A Student’s cognitive dissonance in solving non-routine perimeter problems

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Abstract. Unexplained non-routine problem solving strategies emerging during learning processes encourage students to think deeply about mathematical concepts related to the problems given. In addition, non-routine problems offer a mathematical task never encountered by students before. This article analyses observation results of students’ article are students’ opinions on non-routine problems given, strategies carried out while solving the problems, and accuracies of decisions made to solve the problems. In the study, a qualitative research method comprising analysis of students’ written answers and follow up interviewers is chosen. The subject of the study is a second grade high school students majoring in natural science with an average mathematical achievement, based on her academic records. The results reveal whether relevant cognitive relationships developed by students dissonant.

1. Introduction
Bloom’s Taxonomy revised by Anderson and Kratwohl outlines student cognitive into six aspects, namely remembering, understanding, applying, analyzing, evaluating and creating [1]. Furthermore, those six are classified into low order thinking and high order thinking skills [2]. Low order thinking skills include remembering, understanding, and applying. While high order thinking skills comprise analyzing, evaluating and creating. High order thinking skills can be regarded as the ability to think broadly to find a new challenge [3]. High order thinking skills expect one to apply new information or prior knowledge and manipulate it to reach possible answers in new situations.

Students’ high order thinking skills can be developed, for example, by involving them in problems solving activities. It is possible because problem solving requires mathematical knowledge, strategies, and productive attitudes to respond and resolve problems [4]. Problems given to students in problem solving can be either routine or non-routine. A routine problem can be understood as those that can be solved directly using procedures that have been studied in the classroom. It usually appears in the textbook as an exercise for students to apply procedures that have been learned before. On the other hand, the non-routine problem is something complex so that student requires further thought to solve it. Unexplained non-routine problem solving strategies emerging during learning processes encourage students to think deeply about mathematical concepts related to the problems given. In addition, non-routine problem offers mathematical tasks never encountered by students before. So, it requires students to use heuristic strategies to understand and find solutions [5].

The results of Asman and Markowitz’s research project assert that children are insufficient and not confident in having the aptitudes required for approaching mathematical problems, especially non-
routine ones, in a successful way [6]. In addition, research conducted by Bal and Ersoy concludes that the majority of students have difficulty in solving non-routine problems [7]. Furthermore, Bal and Ersoy also state that the non-routine problem produces individual inconvenience.

Situations that make a person feel any inconvenience will result in cognitive dissonance. This is in accordance with the theory presented by Festinger in 1957. Dissonance can have an impact on the student decision making process while solving the given problems and potentially affects student attitude and the accuracy of the decision made in solving the problem [8]. Festinger revealed that cognitive dissonance theory is part of a relatively new concept called cognition since cognition is part of knowledge a person may have. Cognitive dissonance occurs when two cognitive elements are inconsistent with one another [9]. If two or more elements are related to each other, then the relationship is called a consonant or a dissonant [10]. Relevant cognitive relationships happen when a cognitive element has an impact on other cognitive elements.

A relationship between two cognitive elements is called dissonant if one denies the other. Meanwhile, the relationship between two cognitive elements is called consonant when those two are relevant, not dissonant, and one is followed by the other synchronously [10].

One strategy that can be carried out when solving the non-routine problem is combining two different knowledge or concepts. Therefore, solving a non-routine problem can involve two or more cognitive elements. Based on the previously mentioned definition of cognitive dissonance, it could be inferred that cognitive dissonance may occur in when solving a non-routine problem. In the light of previous research projects revealing that students tend to feel uncomfortable and find difficulties when encountering non-routine problems, the researchers conducted the present study to see the dissonance factor to the dislike, discomfort, and difficulty of students in solving the non-routine problem.

The researchers analyzed the cognitive dissonance of students in solving non-routine problems. The non-routine problem used in the present study is problems related to the perimeter of two-dimensional figures. The purpose of this article is to analyze the cognitive dissonance of students when solving non-routine problems. The results are expected to assist teachers in identifying the dissonance experienced by students and reducing them. Hence, the inconvenience of students in solving non-routine problems can be minimized. For the students knowing what causes the uncomfortable feeling when they solve non-routine problems can help them to overcome it and reduce their cognitive dissonance before solving other non-routine problems.

2. Research Methods

Descriptive research method with a qualitative approach is used in the study. The qualitative research method is used to examine the condition of objects naturally [11]. The object of this study is a student who has cognitive dissonance in solving non-routine problems. Qualitative researchers may collect data from single individuals [11]. The data analysis is inductive and this research project further elaborates on meanings rather than generalizations. It means that the data being analyzed is the actual data which happens to the objects studied which means the data gained deeply so that it uncovers what is visible and invisible.

The Key person of the study is a second-grade high school female student majoring in natural science from one of the private schools in Yogyakarta. She was chosen considering her lack of ability in solving non-routine mathematical problems. She was extrovert and communicative so it’s easy to have an in-depth interview with her. Based on her mathematical ability, she got 75 for cognitive aspect and 75 for skill aspect in the last semester.

In this study, analyze the cognitive dissonance of student focused on three aspects namely, students' responses regarding non-routine problems, students’ strategies in solving non-routine, and the accuracy of decision-making in solving non-routine problems. This analysis revealed students’ dissonant in every aspect which was the focus of this research. In the aspect of student responses regarding non-routine problems, a student have dissonant if she or he feels that the problems are too difficult and she or he is confused by the unknown part. In the aspect of student strategies in solving non-routine problems, the students are said to have dissonant when they use different strategies to
solve questions that have different instructions. For example, a student uses the formula for finding an area to determine the perimeter and student states that there is a different way to determine the perimeter of the regular plane and the irregular plane. In the aspect of decision-making’s accuracy, the student has dissonant when student hesitates about the used strategy and the given answers.

The data in this study were collected through a written test and an interview. The test given was in the form of non-routine problems say which consists of three non-routine problems about determining the perimeter of a plane which can be seen in figure 1, figure 2 and figure 3.

![Figure 1](image1.png)  ![Figure 2](image2.png)  ![Figure 3](image3.png)

**Figure 1.** The image displayed in the non-routine problems number 1.  **Figure 2.** The image displayed in the non-routine problems number 2.  **Figure 3.** The image displayed in the non-routine problems number 3.

From the three figures above, the student is asked to determine the perimeter of each plane. The different of number 1 and number 2 of the non-routine problems is in the length side variables of the plane in problem number 2, the student has to state the plane’s perimeter in an algebra forms. In problem number 3, student is expected to implement Pythagorean concept in determine the length of plane side which unknown before determining its perimeter. The author refers these questions from the junior high school nation exam in 2013/2014.

After the test conducted, the student’s answer sheet was examined to see the sequence of student’s procedure in solving the non-routine problems, the concept that was used in answering the question, and the accuracy of student’s answers in solving non-routine problems. Interviews were conducted to obtain data or further information about the student's answers and opinions about the non-routine problems, student’s opinions regarding the strategies used in solving non-routine problems and the accuracy of decision-making in solving non-routine problems.

Some questions asked in the interview as follows:

**Table 1.** Interview Questions.

| No. | Focused Questions                                      | Questions                                                                 |
|-----|--------------------------------------------------------|---------------------------------------------------------------------------|
| 1   | Student responses regarding non-routine problems       | 1. Are the problems difficult?                                             |
|     |                                                        | 2. Is the perimeter of a plane easy to calculate?                         |
|     |                                                        | 3. Why are those questions difficult?                                      |
| 2   | Strategies in solving non-routine problems             | 1. How do you solve the non-routine problems? (For all questions)         |
|     |                                                        | 2. What is the formula used to solve non-routine problems? (For all questions) |
|     |                                                        | 3. Are the steps used in calculating those planes same with the steps in calculating the perimeter of a square and a rectangle? (Reasons) |
|     |                                                        | 4. Are the steps used in solving problem number 2 same with the steps used for problem number 1? (Reasons) |
|     |                                                        | 5. What are the difference problem number 1 and problem number 2?         |
|     |                                                        | 6. Are the steps used in solving problem number 3 same with the steps used in solving problem number 1 and 2? (Reasons) |
Questions were given to students after solving the non-routine problems. After that, the researchers analyzed the interview result to categorize students' responses to those questions based on the indicators of dissonant students.

3. Results and discussion
The student dissonant in this study was analyzed from student’s works and her opinion regarding non-routine problems given, her opinions about the strategies used to solve non-routine problems, and student’s accuracy in taking the decision in solving non-routine problems. The three-part of the analysis will be described as follows:

3.1. Student responses about the non-routine problems
The first part analyzed from the results is about student responses on the non-routine problems. The student is given the opportunity to observe the non-routine problems before solving them. After she answered the non-routine problems, she was interviewed using questions from Table 1. From the interview, we obtained information that before student solved non-routine problems, she founds no difficulty. This was because the material about plane perimeter has been learnt when the student was in elementary school and also when in junior high school. Furthermore, the student said that to determine the perimeter of a plane, we only have to add all sides of the plane.

There was no problem in early student opinions about non-routine problems. However, after the student completed non-routine problems, she said that the given problems were difficult because there were unknown lengths of the plane sides. The student said that to answer the questions we have to find the unknown value. Problem number 2 was the most difficult of the three problems given. This is because in problem number 2 did not use numbers but letters so it was difficult to determine the perimeter. From the interview conducted with the student, we also knew that student has experienced in solving problem number 3 even there was a difference in length.

The student had two opinions on the non-routine problems given. The first opinion was delivered before student answered the non-routine problems. The student stated that the problems were not difficult. However, the second opinion denied the initial opinion. It shows that the student had dissonant. The student dissonant arose because she found difficulties when solving non-routine problems. Many factors could caused the difficulties. In this study, one of the factors that caused students found difficulties are student understanding of non-routine problems. Student rarely faced non-routine problems so that she was not common in solving them. The student is also accustomed to imitate the steps of solving problems explained by teachers so if there is a slight difference of the new problems given, the student begins to confuse in solving them. Moreover, student’s dissonant about non-routine problems emerged when student confused since there was an unknown part of the problems. Due to this confusion, student refuted her initial opinion which stated that non-routine problems were not difficult.

The student’s dissonant which appeared from the difficulties as described above can be reduced by often giving the student experience in solving non-routine problems. Besides, the student should also be given an opportunity for exploring more problems using her own strategies. The student could also gain experience solving non-routine problems from many references, not only one source.

3.2. Strategies for non-routine problems
The second cognitive dissonance analysis focused on student strategies in solving non-routine problems. The strategies in solving non-routine problems are analyzed for every number of questions. Analysis of non-routine problems strategies of student can be seen from student answer sheets and student interview data. The data obtained will be elaborate as follows.

The student gave 165 cm as the answer of the plane perimeter for problem number 1. The steps that she took to find the answer are shown in Figure 4.

![Figure 4. Student’s Answer for Problem 1.](image)

After examining student answer for problem number 1, the answer was not fit with the rubric. The perimeter of the plane should be 180 cm. From Figure 4 we can see that the student steps in solving the problem is not clear yet so there was a need to conduct an interview to find out what exactly student did.

After conducting the interview, it can be concluded that student used her knowledge about the perimeter. She stated that to determine the perimeter of the first plane has to add all the length of its sides. Thus, student added 15 cm + 15 cm + 15 cm + 30 cm + 40 cm + 50 cm. Of the values that the student added there was 30 cm which was unknown in the problem. So, the researchers asked more how the student came with 30 cm. She said that 30 cm was obtained using the Pythagorean triple. According to her, because there were 40 cm and 50 cm then the length of the other side should be 30 cm. However, when she was asked the reason why using Pythagorean’s theorem, she hesitated whether the shape of the plane was a triangle or not. Then she concluded the plane in problem number 1 was not a triangle but a rectangle which was cut slightly. Although the shape was similar to a rectangle, according to hers the way to determine the perimeter of the shape was different from the perimeter of the rectangle.

The strategy that the student used to determine the perimeter of the plane by adding up all of the side lengths has already been correct. However, the use of the Pythagorean triples in determining the length of the unknown side was incorrect. Thus, the use of the Pythagorean concept was what triggered the dissonant students. In the beginning, she has many doubts about using the strategy of settlement on these numbers. Before being interviewed, the student using the implemented Pythagorean’ concept in finding one of the unknown side lengths. Afterwards, the student changed her reasoning in solving non-routine problem number one.

The student is given the opportunity to answer the same questions again. She is also given the opportunity to observe more deeply about how to determine the perimeter of the plane in question number 1. The student’s worksheet has done for the second time in figure 5 below. The student is asked to observe the plane and the student is allowed to do any method to get the right perimeter.

![Figure 5. Student’s Answer for Problem 1 After Treatment.](image)
The student divides the flat into two parts. So, she get one side of the 25 cm, because she thinks that the unknown side has a length of half of the parallel side which is 50 cm. However, when she is directed to make a helping line so that it resembles the flat build that she often encounters namely a rectangle. She understands that around the flat building can be known by adding 50 cm + 40 cm + 20 cm + 30 cm + 40 cm = 180 cm.

The student answer for problem number 2 was $6ab^2$. The steps that the student took to obtain $6ab^2$ could be seen in figure 6.

![Figure 6. Student’s Answer for Problem 2.](image)

From Figure 6 can be seen that student wrote $3a + 3b + a + b = 6ab^2$. It was similar to what student wrote in figure 1, the step written for problem number 2 was not clear yet. Thus, researchers must figure out how the steps taken by the student can be written like the one on the answer sheet.

Interview carried out revealed information about the steps she took to get the results in Figure 5. The student wrote $3a$ because there were three sides with $a$ length and so were the $b$. After that, she added $3a$ and $3b$ to get $6ab$. However, what she wrote and she said in the interview was different. The student answered $6ab^2$ because she multiplied $6ab$ and $a$. She written answer in the interview was also incompatible with the rubric. The expected answer was $6a + 6b$.

The student is also given the opportunity to answer questions number 2 again. She already understands when answering problem number 2 that student only need to add all the sides with the same method as number 1. The student worksheet for the second time can be seen in Figure 7.

![Figure 7. Student’s Answer for Problem 2 After Treatment.](image)

The student has used the right steps when answering problem number 2. However, she still experiences errors when determining the final result, $12ab$. She still has difficulty to do addition in algebra.

From the results of the written test and interview about problem number 2, it could be seen as dissonant student on algebra operations. The student knew that in algebraic operations there were operations of addition and subtraction. In problem number 2, she was asked to state the perimeter in the form of algebra. This means that student will perform algebraic operations in determining the plane perimeter. However, the student's knowledge about addition and multiplication in algebra does not match with the answers given by the student. Because of this incompatibility caused student’s dissonant and, in the end, student made mistakes in doing algebra operations. To reduce this dissonant, student should enhance her understanding of the algebra operations’s concept.
Figure 8. Student’s Answer for Problem 3.

Figure 8 is a picture of student's answer to problem number 3. The shape of the plane in problem number 3 is different from the previous problems. In problem number 3, to determine the perimeter of a plane, the student must determine one of the unknown sides. The length of the unknown sides can be determined by conducting Pythagorean’s theorem. In the images on problem number 3, there were some given lengths to guide the student in finding the unknown side length.

In the student’s answer sheet shown in Figure 6 can be seen that she wrote 4.8 = 32, 6.8 = 48, and 7.8 = 56. After that, she wrote 32 + 48 + 56 = 136. What she wrote for problem number 3 was still not fit with the rubric. In the rubric the perimeter of the plane was 96 cm, it used the same steps done for problem number 1 and 2. Researchers conducted the interview with her to know the strategies used in solving problem number 3.

From interview, it revealed that how she came up with the answer. She added all sides length and the given lengths to acquire the perimeter for problem number 3. What she wrote in the answer sheet was similar to what she said in the interview session. She also stated that the way to determine the perimeter of the plane in problem number 3 was different from the way of determining the perimeter of other planes because of the different shape.

Figure 9. Student’s Answer for Problem 3.

Figure 9 is a student’s worksheet on question number 3 for the second time. She still make mistakes in solving problem number 3. She has used the Pythagorean’s theorem, but she is confused in determining their hypotheses. From figure 9, she has been able to change the shape of the root. However, she makes the same mistake as problem number 2. She makes a mistake in adding up the root number to determine the perimeter of plane.

Strategies used by student for problem number 3 showed the dissonant in student. Student’s dissonant was revealed from the result of the written test and the student's answers when interviewed. The student simply applied her knowledge of how to determine the perimeter of a plane, which was by adding up all of the lengths of the plane’s sides. Thus, when the student was given the problem like number 3, she directly added all the numbers given on the problems without considering the side length of the given instructions.
In using strategies to solve non-routine problems, student’s dissonant in this study is massive. The student’s dissonant can be seen on any use of strategy in each number of problems. The student’s dissonant on the use of strategies in solving non-routine problems happened for problem number 1 and number 2 since those had the same shape the way to determine the perimeter of those numbers were similar. However, she used a different strategy. She did that because she had dissonant for problem number 1. Thus, there was also a dissonant for the strategy in solving problem number 2. However, the dissonant occurred in using strategies for problem number 2 was not only like what happened in problem number 1, but she had dissonance on another concept namely algebra operations.

3.3. The accuracy of decision making in solving non-routine problems
The second cognitive dissonance analysis focused on student’s strategies in solving non-routine problems. The strategies in solving non-routine problems are analyzed for every number of questions. Analysis of non-routine problems strategies of student can be seen from student’s answer sheets and student’s interview data. The data obtained will be elaborate as follows.

The Student had hesitation in justifying her strategy in solving problems. It can be referred from what the student said about the way to determine the perimeter of all planes in every number was different, according to the form of the shape. However, in the written test, the student stated that to determine the perimeter of the plane for each shape was similar which was by adding up all the lengths of the sides.

The dissonance experienced by the student on the accuracy of this decision making was when the student hesitated whether the procedure to determine the perimeter of the plane was different or similar. Moreover, student also had doubt in using strategy for problem number 1. In addition, at the time of being interview student asked to change the answer because she was not sure with her works in the answer sheet.

4. Conclusion
Cognitive dissonance can be described as a confusing condition that occurs in a person. Such a confusing condition can occur if there is a mathematical concept which used to answer mathematical problems is denied by other mathematical concepts. In addition, a confusing condition, cognitive dissonance can also be described as an uncomfortable condition. An uncomfortable condition happens when the student's opinions of an easy mathematical problem are denied when student tries to solve the problem. It happened in this research. After discussing in the previous section, there are somethings that can be concluded as follows:

- The non-routine problems led to students dissonant
- The instruction to use strategies in solving non-routine problems caused dissonant students
- Student’s dissonant occured on the accuracy of decision making in solving non-routine problems
- Dissonant student happened when there was a lack of understanding concepts to solve non-routine problems.
- A student who has average mathematics ability in study has more than one dissonant when solving non-routine problem.

5. References
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