Quasiclassical negative magnetoresistance of a 2D electron gas: interplay of strong scatterers and smooth disorder

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We study the quasiclassical magnetotransport of non-interacting fermions in two dimensions moving in a random array of strong scatterers (antidots, impurities or defects) on the background of a smooth random potential. We demonstrate that the combination of the two types of disorder induces a novel mechanism leading to a strong negative magnetoresistance, followed by the saturation of the magnetoresistivity $\rho_{xx}(B)$ at a value determined solely by the smooth disorder. Experimental relevance to the transport in semiconductor heterostructures is discussed.

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Magnetotransport in a two-dimensional electron gas (2DEG) has been the subject of intensive research during the last two decades. This interest has been motivated by the progress in preparing high-quality semiconductor heterostructures, opening up new areas in both fundamental physics and applications, see [4] for a review. Within the quasiclassical approach (valid for not too strong magnetic fields $B$), impurity scattering is commonly described by a collision integral in the Boltzmann equation. This leads, for an isotropic system, to the $B$-independent Drude value of the longitudinal resistivity, $\rho_{xx}(B) = \rho_0 = m/e^2 n_e \tau$, where $n_e$ is the carrier density, $m$ the effective mass, and $\tau$ the transport scattering time.

It has become clear, however, that this description is not always valid. In particular, in the case of smooth disorder memory effects induce a strong positive magnetoresistance (MR) [3] followed by an exponential fall-off of $\rho_{xx}(B)$ due to adiabatic localization of drifting electrons [3]. To our knowledge, these effects have not been experimentally observed in the transverse transport in low magnetic fields, since the Shubnikov-de Haas oscillations develop at lower magnetic fields. On the other hand, the memory effects do show up in the composite fermion transport, explaining the peculiar shape of the MR around half-filling of the lowest Landau level [1,3].

In the present paper we study the quasiclassical MR of a 2DEG moving in a random array of rare strong scatterers (modeled by hard disks) and subject additionally to a smooth random potential. Apart from the purely theoretical interest, our work has been motivated by two types of experimental realizations of this problem. The first one is random antidot arrays. Experiments on this kind of structures [3,4] show a strong negative MR which has not been analyzed theoretically. Less obviously, our model is relevant to transport in the unstructured high-mobility 2DEG. To clarify this point, we recall that in order to increase the 2DEG mobility the donors in currently fabricated heterostructures are separated by a large distance $d \gg k_F^{-1}$ (with $k_F$ the Fermi wave number) from the 2DEG plane. It is usually assumed that these remote donors constitute the main source of disorder, inducing small-angle scattering of electrons. It is known, however, that in samples with a wide spacer ($d \geq 70$ nm) large-angle scattering on residual impurities [1] and interface roughness [2] becomes important, limiting the mobility with further increasing width of the spacer. In particular, Ref. [2] concludes that about 50% of the resistivity is determined by such scattering processes, while Ref. [3] finds that for samples with very high mobility this value is as high as 90%.

We thus consider the following two-component model of disorder: (i) randomly distributed hard-core scatterers (which we will term “antidots” or “impurities” below) with density $n_S$ and radius $a$ (where $n_S^{-1/2} \gg a \gg k_F^{-1}$), and (ii) smooth random potential (correlation radius $d$, momentum relaxation rate $\tau_L^{-1}$, transport mean free path $l = v_F \tau_L$). The mean free path for the scattering on antidots is $l_s^{0} = v_F \tau_s^{0} = 1/2 n_S a$, with the corresponding transport mean free path may be somewhat different, $l_s = v_F \tau_s = \gamma l_s^{0}$ with $\gamma \sim 1$ ($\gamma = 3/4$ in the model of specularly reflecting disks). We will set $\gamma = 1$ for qualitative estimates. We will further assume that $\tau_L \gg \tau_s$, so that the zero-$B$ resistivity $\rho_0$ is determined by the hard scatterers, $\tau^{-1} = \tau_L^{-1} + \tau_s^{-1} \approx \tau_s^{-1}$. Finally, we will assume that the motion in the smooth disorder is not adiabatic, i.e. has the form of the guiding center diffusion (rather than drift). The condition for this is $\delta \gg d$, where $\delta$ is the guiding center shift after one cyclotron revolution [see Eq. (2)]. For currently fabricated samples this assumption is usually satisfied in the whole range of applicability of the quasiclassical theory.

We start the analysis of the problem by recalling the results [3,4] for the case when only hard scatterers are present ($\tau_L = \infty$), known as the Lorentz gas. In this limit the resistivity $\rho_{xx}(B)$ reads

$$
\rho_{xx}(B)/\rho_0 = 1 - e^{-2\pi/\omega_c \tau_s^{0}}F(\omega_c \tau_S),
$$

(1)
leading to the mean time $\tau_S^{(0)}$.

The rest of the particles do not hit scatterers at all. In the sequel, we will only consider classically strong magnetic fields, $\omega_c \gg \omega_r \equiv 2\pi/\tau_S^{(0)}$, where the resistivity $\rho$ shows a $1/B$ fall-off. Equation (1) is valid below the percolation threshold, $\omega_c < \omega_{\text{perc}} = 1.67v_FN_S^{1/2}$ (note that $\omega_{\text{perc}}\tau_S \sim n_S^{-1/2}a^{-1} \gg 1$). For larger magnetic fields, $\omega_c \geq \omega_{\text{perc}}$, the resistivity is exactly zero, since the rosette-like families of cyclotron orbits associated with each impurity fail to form an infinite cluster.

Clearly, adding the long-range disorder will increase the diffusion constant $D_{xx}$ and thus to hit the next impurity, $N = \omega_c\tau_S^{(0)}/2\pi$, so that the total return probability is $P \sim (a/\delta)(\omega_c\tau_S)^{1/2}$. This determines the fraction of the area explored twice, implying an effective reduction of the exploration rate and thus a negative correction to the resistivity,

$$\Delta\rho_{xx}/\rho_0 \sim - (a/\delta)(\omega_c\tau_S)^{1/2} \sim - (\omega_c/\omega_0)^2,$$

where $\omega_0 \sim v_F(a/\delta)^{-1/4} \sim \omega_{\text{perc}}(a/\delta)^{-1/4} \ll \omega_{\text{perc}}$.

We turn now to a more rigorous and quantitative derivation. Generalizing the formalism of [2] to the present case, we start from the following Liouville-Boltzmann equation for the distribution function $g(r, \phi)$ of electrons on the Fermi surface,

$$(L_0 + \delta L)g(r, \phi) = \cos(\phi - \phi_E),$$

$$L_0 = v_F n \nabla + \omega_c \frac{\partial}{\partial \phi} - \frac{1}{\tau_L} \frac{\partial^2}{\partial \phi^2},$$

where $n = (\cos \phi, \sin \phi)$ is the unit vector determining the direction of velocity, $v = v_F n$, and $\phi_E$ is the polar angle of the electric field. The operator $L_0$ describes the motion in the smooth random potential, while $\delta L$ corresponds to the scattering on antidots with (random) positions $R_i$. The explicit form of the collision operator $I_R$ (for a hard-wall scatterer) can be found in [3], but we will not need it. We will only use the following properties of $I_R$: (i) it is non-zero only within the distance $a$ from the impurity location $R$, and (ii) its Fourier-transform $I_q$ satisfies $I_q n = (-1/n_S\tau_S)n$.

Expanding in $\delta L$, averaging over the positions $R_i$ of scatterers, and resumming the series, one finds the Green’s function of the Liouville operator, $\langle (L_0 + \delta L)^{-1} \rangle = (L_0 + M)^{-1}$, where $M$ is the “self-energy” operator. The resistivity is expressed in terms of $M$ as follows [2],

$$\rho_{xx} = \frac{m/e^2 n_e}{(\tau_L^{-1} + M_{xx})},$$

where $M_{xx} = \int (d\phi/\pi) \cos \phi M \cos \phi$. One may use a diagrammatic technique analogous to that developed for the Lorentz gas [3] to calculate $M$. The leading term corresponds to a single scattering process, yielding $M^{(1)} = -n_S I_0$ and thus the Drude contribution, $M^{(1)}_{xx} = \tau_S^{-1}$, to the resistivity [2]. The next-order contribution representing the correction induced by the return process is obtained as

$$M^{(2)}_{xx} = -n_S \int \frac{d\phi}{\pi} \cos \phi I_R D^{(1)} I_R \cos \phi,$$

where $D^{(1)} = (L_0 + M^{(1)})^{-1}$ is the electron propagator with the leading-order self-energy included.

Since $\delta \gg a$, the propagator $D^{(1)}(r - r', \phi, \phi')$ describing propagation from the point $r', \phi'$ to the point $r, \phi$
can be replaced in (8) by \( D^{(1)}(0, \phi, \phi') \). Furthermore, we note that once the particle hits an impurity, its guiding center is shifted by an amount of order of the cyclotron radius \( R_c \). As a result, the contribution of such trajectories to \( D^{(1)} \) can be neglected and only non-colliding orbits should be taken into account. Since the motion without collisions is limited by the times \( \sim \tau_S \ll \tau_L \), the particle will return with almost the same direction of velocity, i.e. \( D^{(1)}(0, \phi, \phi') \) will be peaked at \( \phi \approx \phi' \).

We can thus approximate \( D^{(1)}(0, \phi, \phi') \) by \( D^{(1)}_0(\phi - \phi') \), where \( D^{(1)}_0 = \int d\phi D^{(1)}(0, \phi, \phi') \). This quantity is easily found to be

\[
D^{(1)}_0 = \sum_{n=1}^{\infty} \frac{e^{-2\pi n/\omega_c \tau_S^{(0)}}}{(\pi n)^2 v_F \delta} \approx \frac{(\omega_c \tau_S^{(0)})^{1/2}}{(2\pi)^{1/2} v_F \delta}.
\]

Substituting this in (8) and using that \( I_0 n = -(1/n_S \tau_S) n \), we find \( M^{(2)}_{xx} = -(1/n_S \tau_S^2) D^{(1)}_0 \). This implies, according to (9), the negative MR,

\[
\Delta \rho_{xx}/\rho_0 = M^{(2)}_{xx}/\tau_S^{-1} = -\omega_c^2/\omega_0^2, \quad \omega_c \ll \omega_0,
\]

where

\[
\omega_0 = (2\pi n_S)^{1/2} v_F (2\pi l_S/l_L)^{1/4},
\]

in agreement with the above qualitative considerations.

This derivation is valid as long as the correction remains small, i.e. for \( \omega_c \ll \omega_0 \). For stronger magnetic fields, when the quantity \( P \) defined above becomes large, \( P \gg 1 \), it acquires the meaning of the number of returns. It should then be found self-consistently. Specifically, the sum over \( n \) is now cut off at the time \( \tau_S \sim P \tau_S \), since multiple returns to the same area lead to a corresponding increase of the time needed to hit a new impurity. We thus get a self-consistency equation, 

\[
\rho_{xx}/\rho_0 \sim \tau_S/\tau_S^e \sim 1/P \sim (\omega_0/\omega_c)^4, \quad \omega_c \gg \omega_0.
\]

It remains to analyze the conditions of validity of Eqs. (10), (12). First of all, we assumed that the particle finds a new scatterer by exploring the new area in the course of the diffusive motion of the guiding center. There exists, however, a competing mechanism, namely that of the rosette-like motion around a scatterer, which determines the transport in the pure Lorentz gas \( (l_L = \infty) \). Comparing (12) with the Lorentz-gas result \( \rho_{xx}/\rho_0 = 2\pi l_F^2/\omega_c \tau_S \) [see Eq. (4)], we find that the two formulas match at \( \delta \sim a \). The corresponding crossover frequency is \( \omega_{\text{cross}} = v_F (4\pi n_S l_S^2 l_L^{-1})^{1/3} \). Secondly, we assumed \( n_S l_S^2 \gg 1 \), or, equivalently, \( \omega_c \ll \omega_{\text{perc}} \). It is easy to see that in the opposite limit, \( n_S l_S^2 \ll 1 \), the resistivity will be determined by the smooth disorder, with scattering on antidots giving a small \( (\omega_{\text{perc}}/\omega_c)^2 \) correction only, so that \( \rho_{xx}(B) \) will have a plateau with the value \( \rho_{xx}(\omega_c \gg \omega_{\text{perc}}) = m/e^2 n_e \tau_L \).

Comparing the characteristic frequencies, \( \omega_r, \omega_0, \omega_{\text{cross}}, \text{and} \omega_{\text{perc}} \), we conclude that the following three situations can be distinguished, depending on the strength of the smooth disorder (Fig. 1):

(A) \( \omega_0 \ll \omega_r \), or, equivalently, \( l_L/l_S \gg (1/2\pi^2)(n_S l_S^2)^2 \). In this case the smooth disorder hardly affects the Lorentz-gas result (4).

(B) \( \omega_r \ll \omega_0 \ll \omega_{\text{cross}} \ll \omega_{\text{perc}} \), or, equivalently, \( 2.7(n_S l_S^2)^{1/2} \ll l_L/l_S \ll (1/2\pi^2)(n_S l_S^2)^2 \). This is an intermediate situation; the resistivity drops first according to Eqs. (4), (12) and then crosses over at \( \omega_c \sim \omega_{\text{cross}} \) to the Lorentz-gas behavior (4).

(C) \( \omega_r \ll \omega_0 \ll \omega_{\text{perc}} \ll \omega_{\text{cross}} \), or, equivalently, \( l_L/l_S \gg 2.7(n_S l_S^2)^{1/2} \). In this case the Lorentz-gas behavior (4) is completely destroyed, and the results (10), (12) hold in the whole range of \( \omega_c \) below \( \omega_{\text{perc}} \).

For \( \omega_c > \omega_{\text{perc}} \) the resistivity shows in all the cases a plateau, as explained above. On the side of strong magnetic fields this plateau will be modified either by entering into the adiabatic regime at \( \delta \sim d \), meaning \( \omega_c \sim \omega_{\text{ad}} = v_F (4\pi d^2/\ell_L)^{1/3} \) (its implications for the present problem will be considered elsewhere (3)) or by the development of Shubnikov-de Haas oscillations.

It is worth mentioning that while we used the condition \( \tau_L \gg \tau_S \) for the derivation of our main results, a pronounced negative MR will also be observed for \( \tau_L \sim \tau_S \) (which seems to be frequently the relevant situation for high-mobility structures (12)). In this case there is a crossover from \( \rho_{xx}(0) = (m/e^2 n_e)(\tau_L^{-1} + \tau_S^{-1}) \) to \( \rho_{xx}(\omega_c \gg \omega_{\text{perc}}) = m/e^2 n_e \tau_L \) which takes place around \( \omega_c \sim \omega_{\text{perc}} \) (note that \( \omega_0 \sim \omega_{\text{perc}} \) for \( \tau_L \sim \tau_S \)).

We have performed numerical simulations of the MR by fixing parameters of the Lorentz gas \( (\omega_{\text{perc}}/\omega_r = 5.3) \) and the correlation length of the smooth disorder \( (d/a \sim 2.5) \) and changing the strength of the latter. The results shown in Fig. 2 are in good agreement with the analytical predictions. It is seen that a very weak smooth disorder

![FIG. 1. Magnetoresistivity at fixed \( \tau_S \) and different \( \tau_L \): curve A – Lorentz gas \( (\tau_L = \infty) \), curves B and C with \( \tau_L^{(B)} > \tau_L^{(C)} \) correspond to regimes B and C, respectively (see text). The dotted lines denote the asymptotics (10).](image-url)
(giving negligible contribution to \( \rho_0 \)) affects crucially the MR. Some deviations in the values of \( \omega_0 \) from Eq. (11) (see inset) can be attributed to the fact that the conditions \( \omega_s \ll \omega_B \ll \omega_{\text{perc}}, \omega_{\text{ad}} \) are fulfilled only marginally in our simulations.

Finally, let us estimate the characteristic values of \( B \) for existing experiments. Typical parameters in the experiments \( \text{[4]} \) on the antidot arrays were \( n_e = 5 \times 10^{11} \text{ cm}^{-2}, \ n_s = (0.6 \mu \text{m})^{-2}, \ l_S = 1.3 \mu \text{m}, \ l_L = 16 \mu \text{m}. \) This implies the following values for the characteristic magnetic fields (we use the obvious notations \( B_\tau = (mc/e)\omega_\tau \) etc.): \( B_\tau \simeq 0.5 \text{ T}, \ B_0 \simeq B_{\text{perc}} \simeq 0.3 \text{ T}. \) We see that the condition of a dilute antidot array assumed above, \( B_\tau \ll B_{\text{perc}}, \) is not met. Clearly, in this situation the above analytical formulas are not valid quantitatively. On the qualitative level, we can conclude that there should be a strong fall-off of \( \rho_{xx} \) around \( B \simeq 0.3 \div 0.5 \text{ T}, \) in agreement with experimental results \( \text{[4]} \). A similar negative MR was observed in other experiments with disordered antidot arrays \( \text{[5,6,7,8,10,13,17]} \).

We turn now to unstructured high-mobility samples. Using the parameters of \( \text{[3]} \) \( (n_e = 2 \times 10^{11} \text{ cm}^{-2}, \text{ mobility } \mu = 10^7 \text{ cm}^2/\text{V} \cdot \text{s}, \) we find the mean free path \( l \simeq 80 \mu \text{m}. \) Let us assume, following the conclusion of \( \text{[3]} \) that the zero-field mobility is determined by background impurities \( (i.e. \ l \simeq l_s), \) while \( l_L \simeq 10 l_s. \) Using the typical volume concentration of the residual impurities \( \text{[2]}, \) \( n_s^{(3D)} \simeq 2.5 \times 10^7 \text{ cm}^{-3}, \) and the value of the Bohr radius in GaAs, \( a_B \simeq 10 \text{ nm}, \) we estimate the sheet density of strong scatterers as \( n_s \sim a_B n_s^{(3D)} \sim (2 \mu \text{m})^{-2}. \) With these parameters, we find \( B_\tau \simeq 5 \text{ mT}, \ B_0 \simeq B_{\text{perc}} \simeq 60 \text{ mT}. \) The condition of the diluted array of scatterers, \( B_\tau \ll B_{\text{perc}}, \) is now well satisfied. Since \( B_0 \simeq B_{\text{perc}}, \) the Lorentz-gas behavior \( \text{[3]} \) is fully destroyed (the case (C) in our classification), and the negative MR is determined by the interplay of smooth disorder and strong scatterers, as described above. Note that though parametrically \( B_0 \ll B_{\text{perc}} \) for \( l_L \gg l_S, \) in practice their values are very close, since \( B_0/B_{\text{perc}} \simeq (10 l_S/l_L)^{1/2}. \) The predicted negative MR may be used for the experimental determination of the ratio \( l_L/l_S. \) Strong negative MR has been observed in very-high-mobility samples \( \text{[3,4,7]}, \) in qualitative agreement with our theory. A more detailed experimental check of our predictions would be desirable.

In conclusion, we have studied the quasiclassical magnetotransport of a 2DEG with smooth disorder and rare strong scatterers. Interplay of these two types of disorder leads to a novel mechanism of strong negative MR; the latter is shown to saturate with increasing \( B \) at a value determined solely by smooth disorder. The results are relevant to experiments on transport in dilute antidot arrays, as well as in high-mobility heterostructures with background impurities or interface imperfections.

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[1] C.W.J. Beenakker and H. van Houten, Solid State Phys. 44, 1 (1991); D.K. Ferry and S.M. Goodnick, Transport in Nanostructures (Cambridge University Press, 1997).
[2] A.D. Mirlin et al., Phys. Rev. Lett. 83, 2801 (1999).
[3] M.M. Förg et al., Phys. Rev. B 56, 6823 (1997).
[4] A.D. Mirlin, D.G. Polyakov, and P.Wölfle, Phys. Rev. Lett. 80, 2429 (1998); F. Evers et al., Phys. Rev. B 60, 8951 (1999).
[5] E.M. Baskin, L.I. Magarill, M.V. Entin, Sov. Phys. JETP 48, 365 (1978); E.M. Baskin and M.V. Entin, Physica B 249-251, 805 (1998).
[6] A.V. Bobylev et al., Phys. Rev. Lett. 75, 197 (1995); J. Stat. Phys. 87, 1205 (1997).
[7] G.M. Gusev et al., J. Phys. Cond. Matter 6, 73 (1994).
[8] K. Tsukagoshi et al., Phys. Rev. B 52, 8344 (1995).
[9] G. Lüttjering, Ph. D. Thesis (Stuttgart, 1996).
[10] G. Nachtweig et al., Phys. Rev. B 55, 6731 (1997).
[11] P.T. Coleridge, Phys. Rev. B 44, 3793 (1991).
[12] T. Saku, Y. Horikoshi, and Y. Tokura, Jpn. J. Appl. Phys. 35, 34 (1996).
[13] V. Umansky, R. de Picciotto, and M. Heiblum, Appl. Phys. Lett. 71, 683 (1997).
[14] J.M.J. van Leeuwen and A. Weijland, Physica B 38, 457 (1967); A. Weijland and J.M.J. van Leeuwen, Physica 38, 35 (1968).
[15] E.H. Hauge, in Transport Phenomena, ed. by G. Kir- czenow and J. Marro (Springer, Berlin, 1974), p.337.
[16] D.G. Polyakov, F Evers, A.D. Mirlin, and P. Wölfle, in preparation.
[17] J. Smet, private communication; M. Heiblum and V. Umansky, private communication.

FIG. 2. Magnetoresistivity at fixed \( \tau_s \) and different \( \tau_L \) from numerical simulations; \( \tau_L/\tau_s = \infty \) (Lorentz gas, \( \bigtriangleup \)), 111 (\( \Box \)), 70 (\( \square \)), 37 (\( \diamond \)). Inset: \( \omega_0 \) determined from the fit to Eq. (10); the full line corresponds to the analytical result (1).

\[ \tau_0 B \rho_0 \tau_0 \]