A Traditional Quadrilateral NURBS Surfaces Generation Method using MATLAB

JIANG Li a*

Science College of Hunan Agricultural University, Changsha, Hunan, 410128 China

Abstract

NURBS surface is defined as a bi-parameter condition, so you can construct a NURBS surface may be calculated surface points the two aspects of some parameters of the algorithm. Build the way surface grid method has higher accuracy than. Also, when the surface of the normal vector calculation pointed out that the surface point can be got. It is to alleviate the surface of the isometric construction. The results show that NURBS surface generation method is effective and strategy, and by adjusting the produce come quadrilateral surface is feasible to control some of the areas.

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Keywords: NURBS Surface; Generation Method; MATLAB

1. Introduction

Surface generation technology is very important surface sculpture, CAD/CAM and numerical control system. In literature [1], and on the basis of the research, said the surface parameters [2] proposed a way to create a multi-looped surface, this paper discusses the reference [3] express Coons-Gordon NURBS surface decoration, [4, 5] discuss carved on from interpolation point of view. This means that the surface generating surface point of the surface is the control points and its coefficient has. We are not going to discuss how to get control points and coefficient of some of the specific terms surface point. A lot of reference discussion.

2. Introduction to nurbs

Since it was put forward in 1980s, NURBS (NON-Uniform Rational B-Spline) has attained enough recognition and found its application in many areas. It can provide a uniform expression
for standard analytical curves, surfaces (conics, conicoids) and free curves, surfaces (B-Spline, NURBS). The definition of a NURBS curve is as follows [6]:

$$P(u) = \frac{\sum_{i=0}^{n} B_{i,k}(u)W_i}{\sum_{i=0}^{n} B_{i,k}(u)W_i} \tag{1}$$

Here $V_i$ are control points; $W_i$ are coefficients of these points; $B_{i,k}(u)$ is the base function of B-Spline. $B_{i,k}(u)$ can be defined as:

$$B_{i,0}(u) = \begin{cases} 1, & u_i \leq u \leq u_{i+1} \\ 0, & u < u_i \cup u > u_{i+1} \end{cases} \tag{2}$$

$$B_{i,k}(u) = \frac{u - u_i}{u_{i+k} - u_i} B_{i,k-1}(u) + \frac{u_{i+k+1} - u}{u_{i+k+1} - u_{i+1}} B_{i+1,k-1}(u), k \geq 1$$

Here $0/0 = 0$ means if the numerator and denominator are zero at the same time, the result is zero; $u$ is the parameter.

$U = [u_0, u_1, \cdots, u_m]$ is the node vector which is an incremental sequence. Usually the node vector is most like the following sequence.

$$\left[ 0, 0, 0, 0, u_{k+1}, \cdots, u_{m-k-1}, 1, 1, 1, 1 \right]$$

From [6], we can also get the definition of a NURBS surface:

$$P(u, \omega) = \frac{\sum_{i=0}^{n} \sum_{j=0}^{m} B_{i,k}(u)B_{j,l}(\omega)W_{i,j}V_{i,j}}{\sum_{i=0}^{n} \sum_{j=0}^{m} B_{i,k}(u)B_{j,l}(\omega)W_{i,j}} \tag{3}$$

Here $V_{i,j}$ are the control points; $W_{i,j}$ are their coefficients; and $B_{i,k}(u)$, $B_{j,l}(\omega)$ are the base function of B-Spline.

3. NURBS surface generation

From (1) and (3), if the control points and their coefficients are known, the values of $P(u)$ or $P(u, \omega)$ are only decided by the parameter $u$ or $u$ and $\omega$. There are many methods to calculate $P(u)$ or $P(u, \omega)$, de Boor algorithm[7] is the most popular one. If

$$Q(u) = \sum_{i=0}^{n} V_i B_{i,k}(u) \tag{4}$$

Here $V_i$ are control points; $B_{i,k}(u)$ is the base function of B-Spline.

Then, according to de Boor algorithm, $Q(u)$ also can be calculated by

$$Q(u) = C_i^{[k]}(u) \tag{5}$$

$C_i^{[k]}(u)$ is defined as

$$C_i^{[r]}(u) = \begin{cases} V_b, & r = 0, b = i - k, b = i - k + 1, \cdots i \\ \frac{u - u_b}{u_{b+k+1} - u_b} C_i^{[r-1]}(u) + \frac{u_{b+k+1} - u}{u_{b+k+1} - u_{b+1}} C_{i+1}^{[r-1]}(u) \end{cases} \tag{6}$$

For example, if $k = 3$, there will be

$$Q(u) = C_i^{[3]}(u) \tag{5}$$
\[ C_i^{[3]}(u) = \frac{u-u_i}{u_{i+1}-u_i}C_i^{[2]}(u) + \frac{u_{i+1}-u}{u_{i+2}-u_i}C_{i+1}^{[1]}(u) \]

\[ C_i^{[2]}(u) = \frac{u-u_{i-1}}{u_{i+1}-u_{i-1}}C_i^{[1]}(u) + \frac{u_{i+1}-u}{u_{i+2}-u_{i-1}}C_{i+1}^{[1]}(u) \]

\[ C_i^{[1]}(u) = \frac{u-u_{i-1}}{u_{i+2}-u_{i-1}}C_i^{[0]}(u) + \frac{u_{i+2}-u}{u_{i+3}-u_{i-1}}C_{i+1}^{[0]}(u) \]

Where \( C_i^{[0]} = V_i \), \( C_i^{[1]} = V_{i-1} \), \( C_i^{[2]} = V_{i-2} \), \( C_i^{[3]} = V_{i-3} \)

In compliance with above equations, we can calculate out \( P(u) \) and \( P(u, \omega) \).

(1) can be rewritten as:

\[ P(u) = \frac{\sum_{i=0}^{n} B_{i,k}(u)W_i}{\sum_{i=0}^{n} B_{i,k}(u)} = \frac{M(u)}{N(u)} \]  \( \text{(7)} \)

Here \( M(u) = \sum_{i=0}^{n} V_i W_i B_{i,k}(u) \), \( N(u) = \sum_{i=0}^{n} W_i B_{i,k}(u) \) and they can be calculated by de Boor algorithm, so \( P(u) \) can be calculate out.

(3) can be rewritten as:

\[ P(u, \omega) = \frac{\sum_{i=0}^{n} B_{i,k}(u) \sum_{j=0}^{m} B_{j,l}(\omega) W_{i,j}}{\sum_{i=0}^{n} B_{i,k}(u) \sum_{j=0}^{m} B_{j,l}(\omega) W_{i,j}} \]  \( \text{(8)} \)

If one of the two parameters (\( u \) and \( \omega \)) is fixed (for example, the parameter \( \omega \) is fixed as \( \omega_e (e = 0,1,2, \ldots) \)) and at the same time let

\[ F_i^* = \sum_{j=0}^{m} B_{j,l}(\omega) W_{i,j} \]  \( \text{(9)} \)

\[ W_i^* = \sum_{j=0}^{m} B_{j,l}(\omega_e) W_{i,j} \]  \( \text{(10)} \)

Then we get

\[ P(u, \omega_e) = \frac{\sum_{i=0}^{n} B_{i,k}(u) F_i^*}{\sum_{i=0}^{n} B_{i,k}(u) W_i^*} \]  \( \text{(11)} \)

Equation (11) shows that it represents a NURBS curve, just as (1) does. Here this curve is called as a \( u \) Direction NURBS Curve. On the contrary, if the parameter \( u \) is fixed, we can get a \( \omega \) direction NURBS curve. So from the above, if we set a series of values (from 0 to 1) of one of the two parameters in terms of a specific algorithm, say \( \omega \), we can get a series of \( u \) direction NURBS curves. These curves can compose the whole surface (see Fig. 1). Then for each of these curves, if
we fix a series of values of the other parameter, say the parameter \( u \), in terms of a specific algorithm, we can get the values of a series of points which compose the curve. In this paper, we call these points as Surface Points. For each of these points, there are two NURBS curves travel through it. One is the so-called \( u \) direction curve, and the other is the \( \omega \) direction curve.

Fig. 1 shows how to generate a NURBS surface and the Equidistant Surface.

![Figure. 1 Generate a Surface](image)

According the method above given, a NURBS surface is generated with the help of MATLAB, see Fig. 2.

4. Non-quadrilateral nurbs surface genenration

In order to get the come quadrilateral NURBS surface, the researchers put forward some methods. Usually, this method is effective, but most of them are complicated and difficult to use. We know the shape of NURBS surface depends on the control points. So we can adjust the shape of the surface adjustment control points, in order to get a comet quadrilateral NURBS surface. Figure 2 shows, four edges are on the same plane is a horizontal plane. Make quad faces on the same plane, control point matrix can be adjusted. If let in the first row control points, the end of the row, the first column, the last column of the matrix of the key point in the same plane, can cause the quad faces is in the same plane. At the same time, through the adjustment, the control points and shy; Quadrilateral NURBS surface can produce.

If limit the projection of the control point matrix in a triangle area, a triangle NURBS surface can be obtained, just as Fig. 4 and Fig. 5 show.

![Figure 2 NURBS surface](image)

![Figure 3 the projection of figure 2](image)

![Figure 4 a Triangle NURBS surface](image)

![Figure 5 the projection of Figure 4](image)

The calculation precision for the computer, three is not exactly a point secant line rule. This method can be used in engineering area; don't have the strict limit accuracy.

If limit the projection of the control point matrix in a pentagon area, a pentagon NURBS surface
can be obtained, just as Fig. 6 and Fig. 7 show.

Figure 6 a pentagon NURBS surface

Figure 7 the projection of Fig. 6

5. Conclusion

The shape control of NURBS surface is through the control point matrix. By adjusting the appropriate critical control point matrix, come quadrilateral NURBS surface can even constructing circular surface. This is a simple way to get the quadrilateral NURBS surface, though sometimes are not satisfied with precision.

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