The chiral vortical effect (CVE) was derived first for non-interacting massless fermions. Recently, an alternative derivation of the CVE was suggested which relates it to the radiation from the horizon of a rotating black hole. We attempt to generalize the latter derivation to the case of photons and encounter a crucial factor of two difference between the two ways of visualizing the CVE. Reservations and possible explanations are briefly discussed.

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The chiral vortical effect (CVE) refers to the flow of chirality of massless fermions in a rotating medium along the vector of the angular velocity \( \Omega \). For a single right-handed Weyl fermion one finds:

\[
\vec{j}^N = \left( \frac{\mu^2}{4\pi^2} + \frac{T^2}{12} \right) \vec{\Omega}, \tag{1}
\]

where \( \vec{j}^N \) is the current of number of particles, \( \mu \) is the chemical potential conjugated to the charge \( Q^N \) associated with the current \( j^N_{\mu} \), \( T \) is the temperature. In the pioneering paper \([1]\) Eq. (1) is derived in the limit of non-interacting fermions.

A new era in theory of chiral effects began with the paper in Ref. [2] which developed an approach valid in the strong-coupling limit, or for ideal fluids. The basic idea is to rely only on the hydrodynamic expansion and (anomalous) conservation laws. Remarkable enough, Eq. (1) survives the change in the framework. Moreover, the coefficient in front of the \( \mu^2 \vec{\Omega} \) term turns to be directly related to the coefficient in the r.h.s. of the chiral anomaly:

\[
\partial^\alpha j^N_{\alpha} = \frac{1}{32\pi^2} \epsilon^{\alpha\beta\gamma\delta} F_{\alpha\beta} F_{\gamma\delta}, \tag{2}
\]

where \( F_{\alpha\beta} \) is the electromagnetic-field strength tensor. The coefficient in front of the \( T^2 \vec{\Omega} \) term remains undefined within the hydrodynamic approach of Ref [2] but can be fixed, say, within the thermal field theory [3].

Note that although derivation of the \( \mu^2 \)-term in [2] is straightforward, the result is not easy to appreciate. Indeed, the \( \mu^2 \vec{\Omega} \) term survives in absence of external electromagnetic fields while the anomaly [2] vanishes in this limit. The resolution of the puzzle [3] is that in case of ideal fluids there exist extra conservation laws and the fluid-helicity current is separately conserved.

Indeed, the \( \mu^2 \)-term can be generated through the substitution [2]:

\[
e\mathbf{A}_\alpha \rightarrow e\mathbf{A}_\alpha + \mu \cdot \mathbf{u}_\alpha, \tag{3}
\]

where \( \mathbf{u}_\alpha \) is the 4-velocity of an element of the fluid, in the expression [2] for the chiral anomaly. On the other hand, introduction of the chemical potential and/or of the temperature cannot affect the anomaly [2] which is fixed by the short-distance physics. Therefore, the extra terms generated through the substitution [3] can contribute to the \( 4d \) axial current, but not to its divergence. This vanishing of the divergence signifies existence of extra conservation laws, see [2] and references therein.

The relation, if any, of the \( T^2 \)-term in Eq. (1) to the anomalies remained a kind of a mystery [2] until there appeared the paper [3]. The main idea here goes back to papers [9] which relate the Hawking radiation, derived first from the thermodynamic approach to the black-hole physics, to field-theoretic anomalies. In more detail, it is suggested [3] to consider space-time with a boundary imposed by the horizon of a rotating black hole. Then one can check that near the horizon the r.h.s. of the gravitational chiral anomaly:

\[
\nabla^\alpha j^N_{\alpha} = \frac{1}{768\pi^2 \sqrt{-g}} \epsilon^{\alpha\beta\gamma\delta} R_{\alpha\beta\rho\sigma} R^{\rho\sigma}_{\gamma\delta}, \tag{4}
\]

where \( R_{\alpha\beta\gamma\delta} \) is the Riemann tensor, is not vanishing.

It was demonstrated [3] that far off from the horizon, where the r.h.s. of Eq. (4) vanishes, there is a flow of chirality which can be found by integrating the r.h.s. of Eq. (4). This flow of chirality is nothing else but the \( T^2 \vec{\Omega} \) term in Eq. (1) provided that the generic temperature \( T \) is replaced by the Hawking temperature \( T_H \) of the black hole,

\[
T \rightarrow T_H \equiv \frac{a}{2\pi}, \tag{5}
\]

and one keeps only the first term in the expansion in \( \Omega \) (which is to be understood now as the angular velocity at the horizon).

Thus, in case of massless spin 1/2 particles there are two complementary ways of deriving the CVE, that is, in terms of (infinite) flat space and in terms of black-hole physics. In these notes we are considering generalization of this observation to the photonic case.
GRavitational Chiral Anomaly and CVE for Photons

As is well known, the chirality of photons is measured by the “charge” $\int d^3x K_0$ where the current $K^\mu$ is given by:

$$K^\mu = \frac{1}{\sqrt{-g}} \epsilon^{\mu\nu\rho\sigma} A_\nu \partial_\rho A_\sigma ,$$

where $A_\mu$ is the electromagnetic potential. The current is normalized in such a way that the corresponding (axial) charge $Q^A_{\text{photon}} = \pm 1$ for the right- and left-hand polarized photons, respectively. Note also that the charge $Q^A_{\text{photon}}$ is gauge invariant, unlike the current itself.

The current is apparently not conserved, since

$$\nabla_\mu K^\mu \equiv 1/2 F_{\mu\nu} \tilde{F}^{\mu\nu} .$$

However, one can demonstrate (for explanations and further references see, in particular, \cite{10}) that naively the expectation value of the r.h.s. of Eq. (4) for photons propagating in external gravitational field vanishes, $<\nabla_\mu K^\mu >_{\text{naive}} = 0$. In this sense, there is analogy with the standard case of charged massless fermions interacting with external electromagnetic field. Moreover, there exists \cite{10,12} the bosonic chiral gravitational anomaly:

$$<\nabla_\mu K^\mu > = \frac{1}{96\pi^2} R_{\mu\nu\rho\sigma} \tilde{R}^{\mu\nu\rho\sigma} ,$$

where

$$\tilde{R}^{\mu\nu\rho\sigma} = \frac{1}{2\sqrt{-g}} \epsilon^{\mu\nu\gamma\delta} R^{\rho\sigma}_{\gamma\delta} .$$

Now, we are all set to evaluate the chiral vortical effect for photons in terms of the black-hole physics following the logic of the paper \cite{3}.

Indeed, the chiral gravitational anomalies for spin-1/2 and spin-1 massless particles are proportional to the same $RR$ and the effect of the rotating black hole reduces to a universal geometric factor. We are interested now in the spin-dependence of the chiral vortical effect. To elucidate the spin dependence of the CVE it is convenient to compare fermionic and bosonic fields with equal number of chiral degrees of freedom, that is normalize the photonic case to the case of a Weyl spinor. By comparing Eqs. (2) and (3) we immediately conclude

$$\frac{(CVE)_{\text{photons}}}{(CVE)_{\text{Weyl fermions}}} |_{\text{Kubo relation}} = 2 .$$

Note that \cite{13} differs from \cite{11} by a factor of 2 (which is the central point of the current notes). Result \cite{13} is also reproduced within kinetic approach, for details and references see, in particular, \cite{13}.

Let us mention that the gauge invariance of the results obtained remains a subtle point since the current $K^\mu$ is not explicitly gauge invariant. Gauge invariance could be imposed explicitly at each step by introducing non-locality. In particular, for photons on mass-shell there is a well-known expression for the current in the annihilation channel:

$$\kappa_\mu = (\text{const}) \frac{q_\mu}{q^2} F_{\alpha\beta} F^{\alpha\beta} ,$$

where $q_\mu$ is the 4-momentum brought in by the current. Note, however, that in two cases most interesting for applications the current $\kappa_\mu$ reduces in fact to $K_\mu$. Namely,

EVALUATION OF PHOTONIC CVE IN FLAT SPACE

CVE through Kubo-type relation

In its original formulation, the chiral vortical effect does not imply any charge non-conservation. The current flows, generally speaking, through the whole space and the flow of charge is conserved since $\text{div} \tilde{\dot{\Omega}} = 0$.

There exist a few approaches to evaluate the CVE in flat space. Nowadays, the most common way to evaluate the CVE is to reduce it to a retarded, 3d Green’s function using the technique similar to derivation of Kubo relations. In more detail, define the conductivity $\sigma_\Omega$ as:

$$\tilde{J}^\mu = \sigma_\Omega \tilde{\dot{\Omega}} .$$

Then, $\sigma_\Omega$ is an equilibrium quantity given by the behavior of the retarded two-point Green’s function between the current $\tilde{J}_i^N$ and the momentum density $T_{0j}$ at zero frequency $\omega$ and small momenta $k_1$ in presence of rotation:

$$G_R(\omega,k)|_{\omega=0} = i\epsilon_{ij} n^i \sigma_\Omega .$$

Detailed calculations along these lines of the CVE in case of charged spin-1/2 particles within thermal field theory can be found, in particular, in \cite{3,4}.

Eq. (12) can immediately be generalized to the case of photons interacting with external gravitational field, see, in particular, \cite{1,4}. In fact the two-loop contribution to the CVE for spin 1/2 particles factorizes into the product of one-loop chiral anomaly \cite{2} and of the CVE associated with the photonic current $K^\mu$. The corresponding conductivity, $\sigma_\Omega$ for the current $K^\mu$ is expressed now in terms of the commutator between the photonic current $K^\mu$ and the momentum density $T_{0j}$. The result can be summarized as

$$\frac{(CVE)_{\text{photons}}}{(CVE)_{\text{Weyl spinor}}} |_{\text{Kubo relation}} = 2 .$$

The problem is that the flat-space derivation suggests rather that the ratio (14) is equal to 2, not 4.
the evaluation of the CVE in the preceding section refers in fact to the limit \( q_i = 0, \omega \to 0 \). In this limit
\[
\lim_{q_i=0,\omega\to0} \kappa_0 = K_0 ,
\]
and the non-local current \( j^\nu_{\Omega} \) reduces to the same charge density \( K_0 \). In case of the 3d picture, see \( \text{(12)} \) one considers the limit \( \omega = 0, q_3 \to 0 \). In this limit the non-local current \( \kappa_\mu \) reduces to the component \( K_3 \):
\[
\lim_{\omega=0,q_3\to0} \kappa_3 = K_3 ,
\]
the same as used in the thermal field calculations. This remark implies that the apparent lack of gauge invariance inherent to using a polynomial-like current \( K_\mu \) might be not such a problem since in the limits considered the current \( K_\mu \) coincides with non-local, explicitly gauge invariant currents.

Two-loop effects: a point aside

Gravitational chiral anomaly for the photonic current \( K_\mu \) is traditionally used to evaluate a two-loop correction to the gravitational chiral anomaly for the massless fermions. According to \( \text{(11)} \),
\[
\nabla^\alpha j^N_\alpha = \frac{1}{768\pi^2} (1 - \frac{2\alpha_{el}}{\pi}) R^\mu_\nu\alpha\beta R^\mu\nu\alpha\beta .
\]
Refs. \( \text{(3, 4)} \) address the issue of evaluating radiative correction to the conductivity \( \sigma_\Omega \):
\[
\sigma_\Omega = \frac{T^2}{12} \left( 1 - \frac{\alpha_{el}}{\pi} \right) .
\]
Note that the radiative corrections in the r.h.s. of Eqs. \( \text{(17)} \) and \( \text{(18)} \) differ by a factor of 2. At least naively, one would expect the corrections to coincide.

Two remarks are now in order. Concentrate first on the one-loop effects, which determine \( \sigma_\Omega \) and the gravitational anomaly for the chiral photonic current, see Eq. \( \text{(8)} \). Because of the assumed factorization of the two-loop effects algebraically the two formulations (in terms of one-loop and two-loop effects) reduce to each other. However the assumption on the factorization of the two-loop effects could be questioned by itself (for discussion see \( \text{(10)} \)). In this sense, the difference of factor of two between the two evaluations of the CVE for photons on the one-loop level is more significant.

Our second remark is that two-loop effects are absent if one considers the conserved (i.e. non-anomalous) current:
\[
(j^N_{\text{conserved}})_\alpha = (j^N_{\text{naive}})_\alpha + \frac{\alpha_{el}}{2\pi} K_\alpha .
\]
The two-loop corrections (see Eqs. \( \text{(17)}, \text{(18)} \), vanish if one starts from such a definition of the axial current.

To our mind, this observation implies that the hydrodynamics is to be defined rather in terms of the current \( j^N \) than in terms of the original, anomalous current \((j^N_{\text{naive}})_\alpha \). This cancellation of the two-loop effects is not sensitive to the factor of two which we are primarily interested in.

CVE from the Sommerfeld expansion

Consider again the flat-space approach to evaluate the CVE. As has been emphasized in the literature (see, in particular, \( \text{(1, 8)} \)) various chiral conductivities can be reduced to the so-called Sommerfeld integrals. For example:
\[
\tilde{j}^N_\Omega = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} c^2 d\epsilon \cdot \left( \frac{1}{1 + e^{\beta(-\epsilon + \mu + \Omega/2)}} - \frac{1}{1 + e^{\beta(-\epsilon - \mu + \Omega/2)}} \right) ,
\]
where \( \tilde{j}^N_\Omega \) is the absolute value of the current \( j^N_\Omega \) (directed along \( \Omega \)). One can expect that a similar representation is valid for the photonic chiral vortical effect. Such a representation would have an advantage of reducing CVE to an integral from the Bose distribution which refers to particles on mass-shell or physical degrees of freedom (while off mass-shell one should take into account unphysical degrees of freedom).

Thus, one could speculate that the photonic chiral current is given by:
\[
K_\Omega = \frac{1}{2\pi^2} \int_{0}^{\infty} c^2 d\epsilon \cdot \left( \frac{1}{-1 + e^{\beta(-\epsilon + \Omega)}} - \frac{1}{-1 + e^{\beta(-\epsilon - \Omega)}} \right) .
\]
Let us emphasize, however, that we do not claim to have derived this expression. Steps in this direction were made in \( \text{(10)} \). Namely, one can use spinor indices to describe the photonic field and reduce the action to a form which is a close reminiscent of the action of a complex scalar field:
\[
S_{eff} = \int d^4x \bar{A} \nabla^2 A ,
\]
where \( \bar{A} \equiv A^* \) and under the chiral rotations \( A \to e^{i\epsilon} A, \bar{A} \to e^{-i\epsilon} \bar{A} \). Moreover, fields \( A, \bar{A} \) describe on the mass-shell right- and left-handed photons. However, the fields \( A, \bar{A} \) are not Lorentz scalars and the corresponding propagators are gauge dependent. To say the least, further efforts are needed to derive \( \text{(21)} \). Nevertheless the validity of Eq. \( \text{(10)} \) does get extra support through consideration of the fields \( A, \bar{A} \). Imagine, however, that these technical difficulties are overcome and
the CVE for photons does reduce to an integral from the Bose distribution. The problem is that the conjectured expression for $K_0$ would not allow in fact to evaluate the chiral vortical effect. Indeed, there are zero/negative bosonic modes and these modes could condense.

**Sensitivity to the infrared**

Let us reiterate the problem to be confronted now. Eq. (13) for the spin dependence of the chiral vortical effect looks very natural. Indeed, we expand in $\Omega$ and are looking for terms linear in $\Omega$. Since $\Omega$ is coupled to spin, $\delta H = -\hat{\Omega} \cdot \hat{S}$, where $\hat{H}$ is Hamiltonian and $\hat{S}$ is the spin, we expect the ratio \( \frac{P}{3} \) to be equal to two. The reservation is that we assume validity of the perturbative expansion in $\delta H$. Naively, this assumption is safe to make since the Sommerfeld integrals are saturated by energy levels $\epsilon \sim T$ and not sensitive to $\epsilon \sim \Omega$ provided that $\Omega \ll T$. The result (13) obtained within the thermal field theory is consistent with this picture. However if there are zero modes in the limit $\Omega \rightarrow 0$ the effect of these modes is to be treated non-perturbatively, because of the pole in the expression for the Bose-Einstein distribution.

One of the possibilities to fix the infrared cut off is to consider energy levels in rotating medium, i.e. to account for the effect of rotation non-perturbatively, for a recent derivation of the CVE along these lines see [17]. More speculatively, the levels can be found by using the well-known analogy between the magnetic field $\hat{H}$ and “field of rotation” $\hat{\Omega}$. For massless charged fermions of spin 1/2 the Landau levels are given by:

\[
E_n = \pm \sqrt{2H(n + 1/2) + P_3^2 + H\sigma_3},
\]

where $P_3$ is the momentum along the magnetic field and $\sigma_3 = \pm(1)$ is the spin projection onto the direction of the magnetic field. The lowest level with $n = 0$, $P_3 = 0$, $\sigma_3 = -1$ corresponds to the famous zero mode, see, in particular [16], which is crucial to derive the chiral magnetic effect. In case of rotation, however, there is no zero mode for spin-1/2 fermions since the gyromagnetic ratio is two times smaller than in case of the electromagnetic interaction.

For spin-1 particle in rotating medium we would expect that the zero mode comes back. Indeed, the spin is doubled and compensates for the loss of the factor of two in the gravitational case. These expectations can be confronted with explicit evaluation of the CVE effect for non-interacting photons in the rotating medium (see [17] and references therein):

\[
J_{CVE} = \frac{1}{8\pi^2} \int_{\Omega^+}^{\infty} d\omega \int_{-\infty}^{+\infty} dk (\omega + k^2/\omega) \left( \frac{1}{e(\omega-\Omega)/T - 1} - \frac{1}{e(\omega+\Omega)/T - 1} \right),
\]

where $\Omega^+$ satisfies $\Omega^+ > \Omega$ while its exact value depends on an extra infrared sensitive cut off. Namely, one introduces a finite radius $R$ of the rotating cylinder and $\Omega^+ \rightarrow \Omega$ in the limit $R \rightarrow \infty$. Clearly, Eq. (23) exhibits the zero mode (in the limit $R \rightarrow \infty$) which we have just discussed. Keeping $R$ finite regularizes the expression for the current (23). However, the result is sensitive to the region $\omega - \Omega \approx 0$ and Eq. (23) predicts that the CVE for photons is 4/3 times larger than it follows from (13) [17].

Thus, evaluation of the chiral vortical effect for photons requires introduction of infrared-sensitive cuts off. Apparently Eq. (23) corresponds to the case:

\[
1/R \ll \Omega \ll T.
\]

One can speculate that in case of

\[
\Omega \ll 1/R \ll T,
\]

one reproduces the “standard” value (13). If so, then the choice (23) is to be considered a proper infrared completion of the perturbative thermal field theory for the CVE. While an arbitrary infrared regularization, generally speaking, brings in dependence on a particular experimental set up.

Our final remark is that for higher spins, $S \geq 3/2$, one expects the lowest level to be negative.

**TEMPERATURE-ACCELERATION, $T \leftrightarrow a/(2\pi)$, DUALITY?**

**Definition of the duality**

Papers [8, 9] establish a kind of duality between thermal and black-hole descriptions of the chiral vortical effect. In the flat-space approach one introduces temperature and evaluates a two-point function, see Eq. (12). In the other case, one utilizes exclusively zero-temperature field theory in external gravitational (and electromagnetic) fields. There are anomalous terms contributing to 4d current divergences as well as covariant derivative of the energy-momentum tensor. The results are non-distinguishable once one identifies acceleration on the horizon and temperature.

This agreement between the two approaches in case of massless spin-1/2 particles is amusing and gratifying. Moreover, it is tempting to use the duality as a guiding principle in evaluation of loop graphs. So far the comparison of the two approaches resulted in a consistent picture. In these notes, however, we emphasize that in case of photonic CVE the two approaches disagree with each other. From the field-theoretic perspective the weak point of our derivation is lack of analysis of possible role of the so called edge states, for a review and further references see, e.g., [18].
Limit of large spin of massless particles

In conclusion, let us notice that consideration of large-spin limit clarifies the origin of the disagreement between two ways of evaluating the chiral vortical effect for photons, which the central point of our current notes.

Namely, it was shown in [19] that the coefficient in front of the gravitational anomaly grows with spin $S$ of the massless chiral particles as $S^3$:

$$\partial_a K^a_S = (-1)^{2S+1}(2S^3 - S)(\text{const}) \hat{R} \hat{R},$$

where $(\text{const})$ does not depend on the spin and $K^a_S$ is the chiral current for massless particles of spin $S$, analog of the current $K^a$ in the photonic case. The current $K^a_S$ can explicitly be constructed in terms of the Pauli-Lubanski pseudovector $\vec{S}$.

As we discussed above, the $S^3$ dependence of the CVE is difficult to interpret in case of the flat-space evaluation. Indeed, we are evaluating the coefficient in front of the term $J_{CEV} \sim T^2 \Omega$. Basically, one expects then linear dependence on the spin since there is an effective coupling proportional to $\vec{S} \cdot \hat{S}$ and the term linear in $\Omega$ is also linear in the spin $S$. There are subtle points related to the infrared divergences, see the end of the previous section. Nevertheless there is no sign of the $S^3$ dependence of the CVE for higher spins, as exhibited by the gravitational anomaly. And this is the main puzzle we are trying to resolve.

If we turn to the evaluation of the CVE on the field-theoretic side in case of external gravitational field, then we are rather interested in the term of the order $J_{CEV} \sim a^2 \Omega$ where $a$ is the acceleration. A crucial point is that for spin-1/2 fermions acceleration is coupled also to spin, like the angular velocity $\vec{\Omega}$. In more detail, in the equilibrium there is an extra piece, $\delta L$ in the effective interaction:

$$\delta L = \frac{\vec{\Omega} + i \vec{a}}{2} \cdot \vec{\sigma}.$$  

If we generalize Eq. (27) to higher spins, then $\vec{\sigma}$ is to be understood as a generator of rotations and its matrix element over wave function of a higher-spin massless particle is proportional to the spin $S$. Let us also remind the reader that we are considering the case $\vec{a} \parallel \vec{\Omega}$, and that linear in $i \vec{a}$ terms cancelled out.

Clearly, Eq. (27) reproduces the $S^3$ dependence in the limit of large $S$. At least at face value, this observation is in variance with the acceleration-temperature duality described above. Namely, acceleration couples to spin while number of degrees of freedom (which determines thermal effects) does not grow with spin for massless particles. This contradiction is somewhat similar to the reason why the so called neutrino theory of light finally fails if the number of dimension is larger than $d = 2$. Indeed, the wave function of photon can be described in Lorentz indices the same way as if photon is composed of fermions.

However, the number of degrees of freedom for photons is smaller than for two fermions.

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