Influence of the drag coefficient on the maximum pressure of water hammer

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Abstract. The article presents the results of theoretical and experimental studies of the effect of hydraulic friction on the maximum increase in pressure during water hammer. To solve the problem of determining hydraulic friction along a pressure pipe, the telegraph equation was adopted as a mathematical model. As a result of solving the adopted equation, dependence is obtained for calculating the energy loss in a non-stationary process. The calculations of the proposed formula are in good agreement with the experimental data of the author. This proves the legitimacy of the obtained dependence of the author and provides an accurate calculation of pipes for strength during impact and resource saving of pipe material.

1. Introduction

The study of water hammer in pressure of water conduits, including irrigation, hydropower and water supply systems, should be aimed at solving issues related to the refinement of methods for calculating water pipes for water pressure hitting.

At present, issues that need to be worked out on the physical essence of the following processes have not been studied, namely: the influence of the distribution of velocities over the living section, the inertia forces of the pipe walls, as well as the friction forces along the length of the pipeline on the magnitude and nature of the change in maximum pressure during water hammer.

For the first time, a theoretical solution to the water hitting problem taking into account the frictional resistance was given by N.E.Zhukovsky [1]: according to this solution, the entire pressure expended in friction was restored by the end of the first phase of the hitting of pressure. The work of N.E.Zhukovsky that the nature of the forces of friction resistance upon impact remains the same as with steady motion was adopted in further studies. However, this provision requires proof, since it is obvious that the formation of the velocity diagram in the pipeline does not occur instantly; while the hitting processes associated with changes in speed and pressure occur in a very short time interval.

Therefore, in principle, one should proceed from the premise that the resistance forces in a non-stationary process have a slightly different picture than the resistance in a stationary pressure mode, and in any case should be justified using other numerical parameters. Considerable work on the study...
of water hammer was carried out by V. S. Dikarevsky [2]. The author's research [2] led to the conclusion that for pipelines with slow closing of the valve, it is possible, within the accuracy sufficient for practice, not to take into account the change in the ordinate of the maximum impact pressure due to the influence of friction forces. This conclusion, however, the author makes under the assumption that all the resistance of the pipeline is concentrated at the gate valve, which is actually not permissible.

Further, the study of the phenomenon of hydraulic shock, taking into account forces, the friction resistance was done by G.I. Melkonyan [3]. To take into account the friction forces, the author [3] introduced correction factors. However, based on an insufficient number of experiments and lacking theoretical justification, the coefficients adopted turned out to be unrelated to the elasticity of the liquid and the pipe walls, and therefore the calculation method proposed by the authors gives a significant discrepancy between the experimental data and the values of analytical studies. In addition to the work of the experimental - theoretical direction, there are theoretical studies related to the forces of friction upon impact. Briefly, the results are as follows. A. A. Surin [4] accepts that water hammer recovers about 70% of the energy expended by the buildup. This assumption is in our opinion overpriced. The author [4] also offers an approximate method of arithmetic summation for calculating indirect impact, and believes that as the valve closes, the pressure expended on friction is gradually restored. It should be noted that the application of this method to the calculation of pipes with diameters less than 900 mm at speeds greater than 1.3 m/s gives some discrepancy with the experimental data. S. D. Chistopolsky [5] suggests that with a direct impact, half of all the pressure expended on friction is restored. I.A Charny [6], when integrating the equation of unsteady fluid motion in pipes, uses the telegraph equation, that is, the linear law of friction: the solution is given by the contour integration method. MA Mostkov [7], to take into account friction forces in the calculation of hydraulic shock by a numerical method, suggests introducing into the calculation scheme of “knots of resistance force” in which the friction resistance is concentrated. The calculation performed by this method makes it possible to approximately trace step by step the change in pressure in the pipes in individual nodes at different points in time. There are also graphical and analytical methods for calculating water hammer taking into account friction forces [8–15]. However, they are suitable only for relatively simple schemes and, in addition, are also based on the assumption that the nature of the friction forces upon impact remains the same as in the steady state [8–15].

2. Methods

The study of works in the field of water hammer in pressure pipes leads to the conclusion that it is necessary to clarify not only the methods of calculating the shock taking into account the friction forces, but also the magnitude of the resistance itself, which is restored during a hydraulic shock, since it is not at all obvious that the resistance forces in both of these types of motion are the same [16].

The experiment methodology was adopted according to [16, 17]. The following assumptions were made during the experiment: the pump unit shuts off instantly; the check valve plate closure time is zero; the water hammer problem is solved on the basis of the telegraph equation of mathematical physics taking into account the friction forces [16, 17, 18].

To accomplish this task, theoretical methods have been developed to establish the factors affecting the quenching of hitting pressure in pipes, and the coefficient of frictional force resistance along the length of the pipeline is determined to calculate the pressure loss in an unsteady mode during a water hammer.

For experimental verification of the proposed dependencies by us, a setup was created on which shock hydrodynamic pressures were recorded; diagrams of changes in shock pressure obtained by other authors were also used [19–24].
3. Results and Discussion

The experiments show that each subsequent value of the maximum pressure of the shock is less than the previous one, and, consequently, the process of moving the longitudinal waves of the hydraulic shock is decaying [16, 17].

A similar phenomenon occurs during the propagation of electrical vibrations through cables, which allows us to apply the telegraph equation of mathematical physics to the analysis of water hammer [16, 17, 18].

The differential equation of impact, taking into account the friction forces and after not taking into account due to the smallness of the convective term, can be written in the following form:

\[
\frac{dy}{dx} = \frac{1}{g} \frac{d\dot{\vartheta}}{dt} + \frac{\vartheta^2 \dot{\lambda}}{2gD},
\]

(1)

\[
\frac{dy}{dx} = \frac{1}{a^2} \frac{dy}{dt}.
\]

(2)

where \(y\) – pressure; \(\dot{\vartheta}\) – speed; \(\dot{\lambda}\) – coefficient of hydraulic resistance to friction; \(D\) – pipe diameter; \(t\) – time; \(g\) – gravity acceleration; \(a\) – water hammer wave velocity.

In small intervals of the change in velocity, we can assume \(k = \frac{\dot{\vartheta} \dot{\lambda}}{2gD} = \text{const}\), which is equivalent to the assumption of a linear dependence of resistance on speed in this interval. Differentiating equation (1) with respect to \(t\), equation (2) with respect to \(x\), introducing the notation \(c = \frac{kg}{a^2}; b = \frac{1}{a^2}\) and excluding from equation (1, 2) \(\frac{\partial^2 y}{\partial x \partial t}\), we write them in the form of a single equation:

\[
\frac{\partial^2 \vartheta}{\partial x^2} = b \frac{\partial^2 \vartheta}{\partial t^2} + c \frac{\partial \vartheta}{\partial t},
\]

(3)

which represents the so-called telegraph equation [19]. To simplify equation (3), we introduce the function

\[
\vartheta(x, t) = \exp(-\mu t)u(x, t).
\]

The constant must be chosen so that the term containing \(\frac{dn}{dt}\) disappears. Differentiating expression (4) with respect to \(x\) and \(t\) and substituting the values obtained for \(\frac{\partial^2 \vartheta}{\partial t^2}, \frac{\partial^2 \vartheta}{\partial x^2}\) and \(\frac{\partial \vartheta}{\partial t}\) into equation (3), grouping and reducing by \(e^{-\mu t}\), we obtain

\[
\frac{\partial^2 u}{\partial x^2} = \left(b \eta^2 - c \eta\right) u - (2\mu b - c) \frac{du}{dt} + b \frac{\partial^2 u}{\partial t^2},
\]

(5)

To fulfill the above conditions for the absence of a member \(c \frac{\partial u}{\partial t}\), so that when \(\frac{\partial u}{\partial t} \neq 0, 2b\mu - c = 0\)

or \(\mu = \frac{c}{2b}\).

(6)

Substituting the last expression in equation (5), we arrive at the wave equation in the form

\[
\frac{\partial^2 u}{\partial x^2} = -\frac{c^2}{4b} u + b \frac{\partial^2 u}{\partial t^2}.
\]

(7)
Since, as calculations show, 
\[
\frac{\partial^2 \varphi}{\partial x^2} + 2 \mu \frac{d\varphi}{dt} \gg 2 \mu^2 \varphi,
\]
then the magnitude \( \frac{c^2}{4b} \) can be neglected, and in this case, the above equation (7) becomes the usual equation of free undamped vibrations of the string. The general integral of the obtained equation according to the Dalamberbe principle is ethereal
\[
(x, t) = F(x - at) + f(x + at),
\]
Replacing now from (4) the expression \( n = \varphi \exp (\mu t) \) we rewrite the integral in the form
\[
\varphi(x, t) = \exp (-\mu t) [F(x - at) + f(x + at)].
\] (8)
Expression (8) contains a member \( e^{-\mu t} \), decreasing over time and therefore causing damping of oscillations, accelerating with increasing logarithmic decrement \( \mu \).
In view of (7), the shock equations written for pressures and velocities will take the form:
\[
y - y_0 = \exp [F(x - at) + f(x + at)] \exp (-\mu \tau)
\]
\[
y - y_0 = -\frac{g}{a} \exp [F(x - at) + f(x + at)] \exp (-\mu \tau),
\] (9)
where \( \tau \) – water hammer phase.
Considering the problem of the so-called simple pipeline, from the condition of complete reflection of the shock wave from the pool, we obtain the change in the ordinates of pressure from phase to phase in the form
\[
F_1 - F_0 = (y_1 - y_0) \exp (\mu \tau)
\]
\[
F_2 - F_1 = (y_2 - y_0) \exp (2\mu \tau)
\]
\[
F_n - F_{n-1} = (y_n - y_0) \exp (n\mu \tau)
\]
and accordingly to change the speed
\[
F_1 - F_0 = -\frac{a}{g} (\varphi_1 - \varphi_0) \exp (\mu \tau)
\]
\[
F_2 - F_1 = -\frac{a}{g} (\varphi_2 - \varphi_0) \exp (2\mu \tau)
\]
where \( For f \) - forward and backward waves of shock pressure. As a result of addition and subtraction of the corresponding members of the above equations, we obtain:
\[
2F_1 = (y_1 - y_0) - \frac{a}{g} (\varphi_1 - \varphi_0)
\]
\[
2F_1 = \left[ (y_2 - y_0) + \frac{a}{g} (\varphi_2 - \varphi_0) \right] \exp (2\mu \tau)
\]
Equating the last two expressions, as a result of the reduction by \( \exp (\mu \tau) \) determine for the first phase of the impact:
\[
y_1 - y_0 - \frac{a}{g} (\varphi_1 - \varphi_0) = \left[ (y_{i+1} - y_0) + \frac{a}{g} (\varphi_{i+1} - \varphi_0) \right] \exp (\mu \tau),
\]
where can the decrement value be determined

\[ \mu_i = \frac{1}{\tau} \frac{a}{g} \left( \frac{\theta_i - \theta_0}{g} - \left( y_i - y_0 \right) \right) \frac{a}{g} \left( \frac{\theta_{i+1} - \theta_0}{g} + \left( y_{i+1} - y_0 \right) \right). \] (10)

For the case of direct impact (phase of impact corresponds to \( \tau < \frac{2L}{a} \)) we have the boundary condition at the blunt end \( \theta_i = \theta_{i+1} = 0 \), thanks to which the above formula will take the form:

\[ \mu_i = \frac{1}{\tau} \frac{a}{g} \left( \frac{-\theta_0}{g} - \left( y_i - y_0 \right) \right) \frac{-\theta_0}{g} + \left( y_{i+1} - y_0 \right). \] (11)

As indicated (6), \( \mu = \frac{c}{2b} \) or as a result of a value substitution \( c \) or \( b \mu = k \frac{g}{2g} \). In turn \( k = \frac{g}{2g} \frac{\lambda}{D} \), so you can write

\[ \mu = \frac{\lambda_0}{g} \mu \] (12)

Thus, the problem can be reduced to finding the coefficient of hydraulic friction under unsteady conditions, which can be written from (12) as follows:

\[ \lambda_0 = \frac{8r_0}{\theta_0} \mu, \] (13)

where \( r_0 \) – hydraulic radius of pipe.

Having a decrement value \( \mu \), we can find the value of the coefficient of friction resistance with unsteady motion. To determine the values of \( \mu \) during hydraulic shocks, we carried out experiments, which are described, methods and procedures for the experiments in [16,17]. Moreover, diagrams of changes in impact pressures recorded by other authors were also used [19,20]. The experimental setup (Figure 1) consisted of the following elements: pump 1, grade K45 / 55, pressure pipe 6 (diameter \( d = 50 \) mm, length \( L = 250 \) m), after the pump a valve 2 and a plug valve 5 are installed, at the beginning and end the pressure pipe also has two pressure tanks (3.7) with capacities of 200 l and 220 l. Tanks are used to establish various geometric heights of water rise. The values of the working pressure (pressure) and pressure loss were determined using standard pressure gauges 4 and 8 of the grade MO 1227 accuracy class 0.15, which were installed on tanks 3 and 7.

![Figure 1. Experimental setup: 1 – pump; 2, 9 – gate valves; 3, 7 – pressure tanks; 4, 8 – manometers; 5, 10 – cork cranes; 6 – pressure pipe.](image-url)
Using the graph (figure 2), you can find the value \( \mu \), and consequently, the value of the resistance coefficient under unsteady mode \( \lambda_u \).

As shown by experiments conducted by us and other authors [19,20], the value of the coefficient of hydraulic resistance to friction in the unsteady mode turned out to be much larger than its value in the steady state. If for our experiences \( \lambda_u \) was of the order of 0.029, then its value in unsteady mode with a direct impact was equal for a small installation in the range from 0.129 to 4.219. The type of shock pressure diagrams obtained based on the use of the telegraph equation and the decrements calculated from it was checked by us as an example of calculating one of the indicator diagrams recorded on the experimental setup. Value \( \mu \) was determined by the formula (11). Knowing value \( \mu \) by the formula (12), we find the value \( \lambda_u \) and formula \( \Delta H = \frac{\lambda_u}{L} \frac{\partial^2}{2g} \) loss of pressure in unsteady mode. The forward and backward waves of pressure disturbances were determined using the equations below. When deriving the calculation equations, we proceed from the main dependences of the water hammer (9). After some conversions, we get:

\[
\exp(\mu \tau) \left[ y + \frac{a}{g} \right] g - \left( y_0 + \frac{a \rho_0}{g} \right) = 2f(x + at).
\]

If you take the elements of time \( t \) and distances \( x \) from section A to section B so, that \( x + at = \text{const} \), we will have:
\[
    t_1 + \frac{x_i}{a} = t_2 + \frac{x_s}{a} = t_j + \frac{x_i}{a}
\]

Consequently,
\[
    \left[\left(y + \frac{a \vartheta_0}{g}\right) - \left(y_0 + \frac{a \vartheta_0}{g}\right)\exp(\mu \tau_1)\right] = \left[\left(y + \frac{a \vartheta_0}{g}\right) - \left(y_0 + \frac{a \vartheta_0}{g}\right)\exp(\mu \tau_2)\right].
\]

Denote for the inverse function of the wave \( y + \frac{a \vartheta}{g} = \Omega \), then for \( i \) - th, phases:
\[
    \Omega_{b_i} = \left(\Omega_{\lambda_i} - \Omega_{\lambda_0}\right)\exp(-\mu \tau) + \Omega_{b_0}.
\]

Similarly for the direct wave function:
\[
    \pi_{b_i} = \left(\pi_{\lambda_i} - \pi_{\lambda_0}\right)\exp(-\mu \tau) + \pi_{b_0}.
\]

Using the above dependences, it is possible to carry out a numerical calculation of water hammer by the wave function method [7]. The calculation progress is presented in tabular form for an example with the following data: \( \tau_1 = 0.06 \) s, \( \mu = 1.89 \), \( a = 1030 \) m/s, \( \vartheta_0 = 0.3 \) m/s.

**Table 1.** Determination of water hammer parameters in design sections of the pipeline

| Water flow parameter | Time proportion in phases |
|----------------------|---------------------------|
|                      | 0                         | 1                         |
|                      | Section A | Section B | Section A | Section B |
| \( \pi \) [m]        | 5.6        | 6.3        | 67.6      | 6.3       |
| \( \Omega \) [m]     | 67.7       | 68.4       | 67.5      | 69.9      |
| \( H \) [m]          | 37.0       | 37.4       | 67.6      | 37.8      |
| \( \vartheta \) [m/s] | 0.30       | 0.30       | 0.30      | 0.30      |

Using value \( \lambda_0 = \frac{8 \vartheta_0}{\vartheta_0} \mu \) it is also possible to carry out calculations by the methods given in [7].

The calculation results are compared with the experimental value. The comparison establishes a good match for the first most dangerous phases of the impact; subsequent observable ordinates have slightly less value than the calculated ones. This can be explained by the fact that the condition
\[
    \left| 2 \mu^2 \vartheta \right| << \left| 2 \mu \frac{\partial \vartheta}{\partial t} + \frac{\partial^2 \vartheta}{\partial t^2} \right|
\]

Well maintained in the first phases, where a rapid change in speed occurs. In subsequent phases, where fluctuations in speed are reduced due to damping of water hammer, the accepted condition is violated. As is known, the greatest increase in hitting of pressure during water hammer in complex pipelines has a place, as a rule, in the first phases after complete closure.

**4. Conclusions**

Therefore, an accurate determination of the magnitude of the shock pressure in the first phases for water supply systems essentially solves the issue of calculating the strength pipes during impact.
1. As the experiments show, the attenuation of the wave oscillation during water hammer occurs much more intensively than by calculation based on the value of the hydraulic resistance coefficient of the stationary mode.

2. Based on the conducted experiments and processing of experimental works by other authors, the dependence of the decrement of the drag coefficient wave reduction on the water velocity and the elastic properties of pressure pipelines is obtained.

3. The values of the coefficient of hydraulic resistance along the length of the pressure pipe in the unsteady mode turned out to be much larger than its value in the steady state, which can be explained by the following reasons:
   a) the presence or penetration of a certain amount of air during the experiments through the joints of the pipes, which leads to a decrease in the velocity of propagation of the shock wave against its theoretical value and causes a loss of energy associated with a change in the volume of air inclusions;
   b) the release of air from a dissolved state and the reverse dissolution of its impact process with simultaneous dissipation of energy;
   c) an increase in resistances associated with the redistribution of velocities over the cross section over time, especially during the phase of rhythmic oscillations. It should be noted that the question of the type of flow motion during an unsteady process requires further experimental and theoretical research.

4. The presence of even a small amount of air in the pipes reduces the magnitude of the velocity of propagation of the shock wave, and hence the magnitude of the maximum pressure shock.

The proposed method can be recommended when calculating the water hammer in the designed pressure pipelines of pumping stations, hydroelectric power stations and closed irrigation systems to perform verification calculations of existing pressure systems.

Acknowledgements
This article is based on the contractual topic “Application of improved methods in the design of water facilities (No. GTI/56-19)“.

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