Engineering Enhanced Thermoelectric Properties in Zigzag Graphene Nanoribbons

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Abstract

We theoretically investigate the thermoelectric properties of zigzag graphene nanoribbons in the presence of extended line defects, substrate impurities and edge roughness along the nanoribbon’s length. A nearest-neighbor tight-binding model for the electronic structure and a fourth nearest-neighbor force constant model for the phonon bandstructure are used. For transport we employ quantum mechanical non-equilibrium Green’s function simulations. Starting from the pristine zigzag nanoribbon structure that exhibits very poor thermoelectric performance, we demonstrate how after a series of engineering design steps the performance can be largely enhanced. Our results could be useful in the design of highly efficient nanostructured graphene nanoribbon based thermoelectric devices.

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I. INTRODUCTION

The ability of a material to convert heat into electricity is measured by the dimensionless thermoelectric figure of merit \( ZT \) defined by:

\[
ZT = \frac{S^2 G T}{\left( \kappa_e + \kappa_l \right)}
\]  

(1)

where \( S \) denotes the Seebeck coefficient, \( G \) the electrical conductance, \( T \) the temperature, \( \kappa_e \) the electronic and \( \kappa_l \) the lattice parts of the thermal conductance \([1]\). Due to the strong interconnection between the parameters that control \( ZT \), it has been traditionally proved difficult to achieve values above unity, which translates to low conversion efficiencies and limit the applications for thermoelectricity.

The recent advancements in lithography and nanofabrication, however, have lead to the realization of breakthrough experiments on nanostructured thermoelectric devices that demonstrated enhanced performance, sometimes even up to 2 orders of magnitude higher than the corresponding bulk material values. Nanostructures provided the possibility of independently designing the quantities that control the \( ZT \) in achieving higher values. Enhanced performance was demonstrated for 1D nanowires (NWs) \([2, 3]\), 2D thin films, 1D/2D superlattices \([4, 5]\), as well as materials with embedded nanostructuring \([6]\).

Graphene, a recently discovered two-dimensional form of carbon, has received much attention over the past few years due to its excellent electrical, optical, and thermal properties \([7]\). Graphene, however, is not a useful thermoelectric material. Although its electrical conductance is as high as that of copper \([8]\), its ability to conduct heat is even higher \([9]\), which increases the denominator of \( ZT \). To make things worse, as a zero bandgap material, pristine graphene has a very small Seebeck coefficient \([10]\), which minimizes the power factor \( S^2 G \). Nanoengineering, however, could provide ways to increase the Seebeck coefficient and decrease the thermal conductivity as well.

The high thermal conductivity of graphene is mostly due to the lattice contribution, whereas the electronic contribution to the thermal conduction is smaller \([11, 12]\). In order to reduce the thermal conductivity, therefore, the focus is placed on reducing phonon conduction. Recently many theoretical studies have been performed regarding the thermal conductivity of graphene-based structures. Several methods, such as the introduction of vacancies, defects, isotope doping, edge roughness and boundary scattering, can considerably
reduce thermal conductance [13–15]. Importantly, in certain instances this can be achieved without significant reduction of the electrical conductance.

In order to improve the Seebeck coefficient graphene needs to acquire a bandgap. This can be achieved by appropriate patterning of the graphene sheet into nanoribbons [16, 17]. Graphene nanoribbons (GNRs) are thin strips of graphene, where the bandgap depends on the chirality of the edges (armchair or zigzag) and the width of the ribbon. Armchair GNRs (AGNRs) can be semiconductors with a bandgap inversely proportional to their width [16]. Although the acquired bandgap can increase the Seebeck coefficient, when attempting to reduce the thermal conductivity by introducing disorder in the nanoribbon, as described above, the electrical conductivity is also strongly affected [18, 19], and the thermoelectric performance remains low. Zigzag GNRs (ZGNRs), on the other hand, show metallic behavior with very low Seebeck coefficient, but as described in Ref. [19], the transport in ZGNRs is nearly unaffected in the presence of line edge roughness, at least in the first conduction plateau around their Fermi level.

In this work, by using atomistic electronic and phononic bandstructure calculations, and quantum mechanical transport simulation, we show that despite the zero bandgap, the thermoelectric performance of ZGNRs can be largely enhanced. For this a series of design steps are employed: i) Introducing extended line defects (ELDs) as described in Ref. [20] can break the symmetry between electrons and holes by adding additional electronic bands. This practically provides a sharp band edge around the Fermi level and offering a band asymmetry which for thermoelectric purposes it practically constitutes an “effective bandgap”. ii) Introducing background impurities enhances the “effective bandgap”. iii) Introducing edge roughness reduces the lattice part of the thermal conductivity (significantly more than it reduces the electrical conductivity). After such procedure, we demonstrate that the figure of merit $ZT$ can be greatly enhanced and high thermoelectric performance could be achieved.

The paper is organized as follows. In section II we describe the methodology used in our calculations. In section III we present the results for the electronic/phononic structure and transmission of ZGNRs for every step of our design approach (in section III A), and their influence on the thermoelectric coefficients (in section III B). Finally, in section IV we conclude.
II. APPROACH

In linear response regime, the transport coefficients can be evaluated using the Landauer formula [21–23]:

\[ G = \left( \frac{2q^2}{h} \right) I_0 \left[ \frac{1}{\Omega} \right] \]  

\[ S = \left( \frac{k_B}{q} \right) I_1 \left[ \frac{V}{K} \right] \]  

\[ \kappa_e = \left( \frac{2T k_B^2}{h} \right) \left[ I_2 - \frac{I_1^2}{I_0} \right] \left[ \frac{W}{K} \right] \]  

Here, \( h \) is the Planck constant, \( k_B \) is the Boltzmann constant, and \( I_j \) is given by

\[ I_j = \int_{-\infty}^{+\infty} \left( \frac{E - E_F}{k_B T} \right)^j T_{\alpha}(E) \left( -\frac{\partial f}{\partial E} \right) dE \]  

where \( T_{\alpha}(E) \) is the electronic transmission probability, \( f(E) \) is the Fermi function and \( E_F \) is the Fermi-level of the system. Similarly, the lattice contribution to the thermal conductance can be given as a function of the phonon transmission probability [18]:

\[ \kappa_l = \frac{1}{h} \int_{0}^{+\infty} T_{\text{ph}}(\omega)\hbar \omega \left( \frac{\partial n(\omega)}{\partial T} \right) d(\hbar \omega) \]  

where \( n(\omega) \) denotes the Bose-Einstein distribution function and \( T_{\text{ph}}(\omega) \) is the phonon transmission probability [24].

For the electronic structure, the Hamiltonian of the GNRs is described in the standard first nearest-neighbor atomistic tight-binding \( p_z \) orbital approximation. The hopping parameter is set to \(-2.7 \text{ eV}\) and the on site potential is shifted to zero so that the Fermi level remains at 0 eV. This model has been recently used to describe the electronic transport of ELD-ZGNR with double-vacancies and the results are in good agreement with first-principle calculations and experimental studies [20, 25]. To the best of our knowledge, only a few first-principle calculations and experimental studies have been conducted in structures that include ELDs [25–27]. The two main features of the electronic structure, the asymmetry between electrons and holes, and the metallic behavior of the ELD in the graphene ribbon channel have been described in these studies, and are also captured by the tight-binding model as we will demonstrate below.

For the phonon modes, the dynamic matrix is constructed using the fourth nearest-neighbor force constant model [23]. The force constant method uses a set of empirical fitting
parameters and can be easily calibrated to experimental measurements. We use the fitting parameters given in Ref. [28] for graphene-based structures. We assume that this model is still valid under structures that include ELDs. Although verification of its validity for ELD-ZGNRs has not been demonstrated yet, i.e. using first-principle calculations, in Ref. [29] it was shown using DFT simulations that there is little difference between the phonon transmission of carbon nanotube structures with/without ELDs which could justify our model choice. In any case, as we show below, the main influence on the phonon transport in this work originates from edge roughness scattering, which reduces the phonon transmission drastically. The effect of edge roughness scattering is the dominant effect, and that can be captured adequately by the model we employ in this work. The influence of the ELDs on the phonon transmission is much smaller compared to the effect of edge roughness, and therefore we still choose to use the numerically less expensive fourth nearest-neighbor force constant method.

In this work, the fully quantum mechanical non-equilibrium Green’s function formalism (NEGF) is used for transport calculations of both electrons and phonons. The system geometry is defined as a set of two semi-infinite contacts and a channel (device) with length $L$. The device Green’s function is obtained as

$$G_{el}(E) = (EI - H - \Sigma_{s,el} - \Sigma_{d,el})^{-1}$$

for electron calculation, where $H$ is the device Hamiltonian matrix and $E$ is the energy. In the case of phonon transport the Green’s function is given by:

$$G_{ph}(E) = (EI - D - \Sigma_{s,ph} - \Sigma_{d,ph})^{-1}$$

where $D$ is the dynamic matrix and $E = \hbar \omega$ [30]. The contact self-energy matrices $\Sigma_{s/d}$ are calculated using the Sancho-Rubio iterative scheme [31]. The effective transmission probability through the channel can be achieved using the relation:

$$T_{el/ph}(E) = \text{Trace}[\Gamma_s G \Gamma_d G^\dagger]$$

where $\Gamma_s$ and $\Gamma_d$ are the broadening functions of contacts [32].

This method is very effective in describing the effect of realistic distortion in nanostructures, including all quantum mechanical effects. In our calculation, we include long-range
substrate impurities with density of one impurity per 125 nm and edge distortion (rough-
ness) up to four layers in each side of the ribbon’s edge. These are applied only on the device part and not in the contact regions [19].

III. RESULTS AND DISCUSSION

An efficient thermoelectric material must be able to effectively separate hot from cold carriers. The quantity that determines the ability to filter carriers is the Seebeck coefficient. The Seebeck coefficient depends on the asymmetry of the density of states around the Fermi level. In semiconductor the Seebeck coefficient is large, but in a metal where the density of states is more uniform in energy the Seebeck coefficient is small. Metallic ZGNRs also have a small Seebeck coefficient because their transmission is constant around the Fermi level, despite the peak in the DOS at $E = 0$ eV due to the edge states. Recently, however, Bahamon et al. have investigated the electrical properties of ZGNRs that included an ELD (ELD-ZGNRs) along the nanoribbon’s length [20]. It was reported that the ELD breaks the electron-hole energy symmetry in nanoribbons, and introduces an additional electron band around the Fermi level. In such a way an asymmetry in the density of states and the transmission function are achieved which improves the Seebeck coefficient as we will show further down. This particular structure has also been recently experimentally realized [25]. Although the method of fabrication was rather complicated to be able to scale for industrial applications, nevertheless it makes studies on GNRs appropriate and interesting as well.

A. Electronic and Phononic Structure

The changes in the electronic structure of the ZGNRs after the introduction of the ELD are demonstrated in Fig. 1. Figure 1-a shows the atomistic geometry of the pristine ZGNR of width $W \sim 4$ nm (with 20 zigzag edge lines) and Fig. 1-b its electronic structure. The Fermi level is at $E = 0$ eV due to the symmetry between electron and hole bands. Figure 1-c shows the structure of the ELD-ZGNR with the same width. The region in which the ELD is introduced is shown in red color. The ELD changes the hexagons of the GNR to pentagons and octagons after a local rearrangement of the bonding and the introduction of two additional atoms in the unit cell. We use a two parameter notation to describe the
ELD-ZGNR structure throughout this work as ELD-ZGNR($n_1,n_2$), where $n_1$ and $n_2$ are the indices of the partial-ZGNRs above and below the line defect, respectively (i.e. the number of zigzag edge lines of atoms), although in all cases we use $n_1 = n_2$. The bandstructure of the ELD-ZGNR(10,10) is shown in Fig. 1-d. The thick-red line shows a new band that is introduced in the conduction band near the Fermi energy ($E = 0$ eV), which corresponds to the ELD. There are two points that result in the creation of the extra band. Part of the physics behind this is explained by Pereira et al. in Ref. [33]. The first point is that a defect in the graphene system will introduce states that reside close to the Fermi level at $E = 0$ eV. This is similar to the edge states of the ribbons that tend to reside near the Fermi level. The second point again described in Ref. [33], is that an asymmetry in the dispersion between electrons and holes will be created when carbon atoms of the graphene sublattice “A” (or “B”) are coupled with atoms from “A” (or “B”) again. Usually, the atomic arrangement in graphene can be splitted in sublattices “A” and “B”, where atoms from “A” couple to “B” and vise versa. When this happens, the dispersion is symmetric in the first-nearest neighbor tight-binding model. At a defect side such as the ELD we consider, where “A” connects to “A” as seen in Fig. 1-c, such asymmetry can be observed. The fact that the overall bandstructure has additional bands compared to the pristine ribbon is also connected to the two extra atoms in the unit cell.

Moving one step further, in Fig. 1-e we show the geometry of a GNR with two ELDs. We denote this structure as 2ELD-ZGNR($n_1,n_2,n_3$), where $n_1$, $n_2$, and $n_3$ denote the the number of zigzag carbon lines above, within, and below the line defects. Figure 1-f shows the electronic structure of the 2ELD-ZGNR(8,4,8). In this case two additional bands are introduced near the Fermi level as noted by the thick-red lines. In this structure the asymmetry between electron and hole bands around the Fermi level ($E = 0$ eV) is further enhanced.

1st Design Parameter- The Effect of ELD: Figure 2 demonstrates the increase in the asymmetry of the bands around the Fermi level by showing how the transmission changes when one or two ELDs are introduced in the channel. For the pristine ZGNR, the transmission is equal to one, indicating the existence of a single propagating band at energies around the Fermi level (green line). With the introduction of one ELD, the conduction band ($E > 0$ eV) is composed of two subbands, whereas the valence band ($E < 0$ eV) is still composed of one subband. With the introduction of two ELDs, three conduction subbands now appear, but still only one valence subband. As it will be shown below, this asymmetry
will improve the Seebeck coefficient. This constitutes the first design step in improving the thermoelectric performance of ZGNRs.

There is, however, another point worth mentioning. In Fig. 3 we show colormaps of the normalized current spectrum at $E = 0.2$ eV in the cross sections of the ELD-ZGNRs described in Fig. 2. Figure 3-a shows the current spectrum of the ELD-ZGNR(10,10). The current is zero close to the edges of the ribbon and peaks near the center. This is demonstrated more clearly in Fig. 3-d, which shows the current along one atomic chain perpendicular to this channel (blue line). The black line of Fig. 3-d illustrates the current density on the cross section of the pristine ZGNR channel for reference.

The current spectrum for the 2ELD-ZGNR(8,4,8) is shown in Fig. 3-b. The situation is now different since most of the current is confined within the two ELDs. This, however, is the case only when the distance between the ELDs is smaller than the widths of the upper/lower regions. In the case where the width of the middle region similar to the widths of the upper/lower regions, the current is spread more uniformly in the channel as shown in Fig. 3-c for the 2ELD-ZGNR(7,6,7) channel. Figure 3-e shows again the current along one atomic chain in the cross section of these ribbons. The current spectrum is localized in the middle of the channel in the 2ELD-ZGNR(8,4,8) channel (red line) compared to the pristine channel (black line). In a 2ELD-ZGNR(9,2,9) channel with a narrower middle region the current is in general flowing around the ELD regions. The design capability to localize the current spectrum in the middle of the channel away from the edges will prove advantageous in the presence of edge roughness since the current in this case will be less affected. On the other hand, in the case of the 2ELD-ZGNR(7,6,7) channel the current spectrum tends to concentrate more close to the edges (green line).

**2nd Design Parameter- The Effect of Background Positive Impurities:** We next illustrate the possibility of further enhancing the asymmetry between electron and hole transport near the Fermi level by the introduction of positively charged substrate background impurities. The effect of background impurities is included in the Hamiltonian in a simplified way as an effective negative long range potential energy on the appropriate on-site Hamiltonian elements as described in Ref. [19]. A positive impurity in the substrate will constitute a repulsive potential for holes (a barrier for holes but a well for electrons) and will degrade hole transport more effectively than electron transport. Figure 4-a shows
how the transmission of the ELD-ZGNR(10,10) channel (dashed-black line) is affected after the introduction of positive charged impurities in the channel (solid-blue line). Indeed, the transmission of holes below the Fermi level \((E = 0 \text{ eV})\) is degraded. This effect additionally increases the asymmetry of the propagating bands and improves the Seebeck coefficient. On the other hand, the opposite is observed when negative impurities are introduced in the substrate. Negative impurities are a barrier for electrons and reduce their transmission [34], but do not interfere with the hole subsystem as shown in Fig. 4-b. This type of impurities will actually harm the asymmetry and needs to be avoided.

3rd Design Parameter- The Effect of Roughness: In the third step of the design process we introduce the effect of edge roughness. The inset of Fig. 4-c shows the influence of edge roughness on the transmission of the ZGNR(20) of length 125 nm. As also described in previous studies [15, 19], in the first conduction plateau the effect is negligible. In contrast to ZGNR, ELD-ZGNRs as well as 2ELD-ZGNRs are affected by edge roughness. This is because the bandstructure of these GNRs has undergone a band folding, and therefore, the states in the first conduction plateau have lower wave vectors. As the long range defects can induce only small value of momentum transfer, the momentum conservation rule indicates that, in contrast to the ZGNR, the transport of ELD-ZGNRs and 2ELD-ZGNRs will not remain ballistic in the presence of line edge roughness and long range substrate impurities. This is shown in Fig. 4-c, where the transmission of a roughened 125 nm long ELD-ZGNR(10,10) channel (solid-blue line) is reduced by \(~ 25\%\) compared to the ballistic value (dashed-black line). Edge roughness degrades the conductivity of holes and electrons by a similar amount, and therefore, the level of asymmetry around the Fermi level is retained.

Figures 5-a and 5-b illustrate the influence of roughness in ELD-ZGNR channels on their transmission, for channels of different lengths and widths. In this calculation positive impurities are also included. Figure 5-a shows the transmission of edge roughened ELD-ZGNR(10,10) versus energy for the channel lengths \(L = 250, 500, \text{ and } 2000 \text{ nm}\). As the channel length is increased, the transmission drops further compared to the transmission of the ideal channel (black-solid line). This is expected since the channel resistance increases with increasing length. Figure 5-b illustrates the effect of the ribbon’s width on the transmission of ELD-ZGNRs with rough edges. In this case the length is kept constant at \(L = 250 \text{ nm}\), and results for three different ribbon with parameters (10,10), (7,7), and (5,5) are shown. As the width of the ribbon is decreased, the effect of line edge roughness
scattering on the transmission becomes stronger because the carriers reside on average closer to the edges.

It is worth mentioning that the effect of edge roughness on the transmission is much stronger in AGNR than in ZGNR. Although in the case of some AGNRs a bandgap is naturally present and the asymmetry does not need to be created with the introduction of line defects and impurities, the conductance is severely degraded by the roughness which renders this type of ribbon not well suited for transport applications [19]. (Note that edge roughness will be needed in order to reduce thermal conductivity as will be shown below.)

As we mentioned above in Fig. 3, the channel which includes two ELDs can shift the majority of the current spectrum in the region between the two ELDs, and thus farther away from the edges. It is therefore expected that the 2ELD-ZGNR will be less affected by edge roughness scattering than the ELD-ZGNR. A comparison of the transmission of these devices with rough edges is shown in Fig. 6. The transmission of ELD-ZGNR($n_1,n_1$), and two cases of 2ELD-ZGNR, 2ELD-ZGNR($n_2,4,n_2$) and the 2ELD-ZGNR($n_3,6,n_3$) at $E = 0.2$ eV versus their width $W$ are compared. The parameters $n_i$ are adjusted such that the three channels have nearly the same width $W$. The first channel belongs to the category shown in Fig. 3-a, the second in the category of Fig. 3-b, and the third in the category of Fig. 3-c. The third channel as shown in Fig. 3 spreads the current spectrum more uniformly in the channel and is expected to be affected the most from edge roughness. All channels have the same length of $L = 250$ nm. For smaller widths the effect of roughness is strong, and the transmissions of all channels are drastically reduced. Since the 2ELD-ZGNR devices can concentrate the current spectrum around the defect lines as shown in Fig. 3-b and 3-c, they effectively bring it closer to the edges and the reduction is larger for these devices. For larger widths the transmission of the ribbons approaches its ballistic value, which is 2 for the ELD-ZGNR devices and 3 for the 2ELD-ZGNR devices. The transmission of the 2ELD-ZGNR($n_2,4,n_2$) channels increases faster with increasing channel width, because the current spectrum is located farther from the edges which makes it less susceptible to scattering as the width increases. The transmission of 2ELD-ZGNR($n_3,6,n_3$) channel eventually increases close to the ballistic transmission value as the width increases, but it increases more slowly than that of the 2ELD-ZGNR($n_2,4,n_2$) channel.

**Effect of roughness on phonon Transport:** Although the reduction in the electronic transmission of channels with ELDs can be quite strong when considering edge roughness,
the reduction in the lattice part of the thermal conductivity is even stronger. We take advantage of on this effect when attempting to optimize the thermoelectric figure of merit. The phonon transmission for the edge roughened ELD-ZGNR(10,10) channel versus energy is shown in Fig. 7-a. Results for channel lengths $L = 10, 100,$ and $2000$ nm are shown. As expected, the transmission decreases as the length is increased. What is important, however, is that the decrease is much stronger than the decrease of the electron transmission shown in Fig. 5-a. For example, for a channel length of $L = 100$ nm the phonon transmission reduces by more than a factor of 6X, whereas the electronic transmission even at larger length $L = 250$ nm reduces only by $< 30\%$. Interestingly, the same order of reduction of the phonon transmission is observed for the 2ELD-ZGNRs as shown in Fig. 7-b, indicating that the line defect does not affect phonon conduction significantly compared to the effect of edge roughness.

B. Thermoelectric Coefficients

The denominator of the $ZT$ figure of merit consists of the summation of the contributions to the thermal conductivity of the electronic system and the phononic system. In graphene the phonon part dominates the thermal conductivity, whereas the electronic part contribution is much smaller. The situation is different, however, in rough ELD-ZGNRs, in which the phonon thermal conductivity is degraded more than the electronic thermal conductivity. Figure 8 clearly illustrates this effect by showing the ratio of the phonon thermal conductance to the electronic thermal conductance versus the rough channel length. The cases of ELD-ZGNR(10,10) and 2ELD-ZGNR(8,4,8) are shown in dashed-red and dash-dot-blue lines, respectively. For small channel lengths, where transport is quasi-ballistic and roughness does not affect the transmission significantly, $\kappa_l$ is almost 5X larger than $\kappa_e$. As the length of the channel increases and the effect of the roughness becomes significant, the phonon system is degraded more than the electronic system, and the $\kappa_l$ is significantly reduced compared to $\kappa_e$. For lengths $L \sim 100$ nm and beyond, $\kappa_l$ can become even smaller than $\kappa_e$. The trend is the same when considering channels with one or two ELDs. We note that from the inset of Fig. 8 which shows that the ratio of the electrical conductance $G$ over $\kappa_e$ is almost constant, it can be indicated that both $G$ and $\kappa_e$ follow the same trend, as the Wiedemann-Franz law dictates. We mention that the $\kappa_l$ and $\kappa_e$ values used in Fig. 7
are extracted using the corresponding mean free paths (MFPs) for phonons and electrons respectively, defined as described in Ref. [15]

\[
T(E) = \frac{N_{ch}(E)}{1 + \frac{L}{\lambda(E)}}
\]

(10)

where, \(T(E)\) is transmission probability, \(N_{ch}(E)\) is the number of modes at energy \(E\), \(L\) is the given length of the channel, and \(\lambda(E)\) is the mean free path of the carriers. Alternatively, \(\kappa_l\) and \(\kappa_e\) could be extracted from the transmission calculations by using a statistical average over several rough samples for each channel length. The results of both methodologies are in good agreement for the electronic part of the thermal conductivity. For the lattice part, the agreement is good only for the shorter channels, below \(\sim 100\) nm. For larger channel lengths, the phonon transmission is severely reduced which increases the noise in the calculation for extracting the \(\kappa_l\). The values extracted directly from the integration of the phonon transmission could be as much as \(2\)X larger, which could increase the \(\kappa_l/\kappa_e\) by a factor of \(2\)X for the longer channels. In this case the ratio \(\kappa_l/\kappa_e\) will be closer to unity, but this is still a huge advantage compared to devices without roughness.

**Power Factor:** Using the first design step, i.e. the effect of ELDs, we have demonstrated that the transmission of electrons around the Fermi level can be increased (from \(T = 1\) to \(T = 2\) and \(T = 3\) in the presence of one and two ELDs, respectively). An asymmetry is thus created between holes and electrons. This increases both the conductivity and Seebeck coefficient of the channel as shown in Fig. 9. Figure 9-a shows the conductance of the 2ELD-ZGNR(8,4,8) (blue), of the ELD-ZGNR (10,10) (red), and of the pristine nanoribbon (green) at room temperature \(300\) K. As expected, the conductance of the channel with two ELDs is the largest, followed by the channel with one ELD. They are larger than the pristine channel by \(\sim 3\)X and \(\sim 2\)X, respectively. Figure 9-b shows the changes of the Seebeck coefficient after the introduction of the ELDs in the nanoribbon. Due to its metallic behavior and the flat transmission near the Fermi level, the pristine channel exhibits zero Seebeck coefficient. Due to the built asymmetry after the introduction of the ELDs, however, the Seebeck coefficient increases for both channels. The channel with two line defects has the largest asymmetry, and therefore the largest Seebeck coefficient (in absolute values). Finally, the power factor in Fig. 9-c is indeed largely improved in the ELD structures, and especially the 2ELD-ZGNR channel.

In Figure 10 we show the same thermoelectric coefficients for the same structures as in
Fig. 9, but now edge roughness and positive impurities are included in the calculation. The length of the channels in this case is 2000 nm. A similar qualitative behavior is observed as in Fig. 9 for both channels. Quantitatively, however, the conductance in Fig. 10-a is now significantly reduced by a factor of \( \sim 15X \) (the dots correspond to the position of the peak of the power factor of the devices without roughness and impurities in Fig. 9). The Seebeck coefficient in Fig. 10-b, on the other hand increases. Finally, the peak of the power factor in Fig. 10-c reduces only slightly compared to the peak of the power factor of the devices without edge roughness in Fig. 9-c (dots).

**Thermoelectric Figure of Merit:** For the devices that include rough edges, however, as we demonstrated in Fig. 8, the phonon thermal conductivity is drastically reduced compared to the electronic thermal conductivity. A large improvement is therefore expected in the \( ZT \) figure of merit. Figure 11 shows the \( ZT \) figure of merit versus energy at room temperature for the ELD-ZGNR(10,10), the ELD-ZGNR(10,10) with impurities and roughness (red), and the 2ELD-ZGNR(8,4,8) (blue) with impurities and roughness. As indicated, large values of \( ZT \) can be achieved, especially in the case of the device with two ELDs. The phonon lattice conductivity value used in this calculation was extracted using the MFP method. Since as explained above, that value could be 2X lower than the value extracted from direct integration of the ’noisy’ transmission, in the inset of Fig. 11 we show the \( ZT \) versus energy using the \( \kappa_l \) values extracted from the transmission. Indeed the values could be reduced by a factor of \( \sim 2X \), but still peak \( ZT \) values above 2 can be achieved at room temperature, which is comparable and even better than the best thermoelectric materials to date [35]. We note that as shown by Ref. [15] rough ZGNRs can have high \( ZT \) values even without the presence of ELDs. For this however, the asymmetry in the sharp edges of the higher subbands is utilized at energies above 0.5 eV. Those energies however, are too high and can not easily be reached. Finally we mention here that our formalism has considered scattering only by edge roughness and impurity scattering, whereas phonon scattering and dephasing mechanisms are not included. However, as it is shown for 1D NWs [36], the effects of impurity scattering and edge roughness are the most important scattering effects in channels of cross sections below 5 nm, and we expect this to hold also for GNRs as well.
IV. SUMMARY

In this work we present a theoretical design procedure for achieving high thermoelectric performance in zigzag graphene nanoribbon (ZGNRs) channels, which in their pristine form have very poor performance. The fully quantum mechanical non-equilibrium Green’s function technique was used for electron and phonon transport, and tight-binding and force constant methods were used for the electronic and phonon bandstructure descriptions. We show that by introducing extended line defects (ELDs) in the length of the nanoribbon we can create an asymmetry in the density of modes around the Fermi level, which improves the Seebeck coefficient. ELDs increase the electronic conduction modes, which increase the channel conductance as well. The power factor is therefore significantly increased. In addition, we show that by introducing edge roughness the phonon thermal conductivity ($\kappa_l$) is drastically degraded much more than the electronic thermal conductivity ($\kappa_e$), or the electronic conductance ($G$). These three effects result in large values of the thermoelectric figure of merit, and indicate that roughed ZGNRs with ELDs could potentially be used as efficient high performance thermoelectric materials.

Acknowledgments

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FIG. 1: The geometrical structure of (a) ZGNR(n), (c) ELD-ZGNR(n₁, n₂), and (e) 2ELD-ZGNR(n₁, n₂, n₃). The bandstructure of (b) ZGNR(20), (d) ELD-ZGNR(10,10), and (f) 2ELD-ZGNR(8,4,8). The bandstructure of ZGNR(20) is folded for a better comparison. The translation vector length is $a = 0.49$ nm. The $n$, $n₁$, $n₂$ and $n₃$ indicate the number of zigzag edges on the top, bottom, and middle of the ELD regions as indicated.
FIG. 2: The transmission function for three different structures: i) The pristine ZGNR(20), ii) ELD-ZGNR(10,10), and iii) 2ELD-ZGNR(8,4,8).
FIG. 3: Normalized current spectrum at $E = 0.2$ eV for (a) ELD-ZGNR(10,10), (b) 2ELD-ZGNR(8,4,8), and (c) 2ELD(7,6,7). (d) The current in the cross section of ZGNR(20) (black line) and ELD-ZGNR(10,10) (blue line). (e) The current in the cross section of ZGNR(20) (black line), 2ELD-ZGNR(9,2,9) (blue line), 2ELD-ZGNR(8,4,8) (red line), and 2ELD-ZGNR(7,6,7) (green line).
FIG. 4: The effect of (a) positive substrate impurity, (b) negative substrate impurity, and (c) roughness on the transmission of ELD-ZGNR(10,10) with length of 125 nm. Inset of (c): The transmission of ZGNR(20) in the presence of roughness.
FIG. 5: The influence of roughness and positive impurities on the ELD-ZGNR channel. (a) Electronic transmission of ELD-ZGNR(10,10). Rough edges are assumed and the length $L$ is varied. The arrow indicates increasing values of length $L$. (b) Electronic transmission of ELD-ZGNRs with different widths. The length is assumed to be constant at 250 nm and the arrow indicates the direction of decreasing the ribbon’s width. Black-solid and black-dashed lines in (a) and (b): The transmission of the pristine ELD-ZGNR.
FIG. 6: Transmission at $E = 0.2$ eV for three different structures as indicated versus their width. The length is assumed to be constant at 250 nm.
FIG. 7: Phonon transmission probability of (a) ELD-ZGNR(10,10) and (b) 2ELD-ZGNR(8,4,8). Rough edges are assumed and the length \( L \) of the channel is varied. The arrows indicate increasing values of channel length \( L \), 10 nm-green, 100 nm-red, 2000 nm-blue lines. Black lines: The phonon transmission of the channels with line defects but without roughness.
FIG. 8: The ratio of the phononic to the electronic thermal conductivity versus channel length $L$ for the ELD and 2ELD structures as noted. Inset: The ratio of the electronic conductivity to the electronic part of the thermal conductivity versus channel length $L$. 
FIG. 9: (a) Electrical conductance, (b) Seebeck coefficient, and (c) thermoelectric power factor of pristine ZGNR(20), ELD-ZGNR(10,10), and 2ELD-ZGNR(8,4,8) channels with perfect edges. The dots indicate the Fermi energy values at which the peak of the power factor occurs for the ELD and 2ELD channels.
FIG. 10: (a) Electrical conductance, (b) Seebeck coefficient, and (c) thermoelectric power factor of ELD-ZGNR(10,10) and 2ELD-ZGNR(8,4,8) with rough edges and positively charged substrate impurities. The channel length is 2 μm. The dots indicate the Fermi energy values at which the peak of the power factor occurs for the pristine ELD and 2ELD channels of Fig. 9 for comparison purposes.
FIG. 11: The thermoelectric figure of merit $ZT$ for the ELD-ZGNR(10,10) (dashed-red line) and 2ELD-ZGNR(8,4,8) (dash-dot-blue line) channels of length $L = 2 \mu m$. The lattice thermal conductance is extracted from the calculated mean free path. Inset: The same figure of merit $ZT$ but with the lattice thermal conductance extracted by integrating the simulated phonon transmission.