Some diagrams and basic formalism of the low energy kaon-hyperon interaction[

M. G. L. N. Santos† and C. C. Barros, Jr‡
Departamento de Física, CFM,
Universidade Federal de Santa Catarina
Florianópolis SC, CEP 88010-900, Brazil
(Dated: August 16, 2018)

In this work the low energy kaon-hyperon interaction is studied with nonlinear chiral invariant Lagrangians considering kaons, hyperons, and the corresponding resonances in the intermediate states. We show the basic formalism to calculate the total cross sections, angular distributions, and some diagrams of interest.

I. INTRODUCTION

In this work we show the mathematical procedure to calculate the diagrams for the low energy kaon-hyperon (KY) interactions considering a model [1] that has been used in the study of the low energy pion-hyperon (πY) interactions. When studying low energy kaon interactions, obviously the processes have different features and different particles must be considered in the intermediate states.

In this paper we just study the direct diagrams for each baryon resonance, and a complete calculation, taking into account the crossed diagrams and other possible interactions will be published soon.

We show how to calculate the total cross sections and angular distributions for KA and KS interactions using nonlinear chiral invariant Lagrangians in the center-of-mass frame.

We also expect that these calculations allow us to calculate other potentials like the NY, YY and NNY, which are very important in the studies of neutron stars and hypernuclei.

II. THE METHOD

To study the KY interactions we make an analogy with the pN interaction that is very well studied, many models and a large amount of experimental data are available. Here we shall use an effective chiral model for the pN scattering and then extend it to the kaon-hyperon case.

The chiral Lagrangians [2] of the πN interaction with spin 1/2 and 3/2 baryons are given by

\[ \mathcal{L}_{\pi NN} = \frac{g}{2m} (N \gamma_{\mu} \gamma_5 \pi N) \cdot \partial^\mu \varphi , \]

\[ \mathcal{L}_{\pi N\Delta} = g_{\Delta} \left( \bar{\Delta} \left[ g_{\mu\nu} - \left( Z + 1/2 \right) \gamma_\mu \gamma_\nu \right] M N \right) \partial^\mu \varphi, \]

where N, Δ, \varphi are the nucleon, delta, and pion fields with masses m, m_Δ and m_π, respectively. M and \varphi are isospin matrices, and Z is a parameter representing the possibility of the off-shell-Δ having spin 1/2. The parameters g and g_Δ are coupling constants and depend on each intermediary particle.

We calculate the Feynman amplitudes and sum all the isospin contributions to obtain the T_{KY} amplitude, that is given by

\[ T_{KY} = \sum_I T^I P_I, \]

with

\[ T^I = \bar{\pi}(\vec{p}) \left[ A^I + \frac{1}{2} (k^+ \vec{k}^') B^I \right] u(\vec{p}), \]

where the subscript (KY) represents the initial particles, a spin 0 kaon and a spin 1/2 hyperon. P_I are the projector operators of total isospin states, T^I, the respective amplitudes, u(\vec{p}) is a spinor representing the initial baryon with \vec{p} momentum and k is the meson four-momentum. So we can calculate the A_I and B_I amplitudes for each total isospin channel in the scattering.

The scattering matrix for a given isospin state is

\[ M^I = \frac{T^I}{8\pi\sqrt{s}} = G^I + H^I i\vec{\sigma} \cdot \vec{n}, \]

which may be decomposed into the spin-non-flip and spin-flip amplitudes G(\theta) and H(\theta), and then expanded in partial-wave amplitudes

\[ G^I = \sum_{l=0}^{\infty} \left[ (l+1) F^I_{l+} + l F^I_{l-} \right] P_l(\theta), \]

\[ H^I = \sum_{l=1}^{\infty} \left[ F^I_{l-} - F^I_{l+} \right] P_l^{(1)}(\theta). \]

The partial-wave amplitudes are, using the Legendre polynomial’s orthogonality relations

\[ F_{l \pm}^I = \frac{1}{2} \int_{-1}^{1} \left[ P_l(\theta)f_{l \mp}^I(\theta) + P_{l \pm}(\theta)f_{l \mp}^I(\theta) \right] d\theta, \]
where
\[ f_l^i(\theta) = \frac{(E + m)}{8\pi\sqrt{s}} [A^l + (\sqrt{s} - m)B^l], \quad (9) \]
\[ f_l^j(\theta) = \frac{(E - m)}{8\pi\sqrt{s}} [-A^l + (\sqrt{s} + m)B^l], \quad (10) \]
where \( E \) is the hyperon energy in the center-of-mass frame and \( s \) is a Mandelstam variable. At low energies, we can consider just the \( S \) and \( P \) waves in a first approximation, which are described by the subscripts \( l \) (\( l = 0 \) and \( l = 1 \)), in the above expressions.

The obtained amplitudes are real, consequently the unitarity of the \( S \) matrix is violated. So, we unitarize the amplitudes with
\[ F_{l\pm} = \frac{F_{\pm}}{1 - ikF_{\pm}}. \quad (11) \]

In the center-of-mass frame the differential cross sections are
\[ \frac{d\sigma}{d\Omega} = |G|^2 + |H|^2, \quad (12) \]
and integrating this expression over the solid angle we obtain the total cross sections
\[ \sigma_T = 4\pi \sum_l [(l + 1)|F_{l+}|^2 + l|F_{l-}|^2], \quad (13) \]
of the reactions of interest.

III. KA INTERACTION

Since the \( \Lambda \) has isospin 0, in the \( KA \) interaction we must consider \( P_l = 1 \). For the diagram (a) in Fig.1, we have, for spin-1/2 the amplitudes
\[ A_N = \frac{g_{AKN}^2}{4m_\Lambda^2} (m_N + m_\Lambda) \left( \frac{s - m_\Lambda^2}{s - m_N^2} \right), \quad (14) \]
\[ B_N = -\frac{g_{AKN}^2}{4m_\Lambda^2} \left[ \frac{2m_\Lambda(m_N + m_\Lambda) + s - m_\Lambda^2}{s - m_N^2} \right], \quad (15) \]
where \( m_N \) is the nucleon (or a spin-1/2 resonance) mass and \( g_{AKN} \) are the coupling constants.

For Fig.1 (b) the spin-3/2 amplitudes are
\[ A_{N^*} = \frac{g_{AKN}^2}{6} \left\{ \left[ \dot{A} + \frac{3}{2} (m_\Sigma + m_{N^*}) t \right] \left[ \frac{2}{m_{N^*}^2 - s} \right] + a_0 \right\}, \quad (16) \]
\[ B_{N^*} = \frac{g_{AKN}^2}{6} \left\{ \left[ \dot{B} + \frac{3}{2} t \right] \left[ \frac{2}{m_{N^*}^2 - s} \right] - b_0 \right\}, \quad (17) \]
where
\[ \dot{A} = \frac{(m_{N^*} + m_\Lambda)^2 - m_K^2}{2m_{N^*}^2} \]
\[ -2m_\Lambda m_{N^*}(m_\Lambda + m_{N^*})^2 - 2m_K^2 (m_\Lambda + m_{N^*})^2 \]
\[ + 6m_K^2 m_{N^*} (m_\Lambda + m_{N^*}) + m_K^4 \], \quad (19) \]
\[ a_0 = -\frac{(m_\Sigma + m_\Delta)(2m_\Delta^2 + m_\Sigma m_\Delta - m_\Sigma^2 + 2m_K^2)}{m_\Delta^2} \]
\[ + \frac{4}{m_\Delta^2} \left[ (m_\Delta + m_\Sigma)Z + (2m_\Delta + m_\Sigma)Z^2 \right] \]
\[ \times \left[ s - m_\Sigma^2 \right], \quad (20) \]
\[ b_0 = \frac{8}{m_\Delta^2} \left[ (m_\Sigma^2 + m_\Sigma m_\Delta - m_K^2)Z + (2m_\Sigma m_\Delta + m_\Sigma^2)Z \right] \]
\[ + \frac{(m_\Sigma + m_\Delta)^2}{m_\Delta^2} + 4Z^2 \left[ s - m_\Sigma^2 \right]. \quad (21) \]
where \( m_{N^*} \) and \( m_K \) are the spin-3/2 resonance and the kaon masses, respectively, \( g_{N^*} \) are the coupling constants and \( t \) is a Mandelstam variable.

IV. KS INTERACTION

A similar approach has been made in order to study the \( KS \) interaction. The diagrams for the \( N, N^* \) and \( \Delta \) resonances exchanges are shown in Fig.2. For these interactions we have different isospin projectors, \( P_2 = \frac{1}{2} \delta^{ab} + \frac{i}{4} \epsilon_{abc} v^c \) and \( P_3 = \frac{2}{3} \delta^{ab} - \frac{i}{4} \epsilon_{abc} v^c \) for the 1/2 and 3/2 isospin channels, where \( a \) and \( b \) are the isospin states of the \( \Sigma \) hyperon.

For instance in the amplitudes for the \( \Delta \) resonance exchange, considering the relation \( M\delta_M M_\delta = \frac{2}{3} \delta^{ab} + \frac{i}{4} \epsilon_{abc} v^c \), we have
\[ A_\Delta^+ = \frac{g_{AKN}^2}{9} \left\{ \left[ \dot{A} + \frac{3}{2} (m_\Sigma + m_\Delta) t \right] \left[ \frac{2}{m_\Delta^2 - s} \right] + a_0 \right\}, \quad (22) \]
\[ \langle K^0\Sigma^0|T|K^0\Sigma^0 \rangle = \langle K^+\Sigma^-|T|K^+\Sigma^- \rangle = 2 \frac{T_3}{3} + \frac{1}{3} T_{\frac{1}{2}}, \tag{28} \]

\[ \langle K^0\Sigma^0|T|K^+\Sigma^- \rangle = \langle K^0\Sigma^0|T|K^+\Sigma^- \rangle = \frac{\sqrt{2}}{3} (T_3 - T_{\frac{1}{2}}). \tag{29} \]

The amplitudes for each isospin channel are calculated by using the expressions for isospin-1/2

\[ A^+ = A^+ + 2A^-, \tag{30} \]
\[ B^+ = B^+ + 2B^-, \tag{31} \]

and for isospin-3/2

\[ A^\frac{3}{2} = A^+ - A^-, \tag{32} \]
\[ B^\frac{3}{2} = B^+ - B^-, \tag{33} \]

where the last expressions are used just for the \( \Delta \) resonance exchange.

V. CONCLUSIONS

The model for the KY interactions studied in this work, based in non-linear Lagrangians and written in the partial wave formalism, presents the same simplicity that is found in the formulation of the \( \pi N \) and \( \pi Y \) interactions \[1\].

In a further paper we will show the complete results of the observables, cross sections, polarizations and phase-shifts of the low energy KY interactions considering also the exchange of \( \rho \) and \( \sigma \) mesons and the crossed diagrams \[4\], not presented in this work.

Another result of interest is the \( D \)-wave phase-shift for the \( K\Lambda \) interaction at the \( \Omega \) baryon mass, that may be used in the study of the \( CP \) violation \[2\] in the \( K\Lambda \to \Omega \) decay.

[1] C. C. Barros and Y. Hama, Phys. Rev. C 63, 065203 (2001).
[2] H.T. Coelho, T.K. Das and M.R. Robilotta, Phys. Rev. C 28, 1812 (1983).
[3] M. G. Olsson and E.T. Osypowski, Nucl. Phys. B 101, 136 (1975).
[4] M. G. L. N. Santos, Master’s thesis, Universidade Federal de Santa Catarina (2018).
[5] C. C. Barros Jr., Phys. Rev. D 68, 034006 (2003).