Non-Universal Gaugino Masses in the NMSSM

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Abstract

The Next-to-Minimal Supersymmetric Standard Model (NMSSM) provides a natural framework to realize a low-scale supersymmetric (SUSY) model, where a singlet superfield is added to the minimal model to generate a SUSY-scale higgsino mass term with its vacuum expectation value. Due to the presence of the extra singlet field, the vacuum conditions to realize the correct electroweak symmetry-breaking become fairly restrictive especially if we impose universality conditions at the unification scale. In this paper, we show that a non-universal gaugino mass spectrum can significantly relax this restriction even though the scalar masses and trilinear couplings are subject to universality conditions. With the gaugino non-universality, we find that higgsino can be the lightest SUSY particle and its thermal relic abundance can reproduce the observed dark matter density in a wide range of parameter space in which the 125 GeV Higgs-boson mass is obtained. This higgsino-like dark matter may be probed in direct detection experiments. We also find that there is an upper bound on the masses of supersymmetric particles in this scenario, and many model points predict colored particles such as gluino to be within the reach of a future 100 TeV collider. Implications for no-scale/gaugino-mediation models are also discussed.

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1 Introduction

A supersymmetric (SUSY) extension of the Standard Model (SM) has been regarded as the leading candidate for physics beyond the SM. One of the main reasons for this is its ability to naturally stabilize the weak scale against radiative corrections if the SUSY breaking scale lies around the TeV scale. Above this scale, SUSY partners of the SM fields appear and their contribution to the quantum corrections to the SM Higgs mass parameter cancels that from the SM particles. This stability is not spoiled even if extra heavy particles exist at high energies, since the radiative corrections to the Higgs mass parameter are completely screened thanks to SUSY. Furthermore, as it turns out, the presence of the TeV-scale SUSY particles allows the SM gauge coupling constants to unify at a high-energy scale with their perturbativity maintained up to the unification scale ($\approx 2 \times 10^{16}$ GeV). Because of these properties, SUSY theories offer favorable framework to the construction of a more fundamental theory such as grand unified theories (GUTs).

In such a high-energy theory, the parameters in the model are given at the scale of the fundamental theory, say, the GUT scale. Hence, if the model contains a mass parameter which respects the symmetries of the theory, we expect this value to be of the order of the fundamental scale. In the minimal SUSY SM (MSSM), the higgsino mass parameter, which is called the $\mu$ parameter, has such a property [1]. It turns out however that if the $\mu$ parameter is much larger than the SUSY-breaking scale the electroweak symmetry breaking (EWSB) does not occur and thus the model cannot be considered to be realistic. A simple way to evade this problem is to remove this mass parameter from the theory with the help of an extra symmetry. We then add a singlet superfield and couple this to the Higgs superfields so that it generates an effective $\mu$ parameter after it develops a vacuum expectation value (VEV). This setup is dubbed as the Next-to-Minimal SUSY SM (NMSSM) [2, 3].

In the NMSSM, both the singlet and the MSSM Higgs fields can acquire VEVs only after SUSY is broken. This can thus explain why these values are as small as the weak scale if the SUSY-breaking scale is around the TeV scale. In fact, it is known [2, 3] that with an appropriate choice of SUSY parameters we can obtain a desirable vacuum at which the electroweak gauge symmetry is adequately broken so that the observed mass spectrum in the SM is realized. This observation makes the NMSSM a quite promising candidate for the SUSY SM.

This vacuum condition, however, turns out to be highly restrictive once we consider some universality at the input scale. In the case of the MSSM, it has been widely known that even though we assume universality among the parameters at the input scale, as in the Constrained Minimal SUSY SM (CMSSM), we can easily obtain a viable set of model parameters at low energies (for recent studies of such models, see, e.g., Refs. [4–17] and references therein). The NMSSM counterpart of the CMSSM, called CNMSSM, has also been extensively studied so far [18–26]. In this case, the correct electroweak vacuum can be realized only in the limited parameter space, where the universal scalar mass $m_0$ and trilinear parameter $A_0$ satisfy $3m_0 \lesssim |A_0|$ and both are much smaller than the universal gaugino mass $m_{1/2}$. These relations make it rather difficult to obtain the observed value.
of the mass of the SM-like Higgs boson \( m_h \simeq 125 \text{ GeV} \) \cite{26}, and require SUSY-breaking scale to be much higher than the electroweak scale. In addition, the condition \( 3m_0 \lesssim |A_0| \) tends to make stau the lightest SUSY particle (LSP) or even tachyonic. In the region where this is evaded, the LSP is singlino—though in principle this can be a good dark matter candidate, its thermal relic abundance often exceeds the observed value of dark matter abundance, \( \Omega_{\text{DM}} h^2 \simeq 0.12 \) \cite{28}, especially when SUSY-breaking scale is in the multi-TeV range so that the 125 GeV Higgs-boson mass is obtained. Such over abundance requires non-trivial cosmological history like entropy production by a long-lived particle.

To evade these problems, it is often the case that the universality conditions on the singlet scalar masses and/or trilinear terms are relaxed so that the correct electroweak vacuum is easily obtained \cite{29–36}. The introduction of right-handed neutrinos may also improve the situation through renormalization group (RG) effect due to their couplings to the singlet superfield \cite{37}.

In this paper, we discuss another possibility, namely, a constrained NMSSM with non-universal gaugino masses. A non-universal gaugino mass spectrum can be realized in theoretically well-motivated frameworks such as the mirage mediation \cite{38–44}, non-minimal GUT models \cite{45–53}, models with a non-universal gauge kinetic function via string compactifications \cite{54, 55}, and so on, and found to be advantageous for the electroweak naturalness problem \cite{56, 57}. These observations have stimulated many studies on non-universal gaugino masses so far \cite{58–75}. In particular, discussions on mirage mediation in the NMSSM can be found in Refs. \cite{76–78}. The present paper focuses on the implications of non-universal gaugino mass spectrum for the vacuum condition in the NMSSM with universal conditions on scalar masses and trilinear couplings imposed at the unification scale. We find that gaugino non-universality modifies the vacuum condition through RG effects and significantly enlarges the parameter space where the desirable EWSB is realized. With this modification, higgsino turns out to be the LSP and its thermal relic abundance can account for the observed dark matter density in a wide range of the parameter space. This higgsino-like dark matter may be probed in dark matter direct detection experiments. Moreover, by requiring the thermal relic of the LSP to be equal to or smaller than \( \Omega_{\text{DM}} h^2 \), we obtain an upper bound on the mass scale of SUSY particles, and many of the viable parameter points are found to predict colored particles such as gluino to be within the reach of a future 100 TeV collider \cite{79–82}.

This paper is organized as follows. In the next section, we give a brief review on the vacuum conditions in the NMSSM, and discuss the effect of a non-universal gaugino mass spectrum on these conditions. Then, we study the phenomenological implications of this setup in Sec. 3, followed by an analysis devoted to no-scale/gaugino-mediation like spectra in Sec. 4. Section 5 is for conclusion and discussions.
2 Vacuum conditions in NMSSM

2.1 Scalar sector in NMSSM

To begin with, we review the scalar sector in the NMSSM with a particular focus on the vacuum conditions. For previous studies on the scalar sector in the NMSSM, see Refs. [23, 83–91]. Throughout this paper we consider the so-called $Z_3$-invariant NMSSM, which is characterized by the superpotential of the Higgs sector

$$W_{\text{Higgs}} = \lambda S (H_u \cdot H_d) + \frac{1}{3} \kappa S^3,$$

and the corresponding soft terms

$$L_{\text{soft}}^{(\text{Higgs})} = - m^2_{H_u} |H_u|^2 - m^2_{H_d} |H_d|^2 - m^2_S |S|^2 - \left[ \lambda A \lambda S (H_u \cdot H_d) + \frac{1}{3} \kappa A_n S^3 + \text{h.c.} \right].$$

As the name stands for, this theory possesses an exact $Z_3$ symmetry at the Lagrangian level, which is spontaneously broken when the scalar fields develop VEVs. The spontaneous breaking of a discrete symmetry results in the generation of domain walls [92–95], which are cosmologically harmful. To evade this problem, one often introduces Planck-scale suppressed non-renormalizable operators that explicitly break the $Z_3$ symmetry in a proper way that these operators do not induce sizable tadpole contributions at low energies [96–100]. In the presence of such an explicit $Z_3$-breaking term, domain walls become unstable.\footnote{For recent studies on unstable domain walls in the NMSSM, see Refs. [101–104].}

In the following discussion, we implicitly assume such a mechanism, which has no effect on our argument presented in this paper. The scalar potential for the neutral fields is obtained from these Lagrangian terms as

$$V_{\text{neutral}} = m^2_S |S|^2 + (|\lambda S|^2 + m^2_{H_u}) |H_u|^2 + (|\lambda S|^2 + m^2_{H_d}) |H_d|^2 + |\kappa S|^2 - \lambda H_u^0 H_d^0|^2$$

$$+ \frac{1}{8} (g^2 + g'^2) (|H_u^0|^2 - |H_d^0|^2)^2 + \left[ - \lambda A \lambda S H_u^0 H_d^0 + \frac{1}{3} \kappa A_n S^3 + \text{h.c.} \right].$$

In what follows, we take all of the parameters to be real just for simplicity. From this potential, we readily find the tadpole conditions for the scalar fields $H_u$, $H_d$, and $S$:

$$\frac{1}{2} \lambda^2 v_u^2 (v_u^2 + v_s^2) + \frac{1}{8} v_u (g^2 + g'^2) (v_u^2 - v_s^2) + m^2_{H_u} v_u - \frac{1}{2} \lambda \kappa v_d v_s^2 - \frac{1}{\sqrt{2}} \lambda A_\lambda v_d v_s = 0,$$

$$\frac{1}{2} \lambda^2 v_d (v_u^2 + v_s^2) + \frac{1}{8} v_d (g^2 + g'^2) (v_d^2 - v_s^2) + m^2_{H_d} v_d - \frac{1}{2} \lambda \kappa v_u v_s^2 - \frac{1}{\sqrt{2}} \lambda A_\lambda v_u v_s = 0,$$

$$m^2_S v_s + \kappa v_s^3 + \frac{1}{\sqrt{2}} \kappa A_n v_s^2 - \frac{1}{\sqrt{2}} \lambda A_\lambda v_u v_d - \lambda \kappa v_u v_d v_s + \frac{1}{2} \lambda^2 v^2 v_s = 0,$$

where $\langle H_u \rangle = v_u / \sqrt{2}$, $\langle H_d \rangle = v_d / \sqrt{2}$, $\langle S \rangle = v_s / \sqrt{2}$, and $v^2 \equiv v_u^2 + v_d^2 + v_s^2 \approx 246$ GeV. A non-zero VEV of $\langle S \rangle$ gives an effective higgsino mass term, i.e., $\mu$-parameter: $\mu_{\text{eff}} \equiv \lambda \langle S \rangle = \cdots$
\( \lambda v_s/\sqrt{2} \). As usual, we take \( v_u, v_d \), and \( \lambda \) to be positive without loss of generality. The sign of \( \mu_{\text{eff}} \) follows that of \( v_s \), which can be either positive or negative. For later use, we further rewrite Eqs. (4) and (5) in the following form:

\[
\mu_{\text{eff}}^2 + \frac{1}{2} m_Z^2 + \frac{m_{H_u}^2 \tan^2 \beta - m_{H_d}^2}{\tan^2 \beta - 1} = 0 ,
\]

\[
\left[ m_{H_u}^2 + m_{H_d}^2 + 2 \mu_{\text{eff}}^2 + \frac{1}{2} \lambda^2 v^2 \right] \sin 2\beta - 2 \mu_{\text{eff}} B_{\text{eff}} = 0 ,
\]

where \( \tan \beta \equiv v_u/v_d \), \( m_Z = v\sqrt{g^2 + g'^2}/2 \), and \( B_{\text{eff}} \equiv A\lambda + \kappa v_s/\sqrt{2} \).

To see the condition for \( v_s \neq 0 \), let us study the tadpole condition (6) in the limit of \( v_s \gg v_u, v_d \) to keep the first three terms:

\[
\left[ m_S^2 + \kappa^2 v_s^2 + \frac{1}{\sqrt{2}} \kappa A \kappa v_s \right] v_s \simeq 0 .
\]

This has non-zero real solutions if and only if \( A_{\kappa}^2 \gtrsim 8 m_S^2 \), which are given by

\[
v_s^{(\pm)} \simeq \frac{1}{2\sqrt{2}\kappa} \left[ -A_{\kappa} \pm \sqrt{A_{\kappa}^2 - 8 m_S^2} \right] .
\]

At either of them, the scalar potential should be deeper than at the origin so that \( v_s \neq 0 \) is energetically favored. For a negative (positive) \( A_{\kappa} \), the potential value at \( v_s^{(+) = (-)} \) is smaller (larger) than that at \( v_s^{(-)} \), and in both of these cases it is below that at the origin if

\[
A_{\kappa}^2 \gtrsim 9 m_S^2 .
\]

This condition is trivially satisfied if \( m_S^2 < 0 \). If, on the other hand, \( m_S^2 > 0 \), then a rather large \( A_{\kappa} \) is required.

For \( v_s^{(\pm)} \) to be a minimum (not a saddle point), the second Hessian matrix of the scalar potential with respect to the scalar fields should be positive definite. As long as the singlet-doublet mixing is not so large, the curvatures in the \( H_u \) and \( H_d \) directions are similar to those in the MSSM, and thus only the singlet direction is potentially dangerous. The curvature in this direction is read from the masses of the singlet scalar and pseudoscalar, which are respectively given by

\[
m_s^2 \simeq 2\kappa^2 v_s^2 + \frac{\kappa A_{\kappa}}{\sqrt{2}} v_s ,
\]

\[
m_a^2 \simeq -\frac{3\kappa A_{\kappa}}{\sqrt{2}} v_s .
\]

These masses should be positive, which yield additional constraints.

In the NMSSM, due to the complexity of the scalar potential, there are quite a few potential minima other than the desired one we considered above. In this paper, we require this desired minimum to be the global minimum, though this is not necessary
as long as the lifetime of the vacuum is sufficiently larger than the age of the Universe. For a recent study on metastable vacua in the NMSSM, see Ref. [91]. Possibilities of unwanted minima in the NMSSM are considered in Refs. [89–91]. Here, we summarize some important cases among them:

(a) A potentially dangerous minimum may be found along the direction in which both the $F$- and $D$-terms vanish, which occurs when

$$ |H_u^0| = |H_d^0| , \quad \kappa S^2 = \lambda H_u^0 H_d^0 . \quad (14) $$

In this case, the fourth and fifth terms in Eq. (3) vanish. By taking $\lambda$, $H_u^0$, and $H_d^0$ to be real and positive without loss of generality, we have the scalar potential in the form

$$ V_{F,D}(\phi) = \left( m_{H_u}^2 + m_{H_d}^2 + \frac{\lambda}{\kappa} m_S^2 \right) \phi^2 \pm 2\lambda \sqrt{\frac{\lambda}{\kappa}} \left( -A_\lambda + \frac{1}{3} A_\kappa \right) \phi^3 + \frac{2\lambda^3}{\kappa} \phi^4 , \quad (15) $$

where we set $\phi \equiv H_u^0 = H_d^0$. Notice that $\kappa \geq 0$ follows from the second condition in Eq. (14) with this convention. The field $\phi$ can have a non-trivial minimum if

$$ 9 \left[ -A_\lambda + \frac{1}{3} A_\kappa \right]^2 \geq 16 \left( m_{H_u}^2 + m_{H_d}^2 + \frac{\lambda}{\kappa} m_S^2 \right) . \quad (16) $$

If this holds, then we need to make sure that the height of $V_{F,D}$ at the minimum be higher than the potential value at the desired minimum. Generically speaking, if soft masses, especially $m_{H_d}^2$, are large enough compared to the $A$-terms $A_\lambda^2$ and $A_\kappa^2$, this direction is stabilized.

(b) Another direction which may provide a local minimum is along the $D$-flat direction but away from the $F$-flat direction. As shown in Ref. [90], we can obtain a condition similar to Eq. (16) and this direction is again stabilized if scalar masses are sufficiently larger than the $A$-terms. This direction includes a special case where $H_u^0 = H_d^0 = 0$ and $S \neq 0$.

(c) We may also consider the case where the $F$-term vanishes but the $D$-term has a non-zero value. Again, there is a condition similar to Eq. (16) for the presence of a local minimum in this direction [90], and the minimum disappears if scalar masses are larger than the $A$-terms squared. This direction contains two special cases where $S = H_d^0 = 0$ and $H_u^0 \neq 0$, or $S = H_u^0 = 0$ and $H_d^0 \neq 0$. In the former case, the scalar potential has the form

$$ V_{S,D}(H_u^0) = m_{H_u}^2 |H_u^0|^2 + \frac{g_1^2 + g_2^2}{8} |H_u^0|^4 . \quad (17) $$

\footnote{As can be seen from Eqs. (4), (5), and (6), if one of the scalar fields has the zero VEV, then at least one of the other two scalar fields must also have the vanishing VEV unless there are unrealistic particular relations among the parameters in the scalar potential—namely, it is generically not possible that only two among the three scalar fields acquire VEVs.}
This potential has a non-trivial solution if $m_{H_u}^2 < 0$ and the potential value at this minimum is given by

$$V_{S,D}^{(\text{min})} = -\frac{2(m_{H_u}^2)^2}{g^2 + g'^2}.$$  \hfill (18)

As we see, if $|m_{H_u}^2|$ is very large, then this may become the global minimum. In the latter case, on the other hand, we obtain a similar expression to Eq. (17) and find that there is no minimum in this direction since $m_{H_d}^2$ is always positive in the parameter space we are interested in.

(d) We also need to ensure the absence of charge and/or color breaking minima, at which some sfermions acquire non-zero VEVs. To avoid such a minimum, we require [85, 105, 106]

\begin{align*}
A_{u_i}^2 &\leq 3(m_{H_u}^2 + m_{Q_i}^2 + m_{D_i}^2) \quad \text{at scale } \mu \sim A_{u_i}/y_{u_i}, \quad \text{(19)} \\
A_{d_i}^2 &\leq 3(m_{H_d}^2 + m_{Q_i}^2 + m_{U_i}^2) \quad \text{at scale } \mu \sim A_{d_i}/y_{d_i}, \quad \text{(20)} \\
A_{e_i}^2 &\leq 3(m_{H_d}^2 + m_{L_i}^2 + m_{E_i}^2) \quad \text{at scale } \mu \sim A_{e_i}/y_{e_i}, \quad \text{(21)}
\end{align*}

where $A_f$ ($f = u_i, d_i, e_i$) are the trilinear couplings corresponding to the SM Yukawa couplings $y_f$ and $m_f$ stand for the sfermion masses. Again, we see that these conditions are satisfied as long as the $A$-terms are much smaller than the soft masses.

2.2 CNMSSM

Next, we give a brief review on the CNMSSM, which is stringently constrained by the vacuum conditions. In the CNMSSM, the gaugino masses, soft scalar masses, and $A$-terms are respectively taken to be universal at the GUT scale $M_{\text{GUT}}$, which is defined by the condition $g_1(M_{\text{GUT}}) = g_2(M_{\text{GUT}})$:

\begin{align*}
M_1 &= M_2 = M_3 \equiv m_{1/2}, \quad \text{(22)} \\
m_f^2 &= m_{H_u}^2 = m_{H_d}^2 = m_S^2 \equiv m_0^2, \quad \text{(23)} \\
A_f &= A_\lambda = A_\kappa \equiv A_0, \quad \text{(24)}
\end{align*}

where $M_a$ ($a = 1, 2, 3$) are gaugino masses. The absolute value of the singlet VEV $|v_s|$, the singlet trilinear coupling $\kappa$, and $\tan \beta$ are determined such that the tadpole conditions (4), (5), and (6) are satisfied with the correct size of the Higgs VEV $v \approx 246$ GeV. Note that the first and second parameters correspond to the degrees of freedom of $|\mu|$ and $B_\mu$ in the case of the CMSSM. The third parameter, $\tan \beta$, is fixed by the universality condition $m_S^2 = m_0^2$ at the GUT scale. As a result, the free input parameters in the CNMSSM are

$$m_0, \ m_{1/2}, \ A_0, \ \lambda, \ \text{sign}(v_s).$$  \hfill (25)

Thus, the number of free parameters in the CNMSSM is the same as in the CMSSM, where each point in the parameter space is specified by $m_0, m_{1/2}, A_0, \tan \beta, \text{and sign}(\mu)$.  

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It turns out that with the universal conditions (24) it is rather difficult to assure the vacuum conditions given in the previous subsection to be satisfied. To evade the current LHC limits on the masses of SUSY particles as well as to explain the observed Higgs boson mass, generically speaking, we need to take the soft SUSY-breaking parameters in Eq. (25) to be $O(1)$ TeV or larger. Then, for a moderate or large value of $\tan \beta$, Eq. (7) is approximated by

$$
\mu^2_{\text{eff}} = \frac{1}{2} \lambda^2 v_s^2 \simeq -m_{H_u}^2.
$$

(26)

For $m_{1/2} \gtrsim O(1)$ TeV, $m_{H_u}^2$ is driven to be a large negative value through the RG effect by gluino so that $-m_{H_u}^2 \simeq m_{1/2}^2$. Thus, $\mu^2_{\text{eff}} \simeq m_{1/2}^2$ is required from the vacuum condition. If both $\lambda$ and $\kappa$ are sizable, this condition and Eq. (10) imply that a large value of $|A_\kappa|$ or $-m_S^2$ is required so that $|v_s|$ can be as large as $O(m_{1/2})$. We however notice that it is difficult to obtain a large $|A_\kappa|$ at low energies as it is suppressed via the RG effect in the case where $\lambda$ and $\kappa$ are sizable, while it is hard to reconcile a large negative value of $m_S^2$ at low energies with the universality condition $m_S^2 = m_D^2 > 0$ at the input scale. To see this in a more qualitative manner, we show the soft mass parameters at a low energy scale that are relevant to the vacuum conditions as functions of the parameters at the GUT scale:

\begin{align*}
A_\lambda(M_s) &= -0.019M_1 - 0.252M_2 + 0.450M_3 + 0.231A_0, \\
A_\kappa(M_s) &= 0.002M_1 + 0.015M_2 - 0.012M_3 + 0.408A_0, \\
m_S^2(M_s) &= -0.001M_1^2 - 0.010M_2^2 + 0.003M_2M_3 + 0.010M_3^2 \\
&\quad + A_0(0.002M_2 - 0.006M_3) - 0.076A_0^2 + 0.410m_0^2, \\
m_{H_u}^2(M_s) &= 0.010M_1^2 - 0.004M_1M_2 + 0.210M_2^2 - 0.013M_1M_3 - 0.078M_2M_3 - 0.902M_3^2 \\
&\quad + A_0(0.010M_1 + 0.054M_2 + 0.193M_3) - 0.104A_0^2 + 0.095m_0^2, \\
m_{H_d}^2(M_s) &= 0.014M_1^2 - 0.004M_1M_2 + 0.230M_2^2 - 0.003M_1M_3 - 0.062M_2M_3 - 0.603M_3^2 \\
&\quad + A_0(0.011M_1 + 0.062M_2 + 0.159M_3) - 0.123A_0^2 + 0.192m_0^2,
\end{align*}

(27-31)

where we set $\lambda = 0.2$, $\kappa = 0.5$, $\tan \beta = 50$, and $M_s = 7.5$ TeV. We have evaluated the numerical coefficients of the above expressions by solving the two-loop renormalization group equations (RGEs) without imposing the universality conditions in the CNMSSM. It is found from Eq. (30) that the low-energy value of $m_{H_u}^2$ is basically determined by the gluino mass and thus becomes a large negative value when $M_3$ is large. On the other hand, both $A_\kappa$ and $m_S^2$ at low energies tend to be relatively suppressed as seen in Eq. (28) and Eq. (29), respectively, with which $\mu^2_{\text{eff}} \ll -m_{H_u}^2$ and thus the vacuum condition (7) cannot be satisfied.

One might think that a sufficiently large $\mu^2_{\text{eff}}$ can be obtained where $|\kappa| \ll \lambda$, since $|v_s|$ becomes large when $|\kappa|$ is small, as can be seen from Eq. (10). Such a parameter region is, however, disfavored by the vacuum stability condition. To see this, let us compare the height of the potential at a desired vacuum $v_u, v_d, v_s \neq 0$ with $V_{S,D}^{(\text{min})}$ given in Eq. (18).
In the limit of $v_s \gg v_u, v_d$, the former is computed as

$$V_{\text{min}} \simeq \frac{1}{2} m_S^2 v_s^2 + \frac{1}{3\sqrt{2}} \kappa A_\kappa v_s^3 + \frac{1}{4} \kappa^2 v_s^4$$

$$= -\frac{\kappa^2}{4} v_s^4 - \frac{1}{6\sqrt{2}} \kappa A_\kappa v_s^3,$$

(32)

where we have eliminated $m_S^2$ in the last equation using Eq. (10). On the other hand, with the condition (26) $V_{S,D}^{(\text{min})}$ can be approximated by

$$V_{S,D}^{(\text{min})} \simeq -\frac{\lambda^4 v_s^4}{2(g^2 + g'^2)}.$$  

(33)

Thus, $V_{\text{min}} < V_{S,D}^{(\text{min})}$ requires

$$\frac{\lambda^4}{g^2 + g'^2} < \frac{\kappa^2}{2} + \frac{\kappa A_\kappa}{3\sqrt{2} v_s},$$  

(34)

which cannot be satisfied if $|\kappa| \ll \lambda$.

The above difficulties can be avoided in the parameter space where both $\lambda$ and $|\kappa|$ are very small. In this case, both $A_\kappa$ and $m_S^2$ rarely run in the RG flow, and thus $A_\kappa \simeq A_0$ and $m_S \simeq m_0$. The condition (11) then leads to $|A_0| \gtrsim 3m_0$, which assures the existence of a solution for $v_s \neq 0$. Even though $\lambda \ll 1$, the condition (26) may be satisfied by taking $|\kappa|$ to be also very small so that the suppression in $|\mu_{\text{eff}}|$ by a small $\lambda$ is compensated by an enhancement in $|v_s|$ with $|\kappa| < \lambda \ll 1$. In fact, a detailed study performed in Ref. [25] demonstrates that viable model points are found in the parameter region where (a) $|\kappa| < \lambda \ll 1$; (b) $3m_0 \lesssim |A_0|$; (c) $m_{1/2} \gg m_0$; (d) $\tan \beta \gg 1$. The reason for the first two conditions have already been addressed. The condition (c) is required in order to avoid the stau LSP or the presence of a tachyonic particle, which often occurs when $|A_0|/m_0$ is sizable. When the conditions (a–c) are met, the second term in Eq. (8) tends to be much smaller than the sum of the terms in the square brackets of the first term, which then leads to $\tan \beta \gg 1$.

In the phenomenologically viable region found in Ref. [25], the LSP is a singlino-like neutralino. In principle, this can be a good dark matter candidate, but in practice this may cause a problem as its thermal relic abundance tends to be much larger than the observed dark matter density due to its small annihilation cross section. As discussed in Ref. [25], the correct dark matter density is obtained only in the limited region of the parameter space where the lighter stau is degenerate with the singlino LSP in mass so that the dark matter abundance is efficiently reduced via coannihilation [107]. It turns out that the correct relic abundance can be obtained if $m_{1/2} \lesssim 3$ TeV. With such a small $m_{1/2}$, however, it in turn becomes difficult to explain the observed value of the Higgs boson mass, $m_h \simeq 125$ GeV. Indeed, a recent analysis [37] shows that the parameter points consistent with $m_h \simeq 125$ GeV are obtained only for $m_{1/2} \gtrsim 3.5$ TeV, where the stau-coannihilation is no longer sufficiently effective. This result implies that we need some additional mechanism to reduce the dark matter abundance, such as the late-time entropy production.
Figure 1: $m_S^2/m_0^2$ at the GUT scale as a function of the gaugino mass ratio $M_2/M_3$ for different values of $\tan \beta$. Here, we take $m_0 = 1$ TeV, $M_3 = 5$ TeV, $A_0 = -4$ TeV, and $\lambda = 0.2$.

2.3 Effect of non-universal gaugino masses

Now we discuss the effect of non-universal gaugino masses on the vacuum conditions presented in Sec. 2.1 and compare this result with that of the CNMSSM discussed in the previous subsection. As we see above, an obstacle to the vacuum conditions in the CNMSSM is a large negative value of $m_{H_u}^2$ due to the RG effect by gluino. However, this contribution can be canceled by the wino contribution once we allow non-universal gaugino masses [56, 57]. This feature can easily be seen by examining Eq. (30); if we take $M_2 = \text{a few} \times M_3$, the gaugino contributions cancel with each other so that $|m_{H_u}^2|$ is much smaller than that in the case of the universal gaugino masses. Moreover, Eq. (29) shows that a heavy wino gives a negative contribution to $m_S^2$, which again relaxes the constraint from the vacuum conditions with the universality condition $m_S^2 = m_0^2$ at $M_{\text{GUT}}$—the negative wino contribution can make $m_S^2$ run negative at low energies even though it is positive at $M_{\text{GUT}}$, which allows the radiative generation of a non-zero $v_s$.

To see the effect of non-universal gaugino masses on the universality condition $m_S^2 = m_0^2$ at $M_{\text{GUT}}$ in more detail, in Fig. 1, we show $m_S^2/m_0^2$ at the GUT scale as a function of the gaugino mass ratio $M_2/M_3$ for different values of $\tan \beta$. Here, we take $m_0 = 1$ TeV, $M_3 = 5$ TeV, $A_0 = -4$ TeV, and $\lambda = 0.2$. To obtain the GUT-scale value of $m_S^2$, we first determine its SUSY-scale value using the vacuum conditions, and evolve it up to $M_{\text{GUT}}$ according to RGEs; we exploit NMSSMTools 5.3.1 [29, 108, 109] for this purpose. We here regard $\tan \beta$ as a free parameter by relaxing the universality condition $m_S^2 = m_0^2$. The other gaugino mass ratio, $M_1/M_3$, is fixed such that the gaugino masses satisfy the
following relation, which is motivated by the mixed modulus-anomaly mediation \cite{41}:

\[ M_a(M_{\text{GUT}}) = M_0 \left[ 1 + \frac{a b_a g_{\text{GUT}}^2}{16 \pi^2} \ln \left( \frac{M_{\text{Pl}}}{m_{3/2}} \right) \right], \tag{36} \]

where \((b_1, b_2, b_3) = (33/5, 1, -3)\) are the one-loop gauge-coupling beta-function coefficients, \(g_{\text{GUT}}\) denotes the unified gauge coupling, \(M_{\text{Pl}}\) is the reduced Planck mass, \(m_{3/2}\) is the gravitino mass, \(M_0\) denotes the modulus-mediated contribution to the gaugino mass, and \(\alpha\) is an \(O(1)\) constant that is supposed to be determined by the UV physics. We assume this relation for gaugino masses throughout this paper.

It is readily found from Eq. \((36)\) that \(M_1\) is always larger than \(M_2\) for \(M_2/M_3 > 1\), and in particular \(M_1 \approx 2M_2\) for \(M_2/M_3 \approx 3\). From Fig. 1, we find that a small change in \(M_2/M_3\) drastically affects the GUT-scale value of \(m_2^S\). For a given value of \(M_2/M_3\), the universality condition \(m_2^S = m_0^2\) can be satisfied by taking an appropriate value of \(\tan \beta\)—there may be two different choices of \(\tan \beta\) that give \(m_2^S = m_0^2\). The required values of \(\tan \beta\) tend to get larger for a larger \(M_2/M_3\). We however note that the condition \(m_2^S = m_0^2\) is not the only one needed to be satisfied; other conditions such as the stability conditions discussed above should also be satisfied, and in fact these requirements disfavor a small value of \(M_2/M_3\) as we see below.

A non-universal gaugino mass spectrum is advantageous also for the vacuum stability. Since a smaller \(|m_{H_u}^2|\), which can be realized with a large wino mass, results in a larger value of \(V_{\text{min}}^{S,D}\) in Eq. \((18)\), the desired vacuum can easily be deeper than this unwanted minimum. In addition, a large wino mass tends to make the right-hand side in Eq. \((16)\) large compared to the left-hand side, which allows the \(\phi\) direction to be stabilized. In our parameter scanning, NMSMTools 5.3.1 only checks whether there is a deeper minimum along the directions that one of the three fields, \(H_u, H_d\) and \(S\), has a non-zero field value. We expect that deeper minima along the \(F\)-flat and/or the \(D\)-flat directions are absent in the parameter space where the \(M_2/M_3\) is large enough to achieve the correct EWSB vacuum.

Another obstacle to a viable parameter point in the CNMSSM is stau being the LSP or tachyonic, for which the non-universal gaugino mass conditions \(M_1/M_3, M_2/M_3 > 1\) are again helpful. In particular, a larger value of \(M_1\) increases the soft mass of the right-handed stau at low energies through the RG effect. This prevents stau from being the LSP or tachyonic even for a large \(|A_0|\).

In Fig. 2, we show viable and excluded parameter points in the \(M_3-M_2/M_3\) plane for each choice of \(\text{sign}(v_s)\) and \(A_0\). Here we set \(m_0 = 1\) TeV and \(\lambda = 0.2\), and determine \(\tan \beta\)

\[ \frac{M_1}{M_3} = \frac{1}{b_2 - b_3} \left[ (b_1 - b_3) \frac{M_2}{M_3} + b_2 - b_1 \right]. \tag{35} \]

\({}^3\)From this relation, we have

\[ \frac{M_1}{M_3} = \frac{b_2 - b_3}{b_1 - b_3} \left[ (b_1 - b_3) \frac{M_2}{M_3} + b_2 - b_1 \right]. \]

\({}^4\)We however note that our discussion is less affected even though we take a different value of \(M_1/M_3\), since the contribution of the bino mass to RGEs is relatively small as the \(U(1)_Y\) gauge coupling is smaller than the other gauge couplings.
Figure 2: Viable/excluded parameter points in the $M_3$-$M_2/M_3$ plane for each choice of $\text{sign}(v_s)$ and $A_0$. Here we set $m_0 = 1$ TeV and $\lambda = 0.2$, and determine $\tan \beta$ from the universality condition $m_S^2 = m_0^2$. 
from the universality condition \( m_S^2 = m_0^2 \). The viable parameter region is shown in the gray shaded area, while other marks indicate that the corresponding parameter points are excluded for various reasons. There are points at which several solutions for \( \tan \beta \) exist, for which the marks associated to these solutions are overlapped. In Fig. 2a, where we set \( \text{sign}(v_s) \) to be negative and \( A_0 = 0 \), there is no solution compatible with \( m_S^2 = m_0^2 \) for \( M_1 = M_2 = M_3,^5 \) as indicated by the gray crosses. This is consistent with our discussion in Sec. 2.2. Nevertheless, if we allow non-universal gaugino masses, we can then find viable parameter points, though they are restricted to a narrow strip spreading at \( M_2 \gtrsim 4 \text{ TeV} \) and \( M_2/M_3 \sim 2 \). If we flip the sign of \( v_s \), on the other hand, there is no viable parameter point as shown in Fig. 2b. The reason is as follows. If we take \( A_0 = 0 \) and \( M_2 > M_3 \), the low-scale value of \( A_\kappa (A_\lambda) \) tends to be positive (negative), as implied by Eq. (28) (Eq. (27)). Given \( v_s > 0 \), i.e., \( \mu_{\text{eff}} > 0 \), \( B_{\text{eff}} = A_\lambda + \kappa v_s/\sqrt{2} > 0 \) is required in order for the vacuum condition (8) to be satisfied. This is possible only if \( \kappa v_s > 0 \). However, if \( A_\kappa > 0 \) and \( \kappa v_s > 0 \), then the pseudo-scalar mass squared becomes negative as seen from Eq. (13). This means that the vacuum we consider is actually unstable, which is indicated by the green dots in Fig. 2b. The narrow strip observed in Fig. 2a is extended if we take a large \( |A_0| \), as seen in Figs. 2c and 2e, where we set \( A_0 = +M_3 \) and \(-M_3 \), respectively. Now the viable parameter region is considerably extended, since a large \( |A_0| \) makes it easy to obtain a large value of \( v_s \) given by Eq. (10) and thus to satisfy the condition (26). As a side effect, a large \( |A_0| \) may result in a light (or tachyonic) stau as discussed above; we find such points indicated by blue triangles, where the lightest neutralino is not the LSP, for a relatively small \( M_2/M_3 \). As we see, the light/tachyonic-stau problem is evaded for a larger value of \( M_2/M_3 \). For a positive \( v_s \), viable parameter points are still rarely found for \( A_0 = +M_3 \) as shown in Fig. 2d due to similar reasoning given above. For \( A_0 = -M_3 \), on the other hand, we find viable parameter region for \( M_2/M_3 \sim 2.5 \), as seen in Fig. 2d. Motivated by this observation—namely, \( \text{sign}(v_s) = - \) allows larger number of viable parameter points, we always take \( \text{sign}(v_s) = - \) in the following analysis.

All in all, the restrictions from the tadpole conditions, the vacuum stability conditions, and the light/tachyonic stau can significantly be alleviated with non-universal gaugino masses. This opens up new viable parameter regions where phenomenological consequences are quite different from those in the CNMSSM, as we see below.

### 3 Phenomenological implications

Now we discuss the phenomenology of the NMSSM with non-universal gaugino masses. First, we show in Table 1 a typical mass spectrum of this scenario which is consistent with \( m_h \simeq 125 \text{ GeV} \) and \( \Omega_{\text{DM}} h^2 \simeq 0.12 \). Here, all of the output parameters except for \( \tan \beta \) are evaluated at the SUSY scale, while \( \tan \beta \) is given at the \( Z \)-boson mass scale. This mass spectrum is computed again with \texttt{NMSSMTools 5.3.1} [29, 108, 109], while the thermal relic abundance of the LSP, \( \Omega_{\text{LSP}} h^2 \), and its spin-independent and spin-dependent scattering cross sections with proton, \( \sigma_{\text{SI}}^{(p)} \) and \( \sigma_{\text{SD}}^{(p)} \), respectively, are obtained by using \texttt{MicrOMEGAs}

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^5We see from Eq. (36) that if \( M_2 = M_3 \) then \( M_1 = M_3 \), i.e., the universal gaugino mass is obtained.
Table 1: Typical mass spectrum of the NMSSM with non-universal gaugino masses.

| GUT-scale input parameters ($M_{\text{GUT}} = 7.57 \times 10^{15} \text{ GeV}$) |  |
|---|---|---|
| $m_0$ | 1 TeV | $A_0$ | $-5.792$ TeV |
| $M_1$ | $28.67$ TeV | $M_2$ | $15.32$ TeV |
|  |  | $M_3$ | $5.792$ TeV |

| SUSY-scale input parameters |  |
|---|---|---|
| $\text{sign}(v_s)$ | $-$ | $\lambda$ | $0.207$ |

| Output parameters |  |
|---|---|---|
| $\tan \beta$ | $54.61$ | $\kappa$ | $-0.455$ |
| $A_\lambda$ | $-2.548$ TeV | $A_\kappa$ | $-2.668$ TeV |
| $M_1$ | $13.73$ TeV | $M_2$ | $12.75$ TeV |
|  |  | $M_3$ | $10.88$ TeV |

| Mass spectrum |  |
|---|---|---|
| $m_{h_1}$ | $124.3$ GeV | $m_{h_2}$ | $3.092$ TeV |
| $m_{a_1}$ | $3.091$ TeV | $m_{a_2}$ | $4.224$ TeV |
| $m_{\tilde{\chi}_1^0}$ | $1.077$ TeV | $m_{\tilde{\chi}_1^\pm}$ | $1.078$ TeV |
| $m_{\tilde{t}_1}$ | $7.685$ TeV | $m_{\tilde{b}_1}$ | $6.776$ TeV |
|  |  | $m_{\tilde{\tau}_1}$ | $7.620$ TeV |

| Other sfermions: | 9.4–13.1 TeV |

| Dark matter |  |
|---|---|---|
| $\Omega_{\text{LSP}} h^2$ | $0.112$ | $\sigma_{\text{SI}}^{(p)}$ | $1.8 \times 10^{-47}$ cm$^2$ |
|  |  | $\sigma_{SD}^{(p)}$ | $1.1 \times 10^{-45}$ cm$^2$ |

| Couplings at the GUT scale |  |
|---|---|---|---|
| $g_1$ | $0.685$ | $g_2$ | $0.685$ |
| $y_t$ | $0.574$ | $y_b$ | $0.498$ |
| $\lambda$ | $0.302$ | $\kappa$ | $-0.656$ |

[110] implemented in NMSSMTools 5.3.1. For the computation of $\sigma_{\text{SI}}^{(p)}$, we use the latest compilation for the nucleon scalar matrix elements given in Ref. [111]: $f_{T_q}^{(p)} = 0.018(5)$, $f_{T_d}^{(p)} = 0.027(7)$, $f_{T_s}^{(p)} = 0.037(17)$, $f_{T_c}^{(p)} = 0.078(2)$, $f_{T_b}^{(p)} = 0.072(2)$, and $f_{T_t}^{(p)} = 0.069(1)$, where the heavy quark matrix elements are obtained from those for light quarks through an $O(\alpha_s^3)$ perturbative QCD calculation. We have checked that the use of these matrix elements results in an enhancement in $\sigma_{\text{SI}}^{(p)}$ by about 13% compared with that computed with the default values of the nucleon matrix elements exploited in MicrOMEGAs. For $\sigma_{SD}^{(p)}$, we use the default values of the matrix elements in MicrOMEGAs. We also note that the computation of the Higgs masses in the NMSSM suffers from a large theoretical uncertainty; indeed, various public codes predict values of $m_h$ differing by as large as a
few GeV [112]. We use the option 2 in NMSSMTools 5.3.1 to compute the Higgs masses, which are found to be in good agreement with the results of other codes as shown in Ref. [112].

In Table 1, we take $M_2 \simeq 2.5 \times M_3$ and $M_1 \simeq 2 \times M_2$ to realize a non-universal gaugino mass spectrum consistent with the relation (36). As seen in the table, $\kappa$ is negative and its absolute value is larger than $\lambda$, which is again a generic feature as we will see below. Our choice of a negative $A_0$ leads to a negative value of $A_\kappa$ at low energies, which requires a negative $\kappa$ to make $m_a^2$ in Eq. (13) positive since $v_s$ is taken to be negative. As $\lambda < |\kappa|$, higgsino is lighter than singlino, and in fact the neutral higgsino is the LSP in this mass spectrum. Its mass is $\simeq 1$ TeV, with which its thermal relic abundance agrees to the observed DM density. In spite of the universality condition $m_S^2 = m_0^2 > 0$ at the GUT scale, $m_a^2$ at the SUSY scale is negative—thus, the generation of a non-zero VEV of the singlet field is induced radiatively. This is a distinct feature of this setup compared with the CNMSSM, where $m_a^2$ scarcely runs and thus is always positive. The gaugino masses at the SUSY scale are close to each other, similarly to those in the mirage mediation [38–44]. The lightest neutral Higgs boson corresponds to the SM-like Higgs boson, while the other Higgs bosons have masses of a few TeV. The third-generation sfermions are relatively light compared with the other sfermions, but they are much heavier than the LSP. All of the couplings are found to remain perturbative up to the GUT scale $M_{\text{GUT}} \simeq 8 \times 10^{15}$ GeV, which is relatively low compared with the typical GUT scale ($\simeq 2 \times 10^{16}$ GeV) in many SUSY mass spectra. This is basically due to large values of gaugino masses [113], and may result in a large rate of the dimension-six proton decay induced by the exchange of GUT gauge bosons.

Next, we perform a parameter scan to search for mass spectra that are consistent with the observed value of the Higgs mass as well as the DM density. We fix/scan the input parameters as follows:

- $m_0 = 1$ TeV.
- $1$ TeV $< M_3 < 12.5$ TeV.
- $1.5 < M_2/M_3 < 4$.
- $0 < |A_0| < 3 M_3$.
- $0.01 < \lambda < 0.5$.
- $\text{sign}(v_s) = -$.

We then check that each parameter point satisfies the required conditions discussed in Sec. 2.

In Fig. 3, we show scatter plots of $\kappa/\lambda$ against $M_2/M_3$, where Fig. 3a and Fig. 3b are for $A_0 < 0$ and $A_0 > 0$, respectively. The green dots (blue crosses) represent the points where the relic abundance of the neutralino LSP is below (above) the observed value. These parameter points are also required to reproduce the mass of the SM-like
Figure 3: Scatter plots of $\kappa/\lambda$ against $M_2/M_3$. The green dots and blue crosses represent the points where the relic abundance of the neutralino LSP is below and above the observed value, respectively. The points indicated by gray squares predict $m_h$ that is out of the acceptable mass range: $122.1 \text{ GeV} < m_h < 128.1 \text{ GeV}$. Here we fix $m_0 = 1 \text{ TeV}$ and $\text{sign}(v_s) = -$. Higgs boson $m_h \simeq 125 \text{ GeV}$; with the theoretical uncertainty in the computation of $m_h$ taken into account, we regard $122.1 \text{ GeV} < m_h < 128.1 \text{ GeV}$ as the allowed range for $m_h$, following the prescription in NMSSMTools. The gray squares in Fig. 3 correspond to the points where $m_h$ is out of this range. From these plots, we find that the viable parameter points spread over $2 \lesssim M_2/M_3 \lesssim 4$ in the case of a negative $A_0$, while they are localized at $M_2/M_3 \sim 2$ for a positive $A_0$. $|\kappa|$ tends to be larger than $\lambda$ for $A_0 < 0$, especially for those fall into the allowed range of the Higgs boson mass. For $A_0 > 0$, we find some points that predict $|\kappa| < \lambda$, but such points are disfavored due to the overproduction of the LSP. These results imply that higgsino is lighter than singlino in most of the viable parameter points and the LSP is the higgsino-like neutralino. In addition, we find more points that give a negative $\kappa$ than those yield a positive $\kappa$ for $A_0 < 0$, while for $A_0 > 0$ most of the points predict a positive $\kappa$. This is because the sign of $\kappa$ should be equal to the sign of $A_\kappa$ at the SUSY-breaking scale, given $v_s < 0$ and the condition (13). We also note that there are quite a few points with $\kappa/\lambda < -5$ ($\kappa/\lambda > 15$) for $A_0 < 0$ ($A_0 > 0$), which are out of the region shown in this figure.

Next we show the same parameter points as above in the $m_\tilde{g} - M_2/M_3$ plane in Fig. 4, where the meaning of the marks is the same as in Fig. 3. The orange shaded region represents the latest ATLAS bound on the gluino mass, $m_\tilde{g} \gtrsim 2.2 \text{ TeV}$ [114] for the LSP.
Figure 4: Scatter plots in the $m_{\tilde{g}}$-$M_2/M_3$ plane. The meaning of the marks and the choice of $m_0$ and sign($v_s$) are the same as in Fig. 3. The orange shaded region represents the current ATLAS bound on the gluino mass, $m_{\tilde{g}} \gtrsim 2.2$ TeV [114] for the LSP lighter than $\sim 1$ TeV.

These plots show that most of the parameter points consistent with the DM abundance predict the gluino mass to be $3$ TeV $\lesssim m_{\tilde{g}} \lesssim 20$ TeV ($7$ TeV $\lesssim m_{\tilde{g}} \lesssim 23$ TeV) for $A_0 < 0$ ($A_0 > 0$). As a comparison, we note that a future 100 TeV $pp$ collider may probe gluinos with a mass of $\lesssim 13$ TeV [79–82]; thus, many parameter points in the non-universal gaugino mass scenario may be probed in the future, especially if $M_2/M_3 \gtrsim 3$. It is also found that other colored particles such as stop and sbottom may be within the reach of a 100 TeV collider as well for a part of the viable parameter points.

As we see in the above plots, many parameter points give the LSP whose thermal relic abundance is within the observed DM density. As seen from Fig. 3, in this case, $\lambda < |\kappa|$ and thus higgsino is lighter than singlino. Since gaugino masses are rather heavy in our scenario, the LSP in the viable parameter points is always higgsino-like neutralino, and its thermal relic abundance, $\Omega_{\text{LSP}}h^2$, is smaller than the observed value $\Omega_{\text{DM}}h^2 \simeq 0.12$ if its mass is $\lesssim 1$ TeV. Higgsino has couplings with the Higgs bosons via the mixing with singlino, bino and wino, and through these couplings it can scatter off nucleons. According to Fig. 3, $|\kappa|/\lambda$ is $\mathcal{O}(1)$ for most of the viable parameter points, and thus the singlino mass is expected to be of the same order as the higgsino mass. In this case, the higgsino-singlino mixing is sizable, and we expect a detectable value of the LSP-nucleon scattering cross section. If this is the case, we may probe the LSP in the future DM direct detection experiments.

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6 For the results of the ATLAS and the CMS with the fewer data; see, e.g., Refs. [115–117].
Figure 5: Scatter plots of the spin-independent LSP-proton scattering cross sections against the LSP mass. Here, the gray shaded region, black dashed line, and the orange shaded region represent the XENON1T bound [118], the expected sensitivity of the LZ experiment [119], and the neutrino floor [120], respectively.

To study this possibility, in Fig. 5, we show the spin-independent scattering cross section of the LSP with proton, $\sigma_{\text{SI}}$, against the LSP mass. Here, we only show the parameter points that predict $m_h$ to be in the favored range and the relic abundance of the LSP to be $\Omega_{\text{LSP}} h^2 \lesssim 0.12$, where the equality holds for the LSP mass of $\sim 1$ TeV. If $\Omega_{\text{LSP}} h^2 < 0.12$, we expect that the local LSP density is smaller than the observed local DM density. To take this into account, we rescale $\sigma_{\text{SI}}$ by a factor of $\Omega_{\text{LSP}} / \Omega_{\text{DM}}$ in this plot. The gray shaded region is excluded by the latest limit imposed by the XENON1T experiment [118], while the black dashed line represents the expected sensitivity of the LZ experiment [119]. The orange shaded area indicates the neutrino floor [120], i.e., the region which the current strategy of the DM direct searches becomes unable to probe due to the neutrino background. This figure shows that all of the viable parameter points predict the cross section well below the current bound. Future multi-ton scale experiments such as LZ and XENONnT [121] are sensitive to a part of the model parameter points, and many of the rest are in principle able to be probed in the future as they are above the neutrino floor.

4 No-scale/gaugino-mediation type mass spectra

A particularly interesting and more constrained scenario is the case in which $m_0 = A_0 = 0$ holds at the input scale. This possibility is motivated by the so-called no-scale [45, 122–
or gaugino mediation [125, 126] models. In the case of the MSSM, it is known that the no-scale condition \( m_0 = A_0 = B_0 = 0 \) (\( B_0 \) denotes the soft bilinear mass term for the Higgs fields) and the universal gaugino mass condition \( m_{1/2} = M_1 = M_2 = M_3 \) at the GUT scale is so restrictive that viable parameter points cannot be found—it turns out that stau tends to be either the LSP or tachyonic in most of the parameter regions [127]. To alleviate the problem, one often takes the input scale to be much higher than the GUT scale, in a similar manner to the super-GUT models [128, 129], so that the RG effect from the input scale to the GUT scale generates non-zero soft terms at the GUT scale [127, 130–132]. Here, we adopt a different approach to resolving this problem. We show in what follows that by going to the NMSSM and allowing the non-universal gaugino masses, we can find viable parameter points even though all of the soft mass parameters except for the gaugino masses are set to be zero at the GUT scale.

In Table 2, we show an example of the phenomenologically viable mass spectrum of the NMSSM with non-universal gaugino masses and the no-scale condition \( m_0 = A_0 = 0 \). As we see, the predicted values of both \( m_h \) and \( \Omega_{\text{LSP}} h^2 \) are in good agreement with the observed values. Contrary to the case in Table 1, in this case the low-energy value of \( A_\kappa \) is predicted to be positive, as suggested by Eq. (28) with \( A_0 = 0 \) and \( M_2 > M_3 \). Therefore, \( \kappa \) should be positive in order to assure \( m_a^2 > 0 \). The predicted value of the LSP-nucleon scattering cross sections are fairly small and all of the colored particles are rather heavy—for this reason, it is difficult to probe this mass spectrum in the future experiments.

Again, we perform a parameter scan with

- \( m_0 = A_0 = 0 \).
- \( 1 \text{ TeV} < M_3 < 12.5 \text{ TeV} \).
- \( 1.5 < M_2/M_3 < 4 \).
- \( 0.01 < \lambda < 0.5 \).
- \( \text{sign}(v_s) = - \).

The results are summarized in Fig. 6, where we show scatter plots similar to Figs. 3, 4, and 5 in Figs. 6a, 6b, and 6c, respectively. As we see, the viable parameter points are found only on the narrow strip lying around \( 2.0 \lesssim M_2/M_3 \lesssim 2.5 \), and the predicted values of the gluino mass are limited to \( 5 \text{ TeV} \lesssim m_{\tilde{g}} \lesssim 17 \text{ TeV} \). The SI scattering cross sections are too small to be probed in the LZ experiment, but may be reached with a larger detector as quite a few points are above the neutrino floor.

\(^7\)Gaugino non-universality in the framework of the gaugino mediation in the MSSM is discussed in Refs. [63, 65].
Table 2: Typical mass spectrum of the NMSSM with non-universal gaugino masses and the no-scale condition.

| GUT-scale input parameters ($M_{\text{GUT}} = 7.34 \times 10^{15} \text{ GeV}$) |
|-----------------------------|-----------------------------|-----------------------------|
| $m_0$ | $A_0$ | $M_1$ | $M_2$ | $M_3$ |
| 0 | 0 | 28.30 TeV | 16.79 TeV | 7.573 TeV |

| SUSY-scale input parameters |
|-----------------------------|
| sign($v_s$) | $\lambda$ | $\mu_{\text{eff}}$ |
| $-\lambda$ | 0.255 | $-1.031 \text{ TeV}$ |

| Output parameters |
|-------------------|
| $\tan \beta$ | $\kappa$ | $m_{\tilde{S}}$ |
| 20.03 | 0.412 | $-4.851 \text{ TeV}^2$ |
| $A_\lambda$ | $A_\kappa$ | $m_{\tilde{S}}$ |
| $-3.653 \text{ TeV}$ | $3.916 \text{ TeV}$ | $14.69 \text{ TeV}$ |
| $M_1$ | $M_2$ | $M_3$ |
| 13.58 TeV | 13.91 TeV | 14.03 TeV |

| Mass spectrum |
|----------------|
| $m_{h_1}$ | $m_{h_2}$ | $m_{h_3}$ |
| 123.6 GeV | 3.190 TeV | 10.59 TeV |
| $m_{a_1}$ | $m_{a_2}$ | $m_{H^\pm}$ |
| 1.400 TeV | 10.59 TeV | 10.59 TeV |
| $m_{\tilde{\chi}_0^0}$ | $m_{\tilde{\tau}_i^\pm}$ | $m_{\tilde{\tau}_1}$ |
| 1.069 TeV | 1.069 TeV | 9.905 TeV |
| $m_{\tilde{t}_1}$ | $m_{\tilde{b}_1}$ | |
| 10.14 TeV | 12.01 TeV | |
| Other sfermions: | $10.2–15.6 \text{ TeV}$ |

| Dark matter |
|----------------|
| $\Omega_{\text{LSP}} h^2$ | $\sigma^{(p)}_{\text{SI}}$ | $\sigma^{(p)}_{\text{SD}}$ |
| $0.103$ | $1.8 \times 10^{-48} \text{ cm}^2$ | $1.8 \times 10^{-44} \text{ cm}^2$ |

| Couplings at the GUT scale |
|-----------------------------|
| $g_1$ | $g_2$ | $g_3$ |
| 0.684 | 0.684 | 0.686 |
| $y_t$ | $y_\tau$ |
| 0.523 | 0.121 | 0.151 |
| $\lambda$ | $\kappa$ |
| 0.287 | 0.561 |

5 Summary and Discussion

We have discussed the effect of non-universal gaugino masses on the NMSSM with universal soft trilinear couplings and scalar masses. We have found that if $M_2$ is large than $M_3$ by a factor of 2–4 at the GUT scale, then constraints from the tadpole and stability conditions can significantly be alleviated. Contrary to the CNMSSM, $m_{\tilde{S}}^2$ runs negative at low energies, which can be regarded as the NMSSM counterpart of the radiative electroweak symmetry breaking in the MSSM [133–138]. The lighter stau, which tends to be either the LSP or tachyonic in the CNMSSM, can be sufficiently heavy due to the RG effects from large $M_1$ and $M_2$. We note that such features are generic for non-universal gaugino masses.
gaugino masses and thus this type of mass spectrum is useful to assure the desirable EWSB vacuum even though we slightly relax the universality conditions of the soft mass parameters. In the most of the parameter points, the LSP is found to be higgsino-like neutralino, and its thermal relic abundance can account for the observed DM density. We have also shown that many parameter points in the non-universal gaugino mass scenario can be tested at a future 100 TeV pp collider or in dark matter direct searches.

In our analysis, we have assumed that all of the input parameters are real just for
simplicity. Generically speaking, however, some of these parameters can be complex, which then introduce additional CP-violating sources and may generate CP-odd quantities that are experimentally observable. If CP-violating sources exist in the Higgs sector, which can appear at tree level \cite{18, 87, 139–141} and/or at loop level \cite{142–144}, there is a mixing between the CP-even and CP-odd Higgs states. This mixing changes the properties of the Higgs bosons like their masses and couplings to the SM fields. An interesting possibility of the CP-violated NMSSM is the electroweak baryogenesis (EWBG) \cite{145}—in the NMSSM, a strong first-order phase transition might have occurred in the early universe, and the extra CP-violating sources in the NMSSM scalar potential may bring in the successful EWBG \cite{146–153}. Such CP-violating sources can be probed \cite{154} with the measurement of the electric dipole moments (EDMs) of electron, neutron, atoms, and so on. In the case of the non-universal gaugino mass scenario, because of the presence of a relatively light higgsino, a sizable EDM of electron, $d_e$, may be induced via the Barr-Zee diagrams \cite{155}. It is found \cite{156, 157} that $|d_e| \approx 10^{-29–30} \text{ e \cdot cm}$ for $\mathcal{O}(10)$ TeV gaugino masses if the relative CP phase between $\mu_{\text{eff}}$ and $M_2$ is sizable. This is below the current bound imposed by the ACME Collaboration \cite{158}, $|d_e| < 8.7 \times 10^{-29} \text{ e \cdot cm}$, but can be probed in the future experiments as their sensitivities are expected to reach $|d_e| \sim 10^{-30} \text{ e \cdot cm}$ \cite{159, 160}.

From the above analyses, we see that the favored values of $M_2/M_3$ are limited to the range $2 \lesssim M_2/M_3 \lesssim 4$. This consequence may have important implications if we consider a UV completion of this scenario above the GUT scale. For instance, if we assume a specific GUT model to realize a non-universal gaugino mass spectrum, the ratio $M_2/M_3$ is determined to be a particular value \cite{50}, which then constrains viable model parameter space. Moreover, if we consider a concrete GUT model, we can also discuss phenomena associated with the GUT-scale physics, such as proton decay and gauge/Yukawa coupling unification. As we have seen above, the non-universal gaugino mass scenario tends to predict a low GUT scale, and thus proton decay lifetime may be rather short and within the reach of future proton decay experiments. More detailed discussions on the non-universal gaugino mass models in the framework of GUTs will be given on another occasion \cite{161}.

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