Abstract. We have used an atomic $^{87}$Rb BEC to study the emergence of quantum turbulence. The application of an external oscillating magnetic field gradient is used to nucleate vortices and anti-vortices spread over the cloud, in many directions, setting up the conditions for the turbulent regime to arise. Once the turbulence is established one may study it in a variety of its different aspects and we will present and discuss a few of these possibilities.

1. Introduction

Bose-Einstein condensates provide ideal experimental conditions for studying superfluidity [1, 2]. In such a coherent particle field, it follows that the superfluid component is irrotational. However, Nature finds its way to create rotation in these systems to form vortex filaments, which are lines along the condensate where the macroscopic wave function amplitude $\Psi(r)$ vanishes and has a curl-free circulation around it. For a free vortex filament, a stable condition implies that the quantized circulation must have a single quantum value [3]. A collection of vortex filaments in the superfluid may generate interesting spatial distributions, line vortices tangles, producing a turbulent state, named quantum turbulence (QT) [4]. The turbulent flow is necessarily rotational indicating that, inside the quantum fluid, there is only a single possible arrangement of vortex lines. This condition creates special features associated with QT, which makes this phenomenon somewhat different from its classical counterpart, and very interesting to be further investigated.

In the beginning, QT was only investigated in superfluid $^4$He [5], evolving later to $^3$He [6, 7], and, finally, observed in alkali BEC systems [8]. Because of the finite size and flexibility to control interactions in superfluid atomic samples, BECs represent a whole new window to explore the QT phenomenon.

We believe that better understanding of the features associated with QT phenomena may be an alternative path to clarify the overall understanding of its classical counterpart. Perhaps, it may be worthwhile to investigate the details of QT using condensates to help elucidate facts still unclear in dense systems like $^3$He and $^4$He. As stated by Davidson [9] in his recent book: “Despite the fact that the governing equations of turbulence have been known since 1845, there is still surprisingly little we can predict with relative certainty. The situation is reminiscent of the state of electromagnetism before it was transformed by Faraday and Maxwell. A myriad of tentative theories have been assembled, often centred around particular experiments, but there is not much in the way of a coherent theoretical framework”.

In this paper, we review our recent work presenting the emergence of QT in a sample of $^{87}$Rb BEC and some other interesting associated properties. We begin with a brief description of the experimental system used in our lab, and the procedure followed to produce the vortices and achieve the turbulent...
regime. A few characteristics related to the overall macroscopic behavior of the turbulent cloud are described. Some perspectives are also presented.

2. Experimental system

The experimental sequence to produce the BEC to generate the vortices in the condensate can be summarized as follows. First, we produce a $^{87}$Rb BEC containing about $1 \times 10^5$ atoms with a small thermal fraction in a cigar-shaped magnetic trap with frequencies given by $v_r = 210$ Hz and $v_z = 23$ Hz. Once the condensate is obtained and still held in the trap, an oscillating external magnetic field gradient is applied. This field is produced by a pair of anti-Helmholtz coils placed with their axis closely, though not exactly, parallel to the weak trap axis. Since the coils are not perfectly aligned to the condensate axis, the external field gradient around the trapping region has components parallel to each trap eigen axis. This oscillatory scheme was developed to investigate coherent mode excitations in our Bose-condensates. Such excitations are also believed to produce vortices. A combination of shape modification, trap minimum displacement and rotation is coupled to the atoms. The quadrupolar field is applied for periods ranging from 20 to 60 ms, oscillating sinusoidally in time with a frequency of 200 Hz. This external magnetic field gradient has its offset chosen so that during a period of oscillation it goes from zero to a maximum value and then back to zero, never inverting its direction. The maximum value reached by the field gradient characterizes the strength of the excitation and has been varied from zero to 190 mG/cm along the vertical axis. The atoms are left inside the trap for an extra 20 ms wait time, as soon as the external perturbation stops. The measurements are performed via time-of-flight absorption images, with the atomic cloud in free fall expansion. For small amplitudes of the oscillating field as well as shorter excitation periods, we observe dipolar modes, quadrupolar modes, and scissor modes in our BEC but no appearance of vortices. Increasing the values of both parameters produced vortices growing in number with the amplitude and/or the excitation elapsed time.

3. Observation of vortex distribution and turbulence

During the experiments, it was observed that by increasing either the amplitude or perturbation time period we were able to nucleate many vortices [10]. Small amplitudes seem to be only able to make the cloud tilt as indicated in Figure 1. The cloud’s axial motion is a clear indication that the oscillatory excitation is able to transfer angular momentum to atoms. The periodic axis tilting observed in the cloud is associated with the scissor modes. Originally, such a mode was investigated theoretically by Guéry-Odelin [11] and is generated by a sudden rotation of the confining trap. As already mentioned, the external coils produce a combination of rotation and translation of the trapping potential, inducing the modes. The mode frequencies are, naturally, closely related to the trap frequencies. In the case of a classical gas, the tilting oscillation does not present itself as a simple harmonic oscillation. The Thomas-Fermi cloud motion strongly indicates that angular momentum is being coupled to the atoms.

As the oscillation amplitude increases, the vortices start to nucleate. We have observed that, during the excitation, a combination of different collective modes is observed, as shown in Figure 2. Following the studies of Bradley and coworkers [12] the high excitation of collective modes promotes the formation of a thermal cloud which causes surface oscillations to become unstable. Long lived vortices are nucleated in the region between the condensed and the thermal cloud. Initially, they penetrate a small inward radial distance into the superfluid (on the order of the vortex size). As the cloud accumulates more (rotational) energy the vortices are able to fully enter into the condensed fraction. In Figure 3, an absorption image shows the process starting with the first inside vortex. We note that the surrounding peripheral vortices with little penetration into the condensed cloud are clearly visible [10].

The nucleation of vortices in the region interfacing the condensed and the thermal clouds may be equivalent to the Kelvin-Helmholtz instabilities phenomenon, well known in classical fluids. The particular point here is the instability generated by the excitation of the collective modes. Once the vortices are formed, they may be able to penetrate into the condensed fraction, when the conditions are energetically favorable. This has to take place while energy is still being pumped into the system.
Figure 1. Axis tilt oscillations observed for 40 mG/cm of the external field gradient (low amplitude perturbation) after 15 ms of free expansion. The images were acquired from different experimental runs under identical conditions and are taken 0.5 ms apart.

Figure 2. Combination of collective modes for different samples due to excitation coils and time of flight with a time interval of 3.5 ms.

Figure 3. Zoomed boxed area of a typical absorption image (see right upper corner inset) containing vortices nucleated in the interface region between the condensed and the thermal fractions of the cloud.
by the external oscillatory field. We have made many observations in which the perturbed cloud had three singularities in the atomic density. We found out that when three vortices were being held together within the atomic cloud, they are most likely arranged as a triangle or a line. Theoretically, it is possible to justify that in the case of a spatial distribution close to a triangle the vorticity of each element presents the same signal while in the case of linear alignment they constitute a vortex tripole [13]. In the latter case, we have the two outer vortices with a given vorticity sign and the central one with the opposite sign. For the interaction regimes found in our experiment this configuration shall be considered metastable. The tripoles are expected to survive for about 20 ms in our setup [14]. The fast decay associated with the tripoles makes their observation less frequent than the triangular ones for the case of equivalent vorticity for all three vortices. A detailed explanation of the observations of vortex anti-vortex clustering can be found in a recent publication [15].

An interesting fact is that the vortex formation in our experimental setup is not restricted to any preferential direction. In fact, the oscillatory generation of vortices corresponds to a combination of rotation, translation and deformation of the trapping potential in different planes [2]. The key idea is analogous to combining rotations in orthogonal directions. After nucleating orthogonal vortex filaments the distribution just established will evolve into turbulence. The evolution from a regular (superfluid) condensed cloud to the turbulent cloud is shown in Figure 4. The turbulent cloud is characterized by vortex distributions in different directions. Once the turbulent regime is reached there are many studies to be carried out. One of the studies corresponds to the investigation of the analogy of the turbulence onset in a trapped condensate to a flow of the superfluid through a geometry (trapping potential) whose dimension is on the same size as that of a typical BEC. In this case a quantum Reynolds number can be defined and evaluated.

4. Perspectives to turbulence in atomic superfluids
The possibility of studying turbulence in an alternative superfluid to liquid helium is very attractive and may still create new exciting possibilities. First of all the relatively low density of trapped atomic superfluids allows a clearer visualization of the vortex geometry as well as the direct observation of the distribution of vortices and other effects related to few vortices. One important class of these effects is the transverse excitations taking place in the vortices lines. Those oscillations, named Kelvin modes, have already been observed in trapped BECs [16] and are considered to be one of the proofs for the condition of seed sustained turbulence in a superfluid. Vortex reconnections [5], which can change the topology of the vortex lines, always play an important role in quantum turbulence and there is a lack of understanding about the microscopic details previously observed and studied in liquid helium.

Quantum turbulence is a strong non-equilibrium phenomenon representing an essentially non-uniform matter, which poses challenging problems for its theoretical description. But, as pointed out by Yukalov...
[17], fully stationary turbulent states are likely to be obtained in trapped superfluids. This suggests that statistical methods may be used to model QT, creating new and exciting approaches to help understand this interesting phenomenon.

Finally, trapped atomic superfluids are systems of finite size and number. New effects due to the finite aspects of the superfluids will appear in these systems that might not be present in homogeneous systems like helium. These new effects might be remarkable concerning the evolution of the turbulent regime and its decay, generating thermal atoms, which can be easily detected and better studied in their dynamical aspects. The finite size effects may place limits on the maximum number of allowed vortices existing in the superfluid cloud before evolving to turbulence. This fact may have dramatic implications on the dynamical behavior of the turbulent states.

In conclusion, the study and understanding of turbulence in Bose-Einstein condensation of trapped neutral atoms is still at the very beginning, but has a great potential to become an important and exciting research field.

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