Jeans instability of self-gravitating magnetized strongly coupled plasma

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Abstract. We investigate the Jeans instability of self-gravitating magnetized strongly coupled plasma. The equations of the problem are formulated using the generalized hydrodynamic model and a general dispersion relation is obtained using the normal mode analysis. This dispersion relation is discussed for transverse and longitudinal mode of propagations. The modified condition of Jeans instability is obtained for magnetized strongly coupled plasma. We find that strong coupling of plasma particles modify the fundamental criterion of Jeans gravitational instability. In transverse mode it is found that Jeans instability criterion gets modified due to the presence of magnetic field, shear viscosity and fluid viscosity but in longitudinal mode it is unaffected due to the presence of magnetic field. From the curves we found that all these parameters have stabilizing influence on the growth rate of Jeans instability.

1. Introduction

In natural and laboratory situations many objects exist in strongly coupled state viz. neutron star, dwarf star, interior of planets, ultra cold neutral plasma and liquid plasma crystal [1]. In these objects the Coulomb potential energy is much greater that the average thermal energy of the plasma particles. The ratio of the Coulomb potential energy to the thermal energy is called the coupling parameter ($\Gamma$) thus for strongly coupled plasma $\Gamma \gg 1$. In the presence of dust particles, plasma exhibits variety of rich phenomena. Under certain conditions, the charged dust grains may organize themselves in a regular crystal-like structure. For $\Gamma > \Gamma_c$ (where $\Gamma_c$ is the critical value for crystallization) the dust component goes into an ordered crystalline phase which forms plasma crystal [2,3]. The mutual interaction among the dust particles can be represented by the Yukawa (Debye–Hückel) potential $\Phi(r) = \frac{Q^2}{4\pi\varepsilon_0 r} \exp\left(-\frac{r}{\lambda_D}\right)$, where $\lambda_D$ is the Debye length of plasma and Q is the charge of dust particle [4]. The waves and instabilities in strongly coupled plasmas are studied by many authors using the generalized hydrodynamic model. Kaw and Sen [5] have discussed the low frequency modes in strongly coupled dusty plasmas. Rosenberg and Shukla [6] have shown the effect of strong ion correlations on the dispersion relation of possible ion-beam plasma instabilities. Many authors [7-9] have discussed the problem of strongly coupled plasma for the investigation of acoustic

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wave and instabilities both analytically and numerically. However, most papers on collective effects in self-gravitating plasmas are concerned with the problem of the coupling between purely electrostatic and gravitational disturbances. Yaroshenko et al. [10] have discussed the linear coupling of electromagnetic and Jeans modes in self-gravitating plasma streams. Recently, Janaki et al. [11] have investigated the Jeans instability of viscoelastic fluid and found that the threshold for the onset of instability appears at higher wavelength in a viscoelastic medium. Looking to the importance of present problem in astrophysical situations, we investigate the Jeans instability of self-gravitating magnetized strongly coupled plasma.

2. Perturbation equations and dispersion relations

Let us consider a compressible, infinitely conducting viscoelastic strongly coupled plasma embedded in uniform magnetic field $B_0 (0, 0, B)$. In the absence of a magnetic field, the mechanical equations can be described small amplitude, longitudinal sound (compressional) waves with speed $c_s$, the square of which is equal to the derivative of the pressure $p$ with respect to the density $\rho$ at constant entropy $T$. The basic equations of the problems are linearized using the standard linearization process. We suppose all the physical quantities are sum of their equilibrium and perturbed parts $\rho(t), \phi(t), \rho_0, \phi_0, B(t), B_0, \sigma(t)$. The generalized equation of motion for magnetized strongly coupled plasma is given by

$$
\left[ 1 + \frac{\partial}{\partial t} \right] \left[ \rho_0 \frac{\partial \vec{v}}{\partial t} + \rho v_\parallel \vec{\phi} + c_s^2 \vec{\rho} - \frac{1}{4\pi} (\nabla \times \vec{B}) \times \vec{B}_0 \right] = \frac{\sigma}{\rho_0} \nabla^2 \vec{v} + \left( \frac{T}{3} + \frac{\phi}{\rho} \right) \nabla (\nabla \cdot \vec{v}),
$$

(1)

The continuity equation is

$$
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0,
$$

(2)

The Poisson’s equation for self-gravitational potential is written as

$$
\nabla^2 \phi = 4\pi \rho_0 \vec{\rho},
$$

(3)

The equation for magnetic field is given by

$$
\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}).
$$

(4)

For linear instability analysis we now solve equations (1)-(4) using normal mode analysis subject to perturbation $\exp[i(k_x x + k_z z + \sigma \tau)]$ (where $\sigma$ is frequency of the harmonic disturbance and $k_{x,z}$ are the wave numbers in transverse and longitudinal directions to the magnetic field). We obtain three linear equations in terms of velocity components $v_x, v_y, v_z$ which gives a $3 \times 3$ matrix whose vanishing solution yields a general dispersion relation. The general dispersion relation is discussed for transverse and longitudinal mode of propagations.

2.1 Transverse mode of propagations $k \perp B$:

$$
\left\{ (1 + \sigma \tau) \left[ \frac{k^2 \sigma}{\rho_0} + \left( c_s^2 + V_s^2 \right) k^2 - 4\pi \rho_0 \right] + \frac{k^2 \sigma}{\rho_0} \left( \frac{4\sigma}{3} \right) \right\} = 0.
$$

(5)

The above equation shows the dispersion relation of homogeneous magnetized, self-gravitating strongly coupled plasma. In classical limit $\sigma \tau \ll 1$ we recover the classical dispersion relation of self-gravitating, magnetized, viscous plasma. The above dispersion relation has two factors, in which the first factor equating zero gives the non-gravitating, damped viscous mode modified due to relaxation time for the strongly coupled plasma. In the kinetic limit $\sigma \tau \gg 1$, second factor of dispersion relation (5) gives
The condition of Jeans instability can be obtained from the constant term of above dispersion relation which is given by

$$\sigma^2 + \left[ (c_s^2 + V_A^2) + \frac{1}{\tau \rho_0} \left( \frac{\xi + 4 \vartheta}{3} \right) \right] k^2 - 4 \pi G \rho_0 = 0. \quad (6)$$

The system will be unstable for all the wave numbers provided by expression (7). We find that condition of Jeans instability and expression of critical Jeans wave number both are modified due to the presence of Alfven velocity, viscosity, shear viscosity and relaxation time. It is obvious that as the shear viscosity, viscosity or magnetic field in the strongly coupled plasma increases the corresponding critical jeans wave number decreases hence all these parameters have stabilizing influence on the growth rate of Jeans instability.

The effect of shear viscosity and sound speed is studied on the growth rate of Jeans instability of magnetized strongly coupled plasma. We now write dispersion relation (6) in the dimensionless form

$$\sigma^{*2} + \left[ 1 + c_s^{*2} + \xi^{*} \right] k^{*2} - 1 = 0, \quad (8)$$

where \( \sigma^{*} = \sigma / (4 \pi G \rho_0)^{1/2}, k^{*} = k V_A / (4 \pi G \rho_0)^{1/2}, c_s^{*} = c_s / V_A, \) and \( \xi^{*} = (1/ \tau \rho_0) \left( \xi + 4 \vartheta / 3 \right) / V_A^2, \) are the dimensionless parameters.

In figure 1 and figure 2 we have plotted the dimensionless growth rate of Jeans instability against the dimensionless wave number for various values of shear viscosity and sound speed. It is obvious from figure 1 that shear viscosity has damping influence on the growth rate of Jeans instability similar to the classical fluid viscosity as in case of weakly coupled viscous plasma. We find that growth rate of Jeans instability decreases as the shear viscosity of the medium increases. Hence it has stabilizing influence on the growth rate of Jeans instability.

![Figure 1. Effect of shear viscosity on the growth rate of Jeans instability keeping \( c_s^{*} = 0.25 \).](image1.png)

![Figure 2. Effect of sound speed on the growth rate of Jeans instability keeping \( \xi^{*} = 0.25 \).](image2.png)
2.2 Longitudinal mode of propagation $k \parallel B$

In the case when $k_z = k, k_s = 0$ we get the following dispersion relation

$$
\left[ (1 + \sigma \tau) \left( \sigma^2 + V_A^2 k^2 \right) + \frac{k^2 \varrho \sigma}{\rho_0} \right]^2 \times \left[ (1 + \sigma \tau) \left( \sigma^2 + k^2 c_s^2 - 4\pi G \rho_0 \right) + \frac{\sigma k^2}{\rho_0} \left( \xi + \frac{4 \varrho}{3} \right) \right] = 0. \quad (9)
$$

This dispersion relation has two factors. The first factor gives when equated to zero a non-gravitating damped viscous Alfvén mode. In the classical limit ($\sigma \tau << 1$) the above equation shows the dispersion relation for classical viscous magnetized plasma. In kinetic limit ($\sigma \tau >> 1$) we obtain the following dispersion relation

$$
\sigma^2 + V_A^2 k^2 + \frac{k^2 \varrho}{\rho_0} = 0. \quad (10)
$$

The second factor of dispersion relation (9) gives

$$
\left[ (1 + \sigma \tau) \left( \sigma^2 + k^2 c_s^2 - 4\pi G \rho_0 \right) + \frac{\sigma k^2}{\rho_0} \left( \xi + \frac{4 \varrho}{3} \right) \right] = 0. \quad (11)
$$

The above equation shows a gravitating shear viscous mode which is unaffected due to the presence of magnetic field. In the kinetic limit ($\sigma \tau >> 1$) we obtain the following condition of Jeans instability $k < k_{ji} = \left[ 4\pi G \rho_0 / (c_s^2 + \frac{1}{\tau \rho_0} (\xi + 4/3 \varrho)) \right]^{1/2}$. It is clear from the above expression that condition of Jeans instability and expression of critical Jeans wave number are both modified due to shear viscosity but they are unaffected due to the presence of magnetic field. Thus magnetic field play role in the condition of Jeans instability only in transverse mode of propagation and it has no influence in the longitudinal mode of propagation.

3. Conclusion

In the present paper we have investigated the Jeans instability of self-gravitating strongly coupled plasma. The dispersion relation of the problem is obtained which is discussed for both transverse and longitudinal mode of propagations. In the kinetic limits ($\sigma \tau >> 1$) we obtain propagation of both longitudinal and transverse viscoelastic modes. In transverse mode we obtain a self-gravitating shear induced viscous mode modified due to Alfvén velocity. The condition of Jeans instability is modified due to the presence of shear viscosity and Alfvén velocity and both parameters have stabilizing influence on the growth rate of Jeans instability. In longitudinal mode of propagation a self-gravitating unmagnetized shear induced viscous mode. In this mode the condition of Jeans instability depends upon shear viscosity and it is unaffected due to the presence of magnetic field.

4. References

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