Influence of surface sublimation on boundary layer stability at high Mach number

S A Gaponov, A N Semenov and B V Smorodsky
Khristianovich Institute of Theoretical and Applied Mechanics SB RAS, 4/1, Institutskaya str., Novosibirsk, 630090, Russia
gaponov@itam.nsc.ru

Abstract. Results of theoretical investigation of the linear stability properties of Mach 6 boundary layer on a flat-plate with sublimation coating are presented. Naphthalene ($C_{10}H_{8}$) was chosen as a substance for such a coating. This is an organic compound capable to pure sublimation (phase transition from the solid state directly to gas, bypassing the melting stage) at moderate temperatures. Calculations performed on the basis of binary-mixture boundary-layer equations in the approximation of local self-similarity revealed that with increasing flow stagnation temperature the adiabatic surface temperature also increases that causes growth of mass flow rate of naphthalene evaporation. This leads to an increase in the density of the binary mixture (air and sublimation vapors) near the wall. The performed linear stability computations show that such a modification of the boundary layer profiles causes a reduction in the local linear amplification rates of perturbations of the first mode, while disturbances of the second mode are destabilized by the surface sublimation.

1. Introduction

The necessity to study the boundary layer under conditions of mass ablation from a surface is known for a long time. The reason for that is a practical need for thermal protection systems of hypervelocity vehicles (specifically reentry apparatus) by usage of destructible heat-shielding coatings. The importance of such studies is emphasized in the monograph [1]. As it was mentioned there, the thermal protection of the very first reentry spacecraft was greatly exaggerated due to the lack of knowledge of a number of important problems of aerothermodynamics at that time. Boundary-layer laminar-turbulent transition is one of most important problems in this row.

Various issues of transition are extensively investigated around the world. It is generally accepted that, at least in conditions of low external noise, the turbulence onset is determined by the boundary layer instability. The foundations of the theory of stability of compressible boundary layers were laid back in the early fifties [2]. Subsequent studies of the stability of supersonic boundary layers are reviewed in [3-5]. All these studies concerned the boundary layers of a single-component gas.

Stability of boundary layers with chemical reactions was investigated for the first time in [6], where stability of boundary layers of a nonequilibrium dissociating gas was analyzed. The results of these investigations were later summarized in a book [7]. Stability of hypersonic boundary layers with chemical reactions of a more general kind were studied e.g. in [8-10].

Stability and laminar-turbulent transition under conditions of ablation have hardly been studied. At present, only papers [11, 12] are known for the stability of the sharp cone hypersonic boundary layer under conditions of mass ablation from the surface. They have considered only two-dimensional (2D)
perturbations at very high Mach numbers of \( M = 16 \) and 20. The results of those computations have never been compared with experiment. Therefore, the expansion of the theoretical studies initiated in those papers onto the development of three-dimensional (3D) perturbations at moderate Mach numbers seems important, regarding especially possibilities of experimental verification of theory.

High-temperature ablation, which is specific for the conditions of a real reentry flight in the atmosphere, is quite difficult and expensive to reproduce in laboratory conditions. However, sublimation under conditions of moderate temperatures, which could be in principle realized in modern wind tunnels, allows to study physical processes of ablation under simplified conditions, in the absence of chemical reactions and other high-temperature gas-dynamic phenomena.

Surface sublimation supplies boundary-layer flow with a foreign gas (sublimation material vapor) which is injected at a certain mass flow rate. In [13, 14] it was shown that injection of heavy foreign gas (with molecular weight greater than for the air) into the near-wall sub-layer of the supersonic boundary layer through the permeable (porous) surface of the model has favorable influence on the boundary layer stability. Theoretical findings of these papers have later been confirmed by stability and transition experiments performed in continuously running supersonic wind tunnel at Mach number 2 [15, 16]. However, permeable surfaces have natural inherent roughness [17], which to a certain degree diminishes useful influence of the heavy gas injection, because roughness normally contribute to the boundary layer destabilization. That is why extension of the theory [13, 14] to investigate application of smooth sublimation coatings with negligible roughness, which would provide injection of heavy foreign vapors into the boundary layer, seems logic and looks promising.

In [18] boundary value problem for self-similar boundary-layer equations on a sublimation surface has been deduced. Computations of base-flow properties of supersonic Mach=2 flat plate boundary layer have been performed. Linear stability computations of such a sublimation surface boundary layer at Mach 2 has been performed in [19]. It was shown that sublimation leads to noticeable stabilization of the first mode perturbations. Theory developed in [18, 19] will be used in this paper to study influence of surface sublimation on linear stability of hypersonic Mach 6 flat-plate boundary layer in relation to first and second instability modes.

2. Base-flow computations

The flat-plate model is considered, which is immersed into a flow of compressible gas. It is assumed that the model surface is coated by the chemical substance, which is able to sublimate at moderate temperatures (that is capable to phase transition from the solid state directly to vapor bypassing the melting stage). Thus, the vapor of the sublimation material, which is actually a foreign gas with molecular mass \( m_1 \), penetrates from the model surface into the boundary layer of the primary gas with a molecular mass \( m_2 \). The sublimation process is assumed to be slow enough. In this case the shape of the model surface could be treated as unchanged. Then the boundary layer flow can not be considered as a single-component flow anymore, but becomes rather binary-mixture boundary layer. Time-dependent dynamics of such a binary mixture of viscous heat-conducting compressible gases is described by the system of partial differential equations which can be found, e.g., in [7]. From these general equations the system describing 2D stationary supersonic flat-plate boundary-layer of binary gas mixture in the absence of any chemical reactions have been deduced in the approximation of local self-similarity [16]. The equations taking into account sublimation vapor diffusion across the boundary layer and corresponding boundary conditions on the adiabatic sublimation surface have been formulated in [18].

It was shown there that the most important parameter influencing the boundary layer properties is the sublimant injection factor \( f_w = \frac{G_w}{\rho U_e \Re_e} \), where \( G_w \) is mass flow rate of sublimation substance vapors from the surface, \( \rho \) is the mixture density, \( U \) is streamwise velocity. Subscript \( e \) stands for the values taken at the boundary layer outer edge. \( \Re = \rho U_e \delta / \mu_e = \sqrt{\Re_e x} \) - Reynolds number based on the Blasius length scale \( \delta = \sqrt{\mu x / \rho U_e} \); \( \mu \) - viscosity, \( x \) - streamwise coordinate, \( \Re_e \) - unit Reynolds...
number. Mass flow rate of the vapor from the surface $G_w$ was determined by Knudsen-Langmuir and Clapeyron-Clausius equations [20] (also [18, 19]).

The viscosity and thermal conductivity coefficients for the components of the mixture, as well as foreign gas diffusion coefficient, were calculated on the basis of kinetic theory using Lennard-Jones potential [21]. The viscosity and thermal conductivity coefficients for the mixture were calculated using semi-empirical Wilke’s formula. The Eucken-type correction was used for calculation of thermal conductivity of polyatomic gases. In the present paper the boundary-value problem for the binary-mixture boundary-layer equations has been integrated numerically by means of the fourth-order Runge-Kutta numerical algorithm. Shooting technique and nested Newton-Raphson iterations have also been used to satisfy the boundary conditions. Boundary-layer equations, boundary conditions and numerical technique used in the present investigation were presented in a more detail in [18].

3. Linear stability computations

Linear stability theory (LST) for binary-mixture boundary-layer flow has been developed and presented earlier, e.g. in [13,14]. It was applied for the supersonic Mach=2 flat-plate sublimation boundary layer in [19]. It was shown in these papers that after linearization of equations of motion of viscous heat conducting binary gas mixture for wavy disturbances $q(x,y,z,t) = \tilde{q}(y) \exp(i(\alpha x + \beta z - \omega t))$ one can deduce a tenth-order system of ordinary differential equations. Here $(\alpha, \beta)$ are streamwise and spanwise wavenumbers; $\omega = \alpha C + \omega^\prime \delta U_e$ – nondimensional frequency, $f$ – dimensional frequency in Hertz. The sought vector $\tilde{q}(y) = \left(\tilde{u}, \tilde{u}', \tilde{v}, \tilde{w}, \tilde{p}, \tilde{h}, \tilde{h}', \tilde{c}, \tilde{c}'\right)^T$ is composed of fluctuations of the three velocity components, pressure, enthalpy and foreign gas (sublimation vapor) concentration, and their derivatives with respect to $y$, denoted by the prime. In the spatial stability analysis it is assumed that $\omega$ and $\beta$ are real numbers, while $\alpha = \alpha_r + i\alpha_i$ is complex. The spatial wavenumber $\alpha_r$ correspond to perturbation spatial amplification rate. System for the sought vector $\tilde{q}(y)$ together with ten homogeneous boundary conditions [19] for disturbances at the sublimation surface and at the boundary layer outer edge constitute the eigenvalue problem. This problem in the present investigation was numerically integrated by means of the method of orthonormalizations [7]. The streamwise wavenumber $\alpha_r$ was found as an eigenvalue of the problem with maximal $-\alpha_r$. Additional details on stability equations and used numerical procedures can also be found in [16].

4. Results

Boundary layer computations in the present paper have been performed for the flow of air over the flat-plate at hypersonic Mach number $M=6$. It was assumed that the model surface was coated with a layer of substance capable to pure sublimation at moderate temperatures, in the absence of other complication phenomena, such as chemical reactions, dissociation and ionization. Such property has, for example, dry ice, camphor, iodine. In this paper we consider a supersonic boundary layer above the surface coated with a layer of naphthalene ($C_{10}H_8$). It is a chemical compound with molecular mass $m_t = 128.17$ that is 4.4 times heavier than air. Other thermo-physical properties of $C_{10}H_8$ vapors required for computations are given in [19]. The choice of naphthalene for our investigations, in addition to its favorable thermo-physical properties, is explained by the fact that $C_{10}H_8$ is an easily accessible and inexpensive hydrocarbon.

In all presented further computational results it was assumed that the value of stagnation pressure $P_0 = 10$ bar, which is in the operational range of hypersonic wind tunnel Transit-M of ITAM [22]. This value of $P_0$ was kept constant, while stagnation temperature $T_o$ was varied. In the present paper we studied theoretically the supersonic boundary layer on thermally insulated, naphthalene coated, flat-plate with increasing mass flow rate $G_w$ of sublimation vapors. Such an increase of surface material
evaporation can be obtained by increasing sublimation coating temperature $T_w$, that was achieved by raising flow stagnation temperature $T_0$. The computed with adiabatic wall conditions cases are tabulated in table 1, where columns are: case number (used in subsequent figures as curve numbers), flow stagnation temperature $T_0$, surface temperature $T_w$ (adiabatic wall temperature, obtained as numerical solution for a given value of $T_0$) and corresponding flow unit Reynolds number $Re_1$. Case 0 presents result of computations in the absence of sublimation and serves as a reference point for our investigation.

Table 1. The computed with adiabatic wall conditions cases.

| No | $T_0$, K | $T_w$, K | $Re_1$, mio/m |
|----|---------|---------|---------------|
| 0  | 350     | 296     | 13.7          |
| 1  | 378     | 300     | 12.3          |
| 2  | 432     | 310     | 10.1          |
| 3  | 482     | 315     | 8.53          |
| 4  | 563     | 320     | 6.72          |

Figure 1. Binary mixture ($Air+C_{10}H_{18}$ vapour) boundary layer profiles: dimensionless velocity $\bar{U} = \bar{U}(\bar{y})$ (a), temperature $\bar{T} = \bar{T}(\bar{y})$ (b), density $\bar{\rho} = \bar{\rho}(\bar{y})$ (c), and foreign gas concentration $c_1 = c_1(\bar{y})$ (d) versus dimensionless normal coordinate $\bar{y} = y/\delta$; for a number of values of flow stagnation temperature $T_0$ (curves 0-4 are numbered in accordance with table 1).

Consider the base-flow properties. Figure 1 shows computed boundary layer profiles: nondimensional velocity $\bar{U} = \bar{U}(\bar{y})$ (figure 1a), temperature $\bar{T} = \bar{T}(\bar{y})$ (figure 1b), density $\bar{\rho} = \bar{\rho}(\bar{y})$
The overbar denotes dimensionless values normalized by corresponding value at the boundary layer outer edge (e.g. $\bar{U} = U/U_i$); $\bar{y} = y/\delta$. One can see that increase of $T_0$ leads to insignificant reduction of the boundary layer thickness in nondimensional representation, that is seen in streamwise velocity profiles (figure 1a). Temperature profiles $\bar{T} = T(\bar{y})$ given at figure 1b demonstrate noticeable decrease of the dimensionless temperature near the wall $\bar{T}(0)$ with increasing $T_0$, despite that dimensional wall temperature is increasing (table 1). Density profiles (figure 1c) demonstrate much variation: with increasing $T_0$ from 350 K to 563 K the value of the mixture density at the wall $\bar{\rho}_w = \rho(\bar{y} = 0)$ increases in two times: from $\bar{\rho}_w \approx 0.15$ to 0.3. Such an increase of $\rho_w$ according to our previous findings with heavy gas injection through permeable wall [13-16], has a favorable influence on boundary layer stability with respect to first mode perturbations. At figure 1d one can see that growth of the mass flow of the sublimation vapors with increasing $T_0$ leads to an increase of foreign gas concentration at the wall $c_{i,w} = c_i(0)$ and reaches the value of $c_{i,w} \approx 35\%$ for the case 4. However, $c_i(\bar{y})$ decreases rapidly with distance from the surface and at the boundary layer outer edge, determined by the velocity profiles (16 $< y_c < 20$, figure 1a), $c_i$ becomes negligibly small. So, foreign gas injection due to naphthalene coating sublimation can be considered as weak in the presented range of parameters, because sublimant does not go beyond the boundary layer bounds.

![Figure 2](image_url)

**Figure 2.** Dimensional (a) and dimensionless (b) presentation of spatial amplification rates $-\alpha_i$ of 2D perturbations versus frequency $f$ (a) and reduced frequency $F = 2\pi f \mu/\rho U_e^2$ (b); for various values of flow stagnation temperature $T_0$ (curves 0-4, according to table 1), $x = 90$ mm.

Figure 2a presents results of linear stability computations. It shows spatial amplification rates of 2D ($\beta = 0$) perturbations versus frequency for various values of flow stagnation temperature $T_0$ at $x = 90$ mm. Numbers near different curves on this plot correspond to case numbers in table 1. The black curve $\theta$ depicts boundary layer stability results computed at lowest stagnation temperature, without sublimation. One can see that the first instability mode occupies the range of $20 < f < 150$ kHz, with maximal amplification rate of $-\alpha_{i,\text{max}} \approx 7$ m$^{-1}$. Much more unstable second mode is located in the range $150 < f < 280$ kHz with $-\alpha_{i,\text{max}} \approx 43$ m$^{-1}$. Increase of stagnation temperature leads to a monotonous reduction of amplification rates in the low-frequency range $20 < f < 150$ kHz. On can see a complete stabilization of 2D first mode on the sublimation surface in the boundary layer with stagnation temperature $T_0 = 563$ K (case 4, table 1), since amplification rate of all frequencies in this range
becomes negative \((-\alpha_i < 0)\). That means, linear 2D first-mode perturbations are stable and their amplitude decreases downstream.

At the same time influence of surface sublimation on perturbations of second mode is not so evident. However, one should keep in mind that increase of \(T_0\) not only intensifies the evaporation of \(C_{10}H_8\) from the surface, but also leads to a decrease of the Reynolds number (table 1), and these two factors act in opposite directions in relation to second mode. Figure 2b presents similar plot like figure 2a only in nondimensional representation, that allows to eliminate reduction of the Reynolds number. As a result one can see that intensification of surface sublimation causes increase of second mode amplification rates, while the reduced frequency corresponding to maximal amplification perturbation also increases. The latter is explained by the mentioned above thinning out of the boundary layer (figure 1a,b) in the result of increase of the binary-mixture density near the wall (figure 1c). Red arrows at figure 2a,b demonstrate how surface sublimation influences the first and second instability modes in hypersonic Mach 6 boundary layer.

Figure 2 shows results for 2D disturbances only. Now we present an overview of the local linear stability properties of hypersonic binary-mixture boundary-layer on the sublimation surface. Figures 3a,b demonstrate stability diagrams as contour lines of spatial amplification rates on the plane: angle of wave vector orientation \(\chi = \arctan(\beta/\alpha_i)\) – frequency \(f\) [kHz]. Computations have been performed for streamwise station \(x = 90\) mm and stagnation temperature \(T_0 = 563\) K (case 4, table 1). Color-filled area shows linear instability region \((-\alpha_i > 0)\), where amplitude of propagating disturbances
grows downstream. Figure 3a shows stability diagram for the boundary layer in the absence of surface sublimation, while figure 3b presents similar diagram for the plate with naphthalene coating. Color schemes of stability diagrams figures 4a,b are taken the same to make the comparison of the computed results easier.

One can see that without sublimation (figure 3a) the instability domain can be provisionally separated on first \((10 < f < 130 \text{ kHz}, \ 0 \leq \chi < 75)\) and second \((140 < f < 220 \text{ kHz}, \ 0 \leq \chi < 40)\) instability mode areas which merge in one instability region. The second mode has maximal amplification \(-\alpha_{i,\text{max}} \approx 25 \text{ m}^{-1}\) at \(\chi = 0, \ f_{\text{max}} = 170 \text{ kHz}\). This is a 2D perturbation. While the first mode has much lower growth rate maximum \(-\alpha_{i,\text{max}} \approx 8 \text{ m}^{-1}\) at \(\chi \approx 60, \ f_{\text{max}} \approx 50 \text{ kHz}\). That is the most unstable first mode disturbance is 3D, oblique wave. Local growth rate maxima of first and second instability modes are depicted by small circles on stability diagrams.

Comparing figures 3a and 3b one can reveal that under the influence of surface sublimation the instability domain is separated into two regions for the first and second modes. The second mode instability domain occupies approximately the same place \((140 < f < 200 \text{ kHz}, \ 0 \leq \chi < 50)\) however the maximal growth rate is increased in about 75\% to \(-\alpha_{i,\text{max}} \approx 44 \text{ m}^{-1}\) at \(\chi = 0, \ f_{\text{max}} = 170 \text{ kHz}\). While the first mode instability region is considerably shrunk (to \(8 < f < 80 \text{ kHz}, \ 40 < \chi < 75\)) under the influence of surface sublimation. Maximal amplification of \(-\alpha_{i,\text{max}} \approx 3.2 \text{ m}^{-1}\) has now a 3D wave with \(f_{\text{max}} \approx 18 \text{ kHz}\) and \(\chi_{\text{max}} \approx 68\). It is also worth to note that 2D perturbations \((\chi \approx 0)\) of the first mode (which are normally precede the amplification of the second mode) are completely stabilized by the surface sublimation.

Thus, the presented modification of the boundary layer stability diagram (figure 4a \(\rightarrow\) figure 4b) under the influence of naphthalene surface sublimation in adiabatic wall conditions means stabilization of low-frequency first mode perturbations. At the same time, higher frequency disturbances of the second mode, which are normally dominating hypersonic boundary layer transition, are destabilized under the influence of surface sublimation.

\[\text{Figure 4. Spatial amplification rates } -\alpha_{i} \text{ [m}^{-1}\text{]} \text{ for 3D perturbations of the (a) first mode with } f = 50 \text{ kHz and (b) second mode with } f = 170 \text{ kHz, versus angle } \chi; \text{ for various values of } T_{0}, \ x = 90 \text{ mm. Dashed lines – on } C_{10}H_{8} \text{ sublimation coating; solid lines – in the absence of sublimation.}\]

Figures 4a,b demonstrate comparison of spatial amplification rates of 3D disturbances of the selected frequencies \(f = 50\) and \(170 \text{ kHz}\) (with maximal amplification for the first and second modes, figure 3a) versus wave-vector orientation angle \(\chi\) for various values of \(T_{0}\) computed on the sublimation surface (dashed lines) and in the absence of sublimation (solid lines). One can see that at the lowest considered
value of the flow stagnation temperature, there is no noticeable influence of sublimation since solid and dashed lines \( \theta \) merge (figures 4a,b).

First mode perturbation (figure 4a) has the largest amplification rate \(-\alpha_{r,\text{max}} \approx 17 \text{ m}^{-1} \) at \( \chi \approx 60^\circ \). One can see that increase of the stagnation temperature causes monotonous reduction of growth rates for all \( 0 \leq \chi < 72^\circ \). Comparing pairs of solid and dashed curves of the same color and numbers (that means, LST-computed in the flow with the same \( T_0 \)) at figure 4a, one can clearly see consecutive stabilization of first mode perturbations by means of naphthalene coating sublimation. Increase of stagnation temperature from \( T_0 = 350 \text{ K} \) (case \( \theta \), table 1) to \( T_0 = 563 \text{ K} \) (case 4) in the absence of sublimation (solid lines 1-4) causes reduction of the maximal growth rate to \(-\alpha_{r,\text{max}} \approx 10 \text{ m}^{-1} \), i.e. in two times. At the same time, application of naphthalene coating allows to reduce maximal growth rate till \(-\alpha_{r,\text{max}} \approx 2 \text{ m}^{-1} \) (dashed line, case 4, figure 4a) that is five times smaller. So, the first mode undergoes considerable stabilization by the surface sublimation, at least in conditions of figure 4a.

Growth of \( T_0 \) (and therefore reduction of \( \text{Re}_\text{t} \), table 1) leads to the reduction of growth rates of the second mode (solid lines \( 0 \rightarrow 4 \), figure 4b) from \(-\alpha_{r,\text{max}} \approx 36 \text{ m}^{-1} \) at \( \chi = 0^\circ \) to \(-\alpha_{r,\text{max}} \approx 26 \text{ m}^{-1} \). At the same time, on naphthalene sublimation coating increase of \( T_0 \) causes noticeable destabilization of the second mode, up to \(-\alpha_{r,\text{max}} \approx 44 \text{ m}^{-1} \) (dashed line, case 4, figure 4b).

One can see, that local growth rates of the second mode in conditions of performed in this paper investigation are at least two times larger in comparison with the growth rates of the first mode. However, the length of amplification of individual frequencies of the second mode in the downstream direction is usually much shorter than for the first mode, which has much longer way of amplification. Therefore, for a final conclusion about stabilizing-or-destabilizing influence of the sublimation coating on the Mach 6 flat-plate boundary layer it is necessary to perform further LST-computations with application of \( \phi^L \)-method to get more complete information about development of the first and second instability modes in the sublimation boundary layer.

5. Conclusions

Computations of the base-flow and its linear stability have been performed for Mach=6 boundary-layer over flat-plate model with naphthalene \((C_{10}H_{12})\) coating. It has been found that heavy foreign gas injection into the near-wall sub-layer of the boundary layer due to evaporation of the surface material leads to an increase of the binary-mixture (air + naphthalene vapor) boundary-layer density near the sublimation wall. Linear stability computations of such a modified base-flow revealed a possibility of considerable reduction of local amplification rates of 3D disturbances of the first-mode, when the model surface is heated enough by raising flow stagnation temperature. Computations also show that naphthalene sublimation leads to destabilization of 2D second mode. Further LST-calculations with application of \( \phi^L \)-method are desirable to quantify possibilities of such naphthalene coating usage for Mach 6 boundary layer stability and transition control.

Acknowledgments

This research was carried out within the framework of the Program of Fundamental Scientific Research of Russian state academies of sciences in 2013-2020 (project AAAA-A17-117030610125-7, No 0323-2018-0009) and was also supported by Russian Foundation for Basic Research (project 18-01-00070a).

References

[1] Tirsky G A 2011 Hypersonic Aerodynamics and Heat-Mass-Transfer of Reentry Space Vehicles and Planetary Probes (Moscow: FML) (in Russian)
[2] Lin C C 1954 The Theory of Hydrodynamic Stability (Cambridge: CUP)
[3] Mack L M 1969 Boundary layer stability theory Document 900-277 (Rev.A., Pasadena)
[4] Gaponov S A and Maslov A A 1980 Development of Disturbances in Compressible Flow

8
(Novosibirsk: Nauka) (in Russian)

[5] Zhigulev V N and Tumin A M 1987 *Onset of Turbulence. Dynamical Theory of Excitation and Development of Instabilities in Boundary Layers* (Novosibirsk: Nauka) (in Russian)

[6] Petrov G V 1974 *Combustion, Explosion and Shock Waves* 10 719–21

[7] Gaponov S A and Petrov G V 2013 *Stability of the Boundary Layer with Nonequilibrium Gas Dissociation* (Novosibirsk: Nauka) (in Russian)

[8] Malik M R and Anderson E C 1991 *Phys. Fluids A.* 3 803–21

[9] Chang C L, Vinh H and Malik M R 1997 *AIAA Paper* 1997-2012

[10] Johnson H B, Seipp T G, Candler G 1998 *Phys. Fluids* 10 2676–85

[11] Mortensen C and Zhong X 2014 *AIAA J.*, 52 1632–52

[12] Mortensen C and Zhong X 2016 *AIAA J.*, 54 976–96

[13] Gaponov S A and Smorodsky B V 2016 *AIP Conference Proceedings* 1770 030047 https://doi.org/10.1063/1.4963989

[14] Gaponov S A and Smorodsky B V 2017 *AIP Conference Proceedings* 1893 030087 https://doi.org/10.1063/1.5007545

[15] Lysenko V I, Smorodsky B V, Ermolaev Y G and Kosinov A D 2018 *Thermophys. Aeromech.* 25 183–90 https://doi.org/10.1134/S0869864318020038

[16] Lysenko V I, Gaponov S A, Smorodsky B V, Yermolaev Y G and Kosinov A D 2019 *Phys. Fluids* 31 104103 https://doi.org/10.1063/1.5112145

[17] Lysenko V I, Gaponov S A, Smorodsky B V, Yermolaev Y G, Kosinov A D and Semionov N V 2016 *JFM* 798 751–73 https://doi.org/10.1017/jfm.2016.347

[18] Gaponov S A and Smorodsky B V 2019 *Siberian Journal of Physics* 14 25–39 (in Russian) https://doi.org/10.25205/2541-9447-2019-14-1-25-39

[19] Gaponov S A and Smorodsky B V 2019 *AIP Conference Proceedings* 2125 030103 https://doi.org/10.1063/1.5117485

[20] Polezhaev Y V and Yurevich F B 1976 *Thermal Protection* (Moscow: Energiya) (in Russian)

[21] Hirschfelder J O, Curtiss C F and Bird R B 1954 *Molecular Theory of Gases and Liquids* (NY: Wiley)

[22] Bountin D, Maslov A and Gromyko Y 2018 *Phys. Fluids* 30 054103 https://doi.org/10.1063/1.5024025