Soft-gluon resummation and NNLO corrections for direct photon production

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Abstract

The resummation of threshold logarithms for direct photon production cross sections in hadronic collisions is presented. The resummation is based on the factorization properties of the cross section and is formulated at next-to-leading logarithmic or higher accuracy. Full analytical and numerical results for the next-to-next-to-leading order expansion of the resummed cross section are given. A substantial reduction of factorization scale dependence is observed. A comparison to experimental results from the E-706 and UA-6 experiments is presented.
1 Introduction

The production of photons with large transverse momenta has long been recognized as an important probe of hard-scattering dynamics. In particular, the subprocess $qg \rightarrow \gamma q$ offers sensitivity to the gluon distribution function in lowest order. Thus, direct photon production is often cited as having the potential to place strong constraints on the gluon distribution in global fits of parton distributions. In recent years, however, the situation has become more complex. Recent detailed phenomenological studies have shown that it is not possible to describe the existing set of direct photon experimental results using a next-to-leading order QCD calculation [1, 2]. Even with the full freedom of separately adjusting the renormalization and the factorization scales for the distribution and fragmentation functions, one can not correctly describe all of the experimental results [3]. In [2] it was pointed out that for many of the data sets the theoretical predictions fall below the data near the low-$p_T$ end of the $p_T$ range spanned by the data. This has been interpreted [4] as providing evidence for the inclusion of $k_T$-smearing which is due to initial state radiation. However, this interpretation is not completely supported by all of the available data and the analysis of [1] shows that agreement with a subset of the data can be obtained if one considers a restricted set of data taken only on proton targets (so as to remove possible nuclear effects) and also if minimum $p_T$ cuts on the data sets are applied which restrict the comparisons to regions where the scale dependence of the theory is moderate, and, hence, the theoretical predictions are thought to be reliable.

In light of the situation described in the preceding paragraph, efforts are underway to improve the precision of the theoretical predictions for direct photon production. One such type of improvement involves the resummation of large logarithms which result from phase space limitations on real gluon emission near the threshold region for the underlying parton scattering subprocesses. Such corrections are expected to be important in the region of large photon transverse momentum, where $x_T = 2p_T/\sqrt{S}$ approaches one. Nevertheless, it is of interest to examine the effects of such resummation calculations over an extended range of $x_T$.

Recently, threshold resummation for direct photon production has been discussed in Refs. [5, 6, 7, 8]. In Refs. [6, 7], a resummation calculation was presented for the inclusive $E_T$ distribution, $d\sigma/dE_T$, and compared with some of the available fixed target data. The results showed that, as expected, the
corrections are large as $x_T$ approaches the edge of phase space. In addition, the scale dependence of the resulting distributions is reduced compared to that of the next-to-leading order calculation. Finally, in the region containing the data the resummed cross section was nearly the same as the next-to-leading order cross section if the factorization and renormalization scales were chosen to be $\mu = p_T/2$.

Another formalism exists for treating the resummation of threshold logarithms in inclusive processes, a review of which can be found in Ref. [9]. This formalism has been applied to lepton pair, heavy quark, and dijet production at fixed invariant mass and more recently has been extended to single-particle inclusive cross sections [4], including direct photon [4] and $W +$ jet production [10, 11]. In the present paper this formalism is applied to the inclusive invariant cross section for direct photon production. One important feature of this method is that one directly obtains the angular dependence (or rapidity dependence) for the produced photon. The resummation explicitly includes all the leading and next-to-leading logarithms (NLL). For detailed numerical work and comparison to other calculations, the resummed cross section expression is expanded to next-to-leading order (NLO) and next-to-next-to-leading order (NNLO). Matching the resulting expressions to existing complete NLO calculations yields the leading, next-to-leading, and next-to-next-to-leading logarithms (NNLL) through $O(\alpha s^3)$. The NLO expansion agrees fully near partonic threshold with existing exact NLO calculations [12, 13] while our NNLO results provide new predictions. A comparison with recent high statistics fixed target results is also presented.

2 Factorization and resummed cross section

2.1 Factorized cross section

In this section we discuss the application of the threshold resummation formalism described in Ref. [9] to direct photon production. The self-contained presentation follows the techniques of Refs. [5, 10].

The quantity to be calculated is the inclusive invariant cross section for the production of photons in hadronic collisions:

$$h_A(p_A) + h_B(p_B) \rightarrow \gamma(p_\gamma) + X.$$  (2.1)
Fig. 1. (a) Factorization for direct photon production near partonic threshold. (b) The soft-gluon function $S$, in which the vertices $c$ link ordered exponentials, representing the partons in the hard scattering.

The Mandelstam invariants formed form the hadron and photon four-vectors are

$$S = (p_A + p_B)^2, \quad T = (p_A - p_\gamma)^2, \quad U = (p_B - p_\gamma)^2.$$  \hfill (2.2)

The parton-parton scattering subprocesses contributing to direct photon production in lowest order are

$$q(p_a) + g(p_b) \longrightarrow \gamma(p_\gamma) + q(p_f)$$  \hfill (2.3)

and

$$q(p_a) + \bar{q}(p_b) \longrightarrow \gamma(p_\gamma) + g(p_f).$$  \hfill (2.4)

The corresponding Mandelstam invariants constructed from the parton and photon four-vectors are given by

$$s = (p_a + p_b)^2, \quad t = (p_a - p_\gamma)^2, \quad u = (p_b - p_\gamma)^2,$$  \hfill (2.5)

which satisfy $s_4 \equiv s + t + u = 0$ at threshold. The variable $s_4$ is the square of the invariant mass of the system recoiling against the observed photon. It will also be useful to define a corresponding variable, $S_4 = S + T + U$, using the hadronic Mandelstam variables.

The factorized form of the cross section for direct photon production is a convolution of the parton distribution functions $\phi$ with the parton-parton hard scattering cross section:

$$E_\gamma \frac{d^3 \sigma_{h_A h_B \rightarrow \gamma X}}{d^3 p_\gamma} = \sum_{f} \int dx_a dx_b \phi_{f_a/h_A}(x_a, \mu_F^2) \phi_{f_b/h_B}(x_b, \mu_F^2).$$
\[ \times E_\gamma \frac{d^3\bar{\sigma}_{f_a f_b \to \gamma X}}{d^3p_\gamma}(s, t, u, \mu_F, \alpha_s(\mu_R^2)) . \] 

(2.6)

The initial-state collinear singularities have been factorized and absorbed into the \( \phi \)'s at a factorization scale \( \mu_F \), while \( \mu_R \) is the renormalization scale.

The threshold for the parton-parton scattering subprocesses occurs at \( s_4 = 0 \). At this point the photon recoils against a massless parton and receives the maximum amount (one half) of the parton center of mass energy allowed. The values of \( x_a \) and \( x_b \) corresponding to this point are the minimum values which can give rise to a photon of the specified rapidity and transverse momentum. Near threshold the phase space for the emission of real gluons is limited and there is an incomplete cancellation of infrared divergences against virtual gluon emission contributions. Although the remainder is finite, there are large logarithmic corrections. In general, \( \bar{\sigma} \) includes plus distributions with respect to \( s_4 \) at \( n \)th order in \( \alpha_s \) of the type

\[ \left[ \ln^m(s_4/p_T^2) \right]_+, \quad m \leq 2n - 1, \] 

(2.7)

where \( p_T = (tu/s)^{1/2} \) is the transverse momentum of the photon.

To organize these plus distributions we introduce a refactorization in terms of new parton distributions \( \psi \) and a jet function \( J \) that absorb the collinear singularities from the incoming partons and outgoing jet [14, 15], respectively; a soft gluon function \( S \) that describes noncollinear soft gluon emission [14, 13]; and a short-distance hard scattering function \( H \). This refactorization is shown in Fig. 1 for the partonic cross section with the colliding hadrons in Eq. (2.6) replaced by partons \( f_a, f_b \). In our soft-gluon approximation the partons in the hard scattering are represented by eikonal lines (ordered exponentials). The cut diagram in Fig. 1 shows contributions from the amplitude and its complex conjugate, with \( H = hh^* \).

The momenta of the parton lines entering the hard scattering subprocess are then \( x_a p_A \) and \( x_b p_B \), where now \( p_A \) and \( p_B \) denote the momenta of external partons. If soft radiation of total momentum \( k_S \) is emitted, then momentum conservation gives \( x_a p_A + x_b p_B = p_\gamma + p_J + k_S \) where \( p_J \) represents the momentum of everything in the final state excluding the photon and the soft radiation. Using the definitions of the Mandelstam variables presented previously, and neglecting corrections of order \( S_4^2 \), the relation between \( S_4 \)
and $s_4$ can be expressed as

\[
\frac{S_4}{S} = \frac{1}{S} \left[ 2p_J \cdot k_S + p_J^2 + (1 - x_a)2p_A \cdot p_J + (1 - x_b)2p_B \cdot p_J \right]
\]

\[
\equiv w_s + w_J + w_a \frac{(-u)}{s} + w_b \frac{(-t)}{s}
\]

\[
= \frac{s_4}{s} - (1 - x_a) \frac{u}{s} - (1 - x_b) \frac{t}{s},
\]

(2.8)

where the dimensionless $w_i$'s measure the fractional contributions to $S_4/S$ of the gluon emission associated with the functions $\psi$, $J$, and $S$. The refactorized partonic cross section can be written as a convolution

\[
E_\gamma \frac{d^3 \sigma_{f_a f_b \rightarrow \gamma X}}{d^3 p_\gamma} = H \int dw_a dw_b dw J dw_s \psi_{f_a f_a}(w_a) \psi_{f_b f_b}(w_b) J(w_J)
\]

\[
\times S(w_S \sqrt{s}/\mu_F) \delta \left( \frac{S_4}{S} - w_s - w_J - w_a \frac{(-u)}{s} - w_b \frac{(-t)}{s} \right).
\]

(2.9)

If we take moments of the above equation, with $N$ the moment variable, we can write the partonic cross section as

\[
\int \frac{dS_4}{S} e^{-N S_4/S} E_\gamma \frac{d^3 \sigma_{f_a f_b \rightarrow \gamma X}}{d^3 p_\gamma} = \tilde{\psi}_{f_a f_a}(N_a) \tilde{\psi}_{f_b f_b}(N_b) \tilde{J}(N) H \tilde{S}(\sqrt{s}/(N \mu_F)) ,
\]

(2.10)

where

\[
\tilde{\psi}_{f_a f_a}(N_a) = \int dw_a e^{-N_a w_a} \psi_{f_a f_a}(w_a),
\]

(2.11)

and similarly for the other functions, with $N_a = N(-u/s)$ and $N_b = N(-t/s)$ for either partonic subprocess.

Next, we take moments of Eq. (2.6) with the incoming hadrons replaced by partons and, using the last line of Eq. (2.8), we find

\[
\int \frac{dS_4}{S} e^{-N S_4/S} E_\gamma \frac{d^3 \sigma_{f_a f_b \rightarrow \gamma X}}{d^3 p_\gamma} = \int dx_a e^{-N(-u/s)(1-x_a)} \phi_{f_a f_a}(x_a, \mu_F^2)
\]

\[
\times \int dx_b e^{-N(-t/s)(1-x_b)} \phi_{f_b f_b}(x_b, \mu_F^2) \int \frac{ds_4}{s} e^{-N s_4/s} E_\gamma \frac{d^3 \sigma_{f_a f_b \rightarrow \gamma X}(s_4)}{d^3 p_\gamma}
\]

\[
\equiv \tilde{\phi}_{f_a f_a}(N_a) \tilde{\phi}_{f_b f_b}(N_b) E_\gamma \frac{d^3 \sigma_{f_a f_b \rightarrow \gamma X}(N)}{d^3 p_\gamma}.
\]

(2.12)
Comparing Eqs. (2.12) and (2.10) we may then solve for the moments of the perturbative cross section $\hat{\sigma}$:

$$E_\gamma \frac{d^3 \hat{\sigma}_{f_a f_b \rightarrow \gamma X}(N)}{d^3 p_\gamma} = \frac{\bar{\psi}_{f_a / f_a}(N_a) \bar{\psi}_{f_b / f_b}(N_b)}{\phi_{f_a / f_a}(N_a) \phi_{f_b / f_b}(N_b)} J(N) H \tilde{S}(\sqrt{s} / (N \mu_F)). \quad (2.13)$$

The moments of the plus distributions with respect to $s^4$ in $\hat{\sigma}$ give powers of $\ln N$ as high as $\ln^2 n N$. In the next section these logarithms are resummed to all orders in perturbation theory.

### 2.2 Resummed cross section

The resummation of the $N$-dependence of each of the functions in the refactorized cross section, Eq. (2.13), is based on their renormalization properties and has been extensively discussed in Refs. [5, 10, 14, 15, 16]. The resummed cross section in the $\overline{\text{MS}}$ factorization scheme is given by:

$$E_\gamma \frac{d^3 \hat{\sigma}_{f_a f_b \rightarrow \gamma X}(N)}{d^3 p_\gamma} = \exp \left\{ \sum_{i=a,b} \left[ E^{(f_i)}(N_i) \right. \right.$$

$$- 2 \int_{\mu_F}^{2p_i \zeta} d\mu' \left. \left[ \gamma_{f_i}(\alpha_s(\mu'^2)) - \gamma_{f_i}(N_i, \alpha_s(\mu'^2)) \right] \right\}$$

$$\times \exp \left\{ E'_J(N) \right\} H(s, t, u, \alpha_s(\mu_F^2))$$

$$\times \tilde{S}(\alpha_s(s/N^2)) \exp \left[ \int_{\mu_F}^{\sqrt{s}/N} d\mu' \frac{d\mu'}{\mu'} 2 \text{Re} \Gamma_S \left( \alpha_s(\mu'^2) \right) \right],$$

where $\zeta^\mu \equiv p_F^\mu / \sqrt{s}$. The first exponent $E^{(f_i)}(N_i)$, which resums the $N$-dependence of the ratio $\bar{\psi}/\bar{\phi}$, is given by [14, 17, 18]

$$E^{(f_i)}(N_i) = - \int_0^1 dz \frac{z^{N_i-1} - 1}{1 - z} \left\{ \int_0^{1 - z^2} \frac{d\lambda}{\lambda} A^{(f_i)} \left[ \alpha_s \left( \lambda(2p_i \cdot \zeta)^2 \right) \right] \right.$$

$$+ \frac{1}{2} \mu^{(f_i)} \left[ \alpha_s((1 - z)^2(2p_i \cdot \zeta)^2) \right] \right\}, \quad (2.15)$$

where, at next-to-leading order accuracy in $\ln N$, we have

$$A^{(f)}(\alpha_s) = C_f \left( \frac{\alpha_s}{\pi} + \frac{1}{2} K \left( \frac{\alpha_s}{\pi} \right)^2 \right), \quad (2.16)$$
Here \( C_f = C_F = (N_c^2 - 1)/(2N_c) \) for an incoming quark, and \( C_f = C_A = N_c \) for an incoming gluon, with \( N_c \) the number of colors, while

\[
K = C_A \left( \frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{5}{9} n_f,
\]

where \( n_f \) is the number of quark flavors \([13]\).

The anomalous dimensions \( \gamma_f \) and \( \gamma_{ff} \) of the fields \( \psi \) and \( \phi \) \([10, 15, 20]\) are given by

\[
\gamma_q = \frac{3}{4} C_F \frac{\alpha_s}{\pi}; \quad \gamma_{qq} = -\left( \ln N - \frac{3}{4} \right) C_F \frac{\alpha_s}{\pi},
\]

\[
\gamma_g = \frac{\beta_0 \alpha_s}{4 \pi}; \quad \gamma_{gg} = -\left( C_A \ln N - \frac{\beta_0}{4} \right) \frac{\alpha_s}{\pi},
\]

for quark and gluon jets, respectively, where \( \beta_0 = (11C_A - 2n_f)/3 \) is the one-loop coefficient of the \( \beta \) function.

The exponent \( E'_{(J)} \), which resums the \( N \)-dependence of the final-state jet, is given by \([5, 10, 13]\)

\[
E'_{(J)}(N) \equiv \int_0^1 dz \frac{z^{N-1} - 1}{1 - z} \left\{ \int_{(1-z)^2}^{(1-z)} \frac{d\lambda}{\lambda} A_{(J)} \left[ \alpha_s(\lambda s) \right] 
+ B'_{(J)} \left[ \alpha_s((1 - z)s) \right] + B''_{(J)} \left[ \alpha_s((1 - z)^2 s) \right] \right\},
\]

with \( A_{(J)} \) given by Eq. (2.16) and \( B'_{(J)}, B''_{(J)} \) given for quarks by \([5, 10]\)

\[
B'_{(q)} = \frac{\alpha_s}{\pi} \left( -\frac{3}{4} \right) C_F, \quad B''_{(q)} = \frac{\alpha_s}{\pi} C_F \left[ \ln(2\nu_q) - 1 \right],
\]

and for gluons by

\[
B'_{(g)} = \frac{\alpha_s}{\pi} \left( -\frac{\beta_0}{4} \right), \quad B''_{(g)} = \frac{\alpha_s}{\pi} C_A \left[ \ln(2\nu_g) - 1 \right].
\]

The \( \nu_i \) terms are gauge dependent. They are defined by

\[
\nu_i \equiv \frac{(\beta_i \cdot n)^2}{|n|^2},
\]

where \( \beta_i = p_i \sqrt{2/s} \) are the particle velocities and \( n \) is the axial gauge vector, chosen so that \( p_i \cdot \zeta = p_i \cdot n \) for \( i = a, b \) \([3, 10]\).
Fig. 2. One-loop eikonal vertex corrections for partonic subprocesses in direct photon production; the eikonal lines represent the incoming partons and the outgoing jet.

### 2.3 Soft anomalous dimensions

The evolution of the soft function in Eq. (2.14), which follows from the renormalization group equation

\[
\left( \mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} \right) S = -2(\text{Re} \, \Gamma_S) S , \tag{2.24}
\]

is given in terms of \( \Gamma_S \), the soft anomalous dimension. \( \Gamma_S \) is calculated explicitly at one loop by evaluating the eikonal vertex corrections in Fig. 2. The color basis for the hard scattering consists of only one tensor, \( c = T_F \), and there are three eikonal lines connecting at the color vertex. Note that the color structure of the hard scattering for direct photon production is much simpler than for heavy quark or dijet production [9, 14, 15, 21, 22]; hence, here \( \Gamma_S \) is simply a \( 1 \times 1 \) matrix in color space.

The soft anomalous dimension for the process \( q(p_a) + g(p_b) \rightarrow \gamma(p_\gamma) + q(p_J) \) is given by [5, 9, 10]

\[
\Gamma^qg\rightarrow\gamma q = \frac{\alpha_s}{2\pi} \left\{ C_F \left[ 2 \ln \left( \frac{-u}{s} \right) - \ln(4\nu_{qa}\nu_{qJ}) + 2 \right] \\
+ C_A \left[ \ln \left( \frac{t}{u} \right) - \ln(2\nu_g) + 1 - \pi i \right] \right\} . \tag{2.25}
\]

The soft anomalous dimension for the process \( q(p_a) + \bar{q}(p_b) \rightarrow \gamma(p_\gamma) + g(p_J) \) is given by [5, 9, 10]

\[
\Gamma^{q\bar{q}\rightarrow\gamma g} = \frac{\alpha_s}{2\pi} \left\{ C_F \left[ - \ln(4\nu_q\nu_{\bar{q}}) + 2 - 2\pi i \right] \right\} .
\]
\[ + C_A \left\{ \ln \left( \frac{t u}{s^2} \right) + 1 - \ln(2\nu_g) + \pi i \right\} \right\}. \quad (2.26) \]

Note that the soft anomalous dimensions are complex and that they also include the gauge dependent \( \nu_i \) terms; however, all gauge dependence cancels out explicitly in the resummed cross section.

Finally, note that the resummed cross section, Eq. (2.14), can be rewritten in a form more convenient for the calculation of the fixed-order expansions, as

\[
E_{\gamma} \frac{d^3\hat{\sigma}_{f_a f_b \rightarrow \gamma X}(N)}{d^3p_{\gamma}} = H \left( \alpha_s(\mu_R^2) \right) \exp \left[ 2 \int_{p_{\gamma}}^{\sqrt{s}} \frac{d\mu'}{\mu'} \beta(\alpha_s(\mu'^2)) \right] \\
\times \exp \left\{ \sum_{i=a,b} \left[ E_{(i)}(N_i) - 2 \int_{\mu_F}^{\sqrt{s}} \frac{d\mu'}{\mu'} \left[ \gamma_{fi}(\alpha_s(\mu'^2)) - \gamma_{fi}(N_i, \alpha_s(\mu'^2)) \right] \right] \right\} \\
\times \exp \left\{ E_{(j)}(N) \right\} \exp \left[ 2 \int_{\mu_F}^{\sqrt{s}} \frac{d\mu'}{\mu'} \left( \gamma_a(\alpha_s(\mu'^2)) + \gamma_b(\alpha_s(\mu'^2)) \right) \right] \\
\times \tilde{S} \left( \alpha_s(s/N^2) \right) \exp \left[ \int_0^1 \frac{dz}{1-z} \frac{z^{N-1} - 1}{2\Re S \left( \alpha_s((1-z)^2s) \right)} \right]. \quad (2.27) \]

### 3 NNLO expansion of the resummed cross section

The formalism presented in the preceding section results in expressions for the moments of the resummed cross section. An inverse Mellin transform is needed in order to obtain results for the \( p_T \) and \( y \) dependence of the cross section. Such a transform involves integrals over regions of phase space where the argument of the running coupling vanishes and one must adopt a prescription for treating the resulting singularities; different prescriptions may give different results. On the other hand, it is possible to expand the moment expressions in powers of \( \alpha_s \) and perform the inversion order-by-order at fixed \( \alpha_s \). In this way one avoids having to adopt a specific prescription for the inversion. Of course, one does not have the fully resummed result after such a procedure. However, the work of Ref. \[7\] shows that the expansion converges quickly until one reaches the region very near \( x_T = 1 \). Therefore,
in regions relevant to current experiments, the corrections beyond NNLO are expected to be small. At any rate, this is a useful exercise for several reasons. First, one may check the resulting NLO $\mathcal{O}(\alpha_s^2)$ expressions with the corresponding terms from existing complete NLO calculations, thereby obtaining a test of the formalism. Second, one may generate analytical and numerical predictions for the dominant terms near threshold at NNLO. These terms represent the leading, next-to-leading, and next-to-next-to-leading logarithms which would be present in a full $\mathcal{O}(\alpha_s^3)$ calculation.

The results can be cast in the common form given by

$$E_\gamma \frac{d^3 \sigma_{ij}^{\overline{MS}}}{d^3 p_\gamma} = \sigma_{ij}^B(s, t, u)$$

$$= \left\{ \delta(s_4) + \sum_{k=1}^{2} \left( \frac{\alpha_s(\mu_R^2)}{\pi} \right)^k \left[ c_1^k \delta(s_4) + \sum_{l=0}^{2k-1} c_{l+2}^k \left( \ln \left( \frac{s_4}{p_T^2} \right) \right) \right] \right\} (3.1)$$

where, for ease of notation, the subprocess labels have been suppressed on the $c$ coefficients. It should be noted that these coefficients depend in general on $s$, $t$, and $u$. The expressions for the two Born terms are given by

$$\sigma_{qg}^B = -\frac{1}{N_c} \frac{\alpha_s}{s} e_q^2 \frac{t}{s} \left( \frac{s}{t} + \frac{t}{s} \right)$$

(3.2)

and

$$\sigma_{qg}^B = \frac{2C_F}{N_c} \frac{\alpha_s}{s} e_q^2 \frac{(t + u)}{t}$$

(3.3)

where $e_q$ is the charge of a quark of type $q$. The $\mathcal{O}(\alpha_s^2)$ results for the coefficients $c$ are given by

$$c_1^{(qg)} = C_F + 2C_A$$

$$c_2^{(qg)} = -\frac{3}{4} C_F - (C_F + C_A) \ln \left( \frac{\mu_F^2}{p_T^2} \right)$$

$$c_1^{(qg)} = \left( -\frac{\beta_0}{4} - \frac{3}{4} C_F - C_F \ln \left( \frac{-t}{s} \right) - C_A \ln \left( \frac{-u}{s} \right) \right) \ln \left( \frac{\mu_F^2}{p_T^2} \right)$$

$$+ \frac{\beta_0}{4} \ln \left( \frac{\mu_R^2}{p_T^2} \right)$$

(3.4)
and
\[
c_1^3(q\bar{q}) = 4C_F - C_A
\]
\[
c_2^3(q\bar{q}) = -\frac{\beta_0}{4} - 2C_F \ln \left(\frac{\mu_F^2}{p_T^2}\right)
\]
\[
c_1^3(q\bar{q}) = C_F \left(\frac{3}{2} - \ln \left(\frac{p_T^2}{s}\right)\right) \ln \left(\frac{\mu_F^2}{p_T^2}\right) + \frac{\beta_0}{4} \ln \left(\frac{\mu_R^2}{p_T^2}\right).
\] (3.5)

These expressions are equivalent to the corresponding terms in the exact NLO results quoted in Ref. [13]. The comparison is most easily made by rewriting the results in terms of the variables \(v\) and \(w\) used in [13] as is shown in the Appendix. Note that, with respect to the \(\delta(s_4)\) contribution, the expansion reproduces only the scale-dependent terms in \(c_1^1\). The full \(\delta(s_4)\) terms can be simply read off from the exact NLO calculation.

The NNLO corrections in the \(\overline{\text{MS}}\) scheme for the \(qg \to \gamma q\) subprocess are given by the following coefficients:
\[
c_2^2(qg) = \frac{1}{2} (C_F + 2C_A)^2
\]
\[
c_3^2(qg) = -\frac{3}{2} (C_F + 2C_A) \left(\frac{3}{4} C_F + (C_F + C_A) \ln \left(\frac{\mu_F^2}{p_T^2}\right)\right) - \frac{\beta_0}{2} \left(\frac{C_F}{4} + C_A\right)
\]
\[
c_3^3(qg) = (C_F + 2C_A) \left[ c_a^{qq} \ln \left(\frac{\mu_F^2}{p_T^2}\right) + c_1^{qq} \right] + \left[ \frac{3}{4} C_F + (C_F + C_A) \ln \left(\frac{\mu_F^2}{p_T^2}\right) \right]^2
\]
\[
+ \ (C_F + 2C_A) \frac{\beta_0}{4} \ln \left(\frac{\mu_R^2}{p_T^2}\right) + \frac{1}{2} K(C_F + 2C_A) + \frac{3}{16} \beta_0 C_F
\]
\[
- \frac{\pi^2}{6} (C_F + 2C_A)^2
\] (3.6)

with
\[
c_a^{qq} = -\frac{\beta_0}{4} - \frac{3}{4} C_F - C_F \ln \left(\frac{-t}{s}\right) - C_A \ln \left(\frac{-u}{s}\right)
\] (3.7)

and
\[
c_1^{qq} = \frac{\beta_0}{4} \ln \left(\frac{\mu_R^2}{p_T^2}\right) - \frac{7}{4} C_F - \frac{5}{4} N_c \ln^2 \left(\frac{s + t}{s}\right)
\]
\[
- \frac{1}{2} C_F \ln^2 \left(\frac{-t}{s}\right) + \frac{3}{4} C_F \ln \left(\frac{s + t}{s}\right) - \frac{3}{2} C_F \ln \left(\frac{p_T^2}{s}\right)
\]
\[ + \frac{N_c}{T_{qq}} \left[ -\frac{t}{s} \ln^2 \left( \frac{s+t}{s} \right) - \frac{t}{2s} \ln \left( \frac{-t}{s} \right) \right. \\
+ \frac{t}{s} \ln \left( \frac{s+t}{s} \right) + \frac{\pi^2}{4} \left( \frac{-t}{s} \right) \left( \frac{2s+t}{s} \right) - \frac{t}{4s} \left( \frac{2s+t}{s} \right) \ln^2 \left( \frac{-t}{s} \right) \\
+ \frac{t}{2s} \left( \frac{2s+t}{s} \right) \ln \left( \frac{-t}{s} \right) \ln \left( \frac{s+t}{s} \right) \right] \\
+ \frac{C_F}{T_{qq}} \left[ \frac{1}{2} \left( \frac{3s+t}{s} \right) \ln \left( \frac{-t}{s} \right) + \frac{\pi^2}{6} \left( 1 - \frac{4(s+t)}{s} + \frac{5(s+t)^2}{s^2} \right) \\
- \frac{1}{4} \left( \frac{3(s+t)^2}{s^2} + \frac{2t}{s} \right) \ln \left( \frac{s+t}{s} \right) + \frac{1}{2} \left( \frac{3(s+t)^2}{s^2} - \frac{2t}{s} \right) \ln^2 \left( \frac{s+t}{s} \right) \\
+ \frac{1}{2} \left( \frac{(s+t)^2}{s^2} + \frac{t^2}{s^2} \right) \ln^2 \left( \frac{-t}{s} \right) - \left( \frac{(s+t)^2}{s^2} + \frac{t^2}{s^2} \right) \ln \left( \frac{-t}{s} \right) \ln \left( \frac{s+t}{s} \right) \right] . \]

(3.8)

The corresponding coefficients for the $q\bar{q} \rightarrow \gamma g$ subprocess are:

\[
c_5^{q\bar{q}} = \frac{1}{2} \left( 4C_F - C_A \right)^2
\]

\[
c_4^{q\bar{q}} = -3 \left( 4C_F - C_A \right) C_F \ln \left( \frac{\mu_F^2}{p_T^2} \right) - \frac{\beta_0}{2} \left( 5C_F - \frac{3}{2} C_A \right)
\]

\[
c_3^{q\bar{q}} = (4C_F - C_A) \left[ c_a^{q\bar{q}} \ln \left( \frac{\mu_F^2}{p_T^2} \right) + c_1^{q\bar{q}} \right] + \left[ \frac{\beta_0}{4} + 2C_F \ln \left( \frac{\mu_F^2}{p_T^2} \right) \right]^2
\]

\[+(4C_F - C_A) \frac{\beta_0}{4} \ln \left( \frac{\mu_F^2}{p_T^2} \right) + (4C_F - C_A) \frac{K}{2} + \frac{\beta_0}{16} - \frac{\pi^2}{6} (4C_F - C_A)^2 \] (3.9)

with

\[
c_a^{q\bar{q}} = C_F \left[ -\frac{3}{2} - \ln \left( \frac{p_T^2}{s} \right) \right] \] (3.10)

and

\[
c_1^{q\bar{q}} = \frac{\beta_0}{4} \ln \left( \frac{\mu_F^2}{p_T^2} \right) + \frac{1}{2} K - \frac{7}{2} C_F + \frac{\pi^2}{2} \left( C_F - \frac{C_A}{6} \right)
\]

\[+ C_F \ln \left( \frac{-t}{s} \right) \ln \left( \frac{-u}{s} \right) - C_F \ln^2 \left( \frac{p_T^2}{s} \right) - \frac{3}{2} C_F \ln \left( \frac{p_T^2}{s} \right) \]

\[+ \frac{C_F}{2T_{qq}} \left[ -2u \ln \left( \frac{-t}{s} \right) - 2t \ln \left( \frac{-u}{s} \right) + u \ln \left( \frac{-t}{s} \right) + t \ln \left( \frac{-u}{s} \right) \right] \]
\[ + \left( \frac{3u^2 - 2t}{s^2} - \frac{2t}{s} \right) \ln^2 \left( -\frac{u}{s} \right) + \left( \frac{3t^2 - 2u}{s^2} - \frac{2u}{s} \right) \ln^2 \left( -\frac{t}{s} \right) \]

\[ - N_c \frac{t u}{2 T_{qg}} \left( \frac{tu}{s^2} \ln \left( \frac{tu}{s^2} \right) - \frac{t^2 + 2ts}{2s^2} \ln^2 \left( -\frac{t}{s} \right) - \frac{u^2 + 2us}{2s^2} \ln^2 \left( -\frac{u}{s} \right) \right) \cdot \]

\( (3.11) \)

In the above expressions for \( c_1'qg \) and \( c_1'q\bar{q} \), \( T_{qg} \) and \( T_{q\bar{q}} \) are given by

\[
T_{qg} = \frac{(s^2 + t^2)/s^2}{s^2} \\
T_{q\bar{q}} = \frac{(t^2 + u^2)/s^2}{s^2}. \quad (3.12)
\]

Note that, at NNLO, all the NNLL terms, i.e. \( \mathcal{O} \left( \ln(s_4/p_T^2)/s_4 \right) \) terms, have been derived by matching with the exact NLO cross section. The scale-dependent terms at that accuracy can also be derived by the straightforward expansion of the resummed cross section. The result is the same, which provides a nice cross check. As an additional check, an explicit variation of the hadronic cross section with respect to scale yields zero up to \( (1/s_4)^+ \) terms. From the expansion of the resummed cross section we can also derive the \( (1/s_4)^+ \) terms containing squared logarithms of the scale. These terms in the coefficient \( c_2^2(qg) \) are

\[
\ln^2 \left( \frac{\mu_F^2}{p_T^2} \right) \left( C_F + C_A \right) \left[ C_F \left( \ln \left( -\frac{t}{s} \right) + \frac{3}{4} \right) + C_A \ln \left( -\frac{u}{s} \right) + \frac{3\beta_0}{8} \right] \\
- \frac{\beta_0}{2} \left( C_F + C_A \right) \ln \left( \frac{\mu_R^2}{p_T^2} \right) \ln \left( \frac{\mu_F^2}{p_T^2} \right), \quad (3.13)
\]

while the corresponding terms in the coefficient \( c_2^2(q\bar{q}) \) are

\[
\ln^2 \left( \frac{\mu_F^2}{p_T^2} \right) C_F \left[ C_F \left( 3 + 2 \ln \left( \frac{p_T^2}{s} \right) \right) + \frac{\beta_0}{4} \right] - \beta_0 C_F \ln \left( \frac{\mu_R^2}{p_T^2} \right) \ln \left( \frac{\mu_F^2}{p_T^2} \right). \quad (3.14)
\]

Since we don’t reproduce the full \( c_2^2 \) coefficients, we don’t include them in our numerical results in the next section.

Finally, we note that all gauge dependence cancels explicitly in the NNLO expansions, as expected.
4 Numerical results

In order to examine the phenomenological consequences of the NNLO expressions presented in this paper, the $\mathcal{O}(\alpha_s^2)$ program of Ref. [23] is used as a starting point. This includes the lowest order $2 \rightarrow 2$ subprocesses and their virtual corrections, the $\mathcal{O}(\alpha_s^3)$ $2 \rightarrow 3$ subprocesses, and a fragmentation contribution calculated by convoluting $2 \rightarrow 2$ parton-parton scattering subprocesses with appropriate photon fragmentation functions. The NNLO expressions are then added to the output of this program. It is important to note that the fragmentation contribution has not been modified by the inclusion of any resummed terms. On the other hand, for fixed target energies the fragmentation component is generally small in the $p_T$ region spanned by the data. The results discussed in this section have been obtained with the renormalization and factorization scales set equal to each other. In each case two scale choices have been used, $p_T/2$ and $2p_T$. The CTEQ5M [24] parton distributions have been used. In each of the comparisons presented below, the calculation for the invariant cross section has been averaged over the appropriate region of rapidity or transverse momentum as specified for the data set being shown.

The curves in Fig. 3 show the NLO and NNLO predictions for the invariant cross section for direct photon production using a proton beam on a beryllium target. The predictions are compared with the experimental results at $p_{beam} = 530$ GeV/$c$ from the E-706 Collaboration [25]. The theoretical curves have been multiplied by a nuclear correction factor of 1.09 [25]. Comparing the band formed by the solid curves with that formed by the dashed curves shows that the inclusion of the NNLO contributions reduces the scale dependence by about a factor of two. A similar reduction has been noted in [7]. The origin of this reduction has been discussed in both [7] and [26]. Furthermore, over the $p_T$ range shown in Fig. 3 the NNLO corrections for $\mu = p_T/2$ are rather small. Thus, the inclusion of the threshold resummation corrections is unable to bring the predictions into agreement with these data. As noted in the introduction, it has been argued that initial state radiation can lead to non-zero values for the transverse momentum of the colliding partons and that the inclusion of such effects may bring the prediction into agreement with the data. However, to date this has only been treated in a phenomenological manner [4] and a more definitive theoretical treatment is needed before firm conclusions can be drawn. Some recent work in this
The resummation formalism presented in this paper allows one to study...
Figure 4. NLO and NNLO results for direct photon production in hadronic collisions compared to data from the E-706 Collaboration [25] at $p_{\text{beam}}=800$ GeV/c.

Figure 5. NLO and NNLO results for direct photon production in hadronic collisions compared to $p\bar{p}$ data from the UA-6 Collaboration [28].
Figure 6. NLO and NNLO results for direct photon production in hadronic collisions compared to $pp$ data from the UA-6 Collaboration [28].

Figure 7. NLO and NNLO results for the rapidity distribution for direct photon production in hadronic collisions compared to $p\bar{p}$ data from the UA-6 Collaboration [28].
Figure 8. NLO and NNLO results for the rapidity distribution for direct photon production in hadronic collisions compared to pp data from the UA-6 Collaboration [28].

the dependence on both the transverse momentum and the rapidity of the photon. In Figs. 7 and 8 the predictions are compared to the rapidity distributions published by the UA-6 Collaboration [28] for pP and pp collisions, respectively. Comparing the NLO and NNLO results shows that the rapidity dependence of the relative size of the NNLO corrections is mild. In each case the reduced scale dependence is evident.

5 Conclusion

In this paper the application of threshold resummation to direct photon production has been studied using the formalism of Refs. [3 4]. The expressions for the moments of the resummed cross section have been expanded to $\mathcal{O}(\alpha\alpha_s^3)$ and expressions for the leading, next-to-leading, and next-to-next-to-leading logarithmic contributions have been given. The NNLL estimates for the $\mathcal{O}(\alpha\alpha_s^3)$ contributions have been incorporated into an existing next-to-leading logarithm $\mathcal{O}(\alpha\alpha_s^2)$ program and compared to high statistics fixed
target data. The scale dependence of the results is markedly decreased by the inclusion of the NNLO terms. Furthermore, it was found that the size of the corrections is small when the factorization and renormalization scales are chosen to be $\mu = p_T/2$. Nevertheless, even with the inclusion of these threshold corrections, it is still not possible to achieve a good description of all of the fixed target data. Additional study of this process will be required, both theoretically and experimentally.

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A Alternative form of the NNLO expansion

In this Appendix we rewrite our NLO and NNLO expansions in terms of the variables $v = (s + t)/s$ and $w = -u/(s + t)$ used in Ref. [13].

The factorized form of the cross section for direct photon production can be rewritten as:

$$E_\gamma \frac{d^3\sigma_{h_A h_B \rightarrow \gamma X}}{d^3 p_\gamma} = \sum_f \int dx_a dx_b \phi_{f_a/h_A}(x_a, \mu_F^2) \phi_{f_b/h_B}(x_b, \mu_F^2) \times \frac{1}{\pi sv} \frac{d\hat{\sigma}_{f_a h_b \rightarrow \gamma X}}{dv dw}(w, s, v, \mu_F, \alpha_s(\mu_F^2)). \quad (A.1)$$

The threshold region is given by $w = 1$. In general, $\hat{\sigma}$ includes distributions with respect to $1 - w$ at $n$th order in $\alpha_s$ of the type

$$\left[ \frac{\ln^m(1-w)}{1-w} \right]_+, \quad m \leq 2n - 1. \quad (A.2)$$

Before presenting the NNLO expansions of the resummed cross section in terms of the variables $v$ and $w$, we want to emphasize that crossing $t$ and $u$ mixes singular and non-singular terms in $1 - w$. The non-singular terms are required in order to have the correct symmetrization under $t \leftrightarrow u$ away from threshold. Therefore in producing numerical results we use the expansions
in terms of the variable $s_4$ as presented in Section 3, where the symmetry is manifest and there is no mixing between singular and non-singular terms.

It is convenient to express the results in a form similar to that in Eq. (3.1):

$$vw(1-v)s \frac{d\tilde{\sigma}^{\text{MS}}_{ij\rightarrow\gamma+X}}{dv dw} = \tilde{\sigma}^B_{ij}(s,v)$$

$$\times \left\{ \delta(1-w) + \sum_{k=1}^{2} \left( \frac{\alpha_s(\mu^2_R)}{\pi} \right)^k \left[ c_1^k \delta(1-w) + \sum_{l=0}^{2k-1} c_{l+2}^k \left( \frac{\ln(1-w)}{1-w} \right)^l \right] \right\}.$$  \hfill (A.3)

The expressions for the two Born terms are

$$\tilde{\sigma}^B_{qq}(s,v) = \frac{1}{N_c} \pi \alpha_s e_q^2 T_{qq}v$$  \hfill (A.4)

and

$$\tilde{\sigma}^B_{qg}(s,v) = \frac{2C_F}{N_c} \pi \alpha_s e_q^2 T_{qg}$$  \hfill (A.5)

where $T_{qq} = 1 + (1-v)^2$ and $T_{qg} = v^2 + (1-v)^2$.

The coefficients for the NLO terms are as follows:

$$c_3^{qg} = C_F + 2C_A$$

$$c_2^{qg} = C_F \left( -\frac{3}{4} + \ln v \right) - C_A \ln \left( \frac{1-v}{v} \right) - (C_F + C_A) \ln \left( \frac{\mu^2_F}{s} \right)$$

$$c_1^{qg} = \left( -\frac{\beta_0}{4} - \frac{3}{4} C_F + C_A \ln \left( \frac{1-v}{v} \right) \right) \ln \left( \frac{\mu^2_F}{s} \right) + \frac{\beta_0}{4} \ln \left( \frac{\mu^2_R}{s} \right)$$  \hfill (A.6)

and

$$c_3^{qg} = (4C_F - C_A)$$

$$c_2^{qg} = -2C_F \ln \left( \frac{1-v}{v} \right) + C_A \ln(1-v) - \frac{\beta_0}{4} - 2C_F \ln \left( \frac{\mu^2_F}{s} \right)$$

$$c_1^{qg} = C_F \left( -\frac{3}{2} + \ln \left( \frac{1-v}{v} \right) \right) \ln \left( \frac{\mu^2_F}{s} \right) + \frac{\beta_0}{4} \ln \left( \frac{\mu^2_R}{s} \right) .$$  \hfill (A.7)
These results are in full agreement with the exact NLO results in Ref. [13]. Note that the expansion reproduces only the scale-dependent terms in $c_1^1$. The full $\delta(1-w)$ terms can be simply read off from the exact NLO calculation.

The coefficients for the NNLO corrections for the $qg \to \gamma q$ subprocess are:

$$
c_2^2 (gg) = \frac{1}{2} (C_F + 2C_A)^2
$$

$$
c_3^2 (gg) = \frac{3}{2} (C_F + 2C_A) \left[ C_F \left( -\frac{3}{4} + \ln v \right) - C_A \ln \left( \frac{1-v}{v} \right) \right.
$$

$$
- (C_F + C_A) \ln \left( \frac{\mu_F^2}{s} \right) \left] - \frac{\beta_0}{2} \left( \frac{C_F}{4} + C_A \right) \right.
$$

$$
c_3^3 (gg) = (C_F + 2C_A) \left[ c_{a}^{gg} \ln \left( \frac{\mu_F^2}{s} \right) + c_{1}^{gg} \right]
$$

$$
+ \left[ C_F \left( -\frac{3}{4} + \ln v \right) - C_A \ln \left( \frac{1-v}{v} \right) - (C_F + C_A) \ln \left( \frac{\mu_F^2}{s} \right) \right]^2
$$

$$
+ (C_F + 2C_A) \frac{\beta_0}{4} \ln \left( \frac{\mu_R^2}{s} \right) + \frac{1}{2} K(C_F + 2C_A)
$$

$$
+ \beta_0 \left[ \frac{C_F}{4} \left( \frac{3}{4} - \ln v \right) + \frac{C_A}{2} \ln \left( \frac{1-v}{v} \right) \right]
$$

$$
- \frac{\pi^2}{6} (C_F + 2C_A)^2, \quad (A.8)
$$

with [13]

$$
c_{a}^{gg} = -\frac{\beta_0}{4} - \frac{3}{4} C_F + C_A \ln \left( \frac{1-v}{v} \right) \quad (A.9)
$$

and

$$
c_{1}^{gg} = \frac{\beta_0}{4} \ln \left( \frac{\mu_R^2}{s} \right) - \frac{7}{4} C_F
$$

$$
+ \frac{N_c}{T_{qg}} \left[ -\frac{1}{4} (v^2 - 2(1-v)) \ln^2 v + \frac{1}{2} (1-v) \ln(1-v) - (1-v) \ln v
$$

$$
+ \frac{1}{4} \pi^2 (1-v^2) + \frac{1}{4} (1-v^2) \ln^2 (1-v) - \frac{1}{2} (1-v^2) \ln v \ln(1-v) \right]
$$

$$
+ \frac{C_F}{T_{qg}} \left[ \frac{1}{2} (1+2v) \ln(1-v) + \frac{\pi^2}{6} (1-4v + 5v^2) \right]
$$
\[
- \frac{1}{4} (3v^2 - 2(1 - v)) \ln v + \frac{1}{2} \left(3v^2 + 2(1 - v)\right) \ln^2 v
+ \frac{1}{2} \left(v^2 + (1 - v)^2\right) \ln^2 (1 - v) - \left(v^2 + (1 - v)^2\right) \ln v \ln(1 - v) \].
\]

The corresponding NNLO terms for the subprocess \(q\bar{q} \rightarrow \gamma g\) are:

\[
\tilde{c}^2_5(q\bar{q}) = \frac{1}{2} (4C_F - C_A)^2
\]

\[
\tilde{c}^2_4(q\bar{q}) = \frac{3}{2} (4C_F - C_A) \left[-2C_F \ln \left(\frac{1 - v}{v}\right) + C_A \ln(1 - v)
- 2C_F \ln \left(\frac{\mu^2_F}{s}\right) - \frac{\beta_0}{2} \left(5C_F - \frac{3}{2}C_A\right)\right]
\]

\[
\tilde{c}^2_3(q\bar{q}) = (4C_F - C_A) \left[c^q_{\bar{q}} \ln \left(\frac{\mu^2_F}{s}\right) + c^q_1\right]
+ \left[-2C_F \ln \left(\frac{1 - v}{v}\right) + C_A \ln(1 - v) - \frac{\beta_0}{4} - 2C_F \ln \left(\frac{\mu^2_F}{s}\right)\right]^2
+ (4C_F - C_A) \beta_0 \ln \left(\frac{\mu^2_F}{s}\right) + (4C_F - C_A) \frac{K}{2} + \frac{\beta_0^2}{16}
+ \beta_0 \left[C_F \ln \left(\frac{1 - v}{v}\right) + C_A \left(\ln v - 2 \ln(1 - v)\right)\right]
- \frac{\pi^2}{6} (4C_F - C_A)^2,
\]\n
with \[13\]

\[
c^q_{\bar{q}} = C_F \left[-\frac{3}{2} + \ln \left(\frac{1 - v}{v}\right)\right]
\]

and

\[
c^q_1 = \frac{\beta_0}{4} \ln \left(\frac{\mu^2_R}{s}\right) + \frac{1}{2} K - \frac{7}{2} C_F + \frac{\pi^2}{2} \left(C_F - \frac{C_A}{6}\right)
+ \frac{n_f}{6} \ln v - C_F \ln v \ln(1 - v)
+ \frac{C_F}{2T_{q\bar{q}}} \left[v(2 + v) \ln(1 - v) + (1 - v)(3 - v) \ln v
+ (3v^2 + 2(1 - v)) \ln^2 v + (2v + 3(1 - v)^2) \ln^2(1 - v)\right]
\]
\[- \frac{N_c}{2 T_{qg}} \left[ v(1 - v) \ln(1 - v) + \frac{(11 - 16v(1 - v))}{6} \ln v \right. \\
+ \left. \frac{(2v + 3(1 - v)^2)}{2} \ln^2(1 - v) + \frac{v(2 - v)}{2} \ln^2 v \right]. \]

(A.13)

Note that, at NNLO, we have derived all NNLL terms, i.e. all the \( \mathcal{O} \left[ \ln(1 - w)/(1 - w) \right] \) terms, by matching with the exact NLO cross section in Ref. \[13\]. The scale-dependent terms at that accuracy have also been derived by the expansion of the resummed cross section, giving the same result. As an additional check, an explicit variation of the hadronic cross section with respect to scale yields zero up to \([1/(1 - w)]_+\) terms. From the expansion of the resummed cross section we can also derive the \([1/(1 - w)]_+\) terms containing squared logarithms of the scale. These terms in the coefficient \( \tilde{c}_2^{2\langle qg \rangle} \) are

\[
\ln^2 \left( \frac{\mu_F^2}{s} \right) (C_F + C_A) \left[ \frac{3}{4} C_F - C_A \ln \left( \frac{1 - v}{v} \right) + \frac{3\beta_0}{8} \right] \\
- \frac{\beta_0}{2} (C_F + C_A) \ln \left( \frac{\mu_R^2}{s} \right) \ln \left( \frac{\mu_F^2}{s} \right),
\]

(A.14)

while the corresponding terms in the coefficient \( \tilde{c}_2^{2\langle q\bar{q} \rangle} \) are

\[
\ln^2 \left( \frac{\mu_F^2}{s} \right) C_F \left[ C_F \left( 3 - 2 \ln \left( \frac{1 - v}{v} \right) \right) + \frac{\beta_0}{4} \right] - C_F \beta_0 \ln \left( \frac{\mu_R^2}{s} \right) \ln \left( \frac{\mu_F^2}{s} \right).
\]

(A.15)

Finally, we note that all gauge dependence cancels explicitly in the NNLO expansions, as expected.

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