Effects of a classical homogeneous gravitational field on the cavity-field entropy and generation of the Schrödinger-cat states in the Jaynes-Cummings model

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Abstract

In this paper, we examine the effects of the gravitational field on the dynamical evolution of the cavity-field entropy and the creation of the Schrödinger-cat state in the Jaynes-Cummings model. We consider a moving two-level atom interacting with a single mode quantized cavity-field in the presence of a classical homogeneous gravitational field. Based on an su(2) algebra, as the dynamical symmetry group of the model, we derive the reduced density operator of the cavity-field which includes the effects of the atomic motion and the gravitational field. Also, we obtain the exact solution and the approximate solution for the system-state vector, and examine the atomic dynamics. By considering the temporal evolution of the cavity-field entropy as well as the dynamics of the Q-function of the cavity-field we study the effects of the gravitational field on the generation of the Schrödinger-cat states of the cavity-field by using the Q-function, field entropy and approximate solution for the system-state vector. The results show that the gravitational field destroys the generation of the Schrödinger-cat state of the cavity-field.

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1 Introduction

The generation of various quantum states occupies a central position in quantum optics. Of special interest are superpositions of coherent states, known as Schrödinger-cat states [1]. Though formed by states having closest classical analogs, such superposition states may exhibit various nonclassical properties such as squeezing and sub-Poissonian statistics [2] due to the quantum interference between the coherent components. Up to now, several theoretical schemes have been proposed to generate various families of such states. Some of those schemes have been addressed to the generation of the Schrödinger-cat states of the radiation field, for example, in a dispersive medium [3], in a Mach-Zehnder interferometer [4], in a Kerr cell [5], in a nanomechanical resonator [6] and in a microwave cavity [7]. Theoretical proposals for generating atomic cat states [8] and cat states of the motional degree of atoms (ions) [9] have been also considered. Experimentally, the Schrödinger-cat states of the radiation field have been realized in the context of cavity quantum electrodynamics [10] by dispersive coupling between a circular Rydberg atom and the cavity field. Recently, a free-propagating light pulse has been also prepared in a Schrödinger-cat state [11].

In the past two decades, considerable attention has been devoted to the dynamical properties of the field entropy [12] and the description of the Schrödinger-cat states in the Jaynes-Cummings model (JCM) [13]. For the JCM, Phoenix and Knight [14] have shown that, at the atomic inversion half-revival time, due to the quantum interaction between the atom and the field, the field (atomic) entropy reaches its first minimum value, and the Q-function of the field mode bifurcates two blobs having the same amplitude but opposite phase. The asymptotically pure field state at this time is approximately a Schrödinger-cat state. Buzek and Haldky [15] have studied the Schrödinger-cat state of the field in the two-photon JCM. They found that at quarter of the revival time, the field (atomic) entropy tends to zero, and the Q-function also splits into two pieces with the same amplitude but which are out of phase by $\frac{\pi}{2}$. At this time, the field is produced in the Schrödinger-cat state. However, all these theoretical study results are obtained only under the condition that the atomic motion is neglected and the influence of the gravitational field is not taken into account.

In the standard JCM, the interaction between a constant electric field and a stationary (motionless) two-level atom is considered. With the development in the technologies of laser cooling and atom trapping the interaction between a moving atom and the field has attracted much attention [16]. Schlicher and Joshi [17] have investigated the influences of the atomic motion and the field-mode structure on the atomic dynamics. They have shown that the atomic motion and the field-mode structure give rise to nonlinear transient effects in the atomic population which are similar to self-induced transparency and adiabatic effects. Furthermore, the influence of the atomic motion on the field squeezing in the two-photon Jaynes-Cummings model has been analyzed [18]. It has been found that the atomic motion does not destroy the squeezing but decreases the squeezing after a long time. On the other hand, experimentally,
atomic beams with very low velocities are generated in laser cooling and atomic interferometry [19]. It is obvious that for atoms moving with a velocity of a few millimeters or centimeters per second for a time period of several milliseconds or more, the influence of Earth’s acceleration becomes important and cannot be neglected [20]. For this reason, it is of interest to study the temporal evolution of a moving atom simultaneously exposed to the gravitational field and a single-mode cavity-field. Since any quantum optical experiment in the laboratory is actually made in a non-internal frame it is important to estimate the influence of Earth’s acceleration on the outcome of the experiment. By referring to the equivalence principle, one can get a clear picture of what is going to happen in the interacting atom-field system exposed to a classical homogeneous gravitational field [21]. A semi-classical description of a two-level atom interacting with a running laser wave in a gravitational field has been studied [22]. However, the semi-classical treatment does not permit us to study the pure quantum effects occurring in the course of atom-field interaction. Recently, within a quantum treatment of the internal and external dynamics of the atom, we have presented [23] a theoretical scheme based on an su(2) dynamical algebraic structure to investigate the influence of a classical homogeneous gravitational field on the quantum non-demolition measurement of atomic momentum in the dispersive JCM. Also, we have investigated the effects of the gravitational field on quantum statistical properties of the lossless [21] as well as the phase-damped JCMs [24]. We have found that the gravitational field seriously suppresses non-classical properties of both the cavity-field and the moving atom.

The purpose of the present contribution is to examine the effects of the gravitational field on the dynamical evolution of the cavity-field entropy and the creation of the Schrödinger-cat state in the JCM. We consider a moving two-level atom interacting with a single mode quantized cavity-field in the presence of a classical homogeneous gravitational field. The atom undergoes one-photon transition between the nondegenerate states \( |g\rangle \) (ground state) and \( |e\rangle \) (excited state). The plan of this paper is as follows. In section 2, we introduce the physical model and based on an su(2) algebra, as the dynamical symmetry group of the model, we obtain an effective Hamiltonian describing the atom-field interaction in the presence of gravity. In section 3, we derive the reduced density operator of the cavity-field which includes the effects of the atomic motion and the gravitational field. We obtain the exact solution and the approximate solution for the system-state vector, and examine the atomic dynamics. In section 4, we consider the temporal evolution of the cavity-field entropy as well as the dynamics of the Q-function of the cavity-field and examine the effects of the gravitational field on the generation of the Schrödinger-cat states of the cavity-field by using the Q-function, field entropy and approximate solution for the system-state vector. Finally, we summarize our conclusions in section 5.
2 The Jaynes-Cummings model in the presence of a gravitational field

The system considered here consists of a moving two-level atom interacting with a single-mode quantized cavity-field via the one-photon transition processes in the presence of gravity. The total Hamiltonian for the atom-field system in the presence of a classical homogeneous gravitational field and in the rotating wave approximation with the atomic motion along the position vector $\vec{x}$ is given by

$$
\hat{H} = \frac{\hat{p}^2}{2M} - Mg\hat{x} + \hbar\omega_c(\hat{a}^\dagger\hat{a} + \frac{1}{2}) + \frac{1}{2}\hbar\omega_{eg}\hat{\sigma}_z + \hbar\lambda[\exp(-i\vec{q}.\hat{x})\hat{a}^\dagger\hat{\sigma}_- + \exp(i\vec{q}.\hat{x})\hat{a}\hat{\sigma}_+],
$$

(1)

where $\hat{a}$ and $\hat{a}^\dagger$ denote, respectively, the annihilation and creation operators of the field-mode of frequency $\omega_c$, $\vec{q}$ is the wave vector of the running wave and $\hat{\sigma}_\pm$ denote the raising and lowering operators of the two-level atom with electronic levels $\ket{e}, \ket{g}$ and Bohr transition frequency $\omega_{eg}$. The atom-field coupling is given by the parameter $\lambda$ and $\hat{p}$, $\hat{x}$ denote, respectively, the momentum and position operators of the atomic center of mass motion and $g$ is Earth’s gravitational acceleration. It has been shown [21,23] that based on an su(2) algebraic structure, as the dynamical symmetry group of the model and in the interaction picture, the Hamiltonian (1) can be transformed to the following effective Hamiltonian

$$
\hat{H}_{eff} = \hbar\lambda(\sqrt{K}\hat{S}_- \exp(-it\hat{\Delta}_1(\vec{p}, \vec{g}, t)) + \sqrt{K}\hat{S}_+ \exp(it\hat{\Delta}_1(\vec{p}, \vec{g}, t))),
$$

(2)

where the operators

$$
\hat{S}_0 = \frac{1}{2}(\ket{e}\bra{e} - \ket{g}\bra{g}), \hat{S}_+ = \hat{a}\ket{e}\bra{g}, \hat{S}_- = \frac{1}{\sqrt{K}}\ket{g}\bra{e}\hat{a}^\dagger,
$$

(3)

with the following commutation relations

$$
[\hat{S}_0, \hat{S}_\pm] = \pm\hat{S}_\pm, [\hat{S}_-, \hat{S}_+] = -2\hat{S}_0,
$$

(4)

are the generators of the su(2) algebra, the operator $\hat{K} = \hat{a}^\dagger\hat{a} + |\bra{e}\ket{e}|$ is a constant of motion which represents the total number of excitations of the atom-radiation system and the operator

$$
\hat{\Delta}_1(\vec{p}, \vec{g}, t) = \frac{1}{2}(\omega_c - (\omega_{eg} + \frac{\vec{q}.\vec{p}}{M} + \vec{q}.\vec{g}t + \frac{3\hbar q^2}{2M})),
$$

(5)

is the Doppler shift detuning of the atom-field interaction at time $t$ [21,23] which depends on both the atomic momentum and the gravitational field.
3 Dynamical Evolution of the system

In this section, by using the effective Hamiltonian (2), we investigate the dynamical evolution of the system under consideration. At any time $t > 0$, the reduced density operator of the cavity-field is given by

$$\hat{\rho}_f(t) = Tr_a[\hat{\rho}_{a-f}(t)] = \int d^3p \langle \hat{\rho}_{a-f}(t) \rangle \otimes |\hat{p}\rangle,$$

(6)

where the density operator of the atom-field system is

$$\hat{\rho}_{a-f}(t) = \hat{u}^\dagger(t) \hat{\rho}_{a-f}(0) \hat{u}(t).$$

(7)

Here, $\hat{\rho}_{a-f}(0)$ is the initial density operator for the system and $\hat{u}(t)$ is the corresponding time evolution operator for Hamiltonian (2),

$$\hat{u}(t) = \exp(-\frac{it}{\hbar} \int_0^t \hat{H}_{eff}(t') dt').$$

(8)

We assume that initially the radiation field is prepared in a coherent state, the atom is in the excited state $|e\rangle$, and the state vector for the center-of-mass degree of freedom is $|\psi_{c.m}(0)\rangle = \int d^3p \phi(\vec{p}) |\vec{p}\rangle$. Therefore, the initial density operator of the atom-radiation system reads as

$$\hat{\rho}_{a-f}(0) = \hat{\rho}_f(0) \otimes \hat{\rho}_{a}(0) \otimes \hat{\rho}_{c.m}(0),$$

(9)

where

$$\hat{\rho}_f(0) = \sum_{n=0}^\infty \sum_{m=0}^\infty w_n(0)w_m(0) |n\rangle \langle m|,$$

(10)

$$\hat{\rho}_{c.m}(0) = \int d^3p \int d^3p' \phi^*(p') \phi(p) \langle\hat{p}'\rangle \langle\hat{p}\rangle,$$

(11)

with $w_n(0) = \frac{\exp(-\frac{|\alpha|^2}{2})}{\sqrt{n!}}$ and $\phi(p) = \frac{1}{\sqrt{2\pi\sigma_0}} \exp(-\frac{p^2}{2\sigma_0^2})$. After some straightforward calculation, we obtain

$$\hat{\rho}_f(t) = |C(t)\rangle \langle C(t)| + |D(t)\rangle \langle D(t)|,$$

(12)

where

$$|C(t)\rangle = \int d^3p \phi(p) \sum_{n=0}^\infty w_n(0) \sqrt{a_n(\vec{p},\vec{g},t)} \exp\left(\frac{i}{2} E_+ (\vec{p},\vec{g},t) \sqrt{n+1} |n\rangle \otimes |\vec{p}\rangle\right),$$

(13)

$$|D(t)\rangle = \int d^3p \phi(p) \sum_{n=0}^\infty w_{n-1}(0) \sqrt{b_n(\vec{p},\vec{g},t)} \exp\left(\frac{i}{2} E_- (\vec{p},\vec{g},t) \sqrt{n} |n\rangle \otimes |\vec{p}\rangle\right),$$

(14)

with

$$a_n(\vec{p},\vec{g},t) = (1 - i(n + 1)E_+ (\vec{p},\vec{g},t) E_-^2 (\vec{p},\vec{g},t)),$$

(15)
\[ b_n(\vec{p}, \vec{g}, t) = i(n + 1)E_+(\vec{p}, \vec{g}, t)E_-^2(\vec{p}, \vec{g}, t), \]  

(16)

and

\[ E_+(\vec{p}, \vec{g}, t) = \left( \frac{1}{2} + \frac{i}{2} \right) \sqrt{\frac{\pi}{\sqrt{q.g}}} \exp(-i\frac{\Delta_0^2(\vec{p})}{2q.g})(-Erf[i(-1)^{\frac{1}{2}}\frac{\Delta_0(\vec{p})}{\sqrt{2q.g}}]) \] 

\[ + \ Erf[(-1)^{\frac{1}{2}}(\frac{\Delta_0(\vec{p})}{\sqrt{2q.g}} - \sqrt{\frac{q.g}{2} t})], \]  

(17)

\[ E_-(\vec{p}, \vec{g}, t) = \left( \frac{1}{2} - \frac{i}{2} \right) \sqrt{\frac{\pi}{\sqrt{q.g}}} \exp(i\frac{\Delta_0^2(\vec{p})}{2q.g})(-Erf[i(-1)^{\frac{1}{2}}\frac{\Delta_0(\vec{p})}{\sqrt{2q.g}}]) \] 

\[ + \ Erf[i(-1)^{\frac{1}{2}}(\frac{\Delta_0(\vec{p})}{\sqrt{2q.g}} - \sqrt{\frac{q.g}{2} t})], \]  

(18)

in which

\[ \Delta_0(\vec{p}) = \Delta_0 - \frac{q.g}{2M}, \]  

(19)

with

\[ \Delta_0 = \frac{1}{2}[\omega_c - (\omega_{eg} + 3\frac{\hbar q^2}{2M})], \]  

(20)

is time-independent and Erf denotes the error function. As is seen, the reduced density operator of the cavity-field includes the effects of both the atomic motion and the gravitational field. The state vector for the system can be written as

\[ |\psi(t)\rangle = |C(t)\rangle|e\rangle + |D(t)\rangle|g\rangle. \]  

(21)

Equation (21) is an exact solution for the system under consideration. Following the work of Gea-Banacloche [25], for large initial mean photon number (\(|\alpha|^2 >> 1\)), considering the property of Poissonian distribution, we can make some approximations [26]: \(w_n(0) \approx w_{n-1}(0), \sqrt{n} \approx \sqrt{|\alpha|^2[1 + \frac{i|\alpha|^2}{2}]},\) and derive the following approximate solution for the JCM in the presence of a gravitational field

\[ |\psi(t)\rangle \approx |C'(t)\rangle|e\rangle + |D'(t)\rangle|g\rangle, \]  

(22)

where

\[ |C'(t)\rangle \approx \int d^3p \phi(\vec{p}) \sum_{n=0}^{\infty} w_n(0) \sqrt{a_n'(|\vec{p}, \vec{g}, t|)} \exp(i \frac{1}{2} E_+(\vec{p}, \vec{g}, t) \sqrt{n + 1}|n\rangle \otimes |\vec{p}\rangle), \]  

(23)

\[ |D'(t)\rangle \approx \int d^3p \phi(\vec{p}) \sum_{n=0}^{\infty} w_n(0) \sqrt{b_n'(|\vec{p}, \vec{g}, t|)} \exp(i \frac{1}{2} E_+(\vec{p}, \vec{g}, t) \sqrt{n}|n\rangle \otimes |\vec{p}\rangle), \]  

(24)

with

\[ \sqrt{a_n'(|\vec{p}, \vec{g}, t|)} \approx \sqrt{\eta(|\vec{p}, \vec{g}, t|)|\alpha|^2[1 + \frac{i|\alpha|^2}{2|\alpha|^2]}}, \]  

(25)
\[ \sqrt{b_n'(\vec{p}, \vec{g}, t)} \approx \sqrt{-\eta(\vec{p}, \vec{g}, t) |\alpha|^2 [1 + \frac{n + 1 - |\alpha|^2}{2|\alpha|^2}]} , \]  

(26) 

\[ \eta(\vec{p}, \vec{g}, t) = -i E_+ (\vec{p}, \vec{g}, t) E_0^2 (\vec{p}, \vec{g}, t) \]  

and \[ \xi(\vec{p}, \vec{g}, t) = 1 + \eta^{-1} (\vec{p}, \vec{g}, t) \]. This approximate solution is useful in examining the properties and the generation of the Schrödinger-cat states.

By using Eq. (21) we can evaluate the time evolution of the atomic inversion which takes the form

\[ W(t) = \langle \psi(t) | \sigma_z | \psi(t) \rangle = \langle C(t) | C(t) \rangle - \langle D(t) | D(t) \rangle , \]  

(27)

where

\[ \langle C(t) | C(t) \rangle = \int d^3 p |\phi(\vec{p})|^2 \sum_{n=0}^{\infty} w_n(0) w_n^*(0) \sqrt{a_n(\vec{p}, \vec{g}, t) a_n^*(\vec{p}, \vec{g}, t)} \exp\left(\frac{i}{2} (E_+ (\vec{p}, \vec{g}, t) \sqrt{n+1} - E_+^* (\vec{p}, \vec{g}, t) \sqrt{n+1}) \right) , \]  

(28)

\[ \langle D(t) | D(t) \rangle = \int d^3 p |\phi(\vec{p})|^2 \sum_{n=0}^{\infty} w_{n-1}(0) w_{n-1}^*(0) \]  

\[ \times \sqrt{b_n(\vec{p}, \vec{g}, t) b_n^*(\vec{p}, \vec{g}, t)} \exp\left(\frac{i}{2} (E_+ (\vec{p}, \vec{g}, t) \sqrt{n} - E_+^* (\vec{p}, \vec{g}, t) \sqrt{n}) \right) . \]  

The numerical results of the atomic inversion \( W(t) \) are shown in Fig.1 for initial mean photon number \( |\alpha|^2 = 25 \) and three different values of the parameter \( \vec{q} \cdot \vec{g} \). In this figure and all the subsequent figures we set \( q = 10^7 m^{-1} \), \( M = 10^{-26} Kg \), \( g = 9.8 m/s^2 \), \( \omega_{rec} = \frac{\hbar}{2M} = .5 \times 10^6 rad/s \), \( \lambda = 1 \times 10^6 rad/s \), \( \sigma_0 = 1 \) and \( \Delta_0 = 8.5 \times 10^7 rad/s \) [21-22]. Fig.1a displays the case when the gravitational influence is negligible. This means very small \( \vec{q} \cdot \vec{g} \), i.e., the momentum transfer from the laser beam to the atom is only slightly altered by the gravitational acceleration because the latter is very small or nearly perpendicular to the laser beam. Figs.1b and 1c illustrate the case when we consider the gravitational influence for \( \vec{q} \cdot \vec{g} = 0.5 \times 10^7 \) and \( \vec{q} \cdot \vec{g} = 1.5 \times 10^7 \), respectively. It can be seen from Fig.1a that, in the condition of no gravitational influence, the atomic population inversion shows the collapse-revival repeatedly, and the amplitude of Rabi oscillation in each revival period is not the same. Increasing of the gravitational influence (Fig.1b and 1c) induces the population inversion to oscillate so drastically that the phenomenon of collapse-revival is not so clear.

### 4 Dynamical Properties of the cavity-field

In this section, we study the time evolution of the cavity-field entropy as well as the dynamics of the Q-function of the cavity-field and examine the influence of the gravitational field on the generation of the Schrödinger-cat state.
4a. Temporal evolution of the field entropy

Here, we use the field entropy as a measure for the degree of entanglement between the cavity-field and the atom of the system under consideration. The quantum dynamics described by the Hamiltonian (1) leads to an entanglement between the cavity-field and the atom, which will be quantified by the field entropy. As shown by Phoenix and Knight [14] the von Neumann quantum entropy is a convenient and sensitive measure of the entanglement of two interacting subsystems

$$S = -Tr(\hat{\rho} \ln \hat{\rho})$$

(30)

where $\hat{\rho}$ is the density operator for a given quantum system and we have set the Boltzmann constant $k_B = 1$. If the atom is in a pure state, then in a suitable basis the density operator is diagonal and has a single element, unit. For this case, $S = 0$ and if $\hat{\rho}$ describes a mixed state, then $S \neq 0$. Araki and Lieb [27] showed that these entropies for a composite system satisfy the triangle inequality $|S_a - S_f| \leq S \leq S_a + S_f$. Quantum entropies are generally difficult to compute because they involve the diagonalization of large (and, in many cases, infinite dimensional) density matrices. Thus explicit illustration of the triangle inequality is difficult. Phoenix and Knight gave a nice illustration of the triangle inequality in the context of the JCM. In our model, the initial state is prepared in a pure state, so the whole atom-field system remains in a pure state at any time $t > 0$ and its entropy is always zero. However, due to the entanglement of the atom and the cavity-field at $t > 0$, both the atom and the field are generally in mixed states, although at certain times the field and the atomic subsystems are almost in pure states. Since the initial state is a pure state, the entropy $S_f$ or $S_a$, which is referred to as the entanglement of the total system in quantum information, is used to measure the amount of entanglement between the two subsystems. When $S_f = S_a = 0$, the system is disentangled or separable and both the field and atomic subsystems are in pure state. The entropies of the particle and the field, are defined through the corresponding reduced density operators by

$$S_{a(f)} = -Tr_{a(f)}(\hat{\rho}_{a(f)} \ln \hat{\rho}_{a(f)})$$

(31)

provided we treat both separately. Since the trace is invariant under a similarity transformation, we can go to a basis in which the density matrix of the cavity-field is diagonal and then express the field entropy $S_f$ in terms of the eigenvalues $\pi^\pm_f(t)$, for the reduced field density operator. Phoenix and Knight [14] have developed a general method to calculate the various field eigenstates in a simple way. To calculate the eigenvalues of the reduced cavity-field density operator, we write the eigenequation as

$$\hat{\rho}_f(t)|\psi_f(t)\rangle = \pi^\pm_f|\psi_f(t)\rangle$$

(32)
The eigenvalues of $\hat{\rho}_f(t)$ are

$$
\pi^\pm_f(t) = \frac{1}{2} \pm \frac{1}{2}\sqrt{1 - 4|\langle C(t)|C(t)\rangle\langle D(t)|D(t)\rangle - |\langle C(t)|D(t)\rangle|^2},
$$

(33)

where $\langle C(t)|C(t)\rangle$ and $\langle D(t)|D(t)\rangle$ are given, respectively, by (28) and (29), and

$$
\langle C(t)|D(t)\rangle = \int d^3p|\phi(\vec{p})|^2 \sum_{n=0}^{\infty} w_n(0)w_n^*(0)\sqrt{a_n^*(\vec{p},\vec{g},t)b_n(\vec{p},\vec{g},t)},
$$

(34)

The field entropy $S_f(t)$ may be expressed in terms of the eigenvalues $\pi^\pm_f$ for the reduced field density operator as

$$
S_f(t) = -(\pi^+_f(t) \ln \pi^+_f(t) + \pi^-_f(t) \ln \pi^-_f(t)).
$$

(35)

As is seen, the field entropy depends on not only the statistics of the cavity-field but also the atomic motion and the gravitational field.

Now we turn our attention to examine numerically the dynamics of the field entropy. The numerical results of the evolution of the field entropy are shown in Fig. 2 with the same corresponding data used in Fig. 1. Fig. 2a illustrates the case when the gravitational influence is negligible, while the rest figures display the cases when we consider the gravitational influence. It is seen from Fig. 2a that, when the gravitational influence is negligible, the cavity-field entropy evolves periodically in the course of time. This periodic evolution can be attributed to the atomic motion. Increasing the parameter $\vec{q}.\vec{g}$ (Figs. 2b and 2c) results in not only increasing in the amplitude of the field entropy but also occurring fast oscillations in the course of time evolution of the field entropy.

### 4b. Q-function and Schrödinger-cat state

Now, we turn our attention to the dynamics of the quasi-probability distribution Q-function and the analysis of the generation of the Schrödinger-cat states in the JCM in the presence of the gravitational field. The field entropy and the Q-function are very useful tools for analyzing the formation of the Schrödinger-cat state. When the field mode is in a state with the minimum value of the field entropy and the Q-function is composed of two equal peaks, we can infer that such a field-mode state is a Schrödinger-cat state and the state vector of the system can be expressed in a factored form at this time.

The Q-function of the cavity-field is defined in terms of the diagonal elements of the density operator in the coherent state basis,

$$
Q(\beta,\beta^*,t) = \frac{1}{\pi}\langle \beta|\hat{\rho}_f(t)|\beta\rangle,
$$

(36)
where $\beta = X + iY$. By using the explicit expression for the reduced density operator of the cavity-field given by Eq.(12) we obtain

$$Q(\beta, \beta^*, t) = \frac{1}{\pi} \left[ |\langle \beta | C(t) \rangle|^2 + |\langle \beta | D(t) \rangle|^2 \right]$$

(37)

$$= \frac{1}{\pi} \int d^3p |\phi(\vec{p})|^2 \sum_n \sum_m w_n(0) w_m^*(0)$$

$$\times \exp\left(\frac{i}{2} (E_+(\vec{p}, \vec{g}, t) \sqrt{n+1} - E_+^*(\vec{p}, \vec{g}, t) \sqrt{m+1}) \right)$$

$$\times \left( \sqrt{a_n(\vec{p}, \vec{g}, t) a_m^*(\vec{p}, \vec{g}, t)} (\beta n) (\beta^* m) \right)$$

$$+ \sqrt{b_n(\vec{p}, \vec{g}, t) b_{m+1}^*(\vec{p}, \vec{g}, t)} (\beta n + 1) (\beta^* m + 1).$$

In Fig.3 we have sketched the three-dimensional plots of the Q-function of the cavity-field versus $X$ and $Y$ for three values of the parameter $\vec{q} \cdot \vec{g}$. The field in coherent state with mean initial photon number $|\alpha|^2 = 25$ and the atom in the excited state is considered. Furthermore, we choose $t = t_R/2 = 7\pi/2\lambda$ which corresponds to one-half of the revival time of the atomic inversion when the gravitational field is not taken into account [see Fig.1a]. Fig.3a, which corresponds to the case $\vec{q} \cdot \vec{g} = 0$, shows that at $t = t_R/2$, the Q-function splits into two blobs with the same amplitude but opposite phase. At this time, the state vector for the system can be written in a factored form approximately by using (22)

$$|\psi(t = t_R/2)\rangle \approx |\psi_a(t = t_R/2)\rangle \otimes |\psi_f(t = t_R/2)\rangle \otimes |\psi_{c.m}(t = t_R/2)\rangle,$$

(38)

where

$$|\psi_a(t = t_R/2)\rangle = |e\rangle + i |g\rangle,$$

(39)

$$|\psi_f(t = t_R/2)\rangle = \sum_{n=0}^{\infty} n w_n(0) |n\rangle,$$

(40)

$$|\psi_{c.m}(t = t_R/2)\rangle = \int d^3p \phi(\vec{p}) g(\vec{p}, t = t_R/2) |\vec{p}\rangle,$$

(41)

with

$$g(\vec{p}, t) = \left[i f_0^{1/2}(\vec{p}, t) \Delta_0^{3/2}(\vec{p}) \sqrt{|\alpha|^2 (1 + |\alpha|^2)}/2 \right.\right.$$

$$\left. - 2i f_0^{1/2}(\vec{p}, t) \Delta_0^{3/2}(\vec{p}) n \sqrt{|\alpha|^2} [\cos(A_0(\vec{p}, t) + F_0(\vec{p}, t)) \right.$$

$$+ i \sin(A_0(\vec{p}, t) + F_0(\vec{p}, t))],$$

(42)

and

$$f_0(\vec{p}, t) = (\cos(\Delta_0(\vec{p}, t) - 1)^3 + i \sin(\Delta_0(\vec{p}, t)) (\cos(\Delta_0(\vec{p}, t) - 1)$$

$$+ (\cos(\Delta_0(\vec{p}, t) - 1) \sin(\Delta_0(\vec{p}, t))^2 + i \sin^2(\Delta_0(\vec{p}, t),$$

(43)

$$A_0(\vec{p}, t) = \frac{i|\alpha|^{-1} \Delta_0(\vec{p})^{-1}}{2} (1 + \frac{1 - |\alpha|^2}{2 |\alpha|^2})$$

$$\times (\cos(\Delta_0(\vec{p}, t) - 1 - i \sin(\Delta_0(\vec{p}, t))),$$

(44)
\[ F_0(\vec{p}, t) = \frac{im|\alpha|\Delta_0(\vec{p})^{-1}}{4} (\cos(\Delta_0(\vec{p})t) - 1 - i \sin(\Delta_0(\vec{p})t)). \] (45)

Combining this result with the fact that the field entropy at this time tends to zero (Fig.2a), we conclude that at this time the cavity-field is in the Schrödinger-cat superposition of macroscopic states. In Figs.3b and 3c we consider the gravitational influence for \( \vec{q}, \vec{g} = 0.5 \times 10^7 \) and \( \vec{q}, \vec{g} = 1.5 \times 10^7 \), respectively. By comparing Fig.3a with Figs.3b and 3c, we find that the gravitational field plays an important role in the dynamics of the Q-function. In Figs.3b and 3c, two peaks of the Q-function join together and the state vector for the system cannot be written in a factored form. For \( \vec{q}, \vec{g} = 0.5 \times 10^7 \) we have

\[ |\psi(t = t_R/2)\rangle \approx \left( \int d^3p \phi(\vec{p}) \sqrt{\eta_1(\vec{p})} \left( \sum_{n=0}^{\infty} w_n(0) g_{1,n}(\vec{p}) |n\rangle \otimes |\vec{p}\rangle \right) \right) |e\rangle \] (46)

\[ + \ i \left( \int d^3p \phi(\vec{p}) \sqrt{\eta_1(\vec{p})} \left( \sum_{n=0}^{\infty} w_n(0) g_{1,n-1}(\vec{p}) |n\rangle \otimes |\vec{p}\rangle \right) \right) |g\rangle \]

\[ + \ \left( \sum_{n=0}^{\infty} w_n(0) g_{1,n}(\vec{p}) |n\rangle \otimes |\vec{p}\rangle \right) \right) |e\rangle \]

\[ \neq |\psi_{a}(t = t_R/2)\rangle \otimes |\psi_{f}(t = t_R/2)\rangle \otimes |\psi_{c,m}(t = t_R/2)\rangle, \]

where

\[ \eta_1(\vec{p}) = 0.43 \times 10^{-9}(1 - i) \exp(1 \times 10^{-7}i \Delta_0^2(\vec{p})) \times (-e_1(\vec{p}) + e_2(\vec{p}))(-e_1(\vec{p}) + e_3(\vec{p}))^2, \] (47)

\[ g_{1,n}(\vec{p}) = 0.1 \exp[0.5i(0.38 \times 10^{-3}(1 - i) \exp[-10^{-7}i \Delta_0^2(-e_1(\vec{p}) + e_2(\vec{p}))]], \] (48)

with

\[ e_1(\vec{p}) = Erf[-0.3 \times 10^{-3} \sqrt{i} \Delta_0(\vec{p})], \] (49)

\[ e_2(\vec{p}) = Erf[(0.3i \sqrt{i} \Delta_0(\vec{p}) - 17.6i \sqrt{i})10^{-3}], \] (50)

\[ e_3(\vec{p}) = Erf[(-0.3 \sqrt{i} \Delta_0(\vec{p}) + 17.6 \sqrt{i})10^{-3}], \] (51)

and for \( \vec{q}, \vec{g} = 1.5 \times 10^7 \) we have

\[ |\psi(t = t_R/2)\rangle \approx \left( \int d^3p \phi(\vec{p}) \sqrt{\eta_2(\vec{p})} \left( \sum_{n=0}^{\infty} w_n(0) g_{2,n}(\vec{p}) |n\rangle \otimes |\vec{p}\rangle \right) \right) |e\rangle \] (52)
\[
+i\left[\int d^3p\phi(\vec{p})\sqrt{\eta_2(\vec{p})}\left(\sum_{n=0}^{\infty} w_n(0) n g_{2,n-1}(\vec{p})|n\rangle \otimes |\vec{p}\rangle\right)|g\rangle
\]
\[
+\left[\left(\int d^3p\phi(\vec{p})(26\sqrt{\eta_2(\vec{p})} + \eta_2^{-1/2}(\vec{p})\right)
\times \left(\sum_{n=0}^{\infty} w_n(0) n g_{2,n}(\vec{p})|n\rangle \otimes |\vec{p}\rangle\right)|e\rangle
\]
\[
+ 26i\left[\left(\int d^3p\phi(\vec{p})\sqrt{\eta_2(\vec{p})}\left(\sum_{n=0}^{\infty} w_n(0) n g_{2,n-1}(\vec{p})|n\rangle \otimes |\vec{p}\rangle\right)|g\rangle
\right]
\neq \left|\psi_a\left(t = \frac{t_R}{2}\right)\right\rangle \otimes \left|\psi_f\left(t = \frac{t_R}{2}\right)\right\rangle \otimes \left|\psi_{c.m.}\left(t = \frac{t_R}{2}\right)\right\rangle,
\]
where
\[
\eta_2(\vec{p}) = 0.2 \times 10^{-3}(1 - i) \exp(0.33 \times 10^{-7}|\Delta_0^2(\vec{p})|) (53)
\times (-e_1'(\vec{p}) + e_2'(\vec{p}))(e_1'(\vec{p}) + e_2'(\vec{p}))^2,
\]
\[
g_{2,n}(\vec{p}) = 0.1 \exp(0.5i \sqrt{1.22 \times 10^{-3}(1 - i)} \exp[-0.33 \times 10^{-7}|\Delta_0^2(\vec{p})|(-e_1(\vec{p}) + e_2(\vec{p}))]), (54)
\]
\[
\text{with}
\]
\[
e_1'(\vec{p}) = \text{Erf}\left[-0.18 \times 10^{-3}\sqrt{i|\Delta_0(\vec{p})|}\right], (55)
\]
\[
e_2'(\vec{p}) = \text{Erf}\left[(0.18i\sqrt{i|\Delta_0(\vec{p})| - 29.9i\sqrt{i}})10^{-3}\right], (56)
\]
\[
e_3'(\vec{p}) = \text{Erf}\left[(-0.18i\sqrt{i|\Delta_0(\vec{p})| + 29.9i\sqrt{i}})10^{-3}\right]. (57)
\]
Combining this result with the fact that in the presence of gravity the field entropy reaches its maximum value at \(t = t_R/2\) (Figs.2b and 2c) we find that the cavity-field is in a statistical mixture state. Therefore, we can conclude that, in the JCM, if the gravitational field influence is taken into account, there are no Schrödinger-cat states in the course of time evolution of the cavity-field. In other words, the gravitational field destroys the generation of the Schrödinger-cat state.

5 Summary and conclusions

In this paper, we considered a moving two-level atom interacting with a single mode quantized cavity-field in the presence of a classical homogeneous gravitational field. Based on an \(su(2)\) algebra, as the dynamical symmetry group of the model, we obtained the exact and the approximate solutions for the system-state vector, and examined the influence of the gravitational field on the evolution of the atomic inversion, the field entropy and the generation of the Schrödinger-cat state. The results are summarized as follows: 1) The gravitational field leads to
the occurrence of fast oscillations in the atomic population inversion such that the collapse and revivals phenomena can not be identified clearly. 2) Increasing the parameter \( \vec{q} \cdot \vec{g} \) results in not only increasing in the amplitude of the field entropy but also occurring fast oscillations in the course of time evolution of the field entropy. 3) The gravitational field destroys the generation of the Schrödinger-cat state.

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FIGURE CAPTIONS:

FIG. 1 Time evolution of the atomic population inversion versus the scaled time $\lambda t$. Here we have set $q = 10^7 m^{-1}$, $M = 10^{-26} kg, g = 9.8 m/s^2, \omega_{rec} = .5 \times 10^6 rad/s$, $\lambda = 1 \times 10^6 rad/s$, $\Delta_0 = 8.5 \times 10^7 rad/s$;

a) For $\vec{q}.\vec{g} = 0$.

b) For $\vec{q}.\vec{g} = 0.5 \times 10^7$.

c) For $\vec{q}.\vec{g} = 1.5 \times 10^7$.

FIG. 2 Time evolution of the von Neumann entropy $S$ versus the scaled time $\lambda t$ with the same corresponding data used in Fig.1;

a) For $\vec{q}.\vec{g} = 0$.

b) For $\vec{q}.\vec{g} = 0.5 \times 10^7$.

c) For $\vec{q}.\vec{g} = 1.5 \times 10^7$.

FIG. 3 The Q-function of the cavity-field versus $X = \text{Re}(\beta)$ and $Y = \text{Im}(\beta)$ with the same corresponding data used in Fig.1 and $t = \frac{\hbar}{\lambda}$;

a) For $\vec{q}.\vec{g} = 0$.

b) For $\vec{q}.\vec{g} = 0.5 \times 10^7$.

c) For $\vec{q}.\vec{g} = 1.5 \times 10^7$. 