$K^-d \rightarrow \pi \Sigma n$ reactions and structure of the $\Lambda(1405)$

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Abstract. We report on the first results of a full three-body calculation of the $\overline{K}NN-\pi Y N$ amplitude for the $K^-d \rightarrow \pi \Sigma n$ reaction, and examine how the $\Lambda(1405)$ resonance manifests itself in the neutron energy distributions of $K^-d \rightarrow \pi \Sigma n$ reactions. The amplitudes are computed using the $\overline{K}NN-\pi Y N$ coupled-channels Alt-Grassberger-Sandhas (AGS) equations. Two types of models are considered for the two-body meson-baryon interactions: an energy-independent interaction and an energy-dependent one, both derived from the leading order chiral SU(3) Lagrangian. These two models have different off-shell properties that cause correspondingly different behaviors in the three-body system. As a remarkable result of this investigation, it is found that the neutron energy spectrum, reflecting the $\Lambda(1405)$ mass distribution and width, depends quite sensitively on the (energy-dependent or energy-independent) model used. Hence accurate measurements of the $\pi \Sigma$ mass distribution have the potential to discriminate between possible mechanisms at work in the formation of the $\Lambda(1405)$.

1. Introduction

Understanding the structure of the $\Lambda(1405)$ with spin-parity $J^\pi = 1/2^-$ and strangeness $S = -1$ is a long-standing issue in hadron physics. The mass of the $\Lambda(1405)$ is slightly less than the $\overline{K}N$ threshold energy. The $\Lambda(1405)$ can be considered as a $\overline{K}N$ quasi-bound state embedded in the $\pi \Sigma$ continuum [1 2]. Guided by this picture, $\overline{K}N$ interactions which reproduce the mass of $\Lambda(1405)$ and two-body scattering data have been constructed phenomenologically [3 4]. On the other hand, $\overline{K}N$ interactions have been studied for a long time based on chiral SU(3) dynamics [5 6 7]. Between the phenomenological and chiral SU(3) $\overline{K}N$ interactions, subthreshold $\overline{K}N$ amplitudes are quite different [8]. The phenomenological model describes $\Lambda(1405)$ as a single pole of the scattering amplitude around 1405 MeV. The $\overline{K}N$ amplitude from the interaction based on chiral SU(3) dynamics has two poles, one of which located not at 1405 MeV but around 1420 MeV [9 10]. The differences in the pole structure come from the different off-shell behavior, especially as a consequence of the energy-dependence of the $\overline{K}N$ interaction. The $\overline{K}N$ interaction based on chiral SU(3) dynamics is energy-dependent, and its attraction becomes weaker as one moves below the $\overline{K}N$ threshold energy. Hence the (upper) pole of the $\overline{K}N$ amplitude shows up around 1420 MeV. On the other hand, the phenomenological $\overline{K}N$ interaction is energy-independent and strongly attractive so that the pole shows up around 1405 MeV. These differences are enhanced in the so-called few-body kaonic nuclei, such as the strange dibaryon resonance under discussion in the $\overline{K}NN-\pi Y N$
coupled system \[1\] \[11\] \[12\] \[13\] \[14\] \[15\] \[16\] \[17\] \[18\] \[19\] \[20\] \[21\]. How a possible signature of this strange dibaryon resonance shows up in the resonance production reaction is also of interest as it reflects the two-body dynamics of the \(\Lambda(1405)\) \[22\].

One of the possible kaon-induced processes forming the \(\Lambda(1405)\) is \(K^-d \rightarrow \Lambda(1405)n\). The signature of the \(\Lambda(1405)\) was observed in an old bubble-chamber experiment that measured the \(\pi\Sigma\) invariant mass distribution in the \(K^-d \rightarrow \pi^+\Sigma^-n\) reaction \[23\]. A new experiment is planned at J-PARC \[24\]. Theoretical investigations of the \(K^-d \rightarrow \pi\Sigma n\) reaction have previously been performed in simplified models assuming a two-step process \[25\] \[26\] \[27\] \[28\].

In this contribution we examine how the \(\Lambda(1405)\) resonance shows up in the \(K^-\bar{K}\) reaction \[23\]. A new experiment is planned at J-PARC \[24\]. Theoretical investigations of the \(K^-d \rightarrow \pi\Sigma n\) reaction have previously been performed in simplified models assuming a two-step process \[25\] \[26\] \[27\] \[28\]. This is the first calculation of this process which incorporates the full three-body dynamics.

2. Three-body Scattering Equations

Throughout this paper, we assume that the three-body processes take place via separable two-body interactions, which have the following form in the two-body center-of-mass (CM) frame,

\[
V_{\alpha\beta}(\vec{q}_\alpha, \vec{q}_\beta; E) = g_\alpha(\vec{q}_\alpha) \lambda_{\alpha\beta}(E) g_\beta(\vec{q}_\beta),
\]

where \(g_\alpha [g_\alpha(\vec{q}_\alpha)]\) is the relative momentum [form factor] of the two-body channel \(\alpha\); \(E\) is the total energy of the two-body system. With this assumption the amplitudes for the quasi-two-body scattering of an “isobar” and a spectator particle, \(X_{ij}(\vec{p}_i, \vec{p}_j; W)\), are then obtained by solving the AGS equations \[29\] \[30\],

\[
X_{ij}(\vec{p}_i, \vec{p}_j, W) = (1 - \delta_{ij}) Z_{ij}(\vec{p}_i, \vec{p}_j, W) + \sum_{n \neq i} \int d\vec{p}_n Z_{in}(\vec{p}_i, \vec{p}_n, W) \tau_n (W - E_n(\vec{p}_n)) X_{nj}(\vec{p}_n, \vec{p}_j, W).
\]

Here the subscripts \(i, j, n\) specify the reaction channels; \(W\) and \(\vec{p}_i\) are the total scattering energy and the relative momentum of channel \(i\) in the three-body CM frame, respectively; \(Z_{ij}(\vec{p}_i, \vec{p}_j; W)\) and \(\tau_i (W - E_i(\vec{p}_i))\) are the one-particle exchange potential and the two-body propagator.

With the quasi-two-body amplitudes, the scattering amplitudes for the break-up process \(d + K \rightarrow \pi + \Sigma + N\) are obtained as

\[
T_{\pi\Sigma N-Kd}(\vec{q}_N, \vec{p}_N, \vec{p}_K, W) = g_{\pi\Sigma}(\vec{q}_N) \gamma_{\pi\Sigma Y_K} (W - E_N(\vec{p}_N)) X_{Y_Kd}(\vec{p}_N, \vec{p}_K, W) + g_{\pi\Sigma}(\vec{q}_N) \gamma_{\pi\Sigma Y_K} (W - E_N(\vec{p}_N)) X_{Y_Kd}(\vec{p}_N, \vec{p}_K, W) + g_{\pi\Sigma}(\vec{q}_N) \gamma_{\pi\Sigma Y_K} (W - E_N(\vec{p}_N)) X_{Y_Kd}(\vec{p}_N, \vec{p}_K, W) + g_{\pi\Sigma}(\vec{q}_N) \gamma_{\pi\Sigma Y_K} (W - E_N(\vec{p}_N)) X_{Y_Kd}(\vec{p}_N, \vec{p}_K, W),
\]

where \(X_{Y_Kd}(\vec{p}_N, \vec{p}_K, W)\) is the quasi-two-body amplitude anti-symmetrized for two nucleons; the subscripts denote the isobars. The notations for the isobars are \(Y_K = KN\), \(Y_\pi = \pi Y\), \(d = NN\), \(N^* = \pi N\) and \(d_y = Y N\), respectively.

In this contribution we employ the first two terms of Eq. (3) as a first step. These terms emerge directly from the \(\Lambda(1405)\) in the final state interaction; they are the dominant parts of the full T-matrix. Using this T-matrix, the differential cross section of the break-up process \(d + K \rightarrow \pi + \Sigma + N\) is calculated as:

\[
\frac{d\sigma}{dE_n} = (2\pi)^4 \frac{E_d E_K}{W P_K} m_N m_\Sigma m_\pi \sum_{ij} \left| <N\Sigma\pi|T(W)|\bar{K}> \right|^2,
\]
Table 1. Cutoff parameters of $\bar{K}N$-$\pi Y$ interaction.

|                | $\Lambda_{\bar{K}N}^{I=0}$ (MeV) | $\Lambda_{\pi\Sigma}^{I=0}$ (MeV) | $\Lambda_{\bar{K}N}^{I=1}$ (MeV) | $\Lambda_{\pi\Sigma}^{I=1}$ (MeV) | $\Lambda_{\pi\Lambda}^{I=1}$ (MeV) |
|----------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| E-dep          | 1000                            | 700                             | 725                             | 725                             | 725                             |
| E-indep        | 1000                            | 700                             | 920                             | 960                             | 640                             |

where $E_n$ is the neutron energy in the center-of-mass frame of $\pi \Sigma$ defined by

$$E_n = m_N + \frac{p_N^2}{2\eta_N}.$$  (5)

3. Models of Two-body Interaction

We use two-body $s$-wave meson-baryon interactions obtained from the leading order chiral Lagrangian,

$$L_{WT} = \frac{i}{8F_\pi^2} Tr(\bar{\psi}_B \gamma^\mu[[\phi, \partial_\mu \phi], \psi_B]).$$  (6)

Here, we examine two interaction models, both of which are derived from the above Lagrangian but have different off-shell behavior: one is the energy dependent model (E-dep),

$$V_{\alpha\beta}(q', q; E) = -\lambda_{\alpha\beta} \frac{1}{32\pi^2 F_\pi^2} \frac{2E - M_\alpha - M_\beta}{\sqrt{m_\alpha m_\beta}} \left( \frac{\Lambda^2_\alpha}{q'^2 + \Lambda^2_\alpha} \right)^2 \left( \frac{\Lambda^2_\beta}{q^2 + \Lambda^2_\beta} \right)^2.$$  (7)

while the other is the energy independent model (E-indep),

$$V_{\alpha\beta}(q', q) = -\lambda_{\alpha\beta} \frac{1}{32\pi^2 F_\pi^2} \frac{m_\alpha + m_\beta}{\sqrt{m_\alpha m_\beta}} \left( \frac{\Lambda^2_\alpha}{q'^2 + \Lambda^2_\alpha} \right)^2 \left( \frac{\Lambda^2_\beta}{q^2 + \Lambda^2_\beta} \right)^2.$$  (8)

Here, $m_\alpha$ ($M_\alpha$) is the meson (baryon) mass; $F_\pi$ is the pion decay constant; $\lambda_{\alpha\beta}$ are determined by the flavor SU(3) structure of the chiral Lagrangian.

In the derivation of these potentials we have assumed the so-called “on-shell factorization” [6] for Eq. (7) and $q, q' \ll M_\alpha$ for Eq. (8). The cutoff parameters $\Lambda$ are determined by fitting experimental data as shown in Table 1.

In the E-dep model, the $\bar{K}N$ amplitudes have two poles for $l = I = 0$ in the $\bar{K}N$-physical and $\pi\Sigma$-unphysical sheets, corresponding to those derived from the chiral unitary model [10]. On the other hand, the E-indep model has a single pole that corresponds to $\Lambda(1405)$. It is interesting to examine how this difference of the two-body interaction models appears in the neutron energy spectrum of the $K^-d \rightarrow \pi \Sigma n$ reaction.

4. Results and Discussion

In Fig 1, we present the differential cross section of $K^-d \rightarrow \pi \Sigma n$ [Eq. (4)] computed using the E-dep (a) and E-indep (b) models, respectively. We investigate the cross section for initial kaon momentum $p_{K^+} = 1000$ MeV in accordance with the planned J-PARC experiment [24].

Here, we decompose the isospin basis states into charge basis states using Clebsch-Gordan coefficients: the solid curve represents the $K^- + d \rightarrow \pi^+ + \Sigma^- + n$; the dashed curve refers to the $K^- + d \rightarrow \pi^- + \Sigma^+ + n$; the dotted curve represents the $K^- + d \rightarrow \pi^0 + \Sigma^0 + n$ reaction, respectively.
Figure 1. Differential cross section $d\sigma/dE_n$ for $K^-+d \rightarrow \pi^+\Sigma^+n$. The initial kaon momentum is set to $p_{K^-}^{lab} = 1000$ MeV. Panel (a): the E-dep model; Panel (b) the E-indep model. Solid curves: $\pi^+\Sigma^-n$; dashed curves: $\pi^-\Sigma^+n$; dotted curves: $\pi^0\Sigma^0n$ in the final state, respectively.

Figure 2. Contribution of each partial wave component to the differential cross section $d\sigma/dE_n$ for $d+K^- \rightarrow \pi^+\Sigma^-+n$. Panel (a): the E-dep model; Panel (b) the E-indep model. The thick solid curve represents the summation of total orbital angular momentum $L = 0$ to $14$; The thin solid curve represents $L = 0$ only; The dashed curve represents $L = 1$ only; The dotted curve represents $L = 2$ only; The dashed-dotted curve represents $L = 3$ only; The dashed-two-dotted curve represents $L = 4$ only, respectively. The initial kaon momentum is set to $p_{K^-}^{lab} = 1000$ MeV.

We subtract the neutron energy $E_{th}$ at which the amplitudes have the $\bar{K}N$ threshold cusp from the neutron energy $E_n$, i.e. $\bar{K}N$ threshold cusp shows up on the differential cross section at $E_n - E_{th} = 0$. Well defined maxima are found at $E_n \sim 17$-30 MeV for the E-dep model and a peak or bumps at $E_n \sim 32$-38 MeV for the E-indep model, depending in the charge combination of $\pi\Sigma$ in the final state. These peak and bump structures appear about 5 MeV higher in energy than the calculated binding energy of the $\Lambda(1405)$ ($E_B \sim 13$ MeV for the E-dep model and $E_B \sim 28$ MeV for the E-indep model). The magnitude of the differential cross section for the E-dep model is twice larger than that for the E-indep model, and the interference patterns with backgrounds are quite different between these two models. This clear difference in the differential cross section, arising from the model dependence of the two-body interactions, suggests that the $K^-d \rightarrow \pi\Sigma n$ reaction can indeed provide useful information on the $\bar{K}N$-$\pi Y$ system.

Finally, we examine the contribution of each partial wave component for total orbital angular
momentum $L$ to the differential cross section (Fig. 2). We conclude that the $s$-wave component is dominant in the low-energy region, but around the $K\bar{N}$ threshold higher partial waves such as the $p$-wave component become important.

In summary, we have calculated the differential cross sections for $K^- + d \rightarrow \pi + \Sigma + n$ reactions. We have found peak and bump structures in the neutron energy spectrum, and therefore it is possible to observe the signal of the $\Lambda(1405)$ resonance in the physical cross sections. We have also shown that the $K^- d \rightarrow \pi \Sigma n$ reactions are useful for judging existing dynamical models of $K\bar{N}-\pi \Sigma$ coupled systems with $\Lambda(1405)$. Further improvements of the present model to account for the neglected contributions in Eq. (3) and relativistic corrections are under investigation.

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