Research on Admittance Control Method of Industrial Robot Based on SMC Strategy

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Abstract. Aiming at the problem of compliance control of industrial robot, in order to overcome the influence of unfavorable factors such as nonlinearity and strong coupling of industrial robot system on robot compliance control, so as to improve robot control accuracy and avoid damage to robot or workpiece caused by environmental contact force. Therefore, a robot admittance control method based on sliding mode control (SMC) strategy is proposed in this paper. The dynamic model of the robot with parameter uncertainty is established, and the exponential reaching law sliding mode control method is introduced into the robot admittance controller to improve the flexibility of the robot. Finally, the six-axis robot is simulated by the simulation platform to verify the feasibility and effectiveness of the proposed method.

1. Introduction
The application of industrial robots is becoming more and more extensive, greatly improving the efficiency of industrial production. Now the robot has been used in complex operations such as cutting and polishing. It needs to move under the condition of force interaction with the environment or the workpiece, which may cause the end torque to exceed the limit or the workpiece damage due to the contact force. Therefore, in this type of contact operation, the robot needs to adjust the robot's pose to adapt to the contact force according to the size of the contact force, so as to realize the compliant control during the operation of the robot [1-2].

Robot compliance control methods are divided into admittance control and impedance control [3]. The admittance control is based on external force signals as feedback, and the admittance algorithm outputs position signals to control the robot, which is suitable for high stiffness environment and can effectively compensate the unmodeled friction. And the impedance control is to feedback the position signal and output the force signal to control the robot. It is necessary to establish an accurate robot model, which is suitable for the low friction system [4]. This paper studies the robot compliant control in a high-rigidity environment, and the joint friction is difficult to accurately measure and model, so the admittance control method is adopted.

In [5], aiming at the problem of compliance control of robot in high stiffness environment, the dynamic model of Cartesian space robot is established, and an admittance controller is designed to realize the compliance control of robot in high stiffness environment. However, due to the characteristics of non-linearity and strong coupling of industrial robot systems [6], the traditional control law used in ordinary admittance control requires linearization or variable parameter processing, and the control accuracy is not high. And sliding mode control is suitable for nonlinear systems, and can overcome the influence of robot parameter errors, environmental forces and other uncertain factors on the system, so it can be applied to robot control [7]. Therefore, an admittance control algorithm based on exponential reaching law sliding mode is proposed in this paper, which can better deal with the nonlinearity and uncertainty of robot model and improve the performance of robot compliance control [8-9].

2. Denavit-Hartenberg Method (D-H Method) to establish a robot kinematics model

2.1. Forward kinematics equation
The kinematics modeling of the robot is to establish the relationship between the joint variables and the
position and posture of the end effector of robot. Forward kinematics is to calculate the position and posture of the robot end by giving the joint variables of the robot. Inverse kinematics is that the position and posture of the end of the robot is known, which is used to calculate the corresponding joint variables of the robot.

D-H method is adopted to establish the coordinate system for each connecting rod of the six-axis robot, then the position and pose transformation relation of the coordinate system of adjacent connecting rod of the robot can be expressed by the homogeneous transformation matrix $i \rightarrow i+1$:

$$
\begin{bmatrix}
c_{\theta_i} & -s_{\theta_i} & 0 & a_{i-1} \\
s_{\theta_i}c_{\alpha_{i-1}} & c_{\theta_i}c_{\alpha_{i-1}} & -s_{\alpha_{i-1}} & -d_i s_{\alpha_{i-1}} \\
s_{\theta_i} s_{\alpha_{i-1}} & c_{\theta_i} s_{\alpha_{i-1}} & c_{\alpha_{i-1}} & d_i c_{\alpha_{i-1}} \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

(1)

Where, $c$ and $s$ represent cos and sin, respectively. The robot D-H link parameters are expressed as: $a_i$, $\alpha_i$, $d_i$, $\theta_i$ respectively represent the length, torsion angle, link offset and the $i$-th joint rotation angle of the $i$-th link, $i=1,2,...,6$, which are link parameters.

The position and pose of the end of the six-axis robot relative to the base coordinate system can be obtained by multiplying the homogeneous transformation matrix of each connecting rod:

$$
\begin{bmatrix}
n_x \\
n_y \\
n_z \\
p_x \\
p_y \\
p_z
\end{bmatrix} =
\begin{bmatrix}
a_{x-1} & a_{y-1} & a_{z-1} & p_x \\
a_{x-1} & a_{y-1} & a_{z-1} & p_y \\
a_{x-1} & a_{y-1} & a_{z-1} & p_z \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

(2)

Where, $[n,o,a]$ is the Euler Angle matrix of the robot end posture, and $p$ is the spatial coordinate vector of the end position, where:

$$
\begin{align*}
n_x &= c_1(c_2(c_2c_3c_6 - s_2s_6) - s_2s_3c_6) + s_1(s_2c_3c_6 + c_4s_6) \\
n_y &= s_1(c_2(c_2c_3c_6 - s_2s_6) - s_2s_3c_6) - c_1(s_2c_3c_6 + c_4s_6) \\
n_z &= -s_2c_3s_6 - s_2s_3c_6 \\
o_x &= c_1(c_2(c_2c_3s_6 + s_2s_6) + s_1(-s_2c_3s_6 + c_4s_6) \\
o_y &= s_1(c_2(c_2c_3s_6 + s_2s_6) + s_1(-s_2c_3s_6 + c_4s_6) \\
o_z &= s_2c_3s_6 + s_2s_3c_6 \\
o_x &= -c_1(c_2c_4s_6 + s_2c_4s_6) - s_1s_2s_6 \\
o_y &= -s_1(c_2c_4s_6 + s_2c_4s_6) + c_1s_2s_6 \\
o_z &= s_2c_4s_6 - s_2c_5c_6 \\
p_x &= c_1(a_{x-1} + a_{y-1}c_{23} - d_4s_{23} + a_{i-1}) - d_4(c_1(c_2c_3s_6 + s_2s_6) + s_1s_2s_6) \\
p_y &= s_1(a_{x-1} + a_{y-1}c_{23} - d_4s_{23} + a_{i-1}) - d_4(s_1(c_2c_3s_6 + s_2s_6) - c_1s_2s_6) \\
p_z &= -a_1s_{23} - a_2s_2 - d_4c_{23} + d_1 + d_6(s_2c_4s_6 - c_2s_5)
\end{align*}
$$

(3)

Where, $c_i$, $s_i$, $c_{ij}$, and $s_{ij}$ represent $\cos \theta_i$, $\sin \theta_i$, $\cos(\theta_i+\theta_j)$, and $\sin(\theta_i+\theta_j)$ respectively, $i,j=1,2,...,6$.

Letting $x = [p_x, p_y, p_z, p_z]^T$ is the three-dimensional vector of the Cartesian coordinates of the end position of the robot. Therefore, the forward kinematics equation of the robot is the Cartesian coordinate $x$ of the end of the robot obtained from the rotation angle $q$ of the robot joint:

$$
x = T(q)
$$

(4)

2.2 Inverse kinematics equation

Inversely derive the mathematical relationship in the forward kinematics equation, which can be solved by inverse kinematics, that is, the joint rotation angle $q$ is obtained from the Cartesian coordinate $x$:

$$
q = T^{-1}(x)
$$

(5)
From the forward kinematics relationship (4), the Jacobian matrix \( J(q) \) can be obtained:

\[
J(q) = \frac{\partial T(q)}{\partial q} = \frac{\partial x}{\partial q}
\]  

(6)

Then the first and second derivatives of Cartesian coordinates \( x \) are:

\[
\dot{x} = J(q)\dot{q}, \quad \ddot{x} = J(q)\ddot{q} + J(q)\dot{q}
\]  

(7)

Furthermore, the relation between robot moment \( \tau \) and force \( F \) is:

\[
F = J^T(q)\tau
\]  

(8)

3. Robot dynamics modeling

According to the Lagrangian method, the dynamic equation of the \( n \)-joint robot is described as [10]:

\[
M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau
\]  

(9)

Where, \( q, \dot{q}, \ddot{q} \in \mathbb{R}^n \), represent the angle, angular velocity, and angular acceleration vector of each joint, respectively, \( M(q) \in \mathbb{R}^{n \times n} \) is the inertia matrix, \( C(q,\dot{q}) \in \mathbb{R}^{n \times n} \) are centrifugal force and Coriolis moment matrix, \( g(q) \in \mathbb{R}^n \) is the weight moment vector, \( \tau = \tau_{eq} + \tau_{ext} \), \( \tau_{eq}, \tau_{ext} \in \mathbb{R}^n \) are control moment vectors and external moment vectors respectively.

Considering the error of the system model parameters, that is, the system parameter matrix is composed of certain and uncertain parts:

\[
\begin{bmatrix}
M(q) = M_0(q) + \Delta M(q) \\
C(q,\dot{q}) = C_0(q,\dot{q}) + \Delta C(q,\dot{q}) \\
g(q) = g_0(q) + \Delta g(q)
\end{bmatrix}
\]  

(10)

Where, \( M_0, C_0, R_0 \) are the deterministic parts of the parametric matrix, \( \Delta M, \Delta C, \Delta g \) are indeterminate parts and they are bounded.

Let the modeling error be

\[
f = \Delta M(q)\ddot{q} + \Delta C(q,\dot{q})\dot{q} + \Delta g(q)
\]  

(11)

Then the dynamic model of an industrial robot with parameter uncertainty is:

\[
M_0(q)\ddot{q} + C_0(q,\dot{q})\dot{q} + g_0(q) + f = \tau
\]  

(12)

Where, \( f \) is a given \( n \)-dimensional constant vector, and \( |f| \leq \overline{f}, \quad \overline{f} \) is the upper bound of the uncertain term \( f, \quad \overline{f} \) is the lower bound of \( f \).

4. System structure design

In the admittance control method, through an expected dynamic relationship, the external force \( F_{ext} \) at the end of the robot is transformed into simulated force displacement \( x_f \). That is, the end position of the robot will generate a virtual offset of \( x_f \) under the action of external force \( F_{ext} \), and then adjust the given position according to the position offset to control the movement of the robot. The dynamic relationship is described as:

\[
M_0\ddot{x}_f + D_0\dot{x}_f + K_0x_f = F_{ext}
\]  

(13)

Where, \( M_0, D_0 \) and \( K_0 \) respectively represent the positive definite diagonal matrix of expected inertia, damping, and stiffness. Then the transfer function from input \( F_{ext} \) to output \( x_f \) is an ideal mass-spring-damping system (M-S-D) [11]:

\[
G_0(s) = \frac{X_f(s)}{F_{ext}(s)} = \frac{1}{M_0s^2 + D_0s + K_0}
\]  

(14)

The admittance characteristics of the robot can be adjusted by adjusting the parameters of inertia, damping and stiffness.
From (14), the simulated force displacement can be obtained:
\[ x_f = F_{ext} G_d \]  
(15)

From (5), the given joint rotation angle is:
\[ q_d = T^{-1}(x_i) \]  
(16)

Where, \( x_d = x_i - x_v \) is the virtual balance position, \( x_i \) is the given path position.

The given joint angle value \( q_d \) and the joint angle output value \( q \) fed back by the actuator are input to the sliding mode controller (SMC) to obtain the control moment \( \tau \). The control torque is applied to the robot dynamics model to obtain the rotation angle \( q \) of the output joint, and finally the position \( x \) of the robot is obtained from the forward kinematics equation (4). Thus, the system can control the robot to adjust the output position according to the external force, to realize the compliant control. The system structure is shown in figure 1.

![System structure diagram](image)

**Figure 1.** System structure diagram

### 5. Sliding mode controller design

According to the system structure diagram, the input of the controller is the given joint rotation Angle \( q_d \), and the output is the control torque \( \tau \). Therefore, the sliding mode control law is designed for the robot dynamics model with parameter uncertainty.

Because the chattering of conventional sliding mode method is difficult to be weakened, and the robot control system needs a high response speed. Therefore, the sliding mode controller based on the exponential reaching law is used as the admittance controller, and the approaching speed decreases gradually from a large value to 0. In this way, the approach time is shortened and the speed of moving point reaching the switching surface is small, which ensures rapidity and weakens chattering [12].

Let \( e = q_d - q \) be the joint angle error, then the sliding mode switching function is:
\[ s = \dot{e} + Be \]  
(17)

Where, \( B = diag(b_1, b_2, \cdots, b_n) \), \( b_i \) is a constant satisfying Hurwitz's condition, that is \( b_i > 0, i = 1, 2, \cdots, n \).

According to Model (13):
\[ \dot{s} = \dot{e} + B\dot{e} = \ddot{q}_d - \ddot{q} + B\dot{e} = \ddot{q}_d - M^{-1}_0(\tau + f - C_0\ddot{q} - g_0) + B\dot{e} \]  
(18)

Take the exponential reaching law as:
\[ \dot{s} = -\varepsilon\text{sgn}(s) - ks \]  
(19)

Where, \( \varepsilon, k > 0 \), are the parameters of the approach law.

Substituting equation (19) into equation (18):
\[ -\varepsilon\text{sgn}(s) - ks = \ddot{q}_d - M^{-1}_0(\tau + f - C_0\ddot{q} - g_0) + B\dot{e} \]  
(20)

Then the sliding mode control law is:
\[ \tau = M_0(B\ddot{e} + \ddot{q}_d + \varepsilon\text{sgn}(s) + ks) + C_0\ddot{q} + g_0 - f \]  
(21)

According to equation (12), \( f \) is an unknown quantity, and its upper bound is \( \overline{f} \). Design \( f = -\overline{f}\text{sgn}(s) \), then the control law can be written as:
\[ \tau = M_0(B\ddot{e} + \ddot{q}_d + \varepsilon\text{sgn}(s) + ks) + C_0\ddot{q} + g_0 + \overline{f}\text{sgn}(s) \]  
(22)

Substituting equation (22) into equation (18):
Construct the Lyapunov function:
\[ V = \frac{1}{2} s^T s \]  

Then:
\[ \dot{V} = s^T \dot{s} = \dot{s}^T (\epsilon \text{sgn}(s) - ks + M_\alpha^{-1} (\mathbf{F} \text{sgn}(s) - \mathbf{f})) \leq \dot{s}^T (\epsilon \text{sgn}(s) - ks) \leq 0 \]

So the system is gradually stable.

To sum up, the admittance control method designed in this paper introduces an exponential reaching law sliding mode controller, which can achieve compliant control of the terminal position of the robot in the Cartesian coordinate control mode, and avoid the excessive torque or workpiece damage caused by excessive external force in the contact operation of the robot.

6. The simulation verification

The sliding mode admittance control method designed is simulated and verified by MATLAB and RoboticsBox toolbox. The simulation object is a six-axis robot GLUON-6L3 model, and its D-H parameters are shown in Table 1:

| Link | \(a_{\alpha}\)/mm | \(a_{\beta}\)/mm | \(d_i\)/mm | \(\theta_i\) |
|------|-----------------|-----------------|-------------|-------------|
| 1    | 0               | 0               | 105.03      | \(q_1\)     |
| 2    | 0               | 90              | 80.09       | \(q_2\)     |
| 3    | 174.42          | 0               | 0           | \(q_3\)     |
| 4    | 174.42          | 180             | 4.44        | \(q_4\)     |
| 5    | 0               | -90             | -80.09      | \(q_5\)     |
| 6    | 0               | 90              | -44.36      | \(q_6\)     |

The sliding mode admittance controller was established by MATLAB. Let the parameters of sliding mode control law: \(B = \text{diag}(5, 5, 5, 5, 5, 5)\), \(\epsilon = 0.5\), \(k = 5\). Let the feedback force damping system parameters \(M_\alpha = 0.2\), \(D_d = 10\), \(K_d = 30\). The upper bound of the uncertain term is \(\mathbf{F} = [2, 2, 2, 2, 2, 2]^T\). According to D-H parameters, the robot model was established by RoboticsBox toolbox. The given path of the robot terminal from the Cartesian coordinate starting point \([0.0767, -0.3329, 0.1677]^T\) to the terminal point \([0.2941, -0.1396, 0.2767]^T\) and the initial velocity of the robot terminal motion in the spatial coordinate are given to make it move at a uniform speed along this spatial linear path.

Let \(F_{\text{ext}} = 0\) \(t \in [0, 50s]\),

\[ F_{\text{ext}} = \begin{cases} 0 & t \in [0, 50s] \\ 2 & t \in [50, 70s] \\ 0 & t \geq 70s \end{cases} \]

Where, \(F_{\text{ext}}\) is the external force at the end of the robot when it is running.

The simulation results are:

The curve of the rotation angle of each joint of the robot is shown in figure 2. The figure shows that when no external force is applied, the joint rotation angle has a good following performance for the rotation angle corresponding to the given path. At \(t = 50s\), the robot begins to be subjected to external forces, and the corners of the third and fourth joints begin to significantly deviate from the corners of the original path. Other joints also have offsets, but the amplitude is smaller. This is because the external force given by the simulation is the force in the \(-z\) direction, and the third and fourth joints are mainly required to adjust the angle to adapt to the external force. At \(t = 70s\), the external force is removed, and the joints can be restored to the desired angle smoothly.

In addition, since the length \(a_5\) and \(a_6\) of the fifth and sixth connecting rods in the GLUON-6L3 model adopted in simulation are all 0, \(q_5\) and \(q_6\) only affect the Euler angle of the posture of the robot end, and have little influence on its position spatial coordinates. And admittance control is the control of position, and the
posture changes little, so the variation range of $q_5$ and $q_6$ in the simulation is always small.
The curve of the angular velocity of each joint of the robot is shown in figure 3. Within 0~50s, without
external force, the robot joint angular velocity can follow the desired angular velocity after a slight jitter, and
the system has a good performance of angular velocity following. Within 50~70s, under the action of external
force, each joint will shift quickly to adjust the position, especially the angular velocity of the third and fourth
joints changes greatly. When $t \geq 70s$, the external force disappears, the angular velocity changes again
greatly and the angle is adjusted to restore the robot to the desired path.
The space position trajectory of the robot is shown in figure 4, the blue dashed line is the given path, and the
orange solid line is the actual running trajectory of the robot end under sliding mode admittance control. As
shown in the figure, when the external force is not applied, the trajectory is well tracked. When the external
force is applied, the robot can respond quickly and stably and adjust the running path to adapt to the ext
ernal force. After the external force disappears, it will return to the original path smoothly to achieve compliant
control.

![Figure 2. Curve of the rotation angle of each joint.](image)

![Figure 3. Curve of the angular velocity of each joint](image)

![Figure 4. Robot trajectory under sliding mode admittance control.](image)

![Figure 5. The curve of the resultant force on the end of the robot.](image)

The resultant force on the end of the robot is $F_{out}$, then $F_{out} = F - F_{ext}$, $F$ is the output force multiplied by
the robot output torque $\tau$ and the Jacobian matrix $J^T$, and the $F_{out}$ curve is shown in figure 5. After starting to run,$F_{out}$ quickly returns to 0, that is, the resultant force on the end of the robot is 0, and it keeps running at a
constant speed. At the beginning of operation, $F_{out}$ recovers to 0 quickly, namely, the resultant force on the end
of the robot is 0 and it keeps running at a constant speed. After receiving the external force, the balance of
force is broken, and the terminal motion state changes to adapt to the external force. $F_{out}$ changes drastically
first, and then decreases due to the offset of the terminal, and oscillating towards 0. After the external force disappears, the force balance is broken again, and the end position is reversed offset to adjust the force, so that the resultant force at the end is gradually restored to 0.

7. Conclusion
In this paper, the kinematics model of six-axis robot is established by D-H method, and the kinematics solution is carried out. Then a Lagrange dynamic model considering the uncertainty of the system is established, and a sliding mode admittance controller based on the exponential reaching law is designed, which reduces the influence of the nonlinear and uncertain problems of the robot control system on the admittance control. Finally, through MATLAB simulation, the expected path of the robot is given and the external force is suddenly added to verify the compliance control of the robot. The system has good robustness and stability, which verifies the feasibility of the proposed method.

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