Non-Stationary Wind Pressure Prediction Based on A Hybrid Decomposition Algorithm of Wavelet Packet Decomposition and Variational Mode Decomposition

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Abstract. A hybrid decomposition algorithm is proposed for wind pressure prediction on structures based on wavelet packet decomposition (WPD), unit root test, and variational mode decomposition (VMD). In the proposed hybrid decomposition, the wavelet packet decomposition is employed to drop intermittent of the wind pressure. Then, the unit root test is adopted to ensure the stationary of the decomposed subseries. For the non-stationary subseries, the variational mode decomposition is employed to further decrease intermittent of the subseries. For the rest of the stationary subseries, they are reconstructed by frequency, which can be divided into high frequency, middle frequency and low frequency. Finally, the prediction process is established by the extreme learning machine (ELM). The three cases forecasting results indicated that: the proposed decomposition algorithm has the best forecasting accuracy compared to the relative decomposition methods.

1. Introduction
Wind load is one of the main design loads of high-rise structures and long-span structures such as bridges and stadiums. Recent research indicates that the wind pressure on the surface of these structures in typhoon shows the property of non-stationary [1]. Traditional research mainly uses the computer software of finite element analysis to model the wind pressure on the surface of the structures or makes the physical model to conduct the wind tunnel test to determine the wind pressure. With the rising and rapid developing of big data and data-driven technology, utilizing machine learning method can directly and smartly predict the wind pressure on the surface of the structures with data digging technology while avoiding complex modeling, time consuming wind tunnel test. It has great significance on future wind resistant design.

Due to the intrinsic volatility and randomness of non-stationary wind pressure on the surface of buildings, it is difficult to obtain a satisfactory prediction. A great number of researches on structure wind pressure prediction mostly focus on: (a) upgrading prediction models, such as neural network or support vector machine [2-3]; (b) optimizing prediction models, such as hybrid artificial swarm optimization and artificial fish swarm optimization or hybrid particle swarm optimization and firefly algorithm [4-5]; (c) selecting satisfied decomposition for original wind pressure. However, the existing decomposition method is not effective enough for the complicate original wind pressure. This paper proposed a hybrid decomposition method to further reduce the complexity of non-stationary wind pressure in order to improve the prediction accuracy.
2. Hybrid decomposition theory

Figure 1 demonstrates the framework of the proposed hybrid decomposition algorithm. From Figure 1, the detailed proposed algorithm need to state as follows:

- The original wind pressure should be decomposed into a number of subseries by WPD method at first (Step 1 in Figure 1).
- Adopting the unit root test to confirm the stationarity of the obtained WPD subseries. All the subseries will be divided into two types: stationary WPD subseries and non-stationary WPD subseries. For the stationary subseries, they can be reconstructed by its frequency: high frequency stationary WPD subseries, middle frequency stationary WPD subseries and low frequency stationary WPD subseries (Step 2 in Figure 1).
- For the non-stationary WPD subseries, they should be further decomposed by the VMD method (Step 3 in Figure 1).
- The final work of the hybrid decomposition is to construct the hybrid decomposition subseries using the obtained high frequency stationary WPD subseries, middle frequency stationary WPD subseries, low frequency stationary WPD subseries and all the VMD modes.

The following sections are brief description of the methods used in the proposed hybrid decomposition procedure.

2.1. Wavelet theory

Wavelet transform is an ideal tool for processing signal at the level of time-frequency. WPD method is an extension of wavelet decomposition, which can effectively deal with detailed components that cannot be handled with in wavelet decomposition. For this reason, WPD method is used to decompose the wind pressure into relatively stationary subseries and its mathematic formula can be seen as follows:

$$CWT_f(a,b) = \langle f(t), \Psi_{a,b}(t) \rangle = \int_{-\infty}^{\infty} f(t) \cdot \overline{\Psi}(t) \cdot ( (t - b) / a ) \cdot \sqrt{ad} dt$$

(1)

where \( f(t) \) is wind pressure, \( \Psi(t) \) is the wavelet function and \( a, b \) is the scale coefficient and the translation coefficient, respectively. In this paper, the wavelet ‘\text{db10}’ and the level ‘3’ are applied to decompose the original wind pressure. The detailed WPD algorithm can be found in reference [6].

2.2. Unit root test

As a mathematical method, the unit root test is utilized to test whether a section of time series has a unit root or not. If the unit root exists, the time series is non-stationary. This method is widely used in financial data such as stock price and adopted to judge the variability of data. In this paper, EVIEWS

![Figure 1. Framework of hybrid decomposition](image-url)
platform is carried out to test the stationary of the WPD subseries. The detected non-stationary WPD subseries need further decomposition, which is more conducive to improving the accuracy of prediction.

2.3. Variational mode decomposition

Variational mode decomposition (VMD), a new signal time-frequency processing method, is applied to decompose a signal into a number of band-limited modes. A centre frequency of each mode is adaptively determined during the decomposition process. In addition, the number of the VMD modes can be determined before the decomposition. As a result, VMD is utilized to decompose the non-stationary WPD subseries into five modes in order to obtain more relative stationary wind pressure subseries. Actually, the variational mode decomposition can be seen as a constrained optimal variational problem as follows:

\[
\min_{\{u_k, \omega_k\}} \sum_{k=1}^{n} \left\{ \int \left[ \delta(t) + \frac{1}{\pi t} \right] * u_k(t) \right\} e^{-j\omega_k t} \]

(2)

\[
s.t. \sum_k u_k = f_i
\]

(3)

where \(f_i\) is the wind pressure signal, \(u_k\) is the mode and \(\omega_k\) is the centre frequency. The specific decomposition process can be found in reference [7].

2.4. Forecasting Predictor

There are some types of artificial neural network. Extreme learning machine (ELM) is a convenient and feasible single hidden feedforward neural network with fast computational speed and high generalization ability. Owing to the randomly generating parameters without repeatedly adjusting rather than traditional neural network, the ELM has the satisfactory performance both in the learning accuracy and speed. As a result, it is a good wind pressure predictor. The structure of the ELM is illustrated in Figure 2. The complete theory of the ELM method is proposed in reference [8].

3. Forecasting experiment

3.1. Wind pressure cases

There are three cases of actual wind pressure on the surface of different structures: a high-rise building in Qingdao [9], a bridge in Hong Kong [10] and a stadium in Wenzhou [11]. Each case includes 1000 samples shown in Figure 3. Research in [9-11] indicates that the wind pressure on the structure surface is non-stationary. In this paper, the 1st-800th samples of these cases are used to establish the ELM multi-step training sets, while the rest 801st-1000th samples in each case is used to complete practicing sets. In order to evaluate the forecasting accuracy of the proposed HYBRID-ELM model, three other decomposition methods are adopted as comparison: the ELM model, the WPD-ELM model and the VMD-ELM model.
3.2. Forecasting Performance measurement

Four error indexes are used to measure the accuracy of all the adopted decomposition methods, including the Mean Absolute Error (MAE), the Root Mean Square Error (RMSE), the Mean Absolute Percentage Error (MAPE) and the correlation coefficient $R$. The indexes of MAE and RMSE are utilized to quantify the accuracy of forecasting, and the smaller they are, the better the prediction is. While the performance measurement of correlation coefficient $R$ is on the contrary, therefore the larger it is, the better the prediction is. The MAPE is an important index to measure the error of the forecasting. In this study, it has two special ways to reflect the prediction performance: (a) a bar chart of the sum of MAPE index in all multi-step forecasting; (b) the MAPE index promoted percentages of the hybrid decomposition model. The lower the sum as well as the higher the percentage is, the better the performance is. The equations of the four indexes are given as:

$$MAE = \frac{1}{N} \sum_{i=1}^{N} |Y_i - \hat{Y}_i|$$  \hspace{1cm} (4)

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (Y_i - \hat{Y}_i)^2}$$  \hspace{1cm} (5)

$$MAPE = \frac{1}{N} \sum_{i=1}^{N} \frac{|Y_i - \hat{Y}_i|}{Y_i} \times 100\%$$  \hspace{1cm} (6)

$$R = \left( \sum_{i=1}^{N} Y_i \hat{Y}_i \right) / \left( \sqrt{\sum_{i=1}^{N} Y_i^2} \sqrt{\sum_{i=1}^{N} \hat{Y}_i^2} \right)$$  \hspace{1cm} (7)

3.3. Forecasting results

Three cases have been utilized to prove the accuracy of the forecasting based on the proposed hybrid decomposition in this paper. Part b in Figure 4 shows the computational results at 801st-1000th samples of the original wind pressure and part a in Figure 4 shows the sum of MAPE index in 1-step, 3-step and 5-step forecasting. The MAPE index promoted percentages can be calculated as given in Table 2. Other indexes results are indicated in Table 1. From the figures and the tables, it can be clearly analysed that the HYBRID-ELM model has a great forecasting performance among all the models in three cases:

![Figure 4. The results of the multi-step forecasting](image)
### Table 1. Forecasting error index

| Index | 1-step | 3-step | 5-step | 1-step | 3-step | 5-step | 1-step | 3-step | 5-step |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| HYBRID-ELM |       |        |        |        |        |        |        |        |        |
| Case 1 | 1.1504 | 4.2310 | 7.6939 | 0.0967 | 0.3019 | 0.4734 | 0.2755 | 0.8018 | 1.4772 |
| Case 2 | 1.4383 | 5.4591 | 9.6871 | 0.1763 | 0.4543 | 0.6381 | 0.4027 | 1.1428 | 1.9838 |
| Case 3 | 0.9999 | 0.9995 | 0.9985 | 0.9998 | 0.9988 | 0.9977 | 0.9999 | 0.9994 | 0.9983 |

### Table 2. MAPE promoted percentages

| MAPE | HYBRID-ELM vs. ELM | HYBRID-ELM vs. WPD-ELM | HYBRID-ELM vs. VMD-ELM |
|------|-------------------|------------------------|------------------------|
| Case 1 |                 |                        |                        |
| 1-step | 90.32%           | 60.13%                 | 36.56%                 |
| 3-step | 68.42%           | 54.17%                 | 41.05%                 |
| 5-step | 29.02%           | 10.74%                 | 24.26%                 |
| Case 2 |                 |                        |                        |
| 1-step | 89.37%           | 71.37%                 | 45.60%                 |
| 3-step | 80.63%           | 68.98%                 | 59.30%                 |
| 5-step | 71.75%           | 48.76%                 | 49.34%                 |
| Case 3 |                 |                        |                        |
| 1-step | 89.37%           | 71.37%                 | 45.60%                 |
| 3-step | 78.44%           | 43.21%                 | 55.86%                 |
| 5-step | 65.75%           | 39.68%                 | 32.07%                 |

- The MAE and RMSE in HYBRID-ELM model are much lower than other three models for generated forecasting and the correlation coefficient R is little higher in the former model than the other three models. For example, in 3-step forecasting of case 1, the MAE from HYBRID-ELM model to VMD-ELM model is 4.2310, 17.6803, 6.4136 and 9.5025, respectively; the RMSE is 5.4591, 23.9932, 8.147 and 13.0906, respectively; and the correlation coefficient R is 0.9999, 0.9941, 0.9993 and 0.9978, respectively.

- It is easy to find a phenomenon that the HYBRID-ELM model as well as other models has considerably better accuracy in 1-step forecasting than 3-step and 5-step forecasting in all cases.

- The decomposed models have much more effective forecasting accuracy than the undecomposed model and the proposed hybrid decomposed models has much better prediction performance than the single decomposed models. For example, in 5-step forecasting of case 3, the MAE of three decomposed models is 1.4772, 2.8052 and 3.8898, respectively, much lower than the undecomposed model of 4.3837 and the RMSE of three decomposed models is 1.9838, 3.9753 and 2.6389, respectively, still lower than the undecomposed model of 5.9254.

- The HYBRID-ELM model promotes the MAPE index in all cases. For instance, in the 1-step prediction of Case 2, the MAPE promoted percentages of the HYBRID-ELM model by the ELM model, the WPD-ELM model and the VMD-ELM model is 89.37%, 71.37% and 45.60%, respectively; in the
3-step prediction, it is 80.63%, 68.98% and 59.30%, respectively; and in the 5-step prediction, it is 71.75%, 48.76% and 49.34%, respectively. Moreover, part b in Figure 4 proves that the sum of MAPE index in multi-step forecasting of HYBRID-ELM model is considerably lower than the relevant models.

4. Conclusion
In the paper, two time-frequency decomposition methods are combined by the unit root test to further decompose the non-stationary wind pressure. According to the above analysis of the three cases multi-step forecasting results, it can be concluded that the proposed hybrid decomposition algorithm can appropriately decompose the non-stationary wind pressure, effectively reduce the complexity of the original wind pressure and obviously improve the prediction performance.

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