No-signaling, intractability and entanglement

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We consider the problem of deriving the no-signaling condition from the assumption that, as seen from a complexity theoretic perspective, the universe is not an exponential place. A fact that disallows such a derivation is the existence of polynomial superluminal gates, hypothetical primitive operations that enable superluminal signaling but not the efficient solution of intractable problems. It therefore follows, if this assumption is a basic principle of physics, either that it must be supplemented with additional assumptions to prohibit such gates, or, improbably, that no-signaling is not a universal condition. Yet, a gate of this kind is possibly implicit, though not recognized as such, in a decade-old quantum optical experiment involving position-momentum entangled photons. Here we describe a feasible modified version experiment that appears to explicitly demonstrate the action of this gate. Some obvious counter-claims are shown to be invalid. We believe that the unexpected possibility of polynomial superluminal operations arises because some practically measured quantum optical quantities are not describable as standard quantum mechanical observables.

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I. INTRODUCTION

In a multipartite quantum system, any completely positive map applied locally to one part does not affect the reduced density operator of the remaining part. This fundamental no-go result, called the “no-signalling theorem” implies that quantum entanglement [1] does not enable nonlocal (“superluminal”) signaling [2] under standard operations, and is thus consistent with relativity, inspite of the counterintuitive, stronger-than-classical correlations [3] that entanglement enables. For simple systems, no-signaling follows from non-contextuality, the property that the probability assigned to projector $\Pi$, given by the Born rule, $\text{Tr}(\rho \Pi)$, where $\rho$ is the density operator, does not depend on how the orthonormal basis set is completed [4, 5]. No-signaling has also been treated as a basic postulate to derive quantum theory [6].

It is of interest to consider the question of whether/how computation theory, in particular intractability and uncomputability, matter to the foundations of (quantum) physics. Such a study, if successful, could potentially allow us to reduce the laws of physics to mathematical theorems about algorithms and thus shed new light on certain conceptual issues. For example, it could explain why stronger-than-quantum correlations that are compatible with no-signal [7] are disallowed in quantum mechanics. One strand of thought leading to the present work, earlier considered by us in Ref. [8], was the proposition that the measurement problem is a consequence of basic algorithmic limitations imposed on the computational power that can be supported by physical laws. In the present work, we would like to see whether no-signaling can also be explained in a similar way, starting from computation theoretic assumptions.

The Turing machine (TM) represents an abstraction of the principles of mechanical computation. The machine consists of a head and a tape. The head is capable of being in one of a finite number of “internal states” and can read and overwrite a symbol from a finite set, then shifting one block left or right along the tape. It contains a finite internal program that directs its operations. The central problem in computer science is the conjecture that two computational complexity classes, $P$ and $NP$, are distinct in the standard Turing model of computation. $P$ is the class of decision problems solvable in polynomial time by a (deterministic) TM. $NP$ is the class of decision problems whose solution(s) can be verified in polynomial time by a deterministic TM. $\#P$ is the class of counting problems associated with the decision problems in $NP$. The word “complete” following a class denotes a problem $X$ within the class, which is maximally hard in the sense that any other problem in the class can be solved in poly-time using an oracle giving the solutions of $X$ in a single clock cycle. For example, determining whether a Boolean formula is satisfied is $NP$-complete, and counting the number of Boolean satisfactions is $\#P$-complete. The word “hard” following a class denotes a problem not necessarily in the class, but to which all problems in the class reduce in poly-time.

$P$ is often taken to be the class of computational problems which are “efficiently solvable” (i.e., solvable in polynomial
time) or “tractable”, although there are potentially larger classes that are considered tractable such as \textbf{RP}, \textbf{BQP}, the latter being the class of decision problems efficiently solvable by a quantum computer [9]. \textbf{NP}-complete and potentially harder problems, which are not known to be efficiently solvable, are considered intractable in the Turing model. If \textbf{P} \neq \textbf{NP} and the universe is a polynomial– rather than an exponential– place, physical laws cannot be harnessed to efficiently solve intractable problems, and \textbf{NP}-complete problems will be intractable in the physical world.

That classical physics supports various implementations of the Turing machine is well known. More generally, we expect that computational models supported by a physical theory will be limited by that theory. Witten identified expectation values in a topological quantum field theory with values of the Jones polynomial that are \#P-hard [11]. There is evidence that a physical system with a non-Abelian topological term in its Lagrangian may have observables that are \textbf{NP}-hard, or even \#P-hard [12].

Other recent related works that have studied the computational power of variants of standard physical theories from a complexity or computability perspective are, respectively, Refs. [8, 13, 14, 15, 16] and Refs. [8, 13]. Ref. [15] noted that \textbf{NP}-complete problems do not seem to be tractable using resources of the physical universe, and suggested that this might embody a fundamental principle, christened the \textbf{NP}-hardness assumption (also cf. [17]). Ref. [18] studies how insights from quantum information theory could be used to constrain physical laws. We will informally refer to the proposition that the universe is a polynomial place in the computational sense (to be strengthened below) as well as the communication sense by the expression “the world is not hard enough” (WNHE) [19]. In Ref. [8], we pointed out that the assumption of WNHE (and further that of \textbf{P} \neq \textbf{NP}) can potentially give a unified explanation of (a) the observed ‘insularity-in-theoryspace’ of quantum mechanics (QM), namely that QM is \textit{exactly} unitary, linear, and requires measurements to conform to the \(|\psi|^2\) Born rule [14, 20]; (b) the classicality of the macroscopic world; (c) the lack of quantum physical mechanisms for non-signaling superquantum correlations [7].

In (a), the basic idea is that departure from one or more of these standard features of QM seems to invest quantum computers with super-Turing power to solve hard problems efficiently, thus making the universe an exponential place, contrary to assumption. The possibility (b) arises for the following reason. It is proposed that the WNHE assumption holds not only in the sense that hard problems (in the standard Turing model) are not efficiently solvable in the physical world, but in the stronger sense that any physical computation can be simulated on a probabilistic TM with at most a polynomial slowdown in the number of steps (the Strong Church-Turing thesis). Therefore, the evolution of any quantum system computing a decision problem, could asymptotically be simulated in polynomial time in the size of the problem, and thus lies in \textbf{BPP}, the class of problems that can be efficiently solved by a probabilistic TM [21].

Assuming \textbf{BPP} \neq \textbf{BQP}, this suggests that although at small scales, standard QM remains valid with characteristic BQP-like behavior, at sufficiently large scales, classical (‘\textbf{BPP}-like’) behavior should emerge, and that therefore there must be a definite scale– sometimes called the Heisenberg cut– where the superposition principle breaks down [22], so that asymptotically, quantum states are not exponentially long vectors. In Ref. [8], we speculate that this scale is related to a discretization of Hilbert space. This approach provides a possible computation theoretic resolution to the quantum measurement problem. In (c), the idea is that in a polynomial universe, we expect that phenomena in which a polynomial amount of physical bits can simulate exponentially large (classical) correlations, thereby making communication complexity trivial, would be forbidden.

In the present work, we are interested in studying whether the no-signaling theorem follows from the WNHE assumption. The article is arranged as follows. Some results concerning non-standard operations that violate no-signaling and help efficiently solve intractable problems, are surveyed in Sections \textbf{III} and \textbf{IV}, respectively. In Section \textbf{V} we introduce the concept of a polynomial superluminal gate, a hypothetical primitive operation that is inhibited by the assumption of no-signaling, but allowed if instead we only assume that intractable problems should not be efficiently solvable by physical computers. We examine the relation between the above two classes of non-standard gates. We also describe an \textit{constant} gate on a single qubit or qutrit, possibly the simplest instance of a polynomial superluminal operation. A quantum optical realization of the constant gate, and its application to an experiment involving entangled photons generated by parametric downconversion in a nonlinear crystal is presented in Section \textbf{VI}. Physicists who could not care less about computational complexity aspects could skip directly to this Section. They may be warned that the intervening sections will involve mangling QM in ways that may seem awkward, and whose consistency is, unfortunately, not obvious! On the other hand, computer scientists unfamiliar with quantum optics may skip Section \textbf{V} which is essentially covered in Section \textbf{VI} which discusses quantitative and conceptual issues surrounding the physical realization of the constant gate. Finally, we conclude with Section \textbf{VII} by surveying some implications of a possible positive outcome of the proposed experiment, and discussing how such an unexpected physical effect may fit in with the mathematical structure of known physics. We present a slightly abridged version of discussions in this work in Ref. [23].
II. SUPERLUMINAL GATES

Even minor variants of QM are known to lead to superluminal signaling. An example is a variant incorporating nonlinear observables \[24\], unless the nonlinearity is confined to sufficiently small scales \[25, 26\]. In this Section, we will review the case of violation of no-signaling due to departure from standard QM via the introduction of (a) non-complete Schrödinger evolution or measurement, (b) nonlinear evolution, (c) departure from the Born \(|\psi|^2\) rule.

In each case, we will not attempt to develop a non-standard QM in detail, but instead content ourselves with considering simple representative examples.

(a) Non-complete measurements or non-complete Schrödinger evolution. Let us consider a QM variant that allows a non-trace-preserving (and hence non-unitary) but invertible single-qubit operation of the form:

\[
G = \begin{pmatrix} 1 & 0 \\ 0 & 1 + \epsilon \end{pmatrix},
\]

where \(\epsilon > 0\) is a real number. The resultant state \(\sum_x \alpha_x |x\rangle\) must be normalized by dividing it by the normalization factor \(\sqrt{\sum_x |\alpha_x|^2}\) immediately before a measurement, making measurements nonlinear. Given the entangled state \((1/\sqrt{2})(|01\rangle + |10\rangle)\) that Alice and Bob share, to transmit a superluminal signal, Alice applies either \(G^m\) (where \(m \geq 1\) is an integer) or the identity operation \(I\) to her qubit. Bob’s particle is left, respectively, in the state \(\rho_B^{(1)} = \frac{1}{2}(|0\rangle|0\rangle + (1 + \epsilon)^2|1\rangle|1\rangle)\) or \(\rho_B^{(0)} = \frac{1}{2}(|0\rangle|0\rangle + |1\rangle|1\rangle)\), which can in principle be distinguished, the distance between the states being greater for larger \(m\) (cf. Section \[11\]), leading to a superluminal signal from Alice to Bob.

More generally, we may allow non-unitary and irreversible evolution but still conform to no-signaling, provided the corresponding set of operator(s) is complete, i.e., constitutes a partition of unity. Suppose Alice and Bob share the state \(\rho_{AB}\), and Alice evolves her part of \(\rho_{AB}\) locally through the linear operation given by the set \(\mathcal{P}\) of (Kraus) operator elements \(\{E_j \equiv e_j \otimes I_B, j = 1, 2, 3, \ldots\}\) \[11\], where \(I_B\) is the identity operator in Bob’s subspace. Bob’s reduced density operator \(\rho_B'\) conditioned on her performing the operation and after normalization is:

\[
\rho_B' = \mathcal{N}^{-1} \text{Tr}_A \left[ \sum_j E_j \rho_{AB} E_j^\dagger \right] = \mathcal{N}^{-1} \text{Tr}_A \left[ \sum_j E_j^\dagger E_j \rho_{AB} \right] = \mathcal{N} = \text{Tr}_{AB} \left[ \sum_j E_j^\dagger E_j \rho_{AB} \right],
\]

where \(\mathcal{N}\) is the normalization factor. We satisfy the no-signaling condition \(\rho_B' = \rho_B\) only if \(\rho_{AB}\) is unentangled or \(\mathcal{P}\) satisfies the completeness relation

\[
\sum_j E_j^\dagger E_j = I_A,
\]

which guarantees that the operation preserves norm \(\mathcal{N}\). Here \(I_A\) is the identity operator in Alice’s subspace. In the above, non-completeness suffices, and the nonlinearity introduced by renormalizing the wavefunction is not necessary, for the superluminality.

If the system \(A\) is subjected to unitary evolution or non-unitary evolution due to noise, or to standard projective measurements or more general measurements described by positive operator valued measures, the corresponding map satisfies Eq. \[43\], and \(\rho_B' = \rho_B\). For terminological brevity, we call a (non-standard) gate like \(G\), or a non-complete operation \(\mathcal{P}\) that enables superluminal signaling, as ‘superluminal gate’, and denote the set of all superluminal gates by ‘\(C^\infty\)’. For the purpose of this work, \(C^\infty\) is restricted to qubit or qutrit gates. Non-unitary super-quantum cloning or deleting, introduced in Ref. \[27\], which lead to superluminal signaling, are other examples of non-complete operations.

Even at the single-particle level, if the measurement is non-complete, there is a superluminal signaling due to breakdown in non-contextuality coming from the renormalization. As a simple illustration, suppose we are given two observers Alice and Bob sharing a delocalized qubit, \(\cos(\theta/2)|0\rangle + \sin(\theta/2)|1\rangle\), with eigenstate \(|1\rangle\) localized near Alice and \(|0\rangle\) near Bob. With an \(m\)-fold application of \(G\) (which can be thought of as an application of imaginary phase on Alice’s side, leading to selective augmentation of amplitude) on this state, Alice produces the (unnormalized) state \(\cos(\theta/2)|0\rangle + (1 + \epsilon)^m \sin(\theta/2)|1\rangle\), so that after renormalization, Bob’s probability of obtaining \(|0\rangle\) has changed in a context-dependent fashion from \(\cos^2(\theta/2)\) to \(\cos^2(\theta/2)(\cos^2(\theta/2) + (1 + \epsilon)^{2m} \sin^2(\theta/2))^{-1}\). By thus nonlocally controlling the probability with which Bob finds \(|0\rangle\), Alice can probabilistically signal 1 bit of information superluminally.

(b) Nonlinear evolution. As a simple illustration of a superluminal gate arising from nonlinear evolution, we consider the action of the nonlinear two-qubit ‘OR’ gate \(R\), whose action in a preferred (say, computational) basis is given by:

\[
\begin{pmatrix} |00\rangle \pm |11\rangle \\ |01\rangle \pm |10\rangle \\ |01\rangle \pm |11\rangle \end{pmatrix} \xrightarrow{R} \begin{pmatrix} |01\rangle \pm |11\rangle \\ |00\rangle \pm |10\rangle \end{pmatrix}, \quad |\alpha\beta\rangle \xrightarrow{R} |\alpha\beta\rangle.
\]
If the two qubits are entangled with other qubits, then the gate is assumed to act in each subspace labelled by states of the other qubits in the computational basis. Alice and Bob share the entangled state $|\Psi \rangle = 2^{-1/2}(|00\rangle - |11\rangle)$. To transmit a bit superluminally Alice measures her qubit in the computational basis or the diagonal basis $|\pm \rangle \equiv 2^{-1/2}(|0\rangle \pm |1\rangle)$, leaving Bob's qubit's density operator in a computational basis ensemble or a diagonal basis ensemble, which are equivalent in standard QM. However, with the nonlinear operation $R$, the two ensembles can be distinguished. Bob prepares an ancillary qubit in the state $|\gamma \rangle$, and applies a CNOT on it, with his system qubit as the control. On the resulting state he performs the nonlinear gate $R$, and measures the ancilla. The computational (resp., diagonal) basis ensemble yields the value 1 with probability $\frac{1}{2}$ (resp., 1). By a repetition of the procedure a fixed number $m$ of times, a superluminal signal is transmitted from Alice to Bob with exponentially small uncertainty in $m$.

Analogous to Eq. (4), one can define a 'nonlinear AND', which, again, similarly leads to a nonlocal signaling. Even at a single particle level, allowing for non-complete operations, superluminal effects can arise from the nonlinearity due to renormalization.

(c) **Departure from the Born $|\psi|^2$ probability rule.** Gleason’s theorem shows that the Born probability rule that identifies $|\psi|^2$ as a probability measure, and more generally, the trace rule, is the only probability prescription consistent in 3 or larger dimensions with the requirement of non-contextuality. Suppose we retain unitary evolution, which preserve the 2-norm, but assume that the probability of a measurement on the state $\sum_j |\alpha_j\rangle^p$ is of the form $|\alpha_j|^p / \sum_k |\alpha_k|^p$ for outcome $j$, and $p$ any non-negative real number. The renormalization will make the measurement contextual, giving rise to a superluminal signal. One might consider more general evolution that preserves a $p$-norm, but there are no linear operators that do so except permutation matrices.

For example, let Alice and Bob share the two-qubit entangled state $\cos \theta |00\rangle + \sin \theta |11\rangle$ $(0 < \theta < \pi/2)$. The probability for Alice measuring her particle in the computational basis and finding $|0\rangle$ (resp., $|1\rangle$) must be the same as that for a joint measurement in this basis to yield $|00\rangle$ (resp., $|11\rangle$). Therefore Bob's reduced density operator is given by the state $\rho^{(1)} = (\cos^p \theta |0\rangle \langle 0| + \sin^p \theta |1\rangle \langle 1|)/(\cos^p \theta + \sin^p \theta)$. On the other hand, if Alice employs an ancillary, third qubit prepared in the state $|0\rangle$, and applies a Hadamard on it conditioned on her qubit being in the state $|0\rangle$, she produces the state $\frac{\cos \theta}{\sqrt{2}} |000\rangle + \frac{\sin \theta}{\sqrt{2}} |001\rangle + |110\rangle$. The probability that Alice obtains outcomes 00, 01 or 10 must be that for a joint measurement to yield 000, 001 or 110. Along similar lines as in the above case we find that she leaves Bob's qubit in the state

$$\rho^{(2)} = \frac{2^{1-p/2} \cos^p \theta |0\rangle \langle 0| + \sin^p \theta |1\rangle \langle 1|}{{2^{1-p/2} \cos^p \theta + \sin^p \theta}}.$$  

Since $\rho^{(1)}$ and $\rho^{(2)}$ are probabilistically distinguishable, with sufficiently many shared copies Alice can signal Bob one bit superluminally, unless $p = 2$.

**III. EXPONENTIAL GATES**

As superluminal quantum gates like $G$ or $R$ are internally consistent, one can consider why no such operation occurs in Nature, whether a fundamental principle prevents their physical realization. One candidate principle is of course no-signaling itself. Alternatively, since we would like to derive it, linearity of QM may be taken as an axiom. Since all the above non-standard operations involve an overall nonlinear evolution, the assumption of strict quantum mechanical linearity can indeed rule out such non-standard gates. Yet it must be admitted that, from a purely physics viewpoint, assuming that QM is linear affords no greater insight than assuming it to be a non-signaling theory. We would like to suggest that the absence of such operations may have a complexity theoretic basis.

Both superluminal gates as well as hypothetical gates that allow efficient solving of intractable problems involve some sort of communication across superposition branches. In particular, the superluminal gates of Section III can be turned into the latter type of gates, as discussed below.

(a) **Non-complete quantum gates.** It is easily seen that the gate $G$ in Eq. (1) can be used to solve NP-complete problems efficiently. Consider solving boolean satisfiability (SAT), which is NP-complete: given an efficiently computable black box function $f : \{0, 1\}^n \rightarrow \{0, 1\}$, to determine if there exists $x$ such that $f(x) = 1$. With the use of an oracle that computes $f(x)$, we prepare the $(n+1)$-qubit entangled state

$$|\Psi_{nc} \rangle = 2^{-n/2} \sum_{x \in \{0,1\}^n} |x\rangle |f(x)\rangle,$$

and then apply $G^m$ to the second, 1-qubit register, where $m$ is a sufficiently large integer, before measuring the register. In particular, suppose that at most one solution exists. The un-normalized ‘probability mass’ of obtaining outcome $|1\rangle$ becomes 1 (and the normalized probability about 1/2) when $m = n/(2 \log(1 + \epsilon))$, if there is a solution,
or, if no solution exists, remains 0. Repeating the experiment a fixed number of times, and applying the Chernoff bound, we find that to solve SAT, we only require \( m \in O(n) \). For terminological brevity, we will call as ‘exponential gate’ such a non-standard gate that enables the efficient computation of NP-complete problems, and denote by \( E \) the set of all exponential gates, restricted in the present work to qubits and qutrit gates.

(b) **Nonlinear quantum gates.** The nonlinear operation \( R \) in Eq. (1) can be used to efficiently simulate non-determinism. We prepare the state \( |\psi\rangle \) in Eq. (6), where the first \( n \) qubits are called the ‘index’ qubits and the last one the ‘flag’ qubit. There are \( 2^{n-1} \) 4-dim subspaces, consisting of the first index qubit and the flag qubit, labelled by the index qubits \( 2, \cdots, n \). On each such subspace, the first index qubit and flag qubit are in one of the states \(|00\rangle + |11\rangle, |01\rangle + |10\rangle, |00\rangle + |10\rangle\). The operation Eq. (6) is applied \( n \) times, pairing each index qubit sequentially with the flag. The number of terms with 1 on the flag bit doubles with each operation so that after the \( n \) operations, it becomes disentangled and can then be read off to obtain the answer \( 10 \). A slight modification of this algorithm solves \#P-complete problems efficiently, by replacing the flag qubit with log \( n \) qubits and the 1-bit nonlinear OR operation with the corresponding nonlinear counting. The final readout is then the number of solutions to \( f(x) = 1 \) \( 10 \). Applying the nonlinear OR and AND alternatively to the state \( |\psi\rangle \) in Eq. (6) allows one to efficiently solve the quantified Boolean formula problem, which is PSPACE-complete \[30\]. Furthermore, it can be shown that a single particle quantum computer employing the nonlinear (due to renormalization) quantum mechanism mentioned above, enables efficient solution of NP-complete problems \[30\].

(c) **Non-Gleasonian gates.** By employing polynomially many ancillas in the method of (c) in the previous subsection, one can show that non-Gleasonian quantum computers (for which \( p \neq 2 \)) can solve PP-complete problems \[30\] efficiently. Defining BQP, as similar to BQP, except that the probability of measuring a basis state \( |x\rangle \) equals \( |\alpha_x|^p/\sum_y |\alpha_y|^p \) (so that BQP\(_2 = \) BQP), it can be shown that PP \( \subseteq \) BQP\(_p \) for all constants \( p \neq 2 \), and that, in particular, PP exactly characterizes the power of a quantum computer with even-valued \( p \) (except \( p = 2 \)) \[14\].

In view of the connection between the two classes of gates, we now propose, as we earlier did in Ref. \[8\], that the reason for the absence in Nature of the superluminal gates of Section II is WNHE: in a universe that is a polynomial place, exponential gates like \( G \) and \( R \) are ruled out. In the next Section we will consider in further detail the viability of the WNHE assumption as an explanation for no-signaling.

### IV. POLYNOMIAL SUPERLUMINAL GATES

Even though WNHE excludes the type of superluminal gates considered above, for the exclusion to be general, it would have to be shown that every superluminal gate is exponential, i.e., \( C^\prime \subseteq E \). It turns out that this cannot be done, because one can construct hypothetical polynomial superluminal gates, which are superluminal operations that are not exponential. In fact, it is probably true that \( E \subseteq C^\prime \). To see this, let us consider solving the NP-complete problem associated with Eq. (6) via Grover search \[31\], which is optimal for QM \[32\] but offers only a quadratic speed-up, thus leaving the complexity of the problem exponential in \( n \), at least in the black box setting. The optimality proof relies on showing that, given the problem of distinguishing an empty oracle \( \forall x, A(x) = 0 \) and a non-empty oracle containing a single random unknown string \( y \) of known length \( n \) (i.e. \( A(y) = 1 \), but \( \forall x \neq y, A(x) = 0 \), subject to the constraint that its overall evolution be unitary, and linear (so that in a computation with a nonempty oracle, all computation paths querying empty locations evolve exactly as they would for an empty oracle), the speed-up over a classical search is at best quadratic.

Any degree of amplitude amplification of the marked state above the quadratic level would then require empty superposition branches being ‘made aware’ of the presence of a non-empty branch, i.e., a nonlinearity of some sort. Let us suppose Bob can perform a trace-preserving nonlinear transformation \( \rho_j \rightarrow \tilde{\rho}_j \) of the above kind on an unknown ensemble of separable states. Further, let Alice and Bob share an entangled state, by which Alice is able to prepare, employing two different POVMs, two different but equivalent ensembles of Bob. Then, depending on Alice’s choice, his reduced density matrix evolves as \( \rho_B = \sum_j p_j \rho_j \rightarrow \sum_j p_j \rho_j = \rho' \) or \( \rho_B = \sum_k p_k \rho_k \rightarrow \sum_k p_k \rho_k = \rho'' \) where \( (\rho_j, \rho_j) \) and \( (\rho_k, \rho_k) \) are distinct, equivalent ensembles \[33\]. The assumption of linearity is sufficient to ensure that \( \rho' = \rho'' \). This is not guaranteed in the presence of nonlinearity, leading to a potential superluminal signal. In a nonlinearity of the above kind, the result would depend on whether the particular ensemble remotely prepared by Alice has states that include \(|y\rangle\) in the superposition. This would lead to a scenario similar to that encountered with nonlinear gate \( R \) in Section II.

Possibly the simplest examples of polynomial superluminal gates are the non-invertible constant gates, which map any state in an input Hilbert space to a fixed state in the output Hilbert space, and have the form \(|\xi\rangle \otimes \sum_j |j\rangle\), for
some fixed $\xi$. Examples in matrix notation are:

$$Q = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \quad Q' = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$  

(7)

acting in Hilbert space $\mathcal{H}_2 = \text{span}\{0,1\}$ and $\mathcal{H}_3 = \text{span}\{0,1,2\}$, respectively. They have the effect of mapping any input state in $\mathcal{H}_2$ to a fixed (apart from a normalization factor) state $|\xi\rangle$, in this case $|\xi\rangle$ being $|0\rangle$. In Eq. (4), we do not in general require the input and output bases to be the same, nor indeed that the input and output Hilbert subspaces be the same (for example, as with the distinct incoming and outgoing modes of a scattering problem.)

Both $Q$ and $Q'$ are non-complete, inasmuch as $Q^t Q \neq I$ and $(Q')^t Q' \neq I$, and represent superluminal gates. For example, by applying or not applying $Q$ to her register in the state $(1/\sqrt{2})(|01\rangle + |10\rangle)$ shared with Bob, Alice can remotely prepare his state to be the pure state $(1/\sqrt{2})(|0+1\rangle)$ or leave it as a maximal mixture, respectively. Similarly, by choosing to apply, or not, $Q'$ on her half of the state $(1/\sqrt{2})(|11\rangle + |22\rangle)$ shared with Bob, Alice can superluminally signal him 1 bit.

The constant gate is linear and preserves no-re-normalization following its non-complete action. The probability of the occurrence of a constant gate $C$ when it is applied to a state $|\psi\rangle$ is simply given by $||C|\psi\rangle||^2$, per the usual prescription. One consequence is that it could not be used to violate no-signaling without the use of entanglement. As an illustration: in $\mathcal{H}_3$, let the states $|0\rangle$ and $|1\rangle$ be localized near Alice and $|2\rangle$ near Bob. Applying $Q'$ on the state $|\psi\rangle = a|0\rangle + b|1\rangle + c|2\rangle$, Alice obtains the (unnormalized) state $Q'|\psi\rangle = (a+b)|0\rangle + c|2\rangle$. If renormalization were allowed, Alice could nonlocally influence Bob's probability to find $|2\rangle$ to be $|c|^2/(|a+b|^2 + |c|^2)$ or $|c|^2$. However, the linearity of the constant gate requires the interpretation that following her action, Alice can detect the particle with probability $|a+b|^2$, while for Bob, the probability remains $|c|^2$. As clarified later, lack of probability conservation can be interpreted as coherent enhancement or suppression of emission of particles from a source to a detector.

On the other hand, neither $Q$ nor $Q'$ nor a general constant gate is an exponential gate: each of them simply transforms any valid input into a fixed output. Intuitively, this lack of any dependence on the input clearly limits its computational power. The family of constant gates like $Q$ and $Q'$ is simply equivalent to a non-deterministic simulation of a constant function, and can be simulated by the following classical Turing machine pseudocode in polynomial time (in fact, $O(1)$ time): “Read first bit of $x$; if $x \neq \text{NULL}$, output 0; else output NULL”.

Operations $Q$ and $Q'$ in Eq. (7) can be extended to a more general class of polynomial superluminal qubit and qutrit gates

$$Q_2(\phi) = \begin{pmatrix} 1 & e^{i\phi} \\ 0 & 0 \end{pmatrix}, \quad Q_3(\phi_1, \phi_2) = \begin{pmatrix} 1 & e^{i\phi_1} & e^{i\phi_2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$  

(8)

By definition, $Q = Q_2(0)$ and $Q' = Q_3(0,0)$. To see that $Q_2(\phi)$ is a polynomial operation, it suffices to show that it can be simulated using only polynomial amount of standard quantum mechanical resources. Given an arbitrary $(n+1)$-qubit state $|\psi\rangle = |\alpha\rangle|0\rangle + |\beta\rangle|1\rangle$, where $|\alpha\rangle$ and $|\beta\rangle$ are not necessarily mutually orthogonal nor normalized, one first applies a phase gate $\begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$, followed by a Hadamard on the $n$th qubit, followed by a measurement conditioned on the outcome being $|0\rangle$, which happens with probability $(||\alpha||^2 + ||\beta||^2 + |\langle\alpha |\beta\rangle|^2)/2$, irrespective of $n$. If $|\alpha\rangle$ and $|\beta\rangle$ are orthogonal, the simulation succeeds with fixed probability $1/4$. Therefore, the class of problems efficiently solvable using quantum computation equipped with the non-standard family of constant gates is in $\text{BQP}$.

In point of fact, one could be worse off applying a constant gate than not applying it. In Eq. (8), let $|\Psi_{nc}^\phi\rangle$ represent the state derived for the function $f^1(\cdot)$, where $f^1(j) = 1$ for precisely one $j$, and let $|\Psi_{nc}^h\rangle$ represent the state derived for the function $f^0(\cdot)$, where $\forall j, f(j) = 0$. In both cases, upon applying $I \otimes Q$, we obtain the same disentangled state, $2^{-(n/2)}(\sum_j |j\rangle|0\rangle)$. The application of $Q$ causes the distance and hence distinguishability between the two states to diminish, or equivalently, the fidelity between them to increase: $1 = \langle |\Psi_{nc}^h\rangle \otimes Q^t |(\Pi \otimes Q)|\Psi_{nc}^0\rangle\rangle > \langle |\Psi_{nc}^1\rangle \otimes Q^t |\Psi_{nc}^0\rangle\rangle = 1 - O(2^{-n})$.

It is worth noting that the constant gate is quite different from the following two operations that appear to be similar, but are in fact quite distinct. The first operation is a standard quantum mechanical completely positive map, polynomial and not superluminal; the second is exponential and consequently superluminal.

(a) To begin with, a constant gate is not a quantum deleter [23], in which a qubit is subjected to a complete operation, in specific, a contractive completely positive map that prepares it asymptotically in a fixed state $|0\rangle$. The action of a quantum deleter is given by an amplitude damping channel $I[24]$, which has an operator sum representation, respectively

$$\rho_2 \longrightarrow \sum_j E_j \rho_2 E_j^\dagger; \quad \rho_3 \longrightarrow \sum_j E_j \rho_3 E_j^\dagger,$$  

(9)
in the qubit case or when extended to the qutrit case, with the Kraus operators given by Eq. (10a) or (10b), respectively

\[
E_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad E_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix},
\]

\[
E'_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad E'_2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad E'_3 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}.
\]

(10a) (10b)

Unlike in the case of $Q$, $Q'$ or $Q''$, there is no actual destruction of quantum information, but its transfer through dissipative decoherence into correlations with a zero-temperature environment. The reduced density operator of Bob’s entangled system remains unaffected by Alice’s application of this operation on her system. The deleting action, though nonlinear at the state vector level, nevertheless acts linearly on the density operator.

(b) Next we note that the constant gate is quite different from the ‘post-selection’ operation, which is a deterministic rank-1 projection [14]. Verbally, if the constant gate corresponds to the operation “for all input states $|j\rangle$ in the computational basis, set the output state to $|\xi\rangle$, independently of $j$, except for a global phase”, where $|\xi\rangle$ is some fixed state, then post-selection corresponds to the action “for all input states $|j\rangle$, if $j \neq \xi$, then discard branch $|j\rangle$”. Post-selective equivalents of $Q$ and $Q'$ are

\[
Q_{PS} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \quad Q'_{PS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.
\]

followed by renormalization. In particular, whereas the action of $Q$ on the first of two particles in the state $(1/\sqrt{2})(|00\rangle + |11\rangle)$ leaves the second particle in the state $(1/\sqrt{2})(|0\rangle + |1\rangle)$, that of $Q_{PS}$ leaves the second particle in the state $|0\rangle$. It is straightforward to see that post-selection is an exponential operation: acting it on the second qubit of $|\Psi_{nc}\rangle$ in Eq. (8), and post-selecting on 1, we obtain the solution to SAT in one time-step.

The seemingly immediate conclusion due to the fact $C^\circ \not\subseteq \mathcal{E}$ is that the WNHE assumption is not strong enough to derive no-signaling, and would have to be supplemented with additional assumption(s), possibly purely physically motivated ones, prohibiting the physical realization of polynomial superluminal gates.

An alternative, highly unconventional reading of the situation is that WNHE is a fundamental principle of the physical world, while the no-signaling condition is in fact not universal, so that some polynomial superluminal gates may actually be physically realizable. Quite surprisingly, we may be able to offer some support for this viewpoint.

We believe that constant gates of above type can be quantum optically realized when a photon detection is made at a path singularity, defined as a point in space where two or more incoming paths converge and terminate. In graph theoretic parlance, a path singularity is a terminal node in a directed graph, of degree greater than 1.

We describe in Section V an experiment that possibly physically realizes $Q$. In principle, a detector placed at the focus of a convex lens realizes such a path singularity. This is because the geometry of the ray optics associated with the lens requires rays parallel to the lens axis to converge to the focus after refraction, while the destructive nature of photon detection implies the termination of the path. As another example, consider a Mach-Zehnder interferometer where the second beam-splitter is replaced by a detector: the two converging arms are then brought into overlap and detected in the overlap region, without being sent through, as would be the case in a conventional Mach-Zehnder set-up. We find that although conceptually and experimentally simple, the high degree of mode filtering or spatial resolution that these experiments require will be the main challenge in implementing them. Indeed, we believe this is the reason that such gates have remained undiscovered so far.

Our argument here has implicitly assumed that $P \neq NP$. If it turns out that $P = NP$, then even the obviously non-physical operations such as $G$ or $R$ would be polynomial gates, and the WNHE assumption would not be able to exclude them. Nevertheless, the question of existence and testability of certain superluminal gates, which is the main result of this work, would still remain valid and of interest. If polynomial superluminal gates are indeed found to exist (and given that other superluminal gates do not seem to exist anyway), this would give us greater confidence that $P \neq NP$ (or, to be safe, that even Nature does not ‘know’ that $P = NP$!) and that the assumption of WNHE is indeed a valid and fruitful one.

V. AN EXPERIMENT WITH ENTANGLPED PAIRS OF PHOTONS

Our proposed implementation of $Q'$, based on the use of entanglement, is broadly related to the type of quantum optical experiments encountered in Refs. [6], and closely related to an experiment performed in Innsbruck that...
FIG. 1: An ‘unfolded’ version of the Innsbruck experiment (not to scale). A pair of momentum-entangled photons is created by type-I parametric down conversion of the pump laser. Alice’s photon (the signal photon) is registered by a detector behind the Heisenberg lens. Bob’s photon (the idler) is detected behind a double-slit assembly. If the ‘Heisenberg detector’ is placed in the focal plane of the lens (of focal length $F$), it projects Bob’s state into a mixture of plane waves, which produce an interference pattern on Bob’s screen in coincidence with any fixed detection point on Alice’s focal plane. Bob’s pattern in his singles count, being the integration of such patterns over all focal plane points, shows no interference pattern. On the other hand, positioning the Heisenberg detector in the imaging plane can potentially reveal the path the idler takes through the slit assembly, and thus does not lead to an interference pattern on Bob’s screen even in the coincidence counts.

elegantly illustrates wave-particle duality by means of entangled light \[37, 38\]. In the Innsbruck experiment, pairs of position-momentum entangled photons are produced by means of type-I spontaneous parametric down-conversion (SPDC) at a nonlinear source, such as a BBO crystal. The two outgoing conical beams from the nonlinear source are presented ‘unfolded’ in Figure 1. One of each pair, called the ‘signal photon’, is received by Alice, while the other, called the ‘idler’, is received and analyzed by Bob. Alice’s photon is registered by a detector behind a ‘Heisenberg lens’.

Bob’s photon is detected after it enters a double-slit assembly. If Alice’s detector, which is located behind the lens, is positioned at the focal plane of the lens and detects a photon, it localizes Alice’s photon to a point on the focal plane. By virtue of entanglement, this projects the state of Bob’s photon to a ‘momentum eigenstate’, a plane wave propagating in a particular direction. For example, if Alice detects her photon at $f$, $f'$ or $f''$, Bob’s photon is projected to a superposition of the parallel modes 2 and 5, modes 1 and 4, or modes 3 and 6. Since this cannot reveal positional information about whether the particle originated at $p$ or $q$, and hence reveals no which-way information about slit passage, therefore, in coincidence with a registration of her photon at a focal plane point, the idler exhibits a Young’s double-slit interference pattern \[37, 38\]. The patterns corresponding to Alice’s registering her photon at $f$, $f'$ or $f''$ will be mutually shifted. Bob’s observation in his single counts will therefore not show any sign of interference, being the average of all possible such mutually shifted patterns. The interference pattern is seen by Bob in coincidence with Alice’s detection, and cannot be seen by him unilaterally. This is of course expected on account of no-signaling.

If the Heisenberg detector is placed at the imaging plane (at distance $2F$ from the plane of the lens), a click of the detector can reveal the path the idler takes from the crystal through the slit assembly which therefore cannot show the interference pattern even in the coincidence counts. For example, if Alice detects her photon at $l$ (resp., $m$), Bob’s photon is projected to a superposition of the mutually non-parallel modes 4, 5 and 6 (resp., 1, 2 and 3) and, because the double-slit assembly is situated in the near field, can then enter only slit $y$ (resp., $x$). Therefore, Alice’s imaging plane measurement gives path or position information of the idler photon, so that no interference pattern emerges in Bob’s coincidence counts \[37, 38\], and consequently also in his singles counts. This qualitative description of the Innsbruck experiment is made quantitative using a simple six-mode model in the next Subsection.
A. Quantum optical description of the Innsbruck experiment

Here we give a simple, quantitative exposition of the experiment. The state of the SPDC field of Figure[2] is modeled by a 6-mode vector:

$$|\Psi\rangle = (1 + \epsilon \sqrt{6} \sum_{j=1}^{6} a_j^\dagger b_j^\dagger)|\text{vac}\rangle$$  \hspace{1cm} (12)

where $|\text{vac}\rangle$ is the vacuum state, $a_j^\dagger$ (resp., $b_j^\dagger$) are the creation operators for Alice’s (resp., Bob’s) light field on mode $j$, per the mode numbering scheme in Figure[2]. The quantity $\epsilon (\ll 1)$ depends on the pump field strength and the crystal nonlinearity. The coincidence counting rate for corresponding measurements by Alice and Bob is proportional to the square of the second-order correlation function, and given by:

$$R_{\alpha}(z) \propto \langle \Psi | E_{z}^{(-)} E_{z}^{(-)} E_{z}^{(+)} E_{z}^{(+)} |\Psi\rangle = \|E_{z}^{(+)} E_{z}^{(+)} |\Psi\rangle\|^2, \hspace{1cm} (\alpha = f, f'', l, m, \cdots).$$  \hspace{1cm} (13)

where $E_{z}^{(+)}$ represents the positive frequency part of the electric field at a point on Alice’s focal or imaging plane, and $E_{z}^{(\pm)}$ that of the electric field at an arbitrary point $z$ on Bob’s screen. We have:

$$E_{z}^{(+)} = e^{ik_p D} (e^{ik_{r2} \hat{b}_2} + e^{ik_{r3} \hat{b}_3}) + e^{ik_{r}\nu} (e^{ik_{r1} \hat{b}_1} + e^{ik_{r} \hat{b}_4}) + e^{ik_{r}\nu} (e^{ik_{r3} \hat{b}_3} + e^{ik_{r} \hat{b}_6}),$$  \hspace{1cm} (14)

where $k$ is the wavenumber, $r_p$ the distance from the EPR source to the upper/lower slit on Bob’s double slit diaphragm (the length of the segment $\overline{pq}$ or $\overline{pr}$); $r_2$ (resp., $r_3$) is the distance from the lower (resp., upper) slit to $z$. The other two terms in Eq. (14), pertaining to the other two pair of modes, are obtained analogously. We study the two cases, corresponding to Alice making a remote position or remote momentum measurement on the idler photons.

Case 1. Alice remotely measures position (path) of the idler. Suppose Alice positions her detector at the imaging plane and detects a photon at $l$ or $m$. The corresponding field at her detector is

$$E_{m}^{(+)} = e^{ik_{lm}} (\hat{a}_1 + \hat{a}_2 + \hat{a}_3); \hspace{1cm} E_{l}^{(+)} = e^{ik_{lm}} (\hat{a}_4 + \hat{a}_5 + \hat{a}_6),$$  \hspace{1cm} (15)

where $s_m$ (resp., $s_l$) is the path length along any ray path from the source point $p$ (resp., $q$) through the lens up to image point $m$ (resp., $l$). By Fermat’s principle, all paths connecting a given pair of source and image point are equal. Setting $\alpha = l, m$ in Eq. (13), and substituting Eqs. (12), (14) and (15) in Eq. (13), we find the coincidence counting rate for detections by Alice and Bob to be

$$R_{m}(z) \propto \epsilon^2 |e^{ik_{r1}} + e^{ik_{r2}} + e^{ik_{r3}}|^2; \hspace{1cm} R_{l}(z) \propto \epsilon^2 |e^{ik_{r1}} + e^{ik_{r3}} + e^{ik_{rl}}|^2,$$  \hspace{1cm} (16)

which is essentially a single slit diffraction pattern formed behind, respectively, the upper and lower slit. The intensity pattern Bob finds on his screen in the singles count, obtained by averaging $R_{\alpha}(z)$ over $\alpha = l, m$, is thus not a double-slit interference pattern, but an incoherent mixture of the two single slit patterns. A similar lack of interference pattern is obtained by Bob if Alice makes no measurement.

Case 2. Alice remotely measures momentum (direction) of the idler. Alice positions her detector on the focal plane of the Heisenberg lens. If she detects a photon at $f$, $f'$ or $f''$, the field at her detector is, respectively,

$$E_{f}^{(+)} = e^{ik_{r2} s_f} \hat{a}_2 + e^{ik_{r3} s_f} \hat{a}_5 = e^{ik_{r2} f} (\hat{a}_2 + \hat{a}_5),$$  \hspace{1cm} (17a)

$$E_{f'}^{(+)} = e^{ik_{r1}' s_f} \hat{a}_1 + e^{ik_{r3}' s_f} \hat{a}_4 = e^{ik_{r1}' f'} (\hat{a}_1 + e^{ik_{r3}' - r_{f'} f_2} \hat{a}_4),$$  \hspace{1cm} (17b)

$$E_{f''}^{(+)} = e^{ik_{r3} s_2} \hat{a}_3 + e^{ik_{r2} s_2} \hat{a}_6 = e^{ik_{r3} f''} (\hat{a}_3 + e^{ik_{r3}' - r_{f''} f_2} \hat{a}_6),$$  \hspace{1cm} (17c)

where $r_{2f}$ (resp., $r_{f_2}$) is the distance from $p$ (resp., $q$) along the path 2 (resp., 5) path through the lens up to point $f$. The distances along the two paths being identical, $r_{2f} = r_{f_2} \equiv r_f$. The distances $r_{1f}$, $r_{4f}$, $r_{3f''}$ and $r_{6f''}$ are defined analogously. Substituting Eqs. (12), (14) and (17) in Eq. (13), we find the coincidence counting rate is given by

$$R_{f}(z) \propto \epsilon^2 \left[ 1 + \cos(k \cdot [r_2 - r_{5f}]) \right]; \hspace{1cm} (18a)$$

$$R_{f'}(z) \propto \epsilon^2 \left[ 1 + \cos(k \cdot [r_1 - r_{4f} + \omega_{14}]) \right]; \hspace{1cm} (18b)$$

$$R_{f''}(z) \propto \epsilon^2 \left[ 1 + \cos(k \cdot [r_3 - r_{6f''} + \omega_{36}]) \right], \hspace{1cm} (18c)$$

where $\omega_{14} \equiv k(r_{4f} - r_{1f})$ and $\omega_{36} \equiv k(r_{6f''} - r_{3f})$ are fixed for a given point on the focal plane. Each equation in Eq. (18) represents a conventional Young’s double slit pattern. Conditioned on Alice detecting photons at $f$, Bob finds
the pattern $R_f(z)$, and similarly for points $f'$ and $f''$. In his singles count, Bob perceives no interference, because he is left with a statistical mixture of the patterns \(18a\), \(18b\), \(18c\), etc., corresponding to all points on Alice’s focal plane illuminated by the signal beam.

In summary, the set-up of the Innsbruck experiment entails that Bob does not find a double-slit interference pattern in his singles count no matter what Alice does. However, in the coincidence counts he finds the interference pattern if Alice measures (momentum) in the focal plane, and none if she measures (position) in her imaging plane.

### B. The proposed experiment

The experiment proposed here, presented earlier by us in Ref. [39], is derived from the Innsbruck experiment, and therefore called ‘the Modified Innsbruck experiment’. It was claimed to manifest superluminal signaling, though it was not clear what the exact origin of the signaling was, and in particular, which assumption that goes to proving the no-signaling theorem was being given up. The Modified Innsbruck experiment is revisited here in order to clarify this issue in detail in the light of the discussions of the previous Sections. This will help crystallize what is, and what is not, responsible for the claimed signaling effect. In Ref. [40], we studied a version of nonlocal communication inspired by the original Einstein-Podolsky-Rosen thought experiment [1]. Recently, similar experiments, also based on the Innsbruck experiment, have been independently proposed in Refs. [41, 42].

First we present a qualitative overview of the modified Innsbruck experiment. The only material difference between the original Innsbruck experiment and the modified version we propose here is that the latter contains a ‘direction filter’, consisting of two convex lenses of the same focal length $G$, separated by distance $2G$. Their shared focal plane is covered by an opaque screen, with a small aperture $o$ of diameter $\delta$ at their shared focus. We want $\delta$ to be small enough so that only almost horizontal modes are permitted by the filter to fall on the double slit diaphragm. The angular spread (about the horizontal) of the modes that fall on the aperture is given by $\Delta \theta = \delta / G$, we require that $(\delta / G) \sigma \ll \lambda$, where $\sigma$ is the slit separation, to guarantee that only modes that are horizontal or almost horizontal are selected to pass through the direction filter, to produce a Young’s double-slit interference pattern on his screen plane. On the other hand, we don’t want the aperture to be so small that it produces significant diffraction, thus: $\delta \gg \lambda$. Putting these conditions together, we must have

$$1 \ll \frac{\delta}{\lambda} \ll \frac{G}{\sigma} \quad (19)$$

The ability to satisfy this condition, while preferable, is not crucial. If it is not satisfied strictly, the predicted signal
is weaker but not entirely suppressed. The point is clarified further down.

If Alice makes no measurement, the idler remains entangled with the signal photon, which renders incoherent the beams coming through the upper and lower slits on Bob’s side, so that he will find no interference pattern on his screen. Similarly, if she detects her photon in the imaging plane, she localizes Bob’s photon at his slit plane, and so, again, no interference pattern is seen. Thus far, the proposed experiment the same effect as the Innsbruck experiment.

On the other hand, if Alice scans the focal plane and makes a detection, she remotely measures Bob’s corresponding photon’s momentum and erases its path information, thereby (non-selectively) leaving it as a mixture of plane waves incident on the direction filter. However only the fraction that makes up the pure state comprising the horizontal modes passes through the filter. Diffracting through the double-slit diaphragm, it produces a Young’s double slit interference pattern on Bob’s screen. Those plane waves coincident with Alice’s detecting her photon away from focus \( f \) are filtered out and do not reach Bob’s double slit assembly. It follows that an interference pattern will emerge in Bob’s singles counts, coinciding with Alice’s detection at \( f \) or close to \( f \). Thus Alice can remotely prepare inequivalent ensembles of idlers, depending on whether or not she measures momentum on her photon. In principle, this constitutes a superluminal signal.

Quantitatively, the only difference between the Innsbruck and the proposed experiment is that Eq. (14) is replaced by an expression containing only horizontal modes. As an idealization (to be relaxed below), assuming perfect filtering and low spreading of the wavepacket at the aperture, we have:

\[
E_z^{(+)} = e^{ikr_D} \left( e^{ikr_2} b_2 + e^{ikr_5} b_5 \right),
\]

(20)

where \( r_D \) now represents the distance from the EPR source to the upper/lower slit on Bob’s double slit diaphragm (the length of the segment \( \overline{ov_{\text{up}}/ov_{\text{lo}}} \)); \( r_2 \) (resp., \( r_5 \)) is the distance from the upper (resp., lower) slit to \( z \). The other two Detection of a signal photon at or near \( f \) is the only possible event on the focal plane such that Bob detects the twin photon at all. Focal plane detections sufficiently distant from \( f \) will project the idler into non-horizontal modes that will be filtered out before reaching Bob’s double-slit assembly. Therefore, the interference pattern Eq. (18a) is in fact the only one seen in Bob’s singles counts. We denote by \( R^F(z) \), this pattern, which Bob obtains conditioned on Alice measuring in the focal plane. By contrast, in the Innsbruck experiment Bob in his singles counts sees a statistical mixture of the patterns (18a), (18b), (18c), etc., corresponding to all points on Alice’s focal plane illuminated by the signal beam.

When Alice measures in the imaging plane, as in the Innsbruck experiment Bob finds no interference pattern in his singles counts. Setting \( \alpha = l, m \) in Eq. (13), and substituting Eqs. (12), (20) and (15) in Eq. (13), we find the coincidence counting rate for detections by Alice and Bob to be

\[
R_\alpha(z) \propto e^2, \quad (\alpha = l, m),
\]

(21)

which is a uniform pattern (apart from an envelope due to single slit diffraction, which we ignore for the sake of simplicity). It follows that Bob’s observed pattern in the singles counts conditioned on Alice measuring in the imaging plane, \( R^I(z) \), is also the same, i.e., \( R^I(z) \propto e^2 \).

Our main result is the difference between the patterns \( R^I(z) \) and \( R^F(z) \), which implies that Alice can signal Bob one bit of information across the spacelike interval connecting their measurement events, by choosing to measure her photon in the focal plane or not to measure. In practice, Bob would need to include additional detectors to sample or scan the \( z \)-plane fast enough. This procedure can potentially form the basis for a superluminal quantum telegraph, bringing into sharp focus the tension between quantum nonlocality and special relativity.

Considering the far-reaching implications of a positive result to the experiment, we may pause to consider whether our analysis of so far can be correct, and – in the chance (however limited) that it is – how such a signal may ever arise, in view of the no-signaling theorem. It may be easy to dismiss a proof of putative superluminal communication as ‘not even wrong’, yet less easy to spot where the purported proof fails and to provide a mechanism for thwarting the signaling. For one, the prediction of the nonlocal signaling is based on a model that departs only slightly from our quantum optical model of Section VA which explains the original Innsbruck experiment quite well. There have been various attempts at proving that quantum nonlocality somehow contravenes special relativity. The author has read some of their accounts, and it was not difficult to spot a hidden erroneous assumption that led to the alleged conflict with relativity. Armed with this lesson, the present claim will be different in the following three ways:

- We individually discuss, in the following Section, various possible objections to our claim, and demonstrate why each of them fails. By ruling out all the obvious mechanisms for thwarting the signaling, we are led to believe either (a) that there are erroneous but less obvious assumptions that have somehow gone into arriving at the superluminal signaling (more likely), or (b) that there is new physics, associated with the signaling (less likely).

Either way, it is in the spirit of science that we must now rely on experiments to be the final arbiter on the question. If item (a) turns out to be the right scenario eventually, our present exercise could still be instructive.
in yielding new theoretical insights. For example, a proposal for superluminal communication based on light amplification was eventually understood to fail because it violates the no-cloning theorem, a principle that had not been discovered at the time of the proposal was made (cf. [14]).

- **We single out, in the following Section, the key assumption responsible for the superluminality.** This is shown to be Alice’s momentum measurement, which implements a non-complete measurement of the polynomial superluminal type. This makes clear exactly what is the non-standard element at stake, and further makes it easier for the reader to judge whether the proposal is wrong, not even wrong, or— as we believe is the case— worth testing experimentally.

- **We have furnished computation- and information-theoretic grounds for why superluminal gates could be possible.** We have shown how no-signaling could be a nearly-universal-but-not-quite side effect of the computation theoretic properties of physical reality; elsewhere [45], we show how the relativity principle could be a consequence of conservation of information. These ideas suggest that no-signaling is not an exact or fundamental law, but an indication of a deeper computational and informational layer underlying physical reality.

It would no doubt be surprising if such non-complete measurements, which have no place in standard quantum mechanics, turn out to exist. In the last Section, we clarify how they could possibly fit in with known physics. There we will argue that they arise owing to the potential fact that practically measurable quantities resulting from quantum field theory are not described by hermitian operators, at variance with a key axiom of orthodox quantum theory [43].

### VI. THE QUESTION OF EXISTENCE AND ORIGIN OF THE SIGNALING

In the Section, we will consider a number of possible objections to our main result. It might at first be supposed that as the only difference between our set-up in Figure 2 and the Innsbruck experiment (Figure 4), the direction filter must be responsible for the signaling, and that therefore, there must be some unphysical assumption in the way the filter is described to work. For example, it might be supposed that in a legitimate filter, the spreading caused by the aperture would wash out any information about Alice’s choice. Yet, in the case of each objection, we will quantitatively demonstrate why there arises no physical mechanism to thwart the nonlocal signaling, and thus the objection fails. For instance, contrary to the above example claim, we will find that the mode-selection at the filter can be described as a local linear unitary (and hence complete) operation acting on the idler, and thus should not lead to any violation of no-signaling. The signaling arises from some action of Alice, which we identify with her ‘momentum’ measurement, and which we show to realize a noncomplete operation in the subspace of interest. It turns out that the filter only serves the practical purpose of exposing the signaling that would otherwise remain hidden in the averaged pattern that Bob receives. These points are discussed in the following Subsections.

#### A. Effect of spreading at the direction filter

In an actual experiment, the conditions [14] may not hold strictly, with narrow filtering leading to a diffractive spreading of Bob’s photon. It might appear that because the direction filter localizes the photon in momentum space, it would cause a complementary positional spread of the wavefunction, as a result of which Bob should observe a fixed interference pattern always, no matter what Alice does (or does not). However, a closer examination shows that such a spreading only causes a reduction in the visibility— and not a total washout— of the pattern received by Bob in the case of Alice’s focal plane measurement. Thus, the spreading only lowers— but does not eliminate— the distinguishability between the two kinds of pattern that Bob receives. A simple, quantitative explanation of this situation is discussed in the remaining part of this Subsection.

For illustration, suppose we choose $\delta = 10\lambda$, and as a result, nearly only horizontal modes $r_2$ and $r_5$ are selected, but the diffraction is strong. We model this diffraction as a unitary rotation \( \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \) in the space acted on by $\hat{b}_2$ and $\hat{b}_5$, where $\theta$ is determined by the geometry of the filter. In place of Eq. (14) we now have:

\[
E_z^{(+)} = e^{ikrD} \left( e^{ikr_2 (\cos \theta \hat{b}_2 + \sin \theta \hat{b}_5)} + e^{ikr_5 (\cos \theta \hat{b}_5 - \sin \theta \hat{b}_2)} \right). \tag{22}
\]

In case of Alice’s position measurement, we now have in place of Eq. (16)

\[
R_\alpha''(z) \propto \epsilon^2 \left[ 1 \pm \sin(2\theta) \cos(k \cdot [r_2 - r_5]) \right], \quad \text{(with $\pm$ according as $\alpha = l, m$),} \tag{23}
\]
where the interference has arisen because the diffraction at \( o \) has given rise to an amplitude input to both slits. The pattern found by Bob in his singles counts is

\[
R'_p(z) + R'_m(z) \propto \epsilon^2, \tag{24}
\]

which is a constant pattern (ignoring the finite width of the slits), just as when the spreading had been ignored (Eq. 21). On the other hand, in place of Eq. (18), we now obtain

\[
R'_f(z) \propto \epsilon^2 [1 + \cos(2\theta) \cos(k \cdot [r_2 - r_5])], \tag{25}
\]

We recover the case of clearest distinction by setting \( \theta = 0 \) (which corresponds to the zero diffraction limit), but even otherwise, the two cases (24) and (25) are in principle distinguishable in terms of visibility (except in the case \( \theta = \pi/4 \), which is highly unlikely, and in any case, can be precluded by altering \( \delta \) or \( G \)).

### B. Alice’s focal plane measurement implements a constant gate

The state (12) is now represented in a simple way as the unnormalized state

\[
|\Psi^{(1)}\rangle = \frac{\epsilon}{\sqrt{6}} \sum_{j=1}^{6} |j,j\rangle, \tag{26}
\]

where for simplicity the vacuum state, which does not contribute to the entanglement related effects, is omitted, and it is assumed that each mode contains at most one pair of entangled photons (i.e., no higher excitations of the light field). Further because of the direction filter, it suffices to restrict our attention to the state

\[
|\psi^{(2)}\rangle \propto \frac{1}{\sqrt{2}} (|2,2\rangle + |5,5\rangle), \tag{27}
\]

the projection of \( |\Psi^{(1)}\rangle \) onto \( \mathcal{H}_2 \otimes \mathcal{H}_2 \), where \( \mathcal{H}_2 \) is the subspace spanned by \( \{|2\rangle, |5\rangle\} \). Under these assumptions, Alice’s position measurement in this subspace, represented by the operators \( \hat{a}_2 \) and \( \hat{a}_5 \), can be written as the Kraus operators \( \hat{a}_2 \equiv |0\rangle\langle 2| \) and \( \hat{a}_5 \equiv |0\rangle\langle 5| \). Within \( \mathcal{H}_2 \) these operators form a complete set since \( \hat{a}_2^\dagger \hat{a}_2 + \hat{a}_5^\dagger \hat{a}_5 = |2\rangle\langle 2| + |5\rangle\langle 5| = \mathbb{I}_2 \).

Thus, Alice’s measurement on \( |\Psi^{(2)}\rangle \) in the position basis does not nonlocally affect Bob’s reduced density operator in this subspace, which is proportional to \( \mathbb{I}_2/2 \).

On the other hand, if Alice measures momentum, her measurement is represented by the field operator \( \hat{E}_f^{(+)} \) in Eq. 17. We have in the above notation

\[
\hat{E}_f^{(+)} \propto \hat{a}_2 + \hat{a}_5 \equiv |0\rangle\langle 2| + |5\rangle\langle 5|. \tag{28}
\]

This is just the polynomial superluminal gate \( Q \) in Eq. 10, with the output basis given by \( \{|0\rangle, |0^\perp\rangle\} \), where \( |0^\perp\rangle \) is any basis element orthogonal to the vacuum state.

By contrast, Bob’s measurement, which involves no focussing, is complete (which rules out a Bob-to-Alice superluminal signaling). Each element of Bob’s screen \( z \)-basis is a possible outcome, described by the annihilation operator approximately of the form \( \hat{E}_f^{(-)} \propto \hat{a}_2 + e^{i\gamma\hat{a}_5} \), where \( \gamma = \gamma(k,z) \) is the phase difference between the paths 2 and 5 from the slits to a point \( z \) on Bob’s screen. This represents a POVM of the form \( \hat{E}_f^{(-)} \hat{E}_f^{(+)} = (|2\rangle + e^{-i\gamma}|5\rangle)(|2\rangle + e^{i\gamma}\langle 5|) \).

Even though \( \hat{E}_f^{(+)} \) has the same form as Alice’s operator \( \hat{E}_f^{(+)^\dagger} \) as a Kraus operator describing the absorption of two interfering modes at a point \( z \), yet, when integrated over his whole ‘position basis’, Bob’s measurement is seen to form a complete set, for, as it can be shown, \( \int_{-\infty}^{+\infty} \hat{E}_f^{(-)} \hat{E}_f^{(+)} dz = |2\rangle\langle 2| + |5\rangle\langle 5| \). In the case of Alice’s momentum measurement, because the detection happens at a path singularity, a similar elimination of cross-terms via integration is not possible, whence the non-completeness. It is indeed somewhat intriguing how geometry plays a fundamental role in determining the completeness status of a measurement. This has to do with the fact that the direct detection of a photon is practically a determination of position distribution. For example, even in remotely measuring the idler’s momentum, Alice measures her photon’s position at the focal plane. We will return again to this issue in the final Section.
C. Role of the direction filter

A simple model of the action of the perfect direction filter is

\[
D \equiv \sum_{j=2,5} |j\rangle\langle j| + \sum_{j\neq 2,5} |−j\rangle\langle j|
\]

acting locally on the second register of the state of Eq. (20). Here \(|−j\rangle\) can be thought of as a state orthogonal to all \(|j\rangle\)’s and other \(|−j\rangle\)’s, that removes the photon from the experiment, for example, by reflecting it out or by absorption at the filter. It suffices for our purpose to note that \(D\) can be described as a local, standard (linear, unitary and hence complete) operation. Since the structure of QM guarantees that such an operation cannot lead to nonlocal signaling, the conclusion is that the superluminal signal, if it exists, must remain even if the direction filter is absent.

We will employ the notation \(|j + k + m\rangle \equiv (1/\sqrt{3})(|j\rangle + |k\rangle + |m\rangle)\). To see that the nonlocal signaling is implicit in the state modified by Alice’s actions even without the application of the filter, we note the following: if Alice measures ‘momentum’ on the state \(|\psi\rangle\) and detects a signal photon at \(f\), she projects the corresponding idler into the state \(|2 + 5\rangle\). Similarly, her detection of a photon at \(f'\) projects the idler into the state \(|3 + 6\rangle\), and her detection at \(f''\) projects the idler into the state \(|1 + 4\rangle\). Therefore, in the absence of the direction filter, Alice’s remote measurement of the idler’s momentum leaves the idler in a (assumed uniform for simplicity) mixture given by

\[
\rho_P \propto |2 + 5\rangle\langle 2 + 5| + |1 + 4\rangle\langle 1 + 4| + |3 + 6\rangle\langle 3 + 6|.
\]

Her momentum measurement is non-complete, since the summation over the corresponding projectors (r.h.s of Eq. (30)) is not the identity operation \(I_6\) pertaining to the Hilbert space spanned by six modes \(|j\rangle\) (\(j = 1, \ldots , 6\)).

On the other hand, if Alice remotely measures the idler’s position, she leaves the idler in the mixture

\[
\rho_Q \propto |1 + 2 + 3\rangle\langle 1 + 2 + 3| + |4 + 5 + 6\rangle\langle 4 + 5 + 6|.
\]

Here again, her position measurement is non-complete, reflected in the fact that the summation over the corresponding projectors (r.h.s of Eq. (31)) is not \(I_6\).

Since \(\rho_P \neq \rho_Q\), we are led to conclude that the violation of no-signaling is already implicit in the Innsbruck experiment. Yet, since Bob measures in the \(z\)-basis rather than the ‘mode’ basis, in the absence of a direction filter—as is the case in the Innsbruck experiment—Bob’s screen will not register any signal, for the following reason. In case of Alice’s focal plane measurement, the integrated diffraction-interference pattern corresponding to different outcomes will wash out any observable interference pattern. On the other hand, in case of Alice’s imaging plane measurement, Bob’s each detection comes from the photon’s incoherent passage through one or the other slit, and hence—again—no interference pattern is produced on his screen. Thus, measurement at Bob’s screen plane \(z\) without a direction filter will render \(\rho_P\) effectively indistinguishable from \(\rho_Q\). The role played by the direction filter is to prevent modal averaging in case of Alice’s momentum measurement, by selecting one set of modes. The filter does not create, but only exposes, a superluminal effect that otherwise remains hidden.

D. Complementarity of single- and two-particle correlations

It is well known that path information (or particle nature) and interference (or wave nature) are mutually exclusive or complementary. In the two-photon case, this takes the form of mutual incompatibility of single- and two-particle interference [14, 15], because entanglement can be used to monitor path information of the twin particle, and is thus equivalent to ‘particle nature’. One may thus consider single- and two-particle correlations as being related by a kind of complementarity relation that parallels wave- and particle-nature complementarity. A brief exposition of this idea is given in the following paragraph.

For a particle in a double-slit experiment, we restrict our attention to the Hilbert space \(\mathcal{H}\), spanned by the state \(|0\rangle\) and \(|1\rangle\) corresponding to the upper and lower slit of a double slit experiment. Given density operator \(\rho\), we define coherence \(C\) by \(C = 2|\rho_{01}| = 2|\rho_{10}|\), a measure of cross-terms in the computational basis not vanishing. The particle is initially assumed to be in the state \(|\psi_a\rangle\), and a “monitor”, initially in the state \(|0\rangle\), interacting with each other by means of an interaction \(U\), parametrized by variable \(\theta\) that determines the entangling strength of \(U\). It is convenient to choose \(U = \cos \theta I \otimes I + i \sin \theta \text{ CNOT}\), where CNOT is the operation \(I \otimes |0\rangle\langle 0| + X \otimes |1\rangle\langle 1|\), where \(X\) is the Pauli X operator. Under the action of \(U\), the system particle goes to the state \(\rho = \text{Tr}_m[U(|\psi_a\rangle\langle 0|\psi_a|\langle 0|)U^\dagger] = \frac{1}{2} + \frac{1}{2}[(\cos \theta + i \sin \theta) \cos \theta|0\rangle\langle 1| + \text{c.c.}]\), where \(\text{Tr}_m[\cdots]\) indicates taking trace over the monitor. Applying the above formula for coherence to \(\rho\), we calculate that coherence \(C = \cos \theta\). We let \(\lambda_{\pm}\) denote the eigenvalues of \(\rho\). Quantifying the degree of entanglement by concurrence [53], we have \(E = 2\sqrt{\lambda_{-}\lambda_{+}} = \sin \theta\). We thus obtain a trade-off between
coherence and entanglement given by $C^2 + E^2 = 1$, a manifestation of the complementarity between single-particle and two-particle interference.

In the context of the proposed experiment, this could raise the following purported objection to our proposed signaling scheme: as the experiment happens in the near-field regime, where two-particle correlations are strong, one would expect that Bob should not find an interference pattern in his singles counts. Yet, contrary to this expectation, Eq. (18) implies that such an interference pattern does appear. The reason is that in the focal plane measurement, Alice is able to erase her path information in the subspace $H_2$, but, by virtue of the associated non-completeness, she does so in only one way, viz. via the non-complete operation $E_{f}^{(+)}$ associated with her measurement. If her measurement were complete, she would erase path information in more than one way, and the corresponding conditional single-particle interference patterns would mutually cancel each other in the singles count. This is clarified in the following Section.

E. Non-completeness implies lack of complementary measurement

Let us suppose Alice’s measurement at $f$ is replaced by a complete scheme in which her measurement is deferred to a point behind a beam-splitter placed at $f$, whereby the path singularity is removed. The action of such a beam splitter is

\[
\begin{align*}
\hat{a}_2 &\rightarrow \hat{a}_2' \equiv \cos \beta \hat{a}_2 + i \sin \beta \hat{a}_5, \\
\hat{a}_5 &\rightarrow \hat{a}_5' \equiv i \sin \beta \hat{a}_2 + \cos \beta \hat{a}_5.
\end{align*}
\]

Equation (32) now holds since $\hat{a}_2' \hat{a}_2 + \hat{a}_5' \hat{a}_5 = I_2$. From Eqs. (12), (13) and (14), with $\hat{a}_j'$ replacing $\hat{a}_j$ ($j = 2, 5$), we find that the joint probability for detection of a photon in mode $2'$ or $5'$ by Alice and at $z$ by Bob is given by

\[
\begin{align*}
p_{2'}(z) &\propto \langle E_{4}^{(1)} a_2^{(1)} E_{2} a_2' \rangle = \frac{1}{2} (1 + \sin(2\beta) \sin(k(r_5 - r_2))) \\
p_{5'}(z) &\propto \langle E_{4}^{(1)} a_5^{(1)} E_{2} a_5' \rangle = \frac{1}{2} (1 - \sin(2\beta) \sin(k(r_5 - r_2))).
\end{align*}
\]

Tracing over Alice’s outcomes, we find $p_{2'}(z) \propto 1$, and so no superluminal signaling occurs.

In the light of this, let us consider how Alice’s momentum measurement could seemingly be completed. We restrict ourselves to the simplified first-quantization representation of the field state vector Eq. (27). The measurement operator corresponding to her detection at $f$ is $\mathbb{P}_f \equiv E_{f}^{(+)1} E_{f}^{(-)} \equiv \frac{1}{2}(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)$ in view of Eq. (28). One might consider that to make her measurement complete, $\mathbb{P}_f$ should be complemented by

\[
\mathbb{P}'_f \equiv \mathbb{I}_2 - \mathbb{P}_f = \frac{1}{2}(|0\rangle - |1\rangle)(|0\rangle - |1\rangle),
\]

which might be interpreted as the operator corresponding to Alice’s non-detection at $f$. If Alice’s momentum measurement were given by the pair $\{\mathbb{P}_f, \mathbb{P}'_f\}$, clearly no superluminal signal occurs for the reason given above. Here one might consider $\mathbb{P}'_f$ to the operator corresponding to Alice’s non-detection at $f$. Unfortunately, the operator $\mathbb{P}'_f$ in necessarily non-physical, which can be seen in several ways.

Let us consider what $\mathbb{P}'_f$ represents from a quantum optics (second quantization) perspective. Converting from first quantization language, we see that it represents $\hat{a}_2 - \hat{a}_5$. But this does not correspond to the electric field operator at any point on Alice’s side, since these modes meet only at $f$, and since $p$ and $q$ are equidistant from $f$ (along optical rays), the form of the electric field operator at $f$ is $\hat{a}_2 + \hat{a}_5$, which is of course $\mathbb{P}_f$ in the first quantization language. Thus $\mathbb{P}'_f$ is not a valid measurement operator of Alice in the current set-up.

In particular, $\mathbb{P}'_f$ is not the measurement operator that corresponds to her non-detection at $f$. If Alice does not detect her photon at the $f$, then she would in principle detect them elsewhere on the focal plane (points $f'$ and $f''$ in our present model of Figure 2). In our simplified 6-mode picture, they are given by the operators

\[
\begin{align*}
\mathbb{P}_f' &\propto \frac{1}{2} \langle (1) + e^{ik(r_{4'\gamma} - r_{1\delta})}|4\rangle(1) + e^{-ik(r_{4'\gamma} - r_{1\delta})}|4\rangle \\
\mathbb{P}_f'' &\propto \frac{1}{2} \langle (3) + e^{ik(r_{6''\gamma} - r_{3\delta})}|6\rangle(3) + e^{-ik(r_{6''\gamma} - r_{3\delta})}|6\rangle,
\end{align*}
\]

in view of Eqs. (17b) and (17c).

But clearly $\mathbb{P}_f \neq \mathbb{P}_f' + \mathbb{P}_f''$. In fact, the two operators don’t even have the same support. Thus $\mathbb{P}_f$ does not correspond to non-detection at $f$ and could not be used to complete $\mathbb{P}_f$. This strange state of affairs is a consequence of non-completeness as had been clarified in note-in-citation [46]. In other words, Alice’s non-detection at $f$ does not give rise to a complementary interference pattern, but to a non-detection at Bob’s side, too.
F. Polarization and interference

The physical realization of \( Q \) allows us to study the polynomiality of the family of \( Q \)-like gates from a less abstract and more physical perspective. The gate \( Q_2(\pi) \) acting on \((1/\sqrt{2})|(0) + (1)\rangle\) annihilates it. Physically this describes the situation where two converging modes at the path singularity, having the same polarization, interfere with each other destructively, resulting in no particles being observed. This is analogous to the situation of dark fringes in a Young’s double-slit experiment. The quantum optics formalism implies that if the polarizations of the two incoming modes are not parallel, then the polarizations add vectorially (whether in a complete or non-complete configuration), with the resulting intensity being the squared magnitude of the vector sum.

This point is worth stressing, since if it were not so, it could give rise to ‘interferometric quantum computing’ that would allow for efficient solution of hard problems. For example, suppose we have an \( N = 2^n \) dimensional system defined on \( n \) qubits, prepared initially in the state \(|a\rangle \equiv (1/\sqrt{N})(|1\rangle + \cdots + |N\rangle)\). The spatial part of the physical \( n\)-qubit system’s matter wave is now split into two partial waves by an appropriate beam-splitter, and then refocused onto a path singularity. On the second partial wave, before the two partial waves reach the region of spatial overlap, an oracle operation is applied which in a single step inverts the sign of all the kets, except the ‘marked’ state \(|N\rangle\), yielding \(|b\rangle \equiv (1/\sqrt{N})(-|1\rangle - \cdots + |N\rangle)\). According to the above prescription for non-complete detection, the output at the path singularity should be \(|a\rangle + |b\rangle \sim 2|N\rangle/\sqrt{N} \), i.e., an outcome \(|N\rangle\) observed with the exponentially low (in terms of \( \log(N) \), the number of qubits used to realize the state) probability of \( |||a\rangle + |b\rangle||^2 = 4/N \). The physical interpretation is that for the most part, with probability \( (N - 4)/N \approx 1 \) for large enough \( n \), no atoms are observed at the path singularity, whereas the solution state \(|N\rangle\) is observed with probability \( 4/N \). Here non-detection of atoms should be interpreted as suppression of spatial transfer of atoms from their source to the path singularity. This is reminiscent of coherent population trapping, where an atom in a ‘dark state’ remains unexcited because two pathways to excitation destructively interfere [49].

Therefore, if the above oracle operation could be defined so that the marked state is a solution to SAT, the measurement would have to be repeated exponentially large number of times to detect a possible ‘yes’ outcome. Alternatively, exponentially large number of atoms should be used to build up an answer signal of strength \( O(1) \) in polynomial time. Either way, the physical situation is compatible with the WNHE assumption, but not with no-signaling.

The implementation of non-complete measurement of modes of arbitrary polarization gives us further insight into the polynomiality of Nature. It is not difficult to imagine a more complicated rule than plain vector addition for the interference of quantum wavefunctions (say a renormalization following vector addition), which could have been used to boost the above signal, to solve SAT in polynomial time. This would in fact implement the post-selection gate. However, it would have required ‘Nature to compute harder’ than believed to be possible with a Turing machine or equivalent model of computation, in contradiction to the WNHE viewpoint that the universe is a polynomial place.

VII. DISCUSSIONS AND CONCLUSIONS

Considering the far-reaching implications of a positive result to our proposed experiment, we have to remain open to the possibility that there is an error somewhere in our analysis, possibly a hidden unwarranted assumption, the elimination of which would provide a mechanism to prevent the superluminal signaling. However, in support of our claim, it may be noted that our analysis of the Modified Innsbruck experiment is based on a model that works quite well in explaining the results of the original Innsbruck experiment. This suggests that it would be difficult to prohibit the superluminal signal in the model of the Modified experiment without also ending up proscribing two-particle correlations in the model for the Innsbruck experiment.

Furthermore, we have ruled out in Section VII all the (so far as known to us) obvious objections. Hence we are led to believe that any erroneous assumption or application of physical principles, if it exists in our analysis, must be sufficiently subtle. Therefore, it would still be instructive to perform our proposed experimental tests because, even if the tests yield a negative result, provided the outcome is unambiguous, we could re-examine our analysis confident of detecting an erroneous element that is otherwise not obvious. As in the earlier mentioned example of the no-cloning theorem, even this potentially negative result could carry new theoretical insights.

On the other hand, in the surprising event the proposed experiment yields a positive outcome, a number of issues would clearly be brought up. Foremost among them: the apparent violation of locality in standard, linear QM would now emerge as a real violation, and no-signaling would no longer be a universal condition. The issue of ‘speed of quantum information’ [50] would assume practical significance.

A putative positive outcome to either of the proposed experiment would also bolster the case for believing that the WNHE assumption is a basic principle of quantum physics, while undermining the case for no-signaling in QM. It would then follow that intractability, and by extension uncomputability, matter to physics in a fundamental way. This
would suggest that physical reality is fundamentally computational in nature. With this abstraction, physical space would be regarded as a type of information, with physical separation no genuine obstacle to rapid communication in the way it seems to be when seen from the conventional perspective of causality in physics. On the other hand, the barrier between polynomial-time and hard problems would be real. The physical existence of superluminal signals would thus not be as surprising as that of exponential gates. Interestingly, polynomial superluminal operations exist even in classical computation. The Random Access Machine (RAM) model, a standard model in computer science wherein memory access takes exactly one time-step irrespective of the physical location of the memory element, illustrates this idea. RAMs are known to be polynomially equivalent to Turing machines. At the least, WNHE could serve as an informal guide to issues in the foundations ofQM, and perhaps even quantum gravity.

Even granting that the noncomplete gate $Q'$ turns out to be physically valid and realizable, this brings us to another important issue: how would non-completeness fit in with the known mathematical structure of the quantum properties of particles and fields, and why, if true, should it have remained theoretically unnoticed so far inspite of its far-reaching consequences? We venture that the answer has to do with the nature of and relationship between observables in QM on the one hand, and those in quantum optics, and more generally, in quantum field theory (QFT), on the other hand.

It is frequently claimed that QFT is just the standard rules of first quantization applied to classical fields, but this position can be criticized. For example, the relativistic effects of the integer-spin QFT imply that the wavefunctions describing a fixed number of particles do not admit the usual probabilistic interpretation. Again, fermionic fields do not really have a classical counterpart and do not represent quantum observables.

In practice, measurable properties resulting from a QFT are properties of particles—of photons in quantum optics. Particulate properties such as number, described by the number operator constructed from fields, or the momentum operator, which allows the reproduction of single-particle QM in momentum space, do not present a problem. The problem is the position variable, which is considered to be a parameter, and not a Hermitian operator, both in QFT and single-particle relativistic QM, and yet relevant experiments measure particle positions. The experiment described in this work involve measurement of the positions of photons, as for example, Alice’s detection of photons at points on the imaging or focal plane, or Bob’s detection at points on the z-plane, respectively. There seems to be no way to derive from QFT the experimentally confirmed Born rule that the nonrelativistic wavefunction $\psi(x, t)$ determines quantum probabilities $|\psi(x, t)|^2$ of particle positions. In most practical situations, this is really not a problem. The probabilities in the above experiment were computed according to standard quantum optical rules to determine the correlation functions at various orders, which serve as an effective wavefunction of the photon, as seen for example from Eqs. In QFT, particle physics phenomenologists have developed intuitive rules to predict distributions of particle positions from scattering amplitudes in momentum space.

Nevertheless, there is a problem in principle, and leads us to ask whether QFT is a genuine quantum theory. If we accept that properties like position are valid observables in QM, the answer seems to be ‘no’. We see this again in the fact that the effective ‘momentum’ and ‘position’ observables that arise in the above experiment are not seen to be Hermitian operators of standard QM (cf. note). Further, non-complete operations like $\hat{E}_{+}^{(\pm)}$, disallowed in QM, seem to appear in QFT. This suggests that it is QM, and not QFT, that is proved to be strictly non-signaling by the no-signaling theorem.

Since nonrelativistic QM and QFT are presumably not two independent theories describing entirely different objects, but do describe the same particles in many situations, the relationship between observables in the two theories needs to be better understood. Perhaps some quantum mechanical observables are a coarse-graining of QFT ones, having wide but not universal validity. For example, Alice’s detection of a photon at a point in the focal plane was quantum mechanically understood to project the state of Bob’s photon into a one-dimensional subspace corresponding to a momentum eigenstate. Quantum optically, however, this ‘eigenstate’ is described as a superposition of a number of parallel, in-phase modes originating from different down-conversion events in the non-linear crystal, producing a coherent plane wave propagating in a particular direction.

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[9] In complexity theory, RP is the class of decision problems for which there exists a probabilistic TM (a deterministic TM with access to genuine randomness) such that: it runs in polynomial time in the input size. If the answer is ‘no’, it returns ‘no’. If the answer is ‘yes’, it returns ‘yes’ with probability greater than 1/2 (else it returns ‘no’). BQP is the class of decision problems solvable by a quantum TM \[10\] in polynomial time, with error probability of at most 1/2 (or, equivalently, any other fixed fraction smaller than 1/2) independently of input size.
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[19] That is, ‘the universe is not hard enough to not’ be simulable using polynomial resources’. The expression is non-technically related to the statement “The world is not enough” (“orbis non sufficit”), the family motto of, as well as a motion picture featuring, a well known Anglo-Scottish secret agent!
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[29] \mathsf{PSPACE} is the class of decision problems solvable by a Turing machine in polynomial (memory) space possibly taking exponential time.
[30] In complexity theory, \mathsf{PP} is the class of decision problems for which there exists a polynomial time probabilistic TM such that: if the answer is ‘yes’, it returns ‘yes’ with probability greater than 1/2, and if the answer is ‘no’, it returns ‘yes’ with probability at most 1/2.
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[46] We can then define Alice’s (remote) ‘momentum observable’ as \( \hat{P} \equiv f|2 + 5\rangle\langle 2 + 5| + f'|1 + 4\rangle\langle 1 + 4| + f''|3 + 6\rangle\langle 3 + 6| \). Interpreted as a quantum field theoretic observable, \( \hat{P} \) is non-complete because the projectors to its eigenstates \( |2 + 5\rangle \), \( |1 + 4\rangle \) and \( |3 + 6\rangle \) do not sum to \( I_6 \equiv |1\rangle\langle 1| + |2\rangle\langle 2| + |3\rangle\langle 3| + |4\rangle\langle 4| + |5\rangle\langle 5| + |6\rangle\langle 6| \). Similarly, Alice’s non-complete ‘position’ observable is \( \hat{Q} \equiv m|1 + 2 + 3\rangle\langle 1 + 2 + 3| + l|4 + 5 + 6\rangle\langle 4 + 5 + 6| \). But note that \( \hat{Q} \)’s projection into the subspace \( \mathcal{H}_{2.3} \) is indeed a complete observable. \( P \) and \( Q \) are of rank 3 and 2, respectively, which is smaller than 6, the dimension of the relevant Hilbert subspace.

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