Studies of light neutron-excess systems from bound states to continuum

Makoto Ito\textsuperscript{1,2} and Hideaki Otsu\textsuperscript{2}

\textsuperscript{1} Department of Pure and Applied physics, Kansai University, 3-3-35 Yamate-cho, Suita, Osaka 564-8680, Japan
\textsuperscript{2} RIKEN Nishina Center for Accelerator-base Science, RIKEN, Wako, 351-0198, Japan

E-mail: itomk@kansai-u.ac.jp

Abstract. The generalized two-centre cluster model (GTCM), which can handle various single-particle configurations in general two-centre systems, is applied to the light neutron-rich system, $^{12}\text{Be} = \alpha + \alpha + 4N$. We discuss the change of the neutrons' configuration around two $\alpha$ cores as the excitation energy varies. We show that the excess neutrons form various chemical-bonding-like configurations around the two $\alpha$ cores in the unbound region above the $\alpha$-decay threshold. The possibility of $\alpha$-cluster formation in a heavier system with neutron excess, $^{28}\text{Ne}$, is also discussed.

1. Introduction

In the last two decades, developments of experiments with secondary radioactive-ion beams have extensively advanced the studies of light neutron-rich nuclei. In particular, much effort has been devoted to the investigation of molecular structures in Be isotopes. The Be isotopes can be considered as typical examples of two-centre superdeformed systems, which are built on the $\alpha + \alpha$ rotor of $^8\text{Be}$. Theoretically, molecular orbitals (MO), in particular, the $\pi^-$ and $\sigma^+$ orbitals, associated with the covalent bonding in atomic molecules, have been successful in explaining the low-lying states of these isotopes [1].

The MO model can describe many kinds of characteristic properties of these isotopes, but it is essentially limited to the analysis of low-lying bound states, and the theory of the highly excited states above the particle-decay threshold is still an open area. In contrast to the situation of theoretical studies, recent experiments on $^{12}\text{Be}$ revealed the existence of many resonant states [2, 3, 4], which strongly decay into $^6\text{He}_{g.s.} + ^6\text{He}_{g.s.}$ and $\alpha + ^8\text{He}_{g.s.}$. Similar resonances, decaying to He isotopes, have also been observed in other Be isotopes, such as $^{10}\text{Be} = \alpha + ^6\text{He}$ and $^{14}\text{Be} = ^6\text{He} + ^8\text{He}$.

In the present study, we investigate the structural changes appearing in an unbound region of $^{12}\text{Be}$. In order to investigate the continuum states above the particle-decay threshold, the intrinsic structures and their coupling to the scattering states should be treated in a unified manner. For this purpose, we introduce the generalized two-centre cluster model (GTCM) [6]. In this model, the covalent MO configurations can be smoothly connected to the atomic or ionic configurations, in which valence neutrons are localized around one of the $\alpha$ clusters. Furthermore, it becomes possible to describe both the formation of the covalent MO structures and reaction processes. In this report, we investigate the chemical-bonding-like structures based...
on the two α cores in $^{12}$Be, which appear in the continuum energy region. As a natural extension of these considerations, we also discuss the formation of an α cluster in a heavier system that has neutron excess, $^{28}$Ne. In the study of $^{28}$Ne, we assume a simple α+$^{24}$O cluster model and estimate the branching ratio of the α and neutron emission. The possibility of the observation of the α+$^{24}$O structure is discussed.

2. Theoretical framework

The detailed explanation of GTCM has already been published in Ref. [6], and here we only show its formulation briefly. In GTCM, the total wave function of $^{12}$Be is given by a superposition of the basis states $\{\Phi_m^{J^pK}(S)\}$, where

$$
\Phi_m^{J^pK}(S) = P_K^{J^p} A \left\{ \psi_L(\alpha)\psi_R(\alpha) \prod_{j=1}^{4} \varphi_j(m_j) \right\}_S.
$$

(1)

The α-cluster wave function $\psi_n(\alpha)$ ($n=L, R$) is chosen to be the (0s)$^4$ configuration of the harmonic oscillator (HO) centred on the left (L) or right (R) side, with the relative distance parameter $S$ [5]. The single-particle wave functions for the four valence neutrons localized around one of the α clusters is given by a 0p atomic orbital (AO) $\varphi(p_k, i, \tau)$, where the subscript $k$ of $p_k$ may be $x$, $y$ or $z$, the symbol $i$ denotes $L$ or $R$, and $\tau$ ($=\uparrow$ or $\downarrow$) stands for the spin state. The four symbols $m = (m_1, m_2, m_3, m_4)$ comprise the labels $(p_k, i, \tau)$ of the four AO’s. The intrinsic basis functions with full antisymmetrization expressed by $A$ are projected to the eigenstate of the total spin $J$, its intrinsic angular-momentum projection $K$ and the total parity $\pi$ by the projection operator $P_K^{J^\pi}$. In the present study, we discuss the level structure of the $J^\pi=0^+$ states. Thus, only the case of $J=K=0$ is considered. We include all possible AO basis elements which have axial symmetric ($K=0$) configurations.

The total wave function is finally given by taking a superposition over $S$ and $m$

$$
\Psi^{0^+}\equiv\int dS \sum_m C_m^\nu(S) \Phi_m^{0^+}(S).
$$

(2)

The coefficients for the $0^+_\nu$ state ($\nu$th $0^+$), $C_m^\nu(S)$, are determined by solving an eigenvalue problem. As for the nucleon-nucleon (NN) interaction, we use the Volkov No. 2 and the G3RS force for the central and spin-orbit part, respectively. The parameters of the NN interactions and the size of the HO are optimized so as to reproduce the $^4$He+$^4$He threshold energies as much as possible [6].

3. Results

3.1. Results for $^{12}$Be

We identified two bound and four resonant $0^+$ states below and above the $\alpha+^8$He$_{g.s.}$ threshold, respectively. The ground state has a well developed MO configuration of $(\pi_{3/2}^-)^2(\sigma_{1/2}^+)^2$. Its characteristic degrees of freedom are the intercluster relative motion and the single-particle motions of the four valence neutrons. By analysing the properties of the wave functions of the excited states, we find that all the excited states can be characterized in terms of the excitations in the characteristic modes of motion of the ground state. We summarize the identified levels and their intrinsic properties in Fig. 1.

(1) Molecular orbital (MO) excitation mode. The $0^+_\uparrow$ state has the MO configuration, $(\pi_{3/2}^-)^2(\pi_{1/2}^-)^2$, while the $0^+_\downarrow$ state has a hybrid structure of MO and AO configurations. Namely, two of the valence neutrons are localized around individual α’s as $^5$He+$^5$He, which is an AO
structure, and the remaining two neutrons occupy the $\sigma^+$ orbital and rotate around both clusters, which is of MO character. This state can be generated by the excitation of two $\pi^{-}$ MO's into AO's around the two $\alpha$ cores.

(2) Cluster excitation mode. The $0^+_3$ and $0^+_6$ states have large components of the ionic structures of $\alpha$$+^8\text{He}_{g.s.}$ and $^5\text{He}_{g.s.}$$+^7\text{He}_{g.s.}$ type, respectively. These two states are the excitation modes of the $\alpha$$-\alpha$ relative motion from the $0^+_1$ and $0^+_2$ states, respectively.

(3) Double excitation mode. The $0^+_4$ state has the atomic configuration, $^6\text{He}_{g.s.}$$+^6\text{He}_{g.s.}$. It involves a simultaneous excitation of the relative motion of the two $\alpha$ cores and the MO motions of the excess neutrons.

The results in Fig. 1 show that the cluster structures can change from level to level, and various cluster configurations coexist within a small energy interval of about 1 \sim 2 \text{ MeV} in $^{12}\text{Be}$. All the excited states contain a large fraction of the (isoscalar) monopole excitation of the ground ($\nu=1$) state [7] expressed by the overlap

\[
M(IS) = \left\langle \Psi^\nu \left| \sum_{i=1}^{12} r_i^2 \right| \Psi^1 \right\rangle \equiv \left\langle 0^+_\nu \left| \sum_{i=1}^{12} r_i^2 \right| 0^+_1 \right\rangle. \tag{3}
\]

Here, $r_i$ denotes the radial coordinate for the $i$th nucleon measured from the centre of mass of $^{12}\text{Be}$. To assess the magnitude of the monopole strength, it should be compared to the single-particle (S.P.) strength, which is defined as the matrix element of a one-node excitation in a simple HO shell model [8]. All the strengths have been found to be comparable with or 2 \sim 3.5 times larger than the S.P. strength. Among the various $0^+$ states shown in Fig. 1, strong enhancement occurs for the transition to the $0^+_3$ state ($\alpha$$+^5\text{He}_{g.s.}$), which is a direct cluster excitation mode from the ground state. Thus the monopole strength can be used as a probe of a direct cluster excitation from the ground state.

The finding of the monopole strength to be comparable with the S.P. strength is consistent with the analysis in Ref. [8], where the enhancement of the low-lying monopole strength by cluster formation is discussed. It should be noted that in the present system all the monopole strength corresponding to various cluster structures appears at $E_x \leq 20 \text{ MeV}$. That is in marked contrast with a naive mean-field picture, in which a monopole excitation involves a $2\hbar\omega$ ($\sim 35 \text{ MeV}$) jump. This observation may be generalized. In a system with a considerable neutron excess, there are almost degenerate monopole states at much lower energies than expected in a naive shell-model picture, and the transition strengths to these states are comparable with the S.P. strength.
3.2. Discussion of $^{28}$Ne
We also discuss $\alpha$-cluster formation in the heavier system $^{28}$Ne. This nucleus has almost the same $N/Z$ ratio as $^{12}$Be and hence the formation of the ionic $\alpha+^{24}$O state is analogous to the $\alpha+^{8}$He structure appearing in $^{12}$Be. According to the prediction of the threshold rule [9], $\alpha$-cluster formation is expected at excitation energies comparable with the dissociation energy into $\alpha+^{24}$O ($E_x \sim 10$ MeV). In such a heavier system, however, there are two effects which will disturb $\alpha$-cluster formation. First, the level density of $^{28}$Ne is larger by about 2 \sim 4 orders of the magnitude than that of $^{12}$Be. Second, the one-neutron emission channel opens at a lower excitation energy than the $\alpha$ threshold ($E_x \sim 4$ MeV). These two factors strongly suggest that an $\alpha$-cluster state couples to many compound nucleus states, which finally emit a neutron. Therefore, we should consider the competition of $\alpha$ decay and decays into the compound states from the $\alpha$-cluster states.

In the present study, we discuss the decay widths for $\alpha$ and neutron emission and their branching ratios. The escape width of the $\alpha$ decay from the $\alpha$ cluster state, $\Gamma^\alpha_{n\uparrow}$, can be estimated from the $\alpha+^{24}$O cluster wave function. Here we applied the Orthogonalized Condition Model (OCM) to $\alpha+^{24}$O. In this calculation, a simple configuration of $(\lambda, \mu) = (0, 2)$ in the SU(3) representation was assumed for the intrinsic structure of $^{24}$O, and we took into account the excitation of $^{24}$O to the $2^+$ state in a coupled-channels framework. We employed a global $\alpha$ potential for the interaction between $\alpha$ and $^{24}$O, and its derivative is used for the form factor of the coupling potential. We found that the $\alpha$-cluster state appears around the corresponding threshold. From the amplitude of the wave function at the surface and from the penetration factor, we calculated $\Gamma^\alpha_\uparrow$.

As for the width of the decay of the cluster state (CL) of excitation energy $E_x$ into the compound state (CN), $\Gamma_{CN}^\downarrow$, and for the neutron-emission width from CN, $\Gamma^\uparrow_n$, we employ a simple perturbation formula

$$\Gamma_{CN}^\downarrow = 2\pi <CN|V|CL>^2_{\alpha\nu} \rho_{28}(E_x)$$
$$\Gamma^\uparrow_n = \Gamma_{CN}^\downarrow \rho_{27}(E_x)$$

(4)

where $\rho_A(E)$ is the level density of nucleus $A$ at excitation energy $E$. The factor $P_n$ represents the one-neutron emission probability, which is estimated from the level densities of $^{27,28}$Ne ($\rho_{27,28}$) and of the free neutron ($\rho_n$). In Eq. (4), the condition of energy conservation $E_x = E_{27} + \epsilon_n + E_{27+n}^{27}$ is observed. $[E_{27}, \epsilon_n$ and $E_{27+n}^{27}$ denote the internal energy of $^{27}$Ne, the emitted neutron’s kinetic energy and the $^{27}$Ne+n threshold, respectively.] In this estimate, we use the density given in Ref. [10] for $^{27,28}$Ne, while a simple level density calculated from a plane wave is applied to the neutron. $<CN|V|CL>_{\alpha\nu}$ represents the average coupling matrix element of the compound state (CN) and the cluster state (CL). Since there is no theoretical analysis for this average matrix element, we assess its order of the magnitude from the experimental spectra of the $\alpha$ inelastic scattering on $^{12}$Be [4] and $^{24}$Mg [11]. The estimate for the matrix element is in the range of $10^{-3} \sim 10^{-3}$ MeV$^2$.

Table 1. The decay width and branching ratio of the $\alpha$ and neutron emission in the $\alpha+^{24}$O system. The energy and $\Gamma_{\text{tot}}$ are written in units of MeV. See text for details.

| $E_x$ | $J^\pi$ | $\Gamma_{\text{tot}}$ | $P(\alpha)$ | $P(n)$ |
|------|--------|----------------|-------------|--------|
| 13.4 | 0$^+$  | 1.45          | 0.69        | 0.10   |
| 15.9 | 2$^+$  | 0.86          | 0.35        | 0.22   |
In Table 1, the estimated branching ratios are shown. The total decay width is $\Gamma_{\text{tot}} = \Gamma_{\alpha}^\uparrow + \Gamma_{\text{CN}}$, and the $\alpha$ and neutron branching ratios are defined as $P(\alpha) = \Gamma_{\alpha}^\uparrow / \Gamma_{\text{tot}}$ and $P(n) = \Gamma_{n}^\uparrow / \Gamma_{\text{tot}}$. The total decay width of the $J^\pi=0^+$ state is about 1.5 MeV, and the dominant decay mode is the $\alpha$ decay, with a branching ratio of $\sim 0.7$. The ratio is suppressed in the $J^\pi=2^+$ state, but is still substantial ($\sim 0.4$), while the one-neutron emission probability is smaller ($P(n) \sim 0.2$).

These estimations suggest that $\alpha$-decay is dominant even if the one-neutron emission channel opens at low excitation energy. Since in the heavier neutron-rich systems, the one-neutron emission channel always opens at lower energy than the $\alpha$-decay threshold, the $\alpha$ and neutron decays generally compete. Therefore, in the study of $\alpha$-cluster states, coincidence measurements for these two particles are essential. The new facility SAMURAI at RIKEN is appropriate for such measurements, and the identification of $\alpha$-cluster states in $^{28}$Ne will be a breakthrough in the study of $\alpha$-cluster formation under the condition of strong neutron emission.

4. Summary and discussion

In summary, we have studied the exotic structure of the excited states of $^{12}$Be as unbound systems by applying GTCM. Due to the excitation of the cluster and single-particle degrees of freedom, various structures appear at different excitation energies. We found that, above the particle decay threshold, there are several excited states within a small energy range. Each excited state can be characterized by a particular combination of ‘atomic’ and ‘molecular’ excitations of the excess neutrons. All the energy levels which appear close to one another have monopole strengths comparable with or larger than the single-particle strength. As a consequence, in light neutron-rich systems, strong monopole transitions can be observed in the low-energy region. This feature is completely consistent with recent observations [4].

We have also extended our study to the heavier system of $^{28}$Ne and discussed the formation of the ionic $\alpha+^{24}$O configuration. In such systems, there is a possibility that the formation of $\alpha$-cluster resonances may be strongly disturbed by the large level density and the one-neutron emission opening at a much lower energy than the $\alpha$ emission. However, according to the present estimate of the branching ratios, an $\alpha+^{24}$O state generated around the $\alpha$ threshold is expected to have a large $\alpha$-decay width. This is because the estimated coupling matrix element between the $\alpha$-cluster state and compound states is small, and the large enhancement of the level density is compensated for by this suppression of the coupling. In experimental studies of cluster structures in the heavier systems with neutron excess, coincidence measurements for $\alpha$-particle and neutron emissions are strongly desirable. Such measurements are possible with the new facility SAMURAI at RIKEN.

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