Latest KLOE result on the $\sigma(e^+e^- \rightarrow \pi^+\pi^-)$ and its impact on the muon anomaly

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Abstract. The KLOE experiment was the first to exploit Initial State Radiation (ISR) processes to obtain the $e^+e^- \rightarrow \pi^+\pi^- (\gamma)$ cross section below 1 GeV (70\% of the leading order contribution to the muon anomaly) publishing two measurements with the photon in the initial state emitted at small angle in 2005 and 2008, and an independent measurement with the photon emitted at large angle in 2011. These measurements were normalized using luminosity from Bhabha. Here, we present the latest KLOE results on $e^+e^- \rightarrow \pi^+\pi^- (\gamma)$ cross section, obtained by the ratio $\sigma(e^+e^- \rightarrow \pi^+\pi^-) / \sigma(e^+e^- \rightarrow \mu^+\mu^-)$ measurement for a total integrated luminosity of 239.2 pb$^{-1}$. The comparison of the present results with with the previous ones performed by KLOE shows a good agreement, while they are in substantially agreement with the other ones obtained by CMD-2, SND and BaBar experiments. From the cross section we extract the pion form factor and the two-pion contribution in the $\sqrt{s}$ invariant mass range of 592-975 MeV to the muon anomaly confirming the current discrepancy between standard model calculation and experimental measurement.

1. Introduction

The muon anomalous magnetic moment is one of the most precisely measured quantities in particle physics and the actual discrepancy between standard model (SM) prediction and the experimental measurement arouses great attention and expectations of the international scientific community, because such difference between measurement and
calculation could be a signal of new physics. In fact, the measurements of the muon magnetic anomaly $a_\mu = (g_\mu - 2)/2$ performed at the Brookhaven Laboratory have reached a remarkable accuracy of 0.54 ppm: $a_\mu = (11 659 208.0 \pm 6.3) \times 10^{-10}$ [1], differing from Standard Model of about 3.2-3.6 standard deviations as estimated in Refs. [2–5].

A recent evaluation [6] finds a difference between 4.7 and 4.9 standard deviations. The authors of Ref. [7] have proposed an interpretation in terms of Supersymmetry, which can be probed at the Large Hadron Collider, so far without success. Another proposal suggests the existence of a light vector boson in the Dark Matter sector, coupled with ordinary fermions through photon exchange, which is not excluded by present low energy tests of the Standard Model [8, 9]. A new round of measurements of $a_\mu$ will soon begin at Fermilab[10] and JPARC[11], with the aim to considerably reduce the experimental error. To fully exploit the significance of improved measurements it is important to confirm the present estimate of the hadronic corrections and possibly to decrease the corresponding error.

The main source of uncertainty in the Standard Model estimates of $a_\mu$ [2, 3, 5] is due to hadronic loop contributions which are not calculable in perturbative QCD. To lowest order, the hadronic contribution $\Delta^{h, \text{lo}} a_\mu$, can be obtained from a dispersion integral [12, 13] over the “undressed” cross section $\sigma^0(e^+e^- \to \text{hadrons}(\gamma))$. The $e^+e^- \to \pi^+\pi^- (\gamma)$ process contributes some 75% of the $\Delta^{h, \text{lo}} a_\mu$ value and accounts for about 40% of the correction uncertainty.

In the following, we present the latest KLOE results on the $\sigma(e^+e^- \to \pi^+\pi^-)$ cross section and the impact on the $\Delta^{h, \text{lo}} a_\mu$ determination.

2. Measurement of $\sigma_{\pi\pi}$ with ISR

The KLOE detector operates at the DAΦNE $e^+e^-$ collider of the INFN Laboratori Nazionali di Frascati. It consists (Fig. 1) of a huge drift chamber providing high momentum resolution ($\sigma_p/p \leq 0.4\%$) [14] on reconstructed tracks and Pb-scintillating fibers calorimeter with excellent time ($\sigma_t \sim 54 \text{ ps}/\sqrt{E \text{ [GeV]}} \oplus 100 \text{ ps}$) and good energy ($\sigma_E/E \sim 5.7\%/\sqrt{E \text{ [GeV]}}$) resolution [15].

DAΦNE is a $\phi-$ factory running at $\sqrt{s} \approx M_\phi$, and has delivered ca. 2.5 fb$^{-1}$ of integrated luminosity to the KLOE experiment. So far KLOE has reported two measurements of the $\pi^+\pi^-$ cross section between 0.35 and 0.95 GeV$^2$ called KLOE05 [16] and KLOE08 [17] in the following. In addition, about 250 pb$^{-1}$ of data have been collected at $\sqrt{s} \approx 1 \text{ GeV}$, 20 MeV below the $\phi$-meson resonance, from which a new measurement of $\pi^+\pi^-$ cross section was obtained (KLOE10 [18]). Running at energies below the $\phi$-meson resonance reduces considerably the background from the copious $\phi$-meson decay products, including scalar mesons. The total cross section $\sigma_{\pi\pi} \equiv \sigma(e^+e^- \to \pi^+\pi^-)$ as a function of $s'$ (the $e^+e^-$ center-of-mass energy squared after ISR emission) is extracted from the differential cross section $d\sigma(e^+e^- \to \pi^+\pi^-)/ds'$
Figure 1. Schematic view of the KLOE detector with selection regions.

using formula [19]:

\[ s \cdot \frac{d\sigma_{\pi\pi\gamma}}{ds'} = \sigma_{\pi\pi}(s') \cdot H(s', s), \]  

with \( s \) the squared \( e^+e^- \) center of mass energy, and \( H \) the radiator function obtained from theory describing the photon emission in the initial state. An alternative method to extract the \( \pi^+\pi^- \) cross section uses the \( \pi^+\pi^-\gamma/\mu^+\mu^-\gamma \) ratio [20]:

\[ \sigma_{\pi\pi(\gamma)} = \frac{4\pi\alpha^2}{3s'} \left( 1 + 2m_\mu^2/s' \right) \beta_\mu \frac{d\sigma_{\pi\pi\gamma}/ds'}{d\sigma_{\mu\mu\gamma}/ds'}, \]  

with \( m_\mu \) the muon mass, \( \beta_\mu \) the muon velocity in the center-of-mass frame, \( d\sigma_{\pi\pi\gamma}/ds' \), \( d\sigma_{\mu\mu\gamma}/ds' \) the \( e^+e^- \rightarrow \pi^+\pi^-\gamma, e^+e^- \rightarrow \mu^+\mu^-\gamma \) differential cross sections, respectively. In both Eqs.(1) and (2) the Final State Radiation (FSR) terms are neglected, but are taken into account properly in the analyses.

3. Measurement of the pion form factor from the \( \pi\pi\gamma/\mu\mu\gamma \) ratio

The same sample as used for KLOE08 measurement, is analyzed with the small angle photon selection. Equation (2) has been used to extract the pion form factor via a bin-by-bin ratio between the observed pion and muon ISR differential cross sections. This approach has several benefits especially concerning radiative corrections. The integrated luminosity as well as the radiation function \( H \) and the vacuum polarisation cancel completely in the \( \pi\pi\gamma/\mu\mu\gamma \) ratio. However, the ratio of the acceptance enters in eq. (2) giving corrections of the order of few percents. The additional analysis on
muon events has to be performed at subpercent level, not a trivial task especially for the pion/muon separation. The analysis for $\pi\pi\gamma$ is essentially the same as for KLOE08 [17], while the one for $\mu\mu\gamma$ is completely new and just for its relevance will be briefly described below.

The signature for $e^+e^-\rightarrow\mu^+\mu^-\gamma$ events with the photon emitted at a small angle is just two tracks of opposite curvature, the photon being lost in the beam pipe. Four types of events contribute to the above signature: 1: $e^+e^-\rightarrow\mu^+\mu^-\gamma$ 2: $e^+e^-\rightarrow\pi^+\pi^-\gamma$ 3: $e^+e^-\rightarrow e^+e^-\gamma$, and 4: $e^+e^-\rightarrow\pi^+\pi^-\pi^0$. The four reactions can be distinguished kinematically. For the reaction $e^+e^-\rightarrow x^+x^-\gamma$, which is once kinematically overdetermined, we can compute the common mass of particles $x^+$ and $x^-$ by using the following equation: 

$$\left(\sqrt{s} - \sqrt{p_{x^+}^2 + m_{x^+}^2} - \sqrt{p_{x^-}^2 + m_{x^-}^2}\right)^2 - (p_{x^+} + p_{x^-})^2 = 0.$$ 

The four processes give $m_x = m_\mu$, $m_x = m_\pi$, $m_x = m_e$ and $m_x > m_\pi$, respectively. Additional $e\mu$ (and $e\pi$) separation is obtained from a PID estimator $L$ which uses time of flight information and the value and shape of the energy deposits of the two charged particles, of known momentum, in the calorimeter [17]. Fig. 1 shows the fiducial volume we use for muons and unobserved photons which is identical to that used in ref. [17] for $e^+e^-\rightarrow\pi^+\pi^-\gamma$. The computed mass for the two observed particle must satisfy $80 < m_x < 115$ MeV (see Fig. 2).

The $\mu\mu\gamma$ cross section measurement has been performed by subtracting the residual background $N_{\text{bkg}}$ (evaluated by fitting the $m_x$ spectrum of the accepted events with a superposition of Monte Carlo simulation distribution describing signal and concurrent channels, see for more detail [21]) to the observed event count $N_{\text{obs}}$ and dividing it for the selection efficiency $\epsilon(M^2_{\mu\mu})$ and integrated luminosity $\mathcal{L}$:

$$\frac{d\sigma_{\mu\mu\gamma}}{dM^2_{\mu\mu}} = \frac{N_{\text{obs}} - N_{\text{bkg}}}{\Delta M^2_{\mu\mu}} \frac{1}{\epsilon(M^2_{\mu\mu}) \mathcal{L}}.$$ 

The luminosity was evaluated, by using Bhabha events at large angle, to be $239.2$
pb$^{-1}$ with a systematic error of 0.3 %[22]. The upper panel of figure 3 shows in the comparison between the measured $\mu\mu\gamma$ cross section and the QED NLO calculation using the PHOKHARA MC code [20]. The lower panel shows the ratio between the two differential cross section. The average ratio (by using statistical error only) of $0.9981 \pm 0.0015$ shows a good agreement between the experiment and the theoretical evaluations within the estimated uncertainties.

The present analysis (KLOE12) has been checked by using different constraints on the events selection, such as a kinematic fit or tighter cuts on the quality of the charged tracks, all giving to consistent results[21]. Trigger, particle identification and tracking efficiencies have been obtained with data control samples[21].

From the ratio between the $\pi^+\pi^-\gamma$ data published in Ref. [17] and the $\mu^+\mu^-\gamma$ differential cross section described above, we obtain the $e^+e^-\rightarrow\pi^+\pi^-$ bare cross section presented in Fig. 4 (inclusive of FSR, VP effects removed):

$$\sigma_{0 \pi\pi(\gamma)}(s') = \sigma(\pi^+\pi^-, s') (1 + \eta_{\text{FSR}})/ (1 - \delta_{\text{VP}})$$

The pion form factor has been then extracted by using the following formula:

$$|F_\pi(s')|^2 = \frac{3}{\pi} \frac{s'}{\alpha^2\beta^2} \frac{\sigma_{0 \pi}(s') (1 + \delta_{VP})(1 - \eta_{\pi}(s'))}{s}$$

The table summarizing bin-by-bin the results on the bare cross section and the pion form factor are presented in Ref. [21].

The KLOE12 pion form factor compared with the one from KLOE10 and KLOE08, showing good agreement in both cases (see Fig. 5). As one can see in the comparisons reported in Fig. 6, KLOE12 are substantially in agreement with the measurements of CMD-2[23, 24], SND[25] (above the $\rho$-peak KLOE12 is slightly lower), and BaBar[26, 27] (above 0.6 GeV BaBar results are higher by 2-3%) experiments (see also the comparison with KLOE10 reported in Figs. 4 and 5 of Ref. [18]). The new measurement represents

Figure 3. Top: comparison of data (black points) and MCS (blue points) for the $\mu\mu\gamma$ absolute cross section as a function of $M_{\mu\mu}^2$ (GeV$^2$). Bottom: data to MC ratio. The green band shows the systematic error.
Figure 4. The bare cross section from the $\pi^+\pi^-\gamma/\mu^+\mu^-\gamma$ ratio.

Figure 5. Upper panels: pion form factor obtained with $\pi\pi\gamma/\mu\mu\gamma$ ratio KLOE12 (crosses) compared to the previous KLOE10 (left) and KLOE08 (right) analyses (points). Lower panels: the fractional difference of the new measurement and the previous ones.

an important check of previous measurements, because this last one does not rely on the value of the integrated luminosity and on the radiator knowledge.

The bare cross section $\sigma_0^{\pi\pi(\gamma)}$ is used to determine $a_{\pi\pi}$ via a dispersion integral:

$$a_{\pi\pi} = \frac{1}{4\pi^2} \int_{s_{\text{min}}}^{s_{\text{max}}} ds' \sigma_0^{\pi\pi(\gamma)}(s') K(s') ,$$

with $s_{\text{min}}$ and $s_{\text{max}}$ the lower and upper bounds in the present analysis, and $K(s)$ the kernel function described in [28]. By using Eq. (6), we obtain $\Delta a_{\pi\pi} = (385.1 \pm 1.1_{\text{stat}} \pm 2.6_{\text{exp}} \pm 0.8_{\text{th}}) \times 10^{-10}$ in the interval $0.35 < M_{\pi\pi}^2 < 0.95$ GeV$^2$. For each $M_{\pi\pi}^2$ bin contributing to the integral, statistical errors are combined in quadrature.
Figure 6. Left panel: pion form factor obtained with $\pi\pi\gamma/\mu\mu\gamma$ ratio KLOE12 (circles) compared to the CMD-2 (triangles) [23, 24] and SND (stars) [25] experiments as a function of $M_{\pi\pi}^2$. Right panel: dipion cross section KLOE12 (full circles) compared to the BaBar (empty circles) [26, 27] experiments as a function of $M_{\pi\pi}$.

and systematic errors are added linearly. A further advantage in the use of the $\pi\pi\gamma$ to

| Systematic sources                        | $\Delta^{\pi\pi}\alpha_\mu$ |
|-------------------------------------------|-------------------------------|
| Background subtraction                    | 0.6%                          |
| $m_x$                                     | 0.2%                          |
| PID                                       | negligible                    |
| Tracking                                  | 0.1%                          |
| Trigger                                   | 0.1%                          |
| Unfolding                                 | negligible                    |
| Acceptance                                | negligible                    |
| Software Trigger (L3)                     | 0.1%                          |
| Experimental systematics                  | 0.7%                          |
| Vacuum Polarization                       | –                             |
| FSR resummation                           | 0.2%                          |
| Radiator function $H$                     | –                             |
| Theory systematics                        | 0.2%                          |
| Total systematic error                    | 0.7%                          |

Table 1. List of systematic errors on the $\Delta^{\pi\pi}\alpha_\mu$ measurement. Most of them are smaller than the individual uncertainty on $\pi\pi\gamma$ and $\mu\mu\gamma$ due to correlation between the two analyses [21].

The $\mu\mu\gamma$ ratio method consists in a reduction of the systematic error in comparison with that published [17] due to correlations between the $\pi\pi\gamma$ and $\mu\mu\gamma$ analyses and a negligible theoretical uncertainty.

The present $a^{\pi\pi}_\mu$ estimation is compared with previous KLOE results (see Table 2).
These results are consistent with the previous estimation thus confirming the current disagreements of about $3\sigma$ between the $a_\mu$ experimental determination and its Standard Model prediction (see our preliminary estimate in Fig. 7).

| Analysis    | $\Delta a^{\pi\pi}_\mu (0.35 - 0.85 \text{ GeV}^2) \times 10^{10}$ |
|-------------|---------------------------------------------------------------|
| KLOE12      | $377.4 \pm 1.1_{\text{stat}} \pm 2.7_{\text{sys+theo}}$     |
| KLOE10      | $376.6 \pm 0.9_{\text{stat}} \pm 3.3_{\text{sys+theo}}$     |

Table 2. Comparison between the new result KLOE12 and the published KLOE10 and KLOE08.

Figure 7 shows our preliminary estimation based on DHMYZ10 (it has not to be considered as a new theoretical estimate) of the muon anomaly $a_\mu$ also including the KLOE12 measurement in comparison with recent theoretical and experimental evaluations. The aim of this last figure is to resume the recent status on $a_\mu$ estimation an to give an idea of the possible impact of the new KLOE measurements on the present scenario.

4. Conclusions

A evaluation of two-pion cross section, pion form factor, and leading order contribution to muon anomaly by using the $\pi\pi\gamma /\mu\mu\gamma$ ratio measurement (KLOE12) has been
performed, analyzing the 239.2 pb $^{-1}$ KLOE data. The comparison of this measurement with the previous KLOE08[17] and KLOE10[18] showed a good agreement and represents a powerful check, the $\pi\pi\gamma /\mu\mu\gamma$ result not depending on the integrated luminosity and on the radiator function knowledge. KLOE12 are also in substantially agreement with the measurements performed by the CMD-2[23, 24], SND[25], and BaBar[26, 27] experiments. We also presented the evaluation of $\sigma(e^+e^- \rightarrow \mu^+\mu(\gamma))$ normalized to integrated luminosity showing a good agreement with the PHOKHARA MC [20] prediction. The present evaluation of hadronic leading order contribution $a_{hlo}^\mu$ confirms the 3\(\sigma\) discrepancy between SM prediction and experimental measurement [1]. This result stresses on the importance of the expected future improvements on the experimental measurements of the muon anomaly, in program at Fermilab[10] and JPARC[11] laboratories.

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