Deliberations on 11D Superspace for the M-Theory Effective Action

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ABSTRACT

In relation to the superspace modifications of 11D supergeometry required to describe the M-theory low-energy effective action, we present an analysis of infinitesimal supergravity fluctuations about the flat superspace limit. Our investigation confirms Howe’s interpretation of our previous Bianchi identity analysis. However, the analysis also shows that should 11D supergravity obey the rules of other off-shell supergravity theories, the complete M-theory corrections will necessarily excite our previously anticipated spin-1/2 engineering dimension-1/2 spinor auxiliary multiplet superfield. The analysis of fluctuations yields more evidence that Howe’s 1997 theorem is specious when applied to Poincaré supergravity or 11D supergravity/M-theory. We end by commenting upon recent advances in this area.

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1 Introduction

During the middle eighties, some of our research initiated a new direction of study for superspace geometry, i.e., the problem of how to describe higher derivative supergravity theories via the use of on-shell superspace techniques. Prior to our work, this area had received no attention in the physics literature. We proposed that since low-energy superstring effective actions naturally contain a dimensionful parameter $\alpha'$ (in our discussions we used a symbol $\gamma'$ which is proportional to $\alpha'$) it should be expected that supergeometries, associated with superstring and heterotic string effective actions, should themselves become expansions in this dimensionful parameter. Thus was introduced into the literature the notion of the “slope parameter expansion” of supergeometries, with the work of providing a proof-in-principle that this technique could be viable. This proof-in-principle, however, was restricted to the dual formulation of 10D, $N = 1$ supergravity. Our early investigations of the dual theory also marked the initial discovery of type I closed string/type I fivebrane duality by showing that under a duality transformation, the stringy higher mode corrections and the stringy quantum loop corrections are exchanged.

It was not until later work wherein the concept of the “Lorentz gaugino” was introduced, before we were able to extend our proof-in-principle to the actual Chern-Simons related corrections of the low-energy heterotic string effective action. This was possible because the “Lorentz gaugino” maps problems involving 10D supergravity into analogous and often simpler problems involving 10D super Yang-Mills theory. The second of these works also revealed that the “Lorentz gaugino” takes its simplest form (i.e., only proportional to the gravitino curl) when the superspace constraints utilized correspond to what is presently called the “string frame.” The discovery of these special superspace constraints was made well before this terminology existed, so we simply referred to these as “improved supergeometries” and showed that they also existed to type-II theories. The concept of the Lorentz gaugino has also played a key role in component discussions of higher derivative supergravity. A perusal of the initial work by Bergshoeff and de Roo shows that after the discovery of this concept in superspace, these researchers utilized it to elevate the level of understanding of higher curvature supergravity at the component level.

Finally in the early nineties, we returned to this class of problems by showing, in some detail, how to modify 11D supergravity based on a 1980 conjecture. We proposed that this suggestion would be critical for discussions of the 11D supergeometry appropriate to encode information about the low-energy effective action of M-theory. In particular, our assertion implied the presence of a dimension one-half and spin-1/2 multiplet of currents. In fact, we suggested that 4D, $N = 2$ supergravity and/or 4D, $N = 1$ nonminimal supergravity was likely to provide the best paradigms to follow in building the 11D superspace theory. We shall argue in the following that this is precisely what has happened in the period since our work.

Shortly after the introduction of our modified 11D supergeometry, there appeared a paper by Howe. A “theorem” was proposed that implied the equations of motion of 11D supergravity/M-theory follow solely from constraints on (engineering) dimension zero superspace torsions. A misunderstanding of our proposal began his
criticism of our 1996 work by stating, “We conclude with some brief comments on a recent paper [9] by Nishino and Gates in which it is claimed that an off-shell extension of eleven-dimensional supergravity can be constructed in superspace involving dimension one-half superfields.” The penultimate paragraph in our work, however, had stated our actual position, “For although we believe our observation is important, we know of at least two arguments that suggest that there must exist at least one other tensor superfield that will be required to have a completely off-shell formalism.”

Therefore we never proposed that “…an off-shell extension of eleven-dimensional supergravity can be constructed in superspace involving a dimension one-half superfield.” We clearly stated that our $J_\alpha$ tensor was to be regarded as only part of an off-shell theory. We referred to an 11D analog of the non-minimal 4D, $N = 1$ theory [11] and containing an “eleven dimensional analog of the $G_\alpha^a$” as being the most likely candidate for 11D supergravity/M-theory modified supergeometry.

In the following, we present an analysis based on the study of infinitesimal 11D supergravity fluctuations about a flat background that allows us to clarify our position. This analysis will show that; (a.) our 1996 embedding of $J_\alpha$ in the torsion supertensors was incorrect, (b.) permits us to find the correct embedding of $J_\alpha$ into the $d = 1/2$ torsion tensors, (c.) provides us with yet another basis for claiming that the best model for off-shell 11D supergravity is the non-minimal 4D, $N = 1$ theory and (d.) provides a proof invalidating Howe’s 1997 theorem.

Our result will also allow us to suggest modifications to the “X-tensor” approach given recently [12]. Parts of these field strength superfields are shown to be purely “conventional constraints” [11, 13] whose vanishing simply implies the algebraic elimination of parts of the linearized 11D vielbein. We also will give the first explicit supergravity/M-theory currents in both the $J$-tensor and $X$-tensor approaches.

2 The 11D Vielbein: $43 \cdot 2^{37}$ Degrees of Freedom

In order to better understand 11D supergravity, we believe that it is useful to look “behind the geometrical curtain” that has been used in all previous discussions. A step toward this is introducing the following spinorial 11D vielbein parametrization

$$E_\alpha = \Psi^{1/2} \left\{ \exp \left( \frac{i}{2} \Delta \right) \right\}_\alpha^\beta \left( D_\beta + H_\beta^m \partial_m \right) \equiv \Psi^{1/2} N_\alpha^\beta \left( D_\beta + H_\beta^m \partial_m \right),$$

$$\Delta_\alpha^\beta \equiv \left[ i \Psi^a \gamma_a + \frac{1}{2} \Psi^{ab} \gamma_{ab} + i \frac{1}{6} \Psi^{[3]} \gamma_{[3]} + \frac{1}{24} \Psi^{[4]} \gamma_{[4]} + i \frac{1}{120} \Psi^{[5]} \gamma_{[5]} \right]_\alpha^\beta. \quad (1)$$

The factor $N_\alpha^\beta$ above is an element of the SL(32, $\mathbb{R}$) group since $\det (N_\alpha^\beta) = 1$ and $(N_\alpha^\beta)^\dagger = N_\alpha^\beta$. The superfields $\Psi$, $\Psi^a$ through $\Psi^{[5]}$ are forms with respect to the 11D tangent Lorentz space which allows us to easily count the number of their independent components.

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4 We have italicized parts in this paragraph to emphasize what was actually written.

5 Although we believe this is natural parametrization that can be applied to supergravity in other dimensions, this has not appeared to our knowledge previously as applied to 10D and 11D supergravity theories.
The form (but not super-form) superfields can be called holonomy $p$-forms because they define the most general holonomy group acting solely on the spinorial frame. The remaining superfield entering the vielbein is $H_{\alpha m}$ which plays the analogous role to conformal prepotentials in lower dimension superspace supergravity theories.

While the component 11D vielbein $e_{\alpha m}$ contains only 121 bosonic d.f. the component 3-form contains only 165 bosonic d.f. and the component gravitino $\psi_{\alpha}^a$ contains 356 fermionic d.f., the total number of component d.f. in $E_{\alpha M}$ is astronomical with $43 \cdot 2^{36}$ bosonic d.f. and the same number of fermionic d.f.! These are distributed according to the following table,

| Superfield | Bosonic $\cdot$ d.f. | Fermionic $\cdot$ d.f. |
|------------|---------------------|-----------------------|
| $\Psi^a$   | $11 \cdot d_S$      | $11 \cdot d_S$        |
| $\Psi^{[2]}$ | $55 \cdot d_S$      | $55 \cdot d_S$        |
| $\Psi^{[3]}$ | $165 \cdot d_S$     | $165 \cdot d_S$       |
| $\Psi^{[4]}$ | $330 \cdot d_S$     | $330 \cdot d_S$       |
| $\Psi^{[5]}$ | $462 \cdot d_S$     | $462 \cdot d_S$       |
| $H_{\beta m}$ | $352 \cdot d_S$     | $352 \cdot d_S$       |
| $E_{\alpha}$ | $1,376 \cdot d_S$   | $1,376 \cdot d_S$     |

Table 2: Degrees of Freedom in Semi-prepotentials

where $d_S = 2^{31} = 2,147,483,648$.

Whatever choice of parametrization of the spinorial vielbein is made, following the experience of all previous constructed solutions to superspace supergravity constraints, we may define the vectorial frame through the equation

$$E_\alpha \equiv \frac{1}{42} \left( \gamma_\alpha \right)^\alpha_\beta \left[ E_\alpha, E_\beta \right].$$

(2)

This is not just a convenience. Without enforcing such a constraint, the superspace formulation does not contain a unique graviton. Following the exact same arguments that were first given many, many years ago [14], if there are absolutely no constraints

\footnote{d.f. \equiv degrees of freedom}
imposed on the superspace frame fields then there exist a Wess-Zumino gauge in which

\[ E_\alpha^m(\theta, x) = \frac{1}{2} \theta^\beta \left[ i(\gamma^k)_{\alpha \beta} e_k^m(x) + (\gamma^{kl})_{\alpha \beta} f_{kl}^m(x) + i (\gamma^{[5]})_{\alpha \beta} f_{[5]}^m(x) \right] + (\text{higher order } \theta\text{-terms}), \]

\[ E_a^m(\theta, x) = \hat{e}_a^m(x) + (\text{higher order } \theta\text{-terms}), \]  

(3)

and there are seen to be two a priori independent fields, \( e_a^m(x) \) and \( \hat{e}_a^m(x) \), whose transformation laws make both candidates to be identified with the usual component graviton. It is precisely the role of the condition in (2) to insure that these are, in fact, the same field. This is clearly an important physical requirement since all conventional component formulations possess a unique graviton.

It has been known since the early eighties how to write the solution to conditions such as that appearing in (2). Firstly, one notices that the quantities defined by

\[ \hat{E}_\alpha \equiv D_\alpha + H_\alpha d \partial_d, \]

\[ \hat{E}_a \equiv \partial_a + i \frac{1}{16} (\gamma_a)^{\alpha \beta} \left[ D_\alpha H_\beta^d + H_\alpha^c \partial_d H_\beta^c \right] \partial_d \equiv \partial_a + H_a^d \partial_d \equiv \hat{E}_a^m \partial_m, \]  

(4)

imply the equations,

\[ \left[ \hat{E}_\alpha, \hat{E}_\beta \right] = \left[ i (\gamma_\alpha)_{\alpha \beta} + \hat{C}_{\alpha \beta} \right] \hat{E}_c, \quad (\gamma_\alpha)^{\alpha \beta} \hat{C}_{\alpha \beta} = 0, \]  

(5)

where an explicit expression for the purely imaginary quantity \( \hat{C}_{\alpha \beta} \) is given by

\[ \hat{C}_{\alpha \beta} = \left[ \delta_{(\alpha} \gamma \delta_{\beta)}^\delta + \frac{1}{16} (\gamma^\alpha)_{\alpha \beta} (\gamma_\delta)_{\gamma \delta} \right] \left[ D_\gamma H_\delta^m + H_\gamma^b \partial_b H_\delta^m \right] \hat{E}_m^c. \]  

(6)

The quantity \( \hat{E}_m^c \) is the inverse to the superfield \( \hat{E}_a^m \) that appears in (4). The general calculation of the commutator algebra of \( \hat{E}_A \) leads to the remaining relations

\[ \left[ \hat{E}_\alpha, \hat{E}_b \right] = \hat{C}_{\alpha b} \hat{E}_c, \quad \left[ \hat{E}_a, \hat{E}_b \right] = \hat{C}_{a b} \hat{E}_c, \]  

(7)

and it is seen that only the vectorial frame appears on the right hand sides of these equations as also is the case in (4). Explicitly we find

\[ \hat{C}_{a b}^c = ( D_a H_b^m + H_a^e \partial_e H_b^m - \partial_b H_a^m - H_b^e \partial_e H_a^m ) \hat{E}_m^c, \]

\[ \hat{C}_{a b}^c = ( \partial_a H_b^m + H_a^e \partial_e H_b^m - \partial_b H_a^m - H_b^e \partial_e H_a^m ) \hat{E}_m^c. \]  

(8)

It is also an interesting fact that

\[ \text{s det}(\hat{E}_A^M) = \text{det}(\hat{E}_a^m) = \text{det}(\delta_a^m + H_a^m). \]  

(9)

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\[ ^7 \text{We use “superspace conjugation” in order to define reality properties. This permits factors of } i \text{ to appear consistently even though our supercoordinates } (\theta^a, x^m) \text{ for 11D superspace are real. A recent pedagogical discussion of this operation can be found in [15].} \]
The set of superframes $\hat{E}_A^M$ are not the ones we require, however. Comparing (1) with the first line in (4) we see

$$E_\alpha = \Psi^{1/2} N_\alpha^\gamma \hat{E}_\gamma,$$

(10)

and this can be substituted into (3) to derive

$$E_\alpha = \Psi N_a^b \hat{E}_b + i \frac{1}{2} (\gamma_\alpha)^{\alpha\beta} (E_\alpha \ln \Psi) E_\beta + i \frac{1}{16} (\gamma_\alpha)^{\alpha\beta} [E_\alpha N_\beta^\delta] (N^{-1})_\delta^\gamma E_\gamma,$$

(11)

where the real quantity $(N^{-1})_{ab}$ is the matrix inverse to $N_a^b$ defined by

$$N_a^b = -\frac{1}{32} (\gamma_\alpha)^{\alpha\beta} N_a^\gamma N_\beta^\delta [ (\gamma_b)^{\gamma\delta} - i \hat{C}_{\gamma\delta}^b ].$$

(12)

The last factor in (11) has an interesting group theoretical interpretation. If we denote the exterior differential by $d$, then the quantity $\Omega_{\alpha\beta}$ defined by

$$\Omega_{\alpha\beta} \equiv [dN_\alpha^\gamma] (N^{-1})_{\gamma\beta},$$

(13)

with $\Omega_\alpha^\alpha = 0$ is the right-invariant Maurer-Cartan form of $SL(32,\mathbb{R})$ satisfying the condition $d\Omega = \Omega \wedge \Omega$.

The commutator algebra of the set of superframes $E_A^M$ is considerably more complicated than that of the set of superframes $\hat{E}_A^M$. Direct calculation reveals

$$[E_\alpha, E_\beta] = C_{\alpha\beta}^\gamma E_\gamma + i [ (\gamma^c)_{\alpha\beta} - i C_{\alpha\beta}^c ] E_c,$$

(14)

where the full spinor-spinor anholonomy coefficients take the forms

$$C_{\alpha\beta}^c = i \frac{1}{64} \left[ (\gamma^2)_{\alpha\beta} (\gamma^2)^{\kappa\lambda} - \frac{1}{60} (\gamma^5)_{\alpha\beta} (\gamma^5)^{\kappa\lambda} \right] F_{\kappa\lambda}^c,$$

(15)

$$C_{\alpha\beta}^\gamma = i \frac{1}{64} \left[ (\gamma^2)_{\alpha\beta} (\gamma^2)^{\kappa\gamma} - \frac{1}{60} (\gamma^5)_{\alpha\beta} (\gamma^5)^{\kappa\gamma} \right] (E_\kappa \ln \Psi) + \frac{1}{2048} \left[ (\gamma^2)_{\alpha\beta} (\gamma^2)^{\kappa\lambda} - \frac{1}{60} (\gamma^5)_{\alpha\beta} (\gamma^5)^{\kappa\lambda} \right] F_{\kappa\lambda}^c (\gamma_\epsilon)^{\kappa\gamma} (E_\kappa \ln \Psi) + \frac{1}{32} \left[ (\gamma^2)_{\alpha\beta} (\gamma^2)^{\kappa\lambda} - \frac{1}{60} (\gamma^5)_{\alpha\beta} (\gamma^5)^{\kappa\lambda} \right] (E_\kappa N_\delta^\gamma) (N^{-1})_d^\gamma + \frac{1}{3072} \left[ (\gamma^2)_{\alpha\beta} (\gamma^2)^{\kappa\lambda} - \frac{1}{60} (\gamma^5)_{\alpha\beta} (\gamma^5)^{\kappa\lambda} \right] (E_\kappa N_\delta^\gamma) (N^{-1})_d^\gamma,$$

(16)

and the real quantity $F_{\kappa\lambda}^c$ is defined according to

$$F_{\kappa\lambda}^c \equiv N_\kappa^\gamma N_\lambda^\delta [ (\gamma^d)_{\gamma\delta} - i \hat{C}_{\gamma\delta}^d ] (N^{-1})_d^c.$$  

(17)

Although the complete analogs of all the results in (4) can be explicitly calculated, we will defer to some future date such a presentation. The most important point of the non-linear analysis completed above is that it proves that the condition in (2) has a solution in terms of $\Psi_{[p]}$ and $H_{a}^{\mu\nu}$ such that no restrictions are placed on these superfields in spite of the fact that (2) implies

$$(\gamma_b)^{\alpha\beta} C_{\alpha\beta}^c = (\gamma_b)^{\alpha\beta} C_{\alpha\beta}^\gamma = 0.$$  

(18)
Finally, due to the definition (2) and its solution discussed above we see
\[
\text{sdet} \left( E_A^M \right) = \Psi^{-5} \text{det} \left( N_c^b \right) \text{det} \left( \hat{E}_a^m \right) .
\] (19)

A special gauge choice of the expressions in (1) and (2) corresponds to 11D supergeometries which are conformally related to flat 11D geometry superspace
\[
E_{\alpha} = \Psi^{1/2} D_{\alpha} , \quad E_a = \Psi \partial_a + i \frac{1}{16} (\gamma_a)^{\alpha \beta} \Psi^{1/2} \left( D_{\alpha} \Psi^{1/2} \right) D_{\beta} .
\] (20)

If we vary the Ψ-superfield in (20) we find,
\[
\delta S_{E_{\alpha}} = \frac{1}{2} L E_{\alpha} , \quad \delta S_{E_a} = L E_a + i \frac{1}{32} (\gamma_a)^{\alpha \beta} (E_a L) E_{\beta} .
\] (21)
where \( L \equiv (\Psi^{-1} \delta \Psi) \). Interestingly enough, this same result is also obtained by varying Ψ in (10) and (11). We are thus led to additional conclusions. All the remaining superfields (Ψ, · · · , Ψ[5] and H_β^m) are superscale invariant and (21) represents a minimal 11D superspace scale transformation law.

Let us use the second of the results in (20) to take a “peek” at some component fields in this conformal gauge. Taking the limit as \( \theta \rightarrow 0 \), we find
\[
e_{a}^{m}(x) = (\Psi | \delta_{a}^{m} , \quad \psi_{\alpha}^{\beta}(x) = i \frac{1}{32} (\gamma_a)^{\alpha \beta} (D_{\alpha} \Psi |) .
\] (22)

Taking the determinant of both side of the first of these and the “gamma” trace of the second implies yields
\[
\Psi | = \left[ \text{det}(e_{a}^{m}) \right]^{1/11} , \quad D_{\alpha} \Psi | = i \frac{32}{11} (\gamma_a)^{\alpha \beta} \psi_{\alpha}^{\beta}(x) .
\] (23)

These equations show that within this gauge the determinant of the component 11D vielbein and the “gamma-trace” of the component 11D gravitino can reside in the single superfield Ψ. The superfield Ψ has been often called the conformal compensator. We emphasize that all known off-shell constructions of supergravity possess this superfield. Of course, since we have imposed no spinorial differential constraints upon Ψ, there are \( 2^{31} - 1 \) addition bosonic d.f. and \( 2^{31} - 32 \) additional fermionic d.f. that accompanying these fields.

### 3 11D Linearized Vielbein Semi-Prepotential Analysis: Anholonomy Coefficients

In the limit of infinitesimal superfields in (1), we “shift” Ψ according to Ψ \( \rightarrow 1 + \Psi \) and expand the exponential in (1) to first order to find
\[
E_{\alpha} = D_{\alpha} + \frac{1}{2} \left( \Delta_{\alpha}^{\gamma} + \Psi \delta_{\alpha}^{\gamma} \right) D_{\gamma} + H_{\alpha c} \partial_c .
\] (24)

This form is convenient for considerations of infinitesimal supergravity fluctuations about a flat 11D background. In this case, the semi-prepotential \( \Psi^{[\nu]} \)

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8These are semi-prepotentials because they must be subject to a complete set of (presently unknown) differential constraints. Such constraints are discussed later in this paper. The true prepotentials are the solutions to the totality of such differential equations.
and $H^a_m$ are the appropriate variables to consider for a linearized analysis of 11D superfield supergravity. In this linearized theory, the vectorial vielbein defined by (2) and (24) takes the form,

\[ E_a = \partial_a + i \frac{1}{32} \left[ D_\beta (\gamma_a \Delta)^\beta \gamma + (D_\beta \Psi)(\gamma_a)^\beta \gamma \right] D_\gamma \]

\[ + \left[ i \frac{1}{16} (\gamma_a)^{\alpha \beta} (D_\alpha H^c_\beta) + \delta^c_\alpha \Psi - \Psi^c_\alpha \right] \partial_c . \]

By looking at the final line of this equation, we see that the 0-form and 2-form holonomy superfields are indeed the compensators for scale and Lorentz symmetries, respectively.

We can use the linearized superframes to find the relations between these linearized semi-prepotentials and the “geometrical constraints” to be imposed on the 11D superspace anholonomy. To do this, we next compute the graded commutator algebra of the linearized frames given in (24) and (27). Starting with the algebra of the spinorial frames we find

\[ C_{\alpha \beta} = \frac{1}{64} \left[ (\gamma^2)_{\alpha \beta} (\gamma^2)_{\gamma \delta} - \frac{1}{60} (\gamma^5)_{\alpha \beta} (\gamma^5)_{\gamma \delta} \right] [ D_\alpha \Delta^\epsilon + \delta^\epsilon_\delta (D_\gamma \Psi) ] , \]

\[ C_{\alpha \beta} = i (\gamma^c_{\alpha \beta} - \frac{1}{32} (\gamma^d_{\alpha \beta})_\gamma \delta D_\gamma H^c_\delta + 32 \delta^c_\alpha \Psi e - 16 \Psi^c_{\alpha \beta} ] + i \frac{1}{1920} (\gamma^d_{\alpha \beta})_\gamma \delta (\gamma_{\alpha \beta})_\gamma \delta D_\gamma H^c_\delta + 40 \delta^c_\alpha \Psi_{\alpha \beta} ) \]

\[ - \frac{1}{15} \epsilon_{\alpha \beta \gamma} \Psi \right] , \]

where we have utilized one of the Fierz identities noted in (8) and as well the definition of $\Delta_{\alpha \beta}$ given in (9) in the latter of these two equations. In reaching these results, we see that neither spinorial nor bosonic differential restrictions have been imposed upon any of the superfields that appear in (24).

We continue by calculating the commutator algebra of $E_a$ and $E_b$ which leads to the anholonomy coefficients,

\[ C_{\alpha \beta} = i \frac{1}{32} \left[ D_\alpha D_\beta (\gamma_a \Delta)^\delta \gamma \right] - \frac{i}{2} \partial_\alpha \Delta^\gamma \gamma - \frac{i}{2} (\partial_\beta \Psi) \delta^\alpha \gamma + \frac{i}{32} \left[ D_\alpha D_\beta \Psi \right] (\gamma_a)^\delta \gamma , \]

\[ C_{\alpha \beta} = i \frac{1}{16} (\gamma^b_\beta)_{\alpha \gamma} \left[ D_\alpha D_\beta H^c_\gamma + (\partial_\beta H^c_\gamma) + \frac{1}{32} \left[ D_\beta (\gamma_\beta \Delta \gamma^c) \right]_{\beta \gamma} - D_\alpha \Psi^c_\beta \right] \]

\[ + \left[ (D_\alpha \Psi) \delta^c_\beta - \frac{1}{32} (\gamma^c_{\alpha \gamma} \Psi \right] . \]

Finally, we calculate the commutator algebra of two vectorial frames which leads to the anholonomy coefficients,

\[ C_{\alpha \beta} = i \frac{1}{32} \left[ - D_\beta (\gamma_\beta \partial_\beta \Delta)^\alpha \delta^\gamma \right] , \]

\[ C_{\alpha \beta} = - i \frac{1}{16} (\gamma^a_\alpha)^{\beta \gamma} \left[ \partial_\alpha \Delta \gamma^a_\gamma - (\partial_\beta \Psi^c_\beta) - \delta^c_\beta \Psi \right] . \]

After the spin-connections are added to the dimension 1/2 anholonomy coefficients, we can choose to impose the conditions (see also (9) eq.(16))

\[ T_{\alpha \beta} = \ldots + \frac{1}{2} \delta^\alpha_\gamma \gamma J_\beta + \frac{1}{32} (\gamma^a_\alpha)^{\beta \gamma} \gamma J_\delta , \]

\[ T_{\alpha \beta} = \ldots + \gamma J_\alpha \delta^c_\beta - \frac{1}{32}(\gamma^c_\alpha)^{\alpha \beta} \gamma J_\gamma . \]
This is the set of constraints that should replace our $d = 1/2$ torsion set given in 1996 as a partial off-shell construction. The first explicit term in each equation above was absent in our previous work on partial off-shell superspace.

4 How About Howe’s 1997 Theorem

In his 1997 work, Howe introduced “Weyl superspace,” which necessarily treats the scale transformation covariantly, so that the scale transformations of his vielbein (which we denote by $\tilde{E}_A$ in the following to distinguish it from the usual frames) take the forms,

$$\delta_{S} \tilde{E}_\alpha = \frac{1}{2} L \tilde{E}_\alpha , \quad \delta_{S} \tilde{E}_a = L \tilde{E}_a.$$  \hfill (31)

Thus the first point to note about Howe’s Weyl superspace, is that the vectorial frame superfields (after imposing the condition $T_{\alpha\beta}^c = i(\gamma^c)_{\alpha\beta}$) must contain more degrees of freedom than the minimal vectorial frame in (2). This is confirmed by the observation\(^9\) that another vectorial frame ($\tilde{E}_a$) defined by

$$\tilde{E}_a \equiv \tilde{E}_a + i \frac{1}{2\sqrt{4}} (\gamma_a)^{\alpha\beta} \tilde{C}_{\alpha\beta}^\gamma \tilde{E}_\gamma$$  \hfill (32)

possesses the exact same transformation law as the vectorial frame in (20). The second term here removes the extra degrees of freedom contained in the first. This implies that Howe’s vectorial frame has all the degrees of freedom of his spinorial frame plus additional degrees of freedom contained in a unconstrained spinor superfield. Therefore, his Weyl superspace introduces an additional 64 $d_S$ degrees of freedom compared to the minimal set of frame superfields defined by (1) and (2).

In another part of his paper, a process we have long called “degauging” is carried out. Here, he finds a dimension 1/2 spin-1/2 quantity denoted by $K_\alpha$. However, he ultimately dispenses with this quantity with a remark, “...and recognize that it can be removed by a super-Weyl transformation.” This encapsulates our fundamental disagreement with Howe.

We expect that when the totality of M-theory corrections are found, some of them will necessarily activate a spinorial auxiliary field multiplet of dimension 1/2. Our reason for this is somewhat subtle. In some of our first reports on how string corrections modify low-energy effective actions and consequently superspace geometry \([1,2]\) we pointed out that there are in fact two distinct sources of corrections. One occurs by integrating out the higher massive mode corrections to the zero mass field equations and the other occurs from the actual quantum loop corrections to the theory. We noted that there is a way to distinguish between these two types of corrections. Namely, one type of correction breaks scale invariance and the other does not. We also observed that under duality transformations, the roles of the two types of corrections are exchanged.

Now let us ask what happens in the context of known off-shell formulations of superfield supergravity when the supergravity system is coupled to a non-scale invariant system? The answer is well known, the auxiliary fields within the superspace

\(^{9}\)We thank P. Howe for bringing to our attention the existence of this frame.
scale compensator become “active.” All of our investigation of the 11D fluctuations suggest that 11D supergravity has the usual canonical structure observed previously in other prepotential formulations of supergravity theories.

From component level results, the first non-trivial bosonic correction to 11D supergravity/M-theory is expected to be of the form

$$\mathcal{L}(1) \sim (\ell_{11})^6 \left[ e^{-1} R^4 + F^{(4)} \wedge X_{CS}^{LL} \right] , \quad (33)$$

which is not scale-invariant since it possesses a weight of -3. Above we have introduced a parameter denoted by $\ell_{11}$ of dimensions (mass)$^{-1}$. We are therefore led to the expectation that since $\mathcal{L}(1)$ is scale non-invariant, it will activate the auxiliary fields in the 11D conformal compensator, $\Psi$.

By the same token, scale-invariant (in the sense defined in [1, 2]) corrections will not activate this multiplet and it only then becomes irrelevant. Since the correction above is expected to simply be the lowest order one, there should occur higher order terms also. It is perhaps useful to discuss an example of such a higher order correction that is scale-invariant. One such weight zero Lagrangian is given by

$$\mathcal{L}^* \sim (\ell_{11})^9 e^{-1} F^{(4)} R^5 . \quad (34)$$

Should such an operator occur in the M-theory effective action, it is our conjecture that it cannot activate the spin-1/2 multiplet of currents. This is three orders higher than the lowest order correction. So it will be difficult to check this conjecture.

5 4D, N = 8 ↔ 11D, N = 1 Oxidation/Reduction?

Howe’s critique of our work also ignored something else that we stated in our discussion of the partial off-shell description of 11D supergravity. Namely we were very much aware that other tensors (besides $J_\alpha$) seem required to describe a completely off-shell theory. Our concluding remarks explicitly say that we were aware of at least two arguments for the existence of these other tensors. We think it is now useful to especially and explicitly review one of these arguments.

Many years ago [16] a much overlooked result was derived within the context of 4D, N = 8 supergravity in superspace. A study was made of what (if any) unusual space-time differential constraints would be imposed upon the Weyl tensor by assuming the traditional sets of constraints

$$T_{\alpha i \beta j} = 0 , \quad T_{\alpha i}^{\beta} \gamma = i \delta_{\alpha}^{\gamma} \delta_{\beta}^{\gamma} \delta_{\gamma}^{\beta} \quad , \quad (35)$$

used on lower $N \leq 4$ superspace supergravity theories. In distinction to all lower dimensional theories it was found that these conditions alone were sufficient to impose within the linearized approximation

$$\partial^\alpha \partial^\beta w_{\alpha \beta \gamma \delta} = 0 \quad , \quad (36)$$

\footnote{We warn the reader that we only mean this equation symbolically. In fact, the full uncontracted Riemann curvature tensor appears. The quantity $X_{CS}^{LL}$ is the Lorentz Chern-Simons 7-form.}
as a restriction on the linearized Weyl tensor. This is symptomatic of acausal propagation, so we concluded that to avoid this above the \( N = 4 \) barrier, it would be necessary to replace (35) by equations

\[
T_{\alpha_i \beta_j}^c \neq 0, \quad T_{\alpha_i \beta}^c \cdot J^c \neq i \delta_\alpha^\gamma \delta_\delta^\gamma \delta^i_j.
\] (37)

If we “oxidize” this result to eleven dimensions, it seems to nicely match the results of a more recent work \(^1\)11.

The primary assertion in the work of \(^1\)11 is the notion that all the equations of motion for 11D supergravity/M-theory follow from the condition \( T_{\alpha \beta}^c = i (\gamma^c)_{\alpha \beta} \). This has been referred to as “Howe’s Theorem” (e.g., \(^2\)12, \(^7\)17). Let us, however, return to the result in (27). Using the notation of \(^1\)11, we can write (in our conventions)

\[
T_{\alpha \beta}^c = i (\gamma^c)_{\alpha \beta} + \frac{1}{2} (\gamma^{[2]})_{\alpha \beta} X_{[2]}^c + i \frac{1}{120} (\gamma^{[5]})_{\alpha \beta} X_{[5]}^c.
\] (38)

If we perform a dimensional compactification on a torus, this leads to (37). Thus, it was our expectation that some new field strengths could appear in the dimension zero superspace torsions.

Upon comparing this with (27), it is seen that both real field strengths \( X_{[2]}^c \) and \( X_{[5]}^c \) are dependent algebraically on some of the holonomy \( p \)-forms, but are independent of \( \Psi \) and \( \Psi_{[2]} \):

\[
X_{ab}^c = \frac{1}{16} \left[ (\gamma_{ab})^{\gamma \delta} D_\gamma H_{\delta}^c + 16 \delta_{[a}^c \Psi_{b]} - 16 \Psi_{ab}^c \right],
\]

\[
X_{a_1 \ldots a_5}^c = \frac{1}{16} \left[ i (\gamma_{a_1 \ldots a_5})^{\gamma \delta} D_\gamma H_{\delta}^c + \frac{1}{3} \delta_{[a_1}^c \Psi_{a_2 \ldots a_5]} - \frac{2}{15} \epsilon_{a_1 \ldots a_5}^{\gamma} \Psi_{[5]} \right].
\] (39)

The vanishing of parts of the \( X \)-field strengths simply determine \( \Psi_{[1]} \), \( \Psi_{[3]} \), \( \Psi_{[4]} \) and \( \Psi_{[5]} \) in terms of \( H_{a \beta} \). Therefore the solution to these algebraic equations allow us to impose the further conventional constraints on the \( X \)-tensors

\[
X_{ab}^c = 0, \quad X_{[abc]} = 0, \quad X_{a_1 \ldots a_5} = 0, \quad X_{[a_1 \ldots a_5]} = 0.
\] (40)

This leaves only \( \Psi \) and \( H_{a \beta} \) with \( 706 \) \( dS \) degrees of freedom as the true variables that describe the 11D supergravity theory. Here \( H_{a \beta} \) is the gauge field for the Weyl theory degrees of freedom and \( \Psi \) is the Goldstone superfield for breaking superconformal symmetry to super Poincaré symmetry. In obtaining this result, we note that neither spinorial nor bosonic differential constraints are imposed upon \( \Psi \), \( \Psi_{[2]} \) nor \( H_{a \beta} \) due the constraints listed in table three. Finally the authors of \(^1\)11 impose the condition that \( X_{[2]}^c = 0 \) and this is seen to be a genuine restriction on \( H_{a \beta} \). While it is not clear to us why this is preferable to the opposite \( X \)-restriction, it is clear that the opposite choice \( X_{[5]}^c = 0 \) leads to a smaller supergravity multiplet if it is viable.

Above we noted that the \( X \)-tensor superfield strengths are independent of \( \Psi \). In particular, this means that imposing conditions on them cannot impose any conditions on \( \Psi \). This is a direct contradiction of the main result of Howe’s 1997 Theorem because we see that even if \( T_{\alpha \beta}^c = i (\gamma^c)_{\alpha \beta} \), no equations at all are imposed on \( \Psi \). Looking back at the full non-linear expressions for \( X_{[2]}^c \) and \( X_{[5]}^c \) (given in \(^1\)11 and \(^1\)17), these are also seen to be independent of \( \Psi \), so the argument valid for the linearized theory generalizes to the non-linear theory and disproves Howe’s assertion in general.

\(^1\)The quantity \( \Psi_{[2]} \) is the Lorentz compensator and is thus a pure gauge degree of freedom.
A Simple Component-Field Modification

At the level of component fields, the manner in which the spin-1/2 multiplet will manifest itself follows from the general discussion given in “Superspace” [14] (p. 323) which is easily generalized to 11D. The transformation law of the graviton using only the constraint given in (2) can be derived to be of the form

\[ \delta Q e_{a}^{m} = - \epsilon^{\beta} \left[ T_{\beta a}^{\ b} + \psi_{a}^{\gamma} T_{\beta \gamma}^{\ b} \right] e_{b}^{m}, \]

\[ \rightarrow \delta Q e^{-1} = e^{-1} \epsilon^{\beta} \left[ i (\gamma^{a})_{\beta \gamma} \psi_{a}^{\gamma} + \frac{341}{32} J_{\beta} + \frac{1}{2} X[2]^{a} (\gamma^{[2]})_{\beta \gamma} \psi_{a}^{\gamma} \right], \]

(41)

and in the presence of a spin-1/2 dimension 1/2 auxiliary field, the \( J_{\beta} \) term is non-zero. This has a very dramatic effect. The \( J_{\beta} \) term is fermionic but supercovariant. In the on-shell theory there is only one perturbatively supercovariant spinorial quantity upon which it can depend, the field strength of the gravitino.

The condition for a fermionic spin-1/2 dimension 1/2 auxiliary field to occur is that in the presence of the M-theory corrections, the elfbein density contains a purely supercovariant term in its supersymmetry transformation law. In turn this means testing for the presence or absence of the spin-1/2 dimension 1/2 auxiliary field is simpler than one might imagine. We only need a consistent procedure to find the elfbein transformation law with its M-theory corrections. Finally, this equation (41) shows how the “X-tensors” also modify the on-shell graviton transformation laws. The work in [17] should definitively lead to an answer as to what is the form of the transformation law chosen by the lowest order correction in supergravity/M-theory.

Let us now discuss what Peeters, Vanhove and Westerberg [17] have presented. Before doing this, it is perhaps useful to warn the reader that due to their non-canonical field definitions, there are numbers of re-definitions necessary in order to compare their results with canonical ones. As an example of what we mean by “non-canonical field definitions,” their equation (3.36) is a useful reference point. It is clear that by making a redefinition of their gravitino field, all of the terms involving the space-time derivative of the local supersymmetry parameter can be removed from the rhs of this equation. The use of an ordinary superspace formalism is simplest after implementing such re-definitions. We have not implemented such re-definitions, so instead of directly commenting on their results we will use them to motivate the appearance of certain superspace structures useful for future study.

Working in the basis of their equation (3.43) and multiplying it by an elfbein in order to form a trace, we find it suggestive of a \( J \)-tensor of the form

\[ J_{\beta} \approx i a_{0} (\ell_{11})^{6} t_{8}^{[s]} \eta_{[a_{2} \eta_{a} a_{6}^{s}]_{\beta \gamma} W_{a_{1} a_{2}} a_{4} a_{4} W_{a_{3} a_{4}} a_{4} a_{4} \left( \nabla a_{4} a_{4}^{s} \right) \}, \]

(42)

where \( t_{8}^{[s]} \) is a very well known rank-8 tensor (see [17] for a definition) and \( a_{0} \) is some normalization constant. As per our expectations, the non-scale invariant M-theory correction would then have excited the spin-1/2 multiplet of currents.

From our perspective, there is one other very puzzling result of their analysis. Near the end of the last chapter, we noted that Cederwall, Gran, Nielsen and Nilsson
chose $X[2]^a = 0$. We noted that the opposite choice $X[5]^a = 0$ would naturally lead to a smaller supergravity multiplet. Interestingly, Peeters, Vanhove and Westerberg report from their calculation of the commutator algebra (within their approximations) that they find $X[5]^a = 0$. As before, if we look at equation (3.39) of their work and compare it to our equation (41) it is suggestive of

$$X[5]^a \approx -a_1 (\ell_{11})^6 \epsilon_8^{[8]} W_{a_1 a_2 a'_1 a'_2} W_{a_3 a_4 a'_3 a'_4} W_{a_5} a_5 a'_5 \delta_{a_6} \delta_{a'_6}^{[a} \delta_{a'_6}^{a]} .$$

where $a_1$ is a second normalization constant. In their final basis they find $a_0 = a_1 = 0$ [17]. It is a problem for the future to study these terms in a complete supergeometry for various values of the parameters and as well as adding additional terms involving the 4-form field strength to both the $J$ and $X$ currents. Let us also close by emphasizing that the results in [12] and [43] are simply motivated from [17] and we have not independently checked their consistency via use of the superspace Bianchi identities. They do mark, however, the first explicit equations for the lowest order supergravity/M-theory currents given in superspace. This is exactly analogous to the initial presentation of the Yang-Mills/open-string superspace current given in equation (8.3) of the first work in [2].

7 Off-shell 11D Supergravity/M-theory Tensors

We have repeatedly referred to the 4D, $N = 1$ supergravity theory as providing a paradigm for the actual superspace geometrical structure required for the M-theory effective action. Perhaps this point is best illustrated by more detailed comments. It is a demonstrable fact that the following set of solely conventional constraints determine all the 11D supergeometry in terms of the semi-prepotentials $H^{\alpha m}$ and $\Psi$. These constraints are such that only four independent superfields $W_{abcd}$, $X[ab]^c$, $X[5]^a$.

| Superfield Determined | Geometrical Constraint |
|-----------------------|------------------------|
| $E_{\alpha m}$        | $i (\gamma_{\alpha})^{\alpha \beta} T_{\alpha \beta}^b = 32\delta_{a}^b$ |
| $E_{\alpha \mu}$      | $(\gamma_{\alpha})^{\alpha \beta} T_{\alpha \beta}^\gamma = 0$ |
| $\omega_{\alpha}^{de}$| $T_{\alpha}^{\left[de\right]} - \frac{2}{9} \left(\gamma_{de}\right)_{\alpha}^{\gamma} T_{\gamma b}^{b} = 0$ |
| $\omega_{\alpha}^{de}$| $(\gamma_{\alpha})^{\alpha \beta} R_{\alpha \beta}^{\left[de\right]} = 0$ |
| $\Psi^{[1]}$          | $(\gamma_{\alpha \beta})^{\alpha \beta} T_{\alpha \beta}^{b} = 0$ |
| $\Psi^{[3]}$          | $(\gamma_{\alpha \beta})^{\alpha \beta} T_{\alpha \beta}^{\left[c\right]} = 0$ |
| $\Psi^{[4]}$          | $(\gamma_{\alpha \beta \gamma})^{\alpha \beta} T_{\alpha \beta}^{c} = 0$ |
| $\Psi^{[5]}$          | $\frac{1}{12} \epsilon_{[5]}^{abcdef} \left(\gamma_{abcde}\right)^{\alpha \beta} T_{\alpha \beta f}^{a} = 0$ |

Table 3: A Set of 11D SG/M-theory Conventional Constraints

12For example, the work of [12] does not relate the $X[5]^a$ tensor to the field strength superfields of the conventional on-shell 11D theory.
13The critical reader may note that this set of constraints, though equivalent, is slightly different from that which implicitly appears in our previous chapters.
$X_{[a_1 \cdots a_5]}^c$ and $J_\alpha$ are required to describe all the gauge-independent parts of the semi-prepotentials. The definitions of these field strengths are

$$\begin{align*}
W_{abcd} &\equiv \frac{1}{32} \left[ i (\gamma^e \gamma_{abcd}) \gamma^\alpha T_\alpha e^\gamma - \frac{1}{3} (\gamma_{abcd})^{\alpha\beta} \left( \nabla_\alpha T_\beta e^c + \frac{14}{3\cdot363} T_{ak}^k T_\beta l^l \right) \right], \\
X_{[ab]}^c &\equiv \frac{1}{32} (\gamma_{ab})^{\alpha\beta} T_{a\beta c}, \quad X_{[a_1 \cdots a_5]}^c \equiv i \frac{1}{32} (\gamma_{a_1 \cdots a_5})^{\alpha\beta} T_{a\beta c}, \\
J_\alpha &\equiv \frac{4}{33} T_{ab}^b.
\end{align*}$$

Given these field strengths, we can discuss the equations of motion associated with the 11D semi-prepotentials. In the ordinary on-shell theory, $J_\alpha = X_{[ab]}^c = X_{[abcde]}^c = 0$. When general scale non-invariant M-theory corrections are described we must have $J_\alpha \neq 0$ and $X \neq 0$. If there are scale-invariant M-theory corrections, then $J_\alpha = 0$ and $X \neq 0$. Since the $X$-superfields are $\Psi$-independent they don’t impose any equations of motion upon $\Psi$. Its equation of motion arises solely from imposing conditions on $J_\alpha$.

This behavior is exactly like that of 4D, $N = 1$ non-minimal supergravity, a system possessing three off-shell multiplets, $W_{\alpha\beta\gamma}$, $G_\alpha$ and $T_\alpha$. The equations of the on-shell theory are given by $T_\alpha = G_\alpha = 0$. Coupling this supergravity theory to conformal matter only excites one of its off-shell multiplets ($T_\alpha = 0$, $G_\alpha \neq 0$) and coupling it to non-conformal matter excites both ($T_\alpha \neq 0$, $G_\alpha \neq 0$). The known lowest order M-theory corrections are not scale-invariant. Due to this and in analogy to the coupling of non-minimal 4D, $N = 1$ supergravity to non-conformal matter we expect that both the $X$-multiplet of currents as well as the $J$-multiplet of currents to be excited at lowest order in the $\ell_{11}$-expansion of 11D superspace geometry. It is inconsistent to attempt to describe the non-minimal 4D, $N = 1$ supergravity multiplet by setting $T_\alpha = 0$ as a constraint and this is the direct analog of imposing Howe’s Theorem.

We end the section with a warning. It has long been known that supergravity theories must also satisfy additional constraints [18] that go beyond the merely conventional ones such as those in table three. At present the explicit form of these addition constraints are not known (although the work of [12] suggests that $X_{[ab]}^c = 0$ for example). Clearly more detailed investigation is required.

8 Summary Discussion

It is therefore our position that while we inaccurately reported the embedding of our long awaited spin-1/2 current multiplet in our 1996 paper, the part of Howe’s 1997 work on ordinary Poincaré superspace can also be seen to necessarily include such a multiplet exactly as we predicted in 1980, but he concludes that it is irrelevant. We believe that neither of these papers [8, 10] is completely correct nor completely incorrect as explained in the previous chapters.

In our equation (41), we have given a simple component level criterion to test for the presence or absence of the dimension 1/2 multiplet of currents. The spin-1/2 and dimension 1/2 multiplet of currents that we have long anticipated is very closely
related the first spinorial derivative of the eleven dimensional scale compensator. Thus scale non-invariant corrections should activate this multiplet and the super-space torsions and curvatures will respond by the appearance of a spinorial multiplet of currents, and frustrate attempts to eliminate the spin-1/2 multiplet of currents from the geometry of 11D supergeometry. We contend that the yet-to-be completed component level analysis of [17] shows a clear potential to support this position.

Our analysis of the linearized supergravity fluctuations clarifies the circumstances of the applicability of the 1997 Howe Theorem (that posits all 11D supergravity equations follow solely from the condition $T_{\alpha\beta\gamma} = i(\gamma^c)_{\alpha\beta}$). The only way Howe’s Theorem can be true is if $\Psi$ is a pure gauge degree of freedom (like $\Psi_{[2]}$) and in the absence of super-scale symmetry this is impossible. One more interesting implication of our 11D result is that it also cast doubt on a much older version [19] of this very same assertion within the context of the heterotic string effective action. A repeat of this type of analysis for 10D, $N = 1$ superspace yields the same results. With the verification that the $J_\alpha$-tensor is present to accommodate the string corrections in 11D, it would follow that such a tensor must be present for, at least, 10D, $N = IIA$ supergravity also where it must take the form $J_{\alpha i}$. Furthermore this increases the likelihood that such an object appears in the correction to type-I closed and heterotic string modified superspace geometries in the form of $J_\alpha$ (i.e., setting one of the isospin copies of the IIA theory to zero). Thus in addition to the $A_{[3]}$-tensor found [1, 2] some time ago, there would be a $J_\alpha$-tensor and possibly an $X$-tensor in these theories also. In fact, the expressions in (42) and (43) can be easily be interpreted within the confines of 10D, $N = 1$ superspace and presumably describe the same correction to the heterotic and type-I closed effective actions. The appearance of new auxiliary fields may well offer the way around a vexing conundrum [20].

We end by noting our twenty-year old conjecture [3] now has increased chances for validation at last.

“The geometry of space is associated with a mathematical group.”
– Felix Klein

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