Dark matter bars in spinning haloes

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ABSTRACT
We study non-linear response of spinning dark matter (DM) haloes to dynamic and secular evolution of stellar bars in the embedded galactic discs, using high-resolution numerical simulations. For a sequence of haloes with the cosmological spin parameter $\lambda = 0–0.09$, and a representative angular momentum distribution, we analyse evolution of induced DM bars amplitude and quantify parameters of the response as well as trapping of DM orbits and angular momentum transfer by the main and secondary resonances. We find that (1) maximal amplitude of DM bars depends strongly on $\lambda$, while that of the stellar bars is indifferent to $\lambda$; (2) efficiency of resonance trapping of DM orbits by the bar increases with $\lambda$, and so is the mass and the volume of DM bars; (3) contribution of resonance transfer of angular momentum to the DM halo increases with $\lambda$, and for larger spin, the DM halo ‘talks’ to itself, by moving the angular momentum to larger radii – this process is maintained by resonances; and (4) prograde and retrograde DM orbits play different roles in angular momentum transfer. The ‘active’ part of the halo extends well beyond the bar region, up to few times the bar length in equatorial plane and away from this plane. (5) We model evolution of discless DM haloes and haloes with frozen discs, and found them to be perfectly stable to any Fourier modes. Finally, further studies adopting a range of mass and specific angular momentum distributions of the DM halo will generalize the dependence of DM response on the halo spin and important implications for direct detection of DM and that of the associated stellar tracers, such as streamers.

Key words: methods: numerical – galaxies: evolution – galaxies: formation – galaxies: interactions – galaxies: kinematics and dynamics – dark matter.

1 INTRODUCTION
Self-gravitating systems still challenge our understanding. Stellar bars in disc galaxies can form either spontaneously, as a result of the bar instability (e.g. Hohl 1971; Sellwood 1980; Athanassoula 1992; Berentzen et al. 1998), or following interactions with other galaxies (e.g. Toomre & Toomre 1972; Gerin, Combes & Athanassoula 1990; Berentzen et al. 2004), or in stellar disc interactions with halo’s dark matter (DM) clumps (Romano-Diaz et al. 2008). Evolution of stellar bars has been studied and analysed in numerical simulations in non-rotating haloes (e.g. Athanassoula et al. 1983; Sellwood & Sparke 1987; Berentzen et al. 1998; Dubinski, Berentzen & Shlosman 2009). Theoretical works predicted a minor effect of spinning parent haloes on the embedded stellar disc evolution (Weinberg 1985). Contrary to these expectations, recent results indicate that a halo spin has a profound effect on the stellar bar evolution, and affects angular momentum redistribution in disc–halo systems (Saha & Naab 2013; Long, Shlosman & Heller 2014; Collier, Shlosman & Heller 2018).

In previous works, we have focused on the evolution of stellar bars in spinning haloes and the associated angular momentum transfer in the disc–halo system (Long et al. 2014; Collier et al. 2018). Here, we aim to analyse the parent DM halo response to this process. While it is known that DM response is triggered by the formation of a bar in the disc (Tremaine & Weinberg 1984; Weinberg 1985; Athanassoula 2005; Shlosman 2008; Petersen, Weinberg & Katz 2016), its properties in spinning haloes are largely unknown.

Models of disc galaxies in spinning haloes brought up a number of surprises. First, the bar instability appears to accelerate compared to the non-rotating haloes (Saha & Naab 2013; Long et al. 2014). Second, the vertical buckling instability in the bar is more profound, and weakens the bars progressively with increased halo spin. Defining the halo spin as $\lambda = J/J_K$, where $J$ and $J_K$ are the halo angular momentum and its maximal angular momentum, respectively (e.g. Bullock et al. 2001), the important transition range lies in $\lambda \sim 0.03–0.06$ (Collier et al. 2018). Above $\lambda \geq 0.03$, the stellar bar amplitude has difficulty to recover after buckling even over the secular time.

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of evolution. Consequently, the bar braking ability against the DM becomes dramatically weaker. Moreover, for \( \lambda \gtrsim 0.06 \), the bars essentially dissolve, leaving behind a weak oval distortion. Thus, bar evolution in spinning haloes can affect a substantial fraction of disc–halo systems.

Works that did not reach similar conclusions on the importance of halo spin to bar dynamics have either limited their analysis to \( \lambda \lesssim 0.03 \) haloes (Petersen et al. 2016), and hence did not test the relevant range in \( \lambda \), or limited the evolution time to the pre-buckling stage of the bar instability (Saha & Naab 2013). Moreover, cosmological simulations that include a broader range of \( \lambda \) and secular evolution of discs do not have sufficient resolution as achieved in studies of isolated haloes. Their treatment of angular redistribution between the DM haloes and embedded discs are not precise enough and hence can miss this effect altogether.

To fully understand the ramification of this new effect, we need to analyse its dependence on two important dynamical indicators – the distributions of mass and angular momentum in the halo. The former has a universal character (e.g. Navarro, Frenk & White 1996) with various degrees of central mass concentration, which reflects the formation time. Baryons tend to increase the central concentration.

Distribution of angular momentum in the halo has been claimed to be universal as well for pure DM haloes (e.g. Bullock et al. 2001). The addition of baryons can modify this distribution in principle by increasing the angular momentum within the inner halo, which can be measured from haloes in Illustris simulation (Vogelsberger et al. 2014). We note that the model haloes of Collier et al. (2018) used in the present work are representative of \( J \) distribution in baryonic haloes and lie within one \( \sigma \) from its median. The detailed study of bar dynamics for a range in \( J \) and mass distributions in DM haloes is forthcoming.

The basic questions that must be answered when dealing with the DM bar evolution in spinning haloes are as follows. Is the strength of a DM bar affected by the halo spin \( \lambda \)? Are DM bar mass and shape dependent on the halo spin? What type of orbits comprise the DM bar? Are all these orbits absorbing the angular momentum from the disc? What part of the halo absorbs the angular momentum from the disc, and how does the efficiency of DM orbit trapping by the resonances depend on the halo spin? We aim to resolve these questions.

In the previous work, we have confirmed that the time-scale of the stellar bar instability is shortened along the \( \lambda \) sequence, as shown by Saha & Naab (2013) and Long et al. (2014). While the maximal strength of these bars is independent of \( \lambda \), their secular evolution depends strongly on the parent halo spin. Probably the most interesting and unexpected effect of stellar bar evolution in spinning haloes is that the buckling instability of the bar is more destructive with \( \lambda \), and stellar bars have have increasing difficulty to recover their strength after buckling. This transition occurs in the range of \( \lambda \sim 0.03-0.06 \). Close to the upper value and above it, the stellar bars are basically destroyed by the buckling instability and never regrow. Models with spherical, oblate, and prolate haloes have been run and behaved similarly. The DM response to the underlying stellar bar perturbation dies out immediately with its disappearance was shown for the first time in Shlosman (2008).

Observational corollaries of this evolution include decreasing braking of a stellar bar against the DM, and much higher ratios of corotation-to-bar size, \( r_{\text{CR}}/r_{\text{BA}} \gtrsim 2 \) after buckling, well beyond the ratios encountered in \( \lambda = 0 \) haloes, \( r_{\text{CR}}/r_{\text{BA}} \sim 1.2 \pm 0.2 \) (Athanassoula 1992). Furthermore, stellar bar growth experiences difficulties with increasing \( \lambda \), and saturates completely for \( \lambda \gtrsim 0.05 \). The high-\( \lambda \) haloes anticorrelate with the existence of ansae, and exhibit smaller size and mass of the peanut/boxy-shaped bulges.

That angular momentum \( J \) flows from a barred stellar disc to a DM halo is known for quite some time (e.g. Sellwood 1980; Weinberg 1985; Debattista & Sellwood 2000; Athanassoula 2003, 2005; Martinez-Valpuesta, Shlosman & Heller 2006; Berentzen et al. 2007; Weinberg & Katz 2007a,b; Dubinski et al. 2009). That this flow is mediated by the orbital resonances, and especially by the inner Lindblad resonance (ILR), outer Lindblad resonance (OLR), and the corotation resonance (CR) has been established as well (Lynden-Bell & Kalnajs 1972; Weinberg 1985; Athanassoula 2003; Martinez-Valpuesta et al. 2006). Importantly, the parent halo spin strongly affects the angular momentum transfer between the disc and its halo, but this point is required to be investigated further (Collier et al. 2018). One expects that the DM halo orbital structure will help to understand the intricacies of angular momentum flow in the system, many of which remain unclear. Weinberg (1985) has suggested that the increase in the number of prograde particles in the spinning halo (with respect to disc rotation) increases naturally the fraction of halo particles trapped by the main resonances and speeds up the bar instability. But this assumption was never verified in a quantitative analysis. Nor was it verified how the trapping by resonances explains the secular evolution of stellar bars in spinning haloes. We attempt to tackle these issues in this paper and in Collier, Shlosman & Heller (2019).

To quantify the angular momentum flow in the disc–halo systems, one can take a dual approach. It is possible to follow the rate of angular momentum flow with the method designed by Villa-Vargas, Shlosman & Heller (2009, see also Section 2.2; Villa-Vargas, Shlosman & Heller 2010; Long, Shlosman & Heller 2014; Collier et al. 2018). To determine the role of the resonances in this transfer, we refer to the orbital spectral analysis (e.g. Binney & Spergel 1982; Athanassoula 2003; Martinez-Valpuesta et al. 2006; Dubinski et al. 2009). This method allows us to determine the fraction of DM orbits trapped by the resonances. One can apply this method in frozen potentials and integrate the orbit for a fixed and large number of periods. Alternatively, one can do this in the live potential of the system. This, however, has its disadvantages – the number of time periods to integrate along the orbit will be small, leading to unreasonable widening of the resonances. Hence, we follow the former method and use it in order to find the DM orbit trapping efficiency by the resonances as well as amount angular momentum transported by these resonances, comparing it along the \( \lambda \) sequence.

Because we focus on properties of DM haloes with increasing spin, one should ask whether discless haloes with the same \( \lambda \) are stable against spontaneous breaking of the axial symmetry. Stability of pure discless haloes with a non-zero cosmological spin is subject to diverging opinions. Based on Jeans (1919) theorem, Lynden-Bell (1960) has argued that spherical haloes with all particle tangential velocities reversed in the same direction are stable. Of course, Jeans theorem does not capture the elusive bar instability, when the system can lower its energy by breaking the axial symmetry, as in Maclaurin sequence of rotating ellipsoids (e.g. Chandrasekhar 1969; Binney & Tremaine 2008). Hence, Allen, Palmer & Papaloizou (1992) claimed, based on their numerical simulations, that rotating models of spherical, oblate, and prolate \( N \)-body systems become unstable and form ‘triaxial bars’. However, Sellwood & Valluri (1997), after reproducing their initial conditions, found that this instability resulted from an error in the code, and
newly re-run models were completely stable. Though these authors warned that a fast spinning halo – one with all orbits rotating in the same direction, may still become bar unstable.

Furthermore, Kuijken & Dubinski (1994) cautioned against using haloes of large spin after seeing bar formation in oblate Evan’s model systems of \( \lambda = 0.18 \). However, the range of \( \lambda \) used in our work is much lower, \( \lambda \lesssim 0.09 \), and we do not expect that our haloes are unstable. Nevertheless, we test discless spherical haloes with maximal spin which can be obtained in our models, up to \( \lambda = 0.108 \), in Section 3.1. Moreover, we tested these haloes with an embedded frozen disc, to account for changes that can be introduced by the disc gravitational potential. All our discless haloes are stable against bar instability or any other global instability over the time of 10 Gyr.

Hence, we find that limiting the spin to \( \lambda = 0.03 \), as motivated by Petersen et al. (2016) is not warranted. We have limited our analysis to only spherical models. Opaque and prolated haloes modeled by Collier et al. (2018) will be discussed elsewhere.

This paper is structured as follows. Section 2 deals with numerics, including initial conditions and orbital spectral analysis. Section 3 presents our results, starting with diskless DM haloes and switching to disc–halo systems. Section 4 discusses our results and theory corollaries, and we end with conclusions.

2 NUMERICS

2.1 Model setup and initial conditions

We analyse models of disc–halo systems described in Collier et al. (2018), and additional models of isolated DM haloes. Our pure DM halo models and disc–halo models differ only with the DM halo spin, \( \lambda \). The initial conditions have been created using a novel iterative method (Rodionov & Sofnikova 2006; Rodionov, Athanassoula & SofnikovaYa. 2009; Long, Shlosman & Heller 2014; Collier et al. 2018). These models have been run using the N-body part of the GIZMO code originally described in Hopkins (2015). We choose units of distance and mass as 1 kpc and \( 10^{10} M_\odot \), respectively. This leads to the time unit of 1 Gyr. The DM haloes have been modeled with \( N_h = 7.2 \times 10^4 \), and stellar discs with \( N_d = 8 \times 10^5 \). So the ratio of masses of DM particles to stellar particles is close to unity.

For convergence test, we run models with twice the number of particles and obtained similar evolution.

Halo shapes include spherical, oblate, and prolated models, but only the former ones are discussed here. The halo spin has been varied by inverting a fixed fraction of tangential velocities for retrograde DM particles (with respect to the disc rotation), which does not change the solution of the Boltzmann equation (Lynden-Bell 1960; Weinberg 1985; Long et al. 2014; Collier et al. 2018). J of each halo has a lognormal universal distribution (Bullock et al. 2001).

The models contain an exponential disc with density:

\[
\rho_d(R, z) = \frac{M_d}{4\pi h^2 z_0} \exp(-R/h) \text{sech}^2\left(\frac{z}{z_0}\right),
\]

where \( \rho(r) \) is the DM density in spherical coordinates, \( \rho_\ast \) is the (fitting) density parameter, and \( r_\ast = 9 \) kpc is the characteristic radius, where the power-law slope is (approximately) equal to \(-2\), and \( r_c = 1.4 \) kpc is a central density core. We used the Gaussian cutoffs at \( r_\ast = 86 \) kpc for the halo and \( R_t = 6h \sim 17 \) kpc for the disc models, respectively. The halo mass is \( M_h = 6.3 \times 10^{11} M_\odot \), its central mass concentration \( c \sim 90 \) kpc/\( r_c \sim 10 \), and halo-to-disc mass ratio within \( R_t \) is 2.

We follow the notation of Collier et al. (2018) to abbreviate the disc–halo models, namely, \( P \) for prograde spinning haloes, followed by the value of \( \lambda \) multiplied by 1000. The Standard Model is defined as that of a non-rotating spherical halo, P00. Pure DM halo models are denoted as \( H \), followed by 1000x, as in disc–halo models. More details can be found in Collier et al.

2.2 Orbital spectral analysis

To examine the role of resonances in angular momentum transfer within the disc–halo systems, we use the orbital spectral analysis method (Binney & Spergel 1982; Athanassoula 2003; Martinez-Valpuesta et al. 2006; Dubinski et al. 2009). We perform the orbit spectral analysis in the frozen potential. The bar pattern speed is fixed in time as well. All spiral features that have the same pattern speed as the bar, are taken into account.

Our goal here is to quantify the role of the important resonances in angular momentum transfer by statistically sampling the stellar disc and DM halo orbits in our simulations and capture the angular velocity, \( \Omega \), and radial epicyclic frequency, \( \kappa \), for individual orbits, and gain insight into the resonance structure as a whole. Additional frequency, \( \Omega_{res} \), is the bar pattern speed measured for each model at the time when the potential is frozen.

We evolve stellar and DM test particles in frozen gravitational potential of the system at time \( t \), for 50 orbits. By integrating the test particles trajectories, we record the corresponding cylindrical coordinates, \( r, \phi, \) and \( z \). For each orbit, we determine the power spectrum of the representative frequencies, \( \Omega \) and \( \kappa \), by applying the Fourier transform to the \( \phi \) and to \( r \) values of each particle along every time-step in their orbit. Next, we calculate the orbit fraction as a function of normalized dimensionless frequency \( \nu \equiv (\Omega - \Omega_{bar})/\kappa \), binned in \( \Delta \nu = 0.01 \). We choose a sample of DM and stellar particles, 200,000 each, for this analysis. The test particles were randomly chosen from the entire sample of particles. The main resonances of interest are the ILR, the CR, and the OLR. They correspond to \( \nu = 0.5 \), and \(-0.5 \), respectively.

The code is automated to capture thousands of orbits simultaneously. Test particles are identified and sorted by radius, in order to group them in bins of a similar dynamical time. This step is parallelized and run using schwimmbad (Price-Whelan et al. 2017) – an MPI tool for PYTHON.

3 RESULTS

We start with basic analysis of discless DM haloes in order to verify their stability, and continue with haloes hosting stellar discs. To calibrate our simulations, we run a number of new models of spherical discless DM haloes within the same range of \( \lambda \sim 0-0.09 \). Moreover, we have added three additional models: discless haloes with \( \lambda = 0.10 \) and 0.1077, as well as P90 model with frozen stellar disc. The reasons for these additional models are explained in Section 3.1. We also measure the Fourier amplitude of developing DM bars in response to evolving stellar bars, present the results of the orbital spectral analysis, and determine the rates of angular
and the original NFW density profile throughout the 10 Gyr run. We include these results to show the stark contrast of halo evolution and angular momentum flow when the system hosts a stellar disc, e.g. Fig. 10. Hence we consider these discless haloes being stable both dynamically and secularly.

Our results do not contradict the models of Kuijken & Dubinski (1994), which obtained instability for a substantially oblate halo with axial ratio of $c/a = 0.8$. In the NFW haloes, most of the mass is located in the outer shells, and oblateness results in moving DM particles away from the rotation axis. As a result, Kuijken & Dubinski model had $\lambda = 0.18$, almost a factor 2 larger in our haloes with a maximal $\lambda$ of 0.1077. Hence, we have verified that DM haloes within our $\lambda$ range are stable in the absence of stellar bars.

We have run an additional test model that has our DM halo with $\lambda = 0.09$ and a frozen disc potential. Such model with $\lambda = 0$ was run by Petersen et al. (2016) to examine the development of a DM bar without interaction from the stellar bar. However, in our test we push further into domain found by Kuijken & Dubinski (1994) to be unstable, i.e. involving much higher spin. In this respect, our test is more challenging. We find no evidence of instability or bar formation in this halo as well.

### 3.2 Evolution of DM bar amplitude in spinning haloes

Collier et al. (2018) have analysed stellar bar evolution and the angular momentum redistribution in the disc of disc–halo systems with a range in $\lambda$. Here, we focus on the response of DM haloes under these conditions. Discs start axisymmetric, spontaneously break this symmetry and develop stellar bars – a dynamical stage of evolution. These bars buckle vertically and either experience a secular growth or not, depending on the parent DM halo spin. To quantify this evolution in the DM, we measure departure of its density distribution from axial symmetry using Fourier mode amplitudes. The bar amplitude has been defined using the Fourier $m = 2$ mode, namely,

$$A_2 = \frac{1}{A_0} \sum_{i=1}^{N_d} m_i e^{2i\phi},$$

although higher modes are not negligible, we ignore them here. The summation is performed over all disc particles with the mass $m = m_i$ at azimuthal angles $\phi_i$, for $R \leq 14$ kpc, and $|z| \leq 5$ kpc. The amplitude of the $m = 2$ mode has been normalized by the monopole term $A_0$. The radial and vertical limits of summation correspond to the maximal length and well above the vertical thickness of stellar bars in P00 model. To measure amplitudes of DM bars, we followed the same procedure. For an unbiased comparison, we refrained from changing the radial and vertical limits of integration when measuring the amplitude of the DM bars, although DM bars appear shorter and ‘fatter’ than the stellar bars in all models.

Fig. 2 displays the evolution of the $A_2$ amplitudes for both stellar and DM bars, for two models at the extremes of the $\lambda$ sequence, namely $\lambda = 0$ and 0.09, i.e. P00 and P90 models. The associated DM bars are much weaker than stellar bars, only reaching the maximum $A_2 \sim 0.02–0.08$ along the spin sequence, while the stellar bars reach $A_2 \approx 0.45$. Both phases of bar evolution, dynamical, and secular, are highly dependent on the $\lambda$ of the parent haloes, as seen in Fig. 2. Stellar bars in non-rotating and slowly rotating haloes resume growth after buckling. Bar amplitude within faster spinning haloes stagnates and shows no growth after buckling, during their secular evolution stage.
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Figure 2. Evolution of stellar bar Fourier amplitudes, $A_2$, in the P00 and P90 models (solid lanes) and their corresponding DM bars (dashed lanes), normalized by the monopole term, $A_0$. These models represent the extremes of $\lambda$ sequence, i.e. $\lambda = 0$ and 0.09. The limits of integration are described in the text.

Figure 3. Fourier amplitude, $A_2$, normalized by the monopole term, $A_0$, evolution of the DM bars along the $\lambda$ sequence in our models. Note the strong dependency of DM bar maximal strength on $\lambda$ in the bar instability stage, and much weaker vertical spread in in $A_2$ in the secular evolution stage.

Fig. 3 exhibits the evolutionary trends of stellar and DM bars along the $\lambda$ sequence. First, as $\lambda$ increases, the DM bar appears earlier, when compared to lower $\lambda$ models. Second, after buckling of stellar bars, DM bars inside large $\lambda$ haloes do not regrow, similarly to their stellar counterparts, as noted first in Collier et al. (2018).

While the above evolution of DM bars is expected due to the evolution of the associated stellar bars, Fig. 4 displays the idiosyncrasy in their behaviour. Here, we plot the maximal strength of DM and stellar bars, before buckling of stellar bars, normalized by the the maximal strength of the DM or stellar bar in the fiducial P00 model. The red line reflects the behaviour of stellar bars in the fig. 1 of Collier et al. (2018), and stays flat. Meaning that the maximal pre-buckling amplitude of stellar bars is basically independent of $\lambda$. In contrast, the blue line representing the DM bars exhibits a dramatic increase with $\lambda$. For example, the P90 DM bar has its maximal pre-buckling strength amplified by a factor of $\sim 3.4$ compared to P00 model.

We test whether this increase in the maximal amplitudes of DM bars along the halo spin sequence is related to the fraction of prograde populated DM orbits in our models. Table 1 presents the halo spins and a fraction of prograde DM orbits. Comparing the maximal pre-buckling amplitudes of DM bars, we observe a direct dependency of the maximal $A_2$ on the fraction of prograde orbits, $f$, in the DM halo. When $f = 1$, all DM particles are rotating in the same (prograde) direction, and when $f = 0.5$, the halo has $\lambda = 0$.

The amplification we observe in DM bar strength, along the $\lambda$ sequence, is not observed in the stellar bars, because most of the particles in the stellar disc are already on prograde orbits. To investigate the role of the prograde orbits along the halo spin sequence, we resort to the orbital spectral analysis in Section 3.3.

Table 1. Fractions, $f$, of prograde DM orbits in our models.

| Model | $\lambda$ | $f$  |
|-------|-----------|------|
| P00   | 0.00      | 0.50 |
| P30   | 0.03      | 0.62 |
| P60   | 0.06      | 0.76 |
| P90   | 0.09      | 0.88 |

3.3 Spectral orbital analysis for stellar discs and DM haloes

Next, we perform the orbital spectral analysis to determine the distribution of stellar and DM orbits with the normalized frequency $\nu$ for each of the models along the $\lambda$ sequence (Section 2.2). Our goal is to find the fraction of disc and halo orbits trapped at the main resonances, the ILR, CR, and the OLR, as well as at higher resonances.

The orbital spectral analysis has been performed prior to buckling in each model, when stellar bar amplitudes have the same value. This allowed to measure the trapping efficiency at similar stages of bar evolution. Due to dependency of the bar strength on $\lambda$, similar $A_2$
representative models, P00, P30, P60, and P90. The orbital spectral analysis has been performed before buckling, when \( \nu \) is normalized by the total number of sampled orbits, i.e. 200,000. Shown are the frames for the halo (top frames) and disc (bottom frames), for four \( \nu \) values.

The resulting distribution of stellar and DM particles with \( \nu \) is given in Fig. 5. Both DM and stellar particles are concentrated at specific frequencies corresponding to the resonances. For disc stellar particles (the bottom frames), the main trapping corresponds to the ILR, \( \nu = 0.5 \). Smaller fractions are trapped at the CR and the OLR, respectively. Previous work has clarified which resonances are mainly responsible for the angular momentum loss by the disc, and singled out the ILR as the main sponsor (Athanassoula 2003; Martinez-Valpuesta et al. 2006; Dubinski et al. 2009). Note that the trapping fraction of stellar orbits is independent of \( \lambda \).

A completely different picture emerges about the DM halo orbits trapped by the resonances (top frames). The CR resonance in the halo indeed remains the most efficient in trapping the DM orbits for all \( \lambda \), as noted before for \( \lambda = 0 \) models. But the fraction of trapped DM orbits by the CR depends on \( \lambda \), increasing monotonically with the spin. This increase in the efficiency of trapping correlates nicely with the increase in the fraction of prograde orbits in the halo Table 1.

The ILR resonance traps small amount of DM particles, which has been noticed already in Martinez-Valpuesta et al. (2006). In P00, the OLR is weak and the ILR is very weak. Other resonances are completely negligible. What is new here, is that the trapping ability of the ILR increases rapidly with \( \lambda \), much faster than that of the CR. For P90, the ILR is the second important resonance. Nearly the same effect occurs with the OLR. For P60 model, the OLR is barely more significant than the ILR, their roles have been reversed in P90, where the ILR dominates over the OLR. We also note the non-negligible trapping by higher resonances for larger \( \lambda \), especially for inner resonances with \( \nu > 0.5 \), but also for outer resonances with \( \nu < -0.5 \). When counted together, these resonances compete with the three main resonances in trapping efficiency of the DM orbits, especially the \( \nu = 1 \) resonance.

To estimate the contribution to the angular momentum transfer by resonance trapped orbits, we use the orbital spectral analysis for two time snapshots. We measure the angular momentum lost by the stellar bar to the outer disc and to halo during this time interval. The angular momentum lost due to resonant trapping DM orbits is obtained from difference in \( J \) of these orbits at these two snapshots. Finally, we subtract the resonant angular momentum lost from the total \( J \) lost—this \( J \) transfer is attributed to non-resonant interactions. The timing of two snapshots for the orbital spectral analysis is tuned to the moment the discs from the P00 and P90 models had lost the same amount of angular momentum. In the P00 model, we find that \( \sim 50 \) per cent of the total angular momentum lost is due to resonant exchange. In the P90 model, this number is \( \sim 88 \) per cent. These numbers reproduce the values of \( f \) (Table 1). This difference might seem small but it represents the angular momentum lost during a limited time, \( \sim 1 \) Gyr, of a pre-buckling evolution. In the long run of 10 Gyr, the P90 model disc loses much less \( J \) than P00 disc, because the bar is essentially dissolved after buckling and the \( J \) transfer is stopped. Overall, we observed a trend, in models with larger fraction of retrograde DM orbits, we find that the non-resonant \( J \) transfer is more significant, as long as the stellar bar persists.

So, we conclude that the fraction of particles trapped at specific resonances is directly responsible for the angular momentum redistribution in the disc–halo system. Section 4 discusses the measured angular momentum transfer from orbital spectral analysis presented in Fig. 11.

For the first time we show that the efficiency of orbit trapping by the resonances in spinning haloes depends on \( \lambda \) and is directly proportional to the fraction of the prograde DM orbits. At the same time...
Figure 6. Evolution of DM (solid lines) and stellar (dashed lines) bar sizes for models in the range of $\lambda \sim 0–0.09$. The stellar bar sizes have been determined from extension of $x_1$ orbits and measuring the bar ellipticity profile of isodensity contours in the $xy$-plane – to the radius where ellipticity has decreased by 15 per cent from its maximum (Martinez-Valpuesta et al. 2006; Collier et al. 2018). The DM bar size have been obtained using the ellipticity profiles.

3.4 Sizes and masses of DM bars and masses of overall DM response in spinning haloes

Collier et al. (2018) have calculated stellar bar sizes using two methods (Martinez-Valpuesta et al. 2006): measuring the maximal extent of an $x_1$ orbits from the characteristic diagram at each timestep, and by fitting ellipses to disc isodensity contours, obtaining the radial ellipticity profiles, $\epsilon(r)$, and determining where $\epsilon$ falls 15 per cent below its maximal value. For DM bar sizes, we apply the second method in the DM halo equatorial slice.

Fig. 6 shows the evolution stellar and DM bar sizes, $R_b$. One observes substantial differences between these two components. For low $\lambda$ haloes, the DM bars grow monotonically in size but remain a fraction of the corresponding stellar bars. For faster spinning haloes, $\lambda > 0.03$, we observe that DM bar length rivals that of associated stellar bar.

We observe three categories of $R_b$ evolution – growth, saturation, and decline. All closely correlated with their stellar counterparts. For $\lambda < 0.03$, the DM bar sizes grow monotonically. For the range of $\lambda \sim 0.03–0.05$, they saturate. And for the extreme, $\lambda \gtrsim 0.06$, the DM bar sizes decline sharply after the buckling of stellar bars. This corresponds to the dissolution of stellar bars.

We now estimate DM and stellar bar masses. For the stellar bars, we adopt the major semi-axes and ellipticities from Collier et al. (2018). We assume that the vertical thickness of a stellar bar is that of the disc and the peanut(boxy bulge). We count all the stellar masses within this volume.

For the DM bars, we use the above method to calculate their $R_b$ in the $xy$-plane. The ellipticity of the isodensity contour crossing $R_b$ in the equatorial plane provides us with the intermediate semi-axes of DM bars. Similarly, we find ellipticity of the isodensity contour which crosses the $R_b$ point in the $xz$-plane, where $x$-axis is oriented along a stellar bar. This gives us the minor semi-axes of DM bars. As a next step, we calculate the volume of the ellipsoid using its axes, and measure the mass inside this figure. But, we do not expect all the DM orbits within the ellipsoid to be trapped by the stellar bar, thus forming the DM bar.

The shapes of DM bars in our models are displayed in Fig. 7. We plot the surface density of trapped DM particles inside the CR of the stellar bar found from spectral analysis in Fig. 5. The top frames of the figure shows the face-on view of the DM bars. As $\lambda$
increases, the major and intermediate semi-axes of the DM bar also increase. The central region of the face-on P90 and P60 DM bars appear lopsided and dumbbell-shaped.

The bottom frames of Fig. 7 shows the edge-on view of the DM bars in all models. As $\lambda$ increases, the number of orbits trapped at higher $|z|$ increases as well. The increase of the vertical extent of trapped DM region with $\lambda$ is in sharp contrast with the trapped stellar particles in the stellar bar, which is confined to the $z$-extent of the disc, which is geometrically thin.

From orbital spectral analysis, we have obtained the percentage of trapped DM orbits by measuring the fraction of orbits inside the ellipsoid with calculated axes – orbits that are in resonance with the stellar bar. We have excluded all the trapped orbits outside the CR. The resulting DM bar masses are then normalized by the associated stellar bar masses and given in Table 2. Increasing $\lambda$ affects the DM bar sizes and masses substantially, by increasing the number of trapped orbits, while it has no effect or an adverse effect on the stellar bar masses. Therefore, in addition to the increased number of prograde orbits in the same volume, more orbits are trapped away from the equatorial plane, providing non-linear amplification to the DM bar strength. Finally, we note that the DM response involves additional resonant orbits outside the CR. This contribution is shown in the last line of Table 2.

From evolution of the obtained ratios of bar masses (Fig. 8), we note that in the low-$\lambda$ models, P00 and P30, the bar mass ratio drops abruptly with the stellar bar buckling, and continues to increase very slowly thereafter. For high-$\lambda$ models, P60 and P90, the bars basically dissolve, and no further measurements have been performed. Note that P30 resonances trap more DM orbits compared to P00 and hence its DM bar is more massive, and extends further out in $R$ and $z$, as is evident from Fig. 7. In the highest spin halo, P90, the DM bar contributes more than 40 per cent of the stellar bar mass. Our error in the above estimates involves resonant DM orbits which lie outside the volume delineated by $R = 29$ kpc and $|z| = 10$ kpc – altogether $\sim 3$ per cent of resonant DM orbits. For the edge on estimation, we look at the limit of $|y| = 10$ which in the $xy$-plane.

The above trend in the DM bars can be extended to the overall DM response to perturbation by the underlying stellar bar. Fig. 9 displays the full response, which includes the DM bars and associated gravitational wakes which extend from inside the corotation radius to the OLR. Both the extent and the amplitude of the response are growing with the halo spin.

### Table 2. Estimates of DM and stellar bar masses near their maximal strength in the pre-buckling stage, in units of $10^{10} M_\odot$. Also shown are the ratios of DM-to-stellar bar masses. The lower line displays the masses of resonant DM orbits outside the CR, in units of $10^{10} M_\odot$.

|        | P00  | P30  | P60  | P90  |
|--------|------|------|------|------|
| DM bar mass | 0.30 | 0.52 | 0.81 | 1.05 |
| Stellar bar mass | 2.49 | 2.49 | 2.49 | 2.49 |
| Ratio (DM/stellar) | 0.12 | 0.21 | 0.33 | 0.42 |
| Mass of DM response | 0.29 | 0.33 | 0.34 | 0.46 |

### 3.5 Angular momentum transfer for prograde and retrograde DM orbits

The DM halo of the P00 model consists of equal fractions of prograde and retrograde DM orbits, with respect to the disk rotation. These fractions change when moving along the $\lambda$ sequence, as shown in Table 1. As a next step, we analyse contributions of prograde and retrograde DM orbits to the rate of angular momentum transfer along the $\lambda$ sequence.

To calculate the angular momentum flow in the system, we use the method prescribed in Villa-Vargas et al. (2009) and also used in Long et al. (2014) and Collier et al. (2018). We bin the halo into cylindrical shells of $\Delta R = 1$ kpc, parallel to its rotation axis, and create a 2D map of the rate of change of $J$, i.e. $\dot{J}$, in each shell as a function of $R$ and time. The maps are colour coded to show positive transfer of $\dot{J}$ in red and negative transfer of $\dot{J}$ in blue. The colour palette has been normalized the same way for each figure. This method follows the total angular momentum rate of transfer between the disc and the DM halo, though here we show only the halo. As we discuss in Section 4, the $J$ transfer is not only limited to disc-to-halo, but the DM halo also ‘talks’ to itself. The disc has been displayed in Collier et al. (2018).

As a first step, we calculate $\dot{J}$ – the rate of the angular momentum transfer to, away, and within the DM halo for models along the $\lambda$ sequence (Fig. 10, top frames). These frames have been shown already in Collier et al. (2018) in lower resolution. The P00 model halo exhibits a pure absorption of $J$ from the disc. Three resonances appear prominent in its frame – the ILR, CR, and OLR. They move out with time due to the stellar bar slowdown. These resonances can be traced as well after buckling. Most of the transfer happens close to the time when the $A_2$ of the stellar bar is at the maximum of $A_2$ (before and after buckling). In this model of a non-rotating halo, the stellar bar strength recovers after buckling and reaches its pre-buckling maximum at $t \sim 10$ Gyr (e.g. Fig. 2).

In the P30 model, prominent changes occur. The halo ILR region shows no emission but a weak absorption of $J$. The $J$ transfer rate in the OLR region is enhanced and the CR rate stays unchanged and weakens thereafter. In this model, the stellar bar although recovers part of its original strength after buckling, it falls short of the maximal $A_2$ in the pre-buckling stage.
The P60 model shows a dramatic difference when compared with the lower $\lambda$ models. The CR dominates in absorption, while the ILR is now very prominent in emission. This trend continues to the P90 model. In these two models, the stellar bars do not grow after the buckling. In fact, the loss of strength in buckling is substantial, and the bars dissolve into weak oval distortions.

By separating the angular momentum flow for prograde and retrograde DM orbits, we gain some insight into $J$ redistribution in the system. The middle row in Fig. 10 displays the $J$ along the $\lambda$ sequence for prograde orbits only. For the P00, separation into prograde and retrograde orbits underlines the diminishing role of the ILR for former orbits compared to the upper row of all orbits, and appearance of a very weak absorption there. On the other hand, the retrograde orbits contribute massively to $J$ in the ILR region.

With increasing $\lambda$ we observe an increasing emission by the ILR for prograde orbits, decreasing absorption and switching from emission to absorption, with an overall decrease of importance of $J$ flow in the ILR region. This can be explained by a fractional decrease of the retrograde orbits along the $\lambda$ sequence. When comparing Fig. 10 with Fig. 5, we see that the non-negligible increase in halo emission of $J$ coincides with the increase in resonant orbits within the ILR, especially at $v = 1$.

**4 DISCUSSION**

Using high-resolution numerical simulations, we analyse evolution of the DM bars in disc–halo systems over 10 Gyr, which form in response to the stellar bars in the embedded discs. We focus on spinning DM haloes with cosmological spin $\lambda \sim 0–0.09$, and investigate how $\lambda$ affects the evolution of DM bars, their morphology, strength, mass, size, the flow of angular momentum in these systems, and implications for the stellar disc evolution. For this purpose, we use a representative model for DM mass and angular momentum distributions, leaving consideration of a wide range of these to future work.

We start with outlining our main results. First, we find that the maximal strength of induced DM bars depends strongly on the halo spine, while the maximal strength of stellar bars is indifferent to $\lambda$. Hence, the maximal strength of DM bars depends on the fraction of prograde DM orbits in the host haloes. Second, the efficiency of trapping DM halo orbits by the resonances, including the ILR, OLR, and CR, as well as higher resonances, depends on $\lambda$. We show this explicitly by means of the orbital spectral analysis. It remains important to show that increase in the trapping efficiency results in increase in the angular momentum transfer by the resonances, and we address this issue below.

Third, higher resonances, inside the ILR and outside the OLR, become progressively more important in trapping the DM orbits and hence for angular momentum flow in the system with $\lambda$. We quantify this transfer below. The DM haloes not only receive their $J$ from the discs, but actively transfer it to larger radii by means of these resonances. In this respect, the halo ‘talks’ not only to the underlying stellar disc but to itself as well.

Fourth, we analysed the roles of prograde and retrograde DM orbits contribution to $J$. The prograde orbits dominate absorption of $J$ at CR for all $\lambda$, and show an increasing emission of $J$ with $\lambda$. On the other hand, the retrograde orbits absorb at low $\lambda$, and emit $J$ at higher $\lambda$.

The robustness of stellar bars was called into question when halo spin has been introduced (Long et al. 2014; Collier et al. 2018). The halo spin $\lambda$ has a dramatic effect on the dynamical and secular evolution of the stellar bar, and this holds true for DM bars as well. Because the DM bars represent the halo response to the stellar bars perturbation, it is not surprising that they mimic evolution of stellar bars to a larger extent. What is surprising is that the DM response is so non-linear – one cannot predict the amplitude of a DM bar by simply scaling it with the stellar bar. Its behaviour is more complex than this.

The maximal amplitude of stellar response in galactic discs appears to be completely indifferent in its dynamical stage (i.e. pre-buckling) to the host halo spin. This is in a sharp contrast with the DM response – in the range of $\lambda = 0–0.09$, the amplitude of DM bar varies by factor of $\sim 3.5$. What is the reason for such a sharp difference between the stellar and DM responses?

Plausibly, the answer lies in the increase of the number of prograde DM orbits, but this increase is less than a factor of two, as shown by Table 1. Note that even in $\lambda = 0$ halo, 50 per cent of the DM orbits are prograde. Hence, additional cause must be at play. A cause which is absent in the case of a stellar disc.

Orbits in a stellar disc prior to bar instability are largely prograde, and the geometrical thickness of the disc naturally limits their $z$-extent. On the other hand, even for the $\lambda = 0.09$ spherical halo, when almost 90 per cent of DM orbits are prograde, the vertical extent of the DM orbital trapping is not strictly limited, and higher latitude orbits can be affected and trapped, for example by orbits closer to the equatorial plane coupling to the higher $z$-orbits. In this case, we should observe that both the mass and size of a DM bar grow with the spin. This is exactly what is shown in Fig. 7 and
Dark matter bars in spinning haloes

Figure 10. Rate of angular momentum, $J$, emission, and absorption by prograde and retrograde DM halo orbits as a function of a cylindrical radius $R$ and time, along the $\lambda$ sequence. The colour palette corresponds to gain/loss rates in $J$ (i.e. red/blue), using a logarithmic scale in colour. The cylindrical shells binned at $\Delta R = 1$ kpc and extend to $z = \pm 10$ kpc. The top row includes both prograde and retrograde orbits in the DM halo. The middle row – only the prograde orbits, and the bottom row – only the retrograde orbits. The unit of angular momentum transfer rate used in the colour palette is $10 M_\odot \text{kpc km s}^{-1} \text{yr}^{-1}$.

Table 1. Moreover, this can be seen directly in Fig. 10, where loss of angular momentum in P60 and P90 models can be observed for DM orbits lying close to the equatorial plane. This $J$ is absorbed by the higher altitude DM orbits.

We conclude that the main difference in DM response compared to the stellar response lies in the availability of orbits capable of resonating with the perturber and being trapped by numerous resonances. The orbital spectral analysis provides the necessary insight into properties of DM bars, their evolution and the associated intricacies of angular momentum transfer within the disc–halo system. Fig. 5 exhibits monotonous increase in the trapping efficiency with $\lambda$ of the three main resonances and additional higher resonances up to $|\nu| \sim 1.5$ searched by us. This increase can be observed already in P30 compared to P00, and is supported by similar frames in Fig. 7. For higher $\lambda$, the change is much more dramatic.

This progression in DM response with $\lambda$ was not observed by Petersen et al. (2016), because they limited their models by $\lambda = 0.03$, and hence passed over this effect. We have tested all our haloes without and with frozen stellar disc potential, and found that all haloes are stable over 10 Gyr, and that previous claims in the literature about developing instabilities in these systems are not substantiated by our analysis, which extends to $\lambda \sim 0.108$ – the maximal spin attainable in spherical NFW haloes (Section 3.1).

We have compared the properties of DM bars, e.g. their masses, for P00 and P30 models, with Petersen et al. (2016), and find a very good agreement. Specifically, the DM-to-stellar bar mass ratio is $\sim 0.1$ for P00. We find, however, that the DM response goes well beyond the DM bar, and additional DM mass participates in the gravitational wake, as shown in Table 2. For $\lambda \leq 0.03$ models, we do observe a very slow growth for the mass ratio of the bars, after the buckling, again in agreement with Peterson et al. Yet during the buckling instability of stellar bars, we find that this ratio drops substantially. For higher $\lambda$, the secular growth is completely suppressed and both types of bars essentially dissolve during buckling.

In addition to varying in length and mass, we have measured the angular separation between the stellar and DM bars. With stellar bar leading, this angle decreases with $\lambda$, if measured for same strength of stellar bars. This is another indication that DM bars become stronger with $\lambda$ (Fig. 3).

Next, we deal with the efficiency of $J$ transfer from the stellar disc to the DM halo by orbits trapped at the resonances. Specifically, we ask what is the fraction of transferred $J$, attributed to the action of the resonances, compared to the total amount of $J$ transferred over a fixed time interval. To answer this question, we choose two moments of time, $t_1$ and $t_2$ for each model, P00 and P90, when stellar discs lose identical amounts of angular momentum. In other words, we choose these times in such a way that each of the stellar discs have lost the same amount, $\Delta J(P00) = \Delta J(P90)$. Because the total $J$ should be conserved, the DM haloes in both models should absorb the same amount of the angular momentum.

We calculate the amount of $J_{\text{res}}$ absorbed by the halo resonances in each model during this time interval, using our method of orbit spectral analysis. Then find the ratio $J_{\text{res}}/\Delta J$, for each model and compare them. This will provide a rough estimate of the efficiency of $J$ transfer by the resonances as a function of the halo spin $\lambda$.

Note that Lynden-Bell & Kalnajs (1972) have demonstrated that $J$-transfer happens at the resonances. But they did not rule out some contribution from non-resonant torques to this process. On
the other, orbits that are non-resonant at some time could resonate at some other time, making the careful check a difficult task. We limit our conclusions to specific times in the evolution of the system only, leaving the more general conclusions outside the scope of the present work.

Fig. 11 shows the results of our analysis, for P00 and P90 models, and includes the orbital trapping in the stellar discs and DM haloes, as well as calculation of angular momentum change at each resonance, up to $|v| = 1.5$, between $t_1$ and $t_2$. This figure confirms that the stellar orbital trapping is independent of $\lambda$, i.e. it is the same in P00 and P90, even comparing individual resonances. The disc lost most of the angular momentum by the ILR, and absorbed a small amounts of angular momentum by the CR and the OLR.

The halo action is very different. The main resonance absorbing $J$ is the CR, but in P90 it absorbs by far more angular momentum than in P00. Where does the excess of $J$ come from? We note that all the other resonances become much more active in P90 halo compared to P00. Some absorb and some emit $J$, but net $J$ is absorbed. In fact, only resonances inside the CR lose $J$, while the outside CR resonances absorb it. We return to this point below, and see it as a proof that the inner DM halo exchanges $J$ with the DM further out – the halo ‘talks’ to itself.

We now compare the numbers obtained from orbital analysis of Fig. 11. We define the angular momentum unit as $10^{10} M_\odot$ kpc km s$^{-1}$, and all angular momentum values in the following are given in these units. The total $J$ lost by P00 and P90 discs, $\Delta J \approx 278$, is the same by construction. The net resonant transfer using $\Delta J_{\text{res}}$ in P00 halo is $\approx 138$, and in P90 is 246. The net transfer by non-resonant torques in P00 and P90 respectively is about 139 and 31. We can estimate the fraction of $\Delta J$ transferred by the resonances, $\sim 50$ per cent in P00, and 88 per cent in P90. Hence, the efficiency of resonant $J$ transfer is scaling directly with the fraction of prograde orbits in the DM halo, $f$, and therefore with its spin $\lambda$.

An additional interesting point about the angular momentum transfer by the resonances can be observed in the P90 halo frame of Fig. 11. This halo obtained about a net of $\Delta J_{\text{res}} \sim 246$ in resonant transfer. This net angular momentum is made of absorbed $\sim 589$ and emitted $\sim 341$. Where is this excess of absorption coming from? It cannot come from the disc, as this number is quoted above and is smaller. However, Fig. 11 reveals that the excess absorption by the CR and the OLR originates from the emission by the ILR and higher resonances inside the CR. The puzzle is resolved by Fig. 10 – this emission by the inner resonances comes from the inner halo, $<10$ kpc, while absorption is performed by the region of DM halo at larger radii, $\sim 10$–30 kpc. This is what we meant above by the DM halo in faster spinning haloes ‘talks’ to itself.

We have discussed the rates of the $J$ transfer along the $\lambda$ sequence in Section 3.5. Here, we take a closer look at these rates, focusing separately on prograde and retrograde orbits in the DM haloes. The two bottom rows of Fig. 10 show the angular momentum transfer by prograde and retrograde DM orbits, respectively. Moving along the $\lambda$ sequence, in the P00 model, the retrograde orbits primarily gain angular momentum, and doing this more intensely than the prograde ones. When $\lambda$ increases, the prograde orbits switch to losing $J$ within the inner 10 kpc. This change in $J$ is accompanied with an increase of absorption by the DM halo orbits at larger radii. At the same time, the retrograde orbits play a lesser role in the absorption, and switch to emission inside the inner 10 kpc. Moving along $\lambda$, there is a smaller number of retrograde orbits at $t = 0$, by construction. So, the P90 model displays only emission in the inner region, before the stellar DM bars dissolve following buckling.

The total number of DM and stellar particles is conserved in these simulations, but what about the ratio of retrograde to prograde DM orbits, $\alpha$? Fig. 12 shows this ratio for two models with extreme halo spins, P00 and P90, and doing this at $t = 0$ and 10 Gyr.

Clearly, $\alpha$ evolves with time, and in a non-uniform manner. At $t = 0$, $\alpha$ is constant with radius for all models. For P00, where $\alpha = 1$ for $t = 0$, at the end of the simulation, $\alpha \sim 0.9$ and is still relatively constant with radius. But for higher $\lambda$ models, at later time, $\alpha$ develops a profile in $r$, and increases substantially at smaller radii for higher $\lambda$ models. For example, in P90, as the stellar bar (and simultaneously the DM bar) develops and gains strength, the number of prograde orbits decreases within the stellar disc radius, more so in the central region, which becomes prograde. The prograde orbits become retrograde when they are trapped by the bar. The DM bar emits $J$ as these retrograde particles become trapped. We observe this loss in Fig. 10. Within the region of central $\sim 10$ kpc. The ratio $\alpha$ has increased from $\sim 0.12$ at the start of the simulation, to $\sim 0.7$. 

Figure 11. Top: P00 model – fractions of trapped stellar orbits and the change of angular momentum, $\Delta J$, by these orbits (left-hand frame), and fractions of trapped DM orbits and their $\Delta J$ (right-hand frame). Bottom: P90 model – same as P00. Trapping has been calculated for the three main resonances (ILR, CR, and OLR) and for higher resonances, up to $|v| = 1.5$ (see Fig. 5 for more details). The $\Delta J$ units are $10^{10} M_\odot$ kpc km s$^{-1}$. Note, the halo and disc $y$-axes are scaled differently for clarity. Note, the stellar bars exhibit the same trapping ability and lose identical $\Delta J$ in both models. Hence, both P00 and P90 haloes gain the same $\Delta J$, with a notable difference: the resonant $J$ transfer dominates over the non-resonant one in P90, while not in P00 (see the text for more details). 

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One can ask whether the ‘depth’ of the buckling instability, as measured by the stellar $A_2$, depends on the number of retrograde orbits in the halo. Our initial conditions have identical discs embedded in DM haloes of a differing spin. Analysing the resonance trapping by the stellar disc at similar bar strengths inside the spinning haloes (Section 3.3), we found that the trapping efficiency of stellar orbits correlates strongly with $\lambda$. We also measured $A_2$ and found that pre-buckling bar strength does not depend on the halo spin. Why then does the loss of strength by the stellar bar during buckling vary so much with $\lambda$?

Fig. 12 provides some, yet not fully compelling explanation for this effect. Substantial loss of strength by the stellar bar during the buckling instability is accompanied by equal loss of strength by the associated DM bar. The DM orbits de-correlate their orientation during this short time period. This is expected to lead to a sharp reduction in the efficiency of resonance trapping. As shown in Collier et al. (2018), the disc is heated up by the appearance of a large number of low angular momentum orbits released by the bar. DM bars follow this trend and release the DM orbits. A larger fraction of these orbits are retrograde for large $\lambda$, and most of them are found in the stellar bar region, as shown in this figure. Because DM bars strength depends on $\lambda$, and increases with it, this de-correlation will have a stronger effect on higher spin haloes compared to low spin ones, and on the underlying stellar discs.

We note, that models with strong stellar bars are expected to be accompanied by a strong DM bar component. This is true especially for haloes with larger spin, but only for $\lambda > 0$! If the halo spin can be measured, a strong case can be made for aiming direct detection DM experiments at these spinning haloes with strong stellar bars, as we expect a large volume and DM mass to be trapped and accompanying the stellar bar.

Our choice of $J$ distribution with radius is not unique but is severely constrained by the NFW density profile and the halo shape. Within these limits, one can imagine the following $J$ distribution – the inner haloes remains as in P00 model, while the outer halo has the fraction of prograde particles modified, i.e. $f \sim 0.88$, as in P90 model. How does this affect the stellar bar evolution?

To test this, we have analysed the rate of $J$ transfer in our models, and especially the spatial extent of $J$ map in Fig. 10. We find that the part of the DM halo that is active in $J$ redistribution extends to the box of $R \sim 30$ kpc and $z \sim 10$ kpc, few time the stellar bar size. The stellar disc communicates with the halo within this volume, which includes the volume in which the halo ‘talks’ to itself. The angular momentum outside this box has no effect on the interior and so on the stellar disc. Yet, one should exercise caution in this respect.

If $J$ is injected from outside into the halo surrounding the above box, it can and will propagate inwards on a secular time-scale. As this process is irreversible, it reminds us of a ‘diffusion.’ Strictly speaking, it cannot be applied to a collisionless system, but was nevertheless argued by Lynden-Bell (1967), by introducing coarse- and fine-grained distribution functions. We have tested this by spinning up the P00 halo to $f \sim 0.88$, but only outside 40 kpc, and observed the $J$ slow propagation inwards.

In summary, the total DM response to the stellar bar involves mass of the order of the stellar mass, as follows from Table 2.

## 5 Conclusions

We present results of high-resolution numerical simulations of stellar discs embedded in spherical DM haloes, with a range of cosmological spin parameter, $\lambda \sim 0.00–0.09$, and a representative angular momentum distribution. In this work, we focus on the DM response to the developing stellar bars in the underlying galactic discs. This DM halo response evolves as a DM bar, and we investigate its role in the angular momentum transfer in the disc–halo systems. To address an ambiguity regarding global stability for spinning spherical haloes, we have also tested models of discless spinning DM haloes, for $\lambda \sim 0.00–0.108$. The upper limit of $\lambda$ follows from a model with all DM particles on prograde orbits. Moreover, we have tested the model for DM halo with $\lambda = 0.09$ and a frozen stellar disc potential.

Our main conclusions are as follows:

(i) We have shown that discless DM spinning haloes are stable to bar instabilities, and maintain their shape and velocity distributions. We have tested a range of haloes, $\lambda = 0.00–0.108$ and found that all our DM haloes are stable in the absence of embedded stellar disks. We put to rest the idea that these haloes can be globally unstable and develop the $m = 2$ Fourier mode. DM haloes with $\lambda = 0.09$ remained stable even in the presence of an embedded frozen disc potential. Hence all the DM bars obtained in our simulations have been triggered by the developing stellar bars, i.e. have been induced by them.

(ii) The strength of DM bars depends strongly on the DM halo spin, and, therefore, on the fraction of prograde orbits within a

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1This statement should be taken with additional constraints imposed by the halo shape and the NFW profile, as discussed in Section 4.
volume substantially larger than the stellar disc radius. The maximal Fourier amplitude of the DM bars increases by a factor of $\sim 3.4$ when $\lambda$ increases from 0 to 0.09. This is in sharp contrast with stellar bars whose pre-buckling amplitude is independent of $\lambda$. For a disc containing about 98 per cent of its mass within 17 kpc, the DM volume affected lies within a sphere with $\sim 30$ kpc radius.

(iii) The efficiency of resonance trapping of DM orbits by the stellar bar increases with $\lambda$. This includes the main resonances, the ILR, CR, and the OLR, as well as higher resonances, both inside and outside the CR. This was shown by invoking orbital spectral analysis.

(iv) The angular momentum transfer from stellar discs to DM parent haloes is maintained by both resonant and non-resonant orbits. But the efficiency of angular momentum transfer by the resonances increases with $\lambda$, from about 50 per cent at $\lambda = 0$ to $\sim 88$ per cent for $\lambda = 0.09$. Furthermore, different resonances exchange the angular momentum with increasing $\lambda$. For example, the ILR emits $\lambda$ while the CR and the OLR absorb it. In other words, at higher $\lambda$, the halo ‘talks’ to itself by means of exchanging angular momentum, which flows from the inner resonances, e.g. the ILR, to larger radii, e.g. the CR and the OLR. Higher resonances become more important with an increase of halo spin.

(v) Prograde and retrograde DM orbits play different roles in the angular momentum transfer in disc–halo systems. Prograde orbits dominate absorption of angular momentum at the CR for all $\lambda$, while retrograde orbits absorb at low $\lambda$ and emitting the angular momentum at higher $\lambda$. The fraction of retrograde DM orbits increases with time during the bar instability, compared to the initial conditions, where the fraction of retrograde orbits is constant with radius, by construction. This increase is more profound with increasing $\lambda$.

(vi) We find that the mass, length, and shape of DM bars have a strong dependence on the parent halo spin. The most massive, long, and strong DM bars are found in spinning haloes before buckling.

(vii) The existence of the DM bar requires a stellar bar. The angle between the stellar and DM bars decreases with an increasing halo spin. The DM bars lag behind the stellar bars.

(viii) The overall DM response to the underlying stellar bar involves mass of the order of the stellar bar mass. Furthermore, the active part of the DM halo which participates in the angular momentum redistribution in the system constitute a volume of $R \lesssim 30$ kpc and $|z| \lesssim 10$ kpc.

Stellar bars have been studied numerically for about half a century. Yet they still pose unanswered questions. To a large extent they are the prime internal factor which shapes disc galaxy evolution. What makes them even more interesting is that they provide a strong link to host DM haloes. Study of angular momentum exchange between the disc and halo components can shed new light on the evolution of galaxies which is driven by competition between internal and external factors. The current work will be followed by a careful study of the effect of DM mass and angular momentum distributions on the bar evolution.

Cosmological simulations will approach the minimal resolution required to study interactions in disc–halo systems in the near future. Both, galactic discs and their host DM haloes provide multiple tracers of internally and externally driven galaxy evolution. The second release of data from Gaia (Gaia Collaboration et al. 2018) cross-matched with SDSS (Sloan Digital Sky Survey) reveals radius–velocity phase-space correlations originated during mergers which occurred over the last few Gyrs. Numerical simulations of Milky Way-type DM haloes have predicted that such ‘streamers’ will be present in the DM haloes, and accumulate since $z \sim 1$ (Romano-Diaz et al. 2009). Recent mixture model analysis of SDSS–Gaia DR2 catalogue has confirmed this effect (Necib, Lisanti & Belokurov 2018). Additional ‘archaeological’ work will uncover other relics imprinted on the halo kinematics during its buildup history.

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