Estimation model of life insurance claims risk for cancer patients by using Bayesian method

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Abstract. This paper discussed the estimation model of the risk of life insurance claims for cancer patients using Bayesian method. To estimate the risk of the claim, the insurance participant data is grouped into two: the number of policies issued and the number of claims incurred. Model estimation is done using a Bayesian approach method. Further, the estimator model was used to estimate the risk value of life insurance claims each age group for each sex. The estimation results indicate that a large risk premium for insured males aged less than 30 years is 0.85; for ages 30 to 40 years is 3.58; for ages 41 to 50 years is 1.71; for ages 51 to 60 years is 2.96; and for those aged over 60 years is 7.82. Meanwhile, for insured women aged less than 30 years was 0.56; for ages 30 to 40 years is 3.21; for ages 41 to 50 years is 0.65; for ages 51 to 60 years is 3.12; and for those aged over 60 years is 9.99. This study is useful in determining the risk premium in homogeneous groups based on gender and age.

1. Introduction
Lately, people with critical illnesses are expanding. One of them cancer. Ironically, health care costs have also increased rapidly over the past few decades. Things need to be aware, unforeseen events can occur in cancer patients, resulting in cancer patients financially are in need of protection. Therefore, the role of insurance companies is very necessary [2], [8].

Critical illness insurance is devoted to protecting customers' patients with critical illnesses, such as cancer, kidney failure, and heart. Critical illness insurance is different with health insurance [9], [3]. Critical illness insurance provides a number of cash when the customer has been diagnosed with a critical illness [2], [5].

In insurance, the insurer (insurance company), provides financial assistance in the form of insurance money that is called a risk premium to the insured (customer). Determining the risk premiums be reckoned with that the company did not experience a loss [10]. With the increasing number of cancer patients, and the cost of cancer treatment is higher, the number of claims from year to year increasing [12]. This is a problem for the insurer in estimating future claims trends to determine the risk premium. Therefore, in this paper do research on the risk of claims, particularly in cancer patients.

2. Methodology
2.1. Bayesian methods
The person who filed a claim \((X)\) distributed Binomial \((n, \theta_B)\) where \(\theta_B\) is a probability of occurrence \(X\). The values \(\theta_B\) are estimated using Bayesian methods. Maximum likelihood estimation of the random variable \(X\) can be formulated with
\[
\text{est}\, \theta = \frac{x}{n}
\]
Then Bayes’ theorem, the posterior distribution of the value obtained as follows.
\[
f_{\theta_B}(\theta | x) \propto f(x | \theta_B) f(\theta_B)
\]
The posterior distribution contains all the information about \( \theta_B \) that can be used for Bayesian estimation of the parameters \( \theta_B \) the binomial distribution [1]. To estimate the value \( \theta_B \) of a single observation \( X \) (the person filing the claim) with prior distribution \( \theta_B \) the binomial distribution, turned into the beta distribution with parameters \( \alpha \) and \( \beta \), we can examine the form of the posterior distribution of \( \theta_B \) [1]:
\[
f(\theta_B | x) \propto \theta_B^x (1 - \theta_B)^{n-x} \theta_B^{\alpha-1} (1 - \theta_B)^{\beta-1} = \theta_B^{\alpha+x-1} (1 - \theta_B)^{\beta+n-x-1}
\]
The value of \( \theta_B \) given by a data sample \( x = (x_1, x_2, ..., x_n) \) has a loss function \( g(x) \), which minimizes the loss estimate by observing its posterior distribution. The loss function commonly used is the quadratic loss function is defined as follows [11]:
\[
L(g(x); \theta) = [g(x) - \theta]^2
\]
By minimizing the quadratic loss function, \( \theta_B \) value can be expressed as the average (mean) of the posterior distribution as follows:
\[
\theta_B = \frac{\alpha + x}{(\alpha + x) + (\beta + n - x)} = \frac{\alpha + x}{\alpha + \beta + n}
\]
Using the theorem credibility, we can declare the value of Bayesian estimation \( \theta_B \) in the form:
\[
\theta_B = Z\theta + (1-Z)\mu = Z\frac{x}{n} + (1-Z)\mu
\]
where credibility factor \( Z \) is as follows:
\[
Z = \frac{n}{\alpha + \beta + n}
\]
and \( \mu \) is the average of the prior distribution of beta distributions declared by
\[
\mu = \frac{\alpha}{\alpha + \beta}
\]
In practice, there are some situations where a prior value is unknown. So, we use the value of non-informative priors. For example, if \( \theta_B \) is the opportunity of a binomial distribution, and \( \theta_B \) not have any information about the prior distribution, the distribution of which had been distributed prior to the hose Uniform (0,1) would seem appropriate. In this case, the prior distribution is the beta distribution with parameter \( \alpha = 1 \) and \( \beta = 1 \). However, the interval that we take not the interval (0,1), but the interval is more realistic. Set the interval \( (\theta_{\text{min}}, \theta_{\text{max}}) \) to get a good estimate. We denote the value of \( s \) as the average of the prior distribution of beta which is the centre of this interval [1], [11].
\[
s = \frac{x_{\text{min}} + x_{\text{max}}}{2}
\]
We mark as \( \theta_0 \) the more distant boundary from the value of 0.5 of the interval \( (\theta_{\text{min}}, \theta_{\text{max}}) \). Calculate the error \( h_B \) as follows:
\[
h_B = |\theta_0 - s|
\]
and calculate the number of claim \( q \) by using formula as follows:
\[
q = \frac{2p\theta_0(1-\theta_0)}{ph_B^2 - \theta_0(1-\theta_0)}
\]
where \( p \) is number of insured in the insurance company. Then, we estimate the value of the parameter \( \alpha \) and \( \beta \) of beta prior distributions as follows:
\[ \alpha = qs \]  
\[ \beta = q - qs \]  

2.2. Individual Risk Model

To determine the risk premium in homogeneous groups according to age and gender, need to be estimated that many insurance policy issued in the following year by extrapolating the trend of the time series using Statistics program Statgraphics Centurion. To select the most appropriate trend function, we use the procedure Comparison of Alternative Models [9], [7]. As for estimating the value of \( n \) in the following period (in 2009), we use the procedure Forecast. In this research, there are two models of risk, namely the risk model of collective and individual risk models. In the collective risk model, we let \( g_1 < 5\% \), \( g_2 < 7\% \), \( g_3 < 15\% \) is a random variable that the variable determining the amount of the claim. The total of the amount of the claim, denoted by [4]:

\[ S = X_1 + X_2 + X_3 + \ldots + X_N \]  

While the individual risk model, the total of the amount of the claim can be denoted as \( Y_j \). So we can write as follows:

\[ S_n = Y_1 + Y_2 + Y_3 + \ldots + Y_n \]  

where \( Y_j \) indicates the number of claims for individual year \( j \), and \( n \) indicates the period of observation. However, it is possible some risks will not give rise to a claim. Therefore, the value \( Y_j \) may be 0. It will be given two assumptions, namely:

- The number of claims in the year to \( j \) is 0 or 1.
- Possible claims in the year to \( j \) is \( g_1 < 7\% \) \( g_3 < 15\% \).

Based on the above assumptions, \( N_j \sim Bi(1; q_j) \), thus the distribution of \( Y_j \) is compound binomial with individual claims are denoted \( X_j \). Then we can write it as follows:

\[ E(Y_j) = q_j \mu_j \]  
\[ D(Y_j) = q_j (\sigma_j^2 + \mu_j^2) - q_j^2 \mu_j^2 = q_j \sigma_j^2 + q_j (1 - q_j) \mu_j^2 \]  

Where \( \mu_j \) and \( \sigma_j^2 \) is the average and variance of \( X_j \). Then, the average and variance of \( S_n \) are:

\[ E(S_n) = E \left( \sum_{j=1}^{n} Y_j \right) = \sum_{j=1}^{n} E(Y_j) = \sum_{j=1}^{n} q_j \mu_j \]  
\[ D(S_n) = D \left( \sum_{j=1}^{n} Y_j \right) = \sum_{j=1}^{n} D(Y_j) = \sum_{j=1}^{n} \left[ q_j \sigma_j^2 + q_j (1 - q_j) \mu_j^2 \right] \]  

In special cases, when \( Y_j, j = 1, 2, \ldots, n \) is a composite of several distribution and is a random variable. Based on the central limit theorem we can approach \( S_n \) distribution by the normal distribution. Therefore, in this case, we assign the value of the risk premium equal to 95% of the normal distribution with parameters \( \mu = E(S_n), \sigma^2 = D(S_n) \). Large risk premium (RP) can be calculated with the calculation below [6], [4]:

\[ P(S_n \leq RP) = 95\% \]  

\[ P \left( \frac{S_n - E(S_n)}{\sigma S_n} \leq \frac{RP - E(S_n)}{\sigma S_n} \right) = 0.95 \]  
\[ P \left( Z \leq \frac{RP - E(S_n)}{\sigma S_n} \right) = 0.95 \]  

From the standard normal distribution table, it is obtained that
1.96    1.96
\[ \frac{RP - E(S_n)}{\sigma_{S_n}} = 1.96 \quad \Rightarrow \quad RP = \left(1.96 \left(\sigma_{S_n}\right)\right) + E(S_n) \]
\[ RP = \left(1.96 \left(\sqrt{D(S_n)}\right)\right) + E(S_n) \quad \Rightarrow \quad \frac{RP}{\text{person}} = \left(1.96 \left(\sqrt{D(S_n)}\right)\right) + E(S_n) \]

3. Results and Discussion

The data used is in the form of simulation data in the form of insured claims data that have been diagnosed with a critical illness at an insurance company for ten years, from 2005 to 2014.

- For insured men aged less than 30 years

| year | n  | x   | \( \frac{x}{n} \) | \( x \) | \( \alpha \) | \( \beta \) | \( \theta_B \) | Z   | %Z |
|------|----|-----|-----------------|-------|--------|--------|----------|-----|----|
| 2005 | 67 | 1   | 0.015           | 0.041 | 8.124  | 0.005  | 0.891    | 89% |
| 2006 | 108| 4   | 0.037           | 1.041 | 74.124 | 0.014  | 0.590    | 59% |
| 2007 | 123| 2   | 0.016           | 5.041 | 178.124| 0.028  | 0.402    | 40% |
| 2008 | 384| 6   | 0.016           | 7.041 | 299.124| 0.023  | 0.556    | 56% |
| 2009 | 688| 12  | 0.017           | 13.041| 677.124| 0.019  | 0.499    | 50% |
| 2010 | 985| 10  | 0.010           | 25.041| 1,353.124| 0.018 | 0.417    | 42% |
| 2011 | 997| 9   | 0.009           | 35.041| 2,328.124| 0.015 | 0.297    | 30% |
| 2012 | 1456| 12 | 0.008           | 44.041| 3,316.124| 0.013 | 0.302    | 30% |
| 2013 | 1878| 18 | 0.010           | 56.041| 4,760.124| 0.012 | 0.281    | 28% |
| 2014 | 2146| 12 | 0.006           | 74.041| 6,620.124| 0.011 | 0.243    | 24% |
| 2015 | 86 | 1   | 0.015           | 8.124 | 0.005  | 0.891  | 89% |

Based on the percentage of the value of Z obtained in Table 1, it can be concluded that the influence of information in previous years to take into account the value of the insured \( \theta_B \) men aged less than 30 years, namely in 2005, 2006, 2008, and 2009 are very large, in 2007 and in 2010 is large enough, whereas in 2011 until 2014 small.

- For insured women aged less than 30 years

| year | n  | x   | \( \frac{x}{n} \) | \( x \) | \( \alpha \) | \( \beta \) | \( \theta_B \) | Z   | %Z |
|------|----|-----|-----------------|-------|--------|--------|----------|-----|----|
| 2005 | 55 | 1   | 0.018           | 0.041 | 8.124  | 0.005  | 0.871    | 87% |
| 2006 | 89 | 2   | 0.022           | 1.041 | 62.124 | 0.016  | 0.585    | 58% |
| 2007 | 94 | 6   | 0.064           | 3.041 | 149.124| 0.020  | 0.382    | 38% |
| 2008 | 222| 4   | 0.018           | 9.041 | 237.124| 0.037  | 0.474    | 47% |
| 2009 | 587| 2   | 0.003           | 13.041| 455.124| 0.028  | 0.556    | 56% |
| 2010 | 878| 6   | 0.007           | 15.041| 1,040.124| 0.014 | 0.454    | 45% |
| 2011 | 984| 4   | 0.004           | 21.041| 1,912.124| 0.011 | 0.337    | 34% |
| 2012 | 1165| 6 | 0.005           | 25.041| 2,892.124| 0.009 | 0.285    | 29% |
| 2013 | 1456| 12 | 0.008           | 31.041| 4,051.124| 0.008 | 0.263    | 26% |
| 2014 | 1987| 18 | 0.009           | 43.041| 5,495.124| 0.008 | 0.264    | 26% |
| 2015 | 61 | 1   | 0.015           | 8.124 | 0.005  | 0.891  | 89% |

Based on the percentage of the value of Z obtained in Table 2, it can be concluded that the influence of information in previous years to take into account the value of the insured \( \theta_B \) women aged less than 30 years, namely in 2005, 2006 and 2009 was very strong, while in 2007, 2008, 2010 and 2011 is quite large, but in the year 2012 to 2014 is small.
For insured men aged 30 years to 40 years

Table 3. Calculation of Bayesian estimation for men aged 30 years to 40 years

| Year | n   | x   | \(\frac{x}{n}\) | \(\alpha\) | \(\beta\) | \(\theta_B\) | Z   | \(\%Z\) |
|------|-----|-----|-----------------|----------|--------|-----------|-----|--------|
| 2005 | 180 | 6   | 0.033           | 0.041    | 8.124  | 0.005     | 0.957| 96%    |
| 2006 | 354 | 4   | 0.011           | 6.041    | 182.124| 0.032     | 0.653| 65%    |
| 2007 | 487 | 9   | 0.018           | 10.041   | 532.124| 0.019     | 0.473| 47%    |
| 2008 | 555 | 15  | 0.027           | 19.041   | 1,010.124| 0.019   | 0.350| 35%    |
| 2009 | 687 | 18  | 0.026           | 34.041   | 1,550.124| 0.021   | 0.302| 30%    |
| 2010 | 987 | 26  | 0.026           | 52.041   | 2,219.124| 0.023   | 0.303| 30%    |
| 2011 | 1165| 34  | 0.029           | 78.041   | 3,180.124| 0.024   | 0.263| 26%    |
| 2012 | 1548| 50  | 0.032           | 112.041  | 4,311.124| 0.025   | 0.259| 26%    |
| 2013 | 1856| 49  | 0.026           | 211.041  | 5,809.124| 0.027   | 0.237| 24%    |
| 2014 | 1951| 55  | 0.028           | 266.041  | 9,512.124| 0.027   | 0.200| 20%    |

Based on the percentage of the value of Z obtained in Table 3, it can be concluded that the influence of information in previous years to take into account the value of the insured men aged 30 years to 50 years ie in 2005 and 2006 are very large, in 2007 and in 2008 is large enough, whereas in the year 2009 to 2014, small.

For insured women aged 30 years to 40 years

Table 4. Calculation of Bayesian estimation for women aged 30 years to 40 years

| Year | n   | x   | \(\frac{x}{n}\) | \(\alpha\) | \(\beta\) | \(\theta_B\) | Z   | \(\%Z\) |
|------|-----|-----|-----------------|----------|--------|-----------|-----|--------|
| 2005 | 165 | 2   | 0.012           | 0.041    | 8.124  | 0.005     | 0.953| 95%    |
| 2006 | 289 | 3   | 0.010           | 2.041    | 171.124| 0.012     | 0.625| 63%    |
| 2007 | 348 | 6   | 0.017           | 5.041    | 457.124| 0.011     | 0.430| 43%    |
| 2008 | 498 | 14  | 0.028           | 11.041   | 799.124| 0.014     | 0.381| 38%    |
| 2009 | 512 | 16  | 0.031           | 25.041   | 1,283.124| 0.019   | 0.281| 28%    |
| 2010 | 594 | 22  | 0.037           | 41.041   | 1,779.124| 0.023   | 0.246| 25%    |
| 2011 | 789 | 31  | 0.039           | 63.041   | 2,351.124| 0.026   | 0.246| 25%    |
| 2012 | 987 | 46  | 0.047           | 94.041   | 4,050.124| 0.029   | 0.236| 24%    |
| 2013 | 1254| 43  | 0.034           | 140.041  | 5,261.124| 0.034   | 0.227| 23%    |
| 2014 | 1789| 49  | 0.027           | 232.041  | 7,001.124| 0.032   | 0.204| 20%    |

Based on the percentage of the value of Z obtained in Table 4, it can be concluded that the influence of information in previous years to take into account the value of the insured women aged 30 years to 50 years ie in 2005 and 2006 are very large, in the year 2007 and the year 2008 is large enough, whereas in the year 2009 to 2014, small.

For insured men aged 40 years to 50 years

Table 5. Calculation of Bayesian estimation for men aged 40 years to 50 years

| Year | n   | x   | \(\frac{x}{n}\) | \(\alpha\) | \(\beta\) | \(\theta_B\) | Z   | \(\%Z\) |
|------|-----|-----|-----------------|----------|--------|-----------|-----|--------|
| 2005 | 185 | 4   | 0.022           | 0.041    | 8.124  | 0.005     | 0.958| 96%    |
| 2006 | 354 | 6   | 0.017           | 4.041    | 189.124| 0.021     | 0.647| 65%    |
| 2007 | 439 | 9   | 0.021           | 10.041   | 537.124| 0.018     | 0.445| 45%    |
| 2008 | 654 | 9   | 0.014           | 19.041   | 967.074| 0.019     | 0.399| 40%    |
| 2009 | 945 | 12  | 0.013           | 28.041   | 1,612.074| 0.017   | 0.366| 37%    |
| 2010 | 1145| 18  | 0.016           | 40.041   | 2,545.074| 0.015   | 0.307| 31%    |
| 2011 | 1265| 21  | 0.017           | 58.041   | 3,672.074| 0.016   | 0.253| 25%    |
| 2012 | 1548| 24  | 0.016           | 79.041   | 4,916.074| 0.016   | 0.237| 24%    |
| 2013 | 1856| 27  | 0.015           | 103.041  | 6,440.074| 0.016   | 0.221| 22%    |
| 2014 | 2156| 36  | 0.017           | 130.041  | 8,269.074| 0.015   | 0.204| 20%    |
| 2015 | 1664| 72  | 0.016           | 266.041  | 10,389.074| 0.016  | 0.178| 18%    |

Based on the percentage of the value of Z obtained in Table 5, it can be concluded that the influence of information in previous years to take into account the value of the insured \(\theta_B\) men
aged 40 years to 50 years ie in 2005 and 2006 are very large, in 2007 until the year 2010 is quite large, whereas in 2011 until 2014, small.

- For insured women aged 40 to 50 years

Table 6. Calculation of Bayesian estimation for women aged over 50 years

| year | n  | x   | x/n | α   | β   | θ_B | Z   | %Z  |
|------|----|-----|-----|-----|-----|-----|-----|-----|
| 2005 | 176| 1   | 0.006| 0.041| 8.124| 0.005| 0.956| 96% |
| 2006 | 326| 1   | 0.003| 1.041| 183.124| 0.006| 0.639| 64% |
| 2007 | 399| 2   | 0.005| 2.041| 508.124| 0.004| 0.439| 44% |
| 2008 | 587| 9   | 0.015| 4.041| 905.124| 0.004| 0.392| 39% |
| 2009 | 878| 12  | 0.014| 13.041| 1,483.124| 0.009| 0.370| 37% |
| 2010| 1059| 6  | 0.006| 25.041| 2,349.124| 0.011| 0.308| 31% |
| 2011| 1159| 16 | 0.014| 31.041| 3,402.124| 0.009| 0.252| 25% |
| 2012| 1478| 21 | 0.014| 47.041| 4,545.124| 0.010| 0.243| 24% |
| 2013| 1784| 30 | 0.017| 68.041| 6,002.124| 0.011| 0.227| 23% |
| 2014| 2111| 27 | 0.013| 98.041| 7,756.124| 0.012| 0.212| 21% |
| 2015| 125.041| 9,840.124| 0.013|  |  |

Based on the percentage of the value of Z obtained in Table 6, it can be concluded that the influence of information in previous years to take into account the value of the insured θ_B on women aged 40 to 50 years, namely in 2005 and 2006 are very large, in the year 2007 to the year 2010 was quite large, whereas in 2011 until 2014, small.

- For insured men aged 50 years to 60 years

Table 7. Calculation of Bayesian estimation for men aged 50 years to 60 years

| year | n  | x   | x/n | α   | β   | θ_B | Z   | %Z  |
|------|----|-----|-----|-----|-----|-----|-----|-----|
| 2005 | 165| 6   | 0.036| 0.041| 8.124| 0.005| 0.953| 95% |
| 2006 | 354| 4   | 0.011| 6.041| 167.124| 0.035| 0.672| 67% |
| 2007 | 477| 9   | 0.019| 10.041| 517.124| 0.019| 0.475| 48% |
| 2008 | 658| 15  | 0.023| 19.041| 985.124| 0.019| 0.396| 40% |
| 2009 | 978| 18  | 0.018| 34.041| 1,628.124| 0.020| 0.370| 37% |
| 2010| 1523| 26 | 0.017| 52.041| 2,588.124| 0.020| 0.366| 37% |
| 2011| 1648| 34 | 0.021| 78.041| 4,085.124| 0.019| 0.284| 28% |
| 2012| 1853| 50 | 0.027| 112.041| 5,699.124| 0.019| 0.242| 24% |
| 2013| 2157| 49 | 0.023| 162.041| 7,502.124| 0.021| 0.220| 22% |
| 2014| 3245| 55 | 0.017| 211.041| 9,610.124| 0.021| 0.248| 25% |
| 2015| 266.041| 12,800.124| 0.020|  |  |

Based on the percentage of the value of Z obtained in Table 7, it can be concluded that the influence of information in previous years to take into account the value of the insured θ_B men aged 50 years to 60 years, namely in 2005 and 2006 are very large, in 2007 until the year 2010 is quite large, whereas in 2011 until 2014, small.
For insured women aged 50 to 60 years

Table 8. Calculation of Bayesian estimation for women aged 50 to 60 years

| year | n   | x  | x/n | α     | β     | θ_B  | Z    | %Z  |
|------|-----|----|-----|-------|-------|------|------|-----|
| 2005 | 152 | 6  | 0.039 | 0.041 | 8.124 | 0.005 | 0.949 | 95% |
| 2006 | 333 | 4  | 0.012 | 0.041 | 6.041 | 0.038 | 0.675 | 68% |
| 2007 | 398 | 9  | 0.023 | 0.041 | 10.041| 0.020 | 0.447 | 45% |
| 2008 | 599 | 15 | 0.025 | 0.041 | 19.041| 0.021 | 0.402 | 40% |
| 2009 | 977 | 18 | 0.018 | 0.041 | 34.041| 0.023 | 0.396 | 40% |
| 2010 | 1485| 26 | 0.018 | 0.041 | 52.041| 0.021 | 0.376 | 38% |
| 2011 | 1677| 34 | 0.020 | 0.041 | 78.041| 0.020 | 0.298 | 30% |
| 2012 | 1789| 50 | 0.028 | 0.041 | 112.041| 0.020 | 0.241 | 24% |
| 2013 | 2054| 49 | 0.024 | 0.041 | 162.041| 0.022 | 0.217 | 22% |
| 2014 | 2987| 55 | 0.020 | 0.041 | 211.041| 0.022 | 0.240 | 24% |
| 2015 |     |    |   0.041 | 0.041 | 266.041| 0.021 | 0.240 | 24% |

Based on the percentage of the value of Z obtained in Table 8, it can be concluded that the influence of information in previous years to take into account the value of the insured \( \theta_B \) on women aged 50 to 60 years ie in 2005 and 2006 are very large, in the year 2007 to the year 2010 was quite large, whereas in 2011 until 2014, small.

For insured men aged over 60 years

Table 9. Calculation Bayesian estimation for men aged over 60 years

| year | n   | x  | x/n | α     | β     | θ_B  | Z    | %Z  |
|------|-----|----|-----|-------|-------|------|------|-----|
| 2005 | 88  | 6  | 0.068 | 0.041 | 8.124 | 0.005 | 0.915 | 92% |
| 2006 | 120 | 4  | 0.033 | 0.041 | 9.041 | 0.063 | 0.555 | 56% |
| 2007 | 155 | 9  | 0.058 | 0.041 | 10.041| 0.046 | 0.418 | 42% |
| 2008 | 258 | 15 | 0.058 | 0.041 | 19.041| 0.046 | 0.418 | 42% |
| 2009 | 394 | 18 | 0.046 | 0.041 | 34.041| 0.054 | 0.385 | 39% |
| 2010 | 478 | 26 | 0.054 | 0.041 | 52.041| 0.051 | 0.318 | 32% |
| 2011 | 658 | 34 | 0.052 | 0.041 | 78.041| 0.052 | 0.305 | 30% |
| 2012 | 789 | 49 | 0.062 | 0.041 | 112.041| 0.052 | 0.244 | 24% |
| 2013 | 841 | 49 | 0.062 | 0.041 | 162.041| 0.052 | 0.244 | 24% |
| 2014 | 897 | 55 | 0.056 | 0.041 | 211.041| 0.058 | 0.187 | 19% |
| 2015 | 985 | 55 | 0.056 | 0.041 | 266.041| 0.058 | 0.187 | 19% |

Based on the percentage of the value of Z obtained in Table 9, it can be concluded that the influence of information in previous years to take into account the value of the insured \( \theta_B \) men aged over 60 years, namely in 2005 and 2006 are very large, in the year 2007 to in 2010 large enough, whereas in 2011 until 2014, small.

For insured men aged over 60 years

Table 10. Calculation -Eighteen Bayesian estimation for over 60 years

| year | n   | x  | x/n | α     | β     | θ_B  | Z    | %Z  |
|------|-----|----|-----|-------|-------|------|------|-----|
| 2005 | 74  | 6  | 0.081 | 0.041 | 8.124 | 0.005 | 0.901 | 90% |
| 2006 | 95  | 4  | 0.042 | 0.041 | 6.041 | 0.074 | 0.536 | 54% |
| 2007 | 111 | 9  | 0.081 | 0.041 | 10.041| 0.057 | 0.385 | 39% |
| 2008 | 198 | 15 | 0.076 | 0.041 | 19.041| 0.066 | 0.407 | 41% |
| 2009 | 242 | 18 | 0.074 | 0.041 | 34.041| 0.070 | 0.332 | 33% |
| 2010 | 298 | 26 | 0.087 | 0.041 | 52.041| 0.071 | 0.295 | 29% |
| 2011 | 397 | 34 | 0.086 | 0.041 | 78.041| 0.076 | 0.279 | 28% |
| 2012 | 874 | 50 | 0.057 | 0.041 | 112.041| 0.079 | 0.380 | 38% |
| 2013 | 897 | 49 | 0.055 | 0.041 | 162.041| 0.071 | 0.281 | 28% |
| 2014 | 985 | 55 | 0.056 | 0.041 | 211.041| 0.066 | 0.236 | 24% |
| 2015 | 266 | 55 | 0.056 | 0.041 | 266.041| 0.066 | 0.236 | 24% |
Based on the percentage of the value of $Z$ obtained in Table 10, it can be concluded that the influence of information in previous years to take into account the value $\theta_B$ insured women over the age of 60 years, namely in 2005 and 2006 are very large, in the year 2007 to the year 2009 big enough, whereas in the year 2010 to 2014, small.

### Table 11. Calculation of the risk premium for insured males by age group

| Age         | $\theta_B$ | $n_{2009}$ | $E(Y_i)$ | $D(Y_i)$ | The risk premium / person |
|-------------|------------|------------|----------|----------|---------------------------|
| < 30 years  | 0.01       | 2.146      | 179.628  | 1,707.642| 0.85                      |
| 30 to 40 years | 0.027     | 2.156      | 51,861.258 | 49,340,328.857 | 3.58                      |
| 40 to 50 years | 0.016     | 2.156      | 35,770.747 | 37,168,956.545 | 1.71                      |
| 50 to 60 years | 0.020     | 3.245      | 86,276.357 | 110,450,316.148 | 2.96                      |
| > 60 year   | 0.059      | 841        | 24,683.851 | 10,354,477.324 | 7.82                      |

### Table 12. Calculation of the risk premium for insured women by age group

| Age         | $\theta_B$ | $n_{2009}$ | $E(Y_i)$ | $D(Y_i)$ | The risk premium / person |
|-------------|------------|------------|----------|----------|---------------------------|
| < 30 years  | 0.008      | 1.987      | 12,115.687 | 9,045,503.896 | 0.56                      |
| 30 to 40 years | 0.032     | 1.789      | 41,465.275 | 29,037,717.865 | 3.21                      |
| 40 to 50 years | 0.013     | 2.111      | 26,374.527 | 25,957,640.904 | 0.65                      |
| 50 to 60 years | 0.021     | 2.987      | 72,548.771 | 88,639,531.691 | 3.12                      |
| > 60 year   | 0.064      | 985        | 32,052.294 | 12,355,569.095 | 9.99                      |

### 4. Conclusion

Bayesian estimation theory provides a great method for estimating the risk premium for the next period of the claim data information in the previous period. In this study, a large risk premium for insured males aged less than 30 years is 0.85, for those aged 30 to 40 years is 3.58, for those aged 40 to 50 years is 1.71, for those aged 50 to 60 years was 2.96, and for age over 60 years is 7.82. Meanwhile, for insured women aged less than 30 years is 0.56, for those aged 30 to 40 years is 3.21, for those aged 40 to 50 years is 0.65, for those aged 50 to 60 years was 3.12, and for those aged over 60 years is 9.99.

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