QUANTUM MEASUREMENT PROBLEM, DECOHERENCE, AND QUANTUM SYSTEMS
SELFDESCRIPTION

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Abstract

Quantum Measurements regarded in Systems Selfdescription framework for mea-
suring system (MS) consist of measured state S environment E and observer O pro-
cessing input S signal. O regarded as quantum object which interaction with S,E obeys to Schrodinger equation (SE) and from it and Breuer selfdescription formal-
ism S information for O reconstructed. In particular S state collapse obtained if O selfdescription state has the dual structure $L_T = \mathcal{H} \otimes \mathcal{L}_V$ where $\mathcal{H}$ is Hilbert space of MS states $\Psi_{MS}$. $\mathcal{L}_V$ is the set with elements $V^O = |O_j\rangle\langle O_j|$ describing random 'pointer' outcomes $O_j$ observed by $O$ in the individual events. The 'preferred' basis $|O_j\rangle$ defined by $O$ state decoherence via $O$ - E interactions. Zurek’s Existential Interpretation discussed in selfmeasurement framework.

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1 Introduction

Quantum Measurement theory is now well established branch of Quantum Mechanics (QM) with many important application mainly in Quantum Information field. Yet its foundations still actively disputed and most long and hot discussion concerned with the state collapse or the objectification problem. This Quantum Measurement problem, discussed here seems to be still unresolved despite the multitude of the proposed solutions (for the review see [1]). In this paper we study dynamics of the quantum information transfer in the measurement process and resulting from it information restrictions. Really, any measurement quantum or classical is Observer information acquisition about studied system S via direct or indirect interaction with it. In classical case this interaction can be done very small and neglected in the calculations, but in QM its influence can be quite important for obtained measurement outcome [2].

Under observer we mean information gaining and utilizing system (IGUS) of arbitrary structure [2]. It can be both human brain or some automatic device processing the information,
but in all cases it’s the system with some internal degrees of freedom (DF) excited during the information acquisition. The computer information processing or perception by human brain supposedly corresponds to the physical objects evolution which on microscopic level obeys to QM laws. Example of it are electron pulses excited in computer circuits during information bits memorization. Such correspondence for mental processes isn’t proved, but there are now strong experimental evidences that QM successfully describes complex systems including biological ones. Basing on them we concede that QM description is applicable both for microscopic and macroscopic objects including observer O (he, Bob). In the simple model dated back to Wigner O state described by Dirack state vector $|O\rangle$ (or density matrix $\rho$ for other cases) relative to some other quantum observer $O'$ (she; Alice). Class of microscopic measurement theories which account observer quantum effects sometimes called Relational QM (for the review see [3]). Our microscopic model of the measuring system (MS) in general includes the measured state (particle) S, detector D, environment E and quantum observer O which processes and stores the information.

The novel point of this approach is that O must describe consistently also his own quantum state, which corresponds to his impressions. Observer selfdescription in the measurement process (selfmeasurement) can be regarded in the context of the general algebraic and logical problems of selfreference [4]. In this framework Breuer derived the general selfmeasurement restrictions for classical and quantum measurements [5]. Basing on his results we propose here modification of standard QM Hilbert space formalism which account observer selfmeasurement features consistently. Its main feature is the enlargement of QM states set $L_T$ over standard Hilbert space $H$, so that $L_T = H \otimes L_R$, where $L_R$ is linear space, which elements describes O information acquired in the measurement in individual events. This modification conserves standard Schrodinger quantum dynamics, but permit to obtain the subjective state collapse in S measurement.

Here it’s necessary to make some comments on our model premises and review some terminology. In our model we’ll suppose that MS always can be described completely (including Environment E if necessary) by some state vector $|MS\rangle$ relative to $O'$ or by density matrix for mixed cases. MS can be closed system, like atom in the box or open pure system surrounded by electromagnetic vacuum or E of other kind. We don’t assume in our work any special dynamical properties of O internal states beyond standard QM. In this paper the brain-computer analogy used without discussing its reliability and philosophical implications [3]. We must stress that throughout our paper the observer consciousness (OC) never referred directly. Rather in our model observer O can be regarded as active reference frame (RF) which interacts with studied object S changing O internal state and thus storing information about S. Thus S state description ’from the point of view’ of the particular O referred by the terms ’S state in O RF’ or simply ’S state for O’. The terms ’perceptions’, ’impressions’ used by us to characterize observer subjective description of experimental results and defined below in strictly physical terms [7].

2 Selfmeasurement and Quantum States Restrictions

In Von Neuman (vN) measurement scheme S interacts with elementary quantum detector D which final state becomes entangled with S [1]. In this model MS chain ended on D and observer interaction with D supposedly is unimportant. We regard the simple model where O has analogous to D structure and same reaction on S input signal which permit to memorize it, as shown below. We omit detector D in MS chain assuming that S directly interacts with O. It’s possible because if to neglect decoherence the only D effect is the amplification of S signal to make it conceivable for O. For the start we omit also O-E interaction - decoherence, but later
we’ll account it and study its influence. In practice detector D and IGUS O have many internal DFs, but their account doesn’t change principally the results obtained below \[21\]. The example of dynamical model with many DFs gives Coleman-Hepp model described in \[23\].

Let’s consider \(O’\) description of the measurement performed by \(O\) of binary observable \(\hat{Q}\) on S state :

\[
\psi_s = a_1|s_1\rangle + a_2|s_2\rangle
\]

, where \(|s_{1,2}\rangle\) are \(Q\) eigenstates with values \(q_{1,2}\). In our model \(O\) has single effective DF and its states space \(\mathcal{H}\) contains at least three orthogonal states \(|O_i\rangle\) which are the eigenstates of \(Q_O\) ‘internal pointer’ observable. Initial \(O\) state is \(|O_0\rangle\) and MS initial state is :

\[
\Psi_{MS}^{in} = (a_1|s_1\rangle + a_2|s_2\rangle)|O_0\rangle
\]

Let’s assume that S-O measuring interaction starts at \(t_0\) and finished effectively at some finite \(t_1\). In our model MS evolution described by Schrodinger equation (SE). From SE linearity the final state of MS system relative to \(O’\) observer for suitable S–O interaction Hamiltonian \(\hat{H}_I\) will be \([2]\) :

\[
\Psi_{MS} = a_1|s_1\rangle|O_1\rangle + a_2|s_2\rangle|O_2\rangle
\]

to which corresponds the density matrix \(\rho_{MS}\), called also statistical state. It obeys to corresponding Schrodinger-Liouville equation (SLE). Thus \(|O_{1,2}\rangle\) are \(O\) states induced by the measurement of eigenstates \(|s_{1,2}\rangle\). In vN theory the corresponding final S,D state is : \(\Psi_{S,D} = \sum_i a_i|S_i\rangle|D_i\rangle\).

All this states including \(|O_i\rangle\) belongs to MS Hilbert space \(\mathcal{H}'\) defined in \(O’\) RF and formally Hilbert space \(\mathcal{H}\) in O RF can be obtained performing \(\mathcal{H}'\) unitary transformation \(\hat{U}'\) to O c.m.s.. In our case when we regard only internal or RF independent states \(U' = I\) can be taken and thus \(\Psi = \Psi^O\) for arbitrary states in \(O’\) and O RF correspondingly. For \(\rho\) defined on \(\mathcal{H}'\) their set denoted \(L_o; O\) states \(|O_i\rangle \in \mathcal{H}_o\) Hilbert subspace of \(\mathcal{H}\); \(\rho_{O}\) subset on \(\mathcal{H}_o\) is \(L_O \subset L_o\).

Thus QM predicts at time \(t > t_1\) for external \(O’\) MS is in the pure state \(\Psi_{MS}\) of \([2]\) which is superposition of two states for different measurement outcomes. MS state in \(ORF\) \(\Psi^{O}_{MS}\) obtained from \(\Psi_{MS}\) by transformation \(U'\), but as was argued in this case (neglecting space shift) \(\Psi^{O}_{MS}\) coincides with \(\Psi_{MS}\). Yet we know that experimentally macroscopic \(O\) observes some random \(Q_O\) value \(q_{1,2}'\) from which he concludes that S final state is \(|s_1\rangle\) or \(|s_2\rangle\), i.e. S state collapses. Thus SLE violated and can’t be applied to the measurement process. In standard QM with Reduction Postulate S final state described by the statistical ensemble of individual final states for \(O\) described by density matrix of mixed state \(\rho^S_m\):

\[
\rho^S_m = \sum_i |a_i|^2|s_i\rangle\langle s_i|
\]

to which responds in vN model random \(D_i\) pointer outcomes, described by S,D states mixture \([1]\). In our model we can phenomenologically ascribe to MS the corresponding mixed state :

\[
\rho_m = \sum_i |a_i|^2|s_i\rangle\langle s_i||O_i\rangle\langle O_i|
\]

which principally differs from \(\Psi_{MS}\). From \(O\) ‘point of view’ \(\Psi_{MS}\) describes superposition of two contradictory impressions : \(Q = q_1\) or \(Q = q_2\) perpectd simultaneously, which Wigner claimed to be nonsense \([3]\). If observers regarded as quantum objects then this contradiction constitutes famous Wigner ‘Friend Paradox’ for \(O, O’\) \([3]\). Thus MS state relative to \(O\) and \(O’\) looks principally different and even contradictory, but it’s quite difficult to doubt both in correctness of \(O'\) description of MS evolution by Schrodinger equation and in the state collapse experimental observations. We attempt to reconcile this two alternative pictures in the united formalism which incorporate both quantum system descriptions 'from outside' by \(O'\) 'from inside' by \(O\) or selfdescription.
To study it, first one should introduce the relations between IGUS functioning and subjective information (impression). For realistic IGUS $|O_{1,2}\rangle$ can correspond to some excitations of $O$ internal collective DFs like phonons, etc., which memorize this $Q$ information, but we don’t consider its possible physical mechanisms here. Concerning the relations between observer state evolution and his information perception we use the following assumptions: for any $Q$ eigenstate $|s_i\rangle$ after $S$ measurement finished at $t > t_1$ and $O$ ‘internal pointer’ state is $|O_i\rangle$ observer $O$ have the definite impression that the measurement event occurred and input state is $s_i$. This calibration assumption is nontrivial and related to ‘preferred basis’ problem discussed below [16]. If $S$ state is the superposition $\psi_s$ then we’ll suppose that its measurement also result in appearance of some unspecified at this stage $O$ impression which will be obtained below. Note that we don’t suppose any special properties of biological or human systems. In our framework the simplest $O$ toy-model of information memorization is hydrogen-like atom for which $O_0$ is ground state and $O_i$ are the metastable levels excited by $s_i$, resulting so into final $S - O$ entangled state. In this approach ‘internal pointer’ $O_i$ and $O$ memory which normally differs supposed to be the same object, but it isn’t important for our model.

Remind briefly Breuer theorem results which are valid both for classical and quantum measurements [7]. Any measurement of studied system $S_T$ is the mapping of $S_T$ states set $N_T$ on observer states set $N_O$. For the situations when observer $O$ is the part of the studied system, $N_O$ is $N_T$ subset and $O$ state in this case is $S_T$ state projection on $N_O$ called restricted state $R_O$. From $N_T$ mapping properties the principal restrictions for MS states recognition by $O$ were obtained, which is the main result of Breuer theorem. Namely, if for two arbitrary $S_T$ states $\Phi_S, \Phi_S'$ their restricted states $R_O, R_O'$ coincide, then for $O$ this $S_T$ states are indistinguishable. The origin of this effect is easy to understand qualitatively: $O$ has less number of DFs than $S_T$ and so can’t describe completely $S_T$ state. QM introduces additional features connected with observables noncommutativity and nonlocality, some of them will be regarded below, but aren’t important here.

For quantum measurements as $O$ restricted state can be chosen the partial trace of MS state [2]:

$$R_O = Tr_s \rho_{MS} = \sum |a_i|^2 |O_i\rangle\langle O_i|$$  \hspace{1cm} (5)

$R_O$ is in fact $\rho_{MS}$ projection into $\mathcal{H}_O$ defined in $O'$ RF and all $R_O$ belong to $\rho_O$ set $L_O$. Note that in quantum case Breuer theorem doesn’t permit to define restricted states directly; due to it such $R_O$ form is only the phenomenological choice and as argued below isn’t unique possible solution. $R_O$ can be interpreted as $O$ subjective state which describe his perception of MS state after measurement.

$R_O$ components weights are defined in $O$ basis, which will be used in the following calculations:

$$w_j(t) = Tr(\hat{P}_j^O R_O)$$  \hspace{1cm} (6)

where $\hat{P}_j^O$ is $O_j$ projection operator, and $w_j(t_1) = |a_j|^2$ for MS final state.

Note that for MS mixed state $\rho_m$ of [4] the corresponding restricted state is the same $R_m^O = R_O$. This equality doesn’t mean automatically collapse of MS pure state $\Psi_{MS}$, because as Breuer argues the collapse presence must be verified by special procedure applied to individual events. For this purpose it’s important to define the quantum state in the individual events; for pure MS it simply coincides with $\Psi_{MS}$. Note hence that for incoming $S$ mixture [4] MS individual state objectively exists in each event $n$, but differs from event to event and can be described as:

$$\rho^n_m(n) = |O_i\rangle\langle O_i| |s_i\rangle\langle s_i|$$

for random $l(n)$ with probabilistic distribution $P_l = w_l(t_1)$ [3]. This individual state can be initially unknown for $O$, but exists objectively. $\rho^n_m(n)$ differs from statistical state [4] and its restricted state is $R^n_O(n) = R_l = |O_i\rangle\langle O_i|$ also differs from $R_O$ of (3). Due to it the main condition
of Breuer Theorem violated, and this theorem isn’t applicable for this situation. Consequently $O$ can differentiate pure/mixed states 'from inside' in the individual events [7]. It means that Breuer selfmeasurement formalism doesn’t results in the collapse appearance in standard QM even with inclusion of observer in the measurement model [7]. In this framework Straightforward $R_O$ interpretation is that it describes $O$ internal pointer $Q_O$ 'splitting' perceived by $O$ or $O$ impressions superposition in Wigner terms. But it’s principally important that in this ansatz MS state description by $O$ 'from inside' can be really different from description by $O$ 'from outside', due to incompleteness of $O$ selfdescription. Because of this incompleteness $O$ can’t see the difference between physically different pure states with equal $D_{ij} = a_i^*a_j + a_i a_j^*$. This situation can be called partial state collapse. In the formalism reported below we’ll demonstrate that changing $O$ restricted states ansatz it’s possible to get observable $S$ collapse - i.e. after $S$ measurement $O$ can’t differ pure and mixed incoming $S$ states with the same $|a_i|^2$.

It’s possible to rewrite formally MS state for individual event $n$ for $O$ for Breuer ansatz in dual form $\Phi^B(n) = |\phi_D, \phi_I \gg, \phi_I$ is $O$ information about MS state acquired in event $n$. $\phi_I$ is equal to $R_O$ for pure state and $\phi_I = R^m_O(n)$ for mixture. Of course in this ansatz for pure state $\phi_I$ is just $\phi_D$ projection, but in other selfdescription schemes their relation can change and the states duality becomes principally important. Corresponding dual states set is $N_T = L_q \otimes L_V$. It’s possible to define also dual statistical state $|\Theta^B \gg = |\eta_D, \eta_I \gg \gg$ for ensemble, for which $\eta_D = \rho_{MS}$ and $\eta_I = R_O$ for pure state and $\eta_I = \sum P_i |O_i\rangle\langle O_i|$ for the mixture. Its meaning will be discussed below in more detail. For this purpose we’ll use MS interference term (IT) observable :

$$B = |O_1\rangle\langle O_2|s_1\rangle\langle s_2| + j.c.$$  
(7)

In standard QM being measured by $O'$ it gives $B = 0$ for mixed MS state [4], but in general $B \neq 0$ for pure MS state [3]. It evidences that for statistical ensemble the observed by $O'$ effects differentiate pure and mixed MS states. Note that $B$ value principally can’t be measured by $O$ directly, because $O$ performs $Q_O$ measurement and $[Q_O, B] \neq 0$ [20]. We’ll proceed with discussion of selfmeasurement theory, in particular description of ensembles by restricted statistical states after introducing our dual formalism.

### 3 Dual Selfmeasurement Formalism

Breuer analysis is quite valuable, because it shows that even observer inclusion into measurement chain and selfdescription restrictions account doesn’t lead to collapse appearance in standard QM. Moreover it prompts how to modify standard QM formalism so that it can describe state collapse consistently. The main idea is to develop alternative selfdescription formalism for which MS dual state $\Phi$ becomes compatible with $S$ state collapse.

Again as the example we regard MS system consist of $S$ and $O$ described also by external $O'$ not interacting with MS. From that we’ll demand that our QM modification satisfy to three main operational conditions :

i) if $S$ (or any other system) don’t interact with $O$ then for $O$ this system evolves according to Schrodinger equation dynamics (SD) (for example GRW theory breaks this condition [14]).

ii) If $S$ interacts with $O$ (measurement) SD can be violated for $O$, but only in such way that for stand-by $O'$ MS evolution must be described by SD, as follows from condition i).

iii) if input $S$ state approximates to the classical scale then for $O$ and $O'$ the classical limit i.e. objective measurement restored which for $O, O'$ is equivalent.

If all this conditions satisfied and $O$ percept random events for input pure $S$ state this phenomena called the weak (subjective) collapse. It means that in such formalism MS final states relative to $O$ and $O'$ can be nonequivalent [10]. We attempt to satisfy to both this
conditions by modification of quantum state which becomes dual. It results in modification of QM states set, which normally is Hilbert space $\mathcal{H}$. Remind that $\mathcal{H}$ is in fact empirical set choice advocated by fitting QM data. Its modifications were published already of which most famous is Namiki-Pascazio many Hilbert spaces formalism $[1]$. Analogous superselection formalism is well studied in nonperturbative Field theory (QFT) with infinite DF number $[10]$ and were applied for quantum measurement problem $[11, 20]$. To explain our main novel idea for $O$ selfmeasurement let’s consider it first for regarded MS measurement. Alike in regarded example for Breuer ansatz let’s write MS state in dual form $\Phi = |\phi_D, \phi_I \rangle \gg$ for dynamical and $O$ information components, which first will be introduced phenomenologically. The first component of our dual state $\phi_D$ is also equal to standard QM density matrix $\phi_D = \rho$ and obeys always including measurement process to Schrodinger-Liouville equation (SLE) for arbitrary Hamiltonian $\hat{H}_c$:

$$\dot{\phi}_D = [\phi_D, \hat{H}_c] \tag{8}$$

which for pure MS states is equivalent to Schrodinger equation (SE). $\phi_I$, which describes the information acquired by $O$ differs from Breuer $R_O$. It supposed that after S measurement in the individual event $n \phi_I = V^O$ where $V^O = |O_j\rangle\langle O_j|$ is stochastic state with $|j(n)$ probabilistic distribution $P_j = |a_j|^2$ and thus such dual event-state $\Phi(n)$ changes from event to event. Thus $O$ subjective information $\phi_I$ in this phenomenological ansatz is relatively independent of $\rho_{MS}$ and correlated with it only statistically. Clearly for such restricted states ansatz $O$ can’t differ the pure and mixed states with the same $|a_i|^2$. It’s important to stress that this formalism differs principally from standard QM state reduction, despite that both of them results in stochastic final states. Their difference as shown below in principle can be tested experimentally.

Initial state $\phi_D = \rho(t_0)$ defined also by standard QM rules. Before measurement starts $O$ state vector is $|O_0\rangle$ (no information on $S$) and the dual state is $\Phi = |\rho(t_0), V^O \rangle \gg$ where $V^O_0 = |O_0\rangle\langle O_0|$ is initial $O$ information. $\phi_I = V^I_0$ corresponds to $\Psi_{MS}$ branch $|P\psi_{si}\rangle = |O_i\rangle|S_i\rangle$ which describes MS quantum state in $O$ RF.

Complete states set in $O$ RF for this event-states is $N_T = L_q \otimes L_V$ i.e direct product of dynamical and subjective components subsets. Here $L_q$ is density matrices $\rho$ set and $L_V$ is the set of diagonal matrices (vectors) $\phi_I$ for which only one component $\phi_{ij} = 1$ in each event and all others are zeroes. If we restrict our consideration only to pure states as we do below then $N_T$ is equivalent to $\mathcal{H} \otimes L_V$ and the state vector $|\Psi\rangle$ can be used as the dynamical component $\phi_D$.

Of course in this approach the quantum states for $O'$ (and other observers) also has the same dual form $\Phi'$. $O'$ doesn’t interact with MS and due to it MS final state for her is $\phi'D = \rho_{MS}$ of (3) and $\phi_I' = |O_0'\rangle\langle O_0'|$. Her information is the same before and after S measurement by $O$, because $O'$ doesn’t get any new information during it. In this theory $O'$ knows that after S measurement $O$ acquired some definite information $O_i$ but can’t know without additional measurements what this information is. Naturally in this formalism $O'$ has her own subjective spaces $L^O_V$, and in her RF the events states manifold is $N_T' = \mathcal{H}' \otimes L^O_V$ for pure states. From the described features it’s clear that subspace $L_V$ is principally unobservable for $O'$ (and vice versa for $L^O_V, O$), because in this formalism only the measurement of $\phi_D$ component described by eq. (8) permitted for $O'$. But $V^O, V'^O$ can be correlated statistically via special measurement by $O'$ of observables of dynamical component $\phi_D$. For this purpose $O'$ can measure $Q_O$ on $O$ getting the information on $V^O$ content or some other MS observables $S$. In general if in the Universe altogether $N$ observers exists then the complete states manifold described in $O$ RF is $L_T = \mathcal{H} \otimes L_V \otimes L^O_V \otimes L^N_V$ of which only first two subsets are observed by $O$ directly and all others available only indirectly via measurements on $\mathcal{H}$ substrates.

From the dual state $\Phi$ one can derive the dual statistical state for quantum ensembles description, because all the necessary probabilities contained in $\phi_D$. Due to its importance it’s


reasonable to define it separately:

$|\Theta \gg |\eta_D, \eta_I \gg = |\rho, R_V \gg$

where $R_V = \sum P_i |O_i\rangle\langle O_i|$ is the probabilistic mixture of $V^O_i$ states describing statistics of $O$ impressions. Generalization of this ansatz for $O'$ and other observers is straightforward. Complete dual states set is $N_S = L_q \otimes L_R$, where $L_R$ set of diagonal density matrices with $\text{Tr} \eta_I = 1$, but as noticed above $N_S$ is equivalent to $L_q$.

For dual theory $|\Theta \gg$ is analog of quantum state $\rho$ in standard QM which predicts the arbitrary system $S_A$ its statistical properties and $O$ impressions statistics in its measurements. $|\Theta \gg$ evolution for arbitrary system Hamiltonian is most simply expressed by the system of equations for its components:

$$\frac{\partial \eta_D}{\partial t} = [\eta_D, \hat{H}_c]$$

$$P_j(t) = \text{tr}(\hat{P}^O_j \eta_D)$$

$$R_V = \sum |P_i| \langle O_i | O_i \rangle$$

If $S_A$ don’t interact with $O$ (no measurement) then $R_V$ is time invariant and one obtains standard QM evolution for dynamical component $\eta_D = \rho$ - statistical state. Thus our dual states are important only for measurement-like processes with direct system $S_A - O$ interactions, but in such case it’s the analog of regarded MS system.

The first equation of (9) is SLE which becomes for $|\Theta \gg$ the analog of master equation for probabilities $P_j(t)$ describing $\phi_I$ distribution. Due to independence of MS dynamical component $\eta_D$ of $\eta_I$ this MS state $\Theta$ evolution is reversible and no experiment performed by $O'$ on MS wouldn’t contradict to standard QM.

Note that in this theory only $\Phi$ gives complete MS state description in the individual event, from which its future state can be predicted. There is no contradiction that $O$ can know both $\phi_I$ and $\phi_D$. $\phi_D$ information can be send to $O$ by $O'$ and stored in $O$ memory cells different from $|O_0\rangle$.

The time of $V^O_0 \rightarrow V^O_j$ transition for $O$ is between $t_0$ and $t_1$ and can’t be defined in the current formalism with larger accuracy, but it doesn’t very important at this stage. The most plausible assumption is that $O$ perception time $t_p$ in MS measurement has distribution:

$$P_p(t) = c_p \sum \frac{\partial P_i(t)}{\partial t} ; \quad i \neq 0$$

where $c_p$ is normalization constant. Note that this result is compatible with standard QM.

Now let’s compare our model with state reduction in standard QM, which also describes how the state vector correlated with the changes of observer information on $S$ after the measurement. There $S,D$ interaction induces the abrupt and irreversible $S$ state $\psi_s$ change to random $\psi_j$ and in accordance with it detector pointer acquires definite position $D_j$. This process is claimed to be objective, i.e. independent of any observer. Such $S,D$ interaction can’t be described by SLE and needs to introduce alternative dynamics, which can violate quantum states evolution linearity and reversibility. Yet it’s practically impossible to incorporate in QM this two contradictory dynamics consistently. In distinction in dual formalism the dynamical component $\phi_D$ of MS dual state evolves linearly and reversibly in accordance with (8). This is objective evolution in a sense that it described equivalently relative to any observer. Only subjective component $\phi_I$ which describes $O$ subjective information about $S$ changes stochastically after $S$ measurement - i.e $S,O$ interaction which makes this formalism consistent.

Extension of dual formalism on mixed states is obvious and here it presented only for the states interested for measurements of the kind (4). For them from eq. (8) naturally follows
\[ P_j = |a_j|^2 \] which gives \( \phi_I \) distribution. In dual formalism the restricted MS state \( R_t = |O_t\rangle\langle O_t| \) where \( l(n_1) \) in the individual event \( n_1 \) defined by \( w_j \). It differs from restricted state \( R_O \) in standard QM given by \( |1\rangle |2\rangle \), but coincide with restriction of mixed state \( \rho_0^{\text{MS}} \) in the individual event \( n \) if \( l(n_1) = l(n) \). Thus Breuer theorem condition can be fulfilled in dual formalism as expected. Obviously in this formalism \( Q \) coincide both for \( O \) and \( O' \).

For any statistical physical theory it’s necessary to construct corresponding consistent probability ansatz, including Kolmogorov triple. In our dual formalism probabilities \( P_j(t) \) coincides with corresponding standard QM probabilities \( P_j^Q = \text{Tr}(\hat{P}_{Sj}|\psi_s\rangle\langle \psi_s|) \) of particular outcome \( q_j \). Remind that in standard QM any \( q_j \) corresponds to \( D \) pointer position \( D_j \) and the true events for any observer are this \( D \) counts, which mapped on \( Q \) values axe. In our formalism \( q_i \) related to \( O \) impressions \( O_i \) mapped on \( Q \) axe. Thus standard QM probabilities ansatz can be used copiously in dual formalism.

Dual formalism excludes spontaneous \( \phi_I \) jumps without effective \( S,O \) interactions : if \( S \) and \( O \) don’t interact then for \( O \) the same random parameter \( j \) of \( V^O \) conserved. It follows from standard QM transition probabilities for the initial state at \( t_1 \):

\[ P^{ij}_t(t_2) = |\langle \Psi_i|U(t_2-t_1)|\Psi_j\rangle|^2 \]

where \( U \) is unitary evolution operator. If for all \( i \neq j \) \( P^{ij}_t = 0 \) than for \( O' \) MS state is uncertain, but there are no transitions between its components in this time. For \( O \) its arbitrary state is definite i.e. some \( O_j \) and continue to be the same as at \( t_1 \).

To illustrate \( \Phi \) dualstate reversibility let’s consider gedankenexperiment which can be called ‘undoing’ the measurement. Such experiment was discussed by Deutsch for many worlds interpretation (MWI), but we’ll regard its slightly different version. Its first stage coincides with regarded \( S \) state \( |1\rangle \) measurement by \( O \) resulting in the final state \( |2\rangle \). This \( S \) measurement can be undone or reversed with the help of auxiliary devices - mirrors, etc., which come into action at \( t > t_1 \) and reflects \( S \) back in \( O \) direction and make them reinteract. It permits for the final state \( \Psi^{\text{MS}}_t \) obtained at time \( t_1 \) at the later time \( t_2 \) to be transformed backward to MS initial state \( \Psi^{\text{MS}}_0 \). Thus if at \( t_1 < t < t_2 \) \( O \) has information about \( S \) state; at \( t > t_2 \) it’s erased and MS state again is \( |\Psi^{\text{MS}}_0\rangle \). In our dual formalism the subjective event-state component \( \phi_I \) describes \( O \) information on \( S \) after measurement and at \( t > t_1 \) becomes equal to some random \( V^O_j \). But after reversing independently of \( j \) it returns to initial value \( V^O_0 \), according to evolution ansatz \( (\Phi) \) described in previous chapters. If such description of this experiment is correct, as we can believe because its results coincides with Schrodinger MS evolution in \( O' \) RF it follows that after \( q_i \) value erased from \( O \) memory it lost unrestorably also for any other possible observer. If after that \( O \) would measure \( Q \) again obtained by \( O \) new value \( q_j \) will have no correlation with \( q_i \). This consideration demonstrates that in dual formalism to predict future MS evolution in the individual event \( O \) should use both \( \Phi \) components.

In any realistic layout to restore state \( |1\rangle \) is practically impossible but to get the arbitrary \( S,O \) factorized state by means of such reversing is more simple problem and that’s enough for such tests. Despite that under realistic conditions the decoherence processes make this reversing immensely difficult it doesn’t contradict to any physical laws.

If we consider this experiment in standard QM with reduction from \( O \) point of view we come to quite different conclusions. When memorization finished at \( t_1 \) in each event MS collapsed to some arbitrary state \( |s_i\rangle |O_t\rangle \). Then at \( t_2 \) \( O \) undergoes the external reversing influence, in particular it can be the second collision with \( S \) during reversing experiment and its state changes again and such rescattering leads to a new state correlated with \( |s_i\rangle \):

\[ |s_i\rangle |O_t\rangle \rightarrow |s_i'\rangle |O_0\rangle \]

It means that \( O \) memory erased and he forgets \( Q \) value \( q_i \), but if he measure \( S \) state again he would measure \( Q \) again obtained by \( O \) new value \( q_j \) would have no correlation with \( q_i \). This consideration demonstrates that in dual formalism to predict future MS evolution in the individual event \( O \) should use both \( \Phi \) components.
would restore the same $q_i$ value. Its statistical state is

$$
\rho'_m = |O_0\rangle\langle O_0| \sum |a_i|^2 |s'_i\rangle\langle s'_i|
$$

But this S final state differs from MS state (1) predicted from MS linear evolution observed by $O'$ and in principle this difference can be tested on S state without $O$ measurement.

Of course one should remember that existing for finite time intermediate $O$ states are in fact virtual states and differ from really stable states used here, but for macroscopic time intervals this difference becomes very small and probably can be neglected. In any realistic layout to restore state (1) is practically impossible but to get the arbitrary S-O factorized state by means of such reversing is more simple problem and that’s enough for such tests. Despite that under realistic conditions the decoherence processes make this reversing immensely difficult it doesn’t contradict to any physical laws.

The analogy of 'undoing' with quantum eraser experiment is straightforward: there the photons polarization carry the information which can be erased and so change the system state [21]. The analogous experiment with information memorization by some massive objects like molecules will be important test of collapse models. Note that observer [21]. The analogous experiment with information memorization by some massive objects like photons polarization carry the information which can be erased and so change the system state [21]. Such experiment regarded for Coleman-Hepp model in [20] doesn’t introduces any new features in comparison with 'Undoing' and so we don’t discuss it here.

4 Decoherence and Existential Interpretation

The preferred basis (PB) problem importance is acknowledged in quantum measurement theory [1]. In its essence, $\Psi_{MS}$ decomposition on $O,S$ states in general isn’t unique and so any theory must explain why namely $|O_i\rangle$ states appears in final mixture $\rho^m$. In our model PB acquires additional aspects being related to $O$ information recognition. In the previous chapters we made calibration assumption that $|O_i\rangle$ state percepted as $O_i$ value. But it’s not clear why namely such states responds to it and not some other $|O'_j\rangle$ - eigenstates of some $Q'_i$, belonging to another orthogonal basis. For example, it can be $|O_{iz}\rangle = \frac{|O_{i}\rangle + |O_{z}\rangle}{\sqrt{2}}$ for binary subspace.

Yet the situation changes principally, if to account decoherence - i.e. $O$ interaction with environment $E$. It’s widely accepted now that decoherence effects are very important in measurement dynamics, and here some its features essential for us reminded [13, 4]. In the simplest decoherence model $E$ consist of $N$ two-level systems (atoms) independently interacting with $O$ with $H_{OE}$ Hamiltonian, which for arbitrary $E$ states $|E^0\rangle$ at large $t$ gives: $|O_0^E\rangle|E^0\rangle \rightarrow |O_1^E\rangle|E^0_1\rangle$, where $|O^E_1\rangle$ belongs to orthogonal basis $O^E$ of $O$ states. $|E^0_1\rangle$ are $E$ states which aren’t necessarily orthogonal. Tuning specially measurement Hamiltonian $H_I$ one can make two basises equal: $|O^E_1\rangle = |O_i\rangle$ and only this case will be regarded here. If in S measurement at $t < t_1$ $O$–$E$ interaction can be neglected than under simple assumptions it results in final MS-E state:

$$
\Psi_{MS+E} = \sum a_i |s_i\rangle |O_i\rangle |E^0_i\rangle
$$

It was proved that such triple decomposition is unique, even if $|E^0_i\rangle$ aren’t orthogonal [16]. Thus PB problem formally resolved if decoherence accounted and this is essential also for our model.

In addition decoherence results in important consequences for the mentioned perception basis choice. Really the memorized states $|O'_i\rangle$ excited by $S_i$ signals must be stable or at least long-living. But as follows from eq. (11) any state $|O'_i\rangle$ different from one of $|O_i\rangle$ in the short time would split into $|O_i\rangle$ combinations - entangled $O,E$ states superposition. But our calibration condition demands that at least $Q^O$ eigenstates will be conserved copiously and not transferred to any combinations. Thus in this model $O$–$E$ decoherence interaction selects the basis of long-living $O$ eigenstates which supposedly describes $O$ events perception and memorization, i.e. $\phi_I$. 
In general the perception calibration by eigenstates is very important both for our model and for quantum signal recognition studies. It means that if our signal is $Q$ eigenstate transformed to $Q_O$ eigenstate $|O_i\rangle$ then it’s memorized by $O$ for long time. MS-E entanglement to some extent stabilize random $O_i$, because to erase $Q$ value experimentalist, beside $S,O$ should also act on $E$. The analogous signal memorization model for brain neurons was considered by Zurek [24].

Decoherence influence should be accounted in $O$ selfdescription formalism. In Breuer formalism $O$ interaction with $E$ states accounted analogously to $S$ states, so that $R_O = T_{RS,E}\rho$ derived taking trace both on $S$ and $E$ states. For dual formalism analogous approach permits to derive $|\Theta \gg\rangle$, thus defines $\phi_I$ distribution. In the individual events $\phi_I$ correspons to $|O_i\rangle|S_i\rangle|E^{O_i}_I\rangle$ branch. In other aspects decoherence doesn’t change our selfmeasurement model.

Under realistic conditions the rate of $E$ atoms interactions with detector $D$ is very high and due to it in a very short time $t_d$ S,D partial state $\rho_p = T_{RE}\rho$ becomes approximately equal to mixed one, because $\rho_p$ nondiagonal elements becomes very small. This fact induces the frequent claim that the state collapse phenomena can be completely explained by detector state decoherence. This decoherence collapse (DC) theory in its simplest form was proved to be incorrect [18], but more complicated variants expect the study. The main argument against is that to decide in which state the system is it’s necessary to analyse its complete not partial state. Omitting details, for $S,D,E$ it’s always possible to construct $IT$ operator $\hat{B}$ analogous to $\hat{B}$ which expectation value reveals that the system state and consequently $D$ state is pure. Dual formalism approach to collapse is close to DC theory attitude, in which also no additional reduction postulate used. The main difference is that DC theory claims the collapse is objective phenomena [18]. In dual theory $S$ state collapse has relational or subjective character and observed only by observer inside decohering system, while for external $O'$ this system including $E$ is in pure state.

The novel approach to the relation of state collapse and decoherence proposed in Zurek ‘Existential interpretation’ [24]. In this approach IGUS or in our terms observer $O$ also regarded as quantum object included in the measurement chain and $O$ state decoherence via interaction with $E$ ‘atoms’ is quite important. In the regarded simple model $O$ memorization of input signal $S$ occurs in several binary memory cells $|m\rangle_{1,2}$, which is the analog of brain neurons. Alike in the regarded above case this $O$ memory state suffers decoherence from surrounding $E$ ‘atoms’ which results in the system state analogous to (11). Under practical IGUS conditions the decoherence time $t_d$ is also quite small and for time much larger than $t_d S,O$ partial state $\rho_p$ differs from the mixture very little. From that Zurek concludes that $O$ percepts input pure $S$ signal as random measurement outcomes.

Hence such theory conserves all the faults of discussed above DC theory. Really analogously as it was argued for DC theory for external $O'$ the system $S,IGUS,E$ also is in the pure state even at $t \gg t_d$ and IT observable $B$ analogous to $\hat{B}$ which proves it can be constructed. Thus in standard QM framework is incorrect to claim that IGUS percepts random events. But as easy to note Zurek IGUS model doesn’t differs principally from our MS scheme. Due to it dual formalism can be applied to it without significant modifications. In its framework IGUS subjective perception described by $\phi_I$ component of dual state which corresponds to random outcomes for input pure $S$ state. Thus application of dual formalism for Zurek IGUS model supports eventually Existential Interpretation hypothesys. Dual formalism considers effectively only single IGUS DF $|O_i\rangle$, but any real IGUS includes many internal DFs practically unobservable for him (brain molecules, etc.). Account of their unobservability can make dual formalism and Zurek theory much closer practically.
5 Collapse and System Selfdescription

Now we can regard dual formalism in the general selfmeasurement framework and compare it with Breuer selfdescription theory for standard QM. Here decoherence neglected, because as was demonstrated it doesn’t introduces any new features after O basis was chosen. As was noticed Breuer theorem doesn’t permit to derive MS restricted state directly, and QM only demands that it satisfy to selfmeasurement conditions i)-iii) cited above. Thus there is some freedom of restricted states \( R_O \) choice. The simplest such possibility is Breuer ansatz for \( R_O \) is \( \rho_{MS} \) projection into \( \mathcal{H}_O \) defined in \( O' \) RF and all \( R_O \) belong to \( \rho_O \) set \( L_O \). Such choice means in fact that \( O \) restricted states are equivalent ‘from inside’ for \( O \) and outside for \( O' \). Remind that in QM the physical states set is spanned on physical observables - Hermitian operators, which measurement permit to differ this states \([7]\). In QM the states difference in any RF means corresponding observables description and also the experimental measurement procedure. Yet Breuer doesn’t propose it for \( R_O \) and this is the weak point of this theory. Really, \( O' \) can measure all \( O \) observables \( A^O \) and \( \mathcal{H}_O( L_O) \) spanned on them. For \( O \) as RF (observer) it is different and in the regarded model its only observable is \( Q_O \), and due to it \( O \) states set must differ from \( L_O \).

Our stochastic ansatz \( \Phi \) is also consistent with conditions i)-iii), which makes it compatible with standard QM in its realm. Moreover \( \phi_I \) are \( Q_O \) eigenstates and so \( \phi_I \) subset \( L_V \in L_O \) spanned on single observable \( Q_O \), which as was noticed is the only observable for \( O \). In accordance with it even ensemble statistics \( \eta_I \) doesn’t contain any information about others \( O \) or MS observables of MS state. Of course this are just semiqualitative arguments which needs the additional mathematical clarification and without it dual theory still conserves the phenomenological features. But even in this form its importance is in the fact that it describes consistent mathematical formalism of state collapse in the system selfdescription framwork.

Note that \( O \) is endpoint of MS measurement chain which can be regarded as singularity of some kind and MS quantum state description by SLE for external \( O' \) is regularization of this singularity which defines its properties consistently. As shows experience with QFT the singularities can results in appearance of new regimes in particular new stochastic parameters \([10]\). The close analogy to discussed effects seems Spontaneous Symmetry Breaking phenomena in nonperturbative fields interactions.

The interesting interpretation problem of dual QM formalism is: can we tell that in given event other \( \Psi_{MS} \) branches exist beside \( |O_i\rangle|S_i\rangle \) observed by \( O \) ? To decide it note that other branches existence for \( O' \) can be confirmed by by \( B \) measurement on MS and for \( O \) indirectly in discussed ‘undoing’ experiment. But this branches coexistence differs from MWI where each of this branches exists in one of many parallel worlds. In dual theory all this branches coexist in the same single world, but other branches aren’t percepted by \( O \), due to principal incompleteness of \( O \) selfdescription. This isn’t too surprising, because realistic \( O \) has many internal DFs i.e. \( O \) substrates practically unobservable for him \([20]\).

Note that subjective \( \phi_I \) component of \( \Phi \) isn’t the new degree of freedom, but \( O \) additional information about its own observable \( Q_O \) and correspondingly on S observable \( Q \) which doesn’t contained in state vector \( \Psi_{MS} \). \( Q \) information for \( O' \) which don’t interact with S described by \( \Psi_{MS} \) and corresponds to arbitrary uncertainty \( q_{min} < Q < q_{max} \). \( \phi_I \) contains additional information on random \( Q = q_j \) percepted by \( O \) only. In this dual theory Schrodinger dynamics and state collapse coexist, by the price that S signal perception by \( O \) occurs via this new stochastic mechanism. Despite we use term ‘perception’ in our model it doesn’t referred to human brain specifically. We suppose that as \( O \) can be regarded any system which can produce the stable entanglement of its internal state and measured state S. It can be even hydrogen-like atom in the simplest case for which \( O_i \) can be different atomic levels. Its perception corresponds to \( \phi_I \) state component and means that in such formalism \( O \) state differently described by interacting
O 'from inside' and by any other O'.

In the dual theory S state collapse has subjective or relative character and due to it strictly connected with acquired S information. To illustrate relation between state collapse and information transfer we consider several gedanken experiments. Remind that standard QM reduction postulate settles that if Q acquired after measurement the definite value relative to O then its objectively exists also for O' or any other observer, but can be unknown for them. In first experiment at the initial stage O measures Q value of S at t₁ which results in MS state |(2)⟩ for O', but after it Q is measured again by observer O' at t₂ > t₁. The interaction of O' with MS results in entangled state of S, O and O' and so both observers acquire some information about S state. This state vector in our formalism is:

$$\Psi_{MS} = |a_1 s_1⟩|O_1⟩|O'_1⟩ + a_2 |s_2⟩|O_2⟩|O'_2⟩$$  \hspace{0.5cm} (11)

Analogous experiments was discussed frequently due to its relation to EPR-Bohm correlations [25] and here we regard only its timing aspect. Our question is : at what time Q value becomes definite and thus S state collapse occurs for O'? In our formalism at t₁ < t < t₂ observer O already acquired the information that Q value is some q₁, reflected by $\phi_I = |O_i⟩⟨O_i|$. In the same time Q value stays uncertain for O', because relative to her MS state vector is |(2)⟩, and O' + MS dual state for O' is

$$\Phi' = |\Psi_{MS}⟩ ⊗ |O'_0⟩, |O'_0⟩⟨O'_0| \gg$$

When at t > t₂ measurement by O' finished Q value measured by O' coincides with q₁. To check that Q value coincides for O' and O, O' can perform measurement both Q and $Q_O$ which is described by (11) and gives the same result as in standard QM. It don't contradicts to the previous assumption that for O' before t₂ Q was principally uncertain. The reason is that in between O' interacts with S and it makes Q value definite for her. This measurement demonstrates the subjective character of collapse, which happens only after S interaction with particular observer occurs. If collapse occurs according to QM reduction postulate then at t > t₁ MS state relative to O' must be the mixture $\rho_{m}$ of |(4). In our formalism at that time MS state vector relative to O' is pure state $\Psi_{MS}$ of |(2)⟩ which isn't Q eigenstate. To test it experimentally O' can measure $\hat{B}$ on MS at t > t₁ which don't commute with Q. If our theory is correct then $\hat{B} \neq 0$ and thus MS state collapse doesn't occurs at t = t₁.

Remind that the state vector has two aspects : dynamical and informational in which $\Psi$ is O maximal information about the object S [1]. Our formalism extends this aspect on the case when O measures S and can acquire more information about S then 'stand-by' O'. In its framework the state collapse directly related with O information acquisition via interaction with S. The same information after S measurement can be send by O to O' in form of some material signal, for example photons bunch. When O' performs quantum measurement of this signal it result for her into $\Psi_{MS}$ collapse to one of final outcome eigenstates, which in our formalism reflected by $\phi_I$ change. Thus in our theory S state collapse directly related with information transferred to arbitrary O via interaction with S or some intermediate system (signalling).

Relativistic analysis of EPR-Bohm pairs measurement also indicates subjective character of state vector and its collapse [25]. It was shown that the state vector can be defined only on space-like hypersurfaces which are noncovariant for different observers. This results correlate with nonequivalence of different observers in our nonrelativistic formalism. Hence we believe EPR-Bohm correlations deserve the detailed study in this dual framework.

6 Discussion

In this paper the measurement models which accounts observer (IGUS) information processing and memorization regarded. Real IGUSes are very complicated systems with many DFs, but
the main quantum effects can be studied with the simple models. The presented dual formalism demonstrates that probabilistic realization is generic and unavoidable for QM and without it QM supposedly can’t acquire any operational meaning. Wave-particle dualism was always regarded as characteristic QM feature, but in our formalism it has straightforward correspondence in dual states ansatz.

Breuer formalism shows that inclusion of observer as quantum object into measurement scheme doesn’t lead to collapse appearance 7. To make it possible in our approach observers are made nonequivalent in a sense that the physical reality description can be principally different for each of them 8. This nonequivalence reflected by the presence of subjective component \( \phi_I \) available directly only for particular \( O \) and for him the subjective state collapse can be obtained. Our theory indicates that to obtain it it’s necessary also to modify the quantum states set which makes it nonequivalent for different observers but conserves Schrodinger evolution for arbitrary quantum system. Here we note that in this framework the key problem becomes the existence of the objective reality of the physical objects properties independent of particular observer. Our results hints that no such reality is possible and any such property has only subjective reality relative to particular observer.

The natural question arise: does observation of random outcomes \( \phi_I = V^O \) means that before the measurement starts S state can be characterized by some objective ‘hidden parameter’ \( j_S \)? Our formalism is principally different from Hidden Parameters theories where this stochastic parameters influence quantum state dynamics and so differs from SD. Due to it in our model \( \phi_I \) internal parameter \( j \) can be ‘generated’ during S-O interaction and don’t exists objectively before it starts.

As we supposed in the introduction our theory doesn’t need any addressing to to human observer consciousness (OC). Rather in this model \( O \) is active RF which internal state excited by the interaction with the studied object. This approach to the measurement problem has much in common with Quantum reference frames introduced by Aharonov 27.

The ideas close to our dual theory were discussed in QM modal interpretation, but they have there phenomenological and philosophical formulations 1. Now this is the whole class of different theories, of which the most close to us is Witnessing interpretation by Kochen 19. His theory phenomenologically supposed that for apparatus \( A \) measured value \( S \) in pure state always has random definite value \( S_j \), yet no physical arguments for it and no mathematical formalism differ from standard QM were proposed.

Historically the possible influence of observer on measurement process was discussed first by London and Bauer 26. They supposed that OC due to ‘introspection action’ violates in fact Schrodinger equation for MS and results in state reduction. This idea was criticized in detail by Wigner 5. In distinction in our dual theory OC perception doesn’t violate MS Schrodinger evolution from \( O \) point of view. But measurement subjective perception in it also performed by OC and its results partly independent of dynamics due to its dependence on stochastic \( V^O \).

This effect deserves further discussion, but we believe that such probabilistic behavior is general IGUS property not related to OC only.

Dual formalism deserves comparison with different Many Worlds interpretations (MWI) variants, due to their analogy - both are the theories without dynamical collapse 1. MWI is still very popular, despite its serious consistency problems. The most close to our theory seems Everett+brain MWI interpretation in which eq. (4) describes so called observer \( O \) splitting identified with state collapse 28. In its framework assumed that each \( O \) branch describes the different reality - separate Universe and the state collapse is phenomenological property of human consciousness. Obviously this approach has some common points with our models which deserve further analysis. In this terms our theory can be qualified as MDI - Many Descriptions interpretation - stressing that in it the picture of the same Universe for different observers can differ principally.
In general all our experimental conclusions are based on human subjective perception. Assuming the computer-brain perception analogy in fact means that human signal perception also defined by $Q_O$ values. Despite that this analogy looks quite reasonable we can’t give any proof of it. In our model in fact the state collapse have subjective character and occurs initially only for single observer $O$. We present here very simple measurement theory and we don’t regard it as final solution of measurement problem. Yet from its results we believe that it’s impossible to solve it without account of $O$ interaction with measured system at quantum level.

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