$K^-p$ and $\pi^-p$ charge-exchange processes
and $\eta - \eta'$ mixing

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Abstract

The processes of charge-exchanges of $\pi^-$ and $K^-$ on protons at high energies and zero transfers are described on the basis of the parton model. It is shown that at the mentioned kinematic conditions the scattering occurs on the valence quarks only, while scattering on sea quarks leads to disruption of final state. Thereby a substantiation is given of the valence approximation for the specified reaction. On the basis of GAMS-4π data it is shown that the description of $\eta - \eta'$ mixing does not require an introduction of additional states that do not fit into the scheme of basis states $|\eta_N>$ and $|\eta_S>$. The resulting value of the mixing angle is $\phi_p = (36.1 \pm 0.9)^{0}$. 

1 Introduction

The problem of $\eta - \eta'$ mixing is the oldest one existing almost from the time that SU(3)-flavor symmetry was proposed. However, until now this problem has no satisfactory solution [1]. The reason may lie in the model dependence of theoretical description of particular processes [2], or in an unknown global systematic error admitted at the data analysis. Anyway the problem manifests itself as instability in determining the mixing angle [3]. Probable reason is sometimes associated with an admixture of higher-mass states, primarily of a glueboll. However, in reality this does not solve the problem, since inclusion of such states also leads to ambiguous situation, see e.g. [2] and recent works [4, 5, 6].

In this connection it is of great interest an appearance of any new reliable data to include them in the analysis. Recently the data with high statistics were obtained on charge-exchange reactions $\pi^- p \rightarrow \eta(\eta') n$ and $K^- p \rightarrow \eta(\eta') \Lambda$ at $32.5$ GeV/c. [7]. Previously similar data were available at different energies, but with relatively low statistics and only in the case of the former reaction [8, 9, 10]. The data with $K^-$ beams were obtained for the first time, and this gave additional advantages for studying the $\eta - \eta'$ mixing. Unfortunately, the theoretical part of analysis of [7], in our opinion, includes inaccuracies. The purpose of this Letter is to correct detected inaccuracies, and to propose more complete development of the model with the aid of which the mixing of states in the context of given reactions can be investigated.
2 Substantiation of the model of investigation

As a rule, the analysis of mixing of states is carried out in the framework of the naïve quark model. The essence of this model consists in an idea that observable particles are made up of minimal allowed by quantum numbers set of constituent quarks and antiquarks. In particular, isosinglet mesons are made up of neutral on flavors pairs of constituent quarks and antiquarks. So, the $\eta$ and $\eta'$ are considered as linear combinations of pairs of nonstrange and strange quarks and antiquarks,

$$
\begin{pmatrix}
\eta \\
\eta'
\end{pmatrix} = 
\begin{pmatrix}
\cos \phi_p & -\sin \phi_p \\
\sin \phi_p & \cos \phi_p
\end{pmatrix}
\begin{pmatrix}
\eta_N \\
\eta_S
\end{pmatrix}.
$$

(1)

Here

$$
|\eta_N \rangle \simeq \frac{1}{\sqrt{2}} |u \bar{u} + d \bar{d} \rangle, \quad |\eta_S \rangle \simeq |s \bar{s} \rangle,
$$

(2)

and $\phi_p$ is the mixing angle in the nonstrange–strange quark basis. If necessary, the model can be extended by introduction of additional basic states, composed e.g. of a colorless pair of constituent gluons. In the latter case one should introduce a third observable state and proceed to $3 \times 3$ mixing matrix.

In the framework of this model the production of $\eta$ and $\eta'$, or in general case of any isosinglet pseudoscalar state $M^0$, in $\pi^-p$ and $K^-p$ charge-exchange reactions can be represented by diagrams of Fig. 1. The upper and middle-row pairs of diagrams in Fig. 1 represent contributions with quark annihilation and with quark exchange, respectively. The lower pair of diagrams represents both types of contributions with simultaneous production of colorless pair of constituent gluons. In the simplest case the diagrams of Fig. 1 exhaust the list of contributions. We start our analysis with consideration of all mentioned diagrams, without assumption a priori that some contributions are suppressed. In this point we diverge from analysis [7], which takes into consideration only the upper pair of diagrams of annihilation type.

First we notice that if the processes occur at high energies, then all diagrams of Fig. 1 imply hard subprocesses. Really, in all cases the quark or antiquark, being a part of high-energy $\pi^-$ or $K^-$, interacts with a quark of proton in the fixed target. As a result either a large energy is released, resulting in the production of quark-antiquark pair with one of the quarks flying away with high energy and another remaining in the target, or the incident quark join the target knocking out another quark with high energy, see Fig. 1(a,b) and Fig. 1(c,d), respectively. In diagrams Fig. 1(e,f) both processes occur with the difference that the energy is mainly transferred to the pair of constituent gluons rather than to a quark or antiquark. So, since all subprocesses include hard component, they can be considered from the standpoint of perturbative QCD. Actually all they arise in the leading order in the coupling constant $\alpha_s$. In the upper two pairs of diagrams this is obtained by joining two internal quark lines via a gluon line (see Fig. 2 below). In the case of the lower pair the corresponding diagrams are obtained by joining of one gluon to the line with quark annihilation and of another gluon to the line with quark exchange.

In this manner all subprocesses in Fig. 1 arise in the common order in $\alpha_s$. It can easily be seen that they arise in the common order in the $N_c^{-1}$-expansion, as well, where $N_c$ is the number of colors. For the upper two pairs of diagrams this is obvious in view of the fact
Figure 1: Diagrams for processes $\pi^- p \to M^0 n$ (a,c,e) and $K^- p \to M^0 \Lambda$ (b,d,f) in the naive quark model. $M^0$ means any isosinglet pseudoscalar state. Dotted circles symbolically indicate areas of formation of constituent gluons.

that they are planar. In the case of the lower pair of diagrams this can be seen taking into consideration the boundary condition under which the diagrams must be considered. Specifically, one must equate the colors of the initial and final quarks in the cases of mesons and baryons (in the lower pair of diagrams, of baryons only) and consider colorless combinations of quarks (gluons) in mesonic states.

So, both in terms of the counting in powers of $\alpha_s$ and the $N_c^{-1}$-expansion all diagrams of Fig. 1 should have comparable contributions. Nevertheless, the contributions of the lower pair of diagrams are strongly suppressed. This follows from the fact that the lower diagrams correspond to double parton processes, and contributions of such processes amount to about 5% of contributions of single parton processes [12, 13, 14].

Thereby we are coming to consideration of only two pairs of diagrams in the cases of both reaction. Under the assumption that they occur at high energies, for their description

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1 The diagrams of the middle-row pair are reduced to explicitly planar form by flipping over on 180° of the upper parts of diagrams. This gives topologically equivalent representation of the same diagrams [11].
the approach of the parton model is applicable. In relation to the GAMS-4p data \cite{7} we consider this is justified as the data were collected at $\sqrt{s} \approx 8$ GeV. Taking into account the actual distributions of valence quarks in mesons and in proton \cite{15, 16} (at close energy scale) this in average corresponds to $\sqrt{\hat{s}} \approx 2.2$ GeV for the hard subprocesses, which is sufficient for application of the parton model.

Further in this analysis we will be interested in phenomenon of mixing of states only. This prescribes us to consider the processes in the limit of zero transfers. Thereby we escape contributions due to exchanges in the $t$-channel and thus get opportunity for meaningful description of relative quark content in final states. Notice, at high energies additionally the mass differences and kinematic effects are irrelevant. Earlier the mixing phenomenon on the basis of charge-exchange reactions was usually studied in the limit of zero transfers, as well \cite{7, 8}.

Concluding this section we note that subprocesses due to exchanges by soft gluons, in particular in the $t$-channel, give significant contributions, too. Actually they are even stronger than the hard subprocesses, and it is impossible to describe them by perturbative methods. However, they do not affect the formation of flavor in the final state. We will take into account such contribution in the spirit of factorization hypothesis, i.e. we will assume that the soft contributions manifest themselves as a background on which the fast hard-scattering subprocesses evolve. In other words, we will describe the cross-section as the product of the hard-scattering contributions and a soft-interaction constant. At the expense of the “soft” constant, we will refer also contributions arising at small momentum fractions of the colliding particles, occurring at small $\hat{s}$ or $\hat{u}$. Such contributions by its nature are also nonperturbative and parton model is inapplicable for their description. Nevertheless, the contributions referred to the “soft” constant must obey isotopic invariance. At high energies one could expect also independence of the “soft” constant from the quark content of isosinglet mesons in the final state. These properties are sufficient to determine the mixing of states, if contributions of the hard subprocesses are taken into account properly.

3 Analysis of partonic contributions

Now we proceed to the analysis of the hard-scattering subprocesses. In the cases of scattering of the valence and sea quarks the corresponding diagrams are presented by Fig. 2 and Fig. 3, respectively. At first we note that at zero transfers the quarks in the final state are distributed in momenta exactly as the relevant quarks in the initial state are distributed. Really, since partons are massless, in the c.m.s. of colliding partons their momenta after the collision are equal to the momenta before the collision. Clearly, the same is true in the c.m.s. of real particles. Then, repeating the thought “experiment” for measuring the momenta of scattered partons, we conclude that the relevant partons in the initial and final states have identical distributions. So if a valence quark is scattered, then the appropriate quark in the final state can be considered as the valence quark, again. Correspondingly, a sea quark turns into a sea quark. The parton distributions in spectator are unchanged, too. In the case of valence quark scattering this means that the resulting beam of partons is equivalent to that of a real meson. Therefore this parton beam should transform into an observable state of isosinglet meson. The probability of this event is determined by the proportion in which in
Figure 2: Diagrams for hard subprocesses in the cases of scattering of valence quarks. The shaded areas symbolize spectators.

Figure 3: Diagrams for the hard subprocesses in the cases of scattering of sea quarks. The shaded areas symbolize spectators. The sea quarks are indicated by slanted letters.
this meson a given set of valence quarks is presented.

In the case of scattering of sea quark situation is drastically different. Really, as follows
from Fig. 3 the sea quark in the final state appears with a flavor that is not compensated
by flavors of other sea quarks. So the partons in the final state are incorrectly distributed,
not as required in one-particle state. This means that in the result of hard-scattering some
intermediate excitation arises. In the course of the final hadronization it must decay to
multiparticle state. This means that the scattering of sea quarks do not contribute to the
reactions under consideration. Thus we come to the conclusion that in the limit of zero
transfers the charge-exchange reactions can be described in the valence approximation, in
which the presence of sea quarks is ignored. Further we exclude subprocesses with sea quarks
from consideration.

The above observation allows us to estimate relative contributions of the hard-scattering
subprocesses. For this purpose we calculate amplitudes squared for subprocesses of Fig. 2.
(Notice, they do not interfere with each other.) In doing so we keep in mind that the
presence of spectators imply certain conditions at calculating the amplitudes. Namely, the
color indices of quarks must coincide pairwise in the cases of mesons and baryons, and in
the case of mesons the helicity of the quark in the final state must coincide with the helicity
of corresponding quark in the initial state, in order to compensate helicity of the spectator.
Given these conditions, direct calculations lead to the following results:

$$|M_{QA}|^2 = N \times \hat{u}^2 / \hat{s}^2 , \quad |M_{QE}|^2 = N \times \hat{s}^2 / \hat{u}^2 .$$

Here $M_{QA}$ and $M_{QE}$ are the amplitudes for diagrams with quark annihilation and with quark
exchange, respectively, $\hat{s}$ and $\hat{u}$ are Mandelstam variables for subprocesses, $N$ in both cases
is the one and the same constant proportional to $\alpha_s^2$. At $\hat{t} = 0$ we have $\hat{u} = -\hat{s}$. So in the
limit of zero transfer the amplitudes squared in (3) coincide each other. Consequently the
contributions of the hard-scattering subprocesses to the cross-section of the whole of process
coincide, as well.

4 The $\eta - \eta'$ mixing

The above result is the key to further analysis. Based on it and in view of Section 2, we
immediately conclude that at high energies and zero transfers the following relations take
place:

$$\sigma(\pi^- p \to \eta_N n) = \sigma_\pi , \quad \sigma(\pi^- p \to \eta_S n) = 0 , \quad \sigma(K^- p \to \eta_S \Lambda) = \sigma(K^- p \to \eta_N \Lambda) = \sigma_K .$$

Hereinafter we mean $d\sigma/dt|_{t=0}$ under the symbol $\sigma$.

Further, we consider the simplest scheme for $\eta - \eta'$ mixing, which is based on assumption
of completeness of two states. Namely, we assume that the states $|\eta_N >$ and $|\eta_S >$ in (2) can
be considered as superpositions of only two observables states of $\eta$ and $\eta'$. From (4) with
taking into account (1) and (2), we get

$$\sigma(\pi^- p \to \eta' n) = \sigma_\pi \sin^2 \phi_P , \quad \sigma(\pi^- p \to \eta n) = \sigma_\pi \cos^2 \phi_P .$$
The ratio of (6) and (7) is

$$R_{\eta'}/\eta = \tan^2 \phi_p . \quad (8)$$

Analogically, with taking into account factor $1/\sqrt{2}$ in (2), from relations (5) we get

$$\sigma(K^-p \rightarrow \eta'^{'}\Lambda) = \sigma_K \left( \frac{1}{2} \sin^2 \phi_p + \cos^2 \phi_p \right), \quad (9)$$

$$\sigma(K^-p \rightarrow \eta n) = \sigma_K \left( \frac{1}{2} \cos^2 \phi_p + \sin^2 \phi_p \right). \quad (10)$$

The corresponding ratio of the cross-sections is

$$R_{\eta'}/\eta = \frac{1}{2} \tan^2 \phi_p + 1 = \frac{1}{2} + \tan^2 \phi_p . \quad (11)$$

It is worth mentioning that relation (11) is also valid in the cases of reactions $K^-p \rightarrow \eta'(\eta)\Sigma^0$ and $K^-n \rightarrow \eta'(\eta)\Sigma^-$. Now we proceed to the application of our results to the GAMS-4\pi data. We use the following values for the differential cross-section ratios at $t = 0$ [7]:

$$R_{\eta'}/\eta = 0.54 \pm 0.04, \quad (12)$$

$$R_{K}/\eta = 1.27 \pm 0.15. \quad (13)$$

On the basis of (8) and (12), we derive

$$\phi_p = (36.3 \pm 1.0)^0. \quad (14)$$

On the basis of (9) and (13), we get

$$\phi_p = (34.6 \pm 5.6)^0. \quad (15)$$

In the octet-singlet representation this corresponds to $\theta_p = -18.4^0$ and $\theta_p = -20.2^0$, respectively. Recall that the octet-singlet basis is connected with the nonstrange-strange basis by means of rotation on the ideal mixing angle, $\theta_p = \phi_p - \theta_i, \theta_i = \arctan\sqrt{2} (\theta_i \approx 54.7^0)$.

Comparing (14) with (15), we see that the values for the mixing angle obtained on the basis of independent data for $\pi^-$ and $K^-$ beams, excellently coincide with each other. This means that the description of the mixing of $\eta$ and $\eta'$ does not require an introduction of any states that do not fit into the scheme of basic states $|\eta_N>$ and $|\eta_S>$. The average value of the mixing angle, we determine by the average value of the tangent. From (12), (13), (8), (11), we get $\tan \phi_p = 0.73 \pm 0.03$. So the resultant angle is

$$\phi_p = (36.1 \pm 0.9)^0. \quad (16)$$

This corresponds to $\theta_p = (-18.7 \pm 0.9)^0$ in the octet-singlet representation. The latter result can be compared with that of NICE experiment $\theta_p = (-18.2 \pm 1.4)^0$ [8].
5 Conclusion

In this paper an approach is proposed to describe processes of charge-exchanges of \(\pi^-\) and \(K^-\) on protons at high energies in the limit of zero transfers, based on the conceptions of the parton model. Specificity of application of the parton model is that partons after the hard scattering do not leave independently the interaction region, but join the spectator. In fact, this behavior of partons is determined by kinematic conditions under which the processes are considered. However, on the other hand, effectively this implies some restrictions imposed on hard subprocesses. Namely, the scattering of only valence quark leads to production of observable one-particle states, while scattering of sea quark leads to disruption of the final state and ultimately formation of multiparticle state. In essence this means validity of the valence approximation for the description of processes under consideration. On the basis of this property, we have shown equality of contributions with annihilation and exchange of quarks in the hard-scattering subprocesses. This fact was omitted in the previous analysis [7], and contributions of the latter type were not taken into consideration. However such contributions are important in the case of scattering of \(K^-\), as they open an additional channel for production of \(\eta\) and \(\eta'\) in the final state. Finally, on the base of GAMS-4\(\pi\) data [7], we have shown that the description of mixing of \(\eta\) and \(\eta'\) does not require an introduction of any states that do not fit into the scheme of basis states \(|\eta_N>\) and \(|\eta_S>\). The obtained value of the mixing angle is \(\phi_p = (36.1 \pm 0.9)^0\).

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