Modelling of transverse shrinkage of injection-moulded parts using experimental methods and fuzzy logic theory

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Abstract. The method of building a mathematical model of fuzzy structure and the set of its rules, on the basis of experimental results is presented. The developed method is illustrated on the example of modelling the relationship of the transverse shrinkage of HDPE injection-moulded part (moulding) versus the shape of the moulding and the conditions of the injection moulding process. The developed shrinkage model can be helpful at the design stage of the moulding geometry, as well as mould cavity and the runner system. Due to the fuzzy structure, the model can be easily modified and generalised into similar moulding injection processes. It is also a good starting point for the optimisation of the process, aimed at compensating the shrinkage value in the entire volume of the moulding by adjusting the pressure profile in the holding phase of moulding injection cycle.

1. The purpose of mathematical modelling

In principle, two types of mathematical models are distinguished: phenomenological and experimental (behavioural). The purpose of creating the first one is to learn about the internal structure and mechanisms governing the modelled system. In contrast, experimental models are primarily created to give formal description of relationships between causal (input) variables and the effects of their interaction on the system (output variables). Experimental models can be used to approximate the system’s response to input variables with values other than the values used to build the model [1, 2]. Therefore, experimental models are called equivalent: “approximation models” or “cause-and-effect models”.

Approximation models are usually built on the basis of the results of a series of experiments (previously planned or not) in which the values of input variables and the corresponding values of the output signals from the system are measured. Next, the mathematical function is matched to the collected data sets, which approximates the experimental data with the smallest error. For practical reasons, the most popular approximation is a polynomial of 2nd or 3rd degree (mostly). An alternative to the approximation with an explicit equation is the construction of artificial neural network. Models of this form allow approximation of very complex relationships, among others: interactions of input variables and strong nonlinearities inside the modelled system.

A slightly different approach to system modelling may be noticed in case of applying the theory of fuzzy logic. Models with fuzzy structure are something intermediate between phenomenological and experimental models. They allow to aggregate in one model the impartial (but often incomplete) knowledge of the mechanisms governing the modelled system and the results of observing behaviour
of the system, i.e. purely experimental knowledge; thus, so-called “expert knowledge” [3, 4]. The advantage of this approach is the ability to model very complex relationships, and to include in the model many partial relations between variables in the form of the so-called “rules”. The disadvantage of the model constructed in this way is its “fuzziness”, resulting in an approximate estimation of the values of output variables from the system [5]. The reason of this is, that both the knowledge base (rules) of the modelled system and the mechanism of inference from rules operate on approximate values, just like: “very small”, “small”, “close to zero” etc.

The paper presents the original method of building a fuzzy model (its set of rules) on the basis of experimental results. The developed method is illustrated on the example of modelling the relationship of the transverse shrinkage value of HDPE injection-moulded part (moulding) against the shape of the moulding and the conditions of the injection moulding process.

2. Polymer shrinkage phenomenon

The shrinkage (equivalent: after-shrinkage) is a phenomenon of reduction of moulding dimensions due to changes occurring during the moulding injection process and later. The primary shrinkage of thermoplastics occurs in the mould cavity [6, 7]. Changes in the specific volume of the material during cooling and phase transitions (solidification, crystallisation) are the reason in this case. The secondary shrinkage is caused by stress relaxation processes inside the solidified moulding which may go on for dozens of days after injection [8].

Shrinkage is usually an undesirable phenomenon and is the cause of dimensional imprecision of mouldings, surface collapse, distortion and deformation [10, 11]. Therefore, it must be considered already at the stage of designing the geometry of the part produced and the injection mould. The FEM computer simulations of filling the mould cavity with the liquefied polymer are widely used on this stage [12, 13, 14, 15, 16].

The value of shrinkage may be intentionally controlled, mainly during the holding phase of moulding injection cycle [17, 18]. Then, the shrinkage changes are compensated by feeding an additional volume of plastic to the molten core of the moulding. It is possible only until the whole moulding or plastic in the runner or in the gate is completely solidified [7, 18, 19, 20].

3. The general form of shrinkage model

The developed model of shrinkage was necessary for the development of decision algorithms (rules defining the adjusting of the injection process settings) designed for the equalisation of the shrinkage value in the entire volume of the injection moulding [21]. A necessary condition for effective shaping of shrinkage is the knowledge of its dependence on the main control variables of the injection process and the geometry of moulding. It was necessary to develop a model of transverse shrinkage \( S_p \) of an injection moulded part of HDPE in the function of three independent variables: nominal wall thickness \( d_n \), distance from the gate \( x \) and the pressure in the holding phase \( p_d \):

\[
S_p = f(d_n, x, p_d)
\]

where transverse shrinkage \( S_p \) is defined as the ratio of the difference in the dimension of the cavity \( d_n \) and the actual wall thickness \( d \) of the moulding to \( d_n \):

\[
S_p = \frac{(d_n - d)}{d_n} \times 100\%
\]

Due to the complexity of the physical phenomena that determine the shrinkage, as well as their dynamic and multidimensional character, it was decided that the model (1) will be built using experimental techniques.

4. Moulding characteristics

The shrinkage model (1) was developed for a series of thin-walled beams with graduated thickness (table 1, figure 1) produced from HDPE. The beams consisted of up to three sections with intentionally varying thickness along their axis. Five different thickness profiles were chosen:
“constant”, “ascending”, “descending”, “thin–thick–thin” and “thick–thin–thick” according to direction of polymer flow. The mould cavity was filled with plastic through the slit–shape gate located at the shorter side of the beam (figure 1). Two identical symmetrically located cavities and runners were made in the mould.

5. Experimental results

For each of the five types of beams (table 1), a series of injection cycles was made with three different holding pressure values $p_d$: 15, 30 and 60 MPa. The value of transverse shrinkage ($S_p$) was measured at three points located on the longitudinal axis of the beam (bottom of table 1), corresponding to the centres of its three segments of different thickness, spaced from the gate by $x = 25, 75$ and 125 mm, respectively. The results of the measurements are shown graphically in figure 2. Numbers in the table grids lying at the intersections of columns and rows ($d_n$ and $x$) are the mean values of transverse shrinkage $S_p$ measured in the respective beam segments. The location of the number in the table cell indicates which type of moulding (of which thickness profile) the result is from. Each of three tables contains the results obtained for three different holding pressures. Negative values of shrinkage mean that the thickness of the moulding was greater than the corresponding dimension of the cavity (effect of high holding pressure).

| Mark | Dimensions | Thickness profile |
|------|------------|------------------|
| 4–4–4 | ![Diagram](image1) | constant |
| 2–4–6 | ![Diagram](image2) | ascending |
| 6–4–2 | ![Diagram](image3) | descending |
| 2–6–2 | ![Diagram](image4) | thin–thick–thin |
| 6–2–6 | ![Diagram](image5) | thick–thin–thick |

Comparing the shrinkage values $S_p$ of beams segments with the same nominal thickness $d_n$ and distance from the gate $x$, but belonging to mouldings of different thickness profiles (i.e. within one table cell of figure 2), it can be concluded that the values of transverse shrinkage are similar to each other, except for the segments of beams with a 2–4–6 profile (with a grey background). Hence,
shrinkage values of respective beam segments for all four (2–6–2, 4–4–4, 6–2–6, 6–4–2) thickness profiles, i.e. excluding results for 2–4–6 profile, were averaged and shown in figure 3.

**Figure 1.** 3D model of injection moulded mould for shrinkage study.

Variation in transverse shrinkage along the axis of the beam with a thickness profile of 2–4–6 for three holding pressure $p_d$ values is shown in figure 4. Only for this shape of the moulding an untypical tendency of the shrinkage increase versus the increase of the holding pressure was observed (in the middle beam segment of thickness $d_n = 4$ mm).

| $x$ [mm] | $d_n$ [mm] | $p_d = 150$ [bar] | $p_d = 300$ [bar] | $p_d = 500$ [bar] |
|----------|------------|------------------|------------------|------------------|
|          |            | 2,5              | 4                | 6                |
| 25       | 3.3        | 1.5              | 3.8              | 8.4              |
| 75       | 4.3        | 5.9              | 12.6             |
| 125      | 5.3        | 7.9              | 15.0             |

| $x$ [mm] | $d_n$ [mm] | $p_d = 500$ [bar] |
|----------|------------|------------------|
| 25       | 0.2        | -2.1             |
| 75       | -1.0       | 2.1              |
| 125      | 0.9        | -0.1             |

**Figure 2.** (a)-(c) transverse mould shrinkage (mean values in %) against nominal wall thickness $d_n$ and distance from the gate $x$ for three values of packing pressure $p_d$; (d) explanation of results presentation.
Figure 3. Mean value of transverse mould shrinkage $S_p$ for beams segments of following thickness profiles: 2–6–2, 4–4–4, 6–2–6 and 6–4–2, i.e. excluding results for 2–4–6 beam; $d_n$ – nominal thickness of segment, $x$ – distance from the gate, $p_d$ – packing pressure.

Figure 4. Mean value of transverse mould shrinkage $S_p$ for beam of ascending thickness profile 2–4–6; $d_n$ – nominal thickness of segment, $p_d$ – packing pressure.

The results of the experiment showed that the value of transverse shrinkage in beams segments of the same thickness $d_n$ is similar, regardless of the beam thickness profile. The exception is the case of a beam with increasing thickness along polymer flow direction, marked as 2–4–6 (figure 2, numbers on grey background). Therefore, it was decided that the model of shrinkage (1) will be extended by a new independent variable – beam thickness profile $\Delta d_n$ along the flow direction of the polymer in the cavity (i.e. along the beam axis):
\[ S_p(d_n, x, p_d, \Delta d_n) \]  \hspace{1cm} (3)

where \( \Delta d_n \) is the difference in the nominal thickness of the segment located farthest from the gate and the thickness of segment nearest the gate (e.g. for the moulding marked 2–4–6, the thickness profile is \( \Delta d_n = 6\text{mm} – 2.5\text{mm} = 3.5\text{mm} \)).

6. Fuzzy model of transverse shrinkage

The theory of fuzzy logic was used to approximate the experimental results. The model with a fuzzy structure was constructed and its basic elements shown in figure 5 and figure 6. Three linguistic values (LV) were defined for the first three independent variables of model (3): Small, Medium, Big. For the fourth independent variable \( \Delta d_n \) (thickness profile), the following LVs were created: Descending, Const., Ascending. The triangular membership functions were defined – figure 5a and 5b. Figure 5c presents LV and their membership functions for the model output variable \( S_p \).

(a)

![Membership functions for linguistic values](image)

(b)

![Membership functions for \( \Delta d_n \)](image)

(c)

![Membership functions for \( S_p \)](image)

**Figure 5.** Fuzzy variables of transverse shrinkage model \( S_p(d_n, x, p_d, \Delta d_n) \); (a) and (b) independent variables, (c) output variable.

The model’s fuzzy rules have the following general form:

\[
\text{if } d_n \text{ is } \ldots \text{ and } x \text{ is } \ldots \text{ and } p_d \text{ is } \ldots \text{ and } \Delta d_n \text{ is } \ldots \text{ then } S_p \text{ is } \ldots
\]  \hspace{1cm} (4)

where dots in predicates (before “than”) should be replaced with linguistic variables of corresponding independent variables (figure 5a and 5b) and “\( S_p \) is” (the successor) should be followed by LV for a shrinkage (figure 5c). The complete set of model rules \((3 \times 3 \times 3 \times 3 = 81 \text{ rules})\) is presented graphically.
in figure 6. The linguistic value of the shrinkage $S_p$ in the successor of each rule (4) was selected on the basis of experimental results presented in figure 2. The LV of $S_p$ was chosen for which position of the maximum of membership function was the closest to the measured shrinkage value.

The Mamdani method of reasoning (a typical FLC controller) was applied [3,4]. Centre of area (COA) defuzzification algorithms was used because of its good approximation ability.

$$\text{if } d_n \text{ is } "M" \text{ and } x \text{ is } "B" \text{ and } p_d \text{ is } "B" \text{ and } \Delta d_n \text{ is } "D" \text{ then } S_p \text{ is } "2.5"$$

Figure 6. Graphical presentation of 81 fuzzy rules for model $S_p(d_n, x, p_d, \Delta d_n)$. Circles indicate LV in successors of rules which were built based on empirical results for beam of ascending thickness profile 2–4–6 ($\Delta d_n$ is Ascending).

7. Conclusions

The method of building a fuzzy logic model of multiple input variables was presented. The experimental data of modelled relationship was used to build a set of rules for fuzzy model. An advantage of the described approach is simplicity of including partial expert knowledge into model by modifying chosen rules, which reduces the error of model approximation.

The developed method was used to build a fuzzy logic model of transverse shrinkage of HDPE mouldings. The model includes following variables: nominal dimensions of the mould cavity, profile of the wall thickness of the moulding considered along with the flow path of the plastic, the distance from the gate and the holding pressure (single-stage). The fuzzy structure of the constructed model allows its easy modification (generalisation) to similar injection processes.

The developed shrinkage model can be helpful at the design stage of the moulding geometry and the mould runner system. It is also a good starting point for the optimisation of the process, aimed at compensating the shrinkage in the entire volume of the moulding by adjusting the pressure profile in the holding phase of moulding injection cycle.

8. References

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