Anomalous hysteretic behavior in a system of dipolar Bose gases

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Abstract. We investigate the hysteresis characteristics in a system of dipolar Bose gases loaded into a triangular lattice. We perform a large-size cluster mean-field analysis for the corresponding dipolar Bose-Hubbard model in the hard-core boson limit, and show that in varying the chemical potential the system can exhibit a hysteretic behavior without forming the standard “hysteresis-loop” structure. In this anomalous hysteresis, the quantum melting transition from the supersolid (or solid) to superfluid state can occur, but the reverse process (quantum solidification) is impossible.

1. Introduction

Hysteresis is a ubiquitous phenomenon that is encountered in a wide area of physics including magnetic, optical, electronic, chemical, and mechanical physics. The existence of hysteresis is deeply related to properties of the free energy of a system. In first-order phase transitions, there are one or more metastable states in addition to the globally stable state. It is well known that this metastability causes a hysteresis behavior of the system in the input-output (e.g., magnetic field versus magnetization) plane. Usually, as can be seen in the conventional liquid-solid transition, the hysteresis trajectory forms a loop structure, called “hysteresis loop.”

In this work, we report a new type of first-order phase transition phenomena without forming the standard loop structure in the hysteresis. We focus here on a system of dipolar Bose gases loaded into a triangular optical lattice. The physics of dipolar gases has been attracting much attention in recent years because of the realization of ultracold dipolar gases of \textsuperscript{52}Cr atoms, which have large magnetic dipole moments [1, 2], and the efforts made towards the creation of quantum-degenerate gases of heteronuclear polar molecules [3–5]. Additionally, triangular optical lattices have been successfully created by using three standing wave lasers, and the superfluid (SF) to Mott insulator transition of \textsuperscript{87}Rb atoms has been observed in the triangular lattice [6]. When dipolar gases are loaded into a triangular optical lattice, the long-range nature of the interaction between dipoles produces strong frustration in the system. The studies on strongly interacting atoms/molecules on frustrated lattices have great potential to pioneer new and intriguing phenomena and to provide a deeper understanding of geometrical frustration from a new perspective.

In this paper, we will demonstrate that the triangular-lattice system of dipolar bosons exhibits...
an anomalous hysteric behavior under varying the chemical potential by performing a large-size
cluster mean-field analysis for the corresponding lattice boson model.

2. Dipolar (hardcore) Bose-Hubbard model and cluster mean-field method

We consider a system of dipolar bosons interacting with strong on-site interactions in a triangular
optical lattice. The dipoles are assumed to be fully polarized in the direction perpendicular to
the lattice plane. In this case, the system can be well-described by the hardcore Bose-Hubbard
model with isotropic long-range dipole-dipole interactions [7–11]:

\[ \hat{H} = -J \sum_{\langle j,l \rangle} (\hat{a}_j^\dagger \hat{a}_l + \text{h.c.}) + \sum_{j<l} V_{jl} \hat{n}_j \hat{n}_l - \mu \sum_j \hat{n}_j, \]

where \( \hat{a}_j^\dagger \) is the creation operator of a hardcore boson at site \( j \), and \( \hat{n}_j = \hat{a}_j^\dagger \hat{a}_j \) is the occupation number operator. The long-range part of the dipole-dipole interaction can be expressed by

\[ V_{jl} = V d^3 / |r_j - r_l|^3, \]

where \( d \) is the lattice spacing. This system is characterized by three independent parameters: the hopping amplitude \( J \), the dipole-dipole interaction coefficient \( V \), and the chemical potential \( \mu \).

To study the hysteresis characteristics of the model, we first apply the single-site mean-
field (MF) theory and then take into account the effects of quantum fluctuations on the MF
solutions using a large-size cluster mean-field method. The cluster mean-field method [11–14]

is a convenient and powerful tool to get the information about the stationary points of the free
energy not only for the globally stable state but also for metastable and unstable states. In the
cluster mean-field method, we treat a cluster consisting of several sites as a reference system to
estimate the values of mean fields, while the standard MF theory deals with an effective single-
site problem. Naturally, we can get more reliable solutions by using larger-size clusters. Here,
to obtain a sufficiently meaningful result, we employ the ten-site cluster mean-field (CMF-10)
method, in which we use a triangular-shaped cluster consisting of neighboring ten sites. While
we treat exactly the interactions within the cluster, the interactions between the cluster and
the rest of the system are also included via effective fields acting at the cluster edge. The
effective fields are determined self-consistently via the expectation values of the operators \( \langle \hat{a}_j \rangle \) and \( \langle \hat{n}_j \rangle \), in a standard manner [12,13]. Note that, to estimate the expectation values \( \langle \cdots \rangle \), one
should take the average not only over all internal sites within the cluster, but also all possible
choices of how to embed the cluster itself in the background sublattice structure. Additionally,
to determine the boundary lines of first-order transitions, we use the Maxwell construction in

\( (J/V, \chi) \)-plane, where \( \chi = \sum_{\langle j,l \rangle} (\hat{a}_j^\dagger \hat{a}_l + \hat{a}_l^\dagger \hat{a}_j) / M \). Here, \( M \) denotes the number of lattice sites.

3. Ground-state phase diagram and anomalous hysteresis

We show the ground-state phase diagrams obtained by the MF and CMF-10 methods in Figs 1(a)
and (b), respectively. These are symmetric around \( \mu/V = (\mu/V)_0 \equiv 3 \sum_l V_{jl} / 4 \approx 5.517 \),
reflecting particle-hole symmetry of the hardcore boson system. For the simplified model
including only the nearest-neighbor part of dipole-dipole interactions, only the two-sublattice
solid and supersolid phases are found in addition to the standard SF phase [15,16]. In the present
case, many different types of solid (\( \rho = 1/2, 1/3, 1/4, \text{etc.} \)) and supersolid (SS1, SS2, SS3, etc.)
phases appear due to the long-range nature of the interaction. The sublattice structure of each
phase is depicted in Figs. 1(I–IV). The MF result shows that the boundary between the SS1 and
SF phases is given by a straight line of \( J/V \approx 0.194 \) connecting the tips of the two (\( \rho = 1/3 \)
and 2/3) solid phases. On the other hand, when quantum fluctuations are included with the use
of the CMF-10 method, the SS1-SF boundary is no longer straight but shows a “dip” around
\( \mu/V = (\mu/V)_0 \). As we will see below, the presence of this dip is essential for an anomalous
behavior of hysteresis to emerge.
Figure 1. Ground-state phase diagrams of hard-core bosons with dipole-dipole interactions on a triangular lattice in the \((J/V, \mu/V)\)-plane, obtained from (a) MF theory and (b) CMF-10 method. Second and first-order phase transitions are indicated by thin and thick lines, respectively. (c) The magnified view of the low-density region \((\rho \leq 0.5)\) of (b). The dashed curve indicates the limit of metastability of SF phase. (I-VI) Sublattice structures of the arising phases. Because of the particle-hole symmetry, we show only those of the low-density side.

Now, let us discuss the hysteresis in the cycle of decreasing and increasing the chemical potential \(\mu/V\). The transitions among the SF, \(\rho = 1/3\) or \(\rho = 2/3\) solid, and SS1 phases exhibit different hysteretic behaviors in three different ranges of \(J/V\), which are defined by the thresholds \((J/V)_{c1} \approx 0.118\), \((J/V)_{c2} \approx 0.130\), and \((J/V)_{c3} \approx 0.156\). These values of \(J/V\) are marked by the dashed vertical lines in Fig. 1(c). In the first region, \(J/V < (J/V)_{c1}\), a typical hysteresis loop is formed in the \((\mu/V, \rho)\)-plane accompanying the SF-solid transition. In the second region, \((J/V)_{c1} < J/V < (J/V)_{c2}\), another hysteresis loop is formed accompanying the SS1-SF first-order transition in addition to the loop around the SF-solid transition.

Of particular interest is the third region, \((J/V)_{c2} < J/V < (J/V)_{c3}\). In this region, the SF state is always (meta-) stable for any \(\mu/V\) as seen in Fig. 1(c), and this property leads to the anomalous hysteresis in which the transition occurs only unidirectionally. To show this, we plot in Figs. 2 (a-c) the solution curves of CMF-10 for the filling factor \(\rho = \sum \langle \hat{n}_j \rangle / M\), for the solid order parameter \(\rho Q \equiv \sum \langle \hat{n}_j \rangle \exp(iQ \cdot r_j) / M\) with \(Q = (4\pi/3d, 0)\), and for the SF order parameter \(\Psi \equiv \sum \langle \hat{a}_j \rangle / M\) at \(J/V = 0.145\). There are two first-order transitions, namely, the one between the solid (at point b) and SF (x) states and the one between the SS1 (d) and SF (y) states.

To discribe a process of the unidirectional hysteresis, we suppose that the initial state of the system is in a stable solid or SS1 phase located between points b and d in the figures. When decreasing (increasing) \(\mu/V\), although a SF state becomes energetically favorable below point b (above point d), the solid or SS1 state remains metastable until it reaches point f (e). Only when the value of \(\mu/V\) exceeds point f (e), the system is destabilized into the true ground state in the SF phase. This is the process of the quantum melting transition occurring from the solid or SS1 state. Next, we consider the opposite process, namely the “solidification” from a SF state. As already mentioned, the SF state is stable for any \(\mu/V\) in the region \((J/V)_{c2} < J/V < (J/V)_{c3}\). This feature can be seen in Figs. 2(a-c), where the globally stable SF solutions at low and high \(\mu/V\) are connected by the line of metastable SF solution. This means that when we decrease or increase \(\mu/V\) starting from a SF state, the system remains in the SF phase even if the value of \(\mu/V\) enters the region where a solid or SS1 state has the lowest energy. Thus, the transition from the SF state to the solid or SS1 state cannot be induced by varying \(\mu/V\), and it can be achieved only through thermal cycling. It is worth noting that the hysteresis trajectory does not form the conventional loop structure in this surprising hysteresis process.
4. Summary

In summary, we have studied hysteresis characteristics of dipolar Bose gases loaded into a triangular optical lattice. Using a ten-site cluster mean-field method, we have found that the system exhibits an anomalous hysteretic behavior: the first-order quantum melting transition from the two-sublattice supersolid (or solid) state to the SF state can occur only unidirectionally in varying the chemical potential. In this regime, the standard hysteresis-loop structure is not formed in the cycle of decreasing and increasing the chemical potential. This anomalous hysteresis cannot be predicted within the MF (classical) approach, since the boundary of the SS1-SF transition is given by a straight line as shown in Fig. 1(a).

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