Photon Polarization Measurements without the Quantum Zeno Effect

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We consider a photon beam incident on a stack of polarizers as an example of a von Neumann projective measurement, theoretically leading to the quantum Zeno effect. The Maxwell theory (which is equivalent to the single photon Schrödinger equation) describes measured polarization phenomena, but without recourse to the notion of a projective measurement.

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I. INTRODUCTION

The notion of a projective quantum measurement was introduced by von Neumann in his treatise on the mathematical foundations of quantum mechanics. The idea is the following: (i) Every quantum measurement yielding the data “yes” or “no” to an experimental question is described by the projection operators $P = P^\dagger = P^2$ (for “yes”) or $Q = Q^\dagger = Q^2$ (for “no”),

$$P + Q = 1.$$ (1)

(ii) If the initial state of a quantum object before a measurement is $|\Psi_{\text{before}}\rangle$, and if the measurement yields the experimental answer “yes” to a question $P$, then the state of the quantum object after the measurement $|\Psi_{\text{after}}\rangle$ is constructed according to the collapse of the quantum state rule

$$|\Psi_{\text{before}}\rangle \rightarrow |\Psi_{\text{after}}\rangle = \frac{P|\Psi_{\text{before}}\rangle}{\sqrt{\langle\Psi_{\text{before}}|P|\Psi_{\text{before}}\rangle}}.$$ (2)

Eq.(2) is part of a calculation scheme which is not a unitary development of the quantum state during a measurement. The unitary behavior (implicit in the Schrödinger equation for a quantum object) was thought by von Neumann to hold only between measurements.

To see how this works, consider a beam of photons which has passed through an “upwards” polarizer as in Figs.1(a) and (b). If a second polarizer has an axis at an angle of $\theta$ with respect to the upward direction, then one writes the photon wave function $|\uparrow\rangle$ in the basis

$$|\uparrow\rangle = \cos \theta |\left\langle\uparrow\right.\rangle + \sin \theta |\left\langle\downarrow\right.\rangle.$$ (3)

The probability to pass through the second polarizer is then

$$p(\theta) = |\langle\left\langle\downarrow\right.\left.\uparrow\rangle\rangle|^2 = \cos^2 \theta.$$ (4)

The cases shown in Fig.1 below work as follows: In Fig.1(a), the second polarizer is set orthogonal to the first polarizer so that no photons will pass; i.e.

$$p_a(\pi/2) = |\langle\left\langle\left\langle\downarrow\right.\left.\uparrow\rangle\rangle\rangle|^2 = \cos^2(\pi/2) = 0.$$ (5)

This probability can be increased, as in Fig.1(b) below, by placing an intermediate polarizer between the first and last orthogonal polarizers.

In Fig.1(b), a fraction $\cos^2 \theta$ of the $|\uparrow\rangle$ photon beam will pass through the intermediate polarizer and collapse into the state $|\left\langle\downarrow\right.\rangle$. Then, a fraction $\cos^2 (\pi/2 - \theta)$ of the $|\left\langle\downarrow\right.\rangle$ photon beam will pass through the final polarizer and again collapse into the state $|\left\langle\downarrow\right.\rangle$. The above two collapse of the quantum state processes dictate that one multiply probabilities. The final probability that a photon in the upward polarized beam passes through both the intermediate and final polarizers is then given by

$$p_b(\theta) = \cos^2 \theta \cos^2 (\pi/2 - \theta) = (1/4) \sin^2(2\theta).$$ (6)
That an intermediate measurement can increase the probability of passing through a polarizer stack, say \( (1/4) = p_b(\pi/4) > p_b(\pi/2) = 0 \), is an example of what is thought to be a quantum Zeno effect \[3\,13\].

A textbook quantum Zeno effect problem in quantum mechanics \[14\] is shown in Fig.2.

![FIG. 2. A vertical polarized photon beam enters the stack of N=8 polarizers, and a horizontally polarized photon beam leaves the stack.](image)

A beam of photons in state \( |\uparrow\rangle \) is incident on a stack of \( N \) polarizers each of which has a polarization axis rotated by an angle of \( \Delta \theta_N = (\pi/2N) \) relative to the previous polarizer. According to the von Neumann projection postulate \[15\], the quantum state of each photon will collapse \( N \) times, yielding a probability

\[
P_N = \left( \cos^2(\Delta \theta_N) \right)^N = \left\{ \cos^2\left( \frac{\pi}{2N} \right) \right\}^N \tag{7}
\]

of passing through the total stack as pictured in Fig.2. Note, in the formal limit of an infinite number of quantum state collapse events,

\[
\lim_{N \to \infty} P_N = 1, \quad (\text{Quantum Zeno Effect}). \tag{8}
\]

Theoretically every photon passes through the stack. In real experiments, there is always some attenuation of a photon beam passing through many polarizers.

Although the large \( N \) quantum Zeno effect does appear to be verified in the laboratory \[14\], we have a very uneasy feeling about attributing such observations to the rule of collapsing quantum states. After all, long before von Neumann considered projective measurements, Maxwell had developed a reliable electromagnetic radiation theory which perfectly well determined how an electromagnetic wave will (or will not) pass through stacks of polarizers. And not even once did Maxwell feel obliged to discuss the collapse of the electromagnetic radiation field functions. For that matter, neither Bohr nor Einstein \[17\] had ever felt obliged to discuss the so-called wave function collapse.

Our purpose is to discuss why the projection postulate (collapse of the quantum state), certainly may and probably should be eliminated from ones thinking in quantum mechanics. The application of this view towards photon polarization measurements is typical of a more general state of affairs. If one returns to the original Copenhagen interpretation of the quantum mechanical formalism, then the quantum state describes an ensemble of experiments. Each photon certainly does not carry its own wave function which might then undergo a collapse whenever a photon disappears (e.g. whenever a photon is absorbed in a polarizer). The wave function (in reality) describes an ensemble of many photons.

The collapse of the quantum state whenever a photon is absorbed closely resembles the collapse of a probability table (at the automobile insurance company) every time a car crashes. Actually, the probability table at the insurance company stays intact during a car crash. It is the automobile and not the probability that collapses. This constitutes an altogether different pile of scrap metal, even if the cross section for two automobiles to scatter off one another were computed with a quantum mechanical amplitudes to include the small diffraction effects.

In Sec.II, it will be shown how the Schrödinger equation for photons in the vacuum is equivalent to the vacuum Maxwell field equations. In Sec.III, the Schrödinger equation for photons propagating in continuous media (e.g. polarizers) will be explored. The photon wave function follows from the Maxwell electromagnetic theory in continuous media. However, the electromagnetic wave attenuates if the continuous media exhibits dissipation in the form of finite conductivity. The complete quantum system (polarizers plus photons) obeys a big monster Schrödinger equation with a huge number of degrees of freedom and does not at all require the projective measurement formalism of Eq.(2). This feature of quantum mechanics is valid for many systems as will be discussed in Sec.IV.

In Sec.V we compute, for a realistic model of polarizers, the Maxwell equation formulation of the Zeno effect. The classical Maxwell wave computation (without projection operators) yields the same result as the quantum photon Schrödinger amplitude (without projection operators). In an appropriate regime, the projection postulate does indeed give an accurate answer, but this may only be proved by solving the full Maxwell theory without the projection postulate. One might be led to ask why should one ever add to the quantum theory a projection theory of measurements. A hint as to why the projection postulate is so popular might be connected to the fact that the Maxwell theory computation (to be presented) required a fast computer. More than mere speed, we required algorithms sufficient to maintain \(~ 10^3\) significant figure accuracy during the entire calculation of the photon transmission probability. Without recently developed precise numerical algorithms, one might be coerced...
into employing the projection postulate.

In the concluding Sec.VI, we question the utility of replacing the wisdom of experimentalists and engineers who design and build laboratory equipment with a merely philosophical view of projective measurements. It is somewhat unreasonable to represent real laboratory equipment with mathematical projection operators.

II. PHOTON WAVE FUNCTIONS

For a spin $S$ particle [18] one requires three Hermitian matrices $S = (S_1, S_2, S_3)$ obeying

$$[S_i, S_j] = i\epsilon_{ijk}S_k, \quad S_1^2 + S_2^2 + S_3^2 = S(S+1).$$

The photon is a spin one ($S = 1$) particle so we may choose the spin matrices

$$S_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad S_2 = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix},$$

and

$$S_3 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.\quad (11)$$

With $p = -i\hbar \nabla$, the vacuum Schrödinger equation for free photons reads

$$i\hbar \frac{\partial\psi(r,t)}{\partial t} = c(S \cdot p)\psi(r,t).$$

Writing

$$\psi(r,t) = \begin{pmatrix} F_1(r,t) \\ F_2(r,t) \\ F_3(r,t) \end{pmatrix},$$

allows one to cast the photon Schrödinger Eq.(12) into the (perhaps) more simple vector form

$$i\left(\frac{\partial F(r,t)}{\partial t}\right) = c \text{ curl } F(r,t).$$

The vector field $F(r,t)$ should be constrained to obey

$$\text{div} F(r,t) = 0.$$  \hspace{1cm} (15)

Eq.(15) is equivalent to only allowing two independent spin states (say superpositions of only two transverse polarizations), thereby eliminating a third (longitudinal polarized) “zero energy” mode. Finally, Eqs.(14) and (15) take on a clearly recognizable form if one defines the real and imaginary parts of $F(r,t)$ to be, respectively, the electric field $E(r,t)$ and the magnetic field $B(r,t)$ [19] [20] [21]; i.e.

$$F(r,t) = E(r,t) + iB(r,t).$$

The photon Schrödinger Eq.(12) then appears equivalent to the vacuum Maxwell theory; i.e.

$$\left(\frac{\partial E(r,t)}{\partial t}\right) = c \text{ curl } B(r,t),$$

and

$$\left(\frac{\partial B(r,t)}{\partial t}\right) = -c \text{ curl } E(r,t),$$

with the vacuum Gauss law Eq.(15) constraints

$$\text{div} B(r,t) = 0, \quad \text{div} E(r,t) = 0.$$ \hspace{1cm} (19)

It is of interest to discuss the relativistic Lorentz symmetry of the photon Schrödinger Eq.(12). The proof of Lorentz symmetry is actually well known since we have already proved equivalence with the Maxwell theory. However, the discrete Lorentz symmetries are still worthy of further discussion [22–28]. If we consider the classical Lorentz force on a charge equation $f = e(E + (v \times B)/c)$, and then allow the charge to change sign $e \rightarrow -e$, then the force $f$ will remain the same if we also allow $E \rightarrow -E$ and $B \rightarrow -B$. Thus, the electromagnetic field Eq.(16) is odd under charge conjugation,

$$\hat{C}F = -F.$$ \hspace{1cm} (20)

Under time reversal, $E \rightarrow E$ and $B \rightarrow -B$ so that

$$\hat{T}F = F^*.$$ \hspace{1cm} (21)

Finally, under parity $E \rightarrow -E$ and $B \rightarrow B$ so that

$$\hat{P}F = -F^*.$$ \hspace{1cm} (22)

Note

$$\hat{T}\hat{C}\hat{P} = 1,$$ \hspace{1cm} (23)

as would be expected from relativistic wave equations.

III. PHOTON WAVE FUNCTIONS IN CONTINUOUS MEDIA

The Maxwell theory for an electromagnetic disturbance moving through continuous media [29] [30] may be formulated starting from

$$\text{div} E(r,t) = 4\pi \rho(r,t), \quad \text{div} B(r,t) = 0,$$ \hspace{1cm} (24)

and

$$\left(\frac{\partial E(r,t)}{\partial t}\right) = c \text{ curl } B(r,t) - 4\pi J(r,t),$$ \hspace{1cm} (25)

and

$$\left(\frac{\partial B(r,t)}{\partial t}\right) = -c \text{ curl } E(r,t).$$ \hspace{1cm} (26)
It is sufficiently general to supplement Eqs. (24), (25) and (26) by employing a linear causal relationship between the current and the electric field, i.e.

$$J(r,t) = \int_0^\infty \int K(r,r',s)E(r',t-s)\,d^3r'\,ds. \quad (27)$$

If, for a complex frequency $\zeta$ where $\Im m \zeta > 0$,

$$E(r,t) = \Re e\{E_0(r,\zeta)e^{-i\zeta t}\}, \quad (28)$$

$$J(r,t) = \Re e\{J_0(r,\zeta)e^{-i\zeta t}\}, \quad (29)$$

Eqs. (27), (28) and (29) serve to define the non-local conductivity $\sigma(r,r',\zeta)$, i.e.

$$J_0(r,\zeta) = \int \sigma(r,r',\zeta)E_0(r',\zeta)\,d^3r', \quad (30)$$

where

$$\sigma(r,r',\zeta) = \int_0^\infty e^{i\zeta s}K(r, r', s)\,ds. \quad (31)$$

Some workers prefer to complete the Maxwell equations in continuous media by employing a constitutive equation relating the Maxwell displacement field $D(r,t)$ to the electric field $E(r,t)$. In the frequency domain, the dielectric response $\epsilon(r, r', \zeta)$ of the continuous media is defined as

$$D_0(r, \zeta) = \int \epsilon(r, r', \zeta)E_0(r', \zeta)\,d^3r'. \quad (32)$$

Just so long as the various response functions are maintained as non-local (in space and time), which of the many equivalent response functions are actually used in a calculation remains a matter of convenience. For example, Eqs. (30) and (32) are equivalent if we choose

$$\epsilon(r, r', \zeta) = 1\delta(r - r') + \left(\frac{4\pi i\sigma(r, r', \zeta)}{\zeta}\right). \quad (33)$$

The dielectric, magnetic, or conductivity response functions conventionally used to describe continuous media now make an appearance into the Schrödinger equation for a photon moving through the continuous media via the photon “self-energy” insertion. To see what is involved, we may write the Maxwell Eqs. (24) and (25) as

$$\text{div}\,F(r, t) = 4\pi \rho(r, t), \quad (34)$$

and

$$i\left(\frac{\partial F(r, t)}{\partial t}\right) = c \, \text{curl} \, F(r, t) - 4\pi iJ(r, t). \quad (35)$$

where Eq. (16) has been invoked. Employing Eqs. (21), (27) and (35), we find our central result for the photon Schrödinger equation in continuous media

$$i\left(\frac{\partial \Psi(r, t)}{\partial t}\right) = c \, \text{curl} \, F(r, t) +$$

$$\int_0^\infty \int \Sigma(r, r', s)F(r', t - s)\,d^3r'\,ds. \quad (36)$$

where the photon self-energy part is defined as

$$\Sigma(r, r', s) = -2\pi iK(r, r', s)(1 + \dot{T}). \quad (37)$$

Using Eqs. (13) and (36), one may write the photon Schrödinger equation in the form

$$i\hbar\left(\frac{\partial \Psi(r, t)}{\partial t}\right) = c(S \cdot p)\psi(r, t) +$$

$$\hbar \int_0^\infty \int \Sigma(r, r', s)\psi(r', t - s)\,d^3r'\,ds. \quad (38)$$

In principle the calculation of the self energy part $\Sigma(r, r', s)$ is equivalent to the calculation of the conductivity $\sigma(r, r', \zeta)$ as is evident from Eqs. (21), (31), and (37). In a practical calculation involving the photon Schrödinger equation in a polarizer, one should use the conductivity and dielectric response functions as those of the optical engineers who designed the device. Finally, the non-local form of Eq. (38) is shared by many other quantum mechanical systems as we shall now briefly explore.

**IV. NON-LOCAL SCHRÖDINGER EQUATIONS**

Although we do not adhere to the non-unitary collapse of the quantum state Eq. (2) as postulated by von Neumann, we do find the notion of projection operators (also introduced into quantum mechanics by von Neumann) to be more than just convenient. Eq. (1) will thereby be invoked in what follows. For a macroscopic laboratory quantum system we here insist that a monster Schrödinger equation with a huge number of degrees of freedom still holds true,

$$i\hbar\left(\frac{\partial}{\partial t}\right)\Psi(t) > = \mathcal{H}_{\text{tot}}\Psi(t) >. \quad (39)$$

We do not contemplate the ultimate monster wave function of the whole universe, not only because we are not smart enough to find it, but because we do not know how to build an experimental ensemble of universes to test a hypothetical quantum state of our universe. We leave such considerations to people more wise than we are.

We are happy to say something meaningful about a very small physical piece of a laboratory quantum state; e.g. a “projected piece”

$$|\psi(t)\rangle = \mathcal{P} |\Psi(t)\rangle. \quad (40)$$

We leave the rest of the monster wave function,

$$|\phi(t)\rangle = \mathcal{Q} |\Psi(t)\rangle, \quad (41)$$

to the engineers who design the experiment. With the little piece of the wave function Eq. (40) and the big piece of the wave function Eq. (41), we rewrite Eq. (39) as
so we solve for the monster piece

\[ i\hbar \left( \frac{\partial}{\partial t} \right) \left| \psi(t) > \right> = \left( \begin{array}{cc} H & V \\ V^\dagger & H' \end{array} \right) \left| \psi(t) > \right> + V \left| \phi(t) > \right>, \]

and

\[ i\hbar \left( \frac{\partial}{\partial t} \right) \left| \phi(t) > \right> = H' \left| \phi(t) > \right> + V^\dagger \left| \psi(t) > \right>, \]

subject to the causal boundary condition

\[ \left| \phi(t) > \right> = -\left( \frac{i}{\hbar} \right) \int_0^\infty e^{-iH's/\hbar} V^\dagger \left| \psi(t-s) > \right> ds. \]

From Eqs.(43) and (45) we find the general non-local Schrödinger equation for the piece of the wave function that is of interest; It is

\[ i\hbar \left( \frac{\partial}{\partial t} \right) \left| \psi(t) > \right> = H \left| \psi(t) > \right> + V \left| \phi(t) > \right>, \]

where

\[ \left| \phi(t) > \right> = -\left( \frac{i}{\hbar} \right) \int_0^\infty e^{-iH's/\hbar} V^\dagger \left| \psi(t-s) > \right> ds, \]

subject to the causal boundary condition

\[ \left| \phi(t) > \right> = -\left( \frac{i}{\hbar^2} \right) V e^{-iH's/\hbar} V^\dagger. \]

The full monster Schrödinger Eq.(39) for complete wave function \( |\Psi(t) > \) reduces to the non-local in time Schrödinger Eq.(46) for the small subspace piece of the wave function \( |\psi(t) > \). A particular example of Eq.(46) is the photon Schrödinger Eq.(38). The point is that when a photon propagates inside of a continuous media device, say a polarizer, the monster wave function includes both the photon and the polarizer degrees of freedom. In a one photon projected subspace, the quantum state of the polarizer is fixed. In addition to the polarizer, there is but one propagating photon. If the photon is absorbed, then an electronic excitation is produced which heats up the polarizer. The piece of the monster quantum state representing this absorbed photon situation is not contained in the single photon subspace. The single photon subspace wave function is then attenuated due to photon absorption in the polarizer. All of this (including the attenuation of the Maxwell wave in the medium) is described by the one photon Schrödinger equation, without recourse to the projection postulate. Maxwell was so lucky to be able to write down the Schrödinger equation for the photon in matter, including attenuation, without knowing anything about quantum states (be they monster or otherwise).

It is of interest to look for a fixed energy solution of the subspace wave function Eq.(46) of the form

\[ |\psi(t) > = |\psi_E > e^{-iEt/\hbar}. \]

Eqs.(46), (47) and (48) imply

\[ \left\{ H + \Delta(E) - \left( \frac{i\hbar}{2} \right) \Gamma(E) \right\} |\psi_E > = E |\psi_E >, \]

where

\[ \Delta(E) - \left( \frac{i\hbar}{2} \right) \Gamma(E) = V \left( \frac{1}{E - H' + i0^+} \right) V^\dagger. \]

That the effective energy dependent Hamiltonian on the left hand side of Eq.(49)

\[ \mathcal{H}_{eff}(E) = H + \Delta(E) - \left( \frac{i\hbar}{2} \right) \Gamma(E) \]

is not Hermitian, is due to the “Fermi platinum rule” transition rates to leave the small subspace and wander into the remaining monster subspaces. The Fermi platinum rule

\[ \Gamma(E) = \left( \frac{2\pi}{\hbar} \right) V \delta(E - H') V^\dagger \]

is just a little bit more valuable than the more usual perturbative Fermi golden rule, since Eq.(52) is rigorously true to all orders of perturbation theory. The fixed energy \( E \) case corresponds to having a fixed photon frequency \( \omega = (E/\hbar) \) for the electromagnetic problem at hand. Let us return to this problem.

V. MAXWELL THEORY AND POLARIZERS

The Maxwell theory of a transverse wave moving through a polarizer may be formulated following the discussion of Born and Wolf [40]. (i) If a transverse wave propagates in the z-direction, then the wave may be described by four electromagnetic field components, say

\[ \chi(z) = \left( \begin{array}{c} E_1(z) \\ E_2(z) \\ B_1(z) \\ B_2(z) \end{array} \right). \]

If the polarizer is oriented by a rotation angle \( \theta \), as in Fig.1(b), then the dielectric response tensor will have the form

\[ \epsilon(\theta) = \left( \frac{\epsilon_1 + \epsilon_2}{2} + \left( \frac{\epsilon_1 - \epsilon_2}{2} \right) e^{-i\tau_2 \tau_3 e^{i\tau_3 \theta}}, \right) \]

where we have employed the 2 × 2 Pauli matrices

\[ \tau_1 = \left( \begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right), \quad \tau_2 = \left( \begin{array}{cc} 0 & -i \\ i & 0 \end{array} \right), \]

and

\[ \tau_3 = \left( \begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array} \right). \]
The complex principal eigenvalues of the dielectric tensor at frequency $\omega$ have the form

$$\tilde{\varepsilon}_j = \varepsilon_j + \left(\frac{4\pi i \sigma_j}{\omega}\right), \quad j = 1, 2.$$  \hfill (57)

The real parts of the dielectric eigenvalues $\varepsilon_{1,2}$ determine the velocities of the waves polarized in the principal directions. The conductivities $\sigma_{1,2}$ determine the dissipative attenuation of the waves polarized in the principle directions. For a well designed polarizer, a wave with an allowed polarization direction will propagate unattenuated while a wave with a disallowed polarization will be strongly attenuated.

*It is the anisotropic dissipative conductivity that is often modeled by the quantum projective postulate for the case of photon polarization.* On the other hand, conductivity is evidently classical since both Ohm and Maxwell were dead and gone before quantum mechanics had been invented.

For the classical Maxwell calculation, the wave at the beginning ($z = 0$) of the polarizer is related to the wave at the end ($z = a$) of the polarizer by a $4 \times 4$ transfer matrix

$$\chi(a) = \mathcal{M}(\theta, \xi)\chi(0), \quad \xi = \left(\frac{\omega a}{c}\right). \hfill (58)$$

$\chi(z)$ is defined in Eq.(53), and in partitioned matrix form

$$\mathcal{M}(\theta, \xi) = \begin{pmatrix} \cos(\xi \sqrt{\varepsilon(\theta)}) & (\sin(\xi \sqrt{\varepsilon(\theta)})/\sqrt{\varepsilon(\theta)}) \tau_2 \\ \tau_2 \sqrt{\varepsilon(\theta)} \sin(\xi \sqrt{\varepsilon(\theta)}) & \tau_2 \cos((\xi \sqrt{\varepsilon(\theta)}) \tau_2 \end{pmatrix}. \hfill (59)$$

For a stack of $N$ polarizers, each of width $a$ and oriented at angles $(\theta_1, ..., \theta_N)$, one may repeat the above process going from the beginning ($z = 0$) of the stack to the end ($z = Na$) of the stack

$$\chi(Na) = \mathcal{M}_N \chi(0), \quad \xi = \left(\frac{\omega a}{c}\right). \hfill (60)$$

One Multiplies the transfer matrices for each polarizer in the stack

$$\mathcal{M}_{tot,N} = \mathcal{M}(\theta_N, \xi) \cdots \mathcal{M}(\theta_2, \xi) \mathcal{M}(\theta_1, \xi). \hfill (61)$$

Once the total $4 \times 4$ transfer matrix is computed, one may find the transmission amplitudes $T_j(N)$ and reflection amplitudes $R_j(N)$ for the two polarization directions ($j = 1, 2$) by solving the linear algebra problem

$$\mathcal{M}_{tot,N} \begin{pmatrix} 1 + R_1(N) \\ R_2(N) \\ R_2(N) \\ 1 - R_1(N) \end{pmatrix} = \begin{pmatrix} T_1(N) \\ T_2(N) \\ -T_2(N) \\ T_1(N) \end{pmatrix}. \hfill (62)$$

For an incident photon in the allowed 1-polarization direction, the probability that the photon exits a stack of $N$ polarizers (rotated into the allowed 2-polarization direction) is given by the absolute square of the transmission amplitude,

$$P_N = |T_2(N)|^2. \hfill (63)$$

From Eqs.(54), (59), and (61)-(63), with each of the angles in $(\theta_1, ..., \theta_N)$ set equal to $(\pi/2N)$, one may compare the the projection postulate $P_N$ in Eq.(7) to the Maxwell theory, or equivalently the photon Schrödinger Eq.(63), without the projection postulate. The matrix multiplications in Eq.(61) and the linear algebra in Eq.(62) are best left to a computer. The results of such a computation are shown in Fig.3.

![FIG. 3. The Zeno effect probability $P_N$ for a photon to pass through $N$ polarizers: (i)The solid curve (evaluated for integer $N$) is the projection postulate in Eq.(7). (ii) The filled circles represent the Maxwell theory with zero attenuation for the allowed polarization direction. (iii) The filled squares represent the Maxwell theory with a small attenuation even for the allowed polarization direction. The solid curve in Fig.3 is the prediction of the projection postulate Eq.(7). For the Maxwell theory with a reasonable stack of laboratory polarizers, there may be $\sim 20$ wavelengths of light in each polarizer in the stack. Thus we chose the parameter $\xi = (\omega a/c) = 100$ for purposes of illustration. We further choose $\tilde{\varepsilon}_1 = 1$ to describe the polarization direction for which the polarizer would be transparent and the complex $\tilde{\varepsilon}_1 = 1 + 2i$ to describe the polarization direction for which the polarizer would be strongly attenuated (if not absolutely opaque). The resulting computed points are the filled circles of Fig.3. For this case, the projection postulate yields a $P_N$ which is quite close to the full Maxwell theory. For some regimes (perhaps more close to the laboratory situation), the Maxwell theory begins to diverge from the projection postulate for large $N$. In the laboratory all polarization directions are subject to small
programming code which allows for precise numerical algorithms together with publicly available high indeed. Fortunately, Aberth [42] has developed pre-polarizers, the required accuracy turns out to be very and (63) be reliably computed. For realistic laboratory at intermediate stages of the calculation, could Eqs.(62) trix multiplications in Eq.(61). Only with high accuracy was needed for the ma-

ure accurate computations. Much of the required code was thus available. We used this code. However, we re-

for the standard functions and standard linear algebra for the calculation produced complete nonsense [43]. The physical problem was that the Maxwell waves reflect back and forth within the polarizers. Hence, there exist exponentially attenuated terms as well as exponentially amplified terms in the transfer matrix. High accuracy was needed for the matrix multiplications in Eq.(61). Only with high accuracy at intermediate stages of the calculation, could Eqs.(62) and (63) be reliably computed. For realistic laboratory polarizers, the required accuracy turns out to be very high indeed. Fortunately, Aberth [42] has developed precise numerical algorithms together with publicly available programming code which allows for $\sim 10^4$ significant figure accurate computations. Much of the required code for the standard functions and standard linear algebra was thus available. We used this code. However, we required $\sim 10^4$ significant figure computations to produce the reliable plots shown in Fig.3.

VI. CONCLUSIONS

In discussions of the foundations of quantum mechanics there has been a tendency to regard all interference of waves as explicitly due to quantum mechanics. This is only sometimes true, since “waves” as well as “particles” also have classical limits. In many (perhaps most) of the electromagnetic experiments and applications, the Maxwell wave itself is classical. The distinction between a classical wave, a classical particle, and a quantum particle has been discussed simply, yet clearly, by Feynman [43].

(i) A quantum particle is certainly not a classical particle. Quantum particle dynamics dictates amplitude superposition, whereas classical dynamics dictates probability superposition. (ii) A quantum particle is certainly not a classical wave. A classical wave is “smooth” while a quantum particle comes in “lumps”. Photons are lumpy. Photon counters yield one click whenever a photon is de-
tected. One photon can never make two different counters each give you one half of a click. That is much too smooth for quantum mechanics. In quantum mechanics, each photon counter gives one click or no clicks. The photon is certainly not a classical wave. The photon is certainly not a classical particle. The photon is certainly not a little bit classical wave and a little bit classical particle. (iii) The photon is neither a classical wave nor a classical particle nor a little bit of both.

Electromagnetic waves, as described by Maxwell, are classical waves. If a radio station broadcasts a signal, then the signal is (for all practical purposes) smooth. Your radio gets some of the signal, and our radio gets some of the signal. There is enough smoothly distributed classical wave to go around to all of the radios. The classical Maxwell wave is smooth. One detector cannot get a fraction of a photon. Remember, photons come in lumps. One photon can get destroyed in at most one radio receiver. One detector can get any fraction of a classical Maxwell wave. Remember, classical waves are smooth. Yet the Schrödinger equation for photon is the same (in a mathematical formalism) as the Maxwell equations for classical electromagnetic waves. This is because the pho-

ton has no mass and because the photon is a Boson. When very many photons are Bose condensed into a single quantum state, the result is a Maxwell wave which is classical for all practical purposes.

The above picture was not all that clear to Einstein (by his own admission), and we do not claim to know more about photons than did Einstein. According to Einstein, a Maxwell wave hits a half silvered mirror and some of the wave goes to one detector and the rest of the wave goes to a second detector. If the wave represents one photon, then only one counter goes click. The other counter gets whacked with a Maxwell wave. Yet this counter counts nothing. When there is but one photon, only one counter can speak for her. Some Maxwell wave parts turn out to have a photon, and some Maxwell wave parts turn out not have a photon. Some counters go click and some counters remain mute. You never know. And photons are lumpy. It is a very strange shell game according to Einstein [44].

However, the Zeno effect formally attributed to the quantum mechanical projection postulate has here been argued to be merely a classical wave effect. Maxwell would not have had to increase his knowledge of electromagnetic theory by even one iota in order to totally understand the polarization version of the Zeno effect. No quantum mechanics is required, not to even speak of the projection postulate.
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