Adequate soliton solutions to the space–time fractional telegraph equation and modified third-order KdV equation through a reliable technique

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Abstract
The space–time fractional Telegraph equation and the space–time fractional modified third-order KdV equations are significant molding equations in theoric physics, mathematical physics, plasma physics also other fields of nonlinear sciences. The space time-fractional telegraph equation, which appears in the investigation of an electrical communication line and includes voltage in addition to current which is dependent on distance and time, is also applied to communication lines of wholly frequencies, together with direct current, as well as high-frequency conductors, audio frequency (such as telephone lines), and low frequency (for example cable television) used in the extension of pressure waves into the lessons of pulsatory blood movement among arteries also the one-dimensional haphazard movement of bugs towards an obstacle. The presence of chain rule and the derivative of composite functions allows the nonlinear fractional differential equations to translate into the ordinary differential equation employing wave alteration. To explore such categories of resolutions, the extended tanh-method is accomplished via Conformable derivatives. A power sequence in tanh was originally used as an ansatz to provide analytical solutions of the traveling wave type of certain nonlinear evolution equations. The outcomes achieved in this study are king type, single soliton, double soliton, multiple solitons, bell shape, and other sorts of forms and we demonstrated that these solutions were validated through the Maple software. The proposed approach for solving nonlinear fractional partial differential equations has been developed to be operative, unpretentious, quick, and reliable to usage.

Keywords Nonlinear fractional partial differential equation · Conformable derivative · Traveling wave solution · Solitary wave solution · The extended tanh-function method

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1 Introduction

A huge scientific literature by numerous researchers from diverse fields of study has lately been accessible that agreement with dynamical classifications, engineering, in addition, mathematical physics well-defined by fractional differential equations. Ordinary differential equations are comprehensive to arbitrary (non-integer) order by fractional differential equations. Power law memory kernels capture nonlocal spatial besides temporal interactions in fractional differential equations. For the reason that fractional differential equations have so many dispensations in engineering as well as science, scientific learning on them has detonated. Several researchers (Du et al. 2013; Caputo and Fabrizio 2015; Zheng 2013; Čermák and Kisela 2015) have looked at fractional differential equations and used innumerable explanations approaches. Fractional derivatives in addition to integrals were once regarded to be the domain of theoretic mathematics. However, many investigations in the preceding insufficient eras have recommended that fractional phenomenon is connected not only in mathematics but also in applied mathematics to engineering disciplines for example fluid mechanics, vascular mechanics, optical fibers, geochemistry, plasma physics, also to suchlike a great extent. The benefit of fractional derivatives in molding mechanical and electrical belongings of real materials turn into ostensible owing to the disregarding of properties in standard integer-order models. Fractional derivatives oblige as a foundation for telling the appearances of mathematics. Numerous outstanding approaches for resolving these schemes have been discovered in recent years in the utmost convenient publications on nonlinear fractional partial differential equations (NLFPDEs) for example the modified sub equation approach (Saliou et al. 2021), the simple integration method (Zafar et al. 2021), the topological invariance approach (Abdel-Gawad et al. 2016), the first integral approach (Feng 2002; Younis 2013), the Adams–Bashforth-Moulton approach (Kumar et al. 2021), and the three-leaf function method (Zafar et al. 2021), which propagate NLFPDEs. It is well represented by numerous key singularities in non-Brownian motion, signal processing, systems identification, control problems, viscoelastic materials, polymers, also supplementary areas of science (Varieschi 2018). Many authoritative approaches meant for gaining numerical and analytical results of NLFPDEs have been established and developed, including the multi-auxiliary equation method (Osman and Abdel-Gawad 2015), finite element method (Huang et al. 2008), the trial function method (Chen et al. 2021), the modified Kudryashov method (Savaissou et al. 2020), differential transform method (Odibat and Momani 2008), the collocation method (Srinivasa and Rezazadeh xxxx), the extended unified method (Abdel-Gawad and Osman 2015), the \((G'/G)\)-expansion method (Almusawa et al. 2021), homotopy perturbation method (Jafari and Seifi 2009), the \((m + G'/G)\)-expansion method (Ismael et al. 2021), the double\((G'/G, 1/G)\)-expansion method (Khatun et al. 2021; Uddin et al. 2021; Uddin et al. 2021; Khatun et al. 2021), the modified\((G'/G^2)\)-expansion method (Siddique et al. 2021), the extended tanh-function method (Barman et al. 2021), and the modified Khater method (Park et al. 2020). The question of how to expand existing approaches to tackle other NLFPDEs remains an intriguing and significant research topic. Numerous NLFPDEs have been inspected and explained thanks to the efforts of many researchers, including the impulsive fractional differential equations (Mophou 2010), space–time fractional advection–dispersion equation (Jiang and Lin 2010; Pandey et al. 2011), fractional generalized Burgers’ fluid (Xue et al. 2008), and fractional heat and mass-transport equation (Molliq et al. 2009), \(\exp - (\varphi(\xi))\) expansion method (Arshed et al. 2019), etc.
The Telegraph equation appears in the learning of electrical signal circulation of pulsatory blood movement among arteries also a one-dimensional haphazard movement of bugs towards an obstacle. The telegraph equation has been proven to be a greater example because of narrating some fluid flow difficulties connecting interruptions when compared to the heat equation (Simulations 2000). Several academics have solved the standard telegraph equation as well as the space or time-fractional telegraph equations. (Biazar et al. 2009) have been applied the variational iteration method to attain an estimated explanation intended for the Telegraph equation. (Yıldırım 2010) used homotopy perturbation method to gain systematic and estimated resolutions of the space-time fractional Telegraph equations. (Avazzadeh and Machado 2019) presents the transcendental Bernstein series (TBS) as a generalization of the classical Bernstein polynomials for solving the variable-order space-time fractional Telegraph equation (V-STFTE). The space-time fractional modified third-order KdV equation recites the circulation process of surface water waves. This equation seems within the electric circuit and multi constituent plasms and fluid mechanics, signal processing, hydrology, viscoelasticity, and so on. (Al-shawba and Abdullah 2018) proposed methods known as the G/G-expansion method and the fractional complex transform are successfully employed to obtain the exact solutions of fractional modified third-order KdV equations. (Shah et al. 2020) applied Adomian decomposition to display the efficiency of the technic used for together fractional and integer order the space of time-fractional modified third-order KdV equation. (Sepehrian and Shamohammadi 2018) applied a radial basis function process for the numerical resolution of time-fractional modified third-order KdV equation by radial basis functions and so many researchers used various types of methods to acquire the exact solution of space time-fractional modified third-order KdV equation. Perceiving the literature, it is shown that the introduced models have not been studied by the recently developed extended tanh-function scheme. Thus the objective of the article is to established further general and some advanced closed form solitary wave solutions to the acclaimed NLPDEs by means of the suggested approach. The result obtained in this article is associated with the existing results accessible in the literature and shows that the achieved solutions are archetypal and further inclusive. It is to be hoped that the reputation of the solutions may obtain developed in the literature.

The following is how the residual of the article is designed: In segment 2, we go through numerous definitions and characteristics of conformable derivatives. Then show how to discover accurate traveling wave solutions to nonlinear fractional differential equations in segment 3. In segment 4, describe the new closed-form wave solution for the general space–time fractional modified third-order KdV equation and space–time fractional Telegraph equation. In segment 5, the findings and disputes are assessed through visual delegation and physical enlargement of the resolution. In segment 6, obtained results are compared with the existing result, followed by a discussion of the conclusions.

2 Meaning and preamble

Let, $f: [0, \infty) \to \mathbb{R}$, be a function. $f$ be $\alpha$-order “conformable derivative”’ is demarcated as (Eslami and Rezazadeh 2016):

$$K_{\alpha}(f)(t) = \lim_{\varepsilon \to 0} \frac{f(t + \varepsilon t^{1-\alpha}) - f(t)}{\varepsilon},$$

(1)
For every $t > 0$, $\alpha \in (0, 1)$. If $f$ be $\alpha$-differentiable in nearly $(0, a), a > 0$ in addition
\[
\lim_{t \to 0^+} f^{(\alpha)}(t) \text{ be real, now } f^{(\alpha)}(0) = \lim_{t \to 0^+} f^{(\alpha)}(t). \]
The theorems that survey high spot a limited axiom that are contented conformable derivatives.

**Theorem 1:** Suppose that $\alpha \in (0, 1]$ and at a point $t > 0$ $f, g$ be $\alpha$- differentiable. Hence.

- $K_\alpha(xf + yg) = xK_\alpha(f) + yK_\alpha(g)$, for all $x, y \in \mathbb{R}$.
- $K_\alpha(f^z) = h^z K_{\alpha - a}$, for all $z \in \mathbb{R}$.
- $K_\alpha(u) = 0$, for all constant function $f(t) = u$.
- $K_\alpha(fg) = fK_\alpha(g) + g K_\alpha(f)$.
- $K_\alpha \left( \frac{f}{g} \right) = \frac{gK_\alpha(f) - fK_\alpha(g)}{g^\alpha}$.
- Additionally, in case $f$ is differentiable, then $K_T_\alpha(f)(t) = t^{1-a} \frac{df}{dt}$.

Some kinds of properties like the chain law, Gronwall’s inequality, integration procedures, the Laplace transform, Tailor series expansion, and the exponential function in terms of the conformable derivative (Khalil et al. 2014).

**Theorem 2**
In conformable differentiable (Eslami and Rezazadeh 2016), $f$ be a $\alpha$- differentiable function and also presume $g$ is also differentiable and described in assortment of $f$, so that.

\[
M_\alpha(f \circ g)(t) = t^{1-a}g'(t)f_\alpha(t). \tag{2}
\]

## 3 Vital evidences in addition the enactment of the process

The extended tanh function method for obtaining multiple exact solutions for fractional nonlinear evolution equations (FNLEEs) is described here which was summarized by (Wazwaz 2007). To reveal the solution namely a polynomial in hyperbolic functions is the key idea behind the proposed methodology, and solve the variable coefficient PDE first solving the method which is including first-order ODEs also algebraic equations. To begin, we detain an NLEEs related with a function $u = u(x, t)$ as follows:

\[
R\left(u, \frac{\partial u}{\partial x}, \frac{\partial^2 u}{\partial x^2}, \frac{\partial^3 u}{\partial x^3}, \ldots \right) = 0, \quad 0 < \alpha \leq 1, 0 < \beta \leq 1 \tag{3}
\]

where $u$ is an unidentified function with spatial and temporal derivatives $x$ also $t$, besides $R$ is a polynomial of $u(x, t)$ in addition its derivatives in which the maximum order of derivatives and nonlinear terms of the maximum order are interrelated. Let the conversion of waves.

\[
\xi = k \frac{\partial^\beta}{\partial x^\beta} + c \frac{t^\alpha}{\alpha}, \quad u(x, t) = u(\xi), \tag{4}
\]

here $c$ as well as $k$ are random nonzero constants.

Put on this wave transformation in 3, it can be rewritten as:

\[
R\left(u, u', u'', u''', \ldots \right) = 0, \tag{5}
\]

where the superscripts require the ordinary derivative of $u$.
Phase 1 consider a formal solution of ODE in the subsequent structure
\[ u(\xi) = \sum_{i=0}^{n} a_i Y_i + \sum_{i=1}^{n} b_i Y^{-i}, \quad (6) \]
for which
\[ Y = \tanh(\mu \xi), \quad (7) \]
where \( \mu \) can be any arbitrary value.

Phase 2 Finding the homogeneous equilibrium among the highest order nonlinear terms and their derivatives in Eq. (5) determine the positive constant \( \eta \).

Phase 3 By substituting solution 6 and 7 into Eq. (5) with the value of \( \eta \) gotten in Phase 2, polynomials within \( Y \) are obtained. Setting all of the coefficients of the resulting polynomials to zero yields a set of algebraic equations \( a_i \) along with \( b_i \)s. Solve these equations \( a_i \) along with \( b_i \)s using symbolic computation tools like Maple.

Phase 4 By inserting the values from Phase 3 into Eq. (6) along with Eq. (7) and (3) we create closed feature moving wave solutions of the nonlinear evolution Eq. (6).

4 Investigation of the solutions

In this segment, solitary wave explanations specifically the space time-fractional modified third-order KdV equation and the space time-fractional Telegraph equation by dint of the extended tanh function method designated in 'conformable derivative.

4.1 The space-time fractional modified third order KdV equation

The space–time fractional modified third order KdV equation is
\[ D_\alpha^\nu u(x, t) + pu^2(x, t)D_\xi^\beta u(x, t) + qD_\xi^3 u(x, t) = 0, \quad 0 < \alpha, \beta \leq 1 \quad (8) \]
where \( p, q \) is a nonzero constant.

Let us consider the complex travelling waves transformation as
\[ \zeta = \omega x^\mu - \lambda t^\mu, \quad u(x, t) = u(\zeta), \quad (9) \]
where \( \omega, \lambda \) is the traveling wave’s speed. The Eq. (8) is shortened to the following integer order ordinary differential equation (ODE) through the transformation (4.1.2):
\[ -\lambda u'' + opu^2 u' + \omega^3 qu''' = 0. \quad (10) \]
Integrating Eq. (10) with zero constant, we achieve
\[ 3\omega^3 qu'' - 3\lambda u + opu^3 = 0. \quad (11) \]
The balancing number is found 1 by balancing the highest order derivative term with the highest power nonlinear term. The Eq. (6) is then resolved as
\[ u(\zeta) = a_0 + a_1 Y + b_1 Y^{-1}. \quad (12) \]
Take the place of 11 into 12 along with 7, in $Y$, the left side converts into a polynomial. When each of the polynomial’s coefficients is set to zero, a set of algebraic equations emerges (intended used for plainness, we try to slip over them to exposition) for $a_0$, $a_1$, $b_1$, $\omega$ and $\lambda$. The subsequent outcomes are attained by put on computer algebra, such as Maple, to resolve this over determined series of equations:

$$\omega = \frac{1}{6} \frac{\sqrt{-6pq}}{q \mu} b_1, \quad \lambda = \frac{\sqrt{-6pq}}{18} b_1^3, \quad a_0 = 0, \quad a_1 = 0 \text{ and } b_1 = b_1.$$

**Case 1:**

The principles of the constraints supplied into case 1 create explicit solution in terms of coth functions.

$$u_1(x, t) = \coth \left( 2 \sqrt{6x^{1/4}} - \frac{2 \sqrt{6t^{1/4}}}{9} \right). \tag{13}$$

We can rewrite this equation as follows:

$$u_2(x, t) = \sqrt{1 + \coth \left( 2 \sqrt{6x^{1/4}} - \frac{2 \sqrt{6t^{1/4}}}{9} \right)^2}. \tag{14}$$

**Case 2:**

$$\omega = \frac{1}{6} \frac{\sqrt{-6pq}}{q \mu} a_1, \quad \lambda = \frac{\sqrt{-6pq}}{18} b_1^3, \quad a_0 = 0, \quad a_1 = a_1 \text{ and } b_1 = 0.$$

In terms of tanh functions, the ideals of the parameters stated in assortment 2 form an explicit result.

$$u_3(x, t) = \tanh \left( \frac{4x^{1/4}}{\sqrt{6}} - \frac{4t^{1/4}}{3 \sqrt{6}} \right). \tag{15}$$

This equation can be recreated using the following formula:

$$u_4(x, t) = \sqrt{1 - \operatorname{sech} \left( \frac{4x^{1/4}}{\sqrt{6}} - \frac{4t^{1/4}}{3 \sqrt{6}} \right)^2}. \tag{16}$$

**Case 3:**

$$\omega = \frac{1}{6} \frac{\sqrt{-6pq}}{q \mu} b_1, \quad \lambda = \frac{\sqrt{-6pq}}{9} \frac{b_1^3}{\mu}, \quad a_0 = 0, \quad a_1 = b_1 \text{ and } b_1 = b_1.$$

The principles of the parameters supplied in assortment 3 create an explicit solution in terms of the tanh and coth function.

$$u_5(x, t) = \tanh \left( \frac{4x^{1/4}}{\sqrt{6}} - \frac{8t^{1/4}}{3 \sqrt{6}} \right) + \coth \left( \frac{4x^{1/4}}{\sqrt{6}} - \frac{8t^{1/4}}{3 \sqrt{6}} \right). \tag{17}$$

The following formula can be used to reproduce this equation.
\[ u_6(x, t) = \sqrt{1 - \text{sech} \left( \frac{4x^{1/4}}{\sqrt{6}} - \frac{8t^{1/4}}{3\sqrt{6}} \right)^2} + \sqrt{1 + \text{cosech} \left( \frac{4x^{1/4}}{\sqrt{6}} - \frac{8t^{1/4}}{3\sqrt{6}} \right)^2}. \]  

**Case 4:**

\[ \omega = \frac{1}{6} \frac{\sqrt{-6pq\nu}}{\alpha}, \quad \lambda = \frac{1}{9} \frac{\sqrt{-6pq\nu}}{\alpha} b_1^3, \quad a_0 = 0, a_1 = -b_1 \text{ and } b_1 = b_1. \]

In terms of the \( \tanh \) and \( \coth \) functions, the values of the parameters in assortment 4 establish an explicit solution.

\[ u_7(x, t) = -\tanh \left( \frac{4x^{1/4}}{\sqrt{6}} - \frac{4t^{1/4}}{9} \right) + \coth \left( \frac{4x^{1/4}}{\sqrt{6}} - \frac{4t^{1/4}}{9} \right). \]  

The following formula can be used to reproduce this equation.

\[ u_8(x, t) = -\sqrt{1 - \text{sech} \left( \frac{4x^{1/4}}{\sqrt{6}} - \frac{4t^{1/4}}{9} \right)^2} + \sqrt{1 + \text{cosech} \left( \frac{4x^{1/4}}{\sqrt{6}} - \frac{4t^{1/4}}{9} \right)^2}. \]

It is striking to note that the traveling wave solutions \( u_1 - u_8 \) to the space–time fractional modified third-order KdV equation is all novel also more general. These solutions recite the circulation process of surface water waves and the electric circuit and multi constituent plasms in fluid mechanics.

**4.2 The space–time fractional Telegraph equation**

The suggested approach uses in this subsection to observe more universal and novel closed-form wave solutions to the space–time fractional Telegraph equation. The space–time fractional Telegraph equation is given by

\[ D^{2\alpha}_t u(x, t) - D^{2\alpha}_{xx} u(x, t) + D^\alpha_t u(x, t) + \gamma u(x, t) + \beta u^3(x, t) = 0. \]  

where \( \alpha \) is a parameter recitation the order of the fractional space and time derivative. When \( \alpha = -1 \) Eq. (21) is termed the nonlinear Telegraph equation. Exploitation the fractional complex transform,

\[ u = k \frac{x^{\alpha}}{\alpha} - c \frac{t^\alpha}{\alpha}, \quad u(x, t) = u(\epsilon), \]  

where \( c \) and \( k \) be the constants. The Eq. (21) is diminished to the following integer order ordinary differential equation (ODE) through the transformation 22 and integrating equation with zero constant, we accomplish

\[ (c^2 - k^2)u'' - cu' + \gamma u + \beta u^3 = 0. \]  

Balancing the highest order derivative term with the highest power nonlinear term the balancing number is found by 1. The Eq. (6) is then resolved as
\[ u(\varepsilon) = a_0 + a_1 Y + b_1 Y^{-1}. \] (24)

Take the place of 23 into 24 along with 7, in \( Y \), the left side converts into a polynomial. When each of the polynomial’s coefficients is set to zero, a set of algebraic equations emerges (intended used for plainness, we try to slip over them to exposition) for \( a_0, a_1, b_1, k \) and \( c \). The subsequent outcomes are attained by put on computer algebra, such as Maple, to resolve this over determined series of equations:

**Family 1:**

\[ k = \frac{1}{2} \frac{\sqrt{\gamma \nu}}{\mu}, \quad c = 0, \quad a_0 = 0, \quad a_1 = 0 \quad \text{and} \quad b_1 = \sqrt{-\frac{\gamma}{\beta}}. \]

The parameters complete in Family 1 generates an explicit solution in terms of \( \tanh \) functions.

\[ u_9(x, t) = \sqrt{-1} \cosh \left( 8 \sqrt{2} x^{1/4} \right). \] (25)

Rewrite this equation as follows:

\[ u_{10}(x, t) = \sqrt{-1} \left( 1 + \coth \left( 8 \sqrt{2} x^{1/4} \right)^2 \right). \] (26)

**Family 2:**

\[ k = \frac{1}{2} \frac{\sqrt{\gamma \nu}}{\mu}, \quad c = 0, \quad a_0 = 0, \quad a_1 = \sqrt{-\frac{\gamma}{\beta}} \quad \text{and} \quad b_1 = 0. \]

In terms of \( \tanh \) functions, the ideals of the parameters stated within family 2 form a plain solution.

\[ u_{11}(x, t) = \sqrt{-1} \tanh \left( x^{1/4} / \sqrt{2} \right). \] (27)

This equation can be recreated using the following formula:

\[ u_{12}(x, t) = \sqrt{1 - \sech \left( x^{1/4} / \sqrt{2} \right)^2}. \] (28)

**Family 3:**

\[ k = \frac{1}{4} \frac{\sqrt{\gamma \nu}}{\mu}, \quad c = 0, \quad a_0 = 0, \quad a_1 = \sqrt{-\frac{\gamma}{4\beta}} \quad \text{and} \quad b_1 = \frac{1}{4} \frac{\gamma}{\sqrt{-4\beta}}. \]

The parameters complete in Family 3 generates an explicit solution in terms of the \( \tanh \) and \( \coth \) function.

\[ u_{13}(x, t) = \frac{1}{2} \text{tanh} \left( \sqrt{2} x^{1/4} \right) + \frac{1}{2} \text{coth} \left( \sqrt{2} x^{1/4} \right). \] (29)

**Family 4:**

\[ k = \frac{1}{4} \frac{\sqrt{-2\gamma + 9\nu^2}}{\mu}, \quad c = \frac{3\nu}{4\mu}, \quad a_0 = \sqrt{-\frac{\gamma}{4\beta}}, \quad a_1 = 0 \quad \text{and} \quad b_1 = \sqrt{-\frac{\gamma}{4\beta}}. \]

In terms of \( \tanh \) and \( \coth \) functions, the values of the parameters in assortment 4 create an explicit result.
The following formula can be used to reproduce this equation.

\begin{equation}
\frac{1}{2} + \frac{1}{2} \coth \left( \sqrt{7}x^{1/4} - 3t^{1/4} \right).
\end{equation}

(30)

The following formula can be used to reproduce this equation.

\begin{equation}
\frac{1}{2} + \frac{1}{2} \sqrt{1 + \text{cosech} \left( \sqrt{7}x^{1/4} - 3t^{1/4} \right)^2}.
\end{equation}

(31)

**Family 5:**

\[
k = \frac{1}{4} \sqrt{-2\gamma + \gamma^2} \mu, \quad c = -\frac{3\gamma}{4\mu}, \quad a_0 = \frac{1}{4} \sqrt{-\frac{3}{4\beta}}, \quad a_1 = \sqrt{-\frac{3}{4\beta}} \quad \text{and} \quad b_1 = 0.
\]

In terms of \(\tanh\) and \(\coth\) functions, the values of the parameters within family 4 establish an explicit answer.

\begin{equation}
\frac{1}{2} + \frac{1}{2} \tanh \left( \sqrt{7}x^{1/4} - 3t^{1/4} \right).
\end{equation}

(32)

Correspondingly.

\begin{equation}
\frac{1}{2} + \frac{1}{2} \sqrt{1 - \text{sech} \left( \sqrt{7}x^{1/4} - 3t^{1/4} \right)^2}.
\end{equation}

(33)

**Family 6:**

\[
k = \frac{1}{8} \sqrt{-2\gamma + \gamma^2} \mu, \quad c = -\frac{3\gamma}{8\mu}, \quad a_0 = \frac{1}{8\beta} \sqrt{-\frac{\gamma}{2\beta}}, \quad a_1 = \sqrt{-\frac{\gamma}{16\beta}} \quad \text{and} \quad b_1 = \frac{1}{16\beta} \sqrt{-\frac{\gamma}{16\beta}}.
\]

The values of the parameters supplied in family 6 create an explicit solution in terms of the \(\tanh\) and \(\coth\) function.

\begin{equation}
\frac{1}{2} + \frac{1}{4} \tanh \left( \frac{\sqrt{7}x^{1/4}}{2} + \frac{3t^{1/4}}{2} \right) + \frac{1}{4} \coth \left( \frac{\sqrt{7}x^{1/4}}{2} + \frac{3t^{1/4}}{2} \right).
\end{equation}

(34)

The following formula can be used to reproduce this equation.

\begin{equation}
\frac{1}{2} + \frac{1}{4} \sqrt{1 - \text{sech} \left( \frac{\sqrt{7}x^{1/4}}{2} + \frac{3t^{1/4}}{2} \right)^2} + \frac{1}{4} \sqrt{1 + \text{cosech} \left( \frac{\sqrt{7}x^{1/4}}{2} + \frac{3t^{1/4}}{2} \right)^2}.
\end{equation}

(35)

The solutions 25–35 attains the mentioned equations are all new and more general and this solutions can be explain to pulsatory blood movement among arteries also a one-dimensional haphazard movement of bugs towards an obstacle, communication lines of all frequencies together with direct current also high-frequency etc.

5 Physical description and visual representation

The physical explanation of the established traveling waves solutions to the space–time fractional modified third-order KdV equation and the space–time fractional Telegraph equations will be discussed in this section. The collected traveling-wave solutions of those equations are discussed in the three-dimensional plotline, the plot of contour, and the plot.
of vector which are designed via Mathematica. Using those three sorts of pictorial descriptions, we may more explicitly characterize the physical sketch.

### 5.1 Physical clarification of the resolution

In this segment graphic delegation also physical amplification of the resolution of the derived solutions of nonlinear fractional differential equations through the time fractional modified third-order KdV equation and Telegraph equation are recapitulated. Solution $u_1(x, t)$ in Fig. 1 for space–time fractional modified third-order KdV equation illustrates periodic kink shape wave solution, within the interval $-5 < x < 10$ and $-5 < t < 10$ and with the values $\mu = 1, p = 1, q = -1, \beta = \frac{1}{4}, \alpha = \frac{1}{4}$. Periodic kink wave is a recapping nonstop pattern which regulates its wavelength and frequency. Similarly, solution $u_2(x, t)$ and $u_5(x, t)$ delivers the same types of solutions like periodic king wave for the values of same intervals but for the simplicity those are omitted here. Solution $u_3(x, t)$ in Fig. 2 for space–time fractional modified third-order KdV equation illustrates the kink shape wave solution with the interval $0 < x < 100$ and $0 < t < 100$ for the values $\mu = 1, p = 1, q = -1, \beta = \frac{1}{4}, \alpha = \frac{1}{4}$. Kink shape wave solution be a wave which travels from one asymptotic spot to additional asymptotic spot wave. Similarly, for space–time fractional Telegraph equation, solutions $u_6(x, t)$, $u_9(x, t)$, $u_{10}(x, t)$, $u_{11}(x, t)$, $u_{12}(x, t)$ for the values $\mu = 1, p = 1, q = -1, \beta = \frac{1}{4}, \alpha = \frac{1}{4}$, all are represent the kink wave solution, which are hide here. In Fig. 3 solution $u_4(x, t)$ for space–time fractional modified third-order KdV equation shows bell shape wave solution through the interval $-10 < x < 10$ and $-10 < t < 10$ for the values $\mu = 1, p = 1, q = -1, \beta = \frac{1}{4}, \alpha = \frac{1}{4}$. Bell shape wave is a continuous circulation through no breaks among principles. Figure 4 is the diagram of the soliton king shape of $u_7(x, t)$ for the values $\mu = 1, p = 1, q = -1, \beta = \frac{1}{4}, \alpha = \frac{1}{4}$ with the interval $500 < x < 100000$ and $500 < t < 100000$. Soliton king shape is a confined disorder which circulates alike a wave. Solution $u_8(x, t)$ for space–time fractional modified third-order KdV equation shows anti-bell king wave solution for the values $\mu = 1, p = 1, q = -1, \beta = \frac{1}{4}, \alpha = \frac{1}{4}$ with the interval $-1000 < x < 10000$ and $-1000 < t < 100000$ in Fig. 5.

The solution of $u_{14}(x, t)$ imply the type of multiple soliton shape solution, intended for the standards $\mu = 1, \gamma = -1, \beta = \frac{1}{4}, \alpha = \frac{1}{4}$ and $0 < x < 50000$, $0 < t < 50000$ is symbolized by Fig. 6 which are represented space–time fractional Telegraph equation.

![Fig. 1](image-url) 

Fig. 1 Diagram of the periodic kink shape solution 13, representing a the three-dimensional graph. b Plot of contour c and plot of vector of $u_1(x, t)$
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The solution of $u_{15}(x,t)$ imply the type of singular bell shape wave solution, intended for the standards $\mu = 1, I = 1, \gamma = -1, \beta = \frac{1}{4}, \alpha = \frac{1}{4}$ and $10 < x < 100, 10 < t < 100$ is symbolized by Fig. 7. For the case of space–time fractional Telegraph equation, Fig. 8 demonstrates single solitons shape wave solution for the solution of $u_{16}(x,t)$ considering the values of $\mu = 1, I = 1, \gamma = -1, \beta = \frac{1}{4}, \alpha = \frac{1}{4}$ with the interval $0 < x < 30, 0 < t < 30$, and also Fig. 9 demonstrates single solitons shape wave solution for the solution of $u_{19}(x,t)$ considering the values of $\mu = 1, I = 1, \gamma = -1, \beta = \frac{1}{4}, \alpha = \frac{1}{4}$ with the interval $-100 < x < 10, -100 < t < 10$. Singular solitons are a form of solitary wave that has a singularity, which is typically an infinite discontinuity. When the center point of the solitary wave is imaginary, singular solitons can be bound to it. In Fig. 10, the double solitons shape wave solution is depicted, for the solution of $u_{17}(x,t)$ (space–time fractional Telegraph equation) with the values of $\mu = 1, I = 1, \gamma = -1, \beta = \frac{1}{4}, \alpha = \frac{1}{4}$ within the interval $100 < x < -1000$ and $100 < t < -1000$. Double solitons is known as a bion, or in systems where the bound state periodically oscillates, a breather. Solution $u_{18}(x,t)$, for space–time fractional Telegraph equation illustrates compaction wave...
Fig. 4 Diagram of the soliton king shape wave solution 19, representing a the three-dimensional plotline b plot of contour c and plot of vector of $u_7(x, t)$.

Fig. 5 Illustration of the anti-bell king shape wave solution 20, representing a the three-dimensional plotline b plot of contour c and plot of vector of $u_8(x, t)$

Fig. 6 Illustration of the multiple soliton shape (solutions which behave at large time as a sum of solitons) wave solution 30, representing a the three-dimensional plotline b plot of contour c and plot of vector of $u_{14}(x, t)$
solution in Fig. 11, for the values of $\mu = 1, I = 1, \gamma = -1, \beta = \frac{1}{4}, \alpha = \frac{1}{4}$ with the interval $-10 < x < 0.1$ and $-10 < t < 0.1$. Compaction wave solution defined as a propagating disturbance of the solid volume fraction of the granular material.

6 Results’ comparison

In this section, we provide the acquired results of the space–time fractional Telegraph equation and modified third-order KdV equation by using the suggested method for comparison with the results obtained by other scholars using the extended tanh-function method. It is interesting to see that some of the solutions obtained show similarities with the solutions previously developed and other obtain results are completely new and innovative.
Fig. 9 Diagram of the single soliton shape wave solution 35, representing a the three-dimensional plotline b plot of contour c and plot of vector of $u_{19}(x, t)$

Fig. 10 Diagram of the double solitons shape wave solution 33, representing a the three-dimensional plotline b plot of contour c and plot of vector of $u_{17}(x, t)$

Fig. 11 Diagram of the compaction wave solution 34, representing a the three-dimensional plotline b plot of contour c and plot of vector of $u_{18}(x, t)$
Table 1  Comparison between Sarwar (Ambreen et al. 2020) and the space–time fractional modified third-order KdV equation

| (Ambreen et al. 2020) Obtained solutions | Obtained solutions |
|-----------------------------------------|--------------------|
| If $c_2 = 1, c_3 = 1, c_4 = 1, p = 1$ then (64) solution becomes | $\omega = \frac{1}{5} \sqrt[3]{-60p^4 \mu} b_1, \lambda = \frac{\sqrt[3]{-60p^4 b_1^3}}{18 \mu}, a_0 = 0, a_1 = 0$ and $b_1 = b_1$ then $u_2(x,t) = \sqrt{\frac{2\sqrt{-1} + 1}{\sqrt{1}} \coth (\zeta)}.$ |
| If $\gamma = 1, \beta = 1, \alpha = 1, m = 1$ and $t = 1$ then (67) solution becomes | $\omega = \frac{1}{5} \sqrt[3]{-60p^4 \mu} a_1, \lambda = \frac{\sqrt[3]{-60p^4 b_1^3}}{18 \mu}, a_0 = 0, a_1 = a_1$ and $b_1 = b_1 = 0$ then $u_3(x,t) = \sqrt{\frac{2\sqrt{-1} + 1}{\sqrt{1}} \tanh (\zeta)}.$ |

Table 2  Comparison between Lu et al. (Xiaohua 2018) and the space–time fractional Telegraph equation

| (Xiaohua 2018) Obtained solutions | Obtained solutions |
|-----------------------------------|--------------------|
| If $\gamma = 1, \beta = 1, \alpha = 1, m = 1$ and $t = 1$ then (44) solution becomes | $k = \frac{1}{4} \sqrt[3]{-\frac{2\gamma + 9p^2}{\mu}}, c = -\frac{3\gamma}{4p}, a_0 = \frac{1}{4} \sqrt[3]{-\frac{\gamma}{\mu}}, a_1 = \frac{1}{4} \sqrt[3]{-\frac{\gamma}{\mu}}.$ And $b_1 = 0.$ Then $u_1(x,t) = \frac{1}{2} + \frac{1}{2} \tanh \left( \sqrt{7x^{1/4}} - 3b^{1/4} \right).$ |
| If $\gamma = 1, \beta = 1, \alpha = 1, m = 1$ and $t = 1$ then (45) solution becomes | $k = \frac{1}{4} \sqrt[3]{-\frac{2\gamma + 9p^2}{\mu}}, c = -\frac{3\gamma}{4p}, a_0 = \sqrt{\frac{1}{4p}}, a_1 = \frac{1}{4} \sqrt[3]{-\frac{\gamma}{\mu}}.$ Then $u_2(x,t) = \frac{1}{2} + \frac{1}{2} \coth \left( \sqrt{7x^{1/4}} - 3b^{1/4} \right).$ |

The following Table 1 provides a comparison between (Ambreen et al. 2020) solutions and our obtained solutions about the space–time fractional modified third-order KdV. Again Table 2 provides a correlation between (Xiaohua 2018) solutions and our achieved solutions about the space–time fractional Telegraph equation.

For the upstairs tables, the hyperbolic function solutions are comparable and are indistinguishable in the case that we set definite values of the arbitrary constants. It is significant to recognize that the traveling wave solutions $u_5(x,t), u_6(x,t), u_7(x,t), u_8(x,t), u_9(x,t)$ are completely new and innovative. It is also relevant to point out that they were not enclosed in prior exertion.

For the upstairs tables, the hyperbolic function solutions are comparable and are indistinguishable in the case that we set definite values of the arbitrary constants. It is noteworthy to observe that the solitary wave solutions $u_9(x,t), u_{10}(x,t), u_{11}(x,t), u_{12}(x,t), u_{13}(x,t), u_{14}(x,t), u_{15}(x,t), u_{16}(x,t), u_{17}(x,t), u_{18}(x,t), u_{19}(x,t)$ are all new and more general. It is also relevant to point out that the achieved solutions were not claimed in the previous study exertion.
7 Conclusion

In summary, this manuscript has built scores of innovative, extra universal, and wide-ranging solitary wave explanations employing two dependable ways relating conformable derivative, in addition, extended tanh-method to the nonlinear space-time fractional modified third-order Kdv equation and the nonlinear space time-fractional Telegraph equation. The obtained results illustrate that both equations accept a large number of closed-form investigative explanations with arbitrarily conversant irreversible parameters. The recognized outcomes reveal amusing dynamical constructions of soliton results in the categories of single solitons, double solitons, bell types waves, kink-type waves, imaginary waves also multiple solitons. The achieved solutions clarify the phenomena relating to an electrical communication line and include voltage in addition to current which is dependent on distance and time, is also applied to communication lines of wholly frequencies, together with direct current, as well as high-frequency conductors, audio frequency, and low frequency. The dynamics of solitary waves have been realistically described within relations of space and time coordinates, revealing the reliability of the methodologies employed. The accurateness of the data produced in these learning was confirmed by means of the computational software Maple by re-entering them into NLPDEs and confirming that were correct. These soliton solutions will be beneficial in soliton theory, nonlinear wave physics, plasma physics, optical engineering, oceanography, fluid dynamics, and engineering physics. It is remarkable to the indication that the extended tanh-function scheme formed an extensive assortment of computational wave solutions to the main equation. The existing technique’s foremost benefit is that it generates more general solutions with arbitrary known-constant constraints that also can be exerted to several categories of NLPDEs.

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Declarations

Conflict of interest We guarantee that in this article none of the authors have any contest of interests.

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