Quantum computers could perform certain tasks which no classical computer can perform in acceptable times. Josephson junction circuits can serve as building blocks of quantum computers. We discuss and compare two designs, which employ charge or magnetic flux degrees of freedom to process quantum information. In both cases, elementary single-qubit and two-qubit logic gates can be performed by voltage or flux pulses. The coherence time is long enough to allow a series of such operations. We also discuss the read-out, i.e. a quantum measurement process. In the charge case it is accomplished by coupling a single-electron transistor to the qubit.

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1. INTRODUCTION

Quantum computers, if available, could perform certain operations in a massive parallel way, which would lead to an exponential speed-up as compared to the performance of classical computers. Furthermore, medium-size quantum information processing devices could be used in a number of applications such as, for instance, quantum communication and quantum cryptography. Several physical realizations of quantum bits, i.e. the elementary building blocks of quantum computers, have been proposed. The best studied ones are trapped ions, NMR in the liquid state and cavity QED. Here we discuss solid-state nano-electronic realizations. They appear promising since they can be scaled up to large numbers of qubits and are most easily
embedded in electronic circuits.

Josephson junction circuits are particularly suitable for quantum information processing, since they combine the intrinsic coherence of the superconducting state and the possibility to control the circuit dynamics by voltage and magnetic flux pulses. Their fabrication and manipulation are possible by present-day technologies. The dynamical variables in these circuits are the charges on the islands and the phases of the superconducting order parameter. Both are canonically conjugated, and Heisenberg’s uncertainty principle holds for them. Depending on the ratio of two characteristic energies – the typical charging energy, which favors well-defined charges, and the Josephson energy, which favors the phase degree of freedom – the charge or the flux can have a well-defined value, while the other fluctuates strongly. Either charge or phase degrees of freedom can be used to store and process quantum information. Here we describe charge and phase (flux) nano-electronic quantum bits and possible designs of the circuits in both cases.

The basic elements of a quantum computer are the qubits, i.e. two-state quantum systems which can be manipulated, separately for each qubit, by the control of the Hamiltonian. This allows performing single-bit logic gates. Additionally, one needs to be able to couple qubits in a controlled way to perform two-bit logic operations. The whole system should stay coherent for a sufficiently long time, that is, the coupling to the environment needs to be weak. Finally, to read-out the information about the quantum state of the system a quantum measurement should be performed. Below we discuss these steps for the relevant designs.

2. JOSEPHSON-JUNCTION QUANTUM BITS

2.1. Charge qubit

The simplest design of the Josephson qubit using the charge degree of freedom is presented in Fig. 1a. It consists of a superconducting electron box with a low-capacitance Josephson junction, with capacitance $C_J$ and Josephson energy $E_J$, biased by a voltage source through a gate capacitor $C_g$. If the superconducting gap is large enough then at low temperatures odd-parity states are forbidden and the charge on the superconducting island is a multiple of the Cooper-pair charge $2ne$, measured relative to the neutral state. The Hamiltonian of the system is then given by

$$
\mathcal{H} = \frac{(2ne - C_g V_x)^2}{2(C_g + C_J)} - E_J \cos \varphi ,
$$

(1)
where the phase difference across the junction, \( \varphi \), is canonically conjugated to \( n \). The charging energy, with scale \( E_C = e^2/2(C_g + C) \), is chosen to dominate over the Josephson term \( E_J \). In equilibrium at low temperature, \( k_B T \ll E_C \), the system is in the ground state, which away from certain degeneracy points is approximately the charge state with the minimal charging energy. Only near the voltages \( V_{\text{deg}} = (2n + 1)e/C_g \), where the states with \( n \) and \( n + 1 \) Cooper pairs on the island are degenerate, the weak Josephson coupling mixes the charge states strongly. Biased near these voltages the system reduces to a two-state problem, with the Hamiltonian in the basis of charge states \( |n\rangle \) and \( |n + 1\rangle \)

\[
\mathcal{H} = \frac{\varepsilon}{2} \hat{\sigma}_z + \frac{\Delta}{2} \hat{\sigma}_x.
\]

Here \( \varepsilon(V_x) = 2e^2C_{\text{J}}/(V_x - V_{\text{deg}}) \) denotes the difference in charging energy between two relevant charge states, the tunneling amplitude between the states is \( \Delta = -E_J \), and the capacitance of the qubit in the circuit is \( C_{\text{qb}} = C_{\text{J}}^{-1} + C_g^{-1} \).

The first term in the Hamiltonian (2) can be controlled through the gate voltage. The Josephson coupling in the second term can also be controlled if the junction is replaced by the dc-SQUID threaded by a magnetic flux \( \Phi_x \), as shown in Fig. 1b. Then the effective Josephson energy is \( E_J(\Phi_x) = 2E_0^0 \cos(2\pi\Phi_x/\Phi_0) \). With the control over these two parameters one can perform any single-bit logic operation. In the idle state between the operations one keeps the qubit at the degeneracy point \( V_x = V_{\text{deg}} \) and chooses \( \Phi_x = \Phi_0/2 \). Hence the Hamiltonian vanishes, \( \mathcal{H} = 0 \), and the qubit’s state does not evolve in time. To perform an operation, one can switch the flux to a different value for a finite time \( \tau \). The resulting change of the quantum state of the qubit is described by the unitary operator \( \exp(i\hat{\sigma}_x E_J\tau/2) \).
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voltage pulse results in another elementary operation, \( \exp(-i\hat{\sigma}_z \varepsilon(V_x)\tau/2) \). With a series of (no more than three) such operations with proper time spans any \( 2 \times 2 \) unitary operation, that is any single-qubit logic gate can be performed. The typical times of the operations are of order \( \hbar/\Delta \) or \( \hbar/\varepsilon \).

The manipulations of the state of the qubit are similar to the manipulations used in NMR experiments, and various techniques familiar from NMR applications can be employed. For instance, instead of rectangular voltage pulses one can use resonant ac-pulses to induce coherent transitions between the qubit’s states. Then the typical operation time is determined by the amplitude of the ac-pulse and can be optimized. It should be slow enough to make the control easy and fast enough to maximize the number of logic operations performed within the phase coherence time.

2.2. Flux qubit

A controllable two-state quantum system can be realized also in the opposite limit of dominating Josephson coupling. The simplest design is the rf-SQUID (see Fig. 1c) that is a superconducting loop interrupted by a Josephson junction. The phase difference across the junction, \( 2\pi\Phi/\Phi_0 \), is controlled by the flux \( \Phi \) in the loop, which fluctuates around the externally applied value \( \Phi_x \). With the Josephson, charging and magnetic contributions taken into account, the Hamiltonian of the system reads

\[
\mathcal{H} = -E_J \cos \left( \frac{2\pi \Phi}{\Phi_0} \right) + \frac{(\Phi - \Phi_x)^2}{2L} + \frac{Q^2}{2C_J}.
\]

(3)

Here \( L \) is the self-inductance of the loop and \( C_J \) the capacitance of the junction. The charge \( Q = -(i/\hbar)\partial/\partial\Phi \) on the leads is canonically conjugated to the flux \( \Phi \). If the self-inductance is large (\( \beta_L \equiv E_J/(\Phi_0^2/4\pi^2L) \) is slightly larger than 1) and the externally applied flux is close to \( \Phi_0/2 \), the two first terms in the Hamiltonian form a double-well potential near \( \Phi = \Phi_0/2 \). At low temperatures only the lowest states in the two wells contribute to the physics of the system. The reduced Hamiltonian of this two-state system is again given by Eq. (3), where now \( \varepsilon(\Phi_x) = 4\pi\sqrt{6(\beta_L - 1)}E_J(\Phi_x/\Phi_0 - 1/2) \) is the asymmetry of the double well potential, and \( \Delta \) is the tunneling amplitude between the wells. The latter can be controlled through the height of the barrier, which is determined by \( E_J \). This Josephson energy can be controlled, in turn, if the junction can be replaced by the dc-SQUID, as shown in Fig. 1d.

With two external parameters governing the Hamiltonian, elementary \( z \)- and \( x \)-rotations can be performed, as we have seen in the previous subsection. They can be driven either by switching the external flux for a finite time or by resonant pulses.

The rf-SQUID described above was discussed in connection with the
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‘macroscopic quantum coherence’, that is coherent oscillations of a quantum system between two macroscopically different states. However, the requirements of sufficiently large self-inductance and Josephson energy of the junction make the rf-SQUID very susceptible to external noise, and the experiments with the rf-SQUID were not successful so far. To overcome this difficulty Mooij et al. suggested to use a smaller superconducting loop with three junctions (one with controllable critical current). Then the double-well potential is formed by Josephson terms for the junctions, and lower critical currents can be used. As a result the system should stay coherent for a longer time. We shall discuss dephasing effects in Section 4.

3. COUPLING OF THE QUBITS

To perform a quantum computation with a register of qubits two-bit logic operations are necessary. For such an operation, which is a unitary transformation of the quantum state of two qubits, the couplings between the qubits should be controlled individually for each pair. In this section we discuss realization of such interactions for charge and flux qubits.

3.1. Controlled coupling of charge qubits

One possibility to couple charge qubits is to join them in parallel to a common inductor, as pictured in Fig. 2a. Then their dynamics is coupled to the oscillations in the LC-circuit, which is formed by the inductor and the capacitances of all N qubits in parallel. If the frequency $\omega_{LC} = 1/\sqrt{LNC_{qb}}$ of these oscillations is higher than typical qubit frequencies, the oscillatory degrees of freedom are not excited by the qubit manipulations, but still they provide the effective coupling between the qubits.

To clarify the physics of the coupling and to estimate its magnitude, we provide a simple derivation. The current through the inductor, $I$, is given by the contributions of all qubits, $I = \sum_i I_i$. In terms of the flux $\Phi$ through the inductor the Hamiltonian of the oscillations is expressed as $H_{osc} = \frac{\Phi^2}{2L} - I\Phi + \frac{Q^2}{2NC_{qb}}$ or

$$H_{osc} = \frac{(\Phi - LI)^2}{2L} + \frac{Q^2}{2NC_{qb}} - \frac{LI^2}{2}. \quad (4)$$

Here $Q = -(i/\hbar)\partial / \partial \Phi$ is the charge canonically conjugated to $\Phi$. If the time-evolution of the qubits is slow compared to the oscillations, we can use an adiabatic approximation and consider $I$ as constant, and the oscillator remains in its ground state. The energy of the ground state of the first two terms in (4), $\hbar \omega_{LC}/2$, does not depend on $I$, while the last
term provides the current-dependent correction. This correction describes
the effective coupling between the qubits, \(-L(\sum_i I_i)^2/2\). In the basis of
the qubits’ charge states individual current operators can be expressed as
\(I^i = (C_{qb}/C_3)(eE^i_J/h)\hat{\sigma}^i_y\), leading to the interaction term

\[
H_{\text{int}} = -\pi^2 \left(\frac{C_{qb}}{C_3}\right)^2 \sum_{i<j} \frac{E_J(\Phi^i_J)E_J(\Phi^j_J)}{\Phi_0/L} \hat{\sigma}^i_y \hat{\sigma}^j_y .
\]

This coupling can be controlled through external fluxes \(\Phi^i_J\), which bias the
SQUIDs of the qubits. Between the operations we keep all \(\Phi^i_J = \Phi_0/2\),
so that \(E^i_J = 0\) and the coupling is off. During single-bit operations we
switch on \(E^J_J\) for at most one qubit, so that the interaction term is still zero.
When a two-qubit operation is needed, the fluxes are changed for two qubits,
providing the interaction between them for a finite period. This interaction
leads to a non-trivial unitary transformation of the quantum state of two
chosen quantum bits, i.e. to a two-bit logic gate.

This design allows performing both single- and two-bit operations by
controlling the same external parameters (fluxes and voltages) as for indi-
vidual qubits.

### 3.2. Coupling of flux qubits

One possibility to couple flux qubits is to use a flux transformer\(\text{[8]}\) which
provides an inductive coupling between the qubits. Each qubit has two
loops, those of the rf- and dc-SQUIDs in the simplest rf-SQUID design. Any
loop of one qubit can be coupled to any loop of the other, giving rise to
\(\hat{\sigma}^i_z \hat{\sigma}^j_z\), \(\hat{\sigma}^i_z \hat{\sigma}^j_y\) or \(\hat{\sigma}^i_y \hat{\sigma}^j_y\) coupling terms. To turn off this coupling completely,
one would need to have an ideal switch in the flux transformer. This switch
is to be controlled by high-frequency pulses, and the related external circuit
can lead to decoherence effects. An alternative is to keep the interaction on
constantly and use ac driving pulses to induce coherent transitions between
the levels of the two-qubit system (cf. Refs. [1],[3]). A disadvantage of this
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approach is that permanent couplings lead to unwanted 2-qubit correlations between the logic operations.

Here we discuss another design of a many-qubit circuit, which allows to control interactions via flux sources of individual bits. The circuit, shown in Fig. 2b, is similar to the register of charge qubits discussed in the previous subsection. It includes an LC-circuit formed by a loop, with the self-inductance \( L_{\text{osc}} \), interrupted by a small capacitor \( C_{\text{osc}} \). The loop is coupled inductively to the set of flux qubits and mediates interaction between them. Following the derivation in Refs. 1, 9 one can find the effective coupling by integrating out the oscillations which are faster than the qubit’s dynamics. A simple way to obtain the coupling is to notice that in the low-capacitance limit, \( C_{\text{osc}} \to 0 \) (almost decoupled qubits) the effect of the qubits is to establish a voltage drop across the inductor \( L_{\text{osc}} \), given by \( \mathcal{V} = \sum_i M \dot{\Phi}_i / L \). Here for each qubit \( \Phi_i \) is the flux in its loop, \( L \) is the self-inductance of the loop and \( M \) is the mutual inductance with the loop of the LC-circuit. Then the Hamiltonian for the oscillator mode is \( \mathcal{H}_{\text{osc}} = \Phi^2 / 2L_{\text{osc}} + Q^2 / 2C_{\text{osc}} - VQ \), with \( Q \) being the charge conjugated to the flux \( \Phi \) through the LC-circuit. By similar arguments as in the previous subsection we find that the oscillator provides the inter-qubit interaction term \(-C_{\text{osc}}V^2/2\). In limit of the weak coupling to the LC-circuit, the time derivatives of the qubits’ fluxes are given by \( \dot{\Phi}_i = \frac{i}{\hbar} [\mathcal{H}_i, \Phi_i] = \delta \Phi_i \Delta_i \hat{\sigma}^i_+ / \hbar \) where \( \delta \Phi_i \) is the separation between two minima of the potential of the rf-SQUID. Hence, the interaction is given by

\[
\mathcal{H}_{\text{int}} = -\frac{\pi^2}{2L} \left( \frac{M}{L} \right)^2 \sum_{i<j} \frac{\delta \Phi_i \delta \Phi_j}{\Phi_0^2} \frac{\Delta_i \Delta_j}{e^2/C_{\text{osc}}} \hat{\sigma}^i_+ \hat{\sigma}^j_+ .
\]  

(6)

This interaction can be controlled through tunneling amplitudes \( \Delta_i \) of individual qubits. To turn off the interaction, one should put \( \Delta_i \) to zero. The amplitudes can be exponentially suppressed by increasing the barrier heights. (Note that in this case also the fluctuations of \( \Delta_i \) are exponentially suppressed, unlike for the charge qubits, where stronger fluctuations are present.) The absence of the interaction between the operations helps increasing the accuracy of the manipulations.

4. ENVIRONMENT AND DEPHASING

Due to the unavoidable coupling to environmental degrees of freedom the quantum state of the qubits gets entangled with the environment, which leads to dephasing effects. Charge qubits are sensitive to the electromagnetic fluctuations in the external circuit and in the substrate and to background charge fluctuations. Flux qubits are insensitive to the latter. Various sources of dissipation for flux qubits are discussed in Ref. 3. Here we estimate the
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effect of fluctuations in the external circuit.

The control over the voltage or flux requires coupling the quantum system to a dissipative external circuit, which introduces fluctuation effects. The voltage fluctuations in charge qubits or the flux fluctuations in the main loop of flux qubits are coupled to $\sigma_z$-degree of freedom. The external circuit can be parameterized by an effective impedance of the voltage (flux) source, which in turn can be modeled by an oscillator bath in the spirit of the Caldeira-Leggett model. The behavior of a two-state system coupled to an oscillator bath has been discussed in the context of the spin-boson model. The conclusion is that the decay of the off-diagonal elements of the density matrix of the qubit (dephasing) and the relaxation of the diagonal elements to their equilibrium values are described by the two time scales

$$\tau_\phi = \frac{\hbar}{\Delta E} \left( \frac{1}{2} \coth \frac{\Delta E}{2k_B T} \sin^2 \eta + \frac{2k_B T}{\Delta E} \cos^2 \eta \right)^{-1},$$

$$\tau_{\text{relax}} = \frac{\hbar}{\Delta E} \left( \coth \frac{\Delta E}{2k_B T} \sin^2 \eta \right)^{-1},$$

respectively. Here $\hbar/\Delta E$ gives the typical operation time (cf. Section 2) in terms of the level spacing $\Delta E = \sqrt{\epsilon^2 + \Delta^2}$ of the qubit, $T$ is the temperature, and $\tan \eta \equiv \Delta/\epsilon$.

The dimensionless parameter $\gamma$ describes the strength of the dissipation. For charge qubits it is determined by the resistance $R_V$ of the voltage source in units of the large quantum resistance $R_K = \hbar/e^2 \approx 25.8k\Omega$,

$$\gamma_V = \frac{1}{4\pi} \frac{R_K}{R_V} \left( \frac{C_J}{C_{qb}} \right)^2.$$ (9)

A small gate capacitance $C_g \approx C_{qb} \ll C_J$ provides a weak coupling between qubit and environment.

For flux qubits, $\gamma$ is fixed by the impedance $R_I$ of the current source in the input loop, which provides the flux bias,

$$\gamma_I = a \frac{R_I}{R_K} \left( \frac{\Phi_0^2}{4\pi^2 M} \right)^2.$$ (10)

Here $M$ is the mutual inductance of the input loop and the qubit’s loop. The numerical prefactor in Eq. (10) is $a = \pi/(6(\beta_L - 1))$ for the rf-SQUID and is also of the order of unity for the design of Ref. 5. The dephasing is slow for small loops and junctions with low critical currents. Indeed, the argument in the bracket in (10) is proportional to $\Phi_0^2/4\pi^2 LE_J$. While this quantity
should be slightly smaller than one for the rf-SQUID, it can be much larger for the design of Mooij et al.\textsuperscript{5} corresponding to slower dephasing.

The qubit is equivalent to a spin-1/2 particle in the external magnetic field $\varepsilon \hat{z} + \Delta \hat{x}$, while the environment produces an additional fluctuating field in $\hat{z}$-direction. The component of the fluctuating field orthogonal to the external field (with the magnitude proportional to $\sin \eta$) induces transitions between the eigenlevels, while the longitudinal component ($\propto \cos \eta$) leads to random fluctuations of eigenenergies. Only the former process leads to relaxation (8), while both contribute to the dephasing (7). This observation explains the $\eta$-dependence of the decoherence rates (7), (8). The relaxation rate is slow for low-impedance voltage sources and high-impedance current sources. Weak coupling to the external circuit, i.e., small $C_g$ or small $M$, helps further maintaining the coherence.

The effect of fluctuations of the $\sigma_x$-term in the Hamiltonian (in the flux circuit of the dc-SQUID-loop which controls the Josephson coupling in both charge and flux qubits) can be described in a similar way.\textsuperscript{2} Because of the different direction of the fluctuating effective ‘magnetic field’, $\sin \eta$ and $\cos \eta$ should be interchanged in those terms. The effect of these terms is relatively weak for the operation regimes discussed in Refs. \textsuperscript{2} and \textsuperscript{5}.

5. READ-OUT: QUANTUM MEASUREMENT

To complete the quantum computation, one needs to read out the information about the quantum state of the qubits, that is to perform a quantum measurement. For charge qubits this can be accomplished by coupling the qubit capacitively to a single-electron transistor\textsuperscript{12} as shown in Fig. 3. During the computation the SET is kept in the off-state $V_{tr} = 0$, with no dissipative current and no additional decoherence. The only effect of the transistor is a renormalization of the capacitances in the qubit’s circuit. To perform the measurement, the transport voltage is switched to a sufficiently high value and the dissipative current starts to flow. The value of the current in the circuit of the transistor is very sensitive to the charge on the island of the qubit. Monitoring the current, one can extract from the data information about the state of the qubit.

To study this quantum measurement process, we analyzed the time evolution of the density matrix of the coupled system of the qubit and the SET.\textsuperscript{12,9} The system is characterized by three energy scales: the typical Coulomb energy of the transistor, $E_{set}$, the charging energy of the qubit, $\varepsilon(V_x)$, and the Coulomb interaction between the charges of the qubit and the middle island of the SET, $E_{int}$. We choose $E_{set}$ to be the largest energy scale, $E_{set} \gg \varepsilon \gg E_{int}$. The Josephson coupling $E_J(\Phi_x)$ is switched to a small
Fig. 3. Read-out is accomplished by coupling a single-electron transistor capacitively to the qubit.

(or zero) value before the measurement. The transport voltage should be large enough, of order $E_{\text{set}}/e$, to overcome the Coulomb energy gap between two charge states of the SET. The value of this gap is slightly different for different states of the qubit, which leads to different tunneling rates $\Gamma \pm \delta \Gamma$ through the transistor, with $\Gamma$ of order $2\pi \alpha E_{\text{set}}$ and $\delta \Gamma$ of order $2\pi \alpha E_{\text{int}}$. Here $2\pi \alpha \equiv R_K/4\pi^2 R_T \ll 1$ is the dimensionless tunneling conductance of the junctions of the transistor, of the same order in both junctions. For definiteness we consider the limit $E_{\text{int}} \ll \Gamma \ll \Delta E$, where essential features of the evolution can be seen, although the derivation can be performed in a wider parameter range.

To describe the system we single out the most important degrees of freedom: the qubit’s degree of freedom, the charge $Ne$ on the middle island of the SET and the charge $me$ which passed through the transistor after the transport voltage is turned on, that is the time integral of the current. The microscopic degrees of freedom can be traced out, and a closed set of equations can be derived for the elements of the density matrix, $\rho_{ij}(N, m)$ which are diagonal in $N$ and $m$.

The evolution of the system is characterized by three time scales. In the first stage the random tunneling processes in the SET lead to the loss of the phase coherence of the qubit. The off-diagonal elements of the reduced $2 \times 2$ density matrix of the qubit, $\sum_{N,m} \rho_{ij}^\phi(N, m)$, decay to zero on a short time scale of order $\tau_\phi \approx \Gamma/E_{\text{int}}^2$.

At a later stage the information about the qubit’s state can be deduced from the current. The dynamics of the current in the SET can be described by the probability distribution of the number of electrons $m$ which have passed through the transistor: $P(m, t) \equiv \sum_{i,N} \rho_{ii}^\dagger(N, m)$. At the start of the measurement $t = 0$ no electrons have tunneled, and $P(m, t) = \delta_{m,0}$. Then the current begins to flow, and the peak shifts in the direction of positive
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Fig. 4. The evolution of the probability distribution $p(m/t, t)$ for the values of the average current in the SET. The initial state of the qubit is $a = \sqrt{1/4}$, $b = \sqrt{3/4}$. a. $t < t_{\text{meas}}$: a very broad peak due to shot noise. b. $t_{\text{meas}} < t < t_{\text{mix}}$: two peaks separate. c-e. $t \sim t_{\text{mix}}$ and $t > t_{\text{mix}}$: a plateau grows between the two peaks, erasing the information about the initial state of the qubit.

$m$ and widens due to shot noise effects. Since the two charge states of the qubit correspond to different net currents in the SET, after some time the peak splits into two. The separation of the peak centers grows linearly with time, $2\delta\Gamma t$, while their width grows as $\sqrt{\Gamma t}$. Therefore they separate after time $t_{\text{meas}} \approx \Gamma/\delta\Gamma^2$. The weights of the peaks after the separation are given by $|a|^2$ and $|b|^2$ for the initial state of the qubit $a|0\rangle + b|1\rangle$, and a good quantum measurement is realized if $m$ is measured after $t_{\text{meas}}$. As expected, the measurement time is longer than the dephasing time.

This ideal picture gets more complicated at even longer times. At finite, or fluctuating, $E_J$ the eigenstates of the qubit are different for different charges $N_e$ of the SET, and the measurement induces transitions between them. After a long mixing time $t_{\text{mix}} \approx \Delta E^4/(E^2_{\text{int}} E^2_J \Gamma)$ the diagonal elements of the density matrix of the qubit become equal to 1/2, and the information about their initial values is lost completely. At the same time $P(m, t)$ develops a plateau between the two peaks. The weights of the peaks decay exponentially. At longer times $t \gg t_{\text{mix}}$ the plateau transforms into a peak around $m = \Gamma t$. This distribution contains no information about the initial state of the qubit anymore.

The time evolution of $P(m, t)$ is depicted in Fig. 4. For convenience the probability distribution $p(m/t, t)$ of possible values of the quantity $m/t$, the current in the SET averaged over time $t$, is shown. It is related to $P(m, t)$ by $p(m,t) = t \cdot P(\Delta t, t)$ where the prefactor $t$ ensures the normalization of the distribution. If the current is averaged over short times $t < t_{\text{meas}}$, Fig. 4a, the shot noise does not allow to distinguish between close values of the current corresponding to different qubit states (very broad peak around $e\Gamma$). At longer times $t_{\text{meas}} < t < t_{\text{mix}}$, Fig. 4b, two peaks are formed around $e\Gamma_\pm = e(\Gamma \pm \delta\Gamma)$, and the measurement can be performed. At $t \sim t_{\text{mix}}$ a plateau starts to grow between the peaks, and at the very long times $t \gg t_{\text{mix}}$ the plateau takes over and transforms into a narrow peak around $e\Gamma$. 

6. CONCLUSION

We have discussed two possible designs of nano-electronic quantum bits, based on Josephson junction circuits in the charge and flux regime. In both cases the quantum dynamics of the qubits can be controlled through voltages and fluxes (currents). We suggest a way to couple flux qubits which is a dual analog of the approach suggested earlier for charge qubits. Compared to the proposal of Mooij et al., it has the advantage that it allows to control the inter-qubit interaction without introducing new links between the qubits and the external circuit. Apart from that we estimated the dephasing times due to coupling to electromagnetic fluctuations in the external circuit. This contribution has not been discussed earlier for flux qubits. The quantum measurement of the state was discussed in detail for charge qubits. We provided an analysis of the long-time evolution of the probability distribution of current values in the single-electron transistor and of the qubit’s density matrix. The design of a quantum measurement circuit for flux qubits requires further investigation.

Earlier experiments with the superconducting box have demonstrated the quantum nature of the two-state system and superpositions of different charge states. Recently Nakamura et al. observed time-resolved coherent oscillations of the quantum state of a charge qubit. Their experiment is the first observation of the macroscopic quantum coherence (MQC) effect and realizes a single-bit operation (corresponding to an $x$-rotation). The observed phase coherence time of several nanoseconds was shorter than the value $\sim 100$ ns, predicted by (7), (8), (9) because of the use of a simple (but efficient) measuring procedure. Further development of the measurement apparatus (cf. Ref. 7) can render the coherence time longer. In flux systems, the attempts to observe MQC with the rf-SQUID were not successful, but recent developments should make it possible in the near future.

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