Bounces, turnaround and singularities in bimetric gravity.

Salvatore Capozziello\textsuperscript{a,b}, Prado Martín-Moruno\textsuperscript{c}

\textsuperscript{a}Dipartimento di Scienze Fisiche, Università di Napoli “Federico II”, Via Cinthia, I-80126 - Napoli, Italy.
\textsuperscript{b}INFN Sez. di Napoli, Comp. Univ. di Monte S. Angelo, Edificio G, Via Cinthia, I-80126 - Napoli, Italy.
\textsuperscript{c}School of Mathematics, Statistics, and Operations Research, Victoria University of Wellington, PO Box 600, Wellington 6140, New Zealand.

Abstract

In this letter, we consider cosmological solutions of bimetric theory without assuming that only one metric is coupled to gravity. We conclude that any cosmology can be described by fixing the matter content of the space that we are not inhabiting. On the other hand, we show that some conclusions can still be extracted independently of the matter content filling both spaces. In particular, we can conclude the occurrence of some extremality events in one universe if we know that they take place in the other space.

Keywords: Bimetric gravity, Cosmology, Singularities

1. Introduction

The theory of bimetric gravity assumes the existence of two metric tensors interacting with each other \cite{1}. If these metric tensors have kinetic terms of the Einstein-Hilbert form and the equivalence principle is fulfilled, then the interaction between both metric fields would lead to an elegant modification of general relativity, being the existence of the other gravitational sector only measurable by its gravitational effects. One could wonder why this theory has not gained a renewed interest as soon as the impossibility of general relativity to describe our universe at astrophysical and cosmological scales (at least without introducing \textit{ad hoc} new material components) has been suggested by several competing approaches (see for example \cite{2,3,4,5,6,7,8,9}). The principal reason is that bimetric gravity generally presents a Boulware-Deser ghost \cite{10}, which implies an instability of the theory. Nevertheless, it has been recently shown that this undesired ghost can be discarded or controlled by considering particular interactions between the metrics \cite{11,12} (see also \cite{13} for a bigravity version of $f(R)$).

Thus, ghost-free bimetric cosmologies have been considered \cite{14,15,16,20,21} and matched to the observational data with promising results \cite{15,17}. However, most of these studies are restricted to considering only a particular class of models which assume that no material content is present in one of the spaces. In this letter we explicitly consider the behavior of the theory in a cosmological scenario in the most general case, illustrating that, as it could be expected, the dynamics of our Universe would depend on the material content of the other space in this framework. As that hidden matter cannot be observed directly, this fact could be indicating a possible degeneracy of the theory. It seems that this degeneracy could only be cured if the matter content of both sectors is specified from the very beginning using some argument based on fundamental principles, or if localized solutions are also taken into account.

Due to the complexity of analyzing the general theory, one can consider whether at least some information about the cosmology of one sector can be extracted from the knowledge of the behavior of the other universe, even without specifying the matter content of the spaces. It is the main aim of the present letter to show that this is indeed possible regarding the occurrence of extremality events, as bounces, turnaround and singularities.

2. Cosmological solutions of the general theory

The action of the ghost-free bimetric gravity theory found in \cite{11} has an interaction term which is a function of $\gamma = \sqrt{g/f}$. That action can be re-expressed as \cite{18}

$$S = -\frac{1}{16\kappa G} \int d^4x \sqrt{-g} \left[ R(g) + 2 \Lambda \right] + \int d^4x \sqrt{-g} \, L_m$$

$$- \frac{\kappa}{16\pi G} \int d^4x \sqrt{-f} \left[ \tilde{R}(f) + 2 \Lambda \right] + \epsilon \int d^4x \sqrt{-f} \, L_m$$

$$+ \frac{m^2}{8\pi G} \int d^4x \sqrt{-g} \, L_{int}(\gamma),$$

where the interaction Lagrangian is

$$L_{int} = \beta_1 e_1(\gamma) + \beta_2 e_2(\gamma) + \beta_3 e_3(\gamma),$$

with

$$e_1(\gamma) = \text{tr}[\gamma];$$

$$e_2(\gamma) = \frac{1}{2} \left[ \text{tr}[\gamma]^2 - \text{tr}[\gamma^2] \right];$$

$$e_3(\gamma) = \frac{1}{6} \left[ \text{tr}[\gamma]^3 - 3 \text{tr}[\gamma] \text{tr}[\gamma^2] + 2 \text{tr}[\gamma^3] \right],$$

being elementary symmetric polynomials. It can be noted that the effective Newton constant for the $f$-space, $\epsilon G/\kappa$, would be equal to that of the $g$-space only if $\epsilon = \kappa$. Apart from the effective Newton constant, the theory is completely symmetric.
under the interchange of $f$ and $g$ due to the properties of the elementary symmetric polynomials $[11, 18]$. If we consider a cosmological scenario, then, assuming that both metrics have the same sign of spatial curvature $k$, we can write

$$d s^2_f = -dt^2 + a(t)^2 \left( \frac{dr^2}{1 - kr^2} + r^2 \left( d\theta^2 + \sin^2 \theta \, d\phi^2 \right) \right). \quad (6)$$

and

$$d s^2_t = -N(t)^2 \, dt^2 + b(t)^2 \left( \frac{dr^2}{1 - kr^2} + r^2 \left( d\theta^2 + \sin^2 \theta \, d\phi^2 \right) \right). \quad (7)$$

where we are dismissing some special case solutions for particular values of the parameter $\beta$. The modified Friedmann equations of both spaces can be obtained by brute force from the action $\mathcal{S}$ and metrics (6) and (7), noticing that this scenario can be described by the generalized Gordon ansatz $[16]$. These are

$$H^2_g + \frac{k}{a^2} = \frac{m^2}{3} \rho_m + \frac{8\pi G}{3} \rho_m + \frac{\Lambda}{3}, \quad (8)$$

and

$$H^2_t + \frac{k}{b^2} = \frac{m^2}{3k} \rho_t + \frac{8\pi G}{3} \rho_t + \frac{\Lambda}{3}, \quad (9)$$

with

$$\rho = \frac{b}{a} \left( 3\beta_1 + 3\beta_2 \frac{b}{a} + \beta_3 \frac{b^2}{a^2} \right), \quad (10)$$

for the $g$-space, and

$$\mathcal{P} = \frac{a}{b} \left( 3\beta_1 + 3\beta_2 \frac{a}{b} + \beta_1 \frac{a^2}{b^2} \right), \quad (11)$$

for the $f$-space. We have defined the Hubble parameters as $H_2 = a'/a$ and $H_1 = b'/b$ (N b), where $' \equiv d/dt$. The appearance of the factor $1/N(t)$ in the Hubble parameter $H_2$ can be expected by noting that metric (7) is not expressed in terms of the cosmic time of this space. We can define the cosmic time of the $f$-space as

$$\tau(t) = \int N(t) \, dt. \quad (12)$$

Thus, we are using the usual definition of the Hubble parameter also in the $f$-space, that is $H_f \equiv b'/b$ with $' \equiv d/dt$.

In view of action (1), one can note that due to the diffeomorphism invariance the matter stress energy tensor of both spaces is conserved. Thus, defining $w_m(a) = p_m/\rho_m$ and $\overline{w}_m(b) = \overline{p}_m/\overline{\rho}_m$, we have $\rho_m + 3H_2[1 + w_m(a)]\rho_m = 0$ and $\overline{\rho}_m + 3H_1[1 + \overline{w}_m(b)]\overline{\rho}_m = 0$, which can be integrated to obtain

$$w_m(a) = \rho_m \exp \left[ -3 \int_{a_0}^a \left[ 1 + w_m(a) \right] \frac{da}{a} \right], \quad (13)$$

and

$$\overline{p}_m(\overline{b}) = \overline{\rho}_m(\overline{b}) \exp \left[ -3 \int_{b_0}^b \left[ 1 + \overline{w}_m(\overline{b}) \right] \frac{db}{\overline{b}} \right]. \quad (14)$$

respectively. Taking into account the Bianchi identities, the stress energy tensor coming from the interaction term must be also conserved. This leads to $[14, 15, 16]

$$b(t) = N(t) \, \dot{a}(t). \quad (15)$$

Therefore, the Hubble parameter of the $f$-space can be expressed as $H_f = \dot{a}/a$, which implies that the Friedmann equations of both spaces, (8) and (9), are coupled. This fact allowed the authors of References [14] and [15] to solve the system (or to indicate how to obtain the solutions) in the particular case that no material content is considered in the $f$-space. In a similar way, we could, in principle, obtain the solution of the system (8) and (9), taking into account Equations (10), (11), (13), (14) and (15). In the first place, multiplying Equation (9) by $b^2/a^2$, subtracting the resulting expression from Equation (8), inserting Equations (10) and (11), and simplifying the result, we obtain the following algebraic equation:

$$c_4 b^4 + c_5 a b^3 - \frac{\overline{c}}{m^2} \overline{p}_m a b^3 c_2 a^2 b^2 + \frac{c_3}{m^2} \rho_m b a + c_4 a^3 b - c_5 a^4 = 0, \quad (16)$$

where $c_4 = \beta_3/3$, $c_5 = \beta_2 - \Lambda/(3m^2)$, $c_2 = \beta_1 - \beta_3/k$, $c_1 = \Lambda/(3m^2) - \beta_2/k$, $c_0 = \beta_1/3k$, $C = 8\pi G/3$, $\overline{C} = 8\pi G/3k$, and we are simplifying notation by assuming the dependence of both material energy densities in their corresponding scale factors, those are $\rho_m(a)$ and $\overline{\rho}_m(b)$. In the second place, considering particular forms for $w_m(a)$ and $\overline{w}_m(b)$ in Equations (13) and (14), and inserting the results in Equation (16), the LHS of Equation (16) can be considered as a polynomial on $b$. Thus, once $w_m(a)$ and $\overline{w}_m(b)$ are fixed, Equation (15) can be solved to obtain $b$ as a function of $a$, at least in principle (e. g. for $\overline{w}_m = 0$ we have a quartic equation which can be analytically solved [15]). In the third place, the obtained function $b(a)$ can be inserted in Equation (10) and the result in Equation (8), which could be integrated considering again Equation (13). Moreover, up to now, most studies have paid attention only to the physics of one space (see References [16, 19] for two interesting exceptions), probably because if no material content is considered in the other space it cannot describe an inhabited universe. Nevertheless, once we have the functions $a(t)$ and $b(a)$, it is straightforward to obtain $b(t)$. This scale factor can be more properly interpreted when it is expressed in terms of its cosmic time, $\tau(t)$, which can also be done easily using Equations (15) and (12). In summary, this procedure would allow us to know the evolution of both universes once $w_m(a)$ and $\overline{w}_m(b)$ are fixed.

On the other hand, it seems that one could describe any possible cosmology in the $g$-space, i.e. any possible combination of $a(t)$ and $w_m(a)$, by assuming a different matter content in the $f$-space, that is a different $\overline{w}_m(b)$. Thus, one could think that the consideration of some matter content of the $f$-space introduces

\[1\] Those solutions are not compatible with considering both metrics being diagonal in the same coordinate patch. That can be understood noting that the argument presented in [18] regarding the classification of solutions would be valid for any material content of both spaces since it is based on the symmetry of the spaces.

\[2\] Note that in Reference [14] a different definition of $H_f$ is used.
a degeneracy on the theory, and in particular on the cosmology. To illustrate this different approach let us consider that \(a(t)\) and \(w_m(a)\) are fixed and have any desired form. In this case we could obtain \(b(t)\) from Equation (5) taking into account (10) and (13). Thus, from Equation (16) we can obtain

\[
\bar{\rho}_m(b) = \frac{m^2}{c} \left[ \frac{c a b}{a} + c_3 + \frac{c_2 a^2}{b} + \frac{c_1 a^3}{b^2} - \frac{c_0 a^3}{b^3} \right],
\]

which can be considered as a function of \(b\), since \(a(b)\) can be, in principle, obtained from \(b(t)\) and \(a(t)\). Therefore, taking into account Equation (14), this procedure implies that by choosing \(\bar{\rho}_m(b)\) carefully in the \(f\)-space, we can obtain the desired cosmology in the \(g\)-space.

This second approach seems to indicate that the consideration of any matter content in the \(f\)-space introduces a degeneracy in the cosmological solutions of the theory. That is, it seems that there is a loss of predictive power since any cosmology could be described with arbitrary accuracy by fitting a material content which cannot be observed directly. As the model considering no material in the \(f\)-space can fit the observational data \([15, 17]\), one could wonder why should we consider the possible existence of any hidden matter in the \(f\)-space. From a theoretical point of view the symmetry between the two gravitational sectors \([11, 19]\) is one of the nicest characteristics of the theory, and postulating the absence of matter in only one sector from the very beginning would break this symmetry. On the other hand, dismissing the possible existence of this matter content could be an assumption stronger than considering a particular kind of matter, if this assumption is not based in any fundamental principles. Thus, as long as these principles are not elucidated, we are in a situation where it seems that no conclusion about the dynamics in both spaces can be extracted in general. Nevertheless, as we will show, this is not the case.

It must be pointed out that the degeneracy mentioned above is only apparent and it would not be present if one goes beyond the cosmological scenario. In particular, one could obtain the value of \(\bar{\rho}_m(b)\) by fitting the general model with future and more precise data, and test whether the predictions of the resulting theory are fulfilled in other scenarios, for example for black hole solutions. Thus, the impossibility of observing directly the matter content of the other space does not imply a degeneracy of the theory itself due to the coupling of both metrics. Moreover, the predictions of the theory are, of course, different from those of general relativity even without considering perturbations, at least in cases where both metrics are not proportional (see, e.g., Reference \([22]\) for more information about the case where the metrics are proportional to each other).

### 3. General behavior of cosmological solutions

We emphasize that condition (15) is independent of the material content in both gravitational sectors, since it is a consequence of the diffeomorphism invariance of action (1) and the particular symmetry that we are considering in metrics \([6]\) and \([7]\). So, it seems that we could find some relation between the dynamics of both spaces even without assuming any characteristics of the matter content. In particular, we can extract some information about the possible occurrence of extremality events in one space once we know that these events take place in the other space. In the first place, we express \(b\) and its derivatives in terms of its cosmic time. So, taking into account the definition (12) in condition (15), we have

\[
\dot{a}(t) = b'(\tau),
\]

where each scale factor is derived in terms of its cosmic time. In the second place, deriving Equation (15) with respect to \(t\) and using again definition (12), we obtain

\[
\dot{a}(t) = N(t) b''(\tau).
\]

It must be noted that if for a particular \(t \to t\), we would have \(N(t) \to \infty\), then, given Equation (12), that would imply \(\tau \to \infty\). In this case, the region \(t > \tau\), would not be in the interior of the \(f\)-space. On the other hand, in order to have a metric with a well defined Lorentzian signature in the \(f\)-space, we must require \(N(t) \neq 0\). One could again interpret the possibility of attaining \(N(t) = 0\) as considering a region which is not in the interior of both spaces, since \(dt\) would be infinitely large for a finite \(d\tau\), being, therefore, the range of definition of \(\tau\) outside that of \(t\). Thus, it would make no sense trying to extract any conclusion relating the dynamics of both spaces for a vanishing or infinite lapse function.

So, let us assume, for now on, that the particular times that we would refer to are in the range of definition of both spaces, i.e. \(0 < N(t) < \infty\), where we are taking both times pointing in the same direction \(N(t) > 0\). Therefore, we can already conclude that if the cosmology of the \(g\)-space present a bounce on a particular \(t\), that is \(\dot{a}(t) = 0\) and \(\ddot{a}(t) > 0\), then the cosmology of the \(f\)-space would have a bounce on \(\tau = \tau(t)\) (which exists and is finite because we are assuming \(0 < N(t) < \infty\) in the vicinity of \(t\)). This statement is obviously also true in the opposite direction, therefore, the \(g\)-cosmology has a bounce at \(t\) if and only if the \(f\)-cosmology has it at \(\tau\). It is straightforward to see that something equivalent could also be said about turnarounds, which are characterized by \(\dot{a}(t) = 0\) and \(\ddot{a}(t) < 0\). Thus, the \(g\)-cosmology has a turnaround at \(t\), if and only if the \(f\)-cosmology has it at \(\tau\).

On the other hand, the discussion of singularities is more subtle. In this case, in order to be able to extract any conclusion, let us follow the spirit of Reference \([24]\) assuming that in the vicinity of a singularity the scale factor of the \(g\)-space has a generalized power series (Puisieux series \([25]\)) expansion (see also \([26, 27]\)). Thus, if the \(g\)-cosmology is born in a big bang singularity at \(t_0\), then for \(t \in (t_0, t_0 + \delta)\) we could write

\[
a(t) \approx c(t - t_0)^n,
\]

where \(c\) and \(n\) are positive numbers (not necessarily integer numbers) and we are writing only the dominant contribution.

---

\(^3\)That this behavior is indeed possible, at least in principle, can be understood considering the results in \([23]\), where another scenario and bimetric gravity model was considered. The authors show that the conformal diagram of one space could be unable to accommodate all points for which the other space is defined.
Note that $0 < n < 1$ is needed to have a big bang singularity, which is characterized by $a(t) \to 0$, $\dot{a}(t) \to \infty$ and $\ddot{a}(t) \to -\infty$, when $t \to t_*$. Due to Equations (18) and (19), we have $b'(\tau) \to \infty$ and $b''(\tau) \to -\infty$, when $\tau \to \tau_*$. So, assuming that $b(\tau)$ can also be expanded in a generalized power series, we should have

$$b(\tau) \approx d(\tau - \tau_*)^m + C,$$

with $0 < m < 1$ and $d > 0$, to be able to reproduce the desired divergences in $b'(\tau)$ and $b''(\tau)$. Thus, a big bang in the $g$-universe implies also a singular origin for the $f$-universe, although it can born at a vanishing or non-vanishing (finite) size. It can be noted that a similar argument would be valid with a big crunch.

Following a similar procedure, we can consider that the $g$-cosmology ends its evolution in a big rip singularity [28], implying that for $t \in (t_* - \delta, t_*)$

$$a(t) \approx c_1(t_* - t)^{-n_1},$$

or in a big freeze [29]

$$a(t) \approx a_m - c_2(t_* - t)^{n_2},$$

with $c_1, c_2, n_1 > 0$ and $0 < n_2 < 1$. That is because these singularities are both characterized by $\dot{a}(t) \to \infty$ and $\ddot{a}(t) \to \infty$, for $t \to t_*$, being also $a(t) \to \infty$ for the big rip case, whereas $a(t) \to a_m > 0$ for the big freeze. Thus, these singularities differ on the value of the scale factor but not on the value of its derivatives. Therefore, taking into account Equations (18) and (19), we get in both cases

$$b(\tau) \approx \frac{d}{m-1}(\tau_* - \tau)^{-m+1} + C,$$

with $d, m > 0$ to have $b'(\tau) \to \infty$ and $b''(\tau) \to \infty$, when $\tau \to \tau_*$. Nevertheless, we cannot say whether $m > 1$, which would imply a big rip, or $0 < m < 1$, which corresponds to a big freeze. Anyway, we can conclude that a big rip or big freeze doomsday for the $g$-universe implies also the occurrence of a big rip or big freeze at a finite time in the $f$-space.

In conclusion, a systematic classification of singularities at finite and infinite in both $g$ and $f$ spaces is needed in the sense discussed in [30].

4. Discussion and conclusions

In this letter we have pointed out the two different approaches that one could follow when studying cosmological scenarios in bimetric gravity. The first one entails the consideration of a particular matter content in both spaces and completely fixes the dynamics of both universes. Whereas the second one points out that the material content of both spaces is not fixed by fundamental principles, then any desired universe (scale factor) filled with any material can be generated, or reconstructed, by choosing carefully the hidden matter. Nevertheless, we have shown that some general conclusions about the dynamics of both universes can be extracted without restricting the analysis to particular material contents. That is, due to the diffeomorphism invariance, we can say that a bounce (turnaround) of one universe implies the occurrence of such event in the other universe at a corresponding cosmic time. The presence of cosmic singularities in both spaces can also be concluded, since the divergent behavior of the derivatives of both scale factors in terms of their respective cosmic times can be easily related to each other.

Finally, we want to stress that the results presented in this letter apply as long as $0 < N(t) < \infty$ and both metrics can be expressed in the form considered in [6] and [7], that is, when both spaces are inside each other (the range of definition of $t$ is inside that of $\tau$ and vice versa) and the metrics can be taken to be diagonal in the same coordinate patch [3]. It is known that the stress energy tensor associated to the interaction term of one of the spaces should violate the null energy condition (if it is not saturated) when the corresponding stress energy tensor in the other space fulfills it [19]. Thus, we expect that there would be particular cases where the total stress energy tensor of one space violates the null energy conditions whereas the total stress energy tensor of the other space fulfills it, which would necessarily imply that the singularities of both cosmologies, if any, are of a different type. Nevertheless, this behavior should correspond to a situation where the assumptions no longer hold, that is, that singularity would take place in one space outside the range of definition of the other space.

Acknowledgments

The authors wish to thank Matt Visser for useful comments. SC is supported by INFN, iniziativa specifica NA12. PMM acknowledges financial support from the Spanish Ministry of Education through a FECYT grant, via the postdoctoral mobility contract EX2010-0854.

References

[1] C. J. Isham, A. Salam and J. A. Strathdee, “F-dominance of gravity”, Phys. Rev. D 3 (1971) 867.
[2] S. Capozziello and M. Francaviglia, ”Extended Theories of Gravity and their Cosmological and Astrophysical Applications”, Gen. Rel. Grav. 40 (2008) 357 [arXiv:0706.1146 [astro-ph]].
[3] S. Capozziello and V. Faraoni, “Beyond Einstein gravity: A Survey of gravitational theories for cosmology and astrophysics”, Fundamental Theories of Physics, Vol. 170, Springer, 2010 New York.
[4] A. De Felice and S. Tsujikawa, "(f(R) theories", Living Rev. Rel. 13 (2010) 3 [arXiv:1002.4928 [gr-qc]].
[5] S. Capozziello and M. De Laurentis, “Extended Theories of Gravity”, Phys. Rept. 509 (2011) 167 [arXiv:1108.6266 [gr-qc]].
[6] S. Nojiri and S. D. Odintsov, ”Unified cosmic history in modified gravity: from F(R) theory to Lorentz non-invariant models”, Phys. Rept. 505 (2011) 59 [arXiv:1011.0544 [gr-qc]].
[7] K. Bamba, S. Capozziello, S. Nojiri and S. D. Odintsov, ”Dark energy cosmology: the equivalent description via different theoretical models and cosmography tests”, Astrophys. Space Sci. 342 (2012) 155 [arXiv:1205.3421 [gr-qc]].
[8] T. Harko, F. S. N. Lobo, S. Nojiri and S. D. Odintsov, ”f(R, T) gravity”, Phys. Rev. D84 (2011) 024020 [arXiv:1104.2669 [gr-qc]].

These conditions have a similar flavor to those considered in [31] to study horizons in bimetric spacetimes.
[9] T. Harko, T. S. Koivisto, F. S. N. Lobo and G. J. Olmo, “Metric-Palatini gravity unifying local constraints and late-time cosmic acceleration”, Phys. Rev. D 85 (2012) 084016 [arXiv:1110.1049 [gr-qc]].
[10] D. G. Boulware and S. Deser, “Can gravitation have a finite range?”, Phys. Rev. D 6 (1972) 3368.
[11] S. F. Hassan and R. A. Rosen, “Bimetric Gravity from Ghost-free Massive Gravity”, JHEP 1202 (2012) 126 [arXiv:1109.3515 [hep-th]].
[12] S. F. Hassan and R. A. Rosen, “Confirmation of the Secondary Constraint and Absence of Ghost in Massive Gravity and Bimetric Gravity”, JHEP 1204 (2012) 123 [arXiv:1111.2070 [hep-th]].
[13] S. Nojiri and S. D. Odintsov, “Ghost-free $F(R)$ bigravity and accelerating cosmology”, Phys. Lett. B 716 (2012) 377 [arXiv:1207.5106 [hep-th]].
[14] M. S. Volkov, “Cosmological solutions with massive gravitons in the bigravity theory”, JHEP 1201 (2012) 035 [arXiv:1110.6153 [hep-th]].
[15] M. von Strauss, A. Schmidt-May, J. Enander, E. Mortsell and S. F. Hassan, “Cosmological Solutions in Bimetric Gravity and their Observational Tests”, JCAP 1203 (2012) 042 [arXiv:1111.1655 [gr-qc]].
[16] V. Baccetti, P. Martín-Moruno and M. Visser, “Gordon and Kerr-Schild ansatze in massive and bimetric gravity”, JHEP 1208 (2012) 108 [arXiv:1206.4720 [gr-qc]].
[17] Y. Akrami, T. S. Koivisto and M. Sandstad, “Accelerated expansion from ghost-free bigravity: a statistical analysis with improved generality”, arXiv:1209.0457 [astro-ph.CO].
[18] V. Baccetti, P. Martín-Moruno and M. Visser, “Massive gravity from bimetric gravity”, arXiv:1205.2158 [gr-qc].
[19] V. Baccetti, P. Martín-Moruno and M. Visser, “Null Energy Condition violations in bimetric gravity”; arXiv:1206.3814 [gr-qc].
[20] D. Comelli, M. Crisostomi, F. Nesti and L. Pilo, “FRW Cosmology in Ghost Free Massive Gravity”, JHEP 1203 (2012) 067 [arXiv:1111.1983 [hep-th]].
[21] D. Comelli, M. Crisostomi and L. Pilo, JHEP 1206 (2012) 085 [arXiv:1202.1986 [hep-th]].
[22] S. F. Hassan, A. Schmidt-May and M. von Strauss, “On Consistent Theories of Massive Spin-2 Fields Coupled to Gravity”, arXiv:1208.1515 [hep-th].
[23] D. Blas, C. Deffayet and J. Garriga, “Global structure of bigravity solutions”, Class. Quant. Grav. 23 (2006) 1697 [hep-th/0508163].
[24] C. Cattoën and M. Visser, “Necessary and sufficient conditions for big bangs, bounces, crunches, rips, sudden singularities, and extremality events”, Class. Quant. Grav. 22 (2005) 4913 [gr-qc/0508045].
[25] C. Cattoën and M. Visser, “Generalized Puisieux series expansion for cosmological milestones”, Proceedings of MG11 Meeting on General Relativity, Edited by H. Kleinert, R.T. Jantzen and R. Ruffini. Hackensack, World Scientific, 2008. pp. 2057-2059 [gr-qc/0609073].
[26] L. Fernandez-Jambrina and R. Lazkoz, “Geodesic behaviour of sudden future singularities”, Phys. Rev. D 70 (2004) 121503 [gr-qc/0410124].
[27] L. Fernandez-Jambrina and R. Lazkoz, “Classification of cosmological milestones”, Phys. Rev. D 74 (2006) 064030 [gr-qc/0607073].
[28] R. R. Caldwell, M. Kamionkowski and N. N. Weinberg, “Phantom energy and cosmic doomsday”, Phys. Rev. Lett. 91 (2003) 071301 [astro-ph/0302506].
[29] M. Bouhmadi-Lopez, P. F. Gonzalez-Diaz and P. Martín-Moruno, “Worse than a big rip?”, Phys. Lett. B 659 (2008) 1 [gr-qc/0612135].
[30] S. Capozziello, M. De Laurentis, S. Nojiri and S. D. Odintsov, “Classifying and avoiding singularities in the alternative gravity dark energy models”, Phys. Rev. D 79 (2009) 124007 [arXiv:0903.2753 [hep-th]].
[31] C. Deffayet and T. Jacobson, “On horizon structure of bimetric spacetimes”, Class. Quant. Grav. 29 (2012) 065009 [arXiv:1107.4978 [gr-qc]].