Identifying critical nodes in Earth’s relief network

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Abstract

Climate change and global warming threaten our society through impacts on social and natural resources; it is associated with, e.g., rising sea level. Earth’s surface land topography and ocean bathymetry play a central role in the dynamical evolution of seas, especially, regarding to the changes of the sea level. Here we develop an approach based on networks and percolation theory to study the geometrical features of Earth. We find some evidence for abrupt transition occurred during the evolution of the Earth’s relief network, indicative of a continental/cluster aggregation. We apply finite size scaling analysis based on coarse-graining procedure to show that the observed transition is most likely to be discontinuous. Furthermore, we detect the potential influenced areas in response to the sea-level rise. Our analysis is based on high resolution, 1 arc-minute, ETOPO1 global relief records. The framework presented here not only provides a new perspective on identifying the critical nodes of high risk in (near) future sea-level rise due to the global climate changes that have started to govern our Earth, but also facilitates the understanding of the geometrical phase transition which has been shown to occur at the present mean sea level on Earth.

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I. INTRODUCTION

Network science has demonstrated its potential as a useful tool in the study of real world systems, such as, physics, biology, and social systems [1-5]. It has also been successfully applied in climate sciences to construct “climate networks”, in which the geographical locations are regarded as nodes, the similarity between the records of different nodes represents the links [6,7]. Climate networks were used to forecast climate extreme events, such as El Niño and heavy rainfall [8-11]. Detecting and identifying vital and critical nodes in networks plays a significant role in unveiling fundamental organization principles of complex systems [12-14].

The topography or bathymetry of Earth shows complex multifractal structures and scaling properties [15-18], which can be regarded as a consequence of plate tectonic processes. Despite most of the major surface topographic features of Earth can be explained by the plate tectonic theory [19], little is known about how to identify and detect critical regions/locations on Earth’s surface. The surface topography of Earth plays a remarkable role in the dynamical evolution of seas, especially, regarding to the changes of the sea level. Sea level rises approximately 1.2–1.9 mm/yr in the 1900–1993 period [20], which is caused primarily by two factors related to global warming: melting ice sheets (glaciers) and the expansion of seawater as it warms [21].

Percolation theory is an effective tool for understanding the resilience of connected clusters to node breakdowns through topological and structural properties [22-24]. The essence of the analysis is the identification of a system’s different clusters and the connectivity between them. It has been applied to many natural and human-made systems [10 [25-29]. Here, we combine network and percolation theory to identify and detect critical nodes in Earth’s relief network, we find abrupt percolation transitions occurred during the evolution of the network. Furthermore, we detect the potential influence areas in response to sea level rise caused by global warming.

The structure of our paper is as follows: in the next section, we describe and introduce the Data and Methods. In Chapter III, the results are presented and discussed. Finally, in Chap. IV a short summary and outlook are provided.
II. DATA AND METHODS

A. Data

In this study, we use the topographic data of ETOPO1 Global Relief Model. It is used to calculate the Volumes of the World’s Oceans and to derive a Hypsographic Curve of Earth’s Surface, built from global and regional data sets [30]. The resolution is 1 arc-minute, i.e., \(N = 10800 \times 21600\) grid points. The present mean sea level (zero height) is assumed as a vertical datum of the height relief \(h(\phi_i, \theta_i)\), where \(\phi_i, \theta_i\) are the corresponding latitude and longitude of grid point \(i\). The data can be downloaded from [https://www.ngdc.noaa.gov/mgg/global/global.html](https://www.ngdc.noaa.gov/mgg/global/global.html).

B. Spatial networks

We embed the ETOPO1 dataset into a two-dimensional square lattice network. The nodes/sites are sorted in decreasing (increasing) order of height level and then added one by one according to decreasing (increasing) height \(h\); i.e., we first choose the node with the highest (lowest) height, then the second and so on. More specifically, the continental landmasses (ocean) are connected first. Existing clusters grow when a new node connects one cluster to another cluster (as small as a single node).

Based on classical graph theory, a cluster is a subset of network nodes such that there exists at least one path from each node in the subset to another [2, 4]. To detect the clusters in evolving lattice system, we can use the Hoshen–Kopelman algorithm [31] or the efficient Newman–Ziff algorithm [32]. In zonal direction, we used periodic boundary conditions; in meridional direction, free boundary conditions are used. Each node has four nearest neighbors. We denote \(G_m\) as a series of sub-networks. Due to the Earth’s spherical shape, the largest cluster in the spatial relief network is defined as [33],

\[
s_1 = \max \left[ \sum_{i \in G_1} \cos(\phi_i), \cdots, \sum_{i \in G_m} \cos(\phi_i), \cdots \right] \frac{1}{\sum_{i=1}^{N} \cos(\phi_i)}.
\]
III. RESULTS

A. Landmass percolation clusters

Similar to prior work [26], we study the Earth’s topography by means of the percolation theory. Our evolving spatial relief network starts globally with $N$ isolate nodes, the nodes are occupied/added one by one according to their height $h$, we define $p$ as the occupied probability, i.e., the fraction of occupied nodes; the order parameter is defined as the size of the largest cluster $s_1$ [see Eq. 1]. As shown in Fig. (1a), we find that Earth’s relief network undergoes several abrupt and statistically significant phase transitions, i.e., exhibiting a significant discontinuity in the order parameter $s_1$. Our results indicate that there are some critical and vital nodes at the percolation threshold that connect or combine large continental landmasses. In this study, for simplicity and without loss of generalization, we chose the largest five gaps during the evolution of our spatial network [see Fig. (1a)] to identify the critical nodes. The gap of $s_1$ for each time step, is defined as,

$$g(p) \equiv s_1(p) - s_1(p - 1/N).$$

Specifically, $g_1$ indicates the largest gap, $g_2$ indicates the second largest gap, and so forth. We first consider the largest gap and the corresponding percolation threshold. Fig. (1b) shows the network landmass clusters structure in the Earth surface map at the percolation threshold (just before the largest gap that is indicated by the blue arrow with $g_1$). We find that the network, just before this jump, is characterized by four major communities [only clusters with size larger than 0.01 are shown]; the largest one is the Afro-Eurasia continental landmass (indicated by red color); the second largest cluster is the Americas (indicated by yellow color); the third one is located in the Antarctica and the fourth is the Oceania. A critical node $(64.458333 \, ^\circ N, 171.141667 \, ^\circ W)$ connects the largest and second largest cluster at the percolation threshold $p_c \approx 0.321$, with altitude level $h = -43$ m, under the current sea level [see Fig. (2)]. Our results are consist with the conclusions: the order parameter has a sharp dropoff around the zero height level, it is an indicative of a geometrical phase transition at this level [26].

We perform the same analysis for the other four smaller jumps, $g_2, g_3, g_4$ and $g_5$ [as shown in Fig (1a)]. The corresponding network landmass clusters structure are presented in Fig. (3). The criti-
Figure 1. (a) The largest landmass cluster relative size $s_1$ versus the fraction number of nodes/sites, $p$, for real (green) and shuffled (red) Earth’s relief records. $g_1$–$g_5$ indicate the largest five gaps, defined in Eq. 2. (b) Snapshots of the landmass clusters structure of the Earth surface topography network just before the percolation threshold (the largest jump at $p \approx 0.321$). Different colors represent different clusters; the grid resolution is 1 arc-minute; the star indicates the critical node. Only the clusters with relative size larger than 0.01 are shown.
cal nodes are: (30.541667°N, 32.125°E) with altitude level \( h = 12 \) m; (60.575°S, 39.258333°W) with altitude level \( h = -3325 \) m; (8.541667°S, 129.208333°E) with altitude level \( h = -1292 \) m and (32.425°S, 1.641667°W) with altitude level \( h = -4116 \) m, respectively. These critical nodes are connected and merged two large disjoint land clusters (or islands).

For demonstrating that these results are not accidental, we analyzed randomized percolation obtained by reshuffling the relief records. In this way we destroyed the topological structures of Earth and the correlations between geometrical features of nodes. We considered 100 such randomizations and determined the averaged giant cluster \( s_1 \), as shown in Fig. 1(a). Compared with the shuffled data, we find that the percolation in real network exhibits significant discontinuity. Since there is only one realization for the real data, to compare, we also present three independent shuffled records in Fig. S1 [34]. The significant discontinuous jumps are still not observed for each reshuffled network.

**B. Finite size effects**

It has been pointed out that a random network or lattice system always undergoes a continuous percolation phase transition and shows standard scaling features during a random process [35]. The question whether percolation transitions could be discontinuous has attracted much attention recently in the context of interdependent networks [36–38] and the so-called explosive percolation models [39–42]. Interestingly, the dynamic evolution on our Earth’s relief network indicates the possibility of discontinuous phase transition, as shown in Fig. 1(a). To further test the order of the percolation phase transitions, we study the finite size effects of our network by altering the resolution of nodes. We then calculate \( g_1(L) \), the largest gap in \( s_1 \) as a function of network system size \( L \). \( L \) is defined as the number of nodes in zonal direction. If \( g_1(L) \) approaches zero as \( L \to \infty \), the corresponding giant cluster is assumed to undergo a continuous percolation; otherwise, the corresponding percolation is assumed to be discontinuous [43]. The results are shown in Fig. 4(a). It suggests a discontinuous percolation since \( g_1(L) \) tends to be a non-zero constant. For comparison, we also show the continuous results for shuffled data, with a known critical exponents \( \beta/\nu = 5/48 \approx 0.104 \) [24]. Where \( \beta \) is the critical exponent of the order parameter \( s_1 \sim |p - p_c|^{\beta} \), \( \nu \) describes the divergence of the correlation length \( \xi \sim |p - p_c|^{-\nu} \). In addition, we present the
Figure 2. The altitude level, $h$, as a function of the fraction number of nodes/sites, $p$. The vertical dashed line (blue) indicates the percolation threshold $p_c$, just before the largest gap; the horizontal dashed line (blue) indicates the height level, $h = -43$ m, of the critical node.

scaling function of the order parameter at the percolation threshold [just before the largest jump $g_1$], $s_c \sim L^{d_t - d}$, for both real and shuffled data [as shown in Fig. 4(b)]. We find that $d_f - d = 0$, an indicative of discontinuous percolation for our real network; however, for the shuffled case from simulations, $d_f - d \approx -0.104$, which agrees well with the known exponent value for standard percolation on square lattices $d_f = 91/48$ [22, 24]. The dashed lines shown in Fig. 4 are the best fit-lines for the data with R-Square $> 0.98$. The origin of the discontinuity might be explained by the plate tectonic theory: the world is composed of major (minor and micro) tectonic plates. Tec-
Figure 3. Snapshots of the landmass clusters structure of the Earth relief network just before the percolation threshold at (a) the second largest jump with $p \approx 0.285$; (b) the third largest jump with $p \approx 0.578$; (c) the fourth largest jump with $p \approx 0.450$; (d) the fifth largest jump with $p \approx 0.712$. The red color represents the largest landmass cluster, the stars indicate the critical nodes.

tectonic plates are gigantic segments of rock and consist of oceanic (seas and oceans) and continental crusts (land mass) [19]. The nodes in the same tectonic plate tend to be clustered and formed into a cluster first; the critical nodes [usually located in plate boundaries] that connect to different tectonic plates yield the abrupt transitions and discontinuous jumps of the order parameter $s_1$.

Fig. S2 presents the order parameter, $s_1$, as a function of occupied probability $p$ with different network system size $L$, we find that there are no significant finite-size effects for our system since the four curves with $L = 21600, 10800, 5400$ and $2700$ are nearly overlapping. The network landmass clusters structure at the 5 percolation thresholds [corresponding to the largest 5 jumps] with $L = 2700$ is shown in Fig. S3, we find that they have the similar manners compared to the cases with $L = 21600$ [Fig. 1(b) and Fig. 2], which indicates the self-similar fractal patterns of Earth’s surface topography [15, 44].
Figure 4. Finite size effects of the percolation in Earth’s relief network. (a) Log-log plot of the largest gap $g_1$ versus the network system size $L$ for original real data (red) and shuffled data (blue). (b) Log-log plot of the largest landmass cluster relative size $s_c$ at the percolation threshold, versus $L$ for real data (red) and shuffled data (blue). For the real data, the slope seems to approach zero, suggesting a discontinuous phase transition; for the shuffled data, the slope approaches -0.10, which suggests a continuous phase transition with a known critical exponent $\beta/\nu = 5/48$ and $d - d_f = 5/48$. The dashed lines are the best fit-lines with R-Square $> 0.98$.

C. Oceanic percolation clusters

The same analysis is also applied for the oceanic clusters, i.e., the nodes are added one by one according to their height level increasing order. As shown in Fig. 5(a), it also gives rise to a discontinuous jump in the oceanic order parameter at the percolation threshold $p_c \approx 0.379$, with altitude level, $h = -257$ m. This is different from the result, $h = -3640$ m, reported in Ref.[26]. The network oceanic clusters structure, just before the jump, are given in Fig. 5(b), we find that the critical node, (59.908333°S, 161.308333°E), connects the Atlantic+Indian Ocean Plate (colored by red) to the Pacific Plate (colored by yellow). It should be noticed that the manners of the largest cluster for landmass and oceanic are different, e.g., the percolation thresholds, the critical nodes are different. These differences unveil the complex and different structure of Earth’s topography (continents) and bathymetry (ocean). The origin of such phenomenon is possible due to the well-
Figure 5. Similar to Fig. but for the oceanic clusters. (a) The largest oceanic cluster relative size $s_1$ versus $p$. (b) Snapshots of the oceanic clusters structure of the Earth relief network just before the percolation threshold (the largest jump at $p \approx 0.379$). The star indicates the critical node.
known bimodal distribution of Earth’s surface [45].

D. Earth’s topography

The topography of the Earth is complex and its morphology results are from diverse processes, such as, tectonic and erosion. To better understand its topography, we then keep the form of the clusters unchanged \((h = 0 \text{ m})\) and manipulate the height probability density function (MHP), i.e., we take the profile of the present Earth topography by multiplying an i.i.d. random number between \([0, 1]\) for the height of each node. We perform the same percolation analysis to the ensembles with new height profiles. The largest landmass cluster relative size \(s_1\) versus the fraction number of nodes \(p\) for one specific configuration is shown in Fig. 6. We find that, \(s_1\), similar to the real data exhibits abrupt changes around the sea level \(h = 0 \text{ m}\). It suggests the current sea level, may serve as a tipping point at which a percolation transition takes place [26]. We also present other three independent MHP examples in Fig. S4. We find that the manner of \(s_1\) are similar and the discontinuity phenomena are robust.

In addition, we study the percolation on 2d Fractional Brownian motion Surfaces (FBS) with Hurst exponent \(H\) [46]. The parameter \(H\) is usually between 0 and 1, where 0 is very noisy, and 1 is smoother. The Earth’s rough surfaces can be modeled via Fractional Brownian motion [47], and the estimation for the earth continents topography is \(H = 0.66\) [16]. We present the percolation analysis results on FBS with \(H = 0.66\) in Fig. 6, the corresponding 2d FBS structure is shown in Fig. S5. Similar to the real network, we find that \(s_1\) also exhibits abrupt transitions around \(p \approx 0.3\). To further test the order of the percolation phase transitions on FBS, we then use the finite size scaling theory. The largest gap \(g_1(H, L)\) (average) as a function of system size \(L\) are shown in Fig. 7(a). We find that percolation on FBS with \(H = 0.66\) is discontinuous, since \(g_1(H, L)\) tends to be a non-zero constant when \(L\) becomes infinity. The percolation threshold \(p_c(H, L)\) corresponding to the largest gap during the evolution of site percolation is shown in Fig. 7(b). We find the values are robust and agree well with the real data where \(p_c \approx 0.321\) [see Fig. 1]. This indicates that the percolation method can be used as an efficient tool to study the Earth’s topography. We notice that recently people discovered the percolation with Hurst exponents \(H \in [-1, 0]\) is continuous [48, 49]; and our results suggest that the percolation on FBS with Hurst
Figure 6. The largest landmass cluster relative size $s_1$ versus the fraction number of nodes/sites, $p$, for real, shuffled, manipulate the height probability density function (MHP) of Earth’s relief records, and 2d Fractional Brownian motion Surfaces (FBS) with different Hurst exponents. The two dashed lines indicate the sea level (h=0 m) and the well known site percolation threshold $p_c = 0.5927$, respectively.

exponents $H > 0$ is discontinuous. Figs. S6–S8 show our three specific examples of FBS and their corresponding percolation progress with $H = 0.1, 0.5$ and 0.9, respectively. The discontinuous jumps of $s_1$ can significantly be observed.
Figure 7. The percolation on 2d Fractional Brownian motion Surfaces. (a) The average of the largest jump \( g_1(H, L) \) as a function of system size \( L \); (b) the corresponding percolation threshold \( p_c(H, L) \) as a function of \( L \). The dashed line in (b) stands the percolation threshold \( p_c = 0.321 \) for real data. Here, \( H = 0.66 \) is the fractal dimension of the Earth continents topography, following \[16\].

E. Sea level rise

Sea level rise is one of the major consequences of global climate warming \[50\] and the effects in its combination with storm surges and other extreme events can already be observed \[51\]. Peaking global CO2 emissions is crucial for limiting the risks of sea-level rise. It is estimated the median sea-level rise will be between 0.7 and 1.2 m until 2300 within the constraints of the Paris Agreement \[52\]. There is thus a great interest in clarifying/predicting the influenced areas in response to sea level rise. To identify the possible influenced nodes, for simplicity, we compare, in Fig. 8, the topology of the the largest oceanic cluster at the present sea level \( [h = 0 \text{ m}, \text{see Fig. 8(a)}] \) and the sea level at \( h = 1, 2 \text{ m} \). Fig. 8(b) and (c) show adding nodes (red) with the sea level at \( h = 1, 2 \text{ m} \), respectively. One can see that the overall pattern of adding nodes is robust across in the continental boundaries. We find that some regions, for example, northern of north America have a higher probability/risk to be influenced. For ease of comparison, here, we do not plot the coastlines in Fig. 8(b) and (c).
Figure 8. (a) Snapshots of the largest oceanic cluster (blue) at the current sea level, $h = 0$ m; (b) the added nodes (red) at the sea level, $h = 1$ m; (c) the added nodes (red) at the sea level, $h = 2$ m. For ease of comparison, we do not plot the coastlines in (b) and (c).
IV. SUMMARY

In summary, we have developed a framework to study the geometrical topography of Earth. Using percolation analysis we studied the dynamical evolution of the giant landmass (oceanic) cluster, $s_1$, and found the Earth’s relief network exhibits abrupt transitions. We defined the discontinuous jumps of $s_1$ to detect the percolation threshold and identify the critical nodes overall the global. In addition, we find that the percolation in real network is first order based on the finite-size scaling theory. To better understand the Earth’s topography, we analyze the percolation properties on new profiles generated by MHP and FBS. Further, we clarify/predict the influenced nodes (areas) in response to sea level rise, at $h = 1, 2$ m. The study of the earth network system may enrich the understanding of the physical discontinuous phase transitions, it also provides a new perspective on identifying critical nodes in Earth’s relief surface network and can potentially be used as a template to study other complex systems.

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