ORIGIN AND EVOLUTION OF NEUTRON STAR MAGNETIC FIELDS

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This paper intends to give a broad overview of the present knowledge about neutron star magnetic fields, their origin and evolution. An up-to-date overview of the rich phenomenology (encompassing “classical” and millisecond radio pulsars, X-ray binaries, “magnetars”, and “thermal emitters”) suggests that magnetic fields on neutron stars span at least the range $10^8$–$10^{15}$ G, corresponding to a range of magnetic fluxes similar to that found in white dwarfs and upper main sequence stars. The limitations of the observational determinations of the field strength and evidence for its evolution are discussed. Speculative ideas about the possible main-sequence origin of the field (“magnetic strip-tease”) are presented. Attention is also given to physical processes potentially leading to magnetic field evolution.

1 Introduction

Given its role in opening the International Workshop on Strong Magnetic Fields and Neutron Stars, this presentation has the purpose of giving a general overview about what is currently known about neutron star magnetic fields, their origin and evolution. Inevitably, it will contain much of the same material presented by the author in similar reviews in previous years (e.g., Reisenegger 2001a), of which it should be regarded to be an extension and update.

Magnetic fields are most likely the main form of “hair” that allows neutron stars, contrary to black holes, to be distinguished from each other and classified into phenomenologically very different groups. Among single (or non-accreting binary) neutron stars, we distinguish “classical” pulsars, millisecond pulsars, soft gamma-ray repeaters, anomalous X-ray pulsars, and inactive, thermal X-ray emitters (see, e.g., Becker & Pavlov 2002). Binary systems with mass transfer onto a neutron star can be divided into high-mass and low-mass X-ray binaries (according to the companion mass), with substantially different properties. Magnetic fields play an essential role by accelerating particles, by channeling these particles or accretion flows, by producing synchrotron emission or resonant cyclotron scattering, and by providing the main mechanism for angular momentum loss from non-accreting stars. Moreover, evidence is mounting that soft gamma-ray repeaters and anomalous X-
ray pulsars are really only slightly distinct types of very strongly magnetized neutron stars (“magnetars”) in which the magnetic field is the main energy source for the observed radiation.

On the other hand, we actually know surprisingly little about neutron star magnetic fields. In particular, most “measurements” of neutron star magnetic fields are indirect inferences, which are put in doubt both by their inconsistency with other observational evidence and with plausible theoretical models for the physics of their surroundings. Even less is known about the geometry of the magnetic field, its evolution, and its origin, so there is open space for speculation, modelling, and (hopefully) prediction of measurable effects that might test the theoretical ideas.

Thus, I first review the observational “classes” of neutron stars mentioned above, with a special eye on the evidence for the presence and strength of the magnetic fields in each class (§2). In §3, the inferred magnetic fluxes are compared to those of other kinds of stars, and possible connections are discussed. Section 4 surveys the evidence for and against magnetic field evolution, and discusses physical processes which may lead to such evolution. General conclusions are presented in §5.

### 2 Classes of neutron stars and evidence for magnetic fields

#### 2.1 Radio pulsars

Radio pulsars are regularly pulsating sources of radio waves, interpreted as magnetized, rotating neutron stars (Pacini 1967; Gold 1968). Beams of radiation emerging from the poles of a roughly dipolar magnetic field misaligned with respect to the rotation axis appear as pulses every time they sweep the location of the Earth. These pulses reveal rotation periods \( P \) from 1.55 milliseconds (ms) to several seconds. A very tangible illustration of the impressively fast rotation rates (remember that more than a solar mass is participating in this rotation!) is given in “The Sounds of Pulsars”, on the Jodrell Bank Observatory webpage [http://www.jb.man.ac.uk/~pulsar/Education/Sounds/sounds.html](http://www.jb.man.ac.uk/~pulsar/Education/Sounds/sounds.html).

Pulsar rotation periods are observed to lengthen with time \( \dot{P} = dP/dt > 0 \). The simplest model for the spin-down process is to consider the neutron star as a magnetized body of moment of inertia \( I \), rotating in vacuum with angular velocity \( \Omega \) (Ostriker & Gunn 1969). It loses rotational energy due to the time-variation of its magnetic dipole moment vector \( \vec{\mu} \), which rotates at a fixed inclination \( \alpha \) with respect to the rotation axis,
\[-\frac{d}{dt} \left( \frac{1}{2} I \dot{\Omega}^2 \right) = \frac{2}{3c^3} |\vec{\mu}|^2 = \frac{1}{6c^3} B^2 R^6 \Omega^4 \sin^2 \alpha, \quad (1)\]

allowing us to infer the dipole magnetic field strength, \( B[\text{G}] \approx 3.2 \times 10^{19} \sqrt{P[\text{s}] \dot{P}} \) (for radius \( R = 10 \text{ km} \), \( I = 10^{45} \text{g cm}^2 \), and \( \alpha = 90^\circ \)).

This “dipole in vacuum” model is unlikely to be very accurate, as real pulsars are surrounded by a co-rotating, highly conducting magnetosphere, and by an interstellar medium whose plasma frequency is much higher than the expected radiation, which therefore cannot propagate. Somewhat more realistic models (e.g., Goldreich & Julian 1969) tend to roughly confirm the estimate of \( B \) (being less sensitive to \( \alpha \)), so this estimate is generally used. Assuming a constant field strength and moment of inertia, eq. (1) can be integrated backwards in time to give a divergent rotation rate at a time \( \tau = \Omega / \left(-2 \dot{\Omega}\right) = P / (2 \dot{P}) \) before the present (at which the spin parameters are to be evaluated), defining a characteristic “spin-down age” for the pulsar.

In terms of these parameters, radio pulsars fall into two fairly disjoint groups (e.g., Phinney & Kulkarni 1994):

- young \((\tau \sim 10^{3-7} \text{ yr})\), relatively slow \((P \sim 16 \text{ ms to several seconds})\), and strongly magnetized \((B \sim 10^{11-13} \text{ G})\) “classical” pulsars, and
- old \((10^{8-10} \text{ yr})\), fast \((1.55 \text{ to several ms})\), and weakly magnetized \((10^{8-9} \text{ G})\) “millisecond” pulsars.

Confirming that \( \tau \) is related to true age, many of the “youngest” classical pulsars (and none of the millisecond pulsars) are found to be associated with supernova remnants (which disperse after \( \sim 10^5 \text{ yr} \)). On the other hand, the vast majority of pulsars in globular clusters are millisecond pulsars. An additional difference between the two classes is that most millisecond pulsars are found in binary systems (in most cases with old white dwarf companions), whereas the vast majority of classical pulsars are single.

One problem with this general picture is that, in the few cases where it is possible to measure \( \dot{\Omega} \) reliably (all of which are young pulsars), the so-called “braking index,” \( n = \dot{\Omega} \Omega^2 / \dot{\Omega}^2 \), does not agree with the canonical \( n = 3 \) predicted in the dipole spin-down model, but has significantly smaller values, which differ from one pulsar to another (see references in Kaspi et al. 2001). The inclusion of higher multipoles (e.g., quadrupole electromagnetic or gravitational radiation) worsens the problem. This means that the inferred magnetic dipole moment in young pulsars increases with time, possibly connecting young pulsars with moderate inferred dipole moments to the slightly
older “magnetars” with stronger inferred dipoles. Whether this corresponds to a true increase of the star’s dipole moment has not been settled (Blandford 1994).

2.2 X-ray binaries and the “recycling” paradigm

Neutron stars with young, high-mass companions (“high-mass X-ray binaries” or HMXBs) tend to appear as X-ray pulsars, in which the accreted material is presumably channelled by the magnetic field onto the polar caps. In some cases, cyclotron features have been found in the X-ray spectrum, corresponding to magnetic fields $B \sim (1 - 4) \times 10^{12}$ G (e.g., Makishima et al. 1999; Coburn et al. 2002, and references therein). Note that these spectral features (found also in “magnetars” and “thermal emitters”, see below) are the only direct measurements of neutron star magnetic fields, akin to the many measurements of magnetic fields on white dwarfs and other stars. Assuming that these objects differ from (similarly young) classical radio pulsars only by the presence of the nearby companion, this would give evidence that the field of neutron stars is organized on a relatively large scale, so the surface field and the dipole field are of comparable magnitude.

Older, low-mass companions tend to live with non-pulsating neutron stars in “low mass X-ray binaries” (LMXBs), in which the field is presumably not strong enough to channel the accretion flow. In some cases, evidence for a fast rotation (and a non-zero magnetic field) has been found in the form of quasi-periodic oscillations and, more recently, true, highly coherent millisecond pulsations (Wijnands & van der Klis 1998; Galloway et al. 2002; for a review of oscillations in LMXBs, see van der Klis 2000). These give support to the long-standing paradigm that explains the puzzling fast rotation of the evidently old MSPs in terms of “recycling” of these neutron stars in an LMXB (e.g., Bhattacharya & van den Heuvel 1991). The accretion of mass from a Keplerian disk extending almost to the surface of the star would also transfer a large amount of angular momentum, which could spin the star up to the observed, fast rotation. It is speculated that the mass transfer might also lead to the decay of the magnetic field to the low value observed in MSPs. In two “classical” pulsars with relatively fast rotation and relatively weak magnetic field, “incomplete recycling” might have occurred (Lyne et al. 1993, 1996).

2.3 Magnetars

Two intriguing kinds of astronomical objects have in recent years found a likely interpretation as very highly magnetized neutron stars (see Thompson 2000 for a review; Kouveliotou et al. 2003 for a popular account):

AReisenegger1: Workshop SMFNS/ICIMAF - March 20, 2022 4
Soft gamma-ray repeaters (SGRs) are objects which repeatedly emit bursts of gamma-rays, in addition to persistent X-rays. For three of these sources, regular pulses have been observed in the persistent X-ray emission, allowing the measurement of a rotation period and period derivative (Kouveliotou et al. 1998; Hurley et al. 1999).

Anomalous X-ray pulsars (AXPs) show persistent X-ray emission, modulated at a stable, slowly lengthening period. Contrary to the standard, binary X-ray pulsars (§2.3), they show no evidence for a companion star (see Mereghetti & Stella 1995; van Paradijs, Taam, & van den Heuvel 1995; Mereghetti 2000).

Recently, bursts have been detected from two AXPs (Gavriil et al. 2002; Kaspi et al. 2003), making the connection even closer. Differences remain in terms of X-ray spectra, burst frequency, and timing stability, but it is not clear whether there is a dichotomy or just a continuum of properties, with “transition objects” connecting those which are more characteristic of each class. All measured periods lie in the narrow range 5 – 12 s, and objects in both classes have been claimed to be associated with supernova remnants (see Gaensler et al. 2001 for references and a critical discussion), arguing for an interpretation as young neutron stars. Remnant ages are in rough agreement with characteristic ages inferred from spin-down (but see J. Horvath’s presentation in these Proceedings).

The dipole fields inferred from the spin-down rate are $10^{14} - 15$ G (Kouveliotou et al. 1998; Hurley et al. 1999), much larger than in previously known classical pulsars, though radio pulsars with similar inferred dipole fields have recently been found (Camilo et al. 2000; McLaughlin et al. 2003). In addition, features in the X-ray spectra of both SGR 1806-20 (Ibrahim et al. 2002) and AXP 1RXS J170849-400910 (Rea et al. 2003), interpreted as proton cyclotron resonance lines (Ibrahim et al. 2003), indicate $B \sim 10^{15}$ G, in reasonable agreement with the inferred dipole fields.

Perhaps most importantly, the persistent X-ray luminosity of these objects is much larger than their inferred spin-down power. Therefore, unlike the case of radio pulsars, rotation can not be a significant energy source. It has long been suggested that magnetic energy may be the ultimate source of both the bursts and the persistent radiation (Duncan & Thompson 1992; Paczynski 1992; Thompson & Duncan 1995, 1996), but this would still require a total magnetic energy significantly larger than inferred from the dipole field, i.e., a buried and/or disordered magnetic flux. In any case, the strong magnetic field may modify the radiation transport in the surface layers, so that these objects radiate a much larger fraction of their fossil heat in X-rays (as
opposed to neutrinos) than less magnetic neutron stars (van Riper 1988; Heyl & Hernquist 1997a, b). The presence of a light-element atmosphere has a similar, even stronger effect (Chabrier et al. 1997; Heyl & Hernquist 1997a). Our own calculations (M. Riquelme et al., in preparation) show that the Landau quantization of electron and proton energy levels for plausible magnetic fields in these objects are not strong enough to affect the neutrino emissivity (see also Baiko & Yakovlev 1999).

2.4 Thermal emitters

At the opposite end of the range of activity and, perhaps, magnetic field strength are neutron stars detected exclusively through their quiescent, thermal emission in X-rays and (sometimes) optical radiation. Some of these have been found at the centers of shell-type supernova remnants (SNRs; see Pavlov et al. 2002 for a review), whereas other objects are isolated (see Haberl 2003 and references therein). Objects in SNRs have relatively high temperatures \((T \sim 0.2 – 0.7 \text{ keV})\) and small emitting areas (compared to the expected surface area of a neutron star), whereas isolated objects are cooler \((T \sim 0.1 \text{ keV})\) and therefore fainter and only detectable in our immediate Galactic neighborhood. That we nevertheless find several of these objects indicates that they are very abundant, possibly representing the “quiet majority” of neutron stars in our Galaxy.

Beyond these general properties, this “class” of neutron stars appears to be heterogeneous and remains a puzzle. In the source of SNR RCW 103, a substantial, long-term flux variation has been found, with a 6.4 h modulation interpreted in terms of accretion from a faint binary companion (Sanwal et al. 2002a; see Pavlov et al. 2002 for earlier references). Two other SNR sources show periodic variations probably attributable to rotation, with periods \(P \sim 0.2 – 0.4 \text{ s}\) (Hailey & Craig 1995; Zavlin et al. 2000), in the general ballpark of classical pulsars, but quite slow for young pulsars in SNRs. In isolated sources, periods \(P \sim 8 – 23 \text{ s}\) have been found (Haberl et al. 1997; Haberl et al. 1999; Hambaryan et al. 2002; Haberl & Zavlin 2002; Haberl et al. 2003), remarkably similar to those of “magnetars”, whereas \(P\) measurements for one source provide evidence for a somewhat weaker magnetic field (Kaplan et al. 2002; Zane et al. 2002). Some objects show strong spectral features (Sanwal et al. 2002b; Haberl et al. 2003; Bignami et al. 2003), which might be indicating the magnetic field strengths in these objects, although a unique interpretation of the lines (e. g., proton vs. electron cyclotron scattering) is still lacking. On the other hand, the flagship object RX J1856.5-3754 (Walter et al. 1996) has not revealed any evidence for periodicity or spectral features,
in spite of intense observational efforts (Burwitz et al. 2003 and references therein). The same is true for some of its “cousins” (Motch et al. 1999).

Therefore, the origin, evolution, and magnetic field of these objects remain clouded in mystery. I will therefore refrain from discussing them further, only noting that they may well provide crucial input to our general understanding of neutron stars and their magnetic fields in a not too distant future.

3 Origin of the magnetic field

3.1 Stars as \( R - L \) circuits and “flux freezing”

Probably, all stars at all stages of their evolution have some magnetic field, due to electronic currents circulating in their interiors.

Naively, one might expect that such currents should decay over the (microscopic) time scale \( \tau_{\text{coll}} \) in which an average electron transfers its momentum to a more massive particle through a Coulomb (or other) collision. However, any decrease in the current \( I \) implies a decrease of the magnetic flux \( \Phi = cLI \) through the stellar equatorial plane, where \( c \) is the speed of light, \( L \sim R/c \) is the star’s self-inductance, and \( R \) is its radius. [Here and below, I use Gaussian cgs units; see, e. g., Jackson (1975).] According to Lenz’s law, such a flux decline will induce an emf \( \varepsilon = -c^{-1}d\Phi/dt = -LdI/dt \) that tends to keep the current going as prescribed by Ohm’s law, \( \varepsilon = \mathcal{R}I \). The resistance \( \mathcal{R} \) can be estimated in terms of a typical conductivity \( \sigma = n_e e^2 \tau_{\text{coll}}/m_e \) (where \( n_e, -e, \) and \( m_e \) denote the electron concentration, charge, and mass) by \( \mathcal{R} \sim c/(\sigma R) \).

Thus, the star is well described by an electric circuit with an inductance \( L \) and a resistance \( \mathcal{R} \) connected in series, in which the current decays at such a rate that the induced emf is always as strong as required to maintain the instantaneous current against resistive decay. The exponential (“Ohmic”) decay time is thus

\[
\tau_{\text{Ohm}} = \frac{L}{\mathcal{R}} \sim \sigma \left( \frac{R}{c} \right)^2 = n_e r_e R^2 \tau_{\text{coll}},
\]

where \( r_e = e^2/(m_e c^2) \) is the “classical electron radius”. Since the electron concentration is typically high, but specially since stellar radii (even in the very compact neutron stars) are large (e. g., compared to typical laboratory scales), in general \( \tau_{\text{Ohm}} \gg \tau_{\text{coll}} \) by many orders of magnitude. Thus, stellar magnetic fields can persist for very long times, being effectively “frozen” into the plasma.

Essentially the only way of changing the magnetic field configuration is by “deforming the circuit”, i. e., by macroscopic displacements of the plasma,
which can be thought of as carrying the magnetic flux lines along. In particular, when a star changes its radius, it could be expected to preserve its enclosed magnetic flux, changing the magnetic field strength in inverse proportion to its cross-sectional area, $B \propto R^{-2}$.

### 3.2 Kinship

Most, if not all, neutron stars descend from main sequence stars with masses $M_{MS} \gtrsim 8M_\odot$, i.e., O and early B stars, while lower-mass main sequence stars give rise to white dwarfs. A fraction of early-type stars (Ap/Bp stars) have strong, highly organized magnetic fields (see, e.g., the contributions of G. Mathys, J.D. Landstreet, S. Bagnulo, and N. Piskunov in Mathys et al. 2001). The same is true for a fraction of the white dwarfs, which tend to be more massive than their non-magnetic counterparts (Liebert et al. 2003), and therefore plausibly more closely related to neutron stars.

It has long been known that the magnetic fluxes of magnetic white dwarfs and neutron stars are similar (e.g., Ruderman 1972), suggesting a common origin, possibly through flux conservation during the evolution from some progenitor phase. Although much more strongly magnetic objects have been discovered in recent years, the most strongly magnetic main sequence stars (Ap/Bp stars with $R \sim$ few $R_\odot$ and $B \sim 3 \times 10^4$ G; e.g., Landstreet 1992), white dwarfs ($R \sim 10^{-2} R_\odot$, $B \sim 10^9$ G; e.g., Wickramasinghe & Ferrario 2000), and neutron stars (magnetars with $R \sim 10^{-5} R_\odot$ and $B \sim 10^{15}$ G; see §2.2) still turn out to have remarkably similar magnetic fluxes, $\Phi = \pi R^2 B \sim 10^{5.5} R_\odot^2 G$, despite vast differences in size, density, and magnetic field strength. Lower limits on magnetic fluxes can unfortunately not be compared, as the magnetic fields of most non-degenerate stars and white dwarfs are too weak to be detected. Of course, we may also not yet know the most magnetic stars, if they are scarce or manifest themselves phenomenologically in a way we have not yet identified.

### 3.3 “Magnetic strip-tease” hypothesis

Another interesting point is as follows.

Early-type main sequence stars have convective cores and radiative envelopes. The mass of the convective core, $M_{\text{conv}}$, is a strongly increasing function of the total mass of the star, $M_{MS}$ (e.g., Kippenhahn & Weigert 1994). The mass of the eventual compact remnant, $M_{\text{rem}}$, is a much more weakly increasing function of $M_{MS}$ (e.g., Weidemann 2000). As Thompson & Duncan (1993) already realized, the two curves cross at $M_{MS} \approx 3 - 4M_\odot$ (~A0 stars), where $M_{\text{conv}} = M_{\text{rem}} \approx 0.7M_\odot$, a plausible dividing line between
magnetic (massive) and non-magnetic (low-mass) white dwarfs (cf. Liebert et al. 2003).

A possible interpretation is that, during the main sequence phase, the field is generated by a dynamo process in the convective core of most early-type stars. (A coherent, equipartition-strength field filling the convective core of a $4M_\odot$ main sequence star produces approximately the maximum flux estimated above.) If it remains confined to the same region during the later stages of evolution, then low-mass white dwarfs have a magnetized region buried in their interior and covered by an unmagnetized envelope (formerly part of the radiative envelope on the main sequence), whereas massive white dwarfs and neutron stars form exclusively from magnetized material, and therefore have a strong surface field.

Thompson & Duncan (1993) also pointed out that the high metallicity-dependence of the size of the convective core of main sequence stars may account for the fact that the mass of a white dwarf does not uniquely determine whether it is detectably magnetic or not. Of course, some questions would still remain open:

- Why do (only) a small fraction of upper main sequence stars have detectable surface fields?
- Might dynamo-generated fields be transported outward through the stellar envelope, at any time during its evolution from the beginning of the main sequence to the white dwarf stage?

4 Magnetic field evolution

4.1 Observational evidence

Several arguments point toward the possibility of an evolving magnetic field in neutron stars:

1) Generally speaking, young neutron stars appear to have strong magnetic fields $\sim 10^{11-15}$ G (“classical” radio pulsars, “magnetars”, X-ray pulsars), whereas old neutron stars have weak fields $\lesssim 10^9$ G (ms pulsars, low-mass X-ray binaries). If these two groups have an evolutionary connection, their dipole moment must decay. Millisecond pulsars are believed to have been spun up to their fast rotation by accretion from a binary companion, a remnant of which is in most cases still present (e.g., Phinney & Kulkarni 1994). The reduction in the magnetic dipole moment may be a direct or indirect consequence of the accretion process, or just an effect of age.

2) Studies of the pulsar distribution on the $P - \dot{P}$ diagram (analogous to “normal” stellar population synthesis studies on the HR diagram) have
led to the claim that the magnetic torque decays on a time scale comparable to the life span of “classical” pulsars (Gunn & Ostriker 1970). This claim was strengthened by the simultaneous consideration of pulsar space velocities and their spatial distribution perpendicular to the plane of the Galaxy (e.g., Narayan & Ostriker 1990), but was later put in doubt by other authors (e.g., Bhattacharya et al. 1992), whose more careful analysis leads to opposite results.

3) If magnetar emission is powered by magnetic energy (Thompson & Duncan 1996), then the rms magnetic field $\langle \vec{B}^2 \rangle^{1/2}$ must decay.

4) A possible explanation for the “anomalous” braking indices $n < 3$ in young neutron stars is that their magnetic dipole moment increases with time.

In the remainder of this section, I discuss the physical mechanisms that may lead to such an evolution of the magnetic field.

### 4.2 Physics of spontaneous field evolution

The composition of neutron star matter is still highly uncertain (e.g., Lattimer & Prakash 2000), but it seems almost inevitable that it will contain both neutral particles (plausibly neutrons) and charged particles (protons, electrons, and possibly others). All particles are highly degenerate. The relativistic energies of the electrons reduce their cross-section for colliding against protons, and most of the phase space for final states is blocked by the Pauli principle, leading to a high conductivity and consequently to an Ohmic decay time (as in eq. 2) longer than the age of the Universe (Baym, Pethick, & Pines 1969b). Therefore, little diffusion of the magnetic field can occur.

Can the magnetic field move with the fluid matter inside the neutron star, driven by magnetic stresses or buoyancy forces? Not in an obvious way. The matter in an equilibrium neutron star is fully catalyzed, i.e., weak interactions have had time to bring each fluid element into chemical equilibrium, minimizing its free energy by distributing baryon number optimally among different “flavors” of particles. This optimal distribution is density-dependent, giving rise to a mechanically stable composition gradient (Pethick 1992; Reisenegger & Goldreich 1992), regardless of the uncertainties in the composition (Reisenegger 2001b). Even in the simplest and most favorable scenario, in which the matter is mostly neutrons, with a small ($\sim 1\%$) “impurity” of protons and electrons, magnetic stresses of order the “impurity” contribution to the fluid stresses are required to overcome the stabilizing forces, demanding a magnetic field $\gtrsim 10^{17}$ G. At lower field strengths, the magnetic stresses can only build up a small chemical imbalance, and evolve on a timescale determined by the weak interactions that reduce this imbalance.
This leads to the question of whether the magnetic field could move only with the charged particles, leaving the neutral particles behind. This question was addressed in a simple model (Goldreich & Reisenegger 1992) in which protons and electrons move under the effect of electromagnetic forces through a static and uniform neutral background, scattering against each other and against this background. It leads to the following evolution law for the magnetic field,

\[
\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) + \gamma \nabla \times \left( -\frac{\vec{j}}{n_e e} \times \vec{B} \right) - \nabla \times \left( \frac{c}{\sigma} \vec{J} \right), \tag{3}
\]

where \(\vec{v}\) is a weighted average velocity of all charged particles, \(\vec{J} = c\nabla \times \vec{B}/(4\pi)\) is the electric current (due to relative motions of the charged particles), \(n_e\) is the density of protons and electrons, \(e\) is the proton charge, \(c\) is the speed of light, \(\sigma\) is an isotropic conductivity, limited by inter-particle collisions, and \(\gamma\) is a dimensionless factor (\(|\gamma| < 1\)) whose magnitude and sign depends on the relative coupling of protons and electrons to the neutral background. Each term on the right-hand side has a familiar (astro-)physical interpretation, in turn:

1) Advection of the magnetic flux by a flow of charged particles, i.e., ambipolar diffusion, familiar from star formation (e.g., Shu et al. 1987): The bulk flow arises from magnetic stresses or buoyancy forces acting on the charged particles, and is impeded by inter-particle collisions. It can be decomposed into two modes, one curl-free and one divergence-free, the first of which will be choked by the chemical potential gradients it builds up in the charged particles, and can only be effective if weak interactions can reduce these gradients. Since the driving forces are \(\propto B^2\), this term is \(\propto B^3\), becoming much more effective at high field strengths.

2) Advection of the magnetic flux by the electric current, or Hall effect: This is a “passive” or “kinematic” effect, not “driven” by any forces and which by itself does not change the magnetic energy. However, it is nonlinear (\(\propto B^2\)) and could possibly lead to small-scale structures in the magnetic field (Goldreich & Reisenegger 1992), particularly in the solid crust, where ambipolar diffusion cannot occur (see Shalybkov & Urpin 1997; Rheinhardt & Geppert 2002; Hollerbach & Rüdiger 2002 for samples of recent work).

3) The familiar resistive or Ohmic diffusion: Linear in \(B\); it is quite ineffective for a large-scale field, but may play a role in dissipating small-scale structures created by the other (nonlinear) processes.

For the flows disturbing chemical equilibrium (bulk flow and curl-free ambipolar diffusion), the timescale is set by weak interactions, which also
produce the early cooling of neutron stars (through neutrino emission), and which are strongly temperature dependent. Therefore, if $B \lesssim 10^{17}$ G, the only way in which these processes can be effective before the star cools down is to keep it hot by some other mechanism, such as dissipation of magnetic energy (e.g., Thompson & Duncan 1996). However, even this is not guaranteed to work, since most of the dissipated energy will be emitted in the form of neutrinos. If the field is strong enough to create a substantial chemical imbalance ($B \gtrsim 10^{16}$ G), the enhanced neutrino emission may even lead to faster cooling (Reisenegger 1995).

None of these mechanisms appear to be interesting at field strengths and time scales relevant to classical or millisecond radio pulsars, unless the magnetic field is confined to a thin layer in the outer crust of the star, where the conductivity is reduced and a combination of Hall drift and Ohmic dissipation may become effective. In magnetars, the high field strength makes both the Hall drift and the ambipolar diffusion quite fast, and their interaction may lead to interesting dynamics, particularly if the rms interior field is somewhat higher than the inferred dipole field, as required from energetic arguments. The Hall reorganization of the field in the crust may also lead to “Hall fracturing” and therefore to dissipation (Thompson & Duncan 1996), without need of invoking the “Hall cascade” to small scales.

This discussion did not consider the formation of superfluid and superconducting states, which probably occurs early in the evolution of a neutron star (Baym, Pethick, & Pines 1969a), concentrating vorticity and magnetic flux into quantized ropes. At moderate to low temperatures, the neutron star fluid will be much more complicated than in the description given above (Mendell 1998). The effect of these complications on magnetic field evolution are not yet well-understood, though much has been speculated. I will refrain from further discussion of these issues.

4.3 Induced field evolution

External agents may also change the magnetic field of a neutron star:

1) The strong thermal gradient in a cooling protoneutron star can overcome the stratifying effect of the chemical gradient, leading to convection. At the same time, the star has not had time to transport angular momentum and will be differentially rotating. This combination naturally acts as a dynamo, which is an alternative to the “fossil flux” idea (discussed above) to give rise to the magnetic field in neutron stars (Thompson & Duncan 1993). It has the merit of having led to the prediction of the existence of “magnetars” with field strengths $B \sim 10^{15}$ G.
2) The thermal gradient persists for a much longer time in the outer crust of the star, where it may act as a battery, again giving rise to a substantial field (Urpin & Yakovlev 1980; Blandford, Applegate, & Hernquist 1983; Wiebicke & Geppert 1996 and references therein). This may in principle explain an increasing field in a young pulsar, as suggested by the braking index measurements.

3) Accretion from a binary companion is a popular way of decreasing the magnetic dipole moment, although there is no agreement on the physics involved. Perhaps the most interesting candidate process is the burial (“diamagnetic screening”) of the magnetic flux by the accreted, highly conducting plasma (Bisnovatyi-Kogan & Komberg 1975; Romani 1993; Cumming et al. 2001). No full models of this process have been produced so far, and three-dimensional simulations will eventually be needed to make sure that all possible instabilities have been taken into account. If effective, this process still begs the question of why after its completion a minute, but fairly constant fraction of the initial dipole moment is left or regenerated to be detectable in ms pulsars. (The observed ms pulsars do not appear to be the “tip of the iceberg” of a distribution extending down to much lower fields, since the death rate of their LMXB progenitors already can barely account for the detectable ms pulsars; e.g., Phinney & Kulkarni 1994; White & Ghosh 1998.)

5 Conclusions

Research on magnetic fields in neutron stars is undoubtedly in one of its most interesting moments. Little is known about the strength, structure, origin, and evolution of the field, but there seems to be little doubt that it plays a fundamental role in determining the increasingly rich phenomenology of these objects. The coming years will most probably improve our understanding of the “magnetars” and “thermal emitters”, hopefully contributing to a coherent picture of how these new subclasses fit together with the more traditional radio pulsar and X-ray binary groups and with other kinds of stars. Along the way, we may expect surprises in many different wavelength bands and much exciting physics.

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