\{ \begin{array}{l} \xi \geq \zeta_{\text{min}} \\ \max_{K_p, K_i, K_d} \left( \min_{\omega} \left| \frac{1}{M(j\omega, K_p, K_i, K_d)} \right| \right) \end{array} \right. \quad (16)$$

where $K_p$, $K_i$, and $K_d$ are the proportional, integrator and differentiator gains, respectively. The first condition on Equation (16) guarantees an effective transient response of the system while the second condition would confirm the disturbance rejection of the control system.

### 3.1. Powertrain torque controller

According to Figure 2, there is a torque controller in the inner loop of the block diagram, which controls the generated torque by the DC motor. This controller has been designed by using a robust PI controller based on the developed optimization approach. The transfer function of the DC motor is of third-order system, which is of the following form:

$$G_1(s) = \frac{K(s + b_0)}{s^3 + a_2s^2 + a_1s + a_0}, \quad (17)$$

where $K$ is the gain and $a_0$, $a_1$, $a_2$ and $b_0$ are the coefficients of the transfer function, which can be found from the numerical value of the system parameters. The combined open-loop transfer function of the controller system can be written by considering the DC motor transfer function and a generalized PI controller, as

$$L(s) = G_{\text{ic}}(s)G_1(s) = \frac{(K_p s + K_i)}{s} \frac{K(s + b_0)}{(s^3 + a_2s^2 + a_1s + a_0)}, \quad (18)$$

where $G_{\text{ic}}(s)$ represents the transfer function of the PI torque controller. According to Equation (18), one can derive the characteristic equation of the closed-loop system as

$$q_1(s) = 1 + L_1(s) = s^4 + a_2s^3 + (a_1 + KK_p)s^2 + (a_0 + KK_pb_0 + KK_id)s + KK_i b_0 = 0, \quad (19)$$

which is a fourth-order polynomial. The characteristic equation can be written as the product of a set of double poles and the standard form of second-order characteristic equation, as

$$(s + \gamma)^2(s^2 + 2\omega_n s + \omega_n^2) = 0, \quad (20)$$

where $\omega_n$ is the natural frequency and $\gamma$ is the value of the double closed-loop pole, respectively. The damping ratio and natural frequency would be calculated on the basis of the design requirements. According to Equations (19) and (20), the PI controller gains, $K_p$ and $K_i$, and also the closed-loop
pole \( \gamma \) can be calculated as

\[
\gamma = \frac{a_2}{2} - \xi \omega_n, \tag{21}
\]

\[
K_P = \frac{a_n^2 + \gamma^2 + 4\xi \omega_n \gamma - a_1}{k}, \tag{22}
\]

\[
K_I = \frac{\omega_n^2 \gamma^2}{k b_0}. \tag{23}
\]

Furthermore, to guarantee the stability of the closed-loop system, all poles of the characteristic equation should be on the left-hand side of \( s \)-plane, which means that \( \gamma > 0 \), therefore

\[
\beta_1 = \frac{a_2}{2\xi \omega_n} > 1, \tag{24}
\]

where \( \beta_1 \) is the lower band of the stability. Subsequently, by considering the first condition of Equation (16) and (24) one can derive the following equations:

\[
\omega_n = \frac{a_2}{2\xi_{\text{min}} \beta_1}, \tag{25}
\]

\[
\gamma = \frac{a_2}{2} - \xi_{\text{min}} \omega_n. \tag{26}
\]

Finally, according to Equation (16) the gains of the PI controller can be determined by solving the following optimization problem:

\[
\max_{\beta_1 > 1} \min_{\omega} |1 + L_1(j\omega, \beta_1)|. \tag{27}
\]

### 3.2. Cruise controller

The cruise controller has been designed by using the PID controller, which has been optimized on the basis of the same optimization approach as explained in Section 3.1. According to Figure 2, the open-loop transfer function of the cruise controller system must have the following form:

\[
L_2(s) = G_{sc}(s)G_2(s) = \frac{(K_P s + K_I + K_D s^2)}{s} \times \frac{K}{s^2 + b_1 s + b_0}, \tag{28}
\]

where \( G_{sc}(s) \) represents the transfer function of the PID cruise controller and \( G_2(s) \) is the combined transfer functions of the vehicle and powertrain, which is a fifth-order system. The \( K \) is the gain and \( a_0, a_1, a_2, a_3, a_4, b_0 \) and \( b_1 \) are the coefficients of the transfer function, which are all functions of system characteristics. Consequently, the characteristic equation of the closed-loop system would be

\[
q_2(s) = 1 + L_2(s) = s^4 + a_4 s^3 + (a_3 + KK_D) s^2 + (a_2 + KK_D b_1 + KK_P) s + (a_0 + KK_D b_1 + KK_P b_0) s + KK_D b_0 = 0. \tag{29}
\]

The closed-loop characteristic equation can then be written in the following form:

\[
(s + \gamma)^4 (s^2 + 2\xi \omega_n s + \omega_n^2) = 0. \tag{30}
\]

By comparing the coefficients of Equations (30) and (20), one can find

\[
\gamma = \frac{a_4}{4} - \frac{\xi \omega_n}{2}. \tag{31}
\]

The stability condition of the closed-loop system will impose that \( \gamma > 0 \). Therefore, the lower band of the stability for the closed-loop system, \( \beta_2 \), can be derived as

\[
\beta_2 = \frac{a_4}{2\xi \omega_n} > 1. \tag{32}
\]

Then, the first constraint of Equation (16) leads to the following equation:

\[
\omega_n = \frac{a_4}{2\xi_{\text{min}} \beta_1}. \tag{33}
\]

Finally, the gain of the PID controller can be found by solving the following optimization problem:

\[
\begin{align*}
\max_{\beta_2 > 1} \min_{\omega} |1 + L_2(j\omega, \beta_2)| \\
K_P &= \frac{4\gamma \omega_n^2 + 4\gamma^3 + 12\xi \omega_n \gamma^2 - a_2 - KK_D b_1}{K} \\
K_I &= \frac{\omega_n^2 \gamma^4}{K b_0} \\
K_D &= \frac{\omega_n^2 + 6\gamma^2 + 8\xi \omega_n \gamma - a_3}{K}
\end{align*} \tag{34}
\]

### 4. Simulation case studies

In this section, the robustness and the disturbance rejection of the designed cruise control system have been evaluated for two different case studies by running computer simulations. The damping ratio of the vehicle system is \( \xi = 0.8 \), and the numerical values of the vehicle parameters are listed in Table 1. The controller gains have been calculated using the described methodology and the numerical value of the gains for both the torque and the speed controllers have been tabulated in Table 2. For the sake of comparison, the simulation results for the controller tuned reported in reference (Preitl & Bauer, 2007) are also presented, where two PI
Table 1. Numerical values of the parameters.

| Parameter                          | Value     | Parameter                          | Value     |
|------------------------------------|-----------|------------------------------------|-----------|
| Frontal area of vehicle $A_d$      | 2.4 m²    | Armature gain constant $R_a$       | 1         |
| Air drag coeff. $C_d$              | 0.5       | Armature time constant $T_a$       | 0.1       |
| Air density $\rho$                 | 1.225 kg/m³ | Actuator gain constant $K_A$  | 30        |
| Rolling resistance coeff. $C_r$    | 0.0155    | Actuator time constant $T_A$       | 0.03      |
| Wheel radius $R$                   | 0.3 m     | speed sensor sensitivity $K_{ss}$  | 0.02      |
| Total drive ratio $i_{tot}$        | 4.875     | current sensor sensitivity $K_{cs}$ | 0.03   |
| Vehicle mass $M$                   | 1940 kg   | Back emf gain $K_b$                | 2         |
| Total inertia $J_{tot}$            | 8.6 kg·m² | Friction torque gain $K_f$         | 0.1       |

Table 2. Numerical values of the controller gains.

| Controller gain | Cruise controller | Torque controller |
|-----------------|-------------------|-------------------|
| $K_P$           | Proposed          | Preitl and Bauer (2007) |
| $K_I$           | 24.86             | 32.58             |
| $K_D$           | 0.09              | 18.6              |

Figure 5. First case study: constant speed tracking case study. (a) Vehicle longitudinal velocity (m/s), (b) DC motor torque (N.m), (c) DC motor current (amp) and (d) road grade disturbance torque (N.m).
controllers have been employed for both the torque and the cruise controllers. The numerical values of the controller gains in reference (Preitl & Bauer, 2007) have also been included in Table 2.

The First case study is for the constant speed tracking at 30 m/s. Figure 5 depicts the result of this case study. As previously mentioned, the changes in the road grade are considered as disturbance to the control system. According to Figure 5(a), the cruise controller has demonstrated insignificant steady-state error and the vehicle follows the desired speed smoothly and continuously. Also, the disturbance rejection behaviour of the control system is acceptable and the vehicle’s speed is slightly decreased in the presence of the road grade resistance torque. Figure 5(b) shows that the cruise controller has properly adjusted the DC motor torque in order to reject the applied disturbances. Figure 5(d) shows the disturbance torque generated by the road grade, where the vehicle climbed a 20% uphill road. It can be seen that the performance of the proposed cruise controller system is more robust than the proposed controller of reference (Preitl & Bauer, 2007) and the vehicle smoothly tracks the reference input without any overshoot and fluctuations.

The second case study demonstrates the performance of the designed cruise control system during a pre-defined driving cycle. Figure 6(a) depicts the speed tracking of the cruise control system, which shows that the vehicle would closely follow the reference input with an insignificant steady-state error. As illustrated, the vehicle velocities for the case of either with or without disturbance simulations are almost superimposed on each other and not easily distinguishable. The DC motor torque is depicted in Figure 6(b). The negative torque is the braking torque generated by the DC motor. Technically, the powertrain of an electric motor can generate the braking torque by using the electric motor and regenerate the kinetic energy of the vehicle. The armature current of the DC motor is shown in Figure 6(c). The equivalent disturbance torque of the road grade is shown in Figure 6(d). The disturbance torque consists of both positive and negative torque since the vehicle was driven on both
uphill and downhill roads with an average grade of 30%. It can be seen that the performance of the proposed cruise controller system improves without any overshoot than the controller used in reference (Preitl & Bauer, 2007). Moreover, the peak value of the active torque is further reduced in the proposed control system, as shown in Figure 6(b).

5. Conclusion

The comprehensive design of a robust cruise controller for a DC motor-driven EV has been carried out. The robust PID and PI controllers were tuned by using a numerical optimization method. The cruise control system consists of two control layers. The first layer corresponds to the speed control system, which was designed on the basis of a PID controller. The second layer relates to the torque control system, which was implemented in the vehicle system using a PI controller. The detailed design process of the controller has been fully discussed. This proposed control design technique is comprehensive and applicable to a variety of higher order systems. In addition, the modelling of the electric powertrain of the vehicle has been presented. The robustness and disturbance rejection behaviour of the designed controller are evaluated by computer simulations based on two different case studies. The simulation results, using the numerical values from a typical EV, showed that the cruise controller has perfectly compensated for the disturbances, and also exhibited a smooth speed-tracking behaviour, with superior performance, for both transient and steady-state responses.

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