Study of $\gamma\gamma \rightarrow W_L^+W_L^-$ and $\gamma\gamma \rightarrow Z_LZ_L$ reactions with Chiral Lagrangians

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Abstract

We analyze the effects of a strongly interacting symmetry breaking sector of the Standard Model in $\gamma\gamma \rightarrow W_L^+W_L^-$ and $\gamma\gamma \rightarrow Z_LZ_L$ reactions at TeV energies by using Chiral Lagrangians and Chiral Perturbation Theory. We find significant deviations from the Standard Model predictions for the differential cross sections at high invariant mass of the gauge bosons pair. We study the experimental signals that could be obtained in a high energy and high luminosity dedicated $\gamma\gamma$ collider and estimate the sensitivity that such experiments could reach to the values of the effective lagrangian parameters.

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1 Introduction

The interest of applying Chiral Lagrangians and Chiral Perturbation Theory (ChPT) to the study of the self-interactions of the longitudinal gauge bosons $W_L^\pm$ and $Z_L$ at and below TeV energies has long been discussed in literature [1]. The outcome of this discussion is that Chiral Lagrangians are ideally suited for studying, with complete generality and in the most economical way, the properties of the symmetry breaking sector (SBS) in the hypothetical case that it is strongly interacting (SISBS). This is so because the only physical input they use is the symmetry breaking pattern which must at least contain $SU(2)_L \times SU(2)_R \rightarrow SU(2)_{L+R}$ if one wants to recover the global $SU(2)_L \times SU(2)_R$ invariance of the scalar sector of the Standard Model (SM) when the $U(1)_Y$ coupling is set equal to zero, and because it only uses the so far confirmed physical degrees of freedom of the SBS, namely the $W_L^\pm$ and $Z_L$.

The Chiral Lagrangian (ChL) is organized as an expansion in terms of derivatives and external fields, and the different possibilities for the SBS are just reflected in the different values for the parameters $L_i$’s appearing in it [2]. Besides, ChPT provides us with a procedure for expanding observables in powers of $\frac{p^2}{(4\pi v)^2}$, where $4\pi v (v=246 \text{ GeV})$ is the scale that normalizes the contribution of higher-dimensional operators and governs the size of the $W_L^\pm$ and $Z_L$ self-interactions. Since $4\pi v \simeq 3 \text{ TeV}$, this expansion is a very good one at energies well below 3 TeV.

Several studies have already been done on the sensitivity to the values of the $L_i$ parameters that could be reached in the present and future experiments, and to conclude at which level one will be able to discriminate amongst the different symmetry breaking alternatives from an experimental measure of those coefficients. Most of the analyses so far have been dedicated to the process of $V_L V_L$ fusion ($V_L = W_L^\pm$ or $Z_L$) at the pp colliders LHC and SSC, the reason being obviously that the reaction $V_L V_L \rightarrow V_L V_L$ is the genuine one for testing the $V_L$’s self-interactions in exactly the same way as the $\pi\pi \rightarrow \pi\pi$ reaction does for the $\pi$’s self-interactions [3]. The sensitivity to the chiral parameters $L_1$ and $L_2$ that will be reached in LHC and SSC through the $V_L V_L$ fusion processes has been studied in [4]. More recently, ChL’s have also been applied to the process $q\bar{q} \rightarrow V_L V_L$ at LHC and SSC at tree level [5], at one loop [6], and in the leading log approximation [7].
The study of these processes will allow to constrain the numerical value of $L_9$. Finally, $e^+e^-$ colliders have also been used to test the chiral parameters [8-11]. Given the fantastic accuracy that is being achieved at LEPI, a study of the radiative corrections to the standard parameters for LEP physics $\Delta r$, $\Delta \kappa$ and $\Delta \rho$ by means of the ChL has allowed to constrain quite significantly the value of $L_{10}$ [9]. On the other hand, a tree level study in ChPT of the reaction $e^+e^- \rightarrow W^+W^-$ first in [10] and a complete study in ChPT to one loop later in [11] have allowed to search for the expected sensitivities of $L_9$ and $L_{10}$ at LEPII energies and at the proposed $e^+e^-$ colliders at TeV energies.

In this letter, we propose an alternative way of testing the SISBS hypothesis, by means of the $\gamma\gamma \rightarrow W_L^+W_L^-$ and $\gamma\gamma \rightarrow Z_LZ_L$ reactions. The interest of this analysis is two-fold. On one hand, the above reactions complement nicely the study of SISBS with Chiral Lagrangians, as the charched channel is sensitive to the $L_9$ and $L_{10}$ parameters which do not contribute to $V_LV_L \rightarrow V_LV_L$ reactions and the neutral channel is sensitive to the chiral loops. On the other hand, the proposal of a dedicated $\gamma\gamma$ collider up to TeV energies [12], where real photons are obtained from backscattering of laser beams off high energy electrons, opens the possibility of studying these reactions in a very clean and almost background free way [13]. As we will comment later on, the peculiarities of a dedicated $\gamma\gamma$ collider make this experiment optimal for our purpose, since one could reach the energy region of $0.5 \text{ TeV} \leq \sqrt{s} \leq 1 \text{ TeV}$, where the effects of $V_L$’s strong interactions are expected to show up, with high $\gamma\gamma$ luminosities.

The use of $\gamma\gamma$ collisions at TeV energies will be the analogous one to the use of $\gamma\gamma \rightarrow \pi\pi$ reactions at GeV energies. As many studies have shown so far, the latter has turned out to be a quite successful and fruitful laboratory to study the effects of strong $\pi\pi$ rescattering in a very clean way. We show in this work that the study of $\gamma\gamma \rightarrow W_L^+W_L^-$ and $\gamma\gamma \rightarrow Z_LZ_L$ reactions in a dedicated $\gamma\gamma$ collider will certainly provide useful information on the $W_L^\pm$ and $Z_L$ self-interactions at energies ranging from threshold to about 1 TeV. In particular, we will see that the process $\gamma\gamma \rightarrow W_L^+W_L^-$ will allow to constrain quite significantly the values of the chiral parameters $L_9$ and $L_{10}$. Finally, we comment on what in principle could be a very promising signal of a SISBS. It refers to the measure of the total and differential cross section for the $\gamma\gamma \rightarrow Z_LZ_L$ process. The interesting point is that, in the SM, the cross section for this process vanishes at tree level, and the only contributions
come from one loop diagrams that are of order $e^8$. In contrast, if the SBS is strongly interacting and the treatment by means of a ChL applies, the cross section for $\gamma\gamma \to Z_LZ_L$ will be of "enhanced electroweak strength" due, essentially, to the typical effect of strong rescattering of the $Z_LZ_L$ final pair that translates into an overall contribution of the order of $e^4$. Unfortunately, as we will show in this work, this potential effect will not be seen in the $\gamma\gamma$ dedicated colliders with the luminosity considered here because of the lack of statistics.

2 Chiral Lagrangians and $T(\gamma\gamma \to V_LV_L)$ amplitudes

The effective ChL we will work with is based on the symmetry breaking pattern of the global chiral symmetry down to its diagonal subgroup: $SU(2)_L \times SU(2)_R \to SU(2)_{L+R}$. We will consider only effective theories for which the custodial symmetry $SU(2)_V = SU(2)_{L+R}$ is exactly preserved in the SBS up to the explicit breaking due to the gauging of the $U(1)_Y$ symmetry. The effective lagrangian consists of an infinite expansion of terms with increasing number of derivatives of the Goldstone boson fields associated to the chiral symmetry breaking $w^\pm$ and $z$, and/or external fields, namely the $A_\mu$, $W_\mu^\pm$ and $Z_\mu$ gauge fields. The Goldstone boson fields, that will eventually become the longitudinal components of the $W^\pm$ and $Z$ fields, are parametrized by means of an unitary matrix field $U$ belonging to the quotient space $SU(2)_L \times SU(2)_R / SU(2)_V$:

$$U = \exp(i\sigma^aw^a/v), \quad a = 1, 2, 3.$$ (1)

The local $SU(2) \times U(1)_Y$ symmetry of the SM is implemented by replacing the derivatives of the $U$ field by the covariant derivatives defined as:

$$D_\mu U = \partial_\mu U + L_\mu U - UR_\mu$$ (2)

where $L_\mu$ and $R_\mu$ are the left and right handed gauge fields:

$$L_\mu = ig\sigma^aW_\mu^a/2, \quad R_\mu = ig's^3B_\mu/2.$$ (3)

The complete effective lagrangian for the scalar sector and the gauge boson sector of the SM that complies with all the symmetries of the SM, and that
includes terms up to $O(p^4)$ is given by:

$$\mathcal{L} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \mathcal{L}_G + \mathcal{L}_{GF} + \mathcal{L}_{FP}$$

$$\mathcal{L}^{(2)} = \frac{\nu^2}{4} \text{Tr} D_\mu U D^\mu U^\dagger$$

$$\mathcal{L}^{(4)} = L_1 \text{Tr} D_\mu U D^\mu U^\dagger \text{Tr} D_\nu U D^\nu U^\dagger$$
$$+ L_2 \text{Tr} D_\mu U D_\nu U^\dagger \text{Tr} D^\mu U D^\nu U^\dagger$$
$$- i L_3 \text{Tr} (F^{\mu\nu}_R D_\mu U D_\nu U^\dagger + F^{\mu\nu}_L D_\mu U D_\nu U^\dagger)$$
$$+ L_{10} \text{Tr} U^\dagger F^{\mu\nu}_R U F^{\mu\nu}_L$$

$$\mathcal{L}_G = -\frac{1}{2g^2} \text{Tr} F_{L\mu\nu} F^{\mu\nu}_L - \frac{1}{2g^2} \text{Tr} F_{R\mu\nu} F^{\mu\nu}_R$$

with $F^{\mu\nu}_{L,R}$ being the field strengths associated to the left and right gauge fields:

$$i F^{L}_{\mu\nu} = \partial_\mu L_\nu - \partial_\nu L_\mu - [L_\mu, L_\nu]$$
$$i F^{R}_{\mu\nu} = \partial_\mu R_\nu - \partial_\nu R_\mu.$$ 

$\mathcal{L}_{GF}$ is the gauge-fixing term:

$$\mathcal{L}_{GF} = -\frac{1}{2} \{ F^2_\gamma + F^2_Z + 2 F_+ F_- \}$$

where:

$$F_\pm = \frac{1}{\sqrt{\xi}} (\partial_\mu W^{\mu\pm} - M_W \xi w^\pm)$$

$$F_Z = \frac{1}{\sqrt{\xi}} (\partial_\mu Z^\mu - M_Z \xi z) ; \quad F_\gamma = \frac{1}{\sqrt{\xi}} (\partial_\mu A^\mu).$$

We will work in the Landau gauge ($\xi = 0$), where the ghosts and Goldstone bosons decouple from each other and the computations are simpler. It is also in this gauge where the goldstone bosons and ghosts fields remain massless and therefore $\mathcal{L}_{GF}$ and $\mathcal{L}_{FP}$ will explicitly respect the global $SU(2)_L \times SU(2)_R$ Chiral Symmetry. The advantages of using this gauge when dealing with a non–linear sigma model approach to the SM were emphasized in [14], so we refer the reader to those references for more details. Finally the ghost term in the Landau gauge is given by:

$$\mathcal{L}_{FP} = \partial_\mu \eta_0 \partial^\mu \chi_0 + 2 \text{Tr} [\partial_\mu \eta (\partial^\mu \chi + [L^\mu, \chi])]$$

4
where \( \eta(x) = \tilde{\sigma} \tilde{\eta}(x)/2 \); \( \chi(x) = \tilde{\sigma} \tilde{\chi}(x)/2 \) and \( \eta^a(x), \chi^a(x), a = 1, 2, 3 \) are the anticommuting scalar ghosts.

Once the complete Lagrangian is given, the derivation of the Feynman rules needed for the computation of \( \gamma\gamma \to W_L^+W_L^- \) and \( \gamma\gamma \to Z_LZ_L \) amplitudes is straightforward. At this point it is worth mentioning that these Feynman rules are not the same as in the linear version of the SM, the reason being obviously the non-linear realization of the chiral symmetry in the ChL.

In order to compute the amplitudes for the reactions \( \gamma\gamma \to W_L^+W_L^- \) and \( \gamma\gamma \to Z_LZ_L \) we have applied the equivalence theorem \([3,15]\), that in this case states:

\[
T(\gamma\gamma \to W_L^+W_L^-) = -T(\gamma\gamma \to w^+w^-) + O(M_W\sqrt{s})
\]

\[
T(\gamma\gamma \to Z_LZ_L) = -T(\gamma\gamma \to zz) + O(M_Z\sqrt{s})
\]

so that the range of applicability will be restricted to high energies as compared to the gauge boson masses. As we will show below, for the process considered here it turns out to be indeed a very good approximation for energies larger than 400 GeV.

Let us now present the results for the Goldstone boson amplitudes obtained from the ChL to order \( p^4 \) in equations (4–11). We work to the lowest order in the gauge coupling constant and to one loop in the chiral expansion. This means that we keep only contributions in the amplitudes of order \( e^2 \) coming from the ChL to tree level, namely from \( \mathcal{L}^{(2)} \) and \( \mathcal{L}^{(4)} \), and use only \( \mathcal{L}^{(2)} \) to compute the one loop contributions. The contributions from the loops generated by \( \mathcal{L}^{(4)} \) are of higher order in the chiral expansion, i.e. of order \( p^6 \), and we will not consider them here. It is important to emphasize that once we have replaced the external fields by the Goldstone bosons and, whenever we work at order \( e^2 \) in the amplitudes, the only one-loop diagrams from \( \mathcal{L}^{(2)} \) that contribute to our processes are those with just Goldstone bosons circulating in the loops.

The amplitudes written down below are presented in the form:

\[
T = T^{(2)} + T^{(4)}
\]

where we have separated explicitly the contributions from \( \mathcal{L}^{(2)} \) at tree level in \( T^{(2)} \) and the contributions from \( \mathcal{L}^{(4)} \) at tree level plus those from \( \mathcal{L}^{(2)} \) at one loop in \( T^{(4)} \).
We get the following results for the various helicity amplitudes $T_{\lambda_1\lambda_2}$ with $\lambda_1, \lambda_2$ being the initial photon helicities:

$$T_{++}(\gamma\gamma \rightarrow W_L^+ W_L^-) = T_{++}^{(4)}(\gamma\gamma \rightarrow W_L^+ W_L^-) = -16\pi\alpha \frac{\hat{s}}{(4\pi v)^2} \left[ 16\pi^2(L_9 + L_{10}) - \frac{1}{4} \right]$$  \hspace{1cm} (14)

$$T_{+-}(\gamma\gamma \rightarrow W_L^+ W_L^-) = T_{+-}^{(2)}(\gamma\gamma \rightarrow W_L^+ W_L^-) = -8\pi\alpha$$  \hspace{1cm} (15)

and for the neutral channels:

$$T_{++}(\gamma\gamma \rightarrow Z_L Z_L) = T_{++}^{(4)}(\gamma\gamma \rightarrow Z_L Z_L) = -8\pi\alpha \frac{\hat{s}}{(4\pi v)^2}$$  \hspace{1cm} (16)

$$T_{+-}(\gamma\gamma \rightarrow Z_L Z_L) = 0$$  \hspace{1cm} (17)

In the above expressions $\sqrt{\hat{s}}$ is the center of mass energy of the $\gamma\gamma \rightarrow V_L V_L$ subprocess. The rest of the helicity amplitudes are related to the above ones by $T_{--} = T_{++}$ and $T_{-+} = T_{+-}$.

Finally, from these expressions we get the following predictions in ChPT for the total cross sections with unpolarized photons $^1$:

$$\hat{\sigma}^{ChPT}(\gamma\gamma \rightarrow W_L^+ W_L^-) = \frac{2\pi\alpha^2}{\hat{s}} \left\{ 1 + 4 \left[ \frac{\hat{s}}{(4\pi v)^2} \left( 16\pi^2(L_9 + L_{10}) - \frac{1}{4} \right) \right]^2 \right\}$$  \hspace{1cm} (18)

$$\hat{\sigma}^{ChPT}(\gamma\gamma \rightarrow Z_L Z_L) = \frac{\pi\alpha^2}{\hat{s}} \left[ \frac{\hat{s}}{(4\pi v)^2} \right]^2$$  \hspace{1cm} (19)

At this point, it is worth commenting on some aspects of the above results that we find interesting. First of all, the two cross sections turn out to be finite in ChPT at one loop. This means that they are fully predictable and do not need of the renormalization procedure that is usually implemented when computing in ChPT. In the case of the neutral channel, this finiteness is explicitly shown in eq.(19), where potential terms of $O(1/\epsilon)$ from dimensional regularization are absent. The finiteness of the charged channel result

$^1$ We have checked that these expressions coincide with the corresponding ones for $\gamma\gamma \rightarrow \pi^+\pi^-$ and $\gamma\gamma \rightarrow \pi^0\pi^0$ in [16] once the chiral limit is taken. We would like to emphasize here that a formal connection between $V_L$ and $\pi$ physics may only be established if the Landau gauge has been chosen in the former and the chiral limit is taken in the latter.
is understood easily from eq.(18) by noticing the particular combination of the chiral parameters appearing there, namely $L_9 + L_{10}$, and recognizing it as a renormalization group invariant quantity that implies $L_9^r + L_{10}^r = L_9 + L_{10}$, with $L_i^r$ being the renormalized parameters.

Another interesting aspect is the result for the neutral channel being independent of the chiral parameters $L_9$ and $L_{10}$ and only dependent on the dimensionful parameter $v$. In other words, the cross section for $\gamma \gamma \to Z_L Z_L$ in ChPT does not depend on the precise underlying dynamics governing the interactions amongst the longitudinal gauge bosons. Thus, in principle we could use this channel to isolate, and in consequence to test, the part of the $V_L$ self- interactions that is universal, including chiral (Goldstone bosons) loops. The result for the neutral channel is shown in Fig.1a. In contrast, the charged channel does depend on the $L_9$ and $L_{10}$ chiral parameters, so that in principle one could use this channel to test the different alternatives for the SBS and thus to constrain their numerical values when comparing with experimental data.

In order to study the differences between a strongly interacting SBS and a weakly interacting SBS, we have chosen to compare the above predictions in ChPT with the corresponding ones in the SM. As can be seen in Fig.1b and Fig.1c, the total cross section for $\gamma \gamma \to W_L^+ W_L^-$ at energies ranging from 500 GeV to 1.5 TeV, is indeed quite sensitive to the particular values of $L_9 + L_{10}$ with the raising of the cross section in the high energy region becoming apparent for values of $L_9 + L_{10}$ larger than $4/16\pi^2$. An explicit computation of the cross section $\sigma(\gamma \gamma \to W_L^+ W_L^-)$ in the SM at tree level using the equivalence theorem approximation and a comparison with the exact result in the SM teaches us (see Fig.1c) that it is indeed a very good approximation for energies above 400 GeV.

Finally, it should be mentioned that the cross sections predicted in ChPT, eqs.(18) and (19) are not well behaved at high energies since the corrections to the lowest order results grow with energy. This behaviour produces as usual in ChPT a violation of perturbative unitarity in the amplitudes at high energy. This problem is well known to happen in the context of $\gamma \gamma \to \pi \pi$ reactions where many implementations of unitarization have been proposed in the literature [16,17]. The common assumption in these works is to relate the unitarization of $\gamma \gamma \to \pi \pi$ reactions with the corresponding one for the final state $\pi \pi$ interactions. Thus, the energy scale at which unitarity is violated in $\gamma \gamma \to \pi \pi$ is deduced directly from the corresponding one in
$\pi\pi \to \pi\pi$ scattering. A similar analysis for the $\gamma\gamma \to W_L^+W_L^-$ and $\gamma\gamma \to Z_LZ_L$ amplitudes can be performed. The conclusion is that the maximum allowed energy so that the results are compatible with the unitarity bounds is $\sqrt{s} \simeq 1.5$ TeV.

Clearly, in order to make effective such a detailed analysis of the reactions involving longitudinal gauge bosons in the final state one should be able to discern experimentally longitudinal from transverse modes. This may be a hard task in pp supercolliders but we believe it will be feasible in high precision experiments as for instance the $\gamma\gamma$ collider at TeV energies that we consider in this work. In this concern, some strategies for measuring the polarization of the final $W_L$'s by studying the correlation between the $W$ pair decay planes have been proposed in the literature in the context of pp super-colliders [18]. We will not apply these techniques here but prefer to postpone this kind of analysis until a more definite proposal of a dedicated $\gamma\gamma$ collider be done. Instead of doing this, we have computed and compared the total and differential cross sections in the SM at tree level for the three different polarization states of the final $W^+W^-$ pairs. We get the following results for the total cross sections:

\[
\hat{\sigma}(\hat{s})_{LL} = \frac{\pi\alpha^2\beta}{\hat{s}r} \left[ \frac{2(r + 3r^2 + r^3 + r^4)}{(1-r)^2} - \frac{r^2(2 + 3r + 2r^2 - r^3)}{(1-r)^2} \right] \frac{1}{\beta} \ln \left( \frac{1 + \beta}{1 - \beta} \right) \]

\[
\hat{\sigma}(\hat{s})_{LT} = \frac{\pi\alpha^2\beta}{\hat{s}r} \left[ \frac{-32r^2 + 8r^3}{(1-r)^2} + \frac{16r^2 - 8r^3 + 4r^4}{(1-r)^2} \right] \frac{1}{\beta} \ln \left( \frac{1 + \beta}{1 - \beta} \right) \]

\[
\hat{\sigma}(\hat{s})_{TT} = \frac{\pi\alpha^2\beta}{\hat{s}r} \left[ \frac{32 - 60r + 52r^2 - 16r^3 + 4r^4}{(1-r)^2} - \frac{r^2(-20 + 26r - 14r^2 + 2r^3)}{(1-r)^2} \right] \frac{1}{\beta} \ln \left( \frac{1 + \beta}{1 - \beta} \right) \]

where $r = \frac{4M_W^2}{\hat{s}}$ and $\beta = \sqrt{1-r}$.

The numerical results shown in Fig.2a, indicate clearly that the transverse modes dominate by far the total cross section. For instance, $\sigma_{TT}$ at $\sqrt{s} = 1$ TeV is almost three orders of magnitude larger than $\sigma_{LL}$. As announced, this confirms the absolute necessity of isolating the longitudinal from the transverse modes in the proposed $\gamma\gamma$ experiment. One possibility we have found will help in this task is by considering the differential cross section.
with respect to the angular variables. For instance, one may compare the
different patterns in Fig.2b for the three combinations TT, LT and LL in the
variable $\cos \theta$, where $\theta$ is the angle of the $W$ in the center of mass system.
By applying a cut of $|\cos \theta| \leq 0.8$ we see that the cross section for TT
modes gets reduced in almost a two orders of magnitude factor, whereas the
cross section for LL modes remains practically unchanged. Of course, a more
realistic study in terms of the leptonic and/or hadronic final states from the
$W$'s and $Z$'s decays should be done in order to be able to reach a definite
conclusion. For the rest of this work, we will assume that this separation
between the longitudinal and transverse modes can be done.

3 Observable cross sections

$\gamma \gamma \rightarrow W^+_L W^-_L$ and $\gamma \gamma \rightarrow Z_L Z_L$ reactions at TeV energies could, in principle,
be studied in $e^+e^-$ colliders by means of virtual photon fusion processes.
However, the photon-photon luminosity is too small at high energies as to
produce a significant number of $W$ pairs with large invariant mass ($0.5 \leq M_{WW} \leq 1$ TeV). Since it is in this $M_{WW}$ range where the effects we are
discussing are expected to show up, they seem to be hardly detectable in this
type of colliders.

We will analyze instead the experimental signals that one would obtain,
in the case of a SISBS, through $\gamma \gamma \rightarrow W^+_L W^-_L$ and $\gamma \gamma \rightarrow Z_L Z_L$ reactions
in a dedicated $\gamma \gamma$ collider, proposed by Ginzburg et al. [12]. They showed
how to transform a linear $e^+e^-$ or $e^-e^-$ collider into a $\gamma \gamma$ collider with
approximately the same energy and luminosity as the original $e^+e^-$ collider.
Each $\gamma$ beam is obtained by backward Compton scattering of laser light
focused on the $e^-$ beams, near the point where the beams of scattered photons
will finally collide. When laser photons with energy $\omega_0$ scatter off electrons
with high energy $E$ at very small scattering angle, the beam of scattered photons emerges along the incident $e^-$ direction with very small angular
spread, and an energy ($\omega$) spectrum given by [12]:

$$f(x, y) = \frac{2(1 + x)^2(2x^2 - 4xy - 4x^2y + 4y^2 + 4xy^2 + 3x^2y^2 - x^2y^3)}{(1 - y)^2((16 + 32x + 18x^2 + x^3) - 2(8 + 20x + 15x^2 + 2x^3 - x^4)\log(1 + x))}$$  \hspace{1cm} (23)

$$x = \frac{4E\omega_o}{m_e^2} \quad y = \frac{\omega}{E}$$  \hspace{1cm} (24)

The variable $x$ is related to the center-of-mass energy of the Compton process by $s_c = m_e^2(x + 1)$, and $y$ is the energy of the scattered photon in units of $E$. For a given value of $x$, determined by the experimental setup, there is an upper limit for the energy of the scattered photons $y_m$:

$$y \leq y_m = \frac{x}{x + 1}$$  \hspace{1cm} (25)

One would like to increase $x$ (i.e. $E$ or $\omega_o$) in order to get values of $y_m$ as high as possible. However, when $x > 2 + 2\sqrt{2}$, some other processes besides Compton scattering become important in the conversion region, mainly $e^+e^-$ pair production from a laser photon and a high energy scattered photon. In order to avoid these background processes, we have fixed $x$ to the value $2 + 2\sqrt{2}$ which implies $y_m = 0.828$. We have chosen, for definiteness, a $e^+e^-$ collider with integrated luminosity of 10 fb$^{-1}$ and beam energies of 0.5 and 1 TeV (the first case corresponds to the parameters of VLEPP [19].) By taking $x = 2 + 2\sqrt{2}$, these energies will imply using lasers with $\omega_o \simeq 0.3$ and 0.15 eV respectively.

The total observable cross sections for $\gamma\gamma \rightarrow W^+_LW^-_L$ and $\gamma\gamma \rightarrow Z_LZ_L$ reactions in the collider described above are obtained from the corresponding subprocess cross sections by convoluting them with the two photons spectra. For the charged channel:

$$\sigma(\gamma\gamma \rightarrow W^+_LW^-_L) = \int_{0}^{y_m} dy_1 \int_{0}^{y_m} dy_2 \left[ f(x, y_1) \cdot f(x, y_2) \right] \tilde{\sigma}(\gamma\gamma \rightarrow W^+_LW^-_L)$$  \hspace{1cm} (26)

where $\tilde{\sigma}$ is given in ChPT in eq.(18) and in the SM in eq.(20), and $f(x, y_i)$ is the spectrum of the $i$-photon given in eq.(23).

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2 We have taken the approximation of zero scattering angle $\alpha_o = 0$ and negligible beam spread at the focal spot $\rho \ll 1$. The conversion coefficient $\kappa$ is taken here equal to 1.
The differential cross section with the invariant mass of the final gauge bosons pair, $M_{VV}$, is then given by:

$$
\frac{d\sigma}{dM_{WW}} = \left[ \frac{d\mathcal{L}(z)}{dz} \right] \frac{1}{\sqrt{s}} \hat{\sigma}(\gamma\gamma \rightarrow W_{L}^{+}W_{L}^{-})
$$

(27)

where: $z = \frac{M_{WW}}{\sqrt{s}} = \sqrt{\frac{z}{s}}$; $\sqrt{s}$ is the total $e^+e^-$ energy, $\sqrt{\hat{s}}$ is the total $\gamma\gamma$ energy and the spectral photon-photon luminosity is given by:

$$
\frac{d\mathcal{L}(z)}{dz} = 2z \int_{z^{m}/ym}^{ym} dy \frac{f(x, z^{2}/y) f(x, y)}{y}.
$$

(28)

Similar expressions to eqs.(26) and (27) can be found for the neutral channel. Notice that the maximum value for $z$ is fixed to $y_{m}$, and therefore, the maximum value of the final $VV$ pair invariant mass $M_{VV}$ is 0.828 and 1.656 TeV for $\sqrt{s} = 1$ and 2 TeV respectively. The final numerical results for $\frac{d\sigma}{dM_{WW}}$ and $\frac{d\sigma}{dM_{ZZ}}$ are shown in Fig.3 and Fig.4, respectively. Only invariant mass values larger than 0.5 TeV are displayed for the charged channel. In this energy range, a perfect agreement between the exact tree level SM result and the corresponding one by using the Equivalence Theorem is found. The predictions for $\frac{d\sigma}{dM_{WW}}$ in ChPT show a sizeable enhancement over the SM result at high invariant mass $M_{VV}$ values, mainly for the $\sqrt{s} = 2$ TeV $e^+e^-$ collider. We have chosen in the plots, for definiteness, two particular values for $L_{9} + L_{10}$, but the same analysis can be performed for any other choice.

In order to study the sensitivity of these experiments to the values of the Chiral parameters we have computed the number of $W_{L}^{+}W_{L}^{-}$ events that are expected in one year at a dedicated $\gamma\gamma$ collider with parameters $\sqrt{s} = 1$ TeV and 2 TeV and integrated one-year-luminosity of $\mathcal{L} = 10$ fb$^{-1}$. We compare the predicted number of events within an invariant mass range of $0.5 \leq M_{WW} \leq M_{WW}^{max}$ for a given value of $(L_{9} + L_{10})$, $N_{ChPT}(L_{9} + L_{10})$, with the corresponding prediction in the SM, $N_{SM}$, that we take always as a reference number characterizing a weakly interacting SBS. For each chosen value of $(L_{9} + L_{10})$, we define our signal as the difference:

$$
\Delta N = N_{ChPT}(L_{9} + L_{10}) - N_{SM}.
$$

(29)
which represents the excess of $W_L^+W_L^-$ events over the SM prediction at large invariant mass $M_{WW} \geq 0.5$ TeV. For the upper limit of the invariant mass we take the maximum value allowed by unitarity constraints $M_{WW}^{max} = 1.5$ TeV in the $\sqrt{s} = 2$ TeV case and the maximum imposed by $y_m$, $M_{WW}^{max} = y_m\sqrt{s} = 0.828$ TeV in the $\sqrt{s} = 1$ TeV case.

In order to estimate the statistical significance of the effect we are looking for, we have also evaluated the variable $F$, defined as $F = \Delta N/\sqrt{N_{SM}}$, as a function of ($L_9 + L_{10}$). This variable is an estimator of the number of sigmas measuring the confidence level of the hypothesis that ($L_9 + L_{10}$) $\neq 0$ produces a noticeable effect. The results are given in Table 1, which show that this experiment will be indeed very sensitive to the values of ($L_9 + L_{10}$) $\neq 0$. Thus, for instance, if a value for the statistical significance as large as $F = 2$ is required, the values of ($L_9 + L_{10}$) that could be tested in a dedicated $\gamma\gamma$ collider are:

\[
(L_9 + L_{10}) \leq -\frac{0.9}{16\pi^2} \quad \text{and} \quad (L_9 + L_{10}) \geq \frac{1.5}{16\pi^2} \quad \text{for} \quad \sqrt{s} = 2 \text{ TeV}
\]

and

\[
(L_9 + L_{10}) \leq -\frac{2.4}{16\pi^2} \quad \text{and} \quad (L_9 + L_{10}) \geq \frac{3.0}{16\pi^2} \quad \text{for} \quad \sqrt{s} = 1 \text{ TeV}
\]

For the neutral channel, the corresponding number of events that are predicted in one-year running are of just $N_{ChPT} = 1$ at $\sqrt{s} = 1$ TeV and $N_{ChPT} = 6$ at $\sqrt{s} = 2$ TeV. Unfortunately, the lack of statistics will not allow to make any detailed analysis of the contributions of the Chiral loops in this channel.

\section{Conclusions}

The proposed dedicated $\gamma\gamma$ colliders at TeV energies, where real photons are obtained from backward Compton scattering of laser beams off high energy electrons, will offer the possibility of studying the reactions $\gamma\gamma \rightarrow W_L^+W_L^-$ and $\gamma\gamma \rightarrow Z_LZ_L$ in the high invariant mass region. We have profited from this advantage to study their potentiality in testing the Symmetry Breaking Sector of the SM and the longitudinal gauge bosons self-interactions.

Our study by means of a Chiral Lagrangian approach of these reactions show that the neutral channel will probe the pure strong $V_LV_L$ rescattering effects (i.e. Chiral loops) while the charged channel will also probe the
numerical values of the Chiral parameters $L_9$ and $L_{10}$. The charged channel cross section turns out to depend on the renormalization group invariant combination $(L_9 + L_{10})$ and therefore is a fully predictable quantity in ChPT.

As a result of our analysis on the expected sensitivities to $(L_9 + L_{10})$ we have been able to place bounds on their minimum positive and their maximum negative values that will be significantly tested in these experiments.

Of course, a definite conclusion can not be drawn until a complete analysis of the signal and backgrounds including the final decays of the $Z_L$’s and $W_L$’s be done.

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**Figure Captions**

**Fig.1** Predictions for the subprocess $\gamma\gamma \to V_LV_L$ cross sections to order $\alpha^2$ as a function of the $VV$ pair invariant mass $M_{VV}$.

(1a) Chiral prediction for $\gamma\gamma \to Z_LZ_L$ reaction. The SM cross section is zero at tree level.

(1b) Behaviour of the $\gamma\gamma \to W^+_LW^-_L$ cross section in the high $M_{WW}$ region. The Chiral prediction is shown for values of $L_9 + L_{10}$ equal to $1/16\pi^2$ (short-dashed line), $2/16\pi^2$ (dashed line) and $4/16\pi^2$ (long-dashed line), and compared with the SM result.

(1c) Cross sections for the $\gamma\gamma \to W^+_LW^-_L$ reaction. The SM prediction at tree level (solid line) is compared with the result obtained using the Equivalence Theorem approximation (dotts) and the complete Chiral prediction for $L_9 + L_{10} = 4/16\pi^2$ (dashed line).

**Fig.2** Contributions of the different polarization states (LL, LT and TT) of the final $W^+_LW^-_L$ pair to the SM cross sections in $\gamma\gamma \to W^+_LW^-_L$ reactions.

(2a) Total cross section as a function of $M_{WW}$.

(2b) Differential cross section as a function of $\cos \theta$, where $\theta$ is the scattering angle of the final $W$ in the CM-system.

**Fig.3** Differential cross section as a function of $M_{WW}$ for $W^+_LW^-_L$ pair production in the dedicated $\gamma\gamma$ collider described in the text.

(3a) Results for an original $ee$ collider with a CM energy of 2 TeV. The results obtained for the SM in the region $0.5 \leq M_{WW} \leq M_{WW}^{\text{max}}$ are compared with the Chiral prediction for $L_9 + L_{10} = 4/16\pi^2$ (long-dashed line) and $L_9 + L_{10} = -2/16\pi^2$ (short-dashed line). The points correspond to the predictions with the Equivalence Theorem approximation.

(3b) The same as 3.a for an $ee$ collider of 1 TeV CM energy.

**Fig.4** Differential cross section as a function of $M_{ZZ}$ for $Z_LZ_L$ pair production in the dedicated $\gamma\gamma$ collider described in the text. The results are shown for $ee$ colliders with CM energies of 1 (short-dashed line) and 2 (long-dashed line) TeV.
Table 1:

| $\sqrt{s} = 1$ TeV | $\sqrt{s} = 2$ TeV |
|---------------------|---------------------|
| $N_{SM} = 1421$     | $N_{SM} = 1416$     |
| $(L_9 + L_{10}) \times 16\pi^2$ | $(L_9 + L_{10}) \times 16\pi^2$ |
| $N_{ChPT}$ | $F$ | $N_{ChPT}$ | $F$ |
|----------------|----------|----------------|----------|
| - 4 | 1603 | 4.82 | - 4 | 2216 | 21.26 |
| - 3 | 1529 | 2.88 | - 3 | 1890 | 12.60 |
| - 2 | 1476 | 1.46 | - 2 | 1651 | 6.26 |
| - 1 | 1442 | 0.55 | - 1 | 1499 | 2.22 |
| 0  | 1427 | 0.16 | 0  | 1434 | 0.49 |
| 1  | 1432 | 0.29 | 1  | 1456 | 1.07 |
| 2  | 1456 | 0.94 | 2  | 1564 | 3.95 |
| 3  | 1500 | 2.10 | 3  | 1760 | 9.15 |
| 4  | 1563 | 3.78 | 4  | 2042 | 16.64 |