Matter couplings in partially broken extended supersymmetry

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Abstract

We use nonlinear realizations to describe the spontaneous breaking of $N=2$ supersymmetry to $N=1$ in four dimensions. We identify the Goldstone multiplet with an $N=1$ chiral superfield, and show that chiral $N=1$ matter is consistent with the partially broken $N=2$ supersymmetry. We find that the chiral matter can be in any representation of the gauge group; no mirror particles are required. We present the Goldstone action and the general couplings to $N=1$ matter to the first nontrivial order in the scale of symmetry breaking.

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1. Introduction.

In four dimensions, supersymmetric theories can be classified by an integer \( N \) that counts the number of supersymmetries. \( N \) can run from one to eight, but most phenomenological studies focus on \( N = 1 \). This is because \( N = 1 \) is the simplest supersymmetry, and the only one that permits fermions to lie in complex representations of symmetry groups \([1]\). Therefore if supersymmetry is relevant to physics at the weak scale, it is likely to be \( N = 1 \), broken to \( N = 0 \).

One cannot help but wonder whether this \( N = 1 \) supersymmetry might be a remnant of some \( N > 1 \) extended supersymmetry, broken to \( N = 1 \) at a higher scale. There is a general argument which implies that extended supersymmetry cannot be spontaneously broken to \( N = 1 \) in four dimensions. The argument runs as follows \([1]\): Suppose that there are two supersymmetries, one broken and one unbroken. Since one supersymmetry is preserved, its supercharge must annihilate the vacuum. Because of the supersymmetry algebra, the Hamiltonian must also annihilate the vacuum. This implies that the other supercharge must annihilate the vacuum, so the second supersymmetry cannot be broken.

Hughes, Liu and Polchinski \([2, 3]\) showed how to evade this argument. They considered \( N = 2 \) supersymmetry, spontaneously broken to \( N = 1 \), and identified the Goldstone multiplet with a four-dimensional membrane propagating in six-dimensional superspace. They found its invariant action, and demonstrated that this system realizes the partial breaking of extended supersymmetry. This was a remarkable result, but one that was difficult to use because of the membrane approach. In particular, it was not clear how to define chiral \( N = 1 \) matter on the membrane, nor whether an invariant matter action could be constructed.

These shortcomings motivated us to reconsider the partial breaking of \( N = 2 \) supersymmetry. We do not use a membrane, but instead we work in four-dimensional \( N = 1 \) superspace and use the techniques of nonlinear realizations to realize the second supersymmetry. (For early work along these lines, see \([4]\).) This approach keeps the unbroken \( N = 1 \) supersymmetry manifest, and allows \( N = 2 \) invariants to be constructed with the help of the Goldstone multiplet. It permits us to examine the couplings of the Goldstone multiplet to \( N = 1 \) supersymmetric matter, and study any restrictions on \( N = 1 \) matter that come from the second supersymmetry.

In this paper we will show that the Goldstone multiplet of partially broken \( N = 2 \) supersymmetry is described by an \( N = 1 \) chiral superfield, and that the chiral \( N = 1 \) representation is preserved by the second supersymmetry. We will prove that any \( N = 1 \) matter can be consistently coupled to the Goldstone multiplet. We will present the general Goldstone-matter coupling to the first nontrivial order in the supersymmetry breaking parameter. The underlying complex geometry and full nonlinear structure will be discussed elsewhere \([5]\).

By construction, the Goldstone-matter coupling exhibits \( N = 2 \) supersymmetry, spontaneously broken to \( N = 1 \). We will see that when the \( N = 1 \) superpotential is \( R \)-symmetric, the theory also exhibits an \( SO(5, 1) \) symmetry, spontaneously broken to \( SO(3, 1) \times SO(2) \). This extra symmetry provides a hint about the origin of \( R \) symmetry and the partial breaking of extended supersymmetry.
2. The Goldstone multiplet and nonlinear realizations.

The $N = 2$ supersymmetry algebra can be written in the following form,

$$\{Q_\alpha, \bar{Q}_\alpha\} = 2\sigma^{\alpha\dot{\alpha}}_a P_a, \quad \{S_\alpha, \bar{S}_\alpha\} = 2\sigma^{\alpha\dot{\alpha}}_a P_a,$$

$$\{Q_\alpha, S_\beta\} = 2\epsilon_{\alpha\beta} Z, \quad \{Q_\alpha, S_\beta\} = 2\epsilon_{\alpha\beta} Z,$$

where $Q_\alpha$ and $S_\alpha$ are the supersymmetry generators, $P_a$ the four-dimensional momentum operator, and $Z$ is a complex central charge, which can be viewed as translation generator along two additional spacelike directions, $Z = P_1 - iP_5$. In what follows, we take $Q_\alpha$ to be the unbroken $N = 1$ supersymmetry generator and $S_\alpha$ to be its broken counterpart.

In general, spontaneous supersymmetry breaking gives rise to a massless spin-1/2 Goldstone field $\psi_\alpha(x)$. When $N = 2$ supersymmetry is broken to $N = 1$, the Goldstone fermion is part of a massless $N = 1$ supersymmetry multiplet. There are two such multiplets that contain spin 1/2: the chiral multiplet (1/2, 0), and the vector multiplet (1, 1/2). The supersymmetry anticommutator $\{Q_\alpha, S_\beta\} = 2\epsilon_{\alpha\beta} Z$ motivates us to choose the chiral multiplet, whose complex spin-0 field $\phi(x)$ can be interpreted as a Goldstone boson for the central charge generator of the supersymmetry algebra.

Note that the geometrical dimension of a Goldstone field is opposite to that of the corresponding broken generator. Since $[Q] = [S] = 1/2$, $[Z] = 1$, we find $[\psi] = -1/2$, $[\phi] = -1$, which is in accord with the $N = 1$ transformation law $\delta\phi = \epsilon^\alpha \psi_\alpha$. In what follows, we will show that it is consistent to take the chiral $N = 1$ multiplet ($\phi, \psi_\alpha$) to be the Goldstone multiplet of the partially broken $N = 2$ supersymmetry. This result was obtained in [2, 3] using different arguments.

In addition to the physical fields ($\phi, \psi_\alpha$), the off-shell Goldstone multiplet contains a complex auxiliary field $\bar{\xi}(x)$. The $N = 1$ transformation law $\delta\bar{\psi}_\alpha = \epsilon_\alpha \bar{\xi} + \ldots$ implies that the geometrical dimension of $\bar{\xi}$ is 0. As we will see, this field can be interpreted as a Goldstone boson parametrizing the coset $SU(2)/U(1)$, where $SU(2)$ is an automorphism group of $N = 2$ supersymmetry. ($Q_\alpha$ and $S_\alpha$ form an $SU(2)$ doublet, while $P_a$ and $Z$ are $SU(2)$ singlets). Since the auxiliary field equation of motion is $\bar{\xi} = 0$, the Goldstone action explicitly breaks $SU(2)$ to $U(1)$. Nevertheless, as we will see later, it is useful to keep track of this $SU(2)$ group.

The formalism of nonlinear realizations provides a systematic way to study the properties of Goldstone fields. This formalism was first developed for internal symmetries [7] and later generalized to space-time symmetries [1]. The procedure is as follows: Let $G$ be the full symmetry group, and $H$ the unbroken subgroup. The generators of $G$ can be divided into three sets: space-time generators $\Gamma_A$, spontaneously broken internal generators $\Gamma_r$, and the unbroken generators $\Gamma_i$ of the subgroup $H$. The $\Gamma_i$ form an ordinary Lie algebra, $[\Gamma_i, \Gamma_j] = C_{ij}^k \Gamma_k$, while the generators $\Gamma_A$ and $\Gamma_r$ span representations of $H$, $[\Gamma_i, \Gamma_A] = C_{iA}^B \Gamma_B$, $[\Gamma_i, \Gamma_r] = C_{ir}^s \Gamma_s$.

For any group $G$ and subgroup $H$, we choose to parametrize the coset $G/H$ as follows,

$$\Omega = \exp(iX^A \Gamma_A) \exp(i\xi^i \Gamma_i),$$

where the $X^A$ are spacetime coordinates and $\xi^i = \xi^i(X)$ are the Goldstone fields that correspond to the broken generators $\Gamma_r$. Then the group $G$ can be realized on the space-
time coordinates and Goldstone fields in the following way,
\[ g\Omega = \Omega' h, \]
where \( g \in G, \Omega' = \exp(iX'^A\Gamma_A) \exp(i\xi'^r(X')\Gamma_r) \). In this expression, \( h = \exp(i\lambda^i(g, X, \xi)\Gamma_i) \) is an element of \( H \) that is chosen to restore the form of \( \Omega \). With these rules, the coordinates and Goldstone fields transform linearly under \( H \).

Given a field \( \chi(X) \) that transforms linearly under \( H \), one can define a realization of \( G \) on \( \chi(X) \) using the element \( h \),
\[ \chi'(X') = \exp(i\lambda^i(g, X, \xi)\Gamma_i) \chi(X). \]
The generators \( \Gamma_i \) are in the appropriate representation of \( H \). Note that this is a covariant transformation law for all elements \( g \in G \).

To construct an invariant action, it is useful to define connection and vielbein forms with the help of the Cartan one-form, \( \Omega^{-1}d\Omega \). The Cartan form can be expanded with respect to the \( G \) generators
\[ \Omega^{-1}d\Omega = i(\omega^A\Gamma_A + \omega^r\Gamma_r + \omega^i\Gamma_i) \]
to give the forms \( \omega^A, \omega^r \) and \( \omega^i \). From
\[ (\Omega^{-1}d\Omega)' = h(\Omega^{-1}d\Omega)h^{-1} + hdh^{-1}, \]
we see that \( \omega^A \) and \( \omega^r \) transform homogeneously under \( G \), while \( \omega^i \) transforms by a shift.

The vielbein \( E_M^A \) is obtained by expanding the space-time form \( \omega^A \) with respect to the coordinate differentials \( dX^M \),
\[ \omega^A = dX^M E_M^A. \]

The covariant derivatives of the Goldstone fields are found by expanding the Goldstone forms \( \omega^r \) with respect to \( \omega^A \)
\[ \omega = \omega^A D_A \xi^r. \]
Finally, the connection one-form \( \omega^i \) can be used to construct covariant derivatives of the fields \( \chi \),
\[ D\chi = \omega^A D_A \chi = (d + i\omega^i\Gamma_i)\chi. \]
These are the building blocks that can be used to construct actions invariant under \( G \).

3. Goldstone constraints and action.

To apply the above formalism to partially broken \( N = 2 \) supersymmetry, we need to specify the groups \( G \) and \( H \). The group \( G \) must contain the supersymmetry transformations (II), but it can also include various automorphisms of the supersymmetry algebra. The maximal automorphism group is \( SO(5,1) \times SU(2) \), where \( SO(5,1) \) is the \( D = 6 \) Lorentz group. (Under \( SO(5,1) \), the generators \( P_a \) and \( Z \) form a \( D = 6 \) vector, while the supercharges form a \( D = 6 \) Majorana-Weyl spinor). We denote the maximal group \( G \) as
where $SO(3,1) \times SO(2) \subset SO(5,1)$, $U(1) \subset SU(2)$, and $SO(3,1)$ is the $D = 4$ Lorentz group.

In what follows, we take $G/H = G_{\text{max}}/H_{\text{max}}$. As we will see, this choice is consistent with $N = 1$ chirality and Kähler invariance. (In section 5, we will consider other cosets $G/H$. We will see that chirality and Kähler invariance imply $G/H = G_{\text{max}}/H_{\text{max}}$.)

A parametrization of the coset $G_{\text{max}}/H_{\text{max}}$ involves the $N = 1$ superspace coordinates $X^A = (x^a, \theta^\alpha, \bar{\theta}_{\dot{\alpha}})$, as well as the Goldstone superfields $\Phi(x, \theta, \bar{\theta}), \ldots, \Xi(x, \theta, \bar{\theta})$:

\[
\Omega = \exp(i(x^aP_a + \theta^\alpha Q_\alpha + \bar{\theta}_{\dot{\alpha}} \bar{Q}_{\dot{\alpha}})) \exp(i(\Phi Z + \Phi \bar{Z} + \Psi^\alpha S_\alpha + \bar{\Psi}_{\dot{\alpha}} S_{\dot{\alpha}})) \\
\times \exp(i(\Lambda^a K_a + \bar{\Lambda}^{\dot{\alpha}} \bar{K}_{\dot{\alpha}} + \Xi T + \bar{\Xi} \bar{T})).
\]  

Here $\Lambda^a, \bar{\Lambda}^{\dot{\alpha}}$ are the Goldstone superfields associated with the generators $K_a, \bar{K}_{\dot{\alpha}}$ of $SO(5,1)/SO(1,3) \times SO(2)$,

\[
[K_a, K_b] = -2iL_{ab} - 2\eta_{ab} M, \quad [K_a, K_b] = [\bar{K}_a, \bar{K}_{\dot{b}}] = 0,
\]

where $L_{ab}$ and $M$ generate $SO(1,3) \times SO(2)$. Similarly, $\Xi, \bar{\Xi}$ are the Goldstone superfields for the broken generators $T, \bar{T}$ of $SU(2)/U(1)$,

\[
[T, \bar{T}] = 2T_0, \quad [T_0, T] = T, \quad [T_0, \bar{T}] = -\bar{T}.
\]

The symmetry transformations of the Goldstone superfields follow from (\ref{13}) and the relations,

\[
[K_a, P_b] = i\eta_{ab} Z, \quad [K_a, Z] = 0, \quad [K_a, \bar{Z}] = 2iP_a; \\
[K_a, \bar{Q}^{\dot{\alpha}}] = i\bar{\sigma}^{\dot{\alpha}}_a S_\alpha, \quad [K_a, \bar{S}^{\dot{\alpha}}] = -i\bar{\sigma}^{\dot{\alpha}}_a Q_\alpha; \\
[K_a, Q_\alpha] = [K_a, S_\alpha] = 0; \\
[T, Q_\alpha] = 0, \quad [T, S_\alpha] = Q_\alpha, \quad [T, \bar{Q}^{\dot{\alpha}}] = -\bar{S}^{\dot{\alpha}}, \quad [T, \bar{S}^{\dot{\alpha}}] = 0.
\]

In particular, to lowest order in the Goldstone fields, they are:

**Second supersymmetry** ($g = \exp i(\eta S + \bar{\eta} \bar{S})$)

\[
\delta \Phi = 2i\eta \theta, \quad \delta \Psi_\alpha = \eta_\alpha, \quad \delta \Lambda = \delta \Xi = 0.
\]

**$K, \bar{K}$ transformations** ($g = \exp i(rK + \bar{r} \bar{K})$)

\[
\delta \Phi = -r_a (x^a - i \theta \sigma^a \bar{\theta}) + \ldots, \quad \delta \Psi_\alpha = -r_a (\bar{\theta} \sigma^a)_{\alpha}, \\
\delta \Xi = 0, \quad \delta \Lambda^a = r^a + \ldots.
\]

**$T, \bar{T}$ transformations** ($g = \exp i(\beta T + \bar{\beta} \bar{T})$)

\[
\delta \Phi = -\bar{\beta} \theta + \ldots, \quad \delta \Psi_\alpha = i\bar{\beta} \theta^\alpha \\
\delta \Lambda^a = 0, \quad \delta \xi = \beta + \ldots.
\]
The $N=1$ superfields $\Phi(x, \theta, \bar{\theta}), \ldots, \bar{\Xi}(x, \theta, \bar{\theta})$ contain many more components than the physical Goldstone multiplet $\phi(x), \psi^\alpha(x), \bar{\xi}(x)$. Therefore the superfields must be properly constrained. To this end, we note that no $G_{\text{max}}$ tensors of dimension $-1, -1/2$ or 0 can be built from the physical fields. This motivates us to impose the following constraints [9]:

\[
\tilde{D}_a \Phi = 0, \quad \tilde{D}_a \Phi = 0, \quad \tilde{D}_a \Phi = 0 \quad (18)
\]

\[
\tilde{D}^\alpha \Psi^\beta = 0, \quad \tilde{D}^\alpha \Psi^\beta = 0 \quad (19)
\]

Except for the trace part $\tilde{D}^\alpha \Psi_\alpha = 0$, the constraints (19) are necessary to ensure the consistency of (18).

The constraints $\tilde{D}_a \Phi = 0, \quad \tilde{D}_a \Phi = 0 \quad \tilde{D}^\alpha \Psi_\alpha = 0$ allow us to express the Goldstone superfields $\Psi^\alpha, \Lambda^a$ and $\bar{\Xi}$ in terms of a single superfield $\Phi$ [10]. To lowest order, we find

\[
\Psi^\alpha = -\frac{i}{2} D^\alpha \Phi + \ldots; \quad \Lambda^a = -\partial_a \Phi + \ldots; \quad \bar{\Xi} = \frac{1}{4} D^2 \Phi + \ldots. \quad (20)
\]

The Goldstone action is uniquely determined by the requirements of $Q$ and $S$ supersymmetries. To this order, it is

\[
S_g = \frac{1}{a^2} \int d^4x d^2\theta d^2\bar{\theta} \left[ \varphi \bar{\varphi} - \frac{1}{2} (\partial_a \varphi)^2 \bar{\varphi}^2 - \frac{1}{2} (\partial_a \varphi)^2 \varphi^2 - \frac{1}{16} D^a \varphi D_a \varphi \bar{D}^\alpha \varphi + O(\varphi^6) \right]. \quad (22)
\]

It can be rewritten in terms of the original Goldstone superfields as follows,

\[
S_g = \frac{1}{a^2} \int d^4x d^2\theta d^2\bar{\theta} E \left[ \Phi \bar{\Phi} + \frac{1}{2} \Lambda^2 \bar{\Phi}^2 + \frac{1}{2} \bar{\Lambda}^2 \Phi^2 + \Psi^2 \bar{\Psi}^2 + O(\Phi^6) \right]. \quad (23)
\]

In this expression, $E=\text{Ber}(E_{M^{4}})$ is the superdeterminant of the vielbein, and $a$ is a constant of dimension 2 that corresponds to the scale of the second supersymmetry breaking. The action (22), (23) is invariant under $SO(5,1)$, but explicitly breaks $SU(2)$ down to $U(1)$.

4. Matter constraints and invariant action.

The above constraints imply that the chirality condition

\[
\tilde{D}_a \chi = 0 \quad (24)
\]

is consistent for a “matter” superfield $\chi$ in an arbitrary representation of $H_{\text{max}}$,

\[
\{D_\alpha, D_\beta\} \chi = 0. \quad (25)
\]
The solution to the chiral matter constraint (24) for an $H_{\text{max}}$ singlet is given in terms of an arbitrary holomorphic function

$$\chi = \chi(x_L, \theta_L), \quad (26)$$

where

\begin{align*}
x^a_L &= x^a - i\theta\sigma^a\bar{\theta} - i\Psi\sigma^a\bar{\Psi} + 2i\Lambda_b\bar{\theta}\sigma^b\bar{\Psi} + 2\Lambda_a\bar{\Phi} + O(\Phi^4), \\
\theta^a_L &= \theta^a - \Lambda^a(\bar{\Psi}\sigma)^a + O(\Phi^4). \quad (27)
\end{align*}

This set of holomorphic coordinates is closed under $G_{\text{max}}$; it generalizes the ordinary chiral $N=1$ superspace in the presence of the Goldstone superfield.

The $G_{\text{max}}$-invariant kinetic term for $\chi$ is given by

$$S_{\text{kin}} = \int d^4xd^2\theta d^2\bar{\theta} E K(\bar{\chi}i, \chi^j). \quad (28)$$

where $K$ is the Kähler potential. The action (28) is invariant under Kähler transformations $K \to K + F(\chi) + \bar{F}(\bar{\chi})$. This fact follows from a remarkable property of the Goldstone supervolume: an invariant integral of a (covariantly) chiral super field vanishes identically,

$$\int d^4xd^2\theta d^2\bar{\theta} E \chi(x_L, \theta_L) = 0. \quad (29)$$

This can be proven by passing from the real basis $(x, \theta, \bar{\theta})$ to a holomorphic basis $(x_L, \theta_L, \bar{\theta})$. In this basis the superdeterminant is itself holomorphic, that is, it does not depend on $\bar{\theta}$:

\begin{align*}
E_L &= E \times \text{Ber} \frac{\partial(x, \theta, \bar{\theta})}{\partial(x_L, \theta_L, \bar{\theta})} \\
&= 1 - \frac{1}{8} D^2(\Phi D^2\Phi) + O(\Phi^4). \quad (30)
\end{align*}

Because of this property, the superdeterminant $E_L$ can be used as a density for the superpotential term

$$S_{\text{superpot}} = \int d^4x_L d^2\theta_L E_L P(\chi^i). \quad (31)$$

The action (23), (28) and (31) is invariant under $N=2$ supersymmetry for any Kähler potential $K$ and superpotential $P$.

If the matter action (28), (31) is invariant under a rigid internal symmetry,

$$\chi \to e^{i\lambda} \chi, \quad \bar{\chi} \to \bar{\chi}e^{-i\lambda}, \quad (32)$$

it can be gauged by introducing an $N=1$ Yang-Mills superfield $V(x, \theta, \bar{\theta})$ that takes its value in the algebra of the internal symmetry group. Under a gauge transformation, $V$ transforms as follows,

$$e^V \to e^{i\lambda} e^V e^{-i\bar{\lambda}}. \quad (33)$$

Here $\lambda = \lambda(x_L, \theta_L)$ is an arbitrary holomorphic gauge parameter, and $\bar{\lambda}$ is antiholomorphic. The gauged $N=2$ supersymmetric action is obtained by replacing (28) by

$$S_{\text{kin}} = \int d^4xd^2\theta d^2\bar{\theta} E K(\bar{\chi}ie^{-V}, \chi^j). \quad (34)$$
The Yang-Mills action coupled to the Goldstone superfield is given by
\[ S_{\text{YM}} = \int d^4x d^2\theta L \left( E \text{ Tr}(W^a W_a) + c.c. \right) \] (35)
where \( W_a = i \mathcal{D}^2(e^V \mathcal{D}^a e^{-V}) \) is the Yang-Mills field strength. The actions (34) and (35) are invariant under the full group \( G_{\text{max}} \).

It is interesting to note that if \( K \) and \( P \) are such that the corresponding \( N = 1 \) theory is \( R \)-invariant (that is, invariant under rigid transformations \( \theta \rightarrow e^{i\alpha} \theta, \chi^i \rightarrow e^{i\alpha} \chi^i \)), then the resulting \( N = 2 \) theory can be made invariant under the full six-dimensional Lorentz group \( SO(5,1) \). To see this, let us first consider the superpotential. The chiral measure is invariant under both supersymmetries and the \( D = 4 \) Lorentz group, but transforms as \( (−1,−1) \) under the induced \( SO(2) \times U(1) \subset SO(5,1) \times SU(2) \).

\[ d^4x d^2\theta d^2\bar{\theta} \rightarrow e^{-2i\alpha}(d^4x d^2\theta d^2\bar{\theta}). \] (36)

Here \( \alpha = i\bar{\alpha} \Lambda - i\bar{\beta} \Xi + O(\Phi^3) \) is the holomorphic parameter of the induced \( SO(2) \times U(1) \). If the superpotential carries \( R \)-charge 2, we can identify the induced \( SO(2) \) and \( U(1) \) with \( U(1)_R \) on the matter fields. In this way the superpotential term can be rendered invariant under the full group \( G_{\text{max}} \).

The kinetic term is a little more subtle. Its measure is invariant under \( SO(2) \times U(1) \), so it is also invariant under the full group \( G_{\text{max}} \). Since the parameter \( \alpha \) is field-dependent, \( \bar{\alpha} \neq \alpha \), and the action (34) is not invariant under \( SO(5,1) \times SU(2) \) unless there is an analog \( U \) of an abelian gauge superfield to compensate the difference,

\[ U \rightarrow U + i(\alpha - \bar{\alpha}). \] (37)

Then the \( G_{\text{max}} \) invariant kinetic term is
\[ S_{\text{kin}} = \int d^4x d^2\theta d^2\bar{\theta} E K(\chi^i e^{-V - qU}, \chi^j). \] (38)

Such an abelian gauge field is built from the Kähler potentials \( K_{SO(5,1)}, K_{SU(2)} \) for the cosets \( SO(5,1)/SO(1,3) \times SO(2) \) and \( SU(2)/U(1) \), respectively
\[ U = K_{SO(5,1)} - K_{SU(2)}; \quad K_{SO(5,1)} = \Lambda \bar{\Lambda} + O(\Phi^4), \quad K_{SU(2)} = \Xi \bar{\Xi} + O(\Phi^4). \] (39)

To summarize, we have seen that any \( N = 1 \) matter lagrangian can be made \( N = 2 \) supersymmetric with the help of the Goldstone multiplet. If the \( N = 1 \) theory is \( R \)-invariant, the symmetry can be enlarged to include the full \( D = 6 \) Lorentz group \( SO(5,1) \). In this case the matter coupling is \( SU(2) \) symmetric, although the \( SU(2) \) symmetry is explicitly broken down to \( U(1) \) by the Goldstone action itself.

5. A comment on other cosets.

The above construction of the Goldstone multiplet and its matter couplings is based on the coset \( G_{\text{max}}/H_{\text{max}} \). Let us now briefly discuss the other possible cosets \( G/H \) related to \( N = 2 \) supersymmetry. Unlike \( G_{\text{max}}/H_{\text{max}} \), these cosets give rise to dimensionless tensors. Indeed, for \( G \subset G_{\text{max}} \), the algebra of \( G \) does not involve the \( SO(5,1)/SO(3,1) \times SO(2) \)
generators $K, \bar{K}$ or the $SU(2)/U(1)$ generators $T, \bar{T}$ (or both). This implies the existence of at least one dimensionless tensor, either $\mathcal{D}^\alpha \bar{\Psi}^\beta \sim \bar{\sigma}_a^\alpha \bar{\sigma}_a^\beta \phi + \ldots$ (if $K, \bar{K}$ are excluded), or $\mathcal{D}^\alpha \Psi_\alpha \sim D^2 \Phi + \ldots$ (if $T, \bar{T}$ are excluded), or both. The dots denote terms of higher order in the Goldstone fields.

As a result, the usual $G/H$ construction \cite{7} is ambiguous because the dimensionless tensors can be used to modify the covariant derivatives $\mathcal{D}_\alpha, \bar{\mathcal{D}}_\alpha$. One can show \cite{5} that the requirements of covariantly chiral $N = 1$ superfields and Kähler invariance restrict the nonminimal terms in just such a way as to effectively restore the coset with the maximal symmetry $G_{\text{max}}/H_{\text{max}}$.

6. Conclusions.

In this paper we reformulated partially broken $N = 2$ supersymmetry in a manifestly $N = 1$ supersymmetric way. We showed that any $N = 1$ matter can be made $N = 2$ supersymmetric with the help of the Goldstone multiplet. In particular, we found that $N = 1$ chirality is preserved in the presence of the Goldstone superfield. This implies that the fermions of this effective theory can be chiral, and no mirror fermions are necessary.

It is still not clear whether this effective theory discussed here can arise from a theory with linearly realized $N = 2$ supersymmetry. If such a theory exists, the $SO(5, 1)$ symmetry hints that the linear theory should most probably be formulated in six dimensions.

An unusual feature of the theories with partially broken supersymmetry is the existence of two conserved energy-momentum tensors \cite{2, 3}. This follows from the fact that in a supersymmetric theory, an energy-momentum tensor can be obtained by anticommuting the supersymmetry current with itself

$$\left\{ \bar{Q}_a, J^Q_{m\alpha}(x) \right\} = \int d^3 y \left\{ \bar{J}^Q_{\bar{a}0}(\bar{y}), J^Q_{m\alpha}(x) \right\} = 2\sigma_\alpha^{\bar{a}} T_{m\alpha}(x). \quad (40)$$

In the case of partially broken $N = 2$ supersymmetry, there are two conserved supersymmetry currents

$$J^Q_{m\alpha} = -\frac{2i}{a^2} (\sigma^a \bar{\sigma}_m \bar{\psi})_\alpha \partial_a \bar{\phi} + \ldots$$
$$J^S_{m\alpha} = \frac{2i}{a^2} (\sigma^m \bar{\psi})_\alpha + \ldots \quad (41)$$

that give rise to two symmetric conserved energy-momentum tensors $T^Q_{mn}$ and $T^S_{mn}$. The two energy-momentum tensors differ by a constant term,

$$T^S_{mn} = T^Q_{mn} + \frac{1}{a^2} \eta_{mn}. \quad (42)$$

This constant term leads to a new phenomenon in a theory of spontaneously broken symmetry: the algebras of the broken and unbroken supersymmetries differ. This is why the “no-go” arguments of \cite{1} can be avoided. However, the presence of two energy-momentum tensors is a problem when coupling these theories to supergravity. Work along these lines is in progress.

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[9] Similar constraints have been considered in the context of two-dimensional supersymmetry [3].

[10] This is an example of the so-called “inverse Higgs effect.” See E.A. Ivanov and V.I. Ogievetsky, *Teor. Mat. Fiz.* **25** (1975) 164.