Multifaceted dynamics and gap solitons in $\mathcal{PT}$-symmetric periodic structures

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We report the role of $\mathcal{PT}$-symmetry in switching characteristics of a highly nonlinear fiber Bragg grating (FBG) with cubic-quintic-septic nonlinearities. We demonstrate that the device shows novel bi-(multi-) stable states in the broken regime as a direct consequence of the shift in the photonic band gap influenced by both $\mathcal{PT}$-symmetry and higher-order nonlinearities. We also numerically depict that such FBGs provide a productive test bed where the broken $\mathcal{PT}$-symmetric regime can be exploited to set up all-optical applications such as binary switches, multi-level signal processing and optical computing. Unlike optical bistability (OB) in the traditional and unbroken $\mathcal{PT}$-symmetric FBG, it exhibits many peculiar features such as flat-top stable states and ramp like input-output characteristics before the onset of OB phenomenon in the broken regime. The gain/loss parameter plays a dual role in controlling the switching intensities between the stable states which is facilitated by reversing the direction of light incidence. We also find that the gain/loss parameter tailors the formation of gap solitons pertaining to transmission resonances which clearly indicates that it can be employed to set up optical storage devices. Moreover, the interplay between gain/loss and higher order nonlinearities brings notable changes in the nonlinear reflection spectra of the system under constant pump powers. The influence of each control parameters on the switching operation is also presented in a nutshell to validate that FBG offers more degrees of freedom in controlling light with light.

I. INTRODUCTION

In the era of high-speed information exchange, all-optical switches and logic devices are versatile components in all-optical communication systems which have lead to widespread research across different devices such as couplers [1, 2], Bragg gratings [3], ring resonators [4] and so on. Among these devices, FBGs have engrossed an ever-mounting attention as they afford a larger degree of freedom and flexibility to engineer any spectral characteristics of interest [5] and are potential candidates for provisioning, protection, packet switching, and external modulation applications [6] in addition to sensing, dispersion compensation, and filtering functionalities [7]. Since the bandwidth of the device is very narrow, a small change in refractive index introduced into the system via an external signal is sufficient enough to detune the in-built photonic band gap from its resonance wavelength. Thus the structure allows light transmission at the wavelengths which were inhibited to transmit previously. This gives the comprehensive picture of the underlying mechanism to realize all these functionalities including optical switching [8].

Optical bistability is a ubiquitous phenomenon in the framework of nonlinear feedback systems such as Bragg gratings [8], nonlinear Fabry–Pérot cavity [9], ring resonators [10], and even in the case of nonlinear directional couplers with the aid of metamaterials [11]. As the name suggests, any nonlinear variation in input intensity results in two or more stable states for the given incident intensity. Apart from its conventional application in the form of optical switches or memory devices where the two stable states can be customized as binary logic, researchers have also exploited the possibility to build all-optical transistors, limiters, inverters [12], and signal processing devices [13]. In principle, optical bistability emanates itself as a result of light-dependent refractive index changes or absorption changes inside the structure upon which they are categorized as dispersive or absorptive optical bistability [14]. Ever since the breakthrough work by Winful et al. [15], the studies on optical bistability in nonlinear feedback structures primarily focused on understanding the underlying physics behind its operation at various conditions [8, 16] which in turn aided the possibility to realize various nonlinear applications [12]. These studies indicate that the bistable curve or the hysteresis loop can be manipulated at will with the aid of any control parameter originating from the device such as device length, detuning parameter, the strength of modulation [3] or via an external control in the form of probe and pump pulse parameters [12]. Some of these factors will be discussed in detail in this paper in later sections.

A suitable choice of materials is an essential ingredient to realize an all-optical switch with low threshold and high figure of merit (FOM) [17, 18]. In this regard optical properties of many nonlinear glasses have been reported in the literature from both theoretical [19] as well as experimental [17, 18] perspectives. Particularly, chalcogenide glass that accounts for both cubic–quintic and septic nonlinearities has been subjected to intense investigations [17, 22]. The nonlinear coefficient of chalco-
genide is quite larger than silica and is of the order of $10^3$. Thus it can reduce the threshold level considerably. Yosia et al. have reported the formation of non-overlapping bistable states influenced by a phase shifted cubic-quintic grating. It is noteworthy to mention that unlike Kerr nonlinearity driven bistability, it offered two completely different approaches to switch into the high state. Recently, Yosia et al. have demonstrated the soliton switching in nonlinear FBG with higher order nonlinearity $^{13}$. These kinds of studies ensure that higher order nonlinearities when properly exploited can play a remarkable role in the next generation signal processing devices.

Having discussed concisely the general aspects of the device, we now wish to stress the importance of $\mathcal{PT}$-symmetry in the current generation optical devices. In the context of optics, the practicality of inherent loss in any functional device was not considered by the scientific community for many years. But with the advent of parity-time ($\mathcal{PT}$) symmetry concept, these losses are no longer considered to be detrimental by virtue of the delicate balance between amplification and attenuation in the system as in the case of $\mathcal{PT}$-symmetric couplers, $\mathcal{PT}$-metamaterials, $\mathcal{PT}$-microring laser, $\mathcal{PT}$-gratings, $\mathcal{PT}$-laser cavities, etc. Though this notion traces its origin to the field of quantum mechanics way back in 1998, the translation of the concept on to photonic platform paved the way for its major theoretical advancements and the first experimental realization by Rüter et al. on a LiNbO$_3$ waveguide $^{40}$. $\mathcal{PT}$-symmetric photonicics is regarded as the booming field in the last decade or so thanks to some unconventional features possessed by these devices ranging from broadband unidirectional invisibility, coherent perfect absorption, controllable bidirectional laser emission and so forth, which are not viable in the perspective of existing systems. Driven by the luxury that refractive index, gain and loss coefficients can be manipulated at ease, there is an increasing number of contributions in the literature dedicated to potential applications of $\mathcal{PT}$-symmetric systems namely optical isolators, lower power optical diodes, signal processing devices, single mode lasing cavities, soliton switches, and logic devices.

To realize a $\mathcal{PT}$-symmetric periodic structure, it is necessary to maintain an equilibrium between generation and annihilation of photons so that it offers no net amplification. In a nutshell, the complex refractive index should satisfy the condition $n(z) = n^*(-z)$. In analogy to its quantum counterpart, optical $\mathcal{PT}$-symmetric media exhibit phase transition at the so called exceptional point. Hence the operation of the $\mathcal{PT}$-symmetric grating is stable below a critical amount of gain and loss and when it is violated, the grating exhibits exponential energy growth or lasing behavior.

Surprisingly, the inclusion of loss to the traditional structures had an affirmative role in its functionality. For instance, contemporary work by Govinderajan et al. on steering dynamics of $\mathcal{PT}$-symmetric coupled waveguides $^{29}$ has marked an immense reduction in the power requirement for switching and huge amplification of pulse power. The interplay between nonlinearity and $\mathcal{PT}$-symmetry has impacted in a fall in intensity of the bistable threshold as reported by Phang et al. $^{31,32,41}$. They have also reported a switching time of 2.5 ps in one of their works, which hints that these systems are well suited to exploit switching and logic operations. $^{34}$ It is worthwhile to mention that $\mathcal{PT}$-symmetric optical devices are undoubtedly far more competent than systems exhibiting no loss or systems with only gain.

As of now, the switching characteristics in $\mathcal{PT}$-symmetric fiber Bragg gratings is briefly studied only with conventional silica grating with third order nonlinearity. Following these works, we here study the role of defocusing quintic nonlinearity combined with self-focusing cubic and septic nonlinearities on switching of $\mathcal{PT}$-symmetric fiber Bragg gratings. We highlight the role of every individual parameter in dictating the bi- and multi-stable phenomena in both the unbroken as well as broken $\mathcal{PT}$-symmetric regimes. The study also includes the formation of gap solitons corresponding to the resonance peaks of transmission curves in the presence of higher order nonlinearities and $\mathcal{PT}$-symmetry. We have also investigated the effects of nonlinearities and $\mathcal{PT}$-symmetry on the spectra of the system in the presence of constant pump power.

The plan of the paper is as follows. Section II describes the necessary mathematical formulation for the system of interest. Sections III and IV give brief explanations of the switching characteristics in the unbroken and broken $\mathcal{PT}$-symmetric regimes, respectively. Section V reveals the gap soliton formation at resonance peaks. Section VI elucidates the switching characteristics of the system in the presence of constant pump power. Finally in Sec. VII we discuss the significance of the results.

II. THEORETICAL MODEL

We consider a $\mathcal{PT}$-symmetric fiber Bragg grating of period $A$ inscribed on the core of a fiber of refractive index $n_0$ and length $L$. The nonlinearity of the fiber is not merely restricted to cubic nonlinearity but it also takes account of the quintic and septic nonlinearities. The complex refractive index distribution $(n(z))$ profile that describes such a $\mathcal{PT}$-symmetric system is mathematically written as $^{16}$

\begin{equation}
    n(z) = n_0 + n_{1R} \cos \left( \frac{2\pi z}{A} \right) + i n_{1I} \sin \left( \frac{2\pi z}{A} \right) + n_2 |E|^2 + n_4 |E|^4 + n_6 |E|^6.
\end{equation}

The strength of modulation parameter is defined by $(n_1)$, which has both real $(n_{1R})$ and imaginary parts $(n_{1I})$ and the imaginary term stands for gain $(+n_{1I})$ or...
loss \((-n_{1f})\) dictated by the so called \(PT\)-symmetric potential, and the last three terms signify self focusing \((n_2, n_6 > 0)\) and self defocusing \((n_4 < 0)\) nonlinearities. The transverse electric field \(E(z, t)\) inside the FBG is the superposition of two counter propagating modulated modes which can be written mathematically as

\[
E(z, t) = E_f(z, t) \exp[i(\beta_0 z - \omega_0 t)] + E_b(z, t) \exp[-i(\beta_0 z - \omega_0 t)],
\]

where the envelope functions \(E_f(z, t)\) and \(E_b(z, t)\) that are used to describe the electric fields in the forward and backward directions obey the slowly varying envelope (paraxial) approximation (SVEA). The propagation constant of the fiber without grating is given by \(\beta_0 = 2 \pi n_0 / \lambda_0\), where \(\lambda_0\) is the free space wavelength. The Bragg wavelength of the grating is expressed as \(\lambda_b = 2 \pi n_0 \Lambda\). Practically, the Bragg wavelength is taken in the telecommunication regime, which lies at 1.55 \(\mu m\). But it can be chosen anywhere from visible region to infrared by suitably altering the grating period (\(\Lambda\)). Note that the typical values of \(\Lambda\) for a short FBG can vary from 200 nm to 800 nm.

The coupled mode equations, which describe the light propagation in the proposed system, are given by [24, 46, 47]

\[
i \frac{\partial E_f}{\partial z} + \frac{n_0}{c} \frac{\partial E_f}{\partial t} + (k_0 + g_0) \exp^{-i 2 \delta_0 z} E_b
+ \gamma_0 (|E_f|^2 + 2|E_b|^2) E_f
- i \Gamma_0 (|E_f|^4 + 6|E_f|^2|E_b|^2 + 3|E_b|^4) E_f
+ \sigma_0 (|E_f|^6 + 12 |E_f|^4|E_b|^2 + 18 |E_b|^4|E_f|^2 + 4|E_b|^6) E_f = 0,
\]

\[
-i \frac{\partial E_b}{\partial z} - \frac{n_0}{c} \frac{\partial E_b}{\partial t} + (k_0 - g_0) \exp^{i 2 \delta_0 z} E_f
+ \gamma_0 (|E_b|^2 + 2|E_f|^2) E_b
- i \Gamma_0 (|E_b|^4 + 6|E_f|^2|E_b|^2 + 3|E_f|^4) E_b
+ \sigma_0 (|E_b|^6 + 12 |E_f|^4|E_b|^2 + 18 |E_b|^4|E_f|^2 + 4|E_f|^6) E_b = 0.
\]

We adopt the well-known transformation \(E_{f,b} = A_{f,b} \exp(\pm i \delta_0 z)\) and further consider the synchronous approximation (SVEA) to obtain the time independent equations [15]. Hence, the governing equations for the propagation of continuous waves (CW) become

\[
i \frac{d A_f}{dz} + \delta_0 A_f + (k_0 + g_0) A_b + \gamma_0 (|A_f|^2 + 2|A_b|^2) A_f
- \Gamma_0 (|A_f|^4 + 6|A_f|^2|A_b|^2 + 3|A_b|^4) A_f
+ \sigma_0 (|A_f|^6 + 12 |A_f|^4|A_b|^2 + 18 |A_b|^4|A_f|^2 + 4|A_b|^6) A_f = 0,
\]

\[
-i \frac{d A_b}{dz} + \delta_0 A_b + (k_0 - g_0) A_b + \gamma_0 (|A_b|^2 + 2|A_f|^2) A_b
- \Gamma_0 (|A_b|^4 + 6|A_b|^2|A_f|^2 + 3|A_f|^4) A_b
+ \sigma_0 (|A_b|^6 + 12 |A_b|^4|A_f|^2 + 18 |A_f|^4|A_b|^2 + 4|A_f|^6) A_b = 0.
\]

The detuning parameter in the coupled equation is given by \(\delta_0 = (2 \pi n_0) \left( \frac{1}{\lambda_0} - \frac{1}{\lambda_0} \right)\). The stop band of the grating is expressed as \(|\delta_0| \leq k_0\), where \(k_0\) is the strength of coupling between the oppositely traveling fields. Within this band, no propagating modes are supported by the grating and so the light transmission is prohibited [3]. The coefficients of coupling, gain/loss, cubic, quintic, and septic nonlinearities are given by [24, 47]

\[
k_0 = \pi n_{1R}/\lambda_0, \quad g_0 = \pi n_{1L}/\lambda_0, \quad \gamma_0 = 2 \pi n_2/\lambda_0, \quad \Gamma_0 = 2 \pi n_3/\lambda_0, \quad \sigma_0 = 2 \pi n_6/\lambda_0.
\]

From the fundamentals of \(PT\)-symmetry, it is well known that the system is said to be in the unbroken regime if \(k_0 > g_0\). On the other hand, if \(g_0 > k_0\) the system is set to operate in the broken regime. The condition in which \(k_0 = g_0\) is called singularity or the exceptional point.

The following transformation is adopted [46] to obtain normalized coupled mode equations

\[
\frac{1}{z} \frac{d u}{dz} + (k + g) v + \gamma \left( |u|^2 + 2|v|^2 \right) u - \Gamma \left( |u|^4 + 6|u|^2|v|^2 + 3|v|^4 \right) u
+ \sigma \left( |u|^6 + 12 |u|^4|v|^2 + 18 |v|^4|u|^2 + 4|v|^6 \right) u = 0,
\]

\[
\frac{1}{z} \frac{d v}{dz} + (k - g) u + \gamma \left( |v|^2 + 2|u|^2 \right) v - \Gamma \left( |v|^4 + 6|u|^2|v|^2 + 3|u|^4 \right) v
+ \sigma \left( |v|^6 + 12 |u|^2|v|^2 + 18 |v|^2|u|^4 + 4|v|^6 \right) v = 0.
\]

The normalized parameters are given by

\[
\delta = \delta_0 z_0, \quad k = k_0 z_0, \quad g = g_0 z_0, \quad \gamma = \gamma_0 P(0) z_0, \quad \Gamma = \Gamma_0 P(0) z_0, \quad \sigma = \sigma_0 P(0) z_0.
\]

Further output intensity can be defined as \(P_1(L) = |u(L)|^2\). These coupled mode Eqs. (9) and (10) are solved by implicit Runge-Kutta fourth order method with the following boundary conditions [23]

\[
u(0) = u_0, \quad v(L) = 0.
\]

Before proceeding to analyze the switching characteristics in detail, we now investigate the role of nonlinearity and the gain/loss coefficient on the photonic band gap of the device. The existence of such a gap is a typical characteristic of any feedback structure and it is already discussed in the context of conventional FBG and \(PT\)-symmetric cubic FBG in the literature [20, 46, 47].
mathematical expressions for the dispersion relation of a highly nonlinear $\mathcal{PT}$-symmetric FBG can be found by substituting the continuous wave solution into the normalized coupled equations given by (9) and (10). The wave solutions represent $u$ and $v$ in terms of forward and backward wave amplitudes $\psi_{1,2}$ along the length of propagation of the incident field inside the grating and they are given by

$$u = \psi_1 \exp \frac{iq\xi}{\Delta n}, \quad v = \psi_2 \exp \frac{iq\xi}{\Delta n}. \quad (13)$$

Here $\psi_1$ and $\psi_2$ are assumed to be real constants. The ratio of $\psi_2/\psi_1$ is assumed as a new parameter $f$. The sum of $\psi_1^2$ and $\psi_2^2$ gives the total power ($J$) of the propagating wave. The relation between $\psi_{1,2}$ and $J$ is written as

$$\psi_1 = \sqrt{\frac{J}{1 + f^2}}, \quad \psi_2 = \sqrt{\frac{J}{1 + f^2} f}. \quad (14)$$

Hence one obtains the nonlinear dispersion relation between $q$ and $\delta$ by using Eqs. (13) and (14) in (9) and (10) as

$$q = -\frac{k}{2f} (1 - f^2) + \frac{g}{2f} \left(1 + f^2 - \frac{\gamma J}{2} \right) \left(1 - f^2\right) +$$

$$\Gamma f^2 \left(1 - f^2\right) + \frac{3\sigma J^3}{2} \left(1 + f^2 + 2f^2 - 2f^4\right), \quad \delta = -\frac{k}{2f} \left(1 + f^2\right) + \frac{g}{2f} \left(1 - f^2 - \frac{3\gamma J}{2}\right) +$$

$$2\Gamma f^2 \left(1 + f^2\right) - \frac{5\sigma J^3}{2} \left(1 + \frac{3f^2}{(1 + f^2)^2}\right). \quad (15)$$

From Figs. (1a) to (h), one can visualize two branches in the dispersion curve admitted by Eqs. (15) and (16). These two branches correspond to the normal dispersion regime ($f > 0$) and the anomalous dispersion regime ($f < 0$). It should be noted that the dispersion curve pertaining to linear and cubic nonlinearity has already been discussed in Refs. [46, 47] and so we here focus only on quintic and septic nonlinearities. In the presence of quintic nonlinearity, a loop is formed at the lower branch of the $q$ vs $\delta$ curve. Any increase in the value of $\delta$ increases the size of the loop and vice-versa. The loop disappears at lower values of $\Gamma$ as a result of per-}

III. TRANSMISSION PROPERTIES IN UNBROKEN $\mathcal{PT}$-SYMMETRIC REGIME

The switching characteristics of the $\mathcal{PT}$-symmetric FBG have been investigated in the recent work by Liu et al. which reveals that with an increase in gain/loss coefficient the threshold of switching gets higher [15]. Even though this outcome is an undesired one, additional degrees of freedom offered by the inclusion of $\mathcal{PT}$-symmetry should not be taken lightly. The additional features of $\mathcal{PT}$-symmetry need to be retained but not at the cost of the threshold. On the other hand, we find that the inclusion of higher order nonlinearities in the conventional FBG aids in the reduction of the threshold. This is the factor that drove us to study the switching characteristics of a highly nonlinear FBG under $\mathcal{PT}$-symmetric notion. Before we proceed to study the individual effects in detail, we portray the combined effects of $\mathcal{PT}$-symmetry with cubic-quintic-septic nonlinearities on the switching
FIG. 1. (Color online) Nonlinear dispersion curves plotted at $J = 2.5$ for a FBG with $k = 4$. The top panels represent the relation between $q$ and $\delta$ in the presence of cubic-quintic nonlinearities ($\gamma = 2, \Gamma = 4, \sigma = 0$) and the bottom panels correspond to $q$ vs $\delta$ curve in the presence of cubic-quintic-septic nonlinearities ($\gamma = 2, \Gamma = 4, \sigma = 2$). Figures (a) and (h) illustrate the existence of photonic band gap in the absence of $PT$-symmetry. Figures (b) and (g) depict the narrowing of band gap in the unbroken $PT$-symmetric regime ($g = 2$). Figures (c) and (f) are plotted at the exceptional point ($g = 4$). Figures (d) and (e) show the novel dispersion curves in the broken $PT$-symmetric regime ($g = 5$).

FIG. 2. (Color online) (a) Schematic sketch showing different branches of a typical bistable curve with their switch-up (marked as B) and switch-down intensities (marked as E). Here the two stable states are indicated by curves AB and DE and the unstable branch is indicated by BE and (b) Comparison of optical bistable (multistable) characteristics of the unbroken $PT$-symmetric FBG device under different nonlinear regimes for $L = 1, k = 3, g = 1.5$, and $\delta = 0$.

A. Effect of length and coupling coefficient

To understand the role of length of the fiber grating and the strength of coupling coefficient, we first consider a simple case in which only the cubic-nonlinearity is present ($\Gamma = \sigma = 0$). Figure 3(a) shows the input-output characteristics at $g = 1.5, k = 3, \delta = 0$ for three different values of the device length $L = 1, 1.25$, and 1.5. When the length is shorter ($L = 1$), the effective feedback to the system gets reduced and therefore we observe only two stable states in the output with a switch up intensity of 2.603 for $L = 1$. Below this length, the feedback is not sufficient to observe bistability when $k = 3$. The switching up intensity between the three curves featured a slim difference but an increase in $L$ severely influences the hysteresis width. The differences between the

threshold. To do so, numerical simulations were carried out with device parameters $L = 1, k = 3, g = 1.5$, and $\delta = 0$ [38]. The switch-up intensities of cubic, quintic and septic nonlinearities were found to be descending in the order 2.60, 1.77, 1.13 (see Fig. 2(b)) and the corresponding values of switch down intensities are 0.99, 0.49, 0.32 respectively. Such a dramatic decrease in the switch-up intensity and the threshold is provided by the inclusion of higher order nonlinearities alongside the $PT$-symmetry. Thus our system can uniquely combine the pros of the individual systems without imposing any impairments.
switch-up and switch-down intensities for the three different lengths are 1.51, 1.99, 2.28 (approximately). The upper stable branch gets flatter at higher values of length. The strength of the coupling also influences the shape and width of the hysteresis curve. At the given length \((L = 1)\), the lower values of \(k\) results in insufficient feedback to create a bistable state. In the simulations, we fix \(L = 1, g = 1.5, \delta = 0\) and for three different values of \(k\) the bistability curves are plotted in Fig. 3(b). When \(k = 2.5\), the switch up intensity increases to 2.12 and further it goes to 2.603 at \(k = 3\). From this, it is very clear that \(k\) not only intensifies the switch up intensity value but it has a combined influence alongside the device length in increasing the feedback to the system. Hereafter, throughout this paper, the length and the strength of the coupling are fixed at \(L = 2, k = 4\) as it is practically feasible to tune the values of gain/loss coefficient and the detuning parameter with an external control rather than the device length and the inherent coupling coefficient. Note that in view of practical realization, the value of coupling parameter \(k\) (which can be in the range of 1 to 10 cm\(^{-1}\)) goes hand in hand with the length of the grating varying from 1 mm to 10 cm and the optimum value of \(kL\) can attain the values from 1 to 10 \([19]\).

\[\begin{align*}
\text{(a)} & \quad P_1(L) \quad P_1(L) \\
\text{(b)} & \quad P_1(L) \quad P_1(L)
\end{align*}\]

FIG. 4. (Color online) Output intensity \(P_1(L)\) as a function of input intensity \(P(0)\) in the unbroken \(PT\)-symmetric cubic regime \((\Gamma = \sigma = 0)\) of the FBG at fixed values of \(g = 3.75\). Figure (a) is plotted for five different values of detuning parameter at \(\gamma = 1\). Figure (b) is simulated for three different values of cubic nonlinear coefficient at \(\delta = 0\).

B. Role of Kerr (cubic) nonlinearity and detuning parameter

With a clear-cut idea on the role of length and strength of the coupling obtained from the previous section, we directly proceed to study the effect of cubic nonlinearity at a given value of gain/loss coefficient and other device constraints. The importance of the detuning parameter \((\delta)\) on the bistable phenomenon is pointed out graphically in Fig. 4(a). If the signal wavelength is away from the Bragg wavelength, it is said to be detuned and depending on whether it is longer or shorter than the Bragg wavelength, it is designated as negative or positive detuning, respectively. Compared to the operation at the synchronized wavelength (refer the plot when \(\delta = 0\)), detuning in the shorter wavelength reduces the threshold and width of the hysteresis whereas the negative detuning increases the threshold and width of the hysteresis. The switch up intensities for different values of \(\delta = -1, -0.5, 0, 0.5, 0.75\) at \(g = 3.75\) are given by 3.041, 2.413, 1.777, 1.145, and 0.8422, respectively. The series of values in descending order representing the switching intensities look deceiving that one may intend to reduce the threshold by increasing the detuning further. But this will detune the system outside the band edges and hence results in insufficient feedback to create any bistable feature.

The nonlinear parameter of the fiber purely depends on the concentration of the dopant added to the intrinsic one. Hence we can have a variety of silica fibers possessing different values of third order nonlinearity. To elucidate the role of nonlinearity numerically, we set the parameters as \(g = 3.75\) and \(\delta = 0\) and vary \(\gamma\) from 1 in steps of 0.2. As expected, higher the value of nonlinearity, lesser the intensity required to switch between the stable states. In Fig. 4(b), the corresponding values of switch up intensity for \(\gamma = 1, 1.2, 1.4\) are measured as 1.777, 1.481, and 1.269, respectively. Their corresponding switch down intensities are found to be 0.612, 0.509, and 0.436.

\[\begin{align*}
\text{(a)} & \quad \Gamma = 0.6 \quad \Gamma = 1 \quad \Gamma = 1.4 \\
\text{(b)} & \quad \delta = -1.5 \quad \delta = 0 \quad \delta = 1.5
\end{align*}\]

FIG. 5. (Color online) Illustrations of the variations in the output intensity \(P_1(L)\) against input intensity \(P(0)\) of an unbroken \(PT\)-symmetric FBG. The top panels correspond to the cubic-quintic nonlinear regime \(\gamma = \Gamma = 1, \sigma = 0\), while the lower panel features FBG with cubic-quintic-septic nonlinearities \((\gamma = \Gamma = \sigma = 1)\). Figures (a) and (b) show the variation with respect to \(\Gamma\) and \(\delta\), respectively. The bottom panels represent the same for the septic case. The figures on the left and right are plotted at \(g = 2\) and \(g = 3.5\), respectively.
C. Combined effects of cubic-quintic nonlinearities

To illustrate the effect of cubic-quintic nonlinearity on the switching, we set $g = 2$, $\delta = 0$, and $\gamma = 1$ (see Fig. 5(a)). When $\Gamma = 0.6$, the switch up intensity is very high at 2.76 and the hysteresis width is quite large. When $\Gamma$ is increased to unity, the intensity reduces to 2 with a slight reduction in switch down intensity from 0.41 to 0.29. The other key difference between the two curves is that, at $\Gamma = 1$, there are more stable branches than at $\Gamma = 0.6$. The switching intensities between the adjacent stable branches also get reduced. More stable states are visible in Fig. 5(a) when $\Gamma$ is increased to 1.4. The reduction in the switch up intensity, as well as hysteresis width, is analogous to the cubic nonlinearity case and so the thumb rule to reduce the switch up intensity is straightforward to pronounce, that is choose a material with higher nonlinearity regardless of the regime in which it is operated. But the detuning has a dissimilar influence on switch-up intensity compared to the cubic effect. In the presence of cubic nonlinearity alone, the intensity falls off in the positive detuning regime whereas in the presence of an additional defocusing (quintic) nonlinearity, the intensity decreases when operated in the negative detuning regime as seen from Fig. 5(b). The values of switch-up intensities between second and third stable states for $\delta = 1.5$, 0, $-1.5$ are 2.45, 2.059, and 1.539, respectively. These values are measured at $g = 3.5$ and $\gamma = \Gamma = 1$. Interpretation of these outcomes implies that when we include the higher order nonlinearities to the system without imposing any changes to the other parameters, multistable states are observed in its input-output characteristics. These multistable states can be employed in $n$ level pulse amplitude modulation (PAM) scheme to improve the quality of reconstructed signal provided that the intensity of the regenerated signal is stationed in one of these stable states [30]. Compared to binary modulation scheme, PAM-4 offers two times larger transmission capacity. Hence FBG with higher order nonlinearities in the $PT$-symmetric unbroken regime can be used in the all-optical short-haul communication networks.

D. Effect of cubic-quintic-septic nonlinearities

The number of stable states and the threshold of switching in our system can be controlled with ease by carefully adjusting the system parameters. Theoretically, higher order ($2^n$) modulation schemes ($n > 3$) can further improve the transmission capacity in short-haul communication network. If such schemes are commercially feasible in the near future, then chalcogenide based FBG with septic nonlinearity can play a key role to setup $n$-PAM signal regenerators, since it admits more number of stable states. The thumb rule stated in the previous section holds good even in the presence of septic nonlinearity i.e., higher the septic nonlinear coefficient lower the switch-up intensity required for switching and the width of the hysteresis also decreases with increase in $\sigma$ as shown in Fig. 5(c). The switch-up powers between the first stable branch and the second branch for three different values of $\sigma = 0.6, 0.8, and 1.2$ are measured as $1.598, 1.393, and 1.167$, respectively. The increase in the number of stable branches at larger nonlinear coefficients is also similar to the cubic-quintic case. There is no big difference between the switch-down power between the first and second branches when $\sigma$ is varied. But the difference keeps mounting for the succeeding branches on the top of the first bistable curve at higher values of $\sigma$ as evident from Fig. 5(c). In Fig. 5(d) the switch up intensity at $\delta = 0$ is measured as 1.384. Since the septic nonlinearity is a focusing effect, operating the device in the positive detuning regime decreases the switch up intensity (0.6848 at $\delta = 1.5$), whereas in the negative detuning regime the switch up intensity is increased (1.7 at $\delta = -1.5$). So we conclude that the role of detuning in power reduction merely depends on the nature of the higher order nonlinearities whether it is self-focusing or defocusing.

E. Role of gain/loss parameter on various nonlinearities

It has been reported that any increase in the value of gain/loss coefficient ($g$) increases the intensity levels required to switch between the stable states in the presence of cubic nonlinearity [35, 45, 48]. But this is true
only for a certain range of $g$ values. For the values of $g$, closer to the value of $k$ it results in the decrease of the switching intensities as in Fig. 6(a). Next, we look into the effect of $g$ in the presence of higher order nonlinearities. When $g$ is increased gradually in the presence of quintic nonlinearities without violating the unbroken $\mathcal{PT}$-symmetric conditions, the switch-up intensity builds-up to 2.058 ($g = 2.25$) via 1.933 ($g = 1.75$), and 1.702 ($g = 1$) as seen in Fig. 6(b). It does not return to the lower branch at the same intensity when the input intensity is decreased. It sustains in the same branch till the input intensity is reduced below 0.332, 0.26, and 0.19, respectively, for the above mentioned values of $g$. There is a marginal decrease in the switch-up intensity between the first and second stable branches in the above mentioned values.

Figure 7(c) exemplifies the role of gain/loss parameter on the system that incorporates all the higher order nonlinearities ($\gamma = \Gamma = \sigma = 1$) at the Bragg wavelength for three different values of $g=1, 2, 2.25$. The switch-up intensity starts to ascend in the order 1.087, 1.261, and 1.435 and the corresponding values of switch down intensities are measured to be 0.1154, 0.1776, 0.26. It is very obvious from these studies in the unbroken regime that the desired bistable or multistable curves can be manipulated at ease by judiciously adjusting the imaginary part of the complex refractive index.

The phase shifted gratings are well known for exhibiting low intensity switching behaviors compared to other grating structures [5, 51]. The $\mathcal{PT}$-symmetric FBGs can exhibit such a low intensity switching if the direction of incidence of the input pulse is reversed which is not at all feasible in a conventional FBG, since it exhibits same bistable and multistable behaviors in both directions. Thus the switching phenomenon in a $\mathcal{PT}$-symmetric FBG can be termed as nonreciprocal switching in the sense that it can have an entirely different switching dynamics for left and right incidences.

It is noteworthy to mention at this juncture that the parameter $g$ has a dual role in controlling the switching intensities of the system. By dual role we mean that the device exhibits distinguishable OB/OM curves for the same value of $g$ and other system parameters except for left and right light incidence directions. For instance, there is a weak bistable curve formation at $g = 3.75$ in Fig. 7(a) for the right incidence. Moreover, the switching intensities decreases with an increase in $g$ for the right incidence in contrast to the left light incidence. This sort of dual nature of parameter $g$ on the switching intensities persists even in the presence of quintic nonlinearity as shown in Fig. 7(b) and we can observe more than two stable states for input powers less than unity. Such a formation of low power multistable states was not observed in Fig. 6(a). This kind of inverse relationship between the parameter $g$ and the switching intensities and the formation of multistable states at low value of input intensities lasts even with the addition of septic nonlinearity to the system as seen in Fig. 7(b). The reason for such a behavior is apparent from the fact that the increase in $g$ suppresses the absorption of the forward field intensity for right incidence and vice versa.

### IV. BROKEN $\mathcal{PT}$-SYMMETRIC REGIME

#### A. Influence of gain/loss parameter

At the final stage of drafting this manuscript, we found an article which claims that the OB/OM curves cannot occur in the broken $\mathcal{PT}$-symmetric regime [48]. Nevertheless, it has been recently revealed that the broken $\mathcal{PT}$-symmetric regime supports a new type of gap solitons which was further termed as dark gap solitons along with exhibiting novel optical bistability [52]. Hence to analyze the issue further, we look at the switching characteristics of the $\mathcal{PT}$-symmetric FBGs with higher order nonlinearities for the left light incidence. To demonstrate the role of $g$ in the broken regime, the detuning parameter is fixed at $\delta = 0$ and in the first case we neglect the higher order nonlinearities for the sake of simplicity so that only the cubic nonlinearity has a significant role. When $g = 5.2$ and $\gamma = 1.5$, the system admits a desirable bistable curve with the switch up intensity around 0.7443 and the corresponding switch down intensity is measured to be 0.5924 (see Fig. 8(a)). If the value of $g$ is reduced with no changes to the other settings, a significant amount of increase in the switch up intensity is observed. In the broken (cubic) $\mathcal{PT}$-symmetric regime, if the value of $g$ is far from the singularity condition, the gain of the system is enhanced rather than absorption within the medium and it must be stressed that this is the much anticipated outcome in the context of any $\mathcal{PT}$-symmetric regime.
FIG. 8. (Color online) Plots depicting the novel optical bistability in a broken P$PT$-symmetric FBG for different values of $g$ at $\delta = 0$ for left incidence. Here the output and the input intensities of the system are represented by the variables $P_1(L)$ and $P(0)$ respectively. Figure (a) is simulated in the presence of cubic nonlinearity alone ($\gamma = 1$, $\Gamma = \sigma = 0$). Figures (b) and (c) represent the role of $g$ in the presence of cubic-quintic nonlinearities ($\gamma = \Gamma = 1$, $\sigma = 0$). Figure (d) is plotted in the presence of cubic-quintic-septic nonlinearities ($\gamma = \Gamma = 1$, $\sigma = 0.6$), respectively.

optical system. Also, from Fig. 8(a), we observe that the value of gain/loss coefficient plays a vital role in deciding the range of intensities over which output state should remain either in the upper branch or the lower branch of the bistability curve. When $g$ is closer to $k$ ($g = 4.7$) the output state remains in the lower branch for a larger range of input intensity. However, if the gain/loss coefficient value is far away from $k$ ($g = 5.2$), it switches to the upper stable state at relatively lower intensities. With a further reduction in the value of gain/loss parameter ($g$), the system admits multistability at the same input power. This gives an overview of the optimum choice of $g$ parameter to get the desired bi- or multi-stability.

We further portray the effect of variation of parameter $g$ when the quintic nonlinearity is added to the system. Unlike the previous case, the system exhibits more than two stable states even at higher values of gain/loss coefficient. A decrease in $g$ at a given set of values of nonlinearities $\gamma = 1$ and $\Gamma = 1$ brings about notable changes in the switch-up and switch down power levels. The number of stable states in the input-output characteristics of the device decreases with an increase in the value of $g$. Thus $g$ serves as an additional degree of freedom to control the number of stable states for a given set of device parameters. A phenomenal outcome of the system is that it admits a ramp like first stable output state for certain values of $g$ as seen in Fig. 8(b). This is yet another critical outcome of our investigation. These kinds of ramp like bistable states are already observed in plasmon resonance structures [53, 54], graphene based structures [55, 56], silicon waveguide resonators [57] and plexcitonic systems [58]. But this is the very first time such a OB/OM is observed in a $PT$-symmetric FBG device, thanks to the judicious balance between the gain and loss. Analyzing Fig. 8(b) further, one can observe that the range of input intensities over which this ramp behavior is observed decreases when $g$ is reduced. When operated at $g = 4.7, 5.65$, and $6.85$ the system concedes a step like response after a sharp transition from its initial value (see Fig. 8(c)). It is important to note that such a mix of stable states resembling a ramp and step has previously been observed in a complex nano-dimer with a semiconductor quantum dot [58]. These kinds of bistable states with flat slopes can be explored in the construction of signal regenerators [3]. Also, the difference between the two power levels keeps mounting with increase in $g$ which inherently means that the value of $g$ plays a central role in dictating the width of the hysteresis loop. This implies that the same device can be exploited for building applications like switching with low hysteresis width or all optical memories with larger widths, simply by tuning the value of $g$ which is very much feasible compared to other device parameters. The effect of $g$ on the switch-up and switch-down powers persists even at higher values of quintic nonlinearity but the only difference being the increase in the number of stable states as an effect of dominant self-defocusing nonlinearity over the focusing one.

We next consider a condition where all the nonlinearities are taken into account ($\gamma = \Gamma = 1$, $\sigma = 0.6$) and $g$ is varied to find its impact on the switching operation. Similar to the previous case, the system exhibits multiple
nonlinearities (Figure (c) is plotted in the presence of cubic-quintic-septic nonlinearities). Figure (a) looks more or similar to the one that we observed in the previous one and the first stable branch is characterized by larger hysteresis width. The bistability plots in this regime thus give a conclusive evidence that the value of $g$ parameter has a central role in deciding the intensity at which the system switches between the first stable state and the second one shown in Fig. 8(d). At higher values of gain/loss parameter ($g = 5.2$), the second stable state is preceded by ramp like stable state. On the other hand, if the value of $g$ is fixed at 4.6 and the intensity is tuned from zero, there is a sharp increase in the output intensity from zero to an intensity slightly greater than unity. Following the sharp transmission, the output intensity is steady over a large range of input intensities. In the plot we observe that multiple stable branches start to emanate at higher intensities. The width of each stable branch is lower than the previous one and the first stable branch is characterized by larger hysteresis width. The bistability plots in this regime thus give a conclusive evidence that the value of the switching intensity is inversely proportional to the value of $g$. It is clear that at two different sets of values of $g$ there is a drastic change in the behavior of the system which confirms the fact that the system is quite sensitive to smaller variations of gain and loss.

It is noteworthy to mention that the low intensity switching phenomenon for the right incidence can happen even in the broken $\mathcal{PT}$-symmetric regime. To do so, we simulate the system with the same set of parameters as in Fig. 9(a) for the right incidence. The hysteresis curve in Fig. 9(a) looks more or similar to the one that we observed in Fig. 8(a). Nevertheless, the OB phenomenon is observed at very low input intensities. The switch up intensities for different values of $g = 4.7, 5$, and 5.2 are measured to be 0.23, 0.152, and 0.08, respectively. With the addition of quintic nonlinearity into the system, the device can exhibit both ramp and step like first stable states (see Figs. 9(b) and (c)), resembling the curves obtained in Figs. 8(b) and (c), respectively. For $P_0 < 1$, there is no formation of multistable states as seen in Figs. 8(b) and (c), whereas we obtain multiple stable states with very low switching intensities in Figs. 9(b) and (c) for the same value of $P_0$. The same explanation holds good with the inclusion of septic nonlinearity too as observed in Fig. 9(d). The study of OB/OM curves in the broken $\mathcal{PT}$-symmetric regime for the right incidence thus opens a new avenue for fabricating all-optical switches and memory devices which require ultra low switching intensities with different launching conditions.

### B. Effect of variation of detuning parameter

In the previous sections, the effect of variation of parameter $g$ under various nonlinear regimes and the effect of nonlinearity at fixed $g$ were examined by setting the detuning parameter value to zero which implies that the signal wavelength must be synchronized with the Bragg wavelength. But there is a strong correlation between the switching intensities (both up and down) and finite...
To illustrate the effect of detuning parameter, the value of $g$ is fixed at $g = 5$. Similar to the last section, first we study the effect of detuning in the absence of higher order nonlinearities. If the device is operated at $\delta = -0.25$, we get a wide bistable curve with switch-up power of 1.371 and switch-down power of 0.9397 and if the same system is operated at $\delta = 0$, the switch up intensity reduces to 1.173 as seen in Fig. 10(a). This confirms that operating in the negative detuning regime increases the hysteresis width as well as the intensity required to switch between the two stable states. If one intends to reduce the threshold, the device should be operated closer to the band gap or in the positive detuning regime. For instance, if the detuning value is assigned to be $\delta = 0.25$, the intensity to switch between first stable state and second stable state reduces to 0.9741. This intensity value further reduces to 0.7939 for $\delta = 0.5$ with a simultaneous reduction in the hysteresis width. Thus negative detuning regime favors the device’s preference to remain in the first stable branch for a larger range of input intensities whereas the positive detuning tend to keep the output intensity in the upper stable branch for a larger range of input intensities.

To portray the effect of detuning in the presence of both cubic and quintic nonlinearities, we set $\gamma = 1$ and $g = 5$. The positive detuning parameter increases the hysteresis width and reduces the number of stable states in the presence of quintic nonlinearity. The plots depicted in Fig. 10(b) give a conclusive evidence that an increase in positive detuning inflates the difference between switch-up and down intensities which implies that the intensity inside the device is sufficient enough to keep the output state dormant post the switching from its previous state. Hence it is preferable to use this kind of multistable states in the construction of optical memories rather than switches. If we look at the same system working in the negative detuning regime the switch up intensity (between the first two stable states) keeps on deflating and therefore if one intends to construct switches with a cubic-quintic FBG in the broken regime, it is preferable to have signal wavelength longer than the Bragg wavelength provided that it lies within the stop band. Also the number of stable states increases if the device is operated in the negative detuning regime whereas it decreases when operated in the positive detuning regime. Physically, the memory operation can be accomplished by varying the holding-beam input power. As the input intensity varies, the output intensity can stay in one of the stable branches and not in the unstable branch. The set can be accomplished by raising the input intensity beyond the switch-up threshold, whereas the reset operation can be effected by reducing the input intensity beyond switch-down intensity. So the switch-up and down intensities can serve as read and write bias pulse for the memory operation. The memory holding width can be altered by changing the magnitude of the detuning parameter.

In the presence of second focusing (septic) nonlinearity, the negative detuning regime shows the growth in the dormant stable states whereas in the positive detuning regime the switch down intensity decreases in addition to the shrinkage in the hysteresis loop as seen in Fig. 10(c). Operating at longer wavelengths has a marginal impact in the reduction of switch-up intensity, quite similar to those occurring at the shorter wavelengths in the cubic-quintic case with the only difference being the number of stable states above the dormant states is comparably larger and are desirable for multilevel signal processing applications. Though we present only a few applications here, the system’s ability to retain its memory of the past state for longer period can be subjected to detailed investigation in future to build new all-optical devices.

### C. Impact of nonlinear parameters

The nonlinear parameter purely depends on the type of glass material used. From the application perspective, researchers are left with many materials offering a wide range of nonlinearities from very high to low values. The nonlinearity plays a crucial role in deciding the number of stable states and the intensity required to switch between the stable branches. To illustrate this, we first consider a simple case with only cubic nonlinearity. The first bistable curve starts to emerge at $\gamma = 1.4$ in our numerical simulations for $g = 5$ and the intensity for up switching is 1.256 which further reduces to 1.099 with an increase in $\gamma$ (1.6). Any further increase in the value of $\gamma$ gives rise to multistable states as a consequence of increase in the effective feedback to the system as seen in Fig. 11(a). Earlier in the unbroken regime, we stated a thumb rule to reduce the switching intensity which demands the nonlinear coefficient to be high. From our simulations, we confirm that the rule holds good in the broken regime too.

With the addition of quintic nonlinearity to the above system, it admits multistable states even at lower input intensities as seen in Fig. 11(b). To comprehend the role of quintic nonlinearity, we numerically vary $\Gamma$ at a fixed value of gain/loss coefficient ($g = 5$). At $\Gamma = 1$, when the input intensity is slowly varied from zero, there is a linear increase of output intensity below 1.479 above which it switches to the second stable state. The output intensity again starts to vary linearly in the second stable branch for values between 1.479 and 2.396. Above this intensity, the system switches to the third stable branch. If the intensity is decreased, the switch-down intensity required to swap from third branch to the second is 1.582 and to switch back to its initial state, the input intensity must be reduced further to 0.6803. When $\Gamma$ is increased further, the switching intensity between various stable branches tapers off and new stable states appear within the same input intensity ($P_0 = 2$). For instance, when $\Gamma = 1.5$ the switch up intensity to switch between the first and second stable state is measured to be 1.093. The switch up intensity reduces below unity when the
V. EXISTENCE OF GAP SOLITONS

Soliton formation is a universal phenomenon that can occur in any nonlinear optical structures through a delicate interplay between the group-velocity dispersion and the nonlinearity of the structure. The former tends to disperse the energy of the propagating pulse whereas the later is supposed to counter-balance the effect caused by the former by concentrating the energy of the pulse \[ \sigma \]. Many types of solitons are reported theoretically and experimentally in the regular optical fibers which include dissipative [61], vector [62], polarization domain wall [63], dispersion managed, Raman and paired solitons [60], as well as multisolitons [61], which rely on the above mentioned phenomenon. Among them, gap solitons are a special type of solitons formed in the periodic structures which posses photonic band gap as a consequence of periodic variation in the linear dielectric constant. This periodic variation can often be engineered at ease [65] and hence gap solitons are well suited for applications such as optical buffering [66], optical delay lines [67], distributed feedback pulse generator [68], transmission filters [69], and logic gates [70].

The phenomenon of optical bistability in the feedback system is operated at \( \Gamma = 2 \).

Finally, we present the effect of septic nonlinearity in the presence of both cubic and quintic nonlinearities with simulation parameters as \( g = 5 \), \( \gamma = 1.5 \), \( \Gamma = 1 \) and septic nonlinearity (\( \sigma \)) is varied. A straightforward evidence one can get from Fig. 11(c) is that inclusion of the septic nonlinearity cuts down the required intensity to switch between the stable states and the width of the hysteresis is further reduced when compared to the system with the same simulation parameters in its absence. The septic nonlinearity also boosts the number of stable states similar to the other nonlinear effects discussed already. When \( \sigma = 0.8 \), it supports more than five stable branches with each branch possessing a width narrower than the previous one. When \( \sigma = 0.6 \), a series of multistable states appear and the intensities to jump from the preceding state are 1.44 (1 to 2), 1.809 (2 to 3) and the respective switch down intensities are given by 0.9697 and 1.433 as shown in Fig. 11(c). The width of the upper stable branches drastically reduces. But with suitable adjustment in other device parameters, it is possible to increase the visibility of the upper stable states and hence such novel bistable states can lead to the efficient all-optical signal processing by controlling (output) light with (input) light. Also, a possible experimental realization of the kind of structure envisaged in this paper is to identify a suitable material (preferably chalcogenide glass) which can allow the fabrication of alternate regions of gain (actively doped by erbium) and loss (no dopant by considering the intrinsic loss or dopant with high absorption by chromium element) into it and thereby serving as a PT-symmetric periodic structure.

\[ |u|, |v| \]

\[ P(|u|,|v|) = 0 \]

\[ (0) \]

\[ L \]

FIG. 12. Top panels (a) and (b) illustrate the transmission characteristics of conventional and PT-symmetric FBGs, respectively. Center and bottom panels delineate the stationary gap soliton formation at the different resonance peaks for conventional and PT-symmetric systems, respectively when the parameters are kept as \( L = 1.5 \), \( k = 3 \), \( g = 2 \), \( \delta = 1 \), \( \gamma = 4 \), \( \Gamma = 1 \) and \( \sigma = 2 \). Here the blue solid lined curves indicate the forward field and red lined curves refer to backward field intensities. The total field intensities \( (|u|^2 - |v|^2) \) are drawn by black dashed lines.
$sech^2$ solitons. Secondly they reside within this photonic band gap. The formation of such gap solitons in the presence of Kerr effect in the conventional periodic structure has already been investigated by many authors \cite{11, 67, 68}. Recently, the formation of gap solitons has been explored in $\mathcal{PT}$-symmetric periodic structures and some noteworthy properties were highlighted. In particular, it is shown that these phenomenological $\mathcal{PT}$-symmetric structures can support the interesting formation of dark gap solitons \cite{52}. In this section, our aim is to show that the gap solitons can persist even in the presence of higher order nonlinearities in such $\mathcal{PT}$-symmetric systems. The first nonlinear resonance is observed at very low input intensities for a conventional FBG with $L = 1.5, k = 3, \gamma = 4, \Gamma = 1, \sigma = 2$ and $g = 0$ (see Fig. 12(a)). The plot of forward field intensities against the propagation distance at the first peaks reveals that a bright gap soliton like entity corresponding to the resonance value appears. We can observe that the difference between the forward and backward field intensity is marginal in Fig. 12(c). At sufficiently larger input intensities, a second order soliton like entity is formed at the transmission resonance. The peak power of the forward field distribution curve is enhanced whereas the peak power of the backward field distribution is reduced at the successive transmission peaks as seen in Fig 12(d). In both cases the total power remains constant throughout the propagation length.

When $\mathcal{PT}$-symmetry is included to the system ($g = 1$), these nonlinear resonances can still exist in Fig. 12(b))]. The plot also depicts that the $\mathcal{PT}$-symmetry plays a significant role in altering the peak intensities of these nonlinear resonances. The first and second transmission peaks occur at slightly larger input intensities when compared to the conventional case in Fig. 12(c). Similar to the conventional case, the value of the forward field peak is enhanced and the backward field peak is reduced at the second transmission peak as seen in Fig. 12(f). The plot of total intensity (see Figs. 12(c) and (f)) against the propagation distance also shows a bright soliton like entity unlike the conventional case. Motivated by the formation of unique gap soliton obtained in the unbroken regime, we next intend to examine whether these kinds of nonlinear resonances can occur in the broken regime too in the presence of higher order nonlinearities. Numerical simulations turn out a surprising outcome of localized modes at the band gap which resembles a dark soliton in the broken regime as seen in Fig. 13. The $\mathcal{PT}$-symmetry dictates the value of the dip of the dark soliton like entity as in the case of forward field intensity distribution and total power distribution as seen in Figs. 13(b) and (c). On the other hand, one can observe that the simultaneous existence of bright soliton like entity in the plot of backward field distribution against the propagation distance (see Fig. 13(d)), which further can be regarded as the unique outcome of $\mathcal{PT}$-symmetry in such periodic structures.

VI. NONLINEAR REFLECTION SPECTRUM FOR CONSTANT PUMP POWER

In the previous sections, we elaborated the switching exhibited by the $\mathcal{PT}$-symmetric system under different conditions. But in all the systems discussed previously, switching is achieved via optical bi- (multi-) stability under continuous variation in the pump power ($P(0)$). We can also find that the FBG exhibits another type of switching mechanism in the presence of constant pump power ($P(0)$) as a function of detuning parameter ($\delta$) in the literature in both linear \cite{5} as well as nonlinear regimes \cite{3, 19}. These studies are restricted to only conventional FBGs without gain and loss. Hence we are interested in studying this kind of switching behavior in the presence of $\mathcal{PT}$-symmetry. To do so, we fix $L = 0.5$, $k = 2$ and vary the other parameters for the unbroken $\mathcal{PT}$-symmetric regime, whereas we fix $L = 1$ and $k = 2$ for the broken $\mathcal{PT}$-symmetric regime.

A. EFFECT OF VARIATIONS IN THE NONLINEARITY IN THE UNBROKEN $\mathcal{PT}$-SYMMETRIC REGIME

In the absence of any nonlinearity the reflection spectrum is centered at $\delta = 0$. The spectrum is shifted towards longer wavelength when a cubic nonlinearity is added to the system. With further increase in the cubic nonlinearity parameter ($\gamma$), the spectrum is blue shifted \cite{52}. Since quintic nonlinearity is a self-defocusing non-

![FIG. 13. (color online) Plots of (a) transmission (b) and (c) dark-gap solitons formation in the broken $\mathcal{PT}$-symmetric regime and (d) illustrates the bright-gap soliton like entity in the broken regime. In fig (d) the dotted line corresponds to backward field intensity at the first transmission resonance and the solid line indicate the same at the second transmission resonance. The system parameters are chosen as $L = 1$, $\kappa = 2$, $g = 3$, $\delta = 0$ and the values of nonlinear parameters are taken to be $\gamma = \Gamma = \sigma = 1$.](image-url)
Reflection \( = \Gamma = 0 \) 
\( = 0.5 \) 
\( = 1 \) 
\( = 1.5 \) 
\( = 2 \)

FIG. 14. (Color online) Role of nonlinear coefficients on the reflection characteristics under constant pump power of an unbroken \( \mathcal{PT} \)-symmetric FBG at \( g = 0.5 \). Figure (a) represents the simulated results for different values of quintic nonlinear coefficient (\( \Gamma \)) at \( \gamma = 2 \) and \( \sigma = 0 \). Figure (b) is plotted for different values of septic nonlinear coefficient (\( \sigma \)) at \( \gamma = \Gamma = 2 \).

B. Effect of variations in the gain/loss parameter (\( g \)) on the reflection characteristics under constant pump power of an unbroken \( \mathcal{PT} \)-symmetric FBG. Figure (a) represents the simulated results for different values of \( g \) in the presence of cubic-quintic nonlinearities (\( \gamma = \Gamma = 2 \), \( \sigma = 0 \)). Figure (b) is plotted for different values of \( g \) in the presence of cubic-quintic-septic nonlinearities (\( \sigma \)) at \( \gamma = \Gamma = 2 \).

FIG. 15. (Color online) Effect of variation in the gain/loss parameter (\( g \)) on the reflection characteristics under constant pump power of an unbroken \( \mathcal{PT} \)-symmetric FBG. Figures (a), (b), and (c) are plotted at \( g = 3, 4, 5 \) respectively. Throughout the figure the value of cubic nonlinear coefficient is set to \( \gamma = 1 \). The dotted lines (red) represents the cubic nonlinear regime (\( \Gamma = \sigma = 0 \)). The dashed lines (black) indicate the quintic nonlinear regime (\( \Gamma = 2, \sigma = 0 \)). The solid lines (blue) represents the septic nonlinear regime (\( \Gamma = 2, \sigma = 1 \)).

linearity the spectrum is shifted towards shorter wavelength with increase in \( \Gamma \). In addition to shifting, the amount of light which is reflected by the system is reduced slightly with increase in \( \Gamma \) as seen in Fig. 14(a). The same kind of phenomenon is observed in the presence of an additional self-focusing nonlinearity (septic) but the shifting of the spectrum is towards longer wavelength similar to cubic nonlinearity case (see Fig. 14(b)). Thus we can conclude that the shifting of the spectrum towards longer or shorter wavelength is dependent upon the nature of nonlinearities. Note that the effect of nonlinearity on the side lobes of the spectra (spectrum outside the band edges) is minimal.

B. Effect of variations in the gain/loss in the unbroken \( \mathcal{PT} \)-symmetric regime

In the absence of any \( \mathcal{PT} \)-symmetry and nonlinearities the reflection is maximum around the Bragg wavelength. The wavelength at which the peak reflectivity occurs is varied by the presence of nonlinearities [62]. In Figs. 15(a) and (b) we can observe that the reflection peak is slightly off-centered as a consequence of higher order nonlinearity in the system. Any variation in the gain/loss does not shift the spectrum as in the case of nonlinearity. But, the actual effect of variation in the parameter \( g \) depends on the launching conditions. The band gap is more or less symmetric about the center wavelength in the presence of cubic nonlinearity alone (\( \gamma = 1, \Gamma = 0 \)). For left incidence, the reflectivity is suppressed with increase in the gain/loss. When we look into the system with higher order nonlinearities, the same effect persists. Thus we can conclude that irrespective of the type of nonlinearity, the amount of light reflected is reduced by the gain/loss parameter \( g \) for the left incidence.

C. Unique spectra in the broken \( \mathcal{PT} \)-symmetric regime

In the unbroken \( \mathcal{PT} \)-symmetric regime, the reflection characteristics are found to be more or less flat within the stop band. But in the broken \( \mathcal{PT} \)-symmetric regime, the peak of the reflection occurs at a single wavelength instead of the entire stop band as in the case of Fig. 15(a) which is simulated at \( g = 3 \). Any additional nonlinearities in the form of quintic nonlinearity or septic nonlinearity shifts the peak of the reflection towards longer wavelengths. When \( g = 4 \), the reflection spectrum begins to divide into two and thus we can observe two separate
peaks on either side of \( \delta = 0 \) (see Fig. 16(b)). The first peak corresponds to negative detuning values of \( \delta \) and the second peak corresponds to positive detuning values. The reflected power is not the same in these peaks. More power is accumulated at the peak lying on the shorter wavelength side compared to its counterpart. But closer to \( \delta = 0 \) the reflection is minimum. This kind of behavior is unique because in both conventional FBG and unbroken PT-symmetry the reflection is maximum within the stop band and at the band edges the reflection is minimum. Even more interestingly, when \( g = 5 \) the reflection is completely prohibited in the side lobes unlike the other cases and the reflection is maximum at a single wavelength rather than band of wavelengths. Otherwise this can be called as a lasing like behavior, with large reflected intensity at the lasing wavelength. Moreover, the presence of quintic nonlinearity red shifts the spectrum instead of a blue shift in both Figs. 16(b) and 16(c) unlike the previous cases.

VII. CONCLUSION

In this paper we have presented a detailed study on the optical bi- and multi-stability phenomena as well as nonlinear reflection spectra in a highly nonlinear fiber Bragg grating influenced by PT-symmetry conditions. We have reported unique behaviors such as ramp and step like stable states in the broken PT-symmetric regime which were previously believed to exist only in complex optical structures involving plasmons, graphene etc. We affirm that these states are feasible in a simple FBG device with minimal effort by having a balance between gain and loss of the system. We have also found novel optical bistabilities in the broken PT-symmetric regime which can pave a new way to the low power switches via reversal of the direction of light incidence. This confirms that FBG offers a fertile ground to unearth unique nonlinear functionalities in the PT-symmetry broken regime in addition to the unbroken regime. We have also depicted the existence of both dark and bright soliton like entities at the transmission resonances. We would like to leave an endnote that all the numerical experiments presented here deserve further investigations through practical observations, and this could open the door for new generation of multi-functional optical devices including optical switches and memories. To be regarded as the next generation multi-functional devices, any system must address some of the important criteria like size miniature, reduction in cost, low power consumption, design flexibility and so on and our ramifications rely on the last two aspects i.e., optical switches with low switching intensities and flexibility to set-up the desired application simply by tuning one of the control parameters. In addition our system can play a key role to set-up a new generation of all-optical regenerators employing PAM scheme in the near future as our system admits a large number of stable states with low switching intensities which is an ideal requirement for any all-optical systems.

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