On the lifetime of the $2^+_1$ state in $^{10}$C

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The lifetime of the $J^p=2^+_1$ state in $^{10}$C was measured using the Doppler Shift Attenuation Method following the inverse kinematics $p(^{10}$B,n)$^{10}$C reaction at 95 MeV. The $2^+_1$ state, at 3354 keV, has $\tau = 219\pm(7)_{\text{stat}}\pm(10)_{\text{sys}}$ fs corresponding to a $B(E2) \downarrow$ of $8.8(3)\ e^2$fm$^4$. This measurement, combined with that recently determined for $^{10}$Be ($9.2(3)\ e^2$fm$^4$), provides a unique challenge to ab-initio calculations, testing the structure of these states, including the isospin symmetry of the wave functions. Quantum Monte Carlo calculations using realistic two- and three-nucleon Hamiltonians that reproduce the $^{10}$Be $B(E2)$ value generally predict a larger $^{10}$C $B(E2)$ probability but with considerable sensitivity to the admixture of different spatial symmetry components in the wave functions, and to the three-nucleon potential used.

I. INTRODUCTION

A new generation of ab-initio calculations based on realistic nucleon-nucleon forces has deepened our understanding of how nuclei work. Comparison of their predictions to precise new experimental data \cite{1,2} has guided the ab-initio calculations, testing the structure of these states, including the isospin symmetry of the wave functions, and much refined theory, we can re-address this interesting problem at a level of precision which should provide stringent tests of ab-initio calculations.

Conceptually, the $A=10$ mirror nuclei are interesting. $^{10}$Be$_6$ can be thought of as two alpha particles with isospin $T=0$, bound to a correlated pair of s-wave neutrons, which are usually outside the alpha clusters. Similarly $^{10}$C$_4$ consists of two alpha particles surrounded by a loosely bound pair of protons. Naively one might expect the two protons to result in a bigger $B(E2)$ value for $^{10}$C than for $^{10}$Be. For example, a simple classical isoscalar liquid drop model \cite{3} is driven by charge: the carbon isotope has a larger quadrupole moment ($\sim Z$) and the decay strength is expected to be larger in $^{10}$C compared to $^{10}$Be by a factor $(6/4)^2 = 2.25$. In contrast, simple shell models \cite{4,5} always have properly constructed quantum-mechanical wave functions for states and allow the electromagnetic decay to be separated into isoscalar and isovector components. For the $A=10$ system, the isovector contributions are predicted to be small, symmetric, and act to enhance the decay in $^{10}$Be and suppress it in $^{10}$C, resulting in a $^{10}$C decay that should be $\sim$10\% lower than that in $^{10}$Be. The relative $B(E2)$ values in $^{10}$C and $^{10}$Be thus represent an interesting test of nuclear modeling and of the isospin dependence of electromagnetic decays.

Within the $A=10$ system, $^{10}$C is the more exotic partner. It has only one bound excited state, with $J^p=2^+$, at 3354 keV. It becomes unbound at 4006 keV at which point it can disintegrate into $^9$B+$p$. Its mirror partner, $^{10}$Be, has six bound states below a breakup threshold of 6812 keV where the $^9$Be+$n$ channel opens. The first excited state of $^{10}$Be also has $J^p=2^+$ and lies at 3368 keV, a sign that, in excitation energy at least, these configurations are similar, despite the difference in binding energy.

A pioneering Doppler Shift Attenuation Method (DSAM) experiment by Fisher et al., \cite{6} in 1968 was aimed at understanding these issues and testing the intermediate-coupling shell model predictions by measuring the decay rates from the first excited states in $^{10}$C and $^{10}$Be. However, the DSAM technique was new and only $\sim$20\% precision could be achieved. $^{10}$C was found to be slightly more collective, but not in glaring disagreement with the shell model, given the large experimental uncertainties. We have recently remeasured the lifetime of the $2^+_1$ state in $^{10}$Be \cite{7} and determined $B(E2; 2^+_1 \rightarrow 0^+_1) = 9.2(3)\ e^2$fm$^4$. Comparing this with the value $B(E2; 2^+_1 \rightarrow 0^+_1) = 12.2\pm1.9\ e^2$fm$^4$ measured by Fisher et al., for $^{10}$C still supports a larger $B(E2)$ rate in $^{10}$C. However, the carbon value has substantial experimental uncertainties, so now, with far superior experimental tools and much refined theory, we can re-address this interesting problem at a level of precision which should provide stringent tests of ab-initio calculations.
II. EXPERIMENT

The $2^+_1$ state in $^{10}$C was populated in the inverse kinematics $p(^{10}$B, $n)^{10}$C reaction. Beams of $^{10}$B ions of $\sim 1$ pA and 95 MeV were produced by the ATLAS accelerator at Argonne National Laboratory. Targets consisted of thin layers of CH$_2$ on thick backings of copper and gold. $^{10}$C nuclei recoiling along the beam direction were selected by the Argonne Fragment Mass Analyzer (FMA) positioned 90 cm downstream of the target and subtending 1° around the beam direction. The $^{10}$C nuclei were produced at recoil velocities of $\beta = v/c \sim 13\%$ and emerged from the backing target layer with $\beta \sim 10\%$. To satisfy the FMA energy acceptance window, the recoils had to be further slowed down to $\beta \sim 8\%$. This was achieved through a series of titanium degrader foils placed at the entrance to the FMA. $^{10}$C ions with $q = 6^+$ were transported to the focal plane while most non-interacting beam particles were rejected by the FMA. The selection of $q = 6^+$ ions was very effective for suppressing scattered beam particles. The transmitted ions first passed through two PPAC detectors before being stopped 50 cm behind the focal plane in a 30-cm deep, two-electrode ionization-chamber operated at 50 torr. Gamma rays were detected with the Gammasphere array consisting of 100 Compton suppressed HPGe detectors in 16 azimuthally symmetric rings from $\theta = 34^\circ$ to $163^\circ$ relative to the beam direction.

Figure 1(a) shows a typical energy loss ($\Delta E$) versus total energy ($E_{\text{tot}}$) spectrum obtained from the ionization chamber. The locus with the largest $\Delta E$ (solid red circled region) corresponds to direct population of the $2^+_1$ state in $^{10}$C. The large spread in total energy for these recoils stems from energy scattering and straggling in the backing and degrader foils. The wide strip of counts below the $^{10}$C recoils is identified with $^{10}$B scattered beam. The pressure in the ionization chamber was not sufficient to fully stop the highest energy $^{10}$C recoils, resulting in a wrap around feature (punch through) in the $\Delta E$ versus $E_{\text{tot}}$ plot. Direct population of the ground state of $^{10}$C is also observed, although due to the punch through, it falls appears in the same $\Delta E$ versus $E_{\text{tot}}$ area as the $^{10}$B scattered beam. The $\gamma$-ray spectrum obtained by gating on the excited $^{10}$C recoils is given in Fig. 1(b), showing only the 3354-keV $2^+_1 \rightarrow 0^+_1$ transition in $^{10}$C.

The current setup offers several advantages over the prior DSAM measurement performed with a regular-kinematics reaction. To compare the present technique with a normal kinematics DSAM measurement, the original Fisher et al., experiment was also repeated with Gammasphere. This regular-kinematics experiment was performed with a 9.5-MeV proton beam incident on a $^{10}$B target followed by a gold backing. Gamma rays were detected with the Gammasphere array. A comparison of the spectra obtained from the regular- and inverse-kinematics reactions is given in Fig. 2 for the same angle group ($\theta = 130^\circ$) in Gammasphere. The regular kinematics spectrum (Fig. 2a)) is very complicated, with considerable background from reactions of the high-energy protons on the $^{10}$B target, the gold backing and scattering in the target chamber. In contrast, requiring detection of a recoiling $^{10}$C residue by the FMA almost entirely suppresses the background (Fig. 2b)), providing a spectrum where only the $2^+_1 \rightarrow 0^+_1$ transition in $^{10}$C is observed. Selection of the $^{10}$C recoils with the FMA also provides a well-defined angle between the recoil velocity vector and the direction (and subsequent detection) of the $\gamma$-rays. Recoil detection comes at the cost of poorer statistics; however, with almost no background the peak centroids can be reliably determined to 1-2 keV. With the regular-kinematics reaction, the recoiling $^{10}$C ions have a kinetic energy of a few MeV, an energy regime where stopping powers are poorly determined and the $\gamma$-ray energy shifts are only a few keV. In the inverse-kinematics reaction, $^{10}$C nuclei were produced at a very high recoil energy ($E \sim 80$ MeV). This allows the measurement to be performed in a velocity regime where the stopping is 99.99% electronic and most precisely known. The high recoil velocity also produces large Doppler shifts. With the set of Gammasphere angles, the range of forward-shifted to backward-shifted peaks spanned more than 800 keV.

In a DSAM measurement the lifetime is derived from
the difference between production and emission velocities. For the inverse-kinematics reaction, the distribution of recoil products is very forward peaked and there are two solutions for residues moving along the initial beam direction, depending on the direction of the emitted neutron. For this particular experiment, recoils emitted at 0° relative to the beam direction can have recoil velocities of 75 or 82 MeV. The FMA was always set to transmit the higher-energy recoil group. The initial β value at production is first measured using a self-supporting CH₂ target and correcting for the small energy loss in the target. Degrader foils of the same thickness as the backed-target experiments were used to replicate the DSAM measurements and ensure that the FMA entrance conditions were the same for both the self-supporting and backed targets.

To determine the average velocity of the recoils at the time of γ-ray emission, the centroid of the 3354-keV transition was determined for each of the 16 Gammasphere angle groups. The solid curve shows the result of the relativistic Doppler shift formula taking the best-fit value of β. (b) Similar to (a) but normalized to $\sqrt{1-\beta^2}/(1-\beta \cos(\theta))$. Lines include the best fit value of β (solid line), maximum β value allowed by the reaction kinematics (dashed line), and the β value corresponding to the previous lifetime measurement (dotted line).

![Fig. 3](image-url)

FIG. 2: Comparison of γ-ray spectra for ¹⁰C decays from DSAM in (a) regular and (b) inverse kinematics. Both spectra correspond to the $\theta = 130^\circ$ angle group in Gammasphere. Reaction details are included in the figure.

![Fig. 3](image-url)

FIG. 3: (Color online) (a) Measured centroids of the 3354-keV transition for each Gammasphere angle group. The solid curve shows the result of the relativistic Doppler shift formula taking the best-fit value of β. (b) Similar to (a) but normalized to $\sqrt{1-\beta^2}/(1-\beta \cos(\theta))$. Lines include the best fit value of β (solid line), maximum β value allowed by the reaction kinematics (dashed line), and the β value corresponding to the previous lifetime measurement (dotted line).

![Fig. 3](image-url)
detected in the ionization chamber, per beam intensity, per time) of a backed target relative to a commercially made self-supporting CH₂ target of known thickness. A series of degrader foils, identical to the target backing, were placed behind the self-supporting CH₂ target to achieve the same energy recoils into the FMA as with the backed targets. The thickness of the CH₂ layer on the backed targets was measured before and after the DSAM measurement via a relative yield measurement as described above. An approximately 20% reduction in yield was observed. For the DSAM analysis, the average of the thickness before and after the measurement was used and the full range of thickness taken as the uncertainty. As the target layers are thin, do not provide much stopping and the ions move very quickly through them, the target thickness does not contribute greatly to the systematic uncertainty. The characterization of carbon and boron ions slowing in CH₂, copper and gold was taken from the SRIM [11] and MSTAR [12] packages. The two models differ in stopping powers on the order of 3% in the relevant velocity regime. These differences were incorporated into the systematic uncertainty.

The lifetime of the 3354-keV level was measured in five separate experiments. The target characteristics and the extracted lifetimes are summarized in Table I. The weighted mean value is \( \tau = 219 \pm 5(\text{stat}) \pm 10(\text{syst}) \) fs which implies \( B(E2; 2^+ \rightarrow 0^+) = 8.8(3) \) e²fm⁴. This lifetime is substantially longer than the previous value obtained by Fisher et al. [8]. Figure 3 gives a comparison of the current measurements (solid symbols) and those of Ref. [8] (open symbols). Clearly, the data are now much more better constrained for investigating the symmetry of the wave functions, but theoretical guidance is needed to infer the meaning of the result.

### III. THEORY

Empirically, assuming charge symmetry for the wave functions, the transition strengths can be written as

\[
B(E2) = \frac{|M(E2)|^2}{5} = |AT + BT_z|^2
\]

where \( M(E2) \) is the reduced matrix element and we use the convention that \( T_z = +\frac{1}{2} \) for the neutron. The new \(^{10}\text{Be} \) and \(^{10}\text{C} \) data can be used to infer that the isoscalar term is dominant, \( A = 3.00(1) \) e fm², while the isovector term is much smaller, \( B = 0.03(3) \) e fm², a 1% effect. In conventional shell-model calculations, isospin enhancements or effective charges, \( T \) are introduced to account for effects such as core polarization: \( A = A' \epsilon(0) \) and \( B = B' \epsilon(1) \), where \( A' \) and \( B' \) are constants derived for a particular wave function. Very early shell model calculations of Cohen and Kurath [6,13] for the \( A = 10 \) system gave predictions for \( B(E2) \) strengths in the form of Eq. (1). These are included in Fig. 3 (solid black line). These p-shell, mirror-symmetric wave functions provide the correct slope for describing the transition strengths between \(^{10}\text{C} \) and \(^{10}\text{Be} \), however, overestimate the overall magnitude (Fig. 3) due to the use of very simple isoscalar and isovector enhancements, \( \epsilon(0) = 2 \) and \( \epsilon(1) = 1 \). Using isoscalar and isovector enhancements now broadly accepted for p-shell calculations [14,15], \( \epsilon(0) = 1.7 \) and \( \epsilon(1) = 0.6 \), one obtains \(^{10}\text{Be} \) \( B(E2; 2^+_1 \rightarrow 0^+_1) = 9.7 \) e²fm⁴ and \(^{10}\text{C} \) \( B(E2; 2^+_1 \rightarrow 0^+_1) = 9.1 \) e²fm⁴, close to the experimental results. These calculations are included in Fig. 3 (dotted red line).

There are two ways to interpret the dominant isoscalar term along with a nearly zero isovector contribution to the \( B(E2) \) strengths. Conceptually, this may be perceived as the two-alpha cluster in these nuclei being the same in \(^{10}\text{Be} \) and \(^{10}\text{C} \) and their respective valence particles contributing little. This mirror-symmetric interpretation conforms well to the predictions of the standard shell model. One can alternately consider the possibility that mirror symmetry is not preserved; for example, the alpha-cluster spacing is modified, but this effect is offset by differing contributions from the valence particles. This effect can be investigated through the use of more sophisticated models.

### TABLE I: Mean lifetimes from different target and backing combinations determined for the 3354-keV level in \(^{10}\text{C} \). The CH₂ thicknesses are the average of the measured values at the beginning and end of each DSAM measurement.

| Target (CH₂) | Backing | \( \tau \) (fs) | \( \Delta \tau_{\text{stat}} \) (fs) |
|-------------|---------|----------------|----------------|
| 105         | 23.9 Cu | 221 ± 8        |                |
| 150         | 31.0 Au | 215 ± 10       |                |
| 170         | 23.8 Cu | 219 ± 13       |                |
| 80          | 24.0 Au | 198 ± 18       |                |
| 300         | 14.9 Au | 216 ± 23       |                |

FIG. 4: (Color online) \( B(E2; 2^+_1 \rightarrow 0^+_1) \) transition strengths (in e²fm⁴) for \(^{10}\text{C} \) and \(^{10}\text{Be} \). Open symbols are the results of Ref. [8] while solid symbols are the current work and the recent measurement of Ref. [8].
The variational Monte Carlo (VMC) and Green’s function Monte Carlo (GFMC) methods have been very useful in improving our understanding of light nuclei and successful in reproducing the electric quadrupole collectivity in $^{10}$Be, without resorting to the use of any effective charges. Using realistic two- and three-body forces and operators (including explicit charge-symmetry-breaking terms), this is a good approach for exploring $^{10}$C and the symmetry of the $A = 10$ wave functions. More details on the GFMC method of calculating transition strengths are given in Ref. [18].

The VMC calculations use trial wave functions containing non-central, two- and three-body correlation operators acting on an antisymmetrized one-body wave function, $\Phi(JMT T_z)$, which determines the quantum numbers of the state being computed. The $\Phi(JMT T_z)$ wavefunction is expanded in $LS$-basis functions:

$$\Phi(JMT T_z) = \sum_{LS[n]} \beta(2S^{\pm 1}L[n], JTT_z) \Phi(2S^{\pm 1}L[n], JMT T_z),$$

where the amplitudes $\beta(2S^{\pm 1}L[n], JTT_z)$ are found from a diagonalization of the Hamiltonian. For $^{10}$Be, we construct states from the three highest spatial symmetries as denoted by the Young diagram [n] (see Ref. [19]). This gives three basis functions for the $0^+$ ground state: $^1S[442]$, $^3P[4411]$, and $^3P[433]$, while the $2^+$ states have six basis functions: $^1D[442]$, $^1D[442]_t$, $^3P[4411]$, $^3P[433]$, $^3F[4411]$, and $^3F[433]$. Note that there are two linearly independent $^1D[442]$ basis states; the distinction between them is arbitrary and we choose to express them as eigenfunctions of the quadrupole operator, with the subscript indicating the sign of the quadrupole moment. The VMC $E2$ matrix element is

$$M(E2) = \sum_{L'S'[n'],LS[n]} \beta(L'[n'], J = 0, T = 1, T_z) \beta(2S^{\pm 1}L[n], J = 2, T = 1, T_z)$$

$$\times \langle C\Phi(2S^{\pm 1}L[n'], J = 0, T = 1, T_z) | E2 | C\Phi(2S^{\pm 1}L[n], J = 2, T = 1, T_z) \rangle,$$

where $C$ denotes the two- and three-body correlations and there are $3 \times 6$ contributions to the sum. The $E2$ operator does not change spatial symmetry, so the only big contributions are those from $^1D[442]$ or $^1D[442]_t$ to $^1S[442]$, $^3P[4411]$ or $^3F[4411]$ to $^3P[4411]$, and $^3P[433]$ or $^3F[433]$ to $^3P[433]$ (the $C$ do not conserve the spatial symmetry so there are small non-zero matrix elements for the other possibilities). These individual contributions, calculated with wave functions for the AV18 two-nucleon and Illinois-7 three-nucleon potentials (AV18+IL7) [20, 21], are shown in Fig. 5 for isospin-symmetric basis states, i.e., the parameters in $\Phi(2S^{\pm 1}L[n], J = 2, M, T = 1, T_z)$ are independent of $T_z$. The diagonalization of the two $^1D[442]$ states into the quadrupole basis was done for $^{10}$Be and not changed for $^{10}$B and $^{10}$C. As can be seen, these isospin-symmetric calculations can give very different $T_z$ behaviors, depending on the pair of $2S^{\pm 1}L[n]$ states being used. Calculations with basis states containing different variational parameters give very similar results to those in the figure; we believe that the trends shown result from the different $2S^{\pm 1}L[n]$ values of the pairs and thus also would be obtained with other realistic Hamiltonians and even for the corresponding harmonic-oscillator shell model states. This means that the nearly $T_z$-independent $B(E2)$ strengths obtained in the shell model calculation require a specific combination of states.

Figure 5 shows VMC and GFMC calculations of $B(E2)$ values for the $2^+_1$ ($Q < 0$) state assuming isospin symmetric wave functions. The wave functions were computed for $^{10}$Be using the AV18 interaction alone or with the IL2 or IL7 three-body potentials. As shown in Ref. [4], the $2^+_1$ state of $^{10}$Be has a negative quadrupole moment and a strong $E2$ decay to the ground state for all the Hamiltonians. (For AV18 alone, the energies of the two $2^+$ states are nearly degenerate, so we choose to identify the $Q < 0$
Isospin symmetry of the wave functions is certainly only an approximation. Due primarily to the increasing Coulomb potential energy going from $^{10}$Be to $^{10}$C, the $^{10}$C states under consideration are less bound (0.5 MeV vs. 3.5 MeV for the $2^+_1$ state) and, hence, should be more diffuse. This can be studied by performing separate calculations for each nucleus. We have done such calculations for the AV18+IL7 Hamiltonian. The one-body parts of the VMC wave functions are solutions of Woods-Saxon wells plus an average Coulomb potential [11]; the strength of the Coulomb term is proportional to the number of $p$-shell protons. Separate diagonalizations were made for each nucleus, so the $\beta$ are also different. The GFMC propagations are still made in a good isospin basis, but the isoscalar Coulomb potential used reflects the total charge of the nucleus [13]. The results of these calculations are compared with the isospin-symmetric AV18+IL7 calculations in Fig. 7. The independent calculations for $^{10}$C and $^{10}$B are not very different from the isospin-symmetric extrapolations from the $^{10}$Be results. Unfortunately, the already too large value for the $^{10}$C $B(E2)$ is further increased.

In light of the apparent failure of these GFMC calculations to reproduce the $B(E2; 2^+_1 \rightarrow 0^+_1)$ transition strength in $^{10}$C, it is important to consider possible shortcomings of the calculation which could cause the discrepancy. One possibility is that, with the weaker binding of the $2^+_1$ state in $^{10}$C compared to $^{10}$Be, contributions from beyond the $p$-shell might become important. In fact, the VMC trial functions already have a fair admixture of $sd$-shell and higher components due to two-body tensor correlations in the $C$ of Eq. 3 (see Ref. [16]), and these are further enhanced in the GFMC propagation. However, to further test this possibility, we constructed alternative clustered VMC trial functions with explicit $sd$-shell components. These wave functions combine a $^8$Be($0^+$ or $2^+$) core with two final nucleons in $p$- or $sd$-shell orbitals (with appropriate Coulomb terms), all $L$-coupled to give the appropriate total $J^\pi$. The $0^+ (2^+)$ states have four (seven) $p$-shell and eight (seven) $sd$-shell components; separate diagonalizations for the corresponding $\beta$s are made for each $T_z$. For a Hamiltonian containing AV18 and the Urbana IX (UIX) three-body potential [22], the $sd$-shell $\beta$s contribute only 2.5% of the total wave function in the $^{10}$Be $0^+_1$ state, but 21% in the $0^+_2$ state; these numbers increase to 3.4% and 28.9%, respectively, in $^{10}$C. The $2^+_1$ and $2^+_2$ states both have 4% or less $sd$-shell contributions, with only slightly greater amounts in $^{10}$C than in $^{10}$Be. (Interestingly, 96% of the $2^+_1$ state has a $^8$Be($2^+$) core in this construction, clearly indicating it is the $j=2$ member of the $K=0$ rotational band.) Consequently, inclusion of the $sd$-shell components has only a very minor effect on the $B(E2)$ values; in $^{10}$Be using $p$-shell only components gives $B(E2) = 8.6 e^2$fm$^4$, while adding $sd$-shell components raises it to 9.2 $e^2$fm$^4$. The corresponding $B(E2)$ values in $^{10}$C are 9.6 and 11.6 $e^2$fm$^4$, respectively, showing the same moderate change with $T_z$ as the AV18+IL2 and AV18+IL7 Hamiltonians.

![FIG. 6: (Color online) VMC and GFMC calculations of $B(E2)$ strengths for the $2^+_1 (Q < 0)$ state assuming isospin symmetric wave functions. The wave functions were computed separately for each nucleus using different VMC correlations and independent GFMC propagation. VMC results are shown as open symbols and dashed lines; GFMC results are solid symbols and solid lines.](image1)

![FIG. 7: (Color online) VMC and GFMC calculations of $B(E2)$ for the lowest $(Q < 0)$ $2^+$ state with the AV18+IL7 Hamiltonian. The wave functions were computed separately for each nucleus using different VMC correlations and independent GFMC propagation. VMC results are shown as open symbols; GFMC results are solid symbols. The dashed lines show the corresponding isospin symmetric results from Fig. 6.](image2)

state as the $2^+_1$ state.) The reduced matrix elements for $^{10}$C were obtained by interchanging protons and neutrons in the $^{10}$Be wave functions and the $^{10}$B reduced matrix elements are the average of the $^{10}$Be and $^{10}$C, i.e., the wave functions are isospin symmetric with those of $^{10}$Be. There is considerable variation in the $T_z$ behavior of the $B(E2)$ strengths for the different Hamiltonians in the VMC calculations. This is presumably due to the different $\beta(2S+1L^2mL, J=2, T=1, T_z=+1)$ amplitudes from the separate diagonalizations. The GFMC generally preserves, or even enhances, these different trends which suggests a strong sensitivity of the isovector $B(E2)$ to the three-body force.
A more likely possibility is simply that the Hamiltonians tested are not adequate for these transitions. The first priority in theoretically modeling the nuclear Hamiltonian has been obtaining good energies for the states in question – both absolute binding energy of the nucleus and excitation energies of the higher states. These energies are shown in Table II for the various Hamiltonians tested are not adequate for these transitions. The charge radii, quadrupole moments and the B(E2) transition strengths in e²fm².

| A$^\text{Be}$ | Observable | NCSM CDB2k | AV18 | GFMC AV18+IL2 | AV18+IL7 | Expt. |
|------------|------------|------------|------|--------------|----------|------|
| $|E_{gs}(0^+)|$ | 56.5(5) | 50.1(1) | 60.4(4) | 64.1(3) | 64.98 |
| $E_x(2^+_1)$ | 3.6(1) | 2.9(1) | 5.0(4) | 3.4(3) | 3.37 |
| $E_x(2^+_2)$ | 4.8(1) | 3.0(1) | 5.8(4) | 5.3(3) | 5.96 |
| $r_c$ | 2.65(5) | 2.47(1) | 2.33(1) | 2.33(1) | 2.36(2) |
| $Q(2^+_1)$ | 2.65(5) | 2.47(1) | 2.33(1) | 2.33(1) | 2.36(2) |
| $Q(2^+_2)$ | 3.5(3) | 5.8(1) | 2.0(1) | 4.5(1) |
| $B(E2; 2^+_1 \rightarrow 0^+)$ | 9.8(4) | 10.5(4) | 8.1(3) | 8.8(4) | 9.2(3) |
| $B(E2; 2^+_2 \rightarrow 0^+)$ | 0.2(2) | 3.4(2) | 3.3(2) | 1.8(1) | 0.11(2) |

| A$^\text{B}$ | Observable | NCSM CDB2k | AV18 | GFMC AV18+IL2 | AV18+IL7 | Expt. |
|------------|------------|------------|------|--------------|----------|------|
| $|E_{gs}(0^+; 1)|$ | 55.9(5) | 48.3(3) | 64.6(4)* | 62.6(2) | 63.01 |
| $E_x(2^+_1; 1)$ | 2.4(1) | 2.9(4)* | 5.0(5) | 3.6(3) | 3.42 |
| $E_x(2^+_2; 1)$ | 3.0(4)* | 5.8(5)* | 5.2(5) |
| $Q(2^+_1; 1)$ | 5.8(1)* | 5.0(5)* | 5.2(5) |
| $Q(2^+_2; 1)$ | 7.9(1)* | 2.0(1)* |
| $B(E2; 2^+_1; 1 \rightarrow 0^+; 1)$ | 6.7(5)* | 11.4(5)* | 11.4(6) |
| $B(E2; 2^+_2; 1 \rightarrow 0^+; 1)$ | 8.9(4)* | 1.0(1)* |

| A$^\text{C}$ | Observable | NCSM CDB2k | AV18 | GFMC AV18+IL2 | AV18+IL7 | Expt. |
|------------|------------|------------|------|--------------|----------|------|
| $|E_{gs}(0^+)|$ | 51.9(5) | 45.8(3)* | 61.7(4)* | 60.0(2) | 60.32 |
| $E_x(2^+_1)$ | 3.6(1) | 2.7(3)* | 4.7(4)* | 3.2(3) | 3.35 |
| $E_x(2^+_2)$ | 4.3 | 2.8(3)* | 5.4(4)* | 5.1(5) |
| $r_c$ | 2.77(1)* | 2.55(1)* | 2.55(1)* |
| $Q(2^+_1)$ | 1.1(12) | 1.1(12) | 1.1(12) |
| $Q(2^+_2)$ | 10.0(2)* | 4.2(2)* | 4.2(2)* |
| $B(E2; 2^+_1 \rightarrow 0^+)$ | 10(2) | 3.7(5)* | 15.3(6)* | 15.3(1.4) | 8.8(3) |
| $B(E2; 2^+_2 \rightarrow 0^+)$ | 17.0(8)* | 0.0(1)* | 0.2(1) |

We have performed a precise measurement of the lifetime of the first (and only) excited bound state in $^{10}$C. The new measurement implies a matrix element that is considerably smaller than previously reported. It is only 2% different from its mirror transition in $^{10}$Be. GFMC calculations with our best Hamiltonian fail to reproduce this near equality. Different basis states have very different trends of the $B(E2)$ going across this isomultiplet.
Thus, if the wave functions are nearly isospin symmetric, our calculations fail to get the correct mixture of states. Inclusion of explicit sd-shell amplitudes into the trial wave functions, while slightly different in $^{10}$Be and $^{10}$C, appears to be a small effect that does not resolve the discrepancy. The calculations show only a small amount of isospin symmetry breaking. The definite experimental signal of such symmetry breaking for this isovector transition operator would be an observation that the matrix element for the transition between the first $T = 1$, $2^+$ and $0^+$ states in $^{10}$B is not the average of those for $^{10}$Be and $^{10}$C. Such a measurement is underway.

Obtaining a better theoretical result would appear to require a Hamiltonian that produces a very specific combination of spatial symmetry components. It may be possible to develop improved three-nucleon potentials that provide both good energies and transition strengths at the same time. Recently, it has been suggested that using the weak $\beta$-decay of tritium in addition to the three-nucleon binding energies is a good way to select an optimal combination of low-energy constants in chiral N3LO three-nucleon potentials \[28\]. In a similar manner, electromagnetic transitions like the ones studied here may provide a valuable constraint on suitable Hamiltonians, i.e. on the 3-body term.

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