Effects of hybridization and spin–orbit coupling to induce odd-frequency pairing in two-band superconductors

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Abstract The effect of spin-independent hybridization potential and spin–orbit coupling on two-band superconductor with equal-time s-wave interband pairing order parameter is investigated theoretically. To study symmetry classes in two-band superconductors, the Gor’kov equations are solved analytically. By defining spin-singlet and spin-triplet s-wave order parameter due to two-band degree of freedom, the symmetry classes of Cooper pair are studied. For spin singlet case, it is shown that spin-independent hybridization generates Cooper pair that belongs to even-frequency spin singlet even-momentum even-band parity (ESEE) symmetry class for both intraband and interband pairing correlations. For spin-triplet order parameter, intraband pairing correlation generates odd-frequency spin triplet even-momentum even-band parity (ETOT) symmetry class, whereas interband pairing correlation generates even-frequency spin triplet odd-momentum odd-band parity (ETOE) class. For the spin singlet, spin–orbit coupling generates pairing correlation that belongs to odd-frequency spin-singlet odd-momentum even-band parity (OSOE) and even-frequency spin-singlet even-momentum even-band parity (ESEE) for intraband and interband pairing correlation, respectively. In the spin-triplet case for intraband and interband correlation, spin–orbit coupling generates even-frequency spin-triplet odd-momentum even-band parity (ETOE) and even-frequency spin-triplet odd-momentum odd-band parity (ETOE), respectively. Our results are in agreement with previous experimental and theoretical studies.

1 Introduction

Symmetries of order parameter in superconductors affect their physical properties. The total wave function of a pair of fermions, in accordance with the Pauli principle, should be asymmetric under the permutation of orbital, spin and time (or equivalently Matsubara frequency) coordinates [1]. This leads to four classes that allowed combinations for the symmetries of the wave function. This would imply that if the pairing is even in time, spin-singlet pairs have even parity (ESE) and spin-triplet pairs have odd parity (ETO), while if the pairing is odd in time, spin-singlet pairs have odd parity (OSO) and spin-triplet pairs have even parity (OTE). Black-Schaffer and Balatsky [2] have shown that the multiband superconducting...
order parameter has an extra symmetry classification that originates from the band degree of freedom, so-called even-band parity and odd-band parity. As a consequence, Cooper pairs can be classified into eight symmetry classes [3].

Transport properties of multi-band superconductor are qualitatively different from those of the one-band superconductor. For instance, two-band system with the non-magnetic impurity violates Anderson theorem [4]. As a result, lots of efforts have been devoted to understanding the properties of such systems both theoretically and experimentally. For these materials, band symmetry plays an important role. A main hypothesis of the model is the formation of the Cooper pairs inside one energy band and transition of this pair from one band to another which leads to intra- and interband electronic interactions. Multi-band model explained lots of strange physical properties of superconductive systems and were consistent with experimental data. Famous multiband superconductors are $\text{MgB}_2$ [5,6] and the iron-based superconductors [7–9]. The nature of their two bands requires that the multiband approach can be used to describe their properties. On the contrary for cuprates despite their multiband nature, a single-band approach is more appropriate.

From a general symmetry analysis of even- and odd-frequency pairing states, it was shown that odd-frequency pairing always exists in the form of odd-interband (orbital) pairing if there is any even-frequency even-interband pairing present consistent with the general symmetry requirements [10]. The appearance of odd-frequency Cooper pairs in two-band superconductors by solving the Gor’kov equation was discussed analytically [11]. They considered the equal-time $s$-wave pair potential and introduced two types of hybridization potentials between the two conduction bands. One is a spin-independent hybridization potential, and the other is a spin-dependent hybridization potential derived from the spin–orbit interaction.

The effect of random nonmagnetic impurities on the superconducting transition temperature in a two-band superconductor, by assuming the equal-time spin-singlet $s$-wave pair potential in each conduction band and the hybridization between the two bands as well as the band asymmetry, was studied theoretically [11,12]. The effect of single-quasiparticle hybridization or scattering in a two-band superconductor by performing perturbation theory to infinite order in the hybridization term in a multiband superconductor was investigated [13].

The superconducting state of multi-orbital spin–orbit coupled systems in the presence of an orbitally driven inversion asymmetry, by assuming that the interorbital attraction is the dominant pairing channel, was studied [14]. They have shown that in the absence of the inversion symmetry, superconducting states that avoid mixing of spin-triplet and spin-singlet configurations are allowed, and remarkably, spin-triplet states that are topologically nontrivial can be stabilized in a large portion of the phase diagram. The impact of strong spin–orbit coupling (SOC) on the properties of new class superconductors has attracted much attentions. It has been the subject of great theoretical and experimental interest [15,16]. The formation of unexpected multi-component superconductors states allows for superconductors with magnetism and SOC. It was shown that for multi-orbital systems such as the Fe-pnictides SOC coupling is much smaller than the orbit Hund’s coupling [17–19]. In contrast for multiband systems such as Ir-based oxide materials, it was found that the SOC interaction is comparable to the on-site Coulomb interaction [20]. The combined effect of Hund’s and SOC coupling on superconductivity in multi-orbital systems was investigated and it was shown that Hund’s interaction leads to orbital-singlet spin-triplet superconductivity, where the Cooper pair wave function is antisymmetric under the exchange of two orbitals [21]. Combined effect of the spin–orbit coupling and scattering on the nonmagnetic disorder on the formation of the spin resonance peak in iron-based superconductors was also studied [22].
2 Two-band model

The basic physics of multiband superconductors can be obtained by introducing a two-band model. We start with a normal two-band Hamiltonian as \[12\]

\[
\hat{H}_N = \int dr \begin{bmatrix}
\psi_{1,\uparrow}^+(r), & \psi_{1,\downarrow}^+(r), & \psi_{2,\uparrow}^+(r), & \psi_{2,\downarrow}^+(r)
\end{bmatrix} \hat{H}_N(r) \begin{bmatrix}
\psi_{1,\uparrow}(r) \\
\psi_{1,\downarrow}(r) \\
\psi_{2,\uparrow}(r) \\
\psi_{2,\downarrow}(r)
\end{bmatrix},
\]

where

\[
\hat{H}_N = \begin{pmatrix}
\xi_{1k} \hat{\sigma}_0 & (v e^{i\theta} + V) \hat{\sigma}_0 \\
(v e^{-i\theta} + V^*) \hat{\sigma}_0 & \xi_{2k} \hat{\sigma}_0
\end{pmatrix}.
\]

Here \(\psi_{\alpha,\sigma}(r)\) is the annihilation (creation) operator of an electron with spin \((\sigma = \uparrow, \downarrow)\) at the \(\alpha\)th conduction band, \(\xi_{\alpha k} = \hbar^2 k^2 / 2 m_e - \mu_F\) is the dispersion energy of band \(\alpha\), \(m_e\) is the mass of an electron, and \(\mu_F\) is the chemical potential. The spin-independent hybridization potential is a complex number characterized by a phase \(\theta\). \(v e^{i\theta}\) denotes the hybridization between the two bands, which is much smaller than the Fermi energy in the two conduction bands. In the absence of spin flip hybridization, the spin–orbit coupling potential is \(V(k) = \eta \hat{\sigma}_j (\sigma \times \mathbf{k}) = \eta (k_x \sigma_x - k_y \sigma_y)\), where \(\eta\) is the parameter that describes the strength of the Rashba spin–orbit coupling and \(\hat{\sigma}_j\) is the unit vector perpendicular to the superconductor surface. This potential is odd-momentum parity functions satisfying \(V(k) = -V(-k)\). Throughout this paper, Pauli matrices in spin, two-band, particle–hole spaces are respectively denoted by \(\hat{\sigma}_j\), \(\hat{\rho}_j\) and \(\hat{\tau}_j\) for \(j = 1 \sim 3\). Superconducting order parameter in band \(\alpha\) is:

\[
\hat{\Delta}_{\alpha\alpha'}(k) = \begin{pmatrix}
\Delta_{11}(k) & \Delta_{12}(k) \\
\Delta_{21}(k) & \Delta_{22}(k)
\end{pmatrix}.
\]

We focus only on interband superconducting order parameter \((\Delta_{11}(k) = \Delta_{22}(k) = 0)\). The interband s-wave pair potential is defined by \[12\]

\[
\Delta_{12,\sigma\sigma'}(r) = g \langle \psi_{1,\sigma}(r) \psi_{2,\sigma'}(r) \rangle.
\]

Here \(g\) is interband attractive interaction between two electrons. By assuming the spatially uniform order parameter, the Fourier transformation of the pair potential becomes

\[
\Delta_{12,\uparrow\downarrow} = \frac{g}{V_{vol}} \sum_k \langle \psi_{1,\uparrow}(k) \psi_{2,\downarrow}(-k) \rangle.
\]

In the two-band model, for spin singlet the order parameter is symmetric (antisymmetric) under the permutation of band (spin) indices

\[
\Delta_{12,\uparrow\downarrow} = \Delta_{21;\uparrow\downarrow} = -\Delta_{12;\downarrow\uparrow},
\]

But for spin triplet the order parameter is antisymmetric (symmetric) under the permutation of band (spin) indices

\[
\Delta_{12,\uparrow\downarrow} = -\Delta_{21;\uparrow\downarrow} = \Delta_{12;\downarrow\uparrow}.
\]
For simplicity we omit the indices of $\Delta_{\alpha\alpha'}$. The Hamiltonian describing superconductor in the Nambu space can be written as \[11\]
\[
\hat{H}_{S(T)} = \frac{1}{2} \sum_{k} \psi_{k,\sigma}^\dagger \left( \hat{H}_N(k) + \hat{\Delta}_{S(T)} \right) \psi_{k,\sigma},
\]
where the spin-singlet and spin-triplet pair potentials ($\hat{\Delta}_S$ and $\hat{\Delta}_T$) are, respectively, given by
\[
\hat{\Delta}_S = \Delta \hat{\rho}_1 \hat{\sigma}_2,
\]
\[
\hat{\Delta}_T = \Delta \hat{\rho}_2 \hat{\sigma}_1.
\]
For a two-band system, the Bogoliubov–de Gennes Hamiltonian can be described by an $8 \times 8$ matrix reflecting spin, particle–hole and two-band degrees of freedom. In particle–hole space $N_1$, we consider the spin of electron as $\uparrow$ and for hole as $\downarrow$, while in particle–hole space $N_2$, we consider the spin of electron as $\downarrow$ and for hole as $\uparrow$; we can describe the Hamiltonian $\hat{H}_0$ by a $4 \times 4$ matrix \[3,12\]
\[
\hat{H}_0 = \begin{pmatrix}
\xi_k & ve^{i\theta} + V(k) & 0 & \Delta \\
ve^{-i\theta} + V^*(k) & \xi_k & -s_{spin} \Delta & 0 \\
0 & -s_{spin} \Delta & -\xi_k & -ve^{i\theta} - V^*(-k) \\
\Delta & 0 & -ve^{i\theta} - V(-k) & -\xi_k
\end{pmatrix}
\]
here $s_{spin} = -1$ for spin singlet and $s_{spin} = 1$ for spin triplet. To discuss the effects of hybridizations and spin–orbit interaction on the properties of superconductors, we calculate the Green’s functions by solving the Gor’kov equation \[23\]
\[
\left( i\omega_n - \hat{H}_0 \right) \hat{G}_0(k, i\omega_n) = \mathbb{1},
\]
\[
\hat{G}_0(k, i\omega_n) = \begin{pmatrix}
\hat{G}_0(k, i\omega_n) & \hat{F}_0(k, i\omega_n) \\
-s_{spin} \hat{F}_0^\dagger(-k, i\omega_n) & -\hat{G}_0^*(-k, i\omega_n)
\end{pmatrix}
\]
where $\omega_n = (2n + 1) \pi k_B T$ is the Matsubara frequency ($k_B$ is the Boltzmann constant), and $\mathbb{1}$ is the identity matrix in $spin \times band \times particle – hole$ space, $\hat{G}_0$ is a $4 \times 4$ matrix where the diagonal components are normal Green’s function and non-diagonal components are anomalous Green’s function.

2.1 Spin singlet pairing order

According to Eq. (11), the Hamiltonian of a two-band superconductor with spin-singlet configuration in the presence of spin–orbit coupling is
\[
\hat{H}_0 = \begin{pmatrix}
\xi_k & ve^{i\theta} + \eta(k_y - ik_x) & 0 & \Delta \\
ve^{-i\theta} + \eta(k_y + ik_x) & \xi_k & \Delta & 0 \\
0 & \Delta & -\xi_k & -ve^{-i\theta} + \eta(k_y - ik_x) \\
\Delta & 0 & -ve^{i\theta} + \eta(k_y + ik_x) & -\xi_k
\end{pmatrix}.
\]
The matrix form of the normal Green’s function (Eq. 15) can be written as
\[
\hat{G}_0(k, i\omega_n) = \frac{1}{Z_0} \{[(\xi - i\omega_n)(v^2 + \eta^2 k^2) - 2\nu\eta(k_x \sin \theta + k_y \cos \theta) + (\xi + i\omega_n) \times (-\xi^2 - \omega_n^2)]\hat{\rho}_0 + \{(-v \cos \theta - \eta k_y)(-\xi + i\omega_n)^2 + v^2 + \eta^2 k^2
\]
\[
- 2\nu\eta(k_x \sin \theta + k_y \cos \theta)\} \hat{\rho}_1 + \{(v \sin \theta + \eta k_x)(-\xi + i\omega_n)^2 + v^2 + \eta^2 k^2 - 2\nu\eta(k_x \sin \theta + k_y \cos \theta)\} \hat{\rho}_2\}
\]
here
\[
Z_0 = \xi^4 + 2\xi^2(\omega_n^2 - \nu^2) + (\omega_n^2 + \nu^2)^2 - 8i\nu\xi\omega_n(k_x \sin \theta + k_y \cos \theta),
\]
\[
+ 2\cos 2\theta \eta^2 v^2(k_x^2 - k_y^2) - 4\sin 2\theta \eta^2 v^2 k_x k_y + 2\eta^2 k^2(\omega_n^2 - \xi^2) + \eta^4 k^4,
\]
that \(k_x = k \cos \phi\) and \(k_y = k \sin \phi\), where \(\phi\) is the angle between momentum and the \(x\) axis.

The matrix form of the normal Green’s function (Eq. 15) can be written as
\[
\hat{G}_0(k, i\omega_n) = \begin{pmatrix}
G_{11}(k, i\omega_n) & G_{12}(k, i\omega_n) \\
G_{21}(k, i\omega_n) & G_{22}(k, i\omega_n)
\end{pmatrix}
\]
(17)

where
\[
G_{11}(k, i\omega_n) = \frac{1}{Z_0}\{[(\xi - i\omega_n)(v^2 + \eta^2 (k_x^2 + k_y^2)) - 2\nu\eta(k_x \sin \theta + k_y \cos \theta)
\]
\[- (\xi + i\omega_n)(\xi^2 + \omega_n^2)],
\]
\[
G_{12}(k, i\omega_n) = \frac{1}{Z_0}\{[-ve^{i\theta} - \eta(ik_x + k_y)](-\xi + i\omega_n)^2 + v^2 + \eta^2 k^2
\]
\[- 2\nu\eta(k_x \sin \theta + k_y \cos \theta)\},
\]
\[
G_{21}(k, i\omega_n) = \frac{1}{Z_0}\{[-ve^{-i\theta} - \eta(-ik_x + k_y)](-\xi + i\omega_n)^2 + v^2 + \eta^2 k^2
\]
\[- 2\nu\eta(k_x \sin \theta + k_y \cos \theta)\},
\]
\[
G_{22}(k, i\omega_n) = \frac{1}{Z_0}\{[\xi - i\omega_n](ve^{-i\theta} - \eta(-ik_x + k_y))(ve^{i\theta} - \eta(ik_x + k_y))
\]
\[- (\xi + i\omega_n)(\xi^2 + \omega_n^2)\}.
\]

By using Eqs. (12) and (13), the anomalous Green’s function can be obtained as
\[
\hat{F}_0(k, i\omega_n) = \frac{\Delta}{Z_0}\{[2v\xi \cos \theta + 2\eta\omega_n ik_y]\hat{\rho}_0 + \{-(v^2 + \xi^2 + \omega_n^2) + \eta^2 k^2\} \hat{\rho}_1
\]
\[
+ (2\nu\eta(k_x \cos \theta - k_y \sin \theta)) \hat{\rho}_2 + (2v\xi i \sin \theta - 2\eta\omega_n k_x) \hat{\rho}_3\}
\]
(22)

In particle–hole space \(N_1\), the matrix form of the anomalous Green’s function (Eq. 22) is
\[
\hat{F}_0^{N_1}(k, i\omega_n) = \begin{pmatrix}
F_{11,\uparrow\downarrow}(k, i\omega_n) & F_{12,\uparrow\downarrow}(k, i\omega_n) \\
F_{21,\uparrow\downarrow}(k, i\omega_n) & F_{22,\uparrow\downarrow}(k, i\omega_n)
\end{pmatrix}
\]
(23)

where
\[
F_{11,\uparrow\downarrow}(k, i\omega_n) = \frac{\Delta}{Z_0}\{2v\xi e^{i\theta} - 2\eta\omega_n k_x + 2\eta\omega_n ik_y\},
\]
(24)
\[
F_{12,\uparrow\downarrow}(k, i\omega_n) = \frac{\Delta}{Z_0}\{-(v^2 + \xi^2 + \omega_n^2) + \eta^2 k^2 - 2i\nu\eta(k_x \cos \theta - k_y \sin \theta)\},
\]
(25)
\[
F_{21,\uparrow\downarrow}(k, i\omega_n) = \frac{\Delta}{Z_0}\{-(v^2 + \xi^2 + \omega_n^2) + \eta^2 k^2 + 2i\nu\eta(k_x \cos \theta - k_y \sin \theta)\},
\]
(26)
\[
F_{22,\uparrow\downarrow}(k, i\omega_n) = \frac{\Delta}{Z_0}\{2v\xi e^{i\theta} - 2\eta\omega_n k_x + 2\eta\omega_n ik_y\}.
\]
(27)
The matrix form of the anomalous Green’s function in Eq. 29 in particle–hole spaces and \( \rho \) becomes

\[
\tilde{\hat{F}}_{0}^{N_2}(k, i \omega_n) = \left( \begin{array}{c}
F_{11,\uparrow\downarrow}(k, i \omega_n) \\
F_{21,\downarrow\uparrow}(k, i \omega_n)
\end{array} \right) = -\tilde{\hat{F}}_{0}^{N_1}(k, i \omega_n).
\] (28)

In the absence of spin–orbit coupling (\( \eta = 0 \)), the anomalous Green’s function (Eq. 22) becomes

\[
\tilde{\hat{F}}_{0}(k, i \omega_n) = \frac{\Delta}{Z_0} [2\nu \xi \cos \theta \hat{\rho}_0 - (v^2 + \xi^2 + \omega_n^2) \hat{\rho}_1 + 2\nu \xi i \sin \theta \hat{\rho}_3],
\] (29)

here

\[
Z_0 = \xi^4 + 2\xi^2 (\omega_n^2 - v^2) + (\omega_n^2 + v^2)^2.
\] (30)

The matrix form of the anomalous Green’s function in Eq. 29 in particle–hole spaces \( N_1 \) and \( N_2 \) is

\[
\tilde{\hat{F}}_{0}^{N_1}(k, i \omega_n) = \left( \begin{array}{c}
F_{11,\uparrow\downarrow}(k, i \omega_n) \\
F_{21,\downarrow\uparrow}(k, i \omega_n)
\end{array} \right) \frac{\Delta}{Z_0} \left( \begin{array}{cc}
2\nu \xi \cos \theta + 2i \nu \xi \sin \theta & -(v^2 + \xi^2 + \omega_n^2) \\
-(v^2 + \xi^2 + \omega_n^2) & 2\nu \xi \cos \theta - 2i \nu \xi \sin \theta
\end{array} \right)
\]

\[
= \frac{\Delta}{Z_0} \left( \begin{array}{c}
2\xi \nu e^{i\theta} & -(v^2 + \xi^2 + \omega_n^2) \\
-(v^2 + \xi^2 + \omega_n^2) & 2\xi \nu e^{-i\theta}
\end{array} \right),
\] (31)

and

\[
\tilde{\hat{F}}_{0}^{N_2}(k, i \omega_n) = \left( \begin{array}{c}
F_{11,\uparrow\downarrow}(k, i \omega_n) \\
F_{21,\downarrow\uparrow}(k, i \omega_n)
\end{array} \right) \frac{\Delta}{Z_0} \left( \begin{array}{cc}
-2\nu \xi \cos \theta - 2i \nu \xi \sin \theta & (v^2 + \xi^2 + \omega_n^2) \\
(v^2 + \xi^2 + \omega_n^2) & -2\nu \xi \cos \theta + 2i \nu \xi \sin \theta
\end{array} \right)
\]

\[
= \frac{\Delta}{Z_0} \left( \begin{array}{c}
-2\xi \nu e^{i\theta} & (v^2 + \xi^2 + \omega_n^2) \\
(v^2 + \xi^2 + \omega_n^2) & -2\xi \nu e^{-i\theta}
\end{array} \right).
\] (32)

The intraband pairing correlations become

\[
F_{11,\uparrow\downarrow}(k, i \omega_n) - F_{11,\downarrow\uparrow}(k, i \omega_n) = \frac{4\Delta}{Z_0} \xi \nu e^{i\theta},
\] (33)

\[
F_{22,\uparrow\downarrow}(k, i \omega_n) - F_{22,\downarrow\uparrow}(k, i \omega_n) = \frac{4\Delta}{Z_0} \xi \nu e^{-i\theta}.
\] (34)

Hybridization generates \( \rho_0 \) and \( \rho_3 \) components which belong to even-frequency symmetry class. It means that in the presence of interband coupling, hybridization generates even-frequency intra-sublattice pairing in the system. These components belong to even-frequency spin-singlet even-momentum even-band parity (ESEE) symmetry class. This result is in agreement with Eq. (20) presented in Ref [12]. Equations (33) and (34) are in agreement with Eqs. (62) and (63) reported in Ref [3] in the first order of \( \Delta (|\Delta|^2 = 0) \) and equal energy bands (\( \xi_- = 0 \)) and both belong to the (ESEE) symmetry class. The band symmetry generates interband pairing correlation:

\[
[F_{12,\uparrow\downarrow}(k, i \omega_n) - F_{12,\downarrow\uparrow}(k, i \omega_n)] + [F_{21,\uparrow\downarrow}(k, i \omega_n) - F_{21,\downarrow\uparrow}(k, i \omega_n)]
\]

\[
= \frac{4\Delta}{Z_0} (v^2 + \xi^2 + \omega_n^2). \tag{35}
\]
which belongs to (ESEE). This result is in agreement with Equation (65) presented in Ref [3]. In Ref [3] the band asymmetry generates the interband pair correlation as

\[
[F_{12, \uparrow \uparrow}(k, i\omega_n) - F_{12, \downarrow \downarrow}(k, i\omega_n)] - [F_{21, \uparrow \downarrow}(k, i\omega_n) - F_{21, \downarrow \uparrow}(k, i\omega_n)] = \frac{2\Delta}{Z_0} i\omega_n \xi_-, \quad (36)
\]

which belongs to the symmetry (OSEO) class. We considered a two-band superconductor with an equal dispersion energy in each band ($\xi_+ = \xi_-). In this case the interband pairing correlation due to band asymmetry is

\[
[F_{12, \uparrow \uparrow}(k, i\omega_n) - F_{12, \downarrow \downarrow}(k, i\omega_n)] - [F_{21, \uparrow \downarrow}(k, i\omega_n) - F_{21, \downarrow \uparrow}(k, i\omega_n)] = 0. \quad (37)
\]

In the absence of hybridization within the second order of the spin–orbit coupling constant ($\eta$), we obtain

\[
\hat{G}_0(k, i\omega_n) = \frac{1}{Z_0} [(\eta^2 k^2 - (\xi + i\omega_n)^2) (\xi - i\omega_n) \hat{\rho}_0 + \eta (k_y + ik_x) (\xi + i\omega_n)^2 \hat{\rho}_1]. \quad (38)
\]

where

\[
Z_0 = (\xi^2 + \omega_n^2)^2 + 2\eta^2 k^2 (\omega_n^2 - \xi^2). \quad (39)
\]

Equation (22) can be rewritten as

\[
\hat{F}_0(k, i\omega) = \frac{\Delta}{Z_0} [2i\eta\omega_n k_y \hat{\rho}_0 - 2\eta\omega_n k_x \hat{\rho}_3 + (-\xi^2 - \omega_n^2 + \eta^2 k^2) \hat{\rho}_1]. \quad (40)
\]

In particle–hole space $N_1$ and $N_2$, the matrix form of the anomalous Green’s function (Eq. (40) is

\[
\hat{F}_0^{N_1}(k, i\omega_n) = \left( \begin{array}{c} F_{11, \uparrow \downarrow}(k, i\omega_n) F_{12, \downarrow \downarrow}(k, i\omega_n) \\ F_{21, \downarrow \uparrow}(k, i\omega_n) F_{22, \uparrow \downarrow}(k, i\omega_n) \end{array} \right) = \frac{\Delta}{Z_0} \left( \begin{array}{cc} 2i\eta\omega_n k_y - 2\eta\omega_n k_x & -\xi^2 - \omega_n^2 + \eta^2 k^2 \\ -\xi^2 - \omega_n^2 + \eta^2 k^2 & 2i\eta\omega_n k_y + 2\eta\omega_n k_x \end{array} \right), \quad (41)
\]

\[
\hat{F}_0^{N_2}(k, i\omega_n) = \left( \begin{array}{c} F_{11, \downarrow \uparrow}(k, i\omega_n) F_{12, \uparrow \downarrow}(k, i\omega_n) \\ F_{21, \downarrow \uparrow}(k, i\omega_n) F_{22, \uparrow \downarrow}(k, i\omega_n) \end{array} \right) = -\hat{F}_0^{N_1}(k, i\omega_n) = \frac{\Delta}{Z_0} \left( \begin{array}{cc} -2i\eta\omega_n k_y + 2\eta\omega_n k_x & \xi^2 + \omega_n^2 - \eta^2 k^2 \\ \xi^2 + \omega_n^2 - \eta^2 k^2 & -2i\eta\omega_n k_y - 2\eta\omega_n k_x \end{array} \right). \quad (42)
\]

The intraband pairing correlations are

\[
F_{11, \uparrow \downarrow}(k, i\omega_n) - F_{11, \downarrow \uparrow}(k, i\omega_n) = \frac{4\Delta}{Z_0} i\omega_n \eta (k_y + ik_x), \quad (43)
\]

\[
F_{22, \uparrow \downarrow}(k, i\omega_n) - F_{22, \downarrow \uparrow}(k, i\omega_n) = \frac{4\Delta}{Z_0} i\omega_n \eta (k_y - ik_x). \quad (44)
\]

Spin–orbit coupling generates $\rho_0$ and $\rho_3$ components which belong to odd-frequency symmetry class. It means that in the presence of interband coupling, spin–orbit coupling generates odd-frequency intrasublattice pairing in the system. These components belong to odd-frequency spin-singlet odd-momentum even-band parity (OSOE) symmetry class. In Ref [3] the intraband pairing correlation is written as

\[
F_{11, \uparrow \downarrow}(k, i\omega_n) + F_{11, \downarrow \uparrow}(k, i\omega_n) = \frac{\Delta}{Z_3} (\xi_+ - \xi_-) V_3, \quad (45)
\]

\[
F_{22, \uparrow \downarrow}(k, i\omega_n) + F_{22, \downarrow \uparrow}(k, i\omega_n) = \frac{\Delta}{Z_3} (\xi_+ + \xi_-) V_3. \quad (46)
\]
The spin dependent hybridization generates pairing correlations that belong to the ETOE class. The band asymmetry generates interband pairing correlation as

\[ [F_{12, \uparrow \downarrow}(k, i\omega_n) - F_{12, \downarrow \uparrow}(k, i\omega_n)] - [F_{21, \uparrow \downarrow}(k, i\omega_n) - F_{21, \downarrow \uparrow}(k, i\omega_n)] = 0. \] (47)

In Ref [3] for spin–orbit hybridization the band asymmetry generates the interband pair correlation as

\[ [F_{12, \uparrow \downarrow}(k, i\omega) - F_{12, \downarrow \uparrow}(k, i\omega)] - [F_{21, \uparrow \downarrow}(k, i\omega) - F_{21, \downarrow \uparrow}(k, i\omega)] = \frac{2\Delta}{Z_0} i\omega_n \xi_-, \] (48)

which belongs to the odd-frequency spin-singlet even-momentum odd-band parity symmetry (OSEO). The interband pairing correlation due to band symmetry is

\[
[F_{12, \uparrow \downarrow}(k, i\omega_n) - F_{12, \downarrow \uparrow}(k, i\omega_n)] + [F_{21, \uparrow \downarrow}(k, i\omega_n) - F_{21, \downarrow \uparrow}(k, i\omega_n)]
\]
\[
= \frac{4\Delta}{Z_0} (\xi^2 + \omega_n^2 - \eta^2 k^2). \] (49)

This component belongs to even-frequency spin-singlet even-momentum even-band parity (ESEE) symmetry class. For spin singlet, hybridization potential generates ESEE symmetry class due to both intra- and interband correlation, whereas the spin-dependent hybridization potential generates this class only for interband pairing correlation due to band symmetry. In this case the odd-frequency pairing arises only due to intraband pairing correlations for spin-dependent hybridization potential.

### 2.2 Spin-triplet pairing order

By considering Eq. (11), the Hamiltonian of a two-band superconductor with spin-triplet configuration in the presence of spin–orbit coupling is

\[
\hat{H}_0 = \begin{pmatrix}
\xi_k & ve^{i\theta} + \eta(k_y + ik_x) & 0 & \Delta \\
0 & -\Delta & -\xi_k & -ve^{i\theta} + \eta(k_y - ik_x) \\
\Delta & 0 & -\xi_k & 0 \\
ve^{-i\theta} + \eta(k_y - ik_x) & -\xi_k & ve^{i\theta} + \eta(k_y + ik_x) & -\xi_k
\end{pmatrix}. \] (50)

The solution of the anomalous Green’s function within the first order of \( \Delta \) is calculated as

\[
\hat{F}_0(k, i\omega_n) = \frac{\Delta}{Z_0} [(-2i\eta\xi k_x + 2i\omega_n \sin \theta)\hat{\rho}_0 + (2i\nu \eta (k_x \cos \theta - k_y \sin \theta))\hat{\rho}_1
\]
\[+(i(v^2 - \xi^2 - \omega_n^2) - i\eta^2(k_x^2 + k_y^2))\hat{\rho}_2 + (-2\eta k_y - 2i\nu \omega_n \cos \theta)\hat{\rho}_3]. \] (51)

The matrix form of the anomalous Green’s function (Eq. 51) can be written as

\[
\hat{F}_{11, \uparrow \downarrow}(k, i\omega_n) = \frac{\Delta}{Z_0} \left(-2i\nu \omega_n ve^{i\theta} - 2i\eta \xi k_x - 2\eta \xi k_y\right), \] (52)

\[
\hat{F}_{12, \uparrow \downarrow}(k, i\omega_n) = \frac{\Delta}{Z_0} [(v^2 - \xi^2 - \omega_n^2) - \eta^2 k^2 + 2i\nu \eta (k_x \cos \theta - k_y \sin \theta)], \] (53)

\[
\hat{F}_{21, \uparrow \downarrow}(k, i\omega_n) = \frac{\Delta}{Z_0} [(-v^2 + \xi^2 + \omega_n^2) + \eta^2 k^2 + 2i\nu \eta (k_x \cos \theta - k_y \sin \theta)], \] (54)

\[
\hat{F}_{22, \uparrow \downarrow}(k, i\omega_n) = \frac{\Delta}{Z_0} \left(2i\nu \omega_n ve^{-i\theta} - 2i\eta \xi k_x + 2\eta \xi k_y\right). \] (55)
In the absence of spin–orbit coupling ($\eta = 0$), the anomalous Green’s function Eq. (51) becomes

$$\hat{F}_0(k, i\omega_n) = \frac{\Delta}{Z_0} [2v\omega_n \sin \theta \hat{\rho}_0 + i (v^2 - \xi^2 - \omega_n^2) \hat{\rho}_2 - 2i v\omega_n \cos \theta \hat{\rho}_3]. \quad (56)$$

Here

$$Z_0 = \xi^4 + 2\xi^2 (\omega_n^2 - v^2) + (\omega_n^2 + v^2)^2. \quad (57)$$

In particle–hole space $N_1$ and $N_2$, the matrix form of the anomalous Green’s function (Eq. 56) is

$$\hat{F}_0^{N_1}(k, i\omega_n) = \begin{pmatrix} F_{11, \uparrow \downarrow}(k, i\omega_n) & F_{12, \uparrow \downarrow}(k, i\omega_n) \\ F_{21, \uparrow \downarrow}(k, i\omega_n) & F_{22, \uparrow \downarrow}(k, i\omega_n) \end{pmatrix} = \frac{\Delta}{Z_0} \begin{pmatrix} 2v\omega_n (\sin \theta - i \cos \theta) & (v^2 - \xi^2 - \omega_n^2) \\ -(v^2 - \xi^2 - \omega_n^2) & 2v\omega_n (\sin \theta + i \cos \theta) \end{pmatrix}, \quad (58)$$

$$\hat{F}_0^{N_2}(k, i\omega_n) = \begin{pmatrix} F_{11, \downarrow \uparrow}(k, i\omega_n) & F_{12, \downarrow \uparrow}(k, i\omega_n) \\ F_{21, \downarrow \uparrow}(k, i\omega_n) & F_{22, \downarrow \uparrow}(k, i\omega_n) \end{pmatrix} = \hat{F}_0^{N_1}(k, i\omega_n) = \frac{\Delta}{Z_0} \begin{pmatrix} 2v\omega_n (\sin \theta - i \cos \theta) & (v^2 - \xi^2 - \omega_n^2) \\ -(v^2 - \xi^2 - \omega_n^2) & 2v\omega_n (\sin \theta + i \cos \theta) \end{pmatrix}. \quad (59)$$

The intraband pairing correlations become

$$F_{11, \uparrow \downarrow}(k, i\omega_n) + F_{11, \downarrow \uparrow}(k, i\omega_n) = \frac{-4\Delta}{Z_0} i\omega_n v e^{-i\theta}, \quad (60)$$

$$F_{22, \uparrow \downarrow}(k, i\omega_n) + F_{22, \downarrow \uparrow}(k, i\omega_n) = \frac{4\Delta}{Z_0} i\omega_n v e^{i\theta}. \quad (61)$$

Hybridization generates $\rho_0$ and $\rho_3$ which belong to odd-frequency symmetry class. These components belong to odd-frequency spin-triplet even-momentum even-band parity (OTEE) symmetry class. This result is in agreement with Equation (24) presents in Ref [12] In the first order of $\Delta (|\Delta|^2 = 0)$ and equal energy bands ($\xi_0 = 0$), Equations (60) and (61) are coincided with Equation (83) and (84) presented in Ref [3] and both belong to the (OTEE) symmetry class. The band symmetry generates interband pairing correlation as

$$[F_{12, \uparrow \downarrow}(k, i\omega_n) + F_{12, \downarrow \uparrow}(k, i\omega_n)] - [F_{21, \uparrow \downarrow}(k, i\omega_n) + F_{21, \downarrow \uparrow}(k, i\omega_n)]$$

$$= \frac{4\Delta}{Z_0} (v^2 - \xi^2 - \omega_n^2), \quad (62)$$

which belongs to even-frequency spin triplet even-momentum odd-band parity (ETEO) symmetry class. In Ref [3] the interband pairing correlation due to band asymmetry is

$$[F_{12, \uparrow \downarrow}(k, i\omega_n) + F_{12, \downarrow \uparrow}(k, i\omega_n)] + [F_{21, \uparrow \downarrow}(k, i\omega_n) + F_{21, \downarrow \uparrow}(k, i\omega_n)]$$

$$= \frac{2\Delta}{Z_5} i\omega_n \xi_. \quad (63)$$

Thus, the band hybridization generates pairing correlations that belong to the odd-frequency spin triplet even-momentum even-band parity (OTEE) class. Since we considered a two-band superconductor with an equal energy bands, the interband pairing correlation due to band asymmetry is

$$[F_{12, \uparrow \downarrow}(k, i\omega_n) + F_{12, \downarrow \uparrow}(k, i\omega_n)] + [F_{21, \uparrow \downarrow}(k, i\omega_n) + F_{21, \downarrow \uparrow}(k, i\omega_n)] = 0. \quad (64)$$
In the absence of hybridization, we obtain
\[ \hat{F}_0(k, i\omega_n) = \frac{\Delta}{Z_0} [-2i\eta\xi k_x \hat{\rho}_0 - 2\eta\xi k_y \hat{\rho}_3 - i (\xi^2 + \omega_n^2 + \eta^2k^2) \hat{\rho}_2]. \] (65)

The matrix form of the anomalous Green’s function in Equation (65) in particle–hole spaces \( N_1 \) and \( N_2 \) are
\[ \hat{F}_0^{N_1}(k, i\omega_n) = \left( \begin{array}{c} F_{11, \uparrow \downarrow}(k, i\omega_n) \\ F_{12, \uparrow \downarrow}(k, i\omega_n) \\ F_{21, \uparrow \downarrow}(k, i\omega_n) \\ F_{22, \uparrow \downarrow}(k, i\omega_n) \end{array} \right) \]
\[ = \frac{\Delta}{Z_0} \left( \begin{array}{c} -2i\eta\xi k_x - 2\eta\xi k_y - (\xi^2 + \omega_n^2 + \eta^2k^2) \\ (\xi^2 + \omega_n^2 + \eta^2k^2) - 2i\eta\xi k_x + 2\eta\xi k_y \end{array} \right). \] (66)

\[ \hat{F}_0^{N_2}(k, i\omega_n) = \left( \begin{array}{c} F_{11, \downarrow \uparrow}(k, i\omega_n) \\ F_{12, \downarrow \uparrow}(k, i\omega_n) \\ F_{21, \downarrow \uparrow}(k, i\omega_n) \\ F_{22, \downarrow \uparrow}(k, i\omega_n) \end{array} \right) = \hat{F}_0^{N_1}(k, i\omega_n) \]
\[ = \frac{\Delta}{Z_0} \left( \begin{array}{c} -2i\eta\xi k_x - 2\eta\xi k_y - (\xi^2 + \omega_n^2 + \eta^2k^2) \\ (\xi^2 + \omega_n^2 + \eta^2k^2) - 2i\eta\xi k_x + 2\eta\xi k_y \end{array} \right). \] (67)

The intraband pairing correlations are
\[ F_{11, \uparrow \downarrow}(k, i\omega_n) + F_{11, \downarrow \uparrow}(k, i\omega_n) = \frac{-4\Delta}{Z_0} \xi_1(ky + ik_x), \] (68)
\[ F_{22, \uparrow \downarrow}(k, i\omega_n) + F_{22, \downarrow \uparrow}(k, i\omega_n) = \frac{4\Delta}{Z_0} \xi_1(ky - ik_x). \] (69)

Spin–orbit coupling generates \( \hat{\rho}_0 \) and \( \hat{\rho}_3 \) which belong to even-frequency symmetry class. These components belong to even-frequency spin-triplet odd-momentum even-band parity (ETOE) symmetry class. In Ref [3] the intraband pairing correlation is calculated as
\[ F_{11, \uparrow \downarrow}(k, i\omega_n) - F_{11, \downarrow \uparrow}(k, i\omega_n) = \frac{-\Delta}{Z_5} i\omega_n V_3, \] (70)
\[ F_{22, \uparrow \downarrow}(k, i\omega_n) - F_{22, \downarrow \uparrow}(k, i\omega_n) = \frac{\Delta}{Z_0} i\omega_n V_3. \] (71)

The spin dependent hybridization generates pairing correlations that belong to the odd-frequency spin singlet odd-momentum even-band parity (OSOE) class. As mentioned in Ref [3], the interband pair correlation can be written as
\[ [F_{12, \uparrow \downarrow}(k, i\omega_n) + F_{12, \downarrow \uparrow}(k, i\omega_n)] + [F_{21, \uparrow \downarrow}(k, i\omega_n) + F_{21, \downarrow \uparrow}(k, i\omega_n)] = \frac{2\Delta}{Z_5} i\omega_n \xi. \] (72)

Thus, the spin–orbit coupling generates pairing correlations that belong to the odd-frequency spin-triplet even-momentum even-band parity (OTEE) class. In contrast in our formalism, the band asymmetry generates interband pairing correlation as
\[ [F_{12, \uparrow \downarrow}(k, i\omega_n) + F_{12, \downarrow \uparrow}(k, i\omega_n)] - [F_{21, \uparrow \downarrow}(k, i\omega_n) + F_{21, \downarrow \uparrow}(k, i\omega_n)] = 0. \] (73)

The interband pairing correlation due to band symmetry is
\[ [F_{12, \uparrow \downarrow}(k, i\omega_n) + F_{12, \downarrow \uparrow}(k, i\omega_n)] - [F_{21, \uparrow \downarrow}(k, i\omega_n) + F_{21, \downarrow \uparrow}(k, i\omega_n)] = \frac{-4\Delta}{Z_0} (\xi^2 + \omega_n^2 + \eta^2k^2). \] (74)

These components belong to even-frequency spin-triplet even-momentum odd-band parity (ETEO) symmetry class. Thus, for spin triplet, the spin-dependent and spin-independent
hybridization both generates the same symmetry class ETEO due to interband pairing correlation. The odd-frequency pairing arises in the presence of spin-independent hybridization due to intraband pairing correlations.

Our theoretical results obtained for different symmetries of the Cooper pairs due to competing between spin orbit coupling and multi band effect are in a good agreement with the recent theoretical and experimental papers. As examples, various kinds of systems, such as oxide heterostructures \( LaAlO_3/SrTiO_3 \) [24–28], noncentrosymmetric superconductor \( CePt_3Si \) [29], \( Sr2RuO_4 \) [30], heavy fermion \( CeCoIn_5/YbCoIn_5 \) superlattice [26,29], \( Up_3 \) [31], \( Bi_2Se_3 \) [32,33], \( MgB_2 \) and iron pnictides [3,34] have recently been investigated.

3 Conclusion

Within the theoretical model, the existence of odd-frequency pairs in two band superconductors by incorporating both spin-independent hybridization and spin-dependent spin–orbit interaction is investigated. This model also includes both the one-particle hybridization term and all possible intraband and interband superconducting pairing interaction terms in a two-band system.

The normal and anomalous thermal Green’s functions have been calculated in the Nambu formalism as elements of the Fourier transformed \( 4 \times 4 \) matrix Green’s function by taking into account of all possible intraband and interband superconducting interaction terms coupling both bands in the mean field approximation. By assuming that the attractive interaction acts on two electrons with different spins in different conduction bands, different symmetry classes were demonstrated in the presence of hybridization and spin–orbit coupling.

The role of intraband and interband pairing correlations to emerge the odd frequency in a two-band superconductor was examined. For spin singlet, the odd frequency is generated by spin dependent hybridization potential owing to intraband pairing correlations in agreement with the odd frequency generated by the interband pair correlation due to band asymmetry in Ref [3]. On the other hand, for spin triplet the spin-independent hybridization potential generates the odd-frequency pairing due to intraband correlations in agreement with the result of Ref [12].

References

1. M. Sigrist, K. Ueda, Phenomenological theory of unconventional superconductivity. Rev. Mod. Phys. 63, 239 (1991)
2. A.M. Black-Schaffer, A.V. Balasky, Proximity-induced unconventional superconductivity in topological insulators. Phys. Rev. B 87, 220506(R) (2013)
3. Y. Asano, A. Sasaki, Odd-frequency Cooper pairs in two-band superconductors and their magnetic response. Phys. Rev. B 92, 224508 (2015)
4. V.A. Moskalenko, M.E. Palistrant, Two-band model determination of the critical temperature of a superconductor with an impurity. Sov. Phys. JETP 22, 526 (1966)
5. X.X. Xi, Two-band superconductor magnesium diboride. Rep. Prog. Phys. 71, 116501 (2008)
6. J. Nagamatsu, N. Nakagawa, T. Muranaka, Y. Zenitani, J. Akimitsu, Superconductivity at 39 K in magnesium diboride. Nature 410, 63 (2001)
7. Y. Shun-Li, L. Jian-Xin, Spin fluctuations and unconventional superconducting pairing in iron-based superconductors. Chin. Phys. B 22, 087411 (2013)
8. Y. Tanaka, P.M. Shirage, A. Iyo, Disappearance of Meissner effect and specific heat jump in a multiband superconductor. J. Supercond. Nov. Magn. 23, 253–256 (2010)
9. P.O. Sprau et al., Discovery of orbital-selective Cooper pairing in FeSe. Science 357, 75–80 (2013)
10. A.M. Black-Schaffer, A.V. Balasky, Odd-frequency superconducting pairing in multiband superconductors. Phys. Rev. B 88, 10514 (2013)
11. Y. Asano, A.A. Golubov, Greens-function theory of dirty two-band superconductivity. Phys. Rev. B 97, 214508 (2018)
12. Y. Asano, A. Sasaki, A.A. Golubov, Dirty two-band superconductivity with interband pairing order. New J. Phys. 20, 043020 (2018)
13. L. Komendova, A.V. Balatsky, A.M. Black-Schaffer, Band hybridization induced odd-frequency pairing in multiband superconductors. Phys. Rev. B 92, 094517 (2015)
14. Y. Fukaya, S. Tamura, K. Yada, K.Y. Tanaka, P. Gentile, M. Cuoco, Interorbital topological superconductivity in spin-orbit coupled superconductors with inversion symmetry breaking. Phys. Rev. B 97, 174500 (2018)
15. V.P. Mineev, M. Sigrist, Basic theory of superconductivity in metals without inversion center. Lect. Notes Phys. 847, 129–154 (2012)
16. M. Smidman, M.B. Salamon, H.Q. Yuan, D.F. Agterberg, Spin-triplet p-wave pairing in a three-orbital model for iron pnictide superconductors. Rep. Prog. Phys 80, 036501 (2017)
17. P. Lee, X.G. Wen, S, Superconductivity and spin-orbit coupling in non-centrosymmetric materials: A review. Phys. Rev. B 78, 014508 (2009)
18. W.L. Yang et al., Evidence for weak electronic correlations in iron pnictides. Phys. Rev. B 80, 014508 (2009)
19. P. Fazekas, Lecture Notes on Electron Correlation and Magnetism (World Scientific, Singapore, 1999)
20. H. Kuriyama et al., Epitaxially stabilized iridium spinel oxide without cations in the tetrahedral site. Appl. Phys. Lett. 96, 27010 (2010)
21. M.P. Christoph, H.Y. Kee, Identifying spin-triplet pairing in spin-orbit coupled multi-band superconductors. EPL 98, 2 (2011)
22. M.M. Korshunov, Y.N. Togushova, Spin-orbit coupling and impurity scattering on the spin resonance peak in three orbital model for Fe-based superconductors. J. Siberian Fed. Univ. Math. Phys. 11, 998 (2018)
23. L.P. Gor’kov, Theory of superconducting alloys in a strong magnetic field near the critical temerature. JETP 10, 998 (1960)
24. G. Singh, E. Lesne, D. Winkler, T. Claeson, T. Bauch, F. Lombardi, A.D. Caviglia, A. Kalaboukhov, Nanopatterning of weak links in superconducting oxide interfaces. Nanomaterials 11, 398 (2021)
25. L. Lepori, D. Giuliano, A. Nava, C. A. Perroni, Interplay between singlet and triplet pairings in multiband two-dimensional oxide superconductors. arXiv:2107.01100 [condmat.str-el] (unpublished)
26. T. Watanabe, T. Yoshida, Y. Yanase, Odd-parity superconductivity by competing spin-orbit coupling and orbital effect in artificial heterostructures. Phys. Rev. B 92, 174502 (2015)
27. S. Gariglio, M. Gabay, J.-M. Triscone, Research update: conductivity and beyond at the LaAlO3/SrTiO3 interface. APL Mater. 4, 060701 (2016)
28. T.V. Trevisan, M. Schütz, R.M. Fernandes, Unconventional multiband superconductivity in bulk SrTiO3 and LaAlO3 / SrTiO3 interfaces. Phys. Rev. Lett. 121, 27002 (2018)
29. Y. Nagai, H. Nakamura, Multi-band Eilenberger theory of superconductivity: systematic low-energy projection. J. Phys. Soc. Jpn. 85, 074707 (2016)
30. L. Komendova, A.M. Black-Schaffer, Odd-frequency superconductivity in Sr2RuO4 measured by Kerr rotation. Phys. Rev. Lett. 119, 087001 (2017)
31. Christopher Triola, Annica M. Black-Schaffer, Odd-frequency pairing and Kerr effect in the heavy-fermion superconductor UPt3. Phys. Rev. B 97, 064505 (2018)
32. J. Schmidt, F. Parhizgar, A.M. Black-Schaffer, Odd-frequency superconductivity and Meissner effect in the doped topological insulator Bi2Se3. Phys. Rev. B 101, 180512(R) (2020)
33. S. Yonezawa, Nematic superconductivity in doped Bi2Se3 topological superconductors. Condens. Matter 4, 2 (2019)
34. A. Ptok, K.J. Kapcia, P. Piekar, Theory of superconducting alloys in a strong magnetic field near the critical temperature. Front. Phys. 8, 284 (2020)