Hybrid Block Diagonalization for Massive Multiuser MIMO Systems

Weiheng Ni and Xiaodai Dong

Abstract

For a massive multiple-input multiple-output (MIMO) system, restricting the number of RF chains to far less than the number of antenna elements can significantly reduce the implementation cost compared to the full complexity RF chain configuration. In this paper, we consider the downlink communication of a massive multiuser MIMO (MU-MIMO) system and propose a low-complexity hybrid block diagonalization (Hy-BD) scheme to approach the capacity performance of the traditional BD processing method. We aim to harvest the large array gain through the phase-only RF precoding and combining and then digital BD processing is performed on the equivalent baseband channel. The proposed Hy-BD scheme is examined in both the large Rayleigh fading channels and millimeter wave (mmWave) channels. Finally, simulation results demonstrate that our Hy-BD scheme, with a lower implementation and computational complexity, achieves a capacity performance that is close to (sometimes even higher than) that of the traditional high-dimensional BD processing.

Index Terms

Massive MIMO, large scale MU-MIMO, hybrid processing, block diagonalization, limited RF chains, mmWave

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I. INTRODUCTION

To realize the tremendous capacity target of the next generation mobile cellular systems, one promising option is scaling up to massive multiple-input multiple-output (MIMO) systems [1]-[4]. In the massive multiuser MIMO (MU-MIMO) systems, some simple linear pre/post-processing (transmit precoding/receive combining) schemes, such as zero-forcing (ZF) and linear minimum mean-square error (MMSE), are able to approach the optimal capacity performance achieved by the dirty paper coding (DPC) as the number of antennas goes to infinity [5]. Moreover, the ZF processing that cancels the inter-user interference through channel inversion can be generalized as block diagonalization (BD) when the base stations (BSs) and mobile stations (MSs) are both equipped with multiple antennas [6]. For the downlink spatial multiplexing in MU-MIMO systems, the BD method achieves sub-optimal capacity performance; however, it reduces the complexity of the transmitter and receiver structures by providing closed-form precoder and combiner solutions.

In massive MIMO systems, the large array gain is rendered by a massive number of antennas at the order of a hundred or more [2]. Conventional pre-processing is performed through modifying the amplitudes and phases of the complex transmit symbols at the baseband and then upconverted to the passband by going through radio frequency (RF) chains (including the digital-to-analog conversion, signal mixing and power amplifying), which requires that the number of the RF chains is in the range of hundreds, equal to the number of the antenna elements. Post-processing is similar involving a large number of analog receive RF chains and digital baseband operations. This leads to unacceptably high implementation cost and energy consumption.

Recently, enabled by the cost-effective variable phase shifters, a limited number of RF chains have been applied in the MIMO systems [7]-[14]. The analog RF processing provides the high-dimensional phase-only control while the digital baseband processing can be performed in a very low dimension, termed as hybrid processing. Under the limited RF chains constraint, references
and [8] investigate the hybrid processing schemes in the point-to-point (P2P) MIMO systems. A single-stream communication under the Rayleigh fading MIMO channels achieves the full diversity order through the equal gain transmission/combining (EGT/EGC) in [7], while the multiple-stream transmission under MIMO channels is proposed in [8]. In addition, [9] and [10] implement the hybrid processing to the downlink of the massive MU-MIMO systems with single-antenna users. In [9], the near-optimal capacity performance, compared to the full-complexity systems, is achieved through the ZF baseband precoding combined with the EGT processing in the RF domain. Note that this technique also works for the millimeter wave (mmWave) channel. In [10], the phase-only RF precoding are employed to maximize the minimum average data rate of users via a bi-convex approximation approach. Furthermore, in mmWave communications systems, it is possible to build a large antenna array in a compact region and apply hybrid processing technique [11]-[14]. The “dominant” paths in P2P mmWave channels are captured through the hybrid processing in [11] and [12], where the former considers the single-stream transmission while the latter enables the multiple-stream communication. [12] presents a hybrid processing by decomposing the optimal precoding/combining matrix via orthogonal matching pursuit with the transmit/receive array response vectors as the basis vectors. Reference [11] can be regarded as a special one-RF-chain case of reference [12]. On the other hand, in the mmWave MU-MIMO systems, [13] considers the single-antenna users and designs the analog RF precoding based on the transmit beam directions, while the digital processing (matched filter, zero-forcing or Wiener filter) performs on the baseband equivalent channels. With the multiple-antenna users, some baseband processing schemes such MMSE and BD are examined in [14], which, however, neglects the design of the analog RF processing.

In this paper, we consider the downlink communication of a massive MU-MIMO system where the BS and all MSs have multiple antennas. With a limited number (≥ 1) of RF chains in BS and MSs, hybrid processing is applied as an alternative to the traditional high-cost full dimensional RF and baseband processing. This scenario has not been studied in the literature.
yet. We propose to utilize the RF precoding and combining to harvest the large array gain provided by the large number of antennas in the massive MU-MIMO channels, which shares the similar objective with the above references that study the hybrid processing in the MU-MIMO systems. However, the analog RF processing design for the MU-MIMO systems with multiple-antenna MSs accommodating multiple data streams per MS is not available in the literature and the novel BS RF precoder design is based on a newly defined “aggregate intermediate channel”. More specifically, the RF combiners of all the MSs are obtained by selecting some of the discrete Fourier transform (DFT) bases, while the RF precoder of the BS is designed by extracting the phases of the conjugate transpose of the aggregate intermediate channel which incorporates the MS RF combiners and the original downlink channels. With the designed RF precoder and combiners, a low-dimensional BD processing can be performed at the baseband to cancel the inter-user interference, and the whole operation is named the hybrid BD (Hy-BD) scheme. The advantages of such a Hy-BD scheme can be summarized as follows:

1) Low implementation cost and low computation complexity;
2) Applicability to both Rayleigh fading and mmWave massive MU-MIMO channels. Channel state information (CSI) is required but not the information of each individual propagation path;
3) Reduction on the feedback overhead of the RF domain operations.

Simulation results demonstrate that the proposed Hy-BD scheme achieves a capacity performance that is quite close to, sometimes even higher than, that of the full-complexity BD scheme in [6] with a lower implementation and computational cost. The Hy-BD scheme is also examined in the mmWave MU-MIMO communication channels and compared to the spatially sparse precoding/combining method [12] initially proposed for SU-MIMO but extended to MU-MIMO in this paper.
II. SYSTEM MODEL

A. System Model

We consider the downlink communication of a massive multiuser MIMO system shown in Fig. 1, where a base station with \( N_{BS} \) antennas and \( M_{BS} \) RF chains is assumed to schedule \( K \) mobile stations. Each MS is equipped with \( N_{MS} \) antennas and \( M_{MS} \) RF chains to support \( N_S \) data streams, which means total \( KN_S \) data streams are handled by the BS. To guarantee the effectiveness of the communication carried by the limited number of RF chains, the number of the transmitted streams is constrained by \( KN_S \leq M_{BS} \leq N_{BS} \) for the BS and \( N_S \leq M_{MS} \leq N_{MS} \) for each MS.

At the BS, the transmitted symbols are assumed to be processed by a baseband precoder \( B \) of dimension \( M_{BS} \times KN_S \) and then by an RF precoder \( F \) of dimension \( N_{BS} \times M_{BS} \). Notably, the baseband precoder \( B \) enables both amplitude and phase modification, while only phase changes (phase-only control) can be realized by \( F \) since it is implemented by using analog phase shifters. Each entry of \( F \) is normalized to satisfy \( |F^{(i,j)}| = \frac{1}{\sqrt{N_{BS}}} \), where \( |F^{(i,j)}| \) denotes the amplitude of the \((i,j)\)-th element of \( F \). Furthermore, to meet the total transmit power constraint, \( B \) is normalized to satisfy \( \|BF\|_F^2 = KN_S \), where \( \| \cdot \|_F \) the Frobenius norm.

We assume a narrowband flat fading channel model and obtain the received signal of the \( k \)-th MS

\[
y_k = H_k FBs + n_k, \quad k = 1, 2, \ldots, K,
\]

where \( s \in \mathbb{C}^{KN_S \times 1} \) is the signal vector for a total of \( K \) MSs, each of which processes a \( N_S \times 1 \) signal vector \( s_k \). Namely, \( s = [s_1^T, s_2^T, \ldots, s_K^T]^T \), where \( (\cdot)^T \) denotes transpose. And the signal vector satisfies \( \mathbb{E}[ss^H] = \frac{P}{KN_S} I_{KN_S} \), where \( (\cdot)^H \) denotes conjugate transpose, \( \mathbb{E}[: \] denotes expectation, \( P \) is the average transmit power and \( I_{KN_S} \) is the \( KN_S \times KN_S \) identity matrix. \( H_k \in \mathbb{C}^{N_{MS} \times N_{BS}} \) is the channel matrix for the \( k \)-th MS with \( \mathbb{E}[\|H_k\|_F^2] = N_{BS}N_{MS} \), and \( n_k \) is the \( N_{MS} \times 1 \) vector of i.i.d. \( \mathcal{CN}(0, \sigma^2) \) additive complex Gaussian noise. And the processed
received signal at the \( k \)-th MS after combining is given by

\[
\tilde{y}_k = M_k^H W_k^H \mathbf{H}_k \mathbf{F} \mathbf{B}_s + M_k^H W_k^H \mathbf{n}_k, \quad k = 1, 2, \ldots, K,
\]

(2)

where \( W_k \) is the \( N_{MS} \times M_{MS} \) RF combining matrix and \( M_k \) is the \( M_{MS} \times N_S \) baseband combining matrix for the \( k \)-th MS. Since \( W_k \) is also implemented by the analog phase shifters, all elements of \( W_k \) should have the constant amplitude such that \( |W_k^{(i,j)}| = \frac{1}{\sqrt{N_{MS}}} \). We define an equivalent baseband channel for each MS as

\[
\tilde{\mathbf{H}}_k = W_k^H \mathbf{H}_k \mathbf{F}, \quad k = 1, 2, \ldots, K,
\]

(3)

and the entire equivalent multiuser baseband channel can be denoted as

\[
\mathbf{H}_{eq} = \begin{bmatrix}
\tilde{\mathbf{H}}_1 \\
\tilde{\mathbf{H}}_2 \\
\vdots \\
\tilde{\mathbf{H}}_K 
\end{bmatrix} = \begin{bmatrix}
\mathbf{W}_1^H & 0 & \cdots & 0 \\
0 & \mathbf{W}_2^H & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \mathbf{W}_K^H 
\end{bmatrix} \begin{bmatrix}
\mathbf{H}_1 \\
\mathbf{H}_2 \\
\vdots \\
\mathbf{H}_K 
\end{bmatrix} \mathbf{F}.
\]

(4)

Then the processed received signal at the \( k \)-th MS can also be represented as

\[
\tilde{y}_k = M_k^H \tilde{\mathbf{H}}_k \mathbf{B}_s \mathbf{s}_k + \sum_{i=1,i \neq k}^{K} M_k^H \tilde{\mathbf{H}}_k \mathbf{B}_i \mathbf{s}_i + M_k^H W_k^H \mathbf{n}_k, \quad k = 1, 2, \ldots, K,
\]

(5)

where \( \mathbf{B}_k \) is the \(((k-1)N_S+1)\)-th to the \((kN_S)\)-th columns of \( \mathbf{B} \), corresponding to the baseband precoding for \( \mathbf{s}_k \). When the Gaussian symbols are used by the BS, the long-term average spectral efficiency achieved will be

\[
R = \sum_{k=1}^{K} \log_2 \left( \frac{1}{N_{S}} + \frac{P}{KN_S} R_i^{-1} M_k^H \tilde{\mathbf{H}}_k \mathbf{B}_k \mathbf{B}_k^H \tilde{\mathbf{H}}_k^H M_k \right),
\]

(6)

where \( R_i = \frac{P}{KN_S} \sum_{i=1,i \neq k}^{K} M_k^H \tilde{\mathbf{H}}_k \mathbf{B}_i \mathbf{B}_i^H \tilde{\mathbf{H}}_k^H M_k + \sigma^2 M_k^H W_k^H W_k M_k \) is the covariance matrix of both interference and noise.
B. Channel Model

In this paper, based on a general channel matrix set $H = [H_1^T, H_2^T, \cdots, H_K^T]^T$, we aim to seek the BS hybrid precoders $(F, B)$ and the hybrid combiners $(W_k, M_k)$’s for all $K$ MSs through the Hy-BD scheme, which achieves a sub-optimal spectral efficiency for massive MU-MIMO systems by perfectly canceling the inter-user interference. Two kinds of channel models are considered in this paper:

1) large i.i.d. Rayleigh fading channel $H_{rl}$;
2) limited scattering mmWave channel $H_{mmw}$.

In the large Rayleigh fading channel, which is commonly considered in massive MU-MIMO systems, all entries of the channel matrix $H_k$ for the $k$-th MS follow i.i.d. $CN(0, 1)$. On the other hand, a large antenna array is often implemented in mmWave communications to combat the high free-space pathloss [11]-[13]. We adopt the clustered mmWave channel model to characterize the limited scattering feature of the mmWave channel. The mmWave downlink channel for the $k$-th MS $H_k$ is assumed to be the sum of all propagation paths that are scattered in $N_c$ clusters and each cluster contributes $N_p$ paths, which can be expressed as

$$H_k = \sqrt{\frac{N_{BS}N_{MS}}{N_cN_p}} \sum_{i=1}^{N_c} \sum_{l=1}^{N_p} \alpha^k_{il} a_{MS}^k(\theta^k_{il}) a_{BS}^k(\phi^k_{il})^H,$$  

(7)

where $\alpha^k_{il}$ is the complex gain of the $i$-th path in the $l$-th cluster, which follows $CN(0, 1)$. To reflect the sparsity of the mmWave channel, both of $N_c$ and $N_p$ should not be too large. For the $(i, l)$-th path, $\theta^k_{il}$ and $\phi^k_{il}$ are the azimuth angles of arrival/departure (AoA/AoD), while $a_{MS}^k(\theta^k_{il})$ and $a_{BS}^k(\phi^k_{il})$ are the receive and transmit array response vectors at the azimuth angles of $\theta^k_{il}$ and $\phi^k_{il}$ respectively, and the elevation dimension is ignored. Within the cluster $i$, $\theta^k_{il}$ and $\phi^k_{il}$ have the uniformly-distributed mean values of $\theta^k_i$ and $\phi^k_i$ respectively, while the lower and upper bounds of the uniform distribution for $\theta^k_{il}$ and $\phi^k_{il}$ can be defined as $[\theta^k_{\min}, \theta^k_{\max}]$ and $[\phi^k_{\min}, \phi^k_{\max}]$. The angle spreads (standard deviations) of $\theta^k_{il}$ and $\phi^k_{il}$ among all clusters are assumed to be constant, denoted as $\sigma^k_{\theta}$ and $\sigma^k_{\phi}$. Finally, the truncated Laplacian distribution is employed to generate all
the AoDs/AoAs for this mmWave propagation channel matrix, based on the above parameters.

The uniform linear array (ULA) is employed by the BS and MSs in our study, while the Hy-
BD scheme in Section III can directly be applied to arbitrary antenna arrays. For an N-element
ULA, the array response vector can be given by

$$a_{ULA}(\theta) = \frac{1}{\sqrt{N}} \left[ 1, e^{j\frac{2\pi}{\lambda}d\sin(\theta)}, \ldots, e^{j(N-1)\frac{2\pi}{\lambda}d\sin(\theta)} \right]^T,$$

(8)

where $\lambda$ is the wavelength of the carrier, and $d$ is the distance between neighboring antenna
elements. The array response vectors of the BS and MSs can be written in the form of (8).

III. HYBRID BLOCK DIAGONALIZATION

In the MU-MIMO systems, the generalized zero-forcing method (i.e., the traditional BD
scheme) is infeasible to be practically implemented due to the high cost brought by the large
number of RF chains as many as the antennas. By reducing the number of RF chains $M_{BS}(M_{MS})$
to far less than the antenna elements $N_{BS}(N_{MS})$ at both the BS and MSs, we propose to utilize
the RF precoding matrix $F$ at the BS and the RF combining matrix $W_k$ at each MS to harvest the
large array gain provided by the large number of antennas in the massive MU-MIMO channel.

With the found $F$ and all $W_k$‘s, the entire multiuser equivalent baseband channel $H_{eq}$ can be
determined based on (4), which consists of all the equivalent channels for the MSs, namely
$\tilde{H}_k$, $k = 1, 2, \ldots, K$. Finally, a low-dimensional BD processing, involving the design of $B$ and
all $M_k$‘s, can be performed at the baseband.

A. Array Gain Harvesting

Owing to the large number of antennas in the massive MU-MIMO systems, the channel gains
of the equivalent channel $H_{eq}$ can be scaled up through the appropriate phase-only control at the
RF domain, which is called the large array gain. To be noted, each element in $H_{eq}$ represents
the equivalent channel gain from one RF chain at the BS to one RF chain at one MS. To achieve
the high capacity with such a hybrid processing structure, the equivalent channel matrix $H_{eq}$ are desired to have the following properties:

1) Rank sufficiency: $H_{eq}$ should be well-conditioned to support the multi-stream transmission, which means the rank of $H_{eq}$ should be at least $KN_S$;

2) Large array gain: $H_{eq}$ should sufficiently harvest the array gain so that it can provide as large gain for each stream transmission as possible. We propose to pursue the large array gain by enlarging the sum of the squares of the diagonal entries in $H_{eq}$.

By definition, $H_{eq}$ consists of the equivalent channels of all the MSs, namely $\tilde{H}_k = W_k^H H_k F$, $k = 1, 2, \ldots, K$. We design the RF domain processing matrices $W_k$’s and $F$ and construct the equivalent channel $H_{eq}$ by approximately satisfying the above two requirements, which will lead to a suboptimal performance under the hybrid precoding structure, but with significantly low complexity.

Assume that all the RF combiners $W_k$’s are given (the actual design of $W_k$’s will be presented shortly). Define an aggregate intermediate channel given by

$$H_{int} = \begin{bmatrix} W_1^H H_1 \\ \vdots \\ W_K^H H_K \end{bmatrix}_{KM_S \times N_{BS}},$$

and then the baseband equivalent channel is $H_{eq} = H_{int} F$. To harvest the array gain with the RF precoder $F$, similar to the equal gain transmission (EGT) method proposed in [9], we perform the phase-only RF precoding by setting

$$F^{(i,j)} = \frac{1}{\sqrt{N_{BS}}} e^{j\psi_{i,j}},$$

where $\psi_{i,j}$ is the phase of the $(i,j)$-th element of the conjugate transpose of $H_{int}$. This EGT precoding method requires $M_{BS} = KM_S$ RF chains at the BS, which means $F$ is an $N_{BS} \times KM_S$ matrix and $H_{eq}$ should be a square matrix. The entries along the diagonal of the baseband equivalent channel $H_{eq}$ denote the equivalent channel gains in terms of the RF chains, while
the remaining entries indicate the inter-chain interference. We focus on the large array gain design through the RF precoding/combining and leave the interference canceling to the baseband processing in the Hy-BD scheme.

Now let us return to the design of the RF combiners $W_k$’s. Denote the $m$-th column of $W_k$ as $w_k^{(m)}$. As the result of the EGT precoding method, the $((k-1)M_{MS}+m)$-th diagonal entry of $H_{eq}$ is then given by $\| (w_k^{(m)})^H H_k \|_1$, where $\| \cdot \|_1$ denotes the 1-norm of a vector, corresponding to the $m$-th RF chain of the $k$-th MS. We aim to maximize the sum of the squares of diagonal entries of the baseband equivalent channel $H_{eq}$, given by

$$\sum_{k=1}^{K} \sum_{m=1}^{M_{MS}} \| (w_k^{(m)})^H H_k \|_1^2,$$


to pursue the large array gain. Due to the independence of $W_k$’s for all the MSs, maximizing $\sum_{k=1}^{K} \sum_{m=1}^{M_{MS}} \| (w_k^{(m)})^H H_k \|_1^2$ is equivalent to maximizing $\sum_{m=1}^{M_{MS}} \| (w_k^{(m)})^H H_k \|_1^2$ for all $k = 1, \cdots, K$ respectively. Hence, the design of the RF combiners can be obtained by solving

$$\max_{W_k} \sum_{m=1}^{M_{MS}} \| (w_k^{(m)})^H H_k \|_1^2$$

$$\text{s.t. } |W_k^{(i,j)}| = \frac{1}{\sqrt{N_{MS}}}, \forall i, j.$$  \hspace{1cm} (11)

In this paper, instead of solving the non-convex problem (11) directly, we modify the constraints to choose from a set of DFT basis, as explained in details next. Note that $\| (w_k^{(m)})^H H_k \|_1^2 = (\sum_{n=1}^{N_{BS}} |(w_k^{(m)})^H h_k^{(n)}|)^2$, where $h_k^{(n)}$ denotes the $n$-th column of $H_k$. Moreover, the geometric MIMO channel models, including the Rayleigh fading and mmWave channels, can be represented in the form of (7), which means $h_k^{(n)}$ is the linear combination of all the array response vectors of the AoAs. This fact implies that $\| (w_k^{(m)})^H H_k \|_1$ is the sum of the projections of those array response vectors on $w_k^{(m)}$. From this perspective, we first propose to set $w_k^{(m)}$ in the form of array response vector (8) to extract the gain from these projections, namely,

$$d(\omega) = \frac{1}{\sqrt{N_{MS}}} [1, e^{j\omega}, e^{j2\omega}, \cdots, e^{j(N_{MS}-1)\omega}]^T,$$  \hspace{1cm} (12)

$^1$In the Rayleigh fading channel, all AoDs/AoAs of the paths (non-LOS) are uniformly distributed among $[0, 2\pi)$ and the number of paths approaches to infinity.
where \( \omega = \frac{2\pi}{\lambda} d \sin \theta \) denotes the corresponding spatial frequency \([15]\).

Furthermore, to meet the rank sufficiency requirement of \( H_{eq} \), it is desirable that the rank of \( H_k \) is not reduced after it being multiplied by \( W_k \). For this purpose, we require the columns of \( W_k \) to be pairwise orthogonal so that the rank of \( W_k^H H_k \) is lower bounded by \( M_{MS} > N_S \) (the rank of the high-dimensional \( H_k \) is assumed to be larger than \( M_{MS} \)), which means the equivalent channel \( H_{eq} \) is potentially capable of supporting the transmission of \( KM_{MS} > KN_S \) streams.

Considering the form of \( w^{(m)}_k \), we discretize the \( \omega \) into \( N_{MS} \) levels over \([0, 2\pi)\) and construct \( N_{MS} \) bases, given by
\[
D = \{ d(0), d(\frac{2\pi}{N_{MS}}), \ldots, d(\frac{2\pi(N_{MS}-1)}{N_{MS}}) \}
\]
as the candidates from which the \( w^{(m)}_k \) is choosen. As we can see, these bases in \( D \) exactly form an \( N_{MS} \)-dimensional DFT basis set, which simultaneously conforms to the rank sufficiency and large array gain requirements of \( H_{eq} \). Therefore, we finally design the RF combiners by solving
\[
\max_{W_k} \sum_{m=1}^{M_{MS}} \| (w^{(m)}_k)^H H_k \|_1^2 \\
\text{s.t. } w^{(m)}_k \in D, \; m = 1, \ldots, M_{MS}.
\]

(13)

To solve the problem (13), we just need to sort all \( N_{MS} \| d(\omega)^H H_k \|_1 \)’s in the descendant order and then choose the first \( M_{MS} \) \( d(\omega) \)’s as the columns of \( W_k \).

**Remark 3.1:** Omitting the cross-terms of \( \| (w^{(m)}_k)^H H_k \|_1 \), we construct another criterion to achieve the large array gain, given by
\[
\| (w^{(m)}_k)^H H_k \|_1^2 = \left( \sum_{n=1}^{N_{BS}} \| (w^{(m)}_k)^H h^{(n)}_k \| \right)^2 \\
\Rightarrow \sum_{n=1}^{N_{BS}} \| (w^{(m)}_k)^H h^{(n)}_k \|^2 = \| (w^{(m)}_k)^H H_k \|_F^2.
\]

(14)

The large array gain can also be attained by solving
\[
\max_{W_k} \| W_k^H H_k \|_F^2 = \sum_{m=1}^{M_{MS}} \| (w^{(m)}_k)^H H_k \|_F^2 \\
\text{s.t. } w^{(m)}_k \in D, \; m = 1, \ldots, M_{MS}.
\]

(15)
The solution to this maximizing problem (15) can be obtained through the sorting operation similar to that of (13). Notably, the simulation results in Section [IV] demonstrate that such a Frobenius norm based problem achieves a similar spectral efficiency performance in the massive MU-MIMO systems with the 1-norm based solution.

**Remark 3.2:** Based on the selection of the DFT bases, the MSs can avoid a huge amount of computation overhead for obtaining all the phase shift elements. In addition, only $N_S$ phase shift elements per MS is needed to be fed back to the BS, so that to the BS is able to re-construct all the $W_k$’s and calculate the aggregate intermediate channel $H_{int}$ for further processing.

**B. Baseband Block Diagonalization**

In this section, based on the obtained baseband equivalent channel $H_{eq}$, given the found RF processing matrices $W_k$ and $F$, we perform the low-dimensional BD processing with the baseband precoder $B$ and combiners $M_k$’s to cancel the inter-user interference, which forces the interference terms $\tilde{H}_k B_i = 0$ for $i \neq k$ in (5). The spectral efficiency of the MU-MIMO system can be further simplified to

$$R = \sum_{k=1}^{K} \log_2 \left( I_{N_S} + \frac{P}{\sigma^2 KN_S} (M_k^H W_k^H W_k M_k)^{-1} M_k^H \tilde{H}_k B_k B_k^H \tilde{H}_k^H M_k \right).$$

(16)

To obtain the baseband precoder $B = [B_1, B_2, \ldots, B_K]$, where $B_k$ incorporates the precoding vectors for the data streams of the $k$-th MS, we first define $\Pi_k$ as

$$\Pi_k = [\bar{H}_1^T, \ldots, \bar{H}_{k-1}^T, \bar{H}_{k+1}^T, \ldots, \bar{H}_K^T]^T.$$  

(17)

The $B_k$ is supposed to lie in the null space of $\Pi_k$. Denote the rank of $\Pi_k$ as $r_k \leq (K-1) M_{MS}$. Then the singular value decomposition (SVD) of $\Pi_k$ is given by

$$\Pi_k = U_k \Sigma_k \left[ V_k^{(K-1)M_{MS}}, V_k^{(M_{MS})} \right]^H,$$

(18)

where $V_k^{(K-1)M_{MS}}$ consists of the first $(K-1) M_{MS}$ right singular vectors of $\Pi_k$, and $V_k^{(M_{MS})}$ holds the rest $M_{MS}$ ones which are exactly the orthogonal bases of the null space of $\Pi_k$. Then
we know
\[
\tilde{H}_i \tilde{V}_k^{(\text{MS})} = \begin{cases} 
0, & i \neq k \\
\tilde{H}_k \tilde{V}_k^{(\text{MS})}, & i = k 
\end{cases}
\tag{19}
\]

Given the above results, block diagonalization of the baseband equivalent channel matrix to remove inter-user interference is written as
\[
\mathbf{H}_{BD} = \mathbf{H}_{eq} \begin{bmatrix} \tilde{V}_1^{(\text{MS})}, & \cdots, & \tilde{V}_K^{(\text{MS})} \end{bmatrix} 
= \begin{bmatrix} 
\tilde{H}_1 \tilde{V}_1^{(\text{MS})} & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & \tilde{H}_K \tilde{V}_K^{(\text{MS})} 
\end{bmatrix} 
\tag{20}
\]

Until now, all the MSs can perform inter-user-interference-free multi-stream transmission through their own sub-channels (the non-zero block in \( \mathbf{H}_{BD} \)). Further precoding/combining will be performed to achieve each MS’s optimal spectral efficiency based on SVD, given by
\[
\tilde{H}_k \tilde{V}_k^{(\text{MS})} = \mathbf{U}_k \mathbf{\Sigma}_k \tilde{V}_k^H. 
\tag{21}
\]

With the above rank sufficiency requirement, \( \tilde{H}_k \tilde{V}_k^{(\text{MS})} \) is a \( M_{\text{MS}} \)-by-\( M_{\text{MS}} \) full-rank sub-channel matrix which enables \( M_{\text{MS}} \geq N_S \) data streams transmission for the \( k \)-th MS. Therefore, the optimal precoder and combiner on the \( k \)-th effective sub-channel \( \tilde{H}_k \tilde{V}_k^{(\text{MS})} \) should be \( \mathbf{V}_k^{(N_S)} \) and \( \mathbf{U}_k^{(N_S)} \), where \( \mathbf{V}_k^{(N_S)} \) and \( \mathbf{U}_k^{(N_S)} \) are the first \( N_S \) columns of the \( \mathbf{V}_k \) and \( \mathbf{U}_k \) respectively. Finally, the overall baseband precoder is given by
\[
\mathbf{B} = \begin{bmatrix} \tilde{V}_1^{(\text{MS})}, & \cdots, & \tilde{V}_K^{(\text{MS})} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1^{(N_S)} & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & \mathbf{V}_K^{(N_S)} 
\end{bmatrix} 
= \begin{bmatrix} \tilde{V}_1^{(\text{MS})} \mathbf{V}_1^{(N_S)}, & \cdots, & \tilde{V}_K^{(\text{MS})} \mathbf{V}_K^{(N_S)} \end{bmatrix}_{K \times M_{\text{MS}} \times N_S}.
\tag{22}
\]
And the baseband combiner for the \( k \)-th MS is given by \( \mathbf{M}_k = \mathbf{U}_k^{(N_S)} \), \( k = 1, 2, \cdots, K \).

The spectral efficiency achieved by the Hy-BD scheme finally becomes
\[
R = \sum_{k=1}^{K} \log_2 \left( \frac{\mathbf{I}_{N_S} + \frac{P \mathbf{A}(\mathbf{M}_k^H \mathbf{W}_k^H \mathbf{W}_k \mathbf{M}_k)^{-1}}{\sigma^2 K N_S} (\mathbf{\Sigma}_k^{(N_S)})^2}{\mathbf{I}_{N_S}} \right), \tag{23}
\]
where $\Lambda$ is a diagonal matrix that performs water-filling power allocation, and $\Sigma_k^{(N_S)}$ represents the first $N_S \times N_S$ block partition of $\Sigma_k$.

On the other hand, since the RF and baseband processing do not require any information of the propagation paths of the channels $H_k$'s, namely, each term in the summation of (7), the Hy-BD scheme can be performed on any kinds of massive MU-MIMO channels as long as the channel matrices are provided.

IV. Simulation Results

In this section, we evaluate the spectral efficiency achieved by the Hy-BD scheme as well as its performance robustness in the massive MU-MIMO channels.

A. Spectral Efficiency Evaluation

In the simulations of this section, we illustrate the spectral efficiency achieved by the Hy-BD scheme in the massive MU-MIMO systems by comparing it with the traditional high-dimensional baseband BD scheme in large i.i.d Rayleigh fading and mmWave multiuser channels and also with the previously proposed spatially sparse precoding/combining scheme [12] in mmWave channels. The range of the signal-to-noise ratio $\text{SNR} = \frac{P}{\sigma^2}$ is from -40 dB to 0 dB in all processing solutions.

Fig. 2 illustrates the sum spectral efficiency achieved by the traditional BD scheme and our proposed Hy-BD scheme (through solving the 1-norm and Frobenius norm based optimization problems in Section III-A) in the large i.i.d. Rayleigh fading channel. The BS with $M_{BS} = 16$ RF chains is employed to schedule $K = 8$ MSs, each of which processes $N_S = 2$ data streams with $M_{MS} = 2$ RF chains. Furthermore, the BS and MSs are equipped with 256 (16) and 64 (4) antennas respectively. In both $256 \times 16$ and $64 \times 4$ antenna settings, the sum spectral efficiency of the Hy-BD scheme consistently approaches the performance achieved by the traditional BD scheme, however, with lower implementation and computational complexity. In addition, the
Hy-BD designs based on solving the 1-norm problem \((13)\) and the Frobenius norm problem \((15)\) respectively lead to the nearly identical sum spectral efficiency, supporting the statement in Remark 3.1. Notably, the results of the \(64 \times 4\) antenna setting indicate that the Hy-BD scheme is still effective in a small scale antenna system.

In the mmWave MU-MIMO channels, the traditional full-complexity BD and Hy-BD schemes perform in a similar fashion as in the Rayleigh fading channels. Based on the limited number of paths scattered in the mmWave channels, the spatially sparse precoding/combining scheme in \([12]\) can be extended to the hybrid processing in MU-MIMO systems through decomposing the solution to the traditional BD scheme (the precoder \(M_S\) and the MMSE combiners in \([6]\)) via orthogonal matching pursuit where the BS and MSs choose the array response vectors of the corresponding AoDs and AoAs as the basis vectors respectively. Fig. 3 shows the sum spectral efficiency of the above processing schemes. We set the mmWave propagation channel with \(N_c = 8\) and \(N_p = 10\). The range of the mean azimuth angles of AoDs at the BS \(|\theta_k^{\text{max}} - \theta_k^{\text{min}}|\) is \(120^\circ\) while the MSs are assumed to be omni-directional due to the relatively smaller antenna array elements. The angle spreads \(\sigma_k^\theta\)'s and \(\sigma_k^\phi\)'s are all equal to \(7.5^\circ\). Moreover, the BS is set to have \(N_{BS} = 256\) antennas and \(M_{BS} = 16\) RF chains, while \(K = 8\) MSs, with \(N_{MS} = 16\) antennas and \(M_{MS} = 2\) RF chains, all dealing with \(N_S = 2\) data streams. In this scenario, the proposed Hy-BD scheme even achieves slightly higher spectral efficiency than the traditional BD scheme. Note that the traditional BD scheme is a sub-optimal solution for the processing of MU-MIMO systems, and it is possible that the Hy-BD outperforms the traditional BD in some situations. As for the spatially sparse precoding/combining scheme, it lags behind the traditional BD and Hy-BD schemes because the columns of the traditional BD precoding and combining matrices do not directly come from the linear combination of the array response vectors of AoDs/AoAs, the basic forming units of the RF matrices in the spatially sparse scheme \([12]\). This is very different from the P2P scenario that the spatially sparse scheme is designed for, where the columns of the SVD based precoder and combiner can be effectively approached by
the linear combinations of the array response vectors according to the observation 3) in [12]. Even though the number of RF chains is enlarged to $M_{MS} = 4$ and $M_{BS} = 32$, the performance of the spatially sparse precoding/combining scheme is still inferior to the full-complexity BD and Hy-BD schemes.

### B. Robustness Evaluation

In addition to simply demonstrating the spectral efficiency of the Hy-BD scheme under different SNRs, we further examine its performance robustness by changing the multiplexing settings (e.g., the number of data streams supported by each user and the number of users) and introducing the channel estimation error.

For the practical implementation of an MU-MIMO system, the total number of supported data streams is a very important criterion to evaluate the system performance, which depends on the number of supported MSs $K$ and the number of data stream supported by each MS $N_S$, namely, space-division multiple access and spatial multiplexing. In Figs. 4-6, the sum spectral efficiency achieved by the traditional BD scheme and the Hy-BD scheme is checked in a $256 \times 16$ 8-user MU-MIMO system in i.i.d Rayleigh fading channels under different SNRs, where each MS only employs $M_{MS} = N_S$ RF chains ($K \ast N_S$ RF chains at the BS) in the Hy-BD scheme.

In Fig. 4, the number of data streams per MS is set as $N_S = 1, 2, 4$ and the SNR ranges from $-40$ dB to $0$ dB. The gap between the sum spectral efficiency of the traditional BD scheme and the Hy-BD scheme remains minute compared to the absolute sum spectral efficiency. However, the Hy-BD scheme only needs the same number of RF chains as the supported data streams at both BS and MSs (up to $M_{BS} = 32$ and $M_{MS} = 4$), much smaller than that of the traditional BD scheme ($M_{BS} = 256$ and $M_{MS} = 16$). Fig. 5 shows the sum spectral efficiency of both schemes when $N_S$ increases from 1 to 16 and the SNR is set as $-10, -5$ and $0$ dB, which indicates that it is suitable to employ the Hy-BD scheme when the total number of data streams in the MU-MIMO system is not too large, so that the Hy-BD can reach the peak spectral efficiency. As we
can see, the sum spectral efficiency achieved by the traditional BD scheme will be continuously augmented in such a $256 \times 16$ 8-user MU-MIMO system when the number of transmitted data streams increases, since more equivalent parallel channels (characterized by the diagonal elements in $\Sigma$ in \cite{6}) can be utilized to transmit the data streams and the effect of inter-user interference is not dominant in this case. However, in the Hy-BD scheme, the spectral efficiency performance is somewhat compromised once a large quantity of data streams are transmitted. This is because the pursuit of the large array gain slightly introduces the inter-stream interference in the RF domain, which will degrade the system spectral efficiency after the baseband BD processing.

On the other hand, with an increasing SNR, the suitable numbers of the supported data streams $N_S$, corresponding to the peak spectral efficiency, for the traditional BD scheme and Hy-BD scheme are also enhanced. For instance, when SNR = 0 dB, the traditional BD scheme supports up to $K \times N_S = 8 \times 16 = 128$ data streams which is the maximum number of the supported data streams by a $256 \times 16$ 8-user MU-MIMO system with full RF chains. However, the Hy-BD scheme can support about $K \times N_S = 8 \times 8 = 64$ data streams with only 64 and 8 RF chains at the BS and MS respectively.

With the same system configuration as that of Figs. 4 and 5 and the number of data streams per MS set as $N_S = 4$, the number of MSs $K$ increases from 1 to 16 in Fig. 6. In this case, the traditional BD scheme with full RF chain configuration reaches a peak spectral efficiency at a certain $K$. This is because when $K$ grows beyond an optimal value, inter-user interference becomes substantially more severe and the sum spectral efficiency is gradually degraded. As for the Hy-BD scheme with the limited RF chain configuration, the sum spectral efficiency keeps improving when $K$ increases from 1 to 16 (the maximum number of supported data stream is still up to $K \times N_S = 4 \times 16 = 64$). By comparing the results of Figs. 5 and 6 the Hy-BD scheme can be safely recommended for implementation in systems with a large number of MSs, however, each of which deals with a small number of data streams, since it is less vulnerable to the inter-user interference than the traditional BD scheme in this case. As for the case in
Fig. 5 where there are fewer MSs and more data streams per MS, the traditional BD scheme achieves superior performance at the cost of high complexity because it can better process the inter-stream interference than the Hy-BD scheme.

Furthermore, we examine the sum spectral efficiency of both schemes with an increasing $N_S$ or $K$ under different SNRs ($-10, -5$ and $0$ dB) in the mmWave MU-MIMO channels whose propagation characteristics are given in Fig. 3’s settings. The BS and MS configurations are the same as those of Figs. 5 and 6. Here, Fig. 7 illustrates the sum spectral efficiency of both schemes when $N_S$ increases from 1 to 16 with $K = 8$, while Fig. 8 gives the result for the number of MSs $K$ increasing from 1 to 16 with $N_S = 4$. As can be seen, the general trends of the sum spectral efficiency of the traditional BD scheme and the Hy-BD scheme in mmWave channels are consistent with those in Rayleigh fading channels, except that the Hy-BD scheme can perform slightly better in mmWave channels compared with the results in Rayleigh fading channels. It is probably due to the fact that the DFT bases selection (conforming to the forms of AoAs/AoDs array responses of the limited number of paths in mmwave channels) in the Hy-BD scheme essentially captures the dominant paths of the mmWave channels.

The Hy-BD scheme is also tested under the imperfect CSI. With the same system configuration as that of Figs. 4-8, assume the channel for the $k$-th MS, $k = 1, \cdots , K$, is $H_k$, and the corresponding estimated channel is $\hat{H}_k = H_k + E_k$, where $E_k$ is the channel estimation error. All entries in $E_k$ follow the identical complex Gaussian distribution with zero means. We define the normalized channel estimation error power as $\frac{\sum_{k=1}^{K} ||E_k||^2_F}{\sum_{k=1}^{K} ||H_k||^2_F}$. In Fig. 9 the i.i.d Rayleigh fading channel is chosen and the channel estimation error power is set from -40 dB to -6 dB and the SNR for the transmit signal is 0 dB. The solid lines represent the performance with no channel estimation while the dash lines indicate the performance of the two schemes with channel estimation error. As Fig. 9 shows, the thresholds for the channel estimation error power for both schemes are about -10 dB. Once the channel estimation error power reaches this value, the performance will drop drastically. Therefore, as long as we can limit the channel estimation error
for all MSs within this threshold, the Hy-BD scheme can maintain a reasonable performance.

V. CONCLUSION

In this paper, a low-complexity hybrid block diagonalization processing scheme has been proposed for the downlink communication of a massive multiuser MIMO system with the limited number of RF chains. We harvest the large array gain through the phase-only RF precoding and combining and then the BD technique is performed at the equivalent baseband channel. It has been demonstrated that the Hy-BD scheme, with a lower implementation and computational complexity, achieves a capacity performance approaching that of the traditional high-dimensional baseband BD processing. Such a low-complexity, low cost Hy-BD scheme can be a promising option for the practical implementation of a massive MU-MIMO system.

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Fig. 1. System diagram of a massive MU-MIMO system with hybrid processing structure.
Fig. 2. Sum spectral efficiency achieved by different processing schemes in an 8-user MU-MIMO system in i.i.d. Rayleigh fading channels where $N_S = 2, M_{MS} = 2, M_{BS} = 16$. 
Fig. 3. Sum spectral efficiency achieved by different processing schemes in an $256 \times 16$ 8-user MU-MIMO system in mmWave channels where $N_S = 2, M_{MS} = 2(4), M_{BS} = 16(32)$. 
Fig. 4. Sum spectral efficiency achieved by different processing schemes in an 8-user MU-MIMO system in i.i.d. Rayleigh fading channels where $N_S = M_{MS} = 1, 2, 4$ and $M_{BS} = 8M_{MS}$. 
Fig. 5. Sum spectral efficiency achieved by different processing schemes in a $256 \times 16$ 8-user MU-MIMO system in i.i.d. Rayleigh fading channels where $N_S$ increases from 1 to 16 and SNR = $-10, -5, 0$ dB.
Fig. 6. Sum spectral efficiency achieved by different processing schemes in a $256 \times 16$ MU-MIMO system in i.i.d. Rayleigh fading channels where $K$ increases from 1 to 16, SNR = $-10$, $-5$, $0$ dB and $N_S = 4$. 
Fig. 7. Sum spectral efficiency achieved by different processing schemes in a $256 \times 16$ 8-user MU-MIMO system in mmWave channels where $N_S$ increases from 1 to 16 and SNR = $-10, -5, 0$ dB.
Fig. 8. Sum spectral efficiency achieved by different processing schemes in a $256 \times 16$ MU-MIMO system in mmWave channels where $K$ increases from 1 to 16, SNR = $-10, -5, 0$ dB and $N_S = 4$. 
Fig. 9. Sum spectral efficiency achieved by different processing schemes in a $256 \times 16$ 8-user MU-MIMO system in i.i.d. Rayleigh fading channels with channel estimation error where $N_S = 2, M_{MS} = 2, M_{BS} = 16$. 