Effect of Surface Waviness on Aerostatic Bearing with Speed

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Abstract: In this paper, the Reynolds equation is solved by the different finite methods. On the basis of the ideal circle, the effect of surface ripple on the static pressure bearing is considered, and the performance parameters of the gas bearing are obtained by using the Newton iteration method. Considering the influence of different rotational speeds on the gas bearing, the influence of the bearing capacity, the air consumption and the friction coefficient on the static pressure bearing are analyzed, and the effect of the axial and circumferential waviness on the aerostatic bearing is analyzed. The results show that with the increase of the rotational speed and the error value, the bearing capacity becomes smaller and the friction coefficient becomes larger. In the specific error value, as the film thickness increases, the bearing capacity increases first and then becomes smaller. At the same time, the circumferential ripple error has a greater influence on the gas bearing than the axial direction.

Key words: Aerostatic bearing; Error; Capacity; Stiffness; Surface waviness

1. Introduction

Due to the low friction, high speed and pollution-free characteristics of gas bearings, they have been widely used in the production of precision instruments, such as aerospace, circuit board manufacturing, and missile technology.[1] Due to the relatively high compressibility of the gas, it is easy to cause instability when the gas bearing is running, and since the manufacturing error is unavoidable in the process of producing the gas bearing, it is necessary to carry out the gas static pressure bearing in the presence of an error. However, in the general analysis of the aerostatic bearing process, the influence of the rotational speed on the aerostatic bearing is often neglected. This paper will consider the influence of the aerostatic bearing with the waviness in the case of the rotating speed. Figure 1 is the structural diagram of the aerostatic bearing.

The discretization was derived by the nonlinear dimensionless Reynolds equation Newton method, and the iterative rate cutting method was solved by the improved Reynolds equation[2]. A new iterative method was proposed to solve the Reynolds method, and the method was applied to analyze the aerostatic bearing[3]. Zhang[4] used the variable step size iterative method to solve the Reynolds equation, and this method had higher convergence efficiency. The surface error of the aerostatic bearing was analyzed and it was concluded that the large surface error may significantly weaken the bearing stiffness. Liu Wei used numerical analysis of the air hammer phenomenon of the aerostatic bearing[5]. In Wang's[6] paper, the static characteristics of corrugated air bearing in the presence of corrugation in the presence of corrugated bearings were analyzed. Zhang Guanghui’s influence on the
gas static pressure bearing through different speeds and different number of air supply holes. The stability conditions of the aerostatic bearing motion were analyzed and the stability region was obtained.[7, 8] The effects of rotational speed, support stiffness and moment of inertia on the stability of the shafting motion are analyzed. Guo Liangbin analyzed the effects of supply pressure, film thickness and disturbance frequency on the dynamic characteristics of the bearing. Rao Heqing[9] and Sun Yazhou[10] simulated the porous aerostatic bearing and obtained the static performance of the porous bearing. At present, the analysis of gas compressibility in gas aerostatic bearings and changes in the internal flow field ignores the rotational speed. This paper will fully consider the effect of the rotational speed on the aerostatic bearing. The aerostatic bearing will also be analyzed for roundness and cylindricity in different modes of existence.

2. Mathematical model

The theoretical equations of roundness and cylindricity, and the dimensionless waviness error function are as follows:

\[ \delta(x, z) = \left( A\delta_A \sin^2 \frac{2\pi x}{A} + B\delta_B \sin \frac{2\pi z}{B} \right) \]  \hspace{1cm} (1)

A, B are error coefficients, \( \delta_A, \delta_B \) are circumferential and axial error values, \( \lambda_A, \lambda_B \) are the wavelengths of the circumferential and axial corrugations, when there is roundness and cylindricity \( A=1, B=1 \), \( A=0, B=0 \) when there is no roundness and cylindricity.

Gas film thickness:

\[ h = h_m \ast (1 + \varepsilon \cos \theta) - \delta(x, z) \]  \hspace{1cm} (2)

The Reynolds equation for the gas bearing watershed interval is as follows:

\[ \frac{\partial}{\partial x} \left( \frac{h^2 \partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left( h^2 \frac{\partial p}{\partial z} \right) + 12\eta \frac{\partial h}{\partial t} - \rho v^2 = 12\eta \frac{\partial (ph)}{\partial t} + 6\eta \frac{\partial (ph)}{\partial z} \]  \hspace{1cm} (3)

Due to the assumption that the entire bearing is a rigid body when analyzing the orifice static and dynamic pressure environment, the surface deformation is not considered, and the bearing is in a steady state during the entire movement process, so the bearing is stable throughout the process. State of motion, Derivation of dimensionless and elimination time terms by equation (3), we can get:

\[ \frac{\partial}{\partial x} \left( \frac{h^2 \partial q}{\partial x} \right) + \frac{\partial}{\partial z} \left( h^2 \frac{\partial q}{\partial z} \right) + \frac{\partial (ph)}{\partial t} = \lambda_x \frac{\partial (ph)}{\partial x} + \lambda_z \frac{\partial (ph)}{\partial z} \]  \hspace{1cm} (4)

Among them:

\[ \bar{Q} = 24\eta q^2 p \rho V_{a1} / (h_n^3 p_s^2 \rho_s) \]  \hspace{1cm} (5)

\[ \lambda_x = 12\eta V_{a1} / (h_n^3 p_s) \]  \hspace{1cm} (6)

\[ \lambda_z = 12\eta V_{a1} / (h_n^3 p_s) \]  \hspace{1cm} (7)

The dimensionless parameters are as follows:

\[ \bar{h} = h / h_n, \quad \bar{p} = p / p_s, \quad \bar{x} = 2x / l, \quad \bar{z} = 2z / l, \quad \bar{m} = Ap \phi / \left( \frac{2p_0}{\rho_0} \phi \right) \]

\[ \phi = \left\{ \begin{array}{ll} k & \quad p / p_s \leq \beta_k \\ k \left[ \left( \frac{p}{p_s} \right)^{k+1} - \left( \frac{p}{p_s} \right)^{k+1} \right] & \quad \beta_k < p / p_s \leq \beta_k \\ k \left( \frac{k}{k+2} \right)^{(k+1)(k+1)} & \quad \beta_k > \beta_k \end{array} \right. \]  \hspace{1cm} (9a)

\[ Q = \sum_i \bar{Q}_i h_n^3 p_s^2 \rho_s / (24\eta q^2) \]  \hspace{1cm} (9b)

Solve the Reynolds equation and define the boundary conditions:

- Atmospheric boundary conditions: \( p(x=0) = p(x=nx+1) \)
- Symmetrical boundary conditions: \( p(x=0) = p(nx+1) \)
Table 1. Aerostatic bearing parameters

| Name                                      | Model parameter |
|-------------------------------------------|-----------------|
| Axis length                               | l (mm)          |
| Diameter                                  | d (mm)          |
| Average radius gap thickness              | h (μm)          |
| Axial grid number                         | nz              |
| Radial grid number                        | nx              |
| Number of orifices in a single row        | k               |
| Throttle diameter                         | d0 (mm)         |
| Number of orifices                        | N               |
| Air density                               | ρ (kg/m³)       |
| Air viscosity coefficient                 | η (pa•s)        |
| Air specific heat ratio                   | κ               |
| Throttling coefficient                    | φ               |
| Atmospheric pressure                      | p (atm)         |

Table 1 shows the structural parameters and lubrication parameters of the aerostatic bearing. Since the Reynolds equation in a compressible fluid is a nonlinear equation, therefore, the analytical solution cannot be directly obtained. Using Taylor's expanded Newton iteration method, it can be found that the Reynolds equation obtains an analytical solution. Then use the segmentation factor method to gradually reduce the pressure difference. The pressure calculation is gradually converge, and the super-relaxation iteration is mainly used as a method of accelerating iteration in the process of solving. Figure 1 shows the schematic diagram of static pressure bearing structure.

Figure 1. Schematic diagram of static pressure bearing structure

3. Characteristic analysis

The dimensionless bearing capacity is obtained by the static characteristics of the bearing:

\[
\begin{align}
\bar{W}_x &= \sum_i \sum_j \bar{p}_j \Delta \tilde{z} \sin \left( \frac{(i-0.5) \Delta \tilde{x}}{R} \right) \\
\bar{W}_y &= \sum_i \sum_j \bar{p}_j \Delta \tilde{z} \cos \left( \frac{(i-0.5) \Delta \tilde{x}}{R} \right) \\
\bar{W} &= \sqrt{\bar{W}_x^2 + \bar{W}_y^2}
\end{align}
\]

(10a)

(10b)

The frictional force in the circumferential direction and the axial direction is obtained by the flow resistance of the flow rate of the gas in the gas film section.
\[
\begin{align*}
\left\{ \begin{array}{l}
  f_x &= \int_0^{2\pi} \left( \frac{h_n}{2} \frac{\partial p}{\partial x} + \eta \frac{u_x}{h_m} \right) dx dz \\
  f_y &= \int_0^{2\pi} \left( \frac{h_n}{2} \frac{\partial p}{\partial x} \right) dx dz 
\end{array} \right. \\
\end{align*}
\]  

(11)

Using the bearing capacity and friction force, the coefficient of friction of the film area is obtained.

\[
\mu = \sqrt{\frac{f_x^2 + f_y^2}{W}}
\]

(12)

The amount of gas consumed by the entire aerostatic bearing is calculated by using the amount of gas to be dissipated at each eccentricity of each orifice. The amount at its individual orifice:

\[
Q_i = \sum_{i} Q_i h_m^3 p_r^2 \rho / (24\eta l^2 p_a)
\]

(13)

4. Calculation results analysis

Figure 2 is the key parameter: the eccentricity is 0, the rotational speed is 2000 rpm, and the axial and circumferential wavelengths are 10 mm, the dimensionless film thickness at the axial intermediate position and the distribution of dimensionless pressure are obtained.

Figure 2(a) is the circumferential film thickness from 0° to 360°. It can be seen that as the ripple amplitude value increases, the circumferential film thickness increases. Figure 2(b) shows the film thickness distribution in the axial direction. At the same wavelength, as the amplitude of the ripple increases, the axial amplitude also increases.

Figure 2(c) shows the distribution of the bearing in the circumferential direction. The pressure decreases with the increase of the amplitude value in the circumferential direction, and the pressure is asymmetrical in the axial distribution. This asymmetry may be caused by wavelengths and peaks. The location is related. Figure 2(d) shows the distribution of pressure in the axial direction. The pressure value decreases as the amplitude of the ripple increases, and the pressure distribution under various error conditions is more clearly distinguished.

Figure 3 is a dimensionless film thickness and pressure distribution of the aerostatic bearing with an eccentricity of 0.2, a rotational speed of 2000 rpm, and a wavelength of 5 mm in the axial and
circumferential directions.

![Image](a)

![Image](b)

![Image](c)

![Image](d)

**Figure 3.** The dimensionless air film thickness and pressure distribution on the axial and circumferential middle section of the bearing ($\varepsilon=0.2, n=2000\text{rpm}$)

Figure 3(a) is the dimensionless gas film thickness distribution in the circumferential direction. When the circumferential and axial errors are not equal, the film thickness fluctuates back and forth around the ideal circular film thickness, and the amplitude of the fluctuation is The circumferential amplitude increases and becomes larger. Figure 3(b) shows the dimensionless film thickness distribution of the aerostatic bearing in the axial direction. The thickness distribution increases with the increase of the amplitude. Figure 3(c) shows the dimensionless pressure distribution in the circumferential direction. Pressure crossover occurred at approximately 150° and 330°. Figure 3(d) is the axial pressure distribution. The pressure distribution on the left and the right sides is symmetrically distributed, which decreases as the amplitude of the circumferential corrugation increases. At the same time, the pressure is reduced in the middle position between the orifices. In Figure 3, when the circumferential amplitude is continuously increased and the axial amplitude is constant, the pressure is equal in the circumferential direction of 150 to 330, because the rotational speed has a certain shearing force in the circumferential direction. There are extrusion and tensile forces on both sides of the minimum film thickness. Therefore, when considering an aerostatic bearing, the influence of error and rotation speed on the aerostatic bearing is not negligible.

4.1 Influence of circumferential ripple

Figure 4 is an analysis of the bearing capacity, stiffness, total gas consumption and friction coefficient of aerostatic bearings with different circumferential vortex amplitudes at 40,000 rpm under different eccentricities.
Figure 4. Different circular amplitude and performance at different speeds in the case of performance changes(n=4000rpm)

Figure 4(a) is for a single row of 5 orifices. The distribution of the total gas consumption flow rate increases with the increase of the circumferential error amplitude. When the eccentricity increases, the total gas consumption decreases gradually. This may be due to the eccentric direction. The thickness of the gas film becomes smaller. Resulting in an increase in flow resistance in the gas flow area. Which leads to less air consumption, and an increase in the space of the gas film away from the eccentric position, which reduces the flow resistance and increases the gas supply amount. However, in the overall air supply amount, the increase in the air supply amount away from the eccentric position is smaller than the air supply amount in the eccentric direction. Figure 4(b) is a distribution diagram of bearing capacity. As the amplitude of the circumferential error increases, the bearing capacity of the bearing is gradually reduced. As the eccentricity increases, the bearing capacity increases. Figure 4(c) is a graph showing the variation of the friction coefficient of different circumferential ripple error amplitudes in the gas bearing film area. As the circumferential ripple error increases, the friction coefficient becomes larger, which indicates that the larger the error. The friction in the bearing area also increases with the frictional theory, which is consistent with the friction theory. Figure 4(d) is a graph showing changes in the bearing capacity of the gas bearing in the circumferential direction when the ripple amplitude is greater than the axial direction ripple amplitude, and the axial and circumferential wavelengths are equal, and the eccentricity changes. In the case of small eccentricity, the higher the bearing capacity is relatively larger, but as the eccentricity increases to the range of 0.35, the bearing capacity of the high and low speeds is almost equal, and as the eccentricity increases, the high speed. The bearing capacity in the case will be significantly greater than the bearing capacity at low speeds, indicating that the effect of the speed and error on the aerostatic bearing is not negligible.

Figure 5 is the effect of circumferential wavelength on the aerostatic bearing when there are axial and circumferential ripple error amplitudes. Analysis of the aerostatic bearing using different circumferential wavelengths.
Figure 5. Different circumferential wavelengths and performance changes at different speeds

Figure 5(a) is an analysis diagram of the total gas consumption of the bearing at three wavelengths. In the process of increasing the wavelength in the circumferential direction, the increase is more obvious when the wavelength is equal to 15 mm, and the change is smaller in 5 mm and 10 mm. There is a point of equal intersection, and the same phenomenon as in Figure 4(a) appears with the increase of the eccentricity. Figure 5(a) shows that the influence of the circumferential wavelength on the aerostatic bearing is also inconsequential. Figure 5(b) shows the bearing capacity becomes larger as the eccentricity increases. In Figure 5(c), the circumferential wavelength change has a great influence on the friction coefficient of the gas bearing, and the change between 0-15 mm is the first drop. Post-rise phenomenon, this phenomenon shows that the wavelength has a greater influence on the flow resistance in the gas region. Figure 5(d) is a graph showing changes in bearing capacity of different rotational speeds with eccentricity in the case where the amplitudes of the axial and circumferential corrugations are the same and the circumferential wavelength is greater than the axial wavelength. It can be seen from the figure that the low-speed gas bearing has a larger bearing capacity than the high-speed gas bearing in the case of a small eccentricity, but the bearing capacity of the gas bearing increases with the eccentricity increasing to 0.45. As the speed increases, it grows larger. It can be illustrated from Figure 5 that the wavelength of the circumferential corrugation is also important for the effect of the gas bearing on various performance parameters. Larger wavelengths have a greater impact on aerostatic bearings.

4.2 Influence of axial ripple

Figure 6 is the influence of the axial ripple error on the parameters of the aerostatic bearing.

Figure 6. Different axial amplitudes and performance parameters at different rotational speeds
Figure 6(a) shows the variation of the air consumption of different corrugation errors with the change of the eccentricity. The same variation and different circumferential ripple error with the eccentricity. The change is the same, as shown in Figure 4(b). Figure 6(b) is the case where the different axial ripple errors change with the eccentricity. Figure 6(c) shows the variation of the bearing capacity of different axial ripple errors as the eccentricity changes. Figure 6(d) shows that the axial ripple amplitude is greater than the circumferential ripple amplitude. The change of the influence of the eccentricity on the bearing capacity of the gas bearing with the same axial and circumferential wavelengths and different rotational speeds. It can be seen that in the case of small eccentricity, the bearing capacity is almost the same, but in the case where the eccentricity is greater than 0.4, the influence of the rotational speed on the bearing capacity is not negligible, and the change is the same as the change in Figure 4(d). Increasingly, the bearing capacity becomes significantly larger. It can be seen from Figure 6 that the variation of the axial ripple error value has little effect on the friction coefficient and bearing capacity of the gas bearing with the eccentricity change, but in the case of large eccentricity and different rotational speeds. The change of the bearing capacity is relatively obvious.

Figure 7 shows the influence of the axial ripple error on the performance parameters of the aerostatic bearing as the wavelength changes.

**Figure 7.** Different axial wavelengths and changes in performance parameters at different rotational speeds

Figure 7(a) shows the air consumption as a function of the eccentricity. Figure 7(b) shows the friction coefficient. In the case of eccentricity change, Figure 7(c) shows the variation of the bearing capacity of different axial wavelengths with the eccentricity. Figure 7(d) The axial wavelength is greater than the circumferential wavelength, and the different rotational speeds vary with the eccentricity. The curve of change is consistent with the distribution trend and trend of load capacity of Figure 5(d), indicating that in the case of a certain wavelength, the lower the rotation speed, the higher the bearing capacity, when a certain eccentricity is reached. In the case, the higher the rotational speed, the greater the bearing capacity. It can be seen from Figure 7 that the axial change in the axial direction can be found to have a small change in the performance parameters of the gas aerostatic bearing.

5. Conclusions
The numerical simulation of the orifice type static pressure gas bearing is carried out by using the
nonlinear Reynolds equation coupled gas flow coefficient, and the characteristics of the gas aerostatic bearing are analyzed by the ripple error in the analysis. Through the calculation results, the conclusions of gas consumption, friction coefficient and bearing capacity change are obtained.

1. The increase of the circumferential ripple amplitude will lead to an increase in air consumption and friction coefficient, and the bearing capacity will gradually become smaller; for this reason, the circumferential ripple amplitude has a greater influence on the static pressure gas bearing.

2. The amplitude of circumferential ripple amplitude has a great influence on the static pressure gas bearing, but the variation law of the influence is not obvious.

3. Compared with the effect of circumferential corrugation on aerostatic bearings, the influence of axial ripple error is small.

4. As the eccentricity increases, the bearing capacity of the bearing increases continuously. The coefficient of friction in the entire film area gradually becomes 0 as the eccentricity increases, and the gas consumption gradually decreases as the eccentricity increases.

5. It is concluded that the magnitude of the axial ripple error has little effect on the performance of the gas bearing, and the axial ripple error can be appropriately ignored in the manufacturing.

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