1/4 PBGS and Superparticle Actions

F. Delduc\textsuperscript{a}, E. Ivanov\textsuperscript{b}, S. Krivonos\textsuperscript{b}

\textsuperscript{a} Laboratoire de Physique,
Groupe de Physique Théorique ENS Lyon
46, allée d’Italie, F - 69364 - Lyon CEDEX 07

\textsuperscript{b} Bogoliubov Laboratory of Theoretical Physics, JINR,
141 980, Dubna, Moscow Region, Russian Federation

We construct the worldline superfield massive superparticle actions which preserve 1/4 portion of the underlying higher-dimensional supersymmetry. We consider the cases of $N = 4 \rightarrow N = 1$ and $N = 8 \rightarrow N = 2$ partial breaking. In the first case we present the corresponding Green-Schwarz type target superspace action with one $\kappa$-supersymmetry. In the second case we find out two possibilities, one of which is a direct generalization of the $N = 4 \rightarrow N = 1$ case, while another is essentially different.

1. Introduction. The most attractive feature of the description of superbranes based on idea of the partial spontaneous breaking of global supersymmetry (PBGS) \cite{1} - \cite{14} is the manifest off-shell realization of the worldvolume supersymmetry. In this approach, the physical worldvolume multiplets are interpreted as Goldstone superfields realizing spontaneous breaking of the full brane supersymmetry group down to its unbroken worldvolume subgroup. The invariant Goldstone superfields actions, after passing to the component fields, coincide with the static gauge forms of the relevant Green-Schwarz (GS) type actions.

Until present, only the examples of 1/2 breaking of supersymmetry corresponding to the standard BPS $p$-branes and D-branes were treated in the literature on the PBGS. It is interesting to extend the PBGS framework to the 1/4 breaking and other fractional
patterns (see, e.g., [15]-[20] for the discussion of such options at the algebraic level).

In the present talk we describe several examples of the $1/4$ PBGS superfield actions in the simplest case of massive superparticles, namely, the actions corresponding to the PBGS patterns $N = 4 \rightarrow N = 1$ and $N = 8 \rightarrow N = 2$. In the first case we construct a worldline superfield action and show its equivalence to the GS-type target superspace action with one fermionic $\kappa$ symmetry. In the second case we find out two different models. For one of them we find the bosonic part of the worldline superfield action and the corresponding GS-type action with two $\kappa$ symmetries. A common feature of all the cases considered is that the algebras of their underlying spontaneously broken $d = 1$ supersymmetries are dimensionally-reduced forms of $N = 1$ and $N = 2$ $D = 4$ Poincaré superalgebras extended by tensorial central charges [21, 22].

2. $\mathcal{N}=4 \rightarrow \mathcal{N}=1$ PBGS. Our goal here will be to construct a $N = 1, d = 1$ superfield action which respects three extra spontaneously broken supersymmetries. Thus, the minimal multiplet should include at least three $N = 1$ fermionic Goldstone superfields $\psi^i(t, \theta)(i = 1, 2, 3)$ given on $N = 1, d = 1$ superspace with the coordinates $\{t, \theta\}, (\bar{t} = -t, \bar{\theta} = \theta)$. They should inhomogeneously transform under the broken supersymmetries. One can check that this requirement is met in a minimal way and the algebra of transformations gets closed at cost of adding one additional fermionic $N = 1$ superfield $\Upsilon(t, \theta)$. The broken supersymmetry transformations read

$$
\delta \psi^i = \epsilon^i (1 - D \Upsilon) - \epsilon^{ijk} \epsilon^j D \psi^k, \quad \delta \Upsilon = \epsilon^i D \psi^i, \quad (1)
$$

where

$$
D = \frac{\partial}{\partial \theta} + \theta \partial_t, \quad \{D, D\} = 2 \partial_t. \quad (2)
$$
They form the following algebra

\[ \{Q, Q\} = 2P, \quad \{S^i, S^j\} = 2\delta^{ij}P, \quad \{Q, S^i\} = 0. \quad (3) \]

We wish to have a superparticle model with the worldline scalar \( N = 1 \) multiplets containing physical bosonic fields. Thus we are led to introduce bosonic superfields \( v^i \)

\[ \psi^i = \frac{1}{2} \mathcal{D}v^i, \quad (\bar{\psi}^i = \psi^i, \quad \bar{v}^i = -v^i) \quad (4) \]

\[ \delta v^i = -2\epsilon^i (\theta - \Upsilon) + \epsilon^{ijk} \epsilon^j v^k, \quad \delta \Upsilon = \frac{1}{2} \epsilon^i \partial v^i \quad (5) \]

Due to the explicit presence of \( \theta \) in the transformations (5), the anticommutators of \( Q \) and the spontaneously broken supersymmetry generators \( S^i \) acquire active central charges \( Z^i \) in the right-hand side:

\[ \{Q, S^i\} = 2Z^i. \quad (6) \]

The central charge generators act as pure shifts of \( v^i \), suggesting the interpretation of \( v^i \) as Goldstone superfields parametrizing transverse directions in a four-dimensional space where \( Z^i, P \) act as the translation operators.

Surprisingly, the superalgebra (6) cannot be interpreted as a dimensional reduction of the standard \( N = 1 \) Poincaré superalgebra in \( d = 4 \), with \( Z^i, P \) being the components of full 4-momentum. One should proceed not from the standard \( N = 1, D = 4 \) super Poincaré algebra, but from its extension by tensorial central charges \([21, 22, 13, 20]\). The generators \( Z^i \) turn out to partly come from these central charges and partly from the extra components of 4-momentum. Namely,

\[ P = \frac{1}{2} P_0, \quad Z^1 = P_2, \quad Z^2 = -P_1, \quad Z^3 = \frac{i}{4} (T_{22} - T_{22}) \quad (7) \]

\(^1\)Hereafter, we deal with the algebras of the superfield variations. The superalgebras of the supercharges constructed by the PBGS actions following the Noether procedure are different: they inevitably include some constant central charges which are crucial for evading \([1, 2]\) the famous Witten’s no-go theorem \([23]\).
where the original $N = 1$, $D = 4$ superalgebra is defined by the relations

\[
\{Q_\alpha, \bar{Q}_\dot{\beta}\} = 2(\sigma^m)_{\alpha\dot{\beta}}P_m, \\
\{Q_\alpha, Q_\beta\} = 2T_{(\alpha\beta)}, \quad \{\bar{Q}_\dot{\alpha}, \bar{Q}_\dot{\beta}\} = 2\bar{T}_{(\dot{\alpha}\dot{\beta})}.
\]

Let us construct invariant action for the system under consideration. The field $\Upsilon(v)$ is a good candidate for the Lagrangian density

\[
S_v = \int dt d\theta \Upsilon(v),
\]

in view of its transformation property (1). Then the question is how to covariantly express $\Upsilon$ in terms of $\psi^i$ and, further, $v^i$. This can be done rather easily.

The most general ansatz for $\Upsilon$ is as follows

\[
\Upsilon = \psi^i D\psi^i A + \psi^{2i} \psi^i B + \psi^{2i} D\psi^i \psi^i D\psi^i C \\
+ \psi^i \psi^j \psi^j D\psi^i E + \psi^j D\psi^i \psi^i F + \psi^3 \psi^3 \psi^i D\psi^i G,
\]

where $A, B, \ldots, G$ are as yet undetermined functions of $X$, and we use the following notations

\[
\psi^i_t = \partial_t \psi^i, \quad \psi^{2i} = \varepsilon^{ijk} \psi^j \psi^k, \quad \psi^3 = \varepsilon^{ijk} \psi^i \psi^j \psi^k.
\]

Now, using (1), (11) we can write $\delta \psi^i$ in terms of $\psi^i$. Then we explicitly evaluate $\delta \Upsilon$ and require it to be equal to $\epsilon^i D\psi^i$ in accordance with the transformation law (1). After rather lengthy calculations we get the system of algebraic equations for the unknowns $A, \ldots, G$

\[
A = \frac{2}{1 + \sqrt{1 - 4D\psi^i D\psi^i}}, \quad B = \frac{A^2}{2(A - 2)}, \quad C = -\frac{A^4}{2(A - 2)}, \\
E = \frac{A^3}{A - 2}, \quad F = \frac{A^3(A - 4)}{6(A - 2)^2}, \quad G = -\frac{A^5(A - 4)}{6(A - 2)^2}.
\]

The integral (9) with $\Upsilon$ defined by (10), (12) provides us with the action for the system under consideration.
We can greatly simplify this action. First, the $B$ and $C$ terms in (10) can be absorbed into the $F$ term. All the remaining $E$, $F$, $G$ terms can be reduced to the single $A$ term, redefining the superfields $v^i$ as follows

$$v^i \rightarrow \phi^i = v^i + \psi^3 \epsilon^{ijk} \psi^i_j D \psi^k H_1 + \psi^3 \psi^i_1 H_2 + \epsilon^{ijk} \psi^2 j D \psi^k H_3,$$  

(13)

where $H_1$, $H_2$, $H_3$ are some functions of $X$. These functions can be given explicitly, but to know their precise structure is of no need for our purposes. The action in terms of the redefined bosonic superfield $\phi^i$ takes the very simple form

$$S_\phi = \int dt d\theta \frac{2 \xi^i D \xi^i}{1 + \sqrt{1 - 4D \xi^j D \xi^j}}, \quad \xi^i \equiv \frac{1}{2} D \phi^i.$$  

(14)

By construction, it is guaranteed to be invariant.

Thus we have found the correct Goldstone superfields action describing the PBGS pattern $N = 4 \rightarrow N = 1$.

Let us end this section by noting that the bosonic core of the action (14)

$$S^{bos}_\phi = \frac{1}{2} \int dt \left( 1 - \sqrt{1 - \partial_t \phi^i \partial_t \phi^i} \right)$$  

(15)

is the standard massive $D = 4$ particle action in the static gauge.

3. **Target space action with one $\kappa$-supersymmetry.** To clarify the situation with $N = 4 \rightarrow N = 1$ PBGS, we construct the target space action which possesses only one $\kappa$ supersymmetry and reduces to the action (14) in a fixed gauge.

We shall deal with the $N = 4$ superalgebra (6). In accord with the standard strategy of constructing GS-type actions for massive superparticles (see [26, 3, 5, 27, 6]) we introduce bosonic $X^0(t), Y^i(t)$ and fermionic $\Theta(t), \Psi^i(t)$ $d = 1$ fields, the coordinates of a target $N = 4$ superspace, with the standard transformation properties under $N = 4$ supersymmetry (6)

$$\delta X^0 = -\epsilon \Theta - \epsilon^i \Psi^i, \quad \delta Y^i = -\epsilon^i \Theta - \epsilon \Psi^i, \quad \delta \Theta = \epsilon, \quad \delta \Psi^i = \epsilon^i,$$  

(16)
and construct the invariants $\Pi^0, \Pi^i$

$$\Pi^0 = \partial_t X^0 + \Theta \partial_t \Theta + \Psi^i \partial_t \Psi^i, \quad \Pi^i = \partial_t Y^i - \partial_t \Theta \Psi^i + \Theta \partial_t \Psi^i. \quad (17)$$

After some guess-work, the target sigma-model action invariant under the global target space supersymmetry (16), local $t$ reparametrizations and one local fermionic $\kappa$ symmetry was found to have the following form

$$S_{gs} = -\int dt \sqrt{\Pi^0 \Pi^0 - \Pi^i \Pi^i} - \int dt \left( \Theta \partial_t \Theta - \Psi^i \partial_t \Psi^i \right). \quad (18)$$

The $\kappa$ symmetry transformations are given by

$$\delta \Theta = \kappa, \quad \delta \Psi^i = \kappa \frac{\Pi^i}{\Pi^0 - \sqrt{\Pi^0 \Pi^0 - \Pi^i \Pi^i}},$$

$$\delta X^0 = -\Theta \delta \Theta - \Psi^i \delta \Psi^i, \quad \delta Y^i = -\Psi^i \delta \Theta - \Theta \delta \Psi^i, \quad (19)$$

where $\kappa(t)$ is an arbitrary fermionic gauge parameter.

The action (18) possesses only one $\kappa$ supersymmetry and therefore provides a “space-time” realization of the $N = 4 \to N = 1$ PBGS phenomenon.

To prove that there are no any other local fermionic symmetry in (18) apart from $\kappa$-symmetry (19), we need to study the algebra of the constraints in the Hamiltonian formalism. We first introduce the einbein $e(t)$ and rewrite the action (18) as

$$S_{gs} = \int dt L = -\int dt \left[ \frac{1}{2e} \left( \Pi^0 \Pi^0 - \Pi^i \Pi^i \right) + \frac{e}{2} \right]$$

$$-\int dt \left( \Theta \partial_t \Theta - \Psi^i \partial_t \Psi^i \right). \quad (20)$$

Then we compute canonically conjugated variables

$$P^0 = \frac{\Pi^0}{e}, \quad P^i = \frac{\Pi^i}{e}, \quad P_e = 0, \quad \Omega = \left( \frac{\Pi^0}{e} + 1 \right) \Theta - \frac{\Pi^i}{e} \Psi^i,$$

$$\Omega^i = \left( \frac{\Pi^0}{e} - 1 \right) \Psi^i - \frac{\Pi^i}{e} \Theta. \quad (21)$$
The canonical hamiltonian reads

\[ H = -\frac{e}{2}(P^0 P^0 - P^i P^i - 1) . \]  

(22)

There is one primary bosonic constraint, \( P_e \), and four fermionic constraints

\[ \tau^0 = \Omega + (P^0 - 1)\Theta + P^i \Psi^i , \quad \tau^i = \Omega^i + (P^0 + 1)\Psi^i + P^i \Theta . \]  

(23)

When taking the Poisson bracket of the primary bosonic constraint with the canonical hamiltonian, we obtain the secondary bosonic constraint

\[ P^0 P^0 - P^i P^i = 1 . \]  

(24)

We now have to determine which of the fermionic constraints \( \tau^\mu = (\tau^0, \tau^i) \) are first class, and thus generate gauge symmetries, and which are second class. We compute the matrix of the Poisson brackets of the fermionic constraints

\[ \{ \tau^\mu, \tau^\nu \} = C^{\mu\nu} , \quad C = 2 \left( \begin{array}{cc} P^0 - 1 & \bar{P}^\mu \\ \bar{P}^\nu & (P^0 + 1)1 \end{array} \right) , \]  

(25)

where 1 is the 3 × 3 unit matrix. The eigenvalues of \( C \) are easily computed to be \( P^0 + 1, P^0 + 1, P^0 + \sqrt{\bar{P}^2 + 1}, P^0 - \sqrt{\bar{P}^2 + 1} \). On the constraint surface, the last of these eigenvalues vanishes, and the other three remain non zero. Thus, there is only one first class constraint which may be chosen to be

\[ \kappa = \tau^0 - \frac{1}{P^0 + 1} \bar{P} \tau . \]  

(26)

Its Poisson brackets with the constraints read

\[ \{ \kappa, \tau^0 \} = \frac{2(P^0 P^0 - P^i P^i - 1)}{P^0 + 1} , \quad \{ \kappa, \tau^i \} = 0 . \]  

(27)

This constraint generates the unique local fermionic symmetry (19) through the Poisson bracket.
In the static gauge the action (18) reads

$$S_{gs} = - \int dt \left[ \sqrt{(1 + \Psi^i \partial_t \Psi^i)^2 - \partial_t Y^i \partial_t Y^i - \Psi^i \partial_t \Psi^i} \right].$$

(28)

It is straightforward to show that it is related to the component form of the action (14) by a field redefinition.

4. \(n=8 \rightarrow n=2\) PBGS. To construct a superparticle model which would exhibit \(N = 8 \rightarrow N = 2\) PBGS we should, before all, examine how 6 broken supersymmetries could be realized on a set of \(N = 2, d = 1\) superfields. We succeeded in finding two such realizations.

Case I. In the first realization the basic set of \(N = 2, d = 1\) superfields consists of seven bosonic superfields: a general real superfield \(\Phi\) and two conjugated triplets of chiral-anti-chiral superfields \(\bar{v}_i, v^i\):

$$Dv^i = \bar{D}v^i = 0, \quad i = 1, 2, 3,$$

where

$$D = \frac{\partial}{\partial \theta} + \frac{1}{2} \bar{\theta} \partial_t, \quad \bar{D} = \frac{\partial}{\partial \bar{\theta}} + \frac{1}{2} \theta \partial_t,$$

(29)

The broken supersymmetry transformations read

$$\delta v^i = -2 \left( \bar{\theta} - D\Phi \right) \epsilon^i + \epsilon^{jk} e^j D\bar{v}^k, \quad \delta \Phi = \frac{1}{2} \left( \epsilon^i Dv^i + \bar{\epsilon}^i \bar{D}v^i \right).$$

(30)

Together with the manifest supersymmetry, they form the algebra with six central charges \(Z^i, \bar{Z}^i\):

$$\{Q, \bar{Q}\} = P, \quad \{S^i, \bar{S}^i\} = \delta^{ij} P,$$

$$\{Q, S^i\} = 2Z^i, \quad \{\bar{Q}, \bar{S}^i\} = 2\bar{Z}^i.$$  

(31)

The fermionic chiral superfields defined by

$$\psi^i = -\frac{1}{2} Dv^i, \quad \bar{\psi}^i = \frac{1}{2} \bar{D}\bar{v}^i$$

8
are transformed under (30) as
\[ \delta \psi^i = \left(1 - \bar{D}D\Phi\right) \epsilon^i + \epsilon^{ijk} \bar{\epsilon}^j \bar{D}\psi^k, \quad \delta \Phi = \epsilon^i \bar{\psi}^i - \bar{\epsilon}^i \psi^i. \] (32)
So they are Goldstone superfields corresponding to the linear realization of six spontaneously broken supersymmetries with the parameters \( \epsilon^i, \bar{\epsilon}^i \). The bosonic superfields \( \bar{v}^i, v^i \) are the Goldstone ones associated with the spontaneously broken central charges transformations.

Once again, the superfield \( \Phi \), in accord with its transformation properties, can be chosen as the Lagrangian density for this PBGS pattern.

To express \( \Phi \) in terms of the Goldstone superfields \( \psi^i, \bar{\psi}^i \), one can apply the method of ref. [24, 25]. It basically consists in passing to another superfield basis by performing a finite spontaneously broken supersymmetry transformation with the Goldstone fermionic superfields as the parameters. The redefined superfield \( \tilde{\Phi} = \Phi + O(\psi, \bar{\psi}) \) transforms homogeneously and so it can be put equal to zero with preserving covariance under all supersymmetries. This produces equations allowing one to express \( \Phi \) in terms of \( \psi^i, \bar{\psi}^i \).

The straightforward application of this method to the present case yields a rather complicated system of equations. It can be easily solved in the limit of vanishing fermions, yielding the static gauge action for a massive particle in a 7-dimensional space-time as the bosonic part of the full superfield action
\[ S^\text{bos}_{v} = \frac{1}{2} \int dt \left(1 - \sqrt{1 + \partial_t v^i \partial_t \bar{v}^i}\right). \] (33)

The GS formulation for this case is very similar to the case of \( \mathcal{N} = 4 \to \mathcal{N} = 1 \) PBGS. We define the standard realization of \( \mathcal{N} = 8 \) superalgebra (31) in the superspace with seven bosonic \( X^0, Y^i, \bar{Y}^i \) and eight fermionic \( \Theta, \bar{\Theta}, \Psi^i, \bar{\Psi}^i \) coordinates:
\[ \delta \Theta = \epsilon, \quad \delta \Psi^i = \epsilon^i, \quad \delta Y^i = -2\epsilon^i \Theta, \]
\[ \delta X^0 = -\frac{1}{2} \left(\epsilon \bar{\Theta} + \bar{\epsilon} \Theta + \epsilon^i \bar{\Psi}^i + \bar{\epsilon}^i \Psi^i\right). \] (34)
Defining the invariants

\[
\begin{align*}
\Pi^0 &= \partial_t X^0 + \frac{1}{2} \left( \Theta \partial_t \bar{\Theta} + \bar{\Theta} \partial_t \Theta + \Psi^i \partial_t \bar{\Psi}^i + \bar{\Psi}^i \partial_t \Psi^i \right), \\
\Pi^i &= \partial_t Y^i + 2 \Psi^i \partial_t \Theta, \quad \Pi^i \equiv (\bar{\Pi}^i).
\end{align*}
\]

we can construct the unique action

\[
S_{gs} = -\int dt \sqrt{\Pi^0 \Pi^0 - \Pi^i \Pi^i} + \int dt \left( \Theta \partial_t \bar{\Theta} - \Psi^i \partial_t \bar{\Psi}^i \right),
\]

with two \( \kappa \) supersymmetries:

\[
\begin{align*}
\delta X^0 &= -\frac{1}{2} \left( \bar{\Theta} \delta \Theta + \Theta \delta \bar{\Theta} + \bar{\Psi}^i \delta \Psi^i + \Psi^i \delta \bar{\Psi}^i \right), \\
\delta Y^i &= -2 \Psi^i \delta \Theta, \\
\delta \Psi^i &= \frac{\Pi^i \delta \bar{\Theta}}{\Pi^0 + \sqrt{\Pi^0 \Pi^0 - \Pi^i \Pi^i}}.
\end{align*}
\]

The Hamiltonian analysis, which repeats the basic steps of the analysis in the \( N = 4 \rightarrow N = 1 \) case, shows that there are no further gauge fermionic symmetries in the action (36).

In the static gauge, \( X^0 = t, \Theta = 0 \), the action (36) takes the very simple form

\[
S_{gs} = -\int dt \left[ \sqrt{1 + \frac{1}{2} \Psi^i \partial_t \bar{\Psi}^i + \frac{1}{2} \partial_t \Psi^i \bar{\Psi}^i} \right]^2 + \partial_t Y^i \partial_t \bar{Y}^i \\
+ \Psi^i \partial_t \bar{\Psi}^i.
\]

**Case II.** The second realization of \( N = 8, d = 1 \) supersymmetry with six spontaneously broken supersymmetries can be constructed in terms of general bosonic \( N = 2 \) superfield \( \Phi \) and six chiral and anti-chiral Goldstone fermions \( \{ \psi_\alpha, \bar{\psi}_\alpha, \xi, \bar{\xi} \}, \alpha = 1, 2 \)

\[
\bar{D} \psi_\alpha = \bar{D} \xi = 0, \quad D \bar{\psi}_\alpha = D \bar{\xi} = 0,
\]

10
which form two doublets and two singlets with respect to $SO(2)$ automorphism group. The appropriate closed set of the broken supersymmetry transformations reads

$$\delta \xi = \left( 1 + \bar{D}D\Phi \right) \nu + \varepsilon_{\alpha\beta} \bar{\mu}_\alpha \bar{D} \bar{\psi}_\beta ,$$

$$\delta \Phi = \bar{\nu} \xi - \nu \bar{\xi} - \bar{\mu}_\alpha \psi_\alpha + \mu_\alpha \bar{\psi}_\alpha ,$$

$$\delta \psi_\alpha = \varepsilon_{\alpha\beta} \left( \bar{\nu} \bar{D} \bar{\psi}_\beta + \bar{\mu}_\beta \bar{D} \bar{\xi} \right) + \left( 1 - \bar{D}D\Phi \right) \mu_\alpha . \quad (40)$$

To reveal the underlying central-charges extended supersymmetry algebra and to gain physical bosonic fields, we need to pass as before to bosonic superfields. The minimal realization amounts to introducing two real scalar superfields $u_\alpha$:

$$\psi_\alpha = -\frac{1}{2} \bar{D} u_\alpha , \quad \bar{\psi}_\alpha = \frac{1}{2} D u_\alpha . \quad (41)$$

To learn what kind of “prepotential” one should introduce for the remaining Goldstone superfield $\xi$, let us examine the relation between $U(1)$ charges of spinor superfields which follows from (40)

$$q_\xi = -2 q_\psi - q_D . \quad (42)$$

Here $q_D$ is the $U(1)$ charge of the covariant derivative $D$ ($q_D = -1$ if one ascribes the charge +1 to $\theta$). From this relation and eq. (41) it follows that the only way to introduce the bosonic superfield $v$ for $\xi$ is to choose it complex and having the $U(1)$ charge $-2 q_D$

$$\xi = -\frac{1}{2} \bar{D} v , \quad \bar{\xi} = \frac{1}{2} D \bar{v} , \quad D v = \bar{D} \bar{v} = 0 . \quad (43)$$

In terms of $v, \bar{v}$ the supersymmetry transformations become:

$$\delta v = -2 \left( \theta + D\Phi \right) \nu + \varepsilon_{\alpha\beta} \bar{\mu}_\alpha D u_\beta ,$$

$$\delta u_\alpha = \varepsilon_{\alpha\beta} \left( \bar{\nu} D u_\beta + \nu D \bar{u}_\beta + \bar{\mu}_\beta \bar{D} v + \mu_\beta D v \right)$$

$$+ 2 \left( \theta + \bar{D} \Phi \right) \bar{\mu}_\alpha - 2 \left( \bar{\theta} - D\Phi \right) \mu_\alpha ,$$

$$\delta \Phi = -\frac{1}{2} \left( \nu D \bar{v} + \bar{\nu} D v - \bar{\mu}_\alpha D u_\alpha - \mu_\alpha D u_\alpha \right) . \quad (44)$$
Denoting the generators of the broken supersymmetry by $S_\alpha, \bar{S}_\alpha$ and $S, \bar{S}$, and the generators of the manifest $N = 2$ supersymmetry by $Q, \bar{Q}$, one can write the full supersymmetry algebra pertinent to this case as

\[
\{Q, \bar{Q}\} = \{S, \bar{S}\} = P, \quad \{S_\alpha, \bar{S}_\beta\} = \delta_{\alpha,\beta} P,
\]

\[
\{Q, \bar{S}\} = 2\bar{Z}, \quad \{\bar{Q}, S\} = 2Z, \quad \{Q, \bar{S}_\alpha\} = 2Z_\alpha,
\]

\[
\{\bar{Q}, S_\alpha\} = 2Z_\alpha, \quad \{S, \bar{S}_\alpha\} = 2\varepsilon_{\alpha,\beta} \bar{Z}_\beta, \quad \{\bar{S}, S_\alpha\} = 2\varepsilon_{\alpha,\beta} Z_\beta . \tag{45}
\]

Once again, we can take the superfield $\Phi$ as the Lagrangian density. To covariantly express $\Phi$ in terms of the Goldstone fermions or Goldstone bosons, we may again stick to the general method of [24, 25]. However it gives rather complicated equations which we for the time being were unable to solve. We could try to find at least the bosonic part of the action. We get the following quartic equation for the bosonic part of the action which we denote by $X$:

\[
(X^2 - X + a)(X^2 + a - 1) + 2D\xi \bar{D}\xi = 0 , \tag{46}
\]

where

\[
a = D\xi \bar{D}\xi + \bar{D}\psi_\alpha D\bar{\psi}_\alpha .
\]

The general solution of this equation exists (we require it to vanish in the limit when all fields are put equal to zero), but it looks not too illuminating to present it here. In the two limits, $\psi_\alpha = 0$ or $\xi = 0$, it takes the familiar form of the static gauge actions of massive particles moving on some 3-dimensional target manifolds

\[
S_{v}^{\text{bos}} = \frac{1}{2} \int dt \left( 1 - \sqrt{1 + \partial_t u \partial_t \bar{v}} \right) ,
\]

\[
S_{u}^{\text{bos}} = \frac{1}{2} \int dt \left( 1 - \sqrt{1 + \partial_t u_\alpha \partial_t u_\alpha} \right) . \tag{47}
\]

In the generic case there is a non-trivial cross-interaction between the bosonic fields appearing in (47). It can hopefully be interpreted
in terms of intersection of the trajectories of two different superparticles, with the physical worldline scalar multiplets represented by the superfields $u_\alpha$ and $v, \bar{v}$, respectively.

The fact that the bosonic part of the action cannot be written in the standard static gauge Nambu-Goto form seriously obscures the construction of the GS formulation for this case. We believe that the better understanding of this case would be helpful for studying the $1/4$ PBGS systems with higher-dimensional worldvolumes.

5. Conclusions. In this paper we presented, for the first time, the manifestly worldline supersymmetric superparticle actions exhibiting hidden spontaneously broken supersymmetries the number of which is four times the number of the linearly realized manifest ones. We treated in detail the case of $N = 4 \rightarrow N = 1$ partial breaking and discussed some basic features of the more complicated $N = 8 \rightarrow N = 2$ case. The common unusual feature of the superparticle systems considered is that their space-time interpretation is possible only within the superspaces corresponding to higher-dimensional supersymmetries with tensorial central charge generators. It would be of interest to understand whether this is the general property of systems with fractional PBGS.

Acknowledgements. We thank A. Kapustnikov, J. Lukierski, D. Lüst, A. Pashnev, P. Pasti, C. Preitschopf, D. Sorokin, M. Tonin and B. Zupnik for many useful discussions. E.I. and S.K. are grateful to Organizers of XIV-th Max Born Symposium for inviting them to present this talk. This work was supported in part by the PICS Project No. 593, RFBR-CNRS Grant No. 98-02-22034, RFBR Grant No. 99-02-18417, Nato Grant No. PST.CLG 974874 and INTAS Grants INTAS-96-0538, INTAS-96-0308.

References

[1] J. Hughes, J. Polchinski, Nucl. Phys. B 278 (1986) 147.
[2] J. Hughes, J. Liu, J. Polchinski, Phys. Lett. B 180 (1986) 370.

[3] A. Achucarro, J. Gauntlett, K. Itoh, P.K. Townsend, Nucl. Phys. B 314 (1989) 129.

[4] J.P. Gauntlett, K. Itoh, P.K. Townsend, Phys. Lett. B 238 (1990) 65.

[5] E. Ivanov, A. Kapustnikov, Phys. Lett B 252 (1990) 439, B 267 (1991) 541E; ibid B 267 (1991) 175; Int. J. Mod. Phys. A 7 (1992) 2153.

[6] J.P. Gauntlett, C.F. Yastremiz, Class. Quantum Grav. 7 (1990) 2089.

[7] J. Bagger, A. Galperin, Phys. Lett. B 336 (1994) 25.

[8] J. Bagger, A. Galperin, Phys. Rev. D 55 (1997) 1091.

[9] J. Bagger, A. Galperin, Phys. Lett. B 412 (1997) 296.

[10] T. Adawi, M. Cederwall, U. Gran, M. Holm, B.E.W. Nilsson, Int. J. Mod. Phys. A 13 (1998) 4691.

[11] M. Roček, A. Tseytlin, Phys. Rev. D 59 (1999) 106001.

[12] F. Gonzalez-Rey, I.Y. Park, M. Roček, Nucl. Phys. B 544 (1999) 243.

[13] S. Bellucci, E. Ivanov, S. Krivonos, Phys. Lett. B 460 (1999) 348.

[14] E. Ivanov, S. Krivonos, Phys. Lett. B 453 (1999) 237.

[15] M. Berkooz, M.R. Douglas, R.G. Leigh, Nucl. Phys. B 480 (1996) 265.

[16] J.P. Gauntlett, G.W. Gibbons, G. Papadopoulos, P.K. Townsend, Nucl. Phys. B 500 (1997) 133.
[17] A.A. Tseytlin, Class. Quantum Grav. 14 (1997) 2085.

[18] N. Ohta, P.K. Townsend, Phys. Lett. B 418 (1998) 77.

[19] I. Bandos, J. Lukierski, Mod. Phys. Lett. A 14 (1999) 1257;
I. Bandos, J. Lukierski, D. Sorokin, “Superparticle models with
tensorial central charges”, FTUV/99-07, IFIC/99-07, TUW-99-06, HUB-EP-99/15 (hep-th/9904109)

[20] J.P. Gauntlett, C.M. Hull, “BPS States with Extra Supersym-
metry”, QMW-PH-99-13, hep-th/9909098.

[21] J.A. de Azcárraga, J.P. Gauntlett, J.M. Izquierdo, P.K. Townsend, Phys. Rev. Lett. 63 (1989) 2443.

[22] S. Ferrara, M. Porrati, Phys. Lett. B 423 (1998) 255.

[23] E. Witten, Nucl. Phys. B 188 (1981) 513.

[24] E.A. Ivanov, A.A. Kapustnikov, J. Phys. A 11 (1978) 2375; J.
Phys. G 8 (1982) 167.

[25] E.A. Ivanov, A.A. Kapustnikov, Phys. Lett. B 143 (1984) 155;
Nucl. Phys. B 333 (1990) 439.

[26] J.A. de Azcárraga, J. Lukierski, Phys. Lett. B 113 (1982) 170;
Phys. Rev. D 28 (1983) 1337.

[27] P.K. Townsend, Phys. Lett. B 202 (1988) 53.