Hartree-Fock ground state of the composite fermion metal

Piotr Sitko and Lucjan Jacak, Institute of Physics, Technical University of Wrocław, Wyb. Wyspiańskiego 27, 50-370 Wrocław, Poland.

Abstract

Within the Hartree-Fock approximation the ground state of the composite fermion metal is found. We observe that the single-particle energy spectrum is dominated by the logarithmic interaction exchange term which leads to an infinite jump of the single-particle energy at the Fermi momentum. It is shown that the Hartree-Fock result brings no corrections to the RPA Fermi velocity.
1. Introduction

Recently there has appeared a great interest, both theoretical and experimental, in studies of the compressible $\nu = 1/2$ quantum Hall state. Halperin, Lee and Read (HLR) constructed the theory of quantum Hall states at fractions with even denominators. The idea comes from the Jain theory of composite fermions originally stated in the context of the fractional quantum Hall effect (FQHE) \[1\]. In close analogy to the Laughlin wave function Jain proposed simpler and more elegant way of understanding the FQHE phenomenon which consists in attaching an even number ($2f$) of flux quanta to each electron. If one takes an average of fluxes (mean field approximation) will find a partial cancelation of the external magnetic field leading to new incompressible states which results in the FQHE. As it was noticed in Ref. \[1\] such a construction at $\nu = 1/(2f)$ leads to a metal-like state (exact cancelation of fields, i.e. the zero effective field) which is to some extent verified experimentally \[3\].

The full description of the composite-fermions picture is given in terms of the Chern-Simons theory \[4\]. Despite the mean field there are interaction terms including also a three-body term which dominating part is a logarithmic interaction \[5\]. HLR studied the linear response of the system within the random-phase approximation. They found that the Fermi velocity vanishes so that the effective mass is divergent which is in agreement with recent experiments \[3\]. However, it is still uncertain whether the self-energy graphs not included in the RPA will normalize the effective mass \[4, 8\]. In this paper we consider all first order contributions to the single-particle energy within the Hartree-Fock (H-F) approximation.
2. Hartree-Fock ground state

The starting point of our consideration is the Hamiltonian:

\[ H = \frac{1}{2m} \sum_{i=1}^{N} (P_i + eA_i + eA_{i}^{ex})^2 \]  

(1)

where the vector potential from the attached fluxes (Chern-Simons field) is given by

\[ A_i = \sum_j A_{ij} = -\frac{2f}{e} \hbar c \hat{z} \times \sum_{j \neq i} \frac{(r_i - r_j)}{|r_i - r_j|^2}, \]

(2)

\( \hat{z} \) - a perpendicular to the plane unit vector. The Hamiltonian (1) for integer values of \( f \) describes composite fermions in the external magnetic field and for other values of \( f \) – anyons with statistics parameters \( \Theta = (1 + 2f)\pi \) (an exchange of two anyons produces a phase factor of \( e^{i\Theta} \)).

The mean field \( B^s \) is given by the average vector potential:

\[ \bar{A}_i = \sum_p \int |\phi_p(r_j)|^2 A_{ij} d^2r_j. \]

(3)

When the effective field is zero, i.e. \( B^s + B^{ex} = 0 \), we assume that the ground state of the system is given by the plane wave: \( \phi_p(r_j) = \frac{1}{\sqrt{S}} \exp(\frac{i}{\hbar} p r_j) \). Let us recall the expressions for the H-F ground-state energy [9, 10]:

\[ <H_a> = \frac{1}{2m} \sum_p \int d1 \phi_p^+(1)(P_1 + \bar{A}_1 + A_{1}^{ex})^2 \phi_p(1), \]

(4)

\[ <H_b> = -\frac{e}{mc} \sum_{p,q} \int d1d2 \phi_p^+(1)\phi_q^+(2)A_{12}(P_1 + \bar{A}_1 + A_{1}^{ex})\phi_p(2)\phi_q(1), \]

(5)

\[ <H_c> = \frac{e^2}{2c^2m} \sum_{p,q} \int d1d2 \phi_p^+(1)\phi_q^+(2)|A_{12}|^2 \phi_p(1)\phi_q(2), \]

(6)

\[ <H_d> = -\frac{e^2}{2c^2m} \sum_{p,q} \int d1d2 \phi_p^+(1)\phi_q^+(2)|A_{12}|^2 \phi_p(2)\phi_q(1), \]

(7)

\[ <H_e> = \frac{e^2}{c^2m} \sum_{p,q,k} \int d1d2d3 \phi_p^+(1)\phi_k^+(2)\phi_q^+(3)A_{12}A_{13}\phi_p(3)\phi_q(1)\phi_k(2), \]

(8)

\[ <H_f> = -\frac{e^2}{2c^2m} \sum_{p,q,k} \int d1d2d3 |\phi_p(1)|^2 \phi_k^+(2)\phi_k^+(3)A_{12}A_{13}\phi_q(3)\phi_k(2) \]

(9)
where \( d_1 = d^2 r_1 \), sums extend over the Fermi sea and omit elements of two equal momentum variables. It was shown in Ref. [10] that the Hartree-Fock ground-state energy \(< H > = \sum_{i=a}^{f} < H_i >\) of the composite fermion metal is finite and equals:

\[
< H > = (1 + 3 f^2) \frac{N}{2} E_F
\]  

(10)

which confirms that the ground state of the system is the Fermi sea. Let us note that the result (10) agrees with the H-F ground-state energy of composite fermions in the FQHE at filling fractions \( \nu = \frac{n}{2(n+1)} \) in the limit \( n \to \infty \) (\( n \) is a number of occupied Landau levels in the effective field).

The single-particle Hamiltonian is found via the first variation in \( \phi^+ \) of Eqs.(4-9) [9]. From the variation of \(< H_a >\) we find:

\[
H_{HF}^1 \phi_p(1) = \frac{1}{2m} \sum_q \int d^2 \phi^+_q(2) (P_1 + \bar{A}_1 + A_1^{ex})^2 \phi_q(2) \phi_p(1) ,
\]

(11)

\[
H_{HF}^2 \phi_p(1) = \frac{e}{mc} \sum_q \int d^2 \phi^+_q(2) A_{21}(P_2 + \bar{A}_2 + A_2^{ex}) \phi_q(2) \phi_p(1) .
\]

(12)

The variation of the term \(< H_b >\) leads to the following expressions:

\[
H_{HF}^3 \phi_p(1) = -\frac{e}{mc} \sum_q \int d^2 \phi^+_q(2) A_{12}(P_1 + \bar{A}_1 + A_1^{ex}) \phi_q(2) \phi_p(1) ,
\]

(13)

\[
H_{HF}^4 \phi_p(1) = -\frac{e}{mc} \sum_q \int d^2 \phi^+_q(2) A_{21}(P_2 + \bar{A}_2 + A_2^{ex}) \phi_q(2) \phi_p(1) ,
\]

(14)

\[
H_{HF}^5 \phi_p(1) = -\frac{e^2}{mc^2} \sum_q \int d^2 \phi^+_q(2) \phi^+_k(3) A_{32} A_{31} \phi_q(3) \phi_k(2) \phi_p(1) .
\]

(15)

Next terms:

\[
H_{HF}^6 \phi_p(1) = \frac{e^2}{mc^2} \sum_q \int d^2 |\phi_q(2)|^2 |A_{12}|^2 \phi_p(1) ,
\]

(16)

\[
H_{HF}^7 \phi_p(1) = -\frac{e^2}{mc^2} \sum_q \int d^2 \phi^+_q(2) |A_{12}|^2 \phi_q(2) \phi_p(1) ,
\]

(17)

are obtained from the variations of \(< H_c >\) and \(< H_d >\), respectively. From the variation of \(< H_e >\) one has:

\[
H_{HF}^8 \phi_p(1) = \frac{e^2}{mc^2} \sum_q \int d^2 \phi^+_q(3) \phi^+_k(2) A_{12} A_{13} \phi_q(1) \phi_k(3) \phi_p(2) ,
\]

(18)
\[ H_{HF}^0 \phi_p(1) = \frac{e^2}{mc^2} \sum_{q,k} d^2 d^3 \phi_q^+(3) \phi_k^+(2) A_{21} A_{23} \phi_q(1) \phi_k(3) \phi_p(2) , \]  
(19)

\[ H_{HF}^{10} \phi_p(1) = \frac{e^2}{mc^2} \sum_{q,k} d^2 d^3 \phi_q^+(3) \phi_k^+(2) A_{31} A_{32} \phi_q(1) \phi_k(3) \phi_p(2) . \]  
(20)

And the variation of the last term leads to the expressions:

\[ H_{HF}^{11} \phi_p(1) = -\frac{e^2}{2mc^2} \sum_{q,k} d^2 d^3 \phi_q^+(2) \phi_k^+(3) A_{12} A_{13} \phi_q(1) \phi_k(2) \phi_p(1) , \]  
(21)

\[ H_{HF}^{12} \phi_p(1) = -\frac{e^2}{mc^2} \sum_{q,k} d^2 d^3 \phi_q^+(2) |\phi_k(3)|^2 A_{31} A_{32} \phi_q(1) \phi_p(2) . \]  
(22)

In the following we will calculate eigenvalues of the single-particle Hamiltonian

\[ H_{HF} = \sum_{i=1}^{12} H_{HF}^i . \]

Let us consider first constant divergent terms:

\[ \xi_{6HF}^6 = \frac{4f^2 \hbar^2}{m \rho} \int d^2 \frac{1}{|r_1 - r_2|^2} , \]  
(23)

\[ \xi_{11HF}^{11} = -\frac{2f^2 \hbar^2}{m \rho_p S} \sum_{q,k} \frac{1}{|k - q|^2} \]  
(24)

where \( \rho_p = \frac{S}{2\pi \hbar R} \). Identifying \( \pi \varepsilon^2 \) and \( \pi \delta^2 \) with the area per one particle in the real and the momentum space, respectively, we can write (\( \pi R^2 = S \)):

\[ \xi_{6HF}^6 = 4f^2 E_F \lim_{R \to \infty} \ln \frac{R}{\epsilon} = 2f^2 E_F \lim_{N \to \infty} \ln \ln N , \]  
(25)

\[ \xi_{11HF}^{11} = -2f^2 E_F (\lim_{\delta \to 0} \frac{p_F}{\delta} - \frac{1}{2}) = -f^2 E_F (\lim_{N \to \infty} \ln \ln N - 1) \]  
(26)

where \( p_F \) is the Fermi momentum. In \( H_{HF}^{12} \) we recognize the logarithmic interaction exchange term:

\[ \xi_{12HF}^{12} = -\frac{4f^2 \hbar^2}{m \rho_p} \sum_{q \neq p} \frac{1}{|p - q|^2} . \]  
(27)

We have also:

\[ \xi_{4HF}^4 = \frac{p_F^2}{2m} , \]  
(28)

\[ \xi_{2HF}^2 = \xi_{3HF}^3 = \xi_{4HF}^4 = \xi_{5HF}^5 = 0 . \]  
(29)
In remaining terms it is convenient to separate the cases of $|p| < p_F$ and $|p| > p_F$. We obtain

$$\xi_{HF}^7(|p| < p_F) = 2 f^2 \frac{p^2}{2m} - 2 f^2 E_F , \quad \xi_{HF}^7(|p| > p_F) = 4 f^2 E_F \ln \frac{|p|}{p_F} ,$$

(30)

$$\xi_{HF}^8(|p| < p_F) = \xi_{HF}^9(|p| < p_F) = -2 f^2 \frac{p^2}{2m} + 2 f^2 E_F ,$$

(31)

$$\xi_{HF}^8(|p| > p_F) = \xi_{HF}^9(|p| > p_F) = 0 ,$$

(32)

$$\xi_{HF}^{10}(|p| < p_F) = 2 f^2 \frac{p^2}{2m} , \quad \xi_{HF}^{10}(|p| > p_F) = 2 f^2 E_F \frac{p_F^2}{p^2} .$$

(33)

Finally we can write:

$$\xi_{HF}(|p| < p_F) = \frac{p^2}{2m} + 3 f^2 E_F + f^2 E_F \lim_{N \to \infty} \ln N - \frac{2 f^2 E_F}{\pi \rho_p} \sum_{q \neq p} \frac{1}{|p - q|^2}$$

(34)

and

$$\xi_{HF}(|p| > p_F) = \frac{p^2}{2m} + 2 f^2 E_F (2 \ln \frac{|p|}{p_F} + 1 + \frac{p_F^2}{p^2}) + f^2 E_F \lim_{N \to \infty} \ln N - \frac{2 f^2 E_F}{\pi \rho_p} \sum_{q \neq p} \frac{1}{|p - q|^2} .$$

(35)

Let us now perform explicitly the sum in Eq. (34). One finds:

$$\sum_{q \neq p} \frac{1}{|p - q|^2} = \pi \rho_p \lim_{\delta \to 0} (\ln \frac{p_F - p}{\delta} + \ln \frac{p_F + p}{\delta})$$

(36)

For $|p| > p_F$ the sum equals:

$$\sum_{q \neq p} \frac{1}{|p - q|^2} = 4 \rho_p \int_{0}^{\alpha} \psi \frac{p \sin \psi}{\sqrt{p_F^2 - p^2 (\sin \psi)^2}} d\psi$$

(37)

where $\sin \alpha = \frac{p_F}{p}$. To see the most important feature of the single-particle energy spectrum it is enough to use the following estimation:

$$\int_{0}^{\alpha} \psi \frac{p \sin \psi}{\sqrt{p_F^2 - p^2 (\sin \psi)^2}} d\psi < \int_{0}^{\alpha} \frac{\pi}{2} \frac{p \sin \psi}{\sqrt{p_F^2 - p^2 (\sin \psi)^2}} d\psi = \frac{\pi}{4} \ln \frac{p + p_F}{p - p_F}.$$ 

(38)

If only $(p_F - p') > 0$ and $(p'' - p_F) > 0$ are not infinitesimal the dominating contribution to the energy difference $\xi(p'') - \xi(p')$ is greater than

$$\xi(p'') - \xi(p') > 2 f^2 E_F \lim_{\delta \to 0} (\ln \frac{p_F + p'}{\delta} + \ln \frac{p_F - p'}{\delta})$$

(39)
which is divergent. Hence, there is an infinite jump of the single-particle energy at the Fermi momentum.

An infinite gap (or jump) in the single-particle energy spectrum appears to be the characteristic feature of the Hartree-Fock ground state of Chern-Simons systems [8, 11]. However, in the present case this feature has no physical meaning due to the screening of the logarithmic interaction in the RPA [1]. This makes the physics distinct from that of the anyon superconductor where the logarithmic interaction remains unscreened and single-particle excitations are vortices [12].

In the composite fermion metal the RPA Fermi velocity vanishes [1]. One can see in Eqs. (34, 35) that the only H-F contributions to the Fermi velocity comes from $\xi_{1}^{1}\xi^{12}_{HF}$ and $\xi_{12}^{12}_{HF}$. However, the free term $\xi_{HF}^{1}$ and the logarithmic interaction exchange term $\xi_{HF}^{12}$ were included in the analysis of HLR, thus the Hartree-Fock result brings no corrections to the HLR conclusions.

3. Conclusions

The Hartree-Fock single-particle energy spectrum of the composite fermion metal is found. The dominating contribution is the logarithmic interaction exchange term which produces an infinite jump of the single-particle energy at the Fermi momentum. However, this feature has no physical meaning due to the screening of the logarithmic interaction in the RPA. We find also that the Hartree-Fock terms not included in the RPA bring no corrections to the Fermi velocity.
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