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Regular sequences and synchronized sequences in abstract numeration systems. (English) Eur. J. Comb. 101, Article ID 103475, 34 p. (2022)

Summary: The notion of $b$-regular sequences was extended to linear recurring bases by Allouche, Scheicher and Tichy in 2000, and to abstract numeration systems by Maes and Rigo in 2002. Their definitions are based on a notion of $S$-kernel that extends that of $b$-kernel. However, these definitions do not allow us to generalize all of the many characterizations of $b$-regular sequences. In this paper, we present an alternative definition of $S$-kernel, and hence an alternative definition of $S$-regular sequences, which enables us to use recognizable formal series in order to generalize most (if not all) known characterizations of $b$-regular sequences to abstract numeration systems. We then give two characterizations of $S$-automatic sequences as particular $S$-regular sequences. Next, we present a general method to obtain various families of $S$-regular sequences by enumerating $S$-recognizable properties of $S$-automatic sequences. As an example of the many possible applications of this method, we show that, provided that addition is $S$-recognizable, the factor complexity of an $S$-automatic sequence defines an $S$-regular sequence. In the last part of the paper, we study $S$-synchronized sequences. Along the way, we provide a constructive proof, only based on weighted automata, that the formal series obtained as the composition of a synchronized relation and a recognizable series is recognizable. As a consequence, the composition of an $S$-synchronized sequence and an $S$-regular sequence is shown to be $S$-regular. All our results are presented in an arbitrary dimension $d$ and for an arbitrary semiring $\mathbb{K}$.

MSC:

68Qxx Theory of computing
11Bxx Sequences and sets
68Rxx Discrete mathematics in relation to computer science

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