Anomalous Action
in Gauge Invariant, Nonlocal, Dynamical Quark Model

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Abstract

Anomalous sector of chiral Lagrangian is calculated in a gauge invariant, nonlocal, dynamical quark model. The Wess-Zumino term is proved coming from two kinds of sources, one is independent of and another dependent on dynamical quark self energy $\Sigma(k^2)$. $p^6$ and more higher order anomalous sectors come only from $\Sigma(k^2)$ dependent source. After some cancellation, standard Wess-Zumino action is obtained.

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Chiral Lagrangian has successfully described low-energy hadronic process. Within context of chiral Lagrangian approach that incorporates symmetries of QCD, to a certain order in the low-energy expansion, the difference between different underlying theories with same spontaneously broken chiral symmetry is in the values of coefficients in the chiral Lagrangian. To test QCD in terms of chiral Lagrangian at quantitative level, we need to calculate values of these coefficients based on QCD and compare them with those from experiment data. Recently chiral Lagrangian is exactly derived from underlying QCD and coefficients in the chiral Lagrangian are formally expressed in terms of Green’s functions of underlying QCD\textsuperscript{[1]}. Further it was shown that the coefficients for normal part with even intrinsic parity of pseudoscalar meson chiral Lagrangian is saturated by dynamical quark self energy $\Sigma(k^2)$ \textsuperscript{[2]}. While the anomaly contribution \textsuperscript{[3]}, conventionally taken as the main source of the coefficients, are completely cancelled, leaving the $\Sigma(k^2)$ dependent coefficients which vanish when

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the strong interaction is switched off. Although this result is obtained in a special gauge invariant, nonlocal, dynamical (GND) quark model, considering this model can be seen as a simple approximation for the exact derivation of chiral Lagrangian from underlying QCD [4], the result is general. In this paper, we generalize the discussion for even intrinsic parity part of chiral Lagrangian in GND quark model to the anomalous part with odd intrinsic parity of chiral Lagrangian.

Anomalous part of chiral Lagrangian was first derived by Wess and Zumino [5] as a series expansion by directly integrating the anomaly consistency conditions. This Lagrangian was later obtained more directly by Witten [6] by writing an action of abnormal intrinsic parity with a free integer parameter fixed from QCD anomaly. It was subsequently shown by several authors that quantum corrections to the Wess-Zumino classical action do not renormalize the coefficient of the $O(p^4)$ Wess-Zumino term and the one-loop counter terms lead to conventional chiral invariant structures at $O(p^6)$ [7]. Ref. [8] classified all possible terms of chiral Lagrangian at order of $p^6$.

The anomalous sector of chiral Lagrangian has already been discussed in the literature in quite some detail [9], its derivation can be classified into two classes: one is related to integrating back infinitesimal variation of quark functional integration measure [5,10]. We call it the strong interaction dynamics independent approach, since except feature of mathematical beauty, this approach lacks explicit correspondence with interaction. i.e. switching off strong interaction among quarks and gluons seems has no interference to the calculation procedure which only depends on transformation property of functional integration measure and couplings among external fields and quarks. Therefore strong interaction dynamics in this approach has nothing to do with anomalous sector of chiral Lagrangian. This problem become serious since in GND quark model [2] and QCD [4], such kind of terms independent of strong interaction are already proved to be exactly cancelled in normal part of chiral Lagrangian. Whether this cancellation continue to happen at anomalous sector of chiral Lagrangian? If so, we need to know whether there still has Wess-Zumino term? if not, we need to specify the reason to break this cancellation. Another type of derivation depends on strong interaction dynamics through constituent quark mass $M_Q$ [9,11]. Anomalous sector in this approach which vanishes when we switch off strong interaction ($M_Q = 0$) can be calculated either by directly performing loop calculations or computing $M_Q$ dependent determinants of quarks by choice of some proper regularization method. Since a hard constituent quark mass in the theory is only a rough approximation of QCD which causes wrong bad ultraviolet behavior of the theory, a more precise description should be replacing it with momentum dependent quark self energy $\Sigma(k^2)$ which plays an important role in QCD low
energy physics [12]. In terms of $\Sigma(k^2)$, Ref. [13] expressed Wess-Zumino action in terms of quark self energy, but did not explain the relation of their result with that from dynamics independent approach. This relation must be clarified to avoid the double counting of anomalous sector from two different sources mentioned above. Since the relation for normal part from different approaches are already discussed in Ref. [2], in this work we focus on how to calculate anomalous part of chiral Lagrangian in GND quark model as a QCD motivated discussion in which we will express Wess-Zumino action in terms of $\Sigma(k^2)$ and setup its relation with dynamics independent approach. We will show that there are two cancellation mechanisms, different choices of cancellation mechanisms will lead to different, dynamics independent or dependent, approaches of anomalous sector of chiral Lagrangian.

We start from the GND quark model action in Euclidean space $^1$, 

$$S_{\text{GND}}[U, J] \equiv \text{Tr} \ln[\theta + J_\Omega + \Sigma(-\nabla^2)] - \text{Tr} \ln[\theta + J_\Omega] + \text{Tr} \ln[\theta + J].$$

Where Tr is trace for color, flavor, Lorentz spinor and space-time indices. $J_\Omega$ is rotated external field which can be decomposed into scalar, pseudoscalar, vector and axial vector parts, it relate to original source $J$ through a local chiral rotation $\Omega(x)$

$$J_\Omega(x) = -i \bar{\psi}(x) \gamma_5 \Omega(x) - i \bar{\psi}(x) \gamma_5 \Omega(x) + s_\Omega(x) + ip_\Omega(x) \gamma_5$$

$$= [\Omega(x) P_R + \Omega^\dagger(x) P_L] [J(x) + \partial x] [\Omega(x) P_R + \Omega^\dagger(x) P_L].$$

The covariant differential $\nabla^\mu$ is defined as $\nabla^\mu = \partial^\mu - iv_\mu^\Omega(x)$, local goldstone field $U(x)$ is related to rotation field $\Omega(x)$ by $U(x) = \Omega^2(x)$.

The GND quark model given in (1) is expressed on rotated basis, by rotating back to unrotated basis, we get an alternative expression of it,

$$S_{\text{GND}}[U, J] = \text{Tr} \ln[\theta + J + \Pi],$$

with $\Omega$ and $\nabla_\mu$ dependent $\Pi$ field

$$\Pi(x) \equiv [\Omega^\dagger(x) P_R + \Omega(x) P_L] \Sigma(-\nabla^2)[\Omega^\dagger(x) P_R + \Omega(x) P_L].$$

$S_{\text{GND}}$ given in (3) is explicitly dynamics dependent through quark self energy $\Sigma(k^2)$, since goldstone field $\Omega$ couple to external fields through $\Sigma(k^2)$, once $\Sigma(k^2)$ vanish, the interaction disappear by $S_{\text{GND}} \rightarrow \text{Tr} \ln[\theta + J]$, i.e., action becomes a pure external field term.

$^1$We have included normalization factor $N' \equiv \text{Tr} \ln[i\theta + J]$ appeared in (13) but dropped in final expression (18) of Ref. [2] which is discussed in Minkovski space.
To calculate the anomalous part of (3), we introduce a parameter $t$ defined at interval $[0,1]$ and replace original $\Omega(x) \equiv e^{i\pi(x)}$ appeared in $\Pi(x)$ of (3) by $\Omega(t, x) = e^{i\pi(x)}$ with $\Omega(1, x) = \Omega(x)$ and $\Omega(0, x) = 1$. Correspondingly, $\Pi(x)$ is replaced by $\Pi(t, x)$ with $\Pi(1, x) = \Pi(x)$ and $\Pi(0, x) = \Sigma[-(\partial_\mu - iv_\mu(x))^2]$. This together with $\delta \ln \text{Det} A = Tr \delta A A^{-1}$ leads to

$$S_{\text{GND}}[U, J] - S_{\text{GND}}[1, J] = \int_0^1 dt \text{ Tr} \left[ \frac{\partial \Pi(t)}{\partial t} [\theta + J + \Pi(t)]^{-1} \right]$$

$$= N_c \int_0^1 dt \int d^4x \frac{d^4k}{(2\pi)^4} \text{ tr}_f \left[ \frac{\partial \Pi(t, k, x)}{\partial t} [\theta_x - ik^\mu + J(x) + \Pi(t, k, x)]^{-1} \right] ,$$

(5)

where $S_{\text{GND}}[1, J]$ can be taken out, since it is a $U$ field independent pure external field term. $\text{ tr}_f$ is trace for Lorentz spinor and flavor indices and

$$\Pi(t, k, x) \equiv \left[ \Omega(t) P_R + \Omega(x) P_L \right] \Sigma[-(\nabla_x - ik)^2] \left[ \Omega(t) P_R + \Omega(x) P_L \right]_{\Omega(x) \to \Omega(t, x)} .$$

(6)

(5) includes all interaction terms in the GND quark model. We are only interested in its anomalous part which can be shown that it is the part containing only one Levi-Civita tensor $(5)$ includes all interaction terms in the GND quark model. We are only interested in its anomalous part which can be shown that it is the part containing only one Levi-Civita tensor

$$S_{\text{GND}}[U, J] \bigg|_{\text{anomalous part}} = N_c \int_0^1 dt \int d^4x \frac{d^4k}{(2\pi)^4} \text{ tr}_f \left[ \frac{\partial \Pi(t, k, x)}{\partial t} [\theta_x - ik^\mu + J(x) + \Pi(t, k, x)]^{-1} \right] .$$

(7)

The next step we are interested in is to calculate the leading order contribution without external fields. We switch off the external field by taking $J = 0$, then $\nabla_x^\mu$ in (6) becomes

$$\nabla_x^\mu = \partial_\mu + \frac{i}{2} \Omega(t) \partial^\mu \Omega(t, x) + \Omega(t, x) \partial^\mu \Omega(t, x) .$$

We can take momentum (or derivatives) expansion, scale external field $s$ and $p$ as order of $p^2$, $v_\mu$ and $a_\mu$ as order of $p^1$, $U$ and $\Omega$ as order of $p^0$. Then, up to the order of $p^4$, after detail calculation, we find

$$S_{\text{GND}}[U, J] \bigg|_{\text{anomalous part, } J = 0, p^4} \equiv \Gamma^-[U]$$

$$= -\frac{N_c C}{48\pi^2} \int_0^1 dt \int d^4x \epsilon^{\mu\nu\rho\sigma} \text{ tr}_f \left[ U^+(t, x) \frac{\partial U(t, x)}{\partial t} L_\mu(t, x) L_\nu(t, x) L_\rho(t, x) L_\sigma(t, x) \right] ,$$

(8)

with $L_\mu(t, x) \equiv U^+(t, x) \partial_\mu U(t, x)$ and

$$C = \frac{3}{\pi^2} \int d^4k \left[ \frac{4\Sigma^6(k^2)}{k^2 + \Sigma^2(k^2)^5} - \frac{8k^2 \Sigma'(k^2) \Sigma^5(k^2)}{[k^2 + \Sigma^2(k^2)^5]^3} \right]$$

$$= -12 \int_0^\infty dk^2 \left[ \frac{k^2 \Sigma^4(k^2)}{[k^2 + \Sigma^2(k^2)^3]^3} \frac{d}{dk^2} \frac{\Sigma^2(k^2)}{[k^2 + \Sigma^2(k^2)^3]} \right] = 1 .$$

(9)
where by changing the integration variable to \( t = \Sigma(k^2)/k^2 \), the integration can be finished with result value 1 in the case of infinity upper limit of momentum integration and requiring quark self energy satisfies following constraint

\[
\frac{\Sigma(k^2)}{k^2} \rightarrow \begin{cases} \infty & k^2 = 0 \\ 0 & k^2 \rightarrow \infty \end{cases}
\]  

(10)

Note constant \( C \) although is \( \Sigma(k^2) \) dependent, but in a rather wide range of nonzero \( \Sigma(k^2) \) given by (10) is 1,

\[
C = \begin{cases} 1 & \Sigma(k^2) \neq 0 \\ 0 & \Sigma(k^2) = 0 \end{cases}
\]  

(11)

In a five dimension disc \( Q \) with its coordinates \( y^i (i = 1 \ldots 5) \) and 4-dimension space-time boundary, we have

\[
\frac{\partial}{\partial t} \text{tr}_f \left[ L_i(t, y)L_j(t, y)L_k(t, y)L_l(t, y)\right] \epsilon^{ijklm} = 5 \frac{\partial}{\partial y^m} \text{tr}_f \left[ U^\dagger(t, y) \frac{\partial U(t, y)}{\partial t} L_i(t, y)L_j(t, y)L_k(t, y)L_l(t, y)\right] \epsilon^{ijklm},
\]

then

\[
\int_Q d\Sigma^{ijklm} \text{tr}_f \left[ L_i(1, y)L_j(1, y)L_k(1, y)L_l(1, y)\right] L_m(1, y)\]  

\[
= \int d\Sigma^{ijklm} \int_0^1 dt \frac{\partial}{\partial t} \text{tr}_f \left[ L_i(t, y)L_j(t, y)L_k(t, y)L_l(t, y)\right] L_m(t, y)\]  

\[
= 5 \int d^4x \int_0^1 dt \epsilon^{\mu\nu\sigma\rho} \text{tr}_f \left[ U^\dagger(t, x) \frac{\partial U(t, x)}{\partial t} L_{\mu}(t, x)L_{\nu}(t, x)L_{\sigma}(t, x)L_{\rho}(t, x)\right].
\]

This relation leads (8) to Wess-Zumino action

\[
\Gamma^-[U] = -\frac{N_c}{240\pi^2} \int_Q d\Sigma^{ijklm} \text{tr}_f \left[ L_i(1, y)L_j(1, y)L_k(1, y)L_l(1, y)\right] L_m(1, y)\]  

(12)

In principle, we can follow the present procedure given by (7) to continue the calculation for external fields dependent \( p^4 \) order terms, \( p^6 \) order terms, \ldots, etc. We will discuss external fields dependent \( p^4 \) order terms in follows in an alternative way and leave the detail discussion of those more higher order lengthy results elsewhere.

Now we discuss the chiral transformation property of \( S_{\text{GND}} \). Consider following \( U_L(3) \otimes U_R(3) \) local chiral transformation with left and right hands transformation matrices \( V_L(x) \) and \( V_R(x) \)

\[2\text{Result given in Ref. [13] is different with us in the integrand by an extra factor } k^2/[k^2 + \Sigma^2(k^2)].\]
\[ J(x) \rightarrow J'(x) = [V_R(x)P_L + V_L(x)P_R][J(x) + \partial_x][V_R^\dagger(x)P_R + V_L^\dagger(x)P_L] \]
\[ \Omega(x) \rightarrow \Omega'(x) = h(x)\Omega(x)V_L^\dagger(x) = V_R(x)\Omega(x)h^\dagger(x) , \quad (13) \]

with \( h(x) \) depends on \( V_R, V_L \) and \( \Omega \), represents an induced hidden local \( U(3) \) symmetry to keep transformed \( \Omega \) be a representative element at coset class. In Ref. [2], it was shown that the corresponding transformation property for \( J_\Omega \) and \( \Sigma(-\nabla^2) \) are

\[ J_\Omega(x) \rightarrow J'_\Omega(x) = h(x)[J_\Omega(x) + \partial_x]h^\dagger(x) \]
\[ \Sigma(-\nabla^2) \rightarrow \Sigma(-\nabla^2) = h(x)\Sigma(-\nabla^2)h^\dagger(x) . \quad (14) \]

With (13) and (14), we find \( \Pi \) field transforms as

\[ \Pi(x) \rightarrow \Pi'(x) = [V_R(x)P_L + V_L(x)P_R]\Pi(x)[V_R^\dagger(x)P_R + V_L^\dagger(x)P_L] . \quad (15) \]

Then for infinitesimal transformation,

\[ V_R(x) = 1 + i\alpha(x) + i\beta(x) + \cdots \quad V_L(x) = 1 + i\alpha(x) - i\beta(x) + \cdots , \]

we have

\[ \delta[\partial + J] = i[\alpha(x) - \beta(x)\gamma_5][\partial + J] - i[\partial + J][\alpha(x) + \beta(x)\gamma_5] \]
\[ \delta\Pi(x) = i[\alpha(x) - \beta(x)\gamma_5]\Pi(x) - i\Pi(x)[\alpha(x) + \beta(x)\gamma_5] , \]

the corresponding infinitesimal transformation of \( S_{\text{GND}} \) is

\[ \delta S_{\text{GND}} = \text{Tr} \left[ [\partial + J + \Pi]^{-1}\delta[\partial + J + \Pi] \right] \]
\[ = i\text{Tr} \left[ [\partial + J + \Pi]^{-1} \left( [\alpha(x) - \beta(x)\gamma_5][\partial + J + \Pi] - [\partial + J + \Pi][\alpha(x) + \beta(x)\gamma_5] \right) \right] \]
\[ = -2i\text{Tr}[\beta\gamma_5] \]
\[ = -2i \lim_{\Lambda \to \infty} \text{Tr} \left[ \beta\gamma_5 e^{\frac{[\partial + J + \Pi]^2}{\Lambda^2}} \right] , \quad (16) \]

where we have taken the Fujikawa approach [14] in last equality to regularize the anomaly. The detail calculation shows that the \( \Pi \) field, scalar and pseudoscalar part of external fields on the exponential of regulator make no contribution to the result in the limit of infinite \( \Lambda \). The result of calculation for (16) just gives the standard Bardeen anomaly [15],

\[ \delta S_{\text{GND}} = -i \int d^4x \text{tr}_f[\beta(x)\tilde{\Omega}(x)] \]
\[ \tilde{\Omega}(x) = \frac{N_c}{16\pi^2} e^{\mu\nu\mu'} \left\{ V_{\mu\nu}(x) V_{\mu'\nu'}(x) + \frac{4}{3} d_\mu a_\nu(x) d_{\mu'} a_{\nu'}(x) \right\} + \frac{2i}{3} \{ V_{\mu\nu}(x), a_{\mu'}(x) a_{\nu'}(x) \} + \frac{8i}{3} a_\mu(x) V_{\mu'\nu'}(x) a_{\nu}(x) + \frac{4}{3} a_\mu(x) a_{\nu}(x) a_{\mu'}(x) a_{\nu'}(x) \} , \quad (17) \]
where \( V_{\mu \nu} = \partial_\mu v_\nu - \partial_\nu v_\mu - i[v_\mu, v_\nu] \) and \( d_\mu a_\nu = \partial_\mu a_\nu - i[v_\mu, a_\nu] \).

With result (17), we can follow the procedure given by Wess and Zumino [5] to construct corresponding \( p^4 \) order anomalous part action by taking a special local chiral rotation \( \delta_\beta = \delta_\beta \big|_{\alpha = 0} \), and

\[
U(x) = e^{-2i\beta(x)}, \quad e^{\delta_\beta} U(x) = 1 ,
\]

then rotation of action (3) leads

\[
e^{\delta_\beta} S_{\text{GND}}[U, J] = \text{Tr} \ln[\theta + J_\Omega + \Sigma(-\nabla^2)] \bigg|_{\Omega(x) = e^{-i\beta(x)}} ,
\]

therefore

\[
S_{\text{GND}}[U, J] = \left[ 1 - \frac{e^{\delta_\beta}}{\delta_\beta} \delta_\beta S_{\text{GND}}[U, J] + \text{Tr} \ln[\theta + J_\Omega + \Sigma(-\nabla^2)] \bigg|_{\Omega(x) = e^{-i\beta(x)}} \right]
\]

\[
= \left[ i \int_0^1 dt \int d^4x \ e^{i\beta} \text{tr}_f [\beta(x)\tilde{\Omega}(x)] + \text{Tr} \ln[\theta + J_\Omega + \Sigma(-\nabla^2)] \bigg|_{\Omega(x) = e^{-i\beta(x)}} \right] \quad (21)
\]

where the first term is the standard Wess-Zumino term constructed by Wess and Zumino in [5]. While for the second term, vector-like transformation law (14) imply it is invariant under chiral rotation (13) and then there will be no Wess-Zumino term in it, since Wess-Zumino term is not invariant under chiral rotation, it gives chiral anomaly [5]. We can directly verify this by performing the similar computation procedure given in (7), the contribution to \( p^4 \) order anomalous term from \( \Sigma \) dependent term of \( \text{Tr} \ln[\theta + J_\Omega + \Sigma(-\nabla^2)] \) is

\[
\text{Tr} \ln[\theta + J_\Omega + \Sigma(-\nabla^2)] \bigg|_{\Sigma \text{ dependent, anomalous } p^4}
\]

\[
= -2N_c \epsilon_{\mu \alpha \beta} \int_0^1 dt \int d^4k \left\{ \frac{\partial U}{\partial t} \right\} \left[ \frac{\Sigma^2(k^2)\Sigma^2(k^2)}{[\Sigma^2(k^2) + k^2]^4} - \frac{2k^2\Sigma'(k^2)\Sigma(k^2)\Sigma^2(k^2) - k^2}{[\Sigma^2(k^2) + k^2]^4} \right]
\]

\[
\times \left( 2\nabla_\mu \nabla_\nu \nabla_\alpha \nabla_\beta + 2a_\mu a_\nu a_\alpha a_\beta - 2\nabla_\mu a_\nu \nabla_\alpha a_\beta + 2\nabla_\mu a_\nu a_\alpha \nabla_\beta + 2a_\mu \nabla_\nu \nabla_\alpha a_\beta - 2a_\mu \nabla_\nu a_\alpha \nabla_\beta
\]

\[
+ 2\nabla_\mu \nabla_\nu a_\alpha a_\beta + 2a_\mu a_\nu a_\alpha a_\beta \right) + \left\{ \frac{k^2\Sigma^2(k^2)}{[\Sigma^2(k^2) + k^2]^4} - \frac{2k^2\Sigma'(k^2)\Sigma(k^2)\Sigma^2(k^2)}{[\Sigma^2(k^2) + k^2]^4} \right\} \left( 4\nabla_\mu \nabla_\nu \nabla_\alpha \nabla_\beta
\]

\[
+ 2a_\mu a_\nu \nabla_\alpha \nabla_\beta - 2\nabla_\mu a_\nu \nabla_\alpha a_\beta + 4a_\mu \nabla_\nu \nabla_\alpha a_\beta - 2a_\mu \nabla_\nu a_\alpha \nabla_\beta + 2\nabla_\mu \nabla_\nu a_\alpha a_\beta \right) \bigg]\}
\]

\[
= \frac{1}{2} \int_0^1 dt \int d^4x \left[ \frac{\partial U}{\partial t} U^\dag \tilde{\Omega}(t, x) \right] ,
\]

where as done in (9), the momentum integration in above formulae for \( \Sigma \) dependent coefficients can be finished, result in a Bardeen anomaly expressed in terms of rotated external fields in the case of infinity upper limit of momentum integration. \( \tilde{\Omega}(t, x) \) is \( \tilde{\Omega}(x) \) defined in (18) with all \( \Omega(x) \) replaced by \( \Omega(t, x) \). This term exactly cancels \( \Sigma \) independent term of \( \text{Tr} \ln[\theta + J_\Omega + \Sigma(-\nabla^2)] \) which is \( \text{Tr} \ln[\theta + J_\Omega] \) and we will calculate minus of it later in (25).
So our result shows that there is no \( p^4 \) order anomalous term in \( \text{Tr} \ln[\theta + J_\Omega + \Sigma(-\nabla^2)] \), the contribution to anomalous part from this term is at least order of \( O(p^6) \). 

Combine two terms in (21) together, \( p^4 \) and more higher order anomalous terms are all included in \( S_{\text{GND}}[U, J] \).

Compare result (21) with (1), we find the last two terms in (1) is

\[
-\text{Tr} \ln[\theta + J_\Omega] + \text{Tr} \ln[\theta + J] = i \int_0^1 dt \int d^4 x \ e^{-t\beta} \ \text{tr}[\beta(x)\tilde{\Omega}(x)]_{\Omega(x)=e^{-i\beta(x)}} . \tag{23}
\]

This shows that the last two terms in (1) only contribute \( p^4 \) order anomalous terms which are Wess-Zumino action. In fact, we can explicitly get Wess-Zumino action from l.h.s of (23) by taking similar computation procedure in (7). Replacing \( \Omega(x) \) by \( \Omega(t, x) = e^{it\pi(x)} \), we can show that under infinitesimal variation of parameter \( t \to t + \delta t \), \( \delta[\theta + J_\Omega(t)] = i\delta t\{\pi(x)\gamma_5, [\theta + J_\Omega(t)]\} \), which leads,

\[
-\text{Tr} \ln[\theta + J_\Omega] + \text{Tr} \ln[\theta + J] = -\lim_{\Lambda \to \infty} \int_0^1 dt \ \text{Tr} \left[ \frac{\partial J_{\Omega(t)}}{\partial t} [\theta + J_{\Omega(t)}]^{-1} \right]
\]

\[
= -\lim_{\Lambda \to \infty} \int_0^1 dt \ \text{Tr} \left[ \frac{\partial U}{\partial t} U^\dagger \gamma_5 e \frac{[\theta + J_{\Omega(t)}]^2}{\Lambda^2} \right] . \tag{24}
\]

Since we are interested in anomalous part, we only need to collect the one \( \epsilon_{\mu\nu\mu'\nu'} \) dependent terms of above result, the detail calculation gives

\[
\left[ -\text{Tr} \ln[\theta + J_\Omega] + \text{Tr} \ln[\theta + J] \right]_{\text{anomalous part}} = -\lim_{\Lambda \to \infty} \int_0^1 dt \ \text{Tr} \left[ \frac{\partial U}{\partial t} U^\dagger \gamma_5 e \frac{[\theta + J_{\Omega(t)}]^2}{\Lambda^2} \right] = \frac{1}{2} \int_0^1 dt \int d^4 x \ \left[ \frac{\partial U}{\partial t} U^\dagger \tilde{\Omega}(t, x) \right] , \tag{25}
\]

As in (22) \( \tilde{\Omega}(t, x) \) is \( \tilde{\Omega}(x) \) defined in (18) by replacing all \( \Omega(x) \) with \( \Omega(t, x) \). We further focus on zero external fields terms. In this case, we can show that the \( \Omega(t) \) dependent external fields satisfy constraints: \( d^\mu a^\mu_{\Omega} = d^\nu a^\mu_{\Omega} \) and \( V^\mu_{\Omega} = i[a^\mu_{\Omega}, a^\nu_{\Omega}] \). With help of these relations, (25) can be further reduced to \( \Gamma^-[U] \) given in (8) with \( C = 1 \) and with help of discussion after (8) we find it is just Wess-Zumino action with vanishing external fields.

Up to now, we have known that in GND quark model given by (1), for anomalous sector, the last two terms contribute to \( p^4 \) order anomalous term, i.e. Wess-Zumino action, while the first term in (1) which is also the second term in (21), as discussed previously, is invariant under chiral rotation and only contribute to \( p^6 \) or more higher order anomalous terms.

\[\text{\textsuperscript{3}}\text{Since this term in original Wess-Zumino paper [5] is forced to be unity as a normalization constant, the } O(p^6) \text{ anomalous term is dropped out there.}\]
The cancellation discussed for normal part in Ref. [2] also happens here. There are two kinds of cancellation mechanism now. In the first cancellation mechanism, Wess-Zumino term contributed from Σ independent and dependent parts of first term of (1) cancelled each other, leaving only $O(p^6)$ anomalous terms. With this cancellation mechanism, the left the Wess-Zumino term in GND quark model is from the second term of (1). This is just the dynamics independent approach, since the source of result Wess-Zumino action here is independent of strong interaction dynamics induced quark self energy Σ($k^2$). In the second cancellation mechanism, Wess-Zumino term given by second term of (1) is completely cancelled by the Σ independent part of first term, leaving GND quark model with the Σ dependent part of first term as Wess-Zumino action which we have explicitly expressed in (3) and calculated through formulae (5) to (12). This is the dynamics dependent approach. Both approaches generate same Wess-Zumino term. Different approaches are corresponding to different choices of cancellation mechanisms.

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