LINEARIZED FIVE DIMENSIONAL

KALUZA-KLEIN THEORY AS A GAUGE THEORY

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Abstract

We develop a linearized five dimensional Kaluza-Klein theory as a gauge theory. By perturbing the metric around flat and the De Sitter backgrounds, we first discuss linearized gravity as a gauge theory in any dimension. In the particular case of five dimensions, we show that using the Kaluza-Klein mechanism, the field equations of our approach implies both linearized gauge gravity and Maxwell theory in flat and the De Sitter scenarios. As a possible further development of our formalism, we also discuss an application in the context of a gravitational polarization scattering by means of the analogue of the Mueller matrix in optical polarization algebra. We argue that this application can be of particular interest in gravitational waves experiments.

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1.- Introduction

It is known that linearized gravity can be considered as a gauge theory [1]. In this context, one may be interested in the idea of a unified theory of linearized gravity and Maxwell theory. This idea is, however, no completely new since in fact the quest of a unified theory of gravity and electromagnetism has a long history [2]. One can mention, for instance, the five dimensional Kaluza-Klein theory [3] which is perhaps one of the most interesting proposals. The central idea in this case is to incorporate electromagnetism in a geometrical five dimensional gravitational scenario. The gauge properties arise as a result of broken general covariance via a mechanism called spontaneous compactification. Symbolically, one may describe this process through the transition $\mathcal{M}^5 \to \mathcal{M}^4 \times S^1$, where $\mathcal{M}^5$ and $\mathcal{M}^4$ are five and four dimensional manifolds respectively and $S^1$ is a circle. Thus, after compactification the fiber-bundle $\mathcal{M}^4 \times S^1$ describes the Kaluza-Klein scenario. Let us picture this attempt of unification as

$$ em \to g, \quad (1) $$

where $em$ means electromagnetism and $g$ gravity.

In the case of linearized gravity theory the scenario looks different because it can be understood as a gauge theory rather than a pure geometrical structure. Therefore, a unified theory in this case may be understood as an idea to incorporate linearized gravity in a gauge Maxwell context. In other words, one may start from the beginning with a generalized fiber-bundle $\mathcal{M}^4 \times B$, with $B$ as a properly chosen compact space. Thus, we have that this case can be summarized by the heuristic picture

$$ em \leftarrow g. \quad (2) $$

Our aim in this paper is to combine the two scenarios (1) and (2) in the form

$$ em \leftrightarrow g. \quad (3) $$

Specifically, we start with linearized gravity in five dimensions and apply the Kaluza-Klein compactification mechanism. We probe that the resultant theory can be understood as a gauge theory of linearized gravity in five dimensions. Furthermore, we show that by using this strategy one can derive
a unified theory of gravity and electromagnetism with a generalized gauge field strength structure. As an advantage of our formalism, we outline the possibility that optical techniques can be applied to both gravity and electromagnetic radiation in a unified context. Thus, we argue that our results may be of particular interest in the gravitational waves detection.

Technically this article is organized as follows. In section 2, we develop linearized gravity in any dimension. In section 3, we discuss linearized gravity in a 5-dimensional Kaluza-Klein theory and in section 4 we generalize our procedure to a De Sitter background. In section 5, we outline a possible application of our formalism of unified framework of electromagnetic and gravitational radiation on an optical geometry via the Mueller matrix. In appendix A we briefly review the De Sitter background theory and in the appendix B we present a generalization to any dimension of the Novello and Neves work [4].

2.- Linearized gravity in any dimension

Let us consider a $1+d$-dimensional manifold $M^{1+d}$, with associated metric $\gamma_{AB}(x^C)$. We shall assume that $\gamma_{AB}$ can be written in the form

$$\gamma_{AB} = \eta_{AB} + h_{AB},$$

where $\eta_{AB} = \text{diag}(-1, 1, ..., 1)$ and $h_{AB}(x^C)$ is a small perturbation, that is

$$|h_{AB}| << 1.$$

To first order in $h_{AB}$, the inverse of the metric $\gamma_{AB}$ becomes

$$\gamma^{AB} = \eta^{AB} - h^{AB}.$$

Using (4) and (6) we find that the Christoffel symbol and Riemann curvature tensor are

$$\Gamma^A_{CD} = \frac{1}{2} \eta^{AB}(h_{BC,D} + h_{BD,C} - h_{CD,B}),$$

and

$$R_{ABCD} = \partial_A \mathcal{F}_{CDB} - \partial_B \mathcal{F}_{CDA},$$

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respectively. Here, the symbol $\mathcal{F}_{CDB}$ means
\[ \mathcal{F}_{CDB} = \frac{1}{2}(h_{BC,D} - h_{BD,C}). \] (9)

Observe that $\mathcal{F}_{CDB}$ is antisymmetric in the indices $C$ and $D$.

In terms of the quantity $\mathcal{F}_A$ defined by
\[ \mathcal{F}_A = \eta^{CB}\mathcal{F}_{ACB}, \] (10)
and the symbol $\mathcal{F}_{ADB}$, the Ricci tensor $R_{BD}$ reads
\[ R_{BD} = \partial^A\mathcal{F}_{ADB} + \partial_B\mathcal{F}_D. \] (11)

Thus, we get that the Ricci scalar $R$ is given by
\[ R = 2\partial^A\mathcal{F}_A. \] (12)

Substituting (11) and (12) into the Einstein weak field equations in $1 + d$ dimensions
\[ R_{BD} - \frac{1}{2}\eta_{BD}R = \frac{8\pi G_{1+d}}{c^2}T_{BD}, \] (13)
we find
\[ \partial^A\mathcal{F}_{ADB} + \partial_B\mathcal{F}_D - \eta_{BD}\partial^A\mathcal{F}_A = \frac{8\pi G_{1+d}}{c^2}T_{BD}, \] (14)
where $G_{1+d}$ is the Newton gravitational constant in $1 + d$ dimensions.

Let us now define the symbol
\[ F_{ADB} \equiv \mathcal{F}_{ADB} + \eta_{BA}\mathcal{F}_D - \eta_{BD}\mathcal{F}_A, \] (15)
which has the property
\[ F_{ADB} = -F_{DAB}. \] (16)

Thus, by using the expressions (15) we find that the field equations (14) are simplified in the form
\[ \partial^A F_{ABD} = \frac{8\pi G_{1+d}}{c^2}T_{BD}. \] (17)

Since
\[ \partial^A F_{ADB} = \partial^A F_{ABD}, \] (18)
the field equations (17) can also be written as
\[ \partial^A F_{A(BD)} = \frac{16\pi G_{1+d}}{c^2} T_{BD}. \] (19)
where the bracket \((BD)\) means symmetrization of the indices \(B\) and \(D\). It is worth mentioning that in a \(1 + 3\)-dimensional spacetime the field equations (19) are reduced to the ones proposed by the Novello and Neves [4].

3.- Linearized gravity in a five-dimensional Kaluza-Klein theory

In a 5-dimensional spacetime the weak field metric tensor \(\gamma_{AB} = \eta_{AB} + h_{AB}\), where \(\eta_{AB} = \text{diag}(-1, 1, 1, 1, 1)\), can be written in the block-matrix form
\[ \gamma_{AB} = \begin{pmatrix} \eta_{\mu\nu} + h_{\mu\nu} & h_{4\mu} \\ h_{4\nu} & 1 + h_{44} \end{pmatrix}, \] (20)
with \(\mu, \nu = 0, 1, 2, 3\). If one adopts the Kaluza-Klein ansatz, with
\[ h_{\mu\nu} = h_{\mu\nu}(x^\alpha), \] (21)
\[ h_{4\mu} = A_{\mu}(x^\alpha) \] (22)
and
\[ h_{44} = 0, \] (23)
where \(A_{\mu}(x^\alpha)\) is identified with the electromagnetic potential, we discover that the only nonvanishing terms of \(F_{DAB}\) are
\[ F_{\mu\nu\alpha} = \frac{1}{2}(h_{\alpha\mu,\nu} - h_{\alpha\nu,\mu}), \] (24)
\[ F_{4\nu\alpha} = \frac{1}{2}\partial_\nu A_\alpha, \] (25)
and
\[ F_{\mu\nu4} = -\frac{1}{2}F_{\mu\nu}, \] (26)
where \(F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}\) is the electromagnetic field strength. Thus, from (15) we find that the nonvanishing components of \(F_{DAB}\) are
\[ F_{\mu\nu\alpha} = F_{\mu\alpha\nu} + \eta_{\alpha\mu}F_{\nu} - \eta_{\alpha\nu}F_{\mu}. \] (27)
\[ F_{4\alpha} = \frac{1}{2}(\partial_{\nu}A_{\alpha} - \eta_{\alpha\nu}\partial^{\beta}A_{\beta}), \tag{28} \]

\[ F_{\mu4} = -\frac{1}{2}F_{\mu\nu}, \tag{29} \]

and

\[ F_{\mu44} = -F_{\mu}. \tag{30} \]

Now, since all fields are independent of the coordinate \( x^4 \) we see that the field equations (17) can be written as

\[ \partial^{\mu}F_{\mu BD} = \frac{8\pi G_{1+4}}{c^2}T_{BD}. \tag{31} \]

These field equations can be separated as follow

\[ \partial^{\mu}F_{\mu\nu\alpha} = \frac{8\pi G_{1+4}}{c^2}T_{\nu\alpha}, \tag{32} \]

\[ \partial^{\mu}F_{\mu 4\nu} = \frac{8\pi G_{1+4}}{c^2}T_{4\nu}, \tag{33} \]

\[ \partial^{\mu}F_{\mu\nu 4} = \frac{8\pi G_{1+4}}{c^2}T_{\nu 4}, \tag{34} \]

and

\[ \partial^{\mu}F_{\mu44} = \frac{8\pi G_{1+4}}{c^2}T_{44}. \tag{35} \]

Using (28) and (29), we find that (33) and (34) lead exactly to the same field equations, namely

\[ \partial^{\mu}F_{\mu\nu} = 4\pi J_{\nu}, \tag{36} \]

where \( J_{\nu} = \frac{4G_{1+4}}{c^2}T_{\nu 4} \) is the electromagnetic current. Of course, the field equations (32) and (36) correspond to linearized gravity and Maxwell field equations, respectively. If we set \( T_{44} = 0 \) then one can see that the field equation (35) is pure gauge expression. In fact if one assumes the transverse traceless gauge in five dimensions

\[ h^{AB} = 0, \]

\[ \dot{h} = \eta^{AB}h_{AB} = 0, \tag{37} \]
one discover that $F_{\mu4}$ is identically equal to zero.

By completeness let us observe that (37) leads to

$$h_{\mu\nu} = 0,$$

and the Lorentz gauge for $A_\mu$

$$A_\mu = 0.$$ (39)

Consequently one finds that, in the gauge (38) and (39), the field equations (32) and (36) reduces to

$$\Box^2 h_{\mu\nu} = -\frac{16\pi G}{c^2} T_{\mu\nu}$$ (40)

and

$$\Box^2 A_\mu = -4\pi J^\mu,$$ (41)

respectively, where $\Box^2 = \eta^\mu_\nu \partial_\mu \partial_\nu$ is the d’Alembertian operator. Thus, we have found a framework in which the gravitational and electromagnetic waves can be treated in the same footing.

4.- The De Sitter generalization

In order to generalize the formalism described in the previous section to a De Sitter scenario we shall replace the flat metric $\eta_{\mu\nu}$ by the de Sitter metric $f_{\mu\nu}(x^\alpha)$. In this case the perturbed Kaluza-Klein metric $\gamma_{AB}$ takes the form

$$\gamma_{AB} = \left( \begin{array}{cc} f_{\mu\nu} + h_{\mu\nu} & A_\mu \\ A_\nu & 1 \end{array} \right),$$ (42)

whose inverse is given by

$$\gamma^{AB} = \left( \begin{array}{cc} f^{\mu\nu} - h^{\mu\nu} & -A^\mu \\ -A^\nu & 1 \end{array} \right).$$ (43)

By combining the results from section 2 and appendix B it is not difficult to obtain the generalized field equations

$$D^A F_{ABD} - \frac{3}{2f^2} (h_{BD} - hf_{BD}) = \frac{8\pi G}{c^2} T_{BD}.$$ (44)
Thus, using (42) and considering that in five dimensions $d = 4$ and $\Lambda = \frac{6}{l^2}$ we find

$$D^\alpha F_{\alpha\mu\nu} - \frac{\Lambda}{4}(h_{\mu\nu} - hf_{\mu\nu}) = \frac{8\pi G_{1+4}}{c^2} T_{\mu\nu},$$

(45)

and

$$D' F_{\mu\nu} - \frac{\Lambda}{4} A_\mu = 4\pi J_\mu.$$ 

(46)

Here, we have used the fact that $D^4 F_{AB} = 0$ and $f_{4D} = 0$. Observe that in this case the electromagnetic field strength $F_{\mu\nu}$ is given by

$$F_{\mu\nu} = D_\mu A_\nu - D_\nu A_\mu.$$ 

The field equations (45) and (46) are remarkable because if we set

$$m^2 = \frac{\Lambda}{4}$$

(47)

one discovers that, up to factors, both the graviton and the photon have the same mass $m$ and even more intriguing is that such a mass is proportional to the square root of the cosmological constant.

5.- Final remarks

The present work may have a number of interesting developments. In particular, as Novello and Neves [4] have shown, the equation (15) can be derived from the formula

$$\partial^A F_{A(BD)} = 0,$$

(48)

where

$$^* F^A(BD) = \varepsilon^{ABEF} F^D_{EF} + \varepsilon^{ABEF} F^B_{EF},$$

(49)

with $\varepsilon^{ABEF}$ the completely antisymmetric symbol. Thus, one may consider an alternative approach [5] for duality aspects of linearized gravity [6] (see Refs. [7]-[11]) as in the case of Maxwell theory (see Refs. [12]-[13]).

Another source of physical interest of the present formalism is a possible connection with the Randall-Sundrum brane world scenario [14]-[15], with gravitational waves formalism (see [16] and references there in) and with
quantum linearized gravity [17]-[18]. Moreover, our work may be also useful to clarify some aspects about the relation between the mass of the graviton and the cosmological constant, which has been subject of some controversy [19]-[20].

Aside from theoretical developments, the present work opens also the possibility to make a number of applications for linearized gravity arising from the Maxwell theory itself. Let just outline one possibility. In Maxwell theory the concept of polarization scattering is of considerable interest in optical physics (see Ref. [21] and references therein). The subject of interest in this arena is to describe in a complex setting the interaction of polarized waves with a target. It turns out that the useful mathematical tool in the scattering radiation process is the so-called Mueller matrix [22] (see also Ref. [23] and references therein). What seems to be interesting about such a matrix is that its elements refer to intensity measurements only. Let us recall the main ideas for the construction of the Mueller matrix.

A polarized radiation field may be represented by a 2-dimensional complex field

\[
\begin{pmatrix}
E_x \\
E_y 
\end{pmatrix}.
\]

From the components of this vector one may define the Hermitian coherency matrix

\[
C = \begin{pmatrix}
E_x E_x^* & E_x E_y^* \\
E_y E_x^* & E_y E_y^*
\end{pmatrix}
\]

and the vector

\[
g = \begin{pmatrix}
g_0 \\
g_1 \\
g_2 \\
g_3
\end{pmatrix},
\]

where

\[
g_0 = E_x E_x^* + E_y E_y^*,
\]

\[
g_1 = E_x E_x^* - E_y E_y^*,
\]

\[
g_2 = (E_x E_y^* + E_y E_x^*) / \sqrt{2},
\]

\[
g_3 = (E_x E_y^* - E_y E_x^*) / \sqrt{2}.
\]
\[ g_2 = E_x^* E_y + E_x E_y^*, \]  
(55)  
and  
\[ g_3 = i(E_x^* E_y - E_x E_y^*). \]  
(56)  
The Jones matrix \( J \) and the Mueller matrix \( M \) apply to \( c \) and the vector \( g \), respectively as follows

\[ C' = JC \]  
(57)  
and  
\[ g' = Mg. \]  
(58)  
It turns out that \( J \) can be identified with a SU(2) matrix, while \( M \) is a 4 \( \times \) 4 matrix augmented form of \( O_3 \). Of course the matrices \( J \) and \( M \) must be related,

\[ M = \frac{1}{2} Tr J^T \sigma J \sigma, \]  
(59)  
where \( \sigma \) denotes the four Pauli matrices (see Ref. [22] for details).

Let us make the identification

\[ F_{0i\alpha} = E_{i\alpha}. \]  
(60)  
Here \( F_{0i\alpha} \) denotes some of the components of \( F_{ADB} \) according to the expression (15). The idea is now to consider the generalization of (50)

\[ \begin{pmatrix} E_{x\alpha} \\ E_{y\alpha} \end{pmatrix} \]  
(61)  
and to consider the analogue of (53)-(56), namely

\[ G_0 = E_x^\alpha E_{x\alpha} + E_y^\alpha E_{y\alpha}, \]  
(62)  
\[ G_1 = E_x^\alpha E_{x\alpha} - E_y^\alpha E_{y\alpha}, \]  
(63)  
\[ G_2 = E_x^{*\alpha} E_{y\alpha} + E_x^\alpha E_{y\alpha} \]  
(64)  
and

\[ G_3 = i(E_x^{*\alpha} E_{y\alpha} - E_x^\alpha E_{y\alpha}). \]  
(65)  
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In this way, we can apply the Mueller matrix $M$ as in (58); $G' = MG$. In fact, since $M$ contains the information of the intensities of both gravitational and electromagnetic radiation via the quantity $F_{0i\alpha}$ one should expect a broad kind of applications of $M$ in a gravitational wave context.

6.- Appendix A

In this appendix, we shall briefly review the De Sitter (anti-De Sitter) space, which provides us with one of the simplest solutions of Einstein field equations with cosmological constant. For this purpose let us consider the line element

$$ds^2 = \eta_{AB} dx^A dx^B + (dx^{d+1})^2,$$  \hspace{1cm} (66)

along with the constraint

$$\eta_{AB} x^A x^B + (x^{d+1})^2 = l^2,$$  \hspace{1cm} (67)

with $l$ being a constant, the indices $A, B$ running from zero to $(1 + d) - 1$, and $\eta_{AB}$ being a flat metric.

From Eq. (67) we get

$$(dx^{d+1})^2 = \frac{x_A x_B dx^A dx^B}{(l^2 - x_C x^C)}.$$  \hspace{1cm} (68)

Thus, Eq. (66) becomes

$$ds^2 = f_{AB} dx^A dx^B,$$  \hspace{1cm} (69)

where the metric $f_{AB}$ is given by

$$f_{AB} = \eta_{AB} + \frac{x_A x_B}{l^2 - x_C x^C},$$  \hspace{1cm} (70)

The inverse of $f_{AB}$ is found to be

$$f^{AB} = \eta^{AB} - \frac{x^A x^B}{l^2 - x_C x^C},$$  \hspace{1cm} (71)

Using (70) and (71) one discovers that the Christoffel symbols and the Riemann tensor are

$$\Gamma^A_{CD} = \frac{x^A}{l^2} f_{AB}$$  \hspace{1cm} (72)
and
\[ R_{ABCD} = \frac{1}{l^2}(f_{AC}f_{BD} - f_{AD}f_{BC}), \] (73)
respectively. Consequently, we find that \( f_{AB} \) is a solution of the vacuum Einstein field equations with cosmological constant \( \Lambda \),
\[ R_{AB} - \frac{1}{2}f_{AB}R + \Lambda f_{AB} = 0, \] (74)
provided the constants \( \Lambda \) and \( l^2 \) are related by
\[ \Lambda = \frac{(D-2)(D-1)}{2l^2}, \] (75)
where \( D = 1 + d \). Observe that in \( D = 4 \) (75) implies that \( \Lambda = \frac{3}{l^2} \). It is interesting to note that these results are independent of the signature of \( \eta_{AC} \).

7.- Appendix B

In this appendix we shall study the linearized gravity with \( f_{AB} \) as a metric background. Consider the perturbed metric
\[ \gamma_{AB} = f_{AB} + h_{AB}, \] (76)
where \( h_{AB} \) is a small perturbation, that is \( |h_{AB}| \ll 1 \). To first order in \( h_{AB} \) one finds
\[ \gamma^{AB} = f^{AB} - h^{AB}. \] (77)

The Christoffel symbols can be written in the interesting form
\[ \Gamma^A_{CD} = \Omega^A_{CD} + H^A_{CD}, \] (78)
where \( \Omega^A_{CD} \) are the Christoffel symbols associated with \( f_{AB} \) and \( H^A_{CD} \) is defined by
\[ H^A_{CD} = \frac{1}{2}f^{AE}(D_Ch_{DE} + D_Dh_{CE} - D_Eh_{CD}). \] (79)
Here, the symbol \( D_A \) denotes covariant derivatives with \( \Omega^A_{CD} \) as a connection.
By using (78) it is straightforward to check that the Riemann tensor can be written in the form

$$R_{ABCD} = \mathcal{R}_{ABCD} + D_Ch_{BD} - D_DH_{BC}, \quad (80)$$

where $\mathcal{R}_{ABCD}$ is the Riemann tensor associated with $\Omega_{CD}^A$.

With the help of the commutation relation

$$D_C D_D h_{AB} - D_D D_C h_{AB} = -\mathcal{R}^E_{ACD} h_{EB} - \mathcal{R}^E_{BCD} h_{AE} \quad (81)$$

we find that the Riemann curvature tensor (80) can also be written as

$$R_{ABCD} = D_A F_{CDB} - D_B F_{DAB} + \frac{1}{2} \mathcal{R}_{ABCD} h^{AC} + \frac{1}{2} \mathcal{R}_{EBD} h^{ED}. \quad (82)$$

Here, the symbol $F_{CDB}$ takes the form

$$F_{CDB} = \frac{1}{2} (D_D h_{BC} - D_C h_{BD}). \quad (83)$$

Observe that $F_{CDB}$ can be obtained from (9) by replacing ordinary partial derivatives by covariant derivatives. From (82) we get the Ricci tensor

$$R_{BD} = D^A F_{ADB} + D_B F_D + R_{BD} - \frac{1}{2} \mathcal{R}_{ABCD} h^{AC} + \frac{1}{2} \mathcal{R}_{EBD} h^{ED}. \quad (84)$$

where

$$F_A = f^{CB} F_{ACB}. \quad (85)$$

Thus, we find that the Ricci scalar $R$ is given by

$$R = 2D^A F_A + R - R_{BD} h^{BD}. \quad (86)$$

Substituting (84) and (86) into the Einstein weak field equations in $1 + d$ dimensions with cosmological constant

$$R_{BD} - \frac{1}{2} (f_{BD} + h_{BD}) R + (f_{BD} + h_{BD}) \Lambda = \frac{\kappa G_{1+d}}{c^2} T_{BD}, \quad (87)$$

we obtain

$$D^A F_{ADB} + D_B F_D - f_{BD} D^A F_A - \frac{1}{2} \mathcal{R}_{ABCD} h^{AC}$$

$$+ \frac{1}{2} \mathcal{R}_{EBD} h^{ED} + \frac{1}{2} f_{BD} \mathcal{R}_{EF} h^{EF} - \frac{1}{2} h_{BD} R + h_{BD} \Lambda = \frac{\kappa G_{1+d}}{c^2} T_{BD}. \quad (88)$$
Here, we used the fact that
\[ R_{BD} - \frac{1}{2} f_{BD} R + f_{BD} \Lambda = 0. \] (89)

Since we have
\[ R_{ABCD} = \frac{1}{l^2} (f_{AC} f_{BD} - f_{AD} f_{BC}) \] (90)

and
\[ \Lambda = \frac{d(d-1)}{2l^2}, \] (91)

we discover that (88) is reduced to
\[ D^A F_{ADB} + D_B F_D - f_{BD} D^A F_A \]
\[ - \frac{1}{2T}(d-1)(h_{BD} - h f_{BD}) = \frac{8\pi G_{1+d}}{c^2} T_{BD}. \] (92)

Thus, by defining the symbol
\[ F_{ADB} \equiv F_{ADB} + f_{BA} F_D - f_{BD} F_A, \] (93)

we find that the field equations (92) are simplified in the form
\[ D^A F_{ABD} - \frac{1}{2l^2} (d-1)(h_{BD} - h f_{BD}) = \frac{8\pi G_{1+d}}{c^2} T_{BD}. \] (94)

In four dimensions \( d = 3 \) and \( \Lambda = \frac{3}{l^2} \). Therefore (94) becomes
\[ D^A F_{ABD} - \frac{\Lambda}{3} (h_{BD} - h f_{BD}) = \frac{8\pi G_{1+d}}{c^2} T_{BD}. \] (95)

which are the field equations obtained by Novello and Neves [4].

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