A SPECTROSCOPIC ANALYSIS OF THE ENERGETIC TYPE Ic HYPERNOVA SN 1997ef

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Received 2000 May 31; accepted 2000 July 25

ABSTRACT

The properties of the bright and energetic Type Ic SN 1997ef are investigated using a Monte Carlo spectrum synthesis code. Analysis of the earliest spectra is used to determine the time of outburst. The changing features of the spectrum and the light curve are used to probe the ejecta and to determine their composition, verifying the results of explosion calculations. Since synthetic spectra computed using our best explosion model, CO100, are only moderately good reproductions of the observations, the inverse approach is adopted, and a density structure is derived by demanding that it gives the best possible fit to the observed spectrum at every epoch analyzed. It is found that the density structure of model CO100 is adequate at intermediate velocities (5000–25,000 km s$^{-1}$), but that a slower density decline ($\rho \propto r^{-4}$) is required to obtain the extensive line blending at high velocities (25,000–50,000 km s$^{-1}$) that is the characterizing feature of this and other energetic Type Ic Supernovae. Also, the inner “hole” in the density predicted by the model is found not to be compatible with the observed evolution of the spectrum, which reaches very low photospheric velocities at epochs of about 2 months. The “best-fit” density distribution results in somewhat different parameters for the SN, namely an ejecta mass of 9.6 $M_\odot$ (vs. 7.6 $M_\odot$ in CO100) and an explosion kinetic energy of $1.75 \times 10^{52}$ ergs (vs. $8 \times 10^{51}$ ergs in CO100). This revised value of the kinetic energy brings SN 1997ef closer to the value for the “prototypical” Type Ic hypernova SN 1998bw. The abundance distribution of model CO100 is found to hold well. The modified density structure is used to compute a synthetic light curve, which is found to agree very well with the observed bolometric light curve around maximum. The amount of radioactive $^{56}$Ni produced by the SN is confirmed at 0.13 $M_\odot$. In the context of an axisymmetric explosion, a somewhat smaller kinetic energy than that of SN 1998bw may have resulted from the nonalignment of the symmetry axis of the SN and the line of sight. This might also explain the lack of evidence for a gamma-ray burst correlated with SN 1997ef.

Subject headings: supernovae: general — supernovae: individual (SN 1997ef) — line: formation — line: identification — line: profiles

1. INTRODUCTION

SN 1997ef in UGC 4107 was recognized as a peculiar and interesting object as soon as its first spectra were taken by the Harvard-CfA team (Garnavich et al. 1997a, 1997b, 1997c). The spectra displayed very broad features, quite unlike those of any other SN known to date, so that it was not even clear whether what was observed was an absorption or an emission spectrum. Continued observation revealed a light curve typical of a SN deriving from a compact object (a SN Ia or a SN Ic), but much broader than that of a Type Ia or an emission spectrum. Continued observation showed that the SN was therefore in the photospheric epoch. Lines of Ca II, O I, and Fe II were strong, and the Si $\lambda$ 6347, 6371 line appeared to be comparatively weak, so the SN was classified as Type Ic, as also supported by the overall similarity with the spectra of other SNe Ic such as SN 1994I (Filippenko 1997; Millard et al. 1999).

Typical models for a low-mass, low kinetic energy SN Ic (Nomoto et al. 1994; Iwamoto et al. 1994) gave good fits to the light curve, but the synthetic spectra computed using such models, while yielding the correct types of spectral lines, completely failed to reproduce the observed large line width, which we estimated as at least 20,000 km s$^{-1}$ for some of the strongest lines (Iwamoto et al. 2000, hereafter Paper I).

Soon after SN 1997ef, however, the extraordinarily bright and powerful Type Ic SN 1998bw was discovered as the optical counterpart of GRB 980425 (Galama et al. 1998). A great deal of study, both observational and theoretical, was and still is being devoted to this SN, which was soon realized to be a massive Type Ic event of exceptionally large kinetic energy (KE $\sim 3 \times 10^{52}$ ergs) (Iwamoto et al. 1998; Woosley, Eastman, & Schmidt 1999; Branch 2000), where the link with the GRB may lie in a rather asymmetric explosion (MacFadyen & Woosley 1999; Höflich, Wheeler, & Wang 1999; Khokhlov et al. 1999; Maeda et al. 2000). It was only natural then also to analyze SN 1997ef as a powerful hypernova. Paper I presents a detailed study of the light curve and the spectrum using two models, one for a “normal” SN Ic (model CO60: $M_{ej} = 4.4 M_\odot$, KE $= 10^{51}$ ergs) and another for a hypernova (model CO100: $M_{ej} = 7.6 M_\odot$, KE $= 8 \times 10^{51}$ ergs). Both models have a $^{56}$Ni mass of 0.15 $M_\odot$. In Paper I we showed that both models reproduce the light curve reasonably well, but only the more massive and energetic model CO100 can also produce reasonable synthetic spectra, and therefore we opted for the hypernova model and suggested that SN 1997ef may be a somewhat less extreme case than SN 1998bw. A possible connection with a GRB has been searched for in the archives for SN 1997ef. The result was that GRB 971115

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could be compatible with SN 1997ef in position and time of occurrence (Wang & Wheeler 1998), but the statistical significance of the correlation is much weaker than for SN 1998bw and GRB 980425.

In this paper we concentrate on the spectroscopy of SN 1997ef. This is interesting in many respects. First, it is important that we not only fit the spectra and their evolution using one explosion model, but that we also understand which lines are present in the spectrum. Since we have available a sufficiently large number of observed spectra, we can follow the spectral evolution and thus probe different depths in the ejecta and verify the composition as a function of velocity, and also describe how the spectrum changes as a function of luminosity as the SN moves along the light curve. We use for this purpose the Monte Carlo (MC) code originally described in Mazzali & Lucy (1993) and further improved and discussed by Lucy (1999) and Mazzali (2000). The code includes the effect of photon branching, which must be very important in a situation in which the high expansion velocities lead to extensive line blanketing. Also, since the code uses the bolometric luminosity as input, we can compare the required \( L \) values with those of a light-curve calculation relaxing the assumption \( BC = 0.0 \) made in Paper I and thus improve the comparison between model \( L_{bol} \) from light-curve calculations and observed \( V \) magnitude.

Furthermore, we use spectrum synthesis to date the earliest spectra, and thus to determine with some accuracy the time of explosion, which is important for both the GRB connection and the light-curve calculation.

Finally, having noticed that even synthetic spectra obtained from CO100 do not give wonderfully good fits to the observed spectra, we adopt the “inverse” approach and try to determine the density and abundance distribution with velocity by looking for a best fit to the spectra. This method was pioneered by Branch (2000), who suggested for SN 1997ef a much flatter density dependence \( \rho \propto r^{-2} \) than what is predicted by model CO100 (where the density index is \( n = -7 \)). Consequently, for SN 1997ef Branch (2000) obtains a much larger ejecta mass \((6 \, M_\odot \text{ above } v = 7000 \, \text{km s}^{-1})\) and kinetic energy \((3 \times 10^{52} \, \text{ergs above the same velocity})\).

Since we study several spectra we can control \( \rho \) as a function of \( \epsilon = r/t \), and hence build our own \( \rho(r) \). We do confirm that the spectra require a flatter density dependence, but we show that this is only necessary at high velocity \((v > 25,000 \, \text{km s}^{-1})\). Examining spectra at advanced epochs, we also show that the predicted density “hole” at low velocities is not compatible with observations. We therefore derive new values for both \( M_{ej} \) and KE.

We take this process one step further by computing a synthetic light curve from our derived \( \rho(r) \), and show that this gives a very good fit to the observed light curve of SN 1997ef.

In the rest of this paper we first describe how we proceeded to date the spectra, then discuss the analysis of six spectra, reviewing the results and their implications. Then we review the properties of the model for SN 1997ef as we derived it, and discuss how it differs from model CO100, which was computed to fit the light curve, not the spectra. We also show the synthetic light curve obtained from our ad hoc model for \( \rho(r) \). Finally, we review and discuss our findings, including possible reasons why SN 1997ef was a hypernova but no GRB was apparently linked to it.

2. DATING THE SPECTRA

We have selected for modeling six spectra of SN 1997ef obtained by the Harvard-CfA group (Garnavich et al., in preparation; see Paper I, Figs. 8 and 9). We have chosen well-exposed, high S/N spectra, with a large wavelength coverage, and tried to sample the light curve as uniformly as possible. The first three spectra are from epochs around maximum (which was not observed either spectroscopically or photometrically, but was probably reached around 1997 December 10, at \( V \sim 16.5 \). The dates of the spectra are November 29, which is very soon after discovery, December 5, and December 17. All the spectra show very broad lines. They are all early enough that they are sensitive to changes in the kinetic energy and to the assumed epoch of outburst. The next two spectra have dates December 24 and January 1, which is around the time when the SN enters the tail of the light curve. The lines in these spectra become progressively narrower, indicating that the high-velocity part of the ejecta is becoming optically thin. The final spectrum has date 1998 January 26, which is well on the light-curve tail. Line velocities are quite small, suggesting that the spectrum is formed deep in the ejecta. One of the challenges for a good light-curve calculation is to follow the evolution of the photospheric velocity with time. Even model CO100 was not perfect in this respect (see Paper I, Fig. 7).

Since the epoch of maximum was not observed, light-curve models are not very tightly constrained. Since the rise time to maximum depends on the structure of the model and the abundance distribution, the range of allowed parameters could be effectively constrained if at least the time of explosion could be determined, albeit with some uncertainty. This could be attempted using spectrum synthesis and the measured photospheric velocity as determined in Paper I from the velocity of the Si II line. It could be expected that this line is formed somewhat above the photosphere at very early epochs, like in SNe Ia (e.g., SN 1994D; Patat et al. 1996).

Since our synthetic spectra are computed using the luminosity \( L \), the epoch \( t \) and the photospheric velocity \( v_{ph} \) as input, an appropriate value of \( t \) must be found that gives a photospheric radius \( R_{ph} = v_{ph} t \), which, when combined with \( L \), produces temperatures in agreement with the observed spectrum. In particular, although the continuum is not well determined in the very complex spectra of SN 1997ef, spectral lines serve a double role: they depend on the temperature (and hence on \( R_{ph} \) and \( L \)) through their ratios, and on \( v_{ph} \) through their displacement. Not many lines are visible in the earliest spectra, but at least Si II \( \lambda \lambda 6346, 6371 \) and the broad complex, which can most likely be identified as O I \( \lambda \lambda 7777-7778 \), \( 8447 \), and the Ca II IR triplet could be used to guide the calculation of the synthetic spectra.

Models for November 29 show that acceptable fits are found for \( t = 7-9 \) days and \( v_{ph} = 18,000-15,500 \, \text{km s}^{-1} \). Greater epochs require smaller velocities, so that \( R_{ph} \) always has a value of about \( 1.1 \times 10^{11} \, \text{cm} \). This radius, combined with the value of \( L \), which depends almost entirely on the assumed distance, gives a temperature appropriate to get a good overall fit to the spectrum. If we assume, e.g., \( t = 11 \) days, then we find a good overall fit to the spectrum for \( v_{ph} = 13,500 \, \text{km s}^{-1} \), which is too small and is reflected on the line shift. The opposite happens if \( t \leq 7 \) days. In order to strengthen our result, we applied the same technique to the December 5 spectrum, and we found
acceptable fits for $t = 14$–$15$ days and $v_{\text{ph}} = 10,000$–$9500$ km s$^{-1}$. Therefore, the two epochs give consistent results, and we can estimate the time of outburst to have been November 20–22. Since the fits obtained for $t = 9$ days on November 29 and $t = 15$ days on December 5 were particularly good, we selected November 20 as our reference epoch, and also for the comparison with the light curve and the photospheric velocities. We will show the fits in the next sections, when discussing the various epochs. In Figure 1 we show as an example the November 29 spectrum and two synthetic spectra computed as above for $t = 9$ days (dashed line) and $t = 11$ days (dotted line), respectively, using model CO100. The difference in velocity is clearly visible.

These results depend on the selection made for two other quantities, distance and overall ejecta mass, as discussed by Mazzalì & Schmidt (2000) in the context of Type Ia SNe. For all models, we adopted a distance modulus of 33.63, i.e., a distance of 53.1 Mpc, as estimated from a recession velocity of 3450 km s$^{-1}$ (P. Garnavich, 2000, private communication) and $H_0 = 65$ km s$^{-1}$ Mpc$^{-1}$. We assume no extinction [E$(B-V) = 0.0$], which is justified by the absence of a narrow interstellar Na I D line from the spectrum of SN 1997ef at all phases. As for the ejected mass, we already showed in Paper I that this must be large in order to fit both the light curve and the spectra. Therefore in this paper we use the solution found in Paper I, model CO100, as a reference.

3. THE 1997 NOVEMBER 29 SPECTRUM

This is to our knowledge the earliest spectrum available of SN 1997ef. Although it is somewhat noisy and it does not extend very far to the red, it nevertheless includes most of the main features that made SN 1997ef such a peculiar and interesting object. The spectrum (Fig. 2) has essentially no continuum, and it is characterized by four broad minima, at roughly 3900, 4700, 6000, and 7500 Å. The minima are separated by three rather sharp peaks, at 4300, 5200, and 6300 Å, respectively. The minima are so broad and the peaks so strong that the spectrum could be confused for an emission spectrum. Both Branch (2000) and Paper I showed that the real nature of the spectrum is a superposition of absorption lines. We already pointed out that the 'emission peaks' are the result of photon travel in a medium of large line opacity. Photons redshift their way through the envelope until they find a portion of the spectrum where line opacity is low because of the intrinsic distribution in wavelength of the atomic transitions. We call such a region a "line-free window." A large number of photons escape through these regions—all the photons coming from the optically thick region immediately to the blue—and this leads to a large flux and to the observed "peaks." While the width of the peak depends solely on the width of the line-free window, its strength depends both on the flux at the lower boundary (the photosphere) and on the width of the blanketing region that feeds photons to it: the broader the blanketing region, the more photons must reach the window to escape, resulting in a higher peak flux. This can be easily seen by inspection of the spectrum.

In Paper I we showed that a synthetic spectrum based on model CO100 gives a reasonably good reproduction of the observations and allows us to identify the four absorptions as follows: 3900 Å: Ca ii H and K plus Ni ii, Co ii, and Si ii lines; 4700 Å: mostly Fe ii lines; 6000 Å: Si ii $\lambda\lambda 6347, 6371$ plus weaker Si ii lines and Na i D further to the blue; 7500 Å: O i $\lambda 7774$. In spectra computed with model CO100 the O i $\lambda 7774$ and the Ca ii IR triplet are well separated, but unfortunately the observed spectrum does not extend far enough to the red to show the Ca ii region. With reference to that synthetic spectrum, which is shown again here as the dashed line in Figure 2, we notice that none of the absorptions extend quite far enough to the blue. For the two bluer
troughs, which are blends of very many lines, this may possibly be due to the lack of lines resulting from neglecting some element or not using a sufficiently complete line list, but this seems unlikely for the two redder troughs, where a few very strong lines dominate.

The synthetic spectrum has $t = 9$ days, $\log L = 42.13$ (ergs s$^{-1}$), $v_{\text{ph}} = 15,500$ km s$^{-1}$. The observed $V$ magnitude (16.7) and the synthetic one (16.75) agree quite well, but the spectrum has a large bolometric correction (0.28), which indicates that the assumption BC = 0.0 made in Paper I when fitting the light curve is at least weak. The observed Si II line velocity is 19,000 km s$^{-1}$, and the core of the Si II absorption is well reproduced even using model CO100. Therefore, in order to improve the synthetic spectrum, we modified the density only above 20,000 km s$^{-1}$. While the original CO100 model has $\rho \propto r^{-8}$ in the outer part, we made the outer density radial dependence flatter. As a consequence, high-velocity absorption becomes stronger. Of course, this happens first in the strongest lines, the Ca II H and K doublet and IR triplet first and foremost. The strength of these lines increases dramatically, because they come from low excitation levels, which are still highly populated even at the very low densities and temperatures of the outer envelope ($T \sim 4000$ K), and so these lines increase greatly in width. Next come the Si II and O I lines. For these lines, the drop in the level population with radius is much steeper, since they come from levels on average about 10 eV above the ground state, and so they do become broader, but not so noticeably, and their cores do not shift much toward the blue. All other lines, strong and weak, are similarly affected, to a higher or lesser degree depending on the ionization and excitation of the levels.

Since we wanted to introduce as little change from model CO100 as possible, we choose the steepest density profile that gave a good fit to the spectrum. This was obtained with a density power-law index $n = -4$ at $v > 25,000$ km s$^{-1}$. When this change is introduced the Ca II H and K doublet, the Si II line, and the O I $\lambda 7774$ line broaden to the point that the observed absorption troughs are very well reproduced. Further to the red, in the unobserved part of the spectrum, the Ca II IR triplet is predicted to broaden and blend with O I $\lambda 7774$. The contribution of O I $\lambda 8446$, which is also a strong line, is nevertheless small compared to that of Ca II. The blending of O I $\lambda 7774$ and the Ca II IR triplet is an important discriminant for the density structure at high velocities. Fortunately, the December 5 spectrum covers that region. The blue sides of the peaks at 4300 and 5000 Å are also fitted much better, as the increased density causes the emission to set in at higher velocities.

The only region where no improvement is seen is near 4500 Å. The Fe II and Co II lines causing the absorption near 4750 Å are not sufficiently strong to extend to the blue, and there are not enough strong lines that absorb near 4400–4500 Å. It is possible that we are either missing lines or ignoring some relevant element. Interestingly, Branch (2000), who used a code that is very efficient for identifying lines, seemed to have the same problem.

The modification to the outer density profile has only a small influence on the total mass, since it adds only about 0.25 $M_\odot$ in the outer ejecta, but it does lead to a significant increase in the overall kinetic energy, since all the mass is added to the high-velocity shells, where it represents a dramatic increase, of 1–2 orders of magnitude. The kinetic energy above 20,000 km s$^{-1}$ increases from $10^{51}$ to about $10^{52}$ ergs, and the total kinetic energy increases from $8 \times 10^{51}$ to $1.75 \times 10^{52}$ ergs. This is quite a large change, but it is still smaller than the value adopted for SN 1998bw. Also, both the outer slope and the revised KE are, respectively, steeper and smaller than the values suggested by Branch (2000). We will discuss the implications of these revised values later. An increase in KE only makes SN 1997ef more like a hypernova.

The best-fitting ad hoc model has the same input parameters as the spectrum computed using CO100. The increased outer density makes the temperature only slightly higher. The synthetic spectrum has $B = 17.45$, $V = 16.74$, and BC = 0.29, so that $M_{\text{Bol}} = -16.70$. Therefore, comparing $M_{\text{Bol}}$ with $V$ is misleading, since $V$ rises more rapidly than $M_{\text{Bol}}$.

Finally, a word of comment about the presence of He in the spectrum. In SN 1998bw, He i $\lambda 10830$ Å was almost certainly observed in absorption at early epochs (Patat et al. 2000). Its presence could be understood if the He shell was not completely lost before the explosion. Unfortunately, IR spectra are not available for SN 1997ef, so the best place to look for He is He i $\lambda 5868$. This line is very close to Na i D, which is rather strong and contributes to the broad 6000 Å trough with an absorption near 5450 Å. A small He mass does not produce a strong line unless very large departure coefficients are assumed, and in any case a strong He i line is not necessary to fit the spectrum. Therefore, although we cannot be final, we do not think He i $\lambda 5868$ is present in the November 29 spectrum.

4. THE 1997 DECEMBER 5 SPECTRUM

The second spectrum in our analysis has a good S/N and a very good wavelength coverage, extending from 3100 to 8700 Å. It is actually quite similar to the 29 November one, but more detail is visible (Fig. 3), e.g., the structure of the complex 6000 Å absorption and, very importantly, the Ca II
IR triplet is visible, and clearly blended with O I λ7774, which is a strong argument in favor of a “flat” outer density distribution, as discussed above. That the feature measured at 7500 Å is a blend is clear, given the presence of two distinct absorptions, whose wavelengths match those of the two component multiplets, as shown also by the spectrum computed with model CO100.

The synthetic spectrum based on model CO100 was shown in Paper I and again here as the dashed line in Figure 3. It has parameters $t = 15$ days, log $L = 42.15$ (ergs s$^{-1}$), $v_{\text{ph}} = 9500$ km s$^{-1}$. Both $L$ and $R_{\text{ph}}$ have values similar to those on November 29; therefore the temperatures are similar and so are the two spectra, although the photosphere is quite a bit deeper on December 5. This is shown also by the Si II line velocity, which is reduced to 13,000 km s$^{-1}$. The observed $V$ mag on December 5 was 16.5, and the CO100 spectrum reproduces it well ($V = 16.63$). The problem with that model is essentially the same as that discussed for the November 29 model: all troughs do not extend far enough to the blue. Furthermore, O I λ7774 and the Ca II IR triplet do not blend anywhere near as much as they should when the original model is used. In that spectrum the Ca II IR triplet extends only to about 7850 Å, i.e., a Ca II velocity of ~25,000 km s$^{-1}$. If the Ca II IR triplet is to blend with the O I line, it must reach at least 7650 Å in the blue wing, i.e., $v \sim 31,000$ km s$^{-1}$. This is consistent with the modified density structure adopted for the November 29 spectrum. Since $v_{\text{ph}}$ is much lower than the values at which the density has been modified ($v > 20,000$ km s$^{-1}$), we can use the same density structure as for November 29.

The spectrum we computed for the modified structure and the same input parameters as above is shown in Figure 3. The improvements are apparent, and they concern mostly the Ca II IR triplet + O I λ7774 feature in the red and the blue region near 4000 Å, which is improved by the strengthening and broadening of Ca II H and K. Improvements in the trough at 4700 Å are again only marginal, as are those to the Si II line region. Surprisingly perhaps, the Si II line does not become broader, but this can be understood since the high-velocity regions where the density has been enhanced are far above the photosphere and are of too low density now to cause much Si II absorption. The situation is made more complicated by the apparent development of an absorption shoulder near 5900 Å. We cannot offer a clear identification for this shoulder, except for several lines of O I at 6157 Å, but these are not very strong. The shoulder was not present on November 29, nor is it on December 17. Another, stronger shoulder is visible near 5750 Å. This is where Si II λλ5958, 5979 falls, and the synthetic spectra show that this doublet, combined with Na I D further to the red, is strong enough to produce a shoulder stronger than the observed one without resorting to He I. Therefore we can confirm the nondetection of He I λ5876.

Note that the Si abundance in this model was lower than on November 29, which is a strong argument in favor of a “flat” outer density distribution, as discussed above. That the feature measured at 7500 Å is a blend is clear, given the presence of two distinct absorptions, whose wavelengths match those of the two component multiplets, as shown also by the spectrum computed with model CO100.

5. THE 1997 DECEMBER 17 SPECTRUM

This third spectrum has again a limited wavelength coverage, but it is the first one available after December 5, and it is therefore useful to study the evolution of the SN immediately following $V$ maximum. The spectrum, which is displayed in Figure 4 has $V = 16.6$. The overall aspect of the spectrum is similar to the previous two epochs, which indicates that the temperature conditions have probably not changed greatly.

However, closer inspection, and in particular a wavelength comparison of this and the two earlier spectra, reveals that all the main absorption and emission peaks have shifted to the red by 100–200 Å, depending on the wavelength. A similar, but smaller shift also occurred between November 29 and December 5. The immediate interpretation is that we are observing the effect of the inward motion of the photosphere through the ejecta as they expand and thin out. Note that this recession takes place essentially in the Lagrangian mass/velocity frame, and not in the observer’s (radius) frame, much like in the plateau phase of some SNe II. The Si II velocity measured from this spectrum is in fact only 8000 km s$^{-1}$. The change in line

![Figure 4](image_url)
absorption velocity is larger between December 5 and December 17 than it was between November 29 and December 5 because the time interval between the spectra is larger. All absorptions move by about the same amount, and the emissions move as well. This is obvious given our explanation for what the emissions really are: if the blanket-ed regions move, the opacity windows must move along with them, just like in P-Cygni profiles caused by a single line, a situation of which the case of SN 1997ef represents after all just an extreme extension. Looking in more detail, the shoulder which affected the Si II line near 5900 Å on December 5 seems to have disappeared on December 17, leaving serious questions about its identity. The absorption near 5000 Å now shows more structure, with minima at 4800 and 4950 Å and two shoulders at 5100 and 5300 Å, which resembles quite closely the typical profile observed in SNe Ia near maximum and soon thereafter. In SNe Ia the entire feature is attributed mostly to Fe II lines, and this was confirmed in SN 1997ef on both November 29 and December 5.

The synthetic spectrum computed with CO100 is shown as the dashed line in Figure 4. The structure in the Fe II absorption is reproduced reasonably well, and the width problem now does not affect either the feature at 4300 Å or that near 7500 Å. This is probably because the outer high-velocity regions are so far above the photosphere now that they do not really cause any significant absorption, and so the original model CO100 gives an acceptable description of the density structure in the intermediate velocity regions. Unfortunately this spectrum does not cover the Ca II IR triplet, which is predicted not to blend with O I 7774 Å in the CO100 model. The synthetic spectrum was computed with the following parameters: \( t = 27 \) days, \( \log L = 42.20 \) (ergs s\(^{-1}\)), \( v_{ph} = 7500 \) km s\(^{-1}\). The temperatures are lower than on December 5 but comparable to those on November 29, and \( R_{ph} \) is actually larger than on both previous epochs, although the mass above the photosphere is now as large as \( 5 \, M_{\odot} \). Compared to the December 5 model, the abundance of O is reduced and that of S increased, which is the behavior expected as deeper and deeper regions of the ejecta are probed. The reduced velocity is the reason for the deblending of many features. There are two major shortcomings in the model: (1) The position of the peak at 5500 Å is not correctly reproduced. This is clearly the consequence of the absorption at 5400 Å not being correctly reproduced, so that the “optical depth window” starts too far to the blue. This feature is not well reproduced in the spectra of SNe Ia either, so it is quite likely that some Fe-group lines are missing or their strength is not correct. (2) The Si II line is too narrow, or rather the very triangular shape of the blue side of the absorption cannot be reproduced. Only another Si II line is active in that region (Si II \( \lambda 5979 \), observed near 5800 Å), but it is too weak. In any case it would be difficult to explain the observed profile with just one line, so we really have no explanation for what might affect the spectrum.

We have also modeled the spectrum using our modified density distribution (Fig. 4, solid line). As we could expect, since the near-photospheric region is not affected by the modification, at this epoch the two synthetic spectra are essentially identical. The spectrum computed with the modified outer density profile has \( B = 17.73 \) and \( V = 16.66 \). The bolometric correction is still large, \( BC = 0.20 \), and \( M_{\text{Bol}} = -16.87 \). It is very interesting that the model luminosity, as obtained from fitting the spectrum, is actually larger on December 17 than on December 5. The bolometric light curve appears then quite different from both the \( V \) and the \( B \) curves. The large bolometric correction is now caused mostly by the red part of the spectrum, with the synthetic \( R \) band still rising so that the bolometric light curve rises slowly to a rather delayed maximum. Near-IR photometry would have been useful to confirm this prediction.

6. THE 1997 DECEMBER 24 SPECTRUM

The December 24 spectrum is the first of a set of three spectra modeled in this paper that extend the time coverage beyond the near-maximum epochs considered also in Paper I. The December 24 spectrum is shown in Figure 5. It does not extend far to the red, but in the blue it has a good S/N down to 3700 Å, thus covering the Ca II H and K region. The epoch of the spectrum is 35 days after explosion, for which we chose November 20. The spectrum has \( v = 16.8 \), a drop of 0.2 mag in only one week from December 17, and lies therefore well in the declining part of the \( V \) light curve. The Si II line is now quite narrow, and its core has a velocity of 5400 km s\(^{-1}\). This and all other absorptions continue to move to the red. Following the trend already noticeable on December 17, many small features that earlier blended into the broad troughs are beginning to be resolved, marking the progressive inward motion of the photosphere. The broad absorption near 6000 Å is the region that changed the most with respect to December 17. In particular, a separate absorption is now visible at 5700 Å.

We modeled the spectrum using model CO100 with the outer modification introduced as discussed above. As usual, \( L \) and \( v_{ph} \) were selected so as to get as good a fit as possible to the observations. The input parameters for the best model, which is shown as the continuous line in Figure 5, were \( t = 34 \) days, \( \log L = 42.13 \) (ergs s\(^{-1}\)) and \( v_{ph} = 4900 \) km s\(^{-1}\). Note that \( L \) has the same value as on November 29,
indicating that the bolometric light curve is now declining. The photospheric velocity is now very low, as is also indicated by the Si II line velocity, and is actually slightly below the inner edge of the density distribution of model CO100 (see Paper I, Fig. 3). Therefore, in order to produce a reasonable spectrum we had to introduce a second change to the CO100 density structure, extending it down to $v_{\text{ph}}$. We used $\rho \propto r^{-1}$ for this inner extension. This is only a first approximation, but it is obvious that if there is a photosphere there must be opacity right down to it, and this is not compatible with the “density hole” predicted by model CO100 below 5300 km s$^{-1}$. Since the photosphere is so deep, the inclusion of the flat outer density profile derived from the earlier spectra has no effect on the synthetic spectrum. Even when the flat outer density is included, O I \( \lambda 7774 \) and the Ca II IR triplet are predicted to be separated in wavelength. This region is not covered on December 24, but our prediction will be confirmed in the January 1 spectrum (see next section). The small inner extension to the density does not affect the total mass much, so that at this epoch the total mass above the photosphere is $8.4 \, M_\odot$.

A major change in this model is in the abundances. As shown in Figure 7 of Paper I, the oxygen-dominated envelope extends down to 6500 km s$^{-1}$. On December 24 the photosphere is well below this point, and Si, S, and the Fe-group have larger abundances. We fitted the abundances to obtain a best fit, and found that S is the element whose abundance must increase most, at the expense of O. The behavior of the abundances as determined from spectrum synthesis is reviewed in the discussion. Most line identifications are the same as on earlier epochs. The absorption at 5700 Å is due to Na I D. Overall, the synthetic spectrum is a good match to the observed one. The major shortcoming is that the emission peak near 6500 Å is not reproduced. Net emission may be present there, although the wavelength does not match that of the expected line of \([\text{O I}]\) 6300 Å. There is a wavelength coincidence with H$_z$, but this is not expected to be a strong feature in a SN Ic ejecta.

The synthetic spectrum has $B = 17.83$ and $V = 16.71$. The bolometric correction is still large, BC = 0.32, so that $M_{\text{bol}} = -16.70$. Therefore, it appears that bolometric maximum was reached near December 24, much later than $V$ or $B$ maximum. A lot of the flux is still released in the near-IR, in particular in the Ca II IR triplet peak.

7. THE 1998 JANUARY 1 SPECTRUM

The next spectrum in our series is rather close in epoch to the previous one but, with an epoch of 42 days, it captures a moment when the light curve begins to settle on the tail. Although the S/N is only moderately high, the ample wavelength coverage ($\lambda 3400$ to $\lambda 8900$) makes it an interesting spectrum to model. The spectrum is shown in Figure 6. Comparing it to the December 24 spectrum, several signs of development can easily be noticed:

1. The ratio of the three peaks in the blue, at 4000, 4500, and 5500 Å is much steeper, indicating a shift of the flux toward the red, which clearly must be the consequence of a reduced temperature.

2. The absorption complex near 6000 Å has now split into two components. One is centered near 5700 Å and is due to Na I D, which is now sufficiently strong and isolated because of the reduction in strength of neighboring lines coming from more highly excited levels that it gives rise to its own P-Cygni emission, near 5900 Å. To the red of Na I D the Si II line is still strong, near 6250 Å, and is accompanied by a weaker absorption near 6100 Å, which was also present on December 24.

3. Two absorptions are developing at 6800 and 7100 Å.

4. The O I $\lambda 7774$ line and the Ca II IR triplet are now well separated, as was already predicted by our synthetic spectrum on December 24. The O I line is rather narrow, while the Ca II absorption is broader. This is a consequence of the different run of excitation with radius (excitation falls more steeply for O than for Ca, and so does the line optical depth).

5. Emission in the Ca II IR triplet is now very strong, much stronger than the corresponding absorption: this indicates that net emission is beginning to be an important contribution to spectrum formation.

6. The line blueshift is further reduced: the Si II line now has a measured $v = 3600$ km s$^{-1}$. This is now significantly smaller than the inner boundary of the original CO100 density distribution. Since the essentially photospheric nature of the spectrum is proved by the progressive evolution of all features, a viable model must include a low-velocity extension to the density profile.

Therefore, we modeled the spectrum using as a starting point the CO100 density distribution modified at high velocity, as described earlier. This modification has no effect on the emerging spectrum at these later epochs: the maximum observable matter velocity, measured at the blue edge of the Ca II IR triplet is only 23, 000 km s$^{-1}$. We extended the density distribution inward to $v = 3000$ km s$^{-1}$, and tried various density laws below 5000 km s$^{-1}$. Our first conclusion is that the density must be increasing inward: if we use a decreasing density a shell forms near 5000 km s$^{-1}$, which affects the lines; if the density is constant below 5000 km s$^{-1}$, the value near the photosphere is
too low and the lines are too weak. On the other hand, if the density increases too steeply the synthetic lines tend to be too sharp. As a best approximate solution we adopted a power law with $\rho \propto r^{-4}$.

Our final model has parameters $t = 42$ days, $\log L = 42.0$ (ergs s$^{-1}$), $v_{\text{ph}} = 3600$ km s$^{-1}$, which is close to the observed Si II velocity. It is interesting that the simultaneous reduction in both $L$ and $v_{\text{ph}}$ means that the electron temperature $T_e$ ranges between 4500 and 6500 K, which is close to the values it had on previous epochs. This explains why the nature of most line features is unchanged throughout the evolution of the SN. The synthetic spectrum is shown as the thin line in Figure 6. Overall, the quality of the fit is rather good, confirming the correctness of our assumption. Major shortcomings are the following: (1) The strength of the peak at 4500 Å; this is probably due to having somewhat too much opacity in the UV. (2) The strength and position of the peak near 5500 Å; this was a problem also on December 24, and we explained it as the consequence of missing line strength near 5400 Å. (3) Flux is missing in the Ca II IR triplet emission; this is because net emission—following collisional excitation—is not included in our code. The difference between the observed and synthetic peak is to be ascribed to net emission, which is responsible for about half of the total line flux.

Other parts of the spectrum are very well reproduced, e.g., the O I and Ca II IR triplet absorption and the structure of the Si II line region, where the weak absorption at 6000 Å is attributed to Fe II $\lambda 6148$ and the emission near 6500 Å is now correctly reproduced, confirming that the peak on December 24 was probably not [O I] $\lambda 6300$. The weak absorptions at 6800 and 7100 Å correspond to lines of O I $\lambda 7001$ and to several Fe II lines, respectively, but the synthetic lines are too weak. Fe II lines are becoming stronger as the abundance of Fe-group elements is higher, while that of O is further decreased.

The inward extension of the density affects the ejecta mass, adding about 1.5 $M_\odot$ between 3000 and 5000 km s$^{-1}$, but it has only a small effect on the kinetic energy, adding only $2 \times 10^{43}$ ergs in that velocity shell. The synthetic spectrum has $V = 17.16$, which compares well with the observed $V = 17.3$. Synthetic $B = 18.41$, showing that the spectrum is rather red. The $R$ magnitude drops less than both $B$ and $V$, and the bolometric correction is still large, $BC = 0.22$, so that $M_{\text{bol}} = -16.35$, but this neglects the net emission in the Ca II IR triplet, so the actual value is probably smaller.

**8. THE 1998 JANUARY 26 SPECTRUM**

This is the last spectrum available, corresponding to an epoch well on the light-curve tail, with $V \sim 18.0$. The spectrum, shown in Figure 7, shows a clear evolution from January 1. In particular, the flux peak has moved to the red quite noticeably, indicating a temperature drop. Looking at the spectrum in detail, one can notice that the features in the blue have changed little, and so has the region furthest to the red, although the Ca II IR triplet emission is now stronger. On the other hand, the region around 6000 Å has changed quite significantly. The flux peak has moved from 5500 to 6000 Å, the absorptions at 5700 Å (Na I D) and at 6200 Å are much stronger. The Si II line has essentially disappeared, all that is left being a weak feature near 6400 Å, which is too red to be identified with the Si II doublet. In fact, a Si II velocity cannot be measured at this epoch. Still, many features have persisted from earlier epochs, and these have shifted further to the red, indicating the presence of significant amounts of material at very low velocity in the ejecta.

The overall nature of the spectrum is still photospheric, but the incidence of net emission is higher than on January 1, which will limit our ability to fit the spectrum using the MC code. We used the density distribution discussed in the previous section, including both the outer $\rho \propto r^{-4}$ part above 25,000 km s$^{-1}$, which has no effect on the synthetic spectrum, and the inner $\rho \propto r^{-1}$ extension between 3000 and 5000 km s$^{-1}$, but we had to extend the density distribution further inward, so that we could obtain an appropriate temperature and spectrum. In keeping with the hydrodynamical prediction of a "density hole" in the center of the ejecta, we tried to use the largest possible value of the density power-law index allowed by the quality of the synthetic spectrum. We selected a model based on the overall fit, but we could not reproduce the observations as well as on previous epochs. We could accommodate an innermost density law $\rho \propto r$ below 3000 km s$^{-1}$, which is an attempt to include an inner "hole" in density. This extension adds a further 0.5 $M_\odot$ to the ejecta mass, but only less than $10^{40}$ ergs to the explosion kinetic energy. Our best-fit spectrum, shown in Figure 7, has parameters $t = 67$ days, $\log L = 41.80$ (ergs s$^{-1}$) and $v_{\text{ph}} = 1950$ km s$^{-1}$. The temperature is significantly lower than on January 1, with $T_e$ ranging from 6000 to 4000 K. Because of the rather flat density structure at low velocity, the temperature is almost constant ($T_e \sim 6000$ K) at $v \leq 6000$ km s$^{-1}$, which is where the spectrum is formed. This helps to keep the lines sufficiently broad. The abundances are similar to those of January 1, but oxygen is further reduced and S is increased, reflecting the expected trend as smaller and smaller velocities are sampled.

Admittedly, the synthetic spectrum is not a very good fit, but almost all the observed features are at least reproduced. The two emission peaks at 4600 and 5500 Å are much too strong in the model, which affects the synthetic $V$ magni-
Another net emission, which is not reproduced by the synthetic spectrum has $V = 17.85$. The $B$ flux is also somewhat overestimated at $B = 19.06$. Possible reasons for the excessive strength of the synthetic peaks at 4600 and 5400 Å were already given in the previous section. In the $V$ region, an absorption near 5600 Å is also not reproduced. A similar feature was also observed on January 1, near 5650 Å, but it was weaker then, while it was absent on December 24.

The absorption might be attributed to He I at 5876 Å at a velocity of about 13,000 km s$^{-1}$. He may be expected to be found in the ejecta as a leftover of the star's He shell. If He is distributed in a shell, He atoms may only be excited at a rather advanced epoch, when the $γ$-rays and positrons from the decay of $^{56}$Co can penetrate the ejecta and reach the shell. However, the velocity implied by the position of the absorption is low for a hypothetical He shell, and the apparent shift of the feature between January 1 and January 26 does not support this scenario either. Alternatively, He can be produced with $^{56}$Ni in the deepest layer as the result of alpha-rich freezeout (see Paper I, Figs. 5 and 6). In this case He would be found together with Co/Fe, so the He velocity could take any value depending on the $^{56}$Ni mixing. However, in this case nonthermal excitation of He would take place immediately, and we should see the He I line as soon as the photosphere reaches the layer where He is present. If He is located at about 14,000 km s$^{-1}$, we should have seen the He I line as early as December 5. The above problems notwithstanding, as long as no other candidate identification is available He must be regarded as at least a possibility. Observations of He I 10830 Å would certainly help settle this issue. In any case, given the moderate strength of the line, the He mass involved may not have to be large.

The model predicts a moderately strong absorption at 6300 Å, attributing it to a blend of the Si II doublet and, mostly, of Fe II $λ$6417, 6456. This feature is present also in the observed spectrum, but it is much weaker. The reason for the discrepancy is clearly the emission at 6300 Å, which must signal the onset of the corresponding [O I] line. Another net emission, which is not reproduced by the model, is visible near 7300 Å. This is clearly Ca II $λ$7292, 7324. Together with the Ca II IR triplet, these are the only three net emissions in the optical spectrum. This is a typical situation for SNe Ic at these epochs (cf. SN 1987M at 60 days after maximum; Swartz et al. 1993).

Comparing the spectra of SN 1997ef and SN 1987M, apart from the obviously broader lines in SN 1997ef, it is possible to note that in SN 1987M the Fe II lines are weaker and that the candidate He I line is not seen. SN 1987M is thought to have produced roughly as much $^{56}$Ni as SN 1997ef (0.15 $M_⊙$; Nomoto, Filippenko, & Shigeyama 1990; Paper I), but this could be an overestimation. Classical, low-energy SNe Ic are not expected to eject any He, but this might not be the case for SN 1997ef, and it certainly is not for the other hypernova, SN 1998bw, where He I $λ$10830 is observed (Patat et al. 2000). On the other hand, at a comparable epoch the [O I] $λ$6300 emission is much stronger in SN 1998bw than in SN 1997ef, supporting the idea that SN 1998bw came from a more massive progenitor. Further detailed study of the He (and O) content of hypernovae would certainly be worthwhile.

The synthetic spectrum has a small BC (0.01), and $M_\text{bol} = -15.87$. The actual value of BC may be even smaller, and possibly negative, since the model $V$ flux is certainly overestimated by about 0.15 mag, and the net emission in the Ca II IR triplet is not included in our estimate. Therefore, the value of $M_\text{bol}$ has an uncertainty of at least 0.2 mag.

9. THE EXPLOSION PROPERTIES OF SN 1997ef

As we have discussed above at great length, modeling the time evolution of the spectrum of SN 1997ef has revealed two main inconsistencies with the explosion model CO100, which we used in Paper I to reproduce both the light curve and the near-maximum spectra. The basic parameters of the observed and synthetic spectra are recapped in Table 1.

Firstly, the high line velocities observed near maximum in several strong lines, notably the Ca II IR triplet and the Si II doublet, could not be reproduced with the rather steep density profile of CO100, but require a flatter outer density law, $ρ \propto r^{-2}$, at $v > 25,000$ km s$^{-1}$.

Secondly, the fact that the January spectra are still predominantly photospheric in nature, showing low line velocities ($v < 5000$ km s$^{-1}$) is not compatible with the presence in CO100 of an inner "density hole." This "hole" was the result of depositing the kinetic energy at a radius in the progenitor, which results in the ejection of exactly the amount of $^{56}$Ni necessary to reproduce the SN tail luminosity. Matter below that radius, known as the "mass cut," is assumed to fall back onto the compact remnant. Clearly, material must exist at low velocities, and we found that an inner extension below 5000 km s$^{-1}$ with a $ρ \propto r^{-3}$ density law down to 3000 km s$^{-1}$ and a $ρ \propto r$ law below that, to reproduce the inner "density hole," is the best overall solution, although we still cannot get perfect fits of those very late epochs, even ignoring the net emission which is clearly present in some lines.

In Figure 8 we show the density profile of the original CO100 model and our modified one. The outer density extension adds significantly to the kinetic energy ($~10^{52}$ ergs), but only marginally to the ejecta mass (0.25 $M_⊙$), while the inner extension adds about 1.65 $M_⊙$ to $M_{ej}$.

| Date       | Epoch (days) | $L$ (ergs s$^{-1}$) | $v_{ph}$ (km s$^{-1}$) | $σ$(Si II) (R$_{0}$) | log $ρ_{ph}$ (g cm$^{-3}$) | Mass ($M_⊙$) | $T_{eff}$ (K) | $T_{ph}$* (K) | $B_{mod}$ | $V_{mod}$ | $V_{obs}$ | BC | $M_{mod}$ |
|------------|--------------|---------------------|------------------------|----------------------|---------------------------|-------------|----------------|---------------|-----------|-----------|----------|----|----------|
| 1997 Nov 29 | 9            | 42.13               | 15500                  | 19072                | -12.65                    | 6123        | 7725           | 17.45         | 16.74     | 16.60     | 0.29     | -16.60  |
| 1997 Dec 5  | 15           | 42.15               | 9500                   | 13962                | -12.30                    | 6128        | 9447           | 17.35         | 16.58     | 16.65     | 0.40     | -16.65  |
| 1997 Dec 17 | 27           | 42.20               | 7500                   | 8011                 | -12.67                    | 5291        | 6648           | 17.72         | 16.65     | 16.66     | 0.20     | -16.775 |
| 1997 Dec 24 | 34           | 42.13               | 4900                   | 5378                 | -12.71                    | 5602        | 7602           | 17.83         | 16.71     | 16.68     | 0.32     | -16.500 |
| 1998 Jan 1  | 42           | 42.00               | 3600                   | 3775                 | -12.78                    | 5426        | 6830           | 18.41         | 17.16     | 17.3       | 0.22     | -16.250 |
| 1998 Jan 26 | 67           | 41.80               | 1950                   | ...                  | -13.51                    | 5232        | 5701           | 19.06         | 17.85     | 18.0      | 0.01     | -15.775 |

* Temperature of the equivalent blackbody, which reproduces the emergent luminosity after the temperature iteration.
has a negligible effect (\(\sim 2 \times 10^{50} \) ergs) on the explosion kinetic energy. Our modified model has a total \(M_{ej} = 9.5 \ M_\odot\) and KE = \(1.9 \times 10^{52}\) ergs, compared to \(M_{ej} = 7.6 \ M_\odot\) and KE = \(8 \times 10^{51}\) ergs for CO100. These new values reinforce the classification of SN 1997ef as a hypernova, which is based on the KE being larger than \(10^{52}\) ergs.

Since we have a \(\rho \propto r^{-4}\) density law on the outside and an essentially flat law deep inside, and since our density distribution was derived starting from that of CO100, whose relic is the steep density gradient in the intermediate part, one might argue that a \(\rho \propto r^{-4}\) density law might work well throughout the ejecta. We have tested such a possibility, but although it works reasonably well for the outer part, at \(v \sim 20,000 \) km s\(^{-1}\), it gives rise to very sharp-lined spectra at later epochs, when the photosphere is in the flat part of the ejecta, at \(v < 10,000 \) km s\(^{-1}\). Also, joining directly the inner flat part and the outer \(\rho \propto r^{-4}\) part beyond \(10,000 \) km s\(^{-1}\) would give a SN with extremely large \(M_{ej}\) and KE, and the synthetic spectra for the epochs when the photosphere is near \(10,000 \) km s\(^{-1}\) would have very deep lines.

We have to ask ourselves what could give rise to the observed deviation from the hydrodynamical model. The flat outer part may be due to mass loss at high rate during the presupernova stages, or to a transition between the oxygen shell and an outer helium shell, some evidence for which may be present as a weak He I 5876Å line in the two January spectra, although at a velocity (\(\sim 13,000 \) km s\(^{-1}\)) which is smaller than what would be expected. Branch (2000) offered a similar solution, and his ad hoc density law was even flatter (\(\rho \propto r^{-2}\)) than ours. As a result he offers an estimate of the kinetic energy above \(7000 \) km s\(^{-1}\) as \(3 \times 10^{52}\) ergs. Our value above the same velocity is somewhat smaller but comparable, \(1.8 \times 10^{52}\) ergs. Also, he suggests that the ejecta mass above \(7000 \) km s\(^{-1}\) is \(\sim 6 \ M_\odot\), and our value is again smaller but comparable, \(\sim 4 \ M_\odot\).

The location of the inner density cutoff, on the other hand, was determined essentially by demanding that the correct mass of \(^{56}\)Ni be ejected from the progenitor. Clearly, the estimate of the position of the cut-off was incorrect. Although it is possible that this may be due to an incorrect calculation of the progenitor’s structure, it is more tempting to attribute the problem to some asymmetry in the explosion. If the explosion was asymmetric, similar to what has been suggested for the hypernova SN 1998bw in order to explain its connection with a GRB, it is quite possible that most \(^{56}\)Ni was produced near the beam axis, while away from that axis burning would be less efficient, and would terminate at intermediate elements such as Si or S (Maeda et al. 2000). Clearly one then has to place the mass cut deeper in order to achieve the require ejected mass of \(^{56}\)Ni. This is also supported by our derived abundance distribution, which suggests that the S abundance is still increasing at the lowest velocities sampled, about \(2000 \) km s\(^{-1}\). The abundance distribution as derived from our models is displayed in Figure 9, but we must keep in mind that, since we used homogenized compositions above the photosphere, all sharp composition boundaries are smoothed out. On the other hand, no clear spectral evidence—such as sudden changes in the properties of the lines—is visible for strict abundance stratification.

The presence of an inner density core was also suggested in Paper I to explain the observed deviation of the synthetic light curve computed with model CO100 from the observations at advanced phases: the synthetic light curve has a steeper slope than the observed one, indicating that \(\gamma\)-ray trapping is more efficient than in the model. This could be achieved if \(^{56}\)Ni was distributed deeper than it is in CO100.

10. A NEW LIGHT-CURVE MODEL

In order to verify our findings, we used our derived density and abundance distribution and computed a syn-
The Monte Carlo code is based on the simple code developed for SNe Ia described in Cappellaro et al. (1997). Gamma-ray deposition in the expanding ejecta is computed following the random walk of $\gamma$-ray photons adopting a constant $\gamma$-ray opacity $\kappa_\gamma$. Once they deposit their energy, the $\gamma$-rays are assumed to generate optical photons on the spot. The random walk of the optical photons through the ejecta is then also followed in Monte Carlo, assuming a constant optical opacity $\kappa_{\text{opt}}$. The Monte Carlo scheme is able to model efficiently the random walk of photons through the ejecta—and thus to take into account the delay between the emission of a photon and its escaping from the SN nebula, which determines the initial shape of the light curve. The value of $\kappa_{\text{opt}}$ depends on the temperature and composition, and it is possible that it changes with time. Thus we had to find a convenient approximate value for this particular model of SN 1997ef. Since $\kappa_{\text{opt}}$ affects the light curve essentially near maximum, we found that a value $\kappa_{\text{opt}} = 0.08 \text{ cm}^2 \text{ g}^{-1}$ gives a good fit to the light curve around maximum. This value is smaller than for SNe Ia, as might be expected since line opacity is much stronger in the Fe-dominated ejecta of a SN Ia.

The synthetic light curve in Monte Carlo is compared to the one obtained with the radiation hydrodynamics code. As described in Paper I, the code solves a multifrequency radiative transfer equation in the fluid's comoving frame, using the Feautrier method iteratively with an approximate Lambda operator. The energy equation of gas plus radiation and the radiation momentum equation are coupled to the transfer equation in order to follow the evolution of gas temperatures. The code also uses average opacities, and the values used were the same as in the Monte Carlo code.

The synthetic bolometric light curves are shown in Figure 10. Except for a slight difference around maximum, which is probably to be ascribed to Monte Carlo noise, the codes produce very similar light curves. Therefore, it is safe to use the simple Monte Carlo code in calculating bolometric luminosities if a reasonable opacity is chosen.

The synthetic light curves reproduce our derived bolometric curve reasonably well around maximum. The $V$ light curve can consequently be reproduced if the bolometric correction as resulting from our synthetic spectra is taken into account. In both codes a $^{56}\text{Ni}$ mass of 0.13 $M_\odot$ was used. In order to achieve a rapid rise of the light curve, consistent with our dating, we had to distribute $^{56}\text{Ni}$ homogeneously throughout the ejecta. Both light curves reproduce the maximum quite well, but their decline on the tail is much too steep. The observed $V$ light curve after about day 60 appears to follow the $^{56}\text{Co}$ decline rate, implying constant and complete deposition of about 0.07 $M_\odot$ of $^{56}\text{Ni}$. However, our codes predict for those advanced epochs a deposition fraction of less than 0.5 and steadily decreasing with time.

In Figure 11 we compare the observed velocity of the Si II doublet with the run of $v_{ph}$ as derived from the two light-curve codes and with the values used as input for the calculation of the synthetic spectra. The two codes are in good agreement with one another, although the MC values are consistently larger than those of the radiation hydro model. This is probably due to different zoning in the two codes.

However, the observed points are lower than both models, and the difference increases with the epoch. The velocities used for the spectral calculations, on the other hand, start lower than the observed values but slowly approach them. This is understandable, as it can be expected that at early times the Si II line forms above the photosphere since it is very strong. Later, as the line becomes weaker, it forms closer and closer to the photosphere. The fact that the value of $v_{ph}$ tends to flatten out at advanced epochs, and does not...
reproduce the observations, is not new (see, e.g., Iwamoto et al. 1998 on SN 1998bw), and it probably indicates that the assumption of a gray photosphere is not good at advanced epochs. However, the difference at early phases cannot be due to that effect. One possibility is that we have somewhat overestimated the density at the highest velocities in our spectral calculations, where we were guided more by the Ca ii IR triplet than by the Si ii doublet. Note that the velocity derived from model CO100 in Paper I was much smaller than the observed values. Another possibility is that our assumption of a gray opacity again fails in those outermost layers, as shown perhaps by the fact that only the Ca ii IR triplet is active there. Note that all of these remarks apply to a spherically symmetric situation.

Another inconsistency between the light curve and spectral calculations is that if we integrate the Ni mass as the sum of the abundances of Fe, Co, and Ni in the various synthetic spectra we obtain an Fe-group mass of only \( \sim 0.04 \, M_\odot \) above 2000 km s\(^{-1}\), which is much less that what is necessary to power the light curve. It is unlikely that our spectrum synthesis may have given such a large error, especially since the Fe lines are already very strong in most spectra.

So we have a situation where at early times we need more surface \(^{56}\)Ni to power the light curve than we see in the spectra, while at late times \(^{56}\)Ni we have used to reproduce the light curve at peak does not deposit sufficiently to reproduce the light-curve slope and brightness. The late light curve appears to require the complete deposition of \( \sim 0.07 \, M_\odot \) of \(^{56}\)Ni. The two problems may be independent, but they may also be related.

One possibility to explain the late light-curve behavior is that, in a spherically symmetric scenario, additional \(^{56}\)Ni \((\sim 0.05 \, M_\odot)\) could be located deep in the ejecta, below 2000 km s\(^{-1}\). If the density is sufficiently high there the \( \gamma \)-rays produced by the decay of this \(^{56}\)Ni may deposit completely. The presence of very low velocity ejecta would further reduce the mass of the expected compact object, and may be in contradiction with the distribution of \(^{56}\)Ni in models of the progenitor evolution and explosion. This would not affect the situation at early times.

Another possibility is that the ejecta may not be spherically symmetric, in either mass or abundance distribution, or both. If some \(^{56}\)Ni is ejected at low velocities together with other elements in some direction, the \( \gamma \)-rays produced by its decay chain could be efficiently trapped even at advanced epochs. The optical photons produced by the thermalization of \( \gamma \)-rays and positrons in this hypothetical high-density region would only be able to escape at advanced phases, and so they would have little effect on the early time spectra but they could power the late light curve. At the same time, some material has been ejected at high velocity, possibly in a jetlike form, to reproduce the sharp rise of the light curve by high-velocity \(^{56}\)Ni and to explain the broad lines of Si ii and Ca ii \((\sim 20,000 \, \text{km s}^{-1})\) at early epochs. In this scenario, the uniform mixing of \(^{56}\)Ni which was used to reproduce the sharp rise of the light curve might refer only to the \(^{56}\)Ni ejected in the jet, while the low velocity \(^{56}\)Ni would enter play only later.

This scenario might help explaining the discrepancy between the light curve and spectroscopic \(^{56}\)Ni masses. If most of the \(^{56}\)Ni is ejected in a preferred direction (e.g., in the jet that might be observed as a GRB if the viewing angle is favorable), this \(^{56}\)Ni may power the early light curve by depositing its \( \gamma \)-rays in the neighboring ejecta, which have lower velocity, higher density and a smaller Fe-group abundance. However, if we view the event from an angle sufficiently far from the jet direction, the spectrum we see would be produced in a region where the abundances are different from those found in the jet—hence the smaller spectroscopic mass of \(^{56}\)Ni. This is a possibility for SN 1997ef, since there is no positive identification of a GRB counterpart to the SN event.

The possibility that the explosion was asymmetric is in line with existing models of energetic supernova explosions linked to GRBs (MacFadyen & Woosley 1999; Höflich et al. 1999; Khokhlov et al. 1999; Maeda et al. 2000), and it deserves further study with multidimensional radiation hydrodynamics codes.

11. DISCUSSION

We have shown in this paper how spectrum synthesis can be used not only to verify explosion models of SNe, but also to improve on them. Given our findings, it would now be very interesting to see hydrodynamic calculations of the explosion, in one or more dimensions, which reproduce closely the observed bolometric light curve. The derived density distribution, ejecta mass, and kinetic energy may or may not be close to what we have obtained spectroscopically, but in any case such a model would have to be tested also for its ability to reproduce the SN spectral evolution. Our study also shows that fitting an observed \( V \) curve with a synthetic bolometric one may be somewhat misleading, since the bolometric correction can be significant.

We have discussed in § 9 the reason for the difference between the density distribution derived from spectral modeling and that obtained from one-dimensional hydrodynamic calculations of the explosion of the stripped core of a massive star, and remarked that the presence of significant amounts of ejected matter at low velocity may be due to some asymmetry in the explosion. A similar conclusion was reached for the other Type Ic hypernova, SN 1998bw, but in that case it was based on somewhat different evidence, most importantly the connection with a GRB (e.g., Wheeler 2000).

The early-time spectra of SN 1998bw have even broader and more blended lines than those of SN 1997ef, but a clear evolution toward lower line velocities and less line blending is not seen as clearly as in SN 1997ef, while the relatively early development of nebular emission is seen in both objects. This may be understandable since although the ejecta mass of SN 1998bw was larger than that of SN 1997ef, even if we use our upward revised value for the latter, the kinetic energy of SN 1998bw appeared to be much larger (Nomoto et al. 2000). Branch (2000) in a spectroscopic study similar to the one he performed for SN 1997ef, suggested an extremely large KE for SN 1998bw \((\sim 6 \times 10^{52} \, \text{ergs above 7000 km s}^{-1})\), i.e., a factor 2 larger than his value for SN 1997ef above the same velocity). We have performed a preliminary analysis of SN 1998bw using the technique described in this paper, and find a similarly large value (Nakamura et al. 1999, 2000).

If SN 1997ef was also a highly asymmetric explosion, although weaker in energy and ejecting a smaller mass than SN 1998bw, why was a GRB not positively detected? The most likely possibilities are that either the jet axis of SN 1997ef was not oriented exactly along the line of sight, or that a weaker explosion energy reduced the beaming so that
a GRB was not formed. Both scenarios might be able to explain many of the observed features of SN 1997ef, in particular the lower expansion velocities and the smaller measured KE. When we apply a spherically symmetric model, we estimate KE by integrating a one-dimensional model around a sphere. Therefore, if the SN is observed on or very close to the jet axis, like SN 1998bw, KE could be grossly overestimated (Höflich et al. 1999). On the other hand, the difference between the velocity of the spectral lines in the two SNe is real, and so is the difference in luminosity at late times. That SN 1998bw produced more $^{56}\text{Ni}$ is also confirmed by the estimate for the $^{56}\text{Ni}$ mass ($\approx 0.6 \, M_\odot$) obtained from the nebular lines, which does not depend much on the asymmetry (see Danziger et al. 1999; Nomoto et al. 2000). Therefore, even though inclination may be a factor, that there is some intrinsic difference between the two objects appears unavoidable, at least as long as the relative distance estimate is reliable.

Finally, we note that another SN 1997ef–like Type Ic hypernova was recently observed, SN 1998ey, whose spectra appear to be identical to those of SN 1997ef, and for which there is also no observed GRB counterpart (Garnavich, Jha, & Kirshner 1998). This coincidence is striking and calls for the accumulation of more data on Type Ic hypernova candidates.

This work has been supported in part by the Grant-in-Aid for Scientific Research (12640233, 12740122) and COE research (07CE2002) of the Ministry of Education, Science, Culture, and Sports in Japan. It is a pleasure to thank P. Garnavich and J. Danziger for useful communications and the referee, D. Branch, for constructive remarks.

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