Understanding primary school children’s learning on addition of fractions

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Abstract. Fractions are universally known to be difficult to learn. Learning basic fractions may not be sustainable if one is to apply it to more advanced mathematical content knowledge as it is interlinked with other mathematics topics. In the effort to understand how children grasp the learning of fractions additions, a study was carried out with a random sampling of Years 4, 5 and 6 children in Brunei Darussalam. The research participants consisted of 33 children who are currently studying at government and private primary schools in Brunei. A pen-and-paper self-designed test was used as an instrument in collecting the data. The quantitative data was analysed by using a descriptive analysis method. The overall reliability of the self-designed tests indicated that the items have relatively high internal consistency with a Cronbach alpha of .859. The findings revealed that the children’s understanding is stronger on questions with answers regarding proper fractions rather than improper fractions. This could be further explored through future studies employing a qualitative approach.

1. Introduction

Topics on difficulty to teach and learn fractions have been reiterated in literature of Primary Mathematics teaching and learning over more than a decade. Students of all school levels around the world have difficulty to grasp the concept [1-5]. Unfortunately, the topic on fractions is a recurring topic learnt, if not directly related, across at least throughout primary to secondary Mathematics learning. Hence its importance as a basis for later understanding on other related topics such as on decimals, percentages, ratios, rates and percentages at various stages of schooling as well as occupational success [6, 7], as well as in the development of technology [8, 9]; this renders for inevitable sound understanding.

It is imperative to note that a mathematical fraction is a part of a unit of measure; numbers used for measuring, but not for counting. Fractions can be represented in 5 sub-constructs. They can be seen as ‘Part-Whole’ sub-construct, which is referring to a quantity divided into equal sizes [10], as ‘ratio’ sub-construct that is to compare two quantities [9], as an ‘operator’ sub-construct, which is application of a function to a number, as ‘quotient’ sub-construct as a result of division [11] and finally as ‘measure’ sub-construct that is ordered along a number line [11, 12].
2. Background of the study
Harun [13] reported that secondary school students in Form 4, hereafter known as Year 10, when tested, performed very poorly in fractional tasks, which are equivalent to syllabus covered in Years 4 and 5 including the four operations of fractions. This is an indication that their basic understanding on fractions is not sound enough for them to obtain a new and more difficult concept in the same topic, fractions, even though Bruneian children have been exposed to fractions as early as Year 2. By Year 5, they should have had 3 years of exposure to fractions.

To do well in fractions, prerequisite knowledge as basic as knowing how to add, subtract, multiply and divide natural numbers must be mastered first and foremost. Without a sound foundation, children will not be able to operate fractions. Ultimately, having strong foundation and being able to relate other topics with fractions in Mathematics is part and parcel of the elementary Mathematics that children must acquire.

Bailey et al. [6] stated that conceptual understanding and procedural knowledge are essential to enable one to perform well in learning fractions. Trivena, Ningsih, and Jupri [14] concur to this. The procedural knowledge of fractions in arithmetic operations skills includes addition, subtraction, multiplication, and division of fractions. Being able to grasp addition of fractions is the prerequisite to understanding the other three operations of fractions, namely subtraction, multiplication and division of fractions, which are closely intertwined to addition of fractions. Whereas the fraction foundation that is the conceptual understanding is embedded in the process in showing fractions as a magnitude allowing students to compare and sort fractions by size that later on will relate to their ability to make sense of proper and improper fractions. Perhaps children might not face much problem in dealing with proper fractions, but they may in on the latter. Baturo and Cooper [15] found in their study that students’ understanding of mixed numbers and improper fractions as well as to unitise or reunitise them on number lines are still lacking. Steffe and Olive [16] mentioned that improper fractions (fractions that are bigger than one) are conceptually challenging to children.

This brings us to the purpose of the study that is to find out the children’s understanding on addition of fractions especially to those questions with improper fractions as their answers. The study is guided by three research questions:

1. What are Year 5 performances in understanding addition of fractions?
2. Is there any correlation in children’s performances in Set A and Set B questions?
3. Do the results differ in Set A and in Set B questions?

3. Method
A random sampling method was employed in this study. A group of participants of mixed abilities were randomly selected with six children from Year 4, five children from Year 6 and while the rest were from Year 5. The data collection tool for this study was a test administered to participants with the age ranging from 8 to 12 years old either from government or private schools in Brunei Darussalam. The pen-and-paper test was administered to all the children at different times and places depending on the preferences of the parents involved.

The pen-and-paper test consisted of a back-to-back six open-ended questions. The items were developed into three parts. Part 1 consists of fractions additions involving like fractions while Part 2 requires children to add unlike fractions with related multiples. Part 3 involves addition of unlike fractions with related multiples. The test items were generated based on the question exercises given to the children in Year 4 in the National Yes Mathematics textbook. Year 4 was chosen instead of Year 5 as the test would be used prior to the studying of a more complicated addition of fractions, which involved addition of mixed numbers in Year 5. However, there are two categories of possible outcomes from the test: proper and improper fractions. Time taken for the test was noted as well.

Due to the nature of the pilot study and the limitation that culminated during the test, which was taken on multiple dates and in various sites, the potential for any participants to start not according to scheduled time was expected thus, each participant was required to fill in the time they started and the time they finished.
Data was entered into the statistical software known as Statistical Package for the Social Sciences (SPSS). Prior to the analysis, the children were coded from P1 to P33. The time taken by the participants was also recorded as a guide during the main study. All questions were answered and 100% return rate was noted. Although the test was designed into three sections, it was analysed in two sets: Set A and Set B. Set A consists of questions the outcome of which is a proper fraction that is less than one while Set B consists of questions the outcome of which is an improper fraction that is more than one. Non-parametric tests were employed in this study. To answer research questions 2 and 3, non-parametric tests were conducted to see the relation between the test scores of set A and Set B using Spearman correlation while Wilcoxon signed-rank test was conducted to find the differences in children’s performance in set A and Set B respectively. Effect size was also computed. Pallant [17] cited Cohen as saying for z value, the criteria to place effect size is of .1=small effect, .3= medium effect, .5=large effect.

The internal consistency of the items is relatively high with Cronbach alpha of .859, indicating the test to be suitable and adoptable for further studies. For a test that consists of less than 10 questions, obtaining Cronbach alpha >.50 is already acceptable [17].

In terms of the suitability or the validity of the self-designed test, the results from the Year 4 children, who obtained more than 60%, were recorded as the lowest with 33.3%. Year 6 children with the least number of participants performed 80% for those obtaining more than 60%. Year 5 children obtaining more than 60% were revealed to be in between Years 4 and 6 with only 36%. And having been computed as the second group obtaining above 60%, the level of difficulty of the self-designed test was suitable for Year 5, which is not too easy and not too difficult for the Year 5 children as seen in Table 1.

| Year | Total No. of children | No. of children scoring more than 60% | % of scoring more than 60% |
|------|-----------------------|--------------------------------------|---------------------------|
| 4    | 6                     | 2                                    | 33.3% (lowest)            |
| 5    | 22                    | 8                                    | 36%                       |
| 6    | 5                     | 4                                    | 80% (Highest)            |

There was no time allocated for the children to complete the test. However, they were encouraged to try to answer all the questions and recheck their answers. They were asked to record the time of start on the top of the test paper while the first author recorded the time they had completed the test as they submitted their paper. As a result of the recorded time, the first author was able to estimate the time taken for the main study, which was not more than 10 minutes excluding time for instructions, distributing and collecting the test paper. Recorded mean time was only approximately 4.77 minutes.

4. Results and Discussion
4.1. Four types of Year 5 performances in understanding addition of fractions
In answering research question 1, there were four types of students identified in their understanding of addition of fractions. They were Type A, those with secure and sound understanding on addition of fractions, Type B those who had incomplete understanding and yet to develop their understanding on addition of fractions and Type C, those who lacked or had little understanding on addition of fractions. While those in Type D, had no understanding on addition of fractions.

Those in Type A obtained full scores on both Set A and Set B questions while Type C children are those who obtained nothing in either set. Whereas those in Type B were those whom either scored full marks in either set but scored less in the other. Type D children are those who didn’t score anything in both sets as shown in the Table 2. Majority of the participants were of Type C with 36.4%, whereas the relatively small percentage was in Type D with 18.2%. Types C and D were those who are badly in need of help and given more attention to.
Table 2. Types of children’s performances in Set A and Set B.

| Types | Children performance combination in Set A and Set B (Set A, Set B) | Number of children (n = 33) |
|-------|---------------------------------------------------------------|---------------------------|
| Type A | (3, 3)                                                        | 7 (21.2%)                 |
| Type B | (3, 2), (2, 3), (2, 2), (1, 3), (3, 1)                        | 8 (24.2%)                 |
| Type C | (1, 0), (0, 1), (2, 0), (0, 2), (3, 0), (0, 3)                | 12 (36.4%)                |
| Type D | (0, 0)                                                        | 6 (18.2%)                 |

4.2. Correlation of children’s performances in Set A and Set B

Research question 2 focuses on the relation between the children’s performances in Set A and Set B questions. It was found that children’s performances in Set A and Set B were strongly and positively correlated between the two variables: Set A and Set B questions with Spearman correlation, \( r = .73 \), \( n = 33 \), \( p < .001 \), those who scored high in set A tended to score relatively the same high score for set B. Likewise those who performed poorly in Set A tended to do the same in Set B.

4.3. Children’s performances based on types of questions

To answer research question 3, a Wilcoxon signed ranks test was conducted to see if the results of the children’s performance on Set A and Set B questions differed. A Wilcoxon signed ranks test revealed children performances on Set B has statistically significant difference in Set B (Md=1.00, n=33) and Set A (Md=2.00, n=33), \( Z = -2.157, p=0.031, r=0.38 \). The median score confirmed slight differences between both sets, having the children to perform better in questions in Set A than Set B, with moderate effect size (\( r=0.38 \)).

4.4. Discussion

As mentioned in the findings, children who performed 60% and above in the total test, with Types A and B combined were still lower than those of Type C and Type D. This is an indication of weak understanding in this topic and if children’s understanding did not improve as they progress, similar scenario as revealed by Harun [13] would happen. If at this level, the children’s understanding was not rectified, their poor performance would prevail till they reach Year 11 as discussed in Harun’s [13] study. This might jeopardise future Mathematics competencies attained in Brunei.

Some children who scored full marks in Set A but scored nil in Set B indicated that the development of their understanding was not fully developed. They were yet to further develop the new conception pertaining to questions with outcomes more than one (improper fractions) especially in relation to the median in Set B that was lower than in Set A. The findings concurred with findings that had been revealed by both Baturo and Cooper [15] and, Steffe and Olive [16], mentioned earlier.

5. Conclusion

This study indicated significant differences in the children’s performances in Set A and Set B even after being exposed to addition of fractions in Year 4, which is a repetitive topic but also increase in level of difficulty in Year 5. Children performed poorly in Set B questions with outcomes of improper fractions than in Set A. It is suggested to find out whether, it is the type of questions or due to the lack of knowledge of fraction magnitude that have contributed to the lack of performance in additions of fractions of Set B type. A further analysis could be conducted in Set A questions. It would be interesting to see how children who have portrayed understanding and borrow their line of reasoning and share that reasoning with those lacking the understanding.

All in all, regardless of the alarming figure of those who understands addition of fractions in this study (\( n = 33 \)), it is also acknowledged that there are also some who portrayed understanding. This will be the future direction of this study, wherein to pursue it qualitatively. Lastly, this self-designed test can be further used to capture children with understanding of addition of fractions.
Due to the limitation of the sample size in this study, drawing conclusive findings deems impossible. Hence a bigger sample can only be seen as a working hypothesis. Since Mathematics and understanding is complex in nature and cannot be comprehended or deciphered by the naked eyes, this test can be compensated with a qualitative study that can closely address children’s mathematical thinking towards addition of fractions. Perhaps, children should be asked during the qualitative study to check the certainty of their understanding as well.

The limitation found in this pilot study pertaining to the dissemination of the test informs the researchers to conduct the study in schools where the distribution of the test can be conducted in a standardised manner rather than opening it to public. Hence this also serves to inform the approximate potential number of participants’ in the future main study.

Acknowledgement
The authors wishes to express their gratitude to Dr Marlizayati Johari, a Physics lecturer for her guidance in using SPSS, as well as Nor’Azmah and Dk Akmarul for sharing some insights on this study.

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