Magnetized Particle Capture Cross Section for Braneworld Black Hole

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Abstract

Capture cross section of magnetized particle (with nonzero magnetic moment) by braneworld black hole in uniform magnetic field is considered. The magnetic moment of particle was chosen as it was done by de Felice and Sorge (2003) and for the simplicity particle with zero electric charge is chosen. It is shown that the spin of particle as well as the brane parameter are to sustain the stability of particles circularly orbiting around the black hole in braneworld i.e. spin of particles and brane parameter try to prevent the capture by black hole.

Keywords capture cross section

1 Introduction

The study of the test particle motion and fields attracts more attention of the astrophysicists because they play an important role in the study of astrophysical compact objects, particularly black holes. The motion of particles near compact stars and black holes has been widely reviewed by Poisson (2004); Enolski et al. (2011); Hartmann et al. (2011); Hackmann et al. (2011); Kovar et al. (2011); Stuchlik and Hledík (2000). Recently, motion of a particle around the black hole (de Felice and Sorge 2003; de Felice et al. 2004; Preti 2004a). Under these assumptions, one can say that the formation of a highly charged black holes is incredibly in astrophysical conditions (Novikov et al. 1998). There are several observations showing that there are various scenarios where the magnetic fields and general relativity can not be neglected. One of them is the presence of strong magnetic fields in active galactic nuclei. Another scenario is the production of relativistic collimated jets in the inner regions of accretion discs, which can be explained considering magneto-centrifugal mechanisms. The interplay between gravitational and electromagnetic interaction is essential for characteristics of the motion, namely its stability properties. Motivation for these studies arises from the problem of motion and acceleration of test particles. The study of the interaction between particles and electromagnetic fields in curved spacetimes is also of astrophysical interest, such is the case of strong synchrotron radiation emerging galactic cores, which can be explained admitting the existence in those regions of extended and very intense magnetic fields, interacting with ultrarelativistic electrons. Such magnetic fields could originate in the inner part of an accretion disc around the central black hole.

First attempts of building multidimensional models were proposed by Kaluza (1921) in order to unify elec-
tromagnetism with gravity. Then these ideas found reflection in elegant string theory which is the subject of extensive research in modern physics and promise to throw light upon many puzzles of nature. One of the most recent theories including extra dimensions is the braneworld picture of the Universe.

The braneworld model was first proposed by Randall and Sundrum (1999) assuming that our four-dimensional space-time is just a slice of five-dimensional bulk. According to this model only gravity is the force which can freely propagate between our space-time and bulk while other fields are confined to four-dimensional Universe. In this view it is noteworthy to look for effects of the fifth dimension on our world in frame of theory of gravity i.e. general relativity. Possible tools for proving braneworld model should be found from astrophysical objects, namely, compact objects, for which effects of general relativity are especially strong. For example, investigations of cosmological and astrophysical implications of the braneworld theories have been done by Maartens (2004); Campos and Sopuerta (2001); Langlois (2001); Harko and Mak (2003); Gergely (2006); Kovacs and Gergely (2008); Majumdar and Mukherjee (2003). Review of braneworld models is given e.g. in (Maartens 2004).

For astrophysical interests, static and spherically symmetric exterior vacuum solutions of the braneworld models were initially proposed by Dadhich et al. (2000) which have the mathematical form of the Reissner-Nordström solution, in which a tidal Weyl parameter $Q^*$ plays the role of the electric charge squared of the general relativistic solution.

Observational possibilities of testing the braneworld black hole models at an astrophysical scale have been intensively discussed in the literature during the last several years, for example, through the gravitational lensing (Pal and Kar 2005), the motion of test particles (Abdujabbarov et. al 2010; Schee and Stuchlik 2004), and the classical tests of general relativity (perihelion precession, deflection of light, and the radar echo delay) in the Solar System (see Böhmer et al. 2008). The energy flux, the emission spectrum, and accretion efficiency from the accretion disks around several classes of static and rotating braneworld black holes have been obtained by Pun et al. (2008). The complete set of analytical solutions of the geodesic equation of massive test particles in higher dimensional spacetimes which can be applied to braneworld models is provided in the recent paper Hackman et al. (2008).

The structure of electromagnetic field of spherically and slowly rotating magnetized star in a Randall-Sundrum II type braneworld has been considered by Fattoev and Ahmedov (2008); Morozova and Ahmedov (2011) where Maxwell’s equations for the external magnetic field of the slowly rotating star in the braneworld are analytically solved in approximation of small distance from the surface of the star. The braneworld version of the Schwarzschild’s interior solution has been obtained by Ovalle (2010). Plasma magnetosphere surrounding rotating magnetized neutron star in the braneworld has been studied by Morozova et al. (2008). The relativistic quantum interference effects in the spacetime of slowly rotating object in braneworld as the Sagnac effect and phase shift effect of interfering particle in neutron interferometer are derived by Mamadianov et al. (2010). Recently the magnetized particle motion around black hole in braneworld have been considered by one of the authors of this paper Rahimov (2011).

This paper is organized as follow: in section 2 equations of motion of magnetized particles in the braneworld spacetime are formulated. For this we use Hamilton-Jacobi equation which contains new terms being proportional to the polarization tensor (see, e.g. de Felice and Sorge 2003; Preti 2004a) which characterizes magnetization of particle in the equation of motion. Our main aim consists of checking of influence of this parameter to capture cross section of particle by black hole in braneworld. And in section 3 we derive the analytical expression for capture cross section of magnetized particles by braneworld black hole. Section 4 is devoted to the study particles release from capture by black hole. We conclude our results in Section 5.

Throughout, we use a space-like signature (–, +, +, +) and a system of units in which $G = 1 = c$, Greek indices run from 0 to 3, Latin indices from 1 to 3.

### 2 Equation of motion for spinning particle

The braneworld spacetime metric in the spherical coordinates has the following form (Dadhich et al. 2000):

$$ds^2 = -A^2 dt^2 + H^2 dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2,$$

where

$$A^2 = H^{-2} = \left(1 - \frac{2M}{r} + \frac{Q^*}{r^2}\right),$$

$Q^*$ is the bulk tidal charge and $M$ is the total mass of the central black hole immersed in an exterior asymptotically uniform magnetic field $B_0$. The polar axis is chosen along the direction of $B_0$. The Hamilton-Jacobi equation for the magnetized particles motion has the following form (See, e.g. de Felice and Sorge 2003):

$$g^{\mu\nu} \left( \frac{\partial S}{\partial x^\mu} - q A_\mu \right) \left( \frac{\partial S}{\partial x^\nu} - q A_\nu \right) = m^2 + m D^{\mu\nu} F_{\mu\nu},$$

(2)
where \( q \) and \( m \) are charge and mass of particle, \( A_\alpha \) is the potential of the electromagnetic field and \( F_{\mu\nu} \) is the electromagnetic field tensor. Nonvanishing components of the electromagnetic field tensor \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \) are (de Felice and Sorge 2003):

\[
F_{r\varphi} = B_0 r \sin^2 \theta, \quad F_{\varphi r} = B_0 r^2 \sin \theta \cos \theta. \tag{3}
\]

The polarization tensor related to antisymmetric spin tensor \( S^{\mu\nu} \) is:

\[
D^{\mu\nu} = \frac{q}{m} S^{\mu\nu} = \eta^{\mu\nu\rho\lambda} u_\rho u_\lambda , \tag{4}
\]

where \( u^\mu \) is the 4-velocity and \( \mu^\lambda \) is the magnetic moment 4-vector of the particle, \( \eta^{\mu\nu\rho\lambda} \) is the Levi-Civita tensor.

The electromagnetic field tensor can be expressed through the components of magnetic \( B^\alpha \) and electric \( E^\alpha \) fields as

\[
F_{\mu\nu} = \eta_{\mu\nu\alpha\beta} B^\alpha u^\beta + u_\mu E_\nu - u_\nu E_\mu. \tag{5}
\]

Using equations (3), (4) and (5) one can factorize a constant quantity out of the interaction term \( D \cdot F \) in the following form:

\[
D^{\mu\nu} F_{\mu\nu} = 2 \mu^\alpha B_\alpha = 2 \mu B_0 \cdot \eta , \tag{6}
\]

where \( \mu \) is the norm of the magnetic moment. Here \( \eta \) characterizes magnetization of particle in the following form (de Felice and Sorge 2003):

\[
\eta = \beta \left( 1 - \frac{2M}{r} + \frac{Q^*}{r^2} \right) \left( 1 - \frac{2M}{r} + \frac{Q^*}{r^2} - \Omega^2 r^2 \right)^{-\frac{1}{2}}, \tag{7}
\]

where \( \beta = 2 \mu B_0 / m \) and \( \Omega = d\phi / dt \) is the angular velocity of particle, which is measured by observer at infinity.

It follows from the equation (2) that for the radial motion of particles:

\[
m^2 r^4 \left( \frac{dr}{d\tau} \right)^2 = f(r) , \tag{8}
\]

where

\[
f(r) = (E^2 + m^2 - m^2 \eta^2 + 2M(m^2 - m^2 \eta^2) r^4 + (m^2 Q^* - Q^* - L^2)^2 r^2 + 2MLr - Q^* L^2 , \tag{9}
\]

\( E \) and \( L_0 \) are the energy and angular momentum of the test particle, respectively. Dividing equation (6) to \( m^2 \) one can easily get the polynomial in the form:

\[
f_1 = \frac{f(r)}{m^2} = (e^2 + \eta - 1)r^4 + 2M(1 - \eta)r^3 + (m\eta Q^* - Q^* - L^2)r^2 + 2MLr - Q^* L^2. \tag{10}
\]

where \( e = E/m \) and \( L = L_0/m \) is the specific energy and angular momentum of particle, respectively.

In our case particles are characterized by magnetic moment \( \mu \), but particles have no electric charge, that is in equation (2) \( q = 0 \) and \( \eta \) characterizes the magnetization of the particle.

Considering the polynomial \( f(r) \) for \( r \geq r_+ = 1 + \sqrt{1 - Q^*} \), we can obtain the following three types of motion in the \( r \) coordinate (see, e.g. Zakharov 1980, 1991):

1) the polynomial \( f(r) \) has no roots for \( r \geq r_+ \), the particle then falls into the black hole;

2) the polynomial \( f(r) \) has roots \( r_{\text{max}} > r_+ \), for \( \partial f/\partial r > 0 \) and the particle departs to infinity after approaching the black hole;

3) the polynomial \( f(r) \) has a root and \( \partial f/\partial r = 0 \) and the particle takes an infinite proper time to approach the surface \( r = \text{const} \).

For the relativistic particles (\( e \gg 1 \)) we obtain expression

\[
f(\rho) = (e^2 + \eta)\rho^4 + 2(1 - \eta)\rho^3 - (Q^* + l^2)\rho^2 + 2l^2 \rho - Q^* l^2 , \tag{11}
\]

where \( \rho = r/M, l = L_0/M \) and \( \tilde{Q}^* = Q^*/M^2 \).

In the slow motion limit when \( E = m \), we get the following equation:

\[
f_1(\rho) = \eta \rho^4 + 2(1 - \eta)\rho^3 - (\tilde{Q}^* + l^2)\rho^2 + 2l^2 \rho - \tilde{Q}^* l^2. \tag{12}
\]

3 Capture cross section for magnetized relativistic particles of a braneworld black hole

For relativistic particles we can assume \( \eta \approx 1 \), that is the value of spin is large and one can obtain from (11)

\[
\rho^4 + \left( \frac{l^2}{e^2 + \eta} \right) \rho^2 + \left( \frac{2l^2}{e^2 + \eta} \right) \rho - \frac{\tilde{Q}^* l^2}{e^2 + \eta} = 0 , \tag{13}
\]

since \( \tilde{Q}^* \ll l^2 \) one can neglect the fourth term in the left hand side of the equation. It is known that the symmetric polynomials (when \( n = 4 \)) have the form (see, e.g. Kostrikin 1994)

\[
p_k = X_1^k + X_2^k + X_3^k + X_4^k. \tag{14}
\]

Expressing polynomials \( p_k \) in terms of \( s_k \) and using equation of the Newton according to (see, e.g. Kostrikin 1994), we calculate the polynomials and the discriminant of the family \( X_k \) in roots of the polynomial \( f(\rho) \) as

\[
p_1 = s_1 = 0, p_2 = -2s_2, p_3 = 3s_3, p_4 = 2s_2^2 - 4s_4 \]
\[
p_5 = -5s_3s_2, p_6 = -2s_2^3 + 3s_3^2 + 6s_4s_2. \tag{15}
\]
Using again formula of the Newton (see, e.g., Kostrikin 1994) for polynomials one may obtain the following expression

\[ s_1 = 0, s_2 = -\frac{l^2}{e^2 + \eta}, s_3 = -\frac{2l^2}{e^2 + \eta}, s_4 = -\frac{Q^*l^2}{e^2 + \eta}. \] (16)

Introducing the notation \( \alpha = (e^2 + \eta)^{-1} \), we get from (16) the following expressions for the symmetrical polynomials

\[ p_1 = 0, \quad p_2 = 2l^2\alpha, \quad p_3 = -6l^2\alpha, \quad p_4 = 2l^2\alpha(l^2\alpha + 2\tilde{Q}^*), \quad p_5 = -10l^2\alpha, \quad p_6 = 2l^2\alpha(l^2\alpha + 3\tilde{Q}^* + 6). \] (17)

Calculating the determinant for symmetrical polynomials (see, e.g., Kostrikin 1994) we get equation

\[ \ell^2(1 - \tilde{Q}^*) + \ell(36\tilde{Q}^* - 8\tilde{Q}^{*2} - 27) = 16\tilde{Q}^{*3} = 0, \] (18)

where \( \ell = l^2/(e^2 + \eta) \). If \( \tilde{Q}^* = 0 \) and \( \eta = 0 \), we get for the classical particle (around the Schwarzschild black hole) the well known expression \( \tilde{L}^2 = 27 \) or \( \tilde{L}_{cr} = 3\sqrt{3} \).

For the spinning particles we have from (18):

\[ \frac{l^2}{e^2 + \eta} = 27 \quad \text{for} \quad \tilde{Q}^* = 0, \] (19)

and

\[ \frac{l^2}{e^2 + \eta} = 16 \quad \text{for} \quad \tilde{Q}^* = 1. \] (20)

And solving the quadratic equation (18) we get (we neglect the term being proportional to \( Q^2 \)):

\[ \frac{l^2}{e^2 + \eta} = \frac{27 - 36\tilde{Q}^* + 27(1 - 2, 66\tilde{Q}^*)^{\frac{1}{2}}}{2(1 - \tilde{Q}^*)}. \] (21)

From the equation (21) one may obtain the known expression for the classical particle when \( \eta = 0 \). One may see that when \( e/(l + \eta) \) is constant the particles can move along the classical orbits with the smaller energy than the particles with zero spin (\( \eta = 0 \)).

### 4 Release of particles from black hole in braneworld

We will consider now particles release from capture by black hole, that is, we will write the expression for the critical values of impact parameter and angle of deflection of particles trajectory. Let us rewrite the equation for radial motion of test magnetized particles around black hole in braneworld in external asymptotically uniform magnetic field in the following form:

\[ \left(\frac{d\rho}{d\tau}\right)^2 = e^2 - V^2, \] (22)

where:

\[ V^2 = \left[ \frac{1}{\rho^2} - \beta \left( 1 - \frac{2}{\rho} + \frac{\tilde{Q}^*}{\rho^2} \right)^{1/2} \right] \times \left( 1 - \frac{2}{\rho} + \frac{\tilde{Q}^*}{\rho^2} \right)^{-1/2}, \] (23)

is the effective potential of radial motion of magnetized test particle for rest observer (\( \Omega = 0 \)) in the limiting case when impact parameter \( b = l/e \) and neglecting small terms of \( O(1/L^2) \). In Fig. 1 the radial dependence of the effective potential for radial motion of magnetized particle around black hole in braneworld immersed in external asymptotically uniform magnetic field for the different values of the dimensionless brane parameter \( \tilde{Q}^* \) and magnetic parameter \( \beta \) are shown. It is easy to see that orbits of the particles become more stable with increasing of the parameter \( \beta \). As it is seen from the figure the particle coming from infinity and passing by the source will be not captured: it will be reflected and go to the infinity again. The orbits start to be only parabolic or hyperbolic and no more circular or elliptical orbits exist with increasing the dimensionless parameter \( \beta \), i.e., captured magnetized particles by the central object are going to leave the black hole in braneworld. It should be noted that the influence of the magnetic parameter to the trajectory of the magnetized particles is not sufficient near the black hole, it comes to be more sufficient at far distances from the compact object.

It is well known that the classical solution for the effective potential has maximum value at \( \rho = 3 \) and one can see that \( b_{cr} = 3\sqrt{3} \), whereas for the spinning particles we obtain

\[ b_{cr} = \frac{3\sqrt{3}}{\sqrt{(1 - \tilde{\eta})(1 + 3\tilde{Q}^*)}}, \] (24)

where \( \tilde{\eta} = M^2\eta/l^2 \), \( \eta \) is the term used above and \( l^2/M^2 \) is the dimensionless angular momentum.

The proper observer in this coordinate system can calculate the velocity of particle in the following way (see, e.g., Misner et al. 1973); \( \vartheta_\tau = \pm \sqrt{1 - b^2/B^2} \) and \( \vartheta_\phi = b/B \), where \( B = \sqrt{(1 - 2/\rho + \tilde{Q}^*/\rho^2)(1/\rho^2 - \tilde{\eta})} \) and from \( \vartheta_{cr} = \arccos \vartheta_\tau = \arcsin \vartheta_\phi \) we can find the angle \( \vartheta_{cr} \), which between the direction of propagation
5 Conclusion

Here we have derived analytical expressions for capture cross section of magnetized particles by black hole in braneworld immersed in external magnetic field. The expressions for capture cross sections were obtained using the Hamilton-Jacobi formalism. The form of Hamilton-Jacobi equation was chosen as in \cite{Delice2003, Preti2004b}. Such analysis was first performed by Zakharov in Ref. \cite{Zakharov1994} for particles with zero magnetic moment. Extensive analysis of the effective potential of the radial motion for the magnetized test particles around black hole in braneworld has shown that the orbits of the particles become more stable with increasing of the parameter $\beta$. It was shown that the particle coming from infinity and passing by the source will be not captured: it will be reflected and go to the infinity again. Captured magnetized particles by the central object are going to leave the black hole in braneworld. It should be noted that the influence of the magnetic parameter to the trajectory of the magnetized particles is not sufficient near the black hole, it comes to be more sufficient at far distances from the compact object. The influence of the brane parameter is sufficient near the object and brane parameter forces stable circular orbits to shift to the observer at the infinity. The similar result has been obtained by \cite{Abdujabbarov2010} and they have obtained upper limit for the dimensionless brane parameter comparing the theoretical results with the astrophysical data.

One of the main interesting tasks is to find the lower limit for the critical angular momentum of magnetized particles. Particles with angular momentum lower than the critical value will be captured by the black hole in braneworld. It is well known that for the neutral particles the critical value of angular momentum is $L_{cr} = 4M$. In this paper we have shown that in the presence of the brane parameter, the external magnetic field and magnetic moment of the particle this critical value is decreased, i.e. magnetized particle to be captured can escape from the central black hole in braneworld immersed in external magnetic field.

In this paper the angle of direction of particle’s motion, that is, in what direction particles can release from the capture of black hole in braneworld have been also considered. We have found the radial dependence of critical angle $\theta_{cr}$ for particles for the different values of the parameter $Q^*$. Particles directed with angle lower than $\theta_{cr}$ will be captured by the central black hole in braneworld. In this paper we have concluded that in the presence of nonvanishing brane parameter $Q^*$ the critical value of angle decreases. Particle with the magnetic moment are more stable with compare to the neutral...
Fig. 2  The radial dependence of the angle of release for the different values of the brane parameter.

Fig. 3  The particles trajectories. If initially particle’s motion direction is lower than \( \theta \), particle will be captured by the central black hole, otherwise particles can escape from the black hole. Dashed circle represents the event horizon of the black hole.

particles around braneworld black hole immersed in external magnetic field.

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