Velocity Statistics Distinguish Quantum Turbulence from Classical Turbulence

M. S. Paoletti,1 Michael E. Fisher,2 K. R. Sreenivasan,2,3 and D. P. Lathrop1,2

1Departments of Physics, Geology, Institute for Research in Electronics and Applied Physics, and
2Institute for Physical Sciences and Technology, University of Maryland, College Park, MD 20742
3International Centre for Theoretical Physics, Trieste, Italy 3414

By analyzing trajectories of solid hydrogen tracers, we find that the distributions of velocity in decaying quantum turbulence in superfluid 4He are strongly non-Gaussian with 1/ν3 power-law tails. These features differ from the near-Gaussian statistics of homogenous and isotropic turbulence of classical fluids. We examine the dynamics of many events of reconnection between quantized vortices and show by simple scaling arguments that they produce the observed power-law tails.

The pioneering work of Kolmogorov 1, 2 remains the cornerstone of the statistical theory of classical incompressible turbulence. Kolmogorov made two key assumptions: (1) local isotropy and homogeneity prevail, and (2) there exists an inertial range in which turbulent energy is transferred from large to small scales independent of viscosity and generation mechanisms. Dimensional arguments then yield the spectral density \( \tilde{E}(k) = \epsilon k^{−5/3} \), where \( \epsilon \) is the average energy dissipation rate per unit mass and \( \kappa \) is the universal Kolmogorov constant. While experiments have found the effects of intermittency to be important for high-order moments, the correction to the spectral form is quite small. Furthermore, in both experiment 3, 4 and direct numerical simulations 5, 6 the velocity in homogenous and isotropic turbulence is found to exhibit near-Gaussian statistics.

Quantum fluids, however, are typically described as a mixture of two interpenetrating fluids 6, 7, a viscous normal fluid and an inviscid superfluid exhibiting long-range quantum order. There is no conventional viscous dissipation in the superfluid component and vorticity is confined to one-dimensional quantized vortices which possess circulation values that are integer multiples of \( \hbar / m \). Thus, quantum turbulence takes the form of a complex tangle of atomically-thin vortex filaments of quantized strength 7. Dissipation in the superfluid component for 1.70 K < T < 2.05 K is mainly produced by mutual friction 8 between the quantized vortices and normal fluid.

Despite these fundamental differences, there have been notable studies demonstrating similarities between quantum and classical turbulence 9, 10, 11, 12, 13, 14, 15, 16, 17, 18. Even though the quality of the supporting evidence has been questioned 19, it will be summarized here. Experiments on turbulence generated in 4He by two counter-rotating disks observed Kolmogorov energy spectra that were indistinguishable above and below the superfluid transition 14. The Kolmogorov energy spectrum was seen in numerical simulations of the Gross-Pitaevskii equation with small-scale dissipation added to the otherwise energy-conserving dynamics 15. The classical decay of vorticity 10 has been observed in towed grid 10, 16, thermal counterflow 17, and impulsive spin down 18 experiments. In all these studies the flow scales observed were considerably larger than typical intervortex spacings. These results may be attributed to the fact that on such scales the pairwise interactions of quantized vortices are insignificant while the normal and superfluid components become “locked” as a result of mutual friction.

In this Letter, we study the velocity statistics of quantum turbulence generated by a thermal counterflow on length scales between our experimental resolution (∼1 mm) and the typical intervortex spacing (∼0.1-1 mm) 17. On such length scales the interactions of individual quantized vortices are important. Specifically, quantized vortex reconnection 20, 21, 22, 23, 24, where two vortices merge at a point, change topology by exchanging parts, and separate (Fig. 1 and 25), produces high, atypical velocities. By analyzing the trajectories of micron-sized solid hydrogen tracers, we may compute both the velocity statistics of quantum turbulence, and identify and assess the effects of individual reconnection events. Previous studies have shown that hydrogen tracers can be trapped on quantized vortices or, if not near a vortex, move with the normal fluid under the influence of Stokes drag 26, 27, 28.

Our experiments are conducted in a cylindrical cryostat of 4.5 cm diameter using liquid 4He. The long axis of the channel is vertical with four 1.5 cm windows separated by 90°. Particles are produced by injecting a mix-
We characterize the resulting dynamics by analyzing the particle trajectories. Time-varying distributions of the horizontal and vertical velocity components, $v_x$ and $v_z$, computed by forward differences, are shown in Fig. 2 for a typical thermal pulse. The $v_x$ distributions are always peaked near zero. However, as predicted by the two-fluid model, the $v_z$ distributions exhibit a different behavior, since entropy injected by the heater is carried upward ($v_z > 0$) by the motion of the normal fluid. To conserve mass, the superfluid component moves downward opposing the normal fluid motion. The bimodal $v_z$ distributions when the heater is on represent particles with $v_z > 0$ moving upward with the normal fluid owing to Stokes drag while particles with $v_z < 0$ are trapped in the vortex tangle that moves downward. Once the heat pulse ends, the vertical velocities collapse to distributions peaked near zero. The probability distribution functions (pdf) of velocity components derived from all trajectories for times after the heat pulse is turned off ($t > t_{off}$) in Fig. 2 are shown in Fig. 3(a). We focus on the tails of these distributions, which are composed of trajectories with high, atypical velocities; these we attribute to quantized vortex reconnection as explained below.

Near the reconnection moment, quantized vortices move with velocities much higher than the background flow. These reconnections have been experimentally visualized using hydrogen particles and studied numerically and analytically. As discussed in some of these works, the minimum separation distance between reconnecting quantized vortices $\delta(t)$ (Fig. 1) evolves approximately as a square-root in time, i.e. $\delta(t) = A \sqrt{|t - t_0|}$, where $t_0$ is the reconnection moment and $A$ is a dimensionless factor of order unity. Thus, for lengths between the vortex core radius and the typical intervortex spacing we expect the velocity to scale as

$$v(t) \propto |t - t_0|^{-1/2},$$

which grows much larger than typical fluid velocities when $t \to t_0$ (although cut off by the speed of sound).

In the pulsed counterflow experiments, a reconnection event is evidenced by a pair of nearby tracers rapidly approaching or separating. Given the large number of possible particle pairs analyzed ($\sim 10^{10}$) an ad hoc criterion is needed to select likely reconnection events. We define a pair of particles $i$ and $j$ as marking reconnection at time $t$ if the separation $\delta_{ij}(t) = |\mathbf{r}_i(t) - \mathbf{r}_j(t)|$ satisfies

$$\delta_{ij}(t \pm 0.25 s)/\delta_{ij}(t) > 4,$$

where $\mathbf{r}_i(t)$ is the two-dimensional projection of the position of particle $i$ at time $t$ and the plus (minus) sign denotes particles that separated after (approached before) an event, which we label as forward (reversed) events. The duration of 0.25 s is chosen since greater times are dominated by boundary effects and the presence of other vortices. The criterion excludes all but a fraction of possible pairs leaving $\sim 4 \times 10^4$ reconnection events.

The measured separations $\delta(t)$ for four typical forward events are shown in Fig. 4(a). Solid line fits invoking a correction factor to the predicted scaling as

$$\delta^{fit}(t) = A[\kappa(t - t_0)]^{1/2}[1 + c(t - t_0)],$$

FIG. 2: Time-varying pulsed counterflow velocity distributions at $T = 1.90$ K showing (a) $v_x$ and (b) $v_z$ for a portion of a heat pulse of 0.17 W/cm$^2$ with the heater turned off at $t = t_{off}$. White denotes amplitudes with zero probability.
describe the data well [31]. The fits minimize \( \chi^2 = n^{-1} \sum_{i=1}^{n} [(\delta_i - \bar{\delta})/\sigma]^2 \), where \( \delta \) denotes the movie frame, \( \sigma = 4 \mu \text{m} (0.25 \text{ pixels}) \) is an estimate of the uncertainty of the particle positions, and \( n = 15, 20, 25 \) for data collected at 60, 80, or 100 frames per second, respectively. To fit reversed events, we use the same form [32] with \((t - t_0)\) replaced by \((t_0 - t)\). The distributions of \( A \) and \( c \) for forward (reversed) events, determined from fifty heat pulses, are shown in black (red) in Figs. 4(b) and (c). These are calculated only from events with \( \chi^2 < 4 \); about 50% of the pairs satisfying [32] meet this \( \chi^2 \) criterion.

To model the pdf of the velocity derived from particle trajectories, we may use the transformation

\[
Pr_v(v) = Pr_t[t(v)]|dt/dv|,
\]

(4)

where \( Pr_v(v) \) is the probability of observing a velocity between \( v \) and \( v + dv \) at any time while \( Pr_t(t)dt \) is the uniform probability of taking a measurement at a time between \( t \) and \( t + dt \). Hence, accepting the scaling relation [1], we predict for large \( v \) (small \( t \)) the behavior

\[
Pr_v(v) \propto |dt/dv| \propto |v|^{-3}.
\]

(5)

The \( v_x \) and \( v_z \) pdfs derived from all particle trajectories for \( t > t_{off} \) for the same pulse in Fig. 2 are shown in Fig. 3(a). The solid lines are fits to [32] allowing for
a mean flow. To emphasize the distinction with classical turbulence, a velocity pdf from an oscillating-grid experiment in water \[32\] is also shown. Evidently the velocity pdfs in superfluid helium differ drastically from the near-Gaussian velocity pdfs observed experimentally \[33\], and in direct numerical simulations of homogenous and isotropic classical turbulence \[34\]. One must note, however, that tracer particles in superfluid helium respond separately to the normal fluid and the quantized vortices (which are influenced by the normal fluid and superfluid); this is fundamentally different from that in water.

By the same argument, the tails of the pdf for the kinetic energy per unit mass \(E = (v_x^2 + v_z^2)/2\) will be dominated by reconnections. Accepting the relation \[1\], we have \(E(t) \propto |t - t_0|^{-1}\) and so for large \(E\) we expect

\[
\Pr(E) \propto |dt/dE| \propto E^{-2}.
\]

The pdf of \(E\) computed from the data in Fig. 3(a) (which includes all particle trajectories) is shown in Fig. 3(b). The departure from the predicted power-law behavior for low velocities and energies may reasonably be attributed to effects from the boundaries and nearby vortices as well as to the background drift of the normal fluid.

In conclusion we have shown that the velocity statistics of quantum turbulence in superfluid \(^4\)He differ drastically from those for classical turbulence owing to the topological interactions of vortices that are different from those in classical fluids. Previous studies argued that the interactions of magnetic field lines can cause the velocity statistics of magnetohydrodynamic (MHD) turbulence also to differ from classical turbulence. The power-law tails in the distributions of electron energies observed in astrophysical settings (Fig. 3 in \[35\] and Fig. 2 in \[36\]) have been attributed to magnetic reconnection \[35\]. Furthermore, theories for MHD turbulence propose that fractional diffusion may be the dominant transport mechanism \[37\]. Such diffusion is associated with power-law tails in velocity distribution functions. Since reconnection is a principal dissipative mechanism in superfluids near absolute zero, superfluid experiments might provide another “laboratory” for studying strong MHD turbulence, as well as other systems exhibiting one-dimensional topological defects \[37\] such as liquid crystals, superconductors, Bose-Einstein condensates, and cosmic strings.

This work was supported by NSF-DMR, NASA, and CNAM at the University of Maryland. We thank Gregory Bewley for past collaboration and Makoto Tsubota, Nigel Goldenfeld, Christopher Lobb, Marc Swisdak, and James Drake for helpful discussions.

* email address: lathrop@umd.edu

[1] A. Kolmogorov, Dokl. Akad. Nauk SSSR 30, 301 (1941).
[2] A. Kolmogorov, Dokl. Akad. Nauk SSSR 31, 538 (1941).
[3] A. Noullez et al., J. Fluid Mech. 339, 287 (1997).
[4] A. Vincent and M. Meneguzzi, J. Fluid Mech. 225, 1 (1991).
[5] T. Gotoh, D. Fukayama, and T. Nakano, Phys. Fluids 14, 1065 (2002).
[6] R. J. Donnelly, Quantized Vortices in Helium II (Cambridge Univ. Press, Cambridge, UK, 1991).
[7] R. P. Feynman, in Progress in Low Temperature Physics, edited by C. J. Gorter (North-Holland, Amsterdam, 1955), vol. 1, pp. 17-53.
[8] W. F. Vinen, Proc. Roy. Soc. A 242, 493 (1957).
[9] D. C. Samuels, Phys. Rev. B 46, 11714 (1992).
[10] M. R. Smith et al., Phys. Rev. Lett. 71, 2583 (1993).
[11] C. F. Barenghi et al., Phys. Fluids 9, 2631 (1997).
[12] C. Nore, M. Abid, and M. E. Brachet, Phys. Rev. Lett. 78, 3896 (1997).
[13] W. F. Vinen, Phys. Rev. B 61, 1410 (2000).
[14] J. Maurer and P. Tabeling, Europhys. Lett. 43, 29 (1998).
[15] M. Kobayashi and M. Tsubota, Phys. Rev. Lett. 94, 065302 (2005).
[16] S. R. Stalp, L. Skrbek, and R. J. Donnelly, Phys. Rev. Lett. 82, 4831 (1999).
[17] L. Skrbek, A. V. Gordeev, and F. Soukup, Phys. Rev. E 67, 047302 (2003).
[18] P. M. Walmsley et al., Phys. Rev. Lett. 99, 265302 (2007).
[19] I. Procaccia and K. R. Sreenivasan, Physica D in press.
[20] K. W. Schwarz, Phys. Rev. B 31, 5782 (1985).
[21] J. Koplik and H. Levine, Phys. Rev. Lett. 71, 1375 (1993).
[22] A. T. A. M. de Waele and R. G. K. M. Aarts, Phys. Rev. Lett. 72, 482 (1994).
[23] S. Nazarenko and R. J. West, J. Low Temp. Phys. 132, 1 (2003).
[24] G. P. Bewley, M. S. Paoletti, K. R. Sreenivasan, and D. P. Lathom, Proc. Natl. Acad. Sci. U.S.A. 105, 13707 (2008).
[25] See EPAPS Document No. for a video of our first observation of reconnecting quantized vortices.
[26] D. R. Poole et al., Phys. Rev. B 71, 064514 (2005).
[27] G. P. Bewley, D. P. Lathom, and K. R. Sreenivasan, Nature 441, 588 (2006).
[28] M. S. Paoletti et al., J. Phys. Soc. Jpn. in press 77, in press (2008).
[29] We thank Eric Weeks and John Crocker for the particle-tracking algorithm.
[30] See EPAPS Document No. for a video showing the experimental dynamics summarized in Figs. 2 and 3.
[31] A model of the form \(\delta(t) = A(t-t_0)^\alpha\) fits the data equally well and will be discussed in a future publication.
[32] B. W. Zeff et al., Nature 421, 146 (2003).
[33] M. Öieroset et al., Phys. Rev. Lett. 89, 195001 (2002).
[34] G. D. Holman et al., Astrophys. J. 595, 197 (2003).
[35] J. F. Drake et al., Nature 443, 553 (2006).
[36] D. del-Castillo-Negrete, B. A. Carreras, and V. E. Lynch, Phys. Plasmas 11, 3854 (2004).
[37] (a) I. Chuang et al., Science 251, 1336 (1991); (b) G. Blatter et al., Rev. Mod. Phys. 66, 1125 (1994); (c) B. M. Caradoc-Davies et al., Phys. Rev. A 62, 011602 R (2000); (d) M. B. Hindmarsh and T. W. B. Kibble, Rep. Prog. Phys. 58, 477 (1995).