TWO COUNTERMEASURE STRATEGIES TO MITIGATE RANDOM DISRUPTIONS IN CAPACITATED SYSTEMS

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Abstract

We examine a capacitated system exposed to random stepwise capacity disruptions with exponentially distributed interarrival times and uniformly distributed magnitudes. We explore two countermeasure policies for a risk-neutral decision maker who seeks to maximize the long-run average reward. A one-phase policy considers implementation of countermeasures throughout the entirety of a disruption cycle. The results of this analysis form a basis for a two-phase model which implements countermeasures during only a fraction of a disruption cycle. We present an extensive numerical analysis as well as a sensitivity study on the fluctuations of some system parameter values.

Keywords: Capacity analysis, capacitated system, lean, random disruptions, countermeasure policy

1. Introduction and Motivation

Lean manufacturing philosophy and associated business practices have been widely embraced and deployed by global enterprises. Some estimates assert that the shift to JIT scheduling in the US automotive industry has saved companies more than $1 billion a year in inventory costs, alone. While lean manufacturing has substantially boosted operational efficiency, it has also left enterprises operating in an increasingly risk-encumbered environment. Capacity disruptions triggered by forces of nature, property- and process-related hazards, and man-made interventions have proven to be the most profound influence on enterprise risk. As evidenced in 1995, an earthquake hit the port town of Kobe, Japan, razed to the ground 100,000 buildings and shut down Japan's largest port for over two years. In 1999, an earthquake in Taiwan displaced power lines to the semiconductor fabrication facilities responsible for more than 50 percent of the worldwide supplies of certain computer components, and shaved 5 percent off earnings for major hardware manufacturers including Dell, Apple, Hewlett-Packard, IBM, and...
Compaq (Wilcox 1999). In September 2002, longshoremen on the US West Coast were locked out in a labor strike for 11 days, forcing the shutdown of 29 ports. With more than $300 billion of dollars in goods shipped annually through these ports, the dispute caused between $11 and $22 billion in lost sales, spoiled perishables and underutilized capacity (Isidore 2002). In December 2002, a political strike in Venezuela made transnational businesses including GM, BP, Ford, Goodyear and Procter & Gamble halt their manufacturing for the duration of the conflict (Wilson 2003). The recent 2003 outbreak of SARS in China and Singapore forced Motorola to close several plants (Berniker 2003). Man-made disasters are on the rise, from terrorist attacks to computer viruses (Lemos 2003). As a result of the above events, according to a recent survey by A.M. Best Company, Inc. of 600 executives, 69 percent of chief financial officers, treasurers and risk managers at Global 1,000 companies in North America and Europe view property-related hazards—such as fires and explosions—and supply chain disruptions as the leading threats to top revenue sources (A.M. Best Company 2006).

Historically, enterprises have lacked appropriate decision support methodologies and computational tools suitable for addressing risk incurred through capacity disruptions. In academia, traditional research efforts on minimizing the cost of supply chain operations and the focus on leveraging economies of scale often yield results that overconcentrate resources. Such optimal solutions can be very sensitive to parameter fluctuations, caused by supply chain disruptions. The inability to recognize the hidden costs of such overconcentration heightens the risk of increased costs and capacity imbalance. Much of the recent literature focuses on minimizing costs of supply chain operations (see, for example, Barness-Shuster et al. (2002), Cheung & Lee (2002), Milner & Kouvelis (2002), Corbett & DeCroix (2001), Lee et al. (1997)), whereas only a small fraction of the efforts have been dedicated to modeling the impact of various disruptions, such as those affecting demand patterns, supplier and production lead times, prices, imperfect process quality, process yield, and other factors.

One of the most common types of disruption appearing in the literature is that of supply rate changes. An excellent work by Arreola-Risa & DeCroix (1998) explores inventory management of stochastic demand systems, where the product supply is disrupted for periods of random duration. The classic economic order quantity (EOQ) problem with supply disruptions is studied by Parlar & Berkin (1991) and Parlar & Perry (1996) consider a order-quantity/reorder-point inventory models with two suppliers subject to independent disruptions to compute the exact form of the average cost expression. Mohebbi (2003) presents an analytical model for computing the stationary distribution of the on-hand inventory in a continuous-review inventory system with compound Poisson demand, Erlang distributed lead time, and lost sales, where the supplier can assume one of the two “available” and “unavailable” states at any point in time according to a continuous-time Markov chain. Papers addressing both supply disruptions and random demand include (Chao 1987, Parlar 1997, Song & Zipkin 1996). Chao (1987)
proposes a dynamic model concerning optimal inventory policies in the presence of market disruptions, which are often characterized by events with uncertain arrival time, severity and duration. Parlar (1997) considers a continuous-review stochastic inventory problem with random demand and random lead-time where supply may be disrupted due to machine breakdowns, strikes or other randomly occurring events. Song & Zipkin (1996), explore an inventory-control model which includes a detailed Markovian model of the resupply system. A number of papers which address supply and demand changes have been developed in the field of oil stockpiling, as there has been grave concern over the oil supply from the Middle East (Teisberg 1981, Chap & Manne 1982, Murphy et al. 1987). Modeling production rate disruptions (machine failures) has been largely addressed by extending classical economic manufacturing quantity (EMQ) models. Rosenblatt & Lee (1986) derive an EMQ model when the production process is subject to a random deterioration from an in-control state to an out-of control state. Lee (1992) models the defect-generating process in the semiconductor wafer probe process to determine an optimal lot size, which reduces the average processing time on a critical resource. Abboud (1997) presents a simple approximation of the EMQ model with Poisson machine breakdowns and low failure rate. Groenevelt et al. (1992) study an unreliable production system with constant demand and random breakdowns, with the focus on the effects of machine failure and repair on optimal lot-sizing decisions. Assuming exponentially distributed time between failures and instantaneous repair of the machine, authors derive some unique properties of their model compared to the classical EMQ model. Groenevelt et al. (1992) extend their earlier work in Groenevelt et al. (1992) to the case where repair times are randomly distributed and excess demand is lost. Kim & Hong (1997) propose an extension to the model in Groenevelt et al. (1992), which determines an optimal lot size when a machine is subject to random failures and the time to repair is constant. They formulate average cost functions for the optimal lot size, and derive conditions for determining the optimal lot size. Hopp et al. (1989) presents a model that assumes the (s, S) control policy. With Poisson failures and exponential repair times, a cost function is derived. Rahim (1994) presents an integrated model for determining an economic manufacturing quantity, inspection schedule and control chart design of an imperfect production process, where he assumes that the process is subject to the occurrence of a non-Markovian shock having an increasing failure rate. Among other notable examples of such works are Henig & Gerchak (1990), Bielecki & Kumar (1988), Buzacott & Shantikumar (1993). Finally, Abboud (2001) examines a single machine production and inventory system with a deterministic production and demand rate, when the machine is subject to random failures. The author models the production/inventory system as a Markov chain and develops an algorithm to compute the potentials that are used to formulate the cost function.

At this point, we can summarize that research efforts addressing the disruption of supply are still comparatively new and scant. Most of the open literature considering various
types of disruptions focuses on issues of inventory, ordering, production lot sizing, production scheduling, and cost management of inventory, setup, and backorder costs. To the best of our knowledge, there have been no attempts to consider introducing countermeasure policies for mitigating unpredicted capacity disruptions in a capacitated system, and analyze the benefits of such policies for the system manager. Our paper presents an initial attempt to fill the vacuum in this area.

The paper has the following organization. In Section 2, we introduce notation and problem definition. Section 3 presents analysis of a one-phase countermeasure policy, where a risk-neutral decision maker implements countermeasures during the entirety of a disruption cycle, striving to maximize the long-run average reward. These results are used in Section 4 to examine a richer class of policies, where countermeasures are activated during only a fraction of a disruption cycle. In Section 5, we present a numerical analysis for determining the optimal phase threshold and examine the sensitivity of the optimal policy to fluctuations in system parameter values. Finally, Section 6 offers concluding remarks.

2. Notation and Problem Definition

For the rest of this paper, we define throughput as the long-run average of the number of item units per unit time processed by a capacitated system, and the available system capacity at time $t$, $C_t$, is defined as the maximum throughput that system resources are capable of sustaining at $t$. Consider a lean (i.e., no inventory) system with a target (demand adjusted) capacity $C^*$ experiencing periodic random disruptions, each of which may render a full or partial system capacity loss. We assume that disruptions occur one at a time and that the $i^{th}$ occurrence results in an instantaneous loss of magnitude $\Delta C_i$ in the remaining system capacity. Following the $i^{th}$ disruption at time $t_i$, the system capacity remains at level $C_t - \Delta C_i$ until the next disruption unless the remaining capacity falls below a critical level $c$ upon which the system regains all lost capacity back to $C^*$.

For the reason of simplicity, in this paper, we assumed instantaneous recovery. The system is assumed to stochastically regenerate at points of recovery (Figure 1). Capacity dynamics as such can be observed in a number of industrial scenarios including, but are not limited to, (i) shortage of repair personnel and performance degradation caused by failing equipment with a full repair upon a complete failure, (ii) non-self-announcing stepwise system failures, and (iii) gradual equipment phaseout and modernization.

![Figure 1](image_url)
We assume that \(X_i, i = 2, 3, \ldots\) are i.i.d. random variables. The time of the \(n\)th capacity loss is expressed as \(Z_n = \sum_{i=1}^{n} X_i\), \(n = 1, 2, \ldots\) where we define \(Z_0 = 0\). Let \(N_c = \min\{n \mid \sum_{i=1}^{n} \Delta C_i > C^* - x\}\). It then follows that \(N_c\) is the number of capacity disruptions between two successive recovery epochs. As such, \(Y = Z_{N_c}\) is the time between two successive recovery events, which marks the beginning and the end of a regenerative cycle.

A proactive decision maker has a number of mitigation options to reduce the rate of disruptions. When no countermeasures are implemented, he earns \(\pi = \pi \cdot C\) at time \(t\), where \(\pi\) is a time independent price factor minus item unit cost. Therefore, the revenue in each cycle is \(R = \pi \int_0^Y C \, dt = \pi \cdot C\).

We assume that a cost of \(m(\lambda)\) per unit time is incurred to activate and operate a set of countermeasures that would maintain a rate of \(\lambda\) capacity disruptions per unit time. In this paper, we are not concerned with the description of the nature of specific countermeasure options but rather we focus on the analytics of the disruption rate reducing impact that those options have on the system performance. We assume that the decision maker has a risk-neutral utility function (Keeney & Raiffa 1993), and thus, our analysis will be based on the limiting long-run average reward as the criterion for policy assessment.

Let \(\Pi = R - m(\lambda) \cdot Y\) denote the total reward earned in one renewal cycle and \(\Pi_t = \int_0^t R_z \, dz - m(\lambda) \cdot t\) denote the total reward by time \(t\). The long-run average reward converges then to the following (Ross 1996):

\[
\frac{\Pi_t}{t} \to \frac{E(\Pi)}{E(Y)}
\]  

(1)

In this paper, we first consider a one-phase mitigation policy in which countermeasures are activated throughout a regenerative cycle. Later, we will expand the analysis to examine a two-phase model.

3. One-Phase Countermeasure Policy

When countermeasures are engaged throughout the entire cycle, \(E(\Pi) = \pi \cdot E(C) - m(\lambda) \cdot E(Y)\), and hence, we seek to derive the expected cycle length and the expected cycle capacity. We assume that interarrival times \(X_i\) are distributed exponentially with rate \(\lambda\) and that fractional capacity losses \(\alpha_i\) are distributed uniformly over \([0, 1]\). Total capacity per cycle can be expressed as

\[
C = C^* \left[ \sum_{i=1}^{N_c} X_i - \sum_{i=2}^{N_c} \sum_{j=1}^{i-1} \alpha_i X_j \right],
\]  

(1)

whereas the cycle length is \(Y = \sum_{i=1}^{N_c} X_i\). Before we proceed with computing \(E(C)\) and \(E(Y)\), we will need the following result to compute \(E(N_c)\), the expected number of capacity loss events per cycle.

Result 1 Let \(\xi_i, i = 1, \ldots, n\) be i.i.d. uniform \([0, 1]\) random variables. Then

\[
P(\sum_{i=1}^{n} \xi_i \leq u) = u^n / n!
\]

Proof. We prove by induction. For \(n = 1\), the result is trivial. Assuming that the result holds for \(n-1\), note that
We have
\[
P(\sum_{i=1}^{n} \xi_i \leq u) = P(\sum_{i=1}^{n-1} \zeta_i + \xi_n \leq u)
\]
\[
= \int_0^u \int_0^{u-s} dy \frac{s^{n-2}}{(n-2)!} ds
\]
\[
= \int_0^u (u-s) \frac{s^{n-2}}{(n-2)!} ds
\]
\[
= \frac{u^n}{(n-1)!} - \frac{u^n}{n(n-2)!} = \frac{u^n}{n!}.
\]

Now we are in a position to compute \( E(Y) \) using Result 1. Let \( c = (1-\alpha)C^* \) for some \( \alpha \in (0,1) \). Note the equivalency of events
\[\{N_c = n\} \quad \text{and} \quad \{\sum_{i=1}^{n-1} \alpha_i \leq \alpha \text{ and } \sum_{i=1}^{n} \alpha_i > \alpha\}.
\]

Using Result 1 we have,
\[
P(\sum_{i=1}^{n-1} \alpha_i \leq \alpha \text{ and } \sum_{i=1}^{n} \alpha_i > \alpha)
\]
\[
= P(\sum_{i=1}^{n} \alpha_i \leq \alpha) - P(\sum_{i=1}^{n} \alpha_i \leq \alpha)
\]
\[
= \frac{\alpha^{n-1}}{(n-1)!} - \frac{\alpha^n}{n!},
\]

which can be used to obtain,
\[
E(N_c) = \sum_{n=1}^{\infty} n \cdot \left[\frac{\alpha^{n-1}}{(n-1)!} - \frac{\alpha^n}{n!}\right]
\]
\[
= \sum_{n=0}^{\infty} \frac{\alpha^n}{n!} = e^\alpha,
\]

and
\[
E(Y) = \sum_{n=1}^{\infty} E(N_c | N_c = n)P(N_c = n)
\]
\[
= \sum_{n=0}^{\infty} \frac{n}{\lambda} \cdot P(N_c = n) = \frac{1}{\lambda} \cdot E(N_c) = \frac{e^\alpha}{\lambda}.
\]

Computation of \( E(C) \) can be found in the Appendix. We have that
\[
E(C) = \frac{C^*}{\lambda} \left[ \sum_{n=3}^{\infty} P(N_c = n) \cdot \left\{ n - \sum_{k=1}^{n-2} h_k(n, \alpha) - h(n, \alpha) \right\} + P(N_c = 2)(2 - h(2, \alpha)) + (1 - \alpha) \right],
\]

where
\[
h_k(n, \alpha) = \frac{k}{(n-k)(n-1)} \left[ n-k+\alpha \frac{k+1}{n+1} \right],
\]

and
\[
h(n, \alpha) = \frac{n(n-1)}{n} \left[ 1 + \alpha - \frac{\alpha^n}{n} + \frac{\alpha^{n+1}}{n+1} \right].
\]

Using (3), we can compute the long-run average reward in the following way. Define \( \bar{C}(\alpha) \) as
\[
\bar{C}(\alpha) = \sum_{n=3}^{\infty} P(N_c = n) \cdot \left\{ n - \sum_{k=1}^{n-2} h_k(n, \alpha) - h(n, \alpha) \right\} + P(N_c = 2)(2 - h(2, \alpha)) + (1 - \alpha).
\]

Then, the limiting value of long-run average reward is given by the following expression.
\[
\frac{\Pi}{\pi} \rightarrow \frac{E(\Pi)}{E(Y)} = \frac{\frac{\pi \cdot e^\alpha \cdot \bar{C}(\alpha)}{\lambda} - \frac{e^\alpha \cdot m(\lambda)}{\lambda}}{e^\alpha / \lambda}. \tag{4}
\]

In this section, we have considered a one-phase mitigation policy where countermeasures are implemented during the entire disruption cycle. The expression for the limiting long-run average reward (Eq. 4) will serve as a basis for analyzing a two-phase policy in the next section.

### 4. A Two-Phase Countermeasure Policy

Consider the set of policies under which
countermeasures are activated at the beginning of each system cycle and remain in effect as long as system capacity exceeds a certain higher level \( c_i > c \), where \( c_i = (1 - \alpha_i) \cdot C^* \) for some \( \alpha_i \in (0, 1) \). Countermeasures remain deactivated for levels below \( c_i \), where the system becomes exposed to “normal” disruption rate. This model is driven by the idea that from the system manager's viewpoint, it is desirable to stay longer in the “on” zone, closer to the target level \( C^* \) rather than prolong the “off” portion of the cycle. As in Section 2 and 3, \( c \) is the critical lower level that triggers instantaneous capacity recovery (Figure 2). The system is said to be “on” when countermeasures are in effect and “off” otherwise. Long-run average reward of this altered process exhibits the same convergence property. Therefore, it is our interest in this section to compute \( \mathbb{E}[\Pi]/\mathbb{E}[Y] \).

**Figure 2** A realization of the system capacity dynamics for a two-phase policy. Disruption rate during the “on” phase \( \lambda_i \) is smaller than the disruption rate during the “off” phase \( \lambda \).

We first derive the distribution of the initial system capacity for the “off” period in a cycle. The following proposition summarizes the result,

**Proposition 1** Consider a capacitated system in which capacity disruption interarrival times are exponentially distributed with parameter \( \lambda \), fractional stepwise capacity losses follow a uniform distribution on \([0,1]\), and capacity is restored fully and instantaneously upon falling below level \( c \). Suppose that the system is “on” when \( C_i > (1 - \alpha_i) C^* \), “off” otherwise, and \( c = (1 - \alpha_i) C^* \), \( \alpha_i < \alpha \). Then, the distribution of initial system capacity of the “off” period is given by the following,

\[
P(\sum_{i=1}^{N_c} \Delta C_i \leq \alpha C^*) = (\alpha - \alpha_i) e^{\alpha_i}.
\]

**Proof.** We proceed by considering the number of capacity losses during the “on” period, \( N_{c_i} \). Let

\[
\Gamma_k = \sum_{i=1}^{k} \alpha_i.
\]

For \( n \geq 2 \),

\[
P(\sum_{i=1}^{N_c} \Delta C_i \leq \alpha C^* \mid N_{c_i} = n) = P(\Gamma_{n-1} \leq \alpha_i, \Gamma_n \in (\alpha_i, \alpha) \mid \Gamma_{n-1} \leq \alpha_i, \Gamma_n > \alpha_i)
\]

\[
= P(\Gamma_{n-1} \leq \alpha_i, \Gamma_n \in (\alpha_i, \alpha) \mid \Gamma_{n-1} \leq \alpha_i, \Gamma_n > \alpha_i)
\]

\[
= \frac{\left[\int_{0}^{t_s} f_{\Gamma_{n-1}}(s) \cdot P(\alpha_n \in (\alpha_i - s, \alpha - s)) \, ds\right]}{1 - \frac{\alpha_i^{n-1}}{(n-1)!} - \frac{\alpha^p}{n!}}
\]

\[
= (\alpha - \alpha_i) \frac{\alpha_i^{n-1}}{(n-1)!} \left[\frac{\alpha_i^n}{(n-1)!} - \frac{\alpha^p}{n!}\right].
\]

Note that

\[
P(\alpha_i \in (\alpha_i, \alpha) \mid \alpha_i > \alpha_i) = (\alpha - \alpha_i) / (1 - \alpha_i).
\]

Therefore, one can verify by slight modifications in the computations above that (5) holds for \( n = 1 \) as well. Using Result 1, we
obtain
\[ P \left( \sum_{i=1}^{N_c} \Delta C_i \leq \alpha C^* \right) = (\alpha - \alpha) \cdot \sum_{n=1}^{\infty} P(N_{c} = n) \]
\[
\{(\alpha - \alpha) \sum_{n=1}^{\infty} \frac{\alpha^n}{(n-1)!} \}
\]
\[= (\alpha - \alpha) \sum_{n=1}^{\infty} \frac{\alpha^n}{(n-1)!} = (\alpha - \alpha) \cdot e^{\alpha}, \]
which concludes the proof.

Expected cycle reward is the sum of expected returns of the “on” and “off” cycle periods. Let \( Y_t \) and \( Y_s \) denote the length of the “on” and “off” cycle periods, respectively, where \( Y = Y_t + Y_s \) is the length of the cycle. Define
\[
\overline{C}_t(\alpha_t) = \sum_{n=3}^{\infty} P(N_{c_t} = n) \cdot \left\{ n - \sum_{k=1}^{n-2} h_k(n, \alpha_t) \right\} - h(n, \alpha_t) + P(N_{c_t} = 2) \cdot (2 - h(2, \alpha_t)) + (1 - \alpha_t).
\]

Results of the previous section can be readily applied to obtain the expression for expected total capacity, \( C_t \), during the “on” period, which is \( E(C_t) = \overline{C}_t(\alpha_t) \cdot C^* / \lambda_t \), where \( \lambda_t \) is disruption rate during the “on” period. Similarly, we have \( E(Y_t) = e^{\alpha_t} / \lambda_t \). While the length of each cycle is affected by the change in disruption rate, the total number of disruption events in a cycle, \( N_c \), is determined solely by a uniform capacity reduction process that evolves independently from the disruption rate. Therefore, we can deduce immediately using (2) that \( E(N_c) = e^{\alpha} \) and \( E(N_{c_t}) = e^{\alpha_t} \). Then, since capacity disruption rate remains at \( \lambda \) during the “off” period we have,
\[ E(Y_s) = e^{\alpha} / \lambda. \]

Likewise, expected total capacity during the “off” period, \( C_s \), can be readily obtained after considering the initial capacity level in the “off” period, \( C_0 \). Note that
\[ C_s = C_0 \cdot \sum_{i=1}^{N_c-N_{c_t}} X_i - C^* \cdot \sum_{i=2}^{N_c-N_{c_t}} \sum_{j=1}^{i-1} X_\alpha, \]
so that
\[ E(C_s \mid C_0 = (1 - \alpha) \cdot C^*) = C_0 \cdot \frac{E(N_c - N_{c_t} \mid C_0 = (1 - \alpha) \cdot C^*)}{\lambda} \]
\[= C_0 \cdot \frac{\lambda}{\lambda} \cdot \sum_{n=2}^{\infty} P(N_c - N_{c_t} = n \mid C_0 = (1 - \alpha) \cdot C^*) \]
\[\sum_{i=2}^{n} E(\sum_{j=1}^{i-1} \alpha) \mid N_c - N_{c_t} = n). \]
This expression can be simplified to yield the following.
\[ E(C_s \mid C_0) = \frac{C_0 \cdot e^{\alpha - \alpha} - C^*}{\lambda} \]
\[ E(N_c - N_{c_t} \mid C_0 = (1 - \alpha) \cdot C^*) \]
\[ \left[ \sum_{n=3}^{\infty} P(N_c - N_{c_t} = n \mid C_0 = (1 - \alpha) \cdot C^*) \right] \]
\[ \left\{ \sum_{k=1}^{n-2} h_k(n, \alpha - \alpha) + h(n, \alpha - \alpha) \right\} + \]
\[ P(N_c - N_{c_t} = 2 \mid C_0 = (1 - \alpha) \cdot C^*) \cdot h(2, \alpha - \alpha) \]
\[ = \frac{C_0 \cdot e^{\alpha - \alpha} - C^*}{\lambda} \cdot \psi(\alpha, \alpha). \]

We also have
\[ P(N_c - N_{c_t} = n \mid C_0 = (1 - \alpha) \cdot C^*) \]
\[ = \frac{(\alpha - \alpha)^{n-1}}{(n-1)!} - \frac{(\alpha - \alpha)^n}{n!}. \]
In order to compute $E(C_s)$, we need the expression for $E(C_0 \cdot e^{\alpha-\sigma})$, which is derived as follows,

$$E(C_0 \cdot e^{\alpha-\sigma})$$

$$= E(E(C_0 \cdot e^{\alpha-\sigma} \mid N_{c_i}))$$

$$= C^* \cdot \sum_{n=1}^{\infty} P(N_{c_i} = n) \cdot \int_{\alpha_i}^{\alpha_i (1-\alpha)} e^{\alpha-\sigma} \cdot \frac{\alpha_i^{n-1}}{(n-1)!} \left[ \frac{\alpha_i^n}{n!} - \frac{\alpha_i^n}{n!} \right] d\alpha$$

$$= C^* \cdot (\alpha - \alpha_i \cdot e^{\alpha-\sigma}) \cdot \sum_{n=1}^{\infty} \left[ \frac{\alpha_i^{n-1}}{(n-1)!} - \frac{\alpha_i^n}{n!} \right]$$

$$= C^* \cdot (\alpha - \alpha_i \cdot e^{\alpha-\sigma}) \cdot e^{\alpha_i}$$

$$= C^* \cdot (\alpha e^{\alpha_i} - \alpha_i \cdot e^{\alpha_i})$$

Now, we are in a position to obtain $E(C_s)$:

$$E(C_s) = \frac{C^*}{\lambda} \cdot (\alpha e^{\alpha_i} - \alpha_i \cdot e^{\alpha_i})$$

$$- \frac{C^*}{\lambda} \int_{\alpha_i}^{\alpha} e^{\alpha} \cdot \psi(\alpha, \alpha) d\alpha$$

$$= \frac{C^*}{\lambda} \cdot f(\alpha_i, \alpha).$$

This brings us to the following principle proposition.

**Proposition 2** For the capacitated system described in Proposition 1, the long-run average reward converges to the following expression.

$$\Pi \to \frac{E(\Pi)}{E(Y)}$$

The proof follows by substituting expressions for $E(C_s)$, $E(C_f)$, $E(Y)$ and $E(Y_f)$ into (7).

### 5. Numerical Analysis and Sensitivity Study

An optimal two-phase policy maximizes long-run average reward by activating countermeasures that set optimal levels of $\lambda$ and $\alpha_i$. In what follows, we conduct a parametric analysis of the optimal policy behavior (Eq. 6). Note that in computing $f(\alpha_i, \alpha)$ (through $\psi(\alpha, \alpha)$) and $\overline{C}_f(\alpha_i)$, we encounter infinite sums which include terms $P(N_{c_i} - N_{c_i} = n)$ and $P(N_{c_i} = n)$, respectively. These terms represent the probability distribution of the number of disruptions during the “off” and “on” phases, respectively. Since the mean disruption magnitude is strictly positive and $C^*$ (and hence, $c_i$ and $c$) is finite, both of these terms go to zero as $n \to \infty$.

Since the disrupted capacity is regained instantaneously at the end of each cycle, the solution to an optimum policy $(\alpha^*, \alpha^*)$ is trivial. Hence, we analyze the behavior of the optimal $\alpha_i$ as a function of $\alpha$, i.e., $\alpha_i^*(\alpha)$, for fixed values of $\lambda$ and $\lambda_i$. We first observe that $\alpha_i^*(\alpha)$ is monotonically non-decreasing in $\alpha$: as $\alpha$ increases, the cycle time will
increase as well, and so are the periods of lower system capacity. On the other hand, increasing $\alpha_t$ results in longer periods of higher system capacity. This trade-off between benefits and costs of activating countermeasures renders $\alpha_t(\alpha) < \alpha$. In what follows, we use the initial parameter values as shown in Table 1.

The cost of countermeasures is assumed to be of the form $m(\lambda_t) = (\lambda / \lambda_t)^r$. The cost decreases as the disruption rate $\lambda_t$ gets higher, which is used to measure effectiveness of countermeasure technology, and the cost increases in $r$, which is used to model the marginal cost of installing a more effective technology. As Figure 3 illustrates, $\alpha_t^*(\alpha)$ is increasingly decreasing in $r$. We also observe that as $r$ gets larger, $\alpha_t^*(\alpha)$ exhibits a higher sensitivity to per unit changes in $r$. As one can see, in flat regions of Figure 3, reducing the disruption rate is not economically sound.

For a linear cost function ($r = 1$, plot I in Figure 4), $\alpha_t^*(\alpha)$ is increasing in $(\lambda_t / \lambda)$, which implies that the incremental benefits of reducing the rate of capacity disruptions do not warrant the use of countermeasures over extended periods of time. However, if the marginal cost of installing better countermeasures is not constant ($r = 0.001$, plot II in Figure 4), the plots of $\alpha_t^*(\alpha)$ for different values of $(\lambda_t / \lambda)$ intersect. If the marginal cost of a decreased $(\lambda_t / \lambda)$ is relatively small, then $\alpha_t^*(\alpha)$ may be increasing with a more advanced technology (this relationship does not hold for higher values of $\alpha$). However, both plots agree that the rate of increase of $\alpha_t^*(\alpha)$ is higher in the lower region of $(\lambda_t / \lambda)$ values. Therefore, we see that expected increase in countermeasure costs over extended periods outweighs the benefits of better technology.

**Table 1 Initial parameter values**

| $\lambda_t$ | $\pi$ | $\lambda$ | $C^*$ | $r$ |
|-------------|-------|-----------|-------|-----|
| 0.0005      | 1,000 | 0.001     | 1     | 1   |

**Figure 3** A 3-dimensional representation of $\alpha_t^*$ as a function of $\alpha$ and $r$.

**Figure 4** Behavior of $\alpha_t^*(\alpha)$ for different values of $\lambda_t / \lambda$.

Furthermore, we observe that $\alpha_t^*(\alpha)$ is insensitive to changes in maximum capacity $C^*$ in the neighborhood of initial parameter values in Table 1. Common wisdom, however, suggests that $C^*$ shall be positively correlated with the optimum period of activated countermeasures.
Should all items be sold, increasing $C^*$ would lead to higher profits and hence, countermeasures should be engaged for longer periods (plot I in Figure 5, where $C^*$ takes values in $[0, 0.1]$). As $C^*$ approaches 0.1, marginal increase in $\alpha^*_1(\alpha)$ falls off sharply. This suggests that the optimal period of activated countermeasures is insensitive to changes in maximum capacity, if $C^*$ is already high. Also, the region of sensitivity of $C^*$ is a function of the unit profit. As illustrated in the second plot of Figure 5, $\alpha^*_1(\alpha)$ becomes responsive to changes in $C^* \in [0.1, 1.0]$, when $\pi$ gets smaller (this change in sensitivity may be minimal if $C^*$ is already high).

A similar relation exists between $\alpha^*_1(\alpha)$ and $\pi$. Figure 6 illustrates that $C^*$ is insensitive to changes in $\pi$ around the original parameter value of $\pi=1000$ whereas at lower unit profit levels, marginal changes in $\pi$ render larger perturbations in $\alpha^*_1(\alpha)$. Changes in system capacity for low value items may require more radical changes in countermeasure policy. Nevertheless, the region of sensitivity is relatively small for both $\pi$ and $C^*$, which suggests on a larger scale that $\alpha^*_1(\alpha)$ is quite robust to changes in system profitability.

6. Conclusions

In this manuscript, we presented one of initial attempts to fill the vacuum in the existing literature focused on development of active countermeasure policies for managing lean capacitated systems in the presence of random capacity disruptions. The system under consideration experienced stepwise partial capacity disruptions with exponentially distributed interarrival times and uniformly distributed magnitudes, followed by instantaneous recovery. Examples of such capacity dynamics include: (i) shortage of repair personnel and performance degradation caused by failing equipment with a full repair upon complete failure, (ii) non-self-announcing stepwise system failures, and (iii) gradual equipment phaseout and modernization.

We explored two different countermeasure policies for a risk-neutral decision maker, who seeks to maximize the long-run average reward. The initial model considered a one-phase policy,
where countermeasures were implemented during the entirety of a disruption cycle. The results of this model served as a basis to analyze a two-phase strategy, where countermeasures were activated during only a fraction of a disruption cycle. For the latter model, we aimed to determine the optimal threshold when the countermeasures should be disengaged. In this paper, we are primarily concerned with analytics of the impact that countermeasure options can have on the system performance. In practice, the countermeasure options could range from purely technological solutions, such as installation of fire prevention water sprinkler systems, to non-technological decisions that could, for example, alleviate labor strikes or prevent terrorist attacks or political unrest. In this investigation, we considered two forms of the countermeasure cost functions. Our sensitivity analysis for the two-phase policy reveals that as the system profitability increases and the costs of countermeasures become smaller, the optimal countermeasure policy becomes less sensitive to changes in the system parameter values.

In this paper, we did not address the question of the best critical threshold that initiates immediate capacity recovery, as we assumed that the cost associated with administering any level of $\alpha$ is zero. Therefore, the problem of obtaining the optimal pair $(\alpha^*, \alpha^*)$ has a trivial solution (i.e., set $\alpha^* = 0$). Rather, we aimed to find the optimal time in each regenerative cycle when countermeasures should be terminated given a capacity recovery threshold of $\alpha$. Section 5 presented a numerical analysis to determine optimal $\alpha_i^*(\alpha)$ that maximized long-run average reward under various parametric settings. We presented the results of our sensitivity analysis for an exponential cost function. In general, $\alpha_i^*(\alpha)$ was found to be quite sensitive to exponentially increasing cost, as well as capacity and unit profit changes, if the system was already operating with low profit margins. However, as the profitability of the system increased, $\alpha_i^*(\alpha)$ had a robust response to system parameter changes.

In general, capacity disruption risk can be mitigated by reducing the probability of the hazardous events as well as their severity. In this paper, we considered countermeasures that mostly impact the probability of hazardous events rather than their severity. For risks that render partial capacity disruptions, the model recommends implementation of countermeasures during only a fraction of the operational cycle. In many cases, partial capacity disruptions are caused by risks associated with daily operations, such as small fire events and stoppages due to machine failures. For such events, our results can substantiate that certain countermeasures may be cost prohibitive even when they offer significant reduction in the disruption rate. For example, in a manufacturing facility, installation of costly fire extinguishing systems may be disfavored to employee training programs that raise awareness of overall factory cleanliness.

This paper provides one of the initial attempts for providing closed form solutions for optimal countermeasure policies for mitigation of random disruptions in capacitated systems. We hope that the presented models will be further generalized to address similar questions for capacitated systems evolving under more complex capacity dynamics. We also believe that such single-facility models will form a basis
to approach capacity management issues in large enterprise networks.

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Appendix
Computation of \( E(C) \) in Equation 3. We begin by conditioning on \( N_c \). Noting the dependency between \( \Delta C_i \) and \( N_c \), we first compute the conditional density of \( \Gamma_k = \sum_{i=1}^{k} \alpha_i \),
\[
f_{\Gamma_k}(s \mid N_c = n) \quad \text{for} \quad k < n-1 \quad \text{and} \quad n \geq 3.
\]
Note that
\[
f_{\Gamma_k}(s \mid N_c = n) = \int_{s}^{\alpha} \frac{P(\Gamma_k = s, \Gamma_{n-1} = u, \Gamma_n > \alpha)}{P(\Gamma_{n-1} \leq \alpha, \Gamma_n > \alpha)} du.
\] (8)

Since \( \alpha_i \)'s are independent, we can use Result 1 to obtain
\[
P(\Gamma_k = s, \Gamma_{n-1} = u, \Gamma_n > \alpha) = P(\sum_{i=1}^{k} \alpha_i = s)P(\sum_{i=k+1}^{n-1} \alpha_i = u)P(\alpha_n > \alpha - u)
\]
\[
= (1+u-\alpha) \cdot s^{k-1} \cdot \frac{(u-s)^{n-k-2}}{(k-1)!} \cdot \frac{(n-k-2)!}{(n-\alpha)!}
\] (9)
Substituting (9) in (8) and evaluating the integral, we obtain
\[
f_{\Gamma_k}(s \mid N_c = n) = \frac{1}{P(\Gamma_{n-1} \leq \alpha, \Gamma_n > \alpha)} \cdot 
\int_{s}^{\alpha} (1+u-\alpha) \cdot s^{k-1} \cdot \frac{(u-s)^{n-k-2}}{(k-1)!} \cdot \frac{(n-k-2)!}{(n-\alpha)!} du
\]
\[
= \frac{s^{k-1}}{(k-1)!} \cdot \frac{(\alpha-s)^{n-k-1} - (\alpha-\alpha)^{n-k}}{(n-k-1)! - (n-k)!}.
\]

We use this conditional density to compute \( E(\Gamma_k \mid N_c = n) \) as follows:
\[
E(\Gamma_k \mid N_c = n) = \int_{0}^{1} \frac{s^{k-1}}{(k-1)!} \cdot \left[ \frac{(\alpha-s)^{n-k-1} - (\alpha-\alpha)^{n-k}}{(n-k-1)! - (n-k)!} \right] ds.
\]
\[
= k \cdot \frac{n!}{k!(n-k)!} \cdot \frac{1}{n\alpha^{n-1} - \alpha^n} \cdot 
\left[ (n-k-\alpha) \int_{0}^{\alpha} s^k (\alpha-s)^{n-k-1} ds \right.
\]
\[
\left. + \alpha^{n+1} B(k+2, n-k) \right].
\]

Note that both integrals represent beta functions (to see this, make a variable change \( t = s / \alpha \)). Therefore, we arrange the expressions to obtain
\[
E(\Gamma_k \mid N_c = n) = \frac{k \cdot n!}{k!(n-k)!} \cdot \frac{1}{n\alpha^{n-1} - \alpha^n} \cdot 
\left[ (n-k-\alpha) \alpha^n B(k+1, n-k) \right.
\]
\[
\left. + \alpha^{n+1} B(k+2, n-k) \right] = \frac{k}{(n-k) \cdot \frac{n}{\alpha} - 1} \cdot 
\left[ (n-k-\alpha + \alpha \frac{k+1}{n+1}) \right.
\]
\[
\left. = h_k(n, \alpha). \right]
\] (10)
Note that this expression holds for \( k < n-1 \) and \( n \geq 3 \). For \( k = n-1 \) and \( n \geq 2 \),
\[
f_{\Gamma_{n-1}}(s \mid N_c = n) = \frac{f_{\Gamma_{n-1}}(s) \cdot P(\alpha_n > \alpha - s)}{P(\Gamma_{n-1} \leq \alpha, \Gamma_n > \alpha)}
\]
Using the above expression, we eventually obtain

\[ E(\Gamma_{n-1} \mid N_c = n) = \frac{1}{\alpha^{n-1} \cdot n!} \left( \int_0^{s^{n-1}} \alpha^n (n-2)! (1-\alpha+s) ds \right) \]

\[ = \frac{n(n-1)}{n\alpha^{n+1} - \alpha^n} \left[ (1-\alpha) \alpha^n + \alpha^{n+1} \right] \]

\[ = h(n, \alpha). \quad (11) \]

We can now derive expected capacity per cycle. Equation (1) gives

\[ E(C) = E(C^* \cdot E \sum_{i=1}^{n_c} X_i - \sum_{j=1}^{N_c} \sum_{j=1}^{N_c} X_j \alpha_j \mid N_c = n)) \]

\[ = C^* \sum_{n=2}^{n_c} P(N_c = n) \cdot \frac{n \cdot \lambda}{\beta} \]

\[ + E(\sum_{i=2}^{n_c} X_i \alpha_i \mid N_c = n) + P(N_c = 1) \cdot \frac{\lambda}{\beta} \]

\[ = C^* \sum_{n=2}^{n_c} P(N_c = n) \cdot \frac{n \cdot \lambda}{\beta} \]

\[ + E(X_i) \cdot E(\Gamma_{i-1} \mid N_c = n) + \frac{1-\alpha}{\beta} \]

\[ = C^* \sum_{n=2}^{n_c} P(N_c = n) \cdot \frac{n \cdot \lambda}{\beta} \]

\[ + \sum_{i=2}^{n_c} E(\Gamma_{i-1} \mid N_c = n) + (1-\alpha). \]

Finally, substituting equations (10) and (11), we obtain

\[ E(C) = C^* \sum_{n=3}^{\infty} P(N_c = n) \cdot \{n - \sum_{k=1}^{n-2} h_k(n, \alpha) - h(n, \alpha)\} + P(N_c = 2) \cdot (2 - h(2, \alpha)) + (1-\alpha) \]

\[ = \frac{C^*}{\lambda} \sum_{n=3}^{\infty} P(N_c = n) \cdot \{ \sum_{k=1}^{n-2} h_k(n, \alpha) + h(n, \alpha) + P(N_c = 2) \cdot h(2, \alpha) \} \]

\[ = \frac{C^*}{\lambda} \sum_{n=3}^{\infty} P(N_c = n) \cdot \{ \sum_{k=1}^{n-2} h_k(n, \alpha) + h(n, \alpha) + P(N_c = 2) \cdot h(2, \alpha) \} \]

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