Statistical properties of shear deformation of model block media and analogies with natural seismic processes

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Abstract. In this paper, the process of shear deformation of the medium formed by the cubic blocks is investigated experimentally. It is demonstrated that the deformation process is intermittent and accompanied by the radiation of acoustic perturbations. These perturbations obey statistical laws that are inherent in natural seismic processes: the frequency-energy scaling relation (the Gutenberg-Richter law) and the generalized Omori law for temporal decay of aftershocks. There are considered both the monodispersed medium and the mixture of blocks of two sizes. An important issue is the possibility to affect by small perturbations on the process of shear deformation. During the stimulation of block medium by small periodic, the existence of a critical frequency providing the smallest number of critical events in the block medium is identified. We also developed the algorithm for controlling the processes of shear deformation of block medium by means of external perturbations, namely, the designed device with inverse coupling provides the avoiding of large stresses. Taking into account the statistical similarity between shear and seismic processes, these results open up a perspective of the controlled impact on seismic processes.

Keyword: Granular media; Shear; Self-organized criticality; Seismic processes

Introduction

The rock massifs forming the lithosphere are extremely diverse, but there is an important characteristics of rocks that is inherent in almost all of them: it is discreteness. Discreteness is observed in a wide range of scale levels: for instance, the grains in rocks, the size of which is millimeters or their fraction; the pieces of rock that can be observed in quarries or mountains with centimeters or meters in size; the tectonic blocks having the dimensions of tens and hundreds of kilometers, the largest structural elements of Earth’s crust - tectonic plates extended over thousands, or even tens of thousands of kilometers. Block structurization is most brightly manifested on the boundaries of tectonic plates, where the rock materials were destroyed by earthquakes for a long time.

According to the concept proposed by Sadovsky et al., the geophysical medium is a thermodynamically open system consisted of hierarchically embedded blocks. In this system, the long-range spatial and temporal correlations take place causing the emergence...
of dynamical localized structures and other phenomena of self-organization. In particular, the dynamic behavior of the seismically active area is similar to the behavior of a system in the state of self-organized criticality (SOC) \cite{18,19}. On the basis of this concept, a number of models describing the statistical laws of earthquakes have been elaborated \cite{20,26}. In particular, the hierarchical block model taking the SOC state of seismic region into account is developed in \cite{28}.

It is worth noting that similar properties are also endowed with granular media under shear deformation \cite{15}. Experimental studies and computer simulation of deformation of granular massifs indicate that fluctuations of intergranular forces and velocities of granules can many times exceed their mean values \cite{27,29,30} that is common to seismic processes \cite{19} as well. The amplitude of fluctuations depends on the size and shape of the granules, the degree of irregularity of their forms, angularity, granular massif density, and shear speed \cite{31}. During the deformation of granular systems, the long-range correlations \cite{32} are intensified. Moreover, the displacement of granular media occurs intermittently \cite{29,30,33,34} and is accompanied by the radiation of stochastic acoustic disturbances. Investigation of these regularities and responses of granular media on external influences are important for understanding the seismic processes itself.

In this paper, we investigate experimentally the shear deformation of model block media consisted of cubic blocks. The statistical properties of acoustic perturbations released by considered block system, as well as the influence of perturbations on the deformation process are studied in more detail. From the foregoing provisions it follows that the behavior of this system is similar to the behavior of the geomedium in a seismic zone. Therefore, such studies shed the light on the unknown properties of natural seismic processes and possibility of artificial influence on them.

1 Experimental installation for the study of shear deformation of massif formed from cubic blocks

The study of shear deformation process of block media was carried out for both cases of external perturbations and their absence. The experimental installation is the box, which is made from plexiglass and consists of lower fixed part and upper movable one (Fig. 1). The lower part has internal dimensions of 0.2×0.3×0.07 m and is rigidly positioned on the table. The upper part has the same dimension. All contact surfaces of the lower part of box were pre-polished. To provide the directional sliding of the upper part, the guide plates were installed on the lower part. In turn, the upper part of the device is equipped with the limiters controlling the movement in the vertical direction. The piston that can move freely in a vertical direction is located on the block massif.

The movement of the upper part of the box is carried out using a traction device (Fig. 1). This device consists of a gear motor, which is connected to the cable shaft. One end of the rope is attached to the shaft, and the other to the force sensor, which is tightly tied to the front wall of the upper box. The diameter of the shaft is chosen so that it raises 0.1 m of cable for a minute and thus the device displaces the connected top part of the capacity connected to it by 0.1 m.

On the upper part of the box there are three accelerometers measuring the accelerations in three orthogonal directions. The force sensor on the front wall measures the reaction of
The block massif on the shear deformation.

The periodic small disturbances are generated by a low-frequency speaker with the power of 10 Watts in the frequency range of 50-1000 Hz.

To investigate the reaction of the block system under the influence of nonperiodic perturbations, namely, perturbations affecting the system when the traction force exceeds a certain threshold $F^\star$, a control unit, switching on the source of disturbances at moments when the condition $F \geq F^\star$ holds, is used. These perturbations are generated by a special device, which is installed in the center on the upper surface of the piston.

During experiments for preparation of block medium two types of plexiglass cubes having edges $l_1 = 10 \text{ mm}$ and $l_2 = 25 \text{ mm}$ respectively are used. Signals are recorded within one minute. During this time, the upper part of the box is displaced relative to the lower one at a distance of $y = 0.1 \text{ m}$.

## 2 The characteristics of block medium reaction on shear deformation

In the first series of experiments, let us consider how the magnitude of loading of the block medium affects the process of its shear deformation. The medium is studied to consist of 3000 cubes of the identical size of 10 mm. Medium’s response on a displacement of the upper part of box is recorded by means of the force sensor.

The temporal dependence of the traction force $F(t)$ registered with the force sensor is depicted in Fig. 2a. Analysis of these dependencies indicates a tendency to increase the force of reaction $F$ with increasing loading on the piston. The spectra of these dependencies are presented in Fig. 2b. All spectra are similar and close to the power relations with the same degree of power. This points to the similarity of processes at different loading, as well as to the large-scale invariance of the shear deformation process.

During the medium deformation, measurements of three components of acceleration $a(t)$ in acoustic waves generated by the medium are performed. The inset in Fig. 3a shows the
Figure 2: Temporal dependences (a) of the force $F$ at loading $P = 0, 30, 60, 90$ N (curves from bottom to top) and their Fourier spectra (b).

part of the signal $a_y(t)$ in the interval $t \in [49.78; 49.80]$ s recorded by the accelerometer in the direction of movement of the upper part of device. This signal is a sequence of perturbations generated by the block system. Each component $a_q$, $q = \{x, y, z\}$ is integrated in order to obtain the temporal dependence of the velocity component $v_q$.

Next let us introduce the quantities

$$e_i = \int_{t_i^b}^{t_i^e} \sum_{x,y,z} v^2_q(t) dt,$$

where $t_i^b$ and $t_i^e$ are the initial and final moments of the $i$th perturbation, i.e. the boundaries of separate pulse (the inset in Fig. 3(a)). It is clear that these quantities are proportional to the energies of the perturbations. As a result, we obtain the sequence $e_i$ which can be regarded as a sequence of energies of acoustic perturbations.

In particular, the sequence $e_i$ for the experiment when the loading on a box cover is $P = 120$ N is shown in Fig. 3(a). Now the question arises: does this sequence obey the statistical laws that take place for seismic processes?

At first, consider the cumulative complementary distribution of acoustic perturbations for energies. It can be approximated by the power relation

$$N(> e) = C e^{-\beta},$$

where $\beta = 0.82 \pm 0.04$, $C = 1.02 \pm 0.05$. From Fig. 3(b) it follows that the approximation matches the experimental distribution. Therefore, this distribution is of power nature as Gutenberg-Richter’s law. Moreover, the index $\beta$ belongs to the interval 0.8-1.05 which is typical for seismic processes [24].

The statistics of foreshocks and aftershocks caused by earthquakes is an important characteristics of seismic zones and their dynamics. Consider analogous phenomena associated with large emission of energy exceeding the threshold $e^* = 0.02$ (Fig. 4a). It turns out, model aftershocks attenuate according to the Omori law which is valid for natural aftershocks of large earthquakes [35]:

$$N = \frac{k}{(t + c)^p},$$
where the coefficients $k = 0.13 \pm 0.03$, $c = 0.01$, $p = 0.91 \pm 0.06$ (Fig. 4b). The presence of foreshocks and aftershocks testifies to the existence of temporal correlations in the process of generation of perturbations.

The next series of experiments is concerned with the study of influence of medium’s heterogeneity on its deformation. To do this, the mixture of cubic blocks of two sizes $l_1 = 10$ mm and $l_2 = 25$ mm is used. The shear deformation performs at loading $P = 60$ N. Temporal dependences of the traction force (Fig. 5) show that the process of deformation of both monodisperse and disperse media is not significantly different. However, when number of blocks with the edge $l_2 = 25$ mm increases, the stress increases as well. Note that if the bulk ratio of cubes of different volumes equals 0.5, the significant increase in traction force takes place leading to destruction of the cable (Fig. 5d).

3 Influence of external perturbations on shear deformation

According to the previous studies, the behavior of the block medium in the process of shear deformation is enriched by complicated stochastic reactions to shear loading. This response of the system obeys the statistical laws that are inherent in the natural seismic process. The question arises whether it is possible to make a change in the behavior of this complex system with the help of small disturbances?

As above, let us consider the shear deformation of the block medium formed by 3000 cubes of the size $l_1 = 10$ mm. The displacement is carried out with the device described above. The periodic perturbations of the 50-1000 Hz frequency range are injected into the medium with the help of acoustic speaker located on the surface of piston.

To check what kind of signal is entered the medium, an additional accelerometer is installed in the bottom of the piston. Fig. 6a shows the accelerations, measured by this sensor,
Figure 4: Dependence of the average number of acoustic disturbances on time to a major earthquake (a); power approximation of aftershocks (b). Here \( n_0 \) stands for the number of main shocks, \( n \) is the total shock number, and the 95% confidence intervals are shown.

and its Fourier spectrum at the 100 Hz frequency of input periodic signal (Fig. 6b). There is the main maximum in the diagram, whereas the spectral amplitudes of other harmonics are insignificant. Similar spectra have signals received for input periodic perturbations at frequencies of 50, 300, 500, 1000 Hz. From this it follows that the signal passing through the piston is almost not distorted.

Thus, after supporting explanations, let’s return to the process of deforming under the action of traction force \( F \). To analyze the influence of external perturbations, jumps of force are calculated as the difference between adjacent local maxima and minima. These jumps of force are associated with the reaction of the block media to the shear. The constructed distributions of the number of jumps at their intensity for the five frequencies of periodic perturbations are shown in Fig. 7a. From this figure it follows that these distributions are close to power functions and depend on the frequency of perturbations. The dependence of the power index on the frequency is shown in Fig. 7b. It turns out that at a 500 Hz frequency the index has a local maximum. That is, at this frequency, the number of large jumps of force is the smallest. In addition, the maximum values of jumps are the smallest among all frequencies. This allows us to conclude that under these conditions the deformation process is the most smooth and ”soft”, without sharp jumps of force.

The next series of experiments is concerned with studies of influences of external non-periodic perturbations on the shear process. In this case the device with inverse coupling is used in order to send the signal in the medium when the traction force reaches some threshold value \( F^* \). The block medium consists of 3000 cubes of the 10 mm sizes. In all experiments, the rate of deformation was the same \( v = 0.1 \) m/min. Figure 8 shows one of the force impulses generating by the device. The amplitude of the pulse is \( f_m = 44 \) N, lasted for \( \tau = 1 \) ms. The experiments are carried out at \( F^* = 200, 150, 125 \) N.

In all experiments, the medium was loaded with a weight of \( P = 60 \) N. The temporal dependencies of the traction force for these three threshold values are plotted in Fig. 9 which also shows the temporal dependence without external action. From this figure it follows that acting on the medium with external perturbations can avoid large tensions. It should be
Figure 5: Temporal dependences for the traction force when the loading on a plate $P = 60\, \text{N}$ and the medium consists of: a – identical cubes of size 10 mm; the mixture of cubes of sizes 10 mm and 25 mm in proportions: b – $80\% \times 20\%$, c – $70\% \times 30\%$, d – $50\% \times 50\%$.

noted that the reduction of the traction force to a value smaller than the threshold value is often achieved not by a single strike. This is well illustrated by Fig. 9b, c, where the perturbations are repeatedly sent in the medium. The experiments indicate that the use of such a mechanism makes the process of shear deformation smoother and that occurs at lower stresses.

4 Concluding remarks

Summarizing, it should be noted that experiments on the shear deformation of the block medium exhibited statistical similarity with natural seismic processes. The acoustic perturbations released by the block medium during shear deformation obey the power-like distribution similar to Gutenberg-Richter’s law. It turned out that the power index lays in the range which is typical for natural earthquakes. Moreover, for large acoustic disturbances the foreshocks and aftershocks were observed as well. It was shown that model aftershocks attenuate in accordance with the power law with the index close to 1, i.e. Omori’s law, that coincides with the statistical properties of seismic process.

Our experiments with different loads revealed the similarity of processes of shear deformation, as well as for mixtures of blocks of two sizes at different proportions of these blocks. The spectra of traction forces are described by the power relations.

It is found out that the action by weak periodic waves with distinct frequencies on the block medium which is in the state of shear deformation affect the medium’s state. There exists the frequency ($\sim 500\, \text{Hz}$) when maximal effect of periodic stimulus is manifested. The experiments have also shown that acting on the medium with small perturbations when the force of tension of a certain threshold value can be achieved, it is possible to attain smoother deformation. They also revealed that the smaller this threshold, the smoother the deformation. This indicates that the medium becomes smoother.

Finally, it should be noted that the detection of statistical regularities described by power laws testify that the block medium evolves in the state of self-organized criticality. This
Figure 6: The signal recorded by the accelerometer at the bottom of the piston (a) and its spectrum (b).

state has quantitative characteristics which are similar to phenomena in seismic zones. This similarity can open new perspectives to control the seismically active zones due to the possibilities to affect the nonequilibrium block medium via the weak loading.

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Figure 7: The distribution of force jumps at different values of perturbation frequencies \( \Omega \) (a), the dependence of the power index \( \beta \) in the approximation of distributions on the frequency of perturbation (b).

Figure 8: The profile of impulse generated by the device.

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Figure 9: Time dependencies of traction force $F(t)$ during shear deformation of massif loaded by the force $P = 60\text{N}$ and stimulated by periodic signals of magnitude $f^m = 44\text{N}$ when it is chosen the different threshold values: $F^* = 200\text{ N}$ (b), $F^* = 150\text{ N}$ (c), $F^* = 125\text{ N}$ (d). For comparison, the profile of $F(t)$ without external action (a).

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