COST OPTIMIZATION STUDY OF TWO-STEP HELICAL GEARBOXES WITH FIRST STAGE DOUBLE GEAR SETS

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ABSTRACT

One of the important goals of the optimized gearbox design is that the gearbox cost is minimized. However, so far, there has been no research about the design of a Two-step Helical Gearbox (THG) with First Stage Double Gearsets (FSDG) and optimum gear ratios to achieve the lowest cost. This paper presents the results of research on the influence of input parameters on the optimum partial gear ratios of the mentioned gearbox. To do that, a simulation experiment was designed and performed. Moreover, a regression formula for finding the optimum gear ratio was proposed. Evaluation results show that the formula is very suitable for the experimental data.

KEYWORDS: Helical Gearbox, Cost Optimization, Gear Ratio & Optimum Gearbox Design

1. INTRODUCTION

Up to now, the optimal gearbox design has received much attention from researchers. Various studies were conducted in several different directions, such as determining the optimal parameters to reduce gearbox vibration [1, 2], to achieve the smallest gear mass [3-8], the smallest gearbox length [3, 8-13], or the smallest gearbox cross-section [3, 8, 14-20]. Besides, the optimal design of gearboxes has carried out with the mechanical system containing a V-belt [21-26] or a chain drive [27-30]. Studies have also been done with various types of gearboxes such as helical gearboxes [6, 11, 12, 15, 17, 27, 28, 31-33], bevel gearboxes [14, 22, 23, 32, 34-36] or worm gearboxes [30, 37-40]. Recently, a cost optimization study for a three-stage helical gearbox has been introduced [41]. However, so far there has been no research on optimization design for minimum cost of two-step helical gearboxes with first stage double gearsets.

This article presents the results of optimum design of a THG with FSDG. In particular, the influence of main design parameters on the optimum gear ratios has been analyzed by designing and performing a simulation experiment. In addition, the optimum gear ratios can easily be found by proposed regression models.

2. METHODOLOGY

It is the fact that the costs of bearings, gears, shafts, casing is found to have strong influence on the cost of a given gearbox. In this study, the cost of bearings will be ignored because of complex cost determination. Consequently, the cost of a THG with FSDG, $C$, can be determined by the following equations:
In which, $C_g$, $C_{gh}$, and $C_s$ are the cost of gears, gearbox housing and shafts, respectively. It should be noticed that the cost of a gear contains the cost of used materials, machining process, heat treatment, operators, etc. These costs construct the final price of a gear. In terms of commerce, the price of a gear can be usually determined by unit price per kilogram which regularly changes according to markets. In the current study, these cost elements are considered as variables and calculated by the following equations:

$$C_g = c_{g,m} \cdot m_g$$

(2)

$$C_{gh} = c_{gh,m} \cdot m_{gh}$$

(3)

$$C_s = c_{s,m} \cdot m_s$$

(4)

Where, $c_{g,m}$, $c_{gh,m}$, and $c_{s,m}$ are the cost of gears, gearbox housing, and shafts (USD/kg); $m_g$, $m_{gh}$, and $m_s$ are the mass of gears, gearbox housing, and shafts (kg). In this work, $c_{g,m}$, $c_{gh,m}$, and $c_{s,m}$ are variables and $m_g$, $m_{gh}$, and $m_s$ are defined as below:

$$m_g = \rho_1 \cdot \left[ 2 \cdot \left( \frac{\pi e_1 d_{w1}^2 h_{w1}}{4} + \frac{\pi e_2 d_{w21}^2 h_{w2}}{4} + \frac{\pi e_3 d_{w22}^2 h_{w2}}{4} \right) \right]$$

(5)

$$m_{gh} = \rho_2 \cdot \left[ 2 \cdot L \cdot B_1 \cdot 1.5 \cdot S_G + 2 \cdot L \cdot H \cdot S_G + 2 \cdot B_2 \cdot H \cdot S_G \right]$$

(6)

$$m_s = \rho_3 \cdot \frac{\pi}{4} \cdot \left( d_1^2 \cdot l_1 + d_2^2 \cdot l_2 + d_3^2 \cdot l_3 \right)$$

(7)

Wherein, $\rho_1$, $\rho_2$, and $\rho_3$ are the weight density of materials of gear, gearbox housing, and shafts, respectively; $\rho_1 = \rho_3 = 7.82$, $\rho_2 = 7.2$ [42]; $e_1 = 1$ and $e_2 = 0.6$; $L$, $S_G$, $H$, $B_1$, and $B_2$ are the element dimensions of gearbox housing (Figure 1) which can be found by:

$$L = (d_{w11} + d_{w21}/2 + d_{w12}/2 + d_{w22} + 22.5)/0.975$$

[4]

(8)

$$S_G = 0.005 \cdot L + 4.5$$

[4]

(9)

$$H = \max(d_{w21}, d_{w22}) + 6.5 \cdot S_G$$

(10)

$$B_1 = 2 \cdot b_{w1} + b_{w2} + 5 \cdot S_G$$

(11)

$$B_2 = B_1 - 2 \cdot S_G$$

(12)

In addition, $b_{w1}$, $b_{w2}$ are the gear widths; $d_{w11}$, $d_{w21}$, $d_{w12}$, and $d_{w22}$ are the pitch diameters of the pinion and the gear of the first and second stages. These parameters can be calculated by below equations [43]:

$$b_{w1} = X_{ba1} \cdot c_{w1}$$

(13)

$$b_{w2} = X_{ba2} \cdot c_{w2}$$

(14)

$$d_{w11} = 2 \cdot c_{w1} / (u_1 + 1)$$

(15)

$$d_{w21} = 2 \cdot c_{w1} \cdot u_1 / (u_1 + 1)$$

(16)
Cost Optimization Study of Two-Step Helical Gearboxes with First Stage Double Gear Sets

\[ d_{w12} = 2 \cdot \alpha_{w2}/(u_2 + 1) \]  \hspace{2cm} (17)

\[ d_{w22} = 2 \cdot \alpha_{w2} \cdot u_2/(u_2 + 1) \]  \hspace{2cm} (18)

\[ \alpha_{w1} = 43 \cdot (u_1 + 1) \cdot \sqrt[3]{T_{11} \cdot k_{HB}/(\sigma_{HB}^2 \cdot u_1 \cdot X_{m1})} \]  \hspace{2cm} (19)

\[ \alpha_{w2} = 43 \cdot (u_2 + 1) \cdot \sqrt[3]{T_{12} \cdot k_{HB}/(\sigma_{HB}^2 \cdot u_2 \cdot X_{m2})} \]  \hspace{2cm} (20)

\[ d_1 = \sqrt[3]{T_{11}/(0.2 \cdot [t])} \]  \hspace{2cm} (21)

\[ d_2 = \sqrt[3]{T_{12}/(0.2 \cdot [t])} \]  \hspace{2cm} (22)

\[ d_3 = \sqrt[3]{T_{13}/(0.2 \cdot [t])} \]  \hspace{2cm} (23)

\[ l_1 = B_1 + 1.2 \cdot d_1 \] \quad (Figure 1) \hspace{2cm} (24)

\[ l_2 = B_1 \] \quad (Figure 1) \hspace{2cm} (25)

\[ l_3 = B_1 + 1.2 \cdot d_3 \] \quad (Figure 1) \hspace{2cm} (26)

**Figure 1:** Calculated Schema.

In the equations (19) and (20), \( k_{HB} \) is the contacting load coefficient which can be chosen by 1.1 [43].

From the above analysis, for minimizing the reducer cost, the objective function of the optimization problem can be designated as follows:
Minimize $\mathcal{C}$

With the following constraints:

\begin{align}
1 & \leq u_1 \leq 9 \\
1 & \leq u_2 \leq 9
\end{align}

Nevertheless, it is known that $u_x = u_1 \cdot u_2$ is the relation between total gearbox ratio and partial ratios. Hence, the optimization of $u_1$ is sufficient, while the optimum ratio of $u_2$ can be obtained by the expression $u_2 = u_x/u_1$.

### 3. EXPERIMENTAL WORK

#### Table 1: Main Design Factors

| Parameter                      | Code  | Unit | Low | High |
|--------------------------------|-------|------|-----|------|
| Total gear ratio               | $u_{k}$| -    | 10  | 50   |
| Gear face ratio of first step  | $X_{2a1}$| -   | 0.3 | 0.35 |
| Wheel face ratio of second step| $X_{2a2}$| -   | 0.35| 0.4  |
| Allowable contact stress of first step | $A_{S1}$| MPa | 350 | 420  |
| Allowable contact stress of second step | $A_{S2}$| MPa | 350 | 420  |
| Output torque                  | $T_{out}$| Nmm | 100 | 10000|
| Gearbox housing cost           | $C_{gh}$| USD/kg | 1  | 5    |
| Gear cost                      | $C_{g}$| USD/kg | 2  | 9    |
| Shaft cost                     | $C_{s}$| USD/kg | 1.5| 5    |

#### Table 2: Runs of Experiment and Values of Response

| Std Order | Run Order | Center Pt | Blocks | $n_{k}$ | $X_{2a1}$ | $X_{2a2}$ | $A_{S1}$ | $A_{S2}$ | $T_{out}$ | $C_{gh}$ | $C_{g}$ | $C_{s}$ | $u_{1}$ |
|-----------|-----------|-----------|--------|---------|-----------|-----------|---------|---------|----------|---------|--------|--------|--------|
| 117       | 1         | 1         | 1      | 5       | 0.3       | 0.4       | 350     | 420     | 10000    | 5       | 9      | 1.5    | 1.56   |
| 50        | 2         | 1         | 1      | 45      | 0.3       | 0.35      | 350     | 420     | 10000    | 1       | 2      | 1.5    | 6.66   |
| 51        | 3         | 1         | 1      | 5       | 0.35      | 0.35      | 350     | 420     | 10000    | 1       | 9      | 5      | 1.24   |
| 101       | 4         | 1         | 1      | 5       | 0.3       | 0.4       | 350     | 350     | 10000    | 5       | 9      | 5      | 1.42   |
| 31        | 5         | 1         | 1      | 5       | 0.35      | 0.4       | 420     | 420     | 100      | 1       | 2      | 5      | 1.00   |
| 11        | 6         | 1         | 1      | 5       | 0.35      | 0.35      | 420     | 350     | 100      | 1       | 9      | 5      | 1.40   |
| ...       |           |           |        |         |           |           |         |         |          |         |        |        |        |
| 74        | 127       | 1         | 1      | 45      | 0.3       | 0.35      | 420     | 350     | 10000    | 5       | 9      | 5      | 8.99   |
| 120       | 128       | 1         | 1      | 45      | 0.35      | 0.4       | 350     | 420     | 10000    | 5       | 2      | 5      | 5.10   |

To perform the optimization problem which is defined by Equations (27) and (28), a simulation experiment was designed and conducted. For the experiment, a 2-level factorial experiment with ¼ fraction was selected by Minitab@19 and a number of $2^{9-2} = 128$ tests were carried out. In this case, the Taguchi method which reduces the number of test is not used. The screening experiment used herein is not only a simple way to investigate the influence of nine factors (Table 1) on the response, but also propose the mathematical models instead of the Taguchi method. Table 2 shows various input parameters and the response (the optimum gear ratio $u_{1}$).

### 4. RESULTS AND DISCUSSIONS

#### 4.1. Effect of Main Design Factors

Figure 2 describes the effect of main design factors on the optimum gear ratio of the first step $u_{1}$. From this graph, it is clear that $u_{1}$ is greatly influenced by $u_{k}$. It increases sharply when $u_{k}$ increases. Besides, $u_{1}$ has a positive relationship...
with $AS_1$, $CG$, $CGh$, while it has an inverse relation with $X_{ba1}$, $X_{ba2}$, $AS_2$, $T_{out}$ and $CS$.

The relationship between the interactions of main design factors and $u_1$ can be understood by Figure 3. Indeed, it can be seen that in term of interactions, $CG$ and $CS$ have strong influences on the response in both low and high values, such as $HA \ (u_1*CG)$, $HB \ (X_{ba1}*CG)$, $HC \ (X_{ba2}*CG)$, $HD \ (AS_1*CG)$, $HE \ (AS_2*CG)$, $HF \ (T_{out}*CG)$, $HG \ (CG^*CG)$, $JA_1 \ (u_1*CG)$, $JB_1 \ (X_{ba1}*CG)$, $JC_1 \ (X_{ba2}*CG)$, $JD_1 \ (AS_1*CG)$, $JE_1 \ (AS_2*CG)$, $JF_1 \ (T_{out}*CG)$, $JG_1 \ (CG^*CG)$, $JH_1 \ (CG^*CG)$. However, $u_1$ has the strongest impact on $u_1$, but having the minor effect on the it when $u_1$ interact with other input parameters. These
are dominant when $u_t$ varies from 5 to 45 like AB, AC, AD, AE, AF, and AG.

The Normal plot of Standardized Effects was presented in Figure 4. This graph not only describes the relationship of the main design parameters to $u_t$, but it also indicates whether the relationship is positive or inverse. From the Plot, factors $u_1, C_B, C_{gh}, A_S_1$ and interactions AH, HJ, AD, AG, and FH have a positive effect. Besides, factors $C_S, A_S_2, F$ and AJ, AE, FG, and AF are inverse on $u_t$.

![Normal Plot of the Standardized Effects](image)

**Figure 4: Normal Plot of the Standardized Effects.**

### 4.2. Proposed Equation for Calculating $u_1$

To find a model to determine $u_1$, Minitab @ 19 was used for a regression process with two interactive elements and the significance of $\alpha = 0.05$. After neglecting unimportant influence factors, the predictable coefficients for $u_1$ are presented in Table 3. It was noted that the P-values of main design factors and interactions are less than $\alpha = 0.05$. That means these factors are robust influence on $u_1$. As a result, the proposed model for calculating $u_1$ is given as follow:

$$
\begin{align*}
    u_1 &= 0.96 + 0.218 \cdot u_t + 0.07 \cdot X_{m1} + 0.000119 \cdot A_S - 0.000444 \cdot A_S + 10^{-7} \cdot T_{out} + 0.128 \cdot C_{gh} \\
    &\quad - 0.0401 \cdot C_g - 0.191 \cdot C_E - 0.0973 \cdot u_t \cdot X_{pa1} + 0.000163 \cdot u_t \cdot A_S - 0.000248 \cdot u_t \cdot A_S^2 \\
    &\quad + 10^{-7} \cdot u_t \cdot T_{out} + 0.002205 \cdot u_t \cdot C_{gh} + 0.004012 \cdot u_t \cdot C_E - 0.000856 \cdot u_t \cdot C_E \\
    &\quad - 0.000008 \cdot T_{out} \cdot C_{gh} + 0.000004 \cdot T_{out} \cdot C_g - 0.01189 \cdot C_{gh} \cdot C_E + 0.0243 \cdot C_g \cdot C_E \cdot C_E
\end{align*}
$$

(29)

It is found that the test data is very suitable with the proposed equation as all of the values of R-square are more than 98% (Table 3). Therefore, this regression equation is great to use to determine $u_1$. After having $u_1$, the optimal gear ratio $u_2$ can easily be found by $u_2 = u_t / u_1$. 

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Table 3: Coded Coefficients for Proposed Model

| Term   | Effect | Coef   | SE Coef | T-Value | P-Value | VIF |
|--------|--------|--------|---------|---------|---------|-----|
| Constant | 4.4532 | 0.0193 | 231.04  | 0.000   | 0.000   | 1.00|
| ut     | 6.1430 | 3.0715 | 159.35  | 0.000   | 0.000   | 1.00|
| Xba1   | -0.1183 | -0.0591 | 0.0193 | -3.07   | 0.003   | 1.00|
| AS1    | 0.2933 | 0.1466 | 0.0193 | 7.61    | 0.000   | 1.00|
| AS2    | -0.4648 | -0.2324 | 0.0193 | -12.06  | 0.000   | 1.00|
| Tout   | -0.1702 | -0.0851 | 0.0193 | -4.41   | 0.000   | 1.00|
| Cgh    | 0.3042 | 0.1521 | 0.0193 | 7.89    | 0.000   | 1.00|
| Cg     | 0.8508 | 0.4254 | 0.0193 | 22.07   | 0.000   | 1.00|
| Cs     | -0.9055 | -0.4527 | 0.0193 | -23.49  | 0.000   | 1.00|
| ut*Xba1| -0.0973 | -0.0487 | 0.0193 | -2.53   | 0.013   | 1.00|
| ut*AS1 | 0.2200 | 0.1140 | 0.0193 | 5.91    | 0.000   | 1.00|
| ut*AS2 | -0.3470 | -0.1735 | 0.0193 | -9.00   | 0.000   | 1.00|
| ut*Tout| -0.0990 | -0.0490 | 0.0193 | -2.54   | 0.012   | 1.00|
| ut*Cgh | 0.1764 | 0.0882 | 0.0193 | 4.58    | 0.000   | 1.00|
| ut*Cg  | 0.5617 | 0.2809 | 0.0193 | 14.57   | 0.000   | 1.00|
| ut*Cs  | -0.5639 | -0.2820 | 0.0193 | -14.63  | 0.000   | 1.00|
| Tout*Cgh| -0.1636 | -0.0818 | 0.0193 | -4.24   | 0.000   | 1.00|
| Tout*Cg | 0.1236 | 0.0618 | 0.0193 | 3.21    | 0.002   | 1.00|
| Cgh*Cg  | -0.1664 | -0.0832 | 0.0193 | -4.32   | 0.000   | 1.00|
| Cg*Cs   | 0.2977 | 0.1488 | 0.0193 | 7.72    | 0.000   | 1.00|

Model Summary

| S      | R sq | R sq(adj) | R sq(pred) |
|--------|------|-----------|------------|
| 0.219068 | 99.61% | 99.54% | 99.45% |

Table 4: ANOVA for \( t_3 \)

| Source        | DF | Adj SS   | Adj MS   | F-Value | P-Value |
|---------------|----|----------|----------|---------|---------|
| Model         | 19 | 1303.42  | 68.60    | 1442.80 | 0.000   |
| Linear        | 8  | 1270.96  | 156.87   | 3340.85 | 0.000   |
| ut            | 1  | 1267.55  | 1267.55  | 25939.56| 0.000   |
| Xba1          | 1  | 0.45     | 0.45     | 9.41    | 0.003   |
| AS1           | 1  | 2.75     | 2.75     | 57.08   | 0.000   |
| AS2           | 1  | 6.91     | 6.91     | 145.41  | 0.000   |
| Tout          | 1  | 0.93     | 0.93     | 19.48   | 0.000   |
| Cgh           | 1  | 2.96     | 2.96     | 62.28   | 0.000   |
| Cg            | 1  | 23.16    | 23.16    | 487.08  | 0.000   |
| Cs            | 1  | 26.24    | 26.24    | 551.71  | 0.000   |
| 2-Way Interactions | 11 | 32.46    | 2.95     | 62.06   | 0.000   |
| ut*Xba1       | 1  | 0.30     | 0.30     | 6.38    | 0.013   |
| ut*AS1        | 1  | 1.68     | 1.68     | 34.97   | 0.000   |
| ut*AS2        | 1  | 3.85     | 3.85     | 81.04   | 0.000   |
| ut*Tout       | 1  | 0.31     | 0.31     | 6.48    | 0.012   |
| ut*Cgh        | 1  | 1.00     | 1.00     | 20.94   | 0.000   |
| ut*Cg         | 1  | 10.10    | 10.10    | 212.33  | 0.000   |
| ut*Cs         | 1  | 10.18    | 10.18    | 212.38  | 0.000   |
| Tout*Cgh      | 1  | 0.86     | 0.86     | 18.01   | 0.000   |
| Tout*Cg       | 1  | 0.49     | 0.49     | 10.28   | 0.002   |
| Cgh*Cg        | 1  | 0.89     | 0.89     | 18.63   | 0.000   |
| Cg*Cs         | 1  | 2.84     | 2.84     | 59.62   | 0.000   |

Model Summary

| S      | R sq | R sq(adj) | R sq(pred) |
|--------|------|-----------|------------|
| 0.219068 | 99.61% | 99.54% | 99.45% |
4.3. Analysis of Variance

The Analysis Of Variance (ANOVA) is carried out and the results are revealed in Table 4 where the weak influences are eliminated. From the table, it is visualized that the parameters of A, D, E, G, H, J, AE, AH, AJ, and HJ exhibit the F-value superior to 50. These have the static importance as all of the values of R-square are higher than 99%.

4.4. Validating Regression Equation

The error valuation between the experimental data and the regression equation of $u_1$ is designated in Figure 5. It can be seen from Figure 5a that the error contribution is very similar to the normal distribution. Figure 5b shows the relation between the residual and model fitted values. It is found that they are arbitrary data. That means the observation order depends on the control parameters. Furthermore, the random connection between the residual and the observation data is also random.

![Normal Probability Plot](response is $u_1$)

![Versus Order](response is $u_1$)

(a) (b)

**Figure 5:** Estimating Errors between Experiments and Regression Equation of $u_1$.

5. CONCLUSIONS

The present work deals with cost optimization design of THG with FSDG. In this work, the effects of key design factors on the optimal gear ratios were evaluated by designing and performing a simulation experiment. In addition, optimal gear ratios can easily be found using the suggested regression equations. Also, several conclusions are given as follows:

- The effect of $u_2$ on the optimal gear ratio of the first step $u_1$ is much more than that of other main design factors.
- The F-value of main design factors and their interactions of A, D, E, G, H, J, AE, AH, AJ, and HJ are significant for $u_1$.
- The suggested regression equation for calculating $u_1$ is extremely fit with the data of the experiment (all of the values of R-square are higher than 99%).

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