Quantum treatment of laser cooling on weak transitions: multipeaks and bimodal momentum distributions.

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Abstract. The work is devoted to the study of the features and parameters of the momentum distributions of atoms laser cooled on weak optical transitions. It was shown that atoms distributions are described by a bimodal momentum distribution whose characteristics depends on the parameters of the light field. In a strong field a velocity selective coherent population trapping effect is observed.

1. Introduction

The laser is a powerful and precise tool for effectively controlling the translational degrees of freedom of atoms. Currently laser cooling is rapidly developing field of science at the intersection of laser physics and atomic optics [1-3], and has a plurality fundamental and applied applications. In particular, laser cooling is used to obtain and study the Bose-Einstein condensate [4], in the field of quantum computer science [5], atomic nanolithography [6] and interferometry [7]. The combination of laser cooling and modern methods of precision spectroscopy allows the creation of standards for frequency and time, the accuracy of which reaches values of the order of $10^{-17} - 10^{-18}$ [8,9]. To date, various methods for localizing and cooling atoms (and magneto dipole traps, optical lattices, etc.) have become an integral part of modern basic and applied science. This progress would not have been possible without theoretical analysis and consideration of the processes arising from the interaction of atoms with the electromagnetic field.

However, the theoretical description, taking into account the multilevel structure of the atom, the degeneracy of the atomic levels, spontaneous decay, the recoil effect, and the effect of field polarization, is an extremely difficult task. The beginning of its solution was laid in the 70's - 80's by the study of the simplest system: a two-level atom in a resonant light field [1,2,10-12]. An analytical study of this model using the quasiclassical approach (see, for example, [13]), which considers cooling in terms of light-induced forces and their fluctuations (diffusion in momentum space), made it possible to understand many of the basic cooling mechanisms. But, the quasi-classical approach describes only those cases where single photon recoil frequency ($\omega_r / \gamma = h k^2 / 2m$, where $k$ – wave vector, $m$ – mass of atom) is small compared with $\gamma$ - natural linewidth cooling transition, and is not suitable for...
consideration forbidden transitions such as intercombination transitions \( \text{Yb, Mg, Sr, etc} \) used in experiments for ultradeep laser cooling [14-17].

In this paper we will investigate the laser cooling of atoms on weak optical transitions where the recoil \( w_r / \gamma \) is not small parameter. The study will be conducted using an accurate quantum calculation [18,19], because it is in contrast to previously developed methods, allows to obtain the density matrix of atoms with full consideration of recoil effects due to the interaction of atoms with the field photons. Density matrix containing full information about both the internal and external degrees of freedom. This method allows us to correctly take into account both the cooling of atoms localized in the optical potential, and the more heat fraction of atoms that perform above-barrier motion.

Particular attention will be paid to the features of impulse distributions of cold atoms as a function of the intensity of the light field.

2. Formulation of the problem

The field of a standing monochromatic wave has the form:

\[
E = 2E_0 \exp(-i\omega t) \cos(kx),
\]

where \( E_0 \) the amplitude of the light field, and \( \omega \) is the frequency of the light field. The interaction of a two-level atom with the field of a given configuration is considered on the basis of the quantum kinetic equation for the density matrix \( \rho(x_1,x_2,t) \) in the coordinate representation (see, for example, [2]):

\[
\frac{\partial}{\partial t} \rho(x_1,x_2,t) = -\frac{i}{\hbar} \left[ \hat{H}(x_1,x_2,t) \rho(x_1,x_2,t) - \rho(x_1,x_2,t) \hat{H}(x_1,x_2,t) \right] + \hat{\Gamma} \{ \rho(x_1,x_2,t) \},
\]

where \( \hat{H}(x_1,x_2,t) \) - Hamiltonian of atom, \( \hat{\Gamma} \{ \rho(x_1,x_2,t) \} \) - Operational functional that describes the processes of spontaneous relaxation. In subsequent calculations, it is convenient to go from the coordinates \( x, x_1 \) and \( x_2 \) to the new (Wigner) coordinates \( x = (x_1 + x_2) / 2 \) and \( q = (x_1 - x_2) \). This transition is dictated by the convenience of numerical calculations.

In the resonance approximation and in the transition to the basis of the rotating wave the Hamiltonian of the atom becomes independent of time:

\[
\hat{H}(x,q) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + \hat{H}_0 + \hat{V}.
\]

The first term is the operator of the kinetic energy of the atom, the second describes the internal structure of the free atom and depends only on the internal degrees of freedom, and the latter describes the dipole interaction of the atom with the field of the standing light wave (1). In the one-dimensional case, the interaction operator can be represented in the form:

\[
\hat{V} = -\hbar \Omega(x) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},
\]

where \( \Omega(x) = 2\Omega \cos(kx) \), \( \Omega = dE_0 / \hbar \) is the Rabi frequency, and \( d \) is the matrix element of the dipole moment operator of the atom. The operator \( \hat{H}_0 = -\hbar \delta | e > < e | \) can be represented as:

\[
\hat{H}_0 = -\hbar \delta \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix},
\]

where \( \delta = \omega - \omega_0 \) - detuning the frequency of the field from the frequency of the atomic transition. The operator \( \hat{\Gamma} \) in (2) describes spontaneous relaxation taking into account the recoil effect:

\[
\hat{\Gamma} = -\frac{\gamma}{2} \begin{pmatrix} -2f(q) & 1 \\ 1 & 2 \end{pmatrix}.
\]
In the absence of recoil \( f(q \to 0) = 1 \), which corresponds to the conservation of normalization for spontaneous transitions.

As a result, the system of equations to be solved for the elements of a two-point density matrix has the form:

\[
\begin{align*}
\rho_{21}(x_1, x_2) &= -\frac{i}{\hbar} \left( V(1) \rho_{12}(x_1, x_2) - V^*(2) \right), \\
\rho_{12}(x_1, x_2) &= -\frac{i}{\hbar} \left( V(1) \rho_{12}(x_1, x_2) - V^*(2) \right), \\
\rho_{22}(x_1, x_2) &= -\frac{i}{\hbar} \left( V(1) \rho_{12}(x_1, x_2) - V^*(2) \right),
\end{align*}
\]

where \( \rho_{22}, \rho_{11} \) are the populations of the excited and unexcited levels, and \( \rho_{12}, \rho_{21} \) - optical coherences between levels.

A method of solving this equation system with full consideration of recoil and localization of atoms was developed by us in [18,19].

3. Results

Various quantum regimes of laser cooling of two-level atoms were studied for different values of the intensity of the light field of a standing light wave with spatial homogeneous polarization. For these parameters, as already noted, the quasiclassical approach is inapplicable the recoil frequency is large enough. It should be noted that the impulse distribution is essentially nonequilibrium and is not described by the Gaussian distribution, therefore, cannot be characterized in terms of the temperature dependence. The momentum distributions obtained in the results of calculations can be roughly divided into two large groups: multi-peak structures with narrow peaks \( \Delta p < \hbar \) (Fig. 1a) and narrow distributions with wide wings (Fig. 1b).

![Figure 1. Two-structure momentum distributions of cold atoms in quantum regimes: a) The distribution has a clearly expressed narrow peaks which width of order a single photon momentum. b) The distribution has narrow structure of cold atoms and wide wings. Parameters of the problem are located on graphs.](image-url)
To begin with, let us consider the second case, which is more important from the point of view of the obtained ultra-low temperatures of the atomic ensemble. It is seen that momentum distribution of atoms has a complex structure - a narrow peak is located on a wide substrate. Such a uniquely bimodal structure can be approximated with good accuracy by two Gaussian functions (Fig. 2) corresponding to two velocity groups of atoms: for parameters \( \delta / \gamma = -1, \Omega / \gamma = 0.1, \omega_p / \gamma = 0.64 \), the "cold" fraction atoms has a characteristic temperature \( T_1 = 0.67h \gamma \), and the "hot" fraction of atoms with temperature \( T_1 = 26.2h \gamma \).

**Figure 2.** Bimodal momentum distribution of cold atoms, approximated by two Gaussian functions. Parameters of the problem are located on graph.

Similar bimodal distributions arise at different parameters of the light fields for the investigated weak optical transitions. In this case, the proportion of hot and cold atoms, as well as their temperatures depend on the parameters of the light fields (detuning and Rabi frequency).

Depending temperature distributions of cold and hot fractions atoms on the intensity of the light field is shown in Fig. 3a. The temperature of the cold fraction is much lower than the average temperature, defined as \( <p^2>/m \) (\( <p^2> \) is the average square of momentum) and reaches the limiting values \( h \gamma \).

**Figure 3.** Dependences of temperature a) and proportion b) of atoms for “cold” and “hot” fractions of atoms of Rabi frequency. Parameters of the problem are located on graphs.
The main tendency consists in the general heating of the impulse distribution with increasing intensity of the light field, both the temperature of the cold fraction and the temperature of the hot fraction increase. At the same time, the temperature of the hot fraction has two peculiarities of behavior that disrupt the monotony of growth. Both can be explained by the distortions of the shape of the momentum distributions of the atoms. For the region $\Omega/\gamma = 0.2 - 0.4$ a major role in the distribution begin to play narrow pulse-order structure of a single photon, accompanying the central peak, and in the area of strong magnetic fields $\Omega/\gamma = 1.2 - 1.6$ the distribution is distorted so that the most probable impulses for atoms are not zero (Fig. 4), but the corresponding $p = \pm \hbar K$, which somewhat worsens the accuracy of the approximation, because Approximation averages the contribution of emissions.

![Figure 4. Multipeak momentum distribution of cold atoms. Parameters of the problem are located on graph.](image)

The fraction of fractions of cold and hot atoms as a function of the intensity is shown in Fig. 3b. The number of atoms in the cold distribution is maximal in a weak field, and reaches 60%. In strong fields, the contribution of both fractions is equalized and amounts to about 50%. Features of behavior manifest at the same intensity as the temperature graphs.

Such complex multi-peak structures are the second basic mode of cooling atoms on weak clock transitions. This structure is associated with the manifestation of the effect of velocity selective coherent population trapping [20,21] and Fig. 5 illustrates this mechanism. For a two-level atom, this structure exists only in an essentially quantum mode.

![Figure 5. VSCPT in two-level system](image)
The intensity of VSCPT peaks becomes maximum when the detuning and the recoil frequency are equal (in modulus), this leads to the most efficient excitation of the lambda circuit inside which most of the atoms are located. The atoms captured in the Lambda diagram form two distinct narrow peaks located at points with impulses $\pm \hbar k$ (Fig. 6a).

With increasing intensity, an increasing number of such peaks are observed, but along with this, the temperature of the hot fraction of atoms also increases, which makes it impossible to obtain fractions of the cold-atom fractionally distributed over the $\pm n\hbar k$ separated velocities (Fig. 6b).

\begin{align*}
\delta / \gamma &= -0.64 \\
\omega_r / \gamma &= 0.64 \\
\Omega / \gamma &= 0.5
\end{align*}

\begin{align*}
\delta / \gamma &= -0.64 \\
\omega_r / \gamma &= 0.64 \\
\Omega / \gamma &= 2
\end{align*}

**Figure 6.** Momentum distribution contains narrow peaks. a) medium field; b) strong field. Parameters of the problem are located on graphs.

### 4. Conclusion

Laser cooling of two-level atoms in quantum regimes was studied. It is shown that there are two main modes - bimodal distribution, and a multi-peak distribution. The multi-peak distribution is caused by the manifestation of VSCPT and is particularly pronounced when the detuning and the recoil frequency are equal in a strong light field, when 7 or more narrow peaks in the momentum distribution are resolved. The bimodal distribution makes it possible to obtain a cold fraction of atoms whose temperature is $< h\gamma$ (which can reach less than 1 millikelvin on researched clock transitions), with up to 60% of atoms in the cold fraction.
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