Interaction of nonrelativistic electrons with a standing electromagnetic wave is considered. The modulation amplitude of an electron current in the field of a standing or traveling electromagnetic wave is calculated in the quantum approach.

**INTRODUCTION**

Elastic scattering of nonrelativistic electrons in the field of a standing electromagnetic field was first considered by Kapitza and Dirac [1]. Their effect in the field of a strong standing electromagnetic wave was investigated by Fedorov [2]. Gaponov and Miller [3,4] studied the modulation and acceleration of nonrelativistic electrons in the field of counterpropagating waves that are shifted in frequency, and it was noted in [3] that the electron interaction with the standing wave is described by an effective potential that is quadratic in the electromagnetic-field strength.

The emission and absorption in an effective potential, however, were not considered in the cited studies. They were first investigated in [5,6], where the probability of this emission was estimated in first-order perturbation theory. It was shown that in this case the emission probability is proportional to $(V_0/E)^2$, where $V_0$ is the effective potential produced by the standing wave, and $E$ is the electron kinetic energy $E > V_0$.

The electron channeling was investigated in intense standing light wave in [7-9]. In [10] was considered quantum modulation of a current of slow electrons reflected from a vacuum-dielectric interface, and also by elastic reflection of electrons from the surface of a transparent single crystal (Bragg reflection). The TCE spectra and intensity were calculated. Other aspects could be found in [11-62].

One can expect the current-density modulation depth and the emission intensity to increase if the electrons are simultaneously acted upon by a spatially periodic field of a diffraction grating and by a traveling electromagnetic wave. The energy and momentum conservation laws are then simultaneously satisfied on account of the grating quasimomentum. The role of this grating can be assumed, for example, by a standing electromagnetic wave.
We consider in the present paper the modulation (classical and quantum) of an electron current in the field of a standing electromagnetic wave.

**BASIC RELATIONS**

Consider a nonrelativistic electron in the field of a standing linearly polarized wave. We define the wave field by a vector potential

\[ A_i = A_{0i}^z \sin \omega_i t \cos k_i y, \]  

(1)

where \( A_{0i}^z \) is the amplitude of the vector potential, the superscript \( z \) indicates the polarization direction, while \( \omega_i \) and \( k_i \) are respectively the frequency and field vector of waves counterpropagating along the \( y \) axis, and their superposition produces the standing wave.

Let the electron momentum be directed along the \( y \) axis. In this geometry, the interaction of the electron with the wave is due to the term \((eA_i)^z\)\(^2\), and the Schrödinger equation for the particle in the field of the wave (1) is

\[ i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \Delta \Psi + V_0 \cos^2 k_i y \Psi. \]  

(2)

Here \( V_0 = (eA_{0i}^z)^2 / 2mc^2 \) is the effective potential and \( A_{0i}^z = -\varepsilon_i \lambda_i \), where \( \varepsilon_i \) is the electromagnetic field and \( \lambda_i \) is the wavelength.

Note that the high-frequency \((2\omega_i)\) term in the interaction is determined by a phase transformation of a \( \Psi \) function and is disregarded hereafter.

We represent Eq. (2), accurate to an inessential phase shift, in the form

\[ i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial y^2} + \frac{V_0}{2} \cos qy \Psi, \]  

(3)

where \( q = 2k_i \). We add to the standing electromagnetic wave a traveling wave with a vector potential

\[ A_2 = A_{02}^z \sin(\omega_2 t - k_2 z), \]  

(4)
where $A_y^{\omega_2}$ is the amplitude of the vector potential, the superscript $y$ indicates the polarization direction, while $\omega_2$ and $k_2$ are respectively the frequency and the wave vector of the traveling wave $A_y^{\omega_2} = -e_0 c / \omega_2$.

The Schrödinger equation takes now the form

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial y^2} + i \frac{e\hbar}{mc} \sin(\omega_2 t - k_2 z) A_y^{\omega_2} \frac{\partial \Psi}{\partial y} + \frac{V_0}{2} \cos q y \Psi. \quad (5)$$

[We have neglected in (5) the high-frequency ($2 \omega_2$) term determined by a phase transformation of a Psi function and disregarded hereafter, since it contains the small parameter $eA_y^{\omega_2} / mc^2$ compared with the retained terms].

We seek the solution of (5) in the form

$$\Psi(y,t) = \sum_n a_n(y)\Psi_n(y) \exp[ink_2z - iE_n t / \hbar], \quad (6)$$

where $E_n = E_0 + n\hbar \omega_2$.

We seek the wave function $\Psi_n(y)$ in the semiclassical approximation

$$\Psi_n(y) = \exp\left[ \frac{i}{\hbar} \int (p_n^2 - 2meV_0 \cos qy')^{1/2} dy' \right]. \quad (7)$$

Here $p_n = (p_0^2 + 2mn\hbar \omega)^{1/2}$ and $p_0 = (2mE_0)^{1/2}$. We assume hereafter $\omega_2 = \omega$, $k_2 = k$.

The function $a_n(y)$ must be determined. We find it for $E >> V_0$ by using an approximation from [9-10]. In this approximation, first, $(p_n, p) >> p_n - p \pm q$, a condition met in the case of resonance $p_n - p \pm q = 0$, and second, $p_n = p$, which introduces in $a_n(y)$ an error order $\hbar \omega / E$.

Taking this into account, we get

$$\frac{da_n(y)}{dy} \frac{\partial \Psi_n}{\partial y} = -\frac{eE_0}{2\hbar \omega} \left( a_{n+1} \frac{\partial \Psi_{n+1}}{\partial y} - a_{n-1} \frac{\partial \Psi_{n-1}}{\partial y} \right). \quad (8)$$

Expanding $p_n$ in terms of the small parameter $V_0 / E_n$ ($p_0 \equiv p, E_0 \equiv E$) and using expression (7) for $\Psi_n$, we get
\[
\frac{da_n(y)}{dy} = -\frac{e\varepsilon_0}{2\hbar\omega} \left\{ a_{n+1} \exp \left[ \frac{i}{\hbar} (p_{n+1} - p_n) y + i \frac{V_0}{4E} \sin qy \right] \right. \\
\left. - a_{n-1} \exp \left[ \frac{i}{\hbar} (p_{n-1} - p_n) y - i \frac{V_0}{4E} \sin qy \right] \right\}.
\]

For \( V_0 / E_n << 1 \), Eq. (9) takes the simpler form

\[
\frac{da_n(y)}{dy} = -\frac{e\varepsilon_0}{8\hbar\omega} \frac{V_0}{E} \left\{ a_{n+1}(y) \exp \left[ \frac{i}{\hbar} (p_{n+1} - p_n - \hbar q) y \right] \\
- a_{n-1}(y) \exp \left[ \frac{i}{\hbar} (p_{n-1} - p_n + \hbar q) y \right] \right\}.
\]

We have thus obtained for \( a_n(y) \) the finite-difference differential equation (10).

**MODULATION OF THE ELECTRON-CURRENT DENSITY**

We shall solve (10) for the case \( n = +1 \). Assuming that \( |a_{\pm 1}| << 1 \) and \( a_0 \approx 1 \), we obtain, if the interaction is turned-on instantaneously (\( Lq >> 1 \))

\[
a_{\pm 1}(y) = \pm \frac{e\varepsilon_0}{8\hbar\omega} \frac{V_0}{E} \left[ \exp \left( \frac{i}{\hbar} (p - p_{\pm 1} \pm \hbar q) y \right) \right] dy' \\
= \frac{e\varepsilon_0}{4\hbar\omega} \frac{V_0}{E} \exp \left[ \frac{i}{\hbar} (p - p_{\pm 1} \pm \hbar q) \frac{y}{2} \right] \times \frac{\sin \left( \frac{(p - p_{\pm 1} \pm \hbar q) y}{2} \right)}{p - p_{\pm 1} \pm \hbar q}.
\]

It follows from (11) that \( a_1 \) increases with \( y \) if \( p_1 - p = q \) or if \( \hbar \omega << E \), \( \omega / v = q + \pi / L \), where \( v = p/m \) is the electron velocity and \( L = (8\pi \hbar p / p)(E / \hbar \omega)^2 \) is the modulation length \([7]\)'

The analogous condition for the increase of the amplitude \( a_{-1} \) is \( \omega / v = q - \pi / L \). The conditions for \( a_1 \) and \( a_{-1} \) to increase are thus incompatible. That is to say, if the electron beam were ideally monochromatic it would be possible to "tune" it only to absorption \( (a_1 \neq 0, a_{-1} = 0) \) or only to stimulated emission \( (a_1 = 0, a_{-1} \neq 0) \), depending on the relations between the parameters \( \omega, v, \) and \( q \). This asymmetry of the coefficients is a quantum effect manifested only when the length \( l \) of the intersection of the standing wave with the electron beam is much larger than \( L \), since \( L \) contains the Planck constant \( \hbar \). Let us calculate the electron-current density, using expressions (6) and (7) with \( n = \pm 1 \) and taking (11) into account:
\[
\begin{align*}
  j &= j_0 \left\{ 1 - \frac{e\varepsilon_1 V_0}{2\hbar \omega E} \left( \sin \frac{D_{+1}y/2}{D_{+1}} \cos(\varphi - D_{+1}y/2) - \frac{\sin D_{+1}y/2}{D_{+1}} \cos(\varphi - D_{+1}y/2) \right) \right\},
\end{align*}
\]

(12)

where \( j_0 \) is the incident-beam current density, defined by

\[
  j_0 = \frac{I_0}{\pi ab} \exp \left( -\frac{x^2}{a^2} - \frac{z^2}{b^2} \right).
\]

Here \( I_0 \) is the total beam current, and \( a \) and \( b \) are the effective beam widths in the \( x \) and \( z \) directions,

\[
  D_{+1} = \frac{\omega}{v} - q \frac{\pi}{L}, \quad \varphi = \omega t - qy - kz - \frac{V_0}{2E} \sin qy.
\]

A real electron beam, however, is not monochromatic, so that expression (12) for the current density must be averaged over the initial-electron-beam distribution function \( F(v) \) in the velocities. We put

\[
  F(v) = \frac{1}{\sqrt{\pi \Delta v}} \exp \left( -\frac{(v - v_0)^2}{(\Delta v)^2} \right),
\]

where \( \Delta v \) is the electron-velocity scatter and \( v_0 \) is the average electron velocity in the beam \((\Delta v \ll v_0)\). The averaged current can be written in the form

\[
  j = j_0 \left\{ 1 - \frac{e\varepsilon_1 V_0}{4\hbar \omega E} \Re \{ e^{i\varphi} (f_{+1}(y) - f_{-1}(y)) \} \right\}
\]

(13)

where

\[
  f_{+1} = 2 \int_{-\infty}^{\infty} F(v) \frac{\sin D_{+1}y/2}{D_{+1}} \exp (-iD_{+1}x/2) dv = \int_{-\infty}^{\infty} F(v) \exp (-iD_{+1}y') dy'dx
\]

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