NNLO SOFT AND VIRTUAL CORRECTIONS FOR ELECTROWEAK, HIGGS, AND SUSY PROCESSES *

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I present applications of a master formula for next-to-next-to-leading order soft and virtual QCD corrections to various electroweak, Higgs, and supersymmetric processes. They include Drell-Yan and charged Higgs production, single-top production in flavor-changing neutral-current processes, and squark and gluino production.

1 Introduction

Soft and virtual QCD corrections to processes of electroweak or supersymmetric (SUSY) origin can be substantial. The calculations of the cross sections, total or differential, in hadron-hadron and lepton-hadron colliders can be represented in factorized form by

$$\sigma = \sum f \int [\prod_i dx_i \phi_{f/h_i}(x_i, \mu_F^2)] \hat{\sigma}(s, t_i, \mu_F, \mu_R)$$  \hspace{1cm} (1)

with $\sigma$ the physical cross section, $\hat{\sigma}$ the partonic cross section, $\phi_{f/h_i}$ the parton distribution for parton $f$ in hadron $h_i$, and $\mu_F, \mu_R$ the factorization and renormalization scales, respectively.

The perturbatively calculable $\hat{\sigma}$ includes soft and virtual corrections from soft-gluon emission and virtual diagrams. These corrections appear as plus distributions and delta functions in $\hat{\sigma}$. In single-particle-inclusive (1PI) kinematics the plus distributions are $D_1(s_4) \equiv [\ln(s_4/M^2)/s_4]_+$ with $s_4 = s + t + u - \sum m^2$, where $s, t, u$ are kinematical invariants and $m$ the masses of the particles in the

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scattering, and \(M\) any relevant hard scale. In pair-invariant-mass (PIM) kinematics they are \(D_i(z) \equiv \ln^i(1 - z)/(1 - z)\), with \(z = Q^2/s\), where \(Q^2\) is the pair mass squared. Note that \(s_4\) (sometimes called \(s_2\)) → 0 and \(z\) (sometimes called \(x\)) → 1 at threshold.

A unified approach and a master formula for calculating these corrections at next-to-next-to-leading order (NNLO) for any process in hadron-hadron and lepton-hadron colliders have been recently presented in Ref. [1]; they follow from threshold resummation studies [2, 3, 4, 5]. Here I describe various applications to processes which are of electroweak or supersymmetric origin at lowest order.

## 2 NLO and NNLO corrections

I begin by presenting the generalized next-to-leading-order (NLO) master formula for processes with simple color flows, which is appropriate for many electroweak and SUSY processes. The NLO soft and virtual corrections in the MS scheme in either 1PI or PIM kinematics take the form

\[
\hat{\sigma}^{(1)} = \sigma^B \frac{\alpha_s(\mu^2)}{\pi} \{c_3 D_1(x_{th}) + c_2 D_0(x_{th}) + c_1 \delta(x_{th})\},
\]

with \(x_{th}\) the threshold variable \(s_4\) (in 1PI kinematics) or \(1 - z\) (in PIM kinematics), where \(\sigma^B\) is the Born term, \(c_3 = \sum C_f, \ c_2 = T_2 - \sum C_f, \ln(\mu^2/F)\) with

\[
T_2 = 2 \text{Re} \Gamma\,'(1)_{S} - \sum C_f, \ln(\mu^2/F)
\]

and \(c_1 = c_0 + T_1\), with

\[
c_i = \sum C_f, \delta K \ln \left(\frac{-t_i}{M^2}\right) - \gamma(1), \ln \left(\frac{\mu^2}{s}\right) + d_\alpha, \beta_0, \ln \left(\frac{\mu^2}{s}\right).
\]

We note that we sum over incoming partons \(i\) and the \(C_f,\)'s are color factors, \(C_F = 4/3\) for quarks and \(C_A = 3\) for gluons. Also \(\delta K\) is 0 (1) for PIM (1PI) kinematics. \(\Gamma\)'s are soft anomalous dimensions which describe the color exchange in the hard scattering, \(\gamma_i\) are parton anomalous dimensions, \(\beta_0 = (11C_A - 2n_f)/3\) is the lowest-order beta function, and \(d_\alpha\) equals 0,1,2 if the Born cross section is of order \(\alpha_s^0, \alpha_s^1, \alpha_s^2\), respectively. More details are given in Ref. [1].
At NNLO the $\overline{\text{MS}}$ scheme master formula for the soft and virtual corrections is

$$\hat{\sigma}^{(2)} = \sigma^{(2)} (\alpha_s^2 (\mu_R^2) / \pi^2) \hat{\sigma}'^{(2)}$$

with

$$\hat{\sigma}'^{(2)} = \frac{1}{2} c_3^2 D_3(x_{th}) + \left[ \frac{3}{2} c_3 c_2 - \frac{\beta_0}{4} c_3 \right] D_2(x_{th})$$

$$+ \left\{ c_3 c_1 + c_2^2 - \zeta_2 c_3^2 - \frac{\beta_0}{2} T_2 + \frac{\beta_0 c_3}{4} \ln \left( \frac{\mu^2}{s} \right) + \sum_i C_{f_i} K \right\} D_1(x_{th})$$

$$+ \left\{ c_2 c_1 - \zeta c_2 c_3 + \zeta_3 c_3^2 - \frac{\beta_0}{2} T_1 + \frac{\beta_0 c_2}{4} \ln \left( \frac{\mu^2}{s} \right) + 2 \Re \Gamma_3^{(2)} - \sum_i \nu_i^{(2)} \right\}$$

$$+ \sum_i C_{f_i} \left[ \frac{\beta_0}{8} \ln^2 \left( \frac{\mu^2}{s} \right) - \frac{K}{2} \ln \left( \frac{\mu^2}{s} \right) - K \delta_K \ln \left( \frac{-t_i}{M^2} \right) \right] D_0(x_{th})$$

$$+ R_3(x_{th}) \delta(x_{th})$$.

More details and extensions of the master formulas to the more general case of complex color flows are given in Ref. [1].

3 Applications to electroweak and SUSY processes

Using the NNLO master formula I have rederived known NNLO results for Drell-Yan and Higgs production and for $W^+\gamma$ production, and I have produced new results for many other processes [1]. Here I give a few examples.

3.1 The Drell-Yan process, $q\bar{q} \rightarrow V$

The NLO corrections are given by Eq. (2) with $x_{th} = 1 - x = 1 - Q^2 / s$, and

$c_3 = 4C_F, c_2 = -2C_F \ln (\mu^2 / Q^2), c_1 = -(3/2)C_F \ln (\mu^2 / Q^2) + 2C_F \zeta_2 - 4C_F$.

Using Eq. (5) we rederive the NNLO soft and virtual corrections in Ref. [6] and thus also derive previously unknown two-loop anomalous dimensions in Eq. (5). Similar results are given in Ref. [1] for $W^+\gamma$ production, $q\bar{q} \rightarrow W^+\gamma$ [7], and related results are derived in [1] for Standard Model Higgs production, $gg \rightarrow H$.

3.2 Charged Higgs production, $b\bar{g} \rightarrow H^+\bar{t}$

The NLO corrections are given by Eq. (2) and the NNLO corrections by Eq. (5), with

$x_{th} = s_2 = s + t + u - m_{H^+}^2 - m_\tau^2 - m_H^2$, and

$c_3 = 2(C_F + C_A), c_2 = \ldots$
$C_F [\ln(m_H^2/(sm_t^2)) - 1 - \ln(\mu_F^2/s)] + C_A [\ln(m_H^2/(t_1 u_1)) + \ln(\mu_F^2/s)]$, and $c_4^t = \ln(\mu_F^2/s)[C_F \ln(-u_1/m_H^2) + C_A \ln(-t_1/m_H^2) - 3C_F/4 - \beta_0/4] + (\beta_0/4) \ln(\mu_R^2/s)$.

### 3.3 FCNC single-top production, $eu \to et$

Here we consider single-top production mediated via flavor-changing neutral currents (FCNC) through a term in the effective Langrangian of the form $\kappa_{tq\gamma} e i \sigma_{\mu\nu} q F^{\mu\nu}/\Lambda$ [8].

![Figure 1: Born cross section, NLO corrections, and total NLO (Born+NLO corrections) cross section for FCNC single-top production at HERA with $m_t=175$ GeV/c$^2$, $\kappa_{tq\gamma} = 0.1$, and $\sqrt{S} = 300$ GeV. Here $Q = \mu_F = \mu_R$.](image)

The NLO corrections are given by Eq. (2) with $x_{th} = s_2 = s + t + u - m_t^2 - 2m_H^2$, $c_3 = 2C_F$, $c_2 = C_F[-1 - 2 \ln((-u + m_H^2)/m_t^2) + 2 \ln(m_t^2 - t)/m_t^2 - \ln(\mu_F^2/m_H^2)]$, and $c_4^t = [-3/4 + \ln((-u + m_t^2)/m_H^2)] C_F \ln(\mu_F^2/s)$. They stabilize the FCNC single top cross section at HERA as a function of scale (see fig. 1)
The NNLO corrections are given by Eq. (5).

### 3.4 Squark and gluino production

We now consider squark and gluino production. We start with squark-pair production. For the process $\bar{q}q \rightarrow \tilde{q}\tilde{q}$ and $qq \rightarrow \tilde{q}\tilde{q}$ the $c_i$ coefficients are the same as for the $q\bar{q} \rightarrow Q\bar{Q}$ channel in heavy quark pair hadroproduction [1]; for $gg \rightarrow \tilde{q}\tilde{q}$ they are the same as for $gg \rightarrow Q\bar{Q}$.

We continue with gluino pair production. For the process $q\bar{q} \rightarrow \tilde{g}\tilde{g}$ the $c_i$'s are the same as for $q\bar{q} \rightarrow \tilde{q}\tilde{q}$; for $gg \rightarrow \tilde{g}\tilde{g}$ they are the same as for $gg \rightarrow \tilde{q}\tilde{q}$.

Finally we study squark-gluino production, $qg \rightarrow \tilde{q}\tilde{g}$. Here $x_{th} = s_4 = s + t + u - m_{\tilde{q}} - m_{\tilde{g}}$, $c_3 = 2(C_F + C_A)$, $c_2 = -C_F - C_A - 2C_F \ln(-u_1/m^2) - 2C_A \ln(-t_1/m^2) - (C_F + C_A) \ln(\mu_F^2/s)$, and $c_1^a = \ln(\mu_R^2/s) [C_F \ln(-u_1/m^2) + C_A \ln(-t_1/m^2) - 3C_F/4 - \beta_0/4] + (\beta_0/2) \ln(\mu_R^2/s)$, with $m = m_{\tilde{q}}$ or $m_{\tilde{g}}$.

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