Can $B \to J/\psi K(K^*)$ Decays Be Described by Factorization?

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Abstract

The new measurements of $B \to J/\psi K(K^*)$ decays by CDF and CLEO indicate that the production ratio $R$ and the fraction of longitudinal polarization $\Gamma_L/\Gamma$ are smaller than the previous results. In conjunction with the new result of parity-odd transverse polarization in $B \to J/\psi K^*$, we found a minimal modification to the factorization hypothesis: While the data of $B \to J/\psi K^*$ can be accommodated in the factorization approach with nonfactorizable terms $\chi_{A_1} = \chi_{A_2} = \chi_V \equiv \chi$ of order 15%, the result of $R$ measurement requires that the nonfactorizable effect $\chi_{F_1}$ on $B \to J/\psi K$ be slightly larger than $\chi$. Therefore, the effective parameter $a_2^{\text{eff}}$ is not universal even for $B \to J/\psi K(K^*)$ decays. We have generalized the considerations to $B \to \psi(2S)K(K^*)$ and $B_s \to J/\psi \phi$ and found that the predictions are in agreement with currently available data.
The two-body nonleptonic weak decays of $D$ and $B$ mesons are conventionally described by the factorization approach. It has been shown \cite{1, 2} that this approach fails to account for the observed fraction of longitudinal polarization $\Gamma_L/\Gamma$ in $B \to J/\psi K^*$ decays in all commonly used models of form factors and the data of the production ratio $R \equiv \Gamma(B \to J/\psi K^*)/\Gamma(B \to J/\psi K)$ in many known models. The issue of how to test or modify the factorization hypothesis to describe the data of $B \to J/\psi K(K^*)$ has been the subject of many subsequent studies. Recently, CDF \cite{3} and CLEO \cite{4} have presented new measurements of $B \to J/\psi K(K^*)$ decays and for the first time CLEO has measured the parity-odd transverse polarization in $B \to J/\psi K^*$. It turns out that the new results of $\Gamma_L/\Gamma$ and $R$ are smaller than the previous values. The purpose of this Letter is to study the theoretical implications of the new data.

Under the factorization approximation, the hadronic matrix element is factorized into the product of two matrix elements of single currents, governed by decay constants and form factors. For the QCD-corrected weak Hamiltonian

$$\mathcal{H}_{\text{eff}} \propto c_1 \mathcal{O}_1 + c_2 \mathcal{O}_2 = c_1 (\bar{q}_1 q_2)(\bar{q}_3 q_4) + c_2 (\bar{q}_1 q_4)(\bar{q}_3 q_2),$$

(1)

where $(\bar{q}_1 q_2) \equiv \bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2$, the external $W$-emission and internal $W$-emission amplitudes (or the so-called class-I and class-II decay amplitudes) characterized by the parameters $a_1$, $a_2$ are related to the Wilson coefficient functions $c_1$ and $c_2$ via

$$a_1 = c_1 + \frac{c_2}{N_c}, \quad a_2 = c_2 + \frac{c_1}{N_c}. \quad (2)$$

Factorization requires that $a_1$ and $a_2$ be universal; i.e., they are channel-independent in $D$ or $B$ decays. However, we have learned from charm decays a long time ago that the naive factorization approach never works for the decay rate of color-suppressed (i.e., class-II) decay modes, though it might work for class-I decays \cite{5}. The most noticeable example is the ratio $\Gamma(D^0 \to \bar{K}^0\pi^0)/\Gamma(D^0 \to K^-\pi^+)$, which is predicted to be $\sim 0.02$ whereas experimentally it is measured to be $0.55 \pm 0.06$ \cite{6}. This implies that the inclusion of nonfactorizable contributions is inevitable and necessary. Since the amplitudes of $D$, $B \to PP$, $VP$ decays ($P$: pseudoscalar meson, $V$: vector meson) are governed by a single form factor, the effects of nonfactorization amount to a redefinition of the effective parameters $a_1$ and $a_2$ \cite{6}:

$$a_1^{\text{eff}} = c_1 + c_2 \left( \frac{1}{N_c} + \chi_1 \right), \quad a_2^{\text{eff}} = c_2 + c_1 \left( \frac{1}{N_c} + \chi_2 \right),$$

(3)

or

$$a_1^{\text{eff}} = a_1 \left( 1 + \frac{c_2}{a_1} \chi_1 \right), \quad a_2^{\text{eff}} = a_2 \left( 1 + \frac{c_1}{a_2} \chi_2 \right).$$

(4)

For the expressions of the nonfactorizable terms denoted by the parameters $\chi_1$ and $\chi_2$, consider the decay $\bar{B} \to D\pi$ as an example; we obtain \cite{3, 7}

$$\chi_1(\bar{B} \to D\pi) = \chi_{1,F_{BD}^0} = \frac{\langle D\pi|\bar{O}_1|B \rangle}{\langle D\pi|O_1|B \rangle_f} + \frac{a_1}{c_2} \frac{\langle D\pi|O_1|B \rangle_{nf}}{\langle D\pi|O_1|B \rangle_f} \equiv \frac{F_{0}^{BD}(m_{\pi}^2)}{c_{2} F_{0}^{BD}(m_{\pi}^2)} + \frac{a_1}{c_{2}} \frac{F_{0}^{(1)nf}(m_{\pi}^2)}{F_{0}^{BD}(m_{\pi}^2)},$$

(8)
\[ \chi_2(\bar{B} \to D\pi) = \chi_{2,F_0^{BS}} = \frac{\langle D\pi|\bar{O}_2|B\rangle}{\langle D\pi|O_2|B\rangle} + \frac{a_2}{c_1} \frac{\langle D\pi|O_2|B\rangle_{nf}}{\langle D\pi|O_1|B\rangle_{f}} = \frac{F_0^{(8)nf}(m_B^2)}{F_0^{BS}(m_B^2)} + \frac{a_2}{c_1} \frac{F_0^{(1)nf}(m_B^2)}{F_0^{BS}(m_B^2)} \]  

where the subscripts \( f \) and \( nf \) denote factorizable and nonfactorizable contributions, respectively, and use has been made of the Fierz identity

\[ O_{1,2} = \frac{1}{N_c} O_{2,1} + \bar{O}_{1,2} \]

with \( \bar{O}_1 = \frac{1}{2}(\bar{q}_1 \lambda^a q_2)(\bar{q}_3 \lambda^a q_4) \) being a product of the color-octet currents \( (\bar{q} \lambda q') \equiv \bar{q} \gamma_\mu (1 - \gamma_5) \lambda^a q' \). Since the factorizable \( B \to D\pi \) amplitudes are

\[ \langle D\pi|O_1|B\rangle_{f} = f_\pi(m_B^2 - m_D^2) F_0^{BD}(m_\pi^2), \]

\[ \langle D\pi|O_2|B\rangle_{f} = f_D(m_B^2 - m_D^2) F_0^{BP}(m_D^2), \]

we have followed [9] to define the form factors \( F_0^{(1)nf} \) and \( F_0^{(8)nf} \):

\[ \langle D\pi|O_1|B\rangle_{nf} = f_\pi(m_B^2 - m_D^2) F_0^{(1)nf}(m_\pi^2), \]

\[ \langle D\pi|\bar{O}_1|B\rangle = f_D(m_B^2 - m_D^2) F_0^{(8)nf}(m_D^2). \]

Likewise for \( \bar{B} \to D\rho \) and \( D^*\pi \) decays, we have

\[ \chi_1(\bar{B} \to D\rho) = \chi_{1,F^{BD}} \, ; \quad \chi_2(\bar{B} \to D\rho) = \chi_{2,A_0^{BD}}, \]

\[ \chi_1(\bar{B} \to D^*\pi) = \chi_{1,A_0^{BD*}} \, ; \quad \chi_2(\bar{B} \to D^*\pi) = \chi_{2,F^{BS}}. \]

Since \( c_1/a_2 \gg 1 \), it is clear from Eqs.(4) and (5) that nonfactorizable effects are more dramatic in color-suppressed decay modes. As a consequence, a determination of \( \chi_2 \) is more reliable than \( \chi_1 \).

The study of nonfactorizable effects in \( M \to VV \) decay is more complicated as its general amplitude consists of three independent Lorentz scalars:

\[ A[M(p) \to V_1(\varepsilon_1, p_1)V_2(\varepsilon_2, p_2)] \propto \varepsilon_\mu^*(\lambda_1)\varepsilon_\nu^*(\lambda_2)(\hat{A}_1 g^{\mu\nu} + \hat{A}_2 p^\mu p^\nu + i\hat{V} e^{\mu\nu\alpha\beta} p_1^\alpha p_2^\beta), \]

where \( \hat{A}_1, \hat{A}_2, \hat{V} \) are related to the form factors \( A_1, A_2 \) and \( V \), corresponding to \( S-, D- \) and \( P- \)waves, respectively. For \( \bar{B} \to D^*\rho \) we can define various nonfactorizable terms:

\[ \chi_{1,A_1}(\bar{B} \to D^*\rho) = \chi_{1,A_1^{BD*}}, \quad \chi_{2,A_1}(\bar{B} \to D^*\rho) = \chi_{2,A_1^{BD}}, \]

\[ \chi_{1,A_2}(\bar{B} \to D^*\rho) = \chi_{1,A_2^{BD*}}, \quad \chi_{2,A_2}(\bar{B} \to D^*\rho) = \chi_{2,A_2^{BD}}, \]

\[ \chi_{1,V}(\bar{B} \to D^*\rho) = \chi_{1,V^{BD*}}, \quad \chi_{2,V}(\bar{B} \to D^*\rho) = \chi_{2,V^{BD}}. \]

Since \( a \) priori there is no reason to expect that nonfactorizable terms weight in the same way to \( S-, P- \) and \( D- \)waves, namely \( \chi_{i,A_1} = \chi_{i,A_2} = \chi_{i,V} \), it is in general not possible to define an effective \( a_1 \) or \( a_2 \) for \( M \to VV \) decays once nonfactorizable effects are taken into account [10]. However, if nonfactorizable contributions are channel-independent, i.e.,

\[ \chi_{i,F_0} = \chi_{i,F_1} = \chi_{i,A_0} = \chi_{i,A_1} = \chi_{i,A_2} = \chi_{i,V} = \chi, \]

(12)
the effective parameters $a_1^{\text{eff}}$ and $a_2^{\text{eff}}$ given by Eq.(3) or (4) are applicable not only to $M \to PP$, $VP$ but also to $M \to VV$ and they become universal. In this case we have a new factorization scheme with $\chi \neq 0$. Note that the predictions for the ratios of physical quantities (e.g., $\Gamma_L/\Gamma$ and $|P|^2$) in the same class of decay modes in the new factorization method are the same as that in naive factorization (i.e., $\chi = 0$). Empirically, it has been found that the discrepancy between theory and experiment for charm decays is greatly improved if Fierz transformed terms in (3) are dropped \[11, 5\]. It has been argued that this empirical observation is justified in the so-called large-$N_c$ approach in which a rule of discarding subleading $1/N_c$ terms can be formulated \[12\]. This amounts to having universal nonfactorizable terms $\chi_1 = \chi_2 = -1/N_c$ in Eq.(3). In reality, there is no reason to expect a universal $\chi_1$ or $\chi_2$. Since $\chi_2$ comes mainly from color-octet currents, it is natural to expect that \[4, 13\]

$$|\chi_2(D \to VP)| \gtrsim |\chi_2(D \to PP)| > |\chi_2(B \to VP)|,$$  \tag{13}$$

as nonfactorizable soft-gluon effects become more important when final particles move slower, allowing more time for significant final-state interactions. Indeed, the above theoretical expectation is confirmed by the phenomenological analyses of $D$ and $B$ decay data \[7, 13\]. For charm decay, $\chi_2$ is found to be in the range $-0.60 < \chi_2(D) < -1/3 \[13\].

We now come back to $B \to J/\psi K(K^*)$ decays. The general expressions for the fraction of longitudinal polarization $\Gamma_L/\Gamma$ and production ratio $R$ read

$$\frac{\Gamma_L}{\Gamma} \equiv \frac{\Gamma(B \to J/\psi K^*)_L}{\Gamma(B \to J/\psi K^*)} = \left[ a \left( 1 + \frac{c_1}{a_2} \chi_{A_1} \right) - bx \left( 1 + \frac{c_1}{a_2} \chi_{A_2} \right) \right]^2 / \xi,$$

$$R \equiv \frac{\Gamma(B \to J/\psi K^*)}{\Gamma(B \to J/\psi)} = 1.08 \frac{\xi}{z^2 \left( 1 + \frac{c_0}{a_2} \xi \right)^2},$$ \tag{14}\]

where

$$\xi = \left[ a \left( 1 + \frac{c_1}{a_2} \chi_{A_1} \right) - bx \left( 1 + \frac{c_1}{a_2} \chi_{A_2} \right) \right]^2 + 2 \left[ \left( 1 + \frac{c_1}{a_2} \chi_{A_1} \right)^2 + c^2 y^2 \left( 1 + \frac{c_1}{a_2} \chi_{V} \right) \right]^2,$$

$$x = \frac{A_{2}^{BK^*}(m_{J/\psi}^2)}{A_{1}^{BK^*}(m_{J/\psi}^2)}, \quad y = \frac{V_{BK^*}^{V}(m_{J/\psi}^2)}{A_{1}^{BK^*}(m_{J/\psi}^2)}, \quad z = \frac{F_{1}^{BK}(m_{J/\psi}^2)}{A_{1}^{BK^*}(m_{J/\psi}^2)},$$ \tag{15}\]

and $\chi_{A_i}$ denotes $\chi_{2, A_i^{BK^*}}$ etc., for example [see Eq.(5)],

$$\chi_{A_1} \equiv \chi_{2, A_i^{BK^*}} = \frac{A_{1}^{(8)n}(m_{J/\psi}^2)}{A_{1}^{BK^*}(m_{J/\psi}^2)} + \frac{a_2}{c_1} \frac{A_{1}^{(0)n}(m_{J/\psi}^2)}{A_{1}^{BK^*}(m_{J/\psi}^2)}.$$

\[16\]

The analytic expressions of $a$, $b$ and $c$ are given in \[1, 2\]. Numerically,

$$a = 3.164, \quad b = 1.304, \quad c = 0.435.$$ \tag{17}\]
The parity-odd ($P$-wave) transverse polarization measured in the transversity basis [14], which is more suitable for parity analysis, has the form

$$|P|^2 = \frac{|A_\perp|^2}{|A_0|^2 + |A_\parallel|^2 + |A_\perp|^2},$$

(18)

where $A_i$’s are defined by the orientation of the $J/\psi$ polarization vector $\epsilon_{J/\psi}$ in the transversity basis [4]: $A_0$ for $\epsilon_{J/\psi}$ parallel to $\hat{x}$, $A_\parallel$ for $\epsilon_{J/\psi}$ parallel to $\hat{y}$, and $A_\perp$ for $\epsilon_{J/\psi}$ parallel to $\hat{z}$. Since $A_i$’s are related to the amplitudes $H_\lambda$ in the helicity basis via

$$H_\pm = \frac{1}{\sqrt{2}}(A_\parallel \pm A_\perp), \quad H_0 = -A_0,$$

(19)

we find

$$|P|^2 = 2c^2y^2\left(1 + \frac{c_1}{a_2}\chi_v\right)^2 / \xi.$$  

(20)

A measurement of $|P|^2$ will thus provide information on the vector form factor.

We would first like to see if the data of $\Gamma_L/\Gamma$, $R$ and $|P|^2$ can be explained in the factorization approach. As we have stressed in passing, naive factorization does not work for the decay rate of color-suppressed decay modes; nonfactorizable contributions should always be included in order to describe the branching ratios of $B \to J/\psi K(K^*)$. However, if $\chi_{F_1} = \chi_{A_1} = \chi_{A_2} = \chi_V$, we have a new factorization scheme but the predictions of $\Gamma_L/\Gamma$, $R$ and $|P|^2$ in the naive factorization method remains intact since all nonfactorizable terms are canceled out in Eqs.(14) and (20). To proceed, we consider several phenomenological models of form factors: (1) the Bauer-Stech-Wirbel model (BSWI) [15] in which $B \to K(K^*)$ form factors are first evaluated at $q^2 = 0$ and then extrapolated to finite $q^2$ using a monopole behavior for all form factors, (2) the modified BSW model (BSWII) [16], which is the same as BSWI except for a dipole $q^2$ dependence for form factors $F_1$, $A_0$, $A_2$ and $V$, (3) the nonrelativistic quark model by Isgur, Scora, Grinstein and Wise (ISGW) [17] with exponential $q^2$ dependence for all form factors, (4) the model of Casalbuoni et al. and Deandrea et al. (CDDFGN) [18] in which form factors are first evaluated at $q^2 = 0$ using heavy meson effective chiral Lagrangians, which incorporate heavy mesons, light pseudoscalar mesons and light vector mesons, and then extrapolated with monopole behavior. Several authors have derived the $B \to K(K^*)$ form factors from experimentally measured $D \to K(K^*)$ form factors at $q^2 = 0$ using the Isgur-Wise scaling laws based on heavy quark symmetry [19], which are allowed to relate $B$ and $D$ form factors at $q^2$ near $q^2_{\text{max}}$. The $B \to K(K^*)$ form factors are calculated in [1] [IW(i)] by assuming a monopole dependence for all form factors, while they are computed in [20] [IW(ii)] by advocating a monopole extrapolation for $F_1$, $A_0$, $A_1$, a dipole behavior for $A_2$, $V$, and an approximately constant for $F_0$. An ansatz proposed in [2], which we call IW(iii), relies on “soft” Isgur-Wise scaling laws and different $q^2$ behavior of $A_1$ from $A_2$, $V$, $F_1$. 

5
In all above form-factor models, the $q^2$ dependence of form factors is assumed to be
governed by near pole (monopole or dipole) dominance. It is thus important to have a
first-principles or model calculation of the form-factor $q^2$ dependence. In principle, QCD
sum rules, lattice QCD simulations, and quark models allow one to compute form-factor
$q^2$ behavior. However, the analyses of the QCD sum rule yield some contradicting results.
For example, while $A_1^{B\rho}$ is found to decrease from $q^2 = 0$ to $q^2 = 15 \text{ GeV}^2$ in [21], such a
phenomenon is not seen in [22, 23]. Also the sum-rule results become less reliable at large $q^2$
due to a large cancellation between different terms. The present lattice QCD technique
is not directly applicable to the $B$ meson. Additional assumptions on extrapolation from
charm to bottom scales and from $q^2_{\text{max}}$ to other $q^2$ have to be made. As for the quark
model, a consistent treatment of the relativistic effects of the quark motion and spin in a
bound state is a main issue of the relativistic quark model. To our knowledge, the light-
front quark model [24] is the only relativistic quark model in which a consistent and fully
relativistic treatment of quark spins and the center-of-mass motion can be carried out. A
direct calculation of $P \rightarrow P$ and $P \rightarrow V$ form factors at time-like momentum transfer
in the relativistic light-front (LF) quark model just became available recently [25]. The
contributions from valence-quark configuration to $B \rightarrow K(K^*)$ form factors are [25]:

\begin{align}
F_{1}^{BK}(m_{J/\psi}^2) &= 0.66, & A_{1}^{BK^*}(m_{J/\psi}^2) &= 0.37, \\
A_{2}^{BK^*}(m_{J/\psi}^2) &= 0.43, & V_{BK^*}(m_{J/\psi}^2) &= 0.50. \quad (21)
\end{align}

We found that $F_{1}^{BK}$, $A_{1}^{BK^*}$, $A_{2}^{BK^*}$, $V^{BK^*}$ exhibit a dipole behavior, while $A_{1}^{BK^*}$ shows a
monopole dependence in the close vicinity of $q^2 = 0$ [25]. Table I summerizes the predictions
of $\Gamma_L/\Gamma$, $R$ and $|P|^2$ in above-mentioned various form-factor models within the factorization
approach by assuming the absence of inelastic final-state interactions.

The pre-1996 values of $R$ and $\Gamma_L/\Gamma$ are given by

\begin{equation}
R = 1.68 \pm 0.33, \quad \Gamma_L/\Gamma = 0.74 \pm 0.07, \quad (22)
\end{equation}

where the former is the average value of ARGUS [26] and CLEO [27], and the latter is the
combined average of ARGUS: $\Gamma_L/\Gamma = 0.97 \pm 0.16 \pm 0.15$ [26], CLEO: $\Gamma_L/\Gamma = 0.80 \pm 0.08 \pm 0.05$
[27], and CDF: $\Gamma_L/\Gamma = 0.65 \pm 0.10 \pm 0.04$ [28]. It is obvious from Table I that all the
existing models fail to produce a large longitudinal polarization fraction, whereas several

\footnote{In the light-front model calculations, the valence-quark contributions to form factors $A_0$, $A_1$, $V$ depend
on the recoiling direction of the $K(K^*)$ relative to the $B$ meson [25]. Thus the inclusion of the non-valence
configuration arising from quark-pair creation is in principle necessary in order to ensure that the physical
form factors are independent of the recoiling direction. Since the non-valence contribution is most important
only near zero recoil, we expect that $Z$-graph effects on $B \rightarrow K(K^*)$ form factors at $q^2 = m_{J/\psi}^2$ obtained in
the “+" frame, where $K^*$ is moving in the $+z$ direction in the rest frame of the $B$ meson, are not important.
As noted in [25], the behavior of $V^{BK^*}$ in the “+" frame is peculiar in the sense that in general the form
factor in the “+" frame is larger than that in the “−" frame, so we have taken $V^{BK^*}$ from the “−" frame
and other form factors from the “+" frame.}
Table I. Predictions for form-factor ratios $x$, $y$, $z$ and for $\Gamma_L/\Gamma$, $R$ and $|P|^2$ in various form-factor models.

| Model   | $x$   | $y$   | $z$   | $\Gamma_L/\Gamma$ | $R$   | $|P|^2$ |
|---------|-------|-------|-------|--------------------|-------|--------|
| BSWI    | 1.01  | 1.19  | 1.23  | 0.57               | 4.22  | 0.09   |
| BSWII   | 1.41  | 1.77  | 1.82  | 0.36               | 1.63  | 0.24   |
| ISGW    | 2.00  | 2.55  | 2.30  | 0.07               | 1.72  | 0.52   |
| CDDFGN  | 1.00  | 3.24  | 2.60  | 0.37               | 1.50  | 0.30   |
| IW(i)   | 0.98  | 2.56  | 1.74  | 0.45               | 2.89  | 0.31   |
| IW(ii)  | 0.88  | 1.77  | 2.05  | 0.56               | 1.84  | 0.16   |
| IW(iii) | 1.08  | 2.16  | 1.86  | 0.45               | 2.15  | 0.26   |
| LF      | 1.16  | 1.35  | 1.78  | 0.50               | 1.84  | 0.13   |
| pre-1996 expt. | | | | 0.74 ± 0.07 | 1.68 ± 0.33 | – |
| CDF     | | | | 0.65 ± 0.11 | 1.32 ± 0.28 | – |
| CLEO    | | | | 0.52 ± 0.08 | 1.36 ± 0.20 | 0.16 ± 0.09 |

models give satisfactory results for the production ratio. Since the prediction of $\Gamma_L/\Gamma$ in new factorization with $\chi_{A_1} = \chi_{A_2} = \chi_V$ is the same as that in naive factorization, this means that if the longitudinal polarization fraction is as large as 0.74 ± 0.07, then nonfactorizable terms should contribute differently to $S$-, $P$- and $D$-wave amplitudes. Consequently, the effective parameter $a_2^{\text{eff}}$ cannot be defined for $B \to J/\psi K^*$ decays if $\Gamma_L/\Gamma$ is large. In order to produce a large $\Gamma_L/\Gamma$, various possibilities of nonfactorizable contributions to $B \to J/\psi K^*$, e.g., $S$-wave dominance: $\chi_{A_1} \neq 0$, $\chi_{A_2} = \chi_V = 0$, have been explored in [10, 13, 29].

The new results from CDF [28, 3] on the branching-ratio measurements of $B \to J/\psi K (K^*)$ decays give rise to

$$R = 1.32 \pm 0.23 \pm 0.16.$$  \hspace{1cm} (23)

Recently, CLEO II [4] has completed the analysis of all data sample and come out with the results:

$$R = 1.36 \pm 0.17 \pm 0.11,$$
$$\Gamma_L/\Gamma = 0.52 \pm 0.07 \pm 0.04,$$
$$|P|^2 = 0.16 \pm 0.08 \pm 0.04.$$  \hspace{1cm} (24)

Evidently, the new data of $\Gamma_L/\Gamma$ and $R$ tend to decrease. As a consequence, the comparison between theory and experiment is turned the other way around: While several model predictions for $\Gamma_L/\Gamma$ are consistent with CLEO or CDF, almost all models (except for CDDFGN)

\footnote{The previous CLEO result [27]: $\Gamma_L/\Gamma = 0.80 \pm 0.08 \pm 0.05$ is based on a subset of the data used in this complete analysis.}
fail to accommodate a small $R$. It appears that IW(ii) and LF models are most close to the data: their results for $\Gamma_L/\Gamma$ and $|P|^2$ are in agreement with experiment, but the predicted $R$ is too large by 2 standard deviations compared to the experimental central value. Hence, the factorization approach with $\chi_{A_1} = \chi_{A_2} = \chi_\nu$ can account for the longitudinal polarization fraction and parity-odd transverse polarization observed in $B \to J/\psi K^*$. In order to explain the production ratio and $\Gamma_L/\Gamma$ simultaneously, it is clear from Eq.(14) that we need

$$\chi_{F_1} > \chi_{A_1} \sim \chi_{A_2} \sim \chi_\nu,$$

(25)

noting that $c_1/a_2 > 0$. Therefore, the data of $B \to J/\psi K(K^*)$ can be understood provided that the nonfactorizable terms in $B \to J/\psi K^*$ are about the same in $S$-, $P$-, and $D$ waves and that $\chi_{F_1}$ in $B \to J/\psi K$ is slightly larger than that in $B \to J/\psi K^*$. The relativistic LF quark model should be more reliable and trustworthy than the IW(ii) model since the $q^2$ behavior of form factors is directly calculated in the former. We will thus confine ourselves to the LF model in ensuing discussion.

To determine the magnitude of nonfactorizable terms we have to consider the branching ratios. The decay rates of $B \to J/\psi K$ and $B \to J/\psi K^*$ are given by

$$\Gamma(B \to J/\psi K) = \frac{p_c^3}{4\pi} \left| a_{2\text{eff}}^F V_{cs} V_{cb}^* f_{J/\psi} F_{1BK}^B (m_{J/\psi}^2) \right|^2,$$

$$= 1.102 \times 10^{-14}\text{GeV} \left| a_{2\text{eff}}^F V_{1BK}^B (m_{J/\psi}^2) \right|^2, \tag{26}$$

and

$$\Gamma(B \to J/\psi K^*) = \frac{p_c}{16\pi m_B^2} \left| a_{2\text{eff}}^F V_{cs} V_{cb}^* f_{J/\psi} m_{J/\psi} (m_B + m_{J/\psi}) A_{1BK}^B (m_{J/\psi}^2) \right|^2 \times \left[ (a - bx)^2 + 2(1 + c^2 y^2) \right],$$

$$= 1.19 \times 10^{-14}\text{GeV} \left| a_{2\text{eff}}^F A_{1BK}^B (m_{J/\psi}^2) \right|^2 \left[ (a - bx)^2 + 2(1 + c^2 y^2) \right], \tag{27}$$

where $p_c$ is the c.m. momentum, and uses of $f_{J/\psi} = 394$ MeV extracted from $J/\psi \to e^+e^-$ and $|V_{cb}| = 0.038$ have been made. Fitting (26) and (27) to the branching ratios

$$B(B^+ \to J/\psi K^+) = \begin{cases} (1.08 \pm 0.09 \pm 0.09) \times 10^{-3}; \\
(1.01 \pm 0.14) \times 10^{-3},\end{cases}$$

$$B(B^0 \to J/\psi K^0) = \begin{cases} (0.92^{+0.17}_{-0.15} \pm 0.08) \times 10^{-3}; \\
(1.15 \pm 0.23 \pm 0.17) \times 10^{-3};\end{cases}$$

$$B(B^+ \to J/\psi K^{*+}) = \begin{cases} (1.41 \pm 0.20 \pm 0.24) \times 10^{-3}; \\
(1.58 \pm 0.47 \pm 0.27) \times 10^{-3};\end{cases}$$

$$B(B^0 \to J/\psi K^{*0}) = \begin{cases} (1.32 \pm 0.15 \pm 0.17) \times 10^{-3}; \\
(1.36 \pm 0.27 \pm 0.22) \times 10^{-3};\end{cases} \tag{28}$$

For other form-factor models, one can always adjust small nonfactorizable contributions (for example, $\chi_{A_1} > \chi_{A_2} > \chi_\nu$) to accommodate the data, as illustrated in [10, 13, 29]. However, in practice, it is very difficult to pin down $\chi_{A_1}$, $\chi_{A_2}$ and $\chi_\nu$ separately if they are not the same. Our results (25) and (31) correspond to a minimal modification to the factorization hypothesis.

Since we have taken into account the $q^2$ dependence of the fine-structure constant, our values of $f_{J/\psi}$ and $f_{\psi'}$ given below are slightly larger than the values cited in the literature.
where the upper entry refers to the CLEO data \[3\] and the lower entry (except for \(B(B^+ \to J/\psi K^+ )\)) which comes from the PDG \([4]\) to CDF data \([5]\), we find averagely

\[
\begin{align*}
  a_2^{\text{eff}}(B \to J/\psi K) &= 0.30, \\
  a_2^{\text{eff}}(B \to J/\psi K^*) &= 0.26,
\end{align*}
\]

for the lifetimes \([6]\)

\[
\tau(B^0) = 1.56 \times 10^{-12}\text{s}, \quad \tau(B^\pm) = 1.62 \times 10^{-12}\text{s},
\]

and the form factors (21). Using \(c_1(m_b) = 1.12\) and \(c_2(m_b) = -0.27\) for \(\Lambda_{\text{QCD}}^{(5)} = 200\text{ MeV}\), we obtain

\[
\chi_{F_1} = 0.17, \quad \chi_{A_1} \sim \chi_{A_2} \sim \chi_V = 0.14.
\]

Since \(a_2^{\text{eff}}(B \to J/\psi K^{(*)})\) is numerically close to \(c_2(m_b)\), one may tempt to argue that the large-\(N_c\) approach works also for \(B\) decays. However, this possibility is ruled out by the observed constructive interference in charged \(B^\pm \to D^{(*)}\pi(\rho)\) decays \([27]\), which implies a positive ratio \(a_2/a_1\). Therefore, the sign of \(a_2^{\text{eff}}\) in (29) and \(\chi_2\) in (31) is fixed to be positive, in dramatic contrast to the charm case. The result (31) supports the conjecture (13), but the question of why the nonfactorizable term, which lies in the range \(-0.60 < \chi_2 < -1/3\) for charm decay \([13]\), becomes positive in \(B\) decay remains mysterious and baffling. (For a recent attempt of understanding the positive sign of \(a_2^{\text{eff}}\) and \(\chi_2\) in \(B\) decay, see \([30]\).)

With the result (29) or (31) we are ready to discuss \(B \to \psi'K(K^*)\) decays where \(\psi' \equiv \psi(2S)\). The analytic expressions for analogous \(\Gamma'_L/\Gamma', \ R'\) and \(|P'|^2\) are the same as (14), (15) and (20) except that the coefficient 1.08 in (14) is replaced by 2.45 and that the unprimed quantities \(a, \ b, \ c, \ x, \ y, \ z\) are replaced by primed ones:

\[
\begin{align*}
  a' &= 2.051, \quad b' = 0.733, \quad c' = 0.356, \\
  x' &= \frac{A_1^{BK^*}(m_{\psi'}^2)}{A_1^{BK}(m_{\psi'}^2)}, \quad y' = \frac{V_{BK^*}(m_{\psi'}^2)}{A_1^{BK^*}(m_{\psi'}^2)}, \quad z' = \frac{F_1^{BK}(m_{\psi'}^2)}{A_1^{BK^*}(m_{\psi'}^2)}.
\end{align*}
\]

We find in the LF model that

\[
F_1^{BK}(m_{\psi'}^2) = 0.92, \quad A_1^{BK^*}(m_{\psi'}^2) = 0.44, \quad A_2^{BK^*}(m_{\psi'}^2) = 0.55, \quad V^{BK^*}(m_{\psi'}^2) = 0.66. \quad (33)
\]

Using \(f_{\psi'} = 293\text{ MeV}\) determined from \(\psi' \to e^+e^-\), and assuming \(a_2^{\text{eff}}(B \to \psi'K) \sim a_2^{\text{eff}}(B \to J/\psi K)\) and \(a_2^{\text{eff}}(B \to \psi'K^*) \sim a_2^{\text{eff}}(B \to J/\psi K^*)\), we obtain

\[
\begin{align*}
  \mathcal{B}(B^0 \to \psi'K^0) &= 0.50 \times 10^{-3}, \quad \mathcal{B}(B^+ \to \psi'K^+) = 0.52 \times 10^{-3}, \\
  \mathcal{B}(B^0 \to \psi'K^{*0}) &= 0.76 \times 10^{-3}, \quad \mathcal{B}(B^+ \to \psi'K^{*+}) = 0.79 \times 10^{-3},
\end{align*}
\]

and

\[
\Gamma'_L/\Gamma' = 0.33, \quad R' = 1.57, \quad |P'|^2 = 0.15. \quad (35)
\]
Our results for branching ratios agree with the new CDF measurements [3]:

\[
B(B^+ \to \psi' K^+) = (0.68 \pm 0.10 \pm 0.14) \times 10^{-3}, \\
B(B^0 \to \psi' K^{*0}) = (0.90 \pm 0.21 \pm 0.20) \times 10^{-3}. 
\] (36)

Note that our prediction of \( \Gamma'L/\Gamma' \) is quite different from the predicted range \( 0.50 < \Gamma'L/\Gamma' \leq 0.67 \) given in [31].

Finally we turn to the decay mode \( B_s^0 \to J/\psi \phi \). Following [25], the relevant form factors are calculated in the LF model to be

\[
A_1^{B_s \phi}(m_{J/\psi}^2) = 0.35, \quad A_2^{B_s \phi}(m_{J/\psi}^2) = 0.39, \quad V^{B_s \phi}(m_{J/\psi}^2) = 0.49. 
\] (37)

A straightforward calculation yields

\[
\Gamma_L/\Gamma(B_s \to J/\psi \phi) = 0.51, \quad |P|^2(B_s \to J/\psi \phi) = 0.13, 
\] (38)

and

\[
B(B_s \to J/\psi \phi) = 1.29 \times 10^{-3} \] (39)

for \( a_2^{eff} \sim 0.25 \). Experimentally, longitudinal polarization fraction and the branching ratio have been measured by CDF:

\[
\Gamma_L/\Gamma = 0.56 \pm 0.21^{+0.02}_{-0.04} \quad [28], \quad B(B_s \to J/\psi \phi) = (0.93 \pm 0.28 \pm 0.17) \times 10^{-3} \quad [3]. 
\] (40)

To summarize, naive factorization or new factorization with universal nonfactorizable terms cannot explain the data of \( \Gamma_L/\Gamma, R \) and \( |P|^2 \) simultaneously in all existing form-factor models. However, we have shown that a minimal modification to the factorization hypothesis can accommodate the recently available data from CDF and CLEO: the nonfactorizable term \( \chi_{F1} \) in \( B \to J/\psi K \) should be slightly larger than \( \chi_{A1} \sim \chi_{A2} \sim \chi_{V} \) in \( B \to J/\psi K^* \) decays. When generalized to \( B \to \psi(2S)K(K^*) \) and \( B_s \to J/\psi \phi \), our predictions agree with presently available measurements. Generally speaking, the bulk of exclusive decays of heavy mesons can be grossly accounted for by the factorization approach with universal nonfactorizable contribution \( \chi \lesssim -1/3 \) for charm decays and with \( \chi \sim 0.15 \) for bottom decays; that is, the new factorization scheme is very different for charm and bottom decays. Just as the charm case, the data of \( B \to J/\psi K(K^*) \) start to reveal departures from the factorization postulation: the nonfactorizable term \( \chi \) in \( B \) decay is also process dependent and not universal.

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