Dependence of Dripping on the Orifice Diameter in a Leaky Faucet

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(Dated: January 18, 2022)

Abstract

We report the results of experiments that examined the dependence of the dripping dynamics of a leaky faucet on the orifice diameter. The transition of the dripping frequency between periodic and chaotic states was found to depend on the orifice diameter. We suggest a theoretical explanation for these transitions based on drop formation time scales. In addition, short-range anti-correlations were measured in the chaotic region. These too showed a dependence on the faucet diameter. Finally, a comparison was done between the experimental results with a one-dimensional model for drop formation. Quantitative agreement was found between the simulations and the experimental results.

PACS numbers: 47.55.db, 47.52.+j
I. INTRODUCTION

The dripping faucet is an everyday physical system that exhibits rich dynamical behavior. In particular it shows period doubling, strange attractors, and transitions between numerous periodic phases\[1\]. The leaky faucet system covers a wide range of interest, from free surface fluid singularities (drop formation)\[2\] to chaotic (dripping) dynamics\[1, 3\].

Drop formation is an ancient problem\[4\]. Plateau\[5\] was the first to point the importance of surface tension in generating the instability leading to drop break-up. However a full description of drop formation was not available until the end of the last decade, where the use of computer models simulated the formation of single drops\[2, 6, 7, 8, 9, 10, 11\].

The chaotic nature of the dripping faucet was first considered by Rössler\[3\]. His basic prediction concerning the behavior of a leaky faucet as a chaotic system was confirmed experimentally by Shaw\[1, 12\]. Yet the dripping dynamics of the leaky faucet still lacks a quantitative theoretical description. There are two complementing theoretical approaches that describe dripping in a leaky faucet. The first is Shaw’s mass-on-spring model\[1\] or similar models\[13, 14, 15, 16, 17, 18\], which give the basic characteristics of the dynamics of a leaky faucet. However, a full quantitative description of the system can not be deduced from such a model. The other approach is based on solving an approximate one-dimensional form of the Navier-Stokes equations of the system\[2, 6, 7, 8\], or a similar one-dimensional Langrangian based fluid mechanical equations of motion\[9, 10\]. The use of an approximate one-dimensional model is necessary due to the extensive consumption of computer time using the full two-dimensional model. However, it yields a relatively good approximate solution for drop formation\[2\], and also agrees qualitatively with the experimental measurements of the dripping dynamics. Yet several problems still remain using the one-dimensional model as will be outlined below.

The lack of a quantitative description of the dripping dynamics can be pinpointed into several basic questions. One of the main questions is the cause for the chaotic behavior of a leaky faucet, which is not yet clear. Shaw’s model\[1\] induces an unstable dripping state via the increasing influence of the vibrations of the drop (as the flow rate increases) on the dripping dynamics during drop formation. These vibrations are due to the act of the restoring surface tension. On the other hand the coupling between oscillations of the residue due to recoil and drop build-up\[9\] is another interesting option.
Most of the experiments in this field focus on measuring the time intervals between successive drops (dripping frequency). The dripping frequency is measured as a function of the flow rate\[1\]. Beside the obvious choice of flow rate, there are several possible control parameters on the dripping dynamics such as the diameter and shape of the faucet orifice, the surface tension and the viscosity of the fluid\[19\]. However the exact dependence of the dripping dynamics on these parameters has not yet been determined. An exception is temperature. The dependence of the dripping dynamics on temperature was measured and quantified\[21\] (see below).

In the present work we focused on the dependence of the dripping dynamics on the orifice diameter. We focused on the dependence of the transition frequency between chaotic and periodic states on the pipette diameter. Note that the size of the pending drop depends on the orifice diameter. Thus the dripping frequency and dynamics are determined also by the size of the orifice. In addition we compared our experimental results with numerical simulations based on the one-dimensional fluid-mechanical model described by Fuchikami et al.\[10\].

II. ASPECTS OF FAUCET DYNAMICS

The main aspects of the leaky faucet system relevant to our experiments are outlined below:

A. Drop Formation

Drop formation can be separated into three main stages\[9\]:

1. Build up time, during which the drop is formed: \( \tau_f \sim \frac{R}{v_0} \), where \( R \) is the orifice radius, and \( v_0 \) is the fluid velocity.

2. Critical time (criticality), where the drop breaks-off the fluid column: \( \tau_n \sim \sqrt{\frac{R^3 \rho}{\Gamma}} \), where \( \rho \) is the fluid’s density, and \( \Gamma \) is the surface tension.

3. Recoil time of the residual mass after break-up: \( \tau_d \sim \frac{V^{7/12}}{\eta^{1/12} \Gamma^{1/3}} \), where \( \eta \) is the viscosity, and \( V \) is the residual volume.

This separation is not strict - for instance, the recoil and the build-up stages overlap.
B. Dripping Dynamics

The dripping dynamics is influenced by many parameters. The most relevant ones from our experimental point of view are outlined below:

1. *Drop Volume*

The evolution of the drop’s volume prior to break-up is approximately linear with time. It incorporates a small oscillatory term, at least up to dripping rates as high as 10 drops/sec\(^2\). Note that the volume grows linearly even at criticality.

Also, at the periodic state the drop volume was shown to be approximately constant at a fixed flow rate\(^21\). At this state it was claimed that the volume of the drop increased while increasing the flow rate\(^21\).

2. *Faucet Diameter*

It has been shown *qualitatively* that the dripping dynamics depend on the diameter \(d\) and thickness of the orifice for relatively small pipettes \((d \leq 4mm)\)\(^22\). Note that in our experiment all the pipettes have a diameter that is less than 4.5\(mm\). The orifice wall thickness is especially important for thin faucets in which the ratio between the orifice wall and the inner pipette radius is less than 0.2\(^22\). For instance, in these thin faucets satellite drop formation was reduced considerably compared to thicker faucets\(^22\). However, the exact dependence of the dripping dynamics on the orifice diameter is not yet clear. In our experiments the ratio between the orifice wall and the inner pipette diameter was always larger than 0.4, thus we expect that this factor will not influence our results. Note also that in the following the term *faucet diameter* refers to the inner pipette diameter.

3. *Faucet Geometry*

The orifice geometry has a significant effect on dripping, as shown by various experiments\(^22, 23, 24, 25\). For instance, increasing the asymmetry of the orifice (cut angle) yielded a more stable dripping sequence. Increasing the faucet’s inclination softened the transition between one chaotic state to another, from a drastic one (boundary crisis)
to a smoother one \[25\]. In our experiments all the faucets had a single straight (cut angle) shape.

4. **Temperature**

Temperature changes influence the dripping dynamics, resulting in an approximate linear dependence between the effective flow rate and temperature: 

\[ \frac{dQ}{dT} \sim 4 \times 10^{-4} \text{ml/Ksec}, \]

for \(288K < T < 303K\) \[21\]. Note that the measured flow rate does not change due to temperature changes, but lowering the temperature results in a shift of the characteristic dripping patterns to higher flow rates \[26\].

5. **Periodic Perturbations**

The possibility of controlling the dripping dynamics by introducing external perturbations was examined for the leaky faucet \[27, 28\]. A periodic perturbation applied to the leaky faucet changed the dripping dynamics from a stable to a chaotic state \[27\]. Theoretical analysis of the mass-on-spring model combined with fluid-mechanical numerical simulations showed that a periodic perturbation induces discontinuous transitions between chaotic and periodic dripping states \[28\]. However the exact dependence of the dripping dynamics on external periodic perturbations is not yet clear, and deserves further examination.

C. **Transitions between Chaotic and Periodic States**

1. **Theoretical Model**

The transition between stable (periodic) and chaotic states can be attributed to various mechanisms \[29\]. Recently there has been an attempt to explain the origin of such transitions for dripping dynamics \[19\]. The suggested model connects between the transition from dripping to jetting to the transition from a periodic to a chaotic state \[19\]. However this model is mostly relevant to viscous fluids, which is not the case in our experiments.
2. **Transitions at High Flow Rates**

A transition from a chaotic to a stable state at high dripping rates was one of the initial observations done for the leaky faucet \[30, 31, 32\]. It was claimed that such a transition is a result of an inverse Hopf bifurcation near continuous flow \( f \leq 40Hz \) \[30, 31, 32\].

D. **The Reliability of One-Dimensional Simulations of Drop Formation**

The comparison between theory and experiment with respect to dripping dynamics is mostly done with the aid of one-dimensional simulations. A comparison between the more realistic two-dimensional (2D) Navier-Stokes equations and the approximate one-dimensional model revealed several problems with the approximate equations\[11\]. Mainly, the model cannot account for the faucet thickness, which influences drop formation\[11\], mostly for low viscous fluids under high flow rates.

III. **NUMERICAL SIMULATIONS**

The simulation is based on a one-dimensional Lagrangian description of the fluid motion. It aims at reconstructing drop formation for a fluid column stretching from an orifice. The algorithm was first introduced by Fuchikami et al.\[10\]. Briefly, an initial drop is decomposed into many thin disks for which separate equations of motion are written and solved. These equations are obtained from the Lagrangian equation under the influence of gravity, surface tension and viscosity.

The major assumptions incorporated in the model are:

1. The fluid is incompressible.

2. The drop is axisymmetric.

3. The horizontal component of the fluid velocity can be neglected in comparison with the vertical one.

4. The vertical component of the velocity depends only on the vertical coordinate.

5. There is no exchange of fluid between different discs.
The Lagrangian of the system is:

\[ L = E_{\text{kin}} - U_g - U_\Gamma \]  \hspace{1cm} (1)

Where \( E_{\text{kin}} \) is the kinetic energy, \( U_g \) is the gravitational energy, and \( U_\Gamma \) is the surface tension, as defined in reference no. [10]. The resulting Lagrangian equation of motion for the each disc is:

\[ \frac{d}{dt} \frac{\partial L}{\partial \dot{z}_j} = \frac{\partial L}{\partial z_j} + \frac{1}{2} \frac{\partial \dot{E}_{\text{kin}}}{\partial \dot{z}_j} \]  \hspace{1cm} (2)

Where \( z_j \) is the coordinate of disc \( j \), and \( \dot{X} \) represent the time derivative of \( X \). By integrating Eq. (2) we obtain the evolution of the drop in time and space. The integration was done using the 5'th order Runge-Kutta method [33]. After integration, the width of each disc was checked. If the width of some disc (termed the "singular disc") is less than a critical value (i.e. 0.01mm), than a drop is broken off, and the simulation restarts without the discs below the singular disc. Thus it is possible to calculate the time duration of drop formation from the starting point until break-off at a specific flow rate, and therefore to compare it with the experimental results.

IV. EXPERIMENTAL

Our experimental setup consisted of a primary water tank, connected to a laminar tube which was connected at its other end to a pipette through which water dripped. The laminar tube was used to stabilize the water flow. Drops falling from the pipette interrupted a laser beam directed at a photodiode sensor. The interruption caused a significant decrease of the sensor’s voltage, which was continuously recorded as a function of time by a computer. The recorded data was used to calculate the time difference between consecutive drops. The sensor’s resolution was 100,000 scans per second. A schematic drawing of the experimental setup is shown in figure [11]. In addition we used a stroboscope with a CCD camera in order to image drop formation in the periodic state.
V. EXPERIMENTAL RESULTS

The time intervals between consecutive drops were measured while continuously changing the flow rate. The flow rate was measured directly by filling for 30 seconds the funnel, and then weighing it. At the same time the water level (height) in the tank was measured. This procedure was repeated every 5 minutes for the entire experiment - until the tank was emptied. Poiseuille’s law describes the flow rate through a tube under a given pressure difference:

\[ Q = \frac{\Delta P \pi r^2}{8 l \eta} \]  

(3)

Where \( Q \) is the flow rate, \( r \) is the tube (faucet) radius through which the fluid flows, \( \Delta P \) is the pressure difference along the tube, and \( l \) the faucet’s length. Since \( Q \propto \frac{dh}{dt} \), and \( \Delta P \propto h \) where \( h \) is the water level inside the tank, the flow rate is expected to decrease exponentially. Such a decay was measured. Then after an exponential fit based on Poiseuille’s law was made to the measured flow rate as a function of time. The result was used to calibrate our experiments - given an initial water level it was then possible to calculate the exact flow rate from the recorded time that has elapsed since the beginning of the experiment. Note that the calibration was done for each faucet separately since the flow rate depends on the faucet diameter.

The resulting measurement of inter-drop time interval as a function of flow rate is shown in figure 2. Such measurements have been done for several different faucets, each with a specific orifice diameter. The diameters \( d \) of the pipettes ranged between \( 0.9 mm \leq d \leq 4.4 mm \).

Defining \( \Delta T \) as the inter-drop time difference, it is possible to distinguish between three major regions in figure 2. First a low frequency region for which \( \Delta T > 120 msec \), where the dripping period is quasi-stable. This state is roughly stable, interrupted occasionally by repeated structures of unstable states. A chaotic region is seen for \( 50 msec < \Delta T < 120 msec \) where there is a broad distribution of points and occasionally a periodic window with several specific dripping frequencies. Finally, a high frequency state for \( \Delta T < 50 msec \), where a periodic state was measured for \( d \geq 2.3 mm \) pipette diameters.

A quantitative comparison between the different dripping regions was done by calculating the normalized average deviation from the mean time interval between consecutive drops (SD):
\[ SD(\Delta T) = \frac{\sqrt{\Delta T^2 - \Delta T'^2}}{\Delta T} \]  \hspace{1cm} (4)

where \( \Delta T \) and \( \Delta T'^2 \) are the average values of \( \Delta T \) and \( \Delta T'^2 \) for 1000 consecutive drops. Note that changing the number of drops over which the average is made does not change the resulting \( SD \). Also, over 1000 drops the maximal flow rate change is 0.01 ml/sec, which is of the same order of magnitude as the error of the measured flow rate.

The results of these calculations for an experiment using a 4.4mm and a 2.3mm diameter pipettes are shown in figure 3. For clarity we define a chaotic region as one in which \( SD > 0.05 \). Similar to the raw data measurements shown in figure 2, figure 3 also shows three main distinguishable regions: a stable area for \( \Delta T \leq 50 \text{msec} \), a chaotic area between \( 50 \text{msec} \leq \Delta T \leq 120 \text{msec} \), and another stable region for \( \Delta T \geq 120 \text{msec} \). Note that there is a significant difference for the transition frequency from a stable to a chaotic state between the different faucet measurements. This difference also exists for the transition from a chaotic to a periodic state at high flow rates.

A. Dripping Dependence on Orifice Diameter

In order to examine the dependence of the dripping dynamics on the orifice diameter we repeated the \( SD \) calculations for different pipettes. Then we extracted the flow rate at which a transition between chaotic and periodic dripping takes place. Figure 4 shows the dependence of the transition frequency on the pipette diameter \( d \). As seen in figure 4, decreasing the orifice diameter increases the transition frequency from a periodic to a chaotic state. As noted above, for relatively large orifices \((d \geq 2.3 \text{mm})\) there is a transition into a high frequency stable state from a chaotic state. As seen in the inset of figure 4, this transition is also diameter dependent: increasing the orifice diameter decreases the critical dripping frequency at which the transition occurs. It should be noted that all the experiments were repeated several times under different temperatures for each faucet. Note also that the overall change in temperature between different experiments was no more than 10K. The resulting difference in the effective flow rate is much less than the one measured between different pipette experiments. Therefore this change gives a lower limit on the error bars of the measured transition frequencies. Thus, temperature and possible
external periodic noise can not account for the changes in the transition frequencies, and it is concluded that these changes are due to the difference in the faucet diameter.

**B. Anti-correlation Measurements**

Correlations between inter-drop time intervals have been measured for the leaky faucet system [35]. These correlations were claimed to be non-Gaussian and long-ranged [35].

In order to quantify such correlations we define a correlation function:

\[
C(n) = \frac{1}{1000} \sum_m \{dt_m - \langle dt \rangle\} \{dt_{n+m} - \langle dt \rangle\}
\]

where \(dt_n\) is the time difference between \(n\) consecutive drops, the brackets \(\langle X \rangle\) denote the average value of \(X\) (for 1000 consecutive drops), and the sum is over \(m = 1000\) consecutive drops. Note that in \(C(n)\) small \(n(< 5)\) represents short-range correlations, while large \(n(> 10)\) represents long-range correlations. Also, correlations are signified by low values while anti-correlations are emphasized by (absolute large) negative values.

We focused on calculating short-range correlations. The calculations were done for different faucets. A typical calculation of short-range correlations \(C(1)\) is shown in figure 5 for a 4.4mm diameter faucet. As shown in the figure, near the stable/chaotic transitions (\(\Delta T \sim 50\)msec and 110msec) there are strong correlations emphasized by the relatively low value of the correlation function. However, in the center of the chaotic area there are short-range anti-correlations where the correlation function gains a large (absolute) negative value. Moreover, these short-range anti-correlations also depend on the faucet orifice. As can be clearly seen in figure 6, the dripping frequency at which \(C(1)\) is maximal increases while decreasing the orifice diameter. In order to emphasize this point we shifted the correlation function calculations of different faucets on a time scale \(t\) equal to \(t = (2.6 - d) \cdot 8\), where \(d\) is the faucet diameter in millimeters. The result is shown in figure 7, where it can be clearly seen that all measurements collapse into a roughly approximate single curve. However this shifting gives only a qualitative picture since the estimated errors on the shift are large.
C. Dripping Imaging

Several images of drop formation were taken with the aid of a stroboscope and a CCD camera. The resulting images, done using a 4.4 mm pipette diameter at the periodic state, showed that the volume of the drop is approximately constant at this state. In addition, the images showed that the drop volume decreases while increasing the flow rate, contrary to the claim in the literature\cite{21}. In order to address this question quantitatively, we measured the average drop mass as a function of the flow rate for different experiments. The measurements were done by filling the funnel with drops for 30 seconds during which the flow rate did not change considerably. Then the water in the funnel was weighed. At the same time using the computer record of the photo-diode sensor, the number of drops falling from the orifice was counted. Dividing the fluid mass by the number of drops yielded the average drop mass. This procedure was repeated at a period of 5 minutes between each collection for an entire experiment. The result of these measurements is shown in figure 8. As can be clearly seen the average drop mass decreases while increasing the flow rate.

D. Numerical Simulation Results

Numerical simulations were made for a 4.4 mm diameter faucet. The resulting inter-drop time difference for different flow rates was used to calculate both $SD$ and anti-correlations. The $SD$ calculations for both simulations and measurements are shown in figure 9. The comparison between experiment and simulation is relatively good for most dripping rates. The comparison between short range anti-correlations drawn from simulations and measurements is shown in figure 4. Again, the agreement between theory and experiment is relatively good. Hence the approximations made in the numerical simulations have proven to hold also for low viscous fluids.

VI. DISCUSSION AND CONCLUSIONS

The dependence of the dripping dynamics on the orifice diameter was examined. We found that the system is driven to a chaotic state at a lower dripping frequency for larger faucets. Such a dependence of the transition frequency on the faucet diameter can be explained using a time-scale model for the transition between stable and chaotic states.
Specifically, we require that in the chaotic state the ratio between the recoil time $\tau_d$ and the drop build-up time $\tau_f$ should be greater than 1. The transition frequency from a stable to chaotic state is set to the dripping frequency in which both times equalize: $\frac{\tau_d}{\tau_f} = 1$. Note that the dripping frequency $f$ is defined as: $f = \frac{1}{\tau_f + \tau_n}$. The transition frequency dependence on faucet diameter stems from the fact that the above ratio depends on it. The result is shown in figure 10. The theoretical transition frequency decreases while increasing the faucet diameter, roughly consistent with the experimental results that are also shown in the graph.

In addition, we found that the high frequency transition from a chaotic to a periodic state is also faucet (diameter) dependent, where increasing the faucet diameter decreases the transition frequency. Again, we suggest a theoretical explanation for such a transition. Our hypothesis is based on the assumption that when the break-up time $\tau_n$ equals the drop build-up time $\tau_f$, the dripping dynamics stabilize. This is due to the fact that above this dripping frequency the important (larger) time scale is the break-up time $\tau_n$, which is constant for a given faucet. Therefore above this time scale $\tau_n > \tau_f$ the system is in a periodic state. The results are shown in figure 10. Although in this case the agreement between theory and experiment is only partial, it is still significant. This is due to the fact that the tendency of the transition frequency to decrease while increasing the faucet diameter is the same for both experimental results and theoretical predictions. Thus our results indicate that the transition between chaotic and periodic states is due to the interaction between drop formation and recoil, and possibly also to the critical state.

Finally, we measured short-time anti-correlations in the dripping faucet experiment inside the chaotic state. These too were shown to depend on the orifice diameter. We stress that these measurements strengthen the connection that has been drawn above between faucet diameter and dripping dynamics. This is emphasized by the fact that the shape of the (short-range) correlation curve shifts towards higher dripping frequencies while decreasing the faucet diameter. This shift represents also a similar shift of the transition frequencies between chaotic and periodic dripping, probed by a totally different method.

We also compared both short-range correlations and $SD$ calculations with numerical simulations. The comparison showed roughly good agreement between theory and experiments, emphasizing the strength of the one-dimensional simulations in giving a partial quantitative description of dripping dynamics.
Acknowledgments

We would like to thank the generous help of the late Moshe Kaganovich, who inspired all of us. We would also like to thank M. Hirshoren, A. Katz and G. Ben-Yosef for their technical support.

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FIG. 1: Schematic representation of the experimental system.

FIG. 2: Inter-drop time difference as a function of flow rate for a typical experiment. The measurement was done using a 4.4mm diameter faucet.
FIG. 3: Typical calculation of the normalized average deviation (SD). The solid circles represent calculations of SD for measurements done using a 4.4 mm diameter faucet. The hollow squares represent the same calculations for a 2.3 mm diameter faucet. The dashed line at $SD = 0.05$ separates the stable (below) and chaotic (above) dripping regions.

FIG. 4: The dependence of the transition frequency from a periodic to a chaotic state on the orifice diameter. Note the significant increase in the transition frequency while the orifice diameter decreases. The inset shows the dependence of the transition frequency from a chaotic to a periodic state (high frequencies) on the orifice diameter.

FIG. 5: The short-range correlation function $C(m=1)$ calculated for a 4.4 mm diameter faucet. The solid circles represent calculations based on experimental measurements. The hollow squares represent calculations based on the numerical simulations of drop formation using the one dimensional model.

FIG. 6: Measurements of the short-range correlation function $C(m=1)$ for various faucets. The measurements were shifted by a constant value (in the y-axis) for clarity. The hollow squares represent data for 4.4 mm diameter faucet, the solid squares for 2.6 mm, the hollow diamonds for 2.3 mm, the solid diamonds for 2.0 mm, the hollow triangles for 1.9 mm, the solid triangles for 1.4 mm, and the hollow circles for 0.9 mm diameter faucet. Note that no short range anti-correlations were measured for the 0.9 mm diameter faucet.

FIG. 7: Measurements of the short-range correlation function $C(m=1)$ for various faucets. The measurements are shifted linearly as described in the text. The symbols represent measurements of different faucets, as described in the caption of Fig. 8. Note that all measurements merge into an approximate single curve.

FIG. 8: The dependence of the drop’s mass on the flow rate. The measurements were made for the 4.4 mm diameter faucet.
FIG. 9: Comparison between the normalized average deviation (SD) calculations based on experimental results for a 4.4mm diameter faucet (solid squares) and for the one-dimensional model (hollow circles).

FIG. 10: Comparison between measured and estimated dripping frequency transitions. The squares refer to a transition from a chaotic to a periodic state, and the circles to a transition from a stable to a chaotic state. The solid symbols represent theoretical estimations which are explained in the text. The hollow symbols represent measurements. Note that a transition to a high-frequency stable state was not measured for small diameter ($d < 2.0mm$) faucets. Note also that all error bars are included inside the symbols size.
