Consequences of bursty star formation on galaxy observables at high redshifts

Alberto Domínguez,1,2* Brian Siana,1 Alyson M. Brooks,3 Charlotte R. Christensen,4 Gustavo Bruzual,5 Daniel P. Stark6 and Anahita Alavi1

1Department of Physics and Astronomy, University of California Riverside, Riverside, CA 92521, USA
2Department of Physics and Astronomy, Clemson University, Clemson, SC 29634, USA
3Department of Physics and Astronomy, Rutgers, The State University of New Jersey, Piscataway, NJ 08854, USA
4Grinnell College Physics Department, Noyce Science Center, Grinnell, IA 50112, USA
5Centro de Radioastronomía y Astrofísica, UNAM, Campus Morelia, Michoacán 58089, Mexico
6Steward Observatory, University of Arizona, 933 N Cherry Ave, Tucson, AZ 85721, USA

Accepted 2015 May 4. Received 2015 May 4; in original form 2014 August 25

ABSTRACT
The star formation histories (SFHs) of dwarf galaxies are thought to be bursty, with large – order of magnitude – changes in the star formation rate on time-scales similar to O-star lifetimes. As a result, the standard interpretations of many galaxy observables (which assume a slowly varying SFH) are often incorrect. Here, we use the SFHs from hydrodynamical simulations to investigate the effects of bursty SFHs on sample selection and interpretation of observables and make predictions to confirm such SFHs in future surveys. First, because dwarf galaxies’ star formation rates change rapidly, the mass-to-light ratio is also changing rapidly in both the ionizing continuum and, to a lesser extent, the non-ionizing ultraviolet continuum. Therefore, flux limited surveys are highly biased towards selecting galaxies in the burst phase and very deep observations are required to detect all dwarf galaxies at a given stellar mass. Second, we show that a \( \log_{10} \left( \frac{L_\nu (1500 \text{ Å})}{L_{H\alpha}} \right) > 2.5 \) implies a very recent quenching of star formation and can be used as evidence of stellar feedback regulating star formation. Third, we show that the ionizing continuum can be significantly higher than when assuming a constant SFH, which can affect the interpretation of nebular emission line equivalent widths and direct ionizing continuum detections. Finally, we show that a star formation rate estimate based on continuum measurements only (and not on nebular tracers such as the hydrogen Balmer lines) will not trace the rapid changes in star formation and will give the false impression of a star-forming main sequence with low dispersion.

Key words: galaxies: evolution – galaxies: high-redshift – galaxies: starburst.

1 INTRODUCTION

One of the most important results derived from deep galaxy surveys is that star formation in the more massive galaxies (galaxies with stellar masses larger than approximately \( 10^9 \, M_\odot \)) is regulated by gradual processes such as gas exhaustion (e.g. Noeske et al. 2007b). This fact implies that the star formation history (SFH) of these galaxies can be described by slowly varying functions of time (on time-scales larger than approximately 100 Myr). However, stochastic processes are expected to dominate in dwarf galaxies (galaxies with stellar masses lower than approximately \( 10^9 \, M_\odot \)). Specifically, the star formation in dwarf galaxies occurs in only a small number of regions and in a small volume. In such systems, feedback from supernovae can heat and expel gas from a volume comparable to the entire volume of the cold gas region, resulting in a temporary quenching of star formation. Therefore, the SFHs of dwarf galaxies are characterized by frequent bursts of star formation and subsequent quenching (on time-scales of the order of a few Myr; e.g. Hopkins et al. 2014; Shen et al. 2014). This burstiness may be caused mainly by supernovae feedback (e.g. Governato et al. 2012; Teyssier et al. 2013). Bursty SFHs complicate our interpretation of observable properties, and oversimplification of these SFHs may lead to significant biases in determinations of fundamental galaxy properties (e.g. Boquien, Buat & Perret 2014).

Recent studies are showing that low-mass galaxies at high redshift are contributing significantly to the total star formation rate (SFR) density (Alavi et al. 2014). Understanding these low-mass galaxies,
especially at $z \sim 2–3$ when the SFR of the Universe peaked (Reddy & Steidel 2009), is essential for explaining a number of phenomena in the early universe.

First, it is thought that dwarf galaxies reionized the intergalactic medium (IGM) at $z > 7$ and provided a significant fraction of the ionizing background at $2 \lesssim z \lesssim 7$ (Haardt & Madau 2012; Becker & Bolton 2013). Many investigations of escaping Lyman continuum (LyC) from galaxies are necessarily conducted at redshifts of $2 < z < 3.5$ (Vanzella et al. 2010; Mostardi et al. 2013; Nestor et al. 2013), as the IGM becomes more opaque at higher redshift (Prochaska, Worseck & O’Meara 2009). In these studies, one typically assumes an intrinsic LyC flux based on the non-ionizing ultraviolet (UV) flux at approximately 1500 Å (Siana et al. 2007). However, this conversion usually assumes constant star formation for a long duration (longer than 100 Myr). The bursty SFHs of dwarf galaxies will significantly affect the level of LyC flux, the selection of the galaxies, and our interpretation of the global escape fraction of LyC photons. Furthermore, the bursty star formation also has important implications for interpreting the by-products of the LyC (i.e. nebular emission lines) in dwarf galaxies.

Second, among more massive galaxies, there is a tight correlation between star formation and stellar mass. This correlation, called the star-forming main sequence, may exist from the local Universe up to $z \sim 6$ (e.g. Brinchmann et al. 2004; Noeske et al. 2007a; Pannella et al. 2009; Whitaker et al. 2012; Speagle et al. 2014). These observations lead to the interpretation, mentioned above, that star formation in these galaxies is predominantly regulated by gradual processes. Whether this tight relation remains at lower masses is still unclear, although very recently progress has been made by Whitaker et al. (2014) in measuring the average main sequence in lower mass galaxies. Bursty star formation should increase the dispersion in this relation. Unfortunately, the star formation indicators that we typically use, namely UV and infrared (IR) fluxes, vary on much larger time-scales than the star formation in dwarf galaxies. It is therefore possible that we are not able to detect an increased scatter in the SFR at low mass with these traditional indicators.

In this paper, we analyse how bursty SFHs produced by hydrodynamical simulations affect our current knowledge of the two issues stated above: the ionizing photon production and its subsequent effect on observables (e.g. nebular line strengths, searches for escaping LyC), and the appearance of a tight star-forming main sequence. The methodology is based on using a stellar population synthesis code to model the spectral energy distributions (SEDs) of these bursty galaxies, which cover the stellar mass range from $10^7$ to $10^{10} M_{\odot}$ at $z \sim 2$. Physical galaxy properties such as stellar masses and SFRs are also derived by using SED fitting, which are compared with their known values from the model galaxies (e.g. Pacifici et al. 2012).

This paper is organized as follows. In Section 2, we briefly describe our hydrodynamical simulations. Then, Section 3 gives details on the extraction of the galaxy SEDs from the results of the simulations. Later, Section 4 shows the results from our analysis in terms of the ionizing photon production and discusses them. Then, Section 5 analyses the star-forming main sequence. In Section 6, we compare physical galaxy properties from the model SEDs and the SED fitting. Finally, we summarize in Section 7 the main conclusions of our analysis. Throughout the paper, we use the same cosmology assumed by the simulations, i.e. a $\Lambda$ cold dark matter ($\Lambda$CDM) cosmology with $H_0 = 73$ km s$^{-1}$ Mpc$^{-1}$, $\Omega_M = 0.24$, and $\Omega_\Lambda = 0.76$ [according to data from the third release of the Wilkinson Microwave Anisotropy Probe (WMAP3); Spergel et al. (2007)].

2 GALAXY SIMULATIONS

The bursty in situ SFHs used in our analysis, defined as the star formation that occurs within the most massive progenitor of the main halo, are extracted from the hydrodynamical simulations that are described in this section.

2.1 Details of the simulations

The high-resolution simulations used in this work were run using the volume renormalization technique (or zoom-in technique) with the N-body plus smoothed particle hydrodynamics code GASOLINE (Wadsley, Stadel & Quinn 2004). The galaxies were selected to span a range of merger histories. All 11 of the galaxies in this work have been published and discussed more extensively in previous papers. Importantly for this work, earlier papers using these simulated galaxies have shown that they have realistic rotation curves (Christensen et al. 2014), that they match the $z = 0$ stellar mass to halo mass relation derived from abundance matching techniques (Munshi et al. 2013), and that the bursty SFH in the seven lowest mass galaxies leads to the creation of dark matter cores in their central density profiles (Governato et al. 2012). These simulations also match the observed mass–metallicity relation (Brooks et al. 2007), the sizes of galaxy discs (Brooks et al. 2011), and result in a realistic population of satellites in Milky Way-mass galaxies (Zolotov et al. 2012; Brooks & Zolotov 2014). These simulated galaxies are among the highest resolution achieved in cosmological simulations to date. The high resolution allows us to resolve the high-density gas clouds where stars form. With this scheme, we have successfully matched a large range of observed galaxy properties, as mentioned above. An extensive discussion of the star formation and feedback scheme in these simulations is found in Christensen et al. (2012). Below, we summarize the salient information about the simulations.

The spline force softening in these simulations ranges from 65 pc (h2003) to 87 pc (h516 and h799) to 174 pc (all others). High-resolution dark matter particles have masses of $6661 M_{\odot}$ ($1.3 \times 10^5 M_{\odot}$), while gas particles start with $1407 M_{\odot}$ ($2.7 \times 10^4 M_{\odot}$) for the lowest (largest) mass galaxies. Star particles are born with 30 per cent of the mass of their parent gas particle and lose mass through supernova (both Ia and II) and stellar winds. Each of these galaxies has between one and five million dark matter particles within the virial radius at $z = 0$. The simulations not only include metal line cooling and metal diffusion (Shen, Wadsley & Stinson 2010), but also a prescription for the formation (both gas phase and on dust grains) and destruction (primarily photodissociation by Lyman–Werner radiation from nearby stellar populations) of $H_2$ (Christensen et al. 2012). Star formation is tied directly to the presence of $H_2$, as observed (Bigiel et al. 2008, 2010; Leroy et al. 2008; Blanc et al. 2009; Schruba et al. 2011). These simulations also include a uniform UV background that turns on at $z = 9$, mimicking cosmic reionization following a modified version of Haardt & Madau (2001).

Critically for this work, the force resolution of these runs (65–174 pc) allows high-density regions where stars form to be resolved ($\rho \gtrsim 100$ amu cm$^{-3}$, comparable to the average density in giant molecular clouds). Star formation in the simulations is done in 1 Myr increments. That is, every 1 Myr we identify the gas that is cold and dense enough to form stars. For all gas particles that meet this criterion, there is a 10 per cent $\times f_{\text{BH}}$ chance (where $f_{\text{BH}}$ is the molecular fraction of that gas particle) that they will spawn a star particle. The high resolutions allow for a physically motivated stellar and supernova feedback scheme that injects energy locally back into
the interstellar medium (ISM; following Stinson et al. 2006) rather than the adoption of an analytic prescription to model feedback globally. This local injection of energy can heat the surrounding gas such that star formation is completely shut down for a brief period (Stinson et al. 2007). In low-mass dwarfs, this leads to a bursty SFH.

We follow each galaxy back to high $z$, identifying at each output time step the halo progenitor that contains most of the stellar mass at lower $z$. We identify haloes at all steps using the Amiga Halo Finder (AHF; Gill, Knebe & Gibson 2004; Knollmann & Knebe 2009). Two of the haloes (h986.grp2 and h986.grp3) merge with h986.grp1 at $z < 1$, and no longer exist as independent galaxies at $z = 0$.

There are other authors who simulate galaxies using different ISM/feedback recipes but still obtain bursty SFHs similar to ours (Ceverino et al. 2014; Hopkins et al. 2014; Shen et al. 2014; Trujillo-Gómez et al. 2015). Relevant to this paper is the fact that, as galaxy stellar mass decreases, the dispersion in SFR increases, especially on short time-scales ($< 100$ Myr). We have calculated the dispersion of SFRs for our simulated galaxies on different time-scales and find that the trend is similar to the values shown in fig. 12 of Hopkins et al. (2014). Therefore, the conclusions in our paper should broadly be true in other simulations as well.

### 2.2 Simulated star formation rate histories

Fig. 1 shows the in situ SFH of three of our 11 simulated galaxies. The SFHs of the lower mass galaxies are characterized by complicated functions of time featuring short bursts of star formation. These bursts can reach an order of magnitude larger SFR than a rather quiescent state in time-scales of a few Myr or even less. Fig. 1 also shows the ex situ SFHs of the stars that are in the galaxies at $z = 2$. These are stars formed outside the main halo that through mergers become part of the main galaxy by $z \sim 2$. We stress that, as seen in Fig. 1, the ex situ contribution is rather low compared to the in situ star formation, particularly in the lower mass galaxies that are the main targets of this study. Observational results from other authors such as Behroozi, Wechsler & Conroy (2013) also suggest that the vast majority of all stars in low-mass haloes (by $z \sim 2$) are formed in situ.

### 3 MODELLING OF THE GALAXY SPECTRAL ENERGY DISTRIBUTIONS

We use the stellar population synthesis code GALAXEV by Bruzual & Charlot (2003, hereafter BC03) to determine the galaxy SEDs and stellar masses from the simulated SFHs. We note that BC03 accurately trace the mass loss from supernovae. The BC03 stellar masses of our galaxies are reported in Table 1 for $z = 2$ and 0.

| Simulation  | $\log_{10}(M_*/M_\odot)$ | $z = 2$ | $z = 0$ |
|------------|--------------------------|--------|--------|
| h2003.grp1 | 6.97                     | 7.05   |
| h799.grp1  | 7.07                     | 7.64   |
| h516.grp1  | 7.70                     | 8.05   |
| h986.grp3  | 8.29                     | –      |
| h603.grp1  | 8.35                     | 9.45   |
| h986.grp2  | 8.41                     | –      |
| h986.grp1  | 8.51                     | 9.29   |
| h239.grp1  | 9.09                     | 10.25  |
| h285.grp1  | 9.33                     | 10.29  |
| h258.grp1  | 9.43                     | 10.22  |
| h277.grp1  | 9.94                     | 10.33  |

These galaxies span the mass range roughly $10^7 – 10^{10} M_\odot$ at $z \sim 2$.

Our model SEDs are extracted at times when the Universe is from 2.5 to 3.5 Gyr old in steps of 5 Myr (a total of 201 SEDs for each simulated SFH). These ages correspond to redshifts from $z = 2.74$ to 1.97 according to the cosmology of the simulations. Though we only output the SEDs at 5 Myr intervals, it is important to note that we are using SFHs with 1 Myr resolution. This is necessary for capturing star formation changes on short time-scales. The initial mass function (IMF) is assumed to be given by Chabrier (2003) and the metallicity is fixed to $Z = 0.2 Z_\odot$, where $Z_\odot$ is solar metallicity. This metallicity is roughly consistent with the subsolar metallicities measured in $\log_{10}(M_*/M_\odot) < 8$ at $z \sim 2$ (Belli et al. 2013; Henry et al. 2013; Sanders et al. 2015). No dust extinction is included.
We neglect secondary effects of metallicity and dust to isolate the effects of bursty star formation on observable quantities.

From the model SEDs, we derive the UV luminosity in the continuum at two different wavelengths: the ionizing continuum at 900 Å (also known as Lyman continuum, or LyC) and the non-ionizing continuum at 1500 Å, where most observational studies easily detect high-redshift galaxies.

The BC03 code outputs the number of ionizing photons per second \( (N) \), which is used to estimate the H\( \beta \) emission-line luminosity \( L_{\text{H}\beta} \), in erg s\(^{-1}\) as

\[
L_{\text{H}\beta} = 4.757 \times 10^{-13} N
\]

(see Krueger, Fritz-v. Alvensleben & Loose 1995). This luminosity is necessary as a reference from which to derive other emission-line luminosities. In our case, we consider Ly\( \alpha \) and H\( \alpha \), whose ratios are \( L_{\text{Ly}\alpha}/L_{\text{H}\beta} = 22.20 \) and \( L_{\text{H}\alpha}/L_{\text{H}\beta} = 2.87 \) (case B recombination; Osterbrock & Ferland 2006). We assume that all of the ionizing photons are absorbed by hydrogen in the ISM of galaxies — i.e. that the LyC escape fraction is near zero. Although this choice may not be realistic for a subset of galaxies (e.g. Mostardi et al. 2013; Nestor et al. 2013), only very large escape fractions (i.e. larger than approximately 0.5) would significantly affect our results. The rest-frame Ly\( \alpha \) and H\( \alpha \) equivalent widths (EWs) are calculated from the model SEDs as the ratio between the luminosity of the line and the continuum luminosity per unit of wavelength at the central wavelength of the line. We do not apply any correction to account for stellar absorption in H\( \alpha \). However, this choice will not substantially affect our results since H\( \alpha \) is typically small in young galaxies (e.g. Domínguez et al. 2013).

As we mentioned in Section 2, a small fraction of the stellar mass is formed ex situ and becomes part of the galaxy through mergers. Importantly for the subsequent analysis, the stellar mass produced ex situ will not contribute to the LyC or UV photon emission since the star formation occurred long enough before coalescence.

4 STUDY OF THE IONIZING PHOTON PRODUCTION

The UV and H\( \alpha \) luminosities are directly related to star formation. The former, \( L_{\nu}(1500 \text{ Å}) \) or \( L_{\nu}(UV) \), is dominated by light coming from young and massive O and B stars (\( \gtrsim 3 \, M_\odot \)). This indicator is sensitive to the recent SFR given the lifetime of the massive stars that produce it (\( \lesssim 100 \text{ Myr} \)). The latter, \( L_{\text{H}\alpha} \), is produced by ionizing radiation coming from nebular regions heated by extremely massive stars (\( \gtrsim 20 \, M_\odot \)). This indicator is sensitive to the instantaneous SFR since it is dominated by stars with very short lifetime (\( \lesssim 5 \text{ Myr} \)).

According to BC03 models, after an instantaneous burst of star formation, it takes only 5 Myr for \( L_{\nu}(900 \text{ Å}) \) to decrease an order of magnitude, whereas it takes 30 Myr for \( L_{\nu}(1500 \text{ Å}) \). Results of our analysis in terms of the production of ionizing radiation and nebular emission lines are discussed in this section.

4.1 Ionizing photon production

The SFRs of dwarf galaxies are expected to change on time-scales similar to lifetimes of the ionizing O-stars. Therefore, we might expect wide variations in the ionizing photon production rate in these systems.

The ratio between the continuum luminosities \( L_{\nu}(1500 \text{ Å})/L_{\nu}(900 \text{ Å}) \) is typically used to derive the relative escape fraction of ionizing photons and also to convert from observed UV luminosity functions to LyC photon production rates (e.g. Steidel, Pettini & Adelberger 2001; Siana et al. 2007, 2010). This flux ratio is usually assumed as a constant ratio of approximately 7 (or 0.84 in log\( L_{\nu}(1500 \text{ Å})/L_{\nu}(900 \text{ Å}) \)). This value is reached when the equilibrium state is produced under continuous star formation when the number of stars producing LyC and 1500 Å flux is constant (or about 200 Myr after the burst). The log\( 10 \left[ L_{\nu}(1500 \text{ Å})/L_{\nu}(900 \text{ Å}) \right] \) is shown in Fig. 2 as a function of time for three different galaxies. We see that the scatter of log\( 10 \left[ L_{\nu}(1500 \text{ Å})/L_{\nu}(900 \text{ Å}) \right] \) is significantly larger in the low-mass galaxies, varying by nearly an order of magnitude on short time-scales but only around 0.1 dex for the larger mass galaxies.

In Fig. 3, we show the normalized distribution of log\( 10 \left[ L_{\nu}(1500 \text{ Å})/L_{\nu}(900 \text{ Å}) \right] \) for three different stellar mass bins that include all evolutionary stages. From the analysis of our simulations, it is clear that the assumption of
The intrinsic Lyα emission-line EW larger than 30 Å and have emitter for emitting galaxies. In general, non-Lyα emitters require more detailed dust as a function of stellar mass. α-pho

2014 10

emitters are selected at Lyα emitters. Therefore, galaxies selected as Lyα galaxies have log(Mα/Lα) > αEW (e.g. Nestor et al. 2011, Mostardi et al. 2013) will have a higher likelihood of LyC detection than galaxies with lower Lyα EW, even if the LyC escape fractions are similar. Second, low-mass galaxies that have recently turned off their star formation will still be reasonably luminous at 1500 Å, but will not be producing significant LyC photons. In Fig. 3, we can see that 38 per cent of low-mass (Mα < 10^8 M⊙) galaxies have log(Mα/Lα) > 1.3 (or about 20 in linear terms). These galaxies would often result in non-detections in direct LyC surveys, even if the LyC escape fractions of these galaxies were high. Ultimately, the burstiness of star formation as a function of mass will need to be better determined and incorporated into any analysis of LyC observations. We note that in larger mass galaxies (Mα > 10^9 M⊙), it is reasonable to assume a constant Lα/Lα = (900 Å) because the star formation does not typically change much on short time-scales.

Nestor et al. (2011, 2013) find a high ionizing emissivity from Lyα emitters at z = 3.09, possibly explaining the entire ionizing background at that redshift. This finding is initially surprising because only a subset of UV-bright galaxies (Lyα emitters) is enough to explain, at least, a significant fraction of the ionizing background. However, it may be possible that Lyα emitting galaxies are the only ones producing significantly strong LyC. Indeed, this is shown in Fig. 4, where we plot Lyα EW as a function of log(Mα/Lα = (900 Å)). The horizontal line shown in Fig. 4 defines typical Lyα emitters as intrinsic Lyα EW larger than 60 Å if we assume a Lyα escape fraction of 50 per cent. Typically Lyα emitters are selected at Lyα EW larger than 30 Å and have Lyα escape fractions of 30–60 per cent; Gronwall et al. 2007; Ouchi et al. 2008; Nilsson et al. 2009.) As we see in the figure, in general, galaxies that are not Lyα emitters are by definition faint in the LyC. We stress here that all of these Lyα EW predictions are intrinsic measures before scattering by Hα and absorption by dust. Of course, most of the more massive, dusty galaxies will not be Lyα emitters. Therefore, galaxies selected as Lyα emitters may be primarily composed of lower mass galaxies (with lower columns of neutral hydrogen and dust) in a burst phase. Determining exactly when each galaxy will be a Lyα emitter requires more detailed dust modelling and ray tracing, which is beyond the scope of this paper. Nonetheless, we stress that low-mass galaxies may not produce significant LyC about half of the time and selection on high Lyα EW will miss these galaxies. Thus, it may not be surprising that a large fraction of the known ionizing background at high redshift is caused by low-mass galaxies selected as Lyα emitters.

A potential problem in our calculations is the fact that Lyα photons significantly scatter with neutral hydrogen within the galaxy before escaping. This effect may produce a delay in the escape of the photons from the galaxy, which could smooth the Lyα luminosity time variation. However, given the size of a typical dwarf galaxy at z ~ 2 (of the order of a kpc) any delay due to a large number of scatters should be significantly smaller than the 1 Myr resolution of our simulations.

### Table 2

The standard deviation of the quantities listed in column 1 in three different stellar mass bins after removing the stellar mass trend. We are reporting only lower limits for the two lower bins of the SFR because the values for SFR = 0 are not included in the standard deviation calculation.

| Standard deviation | Stellar mass range (M⊙) |
|--------------------|------------------------|
| log10(Lα/1500 Å)   | ≤10^8                  |
|                    | 10^8–10^9              |
|                    | ≥10^9                  |
| log10(Mα/Lα)       | 0.74                   |
| log10(Mα/Lα)       | 0.53                   |
| log10(Mα/Lα)       | 0.97                   |
| log10(Mα/Lα)       | >0.65                  |
| log10(Mα/Lα)       | >0.45                  |
| log10(Mα/Lα)       | 1.03                   |
| log10(Mα/Lα)       | 0.43                   |
| log10(Mα/Lα)       | 0.15                   |
| log10(Mα/Lα)       | 0.10                   |

**Figure 4.** The intrinsic Lyα EW versus Lα/1500 Å/Lα(900 Å) for three galaxies of significantly different stellar mass at different evolutionary stages. These galaxies are our lowest mass galaxy h2003 grp1 (log10(Mα/Lα) = 6.97), the intermediate-mass galaxy h986 grp1 (log10(Mα/Lα) = 8.51), and our largest mass galaxy h277 grp1 (log10(Mα/Lα) = 9.94). The evolutionary stages above the horizontal line are considered typical Lyα emitters. In general, non-Lyα emitter are significantly faint in the LyC. Though many LyC searches of high-redshift galaxies are not targeting low-mass galaxies with low Lyα EW, those galaxies likely do not have strong LyC luminosities anyway, as the star formation may have recently been quenched. For instance, our galaxy of lowest mass is a Lyα emitter for 57 per cent of the time.

log10(Lα/1500 Å) = 0.84 is valid for the larger mass galaxies but not for the lower mass galaxies due to significantly large scatter. The scatter is quantified as the standard deviation of the distribution in three stellar mass bins in Table 2.

This scatter in Lα/1500 Å/Lα(900 Å) in low-mass galaxies will require particular care in interpreting studies of escaping LyC. First, there are some galaxies, caught just when a burst begins, that have large LyC luminosities relative to the non-ionizing 1500 Å luminosities. This will make them easier to detect in direct LyC searches. These same galaxies will have very high intrinsic (unextinguished by dust) Lyα EW (see Fig. 4). Therefore, galaxies selected for high Lyα EW (e.g. Nestor et al. 2011; Mostardi et al. 2013) will have a higher likelihood of LyC detection than galaxies with lower Lyα EW, even if the LyC escape fractions are similar. Second, low-mass galaxies that have recently turned on their star formation will still be reasonably luminous at 1500 Å, but will not be producing significant LyC photons. In Fig. 3, we can see that 38 per cent of low-mass (Mα < 10^8 M⊙) galaxies have log10(Lα/1500 Å/Lα(900 Å)) > 1.3 (or about 20 in linear terms). These galaxies would often result in non-detections in direct LyC surveys, even if the LyC escape fractions of these galaxies were high. Ultimately, the burstiness of star formation as a function of mass will need to be better determined and incorporated into any analysis of LyC observations. We note that in larger mass galaxies (Mα > 10^9 M⊙), it is reasonable to assume a constant Lα/Lα = (900 Å) because the star formation does not typically change much on short time-scales.

Nestor et al. (2011, 2013) find a high ionizing emissivity from Lyα emitters at z = 3.09, possibly explaining the entire ionizing background at that redshift. This finding is initially surprising because only a subset of UV-bright galaxies (Lyα emitters) is enough to explain, at least, a significant fraction of the ionizing background. However, it may be possible that Lyα emitting galaxies are the only ones producing significantly strong LyC. Indeed, this is shown in Fig. 4, where we plot Lyα EW as a function of log10(Lα/1500 Å/Lα(900 Å)). The horizontal line shown in Fig. 4 defines typical Lyα emitters as intrinsic Lyα EW larger than 60 Å if we assume a Lyα escape fraction of 50 per cent. Typically Lyα emitters are selected at Lyα EW larger than 30 Å and have Lyα escape fractions of 30–60 per cent; Gronwall et al. 2007; Ouchi et al. 2008; Nilsson et al. 2009.) As we see in the figure, in general, galaxies that are not Lyα emitters are by definition faint in the LyC. We stress here that all of these Lyα EW predictions are intrinsic measures before scattering by Hα and absorption by dust. Of course, most of the more massive, dusty galaxies will not be Lyα emitters. Therefore, galaxies selected as Lyα emitters may be primarily composed of lower mass galaxies (with lower columns of neutral hydrogen and dust) in a burst phase. Determining exactly when each galaxy will be a Lyα emitter requires more detailed dust modelling and ray tracing, which is beyond the scope of this paper. Nonetheless, we stress that low-mass galaxies may not produce significant LyC about half of the time and selection on high Lyα EW will miss these galaxies. Thus, it may not be surprising that a large fraction of the known ionizing background at high redshift is caused by low-mass galaxies selected as Lyα emitters.

A potential problem in our calculations is the fact that Lyα photons significantly scatter with neutral hydrogen within the galaxy before escaping. This effect may produce a delay in the escape of the photons from the galaxy, which could smooth the Lyα luminosity time variation. However, given the size of a typical dwarf galaxy at z ~ 2 (of the order of a kpc) any delay due to a large number of scatters should be significantly smaller than the 1 Myr resolution of our simulations.

### 4.2 Nebular emission-line equivalent width

Recently, very young galaxies have been found at z > 1 by identifying galaxies with large EW nebular emission lines (Atek et al. 2011, 2014; van der Wel et al. 2011; Stark et al. 2014). In particular, the Hα EW is a proxy of the specific SFR of a galaxy (i.e. the star formation per unit of stellar mass). The rest-frame intrinsic Lyα and Hα EWs are plotted in Fig. 5 as a function of stellar mass. Both EWs have a similar behaviour. Their scatter increases in the low-mass galaxies as a consequence of the burstiness of the SFHs. Table 2 quantifies the dispersion of the Lyα and Hα EWs distribution as the standard deviation in three stellar mass bins. We note that the dispersion of the Hα EW is larger than the dispersion of Lyα EW at all masses. The reason is that when the LyC is high (during a burst of star formation), both Lyα and Hα emission-line...
EWs are maximum values because it is expected that a significantly lower than 30 Å.

Figure 5. The intrinsic Ly α (upper panel) and Hα EW (lower panel) versus stellar mass for our sample of 11 galaxies. Each galaxy is represented by a different colour. A circle is plotted every 5 Myr from times of 2.5 to 3.5 Gyr, which makes it a total of 201 data points for each galaxy. We note that the range of the y-axis is the same for both panels. The simulations predict a large population of low-mass galaxies with Hα EW lower than 30 Å.

luminosities increase by the same factor. However, the rest-UV continuum at 1216 Å also increases strongly at the same time, whereas the continuum near 6563 Å does not change as much as the rest-UV continuum. Therefore, the Hα EW change is more dramatic.

The EWs increase immediately following a new burst of star formation, as the ionizing photon production rate reacts immediately, but the continuum flux takes a longer time to increase. Conversely, a few Myr after the star formation turns off, the LyC disappears but the 1500 Å flux fades much more slowly. Thus, the EWs quickly go to zero in recently quenched systems. We note that the plotted Lyα EWs are maximum values because it is expected that a significant fraction of Lyα photons will be absorbed by dust as it scatters through the interstellar and circumgalactic media.

Two of the most common SFR indicators at high redshift are the UV and Hα luminosities. However, as seen above, the ionizing continuum responsible for producing Hα can change on very short time-scales but the UV continuum takes longer to react. Therefore, we expect that the UV and Hα-derived SFRs may differ significantly depending on when the galaxy is observed (e.g. Glazebrook et al. 1999). Fig. 6 shows the \( \log_{10}[\nu L_\nu(1500\text{ Å})/L_{H\alpha}] \) as a function of stellar mass. The \( \log_{10}[\nu L_\nu(1500\text{ Å})/L_{H\alpha}] \) tends to a value of 2.05 for the larger mass galaxies. This value corresponds to luminosities from a galaxy with constant SFR when the equilibrium in the number of O-stars to UV-emitting stars is reached. We can see in Fig. 6 that dwarf galaxies with bursty SFHs are characterized by \( \log_{10}[\nu L_\nu(1500\text{ Å})/L_{H\alpha}] > 2.5 \) shortly after star formation has been quenched. Both \( L_\nu(1500\text{ Å}) \) and \( L_{H\alpha} \) are typically observed by deep galaxy surveys. Therefore, this figure provides a reliable test for bursty SFHs in star-forming galaxies based on large ratios of the UV continuum at 1500 Å and the Hα emission luminosity. The effect of periodic bursty SFHs on the ratio of UV and Hα luminosity is studied in low-redshift dwarf galaxies by some authors such as Iglesias-Páramo et al. (2004), Boselli et al. (2009), Meurer et al. (2009), and Weisz et al. (2012). These authors use analytical models to explore the amplitude, duty cycle, and duration of bursts of star formation by fitting the observed distribution of UV to Hα ratios. In our analysis, we take an alternative approach using SFHs motivated by results from hydrodynamical cosmological simulations.

4.3 Populating the IMF stochastically

At low SFRs, additional analysis uncertainties may arise due to poor sampling of the high-mass end of the IMF. Individual bursts may also have different IMFs. Stochastic effects on the IMF have been studied by other authors such as Fumagalli, da Silva & Krumholz (2011), Forero-Romero & Dijkstra (2013), and da Silva, Fumagalli & Krumholz (2014) using the code Stochastically Lighting Up Galaxies (SLUG) by da Silva, Fumagalli & Krumholz (2012). According to Forero-Romero & Dijkstra (2013, see their figs 1 and 2) this effect becomes important at SFRs lower than 0.01 M⊙ yr⁻¹. From the examples shown in our Fig. 1, we note that the stellar mass involved in the bursts considered is sufficiently high that individual bursts fairly sample the underlying IMF. This implies that this stochastic effect should not be too large in our sample. Though we expect this to be a minor effect, it will result in a slight decrease in the distribution of Lyα and Hα EWs at the low-mass end in Fig. 5. However, all the main observational conclusions of this paper will not be affected.

5 THE STAR-FORMING MAIN SEQUENCE

Typically, when evaluating trends in SFHs, the SFR is plotted as a function of stellar mass to show the star-forming main sequence. Our theoretical work complements other observational analyses of the main sequence. Here, we choose to analyse directly the luminosities instead of the SFRs to avoid any uncertainties from their conversion.

In Fig. 7, we plot the SFR versus stellar mass. In addition, three other observables are plotted in Fig. 7, Hα luminosity,
UV luminosity density, and optical luminosity density versus stellar mass. One can see that the large variation in SFR in low-mass galaxies is traced very well by $H_\alpha$ as it reacts quickly to the changing SFR. The UV variations are not as large as the SFR variations, and the optical luminosity is essentially unaffected by the SFR variations.

Table 2 lists the standard deviation of these observables in three mass bins. At large mass, the dispersion in SFR is small, and both $H_\alpha$ and UV reasonably trace the SFR variations (though $H_\alpha$ is certainly better). However, at lower masses ($M_* < 10^7 M_\odot$), as the SFR variations increase, the UV dramatically underpredicts the SFR variation. Therefore, our results show that, for low-mass galaxies, a tight correlation of $L_{UV}$ with stellar mass does not necessarily mean a smooth SFH. Rather, it just means that a significant amount of the variance in star formation is on time-scales shorter than it takes for the UV-emitting stars to react. As we study fainter galaxies, we must seek to use star formation indicators that trace shorter time scales such as $H_\alpha$ and other nebular emission lines.

We also emphasize that observationally a flux limited survey will be strongly biased towards galaxies in the burst phase. This is especially true when the selection is made on emission lines because the luminosities change dramatically. For example, Fig. 7 shows that the $H_\alpha$ selection of recent Hubble Space Telescope (HST) Wide-Field Camera 3 (WFC3)/IR grism surveys (WISP, Atek et al. 2010; 3D-HST, Brammer et al. 2012) can detect galaxies at $z \sim 1.4$ in a recent burst at stellar masses larger than $10^7 M_\odot$, but will miss the vast majority of the galaxies at around $10^6 M_\odot$. When the selection is made in the UV continuum the bias will be less severe since the UV scatter is lower but still significant. Fig. 7 shows that the Hubble Ultra-Deep Field (HUDF) depths (Beckwith et al. 2006) can detect galaxies currently in a burst at $10^6 M_\odot$ but the depths need to be an order of magnitude lower to detect all galaxies of that mass. Indeed, UV selection of dwarf galaxies at these redshifts (Alavi et al. 2014) is still finding mostly galaxies which appear to be in a strong and recent burst of star formation (Stark et al. 2014).
Finally, we note that in Fig. 7, we calculate the average of the SFR and each observable for each galaxy by adding all of the individual values at each time step and dividing by the number of time steps (i.e. to get a mean, rather than a geometric mean). The values are plotted with the large star symbols, and are analogous to what observers would obtain when stacking large samples of galaxies in bins of stellar mass. The slope of the average SFR versus $M_*$ relation is near one, consistent with the slope measured by Whitaker et al. (2014) in galaxies with $9.3 < \log_{10}(M_*/M_\odot) < 10.2$ at these high redshifts. However, other authors such as Henry et al. (2013) find a shallower slope of $0.31 \pm 0.08$ in galaxies with $8.2 < \log_{10}(M_*/M_\odot) < 9.8$. We believe this is a natural consequence of a bias towards galaxies in a burst phase when selecting via emission lines. As a result, the lowest mass galaxies detected in that survey are those with the largest SFRs. Such a bias will naturally lead to perceiving a shallower main sequence.

6 COMPARISON WITH PHYSICAL PROPERTIES FROM SED FITTING

In this section, we explore how well SED fitting to broad-band photometry can determine the physical properties of bursty galaxies. In particular, we examine the effects of the incorrect parametrization of the SFH on determining the galaxies’ SFRs and stellar masses.

The code Fitting and Assessment of Synthetic Templates (FAST; see the appendix in Kriek et al. 2009 for details) is used to find the best-fitting SED template from a grid of BC03 models. This fitting procedure is applied to the model galaxies simulating the following photometry: the UV/optical channel (UVIS) of the WFC3 on board the HST, the Advance Camera Survey (ACS)/WFC (F475W, F625W, F775W, and F850LP), and the near-IR with WFC3/IR (F125W and F160W). This is the same photometry used by Alavi et al. (2014) and it spans the rest-UV to rest-optical across the Balmer/4000 Å break. Nebular emission lines are neither included in the photometry nor they considered in the SED templates. Therefore, we are isolating the effects of the bursty SFHs independent of complications due to emission line contributions to broad-band photometry. A signal-to-noise ratio of 20 is assumed for each photometric band, thus noise is not a significant issue. The IMF is assumed to be given by Chabrier (2003). The parameter space explored by FAST is given by $\log_{10}(\text{Age}/\text{Gyr}) = 7$ up to a maximum age given by the age of the Universe, $A_V = 0$ (since our model galaxies do not have any dust extinction), the metallicity is fixed to $Z = 0.2Z_\odot$ and the SFH $\alpha \exp (+t/\tau)$ with $\tau = 0.3$–10 Gyr. Note that we assume that, on average, the SFRs are exponentially rising. This has been suggested to be more accurate than exponentially decreasing SFHs by other authors such as Reddy et al. (2012) in describing star-forming galaxies at high redshifts. Then, physical properties are derived from these fits such as galaxy stellar mass and SFR.

Fig. 8 plots the galaxy SFR versus time for one of our simulated galaxies (h986.grp1, which has a stellar mass at $z = 2$ of $\log_{10}(M_*/M_\odot) = 8.51$). The figure is showing the simulated SFR with a 1 Myr resolution, the SFR from the simulations but averaged over 5 Myr, and the SFR that is estimated from the SED fitting. It is clearly illustrated that the SFR provided by the SED fitting does not react rapidly enough to the bursty variations of the actual SFR as a consequence of being based on the UV continuum. Furthermore, it is clear that the SFH parametrization does not reflect the variations on short time-scales of the actual SFH. Yet, the average SFR on larger time-scales is recovered.

A comparison of the SFR values from the SED fitting and the simulated SFRs is shown in Fig. 9. The average values are calculated by the linear (not geometric) mean. This ensemble average is similar to what a deep-field survey would measure, as it randomly samples many galaxies at different stages of bursting and quenching to derive a SFR density. Interestingly, the average SED-derived SFRs agree well with the actual average SEDs on long time-scales. That is, there is no bias in one direction. These conclusions apply only if we observe the entire population of galaxies at a given mass. If a flux limited survey only selects the brighter galaxies at a given mass, then it will only select the galaxies in a new burst and the derived average SFR would be biased high. As expected, the actual SFR scatter matches the scatter derived by the SED fitting for the larger mass galaxies. However, as expected from the previous result shown in Fig. 8, the scatter in the SED-derived SFR is much lower than the actual SFR in the lower mass galaxies.

In Fig. 10, a comparison between the stellar masses from SED fitting and the stellar masses from the simulations is shown. The
average values are calculated by the geometric mean of the stellar masses. On average, there is a good agreement for the larger mass galaxies but there is some scatter for galaxies with log(M_*/M⊙) ≥ 8. For these lower mass galaxies the SED fitting underestimates the stellar mass by, on average, a factor of 0.2 dex. The reason is that recent bursts (with their high mass-to-light ratios) can completely dominate the high mass-to-light ratio from older generations of stars. Qualitatively our results on the stellar masses are compatible with other analyses such as the one presented by Mitchell et al. (2013). They conclude that stellar masses estimated from SED fitting may be underestimated when considering dwarf galaxies that have undergone a recent burst of star formation. That is also true in our analysis.

7 SUMMARY

The SFHs of dwarf galaxies are thought to be characterized by short bursts followed by rapid quenching due to supernovae feedback. Typically, analysis of the broad-band SEDs and nebular emission lines of high-redshift galaxies does not consider SFHs with short (<10 Myr) time-scale variations. To the contrary, fits of the SFHs and ages are usually parametrized as slowly varying exponential functions (both decreasing and increasing). In this paper, we use the SFHs from numerical simulations of low-mass galaxies with realistic prescriptions for supernovae feedback to consider several important consequences of bursty star formation in these galaxies. Our galaxies are located at the redshift range where the cosmic SFR peaks, namely z ∼ 2–3, although similar principles apply to other redshifts. In particular, we analyse the L_*(1500 Å)/L∗_ν(900 Å) ratio and the Lyα and Hα EWs. The star-forming main sequence is investigated as well in the stellar mass range of around 10^7–10^10 M⊙. We stress that the values reported in this analysis are rest frame and that dust extinction is not included. This fact will not change any of our conclusions. The main points presented in our analysis are briefly summarized in this section.

(i) Because the LyC, L_α(900 Å), is emitted from only the most massive short-lived stars, it accurately traces the rapid and large variations in SFR in the low-mass galaxies. Specifically, the LyC (and nebular emission lines related to the LyC) vary by 1.03, 0.45, and 0.22 dex in the mass ranges of log(M_*/M⊙) < 8, 8 ≤ log(M_*/M⊙) ≤ 9, and log(M_*/M⊙) > 9, respectively. Though the scatter in the non-ionizing UV continuum, L_ν(1500 Å), increases towards lower stellar mass, it does so at a much lower degree (0.43, 0.28, 0.16 dex in the same mass ranges). Finally, the continuum at optical wavelengths, L_ν(5500 Å), and longward increases by only approximately 0.05 dex from the most massive to least massive bins.

(ii) Because the ionizing photons vary on different time-scales than the longer wavelength continuum photons, the ratio of non-ionizing to ionizing continuum can vary significantly in low-mass galaxies, as the SFR variation also increases. This ratio, L_ν(1500 Å)/L_ν(900 Å), is an important number for converting direct LyC detections to LyC escape fractions or for converting observed UV luminosity functions to ionizing photon production rates. A constant value is often assumed. However, we show that this ratio can be lower by a factor of 2 in galaxies with recent bursts and can be very large after recent star formation quenching. Careful consideration of this variation will need to be taken into account in future studies of low-mass galaxies.

(iii) Because of the widely varying luminosities of dwarf galaxies, it is very difficult to obtain complete samples at a given mass in flux limited surveys. Selection of dwarf galaxies near the flux limit of a survey will be biased towards galaxies in the burst phase. This is of particular concern for Hα-selected samples of lower mass galaxies like those selected with WFC3/IR grism spectroscopy. Yet, it is also a concern for UV-selected dwarf galaxies as well. Such selection biases should be carefully considered when attempting to determine average properties of dwarf galaxies at high redshift in future studies.

(iv) The ratio between UV and Hα luminosity is a useful observable that quantifies burstiness in the SFH of dwarf galaxies. As new instruments allow Hα detection in galaxies of lower mass at z ∼ 2, this will be a particularly useful metric with which to quantify bursty star formation. Specifically, log(L_ν(1500 Å))/L_α(900 Å) > 2.5 signifies a recent shutdown of star formation on the time-scale of less than 10 Myr. Determining this ratio in a large sample of dwarf galaxies will allow us to better constrain our models and simulations of supernovae feedback and bursty star formation.

(v) Dwarf galaxies are only significantly producing ionizing photons in their burst phase. And because galaxies in this phase are preferentially selected (via UV continuum or nebular emission lines), it is much easier to find the galaxies responsible for the bulk of the ionizing emission than it is to detect a mass complete sample. Such selection biases and their effects on derived escape fractions or volume ionizing emissivities of dwarf galaxies should be carefully considered.

(vi) While the scatter in L_ν(1500 Å) does increase towards lower stellar masses, the scatter in L_α(900 Å) is three times larger and closely reflects the actual variation in the SFR. Therefore, when characterizing the dispersion in SFR of galaxies with log(M_*/M⊙) < 9, it is important to use a short time-scale tracer of star formation such as Hα, and not the UV-to-optical continuum SEDs.

(vii) We investigate whether an incorrect (slowly varying) parametrization of the SFH results in incorrect derived quantities (e.g. SFRs and stellar masses). We find that, on average, the SFRs derived from SED fitting are correct at all stellar masses. However, the SFR dispersion derived from SEDs is considerably smaller than the true SFR dispersion in these galaxies. These conclusions apply only if we sample the entire population of galaxies at a given mass,
otherwise the SFR will be overestimated as flux limited surveys will be biased towards selecting galaxies in a burst phase.

(viii) On average, for galaxies with $\log(M_*/M_\odot) > 8$, the stellar masses from SED fitting are correct. However, for galaxies with $\log(M_*/M_\odot) < 8$, we find that stellar masses are underpredicted by the SED fitting by a factor 0.2 dex when assuming slowly varying SFHs.

ACKNOWLEDGEMENTS

We are grateful to Stéphane de Barros, Alaina Henry, Philip Hopkins, Jason X. Prochaska, and Marc Rafelski for useful discussions. We also thank the referee for improving the paper. AMB acknowledges support from HST AR-12631, provided by NASA through a grant from the Space Telescope Science Institute, which is operated by the Association of Universities for Research in Astronomy, Incorporated, under NASA contract NAS5-26555.

REFERENCES

Alavi A. et al., 2014, ApJ, 780, 143
Atek H. et al., 2010, ApJ, 723, 104
Atek H. et al., 2011, ApJ, 743, 121
Atek H. et al., 2014, ApJ, 789, 96
Becker G. D., Bolton J. S., 2013, MNRAS, 436, 1023
Beckwith S. V. W. et al., 2006, AJ, 132, 1729
Behroozi P. S., Wechsler R. H., Conroy C., 2013, ApJ, 770, 57
Belli S., Jones T., Ellis R. S., Richard J., 2013, ApJ, 772, 141
Bigiel F., Leroy A., Walter F., Brinks E., de Blok W. J. G., Madore B., Thornley M. D., 2008, AJ, 136, 2846
Bigiel F., Leroy A., Walter F., Blitz L., Brinks E., de Blok W. J. G., Madore B., 2010, AJ, 140, 1194
Blanc G. A., Heiderman A., Gebhardt K., Evans N. J., II, Adams J., 2009, ApJ, 772, 141
Boselli A., Boissier S., Cortese L., Buat V., Gavazzi G., 2009, A&A, 508, L26
Boquien M., Buat V., Perret V., 2014, A&A, 571, A72
Boisselli A., Boissier S., Cortese L., Buat V., Hughes T. M., Gavazzi G., 2009, A&A, 513, A47
Brooks A. M. et al., 2011, ApJ, 728, 51
Brooks A. M., Governato F., Booth C. M., Willman B., Gardner J. P., Zolotov A. et al., 2014, ApJ, 786, 87
Bruzual G., Charlot S., 2003, MNRAS, 344, 1000 (BC03)
Brammer G. B. et al., 2012, ApJS, 200, 13
Brammer G. B. et al., 2013, ApJS, 200, 10
Brinchmann J., Charlot S., White S. D. M., Tremonti C., Kauffmann G., Heckman T., Brinkmann J., 2004, MNRAS, 351, 1151
Brooks A. M., Zolotov A., 2014, ApJ, 786, 87
Brooks A. M., Governato F., Booth C. M., Willman B., Gardner J. P., Wadsley J., Stinson G., Quinn T., 2007, ApJ, 663, L17
Brooks A. M. et al., 2011, ApJ, 728, 51
Bruzual G., Charlot S., 2003, MNRAS, 344, 1000 (BC03)
Ceverino D., Klypin A., Klimek E. S., Trujillo-Gomez S., Churchill C. W., Primack J., Dekel A., 2014, MNRAS, 442, 1545
Chabrier G., 2003, ApJ, 586, L133
Chabrier G., 2003, ApJ, 586, L133
Christensen C., Quinn T., Governato F., Stilp A., Shen S., Wadsley J., 2012, MNRAS, 425, 3058
Christensen C., Governato F., Quinn T., Brooks A. M., Fisher D. B., Shen S., McCleary J., Wadsley J., 2014, MNRAS, 440, 2843
da Silva R. L., Fumagalli M., Krumholz M., 2012, ApJ, 745, 145
da Silva R. L., Fumagalli M., Krumholz M. R., 2014, MNRAS, 444, 3275
Dominguez A. et al., 2013, ApJ, 763, 145
Forero-Romero R. E., Dijkstra M., 2013, MNRAS, 428, 2163
Fumagalli M., da Silva R. L., Krumholz M. R., 2011, ApJ, 741, L26
Gaivóncio M. et al., 2004, ApJ, 600, L93
Gill S. P. D., Knebe A., Gibson B. K., 2004, MNRAS, 351, 399
Glazebrook K., Blake C., Economou F., Lilly S., Colless M., 1999, MNRAS, 306, 843
Governato F. et al., 2012, MNRAS, 422, 1231
Gronwall C. et al., 2007, ApJ, 667, 79

This paper has been typeset from a TeX/LaTeX file prepared by the author.