Large-\(N\) limit of the two-dimensinal Non-Local Yang-Mills theory on arbitrary surfaces with boundary

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Abstract

The large-\(N\) limit of the two-dimensional non-local U(\(N\)) Yang-Mills theory on an orientable and non-orientable surface with boundaries is studied. For the case which the holonomies of the gauge group on the boundaries are near the identity, \(U \simeq I\), it is shown that the phase structure of these theories is the same as that obtain for these theories on orientable and non-orientable surface without boundaries, with same genus but with a modified area \(V + \hat{A}\).

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1 Introduction

The two-dimensional Yang-Mills theory (YM\textsuperscript{2}) is a theoretical laboratory for testing ideas and concepts about four-dimensional QCD and also string theory. Moreover this theory was studied at large N beginning with [1] from a number of viewpoints [2, 3, 4, 5, 6, 7]. This theory has an exact stringy description in the limit of large number N of colours [1, 2, 4, 5]. It was shown that the coefficients of the 1/N expansion of the partition function of SU(N) YM\textsuperscript{2} are determined by a sum over maps from a two-dimensional surface onto the two-dimensional target space. Recently Vafa [8] has shown that topological strings on a class of non-compact Calabi-Yau threefolds is equivalent to two dimensional bosonic U(N) Yang-Mills on a torus. This correspondence come from the recent result on the equivalenc of the partition function of topological strings and that of four dimensional BPS black holes [9]

\[ Z_{BH} = |Z_{top}|^2 \]  

(1)

On the other hand the black hole partition function is given by the partition function of a quantum field theory living on the brane which produces the black hole

\[ Z_{brane} = |Z_{top}|^2 = Z_{BH} \]  

(2)

The partition function of the field theory on the brane reduces, for the ground state sector, to the YM\textsuperscript{2} theory. In [8, 10] the proposal of [9] was made more concrete by considering Calabi-Yau back ground of the form

\[ L_1 \oplus L_2 \rightarrow \Sigma_g \]  

(3)

where \( \Sigma_g \) is a Riemann surface of genus g and \( L_1, L_2 \) are line bundles such that \( \text{deg}(L_1) + \text{deg}(L_2) = 2g - 2 \). In this case, the relevant brane gauge theory reduces to a q-deformed version of two-dimensional YM theory on the Riemann surface \( \Sigma_g \). q-deformed YM\textsuperscript{2} can be regarded as a one-parameter deformation of the standard two-dimensional Yang-Mills theory [11] (to see recent progress in this topic refer to [12, 13, 14, 15]). In this paper we do not study these important and very interesting progress. Only I would like to emphasis on the new progress has been made in YM\textsuperscript{2} theory.

It is well known that YM\textsuperscript{2} is defined by the Lagrangian \( \text{tr}(\frac{1}{4} F^2) \) on a Riemann surface where \( F \) is the field strength tensor. This theory have certain properties, such as invariance under area preserving diffeomorphism and lack of any propagating degrees of freedom [16]. In a YM\textsuperscript{2} one starts from a B-F theory in which a Lagrangian of the form \( \text{itr}(BF) + \text{tr}(B^2) \) is used where \( B \) is an auxiliary field in the adjoint representation of the gauge group. There are, however, the many way to generalized these theories without losing properties. One way is so- called non-local YM\textsuperscript{2} (nlYM\textsuperscript{2}),and that is to use a non-local action for the auxiliary field [17]. It is remarkable that, the action of nlYM\textsuperscript{2} is no extensive. In non-local YM\textsuperscript{2} theories, the solution appear as some infinite summations over the irreducible representations of the gauge group. In the large - N limit, however, these summations are replaced by suitable path integrals over continuous parameters characterizing the Young tableaux, and saddle-point analysis shows that the only significant representation is the classical one, which minimizes some effective action. This continuous parameters characterizing the representation is a constrained, as the length of the rows of the Young tableau is non-increasing. So for small values of the surface area, the classical solution
satisfies the constraint; for large values of the surface area, it does not. Therefore the dominating representation is not the one, which minimizes the effective action. This introduces a phase transition between these two regimes.

In this paper we would like to study the non-local two-dimensional $U(N)$ Yang-Mills (nlYM$_2$) theories on an arbitrary orientable and non-orientable surface with boundaries. It is interesting to test the conjecture (2) for these types of theories, we hope to come back at future to this important problem.

2 Preliminaries

The partition function of a nlYM$_2$ on an orientable two-dimensional surface $\Sigma_{g,n}$ with genus $g$ and $n$ boundaries is as

$$Z_{g,n}(U_1, \ldots, U_n; A) = \sum_R d_R^{2g-n} \chi_R(U_1) \cdots \chi_R(U_n) e^{\omega[-A\Lambda(B)]}.$$ \hspace{1cm} (4)

$A$ is the surface area, $R$’s label the irreducible representation of the gauge group, $d_R$ is the dimension of the representation $R$, and $\chi_R(U)$ is the character of the holonomy $U_j$. $\omega$ is an arbitrary function of $B$, where $B$ is an auxiliary field in the adjoint representation of the gauge group. $\Lambda(B)$ is given in term of the representation $R$ as following

$$\Lambda = \sum_{k=1}^{p} \frac{\alpha_k}{N_{k-1}} C_k(R).$$ \hspace{1cm} (5)

Here $C_k$ is the $k$th Casimir of gauge group, $\alpha_k$’s are arbitrary constant. As one can see from (4), corresponding to each boundary a factor $\chi_R(U_i)/d_R$ appears in the expression for the partition function.

The representation $R$ of the gauge group $U(N)$ is characterized by $N$ integers $l_1$ to $l_N$, satisfying [18]

$$+\infty > l_1 > l_2 > \cdots > l_N > -\infty.$$ \hspace{1cm} (6)

The $k$th Casimir of gauge group is given by

$$C_k(R) = \sum_{i=1}^{N} [l_i^k - (N-i)^k].$$ \hspace{1cm} (7)

The group element $U$ has $N$ eigenvalues $s_1 = e^{i\theta_1}$ to $s_N = e^{i\theta_N}$. The character $\chi_R(U)$ is then

$$\chi_R(U) = \frac{\det \{e^{i\theta_k}\}}{\operatorname{van}(s_1, \ldots, s_N)},$$ \hspace{1cm} (8)

where $\operatorname{van}(s_1, \ldots, s_N)$ is the van der Monde determinant

$$\operatorname{van}(s_1, \ldots, s_N) = \prod_{i<j} (s_i - s_j).$$ \hspace{1cm} (9)

If we expand $\chi_R(U_i)/d_R$ around $U \approx I$ for the group $U(N)$ we obtain (for more details see [18])

$$\ln \left[ \frac{\chi_R(U)}{d_R} \right] = a \sum_i (s_i - 1) + b \sum_i (s_i - 1)^2 + c \left[ \sum_i (s_i - 1) \right]^2 + \cdots,$$ \hspace{1cm} (10)
where

\[ a = \frac{1}{N} \sum l_i - \frac{N - 1}{2}, \]  
\[ b = \frac{1}{2(N^2 - 1)} \left( \sum l_i^2 \right) - \frac{1}{2N(N^2 - 1)} \left( \sum l_i \right)^2 - \frac{1}{2N} \left( \sum l_i \right) + \frac{5N - 6}{24}, \]  
\[ c = -\frac{1}{2N(N^2 - 1)} \left( \sum l_i^2 \right) + \frac{1}{2} \sum l_i^2 + \frac{1}{24}. \]  

\[ 3 \text{ The large-} N \text{ limit of the } U(N) \text{ partition function} \]

In the large-\( N \) limit, one introduces the continuous variables [19]

\[ \phi(x) = -\frac{l_i(x)}{N}, \quad 0 \leq x = \frac{i}{N} \leq 1, \]

which represent the irreducible representation. In the large-\( N \) limit,

\[ \sum_i f(l_i) \rightarrow N \int_0^1 dx f[-N\phi(x)]. \]

So,

\[ a = -N \left[ \int_0^1 dx \phi(x) + \frac{1}{2} \right], \]  
\[ b = \frac{N}{2} \left\{ \int_0^1 dx \phi^2(x) - \left[ \int_0^1 dx \phi(x) \right]^2 + \int_0^1 dx \phi(x) + \frac{5}{12} \right\}, \]  
\[ c = \frac{1}{2} \left\{ -\int_0^1 dx \phi^2(x) + \left[ \int_0^1 dx \phi(x) \right]^2 + \frac{1}{12} \right\}. \]

In the large-\( N \) limit, the discrete eigenvalues \( s_j = e^{i\theta_j} \) are also represented by the eigenvalue density function \( \sigma(\theta) \) with \( \theta \in [-\pi, \pi] \), and one has [20]

\[ \sum_i f(\theta_i) \rightarrow N \int_{-\pi}^{\pi} d\theta \sigma(\theta) f(\theta). \]

So, if \( U \approx I \) we have

\[ s_j - 1 = i\theta_j - \theta_j^2/2 + \cdots, \]

Inserting Eqs.(16, 17, 18) in (10), one obtain

\[ \ln \left[ \frac{\chi_R(U)}{d_R} \right] = -iN^2 Q(U) \left[ \int dx \phi(x) + \frac{1}{2} \right] - \frac{N^2}{2} V(U) \left\{ \int dx \phi^2(x) - \left[ \int dx \phi(x) \right]^2 - \frac{1}{12} \right\}, \]

where

\[ Q(U) = \int d\theta \sigma(\theta) \theta \]  
\[ V(U) = \int d\theta \sigma(\theta) \theta^2 - \left[ \int d\theta \sigma(\theta) \theta \right]^2. \]
For the remaining part of the partition function one has
\[ d^\eta Re\omega[-A\Lambda(B)] = e^{S_0}, \quad \eta = 2 - 2g, \quad \text{(24)} \]
and [21](Note that \( l_i \) in this paper is the same as \( n_i - i + N \) in [21]).

\[ S_0[\phi] = -N^2\Omega\left(A \int_0^1 W[\phi(x)]dx\right) + N^2(1 - g) \int_0^1 dx \int_0^1 dy \log |\phi(x) - \phi(y)|, \quad \text{(25)} \]
where
\[ W[\phi(x)] = \sum_k (-1)^k \alpha_k \phi^k(x). \quad \text{(26)} \]

Here we have redefined the function \( \omega \) as
\[ \omega(-A\Lambda(R)) = -N^2\Omega\left(A \sum_{k=1}^p \alpha_k \hat{C}_k(R)\right) \quad \text{(27)} \]
where
\[ \hat{C}_k(R) = \frac{1}{N^{k+1}} \sum_{i=1}^N \ell_i^k \quad \text{(28)} \]
The large-\( N \) limit of the partition function (4) then becomes the following functional integral
\[ Z_{g,n}(U_1, \ldots, U_n; A) = \int D\phi e^{S[\phi]}, \quad \text{(29)} \]
where
\[ S[\phi] = S_0[\phi] + S'[\phi], \quad \text{(30)} \]
in which
\[ S'[\phi] = -iN^2 Q \left\{ \int dx \phi(x) + \frac{1}{2} \right\} - \frac{N^2}{2} V \left\{ \int dx \phi^2(x) - \left[ \int dx \phi(x) \right]^2 - \frac{1}{12} \right\}, \quad \text{(31)} \]
where
\[ Q = \sum_j Q(U_j), \quad V = \sum_j V(U_j). \quad \text{(32)} \]
The phase structure of the action \( S_0[\phi] \) is equivalent to the phase structure of the following action [21]
\[ S_0'[\phi] = -N^2\hat{A} \int_0^1 W[\phi(x)]dx + N^2(1 - g) \int_0^1 dx \int_0^1 dy \log |\phi(x) - \phi(y)|, \left(\text{(33)}\right) \]
with
\[ \hat{A} = 2A\Omega'\left(A \int_0^1 W[\phi(x)]dx\right). \quad \text{(34)} \]
Defining
\[ q = \int_0^1 dx \left[ \phi(x) + \frac{1}{2} \right], \quad \psi(x) = \phi(x) + \frac{1}{2} - q, \quad \text{(35)} \]
one arrives at
\[ Z_{g,n}(U_1, \ldots, U_n; A) = Z_1 Z_2, \quad \text{(36)} \]
where

$$Z_1 = \exp \left( -\frac{N^2 Q^2}{2A} \right),$$

(37)

and

$$Z_2 = e^{N^2V/24} \int \mathcal{D}\psi \exp \left\{ -\frac{N^2}{2} \left[ 2\hat{A} \int dx W[\psi(x)] + V \int dx \psi^2(x) - \eta \int dx dy \log |\psi(x) - \psi(y)| \right] \right\}. $$

(38)

For the special case $W[\psi(x)] = \frac{\psi^2}{2}$ we have

$$Z_2 = e^{N^2V/24} \int \mathcal{D}\psi \exp \left\{ -\frac{N^2}{2} \left[ (\hat{A} + V) \int dx \psi^2(x) - \eta \int dx dy \log |\psi(x) - \psi(y)| \right] \right\}. $$

(39)

It is seen that the partition function (39) is in fact equal to the partition function on a closed orientable surface with genus $g$ and modified surface $\hat{A} + V$:

$$Z_2 = \hat{Z}_{g,0}(V + \hat{A})$$

(40)

Then

$$\log[Z_{g,n}(U_1, \ldots, U_n; A)] = \frac{N^2}{2} \left( V - \frac{Q^2}{A} \right) + \log[\hat{Z}_{g,0}(\hat{A} + V)].$$

(41)

As one can see, the logarithm of the partition function on a surface with boundaries has been written in terms of the logarithm of the partition function on a surface without boundaries, but with modified area $\hat{A}$. This relation is an approximate and is valid only if the holonomies corresponding to the boundaries don’t differ much from unity. The area-dependence of the partition-function corresponding to a surface without boundaries is known, and it is known that at some specific area ($\pi$ squared for ordinary Yang-Mills[22]) it undergoes a third order transition, that is, at this point the third derivative of the logarithm of the partition function is discontinuous. The remaining term is obviously analytic, so it does not change the discontinuity of the third derivative. Hence the phase structure of the non-local two-dimensional Yang-Mills theory on an orientable surface with boundary come from $\hat{Z}_{g,0}(V + \hat{A})$ and it has third order phase transition only on surfaces with $g = 0$.

Now we consider the similar problem for two-dimensional non-local Yang-Mills theory on a non-orientable surface with area $A$, genus $g$, $s$ copies of Klein bottle, $r$ copies of projective plan, and $n$ boundary. The partition function of this theory is as

$$Z_{g,n}(U_1, \ldots, U_n; A) = \sum_R d_R^{2-(2g+2s+r+n)} \chi_R(U_1) \cdots \chi_R(U_n) e^{\omega[-A\Lambda(B)]},$$

(43)

where the summation is only over self-conjugate representation of the gauge group. This requirement in $U(N)$ means that there is the additional constraint to the sums as

$$n_i = -n_{N-i+1}$$

(44)

This does not modify the actual area of the surface, of course. The point is that $\log[Z_{g,0}(A)]$ is known to have a discontinuity (in its third derivative) at $A = A_c$. From Eq.(41), it is seen that $\log(Z)$ has a discontinuity in its third derivative, at $\hat{A} + V = A_c$. This means that the critical area is $A_c - V$. So there is a phase transition only for $\Sigma_{0,n}$, and the transition occurs at

$$A_c = \pi^2 - V.$$
In the large-N limit, this implies that the continuum variables, \( \phi(x) \), satisfy

\[
\phi(x) = -\phi(1 - x) \tag{45}
\]

So one can define a new function such as

\[
\phi(x) = \psi(x) \quad \text{for} \quad 0 \leq x \leq 1/2; \\
\phi(x) = -\psi(1 - x) \quad \text{for} \quad 1/2 \leq x \leq 1 \tag{46}
\]

Here the function \( \psi(x) \) being defined on the interval \([0, 1/2]\), in which \( \psi(1/2) = 0 \). Then, by applying this constraint to the large-N limit of (43), one can arrive at

\[
Z_{g,n}(U_1, \ldots, U_n; A) = \int D\psi(x) \exp \left( S_0[\psi] + S'[\psi] \right), \tag{47}
\]

where

\[
S_0[\psi] = -N^2 \Omega \left( 2A \int_0^{1/2} W[\psi(x)] dx \right) + 2N^2 (1 - (g + s + r/2)) \int_0^{1/2} dx \int_0^{1/2} dy \log |\psi^2(x) - \psi^2(y)|. \tag{48}
\]

The phase structure of this term is equivalent to the phase structure of the following action,

\[
S_0'[\psi] = -N^2 \hat{A} \int_0^{1/2} W[\psi(x)] dx + 2N^2 (1 - (g + s + r/2)) \int_0^{1/2} dx \int_0^{1/2} dy \log |\psi^2(x) - \psi^2(y)|, \tag{49}
\]

where

\[
\hat{A} = 4A \Omega' \left( 2A \int_0^{1/2} W[\psi(x)] dx \right) \tag{50}
\]

and

\[
S'[\psi(x)] = -N^2 V \int_0^{1/2} \psi^2(x) dx, \tag{51}
\]

which coming from characters of the gauge group. Now we consider the special case \( W[\psi(x)] = \frac{1}{2} \psi^2(x) \), in this case the partition function (47) can be rewritten as

\[
Z_{g,n} = \int D\psi(x) e^{\left[-N^2(\hat{A} + V) \int_0^{1/2} \psi^2(x) dx + 2N^2(1 - (g + s + r/2)) \int_0^{1/2} dx \int_0^{1/2} dy \log |\psi^2(x) - \psi^2(y)|\right]}, \tag{52}
\]

It is seen that this partition function is equal to the partition function on a non-orientable surface with modified area \( \hat{A} + V \), genus \( g \), \( r \) copies of projective plane, \( S \) copies of Klein bottle and without boundaries. This model has third order phase transition [23] on non-orientable surface with boundary, \( g = 0, r = 1, s = 1 \) and modified area \((\hat{A} + V)\).

4 Conclusion

It was shown that the free energy of the U(\( N \)) YM\(_2\) on a sphere with the surface area \( A < A_c = \pi^2 \) has a logarithmic behavior [20]. In [22], the free energy was calculated for areas \( A > \pi^2 \), from which it was shown that the YM\(_2\) on a sphere has a third-order phase transition at the critical area \( A_c = \pi^2 \). For surfaces with boundaries, the situation is much more involved. In these cases, for each boundary, the character of the holonomy of the gauge filed corresponding to that boundary appears in the expression of the partition.
function. If we denote the $j$’th boundary by $C_j$, then each boundary condition is specified by the conjugacy class of the holonomy matrix $U_j = \text{Pexp} \oint_{C_j} dx^\mu A_\mu(x)$. So, the boundary condition corresponding to $C_j$ is fixed by the eigenvalues of $U_j$. These eigenvalues are unimodular, so that each of them is specified by a real number $\theta$ in $[-\pi, \pi]$. In the large-$N$ limit, the eigenvalues of these matrices become continuous and one can denote the set of these eigenvalues (corresponding to the $j$’th boundary) by an eigenvalue density $\sigma_j(\theta), \theta \in [-\pi, \pi]$.

In this paper we have studied the large-$N$ behavior of non-local YM$_2$ on an orientable and non-orientable genus-$g$ surface with $n$ boundaries ($\Sigma_{g,n}$). Here we have restricted ourselves to the cases in which the boundary holonomies $U_j$’s are close to identity. We have shown that the critical behavior of non-local YM$_2$ on $\Sigma_{g,n}$ with area $A(\Sigma_{g,n})$ is the same as a genus-$g$ surface with no boundary ($\Sigma_{g,0}$), but with the area $A(\Sigma_{g,0}) = A(\Sigma_{g,n}) + V(U_1, \ldots, U_n)$. Therefore, it is seen that in the large-$N$ limit, the phase structure of non-local YM$_2$ on $\Sigma_{g>0,n}$ is trivial, while non-local YM$_2$ on $\Sigma_{0,n}$ exhibits a third-order phase transition. Therefore the boundary conditions do not change the structure of the phase transition.

5 Acknowledgements

I gratefully acknowledge helpful discussion with Prof. M. Khorrami.

6 Appendix

It has been shown in [24] that in order to obtain the Yang-tableau density corresponding to the dominant representation, one should solve the generalized Hopf equation

$$\frac{\partial}{\partial t}(v \pm i\pi \sigma) + \frac{\partial}{\partial \theta} G[-i(v \pm i\pi \sigma)] = 0,$$

with the boundary conditions

$$\sigma(t = 0, \theta) = \sigma_1(\theta)$$

$$\sigma(t = \hat{A}, \theta) = \sigma_2(\theta).$$

(54)

Then, if there exists some $t_0$ for which

$$v(t_0, \sigma) = 0,$$

(55)

one denotes the value of $\sigma$ for $t = t_0$ by $\sigma_0$:

$$\sigma_0(\theta) := \sigma(t_0, \theta),$$

(56)

and the desired density satisfies

$$\pi \rho[-\pi \sigma_0(\theta)] = \theta$$

(57)

What is shown is that from this point of view, the non-local theory behaves like a local theory but with a surface area $\hat{A}$ instead of $A$. Note, however, that $\hat{A}$ itself depends on the Yang-tableau density of the dominant representation, through

$$\hat{A} = A\Omega \left( A \int_0^1 W[\phi(x)] dx \right).$$

(58)
or equivalently

\[ \hat{A} = A\Omega'\left( \int dz \rho(z) W(z) \right). \]  

(59)

In [20], the critical area for a Yang-Mills theory on a disk, \( \sigma_1(\theta) = \delta(\theta) \), has been found as:

\[ A_{c}^{-1} = \frac{1}{\pi} \int \frac{d\theta'}{\pi - \theta'} \sigma_2(\theta'). \]  

(60)

For a sphere, \( \sigma_2(\theta) = \delta(\theta) \), and one arrives at the familiar result

\[ A_c = \pi^2. \]  

(61)

These results can be used to obtain the critical area for a non-local Yang-Mills theory on a disk. One can obtain \( \hat{A}_c \) as

\[ \hat{A}_c = \left( \frac{1}{\pi} \int \frac{d\theta'}{\pi - \theta'} \sigma_2(\theta') \right)^{-1}. \]  

(62)

To obtain \( A_c \) from \( \hat{A}_c \), using (59), one needs the critical density \( \rho_c \). Even for the disk, it is not easy to find a closed form for \( \rho_c \) for arbitrary \( \sigma_2 \).

In our problem, where, the holonomies corresponding to the boundaries don’t differ much from unity, \( U \approx I \), \( \sigma(\theta) \) is nonnegligible only at \( \theta \) near zero. Expanding the denominator of (62) up to second order in \( \theta \), one arrives at

\[ \hat{A}_c = \pi^2 \left[ \int \frac{d\theta \sigma(\theta)}{\pi} + \frac{1}{\pi} \int \frac{d\theta \sigma(\theta)}{\pi} \theta + \frac{1}{\pi^2} \int \frac{d\theta \sigma(\theta)}{\pi} \theta^2 + \cdots \right]^{-1} = \pi^2 - \int \frac{d\theta \sigma(\theta)}{\pi} \theta^2 + \cdots. \]  

(63)

This is consistent with our general result (42), since in this case the second term of \( V \) in (23) vanishes:

\[ V \approx \int \frac{d\theta \sigma(\theta)}{\pi} \theta^2 \]  

(64)

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