Recent years we are witnesses of a great progress in calculations of multiloop amplitudes (see e.g. 11-12 and refs. therein) an important part of which is related to the applications and development of the Britto-Cachazo-Feng-Witten (BCFW) approach 3. This first allowed to obtain Britto-Cachazo-Feng (BCF) recursion relations for tree amplitudes in D=4 Yang Mills and N = 4 supersymmetric Yang-Mills (SYM) theory 4,5 and then was developed for the case of superamplitudes of N = 4 SYM 4,10, loop (super)amplitudes and N = 8 supergravity 11,12 (see 11,12 for more references). To lighten the text, below we will mainly omit 'super' in superamplitudes, calling them amplitudes.

This approach was generalized for the tree amplitudes of D=10 SYM model in 13, but then mainly used in the context of type IIB supergravity 14-17 where the presence of complex structure allowed to lighten the 'Clifford superfield' description of amplitudes in 13. The observation that the constrained bosonic spinor helicity variables used in 13 can be identified with spinor moving frame variables of 18-20 (or equivalently, with Lorentz harmonics of 21,22,44) allowed us to simplify it's N = 1 version 25 and also to generalize it to the case of D = 11 supergravity 15. The results of this 11D generalization of the on-shell superfield description of tree amplitudes and of the BCFW recurrent relations for these will be reported in this letter.

The BCFW recursion relations 2 are written for n-particle tree amplitudes \( A^{(n)}(p_1, \ldots, p_n) \) in spinor helicity formalism, in which the information on the (light-like) momentum \( P_{\mu(i)} \) and on helicity of the \( i \)-th external particle is encoded in the bosonic spinor \( \lambda_{\mu(i)} = (\hat{\lambda}_{\mu(i)})^* \). The light-like momentum is defined by Cartan-Penrose representation (see 29 and refs. therein)

\[
P_{\mu(i)} \sigma_{AA}^\mu = 2 \lambda_{\mu(i)} \bar{\lambda}_{\mu(i)} \iff P_{\mu(i)} = \lambda_{\mu(i)} \sigma_{\mu \lambda(i)} \text{, (1)}
\]

where \( \sigma_{\mu \lambda} \) are relativistic Pauli matrices, \( A = 1,2 \) and \( \bar{A} = 1,2 \) are Weyl spinor indices and \( \mu = 0, \ldots, 3 \).

All n-particle amplitudes for the fields of the \( N=4 \) SYM can be described by a superfield amplitude (super-amplitude) 2,10

\[
A^{(n)}(\lambda_{\mu(1)}, \bar{\lambda}_{\bar{\mu}(1)}, \eta_{(1)}; \ldots; \lambda_{\mu(n)}, \bar{\lambda}_{\bar{\mu}(n)}, \eta_{(n)})
\]

depending, besides \( \lambda_{\mu(i)} \) and \( \bar{\lambda}_{\bar{\mu}(i)} \), on the set of n complex fermionic coordinates \( \eta_{(i)} \) and \( \bar{\eta}_{(i)} \) (first introduced in 30),

\[
\eta_{(i)} \eta_{(i)}^P = -\bar{\eta}_{(i)} \bar{\eta}_{(i)}^P, \quad \eta_{(i)} \eta_{(i)} = -\bar{\eta}_{(i)} \bar{\eta}_{(i)}
\]

considering the index \( q = 1, \ldots, 4 \) of the fundamental representation of \( SU(4) \). These superfield amplitudes are multiparticle counterparts of the so-called on-shell superfield

\[
\Phi(\lambda, \bar{\lambda}, \eta^P) = f(-)(\lambda, \bar{\lambda}) + \eta^P \lambda q + \frac{1}{2} \eta^P \eta^P \epsilon_{pq} \lambda^q + \frac{1}{3} \eta^P \eta^P \eta^P \epsilon_{pq} \lambda^q \Phi(\eta^P) f(+)
\]

(2) describing all the states of the linearized SYM provided it obeys the so-called helicity constraint 22,30,

\[
\hat{h} \Phi(\lambda, \bar{\lambda}, \eta) = \Phi(\lambda, \bar{\lambda}, \eta)
\]

(3)

\[
2 \hat{h} := -\lambda^A \frac{\partial}{\partial \lambda^A} + \bar{\lambda}^A \frac{\partial}{\partial \bar{\lambda}^A} + \eta^a \frac{\partial}{\partial \eta^a}
\]

(4)

The n-particle on-shell superfield amplitudes of 4D \( N=4 \) SYM, \( A^{(n)}(\lambda_{(1)}, \bar{\lambda}_{(1)}, \eta_{(1)}; \ldots; \lambda_{(n)}, \bar{\lambda}_{(n)}, \eta_{(n)}) \equiv A^{(n)}(...; \lambda_i, \bar{\lambda}_i, \eta_i; ...), \) should obey the set of n helicity constraints,

\[
\hat{h}(\lambda_i) A^{(n)}(...; \lambda_i, \bar{\lambda}_i, \eta_i; ...) = A^{(n)}(...; \lambda_i, \bar{\lambda}_i, \eta_i; ...) \text{, (5)}
\]

with \( 2 \hat{h}(\lambda_i) := -\lambda^A \frac{\partial}{\partial \lambda^A} + \bar{\lambda}^A \frac{\partial}{\partial \bar{\lambda}^A} + \eta^a \frac{\partial}{\partial \eta^a} \).

We refer to 4,10 for the superfield generalization of the original D=4 BCFW recurrent relations 3, and pass to the 11D generalization of the spinor helicity formalism.

1. Spinor helicity formalism in D=11.

Let us denote the D=11 vector indices by \( a, b, c = 0,1, ..., 9,10 \), spinor indices of \( SO(11,10) \) by \( \alpha, \beta, \gamma, \delta = 1, ..., 32 \) and D=11 Dirac matrices by \( \Gamma_{\alpha \beta} \). In our mostly minus notation, \( \eta^{ab} = \text{diag}(-1,-1,-1,-1) \), both \( \Gamma_{\alpha \beta} \) and the charge conjugation matrix \( C^{\alpha \beta} = -C^{\beta \alpha} \) are imaginary. We will also use the real symmetric matrices \( \Gamma_{\alpha \gamma} = \Gamma_{\alpha \gamma} = \Gamma^2_{\alpha \beta} \) and \( \Gamma_{\alpha \beta} = C^{\alpha \gamma} \Gamma_{\gamma \beta} = \Gamma_{\alpha \beta} \).

The light-like momentum of a massless 11D particle can be expressed by the relations similar to (4),

\[
k_a \Gamma_{\alpha \beta} = 2 \rho^\# v_{\alpha q} v^\#_{\beta q}, \quad \rho^\# v_{\alpha q} \Gamma_{\alpha \beta} v_p = k_a \delta_{qp} \text{, (6)}
\]

in terms of 'energy variable' \( \rho^\# \) and a set of 16 constrained bosonic 32-component spinors \( v_{\alpha q} \), \( q, p = 1, ..., 16 \), which can be identified with D=11 spinor moving frame variables 31,33 or Lorentz harmonics 34. Essentially, the constraints on \( v_{\alpha q} \) are given by Eq. (9) supplemented by \( v_{\alpha q} C^{\alpha \beta} v^\#_{\beta q} = 0 \), and by the requirement that the rank of 32\times16 matrix \( v_{\alpha q} \) is equal to 16. We refer to 32,33 for the complete description and discussion of...
the constraints and gauge symmetries of the spinor moving frame formalism for 11D massless superparticle and only notice that, taking all these into account, the variables \( v_{aq} \) can be considered as homogeneous coordinates on \( S^9 \), the celestial sphere of a D=11 observer,

\[
\{v_{aq}\} = S^9 .
\]  

(7)

The sign superindices \(-\) and \(\#\) \(\equiv ++\), carried by \(v_{aq}^-\) and \(\rho^\#\), characterize their scaling properties with respect to \(SO(1,1)\) gauge symmetry of the spinor moving frame (or Lorentz harmonic) approach to massless (super)particle.

One can check that, due to (6) and \( v_\gamma^a C v_p^- = 0 \), the momentum vector \(k_\gamma\) is light-like, \(k_\gamma k^{\alpha} = 0\), and moreover that the spinor moving frame variables \(v_{aq}\) obey the massless Dirac equation (in momentum representation)

\[
k_\alpha \Gamma^\alpha_{\alpha\beta} v_{aq}^- = 0 \quad \leftrightarrow \quad k_\alpha \Gamma^\alpha_{\alpha\beta} v_{aq}^- = 0 .
\]  

(8)

The 11D counterpart of the 10D spinor helicity variables of (13) are \(\lambda_{aq} = \sqrt{\rho^\# v_{aq}^-}\) the counterpart of the polarization spinor of the 10D fermionic field in D=11 is given by the same helicity spinor but with risen spinor index, \(\lambda^\alpha = \sqrt{\rho^\# v_{aq}^-} = i C^{\alpha\beta} \lambda_\beta \equiv (\lambda_q^\alpha)^+\).

One notices that Eqs. (9) can be written as \(\Gamma^\alpha_{\alpha\beta} \lambda_\beta = 2 \lambda_{aq} \delta_{\alpha q}\) and \(\lambda_{\alpha q} \lambda_{\beta q} = k_\gamma \delta_{\alpha q}\). However, the energy variable \(\rho^\#\) and its canonically conjugate coordinate \(x^\gamma\) play an important role in our construction below. In particular the D=11 counterpart of the on-shell superfields are defined on superspace

\[
\Sigma^{(10|16)} : \quad \{(x^\gamma, v_{aq}^-, \theta^-_q)\} ,
\]  

(9)

with bosonic sector \(R \otimes S^9\) (see (7)) including \(R = \{x^\gamma\}\).

2. D=11 on-shell superfields

The description of linearized 11D supergravity multiplet by superfields in the on-shell superspace (9) was proposed in \cite{34} (and can be reproduced when quantizing the massless 11D superparticle \cite{29}). It was given in terms of a bosonic antisymmetric tensor superfield \(\Phi^{IJK} = \Phi^{[IJK]}(x^\gamma, \theta_q^-, v_{aq}^-)\) which obeys

\[
D_+^a \Phi^{IJK} = 3 i \gamma^{[I} \gamma_{\gamma p}] \Phi^{JK]} , \quad \gamma^{[p} \Phi^{JK]} = 0 .
\]  

(10)

Here \(I, J, K = 1, \ldots, 9, q, p = 1, \ldots, 16\), \(\gamma_{lq}^1 = \gamma_{pq}^1\) are dirac matrices, \(\gamma^J \gamma^I = \delta^{[J} \delta_{I]}_{16 \times 16}\), and

\[
D_+^a = \partial_+^a + 2 i \theta^q_\gamma \partial_\gamma^- = \frac{\partial}{\partial \theta^q_\gamma} + 2 i \theta^q_\gamma \frac{\partial}{\partial x^\gamma} .
\]  

(11)

is the fermionic covariant derivative obeying the d=1, \(N = 16\) supersymmetry algebra \(\{D_+^a, D_+^b\} = 4 i \delta^{pq}_{\gamma \delta} \partial_\gamma^-\).

The consistency of Eq. (10) requires that fermionic superfield \(\Psi_{\gamma p}^I\) satisfies, besides \(\gamma_{lq}^1 \gamma_{pq}^1 = 0\),

\[
D_+^a \Psi_{\gamma p}^I = \frac{1}{16} \left( \gamma^{[IJK} + 6 \delta^{[J} \gamma_{\gamma p]} \right) \partial_\gamma^- \Phi^{JK]} + 2 \partial_\gamma^- H_{IJK} \gamma_{\gamma p}^1 ,
\]  

(12)

with symmetric traceless \(SO(9)\) tensor superfield \(H_{IJ} = H_{\{IJ\}}\), obeying

\[
D_+^a H_{IJ} = i \gamma_{\gamma p}^{[I} \Psi_{\gamma p}^J , \quad H_{IJ} = H_{JI}, \quad H_{II} = 0 .
\]  

(13)

The leading component of this bosonic superfield, \(h_{IJ}(x^\gamma, v_{aq}^-) = H_{IJ} |v_{aq}^- = 0\), describes the on-shell degrees of freedom of the 11D graviton (see \cite{43} for more details).

One can collect all the above on-shell superfields in

\[
\Psi_Q(x^\gamma, v_{aq}^-, \theta_q^-) = \left\{ \Psi_I q, \Phi_{[IJK]}, H_{\{IJ\}} \right\} ,
\]  

(14)

with multiindex \(Q\) taking \(128(=144-16)\) 'fermionic' and \(128=84+44\) 'bosonic values', \(Q = \{Iq, [IJK], ((IJ))\}\).

The set of equations (12), (10) and (13) can be unified in

\[
D_+^a \Psi_Q = \Delta_{Qp} \Psi_{p} ,
\]  

(15)

where the operator \(\Delta_{Qp}\) can be easily read off Eqs. (12), (10) and (13). It contains differential operator \(\partial_\gamma^-\) when \(Q = Iq\) and is purely algebraic otherwise. This difference is diminished when passing to the Fourier images of the superfields with respect to \(x^\gamma\) coordinate, \(\Psi_Q(x^\gamma, v_{aq}^-, \theta_q^-) = \frac{1}{(2\pi)^4} \int dx^\gamma \exp(i \rho^\# x^\gamma) \Psi_Q(x^\gamma, v_{aq}^-, \theta_q^-)\). These obey the same equation (15) but with \(\partial_\gamma^- \rightarrow -i \rho^\#\) and

\[
D_+^a = \partial_+^a + 2 \rho^\# \theta_q^- .
\]  

(16)

As we have already noticed, the set of Eqs. (12), (10) and (13), collected in (15), are dependent. We can choose any of them and reproduce two others from its consistency conditions. Passing to Fourier image makes natural to choose the fermionic superfield as fundamental and to describe the linearized 11D supergravity by the equation

\[
D_+^a \Psi_{p} = -\frac{1}{18} \gamma_{\gamma p}^{[I} \left( \gamma^{JK]} + 6 \delta^{[J} \gamma_{\gamma p]} \right) \partial_\gamma^- \Phi^{JK]} - 2 \delta^{[J} \gamma_{\gamma p]} H_{IJK} \gamma_{\gamma p}^1 .
\]  

(17)

Eqs. (15) (i.e. the set of Eqs. (10), (12) and (13)) and \(\gamma_{\gamma p}^{[I} \Psi_{p}^J = 0\) play the role of D=4 helicity constraint (32). Then it is natural to expect that an on-shell tree superfield amplitude should satisfy essentially the same set of equations for each of the scattered particles.

3. Tree on-shell amplitudes in D=11

The tree on-shell n-particle scattering amplitudes can be described as a function in a direct product of n copies of the on-shell superspace (9)

\[
A_{Q_1 \ldots Q_n}^{(n)}(k_1, \theta_1^-; \ldots; k_n, \theta_n^-) \equiv A_{Q_1 \ldots Q_n}^{(n)}(\ldots; k_1, \theta_1^-; \ldots) \equiv A_{Q_1, \ldots, Q_n}^{(n)} \ldots \rho_{l q}^{\#} v_{l q}^I \theta_{l q}^- \ldots ,
\]  

(18)

carrying n multi-indices \(Q_i = \{Iq, [IJK], ((IJ))\}\) (see (14)). As indicated in (18), for shortness we often write the bosonic argument of the amplitude as \(k_\alpha^{(i)}\) instead of \(\rho_{l q}^{\#} v_{l q}^I \theta_{l q}^-\) (implying that \(k_\alpha^{(i)}\) is expressed in terms
of these by \((\mathbb{I})\), where \(\rho^\#_{(i)}\) is allowed to be negative). We will also omit the arguments of the amplitude when this does not produce a confusion.

The set of equations for the 11D amplitudes, playing the role of D=4 helicity constraints \([\mathbb{I}]\), includes, besides the \(\gamma\)-tracelessness on every "fermionic" multiindex \(I_{(i)}\),
\[
\gamma_{(i)q}^I A_{I_{(i)}q_{(i)}...} = 0 ,
\]
the equation
\[
D_{q(I)}^\pm A_{...Q(i)...} = (-)^{\Sigma_i} \Delta_Q q_{p(i)} A_{...p(i)...} ,
\]
where \(\Delta_Q q_{p(i)}\) is the same as in \([\mathbb{I}]\) \(i.e.\) can be read off \([\mathbb{17}, \mathbb{10}\) and \([\mathbb{13}]\), but acting on variables and indices corresponding to \(l\)-th particle, and \(\Sigma_i\) can be defined as the number of fermionic, \(I_j q_j\), indices among \(Q_1, \ldots Q_{(l-1)}\), \(i.e.\)
\[
\Sigma_i = \frac{1}{2} \sum_{j=1}^{l-1} (1 - (-1)^{\varepsilon(I_j J_i)}) , \quad \varepsilon(I_j J_i) = 1 ,
\]
In particular, when \(Q_l = I_{(i)} p_l\), Eq. \([\mathbb{20}]\) reads
\[
(-)^{\Sigma_i} D_{p(i)}^{\pm(l)} A_{Q_1...I_{(i)}...p_l...} = -2i \rho^\#_{(i)} \gamma_{(i)q_p} A_{(I_{(i)}(I_{(i)})...Q_n} = -2i \rho^\#_{(i)} \gamma_{(i)q_p} A_{(I_{(i)}(I_{(i)})...Q_n} ,
\]
This nilpotent matrix enters also the deformation of the momenta which will also omit the arguments of the amplitude when this
\[
\left(k_{(1)}^a\right)^2 = 0 \quad \left(k_{(n)}^a\right)^2 \Rightarrow \left(k_{(1)}^a\right)^2 = 0 \quad \left(k_{(n)}^a\right)^2 .
\]
This catastrophic change of dependence on \(I_{(i)}(I_{(i)})...Q_n\), \(\theta_{(1)}(\theta_{(1)}: \ldots ; k_{(1)}; \theta_{(1)}: \ldots),\) remains an on-shell amplitude.

In D=4 the deformation of the momenta \([\mathbb{20}]\) results from the following deformation of the bosonic spinors entering the Penrose representation \([\mathbb{1}]\)
\[
\lambda_{(1)}^A = \lambda_{(1)}^A + z \lambda_{(1)}^A , \quad \lambda_{(1)}^A = \lambda_{(1)}^A - z \lambda_{(1)}^A ,
\]
In D=11 \([\mathbb{24}]\) results from the following deformation of the associated spinor moving frame variables
\[
\tilde{v}_{\alpha\eta} = v_{\alpha\eta} + z v_{\alpha\eta} \sqrt{\rho^\#/\rho_{(i)}^\#} ,
\]
\[
\tilde{v}_{\alpha\eta} = v_{\alpha\eta} - z M_{pq} v_{\alpha\eta} \sqrt{\rho^\#/\rho_{(i)}^\#} ,
\]
which enter the Penrose-like constraints \([\mathbb{11}]\),
\[
k_{(1)}^a \Gamma_{a\alpha\beta} = 2 \rho^\#_{(i)} v_{\alpha\eta} v_{\eta\beta} \tilde{v}_{\alpha\eta} ,
\]
\[
k_{(1)}^a \delta_{pq} = \rho^\#_{(i)} v_{\alpha\eta} (\tilde{v}_{\alpha\eta} - \tilde{v}_{\eta\alpha}) ,
\]
\[\quad \text{due to \([\mathbb{24}]\). This nilpotent matrix enters also the deformation rules of the fermionic coordinates}
\[
\tilde{\theta}_{\eta} = \theta_{\eta} + z \theta_{\eta} M_{pq} \sqrt{\rho^\#/\rho_{(i)}^\#} ,
\]
\[
\tilde{\theta}_{\eta} = \theta_{\eta} - z M_{pq} \theta_{\eta} \sqrt{\rho^\#/\rho_{(i)}^\#} .
\]
These can be also written as
\[
\tilde{\theta}_{\eta} = e^{-z D_{\eta}^+ M_{\eta\eta} - z \theta_{\eta} M_{\eta\eta}^+ \theta_{\eta}^*} ,
\]
where the covariant fermionic derivatives \(D_{\eta}^+\) are defined in \([\mathbb{16}]\). Their deformation
\[
D_{\eta}^+ = e^{-z D_{\eta}^+ M_{\eta\eta} - z \theta_{\eta} M_{\eta\eta}^+ \theta_{\eta}^*} \tilde{D}_{\eta}^+ = e^{-z D_{\eta}^+ M_{\eta\eta} + z \theta_{\eta} M_{\eta\eta}^+ \theta_{\eta}^*} D_{\eta}^+ \quad \text{is similar to the deformation of 8d Clifford algebra valued variables in the 10D construction of \([\mathbb{13}]\).}
\]

5. Generalized BCFW recurrent relations for tree amplitudes in \(D = 11\)\)

The deformed tree amplitude is defined as an amplitude depending on deformed momenta and fermionic coordinates. We denote it by
\[
A_{\eta_1...\eta_l} := A_{(n)}^\eta_{\eta_1...\eta_l} (\tilde{k}_{(1)}, \ldots; \tilde{k}_{(l)}, \tilde{\theta}_{(1)}; \ldots; \tilde{\theta}_{(n)}),
\]
\[
A_{\eta_1...\eta_l} := A_{(n)}^\eta_{\eta_1...\eta_l} (\tilde{k}_{(1)}; \tilde{k}_{(2)}, \ldots; \tilde{k}_{(n-1)}; \tilde{\theta}_{(1)}; \ldots; \tilde{\theta}_{(n)}) ,
\]
where in the last line it is assumed that the deformed momenta correspond to $1$-st and $n$-th of the scattered particles (so that $\hat{k}_{(l)}$, $\theta_{(l)} = k_l, \theta_l$ for $l = 2, \ldots, (n-1)$), and the subscript $z$ indicates the parameter used in this deformation \cite{27}--\cite{33}. Notice that deformed amplitudes satisfy, besides the gamma-tracelessness \cite{19}, Eqs. \cite{20} with deformed derivatives \cite{35},

\begin{equation}
\hat{D}_q^{+}(z) \hat{A}_{z} Q_1 \ldots Q_n = (-)^{\Delta_Q q} P_l(z) \hat{A}_{z} Q_1 \ldots P_l \ldots \cdot \ (37)
\end{equation}

In particular,

\begin{equation}
\begin{align*}
(-)^{\Sigma} \hat{D}_q^{+}(\hat{A}_z \ldots [l_1 \ldots l_k]) = 3i\gamma_{[l_1 \ldots l_k]} \hat{A}_z \ldots [l_1 \ldots l_k], \quad (38) \\
(-)^{\Sigma} \hat{D}_q^{+}(\hat{A}_z \ldots ([l_1], [j_1]) = i\gamma_{q \{[l_1]} [\hat{A}_z \ldots [j_1])p \ldots . \quad (39)
\end{align*}
\end{equation}

The proposed BCFW-type recurrent relation for tree superamplitudes of 11D supergravity reads

\begin{equation}
\mathcal{A}^{(n)}_{Q_1 \ldots Q_n} (k_1, \theta_{(1)}; k_2, \theta_{(2)}; \ldots; k_n, \theta_{(n)}) = \\
\sum_{l} \frac{(-)^{\Sigma_{q+1}}}{64(\rho^q(z))^2} D_{q(z)}^{+} \left( \hat{A}_{z}^{(l+1)} Q_1 \ldots Q_l \hat{P}_l(z), \theta^{-}; k_2, \theta_{(2)}; \ldots; k_l, \theta_{(l)}; \hat{P}_l(z), \theta^{-} \right) \times \left( \hat{A}_{z}^{(n-l+1)} Q_1 \ldots Q_n (\sim \hat{P}_l(z), \theta^{-}; k_{l+1}, \theta_{(l+1)}; \ldots; k_{l+n}, \theta_{(n-l+1)}; \hat{P}_l(z), \theta^{-}) \right) |_{\theta^{-} = 0} . \quad (40)
\end{equation}

Here

\begin{equation}
P_l^a = - \sum_{m=1}^{l} k_m^a , \quad (41)
\end{equation}

\begin{equation}
\hat{P}_l(z) = \sum_{m=1}^{l} k_m^a(z) = P_l^a - z q^a , \quad (42)
\end{equation}

\begin{equation}
z_l := P_l^a P_{a l} / (2P_l^b q_b) , \quad (43)
\end{equation}

with $q^a$ obeying \cite{21} and \cite{30} \cite{40}. Eq. \cite{12} implies that $(\hat{P}_l(z))^2 = (P_l)^2 - 2z P_l \cdot q$, so that $\hat{P}_l(z)$ is light-like

\begin{equation}
(\hat{P}_l(z))^2 = 0 , \quad z_l := (P_l)^2 / (2P_l \cdot q) . \quad (44)
\end{equation}

As a result, firstly, both amplitudes in the r.h.s. of \cite{40} are on the mass shell, and secondly we can express $\hat{P}_l(z)$ in terms of associated spinor moving frame variables $v_{*a}(z_l) := v_{*a} P_l(z_l)$ and energy $\pm \hat{\rho}^q(z_l)$ (see \cite{22})

\begin{equation}
\hat{P}_l^a(z_l) \Gamma_{\alpha \beta} = 2\hat{\rho}^q(z_l) v_{*a}(z_l) v_{*\beta}^\dagger(z_l) , \quad \hat{P}_l^a(z_l) \delta_{\alpha \beta} = \hat{\rho}^q(z_l) v_{*a}^\dagger(z_l) \Gamma_{\alpha \beta} v_{*a}(z_l) . \quad (45)
\end{equation}

This $\hat{\rho}^q(z_l)$ enters the denominator of the terms in the r.h.s. of \cite{40} (which is needed to simplify the relation between amplitude and superamplitude).

Actually, the bosonic arguments of the on-shell amplitudes are energies $\hat{\rho}^q(z_l)$ and $v_{*a}(z_l)$ related to light-like momenta $k_{(l)}$ by \cite{29}, and the above $v_{*a}(z_l)$ and $\pm \hat{\rho}^q(z_l)$; just for shortness in \cite{40}, following \cite{13}, we hide this writing instead the dependence on the momenta.

Finally, $D_q^{+}(z_l)$ in \cite{40} is the covariant derivative with respect to $\Theta_q$ constructed with the use of $\hat{\rho}^q(z_l)$ of \cite{35}.

\begin{equation}
D_q^{+}(z_l) = \frac{\partial}{\partial \Theta_q} + 2\hat{\rho}^q(z_l) \Theta_q . \quad (46)
\end{equation}

Notice that the structure of the r.h.s. of \cite{40},

\begin{equation}
D_q^{+} \left( \mathcal{A}_{\ldots J_p} D_q^{+} \mathcal{A}_{J_p} \ldots \right) \left|_{\Theta_q = 0} = \left. \right|_{\Theta_q = 0} \quad (47)
\end{equation}

\begin{equation}
\equiv D_q^{+} \left( \mathcal{A}_{\ldots J_p} D_q^{+} \mathcal{A}_{J_p} \ldots - (-)^{\Sigma_{q}} D_q^{+} \mathcal{A}_{\ldots J_p} \mathcal{A}_{J_p} \ldots \right) \left|_{\Theta_q = 0} \quad (48)
\end{equation}

can be treated as an integration over the fermionic variable $\Theta_q$ in \cite{41} with an exotic measure similar to one used in \cite{33} \cite{36} to construct a worldsheet superfield formulation of the heterotic string (see \cite{37} for formal discussion on superspace measures).

To argue that there is no contribution to the r.h.s. of \cite{40} of a pole at $|z| \to \infty$, we can use the line of arguments presented in \cite{13} for 10D case, which refers on the case when external momenta lays in some 4d subspace of spacetime and on the original proof of \cite{3} which was extended to $N = 8$ supergravity in \cite{9} \cite{11}.

The calculation of sample tree superamplitudes of 11D supergravity with the use of the above BCFW-type recurrent relations \cite{40}, and generalization of these to loop amplitudes will be the subject of subsequent work. See supplemental material to this paper \cite{47} for some technicalities needed to proceed with explicit superamplitude calculations.

**Acknowledgements** This work has been supported in part by the Spanish MINECO grant FPA2012-35043-C02-01, partially financed with FEDER/ERDF (European Regional Development Fund of the European Union), by the Basque Government Grant IT-979-16, and the Basque Country University program UFI 11/55. The author is thankful to Theoretical Department of CERN for hospitality and support of his visits at different stages of this project, and to Luis Alvarez-Gaume, Boris Pio- line, Emeri Sokatchev and Paolo Di Vecchia for useful discussion on related topics during these visits.
