Concerning an old (but still quite alive) rebuttal of the theorem of John Bell

Aurélien . Drezet

Institut Néel, CNRS and Université Joseph Fourier, BP 166, 38042 Grenoble, France

Abstract

In a old paper by G. Lochak, it is claimed that the Bell definition of a hidden variable is in conflict with the formalism of quantum mechanics. This result implies that it is not necessary to invoke non locality to explain the violation of the Bell inequality. A careful analysis of the concept of probability for hidden variables, as defined differently by Bell and Lochak, show that the reasoning and main conclusions of [1] are not correct.

Pacs: 03.65.ud
I. INTRODUCTION

It is well known that de Broglie did not accept the interpretation of quantum mechanics given by the Copenhagen school of Bohr and Heisenberg. De Broglie was particularly convinced that a coherent formulation of quantum mechanics should include the description of a dynamical structure at the fundamental level. In this context the problem of the locality and non locality in quantum mechanics in connection with the Einstein Podolsky Rosen paradox represented certainly for him a serious challenge. In 1976 G. Lochak, collaborator of L. de Broglie, published in the journal Foundation of Physics an argument against the J. S. Bell theorem concerning local hidden-variables theories. This refutation was commented briefly by Bell saying that it may have a mistake in the reasoning of Lochak since the derivation in identifies clearly the necessity of non locality in any hidden variables models. In the words of Bell “It may be that Lochak has in mind some other extension of de Broglie’s theory, to the more-than-one-particle system, than the straightforward generalization from 3 to 3N that I considered. But if this extension is local it will not agree with quantum mechanics and if it agrees with quantum mechanics it will not be local ”. In the past and recently Lochak defended his point of view in various articles and a Book. Additionally Lochak explained to me that de Broglie agreed strongly with his reasoning (this is confirmed in ). Because of that, and because of the importance of the de Broglie conceptions in the quest of a self consistent hidden variable theory, I think that it is necessary to reexamine the thought of Lochak and de Broglie at that time. I want to show in this manuscript (written in 2004 but never published) the origin of the problem by pointing out different mistakes in the analysis by George Lochak. Beside its historical interest, the (secret) motivation of this work is, as say to me my mother many times when I was a child, “to clean the room before building up something new”.

II. REFUTATION OF BELL’S THEOREM

The main idea of is that there is in Bell’s reasoning an hidden statistical assumption which is independent of the locality condition. Indeed, according to Bell the locality condition means that there is an hidden variable λ used to calculate all the different measured quantities. Following Bell this hidden variable must be the initial parameter(s) defining the
complete motion of the system, i.e. considered a long time before that the measurement occurs. Lochak questions this last part of the Bell definition. Can we really use such parameters when calculating experimental quantities? Lochak answers by the negative and his argumentation runs as follow:

Suppose an atomic wave packet propagating along the \( z \) axis and directed on a Stern and Gerlach device such as the one represented on Fig. 1. A long time before its interaction with the magnetic field, the initial state is characterized by its wave function that we suppose to be \(|\psi\rangle = \Psi (x, t) [c_1| \uparrow \rangle + c_1| \downarrow \rangle]\). Here we consider only a spin 1/2 and \( \uparrow, \downarrow \) correspond to the two states of the spin along \( x \). If the magnetic field is aligned with \( x \) the atomic wave packet is separated into two parts with amplitudes proportional to \( c_\uparrow \) and \( c_\downarrow \) respectively. Reproducing several times the same experiment shows naturally that the probability of finding the atom in one of the two exits is proportional to \(|c_\uparrow|^2\) and \(|c_\downarrow|^2\) respectively. Similarly changing the direction \( a \) of the magnetic field implies that the numbers of particles detected in the two exits become proportional to one or the other of the two coefficients \(|c_+a|^2\) and \(|c_-a|^2\) calculated in the new basis \( \pm a \). Due to the fact that the spin observable is two valued we will write in the following \( P(\alpha = \pm 1, a) \) the single particle probabilities involved (\( \alpha \) are the eigenvalue of the projection operator \( \hat{A} = \sigma \cdot a \), where \( \sigma_i \) \((i = 1, 2, 3)\) are Pauli’s Matrixes).

Consider now the hidden variable hypothesis as defined by Bell. In the context of deterministic hidden variable theories, the probability of finding an atom in one of the two exit doors of a Stern and Gerlach beam splitter is given by

\[
P_1(\alpha = \pm 1, a) = \int P_1(\alpha = \pm 1|a, \lambda) \rho(\lambda) d\lambda.
\]

(1)

Here, following Bell, \( \lambda \) is associated with the initial coordinates of the system and we accepted the locality condition \( \rho(\lambda, a) \equiv \rho(\lambda) \) imposing that the initial state is independent of the measurement settings. G. Lochak wrote equivalently

\[
P_1(\alpha = \pm 1, a) = \int_{E_{\alpha,a}} \rho(\lambda) d\lambda
\]

(2)

where \( E_{\alpha,a} \) is a sub-ensemble of the hidden variable space associated with the value \( \alpha \) taken by the observable. Such notation is evident if the system follows a well defined trajectory imposing \( P_1(\alpha = \pm 1|a, \lambda) = \delta_{\alpha, A(\lambda,a)} \) (which by definition of the symbol of Kronecker can only have the two values 0 or 1 depending on \( \lambda \) and \( \alpha \)). This is clearly connected to the hypothesis that the observable \( A(\lambda, a) = \pm 1 \) must be, as in the de Broglie model, functions
FIG. 1: A simple idealized apparatus to measure the spin of an atom of spin 1/2. Prior to enter in
the region of the magnetic field the atom moves along the axis z. After the interaction the particle
is located in one or the other of the two exits corresponding to the two states of the atomic spin
along a) the axis a or b) the axis a'.

of the initial coordinates. Naturally for another orientation of the magnetic field we have

\[ P_1(\beta = \pm 1, b) = \int_{E_{\beta,b}} \rho(\lambda) \, d\lambda. \]  

Now comes the paradox enounced by Lochak: Considering the ensembles intersection \( E_{\alpha,a} \cap E_{\beta,b} \) allow us to define the probability

\[ P_1(\alpha,a,\beta,b) = \int_{E_{\alpha,a} \cap E_{\beta,b}} \rho(\lambda) \, d\lambda \]  

i. e.

\[ P_1(\alpha,a,\beta,b) = \int \delta_{\alpha,A(\lambda,a)} \delta_{\beta,A(\lambda,b)} \rho(\lambda) \, d\lambda \]

\[ = \int P_1(\alpha|a,\lambda) P_1(\beta|b,\lambda) \rho(\lambda) \, d\lambda, \]  

which is identified in [1] with the probability of observing \( \alpha \) and \( \beta \) with the same particle [14]. But quantum mechanics prevents to measure with the same particle values associated to non-commutative observable. As a consequence, the assumption that an unique \( \rho(\lambda) \) exists is erroneous and we must introduce two contextual distributions \( \rho(\lambda,a) \) and \( \rho(\lambda,b) \). This
is in contradiction with the premisses and we must conclude, if we accept the reasoning of Lochak, that the definition of Bell\[2\] is already unadapted for the description of a single spin measurement. In the words of Lochak \textit{“So it was perfectly legitimate and even evident that the initial probability distribution }\rho(\lambda)\text{ does not depend on }a\text{ (or on }b\text{), but this distribution is not and can not be the one that we need for the statistics of measurement results: }\text{We can be sure of this assertion precisely because, if we adopt this initial density }\rho(\lambda)\text{, we obtain a traditional statistical pattern on measurement results, which obviously contradicts the well-known and certainly true statistical results in quantum mechanics”}. The question that we ask immediately is naturally “and what about non locality? does it means that any hidden variable model is necessary non local as seems to impose to conclude the precedent reasoning? Is a single particle measurement also non local?”

The answer given by Lochak is that non locality is not necessary involved for explaining the experimental results. In order to prove that Lochak analyzed the concrete examples of the L. de Broglie and D. Bohm models \[15, 16, 17, 18\] which, in the single particle case, are equivalent and completely local. In these models a neutral single particle with spin 1/2, which is represented by a wave packet having two components

$$\Psi (x, t) = \begin{pmatrix} \psi_\uparrow (x, t) \\ \psi_\downarrow (x, t) \end{pmatrix},$$

(6)
can be described dynamically as a point like object moving with the velocity $v(x, t) = [J/\rho] (x, t)$. Here

$$J (x, t) = \hbar [|\psi_\uparrow (x, t)|^2 \nabla \phi_\uparrow (x, t) + |\psi_\downarrow (x, t)|^2 \nabla \phi_\downarrow (x, t)]$$

(7)

and

$$\rho (x, t) = |\psi_\uparrow (x, t)|^2 + |\psi_\downarrow (x, t)|^2$$

(8)

define the probability current and probability density respectively, and $\phi_\uparrow, \phi_\downarrow$ are the phases of $\psi_\uparrow, \psi_\downarrow$. In presence of a magnetic field the two contributions are oriented in one or the other of the exits \[17, 19, 20\], separating the trajectories associated with the two states $\uparrow$ and $\downarrow$. Naturally, as explained before, any modifications of the magnetic field orientation change the analyzed basis $\uparrow, \downarrow$. Consequently in presence of the Stern and Gerlach apparatus analyzing the spin components along $a$ and $-a$ the density of probability $\rho (x, t) = |\psi_a (x, t)|^2 +$
$|\psi_{-a}(x, t)|^2$ depends explicitly on the orientation of the magnetic field and must be written $\rho(x, t, a)$. In this model we can define an instantaneous spin vector

$$S(x, t, a) = \frac{\Psi^\dagger \sigma \Psi}{\rho(x, t, a)}. \quad (9)$$

The projection $\Sigma(x, t, a) = S(x, t, a) \cdot a$ spans a continuum of values during the interaction with the magnetic field but at end of the measure (i.e. at $t = \infty$) we have $\Sigma = \pm 1$ corresponding to the spin observable $A = \pm 1$. We can naturally define the mean value of the spin projection $\Sigma$ by

$$E_1(\sigma) = \langle \Psi | \sigma \cdot a | \Psi \rangle = \int \Sigma(x, t, a) \rho(x, t, a) \, d^3x. \quad (10)$$

In the de Broglie theory the actual position of the particle is an hidden variable. If we choose to identify the hidden parameter $\lambda$ used by Bell with the coordinates of the particle in the wave packet at $t = +\infty$ then we can justify Eqs. 1, 2, 3, with Eq. 10 but in order to do that we must use the contextual distributions $\rho(\lambda, a)$ instead of the unique $\rho(\lambda)$ postulated in [2] by Bell. The statistical hypothesis of Bell is, following [1], consequently invalidated but the locality is nevertheless preserved. At that point it is important to make a break and to consider the problem of the locality for a single atom in the context of the de Broglie-Bohm theory. This point is fundamental because Lochak didn’t insisted sufficiently on it in [1]. At the beginning of [1] we can read that the postulate of locality “which will not study further in this paper” is not a part of the argumentation. However this postulate is clearly accepted by Lochak as seen in particular in the sentence “there is obviously no reason to suppose any dependance of [the initial] distribution on any future measurement”, and in “So it was then perfectly legitimate and even evident to suppose that the initial probability distribution $\rho(\lambda)$ does not depend on $a$ (or on $b$)”, and again in his discussion with d’Espagnat [10]. We accepted as a fact that the model proposed by de Broglie is local but this need to be verified since in general the motion of the particle is affected by a quantum potential taking into account the wave function [16]. This problem is discussed in [17]. Clearly any modifications of the boundary conditions or of the magnetic fields during the motion of the atomic wave packet will disturb the wave function. Such perturbation propagates into the direction of the particle and affects the motion later on. There is then effectively no spontaneous action at distance. In the present case the two spatial regions occupied by the wave packet and the magnetic field respectively are well separated at a given time $t = 0$ prior to the measurement.
Any instantaneous changes of the orientation of the Stern and Gerlach device at \( t = 0 \) can generate, in principle, a weak electromagnetic impulsion propagating in the direction of the atomic wave packet (a causal signal). The modification of the motion of the particle by this signal can only occur a time \( T = |Z - z(T)|/c \) after that the change in the measuring device is done (\( Z \) is the coordinate of the beam splitter on the \( z \) axis, \( z(T) \) is the coordinate of the atomic wave packet at \( t = T \)). This is particularly clear if we consider that the time evolution of the quantum system is determined by the interaction hamiltonian which is proportional to the local value of the magnetic field \( \mathbf{B}(x, t) \) at the wave packet position:

\[
\begin{aligned}
    i\hbar \partial_t \psi_a(x, t) &= -\frac{\hbar^2 \nabla^2}{2M} \psi_a(x, t) + \mu (\mathbf{B}(x, t) \cdot \mathbf{a}) \psi_a(x, t) \\
    i\hbar \partial_t \psi_{-a}(x, t) &= -\frac{\hbar^2 \nabla^2}{2M} \psi_{-a}(x, t) - \mu (\mathbf{B}(x, t) \cdot \mathbf{a}) \psi_{-a}(x, t)
\end{aligned}
\]

(11) 

(\( M \) is the atomic mass and \( \mu \) the magnetic moment). There is then clearly a local dynamic in this problem and the implicit postulate of Lochak is consequently completely pertinent.

What is the conclusion of Lochak? We saw at a first stage i) that the definition given by Bell of \( \rho(\lambda) \) implies paradoxes already for the single particle problem. This reasoning shows that the definition used by Bell imposes the existence of joint probabilities for non commutating observable in contradiction with the basic rules of quantum mechanics. At a second stage ii) we saw that the model of de Broglie, which is local when limited to a single particle, can explain the result of the single spin experiment. This proves, if we accept the reasoning of Lochak, that the density of probability \( \rho \) can depend on \( \mathbf{a} \) without introducing nonlocality in the hidden variable model. Analyzing the de Broglie model Lochak found the explanation: the variables \( \lambda \) are NOT the initial coordinates of the particle BUT the actual values \( \lambda(t) \) of these coordinates at the time \( t \) of the measurement.

Let go back now to the original two particles problem proposed by Bohm. Two correlated atoms with spin 1/2 are oriented on two settings of Stern and Gerlach devices \( \mathbf{a}, \mathbf{b} \) located apart from each other. Quantum mechanics allows us to define coincidence probabilities such as \( P_{12}(\alpha, \mathbf{a}, \beta, \mathbf{b}) \) where \( \alpha = \pm 1 \) are the eigenvalues of the spin projection operator along \( \mathbf{a} \) of the first atom and similarly \( \beta = \pm 1 \) are the eigenvalues of the spin projection operator along \( \mathbf{b} \) of the second atom. Using the hidden variables definition given by Bell and applying the locality condition we write the coincidence probabilities

\[
P_{12}(\alpha, \mathbf{a}, \beta, \mathbf{b}) = \int P_1(\alpha|\mathbf{a}, \lambda) P_2(\beta|\mathbf{b}, \lambda) \rho(\lambda) d\lambda.
\]

(12)
But now we can repeat the analysis done for the single spin measurement and define the probabilities

\[ P_1(\alpha, a, \alpha', a') = \int P_1(\alpha|a, \lambda) P_1(\alpha'|a', \lambda) \rho(\lambda) d\lambda \]  

(13)

and

\[ P_2(\beta, b, \beta', b') = \int P_2(\beta|b, \lambda) P_2(\beta'|b', \lambda) \rho(\lambda) d\lambda. \]  

(14)

interpreted as the joint probabilities of measuring two non-commutative observable associated with the same particle at the same time. This is forbidden by quantum mechanics and we must conclude that \( \rho(\lambda) \) is non adapted to the description of the experiments. In reality this reasoning is not explicitly present in [1] but can be attached to the work of A. Fine [21]. However Lochak agreed completely with this reasoning which complete his one [10].

Again comes the question: Does this implies necessarily non locality? Lochak answer a second time by the negative. It could be that quantum mechanics is effectively non local but it could be that the hidden variable definition of Bell is simply wrong. Indeed, may be the hidden variable, which are involved in any calculation of observable and probabilities, should be defined at the time of the measurements and should include the local settings \( a, b, ... \). This direct generalization of the single particle case to the many particle problem was the aim of the research made by de Broglie and his group. The fact that de Broglie never succeeded to build such local model can not however be considered as a proof against Lochak’s reasoning.

III. CRITICS AND COUNTER ARGUMENTS

What I want to show now is that there are several errors in the deductions of Lochak.

Consider first the single particle measurement described in the context of the de Broglie model. Lochak proved that we can always define the averages and the probabilities as a function of the actual particle position \( x(t) \) at the time \( t \) of the measurement. If this time tends to \(+\infty\) we can justify all the predictions given by quantum mechanics. However if we consider the model of de Broglie this is not the unique description of the phenomenon [4]. Indeed we can always define univocally the actual position \( x(t) \) measured for example at \( t = +\infty \) by a function of the initial coordinate \( x_0 = \lambda \) of the particle at a time \( t_0 \to -\infty \),
i.e. a long time before that the particle enters in the Stern and Gerlach apparatus. Due to the conservation of probability requirement the number of states defined by $\rho(x_0,t_0)\delta^3x_0$ in the elementary volume $\delta^3x_0$ is naturally identical to $\rho(x(t),t)\delta^3x(t)$ i.e.:

$$\rho(x_0,t_0)\delta^3x_0 = \rho(x(t),t)\delta^3x(t).$$

(15)

This result is of course well known in fluid dynamics where it is associated to the names of Euler and Lagrange (the so called Euler-Lagrange coordinates). Similarly $\Sigma(x(t),t,a)$ can be expressed as a function of the initial coordinates of the particle and can be written $A(x_0,t_0,t,a)$. This is effectively clear if we write $x(t) = F_a(t,x_0,t_0)$ and substitute it in the expression for $\Sigma$. If we consider now the expectation value $\langle \Psi|\sigma\cdot a|\Psi \rangle$, we can write

$$E(\sigma) = \langle \Psi|\sigma\cdot a|\Psi \rangle = \int A(x_0,t_0,t,a)\rho(x_0,t_0)\delta^3x_0.$$

(16)

If we choose $t = +\infty$ then $A = \pm 1$ and we have the complete definition of Bell (with now $\rho(\lambda)$ independent of $a$ as desired). The apparent paradox obtained by Lochak originates from the fact that we can define the observables in function either A) of the initial state or B) of the actual state obtained after the measurement. This two choices A and B are mathematically possible and equivalent as it is in classical fluid mechanics. However, only in the choice A (of Bell) a clear formulation of locality is possible. Indeed, the Bell definition of $\rho(\lambda_0)$ is possible only in a local word but the definitions $\rho(\lambda,a)$ and $\rho(\lambda,a,b)$ of Lochak for a system of one or two particles can be used also in a non local world. The definitions B of Lochak can therefore not help us to take any conclusions concerning nonlocality.

There is obviously a contradiction between our conclusion and the general result of Lochak-Fine concerning the joint probability $P_1(\alpha,a,\beta,b)$.

In order to solve this paradox it is sufficient to realize that, contrarily to what it is claimed in [1], $P_1(\alpha,a,\beta,b)$ has not to be considered as an observable. If we consider the definition Eq. 5 and interpret this number as a joint probability associated with an experimental protocol, we deduce of course that such protocol is realizable if the two measurements are independent and permutable in opposition with the basics rule of quantum mechanics. However, the number $P_1(\alpha,a,\beta,b)$ can always be defined mathematically without any paradox or contradiction and without being necessarily associated with a possible experiment. In fact, the “probability” $P_1(\alpha,a,\beta,b)$ is just a measure of the number of common elementary states which go along the direction $\alpha$ when using device $a$, and along the direction $\beta$ when
using device \( b \), respectively. We can consider the figure 2 as a help: a initial state which is characterized by different possible \( \lambda \) can be represented by an ensemble of point \( \mathbb{E}\{\lambda\} \) characterized by a density \( \rho(\lambda) \). Such ensemble is directly interpreted in the de Broglie model in which the hidden parameters are the positions of the particles. For a given orientation \( a \) of the magnetic field the points of this ensemble will deterministically move in one or the other of the separated regions “+” and “−” associated with the two values of the observable (see figure 2A). By changing the orientation of the field we modify the partition of the whole ensemble going through the beam splitter in the region “+” or “−” (see figure 2B). However the total number of states is conserved and we can formally define the number of points 
\[
\int_{\mathbb{E}_{\alpha,a} \cap \mathbb{E}_{\beta,b}} \rho(\lambda) \, d\lambda
\]
contained in the intersections “\( \pm, \pm \)” and “\( \pm, \mp \)” associated with these two partitions of \( \mathbb{E}\{\lambda\} \) (see figure 2C). The main thing is that these new partitions have not to be considered as physical since they can not be associated with an experimental procedure. This is the essential mistake of Lochak. More precisely since obviously \( \mathcal{P}_1(\alpha, a, \beta, b) \) is smaller than both \( P_1(\alpha, a) \) and \( P_1(\beta, b) \) the number of points contained in \( \mathbb{E}_{\alpha,a} \cap \mathbb{E}_{\beta,b} \) will just contribute partially to the outcomes \([\alpha, a]\) and \([\beta, b]\). Observing only the total number of particles contained in these two outcomes (associated with two complementary and incompatible experiments) we can consequently not say if such or such individual particle was or not contained in \( \mathbb{E}_{\alpha,a} \cap \mathbb{E}_{\beta,b} \). In other words this means that the probability \( \mathcal{P}_1(\alpha, a, \beta, b) \) defined by Lochak is not an observable but an hidden probability contrarily to what it was claimed in [1].

It is important to observe that Lochak was very close of the correct analysis when he said: “One could object: It is true that we can not measure in one experiment two different projection of the spin of [the particle] in the direction \( a \) and \( a' \), but we can conceive, for the same state \( \lambda \) of this particle, two different experiments for the measurement of each of these projections and define the probability \( \mathcal{P}(\alpha, a, \alpha', a') \) of finding \( \alpha \), if we measure the projection \( a \), and \( \alpha' \), if we measure the projection \( a' \)”. However in spite of his similarity this second interpretation of the probability proposed in [1] is in fact different of ours and was rejected by Lochak immediately after its introduction. Indeed for Lochak “two different experiments” mean a temporal succession of two measurements. But it is well know since the work of de Broglie [23] that the results of such succession of experiments depends on the order of the measurement. We should not then in this context be able to write an unique expression for the probability \( \mathcal{P}_1(\alpha, a, \alpha', a') \) defined in the precedent citation of Lochak.
FIG. 2: Partitions of the whole space $E\{\lambda\}$ into two parts $+$ and $-$ due to a measurement of the spin along A) the axis $a$ or B) the axis $b$. The intersection of such two partitions represented on C) is not associated with an individual experience and is just a mathematical definition.

In the words of Lochak “the probability of finding the value $\alpha$ and $\alpha'$ by measuring two spin components $a$ and $a'$ depends on the order of the measurements. But this fact and so the nonexistence of the probability $P_1(\alpha, a, \alpha', a')$ is not a blemish of the theory, it is a consequence of the wave particle dualism: If a hidden-parameter theory contradicts this fact, it will necessary contradict several correct results of usual wave mechanics”. However again there is a misinterpretation and the probability defined by $\int_{E_{\alpha,a}\cap E_{\beta,b}} \rho(\lambda) \, d\lambda$ has not to be associated with a succession of two measurements. We can define naturally and without contradiction the probability of two successive measurements of the spin components using the initial value of the hidden parameter introduced by Bell and we have

$$P_1(\alpha, a, \alpha', a') \int P_1(\alpha|a, \alpha', a', \lambda) P_1(\alpha'|a', \lambda) \rho(\lambda) \, d\lambda. \quad (17)$$

Where $P_1(\alpha = \pm 1|a, \lambda) = \delta_{\alpha,A(\lambda,a)}$ is the probability of finding $alpha$ for the first measurement if we know that the initial value of the hidden parameter is $\lambda$, and $P_1(\alpha|a, \alpha', a', \lambda)$ is the probability of finding $\alpha'$ for the second measurement if we know that the initial value of the hidden parameter is $\lambda$ and if we know that the result of the first measurement was $\alpha$. Clearly we have no reason to have $P_1(\alpha|a, \alpha', a', \lambda) = P_1(\alpha|a\lambda)$ because the order of the two measurements is crucial in the evolution of the dynamical variable. There is then no
reason to identify $P_1(\alpha, a, \alpha', a')$ and $P_1(\alpha, a, \alpha', a')$.

It can be added that our reasoning was centered on local deterministic theories for which we have $P_1(\alpha|\lambda, a) = \delta_{\alpha,A(\lambda,a)}$. However our present refutation can be done in the most general context of local objective theories for which the particle can obey to a statistical hidden dynamic taking into account the detectors. The probabilities $P_1(\alpha|\lambda, a)$ are now general distributions obeying only to the condition $P_1(+|\lambda, a) + P_1(-|\lambda, a) \leq 1$. We have equality if there is non absorption. We see that the simple interpretation in terms of subensemble given by Lochak in Eqs. 14 is not possible. However since the second line of Eq. 5 is still valid our critics are the same: products of probabilities such as $P_1(\alpha|a, \lambda) P_1(\beta|b, \lambda)$ have not to be interpreted necessary as a probability associated with a unique experimental process i.e. that we have not $P_1(\alpha|a, \lambda) P_1(\beta|b, \lambda) = P_1(\alpha, \beta|a, b, \lambda)$ (which by the way does not exist). A mathematical deduction similar to the one by Lochak was done by A. Fine and Lochak refers to his work in his latter publications (See the debate between d’Espagnat and Lochak on this subject[10]) but the conclusion is completely different. Fine concluded that if we use the definition given by Bell of $\rho(\lambda)$ then we must accept that we can define probabilities for non commutating observable. This is identical to the result of Lochak but for Fine this means that locality must be wrong. However this result is not more satisfying since it is based on the same misconceptions of the “joint” probabilities involved. Our critics apply then as well to his reasoning.

Finally I want to make the following remark: i)If we accept that there is a fundamental dynamic describing the reality (deterministic or stochastic) in term of a temporal evolution of certain dynamical parameter, ii) if we accept the principle of locality saying that the initial state in independent of the subsequent measurement, and iii) if we accept the basic rules of the probability calculus (in particular the law of probability conservation), then we can always write expressions such as 1 or 12. The claim of Lochak concerning the possibility to save locality by forbidding the use of the initial density of probability when calculating observable must then be wrong par principe. Fine didn’t make this error but he identified uncorrectly, as Lochak did before him, the probability defined by the formula 5,12,13 with a joint probability associated with an experiment.
IV. CONCLUSION AND REMARKS

What is the conclusion of our reasoning? First we observed that the model of de Broglie considered by Lochak allows the definition of the expectation values as functions of the initial state $\lambda_0$ existing prior to the measurement. We can in this description à la Bell introduce the density of probability $\rho(\lambda_0)$ in the calculation of the observable. This is in contradiction with the claim of Lochak concerning the impossibility to use such distribution in the calculation of the observable. This simple fact implies already that the reasoning of Lochak is wrong. Secondly we found the origin of the paradox observing that the probabilities defined in Eqs. 4,5 and Eqs. 12,13 have not to be considered as associated with a measurement procedures. Indeed quantum mechanics forbids definition of such probabilities in the case of non commutating observable measurement. The probabilities Eqs. 4,5,12,13 are in general only mathematical definitions which have not to be connected with a physical measurement. In other terms these probabilities are hidden. The problem in the deductions done by Lochak is even more fundamental and takes his source in a profound misinterpretation of the definition of a probability space. Indeed if we accept the concepts of dynamics, locality and of conservation of probability we must always be able to express any expectation values as a sum or integral over the initial distribution $\rho(\lambda_0)$ of some initial dynamical parameter $\lambda_0$ describing the quantum system. The locality imposes that the initial state is independent of the observation settings $a, b,...$ and there is consequently no way of finding a hypothetical loophole in the conclusion given by Bell[2].

V. HISTORICAL REMARK

Latter after that this manuscript was completed I become aware of a counter argument of A. Shimony[24] presenting essentially the same idea concerning the meaning of the probability $\mathcal{P}(\alpha, a, \alpha', a')$. Detailing his argumentation Shimony explains indeed that well interpreted $\mathcal{P}(\alpha, a, \alpha', a')$ is “the probability that the particle is in a state such that a measurement of spin in the direction $a$ (if that option were taken by the experimenter) would yield the value $\alpha$, and a measurement of spin in the direction $a'$ (if that option were taken by the experimenter) would yield the value $\alpha'$”. This definition is strictly equivalent to ours if we mean by state a hidden state and if this probability $\mathcal{P}(\alpha, a, \alpha', a')$ is hidden too. How-
ever Shimony claimed just after that the interpretation that he proposes was anticipated but rejected immediately by Lochak himself. The passage of [1] cited by Shimony to prove that is “One could object: It is true that we can not measure in one experiment two different projection of the spin of [the particle] in the direction \( a \) and \( a' \), but we can conceive, for the same state \( \lambda \) of this particle, two different experiments for the measurement of each of these projections and define the probability \( P(\alpha, a, \alpha', a') \) of finding \( \alpha \), if we measure the projection \( a \), and \( \alpha' \), if we measure the projection \( a' \). This objection is however invalid, because the impossibility of a simultaneous measurement of two spin projections of the same particle is not due to a simple incompatibility of instruments: It comes from the fact that the state in which we must put the particle to measure its spin components \( a \) is not the same as the one in which must put it to measure the component \( a' \)”.

Shimony seems not to have realize that for Lochak the probability \( P(\alpha, a, \alpha', a') \) is a real observable and not a hidden probability. This come from the fact that the word state used in the Shimony definition can not refer to a wave function, associated with a experimental procedure, but only to a pure mathematical subdivision of the whole ensemble \( \mathbb{E}\{\lambda\} \) without direct experimental meaning. For Lochak, in opposition, this state should be experimentally accessible and all his critics is erroneously constructed on this basis as we explained already before. Here again a not sufficiently precise definition of the vocabulary used implies some confusions. It then not surprising that in the rest of his comment Shimony could not understand Lochak. The last part of the article of Shimony concerns the example of the Stern and Gerlach measurement described by Lochak in the context of the double solution theory of de Broglie. Shimony made here the hypothesis that the difficulty encountered comes from the fact that the theory of de Broglie takes into account the disturbance by the measuring device. He guessed then that the paradox should be solved if instead of considering the original deterministic hidden variable defined by Bell we used the most general stochastic theories analyzed in [4, 6]. This is unfortunately wrong because the de Broglie model is strictly deterministic.

[1] G. Lochak, Found. Phys. 6,173 (1976). This article was published in french in Epistemological Letters (Bienne) 38, 41 (1975).

[2] J. S. Bell, Physics 1,195 (1964).
[3] A. Einstein, B. Podolsky and N. Rosen, Phys. Rev. 46, 777 (1935).

[4] J. S. Bell, Introduction to the hidden variable question, in: Proceedings of the international school of physics Enrico Fermi, course IL, Academic Press Inc., New York, 1971.

[5] J. Clauser, M. A. Horne, A. Shimony and R. A. Holt, Phys. Rev. Lett. 23, 880 (1969).

[6] a)J. F. Clauser, M. A. Horne, Phys. Rev. D10, 526 (1974).
   
   b)J. F. Clauser, A. Shimony, Rep. Prog. Phys. 41, 1881 (1978).

[7] D. M. Greenberger, M. A. Horne, A. Shimony, and A. Zeilinger, Am. J. Phys. 58, 1131 (1990).

[8] L. Hardy, Phys. Rev. Lett. 72, 781 (1994).

[9] J. S. Bell, Speakable and unspeakable in quantum mechanics, Cambridge University Press, Cambridge, 1987, pp. 63-66. This article reproduces the answer given by Bell to Lochak which was originally published in Epistemological letters (Bienne) 7, 2 (1976).

[10] See G. Lochak, Ann. Fond. Louis de Broglie 26, 5 and 31 (2001) for a complete list of references, and in particular G. Lochak Revue de métaphysique et de morale 1/1983 p. 85 and 1/1985 p. 400.

[11] G. Lochak, Louis de Broglie, Champs-Flammarion, Paris, 1992, pp. 248-249.

[12] L. de Broglie, G. Lochak, J. A. Beswick, and J. Vassalo-Pereira, Found. Phys. 6, 3 (1976).

[13] G. Lochak private communications.

[14] Other equivalent definitions are

\[ P(\pm, a, \pm, b) = \int d\lambda \rho(\lambda) \frac{A(\lambda, a) + 1}{2} \frac{A(\lambda, b) + 1}{2} \]

\[ P(\pm, a, -, b) = \int d\lambda \rho(\lambda) \frac{A(\lambda, a) + 1}{2} \frac{1 - A(\lambda, b)}{2} \]

\[ P(-, a, +, b) = \int d\lambda \rho(\lambda) \frac{1 - A(\lambda, a)}{2} \frac{A(\lambda, b) + 1}{2} \]

\[ P(-, a, -, b) = \int d\lambda \rho(\lambda) \frac{1 - A(\lambda, a)}{2} \frac{1 - A(\lambda, b)}{2} \]

where \( \pm \) refer to the two values \( \pm 1 \) of the observable \( A(\lambda, a) \) and \( A(\lambda, b) \).

[15] a)L. de Broglie, C. R. Acad. Sci. Paris 183 (1926) 447; 185 (1927) 580.

   b)L. de Broglie, Nonlinear wave mechanics, Elsevier, Amsterdam, 1960.

[16] D. Bohm, Phys. Rev. 85, 166 and 180 (1952).

[17] P. R. Holland, The Quantum Theory of Motion, Cambridge University Press, Cambridge, 1993.

[18] D. Bohm, B. J. Hiley and P. N. Kaloyerou, Phys. Rep. 144 (1987) 321.
[19] M. O. Scully, W. E. Lamb, and A. O. Barut, Found. Phys. 17, 575 (1987).

[20] C. Dewdney, P. R. Holland and A. Kyprianidis Phys. Lett. A119, 259 (1986).

[21] A. Fine, Phys. Rev. Lett. 48, 291 (1982).

[22] We have \( P_1(\alpha, a, \beta, b) + P_1(\alpha, a, -\beta, b) = P_1(\alpha, a) \) and similarly \( P_1(\alpha, a, \beta, b) + P_1(-\alpha, a, \beta, b) = P_1(\beta, b) \).

[23] L. de Broglie, La théorie de la mesure en mécanique ondulatoire (interprétation usuelle et interprétation causale), Gauthier-Villars, Paris 1957.

[24] A. Shimony, Epistemological Letters 8, 1 (1976).