Transport properties of a charged drop in an external electric field

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June 17, 2014

Abstract

Transport properties of a small, dense, charged droplet in external field are discussed. A time dependent non-equilibrium distribution function of the transverse, azimuthally symmetric droplet in the external electric field is calculated. With the help of this distribution function, a shear viscosity coefficient in the transverse plane is calculated as well and it is found to be depend on the time of the droplet’s expansion and is very small. An applicability of the results to the description of initial states of quark-gluon plasma obtained in high-energy interactions of nuclei is also discussed.

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1 Introduction

Collisions of relativistic nuclei in the RHIC and LHC experiments at very high energies led to the revelation of a new state of matter named quark-gluon plasma (QGP). At the initial stages of the scattering, this plasma is in a strongly interacting phase and resembles as a liquid referred as a strongly coupled Quark Gluon Plasma (sQGP) [1], whose microscopic structure is not well understood yet [1, 2, 3, 4, 5, 6, 7, 8, 9]. Anyway, the data obtained at the RHIC experiments are in a good agreement with the predictions of the ideal relativistic fluid dynamics, [10, 11], which establish fluid dynamics as a main theoretical tool to describe the collective flow in the collisions. As an input to the hydrodynamical evolution of the particles it is assumed that after a very short time, $\tau < 1 \text{ fm/s}$ [12], the matter reaches a thermal equilibrium and expands with a very small shear viscosity [13, 14].

In the process of the high-energy scattering, the thermal equilibrium may be achieved only for small fireballs [15] of the matter [3, 8, 14, 16]; the colliding system cannot be in a global equilibrium state because the nuclei scattering at high energy is a highly non-equilibrium process [3, 17]. Subsequent expansion of the matter’s hot spot occurs with the constant entropy [18], which justifies the applicability of the hydrodynamical description of the process. This adiabatic expansion continues till the value of the particle’s mean free path becomes comparable with the size of the system. In this stage, instead a liquid, a gas of interacting particles whose density rapidly decreases reveals.

Application of the fluid dynamics to the process of the fireball’s expansion requires some initial conditions among which the most intriguing one is a small value of the shear viscosity/entropy ratio. Perturbative result for the shear viscosity calculations is large, [19], and some new mechanisms of the explanation of the shear viscosity smallness are required. There are different approaches to the possible mechanisms of this smallness, see for example [20, 21, 22]. In this note we propose a some new mechanism of the shear viscosity smallness, similar in some extent to the ideas of [21, 23]. Namely, we consider a situation when during short time of the hydrodynamical expansion the anomalous viscosity dominates the total shear viscosity due the fact that its value is less than the value of usual kinetic ”collision” viscosity during the same time. But the mechanism of the anomalous viscosity smallness in our approach is different from the proposed in [21].

We employ the following main ideas in our calculations. We consider an expansion of small, [25], hot, charged and dense droplet of charged particles. Following by [18], we consider the process of the droplet’s expansion as the process with the constant entropy. Due to the fact that we consider a charged droplet, the distribution function of the system cannot be stationary, it must depend on time. Thereby, the expansion process of the charged droplet with the constant entropy is determined by Vlasov’s equation, [24], which is the main tool for the kinematic description of the process. Therefore, we solve the Vlasov’s equation for the time-dependent, non-equilibrium distribution function which, nevertheless being time-dependent, anyway preserves constant value of the entropy. Moreover, we consider our hot spot in the external field of the other, relativistic particles, see [25], which contributes to the Vlasov’s equation solution. As we will see further, the interaction between the charged droplet and external field is the mechanism responsible for the leading shear viscosity coefficient and as well for it’s smallness in the given framework.

In our calculations, we consider a simplified model with only transverse dynamics of the droplet included and only electrical external field. More complex and precise models with the longitudinal dynamics and magnetic fields included we will investigate some when later, in an additional paper. In the next Section 2, we derive the electromagnetic field potentials created by the relativistically moved charged drop. In Section 3, we write the Vlasov’s equation for the charged drop in the external electrical field, whereas in Section 4, we rewrite the Vlasov’s equation in a new, integral form. Also, in Section 4, we determine a new initial condition for the equation which is different from the given in Appendix A because of the presence of the external electrical field. In further Section, Section 5, we calculate a non-equilibrium distribution function till the second order of the perturbative series, which is formulated and defined in this Section. Section 6 deals with the transverse density profile of the drop and in Section 7, we calculate the transport characteristic of the charged droplet in the external electrical field, including transverse shear viscosity coefficient. The last Section, Section 8, is a conclusion of the paper.
2 An external field of relativistic charged drop

To a first approximation, we consider a charged drop of the matter moving with some velocity. In the frame related with this drop, the distribution function of the droplet’s particles is described by usual Vlasov’s equations:

\[
\frac{\partial f_s}{\partial t} + \vec{v} \frac{\partial f_s}{\partial \vec{r}} + \left( \frac{q}{c} \vec{v} \times \vec{B} \right) \frac{\partial f_s}{\partial \vec{p}} = 0. \tag{1}
\]

with Maxwell’s equations

\[
\nabla \times \vec{E} = 0, \tag{2}
\]
\[
\nabla \times \vec{B} = 0, \tag{3}
\]
\[
\nabla \cdot \vec{E} = 4\pi q n \int f_s(x,t) \, d^3v, \tag{4}
\]
\[
\nabla \cdot \vec{B} = 0. \tag{5}
\]

In this approximation, for the usual Maxwellian initial distribution function, the solution for the self-consistent field is the screened electrostatic potential plus some constant which can be considered further as an initial value of the potential:

\[
\varphi = C_0 e^{-r/r_D} + 4\pi q n r_D^2 \tag{6}
\]

see [25, 26] for example. Here \( C_0 \) is some constant and

\[
r_D^2 = \frac{k_B T_0}{4\pi q^2 n_0} \tag{7}
\]

is Debye length. Redefining the potential’s initial value we obtain finally the following expression for the potential

\[
\varphi = C_0 e^{-r/r_D} \tag{8}
\]

which formally can be regarded as originating from the exchange of ”massive photons” with masses \( M = 1/r_D \).

Now, let’s consider the field of Eq. (8) in the rest frame of some another dense drop of particles, which is in the rest relatively to the first drop. In this frame, the potential Eq. (8) is created by fast moving dense cloud of the particles with the volume \( V_0 \), particle’s density \( n \) and total charge \( q n V_0 \). In this reference frame, therefore, the potential Eq. (8) is described by Proca equation, see [], which has the following form:

\[
\Delta \varphi - \frac{1}{c^2} \frac{\partial^2 \varphi}{\partial^2 t} - \frac{\varphi}{r_D^2} = -4\pi q n_0 V_0 \delta(\vec{r} - \vec{r}_s(t)) \tag{9}
\]

Solution of this equation is well known:

\[
\varphi(\vec{r},t) = \frac{q n_0 V_0}{2\pi^2} \int d^2k_\perp e^{ik_\perp(r_\perp - b)} \int_{-\infty}^{\infty} e^{ikz(z - vt)} dk_z \tag{10}
\]

where \( \vec{r} = (r_\perp, z) \), the position of the moving drop is given by \( \vec{r}_s = (b, vt) \) with \( r_\perp = (r_x, r_y) \), \( b = (b_x, b_y) \) and \( \gamma = \sqrt{1 - \frac{v^2}{c^2}} \). In the relativistic limit when \( v \approx c \) we obtain in the first order expansion over \( \gamma \) for the electrostatic potential:

\[
\varphi(\vec{r},t) = 2 q n_0 V_0 \delta(z - ct) K_0(|r_\perp - b|/r_D) \tag{11}
\]

where \( K_0 \) is Macdonald’s function [29]. Correspondingly, for the vector potential of the electromagnetic field we have:

\[
A_z(\vec{r},t) = \varphi(\vec{r},t), \tag{12}
\]
\[
A_x(\vec{r},t) = A_y(\vec{r},t) = 0. \tag{13}
\]
After the gauge transformation with the gauge function

\[ f = -2 q n_0 V_0 \theta(z - ct) K_0(|r_\perp - b|/r_D) \]  

we finally obtain the potentials of the moving drop in the rest frame of another dense droplet:

\[ \varphi(\vec{r}, t) = A_z(\vec{r}, t) = 0, \]  

\[ A_x(\vec{r}, t) = -2 q n_0 V_0 \theta(z - ct) \partial_z K_0(|r_\perp - b|/r_D), \]  

\[ A_y(\vec{r}, t) = -2 q n_0 V_0 \theta(z - ct) \partial_y K_0(|r_\perp - b|/r_D). \]  

Here \( \theta \) is the Heaviside step function and coordinate dependence of the potentials is factorized by two parts: the \( \theta \) function depends only on the longitudinal \( z \) coordinate whereas the second function depends on the transverse \( x \) and \( y \) coordinates.

### 3 Vlasov’s equation of the charged droplet in the external field

In this section, we consider an interaction between two different charged spots, one of which was created at the first stage of the interaction and stays in the rest and the second is moving towards the first one. The same systems Eq. (1), Eq. (2) may be reformulated as a system of self-consistent field of the first spot in the field of the second. In this case instead Eq. (1) we have:

\[ \frac{\partial f_s}{\partial t} + \vec{v} \frac{\partial f_s}{\partial \vec{r}} + q \vec{E}_{\text{total}} \frac{\partial f_s}{\partial \vec{p}} = 0, \]  

where we again consider the case when \( B = 0 \), and we take

\[ \vec{E}_{\text{total}} = \vec{E}_s - \vec{E}_{\text{ext}} \]  

with \( \vec{E}_s \) as a new self-consistent field and \( \vec{E}_{\text{ext}} \) as a field of the second, incident drop. Maxwell’s equations for the self-consistent field in this case have a following form

\[ \nabla \times \vec{E}_s = 0, \]  

\[ \nabla \cdot \vec{E}_s = 4 \pi q n \int f_s(x, t) d^3v. \]  

The vector potential which determines the \( \vec{E}_{\text{ext}} \) field can be found by using Eq. (16)-Eq. (17) from the previous section. In cylindrical coordinates with the origin in the center of the rest spot, where we take \( r_\perp = r \), we have:

\[ \vec{E}_{\text{ext}} = -\frac{1}{c} \frac{\partial A_r}{\partial t} \hat{e}_r \]  

with the field

\[ A_r(\vec{r}, t) = A_x(\vec{r}, t) \cos \theta + A_y(\vec{r}, t) \sin \theta = -2 q n_0 V_0 \theta(z - ct) \frac{\partial K_0(|r - b|/r_{0D})}{\partial r}. \]  

Defining an overall charge of the incident drop equal to

\[ Q_T = q n_0 V_0, \]  

we have:

\[ A_r(\vec{r}, t) = -2 Q_T \theta(\eta) \frac{\partial K_0(|r - b|/r_{0D})}{\partial r}. \]  

Here we denoted \( \eta = ct \) because of the new rest frame with the origin in the droplet in rest, therefore \( \eta \geq 0 \) determines the evolution of the drop after the interaction with external field at \( \eta = 0 \).

Also, denoting \( \xi_0 = \frac{r}{r_{0D}} \), we obtain:

\[ A_r(\vec{r}, t) = -2 \frac{Q_T}{r_{0D}} \frac{\partial K_0(\xi_0)}{\partial \xi_0} \theta(\eta) = F(r) \theta(\eta). \]
Here we denoted $K_0(\zeta_0) = K_0(|r - r_0D|/r_0D)$ taking the cut-off of the impact parameter $b$ equal to $r_0D$. We underline, that the total charge of the incident drop $Q_T$ depends in general on a total energy of the process in the c.m.f.

In Vlasov’s equation, Eq. (18), we redefine the time variable as $t \rightarrow ct = \tau$:

$$c \frac{\partial f_s}{\partial \tau} + \vec{v} \frac{\partial f_s}{\partial \vec{r}} + q \left( \vec{E}_s - \vec{E}_{\text{ext}} \right) \frac{\partial f_s}{\partial p_r} = 0.$$  \hspace{1cm} (27)

with

$$\vec{E}_s = -\nabla \Phi.$$  \hspace{1cm} (28)

and

$$\vec{E}_{\text{ext}} = \frac{\partial A_c}{\partial \tau} \hat{e}_r = F(r, s) \delta(\tau) \hat{e}_r,$$  \hspace{1cm} (29)

where $F(r, s)$ is the function from Eq. (25). In Eq. (27) there is no $z$ dependence neither in the distribution function nor in the electric field. The vectors $\vec{r}, \vec{p}$ are radial there, therefore Eq. (27) describes an evolution of the distribution function in two dimensional transverse plane as a function of the "time" parameter $\tau$.

4 Integral form of Vlasov equation and new initial conditions

In order to analyze an analytical solution of Eq. (27)-Eq. (29) it is more convenient to rewrite Vlasov’s equation as an integral one. First of all, we rewrite this equation as following:

$$\frac{1}{\zeta_r} \frac{\partial f_s}{\partial \tau} + \frac{\partial f_s}{\partial r} = -\frac{q}{c \zeta_r} (E_{rs} - E_{\text{ext}}) \frac{\partial f_s}{\partial p_r}$$  \hspace{1cm} (30)

with $\zeta_r = v_r/c$. For the l.h.s. of this equation, we find the fundamental solution from the following equation:

$$\frac{1}{\zeta_r} \frac{\partial f_s}{\partial \tau} + \frac{\partial f_s}{\partial r} = \delta(\tau) \delta(r),$$  \hspace{1cm} (31)

this solution is well known, see for example [30], it is given by

$$\delta'(r - \zeta_r \tau).$$  \hspace{1cm} (32)

Thereby, with the help of Eq. (32), we rewrite Eq. (30) as an integral equation:

$$f_s(r, \vec{c}, \tau) = -\frac{q}{c \zeta_r} \int \frac{d\vec{r'} d\tau'}{\delta(\tau - \tau', r - r')} \left( E_{rs}(r', \tau') - E_{\text{ext}}(r', \tau') \right) \frac{\partial f_s(r', \vec{c}, \tau')}{\partial p_r(\zeta_r)} + f_0(r - \zeta_r \tau, \vec{c}),$$  \hspace{1cm} (33)

where the function $f_0(r, \vec{c})$ will be determined later. Inserting Eq. (32) into Eq. (33), we obtain:

$$f_s(r, \vec{c}, \tau) = -\frac{q}{c} \int_0^\tau \frac{d\tau'}{\delta(\tau - \tau', r - r')} \left( E_{rs}(r - \zeta_r (\tau - \tau'), \tau') - E_{\text{ext}}(r - \zeta_r (\tau - \tau'), \tau') \right) \frac{\partial f_s(r - \zeta_r (\tau - \tau'), \vec{c}, \tau')}{\partial p_r(\zeta_r)} + f_0(r - \zeta_r \tau, \vec{c}).$$  \hspace{1cm} (34)

With the help of expression for the self-consistent field

$$E_{rs}(r, \tau) = \frac{4 \pi q n}{r} \int dz \int f_s(z, v_r, v_\theta, \tau) d^2v,$$  \hspace{1cm} (35)

1In order to separate electrostatic potentials belonging to different drops, here we use the $\Phi$ or $\phi$ symbols instead $\varphi$ symbol used in the previous section.

2In our problem a velocity on the z axis is decoupled from the radial and angle velocities, i.e. $f_s \propto G(v_r) f_s(v_\theta, v_r)$ with $\int G(v_r) dv_r = 1$.

3In the following we will denote $\vec{c} = (\zeta, \zeta_0) = (v_r/c, v_\theta/c)$.

4Here and in the following expressions firstly the derivative over $p_r(\zeta_r)$ is taken and after the value of $r - \zeta_r (\tau - \tau')$ is inserting in the expression for distribution function.
and for the external field

\[ E_{\text{ext}}(r, \tau) = -2 \frac{Q_T}{r_{0D}} \frac{\partial K_0(\xi_0)}{\partial \xi_0} \delta(\tau) = 2 \frac{Q_T}{r_{0D}} K_1(\xi_0) \delta(\tau), \]  

we obtain for Eq. (34):

\[ f_s(r, \zeta, \tau) = -4 \pi q^2 n \int_0^r d\tau' \frac{\partial f_s(r - \zeta(\tau - \tau'), \zeta, \tau')}{\partial p_r(\zeta')} \int^{\tau - \zeta(\tau - \tau')}_{\tau - \zeta(\tau - \tau')} dzz \int f_s(z, v', \theta, \tau') d^2v' + \]

\[ + 2 \frac{q Q_T}{c r_{0D}} K_1(\xi_0 - \zeta, \tau) \frac{\partial f_{so}(r - \zeta, \tau, \zeta)}{\partial p_r(\zeta)} + f_0(r - \zeta, \tau, \zeta). \]  

At \( \tau = 0 \) this equation gives an equation for the \( f_{so} \):

\[ f_{so}(r, p_r) - 2 \frac{q Q_T}{c r_{0D}} K_1(\xi_0) \frac{\partial f_{so}(r, p_r)}{\partial p_r(\zeta)} = f_0(r, p_r) \]  

with some initial function \( f_0 \). Therefore, Eq. (37) can be written in simpler form:

\[ f_s(r, \zeta, \tau) = -4 \pi q^2 n \int_0^r d\tau' \frac{\partial f_s(r - \zeta, \tau - \tau', \zeta', \tau')}{\partial p_r(\zeta')} \]

\[ \cdot \int^{\tau - \zeta(\tau - \tau')}_{\tau - \zeta(\tau - \tau')} dzz \int f_s(z, v', \theta, \tau') d^2v' + f_{so}(r - \zeta, \tau). \]  

Equations Eq. (35)-Eq. (39) are full analogues of the initial Vlasov equations with some initial condition defined by Eq. (38).

An analytical solution of the Eq. (38) can be easily found. For the case of non-relativistic radial momenta it is

\[ f_{so}(r, \zeta) = \frac{1}{\Lambda_0 K_1(\xi_0)} e^{\frac{\Lambda_0}{K_1(\xi_0)}} \int_{\zeta}^{\infty} f_0(r, \zeta') e^{-\frac{\Lambda_0}{K_1(\xi_0)}} d\zeta', \]  

where

\[ \Lambda_0 = \frac{2 q Q_T}{m c^2 r_{0D}} \approx \frac{E_{\text{ext}}}{\delta_{\text{kin}}} \]  

is the parameter which depends only on the field of the incident drop. Here \( E_{\text{ext}} \) is a potential energy of the interaction of the particle with the incident drop,

\[ E_{\text{ext}} \propto \frac{q Q_T}{r_{0D}}, \]  

and \( \delta_{\text{kin}} \) is the relativistic kinetic energy of the incident drop. Therefore, there are two different regimes in our problem, the high-energy (relativistic or weak external field limit) one when \( \Lambda_0 < 1 \) and another one of the strong external field when \( \Lambda_0 > 1 \). Solution Eq. (40) is valid in both cases but we are interesting in the high-energy regime when \( \Lambda_0 < 1 \) as \( s \to \infty \). In this case, we search our function in the form of the following series:

\[ f_{so}(r, \zeta) = \sum_{i=0}^{\infty} F_{i}^{so} \Lambda_0^i. \]  

From Eq. (38) or from Eq. (40), in the first two orders of the approximation, we obtain:

\[ f_{so}(r, \zeta) = f_0(r, \zeta) + \Lambda_0 K_1(\xi_0) \frac{\partial f_0(r, \zeta)}{\partial \zeta}. \]  

This form is useful for the case of high-energy (relativistic) limit of the problem for any form of the initial distribution function \( f_0(r, p_r) \).  

\[ \text{Here } s \text{ is a squared total energy of the scattering process in the drop's rest frame.} \]
Coming back to our initial differential formulation of Vlasov equation Eq. (27), we see that Eq. (39) is equivalent to the usual Vlasov equation:

\[ c \frac{\partial f_s}{\partial \tau} + v_r \frac{\partial f_s}{\partial r} + q E_{rs} \frac{\partial f_s}{\partial p_r} = 0 \]  

(45)

with only self-consistent field included. The only difference of the new equation from the usual formulation is the initial conditions, Eq. (38), where the influence of the external field is included. Somehow it is very predictable result. Because of its factorized form, the external field may be considered as an additional source of the perturbation which acts only at initial time of the process of drop’s compression/expansion by the external field.

5 Solution of Vlasov’s equation for ”rigid-body” initial equilibrium distribution.

In this section, we consider the integral equation Eq. (39), obtained above, in the case \( \Lambda_0 \ll 1 \). Solution of our initial Vlasov’s equation, therefore, is given by Eq. (27). Considering this equation as perturbative one, with ”time” \( \tau \) as a small parameter, we write:

\[ f_s(r, \vec{\zeta}, \tau) = \sum_{i=0}^{\infty} f_{si}(r, \vec{\zeta}) \tau^i. \]

(46)

In the first order on \( \tau \) we have

\[ f_s(r, \vec{\zeta}, \tau = 0) = f_{s0}(r, \vec{\zeta}) = f_0(r, \vec{\zeta}) + \Lambda_0 K_1(\xi_0) \frac{\partial f_0(r, \vec{\zeta})}{\partial \zeta_r}, \]

(47)

see Eq. (44). The initial function \( f_0(r, \vec{\zeta}) \) we choose as a rotating ”rigid-body” equilibrium distribution function described in Appendix A. The reason for this choice is that this form of the distribution function is mostly appropriate for the configuration of the fields created in the high-energy scattering, see [31].

5.1 First order term of the distribution function

In order to clarify a solution of our equation Eq. (39), we perform Fourier transform over \( r \) variable of our functions of interest:

\[ f_s(r, \vec{\zeta}, \tau) = \int e^{ikr} f_{sk}(\vec{\zeta}, \tau) dk. \]

(48)

Taking into account the normalization of the initial distribution function,

\[ \int d^2 v f_{s0}(r, \vec{v}) = 1, \]

(49)

see Eq. (A.14), the equation Eq. (39), therefore, acquires the following form:

\[ \int e^{ikr} \left( f_{sk0}(\vec{\zeta}) + f_{sk1}(\vec{\zeta}) \tau \right) dk = -\Lambda \int_0^\tau (r - \zeta_r \tau)^2 d\tau \cdot \frac{\partial}{\partial \zeta_r} \int e^{ik(r-\zeta_r \tau)} f_{sk0}(\vec{\zeta}) dk + \int e^{ik(r-\zeta_r \tau)} f_{sk0}(\vec{\zeta}) dk, \]

(50)

with

\[ \Lambda = \frac{2\pi q^2 \hbar}{rmc^2}. \]

(51)

Thereby, in the first order on \( \tau \), we obtain the following expression for the distribution function:

\[ \tau \int e^{ikr} f_{sk1}(\vec{\zeta}) dk = -\tau r^2 \left( \frac{\partial}{\partial \zeta_r} \int e^{ikr} f_{sk0}(\vec{\zeta}) dk \right) - \tau \zeta_r \int e^{ikr} (i k) f_{sk0}(\vec{\zeta}) dk, \]

(52)

6This expansion is valid in the case of radial symmetry of the drop without z coordinate evolution included.
The validity of our perturbative expansion is controlled by the request that

\[ \tau < 1. \]  

(55)

Inserting expression Eq. (17) in Eq. (53), we obtain the first order distribution function term:

\[ f_{s1}(r, \zeta) = -\zeta \frac{\partial f_0(r, \zeta)}{\partial \zeta} - \zeta \frac{\partial f_0(r, \zeta)}{\partial r}. \]  

(54)

The validity of our perturbative expansion is controlled by the request that

\[ r^2 \Lambda \tau < 1, \]  

all other parameters in the Eq. (54) are small.

Whereas corrections to averaged values of the velocities can be calculated with the help of Eq. (54) distribution function, the corrections to the density profile are zero in this order. Namely, we have:

\[ \int d^2 v f_{s1}(r, \vec{v}) = 0. \]  

(56)

Indeed, integrating Eq. (54) over velocities we obtain that the first term in Eq. (54) is \( \langle v_r \rangle_0 = 0 \) (the sign \( \langle \cdot \rangle_0 \) means averaging over Eq. (A.1) distribution function), second and fourth terms give zero contributions because Eq. (A.1) distribution function is delta-function, and the third term vanishes because we take the derivative over \( r \) on the constant \( n \) from Eq. (A.14).

### 5.2 Second order term of the distribution function

Taking into account Eq. (56), we write the expression for the distribution function in the second order on \( \tau \) as

\[ \tau^2 \int e^{ikr} f_{sk1} dk = \]

\[ = -\Lambda \int_0^r \left( r^2 - 2r\zeta \tau' \right) dr' \int \left( -i k \tau' \left( f_{sk0} + \tau' f_{sk1} \right) + \frac{\partial f_{sk0}}{\partial \zeta} + \tau' \frac{\partial f_{sk1}}{\partial \zeta} \right) \left( 1 - \tau \zeta \tau' \right) e^{ikr} dk + \]

\[ + \frac{\zeta^2 r^2}{2} \int e^{ikr} (-k^2) f_{sk0}(\zeta) dk. \]  

(57)

Combining all \( \tau^2 \) order terms we obtain:

\[ f_{s2}(r, \zeta) = \frac{\Lambda r^2}{2} \frac{\partial f_{s0}(r, \zeta)}{\partial r} + \frac{\Lambda r^2 \zeta}{2} \frac{\partial^2 f_{s0}(r, \zeta)}{\partial \zeta^2} - \frac{\Lambda r^2}{2} \frac{\partial f_{s1}(r, \zeta)}{\partial r} + \Lambda r \zeta \frac{\partial f_{s0}(r, \zeta)}{\partial \zeta} + \]

\[ + \frac{\zeta^2 r^2}{2} \frac{\partial^2 f_{s0}(r, \zeta)}{\partial r^2}. \]  

(58)

Therefore, our distribution function, till \( \tau^2 \) order, has the following form:

\[ f_s(r, \zeta, \tau) = f_{s0}(r, \zeta) - \tau \left( \frac{\Lambda r^2}{2} \frac{\partial f_{s0}}{\partial r} + \zeta \frac{\partial f_{s0}}{\partial \zeta} \right) + \]

\[ + \tau^2 \left( \frac{\Lambda r^2}{2} \frac{\partial f_{s0}}{\partial r} + \frac{\Lambda r^2 \zeta}{2} \frac{\partial^2 f_{s0}}{\partial \zeta^2} + \Lambda r \zeta \frac{\partial f_{s0}}{\partial \zeta} + \frac{\zeta^2 r^2}{2} \frac{\partial^2 f_{s0}}{\partial r^2} - \frac{\Lambda r^2}{2} \frac{\partial f_{s1}}{\partial r} \right). \]  

(60)

To calculate the density profile function, we need to keep in Eq. (60) the only non-vanishing after the integration over velocities terms, which we denote as \( f_s(r, \zeta, \tau) \). These terms are the following:

\[ \tilde{f}_s(r, \zeta, \tau) = f_0(r, \zeta) + \tau^2 \left( \Lambda r \zeta \frac{\partial f_0}{\partial \zeta} + \frac{\zeta^2 r^2}{2} \frac{\partial^2 f_0}{\partial r^2} \right) = f_0(r, \zeta) + \tau^2 f_{s2}(r, \zeta). \]  

(61)

The calculation of the density profile with the help of this distribution function we perform in the next section.
6 Density profile of the drop

As we noticed before, the only non-vanishing time dependent correction to the density profile\footnote{In the following we will denote \( n(r) \) as a density profile and \( n \) as a some constant density value.} we obtain only in the second order on \( \tau \). Namely we have:

\[
n(r) = n_0(r) + \tau^2 n_2(r),
\]

where

\[
n_0 = \int d^2 v f_{s0}(r, \vec{v}) = 1,
\]

and

\[
n_1 = \int d^2 v f_{s1}(r, \vec{v}) = 0,
\]

see Eq. (A.14) and Eq. (56). For the \( n_2 \) correction term we have:

\[
n_2(r) = \int d^2 v \tilde{f}_{s2}(r, \tilde{\zeta}) = \Lambda r \int d^2 v \zeta_r \frac{\partial f_0}{\partial \zeta_r} + \frac{1}{2} \int d^2 v \zeta_r^2 \frac{\partial^2 f_0}{\partial r^2}.
\]

The first term in the r.h.s. of Eq. (65) is easily calculated: taking into account Eq. (A.1) we have the following integral:

\[
\Lambda r \int d^2 v \zeta_r \frac{\partial f_0}{\partial \zeta_r} = - \Lambda r \int d^2 v f_0 = - \Lambda r = - \frac{2\pi q^2 n}{mc^2},
\]

see Eq. (61). In order to calculate the second term in Eq. (65) we write:

\[
\frac{1}{2} \int d^2 v \zeta_r^2 \frac{\partial^2 f_0}{\partial r^2} = \frac{1}{2} \int d^2 v \zeta_r^2 f_0 = \frac{1}{2c^2} \int d^2 v v_r^2 f_0 = \frac{1}{2c^2} \int d^2 v v_r^2 f_0 = \frac{1}{2c^2} \int d^2 v v_r^2 f_0.
\]

Now, we need to calculate the square of the radial velocity averaged over Eq. (A.1) distribution function:

\[
\langle v_r^2 \rangle_0 = \int d^2 v v_r^2 f_0.
\]

We have:

\[
2 \langle v_r^2 \rangle_0 = 2 \left( \frac{m}{2\pi} \right) \int d^2 v v_r^2 \delta \left( \frac{1}{2m} \left( p_r^2 + (p_\theta - m\omega_r)^2 \right) + \psi(r) - kT_\perp \right)
\]

\[
= \left( \frac{m}{2\pi} \right) \int d^2 v \left( v_r^2 + v_\theta^2 \right) \delta \left( \frac{1}{2m} \left( p_r^2 + p_\theta^2 \right) + \psi(r) - kT_\perp \right),
\]

defining a new variable:

\[
u_r^2 = \frac{1}{2m} \left( p_r^2 + p_\theta^2 \right),
\]

after the variable change we get the following expression:

\[
2 \langle v_r^2 \rangle_0 = \left( \frac{2}{m} \right) \int d^2 v \delta(u^2 + \psi(r) - kT_\perp),
\]

which gives with the help of Eq. (A.16):

\[
\langle v_r^2 \rangle_0 = \left( \frac{1}{m} \right) \left( kT_\perp - \psi(r) \right) = \frac{kT_\perp}{m} \left( 1 - \frac{r^2}{r_b^2} \right).
\]

Thereby, we obtain for the second term of Eq. (65):

\[
\frac{1}{2} \int d^2 v v_r^2 \frac{\partial^2 f_0}{\partial r^2} = \frac{1}{2c^2} \frac{\partial^2 f_0}{\partial r^2} \int d^2 v v_r^2 f_0 = \frac{1}{2c^2} \frac{\partial^2 \langle v_r^2 \rangle_0}{\partial r^2} = - \frac{kT_\perp}{mc^2 r_b^2}.
\]

Taking both Eq. (66) and Eq. (73) together we obtain:

\[
n_2(r) = - \left( \frac{kT_\perp}{mc^2 r_b^2} + \frac{2\pi q^2 n}{mc^2} \right) = - \left( \frac{\psi(r)}{mc^2 r^2} + \frac{2\pi q^2 n}{mc^2} \right) = - \frac{1}{mc^2 r^2} \left( \psi(r) + 2\pi q^2 nr^2 \right).
\]
In the first order of our perturbation series we have:
\[ U_p = q \Phi_s = -\pi n q^2 r^2, \]  
(75)
see Eq. (A.10), and taking into account Eq. (A.9) we obtain therefore:
\[ n_2(r) = -\frac{1}{mc^2} \left( \frac{m\omega_r}{2} (\omega_c - \omega_r) + \pi q^2 n \right) \]  
(76)
This form of \( n_2 \) is useful for further analysis of the density profile behavior.

We also can introduce some ”total” potential energy of the particles in the volume defined by \( r_b \):
\[ \hat{U}_p = -\pi n q^2 r^2 b. \]  
(77)
Therefore, the expression Eq. (74) can be also rewritten as
\[ n_2(r) = -\frac{kT_\perp}{mc^2 r_b^2} \left( 1 - 2 \frac{\hat{U}_p}{kT_\perp} \right). \]  
(78)
For the weakly interacting plasma, when
\[ \Gamma = \left| \frac{\hat{U}_p}{kT_\perp} \right| << 1 \]  
(79)
we obtain therefore
\[ n_2(r) \approx -\frac{kT_\perp}{mc^2 r_b^2} = -\frac{\psi(r)}{mc^2 r^2}, \]  
(80)
whereas for the strongly interacting plasma when
\[ \Gamma >> 1 \]  
(81)
we obtain
\[ n_2(r) \approx 2 \frac{U_p}{mc^2 r^2} = -\frac{2\pi q^2 n}{mc^2}. \]  
(82)
It is worth of noting, that in this case, the expression Eq. (82) does not depend on the magnetic field, which comes in Eq. (80) through the \( \psi(r) \) function.

7 Transport properties of the drop

In this section, we calculate the averaged velocities and shear viscosity of the drop with the help of previously found distribution function. Namely, we will calculate the averaged radial velocity
\[ \langle v_r \rangle = \frac{\int d^2 v v_r f_s}{\int d^2 v f_s}, \]  
(83)
the averaged azimuthal flow velocity
\[ \langle v_\theta \rangle = \frac{\int d^2 v v_\theta f_s}{\int d^2 v f_s}, \]  
(84)
and the non-diagonal term of the stress-energy tensor required for the shear viscosity calculation
\[ \sigma_{ij} = nm \int d^2 v (v_i - \langle v_i \rangle) (v_j - \langle v_j \rangle) f_s, \]  
(85)
where \( i, j = r, \theta \).
7.1 Radial velocity calculations

We calculate radial velocity up to the first order of perturbation series only, therefore we take everywhere

\[ \int d^2 v f_s = \int d^2 v f_{s0} = 1 \]

that gives for the first order term of the radial velocity

\[ \langle v_r \rangle = \int d^2 v v_r f_s = \int d^2 v v_r f_{s0} + \tau \int d^2 v v_r f_{s1} = \langle v_r \rangle_{s0}^0 + \tau \langle v_r \rangle_{s1}^1. \] (86)

Here we used the lower subscript "s" as the sign of averaging over the \( f_s \) distribution function and the superscripts denote the orders of the term in the perturbation series.

The calculation of the first term in Eq. (86) is simple. We have:

\[ \langle v_r \rangle_{s0}^0 = \int d^2 v v_r f_{s0} = \int d^2 v v_r \left( f_0(r, \zeta) + \Lambda_0 K_1(\xi_0) \frac{\partial f_0(r, \zeta)}{\partial \zeta} \right). \] (87)

Opening the brackets we obtain:

\[ \langle v_r \rangle_{s0}^0 = \langle v_r \rangle_0 + c \Lambda_0 K_1(\xi_0) \int d^2 v v_r \frac{\partial f_0(r, \bar{v})}{\partial v_r}. \] (88)

Taking into account the properties of our initial distribution \( f_0 \), we obtain finally:

\[ \langle v_r \rangle_{s0}^1 = -c \Lambda_0 K_1(\xi_0) \int d^2 v f_0(r, \bar{v}) = -c \Lambda_0 K_1(\xi_0). \] (89)

We see, that as we assumed for the \( \Lambda_0 \langle 1 \rangle \), our radial velocity is small \( \langle v_r \rangle_{s0}^0 \langle 1 \rangle \). We obtained also, that the value of this velocity fully determined by the influence of the external field, and contrary to the results of Appendix A, the radial velocity is not zero even in zero order.

In the first order of perturbation we have:

\[ \langle v_r \rangle_{s1}^1 = \int d^2 v v_r f_{s1} = -\int d^2 v v_r \left( \Lambda r^2 \frac{\partial f_{s0}}{\partial \zeta} + \zeta_r \frac{\partial f_{s0}}{\partial v_r} \right). \] (90)

Taking into account the full answer for the distribution function in this order Eq. (54), we can rewrite Eq. (81) as

\[ \langle v_r \rangle_{s1}^1 = \int d^2 v v_r f_{s1} = -\int d^2 v v_r \left( \Lambda r^2 \frac{\partial f_0}{\partial \zeta} + \zeta_r \frac{\partial f_0}{\partial v_r} \right), \] (91)

all other terms gives zero contribution to the radial velocity value in this order. The first term in the r.h.s. in Eq. (91) is

\[ \Lambda r^2 \int d^2 v v_r \frac{\partial f_0}{\partial \zeta} = c \Lambda r^2 \int d^2 v v_r \frac{\partial f_0}{\partial v_r} = -c \Lambda r^2 \int d^2 v f_0 = -c \Lambda r^2, \] (92)

whereas the second term is

\[ \int d^2 v v_r \zeta_r \frac{\partial f_0}{\partial v_r} = \frac{1}{c} \frac{\partial}{\partial v_r} \int d^2 v v_r f_0 = \frac{1}{c} \frac{\partial}{\partial v_r} \langle v_r^2 \rangle_0, \] (93)

where \( \langle v_r^2 \rangle_0 \) is given by Eq. (72). Taking both terms together, we obtain:

\[ \langle v_r \rangle_{s1}^1 = c \left( \frac{2kT_r}{mc^2 r_b^2} + \frac{2\pi q^2 n_r}{mc^2} \right) = \frac{cr}{r_b^2} \left( \frac{2kT_{\perp}}{mc^2} + \frac{2\pi q^2 n r_{b, \perp}^2}{mc^2} \right). \] (94)

As in the previous section, we have here two possible different regimes. The first one for the weakly interacting plasma, when

\[ \Gamma = \frac{|\dot{U}_b|}{kT_{\perp}} << 1 \] (95)
we obtain
\[ \langle v_r \rangle_s^1 \approx c \frac{2 k T \perp r}{mc^2 r_b^2} \]  
(96)
whereas for the strongly interacting plasma when
\[ \Gamma >> 1 \]  
(97)
we obtain
\[ \langle v_r \rangle_s^1 \approx c \frac{2 \pi q^2 n r}{mc^2}. \]  
(98)
In both cases, the corrections are small due the large value of \( mc^2 \) in denominators of Eq. (96)-Eq. (98).

The final expression for radial velocity up to the first order, therefore, is the following:
\[ \langle v_r \rangle = - c \Lambda_0 K_1(\xi_0) + \tau \frac{cr}{r_b^2} \left( \frac{2 k T \perp }{mc^2} + \frac{2 \pi q^2 n r_b^2}{mc^2} \right). \]  
(99)
We see, that whereas the first term of expression Eq. (99) is fully determined by the external field and negative (compression process), the first order perturbation term has positive sign and describes the instability of the charged droplet in absence of permanent external electric field.

### 7.2 Azimuthal velocity calculations

Like the radial velocity, we calculate the azimuthal velocity up to the first order:
\[ \langle v_\theta \rangle = \int d^2v v_\theta f_s = \int d^2v v_\theta f_{s0} + \tau \int d^2v v_\theta f_{s1} = \langle v_\theta \rangle_{s0} + \tau \langle v_\theta \rangle_{s1}. \]  
(100)
In the first order we have:
\[ \langle v_\theta \rangle_{s0} = \int d^2v v_\theta f_{s0} = \int d^2v v_\theta \left( f_0(r, \vec{\zeta}) + \Lambda_0 K_1(\xi_0) \frac{\partial f_0(r, \vec{\zeta})}{\partial \zeta} \right) = \int d^2v v_\theta f_0(r, \vec{\zeta}). \]  
(101)
We see that in this order, our answer is simply Eq. (A.7):
\[ \langle v_\theta \rangle_{s0} = \langle v_\theta \rangle_0 = \omega_r r. \]  
(102)
There is no influence of the external field in this order.

In the next order, taking into account the first perturbed term, we have:
\[ \langle v_\theta \rangle_{s1}^1 = \int d^2v v_\theta f_{s1} = - \int d^2v v_\theta \zeta r \Lambda_0 K_1(\xi_0) \frac{\partial^2 f_0(r, \vec{\zeta})}{\partial \zeta r}. \]  
(103)
All other terms of Eq. (54) give zero contribution to the integral Eq. (90). Thereby we have:
\[ \langle v_\theta \rangle_{s1}^1 = - \Lambda_0 K_1 \int d^2v v_\theta \zeta r \frac{\partial^2 f_0(r, \vec{\zeta})}{\partial \zeta r} = \Lambda_0 K_1 \frac{\partial}{\partial r} \int d^2v v_\theta f_0 = \Lambda_0 K_1 \frac{\partial (v_\theta)_0}{\partial r}. \]  
(104)
Taking both terms Eq. (90) and Eq. (101) together we obtain finally:
\[ \langle v_\theta \rangle = \omega_r r + \tau \Lambda_0 K_1 \omega_r. \]  
(105)
In this case the external perturbation leads to the increase of the droplet’s rotation velocity.
7.3 Transverse shear viscosity of the drop

The shear viscosity is defined by the non-diagonal term of the stress-energy tensor. We have:

\[ \sigma_{r\theta} = n m \int d^2 v \left( v_\theta - \langle v_\theta \rangle \right) \left( v_r - \langle v_r \rangle \right) f_s. \]  

(106)

In the zero order of our perturbation series, we have then

\[ \sigma^0_{r\theta} = n m \int d^2 v \left( v_\theta v_r - v_r \langle v_\theta \rangle_s - v_\theta \langle v_r \rangle_s + \langle v_r \rangle_s \langle v_\theta \rangle_s \right) f_{s0} = 0, \]

(107)

as it must be in the case of equilibrium. In the first order of perturbations we have:

\[ \sigma^1_{r\theta} + \sigma^0_{r\theta} = n m \int d^2 v \left( v_\theta v_r - v_r \langle v_\theta \rangle_s + \tau \langle v \rangle_s - \langle v_r \rangle_s \langle v_\theta \rangle_s \right) f_{s1}. \]

(108)

So we obtain:

\[ \sigma^1_{r\theta} = n \tau \left( \langle v_r v_\theta \rangle_s^1 - \langle v_r \rangle_s \langle v_\theta \rangle_s - \langle v_r \rangle_s \langle v_\theta \rangle_s \right), \]

(109)

and for the first integrand:

\[ - \int d^2 v v_\theta v_r \zeta_r \frac{\partial f_0 (r, \vec{\zeta})}{\partial r} = - \frac{\langle v_\theta \rangle_s}{c} \frac{\partial}{\partial r} \int d^2 v v_r^2 f_0 = - \frac{\langle v_\theta \rangle_s}{c} \frac{\partial}{\partial r} (v_r^2)_s. \]

(112)

Integration of this expression gives, for the first integrand:

\[ - \int d^2 v v_\theta v_r \zeta_r \frac{\partial f_0 (r, \vec{\zeta})}{\partial r} = - \frac{\langle v_\theta \rangle_s}{c} \frac{\partial}{\partial r} \int d^2 v v_r^2 f_0 = - \frac{\langle v_\theta \rangle_s}{c} \frac{\partial}{\partial r} (v_r^2)_s. \]

(113)

Taking into account Eq. (100)-Eq. (104), Eq. (101)-Eq. (104) and summing up all terms of Eq. (109) we obtain:

\[ \sigma^1_{r\theta} = - n m \tau \langle v_r \rangle_s \langle v_\theta \rangle_s = n m \tau c \left( A_0 K_1 \right)^2 \frac{\partial \langle v_\theta \rangle_s}{\partial r}. \]

(114)

This expression immediately determines the shear viscosity coefficient in transverse plane:

\[ \eta_{r\theta} = n m \tau c \left( A_0 K_1 \right)^2. \]

(115)

This result is interesting because the obtained value of the viscosity is time dependent, proportional to the density of the drop and stays very small during evolution of the system if \( \tau \) remains small. Similarly to the definition in [21] we can call this viscosity as an anomalous.

Actually, the parameter \( \tau \) in Eq. (115) is not a mean free path as in usual interpretations of the viscosity in different kinematic approximations. The shear viscosity of Eq. (115) is determined by the collective expansion of the charged particles under an influence of the interactions of the particles inside the droplet and some initial perturbation caused by an external field. Thereby, the expression for this anomalous viscosity value is correct till the Vlasov’s approximation is correct in the description of the expanding dense matter. Therefore, the \( \tau \) parameter in Eq. (115) is a "time" of evolution of the droplet in the phase of the hydrodynamic expansion (compression) and the maximum value of this parameter is itself small. We estimate the value of \( \tau \) approximately as:

\[ \tau \propto \hbar / mc, \]

(116)
see for example [18]. Inserting Eq. (116) into Eq. (115) we obtain:

\[
\eta_{\text{max}} \propto \hbar n \left( \Lambda_0 K_1 \right)^2 \approx \hbar n \left( \frac{E_{\text{ext}}}{E_{\text{kin}}} \right)^2,
\]

(117)

where we took \( K_1 \propto 1 \) for all \( r \) of interest. We see, that the obtained viscosity changes from zero value till very small maximum value during the droplet's expansion. Indeed, the only large parameter in Eq. (117) is the density of the droplet \( n \), which can be very large, but \( \Lambda_0 \) is a small parameter of the approach and overall value of the viscosity is proportional to \( \hbar \).

Obviously, the viscosity/entropy ratio in our calculations remains small. Indeed, we consider Vlasov equation, the entropy during the process of the drop's expansion remains constant and it equals to the initial entropy of the drop: \( s = s_0 = \text{const} \[3\] \). Therefore, the ratio \( \eta / s = \eta / s_0 \) changes only because change of the viscosity coefficient \( \eta \) and overall ratio changes from zero to some small value determined by the viscosity coefficient value Eq. (117).

8 Conclusion

In this paper, we considered a process of the compression/expansion of the dense charged droplet in the transverse plane under the influence of the external transverse electrical field. This droplet, which we assume have been created in the very initial stage of the high-energy scattering of hadrons, initially is in the state of the thermal equilibrium. The configuration of external fields in this case is similar to the problem of a plasma confinement by the external fields, see [31]. The smallness of the drop’s size is required in this case, see [25], in order to achieve the thermalization during very short time. The droplet consists of charged particles, the external fields are not constant in time, therefore the droplet cannot stay in the equilibrium state and begins to expand shortly after it’s creation. The very early stage of the expansion of the droplet is hydrodynamical one, [18], the collisions between particles are not important and the expansion happens with constant entropy. Thereby, the non-equilibrium, time-dependent distribution function of the droplet’s evolution is determined by the Vlasov’s equation with the external field included, the expression for this distribution function Eq. (60) in the transverse plane is the important result of the paper.

We solve a classical problem for the distribution function of the non-equilibrium system, therefore, first of all, it is instructive to check the self-consistency of the obtained solution. Beginning from the "rigid body" initial state, Appendix A, we can "guess" the influence of the external electric field on the droplet and compared it with the obtained answers. In the framework of the definitions of the paper, we have that the case \( \Lambda_0 > 0 \) corresponds to the repulsive external force, that means droplet’s external compression, and the case \( \Lambda_0 < 0 \) corresponds to the attractive external force, that means droplet’s external expansion, the case \( \Lambda_0 = 0 \) corresponds to the absence of the interaction. In our case we consider a electromagnetic interactions, but in general, other types of interactions between the droplets are possible in more complex models, see for example [27, 28].

This processes of external compression/expansion are applied to the process of droplet’s expansion under the influence of charged particles inside the drop. From this point of view we see that our results are self-consistent and coincide with the naive expectations from the process. Indeed, the radial velocity answer, Eq. (99), consist of the two terms. The first one is due the external field, depending on the sign of \( \Lambda_0 \) which can be negative, case of compression, or positive, case of expansion. The second term in the Eq. (99) is positive and corresponds to the expansion of the droplet under the influence of the repulsive forces between charged particles in the drop. The azimuthal velocity expression, Eq. (105), also consist of the two terms. The first one is the same as obtained for the initial distribution of Appendix A, the second one is the correction to the initial value due to the external field. We see here, that for the case of compression, the azimuthal velocity increases, and in the case of negative \( \Lambda_0 \) it decreases, as it must be according with the classical point of view. The density profile of the drop remains constant in this first approximation. It changes only in the second order over the "time" \( \tau \) parameter, and this change does not depend on the external field but only on the instability of the charged particles inside the drop. It is also understandable, the external field affects only on the

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8 Here \( s \) is an entropy of the process
initial condition of the problem, and this influence is ”washed out” by the integration over velocities in the calculation of the density profile.

Important result of the performed calculations is the expression for the shear viscosity coefficient Eq. (115). It would be underlined, that a value of the coefficient is determined by the interaction of the droplet with the external field, i.e. due to the non-zero $\Lambda_0$ coefficient which depends on the strange of the interactions between charged droplets. Also, the ”time” parameter in Eq. (115) is the evolution ”time” of the process, not the mean free path, and it’s value is limited by the value of the ”time’s” applicability of the Vlasov’s approximation, Eq. (116), which itself is small. Thereby, our viscosity is time dependent and changing during the drop’s expansion, but it remains very small, also because $\Lambda_0$ determined by Eq. (41) is a perturbative, small parameter of the initial conditions. This mechanism of the viscosity coefficient smallness can be called as an anomalous one, similarly to [21]. The entropy of the process remains constant during the process of the drop’s expansion/compression. It gives immediately, that the ratio viscosity/entropy changes from zero to some value during the process of interest, but due to the smallness of the viscosity, this ratio anyway remains very small.

In general, we considered only the first step in the description of the evolution of dense droplets in the external electromagnetic fields. In order to describe a whole picture of the droplet’s expansion, one should consider a more complex problem, with, first of all, longitudinal dynamics and second, magnetic external field included. This task is important because this hydrodynamical expansion is proved to be an important ingredient in the description of the data obtained in high-energy collisions of protons and nuclei in the LHC and RHIC experiment, [10]. Our current and future calculations, therefore, can provide a microscopic description of the processes which precedes the hydrodynamics evolution of the matter together with the explanations of the smallness of the viscosity coefficients in the process.

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9In the case when $\Lambda_0 = 0$ the viscosity should be determined by the next order in the $\tau$ perturbative expansion of the distribution function and transport coefficients.
Appendix A: A ”rigid-rotor” initial equilibrium distribution function

Properties of initial distribution in our problem are defined by Eq. (38), which, in turn, depends on some another function $f_0(r, \vec{p})$. As $f_0(r, \vec{p})$ we will choose a ”rigid-rotor” equilibrium function:

$$f_0(r, \vec{p}) = \left(\frac{m}{2\pi}\right)^2 \delta(H_\perp - \omega_r P_\theta - k T_\perp) G(p_z),$$  \hspace{1cm} (A.1)

where we have:

$$\int_{-\infty}^{\infty} G(p_z) dp_z = 1;$$  \hspace{1cm} (A.2)

$$\left(\frac{m}{2\pi} \int_{-\infty}^{\infty} dv_\theta \int_{-\infty}^{\infty} dv_r \delta(H_\perp - \omega_r P_\theta - k t_\perp)\right)_{r=0} = 1.$$  \hspace{1cm} (A.3)

the non-relativistic Hamiltonian of the problem is given by

$$H_\perp = \frac{1}{2m} \left(p_r^2 + p_\theta^2\right) + q \Phi_{s0},$$  \hspace{1cm} (A.4)

where in Eq. (A.1)

$$P_\theta = r (p_\theta - m \omega_c r / 2),$$  \hspace{1cm} (A.5)

and

$$\omega_c = \frac{|q| B_0}{m c}$$  \hspace{1cm} (A.6)

is a cyclotron frequency in presence of some initial magnetic field in $z$ direction, see []. An average azimuthal flow velocity

$$\langle v_\theta \rangle_0 = \frac{\int d^3v \nu_\theta f_0}{\int d^3v f_0} = \omega_r r$$  \hspace{1cm} (A.7)

is fully defined by some given angular velocity $\omega_r = \text{const}$ that explains the name ”rigid-rotor” equilibrium for the Eq. (A.1) function.

It is convenient to rewrite argument of delta function in Eq. (A.3) in the following form:

$$H_\perp - \omega_r P_\theta = \frac{1}{2m} \left(p_r^2 + (p_\theta - m \omega_r r)^2\right) + \psi(r),$$  \hspace{1cm} (A.8)

where an effective potential $\psi(r)$ is defined as

$$\psi(r) = \frac{m}{2} \left(\omega_c \omega_r - \omega_r^2\right) r^2 + q \Phi_{s0}.$$  \hspace{1cm} (A.9)

The corresponding Poisson equation for the self-consistent potential $\Phi_{s0}$ can be solved in this case giving

$$\Phi_{s0} = -\pi n q r^2$$  \hspace{1cm} (A.10)

and we can rewrite potential Eq. (A.9) as

$$\psi(r) = \frac{m}{2} \left(\omega^+ - \omega_r\right) \left(\omega_r - \omega^-\right) r^2$$  \hspace{1cm} (A.11)

with

$$\omega^\pm = \frac{\omega_c}{2} \{1 \pm (1 - n s_q)^{1/2} \}$$  \hspace{1cm} (A.12)

and

$$s_q = \frac{8 \pi q^2}{m \omega_c^2}.$$  \hspace{1cm} (A.13)

Now we see that for the distribution function Eq. (A.1) the density profile is constant:

$$n(r) = \left\{ \begin{array}{ll} 1 = \text{const.}, & 0 \leq r \langle r_b \rangle \langle r_b \rangle, \\ 0, & \end{array} \right.$$  \hspace{1cm} (A.14)

with

$$r_b^2 = \frac{2k T_\perp}{m} \frac{1}{\left(\omega^+ - \omega_r\right) \left(\omega_r - \omega^-\right)}.$$  \hspace{1cm} (A.15)
as some maximal radius of non-zero density. With the help of Eq. (A.15), Eq. (A.11) acquires the following form:

\[
\psi(r) = \frac{k T_\perp r^2}{r_b^2}.
\]  

(A.16)

In the following we consider the case when

\[
r_D \leq r_b,
\]  

(A.17)

that makes approximation Eq. (A.11) correct in the region of interest of our problem.
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