Coherent transport of interacting electrons through a single scatterer

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Abstract

Using the self-consistent Hartree-Fock method, we calculate the persistent current of weakly-interacting spinless electrons in a one-dimensional ring containing a single δ barrier. We find that the persistent current decays with the system length (L) asymptotically like \( L^{-1-\alpha} \), where \( \alpha > 0 \) is the power depending only on the electron-electron interaction. We also simulate tunneling of the weakly-interacting one-dimensional electron gas through a single δ barrier in a finite wire biased by contacts. We find that the Landauer conductance decays with the system length asymptotically like \( L^{-2\alpha} \). The power laws \( L^{-1-\alpha} \) and \( L^{-2\alpha} \) have so far been observed only in correlated models. Their existence in the Hartree-Fock model is thus surprising.

**Key words:** one-dimensional transport, mesoscopic ring, persistent current, electron-electron interaction

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Magnetic flux applied through the opening of a mesoscopic conducting ring gives rise to a persistent electron current circulating along the ring [1]. Here we study the persistent current of interacting spinless electrons in a one-dimensional (1D) ring with a single scatterer. For non-interacting electrons the persistent current \( I \) depends on the magnetic flux \( \phi \) and ring length \( L \) as [2]

\[
I = \left( ev_F/2L \right) |\delta_{k_F}| \sin(\phi'), \tag{1}
\]

if \(|\delta_{k_F}| \ll 1. \) In eq. (1) \( \phi' \equiv 2\pi\phi/\phi_0, \phi_0 = h/e \) is the flux quantum, \( \delta_{k} \) is the electron transmission amplitude through the scatterer, \( k_F \) is the Fermi wave vector, and \( v_F \) is the Fermi velocity. For a repulsive electron-electron interaction the spinless persistent current was derived in the Luttinger liquid model [2]. For \( L \to \infty \)

\[
I \propto L^{-\alpha-1} \sin(\phi'), \tag{2}
\]

where \( \alpha > 0 \) depends only on the e-e interaction.

In this work we find similar results in the Hartree-Fock model. We consider \( N \) interacting 1D electrons with free motion along a circular ring threaded by magnetic flux \( \phi = BS = AL \), where \( S \) is the area of the ring, \( B \) is the magnetic field threading the ring, and \( A \) is the magnitude of the vector potential. In the Hartree-Fock model the many-body wave function is the Slater determinant of single-electron wave functions \( \psi_k(x) \). These wave functions obey the Hartree-Fock equation

\[
\left[ \frac{\hbar^2}{2m} \left( -i \frac{\partial}{\partial x} + \frac{2\pi}{L} \frac{\phi}{\phi_0} \right)^2 + \gamma \delta(x) + U_H(x) + U_F(k, x) \right] \psi_k(x) = \epsilon_k \psi_k(x), \tag{3}
\]

with cyclic boundary condition \( \psi_k(x + L) = \psi_k(x) \), where \( m \) is the electron effective mass, \( x \) is the electron coordinate along the ring, \( \gamma \delta(x) \) is the potential of the scatterer, the Hartree potential is given by

\[
U_H(x) = \sum_{k'} \int dx' V(x - x') |\psi_{k'}(x')|^2, \tag{4}
\]

the Fock term is written as an effective potential

\[
U_F(k, x) = -\frac{1}{\psi_k(x)} \times \sum_{k'} \int dx' V(x - x') \psi_{k'}(x') \psi_{k'}(x), \tag{5}
\]

and \( V(x - x') \) is the electron-electron (e-e) interaction.

We solve equation (3) coupled with the potentials (4) and (5) using self-consistent numerical iterations [4]. We obtain numerically the single-particle states \( \psi_k(x) \)
and $\varepsilon_L$, the energy of the Hartree-Fock groundstate, $E$, and eventually the persistent current $I = -\partial E/\partial \phi$.

We present results for the GaAs ring with electron density $n = 5 \times 10^7$ m$^{-1}$, effective mass $m = 0.067 m_0$, and e-e interaction $V(x - x') = V_0 e^{-|x-x'|/d}$,\footnote{where $V_0 = 34$ meV and $d = 3$ nm. The interaction (6) is short-ranged. It emulates screening and allows comparison with correlated models [2,3] which also use the e-e interaction of finite range.}

We study rings with a strong scatters ($|\tilde{t}_{kF}| \ll 1$), for which the asymptotic behavior with $L$ is reachable for not too large $L$ [3]. To show results typical of $|\tilde{t}_{kF}| \ll 1$, we use the $\delta$ barrier with transmission $|\tilde{t}_{kF}| = 0.03$.

Panel a of figure 1 shows in log scale the persistent current $L I(\phi' = \pi/2)$ as a function of $L$. The full line is the power law $L I \propto L^{-\alpha}$ predicted by equation (2). For weak e-e interaction ($\alpha \ll 1$) it holds\footnote{[5] for small $\tilde{t}_{kF}$. We obtain the scaling law (2) including the proportionality constant $\text{const} = eV_F |\tilde{t}_{kF}| d^{\alpha}/2$. This scaling law is presented in panel c. It can be seen that the results of panels b and c are in good accord.}

\[ \psi_k(x = -L/2) = e^{ikx} + r_k e^{-ikx}, \psi_k(x = L/2) = t_k e^{ikx}, \]

where $r_k$ is the reflection amplitude and $t_k$ is the transmission amplitude (analogously for the electrons entering the wire from the right). We have solved this Hartree-Fock problem self-consistently and we have evaluated the Landauer conductance $(e^2/h)|\tilde{t}_{kF}|^2$.

The result is shown in figure 2 together with the square of $L I$ for the equivalent ring. The conductance scales like $L^{-2\alpha}$ and so does the square of $L I$.

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\section*{References}
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