Development of a New Empirical Relation to Assess P-wave Velocity Anisotropy of Rocks

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Abstract Several physical parameters and anisotropy related to rock textural arrangements, schistosity and weakness planes such as cracks and joints affect the P-wave velocity ($V_P$). First, $V_P$ anisotropy of quartz-mica schist as a common type of widespread metamorphic rock was compared with $V_P$ anisotropy of jointed homogeneous limestone specimens to clarify effect of these two different types of anisotropies. The results showed that the $V_P$ anisotropy of quartz-mica schist texture is stronger than the $V_P$ anisotropy of jointed limestone, because all body of quartz-mica schist specimens have $V_P$ anisotropy behavior. Many rocks are anisotropic and degree of anisotropy varies from one to another. Various investigations have been carried out on $V_P$ anisotropy but there is not a unique comprehensive relation to represent the influence of different degrees of anisotropy on the $V_P$ for different rocks. The relation between $V_P$ and angle ($\theta$) between the axis of symmetry (perpendicular to weakness planes) with the wave propagation direction was analyzed for a wide range of anisotropy degrees using the results of nine different types of rocks including: Angouran quartz-mica schist, Golgohar mica schist, amphibole schist, mica-quart schist, Marcellus shale, Withby shale WUK47B, WUK70 and WUK2, and Veroia-Polymylos gneiss. A new simple empirical relation fitted to all groups of results was obtained to assess $V_P$ for different degrees of anisotropies with a good correlation of determination ($R^2 = 0.937$), low RMSE (RMSE = 320 m/s) and low CV (CV = 7.0%). P wave velocity anisotropy can simply be predicted by the developed relation using only two parameters of $V_{P0}$ and $V_{P90}$ or $\varepsilon$ that is the percentage change of $V_{P0}$ with respect to $V_{P90}$. A $V_P$ anisotropy classification diagram was also developed based on the different values of $\varepsilon$.

Keywords P-wave velocity · Anisotropy · Rock · Schist · Shale · New relation

1 Introduction

Seismic waves are called elastic waves of materials, as these waves cause elastic transformation of materials. When passing through the rocks, these waves cause material transformation and the velocity of transformation process varies in different rocks depending on their physical and mechanical properties. A seismic wave propagates outwards from a seismic source at a velocity determined by the physical and mechanical properties of the surrounding rocks. The wave velocity through a homogeneous rock is the same in all
directions away from the source. Seismic wave travelling along ray paths that, in isotropic media, are everywhere perpendicular to wavefronts, but the wave velocity varies from one direction to another in an anisotropic media. Anisotropy in rocks may be either inherent, induced or both (Ramamurthy, 2008). Wave velocity anisotropy comes intrinsically from the lattice-preferred orientations of rock-forming crystals and their textural arrangements (Babuška, 1981; Ben 1998) and weakness planes such as schistosity foliation, the systematic arrangement of micro cracks, fractures, joints and fault zones that are induced by deviatoric stresses and may have a secondary effect on the anisotropy of rocks (Anderson et al., 1974; Nur, 1971; Song and Suh, 2014). The distortional deformations that result from tectonic and orogenic movements can also be the sources of wave velocity anisotropy (Kern, 1978).

Seismic anisotropy has been studied more frequently by researchers to enhance seismic data interpretation and petrophysical properties evaluation of the subsurface rocks. In metamorphic rocks, foliation is an important parameter to produce seismic anisotropy. P-wave seismic data show anisotropy effects that are often difficult to attribute convincingly to anisotropy (Winterstein, 1990).

P-wave propagation within the metamorphic rocks are usually in directions not parallel to inherent rock symmetry because of texture and structure and/or oblique-to-accurate ray paths. To have information about the effects of seismic velocity anisotropy on seismic wave propagation in the Earth’s crust and better understanding of how some parameters control the ultrasonic characteristics of rock mass, laboratory scale seismic measurements are needed (Takanashi et al. 2001; Kim et al. 2012). Laboratory studies show that there is a good relation between the seismic P-wave velocity and foliation orientation (Tatham 1982; Takanashi et al. 2001; Palmer 2001). It is possible to have detailed information on wave propagation by measuring seismic anisotropy velocities of all directions in rocks (Takanashi et al. 2001). It should be noted that measurements of P-wave velocity anisotropy in one plane give a little information about the anisotropic structure of a refractor (Backus 1962; Crampin and Bamford 1977; Crampin et al. 1980; Crampin and Kirkwood 1981). In recent years, the behavior of transversely isotropic rocks, e.g., gneiss, schist, slate, phyllite, shale, mudstone, and layered sandstone, has attracted increased attention (Cardenes et al. 2021; Xu et al. 2017; Li et al. 2017; Sarout and Gueguen 2008; Barton 2007). The anisotropy is one of the most distinct features that must be considered in this kind of rock, and is widely encountered in civil, mining, petroleum, geothermal, and geo-environmental engineering (Ma et al., 2018).

Nowadays, applications of nondestructive method such as ultrasonic wave velocities have increased progressively in geotechnical projects (Wang et al. 2020; Heidari et al. 2020; Liu et al. 2021). These methods are performed to obtain direct information of rock quality and other physical—dynamic and geodynamic parameters. Experimental study on physical and mechanical characteristics of rocks and relationship between these parameters improve understanding of geophysical and petrophysical data (Patella and Patella 2009). Seismic velocity is closely related to rock properties, which are coupled with lithological and physical parameters in a complex manner (Kern 1990; Song and Suh 2014).

Many investigators have carried out to measure the $V_p$ anisotropy for various rock types (Thomsen, 1986; Barton 2007; Xu et al. 2017; Li et al. 2017; Sarout and Gueguen 2008; Rezaei et al. 2019; Cardenes et al. 2021). The wave velocity anisotropy in rocks can be approximated by simple anisotropy (Thomsen 1986) or its modification can be used as approximate method to study velocity anisotropy (Tsvankin 1997). The most common case is transverse isotropy with only one symmetry axis (denoted TI hereafter). Weak TI, P- and S-wave velocities in all directions can be approximated well by calculations based on Thomsen’s anisotropy parameters: $\varepsilon$, $\gamma$ and $\delta$ (Thomsen 1986). Thomsen (1986) has also developed a relation to assess $V_p$ as a function of 0 angle. To assess the relation between $V_p$ and 0 angle using Thomsen’s relation, the results of four parameters $V_{p0}^\varepsilon$, $V_{p0}^\gamma$, $V_{p0}^\delta$, $V_{p0}^{\varepsilon\gamma\delta}$ and $V_{p0}^{\varepsilon\gamma\delta}$ are needed. A unique comprehensive relation with a small number of parameters has not yet been given to represent the influence of different degrees of anisotropy on the $V_p$ for different rocks.

In this study, first wave velocity in isotropic and anisotropic rock media has been described. $V_p$ anisotropy of schistosity in quartz-mica schist as a common type of widespread metamorphic rock has been compared with $V_p$ anisotropy of jointed homogeneous limestone specimens to clarify the effect of these two different types of anisotropies. The relation
between \(V_P\) and \(\theta\) angle has been analyzed using nine
groups of results to find the parameters affecting the
\(V_P - \theta\) angle relation. This made a way to achieve a
reliable simple relation to predict \(V_P\) anisotropy by
less parameters than the Thomsen’s relation, and
development a \(V_P\) anisotropy classification system.

2 Wave velocity in anisotropic rock media

The relationship between stress (\(\sigma_{xyz}\)) and strain (\(\varepsilon_{xyz}\)) in x, y and z directions for vertical transverse isotropic
symmetry (VTI) in rock media can be written as in the
matrix form follows:

\[
\sigma_{xyz} = C_{VTI} \varepsilon_{xyz}
\]

The transverse anisotropic matrix has five non-zero
values distributed among 12 non-zero elements. The
stiffness matrix for vertical transverse isotropic sym-
metry (VTI) is described by 5 independent elastic
constants as follows:

\[
C_{VTI} = \begin{bmatrix}
C_{11} & C_{11} - C_{66} & C_{13} & 0 & 0 & 0 \\
C_{11} - C_{66} & C_{11} & C_{13} & 0 & 0 & 0 \\
C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{bmatrix}
\]

\(C_{11}, C_{33}, C_{44}, C_{66}\) and \(C_{13}\) are the five elastic
stiffness needed to describe a VTI media. The stiffness
matrix is explained using P wave velocity along the
foliation direction (\(V_{p90^\circ}\)), P wave velocity perpen-
dicular to the foliation (\(V_{p0^\circ}\)), S wave velocity along
the foliation direction (\(V_{s90^\circ}\)) and S wave velocity
perpendicular to the foliation (\(V_{s0^\circ}\)) as follows (Winterstein 1990):

\[
C_{VTI} = \begin{bmatrix}
\rho V_{p90^\circ}^2 & \rho (V_{p90^\circ} V_{s90^\circ}) & \rho V_{s90^\circ}^2 & C_{13} & 0 & 0 & 0 \\
\rho (V_{p90^\circ} V_{s90^\circ}) & \rho V_{p90^\circ} V_{s90^\circ} & \rho V_{s90^\circ} V_{p90^\circ} & C_{13} & 0 & 0 & 0 \\
0 & \rho V_{p90^\circ} V_{s90^\circ} & \rho V_{s90^\circ} V_{p90^\circ} & \rho V_{s90^\circ} V_{p90^\circ} & C_{13} & 0 & 0 \\
0 & 0 & 0 & 0 & \rho V_{s90^\circ} V_{p90^\circ} & \rho V_{s90^\circ} V_{p90^\circ} & \rho V_{s90^\circ}^2 \\
0 & 0 & 0 & 0 & 0 & \rho V_{s90^\circ} V_{p90^\circ} & \rho V_{s90^\circ}^2
\end{bmatrix}
\]
\[
\gamma = \frac{C_{66} - C_{44}}{2C_{44}} = \frac{V_{S0^o}^2 - V_{S90^o}^2}{2V_{SP}^2} = \frac{(V_{S90^o} - V_{SP})(V_{S90^o} + V_{SP})}{2V_{SP}^2} \approx \frac{V_{S90^o} - V_{SP}}{V_{SP}}
\]

\[
\delta = \frac{(C_{13} + C_{44})^2 - (C_{33} - C_{44})^2}{2C_{33}(C_{33} - C_{44})} = \frac{(C_{13}/\rho + V_{P0}^2)^2 - (V_{P0}^2 - V_{S0}^2)^2}{2V_{P0}^2(V_{P0}^2 - V_{S0}^2)}
\]

where \( \varepsilon \) = anisotropy parameter that is the percentage change of \( V_{P90^o} \) with respect to \( V_{P0^o} \),

\( \gamma \) = anisotropy parameter that is the percentage change of \( V_{S90^o} \) with respect to \( V_{SP} \),

\( \delta \) = anisotropy parameter, can be viewed as a measure of the anellipticity of the P wave curve (Cholach and Schmitt 2006).

The relation between \( V_{P(\theta)} \) and \( \theta \) angle has been given by Thomsen (1986) as follows:

\[
V_{P(\theta)} = V_{P0^o}(1 + \delta \sin^2 \theta \cos^2 \theta + \varepsilon \sin^4 \theta)
\] (13)

To determine \( \delta \) value or \( V_{P(\theta)} \) as a function of \( \theta \) angle, four parameters of \( V_{P0^o} \), \( V_{P90^o} \), \( V_{S0^o} \) and \( V_{P60^o} \) are necessary to be measured. \( \delta \) can also be calculated in the case of weak anisotropy by measurements the three parameters of \( V_{P0^o} \), \( V_{P90^o} \) and \( V_{S45^o} \) as follow (Thomsen 1986):

\[
\delta = 4\left[ \frac{V_{P45^o}}{V_{P0^o}} - 1 \right] - \left[ \frac{V_{P90^o}}{V_{P0^o}} - 1 \right]
\] (14)

3 Comparison Between Effect of Textural and Jointed Anisotropy on P-wave Velocity

P-wave velocity anisotropy can be affected by orientations of rock forming crystals and their textural arrangements and parameters such as rock fractures, cracks and joints as dominant weakness planes. P-wave velocity anisotropy of quartz-mica schist is compared with the P-wave velocity anisotropy of jointed homogeneous limestone specimens to clarify the difference between effect of textural and jointed anisotropy of rocks on the P-wave velocity. Mica-rich rocks (shales, slates, schists) are often characterized by a strong seismic anisotropy (Cardenes et al. 2021). Quartz-mica schist as a common type of metamorphic rock has a widespread distribution in the world, including Scottish Highlands, Norway, Sweden, China, India, Bohemia, Saxony, Brittany, the Alps, Himalayas, and many parts of North America. The geotechnical challenges associated with quartz-mica schist has been reported in many rock engineering projects (Zhang et al. 2011).

Five groups of cylindrical specimens having 5.4 cm diameter (NX size) were cored at different foliation orientation angles (\( \theta \)) of 0°, 30°, 45°, 60° and 90° by the core drilling machine (Fig. 1) from the quartz-mica schist blocks of Angouran mine. Angouran mine is located in 135 km west of Zanjan city, Northwest of Iran.
Iran. For preparation of each specimen, the coring direction with respect to the foliation plane of the rock block was designed to obtain a cylindrical specimen having a particular foliation angle. The core specimens were cut at both ends by the diamond core cutting machine (Fig. 2) to make perfect contacts for the ultrasonic transducers. The cylindrical specimens having length to diameter ratio of 2.0–2.5 were prepared according to the ISRM standard (International Society of Rock Mechanics 1978). Petrographic studies under polarized microscope show that the samples have well foliated schistosity that consists quartz micaschist more than 50% quartz minerals, and other minerals including biotite, cordierite, kyanite and chlorite (Fig. 3).

With the same way cylindrical specimens having 5.4 cm diameter were cored from homogeneous limestone blocks of Naqadeh limestone mine. Naqadeh is located in 23 km South of Lake Urmia in West Azerbaijan Province of Iran. Five groups of cylindrical limestone rock specimens having joint orientation ($\theta$ angle) of $0^\circ$, $30^\circ$, $45^\circ$, $60^\circ$ and $90^\circ$ were also prepared by the core cutting machine (Figs. 2 and 4). The orientation angles of microscopic pictures of the well foliated schistosity in quartz-mica schist has been compared with orientation angle of jointed limestone specimens in Fig. 5. $V_P$ of 10 groups of cylindrical quartz-mica schist and jointed limestone specimens having different orientation angles ($\theta$) of $0^\circ$, $30^\circ$, $42^\circ$, $60^\circ$ and $90^\circ$ were measured according ISRM standard (ISRM 1978) (Fig. 6). The $V_P$ results of two different textural and jointed anisotropies are given in Table 1. The effect of $\theta$ angle on $V_P$ of foliated schistosity in quartz-mica schist specimens was compared with the effect $\theta$ angle on the $V_P$ of jointed homogeneous limestone specimens. The results showed that the $V_P$ anisotropy of rock texture in quartz-mica schist is stronger than the $V_P$ anisotropy of jointed rock (Figs. 7 and 8). That is because all body of quartz-mica schist specimens have textural anisotropy hence $V_P$ anisotropy behaviour.
Many rocks are anisotropic and degree of anisotropy varies from one to another. Despite several investigations have been carried out on $V_P$ anisotropy but there is not a unique comprehensive relation to represent the influence of different degrees of anisotropy on the $V_P$ for different rocks. The relation between $V_P$ and $\theta$ angle was analyzed for a wide range of degrees of anisotropy using the results of Angouran quartz-mica schist of this study and other eight groups of results including: Golgohar mica schist, amphibole schist, mica-quart schist (Hajiheidari et al. 2016), Marcellus shale (Jin et al. 2018), Withby shale WUK47B, WUK70, WUK2 under the same relatively small axial stress (Zhubayev et al. 2016) and Veroia-Polymylos gneiss (Saroglou et al. 2004). The $\varepsilon$ value varied in a wide range from 0.324 to 1.63 as the maximum value of $\varepsilon$ was 3.6 times of its minimum value. Several relations were applied to find a better function fit to the nine different groups of results. At the end the
Following exponential relation was achieved as a best fit to the results:

\[ V_{P(\theta)} = \frac{V_{P0}}{C14} e^{(K\theta)} \quad (15) \]

where

- \( V_{P(\theta)} \) = P-wave velocity at \( \theta \) angle,
- \( K \) = a constant coefficient for different types of anisotropies.

The relation between \( V_P \) and \( \theta \) angle for nine groups of results with the acceptable correlations are shown in Fig. 9. For better comparison between nine groups of results having different \( \varepsilon \) values, the dimensionless relations between ratio of \( V_P/V_{P0} \) and \( \theta \) angle are also shown in Fig. 10. The values of \( V_{P0}, V_{P90}, \varepsilon \) and \( K \) and correlation of determination \( (R^2) \) of the relations are given in Table 2. The parameter \( K \) varied from one rock (group of results) to another. But the relation between \( K \) and \( \varepsilon \) was obtained with a good correlation \( (R^2 = 0.959) \) as follows (Fig. 11):

**Fig. 6** Testing equipment of ultrasonic wave velocity system
Table 1  Measured P-wave velocity at different directions with foliated schistosity in Angouran quartz-mica schist (QMS) and jointed Naqadeh limestone specimens having 5.4 cm diameter

| Symbolic direction of foliated schistosity in QMS specimens | 0 (Degree) | Specimen No | V<sub>P</sub> (m/s) | V<sub>P</sub> average (m/s) | Standard deviation (m/s) |
|-----------------------------------------------------------|------------|-------------|-----------------|--------------------------|------------------------|
| QMS0-1                                                   | 0          | 3618        | 3737            | 232                      |
| QMS0-2                                                   |            | 3589        |                 |                          |
| QMS0-3                                                   |            | 4005        |                 |                          |
| QMS30-1                                                  | 30         | 3763        | 3790            | 137                      |
| QMS30-2                                                  |            | 3668        |                 |                          |
| QMS30-3                                                  |            | 3938        |                 |                          |
| QMS45-1                                                  | 45         | 3941        | 3887            | 464                      |
| QMS45-2                                                  |            | 3399        |                 |                          |
| QMS45-3                                                  |            | 4322        |                 |                          |
| QMS60-1                                                  | 60         | 5157        | 4974            | 163                      |
| QMS60-2                                                  |            | 4918        |                 |                          |
| QMS60-3                                                  |            | 4846        |                 |                          |
| QMS90-1                                                  | 90         | 5430        | 5639            | 182                      |
| QMS90-2                                                  |            | 5730        |                 |                          |
| QMS90-3                                                  |            | 5758        |                 |                          |

P-wave velocity of jointed Naqadeh jointed limestone specimens with different orientation angles (0) from 0 to 90 degrees

| Symbolic direction of joint in limestone specimens | 0 (Degree) | Specimen No | V<sub>P</sub> (m/s) | V<sub>P</sub> average (m/s) | Standard deviation (m/s) |
|---------------------------------------------------|------------|-------------|-----------------|--------------------------|------------------------|
| Lim0-1                                             | 0          | 3316        | 3336            | 177                      |
| Lim0-2                                             |            | 3522        |                 |                          |
| Lim0-3                                             |            | 3171        |                 |                          |
| Lim30-1                                            | 30         | 3561        | 3328            | 214                      |
| Lim30-2                                            |            | 3283        |                 |                          |
| Lim30-3                                            |            | 3140        |                 |                          |
| Lim45-1                                            | 45         | 3772        | 3505            | 252                      |
| Lim45-2                                            |            | 3470        |                 |                          |
| Lim45-3                                            |            | 3272        |                 |                          |
| Lim60-1                                            | 60         | 3993        | 3699            | 316                      |
| Lim60-2                                            |            | 3737        |                 |                          |
| Lim60-3                                            |            | 3365        |                 |                          |
| Lim90-1                                            | 90         | 4246        | 3963            | 254                      |
| Lim90-2                                            |            | 3886        |                 |                          |
| Lim90-3                                            |            | 3757        |                 |                          |
Using Eqs. 14 and 15, the relation between $V_{P0}$ and $\theta$ angle for all groups of results is as follows:

$$V_{P(\theta)} = V_{P0}e^{(0.0057e + 0.0015)\theta}$$

In Eq. 16, the $V_{P0}$ and $\varepsilon$ vary from one group to another. $V_P$ of nine groups was predicted by Eq. 16. The predicted $V_P$ was compared with the measured $V_P$ in Fig. 12. The correlation of determination ($R^2$), root mean square error (RMSE) and coefficient of variation (CV) of the predicted $V_P$ with measured $V_P$ were determined. The coefficient of variation (CV) known as relative RMSE. It is often expressed as a percentage, and is defined as the ratio of the RMSE to the mean. There is a good correlation between predicted $V_P$ and measured $V_P$ for all groups of results with $R^2 = 0.937$, RMSE = 320 m/s and CV = 7.0%. The obtained new empirical relation can represent the $V_P$ as a function of $\theta$ angle for different types of rock. In addition, the relation between $V_P$ and $\theta$ angle can simply be determined by two parameters of $V_{P0}$ and $V_{P90}$ or $[\varepsilon = (V_{P90}-V_{P0})/V_{P0}]$ (Table 3).

The developed relation was validated using four groups of $V_P$ anisotropy for gneiss, phyllite, schist and slate (Tsidzi 1997). The predicted $V_P$ using developed relation in this study has significant correlation ($R^2 = 0.943$, RMSE = 284 m/s and CV = 5.4%) with the results (Fig. 13). A new $V_P$ anisotropy classification was also developed by the obtained empirical relation using a wide range of $\varepsilon$ values (Fig. 11). Five classes of P-wave velocity anisotropy were defined using different ranges of $\varepsilon$ values including: very weak anisotropy ($\varepsilon < 0.1$), weak anisotropy $(0.1 \leq \varepsilon < 0.3)$, medium anisotropy $(0.3 \leq \varepsilon < 0.7)$, strong anisotropy $(0.7 \leq \varepsilon < 0.1.3)$ and very strong anisotropy $(1.3 \leq \varepsilon)$ values (Fig. 14).
Fig. 9  The best fuction fit to the relation between $V_P$ and $\theta$ angle for different types of rocks

Fig. 10  Relation between $V_P/V_{P0}$ ratio and $\theta$ angle for different types of rocks
Table 2  $V_{P0}$, $V_{P90}$, $\varepsilon$, coefficient $K$ of the obtained relations for 9 groups of results

| No | Type of rock and reference                                         | $V_{P0}$ (m/s) | $V_{P90}$ (m/s) | $\varepsilon$ | $K$   | $R^2$ |
|----|-------------------------------------------------------------------|----------------|----------------|---------------|-------|-------|
| 1  | Golgohar Mica schist (Hajiheidari et al., 2016)                    | 2700           | 7100           | 1.630         | 0.0104| 0.937 |
| 2  | Marcellus shale (Jin et al., 2018)                                | 3108           | 5407           | 0.740         | 0.0066| 0.913 |
| 3  | Veroia-Polymylos gneiss (Saroglou et al., 2004)                    | 2828           | 4839           | 0.711         | 0.0058| 0.856 |
| 4  | Withby shale WUK47B (Zhbayev et al., 2016)                        | 2405           | 3777           | 0.570         | 0.0053| 0.964 |
| 5  | Angouran quartz-mica schist (This study)                          | 3737           | 5639           | 0.509         | 0.0041| 0.804 |
| 6  | Golgohar amphibole schist (Hajiheidari et al., 2016)              | 5800           | 7970           | 0.374         | 0.0031| 0.762 |
| 7  | Withby shale WUK70 (Zhbayev et al. 2016)                          | 3084           | 4218           | 0.368         | 0.0037| 0.971 |
| 8  | Golgohar mica quartz schist (Hajiheidari et al., 2016)            | 5200           | 6800           | 0.308         | 0.0033| 0.836 |
| 9  | Withby shale WUK2 (Zhbayev et al., 2016)                          | 2749           | 3640           | 0.324         | 0.0029| 0.860 |

$K = 0.0057\varepsilon + 0.0015$

$R^2 = 0.959$

Fig. 11 Relation between coefficients $K$ and $\varepsilon$ for different types of rocks

Fig. 12 Comparison between predicted $V_P$ and measured $V_P$ for different groups of results
5 Conclusions

1. The results showed that the $V_P$ anisotropy of rock texture in quartz-mica schist is stronger than the $V_P$ anisotropy of jointed rock. That is because all body of quartz-mica schist specimens have textural anisotropy hence $V_P$ anisotropy behaviour.

2. The obtained relation between $V_P$ and $\theta$ angle with exponential function has a good fit to different groups of results and coefficient K of the exponential function (Eq. 14) varies from one group of results to another and it has significant correlation with $\varepsilon$ value. The developed exponential function is significantly fitted to different groups of results having different $\varepsilon$ values. $V_P$ anisotropy can simply be predicted by the developed relation using only two parameters of $V_{P0}$ and $V_{P90}$ or $\varepsilon$ value [$\varepsilon = (V_{P90} - V_{P0})/V_{P0}$].

3. A new $V_P$ anisotropy classification was also developed using a wide range $V_P$ anisotropy results and obtained new relation in this study. $V_P$ anisotropy as a function of $\theta$ angle varies in five classes including: very weak anisotropy ($\varepsilon < 0.1$), weak anisotropy ($0.1 \leq \varepsilon < 0.3$), medium anisotropy ($0.3 \leq \varepsilon < 0.7$), strong anisotropy ($0.7 \leq \varepsilon < 1.3$) and very strong anisotropy ($1.3 \leq \varepsilon$) values (Fig. 11).

### Table 3

| Rock type | $\theta$ | Measured $V_P$ | $\varepsilon$ | Predicted $V_P$ |
|-----------|---------|----------------|--------------|----------------|
| Gneiss    | 0       | 3956           | 0.290        | 3956           |
|           | 45      | 4211           | 4559         |                |
|           | 90      | 5102           | 5254         |                |
| Phyllite  | 0       | 5090           | 0.181        | 5090           |
|           | 45      | 5130           | 5704         |                |
|           | 90      | 6010           | 6393         |                |
| Schist    | 0       | 5151           | 0.289        | 5151           |
|           | 45      | 5802           | 5935         |                |
|           | 90      | 6641           | 6838         |                |
| Slate     | 0       | 4893           | 0.208        | 4893           |
|           | 45      | 5074           | 5522         |                |
|           | 90      | 5913           | 6231         |                |

Fig. 13 Validation of predicted $V_P$ by the developed relation and the measured $V_P$ of gneiss, phyllite, schist and slate with varying $\theta$ angle.
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Fig. 14 Classification of V<sub>p</sub> anisotropy based on different values of ε.
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