Particle swarm optimizers (PSO) were first introduced by Kennedy and Eberhart as stochastic algorithms which seek optimal solutions to functions through the use of swarm intelligence. The main theme of PSO is that many particles are allowed to explore a function space. As each particle relocates it inputs its coordinates into the objective function for evaluation. Particles are assigned directions and magnitudes for motion based on distances to the best global functional evaluation outcome (g), and/or their individual best locations (p). Traditionally the positions (X) of particles and their velocities (V) are updated as follows,

$$X_i = X_i + V_i$$

$$V_i = V_i + c_1 \cdot \text{runif}(0, 1) \cdot (p_i - x_i) + c_2 \cdot \text{runif}(0, 1) \cdot (g - x_i)$$

Where $i$ denotes a particle identifier, $c$ represents user-defined weighting factors, and runif(0,1) generates a uniformly distributed random number between 0 and 1.

This algorithm has been used with heuristic success for many optimization problems. However, it is not especially robust to local minima in multimodal functions. The merit of the work contained herein is the introduction of a robust PSO algorithm based on the physical process of crystallization.

The Crystallization Particle Swarm Optimizer (CPSO), originally developed for surface plasmon resonance sensor optimizations follows similar formulas. The main deviations from the traditional movement scheme is that local communications were not considered ($c_1 = 0$)
and that the L_1 displacement factor was replaced by a scaling vector (e). The vector e is most usefully related to experimentally attainable resolutions in independent variables.

\[ X_i = X_i + V_i \]

\[ V_i = \pm c_2 \cdot \text{runif}(0, 1) \cdot e_i \]

A scheme which divides particle behavior into three phases based on iteration count was introduced. The phases consist of diffusion, directed motion, and nucleation. The regions for where these phases begin and end are user defined. However, these phases can be visually assessed by a dynamic scalar term which influences the random motion of particles in accordance with the complementary error function (Figure 1). This scalar term, the chaos factor, is implemented by simple multiplication with the velocity of each particle.

![Figure 1: Chaos factor vs iteration progress. Depending on the chaos factors behavior, the phases of the CPSO algorithm are introduced.](image)

When the chaos factor is in its maximal region, the particles randomly relocate from their previous locations without any attraction toward a global best position (diffusion). In
order to make the diffusion phase efficient optimized spatial partitioning methods such as orthogonal nearest neighbor repulsive agent optimization or centroidal voronoi tessellations should be employed [2]. During the directed motion phase, or downward sloping region of the chaos factor, the balance between random motion and motion towards the global best gradually trades until nucleation/convergence inevitably occurs. One final influencing factor was introduced to this algorithm which allowed for less confined searches during the nucleation stage. When particles were within a preset distance (typically 2 e) of the global best their velocity contribution towards the best was muted so that only random motion remained.

1 Results and Discussion

The CPSO algorithm was compared with the standard PSO2011 algorithm [3] introduced by Clerc et al. Both algorithms attempted to find the global minima in multimodal and unimodal test functions with 1000 functional evaluations for each particle (30 replicates). The population size was varied from 5 to 20 in increments of 5 and the resulting optimized values were recorded (Table I). The test evaluated the following functions in $\mathbb{R}^3$: griewank, rosenbrock, rastrigin, and parabola.

Every multimodal function which was optimized by the CPSO algorithm featured a lower mean and standard deviation than the respective trials performed with the PSO2011 algorithm. The order of magnitude discrepancies for the unimodal parabola function can be explained by the scaling parameters employed. Despite the fact that CPSO is limited to such parameters it often returned mean solutions which were orders of magnitude lower than that of a standard algorithm with fewer points.

The efficacy of CPSO is directly linked to the manner in which it handles multimodal function spaces. The diffusion step is hypothesized to be the most important condition in overcoming convergence toward local minima. Future theoretical investigations and tests on this algorithm will be reported elsewhere.
Table I: Mean and standard deviations of results obtained from test functions

| Function | Griewank                   | Rosenbrock                  | Rastrigin                  | Parabola                   |
|----------|----------------------------|-----------------------------|----------------------------|----------------------------|
|          | Population 5 10 15 20      | Population 5 10 15 20      | Population 5 10 15 20      | Population 5 10 15 20      |
| CPSO     | mean 1.0E-04 4.7E-05 4.5E-05 2.4E-05 | mean 0.97 0.47 0.42 0.13  | mean 7.7E-02 4.1E-02 2.3E-02 1.6E-02 | mean 3.8E-04 2.2E-04 1.2E-04 6.9E-05 |
|          | sd 6.8E-05 3.5E-05 3.0E-05 1.7E-05 | sd 0.81 0.67 0.52 0.37     | sd 3.9E-02 3.2E-02 2.1E-02 1.2E-02 | sd 1.8E-04 1.7E-04 1.1E-04 6.8E-05 |
| PSO2011  | mean 0.30 0.15 0.16 0.16 | mean 3.1 1.2 0.85 0.57    | mean 3.7 1.9 1.8 1.6      | mean 1.4E-06 6.5E-09 5.6E-09 5.2E-09 |
|          | sd 0.21 8.9E-02 6.4E-02 5.0E-02 | sd 2.7 1.7 1.5 1.3         | sd 3.0 1.1 2.0 1.1  | sd 7.5E-06 2.9E-09 2.8E-09 3.1E-09 |

References

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