Improved nonsinglet QCD analysis of the fixed-target DIS data

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Abstract

Deep inelastic scattering data on $F_2$ structure function obtained by BCDMS, SLAC and NMC collaborations in fixed-target experiments were analyzed in the non-singlet approximation with next-to-next-to-leading-order accuracy. The strong coupling constant is found to be $\alpha_s(M_Z^2) = 0.1157 \pm 0.0022$ (total exp.error) + $\{+0.0028 \text{ (theor)} \}$, which is seen to be well compatible with the average world value. Results obtained in the present paper confirm those derived in [2] by carrying out similar fits but with systematic errors in BCDMS data taken into account in a different way.

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1 Introduction

Deep inelastic scattering (DIS) of leptons off hadrons is one of the most important processes in studying the parton distribution functions (PDFs) in nucleons. The latter are nowadays basically required to do any reliable calculations for observables.

The present paper closely follows those devoted to similar studies [1] [2] performed at the next- (NLO) and next-to-next-to-leading-order (NNLO) levels, respectively. The difference between analyses done in [1] and [2] is that only a nonsinglet case is dealt with in the latter. In turn, here we consider systematic errors in BCDMS data in a different manner than it was done in [2] in order the study their influence on our results obtained in [1] [2], where $\alpha_s(M_Z^2)$ value was shown to increase when we cut out BCDMS data with the largest systematic errors. Those results have recently been criticized in [3], where it was found that this effect is negligible. The authors of [3] supposed that the $\alpha_s(M_Z^2)$ value increased due to systematic errors being neglected in BCDMS data in the analyses done in [1] [2].

Here we will show that including the systematic errors in BCDMS data does not significantly alter our results derived in [1] [2]. Upon omitting BCDMS data with the largest systematic errors we obtain larger values of the coupling constant normalization $\alpha_s(M_Z^2)$ fitted to the experimental data. Moreover, the effect does not strongly depend on specific cut values, as it was observed earlier in [1] [2].

1 In the present paper we restrict analysis to the large $x$ region. Consequently, the analysis is dubbed the nonsinglet one (simply signaling the absence of gluons) but actually the data on the entire structure function $F_2(x, Q^2)$ will be considered.
DIS structure function (SF) $F_2(x, Q^2)$ is dealt with by analyzing SLAC, NMC and BCDMS experimental data [4–9] at NNLO of massless perturbative QCD. These calculations become possible due to results concerning $\alpha_s^3(Q^2)$ corrections to the splitting functions (the anomalous dimensions of Wilson operators) [10].

As in our previous papers the function $F_2(x, Q^2)$ is represented as a sum of the leading twist $F_2^{pQCD}(x, Q^2)$ and twist four terms

$$F_2(x, Q^2) = F_2^{pQCD}(x, Q^2) \left(1 + \frac{\tilde{h}_4(x)}{Q^2}\right).$$

(1)

In the analyses performed over experimental data various effects and corrections must be taken into account. Here the nuclear effects, target mass and heavy quark threshold corrections and higher twist terms are considered. For details we refer to [1, 11].

There in general are two methods of carrying out a QCD analysis over DIS data: the first one (see e.g. [12–17]) deals with Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) integro-differential equations [18] and lets the data be examined directly, whereas the second one involves considering SF moments and, therefore, allows performing an analysis in the analytic form as opposed to the former. In the present paper we are going to use a cross between these two latter, i.e. analysis is carried out over SF moments $F_k^2(x, Q^2)$ defined as follows

$$M_n^{pQCD/twist2/...}(Q^2) = \int_0^1 x^{n-2} F_2^{pQCD/twist2/...}(x, Q^2) dx$$

(2)

followed by the reconstruction of SF for each $Q^2$ by using a Jacobi polynomial expansion method [19, 20] (for further details see [1, 11]).

2 Theoretical basis and fitting procedure

Analysis aspects concerning $Q^2$-evolution of PDF moments (with the analytic continuation [21] in their coefficient functions and anomalous dimensions), PDF normalization, target mass (TMC) and higher twist corrections (HTCs), as well as nuclear effects remain essentially the same as in our previous work [1] so we refer to it for more details; here let us mention some stuff.

The moments $f_i(n, Q^2)$ at some $Q^2_0$ is a theoretical input to the analysis which is fixed as follows. In the fits of data with the cut $x \geq 0.25$ imposed, only the nonsinglet parton density is worked with and the following parametrization at the normalization point is used (see, for example, [2, 22]):

$$f(n, Q^2) = \int_0^1 dx x^{n-2} \bar{f}(x, Q^2), \quad \bar{f}(x, Q^2) = A(Q^2)(1-x)^{b(Q^2)}(1+d(Q^2)x),$$

(3)

where $A(Q^2)$, $b(Q^2)$ and $d(Q^2)$ are some coefficient functions.

Recall also some salient points of the so-called polynomial expansion method. The latter was first proposed in [23] and further developed in [24]. The Jacobi polynomials were first proposed and subsequently developed in [19, 20] with the purpose of analyzing the experimental data on SFs. They were then heavily used in [22, 25, 26, 27, 28].

With the QCD expressions for the Mellin moments $M_n^k(Q^2)$ (see, for example, [11]) the SF $F_k^2(x, Q^2)$ is reconstructed by using the Jacobi polynomial expansion method:

$$F_k^2(x, Q^2) = x^a(1-x)^b \sum_{n=0}^{N_{max}} \sum_{j=0}^{n} c_j^{(n)}(\alpha, \beta) M_j^{k}(Q^2),$$

where $\Theta_n^{a,b}$ are the Jacobi polynomials and $a, b$ are the parameters fitted. A condition put on the latter is the requirement of the error minimization while reconstructing the
structure functions. MINUIT package [29] is as usual used to minimize two variables; namely, the function $F_2$ itself and its logarithmic “slope” $d \ln F_2(x, Q^2)/d \ln (Q^2/\Lambda^2)$.

Since a twist expansion starts to be applicable only above $Q^2 \sim 1$ GeV$^2$ the global cut $Q^2 \geq 1$ GeV$^2$ on data is imposed throughout.

We use free normalizations of the data for different experiments. For a reference set, the most stable deuterium BCDMS data at the value of the beam initial energy $E_0 = 200$ GeV is used. When other datasets are taken as a reference one, variation in the results is found to be negligible. In the case of the fixed normalization for each and all datasets the fits tend to yield a little bit worse $\chi^2$, just as was observed earlier.

3 Results

As is known a nonsinglet analysis features no gluons taking part in the analysis; therefore, the cut imposed over the Bjorken variable ($x \geq 0.25$) effectively excludes the region where gluon density is believed to be non-negligible.

A starting point of the evolution is $Q_0^2 = 90$ GeV$^2$ for BCDMS and all datasets, and $Q_0^2 = 20$ GeV$^2$ — for combined SLAC and NMC datasets. These $Q_0^2$ values are close to the average values of $Q^2$ spanning the corresponding data. Following our previous papers the maximum number of moments to be accounted for in the analyses is taken to be $N_{\text{max}} = 8$ [20] and the cut $x \leq 0.8$ is imposed everywhere.

3.1 BCDMS data

Analysis starts with the most precise experimental data [7, 8, 9] obtained by the BCDMS muon scattering experiment for large $Q^2$ values. A complete set of data includes 607 points when the cut $x \geq 0.25$ is imposed. As was pointed out earlier the starting point of QCD evolution is $Q_0^2 = 90$ GeV$^2$. The heavy quark thresholds are taken at $Q_f^2 = m_f^2$. An original analysis carried out by the BCDMS collaboration (see also [12]) gave (back then) comparatively small values for the strong coupling constant; for example, $\alpha_s(M_Z^2) = 0.113$ at NLO was quoted in the latter reference.

As in [1, 2] the data with largest systematic errors are cut out by imposing certain limits on the kinematic variable $Y = (E_0 - E)/E_0$ (where $E_0$ and $E$ are lepton’s initial and final energies, respectively [30]). The following $Y$ cuts depending on the limits put on $x$ are imposed:

- $Y \geq 0.14$ for $0.3 < x \leq 0.4$
- $Y \geq 0.16$ for $0.4 < x \leq 0.5$
- $Y \geq Y_{\text{cut}3}$ for $0.5 < x \leq 0.6$
- $Y \geq Y_{\text{cut}4}$ for $0.6 < x \leq 0.7$
- $Y \geq Y_{\text{cut}5}$ for $0.7 < x \leq 0.8$

An impact of experimental systematic errors on the results of QCD analysis is studied for a few sets of $Y_{\text{cut}3}$, $Y_{\text{cut}4}$ and $Y_{\text{cut}5}$ cuts given in Table 1.

Table 1. A set of $Y_{\text{cut}3}$, $Y_{\text{cut}4}$ and $Y_{\text{cut}5}$ values used in the analysis

| $N_{Y_{\text{cut}}}$ | 1   | 2   | 3   | 4   | 5   |
|----------------------|-----|-----|-----|-----|-----|
| $Y_{\text{cut}3}$   | 0.16| 0.16| 0.18| 0.22| 0.23|
| $Y_{\text{cut}4}$   | 0.18| 0.20| 0.20| 0.23| 0.24|
| $Y_{\text{cut}5}$   | 0.20| 0.22| 0.22| 0.24| 0.25|

2This set slightly differs from that presented in [1, 2].
Following the analyses performed in [1, 2], we arrive at similar results: \( \alpha_s \) values for both original and modified (by cuts) datasets are shown in Tables 2 and 3, where a total systematic error is estimated in quadrature by using the method somewhat different from that utilized in our earlier analyses. \( (N_{Y_{cut}} = 0 \) corresponds to the case without \( Y \) cuts). Namely, instead of accounting for those errors by the multiplication procedure (an old approach outlined in [2]), here they are taken altogether in quadrature from the very beginning.

Table 2. NLO \( \alpha_s(M_Z^2) \) values for various sets of \( Y \) cuts

| \( N_{Y_{cut}} \) | number of points | \( \chi^2 \) quad. syst. err. (mult. syst. err.) | \( \alpha_s(M_Z^2) \) ± stat. error quad. syst. err. | \( \alpha_s(M_Z^2) \) ± stat. error mult. syst. err. | total syst. error |
|---|---|---|---|---|---|
| 0 | 607 | 444 (609) | 0.1078 ± 0.0012 | 0.1072 ± 0.0012 | 0.0054 |
| 1 | 502 | 358 (477) | 0.1149 ± 0.0015 | 0.1146 ± 0.0015 | 0.0039 |
| 2 | 495 | 355 (469) | 0.1151 ± 0.0015 | 0.1148 ± 0.0015 | 0.0038 |
| 3 | 489 | 350 (459) | 0.1157 ± 0.0015 | 0.1155 ± 0.0015 | 0.0036 |
| 4 | 458 | 322 (423) | 0.1166 ± 0.0016 | 0.1163 ± 0.0016 | 0.0031 |
| 5 | 452 | 322 (417) | 0.1172 ± 0.0016 | 0.1168 ± 0.0016 | 0.0030 |

Table 3. NNLO \( \alpha_s(M_Z^2) \) values for various sets of \( Y \) cuts

| \( N_{Y_{cut}} \) | number of points | \( \chi^2 \) quad. syst. err. (mult. syst. err.) | \( \alpha_s(M_Z^2) \) ± stat. error quad. syst. err. | \( \alpha_s(M_Z^2) \) ± stat. error mult. syst. err. | total syst. error |
|---|---|---|---|---|---|
| 0 | 607 | 446 (642) | 0.1064 ± 0.0012 | 0.1056 ± 0.0012 | 0.0054 |
| 1 | 502 | 361 (481) | 0.1132 ± 0.0015 | 0.1127 ± 0.0015 | 0.0039 |
| 2 | 495 | 357 (477) | 0.1135 ± 0.0015 | 0.1130 ± 0.0015 | 0.0038 |
| 3 | 489 | 352 (463) | 0.1140 ± 0.0015 | 0.1136 ± 0.0015 | 0.0036 |
| 4 | 458 | 350 (427) | 0.1150 ± 0.0016 | 0.1144 ± 0.0016 | 0.0031 |
| 5 | 452 | 325 (421) | 0.1155 ± 0.0016 | 0.1149 ± 0.0016 | 0.0030 |

Upon the cuts imposed (in what follows we work with a set \( N_{Y_{cut}} = 5 \)), only 452 points left available for analysis. Fitting them according to the procedure outlined above the following results are obtained:

\[
\alpha_s(M_Z^2) = 0.1155 + \begin{cases} 
\pm 0.0016 \text{ (stat)} & \pm 0.0030 \text{ (syst)} & \pm 0.0007 \text{ (norm)} \\
\pm 0.0035 \text{ (total exp. error)}
\end{cases},
\]

where an abbreviation “norm” denotes the experimental data normalization error stemming from the difference of the fits with free and fixed normalizations of BCDMS data subsets [7,8,9] having different values of the beam energy.

Performing the fits of SLAC and NMC experimental data we obtain results, which are very similar to those derived in [2] while fitting SLAC, NMC and BFP data altogether. Therefore, we do not present here results of the analyses with only SLAC and NMC data included, although note that the results are compatible within errors with those given above in [1] based on the analysis of BCDMS data alone. Thus, we can put all the data together and fit them simultaneously.

### 3.2 SLAC, BCDMS and NMC datasets

As in the case of BCDMS data analysis the cuts imposed are \( x \geq 0.25 \) and \( N_{Y_{cut}} = 5 \) (see Table 1). Then, an overall set of data consists of 756 points. A starting point of QCD evolution is again taken at \( Q_0^2 = 90 \text{ GeV}^2 \).
In order to determine the region where perturbative QCD is applicable we start by analyzing the data without a contribution of twist-four terms (that is $F_2 = F_2^{QCD}$) and perform several fits with the cut $Q^2 \geq Q^2_{\text{min}}$ gradually increased. Table 4 demonstrates that the quality of fits appears to be acceptable already at $Q^2 = 2 \text{ GeV}^2$.

Now, the twist-four corrections are added and the data with a global cut $Q^2 \geq 1 \text{ GeV}^2$ is fitted. As in the previous studies [1, 2] it is clearly seen that higher twists improve the fit quality, with an insignificant discrepancy in the values of the coupling constant to be quoted below.

**Table 4. $\alpha_s(M_Z^2)$ and $\chi^2$ in the combined SLAC, BCDMS, NMC analysis**

| $Q^2_{\text{min}}$ | N of points | HTC | $\chi^2(F_2)/\text{DOF}$ | $\alpha_s(90 \text{ GeV}^2) \pm \text{stat}$ | $\alpha_s(M_Z^2)$ |
|---------------------|-------------|-----|---------------------------|---------------------------------------------|------------------|
| 1.0                 | 756         | No  | 1.41                      | $0.1757 \pm 0.0007$                        | 0.1160           |
| 2.0                 | 731         | No  | 1.03                      | $0.1758 \pm 0.0007$                        | 0.1161           |
| 3.0                 | 704         | No  | 0.84                      | $0.1787 \pm 0.0009$                        | 0.1173           |
| 4.0                 | 682         | No  | 0.79                      | $0.1790 \pm 0.0009$                        | 0.1174           |
| 5.0                 | 662         | No  | 0.79                      | $0.1795 \pm 0.0011$                        | 0.1177           |
| 6.0                 | 637         | No  | 0.79                      | $0.1798 \pm 0.0013$                        | 0.1178           |
| 7.0                 | 610         | No  | 0.78                      | $0.1792 \pm 0.0016$                        | 0.1175           |
| 8.0                 | 594         | No  | 0.79                      | $0.1787 \pm 0.0019$                        | 0.1173           |
| 9.0                 | 575         | No  | 0.78                      | $0.1785 \pm 0.0023$                        | 0.1172           |
| 10.0                | 564         | No   | 0.77                      | $0.1765 \pm 0.0026$                       | 0.1164           |
| 1.0                 | 756         | Yes | 0.88                      | $0.1750 \pm 0.0019$                        | 0.1157           |

The following values for PDF parametrization parameters are obtained in the fit corresponding to the last row of the above table:

\[
A(H_2) = 2.61 \pm 0.045, \quad A(D_2) = 2.50 \pm 0.066, \quad A(C) = 3.43 \pm 0.036, \\
b(H_2) = 4.12 \pm 0.019, \quad b(D_2) = 4.15 \pm 0.023, \quad b(C) = 4.12 \pm 0.031, \\
d(H_2) = 5.48 \pm 0.34, \quad d(D_2) = 3.39 \pm 0.22, \quad d(C) = 1.40 \pm 0.14.
\]

They are seen to be very similar to those presented in [2].

Twist-four parameter values are presented in Table 5. Note that these for $H_2$ and $D_2$ targets are obtained in separate fits by analyzing SLAC, NMC and BCDMS datasets taken together.

**Table 5. Parameter values of the twist-four term in different orders**

| $x$ | LO ± stat | NLO ± stat | NNLO ± stat |
|-----|-----------|------------|-------------|
|     | $h_4(x)$ for $H_2$ | $h_4(x)$ for $D_2$ | $h_4(x)$ for $H_2$ | $h_4(x)$ for $D_2$ | $h_4(x)$ for $H_2$ | $h_4(x)$ for $D_2$ | $h_4(x)$ for $H_2$ | $h_4(x)$ for $D_2$ |
| 0.275 | -0.275±0.012 | -0.274±0.021 | -0.251±0.026 | -0.221±0.007 | -0.176±0.024 | -0.136±0.026 |
| 0.35  | -0.269±0.015 | -0.246±0.030 | -0.190±0.025 | -0.378±0.006 | -0.143±0.032 | -0.021±0.015 |
| 0.45  | -0.181±0.016 | -0.125±0.049 | -0.290±0.017 | -0.002±0.010 | -0.181±0.031 | -0.068±0.015 |
| 0.55  | -0.049±0.033 | 0.080±0.082 | -0.316±0.031 | 0.247±0.014 | -0.233±0.059 | 0.043±0.022 |
| 0.65  | 0.326±0.063 | 0.357±0.144 | -0.064±0.076 | 0.563±0.035 | -0.167±0.156 | 0.353±0.059 |
| 0.75  | 0.805±0.124 | 0.513±0.213 | 0.009±0.122 | 0.770±0.070 | -0.156±0.215 | 0.323±0.104 |

HTCs are shown in Figs. 1 and 2, where twist-four corrections obtained at NLO and NNLO are observed to be compatible with each other. This observation does not contradict earlier studies (see Ref. [2] and references and discussions therein). However, at large $x$ the central values of HTCs are a bit decreased at NNLO level.
Figure 1: Twist-four $\tilde{h}_4(x)$ parameter values obtained at LO, NLO and NNLO for hydrogen data (bars show statistical errors).

Nevertheless, an effect of decreasing NNLO HTCs, observed earlier in [25, 26] for $F_3$ SF, here cannot be clearly seen. The reason for $F_3$ HTC reduction is in small values of the higher twist terms for the latter at any order of perturbation theory (see [2, 31] and discussions therein).

Note that the cut imposed on BCDMS data, which effectively increased $\alpha_s$ values (see Tables 2 and 3), essentially improves agreement between theory and experiment. HTCs as being the difference between the twist-two approximation (i.e. pure perturbative QCD contribution) and the experimental data become considerably smaller at NLO and NNLO levels compared with NLO HT terms obtained in [12] and also with the results of analysis with no $Y$-cuts imposed over BCDMS data (see Figs. 5, 6 in [2]).

Finally, using the nonsinglet evolution analyses of SLAC, NMC and BCDMS experimental data for SF $F_2$ with no account for twist-four corrections and the cut $Q^2 \geq 2$ GeV$^2$, we obtain (with $\chi^2/DOF = 1.03$)

$$\alpha_s(M_Z^2) = 0.1161 + \left\{ \begin{array}{l} \pm 0.0003 \text{ (stat)} \pm 0.0018 \text{ (syst)} \pm 0.0007 \text{ (norm)} \\ \pm 0.0020 \text{ (total exp.error)} \end{array} \right. \quad . \quad (5)$$

Upon including the twist-four corrections, and imposing the cut $Q^2 \geq 1$ GeV$^2$, the following result is found (with $\chi^2/DOF = 0.88$):

$$\alpha_s(M_Z^2) = 0.1157 + \left\{ \begin{array}{l} \pm 0.0008 \text{ (stat)} \pm 0.0020 \text{ (syst)} \pm 0.0005 \text{ (norm)} \\ \pm 0.0022 \text{ (total exp.error)} \end{array} \right. \quad . \quad (6)$$

4 Scale dependence

In this section the dependence of the results on the different choice of the factorization $\mu_F = k_F Q^2$ and renormalization $\mu_R = k_R Q^2$ scales is studied. The threshold crossing point is taken at $Q^2_f = m_f^2$. Following [12, 32] we choose three values ($1/2$, $1$, $2$) for the coefficients $k_F$ and $k_R$.

Results are demonstrated in Table 6. Fits are performed with no account for higher twist corrections, the number of points is 731 (SLAC, BCDMS and NMC data), $Q^2_{\text{min}} = 2$
Figure 2: The same as in Fig. 1 for deuterium data.

GeV$^2$ and different datasets are freely normalized. The change in $\alpha_s(M_Z^2)$ value for various $k_F$ and $k_R$ values is denoted by the difference:

$$\Delta \alpha_s(M_Z^2) = \alpha_s(M_Z^2) - \alpha_s(M_Z^2)|_{k_F=k_R=1}$$  \hspace{1cm} (7)

Table 6. $\alpha_s(M_Z^2)$ for a set of $k_F$ and $k_R$ coefficients

| $k_R$ | $k_F$ | $\chi^2(F_2)$ | $\alpha_s(90 \text{GeV}^2) \pm \text{stat}$ | $\alpha_s(M_Z^2)$ | $\Delta \alpha_s(M_Z^2)$ |
|------|------|---------------|---------------------------------|-----------------|-----------------|
| 1    | 1    | 712           | 0.1758 ± 0.0007                  | 0.1161          | 0               |
| 1/2  | 1    | 660           | 0.1729 ± 0.0007                  | 0.1149          | -0.0012         |
| 1    | 1/2  | 587           | 0.1733 ± 0.0006                  | 0.1150          | -0.0011         |
| 1    | 2    | 852           | 0.1808 ± 0.0008                  | 0.1182          | +0.0021         |
| 2    | 1    | 785           | 0.1805 ± 0.0008                  | 0.1180          | +0.0019         |

As seen from this table theoretical uncertainties for the maximum and minimum values of the coupling constant corresponding to $k_i = 2$ and $k_i = 1/2$ ($i = F, R$), respectively, are found to be +0.0028 and −0.0016. It should be noted that we take into account renormalization scale uncertainty in the expressions for the coefficient functions and respective coupling constants analogously to what was done in [33].

Thus, NS evolution analyses of SLAC, NMC and BCDMS experimental data for SF $F_2$ give for $\alpha_s(M_Z^2)$ the following numbers (with no account for HTC, $Q^2 \geq 2 \text{GeV}^2$ and $\chi^2 = 1.03$):

$$\alpha_s(M_Z^2) = 0.1161 + \left\{ \begin{array}{l}
\pm 0.0003 \text{ (stat)} \\
\pm 0.0018 \text{ (syst)} \\
\pm 0.0007 \text{ (norm)} \\
\pm 0.0020 \text{ (total exp.error)} \\
+0.0028 \\
-0.0016 \text{ (theor).}
\end{array} \right.$$

Note that here we take theoretical errors for the factorization and renormalization scales in quadrature. In our previous analyses [1, 2] we considered the cases with $k_F = k_R = 1/2$ and $k_F = k_R = 2$ that corresponded to taking the scales together linearly rather than in quadrature.
5 Conclusions

In this work the Jacobi polynomial expansion method developed in [19, 20] was used to analyze $Q^2$-evolution of DIS structure function $F_2$ by fitting reliable fixed-target experimental data that satisfy the cut $x \geq 0.25$. Based on the results of fitting, the strong coupling constant value is evaluated at the normalization point. Starting with the reanalysis of BCDMS data by cutting off the points with largest systematic errors it is shown that as earlier $\alpha_s(M_Z^2)$ values rise sharply with the cuts on systematics imposed. On the other hand, the latter do not depend on a certain cut within statistical errors. The present results are compatible with those obtained in our earlier paper [2], where systematic errors in BCDMS data were taken into account in a different way. To be more precise, in [2], and in even earlier studies [20, 1], systematics was dealt with as follows: all fits were done with experimental data multiplied by respective systematical errors for $F_2$ separately for each source of uncertainties. Then, the differences between fits with different sources taken into account give rise to the total systematic error derived in quadrature. In the present paper, systematics is dealt with in quadrature rather than in a multiplicative manner right from the start.

Taking into account systematic errors in BCDMS data does not change results of the fits obtained in [2] except for just a single detail: now perturbative QCD (without HTC) is well compatible with the experimental data already at $Q^2 \geq 2$ GeV$^2$.

Here we would like to give some explanations of the absence of the rise in $\alpha_s(M_Z^2)$ value upon cutting out the regions in BCDMS data with larger systematic errors stated in [3]. Note that the values of systematic errors are rather large in the cut out regions but not infinitely large. In the latter case there is no any effect of absence/existence of the cut out regions. One of possible explanations relates with the fact that Ref. [3] includes combined singlet and nonsinglet analyses, where there is some correlation between $\alpha_s(M_Z^2)$ values and the shape of gluon density. So, cutting out BCDMS data with the largest systematic errors could lead in [3] to the shape of gluon density somewhat altered.

It turns out that for $Q^2 \geq 2$ GeV$^2$ the formulæ of pure perturbative QCD (i.e. twist-two approximation along with the target mass corrections) are enough to achieve good agreement with all the data analyzed. The reference result is then found to be

$$\alpha_s(M_Z^2) = 0.1161 \pm 0.0020 \text{ (total exp.error)}, \quad (8)$$

Upon adding twist-four corrections, fairly good agreement between QCD and the data starting already at $Q^2 = 1$ GeV$^2$, where the Wilson expansion starts to be applicable, is observed. This way we obtain for the coupling constant at $Z$ mass peak:

$$\alpha_s(M_Z^2) = 0.1157 \pm 0.0022 \text{ (total exp.error)}. \quad (9)$$

Note that, in a sense, our results are between those obtained in [3, 15] and [16] (a dynamical approach) and, respectively, results derived by MSTW [14] and NN21 [17] groups. They are consistent with those obtained in [13, 16] (a standard fit) and also with studies of the recent data of CMS and ATLAS collaborations done in [34] and [35], respectively (see the recent review [36]). A complete agreement with recent results obtained in lattice QCD [37] is also observed. Our result is slightly below the central world average value

$$\alpha_s(M_Z^2)_{\text{world average}} = 0.1185 \pm 0.0006,$$

presented in [38], but still compatible within errors.

As was already pointed out the values of theoretical uncertainties, given by this dependence of the results for $\alpha_s(M_Z^2)$ are equal to

$$\Delta \alpha_s(M_Z^2)_{\text{theor}} = \begin{cases} +0.0028 \\ -0.0016 \end{cases}. \quad (11)$$
It is seen that the theoretical uncertainties are already comparable with the total experimental error. Nonetheless, further account of even higher corrections is still desirable and the next step is to consider further corrections (i.e. those coming from three loops) in the coefficient functions \[39\], which allows performing N^3LO fits at large \(x\) values, where the contributions of the corresponding four-loop corrections to yet unknown anomalous dimensions should be negligible. Note that several N^3LO fits had already been done in \[26, 27\].

In order to check whether there is effect of the rise in \(\alpha_s(M_Z^2)\) value when \(y\)-cuts are imposed, it is needed to consider combined nonsinglet and singlet analysis of DIS experimental data over an entire \(x\) region. An application of some resummations, like a Grunberg’s effective charge method \[40\] (as it was done in \[28\] at the NLO approximation) and the “frozen” (see \[41\] and references therein) and analytic \[42\] versions of the strong coupling constant (see \[41, 43, 44\] for recent studies in this direction) could also be useful in understanding the subject. These are left for the future investigations.

6 Acknowledgments

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