On the Regge Slopes Intramultiplet Relation

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Abstract

We show that only additivity of inverse Regge slopes is consistent with both the formal chiral limit \( m(n) \to 0 \) and the heavy quark limit \( M(Q) \gg M(n) \), where \( n = u, d, \) and \( m, M \) are current and constituent quark masses, respectively.

Key words: Regge phenomenology, chiral limit, heavy quark limit

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Introduction

It is well known that the hadrons composed of light \((u, d, s)\) quarks populate linear Regge trajectories; i.e., the square of the mass of a state with orbital momentum \( \ell \) is proportional to \( \ell = \alpha' M^2(\ell) + a(0) \), where the slope \( \alpha' \) depends weakly on the flavor content of the states lying on the corresponding trajectory,

\[
\alpha'_{nn} \simeq 0.88 \text{ GeV}^{-2}, \quad \alpha'_{sn} \simeq 0.84 \text{ GeV}^{-2}, \quad \alpha'_{ss} \simeq 0.80 \text{ GeV}^{-2}.
\]  

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In contrast, the data on the properties of Regge trajectories of hadrons containing heavy quarks are almost nonexistent at the present time, although it is established \[1\] that the slope of the trajectories decreases with increasing quark mass (as seen in \((1)\)) in the mass region of the lowest excitations. This is plausibly due to an increasing (with mass) contribution of the color Coulomb interaction, leading to a curvature of the trajectory near the ground state \[3\]. However, as the analyses \[1, 3, 4\] show, in the asymptotic regime of the highest excitations, the trajectories of both light and heavy quarkonia are linear and have the same slope \(\alpha' \simeq 0.9\ \text{GeV}^{-2}\), in agreement with natural expectations from the string model.

Knowledge of Regge trajectories in the scattering region, i.e., at \(t < 0\), and of the intercepts \(a(0)\) and slopes \(\alpha'\) is also useful for many non-spectral purposes, for example, in the recombination \[5\] and fragmentation \[6\] models. Therefore, as pointed out in ref. \[1\], the slopes and intercepts of the Regge trajectories are the fundamental constants of hadron dynamics, perhaps generally more important than the mass of any particular state. Thus, not only the derivation of mass relations \[1, 3, 4, 11\] but also the determination of the parameters \(a(0)\) and \(\alpha'\) of heavy quarkonia is of great importance, since they afford opportunities for better understanding of the dynamics of the strong interactions in the processes of production of charmed and beauty hadrons at high energies, and estimations of their production rates.

If one assumes the (quasi)-linear form of Regge trajectories for hadrons with identical \(J^{PC}\) quantum numbers (i.e., belonging to a common multiplet), one will obtain for the states with orbital momentum \(\ell\)

\[
\ell = \alpha'_{ii}^i m_i^2 + a_{ii}(0), \\
\ell = \alpha'_{jj}^j m_j^2 + a_{jj}(0), \\
\ell = \alpha'_{jj}^j m_j^2 + a_{jj}(0).
\]

Further, the following relation among the intercepts exists:

\[
a_{ii}(0) + a_{jj}(0) = 2a_{ji}(0). \tag{2}
\]

This relation was first derived for \(u(d)\)- and \(s\)-quarks in the dual-resonance model \[11\]. It is satisfied in two-dimensional QCD \[12\], the dual-analytic model \[13\], and the quark bremsstrahlung model \[14\]. Also, it saturates inequalities for Regge trajectories \[15\] which follow from the \(s\)-channel unitarity condition. Hence, it may be considered as firmly established and may transcend specific models.

With \(2\), one obtains from the above three relations, the quadratic mass formula

\[
\alpha'_{ii}^i m_i^2 + \alpha'_{jj}^j m_j^2 = 2\alpha'_{ji} m_{ji}^2. \tag{3}
\]

In contrast to the relation among the intercepts, Eq. \((2)\), a relation among the Regge slopes is not firmly established yet. Two such relations have been proposed in the literature,

\[
\alpha'_{ii} \cdot \alpha'_{jj} = \left(\alpha'_{ji}\right)^2, \tag{4}
\]

\]
which follows from the factorization of residues of the $t$-channel poles \[17, 18\], and

$$\frac{1}{\alpha_{ij}'} + \frac{1}{\alpha_{jj}'} = \frac{2}{\alpha_{ji}'}.$$  \hspace{1cm} (5)

based on topological expansion and the $q\bar{q}$-string picture of hadrons \[13\]. Also, alternative relations have been also proposed which do not agree with either (4) or (5); for example,\[\footnote{In the following, $m(q)$ and $M(q)$ stand for current and constituent masses of quark $q$, respectively.} \]

$$\alpha_{ji}' = \frac{\alpha'}{1 + 0.2 \left( \frac{M(i) + M(j)}{\text{GeV}} \right)^{3/2}},$$  \hspace{1cm} (6)

where $\alpha' \cong 0.88 \text{ GeV}^{-2}$ is the standard Regge slope in the light quark sector. This last was suggested by Filipponi and Srivastava \[19\], and implies that the relation between the slopes is

$$\left( \frac{1}{\alpha_{ii}'} - \frac{1}{\alpha'} \right)^{2/3} + \left( \frac{1}{\alpha_{jj}'} - \frac{1}{\alpha'} \right)^{2/3} = 2 \left( \frac{1}{\alpha_{ji}'} - \frac{1}{\alpha'} \right)^{2/3}.$$  \hspace{1cm} (7)

For light quarkonia (and small differences in the $\alpha'$ values), there is no essential difference between Eqs. (4) and (5); viz., for $\alpha_{ji}' = \alpha_{ii}'/(1 + 2x)$, whereas Eq. (4) gives $\alpha_{jj}' = \alpha_{ii}'/(1 + x)^2 \cong \alpha'/ (1 + 2x)$, i.e., essentially the same result to order $x^2$. Eq. (7), however, differs from (4),(5) already for small $x$: for $i = n$, it gives $\alpha_{jj}' = \alpha_{ii}'/(1 + 2^{3/2} x) \approx \alpha_{ii}'/(1 + 2.83x)$. For heavy quarkonia (and expected large differences from the $\alpha'$ values for the light quarkonia) Eqs. (4),(5) are incompatible; e.g., for $\alpha_{ji}' = \alpha_{ii}'/2$, Eq. (4) will give $\alpha_{jj}' = \alpha_{ii}'/4$, whereas Eq. (5) $\alpha_{jj}' = \alpha_{ii}'/3$. Eq. (7) gives, respectively, $\alpha_{jj}' \approx \alpha_{ii}'/3.83$ in this case. It would be therefore useful to establish whether any of these relations is realized in nature.

In a series of our previous publications \[7, 8, 9, 10\] we chose Eq. (5), since it is much more consistent with (3) than is Eq. (4), when tested by using measured quarkonia masses in Eq. (3). By eliminating the values of the Regge slopes from Eqs. (3),(5), we derived new (higher power) mass relations which hold with high accuracy for all well established meson multiplets, and may be reduced to quadratic formulas by fitting the values of the slopes.

Here we wish to compare Eqs. (4),(5), and (7) in both the chiral and heavy quark limits. We shall show that, although the three pairs of equations, (3),(4), (3),(5) and (3),(7), are consistent in the formal chiral limit $m(n) \to 0$ ($n = u$ or $d$, and we assume SU(2) flavor symmetry: $m(n) \equiv m(u) = m(d)$), only one pair of equations, (3),(5), is consistent in the heavy quark limit $M(Q) \gg M(n)$, thus unambiguously indicating its preferability. The same arguments indicate that any other relation among the slopes
which does not agree with (5) (e.g., Eq. (7)) is inconsistent with the heavy quark limit. This confirms the conclusion, drawn before on the basis of meson spectroscopy [7, 8, 9, 10], that it is Eq. (5) that is realized in the real world.

**Formal chiral limit** $m(n) \to 0$

To consider the formal chiral limit $m(n) \to 0$, we use the following parametrization of the meson masses in terms of the current quark mass, discussed in more detail in refs. [20, 21]:

$$m_{n\bar{n}}^2 = 2Am(n) + B,$$

where $A$ and $B$ are constants within a given meson multiplet, but may be different for different multiplets (note that $B > 0$ for non-Goldstone bosons). As is easily seen, the Regge recurrences of the state (8) have the masses

$$2Am(n) + B + \frac{1}{\alpha_{n\bar{n}}}, \ 2Am(n) + B + \frac{2}{\alpha_{n\bar{n}}}, \ etc.$$

Let us first introduce

$$x_q \equiv \frac{\alpha_{q\bar{q}}}{\alpha_{n\bar{n}}} \leq 1.$$  (9)  

Determining now $\alpha_{q\bar{q}}/\alpha_{n\bar{n}}$ from (4),(5), or (7), Eq. (3) may be cast into the form, respectively,

$$m_{n\bar{n}}^2 + x_q m_{qq}^2 = \sqrt{x_q} m_{q\bar{n}}^2$$  (10)  

with Eq. (4),

$$m_{n\bar{n}}^2 + x_q m_{qq}^2 = \frac{4x_q}{1 + x_q} m_{q\bar{n}}^2$$  (11)  

with Eq. (5), and

$$m_{n\bar{n}}^2 + x_q m_{qq}^2 = \frac{2}{1 + 2^{-3/2} \left(\frac{1}{x_q} - 1\right)} m_{q\bar{n}}^2$$  (12)  

with Eq. (7).

Consider, e.g., Eq. (11). As follows from (8),

$$m_{q\bar{n}}^2 = \frac{1 + x_q}{4} m_{qq}^2 + \frac{1 + x_q}{4x_q} (2Am(n) + B).$$  (13)  

In the formal limit $m(n) \to 0, m_{n\bar{n}}^2 \to B$, and the masses of its Regge recurrences go in this limit to

$$B + \frac{1}{\alpha_{n\bar{n}}}, \ B + \frac{2}{\alpha_{n\bar{n}}}, \ etc.$$
so that all these states again populate Regge trajectory with the initial slope $\alpha'_{nn\bar{n}}$. Thus, the parameter $x_q$ need not change in the formal limit $m(n) \to 0$, and, as follows from (13),

$$m^2_{q_n} \to \frac{1 + x_q}{4} m^2_{q_q} + \frac{1 + x_q}{4x_q} B$$

$$= \frac{1 + x_q}{4} m^2_{q_q} + \frac{1 + x_q}{4x_q} m^2_{n\bar{n}}, \quad (14)$$

which is equivalent to (11). We conclude, therefore, that Eq. (11) holds in the formal chiral limit $m(n) \to 0$. Following a similar procedure, it may be shown that both Eqs. (10) and (12) also hold in this limit. Thus, this limit does not distinguish between the three possible flavor dependent Regge slope relations.

**Heavy quark limit $M(Q) \gg M(n)$**

Consider now the heavy quark limit $M(Q) \gg M(n)$, and start with Eqs. (3),(5). Since the slope decreases with the increasing quark mass, one expects $\alpha_{Q\bar{Q}}', \alpha_{n\bar{n}}' \ll \alpha_{n\bar{n}}$, so that one may neglect $1/\alpha_{n\bar{n}}'$ in comparison with $1/\alpha_{Q\bar{Q}}', 1/\alpha_{Q\bar{Q}}'$ in Eq. (5); it therefore takes on the form

$$\alpha_{n\bar{n}}' \simeq 2\alpha_{Q\bar{Q}}'. \quad (15)$$

Also, the term $\alpha_{n\bar{n}}' m^2_{n\bar{n}}$ is negligible in comparison with $\alpha_{Q\bar{Q}}' m^2_{Q\bar{Q}}, \alpha_{n\bar{n}}' m^2_{Q\bar{Q}}$ in Eq. (3) in this limit, for all three cases we are discussing. Indeed, in any of these cases, in the heavy quark limit $M(Q) \gg M(n)$, $\alpha'$ decreases like $\sim 1/(M(Q))^a$, $0 < a < 2$. This follows from, e.g., Eq. (6), and Eqs. (17),(18) below in the remaining two cases. Therefore, since both $m^2_{Q\bar{Q}}$ and $m^2_{Q\bar{Q}}$ grow like $\sim (M(Q))^2$, it is clear that, as $M(Q) \gg M(n)$,

$$\alpha_{n\bar{n}}' m^2_{n\bar{n}} \ll \alpha_{Q\bar{Q}}' m^2_{Q\bar{Q}}, \alpha_{Q\bar{Q}}' m^2_{Q\bar{Q}}. \quad (16)$$

We now show that, as $M(Q) \gg M(n)$, $\alpha_{Q\bar{Q}}'$ decreases like $\sim 1/(M(Q))^a$, $0 < a < 2$.

For Eq. (5), this follows from the following form of the parametrization of the dependence of the slope on the quark masses, consistent with (5), which will be discussed in more detail elsewhere:

$$\alpha_{ji}' = \frac{4}{\pi} \frac{\alpha'}{1 + \sqrt{\frac{M(i) + M(j)}{2}}} \quad (17)$$

where $\alpha' = \alpha_{n\bar{n}}' \approx 0.88 \text{ GeV}^{-2}$ is the standard Regge slope in the light quark sector. In Table I we present the numerical values of the parameter $x_q$, as defined in (9), $(q = n, s, c, b)$ given by both Eq. (11) in which the measured vector and tensor

\[\text{Also, } \alpha'_{n\bar{n}} m^2_{n\bar{n}} \text{ is fixed at } \approx O(1 \text{ GeV}^2).\]
meson masses are used, and Eq. (17). One sees that the formula (17) is in excellent agreement with experiment.

|   | q | n  | s  | c  | b  |
|---|---|----|----|----|----|
| Eq. (11) | 1  | 0.89 ± 0.02 | 0.50 ± 0.01 | 0.23 ± 0.01 |
| Eq. (16) | 1.001 | 0.889 | 0.499 | 0.231 |

Table I. Comparison of the numerical values of the parameter $x_q$, $q = n, s, c, b$, given by Eq. (11) in which the measured vector and tensor meson masses are used, and Eq. (17) in which the following constituent quark masses are used (in GeV) (as extracted from $S$-wave meson spectroscopy): $M(n) = 0.29$, $M(s) = 0.46$, $M(c) = 1.65$, $M(b) = 4.80$.

For Eq. (4), a search for a similar form of the parametrization of the slope dependence on the quark mass, which would be consistent with both Eq. (4) and data, leads to

$$\alpha_{ji}' = \frac{C}{[M(i)M(j)]^{a/2}}, \quad a \approx 0.32, \quad C \approx 0.59 \text{ GeV}^{1.68},$$

and therefore, $\alpha_{jj}'/\alpha_{ii}' = (M(i)/M(j))^{0.32}$. In Table II we present the numerical values of the parameter $x_q$, as defined in (9), $(q = n, s, c, b)$ given by both Eq. (10) in which again the measured vector and tensor meson masses are used, and Eq. (18). One sees that the formula (18) is in excellent agreement with experiment, as well as Eq. (17).

|   | q | n  | s  | c  | b  |
|---|---|----|----|----|----|
| Eq. (10) | 1  | 0.87 ± 0.02 | 0.59 ± 0.01 | 0.39 ± 0.01 |
| Eq. (18) | 0.996 | 0.860 | 0.571 | 0.406 |

Table I. The same as in Table I, for Eqs. (10),(18).

Although the Regge slope dependence on the quark masses (18), $\alpha_{ii}' \approx \frac{M(i)/M(j)^{0.6}}{[M(i)M(j)]^{1/2}}$, represents certain academic interest, it is not realized in the real world, as well as Eq. (6), as we show below, nor it is well defined in the limit $M(q) \to 0$, in contrast to its counterparts (6) and (17).

With (16), Eq. (3) reduces to

$$m_{QQ}^2 \approx 2m_{Qn}^2m_{Qn}^2,$$

It then follows from (15),(19) that, independent of the numerical values of the slopes,

$$m_{QQ} \approx 2m_{Qn},$$
in agreement with the heavy quark limit \( M(Q) \gg M(n) \). Hence, the pair of equations (3),(5) is consistent in this limit.

A similar procedure for the remaining two pairs of relations, (3),(4) and (3),(7), leads, respectively, to

\[
m_{Q\bar{Q}} \simeq \sqrt{\frac{2\alpha'_{n\bar{n}}}{\alpha'_{Qn}}} m_{Qn},
\]

(21) and

\[
m_{Q\bar{Q}} \simeq 2^{5/4} m_{Qn}.
\]

(22)

Eq. (21) is obtained by neglecting the term \( \alpha'_{n\bar{n}} m_{n\bar{n}}^2 \) in (3), while Eq. (22) is obtained by neglecting both this term in (3) and \( 1/\alpha'_{n\bar{n}} \) in (7). It is seen that Eq. (22) is in clear contradiction with the heavy quark limit, as given by (20). Requiring Eq. (21) to be consistent with (20) leads to the following constraint on the slopes in the heavy quark limit:

\[
\alpha'_{QQ} \rightarrow \frac{\alpha'_{n\bar{n}}}{4}, \quad \alpha'_{Qn} \rightarrow \frac{\alpha'_{n\bar{n}}}{2}.
\]

(23)

Since the pair of equations (3),(7) fails already in the heavy quark limit for mesons, we shall not examine it any further. We shall, however, examine the two remaining pairs in the heavy quark limit for baryons, in order to determine further which is preferable, since Eq. (21) may still be consistent with the heavy quark limit, provided the validity of Eq. (23). Although the slopes of heavy quark trajectories, as extracted from data for this case, do not contradict (23), as seen in Table II, we shall show below that the generalization of Eq. (4) to baryons does contradict the corresponding heavy quark limit.

**Generalization to baryons**

The above analysis may be easily generalized to baryons. In this case, one has two pairs of relations [8], which represent the counterparts of Eq. (3),

\[
\alpha'_{nnn} m_{nnn}^2 + \alpha'_{QQn} m_{QQn}^2 = 2\alpha'_{Qnn} m_{Qnn}^2,
\]

(24)

\[
\alpha'_{Qnn} m_{Qnn}^2 + \alpha'_{QQQ} m_{QQQ}^2 = 2\alpha'_{QQn} m_{QQn}^2,
\]

(25)

and Eqs. (4) and (5), respectively:

\[
\alpha'_{nnn} \cdot \alpha'_{QQn} = \left(\alpha'_{Qnn}\right)^2,
\]

(26)

\[
\alpha'_{Qnn} \cdot \alpha'_{QQQ} = \left(\alpha'_{QQn}\right)^2,
\]

(27)

\[
\frac{1}{\alpha'_{nnn}} + \frac{1}{\alpha'_{QQn}} = \frac{2}{\alpha'_{Qnn}},
\]

(28)
\[
\frac{1}{\alpha'_{Qn}} + \frac{1}{\alpha'_{QQ}} = \frac{2}{\alpha'_{QQn}}.
\]

(29)

We shall now show that only these counterparts of Eq. (5) hold in the heavy quark limit for baryons, not those of Eq. (4).

Consider first Eqs. (24),(28). In the heavy quark limit \(M(Q) \gg M(n)\), by virtue of the arguments given above in the meson case, it is possible to neglect the terms \(\alpha'_{nnn}m_{nnn}^2\) and \(1/\alpha'_{nnn}\) in Eqs. (24) and (28), respectively, in comparison with the remaining terms. Therefore, these equations reduce in this limit, respectively, to

\[
\alpha'_{QQn}m_{QQn}^2 \simeq 2\alpha'_{Qnn}m_{Qnn}^2,
\]

(30)

\[
\alpha'_{Qnn} \simeq 2\alpha'_{QQn},
\]

(31)

so that, independent of the values of the slopes,

\[
m_{QQn} \simeq 2m_{Qnn}.
\]

(32)

Consider now Eqs. (25),(29). By expressing \(\alpha'_{nnQ}m_{nnQ}^2\) and \(1/\alpha'_{nnQ}\) from Eqs. (24) and (28), and using further in (25),(29), one obtains, respectively,

\[
\frac{1}{2}\alpha'_{nnn}m_{nnn}^2 + \alpha'_{QQQ}m_{QQQ}^2 = \frac{3}{2}\alpha'_{QQn}m_{QQn}^2,
\]

(33)

\[
\frac{1}{2\alpha'_{nnn}} + \frac{1}{\alpha'_{QQQ}} = \frac{3}{2\alpha'_{QQn}}.
\]

(34)

Neglecting again the terms containing \(\alpha'_{nnn}\) in these relations, one obtains, independent of the values of the slopes,

\[
m_{QQQ} \simeq \frac{3}{2}m_{QQn}.
\]

(35)

Eqs. (32),(35) imply

\[
m_{QQQ} \simeq \frac{3}{2}m_{QQn} \simeq 3m_{Qnn},
\]

(36)

in agreement with the heavy quark limit \(M(Q) \gg M(n)\).

Now we apply a similar procedure to Eqs. (24)-(27). First, analogously to the meson case, one obtains from (24),(26) in the heavy quark limit for baryons:

\[
\alpha'_{QQn} = \frac{\alpha'_{nnn}}{4}, \quad \alpha'_{Qnn} = \frac{\alpha'_{nnn}}{2}.
\]

(37)

The use of these values of the slopes in Eq. (27) leads to

\[
\alpha'_{QQQ} = \frac{\alpha'_{nnn}}{8}.
\]

(38)
in this limit. Further use of $\alpha'_{QQn}$ and $\alpha'_{QQQ}$ in Eq. (33), which is also valid in this case, and in which we again neglect the term $\alpha'_{nnn}m_{nnn}^2$, results in

$$m_{QQQ} \simeq \sqrt{3}m_{QQn},$$

(39)
in contradiction with the heavy quark limit, as given by (35). Thus, the four equations (24)-(27) do not hold in the heavy quark limit for baryons, and therefore, only additivity of inverse Regge slopes, (5) and (28),(29), is consistent with the heavy quark limit for both mesons and baryons.

By using a parametrization of the baryon masses in terms of the current quark mass which is similar to (8) [21], and repeating the arguments given above in the meson case, one can confirm that the equations (24),(25),(28),(29) also hold in the formal chiral limit $m(n) \to 0$.

Concluding remarks

We close with a brief summary of our results.

We have shown that only additivity of inverse Regge slopes, (5) and (28),(29), is consistent with the formal chiral, $m(n) \to 0$, and heavy quark, $M(Q) > M(n)$, limits for both mesons and baryons. Alternative relations among the slopes, (4),(7) and their baryon counterparts, although consistent in the formal chiral limit, fail in the heavy quark limit.

We note, however, that empirical evidence for unambiguous preference of additivity of the inverse slopes is weak at present. Indeed, in the heavy quark limit for mesons, Eq. (23) gives $\alpha'_{QQ} = \alpha'/4$, and the slope of $b\bar{b}$ trajectory, as seen in Table I, is $\alpha'_{b\bar{b}} = \alpha'/4.32$. Eq. (22) in the same limit gives $m_{QQ} \simeq 2^{5/4}m_{Q\bar{Q}} \approx 2.378m_{Q\bar{Q}}$, in contrast to $m_{QQ} \simeq 2m_{Q\bar{Q}}$ in this limit. Also, in the heavy quark limit for baryons, Eq. (39) gives $m_{QQQ} \simeq \sqrt{3}m_{QQn} \approx 1.732m_{QQn}$, in contrast to $m_{QQQ} \simeq 1.5m_{QQn}$ in this limit. In any of the above three cases, the relative error does not exceed $\sim 15\%$ (this is the same accuracy as for mass relations derived on the basis of Eqs. (3),(4) [8]). This situation is quite similar to the case of replacing an ultrarelativistic theory with $\langle \mathbf{p}^2 \rangle/m^2 \gg 1$ by a medium relativistic one with $\langle \mathbf{p}^2 \rangle/M^2 \approx 1$, while the following remains valid: Even the lowest-order $1/M^2$ expansion is still legitimate with accuracy of the same order as above, $(1 + \langle \mathbf{p}^2 \rangle/M^2)^{1/2} \approx 1 + \langle \mathbf{p}^2 \rangle/(2M^2)$, or $1.41 \approx 1.5$, with $\langle \mathbf{p}^2 \rangle \sim M$. This means an error of $\sim 6\%$ in the total energy and $\sim 20\%$ in the kinetic energy; not excellent but sufficient for many purposes in hadronic physics, and, in particular [24], for hadron spectroscopy. That is why even alternative relations for Regge slopes lead to predictions for hadron masses which do not badly disagree with experiment, especially in the light quark sector where the forms of these relations are almost indistinguishable from each other.
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