Research Article

Study on a Heuristic Wheelset Structure without Rail Corrugation on Sharply Curved Tracks

G. M. Mei 1,2, G. X. Chen 1,2, S. Yan, 2 and R. X. Chen 2,3

1State Key Laboratory of Traction Power, Southwest Jiaotong University, Chengdu, China
2School of Mechanical Engineering, Southwest Jiaotong University, Chengdu, China
3Department of Mechanical and Aerospace Engineering, University of Virginia, Charlottesville, USA

Correspondence should be addressed to G. X. Chen; chen_guangx@163.com

Received 27 April 2021; Accepted 20 July 2021; Published 13 August 2021

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Rail corrugation on low rails of sharply curved railway curves is still a difficult problem to solve worldwide. Nearly all low rails of the sharply curved railway curves incur rail corrugation. In the present study, an active method to remedy rail corrugation was studied. From the viewpoint of the frictional self-excited oscillation of a wheelset-track system causing rail corrugation, the effect of wheelset structures on rail corrugation was studied. Three frictional self-excited oscillation models of wheelset-track systems with different wheelset structures were established, which include a heuristic wheelset structure and two being used in the railway industry. The incidence trends of the self-excited oscillations of these three wheelset-track systems were studied. It was found that the wheelset structure has an important effect on rail corrugation, and that the heuristic wheelset structure can restrain or get rid of rail corrugation. With the parameter sensitivity analysis, it was found that when the friction coefficient between the wheel and rail, rail gauge, rail cant, and sleeper span changes to some extent, the heuristic wheelset structure is robust enough to prevent rail corrugation. The proposed heuristic wheelset structure can be used as a potential solution to rail corrugation on sharply curved tracks.

1. Introduction

The metro transportation system is a good solution to traffic jams in big cities. In China, the metro transportation system has been vigorously developed. The total mileage of the metro transportation lines has reached about 7,500 km by the end of 2020. Government administrators and residents feel the benefits of the subway transportation system. However, the fly in the ointment is that the loud noise generated by the subway transportation system is very disturbing. Most of the loud noise has been identified to be due to rail corrugation as shown in Figure 1. Once rail corrugation is eliminated, the metro transportation system will become much quieter.

In order to suppress and eliminate rail corrugation, railway administrators and researchers have made great efforts since the early part of the last century. In the establishment and development of the rail corrugation theory, Kalousek and Johnson [1], Hempelmann and Knothe [2], Nielsen et al. [3], Gomez et al. [4], Meehan et al. [5], Wu and Thompson [6], and Grassie [7] made outstanding contributions. Researchers generally accept that rail corrugation is governed by the material damage mechanism and the wavelength fixing mechanism [7–9]. Vibration of the vehicle’s unsprung mass on the track stiffness is considered to be the most common wavelength fixing mechanism, and wear is considered to be a dominant material damage mechanism [7]. In the application of rail corrugation control technology, many measures were taken to suppress or eliminate rail corrugation. In the past, the discreet support of the rails was considered to be responsible for rail corrugation. Researchers laid several sections of the railway lines with continuous rail support to validate the suppression of the continuous rail support on rail corrugation. Results showed that the continuous rail support can slow the growth of rail corrugation but does not prevent it completely [10, 11]. Eadie et al. [12] experimentally researched the suppression of the...
friction modifier on rail corrugation. They reported that the friction modifier was used to test four tracks and found that no rail corrugation occurred on three test tracks; rail corrugation only occurred on one of these four test tracks. The friction modifier has been applied in some metro tracks to suppress or eliminate rail corrugation. More recently, Meehan et al. [13] reported that variation of the passing speed can suppress rail corrugation. Collette et al. [14] developed a rotational vibration absorber for the alleviation of rail corrugation.

At present, grinding is most commonly used to remove rail corrugation worldwide. In China’s metro lines, grinding is generally applied to remedy rail corrugation. Few friction modifiers are used to remedy rail corrugation. It is because the costs of the equipment for spraying the friction modifier and maintenance are very expensive. The variation of the passing speed is also difficult to be used to suppress rail corrugation in the metro industry. It is because the distance between two underground stop stations is short, about 800–1200 m, in which the metro train is difficult to largely change its passing speed.

There is a good example in the China’s railway industry. In 2000, the maximum operating speed of freight trains running on China’s main-lines was increased to 120 km/h from 80 km/h. In the initial stage of increasing speed, the empty freight cars with 3-piece bogies were found to easily derail when the running speed of the freight cars is larger than 90–100 km/h. The derailment of the empty freight cars was attributed to severe hunting motion. Since the generation mechanism for the hunting motion of railway vehicles was well understood, increasing the warp stiffness of the three-piece bogies can improve the hunting stability of freight cars with three-piece bogies. After about 2–3 years, the derailment of the empty freight cars with 3-piece bogies at speeds of 100–120 km/h had been solved completely by adding two cross-braced rods between two side frames of each bogie. Since 2003, no derailment event of the empty freight cars with three-piece bogies has occurred due to severe hunting. Inspired by this successful application, the authors hope to develop a practical technology to greatly suppress or eliminate rail corrugation on new metro lines or old metro lines, based on the new research progress in rail corrugation.

The authors carried out many heuristic calculations and have found out a wheelset structure which can greatly suppress or eliminate rail corrugation on sharply curved tracks in theory. In the paper, the authors will mainly introduce the simulation result of the heuristic wheelset structure suppressing rail corrugation. In the introductory section, several methods used to suppress or eliminate rail corrugation are introduced. In Section 2, the rail corrugation prediction method is briefly mentioned, which is based on the friction-induced self-excited oscillation of wheelset-track systems. Three rail corrugation prediction models corresponding to different wheel structures are presented. In Section 3, a parameter sensitivity analysis of rail corrugation of the wheelset-track systems is presented. In Section 4, a discussion on the application of the wheelset-track system with the heuristic wheelset structure is made. The conclusions are drawn in the final section.

2. Prediction Model of Rail Corrugation

2.1. Prediction Method of Rail Corrugation. In the literature, there are many methods available to predict rail corrugation. These methods may roughly be compartmentalized into two categories. One is based on the generation mechanism of the interaction between the wheel and rail due to surface roughness causing rail corrugation [15, 16]. The other is based on the generation mechanism of the stick-slip motion of wheel-rail systems causing rail corrugation [17–19]. Which one of these two categories is more suitable to predict rail corrugation depends on which one of their prediction results is closer to the real rail corrugation events. In China, metro lines and railway main-lines have been equipped with the 60 kg/m-type rails and worn-type tread profile for a long time. Now rail corrugation occurring on China’s metro lines and main lines has a strong regularity. The regularity is described as follows: nearly all low rails of sharply curved tracks whose radii $R \leq 350$ m incur rail corrugation, but the incidence likelihood of rail corrugation on high rails of the same sharply curved tracks is less than 10–15%. Two rails of mild railway curves whose radii are larger than 650–800 m or tangential tracks rarely incur rail corrugation. The incidence likelihood of rail corrugation on two rails of the mildly curved tracks or tangential tracks is less than 10–15% for metro lines, or 3–5% for railway main-lines. There are several publications on the verification of rail corrugation prediction models in the literature [20–22]. After serious consideration, the authors think that the rail corrugation prediction method proposed by Chen is more suitable to predict the above-mentioned rail corrugation regularity, which is based on the mechanism of frictional self-excited oscillation of a wheelset-track system causing rail corrugation [19]. In addition, Chen’s prediction method has been validated partially [22] and the source code of the Chen’s prediction method is open to researchers so that researchers can easily reproduce the prediction result [23–25]. Most recently, Beshbichi and Fourie applied Chen’s method to study rail corrugation [26, 27]. Therefore, Chen’s rail corrugation prediction method was used to estimate the incidence likelihood of rail corrugation in the present study.

2.2. Finite Element Model of the Frictional Self-Excited Oscillation of the Wheelset-Track System. In some cases, the slip of wheels on rails occurs. In the sharply curved track, for example, the creep forces between the outer wheel of the leading wheelset of each bogie and the high rail, and between the inner wheel and the low rail are always saturated. That is to say, the creep forces are equal to the normal forces multiplied by the coefficient of friction. In addition, the creep force may also be saturated when a brake application is abnormally imposed. When the creep force between the wheel and rail gets saturated, the wheelset-track system probably incurs self-excited oscillations, which are governed by the following equation [19]:

$$M_2 \dddot{u} + C_2 \dot{u} + K_2 u = 0,$$  
(1)
where \( \mathbf{u} \) is the node displacement vector, \( \mathbf{M}_r, \mathbf{C}_r, \) and \( \mathbf{K}_r \) are the mass matrix, damping matrix, and stiffness matrix, respectively, which are all asymmetric matrices in the presence of friction.

The eigenvalue equation of equation (1) is expressed as follows:

\[
(\mathbf{M}_r \lambda^2 + \mathbf{C}_r \lambda + \mathbf{K}_r) \phi = 0, \quad (2)
\]

where \( \lambda \) is an eigenvalue vector. The general solution of equation (1) is expressed as follows:

\[
\mathbf{u}(t) = \sum \phi_i \exp(\lambda_i t) = \sum \phi_i \exp(\alpha_i + j \omega_i t), \quad (3)
\]

where \( \phi_i \) is the \( i \)th eigenvector of (2), \( \lambda_i = \alpha_i + j \omega_i \) is the \( i \)th eigenvalue of (2), \( \alpha_i \) and \( \omega_i \) are the real part and imaginary part of the \( i \)th eigenvalue, respectively, \( j \) is the imaginary unit. From (3), it is known that when \( \alpha_i > 0 \), the vibrational displacement vector of nodes will diverge with increasing time. That is, the friction system loses stability.

The general procedure for establishing the frictional self-excited oscillation of the wheelset-track system is described as follows [19]:

1. Establish a full-size solid model of the wheelset-track system.
2. Set the material parameters, mesh the solid model, define contact pairs, and set spring and damper elements to connect the rails and sleepers and the sleepers and the ground.
3. Nonlinear static analysis for applying axle box forces.
4. Nonlinear static analysis to impose the transversal sliding speed on the wheelset.
5. Normal mode analysis to extract natural frequencies of the undamped system.
6. Complex eigenvalue analysis that incorporates the effect of friction coupling.

2.3. Wheelset-Track Systems with Three Different Wheelset Structures. Nowadays, there are three kinds of metro wheel structures used in China. Among these three kinds of metro wheel structures, the majority is characterized by a S-shape web plate as shown in Figure 2(a), and the minority is characterized by plane web plates as shown in Figures 2(b) and 2(c). Figure 2(d) shows a metro wheel structure being used in Europe. It was found that the metro wheel structures used in China are different from those used in Europe. The metro wheel structure being used in Europe is also characterized by a different S-shaped web plate. From Figures 2(a)–2(d), it is seen that there are different wheelset structures in the metro railway industry worldwide. Massi and Lc Baillet [28] and Zarraga et al. [29] revealed that the different structures of friction systems could suppress or remove frictional self-excited oscillation of the friction system in some cases. The authors made many heuristic simulation calculations and have found a heuristic wheelset structure, which can remove frictional self-excited oscillation of the wheelset-track system, then rail corrugation. The heuristic wheelset structure is shown in Figures 3(a) and 3(b). Figures 3(c) and 3(d) depict the wheel structure being used in China’s metro vehicles. Comparing Figures 3(a)–3(b) with Figures 3(c)–3(d), the heuristic wheelset structure is found to be different from the wheelset structures being used in China’s and Europe’s metro vehicles. Figure 4 shows three finite element models of the wheelset-track systems with different wheelset structures. In order to clearly depict the wheelset structures of these three wheelset-track systems in Figures 4(a)–4(c), 1/4 of each wheelset was hidden in each model. Figure 4(d) shows the wheel-rail contact forces of the leading wheelset on the sharply curved track. In Figure 4(d), \( N_z \) and \( N_R \) represent normal contact forces, respectively. \( F_L \) and \( F_R \) stand for lateral creep forces, respectively. \( \delta_L \) and \( \delta_R \) represent the left and right contact angles, respectively. \( K_{RL} \) and \( C_{RL} \) represent lateral stiffness and damping of a single fastener, respectively. \( K_{SV} \) and \( C_{SV} \) stand for vertical stiffness and damping of a single fastener, respectively. \( K_{SL} \) and \( C_{SL} \) represent lateral support stiffness and damping of the subgrade. \( K_{SV} \) and \( C_{SV} \) stand for vertical support stiffness and damping of the subgrade, respectively. The parameters of tracks and vehicles are presented in Table 1. The authors applied SIMPACK to simulate the dynamics wheel-rail interaction when the vehicle negotiates a curved track with a radius of 350 m. The simulation result is listed in Table 2.

3. Results

3.1. Effect of Wheelset Structures on Rail Corrugation. When a metro vehicle negotiates a curved track of radius 350 m with superelevation 120 mm at 55 km/h, the angle of attack of the leading wheelset is about 0.23° according to the SIMPACK simulation. The lateral sliding speed of the leading wheelset relative to the rail: \( V_{sr} = (55000/3.6) \times (0.23 \times 3.14/180) = 61.3 \text{ mm/s} \). When the axle box suspension forces listed in Table 2 act on the axle boxes of three wheelsets, as shown in Figure 4(d), the incidence trend of the self-excited oscillation of the wheelset-track systems is depicted in Figure 5. From Figure 5(a), it is seen that there is not any unstable oscillation in the range of frequency 0–1000 Hz for the heuristic wheelset. From Figures 5(b) and 5(c), it is seen that there are several unstable oscillations, whose effective damping rates are negative, in the range of frequency 0–1000 Hz for the plane-web plate wheelset and the S-shaped web plate wheelset. These results demonstrate that rail corrugation will occur when the wheelset structures shown in Figures 4(b) and 4(c) are used. Figure 6 shows some of their unstable oscillation modes.

The result shown in Figure 5 suggests that rail corrugation can be suppressed or eliminated by applying the heuristic wheelset structure. This is an encouraging result. In the brake squeal research community, it is well known that brake squeal can be suppressed or eliminated by modifying the structure of the brake system, and some practical applications proved to be successful in the suppression and elimination of brake squeal [28, 29]. In the present study, the structure modification method was used to improve the resistance of the wheelset-track systems to rail corrugation. The heuristic wheelset structure shown in Figure 4(a) being
capable of suppressing or eliminating rail corrugation is reasonable from the viewpoint of friction-induced brake oscillation and squeal.

It needs to be mentioned that since the wheelsets and rails were modeled as rigid bodies in SIMPACK simulation; the wheel-rail interaction forces, including creep forces and normal forces for the wheelset shown in Figure 4(a), are the same as for the wheelsets shown in Figures 4(b)–4(c). Normal force $N_L = 46280\, \text{N}$, $N_R = 55120\, \text{N}$ from the SIMPACK simulation, as listed in Table 2. By the finite element simulations for the wheelsets shown in Figures 4(a), 4(b), and 4(c), the normal forces $N_L = 45920$, $45580$, $45350\, \text{N}$, $N_R = 56270$, $56550$, $56850\, \text{N}$, respectively. Therefore, the wheel-rail interaction forces for different wheelset structures are approximately close to each other. But the fatigue strengths of different wheelset structures are probably different from each other. The fatigue strengths of different wheelset structures are beyond the present research scope. This paper will not be involved with them.

3.2. Effect of Rail Gauge on Rail Corrugation. The rail gauge will change the positions of the contact points between wheels and rails, which probably affects the motion stability of the wheelset-track system. Numerical results show that when the rail gauge changes from 1435 mm to 1450 mm, there is no unstable oscillation for the heuristic wheelset-track system. Figure 7 shows the effect of rail gauge on rail corrugation of the wheelset-track systems. Figure 7(a) shows the distribution of the unstable oscillation of the wheelset-track system with the heuristic wheel structure at the rail gauge of 1450 mm. It is seen that there is no negative effective damping rate for the wheelset-track system with the heuristic wheel structure at the rail gauge of 1450 mm, demonstrating that no rail corrugation occurs in this case. Figure 7(b) shows the distribution of the unstable oscillation of the wheelset-track system with the plane web plate wheel at the rail gauge of 1450 mm. It is found that there are still three unstable oscillations in the case. Further, it is found that there are still three unstable oscillations when the rail gauge is equal to 1440 mm and 1445 mm. It is just that the effective damping ratios and their corresponding vibration frequencies change a little bit. Figure 7(c) shows the distribution of the unstable oscillation of the wheelset-track system with the S-shaped web plate wheel at the rail gauge of 1450 mm. It is found that there are still three unstable oscillations of the wheelset-track system in this case. Further, it is found that there are still three unstable oscillations when the rail gauge is equal to 1440 mm and 1445 mm. It is just that the effective damping ratios and their corresponding vibration frequencies change a little bit.

El Beshbichi et al. [26] studied the incidence trend of rail corrugation on European metro lines. In Beshbichi’s study, the parameters of rails, wheels as shown in Figure 2(d), sleepers, and fasteners were from the European practical metro lines. Beshbichi’s conclusion showed that there is no friction-induced self-excited oscillation and no rail corrugation in the frequency range of 0–1000 Hz at the rail gauge of 1435 mm. But when the rail gauge is equal to 1450–1465 mm, there are some frictional self-excited oscillations. That suggests that in this case, the low rail will
incur corrugation. For the wheelset-track system shown in Figure 4(b), there is no unstable vibration at the rail gauge of 1428 mm. That suggests that in this case, rail corrugation will not occur. Therefore, it can be concluded that the gauge has an important effect on rail corrugation.

3.3. Effect of Friction Coefficient on Rail Corrugation. It has been reported that the incidence trend of the frictional self-excited oscillation will increase with increasing friction coefficient [19]. That is to say, that the incidence trend of the frictional self-excited oscillation is the maximum when the
The friction coefficient is equal to the maximum. In actual metro lines, the friction coefficient at the wheel/rail interface falls in the range of 0.25–0.6. Figure 8 demonstrates the effects of the friction coefficient on rail corrugation of the wheelset-track systems. From Figure 8(a), it is found that when the friction coefficient at the wheel/rail interface is equal to 0.6, there is no unstable oscillation of the wheelset-track system with the heuristic wheelset structure in the frequency range of 0–1000 Hz. This indicates that rail corrugation will not occur in this case. Comparing Figure 8(b) with Figure 5(b), it is found that when the rail cant is equal to 1/20, there is an unstable oscillation of the wheelset-track system with the heuristic wheelset structure in the range of frequency 0–1000 Hz. This suggests that rail corrugation will not occur in this case. Comparing Figure 8(c) with Figure 5(c), it is found that when the rail cant is equal to 1/20, there is unstable oscillation of the wheelset-track system with the S-shaped web plate wheel. This indicates that rail corrugation will not occur in these cases.

3.4. Effect of Rail Cant on Rail Corrugation. In actual metro lines, the rail cant may change a little bit. Figure 9 demonstrates the effects of rail cant on rail corrugation of three wheelset-track systems. From Figure 9(a), it is seen that when the rail cant is equal to 1/20, there is not any unstable oscillation of the wheelset-track system with the heuristic wheelset structure in the range of frequency 0–1000 Hz. This is to say, rail corrugation will not occur in this case. Further numerical results show when the rail cant is equal to zero, there is not any unstable oscillation of the wheelset-track system with the heuristic wheelset structure in the range of frequency 0–1000 Hz. From Figures 9(b) and 9(c), it is found that when the rail cant is equal to 1/20, there are still some unstable oscillations of the wheelset-track systems with the plane web plate wheel and the S-shaped web plate wheel. This indicates that rail corrugation will not occur in these cases.

**Figure 4:** Finite element models of three different wheelset-track systems: (a) with the heuristic wheelset, (b) with the plane-web plate wheelset, (c) with the S-shaped web plate wheelset, and (d) wheel-rail contact forces of the leading wheelset on sharply curved tracks.
Furthermore, when the rail cant is equal to zero, there are still some unstable oscillations of the wheelset-track systems with the plane web plate wheel and the S-shaped web plate wheel.

### 3.5. Effect of Different Fasteners on Rail Corrugation

In China’s metro lines, several types of fasteners are applied. Until then, all wheelset-track systems were equipped with a DTVI2-type fastener. The stiffness and damping values of the DTVI2-type fastener were listed in Table 1. Table 3 lists the stiffness and damping values of the three fasteners. Figure 10 shows the effect of the spring bar-type fastener on rail corrugation. From Figure 10(a), it is found that when the spring bar-type fastener is used, there is no unstable oscillation of the wheelset-track system with the heuristic wheel structure in the range of frequency 0–1000 Hz. That is to say, rail corrugation will not occur in this case. More numeral results show that when the Colong egg-type

| Table 1: Main parameters of models 1–3. |
|----------------------------------------|
| **Track**                              |
| Type of rail (kg/m)                    | 60                                      |
| Gauge (mm) for curve radius larger than 200 m | 1435                                    |
| Gauge (mm) for 200 m ≥ R > 150 m       | 1440 or 1445                            |
| Gauge (mm) for 150 m ≥ R > 100 m       | 1445 or 1450                            |
| Superelevation (mm)                    | 120                                     |
| Density of rail (kg/m³)                | 7800                                    |
| Young’s modulus of rail (N/m²)         | 2.1 × 10¹¹                              |
| Poisson’s ratio of rail                | 0.3                                     |
| Length of rail (m)                     | 36                                      |
| Sleeper span (m)                       | 0.625                                   |
| Rail cant                              | 1/40                                    |
| Density of railpad (kg/m³)             | 1300                                    |
| Young’s modulus of railpad (N/m²)      | 8.0 × 10⁷                               |
| Poisson’s ratio of railpad             | 0.45                                    |
| Thickness of railpad (m)               | 0.012                                   |
| Vertical stiffness of a fastener alone (MN/m) | 40.73                                |
| Lateral stiffness of a fastener alone (MN/m) | 8.79                                |
| Vertical damping of a fastener alone (Ns/m) | 9898.70                                |
| Lateral stiffness of a fastener alone (MN/m) | 8.79                                |
| Lateral damping of a fastener alone (Ns/m) | 1927.96                                |
| Density of sleeper (kg/m³)             | 2480                                    |
| Young’s modulus of sleeper (N/m²)      | 1.9 × 10¹¹                              |
| Poisson’s ratio of sleeper             | 0.3                                     |
| Vertical support stiffness from monolithic track-bed (MN/m) | 89                                     |
| Vertical support damping from monolithic track-bed (Ns/m) | 8.98 × 10⁴                             |
| Lateral support stiffness from monolithic track-bed (MN/m) | 50                                     |
| Lateral support damping from monolithic track-bed (Ns/m) | 4.0 × 10⁴                             |
| **Vehicle**                            |
| Travelling speed (km/h)                | 55                                      |
| Wheelbase of bogie (mm)                | 2200                                    |
| Distance between backs of wheel flanges (mm) | 1354                                  |
| Profile of tread                       | LM-type worn profile                    |
| Mass of wheelset (kg)                  | 1640                                    |
| Moment of inertia of wheelset in vertical and rolling axes (kg · m²) | 725                                    |
| Moment of inertia of wheelset in lateral axis (kg · m²) | 100                                    |
| Mass of bogie (kg)                     | 3188                                    |
| Moment of inertia of bogie in longitudinal level axes (kg · m²) | 2040                                    |
| Moment of inertia of bogie in lateral level axes (kg · m²) | 2710                                    |
| Moment of inertia of bogie in vertical axes (kg · m²) | 3460                                    |
| Mass of car body (kg)                  | 35400                                   |
| Moment of inertia of car body in longitudinal level axes (kg · m²) | 56800                                   |
| Moment of inertia of car body in the lateral level axes (kg · m²) | 1970300                                |
| Moment of inertia of car body in vertical axes (kg · m²) | 1970300                                |
| Longitudinal stiffness of primary suspension alone (MN/m) | 9                                      |
| Lateral stiffness of primary suspension alone (MN/m) | 6.5                                    |
| Vertical stiffness of primary suspension alone (MN/m) | 1.6                                    |
| Vertical damping of primary suspension alone (Ns/m) | 150000                                 |
| Vertical stiffness of secondary suspension alone (MN/m) | 0.48                                   |
| Lateral stiffness of secondary suspension alone (MN/m) | 0.2                                     |
| Vertical damping of secondary suspension alone (Ns/m) | 206000                                 |
Table 2: Forces acting on axle boxes and attack angles of the leading wheelset.

| Description                                                  | Value  |
|--------------------------------------------------------------|--------|
| Vertical load acting on outer axle box on curved track (N)   | 61400  |
| Vertical load acting on inner axle box on curved track (N)   | 57400  |
| Lateral load acting on outer axle box on curved track (N)    | 13000  |
| Lateral load acting on inner axle box on curved track (N)    | 16700  |
| Attack angle of leading wheelset (°)                         | 0.23   |
| Attack angle of trailing wheelset (°)                        | 0.005  |
| Contact angle between wheel and high rail (°)                | 31.63  |
| Contact angle between wheel and low rail (°)                 | 1.96   |
| Normal force between wheel and high rail (N)                 | 46208  |
| Normal force between wheel and low rail (N)                  | 55120  |

Figure 5: Distributions of the unstable oscillation of three wheelset-track systems, gauge \( l_g = 1435 \) mm: (a) for the system shown in Figure 4(a), (b) for the system shown in Figure 4(b), and (c) for the system shown in Figure 4(c).

Figure 6: Unstable oscillation mode shapes: (a) for the system shown in Figure 4(b), effective damping ratio \( \xi = 0.0017 \), frequency of unstable oscillation \( f = 216.40 \) Hz; (b) for the system shown in Figure 4(c), \( \xi = 0.0038 \), \( f = 310.48 \) Hz.
Figure 7: Distribution of the unstable oscillation of the wheelset-track systems, \( l_g = 1450 \) mm: (a) for the system shown in Figure 4(a), and (b) for the system shown in Figure 4(b), and (c) for the system shown in Figure 4(c).

Figure 8: Continued.
fastener is used, there is no unstable oscillation of the wheelset-track system with the heuristic wheel structure in the range of frequency 0–1000 Hz. From Figures 10(b) and 10(c), it is seen that when the spring bar-type fastener is used, there are several unstable oscillations for the wheelset-track systems with the plane web plate wheel and the...
S-shaped web plate wheel. That is to say, rail corrugation will occur in these cases. More numeral results show that when the Colong egg-type fastener is used, there are still several unstable oscillations for the wheelset-track systems with the plane web plate wheel and the S-shaped web plate wheel. That is to say, rail corrugation will occur in these cases.

3.6. Effect of the Sleeper Span on Rail Corrugation. In China’s metro lines, the sleeper span generally changes in a range of 580–625 mm. Numerical results show that the sleeper span has little effect on rail corrugation. Figure 11 shows the distribution of the self-excited oscillation of three wheelset-track systems at the sleeper span of 580 mm. From Figure 11(a), it is found that there is no unstable oscillation of the wheelset-track system with the heuristic wheel structure in the range of frequency 0–1000 Hz. That is to say, rail corrugation will not occur at the sleeper span of 580 mm. Figures 11(b) and 11(c) show that there are some unstable oscillations of the wheelset-track systems with the plane web plate wheel and the S-shaped web plate wheel at the sleeper span of 580 mm. Rail corrugation will occur in this case.

| Type          | Spring bar | DTVI2          | Cologne-egg |
|---------------|------------|----------------|-------------|
| VS (MN/m)     | 18.28      | 40.73          | 12.07       |
| VD (Ns/m)     | 6361.29    | 9898.70        | 1361.12     |
| LS (MN/m)     | 9.0        | 8.79           | 7.58        |
| LD (Ns/m)     | 1830.22    | 1927.96        | 974.27      |

Note. VS represents vertical stiffness, VD represents vertical damping, LS represents lateral stiffness, and LD presents lateral damping.
4. Discussion

Researchers have been developing possible remedy methods to suppress and eliminate rail corrugation since the early part of the last century. Several remedy methods such as rail grinding, friction modifier, and running speed variation have been proposed. Practices prove these remedy methods to be useful in the suppression and elimination of rail corrugation. However, these remedy methods are all passive and need significant follow-up maintenance costs. In the present study, an active remedy method, in which the proposed heuristic wheelset is used in place of the plane web plate wheelset and the S-shaped web plate wheelset, was proposed to suppress and eliminate rail corrugation. This active remedy method does not require follow-up maintenance costs. The parameter sensitivity analysis shows that the remedy method proposed in the present study has strong robustness to prevent rail corrugation. This active remedy method can be applied not only in the newly built metro lines but also in old ones. When it is applied in old metro lines, all metro vehicles should be equipped with the heuristic wheelset structure, and rails need to be ground completely. In this case, the proposed active remedy method by using the heuristic wheelset structure is very promising for the control of rail corrugation.

Why the heuristic wheelset-track system is capable of suppressing or eliminating rail corrugation? According to the theory of friction-induced self-excited vibration [30], the stiffness matrix of the motion equation of a friction system is a symmetric matrix in the absence of friction at the interface. In this case, there is no positive real part in all eigenvalues of the eigenvalue equation. That is to say, there is no negative effective damping rate for the friction system. The stiffness matrix of the motion equation of a friction system is an asymmetric matrix in the presence of friction at the interface. When the asymmetry of the stiffness matrix is strong enough, there may be one or several positive real parts in the eigenvalues of the eigenvalue equation. That is to say, there may be one or several negative effective damping rates for the friction system in this case. The authors consider that different wheelset structures can change the asymmetry of the stiffness matrix of the wheelset-track system to some extent. When the heuristic wheelset is used, the asymmetry of the stiffness matrix of the wheelset-track system becomes weak. In this case, there is no negative effective damping rate for the friction system.
damping rate for the heuristic wheelset-track system even though friction is present at the interface.

5. Conclusions
(1) The proposed heuristic wheelset structure is capable of largely suppressing or eliminating rail corrugation without any need to change track and train operation parameters and without any need of follow-up maintenance costs. The heuristic wheelset structure is robust enough to prevent rail corrugation in a large parameter range of tracks.

(2) The wheel web structure has an important effect on rail corrugation.

(3) The gauge has an effect on rail corrugation. Sometimes rail corrugation does not occur at a certain rail gauge, but when the rail gauge is increased, rail corrugation will occur.

(4) The greater the friction coefficient at the wheel-rail interface, the greater the incidence likelihood of rail corrugation is. Changing the value of the fastener stiffness and damping, rail cant, and sleeper span cannot eliminate rail corrugation.

Data Availability
The data are available upon request.

Conflicts of Interest
The authors declare no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Acknowledgments
The authors thank the financial support from the National Natural Science Foundation of China (No. 51775461).

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