Soft photons in semileptonic $B \to D$ decays

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Determination of $V_{cb}$ in exclusive semileptonic decays is crucial consistency check against the $V_{cb}$ determined inclusively. Anticipated precision of $V_{cb}$ at the Super Flavor factory is $\sim 1\%$, with most of the theoretical error due to hadronic form factor uncertainties. However, at this level of precision treating electromagnetic effects becomes inevitable. In addition to virtual photon corrections there are also emissions of real photons which are soft enough to avoid detection. The bremsstrahlung part is completely universal and is accounted for in the experimental analyses. However, the so-called structure dependent contribution, which probes the hadronic content of the process and is infrared finite, has been neglected so far. To this end, we estimated fraction of radiative events which are identified as semileptonic by experiment.

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I. INTRODUCTION

Many efforts have been devoted to experimentally check the validity of Kobayashi-Maskawa (KM) mechanism which predicts that all quark flavor observables agree with the unitary CKM matrix and single CP violating phase. The KM mechanism states that either measuring sides or angles of the unitarity triangle, the apex ($\bar{\rho}, \bar{\eta}$) comes out unique. Value of $V_{cb}$ determines lengths of sides adjacent to the apex, among them also the side opposite to angle $\beta$ which is precisely measured. Current average of inclusive and exclusive determinations is

$$|V_{cb}| = (41.2 \pm 1.1) \times 10^{-3},$$

(1)

where $|V_{cb}|_{\text{excl}} = (38.6 \pm 1.3) \times 10^{-3}$ is significantly lower than $|V_{cb}|_{\text{incl}} = (41.6 \pm 0.6) \times 10^{-3}$. Common lore is that most of theoretical error of the exclusive method stems from the $B \to D$ form factors uncertainties and detection efficiencies. Although inclusive analyses are under better control theoretically and consequently result in more precise result, exclusive method provides a crucial cross-check, since errors are believed to be largely independent for both methods. Future expectation for the exclusive precision is about 1% which could be obtained at Super Flavor factory.

II. DETERMINATION OF $V_{cb}$ IN $B \to D\ell\nu$

A. Theory input

Differential rate of exclusive decay to pseudoscalar $D$ is

$$\frac{d\Gamma}{dw}(B \to D\ell\nu) = \frac{G_F^2 |V_{cb}|^2}{48\pi^3}(m_B + m_D)^2m_D^3(w^2 - 1)^{3/2}G(w)^2,$$

(2)

where $w = v \cdot v'$ is the scalar product of meson velocities. Heavy quark symmetry normalizes the form factor $G(w)$ at the maximum recoil point ($w = 1$), where final state $D$ meson is at rest in the $B$ rest frame. Perturbative $\alpha_s$, $\alpha_{em}$, and nonperturbative ($\Lambda_{QCD}/m_b)^n$ symmetry breaking corrections were also computed and are under control at the maximum recoil point. However, further theoretical insight is required to isolate the value of $V_{cb}$. Allowed phase space shrinks as $w$ approaches 1 and there are very few events recorded in this region. So to infer the experimental

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value of $V_{cb} \times \mathcal{G}(1)$ one has to rely on a particular shape of the form factor to guide the extrapolation down to $w = 1$. In experimental literature it has become customary to use so-called CLN shapes of the form factors \cite{3} which rely on analyticity and unitarity. Measured differential decay rate is then fitted with $V_{cb} \times \mathcal{G}(1)$ and slope $\rho^2$ of the form factor at $w = 1$. In the end theoretical prediction of $\mathcal{G}(1)$ is used to determine $V_{cb}$.

B. Experimental method

Measuring semileptonic $B \rightarrow D \ell \nu$ in $e^+e^-$ collider operating at $\Upsilon(4s)$ resonance one can focus only on events where the tag side momentum is completely reconstructed and ensure that missing invariant mass is peaking at zero, as anticipated for a single neutrino in final state. Kinematical constraints are applied with some tolerance (invariant mass of the tag side is $5.27 - 5.29$ GeV for decay of $B^-$, c.f. \cite{4}) which allows the soft photon events to be included among the semileptonic events.

In this study we set out to study radiative corrections of semileptonic decay $B \rightarrow D \ell \nu$ and in particular what is the ratio of structure dependent (SD) radiative to semileptonic events numbers for given photon energy cut of the experiment. We will keep only the lowest pole contributions in our treatment as they turn out to contribute dominantly due to kinematics. Similar studies were carried out for $K$ meson semileptonic decays using chiral perturbation theory, and as it had turned out SD part was negligible for a typical experimental setup \cite{5}. On the contrary, SD amplitude of $B \rightarrow \mu \nu \gamma$ can lead to $\sim 20\%$ background in a typical experiment measuring $Br(B \rightarrow \mu \nu)$ \cite{5}.

III. INFRARED ELECTROMAGNETIC CORRECTIONS

Electromagnetic effects render all experimentally measured widths a sum of rate of specific process plus rates of radiative events with final state photons which cannot be resolved by the experiment. Such inclusive and infrared (IR) safe quantity is schematically

$$d\Gamma_{\text{exp}}(i \rightarrow f) = d\Gamma(i \rightarrow f) + d\Gamma(i \rightarrow f\gamma)E_{\gamma} < E_{\text{cut}} + d\Gamma(i \rightarrow f\gamma\gamma)E_{\gamma\gamma} < E_{\text{cut}}, \Sigma E_{\gamma\gamma} < E_{\text{cut}}' + \cdots.$$  

The above inclusive width solves the IR problem of electrodynamics by cancelling soft divergences due to virtual photon corrections against real emission. The amplitude of the so-called inner bremsstrahlung (IB) diverges as the photon energy approaches zero and residue of the pole is fixed by the charge of the external leg where the photon is emitted from. In the IR limit photons can only resolve total charge of the emitting particle. Accordingly, Low’s theorem states that leading two terms in momentum expansion of the radiative decay width are fixed from the theorem states that leading two terms in momentum expansion of the radiative decay width are fixed from the nonradiative decay width \cite{6}. These IR divergences are compensated by the corresponding virtual corrections at the same order of $\alpha_{EM}$ at the level of decay width.

However, there are also subleading, IR finite, terms in the $d\Gamma_{\text{exp}}$ which are usually neglected in experimental analyses. These structure dependent photon emissions can resolve structure of charged particles. Consequently, prediction of SD terms require knowledge of additional form factors.

A. Amplitude

We adopt notation established in \cite{7} for semileptonic $K$ decays. Amplitude of $B^- \rightarrow D^0 \ell \nu \gamma$ is

$$A_\mu = \frac{eG_F V_{cb}}{\sqrt{2}} \bar{u}(p_\ell) \left( -\frac{F_\nu(t)}{2p_\gamma q} \gamma_\mu (\not{p}_\ell + \not{q} + m_\ell) + V_{\mu\nu} - A_{\mu\nu} \right) \gamma^\nu (1 - \gamma_5)v(k) \quad (4)$$

which is in the end contracted with the photon polarization. Here $q$, $k$, and $p_\ell$ are the respective photon, neutrino, and lepton momenta. First term in brackets is proportional to

$$F_\nu(t) \equiv i \langle D(p') \mid H_\nu \mid B(p) \rangle, \quad t \equiv (p - p')^2 \quad (5)$$

and represents the photon emission from the lepton leg, whereas $V_{\mu\nu}$ and $A_{\mu\nu}$ are hadronic vector and axial form factors of $B \rightarrow D\gamma$ transition, namely when photon is emitted from hadronic line

$$V_{\mu\nu} - A_{\mu\nu} \equiv \int d^4y e^{iqy} \langle D(p') \mid T [J_\mu(y)H_\nu(0)] \mid B(p) \rangle, \quad H^\nu \equiv \bar{c}\gamma^\nu (1 - \gamma_5)b. \quad (6)$$
Here $J_\mu$ is the electromagnetic current. These form factors obey electromagnetic Ward identities

\begin{align}
q^\mu V_{\mu\nu} &= F_\nu(t), \\
q^\mu A_{\mu\nu} &= 0,
\end{align}

which ensure total amplitude is gauge invariant. Intermediate 1-particle resonances give rise to poles due to excited beauty and charm states. The soft photon part of phase space should be well approximated by lowest pole contributions of $B$, $B^*$ and $D^*$. The $B$-pole satisfies the inhomogeneous Ward identity above and we single it out of $V_{\mu\nu}$

\begin{align}
V_{\mu\nu}^{IB} &= \frac{p_\mu}{p\cdot q} F_\nu(t) \\
V_{\mu\nu}^{SD} &= V_{\mu\nu} - V_{\mu\nu}^{IB}, \quad q^\mu V_{\mu\nu}^{SD} = 0.
\end{align}

Lorentz covariance and Ward identities allow the form factors to be split down into eight scalar functions $V_{1...4}, A_{1...4}$,

\begin{align}
V_{\mu\nu}^{SD} &= V_1 (p'_\mu q_\nu - p'_\nu q_\mu) + V_2 (p_\mu q_\nu - p_\nu q_\mu) \\
&\quad + (p\cdot q p'_\mu - p\cdot q p_\mu) (V_3 p_\nu + V_4 p'_\nu), \\
A_{\mu\nu} &= A_1 \epsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta + A_2 \epsilon_{\mu\nu\alpha\beta} p'^\alpha q'^\beta + (A_3 p_\nu + A_4 p'_\nu) \epsilon_{\mu\nu\beta\gamma} p^\alpha q^\beta q^\gamma.
\end{align}

We saturate the SD part of the amplitude with $D^*$ and $B^*$ resonances, which contribute to $V_{\mu\nu} - A_{\mu\nu}$ as

\begin{align}
\frac{i \langle D | J_\mu | D^*(p' + q) \rangle \langle D^*(p' + q) | V_\nu - A_\nu | B \rangle}{(p' + q)^2 - m_{D^*}^2}, \\
\frac{i \langle D | V_\nu - A_\nu | B^*(p - q) \rangle \langle B^*(p - q) | J_\mu | B \rangle}{(p - q)^2 - m_{B^*}^2}.
\end{align}

The $B^*$ pole is not far in unphysical region and its contribution gets enhanced by factor $1/(m_{B^*}^2 - m_{D^*}^2)$ in the limit $E_\gamma \to 0$. The $D^*$ pole, on the other hand, can be on-shell and we model its contribution by Breit-Wigner shape

\begin{align}
\frac{i \langle D | J_\mu | D^* \rangle \langle D^* | V_\nu - A_\nu | B \rangle}{(p' + q)^2 - m_{D^*}^2 + i m_{D^*} \Gamma_{D^*}}.
\end{align}

The above on-shell $D^*$ contribution is expected to dominate the radiative decay in question. Form factors $V_{\mu\nu}, A_{\mu\nu}$ contain nonperturbative matrix elements, as evident from (9), for which we take quenched lattice results of $B \to D^*$ form factors [8, 9]. Value of $g_{D^* D^*}$ was computed on the lattice with dynamical light quarks [10].

![Figure 1: Slices of phase space in the $D^*$ invariant mass versus momentum transfer $t$ for different photon energies. Horizontal line represents on-shell $D^*$, which is reachable only in the range 50 MeV < $E_\gamma$ < 350 MeV.](image)

The intermediate $D^*$ is kinematically allowed to be on-shell only for photon energies in the range of $\sim [50, 350]$ MeV (see Fig. 1). This resonant enhancement of the soft photon kinematical region originates from relatively small mass splitting between $D^*$ and $D$. Next higher excited charm state lies already above 2.4 GeV and would
result in more energetic photons due to larger mass splitting. Thus we expect that higher resonances would mostly produce photons in the experimentally accessible region.

The contribution of $D^*$ is clearly seen in the $E_\gamma$ spectra of $\mu$ and $\tau$ channels, Figs. 2,3 where roughly half of the width lies in the $E_\gamma < 200$ MeV region.

Figure 2: Left: resonant $D^*$ spectrum of $B^- \to D^0 \mu \nu \gamma$. Right: fraction of misidentified radiative events plotted against the experimental resolution of the photon energy $E_{\text{cut}}$.

Figure 3: Left: resonant $D^*$ spectrum of $B^- \to D^0 \tau \nu \gamma$. Right: fraction of misidentified radiative events plotted against the experimental resolution of the photon energy $E_{\text{cut}}$.

The importance of improving experimental resolution of soft photons detection or including them among backgrounds is clearly seen in Figs. 2, 3. A photon energy cut of 300 MeV in $B^- \to D^0 \mu \nu$ results in $\sim 4\%$ of the recorded events to be fake. This would in turn imply a 2% fake enhancement of $|V_{cb}|$. 
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