Does brane cosmology have realistic principles?

D.H. Coule

School of Computer Sciences and Mathematics, University of Portsmouth, Mercantile House, Hampshire Terrace, Portsmouth PO1 2EG.

Abstract

The maximal symmetry, or Perfect Cosmological Principle (PCP), that prevents AdS type spaces from degenerating into anti-inflationary collapse is argued to be unphysical. For example, the simple requirement that brane-bulk models should be the result of having evolved from even more energetic string phenomena picks out a preferred time direction. We question whether quantum cosmological reasoning can be applied in any meaningful way to obtain, what are essentially, classical constructs. An alternative scheme is to more readily accept the PCP and allow the branes to also become eternal. A perpetually expanding and contracting brane model could be driven by the presence of charged black holes in the AdS bulk, that effectively violates the weak-energy condition as singularities are approached. This can be contrasted with the so-called Ekpyrotic universe which also closely accepts the PCP. This being broken only by occasional collisions between branes, that can then simulate a big bang cosmology.

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Introduction

There is presently a lot of interest in brane theories and their cosmological properties. Typically these models and those including Randall and Sundrum have branes existing in bulk spaces with extra, possibly non-compact, dimensions [1]. We will not be too specific as to which brane model we consider since many variations have been presented. Instead we address some common concerns to aspects that seem generic in many models, for a review and extensive list of references see [2]. For this reason the concerns will be mostly independent of the actual compactification scheme and orbifold symmetry implored. We further only consider models that are effectively five dimensional, with the Planck scales related by

\[ M_4^2 = M_5^3 V \]  

where \( M_5 \) and \( M_4 \) are the 5 and 4 dimensional Planck masses and \( V \) the “volume” of the extra dimension. By having a relatively large \( V \) one can obtain a much smaller 5 dimensional Planck scale, say TeV, compared to the usual 4 dimensional one \( \sim 10^{19} \text{GeV} \) [1,2].

We also particularly have in mind models where positive and negative cosmological constants are both present. A specific example is when the positive tension of the brane is typically balanced with a bulk negative cosmological constant. This results in static or global Anti-de Sitter (AdS) space, or black hole variations thereof, being the bulk space time. This will be argued is unjustified and counter to the requirements of fitting string theory within a consistent cosmological model.

The five dimensional model does not live in isolation. It is the result of earlier evolution from an even higher dimensional phase, presumably starting with 11 dimensional M theory. The spacetime should follow a dynamical evolution and not simply be imposed as static, which implies it has always existed and remains unchanging. The metric might become stabilized by new matter components causing such behaviour. But the metric itself should not be constrained initially in such a way. As the models are presently constructed the Perfect Cosmological Principle (PCP) is imposed on the 5 dimensional bulk theory but only the Cosmological Principle (CP) is believed valid in our actual 4 dimensional brane universe. Recall that while the CP takes our place in space as not distinguishable, the PCP extends this also to time, see eg.[3]. There is a double standard which is similar to an earlier
discussion regarding the distinction between a de Sitter solution, which can also obey the PCP, and a physically inspired inflationary one. For similar reasons that discounted pure de Sitter we wish to discount Anti-de Sitter and replace it with the slightly less symmetric but now more realistic form that only satisfies the Cosmological Principle. This results in, and probably fatally, a rapid collapsing solution, or what we can term anti-inflationary behaviour. Note we avoid the use of the term deflation, since this usually refers to the exponentially collapsing stage of a closed de Sitter model.

De Sitter vs Inflation

In an earlier discussion Ellis and Rothman [4] pointed out that de Sitter space caused by a cosmological constant \( \Lambda \) is unsatisfactory since it displays no preferred direction of time. This is due to the symmetries of the metric involving “isometries transitive on space-time”[5] and constant-time surfaces are arbitrary. However, the presence of matter introduces a preferred time direction which breaks the full de Sitter \( SO(4, 1) \) symmetry in 4 dimensions, to those like Friedmann-Robertson-Walker (FRW), which only involves “isometries transitive on space-like surfaces”. These isometry subgroups are of the form \( SO(4), SO(3, 1) \) and \( E(3) \) for closed, open and flat spaces respectively. The quantum vacuum states of massless scalar fields also display this reduced symmetry [6].

Regardless of a matter component that breaks the full symmetry of the de Sitter group, there is a second reason why the inflationary solution is to be preferred. We require the universe to later have a dynamical evolution described by the FRW metric with a global time parameter [7]. Introducing such a parameter, by fiat, at the end of some static phase would simply reintroduce a causality problem that inflation is supposedly meant to overcome. By starting with an expanding inflationary solution this problem is prevented although at the expense of first choosing a specific choice of coordinates to represent this expansion. In other words a preferred time like direction should
be initially chosen since one is anyway needed in the FRW universe, or any universe with evolution.

In summary, the full de Sitter symmetry should be reduced because a) matter determines a time like direction and b) we require matching to a dynamical space time.

**Anti-de Sitter vs Anti-inflation**

We can make a similar distinction between a pure AdS space and that of a negative $\Lambda$ force that will accentuate gravitational collapse: an anti-inflationary rapid collapse. The AdS space is generally taken as static or global when present in string and supersymmetric theories [8]. The corresponding isometry symmetry group is $SO(n, 2)$ for $n + 1$ dimensional AdS space—see eg. [9]. It can be represented in the static form

$$ds^2 = - \left(k + \frac{r^2}{l^2}\right) dt^2 + \left(k + \frac{r^2}{l^2}\right)^{-1} dr^2 + r^2 d\Sigma_k^2$$

where the negative cosmological constant is defined as $\Lambda = -1/l^2$, with $l$ the curvature scale. For $k = 1, 0, -1$, $d\Sigma_k^2$ represents the unit sphere, plane and hyperbolic space in $n - 1$ dimensions respectively. Strictly speaking the full AdS metric only corresponds to the $k = 1$ case. For the Penrose diagrams of odd dimensional AdS space-times see eg. [10], where $k = 1$ corresponds to $M = -1$ in their notation. There is a slight complication, Brane models mostly take the $k = 0$ form of the metric, or the case $M = 0$ in ref. [10]. This metric, in so-called Poincare form, apart from not entirely covering the manifold has problems when $r = 0$, and is especially susceptible to singularities if any matter component is present [10]. For this reason Brane models typically exclude the $r = 0$ regions of two AdS spaces and then “paste” the resulting manifold together—see eg. [11] for the relevant Penrose diagrams.

For similar reasons why the dynamical inflationary metric was justified we will reason that this AdS symmetry should be reduced to give anti-inflation in actual string models. Firstly, the bulk is never entirely empty since dilatons and moduli fields also propagate there. Such matter fields can be used to introduce a time-like direction into the model just as a scalar field does in inflationary models. Secondly, the 5 dimensional effective theory is the result of some earlier stage evolving from even higher energy scales when the full M theory will presumably be required. A time evolution is already present
before the AdS stage becomes relevant. In the inflationary case the time evolution was strictly required only after the inflationary stage; but this, was argued, made its introduction necessary from the start if inflation was to be useful. Breaking the static nature of the metric reduces the isometry to $SO(n, 1)$ so that the metric is given by $[8, 9, 12]$

$$ds^2 = -dt^2 + \cos^2(t)d\Sigma^2_n$$

with $d\Sigma^2_n$ only now the metric on the unit hyperbolic space in n dimensions. There is a third reason why the reduced symmetry of only homogeneity and isotropy $SO(n, 1)$ can be justified. The anti-inflation spacetime is now globally hyperbolic, and the metric covers the domain of dependence of the, say, $t = 0$ initial (now a Cauchy) surface $[8, 9]$. This is now consistent with the 5 dimensional phase evolving from some earlier regime and which should be entirely determined by it. Unlike the full $k = 1$ AdS space there is no time-like surface at spatial infinity present which prevents a suitable Cauchy surface. Information before could enter the spacetime from the time-like or naked singularity surface. Although, it might be possible to quantize such non-globally hyperbolic metrics by controlling information entering from such a surface $[13]$, such boundary conditions are difficult to justify and more severe than the usual horizon problems of FRW metrics. One is setting up a condition on the surface that will actually hold for all the time the model exists. It is doubtful, that for example quantum cosmology using the Wheeler-DeWitt equation would be sufficient. As presently formulated such schemes generally assume a foliation of space or an implicit time parameter that provides initial condition on a suitable Cauchy surface - see eg.$[14, 15]$. Singularities are usually avoided or regulated in some way. Naked singularities are further discounted as being forbidden by the cosmic censor $[16]$, although this isn’t fully understood directly from quantum gravity. Naked singularities would however be difficult to implement in quantum cosmology. In comparison, quantum cosmology can be easily applied to the case of negative $\Lambda$ within an open FRW ansatz, as for example in ref.$[17]$. This is not very helpful since the model is classically already prone to rapid collapse and suffers from the same problems (eg. horizon and flatness) as any non-inflationary FRW universe.

Now, it might be claimed that “cutting and pasting” the $k = 0$ or $k = -1$ AdS metrics together can still allow a Cauchy surface cf.$[18]$, and there is little advantage in the FRW AdS metric in this regard. Note this options would
only occurs for odd dimensional space-times, since we require the “triangle” Penrose diagram of ref.[10]. However, this is still at the expense of non-causal “cutting and pasting” which is equally problematic. The arguments above can equally be levelled against this “process” that conveniently “knows how to remove the bad parts of the metric” and patch copies together if geodesic completeness [8] is to be maintained. We will generally address our criticism to the use of full AdS space where the lack of a Cauchy surface is explicit, but bear in mind this can also be levelled against these other schemes.

In summary, we can justify the use of the cosmological form of AdS for similar reasons that a de Sitter phase is an over idealized version of inflation; the presence of certain matter fields and evolution from an earlier higher dimensional phase which breaks the full AdS symmetry to that, at best, like FRW. Although, these coordinates do not cover the whole of AdS they are more physically justified for the reasons given. This distinction is similar to the difference between a true black hole formed by collapsing matter and that of the ideal eternally existing static case. The left hand side, or other “universe”, of the Penrose diagram is discounted in the physical collapsing star case, although it is present in the maximally extended Schwarzschild metric [8].

There is now also the added advantage that a Cauchy surface is present at the start of this cosmological AdS behaviour. Losing the static nature seems to be fatal as now gravity is simply being made more attractive and rapid contraction can occur. Explaining, the still highly symmetrical ansatz of homogeneity and isotropy seems an added complication if inflation is not to be invoked. We will next consider some specific implications of using only the cosmological metric form of AdS in brane cosmology and holography ideas.

**Brane cosmology**

We address our concerns to higher dimensional models where positive and negative cosmological constants play a crucial role in the formulism. We will make particular use of the approach of ref. [19]. Although, this does not exclude all possible schemes it does relate to the Randall Sundrum model where a “see-saw” mechanism relates the various cosmological constants. For example, in a 5 dimensional theory one obtains a relation [2,19]

\[ \Lambda_4 = \Lambda_5 + \lambda^2 \]  

(4)

where \( \Lambda_4 \) and \( \Lambda_5 \) are the 4 and 5 dimensional cosmological constants and \( \lambda \)
is the tension of the brane. The Friedmann equation is typically [2,19]

\[ H^2 = \frac{8\pi G}{3} \left( \rho + \frac{\rho^2}{2\lambda} \right) + \Lambda_4 \]  

(5)

The tension should be sufficiently large that the term quadratic in the energy density should be suppressed by the time of nucleosynthesis [2]; and \( \Lambda_4 \) is taken to be negligibly small.

The limitation that result when using the FRW form for AdS means that any large negative cosmological constant cannot be allowed to simply roam free without some other mechanism counteracting the increased gravitational force of attraction. Left alone this would simply cause collapse to zero size. The closed string modes having matter kinetic terms would diverge at such a point and cause a resulting singularity. The brane-bulk model would fail since all the extra dimensions of M theory, or whatever have to be adequately dealt with.

The FRW AdS is also open so that the volume is infinite, although one could take compact versions along the lines of ref.[20]. However, the open geometry will generally cause expansion while the negative cosmological constant will contract. There is no stable equilibrium point as once a collapse ensues the curvature cannot alone cause the Hubble parameter to change sign - see eg. [3]. In order that the bulk be sufficiently long lived, with lifetime \( \sim |\Lambda_5|^{-1/2} \), the cosmological constant must be exceedingly small even after allowing for the different Planck scale in the five dimensional theory. As well as a fine tuning that \( \Lambda_5 \) be sufficiently small, this is incompatible with requiring the volume \( V \) to be only moderately sized.

What is required is that the bulk be stabilized at around \( \sim mm \) scales by some other mechanism that presently seems unknown. We have just pointed out the unjustified nature of imposing the highly symmetrical AdS metric in such a scheme. This also goes against the spirit of inflation, which is to try to deduce our universe regardless of the initial conditions, or at least with only a reduced amount of symmetry - for a review see[21]. It is also rather dubious to still invoke inflation on the brane, since the initial underlying symmetry of the metric i.e. \( SO(4,2) \), does not strictly require, it at least for homogeneity. Using inflation just to provide fluctuations seems extravagant since setting up inflationary conditions generally means singularities were earlier present [22]. Essentially because a cosmological constant only dominates at larger scales.
It is also not clear that the justifications, for considering a displaced scalar field as a source for chaotic inflation, are still valid. But, again requiring the brane fields not to be in equilibrium is in stark opposition to the statically imposed bulk. Although, matter is supposedly constrained on the brane, this distinction cannot be sustained as the Planck scale is approached and higher order string corrections are required.

As an alternative one can try and produce an eternal brane by achieving a bounce that prevents a singularity forming. For a Reissner-Nordstrom AdS bulk the Friedmann equation becomes modified, such that [19]

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3} \left( \rho + \frac{\rho^2}{2\lambda} \right) + \frac{M}{a^4} - \frac{Q^2}{a^6}$$

where $M$ and $Q$ represent the mass and charge of the bulk space. In the Friedmann equation the $M$ term behaves like radiation while the charge $Q$ violates the weak-energy condition. For a perfect fluid equation of state $p = (\gamma - 1)\rho$, a bounce can occur for matter softer than dust, i.e. $\gamma < 1$. For the closed case $k = 1$ the resulting perpetually oscillating model is like some simple version of the Steady State universe - see eg. [23]. The advantage now is that the total space-time is in better harmony with the PCP principle. Since both sectors now, at least approximately, display this principle. Other possible advantages of using such a charged bulk have been considered in ref.[24], particularly that giving $M$ and $Q$ a time dependence might help explain the required value of $\Lambda$. One problem still to overcome is that perturbations will tend to grow rapidly during static or, particularly, collapsing phases [25].

We see the recent paper on Ekpyrotic universe [26] as another attempt of reconciling this dichotomy of the eternal nature of the bulk with the apparent finite lifetime of our brane universe. The model is very convoluted with “our universe brane been sparked by collision with a bulk brane that somehow left another hidden brane”. Although we have grave doubts that this model is at all natural, it also puts the total space-time in better harmony with the PCP. Only the occasional emission of branes apparently breaking this symmetry. Why this emission rate is not too slow or too fast seems one example of requiring extra add-hoc parameters.

**Brane quantum cosmology**

Can quantum cosmology be applied to justify this maximal symmetry? A number of studies have considered Euclidean quantum cosmology to be rele-
vant to this problem and made certain predictions [27], see also some related works [28]. This work appears very muddled: should the bulk be assumed first and then branes created, or should they both come into existence at the same time? In my view it only becomes justified to take the Euclidean solutions when a forbidden region is present so that quantum phenomena or tunneling could possibly be appropriate - see [22] for more discussion of this more limited view. Others, take a more liberal approach and consider Euclidean solutions even in the absence of such a classical forbidden region. This seems particularly the case of how quantum cosmology has been applied to branes in the work [27]. We wish to broadly highlight a number of problems that this seems to cause.

Recall in simple FRW models a forbidden region is only present when positive curvature and violation of the strong-energy condition is present. Otherwise the initial singularity is not isolated and simple classical evolution of the model can occur. The archetypal example is the closed de Sitter model with classical scale factor $a \sim \cosh t$, that might be formed by quantum tunneling from the initial state of zero scale factor. Since in the brane-bulk model there is no violation of the strong-energy condition at least within the bulk, it seems, at best, contrived to invoke quantum cosmology. After all, since the model displays the PCP with unchanging behaviour it presumably has always existed. This would be like doing creation of flat or static de Sitter models. How is the value of the cosmological constant determined? The absence of a forbidden region also makes it add-hoc whether Euclidean or Lorentzian times should be used cf.[27]. This seems exacerbated when static metrics are also present. Why was the 5 dimensional theory taken to be Euclidean at some stage but now it must be Lorentzian: there seems no natural anchor to fix the nature of the time parameter. Neither is the curvature varying from a larger and more justifiably quantum regime.

The related notion of signature change does not seem appropriate either [29]. This was motivated by trying to avoid the need for using Euclidean time when forbidden regions occur at small scale factors [30]. Although global AdS can have positive curvature it does not violate the strong-energy condition and any appropriate surface to change signature upon is not simply constrained to small size. Working with signature change in FRW AdS type spaces is even more fraught since they are curvature dominated at small scale factors.

In another sense quantum cosmology might still explain why this model
exists compared to ones with differing topologies but only if the universe was actually started by tunneling from some other state and was free to tunnel where the action, or whatever, was minimized. This avenue does not seem particularly valid here since we expect the model to be the result of evolution from some earlier higher dimensional string state. In the usual applications of quantum cosmology, for example in scalar field type inflationary models, we assume the classical general relativistic equations are roughly valid till the Planck scale [14]. The unknown Planck region then becomes describable, or bypassed by “instantons”, by simply quantizing the usual classical equations. The theory and resulting equations are never assumed to be superseded with “higher order” corrections, contrary to our present understanding of string theory with its hierarchical ladder of descriptions.

Although the primeval string state could ultimately require explanation from quantum cosmology, the current status of string theory means this state is not yet even formulated. In the meantime, we need to be careful to avoid using quantum cosmology to supposedly predict conditions that result from classical, or actually string theoretic evolution from an earlier higher energy state. Indeed the ultimate string theory would already include quantum effects so further quantum cosmological reasoning would be somewhat unwarranted. But because of our present lack of understanding we are having to re-quantize the low-energy classical string theories to pass back into the unknown regime. We then of course hope they bear some relation to the real string or M theoretic description in such a domain. The cardinal sin is to move oppositely into the entirely or more classical regime. There quantize and obtain Euclidean “instantons”, and apparently obtain that the universe is initially created in such a state. There was once an argument that claimed to predict a small $\Lambda$ for the present universe, but implicitly assumed the universe had just now quantum created itself into existence [31]. This type of error in quantum cosmology seems easy to do, especially with static metrics that by necessity are not being dominated by obvious quantum phenomena. A similar example is that of obtaining the initial state for the alternative string pre-big bang cosmology [32]. Here the starting point is taken to be a near vacuum, classical state. Trying to obtain such a state from quantum reasoning would simply appear erroneous. Other principles

\[ \text{Likewise it is rather perverse to consider signature change in such a regime, when we have no reason to doubt the usual Lapse conventions [30].} \]
such as “string duality” are instead used to justify such a state. But these
are not exact symmetries and are now broken in our, also classical, universe
[33]. This reasoning also places quantum mechanics in a rather subsidiary
role for determining the initial state.

We have already remarked that quantum cosmology might only reason-
ably be used to determine conditions on a suitable Cauchy surface\footnote{Or as remarked it cannot reasonably be expected to “cut and paste” suitably expunged metrics together: this being a highly severe form of horizon problem} which is insufficient for the present case, with static AdS type metrics required. Because we are using quantum cosmology to simulate this unknown string evolution the requirement that a regulated time-like singularity be present would seemingly put severe constraint on the initial primeval string state. The cosmological (brane-bulk) model could not be determined solely by causal evolution from the primeval state. The relevant boundary conditions would be being imposed continuously. This should be contrasted with the usual inflationary models where the boundary conditions are used solely to predict the initial classical values. Whether their underlying principles such as ‘no boundary” or “only outgoing modes”\cite{14,15} can be extended and applied to non-globally hyperbolic spacetimes is an interesting point for further study.

We have only made some general remarks as to how the issues should first be clarified before specific calculations can be meaningfully attempted. But in summary it seems difficult to conceive of how having both positive and negative cosmological constants at the same time could result from some natural quantum fluctuation. If the bulk was already present, the presence of a large negative $\Lambda$ would supposedly suppress the creation of a brane with a large positive $\Lambda$ compared to say, that from an initially regular Minkowski space. Explaining the static bulk itself does not seem an appropriate application for quantum cosmology.

**Holography principle**

We also note that the static AdS metric plays a crucial role in the holography principle of the AdS/CFT correspondence\cite{34}. If such metrics are not physically justifiable the correspondence would only be of purely theoretical interest. The holography principle $S \leq A$ would not of necessity be valid in more realistic cosmological models, for general reviews see\cite{35}. It has already been remarked that in the FRW AdS metric holography principle is violated as collapse occurs\cite{36}. This can be understood since singularities

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take time to evolve and any $S > A$ can be postponed into creating a singularity in the far future of the universe. We generally agree that the generalized second law of thermodynamics is the more fundamental concept for dynamic models [37]. The holography principle only occurring for certain restrictions, such as in static metrics or within the apparent horizon size of FRW metrics where black holes can form “quickly” compared to the dynamical time scale of the background metric. We suspect that these restrictions are built into the formulism of the covariant holographic bound [38]. This takes holography as primary, but it contains a rather convoluted recipe for determining the relevant “screens” to apply the principle to.

**Conclusions**

The brane world models with global AdS spacetimes are motivated by trying to solve the hierarchy problem of particle physics: the vast difference in the energy scales between particles and the Planck scale. We should briefly mention that applying the notion of “varying constants” to brane models [39] is rather in conflict with this basic aim. Increasing the speed of light makes the Planck (mass) scale even larger so exacerbating any existing hierarchy problem [40].

The brane models typically use highly symmetrical AdS spaces and appear to explain our universe by symmetry breaking of the PCP to achieve our universe, some discussion of this possible paradigm shift can be found in ref. [41]. This approach treats supersymmetry as sacrosanct, but it becomes extremely contrived to obtain more usual cosmological models from AdS, de Sitter and Minkowski or other static bulk metrics. It has earlier been pointed [42] out that it is highly restrictive that the bulk possess such a high symmetry that the brane can display Lorentz symmetry $SO(3, 1)$. We have rather emphasized how this is especially exacerbated when negative cosmological constants are present and the maximal symmetry is reduced to those like FRW. This also explains the difficulty of obtaining a dynamical derivation [43] of the “see-saw” expression (4), as any negative $\Lambda$ will always dominate over matter to cause collapse of the bulk.

The general drift in cosmology has rather been to reduce the initial imposed symmetry and instead utilize inflation as a means of securing homogeneity and isotropy to some sufficient degree.

Some partial resolution of the dilemma is to make both brane and bulk eternal by allowing the brane to bounce instead of hitting singularities. This, and the related Ekpyrotic model, at least make the models more consistent.
If the bulk is anyway going to satisfy the PCP one can achieve almost this symmetry on the brane by utilizing charged black holes in the bulk. The more general bulk can spill over and effectively violate the weak-energy condition within the brane. It would also be interesting to consider the rotating AdS bulk case as this probably would put dramatic constraints on the models. One major disadvantage of accepting the PCP is that it would remain forever beyond the remit of quantum notions to ever understand the reason for such a model. Since its eternal existence means it is fundamentally a classical construct.

On the other hand string theory is still very much developing. Using maximally symmetric spaces with BPS states or Calabi-Yau style compactifications is probably too idealized and not yet physically realistic. Just as full de Sitter isometry might not be allowed in more realistic supergravity theories [44], we suspect similar results will also be eventually found in static AdS, especially if the cosmic censor is to be recovered in the low energy limit, or an explanation produced for the “cut and paste” procedure.

If brane type models are to compete, from less structured or random initial conditions, they somehow need to be formulated in more general spacetimes. This makes the use of large negative cosmological constants difficult, if not impossible, as they will generally cause rapid gravitational collapse. Whether brane models with stabilized large dimensions are possible remains an interesting point of contention. It would generally be the case that stabilizing extra dimension at small scales would be easier since more quantum effects, like the Casimir effect, can be invoked. More usual quantum cosmology might then be applied to such schemes - see e.g.[45], or something more exotic like quantum crumpling might be appropriate [46]. Since there are presumably 7 dimensions to be adequately dealt with starting from 11 dimensional M theory; it might just be the case that they are all curled up the same way. This unfortunately could simply be at too small a scale for realistic experimental observation.

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