Confinement and center vortex dynamics in different gauge groups

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Abstract. The random vortex world-surface model is extended to the gauge groups $SU(4)$ and $Sp(2)$. Compared to the $SU(2)$ and $SU(3)$ models studied previously, which reproduce the infrared properties of the corresponding Yang-Mills theories on the basis of a simple vortex world-surface curvature action, new dynamical characteristics become important. In the $SU(4)$ case, an explicit dependence of the vortex effective action on the configuration of the Abelian magnetic monopoles residing on the vortices emerges; in the $Sp(2)$ case, a new “stickiness” contribution to the vortex action serves to drive the deconfinement phase transition towards the correct first-order behavior.

Keywords: Center vortices, infrared effective theory, confinement
PACS: 12.38.Aw, 12.38.Mh, 12.40.-y

INTRODUCTION

The random vortex world-surface model is a concrete realization of the center vortex picture of the strong interaction vacuum [1–7], i.e., the notion that the relevant infrared gluonic degrees of freedom of the strong interaction are closed tubes of quantized chromomagnetic flux. The random vortex world-surface model was initially investigated for $SU(2)$ Yang-Mills theory [8–10], and in this simplest case, the main characteristics of the strongly interacting vacuum were reproduced. Both a confining low-temperature phase as well as a deconfined high-temperature phase are found [8], separated by a second-order deconfinement phase transition; furthermore, the topological susceptibility [9, 11–13] and the (quenched) chiral condensate [10] match the ones extracted from $SU(2)$ lattice Yang-Mills theory quantitatively. Extending the investigation to the $SU(3)$ gauge group [14–16], the deconfinement phase transition exhibits weakly first-order behavior [14], and a $Y$-law for the baryonic static potential is found [15], again matching the corresponding characteristics of $SU(3)$ lattice Yang-Mills theory. Studies of the topological and chiral properties in the $SU(3)$ case are pending.

The aforementioned successes of the $SU(2)$ and $SU(3)$ random vortex world-surface models are obtained on the basis of very simple vortex dynamics, with the action determined purely by world-surface curvature. Accordingly, these models only contain one dimensionless coupling parameter, which is adjusted in practice to reproduce the ratio of the deconfinement temperature $T_c$ to the square root of the zero-temperature string tension $\sigma$ found in the corresponding Yang-Mills theory. Recent efforts have focused on the question of how far this simple picture carries as the gauge group is varied. There are two systematic ways of extending the Yang-Mills gauge group beyond the cases discussed above: On the one hand, one may increase the number of colors $N$ determining the $SU(N)$ gauge symmetry; on the other hand [17–19], the $SU(2)$ group
can alternatively be considered as the smallest symplectic group $Sp(1)$, and the $Sp(N)$ sequence can also be used to generalize $SU(2) = Sp(1)$. Accordingly, random vortex world-surface models for the infrared sectors of both $SU(4)$ and $Sp(2)$ Yang-Mills theory have been constructed [20, 21] and are presented in the following. In both cases, new dynamical characteristics emerge.

The concrete modeling methodology used in the random vortex world-surface model is discussed in detail in [8, 14, 20]. Vortex world-surfaces are composed of elementary squares on a hypercubic space-time lattice. The lattice spacing is a fixed physical quantity related to the transverse thickness of vortices; it represents the ultraviolet cut-off inherent in any infrared effective description. An ensemble of closed vortex world-surfaces is generated by Monte Carlo update. For different underlying gauge groups, random vortex world-surface models differ in two respects: On the one hand, the quantization of vortex flux is determined by the center of the gauge group. When encircling a vortex, a Wilson loop acquires a phase given by one of the nontrivial center elements. Accordingly, in general, several species of center vortices, corresponding to the different nontrivial center elements, can exist. They can merge and disassociate into one another. On the other hand, the models can differ in the effective action governing the vortices.

**SU(4) VORTEX MODEL**

The $SU(4)$ group contains the nontrivial center elements $\{i, -i, -1\}$. Vortex fluxes associated with the elements $i$ and $-i$ are related by an inversion of space-time orientation; therefore, there are altogether only two physically distinct types of center vortices. A vortex generating a phase factor $-1$ when linked to a Wilson loop can branch into two vortices associated with a phase factor $\pm i$ and vice versa.

Correspondingly, $SU(4)$ Yang-Mills theory [22–24] also induces two distinct string tensions, the quark string tension $\sigma_1$ and the diquark string tension $\sigma_2$. The $SU(4)$ Yang-Mills confinement properties thus are characterized by the ratios $\sigma_2/\sigma_1$ and $T_c/\sqrt{\sigma_1}$, as well as the behavior at the deconfinement phase transition, which is about twice as strongly first-order as the one of $SU(3)$ Yang-Mills theory.

To properly model these characteristics, it is necessary to use an effective vortex action which is more complicated than for $SU(2)$ or $SU(3)$, and which can be symbolically represented as

$$S = c_i c_j \times - b \times .$$

(1)

The first term is the curvature term already used in the $SU(2)$ and the $SU(3)$ models. It penalizes configurations in which two vortex squares share a link without lying in the same plane by an action increment $c_i c_j$ depending on the types of vortices participating; in the $SU(4)$ case, there are two vortex types, $i, j \in \{1, 2\}$. Studying a model based only on this term led to the conclusion that it cannot faithfully reproduce the confinement properties of $SU(4)$. When $\sigma_2/\sigma_1$ is tuned to the correct value, the deconfinement phase transition is second-order. Consequently, additional dynamics must be introduced, embodied in the second term in (1), the branching term. It facilitates vortex branchings by weighting links at which 3 or 5 vortex squares meet with an action decrement $b$. 
Using this action, agreement with the $SU(4)$ Yang-Mills confinement characteristics is reached at the physical point [20]

\[ c_1 = 0.45 \quad c_2 = 0.80 \quad b = 0.71. \quad (2) \]

It should be noted that, in Abelian gauges, vortex branching can be associated with Abelian magnetic monopoles [20]. Thus, the above result can be interpreted as implying that a realistic vortex model for $SU(4)$ Yang-Mills theory is only achieved by including a dependence of the vortex dynamics on the configuration of the Abelian magnetic monopoles which reside on the vortices in Abelian gauges [9, 11, 12]. This confirms a corresponding expectation formulated in [25], that Abelian magnetic monopoles begin to play a role in infrared Yang-Mills vortex dynamics as the number of colors $N$ is raised. Note that Abelian magnetic monopoles are also present in $SU(2)$ and $SU(3)$ vortex configurations; however, there is no signature for an independent dynamical role of these monopoles. Their distribution appears to be essentially determined by the dynamics of the vortices on which they reside.

**$Sp(2)$ VORTEX MODEL**

A remarkable property of the $Sp(N)$ sequence of groups is that all members have the same center, $Z(2)$, and allow for the same set of center vortex degrees of freedom. There is only one nontrivial center element, $-1$, and therefore only one type of vortex flux. Nevertheless, the effective vortex actions can be very different for different underlying $Sp(N)$ groups; after all, different cosets would be integrated out in each case if one were to derive the effective vortex action from first principles. Therefore, vortex models for different $Sp(N)$ Yang-Mills theories are by no means forced to display similar confinement characteristics, such as deconfinement transitions of the same order.

Indeed, while $SU(2) = Sp(1)$ Yang-Mills theory exhibits a second-order deconfinement phase transition, the deconfinement transition of $Sp(2)$ Yang-Mills theory is strongly first-order [17, 18]. As above, in order to generate such behavior, new dynamics must be introduced compared to the $SU(2)$ vortex model. The confinement characteristics of $Sp(2)$ Yang-Mills theory can be reproduced using an effective vortex action of the symbolic form

\[ S = c \times \text{[diagram]} \quad + \quad s \times \text{[diagram]} . \quad (3) \]

The first term is the curvature term already discussed above. The second term can be interpreted in terms of a “stickiness” of vortices: When 4 (or even 6) vortex squares meet at a link, this corresponds to 2 (or even 3) intersecting vortex fluxes maintaining contact to one another for a finite space-time length instead of intersecting only at one space-time point. Enhancing such behavior by choosing a negative value for $s$ means that vortices become stickier. Indeed, a first-order deconfinement phase transition of the proper strength, together with the correct value of $T_c/\sqrt{\sigma}$ is achieved at the physical point [21]

\[ c = 0.479 \quad s = -1.745 . \quad (4) \]
CONCLUSIONS

Extending the Yang-Mills gauge group to $SU(4)$ and $Sp(2)$, new dynamics emerge in the corresponding infrared effective vortex descriptions. The $SU(4)$ case exhibits clear signatures of Abelian magnetic monopoles (which are intrinsically present in vortex configurations cast in Abelian gauges) attaining a dynamical significance of their own as the number of colors is raised. This corroborates related arguments put forward in [25]. In the $Sp(2)$ case, a new “stickiness” term in the effective action serves to drive the deconfinement transition towards the correct first-order behavior. While $SU(2) = Sp(1)$ and $Sp(2)$ Yang-Mills theory contain the same center vortex degrees of freedom, the vortex effective actions in the two cases differ and thus naturally lead to different behavior at the deconfinement transition.

Having determined the physical points (2) and (4) of the $SU(4)$ and $Sp(2)$ infrared effective vortex models, the behavior of the spatial string tensions at high temperatures can be predicted [20, 21]. As discussed further in [20, 21], comparison with measurements in the corresponding full lattice Yang-Mills theories can be used to test the validity of the model constructions presented here.

ACKNOWLEDGMENTS

This work was supported by the U.S. DOE under grants DE-FG03-95ER40965 (M.E.) and DE-FG02-94ER40847 (B.S.).

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