EARLY OBJECTS IN THE COSMIC STRING THEORY WITH HOT DARK MATTER

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ABSTRACT

We study the accretion of hot dark matter onto moving cosmic string loops, using an adaptation of the Zeldovich approximation to HDM. We show that a large number of nonlinear objects of mass greater than $10^{12} M_{\odot}$, which could be the hosts of high redshift quasars, are formed by a redshift of $z = 4$.

Key words cosmic strings, cosmology: theory, large-scale structure of Universe

1 Introduction

Topological defect theories provide an alternative to inflation for explaining the origin of the primordial fluctuations which have grown into present-day structures through gravitational instability. Cosmic strings are one-dimensional topological defects possibly formed in a phase transition in the early Universe.

The observed presence of massive nonlinear structures at high redshifts of 3 to 4 has provided stringent constraints on models of structure formation containing hot dark matter (HDM), a candidate for which is a massive neutrino. This is due to the large thermal velocities of the HDM particles, which prevent the growth of perturbations on scales smaller than the free-streaming length. For inflationary models with adiabatic density fluctuations, recent data on the abundance of damped Lyman alpha absorption systems (DLAS) and on the quasar abundance has restricted the fraction of HDM to less than about 30% of the total dark matter present. The scenario of structure formation with cosmic strings is, however, viable even if the dark matter in the universe is hot, because cosmic strings, which provide the seeds for the density perturbations, survive the neutrino free-streaming. In this article we present work on the constraints imposed on the cosmic string theory by the abundance of high redshift quasars.

Searches for high redshift quasars have been going on for some time. The quasar luminosity function is observed to rise sharply as a function of redshift $z$ until $z \approx 2.5$. According to recent results from the Palomar grism survey by Schmidt et al. (1995), it peaks in the redshift interval $z \in [1.7, 2.7]$ and declines at higher redshifts. Quasars (QSO) are extremely luminous, and it is generally assumed that they are powered by accretion onto black holes. It is possible to estimate the mass of the host galaxy of the quasar as a function of its luminosity, assuming that the quasar luminosity corresponds to the Eddington luminosity of the black hole. For a quasar of absolute blue magnitude $M_B = -26$ and lifetime of $t_Q = 10^8$ yrs, the host galaxy mass can be estimated as:

$$M_G = c_1 10^{12} M_{\odot},$$

(1)
where $c_1$ is a constant which contains the uncertainties in relating blue magnitude to bolometric magnitude of quasars, in the baryon fraction of the Universe and in the fraction of baryons in the host galaxy able to form the compact central object (taken to be $10^{-2}$). The best estimate for $c_1$ is about 1. Models of structure formation have to pass the test of producing enough early objects of sufficiently large mass to host the observed quasars.

Besides the quasars themselves, absorption lines in their spectra due to intervening matter can be used to quantify the amount of matter in nonlinear structures at high redshifts. Based on the number density of absorption lines per frequency interval and on the column density calculated from individual absorption lines, the fraction of $\Omega$ in bound neutral gas (denoted by $\Omega_g$) can be estimated. For high-column density systems, the damped Ly-$\alpha$ systems (DLAS), recent observational results are that $\Omega_g(z) > 10^{-3}$ (2). In the most recent results by Storrie-Lombardi et al. (1996) there is evidence for a flattening of $\Omega_g$ at $z \sim 2$ and a possible turnover at $z \sim 3$. The corresponding value for $\Omega$ in bound matter is larger by a factor of $f_{b}^{-1}$, where $f_b$ is the fraction of bound matter which is baryonic, which is about 10% in a flat universe.

In the next section, we give a brief review of the cosmic string scenario of structure formation. Then we study the accretion of hot dark matter onto cosmic string loops, which seed large amplitude local density contrasts already at early times. We use the results to compute the number density of high redshift objects as a function of a parameter $\nu$ which determines the number density of loops in the scaling solution (see Section 2). We demonstrate that for realistic values of $\nu$, the number of massive nonlinear objects at redshifts $\leq 4$ satisfies the recent observational constraint of quasar abundances (see Figure 1). We also comment on the implications of (2). We consider a spatially flat Universe containing HDM and baryons. Units where $c = 1$, a Hubble constant of $H = 50h_{50}\text{km}s^{-1}\text{Mpc}^{-1}$ and a redshift at equal matter and radiation of $z_{eq} = 5750\Omega h_{50}^2$ are used.

2 Cosmic Strings and Structure Formation

Cosmic strings are one-dimensional topological defects which might have been formed in a phase transition in the very early Universe. They are produced for the same reasons as ordinary defects in condensed matter systems, such as vortex lines in superfluid helium, but their subsequent dynamics is governed by relativistic equations. In the simplest model, the symmetry breaking occurring in the phase transition is achieved via a two-component scalar Higgs field $\phi = (\phi_1, \phi_2)$ with symmetry-breaking potential $V(\phi) = \frac{1}{4}\lambda(\phi^2 - \eta^2)^2$, where $\eta$ is the energy scale of the phase transition, and $\lambda$ is the coupling constant. Topological considerations require cosmic strings to have no ends, they are either formed as infinitely long strings or as closed loops. These strings are very thin lines of trapped energy, the energy being the false vacuum energy $V(\phi = 0)$ which is trapped inside the defects after the rest of the Universe has evolved to the true ground state $|<\phi>| = \eta$ with $V(\phi) = 0$. The mass per unit length of the strings is given by the symmetry breaking scale $\eta$, $\mu \sim \eta^2$, and it is the only free parameter of the model (in practice uncertainties due to the complicated evolution of cosmic strings after the phase transition introduce extra phenomenological parameters, which are not fundamental, however). The energy in the defects acts gravitationally on surrounding matter and radiation, and thus provides the origin for the density perturbations, which evolve into today’s large-scale structure.

Most important for structure formation are the infinite strings. They are straight over distances increasing with time of one horizon distance $H^{-1}(t) \sim t$, and form an approximate
random walk on distances larger than that. In this way they can cause structures to be formed on the large scales observed today. The model makes predictions consistent with large-scale structure observations\cite{9,10} and CMB anisotropy measurements\cite{11}. An intriguing fact is that the normalization of the free parameter $\mu$ of the model to these observations, which gives $G\mu \approx 10^{-6}$, requires the phase transition giving rise to the defects to take place at a scale of $\eta \sim 10^{16}\,\text{GeV}$. This is just the scale of grand unification where the three coupling constants of the strong, weak and electromagnetic interactions meet, suggesting the appearance of new physics.

The network of cosmic strings is formed in a phase transition in the early universe about $10^{-35}\,\text{sec}$ after the Big Bang, long before the times relevant for structure formation. Therefore the string network has to be evolved over many orders of magnitude in time until $t_{eq}$ (and subsequently). This is very complicated numerically, and would be impossible to handle analytically, if it were not for the fact that the network of strings quickly evolves towards a scaling solution\cite{8} where the energy density in long strings remains a constant fraction of the total background energy density. This is achieved by intercommutations and self-intersections of the strings leading to the production of small loops, which then decay by emitting gravitational radiation. In this way, some of the energy input into the string network coming from the stretching of the strings due to the expansion of the universe is transferred to the background. According to the scaling solution, the string network looks the same at all times when all distances are scaled by the Hubble radius $H^{-1}(t)$, and this allows the extrapolation over such long time intervals. The scaling solution can be pictured as having a fixed number $M$ of long strings per Hubble volume at any given time, and a distribution of small loops.

Some of the parameters characterizing the string network besides the fundamental one $\mu$ are the long string velocities $v_s$ and initial loop velocities $v_l$, the average number $M$ of strings per Hubble volume, the amount of small-scale structure on the long strings quantified by $(\mu - T)$, where $T$ is the tension of the string, and the constants $\alpha$, $\beta$ and $\nu$ appearing in the next three equations. Small-scale structure is produced mainly when string segments self-interact and split off loops, a process which is characterized by the loop production rate and the size of the produced loops. According to the scaling solution, loops are formed with a radius which is a constant fraction of the horizon,

$$R_f(t) = \alpha t,$$

and length

$$l = \beta R.$$  

The number of loops per unit physical volume present at time $t$ with lengths in the interval between $l$ and $l + dl$ is

$$n(l,t)dt = \nu l^{-2}t^{-2}dt.$$  

Since loops decay by emitting gravitational radiation, there is a lower cutoff value of $l$ for the distribution (5) given by $l_{\text{min}} \sim G\mu t$, $G$ being Newton’s constant. Below $l_{\text{min}}$, the distribution $n(l,t)$ becomes constant.

Numerical simulations\cite{12} indicate that $\alpha \leq 10^{-2}$, $\beta \simeq 10$, and $M \sim 10$. From these values it follows that – unless $\nu$ is extremely large – most of the mass of the string network resides in long strings (where long strings are defined operationally as strings which are not loops with radius smaller than the Hubble radius).

Long strings accrete matter in the form of wakes behind them as they move through space\cite{13,14,15}. Spacetime around a long straight cosmic string can be pictured as locally flat, but with a deficit angle\cite{14} of $8\pi G\mu$. Therefore a string moving relativistically with velocity $v_s$ imparts velocity perturbations to surrounding matter towards the plane swept out by
the string, of magnitude

$$u = 4\pi G\mu \gamma_s v_s$$

creating overdensities in the form of a wake behind the string. If small-scale structure is present on the string, there is in addition a Newtonian force towards the string, proportional to $G(\mu - T)$, the strings move more slowly and accrete matter rather in the form of filaments than wakes.

For HDM, the first nonlinearities about wakes resulting from strings without any small-scale structure form only at late times, at a redshift of about 1 for $G\mu \approx 10^{-5}$, which is obtained when normalizing the model to COBE. Before this redshift, no nonlinearities form as a consequence of accretion onto a single uniform wake. Thus, in the cosmic string and hot dark matter theory, a different mechanism is required in order to explain the origin of high redshift objects. Possible mechanisms related to wakes are early structure formation at the crossing sites of different wakes, small-scale structure of the strings giving rise to wakes, and inhomogeneities inside of wakes. In this article, however, we will explore a different mechanism, namely, the accretion of hot dark matter onto loops.

In earlier work, the accretion of hot dark matter onto static cosmic string loops was studied. It was found that in spite of free streaming, the nonlinear structure seeded by a point mass grows from inside out, and that the first nonlinearities form early on (accretion onto string filaments proceeds similarly). Since loop accretion leads to nonlinear structures at high redshift, we will now investigate this mechanism in detail to see whether enough high redshift massive objects to satisfy the QSO constraints and (2) form.

## 3 Nonlinear mass accreted by a single static loop

We will use the modified Zeldovich approximation to study the accretion of hot dark matter onto moving string loops. The Zeldovich approximation is a first order Lagrangian perturbation theory technique in which the time evolution of the comoving displacement $\psi$ of a dark matter particle from the location of the seed perturbation is studied. The physical distance of a dark matter particle from the center of the cosmic string loop is written as

$$h(q, t) = a(t) [q - \psi(q, t)] .$$

For a point-like seed mass of magnitude $m$ (the string loop in our case) located at the comoving position $q' = 0$, the equation for $\psi$ is

$$\left( \frac{\partial^2}{\partial t^2} + 2 \frac{\dot{a}}{a} \frac{\partial}{\partial t} - 4\pi G\rho(t) \right) \psi(q, t) = \frac{Gm}{a^3q^2} .$$

(7)

This equation describes how as a consequence of the seed mass, the motion of the dark matter particles away from the seed (driven by the expansion of the Universe) is gradually slowed down. If the seed perturbation is created at time $t_i$ and the dark matter is cold, then the appropriate initial conditions for $\psi$ are

$$\psi(q, t_i) = \dot{\psi}(q, t_i) = 0 .$$

(8)

As formulated above, the Zel'dovich approximation only works for cold dark matter, particles with negligible thermal velocities. For HDM, free-streaming prevents the growth of perturbations on scales $q$ below the free-streaming length $\lambda_J(t)$, which is the mean comoving distance traveled by neutrinos in one expansion time

$$\lambda_J(t) = v(t)z(t)t ,$$
where \( v(t) \sim z(t) \) is the hot dark matter velocity. The free-streaming length decreases with time (in comoving coordinates) as \( t^{-1/3} \), so that a scale \( q \) which is initially affected by free-streaming, \( q < \lambda_J(t_i) \), becomes equal to \( \lambda_J \) at some later time \( t_s(q) \), and then remains above it and unaffected by free-streaming at later times.

A correct treatment of the effect of free-streaming requires the solution of the collisionless Boltzmann equation for the neutrino phase-space density. But a simple modification of the Zeldovich approximation for CDM gives the same results as the full treatment. For scales \( q < \lambda_J(t_i) \), Eq. 7 is only integrated from the time \( t_s(q) > t_i \) onwards instead of from \( t_i \), i.e. growth only starts at \( t_s(q) \), and the initial conditions are modified to \( \psi(q, t_i) = 0 \) and \( \dot{\psi}(q, t_s(q)) = 0 \). For scales where \( q > \lambda_J(t_i) \) initially the CDM formalism applies.

We can now define the mass that has gone nonlinear about a seed perturbation as the rest mass inside of the shell which is “turning around”, i.e. for which

\[
\dot{h}(q, t) = 0.
\]

In the case of CDM, this yields a nonlinear mass of

\[
M_{CDM}(t) = \frac{2}{5} \frac{m}{t_i} \left( \frac{t}{t_i} \right)^{2/3}, \tag{9}
\]

and for loops where the accretion is affected by free-streaming,

\[
M_{HDM}(t) = \frac{8}{125} \frac{m^3}{M_{eq}^2} \left( \frac{t}{t_{eq}} \right)^2, \tag{10}
\]

with \( M_{eq} = 2v_{eq}^3t_{eq}/(9G) \), \( v_{eq} \) is the HDM-particle velocity at \( t_{eq} \). From Eq. 5 and 10 we can determine, for any time \( t \), a mass \( M'(t) \) such that for all masses \( M > M'(t) \) accreted by any loop so far, accretion has not been affected by free-streaming, and Eq. 7 is applicable. For \( M < M'(t) \), expression 10 is valid.

## 4 Number density of quasar host galaxies

We can now compute the number density \( n_G(> M_G; t) \) in nonlinear objects heavier than \( M_G \) at high redshift \( z(t) \) in the cosmic string and hot dark matter model (in an analogous way the fraction \( \Omega_{nl} \) of the critical density in such objects can be calculated). By considering only the accretion onto string loops we will be underestimating these quantities.

It can be shown that in an HDM model string loops accrete matter independently, at least before the large-scale structure in wakes turns nonlinear at a redshift of about 1, i.e., later than the times of interest in this paper. Hence, the number density \( n(l, t) \) of loops of length \( l \) given by 5 can be combined with the mass \( M(l) \) accreted by an individual loop to give the mass function \( n(M, t) \). Here, \( n(M, t) \, dM \) is the number density of objects with mass in the interval between \( M \) and \( M + dM \) at time \( t \). This in turn determines the comoving density in objects of mass \( M > M_G \). Since the functional form of \( M(m) \) and hence \( M(l) \) changes at \( M = M' \), the functional forms of the mass function \( n(M) \) will be different above and below \( M' \). Thus

\[
n_G(> M_G; t) = z^{-3}(t) \left[ \int_{M_G}^{M'(t)} dM n_{H}(M) + \int_{M'(t)}^{M_G} dM n_{C}(M) \right]. \tag{11}
\]
Similarly, the fraction of the critical density in objects of mass greater than \( M_1 \), \( \Omega_{nl}(t) \) is

\[
\Omega_{nl}(t) = \int_{M_1}^{M(t)} dM n_H(M)M + \int_{M'(t)}^{M_2(t)} dM n_C(M)M \left[ \frac{6\pi G t^2}{M'(t)^3} \right].
\]

(12)

Note the upper mass cutoff \( M_2(t) \) introduced. It is imposed by our approximation of loops as point masses. For masses greater than \( M_2(t) \), this approximation breaks down, the responsible loops would have too large a radius to be able to accrete matter effectively. \( M_2(t) \) can be estimated by demanding that the mass \( M(t) \) accreted onto a loop exceed the mass in a sphere of radius equal to the loop radius at \( t_i \). The integral in Eq. (11) is dominated at \( M'(t) \), and can be approximated as

\[
n_{G}(> M_G; t) \sim n_C(M', t)M'z^{-3}(t) \simeq \frac{1}{5} \nu(\alpha\beta)^2 \frac{\mu^3}{M'(t)^3} z^{-3}(t).
\]

(13)

In Figure 1, the comoving number density of objects massive enough to be able to host quasars, \( n_{G}(> M_G; t) \), is plotted for \( G\mu = 10^{-6} \), \( v_{eq} = 0.1 \), and the values of the parameters \( \alpha = 10^{-2} \) and \( \beta = 10 \). The upper curve is for static loops, the lower one for initial loop velocities of \( v_i = 0.25 \). The effect of loop motion is to decrease the density of these massive objects, as discussed in the next paragraph. We have to compare our results for \( n_{G}(> M_G; t) \) with the number density of host galaxies, \( n_{G}^{obs} \), inferred from the observed quasar abundance \( n_Q \). Since we assumed a finite quasar lifetime \( t_Q \), the number of host galaxies \( n_{G}^{obs} \) is larger than \( n_Q \) by a factor of \( t_H/t_Q \), where \( t_H \) is the Hubble time at time \( t \),

\[
n_{G}^{obs} = \frac{t_H}{t_Q} n_Q.
\]

(14)

This is also plotted in Fig. 1.

The formalism presented above can be adapted to moving instead of static loops. Loop motion changes the geometry of the accreted object, it becomes more elongated and the transverse scale going nonlinear becomes smaller than for loops at rest. Because of this, free-streaming of the neutrinos can hinder the growth of perturbations for a longer time. The smallest mass accreted at time \( t \) which has been unaffected by free-streaming, \( M'(t) \), is increased compared to the case of static loops, thereby decreasing \( n_{G}(t) \) according to Eq. (13).

The result for the fraction of nonlinear mass accreted by moving loops is

\[
\Omega_{nl}(t) \sim 10^{-2} \nu(\alpha-2)(G\mu)^2v_i^{-1}z(t)^{-2}h_5^{4},
\]

(15)

where \( (G\mu)_{6} \) is defined as \( (G\mu)/10^{-6} \).

In Fig. 1 we have plotted the curves up to a redshift of 5. There is another issue to consider, however. Our expressions for \( n_{G} \) are only valid as long as \( M' < M_2(t) \), due to our approximation of loops as point sources. For loops with an initial velocity of \( v_i = 0.25 \), this condition is satisfied only up to a largest redshift of

\[
z_{max} = \frac{6}{5} \beta G\mu v_{eq} v_i^{-1} v_{eq}^{-1}
\]

(16)

which is equal to

\[
z_{max} = 3h_5^{2}
\]

(17)

for \( v_i = 0.25 \) and \( v_{eq} = 0.1 \). Beyond that redshift, the values of \( n_{G}(t) \) and \( \Omega_{nl} \) are suppressed beyond the results plotted in Fig. 1 and Eq. (13) since only the tail of the loop ensemble with velocities smaller than the mean velocity \( v_i = 0.25 \) manage to accrete a substantial amount of mass. It is interesting to note that, as mentioned in the introduction, Storrie-Lombardi et al. (1996) find a flattening of \( \Omega_g \) at \( z \sim 2 \) and a possible turnover at \( z \sim 3 \).
5 Discussion

We have studied the accretion of hot dark matter onto moving cosmic string loops and made use of the results to study early structure formation in the cosmic string plus HDM model. The loop accretion mechanism is able to generate nonlinear objects which could serve as the hosts of high redshift quasars much earlier than the time cosmic string wakes start becoming nonlinear (which for $G\mu = 10^{-6}$ and $v_s = 1/2$ occurs at a redshift of about 1).

The fraction $\Omega_{nl}(z)$ of the total mass accreted into nonlinear objects by string loops unfortunately depends very sensitively on $\alpha$ and $\nu$. On the other hand, this is not surprising since the power of the loop accretion mechanism depends on the number and initial sizes of the loops, and the scaling relation $\Omega_{nl} \sim \nu \alpha$ is what should be expected from physical considerations.

For the values $\nu = 1$ and $\alpha = -2$ which are indicated by recent cosmic string evolution simulations [12], we conclude that the loop accretion mechanism produces enough large mass protogalaxies to explain the observed abundance of $z \leq 4$ quasars (see Figure 1). Note that the amplitude of the predicted protogalaxy density curves depends sensitively on the parameters of the cosmic string scaling solution which are still poorly determined. Hence, the important result is that there are parameters for which the theory predicts a sufficient number of protogalaxies. Since not all protogalaxies will actually host quasars, and since the string parameters are still uncertain, it would be wrong to demand that the amplitude of the $n_G$ curve agree with that of the observed $n_Q$; rather, it should lie above the $n_Q$ curve.

It is more difficult to make definite conclusions regarding the abundance of damped Lyman alpha absorption systems. In the form of Eq. 15, the condition for the cosmic string loop accretion mechanism to be able to explain the data is also satisfied. However, Eq. 15 refers to the value of $\Omega$ in baryonic matter. The corresponding constraint on the total matter collapsed in structures associated with damped Lyman alpha systems is

$$\Omega_{DL}(z < 3) > f_b^{-1}10^{-3}$$

where $f_b$ is the local fraction of the mass in baryons. From Eq. 5 it follows that the above constraint is only marginally satisfied, and this only if the local baryon fraction $f_b$ exceeds the average value for the whole Universe of about $f_b = 0.1$. But in the cosmic string model with HDM we might expect $f_b$ in nonlinear objects to be enhanced over the average $f_b$ because after $t_{eq}$ baryons are able to cluster during the time that the HDM is prevented from accreting by the free streaming. Thus, cosmic strings may be able to restore agreement with (15) in a natural way. More calculations are required to resolve this issue.

Here we have only reported on the mechanism of forming early nonlinear objects through accretion onto string loops. Another mechanism is through small-scale structure on the long strings leading to the formation of filaments rather than wakes, which has recently been investigated [22]. It was found that this could be the most effective mechanism, and for the maximal possible amount of small-scale structure, $\Omega_{nl} \sim 1$ can be reached already at a redshift of 5. It is clearly important to determine the amount of small-scale structure present on strings.

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Figure 1: Comparison of the number density of host galaxies $n_G^{\text{obs}}$ ("*") marks) inferred from the observed number density of quasars brighter than $M_B = -26$ from the Schmidt et al. (1991,1995) survey ("x" marks) with the number density $n_G$ of protogalaxies of mass greater than $10^{12}M_\odot$ predicted in the cosmic string theory with HDM, for the parameters discussed in the text, and $h_{50} = 1$. The horizontal axis is the redshift.