Interacting electrons in disordered wires: Anderson localization and low-\(T\) transport

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(Dated: May 20, 2005)

We study transport of interacting electrons in a low-dimensional disordered system at low temperature \(T\). In view of localization by disorder, the conductivity \(\sigma(T)\) may only be non-zero due to electron-electron scattering. For weak interactions, the weak-localization regime crosses over with lowering \(T\) into a dephasing-induced “power-law hopping”. As \(T\) is further decreased, the Anderson localization in Fock space crucially affects \(\sigma(T)\), inducing a transition at \(T = T_c\), so that \(\sigma(T < T_c) = 0\). The critical behavior of \(\sigma(T)\) above \(T_c\) is \(\ln(\sigma(T) \propto -(T - T_c)^{1/2})\). The mechanism of transport in the critical regime is many-particle transitions between distant states in Fock space.

PACS numbers: 72.20.-i, 72.15.Rn, 71.30.+h, 73.63.-b

In a pathbreaking paper \(^{1}\) Anderson demonstrated that a quantum particle may become localized by a random potential. In particular, in non-interacting systems of one-dimensional (1D) or two-dimensional (2D) geometry even weak disorder localizes all electronic states \(^{2}\), thus leading to the exactly zero conductivity, \(\sigma(T) = 0\), whatever temperature \(T\). A non-zero \(\sigma(T)\) in such systems may only occur due to inelastic scattering processes leading to dephasing of electrons. Two qualitatively different sources of dephasing are possible: (i) scattering of electrons by external excitations (in practice, phonons) and (ii) electron-electron (e-e) scattering. In either case, at sufficiently high temperatures, the dephasing rate \(\tau^{-1}_\phi\) is high, so that the localization effects are reduced to a weak-localization (WL) correction to the Drude conductivity. This correction behaves as \(\ln \tau\), in 2D, and as \(\tau^{1/2}\) in quasi-1D (many-channel wire) systems \(^{3}\), and thus diverges with lowering \(T\), signaling the occurrence of the strong localization (SL) regime. This prompts a question as to how the system conducts at low \(T\).

For the case of electron-phonon scattering the answer is well known. The conductivity is then governed by Mott’s variable-range hopping (VRH) \(^{4}\), yielding \(\sigma(T) \propto \exp\{-T_0/T\}^\mu\) with \(\mu = 1/(d+1)\), where \(d\) is the spatial dimensionality. In the presence of a long-range Coulomb interaction, the Coulomb gap in the tunneling density of states modifies the VRH exponent, \(\mu = \frac{1}{d} + \frac{1}{\nu}\).

But what is the low-\(T\) behavior of \(\sigma(T)\) if the electron-phonon coupling is negligibly weak and the only purpose of the inelastic scattering is the e-e interaction? Our purpose here is to solve this long-standing fundamental problem, which is also of direct experimental relevance; see, e.g., Refs. \(^{5}\) and \(^{6}\), where the crossover from WL to SL with lowering \(T\) was studied for 1D and 2D systems, respectively. For definiteness, we concentrate on the case of a many-channel 1D system with a short-range interaction. Our results are, however, more general (including single-channel wires, 2D systems, Coulomb interaction), as we discuss in the end of the paper.

It was proposed in \(^{7}\) that the e-e interaction by itself is sufficient to induce VRH at low \(T\). This idea was widely used for interpretation of experimental \(^{8}\) \(^{11}\) and numerical \(^{11}\) results on 2D systems. Further, Ref. \(^{12}\) used bosonization to study the problem in 1D and concluded that transport is of VRH character. These results are, however, in conflict with the argument \(^{13}\) – supported by our analysis – that elementary hops in the low-\(T\) limit are forbidden for \(d < 3\) even for the case of long-range \((1/r)\) Coulomb interaction, since energy conservation cannot be respected when an electron attempts a real transition by exciting an electron-hole pair \(^{14}\).

The situation is particularly interesting in 1D and 2D, where no mobility edge exists, activation to which otherwise might give \(\sigma(T) \neq 0\). If neither VRH nor activation, then what?

Let us now specify the model. We consider a many-channel weakly disordered wire, so that the relevant length scales satisfy \(k_F^{-1} \ll l \ll \xi\), where \(k_F\) is the Fermi momentum, \(l\) the mean free path, and \(\xi \sim \nu \nu D\) the localization length (\(\nu\) is the density of states per unit length and \(D\) the diffusion constant) \(^{12}\) \(^{15}\). The corresponding energy scales are the Fermi energy \(E_F\), the elastic scattering rate \(\tau^{-1}\), and the level spacing in the localization volume, \(\Delta_\xi = 1/\nu \xi\), with \(E_F \gg \tau^{-1} \gg \Delta_\xi\). We will assume a short-range interaction \(U(r - r')\) between electrons, characterized by a dimensionless coupling \(\alpha = \nu U(0)\), where \(U(q)\) is the Fourier transform of \(U(r)\). We assume that \(\alpha \ll 1\), which yields a richer behavior of \(\sigma(T)\) and allows better understanding of underlying physics; the case \(\alpha \sim 1\) (as well as Coulomb interaction) will be discussed in the end.

At sufficiently high \(T\), the conductivity \(\sigma(T) \approx \sigma_D + \Delta\sigma_{\text{WL}} + \Delta\sigma_{\text{AA}}\) is close to its Drude value \(\sigma_D\), with quantum corrections related to the weak localization \((\Delta\sigma_{\text{WL}})\) and to interplay of interaction and disorder (Altshuler-Aronov contribution \(\Delta\sigma_{\text{AA}}\) \(^{16}\)).

\[
\frac{|\Delta\sigma_{\text{WL}}|}{\sigma_D} \sim \int_{q_1} dq \frac{d q}{\pi \nu D q^2} \sim \frac{l_\phi}{\xi} \left( \frac{\Delta_\xi}{\alpha^2 T} \right)^{1/3}.
\]

Here we used the result for the dephasing rate length...
\[ l_\phi = (D\tau_\phi)^{1/2} \text{ due to e-e interaction} \]

\[ \tau_\phi^{-1} \sim \alpha^2 T \int_{l_\phi^{-1}}^1 \frac{dq}{\pi \nu D q^2} \sim \alpha^2 T \frac{l_\phi}{\xi}, \quad (2) \]

The WL correction grows with lowering \( T \) and finally becomes strong \((\Delta \sigma_{WL}/\sigma_0 \sim 1)\) when \( l_\phi \) reaches \( \xi \), or, equivalently, when \( \tau_\phi^{-1} \sim \Delta_\xi \). This happens at \( T \sim T_1 = \alpha^{-2} \Delta_\xi \), marking the beginning of the SL regime. The interaction-induced correction \( \Delta \sigma_{AA}/\sigma_0 \sim (\alpha^2 \Delta_\xi/T^1/2) \) remains small at \( T \sim T_1 \) and thus is of no relevance in the present context. (For \( \alpha \sim 1 \), \( \Delta \sigma_{AA} \) is of order \( \sigma_0 \) at \( T \sim T_1 \) and does not lead to any qualitative changes either.) The subject of our interest is \( \sigma(T) \) for \( T < T_1 \).

In fact, SL does not necessarily mean \( \sigma(T) \) is exponentially small. Specifically, in the high-\( T \) part of the SL regime the transport mechanism – we will call it power-law hopping (PLH) – is analogous to the one identified in \[14\] for the case of inelastic electron-phonon scattering, i.e., hopping over length \( \sim \xi \) in time \( \sim \tau_\phi \). In other words, the dephasing time \( \tau_\phi \) serves in this regime as a lifetime of localized states, which adds an imaginary part \( i/2\tau_\phi \) to the single-particle energies \( \epsilon_\alpha \). This yields

\[ \sigma(T) \sim \sigma_{ac}(\Omega)|_{\Omega=i/\tau_\phi} \sim e^{2\nu e^2/\tau_\phi}, \quad (3) \]

where \( \sigma_{ac}(\Omega) \) is the zero-\( T \) conductivity of noninteracting electrons at frequency \( \Omega \). The crucial point here is that \( \tau_\phi \) in this SL regime can still be calculated via Fermi’s golden rule, as we are going to show. The lowest-order decays process of a localized state \( |\alpha\rangle \) is the transition to a three-particle state – two electrons \(|\beta\rangle, |\gamma\rangle \) and a hole \(|\delta\rangle \), all located within a distance \( \sim \xi \), Fig. 1b. The corresponding matrix element of the interaction is

\[ V_{\alpha\beta\gamma\delta} \equiv V \sim \alpha \Delta_\xi; \quad |\epsilon_\alpha - \epsilon_\beta|, |\epsilon_\gamma - \epsilon_\delta| \lesssim \Delta_\xi, \quad (4) \]

and decays fast for larger energy differences (cf. results for a metallic sample \[15\] with the dimensionless conductance set to \( g \sim 1 \)). Since energies of all the relevant single-particle states are within the window of width \( \sim T \), the level spacing of three-particle states to which the original state \( |\alpha\rangle \) is coupled according to \[14\], reads

\[ \Delta_\xi^{(3)} \sim \Delta_\xi^2/T. \quad (5) \]

Using \[14\], \[15\] and the golden rule, we find

\[ \tau_\phi^{-1} \sim |V|^2/\Delta_\xi^{(3)} \sim \alpha^2 T. \quad (6) \]

Note that this result could also be obtained from \[2\] if \( \xi^{-1} \) is used as the infrared cutoff, as appropriate for the SL regime. The condition of validity of the golden-rule calculation is \( V \gg \Delta_\xi^{(3)} \), or, equivalently, \( \tau_\phi^{-1} \gg \Delta_\xi^{(3)} \). This introduces a new temperature scale \( T_3 = \alpha^{-1} \Delta_\xi \), so that the PLH regime is restricted to the range \( T_3 \ll |

FIG. 1: Diagrams for the golden rule (a) and higher-order decay amplitude (b).

\[ T \ll T_3. \] Combining \[3\] and \[6\], we get the conductivity in this regime.

\[ \sigma(T) \sim e^{2\nu e^2/\alpha^2 T} \sim e^{2\xi T/T_1}. \quad (7) \]

What happens below \( T_3 \)? Simple hops on a distance \( \sim \xi \) are then not sufficient to delocalize electrons. Increasing the distance \( r \) of a hop does not help: the matrix element vanishes exponentially with \( r/\xi \), while the level spacing \( \Delta^{(3)} \) only as a power law. We thus have to analyze higher-order processes by exploring the structure of the theory in the many-body Fock space, similarly to the ideas developed for the problem of a quasiparticle decay in quantum dots \[18\] \[19\] \[20\]. The process of \( n \)-th order represents a transition with excitation of \( n \) electron-hole pairs, \( |\alpha\rangle \rightarrow |B^{(n)}\rangle \equiv |\beta_0\beta_1\beta_2\ldots\beta_n\rangle \) with the energy difference \( \lesssim \Delta_\xi \) for each pair in view of \[14\], see Fig. 2b. Let us estimate the dimensionless coupling of \( n \)-th order (ratio of the matrix element to the level spacing of final states), \( V^{(n)}/\Delta^{(2n+1)} \), which is the \( n > 1 \) generalization of the ratio \( V/\Delta^{(3)} \) considered above. In this estimate, it will be sufficient for us to keep track of factors of the type \( n^n \) (or \( n! \)) and \( (T/T_3)^n \). Factors of the type \( c^n \), where \( c \sim 1 \), will be unessential and thus neglected.

Let us assume that the set of states \( B^{(n)} \) is spread over the length \( m\xi \), with \( n \) pairs in each box of length \( \xi \); later we optimize with respect to \( m \) (for \( 1 \lesssim m \lesssim n \)). The corresponding level spacing can be estimated as

\[ \Delta^{(2n+1)}_{m\xi}/\Delta_\xi \sim [(n/m)\Delta_\xi/T]^n. \quad (8) \]

The matrix element \( V_{\alpha\beta_1\ldots\beta_{n+1}} \equiv V^{(n)} \) is given by

\[ V^{(n)} = \sum_{\text{diagrams}} \sum_{\gamma_1\ldots\gamma_{n-1}} V_1 \prod_{i=1}^{n-1} \frac{V_{i+1}}{E_i - \epsilon_{\gamma_i}}, \quad (9) \]

where the matrix element \( V_1 \) corresponds to the \( i \)-th interaction line, \( \gamma_i \) the virtual states corresponding to internal lines, and \( E_i \) the corresponding energy variable which can be expressed as a linear superposition of \( \epsilon_\alpha \) using the energy conservation. We need to take into account only those contributions where all states forming each matrix element are within a distance \( \sim \xi \) from each other, so that \( V_1 \sim \alpha \Delta_\xi \). Therefore, the summation over each \( \gamma_i \) is effectively taken over a single localization domain, and we can replace it by taking the “optimal” \( \gamma_i \),
with $|E_i - \epsilon_{\gamma_i}| \sim \Delta \xi$. We thus get

$$V^{(n)} / \Delta \xi \sim [M_m^{(n)}]^{1/2} (\alpha \Delta \xi)^n,$$

(10)

where $M_m^{(n)}$ is the number of diagrams contributing to the amplitude of the transition $|\alpha\rangle \rightarrow |B^{(n)}\rangle$. These contributions have random signs, hence the factor $[M_m^{(n)}]^{1/2}$.

To find $M_m^{(n)}$, we first calculate the number of topologically different diagrams, $D^{(n)}$, which satisfies the recursion relation

$$D^{(n)} = \sum_{n_1 + n_2 + n_3 = n-1; \ n_1, n_2, n_3 \geq 0} D^{(n_1)} D^{(n_2)} D^{(n_3)},$$

(11)

with the initial condition $D^{(0)} = 1$. It is easy to show that its solution increases only as $D^{(n)} \sim a^n$, with $a \sim 1$, so that with the required accuracy it can be replaced by unity. Thus $M_m^{(n)} \sim A_m^{(n)}$, the number of allowed permutations of the set $B^{(n)}$ over the final-state lines. To estimate $A_m^{(n)}$, we notice that only electron-hole pairs within the same (or nearby) localization volume can be interchanged, which yields $A_m^{(n)} \sim [(n/m)!]^m \sim (n/m)^n$. Combining this with $b_n$, $A_n$, we finally get

$$V^{(n)} / (2n+1) \sim [\alpha (m/n)^{1/2} T / \Delta \xi]^n.$$

(12)

Let us now analyze the result. First of all, for given $n$ the most favorable case is $m \sim n$, which corresponds to “ballistic” paths. In such a process, an electron makes a many-body transition over the distance $n \xi$, leaving behind a string of $n$ particle-hole pairs, as illustrated in Fig. 2. Second, $V^{(n)} / (2n+1)$ increases with $n$ at sufficiently high $T$ and decreases with $n$ (thus remaining small for all $n$) at low $T$. Therefore, at low $T$ the higher-order processes do not help a localized single-particle state to decay, so that $\sigma(T)$ is exactly zero. In contrast, at high $T$, the increase of the coupling $b_n$ with $n$ guarantees that the golden-rule calculation performed for the WL and PLH regimes is not spoiled by the higher-order effects. The temperature $T_c$ of the transition into the zero-conductivity regime can be immediately estimated from $b_n$, $T_c \sim \Delta \xi / \alpha$. In fact, $b_n$ misses a $\ln \alpha^{-1}$ factor recovered in a more accurate treatment below, Eq. (14).

What helps us to analyze the critical behavior at the transition is that the structure of the theory, when restricted to the optimal (“ballistic”) paths, reduces essentially to that of the Anderson model on the Bethe lattice—a tree with a fixed branching number. Indeed, consider the process shown in Fig. 2 an electron hops to an adjacent localization volume, creating an electron-hole pair, then to the next one and so forth. Clearly, the density of final states increases at each step by the same factor

$$K \sim \Delta \xi / \Delta \xi^{(3)} \sim T / \Delta \xi,$$

(13)

which is the branching number of the Bethe lattice. The Bethe-lattice character of the problem can also be inferred from the exponential dependence of the coupling $b_n$ on $n$ at $m = n$. This should be contrasted with the opposite limit, $m = 1$, corresponding to the case of a quantum dot, where $b_n$ contains an additional $n^{-n/2}$ factor. The latter is related to a decrease of the effective branching number with $n$ in this case, as was noticed in $19, 20$. In other words, the mapping on the Bethe lattice model, oversimplified for a quantum dot (for which it was originally proposed in $18$), turns out to work perfectly in the case of localized states in a wire (or, more generally, in a non-restricted geometry), where going to higher generations in Fock space can be accompanied by the exploration of new regions in real space.

We can now use the results for the Anderson transition on the Bethe lattice that has been studied extensively $15, 16, 17, 18, 19$. For a large branching number $K$ the equation for the transition point reads

$$\Delta / V = 4 \ln K,$$

(14)

where $V$ is the hopping matrix element, and $\Delta$ is the mean level spacing of states of generation $n + 1$ coupled to a given state of generation $n$. Using (4), (13), and $\Delta = \Delta \xi^{(3)}$, we find the transition temperature,

$$T_c \sim \Delta \xi / \alpha \ln \alpha^{-1},$$

(15)

so that $T_c \sim T_3 / \ln \alpha^{-1}$. The critical behavior of $\sigma(T)$ above $T_c$ is governed by that of the decay rate $\tau_\xi^{-1}$, which translates into the imaginary part of the self-energy Im $\Sigma$ for the Bethe-lattice problem. The critical behavior of the latter was found in $10, 23$, yielding

$$\sigma(T) \propto \text{Im} \Sigma \propto \exp\{-c_0[(T - T_c)/T_c]^{-1/2}\},$$

(16)

with $c_0 \sim \ln \alpha^{-1}$ for $\alpha \ll 1$. Near the transition, the local density of states on the Bethe lattice acquires an increasingly more sparse “spatial” structure $10, 23$, so that the transport is governed by processes connecting remote states in Fock space, Fig. 2 which implies a glassy character of the system. When $T \rightarrow T_c$, the length of the particle-hole strings diverges. We term the low-$T$ phase “Anderson-Fock glass” (AFG), since its physics is governed by the Anderson localization in Fock space.

The found behavior of $\sigma(T)$ is illustrated in Fig. 3. If $\alpha \sim 1$, all scales become of the same order, $T_1 \sim T_3 \sim T_c \sim \Delta \xi$, and the range of PLH disappears. In realistic systems, weak coupling to phonons will lead to PLH (for...
\[ \sigma(T) = 0 \quad \text{Eq.(16)} \quad \text{Eq.(7)} \quad \text{Eq.(1)} \]

**FIG. 3:** Schematic behavior of \( \sigma(T) \) on the log-log scale: the weak-localization (WL), power-law hopping (PLH), and Anderson-Fock glass (AFG) regimes, with a localization transition at \( T_c \). Dashed line: PLH and VRH contribution to \( \sigma(T) \) in the case of a weak coupling to phonons.

\[ \alpha \ll 1 \] and VRH below \( T_c \), but with a small prefactor, so that the transition at \( T_c \) should be well observable. Note the peculiar character of the transition: not only the exponential critical behavior \[ 10 \] but also that the ordered (metallic) phase corresponds to \( T > T_c \). An apparent conflict with the Mermin-Wagner theorem is related to the unconventional (functional) nature of the order parameter for Anderson localization \[ 12, 22, 23 \].

Before closing the paper, let us briefly mention a few extensions of our results \[ 24 \].

(i) **1D Single channel.** We have recently shown \[ 25 \] that the notion of WL and dephasing are also applicable to a disordered Luttinger liquid and calculated \( \sigma(T) \) in the WL regime. The low-\( T \) results presented above can then be easily generalized to the single-channel case. An important difference is that \( \tau \) (and thus, \( \xi \)) is strongly renormalized by interaction, \( \tau(T) / \tau \sim (T / E_F)^\alpha \).

(ii) **Coulomb interaction.** For 1D and 2D geometry, the transition survives also for \( 1/r \) Coulomb interaction, since correlated hops of two electrons separated by a large distance \( r \gg \xi \) will not delocalize them. Indeed, the corresponding matrix elements decrease with \( r \) as \( V_{\alpha\beta\gamma\delta} \propto 1/r^2 \), which is not compensated by the increase \( \propto r^d \) of the density of final states for \( d < 3 \) and thus does not help. On the other hand, in 3D such processes will lead to delocalization for any \( T \) \[ 13 \]; the behavior of \( \sigma(T) \) in this case requires a separate study.

(iv) **Creep.** Our approach can be used to analyze the non-linear conductivity \( \sigma(E) \) at weak electric field \( E \).

In conclusion, we have studied the conductivity of interacting electrons in a disordered quantum wire; very similar results hold for a 2D system. In contrast to a popular belief, the e-e interaction is not sufficient to support the VRH transport in the low-\( T \) limit. Instead, the system undergoes a localization transition at temperature \( T_c \), below which \( \sigma(T) = 0 \) (assuming vanishing coupling to phonons). We have shown that the conductivity vanishes as \( \ln \sigma(T) \propto -(T - T_c)^{-1/2} \) as \( T \to T_c \). Transport in the critical regime is governed by many-particle transitions between distant states in Fock space, corresponding to the formation of long strings of electron-hole pairs. For weak interactions, this Anderson-Fock glass phase is separated from the WL regime by an intermediate temperature regime of power-law hopping.

We thank V. Cheianov, D. Maslov, G. Minkov, T. Nattermann, and B. Shklovskii for valuable discussions. We are particularly grateful to I. Aleiner, B. Altshuler, and D. Basko for very useful discussions and criticism of the earlier version of this work \[ cond-mat/0407305(1) \]. This helped us to correct an error in counting of the number of diagrams \( M_{\alpha\beta\gamma\delta}^{(n)} \) [which yielded a spurious double exponential tail of \( \sigma(T) \) in the AFG phase], whose elimination gave a true phase transition at \( T = T_c \), as also found in \[ 26 \]. The work was supported by SPP “Quanten-Hall-Systeme” and CFN of the DFG and by RFBR.

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