Analysis of grinding process with the use of field theory

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Abstract. In this work were analyzed objective laws of material allowance removal that are
common for different schemes of grinding with the use of field theory. Each level of the
working surface of the circle corresponds to a certain scalar quantity, which characterizes the
relative reference surface area of the diamond grains and the total grain width at this level. The
total grains width determines the probability of material removal from the component surface
in the process of grinding under the influence of the field of vectors of instrument’s grains.
Changes which occur in the area of contact between material and grinding wheel lead to
negative divergence of material flow and wheel’s grains. The change in the divergence of the
fields is characterized respectively efficiency of machining and destruction of an instrument’s
working area

1. Introduction
There is a wide variety of types of grinding (external and internal round, flat periphery and by the end
face of the grinding wheel, etc.) [1], which differ from each other by the number of movements,
angular arrangement of the grinding wheel (GW) in the coordinate system of the workpiece.

We should remember that that the common method of grinding was earlier pointed out by Maslov
E. N. [2]. By increasing the diameter of the workpiece and provisional turns of GW to infinity the
transition from the outer cylindrical grinding to the scheme of flat grinding of the periphery of the
circle is possible. In [3] was proposed to take into account a variety of patterns of sanding by means of
the introduction of the equivalent radius \( R\) \( \pm \) \( r R \), where grinding wheel \( R \) and \( r \) radii and
workpiece respectively, while the sign in the denominator accounts for the external and internal
grinding; when \( r = \infty \) equivalent radius corresponds to a flat periphery grinding. Recently, in the new
grinding schemes both by the end and by the periphery of the grinding wheel is impossible to take into
account the change in the kinematic interaction of the wheel and a workpiece by introducing an
equivalent radius. Because variable operating conditions of the grains along with the forming GW are
not taken into account when GW is turning its axis relative to the workpiece. The aim of this work is
to find common patterns for removal of allowance for different grinding schemes.

2. Main content
These studies were performed under certain restrictions imposed on the grinding process (no radial
run-out, vibration). Also, we will not take into account the influence of plastic deformations and brittle
destruction of the material allowance for the formation of the cut section on the grain.
We will consider a process of circular external grinding by the periphery of the circle, taking into account the influence of the characteristics of the tool and processing modes. The process of interaction of the grinding wheel with the material of the workpiece allowance is considered in normal cylindrical coordinate systems. The location of grains on the working surface of the circle in statics can be considered as a non-stationary field with the Poisson distribution in the three-dimensional space of the working surface of the grinding wheel [4]. The distribution of the vertices of the grains along the height \( u \) of the circle working surface is uneven and is adequately described by the Weibull distribution with density \( f(u) \). While grinding the wheel rotates and moves progressively against the workpiece. The velocity vector \( \vec{V}_k \) of an arbitrary point of the grinding wheel working surface consists of the speed of the transitional movement \( \vec{V}_t \) and speed \( \omega \times (R-u) \), determined by its rotation with instantaneous angular velocity around the axis.

Let's consider in a vector field of diamond grains of the tool working surface the moving platform of single width 1mm with the height \( dt_i \), located at a depth of \( t_j \) from the original outer surface of the workpiece with radius \( r \) (Figure 1). The angular position of the site in the coordinate system of the circle is determined by the angle measured from the polar axis. Contact pad with a working surface of the wheel occurs at the point C defined by the angle \( \phi_0 \).

The most protruding grain on the working surface of the circle begins to contact the site addressed at the angular position \( \phi_0 \). The connection for determination of the maximum value of the penetration depth into the treated material of grain, located at a depth \( u \) of the working surface of the wheel at its angular position \( \phi_i \) in the working area, can be obtained by means of the elementary trigonometric transformations

\[
u_{\text{max}}(\phi_i, t_j) = (R - u) + \sqrt{(r - t_j)^2 - a_1^2 \cdot \sin^2 \phi_i - a_1 \cdot \cos \phi_i},
\]

where the angle \( \phi \) varies within the limits \( \phi_0(z,u) \leq \phi_i \leq \phi_1(z,u) \), which define the working contact zone.

Figure 1. The scheme of interaction between the grinding wheel and the workpiece in the circular plunge grinding the periphery of the circle
Diamond grains on the GW complete the process of removing the material at the site in question, the location of which also depends on the depth of \( t_j \). In addition, the interaction of the grains of the circle with the workpiece on the site in question will not take place due to the previous removal of material from it in the cutting zone. Thus, the angles which determine the completion of the treatment \( \phi_\alpha(t) \) is functionally dependent on the depth of the site location on the workpiece. The trajectory from angular position \( \gamma_i \) to \( \gamma_{i+1} \), length \( \Delta s = (r-t_j)(\gamma_{i+1}-\gamma_i) \) the site held during \( \Delta \tau = \Delta s/V_d \). During this time \( \Delta \tau \) through it will take a whole number of grains. A sequence of diamond grains passing through a specified volume represents a flow of events of intensity \( \tilde{H}(\tau) \).

The flow rate determines the average number of events per unit of time. The probability of getting one or another number of grains in a spatial figure does not depend on how many grains fell into any other shape that does not intersect with it. For this stream of grains the terms of the ordinary and a lack of interaction are met. Select a curved parallelepiped on the working surface of the circle with the base area equal to one and variable height \( u_{\text{max}}(\phi,t_j) \). The product of the number of grains in the selected volume in the working surface of the grinding wheel on the vector of their velocity determines the intensity of the flow of grain vertices

\[
\tilde{H}(\phi_i,t_j) = V_K \cdot n_j \int \int f(u)du.
\]

The resulting vector determines the intensity of the flow of grain vertices through a fixed plane in space, normal to the velocity vector of the wheel. In the first approximation, the field of the cutting edges of the abrasive tool can be considered as stationary and depending on the time of operation and the angular location (\( \phi \)) of the base portion of the field on the wheel. The random field of diamond grains on the working surface of the grinding wheel has the properties of homogeneity and isotropy, i.e. the profiles of the designer of normal cross sections of the working surface of the grinding wheel have the same statistical characteristics. To estimate the working part of the profile, the mathematical expectation of the number of grains per unit of the circle length and the mathematical expectation of the relative reference length of the grain profile at an arbitrary level (\( c \)) are used. In accordance with the Cavalieri-Acker-Glagolev principle, the ratio of the total length of the segments cut off by the grains to the total length of the baseline \( L(c) = A(c) \). Thus, the scalar characteristic of the GW field also characterizes the probability total width of the grains at this level \( c \). Thus, in each area of the working surface of the circle a scalar value is set, which determines the field of the scalar value. Through each point of the field passes a single surface of the field level, which takes values from 0 to 0.25 for diamond grinding wheel.

Accepted normal system of cylindrical coordinates \( \varphi, u, z \) complete differential fields

\[
dA_c = \frac{\partial A}{\partial \varphi} d\varphi + \frac{\partial A}{\partial z} dz + \frac{\partial A}{\partial u} du = \nabla A_c \cdot dR_c,
\]

where is the differential of the radius vector of the current point of the

\[
R_d = i(R - u) \cos \varphi + j(R - u) \sin \varphi + kz.
\]

Thus, the field gradient of the cutting surface of the circle

\[
\nabla A_c = \left( i \cos \varphi + j \cos \varphi \right) \frac{\partial A}{\partial u} + \left( -i \sin \varphi + j \cos \varphi \right) \frac{\partial A}{\partial \varphi} + k \frac{\partial A}{\partial z} = \nabla A_c.
\]

The gradient of the field is directed along the normal to the surface of the GW level in the direction of the fastest increase of the scalar function and is equal to the derivative in the direction \( u \). Knowing the gradient of the scalar function at a given point, it is possible to determine the change of the function in the transition from the considered point to any other in its vicinity (Figure 2). When the WK is wear, the heights of the grain vertices decrease, which reduces the gradient of the scalar characteristic of the working surface of the wheel.
Thus, the distribution of grain vertices on the working surface of the circle can be considered as a non-stationary field with Poisson distribution. Each level of the working surface of the circle corresponds to a certain scalar value, which characterizes the relative bearing surface is the surface of diamond grains, the volume fraction of diamond grains and determines the possibility of the removal of the material of the allowance tool on this level. The proposed approach to the description of GW allows generalizing the description of various circles and schemes from the positions of stereology and vector field.

When processing in the vector field of cutting elements GW moves not the elementary area, but the elementary volume of the workpiece \( dV \) (Figure 3), in the removal of the material which is involved diamond grains passing through its other side surfaces. The intensity of the grain flow through the \( k \)-th site is determined by the scalar product of the velocity vectors \( \nu_k \) and the normal vector to this site \( n_k \). If the vectors are directed in the opposite direction, then the flow of grains of the circle on the elementary surface is incoming, otherwise it is outgoing. For round cut-in grinding, the input flows of diamond grains are directed on two surfaces of the volume \( \Delta V_0 \) (surfaces 1 and 11), and the output flows are shaded on two surfaces (2 and 2'). The input flows are directed along the site 1 of size \( (r - t_j) \cdot dt \cdot d\gamma \cdot 1 \) with the normal vector \( n_\varphi \) to the site perpendicular to the velocity vector \( \nu_\varphi \) of the workpiece, and along the site 1, size \( dt \cdot 1 \) and with the normal vector - the collinear velocity vector of the workpiece.

The total number of grains involved in the removal of the \( \gamma_0(t_j), \gamma(t_j) \), allowance material from the volume \( \Delta V_0 \) when it is moved in the cutting zone, determined by the angles 0 (tj), and n(tj), can be found by the following relationship

\[
N_V \left( t_j \right) = \frac{V_\nu}{V_\delta} \left[ \gamma_n \left( t_j \right) \left\{ \cos(\varphi + \gamma) + \sin(\varphi + \gamma) \right\} + u_{\max} \left( \varphi(t_j) \right) f(u) \cdot \left( r - t_j \right) du \right].
\]

For round longitudinal grinding, it is necessary to take into account the flow of cutting grains entering through three surfaces of the elementary volume of the workpiece, and in general, leaving through three other surfaces. The material flow of the workpiece is a vector variable in time and space. The direction of movement is determined uniquely by the velocity vector for each point in space.

![Figure 2](image2.png)  
**Figure 2.** The gradient of the relative reference area of the working surface of the grinding wheel1A1 250x16x32 AC6-4-M2-01-125 / 100 1-new wheel, 2-after processing of workpiece

![Figure 3](image3.png)  
**Figure 3.** Part of the workpiece
\[ \vec{V}_\theta = \vec{\omega}_\theta \cdot r \left\{ t_j, \gamma_i \right\} \]. The scalar characteristic of the material field is determined by the probability of material failure \( P\left( \gamma_i, t_j \right) \) [3]

\[ P\left( \gamma_i, t_j \right) = \exp \left(- \frac{\sum b(\gamma_i, t_j)}{L} \right), \tag{5} \]

where \( \sum b(\gamma_i, t_j) \) is the total width of all single grains that have passed through the basic plot \( L \) at the level of the radial coordinate \( t_j \), counting from the surface of the workpiece, with the angular position of the cross section \( \gamma \).

If we denote the probability of failure of the material in discrete sections \( \gamma \) and \( \gamma_{i+1} \) through \( P_i \) and \( P_{i+1} \), then the transition from the state \( P_i \) to the state \( P_{i+1} \) is carried out under the influence of Poisson grain flow. The probability of failure of the material at the base site of the workpiece is determined by the characteristics of the tool field \( b_{i,i+1} \) in this area of its angular displacement \( P_{i+1} (\gamma, t_i) = P_i (\gamma, t_i) \exp(-b_{i,i+1}(\gamma, t_i)) \). If the velocity vector of an arbitrary point the working surface of the grinding wheel multiplied by the scalar function \( A_c \) field \( GW \) [4], we obtain a vector characteristic of the field of the wheel during processing \( \vec{B}_K = A_c \cdot \vec{V} \). We have a random vector field.

The volume of the material passing through the closed surface \( \sigma \), limiting the three-dimensional surface \( V \), is determined by the formula Ostrogradsky

\[ \oint_B \vec{B}(t, \gamma) \cdot \vec{n} \cdot d\sigma = \iiint_V \nabla \vec{B}(t, \gamma) dV \tag{6} \]

where \( \vec{n}_K \) – ort normal to the surface of the selected elementary volume of the workpiece in the work area.

Outside the working area, where there is no interaction with the workpiece grinding wheel, \( \nabla \vec{B}(t, \gamma) = 0 \). Under the influence of the tool field changes occur in the field of workpiece material. Consider the changes occurring in a small volume of material \( dV_\theta = dt \cdot d\varphi (r - t) \cdot 1 = \left( r - t_j \right) \cdot \omega_\theta \cdot dt \cdot d\tau \) under the influence of the grain field of the grinding wheel. The flow rate of the material field in the area \( dV_\theta \), equal to the difference between the incoming and outgoing flows, determines the amount of allowance to be removed or the processing capacity in this area of the work area. The amount of material removed from an item \( dV_\theta \) can be determined by the following dependencies

\[ dM = dV_\theta \cdot \left( P_{i-1,i} - P_{i,i+1} \right) = dV_\theta \cdot B_i \left( t_j, \gamma_i \right) \left( t - \exp(-b_{i,i+1}) \right). \tag{7} \]

Incoming and outgoing flows of \( dQ_\theta \), material pass through the lateral surface of the three-dimensional surface. The difference between the input and output flows per unit time determines the removal performance of the material in the elementary volume of the cutting zone, where \( dQ = \sum_{k=1}^{n} dQ_i \) – the number of sites that limit the allocated volume \( dV_\theta \). If the height of the investigated elementary volume is equal to the maximum depth of the \( u_{\text{mac}}(\varphi) \), to \( n=5 \), since in this case the input and output streams are not possible from the side of the GW bundle.
Each point of the field corresponds to a certain value of the direction of movement of the elementary volumes of the workpiece in space, which, in turn, corresponds to the direction of the velocity vector. To analyze the working conditions of the circle, the flow of material along the normal to the contact surface of the wheel and the workpiece is most often used [1]. Through each elementary section of the contact of the working surface of the wheel with the workpiece $dF$ passes $dQ_n$, flow equal to the elementary flow of the velocity field of the workpiece material through this section

$$dQ_n = \vec{B}\begin{pmatrix} t_j, \gamma_i \end{pmatrix} \cdot \vec{n} \cdot dF = \vec{V} \cdot \vec{n} \cdot \left[ \exp\left( -\sum b\left( t_j, \gamma_i \right) \right) \right] \cdot dF,$$

(8)

where $\vec{n} = \{-\cos \varphi_k, -\sin \varphi_k, 0\}$ is normal to the cylindrical working surface of the wheel, which is directed to the axis of the grinding wheel; $\vec{V} \cdot \vec{n} = \vec{W}$ - vector product, which determines the normal component of the relative velocity of the workpiece and the tool at each point of contact zone.

The total volume of material supplied to the circle per unit time is determined by the flow of the workpiece velocity field through the contact area $F$

$$Q = \left[ \int_{\varphi_0}^{\varphi_n} W \cdot \exp\left( -\sum b\left( t, \gamma \right) \right) \right] \cdot d\varphi \cdot dz.$$

(9)

3. Summary

Using the provisions of field theory and probability theory have been obtained dependencies that allow us to estimate the distribution of the removing volume of material allowance and probability of removal of the material at any point of contact with the workpiece circle. The obtained dependencies allow us to determine the number of grains involved in the removal of the allowance. This makes it possible to use the obtained dependencies for analysis of the conditions under which a tool is performing, tool operating conditions, determination of cutting forces, selection of the scheme and grinding modes. The obtained dependencies and results can be extended to all grinding schemes.

References

[1] Reznikov A 1977 Abrasive and diamond processing of materials (Moscow: Mechanical Engineering) p 391
This reference has two entries but the second one is not numbered (it uses the ‘Reference (no number)’ style.

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Another reference

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[4] Gusev V V 2007 Progressive technologies and systems of engineer 34 (Donetsk DonNTU) pp 78-83