Thermodynamics via Creation from Nothing: Limiting the Cosmological Constant Landscape

A.O. Barvinsky
Theory Department, Lebedev Physics Institute, Leninsky Prospect 53, 119991 Moscow, Russia

A.Yu. Kamenshchik
Dipartimento di Fisica and INFN, Via Irnerio 46, 40126 Bologna, Italy
L.D. Landau Institute for Theoretical Physics, Kosygin str. 2, 119334 Moscow, Russia

The creation of a quantum Universe is described by a density matrix which yields an ensemble of universes with the cosmological constant limited to a bounded range $\Lambda_{\text{min}} \leq \Lambda \leq \Lambda_{\text{max}}$. The domain $\Lambda < \Lambda_{\text{min}}$ is ruled out by a cosmological bootstrap requirement (the self-consistent back reaction of hot matter). The upper cutoff results from the quantum effects of vacuum energy and the conformal anomaly mediated by a special ghost-avoidance renormalization. The cutoff $\Lambda_{\text{max}}$ establishes a new quantum scale – the accumulation point of an infinite sequence of garland-type instantons. The dependence of the cosmological constant range on particle phenomenology suggests a possible dynamical selection mechanism for the landscape of string vacua.

PACS numbers:

Quantum cosmology \[1, 2\] and Euclidean quantum gravity \[3\] might effectively restrict the landscape of string vacua. This landscape is too big \[4\] to predict either the observed particle phenomenology or large-scale structure formation within string theory itself. Other methods have to be invoked, at least some of them based on the cosmological wavefunction \[5, 6, 7\]. This is dominated by the exponentiated Euclidean action, $\exp(-S_E)$, calculated on the gravitational instanton which is a saddle point of an underlying path integral over Euclidean 4-geometries. This instanton gives rise to Lorentzian signature spacetime by analytic continuation across minimal hypersurfaces. The continuation can be interpreted either as quantum tunneling or as the creation of the Universe from "nothing". Thus, the most probable vacuum of the landscape become weighted by the minima of $S_E$. This might serve as a method of selecting a vacuum from the enormously big string landscape.

An immediate difficulty with this program arises from the infrared catastrophe of small cosmological constant $\Lambda$. The Hartle-Hawking wave function \[8\], which describes nucleation of the de Sitter Universe from the Euclidean 4-dimensional hemisphere, has the form

$$\Psi_{\text{HH}} \sim \exp(-S_E) = \exp(3\pi/2G\Lambda). \quad (1)$$

This diverges for $\Lambda \to 0$ because of unboundedness of the Euclidean gravitational action. Despite some early attempts to interpret it as the origin of a zero value of $\Lambda$ \[8\], this result remains both controversial and anti-intuitive because it disfavors inflation and prefers creation of infinitely large universes. Apart from the tunneling proposals of \[9\] which employ an opposite sign in the exponential of (1) and thus open the possibility for opposite conclusions \[10\], no convincing resolution of this problem has thus far been suggested.

In this Letter we show that Euclidean path integration framework naturally avoids this infrared catastrophe. We attain this result by: i) extending the notion of Hartle-Hawking pure state to a density matrix which describes a mixed quantum state of the Universe and ii) incorporating the nonperturbative back reaction of hot quantum matter on the instanton background \[11\]. These extensions seem natural because whether the initial state of the Universe is pure or mixed is a dynamical question rather than a postulate. We address this question below by embedding both types of states into a unified framework of a density matrix.

A density matrix $\rho[\varphi, \varphi']$ is represented in Euclidean quantum gravity \[12\] by an instanton having two disjoint boundaries $\Sigma$ and $\Sigma'$ associated with its two entries $\varphi$ and $\varphi'$ (collecting both gravity and matter observables). The instanton interpolates between these, thus establishing mixing correlations, see Fig.1. In contrast, for the density matrix of the pure Hartle-Hawking state the bridge between $\Sigma$ and $\Sigma'$ is broken, so that the instanton is a union of two disjoint hemispheres which smoothly close up at their poles (Fig.2).

FIG. 1: Density matrix instanton. Dashed lines depict the Lorentzian Universe nucleating at minimal surfaces $\Sigma$ and $\Sigma'$.

The main effect that we advocate here is that thermal
fluctuations and quantum conformal anomaly destroy the Hartle-Hawking instanton and replace it with one filled by radiation. This is already manifest in classical theory, specifically in the Euclidean Friedmann equation for a scale factor $a(\tau)$, $\dot{a}^2/a^2 = 1/a^2 - H^2 - C/a^4$ (we use spatially closed FRW metric $ds^2 = N^2(\tau) d\tau^2 + a^2(\tau) d^2\Omega(3)$ in the gauge $N = 1$ and express $\Lambda = 3H^2$ in terms of the Hubble constant). The radiation density $C/a^4$ prevents the half-instantons from closing and allows $a$ to vary between two turning points \[ a_{\pm} = (1/\sqrt{2H})(1 \pm \sqrt{1 - 4CH^2})^{1/2}. \] This forces a tubular structure on the instanton which spans (at least) one period of oscillation between $a_{\pm}$, provided the constant $C$ characterizing the amount of radiation satisfies the bound $4H^2C \leq 1$.

The existence of radiation naturally follows from the partition function of this state associated with the toroidal instanton obtained by identifying $\Sigma'$ and $\Sigma$. The radiation back reaction supports the instanton geometry in which it exists. Remarkably, when the vacuum energy and conformal anomaly are taken into account this bootstrap yields a set of instantons -- a landscape -- only in the bounded range of $\Lambda$,

$$\lambda_{\min} < \Lambda < \lambda_{\max}. \quad (2)$$

All values $\Lambda < \lambda_{\min}$ are completely eliminated either because of the absence of instanton solutions or because of their infinitely large positive action. A similar situation holds for $\Lambda > \lambda_{\max}$ -- no instantons exist there, and the Lorentzian configurations in this overbarrier domain (if any) are exponentially suppressed relative to those of $\Sigma$.

To quantify the above picture consider the density matrix given by the Euclidean path integral \[ \rho(\varphi, \varphi') = e^{-\Gamma} \int D[g, \phi] \exp \{ -S_E[g, \phi] \}, \quad (3) \]

where $S_E[g, \phi]$ is the classical action, and the integration runs over gravitational $g$ and matter $\phi$ fields interpolating between $\varphi$ and $\varphi'$ at $\Sigma$ and $\Sigma'$. The statistical sum $\exp(-\Gamma)$ is given by a similar path integral over periodic fields on the torus with identified boundaries $\Sigma$ and $\Sigma'$.

The back reaction follows from decomposing $[g, \phi]$ into a minisuperspace $g_0(\tau) = (a(\tau), N(\tau))$, and the "matter" sector which includes also inhomogeneous metric perturbations on minisuperspace background $\Phi(x) = (\phi(x), \psi(x), A_{\mu}(x), h_{\mu\nu}(x), ...)$. With a relevant decomposition of the measure $D[g, \phi] = Dg_0(\tau) \times D\Phi(x)$, the integral for $\Gamma$ expresses in terms of the effective action $\Gamma[g_0(\tau)]$ of quantized matter on the background $g_0(\tau)$, $\exp(-\Gamma[g_0]) = \int D\Phi(x) \exp(-S_E[g_0, \Phi(x)])$, as

$$e^{-\Gamma} = \int Dg_0(\tau) \exp \left( -\Gamma[g_0(\tau)] \right). \quad (4)$$

Our approximation will be to consider $\Gamma[g_0(\tau)]$ in the one-loop order, $\Gamma[g_0] = S_E[g_0] + \Gamma_{\text{loop}}[g_0]$, and handle \[ \Gamma_{\text{loop}}[g_0] \] at the tree level, which is equivalent to solving the effective equations for $\Gamma[g_0]$.

Remarkably, $\Gamma[g_0]$ is exactly calculable for conformally-invariant fields by a conformal transformation \[ \bar{g}_{\mu\nu} = \frac{1}{\Lambda} g_{\mu\nu} \] relating generic FRW metric $ds^2 = a^2(\tau) d^2\Omega(3)$ to that of a static universe of a unit size, $ds^2 = dt^2 + d^2\Omega(3)$ (these metrics are denoted below as $g$ and $\bar{g}$, while $\eta$ is the conformal time). The total action reads

$$\Gamma[a(\tau), N(\tau)] = \int_{\tau_{\pm}}^{\tau_{\pm}} d\tau \left( -\frac{\dot{a}^2}{N} - Na + N\dot{H}^2a^3 \right) + 2B \int_{\tau_{\pm}}^{\tau_{\pm}} d\tau \left( \frac{\dot{a}^2}{N} - \frac{\dot{\phi}^4}{6Na} \right) + B \int_{\tau_{\pm}}^{\tau_{\pm}} d\tau N/a + F \left( 2 \int_{\tau_{\pm}}^{\tau_{\pm}} d\tau N/a \right). \quad (5)$$

We work in units of $m_P = \sqrt{3\pi/4G}$, and the integration runs between two turning points at $\tau_{\pm}$. The first line is the classical part, the second line is the conformal contribution and the last line is the one-loop action on the static instanton of the metric $\bar{g}$.

The conformal contribution $\Gamma_{\text{loop}}[\bar{g}] - \Gamma_{\text{loop}}[g]$ is determined by the coefficients of $\Box R$, the Gauss-Bonnet invariant $E = R_{\mu\nu\rho\sigma} - 4R_{\mu\nu}R_{\rho\sigma} + R^2$ and Weyl tensor term in the conformal anomaly $g_{\mu\nu}\Delta_{\text{loop}}/\delta g_{\mu\nu} = g^{1/2}(\alpha \Box R + \beta E + \gamma C_{\mu\nu\rho\sigma})/4(4\pi)^2$. Specifically this contribution can be obtained by the technique of \[ \Box \alpha \] : it contains higher-derivative terms $\sim R^2$ which produce ghost instabilities in solutions of effective equations. However, such terms are proportional to the coefficient $\alpha$ which can be put to zero by adding the following finite local counterterm

$$\Gamma_R[g] = \Gamma_{\text{loop}}[\bar{g}] + \frac{1}{2(4\pi)^2} \frac{\alpha}{12} \int d^4x g^{1/2}R^2(g). \quad (6)$$

This ghost-avoidance renormalization is justified by the requirement of consistency of the theory at the quantum level. The contribution $\Gamma_R[g] - \Gamma_R[\bar{g}]$ to the renormalized action then gives the second line of \[ \Box \alpha \] with $B = 3\beta/4$.

The static instanton with a period $\eta_0$ playing the role of inverse temperature contributes $\Gamma_{\text{loop}}[\bar{g}] = E_0 \eta_0 + F(\eta_0)$, where the vacuum energy $E_0$ and free energy $F(\eta_0)$ are the typical boson and fermion sums over field oscillators with energies $\omega$ on a unit 3-sphere $E_0 = \pm \sum_\omega \omega^2/2$, $F(\eta_0) = \pm \sum_\omega \ln(1 \mp e^{-\omega/\eta_0})$. The renormalization \[ \Box \alpha \] which should be applied also to $\Gamma_{\text{loop}}[\bar{g}]$ modifies $E_0$, so that $\Gamma_R[g] = C_0 \eta_0 + F(\eta_0)$, $C_0 \equiv E_0 + 3\alpha/16$. This gives the third line of Eq. \[ \Box \alpha \] with $C_0 = B/2$. This universal relation between $C_0$ and...
\[ B = 3\beta/4 \] follows from the known anomaly coefficients \[17\] and the UV-renormalized Casimir energy in a static universe \[18\] for scalar, Weyl spinor and vector fields respectively:

\[
\alpha = \frac{1}{90} \times \begin{cases} -1 & \text{for } k = 1 \\ -3 & \text{for } k = 2, 3, 4 \\ 18 & \text{for } k = 0 \end{cases}, \quad \beta = \frac{1}{360} \times \begin{cases} 2 & \text{for } k = 1 \\ 11 & \text{for } k = 3 \\ 124 & \text{for } k = 4 \end{cases}, \quad E_0 = \frac{1}{960} \times \begin{cases} 4 & \text{for } k = 1 \\ 17 & \text{for } k = 2, 3, 4 \end{cases}.
\]

It is important that for conformally invariant fields the nonlocal action \[5\] is exact, and contains no other terms of higher order in the curvature.

The effective equation \(\delta \Gamma / \delta N(\tau) = 0\) has the form of the classical equation modified by the quantum \(B\)-term

\[
\frac{\dot{a}^2}{a^2} + B \left( \frac{1}{2} \frac{\dot{a}^4}{a^4} - \frac{\dot{a}^2}{a^2} \right) = \frac{1}{a^2} - H^2 - C \frac{\dot{\tau}}{a^2}.
\]

\[ C = B/2 + F'(\eta_0), \quad \eta_0 = 2 \int_{\tau_-}^{\tau_+} d\tau/a(\tau). \]

Remarkably, the contribution of the nonlocal \(F(\eta_0)\) in \[5\] reduces to the radiation constant \(C\) as a nonlocal functional of \(a(\tau)\), determined by the bootstrap equation \[9\]. Here \(F'(\eta_0) = dF(\eta_0)/d\eta_0 > 0\) is the energy of a hot gas of particles, which adds to their vacuum energy \(B/2\).

### FIG. 3: Instanton domain in the \((H^2, C)\)-plane. Garland families are shown for \(k = 1, 2, 3, 4\). Their sequence accumulates at the critical point \((1/2B, B/2)\).

Periodic instanton solutions of Eqs.\[8\]-\[9\] exist only inside the curvilinear wedge of \((H^2, C)\)-plane between bold segments of the upper hyperbolic boundary and the lower straight line boundary of Fig.\[3\]

\[ 4CH^2 \leq 1, \quad C \geq B - B^2 H^2, \quad BH^2 \leq 1/2. \]

Below this domain the solutions are either complex and aperiodic or suppressed by infinite positive Euclidean action. Above this domain only Lorentzian (overbarrier) configurations exist, but they are again exponentially damped relative to instantons in \[10\].

These properties are based on the fact that the turning points of \[8\] exactly coincide with classical \(a_{\pm}\), but \(a_-\) exists only when \(a^2 > B\), which gives rise to \[10\]. Otherwise, \(a(\tau)\) at the contraction phase becomes complex or runs to zero which violates instanton periodicity. In the latter case a smooth Hartle-Hawking instanton with \(a_-=0\) forms and yields \(\eta_0 \rightarrow \infty\) in view of \(9\), so that \(F(\eta_0) \sim F'(\eta_0) \rightarrow 0\). Therefore, its on-shell action

\[ I_0 = F(\eta_0) - \eta_0 F'(\eta_0) + 4 \int_{\tau_-}^{\tau_+} \frac{a}{a^2} \left( B - a^2 - \frac{B a^2}{3} \right) \]

due to \(B > 0\) diverges to \(+\infty\) at \(\eta_0 = 0\) and completely rules out pure-state instantons \[11\].

Moreover, inside the range \[10\] our bootstrap eliminates the infrared catastrophe of \(\Lambda \rightarrow 0\). Indeed \(\eta_0 \rightarrow \infty\) as \(H^2 \rightarrow 0\), so that due to \(9\) \(C \rightarrow B/2\), but this is impossible because in view of \(10\) \(C \geq B\).

Thus, instanton family never hits the \(C\)-axes of \(H^2 = 0\) and can only interpolate between the points on the boundaries of the domain \[10\]. For a conformal scalar field the numerical analysis gives such a family \[11\] starting at \(H^2 \approx 2.00, \quad C \approx 0.004, \quad I_0 \approx -0.16\), and terminating at \(H^2 \approx 13.0, \quad C \approx 0.02, \quad I_0 \approx -0.09\). The upper point describes the static universe filled by a hot radiation with the temperature \(T = H^2/\pi \sqrt{1 - 2BH^2}\), whereas the lower point establishes the lower bound of the \(\Lambda\)-range.

The upper bound of the landscape follows from the existence of garlands that can be obtained by glueing together into a torus \(k\) copies of a simple instanton \[13\]; see Fig.\[4\]. Their formalism is the same as above except that the conformal time in \[9\] and the integral term of \[11\] should be multiplied by \(k\). As in the case of \(k = 1\), garland families interpolate between the lower and upper boundaries of \[10\]. They exist for all \(k, \quad 1 \leq k \leq \infty\), and their infinite sequence accumulates at the critical point \(C = B/2, \quad H^2 = 1/2B\), where these boundaries merge. Within the \(1/k^2\)-accuracy the upper and lower points of each family coincide and read

\[ H^2(k) \approx \frac{1}{2B} \left( 1 - \frac{\ln^2 k^2}{2k^2 \pi^2} \right), \quad I_0^{(k)} \approx -B \frac{\ln^3 k^2}{4k^2 \pi^2}. \]

With a growing \(k\), garlands become more and more static and cold with \(I(k) \approx 1/(\sqrt{B} \ln k^2) \rightarrow 0\). Contrary to \[10\] the garland action is not additive in \(k\), so that as \(k \rightarrow \infty\) garlands do not dominate the ensemble.

Thus, our Universe is created in a hot mixed state, but its evolution does not contradict the large-scale structure formation. After nucleation from the instanton the Universe expands; its radiation dilutes, so that A starts dominating and generates inflation under an assumption that
everywhere above $\Lambda$ is a composite field (like an inflaton) decaying at the exit by a standard slow-roll scenario.

The ensemble of universes belongs to a bounded range $[2]$ of $\Lambda = 3H^2$. Its infrared cutoff is provided by the radiation back reaction and survives even in the classical limit as $B \to 0$. In contrast, the high-energy cutoff at

$$\Lambda_{\text{max}} = 3m_P^2/2B, \quad m_P^2 \equiv 3\pi/4G,$$  \hspace{1cm} (13)

is the quantum effect of vacuum energy and the conformal anomaly; this generates a new scale in gravity theory.

We have considered only conformal fields which make our model exactly solvable and provide critically important positivity of the constant $B = 3\beta/4$, cf. [7]. Moreover, conformal invariance together with the ghost-avoidance renormalization renders a particular value of the vacuum energy $B/2$ in [9] which yields the upper boundary of $[2]$ exactly at the critical point $1/2B, B/2$ of Fig. [3] Even if non-conformal fields qualitatively preserve the whole picture, they are likely to break this relation. Then if $C_0 < B/2$ all garlands survive, though they saturate at $\Lambda_{\text{max}}$ with a finite temperature. If $B > C_0 > B/2$, their sequence is truncated at some $k$. Finally, if $C_0 > B$ the infrared catastrophe occurs again—the $k = 1$ instanton family hits the $C$-axes at $C_0$.

Conformal invariance can be justified as a good approximation when conformal particles outnumber non-conformal ones. Moreover, their large number $N$ justifies a semiclassical expansion by scaling down the range $[2]$. Indeed, for a single scalar field the latter is determined by Planckian values, $\Lambda_{\text{max}} = 360m_P^2$, which, however, decrease as $1/N$ in view of the simple scaling $C \to NC, B \to NB, F(\eta_B) \to NF(\eta_B)$ and $H^2 \to H^2/N$. Semiclassical expansion can also be justified for large $B = 3\beta/4$ growing with spin, cf. [7], because the domain $[2]$ with $[13]$ shrinks to a narrow subplanckian range when ascending the particle hierarchy.

Though motivated by the string landscape, all the above results hold outside of the string theory context and, as a feedback, suggest a long-sought selection mechanism for the plethora of string vacua. Modulo the details of a relevant $4D$-compactification, this might work as follows. For $B$ growing with $N$ and spin, the upper scale $[13]$ decreases towards the increasing phenomenology scale, and approaches the latter at the string scale $m_P^2$ where a positive $\Lambda$ might be generated by the mechanism of $[20]$. Our conjecture is that at this scale our bootstrap becomes perturbatively consistent, provided $m_P^2/B \approx m_P^2 \ll m_P^2$, and selects from the string landscape a small subset compatible with observations.

Our results hold within the Euclidean path integral [6] which automatically excludes Lorentzian configurations possibly existing above the upper boundary of $[10]$, $4C\Lambda^2 > 1$. However, one can imagine an extended formulation of quantum gravity generalizing $[3]$ to a wider path integration domain. Our conclusions nevertheless remain true. Indeed the effective action scales as $\Gamma_0 \sim -\sqrt{B}, B \gg 1$, and because it is negative our landscape at the scale $m_s$ is weighted by $\exp(\#\sqrt{B}) \approx \exp(\#m_P/m_s) \gg 1$. Therefore it strongly dominates over Lorentzian configurations, the amplitudes of the latter being $O(1)$ in view of their pure phase nature. Thus, our results look robust against possible generalizations of Euclidean quantum gravity.

A.B. thanks H. Tye, C. Deffayet, J. Khoury, R. Woodard, T. Osborn and especially Jim Hartle for thought provoking discussions and helpful suggestions. A.B. was supported by the RFBR grant 05-02-17661 and the grant LSS 4401.2006.2. A.K. was partially supported by the RFBR grant 05-02-17450 and the grant LSS 1757.2006.2.

[1] B.S. DeWitt, Phys. Rev. 160, 1113 (1967).
[2] A. Vilenkin, Phys. Lett. B 117, 25 (1982); Phys. Rev. D 27, 2848 (1983).
[3] J.B. Hartle and S.W. Hawking, Phys. Rev. D 28, 2960 (1983); S.W. Hawking, Nucl. Phys. B 239, 257 (1984).
[4] R. Bousso and J. Polchinski, JHEP 06 006 (2000); M.R. Douglas, JHEP 05, 046 (2003); hep-th/0303194.
[5] H. Ooguri et al., Phys. Rev. D 74, 311 (2006).
[6] H. Firouzjahi et al., JHEP 0409, 060 (2004); S. Sarangi and S.-H. H. Tye, hep-th/0505104; hep-th/0603237.
[7] R. Brustein and S. P. de Alwis, Phys. Rev. D 73, 046009 (2006).
[8] S.W. Hawking, Phys. Rev. D 34, 403 (1984); S.R. Coleman, Nucl. Phys. B 310, 643 (1988).
[9] A.D. Linde, JETP Lett. 60, 211 (1984); A. Vilenkin, Phys. Rev. D 30, 500 (1984).
[10] A.O. Barvinsky and A. Yu. Kamenshchik, Class. Quantum Grav. 7, L181 (1990); Phys. Rev. B 332, 270 (1994).
[11] A.O. Barvinsky and A. Yu. Kamenshchik, JCAP 09, 014 (2006).
[12] D.N. Page, Phys. Rev. D 34, 2267 (1986).
[13] J.J. Halliwell and R.C. Myers, Phys. Rev. D 40, 4011 (1989).
[14] G.V. Lavrelashvili et al., Nucl. Phys. B 329, 98 (1990); A. Yu. Kamenshchik et al., Phys. Rev. Lett. 357, 36 (1995).
[15] M.V. Fischetti, J.B. Hartle and B.L. Hu, Phys. Rev. D 20, 1757 (1979).
[16] E.S. Fradkin and A.A. Tseytlin, Phys. Lett. B 134, 187 (1984); A.O. Barvinsky et al., gr-qc/9510037; P.O. Mazur and E. Mottola, Phys. Rev. D 64, 104022 (2001).
[17] J.S. Dowker and R. Critchley, Phys. Rev. D 16, 3390 (1977); S.M. Christensen, Phys. Rev. D 17, 946 (1978).
[18] J.S. Dowker and R. Critchley, J. Phys. A 9, 535 (1976); S.G. Mamaev et al., Sov. Phys. JETP, 43, 823 (1976); L.H. Ford, Phys. Rev. D 14, 3304 (1976); P. Candela and J.S. Dowker, Phys. Rev. D 19, 2902 (1979).
[19] W. Fischler, D. Morgan and J. Polchinsky, Phys. Rev. D 42 (1990) 4042.
[20] S. Kachru et al., Phys. Rev. D 68, 046005 (2003); S. Kachru et al., JCAP 10, 013 (2003).