Density Profiles of Strongly Interacting Trapped Fermi Gases

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We study density profiles in trapped fermionic gases, near Feshbach resonances, at all $T \leq T_c$, and in the near-BEC and unitary regimes. For the latter, we characterize and quantify the generally neglected contribution from noncondensed Cooper pairs. As a consequence of these pairs, our profiles are rather well fit to a Thomas-Fermi (TF) functional form, and equally well fit to experimental data. Our work lends support to the notion that TF fits can be used in an experimental context to obtain information about the temperature.

There is now strong, but not unambiguous, evidence that fermionic superfluidity has been observed [1, 2, 3, 4, 5, 6] in trapped atomic gases. What is particularly exciting about these systems is that the strength of the pairing interaction can be arbitrarily tuned (via Feshbach resonance effects) so that superfluidity occurs in a regime which goes beyond traditional weak coupling BCS theory, towards Bose-Einstein condensation (BEC) of pre-formed pairs. These experiments are revealing the nature of superconductivity and superfluidity in a hitherto unexplored regime.

Experimental proof of superfluidity is not straightforward, except in the BEC regime where the particle density profiles acquire a bi-modal form. This bi-modality is similar to observations in bosonic atomic systems, which have been systematically studied for the past decade [7]. In the intermediate regime between BCS and BEC, the measured density profiles [8, 9] in traps do not contain any obvious signatures of the condensate. Nevertheless, there is a significant body of circumstantial evidence in support of superfluidity in this regime. This support is based on fast sweep experiments [1, 2], collective mode measurements [3, 4, 5], as well as pairing gap measurements using radio frequency (RF) techniques [6].

The goals of this paper are to compute the particle density profiles at all $T \leq T_c$ (where $T_c$ is the superfluid transition temperature) in the near-BEC and unitary regimes and to address the implications for experiment. For the former our calculation of the contribution from noncondensed Cooper pairs leads to important corrections to estimates of the condensate fraction based on the conventional Gaussian form. For the unitary case, these non-condensed pairs smooth out the, otherwise abrupt, transition between the condensate and the thermal background of fermionic excitations; this explains why the measured density profiles appear to be so featureless [8, 9].

The unitary or strongly interacting Fermi gas has been the focus of attention by the community. Some time ago it was found [10] that the profiles were reasonably well described by a Thomas-Fermi (TF) functional form at zero $T_c$, and in recent work [11] this procedure has been extended to finite temperatures. Importantly, there has been no particular theoretical support for these TF fits. Below $T_c$ all previous theoretical work (which either ignored non-condensed pair states [11, 12] or used a different ground state [13]) has predicted strong deviations from this $T$-dependent TF functional form. It is extremely important, thus, to have a better understanding of the spatial profiles. Our work lends theoretical support to the viewpoint [10] that TF fits (with proper calibration) can be used in an experimental context to obtain information about the temperature. In addition to establishing these TF fits to theory, we compare theory and experiment directly via one dimensional representations of their respective profiles. We demonstrate remarkable agreement in the shapes of the two profiles. One can infer from this comparison, experimental support, albeit indirect, for the presence of a superfluid condensate.

The first generation theories [14] have focused on a BCS-like ground state [15] which is readily generalized to accommodate a "two channel" variant [16] in which there are Feshbach bosonic (FB) degrees of freedom present as well. This approach has met with some initial success in addressing collective mode experiments [17, 18] and pairing-gap spectroscopy [19]. The excitations of this standard ground state [14, 16] consist of two types: noncondensed fermion pairs hybridized with FB, and Bogoliubov-like fermionic excitations having $E_k \equiv \sqrt{(c_k - \mu)^2 + \Delta^2(T)}$, where $c_k$ is the free fermion kinetic energy. Note that the "gap parameter" $\Delta(T)$ is in general different from the superconducting order parameter, $\Delta_{sc}$. Recent RF experiments [5] have been analyzed [19] using our formalism to suggest that the Bogoliubov quasi-particles have an energy gap or pseudogap, which is present well above $T_c$, and, thus, not directly related to the order parameter. Additional support for this "pseudogap" has recently been reported in another very different class of experiments [20].

We summarize the self-consistent equations [21], in the presence of a spherical trap, treated at the level of the local density approximation (LDA) with trap potential $V(r) = \frac{1}{2}m\omega^2r^2$. $T_c$ is defined as the highest temperature at which the self-consistent equations are satisfied precisely at the center. At a temperature $T$ lower than $T_c$ the superfluid region extends to a finite radius $R_{sc}$. The particles outside this radius are in a normal state, with or without a pseudogap. Our self-consistent equations are given in terms of the Feshbach coupling constant $g$ and inter-fermion attractive interaction $U$ by a gap equation

$$1 + \left[ U + \frac{g^2}{2\mu - 2V(r) - \nu} \right] \sum_k \frac{1 - 2f(E_k)}{2E_k} = 0 ,$$

which is imposed only when $\mu_{pair}(r) = 0$. The pseudogap
contribution to \( \Delta^2(T) = \tilde{\Delta}_s^2(T) + \Delta_{pg}^2(T) \) is given by
\[
\Delta_{pg}^2 = \frac{1}{Z} \sum_q b(\Omega_q - \mu_{\text{pair}}).
\] (2)

The various residues, \( Z \) and \( Z_0 \), which are of no particular physical importance here, are described in Ref. [14]. The quantity \( \Omega_q \) is the pair dispersion, and \( \mu_{\text{pair}} \) and \( \mu_{\text{boson}} \) are the effective chemical potentials of the pairs and FB.

We introduce units \( \hbar = 1 \), fermion mass \( m = \frac{1}{2} \), Fermi momentum \( k_F = 1 \), and Fermi energy \( E_F = \hbar \omega(3N)^{1/3} \). In the figures below, we express length in units of the Thomas-Fermi radius \( R_{TF} = \sqrt{2E_F/(m\omega^2)} = 2(3N)^{1/3}/k_F \); the density \( n(r) \) and total particle number \( N = \int d^3r n(r) \) are normalized by \( k_F^3 \) and \( (k_F R_{TF})^3 \), respectively. The density of particles at radius \( r \) can be written as
\[
n(r) = 2n^{0}_b + \frac{2}{Z_0} \sum_q b(\Omega_q - \mu_{\text{boson}})
+ 2 \sum_k \left[ \nu_k^0 (1 - f(E_k)) + \nu_k^2 f(E_k) \right],
\] (3)

where \( n^{0}_b = g^2 \tilde{\Delta}_s^2/\sqrt{\nu - 2\mu + 2V(r)}U + g^2 \) is the molecular Bose condensate which is only present for \( r < R_{sc} \). The important chemical potential \( \mu_{\text{pair}} = \mu_{\text{boson}} \) is identically zero in the superfluid region \( r < R_{sc} \), and must be solved for self-consistently at larger radii. Our calculations proceed by solving numerically the self-consistent equations.

We now decompose the density profiles into components associated with the condensate, the noncondensed pairs and the fermions. This raises a central issue of this paper. What is the most suitable definition of the condensate ratio in a fermionic superfluid? Indeed, in strict BCS theory there are two alternatives [22], one based on the superfluid density, and the other based on associating the condensate with the perturbation of the superfluid state relative to the underlying free Fermi gas. In BCS theory with this second definition, the zero temperature condensate fraction is of order \( T_c/E_F \), far from the value 100%, which one obtains from the superfluid density. Physically, this second definition reflects the fraction of the original Fermi liquid states which are modified substantially by pairing. We explore both of these here, since they are expected to enter in different physical contexts. In the BEC regime (where fermions are absent) there is no distinction between the two decompositions, and thus the condensate fraction is uniquely defined.

In the present approach to the BCS-BEC crossover picture it is relatively straightforward to deduce [14] the (local) superfluid density, \( n_s \equiv n_s(r) \). For the one-channel model we find [14] a simple result for \( n_s \), as well as for the fermionic quasiparticle \( n_{QF} \) and pair contributions \( n_{\text{pair}} \) to the difference \( n - n_s \):
\[
n_s = \frac{\Delta_s^2}{\Delta^2} n_{BCS}(\Delta),
\] (4a)
\[
n_{\text{pair}} = \frac{\Delta_{pg}^2}{\Delta^2} n_s^{BCS}(\Delta) = n_{BCS}(\Delta) - n_s,
\] (4b)

where \( n_{BCS} \) is the usual superfluid density for a gas of fermions within BCS theory, except that here the full excitation gap \( \Delta \) appears instead of the order parameter. In the one-channel case the order parameter \( \Delta_{sc} \) is equal to the Cooper condensate contribution (which we call \( \Delta_s \)). For the broad Feshbach resonances of \( ^{40}K \) and \( ^{6}Li \), these one-channel results are appropriate for the unitary and BCS regimes of the two-channel case. In the BEC regime (\( \mu < 0 \)), the contribution of the fermionic quasiparticles becomes negligible. Therefore, the one-channel equations [14] will be reduced to
\[
n_{QF} = n - n_s^{BCS}(\Delta),
\] (4c)

Interestingly, this result is also valid for the two-channel problem in the BEC regime.

Formally, we can address the second decomposition of the density profiles by writing the particle density in terms of the single particle Green’s function \( G(K) = G_0(K) + G_0(K)\Sigma(K)G(K) \) where \( G_0 \) represents free fermions, and \( K = (i\omega_n, k) \) is the four-momentum. The second term is then further split into the condensed \( (\Delta_s^2) \) and noncondensed \( (\Delta_{pg}^2) \) components. Summing over the Matsubara frequencies \( \omega_n \), and adding the FB contribution one obtains
\[
n_s = \frac{2Z}{\Delta^2} \Delta_s^2, \quad n_{\text{pair}} = \frac{2Z}{\Delta^2} \Delta_{pg}^2, \quad n_{QF} = \frac{2Z}{\Delta^2} \Delta_{QF}^2,
\] (6a)
(6b)
(6c)

which correspond to condensate density, finite momentum pair/boson density, and free fermion density, respectively.

We characterize the crossover regime by the parameter \( 1/k_F a \) where \( a \) is the inter-atomic s-wave scattering length and \( k_F \), the Fermi momentum. A typical density profile for the near-BEC limit with \( 1/k_F a = 3.0 \) is shown in Fig. II for \( T/T_c = 0.5 \). This plot is in the physically accessible near-BEC regime where the bosons are still primarily Cooper...
of radius distributions in the unitary limit, assumed throughout the trap (as is the experimental convention). It should be noted that the inferred fraction of the condensate may be provided evidence for the existence of a condensate. It should provide a significant role. It can be seen from the plots on the left of Fig. 2 that the noncondensed pairs show a relatively flat density distribution for $r \leq R_{sc}$, just as in the near-BEC limit of Fig. 1. This behavior similarly arises from the vanishing of $\mu_{\text{pair}}(r)$ in the superfluid region.

One can see that the condensate has substantially more weight for the $n_{\text{g}}$-based decomposition [Eq. (4), Fig. 2 lower panels]. Conversely the excited fermions for this case are significantly less important, since they experience a large gap in their excitation spectrum. The contribution of these fermions is concentrated at the trap edge. Indeed, for this case, because they represent two sides of the same coin, the fermions and noncondensed bosons generally appear in different regions of the trap. The latter are associated with the regions where the excitation gap is largest (and the fermion states are quasi-bound into “bosonic” like states), and the former are found where the gap is smallest.

In Fig. 3 we compare theory (at $T/T_F = 0.19$) and experiment for the unitary case. The profiles shown are well within the superfluid phase ($T_c \approx 0.3T_F$ at unitarity). This figure presents Thomas-Fermi fits to the experimental profiles (3a) and theoretical (3b) profiles as well as their comparison (3c), for a chosen $R_{TF} = 100 \mu m$, which makes it possible to overlay the experimental data (circles) and theoretical curve (line). Finally Fig. 3d indicates the relative $\chi^2$ or root-mean-square (rms) deviations for these TF fits to theory. This figure was made in collaboration with the authors of Ref. 10. This temperature was chosen to exhibit the TF functional form in the regime where it is most problematic, since $\chi^2$ has a maximum there. The experimental data were estimated to correspond to roughly this same temperature, based on the analysis of Ref. 10. Two of the three dimensions of the theoretical trap profiles were integrated out to obtain a one-dimensional representation of the density distribution along the transverse direction: $\bar{n}(x) \equiv \int dy dz n(r)$.

This figure is in contrast to earlier theoretical studies which predict a kink at the condensate edge [11, 12]. Moreover, in contrast to the predictions of Ref. 13, the curves behave monotonically with both temperature and radius. Indeed, in the unitary regime the generalized TF fitting procedure of Ref. 10 works surprisingly well for our theory with spherical traps, and for anisotropic experiments (shown in Fig. 3d). This is, in part, a consequence of the fact that trap anisotropy effects become irrelevant for these one dimensional projections. These reasonable TF fits apply to essentially all temperatures investi-

![Density profiles](image)
gated experimentally [10], and all temperatures we have studied, including in the normal state.

To probe the deviations from a TF functional form, and to establish the role of the condensate, in Fig. 3, we show the (relative) rms deviation, or $\chi^2$, from the TF fits as a function of $T$. Except at the very lowest temperatures (where the $\chi^2$ is small and the condensate occupies the entire region of the trap), the condensate causes a small, but potentially measurable variation from the TF form. $\chi^2$ increases rapidly below $T_c$ and reaches a maximum around 0.7$T_c$. Here the profile involves two different functional forms: the condensate and the thermal distribution functions. More precisely, the systematic behavior of $\chi^2$ indicates when $T$ has safely surpassed $T_c$.

The reason the Thomas-Fermi fits work well below $T_c$ has to do with the presence of noncondensed pairs. If these bosonic-like states are ignored, (by omitting the top-most shaded regions) in the right panels of Fig. 2 a clear bimodal distribution emerges, as has been predicted in the literature [11][12]. In this case, TF fits would not work well. A residue of this previously discussed bi-modality must be present to some extent in the present plots, but this can only be seen in the derivatives of our profiles. We find a small kink in the first order derivative and a sharp jump in the second order derivative of $n(r)$, reflecting the condensate edge. These features may be more difficult to identify with current experimental techniques. Alternatively, one may infer the presence of a condensate through $\chi^2$ plots from TF fits. These experiments will require substantial averaging over a large number of profiles, however.

In summary, in this paper we have found that, in the near BEC regime, the component profile for the noncondensed bosons, is different from the Gaussian form generally assumed, although the general shape is consistent with experiment. This result which follows because these noncondensed states are in equilibrium with a condensate, should have implications for measurements of the condensate fraction. Near unitarity, we have found that, our calculated density profiles are consistent with experiment and provide strong support for using Thomas Fermi fits to the profiles. One can infer that the condensate is seen, not in the profile shapes, but, presumably, in their temperature calibration.

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