D-Branes in Linear Dilaton Backgrounds

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Abstract

We construct a Dirichlet boundary state for linear dilaton backgrounds. The state is conformally invariant and satisfies Cardy’s conditions. We apply this construction to two dimensional string theory.

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1 Introduction

The study of Dirichlet branes [1] has played an essential role in the understanding of non-perturbative physics of various string vacua. In the strong coupling regime the D-branes become the lightest BPS states, and therefore play the role of the fundamental degrees of freedom in a dual formulation. D-instantons, similarly, are important in giving non-perturbative contributions to scattering amplitudes.

Given this success, one is motivated to study D-branes in string vacua with less symmetries, and sometimes with no clear geometrical interpretation [2]. Some of these string vacua are holographically dual (in the sense of [3]) to non-gravitational theories, and the D-brane spectrum and interactions are important in the dynamics of those theories.

The study of D-brane in general backgrounds is difficult, and a general method of construction is unknown. We review the boundary state approach to the subject in section 2. In particular we review a construction of boundary states due to Cohen, Moore, Nelson and Polchinski [4]. This construction can be useful in conformal field theories where the spectrum has a simple structure.

The examples studied in this paper are conformal field theories with a linear dilaton component. We review such backgrounds in section 3, and construct a Dirichlet boundary state for such backgrounds. (We note that Dirichlet boundary states in linear dilaton background have also been considered in [5]. The boundary state constructed there appears to be conformally invariant, but does not satisfy Cardy’s conditions, defined below.)

In section 4 we demonstrate the construction by explicitly constructing D0-branes and D-instantons for the case of 2d string theory [6]. Those objects are relevant in studying non-perturbative questions in 2d string theory. We describe further applications and directions for future research in the final section. An appendix contains a generalization of the boundary state to incorporate worldsheet supersymmetry.

2 General D-branes

The general construction of a D-brane in an arbitrary string background is not known. In this section we outline an approach to this problem based on
In the process of doing so, we review some well-known facts about open string theories, and introduce our notation.

To make the presentation simpler, we concentrate on the matter part of the CFT. One may choose to work in light-cone variables as in [1], or add a ghost sector and perform a BRST quantization [5]. In a conventional string background, where the ghosts decouple from the matter CFT, their treatment does not depend on the particular background chosen.

The simplest way to construct an open string theory is to impose boundary conditions on the string variables and perform a canonical quantization in the open string sector. The procedure requires a Lagrangian description of the CFT, and is hard to implement in general string backgrounds.

We instead work in the boundary state formalism [5]. In this formalism, we are given a ‘bulk’ CFT, that is, a closed string background. This bulk CFT is specified by a set of bulk operators and their OPEs. These OPE coefficients are required to satisfy nonlinear constraints: associativity and modular invariance on the torus. These conditions guarantee consistency of the theory formulated on an arbitrary Riemann surface.

Given the bulk CFT, we define a D-object to be an open string theory consistent with the bulk CFT. The building blocks of an open CFT, and the conditions they satisfy, were analysed in [9]. A complete specification of an open string theory involves the following elements:

- A boundary state \(|B\rangle\) which is a generalized coherent state in the closed string Hilbert space. To preserve the conformal invariance, essential for having a consistent string theory, one requires:

\[
L_n - \bar{L}_{-n} |B\rangle = 0 \quad (1)
\]

The state \(|B\rangle\) specifies the tree level couplings of the D-object to all closed string fields.

- A set of open string operators (also called boundary operators, or more precisely “boundary condition preserving boundary operators”.)

- Closed string OPEs, open string OPEs, and open-closed OPEs.

These building blocks have to satisfy nonlinear relations [9], the sewing constraints. These conditions guarantee the consistency of the theory on an arbitrary (oriented) Riemann surface with boundaries [9].

In the following, we study boundary states in linear dilaton backgrounds. We emphasize that in principle, the boundary state alone does not specify
the open string theory. One has to supply an open string operator algebra which satisfies the sewing constraints. The existence of such an operator algebra might impose further conditions on the possible boundary states. Furthermore, such an operator algebra might not be unique, in which case distinct D-objects share the same boundary state.

The unique constraint involving only the boundary state is Cardy’s condition \[10\], introduced below.

Our starting point is a solution of the conformal invariance condition (1) based on any Virasoro module in the theory. Suppose we are given a level matched primary of the Virasoro algebra of dimensions \(h_L = h_R = h\). Define

\[
|B\rangle_h = \sum_{I,J} M^{-1}_{IJ} L_{-I} \bar{L}_{-J} |h\rangle
\]  

(2)

Here \(I, J\) are ordered strings of indices \(n_1 \cdots n_r\), and

\[
L_I = L_{n_1} \cdots L_{n_r}.
\]  

(3)

The contravariant form is defined as

\[
M_{IJ} = \langle h| L_I L_{-J} |h\rangle = \langle h| \bar{L}_I \bar{L}_{-J} |h\rangle
\]  

(4)

\(M_{IJ}\) is invertible for any Virasoro module. For degenerate modules, one has to mod out by the null vectors.

It is easy to see that \(|B\rangle_h\) is conformally invariant by showing that \((L_n - \bar{L}_{-n})|B\rangle_h\) is orthogonal to all states in the module based on \(|h\rangle\). Furthermore, by an application of Schur’s lemma, the solution is unique in each Virasoro module \([11]\), \([12]\). A similar construction can be given for any chiral algebra, yielding a boundary state preserving one copy of that chiral algebra \([13]\). In particular, we extend the discussion to include worldsheet supersymmetry in appendix A.

The boundary states \(|B\rangle_h\) are the building blocks of any physical boundary state \(|B\rangle\). The physical boundary state is required to satisfy Cardy’s conditions, which guarantee the existence of open string quantization of the system. We regard this condition as an effective way of finding a basis of the physical boundary states in the theory.

Cardy’s conditions on the boundary state are obtained as follows. The partition function on the annulus can be computed in the closed string sector as

\[
Z_B = \langle B| e^{-2\pi i (L_0 + \bar{L}_0 - \frac{c}{12})} |B\rangle = \langle B| e^{\bar{q} \left(L_0 + \bar{L}_0 - \frac{c}{12}\right)} |B\rangle
\]  

(5)
where $l$ is the closed string modulus of the annulus, and $\tilde{q} = e^{-2\pi t}$.

This partition function can be written in terms of the open string modulus, $q = e^{-\pi t}$ (with $t = \frac{1}{2l}$ being the open string modulus). As a power series in $q$, one can read from $Z_B$ the dimensions and multiplicities of the open string operators. As such, all coefficients in the series must be nonnegative integers. The coefficient of the unit operator must be one (up to an overall multiplicative factor in the boundary state, interpreted as the number of D-branes.)

Finally, in the case of noncompact bosons, one has to allow a slight generalization. Associated with a noncompact boson one has an integration over continuous momentum in either the open string sector or the closed string sector. Such integrations lead to factors which are nonanalytic in either $q$ or $\tilde{q}$. We generalize Cardy’s conditions to allow such factors, since their origin is clear for noncompact bosons. In fact such factors can be helpful in interpreting the boundary state. For a free boson, a Dirichlet boundary state allows an arbitrary momentum in the closed string sector, while in the Neumann boundary state the open strings can carry arbitrary momentum. We generalize this to any noncompact boson, by defining a “Dirichlet” boundary state as having no free momentum for the open strings, corresponding intuitively to open strings with fixed endpoints.

3 Linear Dilaton Backgrounds

Conformal field theories with a linear dilaton appear as an ingredient in many string backgrounds, critical and non-critical. For example, they appear in the NS 5-brane theory \[14\], they provide an explicit Lagrangian formulation of Virasoro minimal models \[15\] and WZW models \[13\], and are an essential part of 2d string theory \[6\]. The construction of the Dirichlet boundary state should apply to all such backgrounds.

We demonstrate the construction for the case of the 2d string theory. The worldsheet theory is Liouville theory coupled to a single boson $X$. The worldsheet action is then

$$L = \partial X \bar{\partial} X + \partial \phi \bar{\partial} \phi + \mu e^{\gamma \phi} + Q \phi R \quad (6)$$

with $Q = 2\sqrt{2}, \gamma = \sqrt{2}$ chosen to define a critical string theory.
The treatment of the free boson is standard, and we focus on the field $\phi$. The states in Liouville theory form a continuum $|p\rangle$, $p \geq 0$ with dimensions $h_p = \frac{1}{2}p^2 + \frac{1}{8}Q^2$. In addition, there is a discrete set of states, the special states, which form a set of measure zero. The states above do not correspond to local operators, and as normalizable modes are the appropriate ones to form a coherent state in the closed string Hilbert space.

For generic $p$, the Virasoro module based on $|p\rangle$ is nondegenerate. So it is straightforward to compute the annulus diagram for the boundary state $|B\rangle_p$ (defined as in eq. (5))

$$p\langle B|e^{-2\pi i(L_0 + \tilde{L}_0 - \frac{c}{12})}|B\rangle_p = \tilde{q}^{p^2} \prod_n (1 - \tilde{q}^{2n})\tilde{q}^{-\frac{1}{24}}$$

The result is identical to the corresponding calculation for a single free boson. One can construct then

$$|B\rangle_D \equiv \sum_p e^{-ipX_0}|B\rangle_p.$$  

To check Cardy’s conditions we compute the annulus diagram with the boundary state $|B\rangle_D$. After modular transformation, we find

$$D\langle B|e^{-2\pi i(L_0 + \tilde{L}_0 - \frac{c}{12})}|B\rangle_D = \prod_n (1 - q^{2n})q^{-\frac{1}{24}}$$

where $q$ is the open string modulus defined above.

Therefore the boundary state $|B\rangle_D$ satisfies Cardy’s conditions. There are no logarithmic factors in the open string channel, as appropriate for a Dirichlet boundary state.

We note that the above boundary state has a similar form to the boundary state of a single free boson. This resemblance is misleading. The Liouville theory is not exactly solvable, and a complete specification of the open string theory would be different from that of D-branes in flat space. However, the boundary state itself is sensitive only to the spectrum, therefore it has a simple form when expressed in terms of Virasoro modules. When expanded in modes of $\phi$, the state looks very different from the usual Dirichlet boundary state in flat space.

We mention in passing that Neumann boundary conditions in linear dilaton backgrounds were discussed in [16].

5
4 D-branes in 2d String Theory

We now turn to the case of 2d string theory, for which we need to include the free boson $X$. For a free boson there are standard boundary states

$$\langle B_N \rangle^{(X)} = \exp \left( - \sum_n \frac{1}{n} \alpha_n \bar{\alpha}_n \right) |\text{vac}\rangle$$ (10)

$$\langle B_D \rangle^{(X)} = \exp \left( \sum_n \frac{1}{n} \alpha_n \bar{\alpha}_n \right) |\text{vac}\rangle$$ (11)

corresponding to Neumann and Dirichlet boundary conditions respectively. Here $\alpha, \bar{\alpha}$ are the left- and right-moving modes of $X$.

We find then the boundary states of 2d string theory by tensoring the state found in the previous section with either $\langle B_N \rangle^{(X)}$ or $\langle B_D \rangle^{(X)}$, yielding a D0-brane or a D-instanton respectively.

The lowest lying modes in the open string sector are the open string tachyon and the center-of-mass operator. To understand the dynamics of the D-brane, one needs to write an effective action for these fields. This is more difficult than in the flat space case, because we have defined the D-brane as a boundary state instead of as a boundary condition on the open string theory.

In the boundary state formalism, one has to solve the sewing constraints on open string OPEs [9], which are nonlinear and generally difficult to solve. In our case, however, we can solve them in the asymptotic weak coupling region by a comparison to flat space D-branes.

In our construction the boundary state has the same form as a flat space D-brane, except that we have used the primaries of the Liouville theory rather than the standard primaries of flat space. Furthermore, in the weakly coupled region, the interactions of the Liouville primaries are the same (to leading order) as the interactions of the free theory. The solution of the sewing constraints is then the same as in flat space and the action is the standard flat space Born-Infeld action.

In particular, the action for the center of mass is

$$\mathcal{L} = \frac{1}{g_s(X_0)} \sqrt{1 - \dot{X_0}^2}$$ (12)

where $X_0$ was defined in eqn. (8). The equation of motion is that of constant acceleration towards the strong coupling region.
In the full Liouville theory, solving the sewing constraints requires more information. By KPZ scaling [17, 6], the leading order action scales as $\frac{1}{\mu}$ since it is obtained from a disc diagram. Since the mass is no longer position dependent, we expect this to describe a static D-brane bound to the wall.

The D-instanton would correspondingly have an action proportional to $\frac{1}{\mu}$ and therefore would produce nonperturbative effects in the spacetime theory. These correspond to the famous $e^{-\frac{1}{\mu}}$ effects found in the matrix model [18]. To perform a detailed comparison, though, one needs to find special quantities for which the perturbative series can be summed.

Consideration of black holes in 2d string theory [19] suggests that Liouville theory has many more states than suggested by the perturbative spectrum. The entropy of black holes suggests in fact a Hagedorn density of states [20], whereas the perturbative spectrum consists only of one 1+1 dimensional field (and some special states). The success of entropy counting in higher dimensions [21] suggests that D-branes and their excitations might provide the required states.

5 Applications

Since linear dilaton backgrounds are ubiquitous in string theory, the boundary state found here has many applications. Most applications would require a generalization to incorporate worldsheet supersymmetry and GSO projection. This is straightforward, and the resulting boundary state is constructed in Appendix A.

It has been recently proposed that string theories backgrounds with a linear dilaton component are holographically dual to nongravitational, nonlocal theories [22, 23, 24, 25]. The earliest and best known example is the near horizon limit of the NS fivebranes, which are holographically dual to the little string theories in 6 dimensions with 16 supercharges [26, 27]. Other examples with fewer supersymmetries and in other dimensions were given in [24, 25].

Our construction can be applied to many such backgrounds since they consist of a linear dilaton part and an often solvable conformal field theory. However, many such backgrounds have a strong coupling region where perturbative string theory is inapplicable. In these cases, the description of the D-brane as a boundary state is useful only asymptotically, in the weak
coupling region.

There are however instances where the weakly coupled string theory is applicable everywhere [25]. This corresponds to a deformation of the linear dilaton background analogous to turning on the cosmological constant in Liouville theory. In such cases, our construction provides an effective way of describing the D-brane spectrum and interactions. These D-branes correspond to nonperturbative states in the holographically dual theory.

In particular, some of the backgrounds in [25] are T-dual to noncompact Calabi-Yau spaces with a slightly deformed singularity [28]. In such cases, the D-brane spectrum might be more transparent in the linear dilaton description of the background.

As mentioned already in section 4, D-branes and D-instantons can shed light on nonperturbative effects in 2d string theory. In particular, counting the microscopic states of black holes in 2d string theory requires an understanding of the quantum mechanics of the D-branes in this background. In addition, $e^{-1/g}$ effects in the matrix quantum mechanics should be related to D-instanton effects in the Liouville theory.

Finally, one can speculate on a holographic relation between the c=1 matrix model and Liouville theory, along the lines of [3]. The world-volume theory on a large number $N$ of D0 branes in a linear dilaton background is a quantum mechanics of $N \times N$ matrices. On the spacetime side, the D0 branes act as a source for the tachyon, and hence the spacetime background around $N$ D0 branes is a linear dilaton background with a nonzero cosmological constant. The analogue of the AdS/CFT conjecture would then state that string theory in this background is holographically described by the matrix quantum mechanics, which is the c=1 matrix model/Liouville theory correspondence. To make this precise, one needs a better understanding of the world-volume action of the D0-branes and the role of the double scaling limit in the spacetime geometry.

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7 Appendix

In section 2, we reviewed a construction of a boundary state preserving conformal invariance and based on a single conformal primary. In order to incorporate worldsheet supersymmetry we need to provide a slight generalization of that construction.

Suppose we are given isomorphic left and right chiral algebras, and left and right primaries $|\Phi_L\rangle, |\Phi_R\rangle$, such that:

$$\Omega|\Phi_L\rangle = |\Phi_R\rangle$$  \hspace{1cm} (13)

where $\Omega$ is the isomorphism between the chiral algebras. Based on the module of $|\Phi_L\rangle \times |\Phi_R\rangle$, one can build a boundary state satisfying

$$(T_\Omega - \tilde{T})|B\rangle = 0$$  \hspace{1cm} (14)

where $T, \tilde{T}$ are any generators of the chiral algebra, and $T_\Omega$ is the image of $T$ under the action of $\Omega$.

The construction is identical to eq (2).

$$|B\rangle = \sum_{I,J} M^{-1}_{IJ} L_{-I} \tilde{L}_{-J} |h\rangle$$  \hspace{1cm} (15)

where here $L_{-I}, \tilde{L}_{-J}$ are products of modes of $T_\Omega, \tilde{T}$ respectively. $M_{IJ}$ is the contravariant form, which is identical for the left and right moving algebras since they are isomorphic.

In the case of $N = 1$ supersymmetry on the worldsheet, one has to consider the supersymmetric extension of the conformal algebra: the NS algebra or the R-algebra. In the NSNS and the RR sector one can apply the above construction and build a boundary state based on each superconformal primary.\footnote{This can work in the R-NS sector only if the NS and R algebras are isomorphic, which is the case when the theory is spacetime supersymmetric, and therefore has a N=2 algebra, and a spectral flow operator which acts as an isomorphism of the NS and R sectors.}
The boundary states $|B\rangle_{NSNS}$ and $|B\rangle_{RR}$ have to be combined in a manner ensuring a tachyon free open string sector. This is equivalent to imposing a GSO projection in the open string sector. Prior to that, one has to apply a GSO projection to the boundary state (i.e. in the closed string channel). We now describe how to achieve that goal. For simplicity we concentrate on the NS-NS sector.

The $N = 1$ superconformal algebra has an automorphism which reverses the sign of the supercharges, leaving the bosonic generators intact. This automorphism is used to perform the GSO projection.

In order to construct a GSO even boundary state one can build boundary states $|B\rangle_\eta$ satisfying

$$\langle L_n - \tilde{L}_{-n} |B\rangle_\eta = 0$$

$$\langle G_r - \eta \tilde{G}_{-r} |B\rangle_\eta = 0$$

where $\eta = \pm 1$. The action of the GSO projection on $|B\rangle_\eta$ is then given by

$$(-)^{FL} |B\rangle_\eta = S_L |B\rangle_{-\eta}$$

$$(-)^{FR} |B\rangle_\eta = S_R |B\rangle_{-\eta}$$

where $S_L, S_R$ are the transformation properties of the primaries. The primary has to be GSO even to be physical, therefore we restrict to $S_L = S_R = 1$. The GSO invariant boundary state is then:

$$|B\rangle = |B\rangle_\eta - |B\rangle_{-\eta}$$

This boundary state couples then only to physical (GSO-even) states.
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