Correlation between lepton flavor violation and $B_{(d,s)} - \bar{B}_{(d,s)}$ mixing in SUSY GUT

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Motivated by the recent measurements of the $B_{s} - \bar{B}_{s}$ mass difference from the DØ and CDF collaborations, we probe new physics effects in the $B_{d} - \bar{B}_{d}$ mixing within the context of the supersymmetric grand unified model (SUSY GUT). We find that new physics effects in $B_{(d,s)} - \bar{B}_{(d,s)}$ mixing lead to the correlated information in the branching fractions of the lepton flavor violating decays, which may serve as a test of the SUSY GUT. We also discuss the implication of such new physics effects on the quark-lepton complementarity in the neutrino mixings.

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INTRODUCTION

The recent measurements for the $B_{s} - \bar{B}_{s}$ mass difference from the DO [1] and CDF [2] collaborations given by

$$17 \text{ ps}^{-1} < \Delta M_{s}^{\text{exp}} < 21 \text{ ps}^{-1} \quad (90\% \text{ CL, DO}),$$

$$\Delta M_{s}^{\text{exp}} = (17.77 \pm 0.10 \pm 0.07) \text{ ps}^{-1} \quad (\text{CDF}),$$

have triggered to probe new physics effects in $b \rightarrow s$ transition. Although the experimental results are consistent with prediction of the standard model (SM) dominated by the $t$–quark exchange in the $B_{s}^{0} - \bar{B}_{s}^{0}$ box diagram, they do not fully exclude all the possible new physics effects in $\Delta B = 2$ transitions. Since flavor changing $b \rightarrow s$ transitions are very sensitive to new physics, it is worthwhile to probe them through the $B_{d} - \bar{B}_{d}$ mixing phenomena [3, 4, 5, 6]. Moreover, the combined analysis of the $B_{s} - \bar{B}_{s}$ mixing and the $B_{d} - \bar{B}_{d}$ mixing may provide clearer hint on the existence of new physics in flavor changing transitions, due to a possible cancellation of hadronic uncertainties.

Probing such a possibility of new physics in flavor changing transitions is the main purpose of this work. As a concrete example of physics beyond the SM, we will consider the supersymmetric grand unified model (SUSY GUT) with heavy right-handed neutrinos where the imprint of large atmospheric neutrino mixing may appear in the squark mass matrices. In this model large Dirac neutrino Yukawa couplings can induce large off-diagonal mixing in the right-handed down type squark mass matrix, and there can exist possible correlation between quark flavor changing processes and lepton flavor violating (LFV) decays, $\tau \rightarrow \mu (e) \gamma$. Here we want to test such new physics in $b \rightarrow s (d)$ processes in the SUSY GUT, to find out the correlated new physics effects in LFV decays as a function of branching fractions of $\tau \rightarrow \mu (e) \gamma$. We examine in detail how new physics effects in $B_{s(d)} - \bar{B}_{s(d)}$ mixing phenomena are correlated with the branching fractions of the LFV decays. Comparing our analysis with future measurements of the branching fractions of LFV decays may serve as a test of the supersymmetric grand unified model. Recently, the similar idea has been proposed in Ref. [7]: The authors have studied a correlation between $B_{d}$ mixing and LFV decay $\tau \rightarrow \mu \gamma$ by assuming that there is no new physics effects in $B_{d}$ mixing and some arbitrary mixing term in the slepton mixing matrix. However, in our approach, we consider not only such a correlation but also the correlation among the ratio of new physics contributions to $B_{d}$ and $B_{s}$ mixings and the corresponding ratio of the branching fractions of LFV decay $Br(l_{i} \rightarrow l_{j} \gamma)$. Thus, our approach is less dependent on arbitrary SUSY input parameters and may also reduce the hadronic uncertainties in those processes. Moreover, interestingly enough, our analysis can lead to an implication of new physics effects in the quark-lepton complementarity [8] between the solar neutrino mixing angle and Cabibbo angle, as will be shown later.

In general, the $B_{q}^{0} - \bar{B}_{q}^{0}$ mass difference is defined as $\Delta M_{q} = 2|\Delta M_{12}(B_{q})| = 2|\langle B_{q}^{0}|H_{\text{eff}}^{\Delta B = 2}|\bar{B}_{q}^{0}\rangle|$, where $H_{\text{eff}}^{\Delta B = 2}$ is the effective Hamiltonian responsible for the $\Delta B = 2$ transition, and the SM prediction [8] is given by

$$M_{12}^{\text{SM}}(B_{q}) = \frac{G_{F}^{2}m_{W}^{2}}{12\pi^{2}}M_{B_{q}}\hat{\eta}^{B_{q}}\hat{B}_{B_{q}}\hat{f}_{B_{q}}^{2}(\hat{V}_{i0}^{B_{q}}\hat{V}_{i0})^{2}L_{0}(x_{i}),$$

where $G_{F}$ is the Fermi constant, $x_{i} = m_{q}^{2}/m_{W}^{2}$, $L_{0}$ is an “Inami-Lim” function [9], $\hat{\eta}^{B_{q}}$ is a short-distance QCD correction, and $f_{B_{q}}$ and $\hat{B}_{B_{q}}$ are non-perturbative parameters from which main theoretical uncertainties arise.

Due to the hadronic uncertainties in the SM prediction for $M_{12}^{\text{SM}}(B_{q})$, various estimates of the SM values of $\Delta M_{q}^{\text{SM}}$ have come out so far consistent. In order to estimate the SM values of $\Delta M_{q}^{\text{SM}}$, in particular, we adopt the
Let us implement two results for the input hadronic parameters, $B_{B_s} f_{B_s}^2$. The first one is from the most recent (unquenched) simulation by JLQCD collaboration \cite{10}, with non-relativistic $b$ quark and two flavors of dynamical light quarks. The second one is from combined results, denoted by (HP+JL)QCD. Lacking any direct calculation of $B_{B_s}$ with three dynamical flavors, it has been suggested to combine the results of $f_{B_s}$ from HPQCD collaboration \cite{11} with that of $B_{B_s}$ from JLQCD. Then, two numerical results for $\Delta M_q^\text{SM}$ are given by \cite{12}

\begin{align}
\Delta M_q^\text{SM} &= [0.52^{+0.21}_{-0.19}] \text{ ps}^{-1} \quad \text{JLQCD} \quad (3) \\
&= [0.69 \pm 0.14] \text{ ps}^{-1} \quad \text{(HP + JL)QCD},
\end{align}

\begin{align}
\Delta M_q^\text{SM} &= [16.1 \pm 2.8] \text{ ps}^{-1} \quad \text{JLQCD} \quad (4) \\
&= [23.4 \pm 3.8] \text{ ps}^{-1} \quad \text{(HP + JL)QCD}.
\end{align}

The experimental result for $\Delta M_q^\text{exp}$ \cite{12} is known to be

$$
\Delta M_q^\text{exp} = (0.507 \pm 0.004) \text{ ps}^{-1}. \quad (5)
$$

The mixing amplitude including new physics contributions can be parameterized in a model independent way as

$$
\mathcal{M}_{12}(B_q) = \mathcal{M}_{12}^\text{SM}(B_q) [1 + R_q] = \mathcal{M}_{12}^\text{SM}(B_q) [1 + r_q e^{i \phi_q}]. \quad (6)
$$

From Eq. (6), we obtain

$$
|R_q|^2 = \left| \frac{\mathcal{M}_{12}(B_q)}{\mathcal{M}_{12}^\text{SM}(B_q)} \right|^2 + 1 - 2 R_e \left( \frac{\mathcal{M}_{12}(B_q)}{\mathcal{M}_{12}^\text{SM}(B_q)} \right). \quad (7)
$$

Using $\left| \frac{\mathcal{M}_{12}(B_q)}{\mathcal{M}_{12}^\text{SM}(B_q)} \right| = \frac{\Delta M_q^\text{exp}}{\Delta M_q^\text{SM}} \equiv \Delta_q$, we get the following relation,

$$
_r_q \equiv |R_q| = - \cos \sigma_q \pm \sqrt{\cos^2 \sigma_q + \Delta_q^2 - 1},
$$

or

$$
(\Delta_q - 1)^2 \leq |R_q|^2 \leq (\Delta_q + 1)^2. \quad (8)
$$

On the other hand, the complexity of the mixing amplitude $\mathcal{M}_{12}(B_q)$ leads to the CP violation. In Eq. (6), the CP phase may be composed of the SM and new physics contributions as

$$
\phi_q = \phi_q^\text{SM} + \phi_q^\text{NP} = \phi_q^\text{SM} + \arg(1 + r_q e^{i \theta_q}). \quad (9)
$$

Then, the CP phase arisen from new physics contribution is expressed by

$$
\sin \phi_q^\text{NP} = \frac{r_q \sin \sigma_q}{\sqrt{(1 + r_q \cos \sigma_q)^2 + (r_q \sin \sigma_q)^2}}. \quad (10)
$$

From the relations Eqs. (8)-(10), we can extract useful information on the new physics effects in the $B_q \bar{B}_q$ mixing by using the experimental results for $\Delta M_q^\text{exp}$ and $\sigma_q$.
mixing. As shown in SUSY SU(5), the off-diagonal elements in the mass matrix of the down-type squarks can be generated through RG running, and is approximately given by [14]

\[
(m_{d_{BB}}^{2})_{ij} \approx -\frac{1}{8\pi^2}(Y_{d}^{i}Y_{d}^{j})(3m_{0}^2 + A_{0}^2)\ln\frac{M_{s}}{M_{GUT}},
\]

where \(m_{0}\) and \(A_{0}\) stand for the universal scalar mass and the universal \(A\)-parameter for the soft SUSY breaking, \(M_{s}\) for the scale where the universality of the scalar mass is imposed and \(M_{GUT}\) denotes the SU(5) breaking scale. Then, \((m_{d_{BB}}^{2})_{32}\) and \((m_{d_{BB}}^{2})_{31}\) contribute to \(B_{s}\) and \(B_{d}\) mixing, respectively.

From the experimental results for \(\Delta M_{s}\) and their SM predictions, one can extract the allowed regions of the parameters \(|R_{q}|\) and \(\sigma_{q}\) by using Eq. (8). Using these constraints for \(|R_{q}|\), we can extract useful information on the relevant LFV radiative decays in SUSY GUT context.

**CORRELATION BETWEEN \(B_{s}^{0} - \bar{B}_{s}^{0}\) MIXING AND LEPTON FLAVOR VIOLATION**

Let us discuss how the new physics effects extracted from measurements of the \(B_{q}\) mixing can be related to the lepton flavor violation in the context of SUSY GUT. In the case that the \(RR\) contribution to \(R_{q}\) dominates over the others in Eq. (11), the new physics contributions to \(R_{q}\) are approximately given by

\[
R_{s} \approx a_{3}(\delta_{RR}^{d})_{23}^{2},
\]

\[
R_{d} \approx a_{3}(\delta_{RR}^{d})_{13}^{2}.
\]

Since the term \((\delta_{RR}^{d})_{ij}\) is proportional to \((Y_{d}^{i}Y_{d}^{j})_{ij}\) in SUSY GUT, we can obtain the following simple relation,

\[
\frac{R_{s}}{R_{d}} \approx \frac{(\delta_{RR}^{d})_{23}^{2}}{(\delta_{RR}^{d})_{13}^{2}} = \frac{(Y_{d}^{i}Y_{d}^{j})_{23}^{2}}{(Y_{d}^{i}Y_{d}^{j})_{13}^{2}}.
\]

Therefore, the origin of new physics effects in the \(B_{q}\) mixing is from the SUSY seesaw in this case.

On the other hand, the SUSY seesaw model we consider can lead to sizable effects on the LFV processes such as \(l_{i} \rightarrow l_{j}\gamma\) due to the new source of lepton flavor violation arisen from the misalignment of lepton and slepton mass matrices, and the branching ratios of the LFV decays depend on the specific structure of the neutrino Dirac Yukawa matrix. In the context of SUSY GUT, this mixing in the charged lepton sector is dictated as same as that of the down type quark sector in Eq. (13). As discussed in [17], the LFV processes in SUSY GUT models can provide a probe of quark-lepton unification. Thus, combining the idea proposed in [17] with the analysis based on \(B_{q}\) mixing, we can further probe quark-lepton unification. The contribution to the branching fractions of the LFV decays due to the slepton mass term is roughly given by [15]

\[
Br(l_{i} \rightarrow l_{j}\gamma) \approx \frac{\alpha^{3} m_{\tilde{q}}^{2} (\delta_{RR}^{d})_{ij}^{2}}{16\pi^{2} M_{s}} \tan^{2} \beta,
\]

where \(m_{S}\) is a supersymmetric leptonic scalar mass scale and we used a rough GUT relation, \((\delta_{RR}^{d})_{ij} \approx (\delta_{LL}^{d})_{ij}\). We remark that such a GUT relationship for the parameter \(\delta\)’s must be corrected down to the typical mass scale of the right-handed Majorana neutrinos \(M_{R}\) because the slepton mass term gets additional corrections due to RG evolution. However, in general, such RG effects as well as corrections from RG evolution down to \(M_{W}\) do not significantly modify the GUT relations for off diagonal elements \((\delta_{LL}^{d})_{ij}\) presented at \(M_{GUT}\). Furthermore, the logarithmic scale dependence is suppressed in the ratio of branching fractions of LFV processes. Relating Eq. (13) to the expression for \(Br(l_{i} \rightarrow l_{j}\gamma)\), we can derive the following simple relation

\[
\left|\frac{R_{s}}{R_{d}}\right| \approx \frac{Br(\tau \rightarrow \mu\gamma)}{Br(\tau \rightarrow e\gamma)}.
\]

Therefore, using the experimentally allowed regions of \(|R_{s}/R_{d}|\), we can predict the ratio of the corresponding LFV processes.

However, recent work reported that the SUSY models with the dominant \(RR\) mixing would be disfavored by the \(\Delta M_{s}\) constraints [8]. In fact, the \(LL\) squark mixing receives renormalization group (RG) effects through the CKM matrix. The evolution from \(M_{s}\) to the weak scale \(M_{W}\) leads to the \(LL\) mixings such as

\[
(\delta_{LL}^{d})_{23} \approx -\frac{1}{8\pi^2}Y_{d_{i}}^{2}V_{el}^{3}m_{0}^{2} + A_{0}^{2}\ln\frac{M_{s}}{M_{W}},
\]

\[
(\delta_{LL}^{d})_{13} \approx -\frac{1}{8\pi^2}Y_{d_{i}}^{2}V_{em}^{3}m_{0}^{2} + A_{0}^{2}\ln\frac{M_{s}}{M_{W}},
\]

where \(Y_{i}\) is the top quark Yukawa coupling, \(m_{0}\) is the typical soft SUSY scale. It is known that the RG evolution from the GUT scale in supergravity scenario induces \((\delta_{LL}^{d})_{23} \approx 0.04 \sim \lambda^{2}\) and \((\delta_{LL}^{d})_{13} \sim \lambda^{3}\). Then, the contribution of the double insertion \((\delta_{LL}^{d})_{23}(\delta_{RR}^{d})_{23}\) in Eq. (11) should not be ignored. In fact, as long as the size of \((\delta_{RR}^{d})_{23}\) is not greater than \(O(\lambda)\), it turns out that the term, \(a_{4}(\delta_{LL}^{d})_{23}(\delta_{RR}^{d})_{23}\), in Eq. (11) can dominate over the term, \(a_{4}[(\delta_{LL}^{d})_{23}]^{2}\), due to \(|a_{4}| \sim 100|a_{1}|\). Keeping only the leading term, \(a_{4}(\delta_{LL}^{d})_{13}(\delta_{RR}^{d})_{33}\), in \(R_{s,d}\), we obtain the following simple relation between the ratio of \(R_{s}/R_{d}\) and the ratio of the branching fractions for LFV processes.

1 But, there exist highly model dependent cases with large mixings in neutrino Dirac Yukawa matrices which may destroy the GUT relation for \(\delta\)’s due to large change from RG evolution [16]. Those cases are not relevant to our work.
decays,
\[
\left| \frac{R_{d}}{R_{s}} \right|^2 \approx \frac{V_{ts}}{V_{td}}^2 \frac{Br(\tau \rightarrow \mu\gamma)}{Br(\tau \rightarrow e\gamma)}, \tag{20}
\]
In the following section, we will perform numerical study in detail. Note that although we keep both contributions proportional to the coefficients \(a_1\) and \(a_4\) in our numerical analysis, the relation Eq. (20) holds reasonably well.

**NUMERICAL ESTIMATES AND DISCUSSIONS**

Let us begin by examining how to extract the allowed regions of \(r_q\) and \(\sigma_q\) based on the SM predictions for \(\Delta M_q^{SM}\). As demonstrated in the introduction, we can estimate the SM predictions for \(\Delta M_q^{SM}\) by using the results of the hadronic parameters from JLQCD collaboration and the (HP+JL)QCD collaboration. In this paper, we only take the estimation by the (HP+JL)QCD collaboration as given by Eqs. (3) and (4), from which we could probe new physics effects in \(B_d\) and \(B_s\) mixing more concretely. As described in Ref. [4], by using Eqs. (6-8) and experimental results for \(\Delta M_q^{exp}\), we can obtain new physics contributions to \(\mathcal{M}_{12}(B_q)\) which is parameterized in terms of \(r_q\) and \(\sigma_q\). First, the values of \(\Delta_q\) are extracted to be
\[
\Delta_d = 0.75 \pm 0.30 \quad \tag{21}
\]
\[
\Delta_s = 0.74 \pm 0.18. \quad \tag{22}
\]

Using these results for \(\Delta_q\) and the relation in Eq. (5), one can obtain the constrained regions of the parameters \(r_{d,s}\) and \(\sigma_{d,s}\), which are presented in Fig. 1. In addition, the parameters \(r_d\) and \(\sigma_d\) can be further constrained through the allowed values of the CP phase of new physics in the \(B_d\) mixing \(\phi_d^{NP}\) which is obtained by the experimental value of the \(B_d\) mixing phase \(\phi_d(=\phi_d^{SM}(=\beta_{SM})+\phi_d^{NP})\) [4]. For \(B_q\) system, in particular, we have used the known constraint on \(\phi_d^{NP}\) given by [4]
\[
\phi_d^{NP}|_{incl} = - (10.1 \pm 4.6)\,^\circ, \quad \tag{23}
\]
which further constrains \(r_d\) and \(\sigma_d\) through the relation of Eq. (10). Note that at present there is no constraint on \(\phi_s^{NP}\).

By using the allowed regions of \(r_{d,s}\) and \(\sigma_{d,s}\) presented in Fig. 1, we can estimate the value of the ratio \(Br(\tau \rightarrow \mu\gamma)/Br(\tau \rightarrow e\gamma)\) through the relation Eq. (20). It appears that the effects of the phases \(\sigma_{d,s}\) vanish away by taking absolute values in Eq. (20), the phase dependence is still imprinted in the absolute value \(|R_q| (= r_q)\), as shown in Fig. 1. Please note that the approximate relation Eq. (20) holds quite well although it keeps only the contributions proportional to the coefficient \(a_4\) in Eq. (12). In Fig. 2, we display the scatter plot of the result for the ratio of branching fractions \(Br(\tau \rightarrow \mu\gamma)/Br(\tau \rightarrow e\gamma)\) vs. the CP phase of new physics \(\phi_s^{NP}\). Note that the values of the CP phase \(\phi_s^{NP}\) are determined in terms of \(r_s\) and \(\sigma_s\) through Eq. (10). As shown in Fig. 2, the value of \(Br(\tau \rightarrow \mu\gamma)/Br(\tau \rightarrow e\gamma)\) decreases as \(\phi_s^{NP}\) approaches zero. We note that the current limits on LFV radiative decays are \(B(\tau \rightarrow e\gamma) < 1.1 \times 10^{-7}\) and \(B(\tau \rightarrow \mu\gamma) < 6.8 \times 10^{-8}\) from BaBar Collaboration [18], and \(B(\tau \rightarrow e\gamma) < 1.2 \times 10^{-7}\) and \(B(\tau \rightarrow \mu\gamma) < 4.5 \times 10^{-8}\) from Belle preliminary report [19]. Therefore, if both LFV radiative decays are observed in near future, we can narrowly constrain the CP phase \(\phi_s^{NP}\), and SUSY GUT, which we now consider, can be ruled out in case that \(Br(\tau \rightarrow \mu\gamma)/Br(\tau \rightarrow e\gamma)\) is determined to be below \(10^{-3}\). Inversely, if we determine the size of \(\phi_s^{NP}\) as well as that of \(r_{s,d}\) narrowly enough, we can predict the value of \(Br(\tau \rightarrow \mu\gamma)/Br(\tau \rightarrow e\gamma)\), and thus our approach can serve as a test of the SUSY GUT.

We also calculate the branching fraction for the LFV decay, \(\tau \rightarrow \mu\gamma\). In Fig. 3, we present the prediction of \(Br(\tau \rightarrow \mu\gamma)\) vs. the CP phase of new physics \(\phi_s^{NP}\).
Eq. (12) leads to independent of the SUSY input parameters. whereas the ratio of the branching fractions is almost \( \frac{Br(\tau \rightarrow \mu \gamma)}{Br(\tau \rightarrow e \gamma)} \) given by Eq. (24) may indicate that the neutrino mixing matrix \( U_{\text{PMNS}} \) parameterized by \( U^\dagger(\lambda)U_{\text{bimax}} \) with CKM-like matrix \( U(\lambda) \) and the bi-maximal mixing matrix \( U_{\text{bimax}} \) is preferred. In summary, motivated by the recent measurements for the \( B_q - \overline{B}_q \) mass difference from the DØ \cite{1} and CDF \cite{2} collaborations, we have probed new physics effects in the \( B_q - \overline{B}_q \) mixing in the context of the supersymmetric grand unified model. We have found that new physics effects in \( B_q(d) - \overline{B}_q(d) \) mixing lead to the correlated information on the branching fractions of the lepton flavor violating decays, which may serve as a test of the SUSY GUT. We have also discussed the implication of such new physics effects on the quark-lepton complementarity in the neutrino mixings.

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