Model of the Movement of Links of the Combined Manipulator

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Abstract. In various industries (mechanical engineering, transport) broad application is found by the combined manipulators which design telescopic arrows and basic and rotary devices with the mobile columns in the longitudinally vertical plane providing increase in a departure of working bodies and the served space enter. Article is devoted to a question of development of mathematical model of the movement of kinematic links of the combined manipulator with a mobile column in the longitudinally vertical plane. The mode of lifting of loads at the synchronous movement of an arrow and column from the provision of a set of freight in transport situation is considered.

1. Introduction
At creation of hoisting-and-transport, loading and other cars the processing equipment in the form of the manipulators combined the gidrofitsirovannykh is widely applied. It is expedient to carry out works on creation and improvement of such cars on the basis of studying of dynamics of elements of a design. Thus methods of mathematical modeling are the most effective. Proceeding from it, the researches on mathematical models of dynamics of the mode of the movement of links of the combined manipulator directed on justification of parameters of kinematics and a design of processing equipment should be considered actual. Justification of the settlement scheme.

2. Theoretical part
The settlement scheme of system "processing equipment - freight" is submitted in figure 1. The considered mode can take place during the operation of the manipulator as processing equipment of forest, road-building, load-lifting and other cars.

After capture by working body freight is tightened to the car by a telescopic arrow retraction of sections, then freight is established by inclusion of hydraulic cylinders of turn of a column and raising of an arrow in transport situation. Thus the arrow makes turn concerning axis K, and a column concerning axis O. An arrow angle of rotation – φ (the relative movement), a column angle of rotation – α (the figurative movement). I reckoned a corner φ – from the extreme lower provision of an arrow; counting of a corner α – from the extreme right provision of a column.
1 – the basic and rotary device; 2, 3, 4 – external, average, internal sections of a telescopic arrow; 5 – hydraulic cylinder of raising of an arrow; 6, 7 – hydraulic cylinders of the mechanism of promotion of sections; 8 – the mechanism of turn of the manipulator in the horizontal plane; 9 – hydraulic cylinder of turn of a column; 10 – column; o₁k = C; oo₂ = l₆ = C1.

In figure 1 the following designations are accepted:

- G₁, G₂, G₃ – the gravity of external, average and internal sections of an arrow;
- Gₐ₁, Gₐ₂, Gₐ₃ – the gravity of hydraulic cylinders of promotion of sections and the mechanism of promotion of sections of an arrow;
- Gₐ₄, Gₐ₅ – the gravity of capture, freight, the rotator;
- Gₐ₆, Gₐ₇, Gₐ₈, Gₐ₉, Gₐ₁₀, Gₐ₁₁ – the sizes of elements of a design of the manipulator.

**Figure 1.** The settlement scheme of system "processing equipment – freight".

3. Development of the equations of the movement of system "processing equipment - freight"

The arrow makes rotation in the ZKX plane, a column – in the Z1OX1 plane. Angles of rotation α and φ unambiguously define provisions of these elements of system in the rotation planes. At the known sizes of an arrow of L and columns the LK provision of any point can be defined through the specified parameters. Proceeding from it, the system can be considered as system with two degrees of freedom (K = 2) with the generalized coordinates α and φ.

For drawing up the equations of the movement of this mechanical system we will use Lagrange's equations of the 2nd sort. According to number of degrees of freedom of system we write down two equations of Lagrange.
\[ \frac{d}{dt} \left( \frac{\partial \tau}{\partial \alpha} \right) - \left( \frac{\partial \tau}{\partial \alpha} \right) = Q_\alpha \]
\[ \frac{d}{dt} \left( \frac{\partial \tau}{\partial \phi} \right) - \left( \frac{\partial \tau}{\partial \phi} \right) = Q_\phi \]  
(1)

where – the generalized forces corresponding to the generalized coordinates, respectively, \( \alpha \) and \( \phi \).

Kinetic energy of the considered system is equal to the sum of kinetic energy of a column and arrow, i.e. the sum of kinetic energy in the relative and figurative movement

\[ T = T_1 + T_2, \]  
(2)

where \( T_1 \) – kinetic energy of the specified mass of a column together with the specified mass of the elements of a design mounted on it (hydraulic cylinders of turn of a column and raising of an arrow and other parts of a hydraulic actuator); \( T_2 \) – kinetic energy of the specified mass of an arrow and freight.

In the course of turn of a telescopic arrow of its section don't move forward, the size of an arrow of \( L \) doesn't change, therefore, provisions of the centers of mass of elements of a design of an arrow concerning axis \( K \) (radiuses of inertia of masses) remain constants. In this case for the purpose of simplification we give definitions of kinetic energy of system of mass of elements of a design of a telescopic arrow to a point of \( C \) – to a rotator subweight point to an arrow. We lead the mass of elements of a design of a column to an axis of fastening of an arrow of \( K \). At determination of the specified mass of elements of a design of an arrow of \( m_{PC} \) we proceed from a condition of equality of kinetic energy of the specified weight to the sum of kinetic energy of masses which it replaces. Therefore

\[ \frac{G_{PC} \ell^2}{2g} \cdot \frac{L^2}{\ell^2} \cdot \left[ (G_1 \frac{\ell_3^2}{\ell^2} + G_2 \frac{\ell_4^2}{\ell^2} + G_3 \frac{\ell_5^2}{\ell^2} + G_{C1} \frac{\ell_2^2}{\ell^2} + G_0 \frac{\ell_4^2}{\ell^2} + G_3 + G_Z + G_y) \right] \]  
(3)

From here the \( C \) mass of an arrow specified to a point

\[ m_{PC} = \frac{G_{PC}}{g} = \frac{G_1}{g} \cdot \frac{\ell_3^2}{\ell^2} + \frac{G_2}{g} \cdot \frac{\ell_4^2}{\ell^2} + \frac{G_3}{g} \cdot \frac{\ell_5^2}{\ell^2} + \frac{G_{C1}}{g} \cdot \frac{\ell_2^2}{\ell^2} + \frac{G_0}{g} \cdot \frac{\ell_4^2}{\ell^2} + \frac{G_3 + G_Z + G_y}{g}, \]  
(4)

\[ m_{PC} = m_1 \ell_3^2 + \frac{m_2 \ell_4^2}{\ell^2} + \frac{m_3 \ell_5^2}{\ell^2} + m_{C1} \frac{\ell_2^2}{\ell^2} + m_{C2} \frac{\ell_3^2}{\ell^2} + m_0 \frac{\ell_4^2}{\ell^2} + m_r + m_Z + m_y \]  
(5)

K mass of a column specified to a point and the elements of a design fixed on it can be determined from the following expression:

\[ \frac{G_{KR} \ell^2}{2g} \cdot \frac{L^2}{\ell^2} = \frac{G_K \ell^2}{2g} \left[ \frac{(0.5l_K)^2}{L^2} + 0.5G_{C3} \ell^2 (0.5l^2)^2 + 0.5G_{C4} \ell^2 (0.5l^2)^2 \right] \]  
(5)

By drawing up expression (6) we proceed from an assumption that the gravity of hydraulic cylinders of \( G_{C3} \) and \( G_{C4} \) is evenly distributed between an arrow and a column, and also between a column and the basis of the basic and rotary device; points of their appendix are, respectively, \( 0.5l \) and \( 0.5l_8 \) from an axis of fastening of an arrow to a column – K point.

From expression (6) \( K \) mass of a column specified to a point and elements of a design is equal

\[ m_{PK} = \frac{G_K}{g} \cdot \frac{(0.5l_K)^2}{L^2} + \frac{0.5G_{C3} \ell^2 (0.5l^2)^2}{L^2} + \frac{0.5G_{C4} (0.5l^2)^2}{L^2} = 0.25 \frac{G_K}{g} + 0.125 \frac{G_{C3}}{g} \cdot \frac{\ell^2}{L^2} + \frac{G_{C4}}{g} \cdot \frac{\ell^2}{L^2} \]  
\( 7 \)

\[ m_{PK} = 0.25 \frac{G_K}{g} + 0.125 \frac{G_{C3}}{g} \cdot \frac{\ell^2}{L^2} + 0.125 \frac{G_{C4}}{g} \cdot \frac{\ell^2}{L^2} \]  
(6)

According to (2) kinetic energy of system

\[ T = T_1 + T_2 = \frac{1}{2} \left( m_{PC} \frac{V_{PC}^2}{\ell^2} + I_{C \phi} \frac{\phi^2}{L^2} \right). \]  
(7)
where \( I_0 \) – the moment of inertia of a column concerning an axis \( O, I_C \) – central moment of inertia of an arrow assembled, \( V_{ac} \) – speed of the absolute movement of a point \( C \) – points of reduction of mass of an arrow.

We will apply the theorem of addition of speeds according to which the absolute speed of a point \( C \) is equal to the geometrical sum of its figurative and relative speeds.

\[
V_{ac}^2 = V_{ec}^2 + V_{rc}^2 + 2V_{ec} \cdot V_{rc} \cos \gamma
\]

where \( V_{ec}, V_{rc} \) – speeds of the figurative and relative movement of a point \( C \),

\( \cos \gamma \) – a corner between the directions of vectors of the figurative and relative movement of a point of Page.

In figure 2 the scheme for determination of speed of the absolute movement of a point \( C \) – points of reduction of mass of an arrow and freight is shown.

\[ V_{ec} = OC \cdot \dot{\alpha}; V_{rc} = L \cdot \dot{\varphi} \]

From a triangle of Construction Department

\[ OC^2 = L_C^2 + L^2 - 2L_C \cdot L \cos(\varphi_H + \varphi) \]

From the same triangle

\[ L_C^2 = L^2 + OC^2 - 2L \cdot OC \cdot \cos C \]

Follows from figure 2 that corners \( \gamma \) and \( C \) are equal, as corners with mutually perpendicular parties. Therefore

\[
\cos \gamma = \cos C = \frac{L^2 + OC^2 - L_C^2}{2L \cdot OC}
\]

Speed of the absolute movement

\[
V_{ac}^2 = \dot{\alpha}^2 \cdot OC^2 + \dot{\varphi}^2 \cdot L^2 + 2\dot{\alpha} \cdot \dot{\varphi} \cdot L \cdot OC \cdot \cos \gamma.
\]

\[
\cos \gamma = \frac{L^2 + OC^2 - L_C^2}{2L \cdot OC} = \frac{L^2 + L_C^2 + L^2 - 2L_C \cdot L \cos(\varphi_H + \varphi) - L_C^2 \cdot \frac{L^2 - L_C \cdot L}{L \cdot OC}}{2L \cdot OC} \cdot \cos(\varphi_H + \varphi)
\]

\[
V_{ac}^2 = \dot{\alpha}^2 \cdot OC^2 + \dot{\varphi}^2 \cdot L^2 + 2\dot{\alpha} \cdot \dot{\varphi} \cdot L \cdot OC \cdot \frac{L^2 - L_C \cdot L}{L \cdot OC} \cdot \cos(\varphi_H + \varphi)
\]

(8)

Taking into account (10) expression of kinetic energy assumes an air

\[
T = \frac{m_{p, K} \cdot L_C \cdot \dot{\alpha}^2}{2} + \frac{m_{p, C} \cdot OC^2 \cdot \dot{\alpha}^2}{2} + \frac{m_{p, C} \cdot L^2 \cdot \dot{\varphi}^2}{2} + m_{p, C} \cdot L \cdot OC \cdot \dot{\alpha} \cdot \dot{\varphi} \cdot \frac{L^2 - L_C \cdot L}{L \cdot OC} \cdot \cos(\varphi_H + \varphi) + \frac{l_C \cdot \dot{\varphi}^2}{2}
\]

(9)

Differentiating expression of kinetic energy (11) on components of the equations of Lagrange (1), we receive system of the equations of the movement of dynamic system of the manipulator in the following look

\[
m_{p, K} \cdot L_C \cdot \dot{\alpha} + m_{p, C} \cdot OC^2 \cdot \dot{\alpha} + m_{p, C} \cdot OC \cdot L \cdot \dot{\varphi} \cdot \frac{L^2 - L_C \cdot L}{L \cdot OC} \cdot \cos(\varphi_H + \varphi) - m_{p, C} \cdot OC \cdot L \cdot \dot{\varphi} \cdot \frac{L^2 - L_C \cdot L}{L \cdot OC} \cdot \sin(\varphi_H + \varphi) = Q_a
\]

(10)

\[
(m_{p, C} L_C^2 + l_C) \dot{\varphi} + m_{p, C} \cdot OC \cdot \dot{\alpha} \cdot L \cdot \frac{L^2 - L_C \cdot L}{L \cdot OC} \cdot \cos(\varphi_H + \varphi) = Q_\varphi
\]
Figure 2. The scheme for determination of speed of the absolute movement of a point $C$ at the synchronous movement of an arrow and column: $\varphi$ $n$ - an initial corner between axes of an arrow and a column; $\varphi$ - an arrow angle of rotation (the relative movement); $\alpha$ - a column angle of rotation (the figurative movement); $\dot{\alpha}$, $\dot{\varphi}$ - angular speeds of a column and arrow, respectively.

For determination of the generalized forces of $Q_\varphi$ and $Q_\alpha$ we will use the principle of possible movements of system in the direction of increase of the generalized coordinates $\varphi$ and $\alpha$ - $\Delta \varphi$ и $\Delta \alpha$. Thus at calculation of the generalized force of $Q_\alpha$ it is accepted $\Delta \alpha = 0$, and at calculation of $Q_\varphi$, $\Delta \varphi = 0$. As active forces the gravity of elements of a design and freight of $G_i$ effort on rods of hydraulic cylinders of $P_C$ and $P$ are accepted. The generalized force is accepted in the form of coefficient in expression of the sum of elementary works of active forces in the direction of possible movement:

$$\delta A_\varphi = Q_\varphi \cdot \Delta \varphi; \ \delta A_\alpha = Q_\alpha \cdot \Delta \alpha.$$  \hspace{1cm} (11)

We will determine the sum of elementary works of active forces in the direction of the generalized coordinate $\varphi$. Thus $\Delta \varphi \neq 0, \Delta \alpha = 0$.

At determination of the generalized force of $Q_\varphi$ we use expressions (4), (5) brought to a point $C$

From the mass of an arrow of $m_{\text{arrow}}$ at the horizontal provision of an arrow.

Then $\sum \delta A_\varphi = (P \ell \sin \beta - m_{P \cdot C} \cdot g \cdot L) \Delta \varphi$ $Q_\varphi = P \ell \sin \beta - m_{P \cdot C} \cdot g \cdot L$  \hspace{1cm} (12)

We will determine the sum of elementary works of active forces and gravity of elements of a design in the direction of the generalized coordinate $\alpha$. Thus $\Delta \varphi = 0$, $\Delta \alpha \neq 0$ in the direction of the generalized coordinate $\alpha$ the active force of $P_C$ and gravity of elements of a design of an arrow and
column make work. For the purpose of simplification of expression of the generalized force of $Q_\alpha$ we will lead the gravity of elements of a design of an arrow, freight and a column to $K$ point. Thus we consider the horizontal provision of an arrow. Reduction of forces to the chosen points we carry out proceeding from a condition of equality of the moments of the specified force to the sum of the moments of the specified forces concerning any point on the plane (Varignon's theorem).

When determining of the arrow brought to a point $K$ gravity we use expressions of the specified mass of an arrow to a point $C$ at the horizontal provision of an arrow (4), (5) $m_{p,c}$.

We will work out the equation of the moments of the given arrow gravity to a point $C$ $G_{p,c}$ and the given arrow gravity to a point $K$ - $G_{g,K}^C$ concerning axis $O$.

\[
G_{g,K}^C = \frac{m_{p,c} g (L + L_K \cdot \cos \alpha_1) = G_{g,K}^C \cdot L_K \cdot \cos \alpha_1}{m_{p,c} g L + m_{p,c} g L_K \cdot \cos \alpha_1} = \frac{m_{p,c} g L}{L_K \cdot \cos \alpha_1} + m_{p,c} g
\]  

(13)

We will define brought to a point $K$ the gravity of elements of a design of a column and hydraulic cylinders of the drive of executive $G_{c}, G_{c4}, G_{c3}$ mechanisms. Thus we will use expressions (7), (8) determination of the specified mass of the specified design elements to $K$ point:

\[
G_{g,K}^C = m_{p,c} g
\]  

(14)

As the figurative movement of system is rotary and at simultaneous rotation of a column and arrow the distance of $OC$ constantly increases, there is a rotary (Coriolis) acceleration $w_C$ and the Coriolis force of inertia $F_C$.

Coriolis acceleration is determined by a formula

\[
\omega_C = 2\dot{\alpha} \cdot \nu_C = 2\dot{\alpha} \cdot \varphi \cdot L
\]

Taking into account the moment from the Coriolis force of inertia the generalized force of $Q_\alpha$ assumes an air

\[
Q_\alpha = P_C \cdot \sin \alpha_2 \cdot \ell g - (G_{g,K}^C + G_{g,R}^C) L_K \cdot \cos \alpha_1 - 2m_{p,c} \cdot \dot{\alpha} \cdot \varphi \cdot L \cdot L_K \cdot \cos \gamma_1.
\]  

(15)

Taking into account expressions (14) and (17) equations of the movement of the considered dynamic system assume the following air:

\[
\begin{aligned}
&\dot{H}_{p,c} = L_K^2 \cdot \dot{\alpha} + m_{p,c} \cdot \dot{\alpha} + m_{p,c} \cdot \nu_C \cdot \nu_C \cdot L \cdot \dot{\varphi} \cdot \left( \frac{L^2_L - L_K^2 \cdot L}{L \cdot \nu_C} \cdot \cos(\varphi_H + \varphi) - m_{p,c} \cdot \nu_C \cdot L \right) \cdot \dot{\varphi}^2 \cdot \left[ \frac{L^2_L - L_K^2 \cdot L}{L \cdot \nu_C} \right] \cdot \sin(\varphi_H + \varphi) = \\
&= P_C \cdot \sin \alpha_2 \cdot \ell g - (G_{g,K}^C + G_{g,R}^C) L_K \cdot \cos \alpha_1 - 2m_{p,c} \cdot \dot{\alpha} \cdot \varphi \cdot L \cdot L_K \cdot \cos \gamma_1.
\end{aligned}
\]  

(16)

4. Conclusion

As a result of the performed work the system of the non-uniform differential equations of the second order which are a basis of mathematical models of the combined manipulator during the work in the mode of lifting of loads at the synchronous movement of an arrow and column is received. The received system of the equations allows to investigate in the course of modeling influence on the level of loads of processing equipment of parameters of the kinematic scheme and also sizes of mass of freight and processing equipment. Besides, changing the frequency of rotation of shaft of pumps of a hydraulic system, we change the speed of the movement of rods of hydraulic cylinders and, therefore, size of absolute speed of a point of reduction of mass of an arrow and freight (a point $C$), the angular
speed and angular acceleration of an arrow and a column. The equations allow to estimate also influence of these factors on the size of loads of elements of a design of processing equipment.

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