QED-Induced Rapidity-Gap Events at the Z Peak*

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ABSTRACT

We study rapidity-gap events in $e^+e^-$ annihilation at the Z boson peak initiated by the emission of a virtual photon. This mechanism is suppressed by the QED coupling constant, but it is enhanced due to a large propagator term from the virtual photon. For typical kinematics, we find a smaller event rate than analogous QCD type gap events. In the small jet-pair invariant mass limit, the QED type events follow a $1 + \cos^2 \theta$ distribution in the jet-pair scattering angle, instead of the $\sin^2 \theta$ distribution of the QCD case.

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Jet events observed at $e^+e^-$ annihilation can be understood as the creation of short-distance quarks and gluons which subsequently materialize into hadrons. As the short-distance colored particles move apart, the rapidity region separating the colored objects is filled by the hadrons. However, as pointed out in Refs. [1,2], a perturbative QCD mechanism exists which generates jet events containing rapidity gaps. The representative diagrams are shown in Fig. 1. Essentially, two color-singlet collinear jet pairs are produced at short distance. The production of hadrons is expected to be suppressed in the rapidity gap region separating the two color-singlet systems. This QCD mechanism is found to give an observable rate at Z-peak for producing gap events\(^3\). Qualitatively, the rapidity gap events constitute a fraction

$$R_{\text{gap}}^{QCD} = \frac{\sigma_{\text{gap}}^{QCD}}{\sigma_{\text{tot}}} \sim \frac{\alpha_s^2 M_1^2}{s} \frac{M_2^2}{s}$$  \hspace{1cm} (1)$$

of the total Z cross section, where $M_1^2$ and $M_2^2$ are the invariant mass of the jet pairs.

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Figure 1. Perturbative QCD mechanisms for generating rapidity gap events. The dashed lines indicate that the produced partons are in color-singlet state. (a) Two final-state quark-antiquark pairs. (b) A quark-antiquark jet-pair and a two-gluon jet-pair.
The recent interest in the study of events containing rapidity gaps has been motivated by the use of these events as possible triggering signals in high-mass scale physics [1,4,5,6]. In this article, we shall study an interesting QED mechanism that contributes additionally to the generation of rapidity-gap events in $e^+e^-$ annihilation at $Z$ peak. One possibility is to replace the gluon propagator in Fig. 1(a) by a photon propagator. This contribution has an identical kinematic dependence of the analogous QCD process, but it is suppressed due to the smallness of the QED coupling constant. In Fig. 2 we show the Feynman diagram in an alternative configuration, where one of the color-singlet jet-pairs is effectively the decay product of a virtual photon. There is no QCD analogy for this configuration since the gluon is a color octet object. Although the new contribution is still suppressed by $\alpha^2_{em}$, the virtual photon propagator can substantially compensate for the coupling constant suppression. In fact, for small invariant-mass jet-pairs, the diagram in Fig. 2 acquires a large enhancement due to the small virtuality carried by the photon. This should be contrasted with the QCD diagram in Fig. 1(a), where the quark and antiquark produced by the virtual gluon are required to go
in opposite directions across the rapidity gap, so that the virtuality carried by the gluon is generally large.

From power counting we expect the QED rapidity gap events to be produced at a rate given by

\[ R_{\text{QED gap}} = \sigma_{\text{gap}}^{\text{QED}} / \sigma_{\text{tot}} \sim \alpha_{\text{em}}^2 \frac{M_2^2}{s}. \] (2)

That is, compared to the QCD case of Eq. (1), the event rate is enhanced by a power of \( s/M_1^2 \), despite the suppression in the coupling constant. In fact, for typical kinematics, the QED contribution can become as large as the \( q\bar{q}gg \) contribution of the QCD case (Fig. 1(b)). (In Ref. [1] we have shown that the \( q\bar{q}q\bar{q} \) contribution of Fig. 1(a) is about an order of magnitude larger than the \( q\bar{q}gg \) contribution.) In principle, the QED diagrams interfere with the QCD diagrams when the quark jets have identical flavor. However, as we shall see, in the small invariant mass limit, the interference effects become unobservable when we integrate over the azimuthal angle of the individual jets with respect to the thrust axis of the jet-pairs. In particular, this interference has no effect on the total cross section. The QED and QCD contributions to the event rate can therefore be calculated separately.

As in Ref. [1], we will neglect quark masses and consider the small jet-pair invariant mass limit: \( M_1^2, M_2^2 \ll s \). The kinematic variables are as specified in Fig. 2, where \( P_1 \) and \( P_2 \) are the four momentum of the jet pairs, and \( x_1, 1 - x_1, x_2, 1 - x_2 \) are the momentum fractions of the individual jets inside the jet pairs. We define the two-component weak charge of a fermion to be

\[ Q_f = \begin{pmatrix} Q_f^L \\ Q_f^R \end{pmatrix} = \begin{pmatrix} \sec \theta_w I_f - \sin \theta_w \tan \theta_w Q_f \\ -\sin \theta_w \tan \theta_w Q_f \end{pmatrix}, \] (3)

where \( \theta_w \) is the weak angle, \( I_f \) the isospin and \( Q_f \) the electric charge of the fermion.
f. Using this notation, the total $e^+e^-$ annihilation cross section around the $Z$ resonance can be conveniently expressed as:

$$\sigma_Z = \frac{\pi}{3} \frac{Q_Z^2 Q_e^2 \alpha_w^2 s}{(s - M_Z)^2 + \Gamma_Z^2 M_Z^2},$$

where

$$Q_Z^2 = \sum_f Q_f^2 = \sum_l Q_l^2 + 3 \sum_q Q_q^2 \simeq 3.771$$

$$(l = e, \mu, \tau; q = u, d, c, s, b),$$

$$Q_f^2 = Q_f^L^2 + Q_f^R^2,$$

$$\alpha_w = \frac{g_w^2}{4\pi} = \frac{e^2}{4\pi \sin^2 \theta_w} \simeq \frac{1}{29.3},$$

$$M_Z, \Gamma_Z = \text{mass and width of the } Z \text{ boson.}$$

We shall later use this cross section to normalize the production rate of rapidity-gap events. This is subject to the same caveat pointed out in Ref. [1]. Namely, initial state radiation induces a substantial correction to the above result for $\sigma_Z$ [7]. However, the same effect is present in rapidity-gap events; thus we expect these effects to largely cancel when we consider ratios of cross sections.

There are three additional Feynman diagrams contributing to QED rapidity gap events similar to the one depicted in Fig. 2. Basically, there are two diagrams with the virtual photon decaying into $q_a\overline{q}_a$ and another two diagrams with the virtual photon decaying into $q_b\overline{q}_b$. The intermediate formulas in the calculation of the helicity amplitudes are complicated and will not be presented here. However, we shall display a particular helicity amplitude in order to facilitate later discussion.

Let us momentarily add up the contributions of the two Feynman diagrams with the virtual photon decaying into $q_a\overline{q}_a$ and designate this sum by $iM^a$. Keeping only leading contribution in the jet-invariant mass $M_1$, we obtain the following
amplitude for all-positive fermion helicities (one right-handed electron line and
two right-handed quark lines):

\[ i\mathcal{M}^a(+) = \frac{4g_w^2 e^2 Q_e^R Q_a^R Q_b \sqrt{s}}{(s - M_Z^2 + i\Gamma_Z M_Z) M_1} \left\{ (1 - x_1) \sqrt{\frac{x_2}{1 - x_2}} \cos^2(\theta/2) e^{i\phi_1} - x_1 \sqrt{\frac{1 - x_2}{x_2}} \sin^2(\theta/2) e^{-i\phi_1} \right\}, \]

where \( g_w \) is the weak coupling constant, \( \theta \) is the polar angle of the jet-pair thrust axis, and \( \phi_1 \) is the azimuthal angle of the \( q_a \) jet with respect to the thrust axis.

The sum of the other two amplitudes where the virtual photon decays into \( q_b q_b \) can be obtained by interchanging \( a \leftrightarrow b, x_1 \leftrightarrow x_2, \phi_1 \leftrightarrow \phi_2 \) in the previous formula. (We will refers to this sum as \( i\mathcal{M}^b \).) Other helicity amplitudes can be deduced from the previous formula by various conjugation operations, but for simplicity we shall omit them here. Notice the presence of the \( e^{\pm i\phi_1} \) dependence. This dependence on the azimuthal angle \( \phi_1 \) is absent for the QCD amplitudes as those in Fig. 1. Therefore, the interference effect between the QED and QCD amplitudes disappear upon integration of the \( \phi_1 \) angle. Similarly, the interference term between \( i\mathcal{M}^a \) and \( i\mathcal{M}^b \) disappear after integrating out \( \phi_1 \) or \( \phi_2 \). In short, interference effects only alter the azimuthal angle distribution of the individual jets with respect to the thrust axis, and these effects become unobservable when we integrate out those azimuthal angles.

The difference in the azimuthal angle dependence in QED and QCD is related the following fact. In the QCD case, the outgoing parton pairs can be perfectly aligned. In this collinear limit the azimuthal angles of the individual jets with respect to the thrust axis are not expected to play a role in the scattering amplitude. In the QED case, the jets originated from the virtual photon are forbidden to be
exactly parallel, since a timelike virtual photon with helicity $\pm 1$ cannot decay into two parallel, massless quarks: some non-collinearity is required in order to carry the photon’s polarization. The appearance of the azimuthal dependence reflects the correlation between the outgoing jets with the event plane formed by the virtual photon and the beam direction.

Considering only $iM^a$, after squaring, averaging and adding the various helicity contributions, counting the color multiplicity of the quarks, symmetrizing the momentum fraction variables, integrating out the azimuthal angle $\phi_1$, and normalizing the cross section with respect to the total $Z$ cross section, we obtain

$$\frac{\sigma_a}{\sigma_Z} = \frac{27}{32} \left( \frac{\alpha_{em}}{\pi} \right)^2 \frac{Q_a^2 Q_b^2}{Q_Z^2} \int \frac{dM_1^2}{M_1^2} \frac{dM_2^2}{s} \frac{dx_1 dx_2 d\cos \theta}{s} \times \left[ x_1^2 + (1 - x_1)^2 \right] \left[ \frac{x_2}{1 - x_2} + \frac{1 - x_2}{x_2} \right] (1 + \cos^2 \theta).$$

(7)

Notice first the $1 + \cos^2 \theta$ term in the above formula. This angular dependence differs from the $\sin^2 \theta$ distribution obtained for QCD induced gap events. In particular, the QED type events can become the dominant contribution for polar angles near the forward and backward beam direction. Notice also the integral of $M_1^2$ has an apparently infrared divergence when $M_1^2 \to 0$. In reality this divergence does not occur due to the physical energy threshold $\tilde{M}_a$ for the production of the quark-antiquark pair $q_a\bar{q}_a$ from a virtual photon. Numerically we will take $\tilde{M}_a = M_\rho, M_\omega, M_\phi, M_{J/\psi}$, and $M_\Upsilon$ as representative values for the threshold energy of producing quark-antiquark pairs of flavor $u, d, s, c$ and $b$. Finally, notice the factorization of the $x_1, x_2$ and $\cos \theta$ dependence. Eq. (7) can therefore be interpreted as the probability of the radiative decay of $Z$ into a quasicollinear quark-antiquark pair and a virtual photon, multiplied by the probability
of a photon "splitting" into a quasi-collinear quark-antiquark pair (represented by the $x_1^2 + (1 - x_1)^2$ term in the equation). The singularity in the $x_2$ and $1 - x_2$ denominators reflects the infrared divergence in the virtual quark propagator when its associated quark or antiquark becomes soft (See Fig. 2). However, as in the QCD case [1,2], when we impose the existence of a rapidity gap, the virtual photon and the other jet-pair are required to go in opposite direction across the rapidity gap, hence the momentum transfer of the virtual quark is generally hard and the infrared divergence is avoided.

Taking now into account contributions from $i\mathcal{M}^b$, adding over all quark flavor combinations, symmetrizing the identical flavors cases, and integrating over $\theta$, we obtain

$$R_{\text{QED}}^{\text{gap}} = \left(\frac{9}{4}\right)^2 \left(\frac{\alpha_{\text{em}}}{\pi}\right)^2 \sum_{i} Q_i^2 Q_{\bar{i}}^2 \int \frac{dM_1^2}{M_1^2} \int \frac{dM_2^2}{s}$$

$$\times \int dx_1 \left[ x_1^2 + (1 - x_1)^2 \right] \int dx_2 \left[ \frac{x_2}{1 - x_2} + \frac{1 - x_2}{x_2} \right].$$

(8)

In principle, we should consider also the QED contribution from diagrams like Fig. 1(a) where the gluon propagator has been replaced by a photon propagator. These diagrams contribute mainly through interference effects with the corresponding QCD diagrams. A detailed analysis reveals that these effects are negligible. (The resulting interference causes only a 0.6% decrease in the QCD-type event rate for the symmetric gap case.)

The limits of the various integrals in the above formula depend on the physical cuts we impose for the selection of events. As in Ref. [1], we first analyze the event rate for a symmetric gap cut case and then repeat the analysis for the asymmetric gap cut case. In the former case, we sum over all events with all the produced jets.
having an absolute rapidity greater than $g/2$ with respect to the jet-pair thrust axis. (This is subject to the same caveat pointed out in Ref. [1]; namely, due to the effect of hadronization process, the hadron fragments of each quark are concentrated within a circle of radius $\sim 0.7$ in the lego plot. The physically observed gap is thus expected to have a width $g_{\text{eff}} \sim g - 1.4$.)

Figure 3. Rapidity gap event rate per million $Z$ in the symmetric gap cut case, for QCD and QED.

Figure 4. Rapidity gap event rate per million $Z$ in the asymmetric gap cut case, for two different values of jet-pair invariant-mass cut ($M_c = 30$ GeV and $M_c = 15$ GeV). The dashed lines are the event rates for QCD-induced gap events, and the solid lines are for QED-induced gap events.
The QED and QCD type gap event rate per million Z events, is shown in Fig. 3 as a function of the symmetric rapidity gap cut \( g \). In Fig. 4 we plot the event rate per million Z events when the gap is not required to be symmetric and the jet-pair invariant masses are required to be less than 30 or 15 GeV. We have used a value \( \alpha_s = 0.13 \) for the strong coupling constant. We see from these figures that in the large rapidity gap region the QED-induced events can constitute a substantial fraction of the QCD type events. For instance, \( R_{\text{gap}}^{\text{QED}} / R_{\text{gap}}^{\text{QCD}} \sim 0.11 \) for \( g = 4 \) in the symmetric gap case. The event rate for a larger gap region is probably too small for present experimental observation. In Ref. [1] we have pointed out that in the QCD case, the \( q\bar{q}gg \) type events are suppressed by a factor 0.159 with respect to the \( q\bar{q}q\bar{q} \) type events in the symmetric gap case. (This relative suppression can be understood on the basis of color factors: at large \( N_C \) the rate for color singlet production \( (q\bar{q}) + (gg) \) is proportional to \( N_C \), whereas the rate for two mesonic dijets \( (q\bar{q}) \) (\( q\bar{q} \)) is of order \( N_C^2 \).) Hence the QED type events can become as important as the \( q\bar{q}gg \) type rapidity gap events in the large gap region \( (g \geq 4) \).

In summary, we have studied QED mechanisms for producing jet event containing large rapidity gaps in \( e^+e^- \) annihilation at the Z peak energy. For typical kinematics, the event rate is found to be small compared to the rate for the corresponding QCD processes. However, the QED-induced rapidity gap events have some distinctive features. For instance, the QED events are distributed as \( 1 + \cos^2 \theta \) in terms of the polar angle \( \theta \) of the thrust axis as opposed to the QCD events which are distributed as \( \sin^2 \theta \). The QCD and QED mechanism also favor different flavor combinations. A process involving a \( (b\bar{b}) \) and \( (c\bar{c}) \) color singlet jet-pair is clearly absent in the QCD \( q\bar{q}q\bar{q} \) mechanism. Also, in the QED case, one of the jet-pairs tends to have a small invariant mass. In fact, the virtual photon easily transforms
into its hadronic components ($\rho, \omega, \phi, J/\psi, \Upsilon$). Thus, a likely signal of the QED mechanism would be a vector meson going in one direction and a hadronic system going in the opposite direction, with a large rapidity gap in between. Events containing a narrow resonance such as the $J/\psi$ recoiling against a dijet system could be particularly interesting since the direction and polarization of the vector meson is revealed through its decay into a lepton pair. Combining with beam polarization, the detailed study of these events can offer non-trivial tests of Standard Model features. Monte Carlo study of the hadronization stage [3] would allow us to distinguish the gap events produced through the described mechanisms from the random fluctuation of hadron fragments. Finally, let us point out that at future linear colliders (energies above the $Z$ mass) the QED-induced rapidity gap events can also come from configurations corresponding to the “radiative tail” of the $Z$. That is, a low mass photon emitted in the initial state takes away the right amount of energy to “restore” the virtual $Z$ into resonance, and two color singlet jet-systems are generated separately as the decay products of the virtual photon and the resonant $Z$.

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