CP/CPT EXPERIMENTS WITH NEUTRAL KAONS
OR
EXPERIMENTAL STUDY OF TWO COMPLEX NUMBERS $\eta_{+-}$ AND $\eta_{00}$

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ABSTRACT

Recent and upcoming Fermilab experiments to probe the origin of CP violation and to search for CPT violation are reviewed for an audience of theoretical particle physics graduate students.

1. CP Violation

I will begin with a quote from J. Cronin, who co-discovered CP violation with Christenson, Fitch, and Turlay in 1964. “The discovery of CP violation was a complete surprise to experimentalists (who) found it as well as to the physics community at large....The experiment that made the discovery was not motivated by the idea that such a violation might exist.” They found that $K_L$ (L for long-lived), thought to be the pure CP-odd mixture of $K^0$ and $\bar{K}^0$, decayed into the CP-even $\pi^+\pi^-$ state at the approximate rate of two per thousand, indicating a small CP-even contamination in the $K_L$. This CP-violation also shows up as the charge asymmetry in the semileptonic decays, and in the existence of other CP-even decay modes such as $\pi^0\pi^0$. (It is interesting to note that no manifestation has been found outside the $K_L$ system yet - not even in its quantum-mechanical cousin $K_S$ (S for short-lived), which is required by the CPT symmetry to have the same proportion of wrong-CP admixture in it.)

On the theoretical side, we cannot yet claim to understand the phenomenon of CP violation. The standard model offers the best explanation as yet through the imaginary phase of the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix. However, the predictions of this hypothesis have so far eluded the vigorous experimental scrutiny described in this article.

Since CP is a very fundamental symmetry, it is not surprising that it has implications in cosmology. Our present understanding of baryogenesis (why matter far outweighs antimatter in the universe) requires CP violation as a necessary ingredient. It is possible that the conventional CP violation, i.e., observed for $K_L$ and presumably originating in the CKM mixing matrix, is sufficient for this purpose but this subject is controversial. If a new source of CP violation is necessary for

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baryogenesis, it would be surprising if the origins of the two types of CP violation were totally dissimilar.

The outline of the CP section of this paper is as follows. I will briefly review how the accommodation of CP violation in the standard model leads to the prediction of a very small ‘direct’ (i.e., occurring in kaon decay as opposed to in kaon mixing) CP violation. Experimental signatures of the direct CP violation can occur in a variety of decay modes in the kaon system. I will first describe the signature in the $2\pi$ decay of the $K_L$, characterized by the ratio $\epsilon'/\epsilon$. Then I will discuss Fermilab fixed-target experiments E731 (concluded) and E832 (expected to run in 1996).

A good reference for this subject is a recent review article on the search for direct CP violation by Winstein and Wolfenstein. This review includes a comprehensive compilation of references on CP violation. In this article, I have avoided duplicating their list of references.

2. Standard Model and Direct CP Violation

The minimal standard model with its three generations allows the generation-mixing CKM mixing matrix to have one non-trivial phase. The coupling between the quarks and the W boson involves the CKM matrix, and the introduction of the phase in the coupling allows CP violating processes to occur. This CP-violating phase appears as a free parameter in the standard model, and its value will have to be inferred from existing and future experimental measurements. To this extent, the standard model incorporates CP violation on a somewhat ad-hoc basis. The second-order box diagram (Fig. 1) shows how the $K^0 - \bar{K}^0$ mixing takes place.

The calculation of the mixing parameter $\epsilon$ is complicated due to QCD corrections, dependence on the quark masses, etc. Due to the uncertainties involved, one cannot translate exactly the measured value of $\epsilon$ into the CP violating phase of the CKM matrix, but $\epsilon$ provides one of the constraints.

Although this hypothesis for explaining $\epsilon$ is somewhat ad-hoc, fortunately there is a prediction. Once the Pandora’s box in Fig. 1 is opened to explain mixing, other diagrams (such as the one shown in Fig. 2, called the gluonic Penguin) come marching right out.

Due to these diagrams, in addition to the CP violating mixing ($\epsilon$) of CP-even kaon in the CP-odd kaon, one also gets CP violation in the decay of the kaon, the so-called ‘direct’ CP violation characterized by $\text{Real}(\epsilon'/\epsilon)$. The calculation of $\epsilon'/\epsilon$ is also plagued by similar difficulties. This is an evolving subject, but most calculations indicate that $\text{Real}(\epsilon'/\epsilon)$ is a small number in the range of $10^{-3}$.

3. Observable Manifestations of Direct CP Violation

Let us consider the CP violating decay mode $K_L \rightarrow 2\pi$. Since the expected value of $\text{Real}(\epsilon'/\epsilon)$ is small, the CP violation in this decay mode is mostly ascribed to the CP-even $\epsilon K_1$ contamination in the CP-odd state $K_2$. The puzzle is how to
tell apart a small direct CP violating $K_2 \to 2\pi$ decay component in the presence of a much more dominant $K_1 \to 2\pi$ component that arises from mixing. A brute force approach would require both the measurement and the prediction of $\epsilon$ accurate to better than $10^{-3}$, which is clearly unrealistic.

Fortunately, the $2\pi$ final state comes in two varieties - $\pi^+\pi^-$ and $2\pi^0$, each a different isospin combination. The CP violating amplitude ratios for these states are defined as follows.

\[
\eta_{+-} \equiv |\eta_{+-}|e^{i\phi_{+-}} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)}
\]

\[
\eta_{00} \equiv |\eta_{00}|e^{i\phi_{00}} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)}
\]

Note that the initial and final states in the numerator and the denominator are physical states. Let us now define the direct CP violating quantity $\epsilon'$. 

\[
\epsilon' = \frac{1}{\sqrt{2}} \frac{A(K_2 \to 2\pi, I = 2)}{A(K_1 \to 2\pi, I = 0)}
\]

$\epsilon'$ as defined above is an abstract quantity in the sense that both the initial and final states in the numerator and the denominator are not physical states but are eigenstates of CP and isospin. Note also that the numerator not only exhibits direct CP violation, but also breaks the $\Delta I = 1/2$ rule. One can now use Clebsch-Gordon coefficients to go from the pure isospin states to the physical $2\pi$ states, and the known CP violation quantities to express the pure CP states $K_1$ and $K_2$ in terms of the weak eigenstates $K_S$ and $K_L$. After all the dust settles, the abstract definition of $\epsilon'$ translates into a small, but experimentally accessible inequality of CP violation in the $\pi^+\pi^-$ and the $\pi^0\pi^0$ decay modes of the $K_L$, normalized to the corresponding CP conserving modes of the $K_S$.

\[
\text{Real}(\epsilon' / \epsilon) \approx 1/6 \left( \frac{|A(K_L \to \pi^+\pi^-)/A(K_S \to \pi^+\pi^-)|^2 - 1}{|A(K_L \to \pi^0\pi^0)/A(K_S \to \pi^0\pi^0)| - 1} \right)
\]

We have ignored some higher order quantities where appropriate, and also taken into account the known suppression due to the $\Delta I = 1/2$ rule. The same expression can also be written as

\[
\text{Real}(\epsilon' / \epsilon) \approx 1/6 \left( \frac{|\eta_{+-}|^2}{|\eta_{00}|} - 1 \right)
\]

4. Search for Direct CP violation

As can be seen from Eq. (4) above, a measurement of $\text{Real}(\epsilon' / \epsilon)$ involves four different decay modes. The branching ratio for the decay modes $K_L \to \pi^+\pi^-$ is compared to the branching ratio for $K_L \to \pi^0\pi^0$, after normalizing to the corresponding
branching ratios for the $K_S$. One may be tempted to look up the four branching ratios and calculate $\epsilon'$, only to find that the double ratio is known to approximately 5% accuracy, whereas the desired precision is in the range of $10^{-3}$. A better (but still challenging) experiment measures the double ratio of the four modes simultaneously, eliminating the common systematic errors. In a nutshell, an experiment to measure $\text{Real}(\epsilon'/\epsilon)$ is a counting experiment for these four decay modes. Difficulty arises in obtaining sufficient statistics while maintaining the precise knowledge of the relative detection efficiencies for the four decay modes. For example, since the lifetimes of the $K_L$ and the $K_S$ are very different, change in detector acceptance along the flight path of the kaons results in different detector acceptance for the $K_L$ and the $K_S$ decays. This difference in acceptance has to be understood very well in order to correctly estimate the actual number of decays that took place in the detector from the number of decays recorded by the detector.

There are two experimental groups currently trying to establish the phenomenon of direct CP violation in the $2\pi$ decay mode, one is based at Fermilab (E731/E832), and the other is based at CERN (NA31/NA48). Both are fixed-target experiments, with different measurement techniques. E731 and NA31 have concluded their analysis, and the next generation experiments E832 and NA48 are expected to be taking data in 1996. I will concentrate on the Fermilab experimental technique in this article.

Fermilab E731 collected data during the 1987/88 Fermilab fixed-target run. Fig. 3 shows the schematic of the E731 detector. The key feature of the experiment is the reduction of systematic error by using two beams simultaneously, one with mostly $K_L$ decays, and the other with mostly $K_S$ decays. Two parallel beams were produced by striking protons on a common target placed approximately 110 meters upstream of the decay volume. Given the large travel distance, almost all the $K_S$'s decayed out and only the $K_L$'s survived in both beams. $K_S$'s were produced by placing a two interaction length Boron Carbide regenerator in one of the beams. The $K_L$ and $K_S$ decays were collected simultaneously, thus greatly reducing the systematic error due to accidental activity, rate dependent effects, etc. The regenerator alternated from beam to beam every proton spill (approximately once a minute) to cancel the errors due to the detector and beam asymmetries. Further details can be found elsewhere.\[6\]

To record the $\pi^+\pi^-$ decay modes, the charged particle tracks and momenta were measured with a spectrometer system consisting of four precision drift chambers and a large aperture analysis magnet placed between the 2nd and the 3rd drift chambers. The position and momentum resolutions of this spectrometer were 0.1mm and $(0.45 + 0.011p/\text{GeV}/c)^\%$, respectively. The charged mode ($\pi^+\pi^-$) trigger was essentially geometric in nature, and made use of scintillator bank signals and the hit information from the two halves of one of the drift chambers.

The energies and positions of the four photons from the $2\pi^0$ decay modes were measured with a fine-resolution electromagnetic calorimeter array of 804 lead-glass blocks. The energy resolution for electrons was $(1 + 5/\sqrt{E/\text{GeV}})^\%$. The photon
energy resolution had an extra one percent constant term due to the fluctuations in the conversion depth of the photon. The gain of each calorimeter block was tracked with a flash-lamp system throughout the run. The trigger for the neutral mode \((2\pi^0)\) was formed by counting the number of clusters in the calorimeter and requiring a significant (30 GeV) energy deposit.

The event sample consisted of 410k events of the CP violating mode \(K_L \to 2\pi^0\), normalized to the 800k \(K_S \to 2\pi^0\) events, and similarly 327k \(K_L \to \pi^+\pi^-\) events normalized to 1060k \(K_S \to \pi^+\pi^-\) events. This sample, after correcting for acceptance, decided the measured value of \(\text{Real}(\epsilon'/\epsilon)\). However, to make sure that the detector was understood properly, much higher statistics samples of non-CP violating modes were also recorded. The \(\pi^+\pi^-\) mode acceptance was scrutinized using several million \(K_L \to \pi^\pm\varepsilon^\mp\nu\) decays, whereas several million \(K_L \to 3\pi^0\) decays served the same function for the \(2\pi^0\) sample. These high statistics decay samples were used extensively for calibration, aperture determination, and acceptance studies.

Once the signal events are reconstructed, one needs to know the detector acceptance to high accuracy. In particular, the detector acceptance difference for \(K_L\) and \(K_S\) has to be understood very well in both neutral and charged modes. This is achieved with the aid of a detailed detector simulation, which has a small number of adjustable parameters such as the kaon production momentum spectrum, proton beam targetting angle, and the kaon beam collimator positions. The accuracy of the simulation was judged by juxtaposing many data distributions against the corresponding predictions obtained from the simulation. Fig. 4, for example, shows the comparison of data and simulation for the decay vertex \((Z)\) of the CP violating \(\pi^+\pi^-\) decays in the \(K_L\) beam. The good quality of agreement between the data and the simulation for the signal modes as well as the high-statistics modes implies that the detector properties are well-understood, i.e., the acceptance is known accurately. The evaluation of \(\text{Real}(\epsilon'/\epsilon)\) is now straightforward. The number of \(2\pi\) decays in the vacuum beam and the regenerator beam were corrected for acceptance and backgrounds. The wavefunction for the coherently regenerated kaons in the regenerator beam is given by \(|K_L > + \rho|K_S >|\), where \(\rho\) is the coherent regeneration amplitude. The acceptance corrected data is used to extract the ratios \(\rho/\eta^+\), and \(\rho/\eta^0\) of the regeneration amplitude to the CP-violating amplitudes in the charged and the neutral mode, respectively. The splitting between the two ratios in a combined fit with common regeneration amplitude gives the value of \(\text{Real}(\epsilon'/\epsilon)\). The value obtained from the entire 1987-88 data sample was \((7.4 \pm 5.2) \times 10^{-4}\), where the error is statistical only.

The five major sources of systematic error were: calibration uncertainties (photons are hard to measure!), uncertainties associated with one of the trigger planes, accidental (rate) effects, background subtractions, and finally acceptance corrections. The combined systematic error in \(\epsilon'/\epsilon\) from these and some other minor sources amounted to \(2.9 \times 10^{-4}\), thus making the result \((7.4 \pm 5.2 \pm 2.9) \times 10^{-4}\).

5. Direct CP Violation - Status and Future
The E731 result, \( \text{Real}(\epsilon'/\epsilon) = (7.4 \pm 6.0) \times 10^{-4} \), is consistent with the null result. The CERN group (NA31) \(^7\) gets a value of \((23 \pm 3.6 \pm 5.4) \times 10^{-4}\) from their 1986, 1988, and 1989 runs. The NA31 value is approximately 3.5 standard deviation away from zero. Given the disagreement between the two results, the errors have to be inflated artificially to achieve unit \(\chi^2\) for the combined result. This procedure gives \(\text{Real}(\epsilon'/\epsilon) = (14 \pm 8) \times 10^{-4}\), which is not significant enough to claim the discovery of direct CP violation in the \(2\pi\) modes.

Both the Fermilab and CERN groups are currently building significantly improved experiments, with the goal of getting the errors down to \(1-2 \times 10^{-4}\). The Fermilab E832 group will use essentially the same technique as in E731. It will have an improved beamline to cleanly deliver more kaons, and a detector with improved resolution that will also be able to handle higher rates. In particular, replacing the Lead-glass calorimeter array by a Cesium Iodide scintillator crystal array will significantly improve the uncertainties associated with the measurement of photons. The CERN NA48 group is planning to use a calorimeter with liquid krypton. Both the calorimeters are expected to withstand the higher rate needed to accumulate greater statistics.

If CP violation is indeed explained by the standard model, the direct CP violation effect in the \(2\pi\) mode could be established with a measurement precision of \(1-2 \times 10^{-4}\) over the next few years.

Direct CP violation is also expected to be seen in other decay modes of the \(K_L\), such as \(\pi^0 e^+e^-\) and \(\pi^0 \nu \bar{\nu}\). In fact, "\(\epsilon'/\epsilon\)" for these modes is expected to be comparable to unity or even higher. However, these decay modes are very rare and experimentally challenging. There is ongoing experimental effort (Fermilab E799) to search for these and other rare decay modes.

6. CPT Symmetry

While CP symmetry is a fundamental symmetry with implications for cosmology, CPT symmetry is even more fundamental - it is difficult to construct a field theory without the CPT symmetry. All local quantum field theories obey CPT symmetry \(^8\), with minimal general assumptions such as Lorentz invariance and asymptotically free states. At the same time, P and CP symmetries have been experimentally dismantled over time, and it is fair to ask whether the CPT symmetry would also be vulnerable to sensitive probes. It is amusing to note that the CPT theorem was published contemporaneously with the famous paper by Lee and Yang \(^9\) bringing down the P symmetry. Conceptually, one might wonder about the status of CPT when the interactions are not local (as in superstring theories), or when the states are not asymptotically free (as in Quantum Chromodynamics). There is also the possibility of CPT symmetry being broken in the as yet unformulated quantum theory of gravity, or by some non-quantum mechanical effects.

The question of possible CPT violation takes on special relevance for the neutral kaon system, which is unique in having exhibited CP violation and also in offering
us a remarkably sensitive interferometer with precision approaching that of the Planck scale. The ratio of the difference of $K_L$ and $K_S$ mass to the $K_L$ mass is approximately $7 \times 10^{-15}$, which is a phenomenally small number.

A good reference on CPT symmetry in the neutral kaon system is a paper by Barmin and colleagues. Although the experimental status has changed significantly since the publication of this article, it is still a good reference for phenomenology in the neutral kaon sector.

7. Search for CPT Violation

CPT symmetry manifests itself in the form of equality of particle-antiparticle masses, lifetimes (or equivalently, total decay rates), moments, etc. For the neutral kaon, CPT symmetry places constraints on the values of the phases of CP violating amplitude ratios $\eta_{+}$ and $\eta_{00}$. Two significant experimental CPT tests have been recently done in the kaon sector. Recall that direct CP violation causes the magnitudes of the amplitude ratios $\eta_{+}$ and $\eta_{00}$ to split (see Eq. (5)). Similarly, CPT violation would cause the phases $\phi_{+}$ and $\phi_{00}$ of these amplitude ratios to split.

CPT symmetry requires that the phase difference $\Delta \phi \equiv \phi_{00} - \phi_{+}$ be significantly less than 1°. A second consequence of CPT symmetry is that these phases should also be very close to the so-called superweak phase $\phi_{sw} \equiv \tan^{-1}(2\Delta m/\Delta \Gamma)$, where $\Delta m$ is the $K_L - K_S$ mass difference, and $\Delta \Gamma$ is the $K_S - K_L$ decay width difference $\Gamma_S - \Gamma_L$.

The $\pi^+\pi^-$ decays from a coherently regenerated kaon beam exhibit an interference pattern due to the non-zero mass difference $\Delta m$, as indicated in the following rate equation.

$$\text{Rate}(\pi^+\pi^-) = |\eta_{+}|^2 e^{-\Gamma_L t} + |\rho|^2 e^{-\Gamma_{st}} + 2|\rho\eta_{+}|e^{-\frac{\Gamma_S + \Gamma_L}{2} t} \cos(\Delta m t + \phi_\rho - \phi_{+} - \phi_{00})$$ (6)

where $\rho \equiv |\rho|e^{i\phi_\rho}$ is the regeneration amplitude. A similar interference is also seen in the $2\pi^0$ decay mode.

$$\text{Rate}(\pi^0\pi^0) = |\eta_{00}|^2 e^{-\Gamma_L t} + |\rho|^2 e^{-\Gamma_{st}} + 2|\rho\eta_{00}|e^{-\frac{\Gamma_S + \Gamma_L}{2} t} \cos(\Delta m t + \phi_\rho - \phi_{00})$$ (7)

The argument of the cosine term in the two equations above shows that the phase difference $\Delta \phi$ can be measured experimentally by looking for a relative shift in the interference patterns for the $\pi^+\pi^-$ and the $2\pi^0$ modes. The equality of $\phi_{+}$ and $\phi_{sw}$, which is the second CPT test, can also experimentally be tested by extracting $\phi_\rho - \phi_{+}$ from the $\pi^+\pi^-$ interference pattern, and then using the value of $\phi_\rho$ obtained using analyticity assumption.

Both Fermilab E731 and its successor experiment E773 have performed these two CPT tests. Fig. 5 shows an example of an extracted interference curve from E773. E731 reported $\Delta \phi = -1.6^{0} \pm 1.2^{0}$, which is consistent with zero, and $\phi_{+} = 42.2^{0} \pm 1.4^{0}$, which is consistent with $\phi_{sw} = 43.4^{0}$, also measured by E731. The preliminary values reported by E773 also indicate no evidence for CPT
violation. The final results from E773 are expected to place CPT limits with $1^0$ error on $\Delta \phi$, and $0.8^0$ error on $\phi_{+-}$. The CPT symmetry appears to be safe for the time being, but data from the next round of $\epsilon'/\epsilon$ experiments will probe the CPT symmetry to an even greater accuracy.

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9. References

1. J.W. Cronin, Nishina Memorial Lecture (unpublished), Nishina Foundation, Tokyo, Japan, September 1993.
2. J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Lett. 13 (1964) 138.
3. G.R. Farrar and M.E. Shaposhnikov, Phys. Rev. Lett. 70 (1993) 2833, and Phys. Rev. D50 (1994) 774.
4. M. Gavela et al., Mod. Phys. Lett. A9 (1994) 795, also P. Huet and E. Sather, SLAC-PUB-6479 (1994), submitted to Phys.Rev. D.
5. B. Winstein and L. Wolfenstein, Rev. Mod. Phys. 65 (1993) 1113.
6. L.K. Gibbons et al., Phys. Rev. Lett. 70 (1993) 1203.
7. G.D. Barr et al., Phys. Lett. B317(1993) 233.
8. G. Luders, Dan. Videns Selsk. Mat.-Fys. Medd. 28 (1954) No. 5.
9. W. Pauli in Niels Bohr and the Development of Physics, ed. W. Pauli (McGraw-Hill, New York, 1955).
10. T.D. Lee and C.N. Yang, Phys. Rev. 104 (1956) 254.
11. V.V. Barmin et al., Nucl. Phys. B247 (1984) 293.
12. L.K. Gibbons et al., Phys. Rev. Lett. 70 (1993) 1199.
13. S.V. Somalwar, American Physical Society Meeting, Crystal City, Virginia, USA (March 1994).
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