Anti-de Sitter space, branes, singletons, superconformal field theories and all that.

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ABSTRACT

There has recently been a revival of interest in anti de-Sitter space (AdS) brought about by the conjectured duality between physics in the bulk of AdS and a conformal field theory on the boundary. Since the whole subject of branes, singletons and superconformal field theories on the AdS boundary was an active area of research about ten years ago, I begin with a historical review, including the “Membrane at the end of the universe” idea. Next I discuss two recent papers with Lu and Pope on $AdS_5 \times S^5$ and on $AdS_3 \times S^3$, respectively. In each case we note that odd-dimensional spheres $S^{2n+1}$ may be regarded as $U(1)$ bundles over $CP^n$ and that this permits an unconventional “Hopf” duality along the $U(1)$ fibre. This leads in particular to the phenomenon of $BPS$ without $BPS$ whereby states which appear to be non-$BPS$ in one picture are seen to be $BPS$ in the dual picture.

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1 Historical review

1.1 Gauged extended supergravities and their Kaluza-Klein origin

In the early 80’s there was great interest in four-dimensional $N$-extended supergravities for which the global $SO(N)$ is promoted to a gauge symmetry $[1]$. In these theories the underlying supersymmetry algebra is no longer Poincare but rather anti-de Sitter (AdS$_4$) and the Lagrangian has a non-vanishing cosmological constant $\Lambda$ proportional to the square of the gauge coupling constant $e$:

$$G\Lambda = -e^2$$ \hspace{1cm} (1.1)

where $G$ is Newton’s constant. The $N > 4$ gauged supergravities were particularly interesting since the cosmological constant $\Lambda$ does not get renormalized $[2]$ and hence the $SO(N)$ gauge symmetry has vanishing $\beta$-function$[2]$. The relation (1.1) suggested that there might be a Kaluza-Klein interpretation since in such theories the coupling constant of the gauge group arising from the isometries of the extra dimensions is given by

$$e^2 \sim Gm^2$$ \hspace{1cm} (1.2)

where $m^{-1}$ is the size of the compact space. Moreover, there is typically a negative cosmological constant

$$\Lambda \sim -m^2$$ \hspace{1cm} (1.3)

Combining (1.2) and (1.3), we recover (1.1). Indeed, the maximal ($D = 4, N = 8$) gauged supergravity $[4]$ was seen to correspond to the massless sector of ($D = 11, N = 1$) supergravity $[5]$ compactified on an $S^7$ whose metric admits an $SO(8)$ isometry and 8 Killing spinors $[6]$. An important ingredient in these developments that had been insufficiently emphasized in earlier work on Kaluza-Klein theory was that the AdS$_4 \times S^7$ geometry was not fed in by hand but resulted from a spontaneous compactification, i.e. the vacuum state was obtained by finding a stable solution of the higher-dimensional field equations $[7]$. The mechanism of spontaneous compactification appropriate to the AdS$_4 \times S^7$ solution of eleven-dimensional supergravity was provided by the Freund-Rubin mechanism $[8]$ in which the 4-form field strength in spacetime $F_{\mu\nu\rho\sigma}$ ($\mu = 0, 1, 2, 3$) is proportional to the alternating symbol $\epsilon_{\mu\nu\rho\sigma}$ $[9]$:

$$F_{\mu\nu\rho\sigma} \sim \epsilon_{\mu\nu\rho\sigma}$$ \hspace{1cm} (1.4)

$^{2}$For $N \leq 4$, the beta function (which receives a contribution from the spin 3/2 gravitinos) is positive and the pure supergravity theories are not asymptotically free. The addition of matter supermultiplets only makes the $\beta$ function more positive $[1]$ and hence gravitinos can never be confined. I am grateful to Karim Benakli, Rene Martinez Acosta and Parid Hoxha for discussions on this point.
A summary of this $S^7$ and other $X^7$ compactifications of $D = 11$ supergravity down to $AdS_4$ may be found in [13]. By applying a similar mechanism to the 7-form dual of this field strength one could also find compactifications on $AdS_7 \times S^4$ [10] whose massless sector describes gauged maximal $N = 4$, $SO(5)$ supergravity in $D = 7$ [11, 12]. Type IIB supergravity in $D = 10$, with its self-dual 5-form field strength, also admits a Freund-Rubin compactification on $AdS_5 \times S^5$ [14, 15, 16] whose massless sector describes gauged maximal $N = 8$ supergravity in $D = 5$ [17, 18].

| Compactification | Supergroup | Bosonic subgroup |
|------------------|------------|------------------|
| $AdS_4 \times S^7$ | $OSp(4|8)$ | $SO(3,2) \times SO(8)$ |
| $AdS_5 \times S^5$ | $SU(2,2|4)$ | $SO(4,2) \times SO(6)$ |
| $AdS_7 \times S^4$ | $OSp(6,2|4)$ | $SO(6,2) \times SO(5)$ |

Table 1: Compactifications and their symmetries.

In the three cases given above, the symmetry of the vacuum is described by the supergroups $OSp(4|8)$, $SU(2,2|4)$ and $OSp(6,2|4)$ for the $S^7$, $S^5$ and $S^4$ compactifications respectively, as shown in Table 1.

1.2 Singletons

Each of these groups is known to admit the so-called singleton, doubleton or tripleton supermultiplets [19] as shown in Table 2.

| Supergroup | Supermultiplet | Field content |
|------------|----------------|---------------|
| $OSp(4|8)$ | $(n = 8, d = 3)$ singleton | 8 scalars, 8 spinors |
| $SU(2,2|4)$ | $(n = 4, d = 4)$ doubleton | 1 vector, 4 spinors, 6 scalars |
| $OSp(6,2|4)$ | $((n_+, n_-) = (2,0), d = 6)$ tripleton | 1 chiral 2-form, 8 spinors, 5 scalars |

Table 2: Superconformal groups and their singleton, doubleton and tripleton representations.

We recall that singletons are those strange representations of $AdS$ first identified by Dirac [20] which admit no analogue in flat spacetime. They have been much studied by Fronsdal.

\[\text{Our nomenclature is based on the } AdS_4, AdS_5 \text{ and } AdS_7 \text{ groups having ranks 2, 3 and 4, respectively, and differs from that of Gunaydin.}\]
and collaborators [21, 22]. Let us first consider $AdS_4$ which can be defined as the four-dimensional hyperboloid
\[ \eta_{ab} y^a y^b = -\frac{1}{a^2} \] (1.5)
in $R^5$ with Cartesian coordinates $y^a$ where
\[ \eta_{ab} = \text{diag}(-1, 1, 1, 1, -1) \] (1.6)
In polar coordinates $x^\mu = (t, r, \theta, \phi)$ the line element may be written
\[ g_{\mu\nu} dx^\mu dx^\nu = -(1 + a^2 r^2) dt^2 + (1 + a^2 r^2)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \] (1.7)
Representations of $SO(3, 2)$ are denoted $D(E_0, s)$, where $E_0$ is the lowest energy eigenvalue (in units of $a$) and $s$ is the total angular momentum. The representation is unitary provided $E_0 \geq s + 1/2$ for $s = 0, 1/2$ and $E_0 \geq s + 1$ for $s \geq 1$. The representations are all infinite dimensional. In the supersymmetric context, all linear irreducible representations of $N = 1 AdS$ supersymmetry were classified by Heidenreich [23]. They fall into 4 classes:
1. $D(1/2, 0) \oplus D(1, 1/2)$
2. $D(E_0, 0) \oplus D(E_0 + 1/2, 1/2) \oplus D(E_0 + 1, 0), E_0 \geq 1/2$
3. $D(s + 1, s) \oplus D(s + 3/2, s + 1/2), s \geq 1/2$
4. $D(E_0, s) \oplus D(E_0 + 1/2, s + 1/2) \oplus D(E_0 + 1/2, s - 1/2) \oplus D(E_0 + 1, s)$.

Class 1 is the singleton supermultiplet which has no analogue in Poincare supersymmetry. Class 2 is the Wess-Zumino supermultiplet. Class 3 is the gauge supermultiplet with spins $s$ and $s + 1/2$ with $s \geq 1/2$. Class 4 is the higher spin supermultiplet. The corresponding study of $OSp(4|N)$ representations was neglected in the literature until their importance in Kaluza-Klein supergravity became apparent. For example, the round $S^7$ leads to massive $N = 8$ supermultiplets with maximum spin 2. This corresponds to an $AdS$ type of multiplet shortening analogous to the shortening due to central charges in Poincare supersymmetry [24]. Two features emerge: (1) $OSp(4|N)$ multiplets may be decomposed into $OSp(4|1)$ multiplets discussed above; (2) In the limit as $a \to 0$ and the $OSp(4|N)$ contracts to the $N$-extended Poincare algebra, all short $AdS$ multiplets become massless Poincare multiplets.

1.3 Singletons live on the boundary

As emphasized by Fronsdal et al [21, 22], singletons are best thought of as living not in the $(d + 1)$-dimensional bulk of the $AdS_{d+1}$ spacetime but rather on the $d$-dimensional $S^1 \times S^{d-1}$ boundary where the $AdS$ group $SO(d - 1, 2)$ plays the role of the conformal
group. Remaining for the moment with our 4-dimensional example, consider a scalar field 
\( \Phi(t, r, \theta, \phi) \) on \( AdS_4 \) with metric (1.7), described by the action

\[
S_{\text{bulk}} = -\frac{1}{2} \int_{AdS_4} d^4x \sqrt{-g} \Phi \left( -g^{\mu\nu} \nabla_\mu \nabla_\nu + M^2 \right) \Phi
\]

Note that this differs from the conventional Klein-Gordon action by a boundary term. Since the scalar Laplacian on \( AdS_4 \) has eigenvalues \( E_0 (E_0 - 3) a^2 \), the critical value of \( M^2 \) for a singleton with \( (E_0, s) = (1/2, 0) \) is

\[
M^2 = \frac{5}{4} a^2
\]

In this case, one can show with some effort [21, 22] that as \( r \to \infty \),

\[
\Phi(t, r, \theta, \phi) \to r^{-1/2} \phi(t, \theta, \phi)
\]

and hence that the radial dependence drops out:

\[
S_{\text{boundary}} = -\frac{1}{2} \int_{S^1 \times S^2} d^3\xi \sqrt{-h} \left[ h^{ij} \nabla_i \phi \nabla_j \phi + \frac{1}{4} a^2 \phi^2 \right]
\]

Here we are integrating over a 3-manifold with \( S^1 \times S^2 \) topology and with metric

\[
h^{ij} d\xi^i d\xi^j = -dt^2 + \frac{1}{a^2} (d\theta^2 + \sin^2 \theta d\phi^2)
\]

This 3-manifold is sometimes referred to as the boundary of \( AdS_4 \) but note that the metric \( h_{ij} \) is not obtained by taking the \( r \to \infty \) limit of \( g_{\mu\nu} \) but rather the \( r \to \infty \) limit of the conformally rescaled metric \( \Omega^2 g_{\mu\nu} \) where \( \Omega = 1/ra \). The radius of the \( S^2 \) is \( a^{-1} \) not infinity.

Most particle physicists are familiar with the conformal group in flat Minkowski space. It is the group of coordinate transformations which leave invariant the Minkowski lightcone. In the case of three-dimensional Minkowski space, \( M_3 \), it is \( SO(3,2) \). In the present context, however, the spacetime is curved with topology \( S^1 \times S^2 \), but still admits \( SO(3,2) \) as its conformal group, i.e. as the group which leaves invariant the three-dimensional lightcone \( h_{ij} d\xi^i d\xi^j = 0 \). The failure to discriminate between these different kinds of conformal invariance is, we believe, a source of confusion in the singleton literature. In particular, the \( \phi^2 \) “mass” term appearing in the action (1.11) would be incompatible with conformal invariance if the action were on \( M_3 \) but is essential for conformal invariance on \( S^1 \times S^2 \). Moreover, the coefficient \( a^2/4 \) is uniquely fixed [31].

So although singleton actions of the form (1.8) and their superpartners appeared in the Kaluza-Klein harmonic expansions on \( AdS_4 \times S^7 \) [25, 26, 27], they could be gauged

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4One sometimes finds the statement in the physics literature that the only compact spaces admitting conformal Killing vectors are those isomorphic to spheres. By a theorem of Yano and Nagano [60], this is true for Einstein spaces, but \( S^1 \times S^2 \) is not Einstein.
away everywhere except on the boundary where the above $OSp(4\vert 8)$ corresponds to the superconformal group \[28\]. One finds an $(n = 8, d = 3)$ supermultiplet with 8 scalars $\phi^A$ and 8 spinors $\chi^A$, where the indices $A$ and $\dot{A}$ range over 1 to 8 and denote the $8_s$ and $8_c$ representations of $SO(8)$, respectively. The $OSp(4\vert 8)$ action is a generalization of (1.11) and is given by \[30\]
\begin{equation}
S_{\text{singleton}} = -\frac{1}{2} \int_{S^1 \times S^2} d^3 \xi \sqrt{-h} [h^{ij} \nabla_i \phi^A \nabla_j \phi^A + \frac{1}{4} g^2 \phi^A \phi^A + i \bar{\chi}^A (1 - \gamma) \gamma^i D_i \chi^A] \tag{1.13}
\end{equation}
where $\gamma = -\gamma_0 \gamma_1 \gamma_2$ and where $D_i$ is the covariant derivative appropriate to the $S^1 \times S^2$ background.

In the case of $AdS_5 \times S^5$ one finds a $(n = 4, d = 4)$ supermultiplet with 1 vector $A_i$, $(i = 0, 1, 2, 3)$, a complex spinor $\lambda^a_+$, $(a = 1, 2, 3, 4)$, obeying $\gamma_5 \lambda^a_+ = \lambda^a_+$ and 6 real scalars $\phi^{ab}$, obeying $\phi^{ab} = -\phi^{ab}$, $\phi^{ab} = e^{abcd} \phi_{cd}/2$. The corresponding action for the doubletons of $SU(2, 2\vert 4)$ is \[31\]
\begin{equation}
S_{\text{doubleton}} = \int_{S^1 \times S^3} \left[ -\frac{1}{4} F_{ij} F^{ij} - \frac{1}{4} a^2 \phi_{ab} \phi^{ab} - \frac{1}{4} \partial_i \phi_{ab} \phi^i \phi^{ab} + i \bar{\lambda}^a_+ a^i D_i \lambda^a_+ \right] \tag{1.14}
\end{equation}
where $F_{ij} = 2 \partial_i A_j$. However, in contrast to the singletons, we know of no derivation of this doubleton action on the boundary starting from an action in the bulk analogous to (1.11).

In the case of $AdS_7 \times S^4$ one finds a $((n_+, n_-) = (2, 0), d = 6)$ supermultiplet with a 2-form $B_{ij}$, $(i = 0, 1, \ldots, 5)$, whose field strength is self-dual, 8 spinors $\lambda^A_+$, $(A = 1, 2, 3, 4)$, obeying $\gamma^7 \lambda^A_+ = \lambda^A_+$ and 5 scalars $\phi^a$, $(a = 1, 2, \ldots, 5)$. The $OSp(6, 2\vert 4)$ tripleton covariant field equations on $S^1 \times S^5$ are \[31\]:
\begin{align*}
(\nabla^i \nabla_i - 4a^2) \phi^a &= 0 \\
\gamma^i D_i \lambda^A_+ &= 0 \\
H_{ijk} &= \frac{1}{3!} \sqrt{-h} e^{ijklmn} H_{lmn} \tag{1.15}
\end{align*}
where $H_{ijk} = 3 \partial_i B_{jk}$. Once again, we know of no derivation of these tripleton field equations on the boundary starting from equations in the bulk.

### 1.4 The membrane as a singleton: the membrane/supergravity bootstrap

Being defined over the boundary of $AdS_4$, the $OSp(4\vert 8)$ singleton action (1.13) is a three dimensional theory with signature $(-, +, +)$ describing 8 scalars and 8 spinors. With the discovery of the eleven-dimensional supermembrane \[32\], it was noted that 8 scalars and 8
spinors on a three-dimensional worldvolume with signature $(-, +, +)$ is just what is obtained after gauge-fixing the supermembrane action! Moreover, kappa-symmetry of this supermembrane action forces the background fields to obey the field equations of $(N = 1, D = 11)$ supergravity. It was therefore suggested [29] that on the $AdS_4 \times S^7$ supergravity background, the supermembrane could be regarded as the singleton of $OSp(4|8)$ whose worldvolume occupies the $S^1 \times S^2$ boundary of the $AdS_4$. Noting that these singletons also appear in the Kaluza-Klein harmonic expansion of this supergravity background, this further suggested a form of bootstrap [29] in which the supergravity gives rise to the membrane on the boundary which in turn yields the supergravity in the bulk. This conjecture received further support with the subsequent discovery of the “membrane at the end of the universe” [34] to be discussed in section 1.8, and the realisation [40] that the eleven-dimensional supermembrane emerges as a solution of the $D = 11$ supergravity field equations.

The possibility of a similar 3-brane/supergravity bootstrap arising for the $SU(2,2|4)$ doubletons on $AdS_5 \times S^5$ and a similar 5-brane/supergravity bootstrap arising for the $OSp(6,2|4)$ triplets on $AdS_7 \times S^4$ was also considered [29]. Ironically, however, it was (erroneously as we now know) rejected since the only supermembranes that were known at the time [11] had worldvolume theories described by scalar supermultiplets, whereas the doubletons and triplets required vector and tensor supermultiplets, respectively. See section 1.6.

Nevertheless, since everything seemed to fit nicely for the $(d = 3, D = 11)$ slot on the brane-scan of supersymmetric extended objects with worldvolume dimension $d$, there followed a good deal of activity relating other super $p$-branes in other dimensions to singletons and superconformal field theories [34, 35, 31, 36, 37, 33, 38, 31, 39]. In particular, it was pointed out [30, 31, 37] that there was a one-to one-correspondence between the 12 points on the brane-scan as it was then known [41] and the 12 superconformal groups in Nahm’s classification [28] admitting singleton representations, as shown in Table 3. The number of dimensions transverse to the brane, $D - d$, equals the number of scalars in the singleton supermultiplet. (The two factors appearing in the $d = 2$ case is simply a reflection of the ability of strings to have right and left movers. For brevity, we have written the Type II assignments in Table 3, but more generally we could have $OSp(p|2) \times OSp(q|2)$ where $p$ and $q$ are the number of left and right supersymmetries [12].) Note that the $d = 6$ upper limit on the worldvolume dimension is consistent with the requirement of renormalizability [38].
Table 3: The brane scan of superconformal groups admitting singletons.

Note, however, that the \((d = 3, D = 11), \OSp(4|8)\) slot (written in boldface) occupies a privileged position in that the corresponding \(D = 11\) supergravity theory admits the \(\AdS_4 \times S^7\) solution with \(\OSp(4|8)\) symmetry, whereas the other supergravities do not admit solutions with the superconformal group as a symmetry. For example, \(D = 10\) supergravity admits an \(\AdS_3 \times S^7\) solution [13, 53], but it does not have the full \([\OSp(8|2)]^2\) symmetry because the dilaton is non-trivial and acts as a conformal Killing vector on the \(\AdS_3\). This is slightly mysterious, since the bulk theory has less symmetry than the boundary theory.

We shall return to this in sections [1.7 and 2.1].

1.5 Many a time and oft

One might hope that a theory of everything should predict not only the \textit{dimensionality} of spacetime, but also its \textit{signature}. For example, quantum consistency of the superstring requires 10 spacetime dimensions, but not necessarily the usual \((9, 1)\) signature. The signature is not completely arbitrary, however, since spacetime supersymmetry allows only \((9, 1), (5, 5)\) or \((1, 9)\). Unfortunately, superstrings have as yet no answer to the question of why our universe appears to be four-dimensional, let alone why it appears to have signature \((3, 1)\). The authors of [37, 36, 44] therefore considered a world with an arbitrary number \(T\) of time dimensions and an arbitrary number \(S\) of space dimensions to see how
far classical supermembranes restrict not only $S + T$ but $S$ and $T$ separately. To this end they also allowed an $(s, t)$ signature for the worldvolume of the membrane where $s \leq S$ and $t \leq T$ but are otherwise arbitrary. The resulting allowed signatures are summarized on the “brane-molecule” of Table 4, where $R$, $C$, $H$ and $O$ denote real $(1 + 1)$, complex $(2 + 2)$, quaternion $(4 + 4)$ and octonion $(8 + 8)$, respectively.

Moreover, it is not difficult to repeat the $AdS$ analysis of section 1.4 for arbitrary signatures, and to show that there is once again a one-to-one correspondence between supermembranes whose worldvolume theories are described by scalar supermultiplets and superconformal theories in Nahm’s classification admitting singleton representations [37, 38].

\[
S \uparrow
\begin{array}{cccccccccc}
11 & .  \\
10 & . & O  \\
 9 & H & H/O & O  \\
 8 & . & H  \\
 7 & . & H  \\
 6 & . & H & O  \\
 5 & C & C/H & H & H & H/O & O  \\
 4 & . & C & H  \\
 3 & . & R/C & H  \\
 2 & . & R & H & O  \\
 1 & . & R & R/C & C & C/H & H & H & H/O & O  \\
 0 & . & . & . & . & C & . & . & H & . & .  \\
\end{array}
\]

\[
0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10 \quad 11 \quad T \rightarrow
\]

Table 4: The brane-molecule

At the time, we posed the obvious question of why the mathematics of supermembranes seems to allow these universes with more than one time dimension whereas the physical world seems to demand just one. This question has recently been answered by Hull [15], who claims that all these possible mathematical signatures are allowed physically and that, despite appearances, they are dual to one another. Hull’s resolution is both radical and conservative at the same time: it is radical in introducing universes with more than one time dimension into physics but conservative in saying that the only many-time universes we need worry about are those that are really one-time universes in disguise!
1.6 Doubletons and tripletons revisited

These early works focussed on scalar supermultiplets because these were the only $p$-branes known in 1988 [11]. However, with the discovery in 1990 of Type $II$ $p$-brane solitons [48, 49, 46, 47, 50], vector and tensor multiplets were also seen to play a role. In particular, the worldvolume fields of the self-dual Type IIB superthreebrane were shown to be described by an $(n = 4, d = 4)$ gauge theory [47], which on the boundary of $AdS_5$ is just the doubleton supermultiplet of the superconformal group $SU(2,2|4)!$ Thus one can after all entertain a 3-brane-doubleton-supergravity bootstrap similar to the membrane-singleton-supergravity bootstrap of section 1.4, and we may now draw the doubleton brane scan of Table 5. Once again, the restriction to $d = 4$ is consistent with renormalizability. Note, however, that the $(d = 4, D = 10), SU(2,2|4)$ slot (written in boldface) occupies a privileged position in that the corresponding $D = 10$ Type $IIB$ supergravity admits the $AdS_5 \times S^5$ solution with $SU(2,2|4)$ symmetry, whereas the other supergravities do not admit solutions with the superconformal group as a symmetry since, as discussed in section 1.7, the dilaton is again non-trivial.

\[
\begin{array}{cccccccc}
D\uparrow \\
11 & . & . \\
10 & . & . & . & . & . & . & . & \textbf{SU}(2,2|4) \\
9 & . & . & . & . & . & . & . & . \\
8 & . & . & . & . & . & . & . & . \\
7 & . & . & . & . & . & . & . & . \\
6 & . & . & . & . & . & . & . & . \\
5 & . & . & . & . & . & . & . & . \\
4 & . & . & . & . & . & . & . & . \\
3 & . & . & . & . & . & . & . & . \\
2 & . & . & . & . & . & . & . & . \\
1 & . & . & . & . & . & . & . & . \\
0 & . & . & . & . & . & . & . & . \\
\end{array}
\]

Table 5: The brane scan of superconformal groups admitting doubletons
Similarly, with the discovery of the M-theory fivebrane \[51\], it was realized \[52\] that the zero modes are described by an \((n_+, n_-) = (2, 0), d = 6\) multiplet with a chiral 2-form, 8 spinors and 5 scalars, which on the boundary of AdS\(_7\) is just the tripleton supermultiplet of the superconformal group \(OSp(6, 2|4)\)! (These zero modes are the same as those of the Type IIA fivebrane, found previously in \[48, 49\]). Thus one can after all also entertain a 5-brane-tripleton-supergravity bootstrap similar to the membrane-singleton-supergravity bootstrap of section 1.4. Thus we may now draw the tripleton brane scan of Table 6.

Note once again, however, that the \((d = 6, D = 11), OSp(6, 2|4)\) slot (written in boldface) occupies a privileged position in that the corresponding \(D = 11\) supergravity admits the \(AdS_7 \times S^4\) solution with \(OSp(6, 2|4)\) symmetry, whereas the other supergravities do not admit solutions with the superconformal group as a symmetry since, as discussed in section 1.7, the dilaton is again non-trivial.

\[
\begin{array}{cccccccccccc}
D \uparrow & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad & \quad 
\end{array}
\]

Table 6: The brane scan of superconformal groups admitting tripletons

With the inclusion of branes with vector and tensor supermultiplets on their worldvol-
ume, another curiosity arises. Whereas the singleton brane scan of Table 3 exhausts all the scalar branes and the triplet brane scan of Table 6 exhausts all the tensor branes, the doubleton brane scan of Table 5 is only a subset of all the vector branes \[14\]. The Type IIB 3-brane is special because gauge theories are conformal only in \(d = 4\). So taking the brane-supergravity bootstrap idea seriously in 1988 would have lead to the earlier discovery of the M-theory fivebrane and Type IIB 3-brane, but not the other Type II branes.
1.7 Near horizon geometry and p-brane aristocracy

More recently, AdS has emerged as the near-horizon geometry of black p-brane solutions in D dimensions. The dual brane, with worldvolume dimension \( \tilde{d} = D - d - 2 \), interpolates between D-dimensional Minkowski space \( M_D \) and \( AdS_{\tilde{d}+1} \times S^{d+1} \) (or \( M_{\tilde{d}+1} \times S^3 \) if \( d = 2 \)). To see this, we recall that such branes arise generically as solitons of the following action [61]:

\[
I = \frac{1}{2\kappa^2} \int d^D x \sqrt{-g} \left[ R - \frac{1}{2}(\partial \phi)^2 - \frac{1}{2(d+1)!} e^{-\alpha \phi} F_{d+1}^2 \right]
\]

(1.16)

where \( F_{d+1} \) is the field strength of a \( d \)-form potential \( A_d \) and \( \alpha \) is the constant

\[
\alpha^2 = 4 - \frac{2dd}{d + \tilde{d}}
\]

(1.17)

Written in terms of the \((d - 1)\)-brane sigma-model metric \( e^{-\alpha/d}\phi g_{MN} \), the solutions are [61, 55]

\[
ds^2 = \Delta^{-\frac{d-2}{d}} (-dt^2 + dx.dx) + \Delta^{-2} dr^2 + r^2 d\Omega_{d+1}^2
\]

\[
e^{-2\phi} = \Delta^\alpha
\]

\[
F_{d+1} = db^d \epsilon_{d+1}
\]

(1.18)

where \( dx.dx \) is the Euclidean \((\tilde{d} - 1)\) metric, and

\[
\Delta = 1 - \left( \frac{b}{r} \right)^d
\]

(1.19)

The near horizon geometry corresponds to \( r \sim b \), and we make the change of variable

\[
r = b \left( 1 + \frac{\lambda}{\tilde{d}} \right)
\]

(1.20)

in which case

\[
ds^2 = [\lambda^{-\frac{d-2}{d}} (-dt^2 + dx.dx) + \left( \frac{b}{\tilde{d}} \right)^2 \lambda^{-2} d\lambda^2 + b^2 d\Omega_{d+1}^2](1 + O(\lambda))
\]

(1.21)

Neglecting the \( O(\lambda) \) terms, as before, and defining the new coordinate

\[
\lambda = e^\frac{d}{b^2} \zeta
\]

(1.22)

we get

\[
ds^2 \sim e^{\frac{d}{b^2}} (-dt^2 + dx.dx) + d\zeta^2 + b^2 d\Omega_{d+1}^2
\]

\[
\phi \sim -\frac{d\alpha}{2b} \zeta
\]

\[
F_{d+1} = db^d \epsilon_{d+1}
\]

(1.23)
Thus for $d \neq 2$ the near-horizon geometry is $AdS_{\tilde{d}+1} \times S^{d+1}$. Note, however, that the gradient of the dilaton is generically non-zero and plays the role of a conformal Killing vector on $AdS_{\tilde{d}+1}$. Consequently, there is no enhancement of symmetry in the near-horizon limit. The unbroken supersymmetry remains one-half and the bosonic symmetry remains $P_{\tilde{d}} \times SO(d+2)$. (If $d = 2$, then (1.23) reduces to

$$ds^2 = (-dt^2 + dx^2 + d\zeta^2) + b^2 d\Omega_3^2$$

$$\phi \sim -\frac{\alpha}{b} \zeta$$

$$F_3 \sim 2b^2 \epsilon_3$$

which is $M_{\tilde{d}+1} \times S^3$, with a linear dilaton vacuum. The bosonic symmetry remains $P_{\tilde{d}} \times SO(4)$.)

Of particular interest are the ($\alpha = 0$) subset of solitons for which the dilaton is zero or constant: the non-dilatonic $p$-branes. From (1.17) we see that for single branes there are only 3 cases:

$$D = 11 : d = 6, \tilde{d} = 3$$

$$D = 10 : d = 4, \tilde{d} = 4$$

$$D = 11 : d = 3, \tilde{d} = 6$$

which are precisely the three cases that occupied privileged positions on the singleton, doubleton and tripleton branescans of Tables 3, 5 and 6. Then the near-horizon geometry coincides with the $AdS_{\tilde{d}+1} \times S^{d+1}$ non-dilatonic maximally symmetric compactifications of the corresponding supergravities. The supersymmetry doubles and the bosonic symmetry is also enhanced to $SO(\tilde{d}, 2) \times SO(d+2)$. Thus the total symmetry is given by the conformal supergroups $OSp(4|8)$, $SU(2, 2|4)$ and $OSp(6, 2|4)$, respectively.

For bound states of $N$ singly charged branes, the constant $\alpha$ gets replaced by

$$\alpha^2 = \frac{4}{N} - \frac{2dd}{d+\tilde{d}}$$

A non-dilatonic solution ($\alpha=0$) occurs for $N = 2$:

$$D = 6 : d = 2, \tilde{d} = 2$$

which is just the dyonic string [54], of which the self-dual string [21] is a special case, whose near-horizon geometry is $AdS_3 \times S^3$. For $N = 3$ we have

$$D = 5 : d = 2, \tilde{d} = 1$$
which is the 3-charge black hole \([69]\), whose near-horizon geometry is \(AdS_2 \times S^3\), and

\[ D = 5 : d = 1, \tilde{d} = 2 \]

which is the 3-charge string \([69]\) whose near-horizon geometry is \(AdS_3 \times S^2\). For \(N = 4\) we have

\[ D = 4 : d = 1, \tilde{d} = 1 \]

which is the 4-charge black hole \([70, 71]\), of which the Reissner-Nordstrom solution is a special case \([65]\), and whose near-horizon geometry is \(AdS_2 \times S^2\) \([66]\).

Thus we see that not all branes are created equal. A \(p\)-brane aristocracy obtains whose members are those branes whose near-horizon geometries have as their symmetry the conformal supergroups. As an example of a plebian brane we can consider the ten-dimensional superstring:

\[ D = 10 : d = 6, \tilde{d} = 2 \]

whose near-horizon geometry is the \(AdS_3 \times S^7\) but with a non-trivial dilaton of section \([1.4]\) which does not have the conformal group \([OSp(8|2)]^2\) as its symmetry, even though this group appears in the \((D = 10, \tilde{d} = 2)\) slot on the singleton branescan of Table 3. In which case, of course, one may ask what role do these singletons play. We shall return to this in section \([2.1]\).

1.8 The membrane at the end of the universe

As further evidence of the membrane/supergravity bootstrap idea, solutions of the combined \(D = 11\) supergravity/supermembrane equations were sought for which the spacetime is \(AdS_4 \times M^7\) and for which the supermembrane occupies the boundary of the \(AdS_4\): the Membrane at the End of the Universe \([34, 35, 67]\).

The bosonic sector of the supermembrane equation is

\[
\partial_i(\sqrt{-h} h^{ij} \partial_j X^N g_{MN}) + \frac{1}{2} \sqrt{-h} h^{ij} \partial_i X^N \partial_j X^P \partial M g_{NP} + \frac{1}{3!} \epsilon^{ijk} \partial_i X^N \partial_j X^P \partial_k X^Q F_{MNPQ} = 0
\]

(1.26)

where

\[
h_{ij} = \partial_i X^M \partial_j X^N g_{MN}
\]

(1.27)

A membrane configuration will have residual supersymmetry if there exist Killing spinors \(\epsilon(X)\) satisfying \([34, 35]\)

\[
\tilde{D}_M \epsilon = 0, \quad \Gamma \epsilon(X) = \epsilon(X)
\]

(1.28)
where $\tilde{D}_M$ is the $D = 11$ supergravity covariant derivative appearing in the gravitino transformation rule and $\Gamma$ is given by

$$\Gamma = \frac{1}{3!} \sqrt{-h} \epsilon^{ijk} \partial_i X^M \partial_j X^N \partial_k X^P \Gamma_{MNP}$$  \hspace{1cm} (1.29)$$

Let us denote the membrane worldvolume coordinates by $\xi^i = (\tau, \sigma, \rho)$. The original membrane at the end of the universe [34, 35] was embedded in the $AdS_4$ geometry as

$$ds^2 = -(1 + a^2 r^2) d\tau^2 + (1 + a^2 r^2)^{-1} dr^2 + r^2 (d\sigma^2 + \sin^2 \sigma d\rho^2)$$  \hspace{1cm} (1.30)$$

and has topology $S^1 \times S^2$. Consequently, the $OSp(4|8)$ singleton action is the one given in (1.11) with its scalar mass terms. Alternatively, one could take as the membrane at the end of the universe to be the near-horizon membrane, which is embedded as

$$ds^2 = e^{4\zeta/b} (-d\tau^2 + d\sigma^2 + d\rho^2) + d\zeta^2$$  \hspace{1cm} (1.31)$$

and has $M_3$ topology. It is still possible to associate an $OSp(4|8)$ action but this time it is defined over $M_3$ and has no scalar mass terms [67, 68]. One can continue to call these fields “singletons”, of course, if by singleton one simply means anything transforming according to the $D(1/2, 0)$ and $D(1, 1/2)$ representations of $SO(3, 2)$. A comparison of these two approaches is discussed in some detail in [37].

1.9 Supermembranes with fewer supersymmetries. Skew-whiffing.

So far we have focussed attention on compactifications to $AdS_{d+1}$ on round spheres $S^{d+1}$ which have maximal supersymmetry, but the supergravity equations admit infinitely many other compactifications on Einstein spaces $X^{d+1}$ which have fewer supersymmetries [13]. Indeed generic $X^{d+1}$ have no supersymmetries at all [1]. We note in this connection the skew-whiffing theorem [13], which states that for every $AdS_{d+1}$ compactification preserving supersymmetry, there exists one with no supersymmetry simply obtained by reversing the orientation of $X^{d+1}$ (or, equivalently, reversing the sign of $F_{d+1}$). The only exceptions are when $X^{d+1}$ are round spheres which preserve the maximum supersymmetry for either orientation. A corollary is that other symmetric spaces, which necessarily admit an orientation-reversing isometry, can have no supersymmetries. Examples are provided by products of round spheres.

---

5Thus in the early eighties, the most highly prized solutions were those with many supersymmetries. Nowadays, bragging rights seem to go those which have none!
The question naturally arises as to whether these compactifications with fewer supersymmetries also arise as near-horizon geometries of p-brane solitons. The answer is yes and the soliton solutions are easy to construct \[56, 57\]. One simply makes the replacement

\[ d\Omega_{d+1}^2 \rightarrow d\hat{\Omega}_{d+1}^2 \]  

(1.32)

in (1.18), where \( d\hat{\Omega}_{d+1}^2 \) is the metric on an arbitrary Einstein space \( X^{d+1} \) with the same scalar curvature as the round \( S^{d+1} \). The space need only be Einstein, it need not be homogeneous \[56\]. (There also exist brane solutions on Ricci flat \( X^{d+1} \) \[56\] but we shall not discuss them here). Note, however, that these non-round-spherical solutions do not tend to \((D - d)\)-dimensional Minkowski space as \( r \rightarrow \infty \). Instead the metric on the \((D - \tilde{d})\)-dimensional space transverse to the brane is asymptotic to a generalized cone

\[ ds_{D-d}^2 = dr^2 + r^2 d\hat{\Omega}_{d+1}^2 \]  

and \((D - d)\)-dimensional translational invariance is absent except when \( X^{d+1} \) is a round sphere. The number of supersymmetries preserved by these \( p \)-branes is determined by the number of Killing spinors on \( X^{d+1} \).

To illustrate these ideas let us focus on the eleven-dimensional supermembrane. The usual supermembrane interpolates between \( M_{11} \) and \( AdS_4 \times \) round \( S^7 \), has symmetry \( P_3 \times SO(8) \) and preserves 1/2 of the spacetime supersymmetries for either orientation of the round \( S^7 \). Replacing the round \( S^7 \) by generic Einstein spaces \( X^7 \) leads to membranes with symmetry \( P_3 \times G \), where \( G \) is the isometry group of \( X^7 \). For example, \( G = SO(5) \times SO(3) \) for the squashed \( S^7 \) \[58, 59\]. For one orientation of \( X^7 \), they preserve \( N/16 \) spacetime supersymmetries where \( 1 \leq N \leq 8 \) is the number of Killing spinors on \( X^7 \); for the opposite orientation they preserve no supersymmetries since then \( X^7 \) has no Killing spinors. For example, \( N = 1 \) for the left-squashed \( S^7 \) owing to its \( G_2 \) holonomy \[58, 13, 59\], whereas \( N = 0 \) for the right-squashed \( S^7 \). However, all these solutions satisfy the same Bogomol’nyi bound between the mass and charge as the usual supermembrane \[56\]. Of course, skew-whiffing is not the only way to obtain vacua with less than maximal supersymmetry. A summary of known \( X^7 \), their supersymmetries and stability properties is given in \[13\]. Note, however, that skew-whiffed vacua are automatically stable at the classical level since skew-whiffing affects only the spin \( 3/2, 1/2 \) and \( 0^- \) towers in the Kaluza-Klein spectrum, whereas the criterion for classical stability involves only the \( 0^+ \) tower \[13\].
In more recent times, both perturbative and non-perturbative effects of ten-dimensional superstring theory have been subsumed by an eleven-dimensional theory \[83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95\], called \(M\)-theory, whose low-energy limit is \(D = 11\) supergravity. In particular, the \(D = 10\) Type \(IIA\) superstring emerges from \(M\)-theory compactified on \(S^1\) \[83, 86, 87\]. In this picture, the resulting Kaluza-Klein modes are Dirichlet 0-branes \[101\] with masses proportional to \(1/\lambda\) in the string metric, where \(\lambda\) is the string coupling constant. They are thus non-perturbative from the Type \(IIA\) perspective. This may also be seen from the fact that perturbative string states carry no Ramond-Ramond \(U(1)\) charge whereas the massive Kaluza-Klein modes are necessarily charged under this \(U(1)\). \(M\)-theory, on the other hand, draws no distinction between perturbative and non-perturbative states. An interesting question, therefore, is whether there is any difference in the status of supersymmetry when viewed either from the perturbative Type \(IIA\) string or from the vantage point of non-perturbative \(M\)-theory.

This rehabilitation of \(D = 11\) supergravity has thus revived an interest in \(AdS_4 \times X^7\) compactifications. In \[72\], for example, such \(M\)-theory vacua with \(N > 0\) supersymmetry were presented which, from the perspective of perturbative Type \(IIA\) string theory, have \(N = 0\). They can emerge whenever the \(X^7\) is a \(U(1)\) bundle over a 6-manifold. The missing superpartners are Dirichlet 0-branes. Someone unable to detect Ramond-Ramond charge would thus conclude that these worlds have no unbroken supersymmetry. In particular, the gravitinos (and also some of the gauge bosons) are 0-branes not seen in perturbation theory but which curiously remain massless however weak the string coupling.

The simplest example of this phenomenon is provided by the maximally-symmetric \(S^7\) compactification \[13\] of \(D = 11\) supergravity. Considered as a compactification of \(D = 11\) supergravity, the round \(S^7\) yields a four dimensional \(AdS\) spacetime with \(N = 8\) supersymmetry and \(SO(8)\) gauge symmetry, for either orientation of \(S^7\). The Kaluza-Klein mass spectrum therefore falls into \(SO(8)\) \(N = 8\) supermultiplets. In particular, the massless sector is described by gauged \(N = 8\) supergravity \[13\]. Since \(S^7\) is a \(U(1)\) bundle over \(CP^3\) the same field configuration is also a solution of \(D = 10\) Type \(IIA\) supergravity \[27\]. However, the resulting vacuum has only \(SU(4) \times U(1)\) symmetry and either \(N = 6\) or \(N = 0\) supersymmetry depending on the orientation of the \(S^7\). The reason for the discrepancy is that the modes charged under the \(U(1)\) are associated with the Kaluza-Klein reduction from \(D = 11\) to \(D = 10\) and are hence absent from the Type \(IIA\) spectrum originating from the massless Type \(IIA\) supergravity. In other words, they are Dirichlet 0-branes and
hence absent from the perturbative string spectrum. There is thus more non-perturbative
gauge symmetry and supersymmetry than perturbative. (Here the words “perturbative”
and “non-perturbative” are shorthand for “with and without the inclusion of Dirichlet 0-
branes”, but note that the Type IIA compactification has non-perturbative features even
without the 0-branes \[72\]). The right-handed orientation is especially interesting because
the perturbative theory has no supersymmetry at all! See Table 7 (where we are using
the notation of \[96\] for \(SU(4)\) representations). It is interesting to note that the \(D = 4\)
massless states in the left-handed vacuum originate from the \(n = 0\) massless level and
\(n = 1, 2\), massive Kaluza-Klein levels in \(D = 10\): whereas in the right-handed vacuum they
originate from \(n = 0, 1, 2, 3, 4\) levels.

| Spin | \(SO(8)\) reps | Left \(SU(4) \times U(1)\) reps | Right \(SU(4) \times U(1)\) reps |
|------|----------------|-------------------------------|-------------------------------|
| 2    | 1              | 1<sub>0</sub>                  | 1<sub>0</sub>                  |
| \(\frac{3}{2}\) | 8<sub>s</sub> | 6<sub>0</sub> + 1<sub>_2</sub> + 1<sub>−2</sub> | 4<sub>1</sub> + 4<sub>−1</sub> |
| 1    | 28             | 1<sub>0</sub> + 15<sub>0</sub> + 6<sub>2</sub> + 6<sub>−2</sub> | 1<sub>0</sub> + 15<sub>0</sub> + 6<sub>2</sub> + 6<sub>−2</sub> |
| \(\frac{1}{2}\) | 56<sub>s</sub> | 6<sub>0</sub> + 10<sub>0</sub> + 10<sub>0</sub> + 15<sub>2</sub> + 15<sub>−2</sub> | 4<sub>1</sub> + 4<sub>−1</sub> + 20<sub>1</sub> + 20<sub>−1</sub> + 4<sub>−3</sub> + 4<sub>3</sub> |
| 0<sup>+</sup> | 35<sub>v</sub> | 15<sub>0</sub> + 10<sub>−2</sub> + 10<sub>2</sub> | 15<sub>0</sub> + 10<sub>−2</sub> + 10<sub>2</sub> |
| 0<sup>−</sup> | 35<sub>c</sub> | 15<sub>0</sub> + 10<sub>_2</sub> + 10<sub>−2</sub> | 1<sub>0</sub> + 20<sub>′0</sub> + 6<sub>2</sub> + 6<sub>−2</sub> + 1<sub>4</sub> + 1<sub>−4</sub> |

Table 7: The massless multiplet under \(SO(8) \rightarrow SU(4) \times U(1)\)

A summary of perturbative versus non-perturbative symmetries is given in Table 8. In
particular, the non-perturbative vacuum may have unbroken supersymmetry even when
the perturbative vacuum has none.

| Compactification | Perturbative Type IIA | Nonperturbative M-theory |
|-----------------|-----------------------|--------------------------|
| Left round \(S^7\) | \(N = 6\) \(SU(4) \times U(1)\) | \(N = 8\) \(SO(8)\) |
| Right round \(S^7\) | \(N = 0\) \(SU(4) \times U(1)\) | \(N = 8\) \(SO(8)\) |
| Left squashed \(S^7\) | \(N = 1\) \(SO(5) \times U(1)\) | \(N = 1\) \(SO(5) \times SU(2)\) |
| Right squashed \(S^7\) | \(N = 0\) \(SO(5) \times U(1)\) | \(N = 0\) \(SO(5) \times SU(2)\) |
| Left \(M(3, 2)\) | \(N = 0\) \(SU(3) \times SU(2) \times U(1)\) | \(N = 2\) \(SU(3) \times SU(2) \times U(1)\) |
| Right \(M(3, 2)\) | \(N = 0\) \(SU(3) \times SU(2) \times U(1)\) | \(N = 0\) \(SU(3) \times SU(2) \times U(1)\) |

Table 8: Perturbative versus non-perturbative symmetries
We cannot resist asking whether this could be a model of the real world in which you can have your supersymmetry and eat it too. The problem with such a scenario, of course, is that God does not do perturbation theory and presumably an experimentalist would measure God’s real world and not what a perturbative string theorist thinks is the real world. Unless, for some unknown reason, the experimentalist’s apparatus is so primitive as to be unable to detect Ramond-Ramond charge in which case he or she would conclude that the world has no unbroken supersymmetry.

2 The new AdS/CFT correspondence

2.1 The Maldacena conjecture

The year 1998 marks a revolution in anti de-Sitter space brought about by Maldacena’s conjectured duality between physics in the bulk of $AdS$ and a conformal field theory on the boundary \[100\]. In particular, $M$-theory on $AdS_4 \times S^7$ is dual to a non-abelian ($n = 8, d = 3$) superconformal theory, Type $IIB$ string theory on $AdS_5 \times S^5$ is dual to a $d = 4$ $SU(N)$ super Yang-Mills theory and $M$-theory on $AdS_7 \times S^4$ is dual to a non-abelian ($n_+, n_- = (2, 0), d = 6$) conformal theory. In particular, as has been spelled out most clearly in the $d = 4$ $SU(N)$ Yang-Mills case, there is seen to be a correspondence between the Kaluza-Klein mass spectrum in the bulk and the conformal dimension of operators on the boundary \[112, 114\].

One immediately recognises that the dimensions and supersymmetries of these three conformal theories are exactly the same as the singleton, doubleton and tripleton supermultiplets of section 1.2. Moreover, both the old and new AdS/CFT correspondences are holographic in the sense of \[120, 121\]. Following Maldacena’s conjecture \[100\], therefore, a number of papers appeared reviving the old singleton-AdS-membrane- superconformal field theory connections \[108, 109, 110, 111, 112, 113, 114, 73, 115, 116, 117, 118, 119\] and applying them to this new duality context. What are the differences?

One curious difference is that, with the exception of the three aristocratic branes, all the slots on the three brane-scans of superconformal field theories corresponded to bulk supergravities whose brane solutions are dilatonic, and hence have a symmetry smaller

---

6A scheme in which you can have all the benefits of unbroken supersymmetry while appearing to inhabit a non-supersymmetric world has also been proposed by Witten \[98\] but his mechanism is very different from ours. In particular, our vacua necessarily have non-vanishing cosmological constant unless cancelled by fermion condensates \[99\].
than the boundary theory. It seems that the branes at the end of the universe do not care about the dilaton because the $r = \text{constant}$ surfaces in (1.30) (or the $\zeta = \text{constant}$ surfaces in (1.31)) possess the full superconformal symmetry even though the bulk $AdS$ solution does not. In other words, they admit the maximal set of conformal Killing vectors even though the the bulk admits less than the maximal set of Killing vectors. This contrasts with the new $AdS/CFT$ conjecture where a non-conformal supergravity solution in the bulk [53] is deemed to be dual to non-conformal field theory on the boundary [107]. It is not obvious at the moment whether this difference is real or apparent and it would be interesting to pursue the matter further.

Secondly, attention was focussed on free superconformal theories on the boundary as opposed to the interacting theories currently under consideration. For example, although the worldvolume fields of the Type $IIB$ 3-brane were known to be described by an $(n = 4, d = 4)$ gauge theory [47], we now know that this brane admits the interpretation of a Dirichlet brane [101] and that the superposition of $N$ such branes yields a non-abelian $SU(N)$ gauge theory [102]. These observations are crucial to the new duality conjecture [100]. For earlier related work on coincident threebranes and $n = 4$ super Yang Mills, see [103, 104, 105, 106]. Let us consider the solution for $N$ coincident 3-branes corresponding to $N$ units of 5-form flux [46, 47]:

$$ds^2 = \Delta^{1/2}(-dt^2 + dx \cdot dx) + \Delta^{-2}dr^2 + r^2d\Omega_5^2$$

$$F_5 = 4Nb^4\epsilon_5 = *F_5$$  \hspace{1cm} (2.1)

where $dx \cdot dx$ is the Euclidean 3-metric, and

$$\Delta = 1 - \frac{Nb^4}{r^4}$$  \hspace{1cm} (2.2)

Instead of regarding the near horizon geometry as an $r \sim N^{1/4}b$ limit we may equally well regard it as large $N$ limit, We find $AdS_5 \times S^5$, but with an $AdS$ radius proportional to $N^{1/4}$. The philosophy is that Type $IIB$ supergravity is a good approximation for large $N$ and that Type $IIB$ stringy excitations correspond to operators whose dimensions diverge for $N \rightarrow \infty$. This makes contact with the whole field of large $N$ QCD. These large $N$, non-abelian features were absent in the considerations of a 3-brane/supergravity bootstrap discussed in section 1.6, as was the precise correspondence between the Kaluza-Klein mass spectrum in the bulk and the conformal dimension of operators on the boundary [112, 114]. Nevertheless, as the present paper hopes to show, there are sufficiently many similarities between the current bulk/boundary duality and the old Membrane at the End of the Universe idea, to merit further comparisons.
It is to the $AdS_5 \times S^5$ case that we now turn. Noting that $T$-duality untwists $S^5$ to $CP^2 \times S^1$, we construct the duality chain $n = 4$ super Yang-Mills $\rightarrow$ Type IIB superstring on $AdS_5 \times S^5$ $\rightarrow$ Type IIA superstring on $AdS_5 \times CP^2 \times S^1$ $\rightarrow$ M-theory on $AdS_5 \times CP^2 \times T^2$. This provides another example of the phenomenon of *supersymmetry without supersymmetry* [72], but this time without involving Dirichlet 0-branes. On $AdS_5 \times CP^2 \times S^1$ Type IIA supergravity has $SU(3) \times U(1) \times U(1) \times U(1)$ and $N = 0$ supersymmetry. Indeed, since $CP^2$ does not admit a spin structure, its spectrum contains no fermions at all! Nevertheless, Type IIA string theory has $SO(6)$ and $N = 8$ supersymmetry. The missing superpartners (and indeed all the fermions) are provided by stringy winding modes. These winding modes also enhance $SU(3) \times U(1)$ to $SO(6)$, while the gauge bosons of the remaining $U(1) \times U(1)$ belong to massive multiplets.

As a preliminary, we shall show how to construct the odd-dimensional unit spheres $S^{2n+1}$ as $U(1)$ bundles over $CP^n$.

### 2.2 Hopf fibrations

The construction, which generalizes the $S^7$ example of section 1.10, involves writing the metric $d\Omega^2_{2n+1}$ on the unit $(2n+1)$-sphere in terms of the Fubini-Study metric $d\Sigma^2_{2n}$ on $CP^n$ as

$$d\Omega^2_{2n+1} = d\Sigma^2_{2n} + (dz + \bar{A})^2.$$  \hfill (2.3)

In fact we may give general results for any metric of the form

$$ds^2 = c^2 (dz + \bar{A})^2 + d\bar{s}^2$$ \hfill (2.4)

on a $U(1)$ bundle over a base manifold with metric $d\bar{s}^2$, where $c$ is a constant. Choosing the vielbein basis $c^z = c(dz + \bar{A})$, $c^i = \bar{e}^i$, one finds that the Riemann tensor for $ds^2$ has non-vanishing vielbein components given by

$$R_{ijk\ell} = \bar{R}_{ijk\ell} - \frac{1}{4} c^2 (\bar{F}_{ik} \bar{F}_{j\ell} - \bar{F}_{i\ell} \bar{F}_{jk} + 2 \bar{F}_{ij} \bar{F}_{k\ell}) ,$$

$$R_{zizj} = \frac{1}{4} c^2 \bar{F}_{ik} \bar{F}_{jk} , \quad R_{ijkz} = \frac{1}{2} c \nabla_k \bar{F}_{ij} .$$ \hfill (2.5)

In all the cases we shall consider, the components $R_{ijkz}$ will be zero, since $\bar{F} = d\bar{A}$ will be proportional to covariantly-constant tensors, such as Kähler forms. The Ricci tensor for $ds^2$ has the vielbein components

$$R_{zz} = \frac{1}{4} c^2 \bar{F}_{ij} \bar{F}_{ij} , \quad R_{ij} = \bar{R}_{ij} - \frac{1}{2} c^2 \bar{F}_{ik} \bar{F}_{jk} \quad R_{zi} = -\frac{1}{2} c \nabla_j \bar{F}_{ij} ,$$ \hfill (2.6)
Applied to our present case, where the unit \((2n + 1)\)-sphere should have a Ricci tensor satisfying \(R_{ab} = 2n \delta_{ab}\), we see that this is achieved by taking the field strength to be given by \(\bar{F}_{ij} = 2J_{ij}\), where \(J_{ij}\) is the covariantly-constant Kähler form on \(CP^n\). Furthermore, the Fubini-Study Einstein metric on \(CP^n\) should be scaled such that its Ricci tensor satisfies \(\bar{R}_{ij} = 2(n + 1) \delta_{ij}\). The volume form \(\Omega_{2n+1}\) on the unit \((2n + 1)\)-sphere is related to the volume form \(\Sigma_{2n}\) on \(CP^n\) by \(\Omega_{2n+1} = dz \wedge \Sigma_{2n}\). Note also that the volume form on \(CP^n\) is related to the Kähler form by

\[
\Sigma_{2n} = \frac{1}{m} J^n .
\]  

(2.7)

### 2.3 \(AdS_5 \times S^5\) untwisted

Let us write the \(AdS_5 \times S^5\) geometry in the form

\[
d s^2 = d s^2(AdS_5) + d s^2(S^5) ,
\]

\[
H_{(5)} = 4m \Omega_{AdS_5} + 4m \Omega_{S^5} ,
\]

(2.8)

where \(\Omega_{AdS_5}\) and \(\Omega_{S^5}\) are the volume forms on \(AdS_5\) and \(S^5\) respectively, \(m\) is a constant, and the metrics on \(AdS_5\) and \(S^5\) satisfy

\[
R_{\mu\nu} = -4m^2 g_{\mu\nu} , \quad R_{mn} = 4m^2 g_{mn}
\]

(2.9)

respectively. Since the unit 5-sphere has metric \(d\Omega_5^2\) with Ricci tensor \(\bar{R}_{mn} = 4 \bar{g}_{mn}\), it follows that we can write

\[
d s^2(S^5) = \frac{1}{m^2} d\Omega_5^2 .
\]

(2.10)

From (2.3), it follows that we can write this as

\[
d s^2(S^5) = \frac{1}{m^2} d\Sigma_4^2 + \frac{1}{m^2} (dz + \bar{A})^2 ,
\]

(2.11)

where \(d\Sigma_4^2\) is the metric on the “unit” \(CP^2\), and \(d\bar{A} = 2J\), where \(J\) is the Kähler form on \(CP^2\).

We may now perform a dimensional reduction of this solution to \(D = 9\), by compactifying on the circle of the \(U(1)\) fibres, parameterized by \(z\). Comparing with the general Kaluza-Klein prescription, for which

\[
d s^2_{10} = d s^2_5 + (dz_2 + \bar{A})^2 ,
\]

\[
H_{(5)} = H_{(5)} + H_{(4)} \wedge (dz_2 + \bar{A}) ,
\]

(2.12)
we see, from the fact that the $S^5$ and $CP^2$ volume forms are related by $\Omega_5 = (dz + \bar{A}) \wedge \Sigma_4$, that the solution will take the 9-dimensional form

$$
\begin{align*}
    ds^2_9 &= ds^2(AdS_5) + \frac{1}{m^2} d\Sigma^2_4, \\
    F_{(4)} &= \frac{4}{m^2} \Sigma_4, \quad F_{(2)} = \frac{2}{m} J.
\end{align*}
$$

(2.13)

(Note that in the dimensional reduction of the 5-form of the type IIB theory, its self-duality translates into the statement that the fields $H_{(5)}$ and $H_{(4)}$ in $D = 9$ must satisfy $H_{(4)} = \ast H_{(5)} = F_{(4)}$.)

We now perform the $T$-duality transformation to the fields of the $D = 9$ reduction of the Type IIA theory. The relation between the IIB and the IIA fields is given in [75]. Thus in the IIA notation, we have the nine-dimensional configuration

$$
\begin{align*}
    ds^2_9 &= ds^2(AdS_5) + \frac{1}{m^2} d\Sigma^2_4, \\
    F_{(4)} &= \frac{4}{m^2} \Sigma_4, \quad F_{(12)}^{(12)} = \frac{2}{m} J.
\end{align*}
$$

(2.14)

The crucial point is that the 2-form field strength $F_{(12)}^{(12)}$ of the IIA variables is no longer a Kaluza-Klein field coming from the metric; rather, it comes from the dimensional reduction of the 3-form field strength in $D = 10$. Indeed, if we trace the solution (2.14) back to $D = 10$, we have the Type IIA configuration

$$
\begin{align*}
    ds^2_{10} &= ds^2(AdS_5) + \frac{1}{m^2} d\Sigma^2_4 + dz_2^2, \\
    F_{(4)} &= \frac{4}{m^2} \Sigma_4, \quad F_{(1)}^{(1)} = \frac{2}{m} J \wedge dz_2.
\end{align*}
$$

(2.15)

The solution has the topology $AdS_5 \times CP^2 \times S^1$. This should be contrasted with the topology $AdS_5 \times S^5$ for the original $D = 10$ solution in the Type IIB framework. Thus the $T$-duality transformation in $D = 9$ has “unravelled” the twisting of the $U(1)$ fibre bundle over $CP^2$, leaving us with a direct product $CP^2 \times S^1$ compactifying manifold in the Type IIA description.

At first sight, the $T$-duality transformation that we have performed has a somewhat surprising implication. We began with a solution on $AdS_5 \times S^5$, which admits a spin structure, and mapped it via $T$-duality to a solution on $AdS_5 \times CP^2 \times S^1$, which does not admit a spin structure (because $CP^2$ does not admit a spin structure). In particular, this means that the spectrum of Kaluza-Klein excitations in the $CP^2 \times S^1$ compactification of Type IIA supergravity contains no fermions at all! The equivalence is restored only when the stringy winding modes are incorporated. Further details may be found in [75].
2.4 Less supersymmetry

Example of Type II B compactifications to AdS5 with less supersymmetry, arising as in section 1.9 from the near-horizon geometry of 3-branes with less supersymmetry, may be obtained by replacing S5 by generic Einstein spaces X5. Examples include: orbifolds of S5 which can preserve N = 4, 2, 0 [72, 73]; non-singular lens spaces S5/Zn which can preserve N = 4, 2, 0 [72, 77] (reducing the supersymmetry using lens spaces was discussed in [74]); Q(n1, n2) spaces which are U(1) bundles over S2 × S2 and which generically have N = 0 but have N = 4 for (n1, n2) = (1, 1) [78, 79] (This leads to one of the gauged (D = 5, N = 4) supergravities discussed in [82]); Tp,q spaces which are cosets [SU(2) × SU(2)]/U(1) and which generically have N = 0 but have N = 2 for (p, q) = (1, 1) [80, 81].

2.5 AdS3 × S3 (un)twisted and squashed

As discussed in section 1.7, the six-dimensional space AdS3 × S3 emerges as the near horizon geometry of the self-dual string [53, 54] or, more generally, the dyonic string [61, 55, 126, 127, 128, 129, 130]. The dyonic string admits the ten-dimensional interpretation [64] of an intersecting NS − NS 1-brane and 5-brane, which in a Type II context is in turn related by U-duality to the D1 − D5 brane system [123, 124, 125]. This geometry plays a part in recent studies of black holes and has attracted a good deal of attention lately following Maldacena’s conjecture. AdS3 is particularly interesting in this regard because the conformal field theory on the boundary is then of the familiar and well-understood 1 + 1 dimensional variety.

In this section, we wish to apply the above Hopf duality techniques to find Type IIA (and hence M-theory) duals of six-dimensional Type IIB AdS3 × S3 configurations obtained by either T4 or K3 compactifications [76]. The novel ingredient is that these can be supported by both NS − NS and R − R 3-forms, in contrast to the AdS5 × S5 example where the 5-form was strictly R − R. This has some interesting and unexpected consequences. Noting that S3 is a U(1) bundle over CP1 ∼ S2, we construct the dual Type IIA configurations by a Hopf T-duality along the U(1) fibre. In the case where there are only R − R charges, the S3 is untwisted to S2 × S1 (in analogy with the previous treatment of AdS5 × S5). However, in the case where there are only NS − NS charges, the S3 becomes the cyclic lens space S3/Zp with its round metric (and is hence invariant when p = 1), where p is the magnetic NS − NS charge. In the generic case with NS − NS and R − R charges, the S3 not only becomes S3/Zp but is also squashed, with a squashing parameter that is related to the values of the charges. Similar results apply if we regard AdS3 as a bundle over AdS2 and T-dualize along the fibre. We note that these Hopf dualities preserve the area of the
horizons, and hence they preserve the black hole entropies.

The dyonic string solution is supported either by the $NS-NS$ 3-form $F_{(3)}^{NS}$ or the $R-R$ 3-form $F_{(3)}^{RR}$. More general solutions can be obtained by acting with the $O(2, 2)$ symmetry of the theory, allowing us, in particular, to find solutions for dyonic strings carrying both $NS-NS$ and $R-R$ charges. This is done in detail in [76], obtaining an $O(2, 2; \mathbb{Z})$ multiplet of dyonic strings.

Near the horizon, even though the above dyonic solutions carry four independent charges, the 3-forms $F_{(3)}^{NS}$ and $F_{(3)}^{RR}$ become self-dual, and the metric approaches that of $AdS_3 \times S^3$. The dilatons $\phi_1$ and $\phi_2$ and the axions $\chi_1$ and $\chi_2$ are constant in the solution, and for simplicity we shall take them to be zero. The remaining equations are solved by taking the metric and 3-forms to be

$$ds_6^2 = ds^2(AdS) + ds^2(S^3) ,$$

$$F_{(3)}^{NS} = \lambda \epsilon(AdS) + \lambda \epsilon(S^3) ,$$

$$F_{(3)}^{RR} = \mu \epsilon(AdS) + \mu \epsilon(S^3) ,$$

where $\lambda$ and $\mu$ are constants, and the metrics on the $AdS_3$ and $S^3$ have Ricci tensors given by

$$R_{\mu\nu} = -\frac{1}{2}(\lambda^2 + \mu^2) g_{\mu\nu} , \quad R_{mn} = \frac{1}{2}(\lambda^2 + \mu^2) g_{mn}$$

respectively. The constants $\lambda$ and $\mu$ are related to the magnetic charges as follows:

$$Q_{NS} \equiv \frac{1}{16\pi^2} \int F_{(3)}^{NS} = \frac{\lambda}{(\lambda^2 + \mu^2)^{3/2}} , \quad Q_{RR} \equiv \frac{1}{16\pi^2} \int F_{(3)}^{RR} = \frac{\mu}{(\lambda^2 + \mu^2)^{3/2}} .$$

We now make use of the fact that the metric $d\Omega_2^2$ can be written as a $U(1)$ bundle over $CP^1 \sim S^2$ as follows:

$$d\Omega_2^2 = \frac{1}{4}d\Omega_2^2 + \frac{1}{4}(dz + B)^2 ,$$

where $d\Omega_2^2$ is the metric on the unit 2-sphere, whose volume form $\Omega_{(2)}$ is given by $\Omega_{(2)} = dB$. (If $d\Omega_2^2$ is written in spherical polar coordinates as $d\Omega_2^2 = d\theta^2 + \sin^2 \theta \, d\phi^2$, then we can write $B$ as $B = \cos \theta \, d\phi$.) The fibre coordinate $z$ has period $4\pi$. Thus the six-dimensional metric given in (2.16) can be written as

$$ds_6^2 = ds^2(AdS) + \frac{1}{\lambda^2 + \mu^2} d\Omega_2^2 + \frac{1}{\lambda^2 + \mu^2} (dz + B)^2 .$$

The four-dimensional area of the horizon is given by

$$A \sim L(\lambda^2 + \mu^2)^{-3/2} ,$$

(2.21)
where $L$ is the contribution from $ds^2(\text{AdS})$ at the boundary at constant time. The field strengths in (2.16) can now be written as

\[
F^{\text{NS}}_{(3)} = \lambda \epsilon(\text{AdS}) + \frac{\lambda}{(\lambda^2 + \mu^2)^{3/2}} \Omega_{(2)} \wedge (dz + B) ,
\]
\[
F^{\text{RR}}_{(3)} = \mu \epsilon(\text{AdS}) + \frac{\mu}{(\lambda^2 + \mu^2)^{3/2}} \Omega_{(2)} \wedge (dz + B) .
\] (2.22)

If we dimensionally reduced on the fibre coordinate we obtain the 5-dimensional metric

\[
ds^2_5 = (\lambda^2 + \mu^2)^{-1/3} ds^2(\text{AdS}) + (\lambda^2 + \mu^2)^{-4/3} d\Omega^2_2 ,
\] (2.23)

while the new dilaton $\varphi$ is a constant, given by

\[
e^{\varphi/\sqrt{6}} = (\lambda^2 + \mu^2)^{-1/3} .
\] (2.24)

Comparing (2.22) with the reduction ansätze $F_{(n)} \to F_{(n)} + F_{(n-1)} \wedge (dz + B)$ for the field strengths, we find that in $D = 5$ we have

\[
F^{\text{NS}}_{(3)} = \lambda \epsilon(\text{AdS}) , \quad F^{\text{NS}}_{(2)1} = \frac{\lambda}{(\lambda^2 + \mu^2)^{3/2}} \Omega_{(2)} ,
\]
\[
F^{\text{RR}}_{(3)} = \mu \epsilon(\text{AdS}) , \quad F^{\text{RR}}_{(2)1} = \frac{\mu}{(\lambda^2 + \mu^2)^{3/2}} \Omega_{(2)} ,
\]
\[
\mathcal{F}_{(2)} = dB = \Omega_{(2)} .
\] (2.25)

We are now in a position to implement the $T$-duality transformation from the Type IIB description to the Type IIA description in $D = 5$. Using the dictionary of [76]. We find

\[
F_{(3)} = \lambda \epsilon(\text{AdS}) , \quad \mathcal{F}_{(2)} = \frac{\lambda}{(\lambda^2 + \mu^2)^{3/2}} \Omega_{(2)} ,
\]
\[
F_{(3)1} = -\mu \epsilon(\text{AdS}) , \quad F_{(2)} = \frac{\mu}{(\lambda^2 + \mu^2)^{3/2}} \Omega_{(2)} ,
\]
\[
\mathcal{F}_{(2)1} = \Omega_{(2)} .
\] (2.26)

From the duality dictionary [76] and (2.24), together with the fact that we are taking $\phi_1 = \phi_2 = 0$ in the original Type IIB solution, it follows that the dilatons in the Type IIA picture will be given by

\[
e^\varphi = (\lambda^2 + \mu^2)^{1/\sqrt{6}} , \quad e^{\phi_1} = e^{\phi_2} = (\lambda^2 + \mu^2)^{1/2} .
\] (2.27)

Finally, we can uplift the Type IIA solution that we have just obtained back to $D = 6$, by retracing the standard Kaluza-Klein reduction steps. Doing so, we find that the six-dimensional metric in the Type IIA picture is

\[
ds^2_6 = (\lambda^2 + \mu^2)^{-1/2} ds^2(\text{AdS}) + (\lambda^2 + \mu^2)^{-3/2} \left[ d\Omega^2_2 + \frac{\lambda^2}{\lambda^2 + \mu^2} (dz' + B)^2 \right] ,
\] (2.28)
where \( B \) is a potential such that \( \Omega_{(2)} = dB \), and the coordinate \( z' \) is related to \( z \) by
\[
z = \frac{\lambda}{(\lambda^2 + \mu^2)^{3/2}} z' = Q_{NS} z' .
\] (2.29)

It is straightforward to verify that the area of the horizon of the metric (2.28) is the same as that before the Hopf \( T \)-duality transformation, given by (2.21). The Type IIA field strengths in \( D = 6 \) are given by
\[
F^{(4)} = -\mu \epsilon(AdS) \wedge (dz + A_{(1)}) , \quad F^{(3)} = \lambda \epsilon(AdS) + \Omega_{(2)} \wedge (dz + A_{(1)}) , \\
F^{(2)} = \frac{\mu}{(\lambda^2 + \mu^2)^{3/2}} \Omega_{(2)} ,
\] (2.30)
where
\[
A_{(1)} = \frac{\lambda}{(\lambda^2 + \mu^2)^{3/2}} B = Q_{NS} B .
\] (2.31)

We find that the charges carried by these field strengths are as follows:
\[
Q^{(3)}_{\text{elec}} \equiv \frac{1}{16\pi^2} \int_{S^3} e^{-\phi_1 - \phi_2} * F^{(4)} = Q_{NS} , \\
Q^{(3)}_{\text{mag}} \equiv \frac{1}{16\pi^2} \int_{S^3} F^{(3)} = 1 , \\
Q^{(4)}_{\text{elec}} \equiv \frac{1}{4\pi} \int_{S^2} e^{\frac{1}{2}\phi_1 - \frac{1}{2}\phi_2} * F^{(4)} = -Q_{RR} , \\
Q^{(2)}_{\text{mag}} \equiv \frac{1}{4\pi} \int_{S^2} F^{(2)} = Q_{RR} .
\] (2.32)

If the fibre coordinate \( z' \) in (2.28) had had the period \( 4\pi \), then the topology of the compact 3-space would have been \( S^3 \). Since it is related to \( z \) as given in (2.29), and \( z \) has period \( 4\pi \), it follows that \( z' \) has period \( 4\pi/Q_{NS} \), and hence the topology of the compact 3-space is \( S^3/Z_{Q_{NS}} \), the cyclic lens space of order \( Q_{NS} \). On the other hand the magnetic charge carried by the field strength \( F^{(3)} \) is equal to 1, having started, in the original solution, as \( Q_{NS} \). Furthermore, we can see from (2.28) that the metric on the lens space is not in general the “round” one, but is instead squashed along the \( U(1) \) fibre direction, with a squashing factor \( \nu \) given by
\[
\nu = \frac{\lambda}{\sqrt{\lambda^2 + \mu^2}} = \frac{Q_{NS}}{\sqrt{Q_{NS}^2 + Q_{RR}^2}} .
\] (2.33)

We could have considered original solutions in which the constant dilatons \( \phi_1 \) and \( \phi_2 \) were non-zero, in which case the original electric and magnetic charges need not have been equal. The lens space after the Hopf \( T \)-duality transformation will then be \( S^3/Z_{Q_{NS}} \). Also, we can generalise the starting point further by consider a solution on the product of \( AdS_3 \) and the lens space \( S^3/Z_n \), rather than simply \( AdS_3 \times S^3 \). (From the lower-dimensional point of view, this corresponds to giving the Kaluza-Klein vector a magnetic charge \( n \) rather than
1.) If we do this, then we find that a Type IIB solution \( \text{AdS}_3 \times S^3/Z_n \) with charges \( Q_{\text{elec}}^{\text{NS}} \), \( Q_{\text{elec}}^{\text{RR}} \) and \( Q_{\text{mag}}^{\text{RR}} \) will result, after the \( T \)-duality transformation, in a Type IIA solution \( \text{AdS}_3 \times S^3/Z_{Q_{\text{mag}}^{\text{NS}}} \) with charges

\[
Q_{\text{elec}}^{(3)} = Q_{\text{elec}}^{\text{NS}}, \quad Q_{\text{mag}}^{(3)} = n, \quad Q_{\text{elec}}^{(4)} = -Q_{\text{elec}}^{\text{RR}}, \quad Q_{\text{mag}}^{(2)} = Q_{\text{mag}}^{\text{RR}}. \tag{2.34}
\]

Although the construction of conformal field theories with background \( R - R \) charges is problematical, there is an exact CFT duality statement in the case of pure NS – NS charge \([76]\). Namely, strings on \( S^3/Z_n \) with 3-form flux \( m \) are dual to strings on \( S^3/Z_m \) with 3-form flux \( n \).

**2.6 BPS without BPS**

The Type IIA configuration (2.15) can be further uplifted to \( D = 11 \):

\[
ds_{11}^2 = ds^2(\text{AdS}_5) + \frac{1}{m^4} d\Sigma_4^2 + dz_1^2 + dz_2^2, \\
F_{(4)} = \frac{1}{m^4} \Sigma_4 - \frac{2}{m} J \wedge dz_1 \wedge dz_2. \tag{2.35}
\]

The topology of this solution is \( \text{AdS}_5 \times CP^2 \times T^2 \). This is just the near-horizon \( (y \sim 0) \) geometry of the \( M \)-theory dual of the full Type IIB 3-brane \([75]\):

\[
ds_{11}^2 = y^{2/3} \left[ H^{-1/3}(dx^\mu dx_\mu + r^{-2}(dz_1^2 + dz_2^2)) + H^{2/3}(dy^2 + d\Sigma_4^2) \right] \\
H = 1 + \frac{1}{4} Q y^4 \\
F_4 = 2Qdz_1 \wedge dz_2 \wedge J + Q \Sigma_4 \tag{2.36}
\]

The interesting observation is that this provides a solution of \( M \)-theory which, according to the transformation rules of \( D = 11 \) supergravity, preserves no supersymmetry. Yet we know in fact that it is BPS because it is just the Type IIB 3-brane in disguise. This is but one example of the more general phenomenon of \( BPS \) without \( BPS \) provided by Hopf duality. This reminds us (if we needed reminding) that there is more to \( M \)-theory than \( D = 11 \) supergravity, and if we knew what the correct equations of \( M \)-theory were we should find that (2.36) is indeed BPS.

One reason these \( M \)-theory duals of Type IIB phenomena are interesting is that, in the AdS/CFT duality, Type IIB supergravity with its Kaluza-Klein excitations is a good approximation for large \( N \), and stringy excitations correspond to CFT operators whose dimensions diverge for as \( N \to \infty \). But since the Hopf \( T \) duality interchanges stringy and Kaluza Klein modes, the \( M \)-theory description may throw light on the finite \( N \) regime.
Many theorists are understandably excited about the AdS/CFT correspondence because of what $M$-theory can teach us about non-perturbative QCD. In my opinion, however, this is, in a sense, a diversion from the really fundamental issue: What is $M$-theory? So my hope is that this will be a two-way process and that superconformal theories will also teach us more about $M$-theory.

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