On the possibility of distinguishing between Majorana and Dirac neutrinos

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Abstract

We clarify that one cannot distinguish between Majorana and Dirac neutrinos in the limit of vanishing neutrino mass. In particular we show, that the forward-backward asymmetry in the reaction $e^+e^- \to \nu\bar{\nu}$ is the same for $\nu_M$ and $\nu_D$, in contrast to recent claims made in the literature.

1 Introduction

If the neutrinos are massive [1], then they are described by either Dirac fields with four independent components or by Majorana fields with only two, $\nu_L$ and $\bar{\nu}_R = (\nu_L)^C$. This possibility of being a Majorana particle arises if the neutrinos have no conserved charges so that one cannot distinguish between particles and antiparticles.

In the strictly massless case two of the standard model Dirac components are sterile to the weak interactions and the Dirac field is equivalent to the two component Weyl fermions describing the Majorana field. Thus there is no distinction between $\nu_M$ and $\nu_D$ [2].

Since the neutrino mass only enters the Lagrangian as $\sum_{a,b} \bar{\psi}_a M_{ab} \psi_b + h.c.$, one would consider the mass as being a perturbation to the massless case and hence expect physical observables to be smooth in the limit $m_\nu \to 0$. In this letter we will show that this is indeed the case for the forward-backward asymmetry, and thereby clarifying some confusion in the recent literature [3].

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2 The massless limit

It has long been known, that there is no distinction between Majorana and Dirac neutrinos, when the neutrino mass is zero, $m_\nu = 0$ [1, 2]. Various authors have described the difference between $\nu_M$ and $\nu_D$ in the massive case [4-8], however, there still seem to be some confusion in the understanding of the $m_\nu \to 0$ limit. Indeed, it was recently claimed [3] that this limit is not smooth and the difference could be seen in the forward-backward asymmetry in the process $e^+ e^- \to \nu \nu$. We will show in the following, that the limit $m_\nu \to 0$ is smooth.

In order to find the differential cross section for the neutral current process $e^+ e^- \to \nu_M \nu_M$ (or $e^+ e^- \to \nu_D \bar{\nu}_D$) depicted in Fig. 1, we can evaluate the expressions for the matrix elements in eqn. (5) in the appendix for the Majorana case and eqn. (6) for Dirac neutrinos. We go to the CM-frame [4] using the coordinate system of Fig. 2, where the left-handed

![Feynmann diagrams and notation for the reaction $e^+ e^- \to \nu_M \nu_M$. For Dirac neutrinos there is only one diagram.](image1)

![Coordinate system and notation for the calculation of the differential cross section for the reaction $e^+ e^- \to \nu \nu$.](image2)

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2 In the charged current processes the limit is also smooth [3].
neutrino always is along the positive z-axis, both for Majorana and for Dirac neutrinos.
With this notation the neutrino and electron four-momenta are:

\[ p_\nu = p_3 = E(1, 0, 0, \beta) \quad \text{and} \quad p_e = p_1 = E_e(1, \sin \theta, 0, \cos \theta) \]

and the polarization four-vector which reduces to the unit spin vector in the particle rest frame is:

\[ a_\nu = a_3 = (\beta \gamma s_z, s_x, s_y, \gamma s_z), \]

with \( \beta = (1 - (m_\nu/E)^2)^{1/2} \) and \( \gamma = E/m_\nu \). In the limit of \( \beta \to 1 \) the Dirac neutrinos have their spins along the z-axis, and the differential cross section at the Z-peak smoothly approaches:

\[
\left. \frac{d\sigma_D}{d\Omega} \right|_{m \to 0} = \sigma_0 \left[ \left( g_L^2 + g_R^2 \right) \left( 1 + \cos^2 \theta \right) + 2 \left( g_L^2 - g_R^2 \right) \cos \theta \right] (1 - s_z)(1 - \overline{s}_z),
\]

(1)

where \( g_L = -1/2 + \sin^2 \theta_W \), \( g_R = \sin^2 \theta_W \) and \( \theta \) is the angle between the electron and the left-handed neutrino:

\[
\sigma_0 = \frac{G_F^2 s}{32\pi^2} \left| \frac{M_Z^2}{s - M_Z^2 + iM_Z\Gamma_Z} \right|.
\]

This is describing left-handed neutrinos (not the anti-neutrinos) going in the positive z-direction. That is, in a given experiment we would primarily measure electrons in the upward direction.

In the Majorana case we should be slightly more careful. Let us for the moment ignore the transverse degrees of freedom, \( s_x \) and \( s_y \), which are unphysical for the massless neutrinos.

Then the Majorana differential cross section at the Z-peak in the \( \beta \to 1 \) limit is:

\[
\left. \frac{d\sigma_M}{d\Omega} \right|_{m \to 0} = \sigma_0 \left[ \left( g_L^2 + g_R^2 \right) \left( 1 + \cos^2 \theta \right) (1 + s_z \overline{s}_z) + 2 \left( g_L^2 - g_R^2 \right) \cos \theta (s_z + \overline{s}_z) \right],
\]

(2)

which can be rewritten as:

\[
\left. \frac{d\sigma_M}{d\Omega} \right|_{m \to 0} = \frac{1}{2} \sigma_0 \left[ \left( g_L^2 + g_R^2 \right) (1 + \cos^2 \theta) (1 - s_z) (1 - \overline{s}_z) \right.
\]

\[
+ \frac{1}{2} \sigma_0 \left[ \left( g_L^2 + g_R^2 \right) (1 + \cos^2 \theta) - 2 \left( g_L^2 - g_R^2 \right) \cos \theta \right] (1 + s_z)(1 + \overline{s}_z).
\]

(3)

\(^3\)A massive majorana neutrino is in general not in a helicity eigenstate, however, the terms including \( s_x \) and \( s_y \) are proportional to \( \cos^2 \theta \) and do therefore not contribute to the forward-backward asymmetry. For inclusion of the transverse degrees of freedom see refs. \([4,5]\).
This expression includes both the positive and the negative helicity states in the upward direction, in contrast to the Dirac result in eqn. (1) where we only had the negative helicity states. One could say, that in the Dirac case we only have $\nu$’s upwards, whereas we have both $\nu$ and $\bar{\nu}$ in the Majorana case. However, in the Majorana case these two terms do not cancel, but add up to give the same cross section as in the Dirac case.

More precisely, the first term of eqn. (3), which is exactly half of the Dirac expression in eqn. (1) (as pointed out in ref. [5]), describes the probability for emitting the left-handed neutrino along the positive $z$-direction, and the second term of eqn. (3) gives the probability for emitting the right-handed neutrino in the positive $z$-direction. However, the definition of the coordinate system in Fig. 2, which is for both Majorana and Dirac particles, demands that the left-handed neutrino is in the positive $z$-direction, and we must therefore interchange $z \to -z$ in the last term of eqn. (3). This also implies that $\cos \theta \to -\cos \theta$, and therefore the right-handed neutrinos contribute the same to $d\sigma_M/d\Omega$ as the left-handed.

Thus the differential cross sections for $\nu_M$ and $\nu_D$ are the same in the massless limit. In particular the forward-backward asymmetry $A_{FB}$, $A_{FB} \approx 0.45$, is the same for Dirac and Majorana particles in the reaction $e^+e^- \to \nu\bar{\nu}$.

### 3 Conclusion

We have calculated the differential cross sections for the interaction $e^+e^- \to \nu\bar{\nu}$ with either Majorana or Dirac neutrinos in the final state. We showed, that these cross sections are identical in the $m_\nu \to 0$ limit. In particular we have explained why the asymmetric terms don’t cancel in the Majorana case, but instead add up to give the same as in the Dirac case.

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A Matrix elements

The differential cross section for the neutral current interaction in Fig. 1 is proportional to the polarization function [6]:

$$\frac{d\sigma}{d\Omega} \sim G_F^2 \ Tr \left( \rho_3 \mathcal{O}_\alpha \rho_4 \mathcal{O}_\beta \rho_2 \mathcal{O}_\alpha \rho_1 \mathcal{O}_\beta \right),$$

where e.g. $\mathcal{O}_\alpha = \frac{1}{2} \gamma^\alpha (1 \pm \gamma_5)$ for Dirac fermions, and because the Majorana field is CPT invariant it has only axial coupling, $\mathcal{O}_\alpha = \gamma^\alpha \gamma_5$ [7]. One must use the polarization density matrix:

$$\rho_i = \frac{1}{2} (\hat{p}_i \pm m)(1 - \gamma_5 \gamma_i),$$  \hspace{1cm} (4)

when the particles are partially polarized and we measure the spins of both the resulting particles in a reaction such as $e^+ e^- \rightarrow \nu \bar{\nu}$. Here we are using the polarization four-vector:

$$a_\mu = (|\vec{p}|/m \ s_\parallel, \vec{s}_\perp, E/m \ s_\parallel) = (\beta \gamma s_\parallel, \vec{s}_\perp, \gamma s_\parallel),$$

which in the particles rest frame reduces to the unit spin vector and obeys $(p \cdot a) = 0$.

For the Majorana case the matrix element is:

$$|A|^2_M =$$

$$2^6 G_F^2 \left[ 2m_e^2 g_L g_R \left( (p_3 \cdot p_4) - m_\nu^2 (a_3 \cdot a_4) - 2m_\nu^2 \right) + m_\nu (g_L^2 - g_R^2) (p_1 \cdot (a_4 - a_3) p_2 \cdot (p_4 - p_3) - p_2 \cdot (a_4 - a_3) p_1 \cdot (p_4 - p_3)) + (g_L^2 + g_R^2) \left( [(p_1 \cdot p_3)(p_2 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3)] (1 + (a_3 \cdot a_4)) - m_\nu^2 (p_1 \cdot a_3)(p_2 \cdot a_4) + (p_1 \cdot a_4)(p_2 \cdot a_3) \right) - (p_1 \cdot p_2) \left( (a_3 \cdot a_4)(p_3 \cdot p_4) - (a_3 \cdot p_4)(p_3 \cdot a_4) + m_\nu^2 \right) + (p_3 \cdot p_4) \left( (a_3 \cdot p_2)(a_4 \cdot p_1) + (a_3 \cdot p_1)(a_4 \cdot p_2) \right) - (p_3 \cdot a_4)(a_3 \cdot p_2)(p_4 \cdot p_1) + (a_3 \cdot p_1)(p_4 \cdot p_2) \right) - (a_3 \cdot p_4) \left( (a_4 \cdot p_2)(p_3 \cdot p_1) + (a_4 \cdot p_1)(p_3 \cdot p_2) \right) \right].$$  \hspace{1cm} (5)
and for the Dirac case it is:

\[ |A|^2_D = \]

\[ 2^6 G_F^2 \left[ 2m^2_{\nu} g_L g_R \left( \left( p_3 \cdot p_4 \right) - m_{\nu}(p_3 \cdot a_4) + m_{\nu}(a_3 \cdot p_4) - m^2_{\nu}(a_3 \cdot a_4) \right) \right] \]

\[ + (g_L^2 - g_R^2) \left[ \left( p_1 \cdot p_3 \right) (p_2 \cdot p_4) - (p_2 \cdot p_3)(p_1 \cdot p_4) \right. \]

\[ - m^2_{\nu} \left( (p_1 \cdot a_3)(p_2 \cdot a_4) - (p_2 \cdot a_3)(p_1 \cdot a_4) \right) \]

\[ + m_{\nu} \left( (p_1 \cdot a_3)(p_2 \cdot p_4) - (p_2 \cdot a_4)(p_1 \cdot p_3) + (p_2 \cdot a_3)(p_1 \cdot p_4) - (p_1 \cdot a_4)(p_2 \cdot p_3) \right) \]

\[ + (g_L^2 + g_R^2) \left[ \left( p_1 \cdot p_3 \right)(p_2 \cdot p_4) + (p_2 \cdot p_3)(p_1 \cdot p_4) \right. \]

\[ - m^2_{\nu} \left( (p_1 \cdot a_3)(p_2 \cdot a_4) + (p_2 \cdot a_3)(p_1 \cdot a_4) \right) \]

\[ + m_{\nu} \left( (p_1 \cdot a_3)(p_2 \cdot p_4) - (p_2 \cdot a_4)(p_1 \cdot p_3) - (p_2 \cdot a_3)(p_1 \cdot p_4) + (p_1 \cdot a_4)(p_2 \cdot p_3) \right) \right]. \]

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