Geometric Scaling in a Symmetric Saturation Model

Stéphane Munier

Università di Firenze
Dipartimento di Fisica, via G. Sansone 1
50009 Sesto Fiorentino, Italy
munier@fi.infn.it

Abstract

We illustrate geometric scaling for the photon-proton cross section with a very simple saturation model. We describe the proton structure function $F_2$ at small-$x$ in a wide kinematical range with an elementary functional form and a small number of free parameters. We speculate that the symmetry between low and high $Q^2$ recently discovered in the data could be related to a well-known symmetry of the two-gluon-exchange dipole-dipole cross section.

1 Introduction

Golec-Biernat and Wüsthoff (GBW) have shown [1] that a saturation model is able to describe the HERA data for the proton structure function $F_2(x, Q^2)$ at small values of Bjorken variable $x$. The agreement of their model with the data is quite good for all available values of $Q^2$, including the lowest ones.

The GBW model is based on a dipole picture of photon-proton interaction [2]. Its main ingredient is that at high energy, a particular frame can be chosen in which the interaction factorizes in two processes well separated in time. First the photon fluctuates in a $q\bar{q}$ pair of given transverse size $r_\gamma$, which then scatters off the proton. The factorized dipole-proton cross section $\sigma^{dp}$ contains the QCD evolution. It only depends on the size of the dipole $r_\gamma$ and on the Bjorken variable $x$. The following parametrization was chosen by GBW:

$$\sigma^{dp}(x, r_\gamma) = \sigma_0 \left(1 - \exp(-r_\gamma^2 Q_s^2(x)/4)\right),$$

(1)

where it was assumed that the transverse momentum scale $Q_s$ had a power-like dependence on $x$:

$$Q_s^2(x) = 1 \text{ GeV}^2 \cdot \left(\frac{x}{x_0}\right)^{-\lambda}.$$ 

(2)

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This choice was motivated by the following considerations. When \( rQ_s/2 \ll 1 \), the model reduces to colour transparency. When one approaches the region \( rQ_s/2 \sim 1 \), the exponential in Eq. (1) takes care of resumming many gluon exchanges, in a Glauber-inspired way. Intuitively, this is what happens when the proton starts to look dark. In the region \( rQ_s/2 \geq 1 \), such a microscopic interpretation is no more valid but the exponential has the virtue to force the cross section to tend to a constant at large \( rQ_s \) (which also means at small values of \( x \) because of the \( x \)-dependence of \( Q_s \)), thus respecting the unitarity constraint through a strict compliance with the Froissart bound. These formulae involve 3 free parameters only, \( \sigma_0, x_0 \) and \( \lambda \). Once those are determined by a fit to the \( F_2 \) data, one can take advantage of the universality of the dipole cross section to extend the model to more exclusive processes. Diffractive structure functions [3] are predicted in a quite satisfactory way. Elastic electro- and photoproduction of vector mesons is also accounted for very successfully [4]. Some attempts were recently made to predict the \( \gamma^* - \gamma^* \) cross section [5].

However, several points remain unsatisfactory in this model, both on the theoretical and on the phenomenological side. First, the form of the dipole cross section is quite ad hoc. Then, we note that for the description of small-\( Q^2 \) data, an arbitrary quark mass of order 140 MeV has to be chosen. On the phenomenological side, the original saturation model fails at describing the log \( 1/x \)-slope of \( F_2 \) at large \( Q^2 \). A revisited version has just been released [6], which takes care of some collinear resummations. The new model is much more successful, but it seems to incorporate less saturation effects since the saturation scale \( Q_s \) is shifted towards a lower value [7]. This might indicate that either saturation effects are there but can be mimicked by a DGLAP evolution, or DGLAP effects are still predominant. In the latter case the success of the original dipole model could be attributed to collinear resummations effectively taken into account in the dipole-proton cross section, though this is not at all apparent.

Our goal in this paper is to go back to the main known features of saturation and to incorporate them in a simple parametrization for the structure function. Our main result is that an exponentiated elementary dipole-dipole cross section (see Eqs. (14,15) and Figs. 2,3,4) fits remarkably well the HERA data. We give some motivation and interpretation for this parametrization in Sec. 2 and we present the comparison to the data in Sec. 3.

2 Formulation

Following GBW, we choose the so-called dipole-frame, in which all the QCD evolution is incorporated in the proton. The photon has just time to fluctuate in a \( q\bar{q} \) pair which subsequently interacts with a highly evolved proton. We describe qualitatively the energy evolution of the proton in subsection (2.1) and the scattering in Secs. (2.2) and (2.3). The model is summarized on Fig.1.
2.1 The saturated proton

When the center-of-mass energy increases (or equivalently when the Bjorken variable $x$ decreases at fixed $Q^2$), more and more quantum fluctuations are revealed in the proton. Indeed, $x$ coincides with the fraction of longitudinal momenta (with respect to the proton momentum), of the probed partons. The lifetime of a partonic fluctuation is proportional to this quantity. Thus going to smaller $x$ means probing shorter time intervals, and thus becoming sensitive to more fluctuations.

The rise of the parton densities with $1/x$ is predicted in QCD for a heavy onium [8]: the density $\tilde{n}(x,k)$ of gluons of longitudinal momentum fraction $x$ and transverse momentum $k$ is found to obey a linear integro-differential equation, the BFKL equation [9, 10]. Its solution shows that the gluon density increases like a power of $x$: $\tilde{n}(x,k) \sim x^{-\lambda}$. Apart from a diffusion term which would appear in a more refined treatment, this rise is independent of $k$. This model is not correct for very small $x$ because the gluons interact with each other and thus each energy level can only accommodate a finite number of them: the density of partons of given transverse momentum should saturate at some point. This feature has been taken into account by a modification of the BFKL equation. This modification consists in supplementing it with a nonlinear term:

$$\frac{\partial \tilde{n}(x,k)}{\partial \log(1/x)} = \frac{\alpha_s N_c}{\pi} K \cdot \tilde{n}(x,k) - \frac{\alpha_s N_c}{\pi} \tilde{n}^2(x,k), \quad (3)$$

where $K$ is the linear BFKL kernel. This is the GLR equation [11]. Several groups have derived saturation equations within QCD (for a review, see Ref. [12, 13]). Among them, Kovchegov was able to write a simple equation [14] which reduces to Eq. (3) when the partons are probed by a dipole of small size. All available approaches to saturation predict that the transverse momenta of the partons are on average shifted to a scale $Q_s$ called the saturation scale such that

$$Q_s^2(x) = \Lambda^2 e^{\lambda \log(1/x)}.$$  \quad (4)

This was confirmed by a numerical study presented in Ref. [15]. Corrections to formula (4) have been computed recently, see Ref. [16]. The curve $Q^2 = Q_s^2(x)$ in the $(x,Q^2)$ plane is called the critical line. The saturation scale $Q_s(x)$ is believed to reach 1 GeV around $x \sim 10^{-4}$: this is what comes out of the GBW phenomenological approach but a direct experimental evidence was also provided in Ref. [17].

It is convenient to pair the gluons in colour dipoles of given size $r$ (which is possible in the large $N_c$ limit [8]). The dipoles are then characterized by a density $n(x,r)$. $1/Q_s(x)$ is their average size. It is also the mean distance between the center of neighbouring dipoles. We shall assume that the evolved proton is a collection of independent dipoles at the time of the interaction whose sizes are distributed around $1/Q_s(x)$. We will approximate the distribution $n(x,r)$ by a delta function of the mean value of the dipole size $\langle r \rangle = 1/Q_s$. Its complete evaluation is presently out of reach. Our confidence in this approximation relies on the observation made in Ref. [18] where it was shown that the data for $\sigma^{\gamma p}$ for any $x \leq 10^{-2}$ and $Q^2$ is function of the combined variable $Q/Q_s(x)$ only, to a very good accuracy.
To fix the normalization of \( n \), we assume that the total area of these dipoles over the area of the proton is a constant ratio, of order 1. This is a kind of boundary condition on the critical line, in the spirit of Refs. [19] and [20] and yields the constraint

\[
\int \frac{d^2r}{r^2} n(x, r) = \frac{Q_s^2(x)}{\Lambda^2} .
\]  

This assumption is supported locally by the saturation equations of the type (3). However, it also means implicitly that throughout the evolution, the area of the proton remains of order \( 1/\Lambda^2 \). This can only result from non-perturbative confinement effects. It was indeed shown [21] that the Kovchegov equation, i.e. local saturation alone would lead to violations of unitarity due to the fast expansion of the nucleon radius. Note that the same hypothesis of a proton of fixed transverse size is contained in GBW model.

The dipole distribution in the proton eventually reads

\[
\int \frac{d^2r}{r^2} n(x, r) = \frac{Q_s^2(x)}{\Lambda^2} \cdot 2\pi r \cdot \delta(r - 1/Q_s(x)) .
\]  

2.2 The scattering

In our model where the proton is represented by a density \( n \) of dipoles, the cross section for dipole-proton scattering reads

\[
\sigma^{\gamma p}(x, Q^2) = \int d^2r_\gamma \int_0^1 dz |\psi_Q(r_\gamma, z)|^2 \int \frac{d^2r}{r^2} n(x, r) \cdot \sigma^{dd}(r_\gamma, r) ,
\]  

where \( \psi_Q \) is the photon wave function on a \( q\bar{q} \) dipole state.

\( \sigma^{dd}(r_\gamma, r) \) is the dipole-dipole cross section computable in perturbative QCD. We only have to consider its lowest order expression, since all the QCD evolution is put in the dipole density \( n \): there is no more room for radiative corrections in \( \sigma^{dd} \). It reads [22]

\[
\sigma^{dd}(r_\gamma, r) = 2\pi\alpha_s^2 r_\gamma^2 \left( 1 + \log \frac{r_\gamma}{r_<} \right) ,
\]  

where \( r_\gamma = \min(r_\gamma, r) \) and \( r_> = \max(r_\gamma, r) \). The different factors that appear in this expression have a straightforward interpretation. This cross-section is the imaginary part of the dipole-dipole elastic amplitude, which requires the exchange of at least two gluons paired in a colour singlet. These gluons must be able to resolve the smallest dipole, hence their transverse momentum is larger than 1/r_\gamma. This is colour transparency. It accounts for the factor \( r_\gamma^2 \). The correcting logarithm, which becomes sizeable when the dipoles have very different sizes, comes from the monopolar Coulomb field, which results in a logarithm in 2 dimensions. Indeed, when \( r_> \gg r_< \), the smallest dipole sees the constituents of the largest one as two well-separated colour charges.

In order to have an equal treatment for the photon and the proton and to get the simplest model, we treat the photon as a dipole of size 1/Q instead of taking account
of the complete probability distribution \( \Phi(r_{\gamma}) = \int dz r_{\gamma}^{2} |\psi_{Q}(r_{\gamma}, z)|^2 \). This picture is relevant for a longitudinal photon whose wave function is sharply peaked (around \( \log r_{\gamma} \sim \log(1/Q_{s}(x)) \)). It is less relevant for transverse photons since their wave function develops a plateau at large \( Q^2 \): this is due to the well-known fact that large size dipole configurations (aligned jet configurations) are always present in the transversely polarized photon. We shall however ignore this difficulty. Indeed we believe it is not essential to our discussion since anyhow we don’t pretend to be able to describe the very large \( Q^2 \) region. In this spirit, we approximate the distribution \( \Phi \) by a Dirac distribution:

\[
\Phi(r_{\gamma}) = N \cdot 2\pi r_{\gamma} \cdot \delta(r_{\gamma} - 1/Q) \, .
\]

(9)

\( N \) is a normalization factor that gives the rate of such a fluctuation \( \gamma^* \rightarrow q\bar{q} \).

We take \( \alpha_s \) fixed and at the scale 1 GeV. The approximation made here is that we stick to a kinematical region close to \( Q_{s} \) where \( |\log(Q/Q_{s}(x))| \ll \log(Q_{s}/\Lambda) \). Injecting Eqs. (9), (6) and (8) into Eq. (7), we obtain:

\[
\sigma^{\gamma^*p}(x, Q^2) = N \frac{1}{\Lambda^2} \frac{Q^2(x)}{Q_{s}^2} \cdot 2\pi \alpha_s^2 \left( 1 + \log \frac{Q}{Q_{s}(x)} \right) \quad \text{for } Q > Q_{s}(x) \quad (10)
\]

\[
\sigma^{\gamma^*p}(x, Q^2) = N \frac{1}{\Lambda^2} \cdot 2\pi \alpha_s^2 \left( 1 + \log \frac{Q_{s}(x)}{Q} \right) \quad \text{for } Q < Q_{s}(x) \, . \quad (11)
\]

These formulae deserve an immediate interpretation. When \( Q > Q_{s}(x) \) (first formula), we are in the usual picture of DIS where a photon of small size probes the parton (dipole) content of the proton. The flux factor is \( N \), the target density is \( \frac{Q^2(x)}{\Lambda^2} \), and the elementary cross section is \( 2\pi \alpha_s^2 \left( 1 + \log(Q/Q_{s}(x))/Q^2 \right) \). This is the hard Pomeron regime: the cross section grows like the exponential of the rapidity, due to the multiplication of dipoles in the proton.

When \( Q < Q_{s}(x) \) instead, the dipoles of the proton are smaller than the dipole of the photon. The use of a perturbative elementary dipole-dipole cross section is justified if \( x \) is very small so that the scale of the coupling constant \( Q_{s}(x) \) is large enough. How large has always been subject to debate. We will assume that \( Q_{s} \sim 1 \) GeV is fine.

The proton looks like a collection of “small” size probes. The picture is inverted with respect to the previous case: \( 1/Q_{s}(x) \), which is both the mean dipole size and their separation, fixes the resolution. The flux factor is \( \frac{Q_{s}^2(x)}{\Lambda^2} \), the target density is \( N \) and the elementary dipole-dipole cross section is \( 2\pi \alpha_s^2 \left( 1 + \log(Q_{s}(x)/Q) \right)/Q_{s}^2 \). The growth of the parton densities with the energy is compensated by the photon-parton cross section, which falls with the energy. We are in the soft Pomeron regime, in which the \( \gamma^* - p \) cross section grows only like a small power of the energy, \( (W^2)^{0.08} \). We note that in this region, the precise relationship between the photon virtuality \( Q^2 \) and the dipole size is not very relevant: corrections to it come as an additional energy-independent log term.

An important property of this parametrization is first that it only depends on \( Q/Q_{s}(x) \) and second that the quantity \( Q/Q_{s}(x) \sigma^{\gamma^*p}(Q/Q_{s}(x)) \) is exactly symmetric under the exchange \( Q \leftrightarrow Q_{s}(x) \). These two features have been observed in the HERA data \[18\].
We would now like to refine the model by including multiple gluon exchanges, which as we will argue, are important when \( Q \sim Q_s(x) \). Throughout we will preserve the symmetry just pointed out.

### 2.3 Accounting for multiple interactions

Usually multiple gluon exchanges are disfavoured by the smallness of the coupling constant and/or by powers of \( Q_s^2(x)/Q^2 \). We can see this by computing the probability that a dipole going through the proton undergoes an interaction. If the dipoles are uniformly spread over the surface of the proton, this probability is

\[
p = \frac{1}{N} \sigma^{\gamma^*p}(x, Q^2) \cdot \frac{\Lambda^2}{\pi} \approx 2\pi \alpha_s^2 \frac{Q_s^2}{\pi Q^2}
\]

as long as it is small enough compared to 1. Note that \( p \) coincides with the packing factor \( \kappa \) usually defined in the context of saturation (see for example [24]).

We first observe that parametrically \( 2\pi \alpha_s^2 \sim 1 \) for \( \alpha_s \) evaluated at the scale \( Q \sim Q_s \sim 1 \) GeV. Then when \( Q_s^2(x) \ll Q^2 \), \( p \) is much less than 1: multiple gluon exchanges are disfavoured by a power of this small factor. When \( Q^2 \sim Q_s^2(x) \) however, \( p \) is of order 1 and it is not justified to neglect multiple gluon exchanges.

In a classical framework, these multiple interactions could be accounted for by modifying the interaction probability in the following way. We note that \( e^{-p} \equiv S^2 \) is the probability of no interaction, where \( S \) is the \( S \)-matrix element for the dipole-proton interaction for a given impact parameter (it is constant over the area of the proton within our approximations). The dipole-proton total cross section is then \( 2\pi/\Lambda^2 \cdot (1 - S) \). Multiplying it by the rate \( N \) of splittings \( \gamma^* \rightarrow q\bar{q} \), we obtain for the total photon-proton cross section

\[
\sigma^{\gamma^*p}(x, Q^2) = N \cdot \frac{2\pi}{\Lambda^2} (1 - S) = \frac{2\pi N}{\Lambda^2} \left( 1 - e^{-p/2} \right). \quad (13)
\]

The exponential is reminiscent of a Glauber resummation which would be well justified for example in the case of a nuclear target. But we must keep in mind that the resummation is theoretically not under control for a proton target and there is up to now no systematic way to do it in QCD.

We choose to keep the exponential form but we add a free parameter \( \nu_> \) in its argument, so that we recover Eq. (11) in the two-gluon exchange limit and the strength of the 4-gluon interaction be fixed by the data. We will check phenomenologically that the result is not very sensitive to \( \nu_> \). Absorbing all dimensionless normalizations in the parameters \( N \) and \( \nu_> \), we obtain the following formula:

\[
\sigma^{\gamma^*p}(x, Q^2) = N \frac{1}{\Lambda^2 \nu_>} \left\{ 1 - \exp \left( -\nu_> \frac{Q_s^2(x)}{Q^2} \left( 1 + \log \frac{Q}{Q_s(x)} \right) \right) \right\}. \quad (14)
\]

Note that this formula is very close to the original GBW model (see Eq. (11)). The main difference is the log present in the exponential, which will play an important role for the description of the data, as we shall see in the following section.
Let us now consider the case $Q < Q_s(x)$. We wish to preserve the symmetry of $Q/Q_s(x) \sigma^{\gamma p}(Q/Q_s(x))$ for $Q \leftrightarrow Q_s(x)$ at large $|\log(Q/Q_s(x))|$. Hence we take the following ansatz for the cross section in this regime:

$$\sigma^{\gamma p}(x,Q^2) = N \frac{1}{\Lambda^2} \frac{Q_s^2(x)}{Q^2} \nu_\nu \left\{ 1 - \exp \left( -\nu_\nu \frac{Q^2}{Q_s^2(x)} \left( 1 + \log \frac{Q_s(x)}{Q} \right) \right) \right\}.$$

We have kept the same parameters except $\nu_\nu$ which is replaced by $\nu_\ell$. Indeed, there is no reason why the resummation should be the same for $Q > Q_s(x)$ and for $Q < Q_s(x)$.

Let us now try to interpret formula (15). The probability that a specific dipole in the proton has one interaction is $p = (\sigma^{\gamma p}(x,Q^2)/n) \cdot Q^2/\pi N$. By interpreting the proton as a single effective dipole, with flux $Q_s^2(x)/\Lambda^2$ and the photon as a set of dipoles with density $N$, we are led to Eq. (15) by the same argument of a Glauber resummation of multiple gluon pair exchanges. Of course, this is only an a posteriori interpretation which has to be taken with great care. Indeed, it is usually argued that for $Q < Q_s(x)$, the dipoles in the proton are strongly correlated.

3 A comparison to the data

In this section, we compare Eq. (14) (for $Q > Q_s(x)$) and Eq. (15) (for $Q < Q_s(x)$) to the recent ZEUS data for the $F_2$ structure function both in the high [25] and low [20] $Q^2$ regime, and for $x \leq 10^{-2}$. We assume the parametrization (2) for the saturation scale $Q_s(x)$. We end up with 5 free parameters to be determined by the fit: $x_0$ and $\lambda$ parametrize $Q_s(x)$, $N$ is the global normalization, and $\nu_\nu$ and $\nu_\ell$ is the strength of the 4 gluon coupling with respect to the 2 gluon coupling, for $Q > Q_s(x)$ and $Q < Q_s(x)$ respectively.

The fit is performed with the 177 experimental points which remain after the selection $x \leq 10^{-2}$ and $Q^2 \leq 150$ GeV$^2$. The $\chi^2$ is 1.09 per degree of freedom. It rises to 1.15 when we take all available $Q^2$ (185 data points). The parameters for $Q_s(x)$ are $\lambda = 0.35$ and $x_0 = 1.88 \cdot 10^{-3}$. The exponent for the rise of the parton densities $\lambda$ is consistent with the one found within other approaches. The saturation scale is 1 GeV already for relatively high $x$ ($\sim 2 \cdot 10^{-3}$), but we cannot make a point of this fact since many neglected effects could shift the saturation scale by a constant factor (for instance the full photon wave function would have such an effect). The exponents $\nu_\nu$ and $\nu_\ell$ are very loosely determined by the data: we find $\nu_\nu = 0.8$ and $\nu_\ell = 0.6$, with an error of about 50% and 100% respectively. If we fix them, to 1 for instance, the quality of the fit does not drop much ($\chi^2 = 1.4$ d.o.f.).

The result of the fit is presented on Figs. 2 and 3. We have added some of the available data points which were not included in the fit and which sit at large $Q^2$ and large $x$. We see a good agreement over the whole $Q^2$, $x$ range. The $x$-slope seems to be reproduced fairly well even at large $Q^2$. However, we slightly overshoot the normalization in this region.

We have also shown, for comparison, the cross section without multiple scatterings, given by Eqs. (19) and (13). We observe that the data is well reproduced for $Q \ll Q_s(x)$.
and for $Q \gg Q_s(x)$, but the resummations are needed around the transition region $Q \sim Q_s(x)$. We note that without the resummation of multiple gluon exchanges through the exponentiation, we would obtain a very poor global fit. However, we have a very good agreement with the data when we fit separately formulae (10) and (11) to the high and low $Q^2$ data, but with very different parameters $x_0$ and $\lambda$ in the two regions.

We plot the quantity $Q/Q_s(x)\sigma^{\gamma^*p}(x,Q^2)$ in Fig. 4. The symmetry under the exchange of $Q$ and $Q_s(x)$ is apparent. Our parametrization reproduces it exactly, by construction. We see that the Glauber resummations help to describe the early turnover seen near $Q^2/Q_s^2(x) \sim 3$.

A final remark concerns the comparison with the Golec-Biernat and Wüsthoff model. We have already noted that in the high-$Q^2$ domain, the main difference between our formula (14) and the GBW model is the log in the exponent. We have checked that this term is crucial to fit the data. In the GBW model of Ref. [1], it is absent from the exponent, but there is an overall log factor coming from the photon wave function.

4 Summary and discussion

We can summarize our model as follows. We have assumed that in the dipole frame the energy evolution of the proton leads to the multiplication of the partons and consequently to the appearance of a transverse momentum scale $Q_s(x)$. When $Q > Q_s(x)$, we have the usual DIS picture, where the photon probes a set of independent partons. The rate of the growth of the parton densities has been parametrized by $Q_s^2(x)/\Lambda^2$. This procedure led us to formula (11). Then, we have taken seriously its extrapolation to small values of $Q^2$, and we have observed that the symmetry of the dipole-dipole cross section by exchange of the dipole sizes is exactly seen in the data for $\gamma^*p$ total cross section. When $Q \sim Q_s(x)$, we have argued that the probability of multiple interactions between the photon and the proton becomes sizeable. A Glauber-like resummation was introduced for $Q > Q_s(x)$ and extrapolated by symmetry for $Q < Q_s(x)$. Although very simple and based on very qualitative ideas, the model agrees quantitatively with the recent high precision ZEUS data.

This model provides an intuitive picture of geometric scaling both in the large $Q$ (hard Pomeron, perturbative) and small $Q^2$ (soft Pomeron, non-perturbative but weakly coupled) regimes based on a symmetry between low $Q^2$ and high $Q^2$ recently observed in the photon-proton cross section.

On the theoretical side, we may observe that our model could be related to unitary models based on Regge theory (see Ref.[27]). In these models, a separation scale of 1 GeV$^{-1}$ between small and large distance is assumed. This could correspond to our “saturation” radius $1/Q_s$ which is of the same order. In our approach, the fan diagrams involving triple Pomeron vertices are hidden in the parametrization of the proton evolution (they are explicitly present in Kovchegov’s approach for instance). The formula (14) may be seen as an eikonalization of fan diagrams. On the other hand, in a recent paper [13] the dipole-proton scattering amplitude was derived in the context of linear BFKL supplemented by absorptive boundary conditions (see also [19]). The final formula looks very much like Eq. (14). It differs only by an anomalous
dimension which we do not take into account in our qualitative approach. The log has also presumably a different origin. However, this gives us confidence that the formula we propose is anyway well rooted in QCD in the restricted perturbative domain \( Q > Q_s(x) \).

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Figure captions

Figure 1. A picture of the photon-proton interaction in the dipole frame

In this frame, the momenta of the photon and of the proton are collinear. The photon splits in a $q\bar{q}$ pair of size $1/Q$. The QCD evolution leads to a set of dipoles of sizes $1/Q_s(x)$ at the proton level at the time of interaction. These two systems scatter through the exchange of colour singlet pairs of gluons. The cross section is depicted for a. the exchange of two gluons, b. the exchange of four gluons.

Figure 2. The $F_2$ structure function for medium $Q^2$

Our parametrization of $F_2$ is shown (solid line) together with the latest published ZEUS data [25]. The fit was done only for $x \leq 10^{-2}$. The dashed line is the simple dipole cross section without taking account of many gluon exchanges (see formulae (10) and (11)). The errors shown for the data are the quadratic sum of the statistic and systematic errors.

Figure 3. The $F_2$ structure function for low $Q^2$

The same in the low $Q^2$ region. The data points are from the ZEUS collaboration [26]. We only show the $Q^2$-bins for which a significant number of data points are available.

Figure 4. Geometric scaling

The scaled photon-proton total cross-section $Q/Q_s(x)\sigma^{\gamma p}(x, Q^2)$ is plotted against the combined variable $Q^2/Q_s^2(x)$. The data are from the ZEUS collaboration [24, 26], and are selected according to $x \leq 10^{-2}$. 
Figure 1
Figure 2
Figure 3
