Different contributions of two shear strength parameters to soil slope stability with limit equilibrium method based slice techniques

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Abstract. This paper presents the different contributions of two shear strength parameters to the factor of safety (FOS) value in limit equilibrium method (LEM)-based slice techniques. Many potential slip surfaces are designed and evaluated for their FOS values with respect to the slip depth in a homogeneous soil slope. This paper found that FOS values contributed only by internal cohesion decrease rapidly and inversely as the slip depth increases. However, FOS values contributed only by internal friction angle increase slowly and linearly as the slip depth increases. Because of the different contributions of cohesion and internal friction angle to slope stability, FOS values decrease and then increase with respect to the slip depth, allowing it to have a minimum value at an intermediate depth. This is the mechanical reason for the existence of a critical slip surface with a minimum FOS value in LEM-based slice techniques in slope stability assessment. Further evaluation of an actual slope shows that these findings are also valid to the presence of groundwater and soil inhomogeneity.

Keywords: slope stability, FOS, limit equilibrium method, internal cohesion, internal frictional angle, landslide

1. Introduction

1.1. General

Slope stability assessment is a necessary task in slope engineering and landslide disaster prevention. It provides a quantitative evaluation of the factor of safety (FOS) [1-5]. Many researchers have investigated the FOS of slopes in the last century and developed three approaches for FOS calculation. They include limited equilibrium method (LEM)-based slice techniques [6], finite element method-based shear strength reduction techniques [7], and limit analysis method-based plastic bounding theorems [8]. This paper adopts the widespread LEM-based slice techniques.

Fellenius [6] proposed the first LEM-based slice technique. Subsequently, many researchers developed this technique with the relaxation of some assumptions in the mechanical models of general soil slopes [9-14]. Slice techniques assume that a potential slip surface (or slip) exists in a slope, and then, the soil mass above the slip is divided into a series of vertical thin slices with a small width. The downward gravitational force or other loadings along the slip are restricted or supported by the internal cohesion and frictional resistance of the slope soil. The limit soil resistance to sliding or shearing is typically estimated with the Mohr-Coulomb shear strength criteria. The overall FOS is...
defined as the ratio of the limit resistance to the downward shear stress at the entire slip surface. Therefore, the FOS value is influenced by soil shear strength parameters, which have been studied by many researchers. These studies were mainly focused on the total soil shear strength [16], as well as the combined cohesion and internal friction angle factor [17, 18].

A slope can have numerous potential slips, and each slip has an FOS value. Hence, an important task in slope stability assessment is to find the critical slip with the minimum FOS value among numerous potential slips, which has also been extensively studied. For example, Chen and Shao [19, 20] developed the gradient descent method for automatically searching the critical slip with the minimum FOS value. Zhu [21] constructed an FOS field to determine the critical slip with the minimum FOS value. Li et al. [22, 23] analyzed an actual landslide in a newly dredged slope for port development in the city of Tianjin and located the failure rupture from a serial possible critical slip.

1.2. Objective of this paper

Many studies on the effect of the two shear strength parameters on slope stability can be found in the open literature [16]. However, there may be a scarcity of studies on the individual effect of soil internal cohesion and the internal frictional angle on slope stability. Meanwhile, many studies have focused on searching and obtaining the critical slip [19-23], but there may also be a lack of mechanical reasons for the existence of a minimum FOS value.

This paper aims to study the different contributions of soil cohesion and friction angle to slope stability and then obtain the mechanical reason for the existence of a critical slip with the minimum FOS value in a soil slope based on the different contributions. A slip depth plot technique is used to organize and plot the calculated FOS values of many potential slips by any LEM methods and algorithms in soil slopes. The conventional Morgenstern-Price (M-P) technique in commercial software Slope/W is used for calculating FOS values of potential slips, and the most widely popular Mohr-Coulomb shear strength criteria is employed to calculate the limit shear strength.

2. Different contributions of two shear strength parameters to soil slope stability in homogeneous soil slope

2.1. Soil slope model

The homogeneous soil slope model is shown in Figure 1. The slope height is 7 m, and the slope angle is 60°. The soil unit weight $\gamma$ is 18 kN/m³. The soil cohesion $c = 20$ kPa, and the soil internal friction angle $\phi = 35^\circ$.

![Figure 1](image.png)

**Figure 1.** Typical homogeneous soil slope model and its potential slips at different depths $d$

2.2. Variations and features of FOS$^\phi$ values with slip depth

As shown in Figure 1, a total of 13 potential slips (nos. ① to ⑬) are examined. They are uniformly distributed in the soil slope. The slip depth is the distance from the slope surface to a point on the slip with a tangent parallel to the slope surface. The slip depths of these 13 potential slips ($d_1$ to $d_{13}$) range from 0.60 to 6.07 m. FOS values associated with these slips are calculated and denoted as FOS$^\phi$ for
the case of a general soil slope. They are plotted against the slip depth \( d \), as the hollow circular marks in Figure 2, and have values ranging from 1.689 to 2.957.

Figure 2 shows that FOS\(^{c\phi}\) values can have a lower bound envelope (i.e., the red dash line) and are the lowest around \( d = 2.07 \) m (slip ③). Additional 26 potential slips are further analyzed with their slip depths between \( d_3 \) and \( d_5 \) and expressed, as the cross marks in Figure 2. Along the lower bound envelop, the local minimum FOS\(^{c\phi}\) value decreases inversely proportionally from \( d = 0.60 \) m to 2.19 m and increases linearly as \( d \) exceeds 2.19 m. The lowest minimum FOS\(^{c\phi}\) value is 1.6734 at \( d = 2.19 \) m. It can be the critical slip for this soil slope.

Hence, the plots in Figure 2 explicitly demonstrate the phenomenon that the soil slope has a critical slip with the minimum FOS value since the local minimum FOS\(^{c\phi}\) value decreases and then increases as \( d \) increases.

![Figure 2](image)

**Figure 2.** Variations and features of FOS\(^{c\phi}\) values with slip depth \( d \) for the slope model in Figure 1

### 2.3. Definitions of FOS values contributed by soil cohesion and internal friction angle separately

For a general soil slope, its FOS can be defined as follows according to linearly Mohr-Coulomb shear strength criterion and Terzaghi effective stress principle.

\[
\text{FOS} = \frac{c'}{\tau_n} + \frac{(\sigma_n - u) \tan \phi'}{\tau_n}
\]

where \( c' \) represents the effective internal cohesion, and \( \phi' \) represents the effective internal friction angle. \( c' \) and \( \phi' \) are normally assumed constants for a given slope [3]. \( \sigma_n \) and \( \tau_n \) are the total normal stress and shear stress on the potential slip, respectively; \( u \) represents the water pore pressure.

Equation (1) indicates that the FOS of a soil slope separately depends on \( c' \) and \( \phi' \). Accordingly, this paper defines two partial FOS terms. One is the FOS\(^c\) related only to the internal cohesion \( c \) (or \( c' \)). The other is the FOS\(^\phi\) related only to the internal cohesion \( \varphi \) (or \( \phi' \)). For each potential slip, its FOS\(^c\) value can be calculated using the same slice techniques by assuming \( \phi' = 0 \) and keeping \( c \) as the original value of the soil slope. Similarly, the FOS\(^\phi\) value of that potential slip can be calculated using the same slice techniques by assuming \( c = 0 \) and keeping \( \varphi \) as the original value of the soil slope.

Hence, the individual contributions of the internal cohesion \( c \) and the internal friction angle \( \varphi \) to the FOS\(^{c\phi}\) value can be evaluated separately and explicitly. The variation features of FOS\(^c\) and FOS\(^\phi\) values with respect to the slip depth \( d \) can be examined. The different contributions of two shear strength parameters to soil slope stability can be obtained based on the linearly Mohr-Coulomb shear strength criterion.
2.4. Different contributions of cohesion and internal friction angle to FOS values

2.4.1. Contribution to FOS by only internal cohesion \( c (\phi = 0) \). The soil slope model in Figure 1 is reassessed by assuming \( c = 20 \) kPa and \( \phi = 0 \). The FOS value contributed by \( c \) only is defined as \( \text{FOS}^c \) and shown in Figure 3. The blue dash line in Figure 3 represents the lower \( \text{FOS}^c \) envelop (or local minimum \( \text{FOS}^c \) value curve). As \( d \) increases, the local minimum \( \text{FOS}^c \) value decreases inversely.

2.4.2. Contribution to FOS by only internal frictional angle \( \phi \) (\( c = 0 \)). The soil slope model in Figure 1 is reassessed by assuming \( c = 0 \) kPa and \( \phi = 35^\circ \). The FOS value only contributed by \( \phi \) is defined as \( \text{FOS}^\phi \) and shown in Figure 3. The green dash lines in Figure 3 represent the lower \( \text{FOS}^\phi \) bound envelop (or local minimum \( \text{FOS}^\phi \) value curve). The local minimum \( \text{FOS}^\phi \) value increases as \( d \) increases.

![Figure 3](image-url). Variations and features of \( \text{FOS}^c \) and \( \text{FOS}^\phi \) values with slip depth \( d \) for the slope in Figure 1.

Table 1. Values and ratios of \( \text{FOS}^c \), \( \text{FOS}^\phi \), \( \text{FOS}^{c+\phi} \), and \( \text{FOS}^{c+\phi} \) in Figures 2 and 3 vs slip depth \( d \)

| \( d \) (m) | \( \text{FOS}^c \) | \( \text{FOS}^\phi \) | \( \text{FOS}^{c+\phi} \) | \( \text{FOS}^c \) \( \text{FOS}^\phi \) | \( \text{FOS}^{c+\phi} \) \( \text{FOS}^{c+\phi} \) |
|----------|-----------------|-----------------|-----------------|------------------|------------------|
| 0.60     | 2.332           | 0.466           | 2.887           | 80.80%           | 16.10%           | 96.90%           | 2.798           |
| 0.92     | 1.715           | 0.507           | 2.222           | 77.20%           | 22.80%           | 100.00%          | 2.222           |
| 1.18     | 1.453           | 0.549           | 1.942           | 74.80%           | 28.30%           | 103.10%          | 2.002           |
| 2.00     | 1.112           | 0.688           | 1.754           | 63.40%           | 39.20%           | 102.60%          | 1.800           |
| 2.07     | 1.015           | 0.707           | 1.689           | 60.10%           | 41.90%           | 102.00%          | 1.722           |
| 2.19     | **0.941**       | **0.711**       | **1.673**       | **56.20%**       | **42.50%**       | **98.70%**       | **1.652**       |
| 2.69     | 0.889           | 0.819           | 1.684           | 52.80%           | 48.60%           | 101.40%          | 1.708           |
| 2.95     | 0.943           | 0.895           | 1.756           | 53.70%           | 51.00%           | 104.70%          | 1.838           |
| 4.40     | 0.934           | 1.239           | 2.070           | 45.10%           | 59.90%           | 105.00%          | 2.173           |
| 4.83     | 0.929           | 1.355           | 2.200           | 42.20%           | 61.60%           | 103.80%          | 2.284           |
| 5.84     | 0.950           | 1.633           | 2.513           | 37.80%           | 65.00%           | 102.80%          | 2.583           |
| 7.07     | 0.980           | 2.023           | 2.957           | 33.10%           | 68.40%           | 101.60%          | 3.003           |

2.5. The mechanical reason for the existence of a critical slip surface

The summation of \( \text{FOS}^c \) and \( \text{FOS}^\phi \) values for each potential slip is defined as follows.

\[
\text{FOS}^{c+\phi} = \text{FOS}^c + \text{FOS}^\phi
\]
FOS$^{c \varphi}$ and FOS$^{c+\varphi}$ values can be compared so that the mechanical reason for the existence of a minimum FOS value can be found.

Table 1 shows the local minimum values of FOS$^{c \varphi}$, FOS, FOS', and FOS$^{c+\varphi}$ and their ratios with respect to $d$, ranging from 0.6 to 7.07 m. As $d$ increases from 0.6 m to 7.07 m.

FOS'/FOS$^{c \varphi}$ decreases from 84.13% to 40.63%, and FOS'/FOS$^{c+\varphi}$ increases from 16.10% to 68.40%. Furthermore, FOS$^{c \varphi}$/FOS$^{c+\varphi}$ is between 104.98% and 96.08% and has an average of 101.79%. Hence, the FOS$^{c+\varphi}$ value is almost equal to the FOS$^{c \varphi}$ value at each $d$.

Hence, the finding that the local minimum FOS$^{c \varphi}$ value decreases and then increases as $d$ increases is caused by the different contributions of $c$ and $\varphi$ in terms of the slip depth $d$. At a shallow depth, $c$ has a larger contribution to FOS$^{c \varphi}$, whereas at a deeper depth, $\varphi$ has a larger contribution to FOS$^{c \varphi}$. Such different contributions of $c$ and $\varphi$ to FOS$^{c \varphi}$ is the mechanical reason for the existence of a critical slip with a minimum FOS value in the soil slope.

3. Effect of soil inhomogeneity and water of the contributions to slope stability

3.1. The inhomogeneous soil slope model

The slope in Figure 4 was a bank fill soil slope at Tianshenqiao Hydroelectric Power Project in Guangxi Province, China, and had a landslide on December 04, 1985 [19]. It has three representative slope angles of 43.2°, 39.2°, and 37.2° from the toe to the crest. The slope is 33.76 m in height and has six different soil layers. From top to bottom, the six layers are (1) fresh fill, clay, and debris mixture; (2) old fill, sand, clay, and debris mixture; (3) quaternary talus and clay with rock fragments; (4) quaternary alluvium, fine sand, and medium sand; (5) quaternary alluvium and gray and dark silty clay; (6) quaternary alluvium, gravels, and sands. Their shear strength and unit weight values are listed in Figure 4. The bedrock consists of tertiary rock, shales, sandstones with limestone intercalation, and its unit weight $\gamma$ is 2.4 g/cm$^3$, with soil cohesion $c = 45.0$ kPa, and the soil internal friction angle $\varphi = 39.2^\circ$.

The actual rupture, shown as the blue line in Figure 4, started from the retaining wall buried by the new fill at the top and extended along the approximate ellipsoid surface to the toe. According to Chen and Shao [19], the actual rupture has an FOS of 0.9170, and the most critical slip has a minimum FOS of 0.8631. The critical slip (i.e., the green slip in Figure 4) is located above the actual rupture (i.e., the blue slip in Figure 4).

![Figure 4. Model of actual bank slope (modified after Chen and Shao [19]), and its potential slips through partial soil layers at different depths $d$](image)

3.2. Variations and features of FOS$^{c \varphi}$ values with slip depth
A total of 76 potential slips are designed and analyzed, as shown in Figure 4. Seventeen potential slips are located in the upper soil layers, and the other 65 potential slips pass through all soil layers in Figure 4. Their numerical FOS values are presented in Figures 5 and 6 and Table 2. Each of FOS<sup>c </sup>φ, FOS<sup>c</sup>c, and FOS<sup>φ</sup>c values has a lower bound envelope with respect to the slip depth d.

Figure 5 shows that the local minimum FOS<sup>c</sup>cφ value along the lower bound envelope decreases inversely as d increases from 0 to 8.69 m and then increases slightly as d increases from 8.69 to 13.00 m. The minimum FOS<sup>c</sup>cφ value is 0.786, and its corresponding critical slip is the red slip in Figure 4. The minimum FOS<sup>c</sup>cφ value in this study (0.786) is less than the FOS value (0.863) of the critical slip in Chen and Shao [19]. The difference between critical and actual slips may be caused by the weakness of the interface between soil and rock, as well as the interface between the soil and wall [24].

![Figure 5](image)

**Figure 5.** Variations and features of FOS<sup>c</sup>cφ with slip depth d for the slope in Figure 5

3.3. Different contributions of cohesion and internal friction angle to FOS values

Figure 6 shows that the local minimum FOS<sup>c</sup>c value along the lower bound envelop decreases inversely as d increases. The local minimum FOS<sup>c</sup>c value along the lower bound envelope increases linearly as d increases.

Table 2 further shows the values of FOS<sup>c</sup>c, FOS<sup>φ</sup>c, FOS<sup>c</sup>cφ, and FOS<sup>c</sup>cφ and their ratios with respect to d for the local and overall potential slips in Figs. 5 and 6. In Table 2, the local minimum FOS<sup>c</sup>c/
FOS_{c+\phi} value decreases from 89.43\% to 39.15\%, whereas the local minimum FOS_{\phi} / FOS_{c+\phi} value increases 10.18\% to 64.63\% as d increases from 0.8 m to 12.63 m. Most importantly, the FOS_{c+\phi} / FOS_{c+\phi} value is between 103.78\% and 98.80\% and has an average of 100.24\%.

Hence, under the inhomogeneous soil condition, the different contributions of c and \phi to FOS_{c+\phi} is also the mechanical reason for the existence of a critical slip with a minimum FOS value in slope.

Table 2. Values and ratios of FOS_c, FOS_\phi, FOS_{c+\phi}, and FOS_{c+\phi} in Figures 5 and 6 vs slip depth d

| d (m) | FOS_c | FOS_\phi | FOS_{c+\phi} | FOS_{c+\phi} / FOS_c | FOS_{c+\phi} / FOS_\phi | FOS_{c+\phi} / (FOS_c + FOS_\phi) |
|-------|-------|----------|--------------|------------------------|-------------------------|----------------------------------|
| 0.78  | 4.955 | 0.557    | 5.489        | 90.27\%                | 10.15\%                 | 100.42\%                         |
| 1.31  | 2.026 | 0.548    | 2.570        | 78.83\%                | 21.32\%                 | 100.16\%                         |
| 2.39  | 1.203 | 0.578    | 1.778        | 67.66\%                | 32.51\%                 | 100.17\%                         |
| 3.65  | 0.713 | 0.596    | 1.303        | 54.72\%                | 45.74\%                 | 100.46\%                         |
| 4.79  | 0.594 | 0.391    | 0.997        | 59.58\%                | 39.22\%                 | 98.80\%                          |
| 5.48  | 0.434 | 0.938    | 1.338        | 32.44\%                | 70.10\%                 | 102.54\%                         |
| 6.34  | 0.45  | 0.395    | 0.854        | 52.69\%                | 46.25\%                 | 98.95\%                          |
| 7.27  | 0.439 | 0.491    | 0.916        | 47.93\%                | 53.60\%                 | 101.53\%                         |
| **8.69** | **0.35** | **0.438** | **0.786** | **44.53\%** | **55.73\%** | **100.25\%** | **0.788** |
| 9.49  | 0.356 | 0.451    | 0.801        | 44.44\%                | 56.30\%                 | 100.75\%                         |
| 12.63 | 0.352 | 0.581    | 0.899        | 39.15\%                | 64.63\%                 | 103.78\%                         |

4. Conclusions

This paper has designed, calculated, and evaluated many potential slips for their FOS values with respect to the slip depth in homogeneous or non-homogeneous soil slopes with or without groundwater. This paper has researched the different contributions of two shear strength parameters to FOS values in LEM-based slice assessment with the Mohr-Coulomb shear strength criterion for general soil slope. The different contribution is the mechanical reason for the existence of a critical slip with a minimum FOS value. This paper has defined FOS_{c+\phi}, FOS_c, FOS_\phi, and FOS_{c+\phi} for the examination and evaluation. This paper found that their variations have features and that they are regulated by the slip depth d.

At first, FOS_{c+\phi} is contributed by both soil cohesion and soil internal frictional angle and can be directly calculated from LEM-based slice techniques. The local minimum FOS_{c+\phi} value decreases and then increases as the slip depth increases, demonstrating the existence of a critical slip with a minimum FOS_{c+\phi} value in the soil slope.

Secondly, FOS_c is contributed by soil cohesion only. The local minimum FOS_c value always decreases as the slip depth increases.

Thirdly, FOS_\phi is contributed by internal soil cohesion only. The local minimum FOS_\phi value always increases as the slip depth increases.

Fourthly, FOS_{c+\phi} equals the direct summation of FOS_c and FOS_\phi values for each potential slip. Numerical results show that the FOS_{c+\phi} value is nearly equal to the FOS_{c+\phi} value for each potential slip.

Hence, this paper has demonstrated that the different contributions of the internal cohesion and internal friction angle with respect to the potential slip depth cause the existence of a critical slip with a minimum FOS value. The contribution from the internal cohesion decreases rapidly and inversely, and the contribution from the internal friction angle increases slowly and linearly as the slip depth increases. FOS_c controls FOS_{c+\phi} with potential slips at shallow depth, whereas FOS_\phi controls FOS_{c+\phi} with potential slips at a deep depth. For a potential slip at an intermediate depth, FOS_{c+\phi} becomes the minimum value and the potential slip becomes the critical slip accordingly.
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