The magnetic fields of millisecond pulsars in globular clusters

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ABSTRACT

Many of the characteristic properties of the millisecond pulsars found in globular clusters are markedly different from those in the Galactic disc. We find that one such physical parameter is the surface magnetic field strength. Even though the average spin periods do not differ much the average surface magnetic field is two–five times larger in the globular cluster pulsars. This effect could be apparent, arising due to one or more of several biases. Alternatively, if future observations confirm this effect to be real, then this could be interpreted as a preferential recycling of pulsars in tight binaries where the mass transfer takes place at high accretion rates.

Key words: magnetic fields – stars: neutron – pulsars: general – globular clusters: general.

1 INTRODUCTION

A radio pulsar is a strongly magnetized rotating neutron star. Discovered serendipitously (Hewish et al. 1968), the pulsars are characterized by their short spin periods (P) and very large inferred surface magnetic fields (B). However, the detection of the 1.5-ms pulsar B1937+21 (Backer et al. 1982) heralded a new genre, that of the radio millisecond pulsars (MSPs). The ranges of the spin periods and the surface magnetic fields of these MSPs place them in a nearly disjoint region of the B–P plane from the normal radio pulsars. Close to two thousand radio pulsars have been detected to date with P ∼ 10^{-3}–10 s and B ∼ 10^{8}–10^{15} G. Amongst these, the MSPs are typically characterized by P <∼ 30 ms, and typical surface field strengths of ∼10^{8}–10^{9} G with characteristic spin-down ages of ∼10^{9} yr.

The connection between these two seemingly disjoint populations of pulsars is realized through the binary association of the neutron stars. An MSP is understood to descend from an ordinary, long-period pulsar that has been spun-up and recycled back as an MSP in a low-mass X-ray binary (LMXB) by mass accretion (Radhakrishnan & Srinivasan 1982). In the particular case of an isolated MSP, like PSR B1937+21, the donor is later destroyed probably by the wind of the newly recycled pulsar itself (Alpar et al. 1982). In general the MSPs are considered to be the end products of LMXBs and intermediate-mass X-ray binaries where the primary neutron star is assumed to have formed by the core-collapse of a massive (M ∼ 8 M☉) star (Bisnovatyi-Kogan & Komberg 1974; Bhattacharya & van den Heuvel 1991).

This scenario for MSP formation has been backed by several observational indications over the years. The strongest support for the connection between MSPs and X-ray binaries comes from the discovery of coherent millisecond X-ray pulsations in SAX J1808.4−3658 (Chakrabarty & Morgan 1998; Wijnands & van der Klis 1998) and subsequently in many other systems (Wijnands et al. 2005). These accreting millisecond X-ray pulsars (AMXP) are understood to be the immediate precursors of the radio MSPs and are expected to turn into those when active mass accretion stops. The identification of PSR J1023+0038 as an MSP which has only recently lost its accretion disc (Archibald et al. 2009) provides almost a direct proof of this theory of MSP formation.

However, it seems that the large number of pulsars detected in globular clusters in the recent years have somewhat different characteristics than the pulsars observed in the Galactic disc. To begin with, the fraction of binary and MSPs in the clusters are ∼40 and ∼92 per cent, respectively, whereas in the disc these fractions are only ∼5 and ∼4 per cent. The size of the MSP population is much larger in the clusters than in the disc. In fact, the globular clusters (about 150 of them) known to orbit the Milky Way contain roughly three orders of magnitude more observed MSPs per unit mass than the Galactic plane, which contains 73 known MSPs (Camilo & Rasio 2005). The conditions prevailing in a globular cluster are rather different from that in the Galactic disc primarily due to the extremely high stellar densities in the clusters. One of the obvious effects of this high density is a dramatic increase in the number of binaries, as well as in the rate of close stellar encounters allowing for many different channels for binary formation. Effectively, these systems provide different ways of pulsar recycling than seen in the disc where the MSPs are mainly formed in primordial low-mass binaries.

Therefore, we can logically expect the MSPs in the disc to be different from the MSPs in the clusters. This is indeed the case. There is a greater proportion of single MSPs in the clusters and the majority of the binary MSPs have very short orbital periods compared to those in the Galactic disc. In fact, many of the cluster binaries have properties similar to those of the rare eclipsing black widow pulsars seen in the Galactic disc population (King, Davies & Beer 2003; Freire 2005). Recently, Bagchi & Ray (2009a,b) have...
investigated the effect of different types of stellar collisions on the orbital parameters of the binary MSPs in globular clusters. They too find the cluster MSPs to be significantly different from those in the disc.

In this work, we look at one of the important intrinsic parameters of the MSPs, namely the surface magnetic field. We find that a subset of slower MSPs in the globular clusters (for which field measurements are available) appear to have surface magnetic fields that are two-to-five times larger compared to their disc counterparts. Even though the spin periods of this subset of cluster pulsars are similar to the spin periods of the MSPs in the disc. In order to understand this fact, we look at the spin period and the magnetic field distributions of the MSPs in the disc and in the globular clusters and compare them to check for implied similarities/differences in their evolutionary histories. We also look at the details of the field evolution itself to see if the physics of the MSP formation in the disc and in the clusters could actually give rise to different surface fields. We examine whether close stellar encounters resulting in binary disruption and/or re-formation of new binaries could be responsible for the higher magnetic fields observed in cluster MSPs. Or if this is an apparent effect caused by other external factors.

To this end, first we discuss the MSP statistics in Section 2. In Section 3 we look at the details of the field evolution and the effect of the cluster conditions on the evolution. Finally, we summarize our conclusions in Section 4.

## 2 THE MILLISECOND PULSARS

### 2.1 The definition

It is well known that the MSPs occupy a distinctly separate region from the normal pulsars in the $B$–$P$ plane. However, defining the MSP population with some accuracy is not simple since this requires separating out a class of neutron stars that have undergone a particular kind of binary evolution. The primary observed quantities of a pulsar are $P$ and $\dot{P}$, and the most important derived quantity using both $P$ and $\dot{P}$ is the dipolar component of the surface magnetic field given by (Manchester & Taylor 1977)

$$B_\text{s} \simeq 3.2 \times 10^{10}(P \dot{P})^{1/2} \text{G},$$

where $P$ and $\dot{P}$ are in units of seconds and ss$^{-1}$. Even though the term millisecond pulsar has traditionally been reserved for recycled pulsars with ultrafast rotation ($P \lesssim 30$ ms) and a low magnetic field ($B \simeq 10^8$–$10^9$ G), the definition has mostly used the condition on $P$. Using $P$ to classify pulsars is somewhat problematic because for a large number of pulsars (particularly for those in the clusters) a measurement of $P$ is either not available or the value is not reliable due to contamination from the proper motion of the pulsar.

The most reasonable criterion to define an MSP should be to use both $P$ and $\dot{P}$ since the evolution of these two parameters are inter-related in the formation process of MSPs. Some effort has indeed been made to use such a relation assuming the MSPs to be recycled through straightforward LMXB evolution (Story, Gonthier & Harding 2007). However the situation is different in globular clusters where, due to a much larger probability of stellar collisions, it is possible for an MSP to have gone through very complex binary evolution. Therefore, in general, it is not easy to separate the MSPs from the normal pulsars using a definitive relation involving both $P$ and $\dot{P}$.

In Fig. 1 we have plotted all known pulsars (with a reliable estimate for $B$) in the $B$–$P$ plane, marking some of the $B$ and $P$ lines relevant for separating out the MSP population. Unfortunately, these simple criteria run into trouble with objects like the 16-ms pulsar J0537–6910 in Large Magellanic Cloud (LMC; Marshall et al. 1998). As this pulsar has a strong magnetic field ($B \sim 10^{11}$) and is also rather young ($\tau \sim 10^4$ yr), it is not likely to be a member of the generic recycled MSP population (this extra-Galactic pulsar is not included in Fig. 1). Therefore as a working hypothesis we shall use the classical definition of $P \lesssim 30$ ms ($\log(P/\text{s}) \lesssim -1.5$), barring such obvious misfits like J0537–6910. This way of defining MSPs does appear to have a certain merit. In Fig. 2 a histogram of the spin period of all known pulsars is made. It appears that the pulsars

**Figure 1.** All known Galactic pulsars, with a reliable $B$ measurement, in the $B$–$P$ plane. The isolated and binary pulsars in the Galactic disc are marked by filled squares and open stars, respectively. The isolated and binary pulsars in the globular clusters are marked by open circles and circles with dots within. The data is from the Australia Telescope National Facility (ATNF) on-line catalogue, taken in 2010 April.
with $P \lesssim 30$ ms may indeed represent a separate class as the period histogram shows a sharp dip around this value of $P$.

### 2.2 The statistics

Interesting facts emerge when the MSP population, as defined above, is subjected to statistical analysis. To begin with, we separate them into two groups – MSPs residing in the globular clusters and those in the Galactic disc. Each of these groups is further divided into isolated and binary pulsars. Table 1 shows the average values of $P$ and $B$ for each of these subpopulations. It needs to be noted that the number of pulsars with a measured value of $B$ can be much smaller than the actual number of pulsars in a given subgroup (see the corresponding numbers in Table 1). This is particularly true of the cluster pulsars. Because of this, we have calculated another average of $P$ using only those pulsars with a known $B$. This average, $P_{av}^{B}$, is expected to be more commensurate with $B_{av}$ than the $P_{av}$ obtained using all the pulsars in the subpopulation. The significance of these quantities shown in Table 1 is discussed below.

#### Table 1. The MSP statistics. In the first column the type of the MSP subpopulation has been indicated. The second and the third column show the average values of the measured spin periods and derived dipolar field strengths, respectively. $P_{av}$ in the fourth column is the average period calculated using only those pulsars for which $B$ measurements exist. The spin periods are in ms and the magnetic fields are in $10^{8}$ G. The numbers in the parentheses denote the numbers of objects available for calculating the averages. The data are from the ATNF on-line pulsar catalogue and Paulo Freire’s catalogue of globular cluster pulsars, taken in 2010 April.

| MSP population (P $\leq$ 30 ms) | $P_{av}$ (ms) | $B_{av}$ ($10^{8}$ G) | $P_{av}^{B}$ (ms) |
|----------------------------------|----------------|------------------------|-------------------|
| Galactic disc                    |                |                        |                   |
| Isolated                         | 5.82 (19)      | 3.11 (18)              | 5.97 (18)         |
| Binary                           | 8.03 (54)      | 9.32 (52)              | 8.45 (52)         |
| All                              | 7.75 (73)      | 7.72 (70)              | 7.82 (70)         |
| Globular clusters                |                |                        |                   |
| Isolated                         | 5.82 (61)      | 14.70 (11)             | 6.16 (11)         |
| Binary                           | 5.58 (68)      | 18.84 (14)             | 8.62 (14)         |
| All                              | 5.70 (129)     | 17.02 (25)             | 7.54 (25)         |

#### Figure 2. Histogram showing the distribution of spin periods ($P$) of all known pulsars. The data correspond to the ATNF on-line catalogue and Paulo Freire’s on-line catalogue of globular cluster pulsars, taken in 2010 April.

The well-known fact that the MSPs in globular clusters are, in general, spinning faster than their disc counterparts is immediately seen from the values in Table 1. However, it can also be seen that the $P_{av}$ for almost all the subgroups are similar except for the binary MSPs in the disc which has a larger average spin period than the rest of the subgroups. In other words, the disc binary MSPs are spinning more slowly than the rest of the MSPs. However, an average difference of $\sim 3$ ms may not indicate anything significant as the standard deviation of $P$ in these subgroups range from $\sim 4$ to 7 ms. Moreover, it is not clear if this could be due to any kind of observational bias. Though it should be noted that the size of the subgroups is more or less similar except for the isolated objects in the disc (about a factor of 3 smaller). Therefore a bias (of any kind) would be stronger for the isolated disc MSPs more than the rest of the groups.

In order to understand the relative nature of the $P$ distributions of the various subpopulations, beyond the simple averages discussed above, we have performed the Kolmogorov–Smirnov (K–S) tests on these populations. In Fig. 3 we compare the cumulative fractional $P$ distribution corresponding to each K–S test. Furthermore, our conclusions from these tests are summarized below.

(i) The isolated and the binary MSPs in the globular cluster itself are the least correlated ($P_{KS} \sim 0.002$) and are in good agreement with that found by Hessels (2009). The result itself is surprising because these two subgroups have very similar average spin periods. Furthermore, the luminosity distributions of isolated and binary globular cluster pulsars have been found to be statistically similar (Hessels et al. 2007). The low correlation is also contrary to expectations because all the pulsars in the clusters are likely to have similar evolutionary histories, strongly influenced by stellar encounters. Since it is quite possible for an isolated MSP to acquire a companion or a binary MSP to lose one given the high rates of stellar collisions in a globular cluster which effectively means that in a globular cluster the phase (isolated or binary) in which an MSP is observed at a given time could be quite temporary.

(ii) The isolated and the binary MSPs in the galactic disc are also not correlated ($P_{KS} \sim 0.2$) indicating major differences in their evolutionary histories. This is supported, to some extent, by the fact that their average spin periods are different (but not by any significant amount, as mentioned earlier).

(iii) Similarly, there is little correlation ($P_{KS} \sim 0.03$) between the spin distributions of the binary populations in the disc and in the clusters. The explanation for this absence of correlation is possibly due to the fact that in the disc the MSP recycling typically happens in primordial binaries whereas in the clusters the binaries could well have formed by recent stellar encounters. Hence, the nature of the associated MSPs could be very different.

(iv) Another surprising result is the very strong correlation ($P_{KS} \sim 0.9$) between the spin distributions of isolated MSPs in the disc and in the clusters. The average spin periods are similar too, which may indicate similar evolution of the isolated MSPs in the disc and the clusters. However these two populations are not expected to be so well correlated. In the disc the isolated MSPs are supposed to evolve in primordial LMXBs and then evaporate their companions. In the clusters, this is certainly a possibility. However, the isolated MSPs are more likely to be results of stellar encounters and consequent disruption of binaries in the clusters. It should also be remembered that the size of one of the subgroups (the disc pulsars) in this test is much smaller than the other and this could introduce some bias in the K–S test.
Figure 3. K–S test for the $P$ distributions of different subgroups of the MSPs. The cumulative fractional distributions are shown for easy comparison. The solid and the dashed lines correspond to the isolated and the binary MSPs in the globular clusters, whereas the dash–dotted and the dotted lines correspond to the isolated and the binary MSPs in the Galactic disc. The number of objects in each subgroup is shown within the parentheses. The K–S probabilities are indicated for each test.

2.2.2 Surface magnetic field

First of all, it should be noted that the $B$ measurement is available only for a small number of pulsars in the globular clusters (25 out of a total of 129). This is because of the difficulty in measuring the $P$ there due to proper motion contamination. In fact, in many cases $P$ is seen to be negative even when the system is non-accreting (and therefore has no reason to spin-up) indicating that the measured value is not at all reliable. In such cases there is no way to estimate $B$. Evidently, it would be relatively easier to measure larger $\dot{P}$. This then automatically introduces a bias, that of preferentially measuring $B$ in systems that have higher field values. On the other hand, almost all the MSPs in the disc have a reliable $B$ measurement. So any comparison between the disc and the cluster MSPs suffers from an inherent bias in this respect.

Given the above problem it is still interesting to see that the magnetic fields of the MSPs in clusters are larger by a factor of 2–5 compared to their disc counterparts. Unlike in the case of $P$ this difference is much larger than the standard deviation in $B$. Yet the corresponding $P_{av}$s are similar in the disc and in the clusters, both for the isolated and the binary MSPs. However, it should be noted that the fraction of MSPs, with field measurements, is much smaller in the clusters compared to the disc (see Table 1). For example, the fractions of MSPs with field measurements are 11/61 (isolated) and 14/68 (binary) in the clusters. During the process of recycling a pulsar is spun-up in the accretion phase. The maximum spin-up, for a given rate of accretion and a given strength of the surface field, is given by the following relation of spin equilibrium (Alpar et al. 1982; Chen & Ruderman 1993):

$$P_{eq} \simeq 1.9 \text{ ms} \left( \frac{B}{10^9 \text{ G}} \right)^{6/7} \left( \frac{M}{1.4 M_\odot} \right)^{-5/7} \times \left( \frac{\dot{M}}{M_{\text{Ed}}} \right)^{-3/7} \left( \frac{R}{10^6 \text{ cm}} \right)^{16/7}. \quad (2)$$

Here $M$ and $R$ denote the mass and the radius of the neutron star, $\dot{M}$ and $M_{\text{Ed}}$ stand for the actual and the Eddington rate of mass accretion and $B$ is the surface magnetic field. When $\dot{M}$ equals $M_{\text{Ed}}$ the above relation defines the spin-up line, i.e. the minimum period to which a pulsar with a given magnetic field can be spun-up. However,
Magnetic fields of MSPs

(i) The binary MSPs in the disc and the clusters have a low correlation ($P_{KS} \lesssim 0.2$), as do the isolated and the binary MSPs in the disc ($P_{KS} \lesssim 0.1$). These results are in conformity of what we have seen for the respective $P$ distributions. And we can conclude that these subgroups have different evolutionary histories, as discussed before.

(ii) Surprisingly, the $B$ distributions of the isolated pulsars in the disc and the clusters have extremely low correlation ($P_{KS} \lesssim 5 \times 10^{-4}$). This is in complete contrast with the high correlation ($P_{KS} \sim 0.9$) of their $P$ distributions. However it should be remembered that for the cluster pulsars only a small number (11 out of 61) of objects have a measured $B$ and hence only those have been used for this test. However, as mentioned earlier the $P_{\text{corr}}$'s of these two groups are quite similar, indicating selection of pulsars with similar $P$ values.

(iii) Another surprise is the very good correlation ($P_{KS} \sim 0.7$) between the isolated and the binary pulsars in the clusters, again in complete contrast with the poor correlation ($P_{KS} \sim 0.002$) of their $P$ distributions. Though once again we need to note that a very small sample of objects (11 out of 61 for the isolated and 14 out of 68 for the binaries) is used for the $P$ test compared to the $B$ test. Whether this indicates a similar $B$ evolution for the groups is difficult to say because somewhat contrarily the $P_{\text{corr}}$'s are different for these two groups.

In summary then we can think of the following possibilities giving rise to the higher surface fields observed in the cluster MSPs.

(i) The higher field observed could simply be due to certain observational biases (preferentially selecting high-$B$ systems, measuring a higher value of $P$ and hence $B$ due to proper motion contamination and so on...).

(ii) It is also possible that the cluster MSPs have higher fields because we are looking at a younger population in the cluster compared to the disc. These young MSPs observed today would slow down with time and migrate towards the right of the $B$–$P$ plane.
Figure 5. K–S test for the $B$ distributions of different subgroups of the MSPs along with the cumulative fractional distributions. The solid and the dashed lines correspond to the isolated and the binary MSPs in the globular clusters, whereas the dash–dotted and the dotted lines correspond to the isolated and the binary MSPs in the Galactic disc. The number of objects in each subgroup is shown within the parentheses. The K–S probabilities are indicated for each test.

Looking at Fig. 4 we can see that if the cluster MSPs move towards the right of the plot by appropriate amount then they may become identical with the current disc population. There is some support to this possibility from the observed AMXPs. The AMXPs turn into radio MSPs as soon as accretion stops in such systems. In principle, their physical properties should be similar to very young radio MSPs. In Table 2 we list the AMXPs and the burst sources that have some estimate of their magnetic fields. These have been plotted in Fig. 4 and we see that in the $B$–$P$ plane the AMXPs occupy the same region as the cluster MSPs.

(iii) On the other hand, the cluster MSPs could really have higher fields compared to the disc MSPs. This is possible only if the different nature of the recycling process in the clusters affects the field evolution significantly. In the next section we use a simple model for the evolution of the magnetic field to see if the stellar dynamics in globular clusters facilitate halting the field decay earlier than is expected in a primordial LMXB.

3 RECYCLING: EVOLUTION OF THE MAGNETIC FIELD

In the standard formation scenario the MSPs are generated by recycling of ordinary pulsars in the LMXBs. Accretion-induced field decay is an integral part of this generic picture. A number of mechanisms, responsible for the evolution of the field, have been suggested. First, the magnetic field may be dissipated in the stellar crust by ohmic decay, accelerated by heating as the accreted plasma impacts upon the star (Konar & Bhattacharya 1997, 1999a; Brown & Bildsen 1998; Urpin, Geppert & Konenkov 1998; Cumming, Arras & Zweibel 2004). On the other hand, if the magnetic field resides in the superfluid core in the form of Abrikosov fluxoids then they may be dragged out of the core by the outward motion of superfluid vortices, as the star spins down (Srinivasan et al. 1990; Jahan Miri & Bhattacharya 1994; Ruderman, Zhu & Chen 1998). This flux would

| Accreting sources | $P_s$ (ms) | $B$ (G) |
|-------------------|-----------|---------|
| IGR J00291+5934$^a$ | 1.67 | $<3 \times 10^8$ |
| Aql X-1 (1908+005)$^b$ | 1.82 | $\leq 10^9$ |
| XTE J1751–305$^c$ | 2.30 | $3–7 \times 10^8$ |
| SAX J1808.4–3658$^d$ | 2.49 | $1–5 \times 10^8$ |
| XTE J1814–338$^e$ | 3.18 | $8 \times 10^8$ |
| XTE J0929–314$^f$ | 5.41 | $<3 \times 10^8$ |
| SWIFT J1756.9–2508$^g$ | 5.49 | $0.4–9 \times 10^8$ |
| KS 1731–260$^h$ | 1.91 | $\lesssim 7 \times 10^8$ |
| EXO 0748–676$^i$ | 1.81 | $\sim 1–2 \times 10^9$ |
| *MXB 1730–335$^j$ | 3.27 | $\sim 4 \times 10^9$ |

The data are taken from $^a$Galloway et al. (2010), Burderi et al. (2007), Torres et al. (2008); $^b$Di Salvo & Burderi (2003), Casella et al. (2008); $^c$Wijnands et al. (2005), Papitto et al. (2008); $^d$Di Salvo & Burderi (2003), Hartman et al. (2008), Cackett et al. (2009); $^e$Papitto et al. (2007); $^f$Galloway et al. (2002), Wijnands et al. (2005); $^g$Krimm et al. (2007), Patruno, Altamirano & Messenger (2010); $^h$Di Salvo & Burderi (2003); $^i$Loeb (2003) and $^j$Masetti et al. (2000).
then be subsequently dissipated in the accretion-heated crust (Konar & Bhattacharya 1999b; Konenkov & Geppert 2001). Third, the magnetic field could also be screened by accretion-induced currents within the crust (Bisnovatyi-Kogan & Komberg 1974; Lovelace, Romanova & Bisnovatyi-Kogan 2005). In particular, the field may be buried under a mountain of accreted plasma channelled on to the magnetic poles (Hameury et al. 1983; Romani 1990; Brown & Bildsten 1998). When the accreted matter is large enough, the mountain spreads laterally, transporting the polar magnetic flux towards the equator and finally dissipating them there (Cumming, Zweibel & Bildsten 2001; Melatos & Phinney 2001; Choudhuri & Konar 2002; Konar & Choudhuri 2004; Payne & Melatos 2004, 2007).

In our earlier investigations we have assumed that the currents supporting the field finally get dissipated in the accretion-heated crust, wherever they originally may have resided (Konar & Bhattacharya 1997, 1999a,b). We adopt the methodology developed in these articles (see Konar 1997 for details) for the present work and assume that purely crustal currents support the observed magnetic field of the neutron star.

The evolution of the magnetic field is governed by the following equation:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) - \frac{c^2}{4\pi} \nabla \times \left( \frac{1}{\sigma} \nabla \times \mathbf{B} \right),$$

(3)

where \(\mathbf{V}\) is the velocity of material movement and \(\sigma\) is the electrical conductivity of the medium. The material movement in the crust due to mass transfer defines \(\mathbf{V}\).

The spin-up of a neutron star, in a binary system, is caused by the angular momentum brought in by the accreted matter. In magnetospheric accretion matter accretes with angular momentum specific to the Alfvén radius. Therefore, the total angular momentum brought in by accretion is

$$J_{\text{accreted}} = \delta M R_A V_A,$$

(4)

where \(\delta M\) is the total mass accreted, \(R_A\) and \(V_A\) are the Alfvén radius and Keplerian velocity at that radius. The final period of the neutron star then is

$$P_{\text{final}} = \frac{2\pi I_{\text{ns}}}{J_{\text{accreted}}},$$

(5)

where \(I_{\text{ns}}\) is the moment of inertia of the neutron star. The average dipolar field of the disc MSPs is \(\sim 10^8\) G, whereas it is \(\sim 10^9\) G for the MSPs in globular clusters (see Table 1). So the minimum mass required to spin a pulsar, with a surface magnetic field of \(\sim 10^9\) G, to \(\sim 5\) ms would be \(\delta M \sim 0.01 - 0.1\) M\(_\odot\). The radius and the crustal mass of a neutron star remain effectively constant for this amount of accreted masses considered, and the corresponding change in the crustal density profile is negligible. We therefore take the mass flux to be the same throughout the crust, equal to its value at the surface. Assuming the mass flow to be spherically symmetric in the deep crustal layers of interest, one obtains the velocity of material movement to be

$$V = - \frac{\dot{M}}{4\pi r^2 \rho(r)} \hat{r},$$

(6)

where \(\dot{M}\) is the rate of mass accretion and \(\rho(r)\) is the density as a function of radius \(r\).

The physical conditions of the crust enter equation (3) through the electrical conductivity, \(\sigma\), which is a function of (i) the density of the current concentration, (ii) the temperature of the crust and (iii) the impurity content of the crustal lattice (negligible for an accretion-heated crust). We have assumed the currents to be concentrated at a density \(\sim 10^{15}\) g cm\(^{-3}\), and the impurity concentration to be \(\sim 0.01\) for a cold crust. These values are consistent with the fact that the magnetic fields of the isolated pulsars do not decay significantly over a time-scale of \(10^6\) yr (Bhattacharya et al. 1992; Hartman et al. 1997).

The temperature of the crust is another important parameter for the field evolution. After the neutron star is born it monotonically cools down through copious emission of neutrinos (Page 1998). When, in the course of binary evolution, the neutron star actually starts accreting mass – the thermal behaviour changes drastically. Accretion releases heat and the crustal temperature quickly (in about \(\sim 10^5\) yr) settles down to a more or less uniform and steady value determined by the accretion rate (Miralda-Escude, Paczynski & Haensel 1990; Zdunik et al. 1992). Though analytical expressions giving the crustal temperature for a given rate of accretion have been obtained by Zdunik et al. (1992) and Urpin & Geppert (1995), the values are too high for temperatures for \(M \geq 2 \times 10^{-10} M_{\odot} \text{yr}^{-1}\). Observations of thermal blackbody radiation from the surface of accreting neutron stars indicate that for higher \(M\) the temperature of the deep crustal layers probably saturate around \(\sim 10^9\) K (see e.g. Brown & Cumming 2009). Accordingly for \(M \geq 10^{-9} M_{\odot} \text{yr}^{-1}\) we assume the crustal temperatures to be in the range \(\sim 10^8 - 5 \times 10^8\) K. It should be noted that we consider only the temperature of the deep crustal layers, where the currents are assumed to be concentrated. And the temperature of the entire crust beyond a density of \(10^{10}\) g cm\(^{-3}\) is practically the same though it drops by almost two orders of magnitude at the outermost layers of the star (Gudmundsson, Pethick & Epstein 1983; Potekhin, Chabrier & Yakovlev 1997).

From the point of view of mass transfer (responsible for the physical process of recycling), pulsars go through three distinct phases of evolution in typical LMXBs. These phases are as follows (Bhattacharya & van den Heuvel 1991; Verbunt 1993; van den Heuvel 1995).

(i) The isolated phase. Though the stars are gravitationally bound, there is no mass transfer. In general, the isolated phase lasts between \(10^8\) and \(10^9\) yr.

(ii) The wind phase. The interaction is through the stellar wind of the companion which is likely to be in its main sequence. In general this phase lasts for about \(10^5 - 10^8\) yr with attendant rates of accretion ranging from about \(10^{-15}\) to \(10^{-12}\) M\(_\odot\) yr\(^{-1}\).

(iii) The Roche contact phase. When the companion of the neutron star fills its Roche lobe a phase of heavy mass transfer ensues. In this phase, the mass transfer rate could be as high as the Eddington rate \(10^{-8} M_{\odot} \text{yr}^{-1}\) for a \(1.4 M_{\odot}\) neutron star, lasting for \(\lesssim 10^8\) yr. However there has also been indications that the low-mass binaries may even spend \(\sim 10^{10}\) yr in the Roche contact phase with a sub-Eddington accretion rate (Hansen & Phinney 1998). For wide binaries, however, the contact phase may last as little as \(10^7\) yr.

Earlier we have followed the evolution of the magnetic field for neutron stars in different types of binaries, assuming all the binaries to be primordial, undergoing standard phases of binary evolution. For our adopted model (Konar & Bhattacharya 1997, 1999a) it has been seen that the field decays rapidly in the initial phase, followed by a slow-down and a final freezing. The initial decay is due to the heating of the crust in which the currents undergo rapid ohmic dissipation. However as the accretion proceeds there is addition of extra material in the outer layers of the star. Since an increase of mass makes a neutron star more compact, in the deeper layers of the crust this induces an inward radial motion. As a result the current carrying layers progressively move into higher density and higher conductivity region. Consequently the decay
sloows down. And when the entire current distribution, responsible for the field, gets assimilated into the highly conducting (time-scale of diffusion larger than the Hubble time) core the decay stops altogether freezing the field at its final value. Understandably there is no further evolution of the field after freezing even if mass accretion continues. Also the higher the accretion rate the sooner the freezing sets in resulting in a higher value of the final surface field.

In globular clusters, however, most binaries are not primordial. For example, the total observed number of LMXBs in globular clusters exceeds their formation rate in the disc by several orders of magnitude, indicating a dynamical origin (Clark 1975). The composition of the binary itself may change, even more than before. Dynamical interactions in the clusters involving at least one neutron star are typically of the two- and the three-body types, as the probability of interactions involving four or more objects would be negligibly small. Similarly, a three-body interaction is essentially between a single star and a binary system. Stellar interaction in the clusters involving a neutron star has been studied by a number of authors (Krolik, Meiksin & Joss 1984; Rasio & Shapiro 1991; Davies, Benz & Hills 1992; Davies & Hansen 1998; Rasio, Pfahl & Rappaport 2000; King et al. 2003). We briefly discuss below the interactions which play an active role in pulsar recycling (see Camilo & Rasio 2005 for a detailed review). It needs to be mentioned that we do not consider the interactions that have no direct effect on the mass transfer phases of a binary pulsar (for example the ‘fly-by’ kind of interaction between a single star and a binary).

3.1 Two-body interactions

The interaction could be a close tidal encounter or a direct physical collision. However, the formation of stable binaries through tidal encounters is not very likely. Therefore, this process is not important for MSP formation. For collisional encounters there can be mainly two types of partners.

Main-sequence star. Typically such a collision leads to the complete destruction of the main-sequence star forming a thick, rapidly rotating envelope around the pulsar. Depending on the amount of accretion the pulsar may be spun-up to millisecond periods or it may be only mildly recycled, either way giving rise to a single recycled pulsar.

Red giant star. Such a collision leads to the formation of a high-eccentricity binary. These collisions provide a natural formation process for eccentric low-mass binary pulsars with white dwarf companions. If the post-collision neutron star–white dwarf binaries retain high eccentricities, then they could decay through gravitational-wave emission and possibly become ultracompact X-ray binaries (UCXB) with $P_h \lesssim 1 \text{ h}$. These are important for pulsar recycling since a number of the known AMXPs are actually members of UCXBs, which are also probably the progenitors of the black-widow MSPs (King et al. 2003).

3.2 Three-body interactions

When a single star interacts with a binary (the neutron star could either be a member of the binary or the single object) the result could be one of the following types of changes to the binary.

(i) Exchange. In this kind of encounter one of the binary components is replaced by the single star. So a single pulsar could acquire a binary companion through this process. Alternatively, a previously formed binary pulsar could interact with another star or binary. This would lead to a new companion for an MSP, or for a non-recycled pulsar, or could release an MSP from a binary, creating a single MSP. Systems with higher mass companions, fast MSPs and very high eccentricities are likely to be the result of such exchange interactions, i.e. the presently observed companion was likely acquired later and is not the donor from which the pulsar was recycled. Exchange interactions between ordinary pulsars and primordial binaries also provide a natural way of forming possible progenitors of UCXBs (King et al. 2003).

(ii) Disruption. Finally, it is possible for the binary to be completely disrupted by its interaction with the single star. This would release a single pulsar. Depending on when the binary is disrupted the pulsar could be an MSP or a mildly recycled pulsar.

(iii) Multiple interactions. In some of the clusters the stellar densities are so high that the interaction time-scale for either exchange or fly-by could be small making it possible for the binaries to undergo multiple interactions. Under the circumstances, a neutron star may actually go through various phases of evolution in a number of binaries sequentially.

Given the above possibilities, a number of different situations can be envisaged where the phases of accretion are rather different from those in typical LMXBs. It has been seen that at least $\sim 10^{-2} M_\odot$ is required to spin a pulsar up to millisecond periods. We have seen earlier that accretion of $\sim 10^{-2} M_\odot$ is also enough for the field to freeze to its final stable value (Konar & Bhattacharya 1997). Because the mass of the crust of a typical $1.4 M_\odot$ neutron star is $\sim 10^{-2} M_\odot$. And when this amount of material is accreted, the entire mass of the original crust containing the current carrying layers get accumulated into the highly conducting core where there can be no dissipation of the field. Once this amount of mass is accreted the field attains its final value even though the star may continue to go through further accretion and spin-up. Below, we list possible situations where $M \gtrsim 10^{-2} M_\odot$ could be accreted on to a neutron star. For our calculations we have evolved the field till the final frozen value is attained.

(A) The entire mass is accreted with $M$ characteristic of the wind phase ($M \sim 10^{-14} \sim 10^{-11}$) in low-mass stars. This situation can arise if the original low-mass binary is disrupted before Roche contact is established. The only way a pulsar could be spun up to millisecond period entirely by wind accretion is if $M \sim 10^{-12} \sim 10^{-11} M_\odot$ yr$^{-1}$, assuming the main-sequence phase of an extremely low-mass companion to last for $10^{-7} \sim 10^{-6}$ yr.

(B) A brief wind phase (or complete absence of it), followed by heavy mass transfer ($M \sim 10^{-10} \sim 10^{-8} M_\odot$) characteristic of the Roche contact phase in which most of actual mass accretion takes place. It needs to be mentioned here that this situation is very similar to typical LMXB evolution, except for a long wind phase with attendant low values of $M$ prior to Roche contact which is realized in most LMXBs. In our earlier investigations we have seen that the final field values attained with or without a phase of wind accretion are not greatly different (Konar & Bhattacharya 1999a). Hence, even though the nature of binary evolution would be different the final $P$, $B$ seen in the resultant MSP would be similar to one processed in a primordial low-mass binary. Now, this kind of mass transfer scenario is possible if an unrecycled (mildly or otherwise) pulsar acquires a partner which is already in an evolved phase (e.g. a red giant star), in a tight binary. It should be noted that collisions with red giant stars are expected to give rise to UCXBs, extremely compact systems that are likely to have mass transfer at high $M$s.

(C) A brief phase of heavy mass transfer, followed by a long interval with low $M$, most of the mass being accreted in the second phase. This situation can arise either through an exchange
interaction (the pulsar being always in the binary) or due to a binary (containing a pulsar) disruption followed by an acquisition of a new partner by the pulsar. We find that this is very similar to case (A) with the initial phase of heavy accretion having hardly any significant effect.

(D) A phase of heavy mass transfer followed by a brief interval of accretion with small $M$ where most of the mass is accreted in the first phase. This is again very similar to case (B).

In summary, it can be said that given the time-scales of binary evolution and stellar collisions the amount of mass required to freeze the magnetic field to its final stable value would be mostly achieved in one particular phase of accretion with a given $M$. In Fig. 6 we plot the evolution of the surface field with time for different values of $M$. It is seen that a higher magnetic field is retained by the pulsar if most of the mass is transferred in a short period with a high $M$ even though initially the field decays faster due to a higher crustal temperature. This happens because the current carrying layers get assimilated into the core (which has effectively infinite conductivity) faster for a higher $M$. A high rate of mass transfer can be realized if the pulsar is in a tight binary. Interestingly, it is already known that the binary MSPs in the clusters have relatively shorter orbital periods (tighter orbits) compared to the disc population. Dynamically too, collisions of single pulsars with red giant stars or exchange interactions are likely to produce UCXBs – again systems where higher values of $M$ can be achieved. So there definitely exist MSP formation channels in the clusters that are conducive of producing high magnetic field MSPs. However, in general, the average field values would be higher than the disc MSPs only if this were the dominant channel of MSP formation in the clusters. In an ongoing work we are looking at this question of relative importance of different channels of MSP formation in the clusters (Bagchi & Konar, in preparation) but the results are not available yet.

4 CONCLUSIONS

In this work, we have compared the MSPs in the Galactic disc with those in the globular clusters vis-à-vis their distribution of the spin period and the surface magnetic field. The statistical nature of the two populations is as follows:

(i) The average spin periods of isolated and binary MSPs in globular clusters as well as isolated MSPs in the Galactic disc are very similar. Though the average spin period of the binary MSPs in the disc is somewhat different, the difference is not very significant.

(ii) The K–S probabilities indicating the extent of correlation of the spin period between various subgroups of MSPs are found to be as follows:

(a) isolated versus binary in the clusters – $P_{KS} \sim 10^{-3}$;
(b) isolated versus binary in the disc – $P_{KS} \sim 10^{-1}$;
(c) isolated in the clusters versus isolated in the disc – $P_{KS} \sim 0.9$;
(d) binary in the clusters versus binary in the disc – $P_{KS} \sim 10^{-2}$.

(iii) Finally, the subset of cluster MSPs, for which field measurements are available, appear to have two–five times higher surface magnetic fields compared to the disc MSPs. Though the MSPs with field measurements are actually a slower subset of the cluster MSPs their average spin periods are similar to the MSPs in the disc which are, in general, slower than the cluster MSPs. We feel the difference in the surface magnetic field could be due to one or several of the possibilities listed below.

(a) There are systematic biases in $P$ (hence $B$) measurement for cluster MSPs.
(b) The cluster MSPs are younger and may evolve to a distribution similar to the disc MSPs with time.
(c) Preferential recycling of MSPs in tighter binaries with high rates of attendant mass transfer may actually result in cluster pulsars retaining higher magnetic fields.

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