Heat transfer coefficient of laminar rotational flow

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Abstract. A model of the distribution of laminar temperature and dynamic boundary layers is considered. An integral relation is derived for the energy equation of the temperature spatial boundary layer, that allows integration over the surface of any shape, which is necessary for determining the thickness of the energy loss., The expressions for the thickness of the energy loss of the temperature spatial boundary layer are defined for the laminar flow. The expressions for the thickness of the energy loss are necessary to determine the local heat transfer coefficients for the characteristic cases of flow, taking into account heat transfer. Analytically, expressions are obtained for determining the local heat transfer coefficient in the form of the Stanton criterion for the rotational flow according to the law of a solid and the rotational flow of a free vortex. Local heat transfer coefficients are determined for laminar rotational flows.

1. Introduction

Consideration of the characteristics of heat transfer in flow-through parts of power units is an important task. At present, taking into account the peculiarities of the flow with heat transfer in the implementation of potential and vortex rotational flow in the flow parts is mainly carried out by the following methods: using empirical equations, numerical and analytical methods for solving partial differential equations [1]. The first method does not always provide the required accuracy of the calculation of the hydrodynamic and thermal characteristics of rotational flows, taking into account heat transfer, and requires additional experimental refinements. This entails quite large time and material costs for the formulation and conduct of research.

Numerical methods are rather difficult to use when carrying out engineering calculations and require their implementation in specialized software. Numerical methods use direct numerical simulation (DNS method) and the Reynolds-averaged Navier-Stokes equations (RANS method). The choice of method depends on the complexity of the problem and the accuracy of the results. The RANS method is quite often used with the use of k-ε and k-ω turbulence models [2-7].

The analytical method allows to obtain analytical dependencies applicable for engineering calculations in a wide range of possible variations of design and operational parameters. Analytical methods, as a rule, were developed for rectilinear uniform flow and have several limitations. One of the problems with the use of the analytical approach is the approximation of the distribution of the profile of changes in temperature and velocity in the temperature and dynamic spatial boundary layers. The use of the velocity and temperature distribution profile in the boundary layer is proposed by V.D.
Ranni [8] and modified by D.L. Turcott [9]. Turcott's analysis of the underlayer took into account the effect of heat transfers on turbulence. Analytical methods for determining the heat transfer coefficients proposed in [10, 11] take into account convective heat transfer and are made for straight turbulent flow. A one-dimensional analytical model, for subcritical conditions, is also proposed by S.R. Shine [12]. In the flow parts of turbo power plants (turbines and centrifugal pumps) there are both turbulent and laminar rotational flows.

2. Object of study
When designing the flow parts of the units and assemblies, it is necessary to consider the temperature change of the working fluid flow along the length of the working channel, since the viscosity parameter is a function of temperature and determines the flow regime and, as a consequence, loss.

Modes of operation are possible, especially of supply units, in which even a slight heating of the working fluid can cause boiling up and, consequently, a drop in performance, as well as a loss of tightness of the unit as a whole. On the other hand, insufficient heating in the flow part of some types of working fluids leads to their high viscosity and decrease in the overall efficiency of the turbine unit.

The main object of study, where the potential and vortex rotational flow is realized, is the structural elements of gas turbines and centrifugal pumps: inlet and outlet devices, cavities between the stator and the impeller, auxiliary hydraulic path [13].

3. Setting a research problem
In the generalized formulation of the problem of fluid flow during heat exchange with the surface of aggregates, such as compressors, expanders, pumps of cryogenic components, etc., it is necessary to take into account the temperature change of the flow along the length of the working channel, since viscosity, as a function of temperature, mainly determines the flow regime and, as a result, hydraulic losses [14].

For the case of the flow of an incompressible fluid, it is necessary and sufficient to jointly solve the equations of motion and energy in the boundary conditions of the spatial boundary layer [15]; for a compressible fluid, the system must be supplemented with an equation of state.

4. Flow with heat transfer in the cavities of rotation
The case of rotational flow is considered taking into account the heat transfers for a liquid (Pr > 1). Taking into account the analysis of the scale of quantities and the absence of internal heat sources, integrating the energy equation along the coordinate within the boundaries of the boundary layer thickness, an expression is obtained for the integral ratio of the spatial boundary layer energy equation (SBL) [16]:

\[
1 \frac{\partial (\delta^{**})}{\partial \varphi} + \frac{1}{H_{\psi}} \frac{\partial (\delta^{**})}{\partial \psi} + \frac{1}{H_{\varphi}H_{\psi}} \frac{\partial H_{\varphi}}{\partial \varphi} \delta^{**} + \frac{1}{H_{\varphi}H_{\psi}} \frac{\partial H_{\psi}}{\partial \psi} \delta^{**} = \frac{\alpha}{\rho C_p U} - \frac{\tau_{\varphi 0}}{\rho C_p} \frac{1 + \varepsilon^2}{\left(T_{\delta} - T_0\right)},
\]

(1)

where \(\delta_{i0}^{**}\) is the thickness of the energy loss of the temperature SBL in the longitudinal direction;

\(\delta_{i0}^{**}\) is the thickness of the energy loss of the temperature SBL in the transverse direction.

The study examines the laminar flow with regard to heat exchange in the cavities of rotation. The features of the temperature and dynamic boundary layer distribution are shown in Figure 1.

The distribution of the laminar dynamic boundary layer is approximated by the velocity distribution function [17]:

\[
\frac{u}{U} = \left[1 - \left(1 - \frac{y}{\delta}\right)^m\right]
\]

(2)
The distribution function of the laminar temperature boundary layer

\[
\frac{T - T_0}{T_\delta - T_0} = \left[ 1 - \left(1 - \frac{y}{\delta_t} \right)^m \right].
\]  

(3)

**Figure 1.** Model of laminar temperature distribution and dynamic boundary layers with Pr > 1

We define the expression for the thickness of the energy loss for the adopted model of the distribution of boundary layers:

\[
\delta_{\text{FIP}} = \int_0^\delta \left[ 1 - \left(1 - \frac{y}{\delta} \right)^m \right] \left[ 1 - \left(1 - \frac{y}{\delta_t} \right)^m \right] dy.
\]

required to determine the local heat transfer coefficient. Then the thickness of the energy loss for the laminar flow is defined as:

\[
\delta_{\text{FIP}} = \frac{\delta_t \left(20\delta_t^2 + 5\delta_t \delta - \delta_t^2 \right)}{30\delta_t^2}.
\]

To solve the problem of heat exchange with the surface, the heat transfer law is written down:

\[
St = \frac{q_0}{\rho C_p U (T_\delta - T_0)} = \frac{\lambda \left( \frac{\partial T}{\partial y} \right)_{y=0}}{\rho C_p U (T_\delta - T_0)} = \frac{\lambda}{\rho C_p U} \left[ \frac{\partial}{\partial y} \left( \frac{T - T_0}{T_\delta - T_0} \right) \right]_{y=0},
\]

where \( \alpha = \frac{q_0}{(T_\delta - T_0)} \) is the heat transfer coefficient.
To obtain an additional equation relating the thickness of the energy loss of the temperature of the SBL and the law of heat transfer, we define the derivative of the temperature boundary layer on the heat exchange surface:

\[
\frac{\partial}{\partial y} \left( \frac{T - T_0}{T_\delta - T_0} \right)_{y=0} = \frac{\partial}{\partial y} \left[ 1 - \left( \frac{y}{\delta_t} \right)^2 \right]_{y=0} = \frac{2}{\delta_t}.
\]

Or considering to the expression for the thickness of the energy loss

\[
\frac{\partial}{\partial y} \left( \frac{T - T_0}{T_\delta - T_0} \right)_{y=0} = \left( \frac{20 + 5r + r^2}{30} \right) \frac{1}{\delta_{t\varphi}^{**}},
\]

where \( r = \frac{\delta_t}{\delta} \).

The heat transfer law for the profile (2), (3) takes the form:

\[
St = \frac{\lambda}{\rho C_p U} \cdot \frac{20 + 5r + r^2}{30} \cdot \frac{1}{\delta_{t\varphi}^{**}}.
\]

The integral relation of the equation of energy SBL for a rotational flow and distribution profiles (2) and (3) written in cylindrical coordinates is determined:

\[
J_e \frac{\partial}{R \delta_{t\varphi}^{**}} + J_e \frac{\partial}{R \delta_{t\varphi}^{**}} = \frac{\lambda}{\rho C_p U \delta_{t\varphi}^{**}} \cdot \frac{20 + 5r + r^2}{30} - \frac{\tau_{\varphi 0} (1 + e^2)}{\rho C_p (T_\delta - T_0)}
\]

The obtained integral relations (4) have been integrated from zero to the current value and the thickness of the mass of the temperature SBL for the rotational flow is determined according to the laws of "solid" and "free vortex".

Substituting the values of the thickness of the energy loss of the temperature SBL in the heat transfer law, we determine the dimensionless heat transfer coefficient in the form of the criterion for the Stanton criterion:

- for laminar rotational flow according to the law of "solid"

\[
St = \sqrt{\frac{J_e}{Pr \cdot Re} \cdot \frac{20 + 5r + r^2}{30}}
\]

- for laminar rotational flow according to the law of "free vortex"
It should be noted that by considering and analyzing the rotational flow according to the “free vortex” law, the vortex degeneration and the transition to the case of a straight flow are possible.

The local heat transfer coefficient is defined as:

\[ \alpha = \rho C_p U \cdot St \]

flow according to the law of a "solid body". From the graph of Figure 2 it can be seen that the theoretical dependences are in good agreement with those given by other authors.

The graphical dependence in Figure 2 is shown as a dimensionless heat transfer coefficient as a Nusselt criterion.

\[ Nu = St \cdot Pr \cdot Re \]

Figure 2. Dependence of the dimensionless heat transfer coefficient of laminar rotational flow at \( Pr = 4,341 \).

In general, the discrepancy between the data obtained from models with a convective component and with affine-like profiles does not exceed 10% (at a discrepancy is 9.14%). The obtained theoretical dependences by other authors as a whole are in the range of permissible values and essentially depend on the Prandtl criterion (temperature and working medium).
5. Conclusions

An integral relation for the energy equation of the temperature spatial boundary layer is derived, which allows to integrate over the surface of any shape, that is necessary for determining the thickness of the energy loss. The expressions for determining the thickness of the energy loss of the temperature spatial boundary layer are necessary to determine the local heat transfer coefficients for the characteristic cases of flow, considering heat exchange.

Analytically, expressions are obtained for determining the local heat transfer coefficient in the form of the Stanton criterion for laminar rotational flow according to the law of a solid and the rotational flow of a free vortex, for the case of $Pr > 1$.

Analytical expressions for heat transfer coefficients are in good agreement with the dependencies of other authors.

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