Medical Image Segmentation Based on Combination Method

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Abstract. This paper studies the data segmentation of medical images based on compressed sensing. We do not use image data as interpolation data, but as constraints to construct surface patches with quadratic polynomial approximation accuracy, study the properties and correlation of pixels, and take these edge points as constraints, and map the discrete image data into continuous mathematical functions, so that the continuous function or point data not only has the shape recommended by the image data, but also has a higher segmentation accuracy, and then realize the fine segmentation of images, which provides a new research idea to solve the difficult problems of spatial data representation, and has good theoretical significance and application value.

1. Introduction

Image segmentation is very important in medical image processing and analysing. There are many articles to study medical image segmentation, such as many fuzzy C-means[1,2], local inhomogeneous intensity clustering[3], level set method[4], anti-noise and bias-field correction[5], deep neural networks[6], wavelet method[7], convolutional neural networks[8], Maximum likelihood estimation[9], superpixel segmentation[10]. These research mainly focuses on the segmentation of gray matter, white matter and cerebrospinal fluid in brain tissue, which named magnetic resonance imaging (MRI). Image quality is easily interfered by illumination, noise and other external factors, and the boundary is fuzzy, which increases the difficulty of image segmentation. Because image segmentation is the key step from image analysis to image understanding, image segmentation separates the target region from the complex background, which simplifies the complexity of processing on the one hand, and provides the basis for the extraction and measurement of image features on the other hand. Therefore, many different segmentation algorithms are born and become a hot field of medical research.

Medical images are collected by Fourier undersampling, which is equivalent to extracting part of Fourier transform coefficients in compressed sensing as observation values to find the most sparse image representation to shorten the imaging time, which provides a large research space for future medical accurate reconstruction.

Image segmentation is to divide the image into several specific and unique regions, each of which has the same pixel strength value. Fuzzy edges and annoying artifacts are usually the most annoying visual interference problems, which affect the accuracy of image segmentation.

Considering the limitation of MR image reconstruction based on compressed sensing, the disordered research objects, inaccurate boundary information, non-uniform intensity value are considered as clustering problems. On the basis of fully studying various segmentation methods[11-15], a new algorithm for automatic segmentation of MR images using piecewise quadratic polynomial surface fitting method[16] is proposed. Firstly, a high-quality image matrix is reconstructed from the sparse matrix obtained from the compressed sensing measurement in the wavelet domain, which is usually the
edge of the image and plays a key role in the visual effect of the image. It is used as a constraint condition to generate the constraint formula. The quadratic polynomial interpolation surface patch is constructed by reverse sampling. The weighted combination of all patches forms the fitting surface that approximates the original surface, thus completing the image data fitting.

The new algorithm shows that the fitting surface has a good approximation accuracy, so it can complete the fine image segmentation.

2. The Basic Idea of Algorithm

The basic idea of this algorithm is to first use wavelet transform technology to enhance the boundary points, and then take these boundary points as constraints to construct piecewise quadratic polynomial surfaces to fit the original scene, so that the constructed surface can better reflect the characteristics of image edge.

2.1. Research on pixel correlation

Starting from the transformation domain space, this paper tries to choose the ideal transformation model by using different methods: Firstly, a \(9 \times 9\) square sub image with each pixel as the center is established, and the discrete wavelet transform is carried out on it. The relationship between the elements of the matrix before and after the transformation is studied. The function mapping between the two matrices is established to find out the difference and simplify the functional relationship, so as to summarize the correlation model between the pixel and its adjacent pixels.

Suppose that \( P \) is an image composed of \( n \times n \) pixels \( P_{ij}(i,j=1,2,...,n) \), \( P \) can be regarded as a sample on the continuous original surface \( F(x, y) \), on the region \([0.5, n+0.5] \times [0.5, n+0.5]\) shown in Figure 1, suppose that each element \( P_{ij} \) is sampled from a unit square, i.e.

\[
P_{ij} = \int_{j-0.5}^{j+0.5} \int_{i-0.5}^{i+0.5} w(x,y)F(x,y)dxdy
\]

(1)

where \( w(x,y) \) is a weight function with \( w(x,y)=1 \). Because in \( P_{ij} \), \( i,j=1,2,...,n \) are integers, formula (1) does not hold in general, but it holds approximately.

Because the image data is sensed from different parts of the human body, the images with different tissues are different, and it is difficult to get the ideal results by single transformation. Therefore, the combined transformation is studied and the corresponding weight function is calculated according to the correlation of pixels.

\[\text{Figure 1. Adjacent region of a pixel} \quad \text{Figure 2. Four directions}\]

2.2. Boundary problem and its solution

Sometimes the bicubic surface patch constructed above is not ideal because it only considers the edge features of the central pixel and the surrounding pixels in the horizontal, vertical and diagonal directions. The actual situation is often more complex, which will be improved from two aspects: one is to use curvelet and other tools to express the geometric features of edges in multi-scale and multi-directional, and to find more abundant information; the other is to modify the assignment method of weight functions.
$w_{ij}$, so that the newly constructed surface is more in line with the contour characteristics. In the case of weak edge, the perceptual data is taken as the constraint, and the piecewise continuous cubic Hermite interpolation is used to solve the breakpoint problem and find the optimal solution, so as to obtain continuous boundary contour and provide guarantee for accurate segmentation.

Wavelet transform is used to enhance the acquisition of boundary points as constraints are. According to the quad-tree structure of wavelet decomposition \cite{17}, the classification of the coarse-scale wavelet coefficients is conducive to the classification of fine-scale wavelet coefficients. This is due to the persistence of the wavelet coefficients. If the wavelet coefficients of the parent node are large, the wavelet coefficients of its child nodes may also be large. So when wavelet coefficients of parent node are assigned a certain label, the child nodes may be also assigned to the same label. Since adjacent wavelet coefficients are of similarity, we can combine neighborhood wavelet coefficients to complete label from the coarse scale to a fine scale, and finally get the label of the most fine-scale images.

The wavelet reconstruction process can make full use of the high-frequency coefficients of each layer, so that the reconstructed image retains good details. Each pixel has a label from the reconstructed image. When the labels of two pixels are the same, the similarity increases, indicating that the distance between the two pixels decreases. For this purpose, a parameter is defined, which is set as follows:

$$\alpha_{ij} = \begin{cases} t & x_i \text{ and } v_j \text{ are of the same labels} \\ 1 & x_i \text{ and } v_j \text{ are of the different labels} \end{cases}$$ (2)

usually $0 < t \leq 1$. In this paper, $t = 0.8$. Defined the distance between two pixels:

$$d_{ij}(x_i, v_j) = ||\alpha_{i,j}^T(y_j - y_i)||^2$$ (3)

where $\alpha_{i,j}$ represents the similarity between $x_i$ and the cluster centroid $v_j$. $\lambda$ is the degrees of freedom parameter, which determines the degree of influence that segmentation results of coarse resolution is to the distance $d$, this paper $\lambda = 1$. $y_i, y_j$ is the pixel’s vector value.

Traditional FCM algorithm, only consider the pixel own gray value, no use of the information of the neighboring pixels. This paper redefine the pixel value, which is the weighted sum of the pixel own gray value and the average gray value of neighboring pixels. It is defined as follows:

$$y_j = \beta \times x_i + (1 - \beta) \times \frac{1}{N_R} \sum_{j \in N(x_i)} x_j$$ (4)

$x_i$ is the pixel’s gray value, where $N_R$ is the number of neighboring pixels, and $N(x_i)$ is the neighboring pixels of the pixel $x_i$. $\frac{1}{N_R} \sum_{j \in N(x_i)} x_j$ represents the average gray value of neighboring pixels. $\beta$ $(0 \leq \beta \leq 1)$ is the weighting factor, which is defined as follows:

$$\beta = \frac{1}{N_R} \sum_{j \in N(x_i)} s_{i,j}$$ (5)

$N_R$ is the number of neighboring pixels, and $N(x_i)$ are the neighboring pixels of the pixel $x_i$. $s_{i,j}$ represents the similarity between $x_i$ and $x_j$. 

2.3. Surface reconstruction

In order to solve the waste of system resources caused by converting two-dimensional image into one-dimensional long vector in compressed sensing, the image is divided into blocks and measured separately with the same measurement matrix (i.e. area sampling). Each block image is reconstructed according to the measured value, and finally the whole image is combined. However, there are some problems such as blocking effect and low reconstruction efficiency. So we try a new algorithm of bicubic polynomial fitting with perceptual data as constraints.

According to the above formula (1), the purpose is to construct a new surface \( f(x,y) \), and make it as close to \( F(x,y) \) as possible, and it has the accuracy of quadratic polynomial, then

\[
f(x,y) = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} P_{i,j} k(x-i,y-j)
\]

where \( k(x,y) \) is a kernel function and usually satisfies the following constraints

\[
\sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} k(x-i,y-j) = 1
\]

\( F(x,y) \) can be any function, while \( P_{i,j} \) are sampling values constrained\(^{[18]} \) by \( F(x,y) \), which do not always coincide with \( F(x,y) \). For convenience of discussion, it is assumed that \( F(x,y) \) is defined on \([0.5, n+0.5] \times [0.5, n+0.5] \), \( w(x,y)=1 \). According to the approximation theory\(^{[11]} \), any continuous function \( \zeta(x,y) \) can be expanded into polynomial Taylor series at a point \((x_c,y_c)\) in the continuous region. The Taylor series is the approximation to \( \zeta(x,y) \) in the neighborhood of \((x_c,y_c)\).

Assuming that \( F(x,y) \) can be approximated by the first three terms of Taylor series, that is, it can be approximated by piecewise quadratic polynomials on its domain of definition, then the process of constructing \( f(x,y) \) is as follows:

For each region \([i-1.5, i+1.5] \times [j-1.5, j+1.5] \) \((i,j=2,3,...,n-1)\), a quadratic polynomial surface patch \( f_{i,j}(x,y) \) is constructed, and its weighted average generates \( f(x,y) \). Let \( u=x-i \), \( v=y-j \), then in the UV plane, \( f_{i,j}(x,y) \) can be written in the interval \([-1.5, 1.5] \times [-1.5, 1.5] \) as follows:

\[
f_{i,j}(x,y) = au^2 + buv + cv^2 + du + ev + f
\]

where \( a, b, c, d, e \) and \( f \) are unknown quantities, they can be determined by the boundary conditions. Because the quality of image edge plays a very important role in vision, the constructed \( f_{i,j}(x,y) \) should reflect the edge features of the image as much as possible. \( d \) and \( e \) are determined by the constrained least square method. Let

\[
G(d,e) = w_1(e-e_1)^2 + w_2(d+\sqrt{2}e_2)^2 + w_3(d-e_3)^2 + w_4(d-e+\sqrt{2}e_4)^2
\]

where \( e_1 = (P_{i,j} + P_{i+1,j})/2 \), \( e_2 = (P_{i+1,j} + P_{i+1,j+1})/2 \), \( e_3 = (P_{i+1,j} + P_{i+1,j})/2 \), \( e_4 = (P_{i+1,j} + P_{i+1,j+1})/2 \). Now define the weight function\(^{[19,20]} \):

\[
w_i = 1/(1+\Delta_i^2), \quad i=1,2,3,4
\]

and define

\[
\Delta_1 = (P_{i,j} + P_{i+1,j})/2, \quad \Delta_2 = (P_{i+1,j} + P_{i+1,j+1})/2, \quad \Delta_3 = (P_{i+1,j} + P_{i+1,j})/2, \quad \Delta_4 = (P_{i+1,j} + P_{i+1,j+1})/2
\]

According to \( f(x,y) = P_{i,j} a/12 - c/12 \) can be obtained. Now in \( f(x,y) \), there are only three unknown parameters \( a \), \( b \), and \( c \), and there are eight pixels around \( P_{i,j} \) \( (P_{i,j}) \) is the center pixel, its neighbor pixels

\[
s_{i,j} = \begin{cases} 1 & \text{if } x_i \text{ and } v_j \text{ are of the same labels} \\ 0 & \text{if } x_i \text{ and } v_j \text{ are of the different labels} \end{cases}
\]
denoted as \( x, x-y, y, \) and \( x+y \). There are four directions for each one, shown in Figure 2), which can be determined by constrained least squares. Let

\[
g_{k,l}(a,b,c) = \int_{-0.5}^{0.5} \int_{-0.5}^{0.5} f_{i,j}(u,v) \, du \, dv = P_{i+k,j+l}
\]

By minimizing the objective function \( G(a,b,c) \), three unknown quantities \( a, b \) and \( c \) can be determined:

\[
G(a,b,c) = \sum_{k,l=0}^{k,l=0} w_{k,l}(g_{k,l}(a,b,c) - P_{i+k,j+l})^2
\]

where \( w_{k,l} \) is weight function \( k,l=1,0,1,k\neq l=0 \).

If \( f_{i,j}(x,y) \) in equation (3) is a linear function along the \( x \) direction, then \( P_{i+1,j} \) and \( P_{i-1,j} \) play an important role in the formation of \( f_{i,j}(x,y) \) along the \( x \) direction, in this case, \( w_{-1,0} \) and \( w_{1,0} \) should have a larger value. Therefore, \( w_{1,0} \) and \( w_{-1,0} \) can be defined by \( w_{1} \) in equation (10), and \( \Delta_i = 2a_1 \) is close to zero when the central pixel changes along the \( x \) direction and approaches to a linear function, \( w_1 \) should be inversely proportional to \( \Delta_i \). Similarly, we can discuss how to define \( w_2, w_3 \) and \( w_4 \).

Then the desired weighted surface \( f(x,y) \) can be generated. The method is to construct a bicubic patch \( B_{i,j}(x,y) \) on each region \([i, i+1] \times [j, j+1] \) \((i,j=1,2,\ldots,n-1)\). For \( i,j=2,3,\ldots,n-2 \), the bicubic patch \( B_{i,j}(x,y) \) on \([i, i+1] \times [j, j+1] \) are generated by the weighted average of \( f_{i,j}(x,y), f_{i+1,j}(x,y), f_{i,j+1}(x,y), f_{i+1,j+1}(x,y) \) as follows[21,22]:

\[
B_{i,j}(x,y) = w_{i,j}(x,y)f_{i,j}(x,y) + w_{i+1,j}(x,y)f_{i+1,j}(x,y) + w_{i,j+1}(x,y)f_{i,j+1}(x,y) + w_{i+1,j+1}(x,y)f_{i+1,j+1}(x,y)
\]

where \( w_{i,j}(x,y) = (1-v)(1-w), w_{i+1,j}(x,y) = v(1-w), w_{i,j+1}(x,y) = (1-v)w, w_{i+1,j+1}(x,y) = v(1-w) \) are weight functions, \( v=x-i, w=y-j \).

3. Experiments

In this section, some experimental results are given. These include the optimization of boundary points and the results of medical image segmentation using combination method.

3.1. Edges constraints

The following figure shows that the optimized wavelet transform coefficients have good advantages in the extraction of medical image boundary points. Here \( t \) in formula (2) is set to 0.8.

![Figure 3](image-url)

Figure 3. The comparison of edge detection results using two methods: (a) an original image area; (b) only Mallat method; (c) new method.

From Figure 3 we found that the results of single method are not satisfactory. The result of the new method is quite good because of the combination of wavelet transform and pixel correlation region
growing, the local maxima of first derivative and the zero-crossing points of second derivative synthetically. This algorithm can eliminate all those pixels might produced by noise approximately. Therefore this method can delete false edge points caused by noise, to get distinct boundaries of medical image, so as to solve the conflict between weak edge characteristic and precise location efficiently.

3.2. Experiments based on combination method
The following experiments show the advantages of this combination method. Here three medical images are used to verify the new algorithm with size 256×256.

The process of comparison is to sample the function $F(x)$, and then construct the approximation function to $F(x)$ based on the sampling points. The used sampling points $P_i$ ($i=0,1,2,...,199$) are defined by the following formula:

$$P_i = \int_{i/200}^{(i+1)/200} F(x) dx$$  \hspace{1cm} (15)

The experimental results are shown in Figure 4. We can see that the combination method is better. From the visual point of view, the latter has better segmentation effects.

4. Conclusion
In this paper, a new method of constructing fitting surface based on wavelet transform image data is proposed. The new method assumes that image data can be represented by piecewise quadratic polynomial surface. A quadratic polynomial approximation surface is defined at each pixel point. The bicubic surface patches on the square corresponding to each four adjacent pixels are generated by the weighted combination of four quadratic polynomial patches, and all bicubic surfaces are pieced together. The fitting surface is an approximation of the original surface and has quadratic polynomial interpolation accuracy. It can well reflect the characteristics of image edge. The experimental results show that the segmentation image obtained by the new method has better visual effect.

The proposed algorithm is very good, there is still a lot of research space in the future. For example, in the control of parameters and the setting of function and so on, we need to continue to explore, rather than choose the fitting function by experience.

![Figure 4. Segmentation results. (a) orign images, (b) FCM method, (c) new method.](image_url)
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