An Improved Iterative Frequency Domain Decision Feedback Equalization for STBC-SC-FDE

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Abstract. This article proposes an improved iterative frequency domain decision feedback equalization (IBDFE) for the unique word (UW) based system of space-time block coded-single carrier frequency domain equalization (STBC-SC-FDE). Compared with the conventional IBDFE, this algorithm uses minimum mean square error (MMSE) equalization and a noise prediction module based on UW instead of the feedback equalization part, while maintaining the feedforward equalization. It is shown from the simulation results that the improved IBDFE can achieve satisfactory performance without iterations.

1. Introduction
The high data rate transmission through wireless environment often undergoes a severe inter-symbol interference (ISI) channel. Several approaches have been proposed to solve this problem. Orthogonal frequency division multiplexing (OFDM) is an efficient solution, but it suffers two inherent drawbacks: high peak-to-average power ratio (PAPR) and sensitivity to frequency offset. Single carrier frequency domain equalization (SC-FDE) have a reduced PAPR when compared to OFDM, while maintaining their convenient low complexity frequency domain equalization [1-4]. Diversity transmission using Alamouti’s space-time block coding (STBC) is a simple and powerful diversity technique [5]. It can be applied with SC-FDE to make better performance.

Zero-forcing (ZF) equalization and minimum mean square error (MMSE) equalization are two classical linear equalization algorithms of SC-FDE. ZF equalization can restore the original spectrum regardless of the channel conditions, but it suffers from significant noise enhancement for transmission over deep frequency-selective fading channels. MMSE equalization minimizes the power sum of the residual ISI (RISI) and additive noise after the equalization and it outperforms ZF equalization with respect to the BER performance of SC-FDE. In [6], Al-Dhahir proposed MMSE frequency domain linear equalization for STBC-SC-FDE system. However, the impact of RISI after MMSE equalization is still severe. A decision feedback equalization with a hybrid structure (H-DFE) for STBC-SC-FDE is proposed in [7], where a feedforward filter is performed in frequency domain and a feedback filter is performed in time domain. It can significantly improve the system performance because of the feedback filter. Nevertheless, it is prone to error-propagation phenomena and its achievable performance improvement is at the expense of increased complexity. An iterative frequency domain decision feedback equalization (IBDFE) [8] which both the feedforward and feedback filter operation in the frequency domain is proposed for STBC-SC-FDE in [9]. It can achieve a lower computational complexity than H-DFE and make more performance by making use of iterations. However, the cross-
correlation function between the detected symbols and the transmitted symbols has to be calculated in each iterations of the feedback.

In this work, an improved iterative frequency domain decision feedback equalization (IBDFE) is proposed for the unique word (UW) based system of STBC-SC-FDE. Compared with the conventional IBDFE, this algorithm uses MMSE equalization and a noise prediction module based on unique word (UW) instead of the feedback equalization part, while maintain the feedforward equalization. The UW based on the system of STBC-SC-FDE which used in this article is proposed by [10].

Notation:Bold upper (lower) letters denote the matrices(column vectors); (·)T and (·)H denote transpose, and Hermitian transpose, respectively; F is the normalized \(N \times N\) DFT matrix with \((p, q)\) th entry \(\sqrt{\frac{1}{N}} e^{-j2\pi pq/N}\) for \(p, q = 0, 1, \ldots, N-1\); \(\theta_{M \times N}\) is an all-zero matrix of size \(M \times N\); \(E[\cdot]\) is the expectation operator; We define \(N \times N\) permutation matrices \(P_{k}\) and \(Q_{k}\) that perform a cyclic shift and a reversed cyclic shift, respectively. For an \(N \times 1\) vector \(y = [y_0, y_1, \ldots, y_{N-1}]^T\), we have \([P_{k}y]_n = y_{(k+n) \mod N}\), \([Q_{k}y]_n = y_{(k-n) \mod N}\), and \((Wy)^* = WP_{k}Q_{k}y^*\).

2. System Model
We consider a UW based STBC-SC-FDE system with two transmit antennas and one receive antenna. The system structure is illustrated in Fig 1. The \(i\) th length- \(N\) block \(x_{m}^{(i)}\) of transmitted symbols from \(m\) th transmit antenna can be partitioned into a length- \(K\) vector \(d_{m}^{(i)}\) of data symbols and a length- \(G\) vector \(u_{m}^{(i)}\) representing the UW as

\[
x_{m}^{(i)} = [d_{m}^{(i)} \ u_{m}^{(i)}]^T
\]

(1)

where \(K\) is the memory order of the channel impulse response (CIR), so the inter-block interference can be removed.

After the channel transmission, the received blocks \(r_{1}\) and \(r_{2}\) can be expressed as

\[
x_{m}^{(i)} = [d_{m}^{(i)} \ u_{m}^{(i)}]^T
\]

(2)

where \(d_{m}^{(i)} = [d_{m}^{(i)} \ \theta_{m \times 1}]^T\) and \(u_{m}^{(i)} = [\theta_{k \times 1} \ u_{m}]^T\).

After the channel transmission, the received blocks \(r_{1}\) and \(r_{2}\) can be expressed as
\[ r_i = r^{(i)} = H^{(i)} x^{(i)}_1 + H^{(i)} x^{(i)}_2 + n_i, \]
\[ r_2 = r^{(i+1)} = H^{(i+1)} x^{(i+1)}_1 + H^{(i+1)} x^{(i+1)}_2 + n_2, \]  
\( i = 0, 2, 4, \ldots \)  

where \( H_m, m = 1, 2 \) are \( N \times N \) circulant channel matrices from the first and second transmit antennas to receive antenna, respectively. \( n_m (m = 1, 2) \) are the length- \( N \) blocks of zero-mean complex additive white Gaussian noise with variance \( \sigma_n^2 \). For simplicity, we omit the superscript \(^{(i)}\). Assuming that the channels are constant over two consecutive blocks, let \( H_m = H_m^{(i)} = H_m^{(i+1)} \), the formula (3) can be rewritten as

\[ r_i = H_i x_i + H_2 x_2 + n_i, \]
\[ r_2 = H_i (-Q_n \tilde{d}_i + \tilde{u}_i) + H_2 (Q_n \tilde{d}_i + \tilde{u}_i) + n_2, \]  

Multiplying \( r_i \) by \( P \) and using the property \( H_m P = PH_m \), we obtain

\[ r'_i = P_k r_i \]
\[ = H_i (-P_k Q_n \tilde{d}_i + P_k \tilde{u}_i) + H_2 (P_k Q_n \tilde{d}_i + P_k \tilde{u}_i) + P_k n_2 \]  

The received blocks \( r_i \) and \( r'_i \) are transformed to the frequency-domain by applying the DFT

\[ R_i = F r_i = A X_1 + A_2 X_2 + N_1 \]
\[ R'_i = F r'_i = A_1 (-\tilde{N}_2) + F P_k \tilde{u}_i + A_2 (\tilde{N}_1 + F P_k \tilde{u}_i) + \tilde{N}_2 \]

where \( \tilde{N}_2 = FP_k n_2 \), \( X_m = FX_m \), and \( A_m = FH_m F'^T \) is a diagonal matrix whose \( (k, k) \) entry is equal to the \( k \) th DFT coefficient of the CIR from antenna \( m \).

After inserting UW, UW is processed to receive data in order to facilitate decoding. It is deducted in detail in [10]. The received symbol before decoder can be given by

\[ R = \begin{bmatrix} R_i \\ R'_i \end{bmatrix} = \begin{bmatrix} A_1 & A_2 \\ A'^T_1 & A'^T_2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} N_1 \\ \tilde{N}_2 \end{bmatrix} \]
\[ = AX + N \]  

where \( A = \begin{bmatrix} A_1 & A_2 \\ A'^T_1 & A'^T_2 \end{bmatrix} \). After decoding, type (7) can be divided into

\[ Y_1 = \tilde{A} X_1 + \tilde{N}_1 \]
\[ Y_2 = \tilde{A} X_2 + \tilde{N}_2 \]  

where \( \tilde{A} = A_1^T + |A_2|^2 \) is a \( N \times N \) diagonal matrix ,
\[ \tilde{N}_1 = A'^T_1 N_1 + A_1 \tilde{N}_2 \]
\[ \tilde{N}_2 = A'^T_2 N_1 - A_1 \tilde{N}_2 \]  

The automatrix of \( \tilde{N}_1 \) and \( \tilde{N}_2 \) can be given by

\[ E \left[ \tilde{N}_1 \tilde{N}_1^H \right] = \left( |A_1|^2 + |A_2|^2 \right) \sigma_n^2 = \tilde{A} \sigma_n^2 \]
\[ E \left[ \tilde{N}_2 \tilde{N}_2^H \right] = \left( |A_1|^2 + |A_2|^2 \right) \sigma_n^2 = \tilde{A} \sigma_n^2 \]  

3. Improved IBDFE Algorithm

3.1. IBDFE Algorithm
Figure 3. The structure of IBDFE algorithm

We can see from type (8) that $Y_1$ and $Y_2$ has the same form, for the convenience of expression, the subscript of $Y_i$ and $Y_j$ is omitted in the following deduction. Using $Y$ for unified expression of $Y_1$ and $Y_2$, and the rest of the variables are also processed by the same way.

The structure of the IBDFE is shown in Figure 3, in which both the feedforward and feedback filters operate in the frequency domain. At iteration $l$, we can obtain from the detection point that

$$Z[l] = C[l]Y[l] + B[l]\hat{X}[l-1]$$

where $\{C_n\}, n=0,1,\ldots,N-1$, represents the coefficients of the FF filter; $\{B_n\}, n=0,1,\ldots,N-1$, represents the coefficients of the FB filter.

As derived in [8], the feedforward equalization coefficients of IBDFE can be obtained as

$$C[l] = \frac{\hat{A}_n}{\hat{A}_n\sigma_n^2 + M_{X_n}(1 - \frac{r_{X_n,\hat{X}_n[-1]}}{M_{\hat{X}_n, X_n}})|\hat{X}_n|^2}$$

where the power of the involved signals in the frequency domain can be given by

$$M_{X_n} = E[|X_n|^2], \quad M_{\hat{X}_n, X_n} = E[|\hat{X}_n[-1]|^2]$$

the correlation between transmitted and detected data sequences

$$r_{X_n,\hat{X}_n[-1]} = E[X_n\hat{X}_n[-1]]$$

The feedback equalization coefficients of IBDFE is

$$B[l] = \frac{r_{X_n,\hat{X}_n[-1]}}{M_{\hat{X}_n, X_n}}[\hat{A}_nC[l] - \gamma[l]]$$

with $n = 0,1,\ldots,N-1$, and

$$\gamma[l] = \sum_{n=0}^{N-1} \hat{A}_nC[l]$$

From type (13)(16), it can be known that the estimation of the correlation factor and the power of the detected data are needed in each iterations of IBDFE, which increases the complexity greatly.

3.2. Improved IBDFE Algorithm

Figure 4 shows the structure of the improved IBDFE algorithm, it uses MMSE equalization and a noise prediction module based on unique word(UW) instead of the feedback equalization part, while maintain the feedforward equalization.
The data after the MMSE equalization can be expressed as

\[ \hat{X} = WY \]  

(18)

where

\[ W = \frac{\hat{A}^H}{\hat{A}^H \hat{A} + \hat{A} \sigma_w^2 / \sigma_x^2} \]  

(19)

After IFFT

\[ \hat{x} = F^H \frac{\hat{A}^H}{\hat{A}^H \hat{A} + \hat{A} \sigma_w^2 / \sigma_x^2} Y \]  

(20)

From type (8)(20), the estimation error of the MMSE equalization can be obtained as

\[ \varepsilon = \hat{x} - x = F^H \frac{\hat{A}^H}{\hat{A}^H \hat{A} + \hat{A} \sigma_w^2 / \sigma_x^2} (\hat{A}X + \hat{N}) - x \]  

(21)

Contrasted with (1), we can obtain that

\[ \hat{x} = \begin{bmatrix} \hat{x}_d \\ \hat{x}_u \end{bmatrix} \]  

(21)

Therefore, we can separate \( \hat{x} \) into data vector \( \hat{x}_d \) and UW vector \( \hat{x}_u \).

\[ \hat{x}_d = F^H_d \frac{\hat{A}^H}{\hat{A}^H \hat{A} + \hat{A} \sigma_w^2 / \sigma_x^2} Y \]  

(22)

\[ \hat{x}_u = F^H_u \frac{\hat{A}^H}{\hat{A}^H \hat{A} + \hat{A} \sigma_w^2 / \sigma_x^2} Y \]  

(23)

where \( F_d \) is the first \( K \) columns of \( F \) and \( F_u \) the last \( G \) columns. The estimation error of the UW part can be written as \( \varepsilon_u \) as \( u \) is known, \( \varepsilon_u \) can be accurately obtained by the receiver.

\[ \varepsilon_u = \hat{x}_u - u \]  

(24)

We can see from [11] that \( \varepsilon_u \) can be used to predict \( \varepsilon_d \). Set the linear MMSE prediction matrix as \( G \), so the minimum mean square error value of the estimation error can be expressed as

\[ J_{\text{min}} = E\left[ \| \varepsilon_d - Ge_u \|^2 \right] \]  

(25)

We can derived from Winner-Hoff matrix[12] that

\[ G = E\left[ \varepsilon_d \varepsilon_u^H \right] \left( E\left[ \varepsilon_u \varepsilon_u^H \right] \right)^{-1} \]  

(26)

The prediction of \( \varepsilon_d \) can be expressed as

\[ \hat{\varepsilon}_d = Ge_u \]  

(27)
By Figure 4, the data after the decision can be expressed as
$$\hat{x}_j = \bar{x}_j - \epsilon_j$$  (28)

We make the assumption that $\hat{x}_j = d$, so we can obtain from the detection point that
$$Z_s = C_s Y_s + B_s X_s$$  (29)

The MSE can be given by
$$MSE = \frac{1}{N} \sum_{n=0}^{N-1} E \left[ \left| z_n - x_n \right|^2 \right]$$  (30)

By applying Parseval’s theorem, $MSE$ can be written as
$$MSE = \frac{1}{N^2} \sum_{n=0}^{N-1} E \left[ C_s Y_s + B_s X_s - X_n \right]^2$$  (31)

Imposing the constraint
$$\sum_{n=0}^{N-1} B_n = 0$$  (32)

To minimize the MSE, we apply the lagrange multiplier method and aim at minimizing the function
$$f(C_s, B_s, \lambda) = \frac{1}{N} \sum_{n=0}^{N-1} E \left[ C_s (\hat{A}_n X_n + \bar{N}_n) + B_s X_n - X_n \right]^2 + \lambda \sum_{n=0}^{N-1} B_n$$  (33)

where $\lambda$ is the Lagrange multiplier. Now, by setting to zero the gradient of (33) with respect to $C_s$, $B_s$, and $\lambda$, respectively, we derive
$$\frac{\partial f}{\partial C_s} = \frac{1}{N^2} \sum_{n=0}^{N-1} 2(C_s \hat{A}_n + B_s - 1) \hat{A}_n + 2C_s \hat{A}_n \sigma_n^2 = 0$$  (34a)
$$\frac{\partial f}{\partial B_s} = \frac{1}{N^2} \sum_{n=0}^{N-1} 2(C_s \hat{A}_n + B_s - 1) + \lambda = 0$$  (34b)
$$\frac{\partial f}{\partial \lambda} = \sum_{n=0}^{N-1} B_n = 0$$  (34c)

We can obtain from (34a)(34b)(34c) that
$$C_s = \frac{\hat{A}_n - C_s \sigma_n^2}{\hat{A}_n + \lambda \sigma_n^2}$$  (35)
$$B_s = -\frac{\hat{A}_n + \sigma_n^2}{2 \sigma_n^2} + 1$$  (36)
$$\lambda = \frac{2 \sigma_n^2}{\frac{1}{N} \sum_{n=0}^{N-1} (\hat{A}_n + \sigma_n^2)}$$  (37)

4. Simulation Results
To demonstrate the performances of the improved IBDFE algorithm, the multipath Rayleigh fading channel was considered. The multipath Rayleigh fading channel which are shown in table 1 are based on the troposcatter channel in north China and the simple-hop communication distance is 300km[13].
**TABLE 1. THE TABLE OF PARAMETERS FOR SCATTER LINK OF 300KM**

| Path num | Delay/us | Path gain | Doppler (HZ) |
|----------|---------|-----------|-------------|
| 0        | 0.0     | 0.2772    | 100         |
| 1        | 0.1     | 0.4130    | 120         |
| 2        | 0.2     | 0.7077    | 110         |
| 3        | 0.3     | 0.8518    | 100         |
| 4        | 0.4     | 0.8184    | 80          |
| 5        | 0.5     | 0.6713    | 90          |
| 6        | 0.6     | 0.4813    | 85          |
| 7        | 0.7     | 0.3055    | 105         |
| 8        | 0.8     | 0.1730    | 86          |

![Figure 5](image_url)

**Figure 5.** The performance of different equalization algorithms in scattering channel

We consider a block transmission with each data block consisting of 256 QPSK modulated symbols, and the UW consists of 32 symbols. The receiver is assumed to have perfect synchronization and channel estimation. Channel coding is not used. Fig. 5 compares the BER performance of three equalization algorithms based on STBC-SC-FDE system. For explicit compare, the QPSK modulation system is also given.

We can see from Fig. 5 that using STBC-SC-FDE system can reduce BER obviously compared with QPSK modulation. This proves the effects of STBC-SC-FDE to combat multipath fading. Compared with MMSE equalization, the IBDFE with three iterations improves the system performance. This is because that the IBDFE adds the feedback part and can make more performance with iterations. However, the performance of the IBDFE with three iterations also can be worse when SNR is low. This is because that when SNR is low, the error-propagation can be worse. We can see from Fig.5 that the improved IBDFE can achieve about 1.2dB gain when BER is $10^{-3}$ compared with the IBDFE with three iterations. What’s more, the proposed algorithm don’t need iterations, which lower the equalization complexity.

**5. Conclusions**

This article proposes an improved IBDFE for the UW based on the system of STBC-SC-FDE. Compared with the conventional IBDFE, this algorithm uses MMSE equalization and a noise prediction module based on UW instead of the feedback equalization part, while maintain the feedforward equalization. It is shown from the simulation results that the improved IBDFE can achieve satisfactory performance without iterations. The applications of the proposed equalization in multiple-input multiple-output (MIMO) areas will be further studied.
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