Heavy vector and axial-vector $D$ mesons in hot magnetized asymmetric nuclear matter

Rajesh Kumar$^a$, Rahul Chhabra$^b$, Arvind Kumar$^c$

Department of Physics, Dr. B R Ambedkar National Institute of Technology Jalandhar, Jalandhar, Punjab 144011, India

Received: 24 May 2020 / Accepted: 23 October 2020 / Published online: 4 November 2020
© Società Italiana di Fisica and Springer-Verlag GmbH Germany, part of Springer Nature 2020
Communicated by Ralf Rapp

Abstract We observed the impact of the finite magnetic field on the in-medium mass and decay constant of isospin averaged vector $D^*(D^{*+}, D^{*0})$ and axial-vector $D_1(D_1^+, D_1^0)$ mesons. The quark and gluon condensates of the nuclear medium at the finite magnetic field, temperature, isospin asymmetry, and density have been obtained by the meson exchange scalar fields within the chiral SU(3) model. The medium attributes modify the scalar and vector density of the nuclear medium and this variation reflects in the in-medium mass and decay constant of spin 1 $D$ mesons. We calculate these observables by comparing the Operator Product Expansion (OPE) and the phenomenological side in the QCD Sum Rules. In the results, we observed a positive mass shift for charged vector and axial-vector $D$ mesons concerning the magnetic field. For neutral vector (axial-vector) $D$ mesons we observed negative (positive) mass shift as a function of the magnetic field. In the application part, we calculate the in-medium partial decay width of the process $D^{*} s(2715/2860) \rightarrow D^{*} K$ by using $^3 P_0$ model. The in-medium effects are incorporated through the in-medium masses of $D^*$ and $K$ mesons.

1 Introduction

The in-medium study of $D$ meson is of great interest to understand the $J/\psi$ suppression phenomenon which is considered as a signature of Quark Gluon Plasma (QGP) formation in Heavy Ion Collisions (HICs) [1]. In addition to $J/\psi$ suppression, other indirect measurements such as jet quenching [2], dilepton enhancements [3–5] and strangeness enhancements [6,7] are used as a signature of QGP formation. In $J/\psi$ suppression, due to color debye screening, the binding radius of charm quark pair becomes greater than the debye screening radius, and hence the binding force cannot keep the charm and anticharm quark together [1,8,9]. The free charm quarks couples with the light quarks in the medium and may form open charm mesons. The properties of $D$ (open charm meson) varies appreciably in the medium due to the presence of light quarks. Higher charmonia state decays to lower charmonium state ($J/\psi$ meson) directly, but if the mass of $D$ mesons decreases appreciably due to medium effects then it will prefer decaying into $D$ mesons and eventually the $J/\psi$ get suppressed [1,8,10]. The in-medium properties of $D$ mesons vary considerably as the light quark condensates vary appreciably in the medium [10]. On the other hand, in hidden charm/bottom mesons (charmonia/bottomonia) the properties do not vary appreciably as it depends upon the interaction of gluon condensates which do not change appreciably in the medium [10,11].

Recently, in HICs the effect of the magnetic field was also found in addition to medium properties such as isospin asymmetry, temperature, and density [12–14]. The strength of generated magnetic field was calculated $eB \sim 2 - 15 m_\pi^2 (1 m_\pi^2 = 2.818 \times 10^{18} \text{ Gauss})$ approximately [10]. The presence of a strong magnetic field urged physicists to find its impact on the phase transition, perturbative and non-perturbative regime of physics [10,13,15–18]. However, the time duration for which the magnetic field sustain is still unclear [10,19]. According to the chiral magnetic effect, the magnetic field interacts with the nuclear matter and due to Lenz’s law, it generates induced current, which affects the electric conductivity of the medium. The change in electric conductivity increases the relaxation time that results in the slow decay of the magnetic field [13,15–17]. The presence of the magnetic field also generates new interesting phenomenon like inverse magnetic catalysis, magnetic catalysis [15]. The effect of all these medium attributes on the properties of hadrons can be...
revealed in the upcoming Heavy Ion Collider (HIC), such as Compressed Baryonic Matter (CBM, GSI Germany), Japan Proton Accelerator Research Complex (J-PARC Japan), Proton AntiProton Annihilation in Darmstadt (PANDA, GSI Germany) and Nuclotron-based Ion Collider Facility (NICA, Dubna Russia) [20].

Various models have been constructed to study the non-perturbative regime of QCD. Some of these models are: Nambu-Jona-Lasinio (NJL) model [21], chiral SU(3) model [10, 18, 22–24], Quark-Meson Coupling (QMC) model [25–30], QCD sum rules (QCDSR) [31–36], and coupled channel approach [37–40]. In the above methodologies, the effect of quantum fluctuations are ignored by using mean field potential. These quantum fluctuations can be included by the Polyakov loop extended NJL (PNJL) model [41–43], Polyakov Quark Meson (PQM) model [44,45], and Functional Remoralization Group (FRG) [46,47]. Furthermore, the partial/full decay width of the heavy mesons have been studied through several models such as elementary meson-emission model [48], 3S1 model [49], flux-tube model [50] and 3P0 model [51].

The effect of the magnetic field on the in-medium properties of vector $D^*$ and axial-vector $D_1$ mesons have not been studied in the literature. In the present work, we apply the combined approach of chiral SU(3) model and QCD sum rules to study the shift in mass and decay constant of these mesons in asymmetric magnetized hot nuclear matter.

At first, using chiral model, we evaluate the magnetic field induced light quark condensates $\langle \bar{q}q \rangle$, and gluon condensates $\langle \frac{1}{3} G^{\mu \nu} G^{\mu \nu} \rangle$, in dense and hot asymmetric nuclear matter. Afterward, we input these condensates in the Borel transformed QCD sum rules to calculate the in-medium mass and decay constant for $D^*$ and $D_1$ mesons. Besides, as an application of in-medium mass shift of vector $D^*$ mesons, using $^3P_0$ model we evaluate the in-medium partial decay width of excited $D_s^*(2860)$ and $D_s^*(2715)$ mesons decaying to vector $D^*$ and pseudoscalar $K$ mesons. The in-medium $K$ meson mass (at finite temperature, density, and magnetic field) for this purpose is calculated using the chiral SU(3) model.

The effect of magnetic field on the in-medium properties of hadrons has been studied by various methods [10,18,19,52–56]. The properties of charmonium [18,52,54,55], bottomonium [54], $\rho$ mesons [57], and $B$ mesons [58,59], were studied in the presence of magnetic field. Using the chiral SU(4) model, the author found an additional positive mass shift in the mass of $D^*$ meson under the influence of the magnetic field [19]. In Ref. [53], using QCD sum rules, Gubler et al. calculated the effect of the finite magnetic field on the mass of $D$ mesons as well as the mixing effect of vector and pseudoscalar $D$ mesons. The decay width of higher charmonia ($\psi(3770), \psi(3686), \chi_{c0}(3414)$ and $\chi_{c1}(3556)$) to $D\bar{D}$ pairs was also studied within the combined approach of $^3P_0$ model and chiral model [10]. In this article, we calculated the impact of the magnetic field on the mass and decay constant of pseudoscalar and scalar $D$ mesons using QCDSR and further used these properties to calculate in-medium partial decay width of excited charmonium states. In Ref. [18,54], we also calculated the in-medium mass of charmonia and bottomonia in the presence of the magnetic field and found an attractive mass shift. Extensive work is done on the in-medium effects without considering the magnetic field [8,60–62]. For instance, the unification of Operator Product Expansion (OPE) and Borel transformation on the current-current correlation function was applied to equate the mass and various condensates [32,33]. The mass splitting between $D$ and $\bar{D}$ mesons was studied using the linear density approximation in QCD sum rules [33]. Using QCD sum rules and chiral SU(3) model, the author evaluated the shift in decay constants and mass of vector and axial-vector mesons [60]. In this article, they observed attractive interaction for vector $D^*$ and $B^*$ mesons whereas, repulsive interaction for axial-vector $D_1$ and $B_1$ mesons in the hadronic medium.

The $^3P_0$ model had been used to study the partial decay widths of different mesons [51,63–77]. It was predicted by the Babar and Belle collaboration that the excited states of $D_s(1968)$, i.e., $D_s^*(2860)$ and $D_s^*(2715)$ are of specific importance having decay width approximately equal to 48 and 115 MeV, respectively. But, the spectroscopic state of $D_s^*$ meson is not yet confirmed. To find the exact quantum numbers, theoretical calculations have been done by observing different decay channels [8,64]. For instance, using $^3P_0$ model, authors investigated various decay channels of $D_s^*(2860)$ and $D_s^*(2715)$ mesons and proposed the possible quantum numbers of $D_s^*(2860)$ and $D_s^*(2715)$ states such as $3−(1^3D_3), 1−(1^3D_1)$ and $1−(2^3S_1)$ (least probable) [8,64,72,77–79]. By comparing the theoretical and empirical results, the study of $D_s^*(2715$ or 2860) $\rightarrow D^* + K$ decay mode will help to reveal the possibility to allocate $D_s^*$ meson quantum spectroscopic states [8,64]. The impact of magnetic field is studied on the $K$ and $\bar{K}$ mesons in cold asymmetric nuclear matter [56]. In this article, authors used chiral SU(3) model to calculate the in-medium mass of $K$ and $\bar{K}$ mesons and compared the results of zero and non zero contributions of anomalous magnetic moment. Moreover, the in-medium properties of $K$ and $\bar{K}$ mesons in zero magnetic field have been studied extensively in the literature [80–83].

The outline of the present paper is as follows: In Sect. 2.1, we will briefly explain the methodology used to calculate the in-medium quark and gluon condensates and the mass of kaons in the nuclear medium. In Sect. 2.2, we will explain the formalism to calculate the effective masses and decay constants of vector and axial-vector $D$ mesons under the effect of the magnetic field. The methodology to calculate the in-medium decay width of excited $D_s^*$ meson will be given in Sect. 2.3. Section 3 will be devoted to discussing the quan-
titative results of the present work and finally, the summary will be presented in Sect. 4.

2 Formalism

In the present work, we use the combination of two non-perturbative QCD techniques, (i) Chiral SU(3) model and (ii) QCD Sum Rules [10,32]. From the first, we extract the magnetically induced light quark and gluon condensates, which are used in the QCD Sum Rules to evaluate the medium effects on the mass and decay constant of vector $D^*$ and axialvector $D_1$ mesons. The medium modified mass of kaons (calculated from the chiral model) and $D^*$ meson are further used in the $^3P_0$ model to calculate the in-medium decay width of $D^*_1$ meson. In the following subsections, we briefly describe the formulation to obtain the results.

2.1 In-medium quark and gluon condensates and mass of $K$ mesons

The chiral $SU(3)$ model includes the basic QCD properties such as trace anomaly (broken scale invariance) and non-linear realization of chiral symmetry [10,18,24,84–87]. In this model, the isospin asymmetry of the medium is introduced by including the scalar isovector field $\delta$ and vector-isovector field $\rho$ [10]. Moreover, the broken scale invariance property is preserved by the inclusion of scalar dilaton field $\chi$ [10,24]. The effect of thermal and quantum fluctuations are neglected in the present model by using mean field approximation [10,19]. The Lagrangian density of this model under mean-field approximation is given as

$$\mathcal{L}_{\text{chiral}} = \mathcal{L}_{\text{kin}} + \sum_{M=S,V} \mathcal{L}_{NM} + \mathcal{L}_{\text{vec}} + \mathcal{L}_{0} + \mathcal{L}_{SB}. \quad (1)$$

Explicitly,

$$\mathcal{L}_{NM} = -\sum_{i} \bar{\psi}_i \left[ m_i^s + g_{\omega i} \gamma_0 \omega + g_{\rho i} \gamma_0 \rho \right] \psi_i, \quad (2)$$

$$\mathcal{L}_{\text{vec}} = \frac{1}{2} \left( m_0^2 \omega^2 + m_0^2 \rho^2 \right) \frac{x^2}{\chi_0^2} + g_4 (\omega^4 + 6 \omega^2 \rho^2 + \rho^4), \quad (3)$$

$$\mathcal{L}_0 = -\frac{1}{2} k_0 \chi^2 \left( \sigma^2 + \chi^2 + \delta^2 \right) + k_1 \left( \sigma^2 + \chi^2 + \delta^2 \right) + k_2 \left( \sigma^2 - \chi^2 - \delta^2 \right) \varsigma + k_3 \chi \left( \sigma^2 - \delta^2 \right) \varsigma + k_4 \chi^4 - \frac{1}{4} \chi^4 \ln \left( \frac{\sigma^2 - \delta^2}{\sigma_0^2 \delta_0} \right) \left( \frac{\chi}{\chi_0} \right)^3, \quad (4)$$

and

$$\mathcal{L}_{SB} = -\left( \frac{\chi}{\chi_0} \right)^2 \left[ m_\pi^2 f_\pi \sigma + \left( \sqrt{2} m_K^2 f_K - \frac{1}{\sqrt{2}} m_\pi^2 f_\pi \right) \varsigma \right]. \quad (5)$$

First term, $\mathcal{L}_{\text{kin}}$ in Eq. (1) represents the kinetic energy term, $\mathcal{L}_{NM}$ denotes the nucleon–meson interaction term, where $S$ and $V$ represents the scalar and vector mesons, respectively. The effective mass of nucleons, $m_i^e$ is given as

$$m_i^e = -(g_{\sigma i} \sigma + g_{\varsigma i} \varsigma + g_{\delta i} \tau_3 \delta). \quad (6)$$

In Eq. (6), $g_{\sigma i}, g_{\varsigma i}$ and $g_{\delta i}$ represent the coupling strengths of non-strange scalar-isoscalar field $\sigma$, strange scalar-isoscalar field $\varsigma$ and scalar-isovector field $\delta$ with nucleons $(i=p,n)$ respectively and $\tau_3$ denotes the third component of isospin. Also, $\sigma_0, \varsigma_0, \delta_0$ and $\chi_0$ represent the vacuum values of the $\sigma, \varsigma, \delta$ and $\chi$ scalar fields, respectively. Furthermore, the term $\mathcal{L}_{\text{vec}}$ of Eq. (1) contains the quartic self-interaction terms and it produces the mass of vector mesons through the interactions with spin-0 mesons. $\mathcal{L}_0$ and $\mathcal{L}_{SB}$ represent the spontaneous chiral symmetry breaking and the explicit chiral symmetry breaking term, respectively.

In the present investigation, the effect of the external magnetic field is incorporated by adding the magnetic induced Lagrangian density to the chiral model’s Lagrangian density [18,19]. Thus, we write

$$\mathcal{L}_T = \mathcal{L}_{\text{chiral}} + \mathcal{L}_{\text{mag}}, \quad (7)$$

where

$$\mathcal{L}_{\text{mag}} = -\bar{\psi}_i \gamma_\mu \gamma_\nu A^{\mu \nu} \psi_i - \frac{1}{4} k_1 \mu_N \bar{\psi}_i \sigma^{\mu \nu} F^{\mu \nu} \psi_i - \frac{1}{4} F^{\mu \nu} F_{\mu \nu}. \quad (8)$$

Where, $\psi_i$ is a wave function of $i$th nucleon, and the middle term represents the tensorial interaction with the electromagnetic field tensor, $F^{\mu \nu}$. Moreover, $\mu_N$ and $k_i$ are the nuclear magneton, given as $\mu_N = \frac{e}{2m_N}$, and anomalous magnetic moment of $i$th nucleon, respectively, where $m_N$ is the nucleon’s vacuum mass. We choose a uniform magnetic field along the $Z$-axis, with the vector potential $A^z = (0, 0, Bx, 0)$.

The coupled equations of motion of the chiral model fields are obtained by putting the total Lagrangian (Eq. 7) of chiral model in Euler–Lagrange equations of motion [18,54] and are given as

$$k_0 \chi^2 \sigma - 4k_1 \left( \sigma^2 + \chi^2 + \delta^2 \right) \sigma - 2k_2 \left( \sigma^3 + 3 \sigma \delta^2 \right) - 2k_3 \chi \sigma \varsigma - \frac{d}{\sqrt{2}} \chi \left( \frac{2 \sigma}{\sigma^2 - \delta^2} + \frac{\chi}{\chi_0} \right) \frac{m_\pi^2 f_\pi}{\sqrt{2}} = \sum g_{\sigma i} \rho_i^e, \quad (9)$$

$$k_0 \chi^2 \varsigma - 4k_1 \left( \sigma^2 + \chi^2 + \delta^2 \right) \varsigma - 4k_2 \chi^3 - k_3 \chi^2 \varsigma \varsigma - \frac{d}{\sqrt{2}} \chi \left( \frac{2 \varsigma}{\varsigma^2 - \delta^2} + \frac{\chi}{\chi_0} \right) \frac{m_\pi^2 f_\pi}{\sqrt{2}} = \sum g_{\varsigma i} \rho_i^e, \quad (10)$$

$$k_0 \chi^2 \delta - 4k_1 \left( \sigma^2 + \chi^2 + \delta^2 \right) \delta - 2k_2 \left( \delta^3 + 3 \sigma \delta^2 \right) + 2k_3 \chi \delta \varsigma$$

$$- \frac{d}{\sqrt{2}} \chi \left( \frac{2 \delta}{\delta^2 - \sigma^2} + \frac{\chi}{\chi_0} \right) \frac{m_\pi^2 f_\pi}{\sqrt{2}} = \sum g_{\delta i} \rho_i^e.$$
\[
\frac{2}{3} d\chi^4 \left( \frac{\delta}{\sigma^2 - \delta^2} \right) = \sum g_{\omega} T_3 \rho_i^\omega, \tag{11}
\]
\[
\left( \frac{x}{x_0} \right)^2 m_{\rho}^2 \omega + g_4 (4\omega^3 + 12\rho^2 \omega) = \sum g_{\omega} \rho_i^\omega, \tag{12}
\]
\[
\left( \frac{x}{x_0} \right)^2 m_{\rho}^4 \rho + g_4 (4\rho^3 + 12\omega^2 \rho) = \sum g_{\rho} T_3 \rho_i^\rho, \tag{13}
\]
and
\[
k_0 x \left( \sigma^2 + \zeta^2 + \delta^2 \right) - k_3 \left( \sigma^2 - \delta^2 \right) \zeta
+ \chi \left[ 1 + \ln \left( \frac{x^4}{x_0^4} \right) \right] + (4k_4 - d) x^3
- \frac{4}{3} d x^3 \ln \left( \frac{\sigma^2 - \delta^2}{\sigma_0^2} \right) \left( \frac{x^4}{x_0^4} \right)
+ \frac{2\chi}{x_0^2} \left[ m_{\rho}^2 f_{\pi} \sigma + \left( \sqrt{2} m_{K}^2 f_{K} - \frac{1}{\sqrt{2}} m_{\rho}^2 f_{\pi} \right) \zeta \right]
- \frac{x}{x_0^2} (m_{\rho}^2 \omega^2 + m_{\rho}^2 \rho^2) = 0, \tag{14}
\]
respectively.

In above, the constants \(m_{\pi}, m_{K}, f_{\pi}, \) and \(f_{K}\) are the masses and decay constants of pions and kaons, respectively and the parameters \(k_i (i = 1 \text{ to } 4)\) are fitted to reproduce the vacuum values of scalar meson fields \([62]\). In addition, the isospin asymmetry is incorporated in the model through definition, \(\eta = \frac{g_{\omega} T_3 \rho_i^\omega}{g_{\rho} T_3 \rho_i^\rho}\). Note that \(\rho_i^\omega\) and \(\rho_i^\rho\) denote the scalar and vector densities of \(i\)th nucleons \((i = n, p)\) in the presence of magnetic field (along Z-direction) \([18, 88, 89]\) and for neutral neutron their expressions are as follows

\[
\rho_{n}^\omega = \frac{1}{2\pi^2} \sum_{\pi = \pm 1} \int_{0}^{\infty} k_{\parallel}^n d\kappa_{\parallel}^n \left( 1 - \frac{s \mu_{N,K} B}{\sqrt{m_{n}^2 + (\kappa_{\parallel}^n)^2}} \right)
\int_{0}^{\infty} dk_{\perp}^n \frac{m_{n}^*}{E_{n}^{n}} \left( f_{k,s}^n + \tilde{f}_{k,s}^n \right), \tag{15}
\]
and
\[
\rho_{n}^\rho = \frac{1}{2\pi^2} \sum_{\pi = \pm 1} \int_{0}^{\infty} k_{\parallel}^n d\kappa_{\parallel}^n \int_{0}^{\infty} dk_{\perp}^n \left( f_{k,s}^n - \tilde{f}_{k,s}^n \right), \tag{16}
\]
respectively. Likewise, for the positively charged proton, the scalar and vector densities are given as follows \([88, 89]\)

\[
\rho_{p}^\omega = \frac{|q_p| B m_{p}^*}{2\pi^2} \left[ \sum_{\pi = \pm 1} \int_{0}^{\infty} \frac{dk_{\parallel}^p}{(k_{\parallel}^p)^2 + (\tilde{m}_{p})^2} \left( f_{k,v,s}^p + \tilde{f}_{k,v,s}^p \right)
+ \sum_{\pi = \pm 1} \int_{0}^{\infty} \frac{dk_{\parallel}^p}{(k_{\parallel}^p)^2 + (\tilde{m}_{p})^2} \left( f_{k,v,s}^p - \tilde{f}_{k,v,s}^p \right) \right], \tag{17}
\]
and
\[
\rho_{p}^\rho = \frac{|q_p| B m_{p}^*}{2\pi^2} \left[ \sum_{\pi = \pm 1} \int_{0}^{\infty} \frac{dk_{\parallel}^p}{(k_{\parallel}^p)^2 + (\tilde{m}_{p})^2} \left( f_{k,v,s}^p + \tilde{f}_{k,v,s}^p \right)
+ \sum_{\pi = \pm 1} \int_{0}^{\infty} \frac{dk_{\parallel}^p}{(k_{\parallel}^p)^2 + (\tilde{m}_{p})^2} \left( f_{k,v,s}^p - \tilde{f}_{k,v,s}^p \right) \right], \tag{18}
\]
respectively, where \(\tilde{m}_{p}\) denotes the magnetic field induced mass, and is given as
\[
\tilde{m}_{p} = \sqrt{m_{p}^2 + 2|q_p| B - s \mu_{N,K} B}. \tag{19}
\]

In the above equations, symbol \(v\) represents the quantized Landau levels. The effective single particle energy of neutron is given by \(\tilde{E}_{n}^v = \sqrt{\left( k_{\parallel}^n \right)^2 + \left( \sqrt{m_{n}^2 + (k_{\perp}^n)^2} - s \mu_{N,K} B \right)^2}\), whereas for proton its expression is given as \(\tilde{E}_{p}^v = \sqrt{\left( k_{\parallel}^p \right)^2 + \left( \sqrt{m_{p}^2 + 2|q_p| B - s \mu_{N,K} B} \right)^2}\). The symbols \(k_i\) and \(s\) denote the anomalous magnetic moment and the spin of the nucleons, respectively. Moreover, \(f_{k,v,s}^n, \tilde{f}_{k,v,s}^n, f_{k,v,s}^p, \tilde{f}_{k,v,s}^p\) denote the Fermi distribution functions at finite temperature for nucleon and their antiparticle, and are shown as

\[
f_{k,s}^n = \frac{1}{1 + \exp \left[ \beta (\tilde{E}_{n}^v - \mu_{n}^*) \right]}, \tag{20}
\]
\[
\tilde{f}_{k,s}^n = \frac{1}{1 + \exp \left[ \beta (\tilde{E}_{n}^v + \mu_{n}^*) \right]}, \tag{21}
\]
\[
f_{k,v,s}^p = \frac{1}{1 + \exp \left[ \beta (\tilde{E}_{p}^v - \mu_{p}^*) \right]}, \tag{22}
\]
\[
\tilde{f}_{k,v,s}^p = \frac{1}{1 + \exp \left[ \beta (\tilde{E}_{p}^v + \mu_{p}^*) \right]}.
\]

In this model, through the explicit symmetry breaking terms, the scalar up and down quark condensates are expressed as \([10]\)

\[
\langle \bar{u}u \rangle_{\rho_N} = \frac{1}{m_u} \left( \frac{\chi}{x_0} \right)^2 \left[ \frac{1}{2} m_{\pi}^2 f_{\pi} (\sigma + \delta) \right], \tag{22}
\]
and
\[
\langle \bar{d}d \rangle_{\rho_N} = \frac{1}{m_d} \left( \frac{\chi}{x_0} \right)^2 \left[ \frac{1}{2} m_{\pi}^2 f_{\pi} (\sigma - \delta) \right]. \tag{23}
\]
respectively. Here, $m_u$ and $m_d$ are the masses of up and down quarks respectively.

As discussed earlier, the dilaton field $\chi$, is incorporated in the model to mimic the trace anomaly property of QCD [24, 62]. The comparison of the trace of the energy momentum tensor of QCD with the trace of energy momentum tensor in chiral model relates the dilaton field to the scalar gluon condensate, which in the limit of massless quarks can be written as

$$T_\mu^\nu = \left(\frac{\beta_{\text{QCD}}}{2g} G_{\mu\nu}^a G_{a\mu\nu}^a\right) \equiv -(1 - d) \chi^4.$$  \hspace{1cm} (24)

For finite quark masses, the above equation modifies to

$$T_\mu^\nu = \sum_i m_i q_i(q_i + \left(\frac{\beta_{\text{QCD}}}{2g} G_{\mu\nu}^a G_{a\mu\nu}^a\right)) \equiv -(1 - d) \chi^4.$$ \hspace{1cm} (25)

In above, the summation term of the energy-momentum tensor is the negative of the explicit chiral symmetry breaking term, $\mathcal{L}_{SB}$, given in the Eq. (5).

To calculate the value of the $d$ parameter, from Eq. (24), we recall the QCD $\beta$ function at one loop level, for $N_c$ colors and $N_f$ flavors and is given by [90, 91]

$$\beta_{\text{QCD}}(g) = -\frac{11N_c g^3}{48\pi^2} + \frac{N_f g^3}{24\pi^2} + O(g^5).$$ \hspace{1cm} (26)

In the above equation, the first term comes from the anti-screening (self-interaction) of the gluons and the second term comes from the screening of quark pairs. For $N_f=3$ and $N_c=3$, Eqs. (24) and (26) calculate the value of $d$ to be 6/33, and for $N_f=2$ and $N_c=3$, the value of $d$ comes out to be 4/33, which is in agreement with the one-loop approximation of the QCD $\beta$ function [62, 90, 91]. In the chiral SU(3) model, the in-medium properties of the open charm mesons is evaluated from the medium condensates which further depends on the dilaton field, $\chi$.

We use the value of $d=0.064$ [62] in the present article, which along with the other medium parameters are fitted to reproduce the vacuum values of meson fields ($\sigma_0, \zeta_0, \delta_0, \chi_0$, $\omega_0$ and $\rho_0$), masses of the $\eta$, $\eta'$ mesons and nucleons [18, 24, 62].

The scalar gluon condensate can be expressed in terms of scalar fields using Eq. (25) and is given as [18]

$$\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^a G_{a\mu\nu}^a \rangle_{\rho_N} = \frac{8}{9} \left[ (1 - d) \chi^4 + \left( \frac{\chi}{\chi_0} \right)^2 \right].$$ \hspace{1cm} (27)

As said before, to calculate the in-medium decay width of $D_s^+$ meson, we need in-medium mass of $K$ mesons. To obtain the mass modification of $K (\bar{K})$ meson, we start from kaon-antikaon interaction Lagrangian density [56]

$$\mathcal{L}_{K(\bar{K})}^\text{int} = -\frac{i}{4f_K^2}[(2\bar{p}\gamma^\mu p + \bar{n}\gamma^\mu n)(K^-(-\partial_\mu K^+) - (-\partial_\mu K^-)K^+) \hspace{1cm} \text{+} \hspace{1cm} \bar{q}(\gamma^\mu p + 2\bar{n}\gamma^\mu n)(K_0^-(\partial_\mu K_0^+ - (\partial_\mu K_0^-)K_0^0))] \hspace{1cm} \text{+} \hspace{1cm} \frac{m_K^2}{2f_K^2}[(\sigma + \sqrt{2}\zeta + \delta)(K^+ K^-) \hspace{1cm} \text{+} \hspace{1cm} (\sigma + \sqrt{2}\zeta - \delta)(K_0^- K_0^0)] \hspace{1cm} \text{+} \hspace{1cm} \frac{m_K^2}{2f_K^2}[(\sigma + \sqrt{2}\zeta - \delta)(\partial_\mu K^-)(\partial^\mu K_0^-) \hspace{1cm} \text{+} \hspace{1cm} (\sigma + \sqrt{2}\zeta + \delta)(\partial_\mu K_0^+)(\partial^\mu K_0^-)] \hspace{1cm} \text{+} \hspace{1cm} \frac{m_K^2}{2f_K^2}(\bar{p}(\partial_\mu K^+)(\partial^\mu K^-) + \bar{n}(\partial_\mu K_0^+)(\partial^\mu K_0^-)).$$ \hspace{1cm} (28)

The first term in the above expression is the vectorial Weinberg Tomozawa interaction term. This term have leading order contribution and is repulsive for $K$ mesons but attractive for $\bar{K}$ mesons [56]. Further, the scalar meson exchange term represents the next to leading order contributions. The parameters $d_1$ and $d_2$ in the above expression are taken as $2.56/m_K$ and 0.73/$m_K$ respectively [56], by fitting the experimental values of the kaon-nucleon ($KN$) scattering length [92].

The total Lagrangian density for kaon-antikaon is given by

$$\mathcal{L}_K = (\partial_\mu \tilde{K})(\partial^\mu K) - m_K^2 K \bar{K} + \mathcal{L}_{K(\bar{K})}^\text{int}.$$ \hspace{1cm} (29)

Now, by performing Fourier transformation on the above expression, we get following dispersion relation for kaons

$$-\omega^2 + |\mathbf{k}|^2 = \Pi_K(\omega, |\mathbf{k}|) = 0.$$ \hspace{1cm} (30)

In above, $\Pi_K$ represents the kaon in-medium self energy and is explicitly given as

$$\Pi_K(\omega, |\mathbf{k}|) = -\frac{1}{4f_K^2}[(2\rho_p + \rho_n) \pm (\rho_p - \rho_n)] \omega \hspace{1cm} \text{+} \hspace{1cm} \frac{m_K^2}{2f_K^2}(\sigma' - \sqrt{2}\zeta' \pm \delta') \hspace{1cm} \text{+} \hspace{1cm} \left[-\frac{1}{f_K}(\sigma' + \sqrt{2}\zeta' \pm \delta') \hspace{1cm} \text{+} \hspace{1cm} \frac{d_1}{2f_K}(\rho_p^s + \rho_n^s) \hspace{1cm} \text{+} \hspace{1cm} \frac{d_2}{4f_K}(\rho_p^s + \rho_n^s) \pm (\rho_p^s - \rho_n^s)\right](\omega^2 - |\mathbf{k}|^2).$$ \hspace{1cm} (31)

where the $\pm$ signs represents the $K^+$ and $K^0$ mesons, respectively. Also, in the above expression $\sigma'$, $\zeta'$ and $\delta'$ denote
the deviation of the field’s expectation values from their vacuum expectation values. In magnetized nuclear matter, the masses for $K$ meson is calculated under the condition $m^*_K = \omega(|k|=0)$. The charged $K^+$ meson in magnetic medium gets additional positive mass shift due to the Landau quantization [10]

$$m_{K^+}^* = \sqrt{m_{K^+}^2 + eB},$$

(32)

where $m_{K^+}$ is the in-medium mass of $K^+$ meson, $e$ is the electrostatic unit of charge and $B$ is the magnetic field along $Z$-direction. On the other hand, for uncharged $K^0$ meson there will not be Landau quantization.

2.2 In-medium mass and decay constant of vector and axial-vector $D$ mesons

QCD Sum Rules are low energy QCD methodology to relate the hadron properties from medium to vacuum [32,34]. These rules are based on the Operator Product Expansion (OPE) and Borel transformation to overcome the divergent behavior of the perturbative expansion [34,36]. In these rules, we have taken into account the next to leading order contributions in the Borel transformed coefficients. We start with a current-current correlator function, which is a Fourier transformation of time ordered product of the meson current, $J(x)$ at some value of nuclear density, $\rho_N$, and temperature, $T$ [10,93]

$$\Pi(p) = i \int d^4x \; e^{ipx} \langle T \left\{ J(x)J^\dagger(0) \right\} \rangle_{\rho_N,T},$$

(33)

where $p$ is the four momentum. In the present work, we have used centroid approximation, in which we have used same rules (degeneracy) for particles and antiparticles. The mass splitting between particle and antiparticle can be studied by considering even and odd sum rules [33]. The isospin average meson currents of the degenerate vector and axial-vector $D$ mesons are given by the relations

$$J_\mu(x) = J^\dagger_\mu(x) = \frac{\bar{c}(x)\gamma_\mu q(x) + \bar{q}(x)\gamma_\mu c(x)}{2},$$

(34)

and

$$J_{5\mu}(x) = J^{5\dagger}_\mu(x) = \frac{\bar{c}(x)\gamma_\mu \gamma_5 q(x) + \bar{q}(x)\gamma_\mu \gamma_5 c(x)}{2},$$

(35)

respectively. Here, the quark operator, $q(x)$ represents the respective light quark content of $D^*(D_1)$ mesons whereas $c(x)$ represents the charm quark operator. The quark composition of $D^{*+}$, $D^{*0}$, $D_1^{*+}$ and $D_1^{*0}$ mesons are $c\bar{d}$, $c\bar{u}$, $c\bar{d}$ and $c\bar{d}$ respectively. The mass splitting of the vector $D^*(D^{*+}, D^{*0})$ and axial-vector $D_1(D_1^{*+}, D_1^{*0})$ isospin doublet will be studied under the effect of isospin asymmetric matter. However, we will see the splitting will not be much visible due to the interference of magnetic field. We can divide current-current correlator in three parts (i) vacuum (ii) static nucleon and (iii) pion bath thermal, i.e.,

$$\Pi(p) = \Pi_0(p) + \frac{\rho_N}{2m_N}T_N(p) + \Pi_{P,B}(p,T).$$

(36)

In the above, $m_N$ denotes the vacuum mass of nucleons, and $T_N(p)$ denotes the forward scattering amplitude. In the QCDSR, the pion bath term is used to incorporate the temperature effects of the medium, but in the present approach, we will introduce the finite temperature effects by the quark and gluon condensates which are calculated through the scalar fields within the chiral model. These meson fields $\sigma$, $\xi$, $\delta$, and $\chi$ are solved as coupled equations under the effect of temperature, magnetic field, density, and isospin asymmetry [18]. Therefore, we neglect the contribution of the thermal term in the correlator function and write

$$\Pi(p) = \Pi_0(p) + \frac{\rho_N}{2m_N}T_N(p).$$

(37)

In the limit of $p \rightarrow 0$, the scattering amplitude, $T_N(p)$ is represented in terms of spectral density which is parametrized in three unknown parameters $a$, $b$, and $c$. From the expression of the phenomenological spectral density and forward scattering amplitude, one can find the scattering length which is given as [93]

$$a_{D^*/D_1} = \frac{a}{f_{D^*/D_1}^2m_{D^*/D_1}^3(-8\pi(m_N + m_{D^*/D_1})).$$

(38)

To get the values of unknown parameters $a$ and $b$, we equate the Borel transformed scattering matrix of OPE side with the Borel transformed scattering matrix on the phenomenological side [32]. By doing this, we get a mathematical relation in the Borel transformed coefficients and unknown parameters $a$ and $b$, which is

$$a_{C_a} + b_{C_b} = C_f.$$  

(39)

The explicit form of Borel transformed coefficients $C_a$, $C_b$ and $C_f$ for vector meson current, $J_\mu(x)$ upto next to leading order contributions are given as [93],

$$C_a = \frac{1}{M^2} \exp \left( -\frac{m_{D^*}^2}{M^2} \right) - \frac{s_0}{m_{D^*}^2} \exp \left( -\frac{s_0}{M^2} \right),$$

(40)

$$C_b = \exp \left( -\frac{m_{D^*}^2}{M^2} \right) - \frac{s_0}{m_{D^*}^2} \exp \left( -\frac{s_0}{M^2} \right),$$

$$C_f = \frac{2m_N(m_H + m_N)}{(m_H + m_N)^2} \left( f_{D^*} m_{D^*} g_{D^* N H} \right)^2 \left\{ \frac{1}{M^2 - m_{D^*}^2 - (m_H + m_N)^2} \right\} \exp \left( -\frac{m_{D^*}^2}{M^2} \right).$$

$$\frac{1}{M^2 - m_{D^*}^2 - (m_H + m_N)^2} \exp \left( -\frac{m_{D^*}^2}{M^2} \right).$$
For the axial-vector current \( J_5 \), the following transformations are used in the vector current, where \( i = a, b, f \)

\[ C_i \rightarrow C_i \left( m_N \rightarrow -m_N, \ m_c \rightarrow -m_c, \ D^* \rightarrow D_1 \right). \]

In above equations, the \( \langle \bar{q}q \rangle_N, \langle \bar{q}iD_\mu q \rangle_N, \langle \bar{q}iD_\mu iD_\nu q \rangle_N \) and \( \langle \frac{\alpha_s}{\pi} G_{\mu\nu}^a G^{a\mu\nu} \rangle_N \) are the nucleon expectation values of quark and gluon condensates, respectively. The fraction \( \frac{1}{M^2} \)

is the Borel operator. Also, the parameter \( s_0 \) appearing in the Eq. (40) is the continuum threshold parameter and its value (8.5 GeV) is chosen to adjust the contributions from OPE side [93–95]. The in-medium quark and gluon condensates can be expressed in the nucleon expectation via relation [8, 10]

\[ \langle O \rangle_N = \frac{2m_N}{\rho_N} \left( \langle O \rangle_{\rho_N} - \langle O \rangle_{\text{vac}} \right), \]

where \( O \) represents any of the four condensate, \( \langle O \rangle_{\rho_N} \)

denotes the medium dependent expectation value and the \( \langle O \rangle_{\text{vac}} \)

denotes the vacuum expectation value. The condensates \( \langle \bar{u}u \rangle_{\rho_N}, \langle \bar{d}d \rangle_{\rho_N} \) and \( \langle \frac{\alpha_s}{\pi} G_{\mu\nu}^a G^{a\mu\nu} \rangle_{\rho_N} \) are calculated from the chiral SU(3) model and are given in Eqs. (22), (23) and (27), respectively. The value of dimension-four, \( \langle \bar{q}iD_\mu q \rangle_{\rho_N} \) and dimension-five, \( \langle \bar{q}iD_\mu iD_\nu q \rangle_{\rho_N} \) condensates are estimated from the parton distribution functions measured in deep inelastic scattering [93, 100]. The value of \( \langle \bar{q}iD_\mu iD_\nu q \rangle_{\rho_N} \) is taken as 0.18 GeV\(^2\) \( \rho_N \) [93, 101] and the \( \langle \bar{q}iD_\mu iD_\nu q \rangle_{\rho_N} \) condensate is calculated from light quark condensates using equations [8, 93, 98]

\[ \langle \bar{q}iD_\mu iD_\nu q \rangle_{\rho_N} + \frac{1}{8} \langle \bar{q}g_\alpha g_\beta G_{\mu\nu} q \rangle_{\rho_N} = 0.3 \text{ GeV}^2 \rho_N, \]

and

\[ \langle \bar{q}g_\alpha g_\beta G_{\mu\nu} q \rangle_{\rho_N} = \lambda^2 \langle \bar{q}q \rangle_{\rho_N} + 3.0 \text{ GeV}^2 \rho_N, \]

where \( \lambda \) is estimated to be 0.5 [8, 93].

Now, in Eq. (39), we have two unknown parameters but one equation. To solve this, we need one more equation. To achieve this, we differentiate the first equation by \( z = \frac{1}{M^2} \).

Therefore, the modified equation becomes

\[ a \frac{d}{dz} C_a + b \frac{d}{dz} C_b = \frac{d}{dz} C_f. \] (46)

From Eqs. (39) and (46), we can write

\[ a = \frac{C_f \left( -\frac{d}{dz} \right) C_b - C_b \left( -\frac{d}{dz} \right) C_f}{C_a \left( -\frac{d}{dz} \right) C_b - C_b \left( -\frac{d}{dz} \right) C_a} \]

\[ b = \frac{C_f \left( -\frac{d}{dz} \right) C_a - C_a \left( -\frac{d}{dz} \right) C_f}{C_b \left( -\frac{d}{dz} \right) C_a - C_a \left( -\frac{d}{dz} \right) C_b}. \] (47)

The in-medium mass shift and shift in decay constant can be expressed in terms of unknown parameters \( a \) and \( b \) via relations [93]

\[ \Delta m_{D^*/D_1}\]

\[ = 2\pi \frac{m_nm_{D^*/D_1} f_{D^*/D_1} m_{D^*/D_1}^2}{m_{D^*/D_1}^2 + f_{D^*/D_1}^2} \]

\[ \times \left( \frac{b}{2m_N} - 2f_{D^*/D_1} m_{D^*/D_1} \Delta m_{D^*/D_1} \right), \]

\[ \Delta f_{D^*/D_1}^a = \frac{1}{2f_{D^*/D_1} m_{D^*/D_1}^2} \]

\[ \left( b \rho_N - 2f_{D^*/D_1} m_{D^*/D_1} \Delta m_{D^*/D_1} \right), \]

\[ \Delta m_{D^*/D_1} = m_{D^*/D_1} + \Delta m_{D^*/D_1}, \]

\[ \Delta m_{D^*/D_1} = m_{D^*/D_1} + \Delta m_{D^*/D_1}. \]

Here, \( m_{D^*/D_1} \) denotes vacuum mass of vector and axial-vector \( D \) mesons. The in-medium mass of charged \( D^{*+} \) and \( D_1^+ \) mesons experiences an additional positive shift in mass through the interaction of the charged particle with a magnetic field, and this effect is called Landau quantization. The modified expression for mass is given as

\[ m_{D^{*+}, D_1^+} = \sqrt{m_{D^{*+}, D_1^+}^2 + |eB|}. \] (51)

As was the case for the neutral \( K \) meson, the neutral vector, and axial-vector \( D \) meson does not experience any additional mass shift due to Landau quantization.
Table 1 Values of different parameters used in the chiral model calculations

| $g_{\sigma N}$ | $g_{\zeta N}$ | $g_{\delta N}$ | $g_{\omega N}$ | $g_{\rho N}$ |
|----------------|--------------|--------------|--------------|--------------|
| 10.56          | -0.46        | 2.48         | 13.35        | 5.48         |
| $\sigma_0$ (MeV) | $\zeta_0$ (MeV) | $\chi_0$ (MeV) | $d$          | $\rho_0$ (fm$^{-3}$) |
| -93.29         | -106.8       | 409.8        | 0.064        | 0.15         |
| $m_{\pi}$ (MeV) | $m_K$ (MeV)  | $f_{\pi}$ (MeV) | $f_K$ (MeV) | $g_4$        |
| 139            | 494          | 93.29        | 122.14       | 79.91        |
| $k_0$          | $k_1$        | $k_2$        | $k_3$        | $k_4$        |
| 2.53           | 1.35         | -4.77        | -2.77        | -0.218       |

Fig. 1 In the above figure the variation of in-medium mass of vector $D^{*}(D^{**}, D^{***})$ mesons
In the above figure the variation of shift in decay constant of vector $D^*(D^{*0}, D^{*+})$ mesons.

2.3 In-medium decay width of $D_s^* \rightarrow D^* K$ meson using $^3P_0$ model

We use $^3P_0$ model to study the effect of mass shift of $D^*$ and $K$ meson on the partial decay widths of $D_s^*(2715)$ and $D_s^*(2860)$ states decaying to $D^*$ and $K$ mesons [8,64]. As discussed earlier, the parent meson $D_s^*$ is proposed to be in the $3^-(1^3D_3), 1^-(2^3S_1)$ spectroscopic states [8,64,72,77–79]. In this model, we assume the creation of quark and anti-quark pair having quantum numbers $0^{++}$. The helicity amplitude is given by [64]

\[
M^{M_{D_s^*},M_{D^*},M_K} = \gamma \sqrt{8E_{D_s^*}E_{D^*}E_K} \\
\sum_{M_{L_{D_s^*}},M_{S_{D_s^*}},M_{L_{D^*}},M_{S_{D^*}},M_{S_K},m} \langle 1m; 1 - m|00 \rangle \\
\times \langle L_{D_s^*}M_{L_{D_s^*}},S_{D_s^*}M_{S_{D_s^*}}|J_{D_s^*}M_{J_{D_s^*}} \rangle \\
\langle L_{D^*}M_{L_{D^*}},S_{D^*}M_{S_{D^*}}|J_{D^*}M_{J_{D^*}} \rangle \langle L_KM_{L_K}S_{K}M_{S_K}|J_KM_{J_K} \rangle
\]
Table 2 In above table, we tabulated the values of in-medium masses of $D^{*+}$, $D^{*0}$, $D^{*+}_1$ and $D^{*0}_1$ mesons (in units of MeV)

| $eB/m_D^2$ | $\eta = 0$ | $\eta = 0.5$ |
|-------------|------------|-------------|
|             | $T = 0$    | $T = 100$   |
|             | $\rho_0$   | $4\rho_0$   |
|             | $\rho_0$   | $4\rho_0$   |
| $m^{**}_{D^{*+}}$ |            |            |
| 3           | 1928       | 1942        |
| 5           | 1935       | 1949        |
| $m^{**}_{D^{*0}}$ |            |            |
| 3           | 1865       | 1886        |
| 5           | 1851       | 1873        |
| $m^{**}_{D^{*+}_1}$ |            |            |
| 3           | 2508       | 2494        |
| 5           | 2519       | 2505        |
| $m^{**}_{D^{*0}_1}$ |            |            |
| 3           | 2536       | 2516        |
| 5           | 2542       | 2522        |

Table 3 In above table, we tabulated the values of in-medium shift in decay constant of $D^{*+}$, $D^{*0}$, $D^{*+}_1$ and $D^{*0}_1$ mesons (in units of MeV)

| $eB/m_D^2$ | $\eta = 0$ | $\eta = 0.5$ |
|-------------|------------|-------------|
|             | $T = 0$    | $T = 100$   |
|             | $\rho_0$   | $4\rho_0$   |
|             | $\rho_0$   | $4\rho_0$   |
| $\Delta f^{**}_{D^{*+}}$ |            |            |
| 3           | -25.43     | -21.73      |
| 5           | -26.31     | -22.61      |
| $\Delta f^{**}_{D^{*0}}$ |            |            |
| 3           | -36.73     | -31.30      |
| 5           | -38.29     | -32.90      |
| $\Delta f^{**}_{D^{*+}_1}$ |            |            |
| 3           | 24.97      | 20.53       |
| 5           | 26.04      | 21.58       |
| $\Delta f^{**}_{D^{*0}_1}$ |            |            |
| 3           | 38.61      | 32.15       |
| 5           | 40.48      | 33.99       |

\[
\langle \phi^{13}_{D^*} \phi^0_K | \phi^{12}_{D^*} \phi^0_0 \rangle \langle \phi^{13}_{S_D^* M_{S_D^*}} \phi^2_{S_K M_{S_K}} | \phi^{12}_{S_D^* M_{S_D^*}} \phi^2_{S_K M_{S_K}} \rangle J_{M_{L_D^*}, M_{L_K}} (k). \tag{52}
\]

In above, $E_{D^*} = m_{D^*_1} - m_{D^*}$, $E_K = \sqrt{m^2_K + k^2_K}$ represent the effective energies of respective mesons. Here $m^{**}_{D^*}$ and $m^{**}_K$ are the in-medium masses of $D^*$ and $K$ mesons respectively. The partial wave amplitude is obtained by transformation of helicity amplitude using Jacob–Wick formula \cite{8, 64}

\[
\mathcal{M}^{IL} (D^{*}_s \to D^{*} K) = \sqrt{2L + 1 \over 2J_{D^*} + 1} \sum_{M_{J_D^*}, M_{J_K}} \langle L0 | J_{M_{J_D^*}} | M_{M_{J_D^*}} \rangle \\
\times \langle J_{M_{D^*}} | M_D^* | J_K \rangle \\
\times \langle J_{M_{D^*}} | M_{M_D^*} | M_K \rangle \langle k_{D^*} \rangle. \tag{53}
\]

In above, $M_{M_D^*} = M_{M_D^*} + M_{J_K}$, $|J_{D^*} - J_K| \leq J \leq |J_{D^*} + J_K|$ and $|J - L| \leq |J_{D^*} | \leq |J + L|$.

The decay width of $D^{*}_s$ meson can be calculated by the following formula

\[
\Gamma = \frac{|k_{D^*}|}{8\pi m_{D^*_s}} \sum_{JL} |\mathcal{M}^{IL}|^2. \tag{54}
\]

Since the decaying meson is assumed to be at rest, the magnitude of the momentum of $D^*$ and $K$ meson is same, i.e., $|k_{D^*}| = |k_{K^*}|$ and is given as

\[
|k_{D^*}| = \sqrt{\left[ m^2_{D^*_s} - (m_{D^*_s} + m_{K^*})^2 \right]} \left[ m^2_{D^*_s} - (m_{D^*_s} + m_{K^*})^2 \right] \over 2m_{D^*_s}. \tag{55}
\]

Thus, the in-medium partial decay widths of different spectroscopic states of $D^*_1 (2715)$ and $D^*_s (2860)$ states decaying to $(D^*, K)$ can be calculated.
2.3.1 $1^- (1^3D_1)$ $c \bar{s}$ state

The harmonic-oscillator wave function of $1^- (1^3D_1)$ $c \bar{s}$ state will be

$$\phi^{a=1; L=2}(k_1, k_2) = \frac{R^{7/2}}{\sqrt{15\pi^{1/4}}} \sqrt{2} \left(\frac{k_1 - k_2}{2}\right) \exp\left[-\frac{1}{8}(k_1 - k_2)^2 R^2\right].$$

(56)

where $R$ denotes the radius of the meson. We have the decay matrix amplitude [64]

$$\mathcal{M}(c \bar{s}(1^3D_1) \rightarrow 1^- + 0^-) = \frac{\gamma \sqrt{8E_D^* E_{D^*} E_K}}{\sqrt{18}} \left[\frac{1}{\sqrt{30}} I_{0,0} + \frac{1}{\sqrt{40}} I_{1,-1} + \frac{1}{\sqrt{40}} I_{1,1}\right],$$

(57)

where

$$I_{0,0} = \frac{|k_{D^*}| \pi^{1/4} R_{D^*}^{2/2} R_{D^*}^{3/2} R_K^{3/2} (\Upsilon - 1)}{(R_{D^*}^2 + R_{D^*}^2 + R_K^2)^{5/2}} \left[(R_{D^*}^2 + R_{D^*}^2 + R_K^2)(\Upsilon^2 - 1)k_{D^*}^2 + 8\right].$$
In the above figure the variation of shift in decay constant of axial-vector $D_{1}(D_{1}^{*}, D_{0}^{*})$ mesons. The symbols $D_{1}^{*}$, $D^{*}$ and $K$ denote $1^{-}(1^{3}D_{1})c\bar{s}$ state, vector $D$ meson and pseudoscalar kaon, respectively.

2.3.2 $3^{-}(1^{3}D_{3})c\bar{s}$ state

The general decay matrix amplitude for $c\bar{s}(1^{3}D_{3}) \rightarrow 1^{-} + 0^{-}$ can be written as

$$I_{1}^{1.1} = I_{1}^{1.1} = \frac{4\sqrt{3}|k_{D^{*}}|^{1/4}R_{D^{*}}^{3/2}R_{K}^{3/2}(\gamma - 1)}{(R_{D^{*}}^{2} + R_{D^{*}}^{2} + R_{K}^{2})^{3/2}}$$

$$\exp\left[-\frac{k_{D^{*}}^{2}R_{D^{*}}^{2}(R_{D^{*}}^{2} + R_{K}^{2})}{8(R_{D^{*}}^{2} + R_{D^{*}}^{2} + R_{K}^{2})}\right]$$

$$\exp\left[-\frac{k_{D^{*}}^{2}R_{D^{*}}^{2}(R_{D^{*}}^{2} + R_{K}^{2})}{8(R_{D^{*}}^{2} + R_{D^{*}}^{2} + R_{K}^{2})}\right]$$

The general decay matrix amplitude for $c\bar{s}(1^{3}D_{3}) \rightarrow 1^{-} + 0^{-}$ can be written as
Fig. 5 The mass splitting $\Delta m_0$ in the pseudoscalar $D^+$ and the masses, $m^0_{D^+}, m^0_{D^-}$ and $m^0_{mes}$ is plotted as a function of nucleonic density at $s_0=5.3$ GeV.

Fig. 6 We have showed the in-medium dependence of $s_0$ parameter on the mass-shift ($\Delta m^*_cen = m^*_cen - m_{vac}$) with respect to vacuum and the in-medium mass ($m^*_cen$) of pseudoscalar $D$ meson in the centroid approximation. Here, $m_{vac}$ is the vacuum mass of $D$ meson.

\[ \mathcal{M}(c\bar{s}(1^3D_3) \rightarrow 1^- + 0^-) = \gamma \sqrt{8E_D} \frac{E^*_K}{\sqrt{12}} \left[ \frac{2}{\sqrt{30}} I_{0,0} + \sqrt{\frac{2}{45}} I_{1,-1} + \sqrt{\frac{2}{45}} I_{1,1} \right], \]

where $I_{0,0}$, $I_{1,-1}$ and $I_{1,1}$ are same as in Eqs. (58) and (59) [64].

2.3.3 $1^- (2^3S_1)$ c$\bar{s}$ state

The harmonic oscillator wave function of the $1^- (2^3S_1)$ c$\bar{s}$ state is given by

\[ \phi_{n=2, L=0}(k_1, k_2) = \frac{1}{\sqrt{4\pi}} \left( \frac{4R^3}{\sqrt{\pi}} \right)^{1/2} \left[ \frac{3}{2} - \frac{R^2}{4} (k_1 - k_2)^2 \right] \exp \left[ -\frac{1}{8} (k_1 - k_2)^2 R^2 \right]. \]

The general decay amplitude of $c\bar{s}(2^3S_1) \rightarrow 1^- + 0^-$ is given as

\[ \mathcal{M}(c\bar{s}(2^3S_1) \rightarrow 1^- + 0^-) = \gamma \sqrt{8E_D} \frac{E^*_K}{\sqrt{12}} \left( \frac{1}{\sqrt{18}} I_{0,0} \right), \]

where

\[ I_{0,0} = \sqrt{\frac{1}{2}} \left( R^2_{D^+} + R^2_{D^*} + R^2_K \right)^{3/2} \]

\[ \left\{ -6(R^2_{D^*} + R^2_{D^*} + R^2_K)(1+\Upsilon) + R^2_{D^*} \left[ 4 + 20 \Upsilon + R^2_{D^*} + R^2_{D^*} + R^2_K \right] \right\} \]

\[ \exp \left[ -\frac{k^2_{D^*} R^2_{D^*} (R^2_{D^*} + R^2_{D^*} + R^2_K)}{8(R^2_{D^*} + R^2_{D^*} + R^2_K)} \right]. \]

In above equations, the variables $R_{D^*}, R_{D^*}$ and $R_K$ represent the radius of respective mesons.

The parameter $\alpha$ is taken to be 1 [64] and the universal parameter $\gamma$ is taken as 6.9 [64, 102]. In addition, the parameter $\Upsilon$ is expressed as $\Upsilon = \frac{R^2_{D^*}}{R^2_{D^*} + R^2_{D^*} + R^2_K}$. 

\[ \oddspace \par\]
3 Numerical results and discussions

In this section, we will discuss our observations of the magnetic field induced mass spectra of vector ($D^{*+}, D^{*0}$) and axial-vector ($D^{*+}_1, D^{0}_1$) mesons. As discussed earlier, the light quark condensates and gluon condensates have been calculated by using the chiral SU(3) model, and the different parameters used in the model are tabulated in Table 1. In present work, the mass and decay constant of $D$ mesons have been calculated using QCDSR. In this calculation, the value of charm quark mass $m_c$, running coupling constant $\alpha_s$, coupling constant $g_{DNH}$ and constant $\lambda$ are approximated to be 1.3 GeV, 0.45, 3.86 and 0.5, respectively [60,93]. Also, the vacuum masses of $D^{*+}, D^{*0}, D^{*+}_1$ and $D^{0}_1$ mesons are taken as 2.010, 2.006, 2.423 and 2.421 GeV, respectively. The vacuum values of the decay constant for vector and axial-vector mesons are taken as 0.270 and 0.305 GeV, respectively. In QCDSR, we use a proper Borel window to ensure the observed properties have minimum variation in their value for a range of Borel parameter. The range of the boundary...
of proper Borel window is selected with a constraint that within this window there should be the least variation in the medium mass and decay constant of the particular meson under investigation [32,94–97]. In the Borel sum rule, the in-medium mass of meson is calculated by the combination of vacuum and medium contributions. The former is incorporated through the Operator Product Expansion method, called as OPE side, and the latter is fetched by the hadronic contributions, i.e. phenomenological side. By using these, the in-medium properties of mesons are reproduced by taking the balanced contribution from the “pole + continuum” part. The least variation of mass with the Borel parameter, \( M \) assures that the mass is not influenced much by the continuum’s perturbative corrections which are related to the convergence of OPE. In particular, the lower boundary of the Borel window is fixed by considering the contribution from the OPE side, whereas, the upper boundary of the Borel window is obtained by imposing the condition that pole contribution should be more than the continuum [93,94,98,99,103]. Following the above approach, in the present investigation contributions from highest order terms should be less than of lowest order terms i.e. \( \langle \bar{q}q \rangle_{\rho_N} \) condensate must dominate over other condensates and as we will see later, these lowest order condensates give maximum contribution to the mass-shift of \( D \) mesons. Therefore, the value of lower boundary \( M_{\text{min}} \) is extracted by using the inequality

\[
0.1 \geq \frac{\Pi^H_{\text{OPE}}(M_{\text{min}})}{\Pi^H_{\text{OPE}}(M_{\text{min}})},
\]

where \( \Pi^H_{\text{OPE}}(M) \) denotes the total OPE side contributions whereas \( \Pi^H_{\text{OPE}}(M) \) represents the OPE contributions from higher terms only. Unlike even-odd QCD sum rules of Ref. [98], no perturbative terms having continuum contribution are present on OPE side in the QCD sum rules used in the present work [32,103]. On the phenomenological side, the continuum contribution in the spectral density are absorbed through the term \( \delta (\omega^2 - s_0) \) [32,94]. The upper boundary \( M_{\text{max}} \) is obtained such that the exponential factor \( e^{-s_0/M^2} \) suppress the continuum contributions substantially and also a proper Borel platform appears in the calculations [93,94]. Following above, the Borel window for masses of \( (D^+,D^{\ast 0}) \) and \( (D_1^+,D_1^{\ast 0}) \) are taken as (4.5–5.5) and (6–7) GeV\(^2\), respectively, whereas the range of Borel window for decay constant of \( (D^+,D^{\ast 0}) \) and \( (D_1^+,D_1^{\ast 0}) \) are taken as (2–3) and (7–9) GeV\(^2\), respectively. We have divided our discussion into two parts.

### Table 4

| \( D_s^*(2715) \rightarrow D^* K \) | \( eB/m_s^2 \) | \( \eta = 0 \) | | \( \rho_0 \) | 4\( \rho_0 \) | \( \rho_0 \) | 4\( \rho_0 \) | \( \eta = 0.5 \) | | \( \rho_0 \) | 4\( \rho_0 \) | \( \rho_0 \) | 4\( \rho_0 \) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| \( 1^- (1^3 D_1) \) | 0 | 5.9 | 8.47 | 5.34 | 6.88 | 5.84 | 6.31 | 5.28 | 5.26 |
| | 6 | 4.84 | 9.52 | 4.41 | 7.47 | 4.22 | 4.03 | 4.67 | 5.63 |
| \( 3^- (1^3 D_3) \) | 0 | 2.73 | 4.49 | 2.37 | 3.35 | 2.69 | 2.95 | 2.33 | 2.26 |
| | 6 | 2.07 | 5.31 | 1.80 | 3.79 | 1.69 | 1.54 | 1.95 | 2.52 |
| \( 1^- (2^3 S_1) \) | 0 | 18.32 | 17.84 | 18.37 | 18.71 | 18.44 | 19.23 | 18.48 | 19.40 |
| | 6 | 18.28 | 16.69 | 18.15 | 18.02 | 18.22 | 19.01 | 18.34 | 19.06 |

### Table 5

| \( D_s^*(2860) \rightarrow D^* K \) | \( eB/m_s^2 \) | \( \eta = 0 \) | | \( \rho_0 \) | 4\( \rho_0 \) | \( \rho_0 \) | 4\( \rho_0 \) | \( \eta = 0.5 \) | | \( \rho_0 \) | 4\( \rho_0 \) | \( \rho_0 \) | 4\( \rho_0 \) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| \( 1^- (1^3 D_1) \) | 0 | 19.84 | 25.82 | 18.56 | 22.60 | 19.82 | 21.69 | 18.53 | 19.40 |
| | 6 | 17.36 | 27.42 | 16.29 | 23.43 | 15.96 | 16.34 | 17.02 | 19.89 |
| \( 3^- (1^3 D_3) \) | 0 | 13.91 | 19.46 | 12.76 | 16.36 | 13.88 | 15.46 | 12.71 | 13.35 |
| | 6 | 11.70 | 21.13 | 10.76 | 17.22 | 10.96 | 10.66 | 11.38 | 13.85 |
| \( 1^- (2^3 S_1) \) | 0 | 10.56 | 7.23 | 11.43 | 9.30 | 10.68 | 10.21 | 11.56 | 11.88 |
| | 6 | 12.20 | 5.98 | 12.92 | 8.41 | 13.29 | 13.98 | 12.53 | 11.18 |
3.1 In-medium mass and shift in decay constant of vector $D^*$ and axial-vector $D_1$ mesons

In Fig. 1 (Fig. 2), we represent the in-medium mass (decay constant) of $D^{*+}$ and $D^{*0}$ mesons in hot and dense isospin asymmetric nuclear medium as a function of the magnetic field. The numerical values of in-medium masses and decay constants are given in Tables 2 and 3, respectively. We observe that, for any constant value of temperature $T$, isospin asymmetric parameter $\eta$ and nuclear density $\rho_N$ of the medium, the finite magnetic field $eB/m^2_\pi$ of the medium causes an enhancement (drop) in the mass (shift in decay constant) of $D^{*+}$ meson, whereas drop (drop) in the mass (shift in decay constant) of $D^{*0}$ meson. For example, in symmetric nuclear medium, at temperature $T=0$ and nuclear density $\rho_N=\rho_0$, the masses (shift in decay constant) of $D^{*+}$ and $D^{*0}$ mesons are observed to be 1928 (−25.43) and 1865 (−36.73) MeV, respectively. For example, at finite magnetic field $eB=3m^2_\pi$, the above values shift to 1935 (−26.31) and 1859 (−38.29) MeV, respectively. Further, we notice that the mass of $D^{*+}$ is more sensitive to the presence of the magnetic field of the medium, as compared to $D^{*0}$ mesons, and this is because of the Landau effect. In this effect, the charged particle couple with the magnetic field and starts revolving in circular levels called Landau levels. This quantization modifies the scalar and vector density of the nucleons and in turn, the quark and gluon condensates [10]. Further, for any constant value of the magnetic field, isospin asymmetric parameter, and temperature of the medium the masses and decay constants of these mesons decrease as a function of the nuclear density of the medium. For example, in cold symmetric nuclear medium and at four times nuclear saturation density, i.e., $\rho_N=4\rho_0$ the masses (shift in decay constant) of $D^{*+}$ and $D^{*0}$ mesons are observed to be 1872 (−40.41) and 1772 (−61.23) MeV, respectively. Also at $eB=5m^2_\pi$ the above values shift to 1877 (−41.63) and 1760 (−64.25) MeV, respectively. Moreover, in the finite magnetic field, the effect of the finite temperature on the in-medium mass and decay constants of $D^{*+}$ and $D^{*0}$ mesons is opposite to that of the nuclear density of the medium. For example, in symmetric nuclear medium, at finite magnetic field of the medium, i.e., $eB=3m^2_\pi$ (5$m^2_\pi$) the masses of...
$D^{*+}$ and $D^{*0}$ mesons are observed as 1942 (1949) and 1886 (1880) MeV, at temperature $T = 100$ MeV and $\rho_N = \rho_0$, respectively. Likewise, at $\rho_N = 4\rho_0$, the above values modify to 1881 (1886) and 1785 (1773) MeV, respectively. We can compare these values at zero temperature situations, as discussed earlier.

Apart from this, finite isospin asymmetry of the medium causes splitting between the in-medium masses (decay constants) of vector $D^*$ mesons. For example, in an isospin asymmetric medium, $\eta = 0.5$, $\rho_N = \rho_0$ and $eB = 3m_\pi^2$, the values of the masses (decay constants) of $D^{*+}$ and $D^{*0}$ mesons observed to be 1923 ($-25.10$) and 1890 ($-30.17$) MeV at $\rho_N = \rho_0$, temperature $T = 0$. Likewise, at $eB = 5m_\pi^2$ the above value change to 1940 ($-24.9$) and 1891 ($-29.96$) MeV, respectively. Evidently, these values are different from the masses and decay constants of $D^{*+}$ and $D^{*0}$ mesons as discussed for the symmetric matter situation. The presence of a magnetic field helps to increase the asymmetry effect of the medium. The scalar/vector density of the proton has direct magnetic field dependence whereas for neutron there is no such dependence. The increase in the magnetic field increases the inequality between the scalar/vector density of the neutron and proton and hence gives rise to the crossover asymmetry effects [10].

In Fig. 3, we observe an enhanced in-medium mass of axial-vector $D_1$ meson in the magnetized nuclear matter as compared to non-magnetized nuclear matter. In particular, in cold symmetric nuclear medium and at nuclear saturation density, the values of the mass of $D_1^+(D_1^0)$ meson observed as 2508 (2536) and 2519 (2542) MeV, respectively, at $eB = 3m_\pi^2$ and $eB = 5m_\pi^2$. In addition to the magnetic field, the finite baryonic density of the medium also causes an increase in the values of the masses of $D_1^+(D_1^0)$ meson. For instance, in cold symmetric nuclear medium and $\rho_N = 4\rho_0$, the above values modifies to 2509 (2573) and 2522 (2584) MeV, respectively. Similar to vector mesons, the impact of the finite temperature of the medium on the in-medium mass of axial-vector mesons is also opposite to that of the density and magnetic field. For example, at $T=100$ MeV, $eB = 3m_\pi^2$ and $eB = 5m_\pi^2$, the masses of $D_1^+(D_1^0)$ meson are observed to be 2494 (2516) and 2505 (2522) MeV, respectively, at $\eta=0$, $\rho_N=\rho_0$. On the other hand, finite isospin asymmetric parameter causes further modification in the masses of $D_1^+$ and $D_1^0$ mesons. As discussed earlier, the crossover effects represent the behav-
ior of quark and gluon condensates which in turn reflects the medium modification of the scalar fields in the nuclear medium [10,18].

In Fig. 4, we have shown the shift in decay constants of axial-vector $D$ mesons. As similar to the masses, the shift in the in-medium decay constants of $D_1^{+}$ and $D_1^{0}$ mesons increase with the increase in magnetic field and these are shown particularly, at $eB=3m_{\pi}^{2}$ and $eB=5m_{\pi}^{2}$, the values of the shift in decay constants of $D_1^{+}$ ($D_1^{0}$) mesons are observed as 24.97 (38.61) and 26.04 (40.48) MeV, respectively, at $T=0$, $\eta=0$ and $\rho_N=\rho_0$. Likewise, at $\rho_N=4\rho_0$ the above values shift to 27.85 (52.50) and 29.42 (56.62) MeV, respectively. Whereas, the role of finite temperature is to decrease the shift in the decay constants. For example, in hot nuclear medium, say at $T=100$ MeV, $\rho_N=\rho_0$ and $\eta=0$, the values of decay constants of $D_1^{+}$ ($D_1^{0}$) mesons are observed to be 20.53 (32.15) and 21.58 (33.99) MeV, at $eB=3m_{\pi}^{2}$ and $eB=5m_{\pi}^{2}$, respectively. The finite isospin asymmetry of the medium also causes the splitting in the in-medium decay constants of $D_1^{0}$ and $D_1^{+}$ mesons. In the present investigation, we found the opposite behavior of the masses and decay constants of vector $D^*$ and axial-vector $D_1$ mesons. By ignoring the additional mass shift due to the Landau effect, we observe that the value of vector $D^*$ meson’s mass and shift in decay constant decreases as a function of magnetic field whereas for axial-vector $D_1$ mesons these in-medium attributes increase. This happens because of the negative sign with the term $\frac{m_c\langle\bar{q}q\rangle_N}{2}$ (see Eq. 41), of the Borel transformed equation. This causes negative and positive values of the scattering length for $D^*N$ and $D_1N$, scattering, respectively in the presence of a finite magnetic field [60,93]. In the absence of a magnetic field, the in-medium mass of heavy vector and axial-vector have also been studied in the literature [60,61,93]. However, in our knowledge, no work is available to date which studies the impact of the magnetic field on the vector and axial-vector $D$ mesons.

As discussed earlier, we used centroid approximation and found the average mass shift of particle and antiparticle of vector and axial vector $D$ mesons, respectively. However, to have an idea about the mass splitting of particle-antiparticles, using even-odd QCD sum rules [33,98] along with the chiral SU(3) model, we calculate the mass splitting between pseu-

---

**Fig. 10** The in-medium decay width of $D_s^*(2715)$ in $3^- (1^3D_3)$ spectroscopic state is plotted with respect to magnetic field for different conditions of the medium.
Fig. 11 The in-medium decay width of $D_s^*(2860)$ in $3^-(1^3D_3)$ spectroscopic state is plotted with respect to magnetic field for different conditions of the medium.

doscalar $D^+$ and $D^-$ meson as a function of the density of nuclear matter. By using medium modified quark and gluon condensates from the chiral model in the expressions of even-odd QCD sum rules of [33,98]. The detailed explanation of the even-odd sum rules for the pseudoscalar case is given in the appendix A. We observed the significant mass splitting between the $D^+$ and $D^-$ meson as a function of medium density. We have plotted the mass splitting in the subplot (a) of Fig. 5. In this figure, we observe the negative in-medium mass-shift which increases substantially with medium density. Also, in Fig. 5, subplot (b), we plot the $m_{D^+}^*$, $m_{D^-}^*$ and the centroid approximation, $m_{cen}^* = \frac{m_{D^+}^* + m_{D^-}^*}{2}$. We observe that the mass of $D^+$ meson first increases and then decreases concerning density. The mass of $D^-$ meson increase with the increase in density. These observations are complimentary to the work done using QCD sum rules (centroid approximation) [10,93], and chiral model [19]. We have also added the Fig. 6 showing effect of $s_0$ medium dependence on the in-medium mass of pseudoscalar $D$ mesons. The medium dependence of $s_0$ parameter is incorporated by the relation, $s_0^\pm = s_0^\pm \pm \frac{\rho}{\rho_0}$ [33], here we use $s_0^\pm \rho$ for positive contribution and $s_0^\pm \rho$ for negative. We observe significant dependence of mass on the continuum threshold parameter. Analogous to the pseudoscalar case, we expect appreciable density dependence of mass splitting between vector/axial vector $D$ meson charged partners as well and will consider it in the future work.

In Fig. 7, the in-medium masses of $K^+$ and $K^0$ mesons are also plotted in hot, dense and asymmetric magnetized nuclear matter. As a function of the magnetic field, we observe a positive shift in the mass of $K^+$ mesons in both low and high density of nuclear matter. On the other hand, for neutral $K^0$ meson, we observe a negative shift in mass for the magnetic field. The negative shift is appreciable in the high density regime. This is because, the self-energy of kaons (see Eq. 31) directly depends upon the scalar and vector density of the nucleons. The charged $K^+$ meson largely depends upon the scalar and vector density of proton whereas the uncharged $K^0$ meson depends upon the neutron’s scalar and vector density. In addition to this, in the presence of a magnetic field, the $K^+$ meson experience an additional positive shift in the mass whereas the uncharged $K$ meson does not experience any additional shift. It may be noted that in the presence of a magnetic...
field the value of isoscalar $\delta$ field no longer remains zero in symmetric nuclear matter [10] which also induce additional asymmetry effect. In our findings at nuclear saturation density, $\eta=0$, $T = 50$ MeV and $eB = 5m_\pi^2$, the medium mass of $K^+ (K^0)$ is observed as 605 (520) MeV. At $\eta = 0.5$, and the same other parameters the value changes to 602 (535) MeV. The Weinberg Tomozawa term (first term of the self-energy), leads to an increase in the mass of kaon and the second term gives attractive contributions. The remaining range terms gives repulsive (third term) and attractive contributions ($d_1$ and $d_2$ terms). Furthermore, we observe a little effect of the temperature on the mass of kaons in low density regime, however, this effect is appreciable in the high density regime. In Ref. [56], the author studied the in-medium mass of $K$ and $\bar{K}$ meson in the asymmetric magnetized nuclear matter using the chiral SU(3) model at zero temperature. In this article, they calculated the in-medium mass of $K$ and $\bar{K}$ meson with and without taking the account of anomalous magnetic moment of nucleons. Now, we will utilize the in-medium mass of $D^*$ and $K$ meson to calculate the magnetic field induced decay width of $D^*_s$ meson.

Now, we will calculate the in-medium decay width of the $D^*_s$ meson into $D^*$ and $K$ mesons. The $D^*_s$ meson is expected to have one of the quantum state $1^{-}(1^3D_1)$, $3^{-}(1^3D_3)$ and $1^{-}(2^3S_1)$ with mass of 2860 or 2715 MeV [8,64]. In the present work, we have calculated the decay width for three different cases for each value of mass. The results will be compared with the theoretical findings and empirical values, which will help in the future to assign a quantum state to parent meson. In the present investigation, we have neglected the medium modifications of the parent meson. To the best of our knowledge, no study has been done to study the in-medium properties of $D^*_s(2715)$ and $D^*_s(2860)$ meson in the presence or absence of a magnetic field in the nuclear medium. In Tables 4 and 5, we have tabulated the decay width of $D^*_s(2715)$ and $D^*_s(2860)$ mesons, respectively at $R_A=2.94$ GeV$^{-1}$ for $1^{-}(1^3D_1)$, $3^{-}(1^3D_3)$ states and 3.2 GeV$^{-1}$ for $1^{-}(2^3S_1)$ state. The values of $R_{D^*}$ and $R_K$ are taken as 2.70 and 2.17 GeV$^{-1}$, respectively [8,64]. Note that the parameter $R_A$ appears in the harmonic oscillator wave functions. To fix its value, one calculates the RMS radius using harmonic oscillator wave function and compare to the value of RMS radius calculated by solving the Schrodinger equation for the
In Fig. 8, for a given value of temperature, density, and isospin asymmetry, we have plotted the decay width of $D_s^*(2715)$ meson in $1^- (1^3D_1)$ state as a function of a magnetic field. We observe a linear decrease in the value of decay width with increasing magnetic field. Further, the value of decay width increases when we move from low density to high density. At $\rho_N=4\rho_0$, we observe the value of decay width increases linearly as a function of the magnetic field. The inclusion of isospin asymmetric effects shows a significant impact on the decay width. The temperature effects are more appreciable in the asymmetric nuclear matter as in the current framework the scalar and vector densities consist of Fermi distribution functions. In Fig. 9, in the same quantum state, when we change the mass of $D_s^*$ meson from 2715 to 2860 MeV, we observe similar behavior of decay width for different medium parameters. The only difference in $D_s^*(2860)$ is that we observe a large value of decay width for each parameter as compared to $D_s^*(2715)$ case.

In Figs. 10 and 11, we have plotted the decay width of $D_s^*$ meson in $3^- (1^3D_3)$ state as a function of the magnetic field, asymmetry, temperature and density for mass values 2715 and 2860 MeV, respectively. At $\rho_N=\rho_0$, we observe the non-zero value of the decay width which decreases linearly as we increase the magnetic field. We observe a similar appreciable effect of density on the decay width of the $D_s^*$ meson. The effect of the magnetic field is more pronounced in the high density regime than the low density regime. Also, the asymmetry effects are more visible in the high temperature regime. This is because the decay width inversely relates to the in-medium mass. If mass of $K$ and $D^*$ meson decrease more in the medium then we observe more increase in the decay width. Furthermore, in the same spectroscopic state, we observed a large value of decay width for $D_s^*(2860)$ as compared to $D_s^*(2715)$ meson.

In Figs. 12 and 13, the decay width of $D_s^*(2715)$ and $D_s^*(2860)$ meson in $1^- (2^3S_1)$ quantum state is shown. In this spectroscopic state, we observe a significant value of decay width of $D_s^*$ meson in both mass states. In the former case, we observe a negligible change in the value of decay width for the magnetic field. The effect of temperature and isospin asymmetry are also negligible in this case. The formula of decay width consists of two terms, polynomial and exponential.
Fig. 14 The in-medium decay width of $D_s^*$ decaying to $D^{*+} K^0$ mesons in various spectroscopic states ($1^-(1^3D_1)$, $3^-(1^3D_3)$ and $1^-(2^3S_1)$) is plotted with respect to magnetic field for $\rho_N = \rho_0$ and $4\rho_0$ at $T = 50$, and $\eta = 0$. We have shown the results for 3% and 5% drop in $D_s^*$ from vacuum value.

This behavior reflects the interplay between these two parts. Moreover, in the latter case, we observe appreciable asymmetric effects in the high temperature regime. We observe less value of decay width as compared to the former case. In this case, we observe that the value of decay width decreases when we move from low density to high density which is opposite to the behavior observed for the $D_s^*$ mesons in the $D$ spectroscopic states. The vacuum decay width of the decay mode $D_s^*$ (in different spectroscopic states) into $D^{*+} K$ and various other modes has been calculated in literature [64]. But the present theoretical and empirical data is not enough to assign a quantum state to the $D_s^*$ meson.

Furthermore, as mentioned earlier in our above discussion of decay width we considered the vacuum masses of $D_s^*(2715)$ and $D_s^*(2860)$ states. To understand the impact of medium modified mass of parent mesons on the in-medium partial decay width we follow the work on the mass modification of ground state $D_s^*$ mesons where in cold symmetric nuclear medium, at $\rho_B = \rho_0$ and $4\rho_0$ a drop of 2.3% and 3.7% with respect to the vacuum mass was observed [9]. If a similar decrease in the masses of $D_s^*(2860)$ states is considered, we observe a further decrease in the decay width of $D_s^*(2715)$ and $D_s^*(2860)$ mesons. In Fig. 14, we represent the impact of the drop in the mass of parent mesons on their in-medium partial decay widths. We show the result consid-
er the drop of 3% and 5%. In the symmetric nuclear matter at zero magnetic fields, for $\rho_B=\rho_0$ and temperature $T=50$ MeV, partial decay widths were observed to be 20, 12 and 9.8 MeV with spectroscopic states $1^-(1^3D_1), 3^-(1^3D_3)$ and $1^-(2^3S_1)$, respectively, when no modification in the masses of parent mesons was considered. However, for the same medium conditions if we allow 3% (5%) drop in the parent meson mass then the above values of decay width shift to 10.19 (6.02), 5.79 (2.81), and 16.72 (18.31) MeV, respectively. Likewise, at $\rho_B=4\rho_0$, for a drop of 3% (5%) in parent meson mass, the values of decay width are observed to be 13.97 (8.63), 8.78 (4.60), and 14.60 (17.77) MeV. These can be compared to the values 19.84 (25.82), 13.91 (19.46), and 10.56 (7.23) MeV at $\rho_N=\rho_0$ ($4\rho_0$), when medium modification of parent mesons was not considered. Clearly, the shift in the parent meson mass has a significant impact on their partial decay width and this can be explored further in near future.

4 Conclusion

To conclude, we studied the medium modification of the masses and the shift in decay constant of vector and axial-vector $D$ meson under the influence of a strong magnetic field. We also studied the effect of isospin asymmetry, temperature, and density alongside the magnetic field. The in-medium mass of $K$ meson is calculated by using the chiral model and to calculate the in-medium mass of $D$ mesons, we used the unified approach of QCD Sum Rule (QCDSR) and chiral SU(3) model. The scalar (vector) density of nucleons modifies differently due to the presence of the magnetic field and isospin asymmetry. The magnetic field interacts differently with charged protons and gives rise to the Landau effect. The non-linear coupled equations of different mesons fields are solved by incorporating the magnetic field induced density which in turn generates the light quark and gluon condensates. We found appreciable effects of a strong magnetic field on the charged vector $D^{*+}$ and axial-vector $D^{*+}_1$ mesons whereas, for uncharged $D$ mesons, the effects were less appreciable. We calculated negative (positive) mass shift for vector (axial-vector) uncharged $D$ mesons and for charged vector and axial-vector $D$ meson, we found a positive shift in mass concerning the magnetic field. The density and temperature effects on these mesons were also appreciable. The isospin asymmetry effects are suppressed by the Landau quantization for the charged meson case whereas for neutral mesons it shows crossover behavior. By utilizing the in-medium mass, we calculated the magnetic field induced decay width of $D^{*+}_s$ meson decaying to $D^{*+}$ and $K^0$ meson via $\gamma P_0$ model and observed prominent modifications in the decay width of $D^{*+}_s$ mesons in $1^-(1^3D_1), 3^-(1^3D_3)$ and $1^-(2^3S_1)$ spectroscopic states. The observed decay probability will be compared with the experimental results of future experiments such as CBM, PANDA, J-PARC and NICA.

Acknowledgements One of the authors, (R.K) sincerely acknowledge the support towards this work from Ministry of Science and Human Resources Development (MHRD), Government of India via Institute fellowship under National Institute of Technology Jalandhar.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors’ comment: Data is represented in form of figures and given in tables in the manuscript.]

Appendix A: Even-odd QCD Sum Rules

QCD Sum Rules are another formalism to study the physics of non-perturbative regime [32–34]. This formalism is based on the Borel transformation which is a mathematical technique to avoid the divergence in the perturbative expansion [34,36]. To relate the mesons properties from medium to the vacuum, we start with the two-point current co-relation function

$$\Pi(p) = i \int d^4 y \, e^{i p y} \left\langle \left( \bar{O} | T \left( j \left( p \right) \right) ^\dagger (0) | \bar{O} \right) \right\rangle$$

(A1)

which represents time-ordered product of the meson currents $j(y)$ and $j^\dagger(0)$. To study the in-medium mass of $D^+$ and $D^-$ mesons separately, we split the co-relation function $\Pi(p_0, p)$ into an even ($e$) and odd ($o$) part as

$$\Pi(p_0, p) = \Pi_e(p_0^2, p) + p_0 \Pi_o(p_0^2, p).$$

(A2)

Explicitly,

$$\Pi_e(p_0, p) = \frac{1}{2} \left( \Pi(p_0, p) + \Pi(-p_0, p) \right) = \Pi_e(-p_0, p),$$

(A3)

and

$$\Pi_o(p_0, p) = \frac{1}{2p_0} \left( \Pi(p_0, p) - \Pi(-p_0, p) \right) = \Pi_o(-p_0, p).$$

(A4)

In the rest frame, the OPE side is derived using pseudoscalar current $j_{D^+} = i\bar{d} \gamma_5 c = j_{D^-} = j_{D^+}^\dagger(x) = i\bar{c} \gamma_5 d$. By application of the Borel transformation operator on the OPE side with the condition of large pole mass of charm quark $m_c$ and $m_d \rightarrow 0$, we obtain

$$B \left[ \Pi_e^{OPE} (\omega^2, p = 0) \right] \left( M^2 \right) = \frac{1}{\pi} \int_{m_c^2}^\infty d\omega \, e^{-\omega^2/M^2} \text{Im} \Pi_{per} (\omega, p = 0) + e^{-m_c^2/M^2} \left( -m_c \langle d \bar{d} \rangle + \frac{1}{2} m_c^2 \frac{M^2}{2M^2} \right) \langle d \bar{d} \rangle G_d,$$
along with other sum rules parameters are mentioned in the in-medium sum rules for the even part up to mass \( M \):

\[
\left[ (\bar{d}D_0^2 - \frac{1}{8} \langle d\sigma G d \rangle) \right], \quad (A5a)
\]

\[
E \left[ \Pi_o^{PE}(q^2, \mathbf{p} = 0) \right] (M^2)
\]

\[
= e^{-m_c^2/M^2} \left( \langle d^1d \rangle - 4 \left( \frac{m_c^2}{2M^2} - \frac{1}{M^2} \right) \right)
\]

\[
(\langle d^1D_0^2 \rangle - \frac{1}{M^2} \langle d^1\sigma G d \rangle), \quad (A5b)
\]

where

\[
\text{Im} \Pi^{per}(p^2)
\]

\[
= \frac{3}{8\pi} \frac{(p^2 - m_c^2)^2}{p^2} \left[ 1 + \frac{4\alpha_s(p^2)}{3\pi} \right] \left[ \frac{9}{4} + 2\text{Li}_2\left( \frac{m_c^2}{p^2} \right) \right]
\]

\[
+ \ln \left( \frac{p^2}{p^2 - m_c^2} \right) \ln \left( \frac{p^2}{p^2 - m_c^2} \right)
\]

\[
+ \frac{3}{2} \ln \left( \frac{m_c^2}{p^2 - m_c^2} \right) + \ln \left( \frac{p^2}{p^2 - m_c^2} \right)
\]

\[
+ \frac{m_c^2}{p^2} \ln \left( \frac{p^2}{p^2 - m_c^2} \right) + \frac{m_c^2}{p^2} \ln \left( \frac{p^2}{p^2 - m_c^2} \right). \quad (A6)
\]

In the above equation, \( \text{Li}_2(y) = -\int_0^y z^{-1}\ln(1 - z)dz \) is the Spence function.

The physical quantity mentioned inside \( \langle \rangle \) in Eqs. (A5b) and (A5a) are known as condensates. The density dependent down quark condensate, \( \langle \bar{d}d \rangle \) and scalar gluon condensate, \( \left( \frac{G}{\pi} G^2 \right) \), are calculated from chiral SU(3) model by Eqs. (23) and (27), respectively and the values of other in-medium quark and gluon condensates are taken from Ref. [33]. The numerical values of these in-medium condensates along with other sum rules parameters are mentioned in the Table 6. In the Borel transformed sum rules of pseudoscalar \( D \) meson, the charm quark mass, \( m_c \) plays a crucial role to intensify the contribution of light quark condensate, \( \langle \bar{d}d \rangle \).

The in-medium sum rules for the even part up to mass dimension 5, can be rewritten in the functional form [33,98]

\[
h (M, s_0, m_c, \mu) \equiv m_+ H_+ e^{-m_c^2/M^2} + m_- H_- e^{-m_c^2/M^2}
\]

\[
= \frac{1}{\pi} \int_{m_0^2}^{\infty} ds e^{-s/M^2} \text{Im} \Pi^{per}(s)
\]

\[
e^{-m_c^2/M^2} \left( -m_c \langle \bar{d}d \rangle \right) + \left( \frac{m_c^3}{2M^4} - \frac{m_c}{M^2} \right)
\]

\[
+ \frac{1}{12} \frac{\alpha_s G^2}{\pi} + \left[ \frac{7}{18} + \frac{1}{3} \ln \left( \frac{m_c^2}{M^4} - \frac{2\gamma}{3} \right) \right]
\]

\[
\left( \frac{m_c^2}{M^2} - 1 \right) - \frac{2m_c^2}{M^2} \left( \frac{\alpha_s}{\pi} \left( \frac{G^2}{v^2} - \frac{G^2}{4} \right) \right)
\]

\[
+ 2 \left( \frac{m_c^2}{M^2} - 1 \right) \langle d^1d \rangle + 4 \left( \frac{m_c^3}{2M^4} - \frac{m_c}{M^2} \right)
\]

\[
\left[ \langle \bar{d}D_0^2 \rangle - \frac{1}{8} \langle d\sigma G d \rangle \right], \quad (A5a)
\]

\[
\left( \langle d^1d \rangle - 4 \left( \frac{m_c^3}{2M^4} - \frac{1}{M^2} \right) \right)
\]

\[
\left[ \langle d^1D_0^2 \rangle - \frac{1}{M^2} \langle d^1\sigma G d \rangle \right), \quad (A5b)
\]

\[
\langle d\sigma G d \rangle \rangle + \frac{1}{12} \frac{\alpha_s G^2}{\pi} + \left[ \frac{7}{18} + \frac{1}{3} \ln \left( \frac{m_c^2}{M^4} - \frac{2\gamma}{3} \right) \right]
\]

\[
\left( \frac{m_c^2}{M^2} - 1 \right) - \frac{2m_c^2}{M^2} \left( \frac{\alpha_s}{\pi} \left( \frac{G^2}{v^2} - \frac{G^2}{4} \right) \right)
\]

\[
+ 2 \left( \frac{m_c^2}{M^2} - 1 \right) \langle d^1d \rangle + 4 \left( \frac{m_c^3}{2M^4} - \frac{m_c}{M^2} \right)
\]

\[
\left[ \langle \bar{d}D_0^2 \rangle - \frac{1}{8} \langle d\sigma G d \rangle \right), \quad (A7)
\]

where \( s_0 \) is the continuum threshold parameter and its density dependence is given by the relation, \( s_0^2 = s_0^2 \pm \frac{\rho}{\rho_0} \) [33].

Now, the in-medium sum rules for the odd part can be written as

\[
s (M, m_c) \equiv H_+ e^{-m_c^2/M^2} - H_- e^{-m_c^2/M^2}
\]

\[
= e^{-m_c^2/M^2} \left( \langle d^1d \rangle - 4 \left( \frac{m_c^3}{2M^4} - \frac{1}{M^2} \right) \right)
\]

\[
\langle d^1D_0^2 \rangle - \frac{1}{M^2} \langle d^1\sigma G d \rangle \rangle. \quad (A8)
\]

Using the above two equations, the in-medium mass of \( D^+ \) and \( D^- \) can be evaluated using the following formula

\[
m_{\pm}^2 = \left[ \frac{1}{4} \left( \frac{s (M, m_c)}{h^2 (M, s_0, m_c, \mu)} + s (M, m_c, \mu) \right) \right]^2
\]

\[
+ \frac{1}{2} \left( \frac{s (M, m_c)}{h^2 (M, s_0, m_c, \mu)} + s (M, m_c, \mu) \right) \frac{1}{\pi} \left( \frac{8}{2} \right)
\]

\[
\left[ \frac{s (M, m_c)}{h^2 (M, s_0, m_c, \mu)} + s (M, m_c, \mu) \right] \right]^2.
\]

\[
(A9)
\]

where \( z = 1/M^2 \), with \( M \) as a Borel mass parameter. Clearly, the in-medium mass is a function of \( M \) and to get a physical valid mass one should choose a proper Borel Window [98]. To reproduce the properties of hadronic medium, there should be balanced contribution from “pole + continuum” ansatz i.e. the perturbative contributions from the continuum part should not influence the result in a larger extent. The higher excitations are controlled by the exponential factor in the perturbative integral which make sure that the continuum contribution to the spectral integral is less than the pole contribution. The expression to calculate maximum Borel mass, \( M_{\text{max}} \) is given as

\[
1 \geq \frac{1}{\pi} \int_{s_0^2}^{\infty} ds \text{Im} \Pi^{per}(s) e^{-s/M_{\text{max}}^2}.
\]

\[
(A10)
\]
For the lower bound $M_{\text{min}}$ of the Borel window, the highest mass dimension terms, should not provide contribution greater than 10% to the OPE side of sum rule. This condition is implied by the following inequality

\[ e^{-m^2/M_{\text{min}}} \left( \frac{m^2}{M_{\text{min}}} (\bar{d} g_\sigma \sigma \bar{d} + 2 \frac{m^2}{M_{\text{min}}} (\bar{d} D^2 g_\sigma \bar{d} - \frac{1}{3} \bar{d} g_\sigma \sigma \bar{d})) \right) \leq \frac{1}{2} \int_{i\epsilon}^{i\infty} ds \text{Im}\Pi_{\text{per}}(s) e^{-s/M_{\text{min}}} + h(M_{\text{min}}, s_0, m, \mu) \]  

(A11)

Further, the mass-splitting between charged partners ($D^+$, $D^-$) can be calculated as

\[ \Delta m^* = m^*_+ - m^*_- \]  

(A12)

References

1. T. Matsui, H. Satz, Phys. Lett. B 178, 416 (1986)
2. J. D. Bjorken, Fermilab-Pub-82/59-THY, Batavia (1982)
3. N. Masera (HELIOS-3 collaboration), Nucl. Phys. A 590, 93c (1995)
4. W.K. Wilson et al., DLS Collaboration. Phys. Rev. C 57, 1865 (1998)
5. D.K. Srivastava, R. Chatterjee, Phys. Rev. C 80, 054914 (2009)
6. A. Capella, Phys. Lett. B 364, 175 (1995)
7. S. Soff et al., Phys. Lett. B 471, 89 (1999)
8. R. Chhabra, A. Kumar, Eur. Phys. J. A 53, 105 (2017)
9. R. Chhabra, A. Kumar, Eur. Phys. J. C 77, 726 (2017)
10. R. Kumar, A. Kumar, Phys. Rev. C 101, 015202 (2020)
11. R. Chhabra, A. Kumar, Phys. Rev. C 98, 025205 (2018)
12. D.E. Kharzeev et al., Nucl. Phys. A 803, 227 (2008)
13. K. Fukushima et al., Phys. Rev. D 78, 074033 (2008)
14. V.V. Skovov et al., Int. J. Mod. Phys. A 24, 5925 (2009)
15. D. Kharzeev et al., Strongly Interacting Matter in Magnetic Fields (Springer, New York, 2013)
16. A. Vilenkin, Phys. Rev. D 22, 3080 (1980)
17. Y. Burnier et al., Phys. Rev. Lett. 107, 052303 (2011)
18. R. Kumar, A. Kumar, Eur. Phys. J C 79, 403 (2019)
19. S. Reddy et al., Phys. Rev. C 97, 065208 (2018)
20. R. Rapp et al., Prog. Part. Nucl. Phys. 65, 209 (2010)
21. Y. Nambu, G. Jona-Lasinio, Phys. Rev. 122, 345 (1961)
22. A. Mishra et al., Phys. Rev. C 69, 015202 (2004)
23. A. Mishra et al., Eur. Phys. J. A 41, 205 (2009)
24. P. Papazoglou et al., Phys. Rev. C 59, 411 (1999)
83. A. Mishra et al., Phys. Rev. C 78, 024901 (2008)
84. S. Weinberg, Phys. Rev. 166, 1568 (1968)
85. S. Coleman et al., Phys. Rev. 177, 2239 (1969)
86. W.A. Bardeen, B.W. Lee, Phys. Rev. 177, 2389 (1969)
87. D. Zschiesche, Description of Hot, Dense, and Strange Hadronic Matter in a Chiral $SU(3)_L \times SU(3)_R \sigma$-Model, Diploma Thesis, Goethe University Frankfurt (1997)
88. A. Broderick et al., Astrophys. J. 537, 351 (2000)
89. A.E. Broderick et al., Phys. Lett. B 531, 167 (2002)
90. J. Schechter, Phys. Rev. D 21, 3393 (1980)
91. H. Gomm, Phys. Rev. D 33, 801 (1986)
92. T. Barnes, E.S. Swanson, Phys. Rev. C 49, 1166 (1994)
93. Z.G. Wang, Phys. Rev. C 92, 065205 (2015)
94. Z.G. Wang, Int. J. Mod. Phys. A 28, 1350049 (2013)
95. K. Morita, S.H. Lee, Phys. Rev. C 85, 044917 (2012)
96. K. Azizi, N. Er, H. Sundu, Eur. Phys. J. C 74, 3021 (2014)
97. D.B. Leinweber, Ann. Phys. 254, 328 (1997)
98. T. Hilger, QCD Sum Rules for D Mesons in Nuclear Matter. Diploma Thesis, Technical University of Dresden (2008)
99. M. Nielsen et al., Phys. Rep. 497, 41 (2010)
100. X. Jin et al., Phys. Rev. C 49, 1 (1994)
101. R. Thomas et al., Nucl. Phys. A 795, 19 (2007)
102. S. Godfrey, N. Isgur, Phys. Rev. D 32, 189 (1985)
103. Z.G. Wang, T. Huang, Phys. Rev. C 84, 048201 (2011)
104. H. G. Blundell, arXiv:hep-ph/9608473