Alpha particle condensation and nuclear rainbow scattering

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Abstract. It is shown that a dilute property of an \(\alpha\) particle condensate can be seen in the Airy structure of nuclear rainbow and prerainbow scattering. The dilute property of the Hoyle state of \(^{12}\text{C}\) with a developed \(\alpha\) cluster structure is discussed by studying refractive \(^3\text{He}^+^{12}\text{C}\) and \(\alpha^+^{12}\text{C}\) scattering.

1. Introduction

Bose-Einstein condensation of \(\alpha\) particles is a challenging subject and recently Tohsaki et al. \cite{1} conjectured that the \(0^+_2\) (7.65 MeV) state of \(^{12}\text{C}\), the Hoyle state, has a dilute density distribution due to Bose-Einstein condensation of three \(\alpha\) particles. The large radius of the Hoyle state due to Bose-Einstein condensation has not been measured directly. When absorption is incomplete a projectile which penetrates deep into the nucleus can carry the information about the nuclear potential. When the nuclear radius is very large due to Bose-Einstein condensation, refractive scattering would greatly differ from the scattering from a nucleus with a normal size. Our approach in this paper is to use a refractive property of the nucleus as a lens.

The sensitivity of the rainbow angle and its associated Airy structure on refractive index \(n\) is seen in the meteorological rainbow. In Fig. 1 Mie scattering from a small water droplet is shown. For a red wave \((n =1.33)\) the primary rainbow appears at about 137.7° and for a violet wave \((n=1.34)\) at 139.5°. As the refractive index increases the rainbow angle and the associated Airy structure is shifted to backward.

On the other hand, different from a meteorological rainbow, a nuclear rainbow is Newton’s zero-order rainbow \cite{2} and has no internal reflection in the medium. However the sensitivity of the rainbow angle and its associated Airy structure on the refractive index, i.e. nuclear potential, can be seen as well. In fact, nuclear rainbow scattering has been powerful in the study of nucleus-nucleus interaction potential when absorption is incomplete \cite{3}. Nuclear rainbow scattering typically occurs at the high energy region where a semiclassical picture holds. In optics refractive index is related to the optical potential as follows:

\[ n(r) = \sqrt{1 - \frac{V(r)}{E_{\text{c.m.}}}}. \] (1)

This shows that the refractive index \(n(r)\) becomes large as the incident energy becomes lower. Rainbow scattering has mostly been studied for elastic scattering in optics and also in nuclear
physics. Inelastic rainbow scattering has not been very clear theoretically and experimentally. Recently it has been shown that the evolution of the Airy structure in inelastic scattering is similar to elastic scattering [4].

The concept of a prerainbow has been proposed to understand the refractive phenomenon at the lower energy region where rainbow-like "Airy" oscillations without a falloff of the cross sections appears in the angular distributions [2]. This prerainbow shows clear oscillations, which are understood as the interference between the subamplitude of the farside component of the internal waves, which penetrate the barrier deep into the inner part of the potential and are reflected at the most internal turning point, and the farside component of the barrier waves, which are reflected at the barrier of the potential at the surface. We show that strong refraction of the Hoyle state can be seen in the prerainbow oscillations in inelastic $^3$He$^{+}$-$^{12}$C scattering and inelastic rainbow scattering of an $\alpha$ particle from $^{12}$C [5, 6].

![Figure 1. Mie scattering intensity profiles for a water droplet (radius 3 mm) at two visible wavelengths. (a) $\lambda=0.65$ $\mu$m (red, $n=1.33$) and (b) $\lambda=0.40$ $\mu$m (violet, $n=1.34$).](image)

2. Double folding model
Kamimura et al. [7] studied the structure of $^{12}$C in the Resonating Group Method by using a three $\alpha$ particle cluster model. The obtained cluster wave functions reproduce many experimental data such as energy levels, $\alpha$ decay widths, electric transition probabilities, electron scattering form factors and have been used successfully in many reactions involving $^{12}$C. The wave function for the Hoyle state is almost completely equivalent to the Bose-Einstein condensate wave function [8]. Therefore in the following coupled channel analysis with the double folding model we use the wave functions of Kamimura et al. [7]. The double folding potential is given
by

\[ V_{ij}(R) = \int \rho_{00}(r_1) \rho_{ij}^{(^{12}\text{C})}(r_2) v_{NN}(E, \rho, r_1 + R - r_2) \, dr_1 \, dr_2, \]

(2)

where \( \rho_{00}(r) \) is the ground state density of a projectile, while \( v_{NN} \) denotes the density-dependent M3Y effective interaction (DDM3Y) \[9\]. \( \rho_{ij}^{(^{12}\text{C})}(r) \) represents the diagonal \((i = j)\) or transition \((i \neq j)\) nucleon density of \(^{12}\text{C}\) calculated by Kamimura et al. \[7\]. The density distribution of \(^3\text{He}\) is taken from Cook et al. \[10\]. In the analysis we introduce the normalization factor \(N_R\) for the real part of the potential and phenomenological imaginary potentials with a Wood-Saxon form factor and a derivative of the Wood-Saxon form factor for each channel.

3. Analysis of \(^3\text{He}^+\!^{12}\text{C}\) scattering

We study elastic and inelastic \(^3\text{He}^+\!^{12}\text{C}\) scattering in the microscopic coupled channel method by taking into account simultaneously the \(0^+_1\) (0.0 MeV), \(2^+_1\) (4.44 MeV), \(0^+_2\) (7.65 MeV), and \(3^-\) (9.63 MeV) states of \(^{12}\text{C}\). Calculated angular distributions at 34.7 MeV and 72 MeV are shown in Fig. 2 in comparison with the experimental data \[11, 12\]. The normalization factor \(N_R = 1.28\) is used and the imaginary potential parameters are adjusted to fit the data. The calculation reproduces the experimental angular distributions for the ground state and the Hoyle state as well as the \(2^+\) and \(3^-\) states \[5\].

![Figure 2](image)

**Figure 2.** Calculated cross sections (solid lines), farside (dotted lines) and nearside (dashed lines) contributions in \(^3\text{He}^+\!^{12}\text{C}\) scattering at (a) 72 MeV and (b) 34.7 MeV are compared with the experimental data (points) \[11, 12\].

The calculated scattering amplitude is decomposed into farside and nearside contributions following Fuller’s prescription. At \(E_L = 72\) MeV the first Airy minimum \(A_1\) for elastic scattering is not seen clearly in the experimental angular distribution. On the other hand, the \(A_1\) minimum for the \(0^+_2\) state is clearly seen in the farside cross sections because the minimum is shifted to a larger angle where the nearside contribution is much smaller. At \(E_L = 34.7\) MeV in Fig. 2(b) the Airy minimum \(A_1\) appears at 60° for elastic scattering and 75° for the \(0^+_2\) state. The latter is shifted to a much larger angle and the Airy minimum is not at all obscured by the nearside.
contributions. For the $0^+_2$ state at $E_L = 34.7$ MeV the nearside contributions are much smaller than the farside contributions compared with the elastic scattering case in the wider range of angles. Thus the difference of the refraction between the ground state and the $0^+_2$ state is much more clearly seen at 34.7 MeV than at 72 MeV.

The absorption is incomplete for the $0^+_2$ state and a more beautiful prerainbow Airy oscillation is seen than for the elastic scattering. The refractive effect of the $0^+_2$ state is more clearly seen in the prerainbow structure at the low incident energy region. Thus it is found that the incident $^3$He is strongly refracted in the Hoyle state in accordance with the picture that the state has a large lens composed of three alpha particles in a dilute density distribution.

4. Analysis of $\alpha + ^{12}$C scattering

In a previous paper [6] we studied $\alpha + ^{12}$C at the high energy region above 139 MeV where a nuclear rainbow with a typical falloff of the cross sections appears in the angular distributions. It was shown that for the Hoyle state the rainbow angle, and therefore the associated Airy structure, is shifted to a larger angle compared with elastic scattering.

![Figure 3](image)

Figure 3. Calculated cross sections (solid lines) in $\alpha + ^{12}$C scattering at $E_L = 110$ MeV are decomposed into farside (dotted lines) and nearside (dashed lines) components and compared with the experimental data (points) [13].

As shown in $^3$He+$^{12}$C scattering at 34.7 MeV, the refractive effect can be seen more clearly at the lower energy region in the prerainbow oscillations. Unfortunately for the $\alpha + ^{12}$C system there are no experimental angular distributions for the Hoyle state in this low energy region. Also it has been known that for the $\alpha + ^{12}$C system a global potential which reproduces elastic scattering angular distributions in a wide range of incident energies including the low energy region where ALAS (Anomalous Large Angle Scattering) appears has not been established. Very recently Dem’yanova et al. [13] measured angular distributions of inelastic scattering to the Hoyle state and elastic scattering at 110 MeV. We have analyzed this data in the same spirit and calculated results are shown in Fig. 3. The calculation reproduces characteristic features of the experimental data. The rainbow angle for the Hoyle state is shifted to a larger angle compared with that for the ground state. The Airy minimum is clearly seen for the Hoyle state. These results confirm the findings of the previous section in the $^3$He+$^{12}$C scattering at 34.7 MeV.

As shown in Fig. 4, by decomposing the total scattering cross sections at 110 MeV into its partial wave contributions, it is found that the peak of the partial cross sections for elastic scattering is located at orbital angular momentum $L = 13$, while the peak for the inelastic scattering to the Hoyle state is located at $L = 18$. This is also consistent with the picture that
Figure 4. Calculated partial cross sections for elastic $\alpha+^{12}\text{C}$ scattering and inelastic scattering in the $0^+_2$ state at $E_L=110$ MeV are shown as a function of orbital angular momentum.

The refraction of inelastic scattering in the Hoyle state takes place at the larger radii than the elastic scattering [14].

5. Summary
We have analyzed the angular distributions of prerainbow and rainbow scattering for the $^3\text{He}+^{12}\text{C}$ system and rainbow scattering for the $\alpha+^{12}\text{C}$ system in a coupled channel method by using a microscopic wave function for the Hoyle state that is almost equivalent to the $\alpha$ particle condensate wave function. The experimental angular distributions are well reproduced by the calculations. It is found that refraction is much stronger for the inelastic scattering to the Hoyle state than that for the elastic scattering and that the reaction takes place at the larger radii for the Hoyle state than for elastic scattering. These results are in agreement with the picture that the Hoyle state has a large lens with a dilute density distribution due to the $\alpha$ particle condensation.

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