A Simplified Dynamical Model for Tuned Wireless Power Transfer Systems

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Abstract – Dynamical models of wireless power transfer (WPT) systems are of primary importance for the dynamical behavior studies and controller design. However, the existing dynamical models usually suffer from high orders and complicated forms due to the complex nature of the coupled resonances and switched-mode power converters in WPT systems. This letter finds that a well-tuned WPT system can be accurately described by a much simpler dynamical model. Specifically, at the tuned condition, the existing dynamical model can be decomposed into two parts. One is controllable and the other one is uncontrollable. The former should be considered in the modeling while the latter can be ignored because it always exponentially converges to zero. For illustration, the recently proposed zero-voltage-switching full-bridge pulse-density modulation WPT system is modeled as an example since such a system can efficiently operate at the tuned condition with soft switching and control capabilities. The derived model was verified in experiments by time-domain and frequency-domain responses.

Index Terms – Dynamical model, wireless power transfer.

I. INTRODUCTION

Wireless power transfer (WPT) systems are becoming widely used in electric vehicles (EVs), consumer electronics, medical devices, factory automations, etc. [2]. Some of these applications suffer from fast parameter changes and require rapid control and protection, e.g. the road-powered EVs [3]. Consequently, the dynamical behaviors of WPT systems need to be well studied and the analytical dynamical models are of primary importance.

The most common dynamical modeling methods for WPT systems include the generalized state-space averaging (GSSA) [4], extended describing functions (EDF) [5], and dynamic phasors [6]. However, the dynamical models derived using these methods suffer from high orders and complicated forms due to the complex nature of the coupled resonances and switched-mode power converters in WPT systems. For example, a simple series-series compensated WPT system shown in Fig. 1, which has 4 resonant elements and 1 filter capacitor, has to be modeled by a 9th-order model [5]. To reduce the model order, previous studies proposed model order reduction techniques and presented 5th-order models for the system shown in Fig. 1 [7]. Furthermore, the experimental results given in [7] imply that the system may be described by an even simpler model, and drive a further study on the dynamical modeling.

II. TUNED WPT SYSTEM

This letter finds that when the system is well tuned, the 5th-order real-valued dynamic phasor model in [7] can be decomposed into two parts. One is controllable and the other one is uncontrollable. The former includes the real part of the transmitter resonant current phasor, the imaginary part of the receiver resonant current phasor, and the dc output voltage. The latter includes the imaginary part of the transmitter resonant current phasor and the real part of the receiver resonant current phasor. The controllable part can be used as a simplified dynamical model for tuned WPT systems.

This letter is an extension of the conference paper: [1] H. Li, J. Fang, and Y. Tang, “Reduced-order dynamical models of tuned wireless power transfer systems,” in IEEE Int. Power Electron. Conf. ECCE Asia, 2018, pp. 337-341. (Corresponding author: Y. Tang.)
The system is said to be tuned when \( u_2(t) \) is synchronized to \( i_{2s}(t) \) with a 180° phase difference (by a zero crossing detector-based phase lock loop), and the resonant frequencies on the two sides: \( \omega_{s1} = \frac{1}{\sqrt{L_1 C_1}} \) and \( \omega_{s2} = \frac{1}{\sqrt{L_2 C_2}} \) both equal the inverter fundamental switching frequency \( \omega_s = 2\pi f_s \). Under this condition, the ac equivalent input impedance of the rectifier is resistive, the phase difference between \( i_{1s}(t) \) and \( i_{2s}(t) \) is 90°, and the inverter has a resistive load. With these features, the dynamical behavior of the system can be described by a simplified dynamical model.

### III. SIMPLIFIED DYNAMICAL MODEL

The 5th-order real-valued dynamical model of the system shown in Fig. 2 can be derived using the method in [7] and expressed as

\[
\begin{align*}
\frac{dI_{1i}(t)}{dt} &= \Delta \omega_1 I_{1i}(t) - \frac{R_1}{L_{o1}} I_{1i}(t) + \frac{\omega M}{L_{o1}} I_{1o}(t) + \frac{S_{1i}(t)}{L_{o1}} V(t) \\
\frac{dI_{1o}(t)}{dt} &= -\Delta \omega_1 I_{1o}(t) - \frac{R_1}{L_{o1}} I_{1o}(t) - \frac{\omega M}{L_{o1}} I_{1i}(t) + \frac{S_{1o}(t)}{L_{o1}} V(t) \\
\frac{dI_{2i}(t)}{dt} &= \Delta \omega_2 I_{2i}(t) - \frac{R_2}{L_{o2}} I_{2i}(t) + \frac{\omega M}{L_{o2}} I_{2o}(t) + \frac{S_{2i}(t)}{L_{o2}} V(t) \\
\frac{dI_{2o}(t)}{dt} &= -\Delta \omega_2 I_{2o}(t) - \frac{R_2}{L_{o2}} I_{2o}(t) - \frac{\omega M}{L_{o2}} I_{2i}(t) + \frac{S_{2o}(t)}{L_{o2}} V(t) \\
\frac{dV(t)}{dt} &= -\frac{S_2(t) I_{1o}(t) + S_{1i}(t) I_{2o}(t) - V(t)}{C_i} - \frac{1}{R_i C_i} \\
\end{align*}
\]

where \( I_{1i}(t), I_{1o}(t) \) and \( I_{2i}(t), I_{2o}(t) \) are the real and imaginary parts of \( I_{1c}(t) \) and \( I_{2c}(t) \), which are the dynamic phasors of \( i_{1s}(t) \) and \( i_{2s}(t) \), respectively; \( V(t) \) and \( V_2(t) \) are the moving averages of \( v_1(t) \) and \( v_2(t) \), respectively; \( S_{1i}(t), S_{2i}(t), S_{1o}(t), S_{2o}(t) \) are the real and imaginary parts of \( S_{1i}(t) \) and \( S_{2i}(t) \), which are the complex-value conversion ratios of the inverter and rectifier, respectively [7]; \( \Delta \omega_1 \) and \( \Delta \omega_2 \) are the beat frequencies given by

\[
\Delta \omega_1 = \omega_s - \omega_1 \quad \text{and} \quad \Delta \omega_2 = \omega_s - \omega_2
\]

\( L_{o1} \) and \( L_{o2} \) are the equivalent inductances given by

\[
L_{o1} = \frac{\omega_s + \omega_1}{\omega_1} L_i \quad \text{and} \quad L_{o2} = \frac{\omega_s + \omega_2}{\omega_2} L_i
\]

Under the tuned condition, \( S_1(t) \) and \( S_2(t) \) depend only on the pulse densities \( d_1(t) \) and \( d_2(t) \), and can be written as

\[
S_1(t) = \frac{2\sqrt{2}}{\pi} d_1(t) \quad \text{and} \quad S_2(t) = j \frac{2\sqrt{2}}{\pi} d_2(t)
\]

Furthermore, \( \omega_{s1} = \omega_{s2} = \omega_s \) yields

\[
\Delta \omega_1 = \Delta \omega_2 = 0
\]

and

\[
L_{o1} = 2L_i \quad \text{and} \quad L_{o2} = 2L_i
\]

With (4)-(6), (1) becomes

\[
\begin{align*}
\frac{dI_{1i}(t)}{dt} &= -\frac{R_1}{2L_i} I_{1i}(t) + \frac{\omega M}{2L_i} I_{1o}(t) + \frac{\sqrt{2}}{\pi L_i} d_1(t) V(t) \\
\frac{dI_{1o}(t)}{dt} &= -\frac{R_1}{2L_i} I_{1o}(t) - \frac{\omega M}{2L_i} I_{1i}(t) \\
\frac{dI_{2i}(t)}{dt} &= -\frac{R_2}{2L_i} I_{2i}(t) + \frac{\omega M}{2L_i} I_{2o}(t) + \frac{\sqrt{2}}{\pi L_i} d_2(t) V_2(t) \\
\frac{dI_{2o}(t)}{dt} &= -\frac{R_2}{2L_i} I_{2o}(t) - \frac{\omega M}{2L_i} I_{2i}(t) - \frac{\sqrt{2}}{\pi L_i} d_2(t) V_2(t) \\
\frac{dV(t)}{dt} &= -\frac{2\sqrt{2}}{\pi C_i} d_1(t) I_{1o}(t) - \frac{1}{R_i C_i} V(t)
\end{align*}
\]

In (7), only the 1st, 4th, and 5th equations include the control inputs \( d_1(t) \) and \( d_2(t) \). Besides, these three equations are all independent to \( I_{1i}(t) \) and \( I_{2i}(t) \), which are included by the 2nd and 3rd equations. Consequently, (7) can be decomposed into a controllable part:

\[
\begin{align*}
\frac{dI_{1i}(t)}{dt} &= -\frac{R_1}{2L_i} I_{1i}(t) + \frac{\omega M}{2L_i} I_{1o}(t) + \frac{\sqrt{2}}{\pi L_i} d_1(t) V(t) \\
\frac{dI_{2i}(t)}{dt} &= -\frac{R_2}{2L_i} I_{2i}(t) + \frac{\omega M}{2L_i} I_{2o}(t) + \frac{\sqrt{2}}{\pi L_i} d_2(t) V_2(t) \\
\frac{dV_2(t)}{dt} &= -\frac{2\sqrt{2}}{\pi C_i} d_1(t) I_{1o}(t) - \frac{1}{R_i C_i} V_2(t)
\end{align*}
\]

and an uncontrollable part:

\[
\begin{align*}
\frac{dI_{1o}(t)}{dt} &= -\frac{R_1}{2L_i} I_{1o}(t) - \frac{\omega M}{2L_i} I_{1i}(t) \\
\frac{dI_{2o}(t)}{dt} &= -\frac{R_2}{2L_i} I_{2o}(t) - \frac{\omega M}{2L_i} I_{2i}(t)
\end{align*}
\]
The uncontrolled part (9) is linear and has a globally stable equilibrium point: \( I_{\text{L1}}(t) = I_{\text{L2}}(t) = 0 \). Therefore, only the controllable part (8) should be considered in the modeling, and it can be used as a simplified dynamical model under the tuned condition.

Moreover, a small-signal model can be derived from (8) to overcome the nonlinearity caused by the terms \( d_2(t)V_2(t) \) and \( d_2(t)I_{\text{L2}}(t) \) as

\[
\frac{d}{dt} \begin{bmatrix} \hat{I}_{\text{L1}}(t) \\ \hat{I}_{\text{L2}}(t) \\ \hat{V}_2(t) \end{bmatrix} = \begin{bmatrix} -\frac{R_1}{2L_1} & \frac{\omega M}{2L_1} & 0 \\ -\frac{\omega M}{2L_2} & -\frac{R_2}{2L_2} & \frac{\sqrt{2}d_2}{\pi L_2} \\ 0 & -\frac{2\sqrt{2}d_2}{\pi C_1} & -\frac{1}{R_1 C_1} \end{bmatrix} \begin{bmatrix} \hat{I}_{\text{L1}}(t) \\ \hat{I}_{\text{L2}}(t) \\ \hat{V}_2(t) \end{bmatrix} + \begin{bmatrix} \sqrt{\frac{2V_1}{\pi L_1}} & 0 \\ 0 & \sqrt{\frac{2V_2}{\pi L_2}} \\ 0 & -\frac{2\sqrt{V_2}}{\pi C_2} \end{bmatrix} \begin{bmatrix} \hat{d}_1(t) \\ \hat{d}_2(t) \end{bmatrix}
\]

(10)

where \( d_2, V_1, V_2, \) and \( I_{\text{L2}} \) are the steady-state values.

### IV. EXPERIMENTAL TESTS

The WPT system and the modulation strategy described in [11] were used for the experiment. The circuit diagram of the system was as shown in Fig. 2. The specific parameters and the corresponding steady-state operating point are listed in TABLE I and TABLE II, respectively.

The first experiment tested the large-signal behaviors by measuring the time-domain responses of \( i_{\text{L1}}(t), i_{\text{L2}}(t), \) and \( u_2(t) \) triggered by the step changes of \( d_1(t) \) and \( d_2(t) \) between 0.5 and 1, as shown in Fig. 3. The step up and down occurred at 0 ms and 0.5 ms, respectively. The transient waveforms of \( i_{\text{L1}}(t), i_{\text{L2}}(t), \) and \( u_2(t) \) coincided with the model predicted envelopes given by \( \sqrt{2}|I_{\text{L1}}(t)|, \sqrt{2}|I_{\text{L2}}(t)|, \) and \( V_2(t) \), which were derived from model (8).

The second experiment tested the small-signal behaviors by measuring the frequency-domain responses of the ripples on \( V_2(t) \) stimulated by the small sine waves injected into \( d_1(t) \) and \( d_2(t) \) at the steady-state operating point. Fig. 4 shows the captured waveforms when the frequency of the injected sine wave was 10 kHz. At this frequency, the magnitude and phase of the output voltage ripples were compared to the sine wave, and the results were plotted in the Bode diagrams (see Fig. 5) as a pair of blue circles. By sweeping the frequency of the sine waves, more results were obtained and they all coincided with the transfer functions derived from model (10).
V. CONCLUSION

This letter proposes a simplified dynamical model for the WPT systems under tuned condition (i.e., the rectifier input impedance is resistive and the two sides’ resonant frequencies both equal the inverter fundamental switching frequency). The condition can be well satisfied by the recently proposed ZVS full-bridge PDM WPT system. Such a system contains 4 resonant elements and 1 filter capacitor, and in this letter, it is modeled by a simple 3rd-order real-valued dynamical model. In contrast, existing dynamical models for the same resonant topology are usually 5th- or 9th-order models with much more complicated forms.

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