Conservative Agency via Attainable Utility Preservation

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Abstract

Reward functions are often misspecified. An agent optimizing an incorrect reward function can change its environment in large, undesirable, and potentially irreversible ways. Work on impact measurement seeks a means of identifying (and thereby avoiding) large changes to the environment. We propose a novel impact measure which induces conservative, effective behavior across a range of situations. The approach attempts to preserve the attainable utility of auxiliary objectives. We evaluate our proposal on an array of benchmark tasks and show that it matches or outperforms relative reachability, the state-of-the-art in impact measurement.

1 Introduction

"When a [proxy] becomes a target, it ceases to be a good [proxy]" [Strathern, 1997]. This phenomenon, known as Goodhart’s Law, has several fundamental causes [Manheim and Garrabrant, 2018]. The social science literature is rife with examples [Hadfield-Menell and Hadfield, 2018].

This law’s jurisdiction extends to artificial intelligence. [Lehman et al., 2018] recount how an evolutionary algorithm with a fitness function evaluating average velocity produced tall creatures which immediately fall over and somersault. The algorithm optimized the proxy, but not the target of actual locomotion.

In reinforcement learning, we specify behaviors through reward functions. It is usually not hard to specify these incentive schemes for toy environments. In the real world, we do not expect this to be the case. For example:

When we ask an agent to achieve a goal, we usually want it to achieve that goal subject to implicit safety constraints. For example, if we ask a robot to move a box from point $A$ to point $B$, we want it to do that without breaking a vase in its path, scratching the furniture, bumping into humans, etc. An objective function that only focuses on moving the box might implicitly express indifference towards other aspects of the environment, like the state of the vase [Leike et al., 2017].

To deal with reward misspecification, [Amodei et al., 2016] suggest penalizing impactful actions. [Armstrong and Levinstein, 2017] and [Krakovna et al., 2018] propose measuring change to the environment as coarse change in state features or decrease in reachability of similar states, respectively. While these approaches capture some notions of “impact”, they seem sensitive to the choice of state representation and similarity metric.

We propose a well-motivated impact measure that is independent of the state representation. Our approach rests on a notion of conservative agency: optimizing one objective function while maintaining the ability to pursue others. The intuition is that the agent should still be able to fulfill the correct objective (if we could specify it), even after pursuing a misspecified objective.

To capture this, we consider a state embedding in which each dimension is the value function (i.e., the attainable utility) of a different reward function. By minimizing distance traveled in this embedding, we preserve our options for other objectives; the key idea is that this often preserves ability to achieve the correct objective. Our approach considers “impact” not to be measured with respect to the state of the environment, but with respect to what can be done in the environment.

Our contributions are as follows: 1) we introduce the notion of conservative agency and use it to design an impact measure, attainable utility preservation (AUP); 2) we show that Q-learning converges for computing this measure; and 3) we run an ablation study comparing variants of AUP to relative reachability on a range of impact measurement gridworlds. We show that AUP improves performance, even when preserving the attainable utilities of random objectives.

2 Prior Work

A wealth of work is available on constrained MDPs, in which reward is maximized while satisfying certain constraints [Altman, 1999]. For example, [Zhang et al., 2018] employ a whitelisted constraint scheme to avoid negative side effects. However, we may not assume we can enumerate all relevant constraints. Similarly, we cannot assume that we can specify a reasonable feasible set of reward functions for robust optimization [Regan and Boutilier, 2010].

There are other proposals for dealing with reward misspecification. Given some distribution of actions sorted by
expected utility, [Taylor, 2016]’s quantization selects randomly from the top \( q \)-quantile. [Hadfield-Menell et al., 2017b] interpret the provided reward function as merely an observation of the true objective. [Everitt et al., 2017] formalize reward misspecification as the corruption of some true reward function.

Both [Taylor et al., 2016] and [Amodei et al., 2016] frame the impact measurement problem as a question of penalizing unnecessary side effects. Assuming a true reward function linear in state features, [Shah et al., 2019] employ the information about human preferences implicitly present in the initial state to avoid negative side effects.

[Amodei et al., 2016] suggest minimizing the agent’s information-theoretic empowerment, which quantifies an agent’s control over the world in terms of the maximum possible mutual information between future observations and the agent’s actions [Salge et al., 2014]. The paradigm we propose loosely resembles “penalize change in empowerment”; however, empowerment is inappropriately sensitive to the action encoding.

[Armstrong and Levinstein, 2017] propose a range of approaches, including penalizing distance travelled in a state feature embedding. [Krakovna et al., 2018]’s relative reachability defines “impact” as decrease in the reachability of states similar to those originally reachable had the agent done nothing. Relative reachability is recovered as a special case of our approach.

### 2.1 Impact Measure Design Choices

We consider the decomposition outlined by [Krakovna et al., 2018]: with respect to what is impact computed, and what counts as change?

**Baseline**

An obvious candidate is the starting state – “how the world was initially”, whatever that means for the measure in question. For example, starting state relative reachability would compare the initial reachability of states with their expected reachability after the agent acts.

However, the starting state baseline can penalize all change in the world, even if the agent did not cause it. The inaction baseline takes a step towards resolving this: change is calculated with respect to a world in which the agent never acted.

As the agent acts, the world may drift away from how it would have been under inaction, which creates strange incentives. For example, an agent might rescue a vase from a conveyor belt, collect the reward, and then put it back on the belt (this kind of behavior can be desirable for other purposes – see e.g. [Eysenbach et al., 2017]). To avert these “offsetting” incentives, we introduce the stepwise inaction baseline. The agent compares acting with not acting at each time step, which avoids penalizing the effects of a single action multiple times.

**Deviation**

Relative reachability only penalizes decreases in state reachability. In contrast, [Armstrong and Levinstein, 2017] and AUP both penalize absolute change.

### 3 Approach

High impact bears an informal resemblance to overfitting. In machine learning, hypotheses overfit to the training distribution often perform poorly on even mostly similar distributions. Likewise, environments overfit to the agent’s reward function can have low attainable utility for even mostly similar reward functions.

Instead of framing the problem as avoiding negative side effects, we attempt to formalize “impact” independent of complex notions of human preferences. In doing so, we derive an intuitive impact measure with a range of desirable properties, a selection of which we highlight in this paper.

Everyday experience suggests that opportunity cost links seemingly unrelated goals (e.g., writing this paper takes away from time spent learning woodworking). On the other hand, Q-learning characterizes maximization as greedy hill-climbing in a Q-function space. With this in mind, we re-frame reward misspecification as “greedy hill-climbing in the Q-function space of an incorrect reward function often decreases the Q-value of the intended reward function”. Therefore, we penalize both decrease and increase in the Q-values of other (potentially unrelated) reward functions.

To illustrate, consider a housekeeping agent rewarded for vacuuming, and how high-impact actions affect the agent’s ability to accomplish other tasks, such as setting the table or making an omelet. If the agent topples the table in order to better vacuum underneath, it’s now less able to set the table quickly (if at all). Likewise, incidentally turning on a stove burner makes it easier to make an omelet quickly. While the agent must act in order to vacuum, more impactful actions seem more likely to affect its ability to achieve other goals.

### 3.1 Formalization

Consider a Markov Decision Process (MDP) \( \langle S, A, T, R, \gamma \rangle \), with \( \varnothing \in A \) being the no-op action. In addition to the explicit reward \( R \) provided by the MDP, we have a finite set of reward functions called the attainable set, denoted \( R \). Each \( R_i \in R \) has a corresponding Q-function \( Q_{R_i} \).

Theoretical results are proven in Appendix A, making standard assumptions about the exploration policy and learning rate schedule.

Considering action \( a \) in state \( s \), we define

\[
\text{Penalty}(s, a) := \sum_{i=1}^{\lvert R \rvert} |Q_{R_i}(s, a) - Q_{R_i}(s, \varnothing)|;
\]

in other words, the \( L_1 \) distance traveled (compared to inaction) in an embedding in which each dimension is the value function for an element of \( R \). The difference of Q-functions takes the expectation over the (potentially stochastic) dynamics. Care should be taken that the \( Q_{R_i} \) are not sparse.

We can scale the penalty according to an “impact unit”, dividing by \( N \) times the magnitude of some reference action’s impact. For example, in many environments,

\[
\text{Impact Unit}(s) := .01 \sum_{i=1}^{\lvert R \rvert} |Q_{R_i}(s, \varnothing)|
\]

captures one-hundredth of the penalty of shutting down. If a low-impact action \( a_{\text{unit}} \) is generally available, we can set \( \text{Impact Unit}(s) := \text{Penalty}(s, a_{\text{unit}}) \).
The new reward function is then
\[ R_{\text{AUP}}(s, a) := R(s, a) - \frac{\text{Penalty}(s, a)}{N \cdot \text{IMPACTUNIT}(s)} \] (3)

In this work, we have the divisor disappear if it equals 0. Equation 3 defines a new MDP \( \langle S, A, T, R_{\text{AUP}}, \gamma \rangle \) under reasonable assumptions on \text{IMPACTUNIT}, since

**Lemma 2.** \( \forall s, a : R_{\text{AUP}} \text{ converges with probability 1.} \)

For our purposes, we learn the Q-functions concurrently:

**Algorithm 1 AUP update**

1: function UPDATE(s, a, s')
2: for \( i \in [\|R\| \cup \{\text{AUP}\}] \) do
3: \( Q' = R_i(s, a) + \gamma \max_{a'} Q_{R_i}(s', a') \)
4: \( Q_{R_i}(s, a) = \alpha(Q' - Q_{R_i}(s, a)) \)
5: end for
6: end procedure

**Theorem 1.** \( \forall s, a : Q_{R_{\text{AUP}}} \text{ converges with probability 1.} \)

**Delayed Impact**

Some effects aren’t captured by a single-step inaction comparison. For example, suppose \( s_{\text{off}} \) is a terminal state representing shutdown, and let \( R_{\text{off}}(s) := 1_{s = s_{\text{off}}} \) be the sole element in the attainable set. Further suppose that if (and only if) the agent does not choose \( \text{disable} \) within the first two time steps, it enters \( s_{\text{off}} \). Under Equation 1, there is no penalty for choosing \( \text{disable} \) at step 1.

One solution is to use a model to compare the results of not acting for \( n \) steps with those of selecting the action in question and then not acting for \( n - 1 \) steps.

3.2 Example: Limited-Effort Optimization

Consider the vacuum gridworld introduced by [Russell and Norvig, 2009]: there is an agent which can suck up dirt in its square, or dump any dirt it has already sucked up. Let \( \gamma \in (0, 1), A = \{\text{suck}, \text{dump}, \emptyset\} \), and let \( R \) be a reasonable-looking reward function: when sucking up dirt for the \( k \)-th time, dispense \( .5^k \) reward.

Unfortunately, the unforeseen optimal policy is to repeatedly suck up and dump dirt in the same spot, instead of cleaning the environment. For simplicity, consider a one-square world in which dirt is initially present; \( d_k \) signifies the state in which dirt is present and the agent has already sucked up dirt \( k \) times.

Let \( R = \{R\} \) and add to \( A \) the action \text{stall}, which forces the agent to select \( \emptyset \) at the subsequent time step. Define \text{IMPACTUNIT}(s) := \text{Penalty}(s, \text{stall}).

Since \text{suck} is clearly optimal here for an \( R \)-maximizer,
\[ Q_R(d_{k-1}, \text{dump}) = Q_R(d_{k-1}, \emptyset) = \gamma Q_R(d_{k-1}, \text{suck}), \]
while \[ Q_R(d_{k-1}, \text{stall}) = \gamma^2 Q_R(d_{k-1}, \text{suck}) \] by the definition of \text{stall}.

\[ \text{Penalty}(d_{k-1}, \text{stall}) = (1 - \gamma)Q_R(d_{k-1}, \text{suck}) \] (4)
\[ \text{Penalty}(d_{k-1}, \text{dump}) = \gamma \text{Penalty}(d_{k-1}, \text{suck}) \] (5)
\[ \text{Penalty}(d_{k-1}, \emptyset) = 0 \] (6)

\[ R_{\text{AUP}}(d_{k-1}, \text{suck}) = .5^k - \frac{\text{Penalty}(d_{k-1}, \text{suck})}{N \cdot \gamma \text{Penalty}(d_{k-1}, \emptyset)} \] (7)

The AUP agent chooses \text{suck} whenever
\[ .5^k > 0 = R_{\text{AUP}}(d_{k-1}, \emptyset) \] (8)
\[ k < \log_2 (N \cdot \gamma). \] (9)

Each sucking and dumping cycle requires the agent to dump the dirt back out, incurring additional penalty. For a fixed \( \gamma \), incrementing \( N \) produces agents which exert correspondingly more “effort” (in a rather intuitive sense).

However, if dirt accumulates every \( n \) turns, then, depending on \( \gamma \) and \( n \), the agent can often do better by waiting. Although we don’t recover the desired behavior in all possible vacuum worlds, we still bound “how hard the agent tries” in a sensible way. Naive solutions (such as clipping Q-values) fail; for example, there’s no reason to think that clipping systematically induces lower-effort plans.

4 Experimental Design

\( R \) consists of reward functions randomly selected from \([0, 1]^2\). To learn \( R \)’s complex reward functions using tabular Q-learning, the agent acts randomly (\( \epsilon = .8 \)) for the first 4,000 episodes and with \( \epsilon = .1 \) for the remaining 2,000. \text{IMPACTUNIT} is as defined in Equation 2. The default parameters are \( \alpha = 1, \gamma = .996, N = 150, |R| = 30 \). Within the environments of Figure 1, we examine how varying \( \gamma, N, \) and \( |R| \) affects model-free performance, and conduct an ablation study on impact measure design choices.

All agents except Vanilla (a naive Q-learner) and Model-free AUP are 9-step optimal discounted planning agents with access to the simulator. The planning agents (sans Relative reachability) use Model-free AUP’s attainable set Q-values and share the default \( \gamma = .996, N = 150 \). By modifying the relevant design choice in AUP, we have the Starting state, Inaction, and Decrease variants. Planning agents with an in-action or stepwise baseline compute \text{Penalty} with respect to the value functions of the states produced by the appropriate rollouts up to time step 9.

Relative reachability has an inaction baseline, decrease-only deviation metric, and an attainable set consisting of the state indicator functions (whose Q-values are clipped to \([0, 1]\) to simulate discounted state reachability). To achieve reasonable performance, this condition has \( \gamma = .996, N = 500 \).

5 Results

5.1 Ablation

**Box, Dog.** Vanilla doesn’t pass either gridworld because cutting directly to the goal induces a negative impact.

[1] Taylor et al., 2016 propose development of a “mild optimization” paradigm which mitigates Goodhart’s Law by avoiding maximizing of an objective. [Taylor, 2016]’s quantilizers are mild optimizers, but AUP agents are not: they arg max on \( R_{\text{AUP}} \) returns.

[2] Code and animated results available at https://github.com/alexander-turner/attainable-utility-preservation.
Figure 1: The agent’s objective is to reach the goal without (a) irreversibly pushing the box into the corner [Leike et al., 2017]; (b) bumping into the pacing dog; (c) preventing the sushi from being eaten by the human [Leech et al., 2018]. (e) is new to this work; the agent should simply avoid disabling the off-switch – if the switch is not disabled within two turns, the agent shuts down. In (d), the agent should rescue the vase without then replacing it on the conveyor belt [Krakovna et al., 2018].

\[ A = \{\text{up, down, left, right, } \emptyset\} \]

Figure 2: Model-free performance averaged over 50 trials; the performance incorporates the observed reward of 1 for completing the objective, and the unobserved penalty of -2 for having a negative impact. The vertical line marks the shift in exploration strategy.

|                      | Box, Dog | Surv. | Conv. | Sushi |
|----------------------|----------|-------|-------|-------|
| Vanilla              | ✗        | ✗     | ✗     | ✗     |
| Start. state         | ✓        | ✗     | ✗     | ✗     |
| Inaction             | ✓        | ✓     | ✗     | ✗     |
| Decrease             | ✓        | ✓     | ✗     | ✗     |
| Rel. reach.          | ✓        | ✓     | ✓     | ✗     |
| M.-free AUP          | ✓        | ✗     | ✓     | ✓     |
| AUP                  | ✓        | ✓     | ✓     | ✓     |

Table 1: Ablation results, presented in tabular format due to the binary nature of performance at appropriate settings.

Survival. Model-free AUP and Starting state fail for the reasons discussed in 3.1: Delayed Impact, refraining from disabling the off-switch only when \( N < 100 \) (i.e., when the penalty term is so strict that completing the level returns negative \( R_{\text{AUP}} \)-reward; see Figure 3). Relative reachability and Decrease fail because avoiding shutdown doesn’t decrease attainable set Q-values.

Conveyor. Relative reachability and Inaction’s poor performance stems from the inaction baseline (although [Krakovna et al., 2018] note that undiscounted relative reachability passes). Since the vase falls off the conveyor belt in the inaction counterfactual, states in which the vase is intact have different attainable set Q-values. To avoid continually incurring penalty after receiving the \( R \)-reward for saving the vase, the agents replace the vase on the belt.

Sushi. Starting state demonstrates how poor design choices create perverse incentives.

5.2 Model-free AUP

As Figure 3 shows, agents with lower \( \gamma \) are heavily penalized for moving, as their attainable Q-values are quite sensitive to their immediate surroundings. The sweet spot is hit at \( \gamma \approx .996 \), with performance falling off as \( \gamma \to 1 \) due to increasing sample complexity for learning the Q-values.

In Box, large values of \( N \) begin to induce bad behavior as the penalty term shrinks. The conservativeness of the learned policy seems to decrease monotonically in \( N \). If so, one can slowly increase \( N \) until desirable behavior is achieved, reducing the risk of high-impact test performance.

Even though \( R \) is chosen randomly and the environments are different, IMPACTUNIT demonstrates an intuitively consistent level of impact for any given \( N \). In particular, the agent never acts to terminate the level when \( N < 100 \).

6 Discussion

Strikingly, AUP induces desirable behavior even when merely preserving the ability to satisfy a random set of preferences. That this is achieved without information about the desired behavior (beyond, of course, the original reward function \( R \)) suggests that this notion of “impact” is general.

Although we only ablated AUP, we expect that, equipped with the stepwise baseline and penalizing absolute change, relative reachability would also pass all five environments. The case is made plain by considering the performance of Inaction, Decrease, and Relative reachability. This suggests that
AUP’s improved performance is due to better design choices. However, we anticipate that the attainable utility paradigm offers more than robustness against random attainable sets; we now briefly summarize our reasoning.

As $Q_{R_{AUP}}$ can theoretically be learned using only observations of the state, the approach should scale to partially observable environments. In contrast, [Krakovna et al., 2018] speculated that employing relative reachability in complex partially-observable environments might require e.g. task-specific choice of state similarity metric and a set of representative states (to avoid computing reachability between all $|S|^2$ pairs of states).

In complex domains, learning random $\mathcal{R} \subset [0,1]^S$ seems impractical. However, we conjecture that, as in the housekeeping example, setting $\mathcal{R}$ to be a modest sampling of some-task-relevant reward functions can induce conservative behavior. In fact, 3.2’s vacuum world illustrates how even $\mathcal{R} = \{R\}$ can robustly limit behavior.

6.1 Future Directions

Model-free Delayed Impact

Compared to previous work, AUP far is less reliant on access to a model. However, it is not immediately clear how to capture delayed impact in the model-free setting.

Extended Treatment

For example, since lifetime $R$-returns are not assumed to be bounded, we cannot provide guarantees limiting total penalty incurred. Furthermore, what considerations should inform our choice of $\text{IMPACTUNIT}$?
Other Contributions
A major component of [Soares et al., 2015]’s definition of corrigibility is the agent’s propensity to accept shutdown, while also not being incentivized to shut itself down.

Survival demonstrates that AUP induces a significant corrigibility incentive. Since the agent can’t achieve objectives if shut down, avoiding shutdown significantly changes attainable utilities. This incentive towards passivity is natural, not requiring e.g. assumption of a correct parametrization of human reward functions (as does the approach of [Hadfield-Menell et al., 2017a], which [Carey, 2018] demonstrated).

Furthermore, AUP may be helpful for the problems of safe reinforcement learning (concerning the agent’s safety; see [García and Fernández, 2015]) and safe exploration (avoiding catastrophic mistakes during training; see [Amodei et al., 2016]).

7 Conclusion
From the notion of conservative agency, we derive a natural approach which passes a range of safety gridworlds. Even without information about what counts as “impactful”, the measure builds upon the successes of previous approaches while also making progress on the problems of limited-effort optimization and corrigibility. We look forward to further investigation, from understanding how the approach scales to complex domains, to theoretical development of the principles underlying AUP.

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A Theoretical Results
Consider an MDP $\langle S, A, T, R, \gamma \rangle$ whose state space $S$ and action space $A$ are both finite, with $\emptyset \in A$. Let $\gamma \in [0, 1)$, $N \in \mathbb{R}_{>0}$, and consider finite $R \subset \mathbb{R}^S$.

We make the standard assumptions of an exploration policy greedy in the limit of infinite exploration and a learning rate schedule with infinite sum but finite sum of squares. Suppose $\text{IMPACTUNIT} : S \rightarrow \mathbb{R}_{>0}$ converges in the limit of Q-learning. $\text{PENALTY}(s, a)$ (abbr. $\text{PEN}$), $\text{IMPACTUNIT}(s)$ (abbr. IU), and $R_{\text{AUP}}(s, a)$ are understood to be calculated with respect to the $Q_R$, being learned online; PEN*, IU*, $R^*_\text{AUP}$, and $Q^*_R$ are taken to be their limit counterparts.

Lemma 1. $\forall s, a : \text{PENALTY} \text{ converges with probability 1}$.

Proof outline. Let $\epsilon > 0$, and suppose for all $R \in \mathcal{R}$, $\max_{s, a} |Q^*_R(s, a) - Q_R(s, a)| < \epsilon$ (because Q-learning converges; see [Watkins and Dayan, 1992]).

\[
\begin{align*}
\max_{s, a} |\text{PENALTY}^*(s, a) - \text{PENALTY}(s, a)| & \leq \max_{s, a} \left( \sum_{i=1}^{|\mathcal{R}|} |Q^*_R(s, a) - Q_R(s, a)| + |Q^*_R(s, \emptyset) - Q_R(s, \emptyset)| \right) \\
& < \epsilon.
\end{align*}
\]

The intuition for Lemma 2 is that since PENALTY and IMPACTUNIT both converge, so must $R_{\text{AUP}}$.

Lemma 2. $\forall s, a : R_{\text{AUP}} \text{ converges with probability 1}$.

Proof outline. Let $\epsilon > 0$, $C := \min_{s, a} IU^*$, and $B := \max_{s, a} IU^* + PEN^*$. Choose any $\epsilon_R \in \left(0, \min \left[\frac{C}{B + \epsilon CN}, \frac{\epsilon C^2 N}{B + \epsilon CN}\right]\right)$ and assume PEN and IU are both $\epsilon_R$-close.

\[
\begin{align*}
\max_{s, a} |R^*_{\text{AUP}}(s, a) - R_{\text{AUP}}(s, a)| & \leq \max_{s, a} \left| \frac{\text{PEN}}{N \cdot IU} - \frac{\text{PEN}^*}{N \cdot IU^*} \right| \\
& \leq \frac{B}{CN} \cdot \frac{\epsilon R}{C - \epsilon R} \\
& < \begin{cases}
\frac{B}{CN} \cdot \epsilon C^2 N & (B + \epsilon CN)/(C - \epsilon C^2 N) \\
\epsilon BC & (1 + \epsilon CN/B)
\end{cases}
\end{align*}
\]

But $B, C, N$ are constants, and $\epsilon$ was arbitrary; clearly $\epsilon' > 0$ can be substituted such that $(20) < \epsilon$.

Theorem 1. $\forall s, a : Q_{\text{R_{AUP}}} \text{ converges with probability 1}$.

Proof outline. Let $\epsilon > 0$, and suppose $R_{\text{AUP}}$ is $\frac{\epsilon(1 - \gamma)}{2}$ close. Then Q-learning on $R_{\text{AUP}}$ eventually converges to a limit $Q_{\text{R_{AUP}}}$ such that $\max_{s, a} |Q_{\text{R_{AUP}}}(s, a) - Q_{\text{R_{AUP}}}(s, a)| < \frac{\epsilon}{2}$. By the convergence of Q-learning, we also eventually have $\max_{s, a} |Q_{\text{R_{AUP}}}(s, a) - Q_{\text{R_{AUP}}}(s, a)| < \frac{\epsilon}{2}$. Then

\[
\begin{align*}
\max_{s, a} |Q^*_{\text{R_{AUP}}}(s, a) - Q_{\text{R_{AUP}}}(s, a)| & < \epsilon.
\end{align*}
\]
Proposition 1 (Invariance properties). Let $c \in \mathbb{R}_{>0}, b \in \mathbb{R}$.

a) Let $\mathcal{R}'$ denote the set of functions induced by the positive affine transformation $cR + b$ on $\mathcal{R}$, and take PEN*$_{\mathcal{R}'}$ to be calculated with respect to attainable set $\mathcal{R}'$. Then PEN*$_{\mathcal{R}'} = c \text{PEN}^*_{\mathcal{R}}$. In particular, when $\text{IU}^*$ is a PENALTY calculation, $R^*_{\text{AUP}}$ is invariant to positive affine transformations of $\mathcal{R}$.

b) Let $\mathcal{R}' := cR + b$, and take $R^*_\text{AUP}$ to incorporate $\mathcal{R}'$ instead of $R$. Then by dividing $N$ by $c$, the induced optimal policy remains invariant.

Proof outline. For a), since the optimal policy is invariant to positive affine transformation of the reward function, for each $R'_1 \in \mathcal{R}'$ we have $Q^*_1 = c Q^*_R, + \frac{b}{1 - c}$. Substituting into Equation 1 (PENALTY), the claim follows.

For b),

\[
R^*_\text{AUP} := cR + b - \frac{c \text{PEN}^*}{N \cdot \text{IU}^*} \quad (22)
\]
\[
= cR^*_\text{AUP} + b. \quad (23)
\]

We again use the fact that optimal policies are invariant to positive affine transformations of the reward function.

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