Six myths on the virial theorem for interstellar clouds

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ABSTRACT
The interstellar medium is highly dynamic and turbulent. However, little or no attention has been paid in the literature to the implications that this fact has on the validity of at least six common assumptions on the virial theorem (VT), which are as follows. (i) The only role of turbulent motions within a cloud is to provide support against collapse; (ii) the surface terms are negligible compared to the volumetric ones; (iii) the gravitational term is a binding source for the clouds since it can be approximated by the gravitational energy; (iv) the sign of the second time-derivative of the moment of inertia determines whether the cloud is contracting ($\ddot{I} < 0$) or expanding ($\ddot{I} > 0$); (v) interstellar clouds are in virial equilibrium (VE) and (vi) Larson’s relations (mean density–size and velocity dispersion–size) are the observational proof that clouds are in VE. However, turbulent, supersonic interstellar clouds cannot fulfill these assumptions because turbulent fragmentation will induce flux of mass, moment and energy between the clouds and their environment, and will favour local collapse while it may also disrupt the clouds within a dynamical time-scale. It is argued that although the observational and numerical evidence suggests that interstellar clouds are not in VE, the so-called ‘virial mass’ estimations, which should actually be called ‘energy-equipartition mass’ estimations, are good order of magnitude estimations of the actual mass of the clouds just because observational surveys will tend to detect interstellar clouds appearing to be close to energy equipartition. Similarly, order of magnitude estimations of the energy content of the clouds is reasonable. However, since clouds are actually out of VE, as suggested by asymmetrical line profiles, they should be transient entities. These results are compatible with observationally based estimations for rapid star formation, and call into question the models for the star formation efficiency based on clouds being in VE.

Key words: turbulence – stars: formation – ISM: clouds – ISM: general – ISM: kinematics and dynamics.

1 INTRODUCTION
Interstellar clouds are thought to be turbulent and supersonic. Their Mach numbers range from 1 ($T \sim 7000–8000$ K, H I clouds) to 10 ($T \sim 10$ K, molecular clouds forming low-mass stars) to 50 ($T \sim 10–50$ K, molecular clouds forming high-mass stars). Since supersonic turbulent motions carry mass and produce large-amplitude density fluctuations, turbulent fragmentation1 is expected to occur in the interstellar medium (ISM).

A useful tool for describing the overall structure of interstellar clouds is the scalar virial theorem (VT). It is obtained by dotting the momentum equation by the position vector and integrating over the volume of interest (Section 2). Being directly derived from the momentum equation, the VT always holds for any parcel of fluid. Virial equilibrium (VE) is a restrictive condition of the VT, and it is defined by the condition that the parcel of fluid under study has a second time-derivative of the moment of inertia equal to zero. It has been invoked extensively to analyse the stability of interstellar clouds. However, since in a trans- or supersonic turbulent ISM, clouds should be redistributing their mass as a consequence of their own turbulent motions, it already seems difficult to achieve VE in a supersonic, turbulent cloud.

Other simplifications, such as the assumption that the cloud is isolated or that the surface terms in the VT are negligible, are frequently made in many (if not in most) astrophysical studies of the VT. Those simplifications have been thought to be applicable to molecular clouds and their substructure for nearly three decades.

1Turbulent fragmentation is defined as the process through which a chaotic velocity field produces a clumpy density structure in the gas within a few dynamical time-scales (see e.g. von Weizsäcker 1951; Sasao 1973; Scalo 1988; Padoan 1995; Ballesteros-Paredes, Vázquez-Semadeni & Scalo 1999a; Klessen, Heitsch & Mac Low 2000; Heitsch, Mac Low & Klessen 2001; Ballesteros-Paredes 2004b).
3.1 First assumption: the kinetic energy is generally a term of support

It is almost universally considered in the literature that the turbulent (or kinetic) energy, $E_{\text{kin}} = \frac{1}{2} \int \rho u^2 \, dV$, provides support to clouds against collapse. While this is true for a system of particles, and partially valid if the kinetic energy is in the form of large-scale expansion and/or rotation, it is by no means certain that all the kinetic energy available will help against collapse in a system where turbulent fragmentation can occur, as discussed below.

This idea has its origin in Chandrasekhar (1951), who proposed that in the analysis of the gravitational instability the turbulent velocity field should be included. In his description, an effective sound speed is introduced, given by

$$c_{\text{eff}}^2 = c_s^2 + \frac{1}{3} u_{\text{rms}}^2,$$

where $c_s$ is the sound speed and $u_{\text{rms}}$ is the velocity dispersion of the turbulent motions (see e.g. Klessen et al. 2000; Mac Low & Klessen 2004, for a review). This description is valid only if (i) turbulent motions are confined to scales much smaller than the size of the cloud (Ballesteros-Paredes et al. 1999a) and (ii) such motions do not produce new, smaller-scale Jeans-unstable density enhancements. The first hypothesis disregards one of the main features of turbulent flows in general (e.g. Kolmogorov 1941; Lesieur 1990), that is, that turbulent motions do not produce Jeans-unstable density enhancements. The second condition is valid only if the turbulent motions are confined to scales much smaller than the size of the cloud (Ballesteros-Paredes et al. 1999a) and (ii) such motions do not produce new, smaller-scale Jeans-unstable density enhancements. The first hypothesis disregards one of the main features of turbulent flows in general (e.g. Kolmogorov 1941; Lesieur 1990), namely, that the largest velocities occur at the largest scales. An attempt to include this fact has been proposed by (Bonazzola et al. 1987), who suggested including the value of the rms velocity dispersion at each scale $l \propto 1/k$, that is,

$$c_{\text{eff}}^2(k) = c_s^2 + \frac{1}{3} (u(k))^2,$$

where $k$ is the wavenumber corresponding to the scale $l$, and $u(k)$ is the energy spectrum $E(k) = CK^{-d}$ as

$$u(k)^2 = \int_{k}^{\infty} E(k) \, dk = \frac{C}{1-\delta} k^{-1-d},$$

where $C$ is a constant and $\delta$ is the spectral index. The second condition, that is, that turbulent motions do not produce Jeans-unstable density enhancements, has the underlying complication that motions at scales larger than $l \sim 1/k$ will be very anisotropic with respect to structures of size $l$. Those modes will produce shear (through vortical modes) or compressions (through compressible modes) to the structures.2 Compressions in particular reduce the local Jeans mass (Sasao 1973; Hunter & Fleck 1982) and can induce local collapse. Thus, a fraction of the turbulent kinetic energy is involved in promoting collapse, rather than opposing it.

By decomposing the velocity field in its solenoidal and compressible components, the kinetic energy modes that provide support to the clouds are those having divergence larger or equal to zero,

$$\nabla \cdot u \geq 0.$$

This includes the solenoidal modes ($\nabla \cdot u = 0$) and the expansional component of the compressible modes ($\nabla \cdot u > 0$). In other words,
the precise result of collapse or support must then reflect the balance between all the agents that favour collapse against those agents that provide support. In the first group, not only should the gravitational energy be included, but the kinetic energy should also be involved in the compressible modes ($E_{kin,v_a<0}$), versus the kinetic energy involved in the expansional and rotational modes ($E_{kin,v_a>0}$).

### 3.2 Second assumption: the surface terms are negligible

It is frequently found in the literature that the surface terms are neglected altogether, especially in observational work (Larson 1981; Myers & Goodman 1988; Fuller & Myers 1992), mainly because there is not a direct way of measuring them observationally, although it is also a common practice in theoretical studies (e.g. Chandrasekhar & Fermi 1953; Parker 1969) and textbooks (e.g. Spitzer 1978; Parker 1979; Shu 1991; Stahler & Palla 2005). This assumption is based on the idea that self-gravitating clouds may be considered isolated because then their internal energies dominate the dynamics. The most notable exception is the thermal pressure, which is frequently invoked for ‘pressure confinement’ (e.g. McCrea 1957; Keto & Myers 1986; Bertoldi & McKee 1992; Yonekura et al. 1997). Although some works have considered, by analogy, the possibility of turbulent pressure confinement by means of the term $\tau_{int}$ (e.g. McKee & Zweibel 1992), such confinement of a cloud is difficult to achieve because the large-scale turbulent motions are anisotropic and will in general distort or even disrupt the cloud (Ballesteros-Paredes et al. 1999a).

Although we cannot measure the surface terms from observations, the possibility that they are as important as their corresponding volumetric terms suggests the investigation of the numerical simulations of the ISM. In fact, (Ballesteros-Paredes & Vázquez-Semadeni 1997) found that for an ensemble of clouds in two-dimensional simulations of the ISM at a kiloparsec scale, the surface terms have magnitudes as large as those of the volumetric ones (for three-dimensional simulations, see also Shadmehri et al. 2002, Tilley & Pudritz 2004 and Dib et al. 2006). This result suggests that, on one hand, either surface or volumetric terms are of comparable importance in shaping and supporting the clouds. On the other hand, it suggests that clouds must be interacting mass, momentum and energy with the surrounding medium. In such an environment, the meaning of thermal or ram pressure confinement is not clear, since motions at all scales must morph and deform the cloud.

### 3.3 Third assumption: the gravitational term is the gravitational energy

The gravitational term entering the VT is written as

\[ W = - \int V \rho \frac{\partial \phi}{\partial x_i} dV. \quad (7) \]

Splitting up the gravitational potential as the contribution from the cloud itself ($\phi_{\text{cloud}}$), plus the contribution from the outside ($\phi_{\text{ext}}$),

\[ \phi = \phi_{\text{cloud}} + \phi_{\text{ext}}, \quad (8) \]

the gravitational term can be written as

\[ W = E_g - \int V \rho \frac{\partial \phi_{\text{ext}}}{\partial x_i} dV, \quad (9) \]

where $E_g = -1/2 \int \rho \phi dV$ is the gravitational energy of the cloud alone, since the volume of integration of $\phi_{\text{cloud}}$ and the volume of the integral coincide (e.g. Chandrasekhar & Fermi 1953). The second term in the right-hand side is usually either implicitly or explicitly assumed to be negligible compared to the first term, and the gravitational term $W$ is then assumed to equal the gravitational energy $E_g$. This is valid only if the distribution of mass is spheroidal, or if the medium outside the cloud is tenuous such that its contribution to the potential is negligible. However, clouds are more similar to irregular fractals with arbitrary shapes (frequently filamentary) than to spheroids (e.g. Falgarone, Phillips & Walker 1991), and the contribution from the external gravitational field may not be negligible, giving rise to tidal torques. Although up to now, there is no observational evaluation of the contribution of the external mass to the gravitational term for any interstellar cloud, mass estimates for HI ‘envelopes’ around molecular clouds are of the same order of magnitude as the mass of the molecular clouds themselves (e.g. Williams & Maddalena 1996; Moriarty-Schieven, Andersson & Wannier 1997). Thus, it is not difficult to realize that the contribution of the second term in the right-hand side of equation (9) to the gravitational term $W$ can be of the same order of magnitude as the gravitational energy $E_g$.

Similar arguments can be made for the interiors of molecular clouds: even though we can approximate their shapes as (triaxial) spheroids (Jijina, Myers & Adams 1999), embedded molecular cloud cores are subject not only to their own self-gravity, but also to the tidal forces from their parental molecular cloud. Thus, it is not clear that the tidal forces represented in the second term of equation (9) will be negligible, and the assumption that the gravitational term equals the gravitational energy of the cloud seems unjustified.

What is the meaning and the effect of the second term of equation (9) on the energy budget of the clouds? It can be seen that it involves the gradient of the external potential. For non-symmetrical distributions of mass, this term is out of balance even if the distribution of mass inside the cloud is symmetric. Thus, this term represents the tidal forces over the mass contained in the volume $V$, and it can be split up into three terms:

\[ \int x_i \rho \frac{\partial \phi_{\text{ext}}}{\partial x_i} dV = \int (\phi_{\text{ext}} x_i \rho) \frac{\partial}{\partial x_i} dV - 3 \int \phi_{\text{ext}} \rho dV - \int \phi_{\text{ext}} x_i \frac{\partial \rho}{\partial x_i} dV. \quad (10) \]

Using the Gauss theorem, the first term on the right-hand side of equation (10) can be interpreted as the gravitational pressure evaluated at the boundary of the cloud,

\[ \int \left( \frac{\partial (\phi_{\text{ext}} \rho x_i)}{\partial x_i} \right) dV = \int_S (\phi_{\text{ext}} \rho x_i) \hat{n} \cdot dS. \quad (11) \]

The second term, by similarity with the gravitational energy, can be interpreted as three times the work done to assemble the density distribution of the cloud against the external mass,

\[ E_{g,ext} = 3 \int \phi_{\text{ext}} \rho dV. \quad (12) \]

Finally, the last term in equation (7),

\[ \int \phi_{\text{ext}} x_i \frac{\partial \rho}{\partial x_i} dV \]

involves the gradient of the density field inside the cloud. Although there is no clear interpretation of this term, it is worthwhile noting that its contribution is null for a homogeneous distribution of mass inside the volume $V$ of integration.

From the numerical point of view, several studies (Ballesteros-Paredes & Vázquez-Semadeni 1997; Ballesteros-Paredes 1999; Shadmehri et al. 2002; Tilley & Pudritz 2004, Dib et al., in preparation) have found that the gravitational term can be negative or
positive. If negative, it will be a confining agent. If positive, its overall action will be to contribute to the disruption of the cloud/core.

3.4 Fourth assumption: the sign of \( \ddot{I} \) defines whether the cloud is collapsing or expanding

It is frequently argued in the literature that if \( \ddot{I} > 0 \), the cloud must be expanding, while \( \ddot{I} < 0 \) implies that the cloud is contracting. Equilibrium is assumed to occur when \( \ddot{I} = 0 \).

This idea rests on the fact that the gravitational energy \( E_g \) has a negative sign, and it is a confining agent, while the internal energies (thermal, kinetic, or magnetic) are positive, and they are assumed to act as supporting agents against collapse. By neglecting the other terms, it can be assumed that if the gravitational energy is larger than the sum of the internal energies, the sign of the right-hand side of equation (2) is negative. Physically, if gravity wins, the cloud collapses.

Although energetically this is true, it is not hard to find an example in which an expanding cloud has a negative second time-derivative of the moment of inertia. Assume a sphere with constant density and fixed mass \( M \). Its moment of inertia is

\[ I = \frac{3}{5} \pi M R^2. \]  

(14)

If its size varies with time, for instance, as a power law,

\[ R = R(t) = R_0 \left( \frac{t}{t_0} \right)^y, \]  

(15)

its second time-derivative is given by

\[ \ddot{I} = \frac{6M}{5} \left( \frac{R_0}{t_0} \right)^2 \left( \frac{t}{t_0} \right)^{2y-2} \left( 2y - 1 \right), \]  

(16)

which is negative if \( 0 < y < 1/2 \), even though it is expanding. In general, \( \dddot{I} \) has been treated in the literature as if it were \( I \).

3.5 Fifth assumption: interstellar clouds are in virial equilibrium

The definition of VE is that the left-hand side of the Lagrangian VT (equation 2) equals zero:

\[ \dot{I}_L = 0 \]  

(17)

(see e.g. Spitzer 1978). Although there are some observational papers showing molecular clouds out of VE (e.g. Carr 1987; Loren 1989; Heyer, Carpenter & Snell 2001), it is frequently encountered in the literature as the VE assumption for MCs and their substructure (e.g. Myers & Goodman 1988; McKee 1999; Krumholz & McKee 2005; Tan, Krumholz & McKee 2006; Ward-Thompson et al. 2006). As mentioned in the Introduction, this statement is based on the (old) idea that all the forces (in the ISM) should be in balance, and the medium should have no net acceleration (see e.g. Spitzer 1978, chapter 11). However, both observational (Jenkins, Jura & Loewenstein 1983; Bowyer et al. 1995; Jenkins & Tripp 2001; Jenkins 2002; Redfield & Linsky 2004) and numerical (Vázquez-Semadeni et al 2003b; Gazol, Vázquez-Semadeni & Kim 2005; Mac Low et al. 2005) studies have found that the ISM is not in strict pressure balance, but exhibits strong pressure fluctuations, and in fact the turbulent ram pressure is significantly larger than the thermal one (e.g. Boulares & Cox 1990).

From an observational point of view, it cannot be demonstrated that clouds are in VE because neither the detailed three-dimensional density structure of molecular clouds nor the time-derivatives for interstellar clouds can be measured observationally. From a theoretical point of view, the VE assumption has strong implications: either supersonically turbulent clouds are not redistributing their mass inside or the way in which the time-derivative of the moment of inertia (\( \dot{I} \)) varies (which can be interpreted as the time-variation of their mass redistribution) is constant. Both statements seem implausible in a highly dynamical, non-linear ISM. (McKee & Zweibel 1992), for instance, have recognized the difficulty of achieving VE in a turbulent medium, because turbulent motions carry fluid elements, redistributing their mass.

From the numerical point of view, fortunately, it is possible to calculate all the terms of the VT for clouds, since we know all the variables involved in the numerical simulations. In order to test the VT, (Ballesteros-Paredes & Vázquez-Semadeni 1997, see also Ballesteros-Paredes et al. 1999a; Ballesteros-Paredes 1999) calculated all the integrals in the Eulerian version of the Virial Theorem for clouds in numerical simulations of the ISM at a kiloparsec scale by Passot, Vázquez-Semadeni & Pouquet (1995). They found that the second time-derivative of the moment of inertia never goes to zero, suggesting that clouds must be transient entities.

McKee (1999) has recognized the difficulty to achieving VE for actual MCs. He proposed two possibilities for assuming clouds in VE: the first one is to average in time the second time-derivative of the moment of inertia. He suggests that \( \dot{I} = 0 \) if the averaging time considered is much larger than the dynamical time-scale of the cloud, that is, if \( t_{\text{avg}} \gg t_{\text{dyn}} \). The second one is that for an ensemble of clouds, some of them may have positive values of \( \dot{I} \), and some others will have negative values, so that VE holds for the ensemble. Regarding the first assumption, there is a hidden assumption behind it: the cloud maintains its identity during several dynamical timescales. However, from the numerical point of view, simulations of the ISM suggest that the clouds are continually morphing and exchanging mass, energy and momentum with their surroundings, so that over \( t \sim t_{\text{dyn}} \) they have changed significantly. On the other hand, observational evidence (Ballesteros-Paredes, Hartmann & Vázquez-Semadeni 1999b; Elmegreen 2000; Hartmann, Ballesteros-Paredes & Bergin 2001; Ballesteros-Paredes & Hartmann 2006) also suggests that the lifetimes of the clouds are not significantly larger than their own dynamical times (i.e. they are transient). Concerning the second assumption, the analysis by Ballesteros-Paredes & Vázquez-Semadeni (1997, see also Ballesteros-Paredes 1999) shows that the moment of inertia spans up to seven orders of magnitude (in absolute value) between the largest and the smallest clouds. Thus, it is not clear that an average for such a large scatter will be representative of the actual dynamics of the clouds. In other words, even if \( \dddot{I} \) did average out to zero for the ensemble, this does not alter the fact that, individually, clouds are not in VE.

3.6 Sixth assumption: Larson’s relationships are the observational demonstration of clouds being in virial equilibrium

The mean density–size and velocity dispersion–size relations first discussed by Larson (1981) are thought to be an observational

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Footnote:

3 In fact, it has to be oscillating around a mean shape without a strong redistribution of mass, in order to achieve the condition \( \dddot{I} = 0 \). A similar assumption is made by (McKee & Zweibel 1992).
Thus, if there is a mean density–size power-law relationship \( \rho \propto R^{\alpha} \), then there should be a velocity dispersion–size relationship of the form
\[
\delta v^2 \propto R^\beta,
\]
with
\[
\beta = \alpha + 2 - \frac{3}{2}.
\]
Several caveats must be mentioned at this point. First of all, this derivation does not mean VE, but energy equipartition, since the only assumption was \( E_k \sim E_m \) (see equation 18). A similar result can be found if the equipartition assumed is valid between the gravitational and magnetic energy. In this case, \( \delta v \) is proportional to the Alfvén speed (see e.g. Myers & Goodman 1988). Secondly, as discussed by Vázquez-Semadeni & Gazol (1995), in the particular case of \( \rho \propto R^{-3}, \delta v \propto R^{1/2} \). However, the pair \( \alpha = -1, \beta = 1/2 \) is not unique. Any pair of values satisfying equation (20) will be consistent with energy equipartition (again, not VE). Finally, it is convenient to recall that the validity of Larson’s relations has been called into question, especially the mean density–size relationship (Kegel 1989; Scalo 1990; Vazquez-Semadeni, Ballesteros-Paredes & Rodríguez 1995; Ballesteros-Paredes & Mac Low 2002). It seems that this relationship is more a consequence of the observational process, in which the dynamical range of the observations is limited below by the minimum sensitivity of the telescopes, and above by saturation of the detectors, optically thick effects and depletion.

4 DISCUSSION: VIRIAL MASS, VIRIAL EQUILIBRIUM AND THE STAR FORMATION EFFICIENCY

As already discussed, there are at least six common assumptions related to the VT which seem to be unjustified for a turbulent ISM. Some questions, such as why Virial Masses are good order of magnitude estimates of the actual mass, or why in principle, the sub- or supercriticality of a molecular cloud core is a good estimation of the dynamical state of such a core, are still valid to ask.

The answer is that those estimations are based on energy equipartition, and not on VE (i.e. \( E_k \sim E_m \sim E_d \)) does not mean that \( I_L \) is negligible when it is compared to the left-hand side of equation (2). Molecular clouds seem to be in approximated equipartition between self-gravity, kinetic and magnetic energy (see e.g. Myers & Goodman 1988) although the super-Alfvénic nature of molecular clouds is still a matter of debate [e.g. Padoan & Nordlund 1999; see also Ward-Thompson et al. (2006) for an observational review and Ballesteros-Paredes et al. (2006) for a theoretical one].

The question thus is why MCs seem to be in energy equipartition? The answer is related to what we identify as a cloud, and to observational limitations: in the first case, clouds with a substantial excess of internal energy will rapidly expand and merge with the more diffuse medium. This is the case of an H II region or supernova explosion expanding within its parental molecular cloud. The amount of energy provided by those events is enough to disrupt their parental environment within a few Myr (e.g. Franco, Shore & Tenorio-Tagle 1994). The excess of internal energy is mostly in the form of ultraviolet photons, which rapidly ionize the molecular gas, reducing even more the life-time scale of what is identified as the parental molecular cloud (see e.g. Mellema et al. 2005).

In the second case, if the cloud has an excess of gravitational energy, its free-fall time velocity is at the most a factor of \( \sqrt{2} \) the velocity dispersion needed for equipartition. This means that the difference between a free-fall collapsing cloud and one in equilibrium is just a factor of 2 in the energy, and a factor of \( \sqrt{2} \) in velocity. Both systems will be, in principle, in order of magnitude energy equipartition. Furthermore, note that in the free-fall collapsing case all the kinetic energy will be observationally identified as energy for support, where it is not actually providing any support. This re-enforces the need to distinguish between the compressible, expansive and vertical modes of the kinetic energy (see Section 3.1).

As discussed above, not all the terms entering the VT can be measured observationally. However, all of them can be measured from numerical simulations. The work by Ballesteros-Paredes & Vázquez-Semadeni (1993, 1997), Shadmehri et al. (2002) and Tilley & Pudritz (2004) on turbulent realizations of MCs shows that all the terms entering the VT are of similar importance. Note that the same problems related to the identification of an expanding cloud are applicable to the numerical work. In this case, what we identify as a cloud are the density enhancements. By definition, vacuumed regions (due either to modelled stellar activity or to the turbulence itself) are no longer considered clouds. As for the collapsing case, strongly self-gravitating clouds will develop a velocity field that is a factor of \( \sqrt{2} \) the value of energy equipartition, if only gravity is acting, and of the order of magnitude if the turbulence is forced. Thus, order of magnitude equipartition between magnetic, kinetic and gravitational energy is valid to assume to predict whether the clouds should be supercritical or subcritical (Bertoldi & McKee 1992; Nakano 1998), or to understand why subcritical cores are unlikely to survive long lifetimes within MCs (Nakano 1998). However, the order-of-magnitude coincidence of the involved energies does not mean that clouds are in equilibrium at all. It should be stressed that the difference is not just semantic (virial mass versus energy-equipartition mass). The difference is conceptual: turbulent clouds cannot be in equilibrium because turbulent fragmentation takes them out of equilibrium, and thus they do not last long. In order to achieve equilibrium [even if it is a time-averaged equilibrium, \( \langle I \rangle = 0 \), as suggested by McKee (1999)], it is necessary to allow the cloud to live for, at least, several crossing times. To give an example, the Taurus Molecular Cloud has a size of \( l \sim 20 \) pc, and a velocity dispersion of \( \delta v \sim 2 \) km s\(^{-1}\). Its lateral crossing time is \( \tau_{\text{cross}} = 1/\delta v \sim 10 \) Myr. In order to achieve equilibrium, Taurus has to live 20–30 Myr without a serious distortion or modification. Such a condition seems difficult to achieve if one realizes that Taurus has a Mach number of the order of 10, and the associated \( H_2 \) gas has Mach numbers of 20–30 in respect to the internal CO gas [see e.g. the velocity–position diagrams of \( H_2 \) and CO in Ballesteros-Paredes et al. (1999b)].

The fact that turbulent clouds cannot be in VE and cannot last several crossing times is in clear contradiction with (Tan et al. 2006), who made theoretical arguments that favour star formation occurring during several dynamical time-scales. However, in their theoretical derivation they applied the star formation rate per free-fall time expression given by equation (30) of (Krumholz & McKee 2005) to a clump in hydrostatic equilibrium. As Krumholz & McKee (2005)
point out, their equation (30) is valid only for large Mach numbers (20–40), while by definition, a clump in hydrostatic equilibrium is subsonic. Thus, the theoretical arguments in favour of slow star formation are wrong.

Another point to be stressed concerns line profiles of H\textsubscript{1}, CO and its isotopes, and even higher density tracers (CS, NH\textsubscript{3}, etc.). If interstellar clouds and/or their cores are not in VE, they must be distorted and disrupted within a dynamical time-scale. How will their line profiles look, and how will they look if they were in equilibrium? Certainly, if turbulence were microscopic (necessary to achieve equilibrium), line profiles should be symmetric. The fact that H\textsubscript{1} and CO clouds (e.g. Hartmann & Burton 1997; Dame, Hartmann & Thaddeus 2001) as well as molecular cloud cores (e.g. Falgarone et al. 1998) exhibit non-symmetric line profiles is probably the best and the only evidence that the term $I_{\text{L}}$ is different from zero.\footnote{Note that in an Eulerian frame of reference, an idealized dustlane in a spiral arm. It is argued that this contribution can be as important as the self-gravitational energy.}

Even a small degree of asymmetry in the line profiles suggests that large-scale motions are present in the observed system, which is a natural consequence of turbulence being a multiscale phenomenon. This does not mean that such a system is collapsing or expanding as a whole, as formerly suggested by (Goldreich & Kwan 1974). Instead, it means that turbulent large-scale motions are present in the system, which should be evolving within a dynamical time-scale.

In this context, it should be stressed that clouds presenting large-scale motions and evolving to form stars rapidly will not necessarily have a high star formation efficiency, as is the common belief since Zuckerman & Evans (1974). Although former models of quasi-static evolution of molecular clouds were proposed to reduce the star formation efficiency, the turbulent models producing local collapse rapidly have small efficiency because gravo-turbulent fragmentation involves only a small fraction of the mass of the system in collapsing regions (Vázquez-Semadeni, Ballesteros-Paredes & Klessen 2003a; Vázquez-Semadeni, Kim & Ballesteros-Paredes 2005). In numerical models, when energy feedback from stars is included, the cloud is blown out rapidly (see e.g. Ballesteros-Paredes 2004a). However, it should be recognized that a detailed quantification of the star formation efficiency in turbulent simulations with open boundary conditions and stellar feedback is needed.

5 CONCLUSIONS

This contribution has discussed the applicability of the six more common assumptions on the VT.

(i) It was shown that the decomposition of the velocity field into its vortical and compressible modes is necessary, since only modes satisfying the condition $\nabla \cdot \mathbf{u} > 0$ provide support, while modes satisfying $\nabla \cdot \mathbf{u} < 0$ foment collapse.

(ii) It was argued that for a supersonic, turbulent ISM, surface terms should not be neglected.

(iv) Using a simple counter-example, it was shown that the sign of the second time-derivative of the moment of inertia does not determine whether the cloud is contracting or expanding. An expanding cloud may very well satisfy the condition $I_{\text{E}} < 0$, and a contracting one may satisfy $I_{\text{E}} > 0$, contrary to the common belief. In other words, $I$ has been treated in the literature as if it were $I_{\text{L}}$.

(v) It was argued that interstellar clouds are not likely to satisfy the VE condition $I = 0$.

(vi) It was shown that Larson’s (1981) relations are not observational proof for clouds being in VE.

(vii) Clouds seem to be in energy equipartition because of either observational limitations or the intrinsic definition of a cloud.

Turbulent fragmentation plays a crucial role for the inapplicability of the VT to interstellar clouds, since it will induce a flux of mass, momentum and energy between the clouds and their environment, and will favour local collapse while disrupting the clouds within a dynamical time-scale. The common assumptions discussed in the present contribution drive our understanding of the dynamical state of the interstellar clouds towards a picture that favours a static ISM. However, they are highly difficult to fulfill if the ISM is highly turbulent, as it was found to be many years ago (e.g. McCray & Snow 1979). Inferences of the star formation efficiency for supersonic (Mach numbers $\sim$ 20–40) clouds in VE living several dynamical time-scales (e.g. Krumholz & McKee 2005; Tan et al. 2006) should be taken with caution.

The lack of observational evidence for clouds being in VE, and the identification of asymmetrical line profiles observed towards interstellar clouds using different tracers are the best evidence of clouds being out-of-equilibrium systems. These facts lead us to the conclusion that clouds should be transient structures, which exchange mass, momentum and energy with their environment. This is precisely the opposite point of view to the old one which says that clouds should be at rest, which is still present in textbooks and papers, but it is consistent with recent observationally based works that favour a scenario of rapid cloud and star formation (see e.g. the reviews by Mac Low & Klessen 2004; Ballesteros-Paredes et al. 2006, and references therein).

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