Dynamics of Nonlinear Excitation of the High-Order Mode in a Single-Mode Step-Index Optical Fiber

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Abstract. This work is concerned with approximate model of higher-order mode nonlinear excitation in a singlemode silica optical fiber. We present some results of simulation for step-index optical fiber under femtosecond optical pulse launching, which confirm ability of relatively stable higher-order mode excitation in such singlemode optical fiber over sufficiently narrow range of launched optical power variation.

1. Introduction

It is well known that optical fibers may operate in a single-mode, multi-mode or few-mode regimes. It depends on ratio between wavelength and optical fiber core diameter as well as optical emission launching conditions and refractive index profile height and form [1 – 5]. For example, in singlemode optical fibers higher-order modes do not satisfy cut-off condition over corresponding operation wavelength band, and here only one fundamental mode is supported under linear propagation conditions, while dozens of guided modes can propagate in large core multimode optical fibers since they satisfy cut-off condition over operation wavelength band. However when laser-based optical emission is launched to the core of multimode optical fiber, only a limited number of modes from the total quality is excited, and optical fiber operates in a few-mode regime.

During high power optical emission propagates over optical fiber, its refractive index profile form is able to change due to nonlinearity factors [6, 7]. In fact this leads to changing of launching conditions and new guided more higher-order modes can be excited and supported by optical fiber in comparison with linear regime [8, 9]. Moreover even for singlemode optical fiber a higher-order mode can satisfy cut-off condition in operation wavelength under nonlinear excitation.

Recently usually nonlinear mode excitation was researched for analysis of fiber optic amplifier mode instability, where optical fiber refractive index profile changes due to heating effects. However the great achievements in the field of femtosecond laser technique permits to consider ability of mode staff management by Kerr nonlinearity [8, 12]. In previous works [13, 14] we demonstrated ability of formulation of necessary conditions for higher-order mode $LP_{11}$ excitation in silica step-index singlemode optical fiber by nonlinear Kerr effect before irreversible changes of optical fiber refractive index would start up to its destruction. However cut-off condition is just necessary but no sufficient for nonlinear mode excitation. Research of this effect requires analysis of process dynamics. This work presents simulation results of nonlinear higher-order mode excitation process dynamics in silica step-index singlemode optical fiber.
2. Generalized solution for numerical simulation of optical pulse propagation along singlemode and multimode optical fibers

Femtosecond optical pulse propagation along multimode optical fiber is usually described by system of coupled nonlinear Schrödinger equations (CNLSE), each of them can be represented in the following form [6, 15–16]:

\[
\frac{\partial A_m}{\partial z} + \beta_{m1} \frac{\partial A_m}{\partial \tau} - j \frac{\beta_{m2}}{2} \frac{\partial^2 A_m}{\partial \tau^2} + \frac{\beta_{m3}}{6} \frac{\partial^3 A_m}{\partial \tau^3} + \frac{\alpha_m}{2} A_m - j \gamma \cdot \sum C_{n,m} |A_n|^2 A_m +
\]

\[+ j B_{n,m} \frac{\partial |A_n|^2}{\partial \tau} - T_R B_{n,m} \frac{\partial |A_n|^2}{\partial m} \bigg] = 0,
\]

where \( A_m \) is optical pulse spatial envelope transferred by \( m \)-th mode in optical fiber; \( \beta_{m1} \) is \( m \)-th mode delay parameter; \( \beta_{m2} \) and \( \beta_{m3} \) are \( m \)-th mode chromatic dispersion parameter; \( \alpha_m \) is \( m \)-th mode second-order chromatic dispersion parameter; \( \alpha_m \) is \( m \)-th mode attenuation parameter; \( C_{n,m} \) and \( B_{n,m} \) are mode coupling coefficients; \( \omega_0 \) is the carrier frequency; \( T_R \) is Raman time constant; \( \tau \) is normalized time; \( z \) is the distance.

In general equation system (1) is widely used for admissible simulation of optical pulse propagation with the pulse width more 10 fs. However under carrier frequency \( \omega_0 \) corresponding to wavelength \( \lambda=800 \) nm typical Raman time constant is \( T_R=5 \) fs [6, 15–17], while for 10-fs and wider optical pulses the equation system (1) is simplified that the last two terms of sum can be neglected [6, 15–18].

Usually for numerical simulation of optical pulse nonlinear propagation the equation system (1) is solved by well known Split-Step Fourier Method (SSFM) [6, 15–18]. Here CNLSE (1) is rewritten in the following form:

\[
\frac{\partial A_m}{\partial z} = \big( \hat{D} + \hat{N} \big) A_m,
\]

where \( \hat{D} \) and \( \hat{N} \) are linear and nonlinear operators taking into account linear and nonlinear optical pulse distortions. Symmetric split-step scheme is usually used with second-order accuracy over integration step \( \hbar \) [15–18]. Solution of (2) is written in the following form:

\[
A_m(z + \hbar, \tau) \approx \exp \left( \frac{\hbar}{2} \hat{D} \right) \exp \left( \hbar \hat{N} \right) \exp \left( \frac{\hbar}{2} \hat{D} \right) A_m(z, \tau),
\]

Linear part \( \exp \left( \hbar \hat{D} \right) / 2 \) is performed in frequency space. It seems clear that the number of equations in CNLSE is defined by the number of modes propagating over researched optical fiber.

3. Higher-order mode nonlinear excitation conditions for singlemode optical fiber

Singlemode optical fibers support only one fundamental mode. For weakly guiding optical fibers it is the mode \( LP_{01} \). Therefore usually under low power of transmitted optical emission that corresponds to linear regime, singlemode optical fiber provides propagation for only one fundamental mode over operation wavelength band. In this case optical signal is transferred by two orthogonal polarization modes which degenerate to one fundamental mode \( LP_{01} \) under ideal symmetry of weakly guiding optical fiber.
In works [13, 14] it was demonstrated that under sufficiently great power level of optical emission more than some threshold value $P_{th}$, when refractive index profile changing due to Kerr nonlinearity should be taken into account and can not be neglected, necessary conditions for higher-order mode $LP_{jj}$ excitation in singlemode step-index optical fibers are provided. Moreover for femtosecond pulses this power level is less than critical power levels $P_{cr}$, when refractive index irreversibly changes occur. It was shown in works [13, 14] the threshold power depends on normalized frequency linearly.

4. Model of pulse propagation along singlemode step-index optical fiber under taking into account higher-order mode nonlinear excitation

In this work simulation of higher-order mode nonlinear excitation dynamics in singlemode optical fiber is based on CNLSE (1) solution by SSFM. Here we utilize parameter

$$\bar{P}_L = \left| A_{01}^2 \right| + \left| A_{11}^2 \right|,$$

where $A_{0j}$ and $A_{1j}$ are envelopes of optical pulse mode components $LP_{0j}$ and $LP_{1j}$. It is supposed that under $P_L<P_{th}$ optical pulse propagation process is described by only one equation written in relation to envelope $A_{0j}$ for the fundamental mode. $P_L>P_{th}$ corresponds to conditions for mode $LP_{1j}$ excitation, and pulse propagation is represented by two CNLSEs for modes $LP_{0j}$ and $LP_{1j}$, while during $P_L>P_{cr}$ optical pulse does not propagate and $A_{0j}=0, A_{1j}=0$ due to optical fiber destruction under such great values of optical power.

During simulation we use algorithm proposed in works [20 – 23] that involves to symmetric split-step scheme of SSFM an additional analysis of researched optical fiber with new refractive index profile deformed due to nonlinearity before nonlinear operator execution. Here parameters of modes $LP_{0j}$ and $LP_{1j}$ are computed by Gaussian approximation. We utilized analytical expressions derived for those modes by taking into account Kerr nonlinearity and represented in works [24 – 26] and developed following model described by following expressions:

$$\left\{ \frac{\partial A_{01}}{\partial z} \hat{D}_0 A_{01} + \hat{N}_{p,01} A_{01} + \hat{N}_{N,01} A_{01} = 0; \quad \bar{P}_L < P_{th}; \right\}$$

$$\left\{ \frac{\partial A_{11}}{\partial z} \hat{D}_1 A_{11} + \hat{N}_{p,11} A_{11} + \hat{N}_{N,11} (A_{01}, A_{11}) = 0; \quad \bar{P}_L > P_{th}; \right\}$$

where

\[
\begin{align*}
\hat{D}A_{01} &= \beta_{01,1} \frac{\partial A_{01}}{\partial \tau} - j \beta_{01,2} \frac{1}{2} \frac{\partial^2 A_{01}}{\partial \tau^2} + \beta_{01,3} \frac{1}{6} \frac{\partial^3 A_{01}}{\partial \tau^3} + \alpha_{01} A_{01}; \\
\hat{D}A_{11} &= \beta_{11,1} \frac{\partial A_{11}}{\partial \tau} - j \beta_{11,2} \frac{1}{2} \frac{\partial^2 A_{11}}{\partial \tau^2} + \beta_{11,3} \frac{1}{6} \frac{\partial^3 A_{11}}{\partial \tau^3} + \alpha_{11} A_{11}; \\
\hat{N}_{01} A_{01} &= j \gamma^2 \left[ A_{01}^2 + j \frac{1}{\omega_0} A_{01} \frac{\partial \left( A_{01}^2 A_{01} \right)}{\partial \tau} - T_k \frac{\partial A_{01}^2}{\partial \tau} \right] A_{01}^2/4 R_{01}; \\
\end{align*}
\]
\[
\hat{N}_{11}A_{11} = j\gamma \left[ A_{11}^2 + j \frac{1}{\omega_0 a_0} \frac{\partial}{\partial \tau} \frac{\partial}{\partial \tau} \right] A_{01}\frac{A_{11}}{4R_{11}^2};
\]
\[
\hat{N}_{01}A_{11} = j\gamma \left[ A_{01}^2 + j \frac{1}{\omega_0 a_1} \frac{\partial}{\partial \tau} \frac{\partial}{\partial \tau} \right] A_{01}\frac{A_{11}}{4R_{01}^2};
\]
\[
\hat{N}_{11}A_{11} = j\gamma \left[ A_{11}^2 + j \frac{1}{\omega_0 a_{11}} \frac{\partial}{\partial \tau} \frac{\partial}{\partial \tau} \right] A_{11}\frac{A_{11}}{4R_{11}^2}.
\]

Here we take into account that mode field radiuses are twice as much as equivalent radiuses of Laguerre-Gaussian modes \(R_{01}\) и \(R_{11}\) [27]. Linear operators are performed in frequency space by fast direct and inverse Fourier transformation.

5. Results of simulation of higher-order mode nonlinear excitation dynamics in a singlemode optical fiber
We performed simulation of \(LP_{11}\) mode nonlinear excitation for step-index silica optical fiber with numerical aperture \(NA=0.16\), GeO\(_2\)-doped core with diameter 8.3 \(\mu m\) and pure SiO\(_2\) outer solid cladding. Total researched optical fiber length is 10 mm. We considered 12-fs optical pulse generating by laser source operating at wavelength \(\lambda=800\) nm injecting to optical fiber core under overfilled launching conditions. Power loss occurring during optical signal propagation over optical fiber was neglected. Example of \(LP_{01}\) and \(LP_{11}\) mode evolution is represents on Fig. 1.

![Figure 1. LP\(_{01}\) and LP\(_{11}\) mode evolution in singlemode optical fiber](image)

Here injected power was 1.1 MW under threshold power 1.0 MW, and we have got stable \(LP_{11}\) mode component under described conditions. However this effect was achieved in sufficiently narrow range of optical power variations. Thus during injected optical power reducing \(LP_{11}\) mode nonlinear excitation conditions were violated, while optical emission intensity quickly achieved threshold value under following improvement of launched optical power. Determination of allowed optical power value range for stable higher-order mode nonlinear excitation requires more detailed researching. Represented results undoubtedly are preliminary due to proposed model is approximate, and it need
experimental verification. However computed results of simulation provide to suppose ability of stable higher-order mode nonlinear excitation in a singlemode optical fiber.

6. Conclusion
We described approximate model for simulation of higher-order mode nonlinear excitation in a singlemode step-index optical fiber and presented some results of this process dynamics computation. In spite of simulation results are preliminary, they demonstrate ability of stable higher-order mode nonlinear excitation in a singlemode optical fiber during femtosecond optical pulse transmission over sufficiently narrow range of injected optical power variation.

7. References
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