Restrictions Imposed by the Wave Function on the Results of Particle Momentum Measurements

N. L. Chuprikov*

Tomsk State Pedagogical University, Tomsk, 634041 Russia

*e-mail: chnl@tspu.edu.ru

Received June 6, 2022; revised July 4, 2022; accepted July 4, 2022

Abstract—It is demonstrated with a particle in the one-dimensional configuration space (OCS) that knowledge of the wave function supposes more than statistical restrictions on the results of measurements. In particular, apart from the probability (density) field in the OCS, the wave function also suggests the existence of two fields that predict two (equiprobable) particle momentum values for each OCS point, and the average of these two momenta at each point is related only to the wave function phase, while their difference (coinciding with the Bohm quantum-mechanical potential) is related only to the wave function amplitude. An analogue of the Heisenberg uncertainty relation is obtained for both fields.

DOI: 10.1134/S1547477122060073

1. INTRODUCTION

At the very beginning of his book [1], John Bell writes “To know the quantum mechanical state of a system implies, in general, only statistical restrictions on the results of measurements.” This thought is complemented by the phrase from [2]: “... nonexistence of definite values of measured quantities before the instant of measurement is a fundamental conclusion of the quantum theory in the Copenhagen interpretation.” Though these statements are not equivalent, they are based on the Born rule, which, as is known, assigns a physical meaning only to the square of the wave function modulus. According to the existing formulation of this rule, at the instant of measurement of any single-particle observable, the measurement result can be any of its eigenvalues and the probability of this event is determined by the square of the wave function modulus in the representation of this observable. According to the Copenhagen interpretation, the simultaneous measurement of the particle coordinate and momentum is fundamentally excluded.

All this makes the consistent physical interpretation of quantum theory impossible because, as a result, researchers face an insolvable dilemma: either, like in the “hydrodynamic” formulation of quantum theory [3–5], introduce definite values of the measured quantities as “local hidden variables,” or, as in the orthodox approach, assume that they appear at the instant of measurement. However, the former contradicts Bell’s theory of hidden variables (see [1]), and, in the latter case, a natural question arises as to what these “measurement-generated” values have to do with the microsystem under investigation. Therefore, David Mermin’s phrase “shut up and calculate!” is quite apropos.

At the same time, this position is ultimately unacceptable: quantum mechanics, with its successful computational tools, suggests the only physical interpretation. As to the situation in question, it arises from the fact that the existing formulation of the Born rule [6] reflects only a small fraction of the restrictions incorporated in the mathematical formalism developed by Born himself for calculating averages of observables whose operators do not commute with the coordinate operator. As will be demonstrated in this work using quantum dynamics of a particle in the one-dimensional configuration space (OCS) as an example, this formalism predicts for each observable not only the average value, but also the coordinate and time function, which we will hereafter refer to as the field of the first initial moment of this observable (or briefly the field of its operator).

In Section 2, fields of the momentum and kinetic and total energy operators of the particle are defined, and their relation to the Schrödinger equation and the wave–particle duality relations is shown. In Section 3, the physical meaning of the momentum operator field and the kinetic energy operator field is discussed. These fields define two particle momentum fields. Thus, though definite particle momentum values are really absent before the instant of measurement, knowledge of the wave function decreases the number of possible momentum values at each OCS point to two. In Section 4, it is shown that both fields satisfy the Heisenberg uncertainty relation.
2. FIELDS OF VALUES OF OBSERVABLES IN THE SCHRODINGER FORMALISM

We begin with the Schrödinger equation that describes quantum dynamics of a particle in the external field \( V(x,t) \)

\[
i\hbar \frac{\partial \psi(x,t)}{\partial t} = \hat{H} \psi(x,t),
\]

where \( \hat{H} = \hat{p}^2/2m + V(x,t) \) is the Hamiltonian, and \( \hat{p} = -i\hbar \frac{\partial}{\partial x} \) is the particle momentum operator. As is known (see [3–5]), if the wave function is written as

\[
\psi(x,t) = R(x,t)e^{iS(x,t)/\hbar} = \sqrt{w(x,t)}e^{iS(x,t)/\hbar},
\]

where \( R(x,t) \) is modulus of the wave function \( (\int_{-\infty}^{\infty} w(x,t)dx = 1) \) and \( S(x,t)/\hbar \) is its (real) phase, the Schrödinger equation will be written as a system of two real equations for the functions \( w(x,t) \) and \( S(x,t) \)

\[
\frac{\partial w}{\partial t} + \frac{1}{m} \left( \frac{\partial w}{\partial x} \right)^2 = 0,
\]

\[
\frac{\partial S}{\partial t} + \frac{1}{2m} \left( \frac{\partial S}{\partial x} \right)^2 + K_R + V = 0;
\]

where the function \( K_R(x,t) \) is defined by the expressions

\[
K_R = -\frac{\hbar^2}{2mR} \left( \frac{\partial^2 R}{\partial x^2} \right) \equiv K_w + U_w,
\]

\[
K_w = \frac{\hbar^2}{8mw^2} \left( \frac{\partial w}{\partial x} \right)^2, \quad U_w = -\frac{\hbar^2}{4mw} \left( \frac{\partial^2 w}{\partial x^2} \right)^2.
\]

Here (2) is the continuity equation. As to Eq. (3), in Bohm’s mechanics, it is treated as the Hamilton–Jacobi equation describing the motion of a particle in the external field \( V(x,t) \) and in the field of the so-called quantum-mechanical potential \( K_R(x,t) \). In this approach, the derivative \( \partial S(x,t)/\partial x \) is considered as the particle momentum and the lines of probability current in the OCS as the particle trajectories. We are going to show that the functions \( \partial S(x,t)/\partial x \) and \( K_R(x,t) \), as well as the lines of the probability current imply another interpretation of the quantum-mechanical potential, see also [7]).

According to Born’s interpretation, the square of the wave function modulus in the representation of any single-particle observable defines only the probability for appearance of eigenvalues of this observable, while the wave function phase has no direct relation to observables. Hence there follows the very conclusion that, in standard quantum mechanics, knowledge of the wave function suggests only statistical restrictions on the results of measurements of observables. However, there is reason to believe that this interpretation covers by no means all the information about observables that is contained in the wave function and the standard formula obtained by Born for calculating averages of observables whose operators do not commute with the coordinate operator. To look into this matter is our main goal.

According to Born, expression \( w(x,t)dx \) gives the probability for observing the particle in the interval \([x, x + dx]\), and the average of its coordinate and any observable \( f \) whose operator commutes with the coordinate operator \( \hat{x} \) is defined as the first initial moment of a random quantity in classical probability theory

\[
\langle f \rangle = \int_{-\infty}^{\infty} f(x,t)w(x,t)dx.
\]

As to the particle momentum and any other observable \( \hat{O} \) whose (self-adjoint) operator \( \hat{O} \) does not commute with the coordinate operator, their averages are defined in the coordinate representation by the integral

\[
\langle \hat{O} \rangle = \int_{-\infty}^{\infty} \hat{\psi}^*(x,t)\hat{O}\psi(x,t)dx,
\]

which, as is assumed in the existing formulation of the Born rule, cannot in principle be reduced to the form (5).

Note, however, that the Born rule puts the self-adjoint operator \( \hat{O} \) into one-to-one correspondence not only with integral, the average of this operator, but also with the integrand function \( \Re[\hat{\psi}^*(x,t)\hat{O}\psi(x,t)] \), which obviously should have the same meaning as the function \( f(x,t)w(x,t) \) in integral (5). Let us make sure of this using the particle momentum, kinetic energy, and total energy operators.

We begin with the momentum operator. Considering expression (1) for the wave function, it is easy to show that

\[
\Re[\hat{\psi}^*(x,t)\hat{p}\psi(x,t)] = \frac{\partial S(x,t)}{\partial x} w(x,t)
\]

\[
\equiv p(x,t)w(x,t).
\]

As is seen, the integrand in formula (6) for the momentum operator is uniquely reduced to the form (5) without any additional assumptions, thus determining not only the average of the momentum operator, but also the function \( p(x,t) = \partial S(x,t)/\partial x \) — the field of the momentum operator (or the field of the first initial moment of the momentum). The average of this field weighted by probability density \( w(x,t) \) is by definition equal to the average of the momentum operator. Therefore, contrary to the existing interpretation of the wave function, its phase has a physical meaning within the axioms of standard quantum mechanics.
Equations (2) and (3) can now be written as
\[
\frac{\partial w}{\partial t} + \frac{1}{m} \frac{\partial (wp)}{\partial x} = 0, \quad (8)
\]
\[-\hbar \omega + K + V = 0, \quad (9)
\]
where the function \( K(x,t) \) is related to the contribution \(-\hbar^2 \frac{\partial^2 \psi}{2m \partial x^2}\) in the Schrödinger equation,
\[
K(x,t) = \frac{1}{2m} [p(x,t)]^2 + K_R(x,t). \quad (10)
\]

If, as in the case of the wave \( e^{ikx-\omega t} \), where \( k \) and \( \omega \) are the constants representing the wave number and the wave frequency respectively, we introduce the wave number field \( k(x,t) \) and the frequency field \( \omega(x,t) \) of the wave function (wave packet) of the general form,
\[
k(x,t) = \frac{1}{\hbar} \frac{\partial S(x,t)}{\partial x}, \quad \omega(x,t) = -\frac{1}{\hbar} \frac{\partial S(x,t)}{\partial t}, \quad (11)
\]
the equality defining the momentum operator field will be written as
\[
p(x,t) = \hbar k(x,t), \quad (12)
\]
which is an analogue of the known de Broglie relation.

The next field to be introduced is the field of the kinetic energy operator \( \hat{K} \). It is easy to show that this field exactly coincides with the function \( K(x,t) \) that enters into (9)
\[
\Re \{ \psi^*(x,t) \hat{K} \psi(x,t) \} = \left( \frac{1}{2m} [p(x,t)]^2 + K_R(x,t) \right) w(x,t) \equiv K(x,t) w(x,t). \quad (13)
\]
Thus, the standard formula
\[
\langle K \rangle = \int_{-\infty}^{\infty} \psi^*(x,t) \hat{K} \psi(x,t) dx
\]
uniquely determines not only the average of the operator \( \hat{K} \), but also the function \( K(x,t) \), which is the field of the kinetic energy operator.

To show characteristic features of this field, we consider the ground state of the quantum harmonic oscillator in the external field \( V = \frac{1}{2} m \omega^2 x^2 \)
\[
\psi(x,t) = \sqrt{\frac{m \omega}{\hbar \pi}} \exp \left(-\frac{m \omega^2 x^2}{2 \hbar} - i \omega t \right). \quad (15)
\]
For this state, \( p(x,t) \equiv 0 \) holds and the functions \( K_w \) and \( U_w \) (see (4)) are defined by the expressions
\[
K_w = \frac{1}{2} m \omega^2 x^2; \quad U_w(x,t) = \frac{\hbar \omega}{2} - m \omega^2 x^2.
\]
Thus, for this state we have
\[
K(x,t) = K_w + U_w = \frac{\hbar \omega}{2} - \frac{m \omega^2 x^2}{2} \equiv E_0 - V(x). \quad (16)
\]

Hence it follows that the function \( K(x,t) \) takes on negative values in classically inaccessible OCS regions where the total particle energy \( E_0 \) is lower than the potential one.

Note that this feature of the field \( K(x,t) \) is not in conflict with quantum mechanics according to which average values of the operator \( \hat{K} \) cannot be negative. First, it is the integrand function \( \Re \{ \psi^*(x,t) \hat{K} \psi(x,t) \} \) in the standard definition that takes on negative values in the OCS region classically inaccessible to the oscillator. Therefore, if integral itself has a physical meaning, the contributions to this integral that correspond to classically inaccessible region also have a physical meaning (see also Section 3). Second, the average value of the field \( K(x,t) \) is knowingly a nonnegative quantity, since the function \( U_w \) (see ), due to which \( K(x,t) \) can take on negative values, is such that
\[
\int_{-\infty}^{\infty} U_w(x,t) w(x,t) dx = 0 \quad \text{for any (normalized) wave function.}
\]

The field \( H(x,t) \) of the operator of the Hamiltonian \( \hat{H} \)
\[
\Re \left[ \psi^*(x,t) \hat{H} \psi(x,t) \right] = \left( \frac{1}{2m} [p(x,t)]^2 + K_R(x,t) + V(x,t) \right) w(x,t) \quad (17)
\]
is defined analogously.

Considering this field, Eq. (9) takes the form
\[
\hbar \omega(x,t) = H(x,t) \equiv \frac{p^2}{2m} + K_R + V. \quad (18)
\]

Differentiating this equation term by term for definitions (11) and (12), we have
\[
\frac{\partial p}{\partial t} + \frac{p}{m} \frac{\partial p}{\partial x} = -\frac{\partial K_R}{\partial x} - \frac{\partial V}{\partial x}. \quad (19)
\]

Equations (8) and (19) make up a closed system of equations for the probability field \( w(x,t) \) and the momentum operator field \( p(x,t) \).

As to Eq. (18), it is an analogue of the Planck–Einstein relation and thus complements equality (12), defining the momentum operator field. These two relations, which connect the corpuscular and wave properties of a quantum particle when its state is described by a wave packet rather than by the de Broglie wave, are exact copies of the known wave–particle duality relations. However, since the meaning of the wave characteristics (wave number and frequency) involved in those relations change in going from the de Broglie wave to
the wave packet, it is natural to expect that the physical meaning of the particle characteristics (momentum and total energy) should also change.

3. FIELDS OF PARTICLE MOMENTUM VALUES

As is known, the modern concept of the single-particle quantum ensemble is based on the (seemingly obvious) assumption that one particle corresponds to each point \( x \) in the OCS, i.e., one system of the single-particle quantum ensemble. However, our analysis shows that this assumption is valid only if the state of the single-particle ensemble is described by the de Broglie wave; in the case of wave packets, when the function \( K_R(x,t) \) is not identically equal to zero, this assumption should be revised.

The point is that the field of the kinetic energy operator \( K(x,t) \) is determined at each OCS point not only by the field of the momentum operator \( p(x,t) \) (see the first contribution in (10)), but also by the field \( K_R(x,t) \), which is not zero when \( p(x,t) \equiv 0 \). In the context of the König theorem, kinetic energy \( K(x,t) \) with two such contributions describes a system of particles rather than a single particle. The contribution \( \{p(x,t)^2\}/2m \) describes the center-of-mass kinetic energy of this system of particles at the point \( x \) at the time \( t \), and the contribution \( K_R(x,t) \) describes the kinetic energy of the motion of the particles in this system relative to its center of mass. That is, the field of the momentum operator \( p(x,t) \) describes not the value of the momentum of a single particle, but the (local) average value of the momentum of the particles in the system at the point \( x \) at the time \( t \). Similarly, the field \( K(x,t) \) describes the (local) average value of the kinetic energy of this system of particles.

All this means that the above assumption is wrong, and each OCS point should be represented in the single-particle ensemble by two rather than one single-particle system (particle) (a different application of the König theorem to this problem can be found in [8]). In other words, with each OCS point at each instant of time one should associate not one particle momentum value, as is done in Bohm’s mechanics, but two values (we denote them by \( p_1(x,t) \) and \( p_2(x,t) \)), which, according to the König theorem, should satisfy the equations

\[
\frac{1}{2} (p_1 + p_2) = p, \quad \frac{1}{2} \left( \frac{p_1^2}{2m} + \frac{p_2^2}{2m} \right) = K \equiv \frac{p^2}{2m} + K_R. \tag{20}
\]

Solutions of these equations are a pair of momentum fields

\[
p_1 = p - \sqrt{2mK_R}, \quad p_2 = p + \sqrt{2mK_R}. \tag{21}
\]

Contrary to Bohm’s mechanics, this formalism excludes the introduction of single-particle trajectories (see also [9]). Now Eqs. (8) and (19) can be written in the form of a closed system of equations for the momentum fields \( p_1 \) and \( p_2 \) and the probability field \( w \)

\[
\frac{\partial w}{\partial t} + \frac{1}{m} \frac{\partial (wp)}{\partial x} = 0,
\]

\[
\frac{\partial p}{\partial t} + \frac{p \partial p}{m} = -\frac{\partial}{\partial x} \left( \frac{p_R^2}{2m} \right) - \frac{\partial V}{\partial x}, \tag{22}
\]

It follows from the continuity equation that trajectories of the center of mass of particle pairs in the OCS are lines of the probability current. From the second equation, it follows that the function \( K_R(x,t) \equiv p_R^2/2m \), which describes kinetic energy of the motion of the particles in the pair relative to the center of mass, plays a dual role: on the one hand, it describes one of two contributions to the field of the kinetic energy operator and, on the other hand, it plays the role of the external field (additional to the field \( V(x,t) \)), in which centers of mass of particle pairs move in the OCS.

According to (19), the inhomogeneous distribution of the kinetic energy \( K_R(x,t) \) in the OCS results in the centers of mass of the particle pairs being forced to leave the OCS regions where this energy is higher for the regions where it is lower. In other words, this mechanism gives rise to probability flows in the OCS, which tend to equalize this distribution, thus causing the wave packet spreading. Stationary distributions arise when the above mechanism is balanced by the action of the external field \( V(x) \).

It should be stressed that it is due to this mechanism that centers of mass of particle pairs (whose kinetic energy, \( p^2/2m \), is always nonnegative) find themselves in classically inaccessible OCS regions. If a pair of particles (and consequently its center of mass) is in the OCS region where the function \( K_R(x,t) \) is negative, this function plays the role of a potential well that binds this pair of particles into an inseparable whole (“atom”). To detect a particle at the point where \( K_R(x,t) < 0 \), the energy \( |K_R(x,t)| \) should be spent for extracting it from this potential well.

4. UNCERTAINTY RELATION FOR MOMENTUM FIELDS

So, while in the standard formulation of the Born rule the momentum of the particle before its observation at one or another point in the OCS can be equal to any of the momentum operator eigenvalues, this is not the case in the new formulation. The proposed approach brings quantum mechanics closer to classical mechanics. First, according to the results obtained, in quantum mechanics one should
distinguish, as in classical mechanics, between classically accessible and classically inaccessible regions of the SSC. Secondly, although at any point in the classically accessible region of the OCS the particle indeed does not have a definite momentum value until the moment of measurement, this uncertainty is now much narrower: in an infinite set of identical experiments, a particle observed with a probability \( w(x,t)dx \) in the interval \([x, x + dx]\) of a classically accessible region can (equiprobably) have only two momentum values, \( p_1(x,t) \) and \( p_2(x,t) \). In a classically inaccessible region, the momentum of the particle (extraction of values, and \( x \)) in the interval \([x, x + dx]\) of a classically accessible region can (equiprobably) have only two momentum values in an infinite set of identical experiments, a particle observed with a probability \( w(x,t)dx \) in the interval \([x, x + dx]\) of a classically accessible region can (equiprobably) have only two momentum values, \( p_1(x,t) \) and \( p_2(x,t) \). In a classically inaccessible region, the momentum of the particle (extraction of which from the potential well requires spending the energy \( |K_p(x,t)| \) is \( p(x,t) \); i.e., it is equal to the momentum of the center of mass of the particle pair.

Let us show that the deviation of fields \( p_1(x,t) \) and \( p_2(x,t) \) from the field \( p(x,t) \) satisfies the Heisenberg inequality. To this end, we calculate the product of “dispersions” \( D_p \) and \( D_x \)

\[
D_p = \int_{-\infty}^{\infty} \left[ (p_1(x,t) - p(x,t))^2 \right] w(x,t)dx,
\]

\[
D_x = \int_{-\infty}^{\infty} (x - <x>)^2 w(x,t)dx,
\]

where \( <x> = \int_{-\infty}^{\infty} xw(x,t)dx \). Considering definitions (21) and (4), as well as the fact that the function \( w(x,t) \) and its derivatives are zero at infinity, we reduce the expression for \( D_p \) to the form

\[
D_p = \frac{\hbar^2}{4} \int_{-\infty}^{\infty} \left[ \frac{1}{w^2} \left( \frac{\partial w}{\partial x} \right)^2 - \frac{2}{w} \frac{\partial^2 w}{\partial x^2} \right] wdx,
\]

\[
= \frac{\hbar^2}{4} \int_{-\infty}^{\infty} \left[ \frac{1}{w^2} \left( \frac{\partial w}{\partial x} \right)^2 \right] wdx.
\]

Further, using the Cauchy–Bunyakovsky theorem for quadratically integrable functions and integrating by parts, we obtain the desired inequality

\[
D_pD_x = \left( \frac{\hbar^2}{4} \int_{-\infty}^{\infty} \left[ \frac{1}{w^2} \left( \frac{\partial w}{\partial x} \right)^2 \right] wdx \right)
\]

\[
\times \left\{ \int_{-\infty}^{\infty} (x - <x>)^2 wdx \right\} \geq \frac{\hbar^2}{4} \int_{-\infty}^{\infty} \left[ \frac{1}{w} \frac{\partial w}{\partial x} \right] (x - <x>)^2 wdx = \frac{\hbar^2}{4}.
\]

This proves the statement.

5. CONCLUSIONS

Knowledge of the quantum-mechanical state suggests more than statistical restrictions on the results of measurements. Together with the Schrödinger equitation for the wave function modulus and phase, the Born formalism for calculating average values of operators of observables actually uniquely determines not only average values of observables, but also fields of the first initial moments of these observables (or fields of operators for short)—the coordinate and time functions. Of key importance for establishing the physical meaning of these fields is the fact that the field of the kinetic energy operator involves two unlike contributions, one determined by the field of the momentum operator, which is only related to the wave function phase, and the other coinciding with the so-called quantum-mechanical potential, which is only related to the wave function modulus.

In other words, in a single-particle quantum ensemble whose state is described by the wave packet, the values of the field of the momentum operator and the field of the kinetic energy operator at any given OCS point at any given instant of time should be associated with two particles rather than one (i.e., with two single-particle systems of the ensemble, in which particles are at that very OCS point at that very instant of time): the first contribution to the field of the kinetic energy operator describes the kinetic energy of the center of mass of this particle pair, and the second describes the kinetic energy of the motion of these particles relative to their center of mass. Thus, the values of the fields of the momentum operator and the kinetic energy operator at this point describe average values of the momentum and kinetic energy of the particles in this pair; knowing these two fields, one can unambiguously determine the fields of momentum values for the particles of the pair at this point.

The kinetic energy of the relative motion of particles in a pair, which coincides with the quantum-mechanical potential, can be negative (this occurs in classically inaccessible regions in the case of bound stationary states of the harmonic oscillator). In these regions, the particle pair (whose position coincides by definition with the center of mass) finds itself in a potential well with the depth equal to the absolute value of the quantum-mechanical potential. At these OCS points, particle pairs form “bound states” and their detection requires spending energy for extracting them from this potential well.

As to the kinetic energy of the centers of mass of particle pairs, it is nonnegative at all points in the OCS. Therefore, there are no OCS regions inaccessible to centers of mass of particle pairs, and trajectories of centers of mass of particle pairs are lines of the probability current. Their motion in the OCS is governed not only by the gradient of the potential energy of a particle in the external field, but also by the gradient of the kinetic energy of the relative motion of particles in pairs: the inhomogeneous distribution of this energy gives rise to flows of centers of mass of particle pairs (probability flows) in the OCS, which tend to
smooth the distribution, and this, in turn, causes the wave packet spreading. It is this mechanism that leads to the centers of mass of the particle pairs finding themselves in classically inaccessible OCS regions. Stationary distributions arise if this mechanism is balanced by the action of the external field.

Summing up, we consider it necessary to point out the following:

• we regard the proposed approach as a further development of the statistical (ensemble) interpretation of quantum mechanics; though this interpretation is unpopular now, it is this interpretation that, in our mind, captures the essence of this theory (see also the recent work [10]);
• a quantum ensemble of one-dimensional single-particle systems in the given state is characterized not only by the field of probability (density) as a function in the OCS, but also by the fields of operators of single-particle observables;
• the values of these fields, including the field of probability, describe not one but two ensemble’s systems at each OCS point; since the ensemble is a single-particle one, this is equivalent to that all these fields describe a pair of (noninteracting) particles at each OCS point; the measurement of these fields implies conducting experiments with beams of (noninteracting) particles;
• momentum values for particles in pairs are uniquely reconstructed from the values of the field of the momentum operator and the field of the kinetic energy operator; the measurement of both momentum fields of particles in pairs implies the conduct of (strictly speaking, an infinite set of) identical single-particle experiments;
• the expressions obtained in the work, which relate two particle momentum fields to the wave function amplitude and phase at each OCS point, indicate that an individual particle possesses wave properties; the existence of these two fields can be regarded as an answer to the question used as a title of [10]: “Does a single electron have wave properties?”;
• lines of the probability current cannot serve as single-particle trajectories of a particle—it is impossible to predict particle trajectories in quantum mechanics; one exclusion is the case where the state of a quantum single-particle ensemble is described by the de Broglie wave (in this case, the field of the kinetic energy operator has only the first contribution, which is related to the field of the momentum operator);
• this approach validates the conclusion (see [2]) about the absence of definite values of observables before the instant of measurement; however, knowledge of the quantum-mechanical state appreciably narrows this indefiniteness. In particular, only two equiprobable particle momentum values are possible at each OCS point before the instant of measurement (nevertheless, despite this radical narrowing of the indefiniteness, these two momentum fields satisfy the Heisenberg uncertainty relation).

REFERENCES
1. J. S. Bell, *Speakable and Unspeakable in Quantum Mechanics* (Cambridge Univ. Press, Cambridge, 1987).
2. A. V. Belinskii and A. A. Klevtsov, “Nonlocal classical ’realism’ and quantum superposition as the nonexistence of definite pre-measurement values of physical quantities,” Phys. Usp. 61, 313 (2018).
3. E. Madelung, “Quantentheorie in hydrodynamischer Form,” Z. Phys. 40, 332 (1926).
4. L. de Broglie, “L’interpretation de la mecanique ondulatoire,” J. Phys. Radium 20, 963 (1959).
5. D. Bohm, “A suggested interpretation of the quantum theory in terms of “hidden” variables. I,” Phys. Rev. 85, 166 (1952).
6. M. Born, “Quantenmechanik der Stossvorgange,” Z. Phys. 38, 803 (1926).
7. G. Grössing, “On the thermodynamic origin of the quantum potential,” Phys. A (Amsterdam, Neth.) 388, 811 (2009).
8. J. Salesi, “Spin and Madelung fluid,” Mod. Phys. Lett. A 11, 1815 (1996).
9. A. V. Belinsky, “Is David Bohm’s quantum mechanics interpretation irrefutable?,” Mosc. Univ. Phys. Bull. 73, 351 (2018).
10. V. A. Bednyakov, “Does a single electron have wave properties?,” Phys. Part. Nucl. Lett. 18, 413 (2021).

Translated by M. Potapov