A Comparative Study of Fault Prognosis Approaches for Timed Stochastic Discrete Event Systems

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Abstract: This paper studies the fault prognosis of timed stochastic discrete event systems. For that purpose, stochastic Petri nets are considered to model both the healthy and faulty behaviors of the system. Assuming that the possible current markings with their associated probabilities are known, the objective is to estimate the probability of faults occurrence within a future time interval. Two approaches, $t_{\max}$-prognosis and $(\rho, \delta)$-prognosis, are presented and compared to deal with this problem. A case study describing a distribution system is proposed to show the applicability of the proposed methods.

Keywords: Discrete-event systems, Fault prognosis, Stochastic Petri nets.

1. INTRODUCTION

Fault prognosis of Discrete Event Systems (DESs) consists in predicting a fault event before its occurrence. It is an important and challenging issue in many disciplines. The objective is to anticipate a fault occurrence in order to take any corrective actions in advance. In the past years, fault prognosis has received considerable attention. In untimed context, (Genc & Lafortune, 2009) is one of the pioneer works that addresses the problem of event prediction (or prognosis) for partially observed DESs. Finite automata models are used to model the system and the notion of predictability was formulated as the capability to deduce future faults occurrences based on actual measurement records. In (Jéron et al., 2008), predictability concerns sequence patterns rather than a single event. In (Kumar & Takai, 2010), the authors consider fault prognosis in decentralized settings such that a global prognosis decision is computed from local decisions. Recent works on fault prognosis focus on some properties such as robustness (Takai 2015) and on guaranteed performance bound of decentralized fault prognosis (Yin & Li, 2016). In timed context, predicting events occurrence has been investigated in (Cassez & Grastien, 2013) for partially observed timed automata. It shows that the explicit consideration of time is crucial for fault prognosis of DESs. Indeed, it gives for example the remaining time before the occurrence of a fault event in order to stop or to reconfigure the system.

Note that the deterministic methods are rather rigid since they only consider the case where the fault occurs without ambiguity. Recently, to overcome this binary analysis, the authors of (Nouioua et al., 2014) and (Chen & Kumar, 2015) studied stochastic failure prognosability of DESs monitored with probabilistic automaton. In (Chen & Kumar, 2015) the notion of $S_m$-Prognosability or $m$-steps Stochastic Prognosability, which is the ability to predict a fault at least $m$-steps prior to its occurrence, is proposed in untimed settings. The problem of fault prognosis of partially observed Petri nets (POMPs) has been introduced in (Lefebvre 2014a, 2014b). Compared to the approach presented here, only the untimed context for the prognosis issue was considered. In (Lefebvre 2014a), the method is based on the probability of firing a specific transition before any other transition using stochastic POPNs. To the best of our knowledge, fault prognosis based on Petri nets (PNs) (Basile et al., 2009) and particularly on stochastic Petri nets (SPNs) was rarely explored.

In this paper, the problem of fault prognosis in timed stochastic DES modeled by SPNs is investigated. This formalism is mainly used for reliability and availability assessment. Here, we are interested in performing fault prognosis within a future time interval. The current state is assumed to be known or estimated. In this last case, the estimation consists in a set of possible markings, each with an associated probability. The state estimation methods have been well-studied in PNs. More recently, these methods are extended to deal with the probabilistic settings (Cabasino et al., 2015; Lefebvre 2014; Ammour et al., 2016b). The probabilistic marking estimation is obtained with respect to (wrt) a measurement sequence issued from the system. From the current possible marking(s), the set of all possible continuations is, in most cases, of infinite cardinality. Hence, a future fault probability cannot be exactly computed. Even if the set is bounded, its determination will require the entire exploration of the marking graph that leads to a prohibitive computation time. The main idea
of the proposed methods is to select a bounded set of the possible continuations to compute an estimation of the probability within a reasonable computational effort. In this way, a first approach denoted by \( l_{\text{max}} \)-prognosis (Ammour et al., 2016a) is proposed. It considers only possible continuations with limited size \( l_{\text{max}} \) from the current markings. The second approach is denoted by \( (p, \delta) \)-prognosis. It is based on two input parameters, the error bound \( p \) and the prognosis time horizon \( \delta \), the objective is to estimate the probability that a fault will occur within a future time \( \delta \) with an estimation error that remains lower than \( p \). Both approaches are detailed and compared.

The remainder of this paper is structured as follows. In Section 2, SPNs and some preliminary results are described. Thereafter, the stochastic fault prognosis is studied in Section 3 where the fault prognosis approaches are detailed and compared. A case study is presented in Section 4 to discuss both approaches. Finally, Section 5 presents some conclusions and perspectives.

2. PRELIMINARIES

2.1. Stochastic Petri nets

Let \( G_s = < P, T, W_{PR}, W_{PO}, \mu > \) be a SPN structure, where \( P = \{ P_1, ..., P_n \} \) is a set of \( n \) places and \( T = \{ T_1, ..., T_q \} \) is a set of \( q \) transitions. \( W_{PR} \in (\mathbb{N})^{n \times q} \) and \( W_{PO} \in (\mathbb{N})^{q \times n} \) are the post and pre incidence matrices and \( W = W_{PO} - W_{PR} \) is the incidence matrix. SPNs are characterized by random firing delays associated with the transitions (Molloy 1982). \( \mu = (\mu_j) \in (\mathbb{R}_+)^q \) is the firing rate vector which characterizes the transition firing periods. \( < G_{s0}, M_{s0} > \) is a SPN model with initial marking \( M_{s0} \) and \( M \in (\mathbb{N})^n \) represents the SPN marking vector. A transition \( T_j \) is enabled at marking \( M \) if and only if (iff) \( M \geq W_{PR}(\cdot, j) \), where \( W_{PR}(\cdot, j) \) is the column \( j \) of the pre incidence matrix. One writes \( M \triangleright T_j \) to denote that the transition \( T_j \) may fire from the marking \( M \). For each transition \( T_j \), the firing periods are given, at marking \( M \), by a random variable (RV) with an exponential probability density function (PDF). The parameter of the RV is \( \mu_j \), where \( \mu_j \) is the firing rate of transition \( T_j \) and \( \mu_j(M) \) stands for its enabling degree at marking \( M \). It is given by \( \mu_j(M) = \min \left\{ \left( \frac{m_k}{w_{PR}(k, j)} \right), P_k \in cT_j \right\} \) where \( cT_j \) is the set of \( T_j \) upstream places denoted by \( P_k \) and \( m_k \) their markings. Finally, \( \lfloor . \rfloor \) stands for the lower rounded value of (.). When \( T_j \) fires once, the marking varies according to \( \Delta M = M' - M = W(\cdot, j) \). This is denoted by \( M[T_j] > M' \). An untimed firing sequence \( \sigma_0 \) of size \( h = |\sigma_0| \) fired at marking \( M(0) \) is a sequence of \( h \) transitions \( \sigma_0 = T(1)T(2) ... T(h) \), with \( T(j) \in T, j = 1, ..., h \) that consecutively fire from \( M(0) \). This leads to the untimed marking trajectory denoted by \( (\sigma_0, M(0)) \). The probability of the untimed trajectory \( (\sigma_0, M(0)) \) is given by (Lefebvre 2014a, 2014b):

\[
P(\sigma_0, M(0)) = \prod_{k=1}^{h} \left( \frac{n_k(M(k-1))\mu_k}{\sum_{j=1}^{q} n_j(M(k-1))\mu_j} \right)
\]

The integer \( x_j(\sigma_0) \) is the number of occurrences of transition \( T_j \) in \( \sigma_0 \) and \( X(\sigma_0) = (x_j(\sigma_0)) \in (\mathbb{N})^q \) is the firing count vector of \( \sigma_0 \). When time is considered, a timed firing sequence will simply be denoted by \( \sigma \). A timed marking trajectory fired at \( M(t_0) \) is then denoted by \( (\sigma, M(t_0)) \) and \( M(t_0) \triangleright \sigma > \) denotes that \( \sigma \) may fire from the marking \( M(t_0) \). Finally, \( T = \{ f_1, ..., f_s \} \) is the set of \( s \) fault classes and the row vector \( F_a = (f_{aj}) \in (\mathbb{N})^{1 \times q} \) assigns the fault class \( f_a \) to some transitions such that \( f_{aj} = 1 \) if \( T_j \) represents a fault of class \( f_a \) else \( f_{aj} = 0 \). Transitions without any fault class are assumed to be healthy and correspond to the expected behaviors.

2.2. Sum of exponential random variables

Consider a set of \( n \) independent RVs \( X_i, i = 1, ..., n \) having exponential PDFs with parameters \( \lambda_i, i = 1, ..., n \) respectively. Let us denote by \( S_n \) the sum of these RVs: \( S_n = \sum_{i=1}^{n} X_i \). In the case where all the parameters \( \lambda_i \) are equal, the RV \( S_n \) is modeled by an Erlang distribution. Now, let us consider the general case where the set of the parameters \( \lambda_i \) is composed of a distinct values \( \{ \lambda_1, \lambda_2, ..., \lambda_n \} \) with multiplicity order \( \{ r_1, r_2, ..., r_n \} \) such that \( r_1 + r_2 + ... + r_n = n \). The cumulative distribution function (CDF) \( F_{\text{sum exp}}(t) \) of the RV \( S_n \) for \( t \geq 0 \) is given by (Amari & Misra, 1997):

\[
F_{\text{sum exp}}(t) = 1 - \left( \prod_{j=1}^{n} \lambda_j \right)^n \prod_{j=1}^{n} \sum_{k=0}^{r_j} \frac{\Psi_k(\beta_j \tau_j^k)}{k! \lambda_j^k} \left( \tau_j \right)^k \left( \frac{\beta_j \tau_j^k}{\lambda_j} \right)^{-k} \quad \text{where } \Psi_k(\alpha) = \frac{(-1)^k}{\alpha^{k+1}} \left( \prod_{j=0}^{k} (\Gamma(j+1) \beta_j^{-j}) \right), \beta_0 = 0, \tau_0 = 1.
\]

3. FUTURE FAULTS PROGNOSIS

To deal with the problem of future fault prognosis, let us first introduce some necessary notations and results that will be used. The current date is denoted by \( t_p \). The set of possible current markings at this date is \( \text{Mark}(t_p) \). Each marking \( M(t_p) \in \text{Mark}(t_p) \) is associated with its probability \( P(M(t_p)) \). The main challenge is to estimate, from these current markings, the probability that a fault of class \( f_a \) will occur within a future time interval \([t_p, t] \).

From a current marking \( M = M(t_p) \), the set of all possible continuations is denoted by \( \text{Post}(M) \) and defined as follows:

\[
\text{Post}(M) = \left\{ \sigma \mid M(\sigma) > \wedge (M \in \text{Mark}(t_p)) \right\}
\]

We also denote by \( \text{Post}_a(M) \) the set of all continuations issued from marking \( M \) that end with a faulty transition of class \( f_a \):

\[
\text{Post}_a(M) = \left\{ \sigma T_j \mid \sigma \triangleright (M(\sigma) > \wedge (M \in \text{Mark}(t_p)) \wedge (F_a.X(\sigma) = 0) \wedge (F_a.X(T_j) > 0)) \right\}
\]
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