HIGH-DENSITY SKYRMION MATTER AND NEUTRON STARS

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ABSTRACT

We examine neutron star properties based on a model of dense matter composed of $B = 1$ skyrmions immersed in a mesonic mean field background. The model realizes spontaneous chiral symmetry breaking nonlinearly and incorporates scale breaking of QCD through a dilaton vacuum expectation value that also affects the mean fields. Quartic self-interactions among the vector mesons are introduced on grounds of naturalness in the corresponding effective field theory. Within a plausible range of the quartic couplings, the model generates neutron star masses and radii that are consistent with a preponderance of observational constraints, including recent ones that point to the existence of relatively massive neutron stars $M \sim 1.7 M_\odot$ and radii $R \sim 12$–14 km. If the existence of neutron stars with such dimensions is confirmed, matter at supranuclear density is stiffer than extrapolations of most microscopic models suggest.

Subject headings: dense matter — equation of state — stars: neutron

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1. INTRODUCTION

Neutron star astronomy, initiated by the serendipitous discovery of the first radio pulsar (Hewish et al. 1968), has since found over 1700 similar “rotation-powered” neutron stars (Manchester et al. 2005). Pulsar timing measurements in radio binaries yield a simple (unweighted) mean neutron star mass $\langle M \rangle \sim 1.4 M_\odot$. In contrast, measurement of general relativistic parameters in neutron star–white dwarf binaries suggest $\langle M \rangle \sim 1.6 M_\odot$, owing to a few exceptionally large inferred neutron star masses in the latter case. Examples include the binary component PSR J0621+1002 (Nice et al. 2007) with $\langle M \rangle = 1.7 M_\odot$, and a more stringent value of $\langle M \rangle \geq 1.6 M_\odot$ at the 2 σ level set by a neutron star binary in the Terzan 5 cluster (Ransom et al. 2005). Constraining neutron star radii is harder, due to uncertainties in atmospheric modeling and distance estimates, but bounds from thermally emitting neutron stars, e.g., RXJ 1856.5-3754 (Walter & Lattimer 2002; Ho et al. 2007), imply that the canonical range of 10–12 km is exceeded. These observations have oriented attention toward “atypical” neutron stars with large mass and possibly large radius.

An independent determination of mass ($2.10 \pm 0.28 M_\odot$) and radius ($13.8 \pm 1.8$ km) of the bursting neutron star in the low-mass X-ray binary (LMXB) EXO 0748-676 has been claimed (Özel 2006) based on an accurate determination of gravitationally redshifted Fe and O absorption lines (Cottam et al. 2002). However, the evolving nature of the source has complicated further observational tests of the same. Furthermore, Doppler tomography of emission lines in the mass-transfer stream between the neutron star and its less massive companion suggests a more canonical value of 1.35 $M_\odot$ for the former (Pearson et al. 2006). NASA’s upcoming Constellation-X mission will improve on such spectral measurements, shrinking systematic errors on mass and radius even further. With due caution on the observational front, the confirmation of a mass $\sim 2.0 M_\odot$ for a neutron star strongly constrains the equation of state (EOS) of dense matter, ruling out the possibility of extreme softening at high densities. It also implies an upper bound on the energy density of observable cold and dense matter (Lattimer & Prakash 2005). A large radius $R \sim 13$–14 km for a 1.4 $M_\odot$ neutron star implies a stiffer symmetry energy at densities of $(1–2)n_0$, where $n_0$ is the saturation density of nuclear matter. These connections between the nuclear physics of dense matter and neutron star observations have been the focus of recent reviews (Steiner et al. 2005; Sedrakian 2007; Page & Reddy 2006; Lattimer & Prakash 2007).

Uncertainties in theoretical aspects of many-body interactions at $n \gtrsim (1–2)n_0$ lead to predictions for the mass versus radius curve that vary widely depending on the EOS, with the maximum mass $M_{\text{max}}$ ranging from 1.4 to 2.7 $M_\odot$ and radius at maximum mass $R_{\text{max}}$ from 9 to 14 km (Lattimer & Prakash 2001). Constraints from astrophysical observations and terrestrial laboratory data have whittled this range down (Li & Steiner 2006) so that most microscopic models of longstanding for nuclei and nuclear matter would struggle to explain the existence of relatively heavy neutron stars ($2 M_\odot$) which also have a large radius ($R \sim 13$ km).

In fact, only a few stiff EOSs, e.g., MPA1 (Müther et al. 1987), MS0 (Müller & Serot 1996), and PAL1 (Prakash et al. 1988), are consistent with the EXO 0748-676 constraint at the 1 σ level, and even they fail if the matter accreted onto the neutron star is helium-rich.\textsuperscript{5} However, while satisfying astrophysical constraints, it is also important to keep in mind constraints from laboratory data on strongly interacting matter around nuclear saturation density. This point was nicely brought out in recent papers by Li & Steiner (2006) and Klähn et al. (2006).

In this context, our goal in this paper is to explore further a recently proposed model of a skyrmion fluid (OBJ; Ouyed & Butler 1999; Jaikumar & Ouyed 2006), which was shown to lead to a very stiff EOS and consequently generate a large maximum mass as well as radius for a neutron star. However, it was pointed

\textsuperscript{5} The largest source of systematic error in extracting the mass and radius of EXO 0748-676 comes from the accreted mass fraction of hydrogen $0.3 < X < 0.7$ (Özel 2006).
out (J. M. Lattimer 2006, private communication) that the rapid rise of the compressibility and symmetry energy just above saturation density in this model puts it at odds with experimental constraints from collective flow data (Danielewicz et al. 2002) and isospin diffusion studies in medium-energy heavy-ion collisions (Tsang et al. 2004). In this work, we determine the extent to which we can satisfy these constraints by extending the skyrmion fluid model to include higher order interactions among the vector mesons that are theoretically motivated by arguments of naturalness in the corresponding effective field theory. The mass and radius predictions of the extended model, henceforth referred to as JOM, are also confronted with constraints set by the observation of X-ray burst oscillations, kHz quasi-periodic oscillations in LMXBs, and thermal emission from neutron stars. We include observational uncertainties wherever they may impact our conclusions. Our phenomenological model is able to satisfy a preponderance of these constraints. We emphasize at the outset that our model, in its current form, has not been investigated for its applicability to nuclei or more complex phases of matter at subsaturation densities. Our EOS presently applies only to infinite nuclear matter and neutron-rich matter in the range (1–5)\(n_\text{th}\).

This paper is presented as follows. In §2 we discuss some conventional EOSs for neutron stars; in §3 we revisit the Ouyed-Butler-Jaikumar (OBJ) model for skyrmion stars and motivate higher order interactions that serve to tune the stiffness of the EOS such that laboratory constraints are met. The main features of the mass versus radius curves are explained in §4. We compare the results obtained in the skyrmion star model to predictions of other neutron star models in light of observational bounds in §5. Our conclusions are in §6.

2. EQUATIONS OF STATE FOR NEUTRON STARS

An EOS for dense matter is a relation between pressure and energy (or baryon) density, usually derived from an underlying microscopic model or effective theory for strong interactions. To apply to neutron stars, it should be able to generate at least a 1.4\(M_\odot\) static neutron star with a radius in the 10–14 km range. The connection to the underlying microscopic theory can be formulated in several ways; examples include relativistic mean field theory, nonrelativistic potential models, and relativistic Dirac-Brueckner-Hartree-Fock theory (Arnett & Bowers 1977; Lattimer & Prakash 2001). As we are concerned with recent findings of relatively heavy neutron stars, we consider three stiff EOSs: MS0 (Müller & Serot 1996), APR (Akmal et al. 1998), and UU (Wiringa et al. 1988).

1. Müller & Serot (1996) used a relativistic theory of pointlike nucleons interacting via mesonic degrees of freedom. These are the neutral scalar (\(\sigma\)) and vector (\(\omega\)) fields, plus the isovector \(\rho\)-meson. In this model, like or unlike meson-meson interactions are encoded by terms that are polynomials in the fields. By demanding a match to the properties of nuclear matter at saturation, they obtained a sequence of EOSs that depend on the coupling constants of the polynomial interactions. The stiffest EOS (MS0) corresponds to vanishing couplings and yields a maximum mass of 2.7\(M_\odot\). This model is consistent with a large neutron star mass and radius \(~\sim\)14 km for static configurations. Presently, the 1 \(\sigma\) limits on the radius of the bursting neutron star source EXO 0748–676 are 13.8 ± 1.8 km. However, there is no fundamental symmetry principle that requires the higher order couplings to vanish. Introducing natural values for these couplings drastically reduces the maximum mass to \(<2.0\ M_\odot\).

2. Akmal et al. (1998) obtained the APR EOS based on the Argonne \(v_{18}\) nucleon-nucleon interaction (Wiringa et al. 1995), Urbana IX three-nucleon interaction (Pudliner et al. 1995), and a relativistic boost term (Forest et al. 1995) as microscopic input. This EOS gives a maximum neutron star mass of 2.2\(M_\odot\) and does not lie within the 1 \(\sigma\) limits on the mass-radius estimate of the neutron star in EXO 0748–676, even assuming the accreted matter is mostly hydrogen.

3. EOS UU is obtained via similar variational methods applied to an older two-nucleon and three-nucleon interaction (Urbana \(v_{14}\) + UVII). This model yields a maximum mass of 2.2\(M_\odot\) at radius 10 km (Wiringa et al. 1988). However, a radius larger than about 11 km is not supported by this EOS for a static neutron star with mass greater than 1.4\(M_\odot\).

Our purpose in selecting and highlighting these EOSs is two-fold. First, these models are representative of complementary philosophies behind constructing an EOS for dense matter: as in (1), forego the connection to laboratory data on vacuum two-nucleon interactions and focus instead on the empirical properties of large nuclei and infinite nuclear matter within a relativistic theory; or as in (2) and (3), insist on a satisfactory description of available data on the structure and interaction of few-nucleon systems (free or bound) in a noncovariant approach. Second, these are among the stiffest EOSs that arise from hadronic degrees of freedom alone. It is possible that hybrid EOSs that allow for quark matter at high density can be almost just as stiff (Alford et al. 2007), but we will not consider quark matter EOSs in this work. These three EOSs are used in our neutron star mass-radius plots (Figs. 4 and 5) and compared with the EOSs for dense skyrmion matter which we now describe.

3. THE SKYRMION FLUID

3.1. Nuclear Matter Phenomenology

Before the advent of quantum chromodynamics (QCD), T. H. R. Skyrme proposed a description of baryons as topological solitons in a mesonic field theory that realizes spontaneous chiral symmetry breaking in nonlinear fashion (Skyrme 1961). This model is now qualitatively supported by studies of large \(N_c\) QCD which suggest that mesonic degrees of freedom are fundamental and baryons arise as solitons. When augmented by the inclusion of low-lying vector mesons (\(m_V \leq 1\ GeV\)) and flavor-symmetry-breaking effects, the Skyrme model can provide a reasonable description of static baryon properties such as mass splittings, charge radii, and magnetic moments (Schecter & Weigel 1999).

In the two-nucleon sector, the problem of finding a sufficiently attractive isoscalar central and spin-orbit force within the Skyrme model at distances 1 fm \(<r<2\ fm\) has held up progress. One solution is to include a dilaton field that mocks up scale breaking in QCD and provides attraction in these channels.

To make progress toward an EOS for a skyrmion fluid, Käbbermann (1997) introduced a self-consistent model that incorporates medium effects through the response of the dilaton\(^6\) \(\sigma\) and the isoscalar \(\omega\)-field to a smooth density distribution obtained by integrating over the collective coordinates of the skyrmion. This is equivalent to an ensemble of noninteracting \(B = 1\) skyrmions, a valid picture up to a separation of 0.8 fm (\(n \leq 5n_\text{th}\)). A subsequent work (Jaikumar & Ouyed 2006) extended this model to asymmetric matter by incorporating the \(\rho\)-meson in the spirit of standard mean field approaches. The Lagrangian for the Skyrme model, augmented by the \(\sigma\)- and \(\omega\)-fields and by including isospin-breaking effects from the \(\rho\)-meson as well as explicit

\(^6\) Since chiral symmetry is broken nonlinearly, the usual \(\sigma\)-field as the chiral partner of the pion does not appear in the theory.
scale-breaking effects from the dilaton and quark masses, is given in JaiKumar & Ouyed (2006). A fit to nuclear matter phenomenology is achieved through additional parameters in the dilaton potential, which is given by Kälbermann (1997) as

\[ V(\sigma) = B[1 + e^{4\sigma}(4\sigma - 1) + a_1(e^{-\sigma} - 1) + a_2(e^\sigma - 1) + a_3(e^{3\sigma} - 1) + a_4(e^{3\sigma} - 1)], \]

where \( B \approx (240 \text{ MeV})^4 \) is related to the Bag constant (the nonperturbative glue that breaks scale invariance in QCD). The six unknown parameters of the model are \( a_1 - a_4 \), \( g_\sigma \) which is the \( \omega-N \) coupling, and \( g_\rho \) which is the \( \rho-N \) coupling. To determine the \( a_i \) values, the following constraints are imposed: the scale anomaly condition \( dV_\rho/d\sigma|_{\sigma=0} = 0 \) which implies that \( a_1 = a_2 + 3a_3 + 3a_4 \); the stationarity with respect to \( \sigma_0 \), viz., \( \partial E/\partial \sigma_0 = 0 \); a binding energy per nucleon of \(-16 \text{ MeV}\) for infinite nuclear matter at saturation density \( (n_0 = 0.16 \text{ fm}^{-3}) \); and a choice of the compressibility \( K \). The variable \( E \) is the energy density of the fluid, and \( \sigma_0 \) is the nonvanishing mean field value of the time component of the \( \sigma \)-field. At saturation, \( \sigma_0 \) is determined by the choice of the effective mass \( M \), which then also fixes \( g_\sigma \). The choice of symmetry energy and effective mass at saturation fixes \( g_\rho \). Once the \( a_i \) values are determined, \( \sigma_0 \) is generally obtained from its equation of motion for an arbitrary density. The \( a_i \) values show very weak dependence on the choice of \( K \) in the range 200–300 MeV, while displaying more sensitivity to the choice of effective mass. Without further modifications, the model displays a sharp rise in the compressibility just above saturation, rising from \( K = 240 \) (our choice) to 2000 MeV for a 10% increase in baryon density. Similarly, the symmetry energy rises too steeply in this range to be consistent with experimental constraints (see § 3.4). These inconsistencies are a consequence of the specific form of the dilaton potential, which is essential to preserve the trace anomaly relation (scale breaking). Therefore, a modification of \( V(\sigma) \) is not desirable. It is the exponential sensitivity of the curvature of the potential \( V(\sigma) \) to the dilaton vacuum expectation value (VEV) that drives the compressibility to large values. One way to address this issue is to view the Skyrmel model in the mean field approximation as an effective field theory of hadrons, so that the Lagrangian can be extended to include higher order terms (meson self-interactions) that parameterize unknown physics at a more microscopic level. This rationale is also employed in Müller & Serot (1996), although their model has an explicit \( \sigma \)-meson, pointlike nucleons, and no scalar-vector mixing, while our model has a dilaton as the only scalar and includes scalar-vector mixing. If the meson fields are viewed as relativistic functionals, the higher order interactions can be thought of as parts of an effective potential that determines their mean field values at a particular density through the stationarity of the effective action associated to the Skyrmel Lagrangian (Furnstahl et al. 1996). They therefore modify the density dependence of the meson fields as well as the properties of the background skyrmion fluid that couples to these fields. We restrict ourselves to quartic self-interactions in the \( \rho \)- and \( \omega \)-fields with coupling constants whose value can be surmised by naturalness. Then, higher order terms such as six- or eight-meson self-interactions do not substantially change the results obtained in the quartic case. In §§ 3.3 and 3.4, we implement this procedure to obtain an acceptable behavior of the compressibility and symmetry energy in dense skyrmion matter.

### 3.2. Mean Field Equations

The additional quartic interactions take the form

\[ \mathcal{L}_\omega = \mathcal{L}_\omega^0 + \mathcal{L}_\omega^{\text{int}}, \quad \mathcal{L}_\omega^{\text{int}} = \frac{\xi}{4} g_\sigma \omega^4, \]

\[ \mathcal{L}_\rho = \mathcal{L}_\rho^0 + \mathcal{L}_\rho^{\text{int}}, \quad \mathcal{L}_\rho^{\text{int}} = \frac{\chi}{4} g_\rho \rho^4, \]

for the \( \omega \)-field and

\[ \mathcal{L}_\rho = \mathcal{L}_\rho^0 + \mathcal{L}_\rho^{\text{int}}, \quad \mathcal{L}_\rho^{\text{int}} = \frac{\chi}{4} g_\rho \rho^4, \]

\[ \mathcal{L}_\rho = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{1}{2} e^{2\sigma} m^2 \omega^2 - g_\sigma n_\sigma, \]

\[ \mathcal{L}_\rho = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{1}{2} e^{2\sigma} m^2 \rho^2 - g_\rho n_\rho, \]

where \( k_F \) is the Fermi momentum of species \( i \) (\( n \) is for neutron and \( p \) is for proton). Then, the Skyrmel Lagrangian is compactly expressed as

\[ \mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_\omega + \mathcal{L}_\rho - V(\sigma), \]

where \( \mathcal{L}_2 \) and \( \mathcal{L}_4 \) involve gradients of the skyrmion profile. For the densities of interest \((n_0 \leq n \leq 5n_0)\) where the approximation of nonoverlapping skyrmions is valid, this profile drops off fast enough that the mean field averaging simply counts the number of nonoverlapping skyrmions is valid, this profile drops off fast enough that the mean field averaging simply counts the number of individual skyrmions in a given volume, equivalent to a non-interacting Fermi gas model. At \( n \geq 5n_0 \), we expect corrections to our mean field model from skyrmion overlap. We also do not expect such a mean field treatment to apply at densities much lower than saturation density, since skyrmions do not form a uniform fluid there. Thus, our model is restricted to \( n_0 \leq n \leq 5n_0 \).

For the \( \omega \)- and \( \rho \)-fields, the equations of motion read as

\[ a_\omega \omega_0^4 + b_\omega \omega_0 + c_\omega = 0, \]

\[ a_\omega = \xi \frac{g_\sigma^4}{6}, \quad b_\omega = e^{2\sigma} m_\omega^2, \quad c_\omega = -g_\omega n, \]

\[ a_\rho \rho_0^4 + b_\rho \rho_0 + c_\rho = 0, \]

\[ a_\rho = \chi \frac{g_\rho^4}{6}, \quad b_\rho = e^{2\sigma} m_\rho^2, \quad c_\rho = g_\rho n_\delta, \]

where \( \omega_0 \) and \( \rho_0 \) denote mean field values and \( \delta = 1-2x \), with \( x = (n_1 - n)/2n \) being the proton fraction of neutron-rich matter (\( x = 1/2 \) for symmetric matter). The magnitudes\(^7\) of the quartic couplings \( \xi \) and \( \chi \) are estimated from the naturalness argument.

\(^7\) We choose the sign of the couplings to be positive, since this guarantees zero mean fields at vanishing source density (baryon/isoospin).
for an effective field theory, viz., that the coefficients of the various terms in the Lagrangian, through a given order of truncation, should be of the same size when expressed in an appropriate dimensionless form. Thus, we find

\[ \xi g^2 \sim 12 \left( \frac{e^{\mu} M_0}{M_0} \right)^2, \quad \chi g^2 \sim 192 \left( \frac{e^{\mu} M_\rho}{M_0} \right)^2. \]  

(12)

We choose the effective mass, given by \( M = e^{\mu} M_0 \), to be 600 MeV at saturation density, so that \( e^{\mu} = 2/3 \) for a bare nucleon mass \( M_0 = 900 \) MeV (neglecting the \( \sim 40 \) MeV contribution from explicit symmetry breaking). Since the model is fit to saturation properties, \( g_{\omega, \rho} \) itself depends on \( \xi \), whose value must be chosen so as to satisfy the naturalness condition above. Furthermore, real solutions to the equation of motion for the dilaton equation (16) cease to exist beyond a small range of couplings. This restricts us to \( 0.1 \leq \xi \leq 0.3 \) and \( 1.0 \leq \chi \leq 2.0 \). These values differ from those in Müller & Serot (1996) due to additional factors of \( e^{2\Delta} \) from the dilaton (metric) that appear in the fitting expressions for \( g_{\omega, \rho} \) and \( g_{\pi} \) and the qualitatively different form of the dilaton potential (it contains all powers in \( \sigma \)). The energy densities corresponding to the vector mean fields are

\[ E_{\omega} = -\frac{a_{\omega} \rho^4}{4} - \frac{b_{\omega} \rho^2_0}{2} - c_{\omega} \rho_0, \quad E_{\rho} = -\frac{a_{\rho} \rho^4}{4} - \frac{b_{\rho} \rho^2_0}{2} - c_{\rho} \rho_0, \]  

(13)

so the total energy density is then

\[ E = E_{\text{kin}} + E_{\omega} + E_{\rho} + V(\sigma), \]

\[ E_{\text{kin}} = \sum_{n, p} \left[ \frac{k_F E_F (E_F^2 + k_F^2)}{8 \pi^2} - \frac{M^4}{8 \pi^2} \ln \left( \frac{k_F + E_F}{M} \right) \right], \]  

(14)

where the kinetic energy \( E_{\text{kin}}(k_F) \) comes from a Lorentz boost of the static skyrmion to momentum \( k_F \) (Kälbermann 1997). In equation (14), \( E_F = (k_F^2 + M^2)^{1/2} \), since the Dirac effective mass is the same for both neutrons and protons. The binding energy is \( E/n - M_0 \) and the pressure is given by

\[ P = n \left[ 0.5 \left( \sum_{n, p} E_F \right) + g_{\omega} \omega_0 - \frac{g_{\rho} \rho_0}{2} \right] - E. \]  

(15)

The effective mass \( M \) is determined at any density from the equation of motion for \( \sigma_0 \)

\[ \sum_{n, p} \frac{k_F^3 E_F}{4 \pi^2} + 4E_{\text{kin}} + \frac{dV}{d\sigma_0} - b_{\omega} \omega_0^2 - b_{\rho} \rho_0^2 = 0. \]  

(16)

3.3. Compressibility

For symmetric matter, \( \delta = 0 \) and \( \rho_0 \) vanishes. The compressibility is defined as

\[ K = 9 \frac{dP}{dn} = 9 \left[ n^2 \frac{\partial^2 (E/n)}{dn^2} + 2n \frac{\partial (E/n)}{dn} \right]. \]  

(17)

| Table 1 | Fit Parameters for the JOM Model for Various \( \xi \neq 0 \) |
|---------|--------------------------------------------------|
| \( \xi \) | \( g_\omega \) | \( a_1 \) | \( a_2 \) | \( a_3 \) | \( a_4 \) |
| 0.0       | 7.54       | -1.699 | -25.780 | 29.237 | -11.464 |
| 0.1       | 6.67       | -1.960 | -27.051 | 29.798 | -11.502 |
| 0.2       | 6.05       | -2.260 | -28.700 | 30.674 | -11.636 |
| 0.3       | 5.57       | -2.602 | -30.747 | 31.878 | -11.870 |

Notes.—The OBJ model has \( \xi = 0 \). In both cases, \( K = 240 \) MeV, B.E. = -16 MeV, \( E_{\text{sym}} = 32 \) MeV, and \( g_{\rho} = 4.917 \).

In the present case, this is equivalent to

\[ K = \left[ \frac{\partial^2 E}{\partial n^2} - \left( \frac{\partial^2 E}{\partial n \partial \sigma} \right) \frac{d\sigma}{dn} \right], \]  

\[ \frac{d\sigma}{dn} = \frac{\partial^2 E / (\partial n \partial \sigma)}{\partial \sigma^2}, \]  

\[ \frac{\partial^2 E}{\partial n^2} = \frac{k_F^2}{3nE_F} + \frac{g_{\rho}^2}{b_{\rho} + 3a_{\omega} \omega_0}, \]

\[ \frac{\partial^2 E}{\partial n \partial \sigma} = \frac{M^2}{E_F} - \frac{2g_{\omega} b_{\rho} \omega_0}{E_F - b_{\omega} + 3a_{\omega} \omega_0}. \]  

(18)

(19)

(20)

For the choice \( K = 240 \) MeV, binding energy = -16 MeV, and \( M = 600 \) MeV at saturation density, the best-fit \( a_i \) values for various \( \xi \) values are listed in Table 1, correcting an unfortunate error in their values quoted for \( \xi = 0 \) in Jaikumar & Ouyed (2006).

For \( \xi = 0 \), at saturation, freedom in choosing the fit parameters \( a_i \) allows for a delicate cancellation between various contributions to the compressibility such that \( K \) is small (\( \sim 200-300 \) MeV). This is achieved largely as a result of fine tuning \( d^2 V/d\sigma^2 \) away from its natural scale \( \sim B \). As we move to slightly higher density, this fine tuning cannot be recovered, since \( V(\sigma_0) \) is exponentially sensitive to changes in the \( \sigma \) VEV. Consequently, the contribution from the \( \omega \)-meson appearing in the \( \partial^2 E / \partial n \partial \sigma \) term dominates, pushing the compressibility to unnaturally large values, as in the OBJ model. For \( \xi \neq 0 \), \( a_3 \neq 0 \), and large \( \omega_0 \), it is clear from equation (19) that \( \partial^2 E / \partial \sigma^2 \), and hence, \( K \) is substantially reduced, as in the JOM model. This effect is reflected in the EOS of symmetric matter, shown in the top panel of Figure 1. By definition, the slope of the pressure-density curve is proportional to the compressibility. The hatched region represents constraints on the EOS of symmetric matter from the analysis of collective flow data in nucleus-nucleus collisions at 1-2 GeV nucleon\(^{-1}\) (Danielewicz et al. 2002). While the \( \xi = 0 \) curve (OBJ) is clearly too stiff, choosing natural values of the coupling (\( 0.1 \leq \xi \leq 0.3 \)) provides a considerable improvement, and the JOM model is able to satisfy the flow constraint for \( \xi \sim 0.25 \). The APR EOS, which has a phase transition at \( n \sim 2\rho_0 \), is also shown for comparison.

3.4. Symmetry Energy

In neutron-rich matter, \( \delta \neq 0 \) and the energy per particle \( \epsilon = E/n \) can be expanded about the symmetric point \( \delta = 0 \) (\( \lambda = 1/2 \)) as

\[ \epsilon(n, \delta) = \epsilon(n, 0) + a_3(n, \sigma_0(n)) \delta^2 + a_4(n, \sigma_0(n)) \delta^4 + \ldots. \]  

(21)
where the ellipses denote higher order terms and

\[
a_2(n, \sigma_0(n)) = \frac{1}{2!} \frac{\partial^2 \xi}{\partial \chi^2} n^0 = \frac{g_s^2 P_F^2}{12 \pi^2 m_n^2 \rho^2} + 1 \frac{P_F^2}{6 E_F},
\]

\[
a_4(n, \sigma_0(n)) = \frac{1}{4!} \frac{\partial^4 \xi}{\partial \chi^4} n^0 = \frac{k_F^6}{648 E_F} \left( \frac{4}{k_F^2} + \frac{3}{k_F^2 E_F^2} + \frac{3}{E_F^4} \right) - \frac{\chi^3}{384} \frac{n^3}{(\rho, g_s^2)^4}.
\]

Studies of neutron matter show that the value of \(x\) for \(\beta\)-equilibrated matter obtained by retaining only \(a_2\) is a good approximation over a wide range of \(n\) (Prakash et al. 1988). Recently, Steiner (2006) has examined the role of the quartic coefficient \(a_4\) at high density, finding the effects on the EOS to be small, although the threshold for the onset of rapid neutrino cooling via the direct Urca process can change considerably. As we are focusing on the EOS in this work, we drop the \(a_4\) term, keeping in mind that were we to retain the \(a_4\) term in equation (21), the correction to the energy difference of symmetric and neutron-rich matter is at the 5%–10% level for densities of interest. Proceeding with \(a_2\) alone, as defined in equation (21), \(\beta\) equilibrium and charge neutrality conditions yield

\[
(3\pi^2 n x)^{1/3} = 4a_2\delta,
\]

which fixes a solution \(x = x_0(\sigma_0)\) for any baryon density. Inverting this relation and solving equation (23) along with the mean field equations for \(\sigma_0, \omega_0,\) and \(\rho_0\), we determine \(x_0(n)\) explicitly.

Fig. 2.—Pressure of \(\beta\)-equilibrated matter (relative to a relativistic gas of the same energy density) as a function of density for the OBJ and JOM EOSs. [See the electronic edition of the Journal for a color version of this figure.]

The bottom panel of Figure 1 shows the density dependence of the symmetry energy \(a_2(\sigma_0(n), n)\). Since \(a_2\) in equation (23) depends on \(\sigma_0\), which takes different values for the same density in symmetric and asymmetric matter, the curves do not begin at 32 MeV (our choice for symmetric matter). We have chosen \(a_2\) from equation (23) to represent the symmetry energy, since it is this quantity that determines the proton fraction of \(\beta\)-equilibrated matter. This is different from the usual identification of the symmetry energy, which is made at \(\delta = 0\). In addition, it must be noted that \(\sigma_0\) and, hence, \(a_2\) now depend on \(\chi\) as well as \(\xi\). For \(\xi = \chi = 0\), a stiff symmetry energy results, which behaves approximately as \(a_2(n) = a_2(n_0)(n/n_0)^{1.26}\) in the range \((1-1.5)n_0\). Li & Steiner (2006) have argued that such a parametrically stiff symmetry energy is inconsistent with isospin-diffusion data in heavy-ion collisions (Rami et al. 2000) and the measured neutron skin thickness of lead (Starodubsky & Hintz 1994). For natural values of the couplings, the symmetry energy softens considerably, behaving approximately as \(a_2(n) = a_2(n_0)(n/n_0)^{0.71}\), which is consistent with the aforementioned experimental studies. While increasing \(\xi\) does not change the density dependence, increasing \(\chi\) softens the symmetry energy at high density. This is a consequence of scalar-vector mixing in our model.

The pressure of neutron-rich matter in \(\beta\)-equilibrium (excluding lepton pressure), scaled to that of a relativistic fluid with the same energy density, is shown in Figure 2. Close to saturation, the pressure is well described by \(P(n, \delta) = P_{\text{sym}}(n) + n^2 \delta^2 d\alpha_2/dn\), where \(P_{\text{sym}}\) is the pressure of symmetric matter. The curves in the bottom panel of Figure 1 imply that \(\delta\) is smaller while \(d\alpha_2/dn\) is...
larger for vanishing couplings, as compared to the case with non-zero couplings. This drives the initial rapid rise of the pressure relative to the energy density for \( \xi = \chi = 0 \). For nonzero couplings, the softer symmetry energy and larger value of \( \delta \) lead to a larger pressure at saturation but a more gradual increase relative to the energy density. Increasing \( \xi \) at fixed \( \chi \) softens the EOS and has a progressively decreasing effect at high densities, where the kinetic contribution to the pressure begins to dominate; hence, the gradual approach to the relativistic limit. Increasing \( \chi \) at fixed \( \xi \) has a more dramatic effect on the pressure and energy density, causing the EOS to exceed the relativistic limit at high densities. This is due to the dynamics dictated by the dilaton potential at high density and scalar-vector mixing in our model. The nucleon’s effective mass increases rapidly with increasing density beyond a certain value of \( \chi \) and the skyrmion shrinks, akin to a strong repulsive force.

4. MASS-RADIUS RELATION

We employ the RNS code (Stergioulas & Freidman 1995; Stergioulas & Freidman 1998) to generate mass versus radius curves for a sequence of static and rapidly rotating neutron stars with the OBJ and JOM EOSs. In both cases, the composition of the star is as follows: (1) \( \frac{\rho}{n_0} \leq 5 \): n, p, e\(^-\) matter in \( \beta\)-equilibrium with nucleonic pressure and energy density given by equations (14) and (15); (2) \( \frac{n}{n_0} \leq 1 \): the BBP EOS (Baym et al. 1971a) for densities below nuclear saturation density matched to the BPS EOS (Baym et al. 1971b) for the low-density nuclear crust of the star. For nonzero couplings (JOM EOS), the maximum mass is not reached until densities larger than \( 5n_0 \), so we had to extend our model to higher densities. Therefore, the maximum masses for the JOM EOS indicated in Figure 3 are to be viewed as extrapolations into a regime where the approximation of nonoverlapping \( B = 1 \) spherically symmetric skyrmions is not likely to hold. A more accurate method would have to first determine the topology and size of deformed or overlapping skyrmions, which we do not attempt in this work.

The variation in the mass-radius curves with the values of \( \xi \) and \( \chi \) reflects the following two facts: (1) the stiffer the EOS at supranuclear densities, the larger the maximum mass; and (2) the larger the pressure in the range \( (1-5) n_0 \), the larger the radius for a \( 1-2 M_\odot \) star. These correlations have been established and emphasized in previous work (Lattimer & Prakash 2001). The extreme stiffness of the symmetry energy for \( \xi = \chi = 0 \) results in large pressures in the range \( (1-5) n_0 \), leading to smaller radii in the JOM model. The maximum mass is also lower with respect to the case when the couplings are set to zero. For the rapidly rotating models, we chose a rotation frequency \( \nu = 600 \) Hz, corresponding to a period of 1.6 ms. In general, the additional centrifugal forces in a rotating star help to counteract the pull of gravity, resulting in larger radii for a given mass.

As pointed out in the context of a different mean field model (Müller & Serot 1996), the inclusion of the additional quartic terms softens the EOS for \( \beta\)-equilibrated neutron-rich matter considerably, making it difficult to obtain a neutron star mass larger than \( 2 M_\odot \). Within our model, while a large radius and mass is possible if these quartic terms are omitted, the requirement of respecting laboratory constraints near saturation density and arguments of naturalness imply that the inclusion of these couplings is essential and its consequences (a lowering of maximum mass and radius) quite general. We now turn to compare our results (along
constraint for EXO 0748-676 rules out the APR and UU EOSs at the highest observed spin frequency (716 Hz) of J1748.

Mass-radius curves for static stars corresponding to some stiff EOSs (APR, UU, and MS0) are plotted along with the ones in this work (OBJ and JOM). The $z = 0.35$ constraint for EXO 0748-676 rules out the APR and UU EOSs at the 1 $\sigma$ level, but allows them at the 2 $\sigma$ level. Dashed lines are 90% confidence limits from observations of quiescent LMXB X7 in 47 Tuc (Heinke et al. 2006). [See the electronic edition of the Journal for a color version of this figure.]

with those from other EOSs) with some current observational bounds on the mass and radius of neutron stars.

5. NEUTRON STAR OBSERVATIONS

In Figures 4 and 5 we show several constraints on a neutron star’s mass and radius that follow from general theoretical principles and observations of neutron star phenomena. Figure 4 is relevant for static or slowly rotating stars. A measurement of the blackbody radiation radius $R_\infty = 16.5$ km (Trümper et al. 2004) and distance estimate $d = 117$ pc for the thermally emitting neutron star RX J1856.5-3754 (Walter & Lattimer 2002) yields the dark grey allowed region, which is a lower bound on the radius, since the blackbody radiator has the smallest emitting area for a given luminosity. Observations of glitches in the spin-down of the Vela pulsar place a one-parameter constraint on the fractional moment of inertia in the crust $\Delta I/I \approx 0.014$ (Link et al. 1999). This parameter is the pressure at the core-crust interface, which takes typical values in the range $0.25 < P$ ($\text{MeV fm}^{-3}$)$^{-1} < 0.65$. The upper and lower limits translate to excluded regions in the top left region of Figure 4 marked Vela-glitch 1 and 2, respectively. Note that the region marked Vela-glitch 2, if taken at face value, rules out even a 1.8 $M_\odot$ star for the APR and UU EOSs up to 1.8 $M_\odot$ stars. Causality of the EOS excludes a (partially overlapping) smaller light grey region in the top left corner. A scaling relation between mass and radius for the minimum allowed spin period (Lattimer & Prakash 2004) combined with the highest observed spin frequency (716 Hz) of J1748—2446ad (Hessels et al. 2006) excludes the region marked “rotation.” Dashed lines show 90% confidence limits on the radiation radius for the thermal source X7 in the globular cluster 47 Tuc (Heinke et al. 2006) arising from atmospheric modeling that includes surface gravity effects consistently. Özeli’s estimate (Özel 2006) of mass and radius based on a redshift measurement $z = 0.35$ (Cottam et al. 2002) and an assumed atmosphere of accreted hydrogen for EXO 0748-676 gives the solid black line, with 1 $\sigma$ and 2 $\sigma$ limits displayed.

We have plotted the mass-radius curve from the JOM ($\xi = 0.2$, $\chi = 1.5$) and OBJ ($\xi = \chi = 0$) EOSs as well as the APR, UU, and MS0 EOSs. Among these, only MS0 and OBJ lie well inside the dark grey allowed region for a star $\geq 1.4 M_\odot$. A smaller distance, still within the above $\pm 12$ uncertainty limits, would also allow the JOM EOS, provided the mass of RX J1856.5-3754 is $\geq 1.8 M_\odot$. The relatively soft EOSs such as APR and UU do not satisfy this constraint coming from RX J1856.5-3754 unless the mass of this object $\geq 2.0 M_\odot$ and the glitch constraint is ignored. Although MS0 and OBJ appear promising in this light, as argued before, they are theoretically incomplete without higher order terms, from the standpoint of an effective field theory. The addition of extra quartic terms to the Lagrangian in the OBJ model, which is essential for the self-consistency of the approximations and the truncation scheme, softens the EOS considerably. This yields the JOM EOS, which comes closest to satisfying all constraints. It is noteworthy that the APR, UU, OBJ, and JOM mass-radius curves are also consistent with recently determined bounds (not shown in Fig. 4) on the radiation radius of neutron stars in the globular cluster M13 (Gendre et al. 2003a) and $\omega$-Centauri (Gendre et al. 2003b) while MS0 is not. In addition, unlike the JOM EOS, the APR and UU EOSs satisfy the observational constraints set by X-ray bursts in EXO 0748-646 only at the 2 $\sigma$ level, but not at the 1 $\sigma$ level.

Therefore, accepting all these observations as accurate, the OBJ and JOM EOSs are unique among the stiff EOSs considered here, in that they are most likely to satisfy all constraints from the aforementioned observations. Unlike the OBJ EOS, however, the JOM EOS can also satisfy constraints from laboratory experiments,

Fig. 4.—Observational constraints on neutron stars. The region allowed by observations of RX J1856.5-3754 (Walter & Lattimer 2002) is shown in dark gray, regions excluded by glitches in the Vela pulsar (Link et al. 1999) by the spin rate of J1748—2446ad (Hessels et al. 2006) and by causality are demarcated in light gray. Mass-radius curves for static stars corresponding to some stiff EOSs (APR, UU, and MS0) are plotted along with the ones in this work (OBJ and JOM). The $z = 0.35$ constraint for EXO 0748-676 rules out the APR and UU EOSs at the 1 $\sigma$ level, but allows them at the 2 $\sigma$ level. Dashed lines are 90% confidence limits from observations of quiescent LMXB X7 in 47 Tuc (Heinke et al. 2006). [See the electronic edition of the Journal for a color version of this figure.]
as demonstrated in § 3. The JOM EOS also implies that the mass of the neutron star RX J1856.5-3754 has to be \( \gtrsim 1.8 \, M_\odot \) and that of the quiescent LMXB X7 has to be \( \geq 1.6 \, M_\odot \).

In Figure 5 we display mass-radius curves for rapidly rotating stars with the same EOSs as in Figure 4. Constraints from kHz quasi-periodic oscillations (QPOs) in the bursting neutron star 4U 1728-34 (Miller et al. 1998; Barret et al. 2006) and practical upper limits on spin frequencies of neutron stars with hadronic EOSs (Lattimer & Prakash 2004) yield the wedge-shaped allowed region in black. We have assumed a fairly typical QPO spin frequency of 300 Hz in obtaining this constraint. We also employ compactness constraints obtained from the analysis of Nath et al. (2002) which assume a two-spot model to explain the X-ray burst oscillations of LMXB 4U 1636-53 (Nath et al. 2002). We assume its mass to be 1.6 \( M_\odot \). All mass-radius curves are for stars rotating with spin frequency \( v = 300 \, \text{Hz} \) (period \( P = 3.3 \, \text{ms} \)). See text for details. [See the electronic edition of the Journal for a color version of this figure.]

6. CONCLUSIONS

We have examined a mean field model for dense nuclear and neutron-rich matter, with direct application to neutron star interiors. The model is derived from the Skyrme Lagrangian, with the inclusion of the dilaton VEV performing a dual role: (1) to mock up scale breaking in QCD and (2) to dial the interaction between the density-dependent vector-meson mean fields and the skyrmion fluid which makes up the dense medium, thus achieving self-consistency. We included quartic self-interactions for the vector mesons to correct an unnaturally rapid rise of the compressibility and symmetry energy. At densities just above saturation, the resulting EOS is in better agreement with laboratory constraints than any EOS that is quite stiff at high density and moderately so near saturation density can satisfy a preponderance of astrophysical and laboratory constraints. The JOM EOS, based on a relativistic mean field theory with scale breaking and symmetry breaking inspired from QCD, provides one such promising example.
given accumulating data that points to the existence of neutron stars with $M \gtrsim 1.7 M_\odot$, although corresponding systematic errors tend to be larger than is the case for stars with $M \sim 1.4 M_\odot$.

The pressure and symmetry energy of dense matter, as well as the mass-radius curves for neutron stars depend sensitively on the (natural) values of the quartic couplings. Increasing the $\omega$-meson self-coupling reduces the compressibility as well as the symmetry energy near saturation density considerably, while increasing the $\rho$-meson self-coupling stiffens the EOS of neutron-rich matter at high density, thereby increasing the maximum mass. The maximum neutron star mass in our model lies between 1.8 and 2.0 $M_\odot$ for static configurations with corresponding radius at maximum mass between 10.5 and 11.5 km. For rapidly rotating stars, these values increase to 2.0–2.3 $M_\odot$ and 11.5–12.5 km, respectively. The minimum spin period for the maximum-mass star lies between 0.66 and 0.71 ms. These macroscopic effects are traceable to changes in the dilaton VEV, which effectively controls the skyrmion size through its interactions with the density-dependent vector-meson mean fields, thereby determining the stiffness of the EOS and highlighting the role of scale breaking in our model.

Confronting our relatively stiff EOS with a set of observational constraints on neutron star mass and radius, we find encouraging agreement. Including commonly used EOSs based on extrapolations of microscopic models of the nucleon-nucleon interaction, we infer that matter is likely to be stiffer at supra-nuclear densities than such models would suggest. Therefore, an accurate and simultaneous determination of mass and radius for the more “extreme” neutron stars, viz., those presently suggestive of high mass and large radius, will be particularly valuable. In addition, constraints from collective flow data in heavy-ion collisions and the isospin dependence of the strong interaction near saturation density are proving to be valuable benchmarks for dense matter studies.

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