Charming penguins in $B \to \pi\pi$ from QCD light-cone sum rules

A Khodjamirian†, Th Mannel, B Melić‡

Institut für Theoretische Teilchenphysik, Universität Karlsruhe, D-76128 Karlsruhe, Germany

We use QCD light-cone sum rules to examine the $B \to \pi\pi$ hadronic matrix element of the current-current operator with $c$ quarks in the penguin topology ("charming penguin") as a potential source of the substantial $O(1/m_b)$ effects. Our results indicate that charming penguins do not generate sizable nonperturbative effects at finite $m_b$. The same is valid for the penguin contractions of the current-current operators with light quarks. The dominant penguin topology effects are predicted to be $O(\alpha_s)$. Still, the nonperturbative effects at finite $m_b$ can accumulate to a visible effect that is illustrated by calculating the CP-asymmetry in the $B_0^\pm \to \pi^\pm\pi^-\gamma$ decay.

1 Introduction

With the first measurements of the direct CP asymmetry in the $B \to \pi\pi$ meson system by the BaBar and Belle collaborations, charmless two-body hadronic $B$ decays have become particularly interesting for constraining the CKM matrix elements. In particular, $B \to \pi\pi$ and $B \to K\pi$ decays can be used to extract the angle $\gamma = \text{arg}(V_{ub}^*V_{cb})$. There are several strategies used to determine the $\gamma$ angle from such decays, mainly based on the isospin and SU(3) relations. Unfortunately, the theoretical accuracy of these relations is limited and it has to be improved by calculating the SU(3) breaking effects [1]. In the direct calculation of the relevant hadronic matrix elements one has also to resort to approximate methods. Apart from the naive factorization, which assumes the vanishing nonfactorizable interactions, there are methods which try to investigate the latter [2,3,4,5]. QCD factorization approach [4] shows that in the $m_b \to \infty$ limit, exclusive $B$-decay amplitudes can be expressed in terms of the factorizable part and calculable $O(\alpha_s)$ nonfactorizable corrections. For phenomenological applications it is important to investigate the subleading effects in the decay amplitudes suppressed by inverse powers of $m_b$. Especially interesting are “soft” nonfactorizable effects, involving low-virtuality gluons and quarks, not necessarily accompanied by an $\alpha_s$-suppression. Quantitative estimates of nonfactorizable contributions, including the power-suppressed $O(1/m_b)$ contributions can be obtained [4,5] using the method of QCD light-cone sum rules (LCSR).

Among the most intriguing effects in charmless $B$ decays are the so called “charming penguins”. The $c\bar{c}$ quark pair emitted in the $b \to c\bar{c}d(l)\bar{s}$ decay propagates in the environment of the light spectator cloud and annihilates to gluons, the latter being absorbed in the final charmless state. In this, so called BSS-mechanism [6] the intermediate $c\bar{c}$ loop generates an imaginary part, contributing to the final-state strong rescattering phase. In QCD factorization approach [4,5], charming penguins are typically small, being a part of the $O(\alpha_s)$ nonfactorizable correction to the $B \to \pi\pi$ amplitude. On the other hand, fits of two-body charmless $B$ decays do not exclude substantial $O(1/m_b)$ nonperturbative effects of the charming-penguin type [6]. Therefore, in [7] we have investigated the effects generated by $c$-quark loops in charmless $B$ decays by using LCSR.

2 Charming penguins in LCSR

The decay amplitude for the $\bar{B}^0 \to \pi^+\pi^-\gamma$ decay is given by the hadronic matrix element $\langle \pi^+\pi^-|H_{\text{eff}}|\bar{B}^0\rangle$ of the effective weak Hamiltonian

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pb}V_{pd}^* \left\{ C_1 O_1^p + C_2 O_2^p + \sum_{i=3,\ldots,10} C_i(\mu) O_i + C_{\gamma\gamma} O_{\gamma\gamma} + C_{8g} O_{8g} \right\}.$$

where $O_1^p = (\overline{d}\gamma_{\mu}\gamma_5 p)(\overline{\pi}\gamma^\mu b)$ and $O_2^p = (\overline{\pi}\gamma_{\mu} p)(\overline{d}\gamma_\mu b)$ are the current-current operators ($p = u, c$ and $\Gamma_\mu = \gamma_\mu (1 - \gamma_5)$), $O_{3-10}$ are the penguin operators, and $O_{8g}$ are the electric dipole and chromomagnetic dipole operator, respectively. Each operator entering Eq. 1 contributes to the $B \to \pi\pi$ decay amplitude with a number of different contractions of the quark lines (topologies). In the discussion we will mainly concentrate on the operator $O_1^c$. For convenience we decompose this operator as $O_1^c = \frac{1}{3} O_2^2 + 2 O_2^c$, extracting the color-octet part $O_2^c = (\overline{\pi}\gamma_{\mu} \gamma_5 c)(\overline{d}\gamma_\mu \gamma_5 b)$.

The LCSR expression for the $B \to \pi\pi$ hadronic matrix element of $O_1^c$ is derived from the procedure described.
in detail in [4, 8]. One starts by introducing the correlation function:

\[ F_{\alpha}(\tilde{O}_{2}) = i^{2} \int d^{4}x e^{-i(p-q)x} \int d^{4}y e^{i(p-k)y} \times \langle 0 | T \{ j_{\alpha}^{(\pi)}(x) \tilde{O}_{2}^{(B)}(0) j_{5}^{(B)}(x) \} | \pi^{-}(q) \rangle = (p-k)_{\alpha} F(s_{1}, s_{2}, P^{2}) + ..., \] (2)

where \( j_{\alpha}^{(\pi)} = \pi_{\alpha} \gamma_{5} d \) and \( j_{5}^{(B)} = \imath m_{B} \gamma_{5} d \) are the quark currents interpolating pion and B meson, respectively. Only the color-octet part of the \( \tilde{O}_{2}^{(B)} \) operator contributes at the leading order. The contributions of \( \tilde{O}_{2}^{(B)} \) have to be considered within higher-order corrections.

The LCSR expression for the hadronic matrix element \( A^{(\tilde{O}_{2})}(\tilde{B} \rightarrow \pi^{+}\pi^{-}) \equiv \langle \pi^{-}(p)\pi^{+}(-q)|\tilde{O}_{2}^{(B)}|\tilde{B}^{0}(p-q) \rangle \) is given by

\[ f_{\pi} f_{B} A^{(\tilde{O}_{2})}(\tilde{B}^{0} \rightarrow \pi^{+}\pi^{-}) e^{-m_{B}^{2}/M_{2}^{2}} = \int_{m_{\eta}^{2}}^{s_{B}^{2}} ds_{2} e^{-s_{2}/M_{2}^{2}} \]

\[ \times \left\{ \int_{0}^{s_{B}^{2}} ds_{1} e^{-s_{1}/M_{2}^{2}} \text{Im}_{s_{1}} \text{Im}_{s_{2}} F(s_{1}, s_{2}, P^{2}) \right\}_{p^{2} \rightarrow m_{\eta}^{2}}, \] (3)

where \( M_{1} \) and \( M_{2} \) are the Borel parameters in the pion and B-meson channels, respectively. The parameter \( s_{B}^{2} \) is the effective threshold parameter of the perturbative continuum in the pion (B-meson) channel. In the sum rule (3) we take the finite \( m_{\eta}^{2} \) corrections into account, but neglect numerically very small corrections of order \( s_{B}^{2}/m_{B}^{2} \). The corresponding sum rule for the hadronic matrix elements of the current-current operator with the light quarks \( \tilde{O}_{1}^{(B)} \), \( A_{\alpha}^{(\tilde{O}_{1})}(\tilde{B}^{0} \rightarrow \pi^{+}\pi^{-}) \equiv \langle \pi^{-}(p)\pi^{+}(-q)|\tilde{O}_{1}^{(B)}|\tilde{B}^{0}(p-q) \rangle \) is easily obtained from LCSR for the \( \tilde{O}_{2}^{(B)} \) operator by putting consistently \( m_{c} \rightarrow 0 \).

We calculate \( F(s_{1}, s_{2}, P^{2}) \) in (3) at large spacelike \( s_{1}, s_{2}, P^{2} \) employing the operator product expansion (OPE) near the light-cone. The corresponding diagrams of \( O(\alpha_{s}) \) are shown in Fig. 1. They contain a c-quark loop, which involves a well known function, producing the perturbative imaginary part, due to the BSS mechanism [5, 9]. The remaining diagrams not shown in Fig. 1 with gluons attached to the virtual b and d lines, do not contribute to the sum rule because their double imaginary parts vanish inside the duality regions \( 0 < s_{1} < s_{B}^{0}, m_{B}^{2} < s_{2} < s_{B}^{B} \).

We proceed by investigating the effect of the soft (low-virtuality) gluons coupled to the c-quark loop. On-shell gluons or light quarks emitted at short distances end up in the multiparticle distribution amplitudes (DA’s) of the pion. These contributions are then of the higher twist and are suppressed by inverse powers of the heavy mass scale with respect to the contributions of 2-particle quark-antiquark DA’s of lower twists. Therefore, for our purposes it is sufficient to consider diagrams with one “constituent” gluon i.e., diagrams involving quark-antiquark-gluon DA’s of the pion. However, for the given correlation function (2), the diagram (Fig. 2a) with one gluon vanishes due to the current conservation in the c-quark loop. Nonvanishing terms with the three-particle DA’s emerge from the diagrams, containing at least one hard gluon in addition to the on-shell gluon, Fig.2b. From the studies of the b \( \rightarrow s\gamma \) matrix elements of \( O_{1,2} \), where similar diagrams with an on-shell photon and virtual gluon have been calculated [9], it could be concluded that the contribution of such diagrams is both, \( \alpha_{s} \) and \( O(1/m_{c}^{2}) \) suppressed with respect to the diagrams in Fig. 1.

Furthermore, from the simple dimension counting we find that another possible contribution, shown in the diagram Fig.3a is suppressed by a factor of \( O(1/m_{c}^{2}\ln(m_{c}^{2})) \). Also, the diagram with two gluons emitted from the c-quark loop (Fig. 3b) is not included in our calculation because it contains DA’s with multiplicity larger than three and it is suppressed with respect to the diagrams in Fig. 1 by at least \( O(1/m_{c}^{2}m_{B}^{2}) \). The presence of \( \ln(m_{c}^{2}) \) and \( m_{c}^{-2} \) in the contributions of Fig 3a and Fig 3b, respectively, indicates that at \( m_{c} \rightarrow 0 \) these terms are divergent and by calculating the contribution of \( O_{1}^{B} \) from such diagrams one will have to consider propagation of the
3 Charming penguins and CP asymmetry

For a numerical estimate of the charming penguin in $B \to \pi\pi$ decay we calculate the ratio of the sum rule (3) to the factorizable amplitude $A_E^{(O^i)}(\bar{B}_d^0 \to \pi^+\pi^-) = i m_{B_d}^2 f_{\pi}^2 f_{\pi}^*(0)$:

$$r^{(O^i)}(\bar{B}_d^0 \to \pi^+\pi^-) \equiv \frac{A^{(O^i)}(\bar{B}_d^0 \to \pi^+\pi^-)}{A_E^{(O^i)}(\bar{B}_d^0 \to \pi^+\pi^-)}$$

$$\approx \frac{2 A^{(O^c)}(\bar{B}_d^0 \to \pi^+\pi^-)}{i m_{B_d}^2 f_{\pi}^2 f_{\pi}^*(0)(LCSR)} .$$

With the parameters taken from [8] 3 and by adding linearly the uncertainties caused by the variation of all parameters, we get the following for the penguin-loop contractions with $c$ and $u$ quarks in the loop:

$$r^{(O^c)}(\bar{B}_d^0 \to \pi^+\pi^-) = \left[ -0.29 \pm 0.56 \right] \cdot 10^{-2} ,$$

$$r^{(O^u)}(\bar{B}_d^0 \to \pi^+\pi^-) = \left[ 0.09 \pm 0.21 \right] \cdot 10^{-2} ,$$

The charming penguin corrections turn out to be very small, not larger than the other nonfactorizable corrections. However, they appear to produce a noticeable effect in the CP asymmetry. Therefore, the CP asymmetry appears also to be a good testing ground for the influence of the $1/m_d$ corrections in the charming penguin contributions.

Penguin contributions influence both the direct and the mixing-induced CP violation in $B \to \pi\pi$. Here we concentrate on the direct CP asymmetry in $B_d^0 \to \pi^+\pi^-$ decay, which is given as

$$\alpha_{CP}^{dir} = \frac{(1 - |\xi|^2)}{(1 + |\xi|^2)}$$

where $\xi = e^{-2i(\delta + \gamma)}(1 + R e^{i\gamma})/(1 + R e^{-i\gamma})$ and $R \equiv -P/(R_0 T)$. Here $T$ is the contribution to the $B \to \pi\pi$ amplitude proportional to $V_{ub}^* V_{cd} = |V_{ub}^* V_{cd}| e^{-i\gamma}$. It contains the tree amplitude, the penguin-loop contractions of the current-current operators $O^u_{1,2}$, and also $V_{ub}^* V_{cd}^*$ proportional penguin operator contributions. The remaining contributions, being proportional to $V_{cb} V_{cd}^*$ are contained in $P$. The penguin-loop contractions of the current-current operators $O^c_{1,2}$ represent the main contribution to this part. The factor $R_0 = |V_{ub}||V_{cd}|/(|V_{ub}||V_{cd}|)$ is the ratio of the CKM matrix elements.

Both $T$ and $P$ amplitudes have strong phases; therefore we have $T = |T| e^{i\delta_T}$ and $P = |P| e^{i\delta_P}$ and the CP
asymmetry for $B^0_d \to \pi^+\pi^-$ as a function of the CKM angle $\gamma$. The upper curve is the result obtained for $m_b \to \infty$. The dark region is the LCSR result, with all uncertainties from the method included (uncertainties in the CKM matrix elements are not taken into account). The light region shows the deviation from the $m_b \to \infty$ limit result.

The hard contribution of the dipole operator $O_3^d$ is highly suppressed in comparison to the phases emerging from the penguin-loop contributions. Therefore, we do not find significant nonperturbative $O(1/m_b)$ corrections. In $m_b \to \infty$ limit our result agrees with the QCD factorization prediction for the penguin contractions. Since the strong phase is generated perturbatively, the CP symmetry in $B^0_d \to \pi^+\pi^-$ is expected to be small. However, at finite $m_b$ we show that $O(\alpha_s/m_b)$ corrections to $a_{CP}^{dir}$ accumulate and can be noticeable.

4 Conclusion

The LCSR estimate presented here for the hadronic matrix element of the current-current operator with penguin topology involving $c$ and $u$ quarks [7] shows that the main contribution to the sum rule stems from the $O(\alpha_s)$ quark loop annihilating to a hard gluon, Fig.1. This justifies the generation of the strong rescattering phases in $B \to \pi\pi$ by the (perturbative) BSS mechanism. The soft-gluon effects, which in the sum rule approach correspond to multiparticle pion DA’s, are suppressed, at least by $O(\alpha_s/m_b^2)$. Therefore, we do not find significant nonperturbative $O(1/m_b)$ corrections. In $m_b \to \infty$ limit our result agrees with the QCD factorization prediction for the penguin contractions. Since the strong phase is generated perturbatively, the CP symmetry in $B^0_d \to \pi^+\pi^-$ is expected to be small. However, at finite $m_b$ we show that $O(\alpha_s/m_b)$ corrections to $a_{CP}^{dir}$ accumulate and can be noticeable.

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