Mass transfer from a giant star to a main-sequence companion and its contribution to long-orbital-period blue stragglers

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ABSTRACT

Binary population synthesis shows that mass transfer from a giant star to a main-sequence (MS) companion may account for some observed long-orbital-period blue stragglers. However, little attention is paid to this blue straggler formation scenario as dynamical instability often happens when the mass donor is a giant star. In this paper, we have studied the critical mass ratio, $q_c$, for dynamically stable mass transfer from a giant star to a MS companion using detailed evolution calculations. The results show that a more evolved star is generally less stable for Roche lobe overflow. Meanwhile, $q_c$ almost linearly increases with the amount of the mass and angular momentum lost during mass transfer, but has little dependance on stellar wind. To conveniently use the result, we give a fit of $q_c$ as a function of the stellar radius at the onset of Roche lobe overflow and of the mass-transfer efficiency during the Roche lobe overflow.

To examine the formation of blue stragglers from mass transfer between giants and MS stars, we have performed Monte Carlo simulations with various $q_c$. The simulations show that some binaries with the mass donor on the first giant branch may contribute to blue stragglers with $q_c$ obtained in this paper but will not from previous $q_c$. Meanwhile, from our $q_c$, blue stragglers from the mass transfer between an asymptotic giant branch star and a MS companion may be more numerous and have a wider range of orbital periods than those from the other $q_c$.

Key words: binaries: close – blue stragglers – stars: evolution.

1 INTRODUCTION

Blue stragglers (BSs) are stars that have remained on the main-sequence (MS) for a time exceeding that expected, for their masses, from standard stellar evolution theory. The existence of BSs indicates an incomplete understanding of stellar evolution and also of star formation within clusters (Stryker 1993). Since they are bright and blue, these objects may affect the integrated spectra of their host clusters by contributing excess spectral energy in the blue and ultraviolet. BSs are therefore important in studies of population synthesis (Xin & Deng 2005). Much evidence shows that these strange objects are relevant to primordial binaries (Ferraro et al. 2003; Davies, Piotto & Angeli 2004; Mapelli et al. 2004). Binary coalescence from a contact binary is a popular hypothesis for single BSs and it is believed that these contact binaries are mainly from case A mass transfer$^1$ (Mateo et al. 1990; Pols & Marinus 1994; Andronov, Pinsonneault & Terndrup 2006; Chen & Han 2008). Mass transfer is another channel to produce BSs from primordial binaries. During Roche lobe overflow (RLOF), the less-massive star accretes some material and goes upward along the MS, if it is still a MS star. The accreting component may then be observed as a BS when it is more massive than the turn-off of the host cluster (Mc Crea 1964; Chen & Han 2004). Previous studies show that both case A and case B mass transfer are only responsible for some short- and mid-period BSs (Leonard 1996; Chen & Han 2004), which are rare in some old open clusters, and case C mass transfer may account for BSs in long-orbital-period spectroscopic binaries. During RLOF, the orbital period will decrease at first, however, if the primary continues to transfer material to the companion after the primary is less massive than the secondary, the binary will become wider and the orbital period will increase. Since dynamical instability often happens when the mass donor is a giant star, there is little work to focus on for this channel to BSs.

As is well known, a fully convective star will increase in radius with mass loss and decrease in Roche lobe radius if it is more massive than its accreting companion (Paczynski 1965). This means that the mass donor will overfill its Roche lobe by an ever increasing amount, leading to mass transfer on a dynamical timescale, the formation of a common envelope (CE) and a spiral-in phase. The critical mass ratio (the mass donor/the accretor) is about

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$^1$ According to the evolutionary state of the primary at the onset of mass transfer, three mass-transfer cases are defined, that is, case A for the primary being on the MS, case B for the primary after the MS but before central He burning, and case C for the primary during or after central He burning (Kippenhahn & Weigert 1967).
exponent $\zeta$. (1) The critical mass ratio

$$q_c = \frac{0.362 + 1}{3(1 - M_2/M_1)}$$

where $M_c$ and $M_1$ are the core mass and the total mass, respectively, of the donor as RLOF begins. (2) The condition for dynamical instability also depends on the amount of mass and angular momentum lost during RLOF. Assuming that the lost mass carries away the same specific angular momentum as pertaining to the mass donor, Soberman, Phinney & van den Heuvel (1997) gave a fitted Roche lobe mass–radius exponent $\zeta_L = -1.7\beta + (2.4\beta - 0.25)\xi$, where $\beta$ is mass-transfer efficiency determined as the mass fraction of the lost mass from the primary accreted by the secondary. We may then obtain the critical mass ratio $q_c$ by setting the adiabatic mass–radius exponent $\zeta = \zeta_L$, where $\xi$ can be fitted from the data of numerical calculations of Hjellming & Webbink (1987) and Soberman et al. (1997) (see also Han et al. 2001). Fig. 1 clearly shows the dependence of $q_c$ on the mass-transfer efficiency $\beta$. (3) Stellar wind prior to RLOF, which will be strongly enhanced due to the tidal interaction with the companion, will increase the core fraction in equation (1) and allow the system to stabilize more easily.

Possibly, a more fundamental problem of such a criterion is that it does not take into account the detailed dynamics of the mass-transfer process. There are some systems, observationally, which should experience dynamical mass transfer while appearing to avoid a CE phase (Podsiadlowski, Joss & Hsu 1992). Several recent full binary evolution calculations also show that the simplistic criterion above is not really appropriate. For example, it is shown by Podsiadlowski, Rappaport & Pfahl (2002) that mass transfer is dynamically stable for all giants up to a mass of about $2 M_\odot$, in the case of giants transferring mass to a neutron star of $1.3/1.4 M_\odot$. The study of Han et al. (2002) showed that $q_c$ may be up to about 1.3 as $\beta = 0$.

In this paper, we will study the critical mass ratio for dynamical stability in mass transfer from a giant star to a MS companion from detailed binary evolutions, and adopt the results to estimate the contribution to BSs from the mass transfer. In Section 2, we simply describe the numerical codes we have employed in the study. The results of the critical mass ratio are shown in Section 3. In Section 4, we show some examples of binaries which eventually form long-orbital-period BSs. The consequences from Monte Carlo simulations are shown in Section 5. Discussions and conclusions are given in Section 6.

2 BINARY EVOLUTION CALCULATIONS

The binary evolution code we employed was originally developed by Eggleton (1971, 1972, 1973), which has been updated with the latest input physics over the last three decades as described by Han, Podsiadlowski & Eggleton (1994) and Pols et al. (1995, 1998). Some characteristics of the code, that is, a self-adaptive non-Lagrangian mesh, the treatment of both the convective and semi-convective mixing as diffusion processes, the simultaneous and implicit solution of both the stellar structure equations and the chemical composition equations, etc., make the code very stable and easy to use. The current code uses an equation of state that includes pressure ionization and Coulomb interaction, recent opacity tables derived from Iglesias & Rogers (1996) and Alexander & Ferguson (1994) (see Chen & Tout 2007), nuclear reaction rates from Caughlan et al. (1985) and Caughlan & Fowler (1988), and neutrino loss rates from Itoh et al. (1989, 1992).

The ratio of mixing length to the local pressure scaleheight $\alpha = l/H_p$ is set to 2, which fits to the Sun (Pols et al. 1998). Convective overshooting is included by incorporating a condition that mixing occurs in a region with $\nabla_r > \nabla_s - \delta_{ov}/(2.5 + 20\xi + 16\xi^2)$, where $\xi$ is the ratio of radiative pressure to gas pressure and $\delta_{ov}$ is a specified constant. $\delta_{ov} = 0.12$ gives the best fit to the observed systems (Schröder, Pols & Eggleton 1997), which corresponds to an overshooting of about $0.25 H_p$. RLOF is included from the boundary condition

$$\frac{dm}{dt} = C\max \left[0, \left(\frac{r_{\text{star}}}{r_{\text{lobe}}} - 1\right)^3\right],$$

where $r_{\text{star}}$ and $r_{\text{lobe}}$ are the radii of the mass donor and its Roche lobe, respectively, $dm/dt$ gives the mass-loss rate of the primary, and $C$ is a constant. In our study, we set $C = 500 M_\odot yr^{-1}$, with which RLOF proceeds steadily and the lobe-filling star overfills its Roche lobe as necessary but never overfills its lobe by much, typically $(r_{\text{star}}/r_{\text{lobe}} - 1) < 0.001$.

If mass transfer is non-conservative, that is, $\beta$ is less than 1.0, some matter will be lost from the system, taking away some angular momentum. In our study, the accretor is a MS star, and it is much

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2 The primary should be more massive than the secondary initially from standard stellar evolution if we expect the binary to be composed of a giant star and a MS companion.
more compact than a giant. Thus, we assume that the lost mass carries away the same specific angular momentum as pertaining to that of the accretor, similar to the case of a compact component. The study of Beer et al. (2007) shows that this assumption is appropriate for MS components.

We only follow the evolution of the primary (initially more massive component) in a binary system when we study the critical mass ratio, since the structure of the primary is the main factor for the formation of CE in this phase. We adopted the solar metallicity ($Z = 0.02$) in our calculations. As we are mainly concerned with BSs which originate from mass transfer, only low- and intermediate-mass binaries are studied here. The initial mass of the primary increases from 1 to 8 $M_\odot$ by step of about $\Delta_m = 0.1$. The stellar radii at the onset of RLOF and the critical mass ratio $q_c$ for $1.41 < q_c < 2.00$ are not plotted to ensure the distinction of the figure.

![Figure 2](https://example.com/figure2.png)

**Figure 2.** Evolutionary tracks for single stars from 1 to 8 $M_\odot$. For primaries in binary systems, the dots (for FGB) and asterisks (for AGB) indicate their positions at the onset of RLOF we studied in the paper. Corresponding stellar radii are presented in Table 1. The AGB cases for $M_1 < 2 M_\odot$ are not plotted to ensure the distinction of the figure.

that at the tip of the FGB. If $M_{1i} \geq 2 M_\odot$, we continue the study on the asymptotic giant branch (AGB) when the stellar radius equals to that at the tip of the FGB, and increase the radius in steps of $\Delta \log R/R_\odot = 0.1$. Fig. 2 shows the positions of the primaries as RLOF begins, corresponding stellar radii are presented in Table 1.

If $M_{1i} < 2 M_\odot$, the code breaks down when the He flash occurs. To investigate the case on the AGB for a mass donor with $M_1 < 2.0 M_\odot$, we constructed some stellar models after the He flash. Considering that some of the mass donors on the AGB may have evolved from some initially more massive stars, we choose the point at the minimum stellar radius at the tip of the FGB for stars with $M_{1i} \geq 2 M_\odot$ (i.e. $\log R/R_\odot = 1.41$) to start our studies and increase the radius in steps of $\Delta \log R/R_\odot = 0.1$. The stellar radii and corresponding $q_c$ are listed in Table 2.

3 The radius of the primary at the onset of RLOF is actually its Roche lobe $R_{\text{crit}}$, since the primary is just filling its Roche lobe at that time. If the mass ratio $q$ is given, the corresponding initial orbital separation $A$ can be calculated from $R_{\text{crit}}/A = 0.49 q^{3/2} [0.6 q^{3/2} + \ln (1 + q^{3/2})]$ (Eggleton 1983). For convenience, we only show the radius of the primary at the onset of RLOF in this paper.

3 For example, a star with $M_{1i} > 2 M_\odot$ becomes less than $2 M_\odot$ because of stellar wind, then the star has not passed through a phase with a high radius before He burning.

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### 3 THE CRITICAL MASS RATIO FOR A GIANT MASS DONOR

As mentioned in Section 1, dynamically unstable RLOF often occurs when the mass donor is a giant at the onset of mass transfer. Fig. 3 shows the mass-transfer rate versus the mass of the primary for a typical binary undergoing dynamically unstable mass transfer. The primary is $1.00 M_\odot$ with a core mass $M_c = 0.344 M_\odot$ at the onset of RLOF, and $\beta = 0$ during the RLOF. From the figure, we see that mass transfer initially occurs on a thermal time-scale. The mass-transfer rate, $\dot{M}$, rises quickly to a level of $10^{-5} M_\odot \text{ yr}^{-1}$ at first and continues to grow more slowly later. After the primary has lost about $0.04 M_\odot$ (point A), the system encounters dynamical instability. The behaviour here is similar to that of radiative mass donors with a mass ratio greater than 3 (Han & Polsiadiowski 2006), but it cannot be explained by the evolution of the entropy profile as in the case of a mass donor with a radiative envelope. It is related to the evolution of binary parameters, which strongly depend on the angular momentum loss associated with the mass loss from the system. Furthermore, in comparison to the delayed dynamical instability in the radiative mass donor, the primary with a convective envelope here loses much less mass during the delayed time and probably has little influence on the final outcome.

The results of our calculations are summarized in Tables 1 and 2. For each zero-age MS (ZAMS), we present the radius of the primary $R/R_\odot$ at the onset of RLOF and the critical mass ratio $q_c$ for dynamically stable RLOF from different mass-transfer efficiency $\beta$. The results demonstrate that RLOF is probably dynamically stable even if the mass donor is more massive than the secondary for $\beta < 1$. In particular, when the mass donor is less than $2 M_\odot$ and fills its Roche lobe on the AGB, RLOF may be dynamically stable even for $\beta = 1$. Since the core is not very degenerate and the envelope is not fully convective when the star is at or near the base of the FGB, $q_c$ is much larger at the base of the FGB than in the following evolutionary phases for each $\beta$.

Figs 4 and 5 present $q_c$ versus log $R/R_\odot - A_0$ for low-mass and intermediate-mass binaries, respectively, when the mass donors are on the FGB. Here, $A_0$ is the log of the radius of the primary at the base of the FGB. We include the result of $M_{1i} = 1.9 M_\odot$ in Fig. 4 (see also Table 1). The results for the mass donor on the AGB are showed in Figs 6 and 7. In all of the figures, we see that a more evolved star (with a larger stellar radius at the onset of RLOF) is
of convection is also different at the two points. This phase.

As shown in Fig. 5, there is a threshold for \( q_c \) when the initial primary mass \( M_1 \) is 3.98 M\(_\odot\). When \( M_1 \leq 3.98 \) M\(_\odot\), \( q_c \) systematically increases with the initial primary’s mass while it is opposite when \( M_1 \geq 3.98 \) M\(_\odot\). This phenomenon is likely to be relevant to the degenerate degree of the core. We examined the degeneracy parameter \( \psi \) in the calculations and found that \( \psi \) becomes negative just at \( M_1 = 3.98 \) M\(_\odot\).

We know that, when central He is exhausted in a star, there exists a He-burning shell and a H-burning shell. The H-burning shell extinguishes first when it burns outwards, and then the He-burning shell also extinguishes. The star begins to contract and the temperature increases until H is ignited. With the increase in temperature, He is also ignited but in a very thin shell. The He burning shell extinguishes first when it burns outwards, and then the He-burning shell also extinguishes. The star begins to contract and the temperature increases until H is ignited.
of equations (4)–(6).

In principle, if the mass donor has two burning shells, the thermal pulse will likely affect the final results. For the cases we studied, however, the influences seem small and can be neglected.

To simply use this results, we fitted \( q_c \) from our calculations. The influence from \( \beta \) is nearly linear and can be fitted as

\[ q_c = q_{c0} - c_0 \beta. \]

For the FGB,

\[ q_{c0} = c_1 + c_2(\log R/R_\odot - A_0) + c_3(\log R/R_\odot - A_0)^2, \]

when \( M_{11} < 2.00 M_\odot \) and \( \log R/R_\odot - A_0 > 0.2 \). In other cases,

\[ q_{c0} = c_4 + c_5(\log R/R_\odot - A_0)^{1/3}. \]

Here,

\[ c_i = c_{i1} + c_{i2} M_1 + c_{i3} M_1^2. \]

The values of \( c_{i1} \) and \( c_{i0} \) are listed in Table 3 for different cases. For the case of the AGB,

\[ q_{c0} = c_1 + c_2(\log R/R_\odot - A_0) + c_3(\log R/R_\odot - A_0)^2, \]

where \( c_i (i = 1, 2, 3) \) are expressed as in equation (6).

The coefficients \( c_i \) and \( c_{0i} \) are listed in Table 4. One may send a request to xuefeichen717@hotmail.com for the FORTRAN code of all the formulae used in this paper.

To compare with the results of the polytropic model, we present the dependence of \( q_c \) on the core mass \( M_c \) as well as on the core mass fraction \( f_c \) for low-mass binaries in Fig. 9, ignoring the points at the base of the FGB for each mass. Here, the core mass is defined as the mass within the hydrogen mass fraction less than 0.1.\(^5\) From the figure, we see a clear relation between \( q_c \) and \( M_c \), but not between \( q_c \) and \( f_c \). \( q_c \) may be fitted as follows:

\[ q_c = q_{c0} - 0.35 \beta, \]

where

\[ q_{c0} = 1.142 + 1.081 M_c - 2.852 M_c^2. \]

The studies above have not included the influences from the stellar wind. Han et al. (2002) included the stellar wind prior to

\(^5\) It is little bit difficult to determine the core in a real star as described in the polytropic model. However, the core mass defined here is a close approximation to that defined in the polytropic model.
RLOF when they studied the $q_c$ for low-mass binaries on the FGB and $\beta = 0$. We have included their results in Fig. 9. The pluses are for binaries with $M_{1i} = 0.8 \, M_\odot$ \cite{6} and the squares for $M_{1i} \geq 1.00 \, M_\odot$ \cite{7} (see table 3 in their paper). We see that all the squares are well along the fitting line of $\beta = 0$, indicating that stellar wind has little influence on $q_c$. However, the mass ratio will decrease due to stellar wind, and RLOF will be more stable, for example, the mass ratio possibly becomes less than $q_c$ at the onset of RLOF and the binary will undergo a stable mass transfer. For the case of $M_{1i} = 0.8 \, M_\odot$, $q_c$ from Han et al. (2002) is obviously larger than the fitting value, which means that RLOF is more stable in binaries with this primary mass. Since the binaries with $M_{1i} = 0.8 \, M_\odot$ have a larger $f_c$ in comparison to those with $M_{1i} \geq 1.0 \, M_\odot$,\footnote{We have not studied the case for $M_{1i} = 0.8 \, M_\odot$ in this paper, because the time-scale is too long for a star with this mass to evolve from ZAMS to giant branch. Meanwhile, it might be more likely that low-mass binaries (i.e. the initial primary less than 1 $M_\odot$) contribute to BSs via coalescence induced by angular momentum loss (Chen & Han 2008), while not from the mass transfer between a giant and a MS.} the high $q_c$ for $M_{1i} = 0.8 \, M_\odot$ here likely indicates some contributions from $f_c$, as is possibly concealed by that from $M_c$ for binaries with $M_{1i} \geq 1.00 \, M_\odot$ (see the discussion in Section 6).

4 EXAMPLES OF THREE BINARIES RESULTING IN BSs

In this section, we present the detailed evolutions for three binaries, which are selected mainly for the reason that we are interested in BSs in the old open cluster M67, where many BSs, including several long-period BSs, have been observed. M67 has a metallicity similar to that of the Sun (Hobbs & Thorburn 1991; Friel & Janes 1993). There are some researches on the age of M67. It may range from 3.2 $\pm$ 0.4 Gyr (Bonatto & Bica 2003) to 6.0 Gyr (Janes & Phelps 1994). The study of VandenBerg & Stetson (2004) derived an age of 4.0 Gyr. In the N-body model of this cluster (Hurley et al. 2005), the authors investigated the behaviour around 4 Gyr. Thus, in the study of Han et al. (2002), the mass donor has a similar core mass but a different stellar mass at the onset of RLOF. Therefore, a high stellar mass means a smaller core mass fraction $f_c$. See table 3 in their paper for details.
we also consider that its age is about 4 Gyr, indicating that the mass of the turn-off $M_{\text{to}}$ is about 1.2–1.3 $M_\odot$, which is the main factor of the cluster we will consider as we choose the binary samples. Basic requirements for the binaries which may contribute to BSs in a cluster via a giant transferring mass to a MS companion are for the masses of both components, that is, the primary should be more massive than the turn-off to ensure that it has left the MS and the secondary should be less than the turn-off if it is still on the MS at the cluster age. The fact that the secondary should accrete certain material before it becomes a BS will give a further constraint on the binary parameters. Obviously, $\beta = 0$ is ruled out since no material is accreted by the secondary in this case. In what follows, we will give three examples and present the evolutionary results in details.

(i) Example 1. RLOF is dynamically stable for initial parameters. As shown in Section 3, $q_c$ decreases with the mass-transfer efficiency $\beta$. This means that, in order to ensure that $\beta$ is large enough to increase the secondary’s mass to be larger than the turn-off, the mass ratio $q$ should be as small as possible. However, $q$ should be larger than unity to confirm that mass transfer occurs between a giant star and a MS companion. We therefore choose a binary of 1.3 + 1.2 $M_\odot$ ($q = 1.08$). Meanwhile, since the orbital period increases with the core mass (or the radius) of the mass donor at the onset of RLOF, a large core mass (or stellar radius) is necessary for long-orbital-period BSs. However, when $M_\text{c} > 0.4 M_\odot$, $q_c$ is less than 1.0 even for $\beta = 0$. We therefore set $M_\text{c} = 0.35 M_\odot$.
The mass-loss rate of the primary to be $1 \times 10^{-4} \text{M}_\odot \text{yr}^{-1}$ at the onset of RLOF. From equations (8) and (9), $\beta = 0.16$ for $q_c = 1.08$. We then set $\beta = 0.1$ in the calculation.

(ii) **Example 2.** RLOF is dynamically unstable for initial parameters, but it will be stable after some mass loss of the primary by stellar wind prior to RLOF. Stellar wind is included only on the giant branch by the mode of Reimers’ (1975):

$$\dot{M}_{\text{wind}} = \frac{4 \times 10^{-13} \eta R L}{M},$$

where $\eta$ is a dimensionless factor. The same binary as example 1 was examined here but with $\beta = 0.5$. In general, $\eta$ is 1/4 (Renzini 1981; Iben & Renzini 1983; Carraro et al. 1996). However, it is expected by many authors that giant stars have much higher mass-loss rates than that of Reimers, especially when the stars are far away from the base of giant branch (e.g. Bloecker 1995). Furthermore, tidal interaction between the two components of the binary also increases the mass-loss rate. Thus, we set $\eta = 2.5^8$ in the calculation. When RLOF begins ($M_e = 0.355 \text{M}_\odot, M_1 = 1.11 \text{M}_\odot (q = M_1/M_2 = 0.93$). We switch the stellar wind off once the mass-transfer rate exceeds the value given by equation (10) and have not included it after RLOF. From equations (8) and (9), we get $q_c = 0.99$ for $\beta = 0.5$ and $M_e = 0.35 \text{M}_\odot$. Thus, RLOF is dynamically stable as it begins.

(iii) **Example 3.** RLOF is dynamically unstable for initial parameters, but it becomes stable after the initially dynamically unstable mass transfer, if we assume that CE has not developed during this phase according to the study of Beer et al. (2007). Beer et al. (2007) argued that a wide range of systems avoid CE phase because mass transfer is super-Eddington even for non-compact companions. The accretion energy released in the rapid mass-transfer phase strips away a large fraction of the giant’s envelope, reducing the tendency to dynamical instability and merging. More constraints are necessary to determine whether the CE is formed or not in this case, but we have little knowledge about it. For simplicity, we assume that CE has not developed in the binary we study here, that is, in a binary with 1.6 + 1.1 M$_\odot$. The code cannot work for the initially dynamically unstable mass transfer because of the high mass-transfer rate $\dot{M}$. We therefore artificially limit the highest $\dot{M}$ to be $1 \times 10^{-4} \text{M}_\odot \text{yr}^{-1}$, and the accretion rate of the secondary $\dot{M}_s$ is equal to $\dot{M}$ as $\dot{M} < 1 \times 10^{-5} \text{M}_\odot \text{yr}^{-1}$, and equal to $1 \times 10^{-5} \text{M}_\odot \text{yr}^{-1}$ as $\dot{M} \geq 1 \times 10^{-5} \text{M}_\odot \text{yr}^{-1}$. From this assumption, $\beta$ ranges from 1.0 to 0.1, then to 1.0 again during the whole RLOF period. The maximum accretion rate of the MS companion here, that is, $1 \times 10^{-5} \text{M}_\odot \text{yr}^{-1}$, is comparable with the typical value of symbiotic stars (Scott & Webbink 1984).

Fig. 10 presents the mass-loss rate to the total mass of the primary for the three binaries. In these figures we see that, after the initial rapid mass loss, the primary loses its material at a rate of about $10^{-6}$–$10^{-8} \text{M}_\odot \text{yr}^{-1}$. Generally this slow mass transfer occurs after the inversion of the mass ratio, that is, the primary is less massive than the secondary. During the slow mass-transfer phase, the radius of the primary again increases but with a much lower level in comparison to that in the rapid mass-loss phase. At the same time, the Roche lobe radius of the primary increases with mass loss, since the orbital period becomes longer and longer after the inversion of $q$. Thus, if CE has not developed in the dynamically unstable phase in a binary such as example 3, it is very likely that CE will never develop in the binary. The material lost from the system in a way similar to stellar wind, and RLOF eventually terminates after most of the primary’s envelope is lost. Note that the mass loss of the system here is different from the ejection of CE. The former needs no orbital energy but the latter needs some. Thus, binaries avoiding CE have long orbital periods.

The behaviours of the secondaries of the three binaries are presented in Fig. 11. Since the secondaries cannot accrete the mass lost from the system in a way similar to stellar wind, and RLOF eventually terminates after most of the primary’s envelope is lost. Note that the mass loss of the system here is different from the ejection of CE. The former needs no orbital energy but the latter needs some. Thus, binaries avoiding CE have long orbital periods.

8 We also examined the cases for $\eta = 1/4$ and 1, and found that both of them are too small to strip enough mass away to lead to $q < q_c$ as RLOF begins.
from the primary in a very short time, they have left thermal equilibrium and evolve in a way similar to pre-MS stars during the initial phase instead of going upward along the MS. Due to the long orbital periods, the secondaries do not overfill their Roche lobes. They finally become normal MS stars with a higher He fraction in the envelope when new thermal equilibrium is established, and evolve similarly to normal single stars with those masses. These rejuvenated stars have much longer time-scales (including the phase of the secondaries with lower masses before accretion) on the MS and have the possibility to be BSs.

The main characteristics of the three binaries are listed in Table 5, from which we may obtain some clues on their parent binaries and evolutionary histories. For example, although the secondary in example 2 has a mass similar to that in example 3 after accretion, the latter is much bluer (Fig. 11) when both of them return to thermal equilibrium since the latter is much less evolved at the onset of RLOF. The two secondaries have different orbital periods, different compact components and different lifetimes, etc., and appear in different age of the cluster, as shown in Table 5.

As an example, we give a colour–magnitude diagram of both of the components as well as the binary system for example 3 in Fig. 12. The combined colour and magnitude of the binary result of the components as well as the binary system for example 3 in Fig. 12. The combined colour and magnitude of the binary result from add-and-subtract calculations of the luminosity of both components. The evolutions of the components prior to RLOF have no differences from a standard stellar evolution and are not plotted in the figure. During the whole mass-transfer phase, the system resembles the characteristics of the primary because the primary is much more luminous than the secondary, but the time-scale is very short. Once the primary becomes cooler, that is, less luminous than its component, the binary is hardly distinguishable from the MS component. It is very difficult to find the clues from the compact component, even from spectral observations because of the long orbital period.

### 5 MONTE CARLO SIMULATION

To investigate BSs from mass transfer between giants and MS companions, we performed a Monte Carlo simulation for a sample of $10^6$ binaries (very wide binaries are actually single stars). A single starburst is assumed in the simulation, that is, all the stars have the same age and metallicity ($Z=0.02$). The initial mass function (IMF) of the primary, the initial mass ratio distribution and the distribution of initial orbital separation are as follows.

(i) The IMF of Miller & Scalo (1979) is used and the primary mass is generated from the formula of Eggleton, Fitchett & Tout (1989):

$$M_1 = \frac{0.19 X}{(1 - X)^{0.35} + 0.032(1 - X)^{1/4}},$$

where $X$ is a random number uniformly distributed between 0 and 1. The mass ranges from 0.8 to 100 $M_\odot$.

(ii) The mass ratio distribution is quite controversial and, for simplicity, we only consider a constant mass ratio distribution (Mazeh et al. 1992):

$$n(q') = 1, \quad 0 \leq q' \leq 1$$

where $q' = 1/q = M_2/M_1$.

(iii) We assume that all stars are members of binary systems and the distribution of separations is constant in log $a$ (where $a$ is separation).

$$n(a) = \begin{cases} \alpha \exp(a/a_0)^n, & a \leq a_0 \\ \alpha \exp(-(a/a_1)^{1/m}), & a_0 < a < a_1 \end{cases}$$

where $a = 0.070, a_0 = 10 R_\odot, a_1 = 5.75 \times 10^6 R_\odot = 0.13 \text{ pc and } m = 1.2$. This distribution gives an equal number of wide binary systems per logarithmic interval and 50 per cent of systems with orbital periods less than 100 yr$^2$ (Han et al. 1995).

It is an assumed distribution inferred from observations of spectrostropic binaries (see a series of papers of R. F. Griffin in The Observatory). Indeed, there are many distributions used in the literature, but they are all flat for wide binaries, leading to similar binary population results. The important thing for the currently adopted distribution is that it implies 50 per cent of stellar systems with orbital periods less than 100 yr. One can simply multiply the results with a coefficient if another percentage is assumed.

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**Table 5.** The characteristics for three different evolutionary examples in the text. $\eta$ = the factor of Reimer’s wind, $M_c$ = the core mass at the onset of RLOF, $\beta$ = the mass fraction of the lost matter of the primary accreted by the secondary, $t_{RLOF}$ = the age at the onset of RLOF, $t_{MS}$ = the age of the secondary when it terminates its MS, and $P_e, M_{1e}$ and $M_{2e}$ are the orbital period, the mass of the primary and the mass of the secondary at the end of RLOF, respectively.

| Binary | Stellar wind | $M_c$ (M$\odot$) | $\beta$ | $P_e$ (d) | $t_{RLOF}$ (Gyr) | $t_{MS}$ (Gyr) | $M_{1e}$ $(M_\odot)$ | $M_{2e}$ $(M_\odot)$ |
|--------|--------------|-----------------|--------|-----------|-----------------|---------------|-----------------|-----------------|
| Example 1 | 1.3+1.2 | 0.0 | 0.356 | 0.1 | 803 | 4.71 | 5.45 | 0.46 | 1.28 |
| Example 2 | 1.3+1.2 | 2.5 | 0.356 | 0.5 | 722 | 4.71 | 5.45 | 0.44 | 1.53 |
| Example 3 | 1.6+1.1 | 0.0 | 0.340 | – | 439 | 2.47 | 4.57 | 0.41 | 1.50 |

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**Figure 12.** The colour and magnitude evolutions for the individual components of the binary 1.6 + 1.1 $M_\odot$ as well as for the whole system. The early evolutions of both components prior to RLOF are not plotted in the figure. The circle and dotted lines are for the primary and the secondary, respectively, and the solid line is the combination of the two components. The numbers are the time sequences of the evolutions.
The rapid binary evolution code developed by Hurley, Pols & Tout (2000, 2002) is employed here. In addition to all aspects of single-star evolution, this code includes many features of binaries, that is, mass transfer, mass accretion, common-envelope evolution, collisions, supernova kicks and angular momentum loss mechanisms, etc. In particular, circularization and synchronization of the orbit by tidal interaction are calculated for convective, radiative and degenerate damping mechanisms. As a comparison, we adopt three criteria obtained in different ways, that is, from a polytropic model (equation 1), from the paper of Hurley et al. (2002) and from this paper.

In the paper of Hurley et al. (2002),

\[ q_c = \frac{1.67 - x + 2(M_c/M)^{\delta}}{2.13}, \]

where \( M_c \) and \( M \) are the core mass and the total mass of the donor, respectively, \( x \) is the exponent of the mass–radius relation at constant luminosity for giant stars and equals to 0.3. This criterion comes from the assumption that the adiabatic mass–radius exponent of a giant star \( \zeta_{ad} = 0nR/0nM \) equals to the Roche lobe mass–radius exponent \( \zeta_L \), where \( \zeta_L \approx 2.13q - 1.67 \) for conservative mass transfer (Tout et al. 1997) and \( \zeta_L \approx -x + 2(M_c/M)^{\delta} \), which is fitted from detailed stellar models.

From the sample, we obtain 303291 binaries which begin RLOF when the primary is a giant or more evolved star via Hurley’s rapid binary evolution code. In the following section, we will give the consequences from different criteria of the dynamical instability for mass transfer from a giant to a MS companion.

5.1 The results from Hurley’s criterion

Among the 303291 binaries undergoing mass transfer from a giant or more evolved star to its companion, most of them eventually experience CE evolution, leaving short-period binaries or mergers of the two components if the CE cannot be stripped away. According to Hurley’s criterion, only 3746 binaries may avoid CE formation and probably show the characteristics of some strange objects, for example, symbiotic stars, BSs, etc., during or after RLOF. Here, we are concerned with the outcomes of BSs.

Most of the 3746 binaries which avoid CE formation begin RLOF when the primaries are on the giant branches, that is, on the FGB for 17 binaries, on the early–AGB (EAGB, after central He burning but before the first thermal pulse) for 415 binaries, and on the thermally pulsing AGB (TPAGB) for 3295 binaries. The simulation shows that, 3139 binaries will pass through BS phase during their lives and 2208 of them have undergone dynamically stable RLOF before they become BSs. This means that RLOF is an important process to increase the secondary’s mass to be larger than the turn-off of a cluster. All the 2208 binaries have long orbital periods (greater than 1000 d) and their mass ratios are less than unity at the onset of RLOF, indicating that the primaries have lost most of their envelopes prior to RLOF (e.g 2164 binaries have a mass donor being on TPAGB).

As a consequence, the BSs from this way also have long orbital periods (greater than 1600 d).

At the age of 4 Gyr, we obtain 96 BSs. Fig. 13 presents the distribution of the orbital period as well as the mass ratio versus the mass of the secondary for 96 binaries which fulfill \( t_1 \leq 4 \) Gyr, \( t_2 \), where \( t_1 \) and \( t_2 \) are the ages at the onset and at the end of BSs, respectively. The circles are for the case at \( t_1 \) and the pluses are for the case at \( t_2 \).

5.2 The results from the polytropic model’s criterion

Using the same binary sample, we examined the evolutionary consequence from the criterion of the polytropic model (see equation 1 in Section 1). As discussed in the paper of Hurley et al. (2002), the value of \( q_c \) from the polytropic model is obviously larger than that of Hurley et al. (2002) with increasing core mass. For example, it is a factor of 2 larger at \( M_c/M_1 = 0.6 \). Thus, more binaries may avoid dynamically unstable RLOF in the criterion of the polytropic model than those in the criterion of Hurley et al. (2002).

From equation (1), 8522 binaries avoid CE formation and 5919 pass through the BS phase during their lives. Among of the 5919 binaries, 4287 have undergone or are just undergoing dynamically stable RLOF before the secondaries become BSs. At the age of 4 Gyr, we obtained 175 long-orbital-period BSs (greater than 1600 d). The distribution of the orbital period as well as the mass ratio versus the mass for the 175 BSs are similar to those from the criterion of Hurley et al. (2002).
Table 6. The BS numbers in M67 obtained from FGB stars transferring matter to MS companions for various $\beta$. The MS time-scale of the secondary after RLOF, $t_{\text{MS}}$, is set to $1 \times 10^8$ yr (from the second to fifth columns) and $5 \times 10^8$ yr (from the sixth to ninth columns), respectively. The final masses of the primary are $M^1_{\text{f, MS}} = M_c$, $M^2_{\text{f, MS}} = M_c + 0.1 M_\odot$, $M^3_{\text{f, MS}} = M_c + 0.2 M_\odot$, and $M^4_{\text{f, MS}} = M_c + 0.4 M_\odot$, where $M_c$ is the core mass of the primary at the onset of RLOF.

| $\beta$ | $M^1_{\text{f, MS}}$ | $M^2_{\text{f, MS}}$ | $M^3_{\text{f, MS}}$ | $M^4_{\text{f, MS}}$ |
|---------|-----------------|-----------------|-----------------|-----------------|
| 0.1     | 414             | 412             | 371             | 319             |
| 0.2     | 169             | 169             | 169             | 169             |
| 0.3     | 31              | 31              | 31              | 31              |
| 0.4     | 2               | 2               | 2               | 2               |

5.3 The results from this paper’s criterion

Now we study the outcomes from the criterion in this paper. Different from the two criteria above, we only evolve binaries to the onset of RLOF using Hurley’s code and obtain the following evolutions from assumptions below.

When we determine whether mass transfer leads to a BS formation or not in a certain cluster, two characteristic parameters of the secondary, that is, the mass $M_\beta$ and the MS lifetimes $t_{\text{MS}}$, are critical. The former is relevant to the initial mass of the secondary and the mass-transfer efficiency $\beta$, while the latter is determined by the central H fraction and the total mass of the secondary after accretion. Meanwhile, the ages at the onset and termination of RLOF are also important. For example, RLOF should start before or at the cluster age $t_{\text{cluster}}$ to ensure that the secondary may accrete some matter, but it cannot stop much earlier than $t_{\text{cluster}}$, or the secondary has likely left the MS. Therefore, only the binaries, which complete RLOF using Hurley’s code and obtain the following evolutions from assumptions below.

Fig. 17, we see that the maximum mass ratio at the onset of RLOF is less than 1.2 for binaries resulting in BSs, which means that the secondary is only slightly less than the turn-off, and then the MS time-scale of the secondary after accretion will not be very long. Meanwhile, it is likely longer than $5 \times 10^8$ yr from Table 5.

Table 7. The BS numbers in M67 obtained from EAGB stars transferring matter to MS companions for various $\beta$.

| $\beta$ | $M^1_{\text{f, MS}}$ | $M^2_{\text{f, MS}}$ | $M^3_{\text{f, MS}}$ | $M^4_{\text{f, MS}}$ |
|---------|-----------------|-----------------|-----------------|-----------------|
| 0.1     | 58              | 56              | 52              | 38              |
| 0.2     | 48              | 47              | 46              | 37              |
| 0.3     | 29              | 29              | 29              | 28              |
| 0.4     | 13              | 13              | 13              | 13              |

Table 8. The BS numbers in M67 obtained from TPAGB stars transferring matter to MS companions for various $\beta$.

| $\beta$ | $M^1_{\text{f, MS}}$ | $M^2_{\text{f, MS}}$ | $M^3_{\text{f, MS}}$ | $M^4_{\text{f, MS}}$ |
|---------|-----------------|-----------------|-----------------|-----------------|
| 0.1     | 97              | 80              | 63              | 29              |
| 0.2     | 150             | 122             | 96              | 50              |
| 0.3     | 175             | 142             | 112             | 61              |
| 0.4     | 175             | 146             | 105             | 58              |
| 0.5     | 157             | 132             | 97              | 45              |
| 0.6     | 111             | 100             | 81              | 66              |
| 0.7     | 68              | 56              | 42              | 15              |
| 0.8     | 39              | 26              | 11              | 4               |
| 0.9     | 32              | 20              | 4               | 0               |
| 1.0     | 33              | 20              | 4               | 0               |

Figure 14. Orbital period versus mass for the BSs in M67 resulting from FGB stars transferring material to MS companions. The final mass of the primary is assumed to be $M_c + 0.2 M_\odot$, where $M_c$ is the core mass of the mass donor at the onset of RLOF. See the text for the details.

BS numbers for different mass donors, that is, the mass donors are FGB stars (FGB+MS), EAGB stars (EAGB+MS) and TPAGB stars (TPAGB+MS), respectively. The corresponding distributions of the orbital period versus the final mass for various $\beta$ are shown in Figs 14–16.

We have not obtained BSs from an FGB star transferring matter to a MS companion in M67 from the criteria of Hurley et al. (2002) and of the polytropic model, since the mass donor has not lost much mass by wind prior to RLOF and the core mass is not very large. Both of these facts make the condition $q < q_c$ difficult to fulfill (see equations 1 and 14). However, from the criterion of this paper, several BSs will be formed in M67 if $\beta \leq 0.4$. From Table 6, we see that $\beta$ strongly affects the contribution to BSs from this evolutionary channel. A small $\beta$ contributes more BSs because...
of the MS companion is the main factor to determine the formation of BS from this way, but the dynamical instability is the crucial factor when $\beta > 0.4$.

From Fig. 14 we see that, for the BSs from mass transfer between an FGB star and a MS companion, their masses increase and the range of the orbital period becomes narrower (mainly some long-orbital-period BSs disappear) with the increasing $\beta$, since a large $\beta$ means more matter accreted by the secondary (leading to a larger final mass) and less angular momentum lost from the system (resulting in a shorter orbital period). Furthermore, a long orbital period indicates a larger stellar radius at the onset of RLOF, and then a smaller $q_\text{c}$, as shown in Section 3. Therefore, with the increase in $\beta$, the RLOF in relatively long-orbital-period binaries firstly changes from dynamically stable to dynamically unstable, and directly leads to the disappearance of long-orbital-period BSs. The orbital period of the BSs from this channel has a wide range, that is, from several days to hundreds of days for $\beta = 0.1$. However, BSs with $P > 100$ d can only be produced when $\beta \leq 0.2$ and their masses are near the turn-off because only a very small fraction of the lost matter from the primary is accreted by the secondary. All the possible candidates contributing to BSs in M67 from FGB+MS are low-mass binaries, since the secondaries from intermediate-mass FGB+MS binaries have left the MS at 4 Gyr. For a 2 M$_{\odot}$ star, the minimum mass of the secondary is about 1.6 M$_{\odot}$ for $\beta = 0.0$ and $M_{\text{c}} = 0.25$ M$_{\odot}$. The stars with this mass have left the MS in the old open cluster.

Different from the case of FGB+MS, the BSs resulting from EAGB+MS and TPAGB+MS have similar orbital periods for various $\beta$ (see Figs 15 and 16). Meanwhile, the mass difference of the BSs from different $\beta$ is also not as obvious as that from FGB+MS binaries. In the cases of EAGB+MS and TPAGB+MS, some BSs have masses similar to those from FGB+MS, and some have lower masses, when $\beta$ is larger than 0.1. The lower mass BSs mix with those from $\beta$ with lower than the given value. Stellar wind may account for this consequence. When the primary evolves from FGB to EAGB, then to TPAGB, its mass becomes less and less due to stellar wind. Correspondingly, the minimum mass of the MS companion for stable mass transfer also becomes smaller. The companions may then have lower masses after RLOF with evolution. As a result, the masses of BSs in Figs 15 and 16 extend to lower values than those in Fig. 14.

From the criteria of the polytropic model and Hurley et al. (2002), only long-orbital-period BSs (with orbital period greater than 1600 d) may be produced from giants transferring matter to the MS companions. Since case A and early case B (the mass donor is during Hertzsprung gap at the onset of RLOF) mass transfer are only responsible for some short- and mid-period BSs (up to 100 d, Chen & Han 2004), there seems to be a period gap of hundreds days from the two criteria above. However, this period gap will not appear from the criterion of this paper. The BSs from different giant binaries cover the period range from several days to thousands of days (see Figs 14–16).

As an example, we show the mass ratio versus the mass of BSs for the case of TPAGB+MS binaries at the onset and at the termination of RLOF in Fig. 17. From the figure, we see that the mass ratio at the onset of RLOF is located in a triangle region, where the upper boundary is determined by the dynamical instability and the lower boundary is determined by the mass of the accretor after the RLOF.

We have not included example 3 in Section 4 in the above study. Systematic investigation for this case is difficult at present, since we have little knowledge of the conditions for binaries to avoid CE during initially dynamically unstable RLOF. The conditions are possibly relevant to the parameters such as the mass (and the core
mass) of the primary, the mass ratio of the components, the orbital period, etc. Furthermore, it may also be affected by the detailed process of mass transfer. The study of the conditions is beyond the scope of this paper. If case 3 is a real case, only a very small fraction (less than 1 per cent) from those with dynamically unstable RLOF is enough to produce BSs in M67. The products here have much longer MS lives in comparison to those from dynamically stable RLOF, since the secondaries in general have much lower mass due to the initially large mass ratio (greater than \( q_c \)) and are less evolved before RLOF. From Table 5, we see that at \( t = 2.47 \) Gyr, a star with \( 1.5 \, M_\odot \) has already formed and it becomes a BS when \( t > 2.5 \) Gyr.\(^{13}\) The material around the BSs might give us some clues on their parent stars.

5.4 Comparison to observations

Several works focus on the BSs in M67 from observations (Milone et al. 1992; Ahumada & Lapasset 1995; Sandquist & Shetrone 2003). In the new catalogue of BSs in open clusters (Ahumada & Lapasset 2007), there are 30 BSs in this old open cluster. Since there is no orbital information for these BSs in Ahumada & Lapasset (2007), the main observational data we adopted here are from some earlier studies.

Milone et al. (1992) reported the radial velocities for 13 BSs in the open cluster M67 according to observations over about 9 yr. Three out of the 13 BSs rotate too rapidly to allow reliable velocity determinations (Latham & Milone 1996). Among the 10 BSs left, only one (F190) shows a short orbital period, about 4.2 d, while five have long-orbital periods in the range from 800 to 5000 d. The other four BSs are considered as single stars by some studies (e.g. Hurley et al. 2001). Three out of the five long-orbital-period BSs have obvious orbital eccentricities and the other two are in near-circular orbits. Sandquist & Shetrone (2003) presented an analysis of the time series photometry of M67 for W UMa systems, BSs and related objects. There are 24 possible BSs observed in their study and most of them show no variation in their light curves. Two BSs, S968 and S1263, which were considered as single stars before, are possible variables in the study of Sandquist & Shetrone (2003).

The BSs resulting from the criterion of Hurley et al. (2002) have orbital periods beyond 1600 d (\( \log P > 3.2 \)) as seen in Fig. 13. However, four of the five observed long-orbital-period BSs have orbital periods less than this value. The simulation from the polytropic model gives a similar result. The BSs from the criterion of this paper may well cover the orbital period range, but it seems that none of the five BSs is from the mass transfer between an FGB and a MS, since the smallest orbital period is 846 d (\( \log P = 2.93 \)), which is far greater than the maximum orbital period resulting from the FGB+MS binaries, as seen in Fig. 14. The BSs with mid orbital period (i.e. from tens of days to hundreds of days) are possibly from FGBs transferring matter to their MS companions, but it needs more observational evidence to confirm the existence of these BSs.

The orbital eccentricity is a puzzle from mass transfer, since the orbit should be circularized by tidal interaction of the two components of a binary, which is just undergoing, or after, RLOF. Some studies considered that dynamical collision is necessary to explain the observed orbital eccentricities. Recently, Bončić Marinović, Glebbeek & Pols (2007) found that, due to the enhanced mass loss of the AGB component at orbital phases closer to the periastron, the net eccentricity growth rate in one orbit is comparable to the rate of tidal circularisation in many cases. They reproduced the orbital period and eccentricity of the Sirius system with this eccentricity enhancing mechanism. Their study provides an explanation for the eccentricities of long-orbital-period BSs without dynamical collision.

6 DISCUSSIONS AND CONCLUSIONS

Our study shows that the critical mass ratio, \( q_c \), for dynamically stable mass transfer between a giant star and a MS companion depends on the stellar radius at the onset of mass transfer. For any given mass, a more evolved star is less stable for RLOF, since the evolutionary time-scale for a more evolved star is shorter and hence the mass-transfer rate is higher than that for less-evolved ones. The results including stellar wind (Han et al. 2002) are consistent with the tendency in this paper, except for cases where \( M_{\text{crit}} = 0.8 \, M_\odot \), which have a larger \( f_c \) and a larger \( q_c \) in comparison to those of \( M_{\text{crit}} \geq 1.0 \, M_\odot \). The behaviour of the cases of \( M_{\text{crit}} = 0.8 \, M_\odot \) here is similar to the consequences of a polytropic model. Meanwhile, from table 3 in the paper of Han et al. (2002), we see that, at a similar core mass, binaries with a low primary mass (or a large \( f_c \)) have larger \( q_c \), also consistent with equation (1) except for the binaries with \( M_{\text{crit}} = 1.9 \, M_\odot \). Our results for \( \beta = 0 \) in this paper match those of Han et al. (2002) (see the upper panel of Fig. 9). Due to the various core masses, however, the dependance of \( q_c \) on \( f_c \) is completely scattered as shown in the upper panel of Fig. 9. So, it is very likely that both \( M_c \) and \( f_c \) have influences on \( q_c \), but the effect is different for binaries of different masses, that is, \( M_c \) dominates the case where the primary’s mass is larger than \( 1 \, M_\odot \) while \( f_c \) is critical for the less-massive ones. The thickness of the envelope might be an important cause here and the transition is possibly a gradual process. The non-monotonic behaviour of \( M_{\text{crit}} = 1.9 \, M_\odot \) results from the low degeneracy degree of the core in primaries with this mass.

From the criterion in this paper, we obtained some BSs with intermediate orbital period when \( \beta \) is less than 0.4. However, there is no one reported at present located in the orbital period range from FGB+MS binaries, as we compare our results to observations. There might be two factors for this contradiction. One is the fact...
that the mass of the BSs formed from such a low $\beta$ is very close to the turn-off of M67, making them hard to distinguish from normal stars around the turn-off (as seen in Fig. 14). The other might be that the value of $\beta$ is substantially larger than 0.1 during mass transfer between FGB stars and MS companions in a real case, so few BSs are produced in this way. Orbital determinations (in the future) might provide some constraints on the value of $\beta$. For example, since the products of FGB+MS have a wide range of orbital periods, from several days to hundreds of days, there would be some BSs with intermediate orbital periods (if $\beta$ is low enough), which are possibly absent from the criterion of Hurley et al. (2002).

The possibility that a MS star becomes a BS via wind accretion was first suggested by Williams (1964). This idea lacked attention for a long time, since an isolated star can hardly accrete enough matter to become a BS. However, it is likely different if the MS star is bound in a binary system where the primary is undergoing a large mass loss. In general, the mass-loss rate of stellar wind is in the range of $10^{-2} M_{\odot}$ yr$^{-1}$ to $10^{-6} M_{\odot}$ yr$^{-1}$ as a star approaches the tip of the AGB, and some fraction of the lost material (up to 10 per cent, Tom, Henri & Alain 1996) is probably accreted by its companion. This means that the secondary may significantly increase its mass when the primary undergoes a large mass loss at the tip of the AGB. Although the mass increase is likely not enough to produce a BS, the secondary still has chances to obtain matter from the primary in the following stable RLOF if it happens. Thus, BSs with very long orbital periods (greater than 1000 d) are likely the consequences of both RLOF and wind accretion.

In this paper we present an example of a binary which avoids CE formation from initially dynamically unstable RLOF and eventually evolves to a BS with a long orbital period. We only assume that a CE has not formed in the initially dynamically unstable RLOF, but no critical conditions are shown. How to discriminate whether a CE has developed or not during this phase is unclear, as mentioned in Section 5. Many parameters, such as the mass (and the core mass) of the primary, the mass ratio of the components, the orbital period, etc., are relevant to this condition, which is also affected by the detailed process of mass transfer. The study of this will be complex and difficult, while interesting. As long as the possibility of this case exists, only a very small fraction of systems with dynamically unstable RLOF may provide an important contribution to BSs.

Mass-transfer efficiency, $\beta$, is an important parameter. Both the criterion of dynamical instability, $q_c$, and the final mass of the accretor, $M_\beta$, (which are the two critical characters to determine the formation of a BS) are relevant to this parameter. From our simulation for M67, the peak of the contribution from FGB+MS binaries is probably about $\beta = 0.1$, with which the BS mass is near 1.3 $M_{\odot}$. With the increase in $\beta$, the orbital period decreases and the BS mass increases, as was explained in Section 5. However, it is unclear what the value of $\beta$ is in binaries. In general, $\beta$ will be very small for RLOF in binaries with a giant mass donor and a compact companion (Han et al. 2002). Thus, $M_\beta$ will not be very large, similar to the turn-off mass.

The detailed evolution calculations in the paper show that $q_c$ decreases with the stellar radii of the primaries at the onset of RLOF, except for cases at or near the base of the giant branch, where the core is not very degenerate and the envelope is not yet fully convective. Non-conservative assumptions will strongly affect $q_c$ while stellar wind before mass transfer has little influence on it. To conveniently use the result we give a fit of $q_c$ as a function of the stellar radius of the primary at the onset of RLOF, and of the mass-transfer efficiency during RLOF. Theoretically, dynamically stable mass transfer occurs once the mass ratio is less than the critical value. However, it is delayed in real binaries. Usually the stable mass transfer occurs after the reversion of the mass ratio.

The Monte Carlo simulations show that some binaries with the mass donor on the FGB, which have no contributions to the BSs from the earlier criteria, will contribute to this population with the criterion obtained in this paper. Meanwhile, from our criterion, the BSs resulting from the mass transfer between an AGB star and a MS companion may be more numerous and have a wider range of orbital periods than those formed from previous criteria. Although the result from our criterion may well cover the observed orbital period range, it seems that none of the five observed long-orbital-period BSs is from the mass transfer between an FGB and a MS companion.

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