A Note on Low-Pass Filter Conditioning for Current-Mode Control

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Abstract

Low-pass filter is a classic control conditioning approach for high frequency current-mode control. However, no existing literature discusses the large-signal stability criterion for the current-mode control with low-pass filters. This paper provides a mathematically rigorous large-signal stability criterion. The result can directly benefit the practical engineering implementation of the low-pass filter in high-frequency current-mode control.

I. Introduction

Low-pass filters, the traditional way to signal condition the interference, stabilize the current control loop by attenuating the amplitude of interference. However, as shown in Fig. 1, the actual inductor current may be distorted. Low-pass filters are traditionally not recommended to cut off before the switching frequency; however, this makes the filters unable to effectively suppress the interference whose spectrum is near or below the switching frequency. Because of our results on the robustness of peak current-mode control, we can allow the cut-off frequency of filters to be well below the switching frequency and still have good performance.

II. System Description

We take constant off-time current-mode control as an example. The current control loop using constant off-time can be modeled as

\[
\begin{align*}
    i_p[n] &= i_p[n - 1] - m_2 t_{off} + m_1 t_{on}[n], \\
    i_c[n] &= h_0(T[n])i_c[n - 1] + \left( i_m(t)u(t) \ast h(t) \right) \bigg|_{t=t_on[n]}, \\
\end{align*}
\]

where \( \ast \) is the convolution operator, \( h(t) \) is the impulse response function of the low-pass filter, \( h_0(t) \) is the zero input response of filter, and \( u(t) \) is the unit step function. Equation (1) captures the low-pass filter response. The variable \( i_m(t) \) represents the current sensor output and filter input; \( i_m(t) \) can be expressed as the additive summation of the inductor current on the bottom switch and interference

\[
i_m(t) = i_p[n - 1] - m_2 T_{off} + m_1 t + w(t).
\]

We exemplify this idea with a first-order low-pass filter with the impulse response function \( h(t) = (e^{-t/\tau})/\tau \) and \( q(t) = e^{-t/\tau} \), where \( \tau \) represents the time constant. A continuous static mapping is a prerequisite for stability. Theorem provides a sufficient condition for the filter to guarantee a continuous static mapping. As long as the time constant satisfies Theorem, the static mapping is continuous. We denote the lower bound of the frequency of interference by \( \omega_l \).

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Fig. 1. Comparison of current-sense voltage with/without the interference and with/without the filter. (—) is for the current-sense voltage with interference and without filter. (···) is for the current-sense voltage with interference and with filter. (- - -) is for the current-sense voltage without interference and without filter. (---) is for the current-sense voltage without interference with filter.

(a) Block diagram.
(b) Root locus without (left)/with (right) filter. — is for the negative feedback; - - - is for the positive feedback.

Fig. 2. Modeling of the current control loop with filter. The block is used in constant on/off-time control.

**Theorem 1.** A current control loop using constant off-time has minimum on time \( T_{on}^{\min} \) and off time \( T_{off} \). The time constant of the first-order low-pass filter is \( \tau \). The interference \( w(t) \) is amplitude and bandwidth-limited. The condition for \( \tau \) to guarantee a continuous static mapping is

\[
\frac{\hat{A}_{ub}}{(1-d)\hat{\tau}} \left( 1 + \frac{d}{\sqrt{1 + (2\pi\hat{\omega}\hat{\tau})^2}} \right) + \frac{b\hat{I}_{\max}}{(1-d)\hat{\tau}} < 1,
\]

where

\[
\hat{T}_{\min}^{\min} = \hat{T}_{on}^{\min} + 1, \quad b = e^{-\frac{\hat{T}_{\min}^{\min}}{\hat{\tau}}} , \quad d = e^{-\frac{\hat{T}_{\min}^{\min}}{\hat{\tau}}}, \quad \hat{\tau} = \frac{\tau}{T_{on}},
\]

\[
\hat{A}_{ub} = \frac{A_{ub}}{m_1T_{on}}, \quad \hat{I}_{\max} = \frac{I_{\max}}{m_1T_{on}}, \quad \hat{\omega}_l = \frac{\omega_lT_{on}}{2\pi}.
\]

**Proof. A. Continuity Theorem for General Linear Filter**

The continuous static mapping is equivalent to the monotonic current sensor output. Therefore, the continuity condition can be equivalently expressed as

\[
(i_p - m_2T_{off}) \frac{d}{dt} [u(t) * h(t)] + i_c q(T_{off}) \frac{dq(t)}{dt} + \frac{d}{dt} \left\{ [m_1 t + w(t)] u(t) * h(t) \right\} > 0,
\]
for all \( t > 0 \).

For all \( t > 0 \), \( u(t) \ast h(t) \) and \( m_1 t u(t) \ast h(t) \) are differentiable

\[
\frac{d}{dt} \left[ m_1 t u(t) \ast h(t) \right] = m_1 u(t) \ast h(t),
\]

\[
\frac{d}{dt} \left[ u(t) \ast h(t) \right] = h(t).
\]

The zero state response of the filter given input signal \( w(t)u(t) \) is

\[
w(t)u(t) \ast h(t) = \left[ w(t) \ast h(t) - \frac{g(0)}{q(0)} q(t) \right] u(t).
\]

where \( \left[ w(t) \ast h(t) \right] u(t) \) is the forced response and the rest is natural response. We denote the forced response as \( g(t) \triangleq w(t) \ast h(t) \). For all \( t > 0 \), \( w(t)u(t) \ast h(t) \) is differentiable

\[
\frac{d}{dt} \left[ w(t)u(t) \ast h(t) \right] = \frac{dg(t)}{dt} u(t) - \frac{g(0)}{q(0)} \frac{dq(t)}{dt} u(t).
\]

By substituting (6), (7), (9) into (5), the continuity condition for the static mapping can be equivalently expressed as

\[
(i_p - m_2 T_{off}) h(t) + i_e q(T_{off}) \frac{dq(t)}{dt} + m_1 u(t) \ast h(t) + \frac{dg(t)}{dt} u(t) - \frac{g(0)}{q(0)} \frac{dq(t)}{dt} u(t) > 0.
\]

B. Continuity Theorem for First-Order Low-Pass Filter

For a first-order low-pass filter with time constant \( \tau \), the impulse response \( h(t) \) and zero input response \( q(t) \)

\[
h(t) = e^{-\frac{t}{\tau}} u(t), \quad q(t) = e^{-\frac{t}{\tau}} u(t).
\]

The frequency response function of the filter is

\[
H(j\omega) = \frac{1}{1 + j\omega \tau}.
\]

From (11) and (12), we can bound \( g \) from the top as

\[
g'(t) = \int_{-\infty}^{+\infty} j\omega W(\omega) H(\omega) e^{j\omega t} d\omega = \int_{-\infty}^{+\infty} \frac{j\omega W(\omega)}{1 + j\omega \tau} e^{j\omega t} d\omega.
\]

\[
|g'(t)| \leq \int_{-\infty}^{+\infty} \frac{|\omega|}{\sqrt{1 + \omega^2 \tau^2}} |W(\omega)| d\omega \leq \frac{A_{ub}}{\tau}.
\]

\[
g(t) = \int_{-\infty}^{+\infty} W(\omega) H(\omega) e^{j\omega t} d\omega = \int_{-\infty}^{+\infty} \frac{W(\omega)}{1 + j\omega \tau} e^{j\omega t} d\omega.
\]

\[
|g(t)| \leq \int_{-\infty}^{+\infty} |W(\omega)| |H(\omega)| d\omega = \int_{-\infty}^{+\infty} \frac{|W(\omega)|}{\sqrt{1 + \omega^2 \tau^2}} d\omega \leq \frac{A_{ub}}{\sqrt{1 + (\omega \tau)^2}}.
\]

\[
g(0) = \int_{-\infty}^{+\infty} W(\omega) H(\omega) e^{j\omega t} d\omega \bigg|_{t=0} = \int_{-\infty}^{+\infty} \frac{W(\omega)}{1 + j\omega \tau} d\omega.
\]
In the worst-case, \( I_p = m_1 T_{\text{off}} \). We substitute (14) and (16) into (5). When \( t > 0 \), (5) can be bounded from below by

\[
\begin{align*}
    i_c q(T_{\text{off}}) \frac{dq(t)}{dt} + m_1 u(t) * h(t) + \frac{dq(t)}{dt} u(t) - \frac{q(0)}{q(0)} \frac{dq(t)}{dt} u(t) = \\
    - e^{-\frac{t}{\tau}} q(T_{\text{off}}) I_c + m_1 (1 - e^{-\frac{t}{\tau}}) + g'(t) + e^{-\frac{t}{\tau}} g(0) \geq \\
    - \frac{I_{\text{max}}}{\tau} \exp \left( - \frac{T_{\text{on}}}{\tau} \right) + m_1 (1 - e^{-\frac{I_{\text{min}}}{\tau}}) - \frac{A_{ub}}{\tau} - \frac{e^{-\frac{I_{\text{min}}}{\tau}}}{\sqrt{1 + (\omega l T)^2}} \frac{A_{ub}}{\tau} = \\
    - \frac{b}{\tau} I_{\text{max}} + m_1 (1 - d) - \frac{A_{ub}}{\tau} \left( 1 + \frac{d}{\sqrt{1 + (\omega l T)^2}} \right), \quad (18)
\end{align*}
\]

where \( b = e^{-\frac{I_{\text{min}}}{\tau}} \), \( T_{\text{min}} = T_{\text{on}}^{\text{min}} + T_{\text{off}} \) and \( d = e^{-\frac{I_{\text{min}}}{\tau}} \). As long as we impose a positive lower bound for (18), the static mapping is continuous. The proof is complete.

We next examine how the filter affects the stability of the current control loop. At the operating point defined by the desired peak inductor current \( I_c \), the actual peak inductor current \( I_p \), the actual valley inductor current \( I_v \), at on time \( T_{\text{on}} \), we linearize system (1) as

\[
\begin{align*}
    i_p[n] &= i_p[n - 1] + m_1 \tilde{t}_{\text{on}}[n], \\
    \tilde{i}_c[n] &= q(T_{\text{on}}) q(T_{\text{off}}) \tilde{i}_c[n - 1] + c_1 \tilde{i}_p[n] + c_2 \tilde{t}_{\text{on}}[n], \quad (19)
\end{align*}
\]

where

\[
\begin{align*}
    c_1 &= u(t) * h(t) \bigg|_{t = T_{\text{on}}}, \\
    c_2 &= - \frac{dq(t)}{dt} \bigg|_{t = T_{\text{on}}} q(T_{\text{off}}) I_c + h(T_{\text{on}}) I_v + \frac{d}{dt} \left[ w(t) u(t) * h(t) \right] \bigg|_{t = T_{\text{on}}}. \quad (20)
\end{align*}
\]

System (19) is represented by the block diagram in Fig. 2(a). The detailed derivations are provided in Appendix A.

The gain term \( K \), pole-zero pair, and feedback gains \( \psi_1 \) and \( \psi_2 \) are introduced as

\[
\begin{align*}
    K &= \frac{1}{1 - e^{-\frac{2\pi}{\tau}}}, \quad (21) \\
    P(z) &= 1 - e^{-\frac{\tau}{z} z^{-1}}. \quad (22) \\
    \psi_1 &= \frac{w(T_{\text{on}})}{\tau} - \frac{e^{-\frac{2\pi}{\tau}}}{\tau} \int_{-\infty}^{+\infty} W(\omega) j\omega d\omega. \quad (23) \\
    \psi_2 &= - \frac{e^{-\frac{\tau}{\tau}} I_c + e^{-\frac{2\pi}{\tau}} (I_p - m_2 T_{\text{off}}). \quad (24)
\end{align*}
\]

Theorem 1 provides the condition for \( \tau \) so that the current control loop is globally asymptotically stable. If \( \tau \) satisfies the condition, the globally asymptotic stability is guaranteed.

**Theorem 2.** A current control loop using constant off-time has a minimum on time \( T_{\text{on}}^{\text{min}} \) and fixed off time \( T_{\text{off}} \). The time constant of the first-order, low-pass filter is \( \tau \). The interference \( w(t) \) is amplitude and
Fig. 3. Block diagram of the current control loop with filter.

bandwidth limited. The bound on \( \tau \) to guarantee the globally asymptotic stability of the current control loop is

\[
\frac{k_0}{\tau} + k_1 \frac{\hat{A}_{ub}}{\tau} + k_2 \frac{\hat{A}_{ub}}{\tau \sqrt{1 + (2\pi \hat{\omega}_l \tau)^2}} < \frac{1}{2},
\]

(25)

and

\[
\frac{k_3}{\tau} \hat{i}_{\text{max}} + \frac{\hat{A}_{ub}}{\tau} + \frac{\hat{A}_{ub}}{\tau \sqrt{1 + (2\pi \hat{\omega}_l \tau)^2}} < \frac{1}{2},
\]

(26)

where

\[
k_0 = \frac{d(\hat{T}_{\text{on}} + \hat{\tau} d - \hat{\tau})}{(1 - d)^2}, \quad k_1 = \frac{1}{(1 - d)}, \quad \hat{T}_{\text{min}} = \hat{T}_{\text{on}} + 1,
\]
\[
b = e^{-\frac{\hat{T}_{\text{min}}}{\tau}}, \quad d = e^{-\frac{\hat{T}_{\text{min}}}{\tau}}, \quad \hat{\tau} = \frac{\tau}{T_{\text{on}}}, \quad \hat{A}_{ub} = \frac{\hat{A}_{ub}}{m_1 T_{\text{on}}},
\]
\[
k_2 = 1 + \frac{(1 + d)d}{(1 - d)^2}, \quad k_3 = \frac{d - b}{(1 - d)^2}, \quad \hat{i}_{\text{max}} = \frac{I_{\text{max}}}{m_1 T_{\text{on}}}, \quad \hat{\omega}_l = \frac{\omega_l T_{\text{on}}}{2\pi}.
\]

(27)

**Proof.** The current control loop with filter can be modeled as the block diagram in Fig. 3 where

\[
K_1 = e^{-\frac{T_{\text{off}}}{\tau}},
\]
\[
K_2 = (e^{-\frac{T_{\text{off}}}{\tau}} - 1),
\]
\[
\xi(\tilde{i}_{\text{on}}) = \left[ I e^{-\frac{T_{\text{off}}}{\tau}} - (I_p - m_2 T_{\text{off}}) \right] \left[ \exp \left( -\frac{T_{\text{on}} + \tilde{i}_{\text{on}}}{\tau} \right) - \exp \left( -\frac{T_{\text{on}}}{\tau} \right) \right]
\]
\[
+ \left[ w(t) u(t) * h(t) \right] \bigg|_{t=\tilde{i}_{\text{on}}+T_{\text{on}}}^{t=T_{\text{on}}}.
\]

(30)

From the small gain theorem,

\[
\|G\|_2 \cdot \|\xi\|_2 \cdot \|F\|_2 < 1.
\]

(31)

\( \tilde{i}_e[n] \) is defined as

\[
\tilde{i}_e[n] = e^{-\frac{T_{\text{off}}}{\tau}} \tilde{i}_e[n] - \tilde{i}_p[n].
\]

(32)

The nonlinear subsystem \( F \) follows

\[
\tilde{i}_e[n] = \exp \left( -\frac{\tilde{i}_{\text{on}}[n] + T_{\text{on}}}{\tau} \right) \tilde{i}_e[n - 1] + u[n].
\]

(33)
We can find the $L_2$ gain of the subsystem $G$ and $F$ as

\begin{align}
\|G\|_2 &\leq \frac{2}{m_1}, \\
\|F\|_2 &\leq \frac{1}{1 - e^{-\frac{T_{\text{off}}}{\tau}}}.
\end{align}

We can find the $L_2$ gain of the nonlinear function $\xi$ by evaluating its derivatives

\begin{equation}
\frac{d\xi}{dt_{\text{on}}} = \psi_1(t_{\text{on}}) + \psi_2(t_{\text{on}}),
\end{equation}

where

\begin{align}
\psi_1(t_{\text{on}}) &= \left. \frac{dg(t)}{dt} \right|_{t=t_{\text{on}}} + \frac{e^{-\frac{t_{\text{on}}}{\tau}}}{\tau} g(0) d\omega, \\
\psi_2(t_{\text{on}}) &= \frac{(i_p - m_2 T_{\text{off}})}{\tau} e^{-\frac{t_{\text{on}}}{\tau}} - \frac{I_c}{\tau} e^{-\frac{t_{\text{on}} + T_{\text{off}}}{\tau}}, \\
g(t) &= \int_{-\infty}^{+\infty} \frac{W(\omega)}{1 + j\omega\tau} e^{j\omega t} d\omega.
\end{align}

$\psi_1(t_{\text{on}})$ can be bounded from above as

\begin{equation}
|\psi_1(t_{\text{on}})| < \frac{A_{ub}}{\tau} + \frac{1}{\sqrt{1 + \omega_1^2\tau^2}} \frac{A_{ub}}{\tau} \triangleq \psi_1^\text{max}.
\end{equation}

To bound $\psi_2(t_{\text{on}})$, we have the following relationship of $i_c$, $i_p$ and $t_{\text{on}}$ in transition as

\begin{equation}
i_c = \left. \frac{[i_p - m_2 T_{\text{off}} + m_1 t + w(t) u(t) \ast h(t)]}{1 - q(t_{\text{on}})q(T_{\text{off}})} \right|_{t=t_{\text{on}}}
= \frac{1 - e^{-\frac{t_{\text{on}}}{\tau}}}{1 - e^{-\frac{T_{\text{off}}}{\tau}}} (i_p - m_2 T_{\text{off}}) + \left(1 + \frac{e^{-\frac{t_{\text{on}}}{\tau}} - 1}{t_{\text{on}}/\tau}\right) \frac{m_1 t_{\text{on}}}{1 - e^{-\frac{T_{\text{off}}}{\tau}}} + \frac{g(t_{\text{on}}) - g(0)e^{-\frac{t_{\text{on}}}{\tau}}}{1 - e^{-\frac{T_{\text{off}}}{\tau}}}.
\end{equation}

We substitute (41) into (38)

\begin{equation}
\psi_2(t_{\text{on}}) = \frac{d' - b'}{(1 - d')\tau} i_c - \frac{d'}{(1 - d')\tau} \left(1 + \frac{d' - 1}{t_{\text{on}}/\tau}\right) m_1 t_{\text{on}}
- \frac{1 - b'}{1 - d'} \frac{g(t_{\text{on}}) - g(0)e^{-\frac{t_{\text{on}}}{\tau}}}{1 - e^{-\frac{T_{\text{off}}}{\tau}}},
\end{equation}

where $b' = e^{-\frac{T_{\text{off}}}{\tau}}$, $T = t_{\text{on}} + T_{\text{off}}$, and $d' = e^{-\frac{t_{\text{on}}}{\tau}}$.

We observe that $\psi_2(t_{\text{on}})$ is a monotonic increasing function of $i_c$ because

\begin{equation}
\frac{\partial \psi_2(t_{\text{on}})}{\partial i_c} = \frac{d' - b'}{(1 - d')\tau} > 0.
\end{equation}

$\psi_2(t_{\text{on}})$ can be bounded from below by substituting $i_c = 0$ and $t_{\text{on}} = T_{\text{on}}^\text{min}$. We denote this lower bound by $\psi_2^\text{min}$.

\begin{equation}
\psi_2(t_{\text{on}}) > - \frac{d}{(1 - d)\tau} \left(1 + \frac{d - 1}{T_{\text{on}}^\text{min}/\tau}\right) m_1 T_{\text{on}}^\text{min} - \frac{1 + d d}{1 - d} \frac{A_{ub}}{\tau} \sqrt{1 + \omega_1^2\tau^2},
\end{equation}

where

\begin{align}
\frac{d}{d t_{\text{on}}} &\triangleq \frac{d}{d t_{\text{on}}} \left(\frac{d - 1}{d - 1} T_{\text{on}}^\text{min}/\tau\right) \frac{m_1 T_{\text{on}}^\text{min}}{1 + d d} \frac{A_{ub}}{\tau} \sqrt{1 + \omega_1^2\tau^2},
\end{align}

and $\frac{d}{d t_{\text{on}}} = \frac{d}{d t_{\text{on}}} \left(\frac{d - 1}{d - 1} T_{\text{on}}^\text{min} / \tau\right)$.
where \( b = e^{-\frac{T_{\min}}{\tau}} \), \( T_{\min} = T_{\text{on}} + T_{\text{off}} \) and \( d = e^{-\frac{T_{\min}}{\tau}} \).

\( \psi_2(t_{\text{on}}) \) can be bounded from above by substituting \( i_c = I_{\max} \) and \( t_{\text{on}} = \infty \). We denote this upper bound by \( \psi_2^{\max} \),

\[
\psi_2(t_{\text{on}}) < \frac{d - b}{(1 - d)\tau} I_{\max},
\]

where \( b = \exp\left(-\frac{T_{\min}}{\tau}\right) \), \( T_{\min} = T_{\text{on}} + T_{\text{off}} \), and \( d = \exp\left(-\frac{T_{\min}}{\tau}\right) \).

We denote the upper bound of the \( L_2 \) gain of \( \xi \) by \( B_{\xi} \),

\[
B_{\xi} = \max \left\{ \psi_2^{\min} - \psi_1^{\max}, \psi_2^{\max} + \psi_1^{\max} \right\}.
\]

From (31), the stability of the current control loop is guaranteed if the filter \( \tau \) satisfies the condition

\[
\frac{2}{m_1} \cdot \frac{1}{1 - d} \cdot B_{\xi} < 1.
\]

III. Conclusion

By applying the 5S framework \[1\] and Small Gain Theorem \[2\], this paper develops two theoretical results for high-frequency current-mode control using low-pass filters: (1) the continuity condition of the static mapping; (2) a large-signal stability criterion of the dynamical mapping. The results allow the cut-off frequency of filters to be well below the switching frequency and still have good performance.
APPENDIX A
LINEARIZED MODEL OF THE CURRENT CONTROL LOOP WITH LOW-PASS FILTER

A. Linearized Model of the Current Control Loop with the General Linear Filter

By assuming the continuity condition, we linearize the model of current control loop as

\[ \tilde{i}_c[n] = q(T_{on})q(T_{off}) \tilde{i}_c[n - 1] + q'(t) \bigg|_{t=T_{on}} q(T_{off}) I_c \tilde{i}_{on}[n] \]

\[ + (u(t) * h(t)) \bigg|_{t=T_{on}} \tilde{p}[n - 1] \]

\[ + (I_p - m_1 T_{off}) (u(t) * h(t)) \bigg|_{t=T_{on}} \tilde{i}_{on}[n] \]

\[ + (m_1 t u(t) * h(t)) \bigg|_{t=T_{on}} \tilde{i}_{on}[n] \]

\[ + (w(t) u(t) * h(t)) \bigg|_{t=T_{on}} \tilde{i}_{on}[n]. \]

(48)

For all \( t > 0^+ \), \( u(t) * h(t) \) and \( m_1 t u(t) * h(t) \) that are differentiable

\[ (m_1 t u(t) * h(t))' = m_1 u(t) * h(t), \] (49)

\[ (u(t) * h(t))' = h(t). \] (50)

By substituting (49), (50), we have

\[ \tilde{i}_c[n] = q(T_{on})q(T_{off}) \tilde{i}_c[n - 1] + (u(t) * h(t)) \bigg|_{t=T_{on}} \tilde{p}[n] \]

\[ + q'(t) \bigg|_{t=T_{on}} q(T_{off}) I_c \tilde{i}_{on}[n] \]

\[ + (I_p - m_2 T_{off}) h(T_{on}) \tilde{i}_{on}[n] \]

\[ + (w(t) u(t) * h(t)) \bigg|_{t=T_{on}} \tilde{i}_{on}[n]. \]

(51)

We denote

\[ c_1 = (u(t) * h(t)) \bigg|_{t=T_{on}} \]

\[ c_2 = q'(t) \bigg|_{t=T_{on}} q(T_{off}) I_c + (I_p - m_2 T_{off}) h(T_{on}), \]

\[ + (w(t) u(t) * h(t)) \bigg|_{t=T_{on}}. \]

(52)

The resulting model is

\[ \tilde{i}_p[n] = \tilde{i}_p[n - 1] + m_1 \tilde{i}_{on}[n], \]

\[ \tilde{i}_c[n] = q(T_{on})q(T_{off}) \tilde{i}_c[n - 1] + c_1 \tilde{i}_p[n] + c_2 \tilde{i}_{on}[n]. \]

(53)

(54)
B. Linearized Model of the Current Control Loop with First-Order Low Pass Filter

The zero state response of the first-order low-pass filter given input signal \( u(t) \) is

\[
    u(t) * h(t) = \left( 1 - e^{-\frac{t}{\tau}} \right) u(t). \tag{55}
\]

The zero state response of the first-order low-pass filter given input signal \( m_1 t u(t) \) is

\[
    m_1 t u(t) * h(t) = m_1 \left( t + (e^{-\frac{t}{\tau}} - 1) \right) u(t). \tag{56}
\]

By substituting (55) and (56) into (57), the equilibrium can be expressed as

\[
    I_c = \frac{\left( (I_p - m_2 T_{off} + m_1 t + w(t)) u(t) * h(t) \right) \bigg|_{t=T_{on}}}{1 - q(T_{on}) q(T_{off})} \\
    = \frac{1 - e^{-\frac{T_{on}}{\tau}} (I_p - m_2 T_{off}) + \left( 1 + \frac{e^{-\frac{T_{on}}{\tau}} - 1}{\frac{T_{on}}{\tau}} \right) m_1 T_{on}}{1 - e^{-\frac{T_{on}}{\tau}}} \\
    + \frac{g(T_{on}) - g^{(0)}(T_{on})}{1 - e^{-\frac{T_{on}}{\tau}}}. \tag{57}
\]

The resulting model is

\[
    \tilde{i}_p[n] = \tilde{i}_p[n - 1] + m_1 \tilde{i}_{on}[n], \\
    \tilde{i}_c[n] = b \tilde{i}_c[n - 1] + c_1 \tilde{i}_p[n] + c_2 \tilde{i}_{on}[n], \tag{58}
\]

where

\[
    c_1 = 1 - d, \\
    c_2 = \frac{b}{\tau} I_c + \frac{d}{\tau} I_v + \frac{w(T_{on})}{\tau} + \frac{e^{-\frac{T_{on}}{\tau}}}{\tau} \int_{-\infty}^{+\infty} \frac{W(\omega)}{j\omega} d\omega, \\
    b = e^{-\frac{T_{on}}{\tau}}, \\
    d = e^{-\frac{T_{on}}{\tau}}. \tag{59}
\]
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