Kinetic modeling of opinion formation of peoples via multiple political parties

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Abstract. - We investigate the opinion formation among the peoples and multiple political parties using the one dimensional relativistic Boltzmann-Vlasov equation for multi-components. A political party is constituted of politicians. The opinion formation depends on self-thinnings of peoples and politicians, and the constraint of the political party over opinions of politicians, when we restrict ourselves to the conciliatory exchange of opinions between two individuals. In particular, shock like profiles are obtained in the distribution of opinions of peoples, when the self-thinking of politicians are absent at the binary exchange of opinions between two politicians in the same political party.

The opinion formation has been studied with great interests in the framework of sociophysics. In previous studies, microscopic models have been proposed by Sznajd \cite{2}, Hegselman and Krause \cite{3}, and Ben-Naim \cite{4}. The master equation, which describes the microscopic motions of opinions, indicates some clustering states of opinions \cite{2} \cite{3} \cite{4}. Meanwhile, the kinetic model of the opinion formation is studied by Toscani and his coworkers \cite{5} using the inelastic Boltzmann equation or partial differential equation (PDE) with strongly nonlinear form, which corresponds to Sznajd \cite{2} model in Ochrombel simplification \cite{6} on a complete graph \cite{7}. The PDE studied by Toscani et al., sets the upper and lower bounds to the strength of the opinion ($m$), namely, $|m| \leq 1$, whereas the diffusion via the self-thinking violates such upper and lower bounds to the strength of the opinion.

Then, Yano and Martin \cite{8} proposed the kinetic model, which always promises the causality of $m$, namely, $|m| \leq 1$ using the relativistic kinetic model. The binary exchange of opinions between two individuals is expressed using the inelastic relativistic Boltzmann equation, whereas the diffusion via the self-thinking is expressed by incorporating the randomly perturbed motion \cite{9} into the inelastic binary collision. In our previous model, the effects of political party on the opinion formation of peoples were expressed using the Vlasov term, because the political party exists as a field, which acts on opinions of peoples as an external force. We, however, know that the treatment of the political party as the external field is insufficient, because the political party is also constituted of individuals, namely, politicians, whose opinions are not always common. The binary exchange of opinions between the people and politician must be expressed using the relativistic kinetic model for
the multi-components, because the weight (mass) of the opinion of the politician must be markedly heavier than that of the people and the diameter of the sphere (the opinion) of the politician also must be larger than that of the people, when we regard the opinion as a hard sphere, because the regime of effects on the opinion of the politician must be markedly larger than that of the people. As a result, we propose the relativistic kinetic model for multi-components to express the binary exchange of opinions between two peoples, politician and people, and two politicians.

In this paper, we restrict ourselves to the conciliatory exchange, namely, compromise of opinions between two agents. Additionally, we assume that politicians who belong to one political party never exchange their opinions with politicians who belong to other political party.

**Relativistic kinetic model for multi-components.** The relativistic kinetic model for multi-components is formulated as

\[
\frac{\partial f_i(t,p)}{\partial t} = \sum_{j=1}^{N} A_{ij} \int_{-\infty}^{\infty} \left[ \frac{1}{J} f_i(t,p') f_j (t,p'') - f_i(t,p) f_j (t,p) \right] g_o dp' \tag{1}
\]

\[
+ A_{ii} \int_{-\infty}^{\infty} \left[ \frac{1}{J} f_i(t,p''') f_i(t,p''') - f_i(t,p) f_i(t,p) \right] g_o dp' + B_i \frac{\partial (p - p_i)}{\partial p} \left( 1 < i \right), \tag{2}
\]

where \(f_i(t,p)\) is the distribution function of the \(i\)-th component, where \(i = 1\) corresponds to the people, \(i \neq 1\) corresponds to \(i\)-th political party. In Eqs. (1) and (2), \(p\) and \(p_\ast\) are momentums of two colliding opinions, which are defined by \(p = M\gamma(m)m\) and \(p_\ast = M_\ast\gamma (m_\ast)m_\ast\) (\(\gamma(m) = 1/\sqrt{1 - m^2}\), Lorentz factor, \(M\): mass of the opinion), and \(t\) is the time. The term in the right hand side of Eq. (1) and the first and second terms in the right hand side of Eq. (2) correspond to relativistic inelastic collisions with the randomly perturbed motion, whereas the total energy \(E = g_o dp' \) and \(\Delta\) is the relativistic inelastic collision with the randomly perturbed motion via the self-thinking. On the other hand, momentums of two colliding opinions, namely, \(p'\) and \(p_\ast'\), change to \(p\) and \(p_\ast\), which are defined by

\[
p' = p + (1 + \alpha) \mu (p_\ast/M_\ast - p/M + \Delta (p_\ast)),
\]

\[
p_\ast' = p_\ast - (1 + \alpha) \mu (p_\ast/M_\ast - p/M + \Delta (p_\ast)), \tag{3}
\]

where \(\alpha\) is the inelasticity coefficient \((0 \leq \alpha \leq 1)\), \(\mu = MMM_\ast / (M + M_\ast)\), and \(\Delta\) is the randomly perturbed motion via the self-thinking. On the other hand, momentums of two colliding opinions, namely, \(p''\) and \(p_\ast''\), change to \(p\) and \(p_\ast\), in which \(p''\) and \(p_\ast''\) are defined by

\[
p'' = p + \mu (1 + \alpha) (p_\ast/M_\ast - p/M + \Delta (p_\ast)) / \alpha,
\]

\[
p_\ast'' = p_\ast - \mu (1 + \alpha) (p_\ast/M_\ast - p/M + \Delta (p_\ast)) / \alpha. \tag{4}
\]

Consequently, the total momentum is conserved by the binary inelastic collision with the randomly perturbed motion, whereas the total energy \((E + E_\ast = \sqrt{M + p^2} + \sqrt{M_\ast + p_\ast^2})\).
is not conserved by the binary collision with the randomly perturbed motion. $J$ in Eq. (1) is the Jacobian, which is defined by

$$J = \left| \det \frac{\partial (p^\alpha, p^\beta)}{\partial (p, p_\ast)} \right|^{-1} = \left| \frac{1}{\alpha} \right| \left( \frac{1}{1 + 1/\alpha} \right) \left( \partial_\alpha \Delta(p, p_\ast) - \partial_\beta \Delta(p, p_\ast) \right)^{-1}. \tag{5}$$

In this paper, we restrict ourselves to $\alpha = 0$, which corresponds to the compromise of two colliding opinions, when $\Delta(p, p_\ast) = 0$.

The significant parameter in the opinion formation is the temperature ($\theta$) in the closed opinion system, because $\theta \to \infty$ means that all $|m| \to 1$, where $|m| = 1$ corresponds to the complete agreement ($m = 1$) or disagreement ($m = -1$) on the single issue, namely, complete decision making. In our relativistic kinetic model, we never postulate the massless particle. Therefore, the individual with the complete decision making, namely, $|m| = 1$, is not considered. We, however, have a question, What is the temperature in the closed opinion system? The possible answer to this question is that the temperature in the closed opinion system is equivalent to the global interest in the single issue. Provided that all the individuals have high interests in the single issue, $|m|$ of all the individuals approximate to unity, namely, complete decision making. Meanwhile, $m$ of all the individuals remain fuzzy state, namely, $|m| \ll 1$, when all the individuals have low interests in the single issue and the political party is absent. Finally, the global interest in the single issue decreases by the binary inelastic collision without the randomly perturbed motion (self-thinking), whereas the global interest increases by the self-thinking, namely, randomly perturbed motion $\Delta(i^j)$, which is the randomly perturbed motion at the binary exchange of opinions between components $i$ and $j$. In this paper, the randomly perturbed motion is formulated as $\Delta(i^j) = \Delta_0(i^j)(2\mathcal{W} - 1)$, where $\Delta_0(i^j)$ is the amplitude of the randomly perturbed motion at the binary exchange of opinions between two components $i$ and $j$, and $0 \leq \mathcal{W} \leq 1$ is the white noise.

Numerical results. In our numerical analysis, we consider two political parties, namely, political parties $A$ and $B$, by assuming the democratic nation. Additionally, physical quantities such as the density, averaged opinion ($\bar{m}$), and global interest ($\theta$) are calculated using Eckart’s decomposition of $N^\alpha = \int_{-\infty}^{\infty} p^\alpha f dp/p^0$ and $T^{\alpha\beta} = \int_{-\infty}^{\infty} p^\alpha p^\beta f dp/p^0$. Finally, Eq. (1) is solved using the direct simulation Monte Carlo (DSMC) method.

Opinion formation under absence of restrictions of political parties. At first, we investigate the opinion formation under the absence of restrictions of political parties, namely, $B_i = 0$ in Eq. (2). Additionally, we consider two cases. One is the opinion formation, when the self-thinking of politicians at the binary exchange of opinions of two politicians, who belong to the same party, is absent. The other is the opinion formation, when the self-thinking of politicians at the binary exchange of opinions of two politicians, who belong to the same party, is considered. $\Delta_{11}^{A}$ and $\Delta_{11}^{B}$ ($\ell = A, B$) are equal to $\Delta_a$. Here, the number of politicians in the political party $A$ ($n_A$) is set as $n_A = 10^{-4}n_1$, in which $n_1$ is the number of peoples, whereas the number of politicians in the political party $B$ ($n_B$) is set as $n_B = 10^{-3}n_1$. Additionally, the mass of the politician in the political party $A$ ($M_A$) is set as $M_A = 10^3M_1$, in which $M_1 = 1$ is the mass of the people, whereas the mass of the politician in the political party $B$ ($M_B$) is set as $M_B = 10^5M_1$. The collision frequency, namely, $A_{ij}$ in Eq. (1) or (2), is calculated using $A_{ij} = A_{11}(d_i/d_1 + d_j/d_1)^2/4$ and $A_{11} = 1$, in which $d_1 = 1$ is the diameter of the people and $A_{11}$ is the collision frequency of the binary exchange of opinions between two peoples. Additionally, we set $d_A = d_4(M_A/M_1)^{1/3} = M_A^{1/3}$ and $d_B = d_4(M_B/M_1)^{1/3} = M_B^{1/3}$.

As initial data, the opinions of peoples are uniformly distributed in the range of $0 \leq |m_1| < 1$, whereas the opinions of politicians, who belong to the political party $A$, are set as $m_A = -0.5$ and the opinions of politicians, who belong to the political party $B$, are set as $m_B = 0.5$. $10^5$ sample opinions of peoples are used to simulate $n_1$ opinions of peoples. 

p-3
The top-left frame of Fig. 1 shows the convergent form of the distribution function of the people, namely, \( f_1(m_1) \) versus \( m_1 \) using three types, namely, \( \Delta_a = 1, 2.5 \) and 5, when the self-thinking of politicians at the binary exchange of opinions of two politicians, who belong to the same party, is not considered, namely, \( \Delta_{\ell}^M = 0 \) \( (\ell = A, B) \), together with convergent forms of equilibrium distribution function of \( m_1 \), namely, Maxwell-Juttner function of \( m_1 \) \( (f_{1M}^M(m_1)) \) versus \( m_1 \), \( f_A(m_A) \) versus \( m_A \) and \( f_B(m_B) \) versus \( m_B \). All the opinions of two political parties converge to the neutral state, namely, \( m_\ell = 0 \) \( (\ell = A, B) \). Opinions of peoples approximate to the decision-making state \((|m_1| = 1)\), as \( \Delta_a \) increases, whereas \( f_1(m_1) \) is different from \( f_{1M}^M(m_1) \) in all the cases of \( \Delta_a = 1, 2.5 \) and 5. Additionally, we find two shock like profiles of \( f_1(m) \) at \(|m_1| \approx 0.71\) in the case of \( \Delta_a = 1 \), \(|m_1| \approx 0.93\) in the case of \( \Delta_a = 2.5 \) and \(|m_1| \approx 0.98\) in the case of \( \Delta_a = 5 \). Such two shock like profiles are obtained as a result of the binary exchange of opinions between the people and politician, who has the neutral opinion \( (m_\ell = 0) \). From Eq. (3), we readily obtain following relation using \( \alpha = 0 \) and \( \mathcal{M}_1 \ll \mathcal{M}_\ell \)

\[
p_1' = p_1 + (1 + \alpha) \mu (p_\ell / \mathcal{M}_\ell - p_1 / \mathcal{M}_1 + \Delta (p_1, p_\ell)),
\]

\[
\simeq p_1 + \mathcal{M}_1 (m_\ell \gamma (m_\ell) - m_1 \gamma (m_1) + \Delta (p_1, p_\ell))
\]

\[
= \mathcal{M}_1 (m_\ell \gamma (m_\ell) + \Delta), \quad \ell = A, B.
\]

(6)

From Eq. (6), \( m_1 \) fluctuates via the self-thinking term \( \Delta \) as a result of the binary collision between the people and politician, when \( m_\ell \) \( (\ell = A, B) \) is temporally constant. As shown in the top-left frame of Fig. 1, the opinion of the political party is neutral, namely, \( m_\ell = 0 \) \( (\ell = A, B) \) in Eq. (6). As a result, \( m_1 \) fluctuates in the range of \(-\Delta_a/\sqrt{1 + \Delta_{\ell}^2} \leq m_1 \leq \Delta_a/\sqrt{1 + \Delta_{\ell}^2}\). Finally, we find that two shock like profiles of \( f_1(m) \) corresponds to two limiting values of \( m_1 \), namely, \(-\Delta_a/\sqrt{1 + \Delta_{\ell}^2} \) and \( \Delta_a/\sqrt{1 + \Delta_{\ell}^2} \), because we obtain \(|m_1| \leq 0.707 \) in the case of \( \Delta_a = 1 \), \(|m_1| \leq 0.928 \) in the case of \( \Delta_a = 2.5 \) and \(|m_1| \leq 0.9805 \) in the case of \( \Delta_a = 5 \).

Next, we investigate the opinion formation, when the self-thinking at the binary exchange of opinions between two politicians, who belong to the same political party, is considered, namely, \( \Delta_{\ell}^M = \Delta_a \) \( (\ell = A, B) \). The top-right frame of Fig. 1 shows \( f_1(m_1) \) and \( f_{1M}^M(m_1) \) versus \( m_1 \) in cases of \( \Delta_a = 1, 2.5 \) and 5. As shown in the top-left frame of Fig. 1, shock like profiles of \( f_1(m_1) \) are dissipated in all the cases of \( \Delta_a = 1, 2.5 \) and 5. As a result, \( f_1(m_1) \), which are obtained by including the self-thinking at the binary exchange of opinions between two politicians, who belong to the same political party, are more similar to \( f_{1M}^M(m_1) \) than \( f_1(m_1) \), which are obtained by neglecting the self-thinking of politicians at the binary exchange of opinions between two politicians, who belong to the same political party, when \( \Delta_a = 1, 2.5 \) and 5. The bottom-left frame of Fig. 1 shows \( f_A(m_A) \) and \( f_{1M}^M(m_A) \) versus \( m_A \) in cases of \( \Delta_a = 1, 2.5 \) and 5, whereas the bottom-right frame of Fig. 1 shows \( f_B(m_B) \) and \( f_{1M}^M(m_B) \) versus \( m_B \) in cases of \( \Delta_a = 1, 2.5 \) and 5. The bottom-left and bottom-right frames of Fig. 1 indicate that \( f_{1M}^M(m_A) \) and \( f_{1M}^M(m_B) \), when \( \Delta_a = 1, 2.5 \) and 5, whereas \( f_A(m_A) \) are markedly noisy in cases of \( \Delta_a = 1, 2.5 \) and 5, because the number of sample opinions of politicians, who belong to the political party A, is 10, which is markedly less than \( 10^5 \) sample opinions of peoples. The smooth profile of \( f_\ell (m_\ell) \) \( (\ell = A, B) \) never set the constant value of \( m_\ell \gamma (m_\ell) \) in Eq. (6), when the self-thinking at the binary exchange of opinions of two politicians, who belong to the same political party, is considered. As a result, the diffusion via the self-thinking at the binary exchange of opinions between two politicians never yield the spectrum profile of \( f_\ell (m_\ell) \) \( (\ell = A, B) \), which leads to two shock like profiles of \( f_1(m_1) \). From above numerical results, opinions of politicians, who belong to the political party A, are indirectly exchanged with opinions of politicians, who belong to the political party B, via the direct exchange of opinions between the people and politician. As a result, opinions of politicians move toward the conciliatory state between two political parties via the direct exchange of opinions between the people and politician, when the restriction of the political party to the
Option formation under restrictions of political parties. We investigate the opinion formation, when the restriction of the political party to politicians is finite, namely, $0 < B_t$ in Eq. (2). The number of politicians in the political party $A$ ($n_A$) and the number of politicians in the political party $B$ ($n_B$) are set as $n_A = n_B = 10^{-3} n_1$. Additionally, the mass of the opinion of the politician in the political party $A$ ($\mathcal{M}_A$) and the mass of the opinion of the politician in the political party $B$ ($\mathcal{M}_B$) are set as $\mathcal{M}_A = \mathcal{M}_B = 5 \times 10^6 \mathcal{M}_1$, in which $\mathcal{M}_1 = 1$. The collision frequency, namely, $A_{ij}$ in Eq. (1) or (2), is calculated using $A_{ij} = A_{11} (d_i/d_1 + d_j/d_1)^2/4$ and $A_{11} = 1$, whereas we set $d_A = d_i (\mathcal{M}_A/\mathcal{M}_1)^{1/3} = \mathcal{M}_A^{1/3}$ and $d_B = d_i (\mathcal{M}_B/\mathcal{M}_1)^{1/3} = \mathcal{M}_B^{1/3}$ using $d_i = 1$ and $\mathcal{M}_1 = 1$. As initial data, the opinions of politicians are uniformly distributed in the range of $0.8 \leq |m|_1 < 1$, whereas the opinions of politicians, who belong to the political party $A$, are set as $m_A = -0.5$ and the opinions of politicians, who belong to the political party $B$, are set as $m_B = 0.5$. The rate of concentration of opinions of politicians to the unified opinion of the political party, namely, $B_t$ ($\ell = A, B$) in Eq. (2) is set as $B_t = 0.1$. The unified opinion of the political party $A$ ($m_A^0$) is set as $m_A^0 = 0.5$, whereas the unified opinion of the political party $B$ ($m_B^0$) is set as $m_B^0 = -0.5$. $10^5$ sample opinions of peoples are used to simulate $n_1$ opinions of peoples. At first, we investigate the opinion formation, when the self-thinking of politicians in the binary exchange of opinions between two politicians, who belong to the same political party, is absent, namely, $\Delta_a = 0$ ($\ell = A, B$), whereas $\Delta_a^{11}$ and $\Delta_a^{1\ell}$ ($\ell = A, B$) are equal to $\Delta_a$. The top-left frame of Fig. 2 shows convergent forms of $f_1(m_1)$ versus $m_1$, $f_A(m_A)$ versus $m_A$ and $f_B(m_B)$ versus $m_B$ together with $f_l^{1\ell}$ $(m_1)$ versus $m_1$ in cases of $\Delta_a = 0.1$ and 0.5. As shown in the top-left frame of Fig. 2, $f_1(m_1)$ are markedly different from $f_1^{1\ell}$ $(m_1)$ in cases of $\Delta_a = 0.1$ and 0.5. $f_A(m_A)$ is spectrum at $m_A = m_A^0 = -0.5$, whereas $f_B(m_B)$ is spectrum at $m_B = m_B^0 = 0.5$. Four shock like profiles of $f_1(m_1)$ are obtained at $m_1 = \pm 0.4308$ and $\pm 0.5608$ in the case of $\Delta_a = 0.1$, and at $m_1 = \pm 0.0771$ and $\pm 0.7329$ in the case of $\Delta_a = 0.5$, which are readily calculated from Eq. (6). Similarly, $N$ unified opinions by $N$ political parties over opinions of politicians yield the $2N$ shock like profiles, when the self-thinking of politicians is absent at the binary exchange of opinions between two politicians in the same political party. $f_1(m_1)$ is the trimodal in the range of $|m_1| < 0.4308$ when $\Delta_a = 0.1$, whereas $f_1(m_1)$ is flat in the range of $|m_1| < 0.0771$ when $\Delta_a = 0.5$. Finally, the increase of the amplitude of the self-thinking, namely, $\Delta_a$, relaxes the concentration of $m_1$ to $m_A^0$ or $m_B^0$, as shown in the top-left frame of Fig. 1. Next, we investigate the opinion formation, when the self-thinking of politicians at the binary exchange of opinions between two politicians, who belong to the same political party, is considered, namely, $0 < \Delta_a^{1\ell}$ ($\ell = A, B$). Here, we consider the effect of the self-thinking of politicians at the binary exchange of opinions of two politicians, who belong to the same political party, using two types of $\Delta_a^{1\ell}$ ($\ell = A, B$), namely, $\Delta_a^{1\ell} = 0.1$ and 0.5 ($\ell = A, B$), whereas $\Delta_a^{11}$ and $\Delta_a^{1\ell}$ ($\ell = A, B$) are equal to $\Delta_a = 0.1$. The top-right frame of Fig. 2 shows convergent forms of $f_1(m_1)$ versus $m_1$ in cases of $\Delta_a^{1\ell} = 0.1$ and 0.5 ($\ell = A, B$) together with the convergent form of $f_1(m_1)$ versus $m_1$ in the case of $\Delta_a^{1\ell} = 0$ ($\ell = A, B$). The increase of the amplitude of the self-thinking of politicians yields more dissipation of four shock like profiles, whereas the trimodal profile obtained using $\Delta_a^{1\ell} = 0$ ($\ell = A, B$) changes to the unimodal profile owing to the increase of dissipation in the case of $\Delta_a^{1\ell} = 0.5$ ($\ell = A, B$). The bottom-left frame of Fig. 2 shows convergent forms of $f_A(m_A)$ versus $m_A$ and $f_B(m_B)$ versus $m_B$ in cases of $\Delta_a^{1\ell} = 0.1$ and 0.5 ($\ell = A, B$). The increase of $\Delta_a^{1\ell}$ yields the increase of politicians, whose opinions are different from the opinion of the political party, namely, $m_A^0$ ($\ell = A, B$). Finally, we investigate temporal evolutions of global interests of peoples and politicians. Here, we plot the temporal evolution of the thermally relativistic measure, namely, $\chi_i = m_i c^2/(k\theta_i)$ ($k$: Boltzmann constant, $i = 1, A, B$) instead of $\theta_i$. Generally, matter with
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the binary exchange of opinions between two politicians in the same political party becomes
global interest of peoples suddenly increases in accordance with sudden increases of global
binary exchange of opinions between two politicians in the same political party. Finally, the
*N* component political parties using the one-dimensional relativistic Boltzmann-Vlasov equation for
in the same political party.

thinking becomes considerable at the binary exchange of opinions between two politicians
the same political party. Such a shock like profile are dissipated, as the amplitude of the self-
thinking of politicians is absent at the binary exchange of opinions between two politicians in
ourselves to the conciliatory exchange of opinions between two agents. In particular, the
citizens, and the constraint of the political party over opinions of politicians, when we restrict
multi-components. The opinion formation depends on the self-thinking of the people, politi-
further decreases at *t* = 22.15 in the case of *D* 1/2 = 0.5 (*ℓ* = *A*, *B*). Such sudden increases
(decreases) of *θ* 1 (χ 1) at *t* = 5.53 and 22.15 are caused by sudden increases (decreases) of
θA (χA) at *t* = 5.53 and θB (χB) at *t* = 22.15 in the case of *D* 1/2 = 0.5 (*ℓ* = *A*, *B*), as shown
in the bottom-right frame of Fig. 2. In our present study, we cannot conclude whether such a
difference between the time of the sudden increase of θA (*t* = 5.53) and that of θB
(*t* = 22.15) is caused by stochastic fluctuations of *f* 1 (*m* 1), *f* A (*m* A) and *f* B (*m* B) involved
with the DSMC method. In either case, sudden transitions of θ 1, θA and θB in the case of
*D* 1/2 = 0.5 are interesting phenomena, which will be investigated in our future study.

Conclusions. — We investigated the opinion formation among the peoples and multiple political parties using the one-dimensional relativistic Boltzmann-Vlasov equation for multi-components. The opinion formation depends on the self-thinking of the people, politicians, and the constraint of the political party over opinions of politicians, when we restrict ourselves to the conciliatory exchange of opinions between two agents. In particular, the shock like profile appears in the distribution of the opinion of the peoples, when the self-thinking of politicians is absent at the binary exchange of opinions between two politicians in the same political party. Such a shock like profile are dissipated, as the amplitude of the self-thinking becomes considerable at the binary exchange of opinions between two politicians in the same political party. *N* unified opinions by *N* political parties over opinions of politicians yield the 2*N* shock like profiles, when the self-thinking of politicians is absent at the binary exchange of opinions between two politicians in the same political party. Finally, the global interest of peoples suddenly increases in accordance with sudden increases of global interests of politicians in two political parties, when the amplitude of the self-thinking at the binary exchange of opinions between two politicians in the same political party becomes considerable.

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1 ≤ χ ≤ 100 is called as thermally relativistic matter, whereas matter with 0 < χ < 1 is
called as thermally ultrarelativistic matter [10]. The bottom-right frame of Fig. 2 shows
temporal evolutions of χ 1 in cases of Δ 1/2 = 0, 0.1 and 0.5 (*ℓ* = *A*, *B*) together with temporal
evolutions of χA and χB in the case of *D* 1/2 = 0.5 (*ℓ* = *A*, *B*). We cannot calculate χA and χB
in cases of *D* 1/2 = 0 and 0.1 (*ℓ* = *A*, *B*), because the modified Bessel function of the second
kind, which is necessary to calculate χℓ (*ℓ* = *A*, *B*), cannot be calculated using the present
numerical algorithm, when 500 ≤ χℓ (*ℓ* = *A*, *B*). The temporal evolution of χ 1 in the case
of Δ 1/2 = 0 (*ℓ* = *A*, *B*) is quite similar to that in the case of *D* 1/2 = 0.1 (*ℓ* = *A*, *B*). χ 1 in the
case of *D* 1/2 = 0.5 (*ℓ* = *A*, *B*) is quite similar to those in cases of *D* 1/2 = 0 and 0.1 (*ℓ* = *A*, *B*)
in the range of 0 ≤ *t* ≤ 5.53, whereas χ 1 in the case of *D* 1/2 = 0.5 (*ℓ* = *A*, *B*) deviates
from χ 1 in cases of *D* 1/2 = 0 and 0.5 (*ℓ* = *A*, *B*) in the range of 5.53 < *t*. In particular, χ 1
further decreases at *t* = 22.15 in the case of *D* 1/2 = 0.5 (*ℓ* = *A*, *B*). Such sudden increases
(decreases) of θ 1 (χ 1) at *t* = 5.53 and 22.15 are caused by sudden increases (decreases) of
θA (χA) at *t* = 5.53 and θB (χB) at *t* = 22.15 in the case of *D* 1/2 = 0.5 (*ℓ* = *A*, *B*), as shown
in the bottom-right frame of Fig. 2. In our present study, we cannot conclude whether such a
difference between the time of the sudden increase of θA (*t* = 5.53) and that of θB
(*t* = 22.15) is caused by stochastic fluctuations of *f* 1 (*m* 1), *f* A (*m* A) and *f* B (*m* B) involved
with the DSMC method. In either case, sudden transitions of θ 1, θA and θB in the case of
*D* 1/2 = 0.5 are interesting phenomena, which will be investigated in our future study.