THERMODYNAMICS OF BLACK HOLES IN PRESENCE OF STRING
INSTANTONS

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Abstract. The coupling of a Nambu-Goto string to gravity allows for Schwarzschild black holes whose entropy to area relation is $S = (A/4)(1 - 4\mu)$, where $\mu$ is the string tension. The departure from $A/4$ universality results from a string instanton which leads to a materialisation of the horizon at the quantum level.

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1 Thermodynamics

The possibility of departing from $A/4$ universality can be understood in thermodynamic terms. Consider the differential mass formula \ref{eq:1} for black holes surrounded by static matter in the form

\begin{equation}
\delta M_{\text{tot}} = \frac{\kappa}{2\pi} \frac{\delta A}{4} + \sum_i \partial_{\lambda_i} H_{\text{matter}} \delta \lambda_i,
\end{equation}

where $M_{\text{tot}}$ is the total mass, $\kappa$ the surface gravity of the hole, $A$ the area of the event horizon and the $\lambda_i$ are all the parameters in the matter action. It is important to realize that the derivation of this identity involves only classical physics and cannot involve the Planck constant. If one admits the Bekenstein assumption \ref{eq:3} that the black hole contributes to the entropy one can interpret Eq. \ref{eq:1} as the expression of the first principle of thermodynamics, $\partial_{\lambda_i} H_{\text{matter}}$ being the generalized forces, by identifying $A/4$ to the entropy up to a multiplicative constant proportional to $\hbar^{-1}$. The temperature will then be proportional to $\hbar$.

It is well known that to compute thermal correlation functions and partition functions in field theory in flat Minkowski spacetime one can use path integrals in periodic imaginary time. The period $\beta$ is the inverse temperature and can be chosen freely. This method was generalized to compute matter correlation functions in static curved backgrounds. For the Schwarzschild black hole, possibly surrounded by matter, the analytic continuation to imaginary time defines a Euclidean background everywhere except at the analytic continuation of the horizon, namely the 2-sphere at $r = 2M$. Gibbons and Hawking \ref{eq:4} extended the analytic continuation to the gravitational action, restricting the hitherto ill-defined path integral over metrics to a saddle point in the Euclidean section. To constitute such a saddle the Euclidean black hole must be regular given that a singularity at $r = 2M$ would invalidate the solution of the Euclidean Einstein equations. This implies a unique Hawking temperature $T_H$ which, in natural units, is always equal to $\kappa/(2\pi)$. Thus Eq. \ref{eq:1} yields the Bekenstein-Hawking area entropy $S$ for the black hole

\begin{equation}
S = \frac{A}{4}.
\end{equation}

The entropy Eq. \ref{eq:2} is not affected by mass surrounding the black hole and would therefore seem to depend only on the black hole mass. This is not the case: a different relation between entropy and area arises when a conical singularity is generated in the Euclidean section \ref{eq:5} at $r = 2M$.

As pointed out by many authors, a conical singularity at $r = 2M$ modifies the Euclidean periodicity of the black hole and hence its temperature. If this singularity would arise from a source term in the Euclidean Einstein equations, Eq. \ref{eq:1} would remain valid and could be written as

\begin{equation}
\delta M_{\text{tot}} = T\delta[(1 - \eta)\frac{A}{4}] + T\frac{A}{4} \delta \eta + \sum_i \partial_{\lambda_i} H_{\text{matter}} \delta \lambda_i,
\end{equation}

for the Schwarzschild black hole, possibly surrounded by matter.
where $\eta$ is the deficit angle and the temperature $T$ is related to the Hawking value $T_H$ by $T = T_H(1 - \eta)^{-1}$. It follows from Eq. (3) that a new generalized force $X_\eta$, conjugate to $\eta$,

$$X_\eta = T \frac{A}{4}$$

must appear and that the entropy of the hole would become

$$S = (1 - \eta) \frac{A}{4},$$

independent of the surrounding matter.

We now show that a deficit angle can be generated by a string instanton and that Eqs. (4) and (5) obtain with $\eta$ determined by the string tension.

## 2 The String Instanton

For simplicity we describe here only the case of a pure black hole of mass $M$. The general case is treated elsewhere [6]. In presence of a Nambu-Goto string the Euclidean action is

$$I = -\frac{1}{16\pi} \int_M \sqrt{g} R + \frac{1}{8\pi} \int_{\partial M} \sqrt{h} K - \frac{1}{8\pi} \int_{(\partial M)_\infty} \sqrt{h_0} K_0 + \mu \int \sigma d^2 \sqrt{\gamma}.$$  (6)

Here $\mu$ is the string tension and $\gamma$ determinant of the induced metric on the world sheet. The latter is taken to have the topology of a 2-sphere. The variation of this action with respect to the metric gives the Einstein equations and the variations with respect to the string coordinates in $\gamma$ give rise to the stationary area condition for the string.

The Einstein equations still admit ordinary black hole solutions corresponding to zero string area. The Euclidean space is regular at $r = 2M$ and the $t$-periodicity is the inverse Hawking temperature [7]

$$\beta_H = 8\pi M.$$  (7)

However there exists a non-trivial solution to the string equations of motion in Euclidean space when the string wraps around the Euclidean continuation of the horizon, a sphere at $r = 2M$. This solution has a curvature singularity at $r = 2M$. Expressing the curvature in the trace of Einstein equations as the product of the horizon times a two dimensional curvature and using the the Gauss-Bonnet theorem for disc topology tell us that there is a conical singularity with deficit angle $2\pi \eta$ such that

$$\eta = 4\mu.$$  (8)
This deficit angle is the sole effect of the string instanton. It raised the temperature from $\beta_H^{-1}$ to $\beta^{-1} = \beta_H^{-1}/(1 - 4\mu)$. 

We now evaluate the free energy of the black hole. The contribution of the string term to the action Eq. (6) exactly cancels the contribution of the Einstein term. The only contributions comes from the boundary terms and one gets

$$F(\beta, \mu) = \beta^{-1}I_{saddle} = \frac{M}{2} = \frac{\beta}{16\pi(1 - 4\mu)}.$$  

From Eq. (9) and the thermodynamic relations $S = \beta^2(\partial F/\partial \beta)_\mu$, $X_4\mu = (\partial F/\partial 4\mu)_\beta$, one recovers Eqs. (4) and (5) with $\eta = 4\mu$.

3 Horizon Materialization at the Quantum Level

The string instanton at $r = 2M$ in Euclidean space does not alter the classical Lorentzian black hole background which remains regular on the horizon. However dramatic effects occur at the quantum level. To illustrate these we consider the toy model consisting of the s-wave component of a free scalar field propagating on the Schwarzschild geometry and we neglect the residual relativistic potential barrier. This amounts to consider a 2-dimensional scalar field propagating on the radial subspace of the 4-geometry. One can then compute the expectation value of the energy-momentum tensor of the scalar field using the trace anomaly [8] and the boundary conditions defined by the temperature $T = T_H(1 - 4\mu)^{-1}$. Using the Kruskal light-like coordinates $(U, V)$ one gets in the vicinity of the horizons

$$\langle T_{UU} \rangle = \frac{\mu}{6\pi U^2} \frac{1 - 2\mu}{(1 - 4\mu)^2}; \quad \langle T_{VV} \rangle = \frac{\mu}{6\pi V^2} \frac{1 - 2\mu}{(1 - 4\mu)^2}.$$  

Thus, the string instanton induces, in an inertial frame, a singularity in the vacuum expectation value of the scalar field energy-momentum tensor on the horizon.

In the original derivation of the Hawking radiation from local field theory [7], the global temperature describing this local equilibrium has the Hawking value Eq. (7) and no singularity appears on the horizon. However if unitarity is to be preserved, as originally suggested by ’t Hooft [9], some kind of materialization appears unavoidable [9, 10]. The fact that the non local effect of the string instanton does both alter the entropy and induce at the quantum level a materialization of the horizon (which is singular in absence of back reaction) points towards the possibility of a retrieval of information through non local effects.
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