Abstract The Orbital Angular Momentum (OAM) wireless communication technology has received much attention in recent years for its natural orthogonality between different OAM states. Combining the OAM technology with the Orthogonal Frequency Division Multiplexing (OFDM) technology, we proposed a new OAM-OFDM wireless communication system to explore a new approach for enhancing the transmission capacity of the wireless communication system. Atmospheric turbulence is an important factor for influencing the capacity of the OAM-OFDM wireless communication system. In consideration of the effect of the atmospheric turbulence, we derived a crosstalk model of the proposed OAM-OFDM wireless communication system. Furthermore, a capacity model of the OAM-OFDM wireless communication system is proposed in consideration of the effect of atmospheric turbulence on both the OAM and OFDM signals. Compared with the conventional OAM wireless communication system, the capacity performance of the proposed OAM-OFDM wireless communication system is significantly improved and the average improvement is 751\%.

Index Terms Orbital angular momentum, capacity, bit error ratio, orthogonal frequency division multiplexing.

I. INTRODUCTION

With the upcoming commercialization of the 5th generation (5G) mobile communication system, more and more wearable devices and other wireless devices will be put into use in the future, which is followed by the challenges of the system capacity and spectrum resource. To solve the challenge of capacity and spectrum resources, a new degree of freedom of wireless resource is needed to further improve the capacity as well as the spectrum efficiency of existing wireless communication systems. Orbital Angular Momentum (OAM) technology [1], due to its spiral wavefront, theoretically infinite states and natural orthogonality between different states have received much attention in recent years in wireless communications systems. It is expected to provide the wireless communication system with a new degree of freedom, thereby increasing the system capacity of wireless communication systems. When the OAM technology is combined with Orthogonal Frequency Division Multiplexing (OFDM) technology, the capacity of the wireless communication system can be improved [2]. But the effect of the atmospheric turbulence on the OAM-OFDM signals is not considered in [2]. However, not only the OAM signals are affected by atmospheric turbulence but also the OFDM signals are affected by atmospheric turbulence during propagation [3]–[6], which in return affects the capacity of the OAM-OFDM wireless communication system. Hence, it is a key issue for enhancing the capacity of the OAM-OFDM wireless communication system in consideration of the atmospheric turbulence effect.

OAM technology has a wide range of applications in wireless communications since OAM technology has been successfully used for wireless communications in the optical frequency bands in 2004 [7]. Wang et al. achieved a spectral efficiency of 95.7 bit/s/Hz and capacity of 2.56 Tbit/s in 193.4 THz by multiplexing OAM beams for wireless communication systems [8]. Huang et al. combined the OAM and polarization, wavelength-division multiplexing technologies in a free space transmission system to attain a capacity of 100.8 Tbit/s [9]. Ge et al. proposed a new OAM spatial modulation (OAM-SM) millimeter wave communication system and achieved 341.6% improvement of energy efficiencies compared with the multi-input multi-output (MIMO)...
millimeter wave communication system [10]. Wang et al. proposed an OAM-based MIMO communication system and the channel capacity of the proposed system outperformed the channel capacity of conventional OAM and MIMO wireless communication systems [11]. When the OAM technology is adopted for wireless communication systems in the optical frequency bands, the OAM beams are influenced by the atmospheric turbulence and the power of the emitted OAM state spreads to the adjacent OAM states. Anguita et al. investigated the crosstalk model of Laguerre-Gauss beams under different atmospheric turbulence regimes to find the optimal set of OAM states for maximizing the capacity [12]. Cheng et al. investigated the conditional probability of non-diffraction Bessel-Gauss beams with the atmospheric turbulence effect and analyzed the capacity of the OAM wireless communication systems [13]. The spiral wavefront of OAM beams is vulnerable in the atmospheric turbulence which leads to the crosstalk issues in OAM beams with different OAM states [14]. In recent years, the usage of OAM in the microwave frequency bands has drawn more and more attention. Thidé et al. were the first to generate OAM beams below 1 GHz in radio frequency band which significantly highlights the potential applying OAM technology for radio wireless communication systems [15]. Tamburini et al. were the first to carry out the experimental test using multiple OAM beams with state 0 and 1 in the microwave frequency band [16], which proved that the transmission capacity of wireless communication systems can be improved obviously by OAM beams. Schemmel et al. proposed a polypropylene spiral phase plate (SPP) to originate the millimeter wave OAM and used a three-dimensional field scanner to measure the millimeter wave OAM [17]. Meanwhile, the millimeter wave OAM was influenced by the atmospheric turbulence [18] and the mode purity of millimeter wave OAM beams changed with propagation distance and OAM state [19].

OFDM technology has a wide range of applications in millimeter wave wireless communication systems [20] and is mainly combined with MIMO technology [21]. Dardari et al. proposed an indoor coded OFDM system at 60 GHz and achieved 155 Mbit/s packet transmission [22]. Cheng et al. investigated a codebook-based beamforming technology for OFDM systems at 60 GHz which significantly reduced the requirement on training data transmissions [23]. Experiments showed that the signal-to-noise ratio (SNR) and signal power were influenced by the atmospheric turbulence in wireless communication systems over optical frequency bands [24]. Wang et al. analyzed the symbol error rate (SER) of OFDM systems under different situations in wireless communication systems considering atmospheric turbulence [25]. The transmission performance of time division multiplexed (TDM) and OFDM signals over optical frequency bands with atmospheric turbulence effects were compared and indicated that the SNR of OFDM systems had 14.7 dB gain in [3]. Bekkali et al. derived a closed-form bit error probability and outage probability expressions for OFDM wireless communication systems considering atmospheric turbulence over optical frequency bands [4]. Djordjevi and the co-authors investigated the low-density parity-check (LDPC) coded OFDM over atmospheric turbulence channels in optical frequency bands and found that the bit error ratio (BER) and spectral efficiency of LDPC coded OFDM outperformed that of the LDPC coded on-off keying (OKK) in wireless communication systems [5]. For the millimeter wave OFDM systems in atmospheric turbulence, the Gamma-Gamma distribution was adopted to model the OFDM signals [6].

In view of the advantage of OFDM technology in spectral efficiency, the OFDM technology can be combined with the OAM technology to enhance the capacity of OAM-OFDM wireless communication systems. However, the effect of atmospheric turbulence on OAM and OFDM signals had been investigated separately in existing studies. As far as we know, the effect of atmospheric turbulence on the performance of OAM-OFDM wireless communication systems has rarely been investigated. In this article, we propose a new OAM-OFDM system, where the effect of atmospheric turbulence on both the OAM and OFDM signals are taken into consideration for analyzing the capacity performance of OAM-OFDM wireless communication systems. The contributions of this article are summarized as follows:

1) The effect of atmospheric turbulence on the performance of OAM-OFDM wireless communication systems is investigated, specifically the effect of atmospheric turbulence on both the OAM and OFDM signals are taken into consideration.

2) A crosstalk model is proposed for the OAM-OFDM transmission system in consideration of atmospheric turbulence. Furthermore, a capacity model of the OAM-OFDM transmission system is proposed for wireless communication in atmospheric turbulence.

3) The impact of the number of subcarriers, the refractive index structure constant, the propagation distance, the subcarrier frequency and the number of OAM states on the capacity of the OAM-OFDM transmission system is analyzed. The analytical results show that the capacity of the OAM-OFDM system decreases with the increase of the refractive index structure constant, the propagation distance and the subcarrier frequency. Moreover, the analytical results show that the BER of the OAM-OFDM system increases with the increase of the refractive index structure constant, the propagation distance, the subcarrier frequency and the OAM states.

The rest of this article is organized as follows. In section II, the system model of the OAM-OFDM system is proposed. Moreover, the function of each part of the proposed system model is explained. In section III, the impact of the atmospheric turbulence on the crosstalk model of OAM-OFDM transmission system is analyzed. In section IV, the capacity model of the OAM-OFDM system considering atmospheric turbulence is proposed for wireless communications. In section V, the analytical results are analyzed and discussed. In the end, conclusions are drawn in Section VI.
II. SYSTEM MODEL
Fig. 1(a) illustrates an OAM-OFDM system whose transmitter contains $L$ OFDM modulation modules. The signals modulated by the OFDM modulation modules are multiplexed with OAM states by the uniform circular array (UCA) which has $U$ antenna elements. The OAM states are denoted as $l_q \in S$, where $1 \leq q \leq L$ and $S = \{-K, -K + 1, \ldots, 0, \ldots, K - 1, K\}$ is the set of the OAM states, $K$ is the maximum OAM state. Without loss of generality, the OFDM modulation modules are configured to correspond the OAM states, i.e., the $q^{th}$ OFDM modulation module corresponds to the OAM state $l_q$. In this case, the total number of OAM states is $L$ and $L = 2K + 1$. Considering the orthogonality among signals of different OAM states, the signals of different OAM states generated by corresponding OFDM modulation modules can share the
same frequency bands in OAM-OFDM systems. Thus the frequency bands can be multiplexed and the spectrum efficiency can be improved for OAM-OFDM transmission systems. The number of subcarriers in each OFDM modulation modules is $M$. Since different OFDM modulation modules occupy the same frequency bands, the frequency of the $m^{th}$ subcarrier in each OFDM modulation module is $f_m \in F = \{f_0, f_2, \cdots, f_m, \cdots, f_{M-2}, f_{M-1}\}$, where $f_m = \frac{m}{T_s} + f_c$, $0 \leq m \leq M-1$, $T_s$ is the original symbol period, $f_c$ is the minimum frequency of OFDM subcarriers. As shown in Fig. 1(a), in the $q^{th}$ OFDM modulation module, the serial to parallel conversion (S/P) module converts a set of serial symbols to $M$ parallel symbols with a low rate. Then the $m^{th}$ symbol of the $M$ parallel symbols is modulated as a frequency domain (FD) signal $X_{m}^{q}$. Later $X_{m}^{q}$ is transformed to the time domain (TD) signal $s_{m}^{q}$ by the inverse fast Fourier transform, i.e., multiplied by $e^{-2\pi i (m/T_s+f_c)t}$. Then the parallel to serial conversion (P/S) module converts the $M$ parallel TD signals $s_{m}^{q}$ to serial signals $s_{OFDM}^{q}$. The cyclic prefix (CP) is added to the serial signal $s_{OFDM}^{q}$ by the CP insertion module afterward and then the digital signal is transformed to analog signal by the digital to analog conversion (A/D) module. In the end, the UCA is adopted to generate the OAM beams with different OAM states. The antenna elements in the UCA is fed with the same OFDM signal generated by the OFDM modulation module $q$ and with a successive delay from element to element such that after a full turn the phase has been incremented by an integer multiple $l_q$ of $2\pi$, where $l_q$ is the OAM state of the transmitted signal. For example, in the $u^{th}$ antenna element of UCA, the phase $u l_q$ is multiplied to the signal. Considering that the UCA is adopted to generate the OAM beams, the OAM state is limited by $|l_q| \leq \frac{u-1}{2}$ [26] to generate a pure rotating phase front and a perfect OAM state.

Fig. 1(b) shows the structure of UCA which is used to transmit and receive the OAM-OFDM signals. The phase $-u l_q$ is multiplied in the $u^{th}$ antenna element of the receive UCA when the signals reach the UCA of the receiver. In the $q^{th}$ OFDM demodulation module, the plane wave after the receive UCA is transformed into the digital signals by the analog to digital conversion (A/D) module and the digital signals get the CP removed by the CP removal module. Then the serial signal $s_{OFDM}^{q}$ is generated in the receiver of the OAM-OFDM system. After that, the S/P module transforms the serial signal $s_{OFDM}^{q}$ into $M$ parallel TD signals and the $m^{th}$ parallel TD signal is $s_{m}^{q}$. On the contrary to the transmitter, the TD signal $s_{m}^{q}$ is transformed into FD signal by the fast Fourier transform, i.e., multiplied by $e^{-2\pi i (m/T_s+f_c)t}$. Finally, the $M$ parallel signals are recovered into the original serial signal by the P/S module.

Without in consideration of the atmospheric turbulence effect, the OFDM signal of the transmitter is given by

$$s_{OFDM}^{q}(t) = \sum_{m=0}^{M-1} s_{m}^{q}(t) = \sum_{m=0}^{M-1} X_{m}^{q} e^{2\pi i (m/T_s+f_c)t},$$  

where $X_{m}^{q}$ is the $m^{th}$ subcarrier signal in the $q^{th}$ OFDM modulation module. Therefore, the signal transmitted by UCA is given by

$$s_{OAM−OFDM}(t) = \sum_{q=0}^{L} \sum_{u=1}^{U-1} \sum_{m=0}^{M-1} X_{m}^{q} e^{2\pi i (m/T_s+f_c)t} e^{il_q u} / U. $$

III. PHASE DISTORTION MODEL OF OAM-OFDM

The field distribution of the OAM-OFDM wave with OAM state $l_q$ and the $m^{th}$ subcarrier frequency $f_m$ is shown in cylindrical coordinate system as [27]

$$E(r, \varphi, z)_{l_q, m} = \frac{-j\mu_0 \omega 4\pi r c l_q}{4\pi r c} J_{l_q} \left( \frac{k_m r D}{2r_c} \right) e^{j k_m r D} r_c e^{j \varphi},$$  

where $r$ denotes the axial distance, $\varphi$ denotes the azimuth angle, $z$ denotes the height; $j$ denotes the current density, $\mu_0$ denotes the magnetic conductivity, $\omega$ denotes frequency, $d$ denotes the electric dipole length. $i$ imaginary unit, $r_c = \sqrt{r^2 + z^2}$, $J_{l_q}$ denotes the Bessel function of first kind of $l_q$ order, $D$ denotes the aperture diameter, the wave number is $k_m = \frac{2\pi l_q}{c}$, the velocity of light is $c = 3 \times 10^8 m/s$.

In this article the propagation scenario is configured in urban ground wireless communication scenarios. In the urban ground wireless communication scenarios, the state of the atmosphere is assumed to be stable. In this case, the atmospheric turbulence can be seen as the weak fluctuation scenarios. Considering that the Rytov variance $\sigma_R^2 = 1.23 C_n^2 k_l^7/\rho_0 11/6 < 1$ is satisfied in weak fluctuation scenarios [28], where $C_n^2$ denotes the refractive index structure constant, $k$ denotes the wave number, $\zeta$ denotes the propagation distance. The Rytov approximation can be used in this article. When the OAM-OFDM waves propagate in urban ground wireless communication scenarios, the phase distortion induced by the atmospheric turbulence is expressed as

$$E (r, \varphi, z) = E(r, \varphi, z)_{l_q, m} e^{i \psi(r, \varphi, z)}.$$  


The Kolmogorov turbulent flow is mainly used in the high frequency communication such as optical communications. In this article, the millimeter wave wireless communication which belongs to the high frequency communication is assumed. So the Kolmogorov turbulent flow is also suitable for wireless communications [18]. Considering the Kolmogorov turbulent flow, the complex phase perturbations \( e^{i(\Psi(r,\varphi,z)+\Psi^*(r,\varphi',z))} \) satisfies [13]

\[
e^{i(\Psi(r,\varphi,z)+\Psi^*(r,\varphi',z))} = \frac{e^{-\frac{2\pi^2 - 2r^2 \cos(\varphi' - \varphi)}{\tilde{r}_0^2}}}{\tilde{r}_0^2},
\]

where \( \tilde{r}_0 = (0.55C_n^2 k^2 z)^{-\frac{3}{5}} \) denotes the spatial coherence length of the spherical wave, \( \Psi^* \) denotes the complex conjugate of \( \Psi \), \( \varphi' \) denotes the azimuth angle in the cylindrical coordinate system after conjugate operations, \( \langle \cdot \rangle \) represent the ensemble average.

The phase distortion causes the power of the transmitted OAM state spreading to the adjoining OAM states. Therefore, the field strength of the OAM-OFDM wave can be expanded as

\[
E(r, \varphi, z) = \frac{1}{\sqrt{2\pi}} \sum_{l=-\infty}^{+\infty} a_l(r, z) e^{il\varphi},
\]

where \( a_l(r, z) \) is expressed as

\[
a_l(r, z) = \frac{1}{\sqrt{2\pi}} \int_0^{2\pi} E(r, \varphi, z) e^{-il\varphi} \, d\varphi = \frac{1}{\sqrt{2\pi}} \int_0^{2\pi} E(r, \varphi, z) e^{-il\varphi} \, d\varphi.
\]

As described by Kolmogorov turbulent flow, atmospheric turbulence is caused by the break of air cells into smaller cells which ends as cells dissipate by viscosity. Due to the fact that transmitted wave-front propagates along a space-varying and time-varying refractive index distribution, the channel is inhomogeneous. Due to the random space-time redistribution of the refractive index, varies of effects on the signals connected with its scintillation or intensity fluctuations and phase fluctuations. So the influence of the atmospheric turbulence can be treated as two independent parts which is the intensity fluctuations and the phase fluctuations. Since the atmospheric scintillation is the main factor that influences the propagation of OFDM signals in atmospheric turbulence [6]. When the OAM beams propagate in the atmospheric turbulence of weak fluctuation conditions, the effect of atmospheric scintillation on the OAM beams can be neglected [29]. The influence of the atmospheric turbulence on the OAM signals can be treated as phase fluctuations, and the influence of the atmospheric turbulence on the OFDM signals can be treated as intensity fluctuations. In this case, the influence of the atmospheric turbulence on the OAM and the OFDM signals can be treated as two independent parts. The influence on the OAM signals arising from the atmospheric turbulence is treated as a pure phase distortion and the influence on the OFDM signals arising from the atmospheric turbulence is treated as an irradiance perturbation. Regardless of the irradiance fluctuation caused by the atmospheric turbulence, when the transmitted OAM state is \( l_q \), the probability density function (PDF) of the OAM-OFDM wave in Kolmogorov turbulent flow with the received OAM state \( l \) and the \( m^{th} \) subcarrier frequency \( f_m \) can be expressed as

\[
f_{l,m}(l_q) = \left| a_l(r, z) \right|^2
\]

\[
= \frac{1}{2\pi} \int_0^{2\pi} \int_0^{2\pi} E_{l_q,m}(r, \varphi, z) E_{l_q,m}^*(r, \varphi', z) e^{-i\hat{\varphi}r} e^{-i\varphi} \, d\varphi \, d\varphi'.
\]

Based on (5), (7) can be derived as (8), shown at the bottom of the page. Based on \( \int \exp \left[ -i\varphi \pm \mu \cos (\varphi - \varphi') \right] \, d\varphi = 2\pi \exp \left( -i\varphi' \right) J_n(\mu) \) [30], where \( J_n(\mu) \) is the modified Bessel function of first kind of order \( n \), (8) can be further derived as

\[
f_{l,m}(l_q) = \left| a_l(r, z) \right|^2
\]

\[
= \frac{\pi}{2} \left( \mu \int_0^{2\pi} \int_0^{2\pi} E_{l_q,m}(r, \varphi, z) E_{l_q,m}^*(r, \varphi', z) e^{-i\varphi} e^{-i\hat{\varphi}r} \, d\varphi \, d\varphi' \right)
\]

\[
= \frac{1}{2\pi} \int_0^{2\pi} \int_0^{2\pi} \left( j\mu \int_0^{2\pi} \int_0^{2\pi} E_{l_q,m}(r, \varphi, z) E_{l_q,m}^*(r, \varphi', z) e^{-i\varphi} e^{-i\hat{\varphi}r} \, d\varphi \, d\varphi' \right) \left( \frac{\pi Dr(m + f_c T_s)}{\pi Dr(m + f_c T_s)} \right)^2
\]

\[
	imes e^{-\left( -\frac{2\pi^2 - 2r^2 \cos(\varphi' - \varphi)}{\tilde{r}_0^2} \right)} \int_{l_q-l}^{l_q} \left( \frac{2r^2}{\tilde{r}_0^2} \right) e^{(l_q-l)(\varphi - \varphi')} \, d\varphi \, d\varphi'.
\]
state \( l \), where \( l \) is one of the OAM states in the OAM state set \( S \), can be expressed as

\[
D_{l,m}(l_q) = \int_{0}^{\infty} f_{l,m}(l_q) \, rdr
\]

\[
= \int_{0}^{\infty} \langle |a_l(r, z)|^2 \rangle \, rdr
\]

\[
= \int_{0}^{\infty} \left( \frac{\mu_{\text{ODU}(m+|l|T_s)}}{\sigma_r T_s} I_{1q} \left( \frac{\pi D_r(m+|l|T_s)}{\sigma_r T_s} \right) \right)^2 \frac{\pi^2}{16} \, rdr \,. \quad (10)
\]

Due to the effect of the atmospheric turbulence, part of the energy of the OAM signals will be redistributed into the adjacent OAM states and the energy leaking to the adjacent OAM states decreases with the increase of the mode interval between the transmitted OAM state and the OAM state which the energy is leaking into. So we define the purity as the ratio of the power of original state in the total power of the OAM-OFDM wave after the propagation in the atmosphere environments [19], [31]. When the transmitted OAM state is \( l_q \), the received OAM state is \( l \), the \( m^\text{th} \) subcarrier frequency is \( f_m \) and the propagation distance is \( z \), then the purity of signal can be described as

\[
P_{l,m}(l_q, z) = \frac{D_{l,m}(l_q, z)}{\sum_{ll=-K}^{K} D_{ll,m}(l_q, z)} \]

\[
= \int_{0}^{\infty} \left( \frac{\mu_{\text{ODU}(m+|l|T_s)}}{\sigma_r T_s} I_{1q} \left( \frac{\pi D_r(m+|l|T_s)}{\sigma_r T_s} \right) \right)^2 \frac{\pi^2}{16} \, rdr \,. \quad (11)
\]

According to the definition of the purity, \( P_{-K,m}(l_q, z) \), \( \ldots \), \( P_{l_q-1,m}(l_q, z) \), \( P_{l_q+1,m}(l_q, z) \), \( \ldots \), \( P_{K,m}(l_q, z) \) is the proportion of the power of the emitted OAM state spreading to the adjoining OAM states. Thus (11) can be used to construct the crosstalk matrices of different subcarrier frequencies in the OAM-OFDM transmission system. Assuming that there is no crosstalk among the OAM-OFDM signals of different subcarrier frequencies. The crosstalk matrices of \( M \) subcarrier frequencies can be considered independently.

Considering that the number of OAM states is \( L \), the dimension of the crosstalk matrices of each subcarrier frequency is \( L \times L \). The crosstalk matrix of the OAM-OFDM transmission system can be described as \( [P_0, \ldots, P_m, \ldots, P_{M-1}] \), where \( P_0, P_m \) and \( P_{M-1} \) respectively stand for the crosstalk matrices of the 0th, the \( m^\text{th} \) and the \((M - 1)^\text{th} \) subcarrier. And \( P_m \) can be represented as (12), shown at the bottom of the page.

Furthermore, the signal-to-interference-and-noise ratio (SINR) of OAM-OFDM channel can be given as

\[
\gamma_{l,m} = \frac{P_{l,m}(l, z)}{\sum_{l_q \in S} P_{l,m}(l_q, z) + \frac{N_0}{P_{TX}}} \,. \quad (13)
\]

where the \( N_0 \) denotes the power of additive white Gaussian noise, \( P_{TX} \) denotes the transmission power. Based on (13), the BER of OAM channels is derived as [12]

\[
ber_{l,m} = \frac{1}{2} \text{erfc} \left( \frac{\sqrt{\gamma_{l,m}}}{2} \right), \quad (14)
\]

where \( \text{erfc}(\cdot) \) denotes the complementary error function. Moreover, the expression in (14) assumes orthogonal signal modulation, such as on-off keying (OOK) or quadrature phase shift keying (QPSK).

IV. CAPACITY MODEL OF OAM-OFDM TRANSMISSION SYSTEMS

The main factor that influences the propagation of OFDM signals in atmospheric turbulence is the atmospheric scintillation. According to the scintillation theory [32], the irradiance of the received OAM-OFDM wave is modeled as \( I = x \cdot y \), where \( x \) arises from large-scale turbulent eddies and \( y \) arises from small-scale turbulent eddies. Assuming that both large-scale and small-scale irradiance fluctuations are governed by gamma distributions [32], the PDFs of \( x \) and \( y \) are respectively expressed as

\[
p_x(x) = \frac{\alpha^{\alpha x}}{\Gamma(\alpha)} \exp(-\alpha x), \quad x > 0, \quad \alpha > 0, \quad (15a)
\]

\[
p_y(y) = \frac{\beta^{\beta y}}{\Gamma(\beta)} \exp(-\beta y), \quad y > 0, \quad \beta > 0, \quad (15b)
\]

where \( \alpha \) and \( \beta \) denote the large-scale and small-scale scintillation parameters respectively given by,

\[
\alpha = \left( \exp \left[ \frac{0.49 \sigma_R^2}{(1 + 1.11 \sigma_R^{12/5})^{7/6}} \right] - 1 \right)^{-1}
\]

\[
P_m = \begin{bmatrix}
P_{-L,m}(z, K) & \ldots & P_{l,m}(K, z) & \ldots & P_{K,m}(K, z) \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
P_{-L,m}(l_q, z) & \ldots & P_{l,m}(l_q, z) & \ldots & P_{K,m}(l_q, z) \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
P_{-L,m}(K, z) & \ldots & P_{l,m}(K, z) & \ldots & P_{K,m}(K, z)
\end{bmatrix}
\]

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\[ \beta = \left( \exp \left[ \frac{0.51\sigma_R^2}{(1 + 0.69\sigma_R^{12/5})^{5/6}} \right] - 1 \right)^{-1}, \]  

(16)

where \( \sigma_R^2 = 1.23C_n^2z^{7/6}z^{11/6} \) denotes the Rytov variance, \( \Gamma(\cdot) \) denotes the Gamma function.

When the value of \( x \) is fixed, the conditional PDF of \( I \) can be derived as

\[ p_x(I|x) = \frac{\beta(\beta x)^{\beta-1}}{x^\beta \Gamma(\beta)} \exp(-\beta x), \quad I > 0, \beta > 0, \ x > 0. \]

(17)

To obtain the unconditional irradiance distribution, we form the average of (17) over the Gamma distribution of (15a). As a consequence, the PDF of \( I \) is derived as a Gamma-Gamma distribution function as follows,

\[ p(I) = \int_0^\infty p_x(I|x)p_x(x)dx = \frac{2(\alpha\beta)^{\alpha+\beta/2}}{\Gamma(\alpha)\Gamma(\beta)} I^{\alpha+\beta/2-1} \cdot K_{\alpha-\beta}(2\sqrt{\alpha\beta}I), \quad I > 0, \]

(18)

where \( K(\cdot) \) is the modified Bessel function of second kind of order \( v \).

Considering (18) and the BER of OAM signals [33], the BER of the OAM-OFDM signals can be expressed as

\[ \langle \text{ber}_l \rangle = \int_0^\infty \frac{1}{2} p(I) \text{erfc}(\sqrt{I - \frac{\gamma_l}{2}}) dI \]

where \( p(I) \) denotes the PDF of the irradiance of the OFDM signals after the propagation in atmospheric turbulence which stands for the effect of the intensity fluctuations and the \( \gamma_l \) denotes the SINR of the OAM channel after the propagation in atmospheric turbulence which stands for the effect of the phase fluctuations. (19) can be further simplified as (20) with the aid of the Meijer G function [34].

\[ \langle \text{ber}_l \rangle = \int_0^\infty \frac{1}{2} p(I) \text{erfc}(\sqrt{\frac{I^2 - \gamma_l}{2}}) dI. \]

(20)

The binary symmetric channel is a widely used channel model in wireless communications and it is widely used in describing the channel of OAM systems. In this article, we assume that each OAM-OFDM channel is a binary symmetric channel, the capacity of OAM-OFDM channel is derived as [12]

\[ C(\langle \text{ber}_l \rangle) = 1 + \langle \text{ber}_l \rangle \log_2 \langle \text{ber}_l \rangle + (1 - \langle \text{ber}_l \rangle) \log_2(1 - \langle \text{ber}_l \rangle). \]

(21)

Then the total capacity of OAM-OFDM transmission systems can be expressed as

\[ C = \sum_{l=-K}^{K} \sum_{m=0}^{M-1} C(\langle \text{ber}_l \rangle) \]

(22a)

with

\[ \langle \text{ber}_l \rangle = \int_0^\infty \frac{1}{2} p(I) \text{erfc}(\sqrt{\frac{I^2 - \gamma_l}{2}}) dI. \]

(22b)

V. ANALYTICAL RESULTS

In this section, the capacity and BER of the proposed OAM-OFDM transmission system are simulated for performance analysis. To make OAM-OFDM transmission systems work in the millimeter wave frequency band and the signals propagate in the atmospheric turbulence of weak fluctuation conditions. The default simulation parameters of OAM-OFDM transmission systems are configured as follows: the propagation distance \( z \) is 2 km, the diameter of UCA is \( D = 0.6m \) [35], the refractive index structure constant is \( C_n^2 = 2 \cdot 10^{-11}m^{-2}r \) [32], the minimum subcarrier frequency is 60 GHz, the subcarrier interval is 1 KHz, the number of subcarriers is 8, the signal-to-noise ratio is \( SNR_0 = 10 \cdot \log(\frac{P_{TX}}{N_0}) = 10dB \) [18], the maximum OAM state is \( K = 5 \) (\( L = 11 \)) in simulations of Fig. 9 and Fig. 10.

**FIGURE 2.** BER with respect to the OAM state under different refractive index structure constants.

Fig. 2 shows the BER with respect to the OAM state under different refractive index structure constants in atmospheric turbulence. When the refractive index structure constant is fixed, the BER increases with the increase of the absolute value of the OAM state for the reason that the OAM signals with higher absolute value of the OAM state are more vulnerable in the atmospheric turbulence. The performance of BER is symmetrical with the OAM state and the symmetrical center is located at the OAM state zero. The BER increases.
with the increase of the OAM state when the OAM state is less than 18. The BER decreases with the increase of the OAM state when the OAM state is larger than or equal to 18 for the reason that the OAM states larger than or equal to 18 have less adjacent OAM states spreading energy. When the value of OAM state is fixed, the BER increases with the increase of the refractive index structure constant since the refractive index structure constant stands for the strength of the atmospheric turbulence.

Fig. 3 shows the BER with respect to the OAM state under different minimum subcarrier frequencies in atmospheric turbulence. When the value of OAM state is fixed, the BER increases with the increase of the minimum subcarrier frequency. The signals with higher frequencies are more sensitive and vulnerable to the effect of the atmospheric turbulence which leads to the higher BER.

Fig. 4 shows the BER with respect to the OAM state under different propagation distances in atmospheric turbulence. When the value of OAM state is fixed, the BER increases with the increase of the propagation distances. With the increase of the propagation distances, not only the attenuation of the OAM-OFDM signals but also the effect of the atmospheric turbulence gets stronger which lead to the higher BER.

Fig. 5 shows the BER with respect to the OAM state under the OAM and OAM-OFDM transmission systems in atmospheric turbulence. When the value of OAM state is fixed, the BER increases with the increase of the number of OAM states. Both the OAM and OFDM signals are influenced in atmospheric turbulence, the influence of the atmospheric turbulence on the OFDM signals can be treated as intensity fluctuations, and the influence of the atmospheric turbulence on the OAM signals can be treated as phase fluctuations. Since the OFDM signals are also influenced in atmospheric turbulence, the OAM-OFDM signals get extra influences in atmospheric turbulence compared with the OAM signals.

The capacity of OAM-OFDM transmission system with respect to the maximum OAM state considering different refractive index structure constants in atmospheric turbulence is analyzed in Fig. 6. When the refractive index structure constant is fixed, the capacity of the OAM-OFDM transmission system increases with the increase of the number of OAM states. For the reason that the increase of the number of OAM states means the increase of the number of OAM signals of different states to be multiplexed which leads to the increase of the capacity of the OAM-OFDM transmission system. When the number of OAM states is fixed, the capacity of OAM-OFDM transmission system increases with the decrease of the refractive index structure constant in atmospheric turbulence. For the reason that the BER increases with the increase of the refractive index structure constant, furthermore the capacity of the OAM-OFDM transmission system decreases with the increase of the BER.

In Fig. 7, the capacity of OAM-OFDM transmission system with respect to the maximum OAM state considering different minimum subcarrier frequencies in atmospheric turbulence is analyzed. When the number of OAM states
is fixed, the capacity of OAM-OFDM transmission system increases with the decrease of the minimum subcarrier frequency in atmospheric turbulence. The BER decreases with the decrease of the minimum subcarrier frequency, which leads to the increase of the capacity of the OAM-OFDM transmission system.

In Fig. 8, the capacity of the OAM-OFDM transmission system with respect to the maximum OAM state considering different propagation distances in atmospheric turbulence is analyzed. When the number of OAM states is fixed, the capacity of OAM-OFDM transmission system increases with the decrease of the propagation distance in atmospheric turbulence. The BER decreases with the decrease of the propagation distance, which leads to the increase of the capacity of the OAM-OFDM transmission system.

In Fig. 9, the capacity of the OAM-OFDM transmission system with respect to the number of subcarriers considering different refractive index structure constants in atmospheric turbulence is analyzed. When the refractive index structure constant is fixed, the capacity of the OAM-OFDM transmission system increases with the increase of the number of subcarriers. For the reason that the increase of the number of subcarriers means the increase of the number of OFDM subcarriers to be multiplexed which leads to the increase of the capacity of the OAM-OFDM transmission system.

Fig. 10 shows the capacity with respect to SNR compared with the OAM, OAM-OFDM and OFDM transmission systems in atmospheric turbulence scenarios. When the SNR is fixed, the capacity of the OAM-OFDM transmission system is larger than the capacity of the OAM and OFDM transmission systems, respectively. At the SNR of 10 dB, the capacity of the OAM-OFDM transmission system is improved by 732% compared with the OAM transmission system and 1058% compared with the OFDM transmission system. Since the number of subcarriers is eight, the capacity of the OAM-OFDM transmission system should be eight times of the capacity of the OAM transmission system ideally. Due to the effect of the atmospheric turbulence, the improvement
OAM-OFDM transmission system with the atmospheric turbulence effect is derived. Analytical results show that the proposed OAM-OFDM transmission system achieves a capacity improvement over the OAM transmission system and the average improvement is 751%. The analytical results also show that the BER of the OAM-OFDM transmission system is larger than the BER of the OAM transmission system. In future works, we plan to develop new channel coding technologies to further reduce the BER of the OAM-OFDM transmission system.

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