Search for the Rare Decays $B^+ \to \mu^+ \mu^- K^+$, $B^0 \to \mu^+ \mu^- K^*(892)^0$, and $B^0_s \to \mu^+ \mu^- \phi$ at CDF

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We search for $b \rightarrow sp^{+}\mu^{-}$ transitions in $B$ meson ($B^{+}$, $B^{0}$, or $B^{0}_{s}$) decays with 924 pb$^{-1}$ of $pp$ collisions at $\sqrt{s} = 1.96$ TeV collected with the CDF II detector at the Fermilab Tevatron. We find excesses with significances of 4.5, 2.9, and 2.4 standard deviations in the $B^{+} \rightarrow \mu^{+}\mu^{-}K^{+}$, $B^{0} \rightarrow \mu^{+}\mu^{-}K^{-}(892)^{0}$, and $B^{0}_{s} \rightarrow \mu^{+}\mu^{-}\phi$ decay modes, respectively. Using $B \rightarrow J/\psi h$ ($h = K^{+}$, $K^{-}(892)^{0}$, $\phi$) decays as normalization channels, we report branching fractions for the previously observed $B^{+}$ and $B^{0}$ decays, $B(B^{+} \rightarrow \mu^{+}\mu^{-}K^{+}) = (0.59 \pm 0.15 \pm 0.04) \times 10^{-6}$, and $B(B^{0} \rightarrow \mu^{+}\mu^{-}K^{-}(892)^{0}) = (0.81 \pm 0.30 \pm 0.10) \times 10^{-6}$, where the first uncertainty is statistical, and the second is systematic. These measurements are consistent with the world average results, and are competitive with the best available measurements. We set an upper limit on the relative branching fraction $\frac{B(B^{0}_{s} \rightarrow J/\psi\phi)}{B(B^{+} \rightarrow \mu^{+}\mu^{-}K^{+})} < 2.6(2.3) \times 10^{-3}$ at the 95(90)% confidence level, which is the most stringent to date.

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The decay of a $b$ quark into an $s$ quark and two muons ($b \to s \mu^+ \mu^-$) is a flavor-changing neutral current process, forbidden at tree level in the standard model (SM) but allowed through highly suppressed internal loops. New physics could manifest itself in a larger branching fraction, a modified dimuon mass distribution, or angular distributions of the decay products different from that predicted by the SM [1, 2, 3, 4]. In this paper, we report branching ratio measurements of exclusive decays, where the $s$ quark hadronizes into a single meson. Reconstructing a final state with only three or four charged final state particles results in smaller backgrounds than those expected in a search for inclusive $B \to X_s \mu^+ \mu^-$ decays. The rare decays, $B^+ \to \mu^+ \mu^- K^+$ and $B^0 \to \mu^+ \mu^- K^*(892)^0$ have been observed at the $B$ factories [5, 6], with branching fractions of $O(10^{-6})$, consistent with SM predictions [7, 8, 9, 10, 11, 12, 13].

The analogous decay in the $B_d^0$ system, $B_d^0 \to \mu^+ \mu^- \phi$, has a predicted branching ratio of $1.6 \times 10^{-6}$, but has not yet been observed [10, 11]. We report branching ratio measurements from 924 pb$^{-1}$ of Collider Detector at Fermilab (CDF) Run II data that pave the way for future studies of larger datasets from which kinematic distributions may be measured. We search for $B \to \mu^+ \mu^- h$ decays, where $B$ stands for $B^+$, $B^0$, or $B_s^0$, and $h$ stands for $K^+$, $K^*(892)^0$, or $\phi$, respectively. The $K^*(892)^0$, referred to as $K^{*0}$ throughout this paper, is reconstructed in the $K^{*0} \to K^+ \pi^-$, decay mode, and the $\phi$ meson is reconstructed as $\phi \to K^+ K^-$. We measure branching ratios relative to $B \to J/\psi h$ decays, followed by $J/\psi \to \mu^+ \mu^-$ decays, resulting in the same final state particles as the rare decay modes. Many systematic uncertainties cancel in the relative branching ratios:

$$\frac{B(B \to \mu^+ \mu^- h)}{B(B \to J/\psi h)} = \frac{N_{\mu^+ \mu^- h}}{N_{J/\psi h}} \frac{\epsilon_{J/\psi h}}{\epsilon_{\mu^+ \mu^- h}} \times \frac{B(J/\psi \to \mu^+ \mu^-)}{B(B \to J/\psi h)},$$

where $N_{\mu^+ \mu^- h}$ is the observed number of $B \to \mu^+ \mu^- h$ decays, $N_{J/\psi h}$ is the observed number of $B \to J/\psi h$ decays, while $\epsilon_{J/\psi h}$ and $\epsilon_{\mu^+ \mu^- h}$ are the combined selection efficiency and acceptance of the experiment for $B \to J/\psi h$ and $B \to \mu^+ \mu^- h$ respectively. Throughout this report, charge conjugate modes are implicitly included.

CDF II is a general purpose detector, located at the Tevatron $p\bar{p}$ collider [18]. Charged particle trajectories (tracks) are detected by the tracking system comprised of a seven-layer double-sided silicon microstrip detector and a drift chamber, both in a 1.4 T axial magnetic field. The silicon detector [19] ranges in radius from 1.3 to 28 cm, and has a single-hit resolution of approximately 15 $\mu$m. The drift chamber [20] provides up to 96 measurements from radii of 40 to 137 cm with a single-hit resolution of approximately 180 $\mu$m. A muon chamber identification system of plastic scintillators and drift chambers [21] is located on the exterior of the detector with a central part covering $|\eta| < 0.6$, and an extended region covering $0.6 < |\eta| < 1.0$, where $\eta$ is the pseudorapidity [22]. In the central (extended) region, muons are detected if their transverse momentum component, $p_T$, is greater than 1.5(2.0) GeV/$c$. Events are selected with a three-level trigger system. The first trigger level requires the presence of two charged particles with $p_T \geq 1.5$ GeV/$c$ ($|\eta| \leq 0.6$) or $p_T \geq 2.0$ GeV/$c$ (0.6 $\leq |\eta|$ $\leq 1.0$), matched to track segments in the muon chambers to form muon candidates. At the second level, a more restrictive selection is made by requiring that the muon candidates have opposite charge, and that their opening angle in the plane transverse to the beamline is less than 120$^\circ$. At the third trigger level, the event is fully reconstructed. The trajectories of the muon candidates in the silicon detector are required to intersect at a point which is displaced transversely from the beamline by at least 100 $\mu$m.

The offline selection begins with a suppression of random combinations of tracks satisfying the selection requirements (combinatoric background), by requiring that all tracks have $p_T > 0.4$ GeV/$c$ and match hits from at least three layers of the silicon detector. The trajectories of a pair of muon candidates that satisfy the trigger requirements, and the tracks that form the hadron candidate, are fitted with the constraint that they originate from a single vertex in 3-dimensional space to form a $B$ candidate. The $\chi^2$ probability of the fit is required to be greater than $10^{-3}$. A $K^+$ candidate is a track assigned the charged kaon mass, and the $K^{*0}$ ($\phi$) candidates are formed from oppositely charged pairs of tracks whose invariant mass must lie within 50 (10) MeV/$c^2$ of the $K^{*0}$ ($\phi$) mass. For all particles, we use the world average values tabulated in Ref. [23]. The ambiguity of the mass assignment in $B^0 \to \mu^+ \mu^- K^{*0}$ decays is handled by choosing the combination whose $K^+ \pi^-$ mass is closer to the $K^{*0}$ mass. In reconstructing the $B$ candidates, the meson containing a strange quark, $h$, is required to have $p_T(h) \geq 1.0$ GeV/$c$, and the $B$ candidate is required to have $p_T(B) \geq 4.0$ GeV/$c$. We require that the distance of closest approach between the flight path of the $B$ candidate and the beamline, $|d_0(B)|$, is less than 120 $\mu$m, to reduce the combinatoric background with no loss of signal.

Normalization mode candidates are identified by having a dimuon invariant mass within 50 MeV/$c^2$ of the $J/\psi$ mass, yielding approximately 12 000 $B^+ \to J/\psi K^+$, 4400 $B^0 \to J/\psi K^{*0}$, and 800 $B_s^0 \to J/\psi \phi$ decays. The $B$ candidate mass distributions of all modes are compatible with a Gaussian of width $\sigma = 20$ MeV/$c^2$. To reduce backgrounds from $B$ decays to mesons contain-
ing $c$ quarks, several vetoes are applied to $B \to \mu^+\mu^-h$ candidates, listed below. We eliminate candidates with a dimuon mass near the $J/\psi(1^{+})$: $2.9 \leq m_{\mu\mu} \leq 3.2$ GeV/$c^2$ or $3.6 \leq m_{\mu\mu} \leq 3.75$ GeV/$c^2$ respectively. $B \to J/\psi(1^{+})h$ decays followed by the radiative decay of the $J/\psi(1^{+})$ into two muons and a photon that is not reconstructed, may have a dimuon mass that passes the $J/\psi(1^{+})$ veto. We reject these events making use of the correlation between the candidate’s invariant mass and the dimuon invariant mass, and require $|m_{\mu\mu} - m_B| - |m_{\mu\mu} - m_{J/\psi(1^+)}| > 100$ MeV/$c^2$. $B \to J/\psi(1^{+})h$ decays with one hadron misidentified as a muon form a potential background to the rare decay search. We reject this class of background by requiring that all combinations of tracks of the candidate have an invariant mass that differs by at least 40 MeV/$c^2$ from the $J/\psi(1^{+})$ mass. We also reject candidates with track pairs with a mass within $\pm 25$ MeV/$c^2$ of the $D^0 \to K^-\pi^+$ decay, or track triplets compatible with a mass within $\pm 25$ MeV/$c^2$ of the $D^+ \to K^-\pi^+\pi^+$ or $D^+_s \to K^+K^-\pi^+$ decays.

We further improve the signal to background ratio by an optimization of the selection based on discriminating variables. For this purpose, Monte Carlo (MC) simulations of the signal processes are used. We generate single $b$ hadrons using the transverse momentum spectrum from $B \to J/\psi X$, measured by CDF [13]. Decays of all $b$ hadrons are simulated using EVTGEN [24], with lifetimes from Ref. [25], allowing for a decay width difference between the $B^0_{s\mu\nu}$ mass eigenstates, $\Delta\Gamma/\Gamma = 0.12 \pm 0.06$ [23]. The decays $B^0 \to J/\psi K^{*0}$ and $B^0 \to J/\psi\phi$ are simulated according to the polarization amplitudes measured by CDF [24]. The dynamics of the rare decay processes are simulated according to the calculations of Ali et al. [11]. For $B^0_{s\mu\nu} \to \mu^+\mu^-\phi$, we assume a mixture of 50% CP-even and 50% CP-odd states. The interactions of final state particles are simulated using a GEANT27 model of the CDF II detector, digitized into the CDF event format, and reconstructed using the same software as in the processing of collision data. The detector simulation includes a full emulation of the CDF trigger system. We find that the transverse momentum spectrum of the $B$ mesons of this measurement are on average higher than in the simulation. This can be explained by the presence of a $B$ baryon component with low transverse momentum in the measurement used as input spectrum of the simulation [13]. To correct for this, the simulated events are weighted by a polynomial function, which is obtained from the ratio of data to MC transverse momentum distribution of the $B$ meson.

We tighten the candidate selection to obtain the smallest expected statistical uncertainty on the measurement of the $B \to \mu^+\mu^-h$ branching ratios by finding the selection with the largest value of $N_{\text{sig}}/\sqrt{N_{\text{sig}} + N_{\text{bkg}}}$. The expected number of signal events, $N_{\text{sig}}$, is determined by scaling the MC yields of the rare decay modes to the yields of the normalization channels in the data, and correcting by the relative branching ratios and selection efficiencies. This procedure requires estimates of the branching ratios of the rare decay modes. For the $B^+ \to \mu^+\mu^-K^+$ and $B^0 \to \mu^+\mu^-K^{*0}$ decays we use the world average measured branching ratios [23], while for $B^0 \to \mu^+\mu^-\phi$ we use the theoretical estimate [12]. The expected number of background events, $N_{\text{bkg}}$, is estimated by extrapolating the candidate yield with an invariant mass $(180 - 300)$ MeV/$c^2$ higher than the $B$ mass, to the signal region. Candidates with an invariant mass lower than the $B$ mass are not suitable for background estimates, since they contain partially reconstructed $B$ decays, not expected in the signal region. We find good discriminating power between signal and background from the following three quantities: $t/\sigma_t$, $\alpha$, and $I$. The significance of the proper decay time, $t/\sigma_t$, is defined as the proper decay time of the $B$ candidate divided by its uncertainty. The angle $\alpha$ is defined as the difference in angle between the $B$ candidate’s momentum vector and the vector from the primary vertex to the $\mu\mu$ vertex. The isolation, $I$, is defined as the transverse momentum carried by the $B$ meson candidate divided by the transverse momentum of all tracks in a cone of $\Delta R = \sqrt{\Delta \eta^2 + \Delta \phi^2} = 1.0$ around the direction of the $B$ meson candidate, including those of the $B$ candidate itself. Here $\Delta \eta$ is the difference in pseudorapidity of the $B$ candidate and each track, and $\Delta \phi$ is the difference in their azimuthal angles. Scanning different combinations of selection thresholds, we find that the optimal values are very similar for the three rare decay modes: $t/\sigma_t \geq 14, \alpha \leq 60$ mrad, and $I \geq 0.6$. Applying the optimized selection requirements to the normalization channels yields the distributions shown in Fig. 1. The shape of the combinatoric background is estimated from a sample with poor vertex quality ($\chi^2$ probability $< 10^{-3}$). The invariant mass distribution of this poor vertex quality sample is fitted with a Gaussian distribution for the remaining signal contribution and an exponential plus a constant to model the background. We estimate the background in the signal region of the tight selection using this functional form, normalized to the number of candidates with invariant mass $(60 - 180)$ MeV/$c^2$ higher than the $B$ mass. Using this method, we find $6361 \pm 82 B^+ \to J/\psi K^+$, $2423 \pm 52 B^0 \to J/\psi K^{*0}$, and $431 \pm 22 B^0_s \to J/\psi\phi$ decays. The invariant mass distributions for the rare decay modes are shown in Fig. 2. In the $\pm 40$ MeV/$c^2$ window around the $B$ mass we find $90 B^+ \to \mu^+\mu^-K^+$, $35 B^0 \to \mu^+\mu^-K^{*0}$, and $11 B^0 \to \mu^+\mu^-\phi$ candidates. From simulation studies we verified that the present selection accepts decays for every kinematically possible dimuon invariant mass, with an efficiency difference not exceeding a factor two, except for the windows around the $J/\psi(1^{+})$ that have been explicitly excluded.

For the signal modes, we evaluate three sources of background: the combinatoric background, hadrons misidentified as muons, and hadrons with misassigned mass. As in the normalization mode, the shape of the combinatoric background is estimated from a sample with poor vertex quality ($\chi^2$ probability $< 10^{-3}$). The
We find an equivalent Gaussian significance of 4.5, 2.9, to fluctuate to the number of observed events or higher, signal window. We find an excess in the predicted background from the number of candidates in the signal region. The resulting backgrounds, and their ratios from Ref. [28, 29] and evaluate the largest difference of 1.0 events for the $B_s$ family coming from misidentified $B_s$ hadrons. We estimate 0.4 fraction of 1.0 events for the $B_s$ channel to 5.1% for the $B_s^0$ mode. The muon trigger efficiency close to the 1.5 GeV/c $p_T$ threshold is poorly known. We find a change in the relative efficiency reaching 1.3% if the minimum $p_T$ is varied by $\pm 100$ MeV/c, covering the range over which the trigger efficiency rises from zero to one. The final state particles of the rare decay modes have approximately 10% more low momentum tracks than those of the normalization channels. The simulation models the track reconstruction efficiency to an accuracy of 2% in the $p_T$ range of 0.4–1.5 GeV/c. We therefore assign a systematic uncertainty on the relative efficiency of 2%×10%=0.2%. We find an uncertainty of 8.3% on the relative efficiency of $B^0$ decays due to the unknown fraction of the short-lived CP-even state in the rare decay mode. We assume that the CP-even fraction is 0.5, and assess systematic uncertainties for the extremes of the fraction valued at 1.0 and 0.0. In addition, the uncertainty on $\Delta \Gamma/\Gamma$ contributes another 2.6%, resulting in a total uncertainty of 8.7% associated with the $B_s^0$ decay width difference. This is the largest systematic uncertainty on the relative efficiency of the $B_s^0$ mode. We evaluate the effect of the uncertainty on the fraction of $J/\psi$ mesons produced with a longitudinal polarization by varying the fraction measured at CDF [28] by $\pm 1\sigma$. The effect is 0.6% for $B^0 \rightarrow J/\psi K^{*0}$ and 0.1% for $B_s^0 \rightarrow J/\psi \phi$.

The statistical uncertainties of yields in the normalization channels are included as systematic uncertainties, which range from 1.3% for the $B^+$ channel to 5.1% for the $B_s^0$ channel. Cabibbo-suppressed $B^+ \rightarrow J/\psi \pi^+$ decays contribute 0.1% to the yield of the normalization channel for the $B^+$ channel. We introduce the full effect into our estimate of the systematic uncertainty without correcting the result. The relative efficiencies between signal

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**FIG. 1:** Invariant mass spectra of (a) $J/\psi K^+$, (b) $J/\psi K^{*0}$ and (c) $J/\psi \phi$ candidates after applying optimized selection requirements.
TABLE I: Systematic uncertainties on the relative efficiency quoted in percent.

| Channel                  | $B^+$ | $B^0$ | $B_s$ |
|--------------------------|-------|-------|-------|
| Theory model             | 1.3   | 3.1   | 1.6   |
| $p_T(B)$ spectrum        | 0.6   | 1.3   | 1.4   |
| Trigger turn-on          | 1.3   | 1.3   | 1.2   |
| Low momentum hadrons     | 0.2   | 0.2   | 0.2   |
| $B_s$ decay width difference | −  | −  | 8.7   |
| Polarization             | −  | 0.6   | 0.1   |
| Norm. channel statistics | 1.3   | 2.1   | 5.1   |
| $B^+ \to J/\psi \pi^+$ contribution | 0.1 | −  | −  |
| MC statistics            | 1.6   | 2.6   | 2.2   |
| Total rel. eff. uncertainty | 2.9  | 5.0   | 10.6  |

TABLE II: Summary of systematic uncertainties quoted in percent.

| Channel                  | $B^+$ | $B^0$ | $B_s$ |
|--------------------------|-------|-------|-------|
| Total rel. eff. uncertainty | 2.9  | 5.0   | 10.6  |
| $B(J/\psi \to \mu^+ \mu^-)$ | 1.0   | 1.0   | 1.0   |
| Background prediction    | 5.2   | 3.1   | 10.2  |
| $B(B \to J/\psi h)$      | 3.5   | 4.5   | 35.5  |

and normalization channels, of 0.71 ± 0.01, 0.74 ± 0.02, and 0.84 ± 0.02 for the $B^+$, $B^0$, and $B_s$ decays, respectively, have uncertainties reaching 2.6%, that arise from the finite size of the MC samples.

The predicted background values depend on the shape of the background function. We evaluate the change in the background yield calculation when using a sample that is similar to the one resulting from the optimal selection, but with less stringent requirements on the three optimization variables instead of the default sample with poor vertex quality. We compare two functional models for the background description, the default exponential plus a constant, and a simpler linear extrapolation function. We find a systematic uncertainty on the background prediction reaching 10% for the $B_s$ channel.

To calculate the systematic uncertainty on the relative branching fraction, the relative efficiency uncertainty is summed in quadrature with the uncertainty of the $J/\psi \to \mu^+ \mu^-$ branching ratio, and with the systematic uncertainty on the number of signal candidates.

To calculate the absolute branching fractions, we use the world average branching fractions of the normalization channels \[23\]. These branching fractions have uncertainties of 3.5%, 4.5%, and 35.5%, respectively, for $B^+$, $B^0$, and $B_s$, which are added in quadrature to the systematic uncertainties on the relative branching ratios.

A summary of the systematic uncertainties is given in Table III.

We calculate relative branching ratios using Eq. 11, and find $B(B^+ \to \mu^+ \mu^- K^+)/B(B^+ \to J/\psi K^+) = (0.59 \pm 0.15 \pm 0.03) \times 10^{-3}$, $B(B^0 \to \mu^+ \mu^- K^{*0})/B(B^0 \to J/\psi K^{*0}) = (0.61 \pm 0.23 \pm 0.07) \times 10^{-3}$, and $B(B_s^0 \to \mu^+ \mu^- \phi)/B(B_s^0 \to J/\psi \phi) = (1.23 \pm 0.60 \pm 0.14) \times 10^{-3}$, where the first uncertainty is statistical, and the second is systematic. We use the world-average branching ratios of the $B^0$ and $B^+$ normalization channels \[23\], resulting in the following absolute branching fractions:

$B(B^+ \to \mu^+ \mu^- K^+) = (0.59 \pm 0.15 \pm 0.04) \times 10^{-6}$, and $B(B^0 \to \mu^+ \mu^- K^{*0}) = (0.81 \pm 0.30 \pm 0.10) \times 10^{-6}$. To obtain an absolute branching ratio for the $B_s^0 \to \mu^+ \mu^- \phi$ decay, we use $B(B_s^0 \to J/\psi \phi) = (1.38 \pm 0.49) \times 10^{-3}$, obtained from correcting the CDF measurement \[30\] for the current value of $f_s/f_d$ \[23\], the $B_s^0$ to $B^0$ production ratio, resulting in $B(B_s^0 \to \mu^+ \mu^- \phi) = (1.70 \pm 0.82 \pm 0.64) \times 10^{-6}$. We find a significance of only 2.4 standard deviations for the $B_s^0 \to \mu^+ \mu^- \phi$ decay mode. Therefore, we choose to set a limit on this decay. We use a Bayesian integration assuming a flat prior \[31\], and find $B(B_s^0 \to \mu^+ \mu^- \phi)/B(B_s^0 \to J/\psi \phi) < 2.60(2.3) \times 10^{-6}$ at the 95(90)% confidence level (C.L.). We also set an upper limit on the absolute branching ratio, taking into account the uncertainty of the branching ratio of the normalization channel \[32\]. We assume that the prior probability density function representing the uncertainty in the normalization channel is a log normal distribution with a mean equal to the central value, and a width parameter equal to the quoted uncertainty of 35.5%. We obtain $B(B_s^0 \to \mu^+ \mu^- \phi) < 6.0(5.0) \times 10^{-6}$ at 95(90)% C.L. The main ingredients and the results are summarized in Table III.

In conclusion, our measurements of the $B^+$ and $B^0$ rare decay modes are consistent with the SM predictions, and with previous measurements \[8\]. The relative limit on $B_s^0 \to \mu^+ \mu^- \phi$ is consistent with the SM predictions, and is the most stringent to date.

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FIG. 2: Invariant mass spectra of (a) $\mu^+\mu^-K^+$, (b) $\mu^+\mu^-K^{*0}$ and (c) $\mu^+\mu^- \phi$ candidates. The superimposed curves are a sum of a single Gaussian with a width of 20 MeV/$c^2$ representing the signal, and a curve representing the background as determined by the procedure described in the text. The curves are not drawn for masses below the signal window, since they are not expected to predict the background level where partially reconstructed $B$ decays contribute.

TABLE III: Summary of the main ingredients and results of this analysis. Where one uncertainty is quoted, the uncertainty is of statistical nature. Where two uncertainties are quoted, the first is statistical, and the second is systematic.

| Decay mode |
|------------|
| $B\to \mu^+\mu^-K^+$ | $B\to \mu^+\mu^-K^{*0}$ | $B\to \mu^+\mu^-\phi$ |
| $N_{ob}$ | 90 | 35 | 11 |
| $N_{bg}$ | 45.3 ± 5.8 | 16.5 ± 3.6 | 3.5 ± 1.5 |
| Gaussian significance | 4.5 $\sigma$ | 2.9 $\sigma$ | 2.4 $\sigma$ |
| $N_{J/\psi}$ | 6361 ± 82 | 2423 ± 52 | 431 ± 22 |
| $\epsilon_{\mu^+\mu^- h/\epsilon_{J/\psi h}}$ | 0.71 ± 0.01 | 0.74 ± 0.02 | 0.84 ± 0.02 |
| $Res B \times 10^3$ | 0.59 ± 0.15 ± 0.03 | 0.61 ± 0.23 ± 0.07 | 1.23 ± 0.60 ± 0.14 |
| $Abs B \times 10^6$ | 0.59 ± 0.15 ± 0.04 | 0.81 ± 0.30 ± 0.10 | 1.70 ± 0.82 ± 0.64 |
| $Res B \ 95(90)\%C.L. \ limit \times 10^3$ | — | — | 2.6(2.3) |
| $Abs B \ 95(90)\%C.L. \ limit \times 10^6$ | — | — | 6.0(5.0) |

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We search for $b \to s \mu^+\mu^-$ transitions in $B$ meson ($B^+$, $B^0$, or $B_s^0$) decays with 924 pb$^{-1}$ of $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV collected with the CDF II detector at the Fermilab Tevatron. We find excesses with significances of 4.5, 2.9, and 2.4 standard deviations in the $B^+ \to \mu^+\mu^-K^+$, $B^0 \to \mu^+\mu^-k^+(892)\phi$, and $B_s^0 \to \mu^+\mu^-\phi$ decay modes, respectively. Using $B \to J/\psi h$ ($h = K^+$, $K^+(892)^0$, $\phi$) decays as normalization channels, we report branching fractions for the previously observed $B^+$ and $B^0$ decays, $B(B^+ \to \mu^+\mu^-K^+) = (0.59 \pm 0.15 \pm 0.04) \times 10^{-6}$, and $B(B^0 \to \mu^+\mu^-K^+(892)^0) = (0.81 \pm 0.30 \pm 0.10) \times 10^{-6}$, where the first uncertainty is statistical, and the second is systematic. These measurements are consistent with the world average results, and are competitive with the best available measurements. We set an upper limit on the relative branching fraction $B(B^0 \to \mu^+\mu^-\phi) / B(B^+ \to J/\psi\phi) < 2.6(2.3) \times 10^{-3}$ at the 95(90)\% confidence level, which is the most stringent to date.

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The decay of a $b$ quark into an $s$ quark and two muons ($b \to s \mu^+ \mu^-$) is a flavor-changing neutral current process, forbidden at tree level in the standard model (SM) but allowed through highly suppressed internal loops. New physics could manifest itself in a larger branching fraction, a modified dimuon mass distribution, or angular distributions of the decay products different from that predicted by the SM [1–4]. In this paper, we report branching ratio measurements of exclusive decays, where the $s$ quark hadronizes into a single meson. Reconstructing a final state with only three or four charged final state particles results in smaller backgrounds than those expected in a search for inclusive $B \to X_s \mu^+ \mu^-$ decays. The rare decays, $B^+ \to \mu^+ \mu^- K^+$ and $B^0 \to \mu^+ \mu^- K^*(892)^0$ have been observed at the $B$ factories [5, 6], with branching fractions of $O(10^{-6})$, consistent with SM predictions [7–14]. The analogous decay in the $B_s^0$ system, $B_s^0 \to \mu^+ \mu^- \phi$, has a predicted branching ratio of $1.6 \times 10^{-6}$ [15], but has not yet been observed [16, 17]. We report branching ratio measurements from 924 pb$^{-1}$ of Collider Detector at Fermilab (CDF) Run II data that pave the way for future studies of larger datasets from which kinematic distributions may be measured. We search for $B \to \mu^+ \mu^- h$ decays, where $B$ stands for $B^+$, $B^0$, or $B_s^0$, and $h$ stands for $K^+$, $K^*(892)^0$, or $\phi$, respectively. The $K^*(892)^0$ refers to $K^{*0}$ throughout this paper, is reconstructed in the $K^{*0} \to K^+ \pi^-$, decay mode, and the $\phi$ meson is reconstructed as $\phi \to K^+ K^-$. We measure branching ratios relative to $B \to J/\psi h$ decays, followed by $J/\psi \to \mu^+ \mu^-$ decays, resulting in the same final state particles as the rare decay modes. Many systematic uncertainties cancel in the relative branching ratios:

$$\frac{B(B \to \mu^+ \mu^- h)}{B(B \to J/\psi h)} = \frac{N_{\mu^+ \mu^- h}}{N_{J/\psi h}} \frac{\epsilon_{J/\psi h}}{\epsilon_{\mu^+ \mu^- h}} \times \frac{B(J/\psi \to \mu^+ \mu^-)}{B(B \to J/\psi h)},$$

where $N_{\mu^+ \mu^- h}$ is the observed number of $B \to \mu^+ \mu^- h$ decays, $N_{J/\psi h}$ is the observed number of $B \to J/\psi h$ decays, while $\epsilon_{J/\psi h}$ and $\epsilon_{\mu^+ \mu^- h}$ are the combined selection efficiency and acceptance of the experiment for $B \to J/\psi h$ and $B \to \mu^+ \mu^- h$ respectively. Throughout this report, charge conjugate modes are implicitly included.

CDF II is a general purpose detector, located at the Tevatron $p\bar{p}$ collider [18]. Charged particle trajectories (tracks) are detected by the tracking system comprised of a seven-layer double-sided silicon microstrip detector and a drift chamber, both in a 1.4 T axial magnetic field. The silicon detector [19] ranges in radius from 1.3 to 28 cm, and has a single-hit resolution of approximately 15 $\mu$m. The drift chamber [20] provides up to 96 measurements from radii of 40 to 137 cm with a single-hit resolution of approximately 180 $\mu$m. A muon chamber identification system of plastic scintillators and drift chambers [21] is located on the exterior of the detector with a central part covering $|\eta| < 0.6$, and an extended region covering $0.6 < |\eta| < 1.0$, where $\eta$ is the pseudorapidity [22]. In the central (extended) region, muons are detected if their transverse momentum component, $p_T$, is greater than 1.5(2.0) GeV/$c$. Events are selected with a three-level trigger system. The first trigger level requires the presence of two charged particles with $p_T \geq 1.5$ GeV/$c$ ($|\eta| \leq 0.6$) or $p_T \geq 2.0$ GeV/$c$ (0.6 $\leq |\eta| \leq 1.0$), matched to track segments in the muon chambers to form muon candidates. At the second level, a more restrictive selection is made by requiring that the muon candidates have opposite charge, and that their opening angle in the plane transverse to the beamline is less than 120$^\circ$. At the third trigger level, the event is fully reconstructed. The trajectories of the muon candidates in the silicon detector are required to intersect at a point which is displaced transversely from the beamline by at least 100 $\mu$m.

The offline selection begins with a suppression of random combinations of tracks satisfying the selection requirements (combinatoric background), by requiring that all tracks have $p_T > 0.4$ GeV/$c$ and match hits from at least three layers of the silicon detector. The trajectories of a pair of muon candidates that satisfy the trigger requirements, and the tracks that form the hadron candidate, are fitted with the constraint that they originate from a single vertex in 3-dimensional space to form a $B$ candidate. The $\chi^2$ probability of the fit is required to be greater than $10^{-3}$. A $K^+$ candidate is a track assigned the charged kaon mass, and the $K^{*0}$ ($\phi$) candidates are formed from oppositely charged pairs of tracks whose invariant mass must lie within 50 (10) MeV/$c^2$ of the $K^{*0}$ ($\phi$) mass. For all particles, we use the world average values tabulated in Ref. [23]. The ambiguity of the mass assignment in $B^0 \to \mu^+ \mu^- K^{*0}$ decays is handled by choosing the combination whose $K^+ \pi^-$ mass is closer to the $K^{*0}$ mass. In reconstructing the $B$ candidates, the meson containing a strange quark, $h$, is required to have $p_T(h) \geq 1.0$ GeV/$c$, and the $B$ candidate is required to have $p_T(B) \geq 4.0$ GeV/$c$. We require that the distance of closest approach between the flight path of the $B$ candidate and the beamline, $|d_0(B)|$, is less than 120 $\mu$m, to reduce the combinatoric background with no loss of signal.

Normalization mode candidates are identified by having a dimuon invariant mass within 50 MeV/$c^2$ of the $J/\psi$ mass, yielding approximately 12 000 $B^+ \to J/\psi K^+$, 4400 $B^0 \to J/\psi K^{*0}$, and 800 $B_s^0 \to J/\psi \phi$ decays. The $B$ candidate mass distributions of all modes are compatible with a Gaussian of width $\sigma = 20$ MeV/$c^2$. To reduce backgrounds from $B$ decays to mesons contain-
ing c quarks, several vetoes are applied to $B \to \mu^+\mu^-h$ candidates, listed below. We eliminate candidates with a dimuon mass near the $J/\psi(\gamma)\to 2.9 \leq m_{\mu\mu} \leq 3.2\text{GeV}/c^2$ or $3.6 \leq m_{\mu\mu} \leq 3.75\text{GeV}/c^2$ respectively. $B \to J/\psi(\gamma)h$ decays followed by the radiative decay of the $J/\psi(\gamma)$ into two muons and a photon that is not reconstructed, may have a dimuon mass that passes the $J/\psi(\gamma)$ veto. We reject these events making use of the correlation between the candidate’s invariant mass and the dimuon invariant mass, and require $|(m_{\mu\mu} - m_B) - (m_{\mu\mu} - m_{J/\psi(\gamma)})| > 100\text{MeV}/c^2$. $B \to J/\psi(\gamma)h$ decays with one hadron misidentified as a muon form a potential background to the rare decay search. We reject this class of background by requiring that all combinations of tracks of the candidate have an invariant mass that differs by at least 40\text{MeV}/c^2 from the $J/\psi(\gamma)$ mass. We also reject candidates with track pairs with a mass within $\pm 25\text{MeV}/c^2$ of the $D^0 \to K^-\pi^+$ decay, or track triplets compatible with a mass within $\pm 25\text{MeV}/c^2$ of the $D^+ \to K^-\pi^+\pi^+$ or $D^0 \to K^+K^-\pi^+$ decays.

We further improve the signal to background ratio by an optimization of the selection based on discriminating variables. For this purpose, Monte Carlo (MC) simulations of the signal processes are used. We generate single $b$ hadrons using the transverse momentum spectrum from $B \to J/\psi X$, measured by CDF [18]. Decays of all $b$ hadrons are simulated using EVTGEN [24], with lifetimes from Ref. [23], allowing for a decay width difference between the $B^\pm$ mass eigenstates, $\Delta \Gamma/\Gamma = 0.12 \pm 0.06$ [25]. The decays $B^0 \to J/\psi K^0$ and $B^0_{\theta} \to J/\psi \phi$ are simulated according to the polarization amplitudes measured by CDF [26]. The dynamics of the rare decay processes are simulated according to the calculations of Ali et al. [11]. For $B^0_{\theta} \to \mu^+\mu^-\phi$, we assume a mixture of 50\% CP-even and 50\% CP-odd states. The interactions of final state particles are simulated using a GEANT [27] model of the CDF II detector, digitized into the CDF event format, and reconstructed using the same software as in the processing of collision data. The detector simulation includes a full emulation of the CDF trigger system. We find that the transverse momentum spectrum of the $B$ mesons of this measurement are on average higher than in the simulation. This can be explained by the presence of a $B$ baryon component with low transverse momentum in the measurement used as input spectrum of the simulation [18]. To correct for this, the simulated events are weighted by a polynomial function, which is obtained from the ratio of data to MC transverse momentum distribution of the $B$ meson.

We tighten the candidate selection to obtain the smallest expected statistical uncertainty on the measurement of the $B \to \mu^+\mu^-h$ branching ratios by finding the selection with the largest value of $N_{\text{sig}}/\sqrt{N_{\text{sig}} + N_{\text{bkg}}}$. The expected number of signal events, $N_{\text{sig}}$, is determined by scaling the MC yields of the rare decay modes to the yields of the normalization channels in the data, and correcting by the relative branching ratios and selection efficiencies. This procedure requires estimates of the branching ratios of the rare decay modes. For the $B^+ \to \mu^+\mu^-K^+$ and $B^0 \to \mu^+\mu^-K^{*0}$ decays we use the world average measured branching ratios [23], while for $B^0_{\theta} \to \mu^+\mu^-\phi$ we use the theoretical estimate [15]. The expected number of background events, $N_{\text{bkg}}$, is estimated by extrapolating the candidate yield with an invariant mass $(180 - 300)\text{MeV}/c^2$ higher than the $B$ mass, to the signal region. Candidates with an invariant mass lower than the $B$ mass are not suitable for background estimates, since they contain partially reconstructed $B$ decays, not expected in the signal region. We find good discriminating power between signal and background from the following three quantities: $t/\sigma_1$, $\alpha$, and $I$. The significance of the proper decay time, $t/\sigma_1$, is defined as the proper decay time of the $B$ candidate divided by its uncertainty. The angle $\alpha$ is defined as the difference in angle between the $B$ candidate’s momentum vector and the vector from the primary vertex to the $\mu\mu$ vertex. The isolation, $I$, is defined as the transverse momentum carried by the $B$ meson candidate divided by the transverse momentum of all tracks in a cone of $\Delta R = \sqrt{\Delta \phi^2 + \Delta \eta^2} = 1.0$ around the direction of the $B$ meson candidate, including those of the $B$ candidate itself. Here $\Delta \phi$ is the difference in pseudorapidity of the $B$ candidate and each track, and $\Delta \eta$ is the difference in their azimuthal angles. Scanning different combinations of selection thresholds, we find that the optimal values are very similar for the three rare decay modes: $t/\sigma_1 \geq 14$, $\alpha \leq 60\text{mrad}$, and $I \geq 0.6$. Applying the optimized selection requirements to the normalization channels yields the distributions shown in Fig. 1. The shape of the combinatoric background is estimated from a sample with poor vertex quality ($\chi^2$ probability $< 10^{-3}$). The invariant mass distribution of this poor vertex quality sample is fitted with a Gaussian distribution for the remaining signal contribution and an exponential plus a constant to model the background. We estimate the background in the signal region of the tight selection using this functional form, normalized to the number of candidates with invariant mass $(60 - 180)\text{MeV}/c^2$ higher than the $B$ mass. Using this method, we find $6361 \pm 82 B^+ \to J/\psi K^+$, $2423 \pm 52 B^0_{\theta} \to J/\psi K^{*0}$, and $431 \pm 22 B^0 \to J/\psi \phi$ decays. The invariant mass distributions for the rare decay modes are shown in Fig. 2. In the $(\pm 40\text{MeV}/c^2)$ window around the $B$ mass we find $90 B^+ \to \mu^+\mu^-K^+$, $35 B^0 \to \mu^+\mu^-K^{*0}$, and $11 B^0_{\theta} \to \mu^+\mu^-\phi$ candidates. From simulation studies we verified that the present selection accepts decays for every kinematically possible dimuon invariant mass, with an efficiency difference not exceeding a factor two, except for the windows around the $J/\psi(\gamma)$ that have been explicitly excluded.

For the signal modes, we evaluate three sources of background: the combinatoric background, hadrons misidentified as muons, and hadrons with misassigned mass. As in the normalization mode, the shape of the combinatoric background is estimated from a sample with poor vertex quality ($\chi^2$ probability $< 10^{-3}$). The
Data

We find an equivalent Gaussian significance of 4.5, 2.9, taking into account the uncertainty on the background. To fluctuate to the number of observed events or higher, in all three channels, and determine the significance by signal window. We find an excess in the predicted background from the number of candidates in the decay modes, respectively. Backgrounds from hadrons misidentified as muons are estimated from simulation. Final state particles from simulated charmless B decays are weighted with the muon misidentification probabilities. The misidentification probabilities for charged kaons and pions are estimated from simulation. Final state particles of the rare decay mode. We assume that the CP-even fraction of the short-lived CP-even state in the rare decay mode. We evaluate the effect of the uncertainty on the fraction of J/ψ mesons produced with a longitudinal polarization by varying the fraction measured at CDF [26] by ±1σ. The effect is 0.6% for B^0 → J/ψ K^+ and 0.1% for B^0 → J/ψ φ.

The statistical uncertainties of yields in the normalization channels are included as systematic uncertainties, which range from 1.3% for the B^+ channel to 5.1% for the B^0 s channel. Cabibbo-suppressed B^+ → J/ψ π^+ decays contribute 0.1% to the yield of the normalization channel for the B^+ channel. We introduce the full effect into our estimate of the systematic uncertainty without correcting the result. The relative efficiencies between signal

![Invariant mass spectra of (a) J/ψK^+, (b) J/ψK^+0 and (c) J/ψφ candidates after applying optimized selection requirements.](image)

**FIG. 1:** Invariant mass spectra of (a) J/ψK^+, (b) J/ψK^+0 and (c) J/ψφ candidates after applying optimized selection requirements.
and normalization channels, of 0.71 ± 0.01, 0.74 ± 0.02, and 0.84 ± 0.02 for the $B^+$, $B^0$, and $B^0_s$ decays, respectively, have uncertainties reaching 2.6%, that arise from the finite size of the MC samples.

The predicted background values depend on the shape of the background function. We evaluate the change in the background yield calculation when using a sample that is similar to the one resulting from the optimal selection, but has less stringent requirements on the three optimization variables instead of the default sample with poor vertex quality. We compare two functional models for the background description, the default exponential plus a constant, and a simpler linear extrapolation function. We find a systematic uncertainty on the background prediction reaching 10% for the $B^0_s$ channel.

To calculate the systematic uncertainty on the relative branching fraction, the relative efficiency uncertainty is summed in quadrature with the uncertainty of the $J/\psi \rightarrow \mu^+\mu^-$ branching ratio, and with the systematic uncertainty on the number of signal candidates.

To calculate the absolute branching fractions, we use the world average branching fractions of the normalization channels [23]. These branching fractions have uncertainties of 3.5%, 4.5%, and 35.5%, respectively, for $B^+$, $B^0$, and $B^0_s$, which are added in quadrature to the systematic uncertainties on the relative branching ratios. A summary of the systematic uncertainties on the relative branching ratios is given in Table II.

We calculate relative branching ratios using Eq. (1), and find $\mathcal{B}(B^+ \rightarrow \mu^+\mu^- K^+)/\mathcal{B}(B^+ \rightarrow J/\psi K^+) = (0.59 \pm 0.15 \pm 0.03) \times 10^{-3}$, $\mathcal{B}(B^0 \rightarrow \mu^+\mu^- K^{*0})/\mathcal{B}(B^0 \rightarrow J/\psi K^0) = (0.61 \pm 0.23 \pm 0.07) \times 10^{-3}$, and $\mathcal{B}(B^0_s \rightarrow \mu^+\mu^- \phi)/\mathcal{B}(B^0_s \rightarrow J/\psi \phi) = (1.23 \pm 0.60 \pm 0.14) \times 10^{-3}$, where the first uncertainty is statistical, and the second is systematic. We use the world-average branching ratios of the $B^0$ and $B^+$ normalization channels [23], resulting in the following absolute branching fractions: $\mathcal{B}(B^+ \rightarrow \mu^+\mu^- K^+) = (0.59 \pm 0.15 \pm 0.04) \times 10^{-6}$, and $\mathcal{B}(B^0 \rightarrow \mu^+\mu^- K^{*0}) = (0.81 \pm 0.30 \pm 0.10) \times 10^{-6}$. To obtain an absolute branching ratio for the $B^0_s \rightarrow \mu^+\mu^- \phi$ decay, we use $\mathcal{B}(B^0_s \rightarrow J/\psi \phi) = (1.38 \pm 0.49) \times 10^{-3}$, obtained from correcting the CDF measurement [30] for the current value of $s_f/s_d$ [23], the $B^0_s$ to $B^0$ production ratio, resulting in $\mathcal{B}(B^0_s \rightarrow \mu^+\mu^- \phi) = (1.70 \pm 0.82 \pm 0.64) \times 10^{-6}$. We find a significance of only 2.4 standard deviations for the $B^0_s \rightarrow \mu^+\mu^- \phi$ decay mode. Therefore, we choose to set a limit on this decay. We use a Bayesian integration assuming a flat prior [31], and find $\mathcal{B}(B^0_s \rightarrow \mu^+\mu^- \phi)/\mathcal{B}(B^0 \rightarrow J/\psi \phi) < 2.6(2.3) \times 10^{-3}$ at the 95(90)% confidence level (C.L.). We also set an upper limit on the absolute branching ratio, taking into account the uncertainty of the branching ratio of the normalization channel [32]. We assume that the prior probability density function representing the uncertainty in the normalization channel is a log normal distribution with a mean equal to the central value, and a width parameter equal to the quoted uncertainty of 35.5%. We obtain $\mathcal{B}(B^0_s \rightarrow \mu^+\mu^- \phi) < 6.0(5.0) \times 10^{-6}$ at 95(90)% C.L. The main ingredients and the results are summarized in Table III.

In conclusion, our measurements of the $B^+$ and $B^0$ rare decay modes are consistent with the SM predictions, and with previous measurements [5, 6]. The relative limit on $B^0_s \rightarrow \mu^+\mu^- \phi$ is consistent with the SM predictions, and is the most stringent to date.

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FIG. 2: Invariant mass spectra of (a) $\mu^+\mu^-K^+$, (b) $\mu^+\mu^-K^{\ast 0}$ and (c) $\mu^+\mu^-\phi$ candidates. The superimposed curves are a sum of a single Gaussian with a width of 20 MeV/$c^2$ representing the signal, and a curve representing the background as determined by the procedure described in the text. The curves are not drawn for masses below the signal window, since they are not expected to predict the background level where partially reconstructed $B$ decays contribute.

TABLE III: Summary of the main ingredients and results of this analysis. Where one uncertainty is quoted, the uncertainty is of statistical nature. Where two uncertainties are quoted, the first is statistical, and the second is systematic.

| Decay mode | $B^{+} \rightarrow \mu^{+}\mu^{-}K^{+}$ | $B^{0} \rightarrow \mu^{+}\mu^{-}K^{\ast 0}$ | $B_{s}^{0} \rightarrow \mu^{+}\mu^{-}\phi$ |
|------------|-----------------------------|-----------------------------|-----------------------------|
| $N_{obs}$  | 90                          | 35                          | 11                          |
| $N_{bkg}$  | 45.3 ± 5.8                  | 16.5 ± 3.6                  | 3.5 ± 1.5                   |
| $N_{sig}$  | 44.7 ± 5.8                  | 18.5 ± 3.6                  | 7.5 ± 1.5                   |
| Gaussian significance | 4.5 $\sigma$ | 2.9 $\sigma$ | 2.4 $\sigma$ |
| $N_{J/\psi h}$ | 6361 ± 82 | 2423 ± 52 | 431 ± 22 |
| $\epsilon_{\phi}/\epsilon_{h}/\epsilon_{J/\psi h}$ | 0.71 ± 0.01 | 0.74 ± 0.02 | 0.84 ± 0.02 |
| Rel $B \times 10^3$ | 0.59 ± 0.15 ± 0.03 | 0.61 ± 0.23 ± 0.07 | 1.23 ± 0.60 ± 0.14 |
| Abs $B \times 10^6$ | 0.59 ± 0.15 ± 0.04 | 0.81 ± 0.30 ± 0.10 | 1.70 ± 0.82 ± 0.64 |
| Rel $B$ 95(90)%C.L. limit $\times 10^3$ | — | — | 2.6(2.3) |
| Abs $B$ 95(90)%C.L. limit $\times 10^6$ | — | — | 6.0(5.0) |

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