Electromagnetic and Axial-Vector Form Factors of the Quarks and Nucleon

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Abstract

In light of the improved precision of the experimental measurements and enormous theoretical progress, the nucleon form factors have been evaluated with an aim to understand how the static properties and dynamical behavior of nucleons emerge from the theory of strong interactions between quarks. We have analysed the vector and axial-vector nucleon form factors ($G_{E,M}^{p,n}(Q^2)$ and $G_A^{p,n}(Q^2)$) using the spin observables in the chiral constituent quark model ($\chi$CQM) which has made a significant contribution to the unraveling of the internal structure of the nucleon in the nonperturbative regime. We have also presented a comprehensive analysis of the flavor decomposition of the form factors ($G_E^q(Q^2)$, $G_M^q(Q^2)$ and $G_A^q(Q^2)$ for $q = u, d, s$) within the framework of $\chi$CQM with emphasis on the extraction of the strangeness form factors which are fundamental to determine the spin structure and test the chiral symmetry breaking effects in the nucleon. The $Q^2$ dependence of the vector and axial-vector form factors of the nucleon has been studied using the conventional dipole form of parametrization. The results are in agreement with the available experimental data.
I. INTRODUCTION

The flavor and spin structure of the nucleon play an essential role in understanding the dynamics of the theory of the strong interaction quantum chromodynamics (QCD). Even after extensive studies, confinement has limited our knowledge and the understanding of hadron internal structure continues to remain a major unresolved problem in high energy spin physics. Ever since the deep inelastic scattering (DIS) experiments discovered the composite nature of the proton, several interesting studies have been carried out experimentally and theoretically to understand the constituents of the nucleon [1, 2]. Even though the DIS with polarized beams and/or targets probe the spin carried by the quarks in the nucleon [3–12], the fundamental question of very small spin (only about 30%) carried by the constituent quarks still remains to be the subject of much controversy. The spin contribution of the strange quarks in the nucleon is a nontrivial aspect and is of intense theoretical interest because of model assumptions and experimental limitations. Further, the results revealed in the famous DIS experiments by the New Muon Collaboration (NMC) [13, 14], Fermilab E866 [15–17], Drell-Yan cross section ratios of the NA51 experiments [18] and HERMES [19] have revealed the presence of sea quarks indicating more subtle dynamics which should be nonperturbative in nature.

The electromagnetic form factors data has been obtained from the cross sections data using the Rosenbluth separation method [20–28], the double polarization experiments by the Jefferson Lab (JLab) [29–32] as well as from the Continuous Electron Beam Accelerator Facility (CEBAF) at JLab [33–37]. Recent experiments at JLab have increased the $Q^2$ range of the form factors [38–40] and have triggered much activity in the determination of the quark flavor contributions to the form factors of the nucleon. Further, during the last few years, the standard electroweak theory has provided a firm basis for the role of weak interaction as a precision probe of the nucleon structure. The study of the electromagnetic structure of the nucleon involves the vector electric and magnetic form factors $G_{E}^{N}(Q^2)$ and $G_{M}^{N}(Q^2)$ whereas the neutral weak interaction between leptons and nucleons involves vector weak form factors $G_{E}^{Z,N}(Q^2)$ and $G_{M}^{Z,N}(Q^2)$ as well as axial form factor $G_{A}^{Z,N}(Q^2)$.

The contribution of strange quarks to the nucleon structure is of special interest because it provides an ideal probe for the virtual sea quarks present in the nucleon. The strange spin polarization $\Delta s$ has received much attention in the past as it corresponds to the value of the
strange axial form factor $G_A^s$ at zero-momentum transfer ($Q^2 = 0$). Over the last decade, several experiments have been proposed to probe the electromagnetic and the weak structure of the nucleon. There is a large effort to look for contribution of the sea quarks in the vector form factors via the parity-violating (PV) electron scattering providing information on the weak structure of the nucleon and their associated quark structure. PV electron scattering measurements which are sensitive to the strange quark contributions but not to the axial-vector form factor have been carried out by various collaborations [41–51]. The DIS of neutrinos or of polarized charged leptons from nucleon and nuclear targets have been used to measure the electromagnetic and axial form factors of the nucleon in the elastic $\nu p$ and $\bar{\nu} p$ scattering from the BNL E734 experiment [52], Fermilab Intense Neutrino Scattering Scintillator Experiment (FINeSSE) [53] at Fermi National Accelerator Laboratory (Fermilab). A determination of the strange form factors through a combined analysis of elastic $\nu p$ and $\bar{\nu} p$ and PV electron scattering is performed in Ref. [54].

Even though many experimental and theoretical efforts have been made to understand the internal structure of the hadrons and origin of the sea quarks [55–87], the effective interaction Lagrangian approach of the strong interactions used in the chiral constituent quark model ($\chi$CQM) [88–97] successfully explains the spin structure of the nucleon [98–108], magnetic moments of octet and decuplet baryons [109, 110], semileptonic weak decay parameters [111, 112], magnetic moments of nucleon resonances and $\Lambda$ resonances [113, 114], quadrupole moment and charge radii of octet baryons [115, 116], etc.. The inclusion of $Q^2$ dependence in the vector and axial-vector form factors can be done through the dipole form of parametrization as

$$G_{V,A}^{p,n}(Q^2) = \frac{g_{V,A}^{p,n}(0)}{(1 + \frac{Q^2}{M_{V,A}^2})^2}, \quad (1)$$

where $M_V$ and $M_A$ are the canonical vector and axial-vector masses respectively. The factors $g_{V}^{p,n}(0)$ are the charge, magnetic moment of the proton and neutron at zero momentum transfer whereas $g_{A}^{p,n}(0)$ is the isovector axial-vector coupling constant of the proton and neutron at zero momentum transfer. In view of the above developments, it becomes desirable to extend the applicability of $\chi$CQM by incorporating $Q^2$ dependence phenomenologically in the proton and neutron form factors whose knowledge would undoubtedly provide vital clues to the nonperturbative aspects of QCD.
II. DIRAC AND PAULI NUCLEON FORM FACTORS

The most general form for the hadronic matrix element of the electromagnetic current operators for a spin $-\frac{1}{2}$ nucleon with internal structure, in terms of the Dirac and Pauli form factors $F_{1,\gamma}^N$ and $F_{2,\gamma}^N (N = p, n)$, satisfying relativistic invariance and current conservation, is expressed as

$$J_{\mu}^{EM} = e \bar{U}(p) \left[ \gamma^\mu F_{1,\gamma}^N(Q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M} F_{2,\gamma}^N(Q^2) \right] U(p),$$

where $M$ is the nucleon mass, $Q^2 = q^2$ is the negative of the square of the invariant mass of the virtual photon in the one-photon exchange approximation in $ep$ scattering. In the static limit $Q^2 = 0$, the form factors give the charge and anomalous magnetic moment as $F_{1,\gamma}^p(0) = 1$, $F_{2,\gamma}^p(0) = \kappa^p$, $F_{1,\gamma}^n(0) = 0$, $F_{2,\gamma}^n(0) = \kappa^n$, of the proton and neutron respectively. The anomalous magnetic moment and the magnetic moment of the proton and neutron are related as $\kappa^p = \mu^p - 1$ and $\kappa^n = \mu^n$ respectively.

The Dirac and Pauli form factors are related to the electric and magnetic Sachs form factors as

$$G_{E,\gamma}^N(Q^2) = F_{1,\gamma}^N(Q^2) - \tau F_{2,\gamma}^N(Q^2),$$

$$G_{M,\gamma}^N(Q^2) = F_{1,\gamma}^N(Q^2) + F_{2,\gamma}^N(Q^2),$$

where $\tau = \frac{Q^2}{4M^2}$. In the static limit $Q^2 = 0$, the electric and magnetic form factors give the charge and magnetic moments of the proton and neutron, respectively as $G_{E,\gamma}^p(0) = 1$, $G_{M,\gamma}^p(0) = \mu^p$, $G_{E,\gamma}^n(0) = 0$, $G_{M,\gamma}^n(0) = \mu^n$.

The quark flavor structure of these form factors can be revealed from the matrix elements of individual quark currents in terms of form factors $F_q^1$ and $F_q^2 (j = u, d, \text{or} s)$

$$J_{\mu}^q = e \bar{U}(p) \left[ \gamma^\mu F_q^1(Q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M} F_q^2(Q^2) \right] U(p).$$

Because of the point-like interaction between electrons and the quark constituents of the nucleon, these nucleon form factors can be expressed in terms of the individual quark flavor contributions with the electric charge of individual quarks as the coupling constants. These quark flavor contributions to the form factors are then global properties of the nucleons. Using the definitions analogous to Eq. (3), we can write

$$G_{E,M}^{p,\gamma}(Q^2) = \frac{2}{3} G_{E,M}^u(Q^2) - \frac{1}{3} G_{E,M}^d(Q^2) - \frac{1}{3} G_{E,M}^s(Q^2),$$

$$G_{E,M}^{n,\gamma}(Q^2) = \frac{2}{3} G_{E,M}^d(Q^2) - \frac{1}{3} G_{E,M}^u(Q^2) - \frac{1}{3} G_{E,M}^s(Q^2).$$

(5)
Here we have assumed charge symmetry for the rotation transformation of \( \frac{\pi}{2} \) in the isospin space between \( p \leftrightarrow n \). The strange form factors in each nucleon are also taken to be the same. This can be used to calculate the flavor decomposition of the form factors by using the well known electromagnetic form factors of the proton and neutron at low \( Q^2 \). The calculation of the strangeness form factors \( G_{E}^{s}(Q^2) \) and \( G_{M}^{s}(Q^2) \) however requires the information from the neutral weak current.

The hadronic matrix element of the neutral weak current operators for a spin\(-\frac{1}{2}\) nucleon can be expressed in terms of the vector form factors \( F_{1}^{N,Z} \) and \( F_{2}^{N,Z} \) as well as the axial form factor \( G_{A}^{N,Z} \) as

\[
J_{\mu}^{NC} = eU(p) \left[ \gamma_{\mu} F_{1}^{N,Z}(Q^2) + \frac{i\sigma_{\mu\nu}q_{\nu}}{2M} F_{2}^{N,Z}(Q^2) + \gamma_{\mu} \gamma_{5} G_{A}^{N,Z}(Q^2) \right] U(p). \tag{6}
\]

The nucleon form factors in terms of the quark flavor contributions to the form factors with the weak electric charge \( e^{Z} \) of individual quarks (\( e^{Z} = \left(1 - \frac{8}{3}\sin^{2}\theta_{W}\right) \) for \( u \) and \( e^{Z} = \left(-1 + \frac{4}{3}\sin^{2}\theta_{W}\right) \) for \( d \) and \( s \) quarks) as the coupling constants and the weak mixing angle \( \theta_{W} \) can be expressed as

\[
G_{E,M}^{p,Z}(Q^2) = \left(1 - \frac{8}{3}\sin^{2}\theta_{W}\right) G_{E,M}^{u}(Q^2) + \left(-1 + \frac{4}{3}\sin^{2}\theta_{W}\right) \left(G_{E,M}^{d}(Q^2) + G_{E,M}^{s}(Q^2)\right), \tag{7}
\]

\[
G_{E,M}^{n,Z}(Q^2) = \left(1 - \frac{8}{3}\sin^{2}\theta_{W}\right) G_{E,M}^{d}(Q^2) + \left(-1 + \frac{4}{3}\sin^{2}\theta_{W}\right) \left(G_{E,M}^{u}(Q^2) + G_{E,M}^{s}(Q^2)\right). \tag{8}
\]

Utilizing the isospin symmetry at leading order as well as the proton and neutron electromagnetic form factors, the up and down quark contributions to the neutral weak form factors can be eliminated to obtain the contribution of strange quarks as \[117\]

\[
G_{E,M}^{s}(Q^2) = \left(1 - 4\sin^{2}\theta_{W}\right) G_{E,M}^{p,\gamma}(Q^2) - G_{E,M}^{n,\gamma}(Q^2) - G_{E,M}^{p,Z}(Q^2). \tag{9}
\]

This clearly shows how the contribution from the strange form factor is related to the electromagnetic form factors as well as the neutral weak form factors. Therefore, the measurement of the neutral weak form factor, in combination with the electromagnetic form factors, will allow the determination of the strange form factor.
III. NUCLEON FORM FACTORS IN THE CHIRAL CONSTITUENT QUARK MODEL ($\chi$CQM)

Since the presence of sea quarks is a nonperturbative in nature, the $\chi$CQM uses the effective interaction Lagrangian approach where the chiral symmetry breaking takes place at a distance scale much smaller than the confinement scale. The dynamics of light quarks ($u, d, s$) and gluons is described by the QCD Lagrangian as

$$\mathcal{L} = i\bar{\psi}_L \not{\partial} \psi_L + i\bar{\psi}_R \not{D} \psi_R - \bar{\psi}_L M \psi_R - \bar{\psi}_R M \psi_L - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu},$$

where $\psi_L$ and $\psi_R$ are the left and right handed quark fields respectively, $M$ is the quark mass matrix, $G^a_{\mu\nu}$ is the gluonic gauge field strength tensor, and $D^\mu$ is the gauge-covariant derivative. Under the chiral transformation ($\psi \rightarrow \gamma^5 \psi$), the mass terms change sign as $\psi_L \rightarrow -\psi_L$ and $\psi_R \rightarrow \psi_R$ and the Lagrangian in Eq. (10) no longer remains invariant.

In case the mass terms are neglected, the Lagrangian will have global chiral symmetry of the $SU(3)_L \times SU(3)_R$ group. The chiral symmetry is believed to be spontaneously broken to $SU(3)_{L+R}$ around the scale of 1 GeV and as a consequence, a set of massless particles (referred to as the Goldstone bosons (GBs)) exist. These GBs are identified with the observed ($\pi, K, \eta$ mesons). Within the region of chiral symmetry breaking scale $\Lambda_{\chi_{SB}}$ and the QCD confinement scale ($\Lambda_{QCD} \simeq 0.1 - 0.3$ GeV), the appropriate degrees of freedom are the constituent quarks, the set of GBs ($\pi, K, \eta$ mesons), and the weakly interacting gluons.

The effective interaction Lagrangian between GBs and quarks in the leading order can now be expressed as

$$\mathcal{L}_{\text{int}} = -\frac{g_A}{f_\pi} \bar{\psi}_L \Phi \gamma^\mu \gamma^5 \psi,$$

where the field $\Phi$ describes the dynamics of octet of GBs. The QCD Lagrangian is also invariant under the axial $U(1)$ symmetry, which would imply the existence of ninth GB. This breaking symmetry picks the $\eta'$ as the ninth GB. The effective Lagrangian describing interaction between quarks and a nonet of GBs, consisting of octet and a singlet, can now be expressed as

$$\mathcal{L}_{\text{int}} = g_8 \bar{\psi} \Phi \psi + g_1 \bar{\psi} \sqrt{3} \eta' \psi = g_8 \bar{\psi} \left( \Phi + \zeta \frac{\eta'}{\sqrt{3}} I \right) \psi = g_8 \bar{\psi} (\Phi') \psi,$$

where $\zeta = g_1/g_8, g_1$ ($g_8$) is the coupling constant for the singlet (octet) GB and $I$ is the $3 \times 3$ identity matrix.

6
The basic idea in the $\chi$CQM \cite{88,89} is the fluctuation process where the GBs are emitted by a constituent quark. These GBs further split into a $q\bar{q}$ pairs, for example,

$$q^{(\uparrow)} \rightarrow GB^0 + q^{(\downarrow)} \rightarrow (q\bar{q})^0 + q^{(\downarrow)}$$ \hspace{1cm} (13)

where $q\bar{q} + q'$ constitute the sea quarks \cite{91,95,98,108}. The GB field can be expressed in terms of the GBs and their transition probabilities as

$$\Phi' = \begin{pmatrix}
\frac{\pi^0}{\sqrt{2}} + \frac{\beta}{\sqrt{6}} + \frac{\zeta'}{\sqrt{3}} & \frac{\pi^+}{\sqrt{2}} + \frac{\beta}{\sqrt{6}} + \frac{\zeta'}{\sqrt{3}} & \alpha K^+ \\
-\frac{\pi^0}{\sqrt{2}} + \frac{\beta}{\sqrt{6}} + \frac{\zeta'}{\sqrt{3}} & \frac{-\pi^+}{\sqrt{2}} + \frac{\beta}{\sqrt{6}} + \frac{\zeta'}{\sqrt{3}} & \alpha K^0 \\
\alpha K^+ & \alpha K^0 & -\beta \sqrt{6} + \frac{\zeta'}{\sqrt{3}}
\end{pmatrix}.$$ \hspace{1cm} (14)

The transition probability of chiral fluctuation $u(d) \rightarrow d(u) + \pi^{+(\cdot)}$, given in terms of the coupling constant for the octet GBs $|g_8|^2$, is defined as $a$ and is introduced by considering nondegenerate quark masses $M_s > M_u,d$. The probabilities of transitions of $u(d) \rightarrow s + K^{+\cdot\cdot}$, $u(d,s) \rightarrow u(d,s) + \eta$, and $u(d,s) \rightarrow u(d,s) + q'$ are given as $a^2 a$, $b^2 a$ and $c^2 a$ respectively. \cite{91-95}. The probability parameters $a^2 a$ and $b^2 a$ are introduced by considering nondegenerate GB masses $M_K, M_\eta > M_\pi$ and the probability $c^2 a$ is introduced by considering $M_{q',o} > M_K, M_\eta$.

The calculations of vector and axial-vector form factors involve the calculations of axial-vector matrix elements of the nucleons using the operator $q^\dagger q^\downarrow$ measuring the sum of the quark with spin up and down as

$$\langle p(n)|q^\dagger q^\downarrow|p(n) \rangle.$$ \hspace{1cm} (15)

Here $q^\dagger q^\downarrow$ is the number operator defined in terms of the number $n^q(q^\dagger)$ of $q^\dagger(q^\downarrow)$ quarks and is expressed as

$$q^\dagger q^\downarrow = \sum_{q=u,d,s} (n^q q^\dagger + n^q q^\downarrow) = n^u q^\uparrow + n^d q^\uparrow + n^d q^\downarrow + n^s q^\downarrow + n^s q^\uparrow,$$ \hspace{1cm} (16)

with the coefficients of the $q^\dagger q^\downarrow$ giving the number of $q^\dagger q^\downarrow$ quarks.

The spin structure of the nucleon after the inclusion of sea quarks generated through chiral fluctuation can be calculated by substituting for each constituent quark

$$q^\dagger q^\downarrow \rightarrow P^q q^\dagger q^\downarrow + |\psi q^\dagger|^2,$$ \hspace{1cm} (17)
where the transition probability of no emission of GB $P^q$ can be expressed in terms of the transition probability of the emission of a GB from any of the $u$, $d$, and $s$ quark as follows

$$P^q = 1 - P^{[q, \ GB]},$$

(18)

with

$$P^{[u, \ GB]} = P^{[d, \ GB]} = \frac{a}{6} \left(9 + 6\alpha^2 + \beta^2 + 2\zeta^2\right), \quad \text{and} \quad P^{[s, \ GB]} = \frac{a}{3} \left(6\alpha^2 + 2\beta^2 + \zeta^2\right).$$

(19)

The probabilities of transforming $q^{\uparrow \downarrow}$ quark after one interaction $|\psi^{q^{\uparrow \downarrow}}|^2$ are expressed by the functions

$$|\psi^{u^{\uparrow \downarrow}}|^2 = \frac{a}{6} \left(3 + \beta^2 + 2\zeta^2\right) u^{\uparrow \downarrow} + ad^{\uparrow \downarrow} + a\alpha^2 s^{\uparrow \downarrow},$$

$$|\psi^{d^{\uparrow \downarrow}}|^2 = au^{\uparrow \downarrow} + \frac{a}{6} \left(3 + \beta^2 + 2\zeta^2\right) d^{\uparrow \downarrow} + a\alpha^2 s^{\uparrow \downarrow},$$

$$|\psi^{s^{\uparrow \downarrow}}|^2 = a\alpha^2 u^{\uparrow \downarrow} + a\alpha^2 d^{\uparrow \downarrow} + \frac{a}{3} \left(2\beta^2 + \zeta^2\right) s^{\uparrow \downarrow}.$$ (20)

The vector and axial-vector form factors of the nucleon at $Q^2 = 0$ are given by the polarized distribution function of the quark $\Delta q$ in the $\chi$CQM defined as

$$\Delta q = q^{\uparrow} - q^{\downarrow},$$

(21)

where $q^{\uparrow}$ ($q^{\downarrow}$) is the probability that the quark spin is aligned parallel or antiparallel to the nucleon spin. This can further be defined in terms of polarized constituent (C) and sea (S) quark distribution functions as

$$\Delta q^{p,n}_C = \Delta q^{p,n}_C^C + \Delta q^{p,n}_C^S, \quad \Delta q^{p,n}_S = \Delta q^{p,n}_S^C + \Delta q^{p,n}_S^S.$$ (22)

Here we have the polarized constituent quark distribution functions for $p$ and $n$ as

$$\Delta u^p_C = \frac{4}{3}, \quad \Delta d^p_C = -\frac{1}{3}, \quad \Delta s^p_C = 0,$$

$$\Delta u^n_C = -\frac{1}{3}, \quad \Delta d^n_C = \frac{4}{3}, \quad \Delta s^n_C = 0,$$

(23)

and the polarized sea quark distribution functions for $p$ and $n$ as

$$\Delta u^p_S = -\frac{a}{3} \left(7 + 4\alpha^2 + \frac{4}{3}\beta^2 + \frac{8}{3}\zeta^2\right), \quad \Delta u^n_S = -\frac{a}{3} \left(2 - \alpha^2 - \frac{1}{3}\beta^2 - \frac{2}{3}\zeta^2\right),$$

$$\Delta d^p_S = -\frac{a}{3} \left(2 - \alpha^2 - \frac{1}{3}\beta^2 - \frac{2}{3}\zeta^2\right), \quad \Delta d^n_S = -\frac{a}{3} \left(7 + 4\alpha^2 + \frac{4}{3}\beta^2 + \frac{8}{3}\zeta^2\right),$$

$$\Delta s^p_S = -a\alpha^2, \quad \Delta s^n_S = -a\alpha^2.$$ (24)
In terms of the polarized distribution functions, the magnetic moment of the nucleon is defined as

$$\mu_{p,n} = \sum_{q=u,d,s} \Delta q_{p,n}^q \mu^q. \quad (25)$$

Apart from the spin of the constituent quarks and spin of the sea quarks, the magnetic moment of a given baryon in the $\chi$CQM also receives contribution from the orbital angular motion of the sea quarks. The total magnetic moment is expressed as

$$\mu_{p,n} = \mu_{p,n}^C + \mu_{p,n}^S + \mu_{p,n}^O, \quad (26)$$

where $\mu_{p,n}^C$ and $\mu_{p,n}^S$ are the magnetic moment contributions of the constituent quarks and the sea quarks respectively coming from the proton and neutron spin polarizations, whereas $\mu_{p,n}^O$ is the magnetic moment contribution due to the rotational motion of the two bodies constituting the sea quarks ($q'$) and GB and referred to as the orbital angular momentum contribution of the quark sea $[91,93]$. 

In terms of quark magnetic moments and spin polarizations, the contributions of constituent quark spin ($\mu_{p,n}^C$), sea quark spin ($\mu_{p,n}^S$), and sea orbital ($\mu_{p,n}^O$) can be defined as

$$\mu_{p,n}^C = \sum_{q=u,d,s} \Delta q_{p,n}^C \mu^q, \quad (27)$$
$$\mu_{p,n}^S = \sum_{q=u,d,s} \Delta q_{p,n}^S \mu^q, \quad (28)$$
$$\mu_{p,n}^O = \sum_{q=u,d,s} \Delta q_{p,n}^C \mu(q\rightarrow q'), \quad (29)$$

where $\mu^q = \frac{e^q}{2M_q}$ ($q = u, d, s$) is the quark magnetic moment in the units of $\mu^N$ (nuclear magneton), $e^q$ and $M_q$ are the electric charge and the mass, respectively, for the quark $q$. $\Delta q_{p,n}^C$ and $\Delta q_{p,n}^S$ can be calculated from Eqs. (23) and (24). The orbital moment for any chiral fluctuation $\mu(q^\uparrow \rightarrow q'^\downarrow)$ can be calculated from the contribution of the angular momentum of the sea quarks to the magnetic moment of a given quark expressed as

$$\mu(q^\uparrow \rightarrow q'^\downarrow) = \frac{e^q}{2M_q} \langle l^q \rangle + \frac{e^{q'} - e^q}{2M_{GB}} \langle l^{GB} \rangle, \quad (30)$$

where

$$\langle l^q \rangle = \frac{M_{GB}}{M_q + M_{GB}}$$
and
$$\langle l^{GB} \rangle = \frac{M_q}{M_q + M_{GB}}, \quad (31)$$
\langle l^q, l^{GB} \rangle \) and \((M_q, M_{GB})\) are the orbital angular momenta and masses of quark and GB respectively. The orbital moment of each process is then multiplied by the probability for such a process to take place to yield the magnetic moment due to all the transitions starting with a given constituent quark, for example

\[
[\mu(u^\uparrow(d^\downarrow) \rightarrow)] = \pm a[\mu (u^\uparrow(d^\uparrow) \rightarrow d^\downarrow(u^\downarrow)) + \alpha^2 \mu (u^\uparrow(d^\uparrow) \rightarrow s^\downarrow] + \left(\frac{1}{2} + \frac{1}{6} \beta^2 + \frac{1}{3} \zeta^2\right)\mu (u^\uparrow(d^\uparrow) \rightarrow u^\downarrow(d^\downarrow))\right],
\]

(32)

\[
[\mu(s^\uparrow \rightarrow)] = \alpha \left[\alpha^2 \mu (s^\uparrow \rightarrow u^\downarrow) + \alpha^2 \mu (s^\uparrow \rightarrow d^\downarrow) + \left(\frac{2}{3} \beta^2 + \frac{1}{3} \zeta^2\right)\mu (s^\uparrow \rightarrow s^\downarrow)\right].
\]

(33)

The above equations can easily be generalized by including the coupling breaking and mass breaking terms, for example, in terms of the coupling breaking parameters \(a, \alpha, \beta\) and \(\zeta\) as well as the masses of GBs \(M_\pi\), \(M_K\) and \(M_\eta\).

IV. \(Q^2\) DEPENDENCE OF NUCLEON FORM FACTORS

The \(Q^2\) dependence of the vector electric and magnetic form factors as well as axial-vector form factors have been experimentally investigated from the PV electron scattering and from the DIS of neutrinos. The conventional dipole form of parametrization has been used to analyse the vector and axial-vector form factors

\[
G_{V,A}^p(Q^2) = \frac{g_{V,A}^p(0)G_{V,A}^D(Q^2)}{(1 + \frac{Q^2}{M_V^2})^2},
\]

where the electric and magnetic form factors of the proton at zero momentum transfer \(g_{V,A}^p(0)\) for \(V = E, M\) correspond to the charge and magnetic moment respectively. \(g_{V,A}^p(0)\) and \(g_{V,A}^n(0)\) are the isovector axial-vector coupling constants of the proton and neutron corresponding to the axial-vector form factors at zero momentum transfer. The vector mass \(M_V\) is taken as \(M_V^2 = 0.71 \text{ GeV}^2\). For the axial mass \(M_A\), we have used the most recent value obtained by the MiniBooNE Collaboration \(M_A^2 = 1.10_{-0.15}^{+0.13} \text{ GeV}^2\) [118, 119].

For the case of proton, both the vector and axial-vector form factors of the proton follow the dipole form of parametrization. The form factors \(G_{E}^p(Q^2), G_{M}^p(Q^2)\) and \(G_{A}^p(Q^2)\) respectively scale with the net charge, magnetic moment \(\mu_p\) and isovector axial-vector coupling...
constant \( g_A^i(0) \) \((i = 0, 3, 8)\) as follows

\[
G_E^p(Q^2) = G_V^p(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{M_V^p}\right)^2},
\]
\[
G_M^p(Q^2) = \mu_p G_V^p(Q^2) = \frac{\mu_p}{\left(1 + \frac{Q^2}{M_V^p}\right)^2},
\]
\[
G_A^i(Q^2) = g_A^i(0) G_A^D(Q^2) = \frac{g_A^i(0)}{\left(1 + \frac{Q^2}{M_A^i}\right)^2}.
\]

(35)

For the case of neutron however, the measurement of form factors raises difficulties because there are no free neutron target available suited for electron experiments. The form factors \( G_E^n(Q^2) \) and \( G_M^n(Q^2) \) respectively scale with the net charge and magnetic moment as follows

\[
G_E^n(Q^2) = \frac{A \tau}{1 + B \tau} G_V^p(Q^2) = \frac{A \tau}{1 + B \tau} \frac{1}{\left(1 + \frac{Q^2}{M_V^p}\right)^2},
\]
\[
G_M^n(Q^2) = \mu_n G_V^p(Q^2) = \frac{\mu_n}{\left(1 + \frac{Q^2}{M_V^p}\right)^2},
\]

(36)

where \( \tau = \frac{Q^2}{4 M_n^2} \) and the parameters \( A \) and \( B \) obtained from the recent fits of root mean square radius are \( A = 1.73 \) and \( B = 4.62 \). The factor \( \frac{A \tau}{1 + B \tau} \) has been introduced to basically account for the condition \( G_E^n(Q^2 = 0) = 0 \).

V. QUARK FLAVOR DECOMPOSITION OF THE FORM FACTORS

Recent measurements of form factors have made it possible to separate out the quark flavor contributions which have been subject of extensive theoretical analysis and are not understood completely by existing models. The quark flavor form factors at \( Q^2 = 0 \) can be calculated in the \( \chi \)CQM and are expressed as

\[
G_E^q(Q^2 = 0) = e^q,
\]
\[
G_M^q(Q^2 = 0) = \mu^q,
\]
\[
G_A^q(Q^2 = 0) = \Delta q.
\]

(37)
The $Q^2$ dependence of these form factors can be calculated using the dipole parametrization in Eq. (34). We have the quark flavor form factors corresponding to the nucleon as

$$G_E^q(Q^2) = e^q G_D^V(Q^2) = \frac{e^q}{\left(1 + \frac{Q^2}{M_V^2}\right)^2},$$

$$G_M^q(Q^2) = \mu_{p,n}^q G_D^V(Q^2) = \frac{\mu_{p,n}^q}{\left(1 + \frac{Q^2}{M_V^2}\right)^2},$$

$$G_A^q(Q^2) = \Delta q G_A^D(Q^2) = \frac{\Delta q}{\left(1 + \frac{Q^2}{M_A^2}\right)^2}.$$  (38)

Out of all the flavor vector and axial-vector form factors, the strangeness form factors have triggered a great deal of interest. The recent measurements by SAMPLE at MIT-Bates [41], HAPPEX [42–45], G0 at JLab [46, 47], A4 at MAMI [48, 49] have observed either one or a combination of electric and magnetic form factors. Recent lattice calculations [120–122] and other phenomenological studies with lattice inputs [123, 124] have also predicted a very small value for the strange contribution.

VI. RESULTS AND DISCUSSION

The probabilities of fluctuations to pions, $K$, $\eta$, $\eta'$ represented by $a$, $a\alpha^2$, $a\beta^2$, and $a\zeta^2$ respectively can be calculated in the $\chi$CQM at $Q^2 = 0$ after taking into account strong physical considerations and carrying out a fine grained analysis using the well known experimentally measurable spin and flavor distribution functions. The parameters are listed in Table I. The table also includes the other input parameters pertaining to quark masses and magnetic moments as well as the GB masses.

In Fig. 1, we have presented the variation of electric ($G_E^p(Q^2)$) and magnetic ($G_M^p(Q^2)$) form factors of the proton with $Q^2$. We find that the charge of $p$ ($G_E^p(Q^2 = 0)$) is 1 and as the $Q^2$ value increases, it falls off very quickly at small values of till $Q^2 \approx 1 \text{ GeV}^2$. The

| Parameter | $a$ | $a\alpha^2$ | $a\beta^2$ | $a\zeta^2$ | $M_{u,d}$ | $M_s$ | $\mu^u$ | $\mu^d$ | $\mu^s$ | $M_\pi$ | $M_K$ | $M_\eta$ | $M_\eta'$ | $A$ | $B$ |
|-----------|----|----------|----------|----------|---------|------|--------|--------|--------|-------|------|-------|-------|-----|----|
| Value     | 0.114 | 0.023 | 0.023 | 0.002 | 330 | 510 | 2 | -1 | -0.65 | 140 | 494 | 548 | 958 | 1.73 | 4.62 |
FIG. 1. (color online). The electric \((G^p_E(Q^2))\) and magnetic \((G^p_M(Q^2))\) form factors of the proton as a function of \(Q^2 (\text{GeV}/c)^2\). The data has been taken from the references mentioned in the legend.

FIG. 2. (color online). The ratio \(G^p_E(Q^2)/G^p_V(Q^2)\) as a function of \(Q^2 (\text{GeV}/c)^2\). The data has been taken from the references mentioned in the legend.

data however falls off steadily from \(\approx 1\) at \(Q^2 \approx 0 \text{GeV}^2\) to \(\approx 0.7\) at \(Q^2 \approx 3 \text{GeV}^2\). For the case of \(G^p_M(Q^2)\), the \(\chi\text{CQM}\) results agree quite well with the data points for \(Q^2 > 0.5 \text{GeV}^2\) and \(Q^2 < 3 \text{GeV}^2\). In the absence of data for \(Q^2 < 0.5 \text{GeV}^2\) and \(Q^2 > 3 \text{GeV}^2\), it is
FIG. 3. (color online). The ratio $G_M^p(Q^2)/\mu_p G_V^D(Q^2)$ as a function of $Q^2\ (\text{GeV}/c)^2$. The data has been taken from the references mentioned in the legend.

difficult to compare the results at these values. The magnetic moment of proton $\mu_p = G_M^p(Q^2 = 0)$ comes out to be $2.80 \mu_N$ which is in fair agreement with data \[127\]. The ratios $G_E^p(Q^2)/G_V^p(Q^2)$, $G_M^p(Q^2)/\mu_p G_V^D(Q^2)$ and $\mu_p G_E^p(Q^2)/G_M^p(Q^2)$ have been respectively presented in Fig. 2 Fig. 3 and Fig. 4. The results for $G_E^p(Q^2)/G_V^D(Q^2)$ are more or less in agreement with the data. Different data shows the value of $G_E^p(Q^2)/G_V^D(Q^2)$ close to 1. Similarly, for the case of $G_M^p(Q^2)/\mu_p G_V^D(Q^2)$ and $\mu_p G_E^p(Q^2)/G_M^p(Q^2)$, a fair agreement with data is obtained. Even though the data varies from 0.95 – 1.05, it stays close to 1. More data for $Q^2 > 4\text{GeV}^2$ may be needed so see if there is some variation from 1. The proton form factors and their ratios have been measured in the polarization experiments, recoil polarization experiments and beam-target asymmetry measurements \[33, 34, 135–142\].

In Fig. 5 we have presented the variation of electric ($G_E^n(Q^2)$) and magnetic ($G_M^n(Q^2)$) form factors of the neutron with $Q^2$. The ratios $G_E^n(Q^2)/G_V^n(Q^2)$, $G_M^n(Q^2)/\mu_n G_V^D(Q^2)$ and $\mu_n G_E^n(Q^2)/G_M^n(Q^2)$ have been respectively presented in Fig. 6 Fig. 7 and Fig. 8. In Fig. 9 we have presented the ratio $G_E^n(Q^2)/G_M^n(Q^2)$. The neutron form factors have been measured in a series of experiments \[144, 145\] and our results are in fair agreement with the available
FIG. 4. (color online). The ratio $\mu_p G_E^p(Q^2)/G_M^p(Q^2)$ as a function of $Q^2 (\text{GeV/c})^2$. The data has been taken from the references mentioned in the legend.

FIG. 5. (color online). The electric ($G_E^n(Q^2)$) and magnetic ($G_M^n(Q^2)$) form factors of the neutron as a function of $Q^2 (\text{GeV/c})^2$. The data has been taken from the reference mentioned in the legend.

experimental data. More data is needed for the profound understanding of the form factors of the neutron.

In Fig. 10, we have plotted the axial-vector form factors $G_A^n(Q^2)$ as a function of $Q^2$...
FIG. 6. (color online). The ratio $G_E^0(Q^2)/G_V^0(Q^2)$ as a function of $Q^2 (\text{GeV}/c)^2$. The data has been taken from the reference mentioned in the legend.

for $i = 0,3,8$. These axial-vector form factors at $Q^2 = 0$ give the axial-vector coupling constants: $g_A^0$ corresponds to the flavor singlet component, $g_A^3$ and $g_A^8$ correspond to the flavor non-singlet components. The present experimental situation for the case of $g_A^0$, $g_A^3$ and $g_A^8$, is summarized as follows [127]:

$$g_A^0 \text{expt} = 0.30 \pm 0.06,$$
$$g_A^3 \text{expt} = 1.267 \pm 0.0025,$$
$$g_A^8 \text{expt} = 0.588 \pm 0.033,$$

(39)

whereas the $\chi$CQM results are given as

$$g_A^0 = 0.519,$$
$$g_A^3 = 1.266,$$
$$g_A^8 = 0.588.$$

(40)

The $Q^2$ dependence of the singlet and non-singlet form factors varies as

$$G_A^3(Q^2) < G_A^8(Q^2) < G_A^0(Q^2).$$

(41)
The strangeness quark contributions to the vector and axial-vector form factors ($G_E^s(Q^2)$, $G_M^s(Q^2)$ and $G_A^s(Q^2)$) have been shown in Fig. 11 as a function of $Q^2$. It is well known that the strange quarks contribute to the internal properties of the nucleon because of the presence of the non-constituent “quark sea” (Eq. 13) which includes the effects of chiral symmetry breaking as well as SU(3) symmetry breaking. From Fig. 11 we find that the magnitude of $G_E^s(Q^2)$, $G_M^s(Q^2)$ and $G_A^s(Q^2)$ fall off with the increasing value of $Q^2$. The explicit strangeness contribution to the magnetic form factor and the axial-vector form factor is very small as compared to the electric form factor. This is in agreement with the small but significant contribution of strangeness in the nucleon as indicated by SAMPLE at MIT-Bates [41], G0 at JLab [46, 47], PVA4 at MAMI [48, 49] and HAPPEX at JLab [42, 45]. A determination of $G_A^s$ at low values of $Q^2$ would have important implications in the determination of strange spin polarization $\Delta s$ which is otherwise zero in the case of nucleon.
VII. SUMMARY AND CONCLUSIONS

To summarize, the electromagnetic and axial-vector form factors of the nucleon ($G_E^n(Q^2)$ and $G_M^n(Q^2)$) have been phenomenologically determined in the chiral constituent quark model ($\chi$CQM) using the spin observables. The $\chi$CQM helps in the understanding the dynamics of the constituents of the nucleon affected by chiral symmetry breaking in terms of the quark flavor contributions to the form factors of the nucleon. Further, in light of the precision data available for increased $Q^2$ range as well as to present a comprehensive analysis of the vector and axial-vector form factors, the calculations have been extended to analyse the $Q^2$ dependence of these quantities using the conventional dipole form of parametrization. The contributions of the quark flavor to the electromagnetic structure of the nucleon have been calculated by combining the electromagnetic and neutral weak vector currents as well as axial current leading to the flavor decomposition of the form factors ($G_E^q(Q^2)$, $G_M^q(Q^2)$ and $G_A^q(Q^2)$). The contribution of strange quarks provides an ideal probe for the virtual sea quarks present in the nucleon particularly the strange spin polarization $\Delta s$ which
FIG. 9. (color online). The ratio $G_n^E(Q^2)/G^n_M(Q^2)$ as a function of $Q^2 \text{(GeV/c)}^2$. The data has been taken from the reference mentioned in the legend.

corresponds to the value of the strange axial form factor $G_A^s$ at zero-momentum transfer ($Q^2 = 0$). Despite considerable efforts in the past few years, the experimental data on $\Delta s$ and $G_A^s$ point out the need for additional refined data. Moreover, the $Q^2$ dependence of $G_A^s$ is also unknown. In the scarcity of precise data at higher $Q^2$ and very low $Q^2$, the results have been compared with the recent available experimental observations.

In conclusion, we would like to state that our results provide important constraints on the future experiments to describe the explicit role of constituent and non-constituent degrees of freedom particularly the strangeness contribution. Different experiments are contemplating the possibility of performing the high precision measurements over a wide $Q^2$ region in the near future which will help in the profound understanding of the nonperturbative properties of QCD.
FIG. 10. (color online). The axial-vector form factors $G_A^0(Q^2)$, $G_A^3(Q^2)$ and $G_A^8(Q^2)$ as a function of $Q^2 (\text{GeV/c})^2$.

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FIG. 11. (color online). The strangeness form factors $G^s_E(Q^2)$, $G^s_M(Q^2)$ and $G^s_A(Q^2)$ as a function of $Q^2$ (GeV/c)$^2$.

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