Temperature Field Reconstruction by Acoustic Based on Newton-Raphson Regularization Iteration

Yanqiu Li, Shi Liu, H. Inaki Schlaberg and Jiye Zhang

ABSTRACT

The principle of three-dimensional temperature field reconstruction by acoustic is introduced. In order to realize the three-dimensional temperature field reconstruction by acoustic, the Newton-Raphson method based on the least square method is developed. Three-dimensional temperature field is reconstructed by regularization iteration method that the regularization term is added according to the features of temperature field distribution. It is shown that the algorithm of Newton-Raphson and regularization iteration can realize the reconstruction of the temperature field.

INTRODUCTION

Acoustic pyrometry technology is very important in many real industrial needs and applications because of the advantages as non-contact acoustic temperature measurement, without interference to measurement objects, easy to realize visual measurement[1-5]. It is the key problem to obtain the quick and accuracy reconstruction algorithm to realize temperature field reconstruction in the region being measured. The main algorithms in two dimensional temperature field reconstruction are least squares, gaussian function and regularization algorithm, algebraic reconstruction algorithm and so on. The algorithms of three dimensional temperature field reconstruction main include the least square and SVD algorithm [6], CGLS and LSQR [7] etc..It is a important research subject to improve the accuracy of the three-dimensional temperature field reconstruction, this paper presents Newton Raphson and regularization iteration method to reconstruct the three-dimensional temperature field, the simulation results show that this method is suitable for the reconstruction of temperature field, that satisfy the requirement of temperature measurement.

Yanqiu Li, Shi Liu, H. Inaki Schlaberg, Jiye Zhang. State Key Laboratory of Alternate Electrical Power System with Renewable Energy Sources, North China Electric Power University, Beijing 102206, China
THE PRINCIPLE OF ACOUSTIC PYROMETRY

The basic principle of acoustic pyrometry is based on that the propagation speed in gas medium is the function of gas temperature[8].

\[ v = \sqrt{\frac{\gamma RT}{M}} = Z\sqrt{T} \]  

(1)

where \( v \) is the propagation speed of acoustic wave in gas medium, m/s; \( R \) is molar gas constant, J/mol•k; \( \gamma \) is gas specific heat ratio, \( T \) is the absolute temperature of gas, K, \( M \) is molar mass of gas, kg/mol, \( Z = \sqrt{\frac{\gamma R}{M}} \), it is a constant to specified gas.

Time of Flight(TOF) that acoustic wave is propagated along a sound ray is represented

\[ b_i = \int_{L_i} \frac{1}{v_j(x,y,z)} dl + n_i = \int_{L_i} a_j(x,y,z) dl + n_i \]  

(2)

in which, \( b_i \) represents the time of flight of the wave along the \( i \)th sound path; \( L_i \) is the \( i \)th sound wave ray transmission path; \( (x,y,z) \) is the location of units, \( v_j(x,y,z) \) is the sound speed of the \( j \)th imaging unit; \( a_j(x,y,z) \) is the slowness of the \( j \)th pixel units (i.e., the reciprocal of velocity), \( n_i \) is the measurement noise. When the number of reconstructed pixel unit and the grid size is small, it can be considered that \( a_j(x,y,z) \) in each imaging unit is approximately constant, then the formula (2) can be simplified as reconstruction model in the form of a matrix equation:

\[ B = LA + n \]  

(3)

in which, \( L \in \mathbb{R}^{M \times N} \) represents the line segment length, \( B \in \mathbb{R}^M \) is the TOF vector measured practically, \( A \in \mathbb{R}^N \) represents the space state factor, i.e. the reciprocal of velocity. \( M \) stands for the total of independent TOF measured across the temperature field, \( N \) is the number of pixels, \( n \in \mathbb{R}^N \) represents the noise vector in TOF measurement data. \( A \) is calculated by proper reconstruction algorithm, and then the temperature \( T(x,y,z) \) is obtained according to (4).

\[ T(x,y,z) = \frac{1}{A(x,y,z)^2 Z^2} \]  

(4)

TEMPERATURE FIELD RECONSTRUCTION BY NEWTON-RAPHSON METHOD

In reconstruction algorithm, the modified Newton-Raphson method based on the least squares technique is most widely used. Newton-Raphson algorithm is an iterative approximation algorithm which is used to find a solution of nonlinear function originally. For the three-dimensional temperature field reconstruction by acoustic
method, it is assumed that there is not measurement error to construct auxiliary function J by the least squares criterion.

\[ J(A) = \frac{1}{2} (LA - B)^T (LA - B) \]  

(5)

where \( J(A) \) is the sum of squares of the time of flight (TOF) differences of calculated and actual measured. The inverse problem of reconstruction of temperature field distribution is to find the distribution of the slowness vector A and obtain the minimum \( J(A) \), which is least-squares problem mathematically. To obtain the minimum of function \( J(F) \), let the partial derivative \( D_j(A) \) that \( J(A) \) is for each element of A is 0, namely

\[ \frac{\partial J(A)}{\partial A_j} = D_j(A) = -\sum_{i=1}^{N} (B_i - L_i A_j) \frac{\partial (L_i A_j)}{\partial A} = 0 \quad j=1,2,...,N \]  

(6)

\[ D_j(A) = -L^T (LA - B) = 0 \]  

(7)

in which A is Jacobian matrix. It is obtained according to the Newton method

\[ A^{(k+1)} = A^{(k)} - [D'(A^{(k)})]^{-1} \cdot D(A^{(k)}) \]  

(8)

where \( D'(A^{(k)}) \) is Hessian matrix \( H(A(k)) \), its calculation equation is

\[ H(A^{(k)}) = D'(A^{(k)}) = L^T L \]  

(9)

it is obtained

\[ A^{(k+1)} = A^{(k)} - (L^T L)^{-1} [L^T (LA^{(k)} - B)] \]  

(10)

This is the Newton -- Raphson algorithm that A (k) should be updated at each iteration. Simple Newton - Raphson algorithm solves the nonlinear problem using the least-squares technique. Jacobi matrix and hessian matrix are ill-posed. A priori information is introduced into the regularization technique to improve the eigenvalues of the solution and make the solving process to be stable. The regularization algorithms commonly used are Tikhonov regularization algorithm, NOSER regularization algorithm, LM regularization algorithm and variation regularization algorithm, etc. This paper uses the Tikhonov regularization algorithm to form the regularization to improve the Newton Raphson algorithm.

Tikhonov regularization algorithm is based on applying damping effect to high order eigenvector and then the reconstruction process of the model parameters is included. It achieves the damping effect to the solution by adding a penalty function in the objective function to achieve the purpose of making the solution stability, at the same time, to a certain extent ensuring the spatial resolution of the solution. The penalty function is added in the objective function during the process of the acoustic temperature field reconstruction.
Then the iteration equation of slowness vector $A$ is

$$A^{(k+1)} = A^{(k)} - (L^T L + \alpha R^T R)^{-1} [L^T (LA^{(k)} - B) - \alpha R^T R]$$  \hspace{1cm} (12)

It is also called regularization iteration method, in which $\alpha$ is a positive constant. It is chosen that $R=I$ and $A_0=0$ in standard Tikhonov regularization method.

It should be pointed out that it is very important to set an appropriate initial value cleverly to solve the nonlinear algebraic equation with Newton-Ralphson method. If the right initial value is set, it can not only guarantee the convergence (i.e., to approach the real value) but also reduce the number of iterations. On the contrary, if the initial value setting is not appropriate, it not only increases the number of iterations, but also makes the iteration divergence (i.e., away from the real value) or circulate (i.e., reciprocate change near the true value). This paper choose the result of the least squares method as initial value to get a better solution.

**NUMERICAL SIMULATION AND DISCUSSION**

In this part, numerical simulation is adopted to verify the feasibility of the Newton-Raphson reconstruction algorithm. A cube space is chosen and 20 acoustic sensors are arranged in three different planes respectively. The whole image reconstruction are divided into 27 sub areas. The bicubic interpolation are adopted after getting the temperature in each subdomain.

Three kinds of classical temperature field are chosen to simulate that are single-peaked symmetric, single-peaked deflection and double peak symmetric, the temperature field distribution are shown in Fig.1.

![Figure 1. Diagrams of model temperature field.](image1)

The temperature field reconstruction results of Newton-Raphson regularization iteration algorithm are shown in Fig.2.

![Figure 2. Diagram of reconstructed temperature field by Newton-Raphson method.](image2)
The root mean square error of reconstruction by Newton-Raphson and Regularization Iteration for three model temperature fields that are single-peaked symmetric, single-peaked deflection and double peak symmetric temperature field are 0.0389, 0.0652 and 0.1256 respectively.

CONCLUSION

The algorithm of temperature field reconstruction is the key problem in temperature field reconstruction by acoustic CT which influences the precision of the reconstruction and practicability of reconstruction technology. In order to find the optimal temperature field reconstruction algorithm, temperature field distribution can be more accurately reconstructed the Newton-Raphson regularized iterative algorithm is used to reconstruct three dimensional temperature field distribution in this paper. Simulation is made using acoustic data and the reconstruction algorithm on temperature field, simulation results show that the algorithm of Newton-Raphson regularization iteration meets the requirement. An effective method of acoustic method for three dimensional temperature field reconstruction is provided.

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