Diverse Temporal Properties of GRB Afterglow

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ABSTRACT

The detection of delayed X-ray, optical and radio emission, "afterglow", associated with γ-ray bursts (GRBs) is consistent with fireball models, where the emission are produced by relativistic expanding blast wave, driven by expanding fireball at cosmogical distances. The emission mechanisms of GRB afterglow have been discussed by many authors and synchrotron radiation is believed to be the main mechanism. The observations show that the optical light curves of two observed gamma-ray bursts, GRB970228 and GRB970508, can be described by a simple power law, which seems to support the synchrotron radiation explanation. However, here we shall show that under some circumstances, the inverse Compton scattering (ICS) may play an important role in emission spectrum and this may influence the temporal properties of GRB afterglow. We expect that the light curves of GRB afterglow may consist of multi-components, which depends on the fireball parameters.

Subject headings: gamma-rays: bursts — radiation mechanisms: non-thermal
1. Introduction

The origin of $\gamma$-ray bursts has been one of the great unsolved mysteries in high energy astrophysics for about 30 years. The recent discovery of fading sources at X-ray and optical wavelengths coincident with the locations of some $\gamma$-ray bursts thus provides a good opportunity to probe the nature of these high energy events (Costa et al. 1997, van Paradijs et al. 1997, Bond et al. 1997). The detection of absorption lines in the optical afterglow of GRB970508 provides the first direct estimate of source distance, constraining the redshift of GRB970508 to the range of $0.8 \leq z \leq 2.3$ (Metzger et al. 1997).

The observed properties of GRB afterglow are broadly consistent with models based on relativistic blast waves at cosmological distances (Mészáros & Rees 1997, Vietri 1997, Waxman 1997a, 1997b, Wijers Rees & Mészáros 1997, Wei & Lu 1997). In fireball models of GRB afterglow, the huge energy released by an explosion ($\sim 10^{52}$ ergs) is converted into kinetic energy of a shell expanding at ultra-relativistic speed. After the main GRB event occurred, the fireball continues to propagate into the surrounding gas, driving an ultra-relativistic shock into the ambient medium. The expanding shock continuously heats fresh gas and accelerates relativistic electrons to very high energy, which produce the delayed emission on timescale of days to months.

There are only two $\gamma$-ray bursts, GRB970508 and GRB970228, related with which afterglows in optical band have been observed. In the fireball model, it is widely believed that afterglow is produced through synchrotron radiation of relativistic electrons, and the effect of inverse Compton scattering can be neglected, which seems to be supported by the observations that the light curves of above two gamma-ray bursts can be described by a simple power law (Galama et al. 1997). However, here we show that under some circumstances, the ICS may have important effect on the emission spectrum and this will affect the light curve of GRB afterglow.
In next section, we use two different methods to calculate the specific intensity of ICS. One is to calculate the emission spectrum directly, the other is to estimate the relative importance of ICS and synchrotron radiation by comparing the energy density of magnetic field and synchrotron radiation. Of course, these two methods give almost the same result. In section 3, we discuss the effect of ICS on the temporal behavior of GRB afterglow, we find that, in general cases, the light curve may be diverse. Finally some discussions and conclusions are given in section 4.

2. The intensity of inverse Compton scattering

The process of inverse Compton scattering has also been discussed by several authors (e.g. Waxman 1997a), but they have not considered the influence of ICS on GRB afterglow, and they think ICS to be not important in GRB afterglow. Here we present a detailed calculation of the intensity of ICS using two different methods.

First, we calculate the emission spectrum of ICS directly. Assuming that after shock acceleration, the electrons have a power law energy distribution

\[ N(\gamma) = k e^{-p} (\gamma_{\text{min}} \leq \gamma \leq \gamma_{\text{max}}), \]

then the photon spectrum of ICS can be expressed as

\[ \frac{dN}{dt dE} = \frac{3}{8} \sigma_T c k_e f(p) E^{-(p+1)/2} \int \epsilon^{(p-1)/2} n(\epsilon) d\epsilon \] (1)

where \( \sigma_T \) is the Thomson cross section, \( f(p) = 2^{p+3} \frac{p^2+4p+11}{(p+3)^2(p+1)(p+5)} \), and \( c \) is the speed of light. We assume that the soft photon spectrum also obeys a power law, \( n(\epsilon) = n_0 \epsilon^{-\alpha} (\epsilon_1 \leq \epsilon \leq \epsilon_2) \). Note that, in our case, the soft photons are produced by synchrotron radiation from these same electrons, so \( \alpha = (p+1)/2 \). Then from eq.(1) we have

\[ \frac{dN}{dt dE} = \frac{3}{8} \sigma_T c k_e f(p) n_0 E^{-(p+1)/2} \ln \left( \frac{\epsilon_2}{\epsilon_1} \right) \] (2)
In the fireball model, the electron density in the comoving frame is \( n_e = \int N(\gamma) d\gamma = \Gamma n_1 \), where \( n_1 \) is the density of surrounding gas and \( \Gamma \) is the bulk Lorentz factor, then we have \( k_e = (p - 1) \Gamma n_1 \gamma_{\text{min}}^{p-1} \). Thus we can obtain the ratio of the specific intensity of ICS at peak energy \( (E = \epsilon_n = \gamma_{\text{min}}^2 \epsilon_m) \) to the synchrotron radiation at peak energy \( (E = \epsilon_m) \)

\[
y = \frac{I_{\text{ICS}}}{I_{\text{syn}}} = \frac{3}{8} \sigma_T (p - 1) n_1 f(p) r \ln(\frac{\epsilon_2}{\epsilon_1}) \tag{3}
\]

where \( \ln(\frac{\epsilon_2}{\epsilon_1}) = 2 \ln(\gamma_{\text{max}} / \gamma_{\text{min}}) \). The minimum and maximum electron Lorentz factors can be estimated by physical processes. For a power law energy distribution, the average electron Lorentz factor \( \bar{\gamma} \sim \gamma_{\text{min}} \) for \( p > 2 \), and it is widely believed that after the shock acceleration, the average electron energy should be \( \bar{\gamma} \sim \xi (m_p / m_e) \Gamma \), where \( m_p (m_e) \) is the mass of proton (electron). The maximum electron energy may be limited by shock acceleration and radiative energy loss. It has been shown that \( \gamma_{\text{max}} \sim 5 \times 10^7 B^{-1/2} \) (Cheng & Wei 1996), where \( B \) is the magnetic field strength, which is usually written as \( B = (\xi_B 8 \pi \Gamma^2 n_1 m_p c^2)^{1/2} \), so \( \ln(\gamma_{\text{max}} / \gamma_{\text{min}}) \sim 10 \). In addition, after the shock acceleration the spectral index of electron distribution \( p \) is typically between 2 and 3 (e.g. Blandford & Eichler 1987), therefore we see that \( y \simeq 10^{-24} n_1 r \).

There is another way to estimate the intensity of ICS, i.e. to calculate the ratio of the synchrotron radiation energy density \( (u_{\text{syn}}) \) to the magnetic energy density \( (u_B) \), \( R = u_{\text{syn}} / u_B \), \( u_B = B^2 / 8 \pi \) and \( u_{\text{syn}} \sim n_e P_{\text{syn}} r / \Gamma c \), where \( P_{\text{syn}} \) is the synchrotron radiation power. This method is simpler and more important, since the value of \( R \) indicates which process is more efficient for electron energy loss, inverse Compton scattering or synchrotron radiation, so it is necessary to calculate the value of \( R \) carefully. It is easy to show that \( R \simeq 10^{-24} \gamma^2 n_1 r \). In the afterglow model, if the fireball expands outward adiabatically, we have

\[
r / r_0 = (t/t_0)^{1/4} \quad \Gamma / \Gamma_0 = (t/t_0)^{-3/8} \tag{4}
\]

where the deceleration radius \( r_0 = r_d = 5 \times 10^{16} (E_{51} / n_1)^{1/3} (\Gamma_0 / 100)^{-2/3} \) (cm), and the
characteristic time \( t_0 \approx \frac{r_0}{2\Gamma_0^2 c} \approx 10^2 (E_{51}/n_1)^{1/3} (\Gamma_0/100)^{-8/3} (s) \) \cite{Meszaros&Rees1997} with \( E_{51} \) being the burst energy in units of \( 10^{51} \) ergs and \( \Gamma_0 \) the initial Lorentz factor. Then we obtain

\[
R = 17 \xi_e^2 n_1^{1/2} E_{51}^{1/2} t_{\text{day}}^{-1/2}
\]

(5)

Obviously, the emission power of ICS is not much smaller than that of synchrotron radiation even for several days after the GRB event, thus the effect of ICS should not be neglected. The ratio of intensities of ICS to synchrotron radiation at peak energy is \( I_{\text{ICS}}/I_{\text{syn}} \sim R \bar{\gamma}^{-2} \sim 10^{-24} n_1 r \), which is in agreement with the previous result.

3. The effect of ICS on GRB afterglow

In the case where the effect of ICS cannot be neglected, the emission spectrum should consist of two components. There exists a critical energy \( \epsilon_c \), below which the spectrum is dominated by synchrotron radiation and above which the spectrum is dominated by ICS, our purpose is to calculate the value of \( \epsilon_c \).

We assume that, in the comoving frame, the synchrotron radiation intensity has the form \( I_\epsilon \propto \epsilon^{-\alpha} \) for \( \epsilon < \epsilon_m \) and \( I_\epsilon \propto \epsilon^{-\beta} \) for \( \epsilon > \epsilon_m \). Since in our situation the soft photons produced through synchrotron radiation are scattered by the same electrons, so the Compton scattered spectrum should have nearly the same form as that of synchrotron radiation, i.e. \( I_\epsilon \propto \epsilon^{-\alpha} \) for \( \epsilon < \epsilon_n \) and \( I_\epsilon \propto \epsilon^{-\beta} \) for \( \epsilon > \epsilon_n \), therefore the total intensity is \( I_\epsilon \propto \epsilon^{-\alpha} \) for \( \epsilon < \epsilon_m \) or \( \epsilon_c < \epsilon < \epsilon_n \), and \( I_\epsilon \propto \epsilon^{-\beta} \) for \( \epsilon_m < \epsilon < \epsilon_c \) or \( \epsilon > \epsilon_n \). Then from eq.(4) and the relation \( I_{\epsilon_m} (\frac{\epsilon_{\epsilon_m}}{\epsilon_{\epsilon}})^{-\beta} = I_{\epsilon_n} (\frac{\epsilon_{\epsilon_n}}{\epsilon_{\epsilon}})^{-\alpha} \) we can obtain the value of \( \epsilon_c \)

\[
\epsilon_c = f(\alpha, \beta) \xi_B^{1/2} \xi_e^{2 - \frac{2\alpha}{3 - \alpha}} \frac{1}{n_1^{3 - \alpha}} \frac{1}{4^{(3 - \alpha)/2}} \left[ \frac{1 + \alpha}{4^{\beta - \alpha}} \right]^{1/2} \left[ \frac{1 - 3\alpha}{4^{\beta - \alpha}} \right]^{1/2} E_{51}^{1/2} t_{\text{day}}^{-3/8} eV
\]

(6)

where \( f(\alpha, \beta) = 20 \times 17^{1 - \frac{1}{3 - \alpha}} 10^{6(1 - \alpha)/3 - \alpha} 8^{2(1 - \alpha)/3 - \alpha} \).

It has been shown that, in the adiabatic case, the electron Lorentz factor \( \gamma_e \propto t^{-3/8} \),
the typical energy \( \epsilon_m \propto t^{-3/2} \), and the comoving specific intensity of synchrotron radiation at peak energy is \( I'_{\epsilon_m} \propto t^{-1/8} \) (Mészáros & Rees 1997). From the relation \( I_{ICS}/I_{syn} \sim R\gamma^{-2} \) it is easy to show that the intensity of ICS at peak energy \( I'_{\epsilon_m} \propto t^{1/8} \) and \( \epsilon_n \propto t^{-9/4} \), then the observed peak flux \( F_{\epsilon_m} \propto t^{2\gamma^5}I'_{\epsilon_m} \propto t^0 \sim \) constant, and \( F_{\epsilon_n} \propto t^{2\gamma^5}I'_{\epsilon_n} \propto t^{1/4} \). Therefore we can conclude that, if our observation is fixed at energy \( \epsilon \), then the observed flux \( F_{\epsilon} \propto F_{\epsilon_m}(\frac{\epsilon}{\epsilon_m})^{-\alpha} \propto t^{-\frac{3}{2}\alpha} \) for \( \epsilon < \epsilon_m \), \( F_{\epsilon} \propto F_{\epsilon_m}(\frac{\epsilon}{\epsilon_m})^{-\beta} \propto t^{-\frac{3}{2}\beta} \) for \( \epsilon_m < \epsilon < \epsilon_c \), \( F_{\epsilon} \propto F_{\epsilon_n}(\frac{\epsilon}{\epsilon_n})^{-\alpha} \propto t^{\frac{1}{4}-\frac{9}{4}\alpha} \) for \( \epsilon_c < \epsilon < \epsilon_n \), and \( F_{\epsilon} \propto F_{\epsilon_n}(\frac{\epsilon}{\epsilon_n})^{-\beta} \propto t^{\frac{1}{4}-\frac{9}{4}\beta} \) for \( \epsilon > \epsilon_n \).

Here the most interesting quantity is the critical energy \( \epsilon_c \), which is dependent on the fireball parameters, i.e. the fireball energy, surrounding gas density, energy fractions in electrons and magnetic field, and the spectral index of synchrotron radiation. In particular, it is easy to show that the effect of inverse Compton scattering is important only for large values of spectral index. As an example, we take \( \alpha = 0.25 \), \( \beta = 1.4 \) (these values are consistent with the observed \( \gamma \)-ray burst spectra), then the value of \( \epsilon_c \)

\[
\epsilon_c = 1.85\left(\frac{\epsilon_{\text{syn}}}{100\text{KeV}}\right)^{4}\left(\frac{\gamma_0}{600}\right)^{0.43}n_{1}^{-1.1}E_5^{0.23}\left(\frac{t}{6\text{day}}\right)^{-1.55}\text{eV}
\]  

(7)

where we have used the relation \( \epsilon_{\text{syn}} = 5.2 \times 10^{-3}\gamma_0^{1/2}n_1^{1/2}\xi_e eV \), which is the peak energy of \( \gamma \)-ray burst spectrum. So we can see that, for the typical values of GRB events, i.e. \( E \sim 10^{51}\text{ergs}, n \sim 1\text{cm}^{-3}, \xi_e \sim 0.3 \), the critical energy \( \epsilon_c \) crosses the optical band for about six days after the burst. Furthermore, we can calculate the peak energy of ICS

\[
\epsilon_n = 1.87\left(\frac{\epsilon_{\text{syn}}}{100\text{KeV}}\right)^{4}\left(\frac{\gamma_0}{600}\right)^{0.43}\left(\frac{\xi_e}{0.1}\right)n_{1}^{-3/4}E_5^{3/4}\left(\frac{t}{60\text{day}}\right)^{9/4}\text{eV}
\]

(8)

Obviously, if we take the same parameters as above, then \( \epsilon_n \) should cross the optical band for about two months, so we expect that the time during which the optical flux varies as \( F \propto t^{\frac{1}{4}-\frac{9}{4}\alpha} \) rather than \( F \propto t^{-\frac{3}{2}\beta} \), and according to the values of \( \alpha \) and \( \beta \), the brightness may decrease very slowly than before.
4. Discussion and conclusion

The detection of $\gamma$-ray burst in the optical and radio bands has greatly furthered our understanding these objects, especially the shape of the light curves of GRB afterglow provide important information on exploring their emission mechanisms. Here we present a detailed calculation to show that the temporal properties of GRB afterglow may be quite diverse due to the effect of inverse Compton scattering.

We have shown that the inverse Compton scattering may play an important role in the GRB afterglow. From eq.(5) one sees that $R \propto t^{-1/2}$, it seems that ICS is more efficient than synchrotron radiation in the early time of GRB afterglow (especially in $\gamma$-ray burst epoch, it seems that ICS dominates the electron energy loss), however, in fact this is not true, since in the early time, the photon energy of synchrotron radiation is so high that in the electron rest frame the photon energy is greatly larger than $m_e c^2$, so the Compton scattering occurs in the extreme Klein-Nishina limit, and thus the scattering is not efficient. It can easily be shown that Compton scattering is efficient only when about $t > 10^3$ s after GRB event.

Here we point out that the light curves of GRB afterglow may consist of four components, which depend on the characteristic energies $\epsilon_m$, $\epsilon_c$, $\epsilon_n$ and the detector frequency $\epsilon$. However, it should be noted that not all the GRB afterglows could contain so many components, the necessary condition for having four components is $\epsilon_c < \epsilon_n$. It is obvious that if $\epsilon_c > \epsilon_n$, the light curves should have three components. In addition, if the ICS efficiency is low enough that there is no intersection between the spectrum of ICS and synchrotron radiation, then the light curves only have two components.

It should be noted that the effect of inverse Compton scattering is strongly dependent on the model parameters, especially on the spectral index of synchrotron radiation, the contribution from inverse Compton scattering is important only for those bursts with large
spectral indices, and the spectral indices may be estimated by the ratio of X-ray flux to the optical flux.

The observations from BATSE show that the spectra of GRBs are rather diverse, varying from burst to burst. For most bursts, their spectra can be described by a broken power law, and the distribution of the spectral indices below and above the peak energy is rather wide. In general, below peak energy the spectral indices ($\alpha$) are mainly between 0 and 0.5, while above peak energy the slopes ($\beta$) are typically between 1 and 1.5 (Band et al. 1993), so we expect that the temporal properties of GRB afterglows (which strongly depend on the parameters $\alpha$ and $\beta$) would be very diverse, and contain multi-components.

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