Kinematic-gravitational ion model of planetary dynamo.

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Abstract. Based on the solution of inverse problem for the main magnetic field of Earth, it is shown that the global sources of magnetic field are situated in the equatorial region of the Earth’s core. The estimates of magnetic moments, volumetric currents and their densities are obtained. The direction of the current flow is determined, which can be explained by the motion of the positively charged core matter under the influence of tidal forces, directed against the basic axial rotation of the Earth. The core is positively charged due to migration of electrons across the core-mantle boundary. For the planets of the Solar system estimates of the tidal forces produced by the Sun and the satellites of the planets are obtained. A close linear relationship is established between the magnitude of tidal forces and the observed magnetic field of the planets. As a result, we arrive to a simple geodynamo mechanism in which the electric current is the motion of weakly positively charged liquid core associated with the action of tidal forces.

1. Introduction
The sources of Earth’s magnetic field are usually approximated to dipoles and current loops [1]. In this paper they are represented by magnetized prisms, filling the two layers in the planet’s core. This allows to transition from magnetization to magnetic moment and volumetric current. This transition is valid due to the equivalence between magnetized objects and currents in [2]. Distribution of effective magnetization within prisms is obtained while solving the inverse magnetometry problem with Z-components of the main magnetic field of the Earth (as per IGRF-2005 model) in geocentric coordinate system as initial condition. Resulting system of 2450 equations with 354 unknowns is resolved using adaptive method [3].

1.1. Magnetization and magnetic field
Magnetic induction potential, created by the magnetized body is defined as [4, 5].

\[
\Delta U(\xi, \zeta, \eta) = \frac{\mu_0}{4\pi} \iiint_B (\mathbf{J}(x, y, z), \text{grad} \frac{1}{r}) dG, \quad (1)
\]
where \( \vec{J}(x, y, z) \) is magnetization (A/m) of a unit of volume \( dG \), with coordinates \( x, y, z \), \( r \) is distance between the unit of volume and the point of observation, and the integral is taken over the entire magnetized volume \( G \).

For numerical integration the volume is divided into rectangular prisms for which magnetization is assumed to be constant. Components of magnetic field within these prisms are calculated via differentiation of equation (1). Resulting system of equations generally is viewed as an ill-posed problem. This was circumvented by using the adaptive method [3].

1.1.1. Adaptive method for solving inverse problem. Key quality of adaptive method is its ability to deal with the systems of equations by refining the unknown values based both on the initial conditions and newly found values for each iteration. This refinement is performed successively for all the equations and is based on the discrepancy between the initial and projected values.

Method is realised in the ADM-3D application for solving magnetometry problems, that is being used for solving various scientific and engineering problems in geophysics.

1.2. Solving the inverse magnetometry problem
The purpose of this work is to come up with a model of the sources for Earth’s magnetic field that results in and accurate representation of actual Earth’s magnetic field. Assuming these sources, as mentioned above, to be uniformly magnetized rectangular prisms, based on the equivalence of currents and magnetized objects [2] the calculation uses two layers of rectangular prisms with 177 prisms in northern and southern hemispheres each. Prisms dimensions in equatorial plane are 400 by 400 km. Effective magnetizations of the prisms are considered the unknown values.

Using even distribution of magnetization for initial condition results in an uneven distribution very similar to the one obtained with no initial magnetization. Root mean squared discrepancies showed significant reductions as iterations went on. This indicates stability and convergence of obtained solution.

1.3. Estimation of magnetic moments and currents
Having obtained effective magnetization \( J \) for each prism and knowing their volume \( V \), we can calculate the magnetic moment [2]:

\[
M = J \ast V \text{ (A*m}^2) \tag{2}
\]

Fig. 1 c, d depict distributions of magnetic moment for prisms in northern and southern hemispheres correspondingly. For reference, figures contain projections of the geographic pole (polus) and certain cities: Tokyo (TOK), Krasnoyarsk (KRS), Yekaterinburg (EKT), London (LON), Ottawa(OTW) in Northern hemisphere and Punta Arenas (PAR) and Canberra (CNB) – in southern.

Obtained values vary between \(-222 \times 10^{18}\) and \(512 \times 10^{18} \text{ A*m}^2\). Total positive magnetic moment for northern hemisphere is \(4.67 \times 10^{22} \text{ A*m}^2\), for southern \(3.54 \times 10^{22} \text{ A*m}^2\). Negative magnetic moment is only observed in a small segment of southern hemisphere and amounts to \(-3.63 \times 10^{21} \text{ A*m}^2\) total. Resulting estimation for the entire Earth’s core \(7.85 \text{ A*m}^2\) is close to the value obtained from evenly magnetized model: \(7.77 \times 10^{22} \text{ A*m}^2\) and estimation of \(10^{23} \text{ A*m}^2\) by [6]. Having obtained the values for magnetic moments we can estimate currents necessary to create them. For rectangular prism \( M = I \ast S \), therefore:

\[
I = \frac{M}{S} \text{ or } I = \frac{J \ast h}{A}, \tag{3}
\]

where \( I \) is total current flowing around the lateral surface of the prism, \( S \) is area of the prism’s base, \( J \) is effective magnetization and \( h \) is the height of the prism.

Fig. 1 (e, f) depict isolines of volumetric currents in northern and southern hemispheres correspondingly that border the sets of prisms with the greatest magnetic moments. The highest current value reaches \(3.2 \times 10^9 \text{ A} \), while current densities lie within the 0.004 - 0.03 \( \text{A*m}^2\) margins. The largest density of volumetric currents can be estimated from the assumption that the current crosses...
the surface equal to the lateral surface of the prism, the area is equal to $d \times h$, where $d$ is the length of the horizontal face of the prism, and $h$ is its height. We obtain the following formula:

$$i = I / (d \times h) = (J \times h) / (d \times h) = J / d \text{ (A/m$^2$)},$$

where $i$ is the current density, which by configuration coincides with the effective magnetization (Fig. 1 a, b), but in magnitude less than it in $d=400000$ times.

Maximum current density reaches 0.004 A/m$^2$.

Previously, an analysis was made of the realism of the model for a larger current density of 0.03 A/m$^2$, which follows from the Kaufmann model.
2. Motion of liquid, charges and currents

It is known [2, 6] that the current density created by moving charges is

\[ i = q \, \nu, \]  

(5)

here \( q \) is the charge density, and \( \nu \) is the velocity of charge motion.

If the velocity \( \nu \) is equal to the velocity of the western drift of the isoline of the MF, equal to 20 km / year [7] (0.635 \times 10^{-3} \, \text{m} / \text{s}), then \( q = 4.7 \, \text{C} / \text{m}^3 \) is sufficient to create this current density, and considering that the unit charge of an electron or ion is equal to \(-1.6 \times 10^{-19} \, \text{C}\), then the required number of charges is \( 2.94 \times 10^{19} \). Taking into account that the unit charge of an electron or ion is equal to \(-1.6 \times 10^{-19} \, \text{C}\), then the required number of charges is \( 2.94 \times 10^{19} \). Considering that the number of atoms in 1 m³ of iron is 0.11 \times 10^{30}, there are \( 3.7 \times 10^9 \) non-charged atoms per charged atom. It is considered that the core consists of plasma. In the ionized environment of the nucleus, the excess of ions is most probable, since electrons, when abundant, have more opportunity to go beyond the nucleus than ions. A possible mechanism for the separation of electrons from the nucleus is given in [8].

The estimation of \( q_\beta \) and \( \nu \) is a topic of special work, but preliminary estimates of some parameters are given in order to show the realism of the assumptions, from which it follows that the MFE is created by currents whose density is low. The velocity of the charges is also not great. But the volume and mass are large, and, consequently, the total charge of the moving liquid core substance.

To create the MF, shown in Fig. 1 (c d), the currents (according to the rule of the borer) should flow clockwise, if you look at the model from the North Pole side. The direction of currents according to the physical definition [6] is the direction of motion of positively charged particles. All loop and dipole models assume a similar motion of currents. Moreover, in the models of the electronic geodynamo, after complex transformations, they arrive at the current flow in a clockwise direction. For example, look at the graphical representation of the E. Bullard model [7]. In the new works we see models with illustrations of current flow in the western direction (that is, clockwise) on the core surface and on the equatorial plane of the nucleus.

A logical and physically sound model is constructed if it is assumed that the high-temperature liquid substance of the nucleus is positively weakly charged. In this case, the direction of motion of the current isolines, i.e. the lines of equal current in Fig. 1c d, can be regarded as trajectories of the motion of not only the current but also the liquid. Analyzing the trajectories (isolines) of the currents in Fig. 1c d, we will try to see this consistency. In this model, in the northern hemisphere, two large anomalies of MF in the core and on the surface create two large fluxes in volume. In the polar region, the flows move in opposite directions and create opposite currents and currents in the longitude sector -45° to +45° counterclockwise. In the outer part of the nucleus, the direction of currents and flows is opposite to the rotation of the planet (Fig. 2). The planet rotates counter-clockwise, if viewed from the North pole. The cause of this fluid motion is the inhibition of the moving shells of the rotating planet (atmosphere, ocean and liquid core) under the action of the gravitational forces of the Moon and the Sun. The braking gravitational force acting on a point with unit mass, located on the surface or inside the planet, is proportional to the distance of this point from the axis of rotation. That is, it will be the largest at the equator of the planet and under the
influence of these forces currents are created in the oceans, reverse to the rotation of the planet (for example, trade winds). Faced with heterogeneities, they deviate in the northern hemisphere to the north, and in the south to the south. And if you look at the currents from the north, they create closed currents, moving clockwise, both in the northern and southern hemisphere.

In the liquid core, the driving tidal force will also be greatest near the equatorial portion near the core-mantle boundary, and the smallest on the axis of rotation. In the southern hemisphere we see a vortex flow of currents creating positive sources of the MF core. Return currents and currents create a zone of return sources. As already noted, their internal MM is an order of magnitude smaller than the fundamental one.

Fig. 2 Direction of currents in the Earth’s core for northern and southern hemispheres.

3. Method for calculating tidal forces and predicting magnetic field strength

We can see that external objects create periodic influence on the rotating planet’s surface. This influence can be described by the maximum value of gravitational influence \( \Delta g_{\text{max}} = \max( \Delta g_{\text{hor}} ) \) and relative angular frequency \( \omega_{\text{rpl}} = \omega_{\text{pl}} - \omega_{\text{sp}} \).

These gravitational waves are responsible for tidal waves and seismic activity, and they create flows of liquids on the global scale not only on the planets’ surface [9], but also in its core [10].

The tidal forces, acting in the core depend on the period of planet’s axial revolution \( T_{\text{pl}} \). We introduce the period relative to the satellite creating \( \Delta g_{\text{hor}} \):

\[
T_{\text{rpl}} = T_{\text{pl}} \left( \frac{T_{\text{sp}}}{T_{\text{pl}} - T_{\text{pl}}} \right),
\]

where \( T_{\text{pl}} \) is the period of the planet’s axial revolution, and \( T_{\text{sp}} \) is the period of satellite orbital rotation.

It is easy to see that when the periods \( T_{\text{pl}} \) and \( T_{\text{sp}} \) coincide, \( T_{\text{rpl}} \rightarrow \infty \), meaning that the satellite is stationary in relation to the planet and does not create horizontal forces on its surface.

Accounting for periods of axial revolution of the planet \( T_{\text{rpl}} \) the following formula is used to estimate the maximum value of the horizontal component of the tidal force \( \Delta F \):

\[
\Delta F = \frac{3^* G m_1 m_2 r}{2 R^2 T_{\text{rpl}}},
\]

where \( G \) is the gravitational constant, \( r \) is the distance between object \( m_1 \) and the planet rotation axis. \( T_{\text{sp}} \) is the rotation period of a fixed point relatively to an external object creating the tidal forces, \( T_{\text{r}} \) is the rotation period of the Earth.

We assume that the induced magnetic field on the planet equator \( B_{\text{pl}} \) is directly proportional to the tidal force. Having estimated \( \Delta F \) and \( B_{\text{pl}} \) on Earth’s equator and \( \Delta F \) on another planet, it is possible to obtain the estimate of the magnetic field on the planet equator relative to Earth.
\[ B_{pl} = B_E \Delta F_{pl} / \Delta F_E \]  

(7)

Full planetary magnetic field on equator for planets with multiple external objects, creating tidal forces:

\[ B_{pl} = B_E(1) + B_E(2) + ... + B_E(n) = \sum_{i=1}^{n} B_E(i), \]

(8)

where, 1, 2…n - is the array of external objects. This allows estimation of magnetic field on equator for all of the planets in the Solar system [11-13].

Table 1. The properties of the planets and satellites as well as the relative magnetic fields on the equator in the prediction and observations

| Planet        | Relative radius \(r_{rel}\) | Source of gravitational field | Source mass, kg | Orbit radius of the source, m | Rotation and revolution periods of planets and satellites, and relative period, days. | Prediction \(\Delta F_{pl} / \Delta F_E\) | Observation \(B_{pl} / B_E\) |
|---------------|-----------------------------|-------------------------------|----------------|-----------------------------|----------------------------------------------------------------------------------|---------------------------------|-----------------------------|
| 1             | 2                           | 3                             | 4              | T_or | T_oc | T_e | T_re |  6                           |  7                           | B_pl / B_E                   | Observation B_pl / B_E       |
| Mercury       | 0.38                        | Sun                           | 1.99E+30       | 5.80E+10 | 58  | 87.9 | 170.5 | 0.012                         | 0.005-0.01                    |
| Venus         | 0.95                        | Sun                           | 1.99E+30       | 1.08E+11 | -244| 225  | 224.7 | -117                         | 352000                        |
| Earth         | 1                           | Sun/Moon                      | 1.99E+30       | 3.64E+08 | 1   | 365  | 1.04  | 1                             | 1                             |
| Mars          | 0.53                        | Sun/Phobos                    | 1.99E+30       | 2.28E+11 | 1   | 687  | 1.04  | 1 -0.47                       | 0.047                         |
| Jupiter       | 11.22                       | Sun                           | 1.99E+30       | 7.80E+11 | 0.4 | 4332 | 0.4    | 0.06                         |
| Io            | 3.38                        | Jupiter/Callisto              | 8.93E+22       | 4.22E+08 | 0.4 | 1.8   | 0.52   | 13.8                         |
| Europa        | 4.83                        | Saturn/Titan                  | 4.3E+22        | 6.71E+08 | 0.4 | 3.6   | 0.45   | 2.11                         |
| Ganymede      | 14.81E+22                   | Jupiter/Callisto              | 14.81E+22      | 10.7E+08 | 0.4 | 7.2   | 0.42   | 1.71                         |
| Callisto      | 10.81E+22                   | Saturn/Titan                  | 10.81E+22      | 18.8E+08 | 0.4 | 16.7  | 0.41   | 0.24                         |
| Total         |                             |                               |                |        |      |      |       | 17.88                        | 14                           |
| Saturn        | 9.47                        | Titan/Thetis                  | 13.5 E+22      | 1.22E+09 | 0.4 | 15.9  | 0.41   | 1.08                         |
| Mimas         | 0.4 E+20                    | Saturn/Titan                  | 0.4 E+20       | 1.85 E+09 | 0.4 | 0.9   | 0.72   | 0.053                        |
| Enceladus     | 1.1 E+20                    | Saturn/Titan                  | 1.1 E+20       | 2.38 E+09 | 0.4 | 1.4   | 0.56   | 0.088                        |
| Thetis        | 6.2 E+20                    | Saturn/Titan                  | 6.2 E+20       | 2.95 E+09 | 0.4 | 1.9   | 0.51   | 0.29                         |
| Dione         | 11 E+20                     | Saturn/Titan                  | 11 E+20        | 3.77 E+09 | 0.4 | 2.7   | 0.47   | 0.26                         |
| Rhea          | 23 E+20                     | Saturn/Titan                  | 23 E+20        | 5.27 E+09 | 0.4 | 4.5   | 0.44   | 0.215                        |
| Total         |                             |                               |                |        |      |      |       | 1.98                         | 0.7                          |
| Uranus        | 4.01                        | Miranda/Miranda               | 0.006E+21      | 1.29E+08 | 0.7 | 1.41  | 1.30   | 0.047                        |
| Miranda       | 0.006E+21                   | Uranus/Neptune               | 0.006E+21      | 1.29E+08 | 0.7 | 1.41  | 1.39   | 0.047                        |
| Ariel         | 1.35E+21                    | Uranus/Neptune               | 1.35E+21       | 1.91E+08 | 0.7 | 2.52  | 0.97   | 0.43                         |
| Umbriel       | 1.17E+21                    | Uranus/Neptune               | 1.17E+21       | 2.66E+08 | 0.7 | 4.14  | 0.84   | 0.158                        |
| Titania       | 3.50E+21                    | Uranus/Neptune               | 3.50E+21       | 4.22E+08 | 0.7 | 5.7   | 0.76   | 0.13                         |
| Oberon        | 3.01E+21                    | Uranus/Neptune               | 3.01E+21       | 5.83E+08 | 0.7 | 13.5  | 0.73   | 0.044                        |
| Total         |                             |                               |                |        |      |      |       | 0.81                         | 0.7                          |
| Neptune       | 3.89                        | Triton/Triton                | 2.14E+22       | 3.54E+08 | 0.8 | -5.8  | 0.7    | 1.44                         |
| Triton        | 2.14E+22                    | Neptune/Neptune              | 2.14E+22       | 3.54E+08 | 0.8 | -5.8  | 0.7    | 1.44                         |

Calculation the correlation between observed and estimated fields produces \(\rho = \text{cov}(B_6 * B_7) / (\sigma_6 * \sigma_7) = 0.996925\).

The proximity of the correlation coefficient to 1 is indicative of the close relation between the observed and the predicted field and justifies the use of the summation of the tidal forces from different external objects.

4. Conclusions

Based on a solution for inverse magnetometry problem a model distribution of magnetic moments in the Earth’s core is obtained.
A model of volumetric currents corresponding to that distribution is established.

A model for generation of global magnetic field based on tidal forces created by external objects, working under following conditions is suggested:
1. Planet has a liquid hot core.
2. Planet rotates on its axis.
3. The presence of one or more external objects (the sun or satellites), creating a tidal forces that cause motion of the liquid on the surface of the planet (trade-wind flow), and in the liquid core. The tidal force is proportional to the mass of the object, the radius of the planet and inversely proportional to the rotation period of the planet relative to external object and to the cube of the distance of the object from the planet.

High correlation of the magnetic field at the equator between the model predictions and observations for planets in the Solar system indicates linear relation between magnetic field and tidal forces.

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