Irrelevance of photon events distinguishability in a class of Bell experiments

Marek Żukowski\textsuperscript{1}, Dagonir Kaszlikowski\textsuperscript{1} and Emilio Santos\textsuperscript{2}

\textsuperscript{1}Instytut Fizyki Teoretycznej i Astrofizyki, Uniwersytet Gdański, PL-80-952 Gdański, Poland
\textsuperscript{2} Departamento de Física Moderna, Universidad de Cantabria, Santander, Spain

(April 1, 2022)

We show that the possibility of distinguishing between single and two photon detection events, usually not met in the actual experiments, is not a necessary requirement for the proof that the experiments of Alley and Shih [Phys. Rev. Lett. 61, 2921 (1988)], and Ou and Mandel [Phys. Rev. Lett. 61, 50 (1988)], are modulo fair sampling assumption, valid tests of local realism. We also give the critical parameters for the experiments to be unconditional tests of local realism, and show that some other interesting phenomena (involving bosonic type particle indistinguishability) can be observed during such tests.

The first Bell-type experiments which employed parametric down conversion process as the source of entangled photons were those reported in refs \[1\] and \[2\]. However, the specific traits of those experiments have led to a protracted dispute on their validity as tests of local realism. In this case the issue was not the standard problem of detection efficiency (which up till now permits a local realistic interpretation of all performed experiments). The trait that distinguishes the experiments is that even in the perfect gedanken situation (which assumes perfect detection) only in 50\% of the detection events each observer receives a photon, in the other 50\% of events one observer receives both photons of a pair while the other observer receives none. The early “pragmatic” approach was to discuss only the events of the first type (as only such ones lead to spatially separated coincidences). Only those were used as the data input to the Bell inequalities in \[1\] and \[2\]. This procedure was soon challenged (see e.g. \[3\], and especially in the theoretical analysis of ref. \[3\]), as it raises justified doubts whether such experiments could be ever genuine tests of local realism (as the effective overall collection efficiency of the photon pairs, 50\% in the gedanken case, is much below what is usually required for tests of local realism). Ten years after the first experiments of this type were made, finally the dispute was resolved \[3\]. It was proposed, to take into account also those “unfavourable” cases and to analyse the entire pattern of events. In this way one can indeed show that the experiments are true test of local realism (namely, that the CHSH inequalities are violated by quantum predictions for the idealised case). The idea was based upon a specific value assignment for the “wrong events” (see further, or \[3\] itself). However, the scheme presented by Popescu et al \[6\] has one drawback. The authors assumed in their analysis that the detecting scheme employed in the experiment should be able to distinguish between two and one photon detections. This was not the case in the actual experiments. The aim of this work is to show that even this is unnecessary, all one needs is the use of the specific value assignment procedure of \[3\].

What is perhaps even more important, problems similar to those sketched above are also shared by the new, potentially highly promising, class of EPR-Bell type experiments, which involve the entanglement swapping procedure \[3\]. Also in this case the first performed experiment did not employ detectors which are able to distinguish between firings caused by two photons and a single photon \[3\]. The entanglement swapping experiments thus far did not violate the visibility threshold for local realism (71\%), however in the future the problem of their relation to the Bell theorem will be of a fundamental importance (as entanglement swapping may find application in future quantum communication schemes \[3\]). The analysis presented in \[3\] can be adapted to describe such experiments, clearly indicating violation of local realism.

Finally, we shall also give prediction of all effects occurring in the experiment. It is quite often overlooked that a kind of Hong-Ou-Mandel dip phenomenon \[10\] can be observed in the experiment.

In the class of experiments we consider (fig. 1) \[1\] a type I parametric down-conversion source \[1\] is used to generate pairs of photons which are degenerated in frequency and polarisation (say \(x\)) but propagate in two different directions. One of the photons passes through a wave plate (WP) which rotates its polarisation by 90°. The two photons are then directed onto the two input ports of a (nonpolarizing) “50 – 50” beamsplitter (BS). The observation stations are located in the exit beams of the beamsplitter. Each local observer is equipped with a polarising beamsplitter \[12\], orientated along an arbitrary axis (which, in principle can be randomly chosen, in the delayed-choice manner, just before the photons are supposed to arrive). Behind each polarising beamsplitter are two detectors, \(D_{1}^{\perp}\), \(D_{1}^{\parallel}\), and \(D_{2}^{\perp}\), \(D_{2}^{\parallel}\) respectively, where the lower index indicates the corresponding observer and the upper index the two exit ports of the polarised beamsplitter (“ + “ meaning parallel with the polarisation axis of the beamsplitter and “ − “ meaning
orthogonal to this axis). All optical paths are assumed to be equal.

FIG. 1. Schematic of the setup. For explanations see the main text.

Let us calculate the quantum predictions for the experiment. We will use the second quantisation formalism, which is very convenient here, since the whole phenomenon observed in the experiment rests upon the indistinguishability of photons.

After the action of the wave-plate one can approximate quantum mechanical state describing two photons emerging from a non-linear crystal along the "signal" and the "idler" beam by

$$|\Psi_0\rangle = a_{1x}^\dagger a_{2y}^\dagger |0\rangle,$$  \hspace{1cm} (1)

where $a_{1x}^\dagger$ and $a_{2y}^\dagger$ are creation operators and $|0\rangle$ denotes the vacuum state. Subscripts $\bar{x}, \bar{y}$ decode the polarisation of the photon (either along $\bar{x}$ or $\bar{y}$ axis). The beamsplitter action can be described by

$$a_{1x}^\dagger = \frac{1}{\sqrt{2}}(ic_{\bar{x}}^\dagger + d_{\bar{x}}^\dagger)$$
$$a_{2y}^\dagger = \frac{1}{\sqrt{2}}(c_{\bar{y}}^\dagger + id_{\bar{y}}^\dagger),$$ \hspace{1cm} (2)

where $c_{\bar{x}}, d_{\bar{x}}, c_{\bar{y}}, d_{\bar{y}}$ are operators describing output modes of the beamsplitter ($c$ stands for the first observer and $d$ for the second one). Thus our state $|\Psi_0\rangle$ changes to:

$$|\Psi\rangle = \frac{1}{2}(ic_{\bar{x}}c_{\bar{y}}^\dagger - c_{\bar{x}}d_{\bar{y}}^\dagger + c_{\bar{y}}d_{\bar{x}}^\dagger + id_{\bar{y}}d_{\bar{x}}^\dagger)|0\rangle$$ \hspace{1cm} (3)

Next comes the action of the polarisers in both beams, which can be described as

$$n_{\bar{x}}^\dagger = \cos(\theta_1)n_{\parallel}^\dagger + \sin(\theta_1)n_{\perp}^\dagger$$
$$n_{\bar{y}}^\dagger = \sin(\theta_1)n_{\parallel}^\dagger - \cos(\theta_1)n_{\perp}^\dagger,$$ \hspace{1cm} (4)

where $n_{\parallel}^\dagger = a_{\parallel}^\dagger$ or $d_{\parallel}^\dagger$, and $n_{\perp}^\dagger$ describes the mode parallel to polarizer's axis and $n_{\parallel}^\dagger$ describes the mode perpendicular to polarizer's axis; $\theta$ is the angle between the $\bar{x}$ axis and polarizer's axis. Thus the final state reaching the detector reads

$$|\psi_{\text{final}}\rangle = \frac{1}{2}[\sin(\theta_1 - \theta_2)|c_{\parallel}^\dagger, d_{\parallel}^\dagger\rangle$$
$$+ \cos(\theta_1 - \theta_2)|c_{\perp}^\dagger, d_{\perp}^\dagger\rangle$$
$$+ \cos(\theta_1 - \theta_2)|c_{\parallel}^\dagger, d_{\perp}^\dagger\rangle + \sin(\theta_1 - \theta_2)|c_{\perp}^\dagger, d_{\parallel}^\dagger\rangle$$ \hspace{1cm} (5)

where e.g. $|c_{\parallel}^\dagger, d_{\parallel}^\dagger\rangle$ denotes one photon in the mode $c_{\parallel}^\dagger$, and one in $d_{\parallel}^\dagger$, whereas $|2c_{\parallel}^\dagger\rangle = \frac{1}{\sqrt{2}}|c_{\parallel}^\dagger, c_{\parallel}^\dagger\rangle$ denotes two photons in the mode $c_{\parallel}^\dagger$.

Let us denote by $P(i, \theta_1; j, \theta_2)$ the joint probability for the outcome $i$ to be registered by observer 1 when her polariser is oriented along the direction that makes an angle $\theta_1$ with the $\bar{x}$ direction and the outcome $j$ to be registered by observer 2 when her polariser is oriented along the direction that makes an angle $\theta_2$ with the $\bar{x}$ direction. Here $i, j = 1, 6$ and have the following meaning:

1. one photon in $D^-$, no photon in $D^+$
2. one photon in $D^+$, no photon in $D^-$
3. no photons
4. one photon in $D^-$ and one photon in $D^-$
5. two photons in $D^+$, no photon in $D^-$
6. two photons in $D^-$, no photons in $D^+$.

The quantum predictions for joint probabilities of those events are given by:

$$P(1, \theta_1; 1, \theta_2) = P(2, \theta_1; 2, \theta_2) = \frac{1}{2}[1 - \cos(2\theta_1 - 2\theta_2)],$$ \hspace{1cm} (6)
$$P(2, \theta_1; 1, \theta_2) = P(1, \theta_1; 2, \theta_2) = \frac{1}{4}[1 + \cos(2\theta_1 - 2\theta_2)],$$ \hspace{1cm} (7)
$$P(5, \theta_1; 3, \theta_2) = P(6, \theta_1; 3, \theta_2) = \frac{1}{8} \sin^2(2\theta_1),$$ \hspace{1cm} (8)
$$P(3, \theta_1; 5, \theta_2) = P(3, \theta_1; 6, \theta_2) = \frac{1}{4} \sin^2(2\theta_2),$$ \hspace{1cm} (9)
$$P(4, \theta_1; 3, \theta_2) = \frac{1}{4} \cos^2(2\theta_1),$$ \hspace{1cm} (10)
$$P(3, \theta_1; 4, \theta_2) = \frac{1}{4} \cos^2(2\theta_2).$$ \hspace{1cm} (11)

Following [5] we associate with each outcome registered by the observer 1 and 2 a corresponding value $a_i$ and $b_j$ respectively, where $a_1 = b_1 = -1$ while all the other values are equal to 1. Let us denote by $E(\theta_1, \theta_2)$ the expectation value of their product

$$E(\theta_1, \theta_2) = \sum_{i,j} a_i b_j P(i, \theta_1; j, \theta_2).$$ \hspace{1cm} (12)

After simple calculations one has:

$$E(\psi_1, \psi_2) = -\frac{1}{2} \cos(\psi_1 + \psi_2) + \frac{3}{4},$$ \hspace{1cm} (13)
where we have put $2\theta_k = (-1)^{k-1}\psi_k$.  

The above formula for the correlation function is valid if one assumes that it is possible to distinguish between single and double photon detection. This is usually not the case. Thus it is convenient to have a parameter $\alpha$ that measures the distinguishability of the double and single detection at one detector ($0 \leq \alpha \leq 1$, and gives the probability of distinguishing by the employed detecting scheme of the double counts). The partial distinguishability blurs the distinction between events 1 and 6 (2 and 5) and thus part of the events of the type 6 are interpreted as of type 1 and are ascribed by the local observer a wrong value, e.g. an event of type 6, if both photons go to the “−” exit of the polariser, can be interpreted as a firing due to a single photon and is ascribed a −1 value. Please note that such events like 1 or 2 in station 1 accompanied by 3 (no photon) at station 2 do not contribute to the correlation function because for any $\alpha \neq 0$ $P(1, \theta_1; 3, \theta_2) = P(2, \theta_1; 3, \theta_2)$.

If the parameter $\alpha$ is taken into account the correlation function acquires the following form:

$$E(\psi_1, \psi_2; \alpha) =$$

$= -\frac{1}{2}\cos(\psi_1 + \psi_2) + \frac{1}{2}\alpha$

$+ \frac{1}{4}(1 - \alpha)(\cos^2 \psi_1 + \cos^2 \psi_2)$ \hspace{1cm} (14)

In this case after the insertion of the quantum correlation function \ref{eq:14} into the CHSH inequality,

$$-2 \leq E(\psi_1, \psi_2; \alpha) + E(\psi_1', \psi_2; \alpha)$$

$+ E(\psi_1, \psi_2'; \alpha) - E(\psi_1', \psi_2'; \alpha)) \leq 2,$

one obtains:

$$-2 \leq -\frac{1}{2}[\cos(\psi_1 + \psi_2) + \cos(\psi_1' + \psi_2')$$

$+ \cos(\psi_1 + \psi_2') - \cos(\psi_1' + \psi_2')] + \alpha$

$+ \frac{1}{4}(1 - \alpha)(\cos^2 \psi_1 + \cos^2 \psi_2) \leq 2.$ \hspace{1cm} (15)

The interesting feature of this inequality is that it can be violated for all values of $\alpha$. What is perhaps even more important, it can be robustly violated even when one is not able to distinguish between single and double clicks at all ($\alpha = 0$). The actual value of the CHSH expression can reach in this case 2.33712 (a numerical result), which is only slightly less than the maximal value for $\alpha = 1$, which is $\sqrt{2} + 1 \approx 2.41421$. Therefore we conclude that in the experiment one can observe violations of local realism even if one is not able to distinguish between the double and single counts at one detector. That is, the essential feature of the method of \ref{eq:14} to reveal violations of local realism in the experiment of this type is the specific value assignment scheme and not the double-single photon counts distinguishability.

The specific angles at which the maximum violation of the CHSH inequality is achieved for $\alpha = 0$ differ very much from those for $\alpha = 1$ (for which the standard result is reproduced), and they read (in radians) $\psi_1 = 2.93798$, $\psi_1' = 4.25513$, $\psi_2 = -0.20241$ and $\psi_2' = 1.11708$.

Let us notice that with the setup of fig.1 one is able to observe effects of similar nature to the famous Hong-Ou-Mandel dip \cite{11}. These are revealed by the probabilities pertaining to the wrong events (8-11). Simply for certain orientations of the polarisers, if the two photons emerge on one side of the experiment only, then they must exit the polarising beamsplitter via a single output port (this effect is due to the bosonic-type indistinguishability of photons, see \cite{11}).

Finally let us discuss what is the critical efficiency of the detection of the experiments of this type. To this end, in our calculations we will use a very simple model of imperfect detections: we insert a beamsplitter with reflectivity $\sqrt{T - \eta}$ in front of an ideal detector, which observes only the transmitted light. This results in the system behaving just like a detector of efficiency $\eta$. If we assume that the incoming light is described by a creation operator $a^\dagger$ then transmitted mode is denoted as $t^\dagger_a$ whereas reflected mode is denoted as $r^\dagger_a$ and one has:

$$a^\dagger = \sqrt{1 - \eta}r^\dagger_a + \sqrt{\eta}t^\dagger_a \hspace{1cm} (16)$$

For instance, if one takes the following part of the state vector \ref{eq:5}:

$$c_{\dagger i}^d t^\dagger_{\dagger i} |0\rangle \hspace{1cm} (17)$$

the beamsplitter model of an imperfect detector transforms this term into:

$$[(1 - \eta)r^\dagger_{\dagger i} t^\dagger_{\dagger i} + \sqrt{\eta(1 - \eta)}r^\dagger_{\dagger i} t^\dagger_{\dagger i} + \sqrt{\eta(1 - \eta)}t^\dagger_{\dagger i} r^\dagger_{\dagger i} + \eta^\dagger t^\dagger_{\dagger i} r^\dagger_{\dagger i} |0\rangle \hspace{1cm} (18)$$

The probabilities now read:

$$P(3, \theta_1; 2, \theta_2) = P(2, \theta_1; 3, \theta_2) \hspace{1cm} (19)$$

$$P(1, \theta_1; 3, \theta_2) = P(3, \theta_1; 1, \theta_2) = \eta(1 - \eta) \hspace{1cm} (20)$$

$$P(1, \theta_1; 1, \theta_2) = P(2, \theta_1; 2, \theta_2) = \frac{1}{4}\eta^2[\sin(\theta_1 - \theta_2)]^2 \hspace{1cm} (21)$$

$$P(2, \theta_1; 1, \theta_2) = P(1, \theta_2; 2, \theta_2) = \frac{1}{4}\eta^2[\cos(\theta_1 - \theta_2)]^2 \hspace{1cm} (22)$$

$$P(5, \theta_1; 3, \theta_2) = P(6, \theta_1; 3, \theta_2) = \frac{1}{8}\eta^2[\sin(2\theta_1)]^2 \hspace{1cm} (23)$$

$$P(4, \theta_1; 3, \theta_2) = \frac{1}{4}\eta^2[\cos(2\theta_1)]^2 \hspace{1cm} (24)$$

$$P(3, \theta_1; 4, \theta_2) = \frac{1}{4}\eta^2[\cos(2\theta_2)]^2 \hspace{1cm} (25)$$

The correlation function, which includes the inefficiency of the detection reads

$$E(\psi_1, \psi_2; \eta, \alpha) = \eta^2 E(\psi_1, \psi_2; \alpha) + (1 - \eta)^2 \hspace{1cm} (26)$$
where $E(\psi_1, \psi_2; \alpha)$ is given by (14). We have tacitly assumed here that the parameters $\alpha$ and $\eta$ are independent of each other (this assumption may not hold for specific technical arrangements). Putting this prediction into CHSH inequality, assuming that $\alpha = 1$ (full distinguishability) we obtain a minimum quantum efficiency needed for violation of local realism equal to 0.91, whereas for other values of $\alpha$ we have: for $\alpha = 0$ $\eta = 0.926$; for $\alpha = 0.5$ $\eta = 0.92$; for $\alpha = 0.75$ $\eta = 0.92$; for $\alpha = 0.875$ $\eta = 0.91$. One should note here that the method of value assignment of [6] is in accordance with the method given by Garg and Mermin [13] for the optimal estimation of required detector quantum efficiency to violate local realism in a Bell-test. Thus the obtained efficiencies are indeed the lowest possible, and show that experiments of this type are not good candidates for a "loophole-free" Bell-test [14], nevertheless due to the fact that the whole observable effect is a consequence of quantum principle of particle indistinguishability such test are very interesting by themselves - they reveal the entanglement inherently associated with this principle.

MZ was supported by the University of Gdansk Grant No BW/5400-5-0202-8. DK was supported by the KBN Grant 2 P03B 096 15.

[1] C.O. Alley and Y.H. Shih, Phys. Rev. Lett. 61, 2921 (1988).
[2] Z.Y. Ou and L. Mandel, Phys. Rev. Lett. 61, 50 (1988).
[3] P.G. Kwiat, P.E. Eberhard, A.M. Steinberg and R.Y. Chiao, Phys. Rev. A 49, 3209 (1994).
[4] P.G. Kwiat, Phys. Rev. A 52 3380 (1995).
[5] L. De Caro and A. Garuccio, Phys. Rev. A 50, R2803 (1994),
[6] S. Popescu, L. Hardy and M. Žukowski, Phys. Rev. A, 56, R4353 (1997).
[7] M. Žukowski, A. Zeilinger, M.A. Horne and A.K. Ekert, Phys. Rev. Lett. 71, 4287 (1993).
[8] J. W. Pan, D. Bouwmeester, H. Weinfurter, A. Zeilinger, Phys. Rev. Lett. 80, 3891 (1998).
[9] S. Bose, V. Vedral, P. L. Knight, Phys. Rev. A 57, 822 (1998).
[10] C.K. Hong, Z.Y. Ou and L. Mandel, Phys. Rev. Lett. 59, 2044 (1987).
[11] C. Hong, L. Mandel, Phys. Rev. A 31, 2409, (1985); A. Casado, A. Fernandez-Rueda, T.W.Marshall, R.Risco-Delgado and E. Santos, Phys.Rev.A 55, 3870 (1997).
[12] Following references [2] and [3] we assume that both local detection stations are equipped with polarising beam-splitters, and each of the output ports is observed by a detector. In the actual experiments [2] and [3] at each station only one of the outputs was monitored.
[13] A. Garg, N.D. Mermin, Phys. Rev. D 35, 3832 (1985)
[14] E. Santos, Phys. Rev. A, 46, 3646 (1992).