Influence of Fluid-Induced Forces on Cavitating Turbopumps Rotordynamics in Forced Whirl Experiment

F. Bertelli¹, A. Pasini¹*, R. Bottai¹
¹Department of Civil and Industrial Engineering, University of Pisa, Pisa, Italy;
E-mail: *angelo.pasini@unipi.it

Abstract. As typically highlighted in the experimental results, in unshrouded axial inducers the fluid-induced rotodynamic forces are significantly dependent on the flow coefficient and do not present a quadratic behavior with respect to the whirl ratio, as commonly reported in radial pumps. Moreover these forces show destabilizing peaks that typically increase with decreasing cavitation number. The present paper illustrates the development of a theoretical model of the rotodynamic system capable of assessing the influence of fluid-induced rotodynamic forces on the dynamic response of turbopumps under cavitating/non-cavitating conditions. The implementation of the fluid-induced rotodynamic forces in the dynamic model has been proposed through the so-called F.I.R.F. model, which is applied to a test set-up for forced whirl experiments, in order to characterize the whirl natural frequencies and the relative mode shapes. The model has been successfully applied to a rotodynamic system including an unshrouded axial inducer and the corresponding computed critical speeds are summarized in the Campbell diagrams pointing out the difference between dry and wet analysis results.

1. Introduction
Operation under limited cavitation and sometimes at supercritical speeds is usually tolerated in order to maximize the performance in rocket propellant feed turbopumps, exposing however the turbopump to the onset of dangerous cavitation-induced fluid dynamic instabilities and rotodynamic forces [1]. The latter are one of the universally recognized and most dangerous sources of vibrations in turbomachines that can affect the impeller itself, and all the components of the machine. Inducers are often employed in liquid rocket turbopumps necessary to prevent the cavitation in the main stages, consequently these components are exposed to cavitation under normal operating conditions. As a consequence of the highly three-dimensional internal flow, the effects of upstream and downstream flow distortions together with the presence of cavitation highly affect the fluid-induced rotodynamic forces, which arise when a whirling inducer interacts with the working fluid. Although these forces have already been extensively studied both in the centrifugal impellers ([2],[5]) and in axial inducers ([5],[8]), the influence of the fluid-induced rotodynamic forces on the turbopump rotordynamics has not yet been investigated in great detail. Starting from one of the still unresolved problems, the present paper addresses the implementation of the fluid induced rotodynamic forces, which are generated by axial inducers, in the rotordynamics of cavitating turbopumps. Based on the available experimental evidence, the obtained continuous spectrum of fluid-induced rotodynamic forces can be very useful to
catch the unlikely foreseeable complexity of these forces and their consequences on stability of axial inducers ([5],[8]). The novelty of the proposed Fluid-Induced Rotordynamics Forces (F.I.R.F.) model is to implement the non-linear forces, which arise during the inducer whirling motion, in the dynamics of the rotor system in a coherent manner with the overall dynamics.

2. Scope of Paper
The key idea of the paper is to integrate in the classical linear rotordynamic analysis the behavior of rotordynamic elements, such as cavitating inducers, that do not present the classical quadratic behavior effectively modelled through the mass, damping and stiffness matrices. In particular, the cavitating inducers differ from the standard behavior because of the fluid-induced rotordynamic forces. In space rocket turbopumps, the integration of such kind of components in the rotodynamic model allows to include in the computation of the critical speeds the effect of the working fluid both in noncavitating and cavitating conditions. So far there is no predictive model capable of estimating the fluid-induced rotordynamic forces in cavitating/noncavitating inducers and therefore the only way to characterize the fluid-induced rotordynamic forces is experimentally through forced whirl experiments that allow to measure the rotordynamic matrix as a function of the operating conditions. In view of an experimental campaign aimed at characterizing the dynamic behavior of cavitating turbopumps through the excitation of the one lobe azimuthal mode by means of a forced whirl motion, the present paper reports the estimation of the critical speeds of the test facility obtained by applying the developed model capable of taking into account the fluid-induced rotordynamic forces. In absence of experimental data for the candidate test item, the rotordynamic matrices of the inducer used in the computation refer to a previous experimental campaign on an analogous inducer.

3. Rotordynamic Modeling
The analytical prediction of the rotor dynamic behavior depends heavily on accurately modeling the physical system and understanding the assumptions and limitations applied on the model and analytical tools employed. Direct Stiffness Method (DSM) is the most used analytical procedure for the analysis of free and forced response of rotordynamic systems. This method uses a discrete mathematical model to reproduce the complex continuous system into a set of simpler, idealized elements placed between two consecutive nodes. By using force-displacement, force-velocity relations and the principle of conservation of momentum, a set of equations can be derived for each components of the system which are then assembled by using geometric displacement constraints. Six DOFs have been used to describe the complete set of vibration: \( q_{(6×1)} = \{x, y, z, \alpha, \beta, \gamma\}^T \), where \( x, y, \alpha, \beta \) identify the lateral vibration while \( z \) and \( \gamma \) identify the axial and torsional vibration respectively. The main components that reproduce the continuous system are modelled as stations, where each consists of thin rigid elements, or lumped mass, connected by shaft segments. In the present work, based on previous research ([11], [12]), the shaft segments are modelled either by massless flexible beam elements or by flexible uniform shaft elements which has a distributed mass. Mass imperfection or misalignment are modelled considering an eccentricity with respect to the axis of rotation. Finally, the bearings are defined as an interconnection component that connects two finite element stations and does not introduce any additional DOFs or finite element stations in the system. The system governing equations of motions for a complete rotordynamic system are obtained through several steps. Once defined the fixed and/or rotating reference frame and divided the real system into a finite degree of freedom, a set of system displacement vector are chosen. Then, the equation of motion for the entire system is written as:

\[
M\ddot{q} + (\Omega G_c + C_b) \dot{q} + \left(\Omega G_c + K_c + K_b\right) q = Q_{u_x}\cos\phi + Q_{u_y}\sin\phi + Q_g + Q_f + Q_{ext} \tag{1}
\]
where all the matrices are real and assembled from the associated components: the stacked mass matrix $M_c$, the stacked gyroscopic matrix $\Omega G_c$, the bearing damping matrix $C_b$, the stacked stiffness matrix $K_c$ and the bearing stiffness matrix $K_b$. The right-hand side of equation (1) contains the unbalance vectors $Q_u$, the gravitational force vectors $Q_g$ and the fluid-induced force vectors $Q_f$. The forcing vectors $Q_{ext}$ contain all other external forces including constant, linear and non-linear forces. It is generally suitable to model the system equation of motion in first-order form by defining the state vector $x = \begin{bmatrix} \dot{q} \\ q \end{bmatrix} T$. In the case that assuming a steady rotational speed ($\Omega = \text{cost}$ and $\phi = \Omega t$) the equation (1) in state-space form is:

\[
\begin{bmatrix}
0 \\
M_c \\
\Omega G_c + C_b
\end{bmatrix}
\begin{bmatrix}
\dot{x}(t)
\end{bmatrix}
+
\begin{bmatrix}
-M_c \\
0 \\
K_c + K_b
\end{bmatrix}
\begin{bmatrix}
x(t)
\end{bmatrix}
=
\begin{bmatrix}
0 \\
Q(t)
\end{bmatrix}
\]

(2)

where the term $Q(t)$ contains all the forcing vectors in the RHS of equation (1).

4. Fluid-Induced Rotordynamic Forces

The prediction of rotordynamic forces in high speed turbomachinery is important in order to evaluate dynamic instabilities thus avoiding system failure. The high rotational speed of the impeller together with mass unbalances, rotor deformations and weight force lead to a whirl motion of the rotor which moves with an eccentric trajectory with respect to its undeflected position. Once whirl motion is established, the interaction between the impeller and the working fluid generates the fluid-induced rotordynamic forces. Although these forces arise due to flow as a consequence of shaft displacement, fluid-induced forces arise also due to presence of flow distorting conditions such as cavitation phenomenon, a condition under which inducers are required to operate. Therefore, an impeller, which is performing a whirl motion within the fluid, undergoes an instantaneous force that can be considered as the sum of two contributions:

\[
F(t) = F_0(t) + F_R(t)
\]

(3)

The force $F_0(t)$, referred to radial force or radial thrust, is a force independent on the whirl motion and perpendicular to the axis of rotation for an ideal perfectly centered rotor and it can be decomposed into: stationary and non-stationary part. While the force $F_R(t)$, referred to so-called fluid-induced rotodynamic force, is an unsteady force caused by the result of eccentric movement of the impeller. As a consequence of eccentric motion, the fluid imparts an unsteady force to the impeller. As shown in figure 1, the deflection of the rotor from the nominal position is evaluated as a function of time by the vector $\varepsilon(t)$, where it has been assumed small and a linear perturbation model will be used to evaluate the rotordynamic forces as:

\[
F_R(t) = [A] \varepsilon(t)
\]

(4)

Referring to the figure 1 (left), let $F(t)$ be the instantaneous force acting on the center of impeller $O$, which will move following the whirl path at a whirl speed denoted by $\omega$, and will undergo a rotational speed denoted by $\Omega$. The dynamic fluid-induced forces in the fixed reference frame can be conveniently expressed in terms of normal and tangential forces with respect to the whirl orbit as shown in figure 1 (left), respectively $F_N$ and $F_T$ by:

\[
F_N = F_R \cdot (\varepsilon / |\varepsilon|) \quad ; \quad F_T = F_R \cdot (\Omega \wedge \varepsilon) / |\Omega \wedge \varepsilon|
\]

(5)

where $F_N$ is defined positive as outward and $F_T$ as positive in the direction of rotational speed.

Possible experimental approach for the evaluation of the rotodynamic matrix $A$, can be carried out through forced experiment [5]. In fact when a circular whirl orbit of a fixed radius
Figure 1. Rotordynamic forces in the perpendicular plan acting on the whirling impeller at its center $O$ that moves on a whirl path (left) and on a fixed and circular whirl orbit (right) [13].

is externally imposed, as schematically shown in figure 1 (right), $F_R$ can be solved into relative circular whirl orbit frame, and the normal and tangential forces with respect to the whirl orbit can be expressed as:

$$F_N = \frac{1}{2} \varepsilon (A_{XX} + A_{YY}) ; \quad F_T = \frac{1}{2} \varepsilon (-A_{XY} + A_{YX})$$ (6)

Experimental investigation, such as the one reported by Jery [2] and Franz [3], shows the skew-symmetry of rotordynamic matrix for circular whirl orbit: $A_{XX} = A_{YY}$ and $-A_{XY} = A_{YX}$. The rotordynamic forces are usually shown by plotting $F_N$ and $F_T$ as functions of the geometry, the operating conditions and the frequency ratio $\omega/\Omega$, in order to characterize the rotordynamic matrix, $A$. In the past years various studies ([2], [3]) have shown, in the case of centrifugal pump, a quadratic variation of forces with $\omega/\Omega$, consequently in the rotordynamic analysis is conventional to decompose the matrix $A$ into added mass, damping and stiffness matrices as a function of frequency ratio $\omega/\Omega$ according to the classical approach proposed by Brennen [8]. In such case, the normal and tangential forces can be written as:

$$F_N = M \left( \frac{\omega}{\Omega} \right)^2 - C \left( \frac{\omega}{\Omega} \right) - K ; \quad F_T = -m \left( \frac{\omega}{\Omega} \right)^2 + C \left( \frac{\omega}{\Omega} \right) + k$$ (7)

where $M_{xx} = M_{yy} = M$ and $M_{xy} = -M_{yx} = m$, $C_{xx} = C_{yy} = C$ and $C_{xy} = -C_{yx} = c$, $K_{xx} = K_{yy} = K$ and $K_{xy} = -K_{yx} = k$. In Jery [2] can be observed the quadratic and linear trend of normalized rotordynamic forces $F_N$ and $F_T$ as a function of frequency ratio in turbopumps impeller under non-cavitating condition. On the other hand, the rotordynamic forces acting on axial inducer are less well understood. Up to the present date, experimental results clearly show the important differences that exist among unshrouded axial inducers and shrouded centrifugal impellers. One of the reasons is that the dynamic response of the tip clearance flows affects the rotordynamics of inducer, therefore the past codes for rotordynamic analysis are not well adapted to deal with deviations from the quadratic trend of rotordynamic forces. In the past years, experimental campaigns focused on the characterization of $F_N$ and $F_T$ in order to better evaluate of axial inducer instabilities. Useful results are reported in Bhattacharyya [3], Pasini et al [8] and Torre et al [14], where it is shown the unclear behaviour of $F_N$ and $F_T$. The quadratic and linear trend of $F_N$ and $F_T$ respectively, are only confirmed for negative frequency ratio in non-cavitating regime as shown by the pink line in figure 2. Under cavitating conditions, several zero-crossing points are observed as the cavitation number is lowered. This results in transition from stable to unstable regime or viceversa (green line in figure 2).
5. F.I.R.F. Model Integration

In all rotodynamic applications, the first consideration in the design of rotor system is the disposition of critical speeds with respect to the reference speed of the machine, in order to avoid critical conditions. Therefore, the free-vibration analysis of equation (2) evaluates the whirl natural frequencies and the relative mode shapes with the purpose of assessing the system operating condition. The determination of operating condition is useful since it identifies whether the system is under sub-critical or super-critical condition. The free-vibration analysis leads to the eigenvalue problem as:

\[ \text{det} \left( \lambda + a^{-1} \cdot b \right) = 0 \]  (8)

In general the determinant of the array \( H(\lambda) \) yields solution \( \lambda_j = \sigma_j + i\omega_j \), where the real part \( \sigma_j \) of the eigenvalue is the damping coefficient and the imaginary part \( \omega_j \) is the damped natural frequency of whirl. In order to understand the influence of fluid-induced rotordynamic forces on the critical speeds it is important to note that in the right-hand side of equation (1) the fluid-induced force vector \( \mathbf{Q}_f \) is defined as:

\[ \mathbf{Q}_f = \mathbf{Q}_{f_0} + \mathbf{Q}_{f_{rd}} + \mathbf{Q}_{f_b} \]  (9)

where \( \mathbf{Q}_{f_0} \) and \( \mathbf{Q}_{f_b} \) is the radial force and buoyancy vector respectively and \( \mathbf{Q}_{f_{rd}} \) is the fluid-induced rotordynamic forces vector. The latter, as discussed above, are function of \( \mathbf{Q}_f = \mathbf{Q}_f (\dot{q}, \ddot{q}, q, \phi, \sigma) \) through the added mass matrix \( \mathbf{M}_f \), the damping matrix \( \mathbf{C}_f \) and the stiffness matrix \( \mathbf{K}_f \), which in turn are functions of whirl ratio \( \omega/\Omega \sim |\dot{q}|/\Omega \). Therefore, substituting into equation (1) the definition of \( \mathbf{Q}_{f_{rd}} \) [1]

\[ \mathbf{Q}_{f_{rd}} = [A]\varepsilon = -(\mathbf{M}_f \ddot{q} + \mathbf{C}_f \dot{q} + \mathbf{K}_f q) \]  (10)

is obtained the equation of motion in state-space form. Since the fluid-induced rotordynamic forces characterize the dynamic system, they are moved in the left-hand side of equation and the state-space form is written as:

\[
\begin{bmatrix}
0 & \mathbf{M}_c + \mathbf{M}_f \\
\mathbf{M}_c + \mathbf{M}_f & \Omega\mathbf{G}_c + \mathbf{C}_b + \mathbf{C}_f
\end{bmatrix}
\begin{bmatrix}
\dot{x}(t) \\
\ddot{x}(t)
\end{bmatrix}
+
\begin{bmatrix}
-(\mathbf{M}_c + \mathbf{M}_f) \\
0
\end{bmatrix}
\begin{bmatrix}
\dot{x}(t) \\
\ddot{x}(t)
\end{bmatrix}
+
\begin{bmatrix}
\mathbf{0} \\
\mathbf{0}
\end{bmatrix}
\begin{bmatrix}
\mathbf{K}_c + \mathbf{K}_b + \mathbf{K}_f
\end{bmatrix}
\begin{bmatrix}
\dot{x}(t) \\
\ddot{x}(t)
\end{bmatrix}
= 
\begin{bmatrix}
\mathbf{0} \\
\mathbf{0}
\end{bmatrix}
\begin{bmatrix}
\dot{f}(t) \\
f(t)
\end{bmatrix}
\]

\[ \text{det} \left( \lambda + a^{-1} \cdot b \right) = 0 \]  (8)

In general the determinant of the array \( H(\lambda) \) yields solution \( \lambda_j = \sigma_j + i\omega_j \), where the real part \( \sigma_j \) of the eigenvalue is the damping coefficient and the imaginary part \( \omega_j \) is the damped natural frequency of whirl. In order to understand the influence of fluid-induced rotordynamic forces on the critical speeds it is important to note that in the right-hand side of equation (1) the fluid-induced force vector \( \mathbf{Q}_f \) is defined as:

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where \( \mathbf{Q}_{f_0} \) and \( \mathbf{Q}_{f_b} \) is the radial force and buoyancy vector respectively and \( \mathbf{Q}_{f_{rd}} \) is the fluid-induced rotordynamic forces vector. The latter, as discussed above, are function of \( \mathbf{Q}_f = \mathbf{Q}_f (\dot{q}, \ddot{q}, q, \phi, \sigma) \) through the added mass matrix \( \mathbf{M}_f \), the damping matrix \( \mathbf{C}_f \) and the stiffness matrix \( \mathbf{K}_f \), which in turn are functions of whirl ratio \( \omega/\Omega \sim |\dot{q}|/\Omega \). Therefore, substituting into equation (1) the definition of \( \mathbf{Q}_{f_{rd}} \) [1]

\[ \mathbf{Q}_{f_{rd}} = [A]\varepsilon = -(\mathbf{M}_f \ddot{q} + \mathbf{C}_f \dot{q} + \mathbf{K}_f q) \]  (10)

is obtained the equation of motion in state-space form. Since the fluid-induced rotordynamic forces characterize the dynamic system, they are moved in the left-hand side of equation and the state-space form is written as:

\[
\begin{bmatrix}
0 & \mathbf{M}_c + \mathbf{M}_f \\
\mathbf{M}_c + \mathbf{M}_f & \Omega\mathbf{G}_c + \mathbf{C}_b + \mathbf{C}_f
\end{bmatrix}
\begin{bmatrix}
\dot{x}(t) \\
\ddot{x}(t)
\end{bmatrix}
+
\begin{bmatrix}
-(\mathbf{M}_c + \mathbf{M}_f) \\
0
\end{bmatrix}
\begin{bmatrix}
\dot{x}(t) \\
\ddot{x}(t)
\end{bmatrix}
+
\begin{bmatrix}
\mathbf{0} \\
\mathbf{0}
\end{bmatrix}
\begin{bmatrix}
\mathbf{K}_c + \mathbf{K}_b + \mathbf{K}_f
\end{bmatrix}
\begin{bmatrix}
\dot{x}(t) \\
\ddot{x}(t)
\end{bmatrix}
= 
\begin{bmatrix}
\mathbf{0} \\
\mathbf{0}
\end{bmatrix}
\begin{bmatrix}
\dot{f}(t) \\
f(t)
\end{bmatrix}
\]
where now, within the term $Q(t)$, are only concentrated the forcing vectors of the problem. Consequently, it is raised the necessity to determine $M_f$, $C_f$ and $K_f$ in order to evaluate the critical speeds of the rotodynamic system. In the case that the trend of the fluid-induced rotodynamic forces is known as a function of whirl ratio and it shows a non-quadratic behavior, the F.I.R.F. model can be integrated in the analysis and it can be used to investigate their influence on the critical speeds [12]. The free-vibration analysis leads to the same eigenvalue problem of equation (8) but in this case the matrix components change dynamically according to the F.I.R.F. model, which is summarized in the following steps. First, the fluid-induced rotodynamic forces must be defined in the whirl ratio range by the experimental data: $f_{exp}(x_{exp})$. In the previous relation, $f_{exp}$ is the dependent variable $F_N$ or $F_T$ while the independent variable $x_{exp}$ is $ω/Ω$. To measure the rotodynamic forces a combined motion of rotation and whirl must be imposed on the impeller shaft. This is accomplished by carrying out the so-called forced whirl experiment which experimentally reproduce the combined motion depicted in Figure 1 (right). A circular whirl motion with fixed eccentricity $|ε|$ and variable whirl frequency $ω$ is imposed to the impeller shaft which runs at constant rotating speed $Ω$. The whirl frequency is varied either in a discrete way between each run, as done in the classical rotodynamic experiments ([6], [7], [9], [11]) or it is varied continuously within a defined whirl ratio range ([11]). Typical results are shown in Figure 2, where both discrete and continuous force measurements for the same impeller are superimposed. Once $f_{exp}(x_{exp})$ is obtained, the experimental range $x_{exp}$ is divided in $n$ parts with $n + 1$ point: $x_1, x_2, ..., x_{n+1}$. For each of $n$ parts it is computed a quadratic regression curve so that the analysis is reduced to the classical approach proposed by Brennen: $a_1, a_2, ..., a_n; b_1, b_2, ..., b_n; c_1, c_2, ..., c_n$. Finally, the quadratic regression curve is defined as follows:

$$y_{interp} = a_{interp} x_{exp}^2 + b_{interp} x_{exp} + c_{interp}$$

where the coefficients can be written as:

$$\begin{align*}
a_{interp} &= a_1 H(-x_{exp} + x_2) + a_2 H(x_{exp} - x_2) H(-x_{exp} + x_3) + \ldots \\
&+ a_{n-1} H(x_{exp} - x_{n-1}) H(-x_{exp} + x_n) + a_n H(x_{exp} - x_n) \\
b_{interp} &= b_1 H(-x_{exp} + x_2) + b_2 H(x_{exp} - x_2) H(-x_{exp} + x_3) + \ldots \\
&+ b_{n-1} H(x_{exp} - x_{n-1}) H(-x_{exp} + x_n) + b_n H(x_{exp} - x_n) \\
c_{interp} &= c_1 H(-x_{exp} + x_2) + c_2 H(x_{exp} - x_2) H(-x_{exp} + x_3) + \ldots \\
&+ c_{n-1} H(x_{exp} - x_{n-1}) H(-x_{exp} + x_n) + c_n H(x_{exp} - x_n)
\end{align*}$$

where $H$ is the heaviside function. Moreover, to include a trend outside of the experimental range:

$$\begin{align*}
a_{interp} &= a_0 H(-x_{exp} + x_1) + a_1 H(x_{exp} - x_1) H(-x_{exp} + x_2) + \ldots \\
&+ a_n H(x_{exp} - x_n) H(-x_{exp} + x_{n+1}) + a_{n+1} H(x_{exp} - x_{n+1}) \\
b_{interp} &= b_0 H(-x_{exp} + x_1) + b_1 H(x_{exp} - x_1) H(-x_{exp} + x_2) + \ldots \\
&+ b_n H(x_{exp} - x_n) H(-x_{exp} + x_{n+1}) + b_{n+1} H(x_{exp} - x_{n+1}) \\
c_{interp} &= c_0 H(-x_{exp} + x_1) + c_1 H(x_{exp} - x_1) H(-x_{exp} + x_2) + \ldots \\
&+ c_n H(x_{exp} - x_n) H(-x_{exp} + x_{n+1}) + c_{n+1} H(x_{exp} - x_{n+1})
\end{align*}$$

where $a_0, b_0, c_0$ is for $x_{exp} < x_1$ and $a_{n+1}, b_{n+1}, c_{n+1}$ is for $x_{exp} > x_{n+1}$. Once the corresponding matrices $M_f$, $C_f$ and $K_f$, as a function of the whirl ratio, are calculated from the extrapolated coefficients of the quadratic curve, the critical frequencies $ω_i^{(n)}$ are evaluated in each whirl ratio range. Finally, the obtained frequencies are filtered with the following relation in order to
evaluate the updated critical frequencies: $x_{\exp}^{(n)} \leq \omega_i^{(n)}/\Omega_i^{(n)} \leq x_{\exp}^{n+1}$. Resort to a quadratic regression on a range of experimental data is due to the return to a classical approach in order to consider the fluid-induced rotodynamic forces. Because of the reduction of the residual error, the $n$ intervals must be increased to improve the fitting curves, however a high value of $n$ increases the discontinuity of the properties of the quadratic curve between two consecutive intervals. Therefore, $n$ is chosen as a compromise since a reduction of the ranges does not correctly approximate the original trend. Figure 2 reports the quadratic regression curve with $n = 10$ as an approximation of rotordynamic force. Considering the regression curve with $n = 5$ the approximation is accurate for a negative whirl ratio but is not faithful for positive value while both other approximations follow the trend correctly. Although the approximation with $n = 20$ is the most precise it turns out to be the one with more discontinuities, which make the analysis more prone to errors as reported in [15].

6. Model Application and Results

Starting from the combination of experimentation and theoretical/numerical modeling, MIT and Unipi jointly focus on the characterization of longitudinal and rotating instabilities in cavitating turbopump inducer. Based on this research project, the present work aims to evaluate the influence of fluid-induced forces on cavitating turbopumps rotordynamics in forced whirl experiment. Consequently, the rotordynamic system needs to be checked in order to avoid the onset of failures during the test with possible fracture of the components. Moreover, the knowledge of the time position of the inducer during its whirl motion in addition to the determination of velocity and pressure field at inlet section allows the study of flow instabilities. The future experimental campaign is based on the experimental set-up of the Cavitating Pump Rotordynamic Test Facility (CPRTF) at SITAEL. The rotordynamic compatibility and the lumped parameter model of the test section of CPRTF will be verified including the F.I.R.F. model into the complete rotordynamic analysis. Details of the rotor system and the analyzed inducer are reported in [15], [12]. Starting from the dry analysis, i.e. without consider the fluid-induced rotodynamic forces, the free vibration analysis evaluated the critical speeds of the CPRTF system, where the forced whirl experiment will be conducted with an harmonic component of the forcing function in the range from 0 to $\pm 2\Omega$ and the intersection of the excitation line with the lines of backward and forward natural whirl modes allows to identify the critical frequencies, which establishes the resonant conditions. In this case the added mass, damping and stiffness matrices do not vary and their influence on the critical speeds is only due to the change of frequency. The dry free-vibration results are shown in Campbell diagram in figure 3 (left) that reports the typical results of centrifugal impellers. The importance of the influence of fluid-induced rotodynamic forces resulting from the interaction between the inducer and the working fluid is now analized. Considering the experimental results under cavitating conditions as reported in Figure 2 and by using the F.I.R.F. model with $n = 10$ the wet free-vibration analysis highlights some important aspects. The effects of continuous updating of added mass, damping and stiffness matrices, now function of $\omega/\Omega$, is clearly visible in figure 3 (right). The Campbell diagram shows how the variation of $M_f$, $C_f$ and $K_f$ influences the critical speeds of the rotor, triggering new eigenvalues with new mode shapes, especially at lower whirl frequency. The area in which the experimental data are accurately approximated is highlighted in green that correspond to the range $|\omega/\Omega| \leq 0.7$. As expected, considering the cavitation phenomena, a general decreasing of critical frequencies is observed. This effect must be taken into account in the design phase since moving the critical frequencies could promote resonance condition during the transient phase.
7. Conclusion

According to the results of the analysis of the proposed F.I.R.F model the following conclusions can be drawn:

- The fluid-induced rotordynamic forces inside a turbopump cannot be considered as external forcing vectors that interact with the intrinsic dynamics of the rotating components because the working fluid elaborated by the pump is part of the rotodynamic components and the intrinsic rotordynamics of the pump are highly affected by the fluid-induced rotordynamics forces (i.e. influence on the critical speeds of the machine).

- If the fluid-induced rotordynamic forces inside a turbopump present a quadratic behavior with respect to the whirl speed described by the corresponding added mass, damping and stiffness matrices (as typically reported in centrifugal impellers), their influence on the critical speeds of the turbomachine is only in terms of change of frequency (no introduction of further critical speeds with respect to the dry case).

- If the fluid-induced rotordynamic forces inside an impeller do not present a quadratic behavior with respect to the whirl speed (as usually shown in unshrouded axial inducers) the problem is no more linear and such kind of fluid-induced rotordynamic forces may not only affect the frequency of the critical speeds but also introduce further critical speeds with respect to the case of dry system.

- The F.I.R.F model proposed in this paper has been implemented to include into the classical linear approach for the computation of critical speeds, based on the eigenvalue problem of the free-vibration analysis, those components, such as noncavitating and cavitating unshrouded axial inducers, that present a non-quadratic behaviour of the fluid-induced rotordynamic forces.

- In the paper, the F.I.R.F model has been successfully applied to a rotodynamic system including an unshrouded axial inducer and the corresponding computed critical speeds confirm that a non-quadratic behaviour of the fluid-induced rotordynamic forces may introduce further critical speeds in the system that are in principle not expected either from the dry case analysis or from the simplified approach to the rotodynamic forces that usually introduce just an added mass to the system associated to the presence of the working fluid.

- Finally, the F.I.R.F model can be easily applied to all the components of the rotodynamic system that do not present a constant value of the mass, damping and stiffness matrices.

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**Figure 3.** Campbell diagram for dry (left) and wet (right) free-vibration analysis [12].
with respect to the whirl frequency (not only unshrouded inducer but also some types of bearings and seals).

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