QCD Axion Kinetic Misalignment: Observational Aspects

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Abstract. When the spontaneous breaking of the Peccei-Quinn (PQ) symmetry occurred, the resulting angular direction of the PQ field, i.e. the axion could have possessed an initial non-zero velocity arising from additional terms that explicitly break the PQ symmetry. This opens up the possibility for smaller values of the decay constant than in the conventional scenario. We elaborate further on the outcome of this “kinetic misalignment” framework, assuming that axions form the entirety of the dark matter abundance. The kinetic misalignment framework possesses a weak limit in which the axion field starts to oscillate at the same temperature as in the conventional scenario, and a strong limit corresponding to large initial velocities which delay the onset of oscillations. We show how this scenario impacts the formation of axion miniclusters, and we sketch the details of these substructures along with potential detecting signatures.
1 Introduction

The QCD axion [1, 2] is a hypothetical pseudo-scalar particle that emerges within the solution to the strong-CP problem proposed by Peccei and Quinn (PQ) [3–5]. In the PQ theory, a new global $U(1)_{PQ}$ symmetry is introduced along with a complex scalar field $\Phi$ that is PQ charged. The axion is the angular direction of $\Phi$ after the spontaneous symmetry breaking (SSB) of the $U(1)_{PQ}$ symmetry. QCD anomalies explicitly break the PQ symmetry, reducing it to an approximate global symmetry. More generally, pseudo-scalar particles that couple derivatively to Standard Model (SM) fields are referred to in the literature as axion-like particles (ALPs) [6]. Axions and ALPs arise in various SM extensions through SSB or from string compactification [7]. Their rich phenomenology allows for numerous experimental approaches which could soon reveal them [8–14]. Along with this theoretical motivation, the QCD axion is also an excellent particle candidate for explaining the missing dark matter (DM) observed [15–18].

It is currently still unknown when the PQ symmetry breaking occurs concerning other events in cosmology such as inflation [19–23]. For instance, if the SSB of the PQ symmetry occurs after inflation (i.e. the post-inflationary scenario), it is accompanied by the formation of defects such as networks of strings and walls which do not inflate away and whose relaxation and decay could be a substantial contribution to the axion budget [24]. These motivations have pushed for consistently refining the computations regarding the production and evolution of the PQ field in the early Universe and to assess the present relic abundance of the QCD axion. Simulating the formation and decay of the network of strings and domain walls on cosmological relevant scales are mainly subject to errors and uncertainties due to the numerical complexity of the simulation [25–30]. Recent works on axion string simulations indicate that during the non-linear transient regime, the relevance of the axion potential is negligible as long as the gradient terms in the full axion field equation dominates [30]. Other recent axion string simulations utilize both the results in Refs. [30, 31] to incorporate the dynamics...
of the non-linear regime and the contribution from domain walls lead to conflicting results. In Ref. [32], the value of the axion mass is expected in the window \( m_a \sim O(40 - 180) \mu\text{eV}, \) while Ref. [33] suggests a heavier axion mass window, \( m_a \sim O(0.2 - 80) \) meV, with the discrepancy that can be traced back on different results of the scaling of the spectral index. The allowed window of QCD axion mass comprising all of the DM abundance significantly alters on invoking non-standard cosmology [34–39] or during an era of primordial black hole domination [40, 41].

Nevertheless, despite the level of computation and the extensive literature concerning the dynamics of the axion, there are still many unknown features. For instance, the dynamics of the PQ field in the early Universe could differ from the standard treatment. The initial conditions for the axion field could be set dynamically because of particular choices [42, 43] or the effect of physics beyond the SM [44, 45]. Contrary to the conventional assumptions, the axion field possesses a non-zero initial velocity in the so-called kinetic misalignment (KM) mechanism [46, 47]. In this case, the PQ symmetry is broken explicitly not only by QCD anomalies but also by the radial direction of the PQ field. For instance, a global symmetry is generally not a fundamental field property and gets spoiled by quantum effects. Nevertheless the quality of the PQ symmetry has to be protected from these effects in order not to jeopardize the solution of the strong-CP problem [48–50], models in which the PQ symmetry is an accidental symmetry explicitly broken by quantum effects have been constructed [51–55]. The additional wiggles in the PQ potential due to the explicit symmetry breaking terms could source the initial non-zero axion rotation and a non-zero PQ charge, which can convert into a matter asymmetry through strong sphaleron processes [56], electroweak sphalerons [57], or in supersymmetric models [58].

As the relic abundance of axions becomes immensely altered for specific benchmark values of its initial velocity, we utilize this property in the context of the mass function of axion miniclusters (AMCs). In this work, we explore how the characteristic mass of AMCs is affected by different axion production mechanisms. We first provide a brief review of the production of axion DM in the standard misalignment and the KM mechanisms. Then, we elaborate on the motivation concerning different regimes of the KM mechanism before analyzing its various impacts on the characteristic minicluster mass function as a function of the axion mass. After AMCs form around matter-radiation equality (MRE), the clumping of these structures proceeds to present time. The merging process of the clumping of the matter is a complicated and numerically extensive process typically comprising of N-body simulations [59]. Even though numerical simulations are in place to be able to make confident predictions, the intention here is to draw attention to a scenario that could serve as a motivation for N-body simulations in the future. Here we employ a semi-analytic approach from the evolution of a linear density contrast such as the Press-Schechter (PS) formalism [60, 61].

The paper is organized as follows. In Sec. 2 we briefly review different production mechanisms for axion mentioning the basic equation of motion that governs axion dynamics. We then move on to the discussion of the central theme of this work which is the KM in Sec. 3, where we first investigate the weak KM limit in Sec. 3.2, while in Sec. 3.3 the strong KM limit is addressed. The impact of KM on axion minicluster mass and some observational consequences are discussed in Sec. 4. Finally, we summarize our findings in Sec. 5.
2 Standard scenario

We consider a SM-singlet complex scalar field $\Phi$, the PQ field, which extends the SM content and which is described by the effective Lagrangian

$$L = L_{\text{QCD}} + |\partial_\mu \Phi|^2 - V(\Phi) + L_{\text{int}},$$

(2.1)

where $L_{\text{QCD}}$ captures all QCD effects in the SM, the PQ field potential responsible for the SSB of $U(1)_{\text{PQ}}$ at the energy scale $v_a$ with coupling $\lambda$, $V(\Phi) = \frac{\lambda^2}{2} \left( |\Phi|^2 - \frac{v_a^2}{2} \right)^2$, (2.2)

and where the term $L_{\text{int}}$ is responsible for the interaction of $\Phi$ with other beyond-SM physics, leading to an effective coupling of the field with gluons and other SM particles. The Lagrangian in Eq. (2.1) is invariant under the continuous shift symmetry $a \rightarrow a + \alpha v_a$, (2.3)

for a generic value of $\alpha$ that corresponds to a rotation in the complex plane $\Phi \rightarrow e^{i\alpha} \Phi$. After SSB, the complex scalar field can be decomposed in polar coordinates as $\Phi = \frac{1}{\sqrt{2}} (S + v_a) e^{i\alpha/v_a}$, (2.4)

where the angular direction is the axion $a$ and the radial direction is the saxion $S$, such that the saxion vacuum mass is $m_S = \lambda \Phi v_a$.

After SSB, the Lagrangian in Eq. (2.1) reads

$$L = L_{\text{QCD}} + \frac{g_s^2}{32\pi v_a} \text{Tr} G_{\mu\nu} \tilde{G}^{\mu\nu} + \frac{1}{2} (\partial_\mu a)^2 - V_{\text{QCD}}(a),$$

(2.5)

where $g_s$ is the QCD gauge coupling of the strong force, and $G_{\mu\nu}$ is the gluon field with dual $\tilde{G}_{\mu\nu}$. The effective QCD axion potential $V_{\text{QCD}}(a)$ arises from the interaction of the axion with QCD instantons around the QCD phase transition [62] and leads to a mass term for the axion, which would otherwise remain massless in the absence of an explicit breaking of the $U(1)_{\text{PQ}}$ symmetry. QCD terms break the continuous symmetry in Eq. (2.3) explicitly, while leaving a residual $Z_N$ discrete shift symmetry with $N$ vacua, $a \rightarrow a + n \pi f_a$, with $n$ a natural number and $f_a = v_a/N$ is the axion decay constant. Here, we set $N = 1$.

Because of this, the exact form of the axion potential is periodic around the temperature at which the QCD phase transition occurs, $T_{\text{QCD}} \simeq 150$ MeV. At high temperatures, the shape of the potential is well approximated by a cosine potential meanwhile, for $T \ll T_{\text{QCD}}$ the shape of the potential is well approximated by its zero temperature prediction which can be computed at the next-to-leading order within perturbation theory [63–65]. Here, we adopt the parametrization

$$V_{\text{QCD}}(a) = \chi(T) \left( 1 - \cos \theta \right),$$

(2.6)

where $\theta(t) \equiv a(t)/v_a$ is the axion angle and $\chi(T)$ is the QCD topological susceptibility. Much of the recent effort has been devoted to the numerical evaluation of the functional form of $\chi(T)$ [66–74]. A fit to the numerical results from lattice simulations is [70]

$$\chi(T) \simeq \chi_0 \times \begin{cases} 1, & \text{for } T \lesssim T_{\text{QCD}}, \\ \left( \frac{T}{T_{\text{QCD}}} \right)^{-b}, & \text{for } T \gtrsim T_{\text{QCD}}, \end{cases}$$

(2.7)
where $\chi_0 \simeq 0.0216\text{ fm}^{-4}$ and $b \simeq 8.16$. At any temperature $T$, the mass of the axion is $m(T) = \sqrt{\chi(T)/f_a}$, so the axion can be effectively regarded as a massless scalar field as long as the QCD effects can be neglected for $T \gg T_{\text{QCD}}$. In the opposite limit $T \ll T_{\text{QCD}}$, the mass squared of the axion at zero temperature is

$$m_a^2 \equiv m^2(T = 0) = \frac{m_um_d}{(m_u + m_d)^2} \frac{m_u^2 f_\pi^2}{f_a^2},$$

(2.8)

where $m_u, m_d$ are the masses of the up and down quarks, $m_u \simeq 140\text{ MeV}$ is the mass of the $\pi$ meson, and $f_\pi \simeq 92\text{ MeV}$ is the pion decay constant. Numerically, this gives $m_a = \Lambda_a^2 / f_a$, with $\Lambda_a = \chi_0^{1/4} \simeq 75.5\text{ MeV}$.

The equation of motion for the axion field obtained from the Lagrangian in Eq. (2.1) is

$$\ddot{\theta} + 3 H(T) \dot{\theta} - \frac{1}{R^2(T)} \nabla^2 \theta + m^2(T) \sin \theta = 0,$$

(2.9)

where a dot is a differentiation with respect to the cosmic time $t$, $R = R(T)$ is the scale factor, and $H(T) \equiv \dot{R}/R$ is the Hubble expansion rate. This expression for the evolution of the axion field is a Klein-Gordon equation in the potential of Eq. (2.6). At any time, the axion energy density is

$$\rho_a(T) = \frac{1}{2} f_a^2 \dot{\theta}^2 + \frac{1}{2} \frac{1}{R^2} (\partial_\mu \theta)^2 + m^2(T) f_a^2 (1 - \cos \theta).$$

(2.10)

Equation (2.9) is solved by evolving the axion field starting from the initial conditions $\theta = \theta_i$ and $\dot{\theta} = \dot{\theta}_i$ which are imposed at temperatures $T \simeq f_a$. We refer to $\theta_i$ as the initial misalignment angle, while the initial velocity is usually set as $\dot{\theta}_i = 0$.1 In the naive estimation of the axion abundance, Eq. (2.9) gets solved by considering super-horizon modes for which the gradient term is negligible. More elaborate treatments have to rely on the lattice simulation of Eq. (2.9) and the interaction of the axion with the radial component of the PQ field. These computations are extremely demanding and lead to conflicting results in the literature. The string network that develops in fully solving Eq. (2.9) has a spectrum that spans all frequencies from the infrared cutoff $\sim H$ to the ultraviolet (UV) cutoff $\sim f_a$, with a spectral index $q$. Current simulations can explore scales down to a size in which the behavior seems to be dominated by the UV spectrum with $q < 1$ [26], however recent results seem to be challenged once even more refined grids are used [30]. In this work, we qualitatively remark the differences between the various misalignment scenarios, for which we rely on solving Eq. (2.9) for super-horizon modes, and we neglect the contribution from strings.

In the misalignment mechanism, the axion field starts to roll about the minimum of the potential once the Hubble friction is overcome by the potential term [15–18]. This occurs around the temperature $T_{\text{osc}}^{\text{mis}}$ defined implicitly as

$$3 H(T_{\text{osc}}^{\text{mis}}) \approx m(T_{\text{osc}}^{\text{mis}}),$$

(2.11)

where generally $m(T_{\text{osc}}^{\text{mis}}) \ll m_a$. We assume the standard radiation-dominated phase, during which the Hubble rate is

$$H(T) = \sqrt{\frac{\pi^2}{90} g_*(T)} \frac{T^2}{M_P},$$

(2.12)

1An alternative production mechanism which is valid for a large initial value of the saxion field is parametric resonance [75–78].
in which $g_*(T)$ is the number of relativistic degrees of freedom at temperature $T$ [79] and $M_P$ is the reduced Planck mass. With this assumption, we obtain (see, e.g. Ref. [34])

$$T_{\text{osc}}^{\text{mis}} \simeq \begin{cases} \left( \frac{10}{\pi^2 g_*(T_{\text{osc}}^{\text{mis}})} M_P m_a \right)^{1/2}, & \text{for } T_{\text{osc}}^{\text{mis}} \lesssim T_{\text{QCD}}, \\ \left( \frac{10}{\pi^2 g_*(T_{\text{osc}}^{\text{mis}})} M_P m_a T_{\text{QCD}}^{b/2} \right)^{2/(4+b)}, & \text{for } T_{\text{osc}}^{\text{mis}} \gtrsim T_{\text{QCD}}. \end{cases}$$ (2.13)

For example, an axion field of mass $m_a \simeq 26 \mu eV$ would begin to oscillate at $T_{\text{osc}}^{\text{mis}} \simeq 1.23 \text{ GeV}$. In the absence of entropy dilution, the axion number density in a comoving volume after the onset of oscillations is conserved,

$$\frac{d}{dt} \left[ \frac{\rho_a(T)/m(T)}{s(T)} \right] = 0,$$ (2.14)

where $s(T) = (2\pi^2/45) g_*(T) T^3$ is the entropy density and $g_*(T)$ is the number of entropy degrees of freedom at temperature $T$ [79]. This last expression gives the present axion density fraction,

$$\Omega_a = \frac{\rho_a(T_\star)}{\rho_{\text{crit}}} \frac{m_a}{m(T_\star)} \frac{g_*(T_\star) T_\star^3}{g_*(T_\star) T_0^3},$$ (2.15)

where $T_\star$ is any temperature such that $T_\star < T_{\text{osc}}^{\text{mis}}$, $T_0$ is the present CMB temperature, and the critical density is given in terms of the Hubble constant $H_0$ as $\rho_{\text{crit}} = 3 M_P^2 H_0^2$. The energy density in Eq. (2.10) is approximated in the limit in which the kinetic energy can be neglected and for a quadratic potential such as

$$\rho_a(T_\star) \simeq \frac{1}{2} m^2(T_\star) f_a^2 \theta_i^2,$$ (2.16)

where $\theta_i$ is the initial value of the misalignment angle at temperatures $T \gg T_{\text{osc}}^{\text{mis}}$. An order of estimate, for $f_a \approx 10^{15} \text{ GeV}$ the correct relic density that matches the observed DM is obtained for initial field values $\theta_i \simeq O(1)$.^2

The left panel of Fig. 1 shows the axion relic abundance $\Omega_a h^2$ as a function of $\theta_i$ for $f_a = 10^{15} \text{ GeV}$, corresponding to the axion mass $m_a \approx 5.7 \text{ neV}$, in the standard misalignment case (i.e., taking $\theta_i = 0$). Since the axion potential is symmetric, hereafter without loss of generality we assume $\theta_i \geq 0$. The black solid and the blue dotted lines in Fig. 1 correspond to the results obtained from the numerical solution of Eq. (2.9) and the analytical solutions in Eq. (2.15), respectively. The analytical solution with the quadratic approximation in Eq. (2.16) underestimates the relic abundance when $\theta_i \approx \pi$, due to the presence of the non-harmonic terms in the QCD axion potential [20, 34, 38, 82]. The red horizontal band showing to $\Omega_a h^2 \simeq \Omega_{\text{DM}} h^2$, where $\Omega_{\text{DM}} h^2 \approx 0.12$ is the DM abundance today from the Planck satellite measurements [83]. The right panel of Fig. 1 depicts the misalignment angle required to produce the whole observed DM abundance in the standard scenario as a function of the QCD axion mass. The slope changes for $m_a \approx 4.8 \times 10^{-11} \text{ eV}$, corresponding to the DM axion mass at which the oscillations in the axion field begins around the QCD phase transition $T_{\text{osc}}^{\text{mis}} = T_{\text{QCD}}$. For $f_a \approx 10^{12} \text{ GeV}$, the initial misalignment angle must be tuned so that $f_a \theta_i^2$.

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^2Astrophysical constraints provide a lower bound on the decay constant requiring $f_a \gtrsim 10^7 \text{ GeV}$ [80, 81].
Figure 1. Standard scenario. Left: Axion relic abundance for $f_a = 10^{15}$ GeV, taking $\dot{\theta}_i = 0$. Right: Misalignment angle required to match the whole observed DM abundance. The two lines show the comparison between numerical (solid black) and analytical (dotted blue).

is approximately constant to give rise to the observed abundance. For $f_a \ll 10^{12}$ GeV, the abundance of cold axions is much smaller than that of DM unless the initial misalignment angle gets tuned to $\theta_i \approx \pi$. In this region of the parameter space, the relevant non-harmonic contributions to the QCD axion potential break the analytic derivation sketched in Eq. (2.16), and a numerical solution of Eq. (2.9) is needed.

3 Kinetic misalignment mechanism

3.1 Overview

So far, we have reviewed the computation of the DM abundance in the standard scenario in which the initial value of the axion field velocity gets set to zero. Recently, the possibility has been considered that the axion field possesses a non-zero initial velocity $\dot{\theta}_i \neq 0$, which is so large that the potential barriers can effectively be ignored, in the so-called kinetic misalignment (KM) mechanism [46, 47].

In the KM mechanism, the axion possesses an initial velocity $\dot{\theta}_i \neq 0$ corresponding to the rotation of the PQ field $\Phi$ in the complex field plane and an overall asymmetry of the PQ charge. At any time, the Noether charge density associated to the shift symmetry of the axion field in Eq. (2.3) is

$$n_\theta = i \left[ \Phi \Phi^* - \Phi^* \Phi \right] = \dot{\theta} f_a^2,$$

with the corresponding yield $Y_\theta \equiv \dot{\theta} f_a^2 / s(T)$. A necessary condition to generate an initial velocity of the axion field consists in the field value of the radial mode $S$ to be initially much larger than the axion decay constant, $S \gg f_a$, as it occurs in the early Universe. Provided that the PQ potential is sufficiently flat, the desired conditions can be realized by either imposing the appropriate initial conditions of inflation, primordial quantum fluctuations, or in supersymmetric models with flat directions in the superpotential. Another possibility to generate the initial misalignment velocity $\dot{\theta}_i \neq 0$ can emerge from axion models where the axion potential becomes tilted by introducing an explicit symmetry breaking term induced
by a higher-dimension potential of the form

$$V_{\text{PQ-break}} = M_P^4 \left( \frac{\Phi}{M} \right)^n + \text{h.c.},$$

where $n > 0$ is an integer and $M$ is a new energy scale lying well beyond the SM. The addition of the potential in Eq. (3.2) would lead to an explicit breaking of the PQ symmetry, and would provide an initial kick to the angular direction of $\Phi$ [46].

Two distinct regions exist for this mechanism: in the weak KM regime, the initial velocity allows the axion to explore a few different minima of the potential, while in the strong KM regime the initial axion velocity is so large that the potential barriers can effectively be ignored, and the onset of coherent oscillations gets delayed. In the following subsections, we discuss DM production in these different regimes.

### 3.2 Weak kinetic misalignment

At high temperatures $T \gg T_{\text{mis osc}}$, the potential term in Eq. (2.9) can be safely neglected and the oscillations in the axion field are damped by the Hubble friction. Therefore, the expression $\dot{\theta} + 3H(T)\dot{\theta} \simeq 0$ predicts $\dot{\theta} \propto R^{-3}$ for any cosmological model. This result can also be obtained from the conservation of the Noether charge $n_\theta$ in Eq. (3.1) [46]. For this reason, if the field possesses a non-zero initial velocity, the kinetic energy term would dominate over the potential energy term and the total energy density would scale as a kination field [84],

$$\rho_a \propto \dot{\theta}^2 \propto R^{-6}. \quad (3.3)$$

Since in a radiation dominated cosmology $H(R) \propto R^{-2}$, the axion field in this configuration would scale as [47]

$$\theta(R) \simeq \theta_i - 2n \pi + \frac{\dot{\theta}_i}{H(R_i)} \left[ 1 - \frac{R_i}{R} \right], \quad (3.4)$$

with $n$ being a natural number that counts how many times the axion crosses a potential barrier, and $\dot{\theta}_i = \dot{\theta}(R_i)$ is the initial misalignment angle. Equation (3.4) can be restated in terms of the the dimensionless redshift-invariant yield $Y_\theta \equiv f_a^2 \dot{\theta}/s(T)$ as

$$\theta(T) \simeq \theta_i - 2n \pi + \frac{Y_\theta}{f_a^2 H(T_i)} \left[ 1 - \left( \frac{s(T)}{s(T_i)} \right)^{1/3} \right], \quad (3.5)$$

where $T_i$ is the photon temperature at $R = R_i$. The standard misalignment scenario in Eq. (2.16) is recovered in the limit $Y_\theta = 0$ and $n = 0$. In this case, $T_{\text{mis}}$ does not vary with respect to the standard misalignment scenario and Eq. (2.11) holds. At later times, the Hubble rate dampens the oscillations and the QCD potential becomes relevant.

In the weak KM regime, the value of the misalignment angle $\theta_i$ in Eq. (3.4) required to match the observed DM abundance is modified by the presence of the initial velocity term $\dot{\theta}_i/H(R_i)$. The parameter space for the KM mechanism is non-trivial and depends on the direction of the velocity $\dot{\theta}_i$ which changes the sign of the yield $Y_\theta$.

Fig. 2 (left panel) shows the dependence of the axion relic abundance $\Omega_a h^2$ on the redshift-invariant yield $Y_\theta > 0$, for a fixed PQ breaking scale $f_a = 10^{15}$ GeV and for the choices $\theta_i = 1$ (black) or $\theta_i = 0$ (blue). The solid and the dotted lines correspond to the numerical solution and its analytical approximation obtained by considering a quadratic QCD axion potential instead of Eq. (2.6). The approximation is in good agreement with the
This behavior halts once the initial kinetic energy is large enough so that the field climbs the top of the potential, $\Omega_a^2 \propto Y_\theta^2$. This behavior halts once the initial kinetic energy is large enough so that the field climbs the top of the potential, $\theta = (1 + 2n)\pi$. A higher value for $Y_\theta$ allows the crossing of the potential barrier, oscillations taking place when the axion starts to roll down the potential, and hence a smaller relic abundance in generated. A minimum for $\Omega_a^2$ occurs when oscillations start at the minimum of the potential where $\theta = 2n\pi$. Increasing values for $Y_\theta$ allow the axion field excursion to cross several potential barriers, and therefore the relic abundance experiences the oscillating behavior shown in Fig. 2. Furthermore, when the kinetic energy completely dominates over the potential, $\Omega_a^2$ loses its dependence on the initial misalignment angle $\theta_i$, and it asymptotically approaches the strong KM regime, see Eq. (3.9) in the following section. The right panel of Fig. 2 shows the results for $\Omega_a^2$ once fixing $\theta_i = 1$ and for the yields $Y_\theta > 0$ (black) or $Y_\theta < 0$ (blue). For positive values of the misalignment angle, the choices $Y_\theta > 0$ and $Y_\theta < 0$ correspond to an axion climbing up or rolling down further in the potential well, respectively. We emphasize that in the case of weak KM, the oscillation temperature is the same as in the standard misalignment scenario, so that there is no delay the onset of oscillations.

We now fix the relic abundance of axions to that of the observed DM, and study the corresponding parameter space $\{\theta_i, Y_\theta\}$ for which this is achieved. This is shown in Fig. 3 for the different choices $f_a = 10^{15}$ GeV (left panel) and $f_a = 10^{12}$ GeV (right panel). In both panels, the black lines correspond to $Y_\theta > 0$, while blue lines correspond to $Y_\theta < 0$. For $f_a = 10^{15}$ GeV, the axion is confined in the potential well containing its minimum and it is not able to explore other minima, i.e. there are only solutions corresponding to $n = 0$ in Eq. (3.4). For $Y_\theta < 0$, the solution features a spike-like behavior, corresponding to the first funnel-shaped region appearing in the right panel of Fig. 2. In the case $Y_\theta < 0$, the axion field has a negative moderate initial velocity that makes it roll down further in the
potential well so that the field value becomes smaller than $\theta_i$ when the oscillations begin; this leads to a suppression in the relic abundance and, as a consequence, a larger $\theta_i$ is required to compensate. A similar behavior is shown for $f_a = 10^{12}$ GeV, however in this case different solutions to Eq. (3.4), corresponding to higher values of $n$, give rise to the observed DM abundance. In this scenario, the axion has enough kinetic energy to explore different minima, and therefore different solutions corresponding to the same initial axion angle appear. As the total axion energy density is dominated by the kinetic term, the new solutions tend to be independent of $\theta_i$.

A similar behavior occurs when plotting the contours describing axion DM over the plane $\{m_a, Y_\theta\}$, see Fig. 4. The left panel shows the region $Y_\theta > 0$ for the initial axion angles $\theta_i = 0$ (blue) and $\theta_i = 1$ (black), whereas the right panel shows the cases for $Y_\theta > 0$ (black) and $Y_\theta < 0$ (blue) assuming $\theta_i = 1$. In order to reproduce the same observed abundance in the KM scenario, the case for $\theta_i = 0$ requires a larger value of $Y_\theta$ compare to the case $\theta_i = 1$. The dotted lines marks the area, to the left of the line, for which the weak KM regime holds, while the strong KM regime applies to the right, see Eq. (3.8) below. All solutions with different initial values of $\theta_i$ (left panel) or the yield $Y_\theta$ (right panel) converge to the solution given by the red solid line in the strong KM regime.

3.3 Strong kinetic misalignment

Contrary to the previous case where an initial velocity allows the axion to explore a few minima, in the strong KM the kinetic energy is so large compared to the potential barrier that the potential is effectively flat. In this scenario, the oscillations in the axion field are delayed with respect to the cases of the standard misalignment and weak KM [46, 47]. The initially dominant axion kinetic energy $K = \dot{a}^2/2$ eventually becomes equal to the maximum of the potential barrier $V_{\text{max}} = 2m^2(T) f_a^2$ at the temperature $T_{\text{osc}}^{\text{skm}}$, defined implicitly by the equality

$$|\dot{\theta}(T_{\text{osc}}^{\text{skm}})| \equiv 2m(T_{\text{osc}}^{\text{skm}}).$$

(3.6)

If at $T = T_{\text{osc}}^{\text{skm}}$ the kinetic energy density is larger than the potential barrier, the axion oscillations are delayed until the kinetic energy falls below the potential energy. This condition

Figure 3. Kinetic misalignment. Parameter space that reproduces the whole observed DM abundance for $f_a = 10^{15}$ GeV (left) and $f_a = 10^{12}$ GeV (right). Black and blue lines correspond to $Y_\theta > 0$ and $Y_\theta < 0$, respectively.
is satisfied for
\[
|\dot{\theta}(T_{\text{osc}}^{\text{mis}})| = |\dot{\theta}(T_i)| \frac{s(T_{\text{osc}}^{\text{mis}})}{s(T_i)} = \frac{|Y_\theta|}{f_a^2} s(T_{\text{osc}}^{\text{mis}}) > 2 m(T_{\text{osc}}^{\text{mis}}) = 6 H(T_{\text{osc}}^{\text{mis}}),
\]
which corresponds to the red dotted line appearing in the panels of Fig. 4. In terms of the yield, this gives
\[
|Y_\theta| > 6 f_a^2 \frac{H(T_{\text{osc}}^{\text{mis}})}{s(T_{\text{osc}}^{\text{mis}})}. \tag{3.8}
\]

To obtain the present relic abundance, we employ the conservation of \(n(T)/s(T)\) from the onset of field oscillations to present time,
\[
\rho_a(T_0) = C \rho_a(T_{\text{osc}}^{\text{skm}}) \frac{m_a}{m(T_{\text{osc}}^{\text{skm}})} \frac{s(T_0)}{s(T_{\text{osc}}^{\text{skm}})} \sim C |\dot{\theta}(T_{\text{osc}}^{\text{skm}})| f_a^2 m_a \frac{s(T_0)}{s(T_{\text{osc}}^{\text{skm}})} = C |Y_\theta| m_a s(T_0), \tag{3.9}
\]
where we used the fact that in the strong KM scenario the axion energy density is completely dominated by the kinetic energy. Although the analytical estimate predicts \(C = 1\), a numerical analysis favors \(C \simeq 2\) [46]. The result in Eq. (3.9) is the red dashed line in Fig. 2, and the red solid lines in Fig. 4. As evident from Eq. (3.9), in this limit the axion DM relic abundance is independent of the initial misalignment angle and the sign of \(Y_\theta\). The transition between the weak and the strong KM regimes occurs at \(f_a \simeq 2.2 \times 10^{11} \text{ GeV}\), corresponding to \(m_a \simeq 26 \mu\text{eV}\).

The KM mechanism allows us to explore the region of the parameter space corresponding to relatively large values of the axion mass.\(^3\) Equations (3.6)–(3.9) allow us to compute the value of \(T_{\text{osc}}^{\text{skm}}\) required to match the observed DM abundance, via the relation
\[
\frac{\sqrt{\chi(T_{\text{osc}}^{\text{skm}})}}{s(T_{\text{osc}}^{\text{skm}})} \simeq \frac{\rho_{\text{DM}}}{2C s(T_0) \sqrt{\chi(T_0)}}, \tag{3.10}
\]
\(^3\)Axions heavier than \(\mathcal{O}(10^{-1})\) eV are in tension with observations from horizontal branch stars and other astrophysical measurements [85].
which, setting $C = 2$, yields a value that is independent on the axion mass,

$$T_{\text{osc}}^{\text{skm}} = \left[ \frac{4 g_{s s}(T_0)}{g_{s s}(T_{\text{osc}}^{\text{skm}})} \frac{\chi_0}{\rho_{\text{DM}} T_{\text{QCD}}^{3/2}} \dot{\theta}_0 \rho_{\text{DM}} T_0^{3/2} \right]^{1/2} \approx 1.23 \text{ GeV}. \tag{3.11}$$

Thus, in the strong KM scenario, axions start to oscillate at a smaller temperature $T = T_{\text{osc}}^{\text{skm}}$ instead of the value obtained in the conventional scenario $T_{\text{osc}}^{\text{mis}}$ given in Eq. (2.13) and, as a consequence, the onset of coherent oscillations is delayed.\(^4\) We have shown the value of the temperature at which the axion field is set into motion as a function of its mass in Fig. 5. The line denoted “Strong KM” is the value given in Eq. (3.11), and the two tilted lines to the left denote the result in Eq. (2.13). A summary of the conditions satisfied by the three different misalignment mechanisms discussed is given in Table 1.

| Mechanism        | Initial velocity | Oscillation temperature |
|------------------|------------------|-------------------------|
| Standard scenario| $\dot{\theta}_i = 0$ | $3H(T_{\text{osc}}^{\text{mis}}) = m(T_{\text{osc}}^{\text{mis}})$ |
| Weak KM          | $\dot{\theta}_i \neq 0$ | $3H(T_{\text{osc}}^{\text{mis}}) = m(T_{\text{osc}}^{\text{mis}})$ |
| Strong KM        | $\dot{\theta}_i \neq 0$ | $|\dot{\theta}(T_{\text{osc}}^{\text{skm}})| = 2m(T_{\text{osc}}^{\text{skm}})$ with $|\dot{\theta}(T_{\text{osc}}^{\text{mis}})| > 2m(T_{\text{osc}}^{\text{mis}})$ |

\(4\) A delay in the onset of oscillations also occurs in the trapped misalignment case [86, 87].
4 Axion miniclusters

4.1 Formation and properties

In the scenario in which the PQ symmetry is spontaneously broken after inflation, the axion field at the time of the onset of oscillations is spatially inhomogeneous over different Hubble patches; its non-zero mass prevents free-streaming from occurring above a specific scale.

The above argument sets the correlation scale between different parts of the axion field as $\sim 1/H(T_{osc})$. The inhomogeneities inherited by AMCs decouple from the mean field value $\bar{\rho}_a$ around MRE, according to the value of the overdensity

$$\delta \equiv \frac{\rho_a}{\bar{\rho}_a} - 1,$$

where $\rho_a$ is the local density in miniclusters. The energy density corresponding to these collapsing AMCs is [88]

$$\rho_{AMC} \approx 140 (1 + \delta) \delta^3 \rho_{eq},$$

where $\rho_{eq}$ is the energy density in DM at MRE.

The comoving size of the fluctuations at the onset of fluctuations is $r_H = 1/(RH)_{osc}$, leading to an AMC of radius $r = r_H \rho_{eq}/\delta$ at the time when the overdensity perturbations decouple from the Hubble expansion and start growing by gravitational instability, rapidly forming gravitationally bound objects [11, 88–92], with the corresponding mass

$$M_0 = \frac{4\pi}{3}(1 + \delta) \rho_{DM} \frac{s(T_{osc})}{s(T_0)} \left(\frac{1}{H(T_{osc})}\right)^3.$$  

The mass scale in Eq. (4.3) corresponds to the heaviest AMCs that are formed at MRE. Bound structures are formed with all masses below $M_0$, down to the smallest physical scales at which the oscillatory behavior of the axion field exerts an effective “quantum” pressure which prevents further clumping. This so-called Jeans length $\lambda_J$ corresponds to the wave number [93, 94]

$$k_J = \frac{2\pi}{\lambda_J} = (16\pi G \rho_{DM} m_a^2 R)^{1/4} \approx 710 \left(\frac{m_a}{\mu eV}\right)^{1/2} \text{pc}^{-1},$$

where the last expression holds at MRE. Perturbations grow for modes $k > k_J$.

Numerically, Eq. (4.3) in different regimes reads

$$M_0 = \begin{cases} 
1.7 \times 10^{-14} M_\odot (1 + \delta) \left(\frac{m_a}{\mu eV}\right)^{-3/2}, & \text{for } T_{osc} \lesssim T_{QCD}, \\
1.1 \times 10^{-10} M_\odot (1 + \delta) \left(\frac{m_a}{\mu eV}\right)^{-\frac{\delta^2}{1+\delta}}, & \text{for } T_{QCD} \lesssim T_{osc} \lesssim T_{osc}^{skm}, \\
2.1 \times 10^{-11} M_\odot (1 + \delta), & \text{for } T_{osc} \gtrsim T_{osc}^{skm},
\end{cases}$$

where the first two lines are found in the case where the onset of oscillations is not delayed by the presence of KM so that Eq. (2.13) holds, while the third line is obtained in the strong KM regime where Eq. (3.11) holds. We stress here that the case $T_{osc} < T_{QCD}$, the first line in Eq. (4.5), generally corresponds to an initial misalignment angle $\theta_i \ll 1$ which is not realized in the post-inflation scenario in which AMCs generally form; we have included this case for
Figure 6. The characteristic minicluster mass, for $\delta = 1$. The two dotted lines correspond to the cases of standard scenario or the weak KM regime, and to the strong KM regime.

illustration purpose only.\(^5\) In fact, in single field inflation the Hubble rate at the end of the inflation epoch is bound as $H_I \lesssim 2.5 \times 10^{-5} M_P$ at 95% confidence level (CL)\(^6\) at the wave number $k_0 = 0.002 \text{Mpc}^{-1}$ which, together with the bound $f_a \lesssim H_I/(2\pi)$ valid for the post-inflationary scenario, implies $m_a \gtrsim 0.6 \mu eV$.

In the post-inflation scenario with the conventional misalignment mechanism employed, the initial misalignment angle averaged over all possible patches is $\theta_i \simeq \pi/\sqrt{3}$, which leads to the result $m_a \simeq (10-100) \mu eV$ and to the AMC mass $M_0 \simeq 10^{-11} M_\odot$ (see the second line in Eq. (4.5)). The KM regime allows achieving different values of the DM axion mass according to the initial velocity, with a wider possibility for the AMC mass ranges. In particular, AMCs can be as heavy as $\sim 10^{-9} M_\odot$. This effect is ultimately due to the delayed onset of oscillations occurring in the KM regime, where the AMC mass is independent of the DM axion mass since the temperature in Eq. (3.11) is constant.

The results in Eq. (4.5) for the AMC mass $M_0$ are sketched as a function of the DM axion mass $m_a$ by the black solid in Fig. 6. The star labels the typical AMC mass obtained with a DM axion of mass $m_a = 20 \mu eV$ using the second line in Eq. (4.5). The first kink to the left corresponds to the change in the behavior of the susceptibility in Eq. (2.7) near $T \sim T_{\text{QCD}}$, while the second kink at heavier axion masses corresponds to the change from the weak to the strong KM regimes, see Fig. 5. The moment where the axion field begins to oscillate coincides with the transition from being frozen at the configuration $\theta = \theta_i$ to an oscillatory behavior with dust-like equation of state $w_a \approx 0$.

In the KM scenario, the axion initially behaves as a kination field with $w_a \approx 1$ and the transition could occur at lower temperatures, i.e. at $T = T_{\text{osc}}^{\text{km}} < T_{\text{osc}}^{\text{mis}}$, as discussed in Sec. 3.3. For this reason, axion miniclusters formed in the strong kinetic regime are heavier than those formed in the standard scenario. The gray band to the left of the plot marks

\(^5\)In the pre-inflation scenario, dense clumps of axions can form by nucleation around primordial black holes\(^9\).
the mass region in the post-inflationary scenario which is excluded by the non-observation of
tensor modes in single-field inflation by the Planck satellite collaboration.

4.2 Growth of structures

At around MRE, axion miniclusters form at scales below the threshold in Eq. (4.3), populat-
ing the decades in mass according to a halo mass function (HMF) \( dn/(d \ln M) \) which provides
the number density \( n \) as a function of the logarithmic mass \( M \). The bottom-up clumping of
axion miniclusters begins already around the time of matter-radiation equality when these
structures form. Recent progress on the merging process has focused on the formation of
axon “minihalos” with a HMF derived from N-body simulations [59, 97], which can be un-
derstood in terms of semi-analytical modeling [60, 61, 98, 99], and is well approximated by
using the standard Press-Schechter (PS) and Sheth-Tormen formalisms.

Even though numerical simulations are in place to be able to make confident predictions,
the intention here is to draw attention to a scenario that could serve as a motivation for N-
body simulations in the future. Here, we intend to estimate how the HMF becomes modified
in the KM case to the standard case, following the PS formalism [100]. The PS formalism is
based on two key assumptions: i) at any time, the density contrast of a spherically-symmetric
overdense region of size \( R \) collapses into a virialized object once it evolves above a critical
overdensity \( \delta_c \). For the critical value of the linear density contrast for spherical collapse
during the matter domination is \( \delta_c \approx 1.686 \). To quantify this criterion, we introduce the
overdensity fuzzed over the spherical region,

\[
\delta_s(x, t) = \int d^3x' \delta(x') W(x + x', R), \tag{4.6}
\]

where \( W(x, R) \) is a kernel function that smooths the spatial overdensity over the spherical
region of radius \( R \). ii) The density contrast is distributed as a normal distribution, specified
by the variance

\[
\sigma^2(z, R) = \int \frac{d^3k}{(2\pi)^3} |\delta_k(R)|^2 T^2(k, z) |W(k, R)|^2, \tag{4.7}
\]

where \( |\delta_k(R)|^2 \) is the power spectrum of the fluctuations, \( T(k, z) \) is the transfer function, and
\( W(k, R) \) is the Fourier transform of the kernel function \( W(x, R) \) that smooths the density
field \( \delta(x) \) over the spherical region of radius \( R \).

The HMF derived from these premises is parametrized as

\[
\frac{dn}{d \ln M} = \frac{\rho_{DM}}{M} f(\sigma) \left| \frac{d\sigma}{d \ln M} \right|, \tag{4.8}
\]

where the multiplicity function \( f(\sigma) \) is defined within the PS formalism as

\[
f(\sigma) = \sqrt{\frac{2}{\pi}} \frac{\delta_c}{\sigma} e^{-\frac{1}{2} \frac{\sigma^2}{\delta_c^2}}. \tag{4.9}
\]

The method described above is extremely efficient in describing the distribution of the high-
end mass spectrum of the HMF. Since our interest is in the heaviest virialized objects that
form within the theory, we specialize the treatment to find an approximate solution for the
maximal mass of the minihalo \( M_{\text{max}}(z) \) at redshift \( z \). The power spectrum of the fluctuations
\( |\delta_k(R)|^2 \) is accessible from lattice simulations in the early Universe [28, 29] and from N-
body simulations [59]. Since we are interested in estimating the maximal mass of axion
structures at redshift \( z \), we adopt the approximation in Ref. [61] of a white noise power spectrum truncated at the comoving scale \( k_{\text{osc}} \) at which coherent field oscillations begin. The normalization of the power spectrum assures that the integral of the power spectrum equates the square of spatial fluctuations averaged over different horizons [101] and it is here set as \( P_0 = (24\pi^2/5) k_{\text{osc}}^{-3} \) [61]. In principle, the transfer function depends on the relative value of \( k \) for the Jeans wavelengths at MRE and today. In practice, these Jeans lengths are too small to yield a significant modification over the white noise spectrum. Here, we use the fact that in the spherical collapse model, the fluctuations collapse and grow to a size \( M \) at redshift \( z < z_{\text{eq}} \) as isocurvature modes with \( \delta(M) \propto a \) so the transfer function can be approximated as a linear scale factor. Finally, we assume a Gaussian kernel function, whose Fourier transform is again a Gaussian function in \( k = |k| \) with the form

\[
W(k, R) = \exp \left( -k^2 R^2 / 2 \right). \tag{4.10}
\]

With this choice, the mass of a structure today that extends to a region of size \( R \) is \( M = (2\pi)^{3/2} \rho_{\text{DM}} R^3 \).

With this parametrization, the standard deviation in Eq. (4.7) is approximated by the analytic function

\[
\sigma(z, R) = \frac{1 + z_{\text{eq}}}{1 + z} \sqrt{\frac{3}{5} \pi \text{erf}(x) - 2x e^{-x^2}} / x^3, \tag{4.11}
\]

where \( x = k_{\text{osc}} R \) and \( \text{erf}(x) \) is the error function. Collapse occurs when \( \sigma(z, R) \geq \delta_c \). More generally, the mass of the heaviest objects that form at redshift \( z \), \( M_{\text{max}}(z) \), is found implicitly from the expression

\[
\sigma(z, M_{\text{max}}(z)) = \delta_c, \tag{4.12}
\]

where \( \sigma^2(z, M) \) is the variance corresponding to Eq. (4.7) once \( R \) is expressed in terms of \( M \). The results for \( z = 0 \) are reported in Fig. 6 (blue line), where the kink at \( m_a \sim 10^{-5} \text{eV} \) corresponds to the effects of the kinetic term in the initial conditions. Whenever there exists a mechanism that grants a large initial kinetic energy for the axion field, the field would begin coherent oscillations in a colder universe, allowing for heavier axion miniclusters at MRE. The clumping of such heavier building blocks also leads to an increased value of the maximal mass, with observational consequences. Note, however, that these large miniclusters would likely not survive tidal stripping from other astrophysical objects such as brown dwarfs and main sequence stars, especially in high-density regions such as the Galactic center [102, 103].

### 4.3 Stripping

It is not guaranteed that AMCs survive tidal stripping from compact objects in galaxies, such as brown dwarfs and stars. In all DM models, tidal interactions destroy small-scale clumps [104, 105], such as axion miniclusters [102]. Recent N-body simulations have proven that the stripping mechanism is crucial for the population of miniclusters in galaxies [103]. In general, it could be expected that the larger and heavier miniclusters produced in the strong KM regime would be more prone to get tidally disrupted by compact objects.

The effect of the encounter of the minicluster with an individual compact object of mass \( M \) with relative velocity \( v_{\text{rel}} \) is that of increasing the velocity dispersion of the bounded axions. An encounter that occurs close enough would deposit sufficient energy so that the minicluster is completely disrupted. This occurs for an impact parameter \( b \) smaller than the
critical value \[106, 107\]
\[
b < b_c \equiv \left( \frac{G M_{\text{AMC}}}{v_{\text{rel}} v_{\text{AMC}}} \right)^{1/2},
\]
where the velocity dispersion of the minicluster is \(v_{\text{AMC}}^2 = G M_{\text{AMC}}/R_{\text{AMC}}\). The probability of disruption for a minicluster moving in a stellar field of column mass density \(S\) is \([102]\)
\[
p_{\text{disr}} = 2\pi S \frac{G R_{\text{AMC}}}{v_{\text{rel}} v_{\text{AMC}}},
\]

In the vicinity of the Solar system, it is generally found \(p_{\text{disr}} = \mathcal{O}(10^{-2})\) and miniclusters generally survive the stripping process. However, this result depends on the density of the minicluster, and not on its mass or radius separately. Since the density of miniclusters given in Eq. (4.2) is related to the spherical collapse model and not to the cosmological history, we generally expect that at the lowest order in which this approximation holds, the probability of disruption would not change among the different scenarios of misalignment mechanisms. In the vicinity of the solar system, Eq. (4.14) yields \(p_{\text{disr}} \approx 2\%\).

### 4.4 Microlensing

We now discuss a possible method to distinguish between the different misalignment scenarios using microlensing. Bound structures made of axions such as miniclusters, minicluster halos, and axion stars can impact lensing from distant sources. For a point-like lens of mass \(M\), the characteristics of the microlensing event are determined by the Einstein radius \([108, 109]\)
\[
R_E(x) = \sqrt{\frac{4 G N M}{c^2} \frac{D_L D_{\text{LS}}}{D_S}} \approx 4.3 \times 10^3 \text{ km}\left[x(1-x) \frac{D_S M}{\text{kpc} 10^{-10} M_\odot}\right]^{1/2},
\]
where \(D_S, D_L, \text{ and } D_{\text{LS}}\) are the distances between the source and the observer, the lens and the observer, and the source and the lens, respectively, and \(x = D_L/D_S\).

Although lensing can occur from axion miniclusters and halos \([61, 103, 110, 111]\), these structures can generally not be modeled as point lenses, as their Einstein radius lies within their mass distribution, so that the internal density profile needs to be known to estimate the lensing power. Extended objects generally lead to weaker limits due to the smaller magnification of the lens \([112, 113]\).

Here, we focus on the lensing of light coming from a distant source when the lensing object is an axion star, which is generally a much more compact object than a minicluster \([114–118]\). Axion stars belong to the class of real scalar field oscillatons \([119–122]\) in which the field occupies the lowest energy state that is allowed by Heisenberg uncertainty principle. An axion star of mass \(M_{\text{as}}\) is generally produced in the dense core of axion miniclusters through the mechanism of gravitational cooling \([123]\) with the relaxation time \(\tau_{\text{as}}\) \([124]\).

Although the decay rate into two photons does not affect significantly the stability of axion stars on a cosmological time scale, self-interactions of the type \(3a \rightarrow a\) can lead to decay of axion stars with a decay rate that depends on \((m_a/f_a)^2\) \([125]\).

An axion star can efficiently lens the light from a distant source, as their radius is typically smaller than their corresponding Einstein radius. Recently, a single microlensing event observed by the Subaru Hyper Suprime-Cam (HSC) collaboration after observing in M31 \((D_S = 770 \text{kpc})\) for 7 hours \([126]\) has been interpreted in terms of an axion star of planetary mass \([127]\) (see also Ref. \([128]\)). However, the result is of difficult interpretation in the standard scenario of the misalignment mechanism, since such massive stars would...
only form for the lightest QCD axions, for which the post-inflation scenario does not hold. Although the AMCs formed in the strong KM regime are generally much more heavy, the axion stars formed within them are not expected to differ much from those of the standard scenario, since the mass of the axion star is only mildly dependent on that of the host AMC as $M_{\text{as}} \propto M_{\text{AMC}}^{1/3}$ [129]. For this reason, the KM regime cannot be invoked to produce the heavy axion stars which are needed to explain the microlensing results in Ref. [127]. One issue with the derivation of the axion star properties is that the formulas used rely on the results of Ref. [129] which are obtained for ultralight axions for which the mass scale greatly differs from that of the QCD axion. Although this can be justified since the set of equations describing the system (the Newton-Poisson equation) possesses a scaling property, a dedicated simulation proving this is yet lacking.

5 Conclusions

An explicit breaking term in the Peccei-Quinn (PQ) symmetry could give rise to a nonzero velocity term for the axion field. This scenario, called kinetic misalignment (KM), has been explored in the literature in relation with baryogenesis. Here, we have discussed further implications for the delayed onset of the oscillations in the axion field that appear in the KM scenario. In the standard scenario, the DM axion mass depends on the relative size of the inflation scale with respect to the axion energy scale: in the pre-inflation regime, the initial misalignment angle can be tuned to achieve a specific mass scale according to the result in Fig. 1, while in the post-inflation regime the mass is fixed by the dynamics of the axion field that yields $\theta_i = O(1)$. In KM scenarios, different values of the DM axion mass can be explored because of the presence of the non-zero initial velocity as a new parameter: in the post-inflation regime, a specific mass scale corresponds to a given choice of $\dot{\theta}_i$ even when fixing $\theta_i = O(1)$ (see Fig. 4).

One aspect that has been explored here is the formation of axion miniclusters (AMCs) and minihalos in KM regimes, as a possible tool that leads to distinctive signatures from the standard scenario. AMCs are generally formed in the post-inflation regime with the typical mass $M_0 \sim 10^{-11} M_\odot$. In KM scenarios, the nonzero velocity term allows for a wider mass range for the AMCs: the mass of the AMCs is larger than what is obtained in the standard scenario because the axion field begins to oscillate in a colder universe with a larger comoving scale, as shown in Fig. 6. In this regime, AMCs are more diffuse and heavier so that, assuming that the fraction of axions in bounded structures is the same, there would be fewer of them and they would be affected by tidal stripping the same way as in the standard scenario. The clumping of heavier AMCs would lead to larger halos of miniclusters, with the typical mass today that could be orders of magnitude above what has been expected so far, and could affect the analysis of the microlensing events from minicluster halos.

Future directions would involve employing a numerical solution of the equation of motion, including the effects of the explicit symmetry breaking. For example, the dynamics of the axion can be resolved by modifying the recent open-source numerical routine MiMeS [130]. The properties of AMCs can only be assessed through more sophisticated analyses that account for the evolution of the PQ field and require a modification of existing open-source codes that are already available [28]. A similar analysis involving the implementation of a Boltzmann solver can be performed in the pre-inflationary scenario, where the KM regime would be additionally constrained by isocurvature fluctuations, as it has been discussed in Ref. [81].
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