String Black Holes as Particle Accelerators to Arbitrarily High Energy

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Abstract We show that an extremal Gibbons-Maeda-Garfinkle-Horowitz-Strominger black hole may act as a particle accelerator with arbitrarily high energy when two uncharged particles falling freely from rest to infinity on the near horizon. We show that the center of mass energy of collision is independent of the extreme fine tuning of the angular momentum of the colliding particles. We further show that the center of mass energy of collisions of particles at the ISCO ($r_{ISCO}$) or at the photon orbit ($r_{ph}$) or at the marginally bound circular orbit ($r_{mb}$) i.e. at $r \equiv r_{ISCO} = r_{ph} = r_{mb} = 2M$ could be arbitrarily large for the aforementioned space-time, which is quite different from the Schwarzschild and the Reissner-Nordstrøm space-time. For non-extremal GMGHS space-time the CM energy is finite and depends upon the asymptotic value of the dilation field ($\phi_0$).

Keywords BSW Effect, ISCO, Photon orbit.

1 Introduction

An interesting effect predicted by Banados, Silk & West (2009) (hereafter BSW) is that rotating black holes may act as particle accelerators with arbitrarily high center-of-mass energy. The process is likely: when two massive dark matter particles with different angular momentum are falling into the black hole and collide near the horizon then the energy in the center of mass frame can become unlimited. Here it is assumed that the particles at infinity are at rest and the collision energy is produced mainly due to the gravitational acceleration. This mechanism is criticized by several authors. Particularly Berti, Cardoso, Gualtieri, Pretorius & Sperhake (2009), pointed out that there is an astrophysical limitations i.e. maximal spin, back reaction effect and gravitational radiation etc. on that center-of-mass(CM) energy due to the Thorn (1974)'s bound i.e. $a/M = 0.998$ ($M$ is the mass and $a$ is the spin of the black hole).

Also Jacobson & Sotiriou (2010) found that CM energy in the near extremal situation for rotating Kerr black hole is $E_{cm} \sim \frac{2}{0.998}$. Lake (2010) calculated the CM energy at the Cauchy horizon of a static Reissner-Nordstrøm black hole which is limited. Grib & Pavlov (2011) investigated the CM energy using multiple scattering process. The collision in the ISCO particles was investigated by the Harada & Kimura (2011) for Kerr black hole. Liu, Chen, Ding & Jing (2011) studied the BSW effect for Kerr-Taub-Nut space-times and showed that the CM energy depends upon both the Kerr parameter ($a$) and the Nut parameter ($n$) of the space-time. McWilliams (2012) studied that the black holes are neither particle accelerators nor dark matter probes. Galajinsky (2013) also demonstrated that the center-of-mass energy in the context of the near horizon geometry of the extremal Kerr black holes and proved that the center-of-mass energy is finite for any value of the particle parameters. Several authors Wei, Liu, Li & Chen (2010); Zaslavskii (2010); Patil & Joshi (2012) have studied the BSW effect for different types black holes and get the unlimited center-of-mass energy.

In this article, we wish to study the BSW effect of the Gibbons-Maeda-Garfinkle-Horowitz-Strominger (hereafter GMGHS) black hole and our aim is to observe what happens this effect precisely in the extremal limit i.e. at $Q^2 = 2M^2 e^{2\phi_0}$.

The dilatonic charged black hole are represented by the “string metric” which is a solution of the effective action of the low energy limit of heterotic string theory.
can be written in the form: Gibbons & Maeda (1988); Garfinkle, Horowitz & Strominger (1991,1992)
\[ ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1}dr^2 + r\left(1 - \frac{Q^2}{Mr}e^{-2\phi_0}\right)[d\theta^2 + \sin^2 \theta d\phi^2] \]  
and
\[ e^{-2\phi} = e^{-2\phi_0}\left(1 - \frac{Q^2}{Mr}e^{-2\phi_0}\right) \]
where \( \phi_0 \) is the asymptotic value of the dilation field, \( M \) represents the mass of the black hole, \( Q \) denotes its magnetic charge and \( \phi \) is the scalar field.

Note that this metric differs from the Reissner-Nordstrom (RN) solutions of the Einstein-Maxwell theory is that it does not have any inner horizon. It also may be noted that this metric is exactly similar to the Schwarzschild metric. But there is a difference in a sense that the area of the two sphere (\( S^2 \)) of constant \( r \) and \( t \) now strictly depends upon the value of \( Q \). This area \( A = 4\pi r(r-b) \) (Here \( r = 2M, b = \frac{Q^2}{Mr}e^{-2\phi_0} \)) goes to zero at the extremal limit i.e. at \( b = 2M \). The other interesting characteristics of this space-time is that there is a curvature singularity occurs at \( r = b \).

But from the string theoretical perspectives this singular nature manifested when \( b = 2M \) which is not important because the string does not couple to the metric \( g_{ab} \) rather to \( e^{2\phi}g_{ab} \).

### 2 ISCOs in GMGHS Black Hole

In this section we will compute the the properties of the circular geodesics for GMGHS black hole in the equatorial plane and also compute the ISCOs for this black hole. To compute the geodesic motion of a test particle in this plane we set \( \dot{\theta} = 0 \) and \( \theta = \text{constant} = \pi/2 \) and follow the reference Pradhan (2012).

Thus the Lagrangian for this motion is given by
\[ 2\mathcal{L} = -(1 - \frac{2M}{r})(u^t)^2 + (1 - \frac{2M}{r})^{-1}(u^r)^2 + r(r-b)(u^\phi)^2. \]

The generalized momenta reads as
\[ p_t = -(1 - \frac{2M}{r})u^t = -E = \text{Const}. \]  
\[ p_\phi = r(r-b)u^\phi = L = \text{Const}. \]  
\[ p_r = (1 - \frac{2M}{r})^{-1}u^r. \]

Here over dot represents differentiation with respect to proper time(\( t \)). Since the Lagrangian of the test particle independent of \( t^t \) and \( \phi^\phi \), so \( p_t \) and \( p_\phi \) are conserved quantities. Solving (5) and (6) for \( u^t \) and \( u^\phi \) we find
\[ u^t = \frac{E}{(1 - \frac{2M}{r})}. \]  
\[ u^\phi = \frac{L}{r(r-b)}. \]
where \( E \) and \( L \) are the energy and angular momentum per unit mass of the test particle.

Therefore the required Hamiltonian reads as
\[ \mathcal{H} = p_t u^t + p_\phi u^\phi + p_r u^r - \mathcal{L}. \]

In terms of the metric the Hamiltonian may be written as
\[ \mathcal{H} = -(1 - \frac{2M}{r})(u^t)^2 + (1 - \frac{2M}{r})^{-1}(u^r)^2 + r(r-b)(u^\phi)^2 - \mathcal{L}. \]

Since the Hamiltonian is independent of \( t^t \), therefore we can write it as
\[ 2\mathcal{H} = -(1 - \frac{2M}{r})(u^t)^2 + (1 - \frac{2M}{r})^{-1}(u^r)^2 + r(r-b)(u^\phi)^2. \]
\[ = -E u^t + L u^\phi + \frac{1}{(1 - \frac{2M}{r})}(u^r)^2 = \epsilon = \text{const}(\#) \]

Here \( \epsilon = -1 \) for time-like geodesics, \( \epsilon = 0 \) for light-like geodesics and \( \epsilon = +1 \) for space-like geodesics. Substituting the equations, (8) and (9) in (13), we obtain the radial equation for any spherically space-time is
\[ (u^r)^2 = E^2 - V_{eff} = E^2 - \left(\frac{L^2}{r(r-b)} - \epsilon\right)\left(1 - \frac{2M}{r}\right). \]

where the standard effective potential for GMGHS space-time denoted as
\[ V_{eff} = \left(\frac{L^2}{r(r-b)} - \epsilon\right)\left(1 - \frac{2M}{r}\right). \]

To compute the circular geodesic motion of the test particle in the Einstein-Maxwell gravitational field, we must have for circular geodesics of constant \( r = r_0 \) and from the equation (13) we have
\[ \frac{dV_{eff}}{dr} = 0. \]  
\[ \mathcal{V}_{eff} = E^2. \]  
and
Thus we obtain the energy and angular momentum per unit mass of the test particle are given by

\[ E_0^2 = \frac{(2r_0 - b)(r_0 - 2M)^2}{r_0(2r_0^2 - (b + 6M)r_0 + 4Mb)}. \]  
(18)

and

\[ L_0^2 = \frac{2Mr_0(r_0 - b)^2}{2r_0^2 - (b + 6M)r_0 + 4Mb}. \]  
(19)

Circular motion of the test particle to be exists when both the energy and angular momentum are real finite, therefore we must have \( 2r_0^2 - (b + 6M)r_0 + 4Mb > 0 \) and \( r_0 > b \).

Again the angular frequency measured by an asymptotic observers for time-like circular geodesics at \( r = r_0 \) same as the angular frequency of Schwarzschild black hole which is given by

\[ \Omega_0 = \frac{u^\phi}{u^t} = \sqrt{\frac{M}{r_0^3}}. \]  
(20)

In general relativity, circular orbits do not exists for all values of \( r \), so the denominator of equations (18,19) real only if \( 2r_0^2 - (b + 6M)r_0 + 4Mb > 0 \). The limiting case of equality gives an circular orbit with indefinite energy per unit mass, i.e. a circular photon orbit. This photon orbit is the innermost boundary of the circular orbit for massive particles. It occurs at the radius

\[ r_c = \frac{1}{4}(b + 6M + \sqrt{b^2 - 20Mb + 36M^2}) \]  
(21)

For extremal GMGHS black hole, the photon orbit occurs at \( r_c = 2M \). Marginally bound circular orbit(MBCO) can be obtained by setting \( E = 1 \), then the radius of marginally bound orbit located at \( r_{mb} = 2M + \sqrt{2M(2M - b)} \). In the limit \( b \to 0 \), we get \( r_{mb} = 4M \) which is the radius of marginally bound circular orbit of Schwarzschild black hole. At the extreme limit \( b = 2M \), marginally bound circular orbit occurs at the radius \( r_{mb} = 2M \) for extremal GMGHS black hole. From astrophysical viewpoint the most important class of orbits are the innermost stable circular orbit(ISCO), which occurs at the point of inflection of the effective potential \( V_{eff} \). Thus at the point of inflection

\[ \frac{d^2V_{eff}}{dr^2} = 0 \]  
(22)

with the auxiliary equation \( \frac{dV_{eff}}{dr} = 0 \). Then the ISCO equation for GMGHS black hole is given by

\[ r_0^3 - 6Mr_0^2 + 6Mbr_0 - 2Mb^2 = 0 \]  
(23)

The real positive root of the equation gives the radius of ISCO at \( r_0 = r_{ISCO} \) which is given by

\[ r_{ISCO} = 2 + Z + \frac{2(2 - \frac{b}{M})}{Z} \]  
(24)

\[ \text{where} \]

\[ Z = \left[ 8 - 6(\frac{b}{M}) + (\frac{b}{M})^2 + \sqrt{(\frac{b}{M})^4 - 4(\frac{b}{M})^3 + 4(\frac{b}{M})^2} \right] \]  
(26)

Now we turn to the derivation of CM energy for the string black hole.

3 CM energy of the collision near the horizon of the string black hole:

In this section, we shall compute the energy in the center-of-mass frame for the collision of two neutral particles coming from infinity with \( \frac{E_1}{m_0} = \frac{E_2}{m_0} = 1 \) and approaching the black hole with different angular momenta \( L_1 \) and \( L_2 \). The center of mass energy is derived by using the formula [Banados, Silk & West (2009)] which is valid in both flat and curved space-time reads

\[ \left( \frac{E_{cm}}{\sqrt{2m_0}} \right)^2 = 1 - g_{\mu\nu}u^\mu_1u^\nu_2. \]  
(27)

where \( u^\mu_1 \) and \( u^\nu_2 \) are the 4-velocities of the two particles, which can be determine from the following equation [31].

For this we need to derive the four velocity of the colliding particles. We assume throughout this work the geodesic motion of the colliding particles are confined to the equatorial plane. Since the space-time has a time-like isometry generated by the time-like Killing vector field \( \xi \) whose projection along the four velocity \( u \) of geodesics \( \xi.u = -E \), is conserved along such geodesics(\( \xi \equiv \partial_t \)). Similarly there is also the 'angular momentum' \( L = \chi.u \) is conserved due to the rotational symmetry(\( \chi \equiv \partial_\phi \)). Thus for the massive particles the components of the four velocity are

\[ u^t = \frac{E}{f(r)} \]  
(28)

\[ u^r = \pm \sqrt{E^2 - f(r) \left( 1 + \frac{L^2}{r(r - b)} \right)} \]  
(29)

\[ u^\theta = 0 \]  
(30)

\[ u^\phi = \frac{L}{r(r - b)}. \]  
(31)
where \( f(r) = 1 - \frac{2M}{r} \).

\[
\begin{align*}
\mu_1 &= \left( \frac{E_1}{f(r)} - X_1, 0, \frac{L_1}{r^2} \right), \\
\mu_2 &= \left( \frac{E_2}{f(r)} - X_2, 0, \frac{L_2}{r^2} \right).
\end{align*}
\]

Therefore using (27), we obtain the center of mass energy for this collision:

\[
\left( \frac{E_{cm}}{\sqrt{2m_0}} \right)^2 = 1 + \frac{E_1 E_2}{f(r)} - \frac{X_1 X_2}{f(r)} - \frac{L_1 L_2}{r(r-b)}.
\]

where

\[
X_1 = \sqrt{E_1^2 - f(r) \left( 1 + \frac{L_1^2}{r(r-b)} \right)},
\]

\[
X_2 = \sqrt{E_2^2 - f(r) \left( 1 + \frac{L_2^2}{r(r-b)} \right)}.
\]

As we have assumed \( E_1 = E_2 = 1 \) previously and substituting \( f(r) = 1 - \frac{2M}{r} \), we obtain finally the center of mass energy near the horizon:

\[
E_{cm} = \sqrt{2m_0} \sqrt{\frac{8M(2M-b) + (L_1 - L_2)^2}{2M(2M-b)}}.
\]

As we know (Banados, Silk & West (2009)), the maximum center-of-mass energy strictly depends upon the value of critical angular momentum such that the particles can reach the event horizon with maximum tangential velocity. Now we compute the critical angular momentum for this black holes in the following way.

In fact, the critical angular momentum and the critical radius can be determined from the effective potential. From the equation (13), at the critical point the effective potential satisfied the conditions as we defined previously by the equations (10) and (17). Therefore using these conditions we get the critical angular momentum for geodesics falling in is

\[- \left( 2M + \sqrt{4M^2 - 2bM} \right) \leq L \leq \left( 2M + \sqrt{4M^2 - 2bM} \right) \]

In the limit \( b \to 0 \), this yields the critical angular momentum for Schwarzschild black hole and the critical values are \( \pm 4M \).

In the limit \( b \to 0 \), the above expression reduce to

\[
E_{cm} = \sqrt{2m_0} \sqrt{\frac{16M^2 + (L_1 - L_2)^2}{4M^2}}.
\]

which is the CM energy for Schwarzschild black hole.

For non-extremal GMGHS spacetimes the CM energy reads as

\[
E_{cm} = \sqrt{2m_0} \sqrt{\frac{8M \left( 2M - \frac{Q^2}{M^2} e^{-2\phi_0} \right) + (L_1 - L_2)^2}{2M \left( 2M - \frac{Q^2}{M^2} e^{-2\phi_0} \right)}}
\]

which shows that the CM energy is finite and depends upon the asymptotic value of the dilation field \( \phi_0 \).

Whenever we taking the extremal limit \( b = 2M \) we get the CM energy near the event horizon \( r = 2M \)

\[
E_{cm} = \sqrt{2m_0} \sqrt{\frac{8M(2M-b) + (L_1 - L_2)^2}{2M(2M-b)}}
\]

which implies that the center-of-mass energy of collision for extremal dilation black hole blows up as we approaches the extremal limit. Thus we get the unlimited C.M. energies. In fact it is independent of the critical values of the angular momentum. This is one of the main results of the paper.

Another important point should be noted here that for extremal GMGHS space-time, the ISCO, circular photon orbit and marginally bound circular orbit coincide with the event horizon i.e. \( r_{ISCO} = r_{ph} = r_{mb} = r_{hor} = 2M \). If we choose the different collision point say ISCO or photon orbit or marginally bound orbit, then the center-of-mass energy will be unlimited for each collision point. This is another interesting features of this space-time.

4 CM Energy of the collision Near the Horizon of the RN Black Hole

In this section we shall compute the particle acceleration and collisions near the outer horizon in the Reissner Nordstrom space-time. Here we shall choose the different collision points, first we choose the collision point at near the event horizon which is a surface of infinite redshift, the second collision point will be at Cauchy horizon and finally we choose the collision point at ISCO. Now we compare the energy obtained for the different collision points.

4.1 Review of Geodesic Motion of RN Spacetime:

The well known metric of RN space-time which is a static, asymptotically flat and spherically symmetric solutions of Einstein- Maxwell equation is given by Chandrasekhar (1983)
\[ ds^2 = -(1 - \frac{2M}{r} + \frac{Q^2}{r^2}) dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} \] 
\[ + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) . \]

The black hole has event horizon which is located at \( r_+ = M + \sqrt{M^2 - Q^2} \) and Cauchy horizon which is located at \( r_- = M - \sqrt{M^2 - Q^2} \). Let us consider \( x^\mu(r) \) represents the trajectory of the moving particles. \( \tau \) is the proper time of the moving particles. We restrict ourselves the geodesic motion of the particles confined on the equatorial plane i.e. \( u^\theta = 0 \) or \( \theta = \pi/2 \). Thus the equatorial time-like geodesics for RN spacetimes are

\[ u^t = \frac{E}{g(r)} \]  \hspace{1cm} (43)
\[ u^r = \pm \sqrt{E^2 - g(r) \left(1 + \frac{L^2}{r^2}\right)} \]  \hspace{1cm} (44)
\[ u^\theta = 0 \]  \hspace{1cm} (45)
\[ u^\phi = \frac{L}{r^2} . \]  \hspace{1cm} (46)

where \( g(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \), \( E \) and \( L \) are represents the energy and angular momentum per unit rest mass of the particle. The radial equation for RN space-time can be rewritten as in terms of standard effective potential

\[(u^r)^2 = E^2 - V_{eff} = E^2 - \left(1 + \frac{L^2}{r^2}\right) \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) . \]

where the standard effective potential for RN space-time is given by

\[ V_{eff} = \left(1 + \frac{L^2}{r^2}\right) \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) . \]  \hspace{1cm} (48)

Let us now compute the CM energy for the two colliding particles of the same rest mass \( m_0 \) in arbitrary Einstein-Maxwell gravitational field. It can be computed by using the formula as given in [Banados, Silk & West (2009)]:

\[ \left( \frac{E_{cm}}{\sqrt{2m_0}} \right)^2 = 1 - g_{\mu\nu}u^\mu_1 u^\nu_2 . \]  \hspace{1cm} (49)

where \( u^\mu_1 \) and \( u^\mu_2 \) are the 4-velocities of the two particles, which can be find from the following equation (40).

\[ u^\mu_1 = \left( \frac{E_1}{g(r)} - Y_1, 0, \frac{L_1}{r^2}\right) . \]  \hspace{1cm} (50)
\[ u^\mu_2 = \left( \frac{E_2}{g(r)} - Y_2, 0, \frac{L_2}{r^2}\right) . \]  \hspace{1cm} (51)

Therefore using (40) one can obtain the center-of-mass energy for this collision:

\[ \frac{d\tau^2}{\left(\sqrt{2m_0}\right)^2} = 1 + \frac{E_1 E_2}{g(r)} - \frac{Y_1 Y_2}{g(r)} - \frac{L_1 L_2}{r^2} . \]  \hspace{1cm} (52)

where

\[ Y_1 = \sqrt{E_1^2 - g(r) \left(1 + \frac{L_1^2}{r^2}\right)} \]  \hspace{1cm} (53)
\[ Y_2 = \sqrt{E_2^2 - g(r) \left(1 + \frac{L_2^2}{r^2}\right)} . \]  \hspace{1cm} (54)

As we assume \( E_1 = E_2 = 1 \) previously and substituting \( g(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \) one could obtain finally the center-of-mass energy near the event horizon \( (r_+) \) for non-extremal RN space-time

\[ E_{cm | r\to r_+} = \sqrt{2m_0} \sqrt{\frac{4r_{+}^2 + (L_1 - L_2)^2}{2r_{+}^2}} . \]  \hspace{1cm} (55)

Near the Cauchy horizon the CM energy for RN space-time is given by

\[ E_{cm | r\to r_-} = \sqrt{2m_0} \sqrt{\frac{4r_{-}^2 + (L_1 - L_2)^2}{2r_{-}^2}} . \]  \hspace{1cm} (56)

In the near extremal limit \( Q^2 = M^2(1 - \epsilon^2) \) for RN black hole, one obtains the CM energy

\[ E_{cm | r\to 1+\epsilon} = \sqrt{2m_0} \sqrt{\frac{4(1 + \epsilon)^2 + (L_1 - L_2)^2}{2(1 + \epsilon)^2}} . \]  \hspace{1cm} (57)

For the extremal RN space-time the value corresponds to CM energy near the horizon is

\[ E_{cm | r\to M} = \sqrt{2m_0} \sqrt{\frac{4M^2 + (L_1 - L_2)^2}{2M^2}} . \]  \hspace{1cm} (58)

We know that for extremal RN black hole the ISCO is at \( r = 4M \) [Maki & Shiraishi (1994)] [Pradhan & Majumdar (2011)]. Thus If we consider the collision point to be ISCO then the corresponding value of CM energy on the ISCO for extremal RN space-time is given by

\[ E_{cm | r\to 4M} = \sqrt[2]{2m_0} \sqrt{\frac{9(26M^2 - L_1 L_2) - \sqrt{112M^2 - 9L_1^2 \sqrt{112M^2 - 9L_2^2}}}{144M^2}} . \]  \hspace{1cm} (59)

5 CM Energy of the collision on the ISCO of the Schwarzschild Black Hole

Here we shall extend our analysis for Schwarzschild black hole and compute the CM energy of the colliding particles on the ISCO. Since the event horizon of
the black hole is a surface of infinite blue-shift and here we shall show how much it different in the value of CM energy if we choose the collision point to be ISCO. This is the main aim of this section. Again we have already described the space-time has symmetry namely time translation symmetry and rotational symmetry. Therefore the energy and angular momentum are conserved quantities along the geodesics. They are denoted by $E$ and $L$. We also confined ourselves on the equatorial plane, so on that plane the equatorial time-like geodesics are described by

$$u^t = \frac{E}{1 - \frac{2M}{r}}$$  \hspace{1cm} (60)$$

$$u^r = \pm \sqrt{E^2 - \left(1 - \frac{2M}{r}\right) \left(1 + \frac{L^2}{r^2}\right)}$$  \hspace{1cm} (61)$$

$$u^\theta = 0$$  \hspace{1cm} (62)$$

$$u^\phi = \frac{L}{r^2}$$  \hspace{1cm} (63)$$

and the components of four velocities are

$$u^\mu_1 = \left(\frac{E_1}{1 - \frac{2M}{r}}, -Z_1, 0, \frac{L_1}{r^2}\right).$$  \hspace{1cm} (64)$$

$$u^\mu_2 = \left(\frac{E_2}{1 - \frac{2M}{r}}, -Z_2, 0, \frac{L_2}{r^2}\right).$$  \hspace{1cm} (65)$$

Since the Schwarzschild space-time is the special case of RN space-time, therefore one can compute the CM energy using the equation (62) as

$$\left(\frac{E_{cm}}{\sqrt{2m_0}}\right)^2 = 1 + \frac{E_1 E_2}{(1 - \frac{2M}{r})} - \frac{Z_1 Z_2}{(1 - \frac{2M}{r})} - \frac{L_1 L_2}{r^2}, \hspace{1cm} (66)$$

where

$$Z_1 = \sqrt{E_1^2 - \left(1 - \frac{2M}{r}\right) \left(1 + \frac{L_1^2}{r^2}\right)}$$  \hspace{1cm} (67)$$

$$Z_2 = \sqrt{E_2^2 - \left(1 - \frac{2M}{r}\right) \left(1 + \frac{L_2^2}{r^2}\right)}$$  \hspace{1cm} (68)$$

As we setting $E_1 = E_2 = 1$ previously, thus one may obtain the CM energy on the ISCO for Schwarzschild black hole is given by

$$E_{cm} \big|_{r=6M} = \frac{\sqrt{2m_0}}{9} \frac{\sqrt{(90M^2 - L_1 L_2) - \sqrt{18M^2 - L_1^2} \sqrt{18M^2 - L_2^2}}}{36M^2}. \hspace{1cm} (69)$$

Schwarzschild black hole are $\pm 4M$. Thus the maximum CM energy occurs for the opposite values of angular momentum i.e. $L_1 = 4M$ and $L_2 = -4M$. Therefore the maximum CM energy on the ISCO for Schwarzschild black hole for these values of angular momentum is

$$E_{cm} \big|_{r=6M} = \frac{\sqrt{52}}{9} m_0. \hspace{1cm} (70)$$

whereas the maximum CM energy at the near horizon is $2\sqrt{5}m_0$.

6 Discussion

In this note, we have investigated the collision of two different particles of different angular momentum with same rest mass and falling towards the spherically symmetric massless dilation black hole and computed the center-of-mass energy for this black hole. Our analysis suggests that for non-extremal GMGHS space-time the CM energy is finite and depends upon the asymptotic value of the field strength.

For extremal GMGHS black holes the CM energy is unlimited and independent of the extreme fine-tuning of the angular momentum of the colliding particles. The another feature of this work is that for extremal GMGHS space-time three orbits namely ISCO, photon orbit and marginally bound orbit coalesce to the horizon [Pradhan (2012)] thus we may vary the collision point but they gives the identical results (infinite energy). Thus in summary, for extremal GMGHS space-time it is shown that the center of mass energy of collision at $r \equiv r_{ISCO} = r_{ph} = r_{mb} = r_{hor} = 2M$ is arbitrarily large.
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