A Tale of Flavor Anomalies and the Origin of Neutrino Mass

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One of the most important discoveries in particle physics is the observation of non-zero neutrino masses, which dictates that the Standard Model (SM) is incomplete. Moreover, several pieces of evidence of lepton flavor universality violation (LFUV) gathered in the last few years hint towards physics beyond the SM. TeV scale scalar leptoquarks (SLQs) are the leading candidates in explaining these flavor anomalies in semileptonic charged and neutral current B-decays, the muon, and the electron magnetic dipole moments that can also participate in neutrino mass generation. In this work, we hypothesize that neutrino masses and LFUV have a common new physics origin and classify the simplest models consisting of SQLs. To this end, we propose a brand new two-loop neutrino mass model that has the potential to resolve some of these flavor anomalies and offer rich phenomenology to test the model in the ongoing and future colliders.

I. INTRODUCTION

The observation of neutrino oscillations [1–7] was the first direct hint that the Standard Model of particle physics is imperfect and must be extended. Lepton flavor universality (LFU), a solid prediction of the SM, can be easily violated in the beyond SM (BSM) models, where the particles preferentially couple to certain generations of leptons. In the last several years, indications of LFU violation (LFUV) have been observed in both $b \to s\ell\nu$ and $b \to c\ell\nu$ processes. Observables associated with these transitions are the well-known ratios:

$$R_{K^{(*)}} = \frac{Br (B \to K^{(*)}\mu^+\mu^-)}{Br (B \to K^{(*)}e^+e^-)},$$

$$R_{D^{(*)}} = \frac{Br (B \to D^{(*)}\tau\bar{\nu}_\tau)}{Br (B \to D^{(*)}\ell\bar{\nu}_\ell)}, \quad \text{with} \quad \ell = e, \mu,$$

respectively.

LFU in the SM predicts the former ratio to be unity with uncertainties less than 1% [8–12]. A deficit in this neutral-current transition has been observed consistently over the years in several experiments [13–16], and LHCb recently updated their measurements [17] that increased the significance of the deviation. Moreover, with yet another observed deviation in $Br (B_s^0 \to \mu^+\mu^-)$ [18–21], the combined significance of the deviation is uplifted to 4.7σ. On the other hand, the $R_{D^{(*)}}$ ratio differs from unity due to the substantial mass difference between tauon and muon. An enhancement of this charged-current transition is reported by several experimental measurements [22–26], which combinedly leads to approximately 3σ deviation from the SM value [27, 28].

Besides, there has been a longstanding tension between the theoretical prediction of the anomalous magnetic dipole moment (AMDM) of the muon $(g-2)_\mu$ and the value measured at the BNL E821 experiment [29]. The FNAL E989 experiment [30] has recently announced its result, which has a smaller uncertainty and is fully compatible with the previous best measurement. Together, these two experiments show a remarkably large deviation with a significance of 4.2σ with respect to the theory prediction [31]. Various new physics models are proposed to explain the observed significant departure. For a most recent review see Ref. [32].

All these anomalies mentioned above are strongly pointing towards physics beyond the SM. Interestingly, the prime candidate to solve these flavor anomalies is leptoquarks (LQs): hypothetical particles that combine the properties of leptons and quarks. The existence of LQs are highly motivated since particles of this type are naturally predicted by Grand Unified Theories [87–91]. Explanation of flavor anomalies requires these particles to have masses of order TeV. Remarkably, these TeV scale scalar leptoquarks (SLQs) can also participate in neutrino mass generation. Due to the renormalizability, in this work, we are interested in extending the particle content of the SM by only scalar (not vector) LQs.

In this work, we hypothesize that neutrino masses and LFUV have a common new physics origin. Motivated by

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1 For a recent review on LQs, see Ref. [86].
2 For both scalar and vector LQs explanations of anomalies related to B-meson decays, see e.g. Refs. [92–190].
3 In this work, neutrinos are assumed to be Majorana particles.
II. CLASSIFICATION OF MODELS

LQs are fields that simultaneously couple to a lepton and a quark. Confining only to the SM fermion content, there are five possible SLQs can couple to a quark-lepton (qℓ) bilinear (since diquark couplings are not required to explain flavor anomalies, we set them to zero). Three of these SLQs can actively participate (color-coded with red) in explaining flavor anomalies: (i) iso-singlet $S_1(3, 1, 1/3)$, (ii) iso-doublet $R_2(3, 2, 7/6)$, and (iii) iso-triplet $S_3(3, 1, 3/3)$. SLQs $S_1$, $R_2$, and $S_3$ couple to $qℓ$ bilinears of the form $\{LL, RR\}$, $\{LR, RL\}$, and $\{LL\}$, respectively. Here, $L$ and $R$ denote left- and right-chiral fermion fields, respectively. Rest of the two SLQs that do not (color-coded with blue) address flavor anomalies but may participate in neutrino mass generation are (iv) iso-singlet $\tilde{S}_1(\bar{3}, 1, 4/3)$ (with $qℓ = \{RR\}$) and (v) iso-doublet $\tilde{R}_2(3, 2, 1/6)$ (with $qℓ = \{RL\}$). Both SLQs $S_1$ and $R_2$ can address anomalies in the $R_D(\gamma)$ ratios, whereas $S_3$ incorporates anomalies in the $R_{KK^{(*)}}$ ratios. The discrepancies in the $R_K - R_{KK^{(*)}}$ ratios can also be alleviated utilizing the $R_2$ LQ that significantly talks to the electrons; for a recent fit, see Ref. [204].

On the other hand, with aspecific choice of Yukawa textures, $S_1$ and $R_2$ both can simultaneously explain $(g - 2)_\mu$ and $(g - 2)_e$ anomalies [61].

This work considers only the simplest radiative neutrino mass models that can partially resolve these flavor anomalies. Simplicity refers to minimal models containing only BSM scalars and no BSM fermions. We classify these models into Class-I, Class-II, and Class-III. Models in Class-I contain a single scalar LQ that can incorporate at most one of the flavor anomalies (i.e., only one of $\{S_1, R_2, S_3\}$ LQs is present). To generate non-zero neutrino masses, a second BSM scalar is introduced, which is not a LQ (color-coded with black). Models in which this second non-LQ field is replaced with another LQ that does not participate (i.e., either $\tilde{S}_1$ or $\tilde{R}_2$) in explaining any of the flavor anomalies are categorized as Class-II. On the other hand, models belonging to Class-III consist of two LQs, both of which have the potential to address at least one of the flavor anomalies. Models in Class-III require a third scalar field (not a LQ) to complete the loop diagram to provide non-zero neutrino masses. We list the simplest models belonging to these three categories in Table. I. In this Table, the model highlighted within the green-shaded band is the newly proposed one in this work.

When building neutrino mass models that belong to Class-I and Class-II, only $S_1$ and $S_3$ are viable options; $R_2$ LQ extended with a second scalar field cannot generate neutrino mass. Model-I and Model-II (Model-III and Model-IV) both share a common feature, which is, non-zero neutrino mass arises via two-loop (one-loop) quantum corrections assisted with a scalar diquark $\omega(3, 1, 2/3)$ (SLQ $\tilde{R}_2$). For Model-I, collider implications of these new colored states are studied in Ref. [206]. The corre-

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4 For classification of effective neutrino mass operators and systematic study of $d = 5$ Weinberg operator, see e.g. Refs. [191–203].
5 For a most recent review on anomalies in the $B$-meson decays, see Ref. [205].
6 Note, however, that by turning off the $LL$ coupling of $S_1$ LQ, neutrino mass appears only at the two-loop order [208, 210]. A model of two-loop neutrino mass can also be constructed using $R_2$ LQ but replacing $S_1$ LQ with a vector-like quark having the same quantum numbers as $u_L$ [211].
lation between neutrino mass and $R_{K^{(*)}}$ anomaly ($R_{K^{(*)}}$ and $B \to K\pi$ anomalies) within Model-II is studied in Ref. [119] (Ref. [150]). Furthermore, by combining Model-I and Model-II, simultaneous explanation of $R_{K^{(*)}}$, $R_{D^{(*)}}$, and $a_\mu$ along with neutrino mass generation is considered in Ref. [160]. Model-III and Model-IV are introduced in Ref. [207], and Model-III was recently studied in Ref. [175] to address the muon $g-2$ in addition to neutrino masses.

On the other hand, models belonging to Class-III consist of three BSM scalars, two SLQs, and another scalar multiplet, which is not a LQ. Two such models in this class already exist in the literature, Model-V, and Model-VI: the non-LQ scalar multiplets are $\chi(3, 1, 2/3)$ and $\Delta(1, 4, 3/2)$ in the former and the latter models, respectively. Interestingly, both these models contain $R_2$ and $S_3$ SLQs and are shown to be compatible with neutrino oscillation data as well as capable of addressing $R_{K^{(*)}}$ and $R_{D^{(*)}}$ anomalies [151, 159, 165]. To this list, we propose a new model, Model-VII, with a particle content of $\{S_1, R_2, \xi(3, 3, 2/3)\}$. The operator leading to non-zero neutrino masses, constructed from BSM states, is shown in Fig. 1. The proposed model has the potential to address some of these flavor anomalies and incorporate neutrino oscillation data. In the following two sections, we work out the details of our new proposal, derive the neutrino mass formula, and briefly discuss the phenomenological implications of this model.

III. PROPOSED MODEL

The new neutrino mass model proposed in this work consists of the following three BSM scalar multiplets (from hereon, we do not use the aforementioned color-code any further):

$$R_2(3, 2, 7/6) \equiv R = \begin{pmatrix} R^{5/3} \\ R^{2/3} \end{pmatrix},$$

$$S_1(3, 1, +1/3) \equiv S = S^{1/3},$$

$$\xi(3, 3, 2/3) \equiv \xi = \begin{pmatrix} \xi^{2/3} \\ \sqrt{2} \xi^{1/3} \\ \xi^{5/3} \end{pmatrix}. $$

Numbers in parentheses stand for quantum numbers of each field under $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge groups.

The SM Higgs is denoted as $H(1, 2, 1/2) = (H^+, H^0)^T$. This scalar field will get a nonzero vacuum expectation value (vev) $v \equiv \langle H \rangle = 174$ GeV during the spontaneous breaking of the electroweak (EW) symmetry. The fermion sector does not change. It contains the same particle content as in the SM

$$L_i(1, 2, -1/2) = \begin{pmatrix} u^L_i \\ e^L_i \end{pmatrix}, \quad Q_i(3, 2, 1/6) = \begin{pmatrix} u^L_i \\ d^L_i \end{pmatrix},$$

$$e^c_i(1, 1, 1), \quad u^c_i(3, 1, -2/3), \quad d^c_i(3, 1, +1/3),$$

where $i$ indicates generation index and $\psi^c \equiv C\bar{\psi}^T$ denotes the charge conjugate of the right-handed field.

Among three new scalars introduced, only $R$ and $S$ can be considered as LQs. They interact with the SM fermions through the following Yukawa interactions

$$\mathcal{L}_{Y}^{\text{new}} = f_{ij}^L u^c_i R \cdot L_j + f_{ij}^R R^c \bar{Q}_j e^c_i + y_{ij}^q S \cdot L_j + y_{ij}^D \bar{Q}_j e^c_i S^c + \text{h.c.}$$

In order to avoid unnecessary cluttered notation, we have used “$\epsilon$” to denote an $SU(2)$ contraction, e.g., $L \cdot Q \equiv L^aQ^b\epsilon_{ab}$ with $\epsilon$ being the antisymmetric tensor ($\epsilon_{12} = -\epsilon_{12} = 1$) and $a, b = 1, 2$ being $SU(2)$ indices.

Note that all terms in Eq. (7) conserve both baryon ($B$) and lepton ($L$) numbers. This can be seen, for instance, by setting $(B, L)$ to $(1/3, -1)$ and $(-1/3, -1)$ for $R$ and $S$ fields, respectively. The nontrivial breaking of lepton number occurs by the simultaneous presence of

$$V \supset \lambda S^c \epsilon^T \xi^c + \mu R^c \epsilon \xi H$$

$$= \lambda S^c \left[ -\sqrt{2} \xi^{-2/3} H^0 + \xi^{1/3} H^0 + \xi^{-5/3} H^+ + \frac{1}{\sqrt{2}} R^{-5/3} \xi^{-2/3} H^+ + R^{-1/3} \xi^{-5/3} H^0 + \frac{1}{\sqrt{2}} R^{-1/3} \xi^{2/3} H^0 \right] + \text{h.c.},$$

breaking $L$ by two units. The two parameters, i.e., $\lambda$ and $\mu$, can be made real by absorbing their respected phases into scalar fields. In contrast to the lepton number, the baryon number remains conserved, forbidding proton decay.

The $\Delta L = 2$ effective operators, depicted in Fig. 1, must contain the product of $\Delta F = 0$ and $\Delta F = 2$ couplings, i.e., any combination of $y^{L,R}f^{L,R}$ so there are four kinds of $\Delta L = 2$ operators that can be generated within this model after integrating out heavy scalar states. It is required that such operators involve gauge bosons, or else they will vanish by $SU(2)$ symmetry. The four operators will contain $(\bar{H}D^uH)$ multiplied by the following combinations:

(i) $(u^c HD_{\mu}L)(LQ)$,
(ii) $(u^c HD_{\mu}L)\bar{e}^c\bar{e}^c$,
(iii) $(e^c D_{\mu}\bar{Q}H)(LQ)$,
(iv) $(e^c HD_{\mu}\bar{Q})\bar{u}^c\bar{e}^c$.

Note that the $SU(2)$ contraction occurs on fields inside parentheses. In addition, the covariant derivative can also act on other fields. All but the operator (iv) will lead to neutrino masses at two-loop level.

Scalar terms in Eq. (8) will cause mixing among $\xi^{1/3} - S^{1/3}$, $\xi^{2/3} - R^{2/3}$, and $\xi^{5/3} - R^{5/3}$ components. The mass matrices of leptoquarks relevant for neutrino mass generation are

$$M^2_{\bar{\xi}/S} = \begin{pmatrix} m_{\bar{\xi}}^2 - \lambda_{\bar{\xi}}u^2 \\ -\lambda_{\bar{\xi}}u^2 \lambda_{\bar{\xi}}v^2 \\ \lambda_{\bar{\xi}}v^2 m_R^2 \end{pmatrix}.$$
parameters, the two mixing angles are given by

\[
\sin 2\theta = \frac{-2\lambda v^2}{m_\xi^2 - m_S^2}, \quad \tan 2\phi = \frac{-\sqrt{2}\mu v}{m_\xi^2 - m_R^2}.
\]  

(13)

Furthermore, the mass eigenvalues of \( \chi_{1/3}^2 \) and \( \chi_{1/3}^1 \) are found to be

\[
M_{1,2}^2 = \frac{1}{2} \left[ m_\xi^2 + m_R^2 \pm \sqrt{(m_\xi^2 - m_R^2)^2 + 2\mu^2 v^2} \right],
\]

(14)

\[
M_{3,4}^2 = \frac{1}{2} \left[ m_\xi^2 + m_S^2 \pm \sqrt{(m_\xi^2 - m_S^2)^2 + 4\lambda^2 v^4} \right].
\]

(15)

Note that \( M_{1,2}^2 \) and \( M_{3,4}^2 \) can be the larger or the smaller of the two mass eigenvalues. They are defined such that

\[
M_1^2 \cos^2 \phi + M_2^2 \sin^2 \phi = M_3^2 \cos^2 \theta + M_4^2 \sin^2 \theta.
\]

(16)

In terms of mass eigenvalues, we come out with an alternative way of writing Eq. (13), namely

\[
\sin 2\theta = \frac{-2\lambda v^2}{M_3^2 - M_4^2} \quad \text{and} \quad \sin 2\phi = \frac{-\sqrt{2}\mu v}{M_1^2 - M_2^2},
\]

(17)

from which both \( \lambda \) and \( \mu \) can be written in terms of mass eigenvalues

\[
\sqrt{2}\lambda v = \frac{-M_1^2 - M_2^2}{v} \sin 2\theta,
\]

(18)

\[
\mu = \frac{-M_1^2 - M_2^2}{v} \sin 2\phi.
\]

(19)

Having rotated the scalars into their mass eigenstates, we now do the same for Yukawa interactions of Eq. (7). Without loss of generality, we can define all couplings in Eq. (7) in the charged lepton mass diagonal basis. If we assume further that the up-type quarks be diagonal as well, Eq. (7) becomes

\[
\mathcal{L} \supset f^{L}_j u^c_i \left[ (U_\psi)_{a2} \chi^a_2 \nu_{L_j} - (U_\psi)_{a2} \chi^a_2 \nu_{L_j} \right] + f^{R}_j \left[ (U_\psi)_{a2} \chi^a_2 u_{L_i} + (U_\psi)_{a2} \chi^a_2 V_{ik} d_{L_k} \right] \nu_{L_j} + y^{L}_j (U_\theta)_{a2} (u_{L_i} \nu_{L_j} - V_{ik} d_{L_k} \nu_{L_j}) \chi^a_2
\]

(20)

where \( V \) is the Cabibbo-Kobayashi-Maskawa mixing matrix. Note that the results are not affected by changing the up-type to the down-type diagonal basis. Since two choices of the Yukawa coupling textures, namely, “up-type” and “down-type” mass-diagonal basis are widely used in the literature, in the following text, we provide neutrino mass formula in both these scenarios.

As one can see from Fig. 1, the neutrino mass generation requires interaction between LQs and W boson, originating from the \( SU(2) \) covariant derivatives

\[
D_\mu R = \left( \partial_\mu - i \frac{g}{2} W^\mu_a \sigma^a - i \frac{g'}{3} g' B_\mu \right) R,
\]

\[
D_\mu \xi = \partial_\mu \xi - i \frac{g}{2} \left( W^\mu_a \sigma^a, \xi \right) - i \frac{g'}{3} B_\mu,
\]

(21)

where \( g, g' \) are the \( SU(2) \) and \( U(1)_Y \) gauge couplings and \( \sigma^a \) are Pauli matrices. Based from Eq. (21), the \( \xi^{2/3} \) \( \xi^{1/3} \) \( W \) vertex can be derived from the triplet kinetic term, that is,

\[
(D_\mu \xi)^{(D^\mu \xi)} \rightarrow i g \left[ \xi^{2/3} \partial_\mu \xi^{1/3} - \frac{7}{6} g' B_\mu \right] W^\mu.
\]

(22)
After rotating the corresponding fields to their mass eigenstates, we obtain

\[
\mathcal{L}_{\text{scalar}}^{cc} = ig(U_\theta)_{a1}(U_\phi)b_1 \\
\times \left[ \partial_\mu \lambda^{a-1/3}_\alpha \lambda^{b-2/3}_\beta - \lambda^{a-1/3}_\alpha \partial_\mu \lambda^{b-2/3}_\beta \right] W^{\nu\mu}.
\] (23)

In this model, we work in the general \( R_\xi \) gauge, so we need to know the LQ interactions with the Goldstone boson. By using Eqs. (8) and (17), the LQs–Goldstone interactions are found to be

\[
\mathcal{L} \supset g \left( \frac{M_b^2 - M_{a+2}^2}{m_W} \right) (U_\theta)_{a1}(U_\phi)b_1 \lambda^{a-1/3} \lambda^{b-2/3} H^+ + \text{h.c.}.
\] (24)

Here the factor of 3 accounts for the exchange of color states inside the loops, whereas \( D_u \) and \( D_t \) are the normalized mass matrices of up-type quarks and charged leptons, respectively

\[
D_u = \text{diag} \left( \frac{m_u}{m_t}, \frac{m_c}{m_t}, 1 \right), \quad D_t = \text{diag} \left( \frac{m_t}{m_\tau}, \frac{m_u}{m_\tau}, 1 \right).
\] (26)

Similarly, in the basis where down-type quark mass matrix is diagonal, we have

\[
(M_\nu)_{ji} = \frac{3 g^2 m_t}{\sqrt{2}(16\pi^2)^2} \left\{ \left[ y_{ki}^d V_{kl}^\ast (D_u)_{kj} f_{kl}^L + f_{kj}^L (D_u)_{kj} V_{kl}^\ast y_{ki}^d \right] \hat{I}_{jkl} + \frac{m_\nu}{m_t} (D_\ell)_{j} \left[ f_{kj}^{R^\ast} V_{kl}^\ast y_{mi}^d + y_{mi}^d V_{kl} y_{mj}^d f_{kj}^{R^\ast} \right] \hat{I}_{jil} \right\}.
\] (27)

One should note that Eqs. (25) and (27) are equivalent, as one can recover the latter, for instance, by redefining \( y^L \rightarrow V^\ast y^L \) and \( f^R \rightarrow V^\ast f^R \). This reflects the basis independence mentioned previously.

The loop integrals shown in Eqs. (25) and (27), i.e., \( \hat{I}_{jkl} \), \( \hat{I}_{jk} \), and \( \hat{I}_{jl} \), indicate the contribution of each subgroup. Each of them is defined as

\[
\hat{I}_{jkl} = (16\pi^2)^2 \left[ I_{(1)}^{(1)} + I_{(1)}^{(2)} + I_{(1)}^{(3)} + I_{(1)}^{(4)} \right],
\]

\[
\hat{I}_{jk} = (16\pi^2)^2 \left[ I_{(2)}^{(5)} + I_{(2)}^{(6)} \right],
\]

\[
\hat{I}_{jl} = (16\pi^2)^2 \left[ I_{(2)}^{(7)} + I_{(2)}^{(8)} \right],
\] (28)

with \( I_{(n)}^{(i)} \) denoting the dimensionless loop function for the \( n \)-th diagram, that is,

\[
(I_{(1)}^{(1)})_{kl} = -(U_\theta)_{a1}(U_\theta)_{a2}(U_\theta)_{b1}(U_\phi)b_2 \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} \left[ g(2q + k) + \frac{q \hat{k}}{k^2} \left( -1 + \frac{k^2 - m_W^2}{k^2 - \xi M_1^2} \right) \right] \frac{1}{k^2 - m_{\text{di}}^2}.
\] (29)
Figure 2. All two-loop diagrams leading to neutrino mass generation.

\[
I_{jj}^{(3)} = - (U_\theta)_{a1}(U_\theta)_{a2}(U_\phi)_{b1}(U_\phi)_{b2} \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} \left[ (2q + \not{k})(\not{q} + \not{k}) + \left( -1 + \xi \frac{k^2 - m_W^2}{k^2 - \xi m_W^2} \right) k \cdot (2q + k) \right] \frac{1}{k^2 - m_W^2}
\]

\[
I_{jj}^{(4)} = (U_\theta)_{a1}(U_\theta)_{a2}(U_\phi)_{b1}(U_\phi)_{b2} \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} \left[ (2q + \not{k})(\not{q} + \not{k}) + \frac{\not{k}(q + \not{k})}{k^2} \left( -1 + \xi \frac{k^2 - m_W^2}{k^2 - \xi m_W^2} \right) \right] \frac{1}{k^2 - m_W^2} \frac{1}{k^2 - m_W^2}
\]

\[
I_{jj}^{(5)} = - (U_\theta)_{a1}(U_\theta)_{a2}(U_\phi)_{b1}(U_\phi)_{b2} \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} \left[ (2q + \not{k})(\not{q} + \not{k}) + \frac{\not{k}(q + \not{k})}{k^2} \left( -1 + \xi \frac{k^2 - m_W^2}{k^2 - \xi m_W^2} \right) \right] \frac{1}{k^2 - m_W^2} \frac{1}{k^2 - m_W^2}
\]

\[
I_{jj}^{(6)} = (U_\theta)_{a1}(U_\theta)_{a2}(U_\phi)_{b1}(U_\phi)_{b2} \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} \left[ (2q + \not{k})(\not{q} + \not{k}) + \frac{\not{k}(q + \not{k})}{k^2} \left( -1 + \xi \frac{k^2 - m_W^2}{k^2 - \xi m_W^2} \right) \right] \frac{1}{k^2 - m_W^2} \frac{1}{k^2 - m_W^2}
\]

\[
I_{jj}^{(7)} = (U_\theta)_{a1}(U_\theta)_{a2}(U_\phi)_{b1}(U_\phi)_{b2} \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} \left[ (2q + \not{k})(\not{q} + \not{k}) + \frac{\not{k}(q + \not{k})}{k^2} \left( -1 + \xi \frac{k^2 - m_W^2}{k^2 - \xi m_W^2} \right) \right] \frac{1}{k^2 - m_W^2} \frac{1}{k^2 - m_W^2}
\]
The cancellation of terms containing the gauge parameter $\xi$ can be inferred directly from Eqs. (29)-(36). To see how this cancellation takes place, it is desirable to express $k \cdot (2q + k)$, which appears in all $W$-mediated diagrams, as

$$k \cdot (2q + k) = [(q + k)^2 - M_b^2 - q^2 + M_{a+2}^2] + M_b^2 - M_{a+2}^2.$$  \hspace{1cm} (37)

Terms inside parentheses will cancel the LQ propagators, particularly those appearing in diagrams 1, 3, and 5. Thus, they will vanish by the orthogonality of LQ mixing matrices. The remaining terms, which are proportional to $M_b^2 - M_{a+2}^2$, will make such loop terms have the same coefficients but opposite signs with the corresponding Goldstone loop integrals, allowing cancellation of $\xi$-dependent terms. For diagram 7, due to momentum switch between $\chi^{1/3}$ and $\chi^{2/3}$ (see Fig. 2), we have instead $k \cdot (2q + k) \rightarrow M_b^2 - M_{a+2}^2$. The cancellation of $\xi$-dependent terms in this case too can be foreseen right away. The gauge parameter cancellation indicates further that all two-loop diagrams presented in Fig. 2 are the complete set of diagrams generating neutrino masses at the lowest order.

We are, then, left with gauge-independent terms. It is straightforward to evaluate the integrals, from which we get

\begin{equation}
\tilde{I}_{jkl} = -\frac{1}{4} \sin 2\theta \sin 2\phi \sum_{a,b=1}^{2} (-1)^{a+b} \frac{1}{M_b^2 - m^2_{a+2}} \int_0^1 dx \int_0^\infty dt \frac{t}{t + m_W^2} \times \left\{ 6x - 5 + \left( \frac{M_b^2 - M_{a+2}^2}{m_W^2} \right) \right\} \left[ \ln \frac{\Delta(x, t; M_b, M_{a+2})}{\Delta(x, t; m_{a+2}, M_{a+2})} - \ln \frac{\Delta(x, t; m_{a+2}, M_{a+2})}{\Delta(x, t; m_{a+2}, m_{a+2})} \right] \\
-4 \left[ A(x; M_b, M_{a+2}) \ln \frac{\Delta(x, t; M_b, m_{a+2})}{m_W^2} + A(x; m_{a+2}, M_{a+2}) \ln \frac{\Delta(x, t; m_{a+2}, M_{a+2})}{m_W^2} \right] \\
- A(x; M_b, m_{a+2}) \ln \frac{\Delta(x, t; M_b, m_{a+2})}{m_W^2} - A(x; m_{a+2}, M_{a+2}) \ln \frac{\Delta(x, t; m_{a+2}, M_{a+2})}{m_W^2} \right\}, \end{equation}

\begin{equation}
\tilde{I}_{j} = \frac{1}{4} \sin 2\theta \sin 2\phi \sum_{a,b=1}^{2} (-1)^{a+b} \frac{1}{M_b^2 - m^2_{a+2}} \int_0^1 dx \int_0^\infty dt \frac{t}{t + m_W^2} \times \left\{ 1 - 6x - \left( \frac{M_b^2 - M_{a+2}^2}{m_W^2} \right) (1 - x) t \ln \frac{\Delta(x, t; M_b, M_{a+2})}{\Delta(x, t; m_{a+2}, M_{a+2})} + 4A(x; m_{a+2}, M_{a+2}) \ln \frac{\Delta(x, t; m_{a+2}, M_{a+2})}{m_W^2} \right\}, \end{equation}

\begin{equation}
\tilde{I}_{j} = -\frac{1}{4} \sin 2\theta \sin 2\phi \sum_{a,b=1}^{2} (-1)^{a+b} \frac{1}{M_b^2 - m^2_{a+2}} \int_0^1 dx \int_0^\infty dt \frac{t}{t + m_W^2} \times \left\{ 1 - 6x - \left( \frac{M_{a+2}^2 - M_b^2}{m_W^2} \right) (1 - x) t \ln \frac{\Delta(x, t; M_{a+2}, M_b)}{\Delta(x, t; m_{a+2}, M_b)} + 4A(x; M_{a+2}, M_b) \ln \frac{\Delta(x, t; M_{a+2}, M_b)}{m_W^2} \right\}, \end{equation}

where only terms relevant to neutrino masses are kept. In those loop integral expressions, we have introduced a
Here, we consider a simple version of this model. Moreover, to solve each of these flavor anomalies, specific scale, as required to address the flavor anomalies, flavor matrices. If the scalar LQs have masses around the TeV scale, as required to address the flavor anomalies, flavor violation restricts many of these couplings to be small. Therefore, to solve each of these flavor anomalies, specific Yukawa texture must be adopted to be consistent with numerous experimental constraints, which is beyond the scope of this work and will be presented elsewhere.\(^{212}\)

A toy model:– The neutrino mass formula given in Eq. (25) (or Eq. (27)) consists of four different Yukawa couplings \(y^{L,R}\) and \(f^{L,R}\), which are a priori arbitrary 3×3 matrices. Theoretical LQs have masses around the TeV scale, as required to address the flavor anomalies, flavor violation restricts many of these couplings to be small. Moreover, to solve each of these flavor anomalies, specific Yukawa texture must be adopted to be consistent with numerous experimental constraints, which is beyond the scope of this work and will be presented elsewhere.\(^{212}\)

Here, we consider a simple version of this model.

Considering the loop integral behavior as discussed above and the suppression of \(m_\tau/m_\mu\), in the case where there is no large hierarchy among Yukawa couplings, it is an excellent approximation to keep only the third generation of quarks. Then the neutrino mass matrix formula, in this case, becomes

\[
(M_\nu)_{ij} \approx \frac{3g^2 m_\mu}{\sqrt{2}(16\pi^2)^2} \left[ y_{3j}^L f_{3i}^L + f_{3j}^L y_{3i}^L \right] \hat{I}_{j33},
\]

where we have also ignored the off-diagonal entries of the CKM matrix. One can see from this formula that the neutrino mass matrix is reduced into a rank two matrix, whose determinant vanishes. Thus, this toy model predicts that one of the neutrinos is massless, although both neutrino mass orderings can be admitted. The neutrino mass matrix in this form can nicely fit oscillation data. It is also worth noting that adding other couplings, as they may be relevant for addressing anomalies, will not jeopardize its ability to fit the data.

We consider a parametrization used in [213, 214] to determine these Yukawa couplings appearing in Eq. (42). To do so, the neutrino mass matrix is diagonalized as follows

\[
M_\nu = U^\dagger \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U, \tag{43}
\]

where \(U\) is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix and \(m_i\) are neutrino mass eigenvalues given by

\[
m_1 = 0, \quad m_2 = \sqrt{-\Delta m_{21}^2}, \quad m_3 = \sqrt{-\Delta m_{31}^2}, \tag{44}
\]

for normal mass ordering and

\[
m_1 = \sqrt{-\Delta m_{32}^2 - \Delta m_{21}^2}, \quad m_2 = \sqrt{-\Delta m_{32}^2}, \quad m_3 = 0, \tag{45}
\]

for inverted mass ordering.

Now the neutrino mass matrix given in Eq. (42) can be re-written as

\[
M_\nu = a_0 \begin{pmatrix} Y_a^T \tilde{m}_Y + Y_b^T \tilde{m}_Y \\ Y_a^T \tilde{m}_Y + Y_b^T \tilde{m}_Y \end{pmatrix}, \tag{46}
\]

\[
a_0 = \frac{3g^2}{\sqrt{2(16\pi^2)^2}}, \quad \tilde{m}_i = m_i \hat{I}_{j33}. \tag{47}
\]

Here, \(Y_a = y_{3i}^L\) and \(Y_b = f_{3i}^L\) are row matrices. Utilizing this form, the two unknown Yukawa coupling matrices can be determined completely by the known values\(^{215}\) of neutrino observables, SM fermion masses, and as a function of scalar masses and mixings that run through the loops,

\[
Y_a^T = \frac{16\pi^2}{3^{1/2}2^{1/4}g} \begin{pmatrix} i r_1 U_{12}^* + r_2 U_{13}^* \\ i r_2 U_{22}^* + r_3 U_{23}^* \\ i r_3 U_{32}^* + r_2 U_{33}^* \end{pmatrix}, \tag{48}
\]

\[
Y_b^T = \frac{16\pi^2}{3^{1/2}2^{1/4}g} \begin{pmatrix} -i r_1 U_{12}^* + r_2 U_{13}^* \\ -i r_2 U_{22}^* + r_3 U_{23}^* \\ -i r_3 U_{32}^* + r_2 U_{33}^* \end{pmatrix}, \tag{49}
\]

where \(r_i = (m_i/m_\mu)^{1/2}\). On the other hand, for inverted ordering, the solution for \(Y^{a,b}\) take the following forms

\[
Y_a = \frac{16\pi^2}{3^{1/2}2^{1/4}g} \begin{pmatrix} r_1 U_{11}^* + i r_2 U_{12}^* \\ r_1 U_{21}^* + i r_2 U_{22}^* \\ r_1 U_{31}^* + i r_2 U_{32}^* \end{pmatrix}, \tag{50}
\]

\[
Y_b = \frac{16\pi^2}{3^{1/2}2^{1/4}g} \begin{pmatrix} r_1 U_{11}^* - i r_2 U_{12}^* \\ r_1 U_{21}^* - i r_2 U_{22}^* \\ r_1 U_{31}^* - i r_2 U_{32}^* \end{pmatrix}. \tag{51}
\]

This parametrization is sometimes useful to fix the undetermined Yukawa parameters of the theory.
V. CONCLUSIONS AND OUTLOOK

Neutrino oscillation was discovered almost 25 years ago, showing that neutrinos have a mass; however, its origin remains unknown. Recently, there have been several pieces of evidence of lepton flavor universality violation that strongly indicate physics beyond the SM. In this work, we hypothesized that neutrino masses and flavor anomalies have a common new physics origin. We provided a classification of simple neutrino mass models with up to three new scalars, of which at least one is a scalar leptoquark that can address at most one of the flavor anomalies. Additionally, we proposed a new two-loop neutrino mass model consisting of scalar leptoquarks $(\bar{3}, 1, 1/3)$ and $(3, 2, 7/6)$ along with a third scalar $(3, 3, 2/3)$. Each of these scalar leptoquarks has the potential to incorporate $R_{D^+}$, $(g - 2)_e$, and $(g - 2)_\mu$ anomalies. The scalar leptoquark $(3, 2, 7/6)$ may also address anomalies in the $R_{D^+}$ ratios via new physics interactions with the electron. A complete study of neutrino mass along with resolving flavor anomalies is beyond the scope of this work and will be studied elsewhere. Since resolution to flavor anomalies requires TeV scale scalar leptoquarks, the proposed model offers rich phenomenology that can be tested in ongoing as well as future colliders.

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