Optimal mission planning for refuelling LEO satellites with peer-to-peer strategy

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Abstract. The problem of optimal mission planning for refuelling LEO satellites with Peer-to-Peer strategy is studied in this paper. Specifically, satellites are distributed over different orbital slots of LEOs. Fuel-sufficient ones are required to exchange fuel with fuel-deficient ones. The purpose of this paper is to find the optimal pairs, as well as the rendezvous trajectories. A Hybrid Optimal Control Model is proposed and the existing rendezvous method is employed to address the model. Numerical simulations will demonstrate the effectiveness of the developed model.

1. Introduction

On-Orbit Refueling (OOR) plays a key role in extending the lifespan of space assets. As the enabling technologies for servicing single spacecraft have been increasing mature (e.g. Orbital Express [1]), the mission planning problem for OOR is issued, in which multiple Servicing Spacecrafts (SScs) are required to refuel multiple satellites. There are number of reported works devoted in On-Orbit Refueling (OOR) mission planning, i.e. developing optimal mission sequence and trajectories for SSs refueling multiple targets, aiming at improving the economic returns and fuel consumptions [2]-[5].

A system of multiple satellites can be served by a single servicing spacecraft in one by one pattern or by a distributed peer-to-peer (P2P) strategy, i.e. satellites exchange fuel amongst themselves in pairs, with the fuel-sufficient satellites providing fuel to the fuel-deficient satellites [6]. P2P strategy is a distributed and robust refueling pattern, which could offer protection against failures to some extent. Systematical studies have been performed on P2P OOR mission planning by Dutta and Tsiotras [7]-[10]. Based on Dutta’s work, Yu et al. solved P2P mission planning problem with time window constraint in [11]. Recently, hybrid optimal control (HOC) theory has been applied to the solution of space mission planning [12].

In this paper, Optimal Mission Planning for Refuelling LEO Satellites with Peer-to-Peer Strategy is studied, a HOC model and the corresponding solution are proposed.

2. Problem description

Problem statement: $2N$ satellites distributed over different orbital slots of a LEO circular orbit are considered, as depicted in figure 1, where $N$ of them are fuel-sufficient, active and called SSc, while the others are fuel-deficient, passive and called OSC. One SSc can exchange fuel with only one OSC. After a fuel exchange takes place between the active and the passive satellite, the active satellite returns to its original available orbital slots. The purpose of this paper is to find the optimal pairs, as well as the rendezvous trajectories.
3. Mathematical description of the HOC problem

Supposing there are $N$ OScs and $M$ SScs, a resource set $Q = \{s_1, s_2, ..., s_M\}$ ($M$ SScs) and a task set $Q_r = \{t_1, t_2, ..., t_N\}$ ($N$ OScs) can be defined. The categorical state space for the problem is $Q = Q_r \times Q$. Each element $q_i = (s_i, t_j) \in Q$ represents a categorical/discrete state. It is clear that each element $q_i = (s_i, t_j) \in Q$ has a definite physical meaning, i.e. the $j$ th OScs would be serviced by the $i$ th SSC.

For this paper, we have a categorical state space as Eq. (1) (0 represents the initial location of the SSC):

$$Q = \begin{cases} 
(1,0) & (1,1) & \cdots & (1,n) \\
(2,0) & (2,1) & \cdots & (2,n) \\
\vdots & \vdots & \ddots & \vdots \\
(n,0) & (n,1) & \cdots & (n,n) 
\end{cases}$$

The directed graph about this problem could be depicted as figure 2:

![Figure 2. Directed graph for P2P on-orbit refueling mission.](image)

A mission plan could be described as $q = \{q_1, q_2, ..., q_n\}$, where $q_i = \{q_{i,0}, q_{i,1}, ..., q_{i,n}\}$, and $(i = 1, 2, ..., n)$. Actually, the initial and final states of the system are definitely defined beforehand, so we just need to find $q_{i,0}, (i^* = 1, 2, ..., n)$ only.

Associated with each state $q \in Q$ is a continuous-time controlled dynamical system: $\dot{x} = f(x, u, t, q)$. In this paper, the dynamical system is the orbital rendezvous. A typical rendezvous process includes four major phases: phasing, far range rendezvous, close range, and mating. In this paper, each phase is completed by two-impulse maneuver, and the decision variable is given as: $u = (t, \Delta v)$, where $t$ describes the maneuver time and $\Delta v$ denotes the velocity increment for each maneuver.

The constraints considered in this paper are:

- $C_1$: Each maneuver should be performed within certain communication time window.
- $C_2$: One SSC can exchange fuel with one and only one OS.

The total velocity increment is expected to be as small as possible. Let $\Delta v_{\text{to}}, \Delta v_{\text{back}}$ represent the maneuver cost for servicing and return respectively, so we can get Eq. (2):

$$J_i = \min \sum_{i=1}^{n^2} (\Delta v_{\text{to}} + \Delta v_{\text{back}})$$

Summing up, the P2P OOR mission planning problem could be modeled as Eq. (3):
\[
\begin{align*}
\text{find} & \quad q, u \\
\text{optimize} & \quad J_1 \\
\text{subject to} & \quad \dot{x} = f(x, u, t, q) \\
& \quad C_1 \cap C_2
\end{align*}
\] 

(3)

4. Solution

To address the HOC model above, two key problems should be considered and resolved:

1. Task assignment: deciding which SSc should complete which task.
2. Trajectory generation: deciding the transferring trajectory for each pair.

A two-level hybrid optimization strategy is proposed to solve the provided P2P refueling problem. The up-level is mission assignment, assigning fuel-deficient satellites to fuel-sufficient satellites in optimal pairs. It is expected to redistribute the fuel as equal as possible. All possible pairs are enumerated to find the best one. The low-level is to find the maneuver time and the corresponding velocity change for each rendezvous segment, minimizing the rendezvous cost. The “orbital rendezvous with complex constraints method” developed in [13] is employed for the lower-level.

5. Simulations

In this section, simulation cases are carried out to demonstrate the effectiveness of the proposed method. 6 satellites are taken into consideration, where 3 satellites are fuel-sufficient (numbered 1, 2, 3) and the other 3 are fuel-deficient (numbered 4, 5, 6). Perturbation are out of consideration. In orbital transfer, only phasing is considered. The orbit radius is \( r = 6878000 \) m. The orbital inclination is 60 degree. The initial RAAN is 60 degree. The time period of planning is 5 days. The initial phase angle and available fuel are given in table 1. 20 ground stations are selected for this case in table 2.

| Satellite number | 1 | 2 | 3 | 4 | 5 | 6 |
|------------------|---|---|---|---|---|---|
| The initial phase angle (degree) | 36 | 100 | 270 | 60 | 90 | 300 |
| Available fuel (unit) | 30 | 36 | 40 | 20 | 25 | 12 |

Table 2. Parameters of ground stations.

| Station number | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|----------------|----|----|----|----|----|----|----|----|----|----|
| Longitude (degree) | 38.54 | 40.132 | 41.727 | 43.223 | 34.250 | 36.842 | 25.476 | 36.443 | 23.482 | 38.537 |
| Latitude (degree) | 73.536 | 85.718 | 94.093 | 113.830 | 91.700 | 106.966 | 106.473 | 113.364 | 103.549 | 113.23 |

| Station number | 28.3 | 24.3 | 50 | 33.7 | 39.6 | 26.5 | 25.6 | 36.6 | 23.9 | 31.6 |
|----------------|------|------|----|------|------|------|------|------|------|------|
| Longitude (degree) | 117.8 | 108.8 | 123.3 | 96.6 | 115.9 | 101.5 | 109.6 | 119.6 | 109.9 | 102.4 |
| Latitude (degree) | | | | | | | | | | |

The solutions calculated by the methodology proposed in this paper are given in table 3. The total maneuver cost is 219.458 m/s.

| SSc | 1 | 2 | 3 |
|-----|---|---|---|
| Time window for the first maneuver (s) | [69710, 70112] | [68722, 69106] | [66164, 66632] |
| Station number | 5 | 14 | 19 |

| OSc | 5 | 4 | 6 |
|-----|---|---|---|
| Time window for the second maneuver (s) | [159606, 160015] | [160128, 160457] | [156434, 156741] |
Simulations show that the model proposed in this paper could address P2P OOR mission planning problem effectively.

6. Conclusions
In OOR mission, P2P is a strategy, in which satellites exchange fuel amongst themselves in pairs. It is a distributed and robust refuelling pattern, offering protection against failures to some extent. The problem of optimal mission planning for refuelling LEO satellites with Peer-to-Peer strategy is studied in this paper. The purpose of this paper is to find the optimal pairs, as well as the rendezvous trajectories. A Hybrid Optimal Control Model is proposed and the existing rendezvous method is employed to address the model. Simulation cases are carried out to demonstrate the effectiveness of the proposed model.

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