Detecting gravitomagnetism with rotation of polarization by a gravitational lens

M. Sereno\textsuperscript{1,2,3}\textsuperscript{*}

\textsuperscript{1}Istituto Nazionale di Astrofisica, Osservatorio Astronomico di Capodimonte, Salita Moiariello, 16, 80131 Naples, Italy
\textsuperscript{2}Dipartimento di Scienze Fisiche, Università degli Studi di Napoli ‘Federico II’, Via Cintia, Monte S. Angelo, 80126 Naples, Italy
\textsuperscript{3}Istituto Nazionale di Fisica Nucleare, Sez. Napoli, Via Cintia, Monte S. Angelo, 80126 Naples, Italy

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**ABSTRACT**

We discuss the effects of an isolated gravitational lens on the rotation of the plane of polarization of linearly polarized light rays, so-called gravitational Faraday rotation, in metric theories of gravity. By applying the thin-lens approximation, we derive simple expressions for extended, rotating astrophysical systems deflecting electromagnetic radiation from background sources. Higher-order corrections and the case of a Kerr black hole are also considered to show how the rotation of polarization is a purely gravitomagnetic effect. Prospects for future detection of gravitational Faraday rotation are examined.

**Key words:** black hole physics – gravitational lensing – polarization – relativity.

**1 INTRODUCTION**

Conceivable post-Newtonian theories of gravity, such as general relativity, predict space–time curvature by mass–energy currents relative to other masses. This feature is known as intrinsic gravitomagnetism (Ciufolini & Wheeler 1995).

The occurrence of several phenomena is due to gravitomagnetism. Small test gyroscopes rotate with respect to distant stars in the vicinity of a spinning body due to its rotation. This is called Lense–Thirring precession. Together with test particles and gyroscopes, photons and clocks also probe some gravitomagnetic phenomena. Bending and time delay of electromagnetic waves are two important phenomena predicted by the theories of gravity. Gravitomagnetic effects due to the spin of an astrophysical object might be detected in gravitational lensing events (Sereno 2002; Sereno & Cardone 2002). The rotation of the lens induces particular signatures on both the position and magnification of multiple images of a background source, also perturbing the microlensing-induced amplification curve (Sereno 2003b).

Another well-known gravitational effect due to space–time curvature by mass currents is the rotation of the plane of polarization for linearly polarized light rays. Under geometric optics, a light ray follows a null geodesic regardless of its polarization state and the polarization vector is parallel transported along the ray (Misner, Thorne & Wheeler 1973). Such a gravitational rotation of the plane of polarization in stationary space–times is a gravitational analogue of the electromagnetic Faraday effect (Piran & Safier 1985; Ishihara, Takahashi & Tomimatsu 1988; Nouri-Zonoz 1999), i.e. the rotation that a light ray undergoes when passing through plasma in the presence of a magnetic field. The gravitational effect is known as gravitational Faraday rotation or the Rytov or Skrotskii effect.

From the original investigations in Skrotskii (1957) and Rytov (1938), the effect of a gravitational field on the plane of polarization of electromagnetic waves has been addressed by several authors (Balazs 1958; Plebanski 1960; Godfrey 1970; Connors, Piran & Stark 1980; Su & Mallett 1980; Piran & Safier 1985; Ishihara et al. 1988; Dyer & Shaver 1992; Nouri-Zonoz 1999; Kopeikin & Mashoon 2002; Sereno 2004). The case of a weak gravitational field has deserved particular attention. In Plebanski (1960), the Maxwell equations in the gravitational field of an isolated physical system were solved and the rotation of the plane of polarization around the propagation vector was considered. In Kopeikin & Mashoon (2002), a formula describing the Skrotskii effect for arbitrary translational and rotational motion of gravitating bodies was derived by solving the equations of motion of a light ray in the first post-Minkowskian approximation. In Sereno (2004), the general formula for the angle of rotation of the plane of polarization of a linearly polarized electromagnetic wave in a stationary space–time was re-obtained with a heuristic approach based on Mach’s principle on dragging of inertial frames, without integrating the equations of motion.

The assumption of the existence of a dynamical space–time curvature, as opposed to a flat space–time of special relativity, is shared among different viable theories of gravity, such as general relativity, Brans–Dicke theory and the Rosen bimetric theory. Metric theories of gravity are defined such that (Will 1993; Ciufolini & Wheeler 1995): (i) space–time is a Lorentzian manifold; (ii) the worldlines of test bodies are geodesics; (iii) the equivalence principle holds in the medium strong form. The usual rules for the motion of particles and photons in a given metric still apply, but the metric may be different from that derived from the Einstein field equations.

A comparison among general relativity and other viable theories of gravity can be made with suitable tests on the basis of higher-order corrections and the case of a Kerr black hole are also considered to show how the rotation of polarization is a purely gravitomagnetic effect. Prospects for future detection of gravitational Faraday rotation are examined.
effects, such as intrinsic gravitomagnetism. To date, results from laser-ranged satellites provide only the demonstration provability of the gravitomagnetic field. In 1995–2002, the Lense–Thirring precession, due to the spin of the Earth, was measured by studying the orbital perturbations of the LAGEOS and LAGEOS II satellites (Ciufolini & Pavlis 1998). Its experimental value agrees to within approximately 20 per cent accuracy with the prediction of general relativity. The NASA Gravity Probe-B satellite should improve this measurement to an accuracy of 1 per cent.

In this paper, we discuss the gravitational Faraday rotation by an isolated rotating gravitational deflector within the standard framework of gravitational lensing (Schneider, Ehlers & Falco 1992; Petters, Levine & Wambganss 2001), when the source of radiation and the observer are remote from the source of the gravitational field (lens). As is known, thanks to some well-known approximations, the effect of the spin of the deflector on the lensing potential can be treated in a quite simple way (Sereno 2002). The lens equation can be modified to consider how some observable quantities, such as the position and the amplification of images, are perturbed by the rotation (Sereno & Cardone 2002; Sereno 2003b). Here, we want to extend such a formalism to the Skrotskii effect and to consider some extended lenses of astrophysical interest. Standard assumptions allow us to compute higher-order approximation terms in the calculation, so that a general treatment of the Skrotskii effect can be performed in metric theories of gravity.

The paper is organised as follows. In Section 2, the weak-field, slow-motion approximation in metric theories of gravity is introduced and the weak-field limit of the gravitational Faraday rotation is investigated. In Section 3, we derive some formulae, using the thin-lens approximation, to treat systems of astrophysical interest. Higher-order correction to the Skrotskii effect are discussed in Section 4; then, the case of light rays propagating in the vacuum region outside a Kerr black hole is considered. Section 5 contains some final considerations.

2 THE WEAK-FIELD LIMIT

Standard hypotheses of gravitational lensing (Schneider et al. 1992; Petters et al. 2001) assume that the gravitational lens is localized in a very small region of the sky and its lensing effect is weak. In Sereno (2003a), an approximate metric element generated by an isolated mass distribution was written in the weak-field regime and slow-motion approximation, up to the post-post-Newtonian order, and with non-diagonal components accounting for effects of gravity by currents of mass. In an asymptotically Cartesian coordinate system, the metric \( g_{\alpha\beta} \) can be expressed as

\[
\mathrm{d} s^2 \simeq \left[ 1 + 2 \frac{\phi}{c^2} + \mathcal{O}(\varepsilon^4) \right] c^2 \mathrm{d} t^2 - \left[ 1 - 2 \frac{\phi}{c^2} + \mathcal{O}(\varepsilon^4) \right] \delta_{ij} \mathrm{d} x^i \mathrm{d} x^j - \frac{8\mu}{c^3} \left( V_i \mathrm{d} x^i \right) (c \, \mathrm{d} t),
\]

(1)

where \( \varepsilon \ll 1 \) denotes the order of approximation. In the above metric element, \( \gamma \) is a standard coefficient of the post-Newtonian parametrized expansion of the metric tensor (Will 1993; Ciufolini & Wheeler 1995), measuring space curvature produced by mass. In general relativity, it is \( \gamma = 1 \); in the Brans–Dicke theory, \( \gamma = (1 + \omega)/(2 + \omega) \). \( \mu \) is a non-standard parameter which quantifies the contribution to the space–time curvature of the mass–energy currents. It measures the strength of the intrinsic gravitomagnetic field (Ciufolini & Wheeler 1995). In general relativity, \( \mu = 1 \); in Newtonian theory, \( \mu = 0 \). Additional terms in a parametrized expansion of the metric element can also be considered (Matzner & Richter 1981; Richter & Matzner 1982). Preferred frame effects, violations of conservation of four-momentum, preferred location effects are not considered in our approximate metric element.

In equation (1), \( \phi \) is the Newtonian potential,

\[
\phi(t, x) \simeq -G \int_{\mathbb{R}^3} \frac{\rho(t', x')}{|x - x'|} \, \mathrm{d}^3 x',
\]

(2)

where \( G \) is the gravitational Newton constant and \( \rho \) is the mass density of the deflector; \( \phi/c^2 \) is of order \( \mathcal{O}(\varepsilon^2) \).

\[
\mathcal{V} = \left[ \mathcal{V} + \mathcal{O}(\varepsilon^2) \right]
\]

is a vector potential taking into account the gravitomagnetic field produced by mass currents. To the lowest order of approximation,

\[
\mathcal{V}_l(t, x) \simeq -G \int_{\mathbb{R}^3} \frac{(\mathbf{v} \times \mathbf{r})(t, x')}{|x - x'|} \, \mathrm{d}^3 x',
\]

(3)

where \( \mathbf{v} \) is the velocity field of the mass elements of the deflector.\(^2\)

We assume that, during the time light rays take to traverse the lens, the potentials in equations (2) and (3) vary by a negligible amount. Then, the lens can be treated as being stationary. In equations (2) and (3), we have neglected the retardation (Schneider et al. 1992).

The polarization vector is dragged along by the rotation of the inertial frames. Once we have introduced the notation

\[
h \equiv g_{00}, \quad A_t \equiv -\frac{g_{0t}}{g_{00}},
\]

the net angle of rotation around the unit tangent three-vector \( \mathbf{k} \) along the path between the source and the observer, in a stationary space–time, reads as (Nouri-Zonooz 1999; Sereno 2004)

\[
\Omega_{sk} = -\frac{1}{2} \int_{\Sigma_{obs}} \sqrt{h} \nabla \times \mathbf{A} \cdot \mathrm{d} \mathbf{x},
\]

(5)

where \( \mathrm{d} \mathbf{x} = \mathbf{k} \, \mathrm{d} l_T \) and \( \mathrm{d} l_T \) is the spatial distance in terms of the spatial metric \( g_{ij} \) (Landau & Lifshitz 1985),

\[
\mathrm{d} l_T^2 \equiv -g_{ij} \, \mathrm{d} x^i \, \mathrm{d} x^j \equiv \gamma_{ij} \, \mathrm{d} x^i \, \mathrm{d} x^j.
\]

(6)

Operations on three-vectors are defined in the three-dimensional space with metric \( g_{\alpha\beta} \).

In Dyer & Shaver (1992), it was shown, using symmetry arguments in a relativistic context, that, for astrophysically interesting cases of both the observer and the source being at large distances from the lens, a non-spherical, non-rotating lens cannot cause a net rotation of the polarization vector. This argument is confirmed by equation (5); the Skrotskii effect is due to mass-currents, i.e. it is a purely gravitomagnetic phenomenon.

As can be seen from equation (5), the order of approximation is determined by off-diagonal components of the metric. The main term is of order \( \mathcal{O}(\varepsilon^4) \). To determine a gravitomagnetic effect to the second leading order, i.e. to \( \mathcal{O}(\varepsilon^5) \), we need to consider terms in \( g_{00} \) and \( g_{0i} \) up to \( \mathcal{O}(\varepsilon^4) \). Higher-order terms would give contributions \( \mathcal{O}(\varepsilon^5) \times \mathcal{V} \sim \mathcal{O}(\varepsilon^5) \) to the Faraday rotation, \( \Omega_{sk} \) is at best \( \sim \mathcal{O}(\varepsilon^5) \) unlike the bending of light, which is \( \sim \mathcal{O}(\varepsilon^4) \). That is why the gravitational Faraday rotation is usually neglected in light propagation analyses.

\(^2\) Bold symbols denote spatial vectors. The contravariant components of spatial three-vectors are equal to the spatial components of the corresponding four-vectors.
In the weak-field limit, $h$ and $A$ are simply related to the gravitational potentials, i.e.,

$$h \simeq 1 + 2 \frac{\phi}{c^2} + \mathcal{O}(\varepsilon^4),$$

$$A_i \simeq 4\mu \varepsilon^5 \left( \frac{3}{V_i} + \frac{5}{V_i} - \frac{3}{V_i} \frac{\phi}{c^2} \right) + \mathcal{O}(\varepsilon^7);$$

the proper arc length reads as

$$d_{\rho} \simeq \left[ 1 - \gamma \frac{\phi}{c^2} + \mathcal{O}(\varepsilon^3) \right] d_{E},$$

where $d_{E} \equiv \sqrt{\delta_{ij} dx^{i} dx^{j}}$ is the Euclidean arc length, whereas the spatial metric reduces to

$$\gamma_{ij} = \left[ 1 - 2 \frac{\phi}{c^2} + \mathcal{O}(\varepsilon^3) \right] \delta_{ij}.$$ \hspace{1cm} (9)

To calculate the gravitational Faraday rotation to order $G^N$, we need the path of the deflected light ray to order $G^{N−1}$ (Fishbach & Freeman 1980; Sereno 2003a). Taking the $x^3$-axis along the unperturbed photon path, to the first order in $G$, the unit propagation vector reads $\hat{k}_{(1)} \equiv \{\mathcal{O}(\varepsilon^2), \mathcal{O}(\varepsilon^2), 1 + \gamma \phi/c^2\}$. The Faraday rotation to order $\mathcal{O}(\varepsilon^5)$ turns out to be

$$\Omega_{sk} \simeq -2\frac{\mu}{c^3} \left\{ \int_{\text{line of sight}} \left[ \frac{3}{V_{2,1}} - \frac{3}{V_{1,2}} \right] d_{E} \right. + \left. \int_{\text{line of sight}} \left\{ \frac{3}{V_{2,1}} - \frac{3}{V_{1,2}} \right\} d_{E} \right\} - \int_{\rho} \left\{ \frac{3}{V_{2,1}} - \frac{3}{V_{1,2}} \right\} d_{E} \right\} + \left. \int_{\text{line of sight}} \left( \frac{3}{V_{1,3}} - \frac{3}{V_{3,1}} \right) \hat{k}_{(1)} \right\} d_{E} \right\} - \frac{2}{c^2} \int_{\text{line of sight}} \left\{ \phi_{1} V_{2,1} - \phi_{2} V_{1,2} \right\} d_{E} \right\} + \int_{\text{line of sight}} \left\{ \phi_{1} V_{2,1} - \phi_{2} V_{1,2} \right\} d_{E} \right\} + \mathcal{O}(\varepsilon^7),$$

(11)

where a comma denotes differentiation and $\rho$ is the spatial projection of the null geodesics. The first term on the right-hand side of equation (11) gives the main contribution to the gravitational Faraday rotation (Sereno 2004). Other terms represent higher-order corrections. The second and the third contributions account for the integration along the deflected path $\rho$. The fourth term in equation (11) derives from the difference between the gravitomagnetic potentials $A$ and $V$.

3 THE THIN-LENS APPROXIMATION

In almost all astrophysical systems, the gravitational lens can be treated as being geometrically thin. The extent of the lens in the direction of the incoming ray is small compared with both the distances between lens and observer and lens and source, so that the maximal deviation of the actual ray from a light path propagating through unperturbed space–time is small with respect to the length scale on which the gravitational field changes. In this picture, gravitational effects are quite local to the lens vicinity.

The lens plane is transverse to the incoming, unperturbed light ray direction. The position vector in the lens plane is the spatial vector $\xi \equiv \{x^1, x^2, 0\}$. We introduce the coordinate $l$ along the $x^1$-axis, so that the lens plane corresponds to $l = 0$. The Born approximation assumes that rays of electromagnetic radiation propagate along straight lines. The integration along the line of sight is accurate enough to evaluate the main contribution to the Skrotskii effect. To this order, we will employ the unperturbed Minkowski metric $\eta_{lm} \equiv (1, -1, -1, -1)$ for operations on vectors.

Let us define the weighted average of a quantity $Q$ along the line of sight,

$$\langle Q \rangle_{\text{line of sight}}(\xi) \equiv \frac{\int Q(\xi, l) dl}{\Sigma(\xi)},$$

where $\Sigma$ is the surface mass density of the deflector,

$$\Sigma(\xi) \equiv \int \rho(\xi, l) dl.$$ \hspace{1cm} (13)

The leading contribution to the Skrotskii effect can be written as (Sereno 2004)

$$\int_{\text{line of sight}} \nabla \times \frac{\mathbf{V}}{\Sigma(\xi)} \left|_{\text{line of sight}} \right| d_{E},$$

(14)

Inserting the expression for the gravitomagnetic potential and performing the integration along the line of sight, we obtain

$$\Omega_{sk} = \frac{4\mu}{c^3} \int \frac{\Sigma(\xi)(L_{\text{line of sight}})_{\text{line of sight}}(\xi, l; \xi)}{|\xi - \xi'|} \left|_{\text{line of sight}} \right| d^2\xi',$$

where $L_{\text{line of sight}}$ is the angular momentum, per unit mass, of a mass element at $\xi'$ with respect a photon with impact parameter $\xi$,

$$\langle L_{\text{line of sight}} \rangle_{\text{line of sight}} = \langle v^1 \rangle_{\text{line of sight}}(\xi^2 - \xi'^2) - \langle v^2 \rangle_{\text{line of sight}}(\xi^1 - \xi'^1);$$

$v^1$ and $v^2$ are the components of $v$ along the $\xi^1$- and the $\xi^2$-axes, respectively.

The angular momentum of the lens can determine a direction of rotation for the polarization vector. When its projection along the line of sight is not null, a handness is introduced in the system and some rotation can be caused. Once the deflector is oriented such that $\langle L_{\text{line of sight}} \rangle_{\text{line of sight}} = 0$, the photon moves as in an equatorial plane and a mirror symmetry about the lens plane holds. Such a symmetry cancels out the effect (Dyer & Shaver 1992).

3.1 Spherical lenses

In most of the astrophysical situations, lenses can be approximated with nearly spherical mass density profiles. Let us consider a spherical deflector in rigid rotation (constant angular velocity $\omega$), when $v = \omega \times x$. We limit ourselves to a slow rotation so that the deformation caused by rotation is negligible and the body has a spherical symmetry.

Taking the centre of the source as the spatial origin of a background inertial frame, the weighted average of the rotational velocity along the line of sight turns out to be

$$\langle v \rangle_{\text{line of sight}} = \left\{ \xi^2 \omega^2 - \xi^1 \omega^2, -\xi^2 \omega^1 \omega_{\text{line of sight}}, \xi^1 \omega_{\text{line of sight}} \right\},$$

(17)

We can, now, evaluate the integral in equation (15); the polarization vector of a linearly polarized light ray with impact parameter $|\xi|$ is rotated by

$$\Omega_{sk} = \frac{4\pi GM}{c^3} \int_{\text{line of sight}} \frac{\partial M_{\text{Eyl}}(> |\xi|)}{d\xi},$$

(18)
The gravitational Faraday rotation is proportional to the mass outside $|\xi|$, and can be significant for lenses with slowly decreasing mass density. $\Omega_{sk}$ can be compared with the contribution to bending of light due to the gravitomagnetic field, $\Omega_{GRM}$ (see equations 15 and 16 in Sereno & Cardone 2002). Typically, $\Omega_{sk}$ and $\Omega_{GRM}$ are of the same order. In contrast to the Faraday effect, the gravitomagnetic contribution to the deflection angle only depends on the component of the angular momentum along the $\xi'$-axis, related to the momentum of inertia of the mass within $|\xi|$ about a central axis, contribute only to $\Omega_{GRM}$ (Sereno & Cardone 2002), the contribution due to the mass outside the impact parameter appears in both $\Omega_{sk}$ and $\Omega_{GRM}$.

Let us consider an isothermal sphere (IS). The IS is a density profile widely used to model systems on very different scales, from galaxy haloes to clusters of galaxies. The surface mass density is

$$\Sigma^{IS} = \frac{\sigma^2}{2G} \frac{1}{\sqrt{|\xi|^2 + \xi^2_\perp}},$$

where $\sigma$ is the velocity dispersion and $\xi_\perp$ is a finite core radius. As the total mass is divergent, we introduce a cut-off radius $R \gg |\xi|$. The gravitational Faraday rotation is

$$\Omega^{IS}_{sk}(|\xi|) = \frac{8\pi G \mu}{c^3} \sigma^2 \rho_{\text{line of sight}} \left( \sqrt{R^2 + \xi^2_\perp} - \sqrt{|\xi|^2 + \xi^2_\perp} \right).$$

In particular, in the inner regions ($|\xi| \ll R$), the above equation reduces to

$$\Omega^{IS}_{sk}(|\xi| \ll R) = \frac{8\pi G \mu}{c^3} \sigma^2 \rho_{\text{line of sight}} R.$$  

In the case of lensing of distant quasars by foreground galaxies, images may form inside the galaxy radius. We can model a typical galaxy as a singular ($\xi_\perp = 0$) IS with $\sigma_c \sim 200 \text{ km s}^{-1}$, $R \lesssim 20 \text{ kpc}$ and $J \sim 0.1 \text{ M}_\odot \text{ kpc}^{-1} \text{ s}^{-1}$, as derived from numerical simulations (Vitvitska et al. 2002). From equation (22), we find $\Omega^{IS}_{sk} \simeq 6 \text{ milliarcsec}$.

Let us consider $\Omega^{IS}_{GRM}$ for an isothermal sphere. It is (Sereno & Cardone 2002),

$$\Omega^{IS}_{GRM}(|\xi| \ll R) = \frac{8\pi G \mu}{c^3} \sigma^2 \rho_{\text{line of sight}} \sqrt{(\omega^2)^2 + (\omega^2_\perp)^2} R.$$  

$\Omega^{IS}_{sk}$ and $\Omega^{IS}_{GRM}$ are of the same order but depend on different components of the angular momentum of the lens.

4 HIGHER-ORDER CORRECTIONS

To determine higher-order terms in the gravitational Faraday rotation, it is enough to consider only the monopole term in the gravitoelectric field but we must go over the dipole moment in the gravitomagnetic potential. We now want to discuss higher-order terms in a rotating black hole.

4.1 The Kerr black hole

The Faraday effect in a Kerr metric, in the weak-field limit, has been already discussed by several authors (Ishihara et al. 1988; Nouri-Zonoz 1999). The right order of approximation in the calculation has been ascertained, but a disagreement on the numerical value still persists. Now, we want to apply our formalism to light rays passing through the vacuum region outside a rotating black hole.

For a source at rest at the origin of the coordinates, assuming that the polar axis of the coordinate system coincides with the rotation axis, the weak-field limit of the Kerr metric in Boyer–Lindquist coordinates reads

$$dx^2 \simeq ds^2_{skh} - \frac{4\mu G J}{c^3 r^2} \sin^2 \theta (r \sin \theta) (dr dt),$$

where $ds^2_{skh}$ is the line element for a spherically symmetric solution and where terms of quadratic and higher order in the angular momentum $J$ have been neglected.

The metric in equation (24) can be expressed in an equivalent isotropic form by introducing a new radius variable, $r$, such as

$$r \simeq \rho \left[ 1 + \frac{GM}{c^2 \rho} + \mathcal{O}(\epsilon^5) \right],$$

where $M$ is the total mass of the black hole. Substituting equation (25) in equation (24) and introducing quasi-Minkovskian coordinates related to $(\rho, \theta, \phi)$ by the usual transformation rules, we obtain the weak-field limit of the Kerr metric in the isotropic form. By arbitrarily orientating the angular momentum $J$, we obtain

$$dx^2 \simeq \left[ 1 - \frac{2GM}{c^2 x} \right] (c dt)^2 - \left[ 1 + \frac{2GM}{c^2 x} \right] \delta_{ij} dx^i dx^j - \frac{8\mu}{c} \left( V_i dx^i \right) (c dt),$$

where $x = \sqrt{\delta_{ij} x^i x^j}$ and

$$V \simeq \frac{3}{V} + \frac{5}{V}.$$  

In the above equations and in what follows, operations on vectors are performed using $\hat{n}_{off}$. Equations (27) and (28) are also valid for large distances from an isolated physical system. As,

$$\nabla \times \mathbf{V}(x) = \frac{G J}{x^3},$$

it is easy to verify that, to order $\mathcal{O}(\epsilon^3)$, there is no gravitational Faraday rotation (Plebanski 1960; Godfrey 1970; Kopeikin & Mashoon 2002). Let us consider higher-order terms.

The unit wave-vector, to order $\mathcal{O}(\epsilon^3)$, reads

$$k_1 = \left\{ - \frac{1 + \gamma}{2} R_{sch} \left( 1 + \frac{l}{\sqrt{l^2 + |\xi|^2}} \right) \frac{|\xi|^2}{|\xi^3|},
- \frac{1 + \gamma}{2} R_{sch} \left( 1 + \frac{l}{\sqrt{l^2 + |\xi|^2}} \right) \frac{|\xi|^2}{|\xi^3|} \cdot 1 + \gamma \frac{\phi}{c^2} \right\}.$$  

where $R_{sch} \equiv 2GM/c^2$ is the Schwarzschild radius of the lens. The deflected path to order $G$ is actually the path in a spherically symmetric metric. To this order, the photon path lies in the plane through source, lens and observer.

After lengthy but straightforward calculations, we can perform the integrations in equation (11). We find

$$\Omega^{Kerr}_{sk} \simeq -\mu \frac{\pi}{4} \frac{c^2}{\epsilon} \frac{J_{\text{line of sight}}}{\epsilon^3} + \mathcal{O}(\epsilon^7).$$

3 Our $J_{\text{line of sight}}$ corresponds to $-am \cos \theta_0$ in Ishihara et al. (1988) and Nouri-Zonoz (1999).

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As is already known (Ishihara et al. 1988; Nouri-Zonoz 1999), the net rotation is of order $O(\varepsilon^2)$.

Only the projection of the angular momentum along the line of sight enters the effect. In Nouri-Zonoz (1999), the integration is performed along the unperturbed path. However, contributions from the deflected path cancel out. The parameter $\gamma$, which measures space curvature produced by mass, does not enter into the effect.

5 CONCLUSIONS

We have discussed the theory of the gravitational Faraday rotation in the weak-field limit in a metric theory of gravity. As a result of a gravitomagnetic field, a net rotation of the polarization plane can occur. Unlike the phenomena of bending of light and time delay of electromagnetic waves, gravitomagnetism appears in the leading term of the rotation angle of the plane of polarization.

Useful formulae have been derived for relevant astrophysical systems. By applying the thin-lens approximation, we obtained quite compact expressions for extended spinning gravitational lenses. The gravitational Faraday increases for lenses with a slowly decreasing mass density. The effect of a gravitomagnetic field is of the same order in both bending of light and rotation of the plane of polarization. In contrast to the gravitomagnetic deflection, the Skrotski effect depends only on the component of the angular momentum along the line of sight. Furthermore, whereas the rotation of the deflector can break the cylindrical symmetry of the deflection angle in the lens plane, no azimuthal dependence enters the gravitational Faraday rotation. A non-null projection of the total angular momentum of the lens along the line of sight introduces a handness in the system but no preferred direction in the alignment of the light beam.

To consider higher-order effects, we have considered light rays propagating outside a rotating black hole. As a result of the order of approximation, the effect on light rays propagating in the vacuum region outside the event horizon of a Kerr black hole has often been missed. Applying a method based on the use of the Walker–Penrose constant up to higher-order terms, Ishihara et al. (1988) first calculated the angle of rotation due to the presence of the spin of the black hole. Nouri-Zonoz (1999) used the 1+3 formulation of stationary space–times and the quasi-Maxwell form of the vacuum Einstein equations to study Kerr spaces. The results in Ishihara et al. (1988) and Nouri-Zonoz (1999) are of the same order but differ by a numerical factor. Here, we have properly considered all the contributions to the gravitational Faraday rotation. In particular, the effect of the curved path has been accounted for. The gravitational Faraday rotation depends linearly on $\mu$, the parameter that quantifies and measures the gravitomagnetic effect. Even at a higher order of approximation, $\mu$ is the only parameter that enters the effects. Other parametrized post-Newtonian parameters, such as $\gamma$, do not appear. For $\mu = 1$, our result agrees with Nouri-Zonoz (1999).

Techniques involving the angular relationship between the intrinsic polarization vectors and the morphological structure of extended radio jets have been proposed to probe the mass distribution of foreground lensing galaxies (Kronberg et al. 1991; Kronberg, Dyer & Röser 1996). The method has then been extended in Surpi & Harari (1999) to consider the effect of weak lensing by large-scale structure. The basic principle underlying these investigations is that the polarization vector of the radiation would not be rotated by the gravitational field of the deflector.

Here, we have considered gravitational Faraday rotation using an astrophysically significant lens models, such as an isothermal sphere. The Skrotski effect turns out to be negligible with respect to modern observational uncertainties. Anyway, high-quality data in total flux density, percentage polarization and polarization position angle at radio frequencies already exist for multiple images of some gravitational lensing systems, such as B0218+357 (Biggs et al. 1999). Given the impressive rate of development of technological capabilities, it is not an unreasonable hypothesis that observational evidence may be obtained in the near future.

Furthermore, polarization measurements of the cosmic microwave background radiation should probe very subtle effects, such as primordial gravity wave signals or a remnant from the reionization epoch. A full understanding of the whole of the foreground contamination turns out to be crucial in the data analysis. So, proper modelling and quantification of the gravitational Faraday effect is necessary.

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