The flight of the bumblebee: solutions from a vector-induced spontaneous Lorentz symmetry breaking model

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Abstract. The vacuum solutions arising from a spontaneous breaking of Lorentz symmetry due to the acquisition of a vacuum expectation value by a vector field are derived. These include the purely radial Lorentz symmetry breaking (LSB), radial/temporal LSB and axial/temporal LSB scenarios. It is found that the purely radial LSB case gives rise to new black hole solutions. Whenever possible, Parametrized Post-Newtonian (PPN) parameters are computed and compared to observational bounds, in order to constrain the Lorentz symmetry breaking scale.

1. INTRODUCTION

Lorentz invariance is clearly one of the most fundamental symmetries of Nature. It is both theoretically sound and experimentally well tested [1, 2], thus playing a leading role in most theories of gravity. Therefore, it is only natural that little attention has been paid to the consequences of explicitly breaking this symmetry.

A more flexible approach to this question admits a spontaneous breaking of this symmetry, instead of an explicit one, analogously to the Higgs mechanism in the Standard Model of particle physics [3, 4, 5, 6]. This can arise if a vector field ruled by a potential exhibiting a minimum rolls to its vacuum expectation value (vev) – this vector field, usually referred to as “bumblebee” vector, thus acquires a specific four-dimensional orientation.

From a theoretical standpoint, a spontaneous Lorentz symmetry breaking (LSB) is possible, for instance, in string/M-theory, arising from non-trivial solutions in string field theory [3, 4] and in noncommutative field theories [7, 8]. A spacetime variation of fundamental coupling constants could also lead to a spontaneous LSB [9]. Experimentally, the violation of Lorentz invariance could be tested in ultra-high energy cosmic rays [10].

The consequences of the “bumblebee” vector scenario were studied in Ref. [11]; in there, three relevant cases were taken into account: the bumblebee field acquiring a purely radial \( v_{\text{vev}} \), a mixed radial and temporal \( v_{\text{vev}} \) and a mixed axial and temporal \( v_{\text{vev}} \). The results were analyzed in terms of the PPN parameters, when possible, prompting for comparison with current and future experimental bounds and effects, for instance, from string theory in a low-energy scenario. These bounds may arise from the observations of the Bepi-Colombo [12] and LATOR [13] missions (see also Ref. [14]) for a discussion on future gravitational experiments).

The action of the bumblebee model is written as

\[
S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\kappa} R + \xi B^\mu B^\nu \varepsilon_{\mu\nu} + \frac{1}{4} B^{\mu\nu}B_{\mu\nu} + V(b^\mu b_\mu + \hat{b}) \right\},
\]

where \( \kappa = 8\pi G, B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \xi \) is a coupling constant and \( b^2 \) sets the bumblebee’s \( v_{\text{vev}} \), since the potential \( V \) driving Lorentz and/or CPT violation is supposed to have a minimum at \( b^\mu b_\mu + \hat{b} = 0 \), i.e. \( V(\phi b^\mu + \hat{\phi}) = 0 \). The particular form of this potential is irrelevant, since one assumes that the bumblebee field is frozen at its \( v_{\text{vev}} \). The scale of \( b_\mu \) should be obtained from string theory or from a low-energy extension to the Standard Model. Hence, one expects \( b_\mu \) to be of order of \( M_{\text{Pl}} \), the Planck mass, or \( M_{\text{EW}} \), the electroweak breaking scale.
2. PURELY RADIAL LSB

In this section, a method to obtain the exact solution for the purely radial LSB is developed. A static, spherically symmetric spacetime, with a Birkhoff metric $g_{\mu\nu} = \text{diag}(1, r^2, r^2, r^2 \sin^2 \theta)$ is considered. It can be easily seen that the Killing vectors of the metric are conserved, showing that radial symmetry is still valid; this enables the construction of a covariantly conserved current associated with the vector bumblebee field [11].

The affine connection derived from the metric $g_{\mu\nu}$ allows for the computation of $b_\mu$, given the non-trivial covariant derivative with respect to the radial coordinate, and taking $b_\mu = \partial_\mu \psi \partial_0 \phi$. Hence, from $\partial_\mu b_\nu = \partial_\mu b_\nu \quad \Gamma^\alpha_{\mu\nu} b_\alpha = 0$, it follows that $b_\psi = \xi^2 b_0 e^\rho$, where the factor $b_\psi$ is introduced for later convenience. As expected, $b^2 = b^\mu b_\mu = b_0^2 = 1$ is constant.

The (spatial) action can be thus written as

$$S_s = \int d^3x \sqrt{-g} \frac{R}{2\kappa} + (\psi')^2 b_0^2 \Psi R_{rr}$$

The determinant of the metric is given by $\sqrt{-g} = r^2 e^{\psi} \phi$; the scalar curvature and the relevant non-vanishing Ricci tensor component are given by $R = 2r^{-2} + (2\rho^2 - 1)e^{-2\phi}$ and $R_{rr} = 2r^{-1}\rho^2$, where the prime stands for derivative with respect to $r$ and we have integrated over the angular dependence. Also, $\xi \phi' R_{rr} = b_0^2 2r^{-1}\rho^2 e^\rho$, where $b_\psi$ is the contravariant radial component of $b_\mu$. By introducing the field redefinition $\Psi = e^{2\phi} r^2$, the action may be rewritten as [15]

$$S_s = \frac{2}{\kappa} \int d^3x \Psi^2 (\beta + b_0^2) \Psi + (1 + b_0^2) r^2 \phi \Psi^0$$

Variation with respect to $\phi$ produces the equation of motion

$$\beta + b_0^2 \Psi + (1 + b_0^2) r^2 \phi \Psi^0 = 0$$

which admits the solution $\Psi(\psi) = \Psi_0 \psi^{-3+L}$, with $L = 3 - (\beta + b_0^2) = (1 + b_0^2) - 2$ and hence $L = \frac{5}{2}$. We thus obtain $g_{rr} = e^{2\phi} = 1 \Psi_0^{-1+L}$. Comparison with the usual Schwarzschild metric yields

$$g_{rr} = 1 \frac{2G_L m}{r} \psi^{-1} \quad \text{and} \quad g_{00} = 1 + \frac{2G_L m}{r} \psi^{-1}$$

where $G_L$ has dimensions $[G_L] = L^2$. In (natural units, where $\hbar = 1$); one can define $G_L = G r_0^L$, where $r_0$ is an arbitrary distance. The $L = 0$ limit yields $G_L = G$ and the usual geometrical mass, $M = \frac{G m}{r}$, with dimensions of length. From now on we express all results in terms of $M$. The event horizon condition is given by

$$g_{00} = 1 + \frac{2M r}{r_0} = 0$$

thus $r_s = (2Mr_0 L)^{1+L}$. The norm-square of the Riemann tensor, $I = R_{\mu\nu\rho\lambda} R^{\mu\nu\rho\lambda}$, is found to be

$$I = \frac{48}{3} L + \frac{17}{12} L^2 + \frac{1}{2} L^3 + \frac{1}{12} L^4 \quad M^2 \quad \frac{r}{r_0} \quad \frac{2L}{r^{6-L}} \quad \frac{5}{3} L \quad \frac{r}{r_0} \quad \frac{2L}{r_0} = 0$$

where $I_0 = 48M^2 r^6$ is the usual scalar invariant in the limit $L = 0$. Since $I(\psi = r_s)$ is finite, the singularity at $r = r_s$ is removable; accordingly, the singularity at $r = 0$ is intrinsic, as the scalar invariant diverges there. One concludes that an axial LSB gravity model admits new black hole solutions with a singularity well protected within a horizon of radius $r_s$. The associated Hawking temperature is

$$T = \frac{\hbar}{k_B 4\pi r_s} = \frac{2M r_0 L^3}{(L+1) r_0^4 L} \quad \text{and} \quad \frac{(2Mr_0 L)^{L} T_0}{(2Mr_0 L)^{L} T_0} = 0$$

where $T_0 = \hbar = 8\pi k_B M$ is the usual Hawking temperature, recovered in the limit $L = 0$. 

Since the obtained metric cannot be expanded in powers of $U = M=r$, a PPN expansion is not feasible. However, a comparison with results for deviations from Newtonian gravity [16], usually stated in terms of a Yukawa potential of the form

$$V_\nu(\nu) = \frac{G\gamma m}{r} \ 1 + \alpha e^{-\lambda r}$$ \quad (9)

yields $G_L^L = G \nu \ 1 + \alpha e^{-\lambda r}$, which, to first order around $r = r_0$, reads

$$G_L^L(r_0) = G \nu (1 + \alpha e^{-\lambda r_0}) \quad (10)$$

so that one identifies $\lambda = r_0$ and $\alpha = L$ (with $G \nu (l \ L) = G \nu r_0^2 = G$). Planetary tests to Kepler’s law in Venus indicate that $\lambda = r_0 = 0.723 AU$ and $L = 3.2 \times 10^3$.

### 3. RADIAL/TEMPORAL LSB

We consider now the mixed radial and temporal Lorentz symmetry breaking. As before, it is assumed that the bumblebee field $B_\mu$ has relaxed to its vacuum expectation value. Provided that one takes temporal variations to be of the order of the age of the Universe $H_0^{-1}$, where $H_0$ is the Hubble constant, a Birkhoff static, radially symmetric metric $g_{\mu\nu} = diag (e^{\phi}, e^{\phi}, e^{\phi} r^2, e^{\phi} r^2 \sin^2(\theta))$ may still be used. The physical gauge choice of a vanishing covariant derivative of the field $B_\mu$ yields $\delta_v = \frac{1}{2} A_0 e^\phi$ and, similarly, $b_0 = \frac{1}{2} A_0 e^\phi$, with $A_0$ and $A_r$ dimensionless constants. As before, $\frac{b^2}{b^2} = b^2 b_\mu = (A^2_0 \ A_0^2) \frac{q}{\xi} \ 1$ is constant.

In the present case, the symmetry $\phi = \rho$ does not hold, as now both a radial and a temporal component for the vector field $v^e$ are present; for this reason, the previous spatial action formalism depicted in Eq. (3) cannot be used. Instead, the full Einstein equations must be dealt with,

$$G_{\mu\nu} = \frac{1}{2} A_0^2 R_{\mu\nu} \ A_0^2 R_{\mu\nu} \ b_b^2 b_\nu R_{\nu\mu} \ b_\mu^2 R_{\nu\mu} \quad (11)$$

Since the bumblebee field has relaxed to its vev and therefore both the field strength and the potential term vanish, the additional equation of motion for the vector field is trivial. The metric Ansatz and the expressions for $b_\mu$ then yield

$$G_{00} = \frac{1}{2} A_0^2 R_{00} \ A_0^2 R_{00} \ b_b^2 R_{00} \ b_\mu^2 R_{\nu\mu} \quad (12)$$

Writing $G_{00} = g_0(\nu)e^{\phi}(\nu) \rho^2 \ G_{rr} = g_r(\nu), R_{00} = f_0(\nu)e^{\phi}(\nu) \rho^2 \ R_{rr} = f_r(\nu)$, where

$$f_0(\nu) = \frac{2}{r^2} \frac{\rho^2}{r^2} \ A_0^2 f_0(\nu) \ A_0^2 f_0(\nu) \ b_b^2 b_\nu R_{\nu\mu}$$

$$g_0(\nu) = \frac{1}{r^2} \frac{\rho^2}{r^2} + \frac{1}{r^2} \ A_0^2 f_0(\nu) \ A_0^2 f_0(\nu) \ b_b^2 b_\nu R_{\nu\mu}$$

the Einstein equations read

$$g_0(\nu) = \frac{1}{2} A_0^2 f_0(\nu) \ A_0^2 f_0(\nu) \ A_0^2 f_0(\nu) \ A_0^2 f_0(\nu) \ A_0^2 f_0(\nu)$$

This is the set of coupled second order differential equations which must be solved, with boundary conditions given by $\phi(\nu) = \rho(\nu) = \phi(\nu) = \rho(\nu) = \phi(\nu) = \rho(\nu) = 0$.

The spontaneous LSB is clearly exhibited; as can be noticed from $g_0 + g_r = f_0 + f_r$, one has $(l + 2\ b^2) \ f_0 = (l + 2\ b^2) \ f_r$; in the unperturbed case $A_0 = A_r = 0, f_0 = f_r$ and the Schwarzschild solution $\phi = \rho$ is recovered from $g_0 + g_r = 0$. This symmetry does not hold in the perturbed case, which produces $f_0 = (l + 2\ b^2) \ f_r$.

An expansion of the metric in terms of $\phi = \phi_{0} + \delta \phi$ and $\rho = \phi_{0} + \delta \rho$ allows for the solving of Eqs. (13), where $\phi_{0}$ is given by the usual Schwarzschild metric, $\phi_{0} = \frac{1}{2} ln(l + 2M=r)$, and $\delta \rho, \delta \phi$ are assumed to be small perturbations. After some algebra, the solution is found to be [11]
\[
\delta \phi = Kr^\alpha \ ; \ \delta \rho \ ' = 1 + \alpha \frac{(A + B)}{2} Kr^\alpha \ ;
\]

where \(A = A_0^2, B = A_r^2, K\) is an integration constant and

\[
\alpha = \frac{G_1 G_2 + P (C_1 + C_2)^2 + 4C_1 C_3}{2C_1} > 0 \ ;
\]

with

\[
C_1 = A + 3B + AB + 9B^2 \ ; \ C_2 = 2 + B \ 3A + 16AB \ 2 \ ; \ C_3 = 2 + B \ 7A + 2 \ ;
\]

One can linearize the exponent \(\alpha\) around \(C_1 = 1\), yielding \(\alpha ' = 2A + B = 2A + 3B\), so that \(\alpha ' = 1\).

After solving the coupled differential Eqs. (13), the non-trivial components of the metric now read

\[
g_{tt} = e^{2(\delta \phi + \delta \rho')} = e^{2Kr^\alpha} \ ; \ \frac{2M}{r} \ ; \ (17)
\]

\[
g_{rr} = e^{2(\delta \phi + \delta \rho')} = e^{\frac{2M}{r}} \ ; \ \frac{2M}{r} \ ;
\]

with the definition \(K_r \quad [1 + \alpha (A + B)=2]K' \quad [1 + \alpha (A + B)=2]K\). Following the algebra of a Lorentz transformation to an isotropic coordinate system, on which all spatial metric components are equal, and then to a quasi-cartesian referential, the resulting metric is [11]

\[
\eta_{tt} = g_{tt} = 1 + 2U \ 1 + \frac{K + K_r}{M} U^2 \ ; \ \eta_{\xi \xi} = 1 + \frac{K + 2K_r}{M} U \ ;
\]

and the PPN parameters may be directly read, yielding \(\beta ' = 1 \quad (K + K_r)=M\) and \(\gamma ' = 1 \quad (K + 2K_r)=M\). Inverting this relation gives \(K = M \quad 2 \beta + \gamma \ K = M \quad \beta \ \gamma\). Hence, a temporal/radial LSB manifests itself linearly on the PPN parameters \(\beta\) and \(\gamma\). A caveat of these results is the clear dependence of the obtained PPN parameters on the free-valued integration constants \(K\) and \(K_r\), instead of the physical parameters associated with the breaking of Lorentz invariance. This reflects the linearization procedure followed in order to obtain the radially symmetric Birkhoff metric solution to the Einstein equations.

The bounds derived from the Nordvedt effect, \(\beta = 1 \quad \gamma = 6 \quad 10^4 [17]\) and the Cassini-Huygens experiment, \(\gamma = 1 + 2A 1 \quad 2B \ 10^5 [18]\), can be used to obtain \(K + K_r < 0.9 \ m\) and \(K + 2K_r = (3 \ l \ 3 \ R) \ 10^6 \ m\). Since, by definition

\[
K_r = 1 + \alpha \frac{(A + B)}{2} K \ ;
\]

with \(\alpha ' = 1, A_r \beta \ 0\), deviations of \(K_r\) from \(K\) are expected to be small. Thus, considering for instance the constraint \(K = K \ j < 0.1\), one gets \(|A + B| < 02\); the limiting case \(K = K\) gives \(K' = (1 \ 1 \ l) \ 10^2\), indicating a perturbation with a very short range (actually, well inside the Sun, so that one should work with the interior Schwarzschild solution). The range of allowed values for these parameters is depicted in Figures 1 and 2.

In the limit \(M \to 0\), Eqs. (18) yield \(\delta_{tt} = e^{2K} \xi\) and \(\delta_{rr} = e^{2K_r} \xi\). An analogy with Rosen’s bimetric theory allows for the PPN parameter \(\alpha_2\) to be obtained, by interpreting this change of the metric component as due to a background metric \(\eta_{tt'}\) [19]. Notice, however, that the vector field no longer rolls to a radial 

vev in the absence of a central mass, since this spatial symmetry is inherited from its presence, so this result should be taken with caution; this said, one obtains \[19] \(\alpha_2 = (\xi = c_G) \ 1 = e^{2K} \xi \ 1 \ 2 \ K \ 1 \ 2 \ K \ M \ x = \xi \ 1 \ (A + B) \alpha = c_G\), which has a radial dependency. Assuming \(\alpha ' = 1\) and considering the spin precession constraint arising from solar to ecliptic alignment measurements [17], one has \(\delta_2 = 1 \ j < 4 \ 10^{-7}\), implying that \(j = B \ j < 4 \ 10^{-7} \ = \ 2.78 \ 10^7 \ m\), where \(r = 6.96 \ 10^6 \ m\) is the radius of the Sun.

Further pursuing this analogy, we remark that, since there is no explicit Lorentz breaking, the speed of light remains equal to \(c\). However, the speed of gravitational waves \(c_G\) is shifted by an amount
and hence it acquires a radial dependence. As stated before, this result is highly simplistic and should be taken with caution, since it lacks a complete treatment of gravitational radiation induced by LSB, taking into account variations of the bumblebee field $B_\mu$ around its vev $b_\mu$.

Finally, notice that, since the radial LSB effects are exact, while the radial/temporal results are not, a direct comparison of these scenarios by taking the $A=0$ limit is not possible.

### 4. AXIAL/TEMPORAL LSB

The anisotropic LSB case is dealt with in this section. As before, we assume that the bumblebee field is stabilized at its vacuum expectation value, which possesses both a temporal and a spatial component; the latter is taken to lie on the $x$-axis, that is, $b_\mu = \kappa \left( \mu_0 b_0 \rho_0 \delta_0 \right)$. Since the radial symmetry of the Scharzschild is clearly broken, one cannot resort to a Birkhoff canonical Ansatz. Instead, the perturbations $h_{\mu\nu}$ to the flat Minkowsky metric must be obtained. To first order in $h_{\mu\nu}$, one has

\[
\frac{e^2}{c} e^{2 (K-K_\tau)} = r \left( 1 + \frac{A+B}{\xi} \right);
\]
\[
R_{00} = \frac{1}{2} \nabla^2 h_{00} \ ; \ R_{0i} = \frac{1}{2} \nabla^2 h_{0i} \ h_{0i} \ ; \ \frac{1}{2} \nabla^2 h_{0k} \ h_{0k} \ ; \ \frac{1}{2} \nabla^2 h_{0j} \ h_{0j} \ ; \ (21)
\]
\[
R_{ij} = \frac{1}{2} \nabla^2 h_{ij} \ h_{00,ij} + h_{ik,\bar{j}} + h_{i\bar{k},j} + h_{i\bar{j},k} + h_{ij,\bar{k}} \ ;
\]

where time derivatives were neglected, since one assumes that \( v \rightarrow c \).

In order to solve the Einstein equations, one first writes the stress-energy tensor for the bumblebee field,

\[
T_{\mu\nu} = \frac{1}{2} b^\alpha b^\beta R_{\alpha\beta} g_{\mu\nu} - b^\alpha b^\mu R_{\alpha\nu} - b^\alpha b^\nu R_{\alpha\mu} \ ;
\]

which has a vanishing trace. From the trace of the Einstein equations, one gets

\[
R_{\mu\nu} = \kappa T_{\mu\nu} + \frac{1}{2} \kappa \ T_B^{\mu\nu} + T_{B\mu\nu} \ ;
\]

To get the \( h_{00} \) component to first order in the potential \( U \), one writes

\[
R_{00} = \frac{1}{2} \ a^2 R_{00} + a^2 R_{11} + 2ab R_{10} - 2a(aR_{00} + bR_{10}) \ ;
\]

which, after a little algebra [11], yields the differential equation

\[
2 + \frac{5a^2}{2 + 5a^2} \ \tilde{\beta} \ \ h_{00,11} + h_{00,22} + h_{00,33} = 0 \ ;
\]

This admits the solution

\[
h_{00} (x,y,z) = \frac{2M}{c_0^2 x^2 + y^2 + z^2} \ ;
\]

where \( c_0^2 = \sqrt{2 + 5a^2} \).

Similarly, the \( h_{i\bar{i}} \) components \((i \neq 1)\) obey

\[
2 \ \tilde{\beta} \ h_{i\bar{i},11} + 4h_{i\bar{i},i} + 2h_{i\bar{i},i\bar{j}} = a^2 \ \tilde{\beta} \ h_{00,11} + 2 + a^2 \ h_{00,\bar{i}} + \bar{a}^2 h_{00,\bar{i}\bar{j}} \ ;
\]

Taking the Ansatz \( h_{i\bar{i}} (x,y,z) = h_{00} (\alpha_1 x, \alpha_2 y, \alpha_3 z) \) and, after some calculation (see Appendix I of Ref. [11]), one can obtain for the coefficients \( \alpha_i \):

\[
\alpha_i^2 = 1 + \frac{2}{2 + 5a^2} \frac{a^2}{\tilde{\beta}} \ h_{00,11} \ ; \ \alpha_i = \alpha_j = 1 \ ;
\]

Hence,

\[
h_{i\bar{i}} (x,y,z) = h_{00} (\alpha_1 x, \alpha_2 y, \alpha_3 z) = \frac{2M}{c_0^2 x^2 + y^2 + z^2} \ ;
\]

with the definition

\[
c_0^2 = c_1^2 c_0^2 = \frac{2}{2 + 5a^2} \frac{a^2}{\tilde{\beta}} \ ;
\]

The \( h_{11} \) component is now computed; a similar calculation leads to the differential equation [11]

\[
h_{11,22} + h_{11,33} = 2M \ \frac{a^2 \ c_0^2}{2 + 5a^2} \ ;
\]

indicating that the solution is a linear combination of \( h_{00} \) and \( h_{22} \). Indeed,
Writing reasoning allows two parameters analogous to the enables to link the proton mass of the proximity to the galactic core. In the present scenario, we note that a radial LSB with the as a test between Mach’s principle and the Equivalence Principle \[21, 22\], relying on the hypothetical effect on the correction to the first order term and hence As expected, the x-axis LSB produces a stronger effect on the magnitude as the average of Since no a quasi-cartesian frame of reference on which, by definition, all diagonal metric components \(g_{ii}\) are equal. However, some PPN-like parameters may be extracted from the results, by noticing that Finally, the \(h_{00}\) component is computed to second order (see Appendix II of Ref. [11]); it can be shown that only a correction to the first order term \(h_{00}^{(1)}\) appears:

The PPN formalism cannot be straightforwardly used to ascertain its effects, since it relies on a transformation to a quasi-cartesian frame of reference on which, by definition, all diagonal metric components \(g_{ii}\) are equal. However, some PPN-like parameters may be extracted from the results, by noticing that

For \(h_{00}\), one gets

Since no \(r^{-2}\) correction appears, the PPN parameter \(\beta\) vanishes in this approach. However, as \(h_{11} \neq h_{22} = h_{33}\), the same reasoning allows two parameters analogous to the \(\gamma\) PPN parameter to be obtained: after neglecting the normalization with respect to \(h_{00}\), one gets

As expected, the x-axis LSB produces a stronger effect on the \(h_{11}\) component. No clear connection can be derived to link \(\gamma\) with \(\gamma_1\) and \(\gamma_2\), due to the aforementioned anisotropy. However, one can take \(\gamma\) to be of the same order of magnitude as the average of \(\gamma_1\) and \(\gamma_2\), integrated over one orbit:

\[\gamma = \frac{1}{2} (\gamma_1 + \gamma_2) \]

where \(e\) is the orbit eccentricity. For low values of \(e\), one gets \(\gamma' = 10^2\). The constraint \(\gamma = 1 + \frac{1}{2} (\gamma_1 + \gamma_2) < 10^5\) then enables \(\gamma' < 10^2\).

A discussion concerning the anisotropy of inertia and its effect in the width of resonance lines has been presented as a test between Mach’s principle and the Equivalence Principle \[21, 22\], relying on the hypothetical effect on the proton mass of the proximity to the galactic core. In the present scenario, we note that a radial LSB with the
galactic core acting as source would amount to an axial LSB in a small region such as the Solar System. The bound $\Delta m_p = m_p \times 3 \times 10^{-22}$, $m_p$ being the proton mass [23], can then be used to obtain

$$\frac{\Delta m_p}{m_p} = 1 + \frac{b^2}{6} \psi_1^2 \left( 1 + \frac{b^2}{2} \right) 3 \times 10^{-22};$$

resulting in the limit $\beta_{\gamma} > 2.4 \times 10^{-11}$, a much more stringent bound than the obtained above.

5. CONCLUSIONS

In this contribution, the solutions of a gravity model coupled to a vector field where Lorentz symmetry is spontaneously broken are studied, and three different relevant scenarios were highlighted: a purely radial, temporal/radial and temporal/axial LSB.

In the first case, a new black hole solution is found, exhibiting a removable singularity at a horizon of radius $r_+ = (2\mathcal{M}_0^2)^{-1/2}$, slightly perturbed with respect to the usual Schwarzschild radius $r_0 = 2\mathcal{M}$. This has an associated Hawking temperature of $T = (2\mathcal{M}_0^2)^{-1/2}T_0$, where $T_0 = \hbar/8\pi G \mathcal{M}$ is the usual Hawking temperature, and protects an intrinsic singularity at $r = 0$. Bounds on deviations from Kepler’s law yield $L \geq 2 \times 10^9$. The temporal/radial scenario produces a slightly perturbed metric that leads to the PPN parameters $\beta_1 = (K + K_0) = M$ and $\gamma_1 = (K + 2K_0) = M$, directly proportional to the strength of the derived effect (given by $K$ and $K_0$). Since $K$ and $K_0$ are integration constants, no constraints on the physical parameters may be derived from the observed limits on the PPN parameters. Also, an analogy with Rosen’s bimetric theory, yields the PPN parameter $\gamma' (A + B) \tilde{\xi}$, $\tilde{\xi}$ being the distance to the central body and $A$ and $B$ parameters ruling the temporal and radial components of the vector field $\text{vev}$.

In the temporal/axial scenario, a breakdown of isotropy is obtained, disallowing a standard PPN analysis. However, the direction-dependent “PPN” parameters $\gamma_1 \equiv b^2 \cos^2 \theta = 2$ and $\gamma_2 \equiv a^2 b^2 \cos^2 \theta = 4$ may be derived, where $a$ and $b$ are respectively the temporal and $x$-component of the bumblebee vector $\text{vev}$; naturally, $\gamma_1 < \gamma_2$ A crude estimative of the PPN parameter $\gamma$ yields $\gamma \approx \beta (1 - \bar{e}) = 4$, where $e$ is the orbit’s eccentricity. Furthermore, a comparison with experiments concerning the anisotropy of inertia produces the bounds $\beta_{\gamma} > 2.4 \times 10^{-11}$.

REFERENCES

1. “CPT and Lorentz Symmetry II”, Ed., V. A. Kostelecký (World Scientific, Singapore, 2002).
2. O. Bertolami, Nucl. Phys. Proc. Suppl. 88, 49 (2000); O. Bertolami in “Decoherence and Entropy in Complex Systems” (Springer-Verlag, Berlin, 2004).
3. V. A. Kostelecký and S. Samuel, Phys. Rev. D 39, 683 (1989); Phys. Rev. Lett. 66, 1811 (1991).
4. V. A. Kostelecký and R. Potting, Phys. Rev. D 51, 3923 (1995).
5. V. A. Kostelecký, Phys. Rev. D 69, 105009 (2004).
6. R. Bluhm and V. A. Kostelecký, hep-th/0412320.
7. S. M. Carroll, J. A. Harvey, V. A. Kostelecký, C. D. Lane and T. Okamoto, Phys. Rev. Lett. 87, 141601 (2001).
8. O. Bertolami and L. Guisado, Phys. Rev. D 67, 025001 (2003); JHEP 0312, O13 (2003); O. Bertolami, Mod. Phys. Lett. A 20, 1359 (2005).
9. V. A. Kostelecký, R. Lehner and M. J. Perry, Phys. Rev. D 68, 123511 (2003); O. Bertolami, R. Lehner, R. Potting and A. Ribeiro, Phys. Rev. D 69, 083513 (2004).
10. H. Sato and T. Tani, Progr. Theor. Phys. 47, 1788 (1972); S. Coleman and S.L. Glashow, Phys. Lett. B 405, 249 (1997); Phys. Rev. D 59, 116008 (1999); O. Bertolami and C.S. Carvalho, Phys. Rev. D 61, 103002 (2000); O. Bertolami, Gen. Relativity and Gravitation 34 707 (2002); R. Lehner, hep-ph/0312093.
11. O. Bertolami and J. Páramos, Phys. Rev. D 72, 044001 (2005).
12. R. Grard, M. Novara and G. Scoon, ESA Bull. 103, 11 (2000); L. Iorio, I. Ciufolini and E. C. Pavlis, Class. Quantum Gravity 19, 4301 (2002).
13. S. G. Turyshhev et al., gr-qc/0505064.
14. O. Bertolami, J. Páramos and S. G. Turyshhev, gr-qc/0601016.
15. M. C. Bento and O. Bertolami, Phys. Lett. B 228, 348 (1999).
16. E. Fischbach and C.L. Talmadge, “The search for non-Newtonian gravity” (Springer, New York 1999).
17. C. M. Will, Living Rev. Rel. 4, 4 (2001).
18. B. Bertotti, L. Iess and P. Tortora, Nature 425, 374 (2003).
19. N. Rosen, J. Gen. Rel. and Grav. 4, 435 (1973).
20. C.M. Will, “Theory and Experiment in Gravitational Physics”, C.M. Will (Cambridge U. P., 1993).
21. S. Weinberg, “Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity” (John Wiley and Sons, New Jersey, 1972).
22. V. A. Kostelecký and C. D. Lane, J. Math. Phys. 40 6245 (1999).
23. S. K. Lamoreaux, J. P. Jacobs, B. R. Heckel, F. J. Raab, and E. N. Fortson, Phys. Rev. Lett. 58, 746 (1987).