An Optimal Initial Foot Position for Quadruped Robots in Trot Gait

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Abstract. Gait planning parameters are important for quadruped robots to walk stably. In this paper, application of inverted pendulum model is widened and a new collision model of quadruped trotting between feet and the ground that can indicate body-attitude changes in walking is established. Influences of initial foot position, walking cyclic period and stride length on walking stability are discussed. The influence varies with different initial foot positions and we get the optimal initial foot position for the quadruped in trot gait with minimum variation in body attitude and maximum walking stability. Finally, we get the optimal initial foot position and walking cyclic period with minimal collision velocity. Simulation results are presented.

Keywords: Quadruped; Trot gait; Walking stability.

1. Introduction
Walking is a common form of locomotion which can move in a variety of terrains. Quadruped Robots offer a good balance between walking stability and mechanical complexity, compared to biped robots and hexapod robots. Research on quadruped robots has been making enormous progress. It is reported that the maximum running speed of MIT Cheetah is 45 km/h. The speed is even faster than human. In order to achieve best mobility, dynamic walking with lower duty factor is preferable to static gaits. More and more quadruped dynamic walking robots have been developed, such as Bigdog, Cheetah, etc. Although great success was achieved, it is still a crucial problem for those researchers that how to maintain dynamic walking stability and increase efficiency in high speed. Raibert introduced virtual legs to simplify modelling of quadruped walking robot and practiced it[1]. Kimura discussed dynamics of quadruped walking robot and concluded that the shorter the period or stride length is, the more stably the quadruped can walk[2]. Sangok presented design principles for legged robots which were implemented in the design of MIT Cheetah and analyzed the high-speed trotting experimental results[3]. Zhang proposed a series of Central Pattern Generators to achieve robust and dynamic trot gait[4]. Other Researchers made enormous efforts on walking stability of quadruped robots too.[5-11]

Two pairs of diagonal legs make standing phase respectively in trot gait and there exists a moment around the supporting axis, which makes the swinging legs touch ground successively[3]. Generally, motion control is used in gait control but force control or hybrid force/position control in contacting with the environment[12]. The difficulty in controlling the motion of walking robot increases greatly since the swinging legs do not touch ground simultaneously. Walking robot will overbalance and fall down if it is out of control. The moment falling around the supporting axis is analyzed and the angle of rotation around the supporting diagonal line is derived according the model.
2. Mechanical Modelling at the Beginning of Walking
As illustrated in Fig. 1, the projection of gravity $G$ in the supporting plane is not on the supporting diagonal line for quadruped robots in trot gait, there exists a moment around the supporting axis ($\theta$ means the angle of rotation). Quadruped walking robot will rotate around the axis as soon as walking starts because of the moment.
For simplicity, we assume:
Four legs of the robot are massless and the mass of the robot is concentrated on the body. Usually the ratio of the mass of legs to the mass of the body is small.

\begin{align*}
\text{Figure 1. Quadruped model rotating around the supporting diagonal line.}
\end{align*}

In order to illuminate dynamics of the robot, according to the assumption, we have this equation by Euler’s theorem within 1st cyclic period in walking:

\begin{equation}
mgl I \dot{\epsilon} = 0
\end{equation}

where $m$ denotes mass of the quadruped walking robot; $g$ denotes acceleration of gravity; $l$ denotes the distance from the projection of the center of gravity (COG) to the supporting diagonal line (see Fig. 2); $I$ denotes the moment of inertia of the robot around the supporting diagonal line and $\epsilon$ denotes angular acceleration of the rotation. Here, joint torques are neglected.

Fig. 2 illustrates the inverted pendulum model used in biped and quadruped walking robots. For quadruped robot walking in trotting gait, we can model the plane constructed by diagonal supporting legs as inverted pendulum (see Fig. 3).

\begin{align*}
\text{Figure 2. Inverted pendulum models in different initial foot positions.}
\end{align*}
We may obtain angular acceleration of the rotation from Eq. 1:

$$\varepsilon = \frac{mgI}{I}$$

(2)

The distance in the walking direction from the projection of the COG in the terrain to the supporting diagonal line is $x$ (see Fig. 2). To study the effects of stride length ($S$) on walking stability, variable $x_1$ ($0 \leq x_1 \leq 0.5$) is selected and $x$ is expressed as $x_1S$; $v(t)$ denotes walking speed; and $l$ is a function of time of walking $t$ because it decreases when the COG moves along the walking direction within 1st period in walking. We have:

$$l(t) = \left(x_1S - \int_0^t v(t) \, dt\right) \sin \alpha$$

(3)

where $S$ denotes stride length, $\alpha$ denotes angle between walking direction and the supporting diagonal line as shown in Fig. 4.

Substituting Eq. 3 into Eq. 2, we obtain:

$$\varepsilon(t) = \frac{mg \sin \alpha}{I} \left(x_1S - \int_0^t v(t) \, dt\right)$$

(4)

Integrating Eq. 4 with $t$, we obtain angular velocity:

$$\omega(t) = \int_0^t \frac{mg \sin \alpha}{I} \left(x_1S - \int_0^t v(t) \, dt\right) \, dt = \frac{mg \sin \alpha}{I} \left(x_1S \int_0^t dt \int_0^t v(t) \, dt\right)$$

(5)

where the initial angular velocity of the rotation $\omega(t)$ is 0 ($t=0$).

For simplicity, coefficient $A$ is selected to represent constants in Eq. 5 and we have:

$$A = \frac{mg \sin \alpha}{I}$$

(6)

$$\omega(t) = A \left(x_1S - \int_0^t dt \int_0^t v(t) \, dt\right)$$

(7)

Integrating Eq. 7 with $t$, we obtain angle of the rotation:

$$\theta(t) = A \int_0^t \left(x_1S - \int_0^t dt \int_0^t v(t) \, dt\right) \, dt = A \left(\frac{x_1St^2}{2} - \int_0^t dt \int_0^t v(t) \, dt\right)$$

(8)

where $\theta$ denotes rotation angle of COG around the supporting diagonal line as illustrated in Fig. 1 and $\theta(t)$ is 0 ($t=0$).

Assuming walking speed is $v$, then substituting $v = S/T$ into Eq. 8, we have:

$$\theta = AS \left(\frac{x_1t^2}{2} - \frac{t^3}{6T}\right)$$

(9)
Where $T$ denotes walking cyclic period of the quadruped robot.
When 1st half of cyclic period in walking ends ($t=0.5T$), we have:

$$\theta = A ST^2 \left( \frac{x_i}{8} - \frac{1}{48} \right) \quad (10)$$

From Eq. 10, we can see that:
Angle of rotation $\theta$ is proportional to $ST^2$. We assume $S=vT$ and get that $\theta$ is proportional to $T^3$ if walking speed $v$ is a constant. As we known that value of $\theta$ is greater, the greater the instability of walking is, the robot will out of control and lose its balance. Walking cyclic period $T$ is most crucial. To maintain most walking stability, we need to minimize value of walking cyclic period $T$ as far as possible.

The initial foot position of the quadruped walking machine largely influences attitude stability and walking stability of the robot. If $x$ is $S/6$ ($x_i=1/6$), angle of rotation (when $t=T/2$) will be 0. It is useful for switching of the supporting legs simultaneously. If $x$ is another value, the angle of rotation is not 0. It will aggravate the collision with the ground. It is a bad thing for completion of gait planning and will increase difficulty in motion control of the robot.

As illustrated in Fig.1, $x$ denotes the distance from movement of supporting legs to body and expressed as $S/n$ ($1 \leq n \leq +\infty$). Walking velocity of the quadruped robot is $V$. $d$ denotes distance from projection of COG in the supporting plane to supporting diagonal line and it is a function of walk time $t$, we have:

$$d(t) = \left( \frac{x}{n} - \int V dt \right) \sin \alpha \quad (11)$$

where $\alpha$ is angle between walking direction and supporting diagonal line as shown in Fig. 4.

From Eq.1, we have:

$$\varepsilon = \frac{mgd(t)}{t} = \frac{mg}{mt^2} \left( \frac{x}{n} - \frac{2St}{T} \right) \sin \alpha = \frac{gS \sin \alpha}{t^2} \left( \frac{1}{n} - \frac{2t}{T} \right) \quad (12)$$

Integrating Eq.12 with $t$, we have:

$$\omega = \frac{gS \sin \alpha}{t^2} \left( \frac{t}{n} - \frac{t^2}{T} \right) \quad (13)$$
We assume that length of the wheel link is $2d$ and $d$ corresponds to the horizontal distance from the swinging feet that first collide with the ground to the supporting diagonal line. At time $t = T/2$, the vertical impact velocity of the body caused by rotating around the supporting axis is:

$$V_2 = \omega d = \frac{gdS \sin \alpha}{l^2} \left( \frac{T}{2n} - \frac{T}{4} \right) = \frac{gdST \sin \alpha}{l^2} \left( \frac{1}{2n} - \frac{1}{4} \right)$$

(14)

As shown in Fig. 4, the composite of moment of rotation $M$ caused by gravity and the component $\tau_2$ in the direction of supporting diagonal line of the joint torque $\tau$ causes the rotation and impact. Here we assume that the velocity of swinging legs relative to the body is 0 at the time of contact. The other component $\tau_1$ only makes the body have the tendency to rotate in the supporting plane constructed by two supporting legs but the rotation does not happen. Therefore, the vertical impact velocity of body to the ground considering body attitude changing is the composite of $V_1$ and $V_2$. For any initial position $x = S/n \ (1 \leq n < +\infty)$, we have:

$$V_i = |V_1 \sin \alpha + V_2| = \frac{S \sin \alpha}{l} \left| \frac{2(n-1)S}{nT} - \frac{gdT}{l} \left( \frac{1}{2n} - \frac{1}{4} \right) \right|$$

(15)

Figure 6. Initial foot position with $x = S/6$.

3. Analysis and Simulation Results

ADAMS® was used to model the quadruped walking machine as illustrated in Fig. 1. In order to study the effects of various initial foot positions on walking stability, different values of initial positions $x$ are selected, such as 0, $S/6$ and $S/4$. Other parameters are the same for the model in simulations. We get data of body attitude at $t=T/2$ in walking, the number of stable walking cyclic periods and the status of variation of body attitude in walking (see Table 1).

Table 1. Simulation results of different initial foot positions of quadruped trotting.

| Initial position | 0  | $S/6$ | $S/4$ |
|------------------|----|-------|-------|
| Pitch angle $(t = T/2)$ | 2.3° | 0.5° | 0.7° |
| Roll angle $(t = T/2)$ | 1.7° | 3.1° | 5.0° |
| No. of stable walking periods | 7  | 20    | 15    |
From simulation results, variations of body attitude (pitch angle and roll angle) are the smallest if initial position \( x \) is \( S/6 \) (see Table. 1). Besides, the number of stable walking periods is the maximum in all initial positions (see Table 1). The results illustrate that initial foot position do influence walking stability of the quadruped walking and there exists an optimal initial foot position. Here, the optimal initial foot position refers to foot position with minimal attitude varying at the end of the supporting ( \( t=\frac{T}{2} \) ) and maximal walking stability.

Next we look for the optimal walking cycle period with minimal impact velocity. Differentiating impact velocity \( V_i \) in Eq. 15 by walking cyclic period \( T \), we have:

\[
\frac{\partial V_i}{\partial T} = \frac{-2S}{T^2} \left( \frac{1}{n} - \frac{1}{n} \right) - \frac{gd}{l} \left( \frac{1}{2n} - \frac{1}{4} \right)
\]

where meaning of each parameter has been described before.

Let \( \frac{\partial V_i}{\partial T} = 0 \), we have:

\[
T_{opt} = \sqrt{\frac{8(n-1)IS}{(n-2)gd}}
\]

where \( T_{opt} \) denotes the optimal walking cyclic period.

We simplified some quadruped walking robot as inverted pendulum model and get:

\( S=225\text{mm} \quad l=500\text{mm} \quad d=200\text{mm} \)

Substitute the above values into Eq. 16, we get the optimal walking periods with different initial foot positions as shown in Figure 7 (The data transilience where \( n=3 \) in the figure is caused by value of parameter \( d \)). From Figure 7, we can see that with foot positions \( (n>3) \), the values of the optimal walking periods are 0.5s or so and similar to the actual walking periods of many high speed quadruped walking robots.

![Figure 7. Optimal walking periods.](image)

4. Conclusion

In this paper, we presented that initial foot position of the quadruped robot in trot gait largely influences walking stability. We found that the angle of rotation around the supporting diagonal line of the quadruped in trot gait is minimal and walking stability is maximal if initial foot position is a fixed value in constant walking speed. The optimal initial foot position also makes it easier to control walking of the robot. If proper buffer gear and vibration absorber are applied, the quadruped walking machine will walk more quickly and stably.

The value of optimal initial foot position got in this paper is true in constant walking speed. If there is any acceleration in walking, we can use the method in this paper to get a new optimal initial position.
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