Arch dam static and dynamic modelling with discrete elements

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Abstract. The safety of concrete arch dams depends to a large extent on the soundness of the foundation rock mass. The large arch dams presently under design and construction apply very significant loads to their abutments and foundations. The assessment of the foundation stability requires the analysis of the potential failure mechanisms defined by the major rock mass discontinuities and joint sets. For dams located in areas of intense seismic activity, the analysis needs to take into account the potential effects of earthquakes, not only on the concrete arch, but also in the nearby rock discontinuities. The discrete element method is a major numerical tools for continuum modelling, allowing the analysis of block systems defined by multiple joint planes. In the paper, the key issues involved in its application to arch dam foundations are reviewed. In particular, the representation of the discontinuities in models intended for failure analysis is discussed. The evaluation of safety factors under static loading is addressed. The framework for seismic analysis of arch dams is also examined, namely the dynamic boundary conditions. Options to analyse the dynamic dam-water interaction and model calibration issues are discussed, based on practical examples.

1. Introduction
Concrete arch dams apply significant loads on the foundation rock mass. The safety of these large structures has always posed a major challenge in rock engineering. The accidents of Malpasset and Vajont have led to important research studies which greatly advanced the discipline of rock mechanics. Presently, new arch dams are being designed and constructed with unprecedented dimensions, requiring further research and developments. The advances in rock mechanics, including field and laboratory experiments and numerical models, provide tools to handle these challenges [1].

Safety assessment of arch dam foundations requires the analysis of potential failure modes defined by the rock mass discontinuities, including major faults and joint sets. Numerical models are presently available to perform those analyses and have been extensively used in dam studies. The Finite Element Method (FEM) provides one approach, in which joint or interface elements are used to represent the non-elastic discontinuities [2]. Alternatively, the Discrete Element Method (DEM) represents the rock mass as an assembly of blocks in interaction along the discontinuities [3]. The DEM class of methods for modelling discontinuous media includes many different techniques (Distinct elements, DDA, AEM, etc.). Both DEM and FEM can address the static and dynamic behavior of arch dam foundations and evaluate their safety under operating conditions and under the action of large earthquakes.

In this paper, the DEM approach will be discussed. This method is particularly appropriate for complex jointing patterns, and typically allows the analysis to continue into the large displacement
A feature common to many DEM codes is an explicit time stepping algorithm, which is employed to reach quasi-static solutions by means of the dynamic relaxation method [4], and also to perform dynamic analysis in the time domain. A key factor in dam foundation studies is the way to represent the rock discontinuities in the numerical model, to be examined in the following section. The major issues raised by static analysis and by seismic loading will then be addressed.

2. Representation of rock discontinuities in numerical models

The classical methods developed by Pierre Londe for safety evaluation of dam foundations in the 1960s were based on a necessarily simplified geometry of the rock discontinuities, since these problems had to be solved by analytical or graphical methods. The software packages presently available for rock mechanics applications are capable of a much more realistic simulation of the rock mass structure. Methods to create Discrete Fracture Networks (DFN), allowing the various joint sets to be generated according to their statistical parameters, are frequently used in the analysis of fluid flow in rock masses [5]. However, for failure analysis, these have to be used with caution, since for non-persistent joint sets, the development of a collapse mechanism may require the breakage of rock bridges. The numerical simulation of fracture propagation in rock remains a difficult problem and an active research topic, different approaches being available. In DEM models, rock blocks may be assumed rigid or deformable. For dam foundation analysis, the latter are preferred as they take into the rock mass deformability in a more detailed manner, for example in the case of heterogeneity throughout the valley banks. Deformable block material may be assumed elastic, the most common case, or may be governed by elasto-plastic or damage constitutive models. But, these may not be appropriate to simulate the localized breakage of rock bridges, which might lead to unsafe results. Therefore, a conservative approach is often used, in which the faults and joints are represented by extended planes, so that kinematically possible failure mechanisms may be created without requiring block breakage [3].

There are particular cases in which the non-persistence of joints may be taken into account in a simplified manner. An example is shown in Figure 1, involving sliding failure under a gravity dam [6]. A set of horizontal joints is included in the model, with rock bridges simulated by randomly placed bonded segments. Breakage of these cohesive contacts allows global failure along one or more horizontal joint planes. For complex joint patterns, it may be more difficult to ensure that kinematically viable failure modes are represented by in the numerical model. This approach also requires a fine discretization, which may become impractical for large 3D models.

![Figure 1](image)

**Figure 1.** (left) Block model with a non-persistent horizontal joint set (rock bridges shown as thick lines); (right) Sliding after failure of some rock bridges.

Research on numerical models of rock fracture has progressed substantially, namely employing the discrete element method [7]. Bonded-Particle Models (BPM), employing systems of circular or spherical particles, have been extensively used in the simulation of lab tests on rock samples and also in studies of fracture around underground excavations [8]. More recently, Bonded-Block Models (BBM) have been developed, using polygonal and polyhedral blocks [9]. This concept is based on a block system in which the potential fracture paths are defined by randomly generated discontinuities, according to the rock mass jointing statistics. However, these detailed models are computationally
demanding and are mostly used in research studies of rock specimens or rock mass regions of limited extent [10].

In practical applications of DEM to dam foundations, a simplified representation of the discontinuities is advisable. It must be realistic, in accordance with the field characterization, yet conservative, following the classical design rules of dam engineering. Major faults that are likely to be involved in the failure mode have to be included in the model, at their known locations. Then, joints belonging to the various sets are added, with sufficient extension so that the model contemplates all the wedge volumes that can potentially slide and trigger structural collapse. An example of this approach was employed in the studies of Foz Tua dam, a 108 m high concrete arch dam, with a crest length of 275 m, recently built in northern Portugal [11]. At the design stage, an initial block model had been created with the code 3DEC [12], including the known faults and a number of joints of each of the main sets. The correct joint orientation was used, but the location was chosen arbitrarily, in order to create a simplified block system. During construction, after the excavation of the arch foundation, the traces of the rock joints were identified (Figure 2). Then, it was possible to update the numerical model geometry, using these traces and extending the joint planes according to the joint orientation, leading to the geometry in Figure 3. The resulting block system is plotted in Figure 4, where the element mesh inside the deformable blocks is also shown. In this model, another conservative option was utilized, which consisted in not extending the block system upstream. This effectively neglects the contribution of the upstream rock mass, corresponding to the assumption that sub-vertical joints in the vicinity of the upstream edge of the dam foundation may open under hydrostatic loads. Mechanical properties are given in Table 1.

| Table 1. Mechanical properties of numerical model |
|-----------------------------------------------|
| Concrete | Young’s modulus (GPa) | 20 | Poisson’s ratio | 0.2 | Density (kg/m$^3$) | 2400 |
| Rock blocks | Young’s modulus (GPa) | 24 | Poisson’s ratio | 0.2 | Density (kg/m$^3$) | 2650 |
| Rock joints | Normal stiffness (GPa/m) | 10 | Shear stiffness (GPa/m) | 2 | Friction angle | 38* |

**Figure 2.** Geological mapping of Foz Tua dam excavation surface.  
**Figure 3.** Downstream view of the main joints included in the 3DEC model.
3. Analysis under static loads

The static analysis of a concrete arch dam starts with the dam construction stage, in which the gravity loads are applied to the independent cantilever blocks. Once the construction stage is completed, the vertical contraction joints are assigned their mechanical properties. In the case studies reported herein, a Coulomb friction model was used. The next stage involves the application of the water pressure loads from the reservoir on the dam upstream face. Then, the joint water pressures are applied in all rock discontinuities and the dam-rock interface, either by running a fluid flow analysis, or by using the design water pressure assumptions. The fluid flow analysis is more realistic, but it always requires some calibration of joint conductivity parameters, which can only be suitably performed after impounding the reservoir, when hydraulic monitoring data becomes available. At the design stage, often simplified conservative water pressure fields are assumed in the joints and dam-rock interface, consistently with the expected effects of the grout and drainage curtains.

Procedures for the evaluation of safety against collapse modes involving the foundation rock mass depend on design practices and national regulatory codes. The method used here is based on the reduction of the strength of rock discontinuities by a progressively increasing factor until failure takes place. In practice, the process is stopped when the dam displacements become too large for the concrete structure to accommodate. In the present studies, rock discontinuities may be assumed to have no tensile or cohesive strength, only frictional shear strength was assumed. The strength reduction factor is applied to the tangent of the friction angle. However, it is possible to consider cohesive strength for faults, and reduce it by the same or a different factor. In this study, the dam-rock interface was also assumed cohesionless, as required by the national regulations for ultimate failure scenarios. Figure 5 shows the result of the joint strength reduction for the Foz Tua dam model [11]. The contours of the magnitude of displacements for a reduction factor of 1.5 are plotted, the maximum value being about 50 mm. From this deformation pattern, we see that the major slip events are associated with the sub-vertical joints in the right bank. No tendency for the development of wedge failure modes is detected at the dam abutments. Therefore, it can be concluded that the orientation of the rock mass joints do not present a significant risk for this dam. The monitoring of dam displacements after the first filling of the reservoir confirms a very satisfactory performance.

![Figure 4. 3DEC block model of arch dam and adjacent rock mass.](image1)

![Figure 5. Displacement magnitude contours (in m) for a joint strength reduction factor RF=1.5.](image2)

4. Analysis under seismic loading

4.1. Modelling assumptions for time domain dynamic analysis

The numerical models for seismic analysis of arch dams have particular requirements to ensure that the dynamic action is properly applied. Time domain analysis is employed, in order to take into account the nonlinear behavior of the structure or the rock mass. The infinite nature of the rock mass
has to be simulated using a finite numerical model with appropriate boundary conditions. This issue, common to FEM and DEM models, has been the topic of significant research [13]. In the 3DEC code, the framework for seismic analysis is based on the free-field model [12], illustrated in Figure 6. It is assumed that the seismic action is represented by a wave propagating vertically upwards from the model base. This dynamic wave may have 3 displacement components. It is applied at the base in the form of a dynamic stress record superimposed on a viscous boundary condition in order to retain the non-reflecting boundary character. The free-field calculations, corresponding to the wave vertical propagation in a horizontally infinite medium, provide the lateral boundary conditions. They give the dynamic stresses to be applied to the main model. There are four 2D free-field meshes on the lateral boundaries of the model, which in turn require their own 1D free-field meshes at the corners. The free-field meshes are linked to the main model by viscous boundaries to ensure that the outgoing waves are not reflected. These viscous elements act on the difference between the free-field velocities and the main model velocities [12].

The model in Figure 6 shows the model of Baixo Sabor dam, a 123 m high arch, with a crest length of 505 m, and a reservoir volume of 1.1x10⁹ m³, built in northeastern Portugal [3]. In dynamic analysis, computation run times are typically long, so numerical models should focus on the fundamental issues. In this case, only the discontinuities of the right-bank had unfavorable orientations. Therefore, they were the only ones included in the model, the remaining rock mass being represented as an equivalent continuum. It should be noted that for dynamic analysis, the dam vertical joints between the cantilevers have to be included, with no tension behavior, as they may open for intense seismic loading. No damping was assumed in the rock mass, the only dissipation mechanism being the frictional sliding on the joints. On the other hand, for the dam, a value of 10% of critical damping at the fundamental frequency was used, in order to account for intense shaking. The joint behavior followed a Mohr-Coulomb model, with only frictional shear strength. An important issue relates to the water pressures in the rock joints during the earthquake, a topic for which limited experimental data exists [14]. Therefore, in this model, the simplest option was adopted, in which the steady-state joint water pressures were assumed unchanged during the dynamic run. The dynamic dam-reservoir interaction issue is discussed in the following section.

**Figure 6.** Model for seismic analysis of arch dam and discontinuous rock mass, with free-field boundary meshes.

**Figure 7.** Model for simulation of forced vibration tests, using a fluid element mesh in the reservoir.

4.2. Dam-reservoir dynamic interaction

For a full reservoir, the interaction of the reservoir water and the concrete arch has a major effect in the response. The classical approach is to use the Westergaard added-mass formulation. However, for realistic results the theoretical masses predicted by this formula need to be scaled. When there are experimental measurements of the dam natural frequencies, the scaling factors can be readily calculated. For the Baixo Sabor dam (Figure 6), the natural frequencies were determined by in situ
forced vibration tests [15], as well as by ambient vibration measurements [16]. The experimental results obtained by the two methods showed a good agreement, and also allowed a validation of the numerical models. In particular, a comparison was made of the Westergaard added-mass model with a more elaborate model using a fluid element representation of the reservoir. The latter model, shown in Figure 7, was used herein only to simulate the forced vibrations tests, during which no nonlinear behaviour was expected, so it was possible to use an equivalent continuum model for the rock mass. As the dynamic action of the vibrator was applied directly on the dam, the outer model boundaries were simply non-reflecting viscous boundaries. In these analyses, the natural frequencies were obtained by applying identification techniques to the numerical output [15].

Table 2 shows the MAC (Modal Assurance Criterion) of the experimental mode shapes of the forced vibration tests and the numerical configurations produced by the fluid element model. The match is very good for the first 6 modes [15]. In order to match the first frequency, the dam concrete Young’s modulus was taken as 36 GPa, a value that is consistent with the experimental test data. In order to test the Westergaard formulation, the model in Figure 7 was changed by removing the fluid elements and including the added-masses in the dam upstream face. The comparison of the natural frequencies is presented in Table 3. In order to match the first frequency, a factor of 0.53 was applied to the added masses. The practical difficulty with the Westergaard model is to determine this factor. In the present case, given the experimental data, the added-masses can be easily scaled. As we see in the table, the added-mass model values for the other frequencies have some errors, with a maximum of about 8% in the first six modes, which may be acceptable for practical purposes. When the model is intended to calculate the concrete dam stresses, then the fluid element formulation is surely more accurate. However, in the present study in which we focus on the behavior of the foundation discontinuities, the added-mass approach was considered to be a sufficiently good approximation. Therefore, it was the added-mass approach that was used in the discontinuous foundation model of Figure 6.

Table 2. MAC matrix for the modal configurations determined experimentally by forced vibration tests and numerically by a model with a fluid element reservoir.

| Numerical Modes (Hz) | Experimental modes (Hz) | 2.44 | 2.57 | 3.34 | 3.93 | 4.78 | 5.37 |
|----------------------|-------------------------|------|------|------|------|------|------|
| 2.44                 | 0.96                    | 0.00 | 0.00 | 0.01 | 0.03 | 0.00 | 0.01 |
| 2.56                 | 0.02                    | 1.00 | 0.01 | 0.04 | 0.01 | 0.00 | 0.03 |
| 3.31                 | 0.07                    | 0.02 | 0.96 | 0.01 | 0.00 | 0.02 |
| 3.92                 | 0.03                    | 0.02 | 0.02 | 0.90 | 0.01 | 0.07 |
| 4.75                 | 0.02                    | 0.00 | 0.02 | 0.03 | 0.88 | 0.02 |
| 5.44                 | 0.01                    | 0.07 | 0.00 | 0.00 | 0.11 | 0.84 |

Table 3. Comparison of the natural frequencies determined with the fluid element reservoir model and the Westergaard added-mass model.

| Modes | Frequencies (Hz) | Difference |
|-------|------------------|------------|
|       | Fluid elements   | Westergaard| (Hz) | (%)  |
| 1     | 2.44             | 2.44       | 0.00 | 0.0% |
| 2     | 2.56             | 2.67       | 0.11 | 4.3% |
| 3     | 3.31             | 3.56       | 0.25 | 7.6% |
| 4     | 3.92             | 4.03       | 0.11 | 2.8% |
| 5     | 4.75             | 4.81       | 0.06 | 1.3% |
| 6     | 5.44             | 5.49       | 0.05 | 1.0% |
4.3. Dynamic slip on rock discontinuities

A series of dynamic simulations were performed with the 3DEC model of Baixo Sabor dam, Figure 6, considering earthquake time records generated for the dam location which were scaled to peak ground accelerations (PGA) ranging from 0.1g to 0.5g [3]. The contours of residual displacement after the dynamic run with the highest intensity are shown in Figure 8. Shearing on the sub-horizontal joint set can be noticed, with small wedges formed by this set and the sub-vertical set. The maximum shear displacement is 79 mm, quite acceptable for the large seismic input, while the maximum displacement at the top of the dam blocks is 118 mm. Figure 9 plots evolution of the maximum joint shear in the model for increasing values of PGA. It can be seen that, in this case, significant slip only takes place for the higher intensity actions.

![Figure 8](image8.png)

**Figure 8.** Permanent displacements (in m) after seismic action with PGA of 0.5g.

![Figure 9](image9.png)

**Figure 9.** Maximum joint shear displacement vs. PGA.

5. Concluding remarks

Presently, numerical models have gained extensive capabilities to analyze the performance of concrete dam foundations, by investigating in detail the key role that rock mass discontinuities play in governing potential deformation and failure modes. Discrete elements are one of the powerful tools available, within a larger group of numerical models of discontinuous media. Two examples were presented of their application to safety evaluation of arch dam foundations considering operating loads and seismic action scenarios.

In rock engineering there are many sources of uncertainty, regarding the geological setting, the rock mass mechanical properties, the hydraulic behavior of rock joints, and so on. Prediction of earthquake response, in particular, remains a challenge, so that more investigation is required, namely involving field studies of the dynamic characteristics of large dam-reservoir-foundation systems. While expanding research and computer resources are allowing the use of larger and more complex models, it is necessary to supply them with accurate field data. More complex geometries or constitutive assumptions always require more and better input data. Therefore, the engineer's judgement remains important in the application of the numerical models so that reliable measures of safety can be obtained.

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