Volume conservation method for the three-dimensional front-tracking method

Shintaro TAKEUCHI* and Grétar TRYGGVASON**

* Department of Mechanical Engineering, Osaka University
2-1 Yamada-oka, Suita, Osaka, 565-0871 Japan
E-mail: shintaro.takeuchi@mech.eng.osaka-u.ac.jp

** Department of Mechanical Engineering, Johns Hopkins University
223 Latrobe Hall, 3400 N. Charles Street, Baltimore, MD, 21218-2681 USA

Received: 30 April 2020; Revised: 24 June 2020; Accepted: 8 July 2020

Abstract
A method to conserve the volume of dispersed components (e.g. bubbles and droplets) in a viscous fluid is proposed for the front-tracking method (Unverdi and Tryggvason, 1992; Tryggvason et al., 2001). The method adjusts the coordinates of each nodal points on the interface (or Lagrangian markers) along the velocity vector. A simplified algorithm determines the new position of the marker independently from those of the surrounding nodes, which allows the volume correction to be accomplished efficiently. The results show that the volume of a deformed fluid particle is kept constant within errors of $O(10^{-7}) \sim O(10^{-6})$. The effects of the time step size and the frequency of the volume correction are investigated. The method is applicable to enclosed structures of non-spherical geometry (e.g. oblate/prolate/spherical-cap fluid particles).

Keywords: Front-tracking method, Volume conservation, Lagrange multiplier

1. Introduction

The dynamics and behaviour of dispersed deformable fluid particles in a viscous fluid flow have been studied extensively during the past decades (Fischer et al., 1978; Keller and Skalak, 1982; Skotheim and Secomb, 2007; Takeuchi et al., 2010; Sugiyama et al., 2011; Rosti et al., 2018; Takeuchi et al., 2018), motivated by interest in industrial and biological applications.

Numerical simulation has been widely used as they produce a detailed picture of the flow, simultaneously and non-invasively, in/outside and on the interface of the dispersed component. Amongst the existing methods, immersed boundary (IB) method (Peskin, 1972, 2003) and front-tracking method (Unverdi and Tryggvason, 1992; Tryggvason et al., 2001) are often employed since these methods represent a sharp interface with Lagrangian markers arranged on a two-dimensional interface, and the interaction between the interface and the surrounding fluids is described by a simple spreading/interpolation (S/I) operation. In the three-dimensional front-tracking method the markers are connected by triangular elements, and shape (i.e., the size and topology) of the triangles is restructured (by addition/deletion/reconnections of elements) when necessary, to facilitate the computation of stress and strain distributions over the interface. Therefore, a growing number of problems involving interacting dispersed components in a flow field have been done using the front-tracking method (Jan and Tryggvason, 1991; Torres and Brackbill, 2000; Shin and Juric, 2002; Yamamoto et al., 2006; Muradoglu and Tryggvason, 2008; Tasoglu et al., 2008; Wang et al., 2013; Gong et al., 2014).

The volume of fluid particles is not always conserved by the front-tracking method (Tryggvason et al., 2001). This is a well-known problem for the original IB method with a smoothed delta function as S/I operator (Peskin, 1972), as the density is constructed from the Lagrangian markers which are moved using the interpolated local
velocity. Using the poorest (and probably the least-recommended) test case of Laplacian-type S/I operator, the relative error in volume was found to be as high as $O(10^{-2})$ per unit time (Cortez and Minion, 2000).

Numerous efforts have been devoted to the improvement of volume conservation of the methods. Bunner and Tryggvason (2002) employed a simple volume conservation for nearly-spherical bubbles. The front nodal points are displaced (very slightly) uniformly to recover the original volume. Increasing the accuracy of the front advection and higher-order S/I function have also been suggested (Peskin and Printz, 1993; Tryggvason et al., 2001). A pressure equation derived from the requirement that the interpolated velocity field (rather than the nodal values) remained incompressible was suggested by Peskin and Printz (1993).

In the present study, we propose an efficient method to correct for volume errors of a fluid particle constant, and the applicability of the method is demonstrated by applying it to a non-spherical buoyant bubble, rising in an initially quiescent fluid.

2. The Front-Tracking Method

We are concerned here with simulations done using front-tracking/finite-volume methods where the governing equations are solved on a fixed, regular mesh, covering both the ambient fluid and the drop or bubble. The interface is marked by connected marker points that are advected with the fluid velocity and a marker function, constructed from the location of the interface is used to set the density and viscosity of the different fluids. The marker points are also used to compute the surface tension. For a description of the method see Unverdi and Tryggvason (1992) and Tryggvason et al. (2001). In the present version of the method, we use a second-order finite difference for the spatial discretisation of all the terms in Navier-Stokes equation and a second-order Adams-Bashforth method for the time marching. The width of the S/I function is set to $2\Delta$, where $\Delta$ is a grid spacing.

One frame from a simulation of a buoyant bubble rising in a periodic domain is shown in Fig. 1. The bubble surface is resolved by triangulated grid and the velocity field, computed on a fixed rectangular grid, is shown in a vertical plane cutting through the middle of the domain.

3. Triangular Elements and Labelling Rule

The volume of a closed region $\Omega$ (with smooth surface $\partial \Omega$) is given by the following formula:

$$\frac{1}{3} \int_{\partial \Omega} \mathbf{s} \cdot \mathbf{n} \, dS ,$$  \hspace{1cm} (1)

where $\mathbf{s}$ is a vector from a point $G \in \Omega$ to an arbitrary point on the surface $P$. The vector $\mathbf{n}$ is a unit outward normal vector to the surface.
As illustrated in Fig. 2(a), a closed surface in three-dimensional space is approximated by a set of triangular elements. The surface element of an infinitesimal triangular element in Fig. 2 is given as follows:

\[ n_e = \frac{(s_j - s_i) \times (s_k - s_i)}{|(s_j - s_i) \times (s_k - s_i)|}, \quad \Delta S_e = \frac{1}{2} |(s_j - s_i) \times (s_k - s_i)| . \]

\[ \therefore n_e \Delta S_e = \frac{1}{2} (s_j - s_i) \times (s_k - s_i) . \]  

(2)

Each node can be labelled with either global or local IDs. Global nodes (within one triangular element \( e \)), \( i, j \) and \( k \), are arranged in a cyclic order around an outward normal vector \( (n_e) : i \rightarrow j \rightarrow k \rightarrow i \). The local nodes must be specified with respect to the element connecting them. In the present paper, the node arrangement is described in a mixture of the global and local IDs, as follows:

\[ i \rightarrow ei_+ \rightarrow ei_- \rightarrow i \quad (= i \rightarrow j \rightarrow k \rightarrow i) \]
\[ j \rightarrow ej_+ \rightarrow ej_- \rightarrow j \quad (= j \rightarrow k \rightarrow i \rightarrow j) \]
\[ k \rightarrow ek_+ \rightarrow ek_- \rightarrow k \quad (= k \rightarrow i \rightarrow j \rightarrow k) . \]  

(3)

In the following discussion, the local and global IDs will be used for the same node interchangeably; when the order of the local nodes and the pivotal global node are of importance, the notation \((i, ei_+, ei_-)\) \((\in \text{element } e)\) is applied; otherwise, simply \((i, j, k)\) or \((e1, e2, e3)\) \((\in \text{element } e)\) is used.

4. Volume Conservation Method

Figure 2(b) shows velocity vectors defined at each node (vertices) of the triangular elements representing \( \partial \Omega \). Within one time step, \( \Delta t \), a nodal point \( s \) is convected to a new position \( s + u \Delta t \) in the front-tracking method. The volume of the region enclosed by the nodes \( s_i + u \Delta t \) \((\in \partial \Omega(t + \Delta t)) ; i = 1, \cdots, N_p \) is not generally conserved, where \( N_p \) is the total number of nodes on \( \partial \Omega \). Therefore, a correction method for the volume is needed. In the present study, the convection velocity \( u_i \) is corrected by a factor \( \alpha_i \):

\[ s_{i|t+\Delta t} = s_{i|t} + \alpha_i u_i \Delta t . \]  

(4)

To simplify the discussion, we assume an explicit Euler time advancement, although a higher order time-update scheme is easily implemented.

The volume of \( \Omega \) at time \( t + \Delta t \) is approximated as follows:

\[ \frac{1}{3} \int_{\partial \Omega} (s + au \Delta t) \cdot n \, dS = \frac{1}{3} \sum_{e} N_e \sum_{n=1}^{3} \frac{1}{2} \sum_{n=1}^{3} s_{en} + \alpha_{en} u_{en} \Delta t \cdot n_e \Delta S_e . \]  

(5)

Here, \( N_e \) is the number of triangular elements covering the region \( \partial \Omega \). Note that \( (\alpha_{en} - 1)u_{en} \Delta t \) \((n = 1, 2, 3)\) is the correction at the node \( en \) at time \( t + \Delta t \). If \( \alpha_i = 1 \) for all the nodal points on \( \partial \Omega \), there would be no corrections of the velocities. Therefore, the least amount of corrections is required by minimising

\[ L \overset{\text{def}}{=} \sum_{i} (\alpha_i - 1)^2 \]
under a constraint
\[
C = \frac{1}{3} \sum_{e=1}^{N_e} \sum_{u=1}^{3} \frac{s_{en} + \alpha_{en} u_{en} \Delta t}{3} \cdot n_e \Delta S_e - V_o = 0
\]
where \(V_o\) is the initial volume of \(\Omega_{t=0}\).

To do this we apply the Lagrange’s multipliers method to the following functional:
\[
\Phi \equiv L - \lambda C
\]
and obtain the following equations:
\[
0 = \frac{\partial \Phi}{\partial \alpha_i} = \sum_j \frac{\partial \Phi}{\partial \alpha_j} \delta \alpha_j + \frac{\partial \Phi}{\partial \lambda} \delta \lambda
\]
\[
\therefore \frac{\partial \Phi}{\partial \alpha_i} = 0
\]
\[
\frac{\partial \Phi}{\partial \lambda} = 0.
\]
Here, \(\delta \alpha_i\) and \(\delta \lambda\) are virtual displacements.

### 4.1. Equations for correction factor \(\alpha_i\) and multiplier \(\lambda\)

From Eq. (7),
\[
2(\alpha_i - 1) - \frac{1}{9} \sum_{e=\Omega_i} R_e = 0
\]
where
\[
R_e = (u_{ei} \Delta t) \cdot n_e \Delta S_e + \frac{\partial (n_e \Delta S_e)}{\partial \alpha_i} (s_{ei} + \alpha_{ei} u_{ei} \Delta t + s_{ei} + \alpha_{ei} u_{ei} \Delta t + s_{ei} + \alpha_{ei} u_{ei} \Delta t)
\]

Notice that the summation in Eq. (9) is only applied to elements that include the node \(i\) as a vertex. The surface element \(n_e \Delta S_e\) after the correction is given as follows:
\[
2n_e \Delta S_e |_{\alpha_i \Delta t} = (s_{ei} + \alpha_{ei} u_{ei} \Delta t - s_i - \alpha_{ei} u_{ei} \Delta t) \times (s_{ei} + \alpha_{ei} u_{ei} \Delta t - s_i - \alpha_{ei} u_{ei} \Delta t)
\]
\[
= (s_{ei} - s_i) \times (s_{ei} - s_i) + (s_{ei} - s_i) \times (\alpha_{ei} u_{ei} \Delta t - \alpha_{ei} u_{ei} \Delta t) + (\alpha_{ei} u_{ei} \Delta t - \alpha_{ei} u_{ei} \Delta t) \times (s_{ei} - s_i)
\]
\[
+ (\alpha_{ei} u_{ei} \Delta t) \times (\alpha_{ei} u_{ei} \Delta t - \alpha_{ei} u_{ei} \Delta t)
\]
\[
\therefore 2 \frac{\partial (n_e \Delta S_e)}{\partial \alpha_i} = (s_{ei} - s_i) \times (-u_{ei} \Delta t) + (-u_{ei} \Delta t) \times (s_{ei} - s_i) + (\alpha_{ei} u_{ei} \Delta t - \alpha_{ei} u_{ei} \Delta t)
\]
\[
= -u_{ei} \Delta t \times (s_{ei} + \alpha_{ei} u_{ei} \Delta t - s_i - \alpha_{ei} u_{ei} \Delta t)
\]

Using Eqs. (11) and (12), Eq. (10) is simplified as follows:
\[
2R_e = 3(u_{ei} \Delta t) \cdot [(s_{ei} + \alpha_{ei} u_{ei} \Delta t) \times (s_{ei} + \alpha_{ei} u_{ei} \Delta t)]
\]
and substituting into Eq. (9) gives:
\[
\alpha_i = 1 + \frac{\lambda}{12} \sum_{e=\Omega_i} (u_{ei} \Delta t) \cdot [(s_{ei} + \alpha_{ei} u_{ei} \Delta t) \times (s_{ei} + \alpha_{ei} u_{ei} \Delta t)]
\]
Equation (14) is a set of non-linear equations with respect to the correction factors. Theoretically, the solution of Eq. (14), \(\alpha_{en} (e = 1, \cdots, N_e; n = 1, 2, 3)\), is obtained as a function of \(\lambda\), and substituting \(\alpha_{en}\) into Eq. (8), the following equation determines the value of \(\lambda\):
\[
9 \frac{\partial \Phi}{\partial \lambda} = \sum_{e=1}^{N_e} (s_{en} + \alpha_{en} u_{en} \Delta t) \cdot n_e \Delta S_e - 9V_o = 0.
\]
These equations could be solved numerically by a recursive approach such as the Newton-Raphson method; first, assuming initial values for \(\lambda\) and \(\alpha_{en}\), then eliminating the residuals by the recursive procedure. This would, however, be very demanding computationally.
4.2. A simple implementation
Assuming that the corrections of the velocities are very small, the second (and third) order terms of $\alpha_\ell (\ell = 1, \cdots , N_d)$ are negligible. Considering that the amount of correction on the velocity at global node $\ell$ is $(\alpha_\ell - 1)u_\ell$, the second-order deviation of the surface area
\[\left| (\alpha_\ell - 1)u_\ell \Delta t \right| \times \left| (\alpha_\ell - 1)u_\ell \Delta t \right|\]
would be negligible. This linearisation reduces the system (Eqs. (14) and (15)) to a set of first-order equations with respect to $\alpha_\ell (\ell = 1, \cdots , N_d)$ and $\lambda$.

Furthermore, if it is assumed that $u_i$ is nearly parallel to $u_{ei}$ (denoted $u_i \parallel u_{ei}$ hereafter) as is likely to be the case for nodes close to each other on a well resolved surface moving almost steadily, the $\alpha_i$ equation (Eq. (14)) can be further simplified with only $u_i, s_{ei}$, and $s_{ei}$, as follows:
\[
\alpha_i = 1 + \frac{\lambda}{12} \sum_{e=1}^{N_e} (s_{ei} \times s_{ei}) ,
\]
which enables evaluation of each correction factor $\alpha_i$ independently. Similarly, the correction factors of the nodes in element $e$ (as in Fig. 2(a) and Eq. (3)) are then given by:
\[
\alpha_i = 1 + \frac{\lambda}{12} E_i , \quad \alpha_{ei} = 1 + \frac{\lambda}{12} E_j , \quad \alpha_{e} = 1 + \frac{\lambda}{12} E_k ,
\]
where
\[
E_m = \sum_{e=1}^{N_e} (s_{em} \times s_{em}) \quad (m = i, j, k) .
\]
In the actual implementation for calculating $E_m$, we loop over the interface elements and store the value $(u_{em} \Delta t) \cdot (s_{em} \times s_{em})$ at the nodal point $m$.

Substituting the above equation (together with the assumption that $u_i \parallel u_j \parallel u_k; i, j, k \in \text{element } e$) into Eq. (11), the surface element area is given by:
\[
2n_e \Delta S_e = \frac{\lambda}{12} f_e + h_e ,
\]
where
\[
f_e \equiv (s_j - s_i) \times (E_i u_k - E_j u_i) \Delta t - (s_k - s_i) \times (E_j u_k - E_i u_j) \Delta t , \quad h_e \equiv (s_j - s_i) \times (s_k - s_i) + (s_j - s_i) \times (u_k \Delta t - u_i \Delta t) + (u_j \Delta t - u_i \Delta t) \times (s_k - s_i) .
\]
With Eqs. (15) and (18), the following quadratic expression of $\lambda$ is obtained:
\[
\begin{align*}
V_o = \frac{1}{9} \sum_{e} \left( \frac{\lambda}{12} (E_i u_i + E_j u_j + E_k u_k) \Delta t + s_j + u_i \Delta t + s_j + u_j \Delta t + s_k + u_k \Delta t \right) \cdot n_e \Delta S_e \\
= \frac{1}{18} \sum_{e} \left( \frac{\lambda}{12} p_e + r_e \right) \cdot \left( \frac{\lambda}{12} f_e + h_e \right) \\
= \left( \sum_{e} \frac{p_e \cdot f_e}{18} \right) \left( \frac{\lambda}{12} \right)^2 + \left( \sum_{e} \frac{p_e \cdot h_e + r_e \cdot f_e}{18} \right) \left( \frac{\lambda}{12} \right) + \sum_{e} \frac{r_e \cdot h_e}{18} ,
\end{align*}
\]
where
\[
p_e \equiv (E_i u_i + E_j u_j + E_k u_k) \Delta t , \quad r_e \equiv s_j + u_i \Delta t + s_j + u_j \Delta t + s_k + u_k \Delta t .
\]
Then, the $\lambda$ is substituted into Eq. (17) to determine the correction factors.

4.3. Result of the volume correction method
A single buoyant bubble rising in an initially quiescent fluid is simulated. The density ratio is set at 20.0, and Morton and Eötvös numbers for the outer fluid are $6.25 \times 10^{-3}$ and 27.1, respectively. The computational domain is rectangular in shape, of size $3.2D_b \times 3.2D_b \times 6.4D_b$ in the two horizontal and vertical directions, where $D_b$ is the initial bubble diameter. A uniform staggered grid with a grid size $\Delta$ is used to resolve the domain.
The boundary conditions are free-slip walls in the horizontal directions and periodic condition in the vertical direction. A single spherical bubble is initially placed near the centre-bottom of the domain. The grid resolution $D_0/\Delta$ is 20 and the time step size is $\Delta t = 2.0 \times 10^{-3}$ (not $\Delta t_0$), which is set at about 1/10 of the viscous time scale based on the grid size. The period of the volume correction $\tau$ is set at 10$\Delta t_0$. Hereafter, $(\Delta t, \tau/\Delta t) = (\Delta t_0, 10)$ is set to the standard case in the present study.

Figure 3(a) shows the instantaneous velocity field around the bubble in a vertical plane cutting through the bubble centroid at steady state ($t = 20$). The gravity vector points downwards in the figure. The bubble Reynolds number at steady state is 228.7, and a spherical-cap bubble is typically observed in this regime (Clift et al., 1978). The bubble shape at steady state ($t = 20$) is compared with the initial spherical shape in Fig. 3(b).

Figure 4(a) compares the time histories of the error in the volume for different time step size and correction period as follows: $(\Delta t, \tau/\Delta t) = (\Delta t_0/2, 20)$ and $(2\Delta t_0, 5)$ to keep the same total number of corrections per unit time as the standard case, and $(\Delta t, \tau/\Delta t) = (\Delta t_0, 2)$ and $(\Delta t_0, \infty)$ to investigate the effect of correction frequency. Note that the last case means no volume correction. For the case of $(\Delta t_0, 10)$, the steady volume error $|1 - V/V_0|$ is $7.0 \times 10^{-7}$, while the uncorrected case shows the average error of $1.8 \times 10^{-2}$ in steady state ($t \geq 4$), suggesting a significant conservation property of the proposed method. The other cases in Fig. 4(a) show that smaller time step size attains smaller error level, and, interestingly, the most frequently-corrected case, $(\Delta t_0, 2)$, shows about the same error level as the standard case.

Figure 4(b) plots the time history of the averaged $\alpha_\ell$ ($\ell = 1, \cdots, N_\ell$) for $(\Delta t, \tau/\Delta t) = (\Delta t_0, 10)$ and compares the standard deviations of $\alpha_\ell$ for the above four correction cases. The average and standard deviations are taken for the total number of the nodal points on the bubble surface (changing from about 2500 to 6000, as it rises). The figure shows that the cases of smaller time step size tend to experience smaller standard deviation of $\alpha_\ell$. The frequent correction case, $(\Delta t_0, 2)$, yields even smaller standard deviations throughout the simulation. However, as observed in Fig. 4(a), the reduced deviation level for $(\Delta t_0, 2)$ seems to have an insignificant effect on the steady volume error.

In addition, our preliminary study for a wide range of viscosity and density ratios shows that $\max_{\ell = 1}^{N_\ell} |\alpha_\ell - 1|$ is commonly found to fall between $5.0 \times 10^{-2}$ and $1.0 \times 10^{-2}$ and that $\lambda$ and $|E_\ell| (\ell = 1, \cdots, N_\ell)$ are around $O(10^0)$ and $O(10^{-5})$, respectively.

5. Concluding Remarks

We have presented a method for the conservation of the volume of a fluid particle in dispersed multiphase flows, where the fluid interface is followed using a front-tracking method. The conservation method employs the Lagrange multipliers, and the nodal correction is determined on each front point so that the total amount of corrections (in terms of $L^2$-norm) on the front nodal positions were minimised with the constraints that volume is conserved. The simplified version of the correction method enabled identification of the correction factor.
Fig. 4 Time histories of (a) the error in the bubble volume normalised by the initial volume, \( |V_t/V_0 - 1| \) and (b) the average correction factor \( \alpha_l (l = 1, \ldots, N_b) \) and the standard deviations. In both figures, the tested cases are indicated by the time increment \( \Delta t \) and correction period \( \tau \); \( (\Delta t, \tau/\Delta t) = (\Delta t_0, 10) \) as the standard case, \((\Delta t_0/2, 20)\), \((2\Delta t_0, 5)\) and \((\Delta t_0, 2)\) together with the no correction case, \((\Delta t_0, \infty)\), for the volume error history, where \( \Delta t_0 = 2.0 \times 10^{-3} \). The discrete changes in the volume error for \((\Delta t_0, \infty)\) are caused by the restructuring process (Tryggvason et al., 2011) of the Lagrangian markers on the bubble surface. The inset figure in (a) shows the decrease trend of the steady errors at the rate of \( \Delta t^2 \).

Independently from those of the neighbouring nodes. The study showed that the time step size \( (\Delta t) \) is more influential on the error level than the frequency \( (\tau^{-1}) \) of the volume correction procedure in the studied period of \( 2 \leq \tau/\Delta t \leq 20 \). The typical error level of the volume was of the order of \( 10^{-7} \) for a spherical-cap bubble with little increase in the overall computational time. These efficient and accurate features facilitate the application of this method to more practical systems with a large population of fluid particles of non-spherical geometry.

Acknowledgements

This work is partly supported by Grants-in-Aid (B) No. 17H03174 of the Japan Society for the Promotion of Science (JSPS), and by the Consortium for Advanced Simulation of Light Water Reactors, an Energy Innovation Hub for Modeling and Simulation of Nuclear Reactors under U.S. Department of Energy Contract No. DE-AC05-00OR22725.

References

Bunner, B. and Tryggvason, G., Dynamics of homogeneous bubbly flows: Part 1. Rise velocity and microstructure of the bubbles, J. Fluid Mech., Vol.466 (2002) pp.17-52
Clift, R., Grace, J.R. and Weber, M.E., Bubbles, Drops, and Particles (1978) Academic Press
Cortez, R. and Minion, M., The blob projection method for immersed boundary problems, J. Comput. Phys., Vol.161 (2000) pp.428-453
Fischer, T.M., Stöhr-Liesen, M. and Schmid-Schünehein, H., The red cell as a fluid droplet: Tank tread-like motion of the human erythrocyte membrane in shear flow, Science, Vol.202 (1978) pp.894-896
Gong, X., Gong, Z. and Huang, H., “An immersed boundary method for mass transfer across permeable moving interfaces”, Journal of Computational Physics, Vol.278 (2014) pp.148-168
Jan, Y. and Tryggvason, G., Computational studies of contaminated bubbles, Dynamics of Bubbles and Vortices Near a Free Surface (eds. Sahin, Tryggvason, Schreyer), ASME/AMD series, Vol.119 (1991) pp.59-64
Keller, S.R. and Skalak, R., Motion of a tank-treading ellipsoidal particle in a shear flow, J. Fluid Mech., Vol.120 (1982) pp.27-47
Muradoglu, M. and Tryggvason, G., A front-tracking method for computation of interfacial flows with soluble surfactants, J. Comput. Phys., Vol.227 (2008) pp.2238-2262
Peskin, C.S., Flow patterns around heart valves: A numerical method, J. Comput. Phys., Vol.10 (1972) pp.252-271

[DOI: 10.1299/mel.20-00216]
Peskin, C.S., The immersed boundary method, Acta Numerica, Vol.11 (2003) pp.479–517
Peskin, C.S. and Printz B.F., Improved volume conservation in the computation of flows with immersed boundaries, J. Comput. Phys., Vol.105 (1993) pp.33-46
Rosti, M.E., Brandt, L., Mitra, D., Rheology of suspensions of viscoelastic spheres: Deformability as an effective volume fraction, Physical Review Fluids, Vol.3 (2018) No.012301
Shin, S. and Juric D., Modeling three-dimensional multiphase flow using a level contour reconstruction method for front tracking without connectivity, J. Comput. Phys., Vol.180 (2002) pp. 427-470
Skotheim, J.M. and Secomb, T.W., Red blood cells and other nonspherical capsules in shear flow: Oscillatory dynamics and the tank-treading-to-tumbling transition, Phy. Rev. Let., Vol.98 (2007) No.078301
Sugiyama, K., II, S., Takeuchi, S., Takagi, S. and Matsumoto, Y., A full Eulerian finite difference approach for solving fluid-structure coupling problems, J. Comput. Phys., Vol.230 (2011) pp.596-627
Takeuchi, S., Yuki, Y., Ueyama, A. and Kajishima, T., A conservative momentum exchange algorithm for interaction problem between fluid and deformable particles, Int. J. Numer. Methods Fluids, Vol.64, Issue 10-12 (2010) pp.1084-1101
Takeuchi, S., Fukuoka, H., Gu, J. and Kajishima, T., Interaction problem between fluid and membrane by a consistent direct discretisation approach, J. Comput. Phys., Vol.371 (2018) pp.1018-1042
Tasoglu, S., Demirci, U. and Muradoglu, M., The effect of soluble surfactant on the transient motion of a buoyancy-driven bubble, Phys. Fluids, Vol.20 040805 (2008)
Torres, D.J. and Brackbill, J.U. The point-set method: front-tracking without connectivity, J. Comput. Phys., Vol.165 (2000) pp.620-644
Tryggvason, G., Bunner, B., Esmaeeli, A., Juric, D., Al-Rawahi, N., Tauber, W., Han, J., Nas, S., and Jan, Y.-J., A front tracking method for the computations of multiphase flow, J. Comput. Phys., Vol.169 (2001) pp.708-759
Tryggvason, G., Scardovelli, R. and Zaleski, S., Direct Numerical Simulations of Gas-Liquid Multiphase Flows, Cambridge University Press, 2011
Unverdi, S.O. and Tryggvason, G., A front-tracking method for viscous, incompressible, multi-fluid flows, J. Comput. Phys., Vol.100 (1992) pp.25-37
Yamamoto, Y., Yamauchi, M. and Uemura, T., Numerical simulation of a droplet taking the effect of surfactant transport on the interface by front-tracking method, Trans. JSME Ser.B, Vol.72-720 (2006) pp.1913-1919
Wang, C., Wang, X. and Zhang, L., Connectivity-free front tracking method for multiphase flows with free surfaces, J. Comput. Phys., Vol.241 (2013) pp. 58-75

[DOI: 10.1299/mel.20-00216] © 2020 The Japan Society of Mechanical Engineers