Evolutionary Conditions in Dissipative MHD Systems Revisited

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The evolutionary conditions of dissipative continuous magnetohydrodynamic (MHD) shocks are studied. We modify Hada’s approach to the stability analysis of the MHD shock waves. The matching conditions between perturbed shock structure and asymptotic wave modes show that all types of the MHD shocks, including intermediate shocks, are evolutionary and perturbed solutions are uniquely defined. We also adapt our formalism to MHD shocks in systems with resistivity but no viscosity, which is often used in numerical simulations, and we show that all types of shocks found in such systems satisfy the evolutionary conditions and perturbed solutions are uniquely defined. These results suggest that intermediate shocks may appear in physical systems.

§1. Introduction

The magnetohydrodynamic (MHD) Rankine-Hugoniot relations possess six shock solutions, representing fast, slow and (four types of) intermediate shocks, which satisfy the entropy condition. Arguments based upon the theory of strictly hyperbolic systems of conservation laws lead to the so-called evolutionary conditions (Lax 1957), and it has been found that all types of the intermediate shocks are non-evolutionary in an ideal MHD system (see Landau and Lifshitz 1960, Jeffery and Taniuti 1964, Kantrowiz and Petschek 1966, Polovin and Demutskii 1990, and references therein). These conditions effectively ruled out the existence of intermediate shocks in nature, since they have no neighboring solutions corresponding to small perturbations.

In spite of the above-mentioned results, a series of numerical experiments on dissipative MHD systems carried out by Wu (1987, 1988, 1990) showed that at least some intermediate shocks are admissible and can be formed through nonlinear steeping from continuous waves. Furthermore, Chao et al. (1993) reported the detection of an interplanetary intermediate shock in the Voyager 1 data.

The dissipative steady solutions that correspond to fast shock and slow shock are coplanar (i.e., the velocity field and the magnetic field are in the same plane everywhere). By contrast, the four types of dissipative steady solutions of intermediate shocks can have non-coplanar structure inside their thin (but a finitely thick) front, and the non-coplanar component of the magnetic flux is limited by a maximum value, which is proportional to the dissipation coefficients. These properties were first pointed out by Wu (1990).

The interactions between intermediate shocks and the Alfvén waves have been the subject of many studies employing nonlinear dissipative MHD simulations (Wu 1988, Wu and Kennel 1992, Markovskii and Skorokhodov 2000, Falle and Komissarov 2001). If the classical evolutionary conditions hold even in such dissipative
systems, intermediate shocks should instantly disappear. However, the results of those works do not show such evolution. Wu (1988) and Wu and Kennel (1992) showed that intermediate shocks survive a finite time after interacting with a rotational discontinuity. Note that such an interaction causes the intermediate shock to become non-coplanar, even outside the shock structure. Therefore, they conjectured that there exists a new class of time-dependent intermediate shocks that do not obey the MHD Rankine-Hugoniot relations, since they violate the condition of coplanarity between upstream and downstream, and that these shocks represent the neighboring states of the intermediate shocks. Wu (1988) also reported that an intermediate shock remains stable in the case of the interaction with small amplitude Alfvén waves. Considering these results, intermediate shocks seem to be evolutionary, but whether an intermediate shock evolves into other shocks and waves depends on the nature of the perturbations. Falle and Komissarov (2001) showed that the lifetime of an intermediate shock is short under noisy conditions. The reason for this is as follows. The magnetic flux inside the structure of an intermediate shock is rotated as a result of interactions with Alfvén waves from the downstream. Then, the intermediate shock eventually breaks up, because there is a maximum magnitude of the non-coplanar component of the magnetic flux that the shock can survive. Thus, they pointed out that intermediate shocks tend to survive for a long time in numerical simulations because the numerical diffusion leads to a large maximum value of the non-coplanar magnetic flux.

The evolutionary conditions for intermediate shocks in dissipative MHD systems were reconsidered by Hada (1994) and Markovskii (1998). Hada (1994) reported that there are additional wave modes that originate in the dissipation, and the number of outgoing waves from a shock front is larger than in the case of an ideal MHD system. He concluded that intermediate shocks are evolutionary, but the set of equations are under-determined, owing to the existence of many dissipative modes. In consideration of these points, he introduced the minimum dissipation principle in order to uniquely define a solution.

In previous studies of the evolutionary conditions in dissipative MHD systems, the continuous structure of the shock front was not taken into account. The analyses employed in those works are based on the linear perturbation theory of discontinuities or weak solutions. However, in dissipative systems, we have to treat an unperturbed shock as a continuous transition layer, and we have to solve differential equations instead of the conservation laws describing an ideal system. In this paper, we examine the evolutionary conditions for continuous MHD shock waves. Section 2 introduces the basic equations that describe the perturbation of the shock structure. In §3, we formulate the evolutionary conditions for continuous shock waves. We adapt our formulation to various MHD shocks in §§4 and 5. In §6, we summarize our results and discuss their implications.

§2. Basic equations for the linear analysis of dissipative MHD systems

The one-dimensional dissipative MHD equations are given by
\[ \frac{\partial \vec{u}}{\partial t} + \frac{\partial \vec{f}}{\partial x} = \frac{\partial \vec{d}}{\partial x}, \quad (2.1) \]

\[ \vec{u} = \begin{pmatrix} \rho \\ \rho v_x \\ \rho \vec{v}_t \\ B_t \\ e \end{pmatrix}, \quad \vec{f} = \begin{pmatrix} \rho v_x \\ \rho v_x^2 + \rho v_x B_t B_x \\ v_x B_t - \vec{v}_t \cdot B_x \\ (e + P) v_x - B_x (\vec{v} \cdot \vec{B}) \end{pmatrix}, \quad (2.2) \]

\[ \vec{d} = \begin{pmatrix} 0 \\ \left( \frac{4}{3} \nu + \mu \right) \frac{\partial v_x}{\partial x} \\ \nu \frac{\partial B_t}{\partial x} \\ \left( \frac{\eta}{\gamma} + \mu \right) v_x \frac{\partial^2 v_x}{\partial x^2} + \nu \vec{v} \cdot \frac{\partial \vec{v}}{\partial x} + \eta B_t \cdot \frac{\partial \vec{B}}{\partial x} + \kappa \frac{\partial}{\partial x} \left( \frac{\rho}{\rho} \right) \end{pmatrix}, \quad (2.3) \]

\[ e = \frac{1}{2} \rho v_x^2 + \frac{p}{\gamma - 1} + \frac{1}{2} B_x^2, \quad P = p + \frac{1}{2} B_x^2, \quad (2.4) \]

where the subscript \( x \) indicates the \( x \)-component and the \( t \) indicates the \( y \)- and \( z \)-components. We use units such that the factor \( 4\pi \) does not appear. Here, \( \nu \) and \( \mu \) are the shear and bulk viscosity coefficients, \( \eta \) is the electric resistivity, and \( \kappa \) is the heat conduction coefficient. In addition to Eq. (2.1), we impose the condition \( \nabla \cdot \vec{B} = 0 \). This implies that \( B_x \) is a constant in the one-dimensional case.

In the following, we consider the steady shock solution of Eqs. (2.1)–(2.4) as an unperturbed state. In this case, without loss of generality, we can choose the coordinate system such that the upstream is \( x = -\infty \), the downstream is \( x = +\infty \) (i.e., \( v_{x,0} > 0 \) everywhere), and the unperturbed velocity and the magnetic field are in the \( x-y \) plane at \( x = \pm \infty \) (coplanarity). We also employ the shock rest frame and assume that the shock structure is located near \( x = 0 \). We call the steady state type 1 if \( v_x \geq c_f \), type 2 if \( c_f \geq v_x \geq c_i \), type 3 if \( c_i \geq v_x \geq c_s \), and type 4 if \( c_s \geq v_x \), where \( c_f, c_s \) and \( c_i \) are the fast, slow and Alfvén (intermediate) speeds, respectively. The steady shock solutions of Eqs. (2.1)–(2.4) have been thoroughly studied (see, e.g., Wu 1990). There are six types of shock solutions, \( 1 \rightarrow 2 \) (fast shock), \( 3 \rightarrow 4 \) (slow shock) and \( 1 \rightarrow 3, 1 \rightarrow 4, 2 \rightarrow 3 \) and \( 2 \rightarrow 4 \) (intermediate shocks), where the numbers before and after the arrow represent the states ahead and behind the shock front.

Let us consider a small perturbation of the steady shock structure. We assume that the perturbation of a physical variable \( g(x,t) \) takes the form

\[ g(x,t) = g_0(x) + \delta g(x) e^{-i\omega t}, \quad (2.5) \]

where the subscript 0 indicates the unperturbed variable, which is a steady shock solution of Eqs. (2.1)–(2.4). Linearizing Eq. (2.1), we obtain the perturbed shock equations:

\[ \frac{d}{dx} \delta \rho = \frac{1}{v_{x,0}} \left( i \omega \delta \rho - \frac{dv_{x,0}}{dx} \delta \rho - \rho_0 \frac{d\delta v_x}{dx} - \frac{d\rho_0}{dx} \delta v_x \right), \quad (2.6) \]
\[
\left(\frac{4}{3} \nu + \mu \right) \frac{d^2}{dx^2} \delta v_x = \frac{d}{dx} \left( v_{x,0}^2 \delta \rho + 2 \rho_0 v_{x,0} \delta v_x + \delta p + B_{y,0} \delta B_y + B_{z,0} \delta B_z \right) - i \omega (\rho_0 \delta v_x + v_{x,0} \delta \rho), \tag{2.7}
\]

\[
\nu \frac{d^2}{dx^2} \delta v_y = \frac{d}{dx} \left( v_{x,0} v_{y,0} \delta \rho + \rho_0 v_{y,0} \delta v_x + \rho_0 v_{x,0} \delta v_y - B_{x,0} \delta B_y \right) - i \omega (\rho_0 \delta v_y + v_{y,0} \delta \rho), \tag{2.8}
\]

\[
\nu \frac{d^2}{dx^2} \delta v_z = \frac{d}{dx} \left( v_{x,0} v_{z,0} \delta \rho + \rho_0 v_{z,0} \delta v_x + \rho_0 v_{x,0} \delta v_z - B_{x,0} \delta B_z \right) - i \omega (\rho_0 \delta v_z + v_{z,0} \delta \rho), \tag{2.9}
\]

\[
\eta \frac{d^2}{dx^2} \delta B_y = \frac{d}{dx} \left( B_{y,0} v_{x,0} + v_{x,0} B_{y,0} - B_{x,0} \delta B_y \right) - i \omega \delta B_y, \tag{2.10}
\]

\[
\eta \frac{d^2}{dx^2} \delta B_z = \frac{d}{dx} \left( B_{z,0} v_{x,0} + v_{x,0} B_{z,0} - B_{x,0} \delta B_z \right) - i \omega \delta B_z, \tag{2.11}
\]

\[
\frac{\kappa}{\rho_0} \frac{d^2}{dx^2} \delta p = \kappa \left( \frac{2}{\rho_0^2} \frac{d}{dx} \left( \frac{d \delta \rho}{dx} \right) + \frac{d^2}{dx^2} \left( \frac{d \rho_0}{dx} \delta \rho \right) \right) - \left( \frac{4}{3} \nu + \mu \right) \frac{d}{dx} \left( v_{x,0} \frac{d \delta v_x}{dx} + \frac{d v_{x,0}}{dx} \delta v_x \right) - \nu \frac{d}{dx} \left( \frac{d v_{y,0}}{dx} + \frac{d v_{y,0}}{dx} \delta v_y + v_{z,0} \frac{d \delta v_z}{dx} \right) + \eta \frac{d}{dx} \left( B_{y,0} \frac{d \delta B_y}{dx} + \frac{d B_{y,0}}{dx} \delta B_y + B_{z,0} \frac{d \delta B_z}{dx} + \frac{d B_{z,0}}{dx} \delta B_z \right) \right. 
\]

\[
\left. + \frac{d}{dx} \left\{ \left( \delta e + \delta p + B_{y,0} \delta B_y + B_{z,0} \delta B_z \right) v_{x,0} + (e_0 + p_0) + \frac{1}{2} B_0^2 \right\} - i \omega \delta e. \right) \tag{2.12}
\]

Equation (2.6) is a first-order ordinary differential equation with respect to \( \delta \rho \), and Eqs. (2.7)–(2.12) are second-order differential equation with respect to \( \delta v_x, \delta v_y, \delta v_z, \delta B_y, \delta B_z, \) and \( \delta p \), respectively. Therefore, we have, in total, 13th-order differential equations. We denote the order of the perturbed equations by \( N = 13 \). If we choose the coplanar intermediate shock solution (i.e., \( v_{z,0} = B_{z,0} = 0 \) everywhere) as the unperturbed state, then Eqs. (2.6)–(2.12) can be separated into two sets of equations. The first set consists of Eqs. (2.6)–(2.8), (2.10), and (2.12), which relate the \( x \) and \( y \) components of the perturbations. This set constitutes, in total, 9th-order differential equations \( (N_{xy} = 9) \). The other set consists of Eqs. (2.9) and (2.11), which relate the \( z \)-components of the perturbations. This set constitutes, in total, 4th-order differential equations \( (N_z = 4) \).

§3. Evolutionary conditions for the continuous shock waves

In this section, we formulate the evolutionary conditions for continuous MHD shock waves. We divide the space into three regions. Two of them are regions far away from the shock front, in which the unperturbed physical variables can be
regarded as constants. We refer to these regions in the upstream and the downstream direction as regions $\mathcal{U}$ and $\mathcal{D}$, respectively. The region between the regions $\mathcal{U}$ and $\mathcal{D}$ is the transition region, where the unperturbed physical variables change continuously. We call this region $\mathcal{T}$. The situation is schematically illustrated in Fig. 1.

As shown in the previous section, the set of perturbed shock equations constitutes, in total, $N$th-order ordinary differential equations. Thus, in order to determine a solution of these equations in the region $\mathcal{T}$, we need $N$ boundary conditions at the edges of the region $\mathcal{T}$. In other words, in the region $\mathcal{T}$, there are $N$ degrees of freedom that we must account for in order to determine a solution of these equations.

In the regions $\mathcal{U}$ and $\mathcal{D}$, unperturbed physical variables are regarded as constants, and we can therefore easily obtain the asymptotic solutions of the perturbed shock equations. Omitting the spatial derivative of the zeroth-order variable, and Fourier transforming Eqs. (2.6)–(2.12) in space ($\partial/\partial x \sim ik$), we obtain the characteristic equation for the asymptotic waves. The determinant of the characteristic matrix provides this characteristic equation:

$$D_{fse} D_i = 0,$$

$$D_{fse} = c_i^2 (\gamma \Omega K V' + 3 i c_a^2 K') + 3 V \{ i c_a^2 K' R + \gamma \Omega K (c_A^2 - c_i^2 + RV'/3)\},$$

$$D_i = (\Omega + i \eta k^2) (\Omega + i \nu k^2) - c_i^2 k^2,$$

where

$$\Omega = \omega - k v_{x,0},$$

$$c_a^2 = \gamma p_0/\rho_0,$$

$$c_i^2 = B_{x,0}^2/\rho_0,$$

$$c_A^2 = (B_{x,0}^2 + B_{y,0}^2)/\rho_0,$$

$$V = k^2 \nu/\rho - i \Omega,$$

$$V' = k^2 (3 \mu - 4 \nu)/\rho - 3 i \Omega,$$

$$R = \eta - i \Omega/k^2.$$
Here, the zeroth-order variables have been evaluated in the regions $\mathcal{U}$ and $\mathcal{D}$, and we have used the fact that the non-coplanar component of the unperturbed variables vanishes ($B_{z,0} = v_{z,0} = 0$) in the asymptotic regions, owing to the coplanarity condition. The solutions of the characteristic equation $D_{fse} = 0$ for $\omega$ as a function of $k$ provide the dispersion relations for the fast, slow, and entropy waves in uniform dissipative medium, and the solutions of the equation $D_i = 0$ provide the dispersion relation for the Alfvén waves.

**Definition of a Mode** As shown below, we use the asymptotic solutions in the regions $\mathcal{U}$ and $\mathcal{D}$ as the boundary conditions for the differential equations (2.6)–(2.12). These conditions are expressed as superpositions of the independent asymptotic solutions. The number of degrees of freedom of the spatial behavior of the perturbation for given $\omega$ is equal to the number of independent asymptotic solutions. Thus, these solutions are obtained by solving the characteristic equation (3.1) for $k$ as a function of given $\omega$. In this paper, we refer to an independent asymptotic solution as a “mode” or “asymptotic mode”. Note that such mode does not represent a solution of characteristic equation for $\omega$ as a function of $k$.

Hada (1994) studied the solutions of $D_i = 0$ in the limit of small dissipation coefficients. He found that in addition to the solutions that correspond to the Alfvén modes, there are other dissipative modes. Details of these solutions are discussed in the next section.

Let us consider the evolutionary condition. If a small amplitude incident (ingoing) wave of frequency $\omega$ from the region $\mathcal{U}$ or $\mathcal{D}$ impinges upon the shock, then the shock front will be perturbed and emit the waves with the same frequency. Let $m$ ($= m_{\mathcal{U}} + m_{\mathcal{D}}$) denote the number of resulting asymptotic modes in the region $\mathcal{U}$ ($m_{\mathcal{U}}$) and $\mathcal{D}$ ($m_{\mathcal{D}}$), excluding the incident wave, i.e. the number of modes that can be emitted or raised at the shock front. In the regions $\mathcal{U}$ and $\mathcal{D}$, the solutions of the perturbed shock equations can be expressed as superpositions of the $m$ asymptotic modes and one incident wave. These asymptotic solutions are determined by $m$ parameters, i.e. the amplitudes of the $m$ asymptotic modes. Note that the amplitude of the incident wave is determined. Thus, in the case

$$N = m,$$  \hfill (3.13)

we can obtain a solution of the perturbed shock equations (2.6)–(2.12) in the region $\mathcal{T}$ that smoothly connects to the asymptotic solutions at the edges of the region $\mathcal{T}$ for an arbitrary incident wave by appropriately choosing $m$ amplitudes of the asymptotic modes. However, if $m$ is less than $N$, we cannot obtain such a solution, and if $m$ is greater than $N$, we can obtain such a solution, but we need additional conditions or constraints in order to uniquely define this solution. Therefore, the condition (3.13) represents the evolutionary condition in the dissipative system.

If the unperturbed shock structure is coplanar, the evolutionary condition (3.13) can be divided into the two conditions

$$N_{xy} = m_{xy},$$  \hfill (3.14)
where \( m_{xy} \) and \( m_z \) are the numbers of asymptotic modes of \( D_{fse} = 0 \) and \( D_i = 0 \), respectively. We call the condition (3.14) the evolutionary condition for the \( x \) and \( y \) components and the condition (3.15) the evolutionary condition for the \( z \)-components. Of course, evolutionary shocks must satisfy both conditions.

§4. The evolutionary condition for the \( z \)-components

In this section we show that, in contrast to an ideal system, all types of intermediate shocks satisfy the evolutionary condition for the \( z \)-components, \( N_z = m_z \). The solutions derived by Hada (1994) of the equation \( D_i = 0 \) for \( k \) under the assumption of small dissipation coefficients are as follows:

\[
k^{(\pm)} = \frac{\omega}{v_{x,0} \pm c_i} + \mathcal{O}(\eta, \nu),
\]

\[
k^{(\pm d)} = \frac{-i}{2\eta \nu} \left[ (\eta + \nu) v_{x,0} \pm \left\{ (\eta + \nu)^2 v_{x,0}^2 - 4 \eta \nu \left( v_{x,0}^2 - c_i^2 \right) \right\}^2 \right]^{1/2} + \mathcal{O}(\eta^0, \nu^0).
\]

Here, the \((+)\) and \((-)\) modes are the usual Alfvén waves, which propagate parallel and anti-parallel to the \( x \)-axis in the fluid rest frame. The \((\pm d)\) modes, with no counterparts in the ideal system, are the so-called “dissipative modes”. The dissipative modes do not propagate, because \( k^{(\pm d)} \) do not have real parts in their primary terms, and they are localized around the shock front. We exclude diverging dissipative modes from our consideration, deeming them to be unphysical asymptotic solutions. Thus, dissipative modes are physical if and only if

\[
\text{Im}[k^{(\pm d)}] > 0 \quad \text{in the region} \ U, \quad \text{or} \quad \text{Im}[k^{(\pm d)}] < 0 \quad \text{in the region} \ D.
\]

If the flow speed is super-Alfvénic in the shock rest frame, then from Eq. (4.2), the imaginary parts of \( k^{(\pm d)} \) are less than zero. Therefore, there are two dissipative modes in the region \( U \), but no dissipative mode in the region \( D \). Contrastingly, if the flow speed is sub-Alfvénic, the imaginary part of \( k^{(+d)} \) is greater than zero and that of \( k^{(-d)} \) is less than zero. Thus, in this case, there is one dissipative mode in each of the regions \( U \) and \( D \).

Let us consider the evolutionary condition for the \( z \)-components. In the case of the fast shock, the flow speeds both in the upstream and downstream regions are super-Alfvénic. Therefore, the \( k^{(\pm)} \) modes propagate in the downstream direction in the region \( D \), and the \( k^{(\pm d)} \) modes are in the region \( U \). In the same way, the flow speeds of the slow shock are sub-Alfvénic in both regions \( U \) and \( D \). In this case, there are \( k^{(-)} \) and \( k^{(+)} \) modes in the region \( U \) and \( k^{(+)} \) and \( k^{(-)} \) modes in the region \( D \). In the case of intermediate shocks, the upstream flow speed is super-Alfvénic, and the downstream flow speed is sub-Alfvénic. There are \( k^{(\pm d)} \) modes in the region \( U \), and \( k^{(+)} \) and \( k^{(-)} \) modes in the region \( D \). The situations are schematically
Fig. 2. Illustration of outgoing ordinary MHD modes and localized dissipative modes, where the bars represent the region \( T \), the arrows represent the propagation of the asymptotic modes, and the triangles represent the localized dissipative modes. The + and − signs before the Alfvén modes denote the propagation direction in the fluid rest frame, and \( c_i = (B_{z0}^2/\rho_0)^{1/2} \) is the Alfvén (intermediate) speed.

illustrated in Fig. 2. In all cases, the MHD shocks satisfy the evolutionary condition for the \( z \)-components, i.e. \( N_z = m_z = 4 \).

§5. Evolutionary conditions in resistive but inviscid MHD systems

Let us consider the case in which there is only resistivity in the dissipation. The one-dimensional basic equation of a weakly ionized gas, a main component of the interstellar medium, can be written as a resistive, inviscid MHD equation under the strong coupling (one-fluid) approximation. Furthermore, Wu (1987, 1990) showed using a one-dimensional simulation of such a system that intermediate shocks are formed through nonlinear steeping from simple waves. Thus, it is meaningful to analyze the evolutionary conditions in a system of this kind. Here, we apply the formalism developed in §3 to resistive, inviscid MHD shocks.

In this system the perturbed basic equations (2.6)–(2.12) become first-order ordinary differential equations with respect to \( \delta \rho, \delta p, \) and \( \vec{\delta}v \) and second-order ones with respect to \( \vec{\delta}B_t \). Thus, we have, in total, 9th-order \((N = 9)\) differential equations \((N_{xy} = 6, N_z = 3\) in the case of coplanar shocks). The number of asymptotic modes is determined by the solutions to the resistive version of Eq. (3.1),

\[
D_{fse} = \Omega \{ c_i^2 \bar{c}_a^2 k^4 - i \eta \bar{c}_a^2 \Omega k^4 - (c_A^2 + \bar{c}_a^2)\Omega^2 k^2 + i \eta \Omega^3 k^2 + \Omega^4 \},
\]  

(5.1)
\[ D_i = \Omega \left( \Omega + i \eta k^2 \right) - c_{i,0}^2 k^2. \]  

Equation (5.2), which contains three asymptotic modes, was studied by Hada (1994). Two of these modes correspond to two Alfvén waves, and the third one corresponds to the dissipative mode. The dissipative mode solution under the assumption of small resistivity is

\[ k^{(d1)} = i \frac{c_i^2 - v_{x,0}^2}{\eta v_{x,0}} + O(\eta^0, \nu^0), \]

which is physical in the upstream region for the super-Alfvénic states (states 1 and 2) and physical in the downstream region for the sub-Alfvénic states (states 3 and 4).

Equation (5.1) contains six asymptotic modes, five of which correspond to two fast waves, two slow waves and one entropy wave, and one of which corresponds to the dissipative mode. The dissipative mode solution under the assumption of small resistivity is

\[ k^{(d2)} = i \frac{c_i^2 c_{a}^2 - (c_A^2 + c_a^2) v_{x,0}^2 + v_{x,0}^4}{\eta v_{x,0} (c_a^2 - v_{x,0}^2)} + O(\eta^0, \nu^0). \]

In the case of the MHD system with resistivity but no viscosity, steady shock structure whose flow velocity is supersonic \((v_x > c_a)\) ahead of the shock front and subsonic \((v_x < c_a)\) behind the shock front is impossible (see Wu 1990). Thus, it is possible for there to be shock structures for the cases \(1 \rightarrow 2, 1 \rightarrow 3, 2 \rightarrow 3\) and \(2 \rightarrow 4, 3 \rightarrow 4\) whose flow speeds are supersonic everywhere in the shock rest frame and \(2 \rightarrow 3, 2 \rightarrow 4, 3 \rightarrow 4\) whose flow speeds are subsonic everywhere in the shock rest frame. The denominator of the right-hand side of Eq. (5.4) is negative in the supersonic state and positive in the subsonic state. The numerator is positive in the state 1 (superfast) and 4 (sub-slow) and negative in the states 2 and 3 (sub-fast and superslow). Therefore, in the supersonic case, the dissipative \((d2)\) mode is physical in the region \(U\) for the state 1 and in the region \(D\) for the states 2 and 3. In the subsonic case, it is physical in the region \(U\) for the states 2 and 3 and in the region \(D\) for the state 4.

The number of asymptotic modes in the asymptotic regions is \(m = 9\) \((m_{xy} = 6, m_z = 3\) in the case of non-coplanar shocks) for all the shocks. We illustrate the situations for supersonic shocks in Fig. 3 and for subsonic shocks in Fig. 4. Therefore, all the shocks that are allowed in an MHD system with resistivity but no viscosity satisfy the evolutionary condition \(N = m = 9\) (or \(N_{xy} = m_{xy} = 6, N_z = m_z = 3\)).

\section{Summary and discussion}

We have studied the evolutionary conditions for continuous MHD shock waves in a dissipative system. We have shown that all types of MHD shocks, even intermediate shocks, satisfy the evolutionary condition for the \(z\)-components, which is not satisfied in an ideal MHD system. In particular, in a resistive, inviscid system, all types of MHD shocks that are allowed in such a system satisfy the evolutionary conditions. We thus conclude that, intermediate shocks have neighboring solutions, and they can survive interactions with arbitrary small amplitude waves.
The difference between the evolutionary conditions of ideal and dissipative systems is that in a dissipative system there are dissipative perturbation modes, which do not propagate and are localized around the shock front. These modes provide degrees of freedom that do not exist in the ideal system. This difference was first pointed out by Hada (1994). Hada (1994) attempted to connect the perturbation by using the Rankine-Hugoniot relations. When this is done, however, the set of equations becomes under-determined, owing to the existence of many dissipative modes. In order to uniquely define solution, Hada (1994) introduced the minimum dissipation principle. In this paper, however, we have shown that perturbations in the asymptotic regions (far from the shock front) can be uniquely connected by solving differential equations, instead of the conservation laws describing the ideal or hyperbolic system, because the shock is not a weak solution but, rather, a continuous solution in a dissipative system. Note that the solution of the differential equations also satisfies the usual conservation laws in the asymptotic regions.

Our approach is similar to that of Wu (1988) and Wu and Kennel (1992). They
studied the interaction between an intermediate shock and a non-linear Alfvén wave whose transverse magnetic field is rotated. They showed using non-linear simulations that there exists a new class of time-dependent intermediate shocks that violate the coplanarity condition even outside the shock structure (i.e., they do not satisfy the MHD Rankine-Hugoniot relations between the upstream and downstream regions). This new class of intermediate shocks may represent neighboring states of the intermediate shocks. Similarly, our approach treats the interaction between the intermediate shock and small amplitude (linear) MHD waves. With this approach, we have found that the intermediate shocks do indeed have neighboring solutions, which describe the time evolution of perturbed intermediate shocks in the linear regime.

The intermediate shock which has maximum non-coplanar magnetic flux inside the shock structure breaks up into other shocks and waves as a result of its interaction with Alfvén waves (Markovskii and Skorokhodov 2000 and Falle and Komissarov 2001). It may be possible to analyze such an unstable phenomenon in terms of linear stability analysis. We expect that our formulation will be used in such analysis.
The existence of intermediate shocks has been debated mainly in the context of interplanetary systems (Chao et al. 1993, Chao 1995). However, it is known that there are C-type MHD shocks in the partially ionized interstellar medium. These shocks have broad structure owing to the friction between neutral and ionized gases. Because it is believed that the lifetime of an intermediate shock is proportional to the width of its front, intermediate shocks are long-lived in the interstellar medium. A quantitative study of such phenomena is left to a future study.

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