Punching strength of conventional reinforced concrete flat slabs

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ABSTRACT

The paper presents comparison between the punching shear calculations from six different codes and two equations from the literature. It utilizes 257 punching tests data collected from the literature. The concrete strengths, $f'c$, range between 12.3 MPa and 68 MPa, the reinforcement ratios range between 0.2% and 5.01%, and the slab depths range between 80 mm and 500 mm. It is found that the smallest error is for CEB-FIP-90 and EC2-2004 while the largest error is for JSCE-2002 and ACI318-19. Also, modifications to one of the equations of the Egyptian reinforced concrete code and two of the equations of ACI318-19 code for calculating the punching strength of flat plates without shear reinforcements are presented. The modified Egyptian and ACI318-19 codes equations for punching strength are compared to the experimental data and good correlations are noticed. The obtained errors are lesser than those of the original codes equations and the average errors are on the conservative side.

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Introduction

International reinforced concrete codes such as ACI318-19, JSCE-2002, ECP-203-2017, BS-8110-97, EC2-2004, and CEB-FIP-90 provide different equations for calculating the punching shear strength of two-way slabs without unbalanced moment and without shear reinforcement which are empirical and based on the available test data. Similar trend is noticed for some of the equations in the literature as those of Elshafey et al. [1].

Mari et al. [2], proposed a punching strength model which considers shear reinforcement. The equation is accurate and simple. It considers the stress of the punching shear reinforcement and the high role of concrete to the punching capacity. The model of Mari et al. [2] can be extended to FRP and steel fiber reinforced concrete and corner and border columns. Theodorakopoulos and Swamy [3], proposed a model for calculating the
ultimate punching capacity. The model is for normal weight and lightweight concrete. This equation assumes that punching occurs under three-dimensional stresses. Finally, failure occurs when the stress reaches the splitting strength. Their model shows good correlation to experimental tests. Alkroosh and Ammash [4] used gene programming to obtain the punching capacity of high strength and normal strength two-way slabs. Their equation was proposed based on 58 punching tests. The obtained function proved that gene programming gives good performance in terms of predicting the punching capacity. Kueres et al. [5], proposed a punching shear design method for column bases and flat plates based on large number of experimental data. Their method is a uniform design method which is applicable for the cases of with or without punching reinforcement. Hamada et al. [6] proposed a simple equation for calculating the punching strength of flat plates. Comparing the calculated punching strength using this equation to test data, a small average error is obtained. Trekin and Pekin [7] presented punching tests for two-way slabs. They also formulated a sequence for calculating the punching capacity which proved to be accurate. Guo and Cheng [8] showed a punching strength formula which considers the flexural reinforcement ratio. Their equation was reasonable when compared to experimental data. Muttoni [9] provided a mechanical explanation for the punching of two-way slabs based on shear crack. He suggested a failure model based on rotation of the slab which proved to be good when compared to experimental results. His equation accounts for the size effect. Muttoni et al. [10] discussed the similarities and differences between squat footings and slender slabs based on the critical shear crack theory. They pointed the shear and bending deformations and their effect on the shear crack. A formula for punching strength was developed. Elshafey et al. [1], evaluated the punching capacity of flat slabs using neural networks and simple new equations. They used 244 test results for internal columns. The results obtained from neural networks are in good agreement with the test data. They used regression analysis and arrived to two new and simple equations which in turn showed very good correlation to test data. Rankin and Long [11] presented a method for evaluating the punching capacity of flat slabs. This method is valid for circular and square columns. Their method considers slab depth factors, concrete strength, reinforcement ratio, and yield strength of reinforcement. A total of 217 test results were used to verify the method and they found that it correlates well with them.

The purpose of this research is to use the available large number of test data to evaluate the punching equations from the international reinforced concrete codes and from the literature. Best fit of 257 punching test data, taken from Elshafey et al. [1], Rankin & Long [11], and Metwally et al. [12] is then used to modify the most common equation for punching shear of the ECP-203-2017 code and two equations of the ACI318-19 code.
Equations for punching shear

The shear stress is calculated by dividing the floor column load by the area of a critical vertical section. The critical section is taken at distance from the column sides differs from one code, or one equation, to another. This punching shear stress due to load has to be lesser than the design values given by the different equations. These design equations are generally function of the column and slab dimensions and concrete’s compressive strength.

ACI318-19 code [13]

Simple equations are proposed in this code. The critical perimeter is at 0.5d from the column faces. According to ACI318-19, the flexural reinforcement ratio has no effect on the punching shear strength. The expressions for the ultimate punching strength given by this code are as follows:

\[ V_c = \frac{1}{6} \left[ \frac{1}{2} + \frac{2}{\beta_c} \right] \sqrt{f_{cu}} \text{MPa} \]

\[ V_c = \frac{1}{12} \left[ \frac{\alpha_s d}{b_o} + 2 \right] \sqrt{f_{cu}} \text{MPa} \]

\[ V_c = \frac{1}{3} \sqrt{f_{cu}} \text{MPa} \]

where

\[ \beta_c = \frac{\text{longside}}{\text{shortside}} \geq 2.0 \]

d = the effective depth, mm

\[ b_o = \text{perimeter of the critical section around the column, mm} \]

\[ \alpha_s = 40 \text{ for interior columns} \]

\[ f_{cu} = \text{cube compressive strength of concrete, MPa} \]

\[ f'_{c} = \text{cylinder compressive strength of concrete, MPa} \leq 68\text{MPa} \]

\[ f'_{c} = 0.80 f_{cu} \]

CEB-FIP-90 code [15]

This code considers the influence of reinforcement ratio. It assumes that the punching strength is proportional to the cubic root of the characteristic compressive strength of concrete. Similar to the EC2-2004 code, the critical
perimeter is at 2d from the column faces. The equation is similar to that of EC2-2004 and is given by:

\[ V_c = 0.18k(100\rho f_{ck})^{1/3}\text{MPa} \]

where

\[ k = \text{size-effect coefficient} \]

\[ k = 1 + \left(\frac{200}{d}\right)^{1/2} \]

\[ \rho = \text{the effective flexural reinforcement ratio at the critical section} \]

\[ f_{ck} = \text{the characteristic cylinder compressive strength of concrete, MPa} \]

**BS8110-97 code [14]**

The critical perimeter for this code is at 1.5d from the column faces. This code considers the effect of the flexural reinforcement ratio. The equations of BS8110-97 are as follows:

\[ V_c = 0.79 \left(100\rho\right)^{1/3} \left(\frac{400}{d}\right)^{1/3} \left(\frac{f_{cu}}{25}\right)^{1/3}\text{MPa} \]

\[ f_{cu} \leq 40\text{MPa} \]

\[ \rho \leq 3\% \]

\[ d \leq 400 \text{ mm} \]

**EC2-2004 code [17]**

The Euro-code 2 calculates the punching strength at relatively large distance from the loaded area which is equal to 2d. This code gives the punching shear strength as proportional to the cubic root of the characteristic compressive strength of concrete. EC2-2004 considers the effect of flexural reinforcement ratio. The equation given by this code is:

\[ V_c = 0.18 k(100\rho f_{ck})^{1/3}\text{MPa} \geq V_{\text{min}} \]

where

\[ k = 1 + \left(\frac{200}{d}\right)^{1/2} \leq 2.0 \text{ with } d \text{ in mm} \]

\[ \gamma_c = \text{material resistance factor for concrete} = 1.0 \text{ for ultimate analysis} \]
\[ V_{\text{min}} = 0.053k^{3/2} \sqrt{f_{\text{ck}}} \text{MPa} \]

**ECP-203-2017 code [16]**

The critical perimeter for this code is at 0.5d from the loaded area. It relates the punching shear strength to the square root of the concrete compressive strength as follows:

\[
V_c = 0.8 \left[ \frac{ad}{b_0} + 0.2 \right] \sqrt{\frac{f_{\text{cu}}}{\gamma_c}} \text{MPa}
\]

\[
V_c = 0.316 \left[ 0.5 + \frac{a}{b} \right] \sqrt{\frac{f_{\text{cu}}}{\gamma_c}} \text{MPa}
\]

\[
V_c = 0.316 \sqrt{\frac{f_{\text{cu}}}{\gamma_c}} \text{MPa}
\]

where

\[ \alpha = 4 \text{ for interior columns} \]

\[ \frac{a}{b} = 1.0 \text{ for circular columns} \]

\[ \gamma_c = 1.0 \text{ for ultimate analysis} \]

\( a \) & \( b \) = smallest & largest dimensions of the column, respectively

**JSCE-2002 code**

The JSCE-2002 code [6] assumes that the punching strength is proportional to the square root of the concrete compressive strength. It considers the effect of flexural reinforcement ratio and assumes that the critical perimeter is at 0.5d from the column faces.

\[
V_c = \beta_d \beta_p \beta_r f_{pcd} \ U_p \frac{d}{1.3} \text{ N}
\]

where

\[ \beta_d = \sqrt[4]{\frac{1}{d}} \]

\[ \beta_p = \sqrt{100\rho} \]
\[ \beta_r = 1 + \frac{1}{1 + 0.25 \frac{b}{d}} \]

\[ f_{pcd} = 0.2 \sqrt{f_c} \]

\( b = \) column perimeter
\( U_p = \) perimeter located at distance \( d/2 \) from column faces

**Equation ‘a’ of Elshafey et al. [1]**

This equation assumes that the critical section is at 0.5d from the loaded area. Equation ‘a’ considers the effect of reinforcement ratio and assumes a power of 0.41 for the concrete compressive strength. It has the following form:

\[ V_c = 0.51 \left(10^{-3}\right) f_c^{0.41} \rho^{0.38} \left(\frac{250}{d}\right)^{0.10} U_p d kN \]

**Equation ‘b’ of Elshafey et al. [1]**

This equation does not assume a specific critical section. Similar to Equation ‘a’, it considers the effect of flexural reinforcement ratio. However, Equation ‘b’ puts a power of 0.34 for the concrete cylinder compressive strength. Its form is as follows:

\[ V_c = 12.30 (c + d)^{0.53} f_c^{0.34} \rho^{0.41} \left(\frac{d}{250}\right)^{1.22} kN \]

where

\( c = \) side length of square columns

**Used experimental results**

The test results reported by Elshafey et al. [1], Rankin and Long [11], and Metwally et al. [12] are used in this study. They consist of 257 specimens which cover a wide range of variables. The flexural reinforcement ratios range between 0.2% and 5.0%, the slab depths vary from 80 mm to 500 mm, and the concrete cylinder compressive strengths vary from 12.3 MPa to 68 MPa. These data are to be used to calibrate and modify the most common equations for the punching capacity of the ECP-203-2017 code and two equations of the ACI318-19 code.
Comparison between the eight equations for punching capacity

The average of the punching strengths from tests divided by the punching strengths from the eight formulae along with the standard deviation for the same are given in Table 1. Table 2 gives the slopes of the trend lines for the relation between punching capacity from tests on the horizontal axis and the punching capacity from the eight equations on the vertical axis. The CEB-FIP-90 and the EC2-2004 codes give the closest average to unity but not on the conservative side. This means that the calculated punching strengths are close to the test values. Also, for these codes the standard deviations are 0.207 and 0.207, respectively, which means intermediate distribution of the results around average. The ACI318-19 code overestimates the average by 28.4% and the JSCE-2002 code overestimates the average by 39.7%. The standard deviations from these two codes are 0.359 and 0.259, respectively. The calculated average for the ECP-203-2017 is 1.200 which lies in the middle among the other codes. In the following section, the paper targets to reduce the 20.0% error of the Egyptian

### Table 1. Statistical data for the values of punching capacity from test/punching capacity from the eight formulae.

| Formula           | Average | Standard deviation |
|-------------------|---------|--------------------|
| ACI318-19         | 1.284   | 0.359              |
| CEB-FIP-90        | 0.980   | 0.207              |
| BS8110-97         | 1.079   | 0.200              |
| EC2-2004          | 0.980   | 0.207              |
| ECP-203-2017      | 1.200   | 0.336              |
| JSCE-2002         | 1.397   | 0.259              |
| Equation ‘a’      | 1.056   | 0.224              |
| Equation ‘b’      | 0.985   | 0.140              |

### Table 2. Slope of the trend line for the relation between punching strength from tests on the x-axis and punching strength from formulae on the y-axis (Figures 1–8).

| Formula           | Slope  |
|-------------------|--------|
| ACI318-19         | 0.963  |
| CEB-FIP-90        | 1.205  |
| BS8110-97         | 0.958  |
| EC2-2004          | 1.205  |
| ECP-203-2017      | 1.031  |
| JSCE-2002         | 0.739  |
| Equation ‘a’      | 1.034  |
| Equation ‘b’      | 1.004  |
code and the 28.4% error of the ACI318-19 code. The slopes of the trend lines in the cases of ECP-203-2017 code and ACI318-19 code are 1.031 and 0.962, respectively, as shown in Figures 1–8 and Table 2. This means that the Egyptian code gives larger values for the punching capacities when compared to the experimental results on the trend lines.

Figure 1. Comparison between punching strength from tests and ACI318-19 (slope of trend line = 0.963).

Figure 2. Comparison between punching strength from tests and CEB-FIP-90 (slope of trend line = 1.205).
Proposed modification to the common ECP-203-2017 and ACI318-19 codes equations

The 257 test data collected from the literature are used in a regression analysis and the following modification is suggested for the common equation for punching capacity of the Egyptian reinforced concrete code:

Figure 3. Comparison between punching strength from tests and BS8110-97 (slope of trend line = 0.958).

Figure 4. Comparison between punching strength from tests and EC2-2004 (slope of trend line = 1.205).
\[ V_c = 0.342 \sqrt{\frac{f_{cu}}{\gamma_c}} \text{ MPa} \]

Figure 9 shows the relationship between the concrete compressive strength and the punching strength for the 257 experimental data. The proposed equation with \( \gamma_c \) taken equals to 1.0 is shown as a dotted line which as clear has an intermediate position among the test data. Figure 10 shows
the relationship between the experimental punching capacity on the horizontal axis and the punching capacity calculated using the proposed modification for the ECP-203-2017 equation, with $\gamma_c$ taken equals to 1.0, on the vertical axis. The slope of the trend line is 1.115 which is higher than the slope in Figure 5, for the original equation, which is 1.031. The average of the experimental punching capacity divided by the punching calculated from the proposed equation is 1.109 and the standard deviation is 0.309. The same

Figure 7. Comparison between punching strength from tests and Equation a (slope of trend line = 1.034).

Figure 8. Comparison between punching strength from tests and Equation b (slope of trend line = 1.004).
values for the original equation are 1.200 and 0.336, respectively. This means that the error for the proposed equation is lesser and the average errors for both equations are on the conservative side.

**Figure 9.** Relationship between compressive strength of concrete, $f'c$, and punching strength from tests for 257 specimens (shown dotted line is the original ECP-203-2017 equation and the solid line is the proposed equation).

**Figure 10.** Comparison between punching strength from tests and proposed modification to ECP-203-2017 (slope of trend line = 1.115).
The same procedure is followed to propose the following two modifications for the ACI318-19 punching strength equations.

\[ V_c = 0.18 \left[ 1 + \frac{2}{f'_c} \right] \sqrt{f'_c} \text{MPa} \]

\[ V_c = 0.36 \sqrt{f'_c} \text{MPa} \]

The average of the experimental punching strength divided by the punching from the proposed modifications to ACI318-19 is 1.178 which is lesser than the average associated with the original equations (1.284). Also, the standard deviation is 0.328 which means lesser dispersion of the data compared to the 0.359 standard deviation of the original equations. Similar to Figures 9 and 11 is plotted. It is clear that the proposed equations have an intermediate position among the experimental data. The experimental punching strength is plotted against the punching strength from the proposed modifications to the ACI318-19 equations. The slope of the trend line is 1.05 which is higher than the slope in Figure 1.

**Conclusions**

1. Eight different equations, six from codes and two from literature, for the punching shear strength of flat plates are evaluated using 257 test
results, collected from different sources, which are the largest data base up to the authors' knowledge.

(2) The equations of the JSCE-2002 and ACI318-19 codes give the largest error when compared to the experimental data. However, the average errors for both are on the conservative side.

(3) The smallest error is obtained for the CEB-FIP-90 and EC2-2004 codes but they are little on the un-conservative side for the average error.

(4) The suggested modification of the ECP-203-2017 shear punching common equation has excellent results compared to 257 test data of wide range of concrete strength, reinforcement ratio, and slab effective depth. The average error of this proposed equation is 10.9% and this error is on the conservative side.

(5) Two modifications are proposed to the punching shear strength equations of the ACI318-19 code. The average error associated with the modified equation is 17.8% and it is on the conservative side.

Disclosure statement

No potential conflict of interest was reported by the authors.

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