Modulated Entanglement Evolution Via Correlated Noises

Brittany Corn† · Ting Yu∗

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Abstract

We study entanglement dynamics in the presence of correlated environmental noises. Specifically, we investigate the quantum entanglement dynamics of two spins in the presence of correlated classical white noises, deriving Markov master equation and obtaining explicit solutions for several interesting classes of initial states including Bell states and X form density matrices. We show how entanglement can be enhanced or reduced by the correlation between the two participating noises.

Keywords

Entanglement dynamics · Correlated noises · Two-qubit model · Entanglement sudden death

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1 Introduction

Many proposed applications in quantum computing [1], quantum communication [2], and quantum cryptography [3] revolve around harnessing the inherent correlation between quantum particles, called entanglement [4]. Although quantum mechanics dictates that these coherence effects are intrinsic in certain systems, even when the atoms or particles are non-local, there is an overall weakening due to coupling to noisy environments [5] that eventually leads to the fast decay of entanglement [6,7,8,9,10], as in the case of amplitude or phase noise, or even the sudden death of entanglement (ESD) in the worst scenarios [11,12,13,14,15]. As such, the study of the controlled entanglement dynamics is of much importance to the prospects of maintaining quantum information [16,17,18,19,20,21,22,23]. Moreover, model systems that theoretically exhibit the rebirth of entanglement have been proposed and discussed in several cases [24,25,26,27].

In this paper, we consider a model system consisting of two uncoupled qubits A and B interacting with stochastic fields, \( f_A(t) \) and \( f_B(t) \), respectively, shown in Fig.1. The setup of the system is quite similar to the two-qubit local dephasing channel of [8] in which the qubits were found to disentangle in a shorter time than their individual atom-field local dephasing times. On the other hand, if the two qubits are coupled to a common environment,
then entanglement was shown to be preserved for a class of initial states living in a subspace called decoherence-free subspace (DFS) \cite{28,29}. By introducing a correlation between the two noisy sources, we can show that the entanglement between the two qubits can be enhanced or reduced by properly choosing the correlation of the two participating external noises.

The paper is organized as follows: We present the specifics of the model system in Section II, which leads to deriving the master equation in the Markov regimes. The entanglement evolution under the correlated noisy sources are studied in Section III. We have shown that dependent on initial states the correlation between two external noises can either enhance or reduce the existing entanglement of the qubit systems. We conclude in Section IV.

2 Qubit Model and the Master Equation

Our model consists of two identical, separated qubits (two-level atoms, spins, excitons, etc.) each having transition frequency \( \omega \) and each coupled to a separate classical random field, \( f_A(t) \) and \( f_B(t) \) respectively. The total Hamiltonian for this system is (setting \( \hbar = 1 \))

\[
H_{\text{tot}} = \frac{\omega}{2} (\sigma_z^A + \sigma_z^B) + f_A(t)\sigma_x^A + f_B(t)\sigma_x^B.
\]

(1)

This Hamiltonian is familiar in condensed matter theory as an extension of the spin-boson model in the semiclassical regime \cite{30}. The classical stochastic fields are assumed to obey the following correlation relations in the Markov approximation:

\[
M[f_A(t)] = M[f_B(t)] = 0,
\]

(2)

\[
M[f_A(t)f_A(s)] = \gamma_A \delta(t-s),
\]

(3)

\[
M[f_B(t)f_B(s)] = \gamma_B \delta(t-s),
\]

(4)

\[
M[f_A(t)f_B(s)] = \Gamma \delta(t-s),
\]

(5)

where \( M[\cdot] \) denotes the ensemble average over the classical stochastic fields and for simplicity we assume \( \gamma_A = \gamma_B = \gamma \). Obviously, \( \Gamma \) determines the respective correlation properties.

![Fig. 1](image)

The model is composed of two uncoupled qubits \( A \) and \( B \) individually coupled to respective stochastic fields, \( f_A(t) \) and \( f_B(t) \), which are then correlated.
The effects of the correlation between the two fields \( f_A(t) \) and \( f_B(t) \), \( M[f_A(t)f_B(s)] \), will be revealed in the study of entanglement between the two qubits.

There has been a lot of work dedicated to quantum entanglement dynamics in the presence of an environmental noise, either in classical or quantum regimes (For some recent progress, e.g., see [31,32,33,34,35,36,37,38,39]). Clearly, having two separated qubits individually coupled to respective external fields would reveal the same Lindblad dynamics for each qubit. That is, individual couplings always cause the irreversible decay of entanglement. However, when there is a correlation between the two classical stochastic fields, we expect to find new entanglement effects between the two qubits. It can be shown that the master equation governing the dynamics of two qubits under the influence of two correlated noises can be derived from the corresponding stochastic Schrödinger equation [40].

\[
\dot{\rho} = -i[H_S, \rho] - 2\gamma_A (\rho - \sigma^A \rho \sigma^A) - 2\gamma_B (\rho - \sigma^B \rho \sigma^B) - 4\Gamma(\sigma^A \sigma^B \rho + \rho \sigma^A \sigma^B - \sigma^A \rho \sigma^B - \sigma^B \rho \sigma^A).
\]

where \( H_S = \frac{1}{2}\omega(\sigma^A + \sigma^B) \) is the Hamiltonian of the two qubits. Upon expanding \( \sigma^A \) and \( \sigma^B \) in terms of \( \sigma^{\pm,\pm} \), we may obtain a more solvable form to the master equation:

\[
\dot{\rho} = -i[H_S, \rho] - \gamma_A (\sigma^A \sigma^A \rho - \sigma^A \rho \sigma^A - \sigma^A \rho \sigma^A - \rho \sigma^A \sigma^A) - \gamma_B (\sigma^B \sigma^B \rho - \sigma^B \rho \sigma^B - \sigma^B \rho \sigma^B - \rho \sigma^B \sigma^B) - \Gamma(\sigma^A \sigma^B \rho - \sigma^B \rho \sigma^A - \sigma^A \rho \sigma^B - \rho \sigma^B \sigma^A) - \Gamma(\sigma^B \sigma^A \rho - \sigma^A \rho \sigma^B - \sigma^B \rho \sigma^A - \rho \sigma^A \sigma^B) + H.C.
\]

At a first glance, Eq. (6) or Eq. (7) immediately displays the dynamics of qubit A in correlation with field \( f_A(t) \), dynamics of qubit B while interacting with field \( f_B(t) \) and the cross terms due to the correlation between the two fields. Without the involvement of \( \Gamma \), the master equation would be the sum of two familiar Lindblad master equations for two qubits, respectively. In that case, the two qubits would evolve separately throughout time and entanglement between the two qubits will deteriorate with time. The correlation between two classical stochastic fields characterized by \( \Gamma \) is expected to affect entanglement evolution in two different ways dependent on the initial states. We will show for some initial states that entanglement can be significantly enhanced by adjusting the cross correlation between the two fields. However, for some other initial states, the cross-correlation works like a catalyst, which may accelerate the decay of entanglement.

### 3 Modulated Entanglement Evolution

Modulated entanglement evolution will of course depend on the state the system was originally in. If the main goal is to try to maintain or improve entanglement, a good initial state would be that which has maximum entanglement, the Bell State, as is presented in cases 3.1 and 3.2. It is also interesting to see how a general X type matrix behaves under these circumstances, as in case 3.3. It is important to note that \( \gamma \) and \( \Gamma \) cannot be arbitrary and must be chosen in a way that preserves positivity of the density matrix (\( \Gamma \leq \gamma \)).
3.1 Bell State \([|++\rangle, |--\rangle]\)

One of the Bell States is described by the pure state vector

\[
|\Psi_0\rangle = \frac{1}{\sqrt{2}} \{|++\rangle + |--\rangle\},
\]

which has the following density operator representation:

\[
\rho = \frac{1}{\sqrt{2}} \begin{pmatrix}
1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 \\
1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 
\end{pmatrix} = \frac{1}{2} \begin{pmatrix}
1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 
\end{pmatrix}.
\]

Inserting \(\rho\) as the initial reduced density matrix, \(\rho(0)\), fuels the following master equation solutions:

\[
\begin{align*}
\rho_{11}(t) &= \frac{1}{8\kappa} e^{-(6\gamma + \kappa)t} [2\gamma(1 - e^{2\kappa t}) + \kappa(2e^{(6\gamma + \kappa)t} + e^{2\kappa t} + 1)], \\
\rho_{22}(t) &= \frac{1}{8\kappa} e^{-(6\gamma + \kappa)t} [-2\gamma(1 - e^{2\kappa t}) + \kappa(2e^{(6\gamma + \kappa)t} - e^{2\kappa t} - 1)], \\
\rho_{23}(t) &= \frac{\Gamma}{\kappa} e^{-(6\gamma + \kappa)t} (e^{2\kappa t} - 1), \\
\rho_{14}(t) &= \frac{1}{2} e^{-4\gamma t},
\end{align*}
\]

where we have defined \(\kappa \equiv \sqrt{4\gamma^2 + 32\Gamma^2}\). It should be noted that this special initial condition leads to the simple relations:

\[
\begin{align*}
\rho_{33}(t) &= \rho_{22}(t), & \rho_{44}(t) &= \rho_{11}(t), & \rho_{32}(t) &= \rho_{23}(t), & \rho_{41}(t) &= \rho_{14}(t), \\
\rho_{12}(t) &= \rho_{13}(t) = \rho_{24}(t) = \rho_{34}(t) = 0, & \rho_{21}(t) &= \rho_{31}(t) = \rho_{42}(t) = \rho_{43}(t) = 0.
\end{align*}
\]

Because the off-diagonal elements initially at zero will remain at zero for all time, the time-dependent reduced density operator will be of the X form with only 4 independent, real terms:

\[
\rho(t) = \begin{pmatrix}
\rho_{11}(t) & 0 & 0 & \rho_{14}(t) \\
0 & \rho_{22}(t) & \rho_{23}(t) & 0 \\
0 & \rho_{32}(t) & \rho_{33}(t) & 0 \\
\rho_{14}(t) & 0 & 0 & \rho_{11}(t)
\end{pmatrix}.
\]

With a solution to the master equation in hand, it is easy to calculate the Concurrency [41], a measurement of entanglement ranging between 0 and 1 evaluated as

\[
C(\rho) = 2 \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\},
\]

where \(\lambda_i\) represent the eigenvalues of the matrix

\[
\rho = \rho(\sigma^A_i \otimes \sigma^B_i) \rho^*(\sigma^A_i \otimes \sigma^B_i),
\]

in descending order. For this \(\rho(t)\), the concurrence throughout time will be

\[
C(\rho(t)) = 2 \max\{0, |\rho_{14}(t)| - \sqrt{\rho_{22}(t)\rho_{33}(t)}\},
\]
and is plotted against $\gamma t$ in Fig. 2 for various values of $\Gamma$.

For all cases of $\gamma$, when $\Gamma = 0$, the system exhibits the sudden death of entanglement well-known to the local-dephasing channel [8]. However, by turning on the correlation $\Gamma$ between the stochastic fields, the concurrence curve moves vertically upward, denoting an increase in the measurement of entanglement. Clearly, we displayed controlled entanglement evolution via correlated environmental noises. We also are faced with the fact that increasing the correlation $\gamma$ of each atom to their respective fields will also induce a faster sudden death of entanglement. By turning on the correlation between the fields to its maximum value, $\Gamma = \gamma$, entanglement is maximally enhanced, as shown in Fig. 2.

3.2 Bell State $(|+\rangle, |-\rangle)$

Let’s now solve the master equation using the other form of the maximally coherent Bell State:

$$\Phi_0 = \frac{1}{\sqrt{2}} \{|+\rangle + |-\rangle\}. \tag{13}$$

The initial reduced density matrix for this state vector is:

$$\rho = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{14}$$

which allows us to solve for the time dependent density matrix:

$$\rho_{12}(t) = \rho_{13}(t) = \rho_{24}(t) = \rho_{34}(t) = \rho_{14}(t) = 0,$$

$$\rho_{21}(t) = \rho_{32}(t) = \rho_{42}(t) = \rho_{43}(t) = \rho_{41}(t) = 0. \tag{15}$$
Concurrence calculated for various values of $\Gamma$ with $\gamma = 1$. For this initial state, entanglement can only be reduced by turning on the correlation between the fields.

Using the same definitions as before

$$\rho_{11}(t) = \frac{1}{4\kappa} e^{-(6\gamma + \kappa)t}[-(2\gamma + 8\Gamma)(1 - e^{2\kappa t}) - \kappa(1 + e^{2\kappa t} - 2e^{(6\gamma + \kappa)t})],$$

$$\rho_{22}(t) = \frac{1}{4\kappa} e^{-(6\gamma + \kappa)t}[(2\gamma + 8\Gamma)(1 - e^{2\kappa t}) + \kappa(1 + e^{2\kappa t} + 2e^{(6\gamma + \kappa)t})],$$

$$\rho_{23}(t) = \frac{1}{4\kappa} e^{-(6\gamma + \kappa)t}[-(4\gamma - 8\Gamma)(1 - e^{2\kappa t}) + 2\kappa(1 + e^{2\kappa t})],$$

$$\rho_{33}(t) = \rho_{22}(t) \quad \rho_{44}(t) = \rho_{11}(t) \quad \rho_{32}(t) = \rho_{23}(t).$$

Making the density operator of a particular X form with 3 independent, real variables throughout all time $t$:

$$\rho(t) = \begin{pmatrix}
\rho_{11}(t) & 0 & 0 & 0 \\
0 & \rho_{22}(t) & \rho_{23}(t) & 0 \\
0 & \rho_{23}(t) & \rho_{22}(t) & 0 \\
0 & 0 & 0 & \rho_{11}(t)
\end{pmatrix} \tag{16}$$

The concurrence of this density matrix (16) is given by:

$$C(\rho(t)) = 2\max\{0, |\rho_{23}(t)| - \sqrt{\rho_{11}(t)\rho_{44}}\}, \tag{17}$$

as plotted in Fig. 3.

This figure displays quite the opposite of that in the previous section. Introducing a correlation between the noise fields, $\Gamma$, actually lowers the concurrence curve and causes a faster decay of entanglement. This is true for all values of $\gamma$, displaying the impossibility for entanglement enhancement for this initial state evolution. This brings forth a very interesting characteristic of entanglement, that even in the Markovian regime where there is no memory of previous times in the system, it is still very sensitive to which initial state is being used. Even though all Bell States produce maximum entanglement, one has the prospect for the rebirth of entanglement and the other does not.
3.3 X state

We can now use a more general approach by utilizing an initial state in the X form which includes the Bell states and Werner states as special cases [42]:

\[ \rho_t = \begin{pmatrix} a & 0 & 0 & w^* \\ 0 & b & z^* & 0 \\ 0 & z & c & 0 \\ w & 0 & 0 & d \end{pmatrix}, \]

(18)

where \( a + b + c + d = 1 \). In this case, the concurrence is

\[ C(\rho_t) = 2 \max \{ |z| - \sqrt{ad}, |w| - \sqrt{bc} \}, \]

(19)

which allows us to account for a large range of initial entanglement conditions.

The solution to the master equation is:

\[ \rho_{11}(t) = \frac{2(a - d)e^{-4\eta} + 1}{4} + \frac{e^{-6\eta}}{4\kappa} [(a - b - c + d) \kappa \cosh \kappa t + (16\kappa z - 2\gamma(a - b - c + d)) \sinh \kappa t], \]

\[ \rho_{22}(t) = \frac{2(b - c)e^{-4\eta} + 1}{4} - \frac{e^{-6\eta}}{4\kappa} [(a - b - c + d) \kappa \cosh \kappa t + (16\kappa z - 2\gamma(a - b - c + d)) \sinh \kappa t], \]

\[ \rho_{33}(t) = \frac{2(c - b)e^{-4\eta} + 1}{4} - \frac{e^{-6\eta}}{4\kappa} [(a - b - c + d) \kappa \cosh \kappa t + (16\kappa z - 2\gamma(a - b - c + d)) \sinh \kappa t], \]

\[ \rho_{44}(t) = \frac{2(d - a)e^{-4\eta} + 1}{4} + \frac{e^{-6\eta}}{4\kappa} [(a - b - c + d) \kappa \cosh \kappa t + (16\kappa z - 2\gamma(a - b - c + d)) \sinh \kappa t], \]

\[ \rho_{23}(t) = \frac{e^{-6\eta}}{\kappa} [(a - b - c + d) \kappa \cosh \kappa t + (16\kappa z + 2\gamma c) \sinh \kappa t] = \rho_{32}(t), \]

\[ \rho_{14}(t) = \frac{e^{-6\eta}}{\kappa} [(a - b - c + d) \kappa \cosh \kappa t + (16\kappa z + 2\gamma c) \sinh \kappa t] = \rho_{41}(t). \]

The density operator describing the mixed states of the system will now remain in the X form throughout time with 6 independent, real terms

\[ \rho(t) = \begin{pmatrix} \rho_{11}(t) & 0 & 0 & \rho_{14}(t) \\ 0 & \rho_{22}(t) & \rho_{23}(t) & 0 \\ 0 & \rho_{32}(t) & \rho_{33}(t) & 0 \\ \rho_{41}(t) & 0 & 0 & \rho_{44}(t) \end{pmatrix}, \]

(20)

such that the concurrence is calculated as

\[ C(\rho(t)) = 2 \max \{ |\rho_{23}(t)| - \sqrt{\rho_{11}(t)\rho_{24}(t)}, |\rho_{14}(t)| - \sqrt{\rho_{22}(t)\rho_{33}(t)} \}. \]

(21)

The X form initial density matrix displays a more general range of control over the modulation of entanglement. As shown below, the correlation may cause entanglement to decay faster initially, but it can compensate the loss of entanglement later. To see this, we choose specific values of the initial X form density matrix

\[ \rho(0) = \frac{1}{3} \begin{pmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}, \]

(22)

and plot the concurrence through time in Fig. 4.
The results demonstrate that the entanglement can be significantly enhanced by the cross-correlation at late times even though the correlation was detrimental to entanglement at early times. More explicitly, it can be shown that at early time $|\rho_{23}(t)| - \sqrt{\rho_{11}(t)\rho_{44}(t)}$ is the dominant term in the calculation of concurrence, resulting in the degradation of entanglement due to the cross-correlation. However, at a certain point $|\rho_{14}(t)| - \sqrt{\rho_{22}(t)\rho_{33}(t)}$ begins to dominate, causing the correlation to enhance the entanglement. This demonstrates our ability to improve entanglement for a wide range of initial states and time scales.

4 Conclusion

In summary, we studied a system of two separated qubits each coupled to a stochastic field which can then be correlated, opening the option for the enhancement or reduction of entanglement between the qubits. By solving the master equation for the qubit dynamics under various initial states and viewing the concurrence as a function of time, the effects of the correlation between the fields, $\Gamma$, became imminent. Of our three cases for initial states, the Bell State, Eq. (9), showed a potential for the regeneration of entanglement. The amount of entanglement this particular case can achieve is dependent on the degree of correlation between the two noises. In the second case we considered the Bell State of Eq. (14) for which correlation $\Gamma$ did not cause the entanglement between the atoms to be regenerated. In fact, it instead reduced the entanglement and caused it to decay at an even faster rate. This illuminates a selectivity that entanglement has toward the initial state of the system, even under the memoryless Markov approximation. It is important then to look at the general X form density matrix of Section 3.3 allowing us to account for various initial entanglement conditions of the system. This case presented the ability to modulate entanglement in either way. The range of maximum enhancement was variable depending on the parameters of the initial matrix, giving us a wide range of control over the improvement of entanglement.

In general, many characteristic properties of entanglement are still unknown. In this model system, we have created a bridge from one qubit to the other through the correlation of the stochastic fields, allowing for entanglement to be modulated. This correlated noise approach can easily be applied to multipartite systems, which in turn might provide an even higher enhancement of entanglement [43,44,45,46]. Finally, it may be worth noting that the relationship between the classical noise model presented here and the fully quantized models discussed in [17,18] is an interesting problem that will be addressed in future publications.
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