Test planning and evaluation of maintainability indices of engineering products

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Abstract. One of the most important issues of maintainability of engineering products is an experimental evaluation of quantitative maintainability indices [1-5]. It is necessary to plan maintenance and repair tests of products to obtain estimates. The analytical dependencies to determine amount of check and determinative tests and value of specified maintenance time for the cases of normal and exponential distribution laws of maintainability indices have been derived in the article. The numerical examples of calculation of amount of tests and evaluation of maintainability indices of engineering products have been represented.

1. Introduction
One of the most important issues of maintainability of engineering products is an experimental evaluation of quantitative maintainability indices [1-5]. It is necessary to plan maintenance and repair tests of products to obtain estimates. Test planning consists in determining conditions of their performance and control [6-10]. Conditions of test performance are based either on real operation of products or on special tests, which are carried out to determine estimates of maintainability indices. If performance conditions of special tests differ from conditions of real operation of products, then testing of a hypothesis of homogeneity of information about derived estimates must perform with the help of significance (goodness-of-fit) tests. Maintainability characteristics can be described by various distribution laws depending on kind of maintenance or repair, consequently, peculiar formulas of quantitative evaluation of maintainability of a product can be derived for each law.

2. Problem statement
The task consists in deriving analytical dependencies to determine amount of check and determinative tests and value of maintenance specified time for the cases of normal and exponential distribution of maintainability indices.

3. Theory
Let us consider simple single sampling inspection of maintainability characteristics of engineering products. Test planning is performed according to acceptance $x_{acc}$ and rejectable $x_{rej}$ levels of maintainability index. Acceptance level is lower than rejectable one for some indices, for example, maintenance or repair time, i. e., $x_{acc} < x_{rej}$.

Single sampling inspection of maintainability index $x$ can be described by equations [4]:

$$x_{acc} \quad \text{and} \quad x_{rej}$$
where $\bar{X}$ means the mean value, which is obtained from the observation results;

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} x_i;$$

$x_i$ means the result of the $i$-th observation from the set of observations $n$; $X_{req}$ means the required value of index; $\alpha$ means the producer risk, i.e. the probability to evaluate a product negatively according to the index under consideration; $\beta$ means the consumer risk, i.e. the probability to evaluate a product positively according to the index under consideration.

Formulas (1) are derived from operating characteristic of single sampling plan of inspection of maintainability index (figure 1). Operating characteristic $P(x)$ is a dependence of probability of positive estimate of a product according to maintainability index under consideration from value of characteristic $x$, which is reached when amount of observations equals $n$ [4]. Then equations (1) have the form of [12]:

$$\alpha = 1 - P(X_{acc}),$$
$$\beta = P(X_{req}).$$

**Figure 1.** Operating characteristic.

Analytical form of dependencies (2) depends on distribution law of maintainability characteristic. Values of amount of observations $n$ and acceptability constant $X_{req}$ can be found from ratios (1) when values of the quantities $x_{acc}$, $x_{req}$, $\alpha$ and $\beta$ are set.

Let us consider single sampling plan of control of maintainability indices for normal and exponential distribution laws of random value.

Maintainability indices of midlife and overhaul repair and also high capacity maintenance indices comply with normal distribution law in most cases. Maintainability indices of daily, weekly, monthly maintenances and also of routine repair comply with exponential distribution law in most cases. Maintainability characteristics can be described by logarithmic-normal law, gamma distribution, Weibull distribution in some cases.

Let us consider the case of normal distribution law of maintainability indices.

Normal law is two-parameter distribution with parameters $\bar{x}$ (mathematical expectation) and dispersion $\sigma^2(x)$. When tests are planned, dispersion is as a rule unknown. Parameter $\nu = \sigma(x) / \bar{x}$ is more often known in practice of test planning. It is variation coefficient, which usually takes on values from 0.1 to 0.4. In this case operating characteristic is determined by formula [12]
\[ P(x) = 1 - F \left( \frac{x - x_{acc}}{\bar{x} \sigma(x)} \right) \]

or

\[ P(x) = 1 - F \left( \frac{(x - x_{req}) \sqrt{n}}{\bar{x} \sigma(x)} \right) \]

where \( \sigma(x) \) means the mean square deviation; \( F(x) \) means the normalized distribution function.

By using ratios (1) and the condition that \( x_{acc} < x_{req} \), let us write:

\[ \alpha = 1 - F \left( \frac{(x_{req} - x_{acc}) \sqrt{n}}{\bar{x} \sigma(x)} \right) \] \hspace{1cm} (3)

\[ \beta = 1 - F \left( \frac{(x_{req} - x_{acc}) \sqrt{n}}{\bar{x} \sigma(x)} \right) \] \hspace{1cm} (4)

Values \( n \) and \( x_{req} \) are determined from formulas (3) and (4) by formulas:

\[ \sqrt{n} = \frac{\nu(x_{acc} - x_{req})}{\sqrt{x_{acc}}} \] \hspace{1cm} (5)

\[ x_{req} = x_{acc} \left( 1 + \frac{\nu}{\sqrt{n}} U_{1-\alpha} \right) \] \hspace{1cm} (6)

\[ x_{req} = x_{req} \left( 1 - \frac{\nu}{\sqrt{n}} U_{1-\beta} \right) \]

where \( U_{1-\alpha} \) and \( U_{1-\beta} \) are the quantiles of normal distribution for probabilities \( 1 - \alpha \) and \( 1 - \beta \).

These quantiles are determined according to table 1 “Values of normal distribution function” of appendix [4].

Values of amounts of check tests \( n \) at \( \alpha = \beta = 0.1 \) and various values of \( \nu = (0.05 - 0.3) \), which were borrowed from [12], are represented in table 1.

**Table 1. Values of amounts of check tests.**

| \( x_{req} \) | \( x_{acc} \) | Values of \( \nu \) | \( \delta_{req} \) | Values of \( \nu \) |
|---------------|---------------|----------------|----------------|----------------|
| \( 0.05 \)   | \( 0.10 \)   | \( 0.15 \)   | \( 0.20 \)   | \( 0.25 \)   |
| 1.1           | 2             | 8             | 17             | 29             | 46             | 66             | 1.5           | –             | –             | –             | –             | 2             |
| 1.2           | 1             | 2             | 5              | 8              | 13             | 18             | 1.6           | –             | –             | –             | –             | 2             |
| 1.3           | –             | 1             | 3              | 4              | 7              | 9              | 1.7           | –             | –             | –             | –             | 2             |
| 1.4           | –             | –             | 2              | 3              | 4              | 6              | 1.8           | –             | –             | –             | –             | –             |

Let us consider the case of exponential distribution law of maintainability indices.

In the case of exponential law distribution density of maintainability index is determined by formula

\[ f(x) = \frac{1}{\bar{x}} \exp \left( -\frac{x}{\bar{x}} \right) = \lambda e^{-\lambda x}, \]

where \( \lambda = 1/\bar{x} \) means the distribution parameter.
Operating characteristic $P(x)$ for this law is distribution $\chi^2$ of random value $2 \sum_{i=1}^{n} \frac{x_i}{\bar{x}}$ with number of degrees of freedom $f = 2n$. Let us use ratios [13] to determine amount of tests $n$ and acceptability constant $t_{m,req}$ on the assumption that $x_{acc} < x_{rej}$:

$$2 \cdot \sum_{i=1}^{n} \frac{x_i}{x_{acc}} = \chi^2_{1-\alpha}(2n); \quad 2 \cdot \sum_{i=1}^{n} \frac{x_i}{x_{rej}} = \chi^2_{\beta}(2n).$$

Whence follows

$$\frac{\chi^2_{1-\alpha}(2n)}{\chi^2_{\beta}(2n)} \leq \frac{x_{rej}}{x_{acc}}. \quad (7)$$

As a result of solving equation (7) minimal amount of tests $n$, at which inequality holds true, can be determined.

In this case value of acceptability constant $t_{m,req}$ can be determined by one of the following ratios:

$$t_{m,req} = \begin{cases} \frac{x_{acc}\chi^2_{1-\alpha}(2n)}{2n} \\ \frac{x_{rej}\chi^2_{\beta}(2n)}{2n} \end{cases} \quad (8)$$

In formulas (7) and (8) $\chi^2_{1-\alpha}(2n)$ and $\chi^2_{\beta}(2n)$ are the quantiles of $\chi^2$-distribution for probabilities $1-\alpha$ and $\beta$ and number of degrees of freedom $f = 2n$.

Let us use formula (5) to find amount of tests at single sampling plan. As variation coefficient equals one at exponential distribution $v = 1$, then we shall get

$$n = \frac{(U_{1-\alpha x_{acc}} + U_{1-\beta x_{rej}})^2}{(x_{rej} - x_{acc})^2}. \quad (9)$$

Values of amounts of check tests $n = f(x_{rej} / x_{acc}, \alpha, \beta)$ for some values $\alpha$, $\beta$ and ratios $x_{rej} / x_{acc}$, which were borrowed from [12], are represented in table 2.

| $\alpha$ | 0.05 | 0.10 | 0.20 | 0.05 | 0.10 | 0.20 | 0.05 | 0.10 | 0.20 |
|---|---|---|---|---|---|---|---|---|---|
| $\beta$ | 0.05 | 0.10 | 0.20 | 0.05 | 0.10 | 0.20 | 0.05 | 0.10 | 0.20 |
| $x_{rej}$ | 1.5 | 70 | 50 | 1.0 | 2.0 | 1.0 | 2.0 | 1.0 | 2.0 |
| $x_{acc}$ | 2.0 | 22 | 18 | 1.4 | 17 | 13 | 10 | 12 | 8 |
| 5.0 | 4 | 3 | 2 | 2 | 2 | 1 | 1 | - | - |

Value of ratio $x_{rej} / x_{acc} \geq 1.5$ is usually accepted when maintainability is checked.

Planning of determinative tests as well as check ones is based on a hypothesis of distribution law of maintainability characteristic being evaluated. Necessary amount of tests can be determined when distribution law of maintainability index $x$ is known from equation [4]

$$P(x_i \leq \bar{x} \leq x_n) = 1 - \alpha, \quad (10)$$

where $\bar{x}$ means the mean value of quantity; $x_i$ and $x_n$ mean its low and high confidence bounds; $(1-\alpha)$ means the confidence probability.
Let us determine amount of determinative tests for the case of normal distribution law of maintainability index.

Mathematical model of determinative tests (10) has the form

\[ P\left\{ (x - \bar{x}) \leq \varepsilon \right\} = 1 - \alpha, \]

where \( \bar{x} \) means the mean value of random value of maintainability index; \( x \) means the current value of random value of maintainability index; \( \varepsilon \) means the absolute error of determining the value \( x \).

The value of absolute error \( \varepsilon \) is determined by formula [12]

\[ \varepsilon = t_\alpha(f) \frac{S(x)}{\sqrt{n}}, \tag{11} \]

where \( t_\alpha(f) \) means the quantile of Student distribution for probability \( \alpha \) and number of degrees of freedom \( f = n - 1 \); \( S(x) \) means the mean square deviation.

\[ S(x) = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2, \]

\[ \bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i. \]

Thus, amount of tests can be determined by setting value \( \varepsilon \) and \( 1 - \alpha \) by formula (11)

\[ n = \left( \frac{t_\alpha(f)S(x)}{\varepsilon} \right)^2, \]

Relative error is often used instead absolute error when amounts of tests are determined

\[ \delta = \frac{\varepsilon}{\bar{x}} = \frac{t_\alpha(f)S(x)}{\bar{x} \sqrt{n}}. \]

In this case amount of tests is found by formula

\[ n = \left( \frac{t_\alpha(f)S(x)}{\delta \bar{x}} \right)^2, \tag{12} \]

As \( S(x) / \bar{x} = \nu \) is the variation coefficient, then equation (12) can be written in such a way

\[ n = \frac{t_\alpha^2(f)\nu^2}{\delta^2} \]

or

\[ \frac{\delta}{\nu} = \frac{t_\alpha(f)}{\sqrt{n}}. \]

Consequently, amount of tests is determined according to the tables for \( t_\alpha(f) / \sqrt{n} \) after setting values of \( \delta, \alpha, \nu \). Such tables are in a number of handbooks of statistical check methods.

Formula (12) is the most comfortable to use when tests are planned, as it allows to stop or continue tests according to the results of tests depending on obtained variation coefficient. If obtained statistical value of variation coefficient will be greater than the set one, than it is necessary to continue tests as long as value of variation coefficient equals or is less than the set one. Value \( \delta / \nu \leq 0.3 \) is the most acceptable for determinative tests.
Values of amounts of determinative tests \( n = f(\delta/v, \alpha) \) for some values \( \delta/v \) and \( \alpha \), which were borrowed from [12], are represented in table 3.

| \( \alpha \) | 0.10 | 0.05 |
| --- | --- | --- |
| 1.0 | 4 | 6 |
| 0.9 | 5 | 7 |
| 0.8 | 6 | 8 |
| 0.7 | 7 | 10 |
| 0.6 | 9 | 13 |
| 0.5 | 10 | 23 |
| 0.4 | 19 | 45 |
| 0.3 | 32 | 98 |
| 0.2 | 70 | 388 |
| 0.1 | 272 |

Let us determine amount of determinative tests for the case of exponential distribution law of maintainability index.

In this case mathematical model (10) has the form

\[
P\left\{ \frac{2S}{\chi_{\alpha/2}^2(f)} < \bar{x} < \frac{2S}{\chi_{\alpha}^2(f)} \right\} = 1 - \alpha ,
\]

where \( S \) means the total observation result \( S = \sum_{i=1}^{n} x_i \) ; \( f = 2n \) means the number of degrees of freedom of \( \chi^2 \)-distribution; \( \chi_{\alpha/2}^2(f) \) , \( \chi_{\alpha}^2(f) \) mean the quantiles of \( \chi^2 \)-distribution with the number of degrees of freedom \( f = 2n \) , which correspond with probabilities \( \alpha / 2 \) and \( 1 - \alpha / 2 \).

From formula (13) low and high confidence bounds equals respectively:

\[
x_{l} = \frac{2S}{\chi_{\alpha}^2(f)} ; \quad x_{h} = \frac{2S}{\chi_{\alpha/2}^2(f)} ,
\]

from whence

\[
\frac{x_{h}}{x_{l}} = \frac{\chi_{\alpha/2}^2}{\chi_{\alpha}^2}.
\]

It is possible to determine unknown value of amount of tests \( n \) by using tables of \( \chi^2 \)-distribution by formula (14) when accuracy of determining maintainability index \( x \) is set in the form of ratio \( x_{h} / x_{l} \) and value of \( \alpha \) is set.

Ratio \( x_{h} / x_{l} \leq 2 \) is recommended when determinative tests are planned. If this ratio exceeds some set value according to the results of tests, than tests should be continued until required value of accuracy will be reached.

Values of amounts of determinative tests \( n \) for some values of ratio \( x_{h} / x_{l} \) and \( \alpha \) are represented in table 4.

| \( \alpha \) | 1.2 | 1.5 | 1.75 | 2.0 | 2.5 | 3.0 | 4.0 | 5.0 |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0.10 | 370 | 66 | 36 | 23 | 13 | 9 | 6 | 4 |
| 0.05 | 495 | 95 | 50 | 33 | 19 | 13 | 8 | 6 |

4. Experiment results
It is necessary to determine amount of check tests and specified value of maintenance time when complex of engineering products being in operation is maintained. It is known that specified time has exponential distribution and the following initial data are known:

\[ t_{m,\alpha} = 100 \text{ hours}, \quad t_{m,\beta} = 120 \text{ hours}, \quad \nu = 0.2, \quad \alpha = \beta = 0.1. \]

We find according to table 1 of appendix [4]: \( U_{1-\alpha} = U_{1-\beta} = 1.282 \). Let us use formula (4) to determine amount of tests

\[
\sqrt{n} = \frac{\nu(t_{m,\alpha} + t_{m,\beta})}{t_{m,\beta} - t_{m,\alpha}} = \frac{0.2 \cdot (100 \cdot 1.282 + 120 \cdot 1.282)}{120 - 100} = 2.82.
\]

Hence \( n = 8 \).

We will calculate value of specified time by formula (5)

\[
t_{m,\text{req}} = t_{m,\alpha} \left(1 + \frac{\nu}{\sqrt{n}} U_{1-\alpha}\right) = 100 \left(1 + \frac{0.2 \cdot 1.282}{2.82}\right) = 109 \text{ hours}.
\]

Thus, if it was obtained according the observation results that mean maintenance time is less than or equals maintenance specified time, then a product corresponds with specified maintainability requirements to an index under consideration, i.e.

\[
\bar{t}_n = \frac{1}{n} \sum_{i=1}^{n} t_{m,i} \leq 109 \text{ hours}.
\]

Let us determine amount of check tests and specified value of maintenance time if it is known that specified time has exponential distribution and the following initial data are known: \( t_{\text{acc}} = 100 \text{ hours}, \quad t_{\text{rej}} = 120 \text{ hours}, \quad \alpha = \beta = 0.1 \).

We find according to table 1 of appendix [4]: \( U_{1-\alpha} = U_{1-\beta} = 1.282 \), then we determine amount of tests by formula (8)

\[
n = \frac{(U_{1-\alpha} t_{\text{acc}} + U_{1-\beta} t_{\text{rej}})^2}{(t_{\text{rej}} - t_{\text{acc}})^2} = \frac{(1.282 \cdot 100 + 1.282 \cdot 120)^2}{(150 - 100)^2} = 197.
\]

As number of degrees of freedom \( n = 197 \), then value of quantile \( \chi^2_{a} \) is determined by formula [14] approximately. When \( n > 30 \)

\[
\chi^2_{a} \approx 0.5 \cdot (U_{1-\alpha} \pm \sqrt{2k-1})^2,
\]

where \( (U_{1-\alpha}) \) means the quantile of normal distribution, which is determined according to table 1 of appendix [4].

In this case number of degrees of freedom \( f = 2(2n) = 798 \).

\[
\chi^2_{a,9} \approx 0.5 \cdot (1.282 + \sqrt{798 - 1})^2 \approx 433.
\]

Then by substituting the initial data in one of formulas (7), we will find value of specified maintenance time

\[
t_{m,\text{req}} = \frac{t_{\text{acc}} \chi^2_{a,9} (2n)}{2(2n)} = \frac{100 \cdot 433}{2 \cdot 197} = 109 \text{ hours}.
\]

Thus, at the same initial data value of specified time is the same in comparison with normal distribution, while number of observations in the case of exponential law is greater considerably: \( n = 197 >> 8 \).
As in the previous example, a product is evaluated positively on maintainability index, if inequality
\[ \bar{t} = \frac{1}{n} \sum_{i=1}^{n} t_{i,m} \leq 109 \text{ hours} \] holds true according to the results of tests.

Let us determine amount of determinative tests to determine maintenance characteristic at the following initial data: \( \alpha = 0.05; \delta/v = 0.2 \). According to table 3 for \( \alpha = 0.05; \delta/v = 0.2; n = 98 \).

Let maintenance requirement be determined in technical specifications in the form of the following initial data: \( x_2 / x_1 = 1.35 \), \( \alpha = 0.10 \). Let us determine necessary amount of determinative tests for set accuracy and confidence.

We choose such values of quantiles according to the tables of \( \chi^2 \)-distribution for \( \alpha = 0.05 \) and \( 1 - \alpha = 0.95 \), at which their ratio will be less than or equals 1.35. In this case we obtain for \( f = 2n = 240 \):

\[ x_{2,0.95}^2 (240) = 205.03, \quad x_{1,0.05}^2 (240) = 276.83. \]

Hence \( \frac{x_{2,0.95}^2 (240)}{x_{1,0.05}^2 (240)} = \frac{276.83}{205.03} = 1.35. \)

Thus, amount of tests equals \( n = 120 \).

5. Conclusion
1) The analytical dependencies to determine amount of check and determinative tests and value of specified maintenance time for the cases of normal and exponential distribution laws of maintainability indices have been derived.
2) The numerical examples of calculation of amount of check and determinative tests and evaluation of maintainability indices of engineering products for the cases of normal and exponential distribution of specified time value have been represented.

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