Contract coordination of emergency supply chain with government participation considering risk aversion of participants

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Abstract. For governments and nonprofit organizations such as the Red Cross, charity federation, through the contract form emergency relief materials reserve not for profit, but to cope with the future the possibility of natural disasters and public health events, the occurrence of natural disasters and public health events has great uncertainty, which makes the demand for emergency relief supplies have great randomness. On the basis of predecessors' research, according to the use of options in the theory of supply chain coordination, to build the mathematical model based on option contract, research shows that option contracts can be achieved under the conditional value at risk of emergency relief supply chain coordination, in order to improve the effectiveness of emergency relief supply chain, with the increase of Co option purchase price, the government will increase by option order quantity q**, Q** way to reduce the initial order quantity q reduce material procurement cost; When the option purchase price Co is fixed, the supplier's risk aversion increases with the decrease of the risk preference coefficient. In this case, the government, as the purchaser, needs to provide a higher option strike price Ce to encourage the supplier to produce according to the optimal output of the supply chain system.

1. Introduction
China is a country with frequent and serious natural disasters, and the two public health emergencies in the 21st century also caused great losses to the country. However, the research on the coordination of the supply chain of emergency and disaster relief materials in China is still insufficient. Option contract is a kind of new contracts, with the aid of financial thought emergency supplies of option contract is to point to in front of the incident happened, the government with emergency supplies manufacturing enterprises signed contracts, contracts require emergency supplies the purchase price and contract term, within the prescribed time limit, the government has the purchase price in accordance with the contract or the right not to purchase emergency supplies. Option contracts stipulate the purchase price and duration of emergency supplies, but do not stipulate whether the government must purchase them.

The main research content of this paper is to consider the contractual coordination of the emergency supply chain of participants' risk avoidance under the premise of government participation, and discuss how to guarantee the supply chain coordination of emergency epidemic prevention supplies and life supplies through option contracts. Many scholars have studied the contract coordination of supply chain from different perspectives. Yefeng Li and Fang Li put forward the emergency supply chain coordination model under the constraint of cost [1]. Chunfeng Dong, Yanping
Meng and others put forward a three-level emergency supply chain coordination with asymmetric production cost information [2]. Jun Tian, Yongling Ge, Congcong Hou proposed a government-led model of emergency material procurement based on real option contracts. [3]. Qun Xiao, Shihua Ma put forward the supply chain repurchase contract under the information asymmetry of promotion effort cost [4]. Liquan Hu, Jun Tian, Yengzhong Feng proposed a production model of emergency supplies based on the production capacity of the supplier [5].

2. Description of the problem and basic assumptions
This chapter will explain and hypothesize the problems studied.

2.1. Problem description
The emergency supplies targeted in this paper are characterized by large demand and short production and preparation period, such as medical masks and protective clothing. In the model, the government and the supplier would enter into option contracts. After both parties determine the relevant contract parameters, the government will determine its initial purchase quantity. This part will be purchased through the wholesale price contract, and the government will be responsible for its own reserve, which is mainly used to deal with the initial demand of emergency supplies in the event of a disaster. At the same time, the government shall also determine the amount of its option orders, stipulating that when the demand for emergency supplies exceeds its own reserves, it shall have the right to require the supplier to supply no more than the quantity ordered by the option at the agreed purchase price.

2.2. Model basic assumptions
In order to facilitate the research, without changing the nature of the problem. The following assumptions are made: (1) the government and the supplier sign a single-period option contract; (2) The expiration date of the contract is the same as the shelf life of the emergency supplies; (3) The government's supplier for physical procurement and the option procurement supplier are the same enterprise; (4) The government and the suppliers operate in full accordance with the market mechanism, and the government cannot interfere in the decision-making of the suppliers by coercive administrative means; (5) Information between the government and the supplier is completely symmetrical, and they are all completely rational decision makers; (6) The demand for emergency supplies is a small probability event. It is assumed that the maximum demand for emergency supplies can be determined based on previous historical data, and its probability distribution function can be estimated. (7) The fixed cost is relatively small in the supplier's production cost. Here, only variable cost is considered, that is, it is assumed that the production cost of each unit of emergency supplies of the supplier is the same. (8) The government and suppliers form a one-to-one two-level supply chain. The option contract they sign is an American option contract with an uncertain time and an uncertain amount of execution, which is aimed at emergency supplies for medical epidemic prevention supplies.

2.3. Symbols that
The main parameter symbols used in this paper and their explanations are as follows:
C: Production cost of emergency supplies of suppliers;
w: Wholesale prices of unit emergency supplies for initial orders paid by the government to suppliers;
Co: Unit option purchase price;
Ce: unit option strike price, Ce > C;
Q: Initial government order quantity;
Q1: Procurement volume (production volume) under the centralized decision-making of supply chain;
q: The amount of options purchased by the government under the option contract;
v: The residual value of the unit's emergency supplies;
h: The cost of the unit's emergency supplies in short supply;
3. Model building
The supply chain system is composed of a leading purchaser (government) and a supplier (producer). The products of epidemic prevention materials in the supply chain have the characteristics of short life cycle, long advance order period and low salvage value. First of all, it is assumed that both participants in the whole supply chain system are completely rational and risk preference is risk neutral. In this decision-making process, the government plays a dominant role, and the whole dynamic game process is under the condition of complete information and perfect information.

3.1. Centralized decision-making model of emergency supply chain under government leadership
Firstly, the centralized decision-making of the emergency supply chain under the leadership of the government is analyzed. In this case, the government as the purchaser and the producer as the supplier act as a unified whole. The whole system is regarded as a coordinated whole. Both the purchaser and the supplier aim to maximize the total benefit of the entire emergency supply chain system. At this time, the production and purchase quantity of the entire supply chain are both Q. First of all, the government purchases the materials of Unit Q from the supplier at unit price w as the initial reserve, and orders the option of Unit Q at the payment cost of (wQ+C_oq). During the exercise of the option contract, when the demand for emergency supplies is 0<x<Q, the government does not need to exercise the option contract, and can obtain the salvage value of v(x-Q). When the demand for emergency supplies is Q<x<Q+q, because the initial order quantity of the government cannot meet the demand, part of the option will be exercised, that is, purchase (x-Q) units of emergency supplies from the supplier, and the payment cost is C_e(x-Q). When the emergency demand is Q+q<x, the government exercises all options, resulting in an out-of-stock cost of h(x-Q-q). Therefore, it can be known that the expected cost of the government is \( \Pi_2 \), the procurement cost of the initial purchase quantity, the royalty paid for the purchase option, the residual value income obtained from the sale of the remaining materials, the procurement cost at the time of execution, and the out-of-stock cost.

\[
\Pi_2 = wQ + C_oq - v \int_0^Q F(x)dx + C_e \left( q - \int_0^{Q+q} F(x)dx \right) + h \left( \mu - Q - q + \int_0^{Q+q} F(x)dx \right) 
\]

When the supply chain makes centralized decisions, the total production and purchase quantity of the supply chain are both Q_1. It can be seen that the government's expected cost function at this time is as follows:

\[
\Pi_2^{**} = wQ^{**} + C_oq^{**} - v \int_0^{Q^{**}} F(x)dx + C_e \left( q^{**} - \int_{Q^{**}}^{Q^{**}+q} F(x)dx \right) + h \left( \mu - Q^{**} - q^{**} + \int_{Q^{**}}^{Q^{**}+q} F(x)dx \right) 
\]

By solving formula (1) and (2), it can be known that the purchase cost of each unit of material is:

\[
C_e + C_o = h - \frac{h - C}{C - v} C_o
\]

Therefore, with the increase of option purchase price C_o, the government will reduce the material purchase cost by increasing the option order quantity Q^{**} and reducing the initial order quantity Q^{**}.

3.2. Decentralized decision-making model of emergency supply chain under government leadership
In the actual situation, the decision-making subject of each supply chain is completely rational, so the whole supply chain is actually in the decentralized decision-making state, and the supply chain system cannot realize the centralized decision-making spontaneously. Under the decentralized decision of option contract, the supplier's expected profit function is:
By solving formula (4), it can be obtained that:

\[
E\Pi_1 = wQ^* + C_o q^* + C_e \left( q^* - \int_{Q^*}^{Q^*+q^*} F(x) \, dx \right) + v \int_{Q^*}^{Q^*+q^*} F(x) \, dx - C(Q^* + q^*)
\]

(4)

By solving formula (4), it can be obtained that:

\[
C_o = C - \frac{C_e(2h - C - v) + (C - h)v}{h - v}
\]

(5)

According to Formula (5), in the emergency supply chain dominated by the government, the option contract can effectively coordinate the normal operation of the entire supply chain, and the option purchase price and the option execution price have an inverse linear relationship.

3.3. An option decision model for supplier risk aversion under CVaR measurement

When participants and suppliers are considered to be risk-averse, the whole emergency supply chain is actually in the decentralized decision-making state. It will determine its optimal decision to maximize its objective function under the CVaR risk measurement criterion. According to the study of Rockefeller and Jurscheuw, CVaR measures the random profit less than the average value of the following part of VaR. Profits above the quantile are not taken into account, and are of concern to risk avoiders. Under the CVaR risk measurement criterion, the decision goal of supply chain is changed from the expected profit maximization to the CVaR function maximization \( CVaR\Pi (Q, q, x) \).

The expected profit model of risk averse suppliers can be constructed as follows:

\[
E \Pi_t (Q, q, x) = \alpha - \frac{1}{\eta} E[\alpha - \Pi_t (Q, q, x)^+]
\]

(6)

\( E \) represents the expected value of the objective function, represents the risk preference coefficient of decision makers in the emergency supply chain, \( \eta \in (0,1) \), the smaller the pie, the greater the risk aversion of suppliers. Alpha is the critical value of the supplier revenue function under the VaR metric. In the formula, \( Q \) is the initial order quantity of the government, \( Q \) is the purchase quantity of the government under the option contract, and \( x \) is the actual market demand for emergency supplies.

At this time, the expected profit of the supplier is:

\[
\Pi_t (Q, q, x) = \int_0^Q \left[ \left( w - C \right) Q + \left( C_e - C + v \right) q \right]^+ f(x) \, dx + \\
\int_Q^{Q+q} \left[ \left( w - C \right) Q + \left( C_e - C \right) q + C_e (x - Q) + v(Q + q - x) \right]^+ f(x) \, dx + \\
\int_{Q+q}^\infty \left[ \left( w - C \right) Q + C_e q + \left( C_e - C \right) q \right]^+ f(x) \, dx
\]

(7)

After solving formula (7), it can be seen that suppliers with a certain risk preference have CVaR function as follows:

\[
CVaR\Pi_t (Q, q, x) = (w - C) Q + C_e q + (C_e - C) q - \frac{C_e - v}{\eta} \int_q^{Q+q} F(x) \, dx
\]

(8)

By taking the derivative of formula (8), we can know that the government's optimal initial order quantity \( Q \) and optimal option purchase quantity \( q \) are formula (9) and formula (10) respectively.

\[
Q = F^{-1} \left[ \frac{\eta(C_o + C_e - w)}{C_e - v} \right]
\]

(9)

\[
q = F^{-1} \left[ \frac{\eta(C_o + C_e - C)}{C_e - v} \right] - F^{-1} \left[ \frac{\eta(C_o + C_e - w)}{C_e - v} \right]
\]

(10)

It can be seen from the above that, under the CVaR measurement criterion, considering the risk aversion of suppliers, in order to maximize the expected profit of the whole emergency supply chain
system and realize the coordination of supply chain, the option contract mechanism formulated by the government must meet the following constraints:

$$C_e = C - C_o + \frac{(h - c)(C_e - v)}{\eta(h - v)}$$  \hspace{1cm} (11)

4. The example analysis
Based on the above analysis and research, this section verifies its effectiveness through numerical examples, which take the supply of COVID-19 emergency epidemic prevention supplies in Hubei Province as the background. Considering that the expression in the above model is relatively complex, matlabR2016a software is used for numerical simulation. Assuming that the demand for emergency relief supplies follows the exponential distribution of [0, 100,000], with the mean value of =10000, and the shortage cost of emergency supplies h=1000, the production cost of emergency supplies C=150, the wholesale price of emergency supplies w =300, and the salvage value of emergency supplies v =50.

For the risk appetite coefficient η of the supplier, the decision-making behavior of the government and the supplier is examined in a descending way. For the convenience of discussion, when Co={4.5,4.8,5.1,5.4,5.7,6.0}, the value of Ce in the cases where η ={0.9,0.8,0.7,0.6,} is shown in Table 1.

| η  | 4.5 | 4.8 | 5.1 | 5.4 | 5.7 | 6.0 |
|----|-----|-----|-----|-----|-----|-----|
| 0.9| 13.15| 12.36| 11.58| 10.79| 10.01| 9.23 |
| 0.6| 13.59| 12.77| 11.94| 11.12| 10.30| 9.48 |
| 0.7| 14.10| 13.24| 12.37| 11.51| 10.64| 9.77 |
| 0.6| 14.72| 13.80| 12.88| 11.96| 11.04| 10.12|

Table 1 shows the influence of different risk preference coefficients of suppliers on option strike price Ce when option purchase price Co is fixed. With the decrease of risk preference coefficient η, option strike price keeps rising.

5. Conclusion
In this paper, the study shows that the option contract can realize the coordination of the supply chain of emergency relief materials under the conditional value-at-risk. With the increase of option purchase price Co, the government will reduce the material purchase cost by increasing the option order quantity Q** and reducing the initial order quantity Q**. When the option purchase price Co is fixed, the supplier's risk aversion increases with the decrease of the risk preference coefficient. In this case, the government, as the purchaser, needs to provide a higher option strike price Ce to encourage the supplier to produce according to the optimal output of the supply.

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