Superheavy magic nuclei in Relativistic Hartree-Fock-Bogoliubov theory

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Abstract. The occurrence of the spherical shell closures in superheavy nuclear systems is explored within the relativistic Hartree-Fock-Bogoliubov (RHFB) theory with density-dependent meson-nucleon couplings. We use the two-nucleon gaps \( \delta_{2n(p)} \) and pairing gaps \( \Delta_{\nu(\pi)} \) to characterize the shell effects. The results depend slightly on the forces used, but the general set of magic numbers beyond \( Z = 208 \) are \( Z = 120, 138 \) and \( N = 172, 184, 228 \) and \( 258 \). Our calculations are in favor of the nuclide \( ^{304}Z = 120 \) as the next spherical doubly magic nucleus. Combined with the bulk properties of symmetric matter, we find that the shell effects are sensitive to the values of both scalar mass and effective mass. These two masses essentially determine the spin-orbit splittings and single-particle level density, respectively. Furthermore, the breaking of relativistic pseudo-spin symmetry influences the level structure and the occurrence of closed shells.

1. Introduction
In this contribution we report on our recent investigations of the possible occurrence of new, not yet observed superheavy nuclei beyond \( Z \geq 120 \). The motivations for this endeavour are based on the new possibilities offered by the Relativistic Hartree-Fock-Bogoliubov (RHFB) theory which has made a major step forward in the recent years [1].

The advantages of a covariant mean field approach for such a study are well established. In the non-relativistic self-consistent mean field method, the central and spin-orbit average potentials depend on different parts of the energy density functional (EDF) and therefore, on independently adjusted parameters of the EDF[2]. In the relativistic self-consistent mean field, on the other hand, all parameters of the EDF contribute to the Lorentz scalar and vector mean fields - \( U_s \) and \( U_v \) - and consequently the central potential \( U_0 = U_s + U_v \), and the spin-orbit potential \( U_{so} = U_s - U_v \) of the nucleonic Dirac equation are not independent. This aspect is quite crucial obviously when one tries to predict magic nuclei in unknown regions of the periodic table.

In the literature, there are quite many versions of effective Lagrangians to be used in the relativistic Hartree-Bogoliubov context, we will call them the Relativistic Hartree-Bogoliubov (RHB) models [3]. Some of the results that we discuss in this contribution are obtained by using representative RHB models, namely the PKDD[4] and DD-ME2[5] Lagrangians. On the other hand, there are also several RHFB Lagrangians which are doing well in describing the general
properties (binding energies, radii, shell closures) of a vast number of nuclei throughout the periodic table, and we wish to include them in this survey of superheavy magic nuclei. Thus, the RHFB Lagrangians PK01, PK02, PKO3[6] and PKA1[7] are also used in this study. All these covariant Lagrangians do not contain originally a term describing the pairing effects. Here, we have used as pairing interaction the original D1S Gogny force[8] modified by an adjusted strength factor $f=0.9$, so that the odd-even mass differences of Pb isotopes are reproduced in our RHFB calculations.

2. Shell closures in the regions $N \geq 160$, $Z \geq 110$

To identify the magic shells it is convenient to look at the two-nucleon gaps - $\delta_{2p}$ for protons and $\delta_{2n}$ for neutrons - because they provide a quantitative measure of the shell effects. These gaps are the differences of two-proton (two-neutron) separation energies between isotones (isotopes) differing by two units:

$$\delta_{2p}(N, Z) = S_{2p}(N, Z) - S_{2p}(N, Z + 2)$$
$$\delta_{2n}(N, Z) = S_{2n}(N, Z) - S_{2n}(N + 2, Z)$$ (1)

Larger values of $\delta_{2p}$ or $\delta_{2n}$ occur whenever $S_{2p}$ or $S_{2n}$ undergo a sudden jump, which is the signal of a magic shell occurrence.

We have thus explored, using the five Lagrangians mentioned in Sec.1, the evolution of $\delta_{2p}$ and $\delta_{2n}$ for the nuclei covering the ranges $110 \leq Z \leq 140$ and $140 \leq N \leq 280$. The calculated values of $\delta_{2p}$ and $\delta_{2n}$ vary from 1 to 5 MeV. As a general result, PKA1-RHFB is the model which predicts the larger two-nucleon gaps (3 to 5 MeV) for $Z = 120, 126, 138$ and $N = 184, 258$. These are the magic numbers corresponding to PKA1. The other Lagrangians also indicate a well marked proton closed shell at $Z = 120$ and $Z = 138$.

Concerning the neutron shells the predictions vary with the particular model Lagrangian employed. At $N=172$ and 228, neutron shell closures are found with the various Lagrangians considered here, although the shell effects appear to be rather weak. Other predicted neutron magic numbers are $N=184$ and 258 (except with PKO2). By examining the predictions of the various models discussed here, one may tentatively conclude that the nuclide $^{304}_{120}184$ could be a doubly magic system, and $^{292}_{120}172$ might be another, less stable candidate.

3. The role of pseudo-spin symmetry and its violation

It is interesting to examine the significance of the magic numbers $Z=120$, $N=184$ in the light of the pseudo-spin symmetry (PSS) [9] and its breaking [7]. The PSS is a property of the solutions of the Dirac equation that one can schematically describe as follows: if the Lorentz scalar and vector potentials, $U_s$ and $U_v$, of the system are such that:

$$U_s(r) + U_v(r) = \text{constant}$$ (2)

at all values of $r$, then the pair of solutions $(nlj)$ and $(n' = n + 1, l' = l - 2, j' = j - 1)$ are degenerate. In finite nuclei, the constant value on the right hand side of Eq. (2) is necessarily zero since both $U_s(r)$ and $U_v(r)$ tend to zero asymptotically. Obviously, the condition (2) is generally not realized in a nuclear system having bound states, because $U_s(r) + U_v(r)$ must be negative inside the nucleus with typical values in the range of -50 MeV near the center, while it tends to zero outside the nuclear region. Thus, the amount of PSS violation will affect the relative positions of the partners $(nlj)$ and $(n'l'j')$, and therefore the $N$ and $Z$ numbers corresponding to filled subshells.

This is illustrated in Fig.1 by the single-particle spectra calculated in the nuclide $^{304}_{120}184$ with the various Lagrangians used in this work. One can see that the PSS is sometimes fairly well
Figure 1. (Color online) Proton (left panel) and neutron (right panel) single-particle levels in the superheavy nucleus $^{304}\text{120}_{184}$. They correspond to RHFB calculations performed with the Lagrangians PKO1 and PKA1, and RHB calculations with PKDD and DD-ME2. The pairing interaction used is the D1S force with a reduction factor $f=0.9$.

obeyed (neutron states $4s1/2 - 3d3/2$) and sometimes strongly broken (neutron states $2h11/2 - 1j13/2$, proton states $3p3/2 - 2f5/2$). In the case of the proton states, the magic number $Z = 120$ just corresponds to the lower partner $2f5/2$ completely filled and the upper partner $3p3/2$ empty. Indeed, for all the models shown in Fig.1 there is a large splitting - of the order of 2 MeV - for the proton pseudo-spin partners $2f5/2 - 3p3/2$, while the spin-orbit splitting of the $3p3/2 - 3p1/2$ proton levels is fairly small.

On the neutron side, we observe that the prediction of PKA1 for the pseudo-spin partners $1j13/2 - 2h11/2$ is very different from that of the 5 other models, with a smaller splitting and a reversed order. We recall that the PKA1 Lagrangian contains an extra degree of freedom as compared to the other models, namely a Lorentz tensor $\rho-N$ coupling. This is probably the reason for the different $1j13/2 - 2h11/2$ neutron splitting.

4. Conclusion
We have explored the regions of the nuclear chart around ($Z=120$, $N=184$) and beyond, in search for doubly closed-shell systems. We have taken advantage of a newly developed tool, the RHFB code in spherical symmetry, and of the most recent meson-nucleon effective Lagrangians adjusted to the properties of known nuclei.

We find that several effective Lagrangians do agree in predicting some new magic numbers, even though the detailed single-particle spectra show some model dependence. In particular, the results from the four Hartree-Fock type of models (PKO1, PKO2, PKO3, PKA1) and from the two Hartree type of models (PKDD, DD-ME2) indicate a well-marked shell closure at $Z=120$, $N=184$. 

$^{304}\text{120}_{184}$
N=184.

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