An integrated deterministic-stochastic approach for predicting the long-term trajectories of COVID-19

Indrajit Ghosh1a, Tanujit Chakraborty2a

*Indian Statistical Institute, Kolkata - 700 108, West Bengal, India

Abstract

The ongoing COVID-19 pandemic is one of the major health emergencies in decades that affected almost every country in the world. As of May 10, 2020, it has caused an outbreak with more than 41,78,000 confirmed infections and more than 2,83,000 reported deaths globally. Due to the unavailability of an effective treatment (or vaccine) and insufficient evidence regarding the transmission mechanism of the epidemic, the world population is currently in a vulnerable position. The daily cases data sets of COVID-19 for profoundly affected countries represent a stochastic process comprised of deterministic and stochastic components. This study proposes an integrated deterministic-stochastic approach to predict the long-term trajectories of the COVID-19 cases for Italy and Spain. The deterministic component of the daily-cases univariate time-series is assessed by an extended SIR (SIRCX) model whereas its stochastic component is modeled using an autoregressive (AR) time series model. The proposed integrated SIRCX-AR (ISA) approach based on two operationally distinct modeling paradigms utilizes the superiority of both the deterministic SIRCX and stochastic AR models to find the long-term trajectories of the epidemic curves. Experimental analysis based on the proposed ISA model suggest that the estimated numbers of cases in Italy and Spain between 11 May, 2020 – 09 June, 2020 will be 10982 (6383–15582) and 13731 (3395–29013), respectively. Additionally, the expected number daily cases on 09 July, 2020 for Italy and Spain are estimated to be 30 (0–183) and 92 (0–602), respectively. These log-term forecasts for Italy and Spain of the coming outbreaks will be very useful for the effective allocation of health care resources to mitigate COVID-19.

Keywords: COVID-19; Integrated approach; Prediction; Extended SIR model; AR model.

1. Introduction

Coronavirus disease 2019 (COVID-19) is a rapidly spreading disease transmitted between people via respiratory droplets. The COVID-19 pandemic is the most significant global crisis since the World War-II that affected almost all the countries throughout the world. The World Health Organization (WHO) declared COVID-19 a “global pandemic” on March 11, 2020. A notable characteristic of COVID-19 is its ability to cause unusually large case clusters.
via superspreading in the absence of any specific antiviral treatment available to cure COVID
(15). The health consequences of the pandemic are devastating and it has also triggered a
global health concern (2). This has bought the scientific community to come up with different
short-term and long-term forecasting models for the better understanding of the pandemic and
mitigate the effects of this.

So far, various mathematical, statistical and machine learning models have been deployed
to predict the disease dynamics and also to assess the efficiency of the intervention strategies in
reducing the burden of COVID-19 (14; 9; 16; 8; 6). In previous studies, mathematical modeling
approaches (e.g. SIR and SEIR models) (13; 11) performed more effectively for predicting the
long-term trajectories of the epidemic whereas stochastic forecasting models (classical ARIMA
and Wavelet-based forecasting model) (4) gave superior real-time short-term forecasts for some
profoundly affected countries. However, a close look at the daily cases time series data sets
of confirmed COVID-19 cases for different countries exhibits both the deterministic trend and
stochastic behavior in the series. This indicates that the predictions of long-term trajectories
of COVID-19 daily cases can be greatly improved by integrating both the deterministic and
stochastic approaches. This study attempts to blend deterministic and stochastic methods for
studying the long term trajectories of COVID-19 cases for Italy and Spain.

Motivated by the above discussion, we propose an integrated deterministic-stochastic model
to describe the long-term trajectories of COVID-19 daily cases. The underlying characteristics
of the epidemic curves show both the deterministic and stochastic nature that intrigued the
need for developing an integrated deterministic-stochastic model to forecast the long-term tra-
jectories of COVID-19 cases. In the first stage of the model, an extended version of SIR model
(SIRCX) is built that preserves the characteristic of the deterministic process in the long run.
The deterministic SIRCX model leaves a certain part of the examined time series unexplained
and we overcome these uncertainties with the help of a stochastic model. In the second stage
of the integrated approach, a stochastic autoregressive (AR) model is applied to analyze the
uncertain behavior of the residuals produced by the compartmental SIRCX model. This newly
introduced integrated SIRCX-AR model, we call it as ISA model, exploits the benefits of two
methodologically contrasting paradigms and overcome their individual shortcomings. The pro-
posed ISA model preserves both the long-term and short-term characteristics of the current
pandemic time series when applied to the daily confirmed cases data sets of Italy and Spain.
This combined approach is also useful for explaining complex autocorrelation structures in the
COVID-19 time-series data and reduce the inductive bias and variances of the individual mod-
els from a modeler’s point of view. Besides that, we estimate the basic reproduction number,
expected total cases after one month and expected daily cases after two months. In the absence
of vaccines or antiviral drugs for COVID-19, these estimates will provide an insight into the
resource allocations for Italy and Spain to keep this epidemic under control.

The remainder of this paper is organized as follows. In Section 2, we discuss the data
sets, data pre-processing steps and the developments of the integrated deterministic-stochastic
approach. In Section 3, the experimental findings are presented. Finally, the limitations of our
methodology along with discussions and future directions of the paper are given in Section 4.
2. Data and Methods

2.1. Data

We focus on the daily figures of confirmed cases for France and Spain. These data sets are retrieved by the Global Change Data Lab\(^1\). All these data sets are collected from the date on which the total number of cumulative daily cases for each of these countries crossed a count of 100. For Spain we collected daily confirmed cases data from March 3 to May 10, 2020 whereas for Italy, it was collected from February 24 up to May 10, 2020. The univariate time series data set for Spain contains a total of 69 observations and 77 observations for Italy. In the next subsections, we discuss the data pre-processing steps followed by the development of the proposed integrated approach.

2.2. Data Pre-processing Stage

In the data pre-processing stage, we verify that whether the time series data sets of Italy and Spain contain long-term memory and have both the deterministic and stochastic components in it or not (5). The long term memory property of a time series is measured using the Hurst exponent (HE). The value of HE lying between 0.5 and 1 proves that the series is sufficiently long. To check the HE for the given data sets, we use \textit{pracma} package in R statistical software. For daily cases data of Spain the value of HE is 0.775 and 0.753 for Italy. Thus, it is confirmed that the data sets have long-term memory. Next, we test if there are deterministic terms in the daily cases data sets using seasonal Mann-Kendall (M-K) trend test with an R package \textit{Kendall}. M-K test is a nonparametric test where the alternative hypothesis corresponds to the existence of a deterministic trend. The two-sided p-value of 0.808 for the series of Italy and 0.856 for the Spain data set confirmed that there exists trends (deterministic nature) in the data. These tests confirmed the underlying characteristic of COVID-19 having both the deterministic and stochastic behaviors and intrigued the need for an integrated deterministic-stochastic approach.

2.3. Proposed Integrated SIRCX-AR (ISA) Model

To find the long-term forecasts of the confirmed cases of COVID-19, we adopt an integrated deterministic-stochastic approaches combining compartmental SIRCX and stochastic AR model. The proposed combined model blends two contrasting modeling paradigms to exploit their individual benefits and overcome their shortcomings. In general, the proposed modeling process consists of three major steps: (a) modeling the deterministic components with an extended SIR (SIRCX) model; (b) evaluation of the SIRCX model residuals; (c) stochastic modeling of residual values by traditional AR model. The daily COVID-19 cases $Q(t)$ comprises both the deterministic ($D(t)$) and stochastic ($P(t)$) components as follows:

$$Q(t) = D(t) + P(t).$$ (2.1)
To predict the long-term trajectories of the COVID-19 cases and better understanding the epidemic curve, the deterministic model is initially used followed by its residuals which consist some unexplained stochastic components \((P(t))\). This assumption holds for the COVID-19 data sets since the residual series produced by the mathematical model has no deterministic terms, as shown in Figure 2. Now, we can estimate both \(D(t)\) and \(P(t)\) from the available daily cases data set. Let, \(\hat{D}(t)\) be the long-term forecasts based on the SIRCX model at time \(t\) and \(P(t)\) represent the residuals containing the stochastic components at time \(t\), obtained from the SIRCX model. Thus, we write

\[
P(t) = Q(t) - \hat{D}(t).
\]

These residuals is remodeled with an AR model and predictions are obtained as \(\hat{P}(t)\). Therefore, we write the combined forecast as:

\[
\hat{Q}(t) = \hat{D}(t) + \hat{P}(t).
\]

The proposed ISA model is a joint approach based on deterministic and stochastic methods. Figure 1 shows the systematic overview of the proposed integrated SIRCX-AR or simply ISA model. Below we discuss about the constituent models (SIRCX and AR) used in the ISA model in details.

![Flow diagram of the proposed Integrated SIRCX-AR (ISA) model](https://doi.org/10.1101/2020.05.13.20101303)

Figure 1: Flow diagram of the proposed Integrated SIRCX-AR (ISA) model
2.3.1. Modeling deterministic components with SIRCX Model

We propose an extension of a generalized SIR modeling (11), namely SIRCX model that reflects the known epidemiology of COVID-19. When basic epidemiological parameters are largely uncertain, dynamical equation based compartmental models can often provide some insights on the long-term dynamics. Here, the total population is assumed to be constant, viz. we assume the birth rate and natural death rate have the same value ($\mu$). We consider five disjoint compartments in the SIRCX model: susceptible $S(t)$, infected $I(t)$, recovered $R(t)$, protected $C(t)$ and isolated $X(t)$. The susceptible individuals can become infected after a successful contact with an infected person. We also assume that isolated persons can not transmit the disease whereas the protected people exercise social distancing and can not become infected. The dynamics of COVID-19 are governed by the following system of equations:

\[
\begin{align*}
\frac{dS}{dt} &= \mu(N - S) - \frac{\beta SI}{N} - \alpha S, \\
\frac{dI}{dt} &= \frac{\beta SI}{N} - (\gamma + \alpha + \xi + \mu)I, \\
\frac{dR}{dt} &= \gamma(I + X) - \mu R, \\
\frac{dC}{dt} &= \alpha S - \mu C, \\
\frac{dX}{dt} &= (\alpha + \xi)I - (\mu + \gamma)X,
\end{align*}
\]

(2.2)

where $N = S + I + R + C + X$. The parameters of system (2.2) are the containment rate ($\alpha$), the infection rate ($\beta$), the recovery rate ($\gamma$), the isolation rate ($\xi$) and the natural death and birth rate ($\mu = (\text{Life expectancy})^{-1}$). Basic Reproduction number (see details in Appendix A) for SIRCX model is found to be $R_0 = \frac{\beta S_0}{N(\gamma + \alpha + \xi + \mu)}$, where $S_0$ is the initial number of susceptibles in the system.

2.3.2. Modeling stochastic components with AR model

Once the deterministic trend is modeled by SIRCX model, we can now remodel the left-out uncertainties (error terms) with an AR model. AR model provides a parsimonious description of a (daily) stationary time-dependent stochastic process in terms of autoregressive terms. AR($p$) denotes the AR model where the parameter $p$ is the order of the model. Since we removed the deterministic component of the series by compartmental model and AR is applied to model the residual components of SIRCX, thus no differencing and moving average terms are required in the model. AR($p$) model can explain the momentum and mean reversion effects of the unexplained error terms that the deterministic model could not capture. AR model can be
mathematically expressed as follows:

\[ Z_t = c + \varepsilon_t + \sum_{i=1}^{p} \phi_i Z_{t-i}, \]

where \( Z_t \) denotes the actual value of the variable under consideration at time \( t \), \( \varepsilon_t \) is the random error at time \( t \), \( c \) is a constant and \( \phi_i \) is the coefficients of the AR model. The ‘best’ fitted AR model is selected based on the Akaike’s information criteria (AIC) (3).

**Remark.** The proposed ISA model combines two contrasting paradigms to forecast long-term trajectories of COVID-19 data. We choose two completely diverse models (SIRCX and AR) for hybridization, one from mathematical epidemiology literature and another from the area of traditional time series forecasting. The newly introduced integrated model will be practically useful for better understanding the dynamics of the complicated COVID-19 pandemic, as shown in the next section.

### 3. Experimental Analysis

In the proposed ISA model, the deterministic trend in the series is first modeled using SIRCX model (2.2). To perform the data fitting, we used a simplex algorithm-based function ‘fminsearchbnd’ (MatLab, R2016a) that minimizes the sum of squared errors (SSE) between simulated indicator

\[ D_{ODE}(t) = \int_{t-1}^{t} [(\alpha + \xi)I] \, dt \]

and reported data. Suppose we have \( M \) independent observations of daily COVID-19 cases, then we minimize \( \sum_{t=1}^{M}(Q(t) - D_{ODE}(t))^2 \), where \( Q(t) \) is daily reported COVID-19 cases.

| Country | Parameter/ICs | Value | Reference |
|---------|---------------|-------|-----------|
| Italy   | \( \beta \)   | 0.3745| Estimated |
|         | \( \alpha \)  | 0.0470| Estimated |
|         | \( \xi \)     | 0.01  | Estimated |
|         | \( \gamma \)  | 0.0213| (6)       |
|         | \( \mu \)     | \( 3.2616 \times 10^{-5} \) | (1)       |
|         | \( N \)       | 6,04,61,826 | (1)       |
|         | \( I(0) \)    | 2345  | Estimated |
| Spain   | \( \beta \)   | 0.3468| Estimated |
|         | \( \alpha \)  | 0.0396| Estimated |
|         | \( \xi \)     | 0.01  | Estimated |
|         | \( \gamma \)  | 0.057 | (10)      |
|         | \( \mu \)     | \( 3.2616 \times 10^{-5} \) | (1)       |
|         | \( N \)       | 4,67,54,778 | (1)       |
|         | \( I(0) \)    | 7912  | Estimated |
The values of the fixed parameters, initials conditions and estimated parameters are reported in Table 1. Using the fitted SIRCX model, we obtain the predicted values and two month ahead forecasts for COVID-19 cases of Italy and Spain.

The leftover residuals obtained from SIRCX model (also see Figure 2) are further modeled with an AR($p$) model by using ‘forecast’ package in R statistical software. The order of AR($p$) model is specified from the autocorrelation function plots (given in Figure 3). AR(5) was fitted to the residual series of Italy data having AIC = 1171.59 and log-likelihood value as -576.79. For Spain, AR(3) was fitted on the residual series with AIC=1168.08 and Log-likelihood value equals -556.23. Using the ‘best’ fitted AR models, two month ahead mean forecasts of the residuals are generated for Italy and Spain along with the confidence intervals.

Finally, the ODE generated forecasts and AR-based residual forecasts were added together to get the final forecasts. The ODE model predictions and ISA model forecasts are then compared using four widely used goodness-of-fit metrics, namely mean absolute percentage error (MAPE), symmetric MAPE (SMAPE), root mean squared error (RMSE) and mean absolute error (MAE). By convention, the models with lower values of goodness-of-fit metrics are capable of giving better forecasts. The formulae to evaluate these performance metrics are as follows:
(a) MAPE = \frac{1}{n} \sum_{i=1}^{n} \frac{|\hat{y}_i - y_i|}{|\hat{y}_i|} \times 100; \quad (b) SMAPE = \frac{1}{n} \sum_{i=1}^{n} \frac{|\hat{y}_i - y_i|}{(|\hat{y}_i| + |y_i|)/2} \times 100; \\
(c) RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2}; \quad (d) MAE = \frac{1}{n} \sum_{i=1}^{n} |y_i - \hat{y}_i|;

where \( y_i \) are reported cases, \( \hat{y}_i \) are the predicted values and \( n \) denotes the total number of observations. The values of these metrics for Italy and Spain are reported in Table 2. From Table 2, we can conclude that the integrated SIRCX-AR model outperformed the individual SIRCX model in the training COVID-19 data sets for Italy and Spain in a large margin.

Table 2: Performance metrics of SIRCX model and ISA model for Italy and Spain

| Country | Metrics | ODE Model | ISA Model |
|---------|---------|-----------|-----------|
| Italy   | MAPE    | 19.09     | 14.99     |
|         | SMAPE   | 20.06     | 15.85     |
|         | RMSE    | 585.99    | 431.58    |
|         | MAE     | 442.82    | 334.77    |
| Spain   | MAPE    | 36.49     | 20.68     |
|         | SMAPE   | 35.88     | 22.72     |
|         | RMSE    | 1186      | 761.94    |
|         | MAE     | 929.17    | 581.92    |

Figure 4: Fitting and long-term forecast of COVID-19 cases in (a) Italy and (b) Spain.

We predicted two month ahead forecasts (long-term) for both Spain and Italy to understand the future outbreaks of the pandemic for Italy and Spain. The plots of actual vs predicted values along with the long-term forecast values are given in Figure 4. Figure 4 depicts that the proposed ISA approach fitted the training data more adequately in comparison with the ODE-based SIRCX model. The band-width presented in Figure 4 shows slow decay in number of future COVID-19 outbreaks in Italy and Spain. The predicted vs actual forecasts suggest that the integrated approach are more accurate than simply relying on the compartmental model. The proposed ISA model assessed the asymmetric risks involved in the series way better than
the individual epidemiological model. The proposed ISA model can easily be updated on a periodic basis (weekly) once more data becomes available.

Using parameter values from Table 1, the basic reproduction numbers for Italy and Spain are found to be 4.78 and 3.25, respectively. These values indicate that COVID-19 spreads very rapidly in a population unless containment strategies such as social distancing and effective isolation of infected individuals are implemented. From the long term trajectories we estimated the expected number of cases in the next 30 days (11 May, 2020 – 09 June, 2020) for the two countries. The estimated numbers of cases in Italy and Spain in the next 30 days are 10982 (6383 – 15582) and 13731 (3395 – 29013), respectively. Furthermore, the expected number of daily cases on 60th day (09 July, 2020) for Italy and Spain is estimated to be 30 (0 – 183) and 92 (0 – 602), respectively. Overall, these two countries have effectively controlled the burden of COVID-19 and are expected to eventually eradicate the disease if similar control interventions are maintained.

4. Limitations and Discussions

The global spread of pandemic COVID-19 poses significant threat to the health-care systems of severely affected countries. The availability of limited data and inadequate epidemiological knowledge make the problem more challenging. Mathematical and statistical forecasting models always try to give insightful information about the future of COVID-19 spread in the community. In this study, we proposed an integrated deterministic-stochastic framework for predicting the long-term trajectories of COVID-19 cases in two profoundly affected countries, namely Italy and Spain. This idea of combining two contrasting paradigms (deterministic and stochastic models) presented in this study is fairly new. We assumed that the time-dependent stochastic process consists of both the deterministic and stochastic components and examined it for the daily cases data sets of COVID-19. The proposed ISA model outperformed the individual SIRCX model in terms of four performance metrics on the training data. Two month-ahead forecasts are provided for Italy and Spain. The estimated numbers of cases in Italy and Spain from 11 May, 2020 to 09 June, 2020 are found to be 10982 (6383–15582) and 13731 (3395–29013), respectively. Furthermore, the expected number of daily cases on 60th day (09 July, 2020) for Italy and Spain is estimated to be 30 (0–183) and 92 (0–602), respectively. These results indicate that COVID-19 will pose less threat on Italy and Spain in near future. However, the current mitigation and control interventions should be maintained as there are high values of basic reproduction numbers for Italy and Spain (4.78 and 3.25, respectively) being observed.

However, there are certain limitations of the proposed integrated framework. While formulating the ODE model we made some simplifying assumptions: (a) the birth rate and death rate of the population are constant, (b) we only consider homogeneous mixing of population, (c) the recovered people will get permanent immunity and (d) the isolated individuals can not transmit the disease and contained people can not get infection. These assumptions can be relaxed in future study. The proposed integrated deterministic-stochastic approach will be best-suited when the peak of the epidemic is passed. As a future scope of study, more complex
statistical models instead of AR model can be employed to model the leftover residuals of the ODE model. Though the model arrived from the analysis of COVID-19 data of Italy and Spain, the proposed ISA model will be well-versed for other profoundly affected countries and also for other similar epidemics.

Appendix A

We determine the condition for disease progression, when most of the people in the community are susceptible ($S$). In this kind of situation, $S$ in the model (2.2) can be replaced by initial number of susceptibles, i.e., $S(0) = S_0$. Therefore, the equation for infected people of the model (2.2) becomes

$$\frac{dI}{dt} = \left[ \frac{\beta S_0}{N} - (\gamma + \alpha + \xi + \mu) \right] I$$

Therefore, we can obtain the solution to the differential equation as $I(t) = I_0 e^{\delta t}$, where $I_0$ is the initial number of infected people. Therefore, we have

$$\delta = \frac{\beta S_0}{N} - (\gamma + \alpha + \xi + \mu) = (\gamma + \alpha + \xi + \mu)(R_0 - 1)$$

where, $R_0 = \frac{\beta S_0}{N(\gamma + \alpha + \xi + \mu)}$ represents the basic reproduction number of the model (2.2). This threshold quantity is one of the most important parameter in the spread of an infectious disease in the community (12). From the expression of $R_0$, it can be noted that isolation rate and containment rates are inversely proportional to $R_0$.

References

[1] Worldometers data repository. https://www.worldometers.info/. Retrieved: 2020-05-10.

[2] Stefano Boccaletti, William Ditto, Gabriel Mindlin, and Abdon Atangana. Modeling and forecasting of epidemic spreading: The case of covid-19 and beyond. Chaos, Solitons and Fractals, 2020.

[3] George EP Box, Gwilym M Jenkins, Gregory C Reinsel, and Greta M Ljung. Time series analysis: forecasting and control. John Wiley & Sons, 2015.

[4] Tanujit Chakraborty and Indrajit Ghosh. Real-time forecasts and risk assessment of novel coronavirus (covid-19) cases: A data-driven analysis. Chaos, Solitons and Fractals, 135, 2020.

[5] Chris Chatfield. The analysis of time series: an introduction. Chapman and Hall/CRC, 2016.
[6] Duccio Fanelli and Francesco Piazza. Analysis and forecast of covid-19 spreading in china, italy and france. *Chaos, Solitons & Fractals*, 134:109761, 2020.

[7] Wei-jie Guan, Zheng-yi Ni, Yu Hu, Wen-hua Liang, Chun-quan Ou, Jian-xing He, Lei Liu, Hong Shan, Chun-liang Lei, David SC Hui, et al. Clinical characteristics of coronavirus disease 2019 in china. *New England Journal of Medicine*, 2020.

[8] Adam J Kucharski, Timothy W Russell, Charlie Diamond, Yang Liu, John Edmunds, Sebastian Funk, Rosalind M Eggo, Fiona Sun, Mark Jit, James D Munday, et al. Early dynamics of transmission and control of covid-19: a mathematical modelling study. *The Lancet Infectious Diseases*, 2020.

[9] Qun Li, Xuhua Guan, Peng Wu, Xiaoye Wang, Lei Zhou, Yeqing Tong, Ruiqi Ren, Kathy SM Leung, Eric HY Lau, Jessica Y Wong, et al. Early transmission dynamics in wuhan, china, of novel coronavirus-infected pneumonia. *New England Journal of Medicine*, 2020.

[10] Leonardo López and Xavier Rodo. A modified seir model to predict the covid-19 outbreak in spain and italy: simulating control scenarios and multi-scale epidemics. *Available at SSRN 3576802*, 2020.

[11] Benjamin F Maier and Dirk Brockmann. Effective containment explains subexponential growth in recent confirmed covid-19 cases in china. *Science*, 2020.

[12] Maia Martcheva. *An introduction to mathematical epidemiology*, volume 61. Springer, 2015.

[13] Faical Ndairou, Ivan Area, Juan J Nieto, and Delfim FM Torres. Mathematical modeling of covid-19 transmission dynamics with a case study of wuhan. *Chaos, Solitons and Fractals*, 135.

[14] Fotios Petropoulos and Spyros Makridakis. Forecasting the novel coronavirus covid-19. *PloS one*, 15(3):e0231236, 2020.

[15] Chen Wang, Peter W Horby, Frederick G Hayden, and George F Gao. A novel coronavirus outbreak of global health concern. *The Lancet*, 395(10223):470–473, 2020.

[16] Joseph T Wu, Kathy Leung, and Gabriel M Leung. Nowcasting and forecasting the potential domestic and international spread of the 2019-ncov outbreak originating in wuhan, china: a modelling study. *The Lancet*, 395(10225):689–697, 2020.