Multi-Brane Recombination and Standard Model Flux Vacua

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In a previous work, we noted that vacua with higher flux could be obtained by recombination of a small set of hidden-sector branes, numbering up to the Kähler degrees of freedom left to be fixed in the problem. Here, we discuss the general construction of Type IIB Standard Model flux vacua in which multiple branes participate in brane recombination in the hidden sector. We present several models with 4, 5 and 10 branes in a stack. They are complete with Pati-Salam gauge group, Standard Model chiral matter, and a hidden sector that combines with nonzero flux to satisfy RR tadpole constraints. We also illustrate a puzzle: within the 4-brane recombination approach, we are unable to find a formal cutoff on the number of Standard Model flux vacua. Nevertheless, we show that phenomenological considerations are sure to cut off the number of vacua, albeit at an extraordinarily large value. We comment on the possibility that more formal constraints could further reduce the number of vacua.

Benefits of Flux Vacua. Recently there has been much interest in flux vacua\textsuperscript{1}, particularly in the question of how many flux vacua one can construct which are “phenomenologically viable” – vacua that are consistent with experimental knowledge.

Of course, it is extremely difficult to find any explicit flux vacuum, let alone those that are phenomenologically viable. Instead, an alternative approach is to begin with a class of flux vacua that retain certain phenomenologically important properties, such as the gauge group and chiral matter content of the supersymmetric Standard Model. Each individual flux vacuum in this class will have moduli with different vevs, resulting in different values for the cosmological constant, as well as for other phenomenological parameters such as \( \frac{\Lambda_{\text{QCD}}}{M_{\text{Planck}}} \), \( M_{\text{higgs}} \), etc. With a large enough number of flux vacua, one can use statistics\textsuperscript{2} to estimate the fraction of the flux vacua in this class that have low-energy phenomenological parameters in a range that is phenomenologically viable. Results could lend support to the existence of a stringy Standard Model Embedding. This embedding can be described as

\[
N_a = 8 : (1,0)(3,1)(3,-1)
N_b = 2 : (0,1)(1,0)(0,-1)
N_c = 2 : (0,1)(0,-1)(1,0),
\]

where \( N_i \) is the number of branes in the \( i \)th stack. Notably, these branes are supersymmetric if the volumes of the second and third tori are equal, \( A_2 = A_3 \). The net D-brane charges of these stacks are

\[
\{Q_D, Q_{D^{7/2}}, Q_{D^{7/4}}, Q_{D^{7/8}}\} = \{72, 8, 2, 2\}
\]

where \( D^{7/4} \) refers to a D7-brane filling space-time and wrapping each of the 3 compact tori except the 4th torus.

This brane embedding by itself does not satisfy the RR-tadpole constraints (i.e., the Gauss’ law constraint that space-filling charge must cancel):

\[
\sum Q_{D^3} = 16, \quad \sum Q_{D^{7/4}} = 16
\]

where the contribution on the right-hand-side is the negative charge carried by the 64 O3- and 12 O7-planes. As such, one must add “hidden sector” charges which can cancel these space-filling charges (and K-theory tadpole constraints\textsuperscript{3}), as well as provide phenomenologically desirable hidden sector dynamics. Fortunately, since RR/NSNS 3-form fluxes also induce space-filling D3-brane charge \( L = \frac{1}{2} \int F_{RR} \wedge G_{\text{NSNS}} \), these fluxes can naturally be incorporated in this story. These fluxes contribute to the superpotential\textsuperscript{5}, which fixes the complex structure moduli (there are 51 of them in this particular compactification) and the axio-dilaton. Further non-perturbative corrections can fix the Kähler moduli, leaving us with supersymmetric vacua with no moduli and

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\[ \text{arXiv:h} \text{ep-th/0604203v3 7 Nov 2006} \]
with Standard Model gauge group and matter content in one sector.

Importance of Large Flux. As shown in \[9\], the number of flux vacua (i.e., the number of solutions to the F-term equations generated by the GVW superpotential) will scale as

\[
N_{\text{vac}} \propto \frac{L^{2n+2}}{n!},
\]

where \(n\) is the number of complex structure moduli. This result is valid in the limit \(L \gg n\). If \(L \sim n\), one instead finds the scaling \(N_{\text{vac}} \sim \exp\sqrt{2\pi(2n+2)N_{\text{flux}}}\).

As such, we see that our task of finding classes of flux vacua as large as possible is rephrased as the task of finding compactifications with canceled RR-tadpoles where the amount of flux we have turned on is as large as possible. Since we have fixed a choice of the visible sector, and our orientifold contributes a fixed negative charge to the tadpole constraints, this problem in turn is rephrased as that of finding hidden sector branes which are supersymmetric, yet contribute as negative a D3-brane charge as that of finding hidden sector branes which are supersymmetric. Since we have fixed a choice of the visible sector, and our orientifold contributes a fixed negative charge to the tadpole constraints at all unless the hidden sector branes contribute negatively to the D3-brane tadpole condition. But the more negative this contribution, the more flux we can turn on to cancel it.

In previous works, various examples of hidden sector branes have been found which are supersymmetric and which contribute negatively enough to the tadpole conditions to allow flux to be turned on. In the examples with the largest amount of flux turned on \[11\], one relied on the process of brane-recombination between two D-branes to generate a supersymmetric hidden sector bound state with a more negative D3-brane charge than otherwise would have been allowed. In that earlier work, the starting argument was made that it would be possible to find flux vacua when one or two additional NSNS tadpole constraints were added, since there were two degrees of freedom among the Kähler moduli that could be tuned to accommodate a supersymmetric solution. Brane recombination relaxes that conclusion. Nevertheless, it was suggested that recombination involving more than 2 D-branes would be unlikely to succeed in this toroidal model, due to the difficulty in satisfying all RR-tadpole constraints.

The purpose of this article is to point out that multi-brane recombination, in particular 4-brane combination, can also be consistent with supersymmetric solutions. Brane recombination involving more than two D-branes can allow for even more negative contributions to the hidden sector, thus permitting much larger classes of flux vacua with Standard Model gauge group and matter content. First, we review the basics of brane recombination before illustrating the efficacy of multi-brane recombination in producing phenomenologically viable solutions with high flux.

Brane Recombination Basics. An undeformed D-brane preserves the same supersymmetry as the orientifold if it satisfies the NSNS tadpole constraint \[12\]

\[
\sum_{i=1}^{3} \tan^{-1}(m_i^a A_i; n_i^a) = 0 \mod 2\pi.
\]

But if this condition is not satisfied, supersymmetry is not necessarily broken; instead, the branes may deform, resulting in a supersymmetric bound state. This process is known as brane-recombination. The result is that we may end up with supersymmetric configurations even if we begin with branes that do not satisfy the NSNS tadpole constraints. This gives us a much larger set of stable objects which we may use in the construction of flux vacua.

In field theory language, the non-cancelation of the NSNS-tadpole implies that there are non-zero Fayet-Iliopoulos terms in the \(N = 1\) effective supergravity \[13\]. For each \(U(1)\) factor arising from the gauge theory on a D-brane, there will be a D-term potential of the form:

\[
V_{D_a} = \frac{1}{2g^2} \left( \sum q_i \phi_i^2 + \xi \right)^2
\]

where \(\xi\) is the FI-term and the scalars \(\phi_i\) arise from strings stretching between branes and charged under this \(U(1)\) (as well as another gauge group, generally). We see that even if the FI-terms are non-zero, scalars with appropriate charges can become tachyonic and get vevs. The total D-term potential then vanishes, and supersymmetry is restored. Of course, quantum effects can correct the FI-term and the Kähler metric, but are not expected to change the form of the D-term potential.

The question of whether or not this brane recombination process can occur thus translates into the question of whether or not there exist enough light scalars with appropriate charges for the D-term potential to relax to zero. Non-vectorlike scalars will arise from strings stretching between two different branes (or between one brane and the orientifold image of itself or another brane). The number of scalars will be determined by the topological intersection number \(I_{ab} = \prod_{i=1}^{3} (n_i^a m_i^b - n_i^b m_i^a)\) between the branes \(a\) and \(b\).

The brane \(a\) and its orientifold image \(a'\) can intersect with any other brane \(c\) or its orientifold image \(c'\), resulting in the following non-vectorlike matter content:

\[
\begin{align*}
ac & : \quad I_{ac} \text{ copies of } (\, \square_c, \square_a) \text{ chirals} \\
ac & : \quad (I_{ac}/2 - 2I_{a,c}) \text{ copies of } \text{sym. chirals} \\
ac & : \quad (I_{ac}/2 + 2I_{a,c}) \text{ copies of } \text{anti - sym. chirals} \\
ac & : \quad I_{ac} \text{ copies of } (\, \square_c, \square_a) \text{ chirals}
\end{align*}
\]

where \(I_{a,c}\) is summed intersection number of \(a\) with each orientifold plane.
A Four-Stack Model. We now present an explicit example of brane-recombination involving 4-stacks of D-branes:

\[
\begin{align*}
N_q &= 2 : (-5,1)(-5,1)(-5,1) \\
N_r &= 2 : (1,-1)(1,3)(1,3) \\
N_s &= 2 : (1,3)(1,3)(1,-1) \\
N_t &= 2 : (1,3)(1,-1)(1,3)
\end{align*}
\] (8)

These branes each satisfy the NSNS tadpole constraints at points in Kähler moduli space, but there exists no point where all of them simultaneously satisfy the tadpole constraints. The effective field theory will thus have non-zero FI-terms ($\xi \neq 0$). The non-vectorlike spectrum arising from the open strings is (in notation $(Q_q, Q_r, Q_s, Q_t)$) given by

\[
\begin{align*}
352(2,0,0,0) & \quad 8(0,2,0,0) \\
8(0,0,2,0) & \quad 8(0,0,0,2) \\
1176(-1,-1,0,0) & \quad 1176(-1,0,-1,0) \\
1176(-1,0,0,-1) & \quad 24(0,-1,-1,0) \\
24(0,-1,0,-1) & \quad 24(0,0,-1,-1) \\
1024(1,-1,0,0) & \quad 1024(1,0,-1,0) \\
1024(1,0,0,-1)
\end{align*}
\] (9)

There are many scalars with either sign of charge under all gauge groups; for any choice of signs of the FI-terms there are enough scalars which become tachyonic and can get vevs, causing the full D-term potential to relax to zero.

One should worry that the superpotential Yukawa couplings could generate a non-vanishing $F$-term. However, we see that brane recombination can occur in a very large subclass without generating such $F$-terms. For example, in the very large volume limit we find $\xi_{q,r,s,t} \propto (-1,1,1,1)$. Brane recombination can occur by only giving large vevs to scalars with charges $(1,-1,0,0)$ and $(0,0,-1,-1)$, in which case gauge invariance implies that no Yukawa coupling can yield dangerous $F$-terms, and $V_F$ is parametrically small (note that this is not a unique recombination method).

The total D-brane charges of this bound state are:

\[
\{Q_{D3}, \tilde{Q}_{D7}\} = \{-244, 4, 4, 4\}
\] (10)

This bound state can be part of the hidden sector for the Pati-Salam left-right model discussed earlier. After brane-recombination, it will preserve the same $N = 1$ supersymmetry as the visible sector and the orientifold plane. The large negative contribution means that we can also turn on fluxes as part of the hidden sector. Since the flux contribution to charge is quantized in units of 32 [3], we can add $N_{flux} = 5$ units. The corresponding number of flux vacua for this model is $[10]^{[10]}$

\[
N_{\text{vac}} \sim 10^{25} \times I
\] (11)

where $I$ is the integral of the vacuum density over the complex structure moduli space (this is a fixed number which depends only on the geometric data, not the brane embedding).

Varying Numbers of Stacks. One can easily see that this construction follows through for varying numbers of stacks. For example, here are five stacks of D-branes which may recombine to form a supersymmetric bound state:

\[
\begin{align*}
N_q &= 4 : (-3,1)(-3,1)(-3,1) \\
N_r &= 4 : (1,-1)(1,4)(1,4) \\
N_s &= 4 : (1,4)(1,4)(1,-1) \\
N_t &= 4 : (1,4)(1,-1)(1,4) \\
N_u &= 2 : (-3,2)(-3,2)(-3,2)
\end{align*}
\] (12)

The total D-brane charges of this system are:

\[
\{Q_{D3}, \tilde{Q}_{D7}\} = \{-150, 4, 4, 4\}
\] (13)

which would allow us to include this bound state as part of the hidden sector, along with $N_{flux} = 2$ unit of flux.

We do not go through the details of presenting the spectrum and showing that brane-recombination can occur, as this is merely a specific example of a much more general class. The point is that the generic magnetized D-brane has non-trivial topological intersection with another generic magnetized brane, thus giving us a large number of charged scalars with varying charges. In general this is sufficient for brane recombination to restore supersymmetry.

As another example, we consider a 10-stack brane model:

\[
\begin{align*}
N_q &= 4 : (2,-1)(2,5)(2,5) \\
N_r &= 4 : (2,5)(2,-1)(2,5) \\
N_s &= 4 : (2,5)(2,5)(2,-1) \\
N_t &= 4 : (1,-1)(1,3)(1,3) \\
N_u &= 4 : (1,3)(1,-1)(1,3) \\
N_v &= 4 : (1,3)(1,3)(1,-1) \\
N_w &= 2 : (-3,1)(-3,1)(-3,1) \\
N_x &= 38 : (-2,1)(-2,1)(-2,1) \\
N_y &= 50 : (-1,1)(-1,1)(-1,1) \\
N_z &= 2 : (-4,1)(-4,1)(-4,1)
\end{align*}
\] (14)

In this case, the total D-brane charges are:

\[
\{Q_{D3}, \tilde{Q}_{D7}\} = \{-428, 8, 8, 8\}
\] (15)

and we can include the recombined bound state in the hidden sector, along with $N_{flux} = 11$ units of flux. Indeed the counting arguments described above suggest $N_{\text{vac}} \sim 10^{37} \times I$ vacua, which is the largest number known in any explicit construction.

An Abundance of Flux Vacua. This result shows that we can get large numbers of flux vacua with a supersymmetric Standard Model visible sector by utilizing bound
states formed from the recombination of multiple branes. In fact, we note a puzzle: an arbitrarily large class of flux vacua with Standard Model visible sector arising from constructions in which the hidden sector has a bound state with very large negative D3-brane charge, canceled by large fluxes (related issues in IIA constructions are discussed in [14]; brane recombination allows one to evade the bounds on flux found in [15]).

Consider the following generalization of our 4 brane stack:

\[
N_q = 2 : \frac{1}{(-x - 1)^2, 1(-x - 1)^2, 1(-x - 1)^2, 1)}
N_r = 2 : (1, -1)(1, x)(1, x)
N_s = 2 : (1, x)(1, x)(1, -1)
N_t = 2 : (1, x)(1, -1)(1, x)
\]

(16)

where \( x \) is a large positive integer. These branes will also individually satisfy the NSNS tadpole constraints at points in the moduli space, though not all at the same point (again yielding \( \xi \neq 0 \)). We can similarly verify that there are enough scalars with appropriate charges for tachyons to form and condense, setting the D-term potential to zero. The \( N = 1 \) field theory analysis thus indicates that the bound state formed after tachyon condensation is supersymmetric. However, the net D-brane charge of this bound state will be

\[
\{Q_{D3}, \tilde{Q}_{D7}\} = \{-(x - 1)^6 + 3, 2, 2, 2\}
\]

(17)

Thus we see that as the integer \( x \) is made arbitrarily large, this bound state will contribute an arbitrary negative charge to the D3-brane tadpole conditions, while still not oversaturating the other D7-brane tadpole conditions. This highly negative D3-brane contribution can be compensated by a large amount of flux, yielding a large number of flux vacua.

This suggests the prospect of an arbitrary number of flux vacua with a Standard Model visible sector. If this were true, then one might worry that there were an arbitrary number of phenomenologically viable string vacua, which could be a significant challenge to predictivity.

However, there is already reason to be suspicious of the field theory analysis\(^1\). The D-term potential basically computes the tree-level worldsheet spectrum and verifies that there are enough tachyons to lower all terms in the potential. This amounts to a stability analysis around the line of marginal stability parameterized in moduli space by \( \xi_{FI} = 0 \). One expects that this analysis will be exact in the limit where \( \xi \) is small. However, in the cases we are interested in, the FI-terms can generically be string scale. This corresponds to having tachyons with string scale masses, in which case it is not necessarily consistent to decouple the excited string modes. However, in similar contexts the low energy effective field theory seems to provide qualitatively correct answers even with string scale tachyons [15].

The essential question is whether additional lines of marginal stability appear far in the interior of the moduli space. If this occurs, one expects the above analysis to be correct (treating the FI-terms as adjustable parameters) until one crosses one of the new lines of marginal stability.

The bound state which we are considering has three positive charges and one negative charge; as such, it satisfies the criterion\(^1\) for undeformed branes to be supersymmetric somewhere in moduli space. But the three positive charges are all much smaller than the magnitude of the negative charge. If their charges were instead zero, then our state would have the charge of an \( D3 \)-brane, and would thus break supersymmetry. One thus might suspect that there is indeed a more subtle constraint on the ratio of charges which is being missed by the D-term analysis.

Indeed, we can already see at least some additional lines of marginal stability must appear in the moduli space. A space-filling D3-brane preserves the same supersymmetry as the orientifold planes. In an \( N = 2 \) compactification on the \( T^6/Z_2 \times Z_2 \) orbifold, the central charge of this D3-brane is real and positive. If a brane bound state preserves the same supersymmetry as the orientifold, we expect that its central charge must also be real and positive. The D-term analysis does not guarantee this; the D-term potential would not notice if the bound state had negative central charge, provided the contribution from fluxes and other branes is positive and large enough to cancel all other negative contributions. At large enough volume, however, the positive contribution to the central charge coming from the D7-branes will dominate. Thus, an additional line of marginal stability must at least separate this large volume region from the region in which the central charge of the state become negative.

So although we suspect that many of the bound states found by the field theory analysis are stable, we know that many are not and we are not sure where the cutoff is. But we can at least place a minimal cutoff on the number of bound states of this type by simply checking by hand if the brane bound state in question has negative central charge. The central charge is given to lowest order by

\[
Z = Q_{D3} + Q_{D71}A_2A_3 + Q_{D72}A_3A_4 + Q_{D73}A_1A_2.
\]

where the \( A_i \) are real Kähler moduli (the volumes of the various tori).

For the bound state in question, \( Q_{D71} = 2 \). If \( Q_{D3} \) becomes increasingly negative, \( Z \) can only remain positive if the volume of the tori become large. But the sizes of these tori are bounded by observation. This places a phenomenological bound on how negative \( Q_{D3} \) can become.

The most generic bounds on the size of extra di-

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\(^1\) We thank M. Douglas for discussions on this point.
mensions arise from gravitational couplings, and per-
mitt negative charges which are extremely large (e.g., |
$Q_{D3}$| $\lesssim 10^{65}$). The flux required to offset this large anti-
D3 brane charge implies from eq. 4 an extraordinarily
large number of flux vacua (e.g., $N_{vac} \sim 10^{6172} \times I$).

Note that we are not attempting to point out this par-
ticular model as an example of a string compactification
with a very large number of vacua. In fact, this particular
string construction was chosen chiefly for its simplicity,
and there are several defects relating to the construc-
tion of SM gauge theories which arise in this $T^6/Z_2 \times Z_2$
construction. In particular, in toroidal examples the con-
straints on gauge couplings will place very tight limits on
the size of extra dimensions. Furthermore, there are con-
straints on the cycles which NSNS fluxes may wrap in
order to avoid anomalies on branes wrapping the same
cycles.[6]. For toroidal models in which all branes wrap
only a few cycles, this may create non-smooth distribu-
tions for the moduli which determine gauge couplings.
But these constraints are not likely to be limiting in in-
tersecting brane models on more general orientifolds with
more cycles in play, where bound states of the same na-
ture as those described here can also arise.

One may also question the extent to which this en-
tire discussion of brane stability is valid in a case where
fluxes are turned on. In particular, one may wonder if
the deformations of the geometry induced by backreak-
tion to the fluxes negates the entire picture of branes
wrapping cycles. But there does not seem to be any rea-
son to believe that this is the case. The global topology
of the orientifolded Calabi-Yau compactification is undis-
turbed by the addition of fluxes (once the RR-tadpoles
are canceled). If global tadpoles are canceled and local
flux energy densities are low (which will be the case in
the large volume limit of interest), there is no apparent
reason for a brane to be unable to wrap a cycle; if field
theory is appropriate, there is no reason why a super-
symmetric field theory configuration (in the presence of
flux energy densities) should fail to be stable. Further-
more, it is amusing to note that we can always choose to
replace our flux entirely by pure D3-branes. In such a
case, of course, we no longer worry about fixing complex
structure moduli. But we will see this very large number
of field theoretic supersymmetric bound states (limited
only by cutoffs on the size of the extra dimensions) with
no complications arising from fluxes.

As we allude to in the conclusions below, the extraor-
dinarily large numbers at play here strain our sensibili-
ties for the amount of charge a stable state can carry.
Nevertheless, we have not yet encountered nor
have been able to devise a rigorous argument that for-
bids these states.

Conclusions. All of the flux vacua solutions we have
discussed have Pati-Salam gauge group and SM chiral
matter content. We have shown that brane recombi-
nation allows us to obtain many more non-zero flux solu-
tions consistent with supersymmetry. The various brane
recombination pathways occurring in the hidden sector
have no direct negative impact on the existence of a visi-
ble sector that contains the right gauge groups and chiral
matter content to be phenomenologically viable.

We have also seen that multi-brane recombination can
lead to an extraordinarily large number of solutions. It is
not clear that these solutions will survive a deeper level
of scrutiny, as such large negative $Q_{D3}$ brane charge and
finite $Q_{D7}$, charges (and more generally additional lines
of marginal stability) could lead to instabilities not cap-
tured by the basic analysis we have performed here.

We emphasize that the considerations behind these
constructions are not expected to be unique to $T^6/Z_2 \times
Z_2$. Bound states of this form may be expected to arise
in generic constructions. As such, it is quite important
to understand what, if any, other formal theoretical con-
straints may place further limits on how negative the
bound state charges can be. One source of constraints
could be a generalized version of II-stability[17]. This
type of analysis is expected to be valid throughout the
moduli space, and thus even in regimes where the FI-
terms are large. Unfortunately, the technology required
to apply this generalization to our framework is not yet
fully known. In particular, it is not known how to formu-
late II-stability in the $N = 1$ framework of orientifolds,
nor in the case where fluxes are turned on. Nevertheless,
it seems a promising avenue for the study of this inter-
esting puzzle. Putting aside the subtleties involving both
orientifolds and fluxes, we have attempted to use the cur-
cently known II-stability framework to determine if the
kind of bound states which we have discussed here are un-
stable to decay into known decay products. Using some
specific examples, our preliminary analysis shows no in-
stability in the bound states which appear stable from the
field theory point of view. Of course, this analysis is far
from definitive, not only because of the aforementioned
subtleties involving fluxes and orientifolds, but also be-
cause we can only check for stability against decay into
a particular set of decay products, namely, the simple
branes which form the bound state. A complete stability
analysis would require one to know all stable objects at
a particular point in moduli space, and then use the for-
malism of II-stability to determine all stable objects at
every other point in moduli space, so that one could be
sure that the putative bound state could not decay into
any possible set of decay products. It is not yet known
how to conduct such an extensive stability analysis, even
in $N = 2$ cases.

The question “how many string vacua can describe the
real world to within experimental precision?” is very phe-
nomenological in nature. It is fascinating to note that
this question might only be accessed by the most formal
aspects of string theory.

Acknowledgments. We gratefully acknowledge A.
Bergman, J. Distler, S. Kachru, R. Reinbacher, E.
Sharpe, G. Shiu, W. Taylor and U. Varadarajan for useful discussions. We are especially grateful to M. Douglas for our many discussions on these and related topics. This work is supported in part by NSF grant PHY-0314712, the Department of Energy, and the Michigan Center for Theoretical Physics (MCTP).

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