Experimental study on phase conjecture based on two-dimensional image and carrying three-dimensional image with mist in electro-holographic reconstruction

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Received: 10 November 2021 / Accepted: 4 July 2022 / Published online: 25 July 2022
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Abstract
A method is proposed for phase conjecture based on the intensity curve of a two-dimensional (2D) image by computing a polynomial equation. The intensity values of the 2D image, which represents the distance between the image detectors and the three-dimensional (3D) scene is converted to phase information by our method. The results of numerical calculation with phase conjecture are analyzed. And what’s more, the numerical reconstruction results with phase information obtained as initial phase factors of a complex object for Fresnel kinoform and dynamic pseudorandom-phase tomographic computer holography (DPP-TCH) are compared. The peak signal-to-noise ratio (PSNR) and correlation coefficient (CC) between the reconstructed images and original object are analyzed. An experimental system is designed for photoelectric holographic reconstruction based on phase-only liquid crystal spatial light modulator (LC-SLM) and mist screen. The electro-optical experimental results indicate that suppressed the speckle noise 3D images that can be observed with naked eye have been obtained.

Keywords Holographic 3D display · Computer holography · Speckle noise · 3D imaging

Holography is an especially excellent form of display, because unlike other imaging technologies, it stores 3D information of an object, including amplitude and phase (Malek et al. 2014). Computer-generated holography is widely used in holography displays, and hence, computer holography and digital holography have been investigated extensively (Nagashima 2003). Computer generated holography can enable not only generation of holograms of photographs conveniently and efficiently with different coding methods, but also fabricating holograms of objects that do not exist or virtual objects through different object function descriptions. In addition, no optical setups are required for computer holography, thus eliminating optical lens error and environmental influences. Therefore,
computer-generated holograms (CGHs) have low noise and higher repeatability. Owing to these advantages, CGHs can simulate spatial filters and transform components and can be satisfactorily applied to holographic 3D virtual displays (Frère and Leseberg 1989; Schw erdtner et al. 2008, 2010). Either the amplitude and phase information, or just the phase information on the hologram plane is loaded on a SLM to reconstruct the 3D diffusive objects.

Image detectors can only gather the intensity information of a light field and cannot directly collect phase information, which contributes to ~75% of the information regarding the objects (Mir et al. 2012), because the oscillation frequency of the light field is relatively high. In studies on phase retrieval (PR), which employ the diffraction of the light field as the model of numerical simulation, the intensity distribution of the emergent plane is obtained by computing the diffraction process of the assumed incident plane, meanwhile, acquiring the final phase distribution of the intensity information that best matches the real scene by comparing the intensity data produced by the real phase with the intensity of the light field emergent plane is research hotspot. R. W. Gerchberg and W. O. Saxton first proposed the concept phase retrieval to solve the problem of electron microscopy imaging (Gerchberg and Saxton 1971; Gerchberg 1972). The result of this algorithm is not very accurate. In order to solve the above problems, many new algorithms based on the Gerchberg–Saxton (GS) PR iteration algorithm were proposed. For example, Xiang (Xiang and Ki-Shu 2000) developed an algorithm using the weighted phase as initial phase data; this algorithm has better accuracy, but there is a trade-off with the rate of convergence. Huang et al. (2010) proposed a method to solve this problem. The PR method which is proposed by Guo Yiming (Guo et al. 2016) is a fast, high precision and method of effective phase retrieval.

The CGHs of 3D objects can be computed by numerous techniques, such as the tomographic approach (Zeng et al. 2015; Wang et al. 2016), Fresnel zone approach (Zhang et al. 2005), and multi-view image synthesizing approach (Abookasis and Rosen 2003). The tomographic method often divides the 3D objects into a series of planar segments. An initial random phase is employed as the phase distribution of the complex amplitude for this effectively allotting the information content over the entire region occupied by the hologram plane.

In this paper, we propose a new method for phase conjecture based on the intensity information of 2D images from 3D scenes by fitting a polynomial, this polynomial is computed by a simple and fast procedure that is already available. The method is based on the gray values along line profiles and phase information profile hidden in real 2D pictures. The phase data obtained based on the 2D images by our method form the phase part of the complex amplitude of the target. Meanwhile, the gray values of the 2D images indicate the amplitude. Thus, complex amplitude is obtained. Instead of Burch encoding, which has low diffraction efficiency (Zhao et al. 2017), kinoform encoding is employed to generate the CGH. Kinoform encoding can adequately eradicate the conjugated images (Zhao et al. 2017), which may severely affect the quality of reconstructed images. The results of numerical calculation and photoelectric via the above mentioned method are analyzed.

Grayscale images whose gray value represent the profiles of phase and gray gradient change turn into phase variation are gleaned from Optoelectronic components CCD (or CMOS) and 3D scene are placed horizontally. 2D intensity image is collected from 3D object which is placed vertically in front of images detectors, light energy distribution function of scene is calculated according to the light energy in CCD. Gray values of every pixel representative the distance between 3D object and CCD. The phase information is calculated by fitting the gray curve obtained by transforming the different
positions of two perpendicular line in 2D intensity image. The continuous phase is quantized to 0 and $2\pi$ according to the gray gradient values. The phase conjecture procedure is vividly illustrated by Fig. 1. The procedure can be written as:

$$
x_h = a_0 + a_1x + a_2x^2
$$
$$
y_v = b_0 + b_1y + b_2y^2
$$

(1)

as the two curves fitted along the horizontal and vertical grayscale images, respectively. where $x_h$ and $y_v$ is horizontal and vertical the phase information. The phase distribution becomes ultimately:

$$
\varphi(m,n) = \exp \left[ i(a_1m + b_1n + a_2^2m^2 + b_2^2n^2) \right]
$$

(2)

where $m$ and $n$ define the sampling number.

The core process of GS algorithm for phase retrieval is to obtain the phase distribution of hologram plane by iterative operations of multiple Fourier transforms and inverse Fourier transformations (Goodman 2005). The basic process of GS algorithm for phase retrieval is shown in Fig. 2.

In the Fig. 2, $\sqrt{I_i}$ and $\sqrt{I_o}$ is the amplitude of input plane and output plane, respectively. $\varphi$ is random phase, or estimated value to dwindle iteration error of initial phase. When normalized mean-square error is lessen than set value. $MSE$ can be written as (Zeng et al. 2015):

![Fig. 1 Flow of phase conjecture](image-url)
where $M$ and $N$ respectively as the total numbers of sampling unit in the direction of $x$ and $y$.

Figure 3 shows geometry system used to compute the CGH of a series of planar segments. The Fresnel diffraction of each segment was calculated by using the single-step Fresnel (SSF) diffraction method—Fourier transform algorithm (Goodman 2005). The contribution of object plane to the hologram plane can be expressed as:

$$MSE = \frac{1}{MN} \sum_{m=1}^{M} \sum_{n=N/2+1}^{N} \left( |f(m, n)|^2 - |f'(m, n)|^2 \right)^2$$  \hspace{1cm} (3)$$

where $M$ and $N$ respectively as the total numbers of sampling unit in the direction of $x$ and $y$.

Figure 3 shows geometry system used to compute the CGH of a series of planar segments. The Fresnel diffraction of each segment was calculated by using the single-step Fresnel (SSF) diffraction method—Fourier transform algorithm (Goodman 2005). The contribution of object plane to the hologram plane can be expressed as:

$$H_i(\epsilon, \eta) = \frac{\exp(\frac{2\pi d_i}{\lambda})}{j \lambda d_i} \exp \left[ \frac{j \pi (\epsilon^2 + \eta^2)}{\lambda d_i} \right]$$

$$\times \iint \left\{ O_i(x_i, y_i) \exp [j \phi_i(x_i, y_i)] \times \exp \left[ j \phi_i(x_i, y_i) \right] \exp \left[ \frac{j \pi (x_i^2 + y_i^2)}{\lambda d_i} \right] \right\} \exp \left( -j \frac{2\pi x_i \epsilon + y_i \eta}{\lambda d_i} \right) dx dy$$  \hspace{1cm} (4)$$

where $(\epsilon, \eta)$ indicates holographic coordinate, $(x_i, y_i)$ represent the coordinate of the $i_{th}$ segment, $\phi_i(x_i, y_i)$ which used to reduce the dynamic range of the object wave spectrum provides the random phase information. The optical distribution from all segments are superimposed on the hologram plane as:

$$H(\epsilon, \eta) = \sum_{i=1}^{n} H_i(\epsilon, \eta) = A(\epsilon, \eta) \exp [j \phi(\epsilon, \eta)]$$  \hspace{1cm} (5)$$

where $A(\epsilon, \eta)$ provides the amplitude of the hologram, $\phi(\epsilon, \eta)$ is the diffusive phase.
Complex amplitude distribution of the 3D scene on the hologram plane can be obtained by tomographic approach mentioned above. In the process of reconstruction, hologram obtained through above method is multiplied by the conjugate reference beam, and then the hologram perform inverse diffract by the Fresnel diffraction formula, at last the reconstructed image at different distances can be obtained.

The phase conjecture process is illustrated by simulated experiments below. A 2D intensity image extracted from 3D sphere is showed as Fig. 4a. Its radius is 0.158 \( \text{um} \). Two perpendicular lines is defined in Fig. 4a, one for each direction \( m \) and \( n \), which are, respectively, the horizontal and vertical directions. Along each of two lines, two gray values lines appear in Fig. 4b and c, and according to Eq. (1), the phase distribution can be obtained easily, as shown in Fig. 4d.

The experimental results show that partial phase distribution of 3D objects can be obtained from 2D images. It is noting that phase conjecture method is applied to conjecture the phase distribution of projection part, while the phase of other perspective 2D images is not valid.

According to the Fig. 2, the result of phase retrieval is shown in Fig. 5a. When the number of iteration reaches 100, the MSE variation according to iterate times is shown in Fig. 5b.

According to the results of the two simulated experiments, it is not difficult to conclude that the phase distribution by proposed method is more close to the actual phase than the result of GS algorithm, which require multiple iterative computation. Difference between the retrieved result and the given phase by different method are shown in Fig. 6a and b respectively. MSE values of proposed method is \( 9.327 \times 10^{-7} \), which is much less than \( 1.5822 \times 10^{-6} \) of GS requires several operations. Not only the complexity the calculation

![Fig. 4 The phase conjecture. a 2D intensity image; b Intensity cure by horizontal line in (a); c Intensity cure by vertical line in (a); d Phase distribution by fitting polynomial equation](image-url)
but also the time required for proposed method are less than that needed for GS algorithm according the Fig. 5b.

Figure 4a as the amplitude and Fig. 4d as the phase of original complex amplitude distribution respectively. So complex amplitude distribution in hologram plane can be obtained by simulating the optical transmission process, and the Fresnel kinoform contributed by the wavefront is shown in Fig. 7a. The intensity and phase distribution of reconstructed images at 350 mm are shown in Fig. 7b and c.

A 3D object composed of two planar segments whose distance is 50 mm is demonstrated DPP-TCH method. The original 3D object and kinoform are shown in Fig. 8. The distance between the letter K and hologram is 400 mm, and the distance between
the letter F and hologram is 350 mm. Superposed results of 200 reconstructed images at 400 and 350 mm are shown in Fig. 8c and d respectively.

As shown in Fig. 7c, reconstructed phase distribution which contains 3D topography information of 3D objects is display, apparently, the reconstructed phase is not only close to initial phase but also its radian is same to initial value. However, DPP-TCH as a kind of technology for displaying 3D objects, it is worth that not all reconstructed image planes are focus, when focusing
on the letter K, the letter F would blur, and vice versa. In order to evaluate the qualities of reconstructed images, CC and PSNR between reconstructed images and original images are utilized. CC and PSNR (Zeng et al. 2015) are defined respectively as follows:

$$CC = \frac{\text{Cov}(I_1, I_2)}{\sqrt{\text{Var}(I_1) \cdot \text{Var}(I_2)}}$$  \hspace{1cm} (7)$$

$$PSNR = 10 \log \left( \frac{\sum_{i=1}^{M} \sum_{j=1}^{N} [I_1(i,j)]^2}{\sum_{i=1}^{M} \sum_{j=1}^{N} [I_1(i,j) - I_2(i,j)]^2} \right)$$  \hspace{1cm} (8)$$

where Cov($I_1, I_2$) indicates covariance operation between $I_1$ and $I_2$, Var($I_1$) is the variance values of $I_1(i,j)$, $I_1(i,j)$ and $I_2(i,j)$ are original and reconstructed image (including phase information) correspondingly.

CC and PSNR curves of the part-images (K) in the reconstructed image at 400 mm from different number of kinoform are shown in Fig. 9a and b.

The CC and PSNR values of proposed method and DPP-TCH are shown in following table. According to Table 1, through comparison with the 200 superposed reconstructed part-image at 400 mm, values of CC and PSNR increase to 0.9866 and 82.8665, respectively. And the elapsed time of DPP-TCH is 482 seconds = $2.41 \times 200$, however the reconstructed image including real phase information could be obtained by single diffraction of which elapsed time is only 3.27 s. The simulated experiment results show that reconstructed phase distribution encompass actual 3D topography information is acquired and speckle noise can be almost removed.

Experimental system for photoelectric holographic reconstruction based on LC-SLM is shown in Fig. 10. The helium–neon laser with wavelength 632.8 nm and power 0.8 mW; The SLM is holoeye Pluto with 1920 × 1080 pixels and 8 – μm pixel pitch; The size

![Table 1](image1)

![Fig. 9](image2)
of mist screen generated from mist generated with frequency 50 Hz and power 25 W is $34 \times 6 \times 120$ mm. The mist screen is shown in Fig. 11.

The incident illumination light which is turn into polarized and parallel light by polarizer and beam expander, respectively, is phase modulated by kinoform loading on the SLM. Figure 12 is the electro-optical reconstructed images which is captured by CCD at 300 mm. It is obvious that the electro-optical reconstruction image carried with cubic mist screen is display in air.

A novel method is proposed for phase conjecture. In this method, the 2D projected image is used to calculate the values of involved parameters by using the phase distribution as the initial phase factors in the object domain. For this purpose, a fitting curve extracted along the profiles defined by two perpendicular lines is used. Unlike the traditional phase retrieval method, the proposed phase conjecture method is simple and efficient. It has been validated as well. No complicated diffraction optical path or iterative operations are involve in this method. The digital experiment results show that the phase distribution of a 3D object can be obtained easily. The results of numerical reconstruction achieved by using phase information as the initial phase factors of a
complex object for Fresnel kinoform and DPP-TCH are compared. The values of CC and PSNR achieved by the proposed method are higher than those for DPP-TCH, the time required for the proposed method is significantly less than that needed for DPP-TCH. Finally, the electro-optical reconstruction generated by the 3D electro-optical holographic display system was studied. However, owing to imperfections attributed to airflow, wind, and inhomogeneous distribution of mist, and because the peak height of the reconstructed form is only 0.158 $\mu m$, the depth cues cannot be detected easily. Future work will aim to improve the 3D reconstructed images.

Funding This work has been supported by the Science and Technology Research Program of Chongqing Municipal Education Commission (Grant No. KJQN202103703), by the Science and Technology Research Program of Chongqing Municipal Education Commission (Grant No. KJZD-K201903701).

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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