LHC luminosity and energy upgrades confront natural supersymmetry models

Howard Baer\textsuperscript{1}, Vernon Barger\textsuperscript{2}, James S. Gainer\textsuperscript{3}, Dibyashree Sengupta\textsuperscript{1}, Hasan Serce\textsuperscript{1} and Xerxes Tata\textsuperscript{3,4}

\textsuperscript{1}Department of Physics and Astronomy, University of Oklahoma, Norman, OK 73019, USA
\textsuperscript{2}Department of Physics, University of Wisconsin, Madison, WI 53706, USA
\textsuperscript{3}Department of Physics and Astronomy, University of Hawaii, Honolulu, HI 96822, USA
\textsuperscript{4}Centre for High Energy Physics, Indian Institute of Science, Bangalore, 560012, India

Abstract

The electroweak fine-tuning measure $\Delta_{\text{EW}}$ allows for correlated SUSY soft terms as are expected in any ultra-violet complete theory. Requiring no less than 3% electroweak fine-tuning implies upper bounds of about 360 GeV on all higgsinos, while top squarks are lighter than $\sim 3$ TeV and gluinos are bounded by $\sim 6 - 9$ TeV. We examine the reach for SUSY of the planned high luminosity (HL: 3 ab\textsuperscript{-1} at 14 TeV) and the proposed high energy (HE: 15 ab\textsuperscript{-1} at 27 TeV) upgrades of the LHC via four LHC collider search channels relevant for natural SUSY: 1. gluino pair production followed by gluino decay to third generation (s)quarks, 2. top-squark pair production followed by decay to third generation quarks and light higgsinos, 3. neutral higgsino pair production with QCD jet radiation (resulting in monojet events with soft dileptons), and 4. wino pair production followed by decay to light higgsinos leading to same-sign diboson production. We confront our reach results with upper limits on superpartner masses in four natural SUSY models: natural gravity-mediation via the 1. two- and 2. three-extra-parameter non-universal Higgs models, 3. natural mini-landscape models with generalized mirage mediation and 4. natural anomaly-mediation We find that while the HL-LHC can probe considerable portions of natural SUSY parameter space in all these models, the HE-LHC will decisively cover the entire natural SUSY parameter space with better than 3% fine-tuning.
1 Introduction

With the discovery of the Higgs boson in 2012\textsuperscript{1}, the CERN LHC has verified the particle content of the Standard Model (SM). In spite of this impressive triumph, many physicists still expect new physics to be revealed at LHC. The primary reason is the instability of the SM Higgs boson mass under radiative corrections if the SM is embedded into a high scale theory (such as string theory). Starting with the SM scalar potential

\[ V = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2, \tag{1} \]

one finds the Higgs mass, including leading radiative corrections cutoff at an energy scale \( \Lambda \) (where new physics degrees of freedom not present in the SM become important), to be

\[ m_h^2 \simeq 2\mu^2 + \delta m_h^2, \]

with\textsuperscript{4}

\[ \delta m_h^2 \simeq \frac{3}{4\pi^2} \left( -\lambda_t^2 + \frac{g^2}{4} + \frac{g^2}{8\cos^2 \theta_W} + \lambda \right) \Lambda^2. \tag{2} \]

Here, \( \lambda_t \) is the top quark Yukawa coupling given in the SM by \( \lambda_t = \frac{g m_t}{\sqrt{2} M_W} \), \( g \) is the \( SU(2)_L \) gauge coupling, \( \theta_W \) is the Weinberg angle and \( \lambda \) is the Higgs quartic coupling in the Higgs boson potential \( (1) \). The quadratic sensitivity of the SM Higgs boson mass to new physics at the high scale \( \Lambda \) embodies the fine-tuning problem of the SM. If the new physics scale \( \Lambda \gg 1 \) TeV, then the free parameter \( \mu^2 \) will have to be accordingly fine-tuned to maintain the Higgs mass at its measured value \( m_h = 125.09 \pm 0.24 \) GeV\textsuperscript{4}. The fine-tuning gets consequently more implausible as the theory cutoff \( \Lambda \) extends significantly beyond the weak scale. The need for large fine-tuning suggests a missing ingredient in the underlying theory because otherwise seemingly independent contributions to the Higgs boson mass would then need to have large unexplained cancellations in order to yield its measured value.

Perhaps the most elegant and compelling resolution\textsuperscript{5} of the fine-tuning problem is to extend the underlying Poincaré spacetime symmetries to the more general superPoincaré group. In the supersymmetrized version of the SM, along with weak scale soft SUSY breaking terms (the so-called Minimal Supersymmetric Standard Model or MSSM\textsuperscript{6}), the quadratic cutoff dependence seen in Eq. (2) is absent, leaving only relatively mild but intertwined logarithmic sensitivity to high scale physics. In addition to including a cure for the divergent Higgs mass, the MSSM receives impressive support from data via several different virtual effects:

- the measured values of gauge coupling constants are consistent with unification under renormalization group running within the MSSM\textsuperscript{7, 8}
- the measured value of the top quark mass is within the range required to trigger a radiatively-driven breakdown of electroweak symmetry\textsuperscript{9}, and

\textsuperscript{1}Quadratic divergences in the SM were studied by Veltman\textsuperscript{2}. While the use of a cutoff as a regulator is not gauge invariant, the coefficient of \( \Lambda^2 \) in Eq. (2) is independent of \( \xi \) in \( R_\xi \) gauges. For subtleties on the regulation scheme dependence of the quadratic sensitivity of the Higgs boson mass to high scale physics, see Ref. [3].
• the measured value of the Higgs mass fall squarely within the narrow window of MSSM prediction[10], and in fact agrees with the radiatively-corrected MSSM $m_h$ calculation provided top squarks ($\tilde{t}_{1,2}$) lie in the TeV range and are highly mixed by TeV-scale trilinear soft terms[11].

The natural MSSM seemingly requires the existence of several superpartners (those that have direct couplings to the Higgs sector) with masses not too far beyond the weak scale as typified by $m_{weak} \simeq m_{W,Z,h} \sim 100$ GeV[6]. So far, searches by LHC experiments have failed to find any superpartners leading to simplified model gluino ($\tilde{g}$) mass limits such as $m_{\tilde{g}} \gtrsim 2$ TeV[12][13] and top-squark ($\tilde{t}_1$) mass limits such as $m_{\tilde{t}_1} \gtrsim 1.1$ TeV[14][15] – along with considerably weaker limits on electroweakly interacting superpartners. The widening mass gap between the weak scale and the soft breaking scale has seemingly sharpened the issue of a Little Hierarchy: how can it be that $m_{weak} \ll m_{soft}$ when the soft breaking scale is supposed to determine the weak scale? Naively, one might expect $m_{weak} \sim m_{soft}$ absent again any fine-tuning. Indeed, early estimates of naturalness or lack of fine-tuning within SUSY models seemed to require $m_{\tilde{g}} \lesssim 350$ GeV and $m_{\tilde{t}_1} \lesssim 400$ GeV for no worse than 3% fine-tuning[16][17][18]. Some more recent naturalness calculations seemed to require three third generation squarks with mass below about 500 GeV[19]. The contrast between these naturalness bounds and current LHC mass limits might indicate a need to fine-tune within the MSSM to maintain $m_{weak} \sim 100$ GeV which in turn may signal some pathology or missing ingredient this time within the SUSY paradigm.

An issue with these estimates is that they ignore the possibility that model parameters–usually taken to be independent in order to parametrize our ignorance of SUSY breaking–should be correlated (inter-dependent) in ultra-violet complete theories. Such correlations can lead to automatic cancellations between terms involving large logarithms: thus, ignoring this possibility can easily lead to large over-estimates of the UV sensitivity of the theory[20][21][22].

To allow for the fact that the underlying model parameters are expected to be correlated, we adopt the very conservative fine-tuning measure, $\Delta_{EW}$[23][24]. The quantity $\Delta_{EW}$ measures how well the weak scale MSSM Lagrangian parameters match the measured value of the weak scale. By minimizing the MSSM weak scale scalar potential to determine the Higgs field vevs, one derives the well-known expression relating the $Z$-boson mass to the SUSY Lagrangian parameters:

$$\frac{m_Z^2}{2} = \frac{m_{H_u}^2}{2} + \Sigma_{d}^d - (m_{H_u}^2 + \Sigma_{u}) \tan^2 \beta - \mu^2 \simeq -m_{H_u}^2 - \Sigma_{u}(\tilde{t}_{1,2}) - \mu^2. \quad (3)$$

Here, $\tan \beta = v_u/v_d$ is the ratio of Higgs field vacuum-expectation-values and the $\Sigma_u$ and $\Sigma_d$ contain an assortment of radiative corrections, the largest of which typically arise from the top squarks. Expressions for the $\Sigma_u$ and $\Sigma_d$ are given in the Appendix of Ref.[24]. Thus, $\Delta_{EW}$ compares the maximal contribution on the right-hand-side (RHS) of Eq. (3) to the value of $m_Z^2/2$. If the magnitudes of the terms on the RHS of Eq. (3) are individually comparable to $m_Z^2/2$, then no unnatural fine-tunings are required to generate $m_Z = 91.2$ GeV. We have shown that once appropriate inter-parameter correlations are properly taken into account[20][21][22], then the traditional fine-tuning measure[16], $\Delta_{BG} \equiv \max_i \left| \frac{\partial \log m_i^2}{\partial \log p_i} \right|$ indeed reduces to $\Delta_{EW}$. 

2
A utilitarian feature of the naturalness calculation is that it leads to upper bounds on sparticle masses which in turn provide targets for present or future colliding beam experiments which seek to discover superpartners or falsify the weak scale SUSY hypothesis. But for which values of $\Delta_{EW}$ is SUSY natural? The original calculations of Barbieri-Giudice used $\Delta_{BG} < 10$, or no less than $\Delta_{BG}^{-1} = 10\%$ fine-tuning. We will, more conservatively, adopt a value $\Delta_{EW} < 30$ (3.3% electroweak fine-tuning) as an upper bound on natural SUSY models\(^2\). That this is a qualitatively different criterion is driven home by the fact that it is possible to have the same model with both $\Delta_{EW} < 30$ and $\Delta_{BG} > 3000$ (if the latter is naively evaluated with multiple uncorrelated soft terms\(^3\)).

Natural models with low electroweak fine-tuning ($\Delta_{EW} \lesssim 30$) exhibit the following features:

- $|\mu| \sim 100 - 350$ GeV\(^2\)\(^2\)\(^3\) (the lighter the better) where $\mu \gtrsim 100$ GeV is required to accommodate LEP2 limits from chargino pair production searches\(^3\).
- $m_{H_u}^2$ is driven radiatively to small– not large– negative values at the weak scale (radiatively-driven naturalness)\(^[23, 24]\).
- The top squark contributions to the radiative corrections $\Sigma^u_{\tilde{t}_1,2}$ are minimized for TeV-scale highly mixed top squarks\(^[23]\). This latter condition also lifts the Higgs mass to $m_h \sim 125$ GeV. For $\Delta_{EW} \lesssim 30$, the lighter top squarks are bounded by $m_{\tilde{t}_1} \lesssim 3$ TeV\(^[24, 27]\).
- The gluino mass, which feeds into the top-squark masses at one-loop and hence into the scalar potential at two-loop order, is bounded by $m_{\tilde{g}} \lesssim 6 - 9$ TeV\(^[24, 27]\) (depending on the details of the model).

These new sparticle mass bounds derived from the $\Delta_{EW}$ measure lie well beyond current LHC search limits and allow for the possibility that SUSY is still natural and still awaiting discovery. The question then is: how far along are LHC SUSY searches on their way to discovering or falsifying supersymmetry? And what sort of LHC upgrade is needed to either discover or falsify natural SUSY? Indeed, recently the European Strategy Study has begun to assess what sort of accelerator (or other experiments) are needed beyond high-luminosity LHC (HL-LHC). One option is to double the field strength of the dipole steering magnets to 16 Tesla. This would allow for an energy upgrade of LHC to $\sqrt{s} = 27$ TeV with an assumed 15 ab\(^{-1}\) of integrated luminosity (HE-LHC). The goal of this paper is to re-examine the SUSY theory/experiment confrontation with a view to informing these questions about future experiments and to examine what collider options are needed to completely probe the natural SUSY parameter space. In doing so, we will confront four different natural SUSY models with updated LHC limits from four SUSY search channels which are deemed most important for discovering/falsifying natural supersymmetry.

The four natural SUSY models we examine here include the following:

- Natural gravity-mediation as exhibited in the two- and three-extra parameter non-universal Higgs model (nNUHM2 and nNUHM3)\(^[28]\). The NUHM2 model has parameter space\(^2\)\(^3\) The onset of fine-tuning for $\Delta_{EW} > 20 - 30$ is visually displayed in Ref. \(^[27]\).\(^3\) We assume that the superpotential $\mu$-term makes the dominant contribution to the higgsino mass.
Natural (generalized) anomaly-mediation or nAMSB adopts the usual AMSB masses but also allows for non-universal bulk Higgs masses $m_{H_u}$ and $m_{H_d}$ as compared to bulk matter scalar masses $m_0$ \[^{[31]}\]. It also includes some bulk trilinear $A_0$ soft term contributions. The parameter space is then $m_0$, $m_{3/2}$, $A_0$, tan $\beta$, $\mu$, $m_A$. The non-universal and trilinear bulk terms allow for $m_h \simeq 125$ GeV while allowing for naturalness in the spectra. For nAMSB, the electroweakinos are oriented such that $\mu < M_2 < M_1 < M_3$ at the weak scale. The LSP in nAMSB is a higgsino-like LSP instead of wino-like as is typically assumed. For greater generality, one may include as well separate first/second versus third generation bulk matter scalar masses $m_0(1,2)$ and $m_0(3)$.

Natural generalized mirage mediation (nGMM) models\[^{[32]}\], in which one expects comparable anomaly- and modulus/gravity-mediated contributions to soft breaking terms. The nGMM parameter space\[^{[33]}\] is $\alpha$, $m_{3/2}$, $c_m$, $c_{m3}$, $a_3$, tan $\beta$, $\mu$, $m_A$, where $\alpha$ parametrizes the relative modulus-to-anomaly-mediation contributions and the $c_m$, $c_{m3}$ and $a_3$ are continuous generalizations of previous discrete parameters related to modular weights. Since gaugino masses unify at some intermediate (mirage) scale $\mu_{\text{mir}} = e^{-8\pi^2/\alpha}m_{\text{GUT}}$, then the gaugino masses are compressed compared to NUHM2(3) so one expects $\mu \ll M_1 \ll M_2 \ll M_3$. As an example, we examine the mini-landscape picture taking $m_{3/2} \simeq m_0(1,2) > 2 \times m_0(3)$ \[^{[30]}\].

These four models have been encoded in Isajet v7.88\[^{[34]}\] which we use for spectra generation and the $\Delta_{\text{EW}}$ calculation. For each of the four models, we scan over the whole parameter space (with tan $\beta$ : 3 – 60) and accept solutions which are consistent with current LHC sparticle mass constraints, with $m_h = 125 \pm 2$ GeV (adopting $\sim \pm 2$ GeV theory error in our Higgs mass calculation). We also require that solutions have $\Delta_{\text{EW}} < 30$ in order to satisfy naturalness—which amounts to a reasonable SUSY model prediction for the magnitude of the weak scale. For the nGMM parameter space, we require $\alpha$ to be positive (real mirage unification) and $\alpha < 40$ so that anomaly mediation is not highly suppressed.

The four most important search channels for natural SUSY at the LHC or its upgrades are the following.

- Gluino pair production $pp \rightarrow \tilde{g} \tilde{g} X$ followed by either two-body gluino decay to top squarks $\tilde{g} \rightarrow \tilde{t}_1 \tilde{t}$, $\tilde{t}_1 \tilde{t}$ or, if these are closed, then gluino three-body decays to mainly third generation quarks\[^{[35]}\]: $\tilde{g} \rightarrow t \tilde{Z}_i$, $b \tilde{Z}_i$ or $t \tilde{W}^+_j + c.c.$.

- Top squark pair production $pp \rightarrow \tilde{t}_1 \tilde{t}_1^* X$ followed by $\tilde{t}_1 \rightarrow t \tilde{Z}_i$ or $b \tilde{W}^+_j$ \[^{[36]}\].
• Higgsino pair production via \( pp \rightarrow \tilde{Z}_i \tilde{Z}_j, \tilde{W}_i \tilde{W}_j \) channels is unlikely to be visible above SM \( Z_j \) background because the signal to background ratio is just 1-2%. However, the \( pp \rightarrow \tilde{Z}_i \tilde{Z}_j \) channel (with contributions from \( pp \rightarrow \tilde{W}_i \tilde{Z}_j \) ), where \( \tilde{Z}_2 \rightarrow \ell \ell \tilde{Z}_1 \) with a soft OS dilepton pair and where the hard initial state radiated jet supplies a trigger, offers a promising search channel for low mass higgsinos with \( m_{\tilde{Z}_{1,2}} \sim 100 - 300 \) GeV. Indeed, the LHC collaborations have presented their first results for this search, and it is especially encouraging that the ATLAS collaboration is able to access a \( \tilde{Z}_2 - \tilde{Z}_1 \) mass gap as small as 2.5 GeV.

• Wino pair production \( pp \rightarrow \tilde{W}_2^\pm \tilde{Z}_3 \) or 4 followed by \( \tilde{W}_2 \rightarrow W \tilde{Z}_{1,2} \) and \( \tilde{Z}_3 \) or 4 \( \rightarrow W^\pm \tilde{W}_1^\mp \). Half the time, this final state leads to a same-sign diboson (SSdB) final state which, when followed by leptonic W decays, leads to same-sign dileptons +MET with very little accompanying jet activity (as opposed to SS dileptons arising from gluino cascade decays). The SSdB signature has very low SM background rates arising mainly from \( ttW \) production.

In Sec. 2 we present our updated reach projections for revised HE-LHC specifications with \( \sqrt{s} = 27 \) TeV and a projected integrated luminosity (IL) of 15 ab\(^{-1}\). In Sec. 3 we examine the four natural SUSY models introduced earlier and present LHC bounds in each of these search channels, and also obtain reach projections for HL- and HE-LHC. We find that while HL-LHC can probe a portion of natural SUSY parameter space, it will take an energy upgrade to the HE-LHC option for a definitive search for natural weak scale SUSY. In Sec. 4 we present a summary and conclusions.

## 2 Updated reach projections of HE-LHC for gluinos and top-squarks

In this section, we update previous HE-LHC reach analyses for top-squark pair production\(^{[42]}\) and gluino pair production\(^{[43,42]}\) in natural SUSY which were performed assuming \( \sqrt{s} = 33 \) TeV and IL= 0.3 – 3 ab\(^{-1}\) to the updated values assigned for the European Strategy report, namely \( \sqrt{s} = 27 \) TeV and IL= 15 ab\(^{-1}\). Along these lines, our first step is to generate updated total production cross sections for our signal processes.

In Fig. 1 we plot the total production cross section for \( pp \rightarrow \tilde{g}\tilde{g}X \) (black) and \( pp \rightarrow \tilde{t}_1\tilde{t}_1^*X \) (orange) at both \( \sqrt{s} = 14 \) TeV (thin solid) and 27 TeV (thick solid). The results are computed at NLL+NLO and the 14 TeV results are taken from the study of Ref. \(^{[44]}\) where we use the gluino pair production results for decoupled squarks. Since Ref. \(^{[44]}\) presents results for \( \sqrt{s} = 13, 14, 33 \) and 100 TeV, we obtain total cross sections for \( \sqrt{s} = 27 \) TeV via interpolation of the 14 and 33 TeV results. Specifically, we fit \( \log \sqrt{s} \) versus \( \log \sigma_{\text{tot}} \) to a quadratic and used the resulting function to obtain \( \sqrt{s} = 27 \) TeV cross sections.

From the results shown in Fig. 1 we see that for \( m_{\tilde{g}} = 2 \) TeV, then the gluino pair production cross section ratio \( \sigma(27)/\sigma(14) \) is 38 while for \( m_{\tilde{g}} = 3.5 \) TeV this ratio increases to \( \sim 394 \). For \( m_{\tilde{t}} = 1 \) TeV, then we find a total top squark pair production ratio \( \sigma(27)/\sigma(14) \) is 12 while for \( m_{\tilde{t}_1} = 2.5 \) TeV then \( \sigma(27)/\sigma(14) \) increases to 83. These ratios clearly reflect the
advantage of moving to higher LHC energies in order to probe more massive strongly interacting sparticles.

2.1 Updated top squark analysis for $\sqrt{s} = 27$ TeV

In Ref. [42], the reach of a 33 TeV LHC upgrade for top-squark pair production was investigated. Here, we repeat the analysis but for updated LHC energy upgrade $\sqrt{s} = 27$ TeV. We use Madgraph [45] to generate top-squark pair production events within a simplified model where $\tilde{t}_1 \to b\tilde{W}_1^+$ at 50%, and $\tilde{t}_1 \to t\tilde{Z}_{1,2}$ each at 25% branching fraction, which are typical of most natural SUSY models [36]. The higgsino-like electroweakino masses are $m_{\tilde{Z}_{1,2}, \tilde{W}_1^\pm} \simeq 150$ GeV. We interface Madgraph with Pythia [46] for initial/final state showering, hadronization and underlying event simulation. The Delphes toy detector simulation [47] is used with specifications as listed in Ref. [42] (which we will not repeat here). We also used Madgraph-Pythia-Delphes for a variety of SM background processes which are listed in Table 1.

In Ref. [42], an optimized set of cuts was found for extracting the signal from a 2.75 TeV top squark over SM backgrounds at $\sqrt{s} = 33$ TeV LHC upgrade. The cuts that were settled upon were

- $n(b - jets) \geq 2$,
- $n(\text{isol. leptons}) = 0$,
- $E_T^{\text{miss}} > \text{max}(1500 \text{ GeV}, 0.2M_{\text{eff}})$,
| process  | $\sigma$ (ab) |
|----------|---------------|
| $bbZ$    | 1.87          |
| $t\bar{t}Z$ | 1.1          |
| $t\bar{t}$ | $4.4 \times 10^{-2}$ |
| $t\bar{t}b\bar{b}$ | $3.3 \times 10^{-2}$ |
| $t\bar{t}t\bar{t}$ | $2.3 \times 10^{-2}$ |
| $t\bar{t}h$ | $1.7 \times 10^{-3}$ |
| total     | 3.07          |

Table 1: Cross sections in $ab$ after cuts, listed in Sec. 2.1, from SM background processes for the top-squark pair production analysis at $\sqrt{s} = 27$ TeV.

- $E_T(j_1) > 1000$ GeV,
- $E_T(j_2) > 600$ GeV,
- $S_T > 0.1$ and
- $\Delta\phi(E_T^{miss}, \text{jet close}) > 30$ deg.

In the above, $M_{eff}$ is the usual effective mass variable, $S_T$ is transverse sphericity and the $\Delta\phi$ cut is on the transverse opening angle between the missing $E_T$ vector and the closest jet (which helps reduce background from boosted tops in $t\bar{t}$ production). The surviving background rates in $ab$ are listed in Table 1. We use the same $K$-factors as listed in Ref. [42] to bring our total background cross sections into accord with various beyond-leading-order calculations. In the present analysis, we have also included the $t\bar{t}Z$ background calculation which was not present in Ref. [42].

Using these background rates for LHC at $\sqrt{s} = 27$ TeV, we compute the $5\sigma$ and $95\%$ CL reach of HE-LHC for 3 and 15 ab$^{-1}$ of integrated luminosity using Poisson statistics. These results are plotted in Fig. 2 along with the top-squark pair production cross section after cuts versus $m_{\tilde{t}_1}$. From the figure, we see the $5\sigma$ discovery reach of LHC27 extends to $m_{\tilde{t}_1} = 2800$ GeV for 3 ab$^{-1}$ and to 3160 GeV for 15 ab$^{-1}$. The $95\%$ CL exclusion limits extend to $m_{\tilde{t}_1} = 3250$ GeV for 3 ab$^{-1}$ and to $m_{\tilde{t}_1} = 3650$ GeV for 15 ab$^{-1}$. We see that $S/B$ exceeds 0.8 whenever we deem the signal to be observable. Of course, somewhat increased reach limits can be obtained in the event of a combined ATLAS/CMS analysis.

### 2.2 Updated gluino analysis for $\sqrt{s} = 27$ TeV

In Ref. [42], optimized cuts were investigated for extracting the signal from a 5.4 TeV gluino over SM backgrounds at a $\sqrt{s} = 33$ TeV LHC upgrade. The optimized cuts were found to be

- $n(b - \text{jets}) \geq 2$,
- $n(\text{isol. leptons}) = 0$,
- $E_T^{miss} > \max(1900 \text{ GeV}, 0.2M_{eff})$,
Figure 2: Plot of top-squark pair production cross section vs. $m_{\tilde{t}_1}$ after cuts at HE-LHC with $\sqrt{s} = 27$ TeV (green curve). We also show the 5$\sigma$ and 95% CL reach lines assuming 3 and 15 ab$^{-1}$ of integrated luminosity (for a single detector).

- $E_T(j_1) > 1300$ GeV,
- $E_T(j_2) > 900$ GeV,
- $E_T(j_3) > 200$ GeV,
- $E_T(j_4) > 200$ GeV,
- $S_T > 0.1$ and
- $\Delta\phi(\vec{E}_T^{\text{miss}}, \text{jet close}) > 10$ deg.

The corresponding backgrounds in ab after cuts are listed in Table 2. The backgrounds are again normalized to recent beyond-leading-order results as detailed in Ref. [42]. We again compute the 5$\sigma$ reach and 95% CL exclusion lines using Poisson statistics for 3 and 15 ab$^{-1}$ of integrated luminosity.

Our results are shown in Fig. 3 where we plot the gluino pair production signal versus $m_{\tilde{g}}$ for a nNUHM2 model line with parameter choice $m_0 = 5m_{1/2}$, $A_0 = -1.6m_0$, $m_A = m_{1/2}$, $\tan \beta = 10$ and $\mu = 150$ GeV with varying $m_{1/2}$. We do not expect the results to be sensitive to this precise choice as long as first generation squarks are heavy. From the Figure, we see that the 5$\sigma$ discovery reach of LHC27 extends to $m_{\tilde{g}} = 4900$ GeV for 3 ab$^{-1}$ and to $m_{\tilde{g}} = 5500$ GeV for 15 ab$^{-1}$ of integrated luminosity. The corresponding 95% CL exclusion reaches extend to $m_{\tilde{g}} = 5300$ GeV for 3 ab$^{-1}$ and to $m_{\tilde{g}} = 5900$ GeV for 15 ab$^{-1}$ of integrated luminosity.
Table 2: Cross sections in ab after cuts, listed in Sec. 2.2, from SM background processes for the gluino pair production analysis at $\sqrt{s} = 27$ TeV.

| process | $\sigma$ (ab) |
|---------|---------------|
| $\bar{b}bZ$ | 0.061 |
| $\bar{t}\bar{t}Z$ | 0.037 |
| $\bar{t}t$ | 0.003 |
| $\bar{t}\bar{t}$ | 0.026 |
| $\bar{t}\bar{t}b\bar{b}$ | 0.0046 |
| $\bar{t}\bar{t}\bar{t}\bar{t}$ | 0.0 |
| $\bar{t}\bar{t}h$ | $8.1 \times 10^{-4}$ |
| total | 0.132 |

Figure 3: Plot of gluino pair production cross section vs. $m_{\tilde{g}}$ after cuts at HE-LHC with $\sqrt{s} = 27$ TeV (green curve). We also show the 5$\sigma$ and 95% CL reach lines assuming 3 and 15 ab$^{-1}$ of integrated luminosity.
3 Confronting natural SUSY models at the LHC and its upgrades

3.1 Gluino pair production

In Fig. 4 we display the results of our scans over parameter space of the nNUMH2, nNUHM3, nAMSB and nGMM models with $\Delta_{EW} < 30$ and with $m_h : 123 - 127$ GeV in the $m_{\tilde{g}}$ vs. $m_{\tilde{Z}_1}$ plane. We also require $m_{\tilde{g}} > 2$ TeV and $m_{\tilde{t}_1} > 1.1$ TeV in accord with recent simplified model mass limits from ATLAS and CMS. The density of points is not to be taken as meaningful. Indeed, in a statistical study of IIB string theory landscape\cite{48}, it is argued that there should exist a power law draw to large soft terms which would not be reflected here but which would then favor larger sparticle masses beyond current LHC reach and $m_h \simeq 125$ GeV. The available natural parameter space can be construed as some boundary enclosing all the natural SUSY scan points in accord with the measured Higgs mass and current LHC sparticle mass constraints.

From Fig. 4 we see that the range of $m_{\tilde{g}}$ extends from about 2 TeV to around $m_{\tilde{g}} \sim 6$ TeV for NUHM2,3 and nGMM models but to significantly higher values for nAMSB. The upper limit on $m_{\tilde{g}}$ occurs because the gluino mass drives top squark soft mass terms to such large values that $\Sigma^u(\tilde{t}_{1,2}) > 30$, leading to a violation of our naturalness criterion. To understand why higher gluino masses are allowed in the nAMSB model, we first note that $m_{\tilde{g}} \gtrsim 6$ TeV occurs only for
negative values of $A_0$. In this case, in order to obtain $m_h$ consistent with its observed value very large negative magnitudes of $A_0$ are required (compared to the positive $A_0$ case). The resulting very large contribution of $A_t$ to their RG evolution then strongly suppresses the weak scale soft top squark mass parameters, allowing correspondingly larger values of $m_{\tilde{g}}$ (vis à vis the other models). The fact that $|M_2|$ is smaller than $|M_3|$ in the nAMSB case also helps. The range of $m_{\tilde{Z}_1}$ varies from 100-350 GeV in accord with the range of $\mu$ which is bounded from below by LEP2 searches for chargino pair production and bounded from above by naturalness in Eq. (3).

We also show by the solid vertical lines around $m_{\tilde{g}} \sim 2$ TeV the results of several ATLAS and CMS simplified model search limits for gluino pair production[12][13]. It is apparent from the plot that a large range of parameter space remains to be explored. The blue dashed line around $m_{\tilde{g}} \sim 2800$ GeV shows the computed 5$\sigma$ reach of high luminosity LHC (HL-LHC) with $\sqrt{s} = 14$ TeV and 3 ab$^{-1}$ of integrated luminosity[19]. While the HL-LHC will somewhat extend the SUSY search via the gluino pair production channel, much of the allowed gluino mass range will remain beyond its reach. We also show with the green (purple) dashed lines the HE-LHC 5$\sigma$ reach (95% CL exclusion region) for gluino pair production as computed in Sec. 2 for $\sqrt{s} = 27$ TeV and 15 ab$^{-1}$ of IL. We see that HE-LHC should probe nearly all of parameter space for the nNUHM2, nNUHM3 and nGMM models while evidently a considerable fraction of nAMSB parameter space would be beyond HE-LHC reach in the gluino pair production channel.

### 3.2 Top squark pair production

In Fig. 5, we show the locus of scan points from the four natural SUSY models in the $m_{\tilde{t}_1}$ vs. $m_{\tilde{Z}_1}$ plane. The $m_{\tilde{Z}_1}$ value is bounded by $\sim 350$ GeV so almost no points occupy the near degeneracy region $m_{\tilde{t}_1} \sim m_{\tilde{Z}_1}$ where much LHC search effort has focussed. We also show the current search limits from ATLAS[13] and CMS[15] as solid red and black contours respectively. These LHC search limits exclude some of natural SUSY parameter space but evidently a large swath of natural SUSY parameter space remains to be explored since top-squark masses may extend up to $m_{\tilde{t}_1} \sim 3.5$ TeV without compromising naturalness.

The ATLAS collaboration projected 95% CL exclusion region for top squarks at HL-LHC[50] is also shown by the black dashed line at $m_{\tilde{t}_1} \sim 1.4$ TeV. While HL-LHC will probe additional parameter space, much of the top squark mass range will lie beyond its reach. The reach of HE-LHC with $\sqrt{s} = 27$ TeV and IL of 15 ab$^{-1}$ was computed in Sec. 2. We show the 5$\sigma$ reach contour as a red dashed line extending out to $m_{\tilde{t}_1} \sim 3.1$ TeV while the 95% CL exclusion region extends to $m_{\tilde{t}_1} \sim 3650$ GeV. The HE-LHC apparently will be able to probe essentially the entire natural SUSY parameter space in the top-squark pair production channel.

In Fig. 6 we show the gluino and top-squark reach values in the $m_{\tilde{t}_1}$ vs. $m_{\tilde{g}}$ plane. The gray shaded region is excluded by the current search limits from CMS[13][15]. In this plane, it is important to note that in the nNUHM2, nNUHM3 and nGMM models, the highest values of $m_{\tilde{g}}$ correspond to the lowest values of $m_{\tilde{t}_1}$ while the highest $m_{\tilde{t}_1}$ values correspond to the lowest $m_{\tilde{g}}$ values. Thus, a marginal signal in one of these channels (due to sparticle masses being near their upper limit) should correspond to a robust signal in the complementary channel. In particular, for nNUHM3 where gluinos might be slightly beyond HE-LHC reach, the top squarks should be readily detectable. The nAMSB model case is different, because as we saw in Sec. 3.1 the very large negative values of $A_0$ needed to obtain the correct value of $m_h$ allow
Figure 5: Plot of points in the $m_{\tilde{t}_1}$ vs. $m_{\tilde{Z}_1}$ plane from a scan over nNUHM2, nNUHM3, nGMM and nAMSB model parameter space. We compare to recent search limits from the ATLAS/CMS experiments (solid contours) and to projected future limits (dashed lines).

Gluino masses in the $6 - 9$ TeV range with modest values of $m_{\tilde{t}_1}$. (The top squark and gluino mass values in the nAMSB model with $A_0 > 0$ are in line with those in the other models.) We see that while gluino pair production might escape detection at the HE-LHC in the nAMSB framework, the top squark signal should be easily visible since $m_{\tilde{t}_1} \lesssim 3$ TeV in this case.

### 3.3 Higgsino pair production

The four higgsino-like neutralinos $\tilde{W}^\pm_1$ and $\tilde{Z}_{1,2}$ are the only SUSY particles required by naturalness to lie not too far above the weak scale, $m_{\text{weak}} \sim 100$ GeV. In spite of their lightness, they are very challenging to detect at LHC. The lightest neutralino evidently comprises only a subdominant part of dark matter and if produced at LHC via $pp \to \tilde{Z}_1\tilde{Z}_1$ would escape detection. In fact, signals from electroweak higgsino pair production $pp \to \tilde{Z}_i\tilde{Z}_j, \tilde{W}_1\tilde{Z}_i, \tilde{W}_1\tilde{W}_1 + X$ ($i, j = 1, 2$) are undetectable above SM backgrounds such as vector boson and top quark pair production because the decay products of the heavier higgsinos $\tilde{W}_1$ and $\tilde{Z}_2$ are expected to be soft. The monojet signal arising from initial state QCD radiation in higgsino pair production events has been evaluated in Ref. [37] and was found to have similar shape distributions to the dominant $pp \to Z j$ background but with background levels about 100 times larger than signal. See, however, Ref. [52].

A way forward has been proposed via the $pp \to \tilde{Z}_1\tilde{Z}_2 j$ channel where $\tilde{Z}_2 \to \ell^+\ell^-\tilde{Z}_1$: a soft opposite-sign (OS) dilepton pair recoils against a hard initial state jet radiation which
Figure 6: Plot of points in the $m_{\tilde{t}_1}$ vs. $m_{\tilde{g}}$ plane from a scan over nNUHM2, nNUHM3, nGMM and nAMSB model parameter space. We compare to projected future search limits from the LHC experiments.

serves as a trigger[38]. Recent searches in this $\ell^+\ell^-j+E_T$ channel have been performed by CMS[39] and by ATLAS[40]. Their resultant reach contours are shown as solid black and blue contours respectively in the $m_{\tilde{Z}_2}$ vs. $m_{\tilde{Z}_2} - m_{\tilde{Z}_1}$ plane in Fig. 7. These searches have indeed begun to probe the most promising portion of the parameter space, since the lighter range of $m_{\tilde{Z}_2}$ masses have some preference from naturalness. The CMS experiment has also presented projected exclusion contours for LHC14 with 300 fb$^{-1}$ and HL-LHC with 3 ab$^{-1}$ shown as the green and purple dashed contours[53]. We see that while these contours can probe considerably more parameter space, much of natural SUSY parameter space lies beyond these projected reaches. So far, reach contours for HE-LHC in this search channel have not been computed but it may be anticipated that HE-LHC will not be greatly beneficial here since $pp \rightarrow \tilde{Z}_1\tilde{Z}_2j + X$ is primarily an electroweak production process so the signal cross section will increase only marginally while QCD background processes like $t\bar{t}$ production will increase substantially: harder cuts may, however, be possible. The nAMSB model inhabits typically a larger mass gap region of the plane since in this model winos are much lighter than in nNUHM2 or nGMM for a given gluino mass. It is imperative that future LHC searches try to squeeze their reach to the lowest $m_{\tilde{Z}_2} - m_{\tilde{Z}_1}$ mass gaps which are favored to lie in the 3-5 GeV region for string landscape projections[48].
3.4 Wino pair production

The wino pair production reaction $pp \rightarrow \tilde{W}_2^\pm \tilde{Z}_4X$ (in nNUHM2,3 and nGMM) or $pp \rightarrow \tilde{W}_2^\pm \tilde{Z}_3X$ (in nAMSB) offers a new and lucrative search channel which is not present in unnatural models where $|\mu| \gg M_{\text{gauginos}}$. The decay modes $\tilde{W}_2^\pm \rightarrow W^{\pm} \tilde{Z}_{1,2}$ and $\tilde{Z}_{3,4} \rightarrow W^{\pm}\tilde{W}_1^\mp$ lead to a same sign diboson (SSdB) plus $\not{E}_T$ final states accompanied by minimal jet activity—just that arising from initial state radiation\[41\]. Thus, the ensuing same-sign dilepton+$\not{E}_T$ signature is quite different from that which arises from gluino and squark pair production where multiple hard jets are expected to be present. The SSdB signature from wino pair production has very low SM backgrounds which might arise from processes like $t\bar{t}W$ production.

In Fig. 8 we show the location of natural SUSY model points in the $m_{\tilde{W}_2}$ vs. $\mu$ plane. The region with large $\mu$ is increasingly unnatural as indicated in the plot. From Fig. 8 we see that the nAMSB model points tend to populate the lower $m_{\tilde{W}_2}$ region, $m_{\tilde{W}_2} \lesssim 1400$ GeV. This is because $M_2 \sim m_{\tilde{g}}/7$ in AMSB models with $m_{\tilde{g}} \lesssim 6 - 9$ TeV from naturalness considerations.

We are unaware of any LHC search limits via the SSdB channel, though this signature should begin to be competitive with the conventional $E_T$ searches for an integrated luminosity of $\sim 100$ fb$^{-1}$ expected to be accumulated by the end of LHC Run 2. The projected HL-LHC reach has been evaluated in Ref. [41] where the $5\sigma$ discovery and 95% CL exclusion dashed contours are shown. Evidently HL-LHC will be able to probe a large part of parameter space for the nAMSB model while only a lesser portion of natural parameter space of nNUHM2, nNUHM3 and nGMM models can be probed. The corresponding reach of HE-LHC has not
been computed for the SSdB channel. But again, since this is an EW production channel, the signal rates are expected to rise by a factor of a few by moving from $\sqrt{s} = 14$ TeV to $\sqrt{s} = 27$ TeV while some of the QCD backgrounds like $t\bar{t}$ production will rise by much larger factors. We also note that because the heavy winos are expected to decay to higgsinos plus a $W^\pm$, $Z$ or $h$ in the ratio 2:1:1\cite{11}, $VV, Vh$ and $hh$ plus $E_T$ signals may be present, possibly with additional soft leptons from higgsino decays. A study of these signals is beyond the scope of the present analysis.

## 4 Summary and conclusions

Our goal, in this paper, was to ascertain what sort of LHC upgrades might be sufficient to either discover or falsify natural supersymmetry. We focused here on natural SUSY spectra consistent with the measured value of the weak scale $m_{\text{weak}} \sim 100$ GeV without a need for implausible fine-tuning of model parameters. Naturalness, after all, remains one of the major motivations for weak supersymmetry and unnatural models seem highly implausible. To this end, we scanned over four different natural SUSY models: nNUHM2, nNUHM3, nAMSB and nGMM. We obtained upper limits on top squark masses ($m_{\tilde{t}} \lesssim 3.5$ TeV), gluino masses ($m_{\tilde{g}} \lesssim 6$ TeV in nNUHM2,3 and nGMM, but $m_{\tilde{g}} \lesssim 9$ TeV in nAMSB) and higgsino and wino masses.

We compared these against current LHC constraints and found large regions of natural
SUSY parameter space remain to be explored. We also compared against the HL-LHC upgrade: the HL-LHC with \( \sqrt{s} = 14 \) TeV and 3 ab\(^{-1}\) of integrated luminosity will explore deeper into natural SUSY parameter space but, barring a SUSY discovery, much of parameter space will remain to be explored. We also updated the HE-LHC reach using the revised energy and integrated luminosity targets as suggested by the ongoing European Strategy study: \( \sqrt{s} = 27 \) TeV and IL= 15 ab\(^{-1}\). For these latter values, we find a HE-LHC reach in \( m_{\tilde{t}} \) to 3200 GeV at 5\( \sigma \) and 3650 GeV at 95\% CL. For the gluino, we find a HE-LHC reach to \( m_{\tilde{g}} = 5500 \) GeV at 5\( \sigma \) and 6000 GeV at 95\% CL. The gluino (top squark) reach is reduced by about 600 GeV (400 GeV) if the integrated luminosity is instead 3 ab\(^{-1}\).

Comparing these values with upper limits from naturalness, we find the HE-LHC is sufficient to probe the entire natural SUSY parameter space in the top-squark pair production channel and also to almost explore nNUHM2,3 and nGMM models in the gluino pair channel. Within these models it is, therefore, very likely that signals from top squark and gluino pair production will be present at the HE-LHC. In the nAMSB model, it appears that gluinos may be beyond the HE-LHC reach.

We also compared the soft OS dilepton+jet signal from higgsino pair production to current and future reach projections for HL-LHC. For this channel, it will be important to explore neutralino mass gaps \( m_{Z_2} - m_{Z_1} \) down to \( \sim 3 \) GeV and higgsino masses up to \( \sim 350 \) GeV for complete coverage. We caution that the energy upgrade of the LHC may not be as beneficial for this discovery channel since QCD backgrounds are expected to rise more rapidly with energy than the EW higgsino pair production signal channel. We also examined the SSdB signature arising from charged and neutral wino pair production. The HL-LHC may explore a portion of – but not all of – natural SUSY parameter space in this channel. It is again unclear whether an energy upgrade will help much in this channel since QCD backgrounds are expected to increase more rapidly than the EW-produced signal channel for an assumed wino mass \( m_{\tilde{W}_\pm} \). We note, though, that there may be signals from wino pair production in \( VV, Vh \) and \( hh + E_T \) channels which may also be interesting to explore.

To sum up: the key theoretical motivation for weak scale supersymmetry as the stabilizer of the Higgs sector still remains, once we acknowledge that model parameters which are usually taken to be independent in spectra computer codes are expected to be correlated in any ultraviolet complete theory. Our final assessment is that the search for natural SUSY will, and should, continue on at LHC and HL-LHC, where more extensive regions of parameter space may be explored. The envisioned HE-LHC upgrade to \( \sqrt{s} = 27 \) TeV and IL= 15 ab\(^{-1}\) seems sufficient to either discover or falsify natural SUSY in the top-squark pair production signal channel, very possibly with an additional signal in the gluino-pair production channel. It is possible that observable signals may also emerge in the wino-pair or higgsino-pair plus monojet search channels as well.

Acknowledgments

This work was supported in part by the US Department of Energy, Office of High Energy Physics. The computing for this project was performed at the OU Supercomputing Center for Education & Research (OSCER) at the University of Oklahoma (OU). XT thanks the Centre
for High Energy Physics, Indian Institute of Science Bangalore, where part of this work was done for their hospitality, and also the Infosys Foundation for financial support that made his visit to Bangalore possible.

References

[1] G. Aad et al. [ATLAS Collaboration], Phys. Lett. B 716 (2012) 1; S. Chatrchyan et al. [CMS Collaboration], Phys. Lett. B 716 (2012) 30.

[2] M. Veltman, Acta Phys. Pol. B12 (1981) 437.

[3] M. Einhorn and D. R. T. Jones, Phys. Rev. D 46 (1992) 5206.

[4] The ATLAS collaboration [ATLAS Collaboration], ATLAS-CONF-2017-046; G. Aad et al. [ATLAS and CMS Collaborations], Phys. Rev. Lett. 114 (2015) 191803 [arXiv:1503.07589 [hep-ex]].

[5] E. Witten, Nucl. Phys. B 188, 513 (1981); R. K. Kaul, Phys. Lett. B 109, 19 (1982).

[6] H. Baer and X. Tata, Cambridge, UK: Univ. Pr. (2006) 537 p.

[7] S. Dimopoulos, S. Raby and F. Wilczek, Phys. Rev. D 24 (1981) 1681.

[8] J. R. Ellis, S. Kelley and D. V. Nanopoulos, Phys. Lett. B 260 (1991) 131; U. Amaldi, W. de Boer and H. Furstenau, Phys. Lett. B 260 (1991) 447; P. Langacker and M. X. Luo, Phys. Rev. D 44 (1991) 817.

[9] L. E. Ibañez and G. G. Ross, Phys. Lett. B110, 215 (1982); K. Inoue et al. Prog. Theor. Phys. 68, 927 (1982) and 71, 413 (1984); L. Ibañez, Phys. Lett. B118, 73 (1982); H. P. Nilles, M. Srednicki and D. Wyler, Phys. Lett. B 120 (1983) 346; J. Ellis, J. Hagelin, D. Nanopoulos and M. Tamvakis, Phys. Lett. B125, 275 (1983); L. Alvarez-Gaumé, J. Polchinski and M. Wise, Nucl. Phys. B221, 495 (1983); B. A. Ovrut and S. Raby, Phys. Lett. B 130 (1983) 277; for a review, see L. E. Ibanez and G. G. Ross, Comptes Rendus Physique 8 (2007) 1013.

[10] M. S. Carena and H. E. Haber, Prog. Part. Nucl. Phys. 50 (2003) 63.

[11] H. Baer, V. Barger and A. Mustafayev, Phys. Rev. D 85 (2012) 075010; A. Arbey, M. Battaglia, A. Djouadi, F. Mahmoudi and J. Quevillon, Phys. Lett. B 708 (2012) 162; L. J. Hall, D. Pinner and J. T. Ruderman, JHEP 1204 (2012) 131.

[12] The ATLAS collaboration [ATLAS Collaboration], ATLAS-CONF-2017-022.

[13] A. M. Sirunyan et al. [CMS Collaboration], Phys. Rev. D 97, no. 1, 012007 (2018) doi:10.1103/PhysRevD.97.012007; A. M. Sirunyan et al. [CMS Collaboration], Eur. Phys. J. C 77, no. 10, 710 (2017) doi:10.1140/epjc/s10052-017-5267-x.

[14] The ATLAS collaboration [ATLAS Collaboration], ATLAS-CONF-2017-037.

[15] A. M. Sirunyan et al. [CMS Collaboration], arXiv:1706.04402 [hep-ex].

[16] J. Ellis, K. Enqvist, D. Nanopoulos and F. Zwirner, Mod. Phys. Lett. A 1 (1986) 57; R. Barbieri and G. Giudice, Nucl. Phys. B 306 (1988) 63.
[17] G. W. Anderson and D. J. Castano, Phys. Rev. D 52 (1995) 1693.
[18] S. Dimopoulos and G. F. Giudice, Phys. Lett. B 357 (1995) 573.
[19] R. Kitano and Y. Nomura, Phys. Rev. D 73 (2006) 095004; M. Papucci, J. T. Ruderman and A. Weiler, JHEP 1209 (2012) 035; C. Brust, A. Katz, S. Lawrence and R. Sundrum, JHEP 1203 (2012) 103.
[20] H. Baer, V. Barger and D. Mickelson, Phys. Rev. D 88 (2013) no.9, 095013.
[21] A. Mustafayev and X. Tata, Indian J. Phys. 88 (2014) 991.
[22] H. Baer, V. Barger, D. Mickelson and M. Padeffke-Kirkland, Phys. Rev. D 89 (2014) 115019.
[23] H. Baer, V. Barger, P. Huang, A. Mustafayev and X. Tata, Phys. Rev. Lett. 109 (2012) 161802.
[24] H. Baer, V. Barger, P. Huang, D. Mickelson, A. Mustafayev and X. Tata, Phys. Rev. D 87 (2013) 11, 115028.
[25] K. L. Chan, U. Chattopadhyay and P. Nath, Phys. Rev. D 58 (1998) 096004;
[26] H. Baer, V. Barger and P. Huang, JHEP 1111 (2011) 031.
[27] H. Baer, V. Barger and M. Savoy, Phys. Rev. D 93 (2016) no.3, 035016.
[28] D. Matalliotakis and H. P. Nilles, Nucl. Phys. B 435 (1995) 115; M. Olechowski and S. Pokorski, Phys. Lett. B 344 (1995) 201; P. Nath and R. L. Arnowitt, Phys. Rev. D 56 (1997) 2820; J. Ellis, K. Olive and Y. Santoso, Phys. Lett. B539 (2002) 107; J. Ellis, T. Falk, K. Olive and Y. Santoso, Nucl. Phys. B652 (2003) 259; H. Baer, A. Mustafayev, S. Profumo, A. Belyaev and X. Tata, JHEP0507 (2005) 065.
[29] O. Lebedev, H. P. Nilles, S. Raby, S. Ramos-Sanchez, M. Ratz, P. K. S. Vaudrevange and A. Wingerter, Phys. Lett. B 645 (2007) 88; M. Badziak, S. Krippendorf, H. P. Nilles and M. W. Winkler, JHEP 1303 (2013) 094.
[30] H. Baer, V. Barger, M. Savoy, H. Serce and X. Tata, JHEP 1706 (2017) 101.
[31] L. Randall and R. Sundrum, Nucl. Phys. B 557 (1999) 79; H. Baer, V. Barger and D. Sengupta, Phys. Rev. D 98 (2018) no.1, 015039.
[32] K. Choi, A. Falkowski, H. P. Nilles, M. Olechowski and S. Pokorski, J. High Energy Phys0411, 076 (2004); K. Choi, A. Falkowski, H. P. Nilles and M. Olechowski, Nucl. Phys. B718, 113 (2005). J. P. Conlon, F. Quevedo and K. Suruliz, JHEP 0508, 007 (2005); K. Choi, K-S. Jeong and K. Okumura, J. High Energy Phys. 0509, 039 (2005).
[33] H. Baer, V. Barger, H. Serce and X. Tata, Phys. Rev. D 94 (2016) no.11, 115017.
[34] ISAJET 7.88, by H. Baer, F. Paige, S. Protopopescu and X. Tata, hep-ph/0312045; Isasugra, by H. Baer, C. H. Chen, R. B. Munroe, F. E. Paige and X. Tata, Phys. Rev. D 51 (1995) 1046.
[35] V. D. Barger, R. W. Robinett, W. Y. Keung and R. J. N. Phillips, Phys. Lett. 131B (1983) 372; H. Baer, X. Tata and J. Woodside, Phys. Rev. D 42 (1990) 1568; H. Baer, C. h. Chen, M. Drees, F. Paige and X. Tata, Phys. Rev. D 58 (1998) 075008.
[36] H. Baer, V. Barger, N. Nagata and M. Savoy, Phys. Rev. D 95 (2017) no.5, 055012.

[37] H. Baer, A. Mustafayev and X. Tata, Phys. Rev. D 89 (2014) no.5, 055007. See also, C. Han, A. Kobakhidze, N. Liu, A. Saavedra, L. Wu and J. M. Yang, JHEP 1402 (2014) 049; P. Schwaller and J. Zurita, JHEP 1403, 060 (2014).

[38] Z. Han, G. D. Kribs, A. Martin and A. Menon, Phys. Rev. D 89 (2014) no.7, 075007; H. Baer, A. Mustafayev and X. Tata, Phys. Rev. D 90 (2014) no.11, 115007; C. Han, D. Kim, S. Munir and M. Park, JHEP 1504 (2015) 132.

[39] CMS Collaboration [CMS Collaboration], CMS-PAS-SUS-16-048.

[40] M. Aaboud et al. [ATLAS Collaboration], Phys. Rev. D 97 (2018) no.5, 052010.

[41] H. Baer, V. Barger, P. Huang, D. Mickelson, A. Mustafayev, W. Sreethawong and X. Tata, Phys. Rev. Lett. 110 (2013) no.15, 151801; H. Baer, V. Barger, P. Huang, D. Mickelson, A. Mustafayev, W. Sreethawong and X. Tata, JHEP 1312 (2013) 013, Erratum: [JHEP 1506 (2015) 053]; H. Baer, V. Barger, J. S. Gainer, M. Savoy, D. Sengupta and X. Tata, Phys. Rev. D 97 (2018) no.3, 035012.

[42] H. Baer, V. Barger, J. S. Gainer, H. Serce and X. Tata, Phys. Rev. D 96 (2017) no.11, 115008.

[43] H. Baer, V. Barger, J. S. Gainer, P. Huang, M. Savoy, H. Serce and X. Tata, Phys. Lett. B 774 (2017) 451.

[44] C. Borschensky, M. Krmer, A. Kulesza, M. Mangano, S. Padhi, T. Plehn and X. Portell, Eur. Phys. J. C 74 (2014) no.12, 3174.

[45] J. Alwall, M. Herquet, F. Maltoni, O. Mattelaer and T. Stelzer, JHEP 1106 (2011) 128; J. Alwall, R. Frederix, S. Frixione, V Herschi, F. Maltoni, O. Mattelaer, H.-S. Shao, T. Stelzer, P. Torrielli and M. Zaro, JHEP 1407 (2014) 079.

[46] T. Sjostrand, S. Mrenna and P. Z. Skands, Comput. Phys. Commun. 178 (2008) 852.

[47] J. de Favereau et al. [DELPHES 3 Collaboration], JHEP 1402 (2014) 057.

[48] M. R. Douglas, hep-th/0405279; H. Baer, V. Barger, H. Serce and K. Sinha, JHEP 1803 (2018) 002.

[49] H. Baer, V. Barger, J. S. Gainer, P. Huang, M. Savoy, D. Sengupta and X. Tata, Eur. Phys. J. C 77 (2017) no.7, 499.

[50] See, e.g. ATLAS Phys. PUB 2013-011; CMS Note-13-002.

[51] H. Baer, A. Lessa, S. Rajagopalan and W. Sreethawong, JCAP 1106 (2011) 031; K. J. Bae, H. Baer and E. J. Chun, Phys. Rev. D 89 (2014) no.3, 031701; H. Baer, V. Barger and H. Serce, Phys. Rev. D 94 (2016) no.11, 115019; K. J. Bae, H. Baer and H. Serce, JCAP 1706 (2017) no.06, 024; H. Baer, V. Barger, D. Sengupta and X. Tata, arXiv:1803.11210 [hep-ph].

[52] T. Han, S. Mukhopadhyay and X. Wang, arXiv:1805.00015 [hep-ph].

[53] Talk by L. Shchutska, Moriond EW 2017. See also Ref. 39.