Temperature distribution in the three-layered upper mantle beneath a continent

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Abstract. Temperature distribution in the upper mantle underneath the continent, as well as temperature distribution in the lower mantle, is obtained. In the continental lithosphere, the solution to the heat transfer equation is obtained in the model of conduction heat transfer with inner heat within the crust. To calculate the temperature distribution in the upper and lower mantle, we use the results of laboratory and theoretical modeling of free convective heat transfer in a horizontal layer heated from below and cooled from above.

1. Introduction

Temperature profile in the upper mantle under the continent is obtained using seismic data with regard to different models of mantle composition [1, 2]. Continental geotherms conforming to $P$-$T$ conditions for mantle xenoliths have been obtained, for example, in [3]. Relying on the solution of the problem of steady state heat conduction, the authors of [4] have found a continental geotherm on the basis of the exponential depth dependence of radiogenic heat production.

In the present paper a convecting upper mantle is considered as a two-layered one. It is constituted by asthenosphere and layer $C$ (mantle transition zone). The continental lithosphere lies above the asthenosphere. Hence, the upper mantle beneath a continent involves lithospheric mantle, asthenospheric layer and layer $C$. Our paper aims at (1) presenting the temperature distribution in high-viscous continental lithosphere under conditions of conduction heat transfer; (2) obtaining the temperature distribution in convecting asthenosphere and layer $C$ beneath the continental lithosphere; and (3) finding the temperature distribution through the lower mantle under free convection conditions. The problems (2) and (3) are solved on the basis of experimental and theoretical modeling of free convection flows in a horizontal viscous liquid layer heated from below and cooled from above.

2. Temperature distribution in the continental lithosphere

We analyze a conductive heat transfer in the continental lithosphere. The distribution of radioactive heat production over the thickness of the crust layer can be approximated by an exponential law [5]:

$$Q = Q_s \exp(-x/x_0),$$

where $Q_s$ is the average radiogenic heat production in surface rocks of a region, $x$ is the depth, and $x_0$ is the length scale for the decrease of radiogenic heat production with depth. In this case, a superadiabatic temperature profile across the thickness of the continent is described by equation [4, 5]:

$$T_a = q_{sm}/\lambda_c + (Q_s x_0^2/\lambda_c)[1 - \exp(-x/x_0)] + T_s,$$

where $q_{sm}$ is the mantle heat flux to the continent, $\lambda_c$ is the thermal conductivity, and $T_s$ is the surface temperature.
At $x < 200$ km an adiabatic temperature gradient is $(\partial T/\partial x)_{\text{ad}} = 0.75$ °C km$^{-1}$, and at $200$ km $<$ $x < 500$ km we have $(\partial T/\partial x)_{\text{ad}} = 0.56$ °C km$^{-1}$ [6]. A distribution of temperature across the thickness of the continent is calculated in terms of superadiabatic temperature: $T = T_{sa} + (\partial T/\partial x)_{\text{ad}} x$.

We find a temperature profile in the continent away from the subduction zone for the following continent parameter values: $q_s = 0.052$ W m$^{-2}$, $Q_1 = 1.82 \cdot 10^{-4}$ W m$^{-3}$, $l_{sa} = 2.2 \cdot 10^4$ m, $\lambda_c = 3$ W m$^{-1}$ °C$^{-1}$, $T_s = 0$, and $l_{\text{cont}} = 2.5 \cdot 10^5$ m, where $q_s$ is the specific heat flux at the Earth's surface, and $l_{\text{cont}}$ is the continental lithosphere thickness. For these values we have $q_{\text{um}} = q_s + Q_s l_{sa} = 0.012$ W m$^{-2}$. The profiles of temperatures $T_{sa}$ and $T$ are calculated up to a depth $x_0 = 250$ km (figure 1). The temperature conditions for Northern Lesotho lherzolites are shown. The temperature of the continental bottom (for $l_{\text{cont}} = 2.5 \cdot 10^5$ m) is equal to $T_{\text{cont}} = 1472$ °C.

![Figure 1. The profiles of temperatures in the upper mantle.](image)

1 – temperature profile across the upper mantle based on results of laboratory and theoretical modeling of free convective currents in a horizontal layer heated from below; 2 – formation conditions of the Northern Lesotho lherzolites [7]; 3 – melting curve of peridotite KLB-1 [8]; 4 – melting curve of the upper mantle matter [9]; 5 – melting points of anhydrous and hydrous basalt [10]; 6 – adiabatic temperature distribution, 7 – melting curve of basalt [11].

3. Temperature distribution in the asthenosphere and layer C beneath a continent

In the continental region, the upper layer is presented by the continental lithosphere. The asthenosphere lies beneath the continental lithosphere. The base of asthenosphere is at a depth of 410 km. Layer C lies beneath the asthenosphere. Such three-layered structure is obtained using seismic velocity measurements [6]. Lithospheric mantle has a high viscosity, namely, its kinematic viscosity is $\nu_{\text{lu}} \approx 10^{18} - 10^{19}$ m$^2$ s$^{-1}$ [5]. The asthenospheric layer is low-viscous in relation to lithosphere ($\nu_s \approx 10^{14}$ m$^2$ s$^{-1}$) [12] as is layer C ($\nu_{C} \approx 4 \cdot 10^{15}$ m$^2$ s$^{-1}$, see below). Heat transfer in the asthenosphere and layer C will be analyzed as a free convection heat transfer. The asthenospheric layer and layer C are separated by the olivine-wadsleyite phase transition boundary (410 km boundary) [13, 14]. Numerical solutions [13] for equal kinematic viscosities on each side of the phase transition have shown that the influence of the phase transition on heat transfer can be neglected.

The thermophysical model of the asthenosphere is a layer heated at the mid-ocean ridge (MOR) axis and cooled from above (at the lithosphere-asthenosphere boundary). The lower boundary of the layer is adiabatic [15]. The relationship between kinematic viscosity and the parameters of the asthenospheric layer is [12]:

$$v = \frac{\beta g}{a} \left( \frac{\lambda_c}{Q_1} \right)^2 \left( \frac{\Delta T_{\text{max}} l_{sa}}{8} \right)^{\frac{3}{2}},$$

where $\alpha$ is the thermal diffusivity coefficient, $\beta$ is the thermal expansion, $g$ is the acceleration due to gravity, $Q_1$ is the amount of heat transported by an ascending flow at the MOR axis (per running meter
of the axis), \( \Delta T_{\text{max}} = T_{\text{max}} - T_{\text{roof}} \), \( T_{\text{max}} \) is the maximal temperature of the upward flow at the roof of the asthenospheric layer, \( T_{\text{roof}} \) is the temperature of the top of the asthenosphere layer, and \( l_a \) is the layer thickness. The amount of heat \( Q_1 \) is equal to \( q_1 X_0 \) where \( q_1 \) is the corresponding heat flux, and \( X_0 \) is the horizontal dimension of the layer \((X_0/l >> 1)\). The kinematic viscosity of the asthenospheric layer is \( \nu_a \approx 10^{14} \text{ m}^2 \text{ s}^{-1} \) [12].

The viscosity of the layer \( C \) can be evaluated when analyzing heat transfer in the layer \( C \) beneath the ocean. The problem of heat transfer in the layer \( C \) is reduced to the problem of free convection heat transfer in the asthenosphere. In this case the relation (1) holds true for the layer \( C \), where \( \Delta T_{\text{max}} = T_2 - T_{\text{min}} \), \( T_2 \) is the temperature at the upper–lower mantle boundary, \( T_{\text{min}} \) is the minimum temperature in the descending flow in a subduction zone, and \( Q_2 = q_2 X_0 \) is the amount of heat entrained by the descending flow in a subduction zone. We accept the following values: \( \beta = (2–5) \cdot 10^5 \text{ K}^{-1}, \lambda = 3.5–4 \text{ W m}^{-1} \text{ K}^{-1}, \rho = 3900 \text{ kg m}^{-3}, a = \lambda c p = (7.5–8.5) \cdot 10^7 \text{ m}^2 \text{ s}^{-1} \) (\( c \) is the heat capacity), \( q_2 = 0.06 \text{ W m}^{-2}, X_0 = 3 \cdot 10^8 \text{ m}, \) and \( \Delta T_{\text{max}} = 850 \text{ K} \) [12, 15]. Then, using (1) we obtain the kinematic viscosity of layer \( C \): \( \nu_C = (2.1–6.0) \cdot 10^{15} \text{ m}^2 \text{ s}^{-1} \).

Away from the contact area between a subducting plate and a continental limb there is a free convective heat transfer in the asthenospheric layer and layer \( C \). A horizontal layer of viscous liquid heated uniformly from below serves as a model of these two layers [5]. The heat transfer in the horizontal layer is described by the following law for the Rayleigh numbers \( Ra = \beta g \Delta T C a/\nu > 10^5 \) [5]:

\[
Nu = 0.1 Ra^{1/3},
\]

where \( \text{Nu} = q_{\text{um}}/\Delta T \lambda \) is the Nusselt number (dimensionless heat transfer coefficient), and \( \Delta T \) is the temperature difference across the layer of thickness \( l \). Using the equality (2), we obtain that the heat flux in the layer is \( q = 0.1 \lambda \Delta T^{3/4} (\beta g/\nu)^{1/4} \). Further analysis has shown that \( Ra_k = 5.2 \cdot 10^5 \) for the asthenospheric layer and \( Ra_C = 4.7 \cdot 10^5 \) for the layer \( C \), i.e., \( Ra > 10^5 \). In accordance with equality (2), the superadiabatic temperature drop in the asthenosphere and layer \( C \) is determined using the relation

\[
\Delta T = \left( \frac{10q_{\text{um}}}{\lambda} \right)^{1/4} \left( \frac{a \nu}{\beta g} \right)^{1/4}.
\]

Heat flux at the 410 km boundary is

\[
q_{410} = q_{\text{um}} \left( \frac{R_E}{R_E - 410} \right)^2,
\]

where \( R_E = 6370 \text{ km} \) is the Earth radius. From the relation (4) for above-obtained \( q_{\text{um}} = 0.012 \text{ W m}^{-2} \) we find that \( q_{410} = 0.0137 \text{ W m}^{-2} \).

We accept the following parameters of asthenosphere beneath the continent: \( \lambda = 3.8 \text{ W m}^{-1} \text{ K}^{-1}, c = 1200 \text{ J kg}^{-1} \text{ K}^{-1}, \rho = 3.2 \cdot 10^3 \text{ kg m}^{-3}, \beta = 3 \cdot 10^5 \text{ K}^{-1}, a = 9.9 \cdot 10^7 \text{ m}^2 \text{ s}^{-1}, \nu_a = 10^{14} \text{ m}^2 \text{ s}^{-1} \) and \( q = q_{410} = 0.0137 \text{ W m}^{-2} \). The relation (3) gives the superadiabatic temperature difference in the asthenospheric layer: \( \Delta T_a = 63 \text{ K} \). As follows from the relation (3) for layer \( C \) for parameters indicated above and \( \nu_C = 3.8 \cdot 10^{13} \text{ m}^2 \text{ s}^{-1} \) the superadiabatic temperature drop is \( \Delta T_C = 156.5 \text{ °C} \).

Temperature at the asthenosphere-layer \( C \) boundary is \( T_{a-c} = T_{\text{cont}} + \Delta T_a + (\partial T/\partial x)_{a-c} \cdot l_a \). For \( T_{\text{cont}} = 1472 \text{ °C}, \Delta T_a = 63 \text{ °C}, (\partial T/\partial x)_{a-c} = 0.56 \text{ °C km}^{-1} \) and \( l_a = 170 \text{ km} \) we obtain \( T_{a-c} = 1630 \text{ °C} \). The temperature at the base of layer \( C \) (upper–lower mantle boundary) \( T_2 = T_{a-c} + \Delta T_C + (\partial T/\partial x)_{a-c} \cdot l_c \), and for \( l_c = 250 \text{ km} \) we obtain \( T_2 = 1932 \text{ °C} \). The average temperature drop across the thermal boundary layer at the heated horizontal plate is \( \Delta T_\text{bl} = 0.5 \Delta T \). The temperature drop across the conduction sublayer of the thermal boundary layer is [5, 16]

\[
\Delta T_{ts} = 0.7 \Delta T_{bl} = 0.35 \Delta T.
\]

The heat flux is determined using the relation

3
The Rayleigh number for the roll layer is

$$q = \lambda \Delta T_{rl} / \delta_{rs},$$

(6)

where $\delta_{rs}$ is the conduction sublayer thickness. Thus, the temperature in the conduction sublayer varies linearly. From relations (3), (5) and (6) it follows that

$$\delta_{rs} = 3.5 \left( \frac{a \nu}{\beta g \Delta T_{rl}} \right)^{\frac{1}{3}}.$$

(7)

Experiments in horizontal liquid layer for Rayleigh numbers $Ra > 5 \cdot 10^6$ have revealed convection rolls adjacent to the heat transfer surface [5]. The near-well layer forms because of unstable stratification of the boundary layer. The Rayleigh number for the roll layer is $Ra_{rl} = \beta g \Delta T_{rl} l_{rl}^3 / \nu \nu = (1.5–2) \cdot 10^4$, where $\Delta T_{rl}$ is the temperature difference across the roll layer, and $l_{rl}$ is the roll layer thickness. The latter is

$$l_{rl} = (Ra_{rl} \nu \nu / \beta g \Delta T_{rl})^{\frac{1}{3}},$$

(8)

where $\Delta T_{rl} \approx \Delta T_{bl}$. For Rayleigh numbers $Ra > 5 \cdot 10^6$ the boundary layer thickness is $\delta_{bl} \approx l_{rl}$ [5].

The data obtained above, i.e., temperature drops $\Delta T_{bl}$ and $\Delta T_{oc}$ as well as temperatures $T_{bl}$ and $T_{oc}$ are used to build the temperature profile in the upper mantle (figure 1). Furthermore, temperature drops $\Delta T_{bl}$ and $\Delta T_{oc}$, as well as the boundary layer thicknesses $\delta_{bl}$ and $\delta_{oc}$ for the asthenospheric layer and $C$ layer, are calculated. Hence, the temperature profiles in the asthenosphere and $C$ layer are built from experimental results on natural convective heat transfer in a horizontal liquid layer heated from below.

4. Temperature distribution in the lower mantle

The temperature of the 670 km boundary is $T_2 = 1916$°C when calculating the distribution of temperature in the continental region situated away from subduction zone (figure 1). Let us evaluate the temperature $T_2$ using relationships for free convection in the lower mantle. Let us consider the lower mantle as spherical layer heated from below at a depth of 2880 km and cooled from above at a depth of 670 km. The mean surface heat flux for continental regions is $q_c = 0.0565 \text{ W m}^{-2}$ and for oceans $q_{oc} = 0.0782 \text{ W m}^{-2}$ [6]. We accept the average heat flux at the Earth's surface as the average heat flux from the lower mantle $q_{lm}$: $q_{lm} = 2/3 q_{oc} + 1/3 q_c = 0.071 \text{ W m}^{-2}$. In accordance with the relation (4), the specific heat flux increases with decreasing depth (radius). From this relation, it follows that the heat flux at the upper–lower mantle boundary (670 km boundary) is $q_2 = 0.089 \text{ W m}^{-2}$ and the heat flux at the base of the lower mantle (2880 km boundary) is $q_1 = 0.236 \text{ W m}^{-2}$.

The Rayleigh number is $Ra = 10^6–10^7$ for the lower mantle and, hence, turbulent free convection flows occur in it [5]. The boundary layer thickness at the roof and the base of the lower mantle is comparable with the thickness of near-wall roll layer. In accordance with experimental data [5], the average superadiabatic temperature is constant outside of the boundary layers.

If the lower mantle is considered as the spherical layer of radius $r_{lm}$, in the case of $l_{bl}/r_{lm} \ll 1$, a boundary layer can be considered in the approximation of the flat horizontal layer. In this case, at the top of the lower mantle we consider the problem on large liquid volume cooling from above (from a horizontal surface). At the base of the lower mantle, we consider the problem on large liquid volume heating from below (from a horizontal surface). In this case the law of heat transfer from a horizontal surface has the form $Nu = 0.18Ra^{\frac{1}{3}}$ [17]. Then the specific heat flux is independent of linear size:

$$q = 0.18\lambda \Delta T_{bl}^{\frac{4}{3}} (\beta g / \nu \nu)^{\frac{1}{3}},$$

(9)

where $\Delta T_{bl}$ is the temperature difference across the boundary layer of thickness $l_{bl}$.

Given the $q$ value one can determine the temperature difference across the boundary layer using the relation (9):

$$\Delta T_{bl} = (q / 0.18\lambda)^{\frac{3}{4}} (\beta g \nu \nu)^{\frac{1}{3}}.$$

(10)
The superadiabatic temperature difference in the lower mantle is

$$\Delta T_{\text{in}} = \Delta T_{\text{bl1}} + \Delta T_{\text{bl2}},$$  \hspace{1cm} (11)

where $\Delta T_{\text{bl1}}$ is the temperature drop across the boundary layer at the base of the lower mantle, and $\Delta T_{\text{bl2}}$ is the temperature drop across the boundary layer at the top of the lower mantle. The thickness of the conduction sublayer is determined using relation (7) and the thickness of the whole thermal boundary layer is obtained from relation (8).

Evaluations of mantle melting curve have shown that the melting point at the core-mantle boundary is $T_{\text{mb}} = 3450 \, ^\circ\text{C}$, when density varies with depth according to the PREM model [6, 18]. This estimate agrees with calculations of the melting temperature [9]. The melting temperature gradient is $(\partial T/\partial x)_{\text{mb}} = 0.5 \, ^\circ\text{C} / \text{km}$. In accordance with the studies of stable thermochemical plume, the difference between the melting temperature of a "dry" mantle and the temperature of the core-mantle boundary is $T_{\text{mb}} - T_{\text{i}} = 30–60 \, ^\circ\text{C}$. Thus, the temperature of the core-mantle boundary is accepted as $T_{\text{i}} = 3400 \, ^\circ\text{C}$. We take the following parameter values for the boundary layer at the top of the lower mantle: $\beta_1 = 3.10^{-5} \, ^\circ\text{C}^{-1}$, $\rho_2 = 5000 \, \text{kg} / \text{m}^3$, $\nu_2 = 8.10^{15} \, \text{m}^2 / \text{s}$, $\lambda_2 = 10 \, \text{W} / \text{m}^2 \cdot ^\circ\text{C}$, $c_2 = 1.2.10^3 \, \text{J} / \text{kg} \cdot ^\circ\text{C}$, $\alpha_2 = 1.667.10^6 \, \text{m}^2 / \text{s}$. Then, according to the relations (5) and (10) for the boundary layer at the upper–lower mantle boundary we obtain $\Delta T_{\text{bl2}} = 281 \, ^\circ\text{C}$ and $\Delta T_{\text{cs2}} = 196 \, ^\circ\text{C}$ for $q_2 = 0.0886 \, \text{W} / \text{m}^2$. In compliance with the above-mentioned estimates of the Rayleigh number we accept $Ra_{\text{bl2}} = 2.10^4$. Using relation (8) we find the boundary layer thickness $\delta_{\text{bl2}} = 154 \, \text{km}$. Using the relation (7) the conduction sublayer thickness $\delta_{\text{cs2}} = 22 \, \text{km}$.

For the boundary layer at the base of the lower mantle we take the same values as for the boundary layer at the top, except for density. We adopt the density $\rho_1 = 5000 \, \text{kg} / \text{m}^3$ [5]. Using the relations indicated above, we obtain $\Delta T_{\text{bl1}} = 66 \, ^\circ\text{C}$, $\Delta T_{\text{cs1}} = 396 \, ^\circ\text{C}$, $\delta_{\text{bl1}} = 154 \, \text{km}$ and $\delta_{\text{cs1}} = 17 \, \text{km}$ for $q_1 = 0.2364 \, \text{W} / \text{m}^2$. The thicknesses of roll layers at the boundaries of the lower mantle are $l_{\text{r1}} \approx \delta_{\text{bl1}}$ and $l_{\text{r2}} \approx \delta_{\text{bl2}}$ (see previous section) and, hence, they are much less than the thickness of the lower mantle.

In accordance with relation (11) the superadiabatic temperature drop is $\Delta T_{\text{in}} = 846 \, ^\circ\text{C}$. The temperature at the top of the lower mantle is

$$T_2 = T_{\text{i}} - \Delta T_{\text{in}} - (\partial T / \partial x)_{\text{ad}} l_{\text{in}}.$$  \hspace{1cm} (12)

Using relation (12) $T_2 = 1935 \, ^\circ\text{C}$ is obtained for the adiabatic temperature gradient in the lower mantle $(\partial T/\partial x)_{\text{ad}} = 0.28 \, ^\circ\text{C} / \text{km}$ and the thickness of the lower mantle $l_{\text{in}} = 2210 \, \text{km}$. The value of $T_2$ corresponds to that, evaluated above with the use of the temperature profile in the continental region. As a result of calculations of temperatures and temperature drops the temperature distribution in the lower mantle is obtained (figure 2).

**Conclusions**

The temperature distribution in the continental lithosphere and upper mantle beneath a continent along with temperature at the upper–lower mantle boundary has been obtained. The temperature profile in the continental lithosphere has been built for the conduction heat transfer with inner radioactive heat sources in the crustal layer. The temperature distribution across the thickness of the continent agrees with temperature conditions for conditions of formation of Northern Lesotho lherzolites.

The laboratory and theoretical modeling of natural convection heat transfer in a horizontal layer heated from below provides a way to construct temperature profiles in the mantle. In addition, the temperature profile in the lower mantle for Rayleigh numbers $Ra = 10^6–10^7$ inherent for turbulent free convection in a horizontal layer has been calculated.
Figure 2. Temperature profiles in the lower mantle.

1 – adiabatic temperature, 2 – mean temperature, 3 – melting curve in the lower mantle [9], 4 – melting point of basalt at the upper–lower mantle boundary [11]. $T_{\text{desc}}$ and $T_{\text{asc}}$ are the respective temperatures of descending and ascending flows of convective cell.

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