The $N/D$ method with non-perturbative left-hand-cut discontinuity and the $^1S_0$ $NN$ partial wave

D.R. Entem

Grupo de Física Nuclear and IUFFyM, Universidad de Salamanca, E-37008 Salamanca, Spain

J.A. Oller

Departamento de Física, Universidad de Murcia, E-30071 Murcia, Spain

In this letter we deduce an integral equation that allows to calculate the exact left-hand-cut discontinuity for an uncoupled S-wave partial-wave amplitude in potential scattering for a given finite-range potential. The results obtained from the $N/D$ method for the partial-wave amplitude are rigorous, since now the discontinuities along the left-hand cut and right-hand cut are exactly known. This solves the open question with respect to the $N/D$ method and the effect on the final result of the non-perturbative iterative diagrams in the evaluation of $\Delta(A)$. A big advantage of the method is that short-range physics (corresponding to integrated out degrees of freedom within low-energy Effective Field Theory) does not contribute to $\Delta(A)$ and it manifests through the extra subtractions that are implemented within the method. We show the equivalence of the $N/D$ method and the Lippmann-Schwinger (LS) equation for a nonsingular $^1S_0$ $NN$ potential (Yukawa potential). The equivalence between the $N/D$ method with one extra subtraction and the LS equation renormalized with one counter term or with subtractive renormalization also holds for the singular attractive higher-order ChPT potentials. The $N/D$ method also allows to evaluate partial-wave amplitudes with a higher number of extra subtractions, that we fix in terms of shape parameters within the effective range expansion. The result at NNLO shows that the $^1S_0$ phase shifts might be accurately described once the electromagnetic and charge symmetry breaking terms are included. Our present results can be extended to higher partial waves as well as to coupled channel scattering.

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the Weinberg counting. The traditional shortcut of the N/D method up to now is that for a given potential the discontinuity $\Delta(A)$ is not known a priori, and the approximation typically made is to calculate it perturbatively. This approach has been pursued by Oller et al using Chiral Perturbation Theory (ChPT) up to NNLO and reproducing low-energy $NN$ phase shifts with good precision [34–37].

The N/D method [33] writes down a $NN$ partial wave as $T(A) = N(A)/D(A)$ and such that $N(A)$ and $D(A)$ have only LHC and right-hand cut (RHC), respectively. From elastic unitarity and the definition of $\Delta(A)$ as the LHC discontinuity of $T(A)$, $D(A)$ and $N(A)$ satisfy along their respective cuts:

$$\text{Im}D(A) = -\rho(A)N(A), A > 0$$
$$\text{Im}N(A) = \Delta(A)D(A), A < L$$

where $L = -m_N^2/4$ and $\rho(A) = M_N\sqrt{A}/4\pi$ is the phase space, with $M_N$ the nucleon mass. In terms of the imaginary parts given in Eq. (1) one can write down in a standard way dispersion relations for the functions $D(A)$ and $N(A)$. The general form of these equations with an arbitrary number of subtractions can be found in Eq.(14) of Ref. [35]. Here we use the N/D method with the number of subtractions necessary to fit the effective range expansion

$$k\cot\delta = -\frac{1}{a} + \frac{1}{2}kr^2 + \sum_{i=2} v_i k^{2i}$$

at a certain order. As in the N/D method the functions $N(A)$ and $D(A)$ are defined up to a constant we have to perform at least one subtraction, which is usually done for $D(A)$ fixing $D(0) = 1$. The equations are:

$$D(A) = 1 + iM_N\sqrt{A}/4\pi^2 \int_{-\infty}^L d\omega_L \frac{D(\omega_L)\Delta(\omega_L)}{(\sqrt{\omega_L} + \sqrt{A})\sqrt{\omega_L}}$$
$$N(A) = \frac{1}{\pi} \int_{-\infty}^L d\omega_L \frac{D(\omega_L)\Delta(\omega_L)}{(\omega_L - A)}$$

We will call this approximation, which has no free parameters, the regular case (or $N/D_{01}$), since for regular interactions the solutions are completely fixed by the potential and must coincide with the ones given by the LS equation.

If we want to fit the scattering length we can take an additional subtraction in $N(A)$ and we obtain the integral equation for the $N/D_{11}$ case, which reads

$$D(A) = 1 + i\omega\sqrt{A} + i\frac{AM_N}{4\pi^2} \int_{-\infty}^L d\omega_L \frac{D(\omega_L)\Delta(\omega_L)}{\omega_L((\sqrt{\omega_L} + \sqrt{A}))}$$

where we see the independence on the subtraction point and the explicit dependence on the scattering length $a$.

If we perform an additional subtraction in $D(A)$ we can fix also the effective range (case $N/D_{12}$). Finally, by taking an additional subtraction in $N(A)$ we fix $\nu_2$ (case $N/D_{22}$). Explicit expressions for these cases can be found in Refs. [35, 38].

It is worth mentioning here two points already discussed in Sec.II.A of Ref. [34]. First, the number of subtractions in $N(A)$ is always less or equal to that taken in $D(A)$ because otherwise some of the RHC integrals in $D(A)$ would be divergent. Second, any possible Castillejo-Dalitz-Dyson pole [39] in the $D(A)$ function can be removed by taking one more subtraction at the same time in $N(A)$ and $D(A)$.

One-pion exchange (OPE) at leading in ChPT for the singlet $^1S_0$ partial wave can be written as

$$V(p',p) = \frac{g_A^2 m_n^2}{4f^2} \frac{m_p^2}{2p'p} Q_0(z) + C_0$$

where $Q_0(z) = \frac{1}{4}\log((z+1)/(z-1))$ and $z = (p'^2 + p^2 + m_N^2)/(2p'p)$. Here the delta-like contribution of the OPE potential has been included in the contact term $C_0$. The latter does not generate contribution to $\Delta(A)$ (as well as any other polynomial that were added to the right-hand side of Eq. (4) corresponding to local terms). The contribution of OPE to $\Delta(A)$ stems entirely from the function $Q_0(z)$ evaluated on-shell, and is given by

$$\Delta_{1\pi}(A) = \theta(L-A)\frac{g_A^2 m_n^2}{4f^2}$$

The once iterated OPE contribution using dimensional regularization has been evaluated by the Munich group [6] and its contribution to the LHC discontinuity, already used within the N/D method in Refs. [34, 35], is
\[
\Delta_{2\pi}(A) = \theta(4L - A) \left( \frac{g_A^2 m^2_\pi}{16 f^2_\pi} \right)^2 \frac{M_N}{A \sqrt{-A}} \log \left( \frac{2\sqrt{-A}}{m_\pi} - 1 \right)
\]  
(5)

We have evaluated the contribution of twice and three-times iterated OPE finding

\[
\Delta_{3\pi}(A) = \theta(9L - A) \left( \frac{g_A^2 m^2_\pi}{4 f^2_\pi} \right)^3 \left( \frac{M_N}{4\pi} \right)^2 \frac{\pi}{4A} \int_{m_\pi}^{2\sqrt{-A} - m_\pi} d\mu_1 \frac{1}{\mu_1 (2\sqrt{-A} - \mu_1)}
\]
\[
\times \theta(\mu_1 - 2m_\pi) \int_{m_\pi}^{\mu_1 - m_\pi} d\mu_2 \frac{1}{\mu_2 (2\sqrt{-A} - \mu_2)}
\]
\[
\Delta_{4\pi}(A) = \theta(16L - A) \left( \frac{g_A^2 m^2_\pi}{4 f^2_\pi} \right)^4 \left( \frac{M_N}{4\pi} \right)^3 \frac{\pi}{4A} \int_{3m_\pi}^{2\sqrt{-A} - m_\pi} d\mu_1 \frac{1}{\mu_1 (2\sqrt{-A} - \mu_1)}
\]
\[
\times \theta(\mu_1 - 3m_\pi) \int_{m_\pi}^{\mu_1 - m_\pi} d\mu_2 \frac{1}{\mu_2 (2\sqrt{-A} - \mu_2)} \theta(\mu_2 - 2m_\pi) \int_{m_\pi}^{\mu_2 - m_\pi} d\mu_3 \frac{1}{\mu_3 (2\sqrt{-A} - \mu_3)}.
\]  
(6)

\[
\Delta_{n\pi}(A) = \theta(n^2 L - A) \left( \frac{g_A^2 m^2_\pi}{4 f^2_\pi} \right)^n \left( \frac{M_N}{4\pi} \right)^{n-1} \frac{\pi}{4A} \prod_{j=1}^{n-1} \theta(\mu_{j-1} - (n + 1 - j)m_\pi) \int_{(n-j)m_\pi}^{\mu_{j-1} - m_\pi} d\mu_j \frac{1}{\mu_j (2\sqrt{-A} - \mu_j)}
\]  
(7)

For a diagram with \( n \) pions we infer from the structure of the evaluated \( \Delta_{m\pi}(A), 1 \leq m \leq 4 \), that

\[
\Delta_{n\pi}(A) = \theta(n^2 L - A) \left( \frac{g_A^2 m^2_\pi}{4 f^2_\pi} \right)^n \left( \frac{M_N}{4\pi} \right)^{n-1} \frac{\pi}{4A} \prod_{j=1}^{n-1} \theta(\mu_{j-1} - (n + 1 - j)m_\pi) \int_{(n-j)m_\pi}^{\mu_{j-1} - m_\pi} d\mu_j \frac{1}{\mu_j (2\sqrt{-A} - \mu_j)}
\]  
(8)

with \( \mu_0 = 2\sqrt{-A} \). This is the formal solution of the IE

\[
\hat{\Delta}(A, \mu) = \Delta_{1\pi}(A) + \left( \frac{M_N}{\pi^2} \right) \theta(\mu - 2m_\pi) \int_{m_\pi}^{\mu - m_\pi} d\mu \frac{\Delta_{1\pi}(A) \Delta(A, \mu)}{\mu (2\sqrt{-A} - \mu)}
\]  
(9)

such that \( \Delta(A) = \hat{\Delta}(A, 2\sqrt{-A}) \). This IE gives the contribution to the LHC discontinuity in the \( ^1S_0 \) partial wave of the iterated OPE contribution when solved for \( \mu \in [m_\pi, 2\sqrt{-A}] \). Notice that, the denominator in the IE never vanishes and the limits of the integration are finite for \( \sqrt{-A} < \infty \). A rigorous proof of Eq. (9) and the generalization to other partial waves and interactions will be given in a forthcoming paper [38]. An important point to notice here is that in the general case, even for singular interactions, \( \Delta(A) \) is finite. The divergent part of the LS equation in the LHC, if present, would affect only the real part of \( T \), which is not an input for the \( N/D \) method.

When \( \sqrt{-A} \gg m_\pi \) we can solve algebraically the IE of Eq. (9) and obtain the asymptotic behavior of the LHC discontinuity which is given by

\[
\Delta(A) = \frac{\lambda \pi^2}{M_N A} \exp \left( -2\pi m_\pi \lambda \text{arctanh} \left( 1 - \frac{m_\pi}{2\sqrt{-A}} \right) \right)
\]  
(10)

with \( \lambda = \frac{g_A^2 m^2_\pi}{4 f^2_\pi} \).

For the physical case with \( g_A = 1.26 \) the twice iterated OPE reproduces closely the non-perturbative results. Because of this we consider the unphysical larger value\(^1\)

\( g_A = 6.80 \) and show in Fig. 1 the comparison of the LHC discontinuity including \( \Delta_{1\pi} \) (green), \( \Delta_{2\pi} \) (blue), \( \Delta_{3\pi} \) (magenta), \( \Delta_{4\pi} \) (light blue), the result of Eq. (9) (red points) and the asymptotic expression Eq. (10) (black points), which is only taken for \( k > m_\pi \). In this case the contributions up to 4 pion exchanges are sizable.

Now that we have the contribution to the LHC discontinuity of the iterated OPE to any order, we can use the \( N/D \) method to calculate the partial-wave amplitude along the physical region \( (A > 0) \). As OPE in the singlet case is a regular interaction we can perform a regular \( N/D \) calculation and compare with the result of the LS equation. As mentioned before, OPE in the \( ^1S_0 \) partial wave is weak, and in order to see the effect of higher-order iterative diagrams in the calculation of \( \Delta(A) \) we change the axial coupling constant to the unphysical value \( g_A = 6.80 \). The results for the phase-shifts are given in Fig. 2 where we can see that now the problem is non-perturbative; even including 4\( \pi \) exchange is not enough to reproduce the phase shift. However with the solution of Eq. (9) we reproduce the calculation of the LS equation. Note that here we take the prescription

\[ a = -23.75 \text{ fm from the LS equation and the } N/D_{01} \text{ case.} \]

\(^{1}\) This value is chosen to reproduce the physical scattering length
of zero phase shift at infinite energy.²

|                  |  $a_s$ (fm) |  $r$ (fm) |  $v_2$ (fm²) |
|------------------|------------|-----------|-------------|
| $\Delta_{1\pi}$ | -23.75     | 8.56      | 15.3        |
| $\Delta_{2\pi}$ | -23.75     | 8.80      | 17.7        |
| $\Delta_{3\pi}$ | -23.75     | 8.88      | 18.4        |
| $\Delta_{4\pi}$ | -23.75     | 8.90      | 18.6        |
| Non-perturbative | -23.75     | 8.90      | 18.7        |

|                  |  $\Delta_{1\pi}$ |  $\Delta_{2\pi}$ |  $\Delta_{3\pi}$ |  $\Delta_{4\pi}$ |
|------------------|------------------|------------------|------------------|------------------|
| N/D₁₁            | 1.66             | 3.53             | 1.80             | -6.89            |
| N/D₀₁            | 5.70 $10^{-2}$   | 8.71 $10^{-2}$   | 13.7             | 47.5             |
| Non Perturbative  | -0.168           | -8.71 $10^{-2}$  | 13.7             | 47.5             |

TABLE I: Effective range parameters for the N/D calculation for the unphysical value of $g_A = 6.80$.

At this point it is interesting to check what we get if we use the N/D method with one subtraction fixing the scattering length to the value obtained from the LS equation. This is given in Fig. 3 where we can see that if we include in $\Delta(A)$ the solution of Eq. (9) we recover exactly the result without subtractions. However in this case the contributions of the iterative diagrams to $\Delta(A)$ look perturbative and with $4\pi$ exchange we almost recover the exact result. Note that here we take the prescription of zero phase shift at threshold.

![Figure 2](image-url)  
FIG. 2: Results for the $^1S_0$ phase-shift up to $k = 400$ MeV with OPE for the N/D₀₁ case and $g_A = 6.80$. The dots show the result of the LS equation while the solid lines are the N/D method including in $\Delta(A)$ contributions of $1\pi$ (green), $2\pi$ (blue), $3\pi$ (magenta), $4\pi$ (light-blue) and the solution of Eq. (9) (red).

![Figure 3](image-url)  
FIG. 3: Results for the $^1S_0$ phase-shift up to $k = 400$ MeV with OPE for the N/D₁₁ case and $g_A = 6.80$. Lines are the same as in Fig. 2.

In Table I we give the first three effective range parameters for both calculations, the regular one and the N/D₁₁. For the latter the parameters vary slowly when more pion exchanges are added. For the regular solution, due to the fact that the system is highly non-perturbative, we do not see a convergent pattern, it will probably show up at a much higher order. However the final result in both prescriptions agrees.

These considerations show how a non-perturbative problem (with respect to the contributions from the LHC discontinuity to the physical partial-wave amplitude) is almost perturbative once one extra subtraction (or more) are taken.

We now compare the results given by the N/D method with those of the LS equation renormalized with one counter term. We use three different regulators. The first one corresponds to a monopole form factor which gives the partial wave projection

$$V_1(p',p) = C_0^1 \frac{\Lambda^2}{2p'p} Q_0(z_\Lambda)$$

where $z_\Lambda = \sqrt{p'^2+p^2+\Lambda^2}$ and has the same analytical properties as the OPE. The second and third ones are Gaussian type form factors

$$V_i(p',p) = C_0^i e^{-\left(\frac{p'}{2n_2}\right)^{2n_2} - \left(\frac{p}{2n_3}\right)^{2n_3}}$$

where $n_2 = 2$ and $n_3 = 3$ and have different analytical properties. In all cases for each value of $\Lambda$ we determine $C_0^i$ to reproduce the $^1S_0$ scattering length $a = -23.75$ fm. We also calculate the LS equation using subtractive renormalization [26, 27]. It has the advantage that the counter term is removed from the equations and they depend explicitly on the low energy constant fixed, e.g. the scattering length $a$.

² Levinson theorem, which is valid for regular interactions, implies that there are two bound states.
To compare with the result of the $N/D$ method we have to make an extrapolation to $\Lambda \to \infty$. In order to do that we plot in Fig. 4 the phase shift for $k = 400$ MeV as a function of $1/\Lambda$. In the figure the dots represent the phase-shifts obtain with the LS equation with the counter terms introduced as in $V_1$ (red), $V_2$ (green) and $V_3$ (blue). These plots show a linear behavior and we make a linear fit to obtain the phase-shift at $1/\Lambda = 0$. The solid black line shows the result from subtractive renormalization [26, 27]. The dashed lines correspond to the results obtained from the case $N/D_{11}$ with the same colors as in Fig. 2. In this scale contributions from $3\pi$ exchange can be observed. The final result agrees very well with the extrapolation made by the fit and this does not depend on the particular regulator used.

Finally, we give in Fig. 5 the phase shifts that result in the $N/D$ method with $\Delta(A)$ calculated from the LO ChPT potential. The cases $N/D_{11}$, $N/D_{12}$ and $N/D_{22}$ are shown. The subtraction constants are fixed using $a = -23.75$ fm, $r = 2.66$ fm and $v_2 = -0.63$ fm$^2$.

We now perform the $N/D$ method calculation with singular interactions given by the NLO and NNLO Chiral Effective Field Theory contributions [6]. At NNLO we use the LEC’s $c_1 = -0.74$ GeV$^{-1}$, $c_3 = -3.61$ GeV$^{-1}$ and $c_4 = 2.44$ GeV$^{-1}$ obtained from the $\pi N$ Roy-Steiner equations matched to ChPT [42]. However, due to the singular character of the interaction not all the $N/D$ equations converge (contrary to the regular case of the Yukawa potential at LO). Indeed, since the interactions are singularly attractive one would expect no solution for the regular case and a well defined solution for the $N/D_{11}$ case, and this is what we obtain. The case $N/D_{12}$ does not converge, which explains from first principles why when trying to fix only $a$ and $r$ no solutions were obtained in the LS studies of Refs. [27, 29]. The $N/D_{22}$ equations give a well defined result. Detailed discussions on the calculation of $\Delta(A)$ and about the convergence of different calculations will be given in a forthcoming paper [38].

In Figs. 6 and 7 we show the results that stem from the calculations employing the NLO and NNLO ChPT potentials, respectively. We use the same values for $a$, $r$ and $v_2$ and the same colors as in Fig. 5, additionally we also include as blue dots the result of subtractive renormalization.

In this work we did not perform a thorough fit of the phase shifts since the electromagnetic and charge symmetry breaking effects, which are important at low energies, has not been included yet. However, this work shows that the $^1S_0$ partial wave could be reproduced with good accuracy with the NNLO ChPT potential within the $N/D_{22}$ case calculation.

A remarkably interesting feature of the $N/D$ method over the LS equation is that one can calculate straightforwardly the scattering amplitude in the whole $A$-complex plane, once $D(A)$ is known along the LHC, cf. Eq. (3). In this way, one could study directly the bound states, virtual states and resonances associated with the calculated

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3 This expectation is based on the fact that a singularly attractive interaction requires to fix a constant relative phase [29, 43].
partial-wave amplitude. The large scattering length in the $^1S_0$ partial wave is a signal of an antibound state. We can calculate the $T$ matrix in the second Riemann sheet $T_{II}(A) = N(A)/D_{II}(A)$ using the analytical continuation of the first Riemann sheet given by

$$T_{II}^{-1}(A) = T^{-1}(A) + 2i\rho(A)$$  \hspace{1cm} (13)$$

where $\rho(A)$ is evaluated such that $\text{Im}(\sqrt{A}) > 0$. So the antibound state is given by the zeros of $D_{II}(A)$. With one subtraction we obtain $-0.070$ MeV at LO, and $-0.067$ MeV for the NLO and NNLO cases. Using more than one subtraction we get $-0.066$ MeV in all cases.

In summary, we have deduced for the first time in the literature an IE to obtain the full discontinuity of a partial-wave amplitude along the LHC, $\Delta(A)$. This is then implemented within the $N/D$ method, with exact discontinuities both along the LHC and RHC. In this way, we obtain the equivalence between the $N/D$ method and the LS equation for a Yukawa potential (regular potential). We also apply the method with the NLO and NNLO ChPT potentials, which are examples of singular attractive interactions [29]. The equivalence of the $N/D$ method with one extra subtraction and the LS equation renormalized with one counter term or with subtractive renormalization holds in this case as well. Within the $N/D$ method one can also include extra subtractions constants; we have done it up to three extra subtractions, which reproduces accurately the $^1S_0$ $NN$ phase shifts of the Granada group analysis [40] when the NNLO ChPT potential is used. This goes definitely beyond the present nonperturbative solution of the LS equation in momentum space renormalized with counter terms, for which theoretical control in the regulator independent case is achieved only when taking one or none counter term [27, 29]. Our results are far reaching and could be of great interest for atomic, molecular, nuclear and particle physics in which singular attractive potentials and short range interactions usually appear.

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