A Production fuzzy Inventory model for deteriorating items Defuzzified by Graded Mean Integration Representation Method

S. K. INDRAJITSINGHA¹, P.N. SAMANTA² and U.K. MISRA³

¹DST INSPIRE fellow, P.G. Department of Mathematics, Berhampur University, Berhampur, Bhanja Bihar-760 007, Odisha (India)
²P.G. Department of Mathematics, Berhampur University, Berhampur, Bhanja Bihar-760 007, Odisha (India)
³Department of Mathematics, NIST, Golanthara, PalurHill, Pin-761 008, Berhampur, Odisha, (India)
Email Address of Corresponding Author: ski.rs.math@buodisha.edu.in
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Abstract

This paper develops a production inventory model for deteriorating items under fuzzy environment. Since holding cost, set-up cost, deterioration cost, deterioration rate, production rate, demand etc. are uncertain in nature; these are carried by pentagonal fuzzy numbers. Graded Mean Integration Representation (GMIR) method is used to defuzzify the total cost function. Numerical example is given to explore the theoretical results and make the comprehensive. Sensitivity analysis with different parameters on the optimal solution is carried to illustrate the effectiveness and behavior of the model.

Key words: Inventory Model, Pentagonal Fuzzy Number, Graded Mean Integration Representation Method, Defuzzification

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1. Introduction

Inventory means physical stock of goods. An inventory fulfills many important functions within the organization. In manufacturing and business operations, the inventory problems are common factors. Harris¹, developed the first inventory model in 1915 taking very few parameters. But, in real life inventory, there are many such parameters like demand, deterioration rate, holding cost, production rate, set-up cost etc. which are uncertain. For solving this type of randomness, researchers traditionally applied probability theory (Covert and
However, in certain situations, uncertainties are due to fuzziness, and such cases are dilated in the fuzzy set theory which was developed by Zadeh in 1965. After the invention of fuzzy set theory, again in 1970, Zadeh and Bellman developed a mathematical model on decision making in fuzzy environment. Hence most of researchers attracted towards the theory of fuzzy sets. Dubois and Prade define some operations on fuzzy numbers. In 1991, Kaufmann and Gupta provided an introduction to fuzzy arithmetic operations and Zimmerman discussed the concept of the fuzzy set theory and its applications.

Some researchers started to apply fuzzy set theory in inventory management problems. Kacprzyk and Staniewski proposed a model on long-term inventory policy-making through fuzzy-decision making model. In 1987, K.S Park developed an economic order quantity model using fuzzy set theory. Yao and Lee proposed a fuzzy inventory model with backorder for fuzzy order quantity. In 1996, Vujosevic et. al. developed an EOQ formula when inventory cost is fuzzy. In 1997, Gen et. al. introduced a fuzzy inventory control model. Chang applied fuzzy triangular number in production inventory model. Syed and Aziz in 2007 applied signed distance method to defuzzify the inventory model without shortage. In 2011, De and Rawat developed a fuzzy inventory model without shortages using triangular fuzzy number. In 2012, Jaggi et. al. explored a fuzzy inventory model for deteriorating items with time-varying demand and shortages in which parameters are treated as triangular fuzzy number. They defuzzified this model by different defuzzification methods. In 2013, Dutta and Kumar developed a fuzzy inventory model for deteriorating items with shortages under fully backlogged condition and defuzzified by this method by different defuzzification methods. Singh and Singh proposed an integrated inventory model from the perspective of a single vendor and multi-buyers for deteriorating items under fuzzy environment and inflation. In that development they consider all costs and inflation as trapezoidal fuzzy number and defuzzified by graded mean integration representation method.

A fuzzy inventory model for deteriorating items with price dependent demand rate was developed by Maragatham and Lakshmidevi in 2014. In which they consider demand as trapezoidal fuzzy number and defuzzified by applying signed distance method. In 2014, Ranganathan and Thirunavukarasu discussed an inventory control model for constant deterioration and logarithmic demand rate under fuzzy environment. Where they defuzzified the total cost function by graded mean integration representation method. Nagar and Surana developed an inventory model for deteriorating items with fluctuating demand using inventory parameters as pentagonal fuzzy numbers. They defuzzified that the model by graded mean integration representation method. In 2015, Kumar and Rajput proposed a fuzzy inventory model for deteriorating items with time dependent demand and partial backlogging in which the demand rate, deterioration rate, backlogging rate are assumed as a triangular fuzzy numbers. They defuzzified the total cost function of that model by signed distance method and centroid method. Ranganathan and Thirunavukarasu formulated a fuzzy inventory model under immediate return for deficient items in which they used triangular fuzzy number. Mishra et. al. proposed an inventory control model of deteriorating items where the deteriorating rate, deteriorating cost, carrying cost and shortages are taken as trapezoidal fuzzy numbers. They defuzzified the model by using graded mean integration representation method. Indrajitsingha et. al. studied a fuzzy economic production quantity model with time dependent demand rate in which demand cost and holding cost are taken as triangular fuzzy numbers and defuzzified the total cost function by using different defuzzification methods.

In 2016, Indrajitsingha et. al. extended a fuzzy inventory model with shortages under fully backlogged in where they defuzzified the total cost function by using signed distance method. Raula et. al. proposed a fuzzy inventory model for constant deteriorating items by using graded mean integration representation method.
in which inventory parameters treated as hexagonal fuzzy numbers. Indrajitsingha et al.\textsuperscript{9} developed a fuzzy economic production quantity model with time dependent demand rate in which the parameters treated as pentagonal fuzzy numbers. They defuzzified the total cost functions by using signed distance method. In 2016, Sahoo et al.\textsuperscript{24} introduced an inventory model with exponential demand and time-varying deterioration in fuzzy approach. Where they used trapezoidal fuzzy numbers as holding cost, deterioration and purchase cost and total cost function was defuzzified by using graded mean integration representation method. Sen et al.\textsuperscript{25} considered a fuzzy inventory model for deteriorating items based on different defuzzification techniques in which parameters treated as triangular fuzzy numbers. Recently, Sahoo et al.\textsuperscript{23} developed a fuzzy inventory model with time dependent demand rate without shortages using pentagonal fuzzy number.

2. Objective and Organizations:

In the present model the cycle length $T$ is divided in to two parts. In the period $(0, T_1)$, the reduction item going on with a constant rate and due to the demand the inventory decreases. In the period $(T_1, T)$, the production is stopped with the same demand rate and the demand decreases to zero. We assumed that the demand rate is constant in both periods. In this paper we developed a fuzzy inventory model for deteriorating items in which all parameters are treated as pentagonal fuzzy number. Single inventory is used. The total cost function is defuzzified by using the graded mean integration representation (GMIR) method.

This paper is organized as follows: In sect. 3, definitions and preliminaries are given. In sect. 4, assumptions and notations of the proposed model are given. Mathematical model in Crisp and Fuzzy sense is formulated in sect. 5. Numerical example is illustrated in sect. 6 to support the proposed model. Sensitivity analysis is carried out by using Matlab R2011b software in sect. 7 followed by conclusion and future of scope.

3. Definitions and Preliminaries:

In order to establish the model we require the following definitions:

**Definition 3.1 (Fuzzy Set)** Let $X$ be a space of points with a generic element $x$ of $X$. Let $\mu: X \rightarrow [0,1]$ be such that for every $x \in X$, $\mu(x)$ is a real number in the interval $[0,1]$, usually called ‘grade of membership’.

We define a fuzzy set $\tilde{A}$ in $X$ as the set of points $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$.

**Definition 3.2** A fuzzy number $\tilde{A} = (a, b, c, d, e)$ where $a < b < c < d < e$ and defined on $R$, is called pentagonal fuzzy number if its membership function is

$$
\mu_{\tilde{A}} = \begin{cases} 
L_1(x) = \frac{x-a}{b-a}, & a \leq x \leq b \\
L_2(x) = \frac{x-b}{c-b}, & b \leq x \leq c \\
1, & x = c \\
R_1(x) = \frac{d-x}{d-c}, & c \leq x \leq d \\
R_2(x) = \frac{e-x}{e-d}, & d \leq x \leq e \\
0, & \text{Otherwise}
\end{cases}
$$

$\alpha$-cut of Pentagonal Fuzzy Number:
Definition 3.3: If \( \tilde{A} = (a, b, c, d, e) \) is a pentagonal fuzzy number then the graded mean representation method of \( \tilde{A} \) is defined as

\[
P(\tilde{A}) = \frac{\int_{0}^{w_{A}} \alpha \left( \frac{L^{-1}(\alpha) + R^{-1}(\alpha)}{2} \right) d\alpha}{\int_{0}^{w_{A}} \alpha d\alpha}
\]

with \( 0 < \alpha \leq w_{A} \) and \( 0 < w_{A} \leq 1 \)

\[
P(\tilde{A}) = \frac{1}{12} [a + 3b + 4c + 3d + e]
\]

4. Assumptions and Notations:

Throughout the manuscripts, we make use of the following assumptions:

i. Single inventory will be used.

ii. Items are produced and added to the inventory.

iii. The lead time is zero.

iv. No shortages are allowed.

v. Replenishment is instantaneous.

vi. Time horizon is finite.

vii. The production rate is proportional to demand rate.

viii. The production rate is always greater than demand rate.

ix. There is no repair of deteriorated items occurring during the cycle.

x. Neglecting the higher power of \( \theta \).

Notations:

\( S_{c} = \) Set-up cost

\( \theta = \) Deterioration rate independent of time, \( 0 < \theta \leq 1 \)

\( T = \) Cycle length
5. Mathematical Formulation:

The status of inventory is shown in Fig. 2 as follows

A machine starts with a constant rate of production and stopped at time $T_1$, and initially, inventory level increases up to time $T_1$ with a constant rate $P_r$. When the production stopped; the inventory level decreases and reduces to zero due to the combined effect of demand and the deterioration. Hence the rate of change of inventory is governed by the differential equations represented as following:

$$\frac{dl_1(t)}{dt} = P_r - \{D + \theta I_1(t)\}, \quad 0 \leq t \leq T_1$$
and
\[
\frac{dl_2(t)}{dt} = -\{D + \theta I_2(t)\}, \quad T_1 \leq t \leq T
\]

Solution of (5.1) and (5.2) with the condition is
\[
I_1(t) = \frac{1}{\theta} (P_r - D)(1 - e^{-\theta t})
\]
and
\[
I_2(t) = \frac{D}{\theta} \left( e^{\theta(T-t)} - 1 \right)
\]

Now, we find $T_1$ by using
\[
I_1(T_1) = I_2(T_1)
\]
\[
\frac{1}{\theta} (P_r - D)(1 - e^{-\theta t}) = \frac{D}{\theta} \left( e^{\theta(T-t)} - 1 \right)
\]
\[
T_1 = \frac{1}{\theta} \log \left\{ 1 + \frac{D}{P_r} \left( e^{\theta T} - 1 \right) \right\}
\]

The total cost is calculated by considering set-up cost, holding cost and deterioration cost.

1. Set-up cost $= S_c$

2. Holding cost per cycle $= h_c \int_0^{T_1} I_1(t) dt + \int_{T_1}^{T} I_2(t) dt$
   \[
   = \frac{h_c}{\theta} [P_r T_1 - DT]
   \]

3. Deterioration cost per cycle $= d_c \int_0^{T_1} \theta I_1(t) dt + \int_{T_1}^{T} \theta I_2(t) dt$
   \[
   = d_c \left( P_r T_1 - DT \right)
   \]

Total cost of the system per unit time is given by
\[
T_c = \frac{1}{T} \left[ \text{Set up cost} + \text{Holding cost} + \text{Deterioration cost} \right]
\]
\[
= \frac{S_c}{T} + \frac{(P_r T_1 - DT)}{\theta T} (h_c + d_c \theta)
\]

Using (5.5) and the assumption (x) we get
\[
= \frac{1}{T} \left[ S_c + \frac{1}{2} (h_c + d_c \theta) DT^2 - \frac{1}{2} (h_c + d_c \theta) \frac{D^2 T^2}{2} \right]
\]

**Fuzzy Model:**

We consider the model in fuzzy approach. Due to vagueness, it is not easy to define all the parameters precisely. Accordingly we assume the parameters $S_c, h_c, d_c, \theta, P_r$ and $D$ in fuzzy environment.

Suppose $\tilde{S}_c = (S_{c1}, S_{c2}, S_{c3}, S_{c4}, S_{c5}), \tilde{h}_c = (h_{c1}, h_{c2}, h_{c3}, h_{c4}, h_{c5}), \tilde{d}_c = (d_{c1}, d_{c2}, d_{c3}, d_{c4}, d_{c5}), \tilde{\theta} = (\theta_{c1}, \theta_{c2}, \theta_{c3}, \theta_{c4}, \theta_{c5}), \tilde{P}_r = (P_{r1}, P_{r2}, P_{r3}, P_{r4}, P_{r5})$ and $\tilde{D} = (D_{1}, D_{2}, D_{3}, D_{4}, D_{5})$ are as pentagonal fuzzy number.
Total cost of the system per unit time in fuzzy environment is given by
\[ (5.7) \; \tilde{T}_c = \frac{1}{T} \left[ S_c + \frac{1}{2} (\tilde{h}_c + \tilde{d}_c \tilde{\theta}) \tilde{D} T^2 - \frac{1}{2} (\tilde{h}_c + \tilde{d}_c \tilde{\theta}) \tilde{D}^2 T^2 \right] \]

We defuzzify the fuzzy total cost \( \tilde{T}_c \) by Graded Mean Integration Representation Method. Thus the defuzzified value of \( \tilde{T}_c \) is
\[ \tilde{T}_{cg} = \frac{1}{12} \left[ \tilde{T}_{c1g1} + 3 \tilde{T}_{c1g2} + 4 \tilde{T}_{c1g3} + 3 \tilde{T}_{c1g4} + \tilde{T}_{c1g5} \right] \]

Where
\[ \tilde{T}_{cigi} = \frac{1}{T} \left[ S_i + \frac{1}{2} (h_i + d_i \theta_i) D_i T^2 - \frac{1}{2} (h_i + d_i \theta_i) D_i^2 T^2 \right], \text{for } i = 1, 2, 3, 4, 5 \]

To minimize the total cost function per unit time \( \tilde{T}_{cg} \), the optimum value of \( T \) can be obtained by solving the differential equation
\[ \frac{d\tilde{T}_{cg}}{dT} = 0, \; \frac{d^2\tilde{T}_{cg}}{dT^2} > 0 \]

i.e.
\[ \frac{d\tilde{T}_{cg}}{dT} = \frac{1}{12} \left[ \frac{d}{dt} \tilde{T}_{c1g1} + \frac{3}{dt} \tilde{T}_{c1g2} + \frac{4}{dt} \tilde{T}_{c1g3} + \frac{3}{dt} \tilde{T}_{c1g4} + \frac{d}{dt} \tilde{T}_{c1g5} \right] = 0 \]

Now
\[ \frac{d\tilde{T}_{cigi}}{dT} = - \frac{S_i}{T^2} + \left[ \frac{1}{2} (h_i + d_i \theta_i) D_i - \frac{1}{2} (h_i + d_i \theta_i) \frac{D_i^2}{P_i} \right], \text{for } i = 1, 2, 3, 4, 5 \]

Thus
\[ \frac{d\tilde{T}_{cg}}{dT} = \frac{1}{12} \left\{ - \frac{S_1 + 3S_2 + 4S_3 + 3S_4 + S_5}{T^2} + \left[ \frac{1}{2} (h_1 + d_1 \theta_1) D_1 + \frac{3}{2} (h_2 + d_2 \theta_2) D_2 \right] \right\} + \left[ \frac{1}{2} (h_1 + d_1 \theta_1) \frac{D_1^2}{P_1} + \frac{3}{2} (h_2 + d_2 \theta_2) \frac{D_2^2}{P_2} + \frac{3}{2} (h_3 + d_3 \theta_3) \frac{D_3^2}{P_3} \right] \]

Since \( \frac{d\tilde{T}_{cg}}{dT} = 0 \), we get
\[ T^2 = \frac{2(S_1 + 3S_2 + 4S_3 + 3S_4 + S_5)}{\left( h_1 + d_1 \theta_1 \right) D_1 \left( 1 - \frac{D_1}{P_1} \right) + 3(h_2 + d_2 \theta_2) D_2 \left( 1 - \frac{D_2}{P_2} \right) + 4(h_3 + d_3 \theta_3) D_3 \left( 1 - \frac{D_3}{P_3} \right) + 3(h_4 + d_4 \theta_4) D_4 \left( 1 - \frac{D_4}{P_4} \right) + (h_5 + d_5 \theta_5) D_5 \left( 1 - \frac{D_5}{P_5} \right) } \]
For optimum value, we have to show that \( \frac{d^2 T_{cg}}{dT^2} > 0 \).

Now
\[
\frac{d^2 T_{cg}}{dT^2} = \frac{1}{12} \left[ \frac{d^2 T_{c1} \text{g}1}{dT^2} + 3 \frac{d^2 T_{c2} \text{g}2}{dT^2} + 4 \frac{d^2 T_{c3} \text{g}3}{dT^2} + 3 \frac{d^2 T_{c4} \text{g}4}{dT^2} + \frac{d^2 T_{c5} \text{g}5}{dT^2} \right] \\
= \frac{1}{6 T^2} [S_1 + 3S_2 + 4S_3 + 3S_4 + S_5] 
\]

Take \( \frac{D_1}{P_1} < 1, \frac{D_2}{P_2} < 1, \frac{D_3}{P_3} < 1, \frac{D_4}{P_4} < 1 \) and \( \frac{D_5}{P_5} < 1 \), then in each case, \( T \) exists. Then for each value of \( T \)
we can see \( \frac{d^2 T_{cg}}{dT^2} > 0 \). Hence we got defuzzified value of total cost function in graded mean integration
representation (GMIR) method i.e. \( dT_{cg} \) is minimum.

6. Numerical Example:
First, we represent the case of vague value as the type of pentagonal fuzzy number. Consider the inventory system with the following parametric values. Suppose \( \bar{S}_c = (50, 52, 54, 56, 58) \),
\( \bar{h}_c = (6, 7, 8, 9, 10) \), \( \bar{\theta} = (0.006, 0.008, 0.010, 0.012, 0.014) \), \( \bar{P}_r = (500, 520, 540, 560, 580) \),
\( \bar{D} = (450, 470, 490, 510, 530) \), \( \bar{d} = (1.1, 1.2, 1.3, 1.4, 1.5) \) are all pentagonal fuzzy number and
We solve this by using Matlab R2011b software. The solution of fuzzy total cost \( \bar{T}_{cg} = 198.1923 \) with cycle
time \( T = 0.5449 \)

7. Sensitivity analysis :
A sensitivity analysis is carried out to study the effect of changes in parameters \( \bar{S}_c, \bar{h}_c, \bar{\theta}, \bar{P}_r, \bar{D} \) and \( \bar{d}_c \).
We use Matlab R2011b software for calculation of the total cost in different defuzzification methods and
plotting the graphs. In this analysis we change the value of a specific parameter keeping all other parameter
remains constant.

| \( \bar{S}_c \)   | Time(Yrs) | Total Cost     |
|------------------|-----------|----------------|
| (46, 48, 50, 52, 54) | 0.5244    | 190.7106       |
| (48, 50, 52, 54, 56) | 0.5347    | 194.4874       |
| (50, 52, 54, 56, 58) | 0.5449    | 198.1923       |
| (55, 57, 59, 61, 63) | 0.5696    | 207.1647       |
| (60, 62, 64, 66, 68) | 0.5932    | 215.7644       |

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Table 2. Sensitivity analysis for $\tilde{h}_c$

| $\tilde{h}_c$ | Time (Yrs) | Total Cost |
|--------------|------------|------------|
| (4, 5, 6, 7, 8) | 0.6290    | 171.7026   |
| (5, 6, 7, 8, 9) | 0.5825    | 185.4211   |
| (6, 7, 8, 9, 10) | 0.5449    | 198.1923   |
| (7, 8, 9, 10, 11) | 0.5138    | 210.1889   |
| (8, 9, 10, 11, 12) | 0.4875    | 221.5368   |

Table 3. Sensitivity analysis for $\tilde{\theta}$

| $\tilde{\theta}$ | Time (Yrs) | Total Cost |
|------------------|------------|------------|
| (0.002, 0.004, 0.006, 0.008, 0.01) | 0.5451    | 198.1280   |
| (0.004, 0.006, 0.008, 0.01, 0.012) | 0.5450    | 198.1601   |
| (0.006, 0.008, 0.01, 0.012, 0.014) | 0.5449    | 198.1923   |
| (0.008, 0.01, 0.012, 0.014, 0.016) | 0.5448    | 198.2244   |
| (0.01, 0.012, 0.014, 0.016, 0.018) | 0.5447    | 198.2565   |

Table 4. Sensitivity analysis for $\tilde{d}_c$

| $\tilde{d}_c$ | Time (Yrs) | Total Cost |
|--------------|------------|------------|
| (0.9, 1.0, 1.1, 1.2, 1.3) | 0.5450    | 198.1675   |
| (1.0, 1.1, 1.2, 1.3, 1.4) | 0.5450    | 198.1799   |
| (1.1, 1.2, 1.3, 1.4, 1.5) | 0.5449    | 198.1923   |
| (1.2, 1.3, 1.4, 1.5, 1.6) | 0.5449    | 198.2046   |
| (1.3, 1.4, 1.5, 1.6, 1.7) | 0.5449    | 198.2107   |

Table 5. Sensitivity analysis for $\tilde{P}_r$

| $\tilde{P}_r$ | Time (Yrs) | Total Cost |
|--------------|------------|------------|
| (460, 480, 500, 520, 540) | 1.1727    | 92.0972    |
| (480, 500, 520, 540, 560) | 0.6904    | 156.4317   |
| (500, 520, 540, 560, 580) | 0.5449    | 198.1923   |
| (520, 540, 560, 580, 600) | 0.4690    | 230.2952   |
| (540, 560, 580, 600, 620) | 0.4209    | 256.5242   |

Table 6. Sensitivity analysis for $\tilde{D}$

| $\tilde{D}$ | Time (Yrs) | Total Cost |
|--------------|------------|------------|
| (430, 450, 470, 490, 510) | 0.4702    | 229.6892   |
| (440, 460, 480, 500, 520) | 0.5026    | 214.8914   |
| (450, 470, 490, 510, 530) | 0.5449    | 198.1923   |
| (460, 480, 500, 520, 540) | 0.6031    | 179.0607   |
| (480, 500, 520, 540, 560) | 0.6896    | 131.5267   |
Fig. 3. Sensitivity analysis for Set-up cost

Fig. 4. Sensitivity analysis for Holding cost

Fig. 5. Sensitivity analysis for Deterioration rate

Fig. 6. Sensitivity analysis for Deterioration cost

Fig. 7. Sensitivity analysis for Production rate

Fig. 8. Sensitivity analysis for Demand rate
All the above observations can be sum up as follows:

i) In table 1 & fig. 3 Shows that, with increase of the value of the parameter of $\bar{S}_c$, keeping all other parameters unchanged, the total cost $\bar{T}_{cg}$ increases with the increase of cycle time $T$.

ii) In table 2, fig. 4, Shows that, with increase of the value of the parameter of $\bar{h}_c$, keeping all other parameters unchanged, the total cost $\bar{T}_{cg}$ increases with the decrease of cycle time $T$.

iii) In table 3, fig. 5, Shows that, with increase of the value of the parameter of $\bar{\delta}$, keeping all other parameters unchanged, the total cost $\bar{T}_{cg}$ increases slowly with the decrease of cycle time $T$.

iv) In table 4, fig. 6, Shows that, with increase of the value of the parameter of $\bar{a}_c$, keeping all other parameters unchanged, the total cost $\bar{T}_{cg}$ increases slowly with the decrease of cycle time $T$.

v) In table 5, fig. 7, Shows that, with increase of the value of the parameter of $\bar{P}_c$, keeping all other parameters unchanged, the total cost $\bar{T}_{cg}$ increases with the decrease of cycle time $T$.

vi) In table 6, fig. 8, Shows that, with increase of the value of the parameter of $\bar{D}$, keeping all other parameters unchanged, the total cost $\bar{T}_{cg}$ decreases with the increase of cycle time $T$.

8. Conclusion and Scope:

In this paper we developed a production fuzzy inventory model of deteriorating items. This model has been developed for single item without shortages. The set-up cost, holding cost, deterioration cost, deteriorating rate, production rate and demand are represented by pentagonal fuzzy numbers. The optimum result of fuzzy model is defuzzified by using graded mean integration representation method. Numerical example is also provided to illustrate the proposed model. As per the Sensitivity analysis the observation narrated in section 7. Due to fuzzy model, we did not get any absurd value. So the decision maker can plan for apply this model to get the optimum value of total cost, and for other related parameters, after analyzing the result. The problem is open. The result can be compared by taking different fuzzy number and defuzzifying in other methods. This can be further developed for different demand also.

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