1 Introduction

The pion-nucleon sigma term \( \Sigma \) has long been a thorn in the side of low energy quantum chromodynamics (QCD) [1,2]. The canonical result \( \Sigma = 64 \pm 8 \) MeV was obtained by Koch [3,4] based on an analysis of pre-1980 \( \pi p \) and \( \pi \pi \) scattering data, KH80 [4,5]. Gasser, et al. [6] later developed an alternative method of extracting \( \Sigma \) which agreed perfectly with Koch when using the same KH80 solution. In the usual picture, the nucleon strangeness parameter is

\[
y/2 = \frac{< N|\bar{s}s|N >}{< N|\bar{u}u + \bar{d}d|N >}
\]

(1)

The canonical \( \Sigma \) result yields \( y = 0.11 \pm 0.07 \), whereby the strange quarks would contribute \( \sim 110 \) MeV to the nucleon mass, an amount considered too large to be physical in light of results from e.g. neutrino scattering [7]. This “sigma term puzzle” spawned a whole generation of \( \pi N \) scattering experiments that have greatly increased the size and the quality of the scattering database.

A long-standing prejudice has been that new and better \( \pi N \) scattering data and an updated analysis ultimately would result in a smaller value for \( y \). With the new generation of experiments almost all completed, our George Washington University/TRIUMF group has sought to extract the \( \Sigma \) term as part of our ongoing \( \pi N \) partial-wave and dispersion relation analysis program, which employs the most up-to-date \( \pi N \) scattering data in our SAID database [8]. Our main conclusion is that contrary to wishful expectation, a thorough analysis of the new data has yielded a larger value, \( \Sigma = 79 \pm 7 \) MeV, which can be understood simply in light of the new experimental information. The sigma term and our analysis will be summarized briefly. Details can be found e.g. in Refs. [4,6,9,10].

2 The Pion-Nucleon Sigma Term

The sigma term \( \hat{\sigma} \) measures the nucleon mass shift away from the chiral \( (m_u = m_d = 0) \) limit, thereby parameterizing the explicit breaking of chiral symmetry in QCD due to the non-zero up and down quark masses. Models of nucleon structure are required to determine \( \hat{\sigma} \). The canonical result \( \hat{\sigma} = 35 \pm 5 \) MeV is due to Gasser [11] based on SU(2) chiral perturbation theory plus meson...
Figure 1: Determination of the $\pi NN$ coupling constant from the H"upper dispersion relation. The y-intercept gives the coupling $g^2/M$, and the left (right)-hand side of the figure is dominated by $\pi^- p$ ($\pi^+ p$) data. This technique is well suited to determine the coupling constant since most systematic effects (e.g. Coulomb corrections) affect each side asymmetrically, “pivoting” the curves about the intercept, hence greatly reducing their effect on $g^2$.

loop corrections. One obtains the strangeness $y$ from

$$\sigma(0) = \frac{\dot{\sigma}}{1 - y}$$

(2)

where the theorem of Brown, Pardee, and Peccei \[12\] relates $\sigma(0)$ to the isoscalar invariant $\pi N$ scattering amplitude $D^+(\nu, t)$ at the “Cheng-Dashen” point \[13\], $\nu = 0, t = 2m^2_{\pi}$:

$$\Sigma = F^2_{\pi} \bar{D}^+(0, 2m^2_{\pi})$$

$$= \sigma(2m^2_{\pi}) + \Delta_R$$

(3)

where

$$\sigma(2m^2_{\pi}) = \sigma(0) + \Delta_\sigma$$

(4)

and $F_{\pi} = 92.4$ MeV is the pion decay constant, $\nu$ is the crossing energy variable, and $t$ is the four-momentum transfer. The “remainder term” $\Delta_R$ is small (<2 MeV \[14\]). The nucleon scalar form factor $\sigma(t)$ shifts by an amount $\Delta_\sigma = 15$ MeV from $t = 0$ to $t = 2m^2_{\pi}$, calculated from a $\pi \pi$ dispersion relation analysis \[8\] and recently confirmed by a chiral perturbation theory calculation \[15\]. The bar over $\bar{D}^+$ indicates that the pseudo-vector Born term has been subtracted.

The Cheng-Dashen point lies outside the physical $\pi N$ scattering region, so the experimental $\bar{D}^+$ amplitude must be extrapolated to obtain $\Sigma$. The most reliable extrapolations are based on dispersion relation (DR) analyses of the scattering amplitudes \[8\]. The Koch result $\Sigma = 64 \pm 8$ MeV was based on hyperbolic dispersion relation calculations \[8\]. More recently, Gasser, et al., (GLLS) \[8\] developed another dispersion theoretic approach based on forward subtracted $\pi N$ dispersion relations. Expanding $D^+(t)$ as a power series in $t$, the experimental sigma term $\Sigma$ can be expressed as

$$\Sigma = F^2_{\pi}(\tilde{d}_{00}^+ + 2m^2_{\pi}\tilde{d}_{01}^+ + \ldots)$$

(5)

$$= F^2_{\pi}(\tilde{d}_{00}^+ + 2m^2_{\pi}\tilde{d}_{01}^+ + \Delta_D)$$

(6)

$$= \Sigma_d + \Delta_D$$

(7)

The GLLS, or truncated, sigma term $\Sigma_d$ is obtained via the subthreshold coefficients $\tilde{d}_{00}^+$ and $\tilde{d}_{01}^+$, calculated from the forward subtracted $\bar{D}^+$ and “derivative” $\bar{D}^+$ dispersion relations, respectively. They can also be determined from the subtraction constants $D^+(0, t) = C^+(0, t)$ in the fixed-t dispersion relation $C^+(\nu, t)$. The intercept of the curve $D^+(0, t)$ yields $d_{00}^+$, whereas the slope at
$t = 0$ yields $d_{01}^+$. The “curvature correction” term $\Delta D = 12 \pm 1$ MeV was determined from a $\pi\pi$ dispersion relation analysis \cite{Koch}. The great advantage of this approach is that $\sigma(0)$ can be obtained simply from $\Sigma_d$ via \cite{6}

$$\sigma(0) = \Sigma_d - (3 \pm 3)\text{MeV}$$

since the correction terms $\Delta\sigma$ and $\Delta D$ almost cancel, both having similar $\pi\pi$ amplitude input \cite{6}.

The analysis of Ref. \cite{6} used the Karlsruhe KH80 \cite{5} $\pi N$ phases as input and fit just the low energy data. Their result was $\Sigma_d \sim 50$ MeV, or $\Sigma \sim 62$ MeV (with $\Delta D = 12$ MeV), in agreement with Koch \cite{4}. Questions regarding the accuracy of the simply from $\Sigma_d$ dispersion relation analysis \cite{6}. The great advantage of this approach is that is sensitive to the smaller and more poorly known higher partial waves than other dispersion relations, were answered by the good agreement which demonstrated the reliability of the approach.

### 3 Analysis Procedure

Solutions from our ongoing $\pi N$ partial-wave and dispersion relation analysis are released when changes to the database and analysis method warrant \cite{8}. Details of our analysis method can be found in Ref. \cite{4,10,16}. An energy-dependent $\pi N$ partial-wave analysis (PWA) is performed on the available data up to 2.1 GeV pion laboratory kinetic energy, applying constraints from forward $C^\pm(\omega)$ and $E^\pm(\omega)$ DRs, as well as fixed-$t$ $B_\pm(\nu,t)$ (in the “Hüper” form \cite{6}) and $C^\pm(\nu,t)$ DRs. These dispersion relations are constrained \cite{4} to be satisfied to within <2% from 30 to 800 MeV for $-0.4 < t < 0.0$ GeV$^2/c^2$. The dispersion integrals use the Karlsruhe KH80 phases from 2.1 to 4.5 GeV and high energy parameterizations above that using forms found in Refs. \cite{4,17}.

Dispersion relations depend on a priori unknown constants e.g. scattering lengths and $g^2$. Our analysis determines these constants by a best fit to the data and the dispersion relations. The coupling $g^2$, the $\pi^- p$ s-wave scattering length $a_{\pi^-p}$, and the p-wave scattering volume $a_{1^+}^+$ were fixed for each fit over a grid of values (for reasons of fit stability), where the combination with the lowest $\chi^2$ yields the final solution. The fitting procedure automatically chooses the best-fit isovector scattering length $a_{0^+}$ and volume $a_{1^+}$, and the subtraction constants $C^\pm(0,t)$. This method enables us to check their sensitivity to various systematic effects, e.g. database changes. The low energy $P_{13}$ partial wave is constrained to follow the expected partial wave dispersion relation behaviour in its Chew-Low approximation form \cite{18}, which our other p-waves satisfy without constraint. As well, the low energy F and higher partial waves, too small to be determined from the $\pi N$ scattering data, are constrained to agree with those calculated by Koch \cite{19} from partial wave projections of fixed-t dispersion relations, which are dominated by t-channel ($\pi\pi$) contributions. This ensures that our higher partial waves satisfy analyticity and unitarity requirements.

### 4 Results

Our main results are summarized in Figs. \cite{1} and \cite{3} and Table \cite{1}. We find for the $\pi NN$ coupling constant $g^2/4\pi = 13.69 \pm 0.07$ (or $f^2 = 0.0757 \pm 0.0004$), stable in our solutions for many years (see \cite{16}). Our coupling constant agrees with most other recent results, in particular the comprehensive $NN$ and $\pi N$ analyses of the Nijmegen group (see Ref. \cite{20} and references cited therein). Note that this result is perfectly consistent with both the Goldberger-Treiman discrepancy \cite{21,22} and the Dashen-Weinstein sum rule \cite{23}, removing a long-standing inconsistency when using the older Karlsruhe value \cite{5} $14.3 \pm 0.3$.

For the s-wave scattering lengths we obtain $3a_{\pi^-p} = 0.261$ m$_{-}\pi^{-1}$ and $3a_{0^+} = 0.260$ m$_{-}\pi^{-1}$, with 1-2% uncertainties. The $\pi^- p$ scattering length agrees with the PSI pionic hydrogen result $3a_{\pi^-p}^{\text{psi}} =$

\footnote{This range has been increased from previous analyses, and in practice the dispersion relations are well satisfied somewhat beyond that range due to the energy dependent partial wave forms}
0.2649±0.0024 m\(^{-1}\), while the isovector scattering length satisfies the Goldberger-Miyazawa-Oehme (GMO) sum rule \(^{22}\) when using our coupling constant and integral \(J_{\text{GMO}} = -1.08 \pm 0.03\) mb\(^{-1}\). The p-wave scattering volume \(a_{1+}^+ = 0.133\) m\(^{-3}\) is consistent with recent analyses of low energy data \(^{25}\), as expected since the resonant \(P_{33}\) partial wave dominates the low energy data and \(a_{1+}^+\).

The dispersion relations are very well satisfied up to about 1 GeV, in general much better than KH80. From the forward \(C^+(\omega)\) and \(E^+(\omega)\) dispersion relations, we obtained the coefficients \(\bar{d}_{00} = -1.30\) m\(^{-1}\) and \(\bar{d}_{01} = 1.19\) m\(^{-3}\), in perfect agreement with the results from the slope and intercept of the \(C^+(0,t)\) subtraction constants at \(t = 0\), shown in Fig. 2. The equivalent \(\bar{d}_{01}\) result from the \(E^+\) dispersion relation, with its \(\sim l^3\) sensitivity to partial waves, and the \(C^+(\nu,t)\) dispersion relation, with its \(\sim l^3\) sensitivity, supports the reliability of our higher partial waves.

Figure 2 shows a polynomial fit to \(C^+(0,t)\) near \(t = 0\), from which \(\bar{d}_{02}, \bar{d}_{03},\) and \(\bar{d}_{04}\) were estimated. The \(\bar{d}_{02}\) coefficient is in perfect agreement with the Karlsruhe result \(^{21}\), while the sum of the higher order terms yield a curvature correction \(\Delta_D > 11\) MeV, in agreement with the \(\pi\pi\) dispersion relation result \(^{22}\) \(12\pm1\) MeV from Ref. \(^{22}\). The amplitude is very small as expected at \(t = m^2_\pi\) (“Adler point”). The overall consistency tends to support our result for the sigma term, \(\Sigma \sim 79 \pm 7\) MeV.

### 4.1 Systematic Checks

Perhaps the most important systematic check is the sensitivity of our results to the scattering database. Around the \(\Delta\) resonance, there is a well known disagreement between the TRIUMF \(\pi^\pm\) differential cross section \(^{22}\) and PSI \(\pi^\pm\) total cross section data \(^{22}\) on the one hand, and the

\(^{2}\)The above increases were also noted in Ref. \(^{24}\)
Solution $\Sigma_d$ [MeV] = "$a_{0^+}$ const. Born $\int D^+$ " $a_{1^+}$ const. Born $\int E^+$

|       | $\Sigma_d$ | $a_{0^+}$ | Born | $\int D^+$ | $a_{1^+}$ | Born | $\int E^+$ |
|-------|------------|-----------|------|------------|-----------|------|------------|
| KH80  | 50         | -7        | +9   | -91        | +352      | -142 | -72        |
| FA01  | 67         | 0         | +9   | -88        | +351      | -136 | -69        |
| difference | 17       | +7        | 0    | +3         | -1        | +6   | +3         |

Table 1: Comparison of $\Sigma_d$ from the Karlsruhe solution KH80 and our recent solution FA01. The change in the $C^+$ subtraction constant ($a_{0^+}$) term, the $E^+$ Born term, and both integral terms are consistent with expectations from, respectively, pionic atom data \[24\], the coupling constant $g^2/4\pi \sim 13.7$ (see Ref. \[20\]), and a narrower $\Delta$ resonance width. Values are rounded. See text for details.

older CERN results \[29\] on the other. Only a small increase 0.07 and 4 MeV was observed in $g^2/4\pi$ and $\Sigma_d$, respectively, for the “CERN-only” database. In practice, both sets are included in the final fit. We found that weeding out large $\chi^2$ data sets had little effect on the result. Also, since the low energy data are consistent with the PSI pionic atom results \[24\], we conclude that there are no large systematic effects from reasonable changes to the current scattering database.

The hadronic amplitudes are corrected for Coulombic effects following the Nordita prescription \[30\], supplemented in this analysis at high energies by extended-source Coulomb barrier factors \[31\]. The current approach improved the agreement with the PSI pionic atom results over our previous Nordita+point-source barrier results; however, neither the coupling constant nor $\Sigma_d$ varied outside the errors when using point- or extended-source barrier factors exclusively, or the Nordita corrections supplemented by either. Moreover, the isospin-violating $\Delta$ resonance is “split” defining “hadronic” = $\Delta^{++}$, consistent with the Nordita definition, but find no difference to our previous approach with “hadronic” = $(\Delta^0 + \Delta^{++})/2$, or with no splitting at all. We conclude that there are also no large systematic uncertainties from our Coulomb correction scheme.

The implementation of our dispersion relation constraints was also checked. We found that every reasonable form for the high energy amplitudes (>2 GeV) yields virtually identical results. Agreement between the forward subtracted and fixed-t unsubtracted dispersion relations is good for reasonable constraints (i.e. typical experimental error \(~\sim\)2%), but suffers if they become too tight, <0.5%. Constraining the low energy $P_{13}$ partial wave to follow the Chew-Low form lowered $\Sigma_d$ by 6 MeV, but once corrected, reasonable deviations caused changes much smaller than our error bar. We also had solutions where the low energy F and higher partial waves were not rigorously constrained to the Koch values \[3\], and no significant difference was found. Furthermore, Olsson \[32\], from a new dispersion relation sum rule, and Kaufmann and Hite \[33\], from an interior dispersion relation analysis, obtained values for $\Sigma$ consistent with our own using an earlier SAID solution. Consequently, we are confident in the reliability of our dispersion relation analysis.

5 Summary

In summary, we have performed a comprehensive partial wave and dispersion relation analysis of the available $\pi N$ scattering data up to 2.1 GeV that includes several improvements upon prior analyses \[1,3\]. For the pion nucleon coupling constant we obtained $g^2/4\pi = 13.69\pm0.07$, consistent with our previous determinations \[1,3\] and the Nijmegen results \[21\]. Our s-wave scattering lengths agree with the latest PSI pionic hydrogen and deuterium results \[24\]. Our $\pi N$ sigma term result is $\Sigma_d = 67 \pm 6$ MeV, or $\Sigma = 79 \pm 7$ MeV, compared to the canonical result $64 \pm 8$MeV from Koch \[3\]. These results have proven robust with respect to the many systematic checks that we have performed. In light of the large nucleon strangeness content $y/2 \sim 0.23$ inferred in the standard

\[3\]The other low and $\Delta$ resonance energy data are fit somewhat better in “TRIUMF+PSI-only” solution
picture, we believe that alternate interpretations of a large sigma term ought to be examined carefully.

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