Orbifold GUT inflation

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Abstract. We consider a scenario of cosmological inflation coming from a
grand unified theory in higher-dimensional orbifold. Flatness of the potential is
automatically guaranteed in this orbifold set-up thanks to the nonlocality of the
Wilson line on higher dimensions and the local quantum gravitational corrections
are exponentially suppressed. The spectral index of scalar perturbation ($n_s$ ≃ 0.92–0.97) and a significant production of gravitational waves are predicted
($r = T/S$ ≃ 0.01–0.12) in the perturbative regime of gauge interaction ($1/g_4^2 = (5–20) \times 2\pi R M_{Pl}$) where the size of compactification is constrained ($R M_{Pl}$ ≃ 20–45) by the measurement of the scalar power spectrum ($\Delta_R \simeq 5 \times 10^{-5}$).

Keywords: extra dimensions, inflation, cosmological applications of theories
with extra dimensions
1. Introduction

Inflation is the best known idea solving several cosmological puzzles of the standard big bang cosmology such as homogeneity, isotropy and flatness of the universe [1]–[3]. Recent precise observations of the cosmic microwave background radiation (CMBR) strongly support the actual existence of the epoch of accelerated expansion in the early universe [4].

In particle physics models, inflation is driven by a single (or multiple) scalar field(s), dubbed inflaton field(s), and the potential for the field is required to be nearly flat to provide enough time for exponential expansion or e-foldings. To protect the flat potential from quantum corrections, models are often constructed in the context of a symmetry principle such as supersymmetry (for a review, see [5]) and an axionic shift symmetry (for a recent review, see [6]). However, it seems that none of the suggested particle physics models are entirely convincing for many reasons. In supersymmetric models, generic supergravity correction can spoil the flatness of the inflaton potential during the inflation since it can induce a large mass correction for the inflaton field [7]. Other models of inflation associated with the shift symmetry are also not fully satisfactory since they often require trans-Planckian fluctuations of the inflaton field \( (\delta \phi \gtrsim M_{\text{Pl}} = \sqrt{8\pi G} = 2.4 \times 10^{18}\text{GeV}) \) [8,9]. A sole symmetry principle may not provide a fully acceptable framework for inflation and there might be something beyond it [10].

A recent attempt made by Arkani-Hamed et al [11] (see also [12]) based on higher-dimensional spacetime could be interesting because the inflaton potential in their consideration is automatically free from quantum gravitational corrections thanks to the nonlocality of the higher-dimensional construction itself. In their original construction, they consider a five-dimensional \( U(1) \) gauge theory on \( M^4 \times S^1 \) where \( M^4 \) denotes the Lorentz spacetime and \( S^1 \) a circle compactification with a length \( L \). First, they consider a nonlocal operator defined by the gauge invariant Wilson line

\[
e^{i\theta} = e^{i \int A_5 \, dy}.
\] (1.1)
Below the $1/L$ scale, the dynamics of the Wilson line field $\theta$ is described by a Lagrangian

$$\mathcal{L} = \frac{1}{2g_4^2 L^2} (\partial \theta)^2 - V(\theta) + \cdots ,$$

(1.2)

where $g_4^2 = g_5^2 / L$ is the effective four-dimensional coupling constant. At the one-loop level, the potential $V(\theta)$ is induced by interactions with the charged bulk fields (of charge $q$) as

$$V(\theta) \simeq \pm \text{Const.} \frac{n \cos (nq\theta)}{n^5}$$

(1.3)

where the sign depends on the spin of the interacting particle. The effective potential essentially has the same form as natural inflation [8,9] with the effective decay constant given by $f_{\text{eff}} = 1/(g_4 L)$. One should note that no dangerous higher-dimensional operator can be generated in a local higher-dimensional theory and the potential can be trusted even when $f_{\text{eff}}$ can be larger than $M_{\text{Pl}}$ in the perturbative regime of gauge interaction\(^1\).

Of course, this original model with $U(1)$ symmetry should be considered as a toy model and we have to construct a realistic model where the standard model gauge group $G_{\text{SM}} = SU(3) \times SU(2) \times U(1)$ is fully considered as an effective field theory. That is the main motivation of the current work.

In this paper, we try to get a potentially realistic model of inflation from a higher-dimensional gauge theory. Having the standard model as a low energy effective theory, we would consider the $SU(5)$ gauge theory as the starting point. Orbifold projection can provide a nice explanation of doublet–triplet splitting [16]. We found that only one particular choice of orbifold projection is available to get the standard model as well as the inflaton field. This will be clarified in the following sections. We would emphasize one more nice property of this orbifold construction: distinguishing from the $U(1)$ toy model, the non-Abelian nature of $SU(5)$ GUT allows the nontrivial one-loop potential of the inflaton field solely coming from the gauge self-interactions. Without introducing any (arbitrary) charged bulk field in the model, no further ambiguity occurs in predicting cosmological observable quantities. Spectral index ($n_s = 0.92–0.97$) and a significant production of gravitational waves is predicted ($r = T/S = 0.02–0.12$). No large non-Gaussianity is expected by the model.

The content of this paper is given as follows. In the next section, we specify the $SU(5)$ GUT model based on $S^1/Z_2$ orbifold compactification. The orbifold boundary condition is chosen in such a way that the standard gauge bosons and a massless scalar particle remain after the orbifold compactification. The massless scalar particle, coming from the fifth component of the gauge boson ($A_5$), defines a Wilson line phase and its radiative potential is calculated at the one-loop level. In section 3, we analyze the potential and show that the slow-roll conditions are nicely satisfied when the gauge coupling is weak. Enough time for e-foldings is obtained. Various cosmological observable quantities such as the spectral index of the scalar perturbations, $n_s$, the ratio of tensor to scalar perturbations, $r = T/S$, and the running of the spectral index, $dn_s/d\ln k$, are fully predicted. In the last section, a summary is given.

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\(^1\) We noticed a reference [13] where the authors claimed that $U(1)$ gauge (without light charged particles) could not be compatible with string theory up to arbitrarily high energy and there should appear a cutoff scale around ($\Lambda \sim g M_{\text{Pl}}$) even if the theory seems perfectly fine as a low energy effective theory.
2. The model

In this section, we specify the model of cosmological inflation coming from the $SU(5)$ GUT gauge theory on the orbifold $(M^4 \times S^1 / \mathbb{Z}_2)$. Let us assume that only pure gauge fields (and gravitons) are propagating through the bulk and all the fermion fields are localized on one of the fixed points of $S^1 / \mathbb{Z}_2$ orbifold\(^2\). The coordinates of five-dimensional spacetime are denoted by $x^M = (x^\mu, y)$, where the indices are given as $(M = 0, 1, 2, 3, 4, \mu = 0, 1, 2, 3)$, respectively. We require that the Lagrangian is single valued and gauge invariant. To respect the orbifold $\mathbb{Z}_2$ condition, boundary conditions for bulk fields should be specified by two parity matrices, $P_0$ and $P_1$, around fixed points, $y = 0$ and $y = \pi R$, respectively\(^3\):

\[
P_0 : A_M(x^\mu, -y) = (-1)^\alpha P_0 A_M(x^\mu, y) P_0^\dagger,

\]

\[
P_1 : A_M(x^\mu, \pi R - y) = (-1)^\alpha P_1 A_M(x^\mu, \pi R + y) P_1^\dagger
\]

(2.1)

where $\alpha = 0(1)$ for $M = \mu(5)$, respectively. The extra $(-1)$ sign is required to preserve the Lorentz invariance. Taking care of overall sign ambiguities, there are essentially two independent ways to break the GUT gauge group, $SU(5)$, down to the standard model gauge group, $SU(3) \times SU(2) \times U(1)$ \([14, 15]\):

- **Choice-I:**

  \[
P_0 = \begin{pmatrix} -I_3 & 0 \\ 0 & I_2 \end{pmatrix}, \quad P_1 = \begin{pmatrix} I_3 & 0 \\ 0 & I_2 \end{pmatrix}.
\]

- **Choice-II:**

  \[
P_0 = \begin{pmatrix} -I_3 & 0 \\ 0 & I_2 \end{pmatrix}, \quad P_1 = \begin{pmatrix} -I_3 & 0 \\ 0 & I_2 \end{pmatrix}.
\]

(2.2)

(2.3)

Here $I_n$ denotes the $n \times n$ unit matrix. Applying the parity matrices to equation (2.1), we could read out the parity assignment for the $SU(5)$ gauge boson in adjoint $5 \times 5$ matrix representation as follows:

- **Choice-I:**

  \[
  A_\mu = \begin{pmatrix} ++ & -- \\ -- & ++ \end{pmatrix}, \quad A_5 = \begin{pmatrix} -- & ++ \\ ++ & -- \end{pmatrix},
  \]

- **Choice-II:**

  \[
  A_\mu = \begin{pmatrix} ++ & -- \\ -- & ++ \end{pmatrix}, \quad A_5 = \begin{pmatrix} -- & ++ \\ ++ & -- \end{pmatrix},
  \]

(2.4)

(2.5)

\(^2\) Actually one-loop generated effective potential for the inflaton, $A_5$, could be induced solely by the gauge self-interaction without any further requirement of charged bulk fields. For simplicity and predictability, here we assumed that all the fermions are localized and do not directly affect the one-loop effective potential at the leading order.

\(^3\) Generically, one could specify three conditions to get an invariant theory on $S^1 / \mathbb{Z}_2$ orbifold. One for translation, $U : y \rightarrow y + 2\pi R$, two for $\mathbb{Z}_2$-orbifold conditions around fixed points, $P_0 : y \rightarrow -y$ and $P_1 : y + \pi R \rightarrow -y + \pi R$, respectively. However, as a transformation $y + \pi R \rightarrow -y + \pi R$ must be exactly the same as a transformation $y + \pi R \rightarrow -(y + \pi R)$ $\rightarrow -y + \pi R$, it follows that $U = P_1 P_0$ or the translation can be obtained by two parity operations.
In either choice, there appear massless gauge bosons, with $(++)$ parity, having exactly the same quantum numbers with the standard model gauge bosons, i.e. gluons, $W^\pm$, $W^3$ and $B$. The main difference between the first and the second choice is the existence of a massless scalar degree of freedom. Only the second choice allows the massless scalar degree of freedom with $(++)$ parity, in $A_5$. Indeed, Kawamura [16,17] took the first choice, to address the doublet–triplet splitting problem of GUT and he could recover the standard model without any light exotic scalar degree of freedom at the lowest level of Kaluza–Klein decomposition. In the first choice the lightest scalar field acquires non-zero mass ($\sim 1/R$) since it has $(+−)$ or $(−+)$ parity instead of $(++)$ parity. However, we need a light scalar field for inflation as well as the standard model gauge fields. Thus our choice is the second one!

This would-be-inflaton scalar field can be written in canonical normalization as follows:

$$\Phi = A_5 \sqrt{\pi R} = f \left( \begin{array}{c} 0 \\ \phi^i \\ 0 \end{array} \right).$$ (2.6)

Here we introduced a mass scale parameter $f = 1/(2\pi Rg_4)$ in such a way that the field $\Phi$ is canonically normalized with a dimensionless angle parameter $\phi$ where the four-dimensional effective coupling is $g_4^2 = g_5^2/\pi R$. One should note that the scale, $f$, can be large at the weak coupling limit, $g_4 \ll 1$. This feature is indeed a unique feature in higher-dimensional gauge theory, in contrast to the case in four-dimensional theory where effective energy scales should be considered less than the Planck scale.

Now let us define a Wilson line phase, $W = Pe^{i \int C A_5 dy}$. The Wilson line can be parameterized by two independent real numbers after taking account of the remaining symmetry after orbifold projection:

$$\phi = \left( \begin{array}{c} \alpha \\ 0 \\ 0 \\ \beta \end{array} \right).$$ (2.7)

The effective potential for $\Phi$ can be evaluated (see [18] for the general formula of the effective potential in 5D $SU(N)$ gauge theory) provided that the full particle spectra of the theory is specified. Here we just turn on the bulk gauge sector without further complication potentially coming from the fermionic sector. After taking care of massless gauge fields and ghost fields, the total effective potential is obtained as

$$V_{\text{eff}}(\Phi) = \frac{1}{R^4} \left( c(\alpha) + c(\beta) + \frac{c(2\alpha) + c(2\beta)}{2} + c(\alpha + \beta) + c(\alpha - \beta) \right)$$ (2.8)

dropping the divergent cosmological constant term$^4$. A dimensionless one-loop function, $c(x)$, is calculated as follows:

$$c(x) = \frac{9}{64\pi^6} \sum_{n=1}^{\infty} \frac{\cos n\pi x}{n^5}. \quad (2.9)$$

The function is periodic, $c(x) = c(x + 2\pi)$, and even, $c(-x) = c(x)$ with respect to inversion, $x \rightarrow -x$, along the compact fifth direction. Let us check the effective potential

$^4$ In this sense, we do not address the cosmological constant problem. We would simply put the potential vanishing at the origin, $V_{\text{eff}}(0) = 0$. 

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more closely (see figure 1). First, we can notice that the global minimum of the potential locates at the origin, \((\alpha, \beta) = (0, 0)\), and the gauge symmetry, \(SU(3) \times SU(2) \times U(1)\), is intact even after taking care of one-loop corrections. If the initial point of the inflaton fluctuation belongs to a local maxima or below the maxima, the field is going slowly down to the global minimum where the standard gauge symmetry is fully recovered. This is an interesting observation since the set-up does not require any fine-tuning in the parameter space or even any arbitrarily extended sector is required beyond the assumption of the minimal \(SU(5)\) GUT with properly chosen boundary conditions.

The cosmological scenario is now obtained as follows. In the beginning, the universe started from a (false) vacuum where the space is a five-dimensional orbifold. A gauge theory of \(SU(5)\) GUT dictated the fundamental symmetry of particle interactions and also provided a scalar field as a part of \(A_5\) component which could play a role as the slowly rolling inflaton field. Below the compactification scale, \(1/R\), heavy modes beyond the standard model were decoupled. The inflaton field slowly ran down to the global minimum where the standard model gauge symmetry was fully recovered\(^5\). After inflation, the usual Higgs phase opened and the standard electroweak symmetry breaking took place with the standard Mexican Hat potential for the Higgs field (see, e.g., [19] for electroweak symmetry breaking from an orbifold gauge theory).

3. E-foldings, slow roll parameters and predictions

Given the inflaton potential, we now predict cosmological observable quantities. Let us first check whether the potential accommodates enough time for 60 e-foldings. To check this, we would analyze the potential along the steepest line along which \(\alpha = \beta\) (see

\(^5\) During the inflation, it is expected that some amount of baryon number could be produced since the VEV of the potential is not vanishing. However, its density could hardly affect the current baryon asymmetry because of dilution. We thank J E Kim for indicating this point.
The one-loop effective potential of $\Phi$ along the steepest line, $\alpha = \beta$. The potential is slow rolling in the weak coupling limit, $g_4 \ll 1/(2\pi RM_{Pl})$.

The number of e-folds, $N_e$, is obtained by a time integration of the Hubble parameter ($H$) and in the slow roll paradigm it can be nicely approximated as follows:

$$N_e = \int_{t_i}^{t_{end}} H \, dt \approx \frac{1}{M_{Pl}^2} \int_{\Phi_{end}}^{\Phi} \frac{V_{eff}}{V'_{eff}} \, d\Phi$$

(3.1)

where $\Phi_{end}$ defines the end of inflation after 60 e-foldings. Here we notice that the number of e-folds is proportional to the square ratio of the fictitious scale, $f$, and the Planck scale:

$$N_e \propto \left(\frac{f}{M_{Pl}}\right)^2.$$

(3.2)

Again, one should notice that this ratio can be much larger than 1 if the coupling is very small, $g_4 \ll 1/(2\pi RM_{Pl})$.

In figure 3, we draw a plot for the number of e-folds as a function of the initial value $a$ where the inflation started. Because the number of e-folds is proportional to $(f/M_{Pl})^2$, we get the larger number of e-folds with the larger value of $(f/M_{Pl})$, as is expected. When the inflation started from the top of the potential, $\Phi_{ini} = \Phi_{top} - \Delta_{Quantum} \Phi$ with the small quantum fluctuation, we could get 60 e-foldings when $f/M_{Pl} \gtrsim 3$.

Let us now check the consistency of the slow-roll paradigm and try to make predictions for cosmological observables. In the slow-roll paradigm, three slow-roll parameters, $\epsilon, \eta$ and $\xi$, determine all the observable quantities such as the spectral index, tensor to scalar perturbation ratio and running of the spectral index. Slow-roll parameters are defined as follows:

$$\epsilon = \frac{M_{Pl}^2}{2} \left(\frac{V'}{V_{eff}}\right)^2,$$

(3.3)

$$\eta = M_{Pl}^2 \left(\frac{V''}{V_{eff}}\right),$$

(3.4)
Figure 3. The number of e-folds, $N_e$, along $\phi \propto (\alpha, \alpha)$ with respect to the initial value of $\alpha$ for the values $f/M_{Pl} = 20, 25, 30, 35$, respectively. For the larger value of $f/M_{Pl}$, it is easier to get the larger number of e-folds because $N_e \sim (f/M_{Pl})^2$.

$$\xi = M_{Pl}^4 \frac{V'_{\text{eff}} V''_{\text{eff}}}{V_{\text{eff}}}$$

(3.5)

where primes denote derivatives with respect to the field $\Phi$. Since the ratio, $f/M_{Pl}$, could become very large as was discussed earlier, slow roll conditions are easily satisfied:

$$\epsilon, \eta, \sqrt{\xi} \sim \left(\frac{f}{M_{Pl}}\right)^{-2} \ll 1.$$  

(3.6)

In figure 4, we draw plots for slow-roll parameters, $\epsilon, |\eta|$ and $|\xi|$, with respect to the ratio $f/M_{Pl}$ in the range of $10–100$ in log scale. It is found that all the slow-roll parameters are sufficiently small in a large range of parameter spaces and we can safely stay in the slow-roll paradigm.

Cosmological measurements provide important information about the structure of the inflaton potential. In particular, observational constraints on the amplitude of scalar perturbations, in the slow-roll framework, imply that

$$\Delta_R \approx \frac{1}{2\sqrt{3\pi M_{Pl}}} \sqrt{\frac{V_{\text{eff}}}{\epsilon}} \approx 5 \times 10^{-5}.$$  

(3.7)

From this constraint, we could read out the size of the extra dimension. In figure 5, the size of the extra dimension, $R$, is calculated in terms of $f/M_{Pl}$.

A standard slow-roll analysis also gives observable quantities such as the spectral index, $n_s$, the relative contribution of gravitation to scalar perturbation, $r = T/S$, and the running parameter of the spectral index, $dn_s/d\ln k$, in terms of the slow-roll parameters to first order as

$$n_s = 1 - 6\epsilon + 2\eta,$$  

(3.8)

$$r = T/S = 16\epsilon,$$  

(3.9)

$$\frac{dn_s}{d\ln k} = -16\epsilon\eta + 24\epsilon^2 + 2\xi^2.$$  

(3.10)
Figure 4. Slow-roll parameters in the orbifold GUT. From the top to the bottom, we draw $\epsilon$, $|\eta|$ and $\xi$ in terms of $f/M_{Pl}$. All of them are small enough and we can consistently rely on the slow-roll paradigm for inflation.

Figure 5. From the measurement of scalar power spectrum, $\Delta_R(k)^2 \approx 5 \times 10^{-5}$, we could find a strict constraint on the size of the extra dimension, $R$. The compactification radius is about 15–45 times larger than the Planck length, $1/M_{Pl}$, in the chosen parameter space for $5 \leq f/M_{Pl} \leq 30$.

Since $dn_s/d\ln k$ has double suppression by a factor of $(f/M_{Pl})^{-4} \ll 1$, it is negligibly small. It is challenging to measure this small amount of parameter within the range of near future sensitivity.

In figure 6, we plot the prediction for the spectral index ($n_s$) and tensor to scalar contribution ($r$) for various values of $(f/M_{Pl})$. With the larger value of $(f/M_{Pl})$, we get the larger spectral index ($n_s \lesssim 0.97$) and larger tensor-to-scalar ratio ($r \lesssim 0.12$). Around the point $f/M_{Pl} \sim 15.0$, the spectral index is almost saturated with the value of 0.97 but the tensor-to-scalar ratio seems to become larger and larger.

Here the spectral index, tensor contribution and the running index are well consistent with the recent WMAP data. Interestingly, the tensor to scalar contribution, $r$, is rather higher than in the usual KKLT type string theory models where $r$ is negligibly small
when the gravitino mass is given in the TeV range \cite{20}. In that sense, the detection of a gravitational contribution will be a nice probe of the orbifold GUT inflation model.

4. Summary

In this paper, we present a model of inflation from the $SU(5)$ orbifold GUT model. The inflation field arises as a consequence of the symmetry transition from a grand unified symmetry, $SU(5)$, to the standard model gauge symmetry, $SU(3) \times SU(2) \times U(1)$, by the orbifold compactification. The advantage of this model is that the inflaton field is a built-in ingredient of the theory and it is automatically free from local quantum gravitational effects because of its higher-dimensional locality and gauge symmetry. Fully radiatively induced inflaton potential is naturally slow-rolling once the theory is weakly coupled during the inflationary era. The spectral index $n_s$ is predicted to be in the range \(0.92 \sim 0.97\) which is fully consistent with the recent observational data. An interesting prediction is that the significant gravitational wave is expected as $r \simeq 0.02 \sim 0.12$ when the model parameter is assumed in the range of $f/M_{\text{Pl}} = 5 \sim 20$. A very small running parameter of the spectral index is expected as well, \( (dn_s/d \ln k \lesssim 0.002) \). We would leave a study to see the coupling unification in the orbifold GUT models in association with the inflationary scenario proposed in the current paper as future work.

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