Chiral Magnetic Effect and Chiral Phase Transition

Wei-jie Fu,†‡ Yu-xin Liu,*†‡ and Yue-liang Wu†‡

1Kavli Institute for Theoretical Physics China (KITPC), Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Science, Beijing 100190, China
2Department of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China
3Center of Theoretical Nuclear Physics, National Laboratory of Heavy Ion Accelerator, Lanzhou 730000, China

We study the influence of the chiral phase transition on the chiral magnetic effect. The azimuthal charge-particle correlations as functions of the temperature are calculated. It is found that there is a pronounced cusp in the correlations as the temperature reaches its critical value for the QCD phase transition. It is predicted that there will be a drastic suppression of the charge-particle correlations as the collision energy in RHIC decreases to below a critical value. We show then the azimuthal charge-particle correlations can be the signal to identify the occurrence of the QCD phase transitions in RHIC energy scan experiments.

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The phase transitions of quantum chromodynamics (QCD), for example the evolution between chiral symmetry breaking and its restoration, the color deconfinement and confinement, have been one of the most active topic in nuclear and particle physics in recent years [1]. Such phase transitions can be driven by the temperature and density of the system. It is then expected that these phase transitions occur and the deconfined quark gluon phase (QGP) is formed in ultrarelativistic heavy-ion collisions [2,3] (for example the current experiments at the Relativistic Heavy Ion Collider (RHIC) and the upcoming experiments at the Large Hadron Collider (LHC)) and in the interior of neutron stars [4–6]. However, the explicit variation behavior of the signals to identify the phase transitions with respect to the temperature and density needs further investigations.

Recently, The STAR Collaboration at RHIC report their measurements of azimuthal charged-particle correlations in Au + Au and Cu + Cu collisions at $\sqrt{s_{NN}} = 200$ GeV. They find a significant signal consistent with the charge separation of quarks along the system’s orbital angular momentum axis [5,8]. The observed charge separation indicates that parity-odd domains, where the parity (P) symmetry is locally violated, might be created during the relativistic heavy-ion collisions [5,11]. The charge separation is related with the so called “chiral magnetic effect” which means that a magnetic field in the presence of imbalanced chirality induces a current along the magnetic field, and therefore results in that positive charge is separated from negative charge along the magnetic field [11].

Now that the chiral magnetic effect can be observed through the measurements of azimuthal charged-particle correlations in the relativistic heavy-ion collisions, a natural question arises, i.e. whether can we detect the properties of the QCD phase transitions, especially the chiral phase transition through the observations of the chiral magnetic effect? To answer this question, we have to study how the chiral magnetic effect or the charge separation effect is influenced by the chiral phase transition. This is our central subject in this letter.

In this work, we will study the chiral magnetic effect and the QCD phase transitions in the 2+1 flavor Polyakov–Nambu–Jona-Lasinio (PNJL) model [12]. The validity of the PNJL model has been confirmed in a series of works by confronting the PNJL results with the lattice QCD data [12–14]. The PNJL model not only has the chiral symmetry and the dynamical breaking mechanism of this symmetry, which are same as the conventional Nambu–Jona-Lasinio model, but also include the effect of color confinement through the Polyakov loop. Therefore, the PNJL model is very appropriate to describe the QCD phase transitions at finite temperature and/or density.

The Lagrangian density for the 2+1 flavor PNJL model is given as

$$\mathcal{L}_{PNJL} = \bar{\psi}(i\gamma_\mu D^\mu - m_0)\psi + G \sum_{a=0}^{8} \left( (\bar{\psi}\tau_a\psi)^2 \right) - K \left[ \det_f(\bar{\psi}(1 + \gamma_5)\psi) \right. $$

$$+ \left. \det_f(\bar{\psi}(1 - \gamma_5)\psi) \right] - U(\Phi, \Phi^*, T), \quad (1)$$

where $\psi = (\psi_u, \psi_d, \psi_s)^T$ is the three-flavor quark field, $D^\mu = \partial^\mu - iA^\mu$ with $A^\mu = \delta_0^\mu A^0$. $A^0 = gA_0^0 \delta_0^\mu = -iA_\mu$. $\lambda_a$ are the Gell-Mann matrices in color space; $m_0 = \text{diag}(m_0^u, m_0^d, m_0^s)$ is the three-flavor current quark mass matrix. In this work, we take $m_0^s = m_0^d \equiv m_0^u$, while keep $m_0^a$ being larger than $m_0^0$. $U(\Phi, \Phi^*, T)$ in the PNJL Lagrangian density is the Polyakov-loop effective potential, which is expressed in terms of the traced Polyakov-loop $\Phi = (\text{Tr}_cL)/N_c$ and its conjugate $\Phi^* = (\text{Tr}_cL)/N_c$. In this work, we use the Polyakov-loop effective potential which is a polynomial in $\Phi$ and $\Phi^*$ [13, given by

$$\frac{U(\Phi, \Phi^*, T)}{T^4} = \frac{b_2(T)}{2} \Phi^* \Phi^3 - \frac{b_3}{6} (\Phi^* \Phi^3)^3 + \frac{b_4}{4} (\Phi^* \Phi^3)^4 (2)$$

where $b_2(T)$ is a polynomial function of temperature $T$. The coefficients $b_2(T)$, $b_3(T)$ and $b_4(T)$ are obtained from lattice QCD simulations.
with

\[ b_2(T) = a_0 + a_1 \left( \frac{T_0}{T} \right) + a_2 \left( \frac{T_0}{T} \right)^2 + a_3 \left( \frac{T_0}{T} \right)^3. \]  

(3)

Parameters in the effective potential are fixed by fitting the thermodynamical behavior of the pure-gauge QCD obtained from the lattice simulations. Their values are \( a_0 = 6.75, \ a_1 = -1.95, \ a_2 = 2.625, \ a_3 = -7.44, \ b_3 = 0.75 \) and \( b_4 = 7.5 \). The parameter \( T_0 \) is the critical temperature for the deconfinement phase transition to take place in the pure-gauge QCD and \( T_0 \) is chosen to be \( 270 \) MeV according to the lattice calculations. Furthermore, we also need to determine the five parameters in the quark sector of the model, which are \( m_0 = 5.5 \) MeV, \( m_0^b = 140.7 \) MeV, \( G_A^2 = 1.835, \ K\Lambda^5 = 12.36 \) and \( \Lambda = 602.3 \) MeV. They are fixed by fitting \( m_{\pi} = 135.0 \) MeV, \( m_K = 497.7 \) MeV, \( m_{\eta'} = 957.8 \) MeV and \( f_\pi = 92.4 \) MeV \[12\].

In the parity-odd domains which are created during relativistic heavy-ion collisions, the number of left- and right-hand quarks is different because of the axial anomaly. In this work we introduce the chiral chemical potential \( \mu_5 \) to study the left-right asymmetry following the method of Ref. \[11\], where the chiral chemical potential \( \mu_5 \) is related with the effective theta angle of the \( \theta \)-vacuum through \( \mu_5 = \partial_\theta/2N_f \) and \( N_f \) is the number of flavor. Consequently, we should add the following term

\[ \bar{\psi}_i \mu_5 \gamma^\mu \gamma^5 \psi_j \]  

(4)

to the Lagrangian density in Eq. \[11\], where \( \mu_5 = \text{diag}(\mu_5^u, \mu_5^d, \mu_5^s) \). Next, we consider the case that a homogeneous magnetic field \( B \) is along the direction of the orbital angular momentum of the system produced in a non-central heavy-ion collision. In the following we denote this direction with \( z \)-direction and particle momentum in this direction with \( p_3 \).

In the mean field approximation, the thermodynamical potential density for the 2+1 flavor quark system under a homogeneous background magnetic field \( B \) and with left-right asymmetry is given by

\[
\Omega = -N_c \sum_{f=u,d,s} \left| q_f \epsilon B \right| \sum_{n=0}^\infty \sum_{s=1}^3 \int \frac{dp_3}{2\pi} \left( E_f - \mu_f - \frac{s|\epsilon_f|}{E_f} \mu_5^f \right) \bigg\{ \frac{T}{3} \ln \left[ 1 + 3 \Phi^* \exp \left[ -3 \left( E_f + \mu_f - \frac{s|\epsilon_f|}{E_f} \mu_5^f \right) / T \right] \right] \\
+ \frac{T}{3} \ln \left[ 1 + 3 \Phi \exp \left[ -3 \left( E_f + \mu_f - \frac{s|\epsilon_f|}{E_f} \mu_5^f \right) / T \right] \right] \bigg\}
\]

(5)

where

\[ |\epsilon_f| = \sqrt{2n|q_f|eB + p_3^2}, \]  

(6)

\[ E_f = \sqrt{2n|q_f|eB + p_3^2 + M_f^2}. \]  

(7)

with \( q_i(i = u,d,s) \) being the electric charge in unit of elementary charge \( e \) for the quark of flavor \( i \) and the constituent mass \( M_i \) reading

\[ M_i = m_0^i - 4G(|\bar{\psi}\psi)_i + 2K(|\bar{\psi}\psi)_i \langle \bar{\psi}\psi \rangle_k, \]  

(8)

and \( \langle \bar{\psi}\psi \rangle \) is the chiral condensate. In Eq. \[5\] we also include the quark chemical potential \( \mu_5 \). The momenta of charged particles in the longitudinal direction, i.e., the \( z \)-direction, are not influenced by the background magnetic field and \( p_3 \) in the expression of the thermodynamical potential density in Eq. \[5\] is continuous; while the momenta in the transverse plane are discretized due to the magnetic field effect. \( |\epsilon_f| \) in Eq. \[6\] is similar to the magnitude of the momentum in free space. \( s \) (for fermion and for anti-fermion is \(-s\)) in Eq. \[5\] is the helicity of particle and we should emphasize that at the lowest order of the transverse quantum number, i.e., \( n = 0 \), the quark spin only has one value in the \( z \)-direction, which means that charged particles in the lowest transverse level are polarized by the external magnetic field; however particles in higher levels, i.e., \( n > 0 \), are not polarized. Therefore, the charge separation effect only comes from quarks in the lowest transverse level.

In order to relate our calculations with observable in heavy-ion collisions, we define \( \Delta_+ \) (\( \Delta_- \)) to be the positive (negative) charge difference in unit of \( e \) (\(-e\)) between on each side of the \( z = 0 \) plane, which is also the reaction plane. Here we use the notations in Ref \[10\]. Taking particles with positive elementary electric charge \( e \) for example, we can express \( \Delta_+ \) as

\[
\Delta_+ = \int d^3x (\bar{\psi}_i \gamma^0 \gamma^5 \psi)_n=0 \\
= \int d^3x (\bar{\psi}_R \gamma^0 \psi_R - \bar{\psi}_L \gamma^0 \psi_L)_{n=0},
\]

(9)

where

\[ \psi_R = \frac{1 + \gamma^5}{2} \psi \quad \text{and} \quad \psi_L = \frac{1 - \gamma^5}{2} \psi. \]  

(10)

The subscript \( n = 0 \) in Eq. \[10\] indicates that only the lowest transverse level states contribute to the \( \Delta_+ \). In the
same way, we can obtain $\Delta_\pm$ for the 2+1 quark system. We take $\Delta_+$ for example once more, which is given as

$$\Delta_+ = V N_+ \frac{e B}{4 \pi^2} \left\{ q_u^2 \int_0^\infty dp_E \frac{p_3}{E_u} \left[ f(E_u - \mu_u) - \frac{p_3}{E_u} \mu_u^u \right] \right. $$

$$+ q_d^2 \int_0^\infty dp_E \frac{p_3}{E_d} \left[ f(E_d + \mu_d) - \frac{p_3}{E_d} \mu_d^d \right] $$

$$\left. - f(E_d + \mu_d + \frac{p_3}{E_d} \mu_d^d) \right\},$$

(11)

where $V$ is the volume of the system, $E_f$ is given by Eq. (7) with $n = 0$, and

$$f(x) = \frac{\Phi^* e^{-x/T} + 2 \Phi e^{-2x/T} + e^{-3x/T}}{1 + 3 \Phi e^{-x/T} + 3 \Phi e^{-2x/T} + e^{-3x/T}}$$

(12)

and

$$\tilde{f}(x) = \frac{\Phi e^{-x/T} + 2 \Phi e^{-2x/T} + e^{-3x/T}}{1 + 3 \Phi e^{-x/T} + 3 \Phi e^{-2x/T} + e^{-3x/T}}.$$

(13)

In fact, we can also obtain Eq. (11) through differentiating the thermodynamical potential in Eq. (5) with respect to the chiral chemical potential $\mu_5$ and summing over the contributions from positive quarks or anti-quarks.

In the same way, differentiating the thermodynamical potential with respect to the quark chemical potential $\mu_5$ and summing over contributions from the three-flavor positive quarks or anti-quarks we obtain the total positive electric charge number $N_+$ in unit of $e$, i.e.,

$$N_+ = V N_+ \frac{e B}{2 \pi} \sum_{n=0}^\infty \sum_{s=\pm 1} \left\{ q_u^2 \int \frac{dp_3}{2 \pi} f(E_u - \mu_u - s \epsilon_u \mu_u^u) \right. $$

$$+ q_d^2 \int \frac{dp_3}{2 \pi} \tilde{f}(E_d + \mu_d - s \epsilon_d \mu_d^d) $$

$$\left. + q_s^2 \int \frac{dp_3}{2 \pi} \tilde{f}(E_s + \mu_s - s \epsilon_s \mu_s^s) \right\}.$$  

(14)

Similarly, the total negative electric charge number $N_-$ in unit of $-e$ can also be obtained.

In the experiments of heavy-ion collisions, the azimuthal charged-particle correlations, i.e., $(\phi_a + \phi_\beta - 2 \Psi_{RP})$, are used to detect the $P$-violating effect [7,8,16]. Here $\phi$ and $\Psi_{RP}$ are the azimuthal angles of the particles and reaction plane, respectively. $\alpha$, $\beta$ represent electric charge $+$ or $-$. With the notation $a_{\alpha \beta} \equiv -(\cos(\phi_a + \phi_\beta - 2 \Psi_{RP}))$, it can be shown that [10]

$$a_{++} = \frac{\pi^2}{16} \frac{\Delta_+^2}{N_+^2}, \quad a_{--} = \frac{\pi^2}{16} \frac{\Delta_-^2}{N_-^4},$$

(15)

and

$$a_{+-} = \frac{\pi^2}{16} \frac{\Delta_+ \Delta_-}{N_+ N_-}$$

(16)

where the azimuthal angle distribution of the charged particles is assumed to be

$$\frac{dN_+}{d\phi} = \frac{1}{2\pi} N_+ + \frac{1}{4} \Delta_\pm \sin \phi.$$

(17)

Since we mainly focus on the influence of the QCD phase transitions, especially the chiral phase transition, on the chiral magnetic effect in this work, we will neglect the screening suppression effect due to the final state interactions [10] and make $\mu_5 = 0$, then we have $a_{++} = a_{--} = -a_{+-}$. Therefore, we only study $a_{++}$ in the following.

Minimizing the thermodynamical potential in Eq. (5) with respect to three-flavor quark condensates, $\Phi$, and $\Phi^*$, we obtain a set of equations of motion. We neglect the influence of the magnetic field on these equations of motion in our numerical calculations, since the magnetic field ($eB = 10^2 \sim 10^4$ MeV$^2$ in the non-central heavy-ion collisions [10]) has little impact on these equations of motion.

![FIG. 1: Correlation $a_{++}$ as function of the temperature calculated in the PNJL model with $\mu_5 = 150$ MeV (left panel) and $\mu_5 = 250$ MeV (right panel). The magnetic field corresponds to $eB = 5 \times 10^2$, $10^3$, and $5 \times 10^3$ MeV$^2$ from top to bottom, respectively.](image-url)
transition (the critical temperature $T_c = 209$ MeV for $\mu_5 = 150$ MeV and $T_c = 185$ MeV for $\mu_5 = 250$ MeV in the PNJL model). From the Fig. 1 one can also find that although the value of $a_{++}$ is proportional to the square of the magnetic field strength, the shape of the curve for $a_{++}$ as function of temperature is almost independent of the magnetic field strength. Furthermore, the cusp at the critical temperature in the curve becomes much sharper with the increase of the chiral chemical potential. With the decrease of the temperature, when the temperature is below $T_c$, chiral symmetry is dynamically broken and quarks obtain masses. Since the axial anomaly can be suppressed by the mass effect, which has been discussed in detail in Ref. [17], the chiral magnetic effect can also be suppressed by large constituent quark masses. Therefore, the azimuthal charged-particle correlations described by $a_{++}$ ($a_{--}$ and $a_{-+}$) defined in Eqs. (15) (16) are quite decreased once the temperature is below the critical temperature. It can be seen from Fig. 1 that, when the temperature is above $T_c$, $a_{++}$ decreases with the increase of the temperature, which is because higher temperature makes it more difficult to polarize quarks with magnetic field and thus suppresses the charge separation effect.

What do our calculated results imply in future energy scanning experiments of heavy-ion collisions? With the decrease of the heavy-ion collision energy, the temperature of the QGP produced in the fireball at early stage is also decreased. Since the magnetic field produced in non-central collisions decays rapidly with time $t$, the observed charge separation mainly carries the information of the QGP at early stage. Therefore, we expect that the azimuthal charged-particle correlations (especially for the same charge correlations, because the opposite charge correlations are suppressed by final state interactions) increase as the collision energy is lowered. However, when the collision energy is lowered to the value that cannot drive the chiral phase transition, it is expected that the azimuthal charged-particle correlations are quite suppressed. Therefore, we can employ the charge separation effect to locate where the QCD phase transitions occur.

In summary, we have studied the influence of the QCD phase transitions on the chiral magnetic effect. The azimuthal charge-particle correlations as functions of the temperature are calculated in the PNJL model. It is found that there is a pronounced cusp in the azimuthal charge-particle correlations around the critical temperature of the chiral phase transition. We predict that there will be a sudden suppression of the charge-particle correlations with the decrease of the collision energy. It indicates that azimuthal charge-particle correlations can be a signal to identify chiral phase transition in the energy scan experiment in RHIC.

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