A Three-Dimensional Strut-and-Tie Model for a Four-Pile Reinforced Concrete Cap

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Abstract

Reinforced concrete pile caps have been traditionally designed with sectional methods in codes and specifications. Since the 1990s, the two-dimensional strut-and-tie model (2D STM) methods, introduced in codes and specifications (Ref. ACI 318-14 and the 8th Edition of the AASHTO LRFD Specifications), have become the design method of choice for pile cap designs. However, sectional methods and 2D STM provisions in current design codes have serious limitations when applied to the design of members with behavior more appropriately captured using three-dimensional (3D) approaches. In this paper, a refined 3D STM method for analysis and design of pile caps is illustrated. In the proposed method, a statically indeterminate 3D STM with diagonal ties is employed to account for the load-carrying capacity in tension of some regions in four-pile caps. The effective strengths of 3D concrete struts and nodal zones are determined by reflecting the effects of the 3D stress states and the degree of concrete confinement provided by reinforcement. The load-carrying capacities of struts and ties are determined using an iterative technique with the axial stresses of concrete struts equal to effective strengths. An extensive comparison between current and proposed methods is conducted using experimental data from 115 reinforced concrete pile caps tested to failure. Besides, several pile caps are designed using the current and proposed methods, and a comparison of the results from the design is conducted.

1. Introduction

Reinforced concrete pile caps in bridges transfer the loads from piers to pile foundation. In design practice, flexural cracks in pile caps are limited to avoid excessive deformations, and shear design is conducted on the basis of strength. Sectional methods for flexure, one-way and two-way shear, are the usual approaches for pile cap designs. In modern codes and standards, it is accepted that these approaches while appropriate for B-regions (regions of the structure where assumptions of plane sections are adequate to describe behavior), they are not suitable for design situations such as in pile cap design where the plane sections assumption does not describe the behavior of the so-called disturbed regions (D-Regions). This is the case of pile caps where the effective depth is greater than the distance measured from the face of the column to the center of the nearest pile.

In D-regions design is carried out using STM methods of current codes and specifications (CSA 2004; fib 2010; ACI 318(ACI 2014); AASHTO 2018) and standards (ACI 445 (ACI 2002) and PCA (Mitchell et al. 2004)). Several studies have demonstrated that the STM approach not only improves the understanding of the behavior but results in more efficient designs. Test results by Clarke (1973) and Suzuki et al. (1998), showed that, when the same amount of reinforcing bar was used, the ultimate strength of the pile caps with bundled flexural reinforcement details was on average 10% higher compared to the grid-type flexural reinforcement. The bundled reinforcement details obtained from the STM method is more efficient when compared to the grid-type flexural reinforcement obtained from a sectional flexural design method. In a study by Adebar and Zhou (1996), the ultimate strengths of forty-eight pile caps (tested to failure) obtained from STM method were compared to that from sectional methods in current design codes. This comparison showed that the calculated ultimate strength of the pile caps obtained using the STM method was less conservative but still safe estimate than obtained using sectional methods.

Using the STM method, it is possible to design the four-pile cap shown in Fig. 1 (ACI 2002, 2010; IABSE 2003; Mitchell et al. 2004; Park et al. 2008). However, as the load transfer mechanism in the four-pile cap is simplified using the model of Fig. 1, it is possible to miss some details of the failure mechanism, and loss accuracy in the ultimate strength predictions of the overall model and in the STM elements that govern ultimate strength and behavior. One more complicating factor is that STM methods in current design codes have been implemented with a focus on 2D structural concrete and extension to members such as the four-pile cap in Fig. 1, whose ultimate strength and behavior is predominantly 3D, is not straightforward.

In this paper, the current STM method in codes and

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specifications is extended to 3D applications with the design of four-pile caps with D-regions. The design application considers the load-carrying capacity of diagonal ties in tension regions and the strength properties of 3D concrete struts and nodal zones. To evaluate and compare the proposed method with current sectional design methods, several design conditions in four-pile caps are considered. In addition, the estimated capacity from the proposed method is compared with the ultimate strengths of 115 reinforced concrete four-pile caps tested to failure.

2. Review of design approaches in four-pile caps

In this section, the existing design methods (CRSI 2008; ACI 2014; AASHTO 2018) and the refined 3D STM method proposed in the present study are reviewed.

2.1 ACI 318 sectional method

The sectional method used in the design of four-pile caps covers the flexural beam and slab design provisions (Section 22.3), the bearing strength provisions (Section 22.8), and the shear design provisions of one-way and two-way slabs (Sections 22.5 and 22.6). For the shear design of one-way slabs, the nominal shear strength provided by the concrete at the critical section, \( cV_w \), is determined using either one of the following equations:

\[
0.17 \lambda \sqrt{f'_c} b_d d
\]

\[
0.16 \lambda \sqrt{f'_c + 17 \rho_v V_u d} b_w d
\]

where the section at the face of the column in a four-pile cap is defined as the critical section, and \( f'_c \), \( b_d \), \( \rho_v \), \( d \), \( V_u \), and \( M_u \) represent the compressive strength of concrete, the web width, the ratio of the area of the flexural reinforcement \( A_r \) to \( b_d d \), a modification factor reflecting the reduced mechanical properties of lightweight concrete, the distance from extreme compression fiber to the center of longitudinal reinforcement, the factored shear force at the critical section, and the factored moment at the critical section, respectively. In Eq. (1), the nominal shear strength should not be greater than \( 0.29 cV_w b_d \) and \( V_d/M_u \) should be less than unity. In the two-way shear design of a four-pile cap, the section along the circumference that is separated from the face of the column a distance of \( d/2 \) is regarded as the critical section. The smallest calculated value based on the following equations is taken as the nominal value of \( V_c \) at the critical section:

\[
V_c = \left( 0.17 + \frac{0.33}{\beta_c} \right) \lambda \sqrt{f'_c} b_d d
\]

\[
V_c = \left( 0.17 + 0.083 \frac{\alpha_d}{d_w} \right) \lambda \sqrt{f'_c} b_d d
\]

\[
V_c = 0.33 \lambda \sqrt{f'_c} b_d d
\]

where \( b_w \) is the perimeter of the critical section, \( \beta_c \) is the ratio of the long to the short dimensions of the pile cap, and \( \alpha_d \) is a constant (equal to 40 for interior columns, 30 for exterior columns, and 20 for corner columns).

2.2 CRSI Sectional design method

The CRSI sectional method for a four-pile cap is quite similar to the ACI 318 sectional method. In the one-way shear design of a four-pile cap, the section at the face of the column is considered as the critical section, and the value of \( V_c \) at the critical section is determined from the following equations:

\[
V_c = V_{c,act} \geq 0.17 \sqrt{f'_c} b_d d \quad \text{for } w/d \geq 1.0
\]

\[
V_c = \frac{d}{w} V_{c,act} \leq 0.83 \sqrt{f'_c} b_d d \quad \text{for } w/d < 1.0
\]

where \( w \) is the distance from the face of the column to the center of the nearest pile, \( b \) is the width of the compression face of the member. Moreover, \( V_{c,act} \) represents the same quantity as \( V_c \) in Eq. (1b) for cases where \( w/d \geq 1.0 \), and the nominal shear stress value of \( V_c \) stated in ACI (1999) for cases where \( w/d < 1.0 \), in accordance to the following equation:

\[
V_c = \left( 3.5 - 2.5 \frac{M_u}{V_u d} \right) 0.16 \sqrt{f'_c + 17 \rho_v V_u d} b_d d
\]

where the values of \( V_c \) and \( 3.5 - 2.5 \frac{M_u}{V_u d} \) must be less than the values of \( 0.5 \sqrt{f'_c} b_d d \) and 2.5, respectively.
When the center of the nearest pile is located within a distance of \( \frac{d}{2} \) from the face of the column, the pile cap is regarded as a two-way slab with the critical section along the circumference of the column. The nominal value of \( c_{V} \) at the critical section is determined by the value of \( c_{V} \) expressed by Eq. (2) for the cases where \( \frac{d}{0.5} \geq w_{d} \) and by Eq. (5) for the cases where \( \frac{d}{0.5} < w_{d} \):

\[
10.172.672 \times 
\]

where \( o_{b} \) is the perimeter of the critical section and \( s_{b} = 4c \) for a square column of width \( c \).

### 2.3 AASHTO and ACI 318 STM methods

The STM design provisions provide the basis in AASHTO (2018) and ACI 318 (2014) for the design of structural concrete D-regions. A pile cap has D-regions due to the concentrated forces from columns and piles. In this paper, the design of the four-pile cap with these specifications was conducted using the statically determinate 3D STM shown in Fig. 1 (Adebar et al. 1990). The effective strengths of concrete struts and nodal zones in the AASHTO and ACI are listed in Tables 1 and 2. In the design, the strength conditions at every steel tie, concrete strut, and nodal zone of the statically determinate 3D STM should be checked.

### 2.4 Refined 3D STM method of present study

Four distinct features, beyond those found in the STM methods of current design codes, are incorporated in the refined 3D STM method of the present study. First, a

![Figure 2: Statically indeterminate 3D STM for four-pile caps.](image-url)
ever, the effective strengths in the current design codes must be used with caution when used outside the strut-and-tie models of 2D structural concrete under similar loading and geometrical conditions as those covered in the experimental and analytical studies.

Since relatively large areas of 2D structural concrete are covered by the concrete struts in simple types of strut-and-tie models, the state of stress in the regions exhibiting severe stress variations along the longitudinal axes of the struts are not properly represented in the current effective concrete strut strengths. Moreover, is questionable whether the current effective strengths of concrete struts can be used in the STM analyses and design of 3D structural concrete as the effects of 3D state of stress and reinforcement details may not have been properly represented in their evaluation. In the proposed method, the effective strengths of 3D concrete struts are determined with special attention to the effects of the 3D states of stress, the strut’s longitudinal length, the deviation angle between the strut orientation and the compressive principal stress flow, and the concrete confinement provided by reinforcing bars. The effective strengths of 3D concrete struts are determined by the procedure and algorithm shown in Figs. 3 and 4. An

**Fig. 3 Procedure for determining effective strength of 3D concrete strut.**

| Step 1 | Select all finite elements located closely at the centerline of each 3-dimensional concrete strut. |
|--------|--------------------------------------------------------------------------------------------------|
| A.     | ![Diagram of strut-and-tie model](RC_Pile_Case)                                                   |
| B.     | Selection of finite elements composing a strut                                                  |

| Step 2 | Determine the principal stresses, principal directions, and failure principal stresses at the failure plane of 3-dimensional concrete. |
|--------|-------------------------------------------------------------------------------------------------------------------------------|
| C.     | ![Diagram of principal stresses](principal_stresses)                                                                             |
| D.     | ![Diagram of principal directions](principal_directions)                                                                         |
| E.     | ![Diagram of failure envelops](failure_envelopes)                                                                               |

| Step 3 | Determine the failure principal stress, $f'_{c,e}$ (eff. strength of finite element), acting parallel to the longitudinal axis of concrete strut. |
|--------|-------------------------------------------------------------------------------------------------------------------------------------|
| F.     | ![Diagram of failure principal stress](failure_principal_stress)                                                                   |
| G.     | ![Diagram of max. comp. failure principal stress](max_comp_failure_principal_stress)                                             |

| Step 4 | Determine the effective strength $f'_{c,e}$ of concrete strut. |
|--------|-----------------------------------------------------------------|
|        | $f_{c,e} = v \times \text{Average of } \left\{ f'_{c,e,i} \right\}$ |
|        | $v = 1.00$ for $f'_{c,e} \leq 28\text{MPa}$                     |
|        | $v = 1.1 - 0.25 f'_{c,e}/70$ for $28 < f'_{c,e} < 70\text{MPa}$     |
|        | $v = 0.85$ for $f'_{c,e} \geq 70\text{MPa}$                      |
example illustrating the proposed method is shown in Section 4.2 of this paper.

Third, since the nodal zone effective strengths in present codes and specifications have been validated with 2D models, effective strengths of nodal zones for 3D strut-and-tie models were developed for the proposed method. Effective stresses for the 3D nodal zones shaped by concrete struts and bearing plates are developed employing concepts introduced in the 3D grid STM method of Yun et al. (2018). The effective strength of the nodal zone confined by compressive members is determined taking into account the effects of 3D state of stress and the degree of confinement. The detailed procedures are illustrated in Fig. 5. In addition, the following strength equations are used for 2D and/or 3D nodal zones, suggested by Bergmeister et al. (1993) and Adebar and Zhou (1996), respectively:

\[ f_{ce} = v_c (A_2 / A_1)^{0.5} f'_c \]  \hspace{1cm} (6)

\[ f_{ce} = 0.85 \beta_n f'_c \]  \hspace{1cm} (7)

where,

\[ \beta_n = 0.60 + 6 \alpha \beta \sqrt{f'_c / f'_c} \]

\[ \alpha = \frac{1}{3} \left( \sqrt{A_2 / A_1} - 1 \right) \leq 1.0 \]  \hspace{1cm} (8)

\[ \beta = \frac{1}{3} \left( h_b / b_b - 1 \right) \leq 1.0 \]

In Eq. (6), \( v_c \) has a value of 0.8 for the case where \( f'_c \leq 28 \) MPa, a value of 0.9 \( - 0.25 f'_c / 70 \) for the case where \( 28 < f'_c < 70 \) MPa, and a value of 0.65 for the case where \( f'_c \geq 70 \) MPa. In the same equation, \( A_1 \) represents the loaded area, and \( A_2 \) represents the area of the lower base of the largest frustum contained wholly within the support. The lower base area \( (A_2) \) has the loaded area for its upper base, and side slopes of one vertical to two horizontal. The parameter \( \alpha \) accounts for the confinement of a concrete strut framing into a nodal zone. The parameter \( \beta \) accounts for the geometry of the concrete strut, where \( h_b / b_b \) is the aspect ratio (height-to-width) of the strut. To calculate the maximum bearing stress for the nodal zone below a column in a pile cap (where two or more compression struts meet), and for the nodal zone above a pile in a pile cap (where only one compression strut is anchored), the aspect ratio of the concrete strut can be approximated to have values of \( 2d / c \) and \( d / d_p \), respectively, where \( d_p \) is the diameter of round pile.

Fourth, since the cross-sectional forces of struts and ties in the statically indeterminate 3D STM are dependant on the axial stiffness of struts and ties, they cannot be determined using equilibrium equations at the nodes. Thus, the cross-sectional forces of struts and ties under design loads are determined using a simple iterative technique that requires the axial stresses of concrete struts to be equal to their effective strengths. The required cross-sectional areas are then determined by dividing the cross-sectional forces of the struts and ties by their effective strengths. The load carrying capacities of the struts and ties are examined by comparing maximum available cross-sectional areas with the required value.

![Fig. 4 Algorithm for considering confinement effect owing to reinforcing bars in determining effective strengths of concrete struts and nodal zones.](image-url)
3. Comparison of design methods

The ultimate strengths of four-pile caps under various design conditions were evaluated to carry out a comparison of the design methods described in Section 2. The range of the distance (between column face and the centerline of piles) to the effective depth ratio (w/d) of the four-pile caps is 0.3-1.0, and the flexural reinforcement ratio is fixed to 0.25%. The geometrical shape, material properties, and reinforcement details of the four-pile caps are shown in Fig. 6.

The results of the evaluations with the procedures...
discussed in Section 2 are summarized in Table 3. In the case of the sectional methods, as shown in Tables 3(a) and 3(b), the minimum value among $P_{a,T1}$, $P_{a,T2}$, $P_{a,S1}$, and $P_{a,P}$ (determined respectively from the flexural strength $M_{a,T1}$, the bearing strength $P_{a,S}$ at the column, the bearing strength $P_{a,P}$ at the pile, the shear strength of the one-way slab $V_{a,1}$, and the shear strength of the two-way shear $V_{a,2}$) was taken as the ultimate strength of the pile cap $P_u$.

In the case of the STM methods using the statically determinate 3D STM, shown in Fig. 1, the minimum value among the values of the strength of concrete strut,

Table 3 Results of the example model of four-pile cap.

(a) ACI 318 Sectional Method

| $w/d$ | $w$ (mm) | $d$ (mm) | $A_s$ (mm$^2$) | Flexure $P_{a,s}$ (kN) | Bearing $P_{a,b}$ (kN) | One-way Shear $P_{a,1}$ (kN) | Two-way Shear $P_{a,2}$ (kN) | Ultimate Load $P_u$ (kN) |
|-------|----------|----------|-----------------|----------------------|---------------------|-----------------|-----------------|------------------|
| 0.3   | 300      | 1000     | 4250            | 3672.0               | 8011.1              | 8109.5          | 8400.6          | 2809.5          |
| 0.5   | 300      | 600      | 2550            | 3672.0               | 8011.1              | 1685.7          | 3492.0          | 1685.7          |
| 0.7   | 300      | 429      | 1820            | 3672.0               | 8011.1              | 1205.3          | 2022.4          | 1205.3          |
| 1.0   | 300      | 300      | 1280            | 3672.0               | 8011.1              | 842.9           | 1164.0          | 842.9           |

(b) CRSI Sectional Method

| $w/d$ | $w$ (mm) | $d$ (mm) | $A_s$ (mm$^2$) | Flexure $P_{a,s}$ (kN) | Bearing $P_{a,b}$ (kN) | One-way Shear $P_{a,1}$ (kN) | Two-way Shear $P_{a,2}$ (kN) | Ultimate Load $P_u$ (kN) |
|-------|----------|----------|-----------------|----------------------|---------------------|-----------------|-----------------|------------------|
| 0.3   | 300      | 1000     | 4250            | 3672.0               | 8011.1              | 13825           | 14436           | 3672.0          |
| 0.5   | 300      | 600      | 2550            | 3672.0               | 8011.1              | 2022.4          | 2022.4          | 3492.0          |
| 0.7   | 300      | 429      | 1820            | 3672.0               | 8011.1              | 1685.7          | 3492.0          | 1685.7          |
| 1.0   | 300      | 300      | 1280            | 3672.0               | 8011.1              | 842.9           | 1164.0          | 842.9           |

(c) ACI 318 STM Method ($w/d = 0.7$, $z = 1.0d$)

| Element | Ele. Type | $f'_s$ (MPa) | $f_w$ (MPa) | $A_{prov}$ (mm$^2$) | $F_{u,str}$ (kN) | $P_{u,str}$ (kN) | $P_u$ (kN) |
|---------|-----------|--------------|-------------|---------------------|-----------------|-----------------|-----------|
| S1      | Strut     | 0.51         | 24.00       | 12.24               | 14151           | 173.2           | 435.7     |
| T1      | Tie       | 1.00         | 400.0       | 910.0               | 1665.7          | 435.7           | 435.7     |
| 1       | CCC       | 0.85         | 24.00       | 20.40               | 90000           | 1836.0          | 1836.0    |
| 2       | CTT       | 0.51         | 24.00       | 12.24               | 90000           | 1836.0          | 1836.0    |

(d) AASHTO STM Method ($w/d = 0.7$, $z = 1.0d$)

| Element | Ele. Type | $f'_s$ (MPa) | $f_w$ (MPa) | $A_{prov}$ (mm$^2$) | $F_{u,str}$ (kN) | $P_{u,str}$ (kN) | $P_u$ (kN) |
|---------|-----------|--------------|-------------|---------------------|-----------------|-----------------|-----------|
| S1      | Strut     | 0.57         | 24.00       | 13.72               | 14151           | 194.1           | 488.4     |
| T1      | Tie       | 1.00         | 400.0       | 910.0               | 1665.7          | 488.4           | 488.4     |
| 1       | CCC       | 0.85         | 24.00       | 20.40               | 90000           | 1836.0          | 1836.0    |
| 2       | CTT       | 0.65         | 24.00       | 15.60               | 90000           | 1836.0          | 1836.0    |
\( P_{\text{utrust}}, \) the strength of steel tie, \( P_{\text{utrust}} , \) and the strength of the nodal zone, \( P_{\text{node}} , \) was taken as the ultimate strength, as shown in Tables 3(c) and 3(d). The maximum available cross-sectional areas of concrete struts and nodal zones in Tables 3(c) and 3(d) were determined based on the ACI 445 (ACI 2002) and PCA (Mitchell et al. 2004) methods, respectively. The difference between the ACI 445 (ACI 2002) and PCA (Mitchell et al. 2004) methods is illustrated in Fig. 7. The backgrounds of the strength equations of the strut, tie, and nodal zones in Table 3(c) are illustrated in Fig. 8.

In the case of the proposed method, the ultimate strength of the four-pile cap was determined in a similar manner by evaluating the strengths \( P_{\text{utrust}} , P_{\text{utie}} , \) and \( P_{\text{unode}} , \) of the statically indeterminate 3D STM shown in Fig. 2. The detailed procedure is indicated in Table 3(e), where the strength \( P_{\text{utie}} \) was evaluated by summing the strengths of the horizontal steel tie \( T_1 \) and the diagonal steel tie \( T_2 \). As the STM is an indeterminate truss structure, the sequential analysis procedure shown in Table 6 is required to evaluate \( P_{\text{utie}} \). However, the strengths of ties \( T_1 \) and \( T_2 \) are directly added for simplicity of strength evaluation. The maximum available cross-sectional areas of concrete tie at T2 location (determined by the ACI 445 (ACI 2002) method) is converted to the steel tie area \( A_{\text{utie}} \).

Refer to Fig. 2 for element numbers; \( u_{\text{strut}}, u_{\text{tie}}, u_{\text{node}} \); refer to Fig. 8; \( \phi_{\text{utrust}}, \phi_{\text{utie}}, \phi_{\text{unode}} \).

For simplicity and comparison of design methods, the maximum available cross-sectional area of concrete tie at T2 location was determined based on the ACI 445 (ACI 2002) method and PCA (Mitchell et al. 2004) methods. The effective strengths of the divided struts S1-1, S1-2, and S1-3 were determined by the procedure and algorithm shown in Figs. 3 and 4.

With the CRSI sectional method, the ultimate strength is governed by the column bearing strength within the value range of \( w/d < 0.48 \) (Fig. 9(a)). As \( w/d \) increases up to 0.70, the ultimate strength was governed by the provision of two-way shear. When \( w/d \) is greater than 0.70, the ultimate strength was governed by flexure. The strengths of flexure, one-way shear, and two-way shear, increase rapidly as \( w/d \) decreases.
gradually. This indicates that a pile cap with greatly increased ultimate strength can be obtained by increasing the pile cap’s effective depth if bearing failure at the column and pile do not control the design. In the case of ACI 318 sectional method, the ultimate strength was governed only by one-way shear in the entire range of \( w/d \), as shown in Fig. 9(b). From the comparison of the sectional methods, the ACI 318 method was shown to yield conservative estimates of strength as \( w/d \) decreases.

The ACI 318 STM method is applied next. In this approach, the ultimate strength was shown to be controlled by the strength of the inclined struts in the entire range of \( w/d \) ratio values (Fig. 9(c)). The ultimate strength in the case of the AASHTO STM method was governed by the strength condition of the Compression-Tension-Tension (CTT) nodal zones within the value range of \( w/d < 0.45 \), as shown in Fig. 9(d). When \( w/d \) is greater than 0.45, the ultimate strength was controlled by the strength condition of the inclined struts. The ultimate strengths of the inclined concrete struts and the CTT nodal zones of the STM methods were found to yield more conservative estimates than those from two-way shear and bearing provisions of the CRSI sectional method. The difference is reduced if greater effective strengths of concrete struts and nodal zones are used. Note that the strengths obtained by the strength conditions of steel ties \( T_1 \) and \( T_2 \), the concrete strut \( S \), and the Compression-Compression-Compression (CCC) nodal zone, \( N \), shown in Fig. 1, were classified as the strengths obtained by the provisions of flexure, two-way shear, and bearing of the sectional methods, respectively, in an effort to compare the ultimate strengths obtained by the STM methods with those obtained by the sectional methods. In addition, the strength obtained by the strength condition of the CTT nodal zone, \( N \), was classified as the strength obtained by the provisions of the two-way shear at the corner rather than on the bearing.

The application of the refined 3D STM method of the present study showed the strength of the inclined concrete struts controlled the design with \( w/d < 0.57 \) (Fig. 9(e)). When \( w/d \) is greater than 0.57, the ultimate strength can be controlled by the strength of the steel ties. The ultimate strength in the proposed method is greater than the ultimate strength of ACI 318 and AASHTO STM methods. The use of greater effective strengths of concrete struts and nodal zones and the

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**Fig. 8 Ultimate strength of pile cap determined by strength conditions of STM’s elements.**
consideration of load-carrying capacities of diagonal ties in tension regions of the four-pile caps are the reasons behind these results. The ultimate strengths of the four-pile caps, determined from Figs. 9(a) - (e), by extracting the minimum loads for each w/d, are plotted in Fig. 9(f).

4. Strength predictions

The ultimate strengths of 115 four-pile cap specimens tested to failure by Clarke (1973), Sabins and Gogate (1984), and Suzuki et al. (1998, 1999, 2000), were evaluated using the sectional methods and the STM methods introduced in Section 2. The characteristics of the materials and geometries of the test specimens are summarized in Table 4.

4.1 Calculated strengths of current approaches

In this section the ultimate strengths of the 115 specimens were determined using the minimum calculated value of those from flexure, one-way shear, two-way shear, bearing strength at the column, and bearing strength at the pile for a given w/d. The same procedures as shown in Tables 3(a) and (b) were used to determine the ultimate strengths of the four-pile caps based on the CRSI and ACI 318 sectional methods. The ultimate strengths were also evaluated using the ACI 318 and AASHTO STM methods. The same proce-
Table 4 Specifications of four-pile caps tested to failure.

| Investigators             | No. of Pile Caps | $w/d$  | $d$ (mm) | Column Size (mm) | Pile Size (mm) | $f'_c$ (MPa) | $f_y$ (MPa) | $\rho$ (%) | Re-bar Layout |
|---------------------------|------------------|--------|----------|------------------|----------------|--------------|-------------|------------|---------------|
| Clarke (1973)             | 13               | 0.25-0.49 | 405      | 200, S           | 200, R         | 18.0-35.0    | 410        | 0.188-0.238 | B, G          |
| Sabnis & Gogate (1984)    | 8                | 0.54-0.58 | 109-117  | 76, R            | 76, R          | 17.9-41.0    | 414-886    | 0.136-1.287 | G             |
| Suzuki et al. (1998)      | 28               | 0.40-0.80 | 150-250  | 250-300, S       | 150, R         | 18.9-31.5    | 345-413    | 0.228-0.564 | B, G          |
| Suzuki et al. (1999)      | 18               | 0.33-0.58 | 250-300  | 250, S           | 150, R         | 25.6-30.9    | 356-383    | 0.106-0.317 | G             |
| Suzuki et al. (2000)      | 30               | 0.29-0.67 | 150-350  | 200-300, S       | 150, R         | 24.5-29.4    | 358-383    | 0.181-0.272 | G             |
| Suzuki et al. (2002)      | 18               | 0.33-0.50 | 300      | 200-300, S       | 150, R         | 20.2-37.9    | 353        | 0.267       | G             |
| Total                     | 115              | 0.29-0.80 | 109-405  | 76-300, 76-200, R| 17.9-41.0      | 345-886      | 0.106-1.287| B, G        |

$w$: distance from the face of column to the center of the nearest pile; R, S, B, G: round, square, bunched, grid

dure as shown in Tables 3(c) - 3(d) and the strength conditions of concrete struts, steel ties, and nodal zones, were used to determine the ultimate strengths. In the application of the STM methods, the statically determinate 3D STM, shown in Fig. 1 (with the variable vertical distance between the lower horizontal tie and the upper horizontal strut), and the effective strengths of concrete struts and nodal zones, shown in Tables 1 and 2, were employed. The ultimate strengths predicted by the sectional and STM methods of the current design codes are summarized in Table 7.

4.2 Calculated strength of proposed method

The statically indeterminate 3D STM shown in Fig. 2 was employed to calculate the capacities of the 115 specimens. The effective strength of concrete struts, the effective strength of nodal zones, and the required cross-sectional areas of struts and ties were determined using the methods in Section 2.4. The ultimate strengths of the caps based on the proposed method could be evaluated using the same procedure illustrated in Tables 3(d) - 3(e). However, since an additional column load in a four-pile cap could be transferred to its supports after the failure of one of the steels or the diagonal ties in the statically indeterminate STM, the ultimate strengths of all pile caps were determined using a sequential failure procedure that considered possible load redistribution in the four-pile caps. The detailed procedure for evaluating the ultimate strengths of the caps is illustrated with the Specimen A4 tested by Clarke (1973). The geometrical shape and reinforcement details of the four-pile cap are shown in Fig. 10.

The first step in the evaluation of the ultimate strength of cap A4 was to conduct a 3D finite element (FE) elastic analysis to obtain the principal stresses and directions. The FE model and the compressive principal stress flows are shown in Fig. 11. The model is composed of eight-node unreinforced concrete brick ele-

Fig. 10 Reinforcement details and geometry of four-pile cap A4.

Fig. 11 Finite element model and compressive principal stress flows of four-pile cap A4.
ments. The external load acting on the column was distributed to the FE nodes of the column by considering the tributary areas of the nodes. The \( x \) - and \( z \) -directional horizontal rollers were placed at the FE nodes located at the interfaces of the piles and the pile cap, and the \( y \) -directional vertical roller was placed at the FE node located at the bottom center of the pile cap. By considering the principal stress flows, the position of the flexural reinforcement, concrete clear cover, and locations of the piles, the struts and ties of the statically indeterminate 3D STM (shown in Fig. 2), were placed.

In the model, the horizontal steel tie \( T_1 \) was placed at the centroid of the flexural bars, and the nodes were located at the centers of the column and the piles. Following the procedures presented in Figs. 3 and 4, the effective strengths of concrete struts (S1-1, S1-2, S1-3) were determined. The procedures are illustrated in Table 5 for the strut S1-2. The finite elements used to determine the effective strength of the strut S1-2 are numbered in Fig. 12. The yield strength of the reinforcing bar was taken as the effective strength of the horizontal steel tie \( T_1 \) and diagonal steel tie \( T_2 \). The effective strengths of the nodal zones were determined using Eq. (6), Eq. (7), and the procedures shown in Figs. 4 and 5.

The ultimate strength of the pile cap A4 was determined by checking the load-carrying capacities of every concrete strut, steel ties, and nodal zones, were determined based on the ACI 445 (ACI 2002) and PCA (Mitchell et al. 2004) methods that consider the geometrical shape of STM and the sizes of loading and bearing plates. The cross-sectional areas of the reinforcing bars placed within the effective width of horizontal steel tie \( T_1 \) were taken as the maximum available areas of tie \( T_1 \). The cross-sectional areas of reinforcing bars outside the effective width of horizontal steel tie \( T_1 \) were converted to the maximum available area of tie \( T_2 \).

To account the tension-stiffening effect even existing after yielding of reinforcing bars (Lee et al. 2011), the cross-sectional areas of maximum available concrete

| Confinement by Re-bars | Finite Ele. No. | Step 1 | Step 2 | Step 3 | Step 4 |
|------------------------|----------------|--------|--------|--------|--------|
|                        | Principal Stress (MPa) | Failure Principal Stress (MPa) | \( f_{es} \) (MPa) | \( (1) \) (MPa) | \( (2) \) (MPa) | \( (3) \) (MPa) |
| Before                 | \( \sigma_{x} \) | \( \sigma_{y} \) | \( \sigma_{z} \) | \( \sigma_{\tau} \) | \( \sigma_{\tau} \) | \( \sigma_{\tau} \) | \( \sigma_{\tau} \) | \( \sigma_{\tau} \) | \( \sigma_{\tau} \) | \( \sigma_{\tau} \) | \( \sigma_{\tau} \) | \( \sigma_{\tau} \) | \( \sigma_{\tau} \) |
| 1                      | -6.34          | -0.24  | 1.23   | -10.21 | -0.39  | 1.98   | -17.05 | -11.34 | (2.50) |
| 2                      | -6.01          | 0.07   | 1.41   | -8.95  | 0.10   | 2.10   | -15.05 | -10.83 |
| 3                      | -4.93          | 0.06   | 1.42   | -7.79  | 0.09   | 2.24   | -13.40 | -10.58 |
| 4                      | -4.81          | 0.31   | 1.60   | -6.93  | 0.45   | 2.31   | -12.05 | -10.58 |
| 5                      | -4.36          | 0.53   | 1.75   | -5.99  | 0.73   | 2.40   | -10.58 | -10.58 |
| 6                      | -4.63          | 0.55   | 1.78   | -6.16  | 0.73   | 2.37   | -10.81 | -10.81 |
| 7                      | -4.18          | 0.53   | 1.70   | -5.89  | 0.75   | 2.39   | -10.42 | -10.42 |
| 8                      | -4.36          | 0.53   | 1.75   | -5.99  | 0.73   | 2.40   | -10.58 | -10.58 |
| 9                      | -4.16          | 0.74   | 1.88   | -5.42  | 0.97   | 2.45   | -9.61  | -9.61  |
| 10                     | -4.12          | 0.94   | 2.07   | -4.93  | 1.13   | 2.48   | -8.77  | -8.77  |
| 11                     | -4.12          | 0.91   | 1.88   | -5.36  | 1.18   | 2.45   | -9.20  | -9.20  |
| 12                     | -4.31          | 1.09   | 2.08   | -5.08  | 1.28   | 2.46   | -8.60  | -8.60  |
| After                  | -6.40          | -0.28  | 1.00   | -11.63 | -0.51  | 1.82   | -19.24 | -14.71 | (1.88) |
|                        | -6.18          | -0.06  | 1.11   | -10.66 | -0.10  | 1.91   | -17.70 | -13.76 |
|                        | -5.10          | -0.04  | 1.10   | -9.48  | -0.07  | 2.05   | -16.04 | -12.04 |
|                        | -5.12          | 0.14   | 1.23   | -8.78  | 0.25   | 2.10   | -14.94 | -14.94 |
|                        | -4.83          | 0.31   | 1.32   | -8.00  | 0.51   | 2.18   | -13.76 | -13.76 |
|                        | -5.09          | 0.31   | 1.34   | -8.17  | 0.50   | 2.15   | -14.00 | -14.00 |
|                        | -4.67          | 0.35   | 1.27   | -8.00  | 0.59   | 2.17   | -13.77 | -13.77 |
|                        | -4.83          | 0.31   | 1.32   | -8.00  | 0.51   | 2.18   | -13.76 | -13.76 |
|                        | -4.85          | 0.50   | 1.39   | -7.63  | 0.79   | 2.19   | -13.20 | -13.20 |
|                        | -5.03          | 0.65   | 1.50   | -7.40  | 0.96   | 2.21   | -12.86 | -12.86 |
|                        | -5.08          | 0.68   | 1.34   | -7.99  | 1.08   | 2.10   | -13.61 | -13.61 |
|                        | -5.56          | 0.84   | 1.43   | -8.05  | 1.21   | 2.07   | -13.75 | -13.75 |

\( f_{es} \): effective strength of finite element; (1): average of \( f_{es} \); (*): standard deviation of \( f_{es} \); (2): \( f_{es} \) within its standard deviation; (3): average of (2)
ties \( A_{\text{tie, req}} \) at the locations of steel ties T1 and T2 are also converted to the additional cross-sectional areas of steel ties \( A_{\text{tie,req}} \) using following equations:
\[
A_{\text{tie,req}} = A_{\text{tie,req}} f_{TS} / f_y
\]  
(9)
where, \( f_y \) is the yield strength of the reinforcing bars in the tie. \( f_{TS} \) is the concrete tensile stress due to the tension stiffening effect, and it can be evaluated by the following equation suggested by Lee et al. (2011):
\[
f_{TS} = (-0.0313 \rho_s^{0.57} d_t + 3.3881 \rho_s^{0.76}) \sqrt{f_y}
\]  
(10)
where, \( \rho_s \) and \( d_t \) are the reinforcement ratio and the rebar diameter in the tie, respectively. Consequently, the cross-sectional area of reinforcing bars in the tie is modified to include the tension-stiffening effect as following:
\[
A_{\text{tie}} = A_{\text{tie}} + A_{\text{tie,req}}
\]  
(11)

The required cross-sectional areas and the axial stiffness of the struts and ties were determined by using the simple iterative technique that requires the axial stresses of concrete struts to be equal to their effective strengths. The required cross-sectional areas of a nodal zone were determined by dividing the cross-sectional forces in the struts and ties framing into the nodal zone by the effective strength of the nodal zone.

### Table 6 Procedure for predicting ultimate strength of pile cap A4 based on proposed method.

| Element No. | \( \beta_c \) | \( f_{ci} \) (MPa) | \( f_{ci} \) (MPa) | \( F_u \) (kN) | \( A_{eq} \) (mm\(^2\)) | \( A_{prov} \) (mm\(^2\)) | \( A_{prov} / A_{eq} \) | Safe/Fail |
|-------------|---------------|-----------------|-----------------|---------------|----------------|----------------|----------------|---------|
| S1-1 | 2.08 | 21.4 | 44.5 | 417.6 | 9382 | 11239 | 1.198 | O |
| S1-2 | 0.65 | 21.4 | 13.9 | 417.6 | 30022 | 132028 | 4.398 | O |
| S1-3 | 0.80 | 21.4 | 17.1 | 417.6 | 24393 | 35312 | 1.448 | O |

\( F_u = \) cross-sec. force under experimental failure load; \( A_{prov} = A_{prov} \) (of the first failure)\( \times 0.998 \times A_{prov} \) (of the first failure)

### Table 6(b) Examination of Failure Strengths of Struts and Ties at the First Failure

| Element No. | \( \beta_c \) | \( f_{ci} \) (MPa) | \( f_{ci} \) (MPa) | \( F_u \) (kN) | \( A_{eq} \) (mm\(^2\)) | \( A_{prov} \) (mm\(^2\)) | \( A_{prov} / A_{eq} \) | Safe/Fail |
|-------------|---------------|-----------------|-----------------|---------------|----------------|----------------|----------------|---------|
| S1-1 | 2.08 | 21.4 | 44.5 | 417.6 | 9382 | 11239 | 1.198 | O |
| S1-2 | 0.65 | 21.4 | 13.9 | 417.6 | 30022 | 132028 | 4.398 | O |
| S1-3 | 0.80 | 21.4 | 17.1 | 417.6 | 24393 | 35312 | 1.448 | O |

### Table 6(c) Examination of Failure Strengths of Nodal Zones

| Node Type | \( \beta_c \) | \( f_{ci} \) (MPa) | \( f_{ci} \) (MPa) | \( F_u \) (kN) | \( A_{eq} \) (mm\(^2\)) | \( A_{prov} \) (mm\(^2\)) | \( A_{prov} / A_{eq} \) | Safe/Fail |
|-----------|---------------|-----------------|-----------------|---------------|----------------|----------------|----------------|---------|
| CCC | 1.60 | 21.4 | 34.2 | P/4 | 308.2 | 9001 | 10000 | 1.111 | O |
| R | 308.2 | 12224 | 11239 | \( \rho_{0.919} \) | O |
| CTT | 1.40 | 21.4 | 30.0 | S1 | 418.6 | 10287 | 31416 | 3.054 | O |
| T1 | 418.6 | 13971 | 35312 | 2.849 | X |
| T2 | 418.6 | 1874 | 0.200 | X |

\( \beta_c \): coefficient of eff. strength of nodal zone; \( F_u = \) cross-sec. force under 100.2% of experimental failure load; \( R = \) support reaction by a pile; \( P = \) applied shear force (= 100.2% of experimental failure load); \( f_{ci} \): coefficient of eff. strength of nodal zone (= \( \beta_c f_y \))
of 1232.8 kN (equal to the sum of the two applied loads), that is, at the maximum load of the statically indeterminate STM. As shown in Table 6(c), the strength of the CCC nodal zone under a load greater than 1133.4 kN (0.919 × 1232.8) was insufficient to transfer the strut forces through the nodal zone. Therefore, 92.1% of the experimental failure load (an equivalent load of 1133.4 kN) of 1230 kN was predicted as the ultimate strength using the proposed method. The ultimate strengths of all the other four-pile caps were calculated using a similar approach.

4.3 Evaluation of calculated against experimental capacity

The ultimate strengths of 115 four-pile caps were calculated using the existing and proposed methods. The results are summarized in Table 7 and plotted against specimen concrete compressive strength in Fig. 13. The ratio of the test to predicted strength based on the CRSI sectional method was calculated as 1.06. However, the strength of 52% of the four-pile caps was overestimated with this method. This outcome was observed in the deep four-pile caps with ultimate strength governed only with this method. This outcome was observed in the deep four-pile caps with ultimate strength governed only with this method. This outcome was observed in the deep four-pile caps with ultimate strength governed only with this method. This outcome was observed in the deep four-pile caps with ultimate strength governed only with this method. This outcome was observed in the deep four-pile caps with ultimate strength governed only with this method. This outcome was observed in the deep four-pile caps with ultimate strength governed only with this method. This outcome was observed in the deep four-pile caps with ultimate strength governed only with this method. This outcome was observed in the deep four-pile caps with ultimate strength governed only with this method. This outcome was observed in the deep four-pile caps with ultimate strength governed only with this method. This outcome was observed in the deep four-pile caps with ultimate strength governed only with this method. This outcome was observed in the deep four-pile caps with ultimate strength governed only with this method.

The ratios of the experimental failure strength to the evaluated strength based on the ACI 318 (ACI 2014) and AASHTO (2018) STM methods were 2.15 and 1.82, respectively. Both methods underestimated the experimental failure strength considerably. The coefficients of variation based on the STM methods were 47.6% and 35.3%, respectively, indicating a considerable variation in the estimation of the capacities when compared with test results. These conservative and inconsistent estimates arise from the load-carrying capacities of diagonal ties in tension regions neglected and from the use of effective strengths of concrete struts and nodal zones which are questionable for this 3D situation. On the other hand, the proposed 3D STM method that incorporates a statically indeterminate STM with appropriate effective strength values of concrete struts and nodal zones estimated the experimental failure strength fairly accurately and consistently as shown in Table 7 and Fig. 13. The ratio of experimental to calculated strength and coefficient of variation (%) were 1.21 and 20.6 respectively using the ACI 445 nodal zone shape. For the PCA case, the same values were 1.19 and 20.6 respectively.

5. Summary and conclusions

A refined 3D STM method that takes into account the load-carrying capacity of diagonal caps in tension regions and the appropriate strength properties of 3D concrete struts and nodal zones for the rational analysis and design of four-pile caps has been illustrated in this paper. The proposed method has been evaluated using the strength at a failure of 115 reinforced concrete pile cap specimens. The calculated capacities were determined using the proposed method, the CRSI and ACI 318 sectional methods, and the ACI 318 and AASHTO STM methods.

Table 7 Ultimate strengths of four-pile caps predicted based on existing and proposed methods.

| Nodal Zone Shape | STM & Effective Strength of Nodal Zone |
|------------------|---------------------------------------|
|                  | Bergmeister et al. (1993) | Adebbar & Zhou (1996) | Yun et al. (2018) |
| ACI 445 (2002)   |                         |                         |                     |
| Model (A), $zd=0.90d$ | 1.25 | 1.26 | 1.25 |
| Model (B), $zd=0.95d$ | 1.21 | 1.24 | 1.18 |
| Model (C), $zd=1.00d$ | 1.28 | 1.35 | 1.21 |
| Total Mean       | 1.23 | 1.27 | 1.21 |
| COV(%)           | 22.3 | 23.3 | 20.6 |
| PCA (2004)       |                         |                         |                     |
| Model (A), $zd=0.90d$ | 1.23 | 1.25 | 1.23 |
| Model (B), $zd=0.95d$ | 1.18 | 1.21 | 1.17 |
| Model (C), $zd=1.00d$ | 1.27 | 1.34 | 1.20 |
| Total Mean       | 1.21 | 1.25 | 1.19 |
| COV(%)           | 21.9 | 22.3 | 20.6 |
The comparison of the CRSI sectional method calculated strengths with tests resulted in an average ratio of experimental to calculated strength of 1.06 with a coefficient of variation of 29.0%. However, the ultimate strengths of more than half of the four-pile caps were overestimated. On the other hand, the calculated ultimate strength from the ACI 318 sectional method significantly underestimated the test strength. The average ratio of experimental to calculated strength and coefficient of variation were 2.25 and 25.8% respectively.

The ACI 318 and AASHTO STM methods underestimated the test strengths. The average ratios of the experimental to calculated strength (and coefficients of variation) were 2.15 (47.6%) and 1.82 (35.3%), respectively. The specimen strengths were estimated accurately and consistently using the proposed method that incorporated an indeterminate truss-type STM. The average ratios of the experimental strength to predicted strength (and coefficients of variation) were in the range of 1.19 - 1.27 (20.6 – 23.3%).

In conclusion, the results of the present study demonstrate both the importance of proper considerations of the tensile strength of the concrete in the form of ties and use of effective stresses that account for the 3D nature of the problem in the case of nodal zones and struts in the 3D STM method. Another important point is the verification of the proposed method using a significant number of tests (115) in the literature as an alternate design method for four-pile caps in structural concrete.

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References
AASHTO, (2018). “AASHTO LRFD bridge design specifications.” 8th Ed. Washington DC: American Association of State Highway and Transportation Officials.
ACI, (1999). “Building code requirements for structural concrete (ACI 318-99) and commentary (ACI 318R-99).” Farmington Hills, MI: American Concrete Institute.
ACI, (2002). “Examples for the design of structural concrete with strut-and-tie models.” SP-208, ACI Committee 445, Farmington Hills, MI: American Concrete Institute.
ACI, (2010). “Further examples for the design of
structural concrete with strut-and-tie models.” SP-273, ACI-ASCE Committee 445, Farmington Hills, MI: American Concrete Institute.

ACI, (2011). “Building code requirements for structural concrete (ACI 318M-11) and commentary.” Farmington Hills, MI: American Concrete Institute.

ACI, (2014). “Building code requirements for structural concrete (ACI 318-14) and commentary (ACI 318R-14).” Farmington Hills, MI: American Concrete Institute.

Adebar, P., Kuchma, D. and Collins, M. P., (1990). “Strut-and-tie models for the design of pile caps: An experimental study.” ACI Struct. J., 87(1), 81-92.

Adebar, P. and Zhou, L., (1996). “Design of deep pile caps by strut-and-tie models.” ACI Struct. J., 93(4), 437-448.

Bergmeister, K., Breen, J. E., Jirsa, J. O. and Kreger, M. E., (1993). “Detailing in structural concrete.” Research Rep. 1127-3 F, Center for Transportation Research, Univ. of Texas at Austin, Austin, TX.

Canadian Standards Association. (2004). “Design of concrete structures for buildings.” CSA A23.3-M04, Rexdale, ON, Canada.

Clarke, J. L., (1973). “Behavior and design of pile caps with four piles.” Rep. No. 42.489, London: Cement and Concrete Association.

fib, (2010). “CEP-FIP model code 2010.” Lausanne Switzerland: Comité Euro-International du Béton, International Federation for Structural Concrete.

CRSI, (2008). “CRSI design handbook 2008.” 10th Edition, Schaumburg IL: Concrete Reinforcing Steel Institute.

IABSE, (1993). “Strut-and-tie model.” IABSE Workshop, University of Stuttgart, New Delhi India: International Association for Bridge and Structural Engineering.

Lee, S.-C., Cho, J.-Y. and Vecchio, F. J., (2011). “Model for post-yield tension stiffening and rebar rupture in concrete members.” Eng. Struct., 33(5), 1723-1733.

Mitchell, D., Collins, M. P., Bhide, S. B. and Rabbat, B. G., (2004). “AASHTO LRFD strut-tie model design examples.” Skokie: Portland Cement Association.

Park, J. W., Kuchma, D. and Souza, R., (2008). “Strength predictions of pile caps by a strut-and-tie model approach.” Canadian J. of Civil Eng., 35, 1399-1413.

Sabnis, G. M. and Gogate, A. B., (1984). “Investigation of thick slab (pile cap) behavior.” ACI J., 81(1), 35-39.

Suzuki, K., Otsuki, K. and Tsubata, T., (1998). “Influence of bar arrangement on ultimate strength of four-pile caps.” Trans. Jpn. Concr. Inst., 20, 195-202.

Suzuki, K., Otsuki, K. and Tsubata, T., (1999). “Experimental study on four-pile caps with taper.” Trans. Jpn. Concr. Insr., 21, 361-368.

Suzuki, K., Otsuki, K. and Tsuchiya, T., (2000). “Influence of edge distance on failure mechanism of pile caps.” Trans. Jpn. Concr. Insr., 22, 327-334.

Yun, Y. M., Kim, B. H. and Ramirez, J. A., (2018). “Three-dimensional strut-tie model approach for structural concrete design.” ACI Struct. J., 115, 15-26.