Investigating fragmentation conditions in self-gravitating accretion discs

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ABSTRACT
The issue of fragmentation in self-gravitating gaseous accretion discs has implications both for the formation of stars in discs in the nuclei of active galaxies, and for the formation of gaseous planets or brown dwarfs in circumstellar discs. It is now well established that fragmentation occurs if the disc is cooled on a timescale smaller than the local dynamical timescale, while for longer cooling times the disc reaches a quasi-steady state in thermal equilibrium, with the cooling rate balanced by the heating due to gravitational stresses. We investigate here how the fragmentation boundary depends on the assumed equation of state. We find that the cooling time required for fragmentation increases as the specific heat ratio \(\gamma\) decreases, exceeding the local dynamical timescale for \(\gamma = 7/5\). This result can be easily interpreted as a consequence of there being a maximum stress (in units of the local disc pressure) that can be sustained by a self-gravitating disc in quasi-equilibrium. Fragmentation occurs if the cooling time is such that the stress required to reach thermal equilibrium exceeds this value, independent of \(\gamma\). This result suggests that a quasi-steady, self-gravitating disc can never produce a stress that results in the viscous \(\alpha\) parameter exceeding \(~0.06\).

Key words: accretion, accretion discs – gravitation – instabilities – stars: formation – galaxies: active – galaxies: spiral

1 INTRODUCTION

It is becoming clearer that self-gravity may play an important role in the dynamical evolution of accretion discs. Active galactic nuclei (AGN) often show rotation curves that depart significantly from a Keplerian profile (Greenhill & Gwinn 1997; Lodato & Bertin 2003; Kondratko et al. 2005), while protostellar disc masses may be a significant fraction of the central object mass during the early stages of star formation (Lin & Pringle 1987; Larson 1984). For example, a recent radio observation of a Class 0 object (a very young protostellar source, with age < 10\(^5\) yrs and mass \(M_\star \approx 0.8M_\odot\), Rodriguez et al. 2005) shows a disc with a estimated mass \(M_{\text{disc}} \approx 0.3 - 0.4M_\odot\) and a very suggestive two armed spiral-like structure. In addition, there are also clues that massive accretion discs can be present around massive protostars (Beltran et al. 2004; Chini et al. 2004).

An important effect of disc self-gravity is that it provides an efficient mechanism for transporting angular momentum outwards, allowing mass to accrete onto the central object (Beltran et al. 1987; Laughlin & Bodenheimer 1994). In protostellar discs, in which the ionization level is expected to be low, thus inhibiting MHD-driven turbulence (Matsumoto & Tajima 1993; Gammie 1996), this may be the dominant mechanism, at least in certain regions of the disc. In general, however, the transport associated with self-gravitating disturbances may not be well described as a simple diffusion mechanism. Balbus & Papaloizou (1991) have shown that, for self-gravitating discs, the energy flux contains non-local terms that they associate with wave energy transport. On the other hand, Lodato & Rice (2004, 2005) have shown that, for a self-gravitating accretion disc in thermal equilibrium, the dissipation arising from gravitational stresses agrees reasonably well with the expectations based on the standard viscous theory (Shakura & Sunyaev 1973; Pringle 1981) (see, however, Mejia et al. 2005 for a different result).

In the simulations performed by Lodato & Rice (2004, 2005) thermal equilibrium is achieved by allowing the disc to heat up through \(PdV\) work and shock dissipation, and cooling it at a prescribed rate. They use a simple cooling term with a cooling time given by \(t_{\text{cool}} = \beta \Omega^{-1}\), where \(\Omega\) is the angular frequency and \(\beta = 7.5\). In thermal equilibrium it can be shown (Pringle 1981; Gammie 2001) that, for a
viscous disc, the parameter \( \alpha \) (Shakura & Sunyaev 1973), that characterises angular momentum transport, and the cooling time, \( t_{cool} \), are related through
\[
\alpha = \frac{4}{9\gamma(\gamma - 1)} \frac{1}{t_{cool}\Omega},
\]
where \( \gamma \) is the ratio of the specific heats.

The ultimate evolution of a self-gravitating accretion disc depends strongly on the rate at which the disc heats up, through the growth of the instability, and on the rate at which it cools. It has been shown, using a local, two-dimensional model (Gammie 2001), that if \( t_{cool} < 3\Omega^{-1} \), the disc will fragment into bound objects rather than evolve into a quasi-steady state, a result that was largely confirmed by Rice et al. (2003) using global, three-dimensional models. This process has been suggested as a mechanism for forming both gaseous planets in protostellar discs (Kuiper 1951; Boss 1998; Mayer et al. 2004), and for forming stars in active galactic discs (Goodman & Tan 2004).

A major obstacle to the formation of objects via disc fragmentation is the requirement that the cooling time be smaller than the local dynamical time (Gammie 2001; Rice et al. 2003). This may be possible in AGN disc (Johnson & Gammie 2003), but appears unlikely in protostellar discs (Rafikov 2005). The cooling time requirement was, however, determined using equations of state with specific heat ratios of \( \gamma = 2 \) (Gammie 2001) and \( \gamma = 5/3 \) (Rice et al. 2003). It has been suggested (Lodato & Rice 2004) that since the stress required to balance the cooling rate (as measured by the viscous \( \alpha \)) depends on the specific heat ratio, \( \gamma \) (see equation 1), the cooling time required for fragmentation may also depend on \( \gamma \). In particular, if there is a maximum stress \( \alpha_{max} \) (in units of the local disc pressure) sustainable by a self-gravitating disc, then equation 1 states that fragmentation should occur if:
\[
\frac{4}{9\gamma(\gamma - 1)} \frac{1}{\alpha_{max}} < t_{cool}\Omega.
\]

In this paper we investigate, using global, three-dimensional simulations, how the fragmentation boundary (as measured by \( t_{cool} \)) varies for different values of \( \gamma \). We also consider various disc masses to study the suggestion by Rice et al. (2003) that the fragmentation boundary may depend on the ratio of the disc mass to the mass of the central object. In §2, we describe the range of cooling times, disc masses, and specific heat ratios, \( \gamma \), that have been considered, and determine, for a given disc mass, the cooling time required for fragmentation. In §3 we discuss these results in light of the relationship between the stresses in the disc (as measured by the viscous \( \alpha \)) and the imposed cooling time. We show that fragmentation is indeed easier in discs with smaller specific heat ratios. We therefore conclude that there is a maximum stress sustainable by a self-gravitating disc and we quantify this maximum stress to be \( \alpha_{max} \approx 0.06 \). In §4 we discuss our results and draw our conclusions.

2 SIMULATION RESULTS

The simulations performed here are very similar to those of Rice et al. (2003) and Lodato & Rice (2004, 2005). The three-dimensional, gaseous discs are modelled using Smoothed Particle Hydrodynamics (SPH) (Monaghan 1992, Benz 1990), a Lagrangian hydrodynamics code. The disc is represented by 250000 SPH particles, while the central star is a point mass onto which gas particles may accrete if they approach to within the accretion radius (here taken to be at a radius of \( R_{acc} = 0.25 \)). In code units, the disc extends from \( R_{in} = 0.25 \) to \( R_{out} = 25 \), and the central object has a mass of \( M_* = 1 \). We consider disc masses of \( M_{disc} = 0.1, 0.25 \) and 0.5, with initial surface density profiles of \( \Sigma \propto r^{-1} \), and with a temperature that has a radial profile of \( T \propto r^{-0.5} \). With these initial surface density and temperature profiles, the Toomre stability parameter, \( Q = c_s\Omega/\pi G \Sigma \), is not initially constant, but decreases with increasing radius. The temperature is therefore normalised such that at the beginning of the simulation the disc is stable, with a minimum \( Q = 2 \) at \( R = 25 \).

Since we are interested in how cooling influences the disc evolution, the disc gas is allowed to heat up due to both PdV work and viscous dissipation, with the viscosity given by the standard SPH artificial viscosity (Monaghan 1992) with \( \alpha_{SPH} = 0.1 \), and \( \beta_{SPH} = 0.2 \). We use an adiabatic equation of state and consider specific heat ratios of \( \gamma = 5/3 \), and \( \gamma = 7/5 \). A particle with internal energy per unit mass \( u_\i \) is then cooled using
\[
\frac{d\langle u \rangle}{dt} = -\frac{u_\i}{t_{cool}}
\]
where \( t_{cool} = 3\Omega^{-1} \).

An important numerical issue to be considered in this context is the role of artificial SPH viscosity. The growth and the saturation of gravitational instabilities depends on the balance between external cooling and internal heating provided by the instability itself. Therefore, we have to be sure that dissipation is dominated by gravitational instabilities rather than by artificial viscosity (which would provide an extra, undesired, stabilization term in the energy balance). It can be shown (Artymowicz & Lubow 1994; Murray 1995) that artificial SPH viscosity scales as \( \alpha_{SPH} \propto \alpha_{SPH}(h) \), where \( h \) is the average SPH smoothing length. We therefore should achieve a high resolution (in order to keep the smoothing length small) and adopt a sufficiently low value of \( \alpha_{SPH} \), while preserving the ability of the code to properly resolve the shocks that arise in the simulation. We have already shown (Lodato & Rice 2004, Appendix) that with the setup described above (\( N = 250000, \alpha_{SPH} = 0.1 \) and \( \beta_{SPH} = 0.2 \)), we are indeed able to properly resolve the shocks and to have an artificial dissipation smaller by more than one order of magnitude with respect to gravitationally induced dissipation. We are therefore confident that artificial viscosity is not going to affect significantly our conclusions.

For each disc mass, and for each \( \gamma \), we have performed a large number of simulations with different values of \( \beta \). We initially start with a \( \beta \) value that should result in fragmentation (Gammie 2001; Rice et al. 2003). We stop the simulation once at least one clump/fragment has formed that has a density 2–3 orders of magnitude greater than the surrounding gas. The densest clump is then tested to check if it is bound. Firstly, we determine the approximate size of the clump, by finding the distance from the center of the clump at which the density has returned to a value comparable with that of the surrounding disk. All the particles within this spherical volume are then assumed to be part of
Figure 1. Surface density structure of discs with masses $M_{\text{disc}} = 0.1$ and with cooling times of $t_{\text{cool}}\Omega = 3$ (top left), $t_{\text{cool}}\Omega = 5$ (top right), $t_{\text{cool}}\Omega = 6$ (bottom left), and $t_{\text{cool}}\Omega = 7$ (bottom right). The logarithmic colour scale in each figure is from $10^2$ g cm$^{-2}$ to $2 \times 10^4$ g cm$^{-2}$. The linear scale is from -25 to 25 for both axes.

In every case in which fragmentation occurred, the densest clump consisted of at least 100 particles, and in some cases as many as 500 particles. This more than satisfies the Jeans criterion (Bate & Burkert 1996), and we are therefore confident that the fragmentation in these simulations is not artificial. Once the clump size has been determined, we then calculate the total thermal energy and the gravitational potential energy. If the net energy is negative the clump is bound, the simulation is stopped, and a new simulation is started with the same initial conditions, but the clump is not artificial. Once the clump size has been determined, we also run the non-fragmenting simulations approximately an outer rotation period longer than the equivalent simulation that did undergo fragmentation.

We repeated the above procedure for disc masses of $M_{\text{disc}} = 0.25$, and $M_{\text{disc}} = 0.5$ and for specific heat ratios of $\gamma = 5/3$ and $\gamma = 7/5$. The results are summarised in Table 1. The columns in Table 1 are the ratio of disc to central object mass, $M_{\text{disc}}/M_*$, the specific heat ratio, $\gamma$, the cooling time, $t_{\text{cool}}\Omega$, and if fragmentation occurs, the total energy (in code units) of the densest clump, $E_{\text{tot}}$, where $E_{\text{tot}}$ is the sum of the thermal energy and gravitational potential energy (Bate et al. 1995). In the earlier work of Rice et al. (2003) there was a suggestion that the cooling time required for fragmentation may depend on the total disc mass, relative to the mass of the central object. The results shown in Table 1 suggest that there is no disc mass dependence. Fragmentation occurs for $t_{\text{cool}}\Omega$ between 6 and 7 when $\gamma = 5/3$ and between 12 and 13 when $\gamma = 7/5$, for all disc masses considered. The reason why there is a difference between Rice et al. (2003) is unclear. Their discs had slightly steeper surface density profiles ($\Sigma \propto r^{-7/4}$ rather than $\Sigma \propto r^{-1}$), and it is possible that their $t_{\text{cool}}\Omega = 5$ simulation that did not fragment, may have done so had it been run for longer.

Table 1. Table showing the results of a series of simulations considering discs with masses between $M_{\text{disc}} = 0.1$ and $M_{\text{disc}} = 0.5$, specific heat ratio of $\gamma = 5/3$ and $\gamma = 7/5$, and various cooling times. These results suggest that the fragmentation boundary does not depend on disc mass, and that for $\gamma = 7/5$ fragmentation may occur for cooling times almost twice the local dynamical time.

| $M_{\text{disc}}/M_*$ | $\gamma$ | $t_{\text{cool}}\Omega$ | $E_{\text{tot}}$ |
|-----------------------|----------|-------------------------|------------------|
| 0.1                   | 5/3      | 3                       | $9.7 \times 10^{-7}$ |
| 0.1                   | 5/3      | 5                       | $1.0 \times 10^{-7}$ |
| 0.1                   | 5/3      | 6                       | $3.8 \times 10^{-5}$ |
| 0.1                   | 5/3      | 7                       | no clumps         |
| 0.1                   | 7/5      | 11                      | $8.8 \times 10^{-7}$ |
| 0.1                   | 7/5      | 12                      | $6.6 \times 10^{-8}$ |
| 0.1                   | 7/5      | 13                      | no clumps         |
| 0.25                  | 5/3      | 5                       | $9.4 \times 10^{-6}$ |
| 0.25                  | 5/3      | 6                       | $3.0 \times 10^{-7}$ |
| 0.25                  | 5/3      | 7                       | no clumps         |
| 0.25                  | 7/5      | 11                      | $8.2 \times 10^{-7}$ |
| 0.25                  | 7/5      | 12                      | $7.2 \times 10^{-7}$ |
| 0.25                  | 7/5      | 13                      | no clumps         |
| 0.5                   | 5/3      | 6                       | $4.9 \times 10^{-5}$ |
| 0.5                   | 5/3      | 7                       | no clumps         |
| 0.5                   | 7/5      | 11                      | $1.9 \times 10^{-5}$ |
| 0.5                   | 7/5      | 12                      | $7.5 \times 10^{-6}$ |
| 0.5                   | 7/5      | 13                      | no clumps         |

Although Table 1 shows that the fragmentation boundary occurs for cooling times longer than that predicted by Gammie (2001), for $\gamma = 5/3$ the required cooling time is still smaller than the local dynamical time. It also shows that as the specific heat ratio decreases, the required cooling time increases and is almost twice the local dynamical time for $\gamma = 7/5$. The fragmentation boundary for a disc mass of $M_{\text{disc}} = 0.25$ and for both of the specific heat ratios considered is shown in Figures 2 and 3. Figure 2 shows the final surface density structures for $M_{\text{disc}} = 0.25$, a specific heat ratio of $\gamma = 5/3$, and for cooling times of $t_{\text{cool}}\Omega = 6$.
The fragmentation boundary is at a cooling time of between \( t_{\text{cool}} \Omega = 6 \) and \( t_{\text{cool}} \Omega = 7 \). The colour scale of the density and the linear scale of the image are the same as in Fig. 1.

![Figure 2. Surface density structure of discs with a mass of \( M_{\text{disc}} = 0.25 \), a specific heat ratio of \( \gamma = 5/3 \), and cooling times of \( t_{\text{cool}} \Omega = 6 \) (left hand panel) and \( t_{\text{cool}} \Omega = 7 \) (right hand panel). The lack of fragmentation in the right hand panel suggests that the fragmentation boundary is at a cooling time of between \( t_{\text{cool}} \Omega = 6 \) and \( t_{\text{cool}} \Omega = 7 \). The colour scale of the density and the linear scale of the image are the same as in Fig. 1.](image)

The boundary value of \( t_{\text{cool}} \Omega = 6 \) (left hand panel) and \( t_{\text{cool}} \Omega = 7 \) (right panel). The \( t_{\text{cool}} \Omega = 7 \) simulation was evolved for almost an outer rotation period longer than the \( t_{\text{cool}} \Omega = 6 \) simulation yet shows no signs of fragmentation. The discs shown in Figure 3 have the same parameters as in Figure 2 except \( \gamma = 7/5 \), and the cooling times are \( t_{\text{cool}} \Omega = 12 \), and \( t_{\text{cool}} \Omega = 13 \). Again there is no sign of fragmentation in the right hand panel which was also evolved for almost an outer rotation period longer than the simulation shown in the left hand panel.

As a further numerical check, we repeated one set of calculations using 125000 particles rather than 250000 particles. We considered only the case where \( M_{\text{disc}} = 0.25 \) and \( \gamma = 5/3 \). The result with 125000 particles was the same as the simulation with 250000 particles. Fragmentation occurred for \( t_{\text{cool}} = 6 \Omega^{-1} \) and did not occur for \( t_{\text{cool}} = 7 \Omega^{-1} \). Therefore, not only do the simulations that fragment satisfy the Jeans Criterion for fragmentation \( \text{[Bate & Burkert, 1997]} \), it appears that the results are resolution independent.

![Figure 3. Surface density structure of discs with a same mass as in Figure 2 but with a specific heat ratio of \( \gamma = 7/5 \), and cooling times of \( t_{\text{cool}} \Omega = 12 \) (left hand panel) and \( t_{\text{cool}} \Omega = 13 \) (right hand panel). The lack of fragmentation in the right hand panel suggests that for \( \gamma = 7/5 \) the fragmentation boundary is at a cooling time of between \( t_{\text{cool}} \Omega = 12 \) and \( t_{\text{cool}} \Omega = 13 \). The colour scale of the density and the linear scale of the image are the same as in Fig. 1.](image)

![Figure 4. The relationship defined by equation (4) for \( \gamma = 2 \) (solid line), \( \gamma = 5/3 \) (short-dashed line) and \( \gamma = 7/5 \) (long-dashed line). The data points show the couples \( (t_{\text{frag}} \Omega, \alpha_{\text{max}}) \) as derived from the simulations: the green squares refer to our simulations, while the blue triangle refers to Gammie (2001). The horizontal green line illustrates the constant values \( \alpha = 0.06251 \).](image)

### 3 A Maximum Value for Gravitational Stresses

Based on the results summarized in Table 1, for every value of \( M_{\text{disc}}/M_* \) and \( \gamma \), we can define a minimum cooling time for which no fragmentation occurs, \( t_{\text{ui}} \) and a maximum cooling time for which fragmentation does occur, \( t_{\text{f}} \).

The boundary value of \( t_{\text{cool}} \Omega = 6 \) for fragmentation can therefore be defined as \( t_{\text{frag}} = 1/2(t_{\text{ui}} + t_{\text{f}}) \), with a corresponding uncertainty given by \( \Delta t_{\text{frag}} = 1/2(t_{\text{ui}} - t_{\text{f}}) \). The stress \( \sigma_{\text{max}} \), corresponding to \( t_{\text{frag}} \), can be computed from equation (4), and the corresponding uncertainty is given by \( \Delta \sigma_{\text{max}} = (\sigma_{\text{max}}/t_{\text{frag}})\Delta t_{\text{frag}} \). The resulting values of \( t_{\text{frag}} \) and \( \sigma_{\text{max}} \) are shown as data points in Fig. 4 together with the curves defined by equation (4), for three values of \( \gamma = 2, 5/3 \) and 7/5. The filled green squares with error bars refer to the simulations presented here. The open blue triangle represent the value found by Gammie (2001) in his local, 2D simulations that assumed \( \gamma = 2 \). This is consistent with our result which suggests that for \( \gamma = 2 \), fragmentation should occur between \( t_{\text{cool}} \Omega = 3 \) and \( t_{\text{cool}} \Omega = 4 \). In fact, it is worth noting that since Gammie’s simulations are 2D, we should not expect a perfect agreement between our 3D results and his ones. This can be partially seen already from Fig. 4.

In particular, care should be taken in considering the role of the adiabatic index \( \gamma \), which has a different physical interpretation in 2D and in 3D. However, as discussed in more detail in Gammie (2001), a mapping is possible between the 2D and the 3D adiabatic indices. In the case of self-gravitating discs, Gammie’s choice of a 2D adiabatic index equal to 2 does correspond to \( \gamma = 2 \) also in 3D (Gammie 2001).

As can be seen, fragmentation occurs at an almost constant value of \( \alpha (\alpha_{\text{max}} \sim 0.06 \), indicated by the horizontal
green line in Fig. 4, thus vindicating the idea that gravitational instabilities cannot provide (in a steady state) a stress larger than \( \alpha_{\text{max}} \). If the dissipation associated with \( \alpha_{\text{max}} \) is not sufficient to balance the cooling rate, then the reaction of the disc is to fragment into bound objects.

4 DISCUSSION AND CONCLUSIONS

In this paper we elucidate the processes that lead to the fragmentation of a massive disc. Our main result is the determination of a maximum value for the stress that can be provided by gravitational instabilities in a quasi-steady state. We then argue that fragmentation will occur whenever the external cooling requires, in order to be balanced by internal heating, a stress larger than this maximum value, that we estimate to be \( \alpha_{\text{max}} \approx 0.06 \) (in units of the local disc pressure). As a consequence, discs with larger values of the ratio of the specific heats will be less susceptible to fragmentation. For \( \gamma = 7/5 \), for example, we estimate the fragmentation cooling time to be between 110\( \Omega^{-1} \) and 120\( \Omega^{-1} \), compared to between 3\( \Omega^{-1} \) and 4\( \Omega^{-1} \) for \( \gamma = 2 \) (Gammie 2001).

We wish to stress that the threshold value for \( \alpha \) that we have found here refers to a quasi-steady state, in which the disc stays in thermal equilibrium and the relevant physical quantities do not change significantly on time scales shorter than the thermal timescale. We have already shown (Lodato & Rice 2005) that very massive discs (with masses comparable to that of the central object) can generate transient strong spiral episodes, with correspondingly large values of the stress \( \alpha \), which, however, do not last for longer than one dynamical timescale (see details in Lodato & Rice 2005).

A further remark is in order, in reference to the possibility of non-local transport in self-gravitating discs. In all our simulations, we did not find any significant evidence for non-local transport of energy due to self-gravity (Lodato & Rice 2004, 2005). If the disc does not fragment, the dissipation provided by the gravitational stresses balances almost exactly the imposed cooling rate. However, this conclusion might depend on the simulation setup and, in particular, on the assumed radial dependence of the cooling time. Mejia et al. (2005) claim to find evidence for non-local energy transport in their simulations that employ a \( t_{\text{cool}} \) constant with radius (rather than \( \propto \Omega^{-1} \), as we do). If this is the case, it might offer a possible escape route for the disc in order to avoid fragmentation. Consider the case where \( t_{\text{cool}} \Omega \) is such that the disc is stable against fragmentation in the inner regions, but would fragment in the outer regions, following our prediction in equation 2 (which is based on the implicit assumption that energy dissipation is viscous).

In such a situation, the inner disc could “help” the outer disc, by heating it up via non-local effects and preventing fragmentation. This process is not viable in the simulations presented here, since here the whole disc is uniformly stable or unstable with respect to fragmentation and increasing the cooling rate of the inner disc via non-local energy transport would make it fragment. Clearly, further simulations of the fragmentation process in discs with a radius-dependent \( t_{\text{cool}} \Omega \) are needed to investigate this issue.

Finally, we note that the fragmentation requirements determined by Rafikov (2005) were calculated assuming the much shorter cooling times of Gammie (2001), and by assuming that the Toomre \( Q \) parameter must be unity or less for fragmentation to commence. Since the actual \( Q \) value required for fragmentation can be slightly greater than 1 (Pickett et al. 1998), and since the required cooling time in discs with \( \gamma = 7/5 \) (as used by Boss (1998, 2002)) can be larger than that predicted by Gammie (2001), the conditions for fragmentation may not be as stringent as those determined by Rafikov (2005).

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