Moduli-mixing racetrack model

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Abstract

We study supersymmetric models with double gaugino condensations in the hidden sector, where the gauge couplings depend on two light moduli of superstring theory. We perform a detailed analysis of this class of model and show that there is no stable supersymmetric minimum with finite vacuum values of moduli fields. Instead, we find that the supersymmetry breaking occurs with moduli stabilized and negative vacuum energy. That yields moduli-dominated soft supersymmetry breaking terms. To realize slightly positive (or vanishing) vacuum energy, we add uplifting potential. We discuss uplifting does not change qualitatively the vacuum expectation values of moduli and the above feature of supersymmetry breaking.

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1 Introduction

String/M theory has been providing a lot of perspectives for particle physics. However, it seems to have an infinite number of vacua and there are many moduli fields labelling them such as complex structure, volume of compact space, positions of branes, and so on. Vacuum expectation values (VEV) of these moduli fields determine various coupling constants and physical scales such as gauge couplings, Yukawa couplings and Planck scale. In order to find a realistic vacuum in string theory, it is necessary and important to determine phenomenologically reasonable values of them. Thus, the moduli stabilization is one of important issues to apply string theory to particle physics as well as cosmology.

Besides the moduli stabilization problem, the origin of supersymmetry (SUSY) breaking is another puzzle which is in general related to the mechanism of stabilizing moduli fields. It seems that some nontrivial mechanism is required in order to realize weak scale SUSY that can solve the gauge hierarchy problem, otherwise the SUSY breaking scale would be typically around the string scale or Planck scale. One of the elegant scenarios is to use field-theoretical dynamics like gaugino condensations \[1\] in the hidden sector, where the SUSY breaking scale can be suppressed by the dimensional transmutation of the gauge theory. A remarkable feature of the gaugino condensation scenario is that it can also generate a potential for moduli fields due to the fact that the gauge coupling is determined by their VEVs.

The so-called racetrack model \[2\] is in most of the case based on the double gaugino condensations, and a modulus field can be stabilized thanks to the different modulus-dependences between two superpotential terms generated by them. However, in a simple setup, e.g. a single-modulus case, the resulting (local) minimum of the potential is a SUSY preserving anti-de Sitter (AdS) vacuum. Then, to be phenomenologically and cosmologically viable, we need at least two things here. One is SUSY breaking and the other is uplifting the vacuum energy to obtain a Minkowski or a slightly de Sitter (dS) background.

Recently a way to achieve these two at the same time has been proposed in Ref. \[3\], which is called the KKLT scenario. This scenario consists of two steps. At the first step, a single modulus is stabilized because of the gaugino condensation and constant term in superpotential induced by three-form flux in type IIB string models, assuming that the other moduli are stabilized through flux compactification \[4\]. Such minimum corresponds to the SUSY AdS vacuum, as mentioned above. At the second step, the uplifting of the vacuum energy is achieved by introducing anti $D3$-branes at the tip of a throat in the Calabi-Yau (CY) threefold which is highly warped due to the existence of three-form flux. Because the flux and anti $D3$-branes preserve different supercharges, SUSY is explicitly broken. This SUSY breaking effects are extensively studied in Ref. \[5\], and it has been shown that the resulting soft SUSY breaking terms of the visible fields have a quite distinctive pattern. (See for their phenomenological aspects Ref. \[6\].) One of important points is that one can study analytically the AdS minimum of the potential before uplifting. That makes it simple to understand the potential minimum with uplifting.

The KKLT scenario is based on a quite simple setup of the type IIB orientifold models. A gauge kinetic function is a mixture of two or more moduli fields in several string models, e.g. weakly coupled heterotic string models \[7, 8\], heterotic M models \[9, 10, 11, 12\], type
IIA intersecting D-brane models and type IIB magnetized D-brane models\cite{13,14}.\footnote{See also Ref.\cite{15} for moduli mixing among Kähler moduli, complex structure moduli and open string moduli in type IIB orientifold models.} In this paper we consider general setup where the gauge couplings in the hidden sector are given by the mixture of some moduli fields (two moduli fields in practice).\footnote{One of two moduli may be frozen around the string scale, e.g. by flux compactification. Such scenario has been studied in Ref.\cite{16}. Here we assume that both moduli remain light.} On top of that, we consider the racetrack model, that double gaugino condensations generate nonperturbative superpotential terms and both of them depend on two moduli fields. Based on the effective four-dimensional (4D) $N = 1$ supergravity (SUGRA) description of such systems, we perform a detailed analysis of the moduli potential and investigate the structure of SUSY breaking and the moduli vacuum values at the local minimum. Similar models have been studied in the literature, in particular through numerical studies. We carry out detail study analytically under certain approximation. The potential minimum without adding an uplifting term corresponds to SUSY breaking AdS vacuum. Such detailed study on the potential minimum is as quite important as similar potential analysis at the first step of the KKLT scenario without uplifting is. Such study makes it possible to understand what would happen in our racetrack model after uplifting. Indeed, we discuss the potential minimum in our model with uplifting is qualitatively the same as one before uplifting. In our model, moduli-dominant SUSY breaking\cite{17,18,19,20} is realized, but the contribution due to anomaly mediation\cite{21} is negligible.

The sections of this paper are organized as follows. In Sec.\ref{sec:mod-mixed-gaug}, we review several string models in which the gauge couplings are given by the mixture of moduli VEVs. Then assuming double gaugino condensations in the hidden sector, we analyze the racetrack model with such moduli-mixed gauge couplings within the framework of effective 4D $N = 1$ SUGRA. First in Sec.\ref{sec:global-mod-pot}, we show the global structure of the moduli potential in such model. We study the stationary points of the potential in Sec.\ref{sec:stationary-pot} and find that the SUSY point is actually a saddle point. Then in Sec.\ref{sec:local-min} we show that there is a SUSY breaking local minimum close to the SUSY saddle point, and estimate the magnitude of SUSY breaking order parameters. In Sec.\ref{sec:non-pan} we discuss the potential minimum after uplifting. In Sec.\ref{sec:phenom} we discuss SUSY breaking phenomenology. Sec.\ref{sec:concl} is devoted to the conclusions and discussions. In Appendix\ref{app:susy-runway}, we show that the SUSY point with $\langle W \rangle = 0$ can be realized only at the runaway vacuum.

## 2 Moduli-mixed gauge couplings

In this paper, we consider general setup for the gaugino condensations where the gauge couplings are given by the mixture of some moduli fields. Such situation occurs when we consider, e.g. heterotic models or type II models with intersecting/magnetized D-branes. In these models, two moduli fields, e.g. the dilaton $S$ and overall Kähler modulus $T$ appear in gauge kinetic function as their linear combination. Here and hereafter, we call them moduli including the dilaton. In this section, we review the heterotic and the magnetized D-brane cases in turn. Note that the proper definition of $S$ and $T$ depends on each string model.
In heterotic (M-)theory on $M_4 \times CY_3(\tilde{x}S^1/Z_2)$, the one-loop gauge kinetic function of strong gauge group, $f_{\text{strong}}$, is given by \[12\]

$$f_{\text{strong}} = S - \beta T + f_{M5},$$

$$\beta \sim \frac{1}{16\pi^2} \int_{CY} J \wedge \left[ \text{Tr} (F^{(2)})^2 - \frac{1}{2} \text{Tr} (R^2) \right],$$

where $J$ is the Kähler form on CY with $h_{1,1} = 1$. This can be seen from ten-dimensional Green-Schwarz term $\int_{M_{10}} B_2 \wedge X_8$ or eleven-dimensional Chern-Simmons term $\int_{M_{11}} C_3 \wedge G_4 \wedge G_4$. The last term $f_{M5}$ represents the contribution from the M5-brane position moduli $Y \sim T x_{11} M_5$ in the orbifold interval that is given by $f_{M5} = \alpha Y^2 / T \sim \alpha T$ and $\alpha \sim \int_{CY} J \wedge *_6 J$. Hence, a gaugino condensation may generate a moduli-mixing superpotential,

$$W_{GC} \sim \exp[- (8\pi^2 / N) f_{\text{strong}}] \text{ for } SU(N).$$

On the other hand, type II models such as intersecting D-brane models or magnetized D-brane models have gauge couplings \[14\] similar to the above heterotic model. For example, in the supersymmetric type IIB magnetized D-brane model on $T^6/(Z_2 \times Z_2)$ orientifold\(^4\) with $h^\text{bulk}_{1,1} = 1$, the gauge kinetic functions are given as

$$f_{mD7} = |m_7| S + |w_7| T,$$

$$f_{mD9} = m_9 S - w_9 T \text{ for } O3/O7 \text{ system},$$

where the coefficients $m_p$, $w_p$ ($p = 7, 9$) $\in \mathbb{Z}$ originate in Abelian magnetic flux contributions $F$ from the world volume and the Wess-Zumino term, and are given by $m_7 = \int_{mD7} F \wedge F$, $m_9 = \int_{mD9} F \wedge F \wedge F$ and $w_9 = \int_{mD9} *_6 J^\text{bulk} \wedge F$ up to a numerical factor. The $w_5$ corresponds to the winding number on a wrapping 4-cycle and magnetic flux contribution, while $w_7$ corresponds to the winding number of D7-brane on the 4-cycle. The signs of $m_9$ and $w_9$ depend on the magnetic fluxes and SUSY conditions. Notice that the Abelian gauge magnetic flux $F$ is quantized on a compact 2-cycle $C_2$ as $\int_{C_2} F \in \mathbb{Z}$ in this case. For example in Ref. [22], one can find negative $m_9$ and $w_9$. In addition, T-duality action can exchange winding number for magnetic number, but the result is similar, that is

$$f_{mD9} = W_9 S - M_9 T \text{ for } O9/O5 \text{ system},$$

\(^3\)The symbol $\tilde{}$ represents a simple direct product or including warp factor such as

$$ds^2_{10} = \Delta(y)g_{\mu\nu}dx^\mu dx^\nu + \Delta^{-1}(y)g_{mn}dy^mdy^n.$$ 

\(^4\)Actually, the $T^6/(Z_2 \times Z_2)$ orbifold, in which the twist action is given by

$$\theta : (z^1, z^2, z^3) \rightarrow (-z^1, -z^2, z^3),$$

$$\omega : (z^1, z^2, z^3) \rightarrow (z^1, -z^2, -z^3),$$

has three Kähler forms in the bulk, that is $h^\text{bulk}_{1,1} = 3$. We here identify the indices of those cycles for simplicity.
where \( W_9, M_9 \in \mathbb{Z} \) are, respectively, the winding number on the 6-cycle, and the winding number on the 2-cycle and magnetic flux contributions given by 
\[
M_9 = \int_{mD9} J_{bulk} \wedge F \wedge F.
\]
Again we have neglected numerical factors.

The gauge coupling on the magnetized brane is written by
\[
\frac{1}{g_{mD9}^2} = |m_9 \text{Re} S - w_9 \text{Re} T|.
\]

The magnetic fluxes can contribute to RR tadpole condition of four-form potential and eight-form potential \cite{22, 23}. In this paper, we will treat \( w_p \) and \( m_p \) as free parameters.

In type IIA intersecting D-brane models, which are T-duals of the above IIB string models, the above expressions of Kähler moduli change for complex structure moduli. However, since there are three- and even-form fluxes, all geometric moduli can be frozen in these type IIA models at low energy as a supersymmetric AdS vacuum.

In orbifold string theory, moduli in twisted sectors, the so-called twisted moduli \( M \) can exist \cite{24}. These modes can contribute to the gauge kinetic function on D-branes near orbifold fixed points,
\[
f_{Dp} = (S \text{ or } T) + \sigma M,
\]
where \( \sigma \) is \( O(0.1)-O(1) \) parameter depending on gauge and orbifold group. These twisted moduli may be stabilized easily due to their Kähler potential \cite{25}, but make little contribution to the gauge coupling because the moduli are related to collapsed cycles of orbifold. Then we may naturally have \( \langle M \rangle \ll 1 \), and neglect contributions of those moduli.

We comment that if a contribution of \( T \) in the gauge coupling \( f = xS \pm yT \) is required to be small compared with \( S \), one needs to tune the ratio \( y/x \) for a few percent. For example in heterotic M-theory, we need to choose a moderate CY model or to tune positions of M5-branes or the magnetic flux and winding number of D-branes.

## 3 Effective 4D \( N = 1 \) SUGRA

Motivated by the moduli-mixed gauge couplings explained in the previous section, we consider a racetrack model with double gaugino condensations at the hidden sector where the gauge couplings depend on two light moduli\(^5\) represented by \( S \) and \( T \). We analyze a scalar potential of the effective 4D \( N = 1 \) SUGRA characterized by the Kähler and the superpotential,
\[
K = -n_S \ln(S + \bar{S}) - n_T \ln(T + \bar{T}),
\]
\[
W = W_1 + W_2,
\]
where \( n_S, n_T > 0 \) and
\[
W_1 = Ae^{-a(S-aT)},
\]
\[
W_2 = -Be^{-b(S+\beta T)}.
\]

\(^5\)If, e.g. there exists three-form flux as in the KKLT model, one of moduli, say \( S \), can be stabilized around the string scale. In this case the modulus field \( S \) should be replaced by the vacuum value \( \langle S \rangle \) in the effective theory, and the analysis is quite different from the one in this paper. Such scenario has been closely studied in Ref. \cite{16}.
We parameterize the effective theory by \( n_S, n_T, a, b, \alpha, \beta, A \) and \( B \) which depend on the hidden gauge groups and the string models explained in the previous section. The scalar potential of this system is written in the standard \( N = 1 \) SUGRA form as

\[
V = e^G (G^{IJ} G_I G_J - 3) = K_{IJ} F^I F^J - 3 e^K |W|^2,
\]

where \( I, J = (S, T) \), \( G = K + \ln |W|^2 \), \( F^I = -e^{K/2} K^{IJ} D_J W \) and

\[
\begin{align*}
D_S W & \equiv G_{S} W = (b - a) A e^{-a(S - aT)} + (K_S - b) W, \\
D_T W & \equiv G_{T} W = (a \alpha + b \beta) A e^{-a(S - aT)} + (K_T - b \beta) W.
\end{align*}
\]

The complex scalar fields \( S \) and \( T \) are written as

\[
S = s + i \sigma_s, \quad T = t + i \sigma_t,
\]

respectively by using four real scalars \( s, t, \sigma_s \) and \( \sigma_t \). The SUSY conditions \( D_S W = D_T W = 0 \) can have a runaway solution with \(|s| \) or \(|t| \rightarrow \infty \) and \( \langle W \rangle = 0 \). (See Appendix A.) This case is outside our interests and we will find a nontrivial vacuum with \( \langle W \rangle \neq 0 \) in the following analysis.

### 3.1 Imaginary directions

First we show that \( \sigma_s \) and \( \sigma_t \) are decoupled from \( s \) and \( t \) by their stationary conditions. The scalar potential \( (1) \) can be written as

\[
V = e^K \left\{ \left( \frac{1}{K_{SS}} (K_S - a) + \frac{1}{K_{TT}} (K_T + a \alpha) - 3 \right) |W_1|^2 \\
\quad + \left( \frac{1}{K_{SS}} (K_S - b) + \frac{1}{K_{TT}} (K_T - b \beta) - 3 \right) |W_2|^2 \\
\quad + r(s, t) (\bar{W}_1 W_2 + \text{h.c.}) \right\},
\]

where

\[
r(s, t) = \frac{1}{K_{SS}} (K_S - a) (K_S - b) + \frac{1}{K_{TT}} (K_T + a \alpha) (K_T - b \beta) - 3,
\]

and we easily find that the following term

\[
\bar{W}_1 W_2 + \text{h.c.} = 2 \text{sign}(AB) |W_1||W_2| \cos [(b - a) \sigma_s + (a \alpha + b \beta) \sigma_t],
\]

is only the source of the potential for \( \sigma_s \) and \( \sigma_t \). The stationary conditions \( \partial_{\sigma_s} V = \partial_{\sigma_t} V = 0 \) fix a linear combination of \( \sigma_s \) and \( \sigma_t \),

\[
(b - a) \sigma_s + (a \alpha + b \beta) \sigma_t = n \pi \quad (n: \text{integer}),
\]

while another combination remains as a flat direction. Along this stationary direction, we also find \( \partial_i \partial_{\sigma_j} V = 0 \) where \( i, j = (s, t) \). That means that there is no mixing between \((s, t)\) and \((\sigma_s, \sigma_t)\). Then we can analyze the stability of \( s, t \) separately from \( \sigma_s, \sigma_t \) in the
following sections. Note that the even or odd \( n \) corresponds to the (local) minimum or maximum of the potential, depending on \( \text{sign}(AB r(s, t)) \), i.e. depending on the vacuum values of \( s \) and \( t \) shown later.

The remaining flat direction is expected to receive typically a mass of \( \mathcal{O}(m_{3/2}/4\pi) \) at the one-loop level with some SUSY breaking. We will not discuss about this flat direction in this paper, although it can play a role in phenomenological and cosmological arguments (see, e.g. Ref. [26] and references therein).

### 3.2 Real directions

Next we show the global structure of the scalar potential \( V(s, t) \) where \( \sigma_s \) and \( \sigma_t \) are already fixed by Eq. (2). The \( F^S \)-flat direction,

\[
D_S W = (K_S - a)Ae^{-a(S-aT)} - (K_S - b)Be^{-b(S+bT)} = 0,
\]

is determined by the curve

\[
t = \frac{1}{a\alpha + b\beta} \left( \ln \frac{B(2bs + ns)}{A(2as + ns)} - (b-a)s \right),
\]

with Eq. (2), which has asymptotic lines

\[
\begin{aligned}
t &\to \frac{-b-a}{a\alpha + b\beta} s + \frac{1}{a\alpha + b\beta} \ln \frac{bB}{aA}, & (s \to \pm \infty, \ bB/aA > 0), \\
s &\to -\frac{ns}{2a}, & (t \to \pm \infty).
\end{aligned}
\]

Similarly, the \( F^T \)-flat direction,

\[
D_T W = (K_T + a\alpha)Ae^{-a(S-aT)} - (K_T - b\beta)Be^{-b(S+b\beta)} = 0,
\]

draws the curve

\[
s = \frac{1}{b-a} \left( \ln \frac{B(-2b\beta t - n_T)}{A(2a\alpha t - n_T)} - (a\alpha + b\beta)t \right),
\]

with asymptotic lines

\[
\begin{aligned}
t &\to -\frac{n_T}{2a\alpha}, & (s \to \pm \infty), \\
s &\to -\frac{a\alpha + b\beta}{b-a} t + \frac{1}{b-a} \ln \frac{bB}{aA} + \frac{1}{b-a} \ln \frac{-\beta}{\alpha}, & (t \to \pm \infty, \ b\beta B/a\alpha A < 0).
\end{aligned}
\]

A certain linear combination of \( D_S W \) and \( D_T W \) is found to be

\[
(a\alpha + b\beta)D_S W - (b-a)D_T W = h(s, t) W,
\]

where

\[
h(s, t) \equiv (a\alpha + b\beta)K_S + (a - b)K_T - ab(\alpha + \beta).
\]
The solution of \( \langle W \rangle = 0 \) corresponds to only the runaway vacuum with \( |s| \) or \( |t| \to \infty \). (See Appendix A) We are not interested in such a runaway solution \( \langle W \rangle = 0 \), and then for \( b \neq a \) and \( a\alpha + b\beta \neq 0 \), one of the SUSY conditions \( D_S W = 0 \) and \( D_T W = 0 \) can be replaced by \( h(s, t) = 0 \) resulting in

\[
t = \frac{n_T (b - a)s}{2ab(\alpha + \beta)s + n_S (a\alpha + b\beta)}.
\]

This curve always passes through the origin of \((s,t)\)-plane, and has asymptotic lines

\[
\begin{align*}
t &\to t_\infty \equiv \frac{n_T (b - a)}{2ab(\alpha + \beta)}, \\
(s &\to \pm \infty), \\
s &\to s_\infty \equiv \frac{-n_S (a\alpha + b\beta)}{2ab(\alpha + \beta)}, \\
(t &\to \pm \infty).
\end{align*}
\]

For \( \sigma_s \) and \( \sigma_t \) satisfying Eq. (2), we have relations,

\[
\begin{align*}
\frac{W_2}{W_1} &= -\frac{a}{b} e^{-\Phi_\perp (s,t)}, \\
\frac{\partial_s W_2}{\partial s W_1} &= -e^{-\Phi_\perp (s,t)}, \\
\frac{\partial_T W_2}{\partial T W_1} &= \frac{\beta}{\alpha} e^{-\Phi_\perp (s,t)}, \\
&\ldots,
\end{align*}
\]

where

\[
\Phi_\perp (s,t) = (b - a)s + (a\alpha + b\beta)t - \ln \frac{bB}{aA}.
\]

Here the ellipsis denotes similar relations for higher derivatives. That implies that our system is almost described by the superpotential \( W \simeq W_1 (W \simeq W_2) \) in the region \( \Phi_\perp (s,t) \gg 1 \) \( (\Phi_\perp (s,t) \ll -1) \) without large hierarchies between parameters,

\[
a/b, \quad \alpha/\beta \sim O(1).
\]

In the band \( -1 \lesssim \Phi_\perp (s,t) \lesssim 1 \), the contributions from \( W_1 \) and \( W_2 \) are comparable, and actually this area is spreading along the asymptotic line of \( D_S W = 0 \) shown in Eq. (3), which corresponds to \( \Phi_\perp (s,t) = 0 \).

In Fig. 1 we show the curves of \( D_S W = 0 \) and \( h(s, t) = 0 \) together with the asymptotic lines (3) and (4) respectively in the \((s,t)\)-plane with Eq. (2), for a parameter choice,

\[
a = \frac{8\pi^2}{N_1}, \quad b = \frac{8\pi^2}{N_2}, \quad \alpha = \frac{N_1}{8\pi^2 n_1}, \quad \beta = \frac{N_2}{8\pi^2 n_2},
\]

and

\[
N_1 = 9, \quad N_2 = 8, \quad n_1 = 1/40, \quad n_2 = 1/8, \\
A = 1.00 \times 10^{-6}, \quad B = 5.00 \times 10^{-6}, \quad n_S = 1, \quad n_T = 3.
\]

The asymptotic values tell us that, when both of \( t_\infty \) and \( s_\infty \) are negative, there is no SUSY point within the physical region \( s, t \geq 0 \) of the moduli space. In this case the scalar potential just has a runaway structure or is unbounded from below, without a
nontrivial stationary point for \( s, t \geq 0 \). When \( t_{\infty} > 0 \) and/or \( s_{\infty} > 0 \), we have a possibility of SUSY stationary point in the region \( s, t > 0 \). Thus, in the following, we consider such case, in which the SUSY stationary point \((s_{\text{SUSY}}, t_{\text{SUSY}})\) can exist within the physical domain \( s_{\text{SUSY}}, t_{\text{SUSY}} > 0 \). Moreover, in the region

\[
|s_{\text{SUSY}}| \gg \left| \frac{n_s(a \alpha + b \beta)}{2a} \right|, \left| \frac{n_s(a \alpha + b \beta)}{2ab(\alpha + \beta)} \right|,
\]

the SUSY point is approximately located at the cross-point of two asymptotic lines (3) and (4), that is

\[
s_{\text{SUSY}} \simeq \frac{1}{b-a} \ln \frac{bB}{aA} - \frac{a \alpha + b \beta}{b-a} t_{\text{SUSY}}, \quad t_{\text{SUSY}} \simeq \frac{n_T(b - a)}{2ab(\alpha + \beta)}.
\]

In addition, if we assume

\[
|s_{\text{SUSY}}| \gg \left| \frac{a \alpha + b \beta}{b-a} \right||t_{\text{SUSY}}|,
\]

we obtain a simple form

\[
s_{\text{SUSY}} \simeq \frac{1}{b-a} \ln \frac{bB}{aA}, \quad t_{\text{SUSY}} \simeq \frac{n_T(b - a)}{2ab(\alpha + \beta)}.
\] (8)

The vacuum energy is negative at this point,

\[
V_{\text{SUSY}} = -3 (m_{3/2}^{\text{SUSY}})^2 < 0,
\]
where
\[ m_3^{\text{SUSY}} \sim \frac{b-a}{b} \left( \frac{aA}{bB} \right)^{\frac{\alpha}{\beta}} \frac{A}{(2s_{\text{SUSY}})^{n_s/2}(2t_{\text{SUSY}})^{n_t/2}}. \] (9)

Note that the gravitino mass is nonvanishing at the AdS SUSY point.

In the following we consider the case that the parameters \( A, B, a, b, \alpha \) and \( \beta \) satisfy\(^6\) \( s_{\text{SUSY}} > 0 \) and \( t_{\text{SUSY}} > 0 \) in Eq. (8). For small values of \( N_1 \) and \( N_2 \), the parameters \( a = 8\pi^2/N_1 \) and \( b = 8\pi^2/N_2 \) are larger than unity, and then \( s_{\text{SUSY}} \) and \( t_{\text{SUSY}} \) tend to be smaller than unity in the unit \( M_{\text{Pl}} = 1 \). Our tree level effective SUGRA analysis is not reliable in this region. For the parameters satisfying, e.g.
\[ a, b \gg 1, \quad b - a \sim \mathcal{O}(1), \quad B/A > a/b, \quad \alpha, \beta \sim \mathcal{O}(1/ab), \] (10)
the approximation (8) is valid, and it is possible with some fine-tuning to realize \( s_{\text{SUSY}}, t_{\text{SUSY}} > 1 \).

4 SUSY saddle point

4.1 General mass formula at SUSY point

In order to investigate that the SUSY point (8) is stable or not, we examine the mass matrix of \( s \) and \( t \) at this point. It is useful to show general results for this matrix, and then we first summarize several formulae related to the mass matrix at the SUSY point.

In general, the first derivative of the scalar potential (1) is
\[ \partial_I V = e^G \left[ G_I \left( G_K G_L G^{K\bar{L}} - 2 \right) + G_{\bar{L}} \left( G_{\bar{K}} G^{K\bar{L}} + G_K \left( \partial_I G^{K\bar{L}} \right) \right) \right]. \]

At the SUSY point where \( G_I = W^{-1} D_I W = 0 \), the second derivatives are shown to be
\[ V_{IJ} = -e^G G_{IJ}, \quad V_{IJ} = -e^G G_{IJ}, \quad V_{IJ} = e^G \left[ -2G_{IJ} + G_{IK} G_{J\bar{L}} G^{K\bar{L}} \right], \] (11)
and Hessian matrices of real parts of complex fields \( I, J \) are given by
\[ V_{IJ} = V_{II} + V_{IJ} + V_{JJ}, \] (12)
where we denoted real parts of \( I, J \) as \( I_R, J_R \). When the Kähler potential is a function of only the real part of complex fields, \( \bar{K} = K(\phi^I + \bar{\phi}^I) \), and the derivatives of superpotential satisfy \( W^{-1} W_I, W^{-1} W_{IJ} \in \mathbb{R} \), we generically have
\[ G_{IJ} = K_{IJ} - \frac{W_I W_J}{W^2} + \frac{W_{IJ}}{W} = \bar{G}_{IJ} \in \mathbb{R}. \] (13)
\(^6\)For the parameters satisfying \( s_{\text{SUSY}} < 0 \) and/or \( t_{\text{SUSY}} < 0 \), the potential has trivial structure in the domain \( s, t \geq 0 \). In this case the moduli can not be stabilized at finite values.
First, for the simplest example, we consider a scalar potential of a single complex scalar field $X$. In the stationary point of a generic potential, we can express mass terms as

$$V^\text{mass} = V_{XX}|x|^2 + \frac{1}{2}(V_{XX}x^2 + V_{X\bar{X}}\bar{x}^2)$$

where $x = X - X_0$ and $\partial_X V|_{X=X_0} = 0$. In the last line we have diagonalized the mass matrix, and find that the following condition

$$V_{XX} > |V_{XX}|,$$

is necessary for a point $X = X_0$ to be a minimum of this potential. In terms of $G_{IJ}$, $G_{IJ}$ and $G_{I\bar{J}}$, the condition (14) becomes

$$|G_{XX}| > 2G_{X\bar{X}}.$$

Next we consider the case with two moduli fields such as $S$ and $T$. We further assume that the Kähler potential is in a separable form,

$$K = K^{(S)}(S + \bar{S}) + K^{(T)}(T + \bar{T}),$$

so that the Kähler metric is diagonal, $G_{IJ} = K_{IJ} = K_{I\bar{J}}\delta_{IJ}$. At the SUSY point, the mass matrix of the real part fields $s = \text{Re} S$ and $t = \text{Re} T$ in the canonical base is expressed by

$$\mathcal{M}^2 \equiv \begin{pmatrix}
\frac{1}{K_{SS}}V_{ss} & \frac{1}{\sqrt{K_{SS}K_{TT}}}V_{st} \\
\frac{1}{\sqrt{K_{SS}K_{TT}}}V_{st} & \frac{1}{K_{TT}}V_{tt}
\end{pmatrix},$$

where each component is given by Eq. (12) under the reality condition (13), and then we find

$$\text{tr} \mathcal{M}^2 = \frac{2e^G}{G_{SS}G_{TT}}\left(G_{TT}G_{SS}^2 + G_{SS}G_{TT}^2 + 2G_{ST}^2 - 4G_{SS}G_{T\bar{T}} - G_{SS}G_{TT} - G_{T\bar{T}}G_{SS}\right),$$

$$\det \mathcal{M}^2 = \frac{4e^{2G}}{G_{SS}G_{TT}^2}\left\{G_{ST}^2 - (G_{SS} - 2G_{SS})(G_{TT} - 2G_{T\bar{T}})\right\} \times \left\{G_{ST}^2 - (G_{SS} + G_{SS})(G_{TT} + G_{T\bar{T}})\right\}.$$

All stable SUSY minima in this case should satisfy $\text{tr} \mathcal{M}^2 > 0$ and $\det \mathcal{M}^2 > 0$. If either $|G_{SS}| \gg |G_{SS}|$ or $|G_{TT}| \gg |G_{TT}|$, we have $\text{tr} \mathcal{M}^2 > 0$. Furthermore, if both $|G_{SS}| \gg |G_{SS}|$ and $|G_{TT}| \gg |G_{T\bar{T}}|$, we have $\det \mathcal{M}^2 > 0$. However, we obtain $\det \mathcal{M}^2 < 0$, e.g. if $|G_{SS}| \gg |G_{SS}|$, $|G_{TT}| \sim |G_{T\bar{T}}|$ and $|G_{SS}G_{TT}| \gg |G_{ST}^2|$. 

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4.2 Stability of moduli at SUSY point

Now we analyze the stability of SUSY stationary point in our model. The mass matrix of the fluctuations of moduli \((s,t)\) around the SUSY point is given by Eq. (15). For the parameters satisfying Eq. (10), we can estimate the mass matrix elements by using the approximation (8) and find

\[
\text{tr}\, \mathcal{M}^2 \simeq \frac{1}{4} |\det \mathcal{M}| \left( m_{3/2}^{SUSY} \right)^{-2} > 0,
\]

\[
|\det \mathcal{M}| \simeq -\frac{128}{n_S} \left( \frac{\ln bB}{aA} \right)^4 \frac{(ab)^2}{(b-a)^4} \left( m_{3/2}^{SUSY} \right)^4 < 0.
\]

Then it is the case that the SUSY stationary point is a saddle point and unstable. Actually in the next section we will find another stationary point near the SUSY point, which is a SUSY breaking (AdS) local minimum. The point here is \(|G_{SS}| \gg |G_{SS}|\) and that the superpotential contributions in \(G_{TT}\) are cancelled by \(G_{ST}^2\) terms in \(|\det \mathcal{M}|\) due to the assumption \(|b-a| \ll |a|, |b|\) in Eq. (10). That leads to \(|\det \mathcal{M}| < 0\).

Finally we comment on a trench of the scalar potential along \(F^S- (F^T-)\) flat direction in the \((s,t)\) moduli space. For a realistic moduli value, \(as, bs \gg 1\), a global SUSY part in the scalar potential \(|W_1/W|^2\) is dominated for this racetrack model, and it is important whether \(\alpha, \beta\) is larger or smaller than 1. For \(|a|, |b| \gg 1 > |\alpha|, |\beta|\) (which is satisfied by Eq. (10)), the inequality \(|W_S| \gg |W_T|\) is satisfied and the curve \(D_S W \approx 0\) determines the structure of the local minimum in the following sense. Along this curve, we obtain \(\Phi_\perp (s,t) \approx 0\), where \(\Phi_\perp\) is defined in Eq. (9). Actually this \(\Phi_\perp\)-direction (approximately \(s\)-direction for \(|a|, |b| \gg 1 > |\alpha|, |\beta|\)) is stabilized with a relatively large mass (with canonically normalized kinetic term),

\[
m_\perp^2 \sim 2 \left| \text{tr}\, \mathcal{M}^2 \right| \gg |m|^2,
\]

due to Eq. (10), where \(m_\parallel^2\) is the mass of the fluctuation \(\Phi_\parallel\) along \(D_S W \approx 0\) perpendicular to \(\Phi_\perp\)-direction satisfying

\[
|m_\parallel^2| \sim \left| \frac{\det \mathcal{M}^2}{\text{tr}\, \mathcal{M}^2} \right| \approx 4 \left( m_{3/2}^{SUSY} \right)^2,
\]

and \((m_\perp^2, m_\parallel^2)\) are the eigenvalues of the mass matrix \(\mathcal{M}^2\). The hierarchy in Eq. (16) illustrates that the potential of our model has a sharp and deep trench along \(D_S W \approx 0\), perpendicular to this \(\Phi_\perp\)-direction. (For \(|a|, |b| \gg |\alpha|, |\beta| > 1\) or \(a|a|, |b| \gg 1 > |a|, |b|\), the curve \(D_T W \approx 0\) determines the local minimum, similarly. However the former hardly satisfies \(s_{SUSY}, t_{SUSY} > 1\). The latter is just the replacement of \(S\) and \(T\) essentially.) Note that along the trench, \(|G_{IJ}| > 2G_{IJ}\) holds for \(i, j\). Then the saddle point condition, \(|\det \mathcal{M}| < 0\), can be simplified as

\[
2 |\det G| < (3 + \text{sign}(G_{IJ}) \text{sign}(\det G)) \left( |G_{SS}| G_{TT} + G_{TT} |G_{SS}| \right),
\]

where \(\det G = G_{SS} G_{TT} - G_{ST}^2\), under the assumption that all \(G_{IJ}\) have the same sign.

The structure of the potential along \(D_S W = 0\) is shown in the \((s,t)\)-plane in Fig. 2 with the parameter choice of Eq. (7). In Fig. 2(a), the trench along \(D_S W = 0\) is shown clearly, and in (b), a region around the SUSY saddle point (as well as the SUSY breaking minimum derived in the next section) is magnified.
5 SUSY breaking local minimum

To find a true (SUSY breaking) local minimum, we consider the case $|a|, |b| \gg |a\alpha|, |b\beta|$ in which there is a sharp and deep trench along the curve determined by $D_SW = 0$. The transverse mode $\Phi_\perp(s, t)$ to this curve can be frozen out as mentioned above, and it is enough to analyze the structure of potential along this curve.

On the curve of $G_S = W^{-1}D_SW = 0$ with Eq. (2), we generically find

$$\partial_t V|_{D_SW=0}(t) = [\partial_t + \partial_t S(t) \partial_s V]_{D_SW=0}(t) = 2p(t) e^G_T|_{D_SW=0}(t),$$

where

$$p(t) = G_{TT} \left( G_T^2 + G_{TT} + \partial_t S(t) G_{ST} \right) + G_{TT}^2 G_T - 2 \right]_{D_SW=0}(t),$$

$$S(t) = s|_{D_SW=0}(t).$$

Then the stationary points other than the SUSY point $(G_T = 0)$ is determined by $p(t) = 0$. In the region with $|2aS(t)| \gg n_S$, the curve $D_SW = 0$ is approximated by the asymptotic line (3),

$$S(t) \simeq -\frac{a\alpha + b\beta}{b - a} t + \frac{1}{b - a} \ln \frac{bB}{aA},$$

along which $\Phi_\perp(s, t) \simeq 0$ in Eq. (3) and then $W_2/W_1 \simeq -a/b$ from Eq. (19). Due to this, the ratios $W_I/W$, $W_{IJ}/W$ are all $t$-independent constants,

$$W_S/W \simeq 0, \quad W_T/W \simeq \frac{\alpha + \beta}{b - a} ab,$$

$$W_{SS}/W \simeq -ab, \quad W_{TT}/W \simeq \frac{a\alpha^2 - b\beta^2}{b - a} ab, \quad W_{ST}/W \simeq -\frac{a\alpha + b\beta}{b - a} ab,$$

which appear in $G_I = K_I + W^{-1}W_I$ and $G_{IJ} = K_{IJ} + W_{IJ}/W - (W_I/W)(W_J/W)$. Then the function $p(t)$ defined above is found to be

$$p(t) \simeq n_T \left( \frac{t}{t_{SUSY}} \right)^2 - 2(n_T - 1) \left( \frac{t}{t_{SUSY}} \right) + n_T - 3,$$

where $t_{SUSY}$ is given in Eq. (8).

The SUSY breaking stationary point is determined by $p(t_{SB}) = 0$ that is easily solved as

$$t_{SB} \simeq t_{SUSY}(1 + \delta_{SB}^t),$$

$$s_{SB} \simeq S(t_{SB}) = s_{SUSY}(1 + \delta_{SB}^s),$$

where $(s_{SUSY}, t_{SUSY})$ is shown in Eq. (8), and

$$\delta_{SB}^t = \sqrt{n_T + 1} - 1,$$

$$\delta_{SB}^s = -\frac{a\alpha + b\beta}{\ln(bB/aA) \delta_{SB}^t}.$$

Note that the other solution $\delta_{SB}^t = -((\sqrt{n_T + 1})/n_T$ corresponds to $t_{SB} \leq 0$ for $n_T \leq 3$ and $0 < t_{SB} \leq t_{SUSY}$ for $n_T > 3$. We will focus on the solution (20) which yields $t_{SUSY} < t_{SB}$, although this additional stationary point may reside in meaningful region for the latter case if $n_T > 3$ is possible.
Figure 2: The logarithm of scalar potential, $\log_{10} (V - (1 + \epsilon)V_{SB})$, plotted in $(s', t')$-plane where $s' = s \cos \vartheta - t \sin \vartheta$, $t' = s \sin \vartheta + t \cos \vartheta$, $\tan \vartheta = -(a\alpha + b\beta)/(b - a)$ and $\epsilon = 0.01$. The parameters are chosen as in Eq. (7). The sharp and deep trench along $D_{SW} = 0$ curve is shown in (a). The SUSY saddle point and the SUSY breaking local minimum is shown in (b).

By noting that

$$G_T|_{D_{SW}=0}(t) \simeq \frac{n_T}{2} \left( \frac{1}{t_{SUSY}} - \frac{1}{t} \right),$$

we can easily examine that this stationary point is a local minimum of the scalar potential, due to the fact that $e^G > 0$, $G_T|_{D_{SW}=0}(t_{SB}) \simeq n_T \delta_{SB}^t/2t_{SB} > 0$ and $p(t_{SB} \pm \epsilon) \gtrsim 0$ in Eq. (17). For the parameters satisfying Eq. (10), we find $\delta_{SB} \lesssim 1$ and $\delta_{SB}^t \ll 1$, and then $t_{SB} \sim t_{SUSY} > 1$, $s_{SB} \simeq s_{SUSY} > 1$.

At the SUSY breaking local minimum, the gravitino mass $m_{3/2}^2 = e^G$ and the order parameter $F_T = -K^{TT}G_T m_{3/2}$ are estimated respectively as

$$m_{3/2} \simeq \frac{e^{-a(\delta_{SB} - a\delta_{SB}^t)}}{(1 + \delta_{SB}^s)^{n_S/2}(1 + \delta_{SB}^t/n_T)^{m_{3/2}}};$$

$$F_T \frac{T + T}{T + T} \simeq -\delta_{SB} m_{3/2};$$

where $m_{3/2}^{SUSY}$ is given in Eq. (9) which can be a TeV scale by tuning parameters, and Eq. (21) has been applied. Note that $F_S$ vanishes within the approximation along $D_{SW} = 0$ asymptotic line ($|2as| > n_S$), and it can receive a nonzero contribution

$$\frac{F_S}{S + S} \simeq \frac{n_S}{as} \frac{F_T}{T + T}.$$

These order parameters generate the vacuum energy

$$V_{SB} = n_S \left| \frac{F_S}{S + S} \right|^2 + n_T \left| \frac{F_T}{T + T} \right|^2 - 3m_{3/2}^2.$$
Table 1: A numerical result of the vacuum values of moduli fields \((s, t)\), the vacuum energy \(V\) and the mass eigenvalues \((m_\perp, m_\parallel)\) evaluated at the SUSY saddle point (labelled by SUSY), at the SUSY breaking AdS local minimum (labelled by SB) and at the uplifted dS minimum (labelled by dS), for the parameter choice of Eq. (7) and the uplifting potential \(2\) with \((n_P, m_P) = (0, 2/3)\). The magnitudes of the SUSY breaking order parameters at the both AdS and dS minima are also shown. The larger (smaller) mass eigenvalue is represented by \(m_\perp (m_\parallel)\) at each point.

\[
\begin{array}{cccccc}
\text{s}_{\text{SUSY}} & \text{t}_{\text{SUSY}} & \text{V}_{\text{SUSY}} & (m_{\perp}^{\text{SUSY}} / m_{3/2}^{\text{SUSY}})^2 & (m_{\parallel}^{\text{SUSY}} / m_{3/2}^{\text{SUSY}})^2 \\
1.41 & 1.18 & -1.79 \times 10^{-26} & 1.11 \times 10^6 & 4.00 \\
\hline
\text{s}_{\text{SB}} & \text{t}_{\text{SB}} & \text{V}_{\text{SB}} & (m_{\perp}^{\text{SB}} / m_{3/2}^{\text{SB}})^2 & (m_{\parallel}^{\text{SB}} / m_{3/2}^{\text{SB}})^2 \\
1.36 & 1.57 & -1.82 \times 10^{-26} & 9.55 \times 10^5 & 7.09 \\
\hline
\text{s}_{\text{dS}} & \text{t}_{\text{dS}} & \text{V}_{\text{dS}} & (m_{\perp}^{\text{dS}} / m_{3/2}^{\text{dS}})^2 & (m_{\parallel}^{\text{dS}} / m_{3/2}^{\text{dS}})^2 \\
1.30 & 1.96 & +1.20 \times 10^{-34} & 8.16 \times 10^5 & 4.15 \times 10 \\
\hline
\text{AdS} & F^S/(S+\bar{S}) & F^T/(T+\bar{T}) & m_3/2 & m_{3/2}^{\text{SUSY}} \\
& -4.38 \times 10^{-15} & -2.77 \times 10^{-14} & 8.28 \times 10^{-14} & 7.72 \times 10^{-14} \\
\hline
\text{dS} & F^S/(S+\bar{S}) & F^T/(T+\bar{T}) & m_{3/2}^{\text{dS}} & D \\
& -1.34 \times 10^{-14} & -6.49 \times 10^{-14} & 9.78 \times 10^{-14} & 2.45 \times 10^{-25}
\end{array}
\]

\[
\sim \left( \frac{(\sqrt{nT+1} - 1)^2}{nT} - 3 \right) m_3^2 < -2m_{3/2}^2.
\]

Therefore we conclude that the local minimum of the moduli-mixing racetrack model \((|2as| \gg n_S)\) provides a SUSY breaking AdS background, which generates the moduli-dominated soft SUSY breaking terms in the visible sector.

A numerical result of the stabilized values of moduli and vacuum energy at the SUSY breaking local minimum (as well as at the SUSY saddle point shown previously and the uplifted local minimum shown later) is shown in Table 1 for the parameter choice of Eq. (7). The SUSY breaking order parameters at the minimum are also shown in the table. The large hierarchy \(|m_\perp / m_\parallel| \gg 1\) originates from the trench structure of the potential explained in the previous section.

### 6 Uplifting

To be phenomenologically viable, we need a Minkowski (or dS) vacuum. Unfortunately we could not realize such vacuum within our effective 4D SUGRA with two moduli. That is rather generic situation. That is because a SUSY point is a good candidate for the potential minimum, but that, in general, leads to the negative vacuum energy
\( V = -3(m_{3/2}^{\text{SUSY}})^2 \) unless \( W = 0 \) is realized at such SUSY point. Furthermore, if such SUSY point is unstable, we would find another SUSY breaking minimum, which has of course a negative vacuum energy \( V < -3(m_{3/2}^{\text{SUSY}})^2 \). Thus, the negative vacuum energy is rather generic problem within the SUGRA framework.

Here, following the original KKLT scenario [3], we consider a simple deformation of this system by introducing additional potential energy,

\[
V_{\text{lift}} = D e^{2K/3}(T + \bar{T})^n P (S + \bar{S})^m P,
\]

which may arise due to, e.g. the existence of anti D3-brane at the tip of warped throat of CY space in type IIB orientifold models, and breaks \( N = 1 \) SUSY explicitly. We fine-tune the value of \( D \) in such a way that the vacuum energy of the previous local minimum vanishes or becomes slightly positive,

\[
V_{\text{dS}} = V + V_{\text{lift}} \geq 0.
\]

For \( s, t \sim 1 \), we have \( D = \mathcal{O}(m_{3/2}^2) \). However, the mode perpendicular to the deep trench \( D_s W = 0 \) has a larger mass \( m_\perp^2 \gg m_{3/2}^2 \). Thus, the vacuum shifts little along this direction. On the other hand, the mode along \( D_s W = 0 \) has a mass comparable with the gravitino mass, i.e. \( m_\parallel^2 = \mathcal{O}(m_{3/2}^2) \). That suggests that the VEV of \( t_{SB} \) as well as \( s_{SB} \) might shift by a factor of \( \mathcal{O}(1) \). However, the third and higher derivatives of the scalar potential at the minimum without uplifting potential are quite large, i.e. \( \partial^3 V/\partial^2 V = \mathcal{O}(a) \). Then, the uplifting potential makes the VEV of \( t \) a small shift of \( \mathcal{O}(0.1) \).

For \( |2aS(t)| \gg n_s \) along \( D_s W = 0 \) asymptotic line [18], we can estimate the vacuum values of \( t \) and \( s \) with the uplifting [25],

\[
t_{\text{dS}} \simeq t_{SB}(1 + \delta_{t_{\text{dS}}}^t),
\]

\[
s_{\text{dS}} \simeq S(t_{\text{dS}}) = s_{SB}(1 + \delta_{s_{\text{dS}}}^s),
\]

at the linear order of \( \delta_{t_{\text{dS}}}^t \). By solving the stationary condition

\[
\partial_t \left( V\big|_{D_s W = 0}(t) + V_{\text{lift}}\big|_{D_s W = 0}(t)\right)\big|_{t = t_{\text{dS}}} = 0,
\]

with the fine-tuning condition

\[
D = \frac{V_{\text{dS}} - V|_{D_s W = 0}(t_{\text{dS})}}{(2t_{\text{dS}})^{n_P - 2n_T/3}(2S(t_{\text{dS}}))^{m_P - 2n_S/3}},
\]

we find, at the linear order of \( \delta_{t_{\text{dS}}}^t \),

\[
\delta_{t_{\text{dS}}}^t = \frac{2n_T/3 - n_P + (2n_S/3 - m_P)\partial_t S(t_{SB}) (t_{SB}/s_{SB})}{2n_T/3 - n_P + (2n_S/3 - m_P)\partial_t S(t_{SB}) (t_{SB}/s_{SB})^2 + n_T + 1 - \sqrt{n_T + 1}},
\]

\[
\delta_{s_{\text{dS}}}^s = -\frac{a\alpha + b\beta}{\ln(bB/aA)} \delta_{t_{\text{dS}}}^t.
\]

\(^8\)In our model, it is impossible to realize \( W = 0 \) at the SUSY point with finite moduli values. See Appendix A.

\(^9\)See for possibilities of uplifting in heterotic M-theory, e.g. Ref. [27] and references therein.
Figure 3: The behavior of the scalar potential $V(s' = s'_{dS}, t')$ including the uplifting potential (25) with $(n_P, m_P) = (0, 2/3)$ and the fine-tuned parameter $D = 2.45 \times 10^{-25}$ (solid curve), as well as $V(s' = s'_{SB}, t')$ without uplifting (dotted curve). The parameters are again chosen as in Eq. (7). The primed-notation $(s', t')$ is defined in the caption in Fig. 2 and $s'_{SB} = s_{SB} \cos \vartheta - t_{SB} \sin \vartheta$, $s'_{dS} = s_{dS} \cos \vartheta - t_{dS} \sin \vartheta$ for the same $\vartheta$. The stabilized values $(s_{SB}, t_{SB})$ and $(s_{dS}, t_{dS})$ are shown in Table 1. The dashed line represents the shift of the minimum.

where $\partial_t S(t) \simeq -(a\alpha + b\beta)/(b - a)$ from Eq. (18) and $V_{dS}/m_{3/2}^2 \ll 1$ has been adopted. For $S$-independent uplifting potential $m_P = 2n_S/3$, the solution is simplified as,

$$
\delta_{dS}^t = \frac{1}{1 + \frac{3(n_T + 1 - \sqrt{n_T + 1})}{2n_T - 3n_P}} (m_P = 2n_S/3),
$$

which is typically $O(0.1)$-$O(1)$ quantity.

At this $dS$ local minimum with the vacuum energy $V_{dS} \geq 0$, the gravitino mass $(m_{3/2}^{dS})^2 = e^G|_{D_SW=0}(t_{dS})$ and the order parameter $F^T$ are estimated respectively as

$$
m_{3/2}^{dS} \simeq e^{-a(\delta_{dS}^t - \alpha \delta_{dS})} (1 + \delta_{dS}^t)^{n_S/2} (1 + \delta_{dS}^t)^{n_T/2} m_{3/2},
$$

$$
\frac{F^T}{T + \bar{T}} \simeq - (\delta_{SB}^t + \delta_{dS}^t + \delta_{SB}^t \delta_{dS}^t) m_{3/2}^{dS},
$$

where $m_{3/2}$ is shown in Eq. (22), and Eq. (21) has been adopted. Again $F^S$ vanishes within the approximation along $D_SW = 0$ asymptotic line ($|2as| \gg n_S$), and it can receive a nonzero contribution (24) as before. Because typically $O(\delta_{SB}^t + \delta_{dS}^t + \delta_{SB}^t \delta_{dS}^t) \sim O(\delta_{SB}^t)$, we find that the SUSY breaking effect induced by the additional explicit breaking term (25) does not change the qualitative structure of SUSY breaking order parameters from the original ones at the local minimum. It can change the ratio $F^T/(T + \bar{T}) m_{3/2}$ by at most a factor depending on mainly $n_T$ and $n_P$. For example, when $n_S = 1$, $n_T = 3$, $n_P = 0$...
and \( m_P = 2/3 \), i.e. \( V_{\text{lift}} = \frac{D}{(T + \bar{T})^m} \), the ratio \( \frac{F^T}{(T + \bar{T})m_{3/2}} \) becomes three times as large as the one in Eq. (23) without uplifting. Then in this dS (Minkowski) minimum, the relation between order parameters again leads to the moduli-dominated SUSY breaking.

Generic form of uplifting potential is not clear still. However, the third and higher derivatives of our scalar potential are quite large as said above. Thus, when the second and higher derivatives of uplifting potential are not large, AdS SUSY breaking vacuum before uplifting does not shift drastically.

Table 1 shows a numerical result of the stabilized values of moduli and vacuum energy at the uplifted dS local minimum by the uplifting potential (25) with \((n_P, m_P) = (0, 2/3)\). The parameters are again chosen as in Eq. (7), and then we fine-tune \( D = 2.45 \times 10^{-25} \) in order to realize a dS (almost Minkowski) minimum. The SUSY breaking order parameters at the uplifted minimum are also shown in the table. The behavior of the total scalar potential including the uplifting term with the same parameter choice is shown in Fig. 3 as well as the original potential without uplifting. Note that the stabilized moduli values for more fine-tuned \( D \) so as to obtain the observed vacuum energy \( V_{\text{dS}} \sim 10^{-120} \) may not be so different from the ones shown in Table 1 for \( V_{\text{dS}} \sim 10^{-34} \).

7 SUSY phenomenology

In this section, we discuss SUSY phenomenology of our results. The size of modulus \( \mathcal{F} \)-term is of \( \mathcal{O}(m_{3/2}) \). Thus, in this type of models, the modulus mediated SUSY breaking is dominant. That is quite different from the original KKLT model and modified model with the single light modulus [5, 16], where anomaly mediation is comparable or rather dominant.

Indeed, dilaton/moduli mediated SUSY breaking has been studied [17]. When there are two and more moduli fields, it is phenomenologically useful to introduce goldstino angles to parameterize practically \( \mathcal{F} \)-terms of moduli without specifying their superpotential [18, 19, 20]. For example, in our model with two moduli \( S \) and \( T \), we introduce the goldstino angle \( \theta \) and parameterize moduli \( \mathcal{F} \)-terms,

\[
\frac{F^S}{S + S} = C \sqrt{\frac{3}{n_S}} m_{3/2} \sin \theta, \quad \frac{F^T}{T + T} = C \sqrt{\frac{3}{n_T}} m_{3/2} \cos \theta, \quad (27)
\]

up to CP phases. The \( \mathcal{F} \)-term scalar potential can be written as

\[
V_F = \frac{n_S |F^S|^2}{(S + S)^2} + \frac{n_T |F^T|^2}{(T + T)^2} - 3m_{3/2}^2, \quad (28)
\]

\[
= 3m_{3/2}^2(C^2 - 1). \quad (29)
\]

The vanishing \( V_F \) corresponds to \( C^2 = 1 \), and such parameter region has often been used. However, we would obtain \( V_F < 0 \) in generic case as discussed in the previous section. That is, we obtain \( C^2 < 1 \), but \( C = \mathcal{O}(1) \). For example, we find \( C \simeq -\sqrt{n_T/3}(\delta s_B + \delta d_S + \delta s_Bd_S) \) from the result in the previous section.

Our model for \( |a|, |b| \gg 1 > |\alpha|, |\beta| \) leads to \( \tan \theta = \mathcal{O}(1/\alpha s) < 1 \). That implies that when the gauge kinetic function of the visible sector is also obtained as \( f_v = S + \gamma T \)

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with $\gamma < 1$, the gaugino mass is smaller than the gravitino mass. Soft scalar masses are naturally of $O(m_{3/2})$ except a particular Kähler metric. On the other hand, when we take $|a|, |b| \gg |\alpha|, |\beta| > 1$, we have $\tan \theta > 1$. Again, the gaugino mass corresponding to the gauge kinetic function $f_v = S + \gamma T$ with $\gamma > 1$ becomes smaller than the gravitino mass. Thus, our model provides a concrete model for such moduli mediated SUSY breaking. In our model, whether $\tan \theta$ larger or smaller than unity, depends on along which direction the deep trench is $D_S W = 0$ or $D_T W = 0$.

At any rate, the gaugino masses seem to be suppressed compared with the gravitino mass $m_{3/2}$, when $\gamma$ corresponds to the parameter region similar to $\alpha, \beta$. As an illustrating example, we use the parameter corresponding to Table 1. In this case, the gaugino mass with the gauge kinetic function $f_v = S + \gamma T$ with $\gamma < 0.1$ is obtained as

$$M_{1/2} = 0.1 \times m_{3/2}. \tag{30}$$

On the other hand, the natural order of soft scalar masses is of $O(m_{3/2})$.

Such spectrum of superpartners would have several phenomenological implications. One of them would realize the focus point [28], which is important from the viewpoints of fine-tuning problem of the minimal supersymmetric standard model and dark matter physics. Such focus point can be realized when left- and right-handed squark masses and the up-sector Higgs soft mass are degenerate, and the gaugino mass as well as the $A$-term is smaller. Such SUSY spectrum can be realized in our model, when the Kähler metric of left- and right-handed quarks and up-sector Higgs fields are degenerate. We need further condition to suppress the $A$-term. The SUSY spectrum of our models have other several interesting aspects, which would be studied elsewhere.

8 Conclusions and discussions

We have investigated supersymmetric models with double gaugino condensations in the hidden sector (racetrack model), where the gauge couplings depend on two light moduli $S$ and $T$. We have analyzed this class of model within the framework of effective 4D $N = 1$ SUGRA, and have shown that there is no stable supersymmetric minimum with finite vacuum values of the moduli fields. The true local minimum of the scalar potential provides a SUSY breaking AdS background as well as reasonable vacuum values of the moduli fields, and generate moduli-dominated SUSY breaking soft terms for the visible fields. This structure of the soft terms may not be affected by the uplifting of the minimum by introducing, e.g. anti D3-branes, which is required in order to obtain phenomenologically viable Minkowski or dS vacuum. During the analysis, we have also derived some general formulae related to the mass matrix of moduli fields at the SUSY point, which would be useful in any case of this kind of analysis.

Although we have mainly analyzed the case with parameters satisfying Eq. (10), the resultant local structure of the scalar potential around the trench (within the physical domain $s, t > 0$) is not changed qualitatively even in the other cases, as far as at least one of the nonperturbative superpotential terms depends on both two moduli, e.g. like the one in the model of Ref. [29] based on heterotic M-theory with open membrane instanton effects, unless there are other effects. Then our result may imply that, due to the saddle
point structure of SUSY stationary point, the anomaly-mediated SUSY breaking scenario or the mixed modulus-anomaly mediated scenario where $m_{3/2}/F^{T,S} \sim 4\pi^2$ are difficult to realize within the moduli-mixing racetrack model in which we have two or more moduli at low energy, even with the KKLT-type uplifting mechanism \cite{5}, which could give smaller moduli-mediated contributions than the anomaly mediation for the single modulus case. Note that such small moduli-contribution is possible in Refs. \cite{5} due to the existence of the stable AdS SUSY minimum.

An additional $D$-term potential with pseudo-anomalous $U(1)$ symmetries would change the situation. Thus, it would be interesting to study the racetrack model with the $D$-term corresponding to the pseudo-anomalous $U(1)$ symmetry, where moduli fields transform non-linearly. However, that is beyond our scope of this paper.

If there exists, e.g. three-form flux in type IIB model as in the KKLT model, one of the moduli $S$ can be stabilized around the string scale. In this case $S$ should be replaced by the vacuum value $\langle S \rangle$ in the effective theory, and the phenomenological consequences are quite different from those in this paper. We have closely studied this possibility in Ref. \cite{16}.

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A Global SUSY vacuum

In this appendix we show that the global SUSY vacuum in our moduli-mixing racetrack model corresponds to a runaway solution of the stationary condition. The global SUSY vacuum should satisfy $W = 0$, $W_S = 0$ and $W_T = 0$ at the same time which yield relations between $s$ and $t$ respectively as

$$
t = -\frac{b-a}{a\alpha + b\beta} s + \frac{1}{a\alpha + b\beta} \ln \frac{B}{A},
$$

$$
t = -\frac{b-a}{a\alpha + b\beta} s + \frac{1}{a\alpha + b\beta} \left( \ln \frac{B}{A} + \ln \frac{b}{a} \right),
$$

$$
t = -\frac{b-a}{a\alpha + b\beta} s + \frac{1}{a\alpha + b\beta} \left( \ln \frac{B}{A} + \ln \frac{b}{a} + \ln \frac{-\beta}{\alpha} \right).$$

Only the possibility that these three equations can be identical is given by the parameter choice,

$$a = b, \quad \alpha = -\beta,$$

that is we obtain

$$W = (A - B)e^{-(b\alpha(S - \alpha T))}.$$
However in this case the vacuum with \( \langle W \rangle = 0 \) corresponds to \( a(s - \alpha t) \to \infty \), that is a runaway solution, and we can not obtain finite vacuum values of moduli. Note that the global SUSY vacuum with finite moduli values may be possible if we have a constant piece in our superpotential as discussed in, e.g. Ref. [30].

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