ABOUT ONE POSSIBILITY OF RELATIVISTIC DESCRIPTION OF POLARIZED DEUTERON FRAGMENTATION

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Abstract

In the framework of the light-front quantum theory developed by Karmanov et al. an analysis of the experimental data on the tensor analyzing power of the nuclear fragmentation of relativistic deuterons with the large transversal momentum proton emission has been made. With the Karmanov’s wave function taken in system in which z-axis directed along the deuteron beam we have managed to explain the existing data without invoking additional to nucleons degrees of freedom.

The experiments with the polarized deuteron beams made in Saclay [1]-[4] and Dubna [5]-[10] have led one to recognize that at the relativistic momenta of deuteron there is something wrong either with the theory of $A(d,p)X$ reactions or with the structure of the deuteron at short distances between nucleons. At first, it was proved out that the experimental dependence of the analyzing power $T_{20}$ on $k$ — internal momentum of nucleons in the deuteron — does not change sign at $k \sim 0.5$ GeV/c, as it followed from theoretical calculations. Further [10], the pion-free deuteron-breakup process $dp \rightarrow ppn$ in the kinematical region close to that of backward elastic $dp$ scattering at a given value of $k$ depends on the incident momentum. This forces one to suggest that in description of this quantity an additional variable is required. This additional variable does not appears in the usual schemes of calculations. At last, the recent measurements of the tensor analyzing power $A_{yy}$ of the breakup of relativistic deuterons on nuclei at large transverse momenta of emitted protons [11, 12] show also that something unusual takes place in the theory of this reaction since the measured $A_{yy}$-values at fixed value of longitudinal proton momentum show a pronounced dependence on the transverse proton momentum, that does not appear in the calculations.

The theoretical considerations of $A(d,p)X$ reaction were carried out in different lines and on the whole the situation with the description of this reaction is contradictory. The most popular of theoretical approaches is one of the light-front dynamics and in this paper we follow it. In general this approach in the approximation of a simple mechanism with the pole in $t$-channel with using the standard deuteron wave functions satisfactorily describes the differential cross section data [13, 14] (see, for example, [15, 16]). On the other hand, the calculations of polarization observables in the same approach [17], as a rule, does not reproduce the experimental data; the exception is the paper [18], where the data on the $T_{20}$ of nuclear fragmentation of relativistic deuterons with the proton emission at 0° are described.
The most simple statement would be to say that this discrepancy between the theory and experiment is due to the oversimplified mechanism of the reaction. But, in our opinion, at the experimental accuracy achieved the possibilities of this simple and thus valuable mechanism have yet not been exhausted.

In all previous papers concerned with the analysis of the polarization observables of the $A(d,p)X$ reaction the deuteron wave function has been presumed to be the superposition of the $S$- and $D$-waves, each represented in the momentum space as a product of angular and radial functions. In particular, this is true for one of relativistic versions of theory — relativistic quantum mechanics [19]. This superposition implies a definite relationship between the transverse and longitudinal components of the momentum of the internal motion of nucleons in a deuteron [17]. However, the dependence of the wave function on the transverse and longitudinal components in the light-front dynamics may be different from that dictated by the $S$- and $D$-wave combination. The attention to this possibility was called in [20, 21], where the relativistic hard collision model of composite hadrons [22] was generalized to the case of relativistic nucleon-nucleus collisions. It is precisely this possibility that is explored in the present paper.

The light-front dynamics [23] has many important benefits for the description of high-energy experiments. Probably the main physical achievement of this approach is the prediction and explanation [24] of behaviour of the ratio of the proton’s elastic electromagnetic form-factors [25]. The light-front dynamics is usually used also for description of the deep inelastic phenomena. A special feature of this dynamics is that the contribution of diagrams going back in time vanish.

Difficulties of the light-front dynamics are in breaking of rotational invariance as a result of selecting a particular direction in space for the orientation of the light front. One point related to this is that angular momentum operators $J_x$, $J_y$ in the light-front dynamics become dynamical operators, i.e. they are dependent on an interaction. This leads to the difficulties associated with the determination of the spin of a composite system. References to papers devoted different aspects of light-front dynamics can be found in the review [26].
These difficulties with rotational invariance were circumvented by Karmanov and coworkers [27]. They found the relativistic deuteron wave function with correct internal spin. This function depends on two vector variables: on the momentum $k$ of nucleons in deuteron in their rest frame and on the extra variable $n$ — the unit normal to the light front surface.

The general statement by Karmanov is: the final results when particles are on mass shell do not depend on the choice of the plane of quantization. But this is right generally speaking only in that case when one makes fully accurate calculations. Really however one makes only approximate calculation and therefore it arises some dependence on the choice of the front light surface. We play on this dependence. Since the direction along deuteron beam is most evidently favoured we direct $z$-axis along beam. As a result the wave function of relativistic deuteron with right spin becomes nontrivially dependent on longitudinal and transverse components of internal momentum and provides new possibilities in the description of experimental data.

Karmanov’s wave function is determined by six invariant functions instead of two ones in the non-relativistic case, each of them depending on two scalar variables $k$ and $z = \cos(kn)$ and has the following form:

$$
\Psi^M_{\sigma_2 \sigma_1} = w^*_{\sigma_2} \psi^M(k, n) \sigma_y w_{\sigma_1},
$$

where $M = 0, \pm 1$ are the projections of spin $J = 1$ on the quantization axis, and

$$
\psi(k, n) = \frac{1}{\sqrt{2}} \sigma f_1 + \frac{1}{2} \left[ \frac{3}{k^2} k(k \cdot \sigma) - \sigma \right] f_2 + \frac{1}{2} \left[ 3 n(n \cdot \sigma) - \sigma \right] f_3 + \frac{1}{2k} [3k(n \cdot \sigma) \\
+ 3n(k \cdot \sigma) - 2\sigma(k \cdot n)] f_4 + \sqrt{\frac{3}{2}} \frac{i}{k} [k \times n] f_5 + \frac{\sqrt{3}}{2k} [k \times n] \times \sigma f_6.
$$

Here $\sigma$ are the Pauli matrices, $w_{\sigma_1(\sigma_2)}$ are the spin functions of non-relativistic nucleons, and $f_1, \ldots, f_6$ are the invariant about rotations functions of the kinematical variables, that

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**Fig. 2.** Parameter $A_{yy}$ of the reaction $^{12}\text{C}(d, p)X$ at 9 GeV/c and a proton emission angle of 85 mr versus the detected proton momentum. Experimental data from [11]. The calculations were made with the Bonn B [32] (dashed curve), Paris [31] (dash-dotted curve) and Karmanov’s (solid curve) deuteron wave functions [27].
define the deuteron state. Here

\[ k = \sqrt{\frac{m_p^2 + p_T^2}{4x(1-x)} - m_p^2}, \quad (\mathbf{n} \cdot \mathbf{k}) = (\frac{1}{2} - x) \cdot \sqrt{\frac{m_p^2 + p_T^2}{x(1-x)}} \],

(3)

where \( x \) is the fraction of the deuteron longitudinal momentum taking away by the proton in the infinite momentum frame.

The invariant amplitude for the reaction \(^1\text{H}(d,p)X\) in the light-front dynamics is as follows

\[ \mathcal{M}_a = \frac{\mathcal{M}(d \rightarrow p_1b)}{(1-x)(M_d^2 - M^2(k))} \mathcal{M}(bp \rightarrow p_2p_3), \]

(4)

where \( \mathcal{M}(d \rightarrow p_1b) \) and \( \mathcal{M}(bp \rightarrow p_2p_3) \) are the amplitudes of the deuteron breakup into the particles \( p_1, b \) and of the reaction \( bp \rightarrow p_2p_3 \), respectively. The ratio

\[ \psi(x, p_{1T}) = \frac{\mathcal{M}(d \rightarrow p_1b)}{M_d^2 - M^2(k)} \]

(5)

is nothing but the wave function in the channel \((b, N)\); here \( p_{1T} \) is the component of momentum \( p_1 \) transverse to the \( z \) axis, and \( M^2(k) \) is given by

\[ M^2(k) = \frac{m_p^2 + p_T^2}{x} + \frac{b^2 + p_{1T}^2}{1-x}, \]

(6)

where \( b^2 \) is a four-momentum squared of the off-shell particle \( b \).

The analyzing power of \( T_{\kappa q} \) is given by

\[ T_{\kappa q} = \frac{\int d\tau \ Sp\{\mathcal{M} \cdot \mathbb{t}_{\kappa q} \cdot \mathcal{M}^\dagger\}}{\int d\tau \ Sp\{\mathcal{M} \cdot \mathcal{M}^\dagger\}}, \]

(7)

where \( d\tau \) is the phase volume element, and the operator \( t_{2q} \) is defined by

\[ < m | t_{\kappa q} | m' > = (-1)^{l-m} < 1 m 1 - m' | \kappa q >, \]

with the Clebsh-Gordan coefficients \( < 1 m 1 - m' | \kappa q > \).

The final expression for the analyzing power has the form

\[ T_{2q}(\frac{p_1o d\sigma}{dp_1})_{un} = \frac{1}{2(2\pi)^3} \frac{I(b,p)}{I(d,p)(1-x)^2} \rho_0(2,q) \sigma(bp \rightarrow X) \]

\[ + \int \frac{dt dp_{2T}}{2y(1-y)} \frac{I(b,p)}{I(d,p)(1-y)} \rho_0(2,q) \frac{p_{1o d\sigma}}{dp_1}(bp \rightarrow p_2p_3) [1 + \mathbb{P} < \sigma >], \]

(8)

where \( I(b,p), I(d,p) \) are the invariant fluxes of the appropriate particles, \( < \sigma > \) is the vector analyzing power of the \( NN \)-scattering, \( \sigma(bp \rightarrow X) \) is the total cross section of the \( NN \)-scattering, \( \mathbb{P} \) is the polarization vector of the nucleon in the deuteron that is characterized by indices \((\kappa, q)\):

\[ \mathbb{P} = Sp(\rho(\kappa,q))/\rho_0(\kappa,q). \]

(9)
The first term in the curly brackets of (8) corresponds to the case when the spectator
proton is detected, and the second term corresponds to the detecting of the proton scat-
tered on the target. The differential cross section for an unpolarized beam entering in (8)
is given by

\[
\left( \frac{p_{10}d\sigma}{dp_1} \right)_{un} = \frac{1}{2(2\pi)^3} \left\{ \frac{I(b,p)}{I(d,p)} \frac{1}{(1-x)^2} \rho_0 \sigma(bp \to p_2p_3) \right. \\
+ \left. \int \frac{dt dp_{2T}}{2y(1-y)} \frac{I(b,p)}{I(d,p)} \rho_0 \frac{p_{10}d\sigma}{dp_1}(bp \to p_2p_3) \right\},
\]

where

\[
\rho_0 = 3[f_1^2 + f_2^2 + f_3^2 + f_2f_3(3z^2 - 1) + 4f_4(f_2 + f_3)z + f_4^2(z^2 + 3) + (f_5^2 + f_6^2)(1-z^2)].
\]

If one introduces the density matrix in the spin space of the nucleon \(b\) at a given deuteron
polarization characterized by indices \((\kappa, q)\)

\[
\rho_{\mu\mu'}(\kappa, q) = \sum_{\nu, M, M'} \psi_M(\nu, \mu)(-1)^{1-M'} < 1 M 1 - M'|\kappa q > \psi_M^*(\nu, \mu') \\
= \rho(\kappa, q) = \frac{1}{2}\rho_0(\kappa, q)(1 + \mathbf{P} \cdot \sigma),
\]

then the density matrices \(\rho_0(\kappa, q)\) may be computed from the relations

\[
\rho_0(\kappa, q) = Sp\{\rho_{\mu\mu'}(\kappa, q)\}.
\]

The results of calculations of tensor analyzing power \(A_{yy}\) of the reaction \(^9\text{Be}(d,p)X\)
at the initial deuteron momentum of 4.5 GeV/c and a proton emission angle of 80 mr are
compared with the experimental data in Fig. 1.

It is seen that the experimental data are qualitatively properly reproduced using the
Karmanov’s relativistic deuteron wave function as opposed to the calculations with the
standard deuteron wave functions \([31, 32]\); the last curves change sign at the proton
momentum \(\sim 3.2\) GeV/c.

In Fig. 2 the experimental data on parameter \(A_{yy}\) of the reaction \(^{12}\text{C}(d,p)X\) at
the initial deuteron momentum of 9 GeV/c and a proton emission angle of 85 mr are
compared with the calculations using different deuteron wave functions. It is seen that
the momentum dependence calculated with the relativistic deuteron wave function is very
close to the experimental points, whereas the curves calculated with the standard non-
relativistic deuteron wave functions are in sharp contradiction with the data.

Since the relativistic deuteron wave function \([27]\) has a rather complicated appear-
ance, the question arises, what terms of this function help to describe qualitatively the
experimental data on the tensor analyzing power of the nuclear fragmentation of the
relativistic deuterons with emission of protons with large transverse momenta? To answer
this question, the calculations of the momentum dependence of the parameter \(A_{yy}\) of the
reaction \(^{12}\text{C}(d,p)X\) at 9 GeV/c and 85 mr have been made, in which the terms \(f_2, ..., f_6\)
of the function \([2]\) have been taken into account successively. The results are shown in
Fig. 3.
It is seen that the two first terms of \( f_i \) give the dominating contribution to the \( A_{yy} \)-dependence, and the remaining terms give only corrections; the role of these corrections increases with the momentum.

It is shown in \[27\] that in the non-relativistic limit the functions \( f_1 \) and \( f_2 \) correspond to the \( S \)- and \( D \)-states of the deuteron. Hence it follows that the relation between the \( k_L \) and \( k_T \) in a moving deuteron differs essentially from that in the non-relativistic case. The method of relativization proposed by Karmanov et al. \[27\] appear to reflect correctly this relation.

Two main conclusions may be made from this investigation. At first, it turns out rather unexpectedly that up to small relative distances corresponding to the internal momenta of nucleons \( k \sim 0.5 - 0.8 \) GeV/c the deuteron can be considered as a two-nucleon system in the light-form of the quantum mechanics. A similar conclusion was made in \[14\] in connection with the measurements of the momentum spectra of protons emitted as a result of fragmentation of 9 GeV/c-deuterons in the region of proton transverse momenta of 0.5 - 1 GeV/c. Secondly, in the fragmentation process the relativistic effects become significant very rapidly, and these effects can be taken into account the most simple way through the use of the light-front dynamics.

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