Students’ thinking process in solving mathematical literacy problem with space and shape content

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Abstract. Thinking process is one of the important activities in solving mathematical literacy problem. Thus, this paper aimed to identify two students' thinking process in solving mathematical literacy problem with space and shape content. This study used descriptive research with a qualitative approach. Research subjects in this study were two tenth grade students of middle school. Data were collected by giving the mathematical literacy problem with space and shape contents and using think aloud. Next, data were analysed by studying the three types of thinking processes: conceptual, semi conceptual, and computational. The results of this study pointed out that the students' thinking process were conceptual and computational. The activities carried out by students could be grouped into four categories, namely 1) stating what is known in the problem with their own language or changing in mathematical sentences, 2) stating what is asked in a problem with their own language or changing in mathematical sentences, 3) making a plan to solve problem completely, and 4) stating the steps taken in solving the problem using the concepts that have been studied. The type of thinking process of students influences the activities carried out by students in solving mathematical literacy problem so that teachers were supposed to consider this aspect when teaching mathematics especially topics that related to mathematical literacy.

Keywords: students, thinking process, mathematical literacy problem, space and shape contents

1. Introduction

The thinking process of students is a very important aspect in learning. The thinking process influences students' ability to understand concepts and solve problems in mathematics. Sanjaya et al [1] said that there is a need to identify students’ thinking process when solving mathematical problems, which could picture the quality of students problem-solving skill. Furthermore, it is very important for educators to know the thinking process of students in solving mathematical problem so that we can know the type and location of the errors [2]. But many students experience problems in thinking process. Jazuli [3] stated that the fundamental problem experienced by our students is the low quality in the mathematical thinking process. In addition, Ngilawajan [4] said that many facts in the field that still show mathematics learning only appear as a monotonous and procedural activity, namely the teacher explains the material, gives an example, assigns students to do the exercises, checks students' answers in passing, then discusses the solution problems which are then emulated by students.
According to Yani, Ikhsan, and Marwan [5] the thinking process of students can run well if there is a role of teachers in helping students to get good and correct results in accordance with what they want. One example of teacher participation is to ask for answers that have been obtained by students according to what is on his mind. Thus, the teacher will find out where the students' understanding of the material being taught, and the teacher can find out the mistakes made by students in solving mathematical problems. Errors made by students can be used as a source of information for teachers to be able to design a learning that is in accordance with the thinking process of students.

The results of preliminary observations conducted by researchers at one of the high schools in Sidoarjo showed that students had difficulty in solving problems and sometimes they complained if faced with mathematical problems (Figure 1). There are also those who solve mathematical problems just by looking at examples of problems in the book without tracing the process, besides that students often despair because they think solving mathematical problems is too difficult.

Figure 1. Result of Preliminary Observation

Zuhri [6] reveals that the thinking process is divided into three types, namely conceptual, semi conceptual, and computational thinking processes. The process of conceptual thinking is a way of thinking that always solves problems by using the concepts that have been owned based on the results of his assessment so far. The semi conceptual thinking process is a way of thinking that tends to solve problems using concepts but lacks understanding of the concept so that the solution is mixed with ways of solving using intuition. Whereas computational thinking process is a way of thinking which in general in solving problems tends to rely on intuition and not using concepts.

Based on the description, it is necessary to have a deeper study of the types of thinking processes of students in solving problem. Researchers chose the mathematical literacy problem because based on the results of international assessment that measured the mathematical literacy, Indonesian students got the using level 2, because Indonesian students had been able to complete or occupy mathematical literacy at level 2 in the Program for International Student Assessment (PISA) task. Mathematical literacy problem used in this study was mathematics problems with real world contexts and to solve it, students must formulate, use, and interpret mathematical knowledge in various contexts.

Moreover, the content chosen in the mathematical literacy problem is the content of space and shape contents because according to Beaton et al. [7] male tend to be better than female in solving problem that include constructive representations of space. Then the results of research by the United
States Department of Education found that male were slightly better than female in mathematics and science. Even so, on average girls are better students and they are significantly better than boys in reading [8]. The differences in the abilities of male and female allow for different processes of thinking between male and female in solving mathematical literacy problem, in line with the results of a study conducted by Azizi [9] that female students' mathematical literacy is better than male students in terms of context, content, process and level.

Based on the results of a study conducted by Beaton et al. [7], Santrock [8], Azizi [9], researchers are interested in conducting research on students' thinking process solving mathematical literacy problem. Thus, this paper aimed to identify two students' thinking process in solving mathematical literacy problem with space and shape content, whether they had conceptual, semi conceptual, or computational thinking processes.

2. Method
This study used the descriptive research with qualitative approach. Research subjects in this study were two tenth grade students of middle school. The selection of research subjects was based on recommendations from classroom teachers to choose students who have the same mathematical abilities and were able to communicate their ideas, so that when thinking aloud students can communicate the strategies used. In addition, classroom teachers chose research subjects that had medium mathematics achievers by considering the mathematics scores.

Data collection to identify students' thinking process was conducted by asked research subjects to solve the mathematical literacy problem. Table 1 showed mathematical literacy problem with space and shape content by adopting from PISA task. Before the mathematical literacy problem was used, the validity was performed through an expert review to maintain the quality of problem, i.e., construct validity, content validity, and face validity.

After students finish mathematical literacy problem, we did think aloud to identify students' thinking process. The method of thinking in this study uses introspection techniques. According to Djwandono [10], introspection was that respondents report their thought processes while working on something. This introspection technique was used to make the subjects examine the thinking process itself. In this study to check the validity of the data we referred to Siswono [11] that the technique of checking the validity of the data could use the rich method. Rich was done by describing the results of the aloud thinking with the results of the student's mathematical literacy problem to obtain the type of thinking process of students in solving mathematical literacy problem.

Data analysis from the results of think-aloud in this study were using qualitative analysis that consist of three stages, namely condensation data, presentation data, and drawing conclusions by referring to the opinions of Miles, Huberman, & Saldana [12]. Then, we used method triangulation to maintain the quality of research by comparing the answers of participants and the results of the think-aloud.
Table 1. Mathematical literacy problem using space and shape contents

| Problem |
|---------|
| Figure 1 below illustrates a staircase with 14 steps and a total height of 252 cm. |

What is the height of each of the 14 steps above?

![Figure 1: Staircase with 14 steps and total height of 252 cm.](image)

| Total depth | Total height |
|-------------|--------------|
| 400 cm      | 252 cm       |

3. Results and Discussion

3.1. Male student’s thinking process in solving mathematical literacy problem with space and shape content

The results of the male student’s answer analysis from mathematical literacy problem with space and shape content using level 2 can be explained as follows.

3.1.1. Able to express what was known in the problem with its own language or change in a mathematical sentence

The answer of the male student in solving mathematical literacy problem part 1 can be seen in Figure 2. From this figure, male student knew that there were 14 steps with total height is 252 cm, and length 400 cm.

![Figure 2: Male Student’s Answer Part 1](image)

Translation:

Known: There are 14 steps, total height: 252 cm

\[ l = 400 \text{ cm} \]

The results think aloud using introspection when male student solving mathematical literacy problem with space and shape content were the male student revealed and wrote completely and correctly what was known in the problem with his own language or changed in mathematical sentences. He said, “First I search for it, here it was there 14 steps, continuing with a total height of 252 cm, and here was 400 cm wide”. This shows that the male student was able to express what was known in the problem with his own language or change in a mathematical sentence.
3.1.2. Able to express what was asked in a problem with their own language or change in a mathematical sentence. The results of the mathematical literacy problem with male student space section 2 can be seen in Figure 3. Male student could know that the problem asked about the height of each of the 14 steps.

![Translation: Asked: What is the height of each of the 14 steps above?](image)

**Figure 3. Male Student’s Answer Part 2**

Based on the results think aloud using introspection when male student solving mathematical literacy problem with space and shape content, the male student revealed and wrote completely and correctly what was asked in the problem with their own language or changed in mathematical sentences. This shows that the male student was able to express what was asked in the problem with was own language or change in a mathematical sentence.

3.1.3. Make a complete solving plan. The results of mathematical literacy problem answers to the space and shape content of male student’ part 3 can be seen in Figure 4. Male student made plan to get the solution by divided the total height with the number of steps.

![Translation: Plan: divide total height by the number of steps](image)

**Figure 4. Male Student’s Answer Part 3**

Following was the conversation between researcher (P) and male (L) when we conducted think aloud using introspection:

L: "The plan was that we will divide from the total height was the same as the number of steps."

P: "Are you sure?"

L: "Yeah."

P: "Why?"

L: "So it's the height of each step, so we have to include the steps too."

From the results think aloud using introspection when male student solving mathematical literacy problem with space and shape content, the male student revealed and wrote a complete and correct completion plan. This shows that the male student made a complete solving plan.

3.1.4. Able to state the steps taken in solving problem using concepts that have been studied. The results of the mathematical literacy problem by thinking aloud using introspection, the answers to the mathematical literacy problem with space and shape content of the male student part 4 can be seen in Figure 5. Male student could write down the procedure well. She stated that the height of each of the 14 steps could be completion by divided total height with the number of steps.
Following was the conversation between researcher (P) and male (L) when we conducted think aloud using introspection:

L: "Then we use the solution method, the total height divided by the number of steps. We enter the total height of 252 cm per number of steps 14, this was calculated 18. So, the height of each stair of 14 steps was 18 cm."

P: "I have already learned about solutions such as this?"

L: "Already."

P: What time?

L: "This division was in the 6th grade of elementary school."

P: "How was it before, how come it seems like it can be done right away?"

L: "This was a bit light. So that's how it was."

The results think aloud using introspection when male student solving mathematical literacy problem with space and shape content were the male student revealed and wrote down the steps to complete correctly in solving the problem using the concepts that had been studied. This shows that the male student was able to state the steps taken in solving the problem using the concepts that have been studied.

3.1.5. Not able to correct mistakes in the answers. From the results of mathematical literacy problem with think aloud using introspection in solving the problem with space and shape content, the male student did not check the answers and did not correct the answers if there was an error in the problem-solving process. This shows that the male student was unable to correct the error of the answer.

Based on the previous explanation about male student’s data in solving problem, i.e. able to express what was known in the problem with its own language or change in a mathematical sentence, able to express what was asked in a problem with their own language or change in a mathematical sentence, make a complete solving plan, able to state the steps taken in solving problem using concepts that have been studied, not able to correct mistakes in answers, the type of male student thinking process was a type of conceptual thinking process [6].

3.2. Female student's thinking process solving mathematical literacy problem with space and shape content. The results of the analysis of mathematical literacy problem by thinking aloud using introspection of female student in solving mathematical literacy problem using content and content with level 2 can be explained as follows.
3.2.1. **Lack of being able to express what was known in a problem with its own language or change in a mathematical sentence.** The results of the answers to problem about the space and shape content of female student’s part 1 can be seen in Figure 6. From this figure, female student wrote down what known from the problem. She used symbol “h” to represent the total height and “L” to represent the length of stair. But it was not common to use symbol “L” (using letter capital) for length because this symbol usually used to represent area.

![Figure 6. Female Student’s Answer Part 1](image)

| Known = h = 252 cm |
|---------------------|
| L = 400 cm          |

**Translation:**

Known = h = 252 cm  
L = 400 cm

From the results of mathematical literacy problem with think aloud using introspection in solving problem with level 2 with space and shape content, female student revealed and wrote incomplete or there was something wrong known in the problem with their own language or changed in mathematical sentences. This shows that the male student was less able to express what was known in the problem with their own language or change in mathematical sentences.

3.2.2. **Not able to state what was asked in a problem with their own language or change in a mathematical sentence.** The results of female student’s working to solving mathematical literacy problem can be seen in Figure 7. From this figure, it could be known that she thought the problem asked about the length.

![Figure 7. Female Student’s Answer Part 2](image)

| Dibahasakan = |
|--------------|
| ?            |

**Translation:**

 Asked = Length?

Following was the conversation between researcher (P) and female (M) when we conducted think aloud using introspection:

P: "What was asked?"
M: "Long."
P: What length? "
M: "Long steps."
P: "Which one?"
M: "This one." (Pointing 14 steps at Picture 1)

Based on the results of mathematical literacy problem with think aloud using introspection in solving mathematical literacy problem with space and shape content using level 2, the female student misrepresented and wrote what was asked in the problem with their own language or changed in mathematical sentences. This shows that the female student was not able to express what was asked in the problem with their own language or change in mathematical sentences.
3.2.3. Not making a solving plan. Next, the answers to problem about the space and shape content of female student’ part 3 can be seen in Figure 8 and the conversation. She made planning of solution by calculated length of steps multiplied the width of the steps was divided by 14.

| Perencanaan | Penyelesaian |
|-------------|--------------|
| Panjang anak tangga $\times$ Lebar anak tangga | 14 |

**Translation:**
Planning of solution
\[
\frac{\text{the length of steps} \times \text{width of steps}}{14}
\]

**Figure 8.** Female Student’s Answer Part 3

Following was the conversation between researcher (P) and female (M) when we conducted think aloud using introspection:
P: "How are you planning?"
M: "Planning for completion, length of steps multiplied the width of the steps was divided by 14. 14 of each of these steps, number 14."
P: "Are you sure that's the plan?"
M: "Yes, sure."
P: "Why are you sure? how come?"
M: "Sure you miss it."

From the results of mathematical literacy problem by thinking aloud using introspection in solving mathematical literacy problem with level 2 space and shape content, the female student revealed and wrote the wrong solution plan. This shown that the female student did not make a solving plan.

3.2.4. Not able to state the steps taken in solving problem using concepts that have been studied. The results of mathematical literacy problem by thinking aloud using introspection can be seen in Figure 9. Female student wrote down the procedure to get the height of each of the 14 steps.

| langkah-langkah: $\frac{P \times L}{14}$ |
|-----------------|-----------------|
| $\frac{252 \times 400}{14}$ |

**Translation:**
Steps: $\frac{P \times L}{14} = \frac{252 \times 400}{14} = \frac{14}{14}$

**Figure 9.** Female Student’s Answer Part 4
Following was the conversation between researcher (P) and female (M) when we conducted think aloud using introspection:

M: "The steps, the length of steps was divided by 14. The length was 252 times 400 divided by 14. This multiplication was immediately put here, then divided by 14."

P: “Why was the multiplication not done?”

M: "Confused, mom."

P: What are you confused?

M: "The truth, mom." (While designate the multiplication calculation done on the answer sheet)

P: What did you use usually?

M: "Use this, but confused, mom."(Pointing the multiplication calculation done on the answer sheet)

The results of mathematical literacy problem with think aloud using introspection in solving mathematical literacy problem with level 2 with space and shape content shown that female student revealed and wrote wrong completion steps in solving problem using concepts that had been studied. This shows that the female student reaching the indicators was not able to state the steps taken in solving the problem using the concepts that have been studied.

3.2.5. Not able to correct mistakes in answers. From the results of mathematical literacy problem by thinking aloud using introspection in solving problem with space and shape content, the female student did not re-examine the answers and did not correct the answers if something went wrong in the problem-solving process. This shows that the female student was unable to correct the error of the answer.

Based on the female student’s data in solving problem, i.e. lack of being able to express what was known in a problem with its own language or change in a mathematical sentence, not able to state what was asked in a problem with their own language or change in a mathematical sentence, not making a solving plan, not able to state the steps taken in solving problem using concepts that have been studied, not able to correct mistakes in answers the type of thinking process for female student was a type of computational thinking process [6].

Moreover, findings on student's thinking process in solving mathematical literacy problem with space and shape content are different from studies conducted by Karim [13] that in solving mathematical problem male students tend to be semi-conceptual types of thinking processes, while female students are more likely to be process types thinking conceptually. This was because in this study, male students tended to have conceptual thinking processes, while female students had computational thinking processes.

4. Conclusions

In solving mathematical literacy problem using space and shape content with level 2, the male student was able to state what was known in the problem with his own language or change in mathematical sentences, able to express what was asked in the problem with his own language or change in mathematical sentences, make a complete plan of completion, able to state the steps taken in solving the problem using the concepts that have been studied, and unable to correct the error of the answer. Thus, the type of process of thinking of male student in solving mathematical literacy problem using space and shape content with level 2 was a type of conceptual thinking process.

On the other hand, female student in solving mathematical literacy problem using space and shape content with level 2 were unable to express what was known in the problem with their own language or change in mathematical sentences, unable to express what was asked in a problem with their own language or change in mathematical sentences, did not make a solving plan, was unable to state the steps taken in solving the problem using the concepts that have been studied, and was unable to correct the error of the answer. Thus, the type of process of thinking of female student in solving mathematical literacy problem using space and shape content with level 2 was a type of computational thinking process.
We suggest using a mathematical literacy problem for another content, for example change and relationship contents or uncertainty and data contents. This was a complete description on students' thinking about the various content used in problem. Moreover, further studies could also use mathematical literacy problem with higher level, namely level 4, level 5, and level 6.

Acknowledgment
We would like to thank Ministry of Higher Education and Research on “Penelitian Dosen Pemula” grant.

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