Computer visualization of automatized calculation the factors of mass transfer the viscous incompressible liquid with contact width variation

L G Varepo¹, A V Panichkin², O V Trapeznikova¹ and I V Nagornova³

¹Omsk State Technical University, 11, Mira ave., Omsk, 644050, Russia
²Sobolev Institute of Mathematics, 13, Pevtsova str., Omsk, 644043, Russia
³Moscow Polytechnic University, 38, B. Semenovskaya str., Moscow, 107023, Russia

E-mail: larisavarepo@yandex.ru

Abstract. The results of using the finite-difference approximation of combined equations for describing the flow of a viscous incompressible liquid between contacting rotating cylindrical surfaces are presented in the work. A quantitative estimation of mass transfer and splitting factors for a viscous incompressible liquid onto a substrate is obtained depending on a contact width variation. A distinctive feature of the proposed approach is the accounting of the structural and microgeometric parameters of surfaces contacting with a viscous incompressible liquid.

1. Introduction

Various approaches to the mathematical formulation of mass transfer and splitting of viscous incompressible liquids that allow to solve the dynamical problem of transmission a viscous incompressible liquid between the surfaces of two cylinders are given in the works of a number of Russian and foreign scientists [1–3].

Modern computer technologies and wide software potential greatly simplified the modeling of such processes. Among the problem elaborations the works of [4–6] are of great scientific interest.

2. Problem definition

Mass flows of an incompressible liquid with mobile barriers are inherent to different technological processes.

An analysis of the previously obtained experimental and theoretical data on the motion of a layer of a viscous liquid between rotating cylinders and its splitting during transfer from one cylinder to another showed that the problem has not been sufficiently studied. Solving such problems is laborious, which is due, first, to the nonlinearity and complexity of the mathematical equations, and second, to the need to determining the free surface when solving the system of partial derivative, which is a characteristic property of flows in layers and films.

Depending on the properties of the viscous incompressible liquid and the rate of layer separation the character of the rupture of the transferred layer also changes. To simplify the concrete problem solving, for example, when transferring a liquid to a porous substrate of one of the contacting cylinders, or a cylinder, it was proposed by many authors to accept conventionally the splitting of the liquid layer divided in half and with a zero absorbency, but there can be an another opinion [7].

The problem solving on the quantitative estimation of mass transfer factors of a viscous incompressible liquid with width variation in the zone of contacting cylinders is, in our opinion, of both practical and scientific interest.
3. The problem solution

To solve the problem, the authors developed an algorithm and its software implementation [8–10].

The most general mathematical model for formulation the flow of a continuous viscous liquid is the Navier-Stokes equations, for the solution of which there are various numerical procedures. Among the most accepted approximation methods, which are of high accuracy, of incompressible flows, the finite elements method is marked out. Nevertheless, finite-difference algorithms are not subject to the above-mentioned method, and in a number of cases they outproduce it in efficiency and accuracy. In this paper, the finite-difference method for solving multidimensional problems is chosen as the main method.

Calculations, for the stated problem situation, were carried out using methods on the finite-difference scheme for equations (1)-(2), using in differential operators of \((\nabla^2 - (\bar{V}, \nabla))\) the operator \(\Lambda\) in (3) and \(\nabla \approx \left(\frac{\Delta_1 + \Delta_{-1}}{2h_x}, \frac{\Delta_2 + \Delta_{-2}}{2h_y}\right)\) with usual orientation of the pattern along the nodal curves for the calculation of convective-diffusion transfer by the example of printing ink transfer between two rotating cylinders [8–12].

\[
\frac{\partial \bar{V}}{\partial t} + (\bar{V}, \nabla) \bar{V} + \frac{1}{\rho} \nabla p = \nabla \nabla^2 \bar{V} + \bar{F}(t, \bar{x}, \bar{V}, \bar{\omega}, \bar{g});
\]

\[
\frac{\partial p}{\partial t} + \nabla \cdot \bar{V} = 0,
\]

\[
\Lambda = \Lambda_0^1 + \Lambda_0^2, \quad \Lambda_0^1 = \frac{\Delta_1 + \Delta_{-1}}{2h_x} + \frac{\nu \Delta_1 \Delta_{-1}}{h_x^2} \quad \text{and} \quad \Lambda_0^2 = \frac{\Delta_2 + \Delta_{-2}}{2h_y} + \frac{\nu \Delta_2 \Delta_{-2}}{h_y^2}, \quad (3)
\]

where \(\bar{V}\) – velocity; \(P\) – pressure; \(\rho\) – liquid density; \(t\) – time; \(\bar{x} = (x, y)\) – coordinate of a point in Euclidean space; \(\nabla\) – gradient operator; \(\nabla^2\) – Laplace operator; \(\nu\) – kinematic viscosity; \(\bar{\omega}\) – angular velocity of rotation; \(\bar{g}\) – vector value of acceleration of the gravitational field; \(\bar{F}\) – components of acceleration from external forces (gravity forces) and forces from transformation of coordinates; \(\Delta_1, \Delta_{-1}, \Delta_2, \Delta_{-2}\) – operators of function shift by step of grid up or down the axes \(x\) and \(y\) with steps of \(h_x\) and \(h_y\).

Specify the conditions of the problem for numerical simulation: \(\omega = 20\ \text{rad/s}\) – angular speed of cylinders rotation, \(P = P_{\text{amb}} = 10^5\ \text{H/m}^2\) – ambient pressure; \(\nu = 0.012\ \text{m}^2/\text{s}\) – kinematic viscosity. Initial dimensions of liquid zone are equal to \(\delta_l = 4\ \mu\text{m}, \delta_c = 5\ \mu\text{m}\) (contact width variation) and \(\delta_S = 2\ \mu\text{m}\) (layer thickness of a liquid), \(\delta = 1\ \mu\text{m}\) (substrate thickness), cylinder radiuses – \(r_1 = r_2 = 0.15\ m\).

A uniform grid with the number of computed nodes by two coordinates \(N_x\) and \(N_y\) equal to 80 is used. The grid size allows to analyze the convergence and behavior of numerical solutions with an accuracy of 1 % at Reynolds numbers of about 1. The iteration step \(\tau\) was chosen in the range from 0.0002 to 0.005 for the considered grid in nondimensional quantities. Calculations of controlled mass transfer factors for a viscous incompressible liquid were performed taking into account the filtration of a liquid into a porous substrate on one of the contacting cylinders. The algorithm of numerical simulation and computer visualization of calculations is shown in Fig. 1.

When calculating the fluid flows between the surfaces to be contacted, the deformation of the surface layers of cylinders and substrate, on which the liquid to be transferred, as the result of changes in liquid pressure under the contact with them was taken into consideration.

Taking into account the infinitesimality of the relative velocities in compressed surfaces with the difference between the internal and external pressures \((P_1 - P_0)\) with the value of boundary deformation accumulating over time \(\Delta r\) for the thickness \(H\) of the substrate layer or the coating of the first cylinder (layer of rubber-textile sheet) with area \(S\) having the elastic modulus \(E\) of the center layer thickness changes \(x_c\) were calculated by equation (4)

\[
\rho HS\ddot{x}_c / 2 = \frac{E}{H} (\Delta r / 2 + x_c) \cdot S + (P_1 - P_0) \cdot S. \quad (4)
\]
Figure 1. An algorithm for numerical simulation of mass transfer the viscous incompressible liquid onto a substrate of a contacting cylinder.
To ensure the automation of the calculation of controlled mass transfer factors the viscous incompressible liquid to a substrate in the zone of two contacting cylinders under the contact width variation, the practical realization of the program on the bases of proposed algorithm is carried out [11].

The results of calculating the controlled factors in given conditions for numerical simulation of studied process are presented (Table 1).

**Table 1. Results of numerical simulation**

| Number of the sample | Contact width variation, μm | Amount of viscous incompressible liquid in a volume layer of a substrate, % | Amount of viscous incompressible liquid in a surface layer of a substrate, % |
|---------------------|-----------------------------|------------------------------------------------------------------|------------------------------------------------------------------|
| 1                   | 4.0                         | 60.98                                                            | 3.08                                                             |
| 2                   | 5.0                         | 66.96                                                            | 3.42                                                             |
| 3                   | 5.0                         | 59.37                                                            | 3.48                                                             |

Computer visualization of numerical simulation of mass transfer the viscous incompressible liquid onto a porous substrate, depending on the changes in contact width variation of two cylinders are in Table 2, depending on the type of substrate – in Table 3.

**Table 2. Computer visualization of mass transfer and splitting the viscous incompressible liquid with various contact width of two cylinders**

| δL = 4 μm | δL = 5 μm |
|-----------|-----------|
| ![Visualization](image1) | ![Visualization](image2) |

Computer visualization of the behavior of the liquid layer between the contacting surfaces in the center of the contact

Computer visualization of liquid behavior between contacting surfaces at the end of contact zone

Computer visualization of complete liquid splitting between contacting surfaces at the end of the contact zone
Table 3. Computer visualization of mass transfer and splitting the viscous incompressible liquid with varying characteristics of substrate

| Microporous substrate (sample 3) | Macroporous substrate (sample 2) |
|----------------------------------|----------------------------------|
| ![Computer visualization](image)  | ![Computer visualization](image)  |

Computer visualization of the behavior of the liquid layer between the contacting surfaces in the center of the contact

Computer visualization of liquid behavior between contacting surfaces at the end of contact zone

Computer visualization of complete liquid splitting between contacting surfaces at the end of the contact zone

4. Conclusion
The developed model provides an accurate method for calculating controlled factors of mass transfer the viscous incompressible liquid onto a porous substrate between contacting cylinders, promote the automation of its control.

Figures of computer visualization widen the concept of the mechanism of studying process, allow to see all the changes that occur in the layer of a viscous incompressible liquid when it is transferred onto a substrate in the zone of two contacting cylinders in dependence on conditions being set, that clearly shows the monitoring stages of this process.

In the figures of computer visualization with increasing contact width variation large deformations appear in the liquid layer, i.e. spreading beyond the edges of the contact zone, and an increase of penetration the viscous incompressible liquid into the volumetric layers of the substrate (Sample 1 and Sample 2, respectively). A similar result is observed when estimating the controlled factors of liquid transferring on a macro and microporous substrate surface.
The proposed solution was tested on transferring the ink (viscous incompressible liquid) onto the paper sheet (substrate) by the offset printing method in the contact zone of the cylinders of the press. The obtained result is interpreted as follows. For a substrate with a microporous (Sample 3), the amount of ink in volume layer of paper sheet is much smaller than to a macroporous (Sample 2) surface. There is a higher concentration of ink pigment in the surface layer for the microporous substrate.

References
[1] Calgaro C, Creusé E, Goudon T, Krell S 2017 *Mathematics and Computers in Simulation* 137 pp 201–225
[2] N C Reis Jr, Griffiths R F, Santos J M 2004 *J. Comput.Phys.* 198 pp 747–770
[3] N C Reis Jr, Griffiths R F, Santos J M 2008 *Applied Mathematical Modelling* 32(3) pp 341–346
[4] Vlachopoulos G, Claypole J, Bould D 2010 Ink mist formation in roller trains: Iarigai proceedings: Advances in printing and media technology. Montreal, Canada 37 pp 227–234
[5] Ozaki Y and Kimura M 2000 *Appita J.* 3 pp 216–219
[6] Koivula H, Preston J S, Heard P J and Toivakka M 2008 *Colloids and Surfaces A: Physicochemical and Engineering Aspects* 317 (1–3) pp 557–567
[7] Fatemeh Ghadiri, Dewan Hasan Ahmed, Hyung Jin Sung, Ebrahim Shirani 2011 *International Journal of Heat and Fluid Flow* 32 (1) pp 308–317
[8] Panichkin A V, Varepo L G 2014 *Springer Proceedings in Physics* 154 pp 79–83
[9] Panichkin A V, Varepo L G, Trapeznikova O V 2016 *IOP: Materials Science and Engineering* 124 pp 1–6
[10] Varepo L G, Panichkin A V 2017 *IOP Conf. Series: Journal of Physics: Conf. Series* (858) 012039 pp 1–6
[11] Varepo L G, Panichkin A V Certif. No 2016617873 of state registration of the program Calculation of the transfer coefficients of a viscous incompressible fluid to a substrate between contacting cylindrical surfaces (Russian Federation); publ. 15.07.2016
[12] Varepo L G, Panichkin A V, Panchuk K L 2017 *Proceedings of Computer and Information Technologies* 11 (161) pp 16–22 (Russian)