Efficiency of Neighbouring Designs for First Order Correlated Models

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ABSTRACT

The comparison of efficiency of Complete and Incomplete Nearest Neighbour Balanced Block Designs over regular block design using average variance, generalized variance and min-max variance with the error term $\varepsilon$ given in the NNBD model follows using first order correlated models. It is observed that, $R_H$ and $R_D$ show increasing efficiency values for direct and neighbour effects (left and right) for MA(1) models. The $R_A$ and $R_G$ show neither increasing nor decreasing efficiency values are observed for direct and neighbouring effects for AR(1) and MA(1) models. In the case of ARMA(1,1) model, neither increasing nor decreasing efficiency values have been observed for average variance and generalized variance. The $R_E$ shows decreasing efficiency values with $\rho$ in the interval 0.1 to 0.4 for direct and neighbouring effects for AR(1), MA(1) and ARMA(1,1) models.

Keywords: Auto Regressive Moving Average, Nearest neighbour, Regular Block Design, Average Variance, Generalized Variance, Min-Max Variance.

1. Introduction

The assumptions in the classical (Fisherian) block model are that the response on a plot to a particular treatment does not affect the response on the neighbouring plots and the fertility associated with plots in a block is constant.
However, in many fields of agricultural research, like horticultural and agro-forestry experiments, the treatment applied to one experimental plot in a block may affect the response on the neighbouring plots if the blocks are linear with no guard areas between the plots. If the treatments are varieties, neighbour effects may be caused by differences in height, root vigor, or germination date especially on small plots, which are used in plant breeding experiments. Treatments such as fertilizer, irrigation, or pesticide may spread to adjacent plots causing neighbour effects. Such experiments exhibit neighbour effects, because the effect of having no treatment as a neighbour is different from the neighbour effects of any treatment. Competition or interference between neighbouring units in field experiments can contribute to variability in experimental results and lead to substantial losses in efficiency. In case of block design setup, if each block is a single line of plots and blocks are well separated, extra parameters are needed for the effect of left and right neighbours. An alternative is to have border plots on both ends of every block. Each border plot receives an experimental treatment, but it is not used for measuring the response variable. These border plots do not add too much to the cost of one-dimensional experiments. The estimates of treatment differences may therefore deviate because of interference from neighbouring units. Neighbour balanced block designs, where in the allocation of treatments is such that every treatment occurs equally often with every other treatment as neighbours, are used for modeling and controlling interference effects between neighbouring plots. Azais et al. (1993) obtained a series of efficient neighbour designs with border plots that are balanced in $v - 1$ blocks of size $v$ and $v$ blocks of size $v - 1$, where $v$ is the number of treatments. Santharam.C & K.N.Ponnuswamy (1997) observed that the performance of NNBD is quite satisfactory for the remaining models. Druilhet (1999) studied optimality of circular neighbour balanced block designs obtained by Azais et al. (1993). Bailey (2003) has given some designs for studying one-sided neighbour effects. These neighbour balanced block designs have been developed under the assumption that the observations within a block are uncorrelated. In situations where the correlation structure among the observations within a block is known, may be from the data of past similar experiments, it may be advantageous to use this information in designing an experiment and analyzing the data so as to make more precise inference about treatment effects (Gill and Shukla, 1985). Kunert et al. (2003) considered two related models for interference and have shown that optimal designs for one
model can be obtained from optimal designs for the other model. Martin and Eccelston (2004) have given variance balanced designs under interference and dependent observations. Tomar and Seema Jaggi (2007) observed that efficiency is quite high, in case of complete block designs for both AR(1) and NN correlation structures. In case of incomplete block designs, designs with AR(1) structure turns out to be more efficient. However, the efficiency of direct effects of treatments is more as compared to neighbour effects under both the structures. Mingyao et al., (2009) studied the optimality of circular neighbor balanced designs for total effects when the one-sided or two-sided neighbor effects are present in the model and the observation errors are correlated according to a First-Order Circular Auto Regressive (AR(1,C)) process.

In this article, we have compared the efficiencies of NNBD and NNBIBD over regular block design using average variance, generalized variance and min-max variance with the error term $\varepsilon$ given in the NNBD model follows AR(1), MA(1) and ARMA(1,1) models. We have investigated the various measures of efficiencies ($R_A$, $R_H$, $R_G$, $R_D$ and $R_E$) of nearest neighbour balanced block design over regular block design using first order correlated models. We have also investigated the various measures of efficiencies ($R_A$, $R_H$, $R_G$, $R_D$ and $R_E$) of nearest neighbour balanced incomplete block design over regular block design using first order correlated models.

2. Model Structures

The designs considered here are assumed to be in linear blocks, with neighbour effects only in the direction of the blocks (say left-neighbour or right-neighbour or both). Because the effect of having no treatment differs from the neighbor effects of any treatment, designs with border plots have been considered, which is, designs with one point added at each end of each block. The border plots receive treatments but are not used for measuring the response variables. The plots, which are not on the borders, are inner plots. The length of a block is the number of its inner plots. It is further assumed that all the designs are circular, that is the treatment on border plots is same as the treatment on the inner plot at the other end of the block.
Let $\Delta$ be a class of binary neighbour balanced block designs with $n = bk$ units that form $b$ blocks each containing $k$ units. $Y_{ij}$ be the response from the $i^{th}$ plot in the $j^{th}$ block ($i = 1,2,\ldots,k; j = 1,2,\ldots,b$). The layout includes border plots at both ends of every block, i.e. at $0^{th}$ and $(k + 1)^{th}$ position and observations for these units are not modeled. The following fixed effects additive model is considered for analyzing a neighbour balanced block design under correlated observations

$$Y_{ij} = \mu + \tau_{(i,j)} + I_{(i-1,j)} + \gamma_{(i+1,j)} + \beta_j + e_{ij} \quad \text{(2.1)}$$

Where $\mu$ the general is mean, $\tau_{(i,j)}$ is the direct effect of the treatment in the $i^{th}$ plot of $j^{th}$ block, $\beta_j$ is the effect of the $j^{th}$ block. $I_{(i-1,j)}$ is the left neighbour effect due to the treatment in the $(i-1)^{th}$ plot of $j^{th}$ block. $\gamma_{(i+1,j)}$ is the right neighbour effect due to the treatment in the $(i+1)^{th}$ plot in $j^{th}$ block. $e_{ij}$ are error terms distributed with mean zero and a variance-covariance structure $\Omega = I_b \otimes \Lambda$ ($I_b$ is an identity matrix of order $b$ and $\otimes$ denotes the kronecker product). The ARMA (1,1) model along with AR(1) and MA(1) and explored the performance of NNBD for $\rho = -0.4(-0.4)0.4$. If the errors within a block follow a AR(1) structure, then $\Lambda$ is a $k \times k$ matrix with $(i,i')^{th}$ entry $(i,i' = 1,2,\ldots,k)$ as $\rho^{(i-i')} - 1 < 1$. The MA(1) structure, then $\Lambda$ is a matrix with diagonal entries as 1 and $(i,i')^{th}$ entry $(i,i' = 1,2,\ldots,k)$ as $\rho$, when $|i-i'| = 1$, otherwise zero Gill and Shukla, (1985). If the errors within a block follow an ARMA(1,1) model then $\Omega = I_b \otimes \Lambda$. Where $I_b$ is an identity
matrix of order $b$ and \( \Lambda = \begin{bmatrix} r_0 & r_1 & r_2 & \ldots & r_{k-1} \\ r_1 & r_0 & r_1 & \ldots & r_{k-2} \\ r_2 & r_1 & r_0 & \ldots & r_{k-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r_{k-1} & r_{k-2} & r_{k-3} & \ldots & r_0 \end{bmatrix} \), where
\[
r_0 = \frac{1 + 2 \rho_1 \rho_2 + \rho_2^2}{1 - \rho_1^2}, \quad r_i = \frac{\rho_1 (1 + \rho_2^2) + \rho_2 (1 + \rho_1^2)}{1 - \rho_1^2}, \quad r_k = \rho_1^k (k-1) \quad \text{for} \quad k \geq 2,
\]
Santharam & Ponnumswamy (1997). The \textbf{NN correlation structure}, the \( \Lambda \) is a matrix with diagonal entries as 1 and off-diagonal entries as \( \rho \).

Model (2.1) can be rewritten in the matrix notation as follows

\[
Y = \mu 1 + \Delta \tau + \Delta_1 l + \Delta_2 \gamma + D' \beta + e
\]  
(2.2)

where \( Y \) is \( n \times 1 \) vector of observations, \( 1 \) is \( n \times 1 \) vector of ones, \( \Delta \) is an \( n \times v \) incidence matrix of observations versus direct treatments, \( \tau \) is \( v \times 1 \) vector of direct treatment effects, \( \Delta_1 \) is a \( n \times v \) matrix of observations versus left neighbour treatment, \( \Delta_2 \) is a \( n \times v \) matrix of observations versus right neighbour treatment, \( l \) is \( v \times 1 \) vector of left neighbour effects, \( \gamma \) is \( v \times 1 \) vector of right neighbour effects, \( D \) is an \( n \times b \) incidence matrix of observations versus blocks, \( \beta \) is \( b \times 1 \) vector of block effects and \( e \) is \( n \times 1 \) vector of errors.

The joint information matrix for estimating the direct and neighbour (left and right) effects under correlated observations estimated by generalized least squares is obtained as follows:

\[
C = \begin{bmatrix}
\Delta (I_b \otimes \Lambda^*) \Delta & \Delta (I_b \otimes \Lambda^*) \Delta_1 & \Delta (I_b \otimes \Lambda^*) \Delta_2 \\
\Delta_1 (I_b \otimes \Lambda^*) \Delta & \Delta_1 (I_b \otimes \Lambda^*) \Delta_1 & \Delta_1 (I_b \otimes \Lambda^*) \Delta_2 \\
\Delta_2 (I_b \otimes \Lambda^*) \Delta & \Delta_2 (I_b \otimes \Lambda^*) \Delta_1 & \Delta_2 (I_b \otimes \Lambda^*) \Delta_2
\end{bmatrix}
\]  
(2.3)

with
\[
\Lambda^* = \Lambda^{-1} - \left( I_k \otimes \Lambda^{-1} I_k \right)^{-1} \Lambda^{-1} I_k I_k^* \Lambda^{-1}
\]
The above $3v \times 3v$ information matrix $(C)$ for estimating the direct effects and neighbour effects of treatments in a block design setting is symmetric, non-negative definite with row and column sums equal to zero. The information matrix for estimating the direct effects of treatments from (2.3) is as follows:

$$C_r = C_{11} - C_{12}C_{22}^{-1}C_{21}$$

(2.4)

where

$$C_{11} = \Delta (I_b \otimes \wedge^*) \Delta'$$

$$C_{12} = \left[ \Delta (I_b \otimes \wedge^*) \Delta'_1 \quad \Delta (I_b \otimes \wedge^*) \Delta'_2 \right]$$

and

$$C_{22} = \left[ \Delta_1 (I_b \otimes \wedge^*) \Delta'_1 \quad \Delta_1 (I_b \otimes \wedge^*) \Delta'_2 \right]
\left[ \Delta_2 (I_b \otimes \wedge^*) \Delta'_1 \quad \Delta_2 (I_b \otimes \wedge^*) \Delta'_2 \right]$$

Similarly, the information matrix for estimating the left neighbour effect of treatments $(C_l)$ and right neighbour effect of treatments $(C_r)$ can be obtained.

**2.1 Construction of Design**

Tomer et al. (2005) has constructed neighbour balanced block design with parameters $v$ (prime or prime power), $b = v(v - 1)$, $r = (v - 1)(v - m)$, $k = (v - m)$, $m = 1, 2, \ldots, v - 4$ and $\lambda = (v - m)$ using Mutually Orthogonal Latin Squares (MOLS) of order $v$. This series of design has been investigated under the correlated error structure. It is seen that the design turns out to be pair-wise uniform with $\alpha = 1$ and also variance balanced for estimating direct $(V_1)$ and neighbour effects $(V_2 = V_3)$.

**3. Comparison of Measures of Efficiency of NNBD**

In this section, we study the behaviour of some estimators of $\rho$ and $\sigma^2_e$. The nearest neighbour balanced block design and regular block design data sets were generated with the following true parameters: $\rho = -0.4$ to 0.4, $\sigma^2_e = 1$, $t = 5, r = 20$ and $t = 6, r = 30$.
The estimation of $\sigma_\varepsilon^2$ based on nearest neighbour balanced block design and regular block design were compared using the following three measures.

**Average Variance Comparison**

Consider the measure

$$R_A = \frac{\sigma_\varepsilon^2 (RBD) \sum_{i=1}^{t-1} \gamma_{RBD}^{-1}(i)}{\sigma_\varepsilon^2 (NNBD) \sum_{i=1}^{t-1} \gamma_{NNBD}^{-1}(i)}$$

where $\sigma_\varepsilon^2 (RBD)$ denotes the estimate of $\sigma_\varepsilon^2$ based on regular block design $\sigma_\varepsilon^2 (NNBD)$ denotes the estimate of $\sigma_\varepsilon^2$ based on NNBD $\gamma_{d(i)}$'s and are nonzero eigen values of the information matrix.

The above measure $R_A$ compares the average variance of elementary treatment contrast when the same data are analysed by regular block design and nearest neighbour balanced block design. It may be noted that the estimates of $\sigma_\varepsilon^2$ and $\rho$ can be different in case of regular block design and nearest neighbour balanced block design. The ratio $\sigma_\varepsilon^2 (RBD)/\sigma_\varepsilon^2 (NNBD)$ could mask the genuine efficiency of NNBD. Therefore, the ratio

$$R_H = \frac{\sum_{i=1}^{t-1} \gamma_{RBD}^{-1}(i)}{\sum_{i=1}^{t-1} \gamma_{NNBD}^{-1}(i)}$$

of harmonic means will also be considered as an index of efficiency.

**Generalised Variance Comparison**

Another way to compare regular block design and nearest neighbour balanced block design is the ratio:

$$R_G = \left[\sigma_{RBD}^2 / \sigma_{NNBD}^2 \right]^{-1} \prod_{i=1}^{t-1} \gamma_{NNBD(i)} \gamma_{RBD(i)}^{-1}$$

of generalized variances of $t-1$ orthonormal treatment contrasts estimated under regular block design and nearest neighbour balanced block design. It may
be noted that $R_G$ is very sensitive to the ratio $\sigma_{RBD}^2/\sigma_{NNBD}^2$. We therefore, consider the ratio

$$R_D = \prod_{i=1}^{t-1} \frac{\gamma_{NNBD(i)}\gamma_{RBD(i)}^{-1}}{\gamma_{NNBD(i-1)}\gamma_{RBD(i-1)}^{-1}}$$

This gives a better comparison of regular block design and nearest neighbour balanced block design.

**Min-max Variance Comparison**

This closeness is measured by the ratio of the smallest nonzero eigen-value to the largest eigen value of the information matrix. Note that this ratio independent of $\sigma^2_e$. For comparing nearest neighbour balanced block design and regular block design, we take the ratio

$$R_E = \frac{\gamma_{NNBD(1)}\gamma_{RBD(t-1)}^{-1}}{\gamma_{NNBD(t-1)}\gamma_{RBD(1)}^{-1}}$$

The tables 3.1, 3.2 and 3.3 show the efficiencies of AR(1), MA(1) and ARMA(1,1) models with $t = 5, r = 20$ and $\alpha = 1$, there is considerable advantage in using NNBD as far as average variance ($R_A$ and $R_G$), generalized variance ($R_H$ and $R_D$) and min-max variance ($R_E$) are concerned. The $R_H$ and $R_D$ show increasing efficiency values, $R_A$ and $R_G$ show decreasing efficiency values for direct effects of treatments for both AR(1) and MA(1) models. In the case of ARMA(1,1) model, neither increasing nor decreasing efficiency values are observed for average variance and generalized variance. The $R_E$ show decreasing efficiency values with $\rho$ in the interval 0.1 to 0.4 for direct and neighbouring effects for AR(1), MA(1) and ARMA(1,1) models.
### Table 3.1: AR(1) - $R_H, R_A, R_D, R_G$ and $R_E$ values for NNBD

$t = 5, r = 20$ and $\alpha = 1$

| AR(1) | $\rho = -0.4$ | $\rho = -0.3$ | $\rho = -0.2$ | $\rho = -0.1$ | $\rho = 0$ | $\rho = 0.1$ | $\rho = 0.2$ | $\rho = 0.3$ | $\rho = 0.4$ |
|-------|---------------|---------------|---------------|---------------|------------|------------|------------|------------|------------|
| $R_H$ | 0.82206       | 0.84056       | 0.84849       | 0.92776       | 0.99731    | 1.07535    | 1.11937    | 1.19711    | 1.33701    |
| $E_l$ | 0.88057       | 0.88385       | 0.87055       | 0.93609       | 0.99731    | 1.07443    | 1.14159    | 1.19428    | 1.38385    |
| $E_y$ | 0.87535       | 0.87293       | 0.90202       | 0.93467       | 0.99731    | 1.07842    | 1.14588    | 1.17232    | 1.44921    |
| $R_A$ | $E_t$         | 1.71980       | 1.62361       | 1.43725       | 1.16062    | 1.0300     | 0.88094    | 0.70476    | 0.67051    |
|       | $E_l$         | 1.04258       | 1.23553       | 1.35437       | 1.05331    | 1.0300     | 1.02668    | 0.95449    | 0.93141    |
|       | $E_y$         | 0.84175       | 0.83609       | 0.77479       | 0.78136    | 1.0300     | 1.02670    | 0.94460    | 0.94350    |
| $R_D$ | $E_t$         | 0.65693       | 0.76922       | 0.80208       | 0.91668    | 0.99742    | 1.06470    | 1.07019    | 1.10153    |
|       | $E_l$         | 0.75975       | 0.80215       | 0.79492       | 0.93214    | 0.99742    | 1.13410    | 1.10125    | 1.13779    |
|       | $E_y$         | 0.75451       | 0.73194       | 0.86888       | 0.93505    | 0.99742    | 1.08569    | 1.11999    | 1.07228    |
| $R_G$ | $E_t$         | 1.37434       | 1.68040       | 1.35863       | 1.14676    | 1.03311    | 0.87221    | 0.67380    | 0.61698    |
|       | $E_l$         | 1.09700       | 1.02896       | 1.28008       | 1.13454    | 1.03311    | 0.98796    | 0.87841    | 0.84006    |
|       | $E_y$         | 0.87294       | 0.74411       | 0.72419       | 0.86790    | 1.03311    | 1.02708    | 1.02430    | 0.83693    |
| $R_E$ | $E_t$         | 0.34195       | 0.41605       | 0.58207       | 0.80513    | 1.02397    | 0.80117    | 0.61089    | 0.51894    |
|       | $E_l$         | 0.50890       | 0.50607       | 0.49748       | 0.90509    | 1.02397    | 0.90788    | 0.71547    | 0.51062    |
|       | $E_y$         | 0.40937       | 0.50946       | 0.57910       | 0.79090    | 1.0297    | 0.77891    | 0.51938    | 0.44272    | 0.35021 |
### Table 3.2: MA(1) - $R_H$, $R_A$, $R_D$, $R_G$ and $R_E$ values for NNBD

$t = 5, r = 20$ and $\alpha = 1$

| MA(1) | $\rho = -0.4$ | $\rho = -0.3$ | $\rho = -0.2$ | $\rho = -0.1$ | $\rho = 0$ | $\rho = 0.1$ | $\rho = 0.2$ | $\rho = 0.3$ | $\rho = 0.4$ |
|-------|----------------|----------------|----------------|----------------|------------|------------|------------|------------|------------|
| $R_H$ | $E_t$           | 0.84249        | 0.86817        | 0.86119        | 0.93935    | 0.99731    | 1.08974    | 1.14109    | 1.24952    | 1.46584    |
|       | $E_l$           | 0.84164        | 0.86526        | 0.86778        | 0.92234    | 0.99731    | 1.11430    | 1.14735    | 1.26867    | 1.54839    |
|       | $E_y$           | 0.85300        | 0.86664        | 0.93185        | 0.96478    | 0.99731    | 1.08162    | 1.18884    | 1.30402    | 1.50626    |
| $R_A$ | $E_t$           | 1.74409        | 1.71577        | 1.38649        | 1.13382    | 1.03300    | 0.85413    | 0.73994    | 0.59917    | 0.45844    |
|       | $E_l$           | 1.44720        | 1.36032        | 1.25010        | 1.22789    | 1.03300    | 0.99040    | 0.85540    | 0.81432    | 0.79324    |
|       | $E_y$           | 0.97037        | 0.89610        | 0.86960        | 0.83836    | 0.83000    | 0.79912    | 0.72593    | 0.76666    | 0.68569    |
| $R_D$ | $E_t$           | 0.72640        | 0.89669        | 0.89790        | 0.92919    | 0.99742    | 1.06906    | 1.08207    | 1.07493    | 1.08956    |
|       | $E_l$           | 0.78649        | 0.81766        | 0.89177        | 0.91675    | 0.99742    | 1.10748    | 1.09527    | 1.13450    | 1.14402    |
|       | $E_y$           | 0.77146        | 0.76634        | 0.89765        | 0.94456    | 0.99742    | 1.06893    | 1.12330    | 1.12611    | 1.19308    |
| $R_G$ | $E_t$           | 1.50378        | 1.47508        | 1.31983        | 1.12156    | 1.03311    | 0.83792    | 0.70167    | 0.51545    | 0.34076    |
|       | $E_l$           | 1.35236        | 1.29648        | 1.20373        | 1.12044    | 1.03311    | 0.98434    | 0.81658    | 0.81763    | 0.73881    |
|       | $E_y$           | 0.98717        | 0.83761        | 0.83769        | 0.82070    | 1.03311    | 0.86120    | 0.81950    | 0.81146    | 0.71732    |
| $R_E$ | $E_t$           | 0.42834        | 0.53675        | 0.58745        | 0.81568    | 1.02397    | 0.74110    | 0.56530    | 0.40386    | 0.24452    |
|       | $E_l$           | 0.63471        | 0.56779        | 0.65418        | 0.90441    | 1.02397    | 0.93008    | 0.66682    | 0.42463    | 0.25251    |
|       | $E_y$           | 0.51057        | 0.52467        | 0.59003        | 0.70308    | 1.02397    | 0.77840    | 0.55752    | 0.36290    | 0.22587    |
Table 3.3: ARMA (1,1) - $R_H$, $R_A$, $R_D$, $R_G$ and $R_E$ values for NNBD

$t = 5, r = 20$ and $\alpha = 1$

| ARMA | $\rho_1 = -0.4 \rho_1 = -0.3 \rho_1 = -0.2 \rho_1 = -0.1 \rho_1 = 0 \rho_1 = 0.1 \rho_1 = 0.2 \rho_1 = 0.3 \rho_1 = 0.4$ |
|------|--------------------------------------------------------------------------------------------------|
| (1,1) | $\rho_2 = -0.4 \rho_2 = -0.3 \rho_2 = -0.2 \rho_2 = -0.1 \rho_2 = 0 \rho_2 = 0.1 \rho_2 = 0.2 \rho_2 = 0.3 \rho_2 = 0.4$ |

| $R_H$ | $E_r$ | 1.73929 | 1.40782 | 1.13165 | 1.03135 | 0.99731 | 1.07079 | 1.25615 | 1.80200 | 2.33113 |
|------|-------|---------|---------|---------|---------|---------|---------|---------|---------|
| $E_l$ |       | 1.27542 | 1.24219 | 1.11838 | 1.00941 | 0.99731 | 1.14190 | 1.33971 | 1.97894 | 2.16379 |
| $E_f$ |       | 1.64768 | 1.14631 | 1.19344 | 1.01525 | 0.99731 | 1.10410 | 1.36069 | 1.86532 | 2.05891 |

| $R_A$ | $E_r$ | 2.38589 | 2.29292 | 1.66082 | 1.25045 | 1.03300 | 0.81184 | 0.49451 | 0.39819 | 0.32413 |
|------|-------|---------|---------|---------|---------|---------|---------|---------|---------|
| $E_l$ |       | 1.38934 | 1.28048 | 1.20192 | 1.28074 | 1.03300 | 0.96485 | 0.41140 | 0.32256 | 0.31241 |
| $E_f$ |       | 1.11417 | 1.14094 | 1.12845 | 1.08510 | 1.03300 | 0.93729 | 0.41176 | 0.31655 | 0.33456 |

| $R_D$ | $E_r$ | 1.24259 | 1.11109 | 0.99616 | 1.00093 | 0.99742 | 1.02257 | 0.99765 | 1.04287 | 1.05567 |
|------|-------|---------|---------|---------|---------|---------|---------|---------|---------|
| $E_l$ |       | 2.21619 | 1.65773 | 1.03140 | 0.97924 | 0.99742 | 1.08982 | 1.05636 | 1.02048 | 1.01697 |
| $E_f$ |       | 1.64380 | 1.62707 | 1.06986 | 0.98238 | 0.99742 | 1.05094 | 1.07005 | 1.07228 | 1.00897 |

| $R_G$ | $E_r$ | 1.70454 | 1.70964 | 1.46198 | 1.21357 | 1.03311 | 0.77528 | 0.39275 | 0.23044 | 0.22034 |
|------|-------|---------|---------|---------|---------|---------|---------|---------|---------|
| $E_l$ |       | 2.41413 | 1.70565 | 1.41117 | 1.24246 | 1.03311 | 0.92084 | 0.82094 | 0.82197 | 0.69714 |
| $E_f$ |       | 1.78736 | 1.76492 | 1.70332 | 1.28235 | 1.03311 | 0.97291 | 0.78334 | 0.81147 | 0.56712 |

| $R_E$ | $E_r$ | 0.25425 | 0.30840 | 0.40455 | 0.71882 | 1.02397 | 0.58190 | 0.29767 | 0.16850 | 0.15650 |
|------|-------|---------|---------|---------|---------|---------|---------|---------|---------|
| $E_l$ |       | 0.49622 | 0.19640 | 0.52302 | 0.70143 | 1.02397 | 0.64260 | 0.34204 | 0.25408 | 0.10192 |

The tables 3.4, 3.5 and 3.6 show the efficiencies of AR(1), MA(1) and ARMA(1,1) models with $t = 6, r = 30$ and $\alpha = 1$, there is considerable advantage in using NNBD as far as average variance ( $R_A$ and $R_G$), generalized
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variance ($R_H$ and $R_D$) and min-max variance ($R_E$) are concerned. The $R_H$ and $R_D$ show increasing efficiency values for direct, left and right neighbour effects for MA(1) models. Whereas neither increasing nor decreasing efficiency values are observed for $R_A$ and $R_G$ for AR(1), MA(1) and ARMA(1,1) models. The $R_E$ show decreasing efficiency values with $\rho$ in the interval 0.1 to 0.4 for direct and neighbouring effects for AR(1), MA(1) and ARMA(1,1) models.

### Table 3.4: AR(1) - $R_H$, $R_A$, $R_D$, $R_G$ and $R_E$ values for NNBD

$t = 6, r = 30$ and $\alpha = 1$

| AR(1) | $\rho = -0.4$ | $\rho = -0.3$ | $\rho = -0.2$ | $\rho = -0.1$ | $\rho = 0$ | $\rho = 0.1$ | $\rho = 0.2$ | $\rho = 0.3$ | $\rho = 0.4$ |
|-------|---------------|---------------|---------------|---------------|-----------|-----------|-----------|-----------|-----------|
| $R_H$ | $E_x$         | 0.77832       | 0.83364       | 1.18472       | 0.94421   | 1.00000   | 1.07791   | 1.16081   | 1.25413   | 1.36637   |
|       | $E_l$         | 1.01559       | 1.19869       | 0.88125       | 0.94508   | 1.00000   | 1.05268   | 1.30270   | 1.28229   | 1.41284   |
|       | $E_\gamma$   | 0.84044       | 0.88062       | 0.85890       | 0.96807   | 1.00000   | 1.08877   | 1.16076   | 1.26551   | 1.38382   |
| $R_A$ | $E_x$         | 2.40263       | 1.61135       | 1.84549       | 1.03443   | 1.00000   | 1.00032   | 0.76598   | 0.63805   | 0.57940   |
|       | $E_l$         | 1.26510       | 1.51088       | 0.98075       | 0.99408   | 1.00000   | 1.12142   | 1.27964   | 1.25184   | 1.36862   |
|       | $E_\gamma$   | 0.86283       | 0.81914       | 0.91018       | 0.87069   | 1.00000   | 1.06665   | 1.21837   | 1.33880   | 1.55820   |
| $R_D$ | $E_x$         | 0.56238       | 0.72942       | 0.83232       | 0.93186   | 1.00000   | 1.06837   | 1.10932   | 1.14256   | 1.16253   |
|       | $E_l$         | 0.98933       | 0.77681       | 0.84631       | 0.93420   | 1.00000   | 1.04625   | 1.12387   | 1.16745   | 1.21139   |
|       | $E_\gamma$   | 0.66824       | 0.72392       | 0.82160       | 0.95574   | 1.00000   | 1.08248   | 1.10874   | 1.14809   | 1.16696   |
| $R_G$ | $E_x$         | 1.73602       | 1.40989       | 1.29655       | 1.02091   | 1.00000   | 0.87209   | 0.73199   | 0.58129   | 0.49297   |
|       | $E_l$         | 1.23239       | 0.97912       | 0.94187       | 0.98263   | 1.00000   | 1.11458   | 1.10398   | 1.13974   | 1.17347   |
|       | $E_\gamma$   | 0.68604       | 0.77824       | 0.87065       | 0.85955   | 1.00000   | 1.06049   | 1.16377   | 1.21458   | 1.31401   |

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|                  | $\rho = -0.4$ | $\rho = -0.3$ | $\rho = -0.2$ | $\rho = -0.1$ | $\rho = 0$   | $\rho = 0.1$ | $\rho = 0.2$ | $\rho = 0.3$ | $\rho = 0.4$ |
|-----------------|---------------|---------------|---------------|---------------|--------------|--------------|--------------|--------------|--------------|
| $R_E$           | 0.20321       | 0.38802       | 0.22031       | 0.74089       | 1.00000      | 0.81287      | 0.56315      | 0.43954      | 0.31437      |
| $E_r$           | 0.37858       | 0.18556       | 0.59450       | 0.78300       | 1.00000      | 0.88736      | 0.49702      | 0.44176      | 0.34283      |
| $E_f$           | 0.28271       | 0.37227       | 0.55862       | 0.79059       | 1.00000      | 0.83293      | 0.55672      | 0.42128      | 0.31865      |

Table 3.5: MA(1) - $R_H$, $R_A$, $R_D$, $R_G$ and $R_E$ values for NNBD

$t = 6, r = 30$ and $\alpha = 1$
### Table 3.6: ARMA(1,1) - $R_H$, $R_A$, $R_D$, $R_G$ and $R_E$ values for NNBD

$t = 6, r = 30$ and $\alpha = 1$

|        | $\rho_1 = -0.4\rho_1 = -0.3\rho_1 = -0.2\rho_1 = -0.1\rho_1 = 0 \quad \rho_1 = 0.1 \quad \rho_1 = 0.2 \rho_1 = 0.3 \rho_1 = 0.4$ |
|--------|----------------------------------------------------------------------------------------------------------------|
|        | $\rho_2 = -0.4\rho_2 = -0.3\rho_2 = -0.2\rho_2 = -0.1\rho_2 = 0 \quad \rho_2 = 0.1 \quad \rho_2 = 0.2 \rho_2 = 0.3 \rho_2 = 0.4$ |
| $R_H$  | $E_\tau$ 1.61665 1.29861 1.09650 0.90622 1.00000 1.09023 1.22625 1.83774 2.15711 |
|        | $E_I$ 1.51890 1.77775 1.14375 1.01721 1.00000 1.07553 1.39113 1.70890 1.98760 |
|        | $E_\gamma$ 0.94790 1.39883 1.11723 1.04661 1.00000 1.10188 1.39116 1.99808 1.99897 |
| $R_A$  | $E_\tau$ 2.86234 2.71432 1.82587 1.28585 1.00000 0.73195 0.45467 0.25593 0.23836 |
|        | $E_I$ 1.35838 1.28848 0.92658 0.92199 1.00000 1.24563 1.27625 1.47355 1.60108 |
|        | $E_\gamma$ 0.97760 0.83002 0.92133 0.88979 1.00000 1.24768 1.40949 1.67064 1.72301 |
| $R_D$  | $E_\tau$ 0.99559 0.89055 0.93523 0.84926 1.00000 1.03193 0.92058 0.90077 0.90214 |
|        | $E_I$ 0.88983 1.43282 1.04002 0.98194 1.00000 1.02545 1.19841 1.19478 1.38984 |
|        | $E_\gamma$ 1.91865 1.14741 0.96832 0.95574 1.00000 1.04569 1.07585 1.18192 1.27654 |
| $R_G$  | $E_\tau$ 1.76278 1.86140 1.55732 1.20503 1.00000 0.84234 0.34133 0.12544 0.22336 |
|        | $E_I$ 0.79579 1.03848 0.84255 0.89002 1.00000 1.18763 1.17720 1.32107 1.31403 |
|        | $E_\gamma$ 0.19787 0.68081 0.79854 0.81254 1.00000 1.18405 1.32204 1.44180 1.53403 |
| $R_E$  | $E_\tau$ 0.14236 0.19702 0.35713 0.48975 1.00000 0.53588 0.23090 0.09060 0.00867 |
|        | $E_I$ 0.13541 0.27493 0.45441 0.62825 1.00000 0.62983 0.36037 0.29192 0.18219 |
|        | $E_\gamma$ 0.41038 0.31932 0.35525 0.61146 1.00000 0.54712 0.24009 0.21890 0.17624 |
4. Comparison of Measures of Efficiency of NNBIBD

In this section, we study the behavior of some estimators of $\rho$ and $\sigma^2_e$. The NNBIBD data sets were generated with the following true parameters: $\rho = -0.4$ to 0.4, $\sigma^2_e = 1$, $t = 5, r = 16$ and $t = 6, r = 25$. The tables 4.1, 4.2 and 4.3 show the efficiencies of AR(1), MA(1) and ARMA(1,1) models with $t = 5, r = 16$ and $\alpha = 1$, there is considerable advantage in using NNBIBD as far as average variance ($R_A$ and $R_G$), generalized variance ($R_H$ and $R_D$) and min-max variance ($R_E$) are concerned. The $R_H$ and $R_D$ show increasing efficiency values with $\rho$ in the interval 0.1 to 0.4 for direct, left and right neighbour effects for AR(1) and MA(1) models. Whereas neither increasing nor decreasing efficiency values are observed for $R_A$ and $R_G$ for both AR(1) and MA(1) models. In the case of ARMA(1,1) model, neither increasing nor decreasing efficiency values are observed for average variance and generalized variance. The $R_E$ shows decreasing efficiency values with $\rho$ in the interval 0.1 to 0.4 for direct, left and right neighbour effects for AR(1), MA(1) and ARMA(1,1) models.

Table 4.1: AR(1) - $R_H$, $R_A$, $R_D$, $R_G$ and $R_E$ values for NNBIBD

$t = 5, r = 16$ and $\alpha = 1$

| AR(1) | $\rho = -0.4$ | $\rho = -0.3$ | $\rho = -0.2$ | $\rho = -0.1$ | $\rho = 0$ | $\rho = 0.1$ | $\rho = 0.2$ | $\rho = 0.3$ | $\rho = 0.4$ |
|-------|---------------|---------------|---------------|---------------|-----------|-------------|-------------|-------------|-------------|
| $R_H$ | $E_\tau$ | 0.77786 | 0.83529 | 0.88792 | 0.92961 | 1.00000 | 1.01727 | 1.09963 | 1.15674 | 1.19348 |
|      | $E_l$   | 0.89672 | 0.83497 | 0.96549 | 0.95583 | 1.00000 | 1.09572 | 1.11314 | 1.14588 | 1.25147 |
|      | $E_\gamma$ | 0.88430 | 0.89980 | 0.94050 | 0.93840 | 1.00000 | 1.05232 | 1.07227 | 1.15538 | 1.24844 |
| $R_A$ | $E_\tau$ | 1.27944 | 1.28998 | 1.13598 | 1.08621 | 1.00000 | 0.88803 | 0.94038 | 0.90180 | 0.86450 |
|      | $E_l$   | 1.01177 | 1.24454 | 1.34190 | 0.97799 | 1.00000 | 0.76243 | 0.93141 | 0.91137 | 1.06250 |
|      | $E_\gamma$ | 1.05091 | 0.65846 | 0.96372 | 0.84897 | 1.00000 | 1.07127 | 1.06240 | 1.19190 | 1.19601 |
Table 4.2: MA(1) - $R_H$, $R_A$, $R_D$, $R_G$ and $R_E$ values for NNBIBD

$t = 5, r = 16$ and $\alpha = 1$

| MA(1) | $\rho = -0.4$ | $\rho = -0.3$ | $\rho = -0.2$ | $\rho = -0.1$ | $\rho = 0$ | $\rho = 0.1$ | $\rho = 0.2$ | $\rho = 0.3$ | $\rho = 0.4$ |
|-------|----------------|----------------|----------------|----------------|------------|------------|------------|------------|------------|
| $R_H$ | $E_\tau$       | 0.86083        | 0.88038        | 0.89597        | 0.96944    | 1.00000    | 1.06681    | 1.11907    | 1.18888    | 1.26490    |
|       | $E_I$          | 0.88099        | 0.83125        | 0.93124        | 1.00291    | 1.00000    | 1.03877    | 1.09574    | 1.23675    | 1.35791    |
|       | $E_\gamma$     | 0.84355        | 0.86391        | 0.91231        | 0.94774    | 1.00000    | 1.02830    | 1.08648    | 1.23418    | 1.35574    |

| $R_A$ | $E_\tau$       | 1.13284        | 1.40250        | 1.19237        | 1.07885    | 1.00000    | 0.97629    | 0.87999    | 0.97257    | 0.83735    |
|       | $E_I$          | 1.10314        | 1.10937        | 1.08170        | 0.75168    | 1.00000    | 0.96844    | 0.94299    | 0.92745    | 0.95085    |
|       | $E_\gamma$     | 0.79164        | 0.71453        | 0.92204        | 0.87113    | 1.00000    | 1.07506    | 1.15091    | 1.47112    | 1.55587    |
| $R_D$ | $E_\tau$ | 0.80087 | 0.83408 | 0.87537 | 0.96886 | 1.00000 | 1.05467 | 1.07995 | 1.09565 | 1.09872 |
|------|---------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $E_I$ | 0.70347 | 0.82745 | 0.91711 | 0.99903 | 1.00000 | 1.01769 | 1.05690 | 1.13925 | 1.14744 |
| $E_\gamma$ | 0.79398 | 0.83028 | 0.89981 | 0.94494 | 1.00000 | 1.02287 | 1.06245 | 1.15902 | 1.17491 |

| $R_G$ | $E_\tau$ | 1.05393 | 1.32875 | 1.16495 | 1.07821 | 1.00000 | 0.96518 | 0.84922 | 0.89630 | 0.68176 |
|------|---------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $E_I$ | 0.88086 | 1.10430 | 1.06528 | 0.74877 | 1.00000 | 0.94880 | 0.90957 | 0.85430 | 0.80347 |
| $E_\gamma$ | 0.74512 | 0.68672 | 0.90942 | 0.86856 | 1.00000 | 1.06938 | 1.12545 | 1.38153 | 1.34835 |

| $R_E$ | $E_\tau$ | 0.63365 | 0.57701 | 0.73024 | 0.99507 | 1.00000 | 0.92787 | 0.71842 | 0.65256 | 0.58474 |
|------|---------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $E_I$ | 0.55367 | 0.90369 | 0.77949 | 0.89515 | 1.00000 | 0.68152 | 0.63403 | 0.51547 | 0.36212 |
| $E_\gamma$ | 0.52558 | 0.66838 | 0.79267 | 0.91481 | 1.00000 | 0.86625 | 0.77192 | 0.56403 | 0.39041 |

Table 4.3: ARMA (1,1) - $R_H$, $R_A$, $R_D$, $R_G$ and $R_E$ values for NNBIBD

$t = 5, r = 16$ and $\alpha = 1$

ARMA (1,1)

| $\rho_1$ | $\rho_2$ | $\rho_3$ | $\rho_4$ | $\rho_5$ | $\rho_6$ | $\rho_7$ | $\rho_8$ | $\rho_9$ |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| -0.4   | -0.3   | -0.2   | -0.1   | 0      | 0.1    | 0.2    | 0.3    | 0.4    |

$R_H$ | $E_\tau$ | 1.71592 | 1.26114 | 1.29357 | 1.01964 | 1.00000 | 1.03598 | 1.14253 | 1.37358 | 1.81046 |
|------|---------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $E_I$ | 2.59153 | 1.49485 | 1.12954 | 1.05501 | 1.00000 | 1.02735 | 1.22523 | 2.01832 | 2.09753 |
| $E_\gamma$ | 1.72077 | 1.38987 | 1.15548 | 1.04394 | 1.00000 | 1.03976 | 1.15657 | 1.67583 | 2.14000 |

$R_A$ | $E_\tau$ | 1.47312 | 1.55894 | 1.86328 | 1.15464 | 1.00000 | 0.76940 | 0.84150 | 0.88997 | 1.27818 |
|------|---------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $E_I$ | 0.72351 | 1.01397 | 0.89693 | 1.05316 | 1.00000 | 0.84368 | 1.01321 | 2.24453 | 2.23429 |

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The tables 4.4, 4.5 and 4.6 show the efficiencies of AR(1), MA(1) and ARMA(1,1) models with \( t = 6, r = 25 \) and \( \alpha = 1 \), there is considerable advantage in using NNBIBD as far as average variance (\( R_A \) and \( R_G \)), generalized variance (\( R_H \) and \( R_D \)) and min-max variance (\( R_E \)) are concerned. The \( R_H \) and \( R_D \) show increasing efficiency values for direct, left and right neighbour effects whereas neither increasing nor decreasing efficiency values are observed for \( R_A \) and \( R_G \) for both AR(1) and MA(1) models. In the case of ARMA(1,1) model, neither increasing nor decreasing efficiency values are observed for average variance and generalized variance. The \( R_E \) show decreasing efficiency values with \( \rho \) in the interval 0.1 to 0.4 for direct, left and right neighbour effects for AR(1), MA(1) and ARMA(1,1) models.

|          | 1.27917 | 0.95827 | 0.88167 | 0.99717 | 1.00000 | 1.15908 | 1.50302 | 2.32502 | 2.45010 |
|----------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| \( E_\gamma \) |         |         |         |         |         |         |         |         |         |
| \( R_D \)  | 1.40212 | 1.05921 | 1.17711 | 1.01432 | 1.00000 | 0.99693 | 0.85306 | 0.50820 |         |
| \( E_\iota \) | 1.87295 | 1.31061 | 1.08719 | 1.04027 | 1.00000 | 0.99888 | 1.08788 | 1.36470 | 1.46250 |
| \( E_\gamma \) | 1.56621 | 1.26519 | 1.10650 | 1.03511 | 1.00000 | 1.01353 | 1.04043 | 1.23131 | 1.35643 |
| \( R_G \)  | 1.20373 | 1.30932 | 1.69553 | 1.14862 | 1.00000 | 0.74039 | 0.71870 | 0.55271 | 0.35879 |
| \( E_\iota \) | 0.52290 | 0.88900 | 0.86330 | 1.03844 | 1.00000 | 0.82030 | 0.89962 | 1.21632 | 0.96622 |
| \( E_\gamma \) | 1.16427 | 0.84279 | 0.84430 | 0.98873 | 1.00000 | 1.12984 | 1.35209 | 1.70830 | 1.52313 |
| \( R_E \)  | 0.39495 | 0.42941 | 0.47639 | 0.90605 | 1.00000 | 0.92787 | 0.44354 | 0.23878 | 0.08087 |
| \( E_\iota \) | 0.21028 | 0.45689 | 0.62885 | 0.80132 | 1.00000 | 0.67754 | 0.43285 | 0.10564 | 0.09841 |
| \( E_\gamma \) | 0.42202 | 0.47483 | 0.63525 | 0.85213 | 1.00000 | 0.73622 | 0.49250 | 0.26196 | 0.09723 |

\( \gamma \)
Table 4.4: $AR(1) - R_H, R_A, R_D, R_G$ and $R_E$ values for NNBIBD

$t = 6, r = 25$ and $\alpha = 1$

|               | $\rho = -0.4$ | $\rho = -0.3$ | $\rho = -0.2$ | $\rho = -0.1$ | $\rho = 0$  | $\rho = 0.1$ | $\rho = 0.2$ | $\rho = 0.3$ | $\rho = 0.4$ |
|---------------|---------------|---------------|---------------|---------------|-------------|-------------|-------------|-------------|-------------|
| $R_H$         |               |               |               |               |             |             |             |             |             |
| $E_\tau \rho$ | 0.75715       | 0.81741       | 0.87770       | 0.94752       | 1.00000     | 1.07554     | 1.13153     | 1.18912     | 1.25529     |
| $E_l \rho$    | 0.78991       | 0.82209       | 0.87333       | 0.98171       | 1.00000     | 1.07280     | 1.13023     | 1.21005     | 1.28822     |
| $E_\gamma \rho$ | 0.76911     | 0.79349       | 0.89559       | 0.92212       | 1.00000     | 1.05652     | 1.14089     | 1.19583     | 1.27012     |
| $R_A$         |               |               |               |               |             |             |             |             |             |
| $E_\tau \rho$ | 1.54026       | 1.31995       | 1.20874       | 1.07690       | 1.00000     | 0.96936     | 0.90038     | 0.85262     | 0.84582     |
| $E_l \rho$    | 1.03521       | 0.93301       | 0.97859       | 1.18969       | 1.00000     | 1.01818     | 1.11520     | 1.16400     | 1.20850     |
| $E_\gamma \rho$ | 0.77878     | 1.16049       | 1.19915       | 0.86793       | 1.00000     | 1.04292     | 1.16891     | 1.16952     | 1.28236     |
| $R_D$         |               |               |               |               |             |             |             |             |             |
| $E_\tau \rho$ | 0.63806       | 0.748000      | 0.84232       | 0.93948       | 1.00000     | 1.06938     | 1.09691     | 1.11265     | 1.12397     |
| $E_l \rho$    | 0.70426       | 0.76875       | 0.84913       | 0.95630       | 1.00000     | 1.06987     | 1.10394     | 1.15359     | 1.18623     |
| $E_\gamma \rho$ | 0.67859     | 0.71308       | 0.86226       | 0.91027       | 1.00000     | 1.05014     | 1.11379     | 1.12729     | 1.14820     |
| $R_G$         |               |               |               |               |             |             |             |             |             |
| $E_\tau \rho$ | 1.29801       | 1.20786       | 1.16001       | 1.06776       | 1.00000     | 0.96381     | 0.87283     | 0.79778     | 0.80211     |
| $E_l \rho$    | 0.92296       | 0.87248       | 0.95148       | 1.15890       | 1.00000     | 1.01540     | 1.08926     | 1.10968     | 1.11283     |
| $E_\gamma \rho$ | 0.68709     | 1.04288       | 1.15452       | 0.85677       | 1.00000     | 1.03662     | 1.14114     | 1.10249     | 1.15926     |
| $R_E$         |               |               |               |               |             |             |             |             |             |
| $E_\tau \rho$ | 0.33526       | 0.46349       | 0.58736       | 0.85204       | 1.00000     | 0.83486     | 0.64981     | 0.54126     | 0.47121     |
| $E_l \rho$    | 0.36148       | 0.52131       | 0.66916       | 0.65587       | 1.00000     | 0.89743     | 0.69088     | 0.58654     | 0.52776     |
| $E_\gamma \rho$ | 0.38828     | 0.38737       | 0.64417       | 0.76051       | 1.00000     | 0.85220     | 0.67219     | 0.55838     | 0.47193     |
Table 4.5: MA(1) - $R_H$, $R_A$, $R_D$, $R_G$ and $R_E$ values for NNBIBD

$t = 6, r = 25$ and $\alpha = 1$

| MA(1) | $\rho = -0.4$ | $\rho = -0.3$ | $\rho = -0.2$ | $\rho = -0.1$ | $\rho = 0$ | $\rho = 0.1$ | $\rho = 0.2$ | $\rho = 0.3$ | $\rho = 0.4$ |
|-------|---------------|---------------|---------------|---------------|------------|------------|------------|------------|------------|
| $R_H$ E_r | 0.79925 | 0.84719 | 0.89198 | 0.93825 | 1.00000 | 1.06842 | 1.14266 | 1.22395 | 1.32585 |
| $E_I$ | 0.82072 | 0.84036 | 0.88665 | 0.92991 | 1.00000 | 1.06064 | 1.15554 | 1.24376 | 1.38881 |
| $E_\gamma$ | 0.78236 | 0.78786 | 0.88938 | 0.94766 | 1.00000 | 1.05418 | 1.13472 | 1.23931 | 1.34378 |
| $R_A$ E_r | 1.43244 | 1.27030 | 1.15913 | 1.10366 | 1.00000 | 0.95051 | 0.85093 | 0.83533 | 0.82640 |
| $E_I$ | 0.80049 | 0.91451 | 0.96212 | 0.91096 | 1.00000 | 1.1134 | 1.08876 | 1.16081 | 1.27921 |
| $E_\gamma$ | 0.96247 | 0.89751 | 0.95033 | 0.95515 | 1.00000 | 1.0572 | 1.16246 | 1.33932 | 1.71044 |
| $R_D$ E_r | 0.70495 | 0.78941 | 0.86471 | 0.93623 | 1.00000 | 1.05878 | 1.09525 | 1.09093 | 1.12666 |
| $E_I$ | 0.73048 | 0.79861 | 0.86714 | 0.92454 | 1.00000 | 1.05143 | 1.11810 | 1.14792 | 1.15737 |
| $E_\gamma$ | 0.62016 | 0.74205 | 0.86729 | 0.94049 | 1.00000 | 1.04549 | 1.09393 | 1.11950 | 1.15923 |
| $R_G$ E_r | 1.26342 | 1.18365 | 1.12369 | 1.10129 | 1.00000 | 0.94193 | 0.81563 | 0.74454 | 0.63992 |
| $E_I$ | 0.71248 | 0.86909 | 0.94095 | 0.90569 | 1.00000 | 1.10170 | 1.05348 | 1.07136 | 0.97393 |
| $E_\gamma$ | 0.76294 | 0.84532 | 0.92673 | 0.94792 | 1.00000 | 1.04206 | 1.12066 | 1.20984 | 1.34825 |
| $R_E$ E_r | 0.42503 | 0.52744 | 0.63889 | 0.92392 | 1.00000 | 0.79331 | 0.58556 | 0.41827 | 0.27592 |
| $E_I$ | 0.42951 | 0.58203 | 0.72463 | 0.86101 | 1.00000 | 0.80398 | 0.62561 | 0.48249 | 0.36456 |
| $E_\gamma$ | 0.52545 | 0.55748 | 0.77617 | 0.80675 | 1.00000 | 0.82280 | 0.60038 | 0.43560 | 0.28557 |
Table 4.6: ARMA(1,1) - $R_H, R_A, R_D, R_G$ and $R_E$ values for NNBIBD

\begin{align*}
\text{ARMA} & \quad \rho_1 = -0.4 \quad \rho_1 = -0.3 \quad \rho_1 = -0.2 \quad \rho_1 = -0.1 \quad \rho_1 = 0 \\
\text{(1,1)} & \quad \rho_2 = -0.4 \quad \rho_2 = -0.3 \quad \rho_2 = -0.2 \quad \rho_2 = 0 \\
& \quad \rho_1 = 0.1 \quad \rho_1 = 0.2 \quad \rho_1 = 0.3 \quad \rho_1 = 0.4 \\
& \quad \rho_2 = 0.1 \quad \rho_2 = 0.2 \quad \rho_2 = 0.3 \quad \rho_2 = 0.4
\end{align*}

\begin{table}[h]
\centering
\begin{tabular}{cccccc}
\hline
 & $R_H$ & $E_t$ & $E_l$ & $E_y$ & $E_r$ & $E_l$ & $E_y$ & $E_r$ & $E_l$ & $E_y$ & $E_r$ \\
\hline
$R_H$ & 1.52232 & 1.25590 & 1.09540 & 1.01367 & 1.0000 & 1.05858 & 1.19610 & 1.39511 & 1.34311 \\
$E_t$ & 1.84867 & 1.31759 & 1.29502 & 1.04689 & 1.0000 & 1.05439 & 1.24908 & 1.74777 & 1.22016 \\
$E_l$ & 1.76384 & 1.79428 & 1.11238 & 1.01631 & 1.0000 & 1.09183 & 1.23092 & 1.57102 & 1.16281 \\
$E_y$ & 1.96092 & 1.65797 & 1.38649 & 1.17350 & 1.0000 & 0.89106 & 0.81524 & 0.83425 & 1.46755 \\
$E_r$ & 0.99938 & 0.89095 & 1.08132 & 0.94590 & 1.0000 & 1.11011 & 1.19732 & 1.36251 & 1.41134 \\
$E_l$ & 0.95179 & 0.97118 & 1.00614 & 0.90295 & 1.0000 & 1.18075 & 1.57642 & 1.30807 & 1.49198 \\
$E_y$ & 1.11823 & 1.02162 & 0.98559 & 0.98581 & 1.0000 & 1.01768 & 0.98571 & 0.79179 & 0.86027 \\
$E_r$ & 1.53911 & 1.16715 & 1.16601 & 1.02631 & 1.0000 & 1.02261 & 1.11011 & 1.51474 & 1.42191 \\
$E_l$ & 1.44149 & 1.24356 & 1.02314 & 0.98868 & 1.0000 & 1.05391 & 1.03416 & 0.88280 & 1.13944 \\
$E_y$ & 1.44042 & 1.34870 & 1.24750 & 1.14125 & 1.0000 & 0.85664 & 0.67185 & 0.47347 & 0.64453 \\
$E_r$ & 0.83203 & 0.78923 & 0.97360 & 0.92730 & 1.0000 & 1.07665 & 1.06453 & 1.91419 & 1.56911 \\
$E_l$ & 0.77785 & 0.67309 & 0.92543 & 0.87841 & 1.0000 & 1.13975 & 1.32443 & 1.73118 & 1.63531 \\
$E_y$ & 0.26028 & 0.33942 & 0.44471 & 0.70276 & 1.0000 & 0.60797 & 0.31515 & 0.13047 & 0.18893 \\
$E_r$ & 0.28127 & 0.41457 & 0.37418 & 0.74233 & 1.0000 & 0.63558 & 0.41272 & 0.36627 & 0.10550 \\
$E_l$ & 0.36160 & 0.17001 & 0.56562 & 0.65460 & 1.0000 & 0.62718 & 0.33230 & 0.12040 & 0.14824 \\
\hline
\end{tabular}
\caption{Table 4.6: ARMA(1,1) - $R_H, R_A, R_D, R_G$ and $R_E$ values for NNBIBD}
\end{table}
5. Results and Conclusion

We have compared the efficiencies of NNBD using average variance, generalized variance and min-max variance when the errors follow first order correlated models. The $R_H$ and $R_D$ show increasing efficiency values for direct, left and right neighbour effects for MA(1) models. The $R_A$ and $R_G$ show neither increasing nor decreasing efficiency values are observed for AR(1), MA(1) and ARMA(1,1) models. The $R_E$ show decreasing efficiency values with $\rho$ in the interval 0.1 to 0.4 for direct and neighbouring effects for AR(1), MA(1) and ARMA(1,1) models.

We have compared the efficiencies of NNBIBD using average variance, generalized variance and min-max variance when the errors follow first order correlated models. The $R_H$ and $R_D$ show increasing efficiency values with $\rho$ in the interval 0.1 to 0.4 for direct, left and right neighbour effects for AR(1) and MA(1) models. Whereas neither increasing nor decreasing efficiency values are observed for $R_A$ and $R_G$ for both AR(1) and MA(1) models. In the case of ARMA(1,1) model, neither increasing nor decreasing efficiency values are observed for average variance and generalized variance. The $R_E$ show decreasing efficiency values with $\rho$ in the interval 0.1 to 0.4 for direct, left and right neighbour effects for AR(1), MA(1) and ARMA(1,1) models.

References

1. Azais, J.M., Bailey, R. A., and Monod, H (1993). A catalogue of efficient neighbour designs with border plots, Biometrics, 49, 1252-1261.
2. Bartlett, M. S (1978). Nearest neighbour models in the analysis of field experiments (with discussion). J.Roy. Statis. Soc. B 40, 147-174.
3. Bailey, R. A (2003). Designs for one-sided neighbour effects, J. Ind. Soc. Agric. Statistics, 56(3), 302-314.
4. Besag, J (1977). Errors-in-variables estimation for Gaussian lattice schemes, J. Roy. Statis. Soc. B 39, 73-78.
5. Druilhet, P (1999). Optimality of neighbour balanced designs, J. Statist. Plan & Inference, 81, 141-152.
6. Gill, P. S. and Shukla, G. K. (1985). Efficiency of nearest neighbour balanced block designs for correlated observations, *Biometrika*, 72, 539-544.

7. Kunert, J., Martin, R. J., and Pooladsaz, S (2003). Optimal designs under two related models for interference, *Metrika*, 57, 137-143.

8. Kunert, J (1987). Neighbour balanced block designs for correlated errors. *Biometrika* 74, 4, 717-724.

9. Martin, J. P. and Chakravarti, I. M (1998). Block designs for first and second order neighbour correlations. *The Ann. of Statist*. 16, 1206-1224.

10. Martin, R. J. and Eccleston, J. A (2004). Variance-balanced designs under interference for dependent observations, *J. Statist Plan & Inference*, 119, 207-223.

11. Mingyao Ai, Gennian Ge and Ling-Yaw chan. (2007), circular neighbor-balanced designs universally optimal for total effects. J. Sci. China Series A: Math., 50, 821-828.

12. R. Senthil Kumar & C. Santharam (2013), Efficiency of Nearest Neighbour Balanced Block Designs using ARMA models, *International Journal of Statistics and Systems*, ISSN: 0973-2675 Volume 8, Number 1, pp. 59-71.

13. R. Senthil Kumar & C. Santharam (2012), Efficiency of Nearest Neighbour Balanced Block Designs for correlated observations (ARMA models), *International Journal of Statistika and Mathematika*, ISSN: 2277-2790 E-ISSN: 2249-8605, Volume 4, Issue 1, pp 01-05.

14. R. Senthil Kumar & C. Santharam (2012), Efficiency of Neighbour Balanced Block Designs for Correlated Observations, *International Journal of Statistika and Mathematika*, ISSN: 2277-2790 E-ISSN: 2249-8605, Volume 3, Issue 3, pp 115-120.

15. Santharam.C & K.N.Ponnuswamy (1997), On the Efficiency of Nearest Neighbour Balanced Block Designs with Correlated Error Structure, *Biometrics*, J. 39, 85-98

16. Santharam.C & K.N.Ponnuswamy (1997), Optimality and Efficiency of Neighbouring Design, *Journal of Indian Society of Agricultural Statistics*, 50(1), 1997: 1-10

17. Tomar, J.S.Jaggi, S., and Varghese, C (2005), On totally balanced block designs for competition effects, *Jour. Applied Statistics*, 32 (1), 87-97.

18. Tomar, J.S.Jaggi, S (2007), Efficient neighbour balanced block designs for correlated observations, *METRON – International Journal of Statistics*, Vol. LXV, n. 2, pp. 229-238.
19. Williams, R.M (1952), Experimental designs for serially correlated observations. *Biometrika*, 39, 151-167.
20. Wilkinson, G. N., Eckert, S. R., Hancock, T. W., and Mayo, O (1983), Nearest Neighbour (NN) analysis of field experiments (with discussion). *J. Roy. Statist. Soc. B* 45,151-2