JET EVOLUTION, FLUX RATIOS, AND LIGHT-TRAVEL TIME EFFECTS

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Received 2003 October 14; accepted 2004 January 16; published 2004 February 13

ABSTRACT

Studies of the knotty jets in both quasars and microquasars frequently make use of the ratio of the intensities of corresponding knots on opposite sides of the nucleus to infer the product of the intrinsic jet speed ($\beta_{\text{jet}}$) and the cosine of the angle that the jet axis makes with the line of sight ($\cos \theta$), via the formalism $I_p/I_r = [(1 + \beta_{\text{jet}} \cos \theta)/(1 - \beta_{\text{jet}} \cos \theta)]^{1 + \alpha}$, where $\alpha$ relates the intensity $I$ as a function of frequency $\nu$ as $I \propto \nu^{-\alpha}$. In the cases in which $\beta_{\text{jet}} \cos \theta$ is determined independently, it is found that the intensity ratio of a given pair of jet to counterjet knots is overpredicted by the above formalism compared with that actually measured from radio images. As an example, in the case of the microquasar Cygnus X-3, the original formalism predicts an intensity ratio of $\sim 185$, whereas the observed ratio at one single epoch is $\sim 3$. Mirabel & Rodríguez have presented a refined approach to the original formalism that involves measuring the intensity ratio of knots when they are at equal angular separations from the nucleus. This method is, however, only applicable where there is sufficient time-sampling (with sufficient physical resolution) of the fading of the jet-knots so that interpolation of their intensities at equal distances from the nucleus is possible. It can therefore be difficult to apply to microquasars and is impossible to apply to quasars. We demonstrate that inclusion of two indisputable physical effects, (1) the light-travel time between the knots and (2) the simple evolution of the knots themselves (e.g., via adiabatic expansion), reconciles this overprediction (in the case of Cygnus X-3 quoted above, an intensity ratio of $\sim 3$ is predicted) and renders the original formalism obsolete.

Subject headings: ISM: jets and outflows — methods: analytical — radiation mechanisms: nonthermal — relativity

1. INTRODUCTION

Relativistic jets are observed in both quasars and microquasars and are often seen to consist of a series of discrete knots moving outward from a central nucleus, believed to correspond to the compact object powering the outflow. Measurements of the proper motions of these knots are often used to constrain properties such as jet speeds and inclination angles and source distance (e.g., Mirabel & Rodríguez 1999; Hjellming & Rupen 1995). The ratio of the intensities of approaching and receding knots (if there is sufficient spatial resolution that these can be accurately identified) has been used (e.g., Saripalli et al. 1997) to constrain their Lorentz factors, via

\[
\frac{S_{\text{app}}}{S_{\text{rec}}} = \left( \frac{\mu_{\text{app}}}{\mu_{\text{rec}}} \right)^{k+\alpha} = \left( \frac{1 + \beta \cos \theta}{1 - \beta \cos \theta} \right)^{k+\alpha},
\]

where $\beta = \nu/c$ is the jet speed, $\theta$ is the inclination angle of the jet axis to the line of sight, $\alpha$ is the spectral index of the emission (defined by $S \propto \nu^{-\alpha}$, where $S$ is the flux density at frequency $\nu$), $S_{\text{app}}$ and $S_{\text{rec}}$ are the flux densities of a corresponding pair of approaching and receding knots, $\mu_{\text{app}}$ and $\mu_{\text{rec}}$ are their proper motions, and $k = 3$ for a jet composed of discrete ejecta.

The luminosities $L(t)$ of the knots change with time $t$, as the knots expand and the magnetic field, and hence the synchrotron emissivity, decreases. Thus the true flux ratio is

\[
\frac{S_{\text{app}}}{S_{\text{rec}}} = \left( \frac{1 + \beta \cos \theta}{1 - \beta \cos \theta} \right)^{k+\alpha} \frac{L_{\text{app}}(t_{\text{app}})}{L_{\text{rec}}(t_{\text{rec}})},
\]

where $t_{\text{app}}$ and $t_{\text{rec}}$ are the times at which light leaves the approaching and receding knots, respectively, in order to arrive at the telescope at the same time. Unless the jet axis is perpendicular to the line of sight, however, the light-travel time between approaching and receding knots will mean that we see the receding jet as it was at an earlier time, when it was more compact and hence intrinsically brighter (but also dimmed in the observer’s frame by its recessional motion, taken into account by the original formalism), compared with the approaching jet seen at the same telescope time. To account for this effect, Mirabel & Rodríguez (1999) proposed that the flux densities used to calculate the ratio should be measured at equal angular separations from the nucleus. This cannot always be implemented in practice, however, since this will require interpolation unless good temporal coverage of the jets is available or unless the jet is a continuous flow, in which case the motion of individual knots cannot be tracked in any case. At early times it may also be difficult to separate the emission from moving jet knots and a fading core if there is insufficient spatial resolution. Moreover, as a result of opacity or the presence of a broken power law, interpolation of the spectrum may not be straightforward if in the observer’s frame we sample...
different parts of the spectrum at any given frequency. In this Letter, we present a method of using the flux ratio from a single image of a source to constrain the jet speeds without resorting to interpolation via the Mirabel & Rodriguez method.

2. FLUX RATIOS

2.1. Simple Scalings

A synchrotron-emitting plasmon in which the particles undergo adiabatic expansion will have a power-law decay in intensity, \( I(t) \propto t^{-\gamma} \), in which case equation (2) becomes

\[
\frac{S_{\text{app}}}{S_{\text{rec}}} = \left(\frac{1 + \beta \cos \theta}{1 - \beta \cos \theta}\right)^{\gamma + \alpha - 3/2} \left(\frac{t_{\text{app}}}{t_{\text{rec}}}\right)^{-\gamma}. \tag{3}
\]

and this scaling will apply to any process that gives a power-law decay in intensity. We consider symmetric approaching and receding jets, in which case after ejection at \( t = 0 \), the epochs at which photons leave corresponding points of the front and back plasmons, \( t_{\text{app}} \) and \( t_{\text{rec}} \), respectively, are related by

\[
\frac{t_{\text{app}}}{t_{\text{rec}}} = \frac{1 + \beta \cos \theta}{1 - \beta \cos \theta}. \tag{4}
\]

So in this simple case equation (3) becomes

\[
\frac{S_{\text{app}}}{S_{\text{rec}}} = \left(\frac{1 + \beta \cos \theta}{1 - \beta \cos \theta}\right)^{k + \alpha - 3/2}. \tag{5}
\]

While this ratio is applicable to any process that gives a power-law decay, in the adiabatically expanding synchrotron case the parameters \( \alpha \) and \( \xi \) are not independent, so that the determination of the flux ratio by equation (5) is actually not introducing an extra parameter.

2.2. Optically Thin Synchrotron Emission and Adiabatic Expansion

The total synchrotron emissivity from a single optically thin jet knot scales as (e.g., Longair 1994)

\[
J(\nu) \propto B^{3/2} N(\gamma) \gamma^2 \nu^{-1/2}, \tag{6}
\]

where \( B \) is the magnetic field strength, \( \gamma \) is the Lorentz factor of an individual electron assumed to be radiating at a single frequency

\[
\nu = \frac{\gamma^2 e B}{2 \pi m_c}, \tag{7}
\]

and \( N(\gamma) \) is the total number of electrons with energies in the range \( (\gamma, \gamma + d\gamma) \) in the plasmon, given by

\[
N(\gamma, t_0) = A \gamma^{-p}, \tag{8}
\]

where \( p \) is the electron index, \( t_0 \) is some arbitrary reference time, and \( A \) is the normalization constant. As the plasmon expands, nonrelativistically, from a radius \( R_0(t_0) \) to \( R(t) \), the electron energy scales as

\[
\gamma = \frac{R}{R_0} \gamma_0, \tag{9}
\]

if synchrotron losses are negligible. The spectrum then evolves according to

\[
N(\gamma, t) = \frac{R}{R_0} N\left(\frac{R}{R_0}, \gamma, t_0\right). \tag{10}
\]

Thus the synchrotron emissivity of an expanding plasmon is given by

\[
J(\nu) \propto \nu^{(1-p)/2} B^{(1+p)/2} R^{1-p}. \tag{11}
\]

As the plasmon expands, the magnetic field strength will decrease, and in the case of a tangled field, we have \( B \propto R^{-1} \), so the plasmon emissivity has a simple dependence on frequency and size given by

\[
J(\nu) \propto \nu^{(1-p)/2} R^{(1-3)p/2}. \tag{12}
\]

The ratio of flux densities as seen by the observer is then

\[
\frac{S_{\text{app}}}{S_{\text{rec}}} = \left(\frac{R(t_{\text{app}})}{R(t_{\text{rec}})}\right)^{3-3p/2} \left(\frac{1 + \beta \cos \theta}{1 - \beta \cos \theta}\right)^{k+3(p+1)/2}. \tag{13}
\]

Although we could take the expansion of the plasmon to be of the form \( R \propto t^\gamma \), it is particularly instructive to look at the case of linear expansion, \( \eta = 1 \), for which equation (13) becomes

\[
\frac{S_{\text{app}}}{S_{\text{rec}}} = \left(\frac{1 + \beta \cos \theta}{1 - \beta \cos \theta}\right)^{k-p}. \tag{14}
\]

This is the flux ratio observed at a given instant by the telescope as opposed to the interpolated flux at equal angular separations. As a simple generic case, emission from a jet composed of discrete ejecta \( (k = 3) \) from a spectrum of electron index \( p = 2 \) will give an exponent of unity for equation (14). By way of contrast, obtaining an interpolated estimate at equal angular separations will, in this case, give an exponent of \( k + \alpha = 3.5 \). The flux ratios to be measured in the two cases would, however, differ, being measured in a single image in the former case and at equal angular separations in the latter. A comparison of both methods in any given source would of course be a useful means of inferring a possible asymmetry in the approaching and receding jets (Atoyan & Aharonian 1997).

2.3. Synchrotron Self-Absorption and Spectral Breaks

When the particle spectrum contains a break or a turnover we must adapt the above discussion. For example, in Cygnus X-3 (Miller-Jones et al. 2004), we observe two discrete knots, one on each side of the central nucleus, with evidence for a turnover due to synchrotron self-absorption. In this case the spectrum will have the form \( J \propto \nu^{3/2} \) below the turnover frequency \( \nu_0 \) when the knot radius is \( R_0 \), while above this frequency the spectrum is optically thin, \( J \propto \nu^{-p/2} \). As the knot expands to a radius \( R \), its emission will then take the self-absorbed form

\[
J(\nu, R) = J_{\text{max}}(R) \left[\frac{\nu}{\nu_*(R)}\right]^{3/2}, \quad \nu \leq \nu_*(R), \tag{15}
\]
while in the optically thin regime the intensity scales as

\[ J_n(R) = J_{\text{max}}(R) \left( \frac{p}{p(R)} \right)^{-\frac{p}{p} - \frac{1}{2}}, \quad p \geq p_c(R). \]  

(16)

The critical frequency beyond which the emissivity becomes optically thin is determined by

\[ p_c(R) = p_0 \left( \frac{R}{R_0} \right)^{-\frac{3p}{2} + \frac{4}{2} - \frac{3}{2}}, \]  

(17)

and the emission at that frequency, which is the peak of the knot spectrum, becomes

\[ J_{\text{max}}(R) = J_0 \left( \frac{R}{R_0} \right)^{-\frac{5p}{2} + \frac{4}{2}}. \]  

(18)

Turning now to the flux ratios observed at a given instant and frequency, it is clear that at sufficiently early and late times we will have two extremes. In the former case, when the observed emission from each knot is optically thick, we will see a \( J \propto R^{5/2} \) spectrum from each and the flux ratio exponent is \( k \). However, it may be difficult to observe actual knots in this regime without mixing in possible nuclear emission. At observed frequency \( \nu \) the emission will become optically thin from the approaching knot when its radius is \( R_1 \), which is determined by

\[ \nu = (1 + \beta \cos \theta) \nu_0 \left( \frac{R_1}{R_0} \right)^{-\frac{3p}{2} + \frac{4}{2} - \frac{3}{2}}, \]  

(19)

if the radii are defined in the observer’s frame. If they are determined in the rest frame of the knot, there is an extra factor of \( \gamma \) on the right-hand side of equation (19). The emission from the receding knot will remain optically thick at this frequency until the approaching knot has a radius of

\[ R_2 = \left( \frac{1 + \beta \cos \theta}{1 - \beta \cos \theta} \right)^{\frac{2p}{3} + \frac{4}{2}} R_1. \]  

(20)

Therefore, when the approaching knot has a radius \( R \) satisfying \( R_1 \leq R \leq R_2 \), the observed emission at frequency \( \nu \) will be a mix of Doppler-boosted optically thin emission from the forward knot and optically thick emission from the receding component. The flux ratio in this regime is now dependent on time, i.e., knot radius, and is given by

\[ \frac{S_{\text{app}}}{S_{\text{rec}}} = \left( \frac{1 + \beta \cos \theta}{1 - \beta \cos \theta} \right)^{\frac{3p}{2} + \frac{4}{2}} \left( \frac{R}{R_1} \right)^{-\frac{3p}{2} + \frac{4}{2}}. \]  

(21)

At \( R = R_1 \) all of the emission becomes optically thin at this frequency, and the flux ratios predicted by equations (14) and (21) are equal. Therefore, the flux ratio exponent drops from a value of \( k \) to \( k - p \) as the front, and then the receding knot become optically thin. During this time the spectrum at frequency \( \nu \) also evolves from \( \nu^{5/2} \) to \( \nu^{-p} \), and the forward knot expands by a factor of \( R_2/R_1 \), where both radii are frequency dependent. The time taken for this expansion is determined by the expansion velocity \( V \) of the knot.

In reality, however, it is unlikely that the turnover in the spectrum would occur at one single frequency. There would be a finite turnover region in which the spectrum evolved from a \( \nu^{5/2} \) power law to \( \nu^{-p} \). Depending on the width of the turnover region and the value of \( \beta \cos \theta \), the receding knot could be in the turnover region of the spectrum by the time the approaching knot had become optically thin. In this case, the above results would not be strictly applicable.

Nonetheless, these general points apply to any source of opacity that changes the spectral shape or indeed to any broken power law that might be attributable to the acceleration mechanism. It presents the possibility that the evolution of the spectrum from flares in microquasars may well be influenced by the light-travel time differences between approaching and receding knots, as outlined in Miller-Jones et al. (2004).

2.4. Caveats

Care should be taken if the expansion mode of the plasmons changes prior to the observation from which the flux ratio is derived. In such a case, for example, a transition from slowed to free expansion (Hjellming & Johnston 1988), the time decay of the flux density would change (in this case steepen with time). In order to use flux ratios to constrain the value of \( \beta \cos \theta \), the flux densities of the approaching and receding knots would then have to be measured when the knots were both in the same expansion regime. Unless the transition radius were known, this would require actually measuring (as opposed to interpolating) the flux densities at equal angular separation from the core. We note that if there is significant deceleration of the expanding plasmons due to interaction with surrounding material (e.g., Hjellming & Han 1995), then \( (R/R_0) \propto (\theta t/\gamma)^{\eta} \), where \( \eta < 1 \), and equation (14) then requires modification. We also draw attention to Fender (2003), which presents caveats to be considered when using proper motions to place limits on the bulk Lorentz factors of jets; any Lorentz factors thus derived are strictly only lower limits.

3. Comparison with Observations

The Very Long Baseline Array observations of Cygnus X-3 presented by Miller-Jones et al. (2004) show a jet that at 5 and 15 GHz appears to be composed of two separating discrete knots, which were interpreted as approaching and receding plasmons. A precession modeling analysis yielded a value \( \beta \cos \theta = 0.62 \pm 0.11 \), and the spectral index of the emission was found to be \( \alpha = 0.60 \pm 0.05 \). Assuming linear expansion of the jet knots, we would thus predict a flux density ratio of 3.2 \pm 1.0. For the last two epochs (2001 September 20 and 21), the measured flux ratios are given in Table 1. While not matching the theoretical prediction perfectly, they are now of the correct order, in contrast with the predictions of the original formalism, which is wrong by 2 orders of magnitude. There are various possible explanations for the slight discrepancy. Most importantly, the measurement of the flux densities themselves was often difficult. It is also possible that the plasmon expansion was not exactly linear with time, which would alter the exponent in equation (14) and change the predicted flux.
density ratio. Moreover, the measured spectral index \( \alpha \) was for the integrated spectrum; the values of \( \alpha \) and \( p \) could in principle differ for the individual jet knots. The quality of the data makes it difficult to interpolate back to the flux densities at equal angular separations in this case, but our best attempts gave flux ratios between 1.58 and 10.63. In such cases, our direct measurement method gives a much more accurate determination of the expected flux ratio, for more meaningful comparison with the jet speeds and inclination angle found by different methods. We note that if the spectral index of the jet material is known, our method requires only a single image to determine the value of \( \beta \cos \theta \), whereas the interpolation method requires at least two images taken at different times. This frees it from the uncertainty inherent in comparing VLBI images, particularly if the imaging is difficult. For a single image, the ratio of two flux densities is set, whereas when comparing different images, in order to be able to interpolate accurately (and hence take an accurate flux density ratio), one has to be confident that one has recovered the same fraction of the true flux density in both images.

This theory could also be applied to the observations of GRS 1915+105 detailed by Mirabel & Rodríguez (1994). They observed discrete radio ejecta moving outward from the nucleus over a period of \( \sim 1 \) month. Again, we take the flux density ratio of their observed knots once they had clearly separated from one another and from the nucleus, and we only compare corresponding pairs of ejecta. From their derived value of \( \beta \cos \theta = 0.323 \pm 0.016 \) and their quoted spectral index of \( \alpha = 0.84 \pm 0.03 \), we predict a flux density ratio of 1.24 \pm 0.05. For the later epochs (1994 April 16, 23, and 30), the measured flux density ratios are 2.33, 2.63, and 1.80, respectively. Again, this is slightly greater than we predict but is of the right order. Underpredicting the flux density ratio implies that the exponent should be larger in equation (14), which requires \( \eta > 1 \), i.e., the knots expand more rapidly with time than \( R \propto t \).

4. CONCLUSIONS

We have considered the evolution of synchrotron bubbles (plasmons) in oppositely directed microquasar jets. We have found that our new formalism can explain the observed flux density ratios in microquasar jets in systems in which the synchrotron bubble model is applicable, such as Cygnus X-3. In contrast, the original formalism considerably overpredicts the observed flux density ratio in observations of this system. In the case of free (linear) expansion, \( (R/R_\infty) \propto (tt_\infty)^{1-p} \), we found that the flux ratios of the approaching and receding plasmons are given by \( S_{app}/S_{rec} = [(1 + \beta \cos \theta)/(1 - \beta \cos \theta)]^{1-p} \).

J. C. A. M.-J. thanks the UK Particle Physics and Astronomy Research Council for a Studentship. K. M. B. thanks the Royal Society for a University Research Fellowship. K. M. B. and P. D. acknowledge a joint British Council/Enterprise Ireland exchange grant.

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Note added in proof.—It has just come to our attention that M. Ribó, E. Ros, J. M. Paredes, M. Massi, & J. Martí (A&A, 394, 983 [2002]) imaged a microquasar candidate, 1RXS J001442.2+580201, with the European VLBI Network and resolved knots on either side of the core. Application of our equation (14) gives a value of \( \beta \cos \theta \) in agreement with that derived from their proper-motion analysis, providing further verification of our method.