Abstract

We give an updated overview of both weak and strong coupling methods to describe the approach to a plasma described by viscous hydrodynamics, a process now called hydrodynamisation. At weak coupling the very first moments after a heavy ion collision is described by the colour-glass condensate framework, but quickly thereafter the mean free path is long enough for kinetic theory to become applicable. Recent simulations indicate thermalization in a time $t \sim 40(\eta/s)^{4/3}/T$ [1], with $T$ the temperature at that time and $\eta/s$ the shear viscosity divided by the entropy density. At (infinitely) strong coupling it is possible to mimic heavy ion collisions by using holography, which leads to a dual description of colliding gravitational shock waves. The plasma formed hydrodynamises within a time of $0.41/T$. A recent extension found corrections to this result for finite values of the coupling, when $\eta/s$ is bigger than the canonical value of $1/4\pi$, which leads to $t \sim (0.41 + 1.6(\eta/s - 1/4\pi))/T$ [2]. Future improvements include the inclusion of the effects of the running coupling constant in QCD.

Keywords: hydrodynamisation, holography

1. Introduction

The creation of a strongly coupled plasma at relativistic nucleus-nucleus collisions is one of the most striking discoveries at RHIC and LHC. One of the hallmarks resulting from this program is the understanding that this plasma is described by viscous relativistic hydrodynamics very quickly, within 1 fm/c. This is surprising since the gradients at that time result in a small longitudinal pressure, even with the small viscosity present. This process of going to a regime described by hydrodynamics is now called hydrodynamisation and depending on the gradients present this can take much shorter than the process of equilibration.

It is profoundly challenging to describe the entire process from collision to hydrodynamics fully within QCD itself. This is partly because at high energy scales a perturbative treatment can be appropriate, while at energy scales of the temperature of the plasma formed a strong coupling picture should be used.

A perturbative treatment is not straightforward, since at higher energies the gluon concentration in nuclei increases. After the collision it is hence natural to expect an overoccupied state of gluons, which act coherently in a state called colour-glass condensate (CGC). At weak coupling the CGC undergoes classical expansion, up to the point where the mean-free path is long enough to allow for a description using kinetic theory. At strong coupling there are currently no theoretical tools to do such a description within QCD itself, although within holography it is possible to try and mimic QCD as close as possible [3].
This talk will review both weak and strong coupling approaches from a far-from-equilibrium initial stage to a plasma described by relativistic hydrodynamics. This includes both the timescale of the process, as well as the evolution of the flow and energy density during the process.

2. Hydrodynamisation at weak and strong coupling

At weak coupling the first stage of a heavy ion collision is described by classical Yang-Mills [4, 5]. After a short time the mean free path of the gluons becomes long enough that evolution using kinetic theory becomes feasible. For pure Yang-Mills this was first achieved in [6], where it was found that for 't Hooft coupling \( \lambda = 10 \) a typical state hydrodynamises within a time of \( \tau \sim 10/Q_s \), with \( Q_s \) the saturation scale. In terms of the temperature at hydrodynamisation and the shear viscosity this translates into
\[
\tau \sim 40(\eta/s)^{4/3}/T \quad [1].
\]
More recently, this has been extended to also include dynamics in the transverse plane [7], which is useful since among else it can shed light on the early time dynamics of radial flow. At early times this pre-flow in the transverse plane grows linearly with time and for any approximately boost-invariant conformal theory is given by [8]
\[
v_\perp = -\frac{\tau}{2} \frac{\nabla_\perp e}{e + P_\perp},
\]
where \( e \) is the energy density profile, and \( P_\perp \) the transverse pressure. For accurate initial hydrodynamic conditions the relevant question is then what the (average) transverse pressure is during the far-from-equilibrium evolution. For strong coupling the transverse pressure starts out high, at \( P_\perp = e \), but decreases fast, giving an average effective pressure of approximately \( P_\perp \sim e/2 \) [9, 10]. In [7] it is instead found that at weak coupling the transverse pressure starts at \( e/2 \) and does not change much at early times. In the end both weak and strong coupling approaches hence give rise to very similar transverse flow.

For hydrodynamisation at strong coupling there have been studies in a homogeneous [15, 16] and boost-invariant setting [17, 18], finding a far-from-equilibrium regime almost immediately followed by a near-equilibrium regime described by quasi-normal modes. These systems were always well described by hydrodynamics within a time of \( 1/T \), with \( T \) the temperature at that time of hydrodynamisation, which is non-trivial for the varying energy density in the boost-invariant case.

Recently, there have been elaborate studies studying similar dynamics in theories of higher derivative gravity [12, 11] (see also [19]). These are especially interesting for heavy ion collision, as these higher derivative terms can correspond to inverse coupling constant corrections on the field theory side, and can
hence give insights into a coupling constant regime more akin to QCD. In these papers higher derivative corrections to $\mathcal{N} = 4$ SYM are considered, as well as Einstein-Gauss-Bonnet theory. Corrections to $\mathcal{N} = 4$ SYM itself are perhaps more interesting from a fundamental point of view, since the field theory is known precisely. Nevertheless, this higher derivative theory does not contain curvature squared ($R^2$) or curvature cubed terms due to the symmetry of this particular theory. For general higher derivative corrections to more general QFTs it is hence more logical to consider a general approach, starting to look at curvature squared theories, which is given by the Einstein-Gauss-Bonnet action:

$$S_{GB} = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g} \left[ R - 2\Lambda + \frac{\lambda_{GB}}{2} \left( R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \right) \right].$$

(2)

The works [12, 11] resolved a puzzle found in [19], where it was found that higher order quasi-normal mode frequencies (QNM, determining relaxation time) received much larger corrections than lower order QNMs. This would quickly make them the dominant QNM, with very long relaxation times. In the examples studied in [12, 11], however, it was found that this effect is really only there when treating $\lambda_{GB}$ perturbatively. When using realistic values, and computing the relaxation time non-perturbatively, the QNMs remain ordered, albeit giving longer relaxation times: $\tau \sim 0.72 \pi T(1 + (\eta/s - 1/4\pi)7.7)$, for the case of $\mathcal{N} = 4$ SYM [12] (Figure 1, left). See also the recent paper [22] for a complete analysis of hydrodynamics and its breakdown in theory dual to Einstein-Gauss-Bonnet.

More realistic for heavy ion collisions is to set up an actual collision, homogeneous in the transverse plane, whereby the initial conditions are given blobs of plasma boosted to the speed of light, while keeping their total energy and longitudinal profile fixed [23, 24, 25]. Since the simplest versions of holography are scale invariant conformal field theories a stationary blob of plasma would start expanding, but the time dilatation from boosting to the speed of light makes them stable (see however [26] for collisions in non-conformal theories). For these ‘holographic heavy ion collisions’ it was found that when the longitudinal profile is thin enough (‘narrow’ shocks) all physics only depends on the energy per transverse area, which we call $\mu$. In this case the plasma is described by hydrodynamics very quickly, within a time of $tT \sim 0.41$, depending slightly on the criterion how accurately hydrodynamics should describe the evolution of the system (we use 10% accuracy for the pressures).

Recently, these collisions have been extended to Einstein-Gauss-Bonnet gravity, treated perturbatively in $\lambda_{GB}$, up to 2nd order. Snapshots of the energy density for both narrow shocks (left, Gaussian with width $\mu w = 0.1$) and wide shocks (right, $\mu w = 1.5$) can be seen in Figure 2. The results clearly illustrate that for the narrow shocks (high energy) there is more energy on the lightcone and less energy in the plasma,
indicating that indeed interactions are weaker and that the collision is hence more transparent. For the wide shocks two effects are visible. Firstly, also here less energy ends up at mid-rapidity, there is less 'pile-up'. Secondly, the energy is flatter, which can also be seen in the narrow case. The latter effect can be explained by the larger viscosity which in this case leads to a larger anisotropy and hence smaller longitudinal pressure, which in turns leads to less acceleration in the longitudinal profile. For these collisions the corrected hydrodynamisation time turns out to be $t \sim (0.41 + 1.6(\eta/s - 1/4\pi))/T$. For wider shocks the time is a bit more involved, since then the width of the shocks provides a more important scale than the temperature and a more elaborate analysis needs to be done.

Another interesting study in the context of holographic heavy ion collisions was done in [27], which studied collisions including a globally conserved U(1) charge. It was found that the charge behaves much like the energy density, including similar (charged) hydrodynamisation times, as well as similar rapidity profiles\(^2\). Including charge, however, gives rise to an interesting opportunity to collide equal energy shocks whereby only one shock is charged, leaving the other shock neutral. This asymmetric set up allows a clean probe of which part of the charge in the plasma comes from which shock (this is true even in the symmetric case, as in our small chemical potential approximation the charge-charge collision is just the sum of two charge-neutral collisions).

Quite strikingly, Fig. 3 shows that after a short time more than 41% of the right-moving charge is in fact moving left, i.e. 41% of the charge 'bounced' off the other shock, as seen in the centre-of-mass frame. This is to be contrasted with a weak coupling picture, where all charge would just move through the shock, perhaps loosing some speed, but not easily changing direction. This hence shows an interesting qualitative difference in the two pictures. From an experimental point of view this 'bounce' is probably not realistic, at least not at top RHIC energies, where indeed most baryon charge is found near the incoming nuclei [28].

Apart from differences in hydrodynamisation times interesting qualitative differences arise at very early times directly after the collision. Conformal invariance and energy conservation implies that in a boost-invariant setting the stress-tensor in ($\tau$, $\eta$, $x_\perp$) coordinates is given by

$$T^{\nu}_{\mu} = \text{diag} \left\{ -\epsilon(\tau), -\epsilon(\tau) - \tau \epsilon'(\tau), \epsilon(\tau) + \frac{1}{2} \tau \epsilon'(\tau), \epsilon(\tau) + \frac{1}{2} \tau \epsilon'(\tau) \right\},$$

with $\epsilon(\tau)$ the energy density and $\tau$ proper time. At early times there are three distinct interesting possibilities, each leading to different pressures:

\(^2\)Quite curiously, the decay of the charge on the lightcone exactly mimics the decay of the energy density. While perhaps natural from a field theory perspective, this is surprising in terms of the gravitational theory, where energy density is determined by non-linear dynamics of curved spacetime and charge evolves according to linear Maxwell equations on top of this geometry.
1. Free streaming of weakly interacting particles gives $\epsilon(\tau) \sim 1/\tau$, which is dominated by the expanding volume. This leads to $P_L = T_\eta^\eta = 0$.

2. In CGC fields act coherently and one has $\epsilon(\tau) \sim \text{const} + O(\tau)$, whereby the fields act coherently. This can lead to a negative longitudinal pressure: $P_L = -\epsilon(\tau)$.

3. At strong coupling the plasma takes time to form, and one has $\epsilon(\tau) \sim \tau^2 + O(\tau^3)$ [29]. This leads to $P_L = -3\epsilon(\tau)$.

Each scenario hence has different qualitative initial time behaviour, especially when focusing on the longitudinal pressure. Nevertheless, for the hydrodynamic initial conditions they may lead to similar conclusions, since at stronger coupling the pressure starts out more negative, but also increases faster.

### 3. Rapidity profile

The study of equilibration and hydrodynamization does not only give information about the time scale of the process. From the simulations it is perhaps even more important to extract the hydrodynamic initial data, which is given by the velocity and temperature fields (or equivalently the energy density in the local restframe). At weak coupling the rapidity profile is studied with the glasma framework in [30] (see also proceedings in this issue, as well as [31]). Even though the glasma does not hydrodynamise by itself, this paper is able to also find an approximately Gaussian rapidity profile, with a width proportional to the inverse coupling $1/\alpha_s$.

At strong coupling it was also found that the rapidity profile is Gaussian to a very good approximation, with width 0.955 [32]. This, however, is the initial rapidity profile at the moment of hydrodynamisation. Hydrodynamic evolution will widen this profile, which was studied by the MUSIC hydrodynamic and freeze-out code in [33]. This allows to directly compare with the experimental pseudo-rapidity distribution as measured by ALICE (Figure 4, left). This pseudo-rapidity profile is clearly wider than the Gaussian of width 0.955, but also significantly narrower than the experimental result. This calculation was a the full result of a completely collision in the simplest holographic setting, at infinitely strong coupling, with no fitting parameters are available.

At this point one has to realise that this computation is not performed in QCD itself, so for a realistic computation one either has to introduce phenomenological fitting parameters (such as promoting the width 0.955 to a parameter), or to consider more advanced versions of holography that are closer to QCD itself. For this latter option the attempt presented in Figure 2 would be such an example. As explained these shocks in Einstein-Gauss-Bonnet gravity have a higher shear viscosity and can in many ways be viewed as being described by a weaker coupling [11]. The rapidity profile is hence of crucial interest and it presented in Figure 4 (right). Indeed at early times the energy density is smaller, since more energy remains on the lightcone due to the weaker coupling. At the same time the rapidity profile is slightly wider, as clearly seen for $\mu T = 1$ and 2. At later times, however, the energy density becomes bigger than the unperturbed theory, and the width of the rapidity profile does not increase as fast as the unperturbed shock collisions. Both features can be understood by realising that this theory has a higher shear viscosity. This leads to higher viscous entropy production, which in turn leads to higher energy densities in the local restframe. Also, with the gradients present the higher viscosity will lead to a larger anisotropy and hence a smaller longitudinal pressure, which explains why in hydrodynamic evolution the rapidity profile does not widen as much as in the canonical collision. Of course these collisions are performed using a single value of $\eta/s$, whereas in QCD the viscosity is expected to vary with the local temperature. In real heavy ion collisions the magnitude of both effects would depend on this variation of the shear viscosity.

### 4. Discussion

The initial stage of heavy ion collisions progresses from weaker coupling at the moment of the collision to strong coupling in the later hydrodynamic regime. In this talk I have reviewed the two main approaches of hydrodynamisation, both currently working at constant coupling. The first approach employs kinetic theory
and is applicable at weak coupling, whereas the holographic approach is valid at strong coupling for theories
that share similarities with QCD.

A somewhat separate question we discussed is the initial condition at the moment the system hydro-
dynamised. This is expected to both depend on the initial condition for the far-from-equilibrium evolution, as
well as the on the evolution itself. At weak coupling the initial condition for the kinetic theory would ideally
come from a first principle computation using classical Yang-Mills theory [4, 5] extended with a rapidity
analysis in glasma such as in [30]. At strong coupling a natural initial condition is the collision of lumps of
energy. These always consist of gravitational waves in the dual gravity theory, but they can be supplemented
by vector (for a conserved charge, [27]), scalar (for a non-conformal theory [26]) or higher order gravity
corrections (for finite coupling corrections [2]).

In future it would be greatly beneficial to have simulations that have a coupling constant evolving with
the energy scale. Treating the entire evolution before hydrodynamisation at weak coupling will likely not
be accurate at later times, where would be strongly coupled with small viscosity (even at $\lambda = 10$ in kinetic
theory the viscosity is still large, with $\eta/s \approx 0.6$ at about $\tau = 1$ fm/c). Treating the entire evolution at
strong coupling, on the other hand, will likely lead to too strong interactions at very early times, which is
illustrated by all the energy and charge ending up around mid-rapidity (Figures 3 and 4). A combination of
both approaches may however lead to a realistic simulation throughout the evolution.

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