Study on hydraulic exciting vibration due to flexible valve in pump system with method of characteristics in the time domain

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Abstract. To analyse the flow characteristics of leakage as well as the mechanism of self-excited vibration in valves, the method of characteristics was used to assess the effect of flexible valve leakage on the self-excited vibration in water-supply pump system. Piezometric head in upstream of the valve as a function of time was obtained. Two comparative schemes were proposed to simulate the process of self-excited vibration by changing the length, the material of the pipeline and the leakage of valves in the above pump system. It is shown that the length and material of the pipe significantly affect the amplitude and cycle of self-excited vibration as well as the increasing rate of the vibration amplitude. In addition, the leakage of the valve has little influence on the amplitude and cycle of self-excited vibration, but has a significant effect on the increasing rate of vibration amplitude. A pipe explosion accident may occur without the inhibiting of self-excited vibration.

1. Introduction
Hydraulic vibration is a “small fluctuation” or transient flow of pressurized water delivery pipeline system which can be analysed by hydraulic impedance method or transfer matrix method [1]. Suo [2] and Zhou et al [3] analysed hydraulic resonance in several hydropower stations and then the methods to reduce vibration were proposed. Yu et al [4] demonstrated the possibility of hydraulic resonance of parallel pump system with air surge tank for water hammer protection. Ye [5] and Zhu et al [6] analysed the piping vibration accident of a pumped storage power station and concluded that self-excited vibration was caused by defects in seal of flexible valve. It is proposed to prevent or eliminate the vibration by using bypass valve or changing water supply mode of the seal of the ball valve. Zhou et al [7, 8] analysed the free vibration of reversible unit operating in unstable region in pumped storage power station, and concluded that the self-excited oscillation was superposed by several oscillation modes whose attenuation factors were positive.

Valve leakage is common in water supply pump system. Valve can be divided into rigid valve and flexible valve according to whether there is a negative slope segment in the characteristic curve of leakage flow of valve. The leakage of flexible valve may cause self-excited vibration and affect the safety of the system [9]. The self-excited vibration caused by leakage of flexible valve in pump system was analyzed by combining with a long-distance water supply project example in this paper. The characteristics of self-excited vibration were obtained to lay a good foundation for the prevention and elimination of self-excited vibration.
2. The flow characteristics of the leakage of valve

If the leakage flow $Q$ of valve increases with the increase of water pressure difference $H$ between the front and back of valve, the valve is called rigid one. The leakage flow

$$ Q = k \sqrt{H} $$

where $k$ is a constant coefficient.

If the leakage area and then the leakage flow $Q$ of valve decrease with the increase of water pressure difference $H$ between the front and back of valve, the valve is called flexible one. The leakage flow

$$ Q = k_1 \sqrt{H} - k_2 H^{\frac{3}{2}} $$

where $k_1 = \pi D \delta_0$ , $k_2 = \frac{\pi D \rho A}{K}$ , $D$ is the diameter of valve. $\delta_0$ is the seal gap when the valve pressure difference is 0. $A$ is the area of sealing cover of valve. $K$ is the elastic coefficient of valve seal. $\rho$ is the density of water. $g$ is the acceleration due to gravity.

The leakage characteristics of rigid valve and flexible valve are shown in figure 1. Figure 1 shows that, for the flexible valve, when $H=0$, $Q=0$; when $H$ increases to $H_c$, $Q$ reaches a maximum; when $H$ continues to increase to $H_m$, $Q=0$; When $H < H_c$, $\frac{dH}{dQ} > 0$ , the slope of valve leakage characteristic is positive, the same trend with the leakage characteristics of rigid valve; when $H_c \leq H \leq H_m$, $\frac{dH}{dQ} < 0$ , the slope of valve leakage characteristic is negative.

![Figure 1. Characteristics of leakage in rigid and flexible valves.](image)

3. Mechanism analysis of self-excited vibration caused by flexible valve

Suppose a simple pipeline system with a reservoir at upstream and a valve at the end, ignoring the hydraulic friction of pipeline, at the valve

$$ \frac{h}{q} = - \frac{a}{gA} \tanh \frac{sl}{a} $$

where $h$ and $q$ are the Laplace transforms of hydraulic head and flow pulsation; $s$ is the complex frequency, $s = \sigma + i \omega$ ; $a$ is the velocity of wave due to water hammer; $A$ is the cross-sectional area of pipe at valve; $l$ is the pipe length and $g$ is the gravitational acceleration.

Linearize the flow characteristic of the valve at the operating point:
\[
\frac{h}{q} = \frac{dH}{dQ} = k
\]  

(4)

By summarizing equations (3) and (4), equation (5) is obtained:

\[
\sigma = \frac{a}{2I} \ln \left| \frac{1 - gAk/a}{1 + gAk/a} \right|
\]

(5)

From equation (5), we can know that when the slope of valve flow characteristic in system is negative, thus \( k < 0 \), \( \sigma > 0 \), the system is unstable and will occur self-excited vibration; when the slope of valve flow characteristic in system is positive, thus \( k > 0 \), \( \sigma < 0 \), the system is stable and will not occur self-excited vibration.

Therefore, when the slope of valve flow characteristic is positive, that is to say, rigid valve or \( H < H_c \) segment of flexible valve, the system is stable, self-excited vibration will not happen; when the slope of valve flow characteristic is negative, that is to say, flexible valve section \( H_c \leq H \leq H_m \), the system will occur self-excited vibration.

4. Analysis of self-excited vibration in pump system

Figure 2 is the structure diagram for a parallel pump system of water supply project, where \( P \) stands for pump, 1 ~ 11 stand for numbers of each pipe section.

![Diagram of pump system](image)

**Figure 2.** Diagram of pump system.

4.1. The establishment of mathematical mode

Assuming such a condition, the pump system stops running and the three valves are all closed. If the valve of No. 3 pipe is a flexible valve and is leaking, the valves of No. 7 pipe and No. 11 pipe are water-tight, the whole system can be simplified as a series pipeline system composed of No. 8, 4 and 3 pipes which have a reservoir at upstream and a valve at the end. The system can be analyzed by the method of characteristics.

4.1.1. The pipeline. The pipeline is segmented by the space step \( \Delta x \), the time step is taken as: \( \Delta t = \frac{\Delta x}{a} \), so a rectangular grid is formed in the \( X \sim t \) plane, and two characteristic line equations about the flow of the point \( i \) \( Q_{pi} \) and piezometric head \( H_{pi} \) can be obtained.

\[
C^+ : H_{pi} = C_p - B_p Q_{pi} \]

(6)

\[
C^- : H_{pi} = C_M + B_M Q_{pi} \]

(7)

Where \( C_p, B_p, C_M, B_M \) can be calculated by the known flow and piezometric head of nodes i-1 and i+1 at the previous moment. So:
\[ C_p = H_{i-1} + BQ_{i-1} \]  
\[ C_M = H_{i+1} - BQ_{i+1} \]  
\[ B_p = B + R|Q_{i-1}| \]  
\[ B_M = B + R|Q_{i+1}| \]  

where \( B = a / (gA) \); \( R = f \Delta x / (2gDA^2) \); \( f \) the coefficient of friction, \( a \) the wave velocity, \( A \) the cross-sectional area of pipe, \( D \) the diameter of the pipe.

### 4.1.2. At the outlet pool

\[ H_p = H_{p0} \]  
\[ Q_p = (H_p - C_M) / B_M \]  

where \( H_{p0} \) is the water level in the outlet pool.

### 4.1.3. At the flexible valve

\[ H_p = C_p - B_pQ_p \]  
\[ Q_p = k_1 \sqrt[3]{H_p - H_w - k_2(H_p - H_w)^2} \]  

where \( H_p \) is the piezometric head at the upstream of valve; \( H_w \) is the hydraulic head at outlet of valve.

Let \( H = H_p - H_w \), by summarizing equations (14) and (15), equation (16) is obtained:

\[ -B_pk_3H_{1}^{3} + H + B_pk_1H_{2}^{1} + H_w - C_p = 0 \]  

Equation (16) takes \( H \) as variable. The value of \( H \) can be obtained by the Newton iterative method, and then \( H_p \) and \( Q_p \) are also obtained.

### 4.2. Example calculation

The water level of the outlet pool is 1420 m and the elevation of the center of valve \( H_w \) is 1260 m. Table 1 lists the pipeline parameters for calculation in the system.

| Number of pipe | l (m) | D (m) | f   | A (m/s) |
|----------------|-------|-------|-----|---------|
| 3              | 15    | 0.219 | 0.025 | 1190    |
| 4              | 1300  | 0.377 | 0.023 | 1180    |
| 8              | 1100  | 0.355 | 0.021 | 350     |

No. 3 & 4 pipes are welded steel pipes.  
No. 8 pipe is M - PVC pipe, its wave velocity due to water hammer is small, \( a = 350 \) m/s.

The characteristic of the flexible is shown in equation (16), letting \( k_1 = 1.8 \times 10^{-4} \), \( k_2 = 7.05 \times 10^{-7} \), \( H = 85 \) m, \( Q_{\text{max}} = 1.1 \times 10^3 m^3 / s \). When the pressure difference between the front and back of valve...
H=160 m, at this time the valve is in the negative slope section, and will cause the self-excited vibration of system, the leakage flow is $Q = 8.5 \times 10^{-4} m^3/s$.

Piezometric head in upstream of valve changing with time is obtained by analyzing the self-excited vibration in the system by the method of characteristics. As figure 3 shows:

![Figure 3. Piezometric head in upstream of valve changing with time.](image-url)

4.3. Analysis of calculation result

The calculation result shows that: the amplitude of vibration caused by flexible valve leakage in the pipeline is relatively small in the initial phase of the oscillation, as shown in figure 3, the oscillation amplitude between 100 s ~ 200 s is only 5.7 m, as time goes by, the oscillation amplitude at valve increases continuously. The calculation shows that the oscillation amplitude can reach 12.1 m in 500 s, after 800 s, that the amplitude increases sharply to 2000 s, and that the oscillation amplitude will reach 142 m.

Jeager et al [10] have used graphical method for the derivation of the hydropower station system: the amplitude of the self-excited oscillation in a simple pipeline is two times the static head. However, the above calculation results show the amplitude of the self-excited vibration of the pump system after a certain period of time has a large gap with the theoretical oscillation amplitude of the hydropower station. Compared with the theoretical hydropower station system, the design flow of the pump system is only 0.043 m$^3$/s, leakage flow is only $8.5 \times 10^{-4} m^3/s$, the No. 8 pipe of pump system is made of PVC pipe, its water hammer wave velocity is 350 m/s, far less than 1180 m/s of steel pipe, it makes the vibration cycle increase.

If the material of No. 8 pipe is changed to steel pipe, the length of No. 8 pipe is changed to 275 m, the length of No. 4 pipe is changed to 325 m and other parameters are unchanged, we call it No. 1 scheme. The results of the calculation are shown in figure 4.

![Figure 4. Piezometric head in upstream of valve changing with time in No. 1 scheme.](image-url)
If the leakage flow of valve two times increases, let $k_1 = 3.6 \times 10^{-4}$, $k_2 = 1.41 \times 10^{-6}$ and the remaining parameters are unchanged, we call it No. 2 scheme. The results of the calculation are shown in figure 5.

![Figure 5](image)

**Figure 5.** Piezometric head in upstream of valve changing with time in No. 2 scheme.

Figure 4 shows that the amplitude of self-excited vibration increases significantly when the length of pipe is shortened and the material of pipe is changed. The amplitude increases to 255 m, the cycle is shortened and the speed of the development of self-excited vibration increases dramatically.

Figure 5 shows that when the valve leakage increases, the amplitude of the self-excited vibration is 105 m, the amplitude and cycle are slightly changed and the speed of the increasing of amplitude increases obviously.

The comparative analysis mentioned above shows that the length and material of the pipe significantly affect the amplitude and cycle of self-excited vibration as well as the increasing rate of the vibration amplitude. In addition, the leakage of the valve has little influence on the amplitude and cycle of self-excited vibration, but has a significant effect on the increasing rate of vibration amplitude.

Although the self-excited vibration amplitude of water pump system is not as big as simple pipe system, but the self-excited vibration caused by flexible valve leakage will still threaten the safety of pipeline. The water hammer pressure is considered 30% of the pressure in working conditions, and the calculation results show that the amplitude of vibration has exceeded the set value of water hammer pressure in 900 s. If the self-excited vibration continues, vibration amplitude will also continue to increase. The accident of explosion of pipe will occur without the holding back of self-excited vibration.

**5. Conclusion**

Valve leakage is very common in pressurized water transfer projects. The leakage flow characteristic curve of flexible valve has a negative slope segment. The theoretical analysis indicates that when the slope of valve leakage flow characteristic is negative, the attenuation factor of hydraulic vibration is greater than 0, the system is not stable, and it will occur self-excited vibration.

The length and material of the pipe significantly affect the amplitude and cycle of self-excited vibration as well as the increasing rate of the vibration amplitude. In addition, the leakage of the valve has little influence on the amplitude and cycle of self-excited vibration, but has a significant effect on the increasing rate of vibration amplitude. When the pump system shut down and the valve closed, the self-excited vibration caused by flexible valve water leakage may lead to an accident of explosion without the inhibiting of self-excited vibration.

Only one self-excited vibration condition is analyzed in the dissertation: the pump system stops running, only a flexible valve is leaking. In fact, there are other conditions needing further study, such
as when all the pumps of parallel pump system stop, the three flexible valves are leaking and when only one pump of the parallel pumps system stops, the flexible valve of its export is leaking, while the rest pumps of the system work.

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