A Model of Endogenous Debt Maturity with Heterogeneous Agents

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Abstract

This paper studies how investor heterogeneity determines equilibrium debt maturity in a model with default. The traditional representative investor assumption is a special case where a weak form of the Modigliani-Miller theorem holds; allocations with either all long- or all short-term debt are equivalent and dominate any interior combination of the two. By contrast, in the general heterogeneous investor setting, debt maturity always matters. A maturity strategy that combines long- and short-term debt is always optimal and dominates either corner solution. Thus, the model highlights the importance of supply frictions in determining equilibrium issuance strategies and is consistent with numerous empirical findings.

Keywords: Debt, heterogeneous agents, general equilibrium, investment, cost of capital

JEL Codes: D92, G11, G12, G31, G32, E22

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1 Introduction

Large and mature corporations typically raise capital by issuing debt of various maturities. Figure 1 shows that over 40% of all investment-grade firms use more than one debt maturity in a given quarter for large debt offerings of generally $1 billion or more.\footnote{Some specific high-profile examples of multiple maturity issuances are the following: IBM issued 5 different maturities over the first 6 months of 2017. In July 2017, AT&T raised $22.5 billion with maturities ranging from 5.5 to 41 years. In 2013, Verizon raised $49 billion with 6 different maturities. Microsoft offered 7 different maturities to raise $19.75 billion in 2016.} This paper develops a model with supply frictions stemming from investor heterogeneity to explain this fact rather than the traditional approach to debt maturity that relies on firm demand frictions.

We introduce heterogeneous and capacity-constrained investors into a standard model with production and default. Investor heterogeneity is modeled à la Geanakoplos (2009) and Fostel and Geanakoplos (2012, 2016) through a continuum of investors with different repayment expectations. The model nests the traditional representative and deep-pocketed investor as a special case and serves as the benchmark. Maturity choice is modeled as a simple tradeoff between repayment incentives arising from short-term debt and the hedging benefit of long-term debt against falling future prices. The model characterizes the optimal debt issuance strategy when borrowers can choose between issuing either all long- or short-term claims, or a combination of the two.

We show two results. First, a “weak” form of the Modigliani-Miller irrelevance theorem holds in the benchmark representative investor setting. In particular, firms are indifferent to financing investment with either long- or short-term claims despite incomplete markets and default. In this special case, the single repayment expectation pins down all risky debt prices irrespective of a firm’s financing choice. Hence, financial policy in either corner solution does not alter the investor’s consumption allocation and does not affect firm budget sets. However, a financial policy that uses a combination of debt maturities does affect the allocation because of debt dilution. Specifically, short-term debt dilutes the value of long-term claims for which the investor requires additional compensation. Thus, the cost of long-term debt is lower when short-term debt is also issued compared with
the cost of long-term debt in the corner solution. Consequently, firm budget sets contract when issuing both long- and short-term claims to the representative investor. The fact that firms are indifferent to either all long- or short-term debt then implies that either corner solution dominates the interior combination.

The main result states that financial policy always matters in a general heterogeneous investor setting. In particular, we show that a combination of both long- and short-term debt is always optimal relative to either corner solution. Investor heterogeneity with capacity constraints introduces as a substitution benefit that is not present in the representative investor setting that counteracts the dilution cost. Thus, the interior debt issuance strategy allows firms to raise capital on better terms at each point in time than in either corner solution, which expands the budget set and allows for more investment.²

The framework is an incomplete markets economy with binomial states and three-periods: 1) an initial state, 2) intermediate states, and 3) terminal states. We introduce investor heterogeneity into a model with repayment enforcement problems. Creditors cannot coerce debtors into repayment, and firm output is pledgeable collateral. For simplicity, ²Note that heterogeneity and capacity constraints micro-founds the upward sloping capital supply curve. Mainly, heterogeneity and capacity constraints imply that prices must fall to clear the market as more debt is issued.
we consider risk neutral creditors with heterogeneous beliefs over the expected value of repayment.\textsuperscript{3} Investors are willing to pay different prices to hold risky debt, and these prices change in intermediate states as uncertainty is either resolved in good states or grows in bad states. Importantly, the prices of risky claims issued at either time 0 or 1 are determined by market clearing, for which investors demand additional compensation to hold risk as more claims are supplied to the market. We characterize the optimal debt issuance strategy between issuing either all long-term debt at time 0, all short-term debt that must be rolled over at time 1, or a combination of the two. To highlight the importance of investor heterogeneity in our model rather than liquidity risk or asymmetric information (see Diamond, 1991, and Flannery, 1986), equilibrium is characterized based on a sufficiently strong termination threat which makes short-term claims issued at time 0 endogenously risk-free.\textsuperscript{4}

The intuition for the results is the following. The representative investor prices default risk from a sequence of short-term claims equivalent to default risk in long-term term claims when issued in isolation. Thus, the two corner issuance strategies are always equivalent. Moreover, the expected marginal cost of long- and short-term debt must be equivalent in any equilibrium where both maturities co-exist. In the benchmark case with a representative investor, issuing both debt maturities introduces a dilution cost to long-term debt compared to the long-term corner solution. Therefore, the marginal cost of both debt maturities must be higher than either respective marginal cost in the corner solutions for the same investment level. Consequently, firms must reduce investment and the scale of their operation if they choose to issue both maturities. Similarly, for a given investment level, the dilution cost of the interior debt issuance strategy is highest as the liability structure converges to only using short-term debt. Therefore, as long-term claims become more diluted, the representative investor requires additional compensation that must equal the

\textsuperscript{3}Interpreting heterogeneity as differing beliefs is not necessary. What is crucial is that investors’ ex ante marginal utility of consumption across states differs. See Back (2010) for a formal statement of model equivalence.

\textsuperscript{4}Liquidity risk is a necessary condition to generate an equilibrium with both long- and short-term claims in Diamond (1991). Proposition 2 in his paper shows that all short-term financing is used when short-term claims are always honored in a non-terminal state.
compensation for holding short-term claims. Again, firms facing higher marginal cost in
the interior solution must reduce investment and the efficient scale of their production.

By contrast, in the heterogeneous investor setting, no single investor necessarily prices
the risk from short-term claims equivalent to long-term claims, which implies that the
corner solutions are no longer equivalent. Importantly, we show that the interior issuance
strategy contains a substitution benefit that counters the dilution cost that is not present
in the representative investor benchmark. Mainly, optimistic investors who want to hold
risk price the dilution cost. Compared to either corner solution, using both debt maturities
allows firms to substitute high marginal cost claims away from relatively less optimistic
investors at time 0 or 1 to more optimistic investors at time 1 or 0. For example, in
the corner solutions, market clearing requires less optimistic investors to hold additional
issuance of long-term (short-term) debt at time 0 (1). The firm can economize on financing
cost by placing a portion of the high marginal cost debt in one period to more optimistic
investors who demand risky assets in the other period. This strategy lowers the marginal
cost of both long- and short-term claims in the interior solution compared to the respective
marginal costs in either corner solution.

The mechanism in our model rests on supply frictions rather than the traditional firm
demand forces such as private information, liquidity risk, and agency frictions (Diamond,
1991 and Zwiebel, 1996). Control rents or inefficient liquidation or both is necessary to
generate equilibria with multiple maturities in these models. More importantly, though
models based on firm demand frictions are consistent with a number of empirical results,
they incorrectly predict that only the most risky firms with access to debt financing will
issue both long- and short-term debt rather than the largest and generally safest firms that
the data suggest. In addition, empirical and survey evidence suggests that firm demand
factors do not significantly affect how large and mature corporations with abundant public
information and analyst coverage raise debt (Graham and Harvey, 2001; Johnson, 2003;
Billet, King, and Mauer, 2007). In fact, Custodio, Ferreira, and Laureano (2013) find that
investor demand or supply factors have far more explanatory power than firm demand
factors in explaining why U.S. corporations use short-term debt in general.

4
The supply-side assumptions we adopt are quite natural for the following reasons: First, the holders of corporate debt consist of numerous different investor types such as insurance companies; hedge, pension, and mutual funds; and the investment banks who underwrite debt issuances. These institutions have different business models and are subject to different regulatory constraints that make them natural holders of different types of assets. Second, multiple investors are generally required to meet both individual and aggregate debt financing needs. For example, Dass and Massa (2014) show that, on average, 18 different institutions hold a given firm’s debt, and many institutions hold multiple maturities issued by the same firm. Additionally, the total outstanding amount of U.S. corporate bond debt is over $5 trillion, an amount that far exceeds any single institution’s funding capacity.

Finally, we consider an extension of the model that prevents debt dilution through secured debt or equivalent covenants. Specifically, we allow long-term debt to be secured by specific firm assets. We first show that the recovery value of long-term debt in a multiple maturity issuance strategy is the same as the recovery value in a long-term only issuance strategy; i.e., there is no dilution effect. We then show that utilizing a mix of long- and short-term debt remains the optimal debt maturity strategy. The reason is that the security interest does not expand firm budget sets when total debt issuance is a linear combination of the two maturities. The security interest simply reallocates the recoverable value of firm assets from short-term claimants to long-term claimants, to which the firm adjusts by issuing more long- and less short-term debt. In essence, the firm undoes the effect of the security interest in long-term debt through its maturity choice and secured debt is irrelevant in the heterogeneous investor case. However, in the representative investor setting, firm financial policy becomes completely irrelevant. The reason is that the interior solution is a linear combination of the two corner solutions, which are equivalent forms of financing.

The organization of the paper is as follows: The related literature is below. Section 2

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5Large corporate loans are also generally arranged through syndicates. Caglio, Darst, and Parolin (2016) show that larger corporates borrow from, on average, eight banks compared with small firms that tend to borrow from one.

6In section 4 and the appendix, we discuss that this extension can be interpreted as a negative pledge covenant, one of the most common covenants found in long-term corporate debt indentures.
introduces the model, agents, the different debt contracts considered. Section 3 presents examples to highlight the main mechanism. Section 4 derives the general solution to the model with the main analytical results and discusses the empirical relevance. Section 5 considers an extension with covenants. Section 6 concludes with a discussion of robustness to alternative assumptions. All proofs that are not obvious from the text are included in the appendix.

Related literature

Our paper introduces production and a role for debt maturity into general equilibrium models of debt financing with investor heterogeneity developed by Geanakoplos (2009), Fostel and Geanakoplos (2008, 2010, 2012, and 2016), and He and Xiong (2012). In these models, short-term financing is the unique equilibrium, and risky assets are in fixed supply. Rampini and Vishwanathan (2010, 2013) study investment in a general equilibrium setting but consider fully collateralized, one-period ahead arrow securities, for which debt maturity plays no role.

Our work is also related to the incomplete markets and heterogeneous agent framework of Allen and Gale (1991). They argue that firms cater to investor needs by splitting claims on cash flows into debt and equity securities. We endogenize investment and introduce a role for maturity to affect firm value. Many elements of our framework are similar to Bisin, Clementi, and Gottardi (2017) who show how to integrate production into general equilibrium analysis with incomplete markets. They analyze the efficiency properties of equilibrium. Our emphasis is on the role that debt maturity plays in the optimal behavior of the firm.

Our approach differs from corporate finance models with private information. Flannery (1986) shows that short-term debt is used to signal firm quality. Diamond (1991, 1993) shows that liquidity risk breaks the reliance on short-term debt and generates cross-sectional differences in maturity choice based on expected future credit ratings. Debt maturity plays a role in optimal contracting models of Hart and Moore (1994, 1995, 1998), but repayment paths are either the fastest or slowest, never a combination of the two. Zwiebel (1996) shows that multiple repayment paths are possible but only for the most risky firms.
for whom debt is a possible financing source. More broadly, dynamic debt models generally depend on firm demand frictions such as distance to default (He and Milbradt, 2016), debt overhang (Diamond and He, 2014), and inability to commit to financing policies (Brunnermeier and Oehmke, 2013).

A related strand of literature clarifies that the Modigliani-Miller irrelevance theorem holds in incomplete markets due to a linearity between the change in the return on firm equity and the return on the assets firms trade (DeMarzo (1988) and Gottardi (1995)). Default and derivative securities introduce a nonlinearity between returns that prevent the perfect substitutability between agents and firms needed for irrelevance. The same reasoning is why the long- and short-term financing regimes are equivalent for the representative agent. However, issuing both debt securities introduces debt dilution, which causes investors to demand additional compensation. An interior optimum requires equivalence between the marginal costs of the different securities, which means that the marginal cost of either security must be higher than if it were the only security issued.

Lastly, the international sovereign debt literature highlights the role that long-term debt maturity plays in smoothing income shocks (Angeletos (2002)). Arellano and Ramnarayan (2012) introduce a maturity choice with the same basic maturity trade off in our model; repayment incentives in future periods against the hedging benefit against falling prices in future bad states. Aguiar, Amador, Hopenhayn, and Werning (2018) study a similar mechanism to ours in this literature by showing how a sovereign’s budget set responds to equilibrium bond prices resulting from its maturity choice.

2 Model

2.1 Time and uncertainty

The model is a three-period production economy with incomplete asset markets. Time is denoted \( t = \{0, 1, 2\} \). Uncertainty is given by a tree of state events \( s \in S \) with root \( s_0 \).

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7See also Jensen and Meckling (1986) and Bolton and Scharfstein (1990, 1998) for additional models on the optimality of debt in the presence of agency frictions.
intermediate states $s \in S$ that take values $\{U,D\}$, and a set of terminal nodes denoted $S_T = \{UU, UD, DU, DD\} \subset S$. Let state realization $U$ be up or a “good” state and $D$ be down or a “bad” state. There is a single durable consumption good available in the economy at $t = 0$, which is the numeraire.

The only uncertainty in the model is an aggregate shock that affects output at $t = 2$. The parameter $A_{s_T}$ captures the effect of the shock to production. The expected value of the shock is conditional on the information revealed at $t = 1$. We assume for simplicity that good news at $t = 1$ resolves uncertainty at $t = 2$ and there is no shock: $A_{UU} = A_{UD} = 1$. Bad news at $t = 1$ raises uncertainty at $t = 2$ about the ability of the firm to repay debts, akin to “scary bad news” in Geanakoplos (2009). Specifically, there is no shock at terminal node $s = DU$, but there is a shock at terminal node $s = DD$, $A_{DD} < A_{DU} = 1$. Note that this uncertainty structure is the same as the simplification made in the continuous time version of Diamond and He (2014). Figure 2 depicts the economy’s state tree.

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8In example 2 of Diamond and He (2014), they assume that asset volatility is state contingent. Specifically, $\sigma_H = \varepsilon > 0 = \sigma_L$ where $\sigma_i$ is asset volatility conditional on state $i$. Clearly uncertainly is resolved when $\sigma_L = 0$. 

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2.2 Debt contracts

There are two types of debt contracts in the economy. Short-term debt matures after one period and long-term debt matures after two periods. All debt contracts are non-contingent and pay zero-coupons. For simplicity, we normalize the repayment value of each contract to 1.

Let the quantity of debt issued at any state and time be \( q_{s_t} \). The quantity of long-term debt issued at \( t = 0 \) is denoted \( q^\ell \) and the market price denoted by \( p^\ell \). Short-term debt may be issued at \( t = 0, 1 \). Let \( q^\varsigma \) denote the quantity of short-term debt issued at \( t = 0 \) and \( q^\varsigma_s \), \( s = U, D \) denote short-term debt issued at \( t = 1 \). The prices of short-term debt at \( t = 0, 1 \) are respectively \( p^\varsigma, p^\varsigma_U, p^\varsigma_D \). Following much of the literature, we assume equal seniority between short- and long-term debt.

The key friction in our model is that agents cannot be coerced to repay debts. As in Rampini and Vishwanathan (2010, 2013) and Fostel and Geanakoplos (2016), the pledgable value of the firm serves as the payment enforcement mechanism. Specifically, creditors have the right to confiscate firm assets up to the value of the promise but nothing more. “Collateral” in our economy will be the firm itself, and can be thought of as the physical assets it produces from its investment decision.

2.3 Agents

We first describe the firm and its objective, followed by the investors’ problem.

2.3.1 Firm

There are a large number of identical price taking firms, which allows us to focus on a representative firm. A manager (equity claimant) operates the firm with access to a two-period decreasing returns to scale production technology.

The production function is denoted by \( f(I; \alpha, A_s) = A_s I^{\alpha}, \alpha < 1 \), where \( I \) is the amount of capital the manager raises at time 0 and puts into production. We follow Diamond (1991) and assume the firm has no cash endowment, does not generate cash flow at \( t = 1 \),
and that new promises issued at \( t = 1 \) do not increase the initial investment \( I \). \(^9\)

The firm maximizes expected profits, choosing investment and the maturity of the debt contracts it issues subject to limited liability.\(^{10}\) Let \( \rho \) denote the portion of debt that is raised long-term, \( \rho = \frac{\ell q}{f} \), and let \( \gamma \) denote the probability of good news. Formally, the firm maximizes the following problem:

\[
\begin{align*}
\max_{I, \rho} & \quad \prod_s \max \left( A_s I^\alpha - q^\ell - q^\varsigma, 0 \right) \\
\text{s.t.} & \quad I = p^\ell q^\ell + p^\varsigma q^\varsigma \\
& \quad 0 \leq \rho = \frac{\ell q}{f} \leq 1 \\
\end{align*}
\]

where \( \prod_s \) is the product of state probabilities along each path from \( t = 0 \) \( \rightarrow \) 2.

At \( t = 1 \), the firm must decide whether it is beneficial to roll over the short-term component of its debt portfolio. The firm repays short-term debt holders by raising \( p^\varsigma q^\varsigma = q^\varsigma, s = \{U, D\} \). For simplicity, we assume the firm can always repay debts conditional on good news at \( t = 1 \) and \( p^\varsigma_U = 1 \). Bad news raises uncertainty about repayment and \( p^\varsigma_D < 1 \) if the firm defaults at \( t = 2 \).

The price of short-term debt issued at time 0 depends on if there is default at \( t = 1 \). If there is no default, short-term debt is initially risk-free and \( p^\varsigma = 1 \). If there is default, \( p^\varsigma < 1 \), the firm makes no profits and all assets are liquidated and distributed to creditors pro rata. If the project is liquidated, creditors receive \( \delta I \) fraction of the assets available within the firm, where \( \delta < 1 \).\(^{11}\) We make the following assumptions on parameters to keep the problem interesting and highlight the importance of supply considerations in our

\(^9\)Alternatively, one could assume that there is an extreme form of limited commitment at the interim date in which no cash flows can be verified at a reasonable cost so debt repayments cannot come from cash flow. Under this alternative, cash flow is independent of how the project is financed. The debt maturity mix will affect the investment cost that generates the cash flow, and management would still issue the types of debt securities to minimize these cost as in our model.

\(^{10}\)We restrict the analysis to debt and abstract away from equity without loss of generality for the maturity structure. Incorporating equity would not change the optimality of the types of debt the firm issues. The reason is the same as Allen and Gale (1991). The firm will want to split the claims it issues into debt and equity. The most optimistic investors will purchase equity and the next most will purchase long-term debt at time 0 and risky short-term debt at time 1. Allowing for equity would simply push down the set of investors that purchase risky debt in each period as the most optimistic would purchase the equity security.

\(^{11}\)The fractional recovery is a stand-in for any number of reasons why liquidation is costly: for example, bankruptcy costs or inefficient second-best use of assets, etc.
mechanism.

**Assumption 1  Parameter restrictions governing default**

- To ensure there is fundamental credit risk in the economy, let \( A_{DD} < \alpha \).
- To prevent liquidation at \( t = 1 \), let \( \delta < \bar{\delta} \), where \( \bar{\delta} \) is defined at the end of Lemma 1 in Appendix A.

As will become clear, the first condition ensures that no debt is mechanically risk free. The second condition implies that intermediate liquidation needs to be costly so that it is not optimal to issue risky short-term debt claims at time 0. One could allow for liquidity risk at time 1, but it would obfuscate the mechanism driving optimal maturity structure relative to existing demand-side theories.\(^{12}\) Furthermore, the parameter restriction is also intuitive. For example, if investors are second-best operators of the firm, or alternatively, bankruptcy costs are low, then the default premium on short-term debt at time 0 will be very small, ensuring its tautological existence.

**Lemma 1  Short-term rollover:** Given assumption 1, if a funding strategy with short-term debt exists, it is unconditionally rolled over \( t = 1 \), and \( p^s = 1 \).

The proof of lemma 1 also demonstrates that a risky short-term funding strategy, while always feasible, is not always optimal relative to a long-term funding strategy. In this sense, the parameter restrictions are not necessary; we maintain them to distinguish the mechanism.

Lemma 1 allows us to simplify the firm’s problem when considering a funding strategy that involves short-term debt. In particular, the roll over condition for short-term debt is given by \( q^s = p^s q_s^s s = U, D \), and the firm becomes:

\[
\max_{l, \rho} \prod \gamma \left( I^\alpha - \rho I \frac{1 - \rho}{p^s} I - \frac{(1 - \rho) I}{p^s} \right) + (1 - \gamma) \gamma \left( I^\alpha - \rho I \frac{1 - \rho}{p_D} I \right)
\]

\(^{12}\)See Diamond (1991) Proposition 2 on the necessity of liquidity risk in models with private information.
where the constraints $\rho = \frac{q^\ell}{q} \quad \text{and} \quad q^\varsigma = p_D^\varsigma q_D^\varsigma$ are substituted to write the problem in terms of choice variables $I$ and $\rho$.

If an interior maximum for $\rho$ exists, the first-order necessary conditions with respect to $I$ and $\rho$, respectively, are

\begin{align*}
\alpha I^{\alpha - 1} \left[ 1 - (1 - \gamma)^2 \right] &= \frac{\rho}{p^\ell} \left[ 1 - (1 - \gamma)^2 \right] + (1 - \rho) \left[ \gamma + \frac{\gamma(1 - \gamma)}{p_D^\varsigma} \right] \\
\left[ \frac{1 - (1 - \gamma)^2}{p^\ell} \right] &= \gamma + \frac{\gamma(1 - \gamma)}{p_D^\varsigma}.
\end{align*}

Equation (2) says that the marginal product of capital in states where the firm makes profits—which occurs with probability $1 - (1 - \gamma)^2$—must equal the maturity-weighted expected marginal cost of debt. The marginal cost of long-term debt is given by $\frac{1 - (1 - \gamma)^2}{p^\ell}$ and the marginal cost of a sequence of short-term bonds is given by $\frac{\gamma + \gamma(1 - \gamma)}{p_D^\varsigma}$. With probability $\gamma$, the firm will issue a sequence of risk-free bonds ($p_U^\varsigma = 1$). With probability $\gamma(1 - \gamma)$, the firm pays a higher short-term borrowing cost $p_D^\varsigma < p_U^\varsigma = p^\varsigma = 1$ per bond to roll over existing claims. Equation (3) says the marginal cost of a long-term bond must equal the expected marginal cost of a sequence of short-term bonds. Intuitively, firms issue debt with the lowest marginal cost. Note that equations (2) and (3) can be combined to express a single necessary condition in terms of either maturity:

\begin{align*}
\alpha I^{\alpha - 1} &= \frac{1}{p^\ell}, \quad (4) \\
\alpha I^{\alpha - 1} \left( 1 - (1 - \gamma)^2 \right) &= \left[ \gamma + \frac{\gamma(1 - \gamma)}{p_D^\varsigma} \right]. \quad (5)
\end{align*}

The above equations are also the first order conditions for an optimum for the corner solutions obtainable by substituting $\rho = 0 \text{ or } 1$ into the firm’s maximization problem.

We now turn to the investors’ problem and show how to solve for equilibrium debt pricing and determine the optimal funding strategy.
2.3.2 Investors

There exists at \( t = 0 \) a continuum of uniformly distributed investors with unit mass, \( h \in H \sim U [0, 1] \), each of whom is endowed with the durable consumption good in all non-terminal states. Investors are risk-neutral, expected utility maximizers that consume at \( t = 2 \), and do not discount the future. Without loss of generality, we assume investors have different priors.\(^{13}\) The uniform distribution is not critical to the optimal issuance strategy we derive. It simply allows one to rank investors according to the likelihood each places on the subsequent state being good, denoted by \( h \). Assuming alternative distributions on beliefs would quantitatively affect how debt is priced, but will not change the outcome that different investors may price debt in different periods, depending on how much long-versus short-term debt is issued. What is important is that investors value debt differently—through some form of heterogeneity—and no single investor can finance the economy’s financing needs.\(^{14}\)

Investors also have access to a riskless storage technology and form portfolios consisting of cash and debt securities issued by firms. The von-Neumann-Morgenstern preferences are given by:

\[
U^h (x_{UU}, x_{UD}, x_{DU}, x_{DD}) = h^2 x_{UU} + h(1-h) x_{UD} + (1-h) h x_{DU} + (1-h)^2 x_{DD}. \tag{6}
\]

Given debt prices, \( (p^f, p^S, p^U, p^D) \), each investor, \( h \in H \), chooses cash holdings, \( \{x^h, x^h_D, x^h_U\} \), debt holdings, \( \{q^{f,h}, q^{S,h}, q^{U,h}, q^{D,h}\} \), and final period consumption decisions, \( \{x^h_s\}, s \in \)...

\(^{13}\)Back (2010) provides a formal proof of the asset pricing equivalence between heterogeneous agents and homogeneous agents with state-dependent utility models. One could assume investors differ in a measure of risk aversion; have different endowments across states, which produces different marginal utilities across states; or have different degrees of “patience.” The critical assumption is the heterogeneity of marginal utilities across investors. We choose to think about beliefs because it is most familiar in these models.

\(^{14}\)If one could, heterogeneity would be immaterial and the investor with the highest marginal valuation for firm assets would fund all investment, which is equivalent to assuming a representative agent.
$S_T$, to maximize utility given by (6) subject to the budget set defined by:

$$B^h \left( p^\ell, p^\varsigma, p^U, p^D \right) = \left\{ \left( x, x_H, q^\ell, q^\varsigma, q^U, q^D, x_s \right)_{h \in H} \right. $$

$$x^h + p^\ell q^h + p^\varsigma q^h = 1,$$

$$x^h_U + p^\varsigma q^h_U = 1 + q^\varsigma U d_U (q^\varsigma) + x^h,$$

$$x^h_D + p^\varsigma q^h_D = 1 + q^\varsigma D d_D (q^\varsigma) + x^h,$$

$$x^h_s = x^h + x^h_U d_U + q^\ell d_s (q^\ell) + q^\varsigma d_s (q^\varsigma), s \in S_T \}.$$  (7)

Each investor uses their initial cash endowment to purchase either type of debt at $t = 0$. The endowment received at $t = 1$ and cash carried forward are used to purchase short-term debt at $t = 1$ or held for final consumption. \(^{15}\) Investors carry all unused cash forward for consumption.

Given that all debt repayments are normalized to $1$ in the upstate, optimists purchase the debt security with the highest yield, i.e., long-term debt at $t = 0$ and risky short-term debt at $t = 1$. \(^{16}\) Relative pessimists will hold safe short-term debt at $t = 0$ and remain in cash at $t = 1$. We now solve for the debt delivery or repayment functions, $d_s (\cdot)$, that determine the expected repayment values and subsequent debt pricing.

### 2.4 Debt repayment

Debt delivers either the full $1$ promise or there is default and creditors receive their pro rata share of firm output.

$$d_{DD} (\cdot) = \begin{cases} 
1, & s \neq DD \\
\frac{A_{DD} q^U}{q^\varsigma + q^\ell}, & s = DD
\end{cases}.$$  (8)

\(^{15}\)Note that the firm is using the proceeds from time 1 short-term debt issuance to repay its initial time 0 short-term liabilities.

\(^{16}\)Given the equal seniority assumption, a basic no-arbitrage argument shows that long-term bonds in the secondary market conditional on $s = D$ must be priced equivalently to short-term bonds issued on the primary market. Hence, long-term bonds will not change hands at $t = 1$ given that optimists already possess them at $t = 0$.  

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Conditional on the issuance of safe short-term debt, both long- and short-term deliveries are given by (8) where \( d_s(q^\ell) = d_s(q^\zeta_s) \), or generically \( d_s(\cdot), s \in S_T \). Equation (8) also provides the recovery value of firm assets for any possible debt maturity equilibria by setting either \( q^\zeta_D = 0 \) for long-term funding or \( q^\ell = 0 \) for short-term funding.

We can now determine how investors price claims on firm cash flows. Perfect competition implies that marginal claimants break even in expectation. Formally, the marginal long-term claimant at \( t = 0 \) determines:

\[
1 - (1 - h_0)^2 + (1 - h_0)^2 d_{DD}(q^\ell) = p^\ell. \tag{9}
\]

Likewise, the marginal short-term claimant at \( t = 1 \) determines:

\[
h_1 + (1 - h_1) d_{DD}(q^\zeta_D) = p^\zeta_D. \tag{10}
\]

In equilibrium, the debt markets must clear at all times. Specifically, investor demand for long-term risky bonds must equal long-term debt supplied by firms:

\[
\frac{1 - h_0}{p^\ell} = q^\ell, \text{ Long-term debt market clearing} \tag{11}
\]

Additionally, the short-term debt market must clear at both \( t = 0, 1 \). Firms issue \( q^\zeta_U = q^\zeta \) in the good state and \( q^\zeta_D = \frac{q^\zeta}{p^D} \) in the bad state to repay short-term creditors. Therefore, conditional on \( s = U \), all short-term debt is risk free and repaid. However, conditional on \( s = D \), the face value of short-term debt must rise to clear the market. Conditional on \( s = D \), investor demand for short-term risky bonds must equal supply:

\[
\frac{1 - h_1}{p^\zeta_D} = q^\zeta_D, \text{ Short-term debt market clearing.} \tag{12}
\]

Note that the general model presented in this section nests the representative investor as a special case. In particular, when \( h_{t=0,1} = \gamma \), all agents have the same expectation over the state tree. The representative investor representation is the benchmark against which the general representation is examined. Before characterizing the optimal issuance
strategy in either case, we present a series of numerical examples to show how investor heterogeneity plays a crucial role in determining firm debt issuance.

3 Examples of investor beliefs and debt maturity choice

We use the following parameters throughout the examples: $A_{DD} = .5$, $\alpha = 0.8$, $\gamma = 0.7$.

Example 1 Representative investor and debt maturity

Long-term only–$\rho = 1$: Substituting $\gamma = h$ into (9), long-term equilibrium debt prices are given by $1-(1-\gamma)^2 1+(1-\gamma)^2 d_{DD}(q^f) = p^f$. Using the firm funding condition, $\frac{1}{q^f} = \frac{P^f}{T}$, solving for $I^{\alpha - 1}$ from (4), and setting $\rho = 1$, the debt delivery function for long-term debt in (8) simplifies to;

$$d_{DD}(q^f) = \frac{A_{DD}}{\alpha}. \quad (13)$$

Intuitively, large technology shocks, a low $A_{DD}$, leave fewer assets available for investors to recover. In addition, investor recovery is higher for more productive firms, low $\alpha$.

To compare outcomes across economies, define the value of the firm output as $V_{\gamma} = I^{\alpha}$ and expected profits as $\Pi_{\gamma} = (1 - [1 - \gamma]^2) \left( V_{\gamma} - q^f \right)$, where the superscript $\ell$ denotes the long-term debt regime and the subscript $\gamma$ denotes the representative investor’s $\gamma$. Table 1 contains the value of the key objects.

Short-term only– $\rho = 0$: Setting $\gamma = h$ in (10), risky short-term debt pricing at $t = 1$ becomes $\gamma 1 + (1 - \gamma) d_{DD}(q^D) = p^D$. Following the same steps laid out in the long-term debt regime, the debt delivery function for short-term debt funding, (8), is

$$d_{DD}(q^D) = \frac{A_{DD}}{\alpha} \left( \frac{p^D \gamma + \gamma(1 - \gamma)}{\gamma + \gamma(1 - \gamma)} \right) < \frac{A_{DD}}{\alpha}. \quad (14)$$

Short-term debt delivery differs from long-term because risky prices are determined at time 1 rather than time 0. In particular, issuing more short-term debt at price $p^D$ to honor expiring claims reduces the per-claim value investors expect to recover in default relative
Table 1: Long-term representative equilibrium

| MC_γ | p_γ | I | V_Γ | Π_Γ | d (q_Γ) |
|------|-----|---|-----|-----|---------|
| (α, A_{DD}, γ) = (0.8, 0.5, 0.7) | 1.034 | .9663 | .2760 | .3570 | .0650 | .625 |

Table 2: Short-term representative equilibrium

| MC_γ | p_D | I | V_Γ | Π_Γ | d (q_D) |
|------|-----|---|-----|-----|---------|
| (α, A_{DD}, γ) = (0.8, 0.5, 0.7) | 1.034 | .8685 | .2760 | .3570 | .0650 | .524 |

to long-term debt. Table 2 provides the values of the key objects for the short-term debt economy.

The example shows that the economies are equivalent from the firm’s perspective. The expected marginal costs are equivalent across the two economies ($MC_γ = MC_Γ = 1.034$) because all agents have the same common information. The representative agent prices long-term debt the same as the expected value of two periods of short-term debt. The expected marginal cost of issuing debt determines the efficient investment scale.

**Example 2 Debt maturity with heterogeneous investors**

Now consider the general heterogeneous investor case described in section 2.3.2 where $h ≠ γ$.

*Long-term only*–Relative to example 1, the only change we make is risky long-term debt pricing is given by (9). Table 3 contains the equilibrium values for the same parameters as example 1.

Notice the value of firm output and profits are higher under the heterogeneous regime: $Π_h = .0660 > Π_Γ = .0650$ and $V_h = .3629 > V_Γ = .3570$. This is because the marginal investor’s prior is higher in equilibrium than the common belief $h = .7182 > γ = .70$, which leads to higher debt prices, expands firm budget sets, and leads to more investment and output.17

*Short-term only*–Relative to example 1, the only change we make that is risky short-term debt pricing is given by (10). The solution to this economy is given in table 4.

17Clearly, one could generate an outcome for which the homogeneous equilibrium would dominate the heterogeneous long-term candidate solution by setting $γ > h$. 

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Table 3: Long-term heterogeneous equilibrium

| $(\alpha, A_{DD}, \gamma) = (.8, .5, .7)$ | $MC^{\ell}_{h}$ | $p^{\ell}$ | $I_{h}$ | $V^{\ell}_{h}$ | $\Pi^{\ell}_{h}$ | $d\left(q^{\ell}\right)$ |
|------------------------------------------|----------------|------------|---------|---------------|----------------|----------------|
| 1.030                                   | .9702         | .7182      | .2817   | .3629         | .0665           | .625           |

Table 4: Short-term heterogeneous equilibrium

| $(\alpha, A_{DD}, \gamma) = (.8, .5, .7)$ | $MC^{\varsigma}_{h}$ | $p^{\varsigma}_{D}$ | $h_{1}$ | $I_{h}$ | $V^{\varsigma}_{h}$ | $\Pi^{\varsigma}_{h}$ | $d\left(q^{\varsigma}_{D}\right)$ |
|------------------------------------------|----------------|----------------|---------|---------|---------------|----------------|----------------|
| 1.0318                                  | .8786         | .7199         | .2800   | .3612   | .0657         | .5286           |

Unlike the representative investor economy, debt maturity is relevant because the expected marginal cost of a sequence of short-term claims need not be equivalent to a long-term claim. In particular, the market-clearing condition given by equation (11) determines the price of long-term debt. Likewise, equation (12) is the market clearing condition for short-term debt. Investors must price both debt maturities with a common expectation given by $h_{0} = h_{1}$ for maturity to be irrelevant. However, there is more uncertainty at $s = D$ than at $s = 0$, which means that the same investor will never price risky short- and long-term debt equivalently.

**Example 3 Optimal debt maturity with heterogeneous investors**

This example shows that spreading risk across time through both long- and short-term claims is the most efficient way to structure debt. Firms choose $0 \leq \rho \leq 1$ to maximize profits encompassing corner solutions if they are optimal. Table 5 contains the solution in this example. All debt pricing is significantly higher when both maturities are issued than in either of the respective corner solutions because both marginal buyers, $h_{0} = .8227$ and $h_{1} = .8690$, are more optimistic. Consequently, firm budget sets expand, allowing for more investment and production. Furthermore, the recovery rate is lower than its fundamental value in the long-term regime. Interestingly, debt dilution raises firm output and profits in the heterogeneous investor economy but always reduces recovery values in the homogeneous investor case. The reason is that short-term debt substitutes for long-term debt and concentrates long-term debt holdings to investors willing to pay higher prices. The substitution effect dominates the dilution effect and all prices rise. We show in the following section that the substitution effect always dominates the dilution effect and a maturity mix
Table 5: Optimal debt maturity in a heterogeneous equilibrium

| Maturity mix | $p^D$ | $p^L$ | $MC$ | $I$ | $\bar{\rho}$ | $h_0$ | $h_1$ | $V$ | $\Pi$ | $d(\cdot)$ |
|--------------|-------|-------|------|----|--------|------|------|----|------|--------|
| Long-term    | 0.9495| 0.9879| 1.012| 0.5752| 0.8227 | 0.8690| 0.3900| 0.0710| 0.6159|
| Short-term   | 0.8786| -     | 1.030| 0.2817| 1.7182 | -    | 0.3629| 0.0665| 0.625 |

is generally the optimal funding regime for firms facing heterogeneous investors.

4 The general debt maturity solution with heterogeneous investors

The examples in the previous section show that investor heterogeneity alone leads to equilibrium outcomes where firm debt maturity choice matters in a novel way. This section formalizes these results and shows how supply frictions are important factors that determine how firms raise debt for investment. In particular, we first show that all long- or short-term debt financing strategies are equivalent for firms facing a representative investor. Second, and by contrast with the first result, it is shown that issuing both long-and short-term claims is never optimal to a representative investor but always optimal with heterogeneous investors. Finally, a corollary result we show is that if short-term debt is issued, it will always be rolled over. By contrast to Diamond (1991), both long- and short-term claims are optimal precisely when short-term debt is safe–i.e., when there is no liquidity risk.

Our first result formalizes that long- and short-term debt are generally equivalent forms of financing in the representative investor economy.

**Proposition 1 Long- and Short-term Debt Equivalence** Assume a representative investor prices all risky claims with the same state probability as the firm, $\gamma$. Consider an economy in which short-term debt is the only financing source. For a given price vector $(p^S,p^S_L)$, let an equilibrium allocation be given by $E^S = (I^*,q^S*,q^S_L*)$. In addition, for a given price vector, $(p^L)$, define an equilibrium allocation in a long-term debt economy by $E^L = (I^*,q^L*)$. The two economies are equivalent for the firm, $I^*|E^S = I^*|E^L$. 


The proof is straightforward. The argument proceeds by noting that equivalent operating scale, $I^*$, implies equivalent first-order conditions in (4) and (5). Then direct substitution of the representative investor debt pricing and debt delivery functions shows that the marginal costs implied by the candidate corner solutions are always equivalent. The details are in the appendix.

One may think that proposition 1 is a consequence of the Modigliani-Miller (MM) irrelevance theorem. However, this is not true. Specifically, the firm strictly prefers either corner solution to any combination of long- and short-term debt, i.e., $0 < \rho < 1$. To see this, solve for (8) without setting $\rho = 0, 1$ to arrive at:

$$d_{DD}(\cdot) = \frac{A_{DD}}{\alpha} \left( \frac{p_D^\xi}{(1-\rho)p^\ell + \rho p_D^\xi} \right) < \frac{A_{DD}}{\alpha}.$$  \hspace{1cm} (15)

The recovery value of all debt contains a dilution factor that represents the loss in long-term claim value due to rolling over short-term debt when prices fall. There is no dilution if short-term debt is not issued, $\rho = 1$. The dilution factor implies that the recovery value of long-term claims monotonically increases in $\rho$, i.e., relative to the long-term corner solution, long-term debt is “maximally diluted” as $\rho \to 0$. This leads to the following result.

**Proposition 2 Debt Dilution and Maturity Relevance**

For a given price vector, $(p^\ell, p^s, p^s_s)$, and maturity choice, $\rho$, let an interior equilibrium allocation in a representative agent economy be given by $E^I = (I^*, q^\ell, q^s, q^s_s)$. An equilibrium allocation given by either corner solution, $E^\xi$ or $E^\ell$, dominates the interior allocation. Specifically, $I^*|_{E^I} < I^*|_{E^\xi, E^\ell}$.

**Proof.** From (15) the recovery value of debt in the interior allocation is less than recovery value of long-term claims in the long-term corner solution given by $A_{DD}/\alpha$. Therefore, the investor must price long-term claims lower in the interior solution than in the corner solution. Moreover, the interior optimality conditions require equivalence of the marginal cost
of long- and short-term claims.\textsuperscript{18} Thus, the marginal cost of either claim must be higher in the interior solution than the marginal cost of a long-term claim in the corner solution. By Proposition 1 both corner solutions are equivalent and have lower marginal costs than the interior solution. \hfill \blacksquare

Proposition 1 shows that both forms of debt financing are independently equivalent despite incomplete markets and default. Proposition 2 clarifies that maturity remains relevant when both maturities co-exist due to the effect of debt dilution on the investor’s consumption allocation and the firm’s budget set. As far as we are aware, the distinction between the effects of debt dilution and the effect of default per se on firm financing choice has not been explicitly shown.

This argument makes it clear that diluting long-term claims by substituting a small portion of all long-term debt for some short-term debt changes the investor’s consumption allocation, and restricts firm budget sets. A related argument explains why short-term only dominates the interior allocation despite improved terms of trade in the market for short-term claims. Suppose the firm issues an interior mix of debt securities with an infinitesimal amount of long-term debt \( q^\ell = \varepsilon \) and \( q^s = \varepsilon \) so that the short-term \( q^s = Q \) only and interior allocations are equivalent \( Q = q^\ell + q^s \). The recovery value of short-term claims are lower in the short-term only allocation, \( \lim_{\rho \to 0} d_{DD}(q^s) \rightarrow d_{DD}(q^s) \). However, the long-term claims are “maximally diluted” as \( \rho \to 0 \). Thus, relative to the corner solution, the marginal cost of long-term claims are much higher, while short-term claims are just marginally less expensive, which contracts the firm’s budget set.

Note from the arguments above that there must be a countervailing force to debt dilution in order for both maturities to improve the equilibrium allocation. In other words, there must be some marginal gain to the firm from reallocating a portion of its debt capital from a single maturity to a combination of the two, \textit{i.e.}, a substitution benefit in addition to the dilution cost. We now show that investor heterogeneity provides one such unexplored source for a substitution benefit between maturities to improve debt-financing costs. To show that the interior solution is the result of investor heterogeneity on the supply side

\textsuperscript{18}See equation (3).
rather than firm liquidity risk as in Diamond (1991), we first characterize when there is no liquidity risk and all short-term debt is endogenously safe.

Short-term debt is “safe” at \( t = 0 \) if and only if it is unconditionally rolled over at \( t = 1 \). The rollover condition states that profits must be greater than or equal to zero after repaying both long- and short-term debts:

\[
I^\alpha \geq q^l + q^s_D. \tag{16}
\]

Focusing on the down-state is without loss of generality because the firm is always better off conditional on \( s = U \) than \( s = D \).\(^{19}\) The price of short-term debt at \( t = 0 \) must be \( p^s = 1 \) if the firm is not liquidated.

**Lemma 2 Short-term Debt Rollover:** For a given quantity vector, \( Q \), short-term debt at \( t = 0 \) is safe if and only if

\[
\alpha \frac{1 - \rho}{(1 - \alpha \rho)} \leq \frac{p^s_D}{p^l} < 1. \tag{17}
\]

Lemma 2 says that there must be a balance between risky debt prices in order for both maturities to exist with safe short-term debt issued at \( t = 0 \). If all risky debt prices were equivalent, firms would choose only short-term debt because it is financed risk free conditional on \( s = U \). Alternatively, if long-term debt prices are significantly higher than short-term debt prices, then incurring the rollover cost of refinancing the short-term component will be too high and firms are better off insulating themselves from short-term roll-over costs.

We now examine the conditions under which (17) holds. Let debt maturity be tilted more toward long-term debt and \( \rho \) approach 1. Substitution into long-term debt does two things: 1) by market clearing, long-term debt prices must fall as more pessimistic investors finance the investment; and 2) it reduces the amount of short-term debt that needs to be rolled over conditional on \( s = D \), leading to higher short-term debt prices. Thus, the price ratio approaches 1 and always holds. Intuitively, firm production rests heavily on long-
term financing, which makes defaulting on a small portion of short-term debt and handing over firm assets extremely costly. This is a standard disciplining argument.

Alternatively, let \( \rho \) approach 0 and short-term debt be the dominant financing source. Substitution into short-term debt causes the price ratio to fall. All claims are honored as long as the ratio of risky debt prices remains greater than a measure of firm production, \( \alpha \). Consider the two limiting cases.

**Case 1** \( \alpha \to 0 \)

Short-term debt is always rolled over as \( \alpha \to 0 \). Intuitively, \( \alpha \) measures the return to a unit of capital input.\(^{20}\) Higher marginal returns to production means that firms issue small amounts of debt they can easily repay.

**Case 2** \( \alpha \to 1 \)

Production is linear when \( \alpha = 1 \). An interior optimum requires that both risky debt prices be equal to 1, hence risk free. However, debt is never risk free \( \forall A_{DD} < 1 \). Therefore, an interior optimum cannot exist when \( \alpha = 1 \).

We can define an upper bound, \( \bar{\alpha} < 1 \), for any \( 0 < \rho < 1 \) by combining cases 1 and 2 for \( \rho \to 0 \) and the fact that Lemma 2 holds for \( \rho \to 1 \). Otherwise, Lemma 2 holds for all parameters in the model. We now state the main proposition of the paper.

**Proposition 3 Multiple debt maturity structure:** Let investors be ranked by their beliefs according to \( h \sim U [0,1] \). Define a vector of debt quantities for the candidate interior solution with \( 0 < \rho < 1 \) as \( Q := (q^\ell, q^\varsigma_0, q^\varsigma_s) \in R^3_{++}, s = \{U,D\} \).

1. **Maturity is in general always relevant:** \( I^*|_{\ell \neq} \neq I^*|_{\varsigma \neq} \neq I^*|_{\ell \neq}, \forall \rho \in [0,1] \).

2. **For** \( \alpha < \bar{\alpha} \), **the optimal debt maturity strategy is characterized by** \( Q := (q^\ell, q^\varsigma_0, q^\varsigma_s) \in R^3_{++}, s = \{U,D\} \). **Moreover, if** \( \bar{\alpha} < 1 \) **that violates Lemma 2, then the optimal debt maturity strategy is always** \( Q := (q^\ell, q^\varsigma_0, q^\varsigma_s) \in R^3_{++}, s = \{U,D\} \).

\(^{20}\)Firm marginal productivity rises as \( \alpha \) falls because \( 0 < I < 1 \).
Proposition 3 says how the firm finances itself always matters in the heterogeneous agent economy. The reason why equivalence never holds is because the marginal buyers are no longer the same, and neither of their belief’s is given by $\gamma$. Proposition 1 shows that equivalence requires that $h_0 = h_1 = \gamma$, which may occur only by coincidence and cannot be in general be guaranteed.

The second part of the proposition says that it is generally optimal to issue both long- and short-term claims to heterogeneous investors when short-term claims are rolled over. The intuition is the following. Moving from all long-term to an interior issuance strategy has two benefits. First, the first marginal short-term claim issued at $s = D$ is effectively priced risk free by optimist $h = 1$. This marginal short-term claim is always cheaper than the marginal long-term claim it replaces. Second, there is more competition among long-term investors for fewer long-term claims at $t = 0$, leading to higher long-term debt prices. Thus, the firm is substituting high marginal cost long-term debt issued at $t = 0$ for low marginal cost short-term debt rolled over at $t = 1$ until, in equilibrium, the marginal costs are equivalent. The cost of substituting into short-term is the indirect dilution effect on long-term claims as in the representative agent setting, which is decreasing in the relative amount of long-term debt issued. Thus, the dilution cost is low for small short-term debt issuances. In sum, the substitution benefits are high while the costs are low for the first marginal short-term claims that supplant long-term claims. Equilibrium occurs as more short-term debt replaces long-term debt and eventually equated the two expected marginal costs.

Now consider moving from issuing all short-term to an interior allocation. In doing so, the firm replaces a high marginal cost short-term claim at $t = 1$ with a low-cost long-term claim financed by an optimist at $t = 0$. Hence, the substitution benefit reflects a reduction in rollover costs in favor of nearly risk free long-term debt. The substitution cost is the high dilution cost contained in the price of the marginal long-term claim. However, unlike in the representative agent case where $\gamma$ expectation requires a high return to hold the diluted claims, the optimist $h_0 \approx 1$ places little weight on default and is willing to hold the risky $\gamma$.

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21 This is a consequence of the fact that $h_1 \approx 1$ when $q_0^\epsilon = \epsilon \approx 0$. 

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claim for a much lower premium. Hence the substitution benefit outweighs the dilution cost. Equilibrium is the point where the cost of long-term debt equals the marginal cost of short-term debt. These arguments imply that it is always cheaper to finance a given investment with a combination of long- and short-term claims. Consequently, interior debt issuance strategies relax firm budget sets and allow for more investment and production than to either corner solution.

Figure 3 is a graphical representation of the benefits and costs of substituting from a single debt maturity to a combination of debt maturities. Using a combination of debt maturities concentrates fewer total long-term claims to investors most willing to hold risk at \( t = 0 \) than a maturity with no short-term debt. Concurrently, the debt issuance needed to ensure short-term debt is rolled over at \( t = 1 \) is also more concentrated to investors with higher willingness to hold risk than if the firm only issued long-term debt.

4.1 Empirical implications

This section briefly discusses the empirical implications of the model. The main prediction is that using a mix of debt maturities reduces financing costs, which allows firms to increase their budget sets. Choi, Hackbarth, and Zechner (2018) show that corporations typically issue debt into, on average, more than three distinct maturity bins, and that those large and mature corporations are more likely to issue multiple debt maturities. Norden, Rooenboom, and Wang (2016) show that borrowing costs are lower and leverage is positively associated with debt granularity i.e., a mix of debt maturities rather than a single debt maturity. Our model is consistent with both results.

Recall that \( \gamma \) is the likelihood that good news arrives in the following period, from the firm’s perspective and that \( \rho (\gamma) \) measures the proportion of long-term debt in firm liability structures. We conjecture that the function \( \rho (\gamma) \) is monotonically decreasing in \( \gamma \). We verify the conjecture with numerical solutions over the entire grid space \((\gamma, A_{DD}) \in (0, 1) \times (0, 1)\)

In the model, firms issue more short-term debt when good news is more likely, higher \( \gamma \). The reason is that the likelihood of rolling over short-term debt at the risk-free rate
Figure 3: Marginal buyer regimes

- Marginal buyer, $h_0$ with $0 < \rho < 1$ for risky long-term debt holders.
- Marginal buyer, $h_D$ with $0 < \rho < 1$ for risky short-term debt holders.
- Marginal buyer, $h = 1$ with $\rho = 0$ for cash holders.

- Risky long-term debt holders, $0 < \rho < 1$.
- Cash, $\rho = 1$.

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increases, which lowers expected rollover costs relative to long-term financing. Figure 4 plots equilibrium values of $\rho$ for the range of $\gamma$. The figure shows that $\rho$ is monotonically decreasing in $\gamma$. This prediction is broadly consistent with Landier and Thesmar (2008) and Graham et al. (2013) who find that management optimism leads to more short-term debt issuance.

Empirical studies measure growth options as the market-to-book value of assets. In our single good numeraire economy, the market value of assets is simply total firm production. The book value of the firm’s asset is the amount of capital it raises to produce, or the book value of its liabilities. The market-to-book value of the firm is given by

$$\text{market-to-book} = \frac{I^\alpha}{I} = I^{(\alpha-1)} = \frac{1}{\alpha p^\ell (\alpha, A_{DD}, \gamma)}.$$  

Equation (18) shows how growth options are endogenous to prices, collateral value via $\alpha$, and optimism or managerial beliefs, $\gamma$. Empirical studies typically treat growth options as exogenous to debt maturity and leverage. Our model suggests why these studies find

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22We use the first order conditions (2) and (3) to derive the market-to-book in terms of the long-term bond price, $p^\ell$. It can also be expressed in terms of short-term bond prices since the expected costs across maturities must be the same in an interior maturity equilibrium.
mixed results. For example Barclay and Smith (1995) and Guedes and Opler (1996) find a negative relationship between growth options and maturity. By contrast, Stohs and Mauer (1996) and Johnson (2003) find a positive relationship and Billet et. al. (2007) find no relationship when controlling for covenants. The prediction that the value of the option depends on how it is financed confounds the identification problem of how growth options affect maturity.

The model features one period of safe short-term debt in any mixed maturity equilibrium because it helps lower financing costs. This intuition rationalizes commercial paper (CP) programs for large safe corporations without much liquidity risk, and is consistent with the “bridge financing” findings of Kahl, Shivdasani, and Wang (2015).

Lastly, our framework shows that supply frictions determine equilibrium financing strategies. This is consistent with the survey results in Graham and Harvey (2001) and Servaes and Tufano (2006) who find very little support for traditional demand factors such as information asymmetries (Flannery (1986)) and debt overhang (Myers (1977)) as driving maturity choice. In addition, Custodio et al. (2013) find that supply-side factors—i.e., investors demand for debt—have more explanatory power in explaining debt maturity than traditional firm demand factors. More broadly, recent empirical studies find that supply-side factors matter a great deal in determining equilibrium debt maturities (Fuertes and Serena (2014), Shin (2014), and IMF (2015)).

5 Protected debt with endogenous maturity

This section presents an extension of the model in which long-term investors can protect their claims from dilution through either specific collateral or covenants. Our notion that long-term claims are protected from dilution means that long-term debt claimants recover the same amount if the firm defaults in any allocation in which the supply of long-term debt is positive. Specifically, the recovery value of long-term debt should be the same in the long-term corner and interior solutions.

We derive two additional results. First, in the heterogeneous agent setting, preventing
debt dilution does not alter the equilibrium allocation. The interior solution remains optimal and is equivalent to the allocation where long-term claims are diluted in section 3. The reason relates to the original logic of Modigliani-Miller turned on its head. Collateral or covenants alter how investor value different claims on firm assets/cash flows, but the firm reverses the price effect through its financial policy by altering its maturity choice. Thus, preventing debt dilution does not alter our main result. Second, preventing debt dilution results in the strict from of Modigliani-Miller in the representative agent setting. Mainly, the interior allocation is equivalent to both the corner solutions and firm financial policy is irrelevant. The reason is that the interior allocation is a linear combination of the corner solutions where firm assets/cash flows are split into two securities with different recovery values, rather than two securities with a common recovery value.

Assume that long-term debt holders negotiate to have assets earmarked as collateral. In particular, $\rho$ portion of firm assets are financed with long-term debt and are apportioned to long-term debt holders in default. Likewise, the remaining $1 - \rho$ portion of firm assets are financed with short-term debt and apportioned to short-term debt holders in default. An alternative interpretation of this setup is that long-term debt contains negative clauses that prevent firms from raising additional capital that may jeopardize the firm’s ability to repay. 23

Under the assumption that $\rho$ fraction of the firm belongs to long-term claimants in default, the recovery value in (8) is split between the holders of different debt maturities:

$$
\begin{align*}
    d_{DD}(q^\ell) &= \frac{\rho A_{DD}^\ell}{q^\ell}, \text{ long-term recovery} \\
    d_{DD}(q^\varsigma_D) &= \frac{(1-\rho)A_{DD}^\varsigma}{q^\varsigma_D}, \text{ short-term recovery}
\end{align*}
$$

Using the definition of $\rho$, the funding constraint and first-order conditions as before, one

---

23Negative pledges are among the most common non-financial covenants found in long-term public debt indentures. The appendix contains a description of the covenant and its relevance.
can show that the debt delivery functions become:

\[
\begin{align*}
    d_{DD}^{\hat{\ell}}(\hat{q}^\ell) &= \frac{A_{DD}}{\alpha} \\
    d_{DD}^{\hat{\xi}}(\hat{q}_D^{\xi}) &= \frac{A_{DD}}{\alpha} \left( \frac{p_T^\gamma(1-\gamma)}{\gamma+\gamma(1-\gamma)} \right).
\end{align*}
\]  

(19)

The hats represent variables in an economy with the covenant. (19) shows that long-term debt holders receive the same value in default irrespective of whether they are the only claimants or if the firm also issues short-term debt. Hence, there is no debt dilution.

**Proposition 4** Consider any \(d_{DD}^{\hat{\ell}}(\hat{q}^\ell)\) defined by (19) and corresponding \(d_{DD}^*(\cdot)\) without a security interest defined by (15). In any debt financing strategy for which \(Q := (q^\ell, q_0^\xi, q_s^\xi) \in \mathbb{R}^3_++\), \(s = \{U, D\}\), the following hold:

- \(d_{DD}^{\hat{\ell}}(\hat{q}^\ell) > d_{DD}^*(\cdot) > d_{DD}^{\hat{\xi}}(\hat{q}_D^{\xi})\) and \(d_{DD}^{\hat{\ell}}(\hat{q}^\ell) = d_{DD}(q^\ell)|_{q^\xi=0}\);

- \(\frac{1-(1-h_0)^2+(1-h_0)^2[d_{DD}(q^\ell)]}{p^\ell} > \frac{1-(1-h_0)^2+(1-h_0)^2[d_{DD}(\cdot)]}{p^\ell}\) any given investor prefers secured long-term debt to unsecured for the same price.

Proposition 4 says that preventing dilution through an explicit security interest in long-term debt raises (lowers) the expected recovery value of long-term (short-term) claims relative to the economy with dilution. Furthermore, any investor prefers the protected to the unprotected long-term claim at the same price. An immediate implication of proposition 4 is that any interior candidate equilibrium contains more long-term debt.

**Corollary 1** Let \(\hat{\rho}\) be the equilibrium portion of long-term debt when collateral values are determined by (19) and \(\rho^*\) be the equilibrium portion of debt issued long-term debt when collateral values are determined by (15). Then, \(\hat{\rho} > \rho^*\) and \(\hat{q}_D^{\xi} < q_D^{\xi^*}\).

Corollary 1 says secured long-term debt leads to more long-term debt in the liability structure, and short-term debt issuance must fall. Hence, consistent with the empirical findings of Billet et. al. (2007), the security interest or covenant acts as a substitute short-term debt.
A consequence of corollary 1 and the fact that total investment is a linear combination of long- and short-term debt issuance, the security interest or covenant protection does not affect firm value. The firm increases the supply of long-term debt while reducing short-term debt supply until the expected cost of the two debt maturities are equivalent.

**Proposition 5** Let \( I^\ast \) be the investment allocation resulting from liability choice \( Q^\ast := (q^f, q^c_0, q^c_s) \in \mathbb{R}^3_{++}, s = \{U, D\}, \forall q \in Q^\ast > 0 \) and price functions, \((p^f, p^c_s)\), as the solution to program (1) with debt deliveries given by (15). Let \( \hat{I} \) be the investment allocation from liability choice \( \hat{Q} := (\hat{q}^f, \hat{q}^c_0, \hat{q}^c_s) \in \mathbb{R}^3_{++}, s = \{U, D\}, \forall q \in \hat{Q} > 0 \) and price functions \((\hat{p}^f, \hat{p}^c_s)\) given debt recovery values in (19). Then, \( I^\ast = \hat{I} \).

The intuition is the following. Debt secured by explicit collateral does not create value. Collateral in long-term debt prevents short-term debt from extracting value from long-term debt, but only at the expense of reducing the value of short-term claims. Consequently, firms adjust their financial policy in response to the new prices investors are willing to pay for different debt securities in a way that leaves prices unchanged. Noting that prices and investment are equivalent in the secured debt equilibrium, it is immediate from proposition 3 that the interior liability strategy remains optimal.

**Corollary 2** The optimal liability choice with secured long-term debt is interior: \( \hat{Q} := (\hat{q}^f, \hat{q}^c_0, \hat{q}^c_s) \in \mathbb{R}^3_{++}, s = \{U, D\}, \forall q \in \hat{Q} > 0 \).

Corollary 2 implicitly states that firm financial policy continues to matter in the heterogeneous agent economy with secured debt. Interestingly, in the representative agent economy, firm financial policy is irrelevant with secured debt. In particular, the interior allocation with secured debt is the same as either corner solution, which we already know to be equivalent from proposition 1. Hence, Modigliani-Miller holds in the strong sense in the representative agent economy with secured debt. The reason is that securing long-term debt re-creates the same long-term and short-term claims available in the two corner solutions. The interior solution becomes a linear combination of the two corner solutions and any interior allocation the firm chooses does not affect the representative investor’s consumption allocation. When short-term dilutes long-term debt, it affects the representative...
investor’s consumption allocation because she holds both claims. Preventing dilution thus restores equivalence.

5.1 Numerical example

We keep $\alpha$ the same as in the previous examples and show how the different values of the technology shock, $A_{DD}$, the good state probability, $\gamma$, and the covenant change the relative amount of long- versus short-term debt given by $\rho$. Table 6 highlights the major effects of the secured debt covenant for various $(A_{DD}, \gamma)$-pairs. The top (bottom) panel contains the endogenous variables for the economy with (without) the covenant. The numbers in red highlight the key changes. Note that all debt prices, investment levels and profits are unchanged across the two panels. The economy with the covenant has more (less) long-term (short-term) debt. The covenant simply tilts the maturity in favor of long-term debt, $\rho \uparrow$, and the firm substitutes away from short-term debt.

The comparative static results are seen comparing rows within each panel. The first two rows of either panel show how variables change as $A_{DD}$ decreases, while the third row shows changes in $\gamma$ for the same $A_{DD}$ as the first row. Comparing rows 1 and 2 in the top panel shows that more down risk at $t = 2$–smaller value of $A_{DD}$–lowers all risky debt prices, investment and profits. The firm re-optimizes its debt maturity more toward long-term debt, $\rho \uparrow$. Comparing rows 1 and 3 of the top panel shows that a decrease in $\gamma$ for the same $A_{DD}$ as in the first row leads to more toward long-term debt, $\rho \uparrow$. Long-term debt prices are lower while short-term debt prices are higher due to the supply effect of substituting between maturities.

6 Discussion and conclusion

This section concludes with a discussion of alternative modeling assumptions. The models makes the simplifying assumption that states are i.i.d. Suppose alternatively that good news is more likely to follow good news than bad news. This assumption alleviates the concern that bad news is effectively “not as bad.” The relative expected cost between
Table 6: Endogenous Variables

| Covenant         | $p^0_0$ | $p^1_0$ | $p^r$ | $q^0_0$ | $q^1_0$ | $q^r$ | $I$   | $\rho$ | $\Pi$ |
|------------------|---------|---------|-------|---------|---------|-------|-------|-------|-------|
| $(A_D, \gamma)=(.5, .8)$ | 1       | .941   | .989  | .145   | .154   | .167  | .311  | .533  | .075  |
| $(A_D, \gamma)=(.2, .8)$ | 1       | .894   | .980  | .136   | .153   | .163  | .297  | .539  | .072  |
| $(A_D, \gamma)=(.5, .5)$ | 1       | .957   | .985  | .107   | .112   | .199  | .304  | .646  | .057  |
| No Covenant      | $p^0_0$ | $p^1_0$ | $p^r$ | $q^0_0$ | $q^1_0$ | $q^r$ | $I$   | $\rho$ | $\Pi$ |
| $(A_D, \gamma)=(.5, .8)$ | 1       | .941   | .989  | .149   | .158   | .163  | .311  | .519  | .075  |
| $(A_D, \gamma)=(.2, .8)$ | 1       | .894   | .980  | .138   | .154   | .162  | .297  | .534  | .072  |
| $(A_D, \gamma)=(.5, .5)$ | 1       | .957   | .985  | .113   | .115   | .197  | .304  | .638  | .057  |

long- and short-term debt would change, but the effect would be a *quantitative* adjustment in the interior value $\rho$ and not affect the equilibrium financing regime. In particular, the state-contingent costs of rolling over short-term debt change due to different repayment probabilities, but as long as the cost of rolling over short-term debt rises in one intermediate state relative to the ex ante long-term issuance cost, issuing both long- and short-term claims will remain optimal. Hence, any structure in which repayment volatility increases in at least one succeeding state will satisfy the condition in Lemma 2.

One could also allow for uncertainty conditional on $s = U$ with $A_{UD} = A_{DU}$, which would be consistent with the formulation in He and Xiong (2012a). This too would only affect the interior value of $\rho$. The additional uncertainty lowers both long-term prices at $t = 0$ and short-term debt prices conditional on $s = U$ at time 1. However, intermediate states would still be characterized by more uncertainty than the initial state; in fact, there would be even more uncertainty. Therefore, the rollover over costs of short-term debt will remain higher than the ex ante costs of issuing long-term debt and satisfy Lemma 2.

The economy itself is purposely simple to highlight the main results. However, the structure can be derived from principles that are more general. For example, Fostel and Geanakoplos (2010) show that when choosing from a menu of projects, agents have the incentive to produce projects with more volatile payouts conditional on bad news. The reason is simple: Uncertainty following bad news is not informative, which implies that price declines in bad intermediate periods are relatively small. Alternatively, if uncertainty were completely resolved after bad news, then prices in bad intermediate periods would
fall much further and reflect the certain bad outcome in the final period. With the collateral constraint imposed by the repayment enforcement frictions in these models, lower intermediate prices limit *ex ante* how much agents can borrow.

Short-term debt issued at time 0 is *endogenously* risk-free. This is not an innocuous outcome, though there are many ways one can argue that short-term debt is informationally insensitive up to some signal (Dang, Gorton, and Holmstrom (2015)). In the model, firms will never issue both long- and short-term debt if there is liquidation along the equilibrium path at time 1 and all firm assets are used to repay creditors. The argument is the following. Risky short-term debt at time 0 must be priced higher than long-term debt because long-term debt allows equity to be retained in 3 of 4 terminal states, while liquidation implies equity retention in only 2 terminal states. Long-term debt does not insulate the firm from liquidation conditional on bad news at time 1. Hence, there is no benefit. The equilibrium maturity choice will depend on the probability of good news, determined by $\gamma$. Short-term (long-term) debt will dominate long-term (short-term) debt when good (bad) news is likely. This is consistent with Diamond (1991) if $\gamma$ is interpreted as a credit rating. This alternative also highlights the fact that firms generally not subject to rollover risk can issue both long and short-term debt, which is consistent with the data.

Finally, we do not allow investors to purchase bonds with leverage as in the collateral equilibrium models developed by Fostel and Geanakoplos. This restriction is not necessary for the result that issuing multiple types of securities is optimal. The reason is that, due to investor heterogeneity, it will remain optimal to issue multiple types of debt securities to cater to investor needs, as in Allen and Gale (1991). The firm will not want to close off the security market to time 1 investors by issuing only long-term debt at time 0, even though allowing for leverage at time 0 will lower long-term spreads and raise prices. Issuing more long-term debt at time 0 implies that less short-term debt needs to be rolled over at time 1, which also raises the prices of short-term debt. The more tilted the issuance becomes to one maturity, the greater is the substitution benefit of the other maturity.
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A Appendix Omitted Proofs

Proof. Lemma 1: We will derive an upper bound on $\delta$ that will equate a debt strategy of issuing all long-term debt and all risky-short-term debt. All $\delta$ less than the upper bound will necessarily lower risky short-term debt prices without altering the all long-term candidate solution, which implies that the all long-term solution always dominates all risky short-term solution. We can then focus the rest of the paper on candidate solutions for which all risky short-term debt is dominated by the all long-term candidate solution.

Let $V_d$ be the firm value function when issuing only risky short-term debt at $t = 0$ that defaults at $s = D$ at time 1. Then $V_d = \gamma (I^\alpha - q_0^s) + (1 - \gamma) \times 0$ as the firm only retains equity at $s = U$ and defaults at $s = D$. If the firm raises all capital short-term, then $I = q_0^s p_0^s$.

The first order condition for a maximum is

$$\alpha I^\alpha - 1 = \frac{1}{p_0^s}, \quad (20)$$

Investors are also perfectly competitive price takers. Because of the connectedness of the set of agents and monotonicity of utility in $h$, there will be a marginal buyer at time 0, $h_0$, who’s expectations price risky short-term debt. All agents $h > h_0$ will purchase short-term debt and all other agents remain in cash. The bond pricing condition is

$$h_0 + (1 - h_0) \delta I = p_0. \quad (21)$$

Rearranging, we get $p_0 = \frac{h_0}{1 - (1 - h_0) \delta}$, which bounds risky short-term bond prices between $h_0$ and 1 as $\delta \to (0, 1)$. Investor recovery in default, $\delta I$, is an increasing function of $\delta$ for any given $I$. Thus, the lower is $\delta$, the lower the price any investor is willing to pay for risky short-term debt. The lower bond price raises the marginal cost of capital, which lowers investment and profits. Finally, market clearing will determine a candidate solution to the risky-short term debt problem:

$$\delta I = (1 - h_0) \quad (22)$$

As $\delta \to 0$, $p_0 \to h_0$, and $I \to (\alpha h_0)^{\frac{1}{1 - \alpha}}$ from the first order condition. Plugging the limiting $I$ into the market clearing condition, we obtain a polynomial that solves for the fixed point:

$$h_0 + (\alpha h_0)^{\frac{1}{1 - \alpha}} - 1 = 0. \quad (23)$$

Clearly there is a unique $h_0 > 0, \forall \alpha \in (0, 1)$ that solves (23) as the left hand side is monotonically increasing in $h_0$ ranging from 0 to $1 + \alpha^{\frac{1}{1 - \alpha}} > 1$.

The candidate solution is the lowest price equilibrium determined by $\delta \to 0$, where $p_0^* = h_0^*$. Now consider the two period long-term investment strategy with value function given by $V_t = 1 - (1 - \gamma)^2 (I^\alpha - q_t)$. The firm raises $I = q_t p_t$, and the first order condition is
\[ \alpha t^{\alpha-1} = \frac{1}{p_t}. \]  

(24)

The optimality condition is the same form as the all short-term problem due do limited liability in default. The marginal investor equation pricing two-period risky long-term debt is

\[ 1 - (1 - h_0)^2 + (1 - h_0)^2 \frac{A_{DD}}{\alpha} = p_\ell. \]  

(25)

The recovery in default term \( \frac{A_{DD}}{\alpha} \) comes from plugging the first order condition for \( I \) and \( I = q_\ell p_\ell \) into the recovery value function \( \frac{A_{DD} I^\alpha}{q_\ell} \). Lastly, market clearing takes the same form as the short-term funding regime:

\[ (1 - h_0) = I. \]  

(26)

The lowest price equilibrium in the long-term regime is when \( A_{DD} = 0 \) and there is no recovery in default, just as in the risky short-term regime. The lowest two-period long-term price is then given by \( p_\ell = 1 - (1 - h_0)^2 = h_0 (2 - h_0) \). Plugging this into the market clearing condition, we obtain a similar polynomial as the short-term regime

\[ h_0 + (\alpha h_0 (2 - h_0))^{\frac{1}{\alpha}} - 1 = 0. \]  

(27)

Comparing (23) with (27), we see that the marginal buyer in the lowest long-term funding regime is always more optimistic than the marginal buyer in the lowest-price short-term funding regime. This reflects the fact that at time 0, any marginal buyer places far less weight on two periods of bad news leading to long-term default, than a single period of bad news leading to short-term default. The highest long-term bond prices will be given by \( \frac{A_{DD}}{\alpha} = 1 \), which implies that \( p_\ell = 1 \). Therefore, both short-term with liquidation and long-term have the same risk-free price when debts are always repaid, but the long-term regime has a higher lowest price equilibrium. Clearly, prices in the two regimes are monotonically increasing in the parameters \( \delta \) and \( A_{DD} \) respectively. Thus, for any \( 0 < A_{DD} < 1 \), there will be at most 1 value of \( \delta \) where the two pricing function cross, if they cross at all. Otherwise, long-term always dominates short-term. Let \( \delta = \bar{\delta} \) be the value when the two pricing functions cross. To find \( \bar{\delta} \), we must compare the value function for the two regimes because the expectations of retaining equity are different due to the firm being profitable in 3 of 4 states in the long-term regime, but only 2 of 4 in the short-term only regime.

Using the respective first order conditions for \( I^* \) and the fact that \( q^* = \frac{\ell}{p^*} \) in both regimes, we can write the equivalence condition for risky short-term and long-term funding \( V_d = V_\ell \) as

\[ (1 + (1 - \gamma)) q_\ell^* = q_0^* \]  

(28)

This says that even if the firm raises the same amount of capital on the same terms in the two regimes in which \( q_0 = q_\ell \), the long-term regime will dominate due to the fact that the firm retains equity in a superset of states with long-term funding. Risky short-term debt
must be less expensive by the factor \((1 + (1 - \gamma))\) than long-term funding, giving us \(\delta = (1 + (1 - \gamma)) \hat{\delta}\). Therefore, for any parameters \((\alpha, A_{DD}, \gamma)\) that determine the solution to the long-term funding regime, if there is a value of \(\delta\) high enough to raise risky short-term prices enough to incentivize issuing short-term debt with default at time 1, then \(\forall \delta < \delta\) long-term funding will dominate. 

**Proof. Lemma 2:** Short-term debt can only be safe for a candidate interior solution if its price at time 0 is equal to 1. In other words, all short-term debt is unconditionally rolled over. We will find under what conditions a candidate interior optimization can be achieved and all debts rolled over assuming \(p^s_0 = 1\). Plugging the first order conditions for an interior maximum are (2) and (3) with \(p^s_0 = 1\) into the necessary rollover condition in (16) immediately gives (17). This is the necessary condition for an interior optimum with short-term debt rollover. 

**Proof. Proposition 1:** The condition for equivalence is that the expected marginal cost of either source is the same. From the F.O.C, equivalence implies

\[
\frac{1}{p^F} = \frac{1}{1 - (1 - \gamma)^2} \left[ \gamma + \frac{\gamma(1 - \gamma)}{p^s_D} \right]. \tag{29}
\]

Equilibrium long-term debt pricing is given by \(p^F = 1 - (1 - \gamma)^2 + (1 - \gamma)^2 \frac{A_{DD}}{\alpha}\), which can be re-written as

\[
p^F = (\gamma + \gamma(1 - \gamma)) \left(1 - \frac{A_{DD}}{\alpha}\right) + \frac{A_{DD}}{\alpha}. \tag{30}
\]

Equilibrium short-term debt pricing is given by \(p^s_D = \gamma + (1 - \gamma) \frac{A_{DD}}{\alpha} \left[\frac{p^F \gamma + \gamma(1 - \gamma)}{\gamma + \gamma(1 - \gamma)}\right]\), which, after some algebra, can be re-written as

\[
p^s_D = \frac{\gamma \left[ \gamma + \gamma(1 - \gamma) \left(1 - \frac{A_{DD}}{\alpha}\right) + \frac{A_{DD}}{\alpha} \right]}{\gamma(1 - \gamma) \left(1 - \frac{A_{DD}}{\alpha}\right) + \gamma}. \tag{31}
\]

Plugging (30) and (31) into (29), equivalence requires that

\[
1 - (1 - \gamma)^2 = \gamma \left[ (\gamma + \gamma(1 - \gamma)) \left(1 - \frac{A_{DD}}{\alpha}\right) + \frac{A_{DD}}{\alpha} \right] + \frac{\gamma(1 - \gamma) \left[ (\gamma + \gamma(1 - \gamma)) \left(1 - \frac{A_{DD}}{\alpha}\right) + \frac{A_{DD}}{\alpha} \right]}{\gamma(1 - \gamma) \left(1 - \frac{A_{DD}}{\alpha}\right) + \gamma}. \tag{32}
\]

Notice that two terms in the fraction in second term on the right hand side, \(\gamma\) and \(\left[ (\gamma + \gamma(1 - \gamma)) \left(1 - \frac{A_{DD}}{\alpha}\right) + \frac{A_{DD}}{\alpha} \right]\), cancel out. After re-writing \(1 - (1 - \gamma)^2 = \gamma + \gamma(1 - \gamma),\)
(29) becomes
\[
\gamma + \gamma (1 - \gamma) = \gamma \left[ (\gamma + \gamma (1 - \gamma)) \left( 1 - \frac{A_{DD}}{\alpha} \right) + \frac{A_{DD}}{\alpha} \right] + (1 - \gamma) \left[ \gamma (1 - \gamma) \left( 1 - \frac{A_{DD}}{\alpha} \right) + \gamma \right] \\
= \gamma (1 - \gamma) \left( 1 - \frac{A_{DD}}{\alpha} \right) + \gamma^2 \left( 1 - \frac{A_{DD}}{\alpha} \right) + \gamma (1 - \gamma) \\
1 = \left( 1 - \frac{A_{DD}}{\alpha} \right) (\gamma + 1 - \gamma) + \frac{A_{DD}}{\alpha} \\
1 = 1.
\]
Hence, the two regimes are always equivalent. ■

**Proof.** Proposition 3: We first show that the long and short-term economies are no longer equivalent, and then show why the interior solution always dominates either corner solution.

First, the long and short-term debt pricing equations in the heterogenous agent economy are given by (9) and (10). Rewrite these similar to Proposition 1 as
\[
p^e = (h_0 + h_0 (1 - h_0)) \left( 1 - \frac{A_{DD}}{\alpha} \right) + \frac{A_{DD}}{\alpha}, \quad \text{and} \\
p^D_D = h_1 + (1 - h_1) \frac{A_{DD}}{\alpha} \left[ \frac{p^D_D \gamma + \gamma (1 - \gamma)}{\gamma + \gamma (1 - \gamma)} \right] \\
= \frac{h_1 (\gamma + \gamma (1 - \gamma)) + (1 - h_1) \frac{A_{DD}}{\alpha} \gamma (1 - \gamma)}{\gamma + \gamma (1 - \gamma) - (1 - h_1) \frac{A_{DD}}{\alpha} + \gamma} \\
= \gamma \left[ h_1 + (1 - \gamma) \left[ h_1 \left( 1 - \frac{A_{DD}}{\alpha} \right) + \frac{A_{DD}}{\alpha} \right] \right].
\]
Plugging these into the equivalence condition, (29), we get
\[
\gamma + \gamma (1 - \gamma) = \gamma \left[ (h_0 + h_0 (1 - h_0)) \left( 1 - \frac{A_{DD}}{\alpha} \right) + \frac{A_{DD}}{\alpha} \right] + \frac{\gamma (1 - \gamma) \left[ (h_0 + h_0 (1 - h_0)) \left( 1 - \frac{A_{DD}}{\alpha} \right) + \frac{A_{DD}}{\alpha} \right]}{\gamma (1 - \gamma) h_1 + (1 - h_1) \frac{A_{DD}}{\alpha} + h_1}.
\]

Inspection of the above equation shows that equivalence requires \( h_0 = h_1 = \gamma \) in order to cancel out the first part of the fraction in the second term on the right. In particular, plug in \( h_1 = \gamma \) into \( \gamma \left[ (1 - \gamma) \left[ h_1 + (1 - h_1) \frac{A_{DD}}{\alpha} \right] + h_1 \right] \) and rearrange to get \( \gamma \left[ (\gamma + \gamma (1 - \gamma)) \left( 1 - \frac{A_{DD}}{\alpha} \right) + \frac{A_{DD}}{\alpha} \right] \), which cancels with the numerator iff \( h_0 = \gamma \). The leftover term from the second term on the right becomes \( (1 - \gamma) \left[ \gamma (1 - \gamma) \left( 1 - \frac{A_{DD}}{\alpha} \right) + \gamma \right] \). And with \( h_0 = \gamma \), the first term on the
right becomes \( \gamma \left( (\gamma + \gamma(1 - \gamma))(1 - \frac{\beta p}{d}) + \frac{\beta p}{a} \right) \) allowing one to arrive at equivalence to the left hand side as in the proof of proposition 1 above. However, the marginal buyer at time 0 will never have the same repayment expectation when uncertainty rises at 1. Therefore, \( h_0 \neq h_1 \) even if by coincidence either happens to equal \( \gamma \).

The second part of the proof is by contradiction and shows that for any investment level, \( I \), that can be financed by either all long- or all short-term debt, financing with both long- and short-term has a lower marginal cost. Suppose maturity is irrelevant, and the same investment plan, \( I \), can be raised through all long or an interior, \( 0 < \rho < 1 \). Maximization requires \( MPK = MC \), which defines investment as a function of prices. Irrelevance implies \( I^* (p^* \ell) = I^* (p^* \ell, p^* s, p^* D) \). By Lemma 2, if \( q^* s > 0 \), then the short-term component of the interior solution is rolled over at time 1 and \( p^* s = 1 \). Equivalence of marginal costs and efficient investment scale across the two funding strategies implies that the amounts of debt issued must also be the same. Let \( \tilde{Q} = \tilde{q}_0^\ell \), and \( p^* \ell = \tilde{p}_0^\ell \) be the long-term candidate equilibrium quantities and prices, and \( \hat{Q} = \hat{q}_0^\ell + \hat{q}_0^s \), and \( \hat{p}_0^\ell, \hat{p}_0^s \) be the interior-candidate equilibrium quantities and prices. Equivalence implies that \( \tilde{Q} = \hat{Q} \Rightarrow \tilde{q}_0^\ell > \hat{q}_0^\ell, \forall \hat{q}_0^s > 0 \). Moreover, since the firm takes prices as given, it must be the case that \( \tilde{p}_0^\ell > \hat{p}_0^\ell \). Market clearing in the long-term corner solution is given by \( (1 - \tilde{h}_0) = \tilde{p}_0^\ell \tilde{q}_0^\ell = I^* \) and in the interior solution given by \( (1 - \hat{h}_0) + (1 - \hat{h}_D) = \hat{p}_0^\ell \hat{q}_0^\ell + \hat{p}_D^\ell \hat{q}_D^\ell = I^* \). Equating the two market clearing conditions for the same \( I^* \) gives \( (1 - \tilde{h}_0) = (1 - \hat{h}_0) + (1 - \hat{h}_D) \). This can only hold if \( \hat{h}_D = 1 \) meaning that \( \hat{q}_D = 0 \)–no short-term debt is issued–or if \( \tilde{h}_0 < \hat{h}_0 \)–the marginal long-term bond buyer in an interior solution is more optimistic than the marginal bond buyer in the corner solution. However, a more optimistic marginal buyer in the interior solution implies \( \tilde{p}_0^\ell < \hat{p}_0^\ell \), which contradicts \( \tilde{q}_0^\ell > \hat{q}_0^\ell, \forall \hat{q}_0^s > 0 \). The same logic also shows that the firm can never be indifferent between a short-term corner solution and the interior. ■

**Proof.** Corollary 1: From Proposition 4 and (19) we know that \( d^D_0 (q^D_0) > d^D_0 (q^D_D) \) for a given \((I^*, \rho^*)\). Suppose the firm does not alter its debt structure and \( \rho^* \) is unchanged. Then, long-term debt prices must rise to reflect greater recovery values, \( \hat{p}^\ell > p^\ell \). But if long-term debt is now cheaper in equilibrium, then the maturity structure for a given \((I^*, \rho^*)\) cannot be optimal and the firm must adjust. Thus \( \rho^* \) must rise. For the second statement that short-term issuance must fall, suppose not and that \( \hat{q}^D_D = q^* D_D \). From Proposition 4, the marginal cost of short-term debt increases because recovery values fall. Hence the expected marginal cost of issuing \( \hat{q}^D_D \) is higher than \( q^* D_D \). Interior optimality requires that the expected marginal costs of the two debt maturities must be the same, \( E[MC] = E[MC^\ell] = E[MC^s] \). Hence, leaving \( q^* D_D \) unchanged results in \( E[MP] < E[MC] \) and cannot be an equilibrium. ■

**Proof.** Proposition 5: Follows immediately from the proof of Corollary 1 and investment optimality in (2) and (3). ■
B Equilibrium Conditions

B.1 Interior Maturity Choice

The ten endogenous variables are \( \left( p_0^\xi, p^\ell, p_D^\xi, q_0^\xi, q^\ell, q_D^\xi, I_0, \rho, h_0, h_D \right) \). The system of equations,

\[
\begin{align*}
p_0^\xi & = \text{time 0 short-term debt price} \\
\alpha f^{a-1} & = \frac{1}{p^\ell}, \text{combined first order} \\
1 - (1-h_0)^2 + (1-h_0)^2 d_{DD} (\cdot) & = p^\ell, \text{long-term debt pricing} \\
h_D + (1-h_D) d_{DD} (\cdot) & = p_D^\xi, \text{short-term debt pricing} \\
\frac{1-h_0}{p^\ell} & = q^\ell, \text{long-term debt market clearing} \\
\frac{1-h_D}{p_D^\xi} & = q_D^\xi, \text{short-term debt market clearing} \\
I & = p^\ell q^\ell + p_D^\xi q_D^\xi, \text{firm funding condition} \\
q_0^\xi & = p_0^\xi q_D^\xi, \text{short-term rollover condition} \\
\rho & = \frac{p^\ell q^\ell}{I}, \text{long-term debt portion} \\
d_{DD} (\cdot) & = \frac{A_{DD}}{\alpha} \left( \frac{p_D^\xi}{(1-\rho) p^\ell + \rho p_D^\xi} \right) \text{debt recovery value.}
\end{align*}
\]

B.2 Long-term Equilibrium

The endogenous variables are in this economy: \( (I, p^\ell, q^\ell, h_0) \), and four equations: (4), (9), (11), and \( I = p^\ell q^\ell \).

B.3 Short-term Equilibrium

The endogenous variables are: \( (I, p_D^\xi, q_D^\xi, h_1) \). Equilibrium is found by simultaneously solving market clearing via (12), the pricing equation, (10), the debt delivery function, (14), and the firm’s funding condition, \( I = p_0^\xi q_0^\xi \).

C Appendix

Here we show that changing the uncertainty structure of the economy does not materially alter the optimal choice to issue both long- and short-term debt. Instead of the structure given by figure 2 where \( \gamma = \gamma|_{s=D} \) let \( \gamma_1 = \Pr (s = U) > \gamma_2 = \Pr (s = DU|s=D) \) so that the likelihood of receive a good state following a bad state is less that receiving an unconditional good state. Breaking the firm’s problem given by (1) into its constituent pieces, we
can write profits as

$$\max_{t_0, \rho} \prod = \left\{ \gamma_1 \left[ t_0^\alpha - \rho \frac{l_0}{p_0^\alpha} - (1 - \rho) \frac{l_0}{p_0^\alpha} \right] + (1 - \gamma_1) \gamma_2 \left[ \frac{t_0^\alpha - \rho \frac{l_0}{p_0^\alpha} - (1 - \rho) \frac{l_0}{p_0^\alpha}}{1} \right] \right\}. $$

This profit expression simply states that conditional on good news at \( t = 1 \), both long- and short-term debt is repaid, and conditional on bad news at \( t = 1 \) long- and short-term debts are repaid only if good news arrives at \( t = 2 \). Notice that the only difference between this problem and the one presented in the main body of the paper is that \( \gamma_2 < \gamma_1 = \gamma \). The first order conditions for a maximum simply become

$$\frac{\gamma_1 + \gamma_2 (1 - \gamma_1)}{p_0^\alpha} = \frac{1}{\rho} \left[ \frac{\gamma_1 + \gamma_2 (1 - \gamma_1)}{p_0^\alpha} \right]$$

$$\alpha t_0^{\alpha - 1} \left[ \frac{\gamma_1 + \gamma_2 (1 - \gamma_1)}{p_0^\alpha} \right] = \frac{\rho \left[ \gamma_1 + \gamma_2 (1 - \gamma_1) \right]}{p_0^\alpha} + \frac{(1 - \rho) \left[ \gamma_1 + \gamma_2 (1 - \gamma_1) \right]}{p_0^\alpha}.$$

Plugging into the other we obtain \( \alpha t_0^{\alpha - 1} = \frac{1}{\rho} \), which of course arises because in equilibrium the marginal cost of a long-term bond must equal the marginal cost of a short-term bond for \( 0 < \rho < 1 \) allowing us to express the first order condition for a maximum as a function of either long- or short-term debt. Let \( A = \gamma + \gamma (1 - \gamma) \) when \( \gamma = \gamma_{s=D} \) and \( B = \gamma_1 + \gamma_2 (1 - \gamma_1) \) from the restated problem above and \( A > B \). Then, \( \forall (t_0, \rho) : \alpha t_0^{\alpha - 1} A > \alpha t_0^{\alpha - 1} B \). This implies that \( \frac{1}{\rho} \big|_A > \frac{1}{\rho} \big|_B \Rightarrow p_0^\alpha \big|_A > p_0^\alpha \big|_B \) at the optimum. In other words, for a given \( \rho \), the firm will only raise the same amount of capital across the two economies if long-term bond prices are higher in the economy with more uncertainty at \( s = D \), which is a contradiction because the firm is less likely to repay debt at \( s = DU \) with in the more uncertainty case. Alternatively, the firm can raise less long-term debt and more short-term debt in the economy with more uncertainty at \( s = D \), leaving total \( l_0 \) unchanged and tilting \( \rho \) more toward short-term debt. This results in lower short-term bond prices and higher long-term bond prices. And by proposition 3, starting from a corner solution, it will always be less costly to balance long- and short-term borrowing costs against one another rather than issuing all long- or short-term debt. The only thing that will change is the relative maturity tilt.

The same logic applies if we were to allow for uncertainty at \( s = U \) and default at \( s = UD \). For this, assume that firm deliver at \( s = UD \) is higher than \( s = DD \), where generically \( d_{UD} (Q) = d_{DD} (Q) + \epsilon < 1 \). This simply reflects the fact that the ultimate shock to collateral is worse in two consecutive bad states than in an up state followed by a down state. The firm’s maximization problem can be split and written as follows:

$$\max_{t_0, \rho} \prod = \left\{ \gamma^2 \left[ t_0^\alpha - \rho \frac{l_0}{p_0^\alpha} - (1 - \rho) \frac{l_0}{p_U^\alpha} \right] + (1 - \gamma) \gamma \left[ \frac{t_0^\alpha - \rho \frac{l_0}{p_0^\alpha} - (1 - \rho) \frac{l_0}{p_U^\alpha}}{1} \right] \right\}. $$

Only two things change in the problem. 1) Debts are no longer repaid conditional on \( s = U \) so that now the first set of repayment states are given by \( \gamma^2 \) rather than \( \gamma \). 2) \( p_U^\alpha \neq 1 \) as
Table 7: Negative pledge covenant

| Negative pledge covenant | Yes  | No   |
|--------------------------|------|------|
| Non-financial            | 14,783 | 11,424 |
| Financial                | 3,117  | 4,825 |
| < 5yr                    | 2,244  | 2,376 |
| 5yr - 30 yr              | 15,284 | 13,401 |
| Total                    | 17,900 | 16,249 |

Source: Covenant data are from Mergent-FISD. Own calculations based on Mergent-FISD data.

it does with full repayment. Taking first order conditions for an interior maximum and plugging in, one can express the same marginal product equals marginal cost as \( \alpha I_0^{y-1} = \frac{1}{p_0} \). And by proposition 3, we know that for any given \( I_0 \) and a candidate corner solution, it is always be cheaper to fund a portion of the investment outlay by substituting into either long or short-term debt rather so that both debt maturities are utilized. QED.

D Negative pledge covenant

Our treatment of protected long-term debt can be thought either as an explicit R pledge or earmark, or the inclusion of a negative pledge covenant that explicitly spells out how long-term debt is secured from short-term debt dilution. The benefit of thinking about negative pledge covenants, as detailed below, is two fold: 1) negative pledges are among the most common covenants found in public debt indentures, 2) given their prominence, surprisingly little is known in the academic literature of their impact. We thus attempt to fill this void with the support of strong practical relevance.

Negative pledges are widely recognized in law and economics (see Bjerre (1999), Wood (2007, 2008)). The covenant stipulates that the firm cannot issue secured debt in the future without securing the current debt issue. For example, Billet et. al. (2007) classify negative pledge covenants as “Secured Debt Restrictions” because they restrict the security of future debt issues. Table III in their paper shows that negative pledges are typically the 3rd or 4th most common covenant, behind cross default or acceleration, asset sale, and merger clauses. Negative pledges are more common than leverage, dividend, and share repurchase restrictions. Table 7 gives a general sense for the basic statistics on types of bonds that contain a negative pledge covenant. They are more prone in medium-to-long-term non-financial corporate indentures.