The growth of massive stars via stellar collisions in ensemble star clusters

M. S. Fujii* and S. Portegies Zwart

Leiden Observatory, Leiden University, NL-2300RA Leiden, the Netherlands

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ABSTRACT

Recent simulations and observations suggest that star clusters form via the assembling of smaller subclusters. Because of their short relaxation time, subclusters experience core collapse much earlier than virialized solo clusters, which have similar properties of the merger remnant of the assembling clusters. As a consequence, it seems that the assembling clusters result in efficient multiple collisions of stars in the cluster core. We performed a series of \(N\)-body simulations of ensemble and solitary clusters including stellar collisions and found that the efficiency of multiple collisions between stars is suppressed if subclusters assemble after they experience core collapse individually. In this case, subclusters form their own multiple collision stars which experienced a few collisions, but they fail to collide with each other after their host subclusters assemble. The multiple collision stars scatter each other and escape, and furthermore the central density of the remnant clusters had already been depleted for the stars to experience more collisions. On the other hand, if subclusters assemble before they experience core collapse, the multiple collisions of stars proceed efficiently in the remnant cluster, and the collision products are more massive than virialized solo clusters and comparable in mass to cold solo clusters.

Key words: open clusters and associations: individual: NGC 3603 – open clusters and associations: individual: Westerlund 1 – open clusters and associations: individual: Westerlund 2 – galaxies: star clusters: general – galaxies: star clusters: individual: R136

1 INTRODUCTION

Young dense star clusters observed in the Milky Way and the Large Magellanic Cloud (LMC), e.g., R136 (Massey & Hunter 1998; Crowther et al. 2010), NGC 3603 (Stolte et al. 2006; Harayama, Eisenhauer & Martins 2008), Westerlund 1 (Clark et al. 2005; Brandner et al. 2008; Gennaro et al. 2011) and Westerlund 2 (Ascenso et al. 2007; Rauw et al. 2007), are good samples for understanding the formation mechanism of dense star clusters. They are massive (\(\sim 10^5 M_\odot\)) and dense (\(>10^5 M_\odot \text{pc}^{-3}\)), and seem to be approaching (or might have experienced) core collapse although they are young (\(<4 \text{ Myr}\)) (Mackey & Gilmore 2003). For example, in R136 in the LMC, its high core density (\(>5 \times 10^3 M_\odot \text{pc}^{-3}\)) (Mackey & Gilmore 2003) and the existence of high-velocity stars (runaway stars) escaping from the cluster (Brandl et al. 2007; Evans et al. 2010; Bestenlehner et al. 2011; Gvaramadze & Gualandris 2011) suggest that it experienced core collapse (Fujii & Portegies Zwart 2011). If such a young massive cluster experiences core collapse, repeated collisions (so-called runaway collisions) of stars, and as a consequence the formation of very massive stars (\(>100 M_\odot\)), are expected (Portegies Zwart et al. 1999; Portegies Zwart & McMillan 2002; Baumgardt & Klessen 2011; Moeckel & Clarke 2011). Such very massive stars formed through multiple stellar collisions could result in the formation of intermediate-mass black holes (IMBHs) (Ebisuzaki et al. 2001).

The formation of IMBHs in dense star clusters via multiple collisions has been studied using \(N\)-body simulations (Portegies Zwart et al. 1999, 2004; Gürkan, Freitag & Rasio 2004; Freitag, Gürkan & Rasio 2006), and the results suggest that IMBHs with \(10^2-10^3 M_\odot\) could be formed in such dense clusters. Including stellar evolution, however, a high mass-loss rate due to the stellar wind of massive stars prevents the growth of the massive stars (Belkus, Van Bever & Vanbeveren 2007; Glebbeek et al. 2009). A very high collision rate is required for such very massive stars to overcome the copious mass-loss and nevertheless leads to the formation of an IMBH (Fujii et al. 2008).

There are some mechanisms to enhance the growth rate of the very massive stars, but the most important factor is the moment of core collapse, \(t_{cc}\). This short but high-density phase is necessary for the cluster to become collisionally dominated, which is critical for the collision rate of stars in the cluster. Earlier collapse times assist an efficient mass accumulation because stars can start multiple collisions before the cluster starts to lose massive stars via stellar evolution.

The core-collapse time is determined by the relaxation time of the virialized star cluster, which with a Salpeter-type mass function is about 20 per cent of the half-mass relaxation time, \(t_{\text{rf}}\) (Portegies
Zwart & McMillan 2002; Heggie & Hut 2003). Massive clusters are unlikely to reach core collapse before the end of the main-sequence lifetime of their most massive stars, which for \( M_\odot \geq 40 \) is \( \sim 3 \) Myr. These clusters can still reach core collapse before the most massive stars leave the main sequence if they are born kinematically relatively cold. A subvirial cluster evolves faster dynamically than a cluster that is born in virial equilibrium; mass segregation in subvirial clusters also proceeds on a shorter time-scale (Allison et al. 2009). Mass segregation as well as core collapse proceeds on the same – dynamical – time-scale, and for subvirial clusters also the increase of the core density proceeds on a shorter time-scale than for virialized clusters. The accelerated dynamical evolution of subvirial clusters enables an efficient mass-growth via multiple collisions of stars.

Mass segregation causes the massive stars to sink to the cluster centre, and consequently to pile up in the cluster core. In the core, these stars find each other and initiate a collision runaway (Portegies Zwart et al. 1999). The consequent mass-growth due to stellar collisions can be quite efficient, in particular if the massive stars are concentrated in the core. Allowing a cluster to be born with some degree of mass segregation also makes the collision runaway more efficient (Ardi, Baumgardt & Minneshige 2008; Goswami et al. 2012), much in the same way as subvirial initial conditions reduce the time of mass segregation, which again leads to an enhanced collision rate (Allison et al. 2009).

Fractal initial conditions and assembling subcluster models also result in an early dynamical evolution similarly to subvirial initial conditions (Aarseth & Hills 1972; McMillan, Vesperini & Portegies Zwart 2007; Allison et al. 2009; Moeckel & Bonnell 2009; Smith et al. 2011; Yu, de Grijs & Chen 2011; Fujii, Saioth & Portegies Zwart 2012). The short relaxation time of subclumps compared to initially massive single clusters causes early mass segregation and core collapse. The memory of such early dynamical evolution is conserved in the merger remnant (McMillan et al. 2007; Fujii et al. 2012, hereafter Paper I), the formation of star clusters by assembling them seems to be an effective way for efficient multiple collisions of stars in young star clusters.

In Paper I, we found that the formation scenario of young dense star clusters via mergers of ensemble subclusters can successfully explain the mature characteristics of young massive star clusters such as R136 in the 30 Dor region. The age of R136 is only 2–3 Myr, but it shows dynamically mature characteristics, such as mass segregation, a high core density, and a wealth of high-velocity escaping stars (Mackey & Gilmore 2003; Brandl et al. 2007; Evans et al. 2010; Bestenhlemer et al. 2011; Gvaramadze & Gualandris 2011). However, the relaxation time of R136 obtained from its current mass and radius is \( \sim 100 \) Myr (Mackey & Gilmore 2003), which is too long to have reached core collapse at its current age. In Paper I, we performed a series of \( N \)-body simulations of ensemble clusters and demonstrated that ‘ensemble’-cluster models can reproduce observations such as the core density, the fraction of high-velocity escapers, and the distribution of massive stars which experienced collisions, but ‘solo’-cluster models, which are initially spherical and virialized, fail to reproduce these observations. Furthermore, these characteristics of the ensemble models are also consistent with the characteristics of other massive young clusters like R136 in the LMC and NGC 3603 in the Milky Way (Crowther et al. 2010).

If young dense clusters formed via assembling subclusters and have experienced core collapse, it is expected that repeating collisions can lead to the formation of very massive stars and possibly even IMBHs. In the observed young dense clusters, however, there is no evidence of IMBHs, but some very massive stars with an initial mass of 100–300 \( M_\odot \) are observed (Stolte et al. 2006; Mengel & Taccon-Garman 2010; Bestenlehner et al. 2011; Gennaro et al. 2011; Roman-Lopes, Barba & Morell 2011).

In this paper, we perform a series of \( N \)-body simulations of solo and ensemble star clusters and demonstrate that the growth of very massive stars through multiple collisions is mediated by star cluster complexes. Our simulations show that the quick dynamical evolution of ensemble clusters does not always result in the formation of extremely high mass stars. When the assembling of clusters proceeds after each subcluster experiences core collapse (‘late-assembling’ case), multiple-collision stars that form in each subcluster fail to coalesce to an extremely massive star, but leads to the formation of several very massive stars. Some of these very massive stars can escape from the cluster as high-velocity stars due to the three-body or binary–binary encounters. When the subclusters assemble before they experience core-collapse (‘early-assembling’ case), the collision rate is enhanced and the assembled cluster forms an extremely massive star of \( \sim 1000 \) \( M_\odot \).

2 METHOD AND INITIAL CONDITIONS

We performed a series of \( N \)-body simulations of solo clusters and ensemble clusters that merge to a single cluster with a mass equal to solo clusters. For the ensemble of subclusters, we adopted two models – A: a King model (King 1966) with a dimensionless concentration parameter, \( W_\odot \), of 2 and the total mass \( M_\odot = 6300 \) \( M_\odot \), and B: a King model with \( W_\odot = 5 \) and \( M_\odot = 2.5 \times 10^5 \) \( M_\odot \). The half-mass radii, \( r_\odot \), of these models are 0.092 and 0.22 pc, and the numbers of particles, \( N \), are 2048 (2k) and 8192 (8k), respectively. The core density is the same for both models (\( \rho_\odot \approx 2 \times 10^5 \) \( M_\odot \) pc \(^{-3} \)). We assumed a Salpeter initial mass function (IMF) (Salpeter 1955) between 1 and 100 \( M_\odot \). We call these models 2kw2 and 8kw5, respectively.

We distribute four or eight of these subclusters in two different initial configurations: spherical and filamentary. The former model stems from clumpy star formation in giant molecular clouds, and the latter is motivated by star formation in a filamentary gas distribution or shocked region of colliding gas in the spiral arms of a galactic disc. The clumpy star formation is initiated by observations of Westerlund 1 (Gennaro et al. 2011) and R136 (Sabbi et al. 2012), and simulations (Bonnell et al. 2011; Saitoh et al. 2011). For the spherical models, we adopted four or eight of models 2kw2 as subclusters, and distributed them randomly in a volume with a radius of \( r_{\max} \) and with zero velocity. We varied \( r_{\max} \) between 1 and 6 pc. For the filamentary models, we initialized eight individual 8kw5 model subclusters. We initialized these subclusters with two different initial mean separations (models e8k8f1 and e8k8f2), but with zero velocity. The initial positions of the subclusters for these models are illustrated in Fig. 1. All runs are summarized in Table 2.

For the solo models, we adopted two more initial conditions with \( M_\odot \) of 5.1 \( \times 10^4 \) and 2.0 \( \times 10^5 \) \( M_\odot \). With the same mass function, these models have 16 384 (16k) and 65 536 (64k) stars and are initialized using King models with \( W_\odot = 6 \) and 8, respectively. In order to obtain the same core density as that of subclusters, their half-mass radii are 0.32 and 1.0 pc, respectively. We call these models 16kw6 and 64kw8, respectively. In Table 1, we summarize the initial conditions, and we present their initial density profiles in Fig. 2. We performed additional simulations of subvirial (cold) initial conditions for 16kw6, and an extra set of simulations in which we reduced the kinetic energy (velocity of each particle) to two-thirds and 10 per cent of the virialized velocity. We call these models s16k-cool and s16k-cold, respectively (see Table 2).
Figure 1. Initial position of ensemble models, e8k8f1 (right-hand panel) and e8k8f2 (left-hand panel). We mimicked filamentary star-forming regions.

Table 1. Models of single clusters.

| Model | N     | $M_c$ (M$_\odot$) | $W_0$ (pc) | $r_h$ (pc) | $\rho_c$ (M$_\odot$ pc$^{-3}$) | $\sigma$ (km s$^{-1}$) | $t_h$ (Myr) | $t_w$ (Myr) | $M_{core}/M_c$ |
|-------|-------|-------------------|------------|------------|-------------------------------|----------------------|------------|------------|----------------|
| 2kw2  | 2048  | $6.3 \times 10^3$ | 2          | 0.097      | $1.7 \times 10^6$             | 11                   | 0.30       | 0.58       | 0.28           |
| 8kw5  | 8192  | $2.5 \times 10^4$ | 5          | 0.22       | $1.7 \times 10^6$             | 15                   | 1.9        | 0.92       | 0.15           |
| 16kw6 | 16384 | $5.1 \times 10^4$ | 6          | 0.32       | $1.7 \times 10^6$             | 17                   | 4.4        | 1.1        | 0.12           |
| 64kw8 | 65536 | $2.0 \times 10^5$ | 8          | 1.0        | $1.6 \times 10^6$             | 19                   | 44         | 18         | 0.053          |

Note. $\sigma$ is the velocity dispersion.

Figure 2. Initial density profiles of single clusters.

The N-body simulations are performed using the sixth-order Hermite scheme with individual time-steps with an accuracy parameter $\eta = 0.15$–0.3 (Nitadori & Makino 2008). We adopted the accuracy parameter to balance speed and accuracy, and the energy error was <0.1 per cent for all runs. Our code does not include special treatment for binaries, but the sixth-order Hermite scheme can handle hard binaries formed in our simulations (see section 2 in Paper I). We took into account collisions of stars with a sticky-sphere approach and mass-loss due to the stellar wind for stars with $>100$ M$_\odot$ with a rate of $5.0 \times 10^{-7}(m/M_\odot)$ M$_\odot$ yr$^{-1}$ (Fujii et al. 2009), which is similar to that obtained in Belkus et al. (2007) and Pauldrach, Vanbeveren & Hoffmann (2012). We neglected the mass-loss from stars with $<100$ M$_\odot$ because it does not affect the results on the short time-scale of our simulations (<5 Myr). The stellar radii are taken from the zero-age main sequence for solar

Table 2. Runs.

| Model      | N$_{cl}$ | Geometry | $(d_{min})$ (pc) | (Sub)cluster | N$_{run}$ |
|------------|----------|----------|------------------|--------------|----------|
| e2k4r3     | 4        | Spherical | 2.5              | 2kw2         | 3        |
| e2k4r6     | 4        | Spherical | 5.1              | 2kw2         | 1        |
| e2k8r1     | 8        | Spherical | 0.51             | 2kw2         | 2        |
| e2k8r3     | 8        | Spherical | 1.3              | 2kw2         | 1        |
| e2k8r5     | 8        | Spherical | 2.8              | 2kw2         | 2        |
| e2k8r6     | 8        | Spherical | 3.3              | 2kw2         | 2        |
| e8k8f1     | 8        | Filamentary | 2.8              | 8kw5         | 1        |
| e8k8f2     | 8        | Filamentary | 4.2              | 8kw5         | 1        |
| s2k        | 1        | –        | –                | 2kw3         | 7        |
| s8k        | 1        | –        | –                | 8kw5         | 6        |
| s16k       | 1        | –        | –                | 16kw6        | 6        |
| s64k       | 1        | –        | –                | 64kw8        | 2        |
| s16k-cool  | 1        | –        | –                | 16kw6        | 2        |
| s16k-cold  | 1        | –        | –                | 16kw6        | 1        |

The models are named according to the following rules: ‘e’ and ‘s’ indicate ensemble and solo models, respectively. For the ensemble models, the following numbers indicate the number of particles of subclusters and the number of subclusters. The last part indicates the initial configuration of subclusters: ‘r’ and the following number indicate spherical geometry and the value of the maximum radius, $r_{max}$, respectively; ‘f’ indicates the filamentary initial configurations (see Fig. 1 for the initial positions of subclusters in these models). For solo models, the number indicates the number of particles. $(d_{min})$ is the averaged distance to the nearest-neighbour subclusters, and $N_{run}$ is the number of runs. s16k-cool and s16k-cold are the same model as s16k, but with the velocity of 67 and 10 per cent of s16k, respectively.
3 SOLO-CLUSTER MODELS

3.1 Virialized solo-cluster models

We describe the results of the initially virialized solo-cluster models, which we will refer to as ‘standard’ models. In Fig. 3 we present the time evolution of the core density for models s2k, s8k, s16k, and s64k. The core densities are calculated using the method of Casertano & Hut (1985). We identify the moment when the cluster reaches the highest core density as the core-collapse time. The core-collapse time measured from the simulations is \( t_{cc} = 0.29 \pm 0.07, 0.71 \pm 0.11, 1.2 \pm 0.13 \) and \( 1.8 \pm 0.0 \) Myr for models s2k, s8k, s16k and s64k, respectively (see also Table 3). The core-collapse time is consistent with those obtained by previous simulations (Gürkan et al. 2004), if we take into account the differences in the mass range of the mass function. As is demonstrated in Gürkan et al. (2004), the core-collapse time scales with the central relaxation time (Heggie & Hut 2003):

\[ t_{cc} = \frac{0.065\sigma_{3D}^3}{G^3(m)\rho_c \ln \Lambda} \]  

Here \( G \), \( \langle m \rangle \), \( \sigma \), and \( \rho_c \) are the gravitational constant, the mean mass of stars, the central velocity dispersion and density, respectively. Here \( \ln \Lambda \) is the Coulomb logarithm. In our simulations, \( t_{cc}/t_{rh} \lesssim 1 \) for models s8k, s16k and s64k, but \( t_{cc}/t_{rh} \lesssim 0.5 \) for model s2k. For model s2k, however, \( t_{cc} \) is shorter than \( t_{rh} \) because the core radius exceeds the half-mass radius. If we adopt a shorter relaxation time, then \( t_{cc}/t_{rh} \sim 1 \) for all the models.

The core collapse of the cluster initiates a collision runaway in the cluster core (Portegies Zwart et al. 1999). In Fig. 4 we present the merger histories of the multiple-collision stars in the solo-cluster simulations s2k, s8k, s16k and s64k. In each model, one primary collision product (PCP) per cluster grows through repeated collisions of stars. In model s2k, the mass-loss due to the stellar wind exceeds the mass-gain by the collisions, and therefore the PCP has lost all gained mass by the end of the simulation (5 Myr). PCPs grow up to the maximum mass \( m_{max} \sim 400 \) M\(_\odot\) via repeating collisions, but by the time it explodes (2–3 Myr; Belkus et al. 2007; Pauldrach et al. 2012) the star is \( \sim 100 \) M\(_\odot\). Here we define \( m_{max} \) as the maximum mass of a star reached during its lifetime as a result of collisions.

PCPs are not the only stars that experienced collisions. In models s8k, s16k and s64k, we find secondary collision products (SCPs). In most cases, SCPs experience only one collision (sometimes a few collisions), but never grow as massive as PCPs, although SCPs

Table 3. Summary of the results.

| Model | \( m_{max}(M_\odot) \) | \( t_{max} \) (Myr) | \( t_{merge} \) (Myr) | \( t_{cc} \) (Myr) | \( m_{SCPs}(M_\odot) \) | \( m_{col}(M_\odot) \) | \( N_{col} \) |
|-------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-------|
| c2k4r3-1 | 287             | 0.45            | 0.2–0.87        | 0.29 ± 0.07     | 375             | 78.3            | 8     |
| c2k4r3-2 | 260             | 0.78            | 0.03–1.2        | 454             | 60.6            | 15               |       |
| c2k4r3-3 | 268             | 0.61            | 0.6–1.3         | 139             | 56.7            | 12               |       |
| c2k4r6   | 238             | 0.57            | 2.2–2.7         | 743             | 46.8            | 16               |       |
| c2k8r1-1 | 998             | 0.80            | 0.03–0.38       | 0.29 ± 0.07     | 0               | 44.2            | 45    |
| c2k8r1-2 | 667             | 1.35            | 0.03–0.32       | 160             | 69.4            | 22               |       |
| c2k8r3   | 530             | 0.86            | 1.0–0.75        | 147             | 80.2            | 14               |       |
| c2k8r5-1 | 334             | 1.11            | 0.47–1.9        | 1192            | 57.0            | 25               |       |
| c2k8r5-2 | 486             | 0.78            | 0.03–2.0        | 651             | 61.2            | 19               |       |
| c2k8r6-1 | 245             | 0.59            | 0.77–2.4        | 1367            | 51.0            | 24               |       |
| c2k8r6-2 | 274             | 0.42            | 0.03–>3         | 970             | 45.5            | 21               |       |
| e8k8f1   | 1310            | 1.40            | 0.4–1.2         | 0.71 ± 0.11     | 268             | 73.0            | 42    |
| e8k8f2   | 659             | 2.28            | 0.8–2.0         | 995             | 88.4            | 33               |       |
| s2k      | 182 ± 21        | 1.3 ± 0.6       | –               | 0.29 ± 0.07     | 16 ± 40         | 53.3            | 4.6   |
| s8k      | 399 ± 60        | 2.2 ± 0.2       | –               | 0.71 ± 0.11     | 149 ± 115       | 63.2            | 11.3  |
| s16k     | 431 ± 54        | 2.6 ± 0.9       | –               | 1.2 ± 0.13      | 54 ± 77         | 65.8            | 13.2  |
| s64k     | 488 ± 57        | 4.4 ± 0.2       | –               | 1.8 ± 0.0       | 0               | 66.0            | 15.5  |
| s16k-cold| 1064            | 0.59            | –               | <0.02           | 0               | 46.6            | 40    |
| s16k-cool| 707 ± 36        | 2.55 ± 0.75     | –               | 0.325 ± 0.075   | 0               | 51.1            | 28.5  |
sometimes exceed our adopted upper limit to the IMF (100 $M_\odot$).
The SCPs end up merging with PCPs (see the bottom right-hand panel in Fig. 4) or just lose their mass by stellar evolution (see the top right-hand panel in Fig. 4). This result agrees with previous numerical simulations (Freitag et al. 2006).

We also find that the time when the PCPs reach their maximum mass $m_{\text{max}}$, $t_{\text{max}}$, is scaled by $t_{\text{rc}}$, and that $t_{\text{max}}/t_{\text{rc}} = 2.3, 2.2, 2.2$ and 2.6 for models s2k, s8k, s16k and s64k, respectively.

3.2 Cold solo-cluster models

Subvirial (cold) initial conditions reach core collapse considerably earlier than virialized ones. Cold models have therefore been suggested to explain the dynamically advanced appearance of observed young star clusters (Allison et al. 2009). In Fig. 5, we present the core-density evolution of models s16k-cool and s16k-cold, which initially have 67 and 10 per cent of the virialized velocity. These models reach core collapse much earlier than virialized models, and as a consequence multiple collisions start earlier and proceed at a higher collision rate. In Fig. 6 we present the mass evolution of the PCPs for models s16k-cold and s16k-cool. Colder initial conditions result in a higher $m_{\text{max}}$ of the PCPs. The high $m_{\text{max}}$ is a result of the high collision rate, which is caused by the high density in the core (see Fig. 5).

By the time the PCPs leave the main sequence (of $\sim 3$ Myr), their masses have been reduced considerably due to stellar mass-loss, which competes with the mass-gain by collisions. In model s16k-cold, the PCP grows quickly in the beginning of the simulation, but after 0.5 Myr the mass-loss rate due to the stellar wind becomes higher than the mass-growth by stellar collisions. In model s16k-cool, the PCP stops growing at $\sim 0.5$ Myr because the mass-growth rate balances the mass-loss rate, and then the PCP maintains its mass until the end of the simulation (3.5 Myr). Although the final masses of the PCPs are comparable in both model s16k-cold and model s16k-cool, $m_{\text{max}}$ of model s16k-cold is twice as massive as that of model s16k-cold.

In Fig. 7 we show $m_{\text{max}}$ of the PCPs for all solo models. The maximum mass of the PCPs in models s8k, s16k and s64k is quite similar ($\sim 400 M_\odot$) irrespective of $M_\text{cl}$. One might expect that more massive clusters contain a larger number of massive stars and therefore a more massive cluster can form a more massive PCP. In our simulation, however, the number of stars which merged into the PCPs and the mean mass of the merged stars are quite similar among these models (see Table 3). By comparing models s8k, s16k and s64k, their $\langle m_{\text{col}} \rangle$ and $N_{\text{col}}$ are quite similar even though their
total cluster masses are different. If the collisions selectively occur among the most massive stars and the numbers of collisions are the same, larger clusters should have a larger mean collision mass \( m_{\text{coll}} \) because larger clusters contain more massive stars. However, the number of massive stars does not simply follow this relation. In Fig. 8, we plot the cumulative number distribution of massive stars with \( m > 50 \, M_\odot \) at the moment in which the mass of the PCP reaches \( m_{\text{max}} \). The numbers of stars with \( M > 50 \, M_\odot \) within \( \sim 0.05 \, \text{pc} \) are similar (\( \sim 20 \)) among models s8k, s16k and s64k and slightly smaller for model s2k. In particular, for models s16k and s64k, the distribution of massive stars preserves the initial distribution in the outer part of the cluster because the half-mass relaxation time \( t_{\text{rh}} \) is longer than that for solo clusters with the same total mass. Their mass-loss rate exceeds their mass-growth rate by stellar collisions. In models s16k and s64k, on the other hand, they have not exhausted their reservoir of massive stars because their half-mass relaxation time is not shorter than the simulation time and therefore some of the massive stars still remain in the outer part of the clusters.

We empirically obtained a relation that \( m_{\text{max}} = 0.02 \, M_\odot \) (dotted line in Fig. 7) for the low-cluster-mass models \( (M_\text{cl} < 2 \times 10^4 \, M_\odot) \) and the cold model. For massive clusters, however, \( m_{\text{max}} \) is smaller than that according to this relation. For the most massive cluster \( (M_\text{cl} = 2 \times 10^5 \, M_\odot) \), \( m_{\text{max}} \) is consistent with the result presented by Portegies Zwart & McMillan (2002), \( m_{\text{max}} = 0.002 \, M_\odot \) (dashed line in Fig. 7).

### 4 ENSEMBLE-CLUSTER MODELS

In Section 3, we demonstrated that the results obtained from our solo-cluster models are consistent with previous numerical studies. In this section, we present the results of ensemble-cluster models, in which subclusters assemble to finally form one single cluster. In ensemble-cluster models, subclusters collapse on a time-scale shorter than that for solo clusters with the same total mass. Their further evolution is dominated by the dynamical evolution of the subclusters before they merge. The conservation of the dynamical states through the mergers (Vesperini, McMillan & Portegies Zwart 2009) drives the further evolution of the cluster merger of the cluster to join the collisions in the core. Similar to model s2k, models s16k-cool and s16k-cold can also gather massive stars from the entire cluster to the cluster centre irrespective of their initial positions. In addition, these subvirial models achieve a higher velocity dispersion (see Fig. 5), which enhances the collision rate. The massive stars in model s16k-cold are more concentrated towards the cluster centre compared with model s16k (see Fig. 8).

Even though for model s2k the PCP can accumulate stars from the entire cluster population of massive stars, their total number and mass still cannot compete with the population of massive stars in the more massive clusters. In these latter models, the maximum mass of the PCP is limited by the reservoir of massive stars, which manages to segregate to the core by the moment of the core collapse. A larger cluster mass therefore does not automatically lead to a massive PCP. As shown in Figs 4 and 6, the mass evolution of the PCPs in models s2k, s8k and s16k-cold shows a clear peak in the middle of the simulation. In the latter phase, when the collision rate decays, their mass-loss rate exceeds their mass-growth rate by stellar collisions. In models s16k and s64k, on the other hand, they have not exhausted their reservoir of massive stars because their half-mass relaxation time is not shorter than the simulation time and therefore some of the massive stars still remain in the outer part of the clusters.
Figure 9. Schematics of two typical assembling processes. Early assembling \((t_{cc} > t_{ens})\): subclusters assemble before they experience core collapse. The merger remnant is more mass segregated than solo clusters which initially have similar properties to the merger remnant because subclusters have a shorter relaxation time than the solo cluster. After their assembling, the remnant cluster collapses and a massive PCP forms. Late assembling \((t_{cc} < t_{ens})\): subclusters experience core collapse and form small PCPs before they assemble. After their assembling, however, the PCPs do not grow efficiently because most of them are scattered from the remnant cluster by binary–binary encounters.

products. As a result, ensemble clusters tend to experience core collapse considerably faster than solo clusters which have initially similar properties to those of the merger remnant of ensemble clusters. In Paper I, we already showed that the quicker dynamical evolution of ensemble clusters can explain the mature characteristics of young dense clusters such as R136 and NGC 3603. Here we use that enhanced dynamical evolution to study the PCPs. The early dynamical evolution of ensemble clusters is similar to that of cold solo clusters. One might expect that ensemble clusters also result in the formation of massive PCPs, but we will show that the early evolution of ensemble clusters is more complicated.

In Fig. 9 we illustrate the schematic evolution of two typical evolutionary paths of ensemble clusters. We find that the most important parameter for the evolution of ensemble clusters is the moment of assembling, \(t_{ens}\), compared to \(t_{cc}\) of subclusters. If \(t_{cc} > t_{ens}\) (‘early assembling’), the PCPs in the remnant cluster grow efficiently by stellar collisions because the short relaxation time of the subclusters drives mass segregation and core collapse faster than solo clusters. This evolution is similar to that of cold solo clusters.

If \(t_{cc} < t_{ens}\) (‘late assembling’), each subcluster experiences core collapse before they assemble and form a PCP per individual sub-cluster. The mass of each PCP is limited by the subcluster mass as we described in Section 3. After the assembling of two or more subclusters, the PCPs formed in the subclusters sink to the centre of the remnant cluster and interact with each other. Most of them, however, are scattered and ejected from the cluster because they tend to reside in hard binaries with a massive companion. The PCPs tend to be in the hardest binaries with the most massive stars when they formed in the subclusters. In each binary–binary encounter following a subcluster merger, two PCPs may collide although they are also ejected without experiencing a collision. Therefore, the majority of the PCP binaries are scattered or ionized, and only one PCP binary survives in the remnant cluster by the time the assembly is completed. The surviving PCP cannot continue to grow in mass because by that time the central density of the assembled clusters has been depleted due to the early dynamical evolution.

4.1 Stellar collisions in ensemble clusters

In Figs 10 and 11, we present the mass evolution of PCPs and SCPs in ensemble clusters. The left-hand and right-hand panels show early- and late-assembling models, respectively. In
**Figure 10.** Top panels: time evolution of the separation between subclusters projected on to the $x$-axis (full curves) and the collisions of PCPs (black dots) for models e2k8r1 (left-hand panel) and e2k8r6-1 (right-hand panel). The positions of the dots show the collision time and the subcluster to which the star initially belongs. Bottom panels: mass evolution of PCPs and SCPs for models e2k8r1 (left-hand panel) and e2k8r6-1 (right-hand panel). The crosses indicate the time when the SCPs merged to PCPs. The arrows indicate the time when subclusters merged. In all panels, the shaded region indicates the core-collapse time with error obtained from the simulations of isolated subclusters.

**Figure 11.** Same as Fig. 10 but for models e8k8f1 (left-hand panels) and e8k8f2 (right-hand panels).
early-assembling models, one massive PCP per remnant cluster grows after the assembling of subclusters. Even though some of the subclusters start forming PCPs before assembling, the PCPs merge after the host subclusters merged. In late-assembling models, on the other hand, each subcluster grows its own PCP, but most of them do not collide with each other even after the assembling of their host subclusters.

We find the reason for the difference between early- and late-assembling cases in the density evolution of these clusters. In Fig. 12 we show the time evolution of the maximum number densities for ensemble and solo clusters. Here we plot the maximum value of the local density, which is calculated using six nearest neighbours. (Note that the maximum local density does not trace the density of one individual subcluster.) In early-assembling cases, the density increases on the core-collapse time-scale of the solo subcluster (model s2k), but the maximum density is higher than that of model s2k and rather comparable to those of the cold models (models s16k-cool and s16k-cold). The evolution after the core collapse is similar to that of the cold models. The density gradually decreases and eventually becomes comparable to that of virialized solo clusters (model s16k).

The density in late-assembling cases also grows on the core-collapse time-scale of the subclusters until a peak is reached at $\sim 0.5$ Myr. The density decreases as quickly as that of the solo subclusters (model s2k), which is different from early-assembling cases. By the end of the simulations, the number density of the late-assembling cases is an order of magnitude lower than in the early-assembling cases. The relatively low density prevents the growth of PCPs in the late-assembled clusters. The effect of the difference in the density can be seen in the number of stellar collisions, $N_{\text{col}}$, in Table 3. In early-assembling models (e2k8r1 and e8k8f1) and the cold solo model (s16k-cold), $N_{\text{col}} = 42 \pm 2$ and $m_{\text{max}} = 1100 \pm 130$, but in late-assembling models (e2k8r5, e2k8r6 and e8k8f2) $N_{\text{col}} = 24 \pm 5$ and $m_{\text{max}} = 400 \pm 150$.

In late-assembling models (e2k8r5 and e2k8r6), the maximum mass of the PCPs is 200–400 $M_\odot$, but the mass of the PCP is similar to those of multiple SCPs, which were PCPs in the subclusters. This feature is consistent with young dense clusters such as R136 in the LMC, which contains five $> 100 M_\odot$ mass stars (Crowther et al. 2010; Bestenlehner et al. 2011), although there is no evidence of any extremely massive stars with $\sim 1000 M_\odot$.

4.2 Maximum mass of PCPs in ensemble clusters

As we show in Section 4.1, early assembling of subclusters results in the formation of a PCP, while late assembling forms a less massive PCP and multiple SCPs as massive as the PCP. In Fig. 13 we present the relation between $m_{\text{max}}/M_\odot$ and $t_{\text{cc}}/t_{\text{enc}}$ of ensemble models, where $t_{\text{cc}}$ is the core-collapse time of the subclusters. Irrespective of the number of subclusters, the maximum mass of the PCPs decreases as the assembling time is delayed.

In the left-hand panel of Fig. 14, we show the relation between $m_{\text{max}}$ of the PCPs and $M_\odot$ for both solo and ensemble clusters. (Note that for the solo clusters, the data are the same as those shown in Fig. 7). The PCP mass of early-assembling models is higher than that of solo clusters with the same mass and as massive as that of the cold model. In late-assembling models, the PCPs are almost as massive as those of the solo clusters with the same mass.

The difference in the maximum mass of PCPs is understood if we take into account all the PCPs and SCPs in the cluster. In the right-hand panel of Fig. 14, we present the total mass of all the PCPs and SCPs in the cluster. The total masses are roughly located on the relation that $m_{\text{max}}/M_\odot = 0.02 M_\odot/t_{\text{cc}}$. This result suggests that the potential maximum mass of the PCPs is 2 per cent of the cluster mass, although the value depends on the IMF and the mass-loss rate due to the stellar wind. The total mass of the SCPS is summarized in Table 3 as $m_{\text{SCP}}$. These SCPS fail to merge with the most massive PCP and their mass will be lost from the cluster by escape or stellar explosion.

In Fig. 15, we plot the radial distribution of PCPs and SCPs, which grows to $> 100 M_\odot$. We combine the results from several runs, separating them in the early- and late-assembling cases. While all the PCPs and SCPs are located in the cluster core in the early-assembling case, $\sim 40$ per cent of them are ejected from the clusters or located in the outskirts of the cluster ($> 10$ pc) in the late-assembling case. The numbers of PCPs and SCPs per cluster are on average 1.75 and 5.8 for the early- and late-assembling cases, respectively. In Fig. 15 we also present the cumulative number distribution of stars with $> 100 M_\odot$ in the R136 region (Crowther et al. 2010; Bestenlehner et al. 2011). The number of such massive stars and their distribution imply that R136 experienced some late
4.3 Central density of remnant clusters

In Fig. 16, we compare the central density of the remnant clusters with the observed density of young massive clusters. Since the core radii and core densities of the simulated clusters are obtained from a rather small number of particles, they show relatively large fluctuations (see Figs 3 and 5). We adopted the core radius of NGC 3603 of 0.14 pc for models e2k4r3, because the mass of these models is comparable to that of NGC 3603 (Harayama et al. 2008). The total mass of models e2k8 is comparable to that of R136, for which we adopted a core radius of 0.4 pc (Selman & Melnick 2012) rather than the observed 0.025 pc (Hunter et al. 1995; Andersen et al. 2009). We argue that the observed value is rather strongly affected by the dynamical evolution of the cluster, and also influenced by the small number of stars in the core; both tend to cause an underestimate of the core radius. In the same figure, we also plot the observed densities of a number of the young clusters listed in Pfalzner (2009) and Anderson et al. (2012). Those observed densities are not measured in the same way as we determine the central density in our simulations, although we tried to mimic the observational technique to measure the core radius as good as possible. Late-assembling models turn out to have a central density which is an order of magnitude lower than in the early-assembling models. Those lower densities very well match the central density in the observations of R136. The central density of virialized solo cluster models evolves quite similarly to the early-assembling simulations.

5 SUMMARY AND DISCUSSION

We performed N-body simulations of solo and ensemble star clusters and found that ensemble clusters evolve through typically two pathways depending on their assembling time compared to the core-collapse time of the subclusters.

In the early-assembling case, the subclusters merge before they experience core collapse individually: $t_{cc} > t_{ens}$. After merging, their
remnant clusters have dynamically mature characteristics (strongly mass segregated and core collapsed) compared to solo clusters. The early-assembling clusters experience mass segregation and core collapse on the time-scale of the subclusters, which is shorter than that of initially large solo clusters; the short relaxation time of subclusters is conserved in the remnant clusters. This dynamically early evolution results in efficient multiple collisions of stars and helps the formation of extremely massive PCPs with $\sim 1000 M_\odot$. The evolution of the early-assembling clusters can be mimicked by solo clusters when those are born subvirial.

In the late-assembling case, the subclusters experience core collapse before they merge into the larger conglomerate: $t_{cc} < t_{rel}$. The dynamically mature characteristics of the merger remnant suppresses the growth of massive stars via stellar collisions. In this case, the subclusters experience core collapse individually and form their own PCPs, but the maximum mass of the PCPs in the subclusters is limited by the total mass of the subclusters. Even after the subclusters assemble, the PCPs stop growing because the central density of the remnant cluster is already depleted due to the quick dynamical evolution of the subclusters. Since the PCPs in subclusters form massive binaries, they interact with each other in the remnant clusters. Some of them (SCPs) collide, but the others are scattered from the cluster by three-body or binary–binary encounters. In our simulations, 40 per cent of the SCPs are ejected from the cluster or scattered to the outskirts of the remnant clusters. The SCPs sometimes escape with a high velocity ($> 30 \text{ km s}^{-1}$) and reach $\sim 100 \text{ pc}$ from the cluster within their lifetime ($\sim 3 \text{ Myr}$). The observed massive high-velocity stars such as VFTS 682 might be formed in this way (see also Paper I).

We also investigated the maximum mass of the PCPs and found that in ensemble clusters, the maximum mass depends on the assembling time of subclusters. In the early-assembling models, the maximum mass of the PCPs is comparable to that of subvirial solo clusters. In the late-assembling models, however, the maximum mass is similar to that of the solo subclusters; the difference is mainly caused by the number of collisions. In the late-assembling models, a larger number of SCPs are ejected from the cluster than in the early-assembling case and the SCPs fail to merge to the PCP.

When the collisions of stars proceed most successfully (in early-assembling and cold solo models), we find that the maximum masses of the PCPs reach $\sim 2$ per cent of the total mass of the clusters even if we take into account the high mass-loss rate due to the stellar wind. Assuming an R136-like cluster of $\sim 5 \times 10^4 M_\odot$, the expected maximum mass is $\sim 1000 M_\odot$. Such an efficient mass-growth might result in the formation of IMBHs. For lower metallicity, the massive stars are predicted to collapse directly to IMBHs (Heger et al. 2003).

In late-assembling cases, however, a smaller PCP and multiple SCPs ($100–400 M_\odot$) are expected to exist inside or around the remnant clusters. These stars are in the mass range of Type Ib/c supernovae (SNe Ib/c) assuming solar metallicity (Heger et al. 2003). In recent observations of dense molecular clouds in the central molecular zone in the Galactic Centre, several expanding shells were found, and the estimated total kinetic energy of them is $\sim 10^{52} \text{ erg}$ (Tanaka et al. 2007; Oka et al. 2012). Especially, three major shells have a kinetic energy of $\sim 10^{51} \text{ erg}$, which corresponds to a hypernova explosion. A young dense massive cluster which is similar to our late-merger models might be embedded in this dense molecular cloud. Furthermore, escaping SCPs will explode up to $\sim 100 \text{ pc}$ from the host cluster. Actually, SN Ib/c associate with star-forming regions (Anderson et al. 2010, 2012; Leloudas et al. 2011; Crowther 2013), and, for example, SN Ic 2007gr is located at $\sim 7 \text{ pc}$ from a young cluster (Crockett et al. 2008).

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REFERENCES

Aarseth S. J., Hills J. G., 1972, A&A, 21, 255
Allison R. J., Goodwin S. P., Parker R. J., de Grijs R., Portegies Zwart S. F., Kouwenhoven M. B. N., 2009, ApJ, 700, L99
Andersen M., Zinnecker H., Moneti A., McCaughrean M. J., Brandl B., Brandner W., Meylan G., Hunter D., 2009, ApJ, 707, 1347
Anderson J. P., Covarrubias R. A., James P. A., Hamuy M., Habergham S. M., 2010, MNRAS, 407, 2660
Anderson J. P., Habergham S. M., James P. A., Hamuy M., 2012, MNRAS, 424, 1372
Ardi E., Baumberg H., Minisini S., 2008, ApJ, 682, 1195
Ascenso J., Alves J., Beletskys Y., Lago M. T. V. T., 2007, A&A, 466, 137
Baumberg H., Klessen R. S., 2011, MNRAS, 413, 1810
Belkus H., Van Bever J., Vanbeveren D., 2007, ApJ, 659, 1576
Bestenlehner J. M. et al., 2011, A&A, 530, L14
Bonnell I. A., Smith R. J., Clark P. C., Bate M. R., 2011, MNRAS, 410, 2339
Brandl B. R., Portegies Zwart S. F., Moffat A. F. J., Chernoff D. F., 2007, in St.-Louis N., Moffat A. F. J., eds, ASP Conf. Ser. Vol. 367, Massive Stars in Interactive Binaries. Astron. Soc. Pac., San Francisco, p. 629
Brandner W., Clark J. S., Stolle A., Waters R., Negueruela I., Goodwin S. P., 2008, A&A, 478, 137
Casertano S., Hut P., 1985, ApJ, 298, 80
Clark J. S., Negueruela I., Crowther P. A., Goodwin S. P., 2005, A&A, 434, 949
Crockett R. M. et al., 2008, ApJ, 672, L99
Crowther P. A., 2013, MNRAS, 428, 1927
Crowther P. A., Schnurr O., Hirschi R., Yusof N., Parker R. J., Goodwin S. P., Kassim H. A., 2010, MNRAS, 408, 731
Ebisuzaki T. et al., 2001, ApJ, 562, L19
Evans C. J. et al., 2010, ApJ, 715, L74
Freitag M., Gürkan M. A., Rasio F. A., 2006, MNRAS, 368, 141
Fujii M. S., Portegies Zwart S., 2011, Sci, 334, 1380
Fujii M., Iwasawa M., Funato Y., Makino J., 2008, ApJ, 686, 1082
Fujii M., Iwasawa M., Funato Y., Makino J., 2009, ApJ, 695, 1421
Fujii M. S., Saitoh T. R., Portegies Zwart S. F., 2012, ApJ, 753, 85 (Paper I)
Gennaro M., Brandner W., 2009, MNRAS, 412, 2469
Glebbeek E., Gaburov E., de Mink S. E., Pols O. R., Portegies Zwart S. F., 2012, A&A, 497, 255
Goswami S., Umbreit S., Bierbaum M., Rasio F. A., 2012, ApJ, 752, 43
Gürkan M. A., Freitag M., Rasio F. A., 2004, ApJ, 604, 632
Gyarmati V. V., Gualandris A., 2011, MNRAS, 410, 304
Harayama Y., Eisenhauer F., Martins F., 2008, ApJ, 675, 1319
Heger A., Fryer C. L., Woosley S. E., Langer N., Hartmann D. H., 2003, ApJ, 591, 288
Heggie D., Hut P., 2003, The Gravitational Million-Body Problem: A Multi-disciplinary Approach to Star Cluster Dynamics. Cambridge Univ. Press, Cambridge
Hunter D. A., Shaya E. J., Holtzman J. A., O’Neil E. J., Jr, Lynds R., 1995, ApJ, 448, 179
Hurley J. R., Pols O. R., Tout C. A., 2000, MNRAS, 315, 543

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Stellar collisions in ensemble star clusters

King I. R., 1966, AJ, 71, 64
Leloudas G. et al., 2011, A&A, 530, A95
Mackey A. D., Gilmore G. F., 2003, MNRAS, 338, 85
McMillan S. L. W., Vesperini E., Portegies Zwart S. F., 2007, ApJ, 655, L45
Massey P., Hunter D. A., 1998, ApJ, 493, 180
Mengel S., Tacconi-Garman L. E., 2009, Ap&SS, 324, 321
Moeckel N., Bonnell I. A., 2009, MNRAS, 400, 657
Moeckel N., Clarke C. J., 2011, MNRAS, 410, 2799
Nitatadi K., Makino J., 2008, New Astron., 13, 498
Oka T., Onodera Y., Nagai M., Tanaka K., Matsumura S., Kamegai K., 2012, ApJS, 201, 14
Pauldrach A. W. A., Vanbeveren D., Hoffmann T. L., 2012, A&A, 538, A75
Pfalzner S., 2009, A&A, 498, L37
Portegies Zwart S. F., McMillan S. L. W., 2002, ApJ, 576, 899
Portegies Zwart S. F., Verbunt F., 1996, A&A, 309, 179
Portegies Zwart S. F., Yungelson L. R., 1998, A&A, 332, 173
Portegies Zwart S. F., Makino J., McMillan S. L. W., Hut P., 1999, A&A, 348, 117
Portegies Zwart S. F., Baumgardt H., Hut P., Makino J., McMillan S. L. W., 2004, Nat, 428, 724
Rauw G., Manfroid J., Gosset E., Nazé Y., Sana H., De Becker M., Foellmi C., Moffat A. F. J., 2007, A&A, 463, 981
Roman-Lopes A., Barba R. H., Morrell N. I., 2011, MNRAS, 416, 501
Sabbi E. et al., 2012, ApJ, 754, L37
Saitoh T. R., Daisaka H., Kokubo E., Makino J., Okamoto T., Tomisaka K., Wada K., Yoshida N., 2011, in Alves J., Elmegreen B. G., Girart J. M., Trimble V., eds, Proc. IAU Symp. 270, Computational Star Formation. Cambridge Univ. Press, Cambridge, p. 483
Salpeter E. E., 1955, ApJ, 121, 161
Selman J. F., Melnick J., 2012, e-print (arXiv:1209.3825)
Smith R., Slater R., Fellhauer M., Goodwin S., Assmann P., 2011, MNRAS, 416, 383
Stolte A., Brandner W., Brandl B., Zinnecker H., 2006, AJ, 132, 253
Tanaka K., Kamegai K., Nagai M., Oka T., 2007, PASJ, 59, 323
Toonen S., Nelemans G., Portegies Zwart S., 2012, A&A, 546, A70
Vesperini E., McMillan S., Portegies Zwart S., 2009, Ap&SS, 324, 277
Yu J., de Grijs R., Chen L., 2011, ApJ, 732, 16

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