Particle Acceleration in Supernova Remnants in the Presence of Streaming Instability and Nonlinear Wave Interactions

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Abstract

The instability in the cosmic-ray (CR) precursor of a SN shock is studied. The level of turbulence in this region determines the maximum energy of accelerated CRs. The consideration is not limited by the case of weak turbulence. It is assumed that Kolmogorov type nonlinear wave interactions together with the ion-neutral collisions restrict the amplitude of random magnetic field. As a result, the maximum energy of accelerated particles strongly depends on the age of a SNR. It can be as high as $10^{17}\ Z$ eV in young SNR and falls down to about $10^{10}\ Z$ eV at the end of Sedov stage ($Z$ is the particle charge). This finding may explain why the SNRs with the age more than a few thousand years are not prominent sources of very high energy $\gamma$-rays. The averaged spectrum of ultrarelativistic CR injected in the interstellar medium is close or somewhat steeper than $E^{-2}$.

1. Introduction

The dependent on energy diffusion coefficient $D(E)$ determines the maximum energy that particles can gain in the process of acceleration. The condition of efficient acceleration is $D(E) \lesssim ku_{sh}R_{sh}$, where $R_{sh}$ is the radius and $u_{sh}$ is the velocity of spherical shock, $k = 0.1$ in the free expansion stage and $k = 0.04$ in Sedov stage, e.g. [3]. The Bohm value $D_B = vr_g/3$ ($v$ is the particle velocity, and $r_g$ is the particle Larmor radius) that is a lower bound of the diffusion along the magnetic field gives $E_{\text{max}} = 1.7 \times 10^{14}\ Z (E_{51}/n_0)$ eV at the end of the free expansion stage when particles reach the highest energy. Here the SN burst with kinetic energy of ejecta $E = E_{51}10^{51}$ erg in the gas with density $n_0\ cm^{-3}$ and the interstellar magnetic field $B_0 = 5 \times 10^{-6}\ G$ is considered.

Analyzing the early stage of SNR evolution when the shock velocity is high, $u_{sh} \sim 10^4\ km\ s^{-1}$, it was found [2] that the CR streaming instability can be so strong that the amplified field $\delta B \geq 10^{-4}\ G$ far exceeds $B_0$. The maximum particle energy increases accordingly. The CR streaming instability is less efficient as the shock velocity decreases with time and the nonlinear wave interactions reduce the level of turbulence at the late Sedov stage [7,10]. This leads to fast diffusion and decreases $E_{\text{max}}$. The effect is aggravated by the possible wave damping on
the ion-neutral collisions [1,6]. In the present work, we consider the acceleration of CR and their streaming instability in a wide range of shock velocities. The level of magnetic field fluctuations is allowed to be arbitrarily large, and the rate of nonlinear wave interactions is assumed to correspond to the Kolmogorov non-linearity. The collisional dissipation is also taken into account. The task is to find the maximum energy of accelerated particles as a function of SNR age.

2. Maximum Energy of Accelerated Particles

In the test particle approximation, the distribution of particles in momentum for high Mach number shocks has the canonical form \( f(p) \sim p^{-4} \). In the case of efficient acceleration, the action of CR pressure on the shock structure causes nonlinear modification of the shock that changes the shape of particle spectrum making it flatter at relativistic energies. So, we assume that the distribution at the shock is of the form \( f_0(p) \sim p^{-4 + a} \) where \( 0 < a < 0.5 \), and value \( a = 0.3 \) is used in the numerical estimates below. The normalization of function \( f_0(p) \) is such that the integral \( N = 4\pi \int dp p^2 f_0(p) \) gives the number density of CR. We assume that the CR pressure at the shock is some fraction \( \xi_{cr} \leq 1 \) of the upstream momentum flux entering the shock front, so that \( P_{cr} = \xi_{cr} \rho u^2_{sh} \). The typical value of \( \xi_{cr} = 0.5 \) and the total compression ratio 7 were found for strongly modified shocks in [3].

The following steady-state equation determines the energy density \( W \) of the turbulence amplified by the streaming instability in the CR precursor upstream of the shock:

\[
    u \nabla W = 2(\Gamma_{cr} - \Gamma_1 - \Gamma_{nl})W. \tag{1}
\]

Here the l.h.s. describes the advection of turbulence by supersonic gas flow. The terms on the r.h.s. of the equation describe respectively the wave amplification by CR, the linear damping of waves in background plasma and the nonlinear wave-wave interactions that limit the amplitude of turbulence. The Kolmogorov-type nonlinearity with the simplified expression \( \Gamma_{nl} = (2C_K)^{-3/2}V_a k A(> k) \approx 0.05V_a k A(> k) \) at \( C_K = 3.6 \) (as given by the numerical simulations [9]) is used in our calculations. Here \( A = \delta B/B_0 \) is the wave amplitude, \( k \) is the wave number. The wave-particle interaction is of resonant character and the resonance condition is \( kr_g = 1 \). The equations for \( \Gamma_{cr} \) and for \( D \) used in our calculations generalize the standard equations [4] derived in the case of weak random field. The details of our consideration can be found in [8].

Fig.1 illustrates the results of calculations of \( E_{max} \) at the Sedov stage of SNR evolution at \( \mathcal{E} = 10^{51} \) erg in the warm interstellar gas with the temperature \( T = 8 \times 10^3 \) K, and the average density \( n_0 = 0.4 \) cm\(^{-3}\). Three solid lines correspond to three cases of wave dissipation considered separately: nonlinear wave interactions; damping by ion-neutral collisions at constant gas density; damping
The particle maximum momentum $p_{\text{max}}$ in units $mc$ as a function of shock velocity $u_{\text{sh}}$.

by ion-neutral collisions when the diffuse neutral gas restores its density after complete ionization by the radiation from SN burst. The maximum energy of protons accelerated by SN shocks at the early Sedov stage is close to $3 \times 10^{14}$ eV (that exceeds the Bohm limit) and decreases to about $10^{10}$ eV at the end of the Sedov stage (that is less than the Bohm limit). In particular, the particle energy is less than $10^{13}$ eV for $t > 3 \times 10^{3}$ yr and this may explain the absence of a TeV $\gamma$-ray signal from many SNRs [5] where $\gamma$-rays could be produced through $\pi^0$ decays if sufficiently energetic CRs were present.

The highest particle energy estimated as $E_{\text{max}} = 2 \times 10^{17} Z(u_{\text{sh}}/3 \times 10^{4}$ km s$^{-1})^2 \xi_{\text{cr}} M_{\text{ej}}^{1/3} n^{1/6}$ eV is reached at the end of the free expansion stage ($M_{\text{ej}}$ is the mass of ejecta in solar masses).

3. Average Spectrum of Injected Cosmic Rays

Let us find the overall spectrum of CR injected to the interstellar space. It is not clear yet how the process of CR exit from the SNRs proceeds. Consider two cases:

A) All accelerated particles leaves the envelope without considerable adiabatic losses. The production of CR in the galactic disk is then $Q = \nu_{\text{sn}} 4\pi \int dt u_{\text{sh}} R_{\text{sh}}^2 f_0$, where $\nu_{\text{sn}}$ is the SN rate in the Galaxy. This gives the approximate scaling $Q \sim \nu_{\text{sn}} \xi_{\text{cr}} E p^{-4}$ (at $p \gg mc$) under the conditions that $\xi_{\text{cr}} = \text{const}$, and that
SNR expansion is adiabatic as in the case of Sedov solution. It is remarkable that the form of average source spectrum \( Q(p) \) is not sensitive to parameter \( a \) at \( a > 0 \) (see also simulations \([3]\)).

\( B) \) The CR with maximum energies can not be confined near the shock and are leaving the precursor because of the \( p_{\text{max}}(t) \) decrease. Other particles which are confined in the envelope experience adiabatic energy changes until the very end of Sedov stage and the exit from a SNR. The runaway particles has the average spectrum of the form \( p^{-4} \), whereas the trapped particles gives approximately \( p^{-4.1} \). The forms of these spectra are not sensitive to \( a \) at \( a > 0 \), and both populations have the comparable number densities.

4. Conclusion

The accounting for non-linear effects in the instability that accompanies the CR acceleration may simultaneously eliminate two difficulties of modern cosmic ray astrophysics. It raises the maximum energy of accelerated particles in young SNR above the standard Bohm limit through the production of strong random magnetic fields and thus helps to explain the origin of galactic CR with energies up to \( 10^{17} \) eV. It also decreases the maximum energy of particles in the late Sedov stage of SNR evolution, which allows us to explain why these objects are not bright in very high energy \( \gamma \)-rays. It is remarkable that even with a flat instantaneous particle spectrum at the shock typical for the strongly nonlinear regime of acceleration, the spectrum of ultrarelativistic particles injected into interstellar space and averaged over the age of SNR is close to \( E^{-2} \) or somewhat steeper.

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5. References

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