Amplification of Hypersound in Graphene with degenerate energy dispersion

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Abstract

Hypersound amplification of acoustic phonons at $q l >> 1$ in Graphene was theoretically studied. The degenerate energy dispersion $\varepsilon(p)$ near the Fermi level and $kT << 1$ were considered. A linear dependence of the absorption coefficient ($\Gamma/\Gamma_0$) on $\omega_q$ was observed where the graph obtained decreases with increasing drift velocity $V_D$. The result qualitatively agreed with an experimentally observed acoustoelectric current via the Weinrich relation in a gate-controlled graphene. For the dependence of $\Gamma/\Gamma_0$ on $V_D/V_s$, the non-linear graph obtained satisfied the Cerenkov effect. When $V_D/V_s < 1$, the absorption increases with increasing $\omega_q$. Similarly, when $V_D/V_s > 1$, the amplification obtained increases with increasing $\omega_q$. For a better understanding of the dependence of $\Gamma/\Gamma_0$ on $\omega_q$ and $V_D/V_s$, a 3D graph was plotted. The values obtained are indicated on the graphs. It is interesting to note from this study that, the hypersound amplification caused by intraband transition in graphene can be analysed for frequencies above $10THz$. This study permit the use of Graphene as hypersound phonon laser (SASER).

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Introduction

Graphene, a member of the Carbon allotropes has exceptional properties for future nanoelectronics \[1, 2, 3, 4\]. It is an ideal two-dimensional electron gas (2DEG) system made up of one layer of Carbon atom having a high electron mobility ($\mu$) at room temperature with high mechanical and thermodynamic stability \[5\]. Several unusual phenomena such as half-integer quantum Hall effect \[6\], non-zero Berry’s phase \[7\], and minimum conductivity \[8\] have been observed experimentally in Graphene. The most interesting property of Graphene is its linear energy dispersion $E = \pm \hbar V_F |k|$ (the Fermi velocity $V_F \approx 10^8 ms^{-1}$) at the Fermi level with low-energy excitation. This makes graphenes applicable in advance electronics and optoelectronic devices such as sub-terahertz Field-effect transistors \[9\], infrared transparent electrodes \[10\] and THz plasmonic deives \[11\]. Currently, among the various studies on Graphene attracting much attention is the generation and detection of hypersound amplification or absorption of acoustic phonons \[12\]. It is known that, when an acoustic phonon passes through a semiconductor, it may interact with various elemental excitations which may lead to amplification or absorption of the phonons. The idea of acoustic wave amplification in bulk material was theoretically predicted by Tolpygo (1956), Uritskii \[13\], and Weinreich \[14\] and in N-Ge by Pomerantz \[15\]. Hypersound generation in bulk \[16\] and low-dimensional materials such as Superlattices \[17, 18, 19, 20\], Cylindrical Quantum Wire \[21\], Quantum Wells \[22\] and Graphenes Nanoribbons (GNR) \[23\] have been studied. Akin to Cerenkov acoustic-phonon emission, when the drift velocity of electrons $V_D$ exceeds the sound velocity
(V_s) of the host material \[24\] lead to amplification of the acoustic-phonons or when \(V_D < V_s\) causes absorption. This has been utilised experimentally to confirm the breakdown of quantum Hall effect \[25\], the generation of coherent phonon-polariton radiation \[26\], and large acoustic gain in coherent phonon oscillators in semiconductors \[27\]. Furthermore, the emission and absorption of acoustic-phonons is used to provide detailed information on the excitation and relaxation mechanisms in semiconductors via deformation potential, where the effect of interactions can be used to determine the physical properties of the material. In particular, acoustic-phonons providing terahertz (10^{12} Hz) hypersonic sources can lead to the attainment of phonon laser or SASER \[28\] \[29\] in graphene via Cerenkov effect which is an intense field of research. Following the works of Nunes and Fonseca \[32\], Zhao et. al \[33\] proposed the possibility of attaining Cerenkov acoustic-phonon emission in Graphene whilst Insepov et. al \[31\], performed experimentally the surface acoustic wave Amplification by D.C voltage supply in Graphene. In this paper, the Cerenkov effect in graphene is archived where \(\frac{V_D}{V_s} > 1\) gives hyper-sound amplification and \(\frac{V_D}{V_s} < 1\) gives absorption of acoustic-phonons. The motivation for this work is to provide the theoretical framework that can lead to the attainment of SASER in Graphene for use as a phonon spectrometer, for generation of high-frequency electric oscillation, and as a non-destructive testing of microstructure and acoustic scanning system. The paper is organised as follows: In theory section, the theory underlying the amplification (Absorption) of acoustic-phonon via Cerenkov effect is presented. In the numerical analysis section, the final equation is analysed and presented in a graphical form. Lastly, the conclusion is presented in section 4.
Theory

We will proceed following the works of [32], here the acoustic wave will be considered as phonons of frequency ($\omega_\mathbf{q}$) in the short-wave region $q l >> 1$ ($q$ is the acoustic wave number, $l$ is the electron mean free path). The kinetic equation for the acoustic phonon population $N_{\mathbf{q}}(t)$ in the graphene sheet is given by

$$\frac{\partial N_{\mathbf{q}}}{\partial t} = \frac{2\pi}{\hbar} g_s g_v \sum_{k,k'} |C_{\mathbf{q}}|^2 \delta_{k,k'+\mathbf{q}} \left\{ (N_{\mathbf{q}}(t) + 1)f_{\mathbf{k}}(1 - f_{\mathbf{k}'})\delta(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k'}} + \hbar \omega_\mathbf{q}) \\
- N_{\mathbf{q}}(t)f_{\mathbf{k'}}(1 - f_{\mathbf{k}})\delta(\varepsilon_{\mathbf{k'}} - \varepsilon_{\mathbf{k}} - \hbar \omega_\mathbf{q}) \right\}$$

(1)

where $g_s = g_v = 2$ account the for spin and valley degeneracies respectively, $N_{\mathbf{q}}(t)$ represent the number of phonons with a wave vector $\mathbf{q}$ at time $t$. The factor $N_{\mathbf{q}} + 1$ accounts for the presence of $N_{\mathbf{q}}$ phonons in the system when the additional phonon is emitted. The $f_{\mathbf{k}}(1 - f_{\mathbf{k}})$ represent the probability that the initial $\mathbf{k}$ state is occupied and the final electron state $\mathbf{k}'$ is empty whilst the factor $N_{\mathbf{q}}f_{\mathbf{k'}}(1 - f_{\mathbf{k}})$ is that of the boson and fermion statistics.

The unperturbed electron distribution function is given by the shifted Fermi-Dirac function as

$$f_{\mathbf{p}} = \left[ \exp(-\beta(\varepsilon_\mathbf{p} - m_\mathbf{p}v_D) - \chi) \right]^{-1}$$

(2)

where $f_{\mathbf{p}}$ is the Fermi-Dirac equilibrium function, with $\chi$ being the chemical potential, $\mathbf{p}$ is momentum of the electron, $\beta = 1/kT$, $k$ is the Boltzmann constant and $V_D$ is the net drift velocity relative to the ion lattice site. In Eqn (1), the summation over $k$ and $k'$ can be transformed into integrals by
the prescription
\[ \sum_{k,k'} \rightarrow \frac{A^2}{(2\pi)^4} \int d^2k d^2k' \]
where \( A \) is the area of the sample, and assuming that \( N_q(t) \gg 1 \) yields
\[ \frac{\partial N_q}{\partial t} = \Gamma_q N_q \tag{3} \]
where
\[ \Gamma_q = \frac{A|\Lambda|^2\hbar q}{(2\pi)^3\hbar V_F \rho V_s} \int_0^\infty k dk \int_0^\infty k' dk' \int_0^{2\pi} d\phi \int_0^{2\pi} d\theta \{[f(k) - f(k')]\}
\]
\[ \delta (k - k' - \frac{1}{\hbar V_F} (\hbar \omega_q - V_D \cdot \hbar \vec{q})) \} \tag{4} \]
with \( k' = k - \frac{1}{\hbar V_F} (\hbar \omega_q - V_D \cdot \hbar \vec{q}) \). \( \Lambda \) is the deformation potential constant, and \( \rho \) is the density of the graphene sheet. At low temperature \( k_B T << 1 \), the distribution function become \( f(k) = \exp(-\beta(\epsilon(k))) \). Eqn(4) can be expressed as
\[ \Gamma_q = \frac{A|\Lambda|^2\hbar q}{(2\pi)^3\hbar V_F \rho V_s} \int_0^\infty k dk \left( k - \frac{1}{\hbar V_F} (\hbar \omega_q - V_D \cdot \hbar \vec{q}) \right) \exp(-\beta \hbar V_F k) \]
\[ - \exp(-\beta \hbar V_F \left( k - \frac{1}{\hbar V_F} (\hbar \omega_q - V_D \cdot \hbar \vec{q}) \right)) \tag{5} \]
Using standard integrals, Eqn(5) can be expressed finally as
\[ \Gamma = \Gamma_0 \left\{ 2 - \beta \hbar \omega_q (1 - \frac{V_D}{V_s}) \right\} \left\{ 1 - \exp(-\beta \hbar \omega_q (1 - \frac{V_D}{V_s})) \right\} \tag{6} \]
where
\[ \Gamma_0 = \frac{A|\Lambda|^2k T \hbar q}{(2\pi)^3 \beta^3 \hbar^4 V_F^4 \rho V_s} \tag{7} \]
Numerical Analysis

The Eqn (6) is analysed numerically for a normalized graph of $\Gamma/\Gamma_0$ against $V_D/V_s$ and $\omega_q$. The following parameters were used $\Lambda = 9eV$, $T = 10K$, $V_s = 2.1 \times 10^6cms^{-1}$ and $\vec{q} = 10^5cm^{-1}$. In Figure 1, the graph for the dependence of $\Gamma/\Gamma_0$ on $\omega_q$ is plotted. The graph was obtained at $V_D/V_s < 1$. The insert shows an experimentally obtained graph of an acoustoelectric current for gate-controlled Graphene [37]. The hypersond absorption graph qualitively agreed with the experimentally obtained graph via the Weinriech relation [36]. In Figure 2a, the dependence of $\Gamma/\Gamma_0$ on $V_D/V_s$ is analysed. From the graph, when $V_D/V_s < 1$, an absorption graph was observed, but when $V_D/V_s > 1$, gave an amplification of hypersond as is indicated in the work of Nunes and Fonseca [32]. To enhanced the observed Amplification (Absorption), a $3D$ graph was plotted for frequencies $\omega_q = 0.2, 0.4, \text{and } 1THz$ (see Figure 2b, 3 and 4). In Figure 2b, the maximum amplification was obtained at $\Gamma/\Gamma_0 = -0.16$ at $\omega_q = 2THz$ for $V_D = 1.1V_s$. For figure 3(a), at $V_D = 1.1V_s$, $\Gamma/\Gamma_0 = -0.34$ whilst in figure 3(b), for $V_D = 1.1V_s$, $\Gamma/\Gamma_0 = -0.08$ was obtained. It is interesting to note that, acoustic-phonon frequencies above $10THz$ can be attained. In Figure 4, at $V_D = 1.1V_s$, gave $\Gamma/\Gamma_0 = -3.17$ which was obtained at $\omega_q = 20THz$

For a gate controlled graphene, with $V_D = 1.1V_s$, the field $E$ can be calculated since $E = V_D/\mu$. The electron mobility $\mu$ in graphene given as $2.0 \times 10^4cm^2/Vs$, $V_s = 2.1 \times 10^5cm/s$ gives $E = 11.5V/cm$. For the source-to-drain voltage, $V_{sd} = V_DL/\mu$, ($L$ being the length from the source to drain electrode in graphene), the in-plane current $I = enV_DL$ ($n$ being the electron density) can be calculated.
Figure 1: Dependence of $\Gamma/\Gamma_0$ on $\omega_q$ insert is the experimental verification of Acousto-electric current versus acoustic phonon frequency

Figure 2: (a) Dependence of $\Gamma/\Gamma_0$ on $\frac{V_D}{V_s}$ for varying $\omega_q$ (left) (b) 3D representation of $\Gamma/\Gamma_0$ on $\frac{V_D}{V_s}$ and $\omega_q$ at 0.2THz (right)
Figure 3: 3D representation of $\Gamma/\Gamma_0$ on $\frac{V_0}{V_s}$ and $\omega_q$ at (a) 0.4THz (left) and at 1THz (right).

Figure 4: A graph of $\Gamma/\Gamma_0$ on $\frac{V_0}{V_s}$ and $\omega_q$ at 2THz.
Conclusion

The generation of hypersound amplification (absorption) of acoustic phonons in a gated controlled graphene is studied. The absorption obtained qualitatively agreed with an experimentally obtained acoustoelectric current in a gate-controlled graphene via the Weinrich relation. For $\frac{V_D}{V_s} > 1$, the hypersound amplification obtained is similar to that of Nunes and Fonseca. For a drift velocity of $V_D = 1.1V_s$, a field of $E = 11.5V/cm$ was calculated. At frequency of $0.2THz$, an amplification of $\Gamma/\Gamma_0 = -3.17$ is attained. From this work, the hypersound studies in graphene offers a much better source of higher phonon frequencies than the homogenous semiconductors which permit the use of graphene as hypersound phonon laser (SASER).

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