Universal Time Tunneling

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How much time does a tunneling wave packet spend in traversing a barrier? Quantum mechanical calculations result in zero time inside a barrier. In the nineties analogous tunneling experiments with microwaves were carried out. The results agreed with quantum mechanical calculations. Electron tunneling time is hard to measure being extremely short and parasitic effects due to the electric charge of electrons may be dominant. However, quite recently the atomic ionization tunneling time has been measured. Experimental data of photonic, phononic, and electronic tunneling time is available now and will be presented. It appears that the tunneling time is a universal property independent of the field in question.

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I. INTRODUCTION

Several theoretical investigations resulted in zero time tunneling, i.e. the time spent inside a barrier is zero [1, 2, 3, 4, 5]. Experimental studies with microwaves have shown that the measured short barrier traversal time is spent at the front boundary of a barrier, whereas actually zero time is spent inside the barrier [6]. For example, as illustrated in Fig. 1 the barrier traversal time was found to be zero in a frustrated total internal reflection (FTIR) experiment with a symmetrical double prisms set-up [7]. The experimental results revealed for the traversal time \( t_\perp = 0 \). Reflected and transmitted signals arrived the detector at the same time. The measured short barrier traversal time is of the order of magnitude of the reciprocal frequency \( T \) of the wave packets [8]. The time \( \tau \) arises at the barrier front boundary, as it is evident from inspecting Fig. 1. This is followed by the fact that reflection and traversal time are equal.

\[
\tau = t_\parallel + t_\perp = t_\parallel
\]

(1)

where \( \tau \) is the measured barrier traversal time and \( \nu \) the carrier frequency of the electromagnetic wave packet.

In the case of a wave packet with a rest mass the traversal time can be written by

\[
\tau \approx \frac{h}{E},
\]

(2)

where \( h \) is the Planck constant and \( E \) is the particle energy.

This universal tunneling time behavior observation was found at the same time by Esposito studying theoretically the traversal time and obtained the modified relation

\[
\tau_A = \frac{1}{\nu} \cdot A,
\]

(4)

where \( A \) is depending on the special barrier and wave packet in question [9].

Later experimental data on phonon and electron tunneling have pointed to a similar barrier traversal time [10, 11]. Quite recently, electronic tunneling data were measured in the ionization process of Helium [12], which also fit in this approximate universal behavior of the traversal time of Eq. 1.

FIG. 1: Sketch of the time components in the investigated symmetrical beam design of double prisms under the condition of FTIR. Reflected and transmitted signals were detected at the same time in spite of the fact that the transmitted beam traveled an additional distance \( d \) due to the gap [7].

II. EXPERIMENTAL DATA

Superluminal tunneling was observed first with microwaves in undersized wave guides by Enders and Nimtz in 1992. Superluminal tunneling has been reproduced at microwave and optical frequencies in photonic band...
TABLE I: Tunneling Time

| Tunneling barriers      | Reference      | \( \tau \) | \( T = 1/\nu \) | \( \tau_A \) |
|-------------------------|----------------|-----------|----------------|-----------|
| frustrated              | Haibel/Nimtz   | 117 ps    | 120 ps         | 81 ps     |
| total reflection         | Balcou/Dutriaux| 30 fs     | 113 fs         | 36.8 fs   |
| at double prisms         | Mugnai et al.  | 134 ps    | 100 ps         | 87 ps     |
| photonic lattice         | Steinberg et al.| 2.13 fs  | 2.34 fs        | 2.02 fs   |
|                         | Spielmann et al.| 2.7 fs    | 2.7 fs         | 2.98 fs   |
| undersized waveguide     | Enders/Nimtz   | 1.30 ps   | 115 ps         | 128 ps    |
| Ionization tunneling      | Eckle et al.   | 6 as      | 15 as          | 25 as     |
| acoustic (phonon) tunneling | Yang et al.   | 0.8 \( \mu \) s | 1 \( \mu \) s | 0.6 \( \mu \) s |

III. DISCUSSION

In 1962 Hartman’s quantum mechanical study on tunneling of a wave packet revealed for opaque barriers a superluminal velocity and a tunneling time independent of barrier length \([1, 11, 14]\). The calculations were confirmed by the above mentioned photonic tunneling experiments, see for instance Refs. \([17, 18, 19, 20]\). Hartman and Nimtz figured an empirical universal relation for the tunneling time out, which was observed with several photonic barriers \([8]\). The empirical relation was theoretically substantiated by Esposito for both particles and evanescent optical modes \([6]\). He has shown that the simple relation \( \tau \approx \frac{1}{\nu} = T \) can be substituted by the general analytical expression of Eq. \( \ref{eq:universal} \) The factor \( A \) is given for the case of a tunneling particle described by the Schrödinger equation as follows,

\[
\tau_A = \frac{\hbar}{\sqrt{E(V_0 - E)}} = \frac{1}{\nu} \cdot \frac{E}{4\pi^2(V_0 - E)},
\]

where \( E \) and \( V_0 \) are the particle energy and the height of the square potential barrier, and \( E = h \nu \).

In the case of FTIR at double prisms the factor becomes

\[
\tau_A = \frac{1}{\nu} \cdot \frac{n_1 \sin^2 \theta}{\pi \cos \theta \sqrt{n_1^2 \sin^2 \theta - n_2^2}}
\]

where \( n_1 \geq n_2 \) are the refractive indices of the prisms and of the gap, respectively. \( \theta \) is the angle of incidence. Schrödinger and Helmholtz equations result in analogous relations for the tunneling time. The factor corrects the \( 1/\nu = T \) value usually by some percent.

Tunneling modes propagate in zero time. This contradicts the often given interpretation of tunneling by the uncertainty relation, where the particle may acquire energy to overcome the barrier. However, this would cause a comparatively long vacuum barrier traversal time, much longer than the observed universal traversal time. The interaction with a square barrier takes place at the barrier’s front and causes the short universal tunneling time presumably for all fields due to the same mathematical tunneling description.
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