Observing Dynamic Deformation of Masonry Wall in Vibration through Digital Photography and Kalman Filter

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Abstract. Masonry structure is one of the common engineering structures, they encounter brittle fractures and even collapse in earthquakes, posing a major threat to the safety of human beings and property. This drives a need to monitor their dynamic deformation during earthquakes and study the seismic performance of masonry structure. This paper uses digital photography to monitor dynamic deformation of masonry structure in vibration. Kalman filter is used to eliminate the random noise and filter the change value of the pixel. Finally, the filtered value is converted into the actual displacement value by the time baseline parallax method. Results show that the relative measurement accuracy is 2.13‰ after using kalman filter to process deformation data. In the early vibration stage, masonry structure develops less deformation and it is in elastic state. The deformation of masonry structures increase gradually with the vibration being stronger. Digital photography can monitor the dynamic deformation of masonry structure in real time and grasp its health status. It can provide technical and data support for studying seismic performance of masonry structure.

Keywords: Digital photography, masonry structure, kalman filter, deformation monitoring.

1. Introduction

Masonry structure is traditional wall material and common building material. And the deformation of masonry structure is characterized by sudden and persistence in the earthquake. It is therefore important to obtain the dynamic deformation to study seismic performance of masonry structure.

However, traditional surveying methods such as physical sensor, GPS [1, 2], the three-dimensional laser scanning measurement [3] allow measurements of static deformation of a target with high precision but, cannot characterize the dynamic deformation over time. These problems can be solved by applying...
digital photography [4-6]. Many scholars such as Kim[7], Abdel-Sayed et al.[8], Yu et al. [9-15] have already used digital photography to monitor engineering structures. But the deformation data is of real-time and small amount and they are obtained by digital cameras. Thus, they are easily affected by random noise such as electronic components. Kalman filter is used to process deformation data to improve measurement accuracy with considering the data characteristics and the factors affecting data accuracy [16, 17]. This paper therefore uses digital photography and kalman filter to obtain the dynamic deformation to study seismic performance of masonry structure.

2. Digital photography and kalman filter

2.1. Time Baseline Parallax Method

The time baseline parallax method is a method to measure the relative change of two dimensional coordinates. Figure 1 illustrates the basic geometric relations between the image plane and the object plane when a deformation point moves from A to B, the deformation value $\Delta X$ and $\Delta Z$ are:

$$\begin{align*}
\Delta X &= \frac{Y}{f} \Delta P_x = M \Delta P_x \\
\Delta Z &= \frac{Y}{f} \Delta P_z = M \Delta P_z
\end{align*}$$

(1)

Where $M$ is the photographic scale, $\Delta X$ and $\Delta Z$ are the horizontal and vertical deformation of deformation point on object plane, $\Delta P_x$ and $\Delta P_z$ are the horizontal and vertical displacement of deformation point on image plane, $Y$ and $f$ are the photographic distance and focus respectively.

![Figure 1. Time baseline parallax](image)

2.2. Kalman filter

According to the experiment characteristics, the data obtained is discrete data. This paper therefore uses the discrete kalman filter to filter deformation data, the mathematical model of kalman filter is expressed as:

$$\begin{align*}
X_{k+1} &= \Phi_{k+1,k} X_k + \psi_{k+1,k} U_k + \Gamma_{k+1,k} \Omega_k \\
L_{k+1} &= B_{k+1,k} X_{k+1} + G_{k+1,k} U_{k+1} + \Delta_{k+1}
\end{align*}$$

(2)
\[ \psi_{k+1,k} = \int_{t_k}^{t_{k+1}} \Phi(t_{k+1}, \tau) C(\tau) d\tau \]

\[ \Gamma_{k+1} = \int_{t_k}^{t_{k+1}} \Phi(t_{k+1}, \tau) F(\tau) d\tau \]

Where
\[ X_k \] and \( L_k \) are the state vector and observation vector at the \( t_k \) moment respectively; \( \Phi_{k+1,k} \) is the state transition matrix from \( t_k \) time to \( t_{k+1} \) time, \( B_{k+1} \) are the observation matrix at the \( t_{k+1} \) moment, \( \Omega_k \) and \( \Delta_k \) are dynamic noise and observation noise at \( t \) moment respectively.

Assume that a deterministic input is out of consideration, the equation (2) can be simplified as:

\[ \begin{align*}
X_{k+1} &= \Phi_{k+1,k} X_k + \Gamma_{k+1,k} \Omega_k \\
L_{k+1} &= B_{k+1} X_k + \Delta_{k+1}
\end{align*} \]

The state estimation of the discrete linear system means to use the observation vector \( L_1, L_2, \ldots, L_k \) to calculate the optimal estimation of the state vector \( X_j \) at \( t_j \) moment based on the relevant mathematical model. The estimator is usually recorded as \( \hat{X}(j/k) \) and it is divided into 3 categories:

1. if \( j = k \), \( \hat{X}(k/k) \) is called the best filter, and we call the calculation process of \( \hat{X}(k/k) \) as kalman filter.
2. if \( j > k \), \( \hat{X}(j/k) \) is called the best filter, and we call the calculation process of \( \hat{X}(j/k) \) as prediction or extrapolation.
3. if \( j < k \), \( \hat{X}(j/k) \) is called the best filter, and we call the calculation process of \( \hat{X}(j/k) \) as smooth or interpolation.

The stochastic model of the system is shown as:

\[ \begin{align*}
E(\Omega_k) &= 0 \\
E(\Delta_k) &= 0 \\
\text{cov}(\Omega_k, \Omega_j) &= D_{\Omega}(k) \delta_{ij} \\
\text{cov}(\Delta_k, \Delta_j) &= D_{\Delta}(k) \delta_{ij} \\
\text{cov}(\Omega_k, \Delta_j) &= 0 \\
E(X_0) &= \mu_x(0) = X(0/0) \\
\text{var}(X_0) &= D_\Delta(0) \\
\text{cov}(\Omega_k, \Delta_j) &= 0 \\
\text{cov}(\Omega_k, \Delta_j) &= 0
\end{align*} \]

Where \( E(\Omega_k) \) is the mathematical expectation of \( \Omega_k \), \( E(\Delta_k) \) is the mathematical expectation of \( \Delta_k \), \( \text{cov}(\Omega_k, \Omega_j) \) is the covariance of \( \Omega_k \) with \( \Omega_j \), \( D_{\Omega}(k) \) is the variance of \( \Omega_k \), \( \text{cov}(\Delta_k, \Delta_j) \) is the covariance of \( \Delta_k \) with \( \Delta_j \), \( E(X_0) \) is the mathematical expectation of \( X_0 \), \( \text{var}(X_0) \) is the variance of \( X_0 \), \( \text{cov}(X_0, \Omega_k) \) is the covariance of \( X_0 \) with \( \Omega_k \), \( \text{cov}(X_0, \Delta_k) \) is the variance of \( X_0 \) with \( \Delta_k \).
If $j = k$, $\delta_j = 1$, and if not, $\delta_j = 0$. Kalman recursive filtering equation is obtained by using successive adjustment method based on the state equation and observation equation and stochastic model:

$$
\begin{align*}
\dot{X}(k/k) &= \hat{X}(k/k-1) + J_k [L_k - Z_k - B_k \hat{X}(k/k-1)] \\
D_\ell(k/k) &= [I - J_k B_k] D_{\ell}(k/k-1)
\end{align*}
$$

(5)

If the deterministic input is out of consideration, the kalman recursive filtering equation is:

$$
\begin{align*}
\dot{X}(k/k) &= \hat{X}(k/k-1) + J_k [L_k - B_k \hat{X}(k/k-1)] \\
D_\ell(k/k) &= [I - J_k B_k] D_{\ell}(k/k-1)
\end{align*}
$$

(6)

Where $\hat{X}(k/k-1)$ is one step predictive value, $\dot{X}(k/k)$ is the filter value of one step predictive value corrected, $D_{\ell}(k/k)$ is the filter value matrix equation, $J_k$ is the filter gain matrix equation.

3. Masonry structure experiment

Manganese steel plate in Figure 2 was used in the experiment. Because it was characterized by strong elasticity and strength, it hardly generated plastic deformation when a hammer impacted steel plate. It was fixed by the awls before the experiment. In addition, the stakes were constructed at a stable place around masonry structure, and reference points were laid on the stakes. Six digital cameras were used, and two cameras were on the north and south side of masonry structure respectively, one camera on the east and west side respectively. In the experiment, we used digital cameras to capture instantaneous deformation of masonry structure when a 25-kilogram-weight hammer falls freely and impacts the steel plate from a height such as 0.3m, 0.6m, 0.9m, 1.2m, 1.5m, 1.8m, 2.1m, 2.4m, 2.7m, 3.0m, 3.3m, 3.6m, 3.9m, 4.2m, 4.5m, 4.8m, 5.1m, 5.4m and 5.4m. We call them Experiment 1 to 19.

For clearly understanding the measurement process, illustration of masonry structure measurement was shown in Figure 2.

![Illustration of masonry structure measurement](image)

Figure 2. Illustration of masonry structure measurement

4. Data processing and discussion

This paper uses kalman filter to eliminate the effect of random noise on data accuracy. The maximum deviation between the kalman filter and monitoring value is 0.16 pixels and the rest is within 0.01 pixels. This suggests that the measurement accuracy can meet the requirements of deformation monitoring after
correcting the digital cameras. The measurement accuracy will be higher after using Kalman filter to eliminate random noise.

Table 1. Actual displacement value (/mm) of deformation point.

| Photo No. | period | dx35 | dz35 | dx36 | dz36 | dx37 | dz37 | dx38 | dz38 |
|-----------|--------|------|------|------|------|------|------|------|------|
| 1         | 1      | -1.01| -0.03| 0.32 | 0.05 | -0.98| 0.04 | -1.08| 0.12 |
| 2         | 2      | -0.22| -1.53| 1.27 | -1.45| -1.68| -0.14| -0.2  | -1.5 |
| 3         | 3      | -0.51| 0.73 | 0.45 | 2.15 | -0.47| 0.51 | -0.93| 0.48 |
| 4         | 4      | -0.18| 0.97 | -0.13| 0.95 | -0.12| 1.03 | -0.07| 1.02 |
| 5         | 5      | -0.59| 2.11 | -0.79| 2.3  | -0.56| 2.04 | -0.75| 2.21 |
| 6         | 6      | 0.09 | 1.59 | -0.04| 3.1  | 0.19 | 3.01 | 0.07 | 3.08 |
| 7         | 7      | 0.5  | 2.56 | 1.97 | 3.05 | 0.5  | 2.54 | -0.9 | 4.45 |
| 8         | 8      | 0.52 | 3.28 | 0.58 | 4.84 | 0.55 | 3.24 | 0.6  | 3.36 |
| 9         | 9      | -0.02| 3.24 | 1.39 | 3.46 | -1.45| 4.68 | -0.05| 3.45 |
| 10        | 10     | -0.17| 5.29 | 0.91 | 4.1  | -0.04| 3.83 | -1.83| 5.52 |
| 11        | 11     | 0.54 | 3.15 | 1.62 | 4.59 | 0.6  | 4.43 | -1.17| 4.42 |
| 12        | 12     | 1.82 | 4.96 | 1.69 | 6.49 | -1.08| 4.94 | 0.23 | 6.47 |
| 13        | 13     | 1.37 | 5.31 | 1.42 | 7.08 | -0.02| 6.64 | 0.03 | 5.53 |
| 14        | 14     | 0.72 | 4.79 | 2.31 | 6.3  | 0.76 | 4.7  | 0.92 | 6.21 |
| 15        | 15     | 2.46 | 6.62 | 2.51 | 6.88 | 1.11 | 5.18 | -0.27| 6.87 |
| 16        | 16     | 2.75 | 8.42 | 4.17 | 7.22 | 1.33 | 7.03 | 1.31 | 7.26 |
| 17        | 17     | 5.38 | 6.9  | 4.07 | 8.36 | 4.03 | 6.78 | 4.16 | 6.8  |
| 18        | 18     | 4.23 | 7.86 | 4.25 | 8.16 | 4.3  | 7.92 | 4.32 | 8.21 |
| 19        | 19     | 4.81 | 7.47 | 5.25 | 9.21 | 4.81 | 7.46 | 3.79 | 9.18 |

Then, the actual displacement value of deformation points in Table 1 are obtained by transforming the values of changed pixels based on time baseline parallax method. We draw the deformation curve (Figure 3) of deformation points to grasp the deformation trend of masonry structure in time. The dx35, dx36, dx37 and dx38 in Table 1 represent actual displacement value of U35, U36, U37 and U38 in X direction respectively. The dz35, dz36, dz37 and dz38 in Table 1 represent actual displacement value of U35, U36, U37 and U38 in Z direction respectively.

Figure 3 (a) shows that the deformation of deformation points increase gradually until masonry structure develops brittle failure as the vibration experiment is carrying out.

![Deformation trend of deformation points](image1)

![Position of deformation points](image2)

Figure 3. Deformation curves of deformation points

However, the deformation curves of U35, U36, U37 and U38 do not represent the partial failure of masonry structure. Their deformations are caused by diagonal crack 1 throughout the masonry structure with reference to Figure 4. The masonry structure on the right side of the crack moves towards the right. Thus, Figure 3 (b) is drawn to study the seismic resistance of masonry structure. As is shown in Figure...
3 (b), red circle represents initial position of deformation points and green cross stands for current position of deformation points after experiment. For clearly analyzing the failure pattern of masonry wall, we connect multiple points of each column into a line.

After analyzing Figure 3 (b), the west of masonry wall moves obviously towards the west, especially the west-middle of masonry wall. This place may develop crossing-cracks. What is more, the east-upper of masonry wall moves towards the east. Diagonal cracks may emerge along the diagonal of masonry wall. In addition, the deformation of the east-down masonry wall is more obvious than the east-upper. Thus, the east-down masonry wall may develop cracks. This appearance is consistent with what is shown in Figure 4.

As is shown in Figure 4, cross crack (Cross diagonal crack) occurs on the west of masonry wall. Diagonal cracks (Diagonal crack 1 and 2) emerge along the diagonal of masonry wall. These experiment phenomena are consistent with the conclusions obtained by analyzing deformation data.

5. Conclusion
Digital photography, converting the pixel change value into actual deformation value through matching the reference photograph and the subsequent photograph based on the reference points, can meet the deformation observation accuracy of masonry structure as it can grasp the 1/1000s dynamic deformation. It is characterized by a photography proportion coefficient of 1.43mm/pixel and relative accuracy is 3‰.

This paper uses kalman filter to eliminate the effect of random noise on data accuracy. The maximum deviation is 0.16 pixels and the rest is within 0.01 pixels. The relative accuracy is 2.13‰ after using kalman filter to eliminate random noise.

After analyzing the deformation on masonry wall, we find some failure patterns of masonry wall: Masonry wall may develop longitudinal through cracks on the side near the hypocenter. Diagonal cracks may occur along the diagonal of masonry wall. Crossing cracks occur on the side far from the hypocenter.

This study proves the potential of using digital photography to monitor dynamic deformation of masonry structure in vibration to study the seismic resistance of masonry structure. It also provides a technical basis to monitor the deformation of masonry structure in real-time. We can warn the safety based on the real-time deformation curve.

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