Lattice QCD simulation of the Berry curvature

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\[ H(p) \Phi(p) = E(p) \Phi(p) \]
Berry Curvature

\[ H(p) \Phi(p) = E(p) \Phi(p) \]

Berry connection

\[ \tilde{A}_\mu(p) = -i \Phi^\dagger(p) \frac{\partial}{\partial p^\mu} \Phi(p) \]

Berry curvature

\[ \tilde{F}_{\mu\nu}(p) = \frac{\partial}{\partial p^\mu} \tilde{A}_\nu(p) - \frac{\partial}{\partial p^\nu} \tilde{A}_\mu(p) \]
\[
H(p) \Phi(p) = E(p) \Phi(p)
\]

**Berry connection**

\[
\tilde{A}_\mu(p) = -i \phi^\dagger(p) \frac{\partial}{\partial p^\mu} \phi(p)
\]

**Gauge connection**

\[
A_\mu(x)
\]

**Berry curvature**

\[
\tilde{F}_{\mu\nu}(p) = \frac{\partial}{\partial p^\mu} \tilde{A}_\nu(p) - \frac{\partial}{\partial p^\nu} \tilde{A}_\mu(p)
\]

**Gauge field strength**

\[
F_{\mu\nu}(x)
\]

Berry (1984)
Berry curvature of fermion spatial momentum

- chiral magnetic effect
- chiral vortical effect
- quantum Hall effect
- topological insulators
Formalism

fermion ground state \( \Phi(p) \)
Formalism

fermion ground state \( \Phi(p) \)

ground state projection

\[
\phi(p, \tau) = \sum_{x, x'} e^{ip \cdot (x-x')} D^{-1}(x, \tau | x', 0) \phi_{\text{init}}
\]

fermion propagator

\[
\rightarrow \Phi(p) \quad \text{in} \quad \tau \rightarrow \infty
\]
Formalism
Formalism

Berry connection

\[ \tilde{A}_\mu(p) \]

Berry link variable

\[ \tilde{U}_\mu(p) = \frac{\Phi^\dagger(p)\Phi(p + \tilde{\mu})}{|\Phi^\dagger(p)\Phi(p + \tilde{\mu})|} \]
Formalism

Berry connection

\[ \tilde{A}_\mu(p) \]

\[ \tilde{U}_\mu(p) = \frac{\Phi^\dagger(p)\Phi(p + \tilde{\mu})}{|\Phi^\dagger(p)\Phi(p + \tilde{\mu})|} \]

Berry curvature

\[ \tilde{F}_{\mu\nu}(p) \]

\[ \tilde{P}_{\mu\nu}(p) = \tilde{U}_\mu(p)\tilde{U}_\nu(p + \tilde{\mu})\tilde{U}_\mu^\dagger(p + \tilde{\nu})\tilde{U}_\mu(p) \]

Berry link variable

\[ \tilde{U}_\mu(p) \]

Berry plaquette

\[ \tilde{P}_{\mu\nu}(p) \]
Example

(2+1)-dim. Wilson fermion

\[ D(x, x') = (ma + 3)\delta_{x,x'} \]

\[-\frac{1}{2} \sum_{\mu=1}^{3} \left[ (1 - \sigma_\mu) U_\mu(x)\delta_{x+\hat{\mu},x'} + (1 + \sigma_\mu) U_\mu^+(x')\delta_{x-\hat{\mu},x'} \right] \]
Example

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2-dim. U(1) Berry link variables

2-dim. U(1) lattice gauge theory
Example

**Berry curvature**

\[ \tilde{F}_{xy}(p) = \text{Im} \, \ln \tilde{P}_{xy}(p) \]

**topological charge density**

\[ F_{xy}(x) \]

**1st Chern number**

\[ N = \frac{1}{2\pi} \sum_p \tilde{F}_{xy}(p) \]

**topological charge**

\[ Q \]
Example

1st Chern number

\[ N = \frac{1}{2\pi} \sum_p \tilde{F}_{xy}(p) \]
Example

1st Chern number

\[ N = \frac{1}{2\pi} \sum_p \tilde{F}_{xy}(p) \]

-6 -4 -2

-1 pole  +3 poles  -3 poles  +1 pole
Example

1st Chern number

\[ N = \frac{1}{2\pi} \sum_p \tilde{F}_{xy}(p) \]

quantum Hall effect

\[ R_{xy} = \frac{2\pi}{e^2} \frac{1}{N} \]
Example

Berry curvature

ma = 0.5

ma = -0.5
Chern number

\[ ma = 0.5 \]

\[ ma = -0.5 \]
Example

Chern number

\[ ma = 0.5 \]

\[ ma = -0.5 \]

\[ N = 0 \]
Example

Chern number

$ma = 0.5$

$N = 0$

$ma = -0.5$

$N = 1$

Ground state
lattice QCD simulation of the Berry curvature:

✓ formulated

✓ checked in (2+1)-dim. Wilson fermion

✓ applicable to realistic systems