Clockwork mirror neutron

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In this paper we address the hierarchy in the neutron-anti-neutron ($n - \bar{n}$) oscillation through clockwork mechanism (CW) with a set of mirror neutrons acting as clockwork gears. This is achieved by coupling Standard Model (SM) neutrons to the zeroth gear while having an explicit mirror baryon number ($\bar{B}$) breaking term at the $N^{th}$ one. The explicit $\bar{B}$ breaking term induces neutron oscillation in the mirror world which propagates to the SM baryon number ($B$) violation through the CW gears. This mirror world is a Standard Model singlet and possess the $\bar{B}$ symmetry that prohibits its decay to SM matter fields. This symmetry is broken to the combination, $B - \bar{B}$, by a scalar field to generate the clockwork. Without introducing large hierarchies among the mirror neutrons, we predict the time period of neutron-anti-neutron oscillation in our world of $\tau_{n\bar{n}} \gtrsim 10^8\text{s}$, which is within the reach of current experiments and the upcoming ones. This mechanism also predicts neutron to mirror neutron transition along with the emission of SM particles, given the phase space is favourable. This could be searched for at neutron stars and will place bounds on the symmetry breaking mechanism in the mirror world. The current constraint on neutron-mirror neutron oscillation, coming from neutron stars, is shown to be satisfied, in the parameter space which predicts a detectable neutron-anti-neutron oscillation in our world. This scenario also predicts double neutron annihilation to produce two mirror neutrons which could also be tested via the luminosity measurements of neutron stars.

I. INTRODUCTION

It is understood that among the accidental symmetries present in Standard Model, the baryon number ($B$) and lepton number ($L$) are individually violated through non-perturbative effects [1] leaving behind $B - L$ conserved. One of the consequences of this symmetry is that it prohibits $\Delta B = 2$, $\Delta L = 0$ processes like neutron-anti-neutron oscillation. On the other hand, there is no reason why the above global symmetries remain conserved on including new physics beyond SM. To that end, the discovery of neutron-anti-neutron oscillation would be a clear evidence of new physics that violate baryon number by two units, without violating lepton number. This is intimately connected with the generation of baryon asymmetry in the universe and if observed, can initiate a low-scale baryogenesis [2].

At low energies, it is worthwhile to look at the Lagrangian in terms of free neutrons and anti-neutrons with baryon number charge, $B = 1$ and $B = -1$ respectively. The neutron-anti-neutron oscillation is then given by the matrix element, $\langle \bar{n}| H_{\Delta B=2} | n \rangle$, with the operator $H_{\Delta B=2} = \epsilon \bar{n}_c n$. Including this, the Lagrangian density of these neutral fermion fields could be written as [3],

$$\mathcal{L} = i \bar{n} \gamma^\mu \partial_\mu n - \frac{m_n}{2} \left[ \bar{n} n + \bar{n}_c n^c \right] - \frac{\epsilon}{2} \left[ \bar{n}_c n + \bar{n}_c n^c \right], \tag{1}$$

where 'n' is the four-component spinor field, identified as the neutron. In the above Lagrangian, the baryon number is violated explicitly through the last term by two units, with the strength $\epsilon$. The neutron-anti-neutron oscillation could be computed from the hadronic matrix element for the $\Delta B = 2$ process as,

$$\langle \bar{n}| H_{\Delta B=2} | n \rangle = -\frac{1}{2} \epsilon \nu^T \bar{C} u_n, \tag{2}$$

where $\nu_n$ and $u_n$ are Dirac spinors for $\bar{n}$ and $n$ respectively. It is straightforward to see that the $\epsilon$ gives the transition probability for a neutron at rest to oscillate to an anti-neutron at time $t$, given by $P_{n \rightarrow \bar{n}}(t) = \sin^2(\epsilon t) e^{-\lambda t}$, where $\lambda$ is the mean lifetime of the free neutron $\lambda^{-1} = \tau_n = 880\text{s}$.

Since both $m_n$ and $\epsilon$ have the same mass dimensions, naively, they could be thought to originate at similar scales. On the contrary, the current limits on the oscillation time period $\tau_{n\bar{n}} \gtrsim 10^8\text{s}$ [4–6], constraints $\epsilon = \frac{1}{\tau_{n\bar{n}}} \lesssim O(10^{-33})\text{GeV}$. Moreover, the upcoming experiments, European Spallation Source (ESS) [8] and DUNE [9] predicts strong improvements to $\tau_{n\bar{n}} \gtrsim 10^9 - 10^{10}\text{s}$. Thus bringing in large hierarchies among the terms in the Lagrangian given in Eq.1.

Such large suppression in the scale of $\epsilon$ seems unnatural in any decent model of particle physics. On the other hand, it is well understood that the clockwork (CW) mechanism has been widely successful in generating large hierarchies, among scales, out of small ones with the help of its gears. In this paper, we aim to explain the large suppression in $\epsilon$ using the Type-1 Clockwork seesaw mechanism [10]. It may be possible to think of clockwork as arising for deconstruction of extra-dimension, with the resulting theory space consisting of a set of non-dynamic fields, but we will restraint from a further detailed comparison.

II. TYPE-1 SEESEAW CLOCKWORK

The success of CW models relies on the so called CW gears, which link and generate amplified coefficients, even though individually the links and their couplings are not hierarchical. For the purpose here, lets consider a set
of left handed \( n_{L(i)} \) (\( i = 1, 2, ..., N \)) and right handed \( n_{R(i)} \) (\( i = 0, 1, ..., N \)) mirror fermion fields, which are Standard Model singlets. Equivalent to the baryon number (\( B \)) in SM, they carry mirror baryon number (\( \bar{B} \)). Note that, in the set of mirror neutrons, the zero mode exists only for the right handed fermions. The Lagrangian density could be written as,

\[
\mathcal{L} = i\bar{n}\gamma^\mu \partial_\mu n - \frac{m}{2} \left( \bar{n}n + \bar{n}^c n^c \right) - y \bar{n} \phi n_{R(0)} - \frac{m_M}{2} \bar{n}_{R(N)} n_{R(N)} + V_{CW} + h.c. \tag{3}
\]

where, \( \phi \) is a scalar field that is charged under both SM baryon symmetry \((U(1)_B)\) and mirror baryon symmetry \((U(1)_{\bar{B}})\). The vacuum expectation value of this scalar field, \( \langle \phi \rangle = v \), breaks \( U(1)_B \times U(1)_{\bar{B}} \rightarrow U(1)_{B-\bar{B}} \) at the SM node. In this interaction term, the Yukawa coupling \('y'\) should be considered as a spurion carrying isospin charge\(^1\). And \('m_M'\) is the explicit mirror baryon number violating term appearing at the \( N^{th} \) gear, which corresponds to the matrix element of neutron-anti-neutron oscillations in the mirror world.

The CW potential, \( V_{CW} \), is given in terms of the neutron gears as,

\[
V_{CW} = \sum_{i=1}^{N} M \bar{n}_{L(i)} n_{R(i-1)} - m \bar{n}_{L(i)} n_{R(i)}, \tag{4}
\]

where \( M \) and \( m \) are spurions that violate the chiral symmetry of the gear fermions. Here, I have assumed slight hierarchy in the parameters, \( M > m \).

Finding the Euler-Lagrange equation of motion by varying with respect to \( n_{L(i)} \) \(^{10}\), we get,

\[
M n_{R(i-1)} = m n_{R(i)}, \tag{5}
\]

where \( i = (1, 2, ..., N) \). Which means,

\[
n_{R(0)} \equiv \frac{m}{M} n_{R(1)} = \left( \frac{m}{M} \right)^2 n_{R(2)} = .... = \left( \frac{m}{M} \right)^N n_{R(N)}. \tag{6}
\]

On diagonalising, and identifying the lightest mode with physical composite state zero mode as \( n^0_{R} \), we get,

\[
n_{R(i)} = \left( \frac{m}{M} \right)^{N-i} n^0_{R}. \tag{7}
\]

Now, the neutron interaction term with gear neutrons in the Lagrangian, in terms of the rotated fields, could be written as

\[
\mathcal{L}_{int} = \bar{\Psi}^c \mathcal{M} \Psi, \tag{8}
\]

where \( \Psi = (n, n^0_R) \) and

\[
\mathcal{M} = \begin{pmatrix} 0 & y_{eff}v \\ y_{eff}^*v & m_M \end{pmatrix}, \tag{9}
\]

where \( y_{eff} = y(\frac{m}{M})^N \). For \( m_M \gg yv \), it is easy to see that the clockwork mechanism now generates a neutron-anti-neutron oscillation term with \( \Delta B = 2 \) for our neutron given as,

\[
\mathcal{L} = i\bar{n}\gamma^\mu \partial_\mu n - \frac{m}{2} \left( \bar{n}n + \bar{n}^c n^c \right) - \frac{\epsilon^2}{2} \left( \frac{m}{M} \right)^{2N} \frac{1}{m_M} \bar{n}^c n + h.c. . \tag{10}
\]

Comparing the above Lagrangian with Eq.1, we can see that the strength of neutron-anti-neutron oscillation as determined in the CW model becomes,

\[
\frac{\epsilon^2}{2} = \frac{(y_{eff}v)^2}{m_M} = y^2 v^2 \left( \frac{m}{M} \right)^{2N} \frac{1}{m_M}. \tag{11}
\]

With Yukawa couplings of \( O(1) \), \( \frac{m}{M} \sim 0.1 \), and \( m_M = 1 GeV \), we get \( \epsilon \sim 10^{-34} GeV \) for 20 gears. This result translates to neutron-anti-neutron oscillation time period of \( 10^{56} \)s, which is within the reach of the current and future experiments \(^{11}\). Note that, in the absence of the explicit mirror baryon number violating term, \( 'm_M' \), the clockwork could not have generated the neutron-anti-neutron transition, while conserving \( B - \bar{B} \) global symmetry. On the other hand, the explicit breaking of the mirror baryon symmetry has now propagated to the explicit breaking of baryon number by two units through the \( B - \bar{B} \) conserving term. In other words, the neutron oscillation in the mirror world has now propagated to our world to generate our \( n - \bar{n} \) oscillations.

From an extra-dimensional point of view, the Yukawa term that conserves \( B - \bar{B} \) is on the infrared (IR) brane, while the term that violate mirror baryon number explicitly term is localised at the ultraviolet (UV) brane. And \( n^0_R \) is the bulk mirror field that mediates the baryon violation from UV brane to IR brane. Thus the neutron converts to a mirror neutron which oscillates among themselves via the UV term, finally converting back to neutron. Thus fabricating a large time period of neutron-anti-neutron oscillations.

### III. CONSTRAINTS

If the lightest mirror neutron, \( n^0_R \), is lighter than the SM neutron, \( B - \bar{B} \) conserving transition from SM neutron can occur and will constrains the parameters of the CW model. Such mirror neutron states can provide a plethora of interesting phenomenological effects as it can constitute the dark matter spectrum\(^{12}\), lead to dark neutron stars \(^{13}\) and astrophysical objects \(^{14}\) and relax the GZK limit of cosmic rays\(^{15}\).
On including a small oscillation of the scalar field about its vacuum expectation value \((\nu+\phi)\), the effective Yukawa term \((y_{eff} = yv(m_{\phi}/M)^N)\) can generate \(n \rightarrow n^R_0 + \phi\) transition, if it is allowed kinematically. Moreover, since the symmetry protecting the stability of mirror neutron is spontaneously broken, processes such as \(n \rightarrow n^R_0 + X\), where \(X\) are the SM photons or light fermion states with masses \(m_{\nu} \lesssim |m_n - m_{\phi}|\), could be generated. For example, the operator term \(L_y = \nu \bar{v}_{ew} n(v_{ew} + h)n_{R(0)}\), where \(h\) is the neutral Higgs field, can give rise to \(n \rightarrow n^R_0 + \gamma + \gamma\), \(n + q \rightarrow n^R_0 + q\) or \(n + e^{-} \rightarrow n^R_0 + e^{-}\) with suppressed amplitudes. Such processes are explored at table-top experiments by looking for the mass difference between the isotopes \((A, Z)\) and \((A - 1, Z)\) but are mostly forbidden due to the binding energy [16], for almost degenerate neutron and mirror neutron.

Though \(n \rightarrow n^R_0 + X\), with \(|m_n - m_{\phi}^R| \lesssim \mathcal{O}(MeV)\), is kinematically forbidden in a stable nuclei, the availability of higher energies facilitate these transitions in a neutron star for \(|m_n - m_{\phi}^R| \lesssim \mathcal{O}(100MeV)\). Where, mirror neutrons are created via a scattering of neutrons on nucleons \(nN \rightarrow n^R_0N, N = (n,p)\), through a Higgs exchange between the Lagrangian terms \(\frac{m_{\phi}}{m_h} \bar{q}q(v_{ew} + h)\) and \(y_{ew} \bar{n}(v_{ew} + h)n_{R(0)}\). The resultant operator, after integrating out the Higgs scalar, responsible for this process could be written as \(C_{nn^R}(n^R_0 n)(\bar{q}q)\), with the Wilson Coefficient given as,

\[
C_{nn^R} = yv\left(\frac{m_{\phi}}{M}\right)^N \frac{1}{m^2_m v_{ew} m_h} ,
\]

where \(m_n\) and \(m_h\) are masses of the light quarks and the Higgs respectively and \(v_{ew}\) is the electroweak vacuum expectation value. The cross-section of the process \(nN \rightarrow n^R_0N\) could be approximated as

\[
\sigma_{n^R_0N} \approx |C_{nn^R}|^2 \left(\frac{n^R_0N}{nN}\right)(\bar{q}q)|nN|^2
\sim |C_{nn^R}|^2 m^2_N \sum_{q=u,d,s} f_q^2 \sigma_{nN} ,
\]

where \(\sigma_{nN}\) is the cross-section for \(nN \rightarrow nN\), computed in the non-relativistic limit using Schrödinger equation [19], and \(f_q = \frac{1}{m_N} (\langle N | m_q \bar{q}q | N \rangle)\) is the nucleon matrix element of the quark scalar operator, whose values are given in [18]. Now, the Wilson Coefficient of the operator responsible for the process \(nN \rightarrow n^R_0N\) could be written as,

\[
C_{nN} \simeq C_{nn^R} m^2_N \sum_{q=u,d,s} f_q .
\]

Thus the effective coefficient of the operator \(n^R_0 n\) for the process \(n \rightarrow n^R_0\), in presence of a nucleon, becomes \(C_{nN^R} v_{ew}\). These effects could be searched for by studying the change in spin period induced by the neutron-mirror neutron transitions, which places a limit on the coefficient of oscillation \(C_{nN^R} v_{ew} \lesssim 10^{-22}GeV\) [17]. Whereas, a luminosity measurement of cold neutron star measuring the temperature coming from the heat released due to \(n \rightarrow n^R_0\) in the core, places a much stronger upper bound of \(C_{nN^R} v_{ew} \lesssim 10^{-26}GeV\) [19] with an oscillation time period of \(\tau \gtrsim 10s\). In the parameter space for which, the neutron-antineutron oscillation lie within the reach of current experiments, for SM, the Wilson coefficient given in Eq.14 could be computed as \(C_{nN^R} v_{ew} \lesssim 10^{-27}GeV\), evading the existing bounds mentioned previously. On the other hand, if the future experiments improve the sensitivity further to constraint the coefficient of oscillation to \(C_{nN^R} v_{ew} \lesssim 10^{-29}GeV\), which corresponds to a time period of \(\tau \gtrsim 10^3s\) [19], a detectable neutron-antineutron oscillation signal would mean that the mirror neutrons either have to be heavier than the SM neutron, thus prohibiting the \(n \rightarrow n^R_0\) transitions, or the \(nN\) scattering should be mediated by via a loop level interaction.

In such scenarios, the double neutron annihilation \((nn \rightarrow n^R_0 n^R_i)\), where \(i = (1, 2, ..., N)\), could be a possible effect to search. A contact Hamiltonian term for this process could be written as,

\[
\mathcal{H}_{nn} = \frac{1}{m^2_{\phi}} y^2 \left(\frac{m}{M}\right)^{2(N-i)} \langle nR_i n)(\bar{n}R_i n)\rangle ,
\]

where, we have assumed that the mass of the \(\phi\) is much higher than the energy available for the process. Note that the Hamiltonian given above suggests that the double neutron annihilation is highly suppressed for process with \((n^R_0, n^R_i)\) final states, but \(nn \rightarrow n^R_0 n^R_i\) could proceed, if enough energy is provided to incoming neutrons such that the phase space is favourable for the process to take place. The cross-section for this process could be approximated as,

\[
\sigma_{nn} \simeq y^4 \left(\frac{m}{M}\right)^{4(N-i)} \frac{m^4_N}{m^4_{\phi}} \sigma_{nN} ,
\]

where \(\sigma_{nN}\) is again the non-relativistic cross-section of \(nN \rightarrow nN\) as before. The maximum momentum carried by these incoming neutrons is determined by the Fermi-momentum, \(p_F \approx 330MeV\). Moreover, since the process creates two mirror neutron at the same place, the Pauli blocking will set in immediately to suppress this process.

### IV. Conclusion

In this paper, we introduced a clockwork mechanism to explain the neutron-anti-neutron oscillation data without bringing in hierarchies. At the quark level the oscillation Hamiltonian is given by a dimension-9 operator suppressed by a scale \(\Lambda \sim \mathcal{O}(500TeV)\). There has been efforts in extra-dimensions to get the required Wilson Coefficient by selectively localising the quark wave profiles [20]. Clockwork, on the other hand, has been understood to be successful in introducing large amplifications of couplings with moderate rotation of its gears. Note that, in this mechanism the explicit mirror baryon number violation happens at the \(N^{10}\) gear and is percolated into SM \(\Delta B = 2\) operator through CW rotations. This
mechanism works once the symmetry that protects the stability of the mirror particles is spontaneously broken, via vacuum expectation value of the scalar field. The breaking of this symmetry is also important for asymmetric cosmological heating of the two sectors because it can be problematic if light degrees of freedom in mirror sector carry the same amount of energy as SM [19, 21]. Upon breaking this symmetry, $n \rightarrow n_0 R + X$ transition is allowed and could in turn explain the neutron lifetime anomaly between the measurements in the beam and bottle experiments. On the other hand, such transitions are highly constrained from experiments that measure change in temperature at the core of the neutron star. Towards the end we also discussed the possibility of double neutron annihilation to mirror neutrons which could also be tested by studying the luminosity of neutron stars given the phase space is favourable.

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