Finite element analysis of a soft mounted 2-pole asynchronous machine concerning vibration stability

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Abstract. The paper describes a finite element calculation concerning vibration stability of a 2-pole asynchronous machine with sleeve bearings and flexible shaft, mounted on soft foundation elements. The finite element model considers the rotor, stator, end shields and bearing housings, oil film stiffness and damping, the electromagnetic interaction between rotor and stator – deduced from an electromagnetic field calculation –, the gyroscopic effect of the rotor and the influence of the foundation stiffness and damping. The critical mode shapes and natural frequencies are calculated for different rotor speeds and the eigenvalues are analyzed to receive the limit of stability. Therefore the paper shows a calculation method to optimize the vibration stability by considering foundation stiffness and damping.

1. Introduction
In the past foundations of rotating machinery for industrial applications were often designed stiff. But nowadays they become softer, because of plant specific requirements. Therefore the foundation influences the vibration system [1, 3] and therefore has to be taken into consideration. Figure 1 shows exemplary a 2-pole asynchronous machine, mounted on soft foundation elements. The 2-pole asynchronous machine shown in figure 1, with a power output of 2 MW, is designed with a flexible shaft and hydrodynamic sleeve bearings, with cylindrical bearing shells. By analyzing the vibration behavior, not only the critical speeds have to be considered, but also the limit of vibration stability, mainly caused by the oil film characteristics.

Figure 1: Soft mounted asynchronous machine: a) view of the shaft end; b) longitudinal section
Raising the rotor speed \( n \) above the limit of stability, self excited vibrations will occur, which disturb the operation [1, 2]. The specialty of asynchronous machines is the additional electromagnetic coupling between rotor and stator by an electromagnetic field [6-9], which also influences the vibration system and therefore the limit of stability. So the whole vibration system, consisting of the rotor, stator, sleeve bearing, electromagnetic field and the foundation has to be taken into consideration.

2. Data of the 2-pole asynchronous machine and the foundation elements
The data of the asynchronous machine and the foundation elements are described in table 1. In this case the 2-pole asynchronous machine is mounted on soft foundation elements. The longitudinal section of the machine and the soft foundation elements are shown in figure 1.

| Notation                     | Symbol | Data          |
|------------------------------|--------|---------------|
| Rated power                  | \( P_N \) | 2000 kW       |
| Number of pole pairs         | \( p \)    | 1             |
| Rated voltage                | \( U_N \)  | 6000 V        |
| Rated frequency              | \( f_N \)  | 50 Hz         |
| Rated torque                 | \( M_N \)  | 6.4 kNm       |
| Rated speed                  | \( n_N \)  | 2990 r/min    |
| Mass of the stator           | \( m_1 \)  | 7200 kg       |
| Mass of the rotor            | \( m_2 \)  | 1900 kg       |
| Type of sleeve bearing       | -       | Side flange bearing |
| Shape of the bearing shell   | -       | Cylindrical   |
| Lubricant viscosity grade of the oil | - | ISO VG 32   |
| Nominal bore diameter of the bearing shell | \( d_b \) | 110 mm       |
| Bearing width                | \( b_b \)  | 81.4 mm       |
| Ambient temperature          | \( T_{amb} \) | 25°C          |
| Lubricant supply temperature of the oil | \( T_{in} \) | 40°C         |
| Mean relative bearing clearance (DIN 31698) | \( \Psi_m \) | 1.6 %         |
| Vertical stiffness of one foundation element | \( c_{iz} \) | 10 kN/mm     |
| Horizontal stiffness of one foundation element | \( c_{iy} = c_{ix} \) | 13 kN/mm     |
| Damping ratio of the foundation element | \( D_f \) | 0.07          |

3. Finite Element model of the soft mounted asynchronous machine
To calculate the natural vibrations and the limit of vibration stability of the soft mounted asynchronous machine a finite element model is deduced, using the finite element program MADYN. For the modeling of the rotor, which consists of a shaft and a rotor core with squirrel cage, linear beam elements (Timoshenko-beam elements) are used, considering bending and shear. The hydrodynamic oil film is considered by the oil film stiffness matrix \( C_v \) and the oil film damping matrix \( D_v \), which contain the oil film stiffness coefficients \( (c_{yy}, c_{yz}, c_{zy}, c_{zz}) \) and the oil film damping coefficients \( (d_{yy}, d_{yz}, d_{zy}, d_{zz}) \). These dynamical coefficients are calculated by solving the REYNOLDS Differential equation [4] for each rotor speed, using the calculation program SBCALC from RENK AG. The structural stiffness of the end shield and the sleeve bearing house is considered as a translational spring element \( (c_{bx}, c_{by}, c_{bz}) \), which is deduced from a separate finite element calculation. The damping of the end shield and bearing house is neglected, because of the low damping ratio. The stator mass \( m_1 \) with its inertias \( (\Theta_{1x}, \Theta_{1y}, \Theta_{1z}) \) is concentrated in the centre of gravity. The stator structure is modeled with rigid beam elements, representing a rigid stator structure. This is possible, because the foundation stiffness is assumed to be much lower than the stator structure, which is typical for a soft mounting. The foundation stiffness \( (c_{bx}, c_{by}, c_{bz}) \) and damping \( (d_{bx}, d_{by}, d_{bz}) \) coefficients are considered by a translational spring and damper element under each machine foot. The magnetic spring constant \( c_m \), which describes the magnetic stiffness, – representing the electromagnetic interaction between rotor and stator – is distributed
over the core length between the rotor and the stator. The magnetic spring constant $c_m$ is derived by a separate finite element calculation (section 4).

**Figure 2:** Finite element model of the soft mounted asynchronous machine

### 4. Finite Element calculation of the magnetic spring constant

To consider the influence of the electromagnetic field, an electromagnetic finite element calculation is deduced, using the two-dimensional electromagnetic field calculation program FEMAG. The electromagnetic model, shown in figure 3a, consists of the stator core containing the three-phase winding, the rotor core with the rotor bars and the cooling slots, the air gap and the rotor shaft. The calculations are done for no-load operation and with rated magnetizing current in the stator winding as static field solution. For the calculation the magnetizing current is impressed as direct current in the stator winding, so no induction in the rotor cage occurs and therefore no field damping by the rotor cage is considered.

**Figure 3:** Finite element field calculation of the 2-pole asynchronous machine with a centric rotor; a) Model; b) Flux lines of the field; c) Flux density (absolute value)
Figure 3b shows the field lines and figure 3c the flux density for a certain field position, with a centric rotor. Afterwards the rotor is displaced by different eccentricities $\varepsilon$ in the two-dimensional model – normal to the field and parallel to the field – and the resulting radial magnetic forces $F_r$ are computed. Without considering field damping effects, these radial magnetic forces are maximal. Field damping leads to lower radial magnetic forces but causes tangential magnetic forces. Mostly these tangential magnetic forces are so low, that they can be neglected for the vibration analysis. The displacement $\varepsilon$ of the rotor is related to the air gap width $\delta_h$, so a relative displacement $\varepsilon = \varepsilon/\delta_h$ is derived. The relative eccentricity is analyzed in the range of $\varepsilon = 0…0.5$. Figure 4 shows, that for rated magnetization current, there is nearly a linear behavior between the relative eccentricity and the radial magnetic force and there is nearly no difference concerning the direction of the eccentricity, up to an eccentricity of about $\varepsilon = 0.2$ [6, 9].

![Graph showing radial magnetic force $F_r$ vs. relative eccentricity $\varepsilon$](image)

**Figure 4:** Radial magnetic force $F_r$ for different relative eccentricities $\varepsilon$ of the rotor, at rated magnetization current (field lines pictured for $\varepsilon = 0.5$)

With raising the relative eccentricity $\varepsilon$ far above 0.2 the radial magnetic force is influenced by the direction of the eccentricity. In practice, the relative eccentricity will be less than $\varepsilon < 0.1$ for this kind of asynchronous machine. Therefore a linear behavior between eccentricity and radial magnetic force can be assumed and a magnetic spring constant $c_m$ deduced [6, 9].

$$c_m = \frac{F_r}{\varepsilon}$$  \hspace{1cm} (1)

The magnetic force is acting in the direction of the eccentricity, so the magnetic stiffness is a “negative” stiffness. Figure 5 shows the influence of the magnetic saturation on the magnetic spring constant $c_m$. The magnetic saturation is described by the ratio of the magnetic induction of the fundamental field $B_f$ to the rated magnetic induction of the fundamental field $B_{PN}$. Up to the rated magnetization current $I_{mN}$, there is nearly a linear relation between the stator current and the magnetic induction. Raising the current above the rated value, the saturation of the iron parts – yoke and teeth – increases strongly and leads to a non-linear behavior. Far above the rated magnetization current the magnetic spring value decreases with rising stator current, because the high saturation leads to an increasing “magnetic” air gap. At the rated conditions, the value of the magnetic spring $c_m$ is here 7.6 kN/mm.

5. Oil film stiffness and damping coefficients

The characteristic of a hydrodynamic oil film is well known [1-5] and is shown in Fig. 6. The movement of the journal shaft is described by the vertical displacement $z_v$ and the horizontal displacement...
The movement of the sleeve bearing housing is described by the vertical displacement $z_v$ and the horizontal displacement $y_b$.

The stiffness and the damping coefficients of the oil film can be calculated by solving the Reynolds-Differential-Equations [4]. The oil film stiffness coefficients ($c_{yy}, c_{yz}, c_{zy}, c_{zz}$) and the oil film damping coefficients ($d_{yz}, d_{zy}, d_{yy}, d_{zz}$) of the sleeve bearings are strongly influenced by the rotor speed $n$, which is shown in figure 7. For this example they are nearly identical for both bearings. Figure 6 shows clearly, that the coupling damping coefficients are equal ($d_{yz} = d_{zy}$) but the coupling stiffness coefficients are not equal ($c_{zy} \neq c_{yz}$). This leads to an asymmetric oil film stiffness matrix $C_v$, which is the reason that vibration instability may occur [1, 2].

![Oil film stiffness matrix](image)

**Figure 6:** Model for hydrodynamic oil film

The coupling coefficients – stiffness coupling coefficients $c_{yz}, c_{zy}$ and damping coupling coefficients $d_{yz}, d_{zy}$ – cause a coupling between vertical and horizontal movement and the vertical oil film force $F_z$ and the horizontal oil film forces $F_y$, which is mathematically described in (2).

$$
\begin{bmatrix}
F_z \\
F_y
\end{bmatrix} =
\begin{bmatrix}
c_{zz} & c_{zy} \\
c_{yz} & c_{yy}
\end{bmatrix}
\begin{bmatrix}
z_v - z_b \\
y_v - y_b
\end{bmatrix} +
\begin{bmatrix}
d_{zz} & d_{zy} \\
d_{yz} & d_{yy}
\end{bmatrix}
\begin{bmatrix}
z_v' - z_b' \\
y_v' - y_b'
\end{bmatrix}
$$

(2)

The stiffness and the damping coefficients of the oil film can be calculated by solving the Reynolds-Differential-Equations [4]. The oil film stiffness coefficients ($c_{yy}, c_{yz}, c_{zy}, c_{zz}$) and the oil film damping coefficients ($d_{yz}, d_{zy}, d_{yy}, d_{zz}$) of the sleeve bearings are strongly influenced by the rotor speed $n$, which is shown in figure 7. For this example they are nearly identical for both bearings. Figure 6 shows clearly, that the coupling damping coefficients are equal ($d_{yz} = d_{zy}$) but the coupling stiffness coefficients are not equal ($c_{zy} \neq c_{yz}$). This leads to an asymmetric oil film stiffness matrix $C_v$, which is the reason that vibration instability may occur [1, 2].

![Oil film stiffness and damping coefficients](image)

**Figure 7:** Oil film stiffness and damping coefficients for different rotor speeds

### 6. Natural vibrations and vibration stability

With the vibration model in figure 2 the natural vibration and the limit of stability can be calculated. Therefore the homogenous differential equation (3), with the mass matrix $M$, the damping matrix $D$, the gyroscopic matrix $G$, the stiffness matrix $C$ and the coordinate vector $q$, has to be solved [1].

$$
M \cdot \ddot{q} + (D + G) \cdot \dot{q} + C \cdot q = 0
$$

(3)

The mathematical ansatz

$$
q = \dot{q} \cdot e^{i\omega}
$$

(4)
with the eigenvalue $\lambda$ and the eigenvector $\mathbf{\hat{q}}$, leads to the eigenvalue equation

$$[\mathbf{M} \cdot \lambda^2 + (\mathbf{D} + \mathbf{G}) \cdot \lambda + \mathbf{C}] \cdot \mathbf{\hat{q}} = 0.$$  \hspace{1cm} (5)

By solving the determination equation the eigenvalues $\lambda_n$ can be computed. The eigenvalues occur mostly conjugated complex [1]

$$\lambda_n = \alpha_n \pm i \cdot \omega_n,$$  \hspace{1cm} (6)

with the real parts $\text{Re}\{\lambda_n\} = \alpha_n$, which describe the decay of each natural vibration and the imaginary parts $\text{Im}\{\lambda_n\} = \omega_n$, which describe the corresponding natural angular frequencies. With this eigenvalues $\lambda_n$ the corresponding eigenvectors $\mathbf{\hat{q}}_n$ can be calculated. Therefore the solution of the homogenous deferential equation can be described by:

$$\mathbf{q} = \sum_n \mathbf{\hat{q}}_n \cdot k_n \cdot e^{\lambda_n t}.$$  \hspace{1cm} (7)

To investigate the vibration stability the real parts $\alpha_n$ of the complex eigenvalues $\lambda_n$ have to be analyzed. If one or more real part gets positive, then vibration instability occurs. To get the limit of stability the rotor speed $n$ is raised until at least one real part gets zero. At the limit of stability, that means at the rotor speed $n_{\text{limit}}$, the undamped mode (with $\alpha_{\text{limit}} = 0$) oscillates with the natural angular frequency of $\omega_{\text{limit}}$, as a self exciting vibration [1, 2].

6.1. Limit of vibration stability without foundation damping

By solving the differential equations and analyzing the real part of the complex eigenvalues, the critical mode shape can be found, which gets instable first of all. The other natural vibrations are not getting instable or only at higher rotor speed. Foundation damping is here neglected ($D_r = 0$), therefore some natural vibration exist, which have a real part near to zero (e.g. the rigid body modes), but they are here not considered, because a marginal foundation damping would last, that these natural vibrations are not getting instable. The mode shape of the critical natural vibration at the limit of stability is shown in figure 8. Rotor and stator are moving on elliptical orbits and both orbits are run through in the same direction as the rotor rotates. Therefore both orbits are elliptical forward whirls. The mode shape shows additionally, that rotor and stator are vibrating contrary to each other.

![Critical mode shape](https://example.com/critical_mode_shape.png)

**Figure 8:** Real part and imaginary part of the complex eigenvalue of the critical mode, depending on the rotor speed, without foundation damping ($D_r = 0$)
To find the limit of stability $n_{\text{limit}}$, the rotor speed $n$ is raised and the real part of the complex eigenvalue is analyzed. The real part of the complex eigenvalue gets zero ($\alpha_0 = \alpha_{\text{limit}} = 0$) at about a rotor speed of $n = n_{\text{limit}} = 4485$ r/min. At this rotor speed, the imaginary part of the complex eigenvalue gets $\alpha_0 = \alpha_{\text{limit}} = 243$ 1/s and the undamped natural frequency becomes $f_n = f_{\text{limit}} \approx 38.7$ Hz. So the limit of stability occurs at a rotor speed of 4485 r/min. Above this rotor speed, vibration instability occurs. For comparison also a rigid mounting is analyzed. Figure 8 shows clearly, that the limit of stability for the rigid mounted machine ($c_{tx} = c_{ty} = c_{tz} \rightarrow \infty$) is reached at a lower rotor speed $n_{\text{limit}} = 3950$ r/min. Here the natural frequency of the critical mode becomes $f_{\text{limit}} = 34.0$ Hz. The vibration shape of the rotor is quite similar to the vibration shape for the soft mounted machine, but without movement of the stator mass, because of the rigid foundation. Therefore the limit of stability can be raised from 3950 r/min to 4485 r/min by putting the asynchronous machine on the soft foundation elements, without considering the damping of the soft foundation elements ($D_t = 0$), which will lead to a still higher limit of stability (section 6.2). The reason for this increase from 3950 r/min to 4485 r/min is the increase of the natural frequency of the critical mode (from 34.0 Hz to 38.7 Hz). The ratio $n_{\text{limit}}/f_{\text{limit}}$ stays nearly the same.

6.2. Influence of foundation damping

In section 6.1 foundation damping was not considered. In this section the influence of the foundation damping on the limit of stability will be shown. In practice, it is often very difficult to derive the real damping of the foundation and often only possible by measurements. But to show the influence of the foundation damping on the limit of stability, different values for the damping coefficients of the foundation are considered here in the example. Referring to [1], the damping coefficients of the foundation under each motor foot $d_{tx}, d_{ty}, d_{tz}$ are here described by the damping ratio $D_t$, the stiffness coefficients $c_{tx}, c_{ty}, c_{tz}$ of the foundation under each motor foot and the stator mass $m_1$, as a rough simplification.

$$d_{tq} = D_t \cdot \sqrt{c_{tq} \cdot m_1} \quad \text{with: } q = x,y,z$$

(8)

Figure 9 shows, that in the range from $D_t = 0...0.15$ the limit of stability $n_{\text{limit}}$ can be raised by increasing the foundation damping. The natural frequency $f_{\text{limit}}$ of the critical mode shape is only marginal influenced by the foundation damping in the analyzed range. For the rated foundation damping ratio ($D_t = 0.07$), the limit of stability can be shifted to a rotor speed of about 4705 r/min. Compared to a rigid mounting the limit of stability can be increased from 3950 r/min to 4705 r/min by putting the machine on the soft foundation elements. But also the natural frequency of the critical mode shape is shifted to higher values, which is also necessary to consider for the operation speed range.

**Figure 9:** Limit of stability $n_{\text{limit}}$ and natural frequency $f_{\text{limit}}$ depending on the foundation damping ratio $D_t$
6.3. Influence of magnetic spring constant $c_m$

Finally the influence of the magnetic spring constant $c_m$ on the limit of stability is analyzed. Figure 10 shows, that with decreasing the magnetic spring constant, the limit of vibration stability is raised. The reason is that a lower magnetic spring constant – e.g. caused by the field damping of the rotor cage – leads to a higher natural frequency of the critical mode shape and therefore to a higher limit of stability. Calculating with the undamped magnetic spring constant is therefore a worst case analysis.

![Figure 10: Limit of stability $n_{limit}$ depending on the magnetic spring $c_m$](image)

7. Conclusion

The paper shows a method for calculating vibration stability of soft mounted asynchronous machines with sleeve bearings and flexible shafts, using the finite element method for the vibration analysis and for the electromagnetic field calculation. It is shown, based on an example, how the limit of vibration stability can be influenced by the foundation stiffness and damping. Additionally the influence of the electromagnetic field on the vibration stability is also derived.

References

[1] Gasch R, Nordmann R and Pfützner H 2002 Rotordynamik (Berlin: Springer)
[2] Rao J S 1991 Rotor Dynamics (New Delhi: Wiley Eastern Limited)
[3] Kirk R G, DeChowdhury P and Gunter E J 1974 The effect of support flexibility on the stability of rotors mounted in plain cylindrical bearings (Proc. IUTAM Symposium Dynamics of Rotors) p 244
[4] Reynolds O 1886 On the Theory of Lubrication (Philosophical Transaction of the Royal Society, Vol. 177, London)
[5] Lund J and Thomsen K 1978 A calculation method and data for the dynamics of oil lubricated journal bearings in fluid film bearings and rotor bearings system design and optimization (ASME, New York) pp 1-28
[6] Seinsch H O 1992 Oberfelderscheinungen in Drehfeldmaschinen (Stuttgart: Teubner)
[7] Belmans R, Vandenput A and Geysen W Calculation of the flux density and the unbalanced magnetic pull in two pole induction machines (Archiv für Elektrotechnik, Vol. 70) pp 151-161
[8] Holopainen T P 2004 Electromechanical interaction in rotor dynamics of cage induction motors (Ph.D. Thesis, VTT Technical Research Centre of Finland, Helsinki University of Technology)
[9] Werner U 2006 Rotordynamische Analyse von Asynchronmaschinen (Dissertation, Technical University of Darmstadt)