Critical Temperature Differences of a Standing Wave Thermoacoustic Prime Mover with Various Helium-Based Binary Mixture Working Gases

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Abstract. Thermoacoustic prime movers are energy conversion devices which convert thermal energy into acoustic work. The devices are environmentally friendly because they do not produce any exhaust gases. In addition, they can utilize clean energy such as solar-thermal energy or waste heat from internal combustion engines as the heat sources. The output mechanical work of thermoacoustic prime movers are usually used to drive a thermoacoustic refrigerator or to generate electricity.

A thermoacoustic prime mover with low critical temperature difference is desired when we intend to utilize low quality of heat sources such as waste heat and sun light. The critical temperature difference can be significantly influenced by the kinds of working gases inside the resonator and stack’s channels of the device. Generally, helium gas is preferred as the working gas due to its high sound speed which together with high mean pressure will yield high acoustic power per unit volume of the device. Moreover, adding a small amount of a heavy gas to helium gas may improve the efficiency of thermoacoustic devices.

This paper presents numerical study and estimation of the critical temperature differences of a standing wave thermoacoustic prime mover with various helium-based binary-mixture working gases. It is found that mixing helium (He) gas with other common gases, namely argon (Ar), nitrogen (N₂), oxygen (O₂), and carbon dioxide (CO₂), at appropriate pressures and molar compositions, reduce the critical temperature differences to lower than those of the individual components of the gas mixtures. In addition, the optimum mole fractions of He gas which give the minimum critical temperature differences are shifted to larger values as the pressure increases, and tends to be constant at around 0.7 when the pressure increases more than 2 MPa. However, the minimum critical temperature differences slightly increase as the pressure increases to higher than 1.5 MPa. Furthermore, we found that the lowest critical temperature difference for He-Ar mixture gas is around 66 °C which is achieved in pressure range of 1.5 MPa - 2.0 MPa and mole fractions of helium of 0.55 - 0.65. The He-N₂ and He-O₂ mixture gases demonstrate almost the same performances, both have the lowest critical temperature difference around 59 °C at pressures of 1.0 MPa - 1.5 MPa and helium’s mole fractions of 0.35 - 0.55. For all tested gases, the lowest critical temperature difference of around 51 °C is provided by He-CO₂ mixture gas at pressures of 0.5 MPa - 1.0 MPa with helium’s mole fractions of 0.15 - 0.40.
1. Introduction
Thermoacoustic prime movers (TAPMs) are energy conversion devices which convert thermal energy (temperature gradient) into acoustic work. The devices employ thermoacoustic effects which are mutual thermodynamic interactions between acoustic wave and solid that possesses a temperature gradient. They have simple structure without any moving parts, and therefore they have a good reliability and low construction and maintenance costs. The TAPMs are environmentally friendly because they do not produce any global warming substances like carbon dioxide gas, and they can utilize clean energy such as solar-thermal energy [1][2] or waste heat from internal combustion engines [3] as the heat sources for their operation. The output mechanical work of TAPMs are usually used to drive a thermoacoustic refrigerator [4] or to generate electricity.[5][6]

Thermoacoustic prime movers still have not achieved the efficiencies as high as those of conventional heat engines. The thermal efficiencies of standing wave TAPMs have so far been limited to 0.20 [7][8] because they employ the intrinsically irreversible thermodynamics cycle. On the other hand, travelling wave TAPMs which are based on inherently reversible Stirling cycle can reach thermal efficiency up to 0.3, but the structures are more complex.[8] However, one of the advantages of thermoacoustic devices is that they are able to work with low quality of heat sources. Hence, although they generally have low thermal efficiency, utilization of waste heat or solar-thermal energy to power the TAPMs would be interesting and beneficial.

The heart of a thermoacoustic device is a stack, which is a porous medium in where the thermoacoustic energy conversions take place. The stack is located inside an acoustic resonator filled by a working gas, as shown in Fig. 1 which is an illustration of a simple standing wave TAPM. By attaching a cold and a hot heat exchangers at both ends of the stack, a temperature gradient along the stack can be built and maintained. A minimum temperature gradient across the stack is required by a TAPM in order to be able to produce acoustic work.

When we intend to utilize the low quality of heat sources, we will need a TAPM which has a low critical temperature gradient. The critical temperature gradient can be significantly influenced by the kinds of working gases inside the resonator and stack’s channels. Generally, practitioners have preferred helium as the working gas because it has high sound speed which together with high mean pressure will yield high acoustic power per unit volume of the devices.[9]

In addition, by adding a small amount of a heavy gas to a light gas, thereby reducing the Prandtl number of the gas below 2/3 typical of pure gases, can improve the efficiency of thermoacoustic devices. The reduction on Prandtl number lowers the viscous dissipation, and has allowed significant improvement in the efficiency of standing wave thermoacoustic refrigerator.[9]

This paper presents a numerical study and estimation of critical temperature differences of a standing wave TAPM with various helium-based binarymixture working gases. This work provides a method to estimate the the critical temperature difference for binary gas mixture with various compositions in a standing wave TAPM with a stack made of a pile of wire-mesh screens. The calculation results would guide us to appropriately choose the working gas and its composition in applications of harnessing low-quality heat sources to power thermoacoustic prime movers.

2. Theory
2.1. Onset and critical conditions
In thermoacoustic prime movers, once the temperature difference across the stack reaches a starting point, the devices will start and maintain an acoustic oscillation by converting heat into acoustic power. This starting point is so-called the onset temperature difference ($\Delta T_{onset}$). Following Atchley, [10] the onset condition in a standing wave thermoacoustic prime mover can
Figure 1. Schematic illustration of a standing wave TAPM. $L_{RES}$ is resonator length, $L_{STK}$ is stack length, and HE refers to heat exchanger.

be analyzed by using the quality factor $Q$ which is expressed as

$$Q = -\frac{\omega \dot{E}_{st}}{\dot{W}}$$

(1)

where $\dot{E}_{st}$ is the stored acoustic energy, $\omega$ is the angular frequency of the oscillation, and $\dot{W}$ is the net power output of the entire device.

As shown in Fig. 1, a thermoacoustic prime mover typically consists of an acoustic resonator, a working gas, a stack, and two heat-exchangers. Hence, $\dot{W}$ can be expressed as $[10][11]

$$\dot{W} = \dot{W}_{RES} + \dot{W}_{HE} + \dot{W}_{STK}$$

(2)

where $\dot{W}_{RES}$ and $\dot{W}_{HE}$ are the dissipations (always negative) in the resonator tube and heat exchangers, respectively, and $\dot{W}_{STK}$ is the net acoustic power out from the stack which can be either positive or negative, depending on the temperature difference ($\Delta T$) imposed across the stack and its position in the standing wave.

As $\Delta T$ increases from zero, the thermal dissipations in the stack decrease. When $\Delta T$ increases further until a certain point, the acoustic generation in the stack exactly balances the dissipations in the stack, $\dot{W}_{STK} = 0$, and the temperature difference is referred as the critical temperature difference ($\Delta T_{crit}$). At this point, however, $\dot{W}$ is still negative and $Q$ is positive and finite.

From the description mentioned above, it is clear that a low critical temperature difference will lead to a low onset temperature difference. In the next subsection, the derivation of an analytical expression of the critical temperature difference for standing wave thermoacoustic prime movers based on the linear thermoacoustic theory is presented.

2.2. Critical temperature difference

An expression for the critical temperature difference ($\Delta T_{crit}$) can be obtained from the linear thermoacoustic theory. The time-averaged acoustic power $d\dot{W}_{STK}$ produced in the length $dx$ of a stack’s channel can be written in complex notation as $[9]

$$d\dot{W}_{STK} = \frac{1}{2} \text{Re} \left[ \tilde{U}_1 \frac{dp_1}{dx} + \tilde{p}_1 \frac{dU_1}{dx} \right]$$

(3)

where $U_1$ is the complex volumetric velocity, $p_1$ is the complex pressure, tilde ($\sim$) sign indicates a complex conjugate, and Re[ ] denotes the real part of a complex number.
For stack’s channels which have regular geometries, Eq. (3) can be written as [9]

\[
\frac{dW_{\text{STK}}}{dx} = -\frac{1}{2} r_v |U_1|^2 - \frac{1}{2r_k} |p_1|^2 + \frac{1}{2} \Re [g\tilde{p}_1 U_1]
\]  

(4)

where \( r_v \) is the viscous resistance per unit length, \( 1/r_k \) is the thermal-relaxation conductance per unit length, and \( g \) is the complex gain (or attenuation) constant for the volume flow rate, which are respectively defined as

\[
r_v = \frac{\omega \rho m}{A} \text{Im}[−f_v],
\]

(5)

\[
\frac{1}{r_k} = \frac{\gamma - 1}{\gamma} \frac{\omega A}{p_m} \text{Im}[−f_k],
\]

(6)

\[
g = \frac{f_k - f_v}{(1 - f_v)(1 - \sigma)} \frac{1}{T_m} \frac{dT_m}{dx}.
\]

(7)

In these equations, \( \rho_m \) is the mean density, \( p_m \) is the mean pressure, \( T_m \) is the mean temperature, \( \gamma \) is the ratio of specific heats, and \( \sigma \) is the Prandtl number of the gas, respectively, \( A \) is the cross sectional area of the stack, \( f_v \) and \( f_k \) are the Rott’s thermoviscous functions that enable the description of the three dimensional phenomena in the stack’s channel.

For the stack’s channel with circular pores geometry, the thermoviscous functions are [9]

\[
f_k = \frac{2J_1[(i - 1)R/\delta_k]}{J_0[(i - 1)R/\delta_k](i - 1)R/\delta_k},
\]

(8)

\[
f_v = \frac{2J_1[(i - 1)R/\delta_v]}{J_0[(i - 1)R/\delta_v](i - 1)R/\delta_v},
\]

(9)

where \( R \) is the pore radius, \( \delta_k \) and \( \delta_v \) are the thermal and viscous penetration depths, respectively, \( J_0 \) and \( J_1 \) are the zeroth- and first-order Bessel functions, respectively. The penetrations dephts are calculated as

\[
\delta_k = \sqrt{2k/\omega \rho c_p},
\]

(10)

\[
\delta_v = \sqrt{2\mu/\omega \rho},
\]

(11)

where \( k \) is the thermal conductivity, \( \mu \) is the dynamic viscosity, \( c_p \) is the isobaric specific heats, and \( \rho \) is the density of the gas.

The critical temperature gradient along the stack \((\nabla_x T_m)_{\text{crit}}\), i.e. \((dT_m/dx)_{\text{crit}}\), can be obtained by letting \( dW_{\text{STK}}/dx = 0 \). By assumming ideal gas and lossless planar acoustic standing wave so that the phase difference between \( p_1 \) and \( U_1 \) is \( \pi/2 \) and hence we have acoustic impedance \( Z(x) = p_1/U_1 = (\rho_m a/A) \tan(\omega x/a) \), we can get

\[
(\nabla_x T_m)_{\text{crit}} = \frac{2\pi}{\lambda} T_m \left[ \frac{\text{Im}[-f_k]}{1-\sigma} + (\gamma - 1) \frac{\text{Im}[-f_k]}{(1-f_v)(1-\sigma)} \tan(\frac{\omega x}{a}) \right].
\]

(12)

Here, \( x \) is the axial location inside the resonator tube measured from the pressure node of the standing wave, \( \lambda \) is the wavelength of the sound wave, and \( a \) is the sound speed in the gas.

We use the short-stack approximation in where the stack length \( (L_{\text{STK}}) \) is much shorter than the acoustic wavelength \( (\lambda) \). The stack is short enough so that the pressure and velocity inside the stack do not vary appreciably, and hence the critical temperature gradient can be considered constant along the stack. Then, the critical temperature difference \( (\Delta T_{\text{crit}}) \) across the stack is simply obtained by multiplying \( (\nabla_x T_m)_{\text{crit}} \) with the stack length. It can be seen from Eq. (12) that the critical temperature difference is influenced by the geometry of the stack’s channels, location of the stack, acoustic wavelength, and properties of the gas which are depending on the gas temperature.
2.3. Properties of binary-mixture gas
The specific heats per unit mass at constant pressure and constant volume for a binary mixture gas can be estimated as

\[ c_{p_{\text{mix}}} = x_1 C_{p_1} + x_2 C_{p_2} \]
\[ c_{v_{\text{mix}}} = x_1 C_{v_1} + x_2 C_{v_2} \]

where \( x_1 \) and \( x_2 \) are the mole fractions of species 1 and species 2, \( C_{p_1} \) and \( C_{v_1} \) are the heat capacities for species 1, \( C_{p_2} \) and \( C_{v_2} \) are the heat capacities for species 2, \( M_1 \) and \( M_2 \) are the molecular weights of species 1 and species 2, respectively. The ratio of specific heats for the binary mixture gas (\( \gamma_{\text{mix}} \)) can thus be computed by the ratio of Eqs. (13) and (14),

\[ \gamma_{\text{mix}} = \frac{c_{p_{\text{mix}}}}{c_{v_{\text{mix}}}} \]

The gas density for a binary mixture gas (\( \rho_{\text{mix}} \)) can be written as a linear combination of the individual component’s densities

\[ \rho_{\text{mix}} = x_1 \rho_1 + x_2 \rho_2 \]

where \( \rho_1 \) and \( \rho_2 \) are the densities of species 1 and species 2, respectively.

The thermal conductivity of a gas mixture (\( k_{\text{mix}} \)) can be approximated by [12]

\[ k_{\text{mix}} = \sum_{i=1}^{n} \frac{x_i k_i}{\sum_{j=1}^{n} x_j \phi_{k,ij}} \]

where

\[ \phi_{k,ij} = \frac{1}{2\sqrt{2}} \left( 1 + \frac{M_i}{M_j} \right)^{-\frac{1}{2}} \left[ 1 + \left( \frac{k_i}{k_j} \right)^{\frac{1}{2}} \left( \frac{M_j}{M_i} \right)^{\frac{1}{4}} \right]^2 \]

and \( i \) and \( j \) are species numbers, \( k_i \) is the thermal conductivity of species \( i \), \( M_i \) is molar mass of species \( i \), \( n \) is number of species (\( n = 2 \) for binary mixture). Similarly, the viscosity of a gas mixture (\( \mu_{\text{mix}} \)) can be approximated by [13]

\[ \mu_{\text{mix}} = \sum_{i=1}^{n} \frac{x_i \mu_i}{\sum_{j=1}^{n} x_j \phi_{\mu,ij}} \]

where

\[ \phi_{\mu,ij} = \frac{1}{2\sqrt{2}} \left( 1 + \frac{M_i}{M_j} \right)^{-\frac{1}{2}} \left[ 1 + \left( \frac{\mu_i}{\mu_j} \right)^{\frac{1}{2}} \left( \frac{M_j}{M_i} \right)^{\frac{1}{4}} \right]^2 \]

and \( \mu_i \) is the viscosity of species \( i \). These approximations for thermal conductivity and viscosity of a gas mixture have a reasonable accuracy.[14] The more accurate calculation methods can be found elsewhere.[15][16]

By using the expressions above, the Prandtl number of the gas mixture (\( \sigma_{\text{mix}} \)) and the sound speed in the gas mixture (\( a_{\text{mix}} \)) can respectively be calculated as

\[ \sigma_{\text{mix}} = \frac{\mu_{\text{mix}} c_{p_{\text{mix}}}}{k_{\text{mix}}} \]
\[ a_{\text{mix}} = \sqrt{\frac{\gamma_{\text{mix}} p_m}{\rho_{\text{mix}}}} \]
3. Calculation Procedure
In this section, it is presented a procedure to calculate the critical temperature difference for a standing wave thermoacoustic prime mover with a straight-closed resonator tube which has been shown in Fig 1. The detail description of the calculation method is given below.

3.1. Specifying the resonator
We suppose that the resonator length ($L_{\text{RES}}$) is 1 m. Both ends of the resonator are closed so that a half-wavelength standing acoustic wave can exist with a pressure node at the middle of the resonator tube. The fundamental frequency of the half-wavelength resonator is

$$f = \frac{a_{\text{mix}}}{2L_{\text{RES}}}.$$  \hspace{1cm} (23)

3.2. Calculating the stack properties
We consider that the stack is made of a pile of stainless-steel wire-mesh screens with mesh number ($n$) of 30 meshes/inches and wire diameter ($d_{\text{wire}}$) of 0.25 mm. The stack length ($L_{\text{STK}}$) is 4 cm and the location ($x$) of the stack’s center is 35 cm from the middle of resonator. The porosity ($\phi$) and hydraulic radius ($r_h$) of the stack can be estimated as [9]

$$\phi = 1 - \frac{\pi nd_{\text{wire}}}{4}$$  \hspace{1cm} (24)

$$r_h = d_{\text{wire}} \frac{\phi}{4(1 - \phi)}$$  \hspace{1cm} (25)

In this work, we use an approximate model of the wire-mesh stack, i.e. it is modeled as an array of circular-cross-section tubes, in where the radius of the tube’s cross-section is represented by the effective circular radius [17]

$$r_{\text{eff}} = \frac{\sqrt{D_h d_{\text{wire}}}}{2}$$  \hspace{1cm} (26)

where $D_h$ is the hydraulic diameter of the wire-mesh stack. This $r_{\text{eff}}$ is therefore the hydraulic radius of the array of circular-cross-section tubes ($r_{h,\text{tube}}$).

3.3. Calculating the properties of the binary-mixture gases
We investigate four helium-based binary-mixture gases as the working medium of the thermoacoustic prime mover, namely helium-argon (He-Ar), helium-nitrogen (He-N$_2$), helium-oxygen (He-O$_2$), and helium-carbon dioxide (He-CO$_2$). The molar mass ($M$) of the individual gases are listed in Table 1. We assume that the mean temperature ($T_m$) of the working gas in the thermoacoustic prime mover is 300 K, and the charged pressures (mean pressures, $p_m$) vary from 0.1 MPa until 3.0 MPa. The properties of the individual gases, such as thermal conductivity and dynamic viscosity, at this temperature and in this pressure range are collected from NIST Chemistry Webbook.[18] The properties of the binary mixture gases are estimated by using equations which have been described in Subsection 2.3.

| Gas    | He  | Ar  | N$_2$ | O$_2$ | CO$_2$ |
|--------|-----|-----|-------|-------|--------|
| $M$ (g/mol) | 4.00| 39.95| 28.01 | 32.00 | 44.01 |

Table 1. Molar mass of the individual gases.
3.4. Calculating the critical temperature difference

To calculate the critical temperature difference, first we compute the fundamental frequency of the resonator by using Eq. (23). Then, we calculate the thermal and viscous penetration depths \( \delta_k \) and \( \delta_v \) by using Eqs. (10) and (11) with \( \omega = 2\pi f \) and adaptations that the thermal conductivity is \( k_{\text{mix}} \), the density is \( \rho_{\text{mix}} \), the isobaric specific heat is \( c_{p_{\text{mix}}} \), and the viscosity is \( \mu_{\text{mix}} \). Next, the thermociscous functions \( (f_k \text{ and } f_v) \) were calculated by using Eqs. (8) and (9) where \( R = r_{\text{hub}} = r_{\text{eff}} \) (see Eq. (26)). After that, by inserting all required quantities which have been found into Eq. (12) we get the critical temperature gradient \( (\nabla_x T_{m})_{\text{crit}} \), and finally by multiplying it with the stack length \( L_{\text{stk}} \), the critical temperature difference \( (\Delta T)_{\text{crit}} \) is obtained. These calculations are carried out for various molar compositions and charged pressures of the gas mixtures.

4. Results and Discussion

Before we discuss about the calculation results for binary mixture gases, it would be good that we present the results for pure individual gases. Figure 2 shows the calculation results of the \( \Delta T_{\text{crit}} \) for each pure individual gases at various charged pressures from 0.1 MPa until 3.0 MPa. It can be seen that for pressures less than 1.0 MPa the \( \Delta T_{\text{crit}} \) of pure helium are much higher than those of Ar, N\(_2\), O\(_2\), and CO\(_2\) gases, while at higher pressures, all individual gases have not so different \( \Delta T_{\text{crit}} \) which are around 80\(^\circ\)C – 100\(^\circ\)C. As can be seen in Eqs.(8)–(12), the dependence of the critical temperature difference on the gas properties is not as clear. It appears in the ratio of specific heats \( \gamma \), Prandtl number \( \sigma \), and implicitly in the thermociscous functions \( f_k \) and \( f_v \).

The calculation results of \( \Delta T_{\text{crit}} \) for binary-mixture gases with various molar compositions at charged pressures from 0.3 MPa until 2.0 MPa are presented in Fig. 3. The left extreme of the abscissa equals 0 is associated with multiple \( \Delta T_{\text{crit}} \) of the individual heavier gases (Ar, N\(_2\), O\(_2\), and CO\(_2\)), whereas the right extreme equals 1 corresponds to \( \Delta T_{\text{crit}} \) of He gas. These all have been summarized in Fig. 2. We can, generally, see in Fig. 3 that the mixing of gases can yield in lower \( \Delta T_{\text{crit}} \) than their values of one or both of components of the gas mixtures. In other words, in this pressure range, there is an optimum molar composition of each gas mixture which gives a minimum critical temperature difference \( (\Delta T_{\text{crit}})_{\text{min}} \), except in the case of Fig. 3(A). We also

![Figure 2](image-url)
found that He-Ar mixture gas always has $\Delta T_{\text{crit}}$ higher than those of other mixture gases for a given molar composition. On the other hand, He-CO$_2$ gas always gives the lowest $(\Delta T_{\text{crit}})_{\text{min}}$ at a given charged pressure. In addition, He-N$_2$ and He-O$_2$ gases demonstrate almost the same $\Delta T_{\text{crit}}$ against variation of molar compositions at a given charged pressure.

In Fig. 3(A) the calculation results for gas pressure of 0.3 MPa is presented. The $\Delta T_{\text{crit}}$ of
pure helium is 233 °C, while those of pure Ar, N₂, O₂, and CO₂ gases are 101 °C, 79 °C, 82 °C, and 57 °C, respectively. This figure shows that mixing helium gas with other heavier gases pulls down the $\Delta T_{\text{crit}}$ below that of pure helium gas, but still higher than those of pure heavier gases. At pressure of 0.5 MPa, as shown in Fig. 3(B), only He-CO₂ curve has a minimum point, i.e. at mole fraction of He of around 0.15 which gives $(\Delta T_{\text{crit}})_{\text{min}}$ of 52 °C. Moreover, at pressures of 0.7 MPa and higher, each $\Delta T_{\text{crit}}$ curves has a minimum point, at different molar compositions, as can be seen in Figs. 3(C)−(F).

It is also found from Figs. 3(C)−(F) that the optimum mole fractions of helium which give the minimum critical temperature differences are shifted to larger values as the pressure increases, and tend to be constant at around 0.7 when the pressure increases to higher than 2 MPa, as can be seen more clearly in Fig. 4. However, as depicted in Fig. 5, the $(\Delta T_{\text{crit}})_{\text{min}}$ slightly increase when the pressure increases to higher than 1.5 MPa. In addition, Fig. 5 reaffirms that the minimum critical temperature differences of helium-based binary-mixture gases are lower than those of pure helium gas.

Table 2. The lowest $(\Delta T_{\text{crit}})_{\text{min}}$ for different binary-mixture gases and the related optimum ranges of charged pressures, optimum mole fractions of helium, and average percentages of $\Delta T_{\text{crit}}$-decreases from their values of single helium gas at the corresponding charged pressures.

| Gas   | Lowest $(\Delta T_{\text{crit}})_{\text{min}}$ (°C) | $p_{\text{m}}$ (MPa) | Mole fraction of He | Average decrease of $\Delta T_{\text{crit}}$ (%) |
|-------|-------------------------------------------------|-----------------------|---------------------|-----------------------------------------------|
| He-Ar | 66                                              | 1.5 - 2.0             | 0.55 - 0.65         | 15                                           |
| He-N₂ | 59                                              | 1.0 - 1.5             | 0.35 - 0.55         | 30                                           |
| He-O₂ | 58                                              | 1.0 - 1.5             | 0.35 - 0.55         | 31                                           |
| He-CO₂| 51                                              | 0.5 - 1.0             | 0.15 - 0.40         | 54                                           |

Table 2 summarizes the lowest $(\Delta T_{\text{crit}})_{\text{min}}$ for various binary-mixture gases together with the related optimum ranges of charged pressures, optimum mole fractions of helium, and average percentages of $\Delta T_{\text{crit}}$-decreases from their values of pure helium gas at the corresponding charged pressures. These results, generally, show that the mixing of helium gas with other common gases yield in significant decreases of the critical temperature differences. The additional advantage
of the gas mixing in this case is that we will need smaller amount of helium gas so that reducing
the construction cost of the thermoacoustic prime mover.

5. Future Works and Applications
To follow up these results, we are now constructing a standing wave TAPM with a wire-mesh
stack, and experiments will be performed to verify the estimation results. The aim is to find a
binary mixture working gas which is experimentally verified giving a low critical temperature
difference. The TAPM with a low critical temperature difference can potentially be applied in
factory area, for example, to harness the waste heat as the heat source in producing the output
work, which in turn can be used to generate electricity or to drive a thermoacoustic refrigerator.

6. Conclusions
Mixing helium gas with other common gases, namely argon, nitrogen, oxygen, and carbon
dioxide, at appropriate pressures and molar compositions, can reduce the critical temperature
differences to lower than those of the individual components of the gas mixtures. The optimum
mole fractions of helium which give the minimum critical temperature differences are shifted to
larger values as the pressure increases, and tend to be constant at around 0.7 when the pressure
increases to higher than 2 MPa. In addition, the minimum critical temperature differences
slightly increase when the pressure increases to higher than 1.5 MPa. Moreover, it is found
that the lowest critical temperature difference for He-Ar mixture gas is around 66°C which is
achieved in the pressure range of 1.5 MPa - 2.0 MPa and mole fractions of helium of 0.55 -
0.65. The He-N\textsubscript{2} and He-O\textsubscript{2} mixture gases demonstrate almost the same performances, both
have the lowest critical temperature difference around 59°C at pressures of 1.0 MPa - 1.5 MPa
and helium’s mole fractions of 0.35 - 0.55. For all tested gases, the lowest critical temperature
difference of around 51°C is provided by He-CO\textsubscript{2} mixture gas at pressures of 0.5 MPa - 1.0 MPa
with helium’s mole fraction of 0.15 - 0.40.

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