3D hydrodynamic simulations of C ingestion into a convective O shell

R. Andrassy,1,2 F. Herwig,1,2 P. Woodward,3,2 C. Ritter1,2

1 Department of Physics & Astronomy, University of Victoria, PO Box 1700 STN CSC, Victoria, BC, V8W 2Y2, Canada
2 Joint Institute for Nuclear Astrophysics, Center for the Evolution of the Elements, Michigan State University, 640 South Shaw Lane, East Lansing, MI 48824, USA
3 LCSE and Department of Astronomy, University of Minnesota, Minneapolis, MN 55455, USA

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ABSTRACT
Interactions between convective shells in evolved massive stars have been linked to supernova impostors, to the production of the odd-Z elements Cl, K, and Sc, and they might also help generate the large-scale asphericities that are known to facilitate shock revival in supernova explosion models. We investigate the process of ingestion of C-shell material into a convective O-burning shell, including the hydrodynamic feedback from the nuclear burning of the ingested material. Our 3D hydrodynamic simulations span almost 3 dex in the total luminosity \( L_{\text{tot}} \). All but one of the simulations reach a quasi-stationary state with the entrainment rate and convective velocity proportional to \( L_{\text{tot}} \) and \( L_{\text{tot}}^{1/3} \), respectively. Carbon burning provides \( 14 - 33\% \) of the total luminosity, depending on the set of reactions considered. Equivalent simulations done on 7683 and 11523 grids are in excellent quantitative agreement. The flow is dominated by a few large-scale convective cells. An instability leading to large-scale oscillations with Mach numbers in excess of \( 0.2 \) develops in an experimental run with the energy yield from C burning increased by a factor of 10. This run represents most closely the conditions expected in a violent O-C shell merger, which is a potential production site for odd-Z elements such as K and Sc and which may seed asymmetries in the supernova progenitor. 1D simulations may underestimate the energy generation from the burning of ingested material by as much as a factor two owing to their missing the effect of clumpiness of entrained material on the nuclear reaction rate.

Key words: stars: massive, evolution, interiors – physical data and processes: hydrodynamics, convection, turbulence

1 INTRODUCTION
Throughout its evolution, various internal layers of a massive star become unstable to convection and mix efficiently. The boundaries of convection zones deep in the interior of evolved stars are usually so stiff that the slow convective eddies, typically characterised by a Mach number in the range \( 10^{-4} \lesssim M_{a} \lesssim 10^{-2} \), cannot directly “overshoot” the boundary by any significant distance (e.g. Roxburgh 1965; Saslaw & Schwarzschild 1965; Hurlburt et al. 1994; Meakin & Arnett 2007b; Woodward et al. 2015). Multi-dimensional numerical simulations of turbulent convection show that mass entrainment from the adjacent stable layers is still possible via a mechanism involving local Kelvin-Helmholtz instabilities in a thin boundary-separation layer formed by large-scale flows (see Sect. 3.1 and Woodward et al. 2015). The standard mixing-length theory (MLT) used in stellar evolution to describe convection does not speak to the properties and efficiency of the convective boundary mixing (CBM). To model this process, simple parametric models are usually adopted. One approach is to extend the fully-mixed convection zone by a distance \( \alpha_{\text{ov}} H_{p} \), where \( \alpha_{\text{ov}} \) is a free parameter and \( H_{p} \) the local pressure scale height (Maeder 1976). Alternatively, CBM can be modelled in a time-dependent way by a diffusion coefficient \( D \propto \exp\left( -\frac{z}{f_{\text{CBM}} H_{p}} \right) \) decaying exponentially with increasing distance \( z \) from the boundary (Freytag et al. 1996; Herwig et al. 1997). Again, \( f_{\text{CBM}} \) is a free parameter usually assumed to be the same for a whole class of (or even all) convective boundaries. For low-mass main-sequence stars asteroseismology has recently provided diagnostics that suggest that exponential CBM is matching observations better than step overshooting (Moravveji et al. 2015), and that the

* E-mail: andrassy@uvic.ca

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CBM parameter for the H-burning cores of low- and intermediate-mass stars is in the range $f_{\text{CBM,Icone}} \approx 0.018 \ldots 0.03$ (Moravveji et al. 2016; Hjørringgaard et al. 2016), in agreement with previously adopted values essentially based on fitting the width of the main-sequence (Herwig 2000; Pignatari et al. 2016).

1D models of massive stars in late stages of their evolution are sensitive to the assumptions made about CBM (Woosley & Weaver 1988; Sukhbold & Woosley 2014; Davis et al. 2017). Davis et al. (2017) present a grid of 1D stellar-evolution models of a 25 $M_{\odot}$ star, in which they vary the $f_{\text{CBM}}$ parameter at a few characteristic points in the star’s evolution after the end of core He burning. They find that the distribution and interactions between convective shells in the model are quite sensitive to $f_{\text{CBM}}$. The core compactness parameter $\xi_5$ (O’Connor & Ott 2011) varies in a non-monotonic way by as much as a factor of two when $f_{\text{CBM}}$ is increased from 0.002 to 0.032 in a few steps. Although this single parameter is likely insufficient to predict if the star will explode (Ugliano et al. 2012; Ertl et al. 2016), such a large uncertainty in core structure at collapse is likely to influence explosion models.

Davis et al. (2017) also document a substantial impact of CBM on pre-supernova yields through two main processes, shell dredge-up or shell mergers. A few cases of convective shell mergers seen in 1D models have been reported in the literature (Rauscher et al. 2002; Tur et al. 2007; Ritter et al. 2018; Clarkson et al. 2018), inspired by a merger of convective C- and O-burning shells in their stellar evolution model of a 15 $M_{\odot}$ star, investigate the unusual nucleosynthesis that occurs when a large amount of C-shell material is rapidly brought into the hot O-shell environment. They conclude, in agreement with earlier reports by Rauscher et al. (2002) and Tur et al. (2007), that the odd-Z elements P, Cl, K, and Sc, which are underproduced in current galactic chemical evolution models, can be synthesised this way. Clarkson et al. (2018) find a highly energetic H-He convective shell interaction event in their model of a 45 $M_{\odot}$ Pop III star when the gap between convective H and He shells closes. Their single-zone calculations of the resulting i-process nucleosynthesis can reproduce certain intermediate-mass-element features in the abundance distribution of the most Fe-poor CEMP-no (C-enhanced metal poor) stars observed to date.

The interaction between two convective shells and their ultimate merger (if it occurs) involve complex physics of convective boundary mixing and convective-reactive flows that the MLT employed in stellar-evolution calculations cannot describe properly. Herwig et al. (2014) investigate H entrainment into a convective He-burning shell during a very late thermal pulse in the post-AGB star Sakurai’s object. The H-rich region above the He-shell flash convection zone is stably stratified in that case, which simplifies the overall problem a bit. Their 3D simulations reveal a new phenomenon that the authors call Global Oscillation of Shell H ingestion (GOSH). The convective shell experiences non-spherical oscillations of large amplitude during a GOSH event, in which the spherically-averaged stratification of composition and entropy is substantially rearranged, and the entrainment rate temporarily increases by two orders of magnitude.

The GOSH phenomenon involves low-order modes of convection, so it is likely to favour convective shells of low aspect ratio. In stellar evolution simulations, the MLT convection responds with a split of the convection zone when the luminosity from the nuclear burning of entrained material reaches a significant fraction of the luminosity that drives the shell convection in the first place. The GOSH is the 3D response of shell convection to this condition. A GOSH-like event might also occur in a massive star when a convective shell entrains some “flammable” material from the overlying stable or convective layers. If such an event is under way at the point of core collapse it could provide the large-scale and large-amplitude seed perturbations that facilitate shock revival in multidimensional supernova explosion models (Couch & Ott 2013, 2015; Müller & Janka 2015; Müller et al. 2017). On the other hand, if it occurs earlier and is energetic enough it might help explain the outbursts (supernova impostors, SN IIn) that have been observed to occur in some supernova progenitors in the last years and decades before the explosion (Smith et al. 2011; Smith & Arnett 2014; Ofek et al. 2014; Arcavi et al. 2017). The energy generated by a shell merger deep in the star’s compact core might be transported to the envelope via waves as proposed by Quataert & Shioide (2012) and later investigated by Shioide & Quataert (2014), Fuller (2017), and Fuller & Ro (2018).

Jones et al. (2017, J17 hereafter) presented a set of idealised 3D hydrodynamic simulations constructed to closely resemble a convective O-burning shell in their 1D model of a 25 $M_{\odot}$ star. They derive a mass entrainment rate at the upper convective boundary of $5.4 \times 10^{-7} M_{\odot} s^{-1}$ for the luminosity of the 1D model. It would only take ~6 days — less than the lifetime of the O shell — for the O-shell convection to reach the bottom of a neighbouring convective C-burning shell at this rate. Some material from the C shell could thus be entrained into the hot O-shell environment or the two shells might even merge, as suggested by the nuclear astrophysics results of (Ritter et al. 2018). As a first essential step towards a full merger simulation, we construct a set of 3D hydrodynamic simulations to investigate the dynamics of C entrainment from a stable layer into a convective O-burning shell. We intend to answer questions like: How strong is the feedback from C burning on the flow in the shell? How does it depend on the O-luminosity of the shell? Is the entrainment process stable? Does it ever lead to a GOSH-like instability? How is the luminosity–entrainment rate relation measured by J17 affected by C burning?

2 METHODS

2.1 1D stellar-evolution model

The 3D simulations described in this work are based on the 1D evolution model of a 25 $M_{\odot}$ star computed by J17 using the MESA code (Paxton et al. 2011, 2013, 2015). The stellar model’s initial metallicity is $Z = 0.02$. Rotation is not considered and the Schwarzschild criterion is used to delineate convective regions. Mixing of chemical species at convective boundaries is modelled using a diffusion coefficient decreasing exponentially with an e-folding length $\frac{1}{f_{\text{CBM}}H_p}$, where $f_{\text{CBM}} = 0.022$ is used for all convective boundaries before core $C$ ignition except the bottom boundaries of burning shells where $f_{\text{CBM}} = 0.005$ is set and $f_{\text{CBM}} = 0.002$ is used for all convective boundaries after core $C$ ignition.
We focus on the first shell O-burning phase that spans approximately from 21 to 4 days before core collapse (Fig. 1). With the start of shell O burning, a convection zone appears and grows in size until its mass reaches 0.66 M$_\odot$. The outward propagation of the upper convective boundary slows down at this point and further growth is limited to an additional 0.08 M$_\odot$ over 7 days, after which convection recedes. At the point of the shell’s maximum extent there is 0.18 M$_\odot$ of stable material separating the O shell from the C shell and there is no mixing between the two in the stellar evolution model.

2.2 3D PPMstar simulations

2.2.1 PPMstar code

We use the PPMstar code of Woodward et al. (2015). It is an explicit Cartesian-grid-based code for 3D hydrodynamics built around the Piecewise-Parabolic Method (PPM; Woodward & Colella 1981, 1984; Colella & Woodward 1984; Woodward 1986, 2007). The code advects the fractional volume of the lighter fluid in a two-fluid scheme using the Piecewise-Parabolic Boltzmann method (PPB; Woodward 1986; Woodward et al. 2015). Thanks to PPB’s use of subcell information, it needs two to three times fewer grid cells along all three axes than PPM to reach the same level of fidelity in the advection of a quantity, like the multifluid mixing fraction, whose value is conserved along stream lines. The code was designed with strong emphasis on parallel efficiency and it has performed past simulations of shell convection on up to 440,000 CPU cores on the NCSA Blue Waters computer (Woodward et al. 2015; Herwig et al. 2014).

2.2.2 3D simulation setup

The initial stratification of our simulations is the same as the one used by J17 and is composed of three polytropes that approximate the stratification of the MESA model. The bottom and top polytropes are stable against convection and the middle one is adiabatic. The C shell and outer layers of the star as well as the inner core are not included. We impose impermeable boundaries at radii of 3.5 M$_\odot$ and 9.2 M$_\odot$. We neglect radiation pressure (∼25% of the total pressure) and the equation of state is that of an ideal gas. Neutrino cooling is not considered. We refer to J17 for more details about the set-up.

The upper stable layer is initially filled with fluid $f_C$ and the rest is fluid $f_O$. There is a smooth transition spanning 0.25 M$_\odot$ between the two fluids. We use the same mean molecular weights as J17 ($\mu_1 = 1.802$ for $f_C$ and $\mu_2 = 1.848$ for $f_O$) to allow direct comparison of entrainment rates. The radial profile of the squared Brunt-Väisälä frequency $N^2$ in this transition layer closely resembles that of the MESA model, see Fig. 6 of J17.

The concentration of $^{16}$O in fluid $f_O$, $X_{16} = 0.382$, is taken directly from the MESA model. Fluid $f_C$ is assumed to be rich in $^{12}$C with the fraction $X_{12} = 0.13$. This concentration is 5 times larger than that in the C shell of the MESA model. We do this to speed up the simulations’ transition to a quasi-stationary state.

J17 used a volume heating term to drive convection. We have implemented an O-burning prescription in the code (see Sect. 2.2.4) and introduced a parameter ($f_{\text{He}}$ in Table 1) to scale the heat output and reach different driving luminosities without having to change the initial stratification. Figure 2 shows that the heating rate distribution is more concentrated to the bottom of the shell in our simulations than in those of J17.

Figure 1. Kippenhahn diagram showing the core structure of the 25 M$_\odot$ MESA model as a function of time until core collapse. Shades of blue show the energy generation rate and grey regions are convective. The light-blue dashed and turquoise dot-dashed lines are the boundaries of the He- and C-free cores, respectively. The red vertical line indicates the model that was used to construct the initial condition of the 3D simulations. The solid portion of that line marks the radial extent of the 3D model.

Figure 2. Comparison of the distribution of the heating rate per unit volume as given by the O-burning prescription used in this work with that given by the volume heating prescription of J17. Both distributions are normalised to give the same luminosity of $10^{11} L_\odot$. The flat part of the entropy profile $A = \rho/\rho'$ is the convection zone.
Table 1: Properties of the 3D PPMstar simulations presented in this work. The first three columns give the run identification code, grid resolution, and the total length of the simulation. If C burning is included the “nuc. net.” column gives which nuclear network was used. Energy output from nuclear reactions is scaled by factors $f_{\text{CO}}$ ($^{16}\text{O}+^{16}\text{O}$), $f_{\text{CC}}$ ($^{12}\text{C}+^{12}\text{C}$), and $f_{\text{CO}}$ ($^{12}\text{C}+^{16}\text{O}$) and the resulting contributions to the total luminosity $L_{\text{tot}}$ are $L_{\text{CO}}$, $L_{\text{CC}}$, and $L_{\text{CO}}$. The last four columns give the entrainment rate $M_e$, rms convective velocity $v_{\text{rms}}$, overturning time scale $\tau_{\text{conv}}$ and a mass-weighted Mach number in the convection zone. The values of $L_{\text{CO}}$, $L_{\text{CC}}$, $L_{\text{CO}}$, $M_e$, $v_{\text{rms}}$, $\tau_{\text{conv}}$, and $\text{Ma}$ correspond to averages over a time window several convective overturns long, which was the same for all variables but varied from run to run. All runs of the D series except for D23 were already presented by J17.

| id | grid | $\tau_{\text{sim}}$ | nuc. net. | $f_{\text{CO}}$ | $f_{\text{CC}}$ | $f_{\text{CO}}$ | $L_{\text{CO}}$ | $L_{\text{CC}}$ | $L_{\text{CO}}$ | $M_e$ | $v_{\text{rms}}$ | $\tau_{\text{conv}}$ | Ma |
|----|------|---------------------|---------|----------------|----------------|----------------|----------------|----------------|----------------|------|---------------|----------------|----|
| D1 | 768$^3$ | 55.2 | - | - | - | - | - | 1.18 (11) | 1.15 (−6) | 38.0 | 3.46 | 9.40 (−3) |
| D2 | 1536$^3$ | 27.3 | - | - | - | - | - | 1.18 (11) | 1.33 (−6) | 39.1 | 3.37 | 9.70 (−3) |
| D3 | 768$^3$ | 37.2 | - | - | - | - | - | 5.91 (11) | 8.07 (−6) | 66.7 | 2.05 | 1.65 (−2) |
| D4 | 768$^3$ | 41.3 | - | - | - | - | - | 1.18 (12) | 1.68 (−5) | 85.8 | 1.65 | 2.13 (−2) |
| D5 | 768$^3$ | 36.2 | - | - | - | - | - | 2.95 (11) | 3.60 (−6) | 52.2 | 2.58 | 1.29 (−2) |
| D6 | 768$^3$ | 43.5 | - | - | - | - | - | 2.95 (12) | 3.83 (−5) | 118 | 1.27 | 2.92 (−2) |
| D7 | 768$^3$ | 43.7 | - | - | - | - | - | 5.91 (12) | 7.94 (−5) | 149 | 1.03 | 3.71 (−2) |
| D8 | 768$^3$ | 53.4 | - | - | - | - | - | 2.95 (10) | 9.50 (−8) | 21.0 | 6.14 | 5.17 (−3) |
| I2 | 768$^3$ | 242 | NET 1 | 1 | 1 | - | 4.27 (10) | 8.30 (9) | - | 5.10 (10) | 3.14 (−7) | 27.9 | 4.74 | 6.85 (−3) |
| I4 | 768$^3$ | 76.3 | NET 1 | 2.7 | 1 | - | 1.16 (11) | 2.84 (10) | - | 1.45 (11) | 1.09 (−6) | 40.3 | 3.34 | 9.89 (−3) |
| I5 | 768$^3$ | 39.5 | NET 1 | 13.5 | 1 | - | 5.79 (11) | 2.54 (11) | - | 8.33 (11) | 9.66 (−6) | 117 | 1.94 | 1.78 (−2) |
| I8 | 768$^3$ | 39.5 | NET 1 | 13.5 | 10 | - | 8.05 (11) | 1.32 (13) | - | 1.40 (13) | 2.24 (−4) | 181 | 0.82 | 4.46 (−2) |
| I11 | 768$^3$ | 46.9 | NET 1 | 2.7 | 1 | - | 1.16 (11) | - | - | 1.16 (11) | 7.53 (−7) | 35.3 | 3.73 | 8.67 (−3) |
| I13 | 768$^3$ | 23.1 | NET 1 | 67.5 | 1 | - | 5.10 (12) | 2.54 (12) | - | 7.63 (12) | 1.07 (−4) | 147 | 1.02 | 3.63 (−2) |
| I14 | 1152$^3$ | 22.4 | NET 1 | 67.5 | 1 | - | 6.26 (12) | 2.87 (12) | - | 9.13 (12) | 1.18 (−4) | 163 | 0.95 | 4.01 (−2) |
| I15 | 1152$^3$ | 58.3 | NET 1 | 2.7 | 1 | - | 1.16 (11) | 3.37 (10) | - | 1.50 (11) | 1.37 (−6) | 41.6 | 3.23 | 1.03 (−2) |
| I16 | 768$^3$ | 73.8 | NET 2 | 2.7 | 1 | 1 | 1.16 (11) | 2.25 (9) | 2.10 (10) | 1.39 (11) | 9.73 (−7) | 39.3 | 3.41 | 9.66 (−3) |
| I17 | 768$^3$ | 27.8 | NET 2 | 67.5 | 1 | 1 | 4.49 (12) | 3.97 (11) | 3.59 (11) | 5.25 (12) | 6.30 (−5) | 139 | 1.06 | 3.44 (−2) |
2.2.3 Simulations

Simulations presented in this work are listed in Table 1 along with some of their global properties. We investigate the luminosity dependence of C ingestion in the series of runs I2, I4, I5, and I13 done on a 768\(^3\) grid. They only differ in the O-luminosity enhancement factor \(f_{\text{OO}}\) (see Table 1). C burning was only turned on at \(t = 74\) min in run I2 by mistake. Runs I14 and I15 are higher-resolution versions of runs I13 and I4, respectively, done on a 1152\(^3\) grid to estimate any resolution dependence. We experimentally turned C burning off in run I12 and enhanced the energy release from C burning by the factor \(f_{\text{CC}} = 10\) in run I11, which are otherwise like runs I4 and I5, respectively. All the runs mentioned so far use the same C-burning reaction network Net 1 (see the next section). We investigate how our results depend on the assumptions about C-burning reactions using runs I16 and I17, which are like runs I4 and I13, respectively, but they use an alternative reaction network Net 2. We also use for the analysis runs D1, D2, D5, D6, D8, D9, and D10 of J17, in which convection is driven by a volume heating term and the burning of the entrained material is not considered. Finally, we have added run D23 to extend the D-series of runs towards even lower luminosities.

2.2.4 Reaction network

Convection in the shell is driven by O burning, which we compute using Eq. 18.75 of Kippenhahn et al. (2012), neglecting electron screening. The oxygen mass fraction \(X_{\text{O}} = 0.382\) is kept constant. The temperature is slightly overestimated in the shell because of our neglect of radiation pressure in the equation of state. We correct the temperature profile before computing energy generation rates using the transformation

\[
\theta = 1.022 T_9^{0.64},
\]

where \(T_9\) is the temperature in units of \(10^9\) K. In order to model the drop in \(T_9\) and \(X_{\text{O}}\) at the bottom of the O shell seen in the MESA model, we further modify the temperature profile using either the transformation

\[
\Theta = \frac{1}{2}(1 - \tanh[200(\theta - 2.24)]) \theta,
\]

or the transformation

\[
\Theta = \frac{1}{2}(1.01 - 0.99 \tanh[0.07(\rho_3 - 1845)]) \theta,
\]

where \(\rho_3\) is the density in units of \(10^3\) g cm\(^{-3}\). Equation 2 is used in low- and medium-luminosity runs I2, I4, I5, I12, I15, and I16, but the shallowness of the temperature profile in the lower stable layer makes \(\Theta\) too sensitive to the relatively large changes in the stratification we see in high-luminosity runs. Equation 3 is much less sensitive to such changes. We use it in runs I11, I13, I14, and I17.

There are a large number of nuclear reactions involved in the burning of the ingested \(^{12}\text{C}\). The rate of the \(^{12}\text{C}+^{12}\text{C}\) reaction scales with the square \(X_{\text{C}}^2\) of the concentration of \(^{12}\text{C}\) and it is thus important when the concentration is high (i.e. when the entrainment rate is high). On the other hand, the \(^{12}\text{C}+^{16}\text{O}\) reaction becomes important at low concentrations (i.e. low entrainment rates) owing to the reaction’s being linear in \(X_{\text{C}}\). We have used two simple reaction networks in our experiments:

- **Net 1**: \(^{12}\text{C}(^{12}\text{C}, \alpha)^{20}\text{Ne}\) followed by \(^{16}\text{O}(\alpha, \gamma)^{20}\text{Ne}\). The rate of the former reaction is computed from the rate of \(^{12}\text{C}+^{12}\text{C}\) given by Caughlan & Fowler (1988) with the neutron and proton branches subtracted according to the branching ratios used by Dayras et al. (1977) and Pignatari et al. (2013). The \(\alpha\) particle is assumed to be immediately captured in the latter reaction. The Q values of the two reactions, 4.62 MeV and 4.73 MeV, respectively, are summed.
- **Net 2**: All channels (\(n, p, \gamma\)) of \(^{12}\text{C}+^{12}\text{C}\) and \(^{12}\text{C}+^{16}\text{O}\). The rates are taken from Caughlan & Fowler (1988) and the Q values are 3.19 MeV and 5.26 MeV, respectively. The subsequent reactions induced by the released neutrons and protons are not considered for simplicity as many temperature-dependent reaction paths are possible.

3 RESULTS

3.1 C entrainment and burning

The flow field in our simulations is dominated by a few large-scale convection cells (Fig. 4). They cause shear flows as they turn over at the upper convective boundary, but, as Woodward et al. (2015) and J17 describe in detail, entrainment does not occur where the shear is the strongest. It rather occurs at places where two neighbouring flows sliding along the boundary collide and are forced back into the convection zone by the global flow topology, dragging slivers of fluid \(\mathcal{T}_C\) along.

According to our assumption, the concentration of \(^{12}\text{C}\) in fluid \(\mathcal{T}_C\) is five times higher than that in the stellar evolution model (see Sect. 2.2.2), so we can see some energy generation due to \(^{12}\text{C}+^{12}\text{C}\) reactions in the upper stable layer in Fig. 4, but the burning is not strong enough to establish a new convection zone on the time scales considered. \(^{12}\text{C}+^{16}\text{O}\) reactions do not contribute in the stable layer, because we only compute reactions involving \(^{12}\text{C}\) from fluid \(\mathcal{T}_C\) and \(^{16}\text{O}\) from fluid \(\mathcal{T}_O\) and there is no fluid \(\mathcal{T}_O\) in the upper stable layer. The concentration of \(\mathcal{T}_C\) drops by orders of magnitude at the upper boundary as the mixing process reduces the buoyancy of the fluid mixture to make it possible for convection to pull it to the bottom of the convection zone. Therefore, C-burning reactions are virtually absent in the upper half of the convection zone and they are only rekindled at the bottom owing to an increase in temperature.

Because the \(^{12}\text{C}+^{12}\text{C}\) reaction rate depends on the square of the concentration \(X_{\text{C}}\) of \(^{12}\text{C}\), the burning time scale in runs with a significant contribution from the \(^{12}\text{C}+^{12}\text{C}\) reaction becomes shorter when the driving luminosity and hence also the mass entrainment rate is increased. The burning time scale is a few times longer than the convective overturning time scale in the low-luminosity run I2, which results in a rather flat fractional volume profile of the entrained C-rich fluid, see Fig. 3. On the other hand, there

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1 Equation 3 also makes sure that \(\Theta\) never drops to zero, which would cause division by zero in the energy generation module if a negligible but non-zero amount of the C-rich fluid got below the convection zone. This sometimes happens when convection becomes too vigorous.
is a significant fractional volume gradient in the convection zone of the high-luminosity run I13, in which the two time scales are comparable. This gradient causes the C-burning layer to be more extended in run I13 than in run I2 (Fig. 3). The burning time scale for the $^{12}$C+$^{16}$O reaction is independent of $X_{12}$.

Figure 4 also shows that there is a lot of large scale structure in the distribution of the entrained material. We show in Sect. 3.5 that these inhomogeneities are the main cause of the asymmetric distribution of the burning rate (Fig. 4) with temperature fluctuations playing a secondary role. The asymmetry is the most pronounced at the upper end of the luminosity range considered.

All runs, with the exception of run I11, reach a quasi-stationary state in which there is a close balance between mass entrainment and burning and the concentration of fluid $T_C$ in the convection zone stays approximately constant. Some selected properties of our simulations are summarised in Table 1. The following three sections present the detailed evolution of the simulations in terms of the entrainment rate, luminosity, and velocity field. We analyse 3D fluctuations of the most relevant quantities in Sect. 3.5 and, finally, we discuss the unstable run I11 in Sect. 3.6.

### 3.2 Entrainment rate

The mass $M_C(t)$ of the C-rich fluid $T_C$ entrained into the convection zone by time $t$ is the sum

$$M_C(t) = M_{C}(t) + M_{B}(t)$$

of the mass $M_{C}(t)$ of $T_C$ present in the convection zone at time $t$ and the mass $M_{B}(t)$ of $T_C$ burnt in the convection zone by time $t$.

To compute $M_{B}(t)$, we employ the method of J17. We first determine the radius $r_{ab}(t)$ of the upper boundary of the convection zone as defined by the position of the steepest decline in the spherically-averaged rms tangential velocity $v_\perp$, and the radius $r_{op}(t) = r_0(t) - H_{eab}(t)$, where $H_{eab}(t) = v_\perp (\partial v_\perp / \partial r)^{-1}$ is the scale height of $v_\perp$ to be evaluated at $r_{ab}(t)$. $M_{B}(t)$ is then the volume integral of the density of fluid $T_C$ inside the radius $r_{op}(t)$.

The burnt mass $M_{B}(t)$ is given by a volume and time integral of the mass burning rate in the convection zone, which is computed from reaction rates in a way analogous to the luminosity computation described in Sect. 3.3. Since the burning is concentrated to the lower part of the convection zone, we simply integrate up to the radius of 7.5 Mm, which is slightly below the initial location of the upper boundary of the convection zone.

Figure 5 shows the entrained mass $M_C(t)$ for runs I2 and I13, which are the least and most luminous of the 768 I-series runs with quasi-stationary C burning, respectively. In run I2, $M_C(t)$ steadily increases until C burning is turned on at $t = 74$ min (see Sect. 2.2.3), after which it takes 3–5 convective overturns to reach a quasi-stationary state with an entrainment rate of $M_C = 3.14 \times 10^{-7} M_{\odot} \text{s}^{-1}$. $M_C$ decreases again very late in the run, which is likely caused by the steepening of the entropy gradient at the upper convective boundary due to the energy release from C burning at the bottom of the upper stable layer (see Sect. 3.1). This effect is negligible in run I13 (Fig. 5), which is 12$\times$ shorter than I2 but, owing to the strong driving of convection, it reaches an entrainment rate so high that all of the upper stable layer is ultimately entrained within the simulated time. The quasi-stationary entrainment rate is $M_C = 1.07 \times 10^{-4} M_{\odot} \text{s}^{-1}$ in I13. In light of the linear dependence of the entrainment rate on the total luminosity discussed in the next paragraph, it is surprising that the entrainment rate is essentially constant while the total luminosity more than doubles between 5 and 14 min of simulation time, see Fig. 7. This might be due to the following opposing effect: as the convective boundary moves further into the stable layer, the entropy jump across the boundary increases, which hinders mass entrainment. The entrainment rate only starts to increase at $t \approx 14$ min when the convective boundary has reached the radius of 8.6 Mm (starting from 8.0 Mm at $t = 0$) and half of the upper stable layer (in terms of radius) has been engulfed by the convection. The scale height $H_{eab}$ of the velocity profile starts increasing at that point. We see the same effect in similarly luminous runs I14, I17, and also in run D10 that does not include C burning, although it only occurs when the boundary has reached 8.75 Mm in D10. A likely explanation is that the outer boundary condition at 9.2 Mm starts to influence the flows at the top of the convection zone when the upper stable layer has become too thin. The last three minutes of run I13 are not shown in Fig. 5, because velocity amplitudes in the upper stable layer become so large in that time interval that the velocity-gradient-based method of locating the convective boundary becomes unusable. We also exclude from the entrainment analysis the last few minutes of runs I14, I17, and D10 for the same reason.

The I-series runs with either C-burning network confirm the linear relation between the entrainment rate and the total luminosity established by the D-series runs of J17, see Fig. 6. Run I11 is close to the scaling relation despite

\[ 2 \] The amount of fluid $T_C$ getting below the convection zone is negligible.
Figure 4. Renderings showing a thin slice through the computational box in terms of the fractional volume $F_V$ of fluid $F_\odot$ (left panels) and energy generation rate from the $^{12}\text{C} + ^{12}\text{C}$ reaction (right panels) in runs I2 and I13. Oxygen burning is initially $67.5\times$ stronger in I13 than in I2 and the two runs differ by a factor of $\sim 170$ in the total luminosity at the points in time corresponding to the renderings. The energy generation rates were computed by post-processing data downsampled from $768^3$ to $192^3$, in which all 3D fluctuations except those in $F_V$ were neglected, see Sect. 3.5.
its unstable nature (see Sect. 3.6 for details). That the entrainment rates in low-luminosity runs fall below the linear trend is likely caused by the limited length of our simulations. One would ideally want to run a simulation of convective boundary mixing at least until all of the initial transition layer between the two fluids has been entrained and a new boundary has been formed, consistent with the properties of the convective flows and of the entrainment process. However, this is very expensive to achieve even on a 768\(^3\) grid when the luminosity is low. The initial transition layer at the upper boundary contains 2.7 \times 10^{-2} M_\odot of fluid \(T_C\), but only 3.3 \times 10^{-3} M_\odot was entrained in the low-luminosity run I2 (5.1 \times 10^{10} L_\odot, see Fig. 5), which involved 3.8 \times 10^6 time steps. The entrainment rate may depend on the assumed structure of the transition layer in such cases. On the other hand, most of the initial transition layer was entrained in runs D5 (5.9 \times 10^{11} L_\odot, see Fig. A1 of J17) and I5 (8.3 \times 10^{11} L_\odot, see Fig. B.1) and the layer was completely engulfed by convection in all runs with \(L_\text{tot} \geq 10^{12} L_\odot\).

### 3.3 Luminosity evolution

We calculate the global burning rates using spherically-averaged profiles of density, temperature, and fractional volume of fluid \(T_C\) in a post-processing step. The spherically-averaged rate of the \(^{12}\text{C}+^{12}\text{C}\) reaction scales with \(\langle X_{12}^2 \rangle = \langle X_{12} \rangle^2 + \langle X_{12} \rangle^2\), where \(\langle X_{12} \rangle^2\) is the standard deviation of the distribution of \(X_{12}\) and the distribution is taken over the full solid angle at a constant radius. We first compute the rate of \(^{12}\text{C}+^{12}\text{C}\) using \(\langle X_{12} \rangle^2\) and then multiply the result by the factor \(\xi = 1 + \langle X_{12} \rangle^2 \langle X_{12} \rangle^2\), where \(\langle X_{12} \rangle^2\) is computed from the standard deviation of the fractional volume of fluid \(T_C\). In this way, we take into account the presence of \(^{12}\text{C}\) clumps in the convection zone, which increases the resulting luminosity by a factor ranging from \(\sim 1.2\) in the least luminous runs to \(\sim 2\) in the most luminous ones compared to the luminosity calculated using the spherical average \(\langle X_{12} \rangle\) only. The presence of clumps is a 3D effect and as such it is neglected in 1D stellar evolution calculations. The non-linear scaling of the \(^{16}\text{O}+^{16}\text{O}\) reaction with the mass fraction \(X_{16}\) of \(^{16}\text{O}\) is inconsequential as the fractional volume of fluid \(T_O\) deep in the convection zone is essentially unity at all times.

The average luminosities are computed for the time intervals during which the entrainment rates were measured (see Figs. 5, B.1, and B.2), and they are summarized in Table 1. The time evolution of the luminosity contributions from O and C burning is shown in Fig. 7 for four characteristic cases. In the low-luminosity (\(\xi_{16} = 2.7\)) run I4, the O-burning luminosity first decreases during the initial adjustment of the stratification and then it levels off. The
C-burning luminosity ($^{12}\text{C}+^{12}\text{C}$) increases until an equilibrium is achieved between the rates of entrainment and burning with C burning providing $\sim 20\%$ of the total luminosity. The I16 case has the same O luminosity but it employs the C-burning network Net 2, which includes the $^{12}\text{C}+^{16}\text{O}$ reaction as well. This reaction dominates C burning when the concentration of $^{12}\text{C}$ in the convection zone is low as is the case in the low-luminosity runs I4 and I16. The contribution of C burning to the total luminosity is $\sim 17\%$ in I16.

Carbon burning is responsible for $\sim 33\%$ of the total luminosity in the more luminous run I13 ($f_{\text{DDE}} = 67.5$, Net 1), but this fraction drops to $\sim 14\%$ when the reaction $^{12}\text{C}+^{16}\text{O}$ is included in the otherwise similar run I17 ($f_{\text{DDE}} = 67.5$, Net 2), see Fig. 7. The reactions $^{12}\text{C}+^{12}\text{C}$ and $^{12}\text{C}+^{16}\text{O}$ are equally important in I17.

The luminosity $L_C$ from the C-burning reactions makes up an increasingly larger fraction of the total luminosity $L_{\text{tot}}$ as $L_{\text{tot}}$ increases from run to run when burning network Net 1 is used, see Fig. 8. When 1152$^3$ runs were performed the $L_C/L_{\text{tot}}$ fractions agree with the corresponding 768$^3$ runs. Because we start our analysis when a quasi-stationary state has been reached (with the exception of the unstable run I11), $L_C$ is close to $qX_{12}M_C$, where $q$ is the energy released per unit mass of $^{12}\text{C}$ burnt and $X_{12} = 0.13$ the mass fraction of $^{12}\text{C}$ in the entrained material (see Sect. 2.2.2). We can then write $L_C/L_{\text{tot}} = qX_{12}M_C/L_{\text{tot}}$ with $M_C/L_{\text{tot}} = 1.36 \times 10^{-17} M_\odot s^{-1} L_\odot^{-1}$ for all runs that follow the linear scaling relation shown in Fig. 6. The $^{12}\text{C}+^{12}\text{C}$ reaction as implemented in Net 1 has $q_{CC1} = 0.390 \text{MeV amu}^{-1}$ (see Sect. 2.2.4), which implies $L_C/L_{\text{tot}} = 34.5\%$. This fraction is close to the measured values for runs I13 (33.3%), I14 (31.4%), and I5 (30.5%), which closely follow the entrainment rate scaling. Runs with $L_{\text{tot}} \lesssim 3 \times 10^{11} L_\odot$ fall below the scaling relation in Fig. 6, which decreases $M_C/L_{\text{tot}}$ and, consequently, also $L_C/L_{\text{tot}}$ for these runs.

As implemented in Net 2, the yields of $^{12}\text{C}+^{12}\text{C}$ and $^{12}\text{C}+^{16}\text{O}$ reactions are $q_{CC2} = 0.133 \text{MeV g}^{-1}$ and $q_{CO} = 0.438 \text{MeV g}^{-1}$. The luminosities $L_{CC}$ and $L_{CO}$ due to the two reactions are almost the equal in run I17 ($f_{\text{DDE}} = 67.5$). The average yield, $q = 0.199 \text{MeV amu}^{-1}$, is almost 2x lower than that in the otherwise similar run I13 ($f_{\text{DDE}} = 67.5$, Net 1), which explains the low value of $L_C/L_{\text{tot}}$ in I17 as compared with I13 (see Fig. 8). Although C burning in run I16 ($f_{\text{DDE}} = 2.7$, Net 2) is dominated by the $^{12}\text{C}+^{16}\text{O}$ reac-
tion, the average yield \( q = 0.358 \text{MeV amu}^{-1} \) is comparable with \( q_{\text{C+1}} = 0.390 \text{MeV amu}^{-1} \) of \(^{12}\text{C}\to^{12}\text{C} \) (Net 1) used in the otherwise similar runs I4 and I15 \((f_{\text{O+}} = 2.7)\). Figure 8 confirms that \( L_{C}/L_{\text{tot}} \) reaches comparable values in runs I4, I15, and I16 \((L_{\text{tot}} \sim 3 \times 10^{11} L_{\odot})\).

An interesting feature of all of the I-series runs with \( L_{\text{tot}} \gtrsim 3 \times 10^{12} L_{\odot} \) is that their luminosity significantly increases in time. This is due to the gradual heating up of the convection zone (by \( \sim 10\% \)) combined with the nuclear reactions’ high temperature sensitivity: quantified as \( \partial \log \varepsilon/\partial \log T \), the typical sensitivities at the bottom of the convection zone are 32 for \(^{16}\text{O}\to^{16}\text{O} \), 19 for \(^{12}\text{C}\to^{12}\text{C} \), and 25 for \(^{12}\text{C}\to^{16}\text{O} \).

### 3.4 Velocity field

To construct the initial condition, we map a 1D hydrostatic stratification onto a 3D Cartesian grid and set all velocity components to zero. The convective flow is driven by the heat released from O burning at the bottom of the shell. An initial transient flow carrying an imprint of the computational grid disintegrates rapidly and the flow becomes fully turbulent after a few convective overturns as described in detail by Woodward et al. (2015) and Jones et al. (2017). This effect can also be seen in the radial velocity spectra shown in Figs. 9 and 11, in which an initially regular pattern of modes quickly disappears and is replaced by a turbulent spectrum.

Figure 10 shows that the velocity field is dominated by large-scale updraughts and downdraughts with complex small-scale turbulent structure in the state of quasi-stationary convection. The velocity distribution changes randomly in time and no particular direction in space is preferred, as expected of convection in the absence of rotation and magnetic field. The large-scale flow pattern remains essentially the same when grid resolution is increased from 768\(^3\) to 1152\(^3\).

Several low-order modes, including an \( l = 2 \) mode, make approximately equal contributions to the velocity field in most of our intermediate- and high-luminosity runs, see runs I5 and I13 in Fig. C.1 and I14 in Fig. 9. The high-luminosity run I17 is an exception discussed below. A strong dipolar component \((l = 1)\), which one would not expect in station-
I2 corners of the simulation cube (dashed lines), occurs when the luminosity runs \( I_2, I_4, I_{12}, I_{15}, \) and \( I_{16} \). The large-scale convection zone approaches the outer boundary condition.

We observe a weak but noteworthy effect in low-luminosity runs \( I_2, I_4, I_{12}, I_{15}, \) and \( I_{16} \). The large-scale updraughts and downdraughts aligned with the diagonals of the simulation cube. The flow remains highly turbulent and it is sometimes difficult to recognise this pattern by eye, but it can nevertheless be identified using spherical harmonic analysis (Fig. 11) and visualised by applying a low-pass filter to the velocity distribution (Fig. 12). It is suspicious that the grid aligned pattern first appears at \( t \approx 80 \text{ min} \) in \( I_2 \), because we turned on C burning in that run at \( t = 74 \text{ min} \), see Sect. 2.2.3. However, we see the same pattern in runs \( I_4, I_{15}, \) and \( I_{16} \) that had C burning on from \( t = 0 \) and even in run \( I_{12} \) that had C burning turned off throughout. It is possible that the geometry of our shell slightly prefers convection cells about as large as those of an \( l = 3 \) mode and, if the convection is weakly driven, these cells align themselves with the \( l = 3 \) mode that is slightly preferred by our Cartesian grid. We are currently investigating such subtle effects that occur in slow flows using a new version of the PPMstar code, which solves for fluctuations around a base state. Our preliminary results indicate that this formulation offers advantages for problems involving weakly driven convection.

Since the grid-alignment occurs in the phase of quasi-stationary convection, one might suspect that this effect could somehow influence the entrainment rate, which we measure in this phase, and it could possibly explain why the entrainment rates are lower than what our scaling law predicts at low luminosities, see Sect. 3.2 and Fig. 6. However, we find that the entrainment rate slightly increases in the early evolution towards the quasi-stationary state in all of the relevant runs, see run \( I_2 \) in Fig. 5, \( I_4 \) in Fig. B.1, and runs \( I_{15} \) and \( I_{16} \) in Fig. B.2. Therefore, if the grid-alignment effect has any influence on the entrainment rate, it most likely causes a slight increase and not a decrease.

Our choice of the C-burning network (\( \text{Net 1 vs Net 2} \)) does not influence the velocity distribution at the low luminosity of runs \( I_4 \) (\( \text{Net 1} \)) and \( I_{16} \) (\( \text{Net 2} \)). Velocity spectra from these two runs are essentially the same, see Fig. C.1. This is not the case for the pair of high-luminosity runs \( I_{13} \) (\( \text{Net 1} \)) and \( I_{17} \) (\( \text{Net 2} \)). \( I_{13} \) switches between modes \( l = 2, 3, 4 \) at random whereas \( I_{17} \) is dominated by an \( l = 3 \) mode throughout most of its evolution. Convection cells in this pattern, however, keep moving around the sphere unlike that in the low-luminosity runs mentioned above. We do not know the cause of this effect.

The fact that we use a realistic O-burning prescription and include the burning of the entrained material in our simulations as opposed to the simulations presented by J17 make surprisingly little difference for time-averaged velocity profiles. Figure 13 compares the low-luminosity run \( I_4 \) with the similar run \( D_1 \) of J17. The only difference can be seen at the transition from the lower stable layer into the convection zone, which is located deeper and is more abrupt in \( I_4 \) than in \( D_1 \). This is qualitatively compatible with the heating rate distribution’s being more concentrated to the bottom of the convection zone in the I series of runs, see Figs. 2 and 3. We can see the same effect in the comparison between the high-luminosity runs \( D_{10} \) and \( I_{13} \) in Fig. 14.

Convective velocity is expected to scale with \( L_{cz}^{1/3} \) on theoretical grounds (Biermann 1932; Porter & Woodward 2000; Müller & Janka 2015, J17), where \( L_{cz} \) is the total luminosity driving convection. We calculate the rms velocity \( v_{rms} = (2E_{kcz}/M_{cz})^{1/2} \), where \( E_{kcz} \) and \( M_{cz} \) are the total kinetic energy and mass of the convection zone, respectively. We average \( v_{rms} \) over the time interval during which the entrainment rate is measured, see Figs. 5, B.1, and B.2. Figure 15 shows that all runs, including the unstable run \( I_{11} \) (see Sect. 3.6 for details), closely follow the expected scaling law and only the least luminous run \( D_{23} \) starts to deviate from it (by \( \sim -16\% \)).

\[ C \text{ ingestion into a convective } O \text{ shell} \]
3.5 Fluctuations

The 3D, time-dependent nature of the flow gives rise to a whole spectrum of fluctuations. Their magnitude depends on the total luminosity, which both determines and depends on the entrainment rate. To quantify these fluctuations, we have used averages measured in 80 radial, space-filling tetrahedra, which we call *buckets* (see Fig. 17 of J17 for their exact distribution).

Figure 16 compares the low-luminosity run I2 with the high-luminosity run I13 in terms of the relative rms fluctuations $X'_{\text{rms}}(r)/X(r)$, where $X$ is the quantity of interest, $X(r)$ the spherical average of $X$ at the radius of $r$, and

$$X'_{\text{rms}}(r) = \sqrt{\frac{1}{N} \sum_{i=1}^{N} [X_i(r) - X(r)]^2},$$

(5)

defines the absolute rms fluctuations, where $N = 80$ is the total number of buckets. Fluctuations in the density $\rho$, pressure $p$, and temperature $T$ are the same within a factor of a few in the convection zone and they increase with increasing luminosity. Pressure fluctuations are much smaller than density fluctuations in the stable layers as expected for buoyancy-driven internal waves.

Density fluctuations in the convection zone are close to $\text{Ma}^2$ in both I2 and I13 (see Table 1 for the Ma values). The contrast in magnitude between the density and temperature fluctuations at the convective boundaries and those in the convection zone becomes larger with decreasing luminosity and Mach number. Meakin & Arnett (2007a) estimate the magnitude of density fluctuations in a convection zone and at its boundaries as $\rho'/\rho \sim \text{Ma}^2 + v_{\text{conv}} N/g$, where $\text{Ma}$ is the convective Mach number, $v_{\text{conv}}$ the convective veloc-
Figure 17. Relative fluctuations in the corrected temperature $\Theta$ (see Eq. 3 and Sect. 2.2.4) with respect to the spherical average $\langle \Theta \rangle$ at $t = 21.3$ min in run I14. The fluctuations are averaged over an XZ slice 1 Mm thick (5% of the simulation box width). The rendering is based on data downsampled from 1152$^3$ to 288$^3$ that were further subject to a lossy compression algorithm. Numerical noise from the compression step dominates the outer portion of the rendering.

We have sub-bucket-scale rms information for the fractional volume only, but it is not included in Fig. 16 for consistency with the other variables shown.

5 We have sub-bucket-scale rms information for the fractional volume only, but it is not included in Fig. 16 for consistency with the other variables shown.

Figure 18. Time evolution of the relative rms fluctuations in the fractional volume $FV$ of fluid $F_{\text{C}}$ at the radius of $r_0 = 4.5$ Mm in runs I4 ($f_{\text{O}} = 2.7$, Net 1), I13 ($f_{\text{O}} = 67.5$, Net 1), I16 ($f_{\text{O}} = 2.7$, Net 2), and I17 ($f_{\text{O}} = 67.5$, Net 2). All spatial scales are considered here as opposed to Fig. 16. Simulation time on the abscissa is normalised for clarity.

(see Sect. 3.3 for the reactions’ temperature sensitivities). For this reason, we did not include them in Fig. 4.

Fluctuations in the fractional volume $FV$ of fluid $F_{\text{C}}$ are large in all of our runs, including the least luminous ones (see Fig. 16 and also Fig. 4). They slightly increase with increasing luminosity, although much less than the fluctuations in $\rho$, $p$, and $T$ do, and they create large-scale asymmetries in the energy generation rate as shown in Fig. 4. The $FV$ fluctuations in runs I16 and I17, which use the C-burning network Net 2, are larger than those in a similar pair of runs I4 and I13, in which Net 1 is used, see Fig. 18.

This is likely a consequence of the fact that a certain concentration $X_{\text{C}}$ of $^{12}$C has to first build up in the convection zone to make the time scale of the $^{12}$C-$^{12}$C reactions ($\sim X_{\text{C}}^4$) considered in Net 1 short enough to balance the rate of mass entrainment whereas the $C$-burning time scale is independent of $X_{\text{C}}$ for the $^{12}$C+$^{16}$O reactions that are also included in Net 2. $^{12}$C+$^{16}$O reactions are responsible for $\sim50\%$ of the $C$-burning luminosity in run I17 and they dominate C burning in run I16, see Sect. 3.3. The $FV$ fluctuations as quantified in Fig. 18 are slightly larger than those in Fig. 16 (compare run I13 shown in both), because all spatial scales are considered (i.e. the index $i$ in Eq. 5 runs over computational grid cells instead of the buckets) in Fig. 18 as opposed to Fig. 16.

3.6 Instability for a case with strong feedback as in O-C shell merger conditions

Depending on the entropy difference between a pair of interacting O- and C-burning convective shells, the rate of mass entrainment into the O shell and the corresponding feedback from C-burning could be even larger than in the simulations discussed so far. In terms of the nucleosynthetic signature of the O-C shell merger, Ritter et al. (2018) show that the production of odd-Z elements increases with increasing entrainment rate and mixing efficiency, and suggest that the
largest enhancements could be reached in a case in which the O and C shells merge completely on the dynamical time scale. An entrainment rate of $\sim 10^{-3} \, \text{M}_\odot \, \text{s}^{-1}$ could be reached assuming near-sonic mass exchange, which we consider an upper limit. To explore this regime within the framework of the current series of runs, we have carried out a variation of the medium-luminosity run I5 ($f_{\text{CC}} = 13.5$) which has an entrainment rate of $\sim 10^{-3} \, \text{M}_\odot \, \text{s}^{-1}$. In order to mimic the energy feedback encountered in a situation closer to a complete merger, we increase the energy yield of C burning by the factor $f_{\text{CC}} = 10$ in run I11. Even taking into account that we increased the amount of C in the ingested material by a factor of five compared to the stellar model value (Sect. 2.2.2), the resulting entrainment rate (see below), which sets the strength of the energy feedback, stays well below the above mentioned upper limit. The degree of asymmetric perturbations reported in this section for run I11 may therefore be considered as a lower limit for the case of a complete O-C shell merger.

Figure 19 shows that the C-burning luminosity ($^{12}\text{C}+^{12}\text{C}$) starts rapidly increasing soon after the onset of entrainment in I11. C burning quickly becomes the dominant source of energy and exponential growth ensues. Strong, large-scale oscillations akin to the GOSH phenomenon described by Herwig et al. (2014) develop after $t \approx 11 \, \text{min}$. In terms of spherical harmonics, modes with $l = 1$ to $l = 4$ are dominant (see Fig. 20) and the upper convective boundary gets significantly deformed (see Fig. 21). The oscillations can also be seen in Fig. 22, which shows at four points in time the Mach numbers $M_a$ and $M_{a\perp}$ corresponding to motions in the radial and tangential direction, respectively, and the relative density fluctuations $(\rho - \langle \rho \rangle)/\langle \rho \rangle$. High values of $M_a$ and $M_{a\perp}$, reaching ~0.3 locally, are concentrated to one half of the renderings at $t = 10.9 \, \text{min}$ and at $t = 12.0 \, \text{min}$. The density fluctuations follow the same pattern. They reach values up to ~15% in some parts of the upper boundary due to the boundary’s large-scale deformation, but they also approach 5–10% in the bulk of the convection zone at $t = 12.0 \, \text{min}$. The Courant number exceeds unity at some point in the simulation volume at $t = 13.5 \, \text{min}$ and the simulation is stopped.

Our standard method of determining the entrainment rate is not applicable to cases of very violent convection associated with strong motions in the upper stable layer as mentioned in Sect. 3.2. In order to characterise the entrainment process throughout run I11, we have slightly modified this method. Instead of using the average velocity profile to define the upper boundary of the convection zone, we use the bucket data (see Sect. 3.5) to measure the radius of the boundary using fractional volume profiles of fluid $\mathcal{F}_C$ in 80 different directions. We define the radius of the upper boundary in each individual bucket as the largest radius inside that the fractional volume does not exceed $\frac{1}{4}$ and we integrate the amount of fluid $\mathcal{F}_C$ inside that radius. We have compared this method with that described in Sect. 3.2 in the time interval from 6.2 min to 9.3 min, in
Figure 22. Time evolution of the Mach numbers $M_a$ and $M_\perp$, corresponding to motions in the radial and tangential direction, respectively, and of the relative density fluctuations $(\rho - \langle \rho \rangle)/\langle \rho \rangle$ in the last three minutes of run I11.
which both methods are applicable. Our standard method yields $M_e = 3.07 \times 10^{-5} M_\odot$ s$^{-1}$ in this time interval. With the new method, we obtain $M_e = 3.61 \times 10^{-5} M_\odot$ s$^{-1}$, i.e. a value only 18% larger. The new method allows us the measure the ultimate entrainment rate achieved in the last two minutes of run I11: $M_e = 2.24 \times 10^{-4} M_\odot$ s$^{-1}$, see Fig. 23.

This entrainment rate is similar to the 1D nucleosynthesis run Sm4 of Ritter et al. (2018) that produces only modest enhancements of odd-Z elements. It is also similar to our high-luminosity runs I13 and I14 ($f_{\text{op}} = 67.5$, no C-burning enhancement), which do not show the runaway effect observed in I11. Although mass entrainment and the subsequent nuclear burning are not spherically symmetric in any of our 3D runs, the use of a 1D mixing prescription for nucleosynthesis post-processing seems more justified in quasi-stationary cases like I13 or I14 than in the unstable case I11.

Although run I11 reaches a luminosity similar to that of runs I13, I14, and I17 (compare Fig. 19 with Figs. 7 and A.1), it differs from them qualitatively. Convection in the latter three runs is quasi-stationary and predominately driven by O burning with O luminosity increasing on the time scale of many convective overturns, following the heating up of the convection zone. The rates of change of the luminosity and entrainment rate are so rapid in I11 that this run cannot be considered quasi-stationary. Figure 23 also shows that $M_e(t)$ in I11, unlike any in other run, is dominated by the entrained fluid’s piling up in the convection zone. The total mass of fluid $M_{\text{CC}}$ entrained by the end of run I11 corresponds to only about 1/5 of the upper stable layer’s initial mass whereas all of that layer gets ultimately entrained in runs I13, I14, and I17. The contribution of the temperature increase to the increase in the burning rates is rather limited in I11 as evidenced by the small ($\lesssim 50\%$) increase in the oxygen luminosity (see Fig. 19), which is a sensitive temperature indicator.

The behaviour of run I11 can be motivated as follows. The linearity of the dependence of the entrainment rate $M_e$ on the total luminosity $L_{\text{tot}}$ (see Fig. 6 and Sect. 3.2) suggests that the amount of energy needed to entrain a constant amount of mass is constant. The scaling relation gives $M_{e,0} = 1.44 \times 10^{-5} M_\odot$ s$^{-1}$ at $L_0 = 10^{11} L_\odot$, so

$$\Gamma = L_0/M_{e,0} = 1.34 \times 10^{17} \text{erg s}^{-1},$$

which can also be expressed as $\Gamma = 0.139$ MeV amu$^{-1}$. Only the mass fraction $X_{12} = 0.13$ of $^{12}\text{C}$ in the entrained fluid represents burnable fuel, so $\Gamma_{12} = \Gamma/X_{12} = 1.07$ MeV amu$^{-1}$ is the amount of energy needed to entrain one atomic mass unit of $^{12}\text{C}$ into the convection zone. Nuclear network Nett releases 9.35 MeV from every single $^{12}\text{C} + ^{12}\text{C}$ reaction (24 amu of $^{12}\text{C}$), which is $q_{\text{CC}} = 0.390 \text{MeV amu}^{-1}$. Since $q_{\text{CC}} < \Gamma_{12}$, the burning of the entrained material normally produces less energy than what was needed to entrain it.\(^6\)

We have, however, increased the energy release from Nett 1 by the factor $f_{\text{CC}} = 10$ in run I11, so $f_{\text{CC}} q_{\text{CC}} = 3.900 \text{MeV amu}^{-1} > \Gamma_{12}$ holds for the amount of energy produced per unit mass of fuel burnt in this run. More energy is produced from the burning of the entrained material than what was needed to entrain it. This closes a positive feedback loop that can explain the exponential growth seen in Figs. 19 and 23.

The main caveat of this hypothesis is that the entrainment rate is rather slow to respond to changes in the total luminosity in other high-luminosity runs, see Sect. 3.2. The e mechanism (see e.g. Kippenhahn et al. 2012) operating in the C-burning layer atop the convection zone may also contribute to the development of large-amplitude oscillations at the upper boundary, but the stability analysis involved in quantifying this contribution is beyond the scope of this paper.

4 SUMMARY AND CONCLUSIONS

We have employed 3D hydrodynamic simulations to investigate the dynamic feedback from the burning of a C-rich material ingested from a stably-stratified layer into a convective O-burning shell in a model of an evolved massive star. All but one of the simulations reach a quasi-stationary state, in which the rates of mass entrainment and burning are in a close balance. Most of our runs use a 768\(^3\) Cartesian grid, but two runs using a 1152\(^3\) grid are in excellent quantitative agreement with their 768\(^3\) counterparts.

The entrainment rate $M_e$ is proportional to the total luminosity, which in our suite of simulations spans almost three orders of magnitude (from $3 \times 10^{10} L_\odot$ to $10^{13} L_\odot$), although $M_e$ starts to deviate from that scaling in the lowest third of the luminosity range. We suggest that these deviations might be caused by the low-luminosity simulations’ being too short for convection to erode the initial transition layer between the two fluids and to form a new boundary consistent with the properties of the convection and of the entrainment process.

Carbon burning contributes between 16% and 33% of the total luminosity in the quasi-stationary state when only $^{12}\text{C} + ^{12}\text{C}$ reactions are considered. $^{12}\text{C} + ^{16}\text{O}$ reactions turn out to be equally important when the luminosity is high ($\approx 10^{13} L_\odot$) and they strongly dominate when the luminosity is low ($\lesssim 10^{11} L_\odot$). The contribution of C burning to the total luminosity is only $\approx 15\%$ almost independently of the luminosity when the network includes both $^{12}\text{C} + ^{12}\text{C}$ and $^{12}\text{C} + ^{16}\text{O}$ reactions. Because we assumed the concentration of $^{12}\text{C}$ in the fluid above the convection zone to be five times

\(^6\) This also holds for nuclear network Nett 2.
higher than that in the C shell of the particular MESA model that we started with, the feedback from C burning would likely be smaller still without that increase.

The velocity field is dominated by spherical harmonic modes \( l \leq 2, 3, 4 \), i.e. there are only a few large convective cells that start the turbulent cascade. Mass entrainment occurs where these large-scale flows run into one another while turning over at the upper convective boundary and they start pulling slivers of the C-rich fluid downwards. Full-sphere simulations are therefore necessary to correctly quantify this process. The rms convective velocity scales with \( U_{\text{rms}}^{1/3} \) as expected and the magnitude of velocity at any given total luminosity \( L_{\text{tot}} \) agrees with simulations of J17 who did not include O and C burning but instead used time-independent volume heating to drive convection in the shell.

Large-scale asymmetries in the distribution of the entrained C-rich material cause significant deviations of the burning rate from spherical symmetry. The quadratic dependence of the \( ^{12}\text{C} \rightarrow ^{12}\text{C} \) reaction rate on the mass fraction of \(^{12}\text{C}\) enhances the C luminosity by up to a factor of two compared with an estimate based on spherical averages when this reaction dominates C burning. On the other hand, fluctuations in the density, pressure, and temperature are smaller than 1% in the convection zone proper. The only exception is run I11, in which we experimentally increased the energy release from our simple C-burning network by a factor of ten to explore the regime closer to a full, dynamic-time-scale merger of the O and C shell. An exponentially-growing instability starts immediately after the onset of C entrainment in I11 and density fluctuations reach \(-5\%\) in the convection zone when a dipolar oscillation has developed in the flow field with a Mach number locally exceeding 0.2. Fluctuations of such magnitude are known to facilitate shock revival in supernova explosion simulations (Couch & Ott 2013, 2015; Müller & Janka 2015; Müller et al. 2017).

If such an instability occurs well before core collapse it might power a SN impostor event assuming that there is a physical mechanism capable of transporting a significant fraction of the kinetic energy contained in the O shell \((\sim 3 \times 10^{56}\text{ erg for } v_{\text{rms}} \sim 200\text{ km s}^{-1})\) to the star’s extended envelope. The entrainment rate reached in the instability, \(2 \times 10^{-4} M_\odot \text{ s}^{-1}\), is close to what is needed to obtain significant production of the odd-Z elements Cl, K, and Sc according to the nucleosynthesis calculations of Ritter et al. (2018). Still, the energy feedback from C ingestion in case I11 is smaller than the energy feedback expected due to the maximum entrainment rate that may be encountered in a full, dynamic O-C shell merger.

We show in Sect. 3.6 that the instability in run I11 could have been caused by the fact that more energy was released from the burning of one unit of entrained mass in I11 than what was needed to entrain that mass. Although this is a direct consequence of our having increased the energy release from C burning in this run, there are a number of energy producing reactions that our simple nuclear network does not include, e.g. those caused by the products of C burning or by the burning of \(^{20}\text{Ne}\) that is abundant in the C shell. Our energy argument suggests that the instability would also occur with less feedback from the burning if the entrainment rate was larger at the same luminosity, which would likely happen if we considered an O shell with a softer upper boundary. Finally, interaction between the convective flows in merging O and C shells (the latter not considered in this work) could contribute to the development of instabilities at the interface as well.

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APPENDIX A: LUMINOSITY CURVES
APPENDIX B: ENTRAINMENT RATE MEASUREMENTS
APPENDIX C: VELOCITY POWER SPECTRA

This paper has been typeset from a TeX/\LaTeX file prepared by the author.
Figure A.1. As Fig. 7, but runs I2, I5, I12, I14, and I15 are shown.
Figure B.1. As Fig. 5, but runs I4, I5, I12, and I14 are shown.
Figure B.2. As Fig. 5, but runs I15, I16, and I17 are shown.
Figure C.1. As Fig. 9, but runs I4, I5, I12, I13, I15, I16, and I17 are shown.