Twist-3 Gluon Fragmentation Contribution to Hyperon Polarization in Semi-Inclusive Deep Inelastic Scattering

RIKU IKARASHI¹, YUJI KOIKE², KENTA YABE¹ and SHINSUKE YOSHIDA³,⁴

¹Graduate School of Science and Technology, Niigata University, Ikarashi 2-no-cho, Niigata 950-2181, Japan
²Department of Physics, Niigata University, Ikarashi 2-no-cho, Niigata 950-2181, Japan
³Guangdong Provincial Key Laboratory of Nuclear Science, Institute of Quantum Matter, South China Normal University, Guangzhou 510006, China
⁴Guangdong-Hong Kong Joint Laboratory of Quantum Matter, Southern Nuclear Science Computing Center, South China Normal University, Guangzhou 510006, China

Abstract

We derive the twist-3 gluon fragmentation function (FF) contribution to the transversely polarized hyperon production in semi-inclusive deep inelastic scattering, $e p \rightarrow e \Lambda^\uparrow X$, in the leading order (LO) with respect to the QCD coupling in the framework of the collinear twist-3 factorization. Together with the known result for the contribution from the twist-3 distribution in the proton and the twist-3 quark FFs for the hyperon, this completes the LO cross section for this process. The constraint relations among the twist-3 FFs are taken into account. The formula is relevant to large-$P_T$ hyperon production in the future Electron-Ion-Collider experiment.
1 Introduction

In a recent paper [1], three of the present authors studied the transverse polarization of hyperons produced in semi-inclusive deep inelastic scattering, $ep \rightarrow e\Lambda'X$. For a large-$P_T$ hyperon production, this process can be analyzed in the framework of the collinear factorization, in which the polarization appears as a twist-3 observable in the absence of a leading twist-2 effect. For $ep \rightarrow e\Lambda'X$, the responsible twist-3 effects are (i) the twist-3 distribution functions (DFs) in the initial proton combined with the twist-2 transversity fragmentation function (FF) for $\Lambda$ and (ii) the twist-3 FFs for the polarized hyperon combined with the twist-2 unpolarized parton DFs in the proton. The twist-3 FFs in (ii) are chiral-even, and both (a) quark and (b) gluon types of twist-3 FFs contribute. In [1], the twist-3 polarized cross section for $ep \rightarrow e\Lambda'X$ from the above (i) and (ii)(a) was derived in the leading order (LO) with respect to the QCD coupling constant. As a sequel to [1], we will derive in this paper the LO cross section from (ii)(b), which completes the LO twist-3 cross section for this process. Since the gluons are ample in the collision environment and the twist-3 quark and gluon FFs mix under renormalization, the effect of (ii)(b) could be as important as (ii)(a). We also remind that the twist-3 fragmentation effect is important to understand the single transverse-spin asymmetry in $p'p \rightarrow \pi X$ [2,3], which shows a similar rising asymmetry at large $x_F$ as the polarization in $pp \rightarrow \Lambda'X$. Our present study has a direct relevance to the hyperon polarization phenomenon in the future Electron-Ion-Collider (EIC) experiment.

Here we make some remarks on the phenomenological use of the twist-3 cross section. As we will see, it contains several unknown nonperturbative functions, determination of which requires global analysis of data for various processes such as $ep \rightarrow e\Lambda'X$, $e^+e^- \rightarrow \Lambda'X$ and $pp \rightarrow \Lambda'X$, etc, combined with an appropriate modelling of those functions. We also recall that in the small-$P_T$ region the transverse-momentum-dependent (TMD) factorization holds for $ep \rightarrow e\Lambda'X$ and $e^+e^- \rightarrow \Lambda'X$, and we anticipate that the two frameworks match in the intermediate region of $P_T$ as for the case of $p'p \rightarrow \ell^+\ell^-X$ [4] and $ep \rightarrow e\pi X$ [5,7]. Information on the TMD functions obtained from the analysis of those small-$P_T$ data will also help to constrain the twist-3 functions owing to the relations between the TMD functions and the twist-3 functions [8,9]. In this connection we mention the recent data on $e^+e^- \rightarrow \Lambda'X$ at Belle [10] and the phenomenological analyses of the data in terms of the TMD factorization [11,14]. These studies will be useful to analyze the EIC data at large-$P_T$ in terms of the twist-3 cross section derived in this work.

The formalism of calculating the twist-3 gluon FFs contribution is very complicated and was completed only recently for a similar process in the pp collision, $pp \rightarrow \Lambda'X$ [15]. Here we apply the method to $ep \rightarrow e\Lambda'X$. Since the kinematics for this process was described in [1], and the method is in parallel to the case for $pp \rightarrow \Lambda'X$ [15], our presentation in this paper will be brief, referring to those papers for the details.

The remainder of this paper is organized as follows: In section 2, we introduce the twist-3 gluon FFs relevant in our study. In section 3, we briefly describe the formalism for calculating the twist-3 gluon FF contribution to $ep \rightarrow e\Lambda'X$ and present the LO cross section. Section 4 is devoted to a brief summary.

2 Twist-3 gluon fragmentation functions

2.1 Three types of twist-3 gluon FFs and $q\bar{q}g$ FFs

Here we list the twist-3 gluon FFs for spin-1/2 hyperon which are necessary to derive the twist-3 cross section for $ep \rightarrow e\Lambda'X$ [6,15]. They are classified into the intrinsic, kinematical and dynamical FFs. First, the intrinsic gluon FFs are defined as the lightcone correlators of the gluon’s field strength $F_s^{\mu\nu}$ with color index $a$ [6,16].

\[
\tilde{g}^{\alpha\beta}(z) = \frac{1}{N^2-1} \sum_X \int \frac{d\lambda}{2\pi} e^{-i\lambda/z} \langle 0 | [\bar{q}w^{\alpha}(\lambda)w^{\beta}(0)]_a | hX \rangle \langle hX | (F_\perp^{\alpha}(\lambda w) [\lambda w, \infty w])_a | 0 \rangle
\]

\[
= -g^{\alpha\beta}_L G(z) - i e^{P_h w^{\alpha\beta}} (S \cdot w) \Delta G(z) + M_h e^{P_h w S_L^{(\alpha w^{\beta})}} \Delta \tilde{G}_{3T}(z) + i M_h e^{P_h w S_L^{(\alpha w^{\beta})}} \Delta \tilde{G}_{3T}(z) + \cdots ,
\]

where $P_h$ is the four momentum of the hyperon with its mass $M_h$. $P_h^{\mu}$ can be regarded as lightlike in the twist-3 accuracy and $w^\mu$ is another lightlike vector satisfying $P_h \cdot w = 1$. $S^\mu$ is the spin vector of the hyperon normalized as $S^2 = -M_h^2$ and can be decomposed as $S^\mu = (S \cdot w) P_h^{\mu} + (S \cdot P_h) w^\mu + M_h S^\mu_\perp$, with the transverse spin vector $S^\mu_\perp (S^2_\perp = -1)$. $g^{\alpha\beta}_L \equiv g^{\alpha\beta} - P_h^\rho w^\beta - P_h^\beta w^\alpha$, $N = 3$ is the number of

Study on this matching will be reported elsewhere.
colors for $SU(N)$ and the ellipse denotes twist-4 or higher. $|h\rangle$ denotes the hyperon state. $[\lambda w, \mu w] \equiv \mathcal{P} \exp \left[ ig f^a_{\mu} d\tau w \cdot A(\tau w) \right]$ is the gauge-link operator which guarantees gauge invariance of the correlation function. We use the convention for the Levi-Civita symbol as $\epsilon^{0123} = 1$. The shorthand notation $\epsilon_{P_{\mu\nu} \alpha \beta} \equiv 2\epsilon^{\mu\nu\alpha\beta}P_{\mu\nu}\epsilon_{\alpha\beta}$, etc. is used, and $\{\alpha\beta\}$ denotes the symmetrization of Lorentz indices. $\hat{G}(z)$ and $\Delta \hat{G}(z)$ are twist-2 unpolarized and helicity FFs, respectively, and $\Delta \hat{G}_{ST}(z)$ and $\Delta \hat{G}_{T}(z)$ are intrinsic twist-3 FFs. All FFs in (1) are defined to be real and have a support on $0 < z < 1$. $\Delta \hat{G}_{ST}(z)$ is naively T-odd, and contributes to the hyperon polarization.

Second, the kinematical gluon FFs are defined from the derivative of the correlation functions for the intrinsic one:

$$\hat{\Gamma}_T^{\alpha\beta\gamma}(z) = \frac{1}{N^2 - 1} \int_X \frac{d\lambda}{2\pi} e^{-i\lambda/z} \langle 0 | [\lambda w,0] F_{\mu\nu}(0) \langle hX | (F_{\mu\nu}^{\alpha\beta}(\lambda w) [\lambda w, \infty w]) \langle 0 | \partial_{\gamma} \rangle,$$

$$-i\frac{M_h}{2} \epsilon_{P_{\mu\nu}\alpha\beta} S_\gamma^{(1)}(z) + \frac{M_h}{2} \epsilon_{P_{\mu\nu}\alpha\beta} S_\gamma^{(1)}(z)$$

$$-i\frac{M_h}{8} \left( \epsilon_{P_{\mu\nu}\alpha\beta} S_\gamma^{(1)} + \epsilon_{P_{\mu\nu}\alpha\beta} S_\gamma^{(1)} \right) \Delta \hat{H}_T^{(1)}(z) + \cdots,$$

where

$$F_{\mu\nu}^{\alpha\beta}(\lambda w) [\lambda w, \infty w]) \langle 0 | \partial_{\gamma}, \lim_{\xi \to 0} \frac{d}{d\xi} F_{\mu\nu}^{\alpha\beta}(\lambda w + \xi)(\lambda w + \xi, \infty w + \xi | 0).$$

There are three twist-3 gluonic kinematical FFs, $\hat{G}_{T}^{(1)}(z)$, $\Delta \hat{G}_{T}^{(1)}(z)$ and $\Delta \hat{H}_T^{(1)}(z)$, which are real functions and have a support on $0 < z < 1$. Among them, $\hat{G}_{T}^{(1)}(z)$ and $\Delta \hat{H}_T^{(1)}(z)$ are naively T-odd contributing to the hyperon polarization, while $\Delta \hat{G}_{T}^{(1)}(z)$ is naively T-even. They can also be written as the $k_T^2/M_h^2$-moment of the TMD FFs.

Third, the dynamical gluon FFs are defined from the 3-gluon correlators. Contraction of color indices with two structure constants for color $SU(N)$, i.e. $-if_{abc}$ and $d_{abc}$, yields two types of FFs:

$$\hat{\Gamma}_F^{\alpha\beta\gamma}(\frac{1}{z_1}, \frac{1}{z_2})$$

$$= \frac{-i f_{abc}}{N^2 - 1} \int_X \frac{d\lambda d\mu}{2\pi 2\pi} e^{-i\lambda/z_1 e^{-i\mu(1/z_2-1/z_1)}} \langle 0 | F_{\mu\nu}(0) | hX \rangle \langle hX | F_{\mu\nu}^{\alpha\beta}(\lambda w) g_{\mu\nu}(\mu w) | 0 \rangle,$$

$$= -M_h \left( \tilde{N}_1 \left( \frac{1}{z_1}, \frac{1}{z_2} \right) \epsilon_{P_{\mu\nu}\alpha\beta} S_{\gamma}^{(1)} + \tilde{N}_2 \left( \frac{1}{z_1}, \frac{1}{z_2} \right) \epsilon_{P_{\mu\nu}\alpha\beta} S_{\gamma}^{(1)} - \tilde{N}_2 \left( \frac{1}{z_2}, \frac{1}{z_1} \right) \epsilon_{P_{\mu\nu}\alpha\beta} S_{\gamma}^{(1)} \right),$$

$$\hat{\Gamma}_F^{\alpha\beta\gamma}(\frac{1}{z_1}, \frac{1}{z_2})$$

$$= \frac{d_{abc}}{N^2 - 1} \int_X \frac{d\lambda d\mu}{2\pi 2\pi} e^{-i\lambda/z_1 e^{-i\mu(1/z_2-1/z_1)}} \langle 0 | F_{\mu\nu}(0) | hX \rangle \langle hX | F_{\mu\nu}^{\alpha\beta}(\lambda w) g_{\mu\nu}(\mu w) | 0 \rangle,$$

$$= -M_h \left( \tilde{O}_1 \left( \frac{1}{z_1}, \frac{1}{z_2} \right) \epsilon_{P_{\mu\nu}\alpha\beta} S_{\gamma}^{(1)} + \tilde{O}_2 \left( \frac{1}{z_1}, \frac{1}{z_2} \right) \epsilon_{P_{\mu\nu}\alpha\beta} S_{\gamma}^{(1)} - \tilde{O}_2 \left( \frac{1}{z_2}, \frac{1}{z_1} \right) \epsilon_{P_{\mu\nu}\alpha\beta} S_{\gamma}^{(1)} \right),$$

where the gauge-link operators are suppressed for simplicity. There are four purely gluonic dynamical FFs, $\tilde{N}_{1,2} \left( \frac{1}{z_1}, \frac{1}{z_2} \right)$ and $\tilde{O}_{1,2} \left( \frac{1}{z_1}, \frac{1}{z_2} \right)$, which are complex functions and have a support on $1/z_2 > 1$ and $1/z_2 > 1/z_1 > 0$. Their real parts are naively T-even, while their imaginary parts are naively T-odd. $\tilde{N}_1 \left( \frac{1}{z_1}, \frac{1}{z_2} \right)$ and $\tilde{O}_1 \left( \frac{1}{z_1}, \frac{1}{z_2} \right)$ satisfy the symmetry relations

$$\tilde{N}_1 \left( \frac{1}{z_1}, \frac{1}{z_2} \right) = \tilde{N}_1 \left( \frac{1}{z_2}, \frac{1}{z_1} \right), \quad \tilde{O}_1 \left( \frac{1}{z_1}, \frac{1}{z_2} \right) = \tilde{O}_1 \left( \frac{1}{z_2}, \frac{1}{z_1} \right).$$

Finally, we introduce other dynamical FFs defined from the quark-antiquark-gluon correlators, which are necessary for the derivation of the twist-3 cross section for $e^{-} \rightarrow e\Lambda X$:

$$\hat{\Delta}_0 \left( \frac{1}{z_1}, \frac{1}{z_2} \right) = \frac{1}{N} \sum_X \int \frac{d\lambda d\mu}{2\pi 2\pi} e^{-i\lambda/z_1 e^{-i\mu(1/z_2-1/z_1)}} \langle 0 | g_{\mu\nu}(\mu w) | hX \rangle \langle hX | \bar{\psi}_j(w) \gamma^\mu \psi_i(0) | 0 \rangle.$$
where $t^a$ is the generators of SU(N) and the spinor indices $i, j$ are shown explicitly. These two functions $\tilde{D}_{FT}(\frac{1}{z_1}, \frac{1}{z_2})$ and $\tilde{G}_{FT}(\frac{1}{z_1}, \frac{1}{z_2})$ are complex functions and have a support on $1/z_1 > 0, 1/z_2 < 0$ and $1/z_1 - 1/z_2 > 1$. Their real parts are naively $T$-even, while the imaginary parts are naively $T$-odd.

### 2.2 Constraint relations among twist-3 gluon FFs

The gluon FFs introduced above are not independent but are subject to the QCD equation-of-motion (EOM) relations and the Lorentz invariance relations (LIRs). The complete set of those relations were derived in [9]. Here we quote those relations which are useful to simplify the twist-3 cross section for $ep \to e\Lambda^+ X$. The relevant EOM relation allows us to express the intrinsic FF in terms of the kinematical and dynamical FFs as

$$\frac{1}{z} \Delta \tilde{G}_{3F}(z) = -3 \tilde{D}_{FT}(z) + \frac{1}{2} \left( \tilde{G}^{(1)}_T(z) + \Delta \tilde{H}^{(1)}_T(z) \right)$$

$$+ \int d \left( \frac{1}{z'} \right) \frac{1}{1/z - 1/z'} \Im \left[ \tilde{N}_1 \left( \frac{1}{z'}, \frac{1}{z} \right) - \tilde{N}_2 \left( \frac{1}{z}, \frac{1}{z'} \right) \right] + \frac{1}{2} \left( \tilde{N}_1 \left( \frac{1}{z'}, \frac{1}{z} \right) - \tilde{N}_2 \left( \frac{1}{z}, \frac{1}{z'} \right) \right) - 2 \tilde{N}_2 \left( \frac{1}{z}, \frac{1}{z'} \right),$$

where $\tilde{D}_{FT}(z)$ is defined as

$$\tilde{D}_{FT}(z) = \frac{2}{C_F} \int_0^{1/z} d \left( \frac{1}{z'} \right) \tilde{D}_{FT} \left( \frac{1}{z'}, \frac{1}{z} \right), \quad \text{with } C_F = \frac{N^2 - 1}{2N}.$$ 

Other relations derived from the LIRs and the EOM relations represent the derivative of the kinematical FFs in terms of other FFs as

$$\frac{1}{z} \frac{\partial \tilde{G}^{(1)}_T(z)}{\partial (1/z)} = -2 \left( 3 \tilde{D}_{FT}(z) - \tilde{G}^{(1)}_T(z) \right)$$

$$+ 4 \int d \left( \frac{1}{z'} \right) \frac{1}{1/z - 1/z'} \Im \left[ \tilde{N}_1 \left( \frac{1}{z'}, \frac{1}{z} \right) - \tilde{N}_2 \left( \frac{1}{z}, \frac{1}{z'} \right) \right]$$

$$+ 2 \int d \left( \frac{1}{z'} \right) \frac{1}{1/z - 1/z'} \Re \left[ \tilde{N}_1 \left( \frac{1}{z'}, \frac{1}{z} \right) + \tilde{N}_2 \left( \frac{1}{z}, \frac{1}{z'} \right) \right],$$

and

$$\frac{1}{z} \frac{\partial \Delta \tilde{H}^{(1)}_T(z)}{\partial (1/z)} = -4 \left( 3 \tilde{D}_{FT}(z) - \Delta \tilde{H}^{(1)}_T(z) \right)$$

$$+ 8 \int d \left( \frac{1}{z'} \right) \frac{1}{1/z - 1/z'} \Im \left[ \tilde{N}_1 \left( \frac{1}{z'}, \frac{1}{z} \right) + \tilde{N}_2 \left( \frac{1}{z}, \frac{1}{z'} \right) \right]$$

$$+ 4 \int d \left( \frac{1}{z'} \right) \frac{1}{1/z - 1/z'} \Re \left[ \tilde{N}_1 \left( \frac{1}{z'}, \frac{1}{z} \right) + \tilde{N}_2 \left( \frac{1}{z}, \frac{1}{z'} \right) \right].$$

The relations (8), (10) and (11) show that the purely gluonic twist-3 FFs are related to the quark-antiquark-gluon FFs, which implies the contribution to $ep \to e\Lambda^+ X$ from the latter needs to be considered together. It’s been shown that the above three relations (8), (10) and (11) are crucial to guarantee the frame independence of the cross section for $pp \to \Lambda^+ X$. Using these relations, we will express the cross section in terms of $\tilde{G}^{(1)}_T, \Delta \tilde{H}^{(1)}_T, \Im \tilde{N}_1, \Im \tilde{O}_1, \Im \tilde{D}_{FT}$ and $\Im \tilde{G}_{FT}$ (see eq. 54 below), which gives the most concise expression for the cross section. We also note that, in principle, the twist-3 kinematical FFs, $\tilde{G}^{(1)}_T$ and $\Delta \tilde{H}^{(1)}_T$, can be also eliminated in terms of the twist-3 dynamical FFs (see eqs. (74) and (75) of [9]).

### 3 Twist-3 gluon FF contribution to $ep \to e\Lambda^+ X$

#### 3.1 Kinematics

Here we briefly summarize the kinematics for the process [1],

$$e(p) + p(p) \to e(p') + \Lambda^+(P_h, S_V) + X,$$ 

(12)
where \( \ell, \ell', p \) and \( P_h \) are the momenta of each particle and \( S_{\perp} \) is the transverse spin vector for \( \Lambda \). With the virtual photon’s momentum \( q = \ell - \ell' \), we introduce the five Lorentz invariants as

\[
S_{ep} \equiv (p + \ell)^2 \simeq 2p \cdot \ell, \quad Q^2 \equiv -q^2, \\
x_{bj} \equiv \frac{Q^2}{2p \cdot q}, \quad z_f \equiv \frac{p \cdot P_h}{p \cdot q}, \quad q_T \equiv \sqrt{-q_{T}^2},
\]

where

\[
q_{\mu}^T \equiv q_{\mu} - \frac{P_h}{p \cdot P_h} p_{\mu} - \frac{p \cdot q}{P_h} P_{\mu}^h
\]

is a space-like momentum satisfying \( q_{\mu} p = q_{\mu} P_h = 0 \). As in [1], we work in the hadron frame [20] (See Fig. 1), where \( p^\mu \) and \( q^\mu \) are collinear and take the following form:

\[
p^\mu = \frac{Q}{2x_{bj}} (1, 0, 0, 1), \quad x_{bj} = \frac{Q^2}{2p \cdot q} (15)
\]

\[
q^\mu = (0, 0, 0, -Q). \quad (16)
\]

Defining the azimuthal angles for the hadron plane and the lepton plane as \( \chi \) and \( \phi \), respectively, as shown in Fig. 1, \( P_h^\mu \) and \( \ell^\mu \) can be written as

\[
P^\mu_h = \frac{z_f Q}{2} \left( 1 + \frac{q_T^2}{Q^2}, \frac{2q_T}{Q} \cos \chi, \frac{2q_T}{Q} \sin \chi, -1 + \frac{q_T^2}{Q^2} \right), \quad (17)
\]

\[
\ell^\mu = \frac{Q}{2} (\cosh \psi, \sinh \psi \cos \phi, \sinh \psi \sin \phi, -1), \quad (18)
\]

where \( \psi \) is defined by

\[
\cosh \psi \equiv \frac{2x_{bj} S_{ep}}{Q^2} - 1. \quad (19)
\]

With this parameterization, the transverse momentum of the hyperon \( P_{hT} \) is given by \( P_{hT} = z_f q_T \).

For the calculation of the cross section, we introduce four axes by

\[
T^\mu \equiv \frac{1}{Q} (q^\mu + 2x_{bj} p^\mu) = (1, 0, 0, 0),
\]

\[
Z^\mu \equiv -\frac{q^\mu}{Q} = (0, 0, 0, 1),
\]
where the actual form in the hadron frame is given after the last equality in each equation. The final hyperon resides in the $XZ$-plane and the transverse spin vector of the hyperon can be written as

$$S_\perp = \cos \theta \cos \Phi_S X^\mu + \sin \theta \cos \Phi_S Z^\mu,$$

where $\theta$ is the polar angle of $\vec{F}_h$ as measured from the $Z$-axis and $\Phi_S$ is the azimuthal angle of $\vec{S}_\perp$ around $\vec{F}_h$ as measured from the $XZ$-plane. From (17), the polar angle $\theta$ is written as

$$\cos \theta = \frac{P_{hz}}{|\vec{P}_h|} = \frac{q_T^2 - Q^2}{q_T^2 + Q^2},$$

$$\sin \theta = \frac{P_{hX}}{|\vec{P}_h|} = \frac{2q_T Q}{q_T^2 + Q^2}.$$

With the kinematical variables defined above, the polarized differential cross section for unpolarized quark DFs $f_1(x)$ from the hadronic tensor $W_{\rho\sigma}(p,q,P_h)$, takes the following form:

$$\frac{d^6\sigma}{dx dy dy_2 dy_3 dy_4 dy_5 dy_6} = \frac{\alpha_{em}^2}{28\pi^2|q_b|^2 S_{q,p} Q^2} z_f \sigma L^{\rho\sigma}(\ell',\ell) W_{\rho\sigma}(p,q,P_h),$$

where $\alpha_{em} = e^2/(4\pi)$ is the QED coupling constant, $L^{\rho\sigma} = 2(\ell'^\rho \ell'^\sigma + \ell'^\sigma \ell'^\rho) - Q^2 g^{\rho\sigma}$ is the leptonic tensor and $W_{\rho\sigma}$ is the hadronic tensor. Although there are two azimuthal angles, $\phi$ and $\chi$, the cross section depends on the relative angle $\varphi \equiv \phi - \chi$ only. Therefore it can be expressed in terms of $S_{q,p}, Q^2, x_f, z_f, q_T^2, \varphi$ and $\Phi_S$.

### 3.2 Hadronic tensor

We now calculate the twist-3 gluon FF contribution to (24) following the formalism developed for $pp \to \Lambda' X$ [15]. It occurs as a nonpole contribution from the hard part as in the case of other twist-3 fragmentation contributions in $ep \to e\pi X$ [21] and $pp \to \Lambda' X$ [15] [22]. We first factorize the twist-2 unpolarized quark DFs $f_1(x)$ from the hadronic tensor $W_{\rho\sigma}(p,q,F_h)$:

$$W_{\rho\sigma}(p,q,F_h) = \int \frac{dx}{x} f_1(x) w_{\rho\sigma}(xp,q,F_h),$$

where $x$ is the momentum fraction of the quark in the proton, and we have omitted the factor associated with the quark’s fractional electric charge as well as summation over quark flavors. Up to twist-3, $w_{\rho\sigma}$ receives contribution from the 2-gluon, 3-gluon and quark-antiquark-gluon correlation functions corresponding to (a)-(e) of Fig. 2

$$w_{\rho\sigma} \equiv w_{\rho\sigma}^{(a)} + w_{\rho\sigma}^{(b)} + w_{\rho\sigma}^{(c)} + w_{\rho\sigma}^{(d)} + w_{\rho\sigma}^{(e)}.$$

where each term can be written as

$$w_{\rho\sigma}^{(a)} = \int \frac{d^3k}{(2\pi)^3} \Gamma_{\rho\sigma}^{(0)\mu\nu}(k) S_{\mu\rho\sigma}(k),$$

$$w_{\rho\sigma}^{(b)} = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \Gamma_{\rho\sigma}^{(1)\mu\nu\lambda}(k,k') S_{\mu\rho\sigma\lambda}(k,k'),$$

$$w_{\rho\sigma}^{(c)} = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \Gamma_{\rho\sigma}^{(1)\mu\nu\lambda}(k,k') S_{\mu\rho\sigma\lambda}(k,k'),$$

$$w_{\rho\sigma}^{(d)} = \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \Delta_{\rho\sigma}^{(1)\lambda}(k,k') S_{\rho\sigma\lambda}(k,k'),$$

$$w_{\rho\sigma}^{(e)} = \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \Delta_{\rho\sigma}^{(1)\lambda}(k,k') S_{\rho\sigma\lambda}(k,k').$$
Here $S_{\mu\nu,\rho\sigma}^{ab}(k)$, $S_{\mu\nu,\lambda,\rho\sigma}^{L(R)}(k,k')$, and $\tilde{S}_{\lambda,\rho\sigma}^{L(R)}(k,k')$ represent the partonic hard parts with $k$ and $k'$ the momenta of partons fragmenting into the final hyperon, and the dependence on $q$ is suppressed for simplicity. $\Gamma_{ab}^{(0)\mu\nu}$, $\Gamma_{Rab}^{(1)\mu\nu\lambda}$, and $\tilde{\Delta}_{R\alpha}^{(1)\alpha}$ denote the fragmentation matrix elements defined as

$$
\Gamma_{ab}^{(0)\mu\nu}(k) = \sum_X \int d^4\xi e^{-ik\cdot\xi} \langle 0|A_\nu^a(0)hX|A_\mu^b(0)\xi|0\rangle,
$$

$$
\Gamma_{Rab}^{(1)\mu\nu\lambda}(k,k') = \sum_X \int d^4\xi d^4\eta e^{-ik\cdot\xi} e^{-i(k'-k)\cdot\eta} \langle 0|A_\nu^a(0)gA^b_\lambda(\eta)hX|A_\mu^c(0)\xi|0\rangle,
$$

$$
\tilde{\Delta}_{R\alpha}^{(1)\alpha}(k,k') = \sum_X \int d^4\xi d^4\eta e^{-ik\cdot\xi} e^{-i(k'-k)\cdot\eta} \langle 0|gA_\alpha^a(\eta)hX|\psi_i(0)\bar{\psi}_j(\xi)|0\rangle,
$$

$$
\tilde{\Delta}_{R\alpha}^{(1)\alpha}(k,k') = \sum_X \int d^4\xi d^4\eta e^{-ik\cdot\xi} e^{-i(k'-k)\cdot\eta} \langle 0|\psi_i(0)\bar{\psi}_j(\xi)hX|gA_\alpha^a(\eta)|0\rangle.
$$

The contribution with two parton lines in the left (right) of the cut in Fig. 2(b)-(e) are characterized by the symbol $L$ ($R$) in the hard parts and the fragmentation matrix elements. The superscripts (0) and (1) indicate the order of the gauge coupling $g$ corresponding, respectively, to the 2-parton and 3-parton correlation functions. The factor 1/2 in Eq. (28) and Eq. (29) takes into account the exchange symmetry in the corresponding matrix element. In Eq. (30) and Eq. (31), the hard parts and the fragmentation matrix elements are matrices both in color and spinor spaces for the quark and $\bar{\tau}$ indicates trace over both indices. The hard parts and the fragmentation matrix elements satisfy

$$
\Gamma_{Rab}^{(1)\mu\nu\lambda}(k,k') = \Gamma_{Labc}^{(1)\mu\nu\lambda}(k',k)*,
$$

$$
\tilde{\Delta}_{R\alpha}^{(1)\alpha}(k,k') = \gamma^\nu \tilde{\Delta}_{L\alpha}^{(1)\alpha}(k',k)\gamma^\lambda \tilde{\Delta}_{L\alpha}^{(1)\alpha}(k,k'),
$$

$$
S_{\mu\nu,\lambda,\rho\sigma}^{L(R)}(k,k') = S_{\mu\nu,\lambda,\rho\sigma}^{L(R)}(k',k)*\Gamma_{Lbc}^{(1)\mu\nu\lambda}. \tilde{\Delta}_{R\alpha}^{(1)\alpha}(k,k') = \gamma^\nu \tilde{\Delta}_{R\alpha}^{(1)\alpha}(k',k)*\gamma^\lambda \tilde{\Delta}_{R\alpha}^{(1)\alpha}(k,k').
$$

We thus have

$$
\omega_{\rho\sigma} = \omega_{\rho\sigma}^{(a)} + 2\Re \omega_{\rho\sigma}^{(b)} + 2\Re \omega_{\rho\sigma}^{(d)}.
$$

To extract the twist-3 contribution to $ep \rightarrow e\Lambda^+ X$ we apply the collinear expansion to the hard part, $S_{\mu\nu,\rho\sigma}^{ab}$, $S_{\mu\nu,\lambda,\rho\sigma}^{L(R)}$, and $\tilde{S}_{\lambda,\rho\sigma}^{L(R)}$, with respect to the momenta $k$ and $k'$ around $P_h/z$ and $P_h/z'$, respectively, taking into account of the following Ward-Takahashi (WT) identities [15]:

$$
k^\mu S_{\mu\nu,\rho\sigma}^{ab}(k) = k^\nu S_{\nu\sigma,\rho\nu}^{ab}(k) = 0,
$$

$$
k^\mu S_{\mu\nu,\lambda,\rho\sigma}^{L}(k,k') = if^{abc}/N^2 - 1 S_{\lambda,\rho\nu,\sigma}^{L}(k'),
$$

$$
k^\mu S_{\mu\nu,\lambda,\rho\sigma}^{R}(k,k') = 0,
$$

$$
(k'-k)^\lambda S_{\mu\nu,\lambda,\rho\sigma}^{L}(k,k') = -if^{abc}/N^2 - 1 S_{\mu,\nu,\rho\sigma}^{L}(k'),
$$

$$
(k-k')^\alpha \tilde{S}_{\lambda,\rho\sigma}^{L}(k,k') = 0,
$$

where $S_{\mu\nu,\rho\sigma}(k) \equiv \delta^{\sigma\eta} S_{\mu\nu,\eta,\rho\sigma}(k)$. We note that unlike the case for $pp \rightarrow \Lambda^+ X$ no ghost-like terms appear in the WT identities [38, 12] for the present case. This way, we obtains the hadronic tensor $\omega_{\rho\sigma}$ in terms of the gauge invariant FF's as (See eq. (51) of [15] and eq. (56) of [21])

$$
\omega_{\rho\sigma} = \Omega_{\mu}^{\alpha} \Omega_{\nu}^{\beta} \int \left( \frac{1}{z} \right) z^2 \tilde{\Gamma}_{\mu\nu}(z) S_{\alpha\beta,\rho\sigma}^{(1)} \left( \frac{1}{z} \right)
$$

$$
- \Omega_{\mu}^{\alpha} \Omega_{\nu}^{\beta} \Omega_{\lambda}^{\gamma} \int \left( \frac{1}{z} \right) z^2 \tilde{\Gamma}_{\mu\nu\lambda}(z) \frac{\partial S_{\alpha\beta,\rho\sigma}}{\partial k^\gamma} \bigg|_{k=P_h/z}
$$

$$
+ \Re \left[ \Omega_{\mu}^{\alpha} \Omega_{\nu}^{\beta} \Omega_{\lambda}^{\gamma} \int \left( \frac{1}{z} \right) d \left( \frac{1}{z} \right) z z' z' \frac{1}{z - 1/z'} \right].
$$
Figure 2: Cut diagrams for the twist-3 gluon fragmentation contribution to $ep \rightarrow e\Lambda^+X$. In each diagram, the lower blob represents the unpolarized quark distribution, the middle one represents the partonic hard cross section and the upper one represents the fragmentation matrix elements for the final hyperon.

\[
\times \left\{ -i f_{abc}N^{-1} \hat{\Gamma}^{\mu\nu\lambda}_{FA} \left( \frac{1}{z'}, \frac{1}{z} \right) + \frac{Nd_{abc}}{N_2 - 4} \hat{\Gamma}^{\mu\nu\lambda}_{FS} \left( \frac{1}{z'}, \frac{1}{z} \right) \right\} S_{abc} \right\}
\]

where $\hat{\Gamma}^{\mu\nu\lambda}$, $\hat{\Gamma}^{\mu\nu\lambda}_{FA}$, $\hat{\Gamma}^{\mu\nu\lambda}_{FS}$ are given by (1), (2), (4), (5) and (7). For the hard part we have used the notation $S_{\alpha\beta,\rho\sigma}$ for $S_{\alpha\beta,\rho\sigma}^L$ and $S_{\alpha\beta,\rho\sigma}^R$ for $S_{\alpha\beta,\rho\sigma}^R$, etc, suppressing $P_\mu$ for short. In the last term of (43), $\tilde{S}_L^{\rho\sigma}$ is defined from $S_{abc}^{\rho\sigma}$ in (30) by $\left( \tilde{S}_L^{\rho\sigma} \right)_{rs} = \frac{1}{2N} \tilde{P}_s \tilde{S}_L^{\rho\sigma}$, where $r, s$ indicates the color indices for the quark, and $\text{Tr}_s$ denotes the trace in the spinor space. The LO diagrams for the hard parts of Figs. 2 (a), (b) and (d) are, respectively, shown in Figs. 3, 4 and 5. It is easy to show that the hadronic tensor $w_{\rho\sigma}$ satisfies the electromagnetic gauge invariance, $q^\rho w_{\rho\sigma} = q^\sigma w_{\rho\sigma} = 0$, owing to the WT identity in QED.

3.3 Spin dependent cross section

The calculation of $L_{\rho\sigma} W_{\rho\sigma}$ in (24) can be done in the same way as \[1\]: $W_{\rho\sigma}$ can be expanded in terms of the six tensors 20 $\gamma_k^{\rho\sigma}$ ($k = 1, \cdots, 4, 8, 9$) defined by

\[
\begin{align*}
\gamma_1^{\rho\sigma} &= X^\rho X^\sigma + Y^\rho Y^\sigma, & \gamma_2^{\rho\sigma} &= g^{\rho\sigma} + Z^\rho Z^\sigma, & \gamma_3^{\rho\sigma} &= T^{\rho} X^\sigma + X^{\rho} T^\sigma, \\
\gamma_4^{\rho\sigma} &= X^\rho X^\sigma - Y^\rho Y^\sigma, & \gamma_8^{\rho\sigma} &= T^\rho Y^\sigma + Y^{\rho} T^\sigma, & \gamma_9^{\rho\sigma} &= X^\rho Y^\sigma + Y^{\rho} X^\sigma. \\
\end{align*}
\]

By introducing the inverses of $\gamma_k^{\rho\sigma}$, $\tilde{\gamma}_k^{\rho\sigma}$ satisfying $\gamma_k^{\rho\sigma} \tilde{\gamma}_k^{\rho\sigma} = \delta_{kk'}$, as

\[
\begin{align*}
\tilde{\gamma}_1^{\rho\sigma} &= \frac{1}{2} (2T^{\rho} T^\sigma + X^{\rho} X^\sigma + Y^{\rho} Y^\sigma), & \tilde{\gamma}_2^{\rho\sigma} &= T^{\rho} T^\sigma, & \tilde{\gamma}_3^{\rho\sigma} &= -\frac{1}{2} (T^{\rho} X^\sigma + X^{\rho} T^\sigma), \\
\tilde{\gamma}_4^{\rho\sigma} &= \frac{1}{2} (X^{\rho} X^\sigma - Y^{\rho} Y^\sigma), & \tilde{\gamma}_8^{\rho\sigma} &= -\frac{1}{2} (T^{\rho} Y^\sigma + Y^{\rho} T^\sigma), & \tilde{\gamma}_9^{\rho\sigma} &= \frac{1}{2} (X^{\rho} Y^\sigma + Y^{\rho} X^\sigma), \\
\end{align*}
\]
$S(z) \sim \begin{bmatrix} \hat{p}_d & q & p \end{bmatrix} \begin{bmatrix} \hat{p} & q & p \end{bmatrix}$

$\times \begin{bmatrix} \hat{p}_d & q & p \end{bmatrix} \begin{bmatrix} \hat{p} & q & p \end{bmatrix}$

$A$

where (1), (2), (4), (5) and (7) into (43), we find that the cross section (24) takes the following structure:

Structure functions with different azimuthal dependences which are carried by the $\psi$ with $\tilde{p}$

$A$

Figure 3: The lowest order Feynman diagrams for $S_{\alpha\beta,\rho\sigma} \left( \frac{1}{z} \right)$ in (43). We set $\hat{p} \equiv x p$ and $\hat{P}_h \equiv P_h/z$. The symbol $\otimes$ indicates the fragmentation to the final hadron and $p_d$ is the momentum of an unobserved parton in the final state.

$W^{\mu\nu}$ can be expanded as

$$W^{\mu\nu} = \sum_{k=1, \ldots, 4, 8, 9} T^{\mu\nu}_{k}[W_{\rho\sigma} T^{\rho\sigma}_k].$$

Then one obtains

$$L^{\mu\nu}W_{\mu\nu} = \sum_{k=1, \ldots, 4, 8, 9} [L^{\mu\nu} T^{\mu\nu}_k][W_{\rho\sigma} T^{\rho\sigma}_k] = Q^2 \sum_{k=1, \ldots, 4, 8, 9} \omega_k(\phi - \chi)[W_{\rho\sigma} T^{\rho\sigma}_k],$$

where $\omega_k(\varphi) \equiv L^{\mu\nu} T^{\mu\nu}_k/Q^2$ are given by

$$\omega_1(\varphi) = 1 + \cosh^2 \psi, \quad \omega_2(\varphi) = -2, \quad \omega_3(\varphi) = -\cos \varphi \sinh 2\psi, \quad \omega_4(\varphi) = \sin \varphi \sinh 2\psi, \quad \omega_5(\varphi) = \sin 2\varphi \sinh^2 \psi,$$

with $\psi$ defined in (19). From (47) and (48), one sees the cross section can be decomposed into the five structure functions with different azimuthal dependences which are carried by the $\omega_k(\varphi)$s. Substituting (1), (2), (4), (5) and (7) into (43), we find that the cross section (24) takes the following structure:

$$d^6\sigma \over dx_dQ^2dz_ddq_d^2d\phi_d\chi = \frac{\alpha_s^2\alpha_s M_h}{16\pi^2 z_b S_c^2 Q^2} \sum_k \omega_k(\varphi)S_k \int dx \left( \frac{1}{z} \right) f_1(x)\delta \left( \frac{q^2}{Q^2} - \left( 1 - \frac{1}{z} \right) \left( 1 - \frac{1}{z} \right) \right)$$

$$\times \left\{ \begin{array}{l} \frac{1}{z} \Delta Gh_T(z) \hat{\sigma}^{k}_{int} + \tilde{G}^{(1)}_T(z) \hat{\sigma}^{k}_{DG} + \frac{1}{z} \frac{\partial \tilde{G}^{(1)}_T(z)}{\partial(1/z)} \hat{\sigma}^{k}_{DG} + \Delta \tilde{H}^{(1)}_T(z) \hat{\sigma}^{k}_{DG} + \frac{1}{z} \frac{\partial \Delta \tilde{H}^{(1)}_T(z)}{\partial(1/z)} \hat{\sigma}^{k}_{DH} \\
\end{array} \right.$$
Figure 4: The lowest order Feynman diagrams for $S_{\alpha \beta \gamma, \rho \sigma}^{Labc} \left( \frac{1}{z'}, \frac{1}{z} \right)$ in (49). We set $\hat{P}_h = P_h / z'$. Three crosses ($\times$) on the quark line in the upper diagrams indicates that the virtual photon line with a cross at one end needs to be attached to one of these crosses, and all three diagrams have to be included. Thus the number of diagrams for $S_{\alpha \beta \gamma, \rho \sigma}^{Labc}$ is $(3 + 3) \times 2 = 12$. The meaning of the other symbols are the same as that in Fig. 3.

\[
\int d \left( \frac{1}{z'} \right) \frac{2}{C_F} \left[ \hat{\Delta}_{FF} \left( \frac{1}{z'}, \frac{1}{z} - \frac{1}{z} \right) \left( \sigma_{DF} + \frac{1}{z} \frac{1}{z} - \frac{1}{z} \frac{1}{z} \sigma_{DF2} + \frac{z'}{z} \frac{z'}{z} \sigma_{DF3} \right) \right. \\
+ \left. \frac{1}{1 - (1 - q^2_f/Q^2)z_f/z'} \sigma_{DF4} + \frac{1}{1 - (1 - q^2_f/Q^2)z_f(1 - 1/z')} \sigma_{DF5} \right] + 3 \hat{G}_{FT} \left( \frac{1}{z'}, \frac{1}{z} - \frac{1}{z} \right) \left( \sigma_{GF} + \frac{1}{1} \frac{1}{z} - \frac{1}{z} \sigma_{GF2} + \frac{z'}{z} \frac{z'}{z} \sigma_{GF3} \right) \\
+ \left. \frac{1}{1 - (1 - q^2_f/Q^2)z_f/z'} \sigma_{GF4} + \frac{1}{1 - (1 - q^2_f/Q^2)z_f(1 - 1/z')} \sigma_{GF5} \right] \right] \right], (49)
\]

where

\[
S_{1,2,3,4} \equiv \sin \Phi, \quad S_{8,9} \equiv \cos \Phi, \quad \hat{x} = x_b / x, \quad \hat{z} = z_f / z,
\]

and we have set

\[
\hat{N}_3 \left( \frac{1}{z'}, \frac{1}{z} \right) \equiv -\hat{N}_2 \left( \frac{1}{z} - \frac{1}{z'} \right), \quad \hat{O}_3 \left( \frac{1}{z'}, \frac{1}{z} \right) \equiv \hat{O}_2 \left( \frac{1}{z} - \frac{1}{z'} \right) \quad (51)
\]

for convenience. Partonic hard parts for each FF can be computed from the corresponding diagrams, Figs. 3, 4, and 5. We have reached the form (49) based on the observation that the $z'$-dependence of the hard parts for the dynamical FFs appears in the cross section only through the factors explicitly shown in (49) (See Appendix C of [15]), and hence we can define all the partonic hard cross sections $\hat{\sigma}$'s in (49) as the functions of $\hat{x}$, $\hat{z}$, $Q$ and $q_T$. In addition we found by explicit calculation of the LO diagrams that

\[
\hat{\sigma}_{i,k}^{\pm(3)} = \hat{\sigma}_{i,k}^{\pm(1)} \quad (52)
\]

\[
\hat{\sigma}_{DF}^{k} = \hat{\sigma}_{GF}^{k} = 0. \quad (53)
\]

In order to transform the cross section (49) into a more concise form, we note the following points: (I) Owing to the symmetry property under $1/z' \leftrightarrow 1/z - 1/z'$ of $\hat{N}_1$ and $\hat{O}_1$ (6) and the relations (51), the
Figure 5: The lowest order Feynman diagrams for $\tilde{S}^L_{\alpha,\rho \sigma} \left( \frac{1}{z^2}, \frac{1}{z^2} - \frac{1}{z} \right)$. The meaning of the symbols is the same as that in Fig. 4. The total number of diagrams for $\tilde{S}^L_{\alpha,\rho \sigma}$ is $(4 + 2) \times 2 = 12$.

terms of $\hat{\sigma}^{(3)}_{\pm}$ and $\hat{\sigma}^{(4)}_{\pm}$ can be combined, respectively, with those of $\hat{\sigma}^{(1)}_{\pm}$ and $\hat{\sigma}^{(2)}_{\pm}$, taking into account the relation (52). (II) Using (8), (10) and (11), one can eliminate the intrinsic FF and the derivative of the kinematical FFs in favor of the kinematical and the dynamical FFs. This way we finally obtain the twist-3 gluon FF contribution to $ep \rightarrow e\Lambda^+ X$ as:

$$
d^0\sigma \overline{d}x_{bj}dQ^2dz_f dq^2 d\phi d\chi = \frac{\alpha_s^\rho M_h}{16\pi^2 z^2 j_b \delta_{\rho \sigma} Q^2} \sum \alpha_b (\phi - \chi) S_k \int_{x_{\min}}^1 \frac{dx}{x} \int_{z_{\min}}^1 \frac{dz}{z} z^2 f_1(x) \delta \left( \frac{q^2}{Q^2} - \left(1 - \frac{1}{x}\right) \left(1 - \frac{1}{z}\right) \right) \times \left\{ \tilde{G}^{(1)}_T(z) \hat{\sigma}^b_T + \Delta \tilde{H}^{(1)}_T(z) \hat{\sigma}^b_H \right. \\
+ \int d \left( \frac{1}{z^2} \right) \left[ \frac{1}{1/z - 1/z'} \mathcal{G} \right] \left( \frac{1}{z'}, \frac{1}{z}, \frac{1}{z} \right) \hat{\sigma}^k_{N1} + \hat{\sigma}^k_{N2} + \hat{\sigma}^k_{N3} \\
+ \frac{1}{1/z - 1/z'} \mathcal{G} \left( \frac{1}{z'}, \frac{1}{z}, \frac{1}{z} \right) \hat{\sigma}^k_{N1} + \hat{\sigma}^k_{N2} + \hat{\sigma}^k_{N3} \\
+ \frac{1}{1/z - 1/z'} \mathcal{G} \left( \frac{1}{z'}, \frac{1}{z}, \frac{1}{z} \right) \hat{\sigma}^k_{N1} + \hat{\sigma}^k_{N2} + \hat{\sigma}^k_{N3} \\
+ \left. \int d \left( \frac{1}{z^2} \right) \frac{2}{c_F} \left[ \mathcal{G} \left( \frac{1}{z'}, \frac{1}{z}, \frac{1}{z} \right) \hat{\sigma}^k_{DO1} + \hat{\sigma}^k_{DO2} \right] \right) \\
+ \frac{1}{1/z - 1/z'} \mathcal{G} \left( \frac{1}{z'}, \frac{1}{z}, \frac{1}{z} \right) \hat{\sigma}^k_{DO1} + \hat{\sigma}^k_{DO2} + \hat{\sigma}^k_{DO3} \\
+ \left. \int d \left( \frac{1}{z^2} \right) \frac{2}{c_F} \left[ \mathcal{G} \left( \frac{1}{z'}, \frac{1}{z}, \frac{1}{z} \right) \hat{\sigma}^k_{DF1} + \hat{\sigma}^k_{DF2} \right] \right) \\
+ \frac{1}{1/z - 1/z'} \mathcal{G} \left( \frac{1}{z'}, \frac{1}{z}, \frac{1}{z} \right) \hat{\sigma}^k_{DF1} + \hat{\sigma}^k_{DF2} + \hat{\sigma}^k_{DF3} \\
+ \frac{1}{1 - \left(1 - q^2\right)/Q^2} z_f/z' \hat{\sigma}^k_{DF4} + \frac{1}{1 - \left(1 - q^2\right)/Q^2} z_f/z' \hat{\sigma}^k_{DF5} \right) \right.$$
\[ + \Im \tilde{G}_{FF} \left( \frac{1}{z'}, \frac{1}{z} - \frac{1}{z} \right) \left( \frac{1}{z} \frac{1}{1/z' - 1/z' \hat{\sigma}_{GF2} + \frac{z'}{z} \hat{\sigma}_{GF3}} \right) + \frac{1}{1 - (1 - q_{TF}^2/Q^2) z_f} \hat{\sigma}_{GF4} + \frac{1}{1 - (1 - q_{TF}^2/Q^2) z_f (1/z' - 1/z') \hat{\sigma}_{GF5}} \right) \],

where the lower limits of \( x \) and \( z \) are, respectively, given by \( x_{min} = x_{bj} \left( 1 + \frac{z_f}{1 - z_f} \frac{\hat{q}_{TF}^2}{Q^2} \right) \) and \( z_{min} = z_f \left( 1 + \frac{x_{bj}}{1 - x_{bj}} \frac{\hat{q}_{TF}^2}{Q^2} \right) \). The partonic hard cross sections which appear newly in (54) are defined from those in (49) as

\[
\hat{\sigma}^k_G = \frac{1}{2} \hat{\sigma}_{int}^k + \hat{\sigma}_{N,DG}^k + 2 \hat{\sigma}_{DG}^k,
\]

\[
\hat{\sigma}^k_H = \frac{1}{2} \hat{\sigma}_{int}^k + \hat{\sigma}_{N,DH}^k + 4 \hat{\sigma}_{DH}^k,
\]

\[
\hat{\sigma}^k_{N1} = 2 \hat{\sigma}_{int}^k + 4 \hat{\sigma}_{DG}^k + 8 \hat{\sigma}_{DH}^k,
\]

\[
\hat{\sigma}^k_{N2} = \hat{\sigma}^k_{int} + 8 \hat{\sigma}_{DH}^k + \frac{1}{2} (\hat{\sigma}_{2,k}^-(1) - \hat{\sigma}_{3,k}^-(1)),
\]

\[
\hat{\sigma}^k_{N3} = -\hat{\sigma}^k_{int} - 4 \hat{\sigma}_{DG}^k + \frac{1}{2} (\hat{\sigma}_{2,k}^-(1) - \hat{\sigma}_{3,k}^-(1)),
\]

\[
\hat{\sigma}^k_{D1} = 2 \hat{\sigma}_{DG}^k + 4 \hat{\sigma}_{DH}^k + \frac{1}{2} (\hat{\sigma}_{2,k}^-(2) - \hat{\sigma}_{3,k}^-(2)),
\]

\[
\hat{\sigma}^k_{D2} = -\hat{\sigma}^k_{int} - 2 \hat{\sigma}_{DG}^k + 2 \hat{\sigma}_{DH}^k,
\]

and others are the same as those appearing in (49). Although \( \hat{\sigma}_{DF}^k = 0 \) as shown in (53), \( \hat{\sigma}_{DF1}^k \) term appears in (54) due to the relations (8), (10) and (11).

To write down the partonic hard cross sections in (54), we further take into account the following relations:

\[
\hat{\sigma}_{O1}^k = \hat{\sigma}_{1,k}^+, \quad \hat{\sigma}_{O2}^k = \frac{1}{2} (\hat{\sigma}_{2,k}^+ + \hat{\sigma}_{3,k}^+),
\]

\[
\hat{\sigma}_{O3}^k = \frac{1}{2} (\hat{\sigma}_{2,k}^+ + \hat{\sigma}_{3,k}^+),
\]

\[
\hat{\sigma}_{D1}^k = \frac{1}{2} (\hat{\sigma}_{1,k}^{+(2)} + \hat{\sigma}_{1,k}^{+(4)}),
\]

\[
\hat{\sigma}_{D2}^k = \frac{1}{2} (\hat{\sigma}_{2,k}^{+(2)} + \hat{\sigma}_{3,k}^{+(4)}),
\]

\[
\hat{\sigma}_{D3}^k = \frac{1}{2} (\hat{\sigma}_{2,k}^{+(4)} + \hat{\sigma}_{3,k}^{+(2)}),
\]

\[
\hat{\sigma}_{DF1}^k = -\hat{\sigma}_{int}^k - 2 \hat{\sigma}_{DG}^k - 4 \hat{\sigma}_{DH}^k,
\]

and the relations (70) and (71) are obvious from (64), (65), (57), and (69), and (72) and (73) are obtained by explicit calculation of the LO diagrams.

Then the independent hard cross sections are given as follows.
\[
\begin{align*}
\hat{\sigma}_1^G &= C_F \frac{2Q^2}{q_f^2} \left( -1 + \hat{z} \right)^2 \hat{x} \hat{x} \hat{x} \hat{x}, \\
\hat{\sigma}_2^G &= C_F \frac{8}{q_f} \hat{x} (-1 + \hat{z}), \\
\hat{\sigma}_3^G &= C_F \frac{2Q}{q_f^2} \left( -1 + \hat{z} \right) \hat{x} \hat{x} \hat{x} \hat{x}, \\
\hat{\sigma}_4^G &= \frac{1}{2} \hat{\sigma}_2^G, \\
\hat{\sigma}_5^G &= C_F \frac{2Q}{q_f^2} \left( -1 + \hat{z} \right) \left( -1 + \hat{x} \right), \\
\hat{\sigma}_6^G &= C_F \frac{4}{q_f} \hat{x} (-1 + \hat{z}).
\end{align*}
\] (74)

\[
\begin{align*}
\hat{\sigma}_1^H &= -C_F \frac{4Q^2}{q_f^2} \left( -1 + \hat{z} \right)^2, \\
\hat{\sigma}_2^H &= 0, \\
\hat{\sigma}_3^H &= -C_F \frac{2Q}{q_f^2} \left( -1 + \hat{z} \right) \left( -1 + \hat{z} \right), \\
\hat{\sigma}_4^H &= -C_F \frac{4}{q_f} \left( -1 + \hat{z} \right), \\
\hat{\sigma}_5^H &= \hat{\sigma}_6^H, \\
\hat{\sigma}_6^H &= \hat{\sigma}_7^H.
\end{align*}
\] (75)

\[
\begin{align*}
\hat{\sigma}_1^{N_1} &= C_F \frac{8Q^2}{q_f^2} \left( -1 + \hat{z} \right)^2 \hat{x} \hat{x} \hat{x} \hat{x}, \\
\hat{\sigma}_2^{N_1} &= C_F \frac{32}{q_f} \hat{x} (-1 + \hat{z}) \hat{x}, \\
\hat{\sigma}_3^{N_1} &= C_F \frac{8Q}{q_f^2} \left( -1 + \hat{z} \right)^2 \left( -1 + \hat{z} \right), \\
\hat{\sigma}_4^{N_1} &= -C_F \frac{8}{q_f} \left( -1 + \hat{z} \right), \\
\hat{\sigma}_5^{N_1} &= -C_F \frac{8}{q_f^2} \left( -1 + \hat{z} \right)^2, \\
\hat{\sigma}_6^{N_1} &= -C_F \frac{8}{q_f} \left( -1 + \hat{z} \right) \hat{x} \hat{x} \hat{x} \hat{x}.
\end{align*}
\] (76)
\[
\begin{align*}
\sigma_{N_2}^1 &= -C_F \frac{4Q^2}{q_T^3} \frac{\left(-1 + \hat{z}\right)^2(-1 + (1 + 3\hat{z})\hat{x})}{\hat{x}^3}, \\
\sigma_{N_2}^2 &= -C_F \frac{8}{q_T} \frac{\hat{x}(1 + \hat{z})}{\hat{z}}, \\
\sigma_{N_2}^3 &= -C_F \frac{2Q}{q_T^2} \frac{(-1 + \hat{z})(-3(1 + \hat{x}) + \hat{z}(3 + 4\hat{z}))}{\hat{z}^2}, \\
\sigma_{N_2}^4 &= -C_F \frac{4}{q_T} \frac{(-1 + \hat{z})(2 + \hat{x})}{\hat{z}}, \\
\sigma_{N_2}^8 &= -C_F \frac{2Q}{q_T^2} \frac{(-1 + \hat{z})(-3 - \hat{x} + \hat{z}(3 + 2\hat{x}))}{\hat{z}^2}, \\
\sigma_{N_2}^9 &= \sigma_{N_2}^4,
\end{align*}
\]

(77)

\[
\begin{align*}
\sigma_{N_3}^1 &= -C_F \frac{4Q^2}{q_T^3} \frac{\left(-1 + \hat{z}\right)^2(3\hat{z}(2 - 3\hat{x})\hat{x} + \hat{x}(1 + \hat{x}) + 2\hat{z}^2(1 - 6\hat{x} + 6\hat{x}^2))}{\hat{x}^3}, \\
\sigma_{N_3}^2 &= -C_F \frac{8}{q_T} \frac{\hat{x}(1 + \hat{z})(-3 + 4\hat{z})}{\hat{z}}, \\
\sigma_{N_3}^3 &= -C_F \frac{2Q}{q_T^2} \frac{(-1 + \hat{z})(1 + \hat{z}(7 - 20\hat{x}) + 5\hat{x} + 8\hat{z}^2(-1 + 2\hat{x}))}{\hat{z}^2}, \\
\sigma_{N_3}^4 &= \frac{1}{2} \sigma_{N_3}^3, \\
\sigma_{N_3}^8 &= -C_F \frac{2Q}{q_T^2} \frac{(-1 + \hat{z})(1 - \hat{x} + \hat{z}(-1 + 2\hat{x}))}{\hat{z}^2}, \\
\sigma_{N_3}^9 &= -C_F \frac{4}{q_T} \frac{\hat{x}(1 + \hat{z})}{\hat{z}},
\end{align*}
\]

(78)

\[
\begin{align*}
\sigma_{D_{N_1}}^1 &= C_F \frac{2Q^2}{q_T^3} \frac{\left(-1 + \hat{z}\right)^2(2\hat{z}(1 - 3\hat{x})\hat{x} + (1 + \hat{x})^2 + \hat{z}^2(1 - 6\hat{x} + 6\hat{x}^2))}{\hat{x}^3}, \\
\sigma_{D_{N_1}}^2 &= \frac{1}{4} \sigma_{D_{N_1}}^2, \\
\sigma_{D_{N_1}}^3 &= C_F \frac{4Q}{q_T^2} \frac{(-1 + \hat{z})^2(-1 - \hat{x} + \hat{z}(-1 + 2\hat{x}))}{\hat{z}^2}, \\
\sigma_{D_{N_1}}^4 &= -C_F \frac{4}{q_T} \frac{(-1 + \hat{z})(1 + \hat{x} - \hat{x}\hat{z})}{\hat{z}}, \\
\sigma_{D_{N_1}}^8 &= \frac{1}{2} \sigma_{D_{N_1}}^3, \\
\sigma_{D_{N_1}}^9 &= \frac{1}{2} \sigma_{D_{N_1}}^9,
\end{align*}
\]

(79)
\[
\begin{align*}
\hat{\sigma}_{DN3}^1 &= -C_F \frac{4Q^2 (-1 + \hat{z})^2 (1 + 2\hat{z}(2 - 3\hat{x})\hat{x} + \hat{z}^2 (1 - 6\hat{x} + 6\hat{x}^2))}{\hat{x}\hat{z}^3}, \\
\hat{\sigma}_{DN3}^2 &= -2\hat{\sigma}_{DN1}^2, \\
\hat{\sigma}_{DN3}^3 &= -C_F \frac{8Q (-1 + \hat{z})^2 (-\hat{x} + \hat{z}(-1 + 2\hat{x}))}{\hat{z}^2}, \\
\hat{\sigma}_{DN3}^4 &= -\hat{\sigma}_{DN1}^4, \\
\hat{\sigma}_{DN3}^8 &= 0, \\
\hat{\sigma}_{DN3}^9 &= 0,
\end{align*}
\]  
\begin{align*}
\hat{\sigma}_{O1}^1 &= -C_F \frac{8Q^2 (-1 + \hat{z})^2 (-1 - \hat{x} + \hat{z}(-2 + 3\hat{x}))}{\hat{z}^3}, \\
\hat{\sigma}_{O1}^2 &= -C_F \frac{16 \hat{x}(-1 + \hat{z})}{\hat{z}}, \\
\hat{\sigma}_{O1}^3 &= -C_F \frac{4Q (-1 + \hat{z})(-1 - 3\hat{x} + \hat{z}(-1 + 4\hat{x}))}{\hat{z}^2}, \\
\hat{\sigma}_{O1}^4 &= \frac{1}{2} \hat{\sigma}_{O1}^0, \\
\hat{\sigma}_{O1}^8 &= -C_F \frac{4Q (-1 + \hat{z})(-1 - \hat{x} + \hat{z}(-1 + 2\hat{x}))}{\hat{z}^2}, \\
\hat{\sigma}_{O1}^9 &= \frac{1}{2} \hat{\sigma}_{O1}^0,
\end{align*}
\]  
\begin{align*}
\hat{\sigma}_{O2}^1 &= C_F \frac{4Q^2 (-1 + \hat{z})^2 (1 + \hat{x} + 3\hat{z}(2 - 3\hat{x})\hat{x} + 2\hat{z}^2 (1 - 6\hat{x} + 6\hat{x}^2))}{\hat{x}\hat{z}^3}, \\
\hat{\sigma}_{O2}^2 &= -C_F \frac{8 \hat{x}(-1 + \hat{z})(3 - 2\hat{z})}{\hat{z}}, \\
\hat{\sigma}_{O2}^3 &= C_F \frac{2Q (-1 + \hat{z})(1 + \hat{z}(5 - 16\hat{x}) + 7\hat{x} + \hat{z}^2 (-4 + 8\hat{x}))}{\hat{z}^2}, \\
\hat{\sigma}_{O2}^4 &= \frac{1}{2} \hat{\sigma}_{O2}^2, \\
\hat{\sigma}_{O2}^8 &= \frac{1}{2} \hat{\sigma}_{O2}^0, \\
\hat{\sigma}_{O2}^9 &= \frac{1}{2} \hat{\sigma}_{O2}^1,
\end{align*}
\[
\begin{align*}
\begin{cases}
\hat{\sigma}^1_{DF} = \frac{C_F Q^2}{N q_T^2} \frac{(-1 + \hat{z})(1 + \hat{x} - \hat{z}\hat{x})}{\hat{x}\hat{z}^2}, \\
\hat{\sigma}^2_{DF} = 0, \\
\hat{\sigma}^3_{DF} = -\frac{C_F}{N q_T^2} \frac{Q}{\hat{z}}, \\
\hat{\sigma}^4_{DF} = -\frac{C_F}{N q_T}, \\
\hat{\sigma}^5_{DF} = \hat{\sigma}^3_{DF}, \\
\hat{\sigma}^6_{DF} = \hat{\sigma}^4_{DF}, \\
\hat{\sigma}^7_{DF} = -\frac{C_F}{N q_T} \frac{1}{x}, \\
\hat{\sigma}^8_{DF} = \hat{\sigma}^9_{DF}.
\end{cases}
\end{align*}
\] (83)

\[
\begin{align*}
\begin{cases}
\hat{\sigma}^1_{DF} & = -\frac{C_F}{N q_T} \frac{1}{\hat{z}} \frac{3\hat{z}(1 - 2\hat{z})\hat{x} + \hat{x}(1 + \hat{x}) + \hat{z}^2(1 - 6\hat{x} + 6\hat{x}^2)}{\hat{x}\hat{z}^2}, \\
\hat{\sigma}^2_{DF} & = -\frac{C_F}{N q_T^2} \frac{4q_T}{Q} \hat{z}^2, \\
\hat{\sigma}^3_{DF} & = -\frac{C_F}{N Q} \frac{1}{\hat{z}} \frac{\hat{x}(-1 - 2\hat{z} + \hat{z}(-2 + 4\hat{x}))}{\hat{x}\hat{z}^2}, \\
\hat{\sigma}^4_{DF} & = -\frac{C_F}{N q_T} \frac{1}{\hat{z}} \frac{(-1 + \hat{x})(-1 + 2(-1 + \hat{z})\hat{x})}{\hat{x}\hat{z}^2}, \\
\hat{\sigma}^5_{DF} & = \frac{C_F}{N q_T} \frac{1}{\hat{z}} \frac{\hat{x}}{\hat{z}}, \\
\hat{\sigma}^6_{DF} & = \frac{C_F}{N q_T} \frac{1}{\hat{z}} \frac{1 - 1 + \hat{x}}{\hat{x}\hat{z}^2},
\end{cases}
\end{align*}
\] (84)

\[
\begin{align*}
\begin{cases}
\hat{\sigma}^1_{DF} & = \frac{C_F}{N q_T} \frac{Q^2}{q_T^2} \frac{(-1 + \hat{z})(-1 + \hat{z} + 5\hat{x} - 6\hat{z}\hat{x} - 6\hat{x}^2 + 6\hat{x}^2)}{\hat{x}\hat{z}^2} + \frac{C_F}{q_T} (\hat{x} - 1)^2(1 + \hat{x}) - \hat{z}(\hat{x} - 1)^2(1 + 6\hat{x}) - \hat{z}^3(1 - 6\hat{x} + 6\hat{x}^2) + \hat{z}^2(1 + \hat{x} - 6\hat{x}^2 + 6\hat{x}^3)\frac{\hat{x}}{\hat{x}\hat{z}^2}, \\
\hat{\sigma}^2_{DF} & = \frac{C_F}{N q_T^2} \frac{4q_T}{Q} \frac{\hat{z}(1 - \hat{z} - \hat{z}\hat{x})}{\hat{x}\hat{z}^2} - \frac{C_F}{Q} \frac{4q_T}{\hat{z}^2} \frac{\hat{z}(1 - \hat{z} - \hat{z}\hat{x})}{\hat{x}\hat{z}^2}, \\
\hat{\sigma}^3_{DF} & = \frac{C_F}{N q_T} \frac{2Q}{q_T} \frac{(-1 + \hat{z})(-\hat{x} + \hat{z}(-1 - 2\hat{x}))}{\hat{x}} + \frac{2C_F}{Q} \frac{(\hat{z} + \hat{z}^2 + \hat{x} - 2\hat{z}\hat{x} - 2\hat{x}^2\hat{x} - \hat{x}^2 + 2\hat{x}\hat{x}^2)}{(-1 + \hat{z} + \hat{x})}, \\
\hat{\sigma}^4_{DF} & = \frac{C_F}{N q_T} \frac{1}{\hat{x}} \frac{(1 + \hat{x} + 2\hat{z}\hat{x} - \hat{z}(1 + 2\hat{x}))}{\hat{x}} - \frac{C_F}{q_T} \frac{2(\hat{z} + \hat{z}^2 + \hat{x} - 2\hat{z}\hat{x} - 2\hat{x}^2\hat{x} - \hat{x}^2 + 2\hat{x}\hat{x}^2)}{(-1 + \hat{z} + \hat{x})}, \\
\hat{\sigma}^5_{DF} & = \frac{C_F}{N q_T} \frac{2}{\hat{z}} - \frac{C_F}{N Q} \frac{\hat{z}(-1 + \hat{z})}{\hat{x}}, \\
\hat{\sigma}^6_{DF} & = \frac{C_F}{N q_T} \frac{1}{\hat{z}} \frac{(1 - \hat{x} + \hat{z}(-1 - 2\hat{x}))}{\hat{x}} - \frac{C_F}{q_T} \frac{2(\hat{z} + \hat{z}^2 + \hat{x} - 2\hat{z}\hat{x} - 2\hat{x}^2\hat{x} - \hat{x}^2 + 2\hat{x}\hat{x}^2)}{(-1 + \hat{z} + \hat{x})}.
\end{cases}
\end{align*}
\] (85)
\[
\begin{align*}
\dot{\sigma}_{DF5}^1 &= -\frac{C_F}{N} \frac{Q}{q_T} \left( -1 + \hat{x} \right)^2 (1 + \hat{x} + 6 \hat{z}^2 \hat{x} - \hat{z}(1 + 6 \hat{x})) \\
&- \frac{C_F}{q_T} \left( \hat{x} - 1 \right)^2 (1 + \hat{x} - \hat{z}(\hat{x} - 1)^2 (1 + 6 \hat{x}) - \hat{z}^3 (1 - 6 \hat{x} + \hat{z}^2) + \hat{z}^2 (1 + \hat{x} - 6 \hat{x}^2 + 6 \hat{x}^3) \\
\dot{\sigma}_{DF5}^2 &= \frac{C_F}{N} \frac{4q_T}{Q} (-1 + \hat{x}) \hat{x} + C_F \frac{4q_T}{Q^2} \frac{\hat{z}(1 + \hat{x} - \hat{z}) \hat{x}}{(-1 + \hat{z} + \hat{x})} \\
\dot{\sigma}_{DF5}^3 &= \frac{C_F}{N} \frac{2}{Q} \frac{(-1 + \hat{x})(-\hat{x} + \hat{z}(-1 + 2 \hat{x}))}{\hat{z} (-1 + \hat{z} + \hat{x})} + 2C_F \frac{(\hat{z} + \hat{z}^2 + \hat{x} - 2 \hat{z}^2 \hat{x} - 2 \hat{x}^2 \hat{x} - \hat{x}^2 + 2 \hat{x}^2)}{Q (-1 + \hat{z} + \hat{x})}, \\
\dot{\sigma}_{DF5}^4 &= \frac{C_F}{N} \frac{1}{q_T} \frac{(-1 + \hat{x})(-1 + \hat{z} + \hat{x} - 2 \hat{z} \hat{x} - 2 \hat{x}^2 + 2 \hat{x} \hat{x}^2)}{\hat{x} (-1 + \hat{z} + \hat{x})} + C_F \frac{2}{q_T} \frac{(-1 + \hat{x})(-1 + \hat{z} + \hat{x}(-1 + 2 \hat{x}))}{\hat{x} (-1 + \hat{z} + \hat{x})}, \\
\dot{\sigma}_{GF2}^1 &= \frac{C_F}{N} \frac{Q^2}{q_T} \frac{(1 + \hat{z})(1 - \hat{x} + \hat{z})}{\hat{x} \hat{z}^2}, \\
\dot{\sigma}_{GF2}^2 &= 0, \\
\dot{\sigma}_{GF2}^3 &= \dot{\sigma}_{DF2}^3, \\
\dot{\sigma}_{GF2}^4 &= \dot{\sigma}_{DF2}^3, \\
\dot{\sigma}_{GF2}^8 &= \dot{\sigma}_{DF2}^3, \\
\dot{\sigma}_{GF2}^9 &= \dot{\sigma}_{DF2}^3,
\end{align*}
\]
In this paper we have studied the transversely polarized spin-1/2 hyperon production in SIDIS, $ep \rightarrow e\Lambda^\uparrow X$. Specifically, we have derived the LO twist-3 gluon FF contribution to the polarized cross section. Since the twist-3 gluon FFs are related to the $qgq$-FFs through the EOM relations and the LIRs, we consistently took into account the latter contribution together. This has completed the twist-3 LO cross section for this process together with the results for the contribution from the twist-3 DF and the twist-3 quark FFs derived in [1]. The final result for the cross section is given in (54). It consists of five components with different azimuthal structures as

$$
\frac{d^6\sigma}{dxdydzdQ_2d\phi d\chi} = F_1 \sin \Phi_S + F_2 \sin \Phi_S \cos \varphi + F_3 \sin \Phi_S \cos 2\varphi + F_4 \cos \Phi_S \sin \varphi + F_5 \cos \Phi_S \sin 2\varphi,
$$

(91)

4 Summary

In this paper we have studied the transversely polarized spin-1/2 hyperon production in SIDIS, $ep \rightarrow e\Lambda^\uparrow X$. Specifically, we have derived the LO twist-3 gluon FF contribution to the polarized cross section. Since the twist-3 gluon FFs are related to the $qgq$-FFs through the EOM relations and the LIRs, we consistently took into account the latter contribution together. This has completed the twist-3 LO cross section for this process together with the results for the contribution from the twist-3 DF and the twist-3 quark FFs derived in [1]. The final result for the cross section is given in (54). It consists of five components with different azimuthal structures as

$$
\frac{d^6\sigma}{dxdydzdQ_2d\phi d\chi} = F_1 \sin \Phi_S + F_2 \sin \Phi_S \cos \varphi + F_3 \sin \Phi_S \cos 2\varphi + F_4 \cos \Phi_S \sin \varphi + F_5 \cos \Phi_S \sin 2\varphi,
$$

(91)
where $\phi = \phi - \chi$ is the relative azimuthal angle between the lepton ($\phi$) and the hadron ($\chi$) planes and $\Phi_S$ is the azimuthal angle of the transverse spin vector of $\Lambda^+$ measured from the hadron plane with the structure functions $F_{1,2,3,4,5}$ written as convolution of the twist-3 FFs and the quark DF in the proton and the partonic hard cross sections. The LO cross section given in [1] and the present study contains several unknown nonperturbative functions, and their determination requires global analyses of many twist-3 processes in which the same twist-3 functions appear. Information from analyses of small-$P_T$ data in terms of the TMD factorization is also of great help to constrain some of the twist-3 functions. In any case our twist-3 cross section formula is the starting point of analyzing the large-$P_T$ hyperon polarization in SIDIS which we hope to be measured in the future EIC experiment.

Acknowledgments

This work has been supported by the establishment of Niigata university fellowships towards the creation of science technology innovation (R.I.), the Grant-in-Aid for Scientific Research from the Japanese Society of Promotion of Science under Contract Nos. 19K03843 (Y.K.) and 18J11148 (K.Y.), National Natural Science Foundation in China under grant No. 11950410495, Guangdong Natural Science Foundation under No. 2020A1515010794 and research startup funding at South China Normal University (S.Y.).

References

[1] Yuji Koike, Kazuki Takada, Sumire Usui, Kenta Yabe, and Shinsuke Yoshida. Transverse polarization of hyperons produced in semi-inclusive deep inelastic scattering. *Phys. Rev. D*, 105(5):056021, 2022.

[2] Koichi Kanazawa, Yuji Koike, Andreas Metz, and Daniel Pitonyak. Towards an explanation of transverse single-spin asymmetries in proton-proton collisions: the role of fragmentation in collinear factorization. *Phys. Rev. D*, 89(11):111501, 2014.

[3] Leonard Gamberg, Zhong-Bo Kang, Daniel Pitonyak, and Alexei Prokudin. Phenomenological constraints on $A_N$ in $p^p \rightarrow p X$ from Lorentz invariance relations. *Phys. Lett. B*, 770:242–251, 2017.

[4] Xiangdong Ji, Jian-Wei Qiu, Werner Vogelsang, and Feng Yuan. A Unified picture for single transverse-spin asymmetries in hard processes. *Phys. Rev. Lett.*, 97:082002, 2006.

[5] Xiangdong Ji, Jian-Wei Qiu, Werner Vogelsang, and Feng Yuan. Single-transverse spin asymmetry in semi-inclusive deep inelastic scattering. *Phys. Lett. B*, 638:178–186, 2006.

[6] Yuji Koike, Werner Vogelsang, and Feng Yuan. On the Relation Between Mechanisms for Single-Transverse-Spin Asymmetries. *Phys. Lett. B*, 659:878–884, 2008.

[7] Jian Zhou, Feng Yuan, and Zuo-Tang Liang. Hyperon Polarization in Unpolarized Scattering Processes. *Phys. Rev. D*, 78:114008, 2008.

[8] Koichi Kanazawa, Yuji Koike, Andreas Metz, Daniel Pitonyak, and Marc Schlegel. Operator Constraints for Twist-3 Functions and Lorentz Invariance Properties of Twist-3 Observables. *Phys. Rev. D*, 93(5):054024, 2016.

[9] Yuji Koike, Kenta Yabe, and Shinsuke Yoshida. Exact Relations for Twist-3 Gluon Distribution and Fragmentation Functions from Operator Identities. *Phys. Rev. D*, 101(5):054017, 2020. [arXiv:1912.11199 [hep-ph]]

[10] Y. Guan et al. Observation of Transverse $\Lambda/\bar{\Lambda}$ Hyperon Polarization in $e^+e^-$ Annihilation at Belle. *Phys. Rev. Lett.*, 122(4):042001, 2019.

[11] F. Murgia U. D’Alesio and M. Zaccheddu. First extraction of the $\Lambda$ polarizing fragmentation function from Belle $e^+e^-$ data. *Phys. Rev. D*, 102(5):054001, 2020.

[12] Daniel Callos, Zhong-Bo Kang, and John Terry. Extracting the transverse momentum dependent polarizing fragmentation functions. *Phys. Rev. D*, 102(9):096007, 2020.

[13] Kai-Bao Chen, Zuo-Tang Liang, Yan-Lei Pan, Yu-Kun Song, and Shu-Yi Wei. Isospin Symmetry of Fragmentation Functions. *Phys. Lett. B*, 816:136217, 2021.
[14] Kai-bao Chen, Zuo-tang Liang, Yu-kun Song, and Shu-yi Wei. Longitudinal and transverse polarizations of Λ hyperon in unpolarized SIDIS and e+e- annihilation. *Phys. Rev. D*, 105(3):034027, 2022.

[15] Yuji Koike, Kenta Yabe, and Shinsuke Yoshida. Twist-3 gluon fragmentation contribution to polarized hyperon production in unpolarized proton-proton collision. *Phys. Rev. D*, 104(5):054023, 2021.

[16] P. J. Mulders and J. Rodrigues. Transverse momentum dependence in gluon distribution and fragmentation functions. *Phys. Rev. D*, 63:094021, 2001. [hep-ph/0009343].

[17] Kenta Yabe, Yuji Koike, Andreas Metz, Daniel Pitonyak, and Shinsuke Yoshida. Twist-3 Gluon Fragmentation Contribution to the Polarized Hyperon Production in Unpolarized Proton–Proton Collision. *JPS Conf. Proc.*, 26:021016, 2019.

[18] Yabe Kenta, Yuji Koike, Andreas Metz, Daniel Pitonyak, and Shinsuke Yoshida. Twist-3 fragmentation contribution to polarized hyperon production in unpolarized proton-proton collision. *PoS*, SPIN2018:192, 2019.

[19] Leonard Gamberg, Zhong-Bo Kang, Daniel Pitonyak, Marc Schlegel, and Shinsuke Yoshida. Polarized hyperon production in single-inclusive electron-positron annihilation at next-to-leading order. *JHEP*, 01:111, 2019. [arXiv:1810.08645 [hep-ph]]

[20] Rui-bin Meng, Fredrick I. Olness, and Davison E. Soper. Semiinclusive deeply inelastic scattering at electron - proton colliders. *Nucl. Phys. B*, 371:79–110, 1992.

[21] Koichi Kanazawa and Yuji Koike. Contribution of twist-3 fragmentation function to single transverse-spin asymmetry in semi-inclusive deep inelastic scattering. *Phys. Rev. D*, 88:074022, 2013. [arXiv:1309.1215 [hep-ph]]

[22] Yuji Koike, Andreas Metz, Daniel Pitonyak, Kenta Yabe, and Shinsuke Yoshida. Twist-3 fragmentation contribution to polarized hyperon production in unpolarized hadronic collisions. *Phys. Rev. D*, 95(11):114013, 2017. [arXiv:1703.09399 [hep-ph]]