On two weak CC $\Delta$ production models

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ABSTRACT
We perform a detail analysis of two models of neutrino CC $\Delta$ production on free nucleons. First model is a standard one based on nucleon-Delta transition current with several form-factors. Second model is a starting point for a construction of Marteau model with sophisticated analytical computations of nuclear effects. We conclude that both models lead to similar results.

PACS numbers: 13.15+g, 25.30.Pt

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1. INTRODUCTION

Future precise neutrino measurements e.g. of $\theta_{13}$ require better understanding of neutrino interactions in few GeV region [1]. In general, in description of neutrino-nucleon interaction vertex three processes are distinguished: quasielastic, single pion production (resonance excitation region) and more inelastic processes taken into account by the formalism of deep inelastic scattering. Weak single pion production is therefore a part of more complicated dynamics. It gives an important contribution to cross section in 1 GeV region and has to be treated with care. In the past it was a subject of many theoretical studies [2]. A sample of existing experimental data is not conclusive as measurements were made with a typical precision of 20-25% [3]. From a point of view of Monte Carlo simulation codes there seems to be an agreement that Rein-Sehgal [4] model is most reliable. It includes contributions from 18 resonances with masses up to 2 GeV, their interference terms together with a non-resonance background. Recent developments in quark-hadron duality suggest however that there is no need to consider so many resonances: contributions from most of them can be described in average by suitably modified PDF’s [5]. When reaction takes place on nuclear targets resonance contributions are additionally smeared out by Fermi motion. A conclusion is that probably only the $\Delta$ excitation has to be treated independently [6].

One way to describe $\Delta$ excitation is to construct a current $< \Delta | J^\mu | N >$ with phenomenological form-factor constrained by CVC and PCAC arguments [7]. There have been also attempts to calculate such form-factors from first principles in the quark model [8]. A precision with which form-factors are known cannot be better then experimental uncertainties.

Few authors tried to discuss nuclear effects in single pion production in a framework of more systematic theoretical schemes. One of such models was developed by Marteau [9]. It includes: Fermi motion, Pauli blocking, elementary 1p-1h, 1$\Delta$-1h and 2p-2h excitations, modification of $\Delta$ width in a nuclear matter, RPA corrections and finite volume effects. Before all the nuclear effects are taken into account, the model is based on simplified dynamical assumptions about CC neutrino $\Delta$ excitation on free nucleons. These simplifications are necessary in order to perform calculations of nuclear effects in compact and elegant way.

![Figure 1. Total cross section for $\pi^+$ production via $\Delta^{++}$ excitation. Experimental points are taken from [3]. No constraints on hadronic invariant mass are imposed.](image-url)
Figure 2. Total cross section of $\pi^+$ production via $\Delta^{++}$ excitation with a restriction on invariant hadronic mass $W \leq 1.4\text{GeV}$. Experimental points are taken from [3].

Figure 3. Total cross section of $\pi^+$ production via $\Delta^{++}$ excitation with a restriction on invariant hadronic mass $W \leq 1.6\text{GeV}$. Experimental points are taken from [3].

2. RESULTS

Marteau model [2] is a sophisticated model containing in one theoretical scheme neutrino CC quasielastic and $\Delta$ excitation reactions on nuclei. We present below its basic assumption for the differential cross section for CC neutrino $\Delta^{++}$ excitation on free nucleon:

$$d^2\sigma^\Delta_{M}\left[\cos^2\theta_e qM^3_{\Delta}\Gamma_{\Delta}\right] = \frac{G^2\cos^2\theta_e M^3_{\Delta}}{24\pi^2E(M + \omega)}\left((M + \omega)^2 - q^2 - M^2_{\Delta}\right)^2 + M^2_{\Delta}\Gamma^2_{\Delta}$$

In order to avoid a confusion we call this model pre-Marteau model. The above formula is derived in Appendix A, also the notation is explained there in detail.

In this paper we compare predictions of pre-Marteau model with the model for neutrino CC $\Delta$ excitation based on nucleon-$\Delta$ transition current with several form-factors. In the latter model [7] (we call it Form-Factor Model) one calculates cross section in a standard way. Straightforward

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computations lead to expression:

\[
\frac{d^2 \sigma_{F}^{\Delta^{++}}}{dW} = \frac{G^2 \cos^2 \theta_\Delta}{64 \pi^2 E^2} \frac{L_{\mu\nu} H^{\mu\nu} q d\omega dq}{(W - M_{\Delta}^2) + \Gamma_{\Delta}^2/4} \tag{2}
\]

In order to calculate \(H^{\mu\nu}\) one introduces nucleon-\(\Delta\) transition current \[15\]

\[
J^\alpha = \sqrt{3} \frac{\tilde{\Psi}_\mu(p')}{(C^Y_3(Q^2) - (g^{\alpha\alpha} \hat{q} - q^\mu \gamma^\alpha)} + C_4^Y (Q^2) M^2 (g^{\mu\alpha} q \cdot p' - q^\mu p'^\alpha) \gamma_5 + \frac{C_4^A (Q^2)}{M^2} (g^{\alpha\alpha} q \cdot p' - q^\mu p'^\alpha) \gamma_5 + (3) \frac{C_5^A (Q^2)}{M^2} q_{\mu} \gamma^\alpha u(p).
\]

where \(q^\mu = p'^\mu - p^\mu\). A factor \(\sqrt{3}\) is present in the current for \(\Delta^{++}\) production. In our numerical calculations we used the form factors from \[13\]. Equations (1) and (2) as they are written very similar. In fact the kinematics is the same in both cases. The inequivalence of two expressions comes from hadronic tensors \((\hat{H}^{\mu\nu})_{\bar{E}=0}\) which are calculated in different ways.

A comparison of predictions of two models was done for a free nucleon target because the existing experimental data applies to this situation. We compared total cross section in the energy range up to 5 GeV with data from \[3\]. We presented plots without bound on the invariant hadronic mass and with bounds 1.4 GeV and 1.6 GeV as such experimental data is available.

We made a decomposition of the total cross section into two parts: "transverse" and "longitudinal" according to spin-isospin operators present in the hadronic current. In the pre-Marteau model longitudinal operators are present only in \(H_{00}, H_{33}, H_{03}\) and transverse only in \(H_{11}, H_{12}\) (in the frame \(\tilde{q} = (0, 0, q)\)). Therefore we define ”longitudinal” and ”transverse” parts as those coming from corresponding terms in \(L_{\mu\nu} H^{\mu\nu}\).

For two theoretical models we performed also a comparison of differential cross sections for neutrino energy of 1 GeV and 2 GeV of: invariant mass, energy transfer and \(\cos \theta_\Delta\), an angle between momenta of incident neutrino and \(\Delta\).

Our conclusions are the following:

i) For neutrino energy lower than \(E_{\nu} = 5\) GeV (Fig. 1) pre-Marteau model gives rise to higher
values of the total cross section but both models approximately agree with the existing experimental data.

ii) For $E_\nu = 5 \text{GeV}$ Form-Factor Model predicts the values of the total cross section greater than pre-Marteau model. Pre-Marteau model cross section becomes approximately constant while Form-Factor Model cross section still increases with neutrino energy.

iii) With constraints on the value of invariant hadronic mass (Figs 2, 3) both models give rise to predictions similar in shape and in agreement with experimental data. Values of total cross section of pre-Marteau model are about 20% higher than predicted by Form-factor model.

iv) Contributions from "longitudinal" and "transverse" contributions (Fig. 4) to the total cross section are similar in both models.

v) Invariant hadronic mass distributions (Fig. 5, 6) are very similar, the only difference is in scale and comes from different values of the total cross section. For higher values of neutrino energy Form-Factor Model gives rise to greater contribution from higher values of $W$. This is why cuts on the invariant mass are more restrictive for that model.

vi) Differential cross sections of energy transfer (Fig. 7, 8) for both models are similar in shape. There is an surprising decline of differential cross section in energy transfer at about $\nu = 0.7 \text{GeV}$ for $E_\nu = 1 \text{GeV}$ (Fig. 7) and at about $\nu = 1.7 \text{GeV}$ for $E_\nu = 2 \text{GeV}$ (Fig. 8) present in predictions of both models. A possible explanation is kinematical in origin. For a fixed value of energy transfer $\nu$ the integration domain in momentum transfer $q$ is

$$q \in \left(\sqrt{\nu^2 - m^2 + 2E(E - \nu) - \rho}, \min\left(\sqrt{\nu^2 - m^2 + 2E(E - \nu) + \rho}, \sqrt{\nu^2 - m_{\pi}^2 + 2M(\nu - m_\pi)}\right)\right).$$

where

$$\rho \equiv 2E\sqrt{(E - \nu)^2 - m^2}.$$
Figure 8. Differential cross section of $\cos \Theta_\Delta$ for $\pi^+$ production via $\Delta^{++}$ excitation. Neutrino energy is $E_\nu = 1$ GeV.

to decrease quickly with $\nu$ because the first argument in $\min$ becomes smaller then the second one.

vii) In the differential cross section of $\cos \theta_\Delta$ ($\theta_\Delta$ is angle between $\Delta$ and incident neutrino momenta) (Fig. 9, 10) one can notice maxima at values $\cos \theta_\Delta \sim 0.875$ ($\theta_\Delta \sim 29^o$) for $E_\nu = 1$ GeV (Fig. 9) and $\cos \theta_\Delta \sim 0.775$ ($\theta_\Delta \sim 39^o$) for $E_\nu = 2$ GeV (Fig. 10). The maxima are present in predictions of both models. For neutrino energy $E_\nu = 2$ GeV (Fig. 12) there is also a second maximum in the forward direction. The shape of differential cross-sections is similar to the one derived in [13].

Our final conclusion is that pre-Marteau model for $\Delta^{++}$ excitation leads to close to standard behavior of $\pi^+$ production cross section and it is legitimate to use it in sophisticated computations of nuclear effects and in MC codes.

**APPENDIX A**

A logic of the Marteau model can be understood if one starts from the differential cross section for quasi-elastic process $\nu_\mu \ n \rightarrow \mu^- p$ in the Fermi gas model [14]:

\[
d^2\sigma^{qel}_{FG} = \int d^3pdq\theta(k_F - |p|)\theta(|p| + q - k_F) \\
\times \delta(E + E_p - E_{k_F} - E_{p'}) \\
\times \frac{G^2 \cos^2 \theta_c N_{k_F} q M M'}{16E_pE_{p'}E^2}L_{\mu\nu}H^{\mu\nu}. \tag{5}
\]

$k^\mu, k'^\mu, p^\mu, p'^\mu$ denote 4-momenta of: neutrino, charged lepton, target and recoil nucleons. $M$ and $M'$ are masses of target and ejected nucleons. In the case of quasielastic process they are taken as equal. $q^\mu = k^\mu - k'^\mu = (\nu, q)$ is energy and momentum transfer. $L_{\mu\nu}$ and $H^{\mu\nu}$ are leptonic and hadronic tensors:

\[
L_{\mu\nu} = 8\left(k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu}(k \cdot k')\right) \\
+ i\epsilon_{\mu\nu\alpha\beta}k'^\alpha k^\beta \tag{6}
\]

\[
H^{\mu\nu} = \frac{1}{8M M'}Tr\left(\Gamma^\mu(\hat{p} + M)\Gamma^\nu(\hat{p'} + M')\right), \tag{7}
\]

\[
\hat{\Gamma}^\nu = \gamma^0(\Gamma^\mu)\gamma^0, \tag{8}
\]
\[ \Gamma^\mu = F_1(Q^2)\gamma^\mu + iF_2(Q^2)\sigma^{\mu\nu} \frac{q_\nu}{2M} \]
\[ + G_A(Q^2)\gamma^\mu \gamma_5 + G_F(Q^2)\gamma^\mu \frac{q_\mu}{2M} \]  
\[ F_1, F_2, G_A \text{ and } G_F \text{ are standard form-factors.} \]
\[ N_{kF} = \frac{3A}{8\pi k_F^3} = \frac{2N}{(2\pi)^3}. \]
\[ \text{We assume nucleus of atomic number } A \text{ to contain equal numbers of protons and neutrons.} \]

In a good approximation (since we will actually calculate cross section for a reaction on free nucleons our derivation becomes exact; we perform all the steps of computations starting from the Fermi gas in order to check normalization factors) in \( \frac{1}{\hat{p} \cdot \hat{q}} H^{\mu\nu} \) we put \( \hat{p} = 0 \) (thus \( \hat{p} = \hat{q} \)). The dependence on \( \hat{p} \) factorizes and we define:
\[ \int d^3p\theta(k_F - |\hat{p}|)\theta(|\hat{p} + \hat{q}| - k_F) \]
\[ \times \delta(E + E_\hat{p} - E_\hat{q} - E_{\hat{p}'}) \]
\[ = -\frac{V}{2Nk_F^3} \mathcal{I} m\Pi_{N-h}(\nu, \bar{q}). \]

The expression for the cross section takes form:
\[ d^2\sigma_{PG}^{\text{rel}} = -\mathcal{I} m\Pi_{N-h}(\nu, \bar{q}) \]
\[ \times \frac{G^2 \cos^2 \theta_{\theta M} qV}{32\pi^2 E^2 E_{\bar{q}}} L_{\mu\nu}(H^{\mu\nu}) = 0 d\nu dq \]

In the original Marteau approach non-relativistic nucleon’s kinematics is used and \( \mathcal{I} m\Pi_{N-h}(\nu, \bar{q}) \) is a Lindhard function, the particle-hole polarization tensor, an object which accounts for Fermi motion and Pauli blocking [17] (in [9] higher order corrections in \( \frac{q}{M} \) are considered).

In the limit \( \hat{p} = 0 \) using non-relativistic decomposition of
\[ J^\mu = \mathcal{P}(\bar{q})\Gamma^\mu u(\bar{q}), \]
one can identify in
\[ H^{\mu\nu} = \frac{1}{2} \sum_{\text{spins}} J^\mu(J^\nu)^* \]
terms coming from different spin operators. For example we calculated [16]:
\[ J^0 = N_{p'}\phi_{s'}^\dagger \left( 1 - F_1 - \frac{q^2}{M' + E_{p'}} \right) \]
\[ + \hat{q} \cdot \hat{q}' \frac{G_A - \nu G_F}{M' + E_{p'}} \phi_s \]
where \( N_{p'} = \sqrt{E_{p'} + M'} \) and \( \phi's \) describe non-relativistic spinors.

Marteau \( \Delta \) excitation model is defined by [18]:
(i) substitution \( M' = M_{\Delta} \);
(ii) multiplication of form-factors by the numerical factor 4.78 = \( \sqrt{1 - E_{\Delta_m}^2} \),
(iii) elimination of “charge” terms (spin operator is necessary to produce a particle of spin 3/2) from \( H^{\mu\nu} \) - we call the new tensor \( \tilde{H}^{\mu\nu} \) with the numerical factor 4.78 included in its definition;
(iv) substitution of \( \Pi_{N-h} \) by \( \Pi_{\Delta-h} \) the polarization tensor for \( \Delta \)-hole excitation:
\[ \tilde{I} m\Pi_{\Delta-h}(\nu, \bar{q}) = -\frac{32}{9} \frac{1}{(2\pi)^3} \int d^3p\theta(k_F - |\hat{p}|) \]
\[ \times \frac{M_{\Delta}^2 \Gamma_{\Delta}} {W^2 - M_{\Delta}^2 + M_{\Delta}^2 \Gamma_{\Delta}} \]
\[ \text{(the factor } \frac{32}{9} \text{ comes from summation over isospin and spin degrees of freedom),} \]
\[ W^2 = (E_{\hat{p}} + \nu)^2 - (\hat{p} + \bar{q})^2 \]
\[ \text{(v) inclusion of RPA correlations and local density effects.} \]

In our derivation we restricted ourselves to steps (i-iv) and we obtained a model for \( \Delta \) excitation on nuclei in the free Fermi gas approximation:
\[ d^2\sigma_{PG}^{\text{rel}} = -\frac{G^2 \cos^2 \theta_{\theta M} qV M_{\Delta} \bar{q}} {32\pi^2 E^2 E_{\bar{q}}} \]
\[ \times \tilde{I} m\Pi_{\Delta-h}(\nu, \bar{q}) L_{\mu\nu}(\tilde{H}^{\mu\nu}) \]
\[ = 0 d\nu dq. \]

In the limit \( k_F \to 0 \) we obtained pre-Marteau model (in this limit target nucleon is at rest and
our evaluation of $H_{\mu\nu}^{\mu\nu}$ becomes exact):

$$\mathcal{I} m \Pi_{\Delta-h} \rightarrow -\frac{8A}{9W} \frac{M_\Delta^2 \Gamma_\Delta}{((M + \nu)^2 - q^2 - M_\Delta^2)^2 + M_\Delta^2 \Gamma_\Delta^2}. \quad (18)$$

Finally (after $k_F$’s get properly cancelled)

$$d^2 \sigma^\Delta = \frac{G^2 \cos^2 \theta_c M_\Delta^4 \Gamma_\Delta Aq}{36\pi^2 E^2(M + \nu)} \times \frac{L_{\mu\nu}(\tilde{H}^{\mu\nu})_{\mu\nu}}{((M + \nu)^2 - q^2 - M_\Delta^2)^2 + M_\Delta^2 \Gamma_\Delta^2} \quad (19)$$

$\Gamma_\Delta$ is defined as $(\Gamma_0 = 115 MeV)$

$$\Gamma_\Delta = \Gamma_0 \frac{q_{cm}(W)^3}{q_{cm}(M_\Delta)^3} \frac{M_\Delta}{W}, \quad (20)$$

where $q_{cm}(W)$ is the pion momentum in $\Delta$ (of mass $W$) rest frame.

Pre-Marteau model provides a prediction for an overall $\Delta$ production i.e. for a sum over isospin degree of freedom. Without nuclear effects relative probabilities to produce isospin states is given by a ratio of Clebsh-Gordan coefficients. Thus for neutrino induced reaction the probability to produce $\Delta^{++}$ is three times the probability to produce $\Delta^+$. In this paper we present a comparison for $\Delta^{++}$ production. It is because in the measurements of the invariant hadronic mass distribution for the process $\nu_\mu n \rightarrow \mu^- p \pi^0$ there is a sharp resonance peak at $W \sim 1.2$ GeV while in the channels $\nu_\mu p \rightarrow \mu^- n \pi^+$ and $\nu_\mu p \rightarrow \mu^- p \pi^0$ peaks are smeared out [3]. It is clear that correct description of the last two channels requires an addition of non-resonant contribution or/and contributions from other resonances while in the first one $\Delta^{++}$ production cross section can be meaningfully compared with the data. Prediction for $\Delta^{++}$ production per nucleon is thus obtained by dividing [13] by $\frac{3}{4}$ and multiplying by $\frac{5}{4}$. The final formula for $\Delta^{++}$ excitation cross section per proton in the pre-Marteau model reads:

$$d^2 \sigma_{\Delta}^{\Delta^{++}} = \frac{G^2 \cos^2 \theta_c q M_\Delta^4 \Gamma_\Delta}{24\pi^2 E^2(M + \nu)} \times \frac{L_{\mu\nu}(\tilde{H}^{\mu\nu})_{\mu\nu}}{((M + \nu)^2 - q^2 - M_\Delta^2)^2 + M_\Delta^2 \Gamma_\Delta^2}. \quad (21)$$

Acknowledgments

The author (supported by KBN grant 344/SPB/ICARUS/P-03/DZ211/2003-2005) thanks Krzysztof Graczyk for useful conversations.

REFERENCES

1. Proceedings of the First International Workshop on Neutrino-Nucleus Interactions in the Few GeV Region, Nucl. Phys. B (Proc. Suppl.) 112 (2002) NOVEMBER 2002.
2. S.L. Adler, Annals of Physics 50, (1968) 189-311; G.L. Fogli, G. Nardulli, Nucl. Phys. B160 (1979) 116.
3. S.J. Barish et al, Phys. Rev. D19 (1979) 2511; G.M. Radecky et al, Phys. Rev. D25 (1982) 1161.
4. D. Rein, L.M. Sehgal, Ann. Phys. 133, (1981) 79.
5. A. Bodek, U.K. Yang, Nucl. Phys. (Proc. Suppl.) 112 (2002) 70.
6. D. Casper, Nucl. Phys. (Proc. Suppl.) 112 (2002) 161.
7. C.H. Albright, L.S. Liu, Phys. Rev. Lett. 13 (1964) 673; C.W. Kim, Nuovo Cimento 37 (1965) 2858; S.M. Berman, M. Veltman, Nuovo Cimento 38 (1965) 5573.
8. J. Liu, N.C. Mukhopadhyay, L. Zhang, Phys. Rev. C52 (1995) 158.
9. J. Marteau, Eur. Jour. Phys. A5 (1999) 183; J. Marteau, J. Delorme, M. Ericson, Nucl. Phys. A 663 (2000) 783; J. Marteau, J. Delorme, M. Ericson, Nucl. Instrum. Meth. A 451 (2000) 76; J. Marteau, Nucl. Phys. (Proc. Suppl.) 119 (2002) 98, 203.
10. K2K Collaboration (M.H. Ahn et al.), Phys.Rev.Lett. 90 (2003) 041801; FINEsSSE Collaboration (L. Bugel et al.), A proposal for a near detector experiment on the booster neutrino beamline: FINEsSSE: Fermilab Intense Neutrino Scattering Scintillator Experiment, [hep-ex/0402007]
11. D. Rein, L.M. Sehgal, Nucl. Phys. B 223 (1983) 29.
12. P. Lipari, The primary protons and the atmospheric neutrino fluxes, in Venice
1999, Neutrino telescopes, vol. 1, 245-274, hep-ph/9905506; S. Nussinov, R. Shrock, Phys. Rev. Lett. 86 (2001) 2223; C. Bleve, G. Co, I. De Mitri, P. Bernardini, G. Mancarella, D. Martello, A. Surdo, Astropart. Phys. 16 (2001) 145; C. Maieron, M.C. Martinez, J.A. Caballero, J.M. Udias, Phys. Rev. C 68 (2003) 048501.
13. L. Alvarez-Ruso, S.K. Singh, M.J. Vicente Vacas, Phys. Rev. C 57 (1998) 2693; L. Alvarez-Ruso, E. Oset, S.K. Singh, M.J. Vicente Vacas, Nucl. Phys. A 663 (2000) 837.
14. R.A. Smith, E.J. Moniz, Nucl. Phys. B43 (1972) 605.
15. C.H. Llewellyn Smith, Phys. Rep 3, no 5 (1972) 261.
16. T. Ericson, W. Weise, Pions and Nuclei, Clarendon Press, Oxford, 1988; H. Sugawara, F. von Hippel, Phys. Rev. 172 (1968) 1764.
17. A.L. Fetter, J.D. Walecka, Quantum Theory of Many - Particle System, McGraw-Hill, New York 1971.
18. J. Marteau, De l'effet des interactions nucléaires dans les reactions de neutrinos sur des cibles d'oxygene et de son role dans l'anomalie des neutrinos atmospheriques, PhD Thesis, Lyon.