New Test of the Gravitational $1/r^2$ Law at Separations down to 52 $\mu$m

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We tested the gravitational $1/r^2$ law using a stationary torsion-balance detector and a rotating attractor containing test bodies with both 18-fold and 120-fold azimuthal symmetries that simultaneously tests the $1/r^2$ law at two different length scales. We took data at detector-attractor separations between 52 $\mu$m and 3.0 mm. Newtonian gravity gave an excellent fit to our data, limiting with 95% confidence any gravitational-strength Yukawa interactions to ranges $< 38.6$ $\mu$m.

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Testing gravity at the shortest attainable distances is interesting for many reasons. String theory’s unification of gravity with the other 3 fundamental interactions inherently involves extra gravitational space dimensions as well as many nominally-massless scalar particles (dilaton and moduli). Both of these features would violate the gravitational inverse-square law (ISL) \[1, 2\], as would a second, heavy graviton\[3\]. New phenomena could occur below the length scale associated with dark energy $\lambda_d = \sqrt{\hbar c/\rho_d} \approx 85$ $\mu$m where $\rho_d \approx 3.8$ keV/cm$^3$ is the observed density of dark energy\[4, 5\]. Suggestions that dark matter may consist of ultra-low-mass scalar and vector bosons\[6, 7\], whose exchange interaction would violate the ISL, provide further motivation for exploring this regime. It is customary to interpret ISL data as constraining an additional Yukawa interaction

$$V(r) = V_N(r)[1 + \alpha \exp(-r/\lambda)] ,$$

where $V_N(r)$ is the familiar Newtonian potential. This form is obviously valid for scalar or vector boson-exchange interactions and is a reasonable approximation for the effects of extra dimensions as long as the minimum separation attained in the experiment is greater than the size of the largest extra dimension\[8\].

Precise studies of gravity at length scales below 100 $\mu$m are challenging because the tiny gravitational forces exerted by appropriately-sized test bodies are easily “polluted” by extraneous effects. Here we report results from two latest generations of the Eötvös rotating-attractor torsion-balance ISL tests. In these tests, harmonic torques, exerted on a detector pendulum by a rotating attractor, are studied as functions of separation $s$ between the facing surfaces of the detector and attractor test bodies. Our new device offers significant improvements over those used previously\[5, 9, 11\]. The new test-body design (see Fig. 1) has both 18-fold and 120-fold azimuthal symmetries and tests the ISL at two different length scales at once. The 50%-transparent hole pattern maximizes the signals for a given test-body diameter. Furthermore, the Fourier-Bessel expansion provides nearly-analytic (a single numerical integration) solutions for Newtonian, Yukawa and dipole-dipole torques\[12, 14\].

In our device, the primary science signals are torques varying at $18\omega$ and $120\omega$ where $\omega$ is the attractor rotation frequency. Figure 1 shows the predicted Newton and Yukawa torques as functions of $s$. Note that as...
s increases, the torques decay exponentially with scale lengths inversely proportional to the azimuthal symmetry number \(n\). We calibrated the torque scale (see Fig. 2) by the gravitational interaction between the 3 small spheres on the detector and 3 large spheres on an external turntable. The separation between the 2 sets of spheres was comparable to those used in measuring Newton’s constant \([13]\) and in a regime where independent experiments\([10]\) have verified the \(1/r^2\) law at the \(10^{-3}\) level; our work can be viewed as percent-level measurements of \(G_N\) at separations down to about 50 \(\mu\)m.

Our raw data consist of torque measurements at a set of 3-dimensional displacements \(\vec{\zeta} = (x, y, s)\) between the detector and attractor test bodies. Each data point comprised \(\theta\) (an autocollimator measurement of the detector twist angle), \(\phi\) (the turntable angle from a high-resolution encoder), the capacitance between the detector and the electrostatic shield (a key element in determining \(s\)), plus a dozen other parameters such as apparatus tilts, various temperatures, etc. Because the Fourier-Bessel hole pattern repeats every 60 degrees the data streams were cut into 60 degree segments typically containing 680 points. The \(\theta(\phi)\) data in each cut were fit with harmonic terms and low-order polynomial drift\([5, 10]\). The data-taking cadence and \(\omega\) were set so that each cut contained integral numbers of data points and free-torsional oscillations and that the 18\(\omega\) and 120\(\omega\) signals lay in low-noise regions of the torque power spectrum (see bottom panel in Fig. 2). Harmonic torques \(N_{n\omega}\) were inferred using \(N_{n\omega} = \theta_{n\omega}/\omega_0^2\) where \(\theta_{n\omega}\) was the harmonic amplitude corrected for pendulum inertia plus electronic and digital-filter\([12]\) time constants, \(I = (91.7 \, \text{g cm}^2\) in generation 2) was the detector’s rotational inertia computed from a detailed numerical model, and \(\omega_0 \approx 0.0184 \, \text{s}^{-1}\) was the detector’s free-oscillation frequency. (Uncertainties in \(I\) have no effect on our results because \(I\) appears in both the Fourier-Bessel and calibration-sphere torques.) Electrostatic “patch” effects\([17]\) altered \(\omega_0\) at small \(s\) so, before and after each science run, \(\omega_0\) was measured in “sweep runs” where the free-oscillation amplitude (typically \(\sim 4 \, \mu\text{rad}\)) was increased to about 20 \(\mu\text{rad}\); this gave more precise values for \(\omega_0\) and also provided corrections for small nonlinearities in the apparatus angle scale. As shown in Fig. 3, accurate values for the displacement, \(\vec{\zeta}\), were obtained with the aid of micrometers on the x,y,z stage that supported the torsion fiber. Measurements of the 120\(\omega\) gravitational torques as functions of x and y gave the horizontal displacement, while electrical capacitances as functions of z were used to obtain the vertical displacement. The main challenges were fabricating and then positioning with \(\mu\text{m}\) accuracy 5.5 cm-diameter objects (one of them suspended from an 83 cm-long torsion fiber), minimizing stray electrostatic and magneto-static effects, dealing with seismic vibrations, and eliminating dust particles that can prevent taking data at small \(s\). The detector was gold-coated and surrounded by a rigid, almost hermetic, gold-coated electrostatic shield; its key element was a 10\(\mu\text{m}\)-thick, tightly-stretched gold-coated beryllium-copper foil located between the detector and attractor. Seismic vibrations coupled to patch fields substantially increased the noise when the pendulum-foil separation was less than 30 \(\mu\text{m}\). Multi-layer \(\mu\text{m}\)etal shields isolated the detector from external fields as well as from the turntable motor. Instrumental temperature variations during a run were controlled at the \(\pm 5\) mK level.

In our first generation work\([12]\), the test bodies were
cut from 50 µm thick tungsten foils by electric discharge machining and were kept flat by attaching them to Pyrex glass annuli using Dow Integral E100 adhesive film. They were then coated with gold and mounted on the pendulum frame and attractor turntable. The much smaller test body scale compared to our earlier work[5, 10] required new instrumentation (SmartScope[13]) for characterizing and aligning the test bodies, more precise turntable control, and new electrostatic shields that provided better access for removing dust particles. Otherwise the instrument and general principles of the analysis were the same as in Ref. [5]. This work resolved at ≈ 50σ the 18ω gravitational signal between two objects with masses of only 200 mg. To our knowledge, this was easily the smallest mass-scale for which the gravitational interaction had been resolved[18]. We did not publish that result, which proved separations between 57 µm and 2.00 mm, because there were hints of a subtle systematic effect that we were unable to identify.

Our second generation work[19], whose results we present here, used platinum test bodies with detector and attractor thicknesses of 54 and 99 µm, respectively. These were epoxied to BK7 glass annuli using a technique, similar to that described in Ref. [20], that filled the hole pattern with glue so that the test-body faces were flat to within ±2.3 µm and ±1.5 µm. In addition, we modified the vacuum vessel and pumping system to accommodate an in situ technique, similar to that described in Ref. [20], that filled the 120-fold and 120-fold hole patterns in the detector with masses of only 200 mg. To our knowledge, this was easily the smallest mass-scale for which the gravitational interaction had been resolved[18]. We did not publish that result, which proved separations between 57 µm and 2.00 mm, because there were hints of a subtle systematic effect that we were unable to identify.

We made the usual checks for systematic effects[5, 10] by varying temperatures and their gradients, impressing electrical and magnetic fields on the detector and attractor, etc. The only significant systematic effect arose from the magnetic susceptibility of the platinum test bodies. An ambient magnetic field will slightly magnetize the test body faces. When error bars are not shown this and later figures, the uncertainties are smaller than the symbols.

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3rd harmonic of the 18ω signal) torques. The Fourier-Bessel framework provides semi-analytic solutions for the magnetic dipole-dipole[14] torques in azimuthal[14] or axial[19] geometries. Fourier-Bessel spin-spin predictions with a single adjustable scale parameter agree with our observations. The 120ω signal had the only significant (roughly 1%) magnetic contribution which we subtracted using the data in Fig. 4. The resulting “gravitational” torques are shown in Fig. 5.
filled the holes. SmartScope measurements of attractor runout and tilt, plus capacitance measurements \cite{12} of pendulum tilt) were so well constrained that the corresponding uncertainties in the predicted torques were negligible. The uncertainties in 5 parameters, \(x_0, y_0\) and \(s_0\) (the sum of the thicknesses of the isolation foil and the glue films on the faces of the detector and attractor test bodies), a surface-roughness correction, and the autocolimator angle scale \(\gamma\), had noticeable effects on the predicted torques. We allowed those parameters to float in fitting our torques but added terms to \(\chi^2\) to constrain them by SmartScope measurements and the data shown in Figs. 2 and 3. We accounted for the uncertainties in \(\zeta\) by minimizing

\[
\chi^2(\lambda) = \sum_{j=1}^{95} \sum_m \left[ N_m(\vec{\zeta}_j) - \tilde{N}_m(\vec{\zeta}_j, \eta_j, \lambda) \right]^2 + \sum_{i=1}^{5} \left[ \frac{\eta_i^{\text{exp}} - \eta_i}{\delta \eta_i^{\text{exp}}} \right]^2
\]

where \(\delta s_j\) is the error in \(s\) arising from uncertainties in the measured capacitances.

We first tested the Newtonian model (\(\lambda = \infty\)) and, as shown in the top panel of Fig. 3, obtained an excellent fit: \(\chi^2 = 275.0\) with \(\nu = 285\). We then tested for a single additional Yukawa term by finding the constraints on \(\alpha\) for 66 assumed values of \(\lambda\) between 5 \(\mu\)m and 9 mm; \(\eta\) was allowed to vary independently at each \(\lambda\). None of these Yukawas improved \(\chi^2\) at the 2\(\sigma\) level (\(\Delta \chi^2 = 6.2\));

FIG. 4: (color online) **Top plot:** effect of an applied vertical magnetic field \(B_z\) at \(s = 72\ \mu\)m. Points on the solid and dashed lines were taken with the outermost magnetic shield removed and in place, respectively. The lines are parabolic fits. The field at the detector vanishes at \(B_z = 65\ \mu\)T. The horizontal green band shows the the measured \(s = 72\ \mu\)m torque in our science data. The \((0.0165 \pm 0.0054)\) fN m difference between the \(B_z = 0\) and \(B_z = 65\ \mu\)T torques is the magnetic contribution to the torque. **Bottom plot:** Systematic effect of a \(B_z\) field as a function of \(s\). Points show the difference between torques at a strong applied field \((B_z = -250\ \mu\)T: outermost shield removed) and a nulled field \((+65\ \mu\)T: all shields in place). The smooth curves show our Fourier-Bessel calculations of the spin-spin interaction between the induced magnetizations in the detector and attractor test bodies; a single normalization reproduces the 18\(\omega\), 54\(\omega\) and 120\(\omega\) effects.

FIG. 5: (color online) **Top plot:** 18\(\omega\), 54\(\omega\) and 120\(\omega\) torques corrected for the magnetic systematic. Unless shown otherwise, uncertainties are smaller than the plot symbols. Data points with essentially the same \(s\) are combined for display purposes only. The Newtonian fit is shown and has \(P = 0.654\). Adding a Yukawa term did not improve the fit appreciably. **Bottom plot:** corresponding 95% confidence upper limits on \(|\alpha|\) from this and previous works \cite{5, 10, 16, 21, 26, 29}.  

the best fit $\Delta \chi^2 = 3.3$ occurred for $\lambda = 7.1\mu m$. The bottom panel in Fig.3 displays our $2\sigma$ constraints on $|\alpha|$ (constraints on $+\alpha$ and $-\alpha$ are given in Supplemental Material[27]). We find that any gravitational-strength Yukawa interaction must have $\lambda < 38.6\mu m$. This implies that the dilaton[28] or heavy graviton[3] mass, and the radion unification[2, 11] mass must be greater than 5.1 meV and 7.1 TeV, respectively, and that the largest extra dimension[2] must have a toroidal radius less than 30 $\mu m$. These are the tightest existing lab constraints on “string inspired” new gravitational phenomena.

Environmental vibrations prevented us from probing separations smaller than 52 $\mu m$ and increased the noise in the smaller separation data. We are now implementing an active system to reduce vertical vibrations.

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