Optimized SCMA Codebook Design using Rotation based Sub-divided Constellation for Overloaded System

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Abstract—5G in wireless communication aims at deploying massive connectivity. Sparse Code Multiple Access (SCMA), proves to be an emerging candidate with multidimensional codebooks proposing high shaping gain and advanced multiuser detection. A framework for designing a codebook for 200\% overloaded SCMA system with system model is presented in this paper. This article aims at delineating a codebook by subdividing the Mother Constellation (MC) and rotating it to achieve a better Symbol Error Rate (SER) performance over higher values of SNR. Different Codebook designs are taken into consideration for comparing with the sub-constellation based 200\% overloaded SCMA.

Keywords — 5G wireless communication, LDPC, MPA, NOMA, SCMA, SER.

I. INTRODUCTION

SCMA, technique is a code-domain scheme of NOMA, being proposed recently [1]. It proves its efficacy over low receiver complexity improving spectral efficiency and ameliorates the performance, as juxtaposed to Low Density Signature (LDS) [2]. With the help of, a predefined set of codebooks, the channel encoded bits are delineated into multidimensional code-words, which are decoded through message passing algorithm (MPA). SCMA has been endorsed to curtail the inter user interference at the receiver. SCMA combines two technologies to maximize the number of users accommodated, in each of the time and frequency resource in shape of code-division process [3]. High shaping gain is procured in this system as compared to LDS.

The efficacy of SCMA system is predominantly determined by the codebook design of the users, thus many codebooks have been designed to outperform the existing system. The conventional SCMA codebook [4] was designed to outperform the LDS system. In this, the mother constellation (MC) points were structures in a straight line to setup the codebook of one user, whereas the other codebooks were designed by mere rotating the MC by a suitable, phase rotation angle [5]. However the system
performance in terms of BER needs to be enhanced for the higher modulation systems, also the Minimum Euclidian Distance (MED) for constellation points also needs to be considered while designing its codebook. Thus, a cognitive technology based SCMA system, characterized by spectrum hole sensing, was preferred to be adopted by the transmitting subcarriers [6]. Further, to maximize the MED, TCM was used to design the constellations [7]. This method helped in improving the system performance, but at the cost of complexity. Then, a 16-Quadrature Amplitude Modulation (QAM) was initiated in the form of star -ring structure [8] [9]. The efficacy of the codebooks was determined on the Gaussian channel [10] that may introduce the maximum noise incorporated in the channel. Along-with, the design of codebook another approach was studied to upgrade the system, was minimizing the complexity in the decoding of the received codebooks [11]. In [12] QAM constellation was studied, to determine the constellation points using befitting rotation angles. In [13] Golden Angle Modulation (GAM) was proposed to construct a worthy multidimensional constellation. Another, design of codebook was introduced by [14], using differential evolution to reduce the symbol error rate (SER).

This paper aims at exploring a suboptimal design of codebook for SCMA using 16 –QAM for 200% overloaded system, means at the rate of 2 given by the ratio of the number of users and resources. The design is based on exploiting the optimized mother constellation used by 16 QAM, which is further divided into sub-constellations of MC. The design of MC is inspired from the star QAM Constellation [15] that intends the enhancement of SER for higher values of SNR. The uplink and the downlink SCMA system for codebook generation and detection is studied in [16], that also has a good impact over the system parameters.

Section II, explores the system model for the proposed SCMA, whereas the Section III, explains the design parameters and the technique, to construct a sub-optimal codebook. Section IV, explores the possible results of the simulations done for several codebooks designed along-with the proposed one. Finally, the paper is concluded in Section V.

II. SYSTEM MODEL

The SCMA model for J=10 users transmitting over K=5 resources has been depicted in the Fig.1, sample codebooks for J=1 and 10 has been introduced. Users transmitting on five Resource Elements (RE) are seen as (K, J, L, M ) = (5, 10, 2, 4). The codebook of dedicated user is unique, together creating, all 4 × 5, complex matrices. In order to transmit, 2-bit information each time through the binary bits, four possible combinations will be derived as [00, 01, 10, 11], in accordance, with the 4 codewords of a codebook. In addition, the codeword is the column vector of the complex matrix. Let us assume, 10 users transmit 10 combinations (1, 1), (0, 1) (1, 0), (1, 1) (0, 1), (0, 0), (1, 1), (0, 1), (1, 0), and (0, 0) simultaneously, which correlate to the 4, 2, 3, 4, 2, 1, 4, 2, 3, 1 columns of the codebook. The coloured cells depicted in the matrix, with complex vectors, represents different non-zero values, whereas the white colour represents a blank matrix. The overloading ratio is described as, the ratio of number of users to the resource elements, \( \lambda = J/K \). Total, M-codewords combine to form a codebook. The procedure of encoding [17] is stated by the equation (1)

\[
b \in B^{\log_2(M)} \rightarrow x
\]
x represents complex number set and $x \subset \mathbb{C}_K$, where C describes the cardinality. M designates the number of constellation points catering each user on a dedicated subcarrier. b describes the bits given by $b_j = (b_{1j}, \ldots, b_{mj})$, which are converted to K-dimensional codewords $c_j = (c_{1j}, \ldots, c_{mj})$. The non-zero elements in the designed codebook is represented by L, the codewords will be sparse if $L < K$. Thus it can be interpreted that each codeword has $K - L$ zeros in it. The mapping matrix is mentioned in Fig.2, denoted by $F = [F_1, \ldots, F_J]$, determining the dimensions of the specified non-zero elements [18] [19].

$$F = \begin{bmatrix}
1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\
\end{bmatrix}$$

These codewords are transmitted across the channel and the received signal is expressed as given in (2) with $n$ as Gaussian noise and $h$ as channel characteristics.

$$r = \sum_{j=1}^{J} \text{diag}(h_j)c_j + n$$

(2)

The codewords are obtained by making use of joint optimum technique through MAP on the signal retrieved, $r$, given by [20].

III. CODEBOOK DESIGN
This paper works on the codebook design for based on J=10 and K=5 in conjunction with the design of factor matrix as depicted in Fig.2 with d_r = 4 and d_v = 2. It describes the location of resource elements or sub-carriers with respect to users. RE1 can be accessed by J1, J6, J7 and J10 (users) similarly, J1 accesses RE1 and RE2 (Sub-carriers) shown in Fig.3. Thus to define codebook, for each user two symbols are taken into consideration, resulting into 2-subconstellations [21].

Considering, transmission over the AWGN channel, the optimal codebook is constructed using Algorithm as followed in the sub-sections.

A. Optimization of MC for SCMA

Considering star- 16 QAM design, seen in Fig.4, the codebook is designed as explained in Algorithm where r1 is computed as the distance of 1st ring from the centre. Also α=3, β=1.587, θ=22.5° is considered from [8], [9] & [22]. Thus r2, r3 and r4 are computed as r1=0.63, r2= β * r1 = 1, r3=3*0.63, r4= α * r2 = 3.

S_a and S_b denotes the coordinates of set of constellation points of the first and second symbol for each user. \n\[ S_a = (\alpha x_2, \alpha x_1, x_2, x_1) \]
\[ S_b = - S_a \]  \hspace{1cm} (3)

Algorithm: Rotation based Sub-divided constellation for Codebook design in Gaussian channel using star 16-QAM

1. Step1:
   \[ r1= \text{the distance of 1st ring from centre.} \]
   \[ r2= \beta \ast r1 \]
   \[ r3= \alpha \ast r1 \]
   \[ r4= \beta \ast r3 \]
   Compute r1, α & β

2. Step 2: To compute each sub-constellation points, the points elevated by θ are computed as
   \[ R = e^{i\theta} \ast r \]

3. Step 3: Computing rotation angle
   \[ \phi = \gamma \ast \Delta \]
   \[ S = C \ast e^{i\phi} \]

4. Step 4: Sub-constellations occupies the factor matrix using latin structure for user allotment.

5. Step5: Design Metric Calculations for each user.
   \[ d_{min} = \sqrt{(S_i_i)^2 - (S_j_j)^2} \]

These are the generalized set of points for M=4, for star QAM structure.

B. Generation of Sub-constellation

![Fig.4: Star Constellation design for 16-QAM](image)
Generally, 2 sub-constellations are derived after the first-order division. Then, we implement successive divisions, performed on the RE. We further divide the MC until sufficient sub-constellations are derived. Thus, MC is further sub divided into 4 sub-constellations as shown in Fig. 5 showcasing alternate rings involved in each sub-constellation points elevated by angle $\theta$ are given by $R_i$ where ($i = 1,2,3,4$). Thus, $C_i$ can be computed as:

$$R_i = e^{i\theta} \cdot r_i$$ (4)

Fig 5. shows the sub-constellations with points present in the alternate rings such that each ring has 2 points, thus dividing 16 point constellation into four 4-point sub-constellations.

For ex: Let us consider the third sub-constellation denoted by $C_3$ we have 4 points in the constellation, 2-points in first ring and 2-points in 3rd ring. Points in 3rd ring are represented as $r_3$ and $-r_3$, while the points in ring are calculated as $R_1$ & $-R_1$ given by (5)

$$R_1 = (\cos(22.5) + j \sin(22.5)) \cdot r_1$$ (5)

Fig.5: Division of MC into 4 sub-constellations

C. Computation of rotation angle

Step 3 involves setting of the optimization angle $\phi_i$ [9], from the perspective of the resource element given by (6) where $\Delta$ is rotation angle and $\gamma$ is an integer assuring ($0 \leq \gamma \leq d_r - 1$).

$$\phi_i = \gamma \cdot \Delta$$ (6)

For ex: $d_r = 4$, $\gamma = 0,1,2,3$ and $\Delta = \pi/6$, hence values of $\phi$ will be, $\phi_0 = 0$, $\phi_1 = \pi/6$, $\phi_2 = 2\pi/6$ and $\phi_3 = 3\pi/6$. Each sub-constellation $S_i$ is rotated with the respective rotation angle given by

$$S_i = C_i \cdot e^{i\phi}$$ (7)

D. User allotment with sub-constellation
Step 4: The sub-constellations occupy the factor matrix as shown in Fig. 6 in the form of latin structure. Using the above calculations and latin structured factor matrix [23] [24], the codebook is designed for each user.

E. Design metric Calculations

Step 5: The bit labelling follows the generalized rules to analyse the design-metric for the codebook constructed for each of the users [25], [26]. The Minimum Euclidian Distance (MED) \( d_{\text{min}} \), for 16-star QAM MC, is 0.39 [8]. The point in subdividing the MC into sub-constellations is to accelerate the MED of the Sub-constellations S1, S2, S3and S4, that can be analysed by (8), where S represents the set of constellation points in sub-constellation, for \( i=1, 2, 3, 4 \) sub-constellations and 1 and 2 represents two distinct vectors in each sub-constellation It represents the MED between the two consecutive symbol vectors in a given subset of constellation, that results into 2.11 for S2 and S4.

\[
d_{\text{min}} = \sqrt{(S_i)^2 - (S_{i+1})^2}
\] (8)

IV. SIMULATION RESULTS

In this paper, \( \lambda = 2 \) overloading factor is considered, where \( J=10 \), \( K = 5 \), \( d_t = 4 \) and \( d_e = 2 \) is considered. The designed codebook is transmitted and simulated over the AWGN channel for evaluating the performance of the proposed codebook design. The symbol error rate is calculated for different values of SNR using Monte Carlo simulations [27]. The SER for AWGN channel [28] is given by (9)

\[
P_e = \frac{1}{\pi} \sum_{\theta_0}^{\pi} \exp\left[-\frac{r^2 \sin^2(\theta_0)}{2\sigma^2 \sin^2(\theta_0 + \theta)}\right] d\theta
\] (9)

The SER performance [14] for sample value of SNR is compared with, the different codebooks mentioned in the Table 1. The conventional codebook [1] is designed for 6 users and 4 RE, thus at 1.5 overloading rate for which the SER performance at 18 dB is 3.2x10^{-4} with MED as 1. The second design used for comparison referred, the sub-constellation based approach for, overloading rate, \( \lambda = 1.5 \) gives SER, 6.3x10^{-4} for 18dB with MED as 0.39 [15]. The same approach is considered for designing codebook for \( \lambda = 2 \) as the third design of codebook for comparison, with the proposed design, using rotation based sub-divided constellation approach to analyse the SER performance. The Table depicts better performance of the proposed codebook over the other designs, in terms of SER without compromising the MED, 2.11 for the sub-constellation design. The BER can be derived from the equation of SER for higher values for SNR by \( P_e / \log_2(M) \). The comparison between different codebook designs, including the proposed one, in term of SER performance for different values of SNR is shown in Fig.7.

| Sr. No | Codebook designs          | J(users) | K(RE) | MED   | SER for 18dB SNR |
|--------|---------------------------|----------|-------|-------|-----------------|
| 1      | Conventional SCMA         | 6        | 4     | 1     | 3.2x10^{-4}     |
| 2      | SCMA with Sub-constellation | 6        | 4     | 2.11  | 6.3x10^{-4}     |
| 3      | SCMA with Sub-constellation | 10       | 5     | 2.11  | 7.42x10^{-4}    |
| 4      | Proposed SCMA             | 10       | 5     | 2.11  | 2.1x10^{-5}     |
V. CONCLUSION

This paper extended a suboptimal approach towards the multiuser, SCMA codebook design for 200% overloaded system. The MC and its sub-constellations are designed aiming to minimize the SER, attain better accuracy and decoding capacity. SER analysis is done, as it helps in detecting the signal, in the interference-limited environment. The major steps, towards the design of proposed codebook, which are explained in the algorithm, by, initially identifying the MC based on star- QAM constellation design. An acceptable generating the mapping matrix is effectuated, which will divide the main constellation, further, to shape up sub-constellations. For improvised system, befitting rotation angles are added. Hence, the proposed method effectively reduces the SER than the conventional methods with better decoding accuracy, thus enhancing the SNR gains. Consequently, the proposed method can be effectively utilized for designing SCMA codebooks for futuristic 5G wireless communication using the concepts of Artificial Intelligence along with its varied applications [29].

![Fig. 7 SER performance comparison between SCMA codebooks](image)

**Data availability statement**

The data that support the findings of this study are available from the corresponding author upon reasonable request.

**Declarations**

1. **Funding** - Not Applicable
2. **Conflicts of interest** –
   
   The authors of this article, certify that they have NO affiliations with or involvement in any organization or entity with any financial interest (such as honoraria; educational grants; participation in speakers’ bureaus; membership, employment, consultancies, stock ownership, or other equity interest; and expert testimony or patent-licensing arrangements), or non-financial interest (such as personal or professional relationships, affiliations, knowledge or beliefs) in the subject matter or materials discussed in this manuscript.

3. **Availability of data and material** – available from the corresponding author upon reasonable request
4. **Code availability** – MATLAB
5) Authors’ Contribution

A. Madhura Kanzarkar
B. Dr.M.S.S. Rukmin
C. Dr.Rajeshree Raut

- A.B. and B.C. conceived of the presented idea.
- A.B. developed the theory and performed the computations. C. verified the analytical methods. B.C. encouraged A.B. to investigate sub-divided constellation and supervised the findings of this work. All authors discussed the results and contributed to the final manuscript. A.B. and B.C. carried out the experiment.
- A.B. wrote the manuscript with support from C.
- A.B. developed the theoretical formalism, performed the analytic calculations and performed the numerical simulations. Both A.B and B.C. authors contributed to the final version of the manuscript.
- A.B., B.C. conceived and planned the experiments and carried out the simulations.
- A.B., B.C contributed to sample preparation of codebook design and contributed to the interpretation of the results.
- A.B. took the lead in writing the manuscript. All authors provided critical feedback and helped shape the research, analysis and manuscript. A.B. and B.C. designed the model and the designed the algorithm and analysed the data.
- A.B. and C. carried out the implementation. A.B. performed the calculations. A.B. wrote the manuscript with input from all authors. C. conceived the study and was in charge of overall direction and planning.
- A.B. designed and performed the experiments, derived the models and analysed the data and C. helped to carry out the simulations to determine SER.
- All authors discussed the results and commented on the manuscript. A.B. and B.C. contributed to the design and implementation of the research, to the analysis of the results and to the writing of the manuscript.
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