Experimental investigations and development of physico-mathematical model of an aerodynamic surface icing in air flow carrying droplets and crystals

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Abstract. The icing of aircraft parts during flight in mixed clouds (containing droplets and crystals) is one of important and, at the same time, insufficiently studied cases. In the present work, the effect of melting ratio on icing intensity of wing leading edge is experimentally studied. A physical and mathematical model of crystals accretion on the surface due to their deformation and destruction is proposed.

1. Experimental setup and results
The icing intensity (average linear growth rate of ice layer thickness) under mixed conditions on the wing model was studied in performed tests. The experiment was carried out as follows. The wing model at zero angle of attack was placed in an air-crystalline flow with specified parameters (velocity \( V \), temperature \( T \), liquid water content LWC and ice water content IWC) during a given time \( t \). All the process was recorded on a video camera in the side view, the camera installation setup is shown in figure 1. It is important to note that shooting in this mode requires accurate installation of the camera – the optical axis should be perpendicular to the end of the model. The experimental data are shown in table 1. The icing intensity \( S \) is defined as the ratio of the ice thickness at the model front point to the ice growth time.

![Camera installation setup. Ice is marked in blue.](image-url)

Figure 1. Camera installation setup. Ice is marked in blue.
Table 1. Icing intensity experimental data

| Run | V (m/s) | T (°C) | LWC (g/m³) | IWC (g/m³) | t (s) | S (µm/s) | β (deg.) |
|-----|---------|--------|------------|------------|------|----------|---------|
| 1   | 40      | –12    | 1.4        | 0          | 180  | 38       | -       |
| 2   | 40      | –12    | 1.4        | 9.0        | 180  | 42       | -       |
| 3   | 40      | –12    | 1.4        | 5.4        | 180  | 53       | 113     |
| 4   | 40      | –12    | 1.4        | 2.7        | 180  | 55       | 121     |
| 5   | 80      | –12    | 0.7        | 0          | 120  | 39       | -       |
| 6   | 80      | –12    | 0.7        | 2.1        | 120  | 39       | -       |
| 7   | 80      | –12    | 0.7        | 5.2        | 120  | 36       | -       |
| 8   | 80      | –6     | 0.7        | 0          | 120  | 31       | -       |
| 9   | 80      | –6     | 0.7        | 1.8        | 120  | 35       | 137     |
| 10  | 80      | –6     | 0.7        | 1.0        | 180  | 27       | -       |

Figure 2 shows photographs of the resulting ice shapes for some runs from table 1. From the figures, one can see the specific differences between runs without crystals (liquid droplet icing) and with crystals (mixed conditions) in the flow. Ice formed in a purely drip flow is more transparent and has a perceptible roughness that grows downstream from the leading edge. The ice obtained in the mixed drop-crystalline flow is more matte, at high speeds its surface becomes smoother due to erosion. A feature of icing in some regimes is the formation of wedge-shaped ice with a vertex at the front point. Such ice shapes are usually characterized by the wedge angle $\beta$ [1]. In figure 2e, the photo of ice is shown with the wedge angle measurement. The data on the wedge angle for the remaining regimes are shown in table 1. One can see that lowering the temperature, increasing ice water content IWC or flow velocity leads to a decrease in the wedge angle, making the ice wedge sharper.

**Figure 2.** Ice shapes for runs 5 – 10 from table 1, a) – f), respectively.
The dependence of the ice thickness on the critical line versus time is close to linear. The icing intensity has a complex dependence on flow parameters. In particular, the relative increase in IWC in total water content non-linearly affects the icing intensity at different temperatures – reduces the amount of ice formed at high flow velocities and increases at small.

Ice shapes corresponding to the table 1 runs were measured using the digital image tracing technique. It is as follows. In the image taken according to the setup shown in figure 1, the outline of ice formation and scale markers were drawn using a graphics tablet. Further, according to these data, using a special program, parameters were converted into metric units and a dimensional grid was applied. An example of a processed ice shape image is shown in figure 3. The thickness of the mesh cell is 1 mm.

![Figure 3. Ice build-up image of run 3 from table 1 and its processing.](image)

2. Experimental results analysis and discussion

To analyze the effect of the experimental parameters on the resulting ice, the data were presented in a dimensionless form. Figure 4 shows the dependence of the reduced icing intensity $\chi = \frac{S\rho}{(u_s TWC)}$ on the melting ratio $\zeta = \frac{LWC}{TWC}$, where $TWC = LWC + IWC$ (TWC – total water content), $\rho$ – ice density. The graph shows the table 1 runs data obtained at TsAGI (rows 1–3) and the data from the Technical University of Braunschweig (TUBS) [1] (rows 4–6). One can see that a series of experiments with different temperatures and velocities are well resolved and fall on different curves. TUBS results were obtained on a NACA0012 test model with a chord length of 500 mm at a flow velocity of 40 m/s, temperatures of 0°C (row 4), –5°C (row 5) and –15°C (row 6) and TWC in the range from 6.4 up to 17 g/m$^3$. The test models under consideration are characterized by almost identical collection efficiency coefficients of crystals at a critical point.

A comparison of the TsAGI and TUBS runs carried out at a flow velocity of 40 m/s (rows 1, 4–6) shows that the sets of points related to different temperatures have similar behavior depending on the melting ratio. The lower the temperature at which a series of tests was carried out, the lower the points of the corresponding series on the graph lie. However, comparing the results obtained at flow velocity of 80 m/s (rows 2 and 3), one can see that the temperature dependence becomes inverted in comparison with the results obtained at flow velocity of 40 m/s. The points of row 3 for a temperature $T = –6$°C lie lower than the points of row 2 corresponding to temperature $T = –12$°C. The mass median diameter of the crystals in both cases is in the range 130–200 μm, but the crystals were obtained in substantially different ways. In TUBS experiments, crystals were condensed in a cloud chamber under conditions close to natural. In TsAGI experiments, crystals were obtained by grinding an ice block. Despite these significant features, it can be seen that the data obtained at TsAGI correlate well with the TUBS data.
The influence of velocity can be traced by the behavior of rows 5 and 3, which relate to the results obtained at flow velocities $V = 40$ and $80$ m/s and temperatures $T = –5$ and $6^\circ$C respectively, as well as rows 1 and 2 for the results at velocities $V = 40$ and $80$ m/s and temperature $T = –12^\circ$C. Rows 5 and 3 are well resolved on the graph, and one can see that the series data are lower with higher velocity. However, the data of rows 1 and 2 lie rather closely and obtaining a general dependence requires additional experiments. Nevertheless, the erosion phenomenon can be called one of the factors in reducing the given icing intensity with increasing speed.

Figure 4. Dependence of the reduced icing intensity $\chi$ on the model spreading line versus the melting ratio $\zeta$ for TsAGI runs (table 1): row 1 ($V = 40$ m/s, $T = –12^\circ$C), row 2 ($V = 80$ m/s, $T = –12^\circ$C), row 3 ($V = 80$ m/s, $T = –6^\circ$C); and TUBS runs on NACA0012 test model: row 4 ($V = 40$ m/s, $T = 0^\circ$C), row 5 ($V = 40$ m/s, $T = –5^\circ$C), row 6 ($V = 40$ m/s, $T = –15^\circ$C).

3. Physical and mathematical model of discrete crystalline icing

In this paragraph we consider the physical and mathematical model of individual ice crystals accretion on the surface due to their partial destruction in the case of icing in fully crystalline conditions. Similar ideas were developed, for example, in [2]. The authors of this article in [3] proposed the model of the ice crystal collision with a dry solid [2] and estimated the mass of the crystal remaining on the surface. In this process, the important role is played by the effective values during compression $\sigma_{nc}$ and shear $\sigma_{se}$ of the yield strengths $\sigma_i$ and $\sigma_w$ (ice and material of the streamlined body), determined by the formula

$$
\sigma_{je}^{-1} = \sigma_{ji}^{-1} + \sigma_{w}^{-1}, \quad j \equiv n, \tau.
$$

In particular, if the streamlined body is absolutely rigid ($\sigma_{rw} \rightarrow \infty$), only the ice crystal is destroyed. If the latter hits the ice formed on the body, then $\sigma_{je} = \sigma_{ji}/2$.

For the ratio $\xi = m/m_0$, where $m_0$ is the mass of the crystal before the collision, $m$ is the mass remaining on the surface after impact and destruction of the crystal, the formula is used

$$
\xi = 1 - \exp\left[-\frac{\rho_i}{2} \left( \frac{v_n^2}{\sigma_{nc}} + \frac{v_t^2}{\sigma_{te}} \right) \right]
$$
Here $v_n$ and $v_\tau$ are the normal and tangential components of the impact velocity, $\rho_i$ is the ice density. At low impact velocities ($v \to 0$), the particle does not collapse and $\xi = 0$; at large ($v \to \infty$), the particle completely destroys and its entire mass remains on the surface.

The values of yield strengths upon impact depend on the velocity and exceed the values measured in quasistatic conditions given in the handbooks. In [2], the following approximation formula was proposed:

$$\sigma_n = \sigma_{n0} \exp \left(0.9 \frac{v_n}{v_{nc}} \right)$$

where $\sigma_{n0}$ is the yield strength at low impact velocities, $v_{nc} = \sqrt{\rho_i / \sigma_{nc}}$.

For the dependence of the ice yield strength at shear $\sigma_\tau$ on the tangential impact velocity $v_\tau$ in [3], a similar formula with the corresponding characteristic velocity $v_{xc} = \sqrt{\rho_i / \sigma_{xc}}$ was used.

In [3], the impact of crystals on the body was considered as a continuous mass flow, while the traditional assumption of “instantaneous” adhesion of the destroyed mass at the impact point and the formation of a smooth ice surface was used. In this article, a discrete-crystalline model is developed, in which the act of interaction of an individual crystal with a surface is considered. Such a consideration leads to the appearance of a resultant tuberous structure.

We assume, as is often done, that crystals have a spherical shape. Suppose that upon impact, a spherical segment breaks off the crystal (figure 5). With a known fraction $\xi$ of the remaining crystal mass, the height $h_m$ of this segment is given by the formula (from the solution of the geometric problem):

$$\frac{h_m}{a} = 1 + 2 \cos \left\{\frac{1}{3} \left[ 2\pi - \arctan \left( \frac{2\xi - 1}{1 - 2\xi} \right) \right] \right\},$$

where $a$ is the radius of the crystal.

We assume that with an inclined incidence, this segment is deformed in the direction of the component of the impact velocity $v_\tau$ so that the displacement of the elementary layer $\Delta y$ located at a height $y$ above the point of impact is equal to $s(y) = \beta y$. Where $\beta = v_\tau \tau_{coll} / h_m$ is proportionality coefficient and $\tau_{coll} \sim h_m/v_{nc}$ is crystal disintegration time.

**Figure 5.** The scheme of interaction of the crystal with the surface: a) the impact of the crystal on the surface (the shattered segment is shaded); b) deformation of the segment upon impact; c) the shape of the formed ice hillock.

Thus, in the coordinate system $x, y, z$ with the beginning at the point of impact, the equation of the remaining deformed crystal lower surface ($y < h_m$) has the form:
Let us assume that the ice hillock formed on the surface has the shape of this inverted segment. Then the equation of the hillock upper surface has the form

$$ (x - \beta y)^2 + (y - a)^2 + z^2 = a^2. $$

Successive collisions of crystals are accompanied by their sliding along a knobby surface and corresponding increments $\Delta h_i$. The direction of crystal sliding is determined by the differential geometry formulas.

The discrete-crystalline model was tested on the example of a circular cylinder with a radius of 15 mm, streamlined by a transverse air stream with a velocity $u_\infty = 50$ m/s. The crystals had a spherical shape, their radius was 500 μm. IWC was 10 g/m$^3$.

Figure 6 shows the results of the ice growths calculation obtained by the discrete-crystalline model after 60 s of icing. Figure 6a gives the ice profile in the central section AA of the computational domain, and Figure 6b gives the surface topography when viewed from the front. Figure 6c shows changes in the thickness of the ice around the circumference of the cylinder.

**Figure 6.** The results of calculations by a discrete-crystalline model: a) a cylinder with an accumulated layer of ice in the AA section; b) the relief of the ice thickness on the frontal surface; c) the angular dependences of the thickness: a solid thin line – in section AA, thick – a thickness averaged over all sections parallel to AA; dashed line is the thickness found under the assumption of a continuous flow of mass of crystals.

**Conclusion**
Experimental studies of ice formation on the leading edge of the wing profile model for various flow parameters were carried out. Measurements of icing intensity (average linear growth rate of the ice layer), the resulting ice shapes, the wedge-angle of the ice layer for various experimental conditions were made. It is shown that at flow velocity of 80 m/s, the effects of erosion become significant, and a decrease in temperature, an increase in IWC or velocity leads to a decrease in the wedge-angle, making the ice wedge sharper.
An analysis of the experimental results was performed, the data are presented in the form of dependency of dimensionless parameters, namely, the dependency of reduced icing intensity on the wing model critical line, as a function of melting ratio. The results obtained in TsAGI and TUBS are compared. It is shown that, despite the fundamentally different methods for producing crystals, a good correlation of the data is observed. However, to explain all the observed phenomena, further research in wider range of parameters is required. The data obtained can be used to verify numerical models.

To simulate the icing of aircraft under pure crystalline conditions, a stochastic model of individual crystals interaction with body surface was developed and tested using the example of circular cylinder icing.

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