Holographic Charge Density Waves

Alexandros Aperis*, Panagiotis Kotetes*,†, Eleftherios Papantonopoulos*,
George Siopsis†, Petros Skamangkas*, and Georgios Varelogiannis*

*Department of Physics, National Technical University of Athens, GR-15780 Athens, Greece
†Department of Physics and Astronomy, The University of Tennessee, Knoxville, TN 37996 - 1200, USA

We discuss a gravity dual of a charge density wave consisting of a $U(1)$ gauge field and two scalar fields in the background of a Schwarzschild-AdS$_5$ black hole together with an antisymmetric field (probe limit). Interactions drive the system to a phase transition below a critical temperature. We numerically compute the ground states characterized by modulated solutions for the gauge potential corresponding to a dynamically generated unidirectional charge density wave in the conformal field theory. Signatures of the holographic density waves are retrieved by studying the dynamical response to an external electric field. We find that this novel holographic state shares many common features with the standard condensed matter version of charge density wave systems.

PACS numbers: 11.25.Tq, 04.70.Bw, 71.45.Lr, 71.27.+a

The AdS/CFT correspondence has become a powerful tool in studying strongly coupled phenomena in quantum field theory using results from a weakly coupled gravity background. According to this correspondence principle [1], a string theory on asymptotically AdS spacetime is dual to a conformal field theory on the boundary. Recently, following a more phenomenological approach, a plethora of condensed matter phenomena have been studied within this holographic principle, like the properties of superfluids and superconductors both conventional [2] and unconventional [3], Fermi liquid behaviour [4], non-linear hydrodynamics [5], quantum phase transitions [6] and transport [7]. Thus, classical gravity theories have been transformed into a laboratory for exploring condensed matter physics from a different perspective [8].

The most elucidated example of the application of the AdS/CFT correspondence to condensed matter physics is the holographic superconductor. The gravity dual of a superconductor consists of a system with a black hole and a charged scalar field coupled to a Maxwell field. The coupling induces a negative effective mass for the scalar field leading to an instability [9], which allows to the black hole to have scalar hair at temperatures smaller than a critical temperature [2]. In the boundary dual theory this corresponds to a phase transition at a finite chemical potential, where the operator dual to the charged scalar field condenses. Considering fluctuations of the vector potential and calculating the dynamical conductivity, are crucial for unveiling the properties of the condensed state.

Apart from superconductivity, understanding real condensed matter systems demands the introduction of additional ordered states and their holographic interpretation. The most important of them are undoubtedly charge and spin density waves (CDW and SDW) [10]. The development of these states corresponds to the spontaneous modulation of the electronic charge and spin density, below a critical temperature ($T_c$). Density waves are widely spread in different classes of materials and one may distinguish among them, either orbitally [11] or Zeeman driven [12] field-induced CDWs, confined [13] and even unconventional density waves [14]. The latter play a special role in condensed matter physics. Characteristic representatives are the d-density wave (DDW) [15] and most recently its chiral (d+id DDW) version [16], both of which have been proposed as prominent candidates for explaining the still unidentified pseudogap phase of the hole-doped cuprates [15, 16] and certain ‘hidden’ orders in heavy fermion compounds [16, 17]. Moreover, density waves are also significant due to their strong tendency to compete or coexist with superconductivity [18, 19].

In this Letter, we put forward a gravity dual of a conventional charge density wave. Our theory consists of the modulus and phase of a complex scalar field, a Maxwell field and an antisymmetric field. We numerically obtain ground states characterized by modulated solutions for the scalar potential corresponding to a CDW in the boundary dual theory. The solution for the scalar hair corresponds to a condensed dual operator which follows the usual Bardeen-Cooper-Schrieffer (BCS) dependence in the vicinity of $T_c$. Signatures of the holographic CDWs are retrieved by studying the collective modes and the dynamical response to an external electric perturbation. We find, in the dual boundary theory, a gapped phason-polariton collective mode [10, 20] with no Fröhlich supercurrent [21], which clearly shows that our CDW is intrinsically commensurate and non-sliding, a novel property stemming from the gravity-imposed pinning of the scalar potential to the phason. Our starting point for constructing the holographic CDW, is the following Lagrangian density in 3+1 dimensions

$$\mathcal{L} = \frac{1}{16\pi G} \left[ R + \frac{6}{L^2} - \frac{1}{6} H^2 \right]$$

$$- \frac{1}{4} F^2 - \frac{1}{2} (\partial_{\mu} \Phi)^2 - \frac{1}{2} m^2 \Phi^2 - \frac{\lambda_1}{2} \Phi^2 (\partial_{\mu} \partial^\mu)^2$$

$$- \frac{1}{2} \lambda_2 \Phi^2 (B - d\omega^{(2)})(A - d\omega^{(1)}) d\partial, \quad (1)$$

where $A_\mu$ is the Maxwell gauge field of strength $F = dA$, $B_{\mu\nu}$ is an antisymmetric field of strength $H = dB$ and
\(\omega^{(1)}, \omega^{(2)}\) are auxiliary Stückelberg fields. The last term is a topological interaction (independent of the metric). The presence of the Stückelberg fields is needed for the above Lagrangian to be gauge invariant as it can be easily checked if the following gauge transformations \(A \rightarrow A + dx^{(1)}, \omega^{(1)} \rightarrow \omega^{(1)} + \chi^{(1)}\), \(B \rightarrow B + dx^{(2)}\) and \(\omega^{(2)} \rightarrow \omega^{(2)} + \chi^{(2)}\) are applied to Eq. 11. We shall fix the gauge by choosing \(\omega^{(1)} = 0, \omega^{(2)} = 0\). Apart from the above gauge symmetries, our model is characterized by an additional global \(U(1)\) symmetry, \(\vartheta \rightarrow \vartheta + \alpha\), that corresponds to the translational invariance of this global symmetry. This global \(U(1)\) symmetry will be spontaneously broken for \(T < T_c\) due to impurities.

By varying the other fields, \(A_\mu, B_{\mu\nu}, \Phi\) and \(\vartheta\), we obtain the field equations, respectively,

\[
\nabla_\mu F^{\mu\nu} = \frac{1}{2} \lambda_2^2 J^\nu \\
\nabla_\mu H^{\mu\nu\rho} = 8\pi G \lambda_2^2 J^{\nu\rho} \\
\n\Box \Phi - m^2 \Phi = \lambda_2^2 (\partial_\mu \vartheta)^2 \Phi + \lambda_2^1 A_\mu J^\mu \\
\n\nabla^\mu (\Phi^2 \partial_\mu \vartheta) = \frac{\lambda_2^3}{2 \lambda_2^1} \nabla_\mu K^\mu
\]

where

\[
J^\mu = \Phi^2 \varepsilon^{\mu\nu\rho\sigma} B_{\nu\rho} \partial_\sigma \vartheta \\
J^{\mu\nu} = \Phi^2 \varepsilon^{\mu\nu\rho\sigma} A_\rho \partial_\sigma \vartheta \\
K^\mu = \Phi^2 \varepsilon^{\mu\nu\rho\sigma} B_{\nu\rho} A_\sigma
\]

To set the boundary conditions for the various fields, write the metric near the boundary of the asymptotically AdS space in terms of Poincaré coordinates as

\[
ds^2 \approx \frac{dr^2}{r^2} + r^2 (-dt^2 + dx^2 + dy^2)
\]

where we set \(L = 1\). Suppose that \(A_\mu = V\) and \(B^x y = -B^{y x}\). \(B^{x y} = -B^{y x}\) are the only non-vanishing components of \(A_\mu\) and \(B_{\mu\nu}\), respectively. We shall also concentrate on static solutions. In the asymptotic region \(r \to \infty\), the field equations \(\Box\) become

\[
V'' + \frac{2}{r} V' \approx 0 \\
B^{x y'} + \frac{4}{r} B^{x y} - \frac{1}{r^2} \partial_x B^{y x} \approx 0 \\
\Phi'' + \frac{4}{r^2} \Phi' - m^2 \Phi \approx 0 \\
\vartheta'' + 2 \left[ \frac{2}{r} + \frac{\Phi'}{\Phi} \right] \vartheta' \approx 0
\]

where we have used the notation \(\partial_r \equiv \prime\). The solutions are

\[
V = V^{(0)} + \frac{V^{(1)}}{r} + \ldots \\
B^{x y} = B^{(0)} + O(r^{-2}) , \quad B^{x y} = \frac{\partial_x B^{(0)}}{r^3} + \frac{B^{(4)}}{r^4} + O(r^{-5}) \\
\Phi = \frac{\Phi^{(\pm)}}{r^3} + \ldots , \quad \vartheta = \vartheta^{(0)} + O(r^{-1})
\]

where \(\Delta_\pm = \frac{3}{2} \pm \sqrt{\frac{3}{4} + m^2}\). The constant term in the expansion is the source of the corresponding field. Notice that if the source of the \(B^{\mu\nu}\) field \(B^{(0)}\) is spatially
dependent, there is an operator in the CFT dual to $B^{xy}$ that condenses. Moreover, in $B^{xy}$, the coefficient of $r^{-4}$ is arbitrary and represents the vacuum expectation value of a dual operator in the CFT. The latter is similar to the metric giving rise to the stress energy tensor of the boundary CFT. In what follows, we shall concentrate on the simple case in which both $\partial_x B^{(0)}$ and $B^{(4)}$ vanish.

We wish to solve the field equations in the probe limit which is obtained by letting Newton’s constant $G \to 0$. In this limit, the Einstein equations (2) and the equations for the $B^{\mu \nu}$ field (9) decouple. The latter become

$$\nabla_\mu H^{\mu \nu} = 0.$$ 

These equations can be solved by fixing any two-dimensional plane, therefore, by choosing the solution

$$B^{xy} = -B^{yr} = 1$$

with all other components vanishing, we shall fix the $(r, y)$ plane. This also allows us to set $B^{xy} = 0$ consistently in this limit on account of (7). We stress here that condensing does not change the particle number.

Having obtained the metric and the $B^{\mu \nu}$ field, the other field equations come from the Lagrangian

$$\mathcal{L}' = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} - \frac{1}{2} \partial_{\mu} \Phi \partial^{\mu} \Phi - \frac{1}{2} m^2 \Phi^2 - \frac{1}{2} \lambda^2 \Phi^2 \partial_{\mu} \partial^{\mu} \Phi - \lambda^2 \Phi^2 \varepsilon_{\mu \nu \rho \sigma} A^{\mu} \partial^{\nu} \Phi. \quad (10)$$

The resulting field equations are independent of $G$ and therefore well-defined in the probe limit $G \to 0$. In fact, in this limit, the last term is of the chiral anomaly type in a $t - x$ spacetime, with the fields depending parametrically on the radial coordinate. This Lagrangian also appears in the low energy effective action of the usual $(1+1)$-D CDWs [24], providing further support to the phenomenological origin of our theory.

The direction of the modulation is defined by the non zero components of $B^{\mu \nu}$. Therefore, the Lagrangian appearing in Eq. (10) shows that the direction of the modulation is parallel to the $x$-axis and it can be used to generate any possible profile of modulated charge density in the conformal field theory. Our aim is to show that the scalar potential $A_t \equiv V(r, x)$ asymptotically is of the form $V(r, x) \sim \mu(x) - \frac{\rho(x)}{r} + \ldots$, with $\mu(x) \sim \cos(kx)$ and $\rho(x) \sim \cos(kx)$. According to the AdS/CFT dictionary these two terms of this expansion will correspond in the dual boundary theory to a chemical potential $\mu(x)$ and a charge density $\rho(x)$, which are modulated. Consequently, all the fields will be $x$-dependent. The chemical potential will be dynamically generated. Thus, we shall be working with a canonical ensemble of fixed particle number. This is possible in our case because the operator that condenses does not change the particle number.

Varying the Lagrangian (10) we obtain the field equations

$$\Phi'' + \left(\frac{f'}{f} + 2 \frac{r}{r^2} \right) \Phi' + \frac{1}{r^2 f^2} \partial^2 \Phi - \left\{ \frac{m^2}{f} \right\} \Phi = 0,$$

$$\varphi'' + \left(\frac{f'}{f} + 2 \frac{1}{r} \frac{\Phi'}{\Phi} \right) \varphi' + \frac{1}{r^2 f^2} \partial^2 \varphi + \frac{2}{r^2 f^2} \left\{ \frac{\partial_{\mu} \Phi}{\partial_{\mu} \varphi} \right\} \varphi = 0,$$

$$V'' + \frac{2}{r^2 f^2} V + \frac{1}{r^2 f^2} \partial^2 V + \frac{2}{r^2 f^2} \partial_{\mu} \Phi \partial_{\mu} \varphi = 0,$$ 

which are also obtained from (9) with the background choices (5) and (9). In the first field equation there is a direct coupling of the scalar potential $V$ with the phase $\varphi$. This has important consequences as we will discuss in the following.

Suppose that the $x$-direction has length $L_x$. Then assuming periodic boundary conditions, the minimum wavevector is

$$k = \frac{2\pi}{L_x}. \quad (12)$$

Taking Fourier transforms, the form of the field equations suggests that it is consistent to truncate the fields by including $(2n + 1)k$-modes for $\varphi$ and $V$ and $2nk$ modes for $\Phi$, where $n \in \mathbb{Z}$. Thus

$$\varphi(r, x) = \sum_{n=0}^{\infty} \varphi_{(2n+1)k}(r) e^{i(2n+1)kx} + c.c.$$ 

$$V(r, x) = \sum_{n=0}^{\infty} V_{(2n+1)k}(r) e^{i(2n+1)kx} + c.c.$$ 

$$\Phi(r, x) = \frac{1}{2} \sum_{n=0}^{\infty} \Phi_{2nk}(r) e^{2nkx} + c.c. \quad (13)$$

We shall consider only the contributions of the leading $(n = 0)$ terms $\varphi_{2k}$, $V_{2k}$ and $\Phi_0$. Higher overtones can be included successively. At each step, we need to add equations for the new overtones which also introduce corrections to the lower Fourier modes already calculated in the previous step. Evidently, at each step the complexity of the system of coupled equations for the overtones of the three fields increases. We plan on carrying out this systematic procedure elsewhere. Ignoring overtones, the
field equations read
\[
\Phi''_0 + \left(\frac{f'}{f} + \frac{2}{r} + 2\right)\Phi_0 - \left\{ \frac{m^2}{f} + 2\lambda_1^2 \left( \partial_k \partial'_k - \frac{k^2}{r^2 f} \right) \right. \\
- 2ik\lambda_2^2 \frac{V_k \partial_{-k} - V_{-k} \partial_k}{f^2} \left. \right\} \Phi_0 = 0 , \tag{14}
\]
\[
\partial''_{\pm k} + \left[ \frac{f'}{f} + 2 \left\{ \frac{1}{r} + \Phi_0 \right\} \right] \partial'_{\pm k} - \frac{k^2}{r^2 f} \partial_{\pm k} = 0 , \tag{15}
\]
\[
V''_{\pm k} + \frac{2}{r} V'_{\pm k} - \frac{k^2}{r^2 f} V_{\pm k} \pm \frac{ik\lambda_2^2 k \partial_{\pm k} \Phi_0}{f} = 0 . \tag{16}
\]
We shall solve the above equations in the case where the mass of the scalar field is \( m^2 = -2 \), which is above the Breitenlohner-Freedman bound \cite{25}. We shall also fix the parameters \( \lambda_1 = \lambda_2 = 1 \). In order to find the boundary conditions at the horizon we take the near horizon limit \( r \to r_h \), where \( f(r) \approx f'(r_h)(r - r_h) \), and extracting the parts that become singular in the above equations we find that \( V_{\pm k}(r_h) = \partial_{\pm k}(r_h) = 0 \), \( \Phi_0(r_h) = m^2 \Phi_0(r_h)/3r_h \) and \( V_{\pm k}(r_h) = \mp i k V_k(r_h)/(3r_h^2) \). At the boundary, we have asymptotically \( \Phi_0(r) = \frac{(2c^2)}{r^4} \), where \( c = 1,2 \) in general, on account of \( (7) \). In this work we focus on \( c = 1 \) leaving the other case to a future study.

The fields \( \vartheta \) and \( V \) are also normalizable and behave asymptotically as in \( (7) \) with \( c = 1 \). The coefficients in their respective expansions have a sinusoidal dependence on \( x \). Thus we obtain in the dual boundary theory a single-mode CDW with a dynamically generated charge density of the form \( \rho(x) = V^{(1)} \sim \cos(kx) \). This case is important because a lot of materials exhibit either a single-mode CDW or only one dominant wave-vector. Moreover, the resulting model is numerically tractable and can provide direct insight on the phase transition and the response of this novel holographic state.

Having determined the behaviour of the fields both at the horizon and the boundary, we solve the five field equations \((13)-(16)\) numerically setting \( k = 1 \). We clearly obtain a second-order phase transition with a modulated scalar potential. Notice that a spatially modulated phase transition has been also observed in a five-dimensional Maxwell theory with a Chern-Simons term \cite{26}. In Fig.1 we plot the temperature dependence of the condensate \( \langle O_1 \rangle \) near the critical temperature. By fitting with \( \left| T - T_c \right|^\beta \), we find that the critical exponent for the transition is 0.5, i.e. of the BCS type.

The critical temperature is controlled by the value of the wave-vector \( k \). Its dependence on \( k \) can be deduced from the scaling symmetry of the system under the transformation \( t \to \xi t, x \to \xi x, y \to \xi y \), \( r \to r/\zeta \), \( V \to V/\zeta \). It follows that \( k/r_h \) is scale-invariant, therefore
\[
T_c \propto k \tag{17}
\]
We have checked the validity of this conclusion numerically. Eq. \((17)\) should be contrasted with the case of a holographic superconductor in which \( T_c \propto \sqrt{\mu} \).

![FIG. 1: Temperature dependence of the condensate (solid line). The dashed line is the BCS fit to the numerical values near \( T_c \). We find \( \langle O_1 \rangle \approx 8.5T_c(1 - T/T_c)^{1/2} \) near \( T \to T_c \).](image)

Also, notice that when the condensate \( \langle O_1 \rangle \) is zero, i.e. for temperatures above the critical temperature, the modulated chemical potential and the charge density vanish, and that they become non zero as soon as the condensate becomes non zero, i.e. when the temperature is lowered below \( T_c \). Therefore, the modulated chemical potential and the charge density are spontaneously generated and do not constitute fixed parameters of controlling \( T_c \), contrary to what happens in holographic superconductors \cite{22} with their homogeneous analogs \( \mu, \rho \). We also have to remark that the numerical solution of these five equations becomes unstable at low temperature. In fact, the fields diverge for \( T/T_c \lesssim 0.4 \). This numerical instability indicates that the probe limit breaks down at low temperatures and backreaction effects on the bulk metric become significant. A further analysis away from the probe limit is certainly needed.

We proceed with examining the collective excitations and response in the dual boundary theory by considering fluctuations of the electromagnetic and phason fields of the form \( A_\pm(r, x) \to \tilde{A}_\pm(r, x)e^{i(q x - \omega t)} \), \( V(r, x) \to V(r, x) + \tilde{V}_{\omega, q}(r, x)e^{i(q x - \omega t)} \) and \( \vartheta(r, x) \to \vartheta(r, x) + \tilde{\vartheta}_{\omega, q}(r, x)e^{i(q x - \omega t)} \). In the Lorentz gauge, we obtain the equations
\[
\tilde{\vartheta}'_{\omega, q} + \left[ \frac{f'}{f} + 2 \left( \frac{1}{r} + \Phi' \right) \right] \tilde{\vartheta}_{\omega, q} \tag{18}
\]
\[
+ \left( \frac{\omega^2}{f} - \frac{q^2}{r^2} \right) \tilde{V}_{\omega, q} + \left( \frac{\omega}{f} - \frac{q^2}{r^2} \right) \tilde{\vartheta}_{\omega, q} = 0 , \tag{19}
\]
\[
\tilde{A}'_{\omega, q} + \frac{f'}{f} \tilde{A}_{\omega, q} + \left( \frac{\omega^2}{f} - \frac{q^2}{r^2} \right) \tilde{A}_{\omega, q} = 0 . \tag{20}
\]
In order to get an insight of the response of the our system to these fluctuations we take the limit \( r_h \to 0 \). After
setting $z = 1/r$ and Fourier transforming $z \to p$, we obtain
\[ (\omega^2 - \xi^2 - p^2) \tilde{V}_{\omega,q}(p) + i q \tilde{V}_{\omega,q}(p) + i \omega \tilde{A}_{\omega,q}(p) = 0, \quad (21) \]
\[ -i \omega (\Omega_1)^2 \tilde{A}_{\omega,q}(p) + \omega p^2 \tilde{A}_{\omega,q}(p) = 0, \quad (22) \]
\[ -i \omega (\Omega_1)^2 \tilde{A}_{\omega,q}(p) + (\omega^2 - \xi^2 - p^2) \tilde{A}_{\omega,q}(p) = 0. \quad (23) \]
The above homogeneous system of equations defines completely the dynamics of the fluctuations through simply setting its determinant equal to zero. Hence, we get
\[ \omega^2 = q^2 + p^2, \quad \omega^2 = q^2 + p^2 + \omega_g^2 \pm \omega_g \sqrt{\omega^2 + 2p^2}. \quad (24) \]
where we have defined $\sqrt{2\omega_g} = \langle \Omega_1 \rangle$. Each Fourier $p$-mode of $\tilde{V}_{\omega,q}, \tilde{V}_{\omega,q}$, and $\tilde{A}_{\omega,q}$ will give a solution of the form $e^{i \xi p}$, with $p = p(\omega, q)$. Near infinity $e^{i \xi p} \sim 1 + i \xi p$, and according to the AdS/CFT correspondence, the first term with $p = 0$ corresponds to a source and the second to a current in the boundary theory. This implies that the dispersions of the collective modes must be calculated for $p = 0$, while for $p \neq 0$ we obtain the corresponding current that controls the response of the system such as the conductivity.

For $p = 0$, we obtain from Eq. (24) three energy dispersions $\omega = q, \omega = q, \omega = \sqrt{(\Omega_1)^2 + q^2}$, respectively.

The dynamical conductivity ($q = 0$) is dominated by strongly hybridized $\tilde{V}_{\omega,0}(p), \tilde{A}_{\omega,0}(p)$ fluctuations and is defined from Ohm’s law, according to the AdS/CFT correspondence, as $\sigma(\omega) = \tilde{A}_{\omega,0} / i \omega \tilde{A}_{\omega,0}$, where $\tilde{A}_{\omega,0}$ and $\tilde{A}_{\omega,0}$ are determined from the asymptotic expansion
\[ \tilde{A}_{\omega,0}(r) = \tilde{A}_{\omega,0}^{(0)} + \tilde{A}_{\omega,0}^{(1)} + \cdots. \]
The Fourier expansion $\tilde{A}_{\omega,0}(r) = \sum_p \tilde{A}_{\omega,0,p}(p)e^{i \xi p}$, implies that asymptotically
\[ \tilde{A}_{\omega,0}^{(1)} = i \sum_p p \tilde{A}_{\omega,0,p}(p) \quad \text{and} \quad \tilde{A}_{\omega,0}^{(0)} = \sum_p \tilde{A}_{\omega,0,p}(p). \]
Furthermore, we obtain from Eq. (24), for $q = 0$ in the second equation, that $p_{\pm, \pm} = \pm \sqrt{\Omega_1^2} (\Omega_1^2 / \omega_g)$, which correspond to two different branches for propagation direction (first sign). By choosing only the ingoing contributions $p_{-\pm, \pm}$ we obtain the dynamical conductivity
\[ \sigma(\omega) = \sum_{\pm} c_{\pm} \frac{e^{i \omega_{\pm} \Phi}}{\omega} \]
where $c_{\pm} = \tilde{A}_{\omega,0,p_{\pm, \pm}} / \tilde{A}_{\omega,0}^{(0)}$.

The factors $c_{\pm}$ will be determined by demanding that $\lim_{\omega \to \infty} [\sigma(\omega) - 1] \to 0$ faster than $1/\omega$, in order for the Kramers-Kronig relations to hold. The condition implies $c_{\pm} = \frac{1}{2}$ which we have verified numerically that also holds for finite temperatures. Kramers-Kronig relations require the fulfillment of the Ferrell-Glover-Tinkham (FGT) sum rule, dictating that $\int d\omega \text{Re}[\sigma(\omega)]$ is a constant independent of temperature. In the superconducting case, the FGT sum rule demands the presence of $\delta(\omega)$ in $\text{Re}[\sigma(\omega)]$, giving rise to a supercurrent. In our case, this rule is satisfied exactly, without the need of $\delta(\omega)$. The latter reflects the absence of the Fröhlich supercurrent, which may be attributed to the commensurate nature of the CDW.

We now present numerical results for the dynamical conductivity in the vicinity of $T_c$. In this limit, we are left to solve the system of the coupled equations Eq. (18) and (20). The stable ingoing boundary conditions at the horizon read $\tilde{A}_{\omega,0}(r_h) \propto \sum_p f^{-i \sqrt{\omega (\pm r_h \Phi) / 3r_h}}$ and $\tilde{A}_{\omega,0}(r_h) \propto \sum_p i \Phi f^{-i \sqrt{\omega (\pm r_h \Phi) / 3r_h}}$. On the first panel of Fig. 2 we plot the real conductivity versus $\omega / \langle \Omega_1 \rangle$ for different temperatures, while on the second, the same data are shown versus $\omega / T$. Notice how
the phason-polariton interplay results in a ‘dip’ structure which gets sharper as temperature decreases. Moreover, this feature moves to higher frequencies as the condensate value increases for lower temperatures. Upon raising the temperature, the ‘dip’ CDW fingerprint washes out, disappearing totally at $T = T_c$ where the normal state is restored.

In conclusion, we have proposed a holographic charge density wave. The dual boundary theory has the characteristic features of a commensurate CDW, with a gapped phason mode and the absence of a Fröhlich supercurrent. The commensurate behaviour due to the locking of the relative phase of the scalar potential and the phase field is an intrinsic property of the system and may be attributed to the presence of the black hole in the bulk. Moreover, the dynamical conductivity shows phason-polariton mixing. The properties of the holographic charge density wave certainly need further investigation, as for example the possibility of a multi-$k$ CDW, the effect of impurities, the response to an external magnetic field and its competition to superconductivity. Moreover, it would be interesting to examine this model away from the probe limit and whether this phenomenological model can be obtained from a consistent truncation of string theory.

Acknowledgements: We are grateful to A. Kehagias, G. Koutsoumbas, T. Kolyvaris, H. Soda, M. Tsoukalas, J. Zaanen and V. Zamarías for stimulating discussions. P.K. acknowledges financial support from the EU project NanoCTM. G. S. is supported by the US Department of Energy under grant DE-FG05-91ER40627. P. S. acknowledges support from the Operational Program “Education and Lifelong Learning” of the National Strategic Reference Framework (NSRF) - Research Funding Program: Heracleitus II, co-financed by the European Union (European Social Fund - ESF) and Greek national funds.

[1] J. Maldacena, Adv. Theor. Math. Phys. 2, 231 (1998); Int. J. Theor. Phys. 38, 1113 (1999); E. Witten, Adv. Theor. Math. Phys. 2, 253 (1998).
[2] S. A. Hartnoll, C. P. Herzog, and G. T. Horowitz, Phys. Rev. Lett. 101, 031601 (2008); J. High Energy Phys. 0812, 015 (2008); G. Siopsis, and J. Therrien, arXiv:1003.3275.
[3] S. S. Gubser and S. S. Pufu, JHEP 0811, 033 (2008).
[4] M. Cubrovic, J. Zaanen and K. Schalm, Science 325, 439 (2009).
[5] S. Bhattacharyya et al, JHEP 0802, 045 (2008).
[6] N. Iqbal et al, Phys. Rev. D 82, 045002 (2010).
[7] C. P. Herzog et al, Phys. Rev. D 75, 085020 (2007).
[8] G. Koutsoumbas, E. Papantonopoulos and G. Siopsis, JHEP 0907, 026 (2009); Q. Pan, B. Wang, E. Papantonopoulos, J. Oliveira and A. B. Pavan, Phys. Rev. D 81, 106007 (2010).
[9] S. S. Gubser, Class. Quant. Grav. 22, 5121 (2005); S. S. Gubser, Phys. Rev. D 78, 065034 (2008).
[10] G. Gruner, Rev. Mod. Phys. 60, 1129 (1988); Rev. Mod. Phys. 66, 1 (1994).
[11] L. P. Gorkov and A. G. Lebed, J. Phys. Lett. 45, L433 (1984); M. Héritier, G. Montambaux and P. Lederer, J. Phys. Lett. 45, L943 (1984).
[12] A. Aperis et al, Europhys. Lett. 83, 67008 (2008).
[13] G. Varelogiannis and M. Héritier, J. Phys.: Condens. Matter 15, L673 (2003).
[14] P. Thalmeier, Z. Phys. B 100, 387 (1996); C. Nayak, Phys. Rev. B 62, 4880 (2000).
[15] S. Chakravarty et al, Phys. Rev. B 63, 094503 (2001).
[16] P. Kotetes, and G. Varelogiannis, Phys. Rev. Lett. 104, 106404 (2010); P. Kotetes, A. Aperis and G. Varelogiannis, arXiv:1002.2719.
[17] H. Ikeda, and Y. Ohashi, Phys. Rev. Lett. 81, 3723 (1998); P. Chandra et al, Nature 417, 831 (2002); B. Dóra et al, Phys. Rev. B 71, 172502 (2005).
[18] A. M. Gabovich et al, Supercond. Sci. Technol. 14, R1 (2001).
[19] A. Aperis, G. Varelogiannis, and P. B. Littlewood Phys. Rev. Lett. 104, 216403 (2010).
[20] P. B. Littlewood, Phys. Rev. B 36, 3108 (1987).
[21] P. A. Lee, T. M. Rice, and P. W. Anderson, Phys. Rev. Lett. 31, 462 (1973).
[22] O. Aharony, Y. Oz and Z. Yin, Phys. Lett. B 430, 87 (1998); S. Minwalla, JHEP 9810, 002 (1998); E. Halyo, JHEP 9804, 011 (1998).
[23] S. A. Hartnoll and C. P. Herzog, Phys. Rev. D 77, 106009 (2008).
[24] Z. b. Su and B. Sakita, Phys. Rev. Lett. 56, 780 (1986); B. Sakita and K. Shizuya, Phys. Rev. B 42, 5586 (1990); V. M. Yakovenko and H. S. Goan, Phys. Rev. B 58, 10648 (1998).
[25] P. Breitenlohner and D. Z. Freedman, Annals Phys. 144, 249 (1982).
[26] S. Nakamura, H. Ooguri and C. S. Park, Phys. Rev. D 81, 044018 (2010); H. Ooguri and C. S. Park, arXiv:1007.5737.