3D network modelling of fracture processes in fibre-reinforced geomaterials

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Abstract

The width of fracture process zones in geomaterials is commonly assumed to depend on the type of heterogeneity of the material. Still, very few techniques exist, which link the type of heterogeneity to the width of the fracture process zone. Here, fracture processes in geomaterials were numerically investigated with structural network approaches, whereby the heterogeneity in the form of large aggregates and low volume fibres is modelled geometrically as poly-dispersed ellipsoids and mono-dispersed line segments, respectively. The influence of aggregates, fibres and combinations of both on fracture processes in direct tensile tests of periodic cells was investigated. For all studied heterogeneities, the fracture process zone localised at the start of the softening regime into a rough fracture. For aggregates, the width of the fracture process zone was greater than for analyses without aggregates. Fibres also increased the initial width of the fracture process zone and, in addition, resulted in a widening of this zone due to fibre pull out.

1 Introduction

Many structures made of geomaterials exhibit failure processes which are influenced by the heterogeneity of the material at an intermediate (meso-) scale. For instance, the type of coarse aggregates in concrete influences stiffness, strength and fracture energy of the material. For fibre reinforced cementitious materials, fibre type and geometry strongly influence the tail of the stress-crack opening curve (Naaman et al., 1991; Li and Wu, 2007). Therefore, modelling approaches which link the geometry, spatial distribution and mechanical response of individual constituents at the meso-scale to the structural response are attractive. Furthermore, detailed investigations based on experiments and computational modelling of the mechanical interaction of individual constituents can contribute to further understanding of failure
processes at larger scales.

Numerical approaches based on nonlinear fracture mechanics (NLFM) (Dugdale, 1960; Barenblatt, 1962) are commonly used to predict the failure of structural components of practical size, since the length of the fracture process zone is too large (with respect to the size of the structural component) for linear elastic fracture mechanics (LEFM), but too small for plastic limit load analysis to be applicable. Here, fracture process zone is defined as the zone in which energy is dissipated at a certain stage during the fracture process. Within computational frameworks, such as the finite element method and discrete stiffness approaches, NLFM is applied in the form of cohesive-crack and crack-band models. In cohesive-crack models, the displacement field across the fracture process zone is replaced by a displacement jump representing the crack opening and stresses are determined from a stress-crack opening law (Hillerborg et al., 1976; Carol et al., 1997). In crack-band models, the displacement jumps are transformed into cracking strains, so that the stress is calculated using stress-strain laws taking into account the size of the regions in which strains localise (Bažant and Oh, 1983). This size is usually a function of the element size, so that the load-displacement curves obtained with this approach are mesh-independent (Jirásek and Bauer, 2012). Discrete approaches describe both elastic and inelastic responses by means of force-displacement laws between discrete bodies (Schlangen and van Mier, 1992a,b; Bolander et al., 2000). Often, these force-displacement laws are chosen to be very similar to crack band approaches (Grassl and Bolander, 2016). These different computational NLFM approaches can model the length of the fracture process zone along the fracture, but not its width.

Continuum mechanics is an alternative to nonlinear fracture mechanics, where the fracture process zone is represented by localised but regular fields of displacements. This is achieved by including a length parameter in continuum models (Pijaudier-Cabot and Bažant, 1987; Bažant and Jirásek, 2002). Maintaining a regularised displacement field during fracture simulations provides mesh-independent solutions upon mesh refinement. However, the length parameter influences the numerically predicted peak load and deformation capacity of structures (Xenos and Grassl, 2016). Therefore, this parameter should be chosen so that the localised field of displacements matches the width of the fracture process zone of the material (Xenos et al., 2015).

The fracture process zone in heterogeneous materials such as concrete has been investigated experimentally and numerically. Experimental studies for fracture in plain concrete in Mihashi et al. (1991); Mihashi and Nomura (1996); Otsuka and Date (2000); Grégoire et al. (2015) showed that the fracture process zone consists of a narrow band of high dissipation surrounded by a wider region of low dissipation. Fracture surface measurements were also performed to provide further insight into the link between roughness and fracture behaviour Lange et al. (1993); Mourot et al. (2006); Morel et al. (2008); Ponson et al. (2006). In Grassl and Jirásek (2010); Grassl et al. (2012); Grégoire et al. (2015); Xenos et al. (2015), information
about the width of the fracture process zone was determined numerically using two-dimensional structural network approach for the meso-scale of plain concrete consisting of coarse aggregates embedded in a mortar matrix. Numerical models for fibre reinforced concrete, in which fibres were modelled discretely, were proposed in Bolander and Saito (1997); Leite et al. (2004, 2007); Kabele (2007); Radtke et al. (2010); Kunieda et al. (2011); Schauffert and Cusatis (2011); Caggiano et al. (2012); Montero-Chacón et al. (2013); Kang et al. (2014); Zhan and Meschke (2016); Kang and Bolander (2017); Montero-Chacón et al. (2017). Most of these studies on fibre reinforced composites aimed at predicting the influence of fibres on stiffness, strength and ductility. There is less information available on how fibres affect the spatial distribution of dissipated energy at the meso-scale.

The aim of this work was to obtain more information about fracture processes in geomaterials at the meso-scale by using a three-dimensional structural network model. The meso-structure of geomaterials was idealised to consist of a matrix with coarse aggregates, interfacial transition zones (ITZs) between matrix and aggregates, and fibres. Periodic direct tension analysis were performed and the effect of aggregates and fibres on the stress-displacement curves and spatial distribution of energy dissipation was investigated.

2 Method

The present numerical approach for obtaining information on fracture processes in fibre-reinforced quasi-brittle materials relies on periodic meso-structure generation, periodic network modelling of the material response, and roughness evaluation of the fracture patterns obtained from the network modelling. In the following sections, the individual modelling techniques are described in more detail.

2.1 Periodic meso-structure generation

The meso-structure of concrete was modelled as coarse aggregates and fibres embedded in a mortar matrix. Aggregates and fibres were idealised as poly-dispersed ellipsoids and mono-dispersed line segments, respectively. They were periodically arranged in a computational cell representing the meso-structure of the material. For a given volume fraction of ellipsoids, Fuller’s grading curve was used to determine the size distribution of ellipsoids (Figure 1a). The total volume of ellipsoids was divided into intervals using sieve sizes. For each volume interval, the upper and lower sieve sizes are $m$ and $n = m/2$, respectively. Here, $m$ is smaller than or equal to the maximum sieve size $d_{a,\text{max}}$ and $n$ is greater than or equal to the minimum sieve size $d_{a,\text{min}}$. Starting with the volume interval obtained with the largest pair of sieve sizes, ellipsoids were generated randomly with radii $s_3 > s_2 > s_1$ so that they fit through the square sieve
size \( m \), but not \( n \) (Figure 1b) as proposed in Slowik and Leite (1999); Leite et al. (2007) and further investigated in Mehrotra (2011). Line segments were assumed to be of uniform length \( l_f \). For a volume fraction \( \rho_f \), the number of fibres was calculated as \( n_f = 4\rho_f V/(\pi d_f^2 l_f) \), where \( V \) is the volume of the unit cell and \( d_f \) is the diameter of the fibres. The input parameters for the meso-structure generation are the volume fraction of ellipsoids \( \rho_a \), the maximum and minimum sieve sizes \( d_{a,\text{max}} \) and \( d_{a,\text{min}} \), respectively, the volume fraction of line segments \( \rho_f \), fibre length \( l_f \) and the diameter of fibres \( d_f \). Only ellipsoids greater than the sieve size \( d_{a,\text{min}} \) were generated, as indicated by the shaded region in Figure 1a.

Next, ellipsoids and line segments were placed in the periodic cell by a random sequential addition approach (Feder, 1980) so that the centroids of ellipsoids and line segments are within the cell. Attention was paid so that the random orientation of ellipsoids and line segments were uniformly generated within the volume (Muller, 1959). For every randomly placed object, overlap with previously placed objects was checked. If overlap was avoided, the object was placed in the cell and 26 mirror objects in the adjacent cells were generated by shifting the object to the adjacent periodic cells. If overlap was detected, a new random position and orientation was generated. This process was repeated until all objects were placed in the cell. For the overlap check between ellipsoids, the algebraic system of equations in Wang et al. (2001) was used (Figure 2). Compared to the overlap check for spheres, solving this system of equations was slow. Therefore, outer and inner bounding spheres of the ellipsoids were used to exclude any unnecessary checks of ellipsoids. If the outer bounding spheres of two ellipsoids do not overlap, the two ellipsoids themselves do not overlap (Figure 2a). If the inner bounding spheres of two ellipsoids overlap, the two ellipsoids overlap (Figure 2c). Only if the outer bounding spheres overlap and the inner spheres do not overlap, the overlap check of two ellipsoids was performed (Figure 2b). This simple method based on
bounding outer and inner spheres required significantly less time than applying the method in Wang et al. (2001) to all ellipsoids.

For combinations of ellipsoids and line segments, only overlaps between ellipsoids, and ellipsoids and line segments were checked. For combination of ellipsoids and line segments, the coarse ellipsoids were placed first, followed by line segments and finally fine ellipsoids.

Examples of generations of ellipsoids with $d_{a,\text{max}} = 16$ mm, $d_{a,\text{min}} = 8$ mm and $\rho_a = 0.8$, line segments with $l_{\ell} = 30$ mm, $d_{\ell} = 0.75$ mm and $\rho_{\ell} = 0.01$, and a combination of ellipsoids and line segments with $d_{a,\text{max}} = 16$ mm, $d_{a,\text{min}} = 8$ mm, $\rho_a = 0.8$, $l_{\ell} = 30$ mm, $d_{\ell} = 0.75$ mm and $\rho_{\ell} = 0.01$ are shown in Figure 3 for a cell with an edge length of 100 mm. The fibre diameter $d_{\ell}$ is only required to calculate the number of line segments to be placed, but not for the placement itself. Here, $\rho_a = 0.8$ is the total volume fraction of ellipsoids, which is significantly greater than the generated volume fraction between the sieve sizes 16 and 8 mm.
2.2 Periodic network modelling

The fracture processes at the meso-scale were modelled for a periodic cell subjected to direct tension with a three-dimensional irregular network of discrete structural elements. The random network generation follows the work in Yip et al. (2005), which was recently extended to dual structural transport problems in Grassl and Bolander (2016). For the network generation, random points are placed in the cell using a sequential addition approach enforcing a minimum distance \( d_{\text{min}} \) between the points (Feder, 1980). These points are used for dual Delaunay and Voronoi tessellations resulting in randomly arranged tetrahedra and polyhedra. In Figure 4a, one of these tetrahedra with a common facet of polyhedra belonging to two vertices of the tetrahedron is shown. The network elements were placed on the edges of the tetrahedra. The mid-crosssections of the network elements were set equal to the common facets of the Voronoi cells associated with the element nodes (Yip et al., 2005). The network elements have six degrees of freedom at each node which are linked by rigid body kinematics to displacement jumps at the centroid of the mid-crosssection. These displacement jumps are then related to corresponding stress components using constitutive models described in Section 2.3.

The information of the spatial arrangement of ellipsoids was mapped onto the network. According to the position of network elements with respect to ellipsoids, network elements were given the properties of matrix, interfacial transition zone (ITZ) and aggregate. Network elements with both nodes positioned within an ellipsoid were given stiff elastic properties representing aggregates. Elements with both nodes located in the matrix were given properties of mortar with corresponding elastic properties, and strength and fracture energy. Finally, for elements with one node in an ellipsoid and another one in the matrix or another ellipsoid, the properties of ITZ were used, which were characterised by lower strength and lower fracture energy than those of the matrix. The stiffness of ITZ elements were determined by the harmonic mean of the stiffnesses of matrix and aggregate.
Figure 5: Modelling fibres: a) 3D frame element for fibres and b) link element for the interaction between fibres and matrix.

The fibres were idealised as linear elastic structural frame elements (McGuire et al., 2000), which were placed on the positions of the line segments (Figure 5a). Interactions between the frame elements representing fibres and the background network representing matrix and ITZ were modelled by means of link elements as described in Yip et al. (2005) (Figure 5b). This type of link elements was originally used for the modelling of bond in reinforced concrete (Ngo and Scordelis, 1967), and was more recently applied to network models in Bolander and Saito (1997); Montero-Chacón et al. (2017). Rigid body kinematics were used to determine, from the nodal degrees of freedom of the link and frame elements, the displacement jump at the node of the frame element (Figure 5b). These displacements are orientated in the axial and two orthogonal directions of the frame element. An elasto-plastic model described in Section 2.3 was used to describe the slip between the frame elements and the background network in the axial direction of the frame element.

Reduction of the embedded length due to pullout of the fibres as discussed in detail in Naaman et al. (1991) was not modelled here, since only small displacements with respect to the pull-out length were considered. Computationally more efficient semi-discrete approaches described in Kang et al. (2014); Kang and Bolander (2017) would be well suited to describe the full pull-out process, since these approaches incorporate important features of the fibre-matrix interaction without modelling individual degrees of freedom.

Periodicity with respect to cell boundaries was introduced for both network geometry and boundary conditions. This was achieved by using a method that was originally proposed in Grassl and Jirásek (2010) for two-dimensional analyses and then extended to three dimensions in Athanasiadis et al. (2018) for hydro-mechanical problems. For every random point placed in the cell, 26 periodic image points in the adjacent cells were created. The two dual tessellations were then performed for the points in the cell and the periodic image points. In the resulting network, elements cross the boundaries of the cell. In Figure 6, the periodic cell with two out of 26 adjacent cells is shown.
As an example, elements $I' - J$ and $I - J'$ cross the boundary of the cell. These elements were used for computing the response of the periodic cell. However, only degrees of freedom (DOF) of nodes located inside the periodic cell were determined. For nodes outside the periodic cell ($I'$ and $J'$), which belong to elements crossing the boundary, the DOF were determined from those of the periodic image nodes inside the cell ($I$ and $J$, respectively) and six average strain components ($\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}, \varepsilon_{xy}, \varepsilon_{yz}, \varepsilon_{xz}$), which were applied to the cell. With these average strain components and the work conjugated stress components ($\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{yz}, \sigma_{xz}$), the loading of the periodic cell was controlled. This approach has the advantage that fracture planes can occur anywhere in the periodic cell (with the normal of the plane being the direction of loading) and were not strongly influenced by the boundaries of the cell which would be the case for techniques in which the elements are aligned with respect to the cell boundaries. Analyses of boundary value problems without the use of periodic boundary conditions would normally require strengthening of the material close to the ends of the specimen to avoid fracture to occur at the boundaries. A detailed description of this formulation can be found in Grassl and Jirásek (2010) and Athanasiadis et al. (2018). This approach was applied to both the background network and the frame and link elements. An example of the background network representing the three phases of matrix, aggregates and ITZ is shown in Figure 7a. Fibres with their corresponding link elements are shown in Figure 7b.

### 2.3 Constitutive models

The constitutive response of the background network representing aggregates, matrix and ITZs were modelled by linear elasticity and damage mechanics. For matrix and ITZ, a scalar damage model was used of the form

$$\sigma = (1 - \omega) D_0 \varepsilon$$

(1)
where $\sigma$ and $\varepsilon$ are the stress vector and strain vector, respectively, $D_e$ is the elastic stiffness matrix and $\omega$ is the damage parameter ranging from 0 (undamaged) to 1 (fully damaged). For a detailed description of this constitutive model, see Grassl and Bolander (2016).

By using the special network generation in Section 2.2 and choosing the stiffness matrix $D_e$ so that the axial stiffness component is equal to the shear stiffness components, the stress and strain fields were elastically homogeneous and produced zero Poisson’s ratio (Yip et al., 2005). The onset of damage is determined by an equivalent strain expression which gives an ellipsoidal strength envelope in the stress space with the shear and compressive strength being greater than the tensile strength. The damage variable was determined from an exponential softening stress-crack opening curve ($\sigma-w_c$) with tensile strength $f_t$ and crack opening threshold $w_f$ (Figure 8). The area under the stress-crack opening curve is the fracture energy $G_F$. With this approach, the resulting load-displacement curves of tensile fracture simulations are independent of the element length, if it is chosen to be sufficiently small. The other parameters of this constitutive model were set to the default values described in Grassl and Bolander (2016). The dissipated energy rate $\dot{d}$ in the constitutive model of the network elements was computed as

$$\dot{d} = \dot{\omega} \frac{1}{2} : D_e : \varepsilon$$

This energy dissipation was used to present the fracture process zone in Section 4. Aggregates were assumed to be elastic. However, fracture in aggregates could be simulated in future studies with this approach, since aggregates were discretised by multiple network elements.

Fibres were modelled to be elastic with a Young’s modulus $E_s$. For the links between the fibres and the network model, an elasto-plastic model in the tangential direction of the fibre was used which is
Figure 8: Constitutive models for a) softening in the matrix and b) bond-slip.

illustrated in Figure 8b. Here, \( \tau_0 \) is the limit stress at which plastic slip occurs. The modulus \( E_b \) controls the elastic response of the link. In the analyses, \( E_b \) was set to a large enough value so that the results were not influenced significantly by it, but small enough so that no numerical problems were created. The dissipated energy rate \( \dot{d} \) for the constitutive model of the link element is

\[
\dot{d} = (\varepsilon - \varepsilon_p) E_b \dot{\varepsilon}_p
\]

Here, \( \dot{\varepsilon}_p \) is the rate of the plastic slip.

### 2.4 Roughness evaluation

The fracture processes were analysed by evaluating the evolution of spatial distribution of dissipated energy. For the present evaluation, both dissipation due to damage in the structural network elements, as well as dissipation due to plastic slip in the link elements were considered. To each element in which energy is dissipated, a crosssectional area with a centroid as shown in Figure 9 was associated. For the elements used for the background network, these are the mid-crosssections with the centroid \( C \) shown in Figure 4b. For the link elements, the crosssectional area is \( A_b \) shown in Figure 5b and the centroid is the node of the frame element to which the link element is connected (node \( H \) in Figure 5b).

Firstly, the mean of all heights of centroids of crosssections was calculated as

\[
\bar{z} = \frac{\sum_{i=1}^{N} w_i z_i}{N}
\]

Here, \( z \) was measured in the direction of the applied tensile strain with the bottom of the cell used as
the origin. Furthermore, $w_i$ were the weights of the individual crosssections, which were calculated as

$$w_i = \frac{A_i \Delta d_i}{\sum_{k=1}^{N} A_k \Delta d_k}$$

(5)

where $A_i$ and $\Delta d_i$ were the area and increment of dissipation per unit area, respectively, of the facet $i$. Then, the standard deviation $\Delta h$ was calculated as

$$\Delta h = \sqrt{\frac{\sum_{i=1}^{N} w_i (z_i - \bar{z})^2}{\sum_{i=1}^{N} w_i}}$$

(6)

This standard deviation is a measure related to the width of the fracture process zone, which takes into account the spatial arrangement and intensity of the dissipation events. It is smaller than the total width of the fracture process zone, which is simply defined as the zone in which energy is dissipated, but does not provide information about the intensity of these events. For a localised crack surface with equal energy dissipation in all elements whose crosssections form this surface, the measure used is equal to the standard deviation of the roughness distribution of the crack surface, which can be determined experimentally as described in Xenos et al. (2015). Because of this geometrical link to the fracture surface, the method was called here roughness evaluation. Nevertheless, for energy dissipation in overlapping zones and fibres, it would not be possible to determine the value of $\Delta h$ experimentally by means of evaluation of the roughness of the surface alone.

3 Analyses

The network modelling approach described in Section 2 was applied to analyse fracture in cubic periodic cells of an edge length of 100 mm subjected to direct tension as shown in Figure 10. For this setup, the average strain in the axial $y$-direction ($\varepsilon_{yy}$) was monotonically increased, which resulted in a reactive stress component in the $y$-direction ($\sigma_{yy}$), which in the presentation of the results is called $\sigma$. All other
average stress components \((\sigma_{xx}, \sigma_{zz}, \sigma_{xy}, \sigma_{yz} \text{ and } \sigma_{xz})\) were kept equal to zero. The analyses were performed quasi-statically with an incremental-iterative approach (see e.g. de Borst et al. (2012)). The iterative part was based on a modified Newton method using the secant stiffness for the damage model for matrix and ITZ, and the elastic stiffness for the elasto-plastic model for the links between fibres and background network.

Four groups of analyses were carried out. For each group, ten random generations of background networks and meso-structures were performed. The network was generated with a minimum distance \(d_{\text{min}} = 3 \text{ mm}\) between the randomly placed points. The first group of analyses consists of a network representing matrix without any meso-scale features explicitly incorporated. In the second group of analyses, the network of elements represented matrix, aggregates and ITZs. For these analyses, the volume fraction of aggregates generated with the techniques described in Section 2.1 was \(\rho_a = 0.8\) with a maximum and minimum sieve size \(d_{a,\text{max}} = 16 \text{ mm}\) and \(d_{a,\text{min}} = 8 \text{ mm}\), respectively. In the third group of analyses, fibres with a length \(l_f = 3 \text{ cm}\), a diameter \(d_f = 0.75 \text{ mm}\) and a fibre volume fraction of \(\rho_f = 0.01\) were used. Finally, the fourth group consisted of combinations of aggregates and fibres with the same input as for the analyses with only one phase. The input parameters for the different phases of the background network are shown in Table 1. These input values were in the typical range of values used for meso-scale analyses of concrete in the literature (Grassl et al., 2012), where it was shown that they provide good agreement with experimental results. For the fibres, a modulus of \(E_f = 200 \text{ GPa}\) was used. The modulus and limit stress of the link elements was set to \(E_b = 3000 \text{ GPa}\) and \(\tau_0 = 4 \text{ MPa}\), respectively.
Table 1: Input values for the background network. The modulus $E$ of ITZ was determined as harmonic mean of moduli of matrix and particle.

| Phase | $E$ [GPa] | $f_t$ [MPa] | $G_F$ [J/m$^2$] |
|-------|-----------|-------------|-----------------|
| Matrix | 30        | 3           | 100             |
| Particle | 90        | -           | -               |
| ITZ   | 57.1      | 3.25        | 50              |

4 Results

The results of the direct tension analyses of the four groups of material setups were shown in the form of stress-displacement curves, spatial patterns of dissipated energy and roughness-displacement curves. The displacement was determined as the average strain multiplied by the cell length $a$ (Figure 10). For the stress-displacement and roughness-displacement curves, the mean of the quantities of random analyses were shown.

The mean stress-displacement curves for four groups of material setup are shown in Figure 11. For the plain configuration with matrix material only, the stress-displacement curve showed the typical response of quasi-brittle materials subjected to direct tension. In the pre-peak, the response is linear elastic in the first part and then exhibits small non-linearities just before the peak. The post-peak regime shows steep softening, which then flattens with the average stress approaching zero. The peak stress is greater than the input tensile strength, because the stress in the network elements consists of combinations of axial and shear components. With the ellipsoidal strength envelope used, the combined normal and shear stress components result in a greater strength than a pure tensile stress component. The addition of aggregates strongly reduces the peak stress because of the weak ITZs between aggregates and matrix. Furthermore, the initial stiffness was slightly increased due to the greater stiffness of the aggregates. If instead of aggregates only fibres are added to the matrix, the peak stress is only slightly increased compared to the plain peak stress. However, the tail of the stress-displacement curve is strongly influenced by the presence of the fibres with a significant bridging stress present after the initial softening. For combinations of aggregates and fibres, the fibres cause again a small increase of the peak stress compared to the aggregate only case and result in a similar bridging stress at the ultimate displacement applied in the analyses.

For the analyses involving fibres, the peak and bridging stresses were compared to empirical estimates reported in Naaman (1987). For the peak values of the stress of the analyses with fibres, the peak stress is

$$
\sigma_{cc} = \sigma_{mu} (1 - \rho_f) + \alpha_1 \alpha_2 \tau_0 \rho_f \frac{l_f}{d_f} \tag{7}
$$

Here, $\alpha_1$ and $\alpha_2$ are factors taking into account the fibre orientation and fraction of bond strength
Figure 11: Meso-scale analysis: Mean stress versus displacement for four groups of material setups (plain, aggregates, fibres and aggregates+fibres). The symbols refer to stages for which the crack patterns are shown in Figures 12 and 13. Furthermore, the lines refer to empirical estimates in (7) and (8).

mobilised, respectively. Furthermore, $\sigma_{mu}$ is the peak stress of the material without fibres. The stress after cracking is estimated as

$$\sigma_{pc} = 4\lambda_1\lambda_2\tau_0\rho_l \frac{L}{d_f}$$

(8)

where $\lambda_1$ and $\lambda_2$ are factors for average pullout length and postcracking orientation efficiency, respectively. These expressions were compared to the numerical results in Figure 11 using $\alpha_1 = 0.5$, $\alpha_2 = 0.2$, $\lambda_1 = 0.25$ and $\lambda_2 = 0.5$, which are typical values for the type of fibres used. The values for $\sigma_{mu}$ were obtained for the corresponding analyses without fibres.

All the global stress-displacement curves in Figure 11 exhibit softening which is usually accompanied by localisation of displacements. Detailed information about the localisation process was studied in the form of spatial distribution of mid-cross-sections at which energy dissipation occurs. The dissipation patterns for the four groups of analyses are shown in Figures 12 and 13 for stages at peak and in the post-peak, respectively, for one random analysis.

The corresponding stages are marked in Figure 11. At stage 1 at peak, the dissipation rate is distributed in the entire specimen (Figure 12). For plain and fibre analyses, the dissipated energy was distributed uniformly. For analyses involving aggregates, the distribution was more heterogeneous, because at the position of the elastic aggregates no dissipation occurs. At stage 2 in the softening regime, the rate of
Figure 12: Meso-scale analyses: Crack patterns of direct tension analysis at stage 1 marked in Figure 11. Orange (online version) polygons refer to mid crosssections in which damage increases at this stage of the analysis.

Figure 13: Meso-scale analyses: Crack patterns of direct tension analyses at stage 2 marked in Figure 11. Orange (online version) polygons refer to mid crosssections in which damage increases at this stage of the analysis.
dissipation was strongly localised (Figure 13). The y-position of the localised region of rate of dissipation differs from the analysis to analysis because of the periodic cell used. For all groups, the localised zone is rough. For the plain analyses, this is due to the irregular background network used. For the other groups, the roughness of the zone of dissipated energy was also influenced by the heterogeneity in the form of aggregates and fibres. For instance, the spatial distribution of energy for the aggregate analyses in Figure 13 appears to be wider than for the plain case. These plots of dissipation rate are from only one random analysis of each group. Also, all mid-crosssections at which energy was dissipated at this stage of the analysis were shown without discriminating between the amount of energy that is dissipated at the crosssections.

For a quantitative representation of the evolution of the zone of rate of dissipated energy, the roughness measure described in Section 2.4 was used. The mean of the measure of the width of the fracture process zone $\Delta h$ in (6) versus displacement was shown in Figure 14. The symbols in the figure refer to the two stages at which the crack patterns were shown in Figures 12 and 13. The overall roughness evolutions for the four groups of analyses are overall very similar. At the start of the analysis, no energy is dissipated, so that $\Delta h$ is not defined. For the uniformly distributed cracking in pre-peak regime, $\Delta h$ is approximately equal to 30 mm. This value agrees well with the theoretical value for the standard deviation of a uniform distribution over the cell size, i.e. the interval from 0 to 100 mm, which is $100/\sqrt{12} = 28.9$ mm. At the start of the post-peak regime, the width of the fracture process zone dropped down to values less than 5 mm for all groups of analyses. This drop occurred in the initial part of the softening regime at a stage at which little energy had been dissipated.

A detail of the evolution of $\Delta h$ after the drop is shown in Figure 15. The roughness $\Delta h$ is the smallest for the analyses with only the matrix material. After the abrupt drop, $\Delta h$ remains almost constant. Adding aggregates results in an increase of the roughness compared to the plain case. Again, the value remains constant after the drop. If, instead of aggregates, fibres are added to the background lattice, the roughness is again greater than for the plain case. However, roughness is not constant with increasing displacement. Instead, it increases with increasing displacement. The same trend is observed if aggregates and fibres are combined. This increase is due to the energy dissipated by the slip between fibre and matrix defined in (3). Before the abrupt drop, there is no energy dissipation due to fibre slip. Only once the crack has formed, the slip between fibres and matrix starts. In the present approach, fibre pull out is not modelled, which means that the embedded length of fibres does not change. Consequently, it is expected that for the analyses involving fibres, $\Delta h$ would reach a constant value once all fibres crossing the localised zone of displacements are significantly stressed so that they dissipate energy along their short embedded length, and the damage in the matrix so high that the energy dissipation in the matrix is insignificant. If fibre pullout would be taken into account as well, the dissipated energy should eventually reduce to zero once all fibres are pulled out. For a fibre length of 3 cm as used in this study, this point would be reached.
Figure 14: Meso-scale analysis: Measure of width of fracture process zone $\Delta h$ versus displacement $\delta$ for random analyses with aggregates, and with aggregates and fibres. The symbols refer to stages for which the crack patterns are shown in Figures 12 and 13.

Figure 15: Meso-scale analysis: Stress versus displacement analyses with aggregates and aggregates with fibres. The symbols refer to stages for which the crack patterns are shown in Figures 12 and 13.
when a displacement of 1.5 cm was applied to the specimen, which is 100 times higher than the maximum displacement considered here.

The evolution of dissipated energy for the four groups of analyses is shown in Figure 16. The symbols refer to the two stages at which the dissipation patterns are shown in Figures 12 and 13. Here, stage 1 marks the peak of the stress-displacement curves shown in Figure 11. For all analyses, the dissipation in the pre-peak regime is very small. For plain and aggregates only cases, the majority of dissipation occurs in the first part of the post-peak regime and then approaches a constant value. For the analyses with fibres, the initial dissipation in the very first part of the post-peak regime is slightly less than for the analysis without fibres. However, this difference is very small. In the later stage of the post-peak regime, the fibres contribute significantly to the dissipation, so that the overall dissipation of the analyses with fibres is much greater than for aggregates only. Only fibres, which cross the localised zone shown in Figure 13, were stretched sufficiently to contribute to the dissipated energy.

![Figure 16: Meso-scale analysis: Dissipated energy $D$ versus displacement $δ$ for the four groups of analyses (plain, aggregates, fibres and aggregates+fibres). The symbols refer to stages for which the crack patterns are shown in Figures 12 and 13.](image)

The interplay of energy dissipation in the different phases (matrix, ITZ and slip between fibres and matrix) is illustrated for the four groups of analyses in Figure 17. From this figure, it can be seen that fibres only contribute to the dissipation in the post-peak regime of the stress-displacement curve in Figure 11. Furthermore, the matrix material dissipates more energy if fibres are present, which is due to the generation of multiaxial stress states in the material. The dissipation within the ITZs is not affected by the presence of the fibres, since the majority of energy dissipation in the ITZs occurs early in the
Figure 17: Meso-scale analysis: Dissipated energy $D$ versus displacement $\delta$ for the four groups of analyses (plain, aggregates, fibres and aggregates+fibres) in the three phases of material in which energy is dissipated (matrix, ITZ, link be. The symbols refer to stages for which the crack patterns are shown in Figures 12 and 13.

fracture process before the fibres are activated. Furthermore, fibres were placed so that no overlap with aggregates occurred. Consequently, the ITZs which are located at the interface between aggregates and matrix would not be expected to be strongly influenced by fibres.

5 Conclusions

Network meso-scale analyses of fracture processes of periodic cells subjected to direct tension were performed with the aim to investigate the link between material heterogeneity and width of the fracture process zone. The meso-structures studied here consisted of a quasi-brittle matrix materia with aggregates, fibres and combinations of aggregates and fibres. For all material configurations, the width of the fracture process zone reduces abruptly after the peak load to the width of a rough crack. This strong localisation happens very early in the post-peak regime at a stage at which very little energy has been dissipated during the fracture process. For material configurations which included only matrix and aggregates, the width of the fracture process zone remained constant after the abrupt drop. For material configurations with fibres, the width of the fracture process zone increased after the drop since the slip between fibres and matrix contributes to the energy dissipation.
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