Comparative analysis of the reliability of electric locomotives based on semi-Markov models of restorable systems

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Abstract. Based on the general theory of semi-Markov recovery processes and their application to Markov recovery processes, the applicability of stationary reliability indicators to assessing the reliability of electric locomotives with an asynchronous motor, the functioning of which is considered as a superposition of independent alternating semi-Markov processes, is shown. By calculating the main indicators of system reliability in a steady state, a comparative analysis of the operational reliability of electric locomotives with an asynchronous DC and AC motors, used respectively on the Azerbaijan and Kazakhstan railways, was carried out.

1. Introduction
The foundations of the theory of semi-Markov processes (SMP) and their applied possibilities are described in detail in the monograph [1], in which classes of semi-Markov processes were considered that simulate the behavior of recoverable systems with fully accessible (unrestricted) recovery and are implemented as a superposition of independent semi-Markov processes with a finite number of states, in particular, a superposition of alternating renewal processes with constraints such as the exponentiality of the original distributions.

In [2], superpositions of both independent and dependent SMPs with a common phase space are introduced as SMPs that simulate the functioning of restored systems with various types of redundancy, a limited and unlimited number of recovery devices (RD). In principle, in this class one can study any of the systems given, for example, in the reference book [3] without additional restrictions such as the exponentiality of the original distributions. As in the well-known work [4], it is noted that the applied capabilities of the SMP are limited for systems with a finite number of states over a long time interval.

An effective way of specifying the SMP is Markov renewal processes (MRPs) - two-dimensional Markov chains, the first component of which (in turn, being a Markov chain) describes the state of the system, and the second fixes the sojourn times of this system in states. The semi-Markov property of the system states is that the residence time in the state depends only on the given (and possibly the next) state and does not depend on the previous evolution of the system. Generally speaking, the physical state of a real system does not possess the semi-Markov property. However, if in a certain way “expand” the physical state \(\{l_1, ..., l_n\} (n – the number of possible states of the system at each moment of time), adding to it some vector \(x = \{x_1, ..., x_n\}, x_i \geq 0,\) then in many cases it is possible to
achieve that the new extended state \( \{ l_1, ..., l_n, x_1, ..., x_n \} \) will already have a semi-Markov property. This allows us to describe the functioning of a number of restored systems using PMB [5]. The problem of modeling complex technical recoverable systems using semi-Markov processes remains relevant at the present time, being the subject of research by scientists from the union of independent states (UIS) countries [6-8] and far abroad [9-13].

In this work, to analyze the reliability of operation of electric locomotives with an asynchronous motor, an approach based on semi-Markov processes and Markov recovery processes is used, the mathematical apparatus for studying which was developed in [1] and was further developed in [2,14,15]. By calculating the main reliability characteristics of restored systems in a steady state (stationary availability factor, mean time between failures and mean time to recovery), a comparative analysis of the operational reliability of electric locomotives with DC and AC induction motors, currently used at the Azerbaijan and Kazakhstan Railways, was carried out.

2. Processes of Markov recovery

A jump-like homogeneous Markov process can be constructively given by a stochastic kernel that determines the transition probabilities of the Markov chain, which specifies changes in the states of the process, and a non-negative function on states that specifies the parameters of exponentially distributed random variables that determine the residence times in the states of the process between neighboring jumps.

The concept of semi-Markov processes is a generalization of this construction of jump-like processes. The change in the states of these processes is also controlled by a discrete Markov chain called an embedded Markov chain (EMC), and the sojourn times in states between adjacent jumps have arbitrary distribution functions that depend on the state of the process. Thus, for the constructive specification of semi-Markov processes, the initial material is the sequence \( \{ \xi_n, \Theta_n, n \geq 0 \} \), the first component of which \( \{ \xi_n, n \geq 0 \} \) forms a homogeneous Markov chain that fixes the state of the process at the nth step (after the nth jump), and the second component \( \Theta_n \geq 0 \) records the time the system is in the state \( \xi_n \).

The two-component sequence \( \{ \xi_n, \Theta_n, n \geq 0 \} \) is a homogeneous Markov chain, the transition probabilities of which do not depend on the values of the second component (the residence time in the previous state does not affect the residence time in this state and the change in this state). Such a Markov chain \( \{ \xi_n, \Theta_n, n \geq 0 \} \) is called the Markov renewal process. The times \( \Theta_n \) turn out to be conditionally independent random variables for a fixed trajectory sojourn of the embedded Markov chain \( \{ \xi_n, n \geq 0 \} \) [11].

According to Kolmogorov’s extension theorem (see, for example, [16]), a random process with a fixed phase space is uniquely (in a probabilistic sense) determined by specifying all its finite-dimensional distributions.

In the case of Markov chains with discrete time and phase space \( (Y,Y) \), where \( (Y,Y) \) is an arbitrary measurable space, all finite-dimensional distributions are calculated using the initial distribution \( p_0(B) \), \( BeY \), and transition probabilities \( P(y,B), yeY, BeY \). So it is enough to know only \( p_0(B), BeY, \) and \( P(y,B), yeY, BeY \) in order to completely define a Markov chain with discrete time. Let’s \( Y = Z \times [0, \infty), Y = Z \times \sigma \), where \( Z-\sigma \)-algebra of subsets from \( Z \), and \( \sigma-\sigma \)-algebra of Borel sets on \([0, \infty) \); \( \times \) is the sign of the Cartesian product of spaces.

Consider a two-component Markov chain \( \{ \xi_n, \Theta_n, n \geq 0 \}, \xi_n \in Z, \Theta_n \in[0, \infty) \), for which \( P(\xi_0 \in B) = p_0(B), BeZ; P(\xi_{n+1} \in B, \Theta_{n+1} \leq t | \xi_0, \Theta_0, \xi_1, \Theta_1, ..., \xi_n = Z, \Theta_n) = P(\xi_{n+1} \in B, \Theta_{n+1} \leq t | \xi_n = Z) \) moreover \( P(\xi_{n+1} \in Z, \Theta_{n+1} < \infty | \xi_n = Z) = 1 \), where \( p_0(\cdot) \) - some distribution on \( Z \). A characteristic feature of the Markov chain \( \{ \xi_n, \Theta_n, n \geq 0 \} \) expressed in the fact that the dependence of the probability of an event \( \{ \xi_{n+1} \in B, \Theta_{n+1} \leq t \} \) from the past is realized only depending on the first component \( \xi_n \) at the previous moment.
3. Semi-Markov processes

When modeling the evolution of refurbished technical systems (RTS), it is of interest to calculate the characteristics not only at the moments of state change, but also at any current moment of time \( t \).

With every MRP \( \{\xi_n, \Theta_n, n \geq 0\} \), which is given by the right-hand sides (1) and (2), is connected with (at least one) random process with continuous time \( \xi(t) \) with the same as for \( \{\xi_n, n \geq 0\} \) phase space.

In order to associate a single random process with continuous time with the indicated PMW, we define on the trajectory \( \xi_0, \xi_1, \xi_2, \ldots \) nested in the given MRP Markov chain \( \{\xi_n, n \geq 0\} \) family of conditionally independent random variables (RV) \( \xi_{\xi_n} \xi_{n+1} \) with conditional distribution functions (DF) [1]:

\[
P(\Theta_{n+1} \leq t | \xi_n, \xi_{n+1}) = F_{\xi_{\xi_n} \xi_{n+1}}(t).
\]

Through RV \( \xi_{\xi_n} \xi_{n+1} \) the so-called counting process is introduced:

\[
\nu(t) = \max \{ n : \sum_{k=0}^{n} \xi_{\xi_k} \xi_{k+1} \leq t \} ;
\]

Random process \( \xi(t) \equiv \nu(t) \) where \( \{\xi_n, n \geq 0\} \) Markov chain embedded in PMW \( \{\xi_n, \Theta_n, n \geq 0\} \), called semi-Markov process (SMP) [1].

3.1. Superposition of independent semi-Markov processes

Consider a system of \( n \) independently operating elements (subsystems), the functioning of each of which is described by the SMP \( \xi^i(t) \) with arbitrary phase space \( (Z^i, Z^i) \), \( i = 1, n \) (processes \( \xi^i(t) \)) collectively independent, \( Z^i \) - \( \sigma \) - algebra of subsets \( Z^i \) (\( i = 1, n \)). For every \( Z^i \) there is a partition \( Z^i = Z^i_1 \cup Z^i_0, Z^i_1 \cap Z^i_0 = \emptyset, i = 1, n \) where \( Z^i_1 \in Z^i \); is interpreted as a set of working states of the \( i \)-th element; \( Z^i_0 \in Z^i \) - set of failure states of the \( i \)-th element.

Let \( \xi^i_0(t) = 1 \) if \( \xi^i_0(t) \in Z^i_1(t); \xi^i_0(t) = 0 \) if \( \xi^i_0(t) \in Z^i_0(t); \xi^i(t) = (\xi^i_0(t), \xi^i_1(t), \ldots, \xi^i_n(t)) \)

Vector process \( \xi(t) \) characterizes the state of the system at the moment of time according to the states (operability or inoperability) of its elements. The set of possible values of the process \( \xi(t) \) is the set of different binary vectors \( d = (d_1, d_2, \ldots, d_n) \) from \( (0,0, \ldots, 0) \) to \( (1,1, \ldots, 1) \). We denote it by \( D = \{d\} \). Consider the concept of a system failure: on the set of binary vectors \( D \), the function \( g(d) \) such that

\[
g(d) = \begin{cases} 1, & \text{if the system is operational for a given combination of states its elements} \\ 0, & \text{otherwise} \end{cases}
\]

The set of values of the vector \( d \) for which the system is operational is denoted by \( D_1 \), and the set of values at which the system is inoperable is denoted by \( D_0 \), i.e. \( D_1 = \{d: g(d) = 1\} \), \( D_0 = \{d: g(d) = 0\} \). By assumption \( D = D_1 \cup D_2, D_1 \cap D_0 = \emptyset \). Let us investigate the reliability characteristics of such a system, in particular, the average stationary (steady-state) residence times of the process \( \xi(t) \) in sets \( D_1 \) and \( D_0 \) (mean time between failures and mean time to recover the system, respectively) and the limit value at \( t \to \infty \) the likelihood of catching the process \( \xi(t) \) in the set \( D_1 \) (stationary availability factor).

By introducing additional components \( u^i(t) \) the problem can be reduced to the study of the characteristics of the CMP. Let be \( u^i(t) \) - lack of progress \( \xi^i(t) \) at the moment \( t \), i.e. \( u^i(t) = t - \sup \{ u : u \leq t, \xi^i(t) \neq \xi^i(t) \} \). Then the process is undershot \( \xi^i(t) \) at time \( t \) of the closest “in the past” to time \( t \) jump of one of the processes \( \xi^i(t), k = 1, n \), equal to \( u^k(t) = u^i(t) - u(t) \), \( u(t) = \min_{i=1,2,\ldots,n} u^i(t) \).

Superposition of (semi-Markov) processes \( \xi^i(t), i = 1, n \), called [2] \( 2n \)- component process

\[
\xi(t) = \left( \xi^1(t), \ldots, \xi^n(t); u^1(t), \ldots, u^n(t) \right)
\]

The availability factor, mean time between failures and mean recovery time in the stationary mode of the PMP \( \xi(t) \) can be determined, respectively, by the following formulas [2]:
\[ K_T = \sum_{d \in D_1} \prod_{i=1}^{n} T_{d_i}^{(i)} \left( \prod_{i=0}^{n} T_1^{(i)} + T_0^{(i)} \right)^{-1} \]  

(1)

\[ T_1 = \sum_{d \in D_1} \prod_{i=1}^{n} T_{d_i}^{(i)} \left( \sum_{d \in D_0} \prod_{i=1}^{n} T_{d_i}^{(i)} \sum_{j \in I(d)} \frac{1}{T_0^{(j)}} \right)^{-1} \]  

(2)

\[ T_0 = \sum_{d \in D_0} \prod_{i=1}^{n} T_{d_i}^{(i)} \left( \sum_{d \in D_0} \prod_{i=1}^{n} T_{d_i}^{(i)} \sum_{j \in I(d)} \frac{1}{T_0^{(j)}} \right)^{-1} \]  

(3)

where \( T_1^{(i)} \) is mean time between failures of the \( i \)-th element; \( T_0^{(i)} \) is average recovery time of the \( i \)-th element; \( T_{d_i}^{(i)} = T_1^{(i)} \) for \( d_i = 1 \) and \( T_{d_i}^{(i)} = T_0^{(i)} \) for \( d_i = 0 \); \( D_0 \) is the set of vectors \( d \in D_0 \) such that a change in the value of some one component from zero to one transforms the vector \( d \) into the set \( D_1 \); \( I(d) \) is the set of numbers of vector components \( d \in D_0 \), changing the value of each of which from zero to one transforms the vector \( d \) into the set \( D_1 \).

Formulas (1) - (13) give exact relations for calculating the stationary reliability indicators of systems with independently functioning elements, and \( K_T = T_1 / (T_1 + T_0) \) [3].

4. System "p of n" with shutdown after failure

Consider a system of \( n \) independently functioning elements. Suppose that the functioning of each \( i \)-th element is described by an alternating recovery process with uptime (RPU) \( \alpha_1^{(i)}(P\{\alpha_1^{(i)} \leq t\} = F_1^{(i)}(t) \) and recovery time (RT) \( \alpha_0^{(i)}(P\{\alpha_0^{(i)} \leq t\} = F_0^{(i)}(t) \). We will consider the system to be operational when it works at least \( p(1 \leq p \leq n) \) elements (in the example of the functioning of a railway locomotive, which we consider below, \( p = n \)). For definiteness, we assume that at the initial moment of time \( t = 0 \) all elements begin to work. Let us determine the main reliability characteristics of the system: stationary availability factor \( K_T \), mean time between failures \( T_1 \) and mean recovery time \( T_0 \).

Suppose the following conditions are met:

A1. Random variables (RV) \( \alpha_1^{(i)} \) – RPU elements, \( \alpha_0^{(i)} \) – RT elements, \( i = 1, n \) (\( n \) – number of elements in the system) are independent in the aggregate and have limited average \( 0 \leq M\alpha_1^{(i)} = T_1^{(i)} < \infty, 0 \leq M\alpha_0^{(i)} = T_0^{(i)} < \infty \).

A2. Distribution functions (DF) RV \( \alpha_1^{(i)} \) \( \alpha_1^{(i)} \) and \( \alpha_0^{(i)} \) respectively, are absolutely continuous with respect to the Lebesgue measure.

When conditions are met \( A_1, A_2 \) right [2] following expressions for \( K_T, T_1 \) and \( T_0 \) respectively, equivalent to expressions (1)-(3):

\[ K_T = \sum_{|d| \leq p} \prod_{i=1}^{n} T_{d_i}^{(i)} \left( \prod_{i=0}^{n} T_1^{(i)} + T_0^{(i)} \right)^{-1} \]  

(4)

\[ T_1 = \left( \sum_{|d| \leq p} \prod_{i=1}^{n} T_{d_i}^{(i)} \right) \left( \sum_{|d| \leq p} \prod_{i=1}^{n} T_{d_i}^{(i)} \sum_{j:d_j=1} 1/T_1^{(j)} \right)^{-1} \]  

(5)

\[ T_0 = \left( \sum_{|d| \leq p} \prod_{i=1}^{n} T_{d_i}^{(i)} \right) \left( \sum_{|d| \leq p} \prod_{i=1}^{n} T_{d_i}^{(i)} \sum_{j:d_j=0} 1/T_0^{(j)} \right)^{-1} \]  

(6)
Let us show the application of formulas (4) - (6) for calculating the stationary reliability indicators of electric locomotives with an asynchronous traction motor in order to compare the reliability of electric locomotives with DC and AC motors used, respectively, on the Azerbaijani and Kazakhstan railways. The calculations will be carried out on the basis of the available data for 2016 in the first case for malfunctions of a small toothed gear \((i = 1)\), motor axial bearing \((i = 2)\), anchor bearing \((i = 3)\), traction motor \((i = 4)\) and in the second case is due to malfunctions of a small toothed gear \((i = 1)\), compressor \((i = 2)\), pantograph \((i = 3)\), traction motor \((i = 4)\). Let us denote by \(\alpha_{0,j}^{(i)}\) and \(\alpha_{1,j}^{(i)}\) – the recovery times (RT) and the failure-free operation times (FOT) of the \(i\) - th element of the \(j\) - th electric locomotive.

Let there be the following data on the downtime (in hours) of DC electric locomotives operated in 2016 on the Azerbaijan Railway (AR) for \(j\) - electric locomotives \((j = 1, \ldots, 11)\):

1) VL11-274: \(\alpha_{0}^{(1)} = 78.4\); 2) VL11-341: \(\alpha_{0}^{(2)} = 98.5\); 3) VL11-351: \(\alpha_{0}^{(3)} = 85.4\); 4) VL11-353: \(\alpha_{0}^{(4)} = 58\); 2) VL11-364: \(\alpha_{0}^{(5)} = 74\); 6) VL11-402: \(\alpha_{0}^{(6)} = 110.25\); 10, 15: \(\alpha_{0}^{(7)} = 21.4\); 7) VL11-460: \(\alpha_{0}^{(8)} = 89.7\); 8) VL11-461: \(\alpha_{0}^{(9)} = 342\); 9) VL11-462: \(\alpha_{0}^{(10)} = 56.5\); 11) VL11-464: \(\alpha_{0}^{(11)} = 39\); 59.4.

Relevant data on the downtime of AC electric locomotives in operation in 2016 (on the Kazakhstan Railways (KR)):

1) KZ18: \(\alpha_{0}^{(1)} = 8\); \(\alpha_{0}^{(2)} = 4\); 2) KZ 9: \(\alpha_{0}^{(3)} = 40\); \(\alpha_{0}^{(4)} = 46\); 3) KZ 12: \(\alpha_{0}^{(5)} = 14\); \(\alpha_{0}^{(6)} = 5\); \(\alpha_{0}^{(7)} = 64\); 4) KZ 15: \(\alpha_{0}^{(8)} = 48\); \(\alpha_{0}^{(9)} = 22\); 5) KZ 17: \(\alpha_{0}^{(10)} = 200\); 6) KZ 18: \(\alpha_{0}^{(11)} = 72\); 7) KZ 20: \(\alpha_{0}^{(12)} = 150\); \(\alpha_{0}^{(13)} = 19\); \(\alpha_{0}^{(14)} = 13\); 8) KZ 22: \(\alpha_{0}^{(15)} = 8\); \(\alpha_{0}^{(16)} = 4\); 9) KZ 27: \(\alpha_{0}^{(17)} = 45\); \(\alpha_{0}^{(18)} = 10\); \(\alpha_{0}^{(19)} = 15\); 10) KZ 25: \(\alpha_{0}^{(20)} = 25\); \(\alpha_{0}^{(21)} = 15\); 11) KZ 34: \(\alpha_{0}^{(22)} = 8\); \(\alpha_{0}^{(23)} = 11\).

According to the data on the Azerbaijan Railways (shortly AR) we have (taking into account 365x12x24 = 8760 working hours per year), denoting through \(T_{0,j}^{(i)}\) and \(T_{1,j}^{(i)}\) respectively, the average VR and the average FOT of the \(i\)-th element of the \(j\)-th electric locomotive:

\[
T_{0,1}^{(i)} = 4340.8 \quad (i = 2); \quad T_{0,2}^{(i)} = 78.4 \quad (i = 0); \quad T_{0,4}^{(i)} = 2125.8 \quad (i = 14); \quad T_{0,2}^{(i)} = 0.7 \quad (i = 2); \quad T_{0,3}^{(i)} = 98.5 \quad (i = 0); \quad T_{0,4}^{(i)} = (38 + 60.25)/2 = 49.125; \quad T_{0,4}^{(i)} = 171.3 \quad (i = 14); \quad T_{0,3}^{(i)} = 46.7 \quad (i = 0); \quad T_{0,3}^{(i)} = 62; \quad T_{0,3}^{(i)} = 28; \quad T_{0,4}^{(i)} = 4321 \quad (i = 14); \quad T_{0,4}^{(i)} = 118; \quad T_{0,4}^{(i)} = 0 \quad (i = 2); \quad T_{0,4}^{(i)} = 4343 \quad (i = 14); \quad T_{0,4}^{(i)} = 74; \quad T_{0,4}^{(i)} = 0 \quad (i = 3); \quad T_{0,4}^{(i)} = 21317 \quad (i = 14); \quad T_{0,4}^{(i)} = 105875\frac{1}{7}; \quad T_{0,4}^{(i)} = 214; \quad T_{0,4}^{(i)} = 7; \quad T_{0,4}^{(i)} = 433515 \quad (i = 14); \quad T_{0,4}^{(i)} = 89.7; \quad T_{0,4}^{(i)} = 0 \quad (i = 2); \quad T_{0,4}^{(i)} = 4209 \quad (i = 14); \quad T_{0,4}^{(i)} = 342; \quad T_{0,4}^{(i)} = 0 \quad (i = 2); \quad T_{0,4}^{(i)} = 288113 \quad (i = 14); \quad T_{0,4}^{(i)} = 58.3; \quad T_{0,4}^{(i)} = 0 \quad (i = 2); \quad T_{0,4}^{(i)} = 281783 \quad (i = 14); \quad T_{0,4}^{(i)} = 0; \quad T_{0,4}^{(i)} = 0.7 \quad (i = 14); \quad T_{0,4}^{(i)} = 56.5; \quad T_{0,4}^{(i)} = 214515 \quad (i = 14); \quad T_{0,4}^{(i)} = 81; \quad T_{0,4}^{(i)} = 49.2; \quad T_{0,4}^{(i)} = 0.
\]

Since for the considered system with disconnection of elements \(T_{1,j}^{(1)} = T_{1,j}^{(2)} = T_{1,j}^{(n)} = \text{const} = T_{1,j}^{\text{syst}}\), the \(T_{1,j}^{\text{syst}}\) – mean (for failures) mean time between failures of the system, determined by the expression \(T_{1,j}^{\text{syst}} = \left(8760 - T_{0,j}^{\text{syst}}\right)/\left(r_{j} + 1\right)\), where \(r_{j}\) – number of refusals \(i\)-th element, then the values \(T_{1,j}^{\text{syst}} / n\) and \(T_{0,j}^{\text{syst}} / n\) can be interpreted as the mean (by elements) MTBF and system recovery time and denote \(T_{1,j}^{\text{elem}} = T_{1,j}^{\text{syst}} / n\) and \(T_{0,j}^{\text{elem}} = T_{0,j}^{\text{syst}} / n\). Thus, denoting the parameters \(K_{r,j}\), \(T_{r,j}\) and \(T_{0,j}\) in (11)-(13) as system parameters \(K_{r,j}^{\text{syst}}\), \(T_{r,j}^{\text{syst}}\) and \(T_{0,j}^{\text{syst}}\), we come to the conclusion that for the system \(j\) with disconnection the relations...
\[ K_{l,j}^{\text{syst}} = K_{l,j}^{\text{elem}}, \quad T_{l,j}^{\text{syst}} = T_{l,j}^{\text{elem}}, \quad T_{0,j}^{\text{syst}} = T_{0,j}^{\text{elem}} \]

and formulas (13) - (15) are equivalent to the following simple formulas

\[ K_{l,j} = T_{l,j}^{\text{syst}} / (T_{1,j}^{\text{syst}} + T_{0,j}^{\text{elem}}), \quad T_{1,j} = T_{l,j}^{\text{syst}} / n = T_{l,j}^{\text{elem}} \]

\[ T_{0,j} = T_{0,j}^{\text{syst}} / n = T_{0,j}^{\text{elem}} \]

This means that for the considered system with disconnection of elements, the average stationary reliability characteristics of the system are equal to the corresponding characteristics of the elements.

Applying formulas (7) - (9) to the data on the AR, we get:

1) \( K_{l,1} = 0,982; \quad T_{1,1} = 1085,2; \quad T_{0,1} = 19,6; \quad 2) \ K_{l,2} = 0,935; \quad T_{1,2} = 531,45; \quad T_{0,2} = 36,9; \quad 3) \ K_{l,3} = 0,926; \quad T_{1,3} = 428,825; \quad T_{0,3} = 34,175; \quad 4) \ K_{l,4} = 0,973; \quad T_{1,4} = 1080,25; \quad T_{0,4} = 29,5; \quad 5) \ K_{l,5} = 0,983; \quad T_{1,5} = 1085,75; \quad T_{0,5} = 18,5; \quad 6) \ K_{l,6} = 0,9; \quad T_{1,6} = 532,295; \quad T_{0,6} = 58,287; \quad 7) \ K_{l,7} = 0,98; \quad T_{1,7} = 1083,787; \quad T_{0,7} = 22,425; \quad 8) \ K_{l,8} = 0,925; \quad T_{1,8} = 1052,25; \quad T_{0,8} = 85,5; \quad 9) \ K_{l,9} = 0,96; \quad T_{1,9} = 720,282; \quad T_{0,9} = 19,43; \quad 10) \ K_{l,10} = 0,9; \quad T_{1,10} = 704,457; \quad T_{0,10} = 76,625; \quad 11) \ K_{l,11} = 0,923; \quad T_{1,11} = 536,287; \quad T_{0,11} = 44,85.

Now calculating the average (over all \( n = 11 \) electric locomotives) values of the coefficients \( K_{l}, T_{1} \) and \( T_{0} \), i.e.

\[ \bar{K}_{l} = \frac{1}{n} \sum_{j=1}^{n} K_{l,j}, \quad T_{1} = \frac{1}{n} \sum_{j=1}^{n} T_{1,j} \quad \text{and} \quad T_{0} = \frac{1}{n} \sum_{j=1}^{n} T_{0,j}, \]

will get

\[ \bar{K}_{l} = 0,944; \quad \bar{T}_{1} = 303,71; \quad \bar{T}_{0} = 40,48 \quad (10) \]

According to the data on the Kazakhstan Railway (KR), as a result of the calculation, we obtain:

\[ \bar{K}_{l} = 0,965; \quad \bar{T}_{1} = 665,56 \quad \bar{T}_{0} = 17,52. \quad (11) \]

Comparing the average indicators (10) and (11), we come to the following conclusion: although the mean time between failures \( T_{1} \) for DC electric locomotives of AR is more than for AC electric locomotives of KR, by about 140 hours, their average availability factor \( \bar{K}_{l} \) is less 2% and the average recovery time is 13 hours longer, which indicates the greater reliability of AC electric locomotives in comparison with DC electric locomotives.

5. Conclusion

The operation of electric locomotives with an induction motor based on data on unscheduled repairs associated with violations of individual elements of an electric locomotive can be represented as a recoverable system with disconnection after failure of one element (or several) of all remaining elements. Under the conditions \( A_{1} \) and \( A_{2} \) for the obtained semi-Markov process of restoration, the formulas for calculating the reliability indicators in the steady state (stationary) mode are applicable.

To calculate these indicators, simplified formulas are obtained in the work, which can be used for a comparative analysis of the reliability levels of various types of complex systems with shutdown in case of failures, having non exponential distribution functions of the residence times in states.

References

[1] Korolyuk V S Turbin A F 1976 Semi-Markov processes and their applications(Kyiv: Naukova Physics) p 183.

[2] Korlat A N Kuznetsov V N Novikov M M and Turbin A F 1991 Semi-Markov models of recoverable systems and queuing systems (Chisinau: Shtints) p 276.
[3] Kozlov B A and Ushakov I A 1975 *Handbook for calculating the reliability of radio electronics and automation equipment* (Moscow: Sov.radio).

[4] Beichelt F and Franken P 1988 *Reliability and maintenance: Mathematical approach* translated from it. (Moscow: Radio and communication).

[5] Silvestrov D S 1980 *Semi-Markov processes with a discrete set of states* (Moscow: Sov.radio) p 272.

[6] Kopp V Ya Obzherin Yu E and Peschansky A I 2000 *Stochastic models of automated production systems with time redundancy* (Sevastopol) p 285.

[7] Zeleny O V Nosovskiy A V and Stadnik O A 2007 Semi-Markov models in the problems of assessing the reliability and risk from NPP operation *Problems of safety of nuclear power plants I of Chernobyl* 7 pp 30-40.

[8] Borisevich A V and Dyakin N V 2015 Semi-Markov model for assessing the reliability indicators of an uninterruptible power supply of a data center *Modern scientific research and innovations* 8 Part I pp 1-8.

[9] Papadoponlou A and Vassilou R-C G 1999 Cantinuous Time Non-Homogeneous Semi-Markov Systems Semi-Markov Models and Applications Kluwer Acad. Publ. pp 241-251.

[10] Limnios N and Oprisan 2001 *Semi-Markov Processes and Reliability* (Boston, Birkhauser).

[11] Grabski F 2003 *The Reliability of an Object with Semi-Markov Failure Rate* Appl.math.Comput. 135 pp 1-16.

[12] Grabski F 2007 Applications of Semi-Markov processes in reliability *RTA* 3-4 December-Special Issue pp 60-75.

[13] Grabski F 2011 Semi-Markov failure rates processes *Applied Mathematics and Computation* 217 pp 9956-9965.

[14] Korolyuk V S Turbin A F 1978 *Phase enlargement of complex systems* (Kyiev: Vishcha school) p 110.

[15] Korolyuk V S and Turbin A F 1982 *Markov recovery processes in problems of system reliability*. (Kyiev: Naukova Physics) p 235.

[16] Gikhman I I and Skorokhod A V 1965 *Introduction to the theory of stochastic processes* (Fizmatgiz, Moscow)