Kinematic analysis anthropomorphic gripper with group drive

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Abstract. The advantages of anthropomorphic robotics include the ability to work in a human-oriented environment and the performance of complex tasks that usually only a person can cope with. One of the most important tasks in this area is the development of powerful anthropomorphic grippers. For several reasons anthropomorphic grippers with group drives are promising, in which movement from several joints is performed from one drive. The purpose of the article is a kinematic analysis of one of the possible design of such a gripper. In the article solved the direct and inverse problems of kinematics in an analytical form using the geometric method. The dependences of the Cartesian coordinates of the joints of the fingers on the joint coordinate describing the motion of a group drive were obtained for solving the direct kinematic problem. The dependences between the joint coordinate of a group drive and desired position of the finger joints or the orientation of its links were obtained for solving the inverse problem of kinematics. The obtained results can be used further for performing dynamic analysis, parametric synthesis of the design and synthesis a control system for an anthropomorphic gripper with group drive.

1. Introduction

This article is a rewritten and extended version of the conference materials presented in [1].

Nowadays, a development of technologies to increase the functioning efficiency of the anthropomorphic robotic systems is the one of a problem in the robotics field. The need for an increasing efficiency is due to the insufficient level of development the modern anthropomorphic robots to replace a person in complex jobs and situations involving a risk to life and occurring in conditions not suitable for life (space, rescue, aggressive zones). It is necessary to develop and improve the robot actuators – the anthropomorphic manipulators – to perform the targeted operations. An anthropomorphic manipulator consists of the nodes, connected by hinges, driven by electric motors, and an anthropomorphic grip.

The existing needs of the anthropomorphic manipulators application require the development of manipulator grips to the capabilities of the human hand, while the kinematic and force parameters of the grip must ensure work with objects and tools designed for human capabilities.

The creation of the anthropomorphic robots by space execution is an unconditional trend in scientific and practical aspects [2]. In flight operations, the actions of astronauts outside of the seals are defined, including: taking the working tool out of the pile, preparing it for work, docking-undocking the cable connectors, working with the spanners, nippers, fastening and unfastening the carbines. The above operations are performed using fine motor skills [3]. It is necessary to have not less than fifteen degrees
of mobility to be able to perform similar actions anthropomorphic manipulator. Such operations are performed in conditions of limited hardware resources. Compliance with the requirements of such conditions is possible due to the use of new approaches to the construction and operation of anthropomorphic grip. One such approach is the use of a group drive.

The anthropomorphic grip contains five executive groups of links (EGL). Each EGL has three links. The group drive is used to ensure the movement of links around the parallel axes. The implementation of the cable transfer option provided the creation of grippers successfully used in the robots of the AR-600 (601), SAR-400 (401), FEDOR (figure. 1). The number of active drives is reduced to eight with a total number of degrees of mobility equal to seventeen in the developed and tested versions of the construction the transmission motion systems EGL using differential mechanisms. In that case, a sequence of motion of the output links is provided, sufficient to perform actions corresponding to fine motor behavior [4].

![Figure 1. General view of anthropomorphic capture of AR-600 with group drive.](image)

The purpose of this work is a kinematic analysis of anthropomorphic capture with three degrees of freedom in each finger using a group drive based on solving the direct and inverse problems of kinematics. The novelty of the work lies in the kinematic analysis of an anthropomorphic gripper, in which the joint coordinates in degrees of freedom are cross-dependent, and the kinematic chain is closed. To achieve this goal it is necessary to solve the following research problems:

- To carry out a critical analysis of the literature on the research topic.
- To derive analytical relations for determining the spatial position of the finger links of the anthropomorphic gripper of the robot based on the solution of the direct kinematics problem.
- Derive analytical relations to determine the joint coordinates of the group drive based on the specified Cartesian coordinates of the links of the finger anthropomorphic gripper of the robot based on the solution of the inverse problem of kinematics.

Let analyze the literature on the research topic to complete the first task. The variety of literature on the subject of the research is presented by numerous developments in this field and is due to the urgency of using these developments in the modern world. Problems arising in the development of solutions for
determining the spatial position and orientation of the anthropomorphic grip of the robot on the basis of solving the direct and inverse kinematics problems are considered in the following papers.

Anthropomorphic grippers are used in most robot designs [5–8], as basic functional operations are performed by capturing objects of various shapes and sizes. The creation of anthropomorphic grips implies the maximum likeness of the kinematic diagram of the device with the human hand. However, the realization of all degrees of mobility using modern executive mechanisms is a complex and interesting task.

The researches aimed at developing new gripping devices are presented in papers [9–11]. In paper [9], a non-anthropomorphic grip of a robot for performing tasks, like a human hand, was carried out. The gripping device is represented by a kinematic chain with a tree structure with five branches that have three joint joints. The authors formulated the equations of direct kinematics of relative displacements for each successive chain in the apparatus of dual quaternions. The path of the hand is planned using a hybrid global numerical solver that combines the genetic algorithm and the local Levenberg-Marquardt optimizer. In paper [10], the authors proposed a new grip device with the possibility of reconfiguration. The device has two degrees of freedom on each finger and can support a sufficient payload for production operations. At the same time, a simple kinematics makes it possible to quickly determine the spatial position of the grip links. The authors detail all the principles and concepts used to design this grip. The physical models are given as the result of the project. In paper [11], the authors describe the construction of a gripping device for processing heavy steel pipes with varying physical properties, such as a diameter, a mass and a length. This exciting device is an alternative solution for expensive and complex anthropomorphic seizures. The research focuses on the development of hardware and conceptual design issues of the device.

The researches aimed at the kinematic and dynamic analysis of the gripping device are given in papers [12–18]. In paper [12], the authors presented a hybrid kinematic model of a hand prosthesis that takes into account the different positions of the hand in accordance with the conditions of interaction with the environment. The presented model uses the positions of the phalanges of the finger, calculated using the Denavit-Hartenberg method, mixed with the representation of quaternions. Such an approach makes it possible to level out the singularities of the transformation matrices and to reduce the number of Denavit-Hartenberg parameters. The kinematic and dynamic finger movements are evaluated using an experimental setup with mechanical parts created by 3D printing and various drives.

In paper [13], the authors gave the results of analysis and research of the kinematic model of anthropomorphic grip with 22 degrees of freedom. They described the process of computer modeling and experimental research of the effectiveness of the grip various objects. The presented analysis technique makes it possible to compare different variants of kinematic grip schemes and to determine the adequacy of the choice of a specific kinematic scheme for optimal grip of a given set of objects. This approach is important in the development of manipulator grip, especially when there are restrictions on the number of controlled degrees of freedom. For example, the task of reducing weight and, accordingly, the number of degrees of freedom is relevant in the development of bionic prostheses. In paper, the authors gave a comparison of two kinematic diagrams for capturing a set of geometric primitives: the human diagram - the thumb is opposite to the little finger and the monkey's hand diagram - the thumb is opposed to the middle finger. In paper [14], the authors carried out a research of the process of modeling robots and optimizing their structure. This process is illustrated by the example of studying the robot grip mechanism, which has a structure with a closed loop and a single degree of freedom (DOF). The authors pursued the goal of conducting a detailed grip study to provide an in-depth step-by-step demonstration of the design process and to illustrate the interactions between its stages. Firstly, a geometric model is established that allows one to determine the spatial position of the final effector and generalized coordinates. The Jacobi matrix is determined on the basis of this for calculating the parameters of the device kinematic model. The dynamic model is determined using the Lagrange equations.

In paper [15], the authors presented a method for describing the kinematics of robot grip for work in indeterminate environments. The goal of the authors is to improve the kinematic scheme of the gripping
device in order to enable the grip of large objects with the minimum necessary effort for working in space. In paper [16], the authors presented the research of the kinematic and dynamic properties of the previously developed gripping device. They analyzed the objects of various shapes and sizes for grip and manipulation based on a modified version of the Grubler formula. In paper [17], the authors considered the applications of a group drive that implements the movement of elements in kinematic pairs with parallel axes of rotation. The authors raised an analytical research of the mechanism of group drive, the expression of geometric relationships in vector form was compiled for kinematic analysis, and then a system of scalar equations was obtained. As a result, the angular graphs change from the stroke of the slider, the position plans and the trajectories of the node points of the mechanisms are created and their angular velocities are determined. These speed plans allow you to get the permissible load on the working group of the elements. In paper [18], the authors consider the problem of reorienting the spatial position of the links of the gripping device with the grip object. A simple grip is presented, which can reorient the position repeatedly based on the solution of the direct and reverse kinematics problems without the use of high-precision contact sensors.

The researches aimed at studying the reliability of the griped object and manipulating it are presented in papers [19–22]. In paper [19], the authors presented a new solution for the management and control of five-finger anthropomorphic grip designed to assemble industrial robot equipment. The solution is based on the Motion Leap device and the software module: HandCommander, HandProcessor and HandSIM. The object to be captured is recognized using the SpatialVision application based on image analysis, and then the 3D model is loaded into the GraspIT application. The user's gesture is recognized and sent to the grip test module and the RoboHand component to grip the preconfigured objects. The object is griped in a physical environment by the RoboHand component, an anthropomorphic grip with five fingers. In paper [20], the authors do research of the reliability the grip of the object. The solution is proposed by developing an intelligent self-locking mechanism installed parallel to the drive that starts automatically when the object is grip. This design uses an adjustable power distribution between the grip and the brake via a differential gear. The advantages of adaptive and strong coupling and energy-saving capabilities of the proposed model are demonstrated experimentally with the help of a prototype gripper. In paper [21], the authors consider the task of adaptability of the gripping device for capturing objects of various shapes. The solution is based on the decomposition of the problem into four stages: an identification of the size and a shape of the object, a determination of the initial spatial position of the grip, a calculation of the trajectory of the motion of the grip links, a calculation of the speed of the links for capturing the object. In paper [22], the authors proposed a method of manipulating a griped object by planning a trajectory of motion based on graph theory. The emphasis is on the operation of capturing small objects, corresponding to the fine motor skills of the human hand.

In [23], a method for eliminating the delay that occurs when controlling the drives of an anthropomorphic manipulator based on solving the inverse dynamic problem with a copy control type in real time is proposed. It is also proposed to use for the movement planning not measured, but predicted values of the generalized coordinates of the operator’s hand. Based on the measured values of the generalized coordinates of the hand of the operator, time series are formed and their prediction is performed.

In paper [24], the authors analyze the problems of copy control. To solve the problem, it is proposed to exchange defining the motion law of the operator’s hand by its predictive estimate obtained using the updated Brown method.

Analysis of existing solutions reflects the individuality of using the developed methods for a specific situation and device. Thus, the kinematic analysis of the design of anthropomorphic gripper with group drive is an important task.
2. Results

2.1. Formulation of the problem

Figure 2 shows the general view, construction and kinematic scheme of a finger of the considered anthropomorphic gripper [2].

An anthropomorphic gripper consists of a base 1 and a group of executive units 2 (Figure 1). Each group of executive links includes the first link 3, connected to the base by means of a rotational kinematic pair $A$, the second link 4, connected in the first link by a rotational kinematic pair $B$, and the third link 5, connected to link 4 by a rotational kinematic pair $C$ (figure 2).

The main feature of the capture is a motion transmission system, representing a lever mechanism with a common drive and kinematically dependent motion. Movement from the common drive is transmitted to all links of the anthropomorphic finger through rotational kinematic pairs. This feature distinguishes it from other common constructions of anthropomorphic gripper and makes it necessary to perform kinematic analysis.

Figure 2. The design of the finger anthropomorphic gripper.

For kinematic analysis, it is necessary to derive an analytical relationship between the position and orientation of the links of the finger of anthropomorphic capture and the generalized coordinate characterizing the position of the group drive. That is, to derive an analytical solution of the direct problem of kinematics. These ratios will be necessary in the future when creating a control system for this gripper.

On the other hand, it is necessary to solve the inverse problem of kinematics in an analytical form. That is, derive the relationship between the desired position and orientation of the links and the joint coordinate of the drive. These ratios will further allow the dynamic analysis of an anthropomorphic gripper and a parametric synthesis of an anthropomorphic gripper with specified properties.

2.2. Solution of the direct kinematics problem

When solving the direct problem of kinematics, we will assume that the lengths of all the links are known:

$$l_{OA}, l_{AB}, l_{BC}, l_{CD}, l_{DK}, l_{KN}, l_{NM}, l_{MH}, l_{LA}, l_{LG}, l_{BD}, l_{GB}, l_{DE}, l_{CE}.$$  \(1\)

The length of these links can be determined on the basis of the design of the finger of an anthropomorphic gripper, and there is no need to define them as part of the solution of the analysis problem.

The finger of the anthropomorphic gripper of the considered structure is a flat mechanism, therefore it will consider its movement in the corresponding plane. Without loss of generality, a transition to a three-dimensional space can be made; for this, all coordinates must be multiplied by a matrix of homogeneous transformations describing the position of the selected coordinate system relative to a
more global coordinate system. This transformation matrix can be obtained, for example, by solving a direct kinematics problem for a manipulator onto which the anthropomorphic gripper in question is being installed. The use of a three-dimensional coordinate system is redundant and leads to unnecessary calculations. Also, the calculation using the Denavit-Hartenberg representation is complicated by the fact that this circuit is closed.

The purpose of the calculation is to determine the Cartesian coordinates of the finger joints, the points $A, B, C, D$. It is also of particular interest to find the angles of rotation of the links relative to the basis of an anthropomorphic gripper.

To calculate the coordinates, we introduce the global coordinate system associated with the base. The center of the coordinate system is located at the point $O$, the axis $Ox$ is directed along the link $OA$, and the axis $Oy$ is directed along the link $OK$. Since at this stage the kinematic analysis is performed, the design of the finger anthropomorphic gripper in figure 2 can be depicted as an equivalent scheme shown in figure 3. This scheme allows us to more clearly explain the logic of the calculations.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3.png}
\caption{The kinematic scheme of the finger anthropomorphic gripper.}
\end{figure}

With the selected location of the coordinate system, the coordinates of the part of the points are known:

\begin{align}
O_x &= 0; \\
O_y &= 0, \quad (2) \\
K_x &= 0, \\
K_y &= l_{OK}; \\
A_x &= l_{OA}, \quad (6) \\
A_y &= 0.
\end{align}

Also known angles between rigidly connected links: $\angle OAL, \angle OBD, \angle CBG, \angle ECP, \angle MAB$.

As a joint coordinate, we will consider the distance between points $K$ and $M$, determined by the translational kinematic pair $N$.

Perform the calculation of the Cartesian coordinates of the nodal points of the finger of the anthropomorphic manipulator sequentially from the leading pair $N$ to the working end $D$.

$\angle MAK$ can be calculated by the cosine theorem for a triangle $AKM$:

\begin{align}
l_{RM}^2 &= l_{AK}^2 + l_{AM}^2 - 2 \ast l_{AK} \ast l_{AM} \ast \cos \angle MAK, \quad (8) \\
\angle MAK &= \arccos \left( \frac{l_{AK}^2 + l_{AM}^2 - l_{RM}^2}{2 \ast l_{AK} \ast l_{AM}} \right). \quad (9)
\end{align}
\[ \angle OAK = \arctg \left( \frac{l_{OK}}{l_{OA}} \right), \]  
(10)

\[ \varphi_{AB} = \pi - \angle MAK - \angle OAK - \angle BAM = \]  
(11)

\[ = \pi - \arccos \left( \frac{l_{AK}^2 + l_{AM}^2 - l_{KM}^2}{2 \cdot l_{AK} \cdot l_{AM}} \right) - \arctg \left( \frac{l_{OK}}{l_{OA}} \right) - \angle BAM, \]  
(12)

of rotation of the link AB relative to the base coordinate system. Further, for the orientation angles of the links, similar designations are used.

For a given orientation angle, the coordinates of point B can be found by the formula:

\[ B_x = l_{OA} + l_{AB} \cdot \cos (\varphi_{AB}) = \]  
(13)

\[ = l_{OA} + l_{AB} \cdot \cos (\pi - \angle MAK - \angle OAK - \angle BAM) = \]  
(14)

\[ = l_{OA} + l_{AB} \cdot \cos (\pi - \arccos \left( \frac{l_{AK}^2 + l_{AM}^2 - l_{KM}^2}{2 \cdot l_{AK} \cdot l_{AM}} \right) - \arctg \left( \frac{l_{OK}}{l_{OA}} \right) - \angle BAM), \]  
(15)

\[ B_y = l_{OB} \cdot \sin (\varphi_{AB}) = \]  
(16)

\[ = l_{AB} \cdot \sin (\pi - \arccos \left( \frac{l_{AK}^2 + l_{AM}^2 - l_{KM}^2}{2 \cdot l_{AK} \cdot l_{AM}} \right) - \arctg \left( \frac{l_{OK}}{l_{OA}} \right) - \angle BAM). \]  
(17)

Calculate the Cartesian coordinates of a point C. According to the cosine theorem for a triangle \( \triangle ABL \):

\[ \angle ABL = \arccos \left( \frac{l_{AB}^2 + l_{BL}^2 - l_{AL}^2}{2 \cdot l_{AB} \cdot l_{BL}} \right). \]  
(18)

According to the cosine theorem for a triangle \( \triangle LBG \):

\[ \angle LBG = \arccos \left( \frac{l_{LB}^2 + l_{BG}^2 - l_{LG}^2}{2 \cdot l_{LB} \cdot l_{BG}} \right). \]  
(19)

\[ \varphi_{BC} = \varphi_{AB} + \pi - \angle LBG + \angle ABL - \angle CBG = \]  
(20)

\[ = \pi - \angle MAK - \angle OAK - \angle BAM + \pi - \angle LBG + \angle ABL - \angle CBG = \]  
(21)

\[ = 2\pi - \angle MAK - \angle OAK - \angle BAM - \angle LBG + \angle ABL - \angle CBG = \]  
(22)

\[ = 2\pi - \arccos \left( \frac{l_{AK}^2 + l_{AM}^2 - l_{KM}^2}{2 \cdot l_{AK} \cdot l_{AM}} \right) - \arctg \left( \frac{l_{OK}}{l_{OA}} \right) - \angle BAM + \]  
(23)

\[ + \arccos \left( \frac{l_{LB}^2 + l_{BG}^2 - l_{LG}^2}{2 \cdot l_{LB} \cdot l_{BG}} \right) - \angle CBG - \arccos \left( \frac{l_{LB}^2 + l_{BG}^2 - l_{LG}^2}{2 \cdot l_{LB} \cdot l_{BG}} \right). \]  
(24)

In this case, the coordinates of the point C can be found by the formulas:

\[ C_x = B_x + l_{BC} \cdot \cos (\varphi_{BC}) = \]  
(25)

\[ = l_{OA} + l_{AB} \cdot \cos (\varphi_{AB}) + l_{BC} \cdot \cos (\varphi_{BC}) = \]  
(26)
\[ l_{DA} + l_{AB} \cdot \cos \left( \pi - \arccos \left( \frac{l_{2A}^2 + l_{2AM}^2 - l_{2KM}^2}{2 \cdot l_{2A}^2 \cdot l_{2AM}} \right) - \arctg \left( \frac{l_{OK}}{l_{OA}} \right) - \angle BAM \right) + \]

\[ + l_{BC} \cdot \cos (2\pi - \arccos \left( \frac{l_{2A}^2 + l_{2AM}^2 - l_{2KM}^2}{2 \cdot l_{2A}^2 \cdot l_{2AM}} \right) - \arctg \left( \frac{l_{OK}}{l_{OA}} \right) - \angle BAM + \]

\[ + \arccos \left( \frac{2l_{AB}^2 + l_{2BL}^2 - l_{2AL}^2}{2 \cdot l_{2AB} \cdot l_{2BL}} \right) - \angle CBG - \arccos \left( \frac{l_{2B}^2 + l_{2BG}^2 - l_{2EG}^2}{2 \cdot l_{2LB} \cdot l_{2BG}} \right), \]

\[ C_y = B_y + l_{BC} \cdot \sin (\varphi_{BC}) = \]

\[ = l_{OB} \cdot \sin (\varphi_{AB}) + l_{BC} \cdot \sin (\varphi_{BC}) = \]

\[ = l_{AB} \cdot \sin (\pi - \arccos \left( \frac{l_{2A}^2 + l_{2AM}^2 - l_{2KM}^2}{2 \cdot l_{2A}^2 \cdot l_{2AM}} \right) - \arctg \left( \frac{l_{OK}}{l_{OA}} \right) - \angle BAM + \]

\[ + l_{BC} \cdot \sin (2\pi - \arccos \left( \frac{l_{2A}^2 + l_{2AM}^2 - l_{2KM}^2}{2 \cdot l_{2A}^2 \cdot l_{2AM}} \right) - \arctg \left( \frac{l_{OK}}{l_{OA}} \right) - \angle BAM + \]

\[ + \arccos \left( \frac{2l_{AB}^2 + l_{2BL}^2 - l_{2AL}^2}{2 \cdot l_{2AB} \cdot l_{2BL}} \right) - \angle CBG - \arccos \left( \frac{l_{2B}^2 + l_{2BG}^2 - l_{2EG}^2}{2 \cdot l_{2LB} \cdot l_{2BG}} \right), \]

Similarly, we calculate the Cartesian coordinates of the point D and the orientation angle of the link CD:

\[ \angle BCD = \arccos \left( \frac{l_{2BC}^2 + l_{2CD}^2 - l_{2BD}^2}{2 \cdot l_{2BC} \cdot l_{2CD}} \right). \]

According to the cosine theorem for a triangle \( \triangle LBG \):

\[ \angle DCE = \arccos \left( \frac{l_{2DC}^2 + l_{2CE}^2 - l_{2DE}^2}{2 \cdot l_{2DC} \cdot l_{2CE}} \right), \]

\[ \varphi_{CD} = \varphi_{AB} + \varphi_{BC} + \pi - \angle BCD + \angle DCE - \angle PCE = \]

\[ = \pi - \angle MAK - \angle OAK - \angle BAM + \pi - \angle LBG + \angle ABL - \angle CBG + \]

\[ + \pi - \angle BCD + \angle DCE - \angle PCE = \]

\[ = 3\pi - \angle MAK - \angle OAK - \angle BAM - \angle LBG + \angle ABL - \angle CBG - \angle BCD + \angle DCE - \angle PCE = \]

\[ = 3\pi - \arccos \left( \frac{l_{2A}^2 + l_{2AM}^2 - l_{2KM}^2}{2 \cdot l_{2A}^2 \cdot l_{2AM}} \right) - \arctg \left( \frac{l_{OK}}{l_{OA}} \right) - \angle BAM + \]

\[ + \arccos \left( \frac{2l_{AB}^2 + l_{2BL}^2 - l_{2AL}^2}{2 \cdot l_{2AB} \cdot l_{2BL}} \right) - \angle CBG - \arccos \left( \frac{l_{2B}^2 + l_{2BG}^2 - l_{2EG}^2}{2 \cdot l_{2LB} \cdot l_{2BG}} \right) + \]

\[ + \arccos \left( \frac{2l_{BC}^2 + l_{2BD}^2 - l_{2AL}^2}{2 \cdot l_{2BC} \cdot l_{2BD}} \right) - \angle CBG - \arccos \left( \frac{l_{2D}^2 + l_{2CE}^2 - l_{2DE}^2}{2 \cdot l_{2DC} \cdot l_{2CE}} \right), \]

In this case, the coordinates of the point D can be found by the formulas:

\[ D_x = B_x + C_x + l_{CD} \cdot \cos (\varphi_{CD}) = \]

\[ = l_{OA} + l_{AB} \cdot \cos (\varphi_{AB}) + l_{BC} \cdot \cos (\varphi_{BC}) + l_{CD} \cdot \cos (\varphi_{CD}) = \]

\[ = l_{OA} + l_{AB} \cdot \cos \left( \pi - \arccos \left( \frac{l_{2A}^2 + l_{2AM}^2 - l_{2KM}^2}{2 \cdot l_{2A}^2 \cdot l_{2AM}} \right) - \arctg \left( \frac{l_{OK}}{l_{OA}} \right) - \angle BAM \right) + \]
anthropomorphic gripper can be optimized. Based on the solution of the inverse problem of kinematics obtained below, the design of an arbitrary independent points of space is impossible, since they are kinematically connected. However, finding finger joints to solve the inverse problem of kinematics, it is necessary to find the dependence of the value of the generalized coordinate on the desired position of one of the finger joints. Finding finger joints through the derived analytical dependences allow us to calculate the Cartesian coordinates of the finger joints of an anthropomorphic gripper, as well as to calculate the angles of rotation of its links relative to the absolute coordinate system.

2.3. The solution of the inverse problem of kinematics

To solve the inverse problem of kinematics, it is necessary to find the dependence of the value of the generalized coordinate on the desired position of one of the finger joints. Finding finger joints in arbitrary independent points of space is impossible, since they are kinematically connected. However, based on the solution of the inverse problem of kinematics obtained below, the design of an anthropomorphic gripper can be optimized.

Consider the equations of the direct problem of kinematics in abbreviated form:

\[ B_x = l_{OA} + l_{AB} \cos(\varphi_{AB}), \]
\[ B_y = l_{AB} \sin(\varphi_{AB}), \]
\[ C_x = l_{OA} + l_{AB} \cos(\varphi_{AB}) + l_{BC} \cos(\varphi_{AB} + \pi - \angle LBG + \angle ABL - \angle CBG), \]

The derived analytical dependences allow us to calculate the Cartesian coordinates of the finger joints of an anthropomorphic gripper, as well as to calculate the angles of rotation of its links relative to the absolute coordinate system.
\[ C_y = l_{OB} \sin(\varphi_{AB}) + l_{BC} \sin(\varphi_{AB} + \pi - \angle LBG + \angle ABL - \angle CBG), \]

\[ D_x = l_{OA} + l_{AB} \cos(\varphi_{AB}) + l_{BC} \cos(\varphi_{AB} + \pi - \angle LBG + \angle ABL - \angle CBG) + \]

\[ + l_{CD} \cos(\varphi_{AB} + \varphi_{BC} + \pi - \angle BCD + \angle DCE - \angle PCE), \]

\[ D_y = l_{AB} \sin(\varphi_{AB}) + l_{BC} \sin(\varphi_{AB} + \pi - \angle LBG + \angle ABL - \angle CBG) + \]

\[ + l_{CD} \sin(\varphi_{AB} + \varphi_{BC} + \pi - \angle BCD + \angle DCE - \angle PCE). \]

As follows from the equations, this design can not provide an arbitrary mutual position of points in space, as, incidentally, the hand of a human. To achieve certain positions, it is necessary to displace the entire hand in space. Joint coordinate \( l_{KM} \) enters the equation only implicitly as part of \( \varphi_{AB} \). Express \( \varphi_{AB} \) from equation. For a given coordinate value \( B_x \):

\[ \varphi_{AB} = \arccos \left( \frac{B_x - l_{OA}}{l_{AB}} \right). \]  

coordinate \( B_y \):

\[ \varphi_{AB} = \arcsin \left( \frac{B_y}{l_{AB}} \right). \]

Let

\[ \alpha = \pi - \angle LBG + \angle ABL - \angle CBG. \]  

For the \( C_x \) coordinate transforming, we get:

\[ C_x - l_{OA} = l_{AB} \cos(\varphi_{AB}) + l_{BC} \cos(\varphi_{AB} \cos(\varphi_{AB}) \cos(\alpha) - l_{BC} \sin(\varphi_{AB}) \sin(\alpha), \]

\[ \cos(\varphi_{AB}) \left( l_{AB} + l_{BC} \cos(\alpha) \right) - (C_x - l_{OA}) = l_{BC} \sin(\varphi_{AB}) \sin(\alpha). \]

Since the gripper is anthropomorphic, we can accept the restriction that \( \varphi_{AB} \in [0; \pi/2] \). In this case, the following transformation is valid:

\[ \cos(\varphi_{AB}) \left( l_{AB} + l_{BC} \cos(\alpha) \right) - (C_x - l_{OA}) = l_{BC} \sqrt{1 - \cos^2(\varphi_{AB})} \sin(\alpha). \]

After squaring:

\[ \cos^2(\varphi_{AB}) - 2 \cos(\varphi_{AB}) \left( l_{AB} + l_{BC} \cos(\alpha) \right) (C_x - l_{OA}) + \]

\[ +(C_x - l_{OA})^2 = l_{BC}^2 \sin(\alpha) \left( 1 - \cos^2(\varphi_{AB}) \right). \]

After bringing such an equation finally takes the form:

\[ \cos^2(\varphi_{AB}) \left( l_{AB} + l_{BC} \cos(\alpha) \right)^2 + l_{BC}^2 \sin^2(\alpha) + \cos(\varphi_{AB}) \]

\[ * \left( -2(l_{AB} + l_{BC} \cos(\alpha)) \right) + (C_x - l_{OA})^2 - l_{BC}^2 \sin^2(\alpha). \]

The choice of one of the solutions must be made on the basis of the limitation \( \varphi_{AB} \in [0; \pi/2] \). The same way \( \varphi_{AB} \) can be expressed from other equations for Cartesian coordinates.

Thus, the equations obtained in the course of solving the direct problem of kinematics allow us to find the solution of the inverse problem of kinematics.

3. Discussion

The kinematic analysis carried out in the article made it possible to derive, in an analytical form, the solutions of the direct and inverse kinematics problems for a group-driven anthropomorphic gripper. The obtained solution of the direct kinematic problem allows to perform further dynamic analysis of an anthropomorphic gripper, as well as modeling the interaction of an anthropomorphic gripper with...
objects of manipulation. The obtained analytical solution of the inverse kinematics problem will allow organizing a control system for obtain control laws and the values of the generalized coordinate describing the operation of the group-driven anthropomorphic gripper. Thus, the objectives of the study completed.

4. Conclusions
The article presents a kinematic analysis of a finger of an anthropomorphic group-driven gripper. The obtained analytical solution of the direct problem of kinematics allows, by the known value of the generalized coordinate characterizing the drive, to find the Cartesian coordinates of the finger joints, as well as the angles of orientation of its links. An analytical solution was also obtained for the inverse kinematics problem, which allows us to find the necessary value of the attached coordinate of the group drive from the coordinates of the finger joints. The results can be used to create a control system for an anthropomorphic gripper with a similar structure, perform dynamic analysis, and also optimize the structure. These actions are planned in future articles.

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