Can the Type-IIB axion prevent Pre-big Bang inflation?

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We look at the possibility of superinflationary behavior in a class of anisotropic Type-IIB superstring cosmologies in the context of Pre-big Bang scenario and find that there exists a rather narrow range of parameters for which these models inflate. We then show that, although in general this behavior is left untouched by the introduction of a Ramond-Ramond axion field through a $SL(2,\mathbb{R})$ rotation, there exists a particular class of axions for which inflation disappears completely. Asymptotic past initial conditions are briefly discussed, and some speculations on the possible extension of Pre-big Bang ideas to gravitational collapse are presented.
The Pre-big Bang scenario developed recently within the context of string theory represents an interesting alternative to the standard inflationary paradigm \[1\]. In this approach a massless scalar field, the dilaton, drives an accelerated expansion towards what we normally think of as the initial cosmological singularity. It is believed that near the singularity string non-perturbative physics takes over and ensures a smooth transition to the next phase where the standard Friedman-Robertson-Walker expansion would take place. While until very recently most investigations were concentrated on the details of this transition, the so-called graceful exit problem \[2\], it has been lately realized that the problem of the susceptibility of the Pre-big Bang phase to initial conditions remains the cornerstone of the scenario. Intuitively, the Pre-big Bang phase may be robust to small irregularities induced by anisotropies and inhomogeneities. This belief is based on the fact that during the Pre-big Bang phase the cosmological model undergoes a period of accelerated expansion so that irregularities, if any, are likely to be ”inflated away”. In this spirit, Veneziano and collaborators \[3\][4] have shown that sufficiently smooth regions of spacetime undergo inflation during the Pre-big Bang phase in spite of the presence of small deviations from homogeneity and isotropy. On the other hand, Turner and Weinberg \[5\] have looked at the effects of the curvature on the Pre-big Bang inflation and have found that large spatial curvature may postpone the onset of inflation, interpreting this result as a restriction on the possible initial conditions. To further study the effects of anisotropy and inhomogeneity a number of exact anisotropic and inhomogeneous solutions have been found \[6\][7] and general algorithms to construct these solutions were designed \[8\].

In a recent paper Clancy et al. \[9\] considered a class of exact anisotropic cosmological solutions which contains Bianchi types III, V and VI\(_h\) models focusing their study on the combined effects of spatial anisotropy and curvature on the Pre-big Bang period. They have found that large regions in the parameter space exist where the occurrence of inflation is prevented, and that the conditions for successful inflation, as compared with the negatively curved FRW model, are stringent.

Now, although the Pre-big Bang scenario is string theory induced, surprisingly, most of the work in Pre-big Bang cosmology is done in the so-called Brans-Dicke sector, where the only addition to general relativity is the presence of a dilaton field. Yet, the low energy sector of the string theory may generally include various massless fields, and for example in the case of the Type-IIB superstring one expects to find along with the scalar dilaton a pseudoscalar axion field \(\chi\) (among other fields) in the Ramond-Ramond sector of the
theory. This pseudoscalar field is rather interesting due to its peculiar coupling to the
dilaton and its presence, in principle, might play a crucial role in damping Pre-big Bang
inflation.

The main purpose of this letter is to study the possible effects derived from the
presence of the pseudoscalar axion field in the Type-IIB superstring spectrum on the
occurrence of the Pre-big Bang inflation in anisotropic cosmological models. To do so, we
consider an exact class of anisotropic Kantowski-Sachs cosmologies; first, in the presence of
the dilaton field only, and then adding the pseudoscalar axion field into the picture. In the
case of the dilaton field alone, we confirm the behaviour found in [9] in that the range of
the parameters producing inflationary behaviour is quite narrow. The introduction of the
pseudoscalar axion field, however, may produce a devastating effect on inflation reducing
the range of the possible parameters to an empty set. Fortunately for the scenario, the
effect is restricted to a special class of axions. Our test cosmologies, Kantowski-Sachs
models, are of interest in this context for they combine the effects of anisotropy and non-
trivial curvature, furnishing the conclusions we arrive at with quite a general character.
On the other hand, these models are simple enough so that one can integrate the exact
solutions explicitly and study their behavior analytically.

We start with the following family of Kantowski-Sachs metrics in string frame

$$ds^2 = -\left(\frac{2\eta}{t} - k\right)^{-\alpha} dt^2 + \left(\frac{2\eta}{t} - k\right)^{\beta} dx^2 + \left(\frac{2\eta}{t} - k\right)^{1-\alpha} t^2 \left[d\theta^2 + f_k(\theta)^2 d\varphi^2\right]$$ (1)

coupled to the dilaton field

$$\phi = \frac{1}{4}(\beta - \alpha) \log \left(\frac{2\eta}{t} - k\right).$$ (2)

This is an exact solution of dilaton gravity provided the constants $\alpha$ and $\beta$ satisfy

$$\alpha^2 + \beta^2 = 2.$$

Here $k$ is the spatial curvature ($k = -1, 0, 1$) and

$$f_k(\theta) = \begin{cases} 
\sin \theta & k = 1 \\
\theta & k = 0 \\
\sinh \theta & k = -1
\end{cases}.$$

The family of metrics defined by eqs. (1) and (2) can be readily obtained by starting with
vacuum Kantowski-Sachs cosmologies (corresponding here to $\alpha = \beta = 1$) and dressing
those with a massless scalar field with the subsequent frame change along the first steps
of a general algorithm given in [8].

Incidentally, the solution (1) is equivalent to the one recently given by Barrow and
Dabrowski [9], who used different parametrization and integrated the equations of motion
directly. The explicit simple solution integrated by these authors (eqs. 3.9-3.12 of ref. [7])
are obtained just by setting $\alpha = 0$ in eq. (1) and taking into account the different
normalization of the dilaton fields.

In ref. [10] the scale factor duality of anisotropic Kantowski-Sachs models was studied
with the result that the only discrete symmetry of the dynamical equations consisted in
inverting the scale factor in the $x$-direction. In (1) and (2) this corresponds to flipping the
sign of $\beta (\beta \rightarrow -\beta)$.

At this point we feel that it is important to clarify some differences between scale
factor duality and T-duality. In scale factor duality the inversion of one or more scale
factors in the metric leaves the equations of motion invariant (or equivalently modifies the
low-energy action by a total derivative). T-duality, on the other hand, is a transformation
of the metric, not necessarily only of the scale factors, leading to a new solution of the
low-energy field equations which corresponds to an equivalent string theory. In the metric
(1) we have four Killing vectors

$$\xi_1 = \sin \varphi \partial_\theta + \frac{f_k'(\theta)}{f_k(\theta)} \cos \varphi \partial_\varphi$$

$$\xi_2 = \cos \varphi \partial_\theta - \frac{f_k'(\theta)}{f_k(\theta)} \sin \varphi \partial_\varphi$$

$$\xi_3 = \partial_\varphi$$

$$\xi_4 = \partial_x.$$  

It is straightforward to see that the Killing vector $\xi_4$ commutes with all the others. Compactifying the $x$ direction we have, in the particular case of the positively curved Kantowski-
Sachs model ($k = 1$), that the total isometry group is $U(1)_x \times SO(3)$, and its maximal
abelian subalgebra is generated by $\partial_x$ and $\partial_\varphi$. T-dualizing with respect to this pair of
commuting Killing vectors we arrive at a dual manifold in which the only Killing vectors
left are $\partial_{\tilde{x}}$ and $\partial_{\tilde{\varphi}}$, with $\tilde{x}$ and $\tilde{\varphi}$ being the dual coordinates. Any other isometry not
commuting with $\xi_3$ and $\xi_4$ will be lost in the sense that it will not be locally realized in the
dual manifold any longer [11]. Therefore, the degree of homogeneity may be reduced by
just performing a T-duality transformation \[8\). Consequently, for the model described by the line element (1) with a cyclic \(x\) coordinate, the only T-duality transformation leaving the spatial symmetry group intact is the one performed along the compactified isometric direction \(x\), which in this case is formally identical to the scale factor duality of the metric (1). In the case of scale factor duality, however, there is no need to impose the compactness of the \(x\)-coordinate since the “dual” model does not necessarily have to be equivalent to the original one as complete string theories.

We now look at the possible range of the parameters leading to inflationary periods for the metric (1). Due to the invariance of the low-energy field equations under time reversal, and in order to keep expressions simple, we will use positive time to label the Pre-big Bang phase, so that the approach to the Big-bang singularity from this phase corresponds to \(t \to 0^+\) and the asymptotic past infinity is mapped into the asymptotic future \(t \to \infty\). By expanding the solution near \(t = 0\) and using co-moving time \(T\) we find the following form for the metric

\[
ds^2 = -dT^2 + a_1^2 T^{-\frac{2\beta}{\alpha+2}} dx^2 + a_2^2 T^{\frac{\alpha+1}{\alpha+2}} [d\theta^2 + f_k(\theta)^2 d\phi^2]
\]

with \(a_1^2, a_2^2\) two constants and the dilaton being

\[
\phi(T) = -\frac{1}{2} \frac{\beta - \alpha}{\alpha + 2} \log T.
\]

Close to \(T = 0\) we find that the scalar curvature scales with time as \(R \approx T^{-2}\), while the behavior of the dilaton does depend on the particular values of \(\alpha\) and \(\beta\) and the effective string coupling in this limit is

\[
geff = e^{\phi(T)} \approx T^{-\frac{1}{2} \frac{\beta - \alpha}{\alpha + 2}}.
\]

When \(\alpha > \beta\), \(geff \to 0\) and we have that near \(T = 0\) the string theory is weakly coupled in spite of having a curvature singularity. In the other case \(\alpha < \beta\) we have the reverse situation in which both the curvature and the effective string coupling blow up when the limit \(T = 0\) is approached. The degenerate case \(\alpha = \beta = \pm 1\) corresponds to have a vanishing dilaton. Since one of the characteristics of the Pre-big Bang scenario is that the approach to the curvature singularity from the Pre-big Bang phase is described by a
Fig. 1: The thick line represents the set of parameters for which inflation is possible.

strongly coupled string theory, we will restrict in the following our attention to the latter case $\beta > \alpha$.

Let us now define the averaged scale factor

$$\tilde{R}(T) = [a_1 a_2^2]^{1/4} T^{\frac{2\alpha - \beta + 2}{3(\alpha + 2)}}$$

Pre-big Bang cosmology is characterized by the superinflationary behavior achieved in the case of negative exponents of the scale factor. Thus, the conditions for “averaged” inflation are

$$\frac{2\alpha - \beta + 2}{3(\alpha + 2)} < 0, \quad \alpha^2 + \beta^2 = 2$$

In fig. 1 we represent the points in parameter space satisfying both conditions. For all points on the circumference lying above the line $2\alpha - \beta + 2 = 0$ the exponent of the co-moving time in the averaged scale factor will be negative and thus will represent metrics undergoing accelerated expansion as $T$ approaches zero. On the other hand in fig. 2 we plot those points for which both scale factors undergo inflation in the $T \to 0$ limit.

To introduce the Type-IIIB axion into the picture we perform a $SL(2, \mathbb{R})$ rotation on the graviton-dilaton system (1), (2). From [8] we find the new metric, dilaton and axion to be

$$ds'^2 = [d^2 + c^2 e^{-2\phi(t)}]^{1/2} ds^2$$

$$e^{-\phi'(t)} = \frac{e^{-\phi(t)}}{[d^2 + c^2 e^{-2\phi(t)}]^{1/4}}$$

$$\chi'(t) = \frac{bd + ac e^{-2\phi(t)}}{[d^2 + c^2 e^{-2\phi(t)}]^{1/4}}, \quad (3)$$
where $a, b, c, d \in \mathbb{R}$ and $ad - bc = 1$. Since as the result of the rotation (3) the metric changes conformally, it can affect in principle the inflationary behavior near the singularity. Notice, however, that four-dimensional $SL(2, \mathbb{R})$ transformations do not invert the string coupling constant, as it is the case in ten-dimensions (cf. [12]).

Let us first analyze the family of transformations with $d = 0$, for which the metric transforms non-trivially near the singularity. The conformal transformation of the metric is just given by

$$ds'_2 = |c|e^{-\phi(t)}ds^2.$$ 

Passing to co-moving time and expanding for small $T$ we have

$$ds'_2 = -dT^2 + b_1^2T^{-2}\frac{\alpha + 3\beta}{3\alpha + 3\beta + 8}dx^2 + b_2^2T^{-2}\frac{3\alpha + 3\beta + 4}{3\alpha + 3\beta + 8}[d\theta^2 + f_k(\theta)^2d\varphi^2]$$

where we have absorbed the $c$-dependence by redefining the constants $a_1^2$ and $a_2^2$. Evaluating the average scale factor we find

$$\tilde{R}'(T)_{d=0} = [b_1b_2^2]^{\frac{1}{2}}T^{-\frac{8\alpha + 5\beta - \beta}{3\alpha + 3\beta + 8}}$$

(4)

for all $\alpha, \beta$ such that $\alpha^2 + \beta^2 = 2$.

To have inflation the exponent of the averaged scale factor $\tilde{R}'(T)$ in (4) must be negative. Plotting these conditions in fig. 3 we find that there are no allowed values of the parameters $\alpha$ and $\beta$ for which the model undergoes inflation as $T \to 0$. We thus arrive at the conclusion that the introduction of the Type-IIB axion via a $SL(2, \mathbb{R})$ rotation of the
Fig. 3: $SL(2, \mathbb{R})$-induced dilatons with $d = 0$ remove averaged inflation.

graviton-dilaton system with $d = 0$ completely topples the Pre-big Bang inflation in the family \((1)\) of Kantowski-Sachs metrics.

Fortunately, things are different when we introduce the axion using a $SL(2, \mathbb{R})$ with $d \neq 0$. The $T \to 0$ behavior of the conformal factor multiplying the metric will depend on the limit of the dilaton field $\phi(T)$ close to the singularity. Since we are restricting ourselves to the case $\alpha < \beta$, the dilaton field blows up as $T = 0$ is approached, and the effect of the $SL(2, \mathbb{R})$ rotation on the metric is just equivalent in this limit to a constant rescaling of the metric tensor. Reabsorbing constant factors we arrive at a metric equivalent to the one we started with

$$ds^2_{d \neq 0} = -dT^2 + b_1^2 T^{-\frac{2\beta}{\alpha+2}} dx^2 + b_2^2 T^{2\frac{\alpha-1}{\alpha+2}} [d\theta^2 + f_k(\theta)^2 d\varphi^2],$$

where, as above, $\alpha^2 + \beta^2 = 2$ and $\alpha < \beta$. Thus, generically, the conditions for inflation are left unchanged by the presence of a non-trivial pseudoscalar axion field. We will not dwell here on the influence of the axion field on the onset of inflation \[5\], but it is clear that even when $d \neq 0$ the axion might play a role in determining the period of time in which inflation may occur away from $T \to 0^+$. 

Another interesting question to consider is the asymptotic past behavior of the cosmological solutions under study. As we have already mentioned, field equations are symmetric under time reversal, and the asymptotic past corresponds to taking $t \to \infty$ in eqs. \[1\] and \[2\]. Recently, Buonanno et al. \[4\] have suggested that the Milne Universe could represent a past attractor for the cosmological models with negative spatial curvature, provided departures from homogeneity and isotropy are sufficiently small. In our view, it is important to stress that the key ingredient in the whole argument must be the smallness and special
character of these deviations, for one cannot really expect a generic cosmological model
to approach Milne geometry in the asymptotic past in the presence of the massless fields
typical of string theory (see as well [13]). In fact, one may invoke the same arguments used
in ref. [14] to discard Misner’s chaotic cosmology program on the grounds that the measure
of anisotropic cosmological solutions, let alone the inhomogeneous ones, approaching the
isotropic regime (Milne in this case) is zero. The models we have looked at with $k = -1$
fall into the class with sufficiently isotropic initial data in the sense of ref. [4], since they
approach a wedge of Minkowski space-time in the distant past where the dilaton is con-
stant. This is not a suprise after all, because these models are in fact of Bianchi type III,
belonging to the class of cosmologies recently studied in ref. [8].

For the spatially flat case ($k = 0$), we are dealing with a locally rotationally symmetric
Bianchi type I model and, in this case, the line element does not approach a vacuum
solution in the infinite past, as can be seen from the fact that the dilaton blows up in that
limit.

The case $k = 1$ represents quite a different physical situation. Though these models
seem rather unsuitable to describe a Pre-big Bang universe due to the fact that they
collapse at a finite time in the past, yet the solution might be of particular interest in the
study of gravitational collapse. In the special case when $\alpha = \beta = k = 1$ the metric (1) is
just the Schwarschild solution inside the horizon

$$ds^2 = -\frac{dt^2}{(2\eta - t - 1)} + \left(\frac{2\eta}{t} - 1\right) dx^2 + t^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$

in the presence of a constant dilaton field. One may wonder whether it is possible to
apply a kind of Pre-big Bang “graceful exit” solution in order to regularize within string
theory the semiclassical black hole singularity at $t = 0$ ($r = 0$ in Schwarschild coordinates).
Then, the solution across the singularity would be related by scale factor duality and time
reversal to the solution (3) an will be given by ($t < 0$)

$$ds^2 = -\frac{-dt^2 + dx^2}{(2\eta - t - 1)} + t^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$

along with a non-trivial dilaton field

$$\phi = -\frac{1}{2} \log \left(\frac{2\eta}{-t} - 1\right).$$
A potential problem with this picture is that the dilaton goes to minus infinity as \( t \to 0^- \), thus resulting in a weakly coupled string theory near the singularity. This makes it difficult, though not impossible, to invoke non-perturbative string physics as the smoothening mechanism near \( t = 0 \). The problem might be bypassed, in principle, for spherically symmetric black holes in more than six space-time dimensions, where a \( SL(2, \mathbb{R}) \) transformation introducing the axion may invert the string coupling. At any rate, the presence of a curvature singularity implies that the perturbative expansion in powers of \( \alpha' \) breaks down and some kind of non-perturbative worldsheet physics has to be incorporated.

To summarize, we have presented an exact and simply parametrized class of Kantowski-Sachs cosmological models in dilaton gravity and studied them in the context of the Pre-big Bang scenario. We have found that there is a narrow range of parameters leading to Pre-big Bang inflation and that the introduction of a particular class of Type-IIB axions via a \( SL(2, \mathbb{R}) \) rotation may reduce the range of parameters compatible with inflation to an empty set. Generically, though, it seems that inflation will survive the presence of the Type-IIB axion. We have also briefly discussed the asymptotic past behavior of these models, and have concluded that the open models \((k = -1)\) have a Milne geometry as a past attractor. Contemplating with the possibility of extending the ideas of Pre-big Bang scenario to a closed case \((k = 1)\), we were led to speculate about the fate of space-time beyond the Schwarzschild singularity. In this line of thought, provided there exists a graceful exit solution to overpass the Big Bang singularity there is no reason to discard a similar mechanism to smoothly extend space-time beyond the singularities arising in gravitational collapse.

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