The H-Dibaryon and the Hard Core

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The H dibaryon, a single, triply magic bag containing two up, two down and two strange quarks, has long been sought after in a variety of experiments. Its creation has been attempted in $K^-$, proton and most recently in relativistic heavy ion induced reactions. We concentrate on the latter, but our conclusions are more generally applicable. The two baryons coalescing to form the single dibaryon, likely $\Lambda\Lambda$ in the case of heavy ions, must penetrate the short range repulsive barrier which is expected to exist between them. We find that this barrier can profoundly affect the probability of producing the H state, should it actually exist.

25.75, 24.10.Lx, 25.70.Pq

I. INTRODUCTION

Since the H-dibaryon was proposed by R. Jaffe \cite{Jaffe} as a likely candidate for a viable six quark bag (uuddss), there have been a variety of experiments proposed and carried out to locate it, which as yet have been unsuccessful. Some of these experiments involve production via $[K^-, K^+]$ and proton induced reactions on light targets \cite{Kadkhoda}. More recently strangeness-rich heavy ion collisions \cite{Naumann} have been thought to offer a preferred mechanism for generating the H. The H can also be viewed in the limit of ideal $SU_f(3)$ symmetry as the doubly strange, maximally symmetric, singlet combination of the octet baryons. However, given the mass gap existing between the $\Lambda$ and the more massive $\Sigma$ and $\Xi$ states, it is probable that H consists mainly of $|\Lambda\Lambda\rangle$. Consequently its lifetime would be expected to be closer to one-half that of the $\Lambda$, than to the $\sim 10^{-9}$ s value suggested by some authors \cite{Yuan}.

Definitive observation of a double $\Lambda$ hypernucleus is often considered antithetical to the existence of the H; one reason being that the two strange baryons, kept captive by their affinity to the normal nucleons would quickly fall into the lower energy dibaryon state. Of the recorded observations of such nuclei \cite{Hagemann, Hagedorn} only the latter, performed at KEK, seems a good candidate \cite{Hagedorn}. This proposition, that the existence of doubly strange hypernuclei rules out the existence of the H, need not be valid, and can become prejudicial. A hybrid state consisting partially of a six-quark bag and partly of a double $\Lambda$ doorway state, attached to a nucleus might be identifiable in experiments involving the production of quite light doubly strange hypernuclei \cite{Garrido}. We will have more to say about this possibility in what follows.

To our knowledge all theoretical estimates of production rates in heavy ion collisions \cite{Bartholomew}, irrespective of mechanism, have generally overlooked the possible existence of a hard core in the baryon-baryon interaction at short distances. As we will show, under reasonable assumptions about the hard core, this can lead to quite appreciable suppression of H production. Here, we introduce this device into the framework of production via heavy ion collisions. A previous calculation \cite{Bartholomew} suggested a high formation probability, $\sim 0.07$ H per central Au+Au collision. A recent AGS experiment, E896 \cite{E896}, is presently analyzing some 100 million central Au + Au events and could, in the light of this earlier prediction of the formation rate, provide a definitive search for the H. It is our present intention to at least semi-quantitatively understand the extent the hard core might interfere with this hope.

We treat the short range baryon-baryon force in a transparent and heuristic fashion. The best evidence from doubly strange nuclei \cite{Hagemann, Hagedorn} suggests that the H, if it exists at all, is rather weakly bound, with binding less than 20 MeV and probably considerably less. We use this to justify handling the coalescence of $\Lambda$ pairs in relativistic ion collisions much in the manner we previously employed for the deuteron \cite{Kahana}, also a weakly bound system and likely to form only after all np-constituents have ceased high energy cascading. This newer calculation of coalescence, in Ref \cite{Kahana}, was not employed in earlier work on the H \cite{Kahana}, and in fact had yielded a somewhat reduced formation rate for the H, by some 50$, in the absence of a hard core. The results arrived at in the present work suggest a more dramatic suppression. This conclusion should hold true for either the pure six quark bag proposed by Jaffe or for hybrid states, in both the relativistic heavy ion and proton induced environments if the hard core indeed exists. We refer simply to the repulsive potential as a hard core, although in practice we employ a repulsive potential with finite height.
II. COALESCENCE AND THE HARD CORE

Coalescence is treated quantum mechanically in the more recent approach \[12\] by calculating the overlap of the wave packets of the initial combining pair with an outgoing packet for the final bound state. The cascade in which this coalescence estimate is embedded provides the distributions of both relative momentum and relative position required for determining the degree of overlap. The overlap integral, squared to produce a probability, is then part of a factorised version of dibaryon production. The combining pair of particles may form a bound state only after each ceases to interact in the cascade, as was indicated previously.

However, for two strange baryons to coalesce they must first penetrate their mutual, repulsive, core. Such a core has a negligible effect on the deuteron which is spatially rather extended. This cannot be so for the H: not if this object consists at least partially of six quarks in a bag comparable in size to that for a single baryon, where short range repulsion can play a considerable role. H formation from two $|ΛΛ>$ could be viewed as proceeding in two steps: first a merging into a broad, deuteron-like doorway state and the second, barrier penetration into a single compact dibaryon, with the hard core repulsion forming the barrier. The overall rate for H production is then found to be the product of the usual coalescence probability and a barrier penetration post-factor. Naturally there are unknowns in such a calculation, one being the effective range at which combining baryons dissolve into a single bag, another being the nature of the short range forces (the hard core). The first we treat as a parameter; the second we approximate by using the $NN$ Bonn potential [13], limiting ourselves to including its shortest ranged components, due to $ω$ and $σ$ exchange.

Our approach should apply equally well to a hybrid model in which the H is a combination of a deuteron-like $|ΛΛ>$ state and a, presumably smaller, six quark bag. The wave function would then in principle appear as:

$$\Psi = α|ΛΛ > + β|g^b >,$$

with $α$ and $β$ representing amplitudes for the two-baryon and six-quark components of the hybrid state.

Whatever the actual composition of the physical dibaryon, it must have some minimal six quark bag presence, or else the relatively weak $Λ − Λ$ force could not lead to binding. The pure Jaffe-like state [1] corresponds to $β = 1$, but our coalescence calculation is in principle independent of this parameter. The procedure we follow to estimate the effect of the repulsive core on entry from a doorway $Λ − Λ$ state into the final H state would be applicable to either the pure bag or hybrid state cases. The only question is the precise nature of the hard core itself. We have stated our assumptions clearly above.

The barrier penetration calculation is described here in full detail. While the cascade and coalescence formalisms are referenced elsewhere [12]. In a standard single meson exchange model (OBEP) [13] of the nuclear two body interaction the hard core arises from $ω$ exchange. In transferring this force to the $ΛΛ$ system one should, however, probably not scale by the numbers of non-strange quarks. This component of the force is expected to be essentially flavour-independent. To the Bonn potential [13] one must then add a term due to the exchange of a $φ$ meson between the $s$ quarks. This observation suggests there should be little or no modification of the nucleon-nucleon hard core in applying it to the two $Λ$ system. We thus assume an intermediate to short range force exists of the form:

$$V(r) = V_ω(r) + V_σ(r),$$

where

$$V_i(r) = g_i(1/r)exp(-m_i r).$$

The strong intermediate range $σ$ attraction reduces the effect of the hard core, while the longer range parts of the potential are assumed to play a negligible role in coalescence. The two baryons will approach to some outer radius $b_i$ in fact a classical turning point, before being faced with the strong short range repulsion produced by the $ω$. At some smaller radius $a_i$, representing the separation of $Λ$-cluster centers, the two baryons are imagined to dissolve into a six quark bag. The calculation is especially sensitive to this ‘critical separation radius’ $a$. Although our final results on barrier penetration are consequently somewhat uncertain, it will become apparent that one thing one cannot do, is to ignore them while there remains any reason to believe that a short range repulsion exists.

III. PENETRATION FACTOR

We appeal to the WKB [14] method for an estimate of the barrier penetration factor. We require some picture of the inter-baryon potential, from the larger separations in the initial doorway state down to the inner reaches of the final multi-quark bag and for $r ≤ a$ take
\[ V(r) = -V_0, \quad (4) \]

while for \( r \geq a \)
\[ V(r) = V_\omega(r) + V_\sigma(r), \quad (5) \]
as specified in Equation 3 above. Thus, inside the radius \( a \) the two baryons, by fiat, melt into a bag. The probability of barrier penetration in this effective two body model can then be determined by calculating the WKB approximation for the transmission coefficient at relative energy \( E \):

\[ T(E) = 4 \sqrt{\frac{(V(b) - E)(E - V_0)}{V(b) - V_0}} \exp(-2\tau), \quad (6) \]

where
\[ \tau = \int_a^b dr \sqrt{2m(V(r) - E)}. \quad (7) \]

As advertised the upper limit, \( b \), of the integral for \( \tau \) is a turning point defined implicitly by
\[ E = V(b), \quad (8) \]
while \( a \) represents the outer separation at which the six quark bag forms. The non-relativistic calculation of transmission performed here is probably adequate, given that coalescence into a relatively weakly bound system will not proceed at very high baryon-baryon relative momentum.

**IV. COALESCENCE MECHANISM**

The rest of the calculation is straightforward, given the existence of a previously constructed heavy ion two-nucleon coalescence code \[12,15\]. The transmission coefficient \( T(E) \) is inserted into this heavy ion simulation at a point after the formation of a broad doorway state for the two strange baryons. That is to say, the \( H \) formation probability is taken as the product of \( T(E) \) and a coalescence factor for the doorway state as defined in Reference \[12\]. The size and structure of this state play only a minor role provided the turning point \( b \) is within its confines. There are two modes for operation of this code, labeled static and dynamic, both of which are described in Reference \[12\]. The dynamic code is self-contained, providing an internal estimate of the spatial spreading of the individual baryon wave packets, occasioned by interactions within the collision medium. The static mode produces essentially identical results provided the wave packet size, assigned as a fixed value in static coalescence, is appropriately tuned.

In the earlier work \[12\], a satisfactory agreement of the dynamic model with known \( Si + Au \) deuteron single and double differential cross-sections was demonstrated, and \( Au + Au \) predictions made. Only very preliminary data for deuterons from AGS \( Au + Au \) collisions existed at that time. In Fig(1) we have compared these 1996 dynamic coalescence calculations for deuteron production in \( Au + Au \) collisions at 11.6 GeV/c, with very recently submitted data from E866 \[16\]. Considering the nature of both experiment and theory, this prediction of absolute deuteron yield must be considered as a triumph. There are in the dynamic simulations no adjustable parameters. In light of this, and to minimise computer time needed to produce sufficient statistics for the much rarer \( H \) dibaryon, we have performed the coalescence estimates in this work using the static treatment, adjusted to agree with the dynamic normalisation and perforced with the deuteron data. This procedure does not appreciably affect \( H \) production estimates and permits us to more efficiently examine bag size parameters \( a \) where the hard core suppression can be rather large and the \( H \) yield truly small.

The deuteron prediction gives one great confidence in our treatment of the coalescence mechanism and lends credence to the use of a similar approach in estimating the creation of the elusive \( H \).

**V. HEAVY ION PRODUCTION OF THE H**

We consider \( Au + Au \) collisions at an incident energy of 10.6 GeV per nucleon. The actual beam energy in E896 \[3\] is 11.6 GeV but the use of a thick target reduces this to a lower effective average. The energy dependence, although appreciable and quoted, is by no means the most critical variable encountered in this simulation. The dependence on
the $\Lambda - \Lambda$ separation, $a$, at the moment of dissolution into a bag easily wins that title. We have also examined the dependence of the results on the size $r_h$ of the H-doorway state and $r_{sp}$ the spatial size of the wave packets in the static coalescence model. Neither prove to be of much consequence. In practice, $r_{sp}$ is chosen to assure agreement with deuteron yields from dynamic coalescence. In the present simulations this occurs for $r_{sp} \sim 1.5 - 2.0 \text{ fm}$, an eminently reasonable value.

The Bonn inspired prescription we described for the $\Lambda \Lambda$ potential reduces numerically to:

$$V(r) = 3.94 \frac{\exp(-3.97r)}{r} - 1.44 \frac{\exp(-2.97r)}{r} \text{GeV},$$

with $r$ measured in Fermis. The resulting short distance potential is graphed in Fig(2).

Our chief results are presented in Fig(3), indicating the variation of the numbers of H-dibaryons produced in central events with $a$. This latter parameter must not be thought of as an effective hard core radius for the $\Lambda - \Lambda$ interaction. The underlying quark-quark forces may also be viewed as possessing a repulsive short range component due to the exchange of vector mesons [17]. Even with complete overlap of the parent baryons, i.e. $a = 0$, the average inter-quark separation is, for a uniform spatial distribution, still comparable to the parent radius $r \sim 0.8 \text{ fm}$, i.e. considerably greater than any conceivable fixed hard core radius. We have considered baryon centers between 0.2 fm and 0.5 fm apart, whereas reasonable values probably lie between 0.2 fm and 0.3 fm, where the baryon overlap region is near 80% of the volume of a single baryon.

Even at the largest separations, H suppression due to the repulsive forces is not ignorable, but for the smallest values of $a$ the observation of the H, should it exist, becomes problematic. Early analysis [13] of the actual experimental setup using the simulation ARC [12], suggested a neutral background comparable with the initial estimate of 0.07 H’s per central collision [10]. Thus, for baryon separations of 0.20 fm to 0.35 fm one would need to achieve tracking sensitivities of $10^{-4}$ to $10^{-6}$ relative to background. In our worst case scenario, at $a = 0.2$, one is still left with perhaps a few thousand dibaryons produced in the E896 [3] sample, but immersed in what may prove to be a daunting background.

We have also considered variations due to bombarding energy and H-doorway radius. These are easily understood, if not noteworthy. A decrease of collision momentum from 14.6 to 10.6 GeV/c results in close to a 30% reduction in H’s produced per central event, while the yield is quite insensitive to the doorway radius.

VI. CONCLUSIONS

Short range repulsion between strange baryons can profoundly hinder coalescence into objects whose very existence depends on the presence of important bag-like structure. This lesson is even more applicable to the many H-searches initiated using ($K^-, K^+$) reactions [2], since these generally involve even lower relative energies $E$, and a consequent increased difficulty in barrier transmission.

There is perhaps one way to circumvent this frustrating roadblock in the discovery of the lightest of all possible strangelets. In the event a pair of strange dibaryons are attached, through a $[K^-, K^+]$ reaction, to a light nucleus, a hybrid H may form, and itself remain bound to the nucleus. An optimum final nucleus might be $^5_{\Lambda\Lambda}H$ [9,19]. The extra nucleons in this five particle system keep the captured $\Lambda$ pair together for some 100 picoseconds, this being far more than enough time for penetration of what then would constitute a rather modest, 1 - 2 GeV barrier. Any evidence that the $\Lambda\Lambda$ pairing energy substantially exceeded the 2 - 3 MeV or so expected from the known $\Lambda\Lambda$ interaction [21] would indicate the presence of a hybrid bag+doorway state. Appreciable observed decay into the $\Sigma^-p$ channel would strengthen such a conclusion.

Unfortunately, the very same repulsive forces which made coalescence into a bound H state difficult, may also, at the quark level, destroy the existence of the state. Only a detailed microscopic calculation can begin to answer this question, for example in a model in which quarks rather than baryons exchange mesons [17]. It is, in this context, disturbing that the search for the only ‘strangelets’ which we are certain do exist, i.e. doubly strange hypernuclei, may be discontinued despite the present finding of only a single good candidate [5].

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FIG. 1. Comparison of E866 data, recently submitted to Phys. Rev. C. (1999), with absolute predictions of the Dynamic Coalescence model from 1996. The agreement is excellent considering the conditions of both theory and experiment. A future work will examine the entire data set, including an attempt to insure that the experimental definition of centrality is more explicitly built into the theoretical simulation. For present purposes the 5 – 10% difference in normalisation between data and calculation is of no consequence.
FIG. 2. The short range Λ-Λ potential taken from the σ and ω exchange parts of the Bonn potential. At some separation a the two strange baryons are assumed to dissolve into a single bag, represented here by a shallow, attractive, effective potential. Barrier penetration from the di-Λ doorway state is represented by the dashed line.
FIG. 3. Absolute H Production per Central Au + Au event. The precise separation of di-Λ centers, $a$, at which a single six quark bag forms is of course not known, but a reasonable value is likely less than 0.3 fm. Even for complete baryon overlap, the average distance between constituent quarks is greater than the conventional nucleon-nucleon hard core radius of 0.4 fm. The assumed energy for the collision is an average 10.6 GeV to account for averaging by a thick target. The energy dependence is not strong enough to alter the displayed results appreciably. Centrality is defined by $b \leq 2$, but again calculation indicates only a weak dependence on impact parameter.
FIG. 4. H Suppression. Suppression is here taken relative to the older calculation of 0.07 per central event \([10]\) since this gives one an approximate suppression relative to an early estimate of background \([18]\). Again the abscissa is the assumed separation \(a\) at which a bag forms.