**$^{10}\text{B}+\alpha$ states with chain-like structures in $^{14}\text{N}$**

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I investigate $^{10}\text{B}+\alpha$ cluster states of $^{14}\text{N}$ with a $^{10}\text{B}+\alpha$ cluster model. Near the $\alpha$-decay threshold energy, I obtain $K^\pi = 3^+$ and $K^\pi = 1^+$ rotational bands having $^{10}\text{B}(3^+)+\alpha$ and $^{10}\text{B}(1^+)+\alpha$ components, respectively. I assign the band-head state of the $K^\pi = 3^+$ band to the experimental $3^+$ at $E_x = 13.19$ MeV of $^{14}\text{N}$ observed in $\alpha$ scattering reactions by $^{10}\text{B}$ and show that the calculated $\alpha$-decay width is consistent with the experimental data. I discuss an $\alpha$-cluster motion around the $^{10}\text{B}$ cluster and show that $^{10}\text{B}+\alpha$ cluster states contain significant components of a linear-chain $3\alpha$ configuration, in which an $\alpha$ cluster is localized in the longitudinal direction around the deformed $^{10}\text{B}$ cluster.

**I. INTRODUCTION**

It is known that cluster structures appear in various nuclei including unstable nuclei (for instance, $^{14}\text{B}$ and references therein.) For cluster states having an $\alpha$ cluster around a core nucleus, well-known examples are $^{18}\text{O}+\alpha$ states in $^{20}\text{Ne}$ and $^{12}\text{C}+\alpha$ states in $^{16}\text{O}$ [6]. Recent experimental and theoretical studies have revealed many cluster resonances in highly excited states near the $\alpha$-decay threshold also in unstable nuclei, for instance, $^{4-8}\text{He}+\alpha$ states in Be isotopes [1, 2, 3, 20], $^{10}\text{Be}+\alpha$ states in $^{14}\text{C}$ [23, 30, 53], $^{14}\text{C}+\alpha$ states in $^{18}\text{O}$ and their mirror states [31, 40], $^{16}\text{O}+\alpha$ states in $^{20}\text{Ne}$ [38, 42].

Multi-$\alpha$ cluster states such as cluster gas and linear-chain states of $\alpha\alpha$ systems are also interesting topics. The $\alpha$-cluster gas was proposed by Tohsaki et al. to describe $3\alpha$ cluster structure of $^{12}\text{C}(0^+_2)$ [10] and extended to excited states of $^{12}\text{C}$ and other nuclei [17, 43]. The linear-chain $\alpha\alpha$ state was originally proposed for $^{12}\text{C}(0^+_2)$ by Morinaga in the 1950-60s [50, 51]. However, in the 1970s, this picture was excluded at least for $^{12}\text{C}(0^+_2)$ having a larger $\alpha$-decay width than the one expected from the linear-chain structure [52]. In spite of many discussions for several decades, existence of linear-chain $\alpha\alpha$ states have yet been confirmed and it is still an open problem to be solved. It is naively expected that the linear-chain configuration is not favored in an $\alpha\alpha$ system because it costs much kinetic energy to keep $\alpha$ clusters in a row. It means that some mechanism is necessary to form the linear-chain structure. In progress of physics of unstable nuclei since the 1990s, it was proposed for neutron-rich C isotopes that excess neutrons may stabilize the linear-chain structure [1, 8]. Itagaki et al. analyzed stability of a $3\alpha$-chain configuration surrounded by excess neutrons in molecular orbitals against the bending motion and suggested that the linear-chain structure can be stable in $^{16}\text{C}$ but unstable in $^{12}\text{C}$ and $^{14}\text{C}$ [53]. More recently, Suhara and the author predicted a rotational band with a linear $3\alpha$ configuration in excited states of $^{15}\text{C}$ near the $\alpha$-decay threshold [54]. They pointed out that the orthogonal condition to lower states is important for the stability of the linear-chain structure. The linear-chain structure is expected to be more favored in high spin states because of stretching effect in rotating systems as suggested in $^{15}\text{C}$ [1] and $^{16}\text{O}$ [55].

According to analysis in Refs. [54, 56], linear-chain states of $^{14}\text{C}$ are found to have a $2\alpha + 2\alpha$ correlation and are interpreted as $^{10}\text{Be}+\alpha$ structures, where the $^{10}\text{Be}$ cluster is a prolate deformed state containing a $2\alpha$ core and an additional $\alpha$ cluster is located in the longitudinal direction of the $^{10}\text{B}$ cluster. Similarly, the linear-chain state of $^{15}\text{C}$ suggested in Ref. [1] also shows a $^{11}\text{Be}+\alpha$ cluster structure with a prolate deformed $^{11}\text{Be}$ cluster and an $\alpha$ cluster in the longitudinal direction. It means that, the linear-chain states in these neutron-rich C tend to have the $2\alpha$ correlation, and therefore $3\alpha$ linear-chain structures are expected to be found in Be+$\alpha$ cluster states.

In this paper, I focus on $^{10}\text{B}+\alpha$ cluster states in excited states of $^{14}\text{N}$. In experimental energy levels of $^{14}\text{N}$ near the $\alpha$-decay threshold, $J^\pi = 3^+$ and $1^+$ resonances were observed by $^{10}\text{B}$ scattering reactions by $^{10}\text{B}$ [57]. These resonances are expected to be $^{10}\text{B}+\alpha$ cluster states because of significant $\alpha$-decay widths. In analogy to $^{10}\text{Be}+\alpha$ cluster states, it is interesting to investigate whether $^{10}\text{B}+\alpha$ states with the dominant linear-chain structure exist. The ground state ($3^+$) and the first excited state ($1^+$) of $^{10}\text{B}$ can be described by the deformed state with a $2\alpha$ core surrounded by $pn$ as discussed in Refs. [4, 58]. If a $^{10}\text{B}+\alpha$ cluster state has an $\alpha$ cluster in the longitudinal direction of the deformed $^{10}\text{B}$ cluster, the $^{10}\text{B}+\alpha$ cluster state can be interpreted as a kind of linear-chain state that contains dominantly $3\alpha$ clusters arranged in a row.

My aim is to study $^{10}\text{B}+\alpha$ cluster states of $^{14}\text{N}$ and discuss $3\alpha$ configurations, in particular, the linear-chain component in $^{10}\text{B}+\alpha$ cluster states. I calculate $^{10}\text{B}(3^+) \otimes L_\alpha$ and $^{10}\text{B}(1^+) \otimes L_\alpha$ components and evaluate partial $\alpha$-decay widths of $^{10}\text{B}+\alpha$ cluster states. To discuss stability of the linear-chain $^{10}\text{B}+\alpha$ structure, I analyze angular motion of an $\alpha$ cluster around the deformed $^{10}\text{B}$ cluster, i.e., rotation of the $^{10}\text{B}$ cluster.

This paper is organized as follows. In Sec. III I explain the formulation of the present $^{10}\text{B}+\alpha$ cluster model. In Sec. IIII calculated positive-parity states and $E2$ transition strengths of $^{14}\text{N}$ are shown. I discuss $\alpha$ cluster motion
around $^{10}$B($3^+$) and $^{10}$B($1^+$) in Sec. IV. Finally, a summary is given in Sec. V.

II. FORMULATION OF THE $^{10}$B+$\alpha$ CLUSTER MODEL

A. Description of the $^{10}$B cluster

For the $^{10}$B cluster in the present $^{10}$B+$\alpha$ cluster model, I adopt a $2\alpha + (pn)$ wave function which can reasonably describe features of the ground ($J^\pi = 3^+$) and first excited ($1^+$) states of $^{10}$B as discussed in Ref. [58]. The $2\alpha + (pn)$ wave function is given by a three-body cluster wave function, where $\alpha$ clusters and a dinucleon $(pn)$ cluster are written by $(0s)^2$ and $(0s)^2$ harmonic oscillator configurations, respectively, as

$$ \Phi_{2\alpha+pn}(R_1, R_2, R_3) = A\{\Phi_\alpha(R_1)\Phi_\alpha(R_2)\Phi_{pn}(R_3)\}, $$

(1)

$$ \Phi_\alpha(R) = \psi_{p\uparrow}(R)\psi_{p\downarrow}(R)\psi_{n\uparrow}(R)\psi_{n\downarrow}(R), $$

(2)

$$ \psi_\sigma(R) = \varphi_0(R)\chi_\sigma, $$

(3)

where $A$ is the antisymmetrizer for all nucleons and $\varphi_0(R)$ is the spatial part of the single-particle wave function of the $0s$ orbit around $R$;

$$ \varphi_0(R) = \left(\frac{2\nu}{\pi}\right)^{3/4}\exp\left\{-\nu(r-R)^2\right\}, $$

(5)

and $\chi_\sigma$ is the spin-isospin wave function for $\sigma = p \uparrow, p \downarrow, n \uparrow,$ and $n \downarrow$. For the $^{10}$B cluster, I set 2 $\alpha$ clusters in the $z$ direction as $R_1 - R_2 = (0, 0, d_{2\alpha})$ with $d_{2\alpha} = 3$ fm and a spin-aligned $pn$ cluster on the $x$-$y$ plane at the distance $d$ from the $2\alpha$ center as $R_3 - (R_1 + R_2)/2 = (d\cos\phi, d\sin\phi, 0)$. I write the $^{10}$B wave function localized around $X_B \equiv (4R_1 + 4R_2 + 2R_3)/10$ as $\Phi^{^{10}B}(X_B; d, \phi)$ with the center position $X_B$ and the distance and angle parameters, $d$ and $\phi$, for the $pn$ cluster position. In the $^{10}$B+$\alpha$ cluster model, I superpose the $^{10}$B wave functions with $d = 1, 2$ (fm) and $\phi_j = \frac{\pi}{4}(j - 0.5) (j = 1, \ldots, 8)$. Parity ($\pi$) and $K$ ($I_z$) projections of the subsystem $^{10}$B can be approximately done by the $\phi_j$ summation;

$$ \Phi^{^{10}B(I_z^\pi)}(X_B; d) = \sum_j c_j \Phi^{^{10}B}(X_B; d, \phi_j) $$

(6)

with $c_j = \exp(i(I_z - 1)\phi_j)$ and $\pi = (-1)^{I_z - 1}$. Here, $I_z$ is the $z$ component of the total angular momentum $I$ of $^{10}$B and is given by a sum of the aligned intrinsic spin $S_z = 1$ and the orbital $\phi$ rotation of the $pn$ cluster. It is clear that $\phi_j$ superposition with given coefficients $c_j$ is equivalent to $I_z$ mixing.

B. $^{14}$N wave function in the $^{10}$B+$\alpha$ model

A $^{10}$B+$\alpha$ wave function is written using the $^{10}$B wave function $\Phi^{^{10}B}(X_B; d, \phi)$ and the $\alpha$-cluster wave function $\Phi_\alpha(X_\alpha)$ as

$$ \Phi^{^{10}B+\alpha}(D_\alpha, \theta_\alpha; d, \phi) = A\{\Phi^{^{10}B}(X_B; d, \phi)\Phi_\alpha(X_\alpha)\}, $$

(7)

where $X_\alpha - X_B = (D_\alpha \sin\theta_\alpha, 0, D_\alpha \cos\theta_\alpha)$. Here the distance $D_\alpha$ and the angle $\theta_\alpha$ indicate the $\alpha$-cluster position relative to the deformed $^{10}$B cluster (see Fig. I). The center of mass position is taken to be $4X_\alpha + 10X_B = 0$ so as to decouple the center of mass motion and the intrinsic wave function. Wave functions for the $J^\pi$ states of $^{14}$N are expressed by superposition of the $J^\pi$-projected wave functions as

$$ \Psi^{^{14}N(J_z^\pi)} = \sum_K \sum_{D_\alpha, \theta_\alpha} \sum_{d, \phi} C(K, D_\alpha, \theta_\alpha, d, \phi) \hat{P}_{MK}^{J_z^\pi} \Phi^{^{10}B+\alpha}(D_\alpha, \theta_\alpha; d, \phi), $$

(8)

where $\hat{P}_{MK}^{J_z^\pi}$ is the parity and total angular momentum projection operator. Coefficients $C(K, D_\alpha, \theta_\alpha, d, \phi)$ are determined by diagonalizing Hamiltonian and norm matrices. I take $D_\alpha = \{2, \ldots, 6\}$ (fm), $\theta_\alpha = \{0, \pi/4, \pi/2\}$, $d = \{1, 2\}$ (fm), and $\phi = \frac{\pi}{4}(j - 0.5) (j = 1, \ldots, 8)$. In the present paper, I calculate positive-parity ($\pi = +$) states of $^{14}$N.
In Eq. (8), coupling of \( I \) (the spin of the \(^{10}\text{B} \)) and \( L_\alpha \) (the orbital angular momentum of the \( \alpha \)-cluster relative to the \(^{10}\text{B} \)) is implicitly described by the \( J^\pi \) projection, \( K \) mixing, and \( \theta_\alpha, \phi \) summations. As shown in Fig. 1, \( L_\alpha \) couples with \( I \) to the total angular momentum \( J = L_\alpha + I \). The \( z \) component, \( J_z = I_z + L_{\alpha z} \), is the so-called \( K \) quantum. Strictly speaking, \( L_\alpha = 0, 2 \) \((S, D\text{-wave})\) mixing is approximately taken into account by the summation of \( \theta_\alpha = \{0, \pi/4, \pi/2\} \) but higher \( L_\alpha \) \((\geq 4)\) mixing cannot be controlled in the present calculation because of the finite number of mesh points for \( \theta_\alpha \).

![Schematic figures for \(^{10}\text{B} + \alpha \) configurations. (a) Parameters of the \(^{10}\text{B} + \alpha \) cluster model. (b) Longitudinal configuration for \( \theta_\alpha \approx 0 \), (c) transverse configuration for \( \theta_\alpha \approx \pi/2 \) without \( \alpha \)-alignment, and (d) \( \alpha \)-aligned transverse configuration.](image)

### C. Overlap function and \( \alpha \)-cluster probability

In order to evaluate \(^{10}\text{B}(3^+) \otimes L_\alpha \) and \(^{10}\text{B}(1^+) \otimes L_\alpha \) components at a channel radius \( (D_\alpha) \), I consider the \( L_\alpha L_{\alpha z} \) projected \(^{10}\text{B}(I^*_\pi) + \alpha \) wave function,

\[
|J^\pi K; ^{10}\text{B}(I^*_\pi); D_\alpha, L_\alpha L_{\alpha z} \rangle = n_\theta \sum \omega(\theta_\alpha)Y_{L_\alpha z}^\alpha(\theta_\alpha)B^{I^*_\pi}_{MK}\Phi_{i^{10}\text{B}(I^*_\pi)+\alpha}(D_\alpha, \theta_\alpha),
\]

(9)

\[
\Phi_{i^{10}\text{B}(I^*_\pi)+\alpha}(D_\alpha, \theta_\alpha) = A \{\Phi_{i^{10}\text{B}(I^*_\pi)}(X_B; d)\Phi_{\alpha}(X_\alpha)\},
\]

(10)

with \( I_z = \{1, 3\} \), \( \pi = +, K = I_z + L_{\alpha z} \), \( X_\alpha - X_B = (D_\alpha \sin \theta_\alpha, 0, D_\alpha \cos \theta_\alpha) \), and \( 4X_\alpha + 10X_B = 0 \). \( Y_\lambda^\alpha \) is the spherical harmonics. \( \Phi_{i^{10}\text{B}(I^*_\pi)+\alpha}(D_\alpha, \theta_\alpha) \) is the wave function for the \( \alpha \)-cluster at \( (D_\alpha, \theta_\alpha) \) around the \( I_z \) projected \(^{10}\text{B} \) cluster, for which I fix \( d = 2 \) fm in the present analysis. \( n_\theta \) is determined from the normalization condition \( \langle JK; ^{10}\text{B}(I^*_\pi); D_\alpha, L_\alpha L_{\alpha z}|JK; ^{10}\text{B}(I^*_\pi); D_\alpha, L_\alpha L_{\alpha z} \rangle = 1 \). In Eq. (9), the \( L_\alpha L_{\alpha z} \) projection is approximately performed by the summation \( \theta_\alpha = \sum \omega(i) (i = 0, \ldots, N_\theta) \) with the weight function \( \omega(\theta_\alpha) = \int_{\min[\theta_\alpha, \pi/2]}^{\max[\theta_\alpha, \pi/2]} \sin \theta d\theta \). I perform only \( L_\alpha = 0, 2 \) projections because \( L_\alpha \geq 4 \) projections are not possible for the present \( N_\theta = 4 \) case. I calculate the squared overlap of the \(^{14}\text{N} \) wave function with the above wave function, \( \langle JK; ^{10}\text{B}(I^*_\pi); D_\alpha, L_\alpha L_{\alpha z}| \Psi_{^{14}\text{N}(J^*_\pi)} \rangle^2 \). Assuming that the \( 3^+_1 \) and \( 1^+_1 \) states of the \(^{10}\text{B} \) cluster are approximately described by the \( I_z \) projected \(^{10}\text{B} \) wave functions, \(^{10}\text{B}(I^*_\pi = 3^+) \) and \(^{10}\text{B}(I^*_\pi = 1^+) \), respectively, I approximately estimate the \(^{10}\text{B}(I^*_\pi) \otimes L_\alpha \) components as

\[
P_{i^{10}\text{B}(I^*_\pi)\otimes L_\alpha}(D_\alpha) \approx \sum_{L_{\alpha z}} \langle JK| I_z L_\alpha L_{\alpha z} \rangle \langle JK; ^{10}\text{B}(I^*_\pi); D_\alpha, L_\alpha L_{\alpha z}| \Psi_{^{14}\text{N}(J^*_\pi)} \rangle^2
\]

(11)

with \( I_z = I \), where \( \langle JK| I_z L_\alpha L_{\alpha z} \rangle \) is the Clebsch-Gordan coefficient.
I also calculate $\alpha$-cluster probability at $(D_\alpha, \theta_\alpha)$ around the $I_2$ projected $^{10}\text{B}$ cluster as

$$P(JK;^{10}\text{B}(I_2^z); D_\alpha, \theta_\alpha) = |\langle JK; D_\alpha, \theta_\alpha;^{10}\text{B}(I_2^z) | \Psi_{14\text{N}(J^z_\alpha)} \rangle|^2,$$

$$|JK;^{10}\text{B}(I_2^z); D_\alpha, \theta_\alpha) = n_0 \hat{P}_{MK} \Phi_{10\text{B}(I_2^z)+\alpha}(D_\alpha, \theta_\alpha).$$

The probability $P(JK;^{10}\text{B}(I_2^z); D_\alpha, \theta_\alpha)$ is useful to discuss geometric configurations of $3\alpha$ clusters in $^{10}\text{B}+\alpha$ cluster states in the strong coupling picture. For instance, $P(JK;^{10}\text{B}(I_2^z); D_\alpha, \theta_\alpha)$ for $\theta_\alpha \sim 0$ means the component of the “longitudinal” configuration, where the $\alpha$ cluster is localized in the longitudinal direction of the deformed $^{10}\text{B}(I_2^z)$ cluster. This configuration corresponds to the linear-chain structure as $3\alpha$ clusters are arranged in a row as shown in Fig. 1(b). Because of the axial symmetry, the longitudinal configuration contains only $K = I_z$ ($L_{\alpha z} = 0$) component. For $\theta_\alpha \sim \pi/2$, $P(JK;^{10}\text{B}(I_2^z); D_\alpha, \theta_\alpha)$ indicates the component of the “transverse configuration” for the $\alpha$ cluster in the transverse direction of the deformed $^{10}\text{B}(I_2^z)$ cluster. The transverse configuration contains $K \neq I_z$ components corresponding to the alignment of $L_{\alpha}$ to the spin of the $pn$-cluster ($I_z$) in the $^{10}\text{B}$ cluster as well as the $K = I_z$ component (see Fig. 1(b) and (c)).

### III. RESULTS

![Diagram of energy levels](image)

**FIG. 2**: Positive-parity energy levels of $^{14}\text{N}$ obtained by the $^{10}\text{B}+\alpha$ cluster model compared with experimental levels taken from $^{[^6]}$. $^{10}\text{B}+\alpha$ cluster states in the $K^\pi = 3^+$ band and those in the $K^\pi = 1^+$ band are labeled by asterisk and down-triangle symbols, respectively.

I adopt the two-body effective nuclear interactions used in Ref. $^{[^8]}$ which are adjusted to describe low-lying energy levels of $^{10}\text{B}$. Namely, I use the Volkov central force $^{[^59]}$ with the Bartlett, Heisenberg, and Majorana parameters $b = h = 0.06$ and $m = 0.60$ and the G3RS spin-orbit force $^{[^60]}$ with the strength $u_1 = -u_{11} = 1300$ MeV, and the Coulomb force approximated by seven Gaussians. Using these interactions, Energies of $^{10}\text{B}$ are obtained to be $-54.3$ MeV for the ground state ($3^+$) and $-53.4$ MeV for the first excite state ($1^+$) with the $2\alpha + pn$ cluster model by superposing $\sum_{J,z} \sum_{d} F_{MK} \Phi_{10\text{B}}(X_B = 0; d, \phi = 0)$ with $d = 1, 2, 3$ fm. Though the calculation underestimates the experimental binding energy (64.75 MeV), it reproduces the spin parity of the ground state ($^{10}\text{B}(3^+)$), and also the calculated excitation energy $E_x = 0.9$ MeV of the $1^+$ state reasonably agrees to the experimental value $E_x = 0.72$ MeV for $^{10}\text{B}(1^+)$. Using the $^{10}\text{B}+\alpha$ cluster wave function in Eq. $^{[^8]}$, I calculate positive-parity states of $^{14}\text{N}$. Properties of the ground state $^{14}\text{N}(1^+_{g.s})$ are reasonably reproduced by the present calculation. Namely, the binding energy B.E. = 102.6 MeV, the magnetic moment $\mu = 0.36 (\mu_N)$, and the electric quadrupole moment $Q = 2.4 (\text{efm}^2)$ of $^{14}\text{N}(1^+_{g.s})$, obtained by the present calculation reasonably agree to the experimental data (B.E. = 104.66 MeV, $\mu = 0.4038 (\mu_N)$, and $Q = 1.93(8) (\text{efm}^2)$). The calculated energy spectra are shown in Fig. 2. The $\alpha$-decay threshold is much higher than the $\beta$-decay threshold.
FIG. 3: (Color online) $E_2$ transition strengths calculated by the $^{10}$B+$\alpha$ cluster model for (a) $J^+ \rightarrow J^+ - 1$ and (b) $J^+ \rightarrow J^+ - 2$ transitions with $B(E2) \geq 15e^2fm^4$. Asterisk and down-triangle symbols show $^{10}$B+$\alpha$ cluster states in the $K\pi=3^+$ and $K\pi=1^+$ bands, respectively.

in the present calculation than the experimental threshold. In other words, the ground and some low-lying states of $^{14}$N show too deep binding from the $\alpha$-decay threshold compared with the experimental data. The significant overestimation of the $\alpha$-decay threshold is a general problem in microscopic calculations with density-independent two-body effective interactions as found for $^{14}$C and O isotopes [6, 32, 54]. One of the origins of this problem is a difficulty in reproducing systematics of binding energies in a wide mass-number region with such effective interactions. In this paper, I mainly investigate $^{10}$B+$\alpha$ cluster states near the $\alpha$-decay threshold and discuss their features. In the calculated energy levels near the threshold, I obtain several excited states having significant component of a spatially developed $\alpha$ cluster around the $^{10}$B cluster. From remarkable $E2$ transitions, I assign the $^{10}$B+$\alpha$ cluster states to a $K\pi=3^+$ band of $J\pi=3^+, 4^+$, and $5^+$ states, and a $K\pi=1^+$ band of $J\pi=1^+, 2^+, 3^+, 4^+$, and $5^+$ states. The former and the latter bands are shown by asterisk and down-triangle symbols in Fig. 2. The $K\pi=3^+$ band has the significant $^{10}$B(3$^+$)+$\alpha$ component, whereas the $K\pi=1^+$ band contains the $^{10}$B(1$^+$)+$\alpha$ component. More details of structure of these states are discussed in the next section.

Figure 3 shows $E2$ transitions with $B(E2) \geq 15e^2fm^4$ for $J \rightarrow J-1$ and $J \rightarrow J-2$ transitions. In-band transitions for the $K\pi=3^+$ and $\pi=1^+$ $^{10}$B+$\alpha$ bands are rather strong because of the developed cluster structures, though $E2$ strengths are somewhat fragmented into neighboring states.
IV. DISCUSSION

$^{10}\text{B}+\alpha$ cluster states in the $K^\pi = 3^+$ and $K^\pi = 1^+$ bands have maximum amplitudes of $\alpha$-cluster probability around $D_\alpha = 5$ fm. In this section, I focus on angular motion of the $\alpha$-cluster at $D_\alpha = 5$ fm. I first investigate the angular momentum coupling of the $\alpha$-cluster ($L_\alpha$) and the $^{10}\text{B}$ cluster ($I$) in a weak coupling picture and estimate $\alpha$-decay widths. Then, I discuss geometric configurations of $^{10}\text{B}+\alpha$ cluster states in the strong coupling picture by analyzing $\theta_\alpha$-dependence of the $\alpha$-cluster probability around the deformed $^{10}\text{B}$ cluster.

A. $D_\alpha$-fixed calculation

In the present calculation, radial motion of the $\alpha$ cluster is described by superposing $^{10}\text{B}+\alpha$ wave functions for $D_\alpha = 2, \ldots, 6$ fm. Instead of the full model space in Eq. (8) including $D_\alpha = 2, \ldots, 6$ fm wave functions, I also perform a similar calculation using the $D_\alpha$-fixed model space

$$
\Psi_{14N(J^\pi)}^{D_\alpha=5} = \sum_{K} \sum_{\theta_\alpha} \sum_{d,\phi} C(K,\theta_\alpha, d, \phi) \hat{P}_{M_K} \Phi^{\alpha}(D_\alpha, \theta_\alpha; d, \phi),
$$

where I fix $D_\alpha = 5$ fm and take $\theta_\alpha = \{0, \pi/8, \pi/4, 3\pi/8, \pi/2\}$, $d = \{1, 2\}$ (fm), and $\phi = \frac{\pi}{4}(j - 0.5) \ (j = 1, \ldots, 8)$. In the $D_\alpha = 5$ fm fixed calculation, I find the states near the threshold energy corresponding to $^{10}\text{B}+\alpha$ cluster states in the $K^\pi = 3^+$ and $K^\pi = 1^+$ bands, but do not obtain lower states below the threshold because of the truncation of the model space. Energy levels of the $K^\pi = 3^+$ and $K^\pi = 1^+$ bands obtained with the full and $D_\alpha$-fixed calculations are shown in Fig. 4. The calculated energies are measured from the $\alpha$-decay threshold. The experimental levels observed by $\alpha$ elastic scattering by $^{10}\text{B}$ are also shown in the figure. The level structures of the $K^\pi = 3^+$ and $K^\pi = 1^+$ bands are essentially consistent between the full and $D_\alpha$-fixed calculations, though about 2 MeV global shift is found for the $K^\pi = 3^+$ band between two calculations.

![FIG. 4: Energies of $^{10}\text{B}+\alpha$ cluster states obtained by the full and $D_\alpha$-fixed calculations and those observed by the experiment of $^{10}\text{B}(\alpha, \alpha)^{10}\text{B}$ reactions [57]. Energies are measured from the $\alpha$-decay threshold.](image)

B. $\alpha$-cluster probability and $\alpha$-decay widths

In Table II I show $L_\alpha$ components ($P_{10B(\pi)} \otimes L_\alpha$ in Eq. (11)) at $D_\alpha = 5$ fm coupled with $^{10}\text{B}(3^+)$ and $^{10}\text{B}(1^+)$ in $^{10}\text{B}+\alpha$ cluster states obtained by the full and $D_\alpha$-fixed calculations. In the result of the $D_\alpha$-fixed calculation, $K^\pi = 3^+$ band states are dominated by the $^{10}\text{B}(3^+) \otimes L_\alpha$ component, whereas $K^\pi = 1^+$ band states contain dominantly the $^{10}\text{B}(1^+) \otimes L_\alpha$ component. In the result of the full calculation, the $K^\pi = 3^+$ and $K^\pi = 1^+$ band states still contain significant $^{10}\text{B}(3^+) \otimes L_\alpha$ and $^{10}\text{B}(1^+) \otimes L_\alpha$ components, respectively, except for the $1^+(K^\pi = 1^+)$ state, though the absolute amplitude of the dominant component decreases because of radial motion and state mixing. The $1^+(K^\pi = 1^+)$ state obtained by the full calculation shows a feature quite different from that obtained by the $D_\alpha$-fixed calculation. In the $D_\alpha$-fixed calculation, the $1^+(K^\pi = 1^+)$ state is approximately described by the pure $^{10}\text{B}(1^+) \otimes (L_\alpha = 0)$ state, where the orbital angular momentum ($L_\alpha$) of the $\alpha$ cluster weakly couples to the spin ($I$) of the $^{10}\text{B}$ cluster. However, in the full calculation, the $1^+(K^\pi = 1^+)$ state does not show the weak coupling feature but has $^{10}\text{B}(1^+) \otimes (L_\alpha = 0)$, $^{10}\text{B}(1^+) \otimes (L_\alpha = 2)$, and $^{10}\text{B}(3^+) \otimes (L_\alpha = 2)$ components with the same order showing a strong coupling feature.
Figure 5 shows \( L_\alpha \) components \( (P_{10B(I^\pi)\otimes L_\alpha}) \) at \( D_\alpha = 5 \text{ fm} \) of \( J^\pi \) states in the \(^{14}\text{N}\) spectra obtained by the full calculation. The \(^{10}\text{B}(3^+)\otimes (L_\alpha = 0) \) and \(^{10}\text{B}(3^+)\otimes (L_\alpha = 2) \) components concentrate at the \( 3^+(K^\pi = 3^+) \) and \( 4^+(K^\pi = 3^+) \) states, respectively, though the components are fragmented into other states. The \( 5^+(K^\pi = 3^+) \) state shows rather strong state mixing. The \(^{10}\text{B}(1^+)\otimes (L_\alpha = 2) \) component concentrates at the \( 2^+(K^\pi = 1^+) \) and \( 3^+(K^\pi = 1^+) \) states, whereas, the \(^{10}\text{B}(1^+)\otimes (L_\alpha = 0) \) component feeds lower \( 1^+ \) states of \(^{14}\text{N}\).

In the experiment of \(^{10}\text{B}(\alpha, \alpha')^{10}\text{B} \) reactions \(^{55} \), the \( 3^+ \) state at \( E_\gamma = 1.58 \text{ MeV} \) \( (E_\pi = 13.19 \text{ MeV}) \) with the width \( \Gamma = 0.065 \text{ MeV} \) is strongly populated. In the analysis of Ref. \(^{57} \), this state is described well by the dominant (almost 100\%) \( S \)-wave \( \alpha \)-decay indicating the significant \(^{10}\text{B}(3^+) \) \( (L_\alpha = 0) \) component of the \( 3^+ \) state. The \( 1^+ \) state at \( E_\gamma = 2.11 \text{ MeV} \) \( (E_\pi = 13.72 \text{ MeV}) \) is weakly populated in \(^{10}\text{B}(\alpha, \alpha')^{10}\text{B} \) reactions, whereas its \( \alpha \)-decay into the first excited state of \(^{10}\text{B}(1^+) \) was observed in \(^{10}\text{B}(\alpha, \alpha' \gamma)^{10}\text{B} \) reactions \(^{62} \). These experiments suggest that the \( 1^+ \) state would contain \(^{10}\text{B}(1^+) \otimes (L_\alpha = 0) \) and \(^{10}\text{B}(3^+) \otimes (L_\alpha = 2) \) components.

From the experimental \( \alpha \)-decay properties, I tentatively assign the theoretical \( 3^+(K^\pi = 3^+) \) and \( 1^+(K^\pi = 1^+) \) states having \(^{10}\text{B}+\alpha \) cluster structures to the experimental \( 3^+ \) \( (E_\exp = 1.58 \text{ MeV}) \) and \( 1^+ \) \( (E_\exp = 2.11 \text{ MeV}) \) states, though the band-head energies \( E_\gamma(3^+; K^\pi = 3^+) = -0.2 \text{ MeV} \) and \( E_\gamma(1^+; K^\pi = 1^+) = 2.0 \text{ MeV} \) obtained by the full calculation do not necessarily agree to the experimental energies (see Fig. 2). I estimate partial \( \alpha \)-decay widths for \( B(I^\pi)\otimes L_\alpha \) channels from \( P_{10B(I^\pi)\otimes L_\alpha}(D_\alpha = a) \) \((a \text{ is the channel radius})\) as follows. Using the approximate evaluation of the reduced width amplitude proposed in Ref. \(^{63} \), the reduced width \( \gamma_\alpha^2(a) \) is calculated as

\[
\gamma_\alpha^2(a) = \frac{\hbar^2}{2\mu a} \left( \frac{\nu}{2\pi A_1 A_2} \right)^{1/2} P_{10B(I^\pi)\otimes L_\alpha}(D_\alpha = a),
\]

and the partial \( \alpha \)-decay width \( \Gamma_{10B(I^\pi)+\alpha} \) for \( L_\alpha = l \) is calculated as

\[
\Gamma_{10B(I^\pi)+\alpha} = 2 P_l(a) \gamma_\alpha^2(a),
\]

\[
P_l(a) = \frac{ka}{F_l^2(ka) + G_l^2(ka)}.
\]
where $k = \sqrt{2\mu E/\hbar}$, and $F_l$ and $G_l$ are the regular and irregular Coulomb functions, respectively. Here I use the momentum $k$ of the energy $E = E_{r}^{(\text{adjust})}$ which is phenomenologically adjusted to the experimental energy position because it is difficult to quantitatively predict the energy position in the present calculation. Namely, I adjust the band-head energies of the $K^\pi = 3^+$ and $K^\pi = 1^+$ bands to the experimental energy positions $E_{r}^{\text{exp}}(3^+) = 1.58$ MeV and $E_{r}^{\text{exp}}(1^+) = 2.11$ MeV, by a constant shift for each band as

$$E_{r}^{(\text{adjust})}(J^+; K^\pi = 3^+) = E_r(J^+; K^\pi = 3^+) - E_r(3^+; K^\pi = 3^+) + E_{r}^{\text{exp}}(3^+),$$

(18)

$$E_{r}^{(\text{adjust})}(J^+; K^\pi = 1^+) = E_r(J^+; K^\pi = 1^+) - E_r(1^+; K^\pi = 1^+) + E_{r}^{\text{exp}}(1^+).$$

(19)

Calculated partial $\alpha$-decay widths are shown in Table I I calculate widths for $L_\alpha = 0$ and $L_\alpha = 2$ channels. $\alpha$-decay widths obtained by the full calculation are several times smaller than those obtained by the $D_\alpha$-fixed calculation because of the suppression of the $\alpha$-cluster probability as shown previously. As a result, the $\alpha$-decay width of the $3^+(K^\pi = 3^+)$ state reduces to be $\Gamma_\alpha = 0.05$ MeV with the dominant $^{10}\text{B}(3^+) \otimes (L_\alpha = 0)$ decay, which is quantitatively consistent with the experimental observation ($\Gamma_\alpha \sim \Gamma = 0.065(10)$ MeV) [57]. For the $1^+(K^\pi = 1^+)$ state, I obtain a small $\alpha$-decay width $\Gamma_\alpha = 0.01$ MeV with the dominant $^{10}\text{B}(1^+) \otimes (L_\alpha = 0)$ decay. This result seems consistent with the weak population in the $\alpha$ elastic scattering [57] and the fact that the $1^+$ state was observed in $^{10}\text{B}(\alpha,\alpha')^{10}\text{B}$ reaction [62]. However, experimental information of partial $\alpha$-decay widths is not enough to confirm the present assignment of the $1^+(K^\pi = 1^+)$ state. The calculated $\alpha$-decay width is much smaller than the experimental total width, $\Gamma = 0.16(2)$ MeV, of the $1^+$ state at 2.11 MeV. I should comment that, because the $^{10}\text{B}(1^+) \otimes (L_\alpha = 0)$ component is fragmented into neighboring states as shown in Fig. 5, an effectively large width could be observed for the $1^+(K^\pi = 1^+)$ state.

TABLE I: $^{10}\text{B}(I^\pi) \otimes (L_\alpha = 0, 2)$ components, $P_{\alpha \otimes L_\alpha}^{(\alpha)}(D_\alpha = 5$ fm), of $^{10}\text{B}+\alpha$ cluster states in the $K^\pi = 3^+$ and $K^\pi = 1^+$ bands obtained by the full and $D_\alpha$-fixed calculations.

| $K^\pi$ | $P_{\alpha \otimes L_\alpha}^{(\alpha)}(D_\alpha = 5 fm)$ | $P_{\alpha \otimes L_\alpha}^{(\alpha)}(D_\alpha = 5 fm)$ |
|---------|------------------|------------------|
| $L_\alpha = 0$ | $L_\alpha = 2$ | $L_\alpha = 0$ | $L_\alpha = 2$ |
| $3^+(K^\pi = 3^+)$ | 0.21 | 0.10 | 0.04 |
| $4^+(K^\pi = 3^+)$ | 0.23 |
| $5^+(K^\pi = 3^+)$ | 0.14 |
| $1^+(K^\pi = 1^+)$ | 0.03 | 0.05 | 0.09 |
| $2^+(K^\pi = 1^+)$ | 0.02 | 0.25 |
| $3^+(K^\pi = 1^+)$ | 0.00 | 0.02 | 0.37 |
| $4^+(K^\pi = 1^+)$ | 0.01 |
| $5^+(K^\pi = 1^+)$ | 0.14 |
| $D_\alpha$-fixed cal. | |
| $3^+(K^\pi = 3^+)$ | 0.57 | 0.25 | 0.01 |
| $4^+(K^\pi = 3^+)$ | 0.73 |
| $5^+(K^\pi = 3^+)$ | 0.75 |
| $1^+(K^\pi = 1^+)$ | 0.02 | 0.89 | 0.05 |
| $2^+(K^\pi = 1^+)$ | 0.01 | 0.78 |
| $3^+(K^\pi = 1^+)$ | 0.10 | 0.13 | 0.74 |
| $4^+(K^\pi = 1^+)$ | 0.00 |

C. Angular motion of the $\alpha$ cluster around the deformed $^{10}\text{B}$ cluster

I here discuss angular motion of the $\alpha$-cluster around the deformed $^{10}\text{B}$ cluster by analyzing $\theta_\alpha$ dependence of $\alpha$-cluster probabilities. Discussions in this section are based on the strong coupling picture, which is somehow different from the previous discussion based on the $L_\alpha$ decomposition in the weak coupling picture. I show energies of $\Phi_{\alpha \otimes L_\alpha}^{(\alpha)}(D_\alpha, \theta_\alpha)$, in which the $\alpha$ cluster is localized at $(D_\alpha, \theta_\alpha)$ around the $I_\pi$ projected $^{10}\text{B}$ cluster. In Fig. 5 intrinsic energies before parity and angular-momentum projections of $\Phi_{\alpha \otimes L_\alpha}^{(\alpha)}(D_\alpha, \theta_\alpha)$ for $I_\pi^\alpha = 3^+$ and $1^+$ are
TABLE II: Partial α-decay widths of $^{10}$B+$\alpha$ cluster states in the $K^\pi = 3^+$ and $K^\pi = 1^+$ bands obtained by the full and $D_\alpha$-fixed calculations. Energies of the band-head states of the $K^\pi = 3^+$ and $K^\pi = 1^+$ bands are adjusted to the experimental resonance energies of the $3^+$ state at 1.58 MeV and the $1^+$ state at 2.11 MeV. The sum ($\Gamma_{10B+\alpha}(L_\alpha \leq 2)$) of partial widths of decay channels $^{10}$B$(3^+) \otimes (L_\alpha \leq 2)$ and $^{10}$B$(1^+) \otimes (L_\alpha \leq 2)$ is also shown. The unit is MeV.

| $J^\pi$ | $E_r^{(adj)}$ | $\Gamma_{10B(3^+) + \alpha}$ | $\Gamma_{10B(1^+) + \alpha}$ | $\Gamma_{10B(3^+) + \alpha}(L_\alpha \leq 2)$ |
|--------|--------------|-------------------------------|-------------------------------|----------------------------------|
| full cal. |              |                               |                               |                                  |
| $3^+(K^\pi = 3^+)$ | 1.58 | 0.04 | 0.00 | 0.00 | 0.05 |
| $4^+(K^\pi = 3^+)$ | 2.43 | 0.06 | 0.00 | 0.00 | 0.06 |
| $5^+(K^\pi = 3^+)$ | 3.87 | 0.16 | 0.16 | 0.16 | 0.16 |
| $1^+(K^\pi = 1^+)$ | 2.11 | 0.00 | 0.01 | 0.00 | 0.01 |
| $2^+(K^\pi = 1^+)$ | 3.35 | 0.02 | 0.09 | 0.11 | 0.11 |
| $3^+(K^\pi = 1^+)$ | 3.23 | 0.00 | 0.12 | 0.13 | 0.13 |
| $4^+(K^\pi = 1^+)$ | 4.60 | 0.01 | 0.01 | 0.01 | 0.01 |
| $5^+(K^\pi = 1^+)$ | 6.31 | 0.36 | 0.36 | 0.36 | 0.36 |
| $D_\alpha$-fixed cal. |           |                               |                               |                                  |
| $3^+(K^\pi = 3^+)$ | 1.58 | 0.12 | 0.01 | 0.00 | 0.13 |
| $4^+(K^\pi = 3^+)$ | 2.95 | 0.41 | 0.41 | 0.41 | 0.41 |
| $5^+(K^\pi = 3^+)$ | 4.27 | 1.07 | 1.07 | 1.07 | 1.07 |
| $1^+(K^\pi = 1^+)$ | 2.11 | 0.00 | 0.10 | 0.11 | 0.11 |
| $2^+(K^\pi = 1^+)$ | 3.61 | 0.01 | 0.41 | 0.42 | 0.42 |
| $3^+(K^\pi = 1^+)$ | 3.88 | 0.19 | 0.15 | 0.51 | 0.85 |
| $4^+(K^\pi = 1^+)$ | 6.77 | 0.01 | 0.01 | 0.01 | 0.01 |

Plotted on the $(x, z) = (D_\alpha \sin \theta_\alpha, D_\alpha \cos \theta_\alpha)$ plane. The energy curves for $D_\alpha = 5$ fm are also shown as functions of $\theta_\alpha$. In the $D_\alpha \geq 5$ fm region, the contour of the energy surface on the $(x, z)$ plane is deformed in the longitudinal ($\theta_\alpha = 0$) direction because of the oblate deformation of the $^{10}$B cluster meaning that the $\alpha$ cluster at the fixed distance $D_\alpha = 5$ fm feels an attraction in the longitudinal direction. In other words, in the intrinsic system, the $\alpha$ cluster at $D_\alpha = 5$ fm energetically favors the longitudinal direction to form the linear $3\alpha$ configuration rather than the transverse direction to form the $\triangle 3\alpha$ configuration. In the $D_\alpha \leq 3$ fm region, the $\alpha$ cluster feels an effective repulsion in the longitudinal direction because of the Pauli blocking from the $^{10}$B cluster whereas it feels an attraction in the transverse ($\theta_\alpha = \pi/2$) direction.

In contrast to the intrinsic energy behavior, $\theta_\alpha$ dependence of $J^\pi$-projected energy is not trivial because the energy is affected by not only potential energy but also kinetic energy of angular motion, i.e., rotational energy. Figure 7 shows energies of $JK$-projected states $P_{MK} \Phi_{MK}(D_\alpha, \theta_\alpha)$ at $D_\alpha = 5$ fm for $K = I_z$, which corresponds to the $L_{\alpha z} = 0$ projection. In high J states, the longitudinal direction ($|\theta_\alpha| \lesssim \pi/8$) is energetically favored than the transverse direction ($|\theta_\alpha - \pi/2| \lesssim \pi/8$) because of the larger moment of inertia (m.o.i.) of the longitudinal configuration than that of the transverse configuration for the $L_{\alpha z} = 0$ projection. However, in the lowest spin state ($JK = 11$), the energy almost degenerates in a wide region of $\theta_\alpha$ because the kinetic energy is smaller for the transverse configuration than the longitudinal configuration because of the phase space factor $\sin \theta_\alpha$ in the $L_{\alpha z} = 0$ projection. This energy degeneracy results in the $L_{\alpha z} = 0$ ($S$-wave) dominance in the $1^+(K^\pi = 1^+)$ state obtained by the $D_\alpha$-fixed calculation.

Figures 8 and 9 show energies of $JK$-projected states at $D_\alpha = 5$ fm for $K \neq I_z$. Note that the $K \neq I_z$ projection corresponds to the $L_{\alpha z} \neq 0$ projection, and $K > I_z$ means the $L_{\alpha z}$ alignment to the $z$ direction (see Fig. 4(c)). For instance, the $L_{\alpha z}$-aligned state for $L_{\alpha z} = 2$ ($D$-wave) is the $K = I_z + 2$ state. As shown in Figs. 8(a)-c) and 9(a)-d), $L_{\alpha z}$-aligned states energetically favor the transverse configuration because of the larger m.o.i. than that of the longitudinal configuration in the $L_{\alpha z} = 2$ projection. Figures 8 and 9 also show the $\alpha$-cluster probability $P(JK; ^{10}$B$(I_z^\pi); D_\alpha, \theta_\alpha)$ at $D_\alpha = 5$ fm in the $^{10}$B+$\alpha$ cluster states obtained by the $D_\alpha$-fixed and full calculations. Let me first discuss the result obtained by the $D_\alpha$-fixed calculation (Figs. 8(d)-(f) and 9(e)-(h)). In the $K^\pi = 3^+$ band states (Fig. 8(d)-(f)), the $J^\pi = 3^+$ state contains dominantly the longitudinal configuration ($|\theta_\alpha| \lesssim \pi/8$) rather than the transverse configuration ($|\theta_\alpha - \pi/2| \lesssim \pi/8$) as expected from the $JK$-projected energy curve for $K = I_z$. As $J$ goes up to $J = 5$, the $L_{\alpha z}$-aligned component ($K = 5$) of the transverse configuration becomes large corresponding to the alignment of the orbital angular momentum $L_{\alpha z}$ of the $\alpha$ cluster to $I_z = 3$ (the spin of $(pm)$ cluster in the $^{10}$B cluster).

In the $K^\pi = 1^+$ band states (Fig. 9(e)-(h)), the $J^\pi = 1^+$ state shows the $\alpha$-cluster probability distributed widely.
in the $0 \leq \theta_\alpha \leq \pi/2$ region indicating the dominant $L_\alpha = 0$ ($S$-wave) component. As $J$ increases, the longitudinal component becomes dominant compared with the transverse component. The alignment of $L_\alpha$ (the orbital angular momentum of the $\alpha$ cluster) and $I_z$ is not so remarkable for $^{10}\text{B}(I_z^+ = 1^+)$ differently from the $^{10}\text{B}(I_z^+ = 3^+)$. Next, let me look into the result of the full calculation shown in Figs. 8(g)-(i) and 9(i)-(l). Compared with the $D_\alpha$-fixed calculation, transverse components tend to be relatively more suppressed than longitudinal components in the full calculation. Note that the longitudinal ($\theta_\alpha = 0$) component is not dominant but is $30\sim 40\%$, which is comparable to the $\theta_\alpha = \pi/4$ component. It indicates that $^{10}\text{B}+\alpha$ cluster states are different from the ideal linear configuration of a classical picture but they show significant quantum fluctuation in the angular ($\theta_\alpha$) motion and are regarded as the chain-like configuration that has relatively enhanced longitudinal components with suppressed transverse components.

The origin of the suppression of transverse components in $^{10}\text{B}+\alpha$ cluster states in the full calculation can be described by orthogonality to lower states which contain transverse components with $D_\alpha < 5$ fm. As shown in Fig. 6 for the energy surface on the $(D_\alpha, \theta_\alpha)$ plane, an energy pocket exists in the transverse direction ($\theta_\alpha \sim \pi/2$) around $D_\alpha \sim 2$, and therefore, transverse components contribute to low-lying $^{14}\text{N}$ states. Although the low-lying states are compact states containing mainly configurations with small $D_\alpha$, transverse components with $D_\alpha = 5$ fm somewhat feed the low-lying states. As a result of the feeding of lower states, transverse components in the $^{10}\text{B}+\alpha$ cluster states near the threshold are suppressed. Figures 10 and 11 show the $\alpha$-cluster probability $P(J K; ^{10}\text{B}(I_z^+); D_\alpha, \theta_\alpha)$ for $\theta_\alpha = 0$ at $D_\alpha = 5$ fm and that for $\theta_\alpha = \pi/4$ and $\pi/2$ at $D_\alpha = 4$ fm. As seen in 10(a)-(c) for $^{10}\text{B}(I_z^+ = 3^+)$, the longitudinal ($\theta_\alpha = 0$) component of $^{10}\text{B}(I_z^+ = 3^+)+\alpha$ shows the largest amplitude at the $K^\pi = 3^+$ band states (labeled by asterisks) and some fragmentation into neighboring states. Similarly, the longitudinal component of $^{10}\text{B}(I_z^+ = 1^+)+\alpha$
concentrates on the $K^\pi = 1^+$ band states (see Fig. 7(a)-(e)). On the other hand, transverse components feed states lower than $^{10}\text{B}+\alpha$-cluster states as seen in Fig. 7(d)(f) and Fig. 11(f)(g). Consequently the $\alpha$ cluster in $^{10}\text{B}+\alpha$-cluster states near the threshold tends to avoid transverse configurations so as to satisfy orthogonality to lower states. This mechanism is consistent with the discussion of Ref. [54] for linear-chain $3\alpha$ states in $^{14}\text{C}$.

V. SUMMARY

I calculated positive-parity states of $^{14}\text{N}$ with the $^{10}\text{B}+\alpha$ cluster model and investigated $^{10}\text{B}+\alpha$ cluster states. Near the $\alpha$-decay threshold energy, I obtained the $K^\pi = 3^+$ and $K^\pi = 1^+$ rotational bands having the developed $\alpha$ cluster with the $^{10}\text{B}(3^+)$ and $^{10}\text{B}(1^+)$ cores, respectively. I assigned the $3^+(K^\pi = 3^+)$ state in the present result to the experimental $3^+$ at $E_r = 1.58$ MeV observed in $\alpha$ scattering reactions by $^{10}\text{B}$, and showed that the calculated $\alpha$-decay width agrees to the experimental width.

I analyzed the component of the longitudinal configuration having an $\alpha$ cluster in the longitudinal direction of the deformed $^{10}\text{B}$ cluster, which corresponds to a linear-chain $3\alpha$ structure with valence nucleons. In the spectra
of $^{14}$N, the linear-chain component concentrates at the $^{10}$B+α cluster states in the $K^\pi = 3^+$ and $K^\pi = 1^+$ bands. However, the $^{10}$B+α cluster states are different from the ideal linear configuration of a classical picture but they show significant quantum fluctuation in the angular ($\theta_\alpha$) motion and are regarded as the chain-like configuration that has relatively enhanced longitudinal components and suppressed transverse components. The orthogonality to low-lying states plays an essential role in the suppression of the transverse component.

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FIG. 10: $\alpha$-cluster probability $P(JK;^{10}B(I^p_\alpha)); \theta_\alpha$ for $I^p_\alpha = 3^+$. $D_\alpha$ is taken to be $D_\alpha = 5$ fm for $\theta_\alpha = 0$, and $D_\alpha = 4$ fm for $\theta_\alpha = \pi/4$ and $\pi/2$. Asterisk and down-triangle symbols show $^{10}B+\alpha$ cluster states in the $K^\pi = 3^+$ and $K^\pi = 1^+$ bands, respectively.

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FIG. 11: \( \alpha \)-cluster probability \( P(JK; \beta \alpha) \) for \( I^+_\pi \) states. \( \beta \alpha \) is taken to be \( \beta \alpha = 0 \) for \( \theta \alpha = 0 \) and \( \beta \alpha = 4 \) fm for \( \theta \alpha = \pi/4 \) and \( \pi/2 \). Asterisk and down-triangle symbols show \( \beta \alpha \)-cluster states in the \( \Lambda \pi \) and \( \Lambda \pi \) bands, respectively.

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