Cosmological evolution in a two-brane warped geometry model

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We study an effective 4-dimensional scalar-tensor field theory, originated from an underlying brane-bulk warped geometry, to explore the scenario of inflation. It is shown that the inflaton potential naturally emerges from the radion energy-momentum tensor which in turn results into an inflationary model of the Universe on the visible brane that is consistent with the recent results from the Planck’s experiment. The dynamics of modulus stabilization from the inflaton rolling condition is demonstrated. The implications of our results in the context of recent BICEP2 results are also discussed.

I. INTRODUCTION

The standard cosmological paradigm, while successful in describing our observable Universe, is plagued with horizon and flatness problems. Moreover, despite being able to explain the large scale structure formation due to some seed fluctuations in our Universe, standard cosmology fails to provide a mechanism that can produce such seed fluctuations. Inflationary models are at present the only way to provide solutions for these shortcomings in standard cosmology [1]. According to this paradigm, the Universe at an early epoch experienced an exponentially rapid expansion for a very brief period due to some apparently repulsive gravity-like force. Such a scenario not only can successfully address the horizon and flatness problems but at the same time, provides a theoretical set up to produce the primordial fluctuations which later may act as a seed for large scale structure formation in the Universe. Amazingly the predicted primordial fluctuations in any inflationary model [2] can be tested accurately through the measurement of temperature anisotropies in the Cosmic Microwave Background Radiation as recently done by Planck experiment [3]. The construction of a viable models for inflation, which are consistent with cosmological observations like Planck experiment, therefore is of utmost importance and is a subject of study of the present work.

Among various models for inflation, the models with extra dimensions have been discussed by many authors [4]. Such models are independently considered in particle phenomenology due to their promise of resolving the well-known naturalness/fine tuning problem in connection with stabilising the mass of Higgs boson against large radiative corrections [5].

In this context, the 5-dimensional warped geometry model due to Randall and Sundrum (RS) [6] is very successful in offering a proper resolution to the naturalness problem without incorporating any intermediate scale other than Plank/quantum gravity scale. The radius associated with the extra dimension in this model (known as RS modulus) acts as a parameter in the effective 4-dimensional theory and from a cosmological point of view, such a modulus can be interpreted as a scalar field which, due to its time evolution, may drive the scale factor of our universe before getting stabilized to a desired value. The well-known methodology to extract an effective or induce theory on a 3-brane from a 5-dimensional warped geometry model is demonstrated in [7] where using the Gauss-Codazzi equation with appropriate junction condition in a two-brane warped geometry model and implementing a perturbative expansion in terms of the

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brane-bulk curvature ratio, the effective Einstein’s equation is obtained on a lower dimensional hypersurface. This eventually results in the form of a scalar-tensor gravity theory in our brane (known as visible brane) [7].

Here we try to explore the role of such a scalar-tensor theory in stabilizing the modulus of the bulk geometry as well as to generate inflation in the visible brane which is consistent with the results obtained by Planck. We show that an effective potential for the modulus field (often called radion) automatically emerges from the construction of the model. The stabilization requirements put further constrains on various parameters of the modulus/scalar potential. After deriving these constraints we study the cosmological evolution in the Einstein frame in presence of such a potential and show that it gives a viable model for inflation with required number of e-folds (to solve the horizon and flatness problem) and also gives a primordial fluctuations which is perfectly consistent with the Planck results. The inflation is shown to end with the modulus attaining its stable value. Hence our set up not only provides a mechanism to stabilise the modulus in the bulk but also provides a viable model for inflation which is consistent with the recent observational results.

The structure of the paper is as follows: in section II, we briefly review the Shiromizu and Koyama set up [7] for the low energy effective gravity in curved branes; in section III, we investigate the constraints on the potential that is necessary for moduli stabilisation; in section IV, we describe the inflationary behaviour in our model and constrain it using the recent result from the Planck experiment; in section V, we briefly comment about the recent BICEP2 results; we end with conclusions in section VI.

II. LOW ENERGY EFFECTIVE GRAVITY IN PRESENCE OF CURVED BRANES

We start with a configuration which contains two 3-branes embedded in a five dimensional $z_2$ symmetric ADS spacetime containing a bulk cosmological constant ($\Lambda_5$). The branes are located at two orbifold fixed points. One has positive tension and is placed at $y = 0$ in the fifth dimension (the ”hidden brane”) while the other has negative tension and is placed at $y = r\pi$ (the ”visible brane”), $r$ being the distance between the two branes.

Next we assume the five dimensional action as [6]:

$$S = \frac{1}{2\kappa^2} \int d^5x\sqrt{-g} \left( R + \frac{12}{l^2} \right) - \sum_{i=1,2} V_i \int d^4x\sqrt{-g^i} + \sum_{i=1,2} \int d^4x\sqrt{-g^i} L^i_{\text{matter}}. \quad (1)$$

Here $\kappa^2$ is the five dimensional gravitational constant, $l$ is the bulk curvature radius which is related to the bulk cosmological constant as $l = \sqrt{-\frac{3}{\kappa^2\Lambda_5}}$. $V_1$ and $V_2$ are the tensions of the hidden and the visible branes. The 5D line element is taken as:

$$ds^2 = e^{2\phi} dy^2 + g_{\mu\nu}(y,x^5)dx^\mu dx^\nu. \quad (2)$$

Fixing the bulk curvature scale to be $L$, we define a parameter $\epsilon = (\frac{L}{l})^2$ and assume $\epsilon << 1$ which is legitimate as the scale of the cosmological evolution in brane is considerably smaller than the Planck’s scale namely the natural scale for the bulk curvature. This ensures that the classical solutions of the effective Einstein’s equation can be trusted. One can perturbatively expand the extrinsic curvature of the brane at fixed $y$ in terms of $\epsilon$. At the zeroth order one retrieves the RS model with the corresponding brane tensions:

$$\frac{1}{l} = \frac{1}{6}\kappa^2 V_1 = -\frac{1}{6}\kappa^2 V_2. \quad (3)$$

The Einstein tensor can be calculated from the given action (see [7] for detail derivation) and in the first order, one can get the effective Einstein’s equation on the visible brane as:

$$G^\mu_\nu = \kappa^2 \frac{T^\mu_\nu}{\Phi} + \kappa^2 \frac{(1 + \Phi)^2}{\Phi} T^\mu_\nu + \frac{1}{l} \left( D^\mu D^\nu \Phi - \delta^\mu_\nu D^2 \Phi \right) + \frac{\omega(\Phi)}{\Phi^2} \left( D^\mu \Phi D^\nu \Phi - \frac{1}{2} \delta^\mu_\nu (D^\alpha \Phi D^\alpha \Phi) \right), \quad (4)$$
where \( \Phi = \exp^{2d_0(x)/l} - 1 \) and \( \omega(\Phi) = -\frac{3}{2l+\Phi} \). Here \( d_0(x) = \int_0^\pi dy e^{\delta(y,x)} \) is the proper distance between the two branes and is the modulus field in the effective 4-dimensional theory. \( T_{1\mu}^\nu \) and \( T_{2\mu}^\nu \) are the respective energy momentum tensors in the hidden and the visible branes.

If we now assume that the two branes are endowed with only cosmological constants i.e \( T_{2\mu}^\nu = \Lambda_2 \delta^\nu_\mu \) and \( T_{1\mu}^\nu = \Lambda_1 \delta^\nu_\mu \) then for a spatially flat FRW metric with scale factor \( a(t) \), the Einstein equations in the visible brane are given by:

\[
3H^2 = \frac{1}{2} \omega(\Phi) \left( \frac{\dot{\Phi}}{\Phi} \right)^2 - 3H \left( \frac{\dot{\Phi}}{\Phi} \right) - \frac{\kappa^2 \Lambda_2}{l} (1 + (1 + \Phi)^2 \left( \frac{\Lambda_1}{\Lambda_2} \right)) \tag{5}
\]

\[
2\dot{H} + 3H^2 = -\frac{1}{2} \omega(\Phi) \left( \frac{\dot{\Phi}}{\Phi} \right)^2 - \frac{\dot{\Phi}}{\Phi} - 2H \left( \frac{\dot{\Phi}}{\Phi} \right) - \frac{\kappa^2 \Lambda_2}{l} (1 + (1 + \Phi)^2 \left( \frac{\Lambda_1}{\Lambda_2} \right)) \tag{6}
\]

where \( H = \frac{\dot{a}}{a} \) is the Hubble parameter. The term

\[
U(\Phi) = -\frac{\kappa^2}{l} \Lambda_2 \left( 1 + (1 + \Phi)^2 \left( \frac{\Lambda_1}{\Lambda_2} \right) \right) \tag{7}
\]

can be interpreted as the potential for the scalar field \( \Phi \) in this model.

### III. CONSTRAINTS ON THE FORM OF THE POTENTIAL

To study the dynamical evolution of our system, it is convenient to write the equations in the Einstein frame which can be obtained using the following conformal transformations:

\[
\begin{align*}
\tilde{g}_{\mu\nu} &= \Phi g_{\mu\nu} \\
\tilde{a}^2 &= \Phi a^2 \\
\tilde{d} \tilde{\tau}^2 &= \Phi d\tau^2
\end{align*} \tag{8}
\]

We use a field redefinition \( \Phi \to \psi \), such that

\[
\left( \frac{d\psi}{d\Phi} \right)^2 = \frac{3}{4} \frac{1}{\Phi^2 (1 + \Phi)} \tag{9}
\]

On solving the above equation we arrive at,

\[
\psi = \pm \frac{\sqrt{3}}{2} \ln \left| \frac{\sqrt{1 + \Phi} - 1}{\sqrt{1 + \Phi} + 1} \right| + c_1
\]

where \( c_1 \) is a constant of integration. We can also write Jordan frame field \( \Phi \) in terms of field \( \psi \) in the Einstein frame as

\[
\Phi = \frac{4\alpha}{(1 - \alpha)^2} \tag{10}
\]

where

\[
\alpha = Ke^{\pm \sqrt{3} \psi} \tag{11}
\]

with \( K = e^{\mp \frac{2}{3} c_1} \).
Potential $V(\psi)$ in Einstein frame is now related to Potential $U(\Phi)$ in Jordan frame as

$$V(\psi) = \frac{U(\Phi)}{2F^2(\Phi)}. \quad (12)$$

We further define two parameters

$$A = \frac{\kappa^2 \Lambda_2^2}{2l}, \quad B = \frac{\Lambda_1}{\Lambda_2}. \quad (13)$$

In terms of the parameters $A, B$ and $K$, $V(\psi)$ has the form:

$$V(\psi) = -A \left( \frac{(1 - Ke^{\pm \sqrt{3}\psi})^4 + (1 + Ke^{\pm \sqrt{3}\psi})^4 B}{16K^2 e^{\pm \sqrt{3}\psi}} \right) \quad (14)$$

Moreover with these transformations, the Einstein equations (5), (6) simplifies to

$$3\dot{H}^2 = \dot{\psi}^2 + 2V(\psi) \quad (15)$$

$$2\dot{H} + 3\dot{H}^2 = -\dot{\psi}^2 + 2V(\psi). \quad (16)$$

There are several restrictions on the form of the potential so that the model can simultaneously address the following issue:

Firstly to achieve the stabilisation of the brane motion, the potential should have a minimum and field value at this minimum should be non-zero to avoid any brane collision. Secondly, the potential should also satisfy the necessary slow-roll conditions to trigger the inflation on the visible brane and finally the spectrum of the primordial fluctuations produced in this case should also be consistent with its recent measurement by Planck experiment. We address these issues one by one to ascertain the viability of the model.

First let us examine the extremum of the potential given by equation (11). The equation $\frac{dV}{d\psi} = 0$ gives the following set of conditions:

$$e^{-2\beta \psi_{min}} = 0 \quad (17)$$

$$Ke^{\beta \psi_{min}} = -1 \quad (18)$$

$$Ke^{\beta \psi_{min}} = 1 \quad (19)$$

$$\psi_{\pm} = \frac{1}{\beta} \ln \left[ \frac{1 + B}{1 - B} \right] \pm \sqrt{\frac{(1 - B)^2 - 1}{(1 + B)^2}} \quad (20)$$

where $\beta = \pm \frac{2}{\sqrt{3}}$. The first and the third conditions result the corresponding $\Phi$ in the Jordan frame to be either infinity or zero. Neither of these are acceptable as they imply infinite or zero separation between the two branes. The second condition is not possible as $K$ is strictly positive. So the acceptable $\psi_{min}$ is given by equation (20). Also it is easy to check that $-1 < B < 0$ is necessary in order to have a real $\psi_{min}$. Now if one further calculates $\frac{d^2V}{d\psi^2}$, one gets

$$\frac{d^2V}{d\psi^2}(\psi = \psi_{min}) = 2\beta^2 \frac{AB}{1 + B}. \quad (21)$$

In order to have a minimum of the potential at $\psi = \psi_{min}$, one further needs $A < 0$. This, together with the condition on $B$, implies that $\Lambda_2$ should be negative and $\Lambda_1$ should be positive which is similar to the Randall-Sundrum set up of warped geometry.
Further, it is easy to show that at the minimum, 

\[ V_{\text{min}} = -\frac{AB}{1+B} < 0. \]  

But the cosmological observations actually is consistent with a de-Sitter Universe. Hence we need to add an uplifting term \( V_1 \) in our potential with the condition such that 

\[ V_1 > \frac{AB}{1+B}. \] 

This feature is similar to the de-Sitter lifting by fluxes in KKLT model \cite{9} where the supersymmetry preserving ADS minima is lifted to a de-Sitter one from the energy of the background fluxes of higher form tensor fields in type IIB string-based \( N = 1 \) supergravity model in presence of brane and anti-brane. Presence of anti-brane breaks the supersymmetry giving rise to a positive definite vacuum energy by compensating the ADS value of the scalar potential at the minimum. The mechanism here is to generate some extra energy from background fluxes which show up in the scalar potential.

We should stress that adding this uplifting term does not disturb the process of radion stabilisation and the subsequent solution of the hierarchy problem.

So the final form of the potential is 

\[ V(\psi) = -A \left( \frac{(1 - K e^{\beta \psi})^4 + (1 + K e^{\beta \psi})^4 B}{16 K^2 e^{2\beta \psi}} \right) + V_1, \] 

where \( A < 0, -1 < B < 0. \)

IV. INFLATION

We now study the inflationary solution induced by the potential as given in equation (24). We define two new variables \( \chi = \sqrt{2} M_{\text{pl}} \psi \) and \( V(\chi) = 2 M_{\text{pl}}^2 V(\psi) \), where \( M_{\text{pl}} \) is the reduced Planck mass. The form of the potential \( V(\chi) \) is shown in figure 1. As \( \chi \) rolls over from the flat part near the region \( \chi = 0 \) towards the minimum at the either side, inflation continues to occur.

The equations (15) and (16) now become,

\[ 3H^2 = \frac{1}{M_{\text{pl}}^2} \left( \frac{\dot{\chi}^2}{2} + V(\chi) \right) \] 

\[ 2\dot{\mathcal{H}} + 3H^2 = \frac{1}{M_{\text{pl}}^2} \left( -\frac{\dot{\chi}^2}{2} + V(\chi) \right) \] 

We further define slow roll parameters as \cite{2},

\[ \epsilon_V = \frac{M_{\text{pl}}^2}{2} \left( \frac{1}{V(\chi)} \frac{dV(\chi)}{d\chi} \right)^2 \] 

and

\[ \eta_V = M_{\text{pl}}^2 \left( \frac{1}{V(\chi)} \frac{d^2V(\chi)}{d\chi^2} \right). \]
Inflation takes place when $\epsilon \ll 1$ and $|\eta| \ll 1$ which are also called the slow-roll conditions. The inflation ends when any one of these conditions breaks down. In figure (2), we show the behaviour of the slow-roll parameters for the potential shown in figure (1). The inflation ends at $\chi_{\text{end}} \sim 5.52 M_{\text{pl}}$ and hence it ends before the scalar field settles down at its minima at $\chi_{\text{min}} \sim 5.61 M_{\text{pl}}$.

The total amount of inflation is measured through the number of efoldings $N$, defined as [2]

$$N = \ln \frac{a(t_e)}{a(t)} = \int_t^{t_e} H dt.$$ (29)

To solve the flatness problem in standard cosmology, we need at least 70 efolds of inflation. For the potential shown in figure (1) and with $\chi_{\text{end}} \sim 5.52 M_{\text{pl}}$, the required initial value of $\chi$ is $\chi_{\text{ini}} \sim 1.02 M_{\text{pl}}$ to achieve the desired number of efolding. The figure (1) clearly depicts that at such a $\chi_{\text{ini}}$, the field is initially displaced slightly from the flat part of the potential. After that it slowly rolls down and one gets enough number of efolds before it finally settles at the minimum of the potential $V(\chi)$.

The relevant observational quantities related to the spectrum of the primordial fluctuations are [2, 3]

$$r \approx 16\epsilon_V,$$
$$n_s \approx 1 - 6\epsilon_V + 2\eta_V,$$
$$A_s \approx \frac{V(\chi)}{24\pi^2 M_{\text{pl}}^2 \epsilon_V},$$ (30)

where $r$ is the tensor to scalar ratio, $n_s$ is the scalar spectral index and $A_s$ is amplitude of the scalar fluctuation. To comply with our purpose all these quantities here must be calculated at the time of Hubble exit $k_s = a_s H_s$. When the scale $k_s$ leaves the Hubble radius, the number of efolding before the end of inflation, $N_*$, is given by
So the quantities $r$, $n_s$ and $A_s$ should be estimated at $N_\star$. The value of $N_\star$ depends crucially on the reheating mechanism. For reasonable inflationary models, one can show that $50 < N_\star < 60$. In our case we take $N_\star = 55$ which is consistent with the Planck’s analysis.

The constraints obtained by the Planck’s measurements are as follows:

\begin{align*}
\frac{r}{n_s} & < 0.11 \\
    n_s & = 0.9603 \pm 0.0073 \\
\ln(10^{10} A_s) & = 3.089_{-0.027}^{+0.024}
\end{align*} \tag{32}

To start with, we fix $K = 1$ without any loss of generality (this is an arbitrary integration constant). To find out the values of parameters (there are three independent parameters e.g $A$, $B$ and $V_1$) for which inflation happens and satisfies Planck constraints (eqn (32)), we proceed as follows: for different values of $A$, we choose a range of values for $V_1$ and $B$. We choose random points in the given range and see if inflation happens with enough number of e-folds and also it ends before the minimum of the potential. If the point does not satisfy these two conditions then we discard them otherwise we calculate values of $A_s$, $n_s$ and $r$ for those points in parameter space and check whether they satisfy the constraints given by Planck as mentioned in eqn (32). The corresponding results are shown in figure (3).

Further in figure 4, we show the allowed regions for our model in the $n_s - r$ plane together with the Planck constraints. We show this for two particular values of $A$, e.g $A = -0.6 \times 10^{-12} M_{pl}^2$ and $A = -1.0 \times 10^{-12} M_{pl}^2$. As usual, we fix
FIG. 3: The allowed region in $B - V_1$ plane for $K = 1$. Grey dots represent the points for which inflation successfully happens but do not satisfy Planck constraints. Black dots do satisfy Planck constraints given in eqn (32). Values of $A$ are $A = -0.6 \times 10^{-12} M_{pl}^2$ (top left), $-0.8 \times 10^{-12} M_{pl}^2$ (top right), $-1.0 \times 10^{-12} M_{pl}^2$ (bottom left) and $-1.2 \times 10^{-12}$ (bottom right).

$K = 1$ without any loss of generality. One can see the allowed regions for our model is very much inside the Planck’s 68% confidence region.

In the Table I, we give different values of model parameters which satisfies the Planck constraints and the corresponding values of $d_0/l$ and two cosmological constants at the visible and hidden branes.

V. BICEP2 RESULTS FOR GRAVITY WAVES

CMB polarization is one of the most important observational signatures that can give important clues about the physics of very early Universe. The E-Mode polarization was first detected by DASI in 2001. But the B-mode polarization which is a clear evidence of primordial gravitational waves generated during inflation has not been detected until recently. Just few months before, BICEP2 experiment has announced the detection of the B-mode signal for the CMB polarization thereby confirming the existence of the primordial gravitational waves. Their measured value for the tensor-to-scalar ratio $r$ turns out to be $r = 0.2^{+0.07}_{-0.05}$ where as they rule out zero tensor fluctuation at 7σ confidence level. This result brings in huge conflict with the Planck results on measurement of $r$ which is $r < 0.11$. But the contribution from Galactic foregrounds to this B-mode signal has been an issue which has to be settled. People have shown that although the BICEP2 data is consistent with $r = 0.2$ with negligible galactic foreground, it is
FIG. 4: Top-left Figure: Plot for $n_s$ vs $r$ for $A = -0.6 \times 10^{-12} M_{pl}^2$, $K = 1.0$. Bottom-left Figure: Same as above but with $A = -1.0 \times 10^{-12} M_{pl}^2$. In both figures, $N = 55$. The right figures are enlarged version of the above figures. The red regions are same as the black regions in Figure 3 for corresponding $A$ values.

also consistent with negligible $r$ with significant polarization due to dust [11, 12]. Just recently by using the Planck HFI polarization data for 100 to 353 GHz, Adam et al. [16] have shown that polarization signal due to dust over the multipole $40 < l < 120$ is roughly the same as that obtained by BICEP2 over this $l$ range. This shows that it is entirely possible that the polarization signal BICEP2 has measured is not due to the primordial gravitational wave but due to dust. As this issue is still not settled, we have not considered the BICEP2 results in our analysis. We believe, we should wait till we get the results from ongoing joint analysis of Planck and BICEP2.

However we should stress that it is difficult to get high value of $r$ in the present model. But added contributions coming from cosmic defects [13], primordial magnetic fields [14] as well as cosmic birefringence caused by the coupling between scalar field and the CMB photons through Chern-Simons term can cause an enhancement to the total contribution for the tensor components.

VI. CONCLUSION

To summarize, we study a scalar tensor theory that can explain the moduli stabilisation in the bulk geometry as well as can produce an inflationary Universe in the visible brane which is consistent with the recent measurements by Planck experiment [3]. The scalar tensor theory can naturally arise as an effective 4-dimensional theory through perturbative corrections of the brane curvature in a two-brane RS-like set up as obtained earlier by Shiromizu and
Model Parameters & Cosmological Parameters & Corresponding Brane Distance & Cosmological constants

| A (in units of $10^{-12} M_{pl}^2$) | B (in units of $10^{-12} M_{pl}^2$) | $V_1$ | $n_s$ | r | ln(10$^{10} A_s$) | $d_0/l$ | $\Lambda_1$ (in units of $10^{-12} M_{pl}^4$) | $\Lambda_2$ (in units of $10^{-12} M_{pl}^4$) |
|---|---|---|---|---|---|---|---|---|
| -1.2 | -0.96 | 50 | 0.955 | 0.003 | 3.09 | 0.02041 | 57.6 | -60 |
| -1.0 | -0.97 | 55 | 0.959 | 0.003 | 3.09 | 0.01523 | 64.7 | -66.7 |
| -0.8 | -0.98 | 55 | 0.961 | 0.003 | 3.08 | 0.01010 | 78.4 | -80 |
| -0.6 | -0.985 | 57 | 0.962 | 0.003 | 3.07 | 0.00756 | 78.8 | -79.9 |

TABLE I: Table for different values of Model parameters A, B and $V_1$ and corresponding values of cosmological parameters $n_s$, r and ln(10$^{10} A_s$). The allowed values for the proper distance between two branes $\frac{d_0}{l}$ and the values of the cosmological constant in the two branes are also listed.

The potential for the inflaton field is not an ad hoc one but emerges from the construction of the model through the effective energy momentum tensor of the modulus field. The dynamics of radion facilitates the inflation and thus offers a natural explanation for the origin of inflation. Such an inflaton field ( i.e. the radion ) needs to be stabilised and stabilisation of the radion in turn is related to the scalar field sitting at the minimum of the potential. To inflict a de-Sitter character to this minimum, we have to add an uplifting term to the potential which is similar to the de-Sitter lifting by adding fluxes in the KKLT set up. We show that one gets enough e-folding in this model to solve the flatness and horizon problems. Moreover the primordial fluctuations produced by the inflaton field is consistent with the Planck’s measurements for $n_s$, r and $A_s$. Hence the present set up not only provides a viable inflationary scenario which is consistent with the Planck data but also offers a possible resolution to the modulus stabilisation mechanism concomitantly.

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