Efficient 2D Modeling of Gravity Anomaly in Space-wavenumber Mixed Domain

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Abstract. Forward modeling is the basis of inversion. In order to implement refined inversion imaging and quantitative interpretation of gravity exploration, a high-efficiency and high-precision numerical simulation method of gravity anomalies in mixed space-wavenumber domain is proposed in this paper. By performing one-dimensional Fourier transform, the method transforms the partial differential equations of the two-dimensional space domain problem of gravity anomaly into one-dimensional differential equations with different wave numbers independent of each other. The depth direction is reserved as the spatial domain, and the shallow grid can be properly encrypted and the deep grid to be properly sparse, which taking account of the calculation accuracy, calculation efficiency and simulation of complex terrain. The two-dimensional gravity anomaly forward modeling in the mixed space wavenumber domain makes full use of the fast Fourier transform and the chase method to solve the five-diagonal equations quickly, and realizes the numerical simulation of gravity anomaly with high efficiency and high precision.

1. Introduction
Gravity exploration is a well-known geophysical exploration method, it has been widely used in deep geological structure delineation, environment and engineering problems, due to its advantages of low cost, high efficiency and large exploration depth. The forward calculation methods of gravity anomaly can be divided into spatial domain method and frequency domain method\cite{1,2}. The spatial domain method mainly includes analytical and numerical method. The analytical method is simple in principle and high in accuracy. However, for complex bodies, the analytic formula is complex or cannot provide strict analytical expressions. Moreover, the spatial domain method is inefficient in calculating a large number of position point anomalies. Recently, the frequency-domain method has gradually become the preferred method for complex forward modeling because of its simple spectral expression and high computational efficiency. Chai (1997) developed the offset sampling method into a complete theoretical system, which effectively improved the accuracy of Fourier transform. Tontini et al. (2009) implemented forward modeling of gravity anomaly in 3D full-space frequency domain, and studies the relationship between FFT edge broadening method and error. Wu and Tian (2014) used Gauss-FFT to improve the accuracy of the inverse Fourier transform, reducing the influence of forced periodization and boundary oscillation effect caused by fast Fourier transform. In order to take the advantages of the accuracy and efficiency of spatial domain method and frequency domain method, this paper proposes a forward modeling method of spatial wavenumber mixed domain for two-dimensional gravity anomalies with arbitrary density distribution. Firstly, one-dimensional Fourier transform is applied, which transforms the partial differential equations of the
two-dimensional space domain problem of gravity anomaly into one-dimensional differential equations with different wave numbers independent of each other. Then, the amount of calculation decreased significantly, due to the one-dimensional differential equations are independent of each other between different wave numbers which can be parallel performed. This method keeps vertical as spatial domain, and can be used to the simulation of complex terrain with high accuracy and efficiency since the encrypted vertical shallow grid generation and the sparse deep grid generation. The finite element method is used to solve the one-dimensional partial differential equations, and the chase method satisfied by different wave numbers further improves the efficiency. The results show that our method is much more efficient than present algorithm, and can provide high accuracy gravity anomaly fields with relative errors below 0.01% compared with the analytical solution.

2. Algorithm

2.1. Basic theory

The gravity potential which satisfies 2D Poisson equation (Blakely, 1996) can be described as

\[ \nabla^2 U(x, z) = -4\pi G \rho(x, z) \]  

In the above formula, \( U \) stands for gravity potential in space domain, \( G \) is the Newton's gravitational constant, \( \rho \) is anomaly residual density.

By applying the one-dimensional Fourier transform of equation 1 in the x direction, the mixed domain of spatial wavenumber can be represented

\[ \frac{1}{c^2} \hat{U}(k, z) - k^2 \hat{U}(k, z) = -4\pi G \hat{\rho}(k, z) \]  

Equation 2 is a one-dimensional partial differential equation satisfying the gravitational potential in the space-wavenumber mixed domain, where \( \hat{U} \) denotes gravity potential in the mixed domain, \( \hat{\rho} \) is anomaly residual density in the mixed domain, \( k \) is the spatial wave number.

In a source free region, the general solution of equation 2 can be denoted as

\[ \hat{U} = Ae^{ik} + Be^{-ik} \]  

Where \( A \) and \( B \) are arbitrary constants, and \( z_{\text{min}} \) and \( z_{\text{max}} \) are the upper and lower boundaries, respectively, in the Cartesian coordinate system (Figure 1).

![Fig.1](image)

Fig.1 The anomaly model of density and the upper and lower boundaries for modeling.

\[ \frac{\partial \hat{U}}{\partial z} \bigg|_{z_{\text{min}}} = k \hat{U} \]

\[ \frac{\partial \hat{U}}{\partial z} \bigg|_{z_{\text{max}}} = -k \hat{U} \]  

With simultaneous equations 2 and 4, we can obtain the boundary value problem satisfied by gravity potential in the mixed domain of spatial wave Numbers. Then based on the variational
The equivalence of a variational problem to the boundary value problem shown in equation 5 can be derived as:

\[
\begin{align*}
F(\mathbf{U}) &= \int \left[ \frac{\partial^2 U}{\partial z^2} + \left( k^2 \rho \right)^2 \right] \, dz + k^2 \left[ (U)_{z_{\text{max}}} - (U)_{z_{\text{min}}} \right] \\
\delta F(\mathbf{U}) &= 0
\end{align*}
\]

(5)

For this equation, we use the one-dimensional finite element method of quadratic interpolation to solve. Then through the relation between the gravitational field and the gravitational potential, it is easy to calculate the gravitational field of the mixed space-wavenumber domain[4]. Finally, we use Gauss-FFT to invert the gravitational potential and gravitational field in the mixed domain of space wave number, and the gravitational potential and gravitational field in the space domain can be obtained[5].

2.2. Algorithm instance

In order to check the correctness and robustness of the 2D numerical algorithm in the mixed space-wavenumber domain, we design a simple constant density model as shown in figure 2, which consists of 200×200 cells with equal intervals and extends from -1000 m to 1000 m in the x direction, from 0 m to 1000 m in the z direction. This source region has a rectangular cylinder density anomaly with a residual density contrast of 1000 kg/m3. It extends from -500 m to 500 m in the x direction, from 400 m to 600 m in the z direction. The value of the background density is 0 kg/m3. The analytical solution of gravity anomaly[4], the numerical solution and the relative errors of each observation point with the ground position are respectively shown in figure 3. As figure 3 shows that the numerical solutions of the x and z components of the gravitational field are in good agreement with the analytical solutions, and the maximum relative error of the ground position is less than 0.01%. The relative error of the numerical and analytical solutions of gravity field is small, which verifies the correctness of the proposed algorithm, and the accuracy is high.

Fig.2 Schematic diagram of two-dimensional model

Fig.3 The numerical and analytical relative errors of the gravity fields at z=0
Computational efficiency is an important index to evaluate the numerical simulation method. Therefore, we compare the algorithm in this paper with the traditional spatial domain method[1] to calculate for the same model in the same computational environment (four CPUs of Inter Core i7, 3.20 GHz, 8 GB RAM), and the code was written in FORTRAN. Figure 4 shows the curve of the calculation time of a ground observation line calculated by the algorithm in this paper and the traditional spatial domain accumulation algorithm with the number of grid subsections. With the increase of the number of mesh generation, the calculation time increases linearly, but it is obvious that the present algorithm is more efficient.

![Fig.4 Comparison of the run time cost of the present algorithm and The Traditional spatial domain accumulation algorithm](image)

3. Conclusions
In this paper, an efficient 2D forward method of gravity anomaly is proposed. By designing a simple two-dimensional body model, the correctness and reliability of the method in this paper are verified by comparing numerical and analytical solutions. Compared with the traditional spatial domain method, it is verified that the method has high computational efficiency. Together this algorithm has high computational accuracy and efficiency, which is of practical significance for improving the efficiency of inversion imaging of two-dimensional model gravity anomalies.

References
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