Abstract
I discuss viscosity corrections to thermal effects on the static QCD potential within hard-thermal loop resummed perturbation theory and for a strongly coupled, large-$N_c$ conformal field theory dual to five-dimensional Gauss-Bonnet gravity. I also present model predictions for quarkonium binding energies in the deconfined phase and for suppression of $R_{AA}(\Upsilon \rightarrow e^+ e^-)$.

The potential between a very heavy $Q\bar{Q}$ pair in a color-singlet state is approximately given by Coulomb attraction at short distances and linear confinement at large separation [1],

$$V_{Q\bar{Q}}(r) = -\frac{\alpha_s C_F}{r} + \sigma r. \quad (1)$$

Here, $\sigma \approx 1 \text{ GeV/fm}$ denotes the string tension in SU(3) gauge theory. (The string is in fact screened in full QCD with dynamical fermions due to “string breaking” at $r \gtrsim 1/\Lambda_{\text{QCD}}$.) At $m_Q \rightarrow \infty$, bound states have small radii and hence the Coulomb attraction dominates. If $\alpha \equiv \alpha_s C_F \ll 1$, the binding energy of the ground state $|E_{\text{bind}}| \sim \alpha^2 m_Q$ is much smaller than the quark mass $m_Q$ and hence the velocity of the quarks in the bound state is small, $v \ll 1$. Furthermore, the Bohr radius $a_0 \sim 1/(\alpha m_Q) \gg 1/m_Q$; the bound quarks are therefore not localized in a region on the order of their Compton wavelength. These observations suggest that to first approximation quarkonium states can be understood from non-relativistic potential models [2] such as (1). A framework for systematic improvements of this simple picture is offered by an effective field theory (potential non-relativistic QCD - pNRQCD) obtained from QCD by integrating out modes above the scales $m_Q$ and then $m_Q v$, respectively [3].

At high temperature, the deconfined phase of QCD exhibits screening of static color-electric fields [4]. Hence, quarkonium states should dissociate once the Bohr radius exceeds the screening length $a_0$ [5]. In recent years, a big effort has been made by various groups to test the validity of potential models at finite temperature, to compute thermal modifications of the potential, and to obtain quarkonium spectral functions and meson current correlators via first-principle QCD calculations performed numerically on a lattice. We refer to ref. [6] for a summary and review. A qualitatively new contribution to the static potential which arises at finite temperature is the imaginary part due to Landau damping of the static gluon exchanged by the heavy quarks [7].

Here, we focus on non-equilibrium effects in a plasma which exhibits a local anisotropy. This arises in heavy-ion collisions due to anisotropic hydrodynamic expansion of a plasma with non-vanishing shear viscosity. The phase-space distribution of thermal excitations is given by

$$f(p) = f_{\text{iso}}(p) \left[ 1 - \xi \frac{(p \cdot n)^2}{2pT_m} (1 \pm f_{\text{iso}}(p)) \right]. \quad (2)$$

**Reference:**

[1] A. Dumitru, *Quarkonium in a viscous QGP*, arXiv:0907.0320v2 [hep-ph] 16 Sep 2009.

**References:**

[2] A. Dumitru, *Quarkonium in a viscous QGP*, arXiv:0907.0320v2 [hep-ph] 16 Sep 2009.
\( f_{\text{iso}}(p) \) is either a Bose distribution or a Fermi distribution, respectively. The correction \( \delta f \) to the equilibrium distribution follows from viscous hydrodynamics / kinetic theory for a fluid element expanding one-dimensionally along the direction \( \mathbf{n} \); the anisotropy parameter \( \xi \) is proportional to the ratio \( \eta/s \) of shear viscosity to entropy density and to the gradient of the flow velocity.

To derive the potential in a plasma described by the momentum distribution (2) one first computes the corresponding retarded and symmetric “hard thermal loop” resummed gluon propagators in the static limit. The one-gluon exchange potential follows essentially from its Fourier transform. Its real part is given by [8, 9]

\[
V(r) = V_{\text{iso}}(r) (1 + \xi F(\hat{r}, \theta))
\]

Here, \( \hat{r} \equiv r m_D \) with \( m_D(T) \) the screening mass in an isotropic medium, and \( V_{\text{iso}}(r) = -\frac{\alpha}{r} \exp(-\frac{\hat{r}}{r}) \) is the well-known Debye-screened Coulomb potential. The viscosity-dependent correction in eq. \( 3 \) reduces thermal screening effects as compared to an ideal \( (\xi = 0) \) plasma.

The potential (5) may fail to reproduce the dominant \( T \)-dependence of the binding energies in the phenomenologically relevant range \( T/T_C = 1 - 3 \), even for very large quark mass. In this range, the “interaction measure” \( (e-3p)/T^4 \) in SU(3)-YM (or QCD) is large. The free energy of a static \( Q\bar{Q} \) pair at infinite separation behaves as [11]

\[
F_{\infty}(T) \approx \frac{a}{T} - bT,
\]

with \( a \approx 0.08 \text{ GeV}^2 \) a constant of dimension two and \( b \) a dimensionless number. The second term is usually identified with an entropy contribution which should be removed. The first term, however, corresponds to a non-vanishing \( V_{\infty}(T) \sim a/T \) tied to the presence of an additional dimensionful scale besides \( T \). In fact, for very small bound states, the temperature dependence of the short-distance potential is much smaller than that of the continuum threshold \( V_{\infty}(T) \) [9]. Note that the binding energy of a quarkonium state is defined relative to the potential at infinity:

\[
E_{\text{bind}} = \langle \Psi | \hat{H} - V_{\infty} | \Psi \rangle - 2m_Q.
\]

Figure 1: Left: Binding energies for the 1S states of charmonium (lower curves) and bottomonium (upper curves) for two values of the plasma anisotropy parameter \( \xi \). The straight line corresponds to \( T \). Right: 1P state of bottomonium.

In ref. [9] a temperature and viscosity dependent interpolation between the short-distance potential (3) and \( V_{\infty} \) has been put forward. The bound states were obtained from the corresponding Schrödinger equation [10]. The binding energies of the charmonium and bottomonium ground
states are shown in Fig. [11] and, as expected, they decrease towards higher $T$. As already mentioned above, this turns out to be largely due to the decreasing continuum threshold $V_{\text{cont}}(T)$. The wave function of the $b\bar{b}$ ground state, for example, is affected little by the medium (for $T \lesssim 2.5T_c$ and $\xi \lesssim 1$) even as $|E_{\text{bind}}|$ drops well below $T$. The figure also shows that $|E_{\text{bind}}|$ increases with the anisotropy $\hat{\xi}$. This can be understood as reduced screening of both the Coulomb and the string potentials at higher viscosity (and fixed $T$). In the future, improved methods should be employed to determine the properties of bound states; in particular, many-body interactions should be taken into account by solving the Schrödinger equation for the non-relativistic Green’s function [12] (incl. threshold effects).

The potential in an anisotropic plasma carries angular dependence. States with non-zero angular momentum then split according to the projection $\mathbf{L} \cdot \mathbf{n}$. For the 1P state of bottomonium, for example, the splitting is estimated to be on the order of 50 MeV; at $T = 200$ MeV, the occupation number of states with $\mathbf{L} \cdot \mathbf{n} = 0$ should be about exp(50/200) = 1.3 times higher than that of states with $\mathbf{L} \cdot \mathbf{n} = \pm 1$.

In the static limit, the retarded and advanced HTL propagators are real. The symmetric propagator, however, is purely imaginary. The static potential therefore also develops an imaginary part. At short distances, $\hat{r} \ll 1$, and including the leading viscosity correction, it is given by [13]

$$i \text{Im } V(\mathbf{r}) = -i \frac{g^2 C_F T}{4\pi} \hat{r}^2 \ln \frac{1}{\hat{r}} \left( \frac{1}{3} - \xi^3 - \frac{3 - \cos(2\theta)}{20} \right).$$

(6)

This leads to a non-vanishing width of the bound states; for a Coulomb wave function,

$$\Gamma(T, \xi) = -\int d^3\mathbf{r} |\Psi(r)|^2 \text{ Im } V(\mathbf{r}) = \frac{16\pi T m_Q^2}{g^2 C_F M_Q^2} \left(1 - \frac{\xi}{2}\right) \ln \frac{g^2 C_F M_Q}{8\pi m_D}.$$  

(7)

$\Gamma_T$ is on the order of tens of MeV, to be compared to the electromagnetic decay width $\Gamma_{T \to e^+e^-} \approx 1$ keV. Hence, $\Upsilon$ states which may form in the plasma at $T/T_c \sim 1 - 2$ are very hard to observe in the $e^+e^-$ channel as the branching ratio $\Gamma_{T \to e^+e^-}/\Gamma_T$ is much smaller than in vacuum. This effect would contribute to a possible $\Upsilon \to e^+e^-$ suppression in central Au+Au collisions at RHIC [14].

It is interesting to compare to a strongly coupled theory. Using the gauge-gravity duality, the static potential (or Wilson loop) [13] and thermal effects at short distances [16] have been computed in $N = 4$ supersymmetric Yang-Mills at large (but finite [17]) $\lambda$'t Hooft coupling $\lambda = g^2 N$ and $N \to \infty$. At $T = 0$,

$$V_{Q\bar{Q}}(r) = -\frac{4\pi^2}{\Gamma(1/4)^4} \frac{\sqrt{A}}{r}.$$  

(8)

The $\sim 1/r$ behavior follows from conformal invariance of the theory. Also, the potential is non-analytic in $\lambda$. Clearly, the coupling should not be very large or else the properties of the resulting bound states are qualitatively different from the $\Upsilon$ etc. states of QCD (numerically, $4\pi^2/\Gamma(1/4)^4 \approx 0.23$).

Effects due to a hot, viscous medium may be investigated in a theory dual to five-dimensional Gauss-Bonnet gravity which leads to [18]

$$V_{Q\bar{Q}}(r) = -\frac{2\sqrt{\lambda}}{r} \left( \frac{\Gamma(3/4)}{\Gamma(1/4)} \right)^2 \left[ 1 - \frac{576\pi^2 \eta T^4}{5 \eta' (1 + \eta')} \left( \frac{\Gamma(5/4)}{\Gamma(3/4)} \right)^2 \right],$$  

(9)

where $\eta' \equiv \sqrt{4\pi \eta/3} \gtrsim 1$. The second term in the square bracket is the leading “thermal screening” correction at small $rT$. In qualitative agreement with eq. (3), the potential decreases (in
magnitude) as $T$ increases but thermal effects diminish as $\eta/s$ increases. However, note that the strong coupling result (9) predicts a more rapid disappearance of temperature effects as $m_Q \to \infty$; for a parametric estimate of the thermal shift of the vacuum binding energy replace $r$ by the Bohr radius $1/(\sqrt{\lambda m_Q})$. The quartic dependence on $rT$ originates from the behavior of the AdS metric near the horizon.

The free energy of a single static quark ($=F_\infty/2$) in the conformal theory dual to GB gravity is equal to

$$F_Q = -\frac{\sqrt{\lambda}}{1 + \eta'} T. \quad (10)$$

Hence, $F_Q$ decreases in magnitude with increasing viscosity. This is qualitatively similar to the behavior in resummed perturbation theory (9). Both produce pure entropy contributions ($0 \sim F \sim T$) only and so the potential energy of the quark in the plasma vanishes once that is removed. It will be interesting to analyze strongly-coupled theories with broken conformal invariance which reproduce the trace anomaly of QCD and $F_\infty$ from eq. (5) above at $T/T_C = 1-3$.

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