A Dynamical Instability of Thin Layers

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We show that a generic relativistic membrane with in-plane pressure and surface density having the same sign is unstable with respect to a series of warping mode instabilities with high wave numbers. We also examine the criteria of instability for commonly studied exotic compact objects, such as gravastars, AdS bubbles and wormholes. For example, a gravastar which satisfies the weak energy condition turns out to be dynamically unstable. A thin-layer black hole mimicker is stable only if it has positive pressure and negative surface density (such as a wormhole), or vice versa.

Introduction. The detection of binary black hole mergers with ground-based gravitational-wave (GW) detectors [1–4], images of the supermassive black holes M87 & Sgr A* with radio interferometry [5, 6], and the observation of S stars orbiting around a small dark region in the galactic center [7], all point to the existence of black holes, of which a description was first obtained by Karl Schwarzschild more than one hundred years ago using General Relativity (GR). The study of black holes is not limited to astrophysics and GR, but also plays a role in other major areas of physics, such as quantum fields and strings, condensed matter physics and quantum information. Because of its unparalleled conceptual and observational importance, it is paramount to test the more refined features of black holes against all viable alternatives allowed by the laws of nature. Any signal, e.g. ringdown quasinormal modes [8, 9], that favors a black hole mimicker over black holes themselves would represent a fundamental breakthrough/revolution in physics. In the coming decades the third-generation ground-based GW detectors [10, 11], the space-borne GW detectors [12, 13] and the next-generation Event Horizon Telescope, will likely improve the precision of such tests by orders of magnitude.

Horizonless compact objects are important candidates for black hole mimickers [14]. One class of them, such as boson stars, has smooth distributions of matter/fields that are convenient for stability analysis and numerical simulations. However, it appears difficult to construct stable configurations of these compact stars that approach the compactness of black holes. For example, a fluid star with causal equation of state can achieve maximum compactness at around \( M/R \lesssim 0.355 \) [15, 16], with \( M \) being the mass and \( R \) being the radius measured from the surface area. The bound for boson stars is around 0.44 [17]. There are proposals for constructing compact stars with anisotropic stress [18–24] to increase the maximum compactness, but they often feature problems such as superruminal sound speed, violation of energy conditions and lack of stability analysis. A recent study showed that the bound can be improved to \( \sim 0.376 \) by including various prescriptions of elastic stress [25].

Another class of compact objects, e.g. gravastars and wormholes, often include a (or more) membrane(s) that separates regions of spacetime. This type of construction allows the transition to the exterior spacetime, which is the same as the black hole spacetime, to be arbitrarily close to the horizon of the corresponding black hole. Therefore, these models can have compactness arbitrarily close to those of black holes. In addition, the membrane may (partially) reflect outgoing GWs, leaving a novel echo-like signal in the ringdown stage of binary mergers [26]. The stability analysis of these objects, on the contrary, is generally less well studied than the smooth compact objects.

In this work, we present a perturbation study of membranes with nontrivial energy and stress. We find that if the signs of in-plane pressure and the surface density in the membrane are the same, there is a generic warping instability for modes with sufficiently high wave numbers. We apply these results to commonly studied compact objects, and find that a significant portion of parameter space of gravastars — which are usually modeled by a de Sitter interior and Schwarzschild exterior with a spherical shell of matter at the boundary — and AdS bubbles (with Anti de Sitter interior) are dynamically unstable. Static thin-shell wormholes always have positive pressure and negative surface energy, so that they are free from these instabilities. Therefore requiring membranes to have negative pressure (for positive surface density) and positive pressure (for negative density) becomes a powerful qualifer for the stability of compact objects. Throughout this work, we adopt the geometric unit that \( c = G = 1 \).

Membrane instability. For a membrane with positive pressure, any local vertical displacement results in an “anti-restoring” force that pushes the mass element away from equilibrium. On the other hand, the gravitational attraction from surrounding mass elements tends to bring it back to equilibrium. We shall show that the anti-spring force always wins in the eikonal limit, resulting in a series of instabilities with high wave number. To illustrate the basic picture, we present the analysis in the Newtonian regime and then follow with the discussion in the relativistic case.

Consider a membrane placed in the \((x − y)\) plane, with surface density \( \sigma \) and surface pressure \( P \). The displacement field \( \xi(x, y) \) can be generally decomposed as

\[
\xi = \xi_x \hat{e}_x + \xi_y \hat{e}_y + \xi_z \hat{e}_z.
\]
We use $\delta$ to denote Eulerian perturbations and $\Delta$ to denote Lagrangian perturbations. For example, the Eulerian density fluctuation is given by $\delta \rho = - \nabla \cdot (\rho \xi)$, where $\nabla$ here operates on the two horizontal directions, and the Lagrangian density perturbation is given by $\Delta \rho = \delta \rho + \xi \cdot \nabla \rho$. For the purpose of this analysis, we only need to consider the case with $\xi$ being nonzero, so that $\delta \rho = \Delta \rho = 0$.

The equation of motion for three-dimensional fluid elements, in terms of Lagrangian variables, can be written as

$$\rho_0 \left( \frac{\partial^2 \xi}{\partial t^2} + \nabla \Delta U - (\nabla \cdot \xi) \nabla U_0 \right) = -\nabla \cdot t,$$  \hspace{1cm} (2)

where $\rho_0$ is the unperturbed mass density, $U_0$ is the unperturbed gravitational potential and $t$ is the stress tensor so that the right hand side represents the hydrodynamical force acting on the fluid element. In other words, the left hand side of the equation is the kinetic term and the right hand side of the equation represents the external force. Similarly, for a mass element on a disk, we can write down the equation of motion as

$$\sigma_0 \left( \frac{\partial^2 \xi}{\partial t^2} + \nabla \Delta U - (\nabla \cdot \xi) \nabla U_0 \right) = F_{\text{disk-in}} + F_{\text{disk-out}},  \hspace{1cm} (3)$$

where $\sigma_0$ is the unperturbed surface mass density and $\Delta U$ is the Lagrangian potential perturbation. Since we only consider the vertical displacement, $\xi$ is divergence-free $\nabla \cdot \xi = 0$, i.e., there are no density perturbations. In the equilibrium case, $U_0$ satisfies $\nabla^2 U_0 = 0$ except at the disk plane, where the vertical derivative is discontinuous:

$$\left. \frac{\partial U_0}{\partial z} \right|_+ - \left. \frac{\partial U_0}{\partial z} \right|_- = 4\pi \sigma_0.  \hspace{1cm} (4)$$

The right hand side of Eq. (3) comprise two components of disk forces. The in-plane component is generated by the pressure variation and the tilt of the disk plane:

$$F_{\text{disk-in}} = -\nabla \Delta P + (\nabla P \cdot \Delta n) \hat{e}_z,$$  \hspace{1cm} (5)

where $n = \hat{e}_z - \partial_i \xi \hat{e}_i - \partial_j \xi \hat{e}_j = \hat{e}_z + \Delta n$ is the normal vector to the disk. The pressure perturbation is related to the density perturbation through the disk equation of state: $\Delta P / P = \Gamma_1 \Delta \sigma / \sigma$ [27], where $\Gamma_1$ depends on the equation of state and the nature of the perturbation (e.g., adiabatic or isothermal). Therefore, the Lagrangian pressure perturbation is zero for vanishing $\Delta \sigma$. The off-plane disk force is due to the warping of the disk. If we imagine the local disk surface has a radius of curvature $R$, then the magnitude of out-of-plane force is just $2P/R$. For general mean curvature $\kappa$, we have

$$F_{\text{disk-out}} = -P \kappa n, \quad \text{with} \quad \kappa = \frac{\partial^2 \xi_z}{\partial x^2} + \frac{\partial^2 \xi_z}{\partial y^2}.  \hspace{1cm} (6)$$

In order to compute the potential perturbation $\Delta U$, in particular, its value and derivatives on the disk plane, we make a coordinate transformation so that $\zeta = z - \xi_z$, with $x,y$ coordinates unchanged. The disk is mapped to the $\zeta = 0$-plane in this new coordinate system, which is more convenient for solving the boundary value problem. Using

$$\frac{\partial}{\partial x^*} = \frac{\partial}{\partial x} + \partial_x \xi \frac{\partial}{\partial \zeta}, \quad \frac{\partial}{\partial y^*} = \frac{\partial}{\partial y} + \partial_y \xi \frac{\partial}{\partial \zeta}, \quad \frac{\partial}{\partial \zeta^*} = \frac{\partial}{\partial \zeta},  \hspace{1cm} (7)$$

the original Laplace equation $\nabla^2 U = 0$ (for $\zeta \neq \xi_z$) becomes

$$\nabla^2 U = 2\partial_\zeta \xi \frac{\partial^2 U}{\partial \zeta^* \partial \zeta} + 2\partial_\zeta \xi \frac{\partial^2 U}{\partial y^* \partial \zeta} + \left( \partial^2_\zeta \xi_z + \partial^2_\zeta \xi_z \right) \frac{\partial U}{\partial \zeta^*},  \hspace{1cm} (8)$$

with the matching conditions that $\partial_\zeta U|_+ - \partial_\zeta U|_- = 4\pi \sigma$ and $U|_+ = U|_-$. Because $\xi$ is an infinitesimal displacement, we can write $U$ as $U_0 + U_1$, with $U_0$ satisfying $\nabla^2 U_0 = 0$ together with $\partial_\zeta U_0|_+ - \partial_\zeta U_0|_- = 4\pi \sigma$ and $U_0|_+ = U_0|_-$. The solution of $U_0$ is obviously known, and $U_1$ may be obtained by solving

$$\nabla^2 U_1 = 2\partial_\zeta \xi \frac{\partial^2 U_1}{\partial \zeta^* \partial \zeta} + 2\partial_\zeta \xi \frac{\partial^2 U_0}{\partial y^* \partial \zeta} + \left( \partial^2_\zeta \xi_z + \partial^2_\zeta \xi_z \right) \frac{\partial U_0}{\partial \zeta^*},  \hspace{1cm} (9)$$

with $\partial_\zeta U_1|_+ - \partial_\zeta U_1|_- = 0$ and $U_1|_+ = U_1|_-$, so that $U_1$ is completely regular in the entire spacetime. In particular, $U_1$ evaluated on the disk surface can be mapped back to $\Delta U$ with $\Delta U := U(\zeta) - U_0(\zeta) = U_1(\zeta) - U_0(\zeta)$, and $\nabla \Delta U$ is the gravitational backreaction described in Eq. (3).

At this point, we consider a planar mode with $\xi \propto e^{ikx}$ in the eikonal limit, that is, $|k| \gg 1$. The right hand side of Eq. (3) is dominated by $F_{\text{disk-out}}$, which is proportional to $k^2$. On the other hand, as $U_1$ is also proportional to $e^{ikx}$ and the source term for $U_1$ in Eq. (9) is dominated by the term proportional to $k^2$, we have $U_1 \propto k^0$, $\nabla U_1 \propto k$ and $\partial_\zeta U_1 \propto k$ ($\partial_\zeta \propto k$ as 1/k is the only length scale in the problem). So the gravitational restoring force is subdominant compared to the anti-restoring force by the warping disk. The dispersion relation is approximately (with $\partial_\zeta \rightarrow -i\omega$)

$$\omega^2 \approx -P/\sigma_0 k^2,  \hspace{1cm} (10)$$

which leads to exponential mode growth if $P/\sigma_0 > 0$.

**Relativistic case.** In the relativistic limit, we consider a model problem for compact objects with a membrane: we consider an infinite membrane with surface mass density $\sigma$ and surface pressure $P$, which is a good approximation for perturbations of (spherical) compact objects in the eikonal limit. We shall derive the equation of motion of the membrane.

If we consider the spacetime of a gravastar or a thin-shell wormhole, the metric can be expressed as diag[$-f(r), 1/h(r), r^2, r^2 \sin^2 \theta$], with different prescriptions for $f(r)$ and $h(r)$. As we focus on perturbations of small wavelength, we can zoom in on the neighborhood of any point on the membrane, and rewrite the metric as

$$ds^2 = g^{(0)}_{\mu\nu}dx^\mu dx^\nu = -U(z)dt^2 + U_z(z)dz^2 + U_p(z)(dx^2 + dy^2),  \hspace{1cm} (11)$$
where $x = \theta \cos \phi$ and $y = \sin \phi$. This local representation of the membrane metric is generic. The Israel boundary conditions on the membrane relate the extrinsic curvature $K_{\alpha\beta}$ to the surface-layer property by \cite{28} ($d\tau = \sqrt{U}dt$)

$$K^\alpha_{\;\;|\alpha} = \frac{1}{\sqrt{U}} \frac{U_p'}{2U} = -4\pi\sigma,$$

$$K^\alpha_{\;\;|\alpha} = \frac{1}{\sqrt{U}} \frac{U'_p}{2U} = 8\pi(P + \sigma/2)$$

where $|$ indicates the difference between $0_+$ and $0_-$ on the membrane in the $\tau$-direction. Since we can always rescale $z$ in the vertical/radial direction, in the rest of the discussion we shall set $U_\tau = 1$.

Let us now assume the membrane is perturbed with vertical displacement $\xi_\tau = \xi(x, y, t)$. The membrane stress energy tensor is given by

$$\tau^{\mu\nu} = \delta(z - \xi)[(\sigma + P)u^\mu u^\nu + \rho g^{\mu\nu} + n^\mu n^\nu],$$

where $n$ is the normal vector of the membrane. It is given by $e_z(1 - h_{zz}/2) - \sum_{\alpha=x,y}(e_\alpha + h_{\alpha z}e_z)/\sqrt{g_{\alpha\alpha}}$ and $e_z$ is given by $\partial_\tau h_{zz}$ (similarly for $e_x$ and $e_y$), where $h_{\mu\nu}$ is sourced by the membrane motion (compare with the right hand side of Eq. (8)). In order to derive the equation of motion for $\xi_\tau$, we transform to the coordinate system with $z' = z - \xi, \tau' = \tau, x' = x, y' = y$, such that the membrane is mapped back to the "equatorial" plane in the new coordinates. The spacetime metric in the new coordinates can be written as $g_{\mu\nu} = g^{(0)}_{\mu\nu} + \xi_{\mu y} + \xi_{y\mu} + h_{\mu\nu} = g^{(0)}_{\mu\nu} + \tilde{h}_{\mu\nu}$, where $|$ represents the covariant derivative with respect to $g^{(0)}_{\mu\nu}$.

The gravitational perturbation is more conveniently computed in the original $(t, z, x, y)$ coordinate system:

$$\tilde{h}_{\mu\nu,\sigma} + 2R_{\mu\rho\sigma\nu}\tilde{h}^{\rho\sigma} = 0,$$

with the trace-reversed $\tilde{h}_{\mu\nu} := h_{\mu\nu} - \frac{1}{2}h g^{(0)}_{\mu\nu}$ and assuming the Lorenz gauge condition $\tilde{h}_{\mu z,|\mu} = 0$, which is preserved along the evolution driven by the wave equations if it is initially satisfied. The waves should be outgoing at infinity and the matching condition at the membrane leads to

$$\int_{-}^{+} dz \tilde{h}_{\mu z,|\mu} = \tilde{h}_{\mu z,|\mu} = 8\pi \delta \tau_{\mu
u}$$

with $\delta \tau_{\mu\nu} = P\delta_\tau \xi_{\mu}/U_p, \delta \tau^{\mu\nu} = P\delta_\tau \xi_{\mu}/U_p, \delta \tau_{\mu\nu} = \sigma \xi_{\mu}/U_p$. The metric functions are continuous across the membrane, and only $\partial_\tau h_{\mu\nu}$ may be discontinuous (we shall assume a simple setup with reflection symmetry, where $\partial_\tau h_{\mu z,|\mu} = \partial_\tau h_{\mu z,|\mu} = 0, \partial_\tau h_{\mu z,|\mu} = 0$, but the final result does not rely on this assumption). In the eikonal limit, $\partial_\tau, \partial_z, \partial_x, \partial_y$ all scale as $k$, which suggests that the boundary value for $\tilde{h}_{\mu\nu} = O(k)^2$ and $\partial_\tau h_{\mu\nu} = O(k)$. Their interior value should have similar scaling laws following the wave equation in Eq. (14). (Such coupled wave equations in Lorenz gauge can be solved numerically in Schwarzschild spacetime \cite{29}, or perturbatively with WKB method because the separation of scales in $1/k$ and the curvature radius of the background spacetime $\sim U/U'$.)

The equation of motion for $\xi$ is given by $T^c_{\nu,uv} = 0$. We integrate it from lower side to the upper side of the membrane ($z' = 0_+ \rightarrow 0_-$), which becomes (evaluated at $z' = 0$)

$$(\sigma + P)(u^c u^\nu)_{,\nu} = P(n^c n^\nu)_{,\nu},$$

or, more explicitly,

$$\frac{\sigma + P}{2U}(2\tilde{h}_{xz,|z} - \tilde{h}_{zz,|z}) = -\frac{P}{2U}p_{zz} \tilde{h}_{zz,|z} + \frac{P}{2U}p_{xz,|z} \tilde{h}_{xz,|z} + \frac{p_{zz,|z}}{U_p} \tilde{h}_{zz,|z}.$$ (17)

By noticing that $\tilde{h}_{\mu\nu} = h_{\mu\nu} + \xi_{\mu y} + \xi_{y\mu} + h_{\mu\nu} = g^{(0)}_{\mu\nu} + \tilde{h}_{\mu\nu}$, the equation reduces to

$$\frac{\sigma - \rho}{U} \xi_{,x} + \frac{P}{U_p}(\xi_{,xx} + \xi_{,yy}) = -\frac{\sigma}{U} h_{zz,|z} - \frac{P h_{xx,|z}}{U_p} - \frac{P h_{y\nu,|z}}{U_p}.$$ (18)

It is clear that the $\xi_{xx} + \xi_{yy}$ terms here provide the anti-spring force that potentially drives the instability. However, to fully address the mode dispersion relation, we also need to account for the gravitational backreaction. The relevant terms in the eikonal limit are described by the terms on the right-hand side, which all scale as $k$ according to the discussion under Eq. (15). Therefore similar to the Newtonian case, the relativistic anti-spring force effect scales as $k^2$ and gravitational backreaction scales as $k$. In the eikonal limit, we therefore find

$$\omega^2 \approx -\frac{U(0)P}{U_p(0)\alpha} k^2,$$

which signals an instability if $P/\sigma > 0$. This result can be straightforwardly extended to cases for which the upper and lower spacetime have different cosmological constants.

**Gravastars.** A gravastar comprises a spherical membrane, which separates an inner de Sitter spacetime and an outer Schwarzschild spacetime. If the inner region is an anti-de Sitter (AdS) spacetime, it is usually referred as AdS Bubbles \cite{30, 31}. Defining $\rho$ as the "energy density" or cosmological constant in the inner space, $a$ as the radius of the membrane, $\sigma$ as the membrane surface energy density and $P$ as its pressure, the total mass $M$ of the spacetime is given by (following the notation in \cite{32})

$$M = M_s + M_\ast \sqrt{1 - \frac{2M_s}{a} + \frac{M_\ast^2}{2a}},$$

where $M_s = 4\pi a^3/3$ is the thin-shell mass and $M_\ast = 4\pi a^3/3$ is the volume energy within the shell. The pressure within the shell is related to these masses through

$$P = \frac{1}{8\pi a} \left[ \frac{1}{\sqrt{1 - 2M_s/a}} - \frac{1}{\sqrt{1 - 2M_\ast/a}} \right]$$

$$= \frac{1}{8\pi a} \left[ \frac{2M_s/a - 1}{\sqrt{1 - 2M_s/a}} + \frac{1}{\sqrt{1 - 2M_\ast/a}} \right].$$ (21)
To ensure meaningful values for $P$, we require that $M/a \leq 1/2$ and $M_1/a \leq 1/2$. If the gravastar satisfies the weak energy, the surface density $\sigma$ and $M_1$ are both positive. We notice that $M \geq M_1 > 0$ according to Eq. (20) and the function $(1 - x)/\sqrt{1 - 2x}$ is a monotonically increasing function in $x$ for $0 \leq x \leq 1/2$. From the second line of Eq. (21), it is straightforward to see that the pressure is always positive. Intuitively it can be viewed as a consequence of the outer spacetime squeezing the inner spacetime, as the outer spacetime has larger effective pressure than the inner spacetime (also with the self-gravitation of the membrane). Although the analysis in the previous section was with topology $\mathbb{R}^2$ while the membrane of gravastars has topology $S^2$, this distinction is irrelevant as we consider local perturbations in the eikonal limit. This simple observation, together with the analysis of the warping mode instabilities, immediately suggests that gravastars satisfying the weak energy condition are unstable. The actually instability timescale is determined by Eq. 19 depending on the prescription of $P$ and $\sigma$.

In the more general setting, as we consider both de Sitter and AdS interior and surface density with arbitrary sign, the warping instability is in operation for part of the gravastars and AdS Bubbles, as shown in Fig. 1. The pressure of gravastars is always positive and the surface density of an AdS Bubble is always positive.

The modal stability of gravastars was initially studied in [32], which explicitly computed the quasinormal mode frequency for $\ell = 2$ axial and polar perturbations. However, the analysis in [32] treats the membrane as the provider of the matching condition between the inner and outer spacetime, in the same spirit as Eq. (4), but did not incorporate the membrane oscillations into the coupled mode equations [33]. It is indeed the membrane modes that destabilize the whole system in the eikonal limit.

**Wormholes.** There are other horizonless compact objects generally considered in the literature as black hole mimickers, or as candidates sourcing gravitational wave echoes. For example, for a static thin-shell wormhole with two Schwarzschild solutions of the same mass $M$ attached at radius $r_0$ [26, 28], the corresponding thin-shell pressure and density at the wormhole throat are

$$P = \frac{1}{4\pi r_0} \frac{1 - M/r_0}{\sqrt{1 - 2M/r_0}}, \quad \sigma = -\frac{1}{2\pi r_0} \sqrt{1 - 2M/r_0} \tag{22}$$

so that the pressure is positive and the density is negative, which means static thin-shell wormholes are free from the warping instability. On the other hand, the empty shell models (which have positive surface energy density) as considered in [34, 35] naturally require positive in-shell pressure to support against gravity, which are all unstable with warping perturbations in the eikonal limit [36].

**Discussion.** One may imagine various ways to “cure” the gravastar systems so that they are free from warping instabilities. One possible way is to add additional rigidity against warping for the membrane, e.g., a new term in the action with

$$S = \alpha \int d^3 \xi \ K^{ij} K_{ij} \tag{23}$$

where $\xi$ is the parametrization for the “world tube” of the membrane, $K^{ij}$ is the extrinsic curvature and $\alpha$ is a positive constant characterizing the rigidity. A possible caveat is that such an additional term in the action may lead to higher-order derivative terms in the equation of motion, which may raise concerns regarding well-posedness issues of the problem. Moreover, adding dissipation to the system does not cure the instability. This is because the anti-spring causes run-away behavior of the displacement instead of oscillations. If the displacement were to saturate at some value, the dissipation becomes zero as there is zero velocity, but the anti-spring force continues to drive the displacement to larger values, i.e., there is no saturation point. On the other hand, if we replace the membrane with a shell of matter of thickness $d$, this can remove the instability. The thickness essentially adds a spatial frequency cutoff $k \sim 1/d$ in the above analysis. The caveat is that $d$ has to be sufficiently large so that the anti-spring in Eq. (10) becomes sub-dominant. Note that in this case the prescription for the matter equation of state also becomes important to determine the vertical (or radial) structure of the matter shell.

A membrane with density and pressure having the same sign generically prefers configurations with higher surface curvature as they are associated with a lower energy state, if gravitational backreaction is neglected. For example, a membrane with an ellipsoidal shape has lower potential energy than that with a spherical shape. Mathematically the potential energy is $\propto Q^2/(2\lambda)$ where $Q_{ij}$ is the mass quadrupole moment and $\lambda$ the tidal Love number. Negative potential energy means that $\lambda$ is negative. Even with gravitational backreaction included, if it is weaker than the anti-spring force such that the potential energy is still negative, the Love number $\lambda$ will also be negative [37]. Therefore the warping instability...
is connected to the negativity of tidal Love numbers, which applies to generic deformations with any $\ell \geq 2$. In the eikonal limit, the tidal Love number $\lambda_\ell$ has to be negative if $P/\sigma > 0$.

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