Compton Scattering and Generalized Polarizabilities

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Abstract. In recent years, real and virtual Compton scattering off the nucleon have attracted considerable interest from both the experimental and theoretical sides. Real Compton scattering gives access to the so-called electromagnetic polarizabilities containing the structure information beyond the global properties of the nucleon such as its charge, mass, and magnetic moment. These polarizabilities have an intuitive interpretation in terms of induced dipole moments and thus characterize the response of the constituents of the nucleon to a soft external stimulus. The virtual Compton scattering reaction $e^- p \rightarrow e^- p \gamma$ allows one to map out the local response to external fields and can be described in terms of generalized electromagnetic polarizabilities. A simple classical interpretation in terms of the induced electric and magnetic polarization densities is proposed.

We will discuss experimental results for the polarizabilities of the proton and compare them with theoretical predictions.

INTRODUCTION AND OVERVIEW

Real Compton scattering (RCS), $\gamma(q, \varepsilon(\lambda)) + N(p, s) \rightarrow \gamma(q', \varepsilon'(\lambda')) + N(p', s')$, has a long history of providing important theoretical and experimental tests for models of nucleon structure (see, e.g., Refs. [1, 2, 3] for an introduction). Based on the requirement of gauge invariance, Lorentz invariance, crossing symmetry, and the discrete symmetries, the famous low-energy theorem of Low [4] and Gell-Mann and Goldberger [5] uniquely specifies the terms in the low-energy scattering amplitude up to and including terms linear in the photon momentum. The coefficients of this expansion are expressed in terms of global properties of the nucleon: its mass, charge, and magnetic moment. In principle, any model respecting the symmetries entering the derivation of the LET should reproduce the constraints of the LET. It is only terms of second order which contain new information on the structure of the nucleon specific to Compton scattering. For a general target, these effects can be parameterized in terms of two constants, the electric and magnetic polarizabilities $\alpha$ and $\beta$, respectively [6].

The scattering amplitude may be parameterized in terms of six independent functions $A_i$ depending on the photon energy $\omega$ and the scattering angle,$$
T = \vec{\varepsilon}'^* \cdot \vec{\varepsilon} A_1 + \vec{\varepsilon}'^* \cdot \vec{q} \vec{\varepsilon} \cdot \vec{q}' A_2 + i \vec{\sigma} \cdot \vec{\varepsilon}'^* \times \vec{\varepsilon} A_3 + \cdots .
$$

In the forward and backward directions only two functions, namely, $A_1$ and $A_3$, contribute. For example, the Taylor series expansion of $A_1$, for the proton, is given by

$$A_1 = -\frac{e^2}{m} + 4\pi(\alpha + \beta z) \omega^2 - \frac{e^2}{4m^3}(1-z) \omega^2 + [\omega^4],$$

(1)
where \( z = \cos(\theta) \). The leading-order term is given by the Thomson term and the forward and backward amplitudes are sensitive to the combinations \( \alpha + \beta \) and \( \alpha - \beta \), respectively. The sum of the polarizabilities is constrained by the Baldin sum rule [7],

\[
\alpha + \beta = \frac{1}{2\pi^2} \int_{\omega_{\text{thr}}}^{\infty} \frac{\sigma^{\text{tot}}(\omega)}{\omega^2} d\omega,
\]

where \( \sigma^{\text{tot}}(\omega) \) is the total photoabsorption cross section. A fit to all modern low-energy experiments [8, 9, 10, 11] and the sum rule relation \( \sigma_p + \beta_p = (13.8 \pm 0.4) \) leads to the result [11]

\[
\alpha_p = 12.1 \pm 0.3_{\text{stat}} \pm 0.4_{\text{syst}} \pm 0.3_{\text{mod}}, \quad \beta_p = 1.6 \pm 0.4_{\text{stat}} \pm 0.4_{\text{syst}} \pm 0.4_{\text{mod}}.
\]

Clearly, the electric polarizability \( \alpha_p \) dominates over the small magnetic polarizability \( \beta_p \), the smallness of which is thought to result from a delicate cancellation between the paramagnetic \( \Delta \) contribution and a nearly equally strong diamagnetic term. Although, for example, soliton models of the nucleon have predicted such a destructive interference for a long time [12, 13], the precise microscopic origin of the relatively large diamagnetic contribution is still under debate.

Information on the neutron has been obtained via low-energy neutron-\(^{208}\)Pb scattering [14], quasi-free Compton scattering \( \gamma d \to \gamma' np \) [15],

\[
\alpha_n = 12.0 \pm 1.5_{\text{stat}} \pm 2.0_{\text{syst}},
\]

and elastic \( \gamma d \) scattering [16]

\[
\alpha_n = 8.8 \pm 2.4_{\text{stat}} \pm 3.0_{\text{syst}}, \quad \beta_n = 6.5 \pm 2.4_{\text{stat}} \pm 3.0_{\text{syst}}.
\]

A recent re-analysis of the sum rule yields \( \alpha_n + \beta_n = (15.2 \pm 0.5) \) [17].

New extractions using effective field theory for the nucleon as well as the deuteron have yielded for the proton polarizabilities at \( \mathcal{O}(p^3) \) in chiral perturbation theory [18]

\[
\alpha_p = 12.1 \pm 1.1_{\text{stat}} \pm 0.5_{\text{mod}}, \quad \beta_p = 3.4 \pm 1.1_{\text{stat}} \pm 0.1_{\text{mod}},
\]

and for the isoscalar nucleon polarizabilities

\[
\alpha_N = 13.0 \pm 1.9_{\text{stat}} \pm 3.9_{\text{mod}},
\]

and a \( \beta_N \) that is consistent with zero within sizeable error bars. Another calculation including explicit \( \Delta \) degrees of freedom obtained [19, 20]

\[
\alpha_p = 11.04 \pm 1.36, \quad \beta_p = 2.76 \pm 1.36,
\]

\[
\alpha_N = 12.6 \pm 1.4_{\text{stat}} \pm 1.9_{\text{syst}}, \quad \beta_N = 2.3 \pm 1.7_{\text{stat}} \pm 0.3_{\text{syst}}.
\]

For a recent review on the status of the spin polarizabilities, see Ref. [3].

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\(1\) The polarizabilities \( \alpha \) and \( \beta \) are given in units of \( 10^{-4} \) fm\(^3\).
VIRTUAL COMPTON SCATTERING AND GENERALIZED POLARIZABILITIES

As in all studies with electromagnetic probes, the possibilities to investigate the structure of the target are much greater if virtual photons are used, since the energy and the three-momentum of the virtual photon can be varied independently. Moreover, the longitudinal component of current operators entering the amplitude can be studied. The amplitude for virtual Compton scattering (VCS) off the proton, \( T_{VCS} \), is accessible in the reaction \( e^- p \to e^- p \gamma \). Similarly to Eq. (1), \( T_{VCS} \) can be expressed in terms of eight transverse and four longitudinal amplitudes. Model-independent predictions, based on Lorentz invariance, gauge invariance, crossing symmetry, and the discrete symmetries, have been derived in Ref. [21]. Up to and including terms of second order in the momenta \( |\vec{q}| \) and \( |\vec{q}'| \), all functions \( A_i \) are completely specified in terms of quantities which can be obtained from elastic electron-proton scattering and RCS, namely \( m, \kappa, G_E, G_M, r_E^2, \alpha, \) and \( \beta \). After dividing the amplitude \( T_{VCS} \) into a gauge-invariant generalized pole piece \( T_{pole} \) and a residual piece \( T_R \), the so-called generalized polarizabilities (GPs) of Ref. [22] result from an analysis of the residual piece in terms of electromagnetic multipoles. A restriction to the lowest-order, i.e. linear terms in \( \omega' \) leads to only electric and magnetic dipole radiation in the final state. Parity and angular-momentum selection rules, charge-conjugation symmetry, and particle crossing generate six independent GPs [22, 23, 24].

Similarly as elastic electron scattering allows one to map out the spatial distribution of charge and magnetization inside the nucleon, the generalized polarizabilities parameterize a local response of a system in an external field. For example, if the nucleon is exposed to a static and uniform external electric field \( \vec{E} \), an electric polarization \( \vec{P} \) is generated which is related to the density of the induced electric dipole moments, \( \mathcal{P}_i(\vec{r}) = 4\pi \alpha_{ij}(\vec{r}) E_j \). (4)

The tensor \( \alpha_{ij}(\vec{r}) \), i.e. the density of the full electric polarizability of the system, can be expressed as [25]

\[
\alpha_{ij}(\vec{r}) = \alpha_L(r) \delta_{ij} + \alpha_T(r) (\delta_{ij} - \hat{r}_i \hat{r}_j) + \frac{3}{r^3} \int_r^\infty \left[ \alpha_L(r') - \alpha_T(r') \right] r'^2 \, dr',
\]

where \( \alpha_L(r) \) and \( \alpha_T(r) \) are Fourier transforms of the generalized longitudinal and transverse electric polarizabilities \( \alpha_L(q) \) and \( \alpha_T(q) \), respectively. The definition of the generalized dipole polarizabilities of Ref. [25] has been obtained from a fully covariant framework as opposed to the multipole decomposition of Ref. [22]. In particular, it is important to realize that both longitudinal and transverse polarizabilities are needed to fully recover the electric polarization \( \mathcal{P} \). The left panel of Fig. 1 shows the induced polarization inside a nucleon as calculated in the framework of heavy-baryon chiral perturbation theory at \( \mathcal{O}(p^3) \) [26] and clearly shows that the polarization, in general, does not point into the direction of the applied electric field.

Similar considerations apply to an external magnetic field. Since the magnetic induction is always transverse (i.e., \( \vec{\nabla} \cdot \vec{B} = 0 \)), it is sufficient to consider \( \beta_{ij}(\vec{r}) = \beta(r) \delta_{ij} \) [25]. Then the magnetization \( \mathcal{M} \) induced by the uniform external magnetic field is given in
terms of the density of the magnetic polarizability as $\vec{M}(\vec{r}) = 4\pi\beta(r)\vec{B}$ (see right panel of Fig. 1).

**FIGURE 1.** Left panel: Scaled electric polarization $r^3\alpha_1 [10^{-3} \text{ fm}^3]$ [26]. Right panel: Generalized magnetic polarizability $\beta(q^2)$ and density of magnetic polarizability $\beta(r)$. Dashed lines: contribution of pion loops; solid lines: total contribution; dotted lines: VMD predictions normalized to $\beta(0)$ [25].

The first results for the two structure functions $P_{LL} - P_{TT}/\varepsilon$ and $P_{LT}$ at $Q^2 = 0.33 \text{ GeV}^2$ were obtained from a dedicated VCS experiment at MAMI [27]. Results at higher four-momentum transfer squared $Q^2 = 0.92$ and $Q^2 = 1.76 \text{ GeV}^2$ have been reported in Ref. [28]. Additional data are expected from MIT/Bates for $Q^2 = 0.05 \text{ GeV}^2$ aiming at an extraction of the magnetic polarizability [29]. Moreover, data in the resonance region have been taken at JLab for $Q^2 = 1 \text{ GeV}^2$ [30] which have been analyzed in the framework of the dispersion relation formalism of Ref. [3, 31].

Table 1 shows the experimental results of [27] in combination with various model calculations. Clearly, the experimental precision of [27] already allows for a critical test of the different models. Within ChPT and the linear sigma model, the GPs are essentially due to pionic degrees of freedom. Due to the small pion mass the effect in the spatial distributions extends to larger distances (see also right panel of Fig. 1). On the other hand, the constituent quark model and other phenomenological models involving Gauß or dipole form factors typically show a faster decrease in the range $Q^2 < 1 \text{ GeV}^2$.

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### TABLE 1

Experimental results and theoretical predictions for the structure functions $P_{LL} - P_{TT} / \epsilon$ and $P_{LT}$ at $Q^2 = 0.33 \, \text{GeV}^2$ and $\epsilon = 0.62$. * makes use of symmetry under particle crossing and charge conjugation which is not a symmetry of NRCQM.

| Source          | $P_{LL} - P_{TT} / \epsilon$ [GeV$^{-2}$] | $P_{LT}$ [GeV$^{-2}$] |
|-----------------|------------------------------------------|----------------------|
| Experiment [27] | $23.7 \pm 2.2_{\text{stat}} \pm 4.3_{\text{syst}} \pm 0.6_{\text{syst.norm.}}$ | $-5.0 \pm 0.8_{\text{stat}} \pm 1.4_{\text{syst}} \pm 1.1_{\text{syst.norm.}}$ |
| LSM [32]        | 11.5                                     | 0.0                  |
| ELM [33]        | 5.9                                      | -1.9                |
| HBChPT [34]     | 26.0                                     | -5.3                |
| NRCQM [35]      | 19.2$^{\pm 14.9^\circ}$                 | -3.2$^{\pm 4.5^\circ}$ |

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