Mixed LFM Signal Estimation Based on Radon-Wigner Transform and Matching Pursuit

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Abstract. Parameter estimation of mixed signals is a key problem in electronic reconnaissance. Based on Radon-Wigner transform (RWT) and Matching Pursuit (MP) algorithm, a parameter estimation method for mixed LFM signals is proposed in this paper. The core of the method is to separate signal components from the mixed signal and estimate their parameters one by one. Firstly, a rough parameter estimation of the strongest signal is obtained by RWT. After that an optimized estimation based on MP algorithm is performed to fine-tune the estimation result. Then, the strongest signal component is reconstructed with the optimized estimation, and it is separated from the mixed signal. Therefore, by iteratively estimating and separating the stronger signal within the residual mixed signal, all of the signal components can be precisely estimated. Experimental results show that the proposed method is able to achieve satisfactory performance on a lower signal-to-noise ratio.

1. Introduction
Linear frequency modulation (LFM) signals have excellent performance in high range-speed resolution and good anti-interference performance [1-2], and they are widely applied in fire-control radar, sonar, and missile fuze. However, with the fast development of radar technology, electromagnetic environment become increasingly complicated, which brings difficulty for reconnaissance receivers to separate mixed LFM signals and estimate their parameters simultaneously. Therefore, the parameter estimation of mixed LFM signals has received widespread attention.

Traditional parameter estimation methods, such as Fractional Fourier transform [3] and traditional time-frequency method [4], are hard to realize precise mixed LFM signal separation and parameters estimation. In this paper, an effective method for LFM signal separation and estimation is proposed. Different from traditional methods for time domain and frequency domain, the proposed method utilizes the Radon-Wigner transform (RWT) [5] to realize a rough estimation, and then uses the matching pursuit (MP) algorithm to achieve a precise estimation. For a single LFM signal, the rough estimation of its parameters can be estimated by RWT firstly. Then, the MP algorithm is used to search a more precise estimation around the rough results, and the estimation can be optimized. For mixed LFM signals, the parameters of the strongest signal component with the mixed signal can be estimated by this method. Then, we can reconstruct the component and separate it from the mixed signal, and then we can continue to estimate the strongest component within the residual mixed signal and separate it. By iteratively estimating and separating the strongest signal component from the residual signal, all the signal components within the mixed signal can be estimated and separated. The theory analysis and simulation results show that the proposed method can effectively estimate and separate mixed signals.
2. The estimation method based on RWT and MP

2.1. The Radon-Wigner transform
The Radon-Wigner transform is a projection transform of linear integral, which is based on Radon transform (RT) and Wigner-Ville distribution (WVD). The integral equation of RWT can be described as

\[
R(\alpha, \beta) = \frac{1}{\sin \beta} \int_{-\infty}^{+\infty} W(t, f_0 + at)dt \bigg|_{f_0 = \alpha \sin \beta, a = -\cot \beta}
\]

(1)

where \( \alpha \) and \( \beta \) are parameters of the integral path. \( f_0 \) is the initial frequency and \( a \) is the frequency-modulating slope. Figure 1 (a)–(c) show the waveform, the WVD and the RWT of an LFM signal separately. It can be seen from Figure 1 that the WVD of an LFM signal is a straight line with definite slope, while in RWT domain, the LFM signal shows a peak. According to the definition of RWT shown in Eq (1), the basic parameters of the LFM signal can be estimated based on the coordinate values of the peak.

2.2. The parameter estimation method
In this section, the parameter estimation method for mixed LFM signals is introduced. There are four main steps for estimating and separating the components of the mixed LFM signals, including the rough estimation, the optimized estimation, the signal reconstruction and the signal separation.

Given a signal \( S(t) \) that mixed by \( n \) LFM signal components, our task is to estimate the initial frequency, the frequency-modulating slope and the end frequency for every component and separate the signal components from the mixed signal. For simplicity, an LFM signal component is expressed as \( s(t) \). The process for estimating and separating every LFM signal component can be described as follows:
Step 1. Rough estimation. In the \( k \)-th iteration, we first compute the RWT of the mixed signal \( S^{(k)}(t) \). It should be noted that \( S^{(1)}(t) \) equals to the original mixed signal \( S(t) \). Then, as every LFM signal component corresponds to a peak in the RWT domain, we should search the highest peak that corresponds to the strongest component. According to Eq (1), the rough estimation of initial frequency \( f_s^{(k)} \) and frequency-modulating slope \( a^{(k)} \) can be computed by the coordinate values of the highest peak. Then, the end frequency \( f_e^{(k)} \) can also computed based on \( f_s^{(k)} \) and \( a^{(k)} \).

Step 2. Optimized estimation. The MP algorithm is used to search the more precise estimation around \( f_s^{(k)} \) and \( a^{(k)} \). It’s an optimization problem that can be described as

\[
\arg\min_{\delta_f} C_k(a, f_s) = \|S^{(k)}(t) - \exp[-j2\pi(f_s^{(k)} + a^{(k)} t^2)]\|^2
\]

where \( f_s^{(k)} \) and \( a^{(k)} \) are probable values in the searching scope \( (f_s^{(k)} - \delta_f, f_s^{(k)} + \delta_f) \) and \( (a^{(k)} - \delta_a, a^{(k)} + \delta_a) \). The searching speed and accuracy depend on the step length and the searching scope.

Step 3. Signal reconstruction. The strongest signal component \( s^{(k)}(t) \) can be reconstructed based on the optimized estimation. Then, we could separate \( s^{(k)}(t) \) from \( S^{(k)}(t) \), and the updated residual signal \( S^{(k+1)}(t) \) is obtained:

\[
S^{(k+1)}(t) = S(t) - \sum_{i=0}^{k} s^{(i)}(t)
\]

Step 4. Threshold Judging and Iteration. If the energy of residual signal \( S^{(k+1)}(t) \) reach the threshold, the algorithm will be ended, otherwise the algorithm turns to Step 1 and start a new iteration. The threshold judging can be represented as

\[
E_{S(t)} \begin{cases} \leq \xi E_{S(t)} & \text{stop iteration} \\ > \xi E_{S(t)} & \text{turn to step(1)} \end{cases}
\]

where \( \xi \) is the parameter of the judging threshold.

3. Experiment and analysis
To validate the effectiveness of the proposed method, a mixed signal that contains three LFM signal components is used for estimation. The length of three signals is 128 points. The parameters sets \( (a, f_s, f_e) \) of each signal are \((0.6, 0.05, 0.35), (0.3, 0.1, 0.25), (-0.3, 0.3, 0.15)\), where \( f_s \) and \( f_e \) are normalized frequencies. The step length of \( f_s \) in MP algorithm is 0.001, while that of \( a \) is 0.001. The threshold parameter \( \xi \) is 0.01. We conducted the experiments under different SNR. The SNR is set to -10 dB, -5 dB, 0 dB and 5 dB. The experimental results are shown in Table 1.

It can be seen from the Table 1 that the average estimation accuracy of the estimation is satisfactory under different SNR. Meanwhile, the average estimation error decreases with the SNR improved.

| SNR (dB) | -10  | -5  | 0   | 5   |
|---------|------|-----|-----|-----|
| LFM1    | \(a\) | 0.6394 | 0.5912 | 0.6089 | 0.6028 |
|         | \(f_s\) | 0.0398 | 0.0615 | 0.0432 | 0.0491 |
|         | \(f_e\) | 0.3595 | 0.3521 | 0.3517 | 0.3505 |
|         | \(a\) | -0.3246 | -0.3508 | -0.3268 | -0.3052 |
| LFM2    | \(f_s\) | 0.3225 | 0.3125 | 0.3085 | 0.3015 |
|         | \(f_e\) | 0.1602 | 0.1391 | 0.1451 | 0.1489 |


Table 1: Parameters of LFM signals

| Signal | a     | fs    | f0   |
|--------|-------|-------|------|
| LFM3   | 0.2580| 0.2680| 0.3170 |
|        | 0.3007| 0.1115| 0.1065 |

Average Accuracy 90.95% 92.04% 95.46% 99.34%

Figure 2 shows the WVD and the RWT of the mixed signal while SNR is 5dB. It can be seen from the Figure 2(a) that lines in the WVD are fuzzy, and it’s hard to find three lines in the figure. Figure 2(b) is the RWT of the mixed signal, and it is also hard to make sure the number of peaks.

![Figure 2. The WVD and RWT of the mixed LFM signal](image)

Then the proposed method is applied to separate and estimate signal components within the mixed signal. The strongest peak in the RWT domain is found after the rough estimation and optimized estimation. The parameters of the strongest LFM signal component \( s^{(1)}(t) \) are estimated as \( (0.6028, 0.0491, 0.3505) \). Figure 3 shows the WVD and the RWT of the residual signal \( S_1(t) \) after separating the strongest LFM component from the \( S(t) \). It can be seen from the Figure 3(a) that, compared to Figure 2, the lightest line in Figure 2(a) is cleared. The strongest peak in RWT domain is also disappeared.

![Figure 3. The WVD and RWT of the residual signal \( S_1(t) \)](image)

After the first iteration, the strongest component within the residual signal \( S^{(2)}(t) \) is estimated based on the peak searching and parameter estimating of the Figure 3(b). The parameters of \( s^{(2)}(t) \) are estimated as \( (-0.3052, 0.3015, 0.1489) \). Then we continue to separate \( s^{(2)}(t) \) from \( S^{(2)}(t) \), and the
updated residual signal $S^{(3)}(t)$ is obtained. Figure 4 shows the WVD and the RWT of $S^{(3)}(t)$. It can be seen from the Figure 4 (a) that the lighter line in Figure 3(a) is cleared. The stronger peak in Figure 3(b) is also disappeared.

Finally, the third LFM signal component $s^{(3)}(t)$ is estimated based on the peak at the RWT domain. The estimated parameters are (0.3007, 0.0997, 0.2500). After three iterations, the energy of the residual signal reach to the judging threshold, and three LFM signals are precisely estimated and separated.

4. Conclusion
In this paper, a parameter estimation method for mixed LFM signal is proposed. A two-stage optimization strategy is designed based on the RWT and MP, therefore the method is able to realize a precise estimation of signal components. Experimental results show that the method is able to precisely estimate and separate mixed LFM signals under different SNR.

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