Performance of an adjustable, threaded inertance tube

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Abstract. The performance of the Stirling type pulse tube cryocooler depends strongly on the design of the inertance tube. The phase angle produced by the inertance tube is very sensitive to its diameter and length. Recent developments are reported here regarding a n adjustable inertance device that can be adjusted in real time. The inertance passage is formed by the root of a concentric cylindrical threaded device. The depth of the threads installed on the outer screw varies. In this device, the outer screw can be rotated four and half turns. At the zero turn position the length of the passage is 1.74 m and the hydraulic diameter is 7 mm. By rotating the outer screw, the inner threaded rod engages with additional, larger depth threads. Therefore, at its upper limit of rotation, the inertance passage includes both the original 1.74 m length with 7mm hydraulic diameter plus an additional 1.86 m length with a 10 mm hydraulic diameter. A phase shift change of 24° has been experimentally measured by changing the position of outer screw while operating the device at a frequency of 60 Hz. This phase angle shift is less than the theoretically predicted value due to the presence of a relatively large leak through the thread clearance. Therefore, the distributed component model of the inertance tube was modified to account for the leak path causing the data to agree with the model. Further, the application of vacuum grease to the threads causes the performance of the device to improve substantially.

Keywords: Adjustable inertance tube, pulse tube cryocooler

1. Introduction

The pulse tube cryocooler has the advantage over the Stirling cryocooler of having no moving parts at the cold end, resulting in high reliability, long life and low vibration at the cold head. However, the thermal efficiency and cooling capacity of a basic pulse tube cryocooler is lower than that of the Stirling cryocooler in part due to the challenge of producing an optimum phase between the mass flow and pressure oscillations at the cold end. In order to improve the phase angle, Mikulin [1] introduced an orifice between the pulse tube and the reservoir and Zhu [2] proposed the idea of a double inlet connecting the pulse tube warm end to the linear compressor outlet; both modifications increased the pulse tube efficiency and cooling performance significantly. However, both the orifice and double inlet modifications result in a mass flow oscillation that always leads the pressure oscillation, limiting the cooling performance of pulse tube. Kanao[3] adopted a long neck tube connecting the pulse tube to the reservoir, which proved to have larger phase shifting ability than that of either the orifice or the double inlet; this long tube is referred to as an inertance tube (IT) because it takes advantage of the inertia of the fluid in the passage. Many researchers [4-14] have worked on improving the phase shifting mechanism of the inertance tube since it was first introduced in 1994. Previously, a fixed diameter and fixed length inertance tube has been used which produces a fixed phase angle at the cold end. However, the fixed inertance tube often does not provide the optimal phase angle and it is difficult to optimize the design of this component without expensive and time-consuming experimental iterations. Therefore, Pfotenhauer and Nellis [15, 16] suggested the use of an adjustable inertance tube, which allows the passage diameter and length to be changed in real time. The present paper continues the progress report of this research by discussing recent theoretical and experimental results for a novel inertance tube formed by the roots of two threaded concentric cylinders so that the geometric characteristics of the inertance passage can be adjusted by rotating the device causing the engagement or disengagement of threads. One design challenge for this type of device is the presence of leakage along the thread clearance.

There are four sections in this paper including the introduction. The second section discusses modifications to the distributed component model reported by Schunk [10] to account for thread leakage as well as the presence of multiple sections of passage that have different geometric characteristics. The third section illustrates the experimental results for this adjustable inertance tube. The simulation results from the modified distributed component model are compared to the experimental data and show good agreement. Vacuum grease
is used in the experiment to reduce the thread leakage and the resulting experimental data shows that the ability of the device to adjust the phase angle is improved. The last section briefly discusses the performance of the adjustable inertance tube and some possible future improvements to this device.

2. Modified distributed component model

The schematic of the adjustable inertance tube is depicted in Figure 1. The channel formed by roots of the threads between an inner and outer screw creates the adjustable inertance tube. By turning the outer screw from its top position Figure 1(a) to its bottom position Figure 1(b), the length of the small diameter passage remains constant while the portion of the large diameter tube will be reduced. There is some leakage through the clearance that is present between the two threads and this leakage has been shown to reduce the performance of the device relative to shifting the phase. Therefore, the distributed component model has been modified in order to account for the effect of the leaks.

The distributed component model separates the inertance tube into several subsections or nodes, each node consists of a fluid resistance, compliance, and inertia [10]. Previous models have not included leakage since a copper or stainless steel pipe is typically used for the inertance tube. However, in the adjustable inertance tube shown in Figure 1 there is a leak passage associated with the engagement of the screw threads. Even a small clearance may result in a relatively large leakage area due to the long helical length of the threads; therefore, it is important to consider the effect of leakage on the phase shift performance. A schematic of the fluid
impedance network including the additional components that are used to model the leakage path (which include both an entrance effect and frictional loss) is shown in Figure 2; note that the additional fluid components are shown in dashed line boxes in Figure 2. The details regarding the calculation of the fluid resistance, compliance and inertia are available in the literature [10, 15]. Here, we will confine our discussion to the leakage and entrance effect resistances that have been added to the fluid network. The leakage and entrance effect resistances are in series and they are together placed in parallel with the compliance and the rest of the impedance, as depicted in Figure 2.

The resistance of the leak is given by:

\[ R_{lk} = \frac{V_{lk} f_{lk} L_{lk} N}{2 D_{lk} th_{lk} W_{lk}} \]  

where \( V_{lk} \) is the velocity through the crack, \( L_{lk} \) (\( L_{small,lk} \) and \( L_{large,lk} \) in Figure 1(a)) are the engagement depths for each screw, (separate dimensions for the large thread region \( L_{large,lk} \) and the small thread region \( L_{small,lk} \) are listed in Table 1); \( N \) in Figure 2 is the number of subsections used to model the total inertance tube, half are used for the large thread region and half for the small thread region; \( D_{lk} \) is the hydraulic diameter of the leak channel, which is calculated as four times the cross sectional area divided by the wetted perimeter of the leakage path. \( th_{lk} \) is the gap thickness between the threads of the two screws, and \( W_{lk} \) is the width of the leak gap, which is the helical thread length from the inlet to the outlet of each section. These dimensions are depicted in Figure 2 and their values are listed in Table 1. Since the maximum Reynolds number going through the leak passage is less than 1400 for the conditions considered here \( f_{lk} \) is the friction factor associated with the developing region for laminar flow in a rectangular duct given by the equation below[17]:

\[ f_{lk} = 4 \left[ \frac{3.44}{L^*} + \frac{1.25}{4L^*} \right] \left[ \frac{3.44}{L^*} + \frac{4}{\sqrt{L^*}} \right] + \frac{0.00021 \left( L^* \right)^2}{\frac{1}{L^*} + \left( L^* \right)^2} \]  

where \( f_{lk} \) is 96/\( Re_{lk} \), as the aspect ratio of the leak passage approaches 0[17]. \( Re_{lk} \) is the Reynolds number for this leakage:

\[ Re_{lk} = \frac{\rho V_{lk} D_{lk}}{\mu} \]  

The parameter \( \rho \) is the gas density and \( \mu \) is the viscosity of the working fluid. The gas properties are determined assuming room temperature and the average or charge pressure measured in the adjustable inertance tube. \( L^+ \) is defined in the following equation:

\[ L^+ = \frac{L_{lk}}{D_{lk} Re_{lk}} \]  

Also, in order to determine the entrance effect from the inertance tube to the leak passage, an orifice equation is used[18].

\[ Z_{out} = \frac{\sqrt{\Delta P_{out}}}{\sqrt{2 \rho C_{out} F_{out} A_{out}}} \]  

where \( \Delta P_{out} \) is a pressure difference through the leak passage, \( A_{out} \) is the cross section area of the leak, calculated as the product of the gap thickness and the width of the leak within the subsection, \( C_{out} \) is the discharge coefficient given by the equation below[18]

\[ C_{out} = 0.5959 + 0.0312 \beta^{2.1} - 0.184 \beta^4 + 91.71 \beta^{2.5} Re_{lk}^{-0.75} \frac{0.0389 \beta^4}{1 - \beta^4} - 0.0158 \beta^3 \]  

\[ \beta = \frac{D_{lk}}{D_p} \]  

\( D_p \) is the hydraulic diameter of the threaded helical section, calculated by using equation (8);
\[ D_p = \frac{4 \cdot h_{IT} \cdot (W_{\text{small,IT}} + W_{\text{large,IT}})}{h_{IT}^2 + (W_{\text{small,IT}} + W_{\text{large,IT}})} \]  

(8)

F_{\text{out}} \text{ is given by the equation (9):}

\[ F_{\text{out}} = \sqrt{\frac{1}{1 - \beta^4}} \]  

(9)

The calculation procedure used for the modified distributed component model is iterative. The total inertance tube, including both the large diameter and small diameter inertance tubes in series, is divided into n nodes; half of the nodes for the large diameter tube the other half for the small diameter tube, having the leak and entrance effect resistance in each node. The starting point for the iteration process includes an estimate for the Reynolds numbers, the velocity through the leak and the pressure difference across the leak at each node. The iteration process begins with the known impedance values for each node and the pressure amplitude at the interface between the pulse tube and the inertance. Then, the fluid network is solved in order to determine the new Reynolds numbers, new velocities through the leaks and new pressure difference across the leaks. If the differences between these values and the values from the previous iteration are smaller than a tolerance limit \( \epsilon \), typically set to about 1e-11 for the Reynolds number, 1e-11 m/s for the leak velocity and 1e-11 pa for the leak pressure difference, then the calculation process is terminated and the acoustic power and phase angle at the pulse tube interface are calculated.

### Table 1. Basic geometry parameters of the cylindrical adjustable inertance tube

| Variables   | Values   | Variables   | Values   | Variables   | Values   |
|-------------|----------|-------------|----------|-------------|----------|
| L_{\text{small,IT}} | 1.74 m | L_{\text{large,IT}} | 0-1.86 m | W_{\text{small,IT}} | 6.35e-3 m |
| W_{\text{large,IT}} | 15.24e-3 m | h_{IT} | 7.62e-3 m | V_{\text{res}} | 1.5e-3 m^3 |
| L_{\text{small,lk}} | 13.7e-3 m | L_{\text{large,lk}} | 4.8e-3 m | th_{lk} | 15.5e-6 m |

Table 1 summarizes the geometric parameters of the cylindrical adjustable inertance tube that was fabricated. The adjustable inertance tube has two different dimension channels in series; one channel is 6.35e-3 m in width and 7.62e-3 m in height, with a length of 1.74 m; the other channels is 15.24e-3 m in width and 7.62e-3 m in height with a length of 1.86 m. The hydraulic diameter of small and large inertance tubes is 7 mm and 10 mm respectively. The length of the large diameter tube will decrease as the outer screw rotates from top to the bottom, so that it can change the total length while the pulse tube is operating. The gap thickness is assumed to be 15.5e-6 m in the table. The thread engagement length is treated as the length of the leak passage, which are 13.7 mm and 4.8 mm for the small and large diameter tubes, respectively.

![Figure 3. Phase angle shift calculated by Distributed Component Model](image-url)
Figure 3 shows the phase angle shift predicted by the Modified Distributed Component Model (MDCM) as a function of the length of the large diameter inertance tube; changing this value from 0 to 1.86 m corresponds to turning the outer screw from 0 to 4.5 turns. Figure 3 is generated assuming that there are no leaks between two screws. Note that the phase angle shift at 60 Hz can be adjusted from $60^\circ$ to $-60^\circ$, when the outer screw rotates from 0 turns to 4.5 turns (bottom to the top). Therefore, the adjustable inertance tube is predicted to have very good phase shift ability for the pulse tube cryocooler. However, the thread leakage along the adjustable inertance tube deteriorates the phase shifting performance. Figure 4 illustrates the phase angle shift versus length of the large diameter inertance tube at the same working condition predicted by the MDCM after adding the leak and entrance resistance into the fluid network shown in Figure 2. The $15.5e-6$ m clearance leak dramatically reduces the phase angle adjustment; the phase angle will only change from $0^\circ$ to $-25^\circ$ as the length of the large diameter inertance tube increases from 0 m to 1.86 m. Also, there seems to be essentially no phase angle change at lower frequencies according to the MDCM when the leakage is present.

3. Experimental results

The Adjustable Inertance Tube (AIT) is mounted on the top of a linear compressor, (60 Amp, Q Drive model 2S297W TwinSTAR Pressure Wave generator). A mass flow meter operating by measuring the pressure difference across a copper screen pack is installed at the inlet of the adjustable inertance tube. A detailed description of this mass flow meter can be found in reference [13]. Figure 5 displays the 3D drawing (left) and physical photo (right) of the experiment. The fin heat exchanger around the reservoir dissipates the acoustic power as heat to the surrounding air. The reservoir connected to the exit of the inertance tube has an approximate total volume of 2 liters. The adjustable inertance tube is formed by the channel between the threads of the two screws. Stops are located at each end of the channel, which force the fluid to flow into the inertance tube and exit from the inertance tube into the reservoir. Nitrogen at an average charge pressure of 300 psig was used in the experiment.

Figure 6 shows the measured phase angle as a function of the position of the outer screw when the device is operated at 60 Hz. There are three curves shown in Figure 6. The triangle symbols on the solid line indicate the phase angle predicted by the MDCM under these conditions with a clearance gap of $15.5e-6$ m in the model. The phase angle increases from $-1.5^\circ$ to $-1^\circ$ and then decreases to $-25^\circ$ as the length of the large diameter inertance tube increases from 0 m to 1.86 m, almost a $24^\circ$ phase angle change. The rectangular symbols on a dashed line illustrate the measured phase angle during our initial testing. Because of the leakage existing between the threads, the phase angle between the mass flow and pressure at the inlet of the inertance tube stays almost constant and equal to approximately $-20^\circ$. It is easy to imagine that the nitrogen flow essentially shortcuts to the reservoir directly after entering the inertance tube. Therefore, no matter what position the outer screw is in, the phase angle remains constant. Because there is a large leakage between the two screws observed in the experiment, vacuum grease was placed on the threads in order to seal the leaks. The experimental data with vacuum grease on the threads corresponds to the circle symbols on the dashed line in Figure 6; these experimental measurements agree with the MDCM predictions quite well both in magnitude and trend, The

![Figure 4. Phase angle shift computed by Modified Distributed Component Model](image-url)
phase angle increases from -10° to -5° and then decreases to -25° as the outer screw is rotated from its bottommost position to its topmost position. The uncertainties shown in the measured phase angle are calculated from the standard deviation of several phase angle measurements. Each pressure wave was measured at least 40 times allowing the phase angle shift to be calculated 40 times at each condition. By applying the vacuum grease on the threads, the clearance gap in the experiment is effectively decreased to 15.5e-6 m

Figure 5. Configurations of cylindrical adjustable inertance tube experimental setup

Figure 6. Phase angle shifts with length of inertance tube at 60 Hz

Figure 7 shows the same information at 45 Hz and 30 Hz. For the 45 Hz case, there is still an 8° phase angle shift predicted by the MDCM (triangle solid curve), while the vacuum grease experimental data (circle dash curve) shows the same trend. The data without vacuum grease scenario again does not predict the phase angle shift because of the huge leak between the threads. At 30 Hz (and lower frequencies), the discrepancies between the simulation and test become large and neither simulation nor experiment show a substantial phase shift.
Figure 7. Phase angle shifts with the length of inertance tube at 45 Hz and 30 Hz

Figure 8. Phase angle change with the length of inertance tube at high frequencies

Figure 8 shows the change of phase angle at higher frequencies. If the vacuum grease is applied to the surface of the threads, the MDCM fits the experimental data well at all frequencies tested above 45 Hz. In Figure 8, the dashed lines represent the data from the experiment, and the solid curve indicates the phase angle shift trend calculated by the model with a 15.5e-6 m clearance gap.

4. Conclusion and discussion
The performance of the adjustable inertance tube is verified both by the experimental data and model simulation. From the experiment, the largest phase angle shift, which is 24°, occurs at 60 Hz when vacuum grease is applied to the surface of the threads. The modified distributed component model captures the same trend when the clearance gap is set as 15.5e-6 m in the model. If there were no leaks between the threads, the phase angle would shift from 60° to -60° according to the MDCM. Therefore, in order to effectively use this cylindrical adjustable inertance tube with a pulse tube cryocooler it is necessary to reduce the leakage effect in future iterations. One method of accomplishing this is to increase the engagement length between the two threads. This device has an engagement length of 13.7 mm for the small diameter inertance tube and only 4.8 mm for the large diameter inertance tube. An alternative method for reducing leakage may be a hard coating anodizing process applied to both the inner screw surface and the outer screw surface so that the clearance gap is reduced.

The cylindrical adjustable inertance tube can shift the phase angle at the inlet of the inertance tube by rotating the outer screw from the bottommost position to the topmost position. It can help improve the performance of the pulse tube cryocooler by changing the phase angle in the real time during operation.
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