Lyapunov Based Frugal Incentive Mechanism for Periodical Mobile Crowdsensing

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Abstract—Mobile crowdsensing (MCS) has been intensively explored recently due to its flexible and pervasive sensing ability. Although many incentive mechanisms have been built to attract extensive user participation, Most of these mechanisms focus only on independent task scenarios, where the sensing tasks are independent of each other. On the contrary, we focus on a periodical task scenario, where each user participates in the same type of sensing tasks periodically. In this work, we consider the frugal payment problem in a general time-dependent and location-aware periodical MCS system, taking the long-term user participation incentive into explicit consideration. We study the problem systematically under both semi-online (the intra-period interactive process is synchronous while the inter-period interactive process is sequential and asynchronous during each period) and online user arrival models (the previous two interactive processes are sequential and asynchronous). In particular, we first propose a Lyapunov-based semi-online frugal incentive mechanism, which converges asymptotically to the optimal benchmark performance. Moreover, we also present Lyapunov-based online frugal incentive mechanism, which satisfies the constant frugality. Besides, the two mechanisms can also satisfy computational efficiency, individual rationality and truthfulness. Through extensive simulations, we evaluate the performance and validate the theoretical properties of our online mechanisms.

Index Terms—periodical crowdsensing, long-term incentive, Lyapunov optimization, frugal payment, submodular function

1 INTRODUCTION

The proliferation of mobile devices (e.g., smartphones) with rich embedded sensors has led to revolutionary new sensing paradigm, often known as mobile crowd sensing (MCS) [1]. Due to the low deploying cost and high sensing coverage, this new paradigm has enabled numerous novel MCS applications, such as air pollution [2–4] for environment monitoring, Nericell [5], SignalGruru [6], and VTrack [7] for providing omnipresent road traffic information, Gigwalk [8] for providing images of store interiors. While participating in MCS applications, smartphone users consume their own resources such as battery and computing power, and disclose their locations with potential privacy threats. Thus, incentive mechanisms are necessary to provide participants with enough rewards for their participation costs. Most of existing incentive mechanisms [9–13] mainly focus on independent task scenarios, where the sensing tasks are independent of each other. However, in practice, there are many periodical sensing task scenarios where each user participates in the same type of sensing tasks periodically, and timely data are valuable whereas outdated data are worthless. Compared with the independent MCS applications, the periodical MCS applications have several unique challenges. Firstly, smartphone users not only consume previous direct sensing cost, but also incur certain extra indirect cost like certain energy and transmission cost when not performing the sensing task. In this case, the above mechanisms for the independent task scenario may not be enough to guarantee the long-term continuous participation of users for the periodical MCS applications. Substantial (preferably monetary) compensation is necessary to drive the periodical MCS applications. Secondly, many sensing tasks are location-aware and time-dependent, and involve spatio-temporal context. The spatio-temporal relation of the periodical MCS applications requires that the incentive mechanism design takes the time relation and spatial coverage into consideration, which we call as the users’ selection issue. All of these bring the incentive issue to the periodical MCS applications.

Generally, in the periodical MCS applications, there are two classes of incentive mechanisms with different objectives: one is to maximize the utility of the platform (the periodical MCS organizer) under a specific budget constraint, e.g., the total value of all services that can be completed by selected users, and the other is to minimize the platform’s total payment under the condition that the specific services can be completed. Although the budget feasible incentive mechanisms for periodical MCS applications have been investigated recently [14], [15], in this paper we focus on the incentive mechanisms under the framework of frugality. Specially, we investigate the case where the value function of selected users is monotone submodular. This case can be applied in many real scenarios. For instance, many periodical MCS applications [2–4] aim to select users to collect sensing data so that the roads in a given region can
be covered before a specified deadline, where the coverage function is typically monotone submodular. Based on the case, we consider a long-term incentive problem imposed by periodical MCS applications: the platform aims to select a subset of users before each period’s deadline, so that the total payment to the selected users is minimized under the condition that the specific service in each period can be completed. The problem is combinatorial in nature due to the budget limited selection process, but it is made far more challenging due to the time-relation non-deterministic sensing values and long-term participation incentives of individual users.

To this end, we first adopt an online learning approach to acquire the statistical information about the sensing values throughout the selection process to tackle the time-dependent and spatio-coverage problems. Besides, we introduce Lyapunov based optimization to guarantee the long-term participation constraint of the periodical MCS applications. In summary, our main contributions in this paper are summarized as follows:

- We establish a connection between the queue stability condition and the user participatory constraint, and propose a Lyapunov based allocation probability of each user to guarantee the long-term user participation.
- We consider a submodular case and design the semi-online and online incentive mechanisms for achieving each given service requirement of periodical MCS applications. In particular, the semi-online incentive mechanism converges asymptotically to the optimal benchmark performance. The online incentive mechanism satisfies the constant frugality. Besides, the two mechanisms can also satisfy computational efficiency, individual rationality and truthfulness.
- Through extensive simulations, we evaluate the performance and validate their theoretical properties.

The rest of the paper is organized as follows. In Section 2 we briefly discuss the related work and motivation. In Section 3 we present our system model and our design goals. In Section 4 we design a regret-minimization incentive mechanism for periodic crowd sensing application with homogenous and heterogeneous participation costs, followed by the theoretical analysis in Section 5 and performance evaluation in Section 6. Finally, Section 7 presents concluding remarks.

2 RELATED WORK

In terms of the manner of data collection for the MCS applications, there are two classes of the MCS scenarios: the independent MCS scenario and periodical MCS scenario. For the independent MCS scenario, the authors of [12], [13] present offline incentive mechanisms and the posted price model respectively for a simple additive utility function. The authors of [10] considered the submodular utility function for offline truthful incentive mechanism design from the platform-centric and user-centric model respectively. The authors of [16], [17] design online incentive mechanisms based on the pricing model and bidding model respectively to maximize the submodular utility function. However, all these studies only apply for the independent sensing task scenarios.

To the best of our knowledge, [14], [15] are the only results that explicitly study the long-term participation incentive in the periodical MCS scenario. To incentivize the continuous participation of users, they introduce a virtual credit for lowering the bids of users who lost in the previous auction rounds, thereby increasing their winning probabilities in the future auctions. The authors of [13] designed a recurrent greedy reverse auction incentive mechanism that chosen a representative subset of the users according to their location under a fixed budget constraint. They consider continuous crowd sensing task scenarios, but their mechanisms only apply for the budget-limit scenarios and also neglect the truthfulness and optimality of the proposed auction.

In this work, to complement the literatures, we will study the long-term incentive of periodical crowd sensing applications under the service constraints, joint with rigorous truthfulness and optimality analysis in the many users’ selection scenario.

3 SYSTEM MODEL AND PROBLEM FORMULATION

3.1 System Model

Specifically, we focus on a general location-aware periodical MCS applications with the goal to monitor some spatial phenomenon (e.g., road traffic information), where the sensing data in different time and/or
locations may have different values. We consider the following periodical MCS system model illustrated in Fig. 1. The system consists of a MCS platform that resides in the cloud, a requester like an urban management department, and many mobile device users $\mathcal{U} = \{1, 2, \ldots, N\}$ that may arrive at the area of interest (AoI) (denoted by a set $\mathcal{I} = \{1, 2, \ldots, I\}$) in future campaigns and are connected to the cloud by cellular networks (e.g., GSM/3G/4G) or WiFi connections. The requester posts a MCS application with series of services $R$ to the platform, which conducts its task that often lasts for a certain amount of time slots (e.g., tens of days). The periodical MCS application consists of a set $T = \{1, 2, \ldots, T\}$ of $T$ periods, one of which has $\tau$ time slots. A series of MCS campaigns are conducted periodically at the beginning of each period. We assume that the number of available users in each period is enough for completing the required services. In terms of the nature of the MCS application, the assumption holds obviously.

Each user has a certain sensing region in each period, depending on factors such as his location, mobility, device type, and local terrain. Let $r_n[t] = \{r_{n,i}[t], i \in \mathcal{I}\}$ denote the total sensing region of user $n$ in period $t$, where $r_{n,i}[t] \in \{0, 1\}$ means whether a PoI $i$ is located in the sensing range of user $n$ in period $t$. The sensing region $r_n[t]$ of each user $n$ changes dynamically in different periods due to the mobility of users. When the user $n$ is selected by the platform in period $t$, i.e., $\phi_n[t] = 1$, it performs data sensing and uploads his sensing data with the value $v_n[t] = \phi_n[t] \sum_{i=1}^{I} r_{n,i}[t] \cdot \omega_i[t]$ to the platform [18], where $\omega_i[t]$ denotes the weight of PoI $i$. Meanwhile, each selected user $n$ also incurs a sensing cost $c_n[t]$ corresponding to the sensing value. Receiving the sensing value, the platform offers a payment $p_n[t]$.

In this paper, for periodical MCS applications, at the beginning of each period, many users will be selected for achieving the service quality requirement of each period, i.e., minimizing the platform’s total payment under the condition that each specific service is completed before each period’s deadline. Assume that the allocation vector $\phi[t] = \{\phi_n[t], n \in \mathcal{U}\}$ at period $t$, where $\phi_n[t] \in \{0, 1\}$ indicates whether user $n$ is selected for sensing. Then the sum of all selected users’ sensing costs $C_{\phi[t]} = \sum_{n=1}^{N} \phi_n[t] \cdot c_n[t]$. Furthermore, without loss of generality, let the coverage requirement of each PoI be 1, then if the total sensing value $V_{\phi[t]} = \sum_{i=1}^{I} \chi_i[t] \cdot \omega_i[t]$ at period $t$ [18], where $\chi_i[t]$ denotes the indicator function that returns 1 if $\sum_{n=1}^{N} \phi_n[t] \cdot r_{n,i}[t] \geq 1$ and 0 otherwise. The rest of the variables are defined in Table 1. In the following, we formalize our problem.

### 3.2 Problem Formulation

Under our service-limit periodical MCS framework (to be elaborated later), at each period $t$, the platform can periodically select a subset of available users as winners to complete the sensing task with the service constraint until each period’s services are completed. For the service-limit periodical MCS, a desired objective is to find an optimal selecting policy $\Phi^*$ such that $\sum_{i \in \mathcal{I}} p_i$ is minimized while achieving the quality of required services in each period. In terms of combinatorial optimization, $\Phi^*$ is an optimal solution to the following integer linear programming (ILP) problem:

$$\min \frac{1}{T} \sum_{t=1}^{T} \sum_{n=1}^{N} \phi_n[t] \cdot p_n[t] \quad \text{(ILP)}$$

s.t. $V_{\phi[t]} \leq R[t], \quad t \in \{1, T\}$ \quad (1)

$$d_n(\phi_n) = \frac{1}{T} \sum_{t=1}^{T} \phi_n[t] \geq A_n, \quad n \in \{1, N\} \quad (2)$$

$$\phi_n[t] \leq y[t] \quad (3)$$

$$\sum_{t=1}^{T} y[t] = y[t + 1] \quad (4)$$

$$y[t] \in \{0, 1\}, \quad \phi_n[t] \in \{0, 1\}, \quad t \in \{1, T\}$$

The first constraint (1) is to satisfy the service requirement in each period. In the second constraint (2), $A_n(\phi_n)$ is the time average allocation probability of user $n$, depending on the allocations of user $n$ in all slots, and $A_n$ is the participation threshold of user $n$, which means that user $n$ will drop out of the periodical MCS system, if his allocation probability (of being selected as a winner) is smaller than the participation threshold. Thus, the second constraint (2) ensures the active participation of users. The third constraint (3) is artificially introduced to force these non-zero elements appearing only at the beginning of the period sequence: it confines the problem dimension without sacrificing generality. The variable $y[t] \in \{0, 1\}$ denotes whether the sensing task is performed for the $t$-th period. Obviously, the number of non-zero $y[t]$ is bounded by $B/C_{\min}$, where $C_{\min}$ is the minimum cost provided by the platform in all periods. The fourth constraint (4) states that users are selected only when the task is performed.

Note that equation (ILP) is a binary integer programming. If the complete future information is known, it can be effectively solved by many classic methods [19]. However, in the our problem, the expected sensing values and users’ bids are not known to the platform, since mobile users move randomly, hence the sensing region of each user $n$ also changes randomly across different periods. Besides, the value function of the service in the first constraint (1) is submodular. The authors of [10] show that the selection problem of the value function without the long-term participation constraint is $NP$-hard, hence it is obvious that equation (ILP) with the long-term participation constraint is also $NP$-hard. In this paper, it only serves as a benchmark in our performance evaluation, whose solution is $p^*$ and the corresponding optimal allocation vector $\Phi^*$. To overcome this difficulty, we will adopt an eco-
TABLE 1

Summary of Notations

| Variable | Description |
|---------|-------------|
| n, N   | index variable and maximal index value of users |
| i, ω, I | index variable, weight and maximal index of PoI |
| t, T   | index variable and maximal index of periods |
| U, I, T | set of users, PoIs, and periods |
| r_n[t], e_n[t] | sensing area and value of user n in period t |
| C[t], V[t] | cost and sensing values in each period t |
| R, R' | number of required and stage required services |
| B, B' | budget and stage budget derived in each period |
| c, τ | number of stages and time slots in each period |
| J, J' | set of winners and sampled users in each period |
| φ[t], φ*[t] | user n bid, cost, and adjusted bid |
| b_n, c_n, e_n | user n bid, cost, and adjusted bid |
| A, d | long-term and average participation constraint |
| L(q_n), q_n | user n’s queue and his queue’s drift |
| Δ, Λ | queue drift and drift-plus-payment |
| δ, η | participation control parameter and frugal ratio |
| p, p*, P | near, optimal and total payments of each period |


dynamic interaction auction approach in each period to solve the NP problem, taking the long-term user participation incentive into explicit consideration. Specifically, since the value function of a required service is the submodular, the platform needs to design a mechanism $\mathcal{M} = (\phi, p)$, which consists of an allocation function $\phi : \mathcal{R}^+_N \to 2^{[N]}$ and a payment function $p : \mathcal{R}^+_N \to \mathcal{R}^+_N$. The allocation function $\phi$ is an indicator function that returns 1 if user $n$ is allocated and 0 otherwise. The utility of user $n$ is $p_n[t] - c_n[t]$ if it is selected in period $t$, i.e., $\phi_n[t] = 1$, 0 otherwise. The payment function $p[t]$ returns a vector $(p_1[t], \ldots, p_N[t])$ of payments to the users. The goal of the platform at period $t$ is to minimize its payment while the required service $R[t]$ is completed and the long-term participation constraint is achieved to compensate the indirect extra cost, i.e., $\min \sum_{n=1}^{N} \phi_n[t] \cdot p_n[t]$, subject to $V_{\phi[t]}[t] \leq R[t]$ and $\frac{1}{T} \sum_{t=1}^{T} \phi_n[t] \geq A_n$, where $V_{\phi[t]}[t]$ is monotone submodular, illustrated in the following definition and theorem.

**Definition 1 (Submodular Function):** Let $\mathcal{U}$ be a finite set, a function $V : 2^\Omega \to \mathcal{R}$ is submodular if $V(J \cup \{n\}) - V(J) \geq V(H \cup \{n\}) - V(H), \forall J \subseteq H \subseteq \Omega$, where $\mathcal{R}$ is the set of reals.

**Theorem 1:** The value function $V_{\phi[t]}(J)[t]$ is monotone submodular.

The detailed proof of Theorem 1 is given in Appendix A.

In the following, we respectively solve the above ILP problem for the semi-online scenario, where all of participating users periodically report their profiles to the platform synchronously at the beginning of each period, and then the platform allocates tasks to a subset of users by considering the profiles of all users at once, and the online scenario, where both the intra-period and inter-period interactive processes are sequential and asynchronous.

4 Semi-Online and Online Frugal Incentive Mechanism for Periodical MCS

In this section, we first introduce Lyapunov optimization [20], which is a widely used technique for solving online optimization problems with time average constraints, to tackle the long-term incentive issue, i.e., the second constraint (2) in equation (ILP). Then according to the Lyapunov optimization, we respectively design semi-online and online frugal incentive mechanism for periodical MCS applications.

4.1 Lyapunov optimization

Since the stability of queues is a key idea of Lyapunov optimization technique, we establish a connection between the queue stability condition and the user participatory constraint as follows to guarantee that the time average constraints in equation (ILP) hold. We assume that each user $n$ has a queue to buffer his allocation request (the requirement of the user participatory constraint) from the platform. Thus, the backlog of a queue denotes the total number of requests in the queue, i.e., the total number of future periods that the user should be selected as a winner so as to meet his participatory constraint. More specifically, let $A_n$ be the arrival rate of user $n$, i.e., his participatory constraint from the platform, and $p_n[t]$ be his departure rate at period $t$, which means whether user $n$ is selected as a winner in this period. Let $q_n[t]$ denote the queue backlog of user $n$ in period $t$. The dynamic equation for user $n$’s queue is given as follows.

$$q_n[t + 1] = [q_n[t] - \phi_n[t]]^+ + A_n,$$

where $[x]^+ = \max(x, 0)$. We say his queue $q_n$ is stable, if $\lim_{t \to \infty} \frac{x[t]}{t} = 0$ with the probability 1 according to Definition 2.2 in theorem 4.1 in [20].

To ensure the above queue stability, we introduce the Lyapunov drift, i.e., the change of Lyapunov function from one period to the next. Let $L(q[t]) = \frac{1}{2} \sum_{n=1}^{N} (q_n[t])^2$, where $q_n[t]$, $n \in \mathcal{U}$ denotes the queue backlog vector of all available users. So the Lyapunov drift can be denoted as follows.

$$\Delta(q[t]) = E\{L(q[t + 1]) - L(q[t])|q[t]\}. $$

If a policy greedily minimizes the Lyapunov drift $\Delta(q[t])$ in each period $t$, then all backlogs are consistently pushed towards a low level, which potentially maintains the stabilities of all queues (i.e., the participatory constraints of all available users hold).

4.2 Semi-Online Frugal Mechanism Design

In this section, we present a semi-online frugal incentive mechanism under a series of service constraints and long-term participation constraint, satisfying the previous desirable properties.
To stabilize the queues, i.e., satisfying the long-term participation constraint, while minimizing the total payment, we need to use an allocation policy that greedily minimizes the following drift-plus-payment, i.e., $\Lambda(q[t]) = \Delta(q[t]) + \delta P[t]$, where $\delta \geq 0$ (a non-negative control parameter that is chosen to achieve a desirable tradeoff between the queue backlog and optimality) and let $P[t] = \sum_{n=1}^{N} \phi_{n}[t] \cdot p_{n}[t]$ for convenience. Moreover,

$$\Lambda(q[t]) \leq \frac{1}{2} \sum_{n=1}^{N} (\phi_{n}[t]^{2} + A_{n}^{2} + 2q_{n}[t] \cdot (A_{n} - \phi_{n}[t])) - \delta P[t] \leq G + \sum_{n=1}^{N} q_{n}[t] \cdot (A_{n} - \phi_{n}[t]) - \delta P[t],$$

where $G = \sum_{n=1}^{N} \frac{1}{2} (1 + A_{n}^{2})$ [18]. The first inequality holds because $(\phi - \phi^+ + M)^2 \leq \phi^2 + M^2 + 2q \cdot (M - \phi)$ [18]. The second inequality is obtained because $\phi_{n}[t]^2 \leq 1$. It is easy to derive that minimizing the upper bound of the drift-plus-payment is equivalent to minimizing the drift-plus-payment itself by using the Lyapunov optimization theory. Removing the constants in the upper bound expression, we can derive that minimizing the above upper-bound of the drift-plus-payment is equivalent to minimizing the expression $P[t] - \sum_{n=1}^{N} \frac{1}{\delta} (q_{n}[t] \cdot \phi_{n}[t])$ [18].

For crowd sensing applications in the offline scenario, the authors of [9, 10, 21] apply the proportional share allocation rule proposed in [9] to address the extensive user participation issue. However, the mechanism only applies for the offline scenario with the budget constraint. To address this problem, we present a service-constraint semi-online incentive mechanism that satisfies the previous desirable properties. Illustrated in Algorithm 1 our mechanism consists of two phases: the winner selection phase and the payment determination phase.

From Definition 1 we can know the utility function $V$ is submodular and derive the following sorting according to increasing marginal contributions relative to their adjusted bids $e_{n}[t] = b_{n}[t] - q_{n}[t]/\delta$ from users’ set to find the largest $k$ satisfying $V(J[t] \cup k)< R[t]$. In the following, for convenience, we drop the variable $t$.

$$V_{1}/e_{1} \geq V_{2}/e_{2} \geq \cdots \geq V_{|J|}/e_{|J|},$$

where $V_{k}$ denotes $V_{k}|S_{k-1} = (V(J_{k-1} \cup \{k\}) - V(J_{k-1}))$, $J_{k} = \{1, 2, \ldots, k\}$, and $J_{0} = \emptyset$. To calculate the payment of each user, we sort the users in $\mathcal{U} \setminus \{n\}$ similarly as follows:

$$V_{n_{1}}(\mathcal{H}_{0})/e_{n_{1}} \geq V_{n_{2}}(\mathcal{H}_{1})/e_{n_{2}} \geq \cdots \geq V_{n_{m-1}}(\mathcal{H}_{m-2})/e_{n_{m-1}}.$$  

Algorithm 1 SFP // Semi-online Frugal Incentive mechanism for Periodical MCS

**Input:** User set $\mathcal{U}$, the service constraint $R[t]$, $q[0]$, $\delta$.

**Output:** The set of winners $J$.

1. for each period $t = 0, 1, \ldots, T$ do
2. // Phase 1: Winner selection
3. for each user $j \in \mathcal{U}$ do
4. Update $e_{j}[t] \leftarrow b_{j}[t] - q_{j}[t]/\delta$;
5. end for
6. $J[t] \leftarrow \emptyset$; $J^* \leftarrow \arg \max_{J \subseteq \mathcal{U}} V_{J}(J[t])/e_{J}$;
7. while $V(J[t])/e_{J} < R[t]$ do
8. $J[t] \leftarrow J[t] \cup J^*$;
9. $J^* \leftarrow \arg \max_{J \subseteq \mathcal{U}\setminus J[t]} V_{J}(J[t])/e_{J}$;
10. end while
11. $B[t] \leftarrow \sum_{J \subseteq \mathcal{U}} b_{J}[t]$;
12. // Phase 2: Winner selection under budget $B$
13. $J \leftarrow \emptyset$; $J^* \leftarrow \arg \max_{J \subseteq \mathcal{U}} V_{J}(J)/e_{J}$;
14. while $V_{J}(J)/e_{J} \geq V(J \cup J^*)/B$ do
15. $J \leftarrow J \cup J^*$;
16. $J^* \leftarrow \arg \max_{J \subseteq \mathcal{U}\setminus J} V_{J}(J)/e_{J}$;
17. end while
18. // Phase 3: Payment determination
19. for each user $n \in \mathcal{U}$ do
20. $p_{n}[t] \leftarrow 0$;
21. end for
22. for each user $n \in J[t]$ do
23. $\mathcal{U} \leftarrow \mathcal{U} \setminus \{n\}$; $\mathcal{H} \leftarrow \emptyset$;
24. repeat
25. $n_{j} \leftarrow \arg \max_{J \subseteq \mathcal{U}\setminus \mathcal{H}} V_{J}(\mathcal{H}|J)/e_{J}$;
26. $\hat{p}_{n}[t] \leftarrow \max\{p_{n}[t], \min\{e_{n}(J)|J, \eta_{n}(J)|J\}\}$;
27. $\mathcal{H}_{j-1} \leftarrow \mathcal{H}$; $\mathcal{H} \leftarrow \mathcal{H} \cup \{n_{j}\}$
28. until $V(\mathcal{H})/e_{\mathcal{H}} \geq R[t]$;
29. $p_{n}[t] \leftarrow \hat{p}_{n}[t] + q_{n}[t]/\delta$;
30. end for
31. // Phase 3: Update Rule
32. for $i = 1$ to $N$ do
33. $q_{n}[t] \leftarrow [q_{n}[t-1] - \hat{p}_{n}[t-1]]^{+} + A_{n}$;
34. end for
35. end for
36. return $(J, p)$;

The marginal value of user $n$ at the position $j$ is $B_{n_{j}}(\mathcal{H}_{j-1})/V(H_{j})$, where $B = \sum_{j \subseteq J} b_{J}$. Assume that $k'$ to be the position of the last user $n_{j} \in \mathcal{U} \setminus \{n\}$, such that $V(H_{k'}) < R$. To guarantee the truthfulness, each winner should be given the payment of the critical value. This indicates that user $n$ can not win the auction if it reports higher than this critical value. More details are given in Algorithm 1 where $e_{n_{j}} = V_{n_{j}}(\mathcal{H}_{j-1})/V_{n_{j}}(\mathcal{H}_{j-1})$ and $\eta_{n_{j}} = V_{n_{j}}(\mathcal{H}_{j-1})B/V(\mathcal{H}_{j-1} \cup \{n\})$. In the following, based on the SFP mechanism, we design an online frugal mechanism.
4.3 Online Frugal Mechanism Design

In this section, we present an online frugal mechanism under a series of service constraints, satisfying all desirable properties. To facilitate understanding, it is also assumed that users arrive in a sequential order. But our mechanism can easily apply generally or be extended to an random online scenario.

An online frugal mechanism needs to handle several challenges below. First, the users’ costs are unknown and need to be elicited in a truthful reporting manner. Second, a series of required services should be completed before each period’s deadline. Third, the long-term user participation need to be guaranteed to support the periodical MCS applications. Finally, the mechanism needs to tackle the online arrival of the users. To achieve good frugality, previous online solutions for the independent MCS applications and generalized secretary problems [12], [22]–[24] is via sampling: the first batch of the input is rejected and used as a sample which enables making an informed decision on the rest of the users. Since users are likely to be discouraged to sense data knowing the pricing mechanism will automatically reject their bid. In other words, those users arriving early have no incentive to report their bids to the platform, which may delay the users’ completion or even lead to task starvation, i.e., the consumer sovereignty issue in economics. Although the author of [24] adopts a multi-stage sampling-accepting process, it applies Dynkin’s algorithm [25] for the classic secretary problem at the initial stage. Obviously, this solution also cannot ensure the above task-starvation issue, since Dynkin’s algorithm adopts a two-stage sampling-accepting process.

To address the above challenges, we introduce the previous Lyapunov optimization theory and a multi-stage sampling-accepting process for each period to design a frugal online incentive mechanism. We first apply multi-stage sampling-accepting process within each period. Then we adjust users’ bids to satisfy the long-term user participation constraint based on the previous result of Lyapunov optimization. For each period, we divides it into multiple stages. At each stage, based on the above submodularity, the mechanism maintains a density threshold which is used to decide whether to accept the users’ bids. The mechanism dynamically increases the sample size and learns a budget that are enough to allocate users for fulfilling the required services, then apply this budget to compute a density threshold by applying budget feasible mechanisms, and finally apply this density threshold for making further decisions.

Specifically, our mechanism (see Algorithm 2) periodically iterates over $\zeta[t] \in \{0, 1, \ldots, \lceil \log \tau \rceil \}$ for each period and at every time step $\zeta$, a required stage-service of $R'[t] = R[t]/2^\zeta$ is applied to complete each period’s sensing services (illustrated in Fig. 2). This means that $R'[t]$ services should be completed before the end of this stage for each period $t$. Finally, the required services $R[t]$ should be completed before the end of the period’s deadline $\tau$. At the beginning of the mechanism, we introduce a small value $\varepsilon$ as initial density threshold for each period. We assume that the marginal value of user $n$ ($n \notin J$) is $V_n(J) = V(J \cup \{n\})$, where $J$ is a winners’ set. In the sequel, as long as the arrival user’s marginal density $V_n(J[t] \cup \{n\})/\delta$ is not less than the current threshold density value $\rho^*$ and the budget has not been exhausted, the mechanism allocates service to it, where $e_n[t] = b_n[t] - q_n[t]/\delta$. 

Algorithm 2 OFP // Online Frugal incentive mechanism for Periodical MCS

Input: Service constraint $R$, sensing task deadlines $\tau$, $q[0], \delta$.

1. for each period $t = 0, 1, \cdots, T$
2. \hspace{1em} for each user $j \in U$
3. \hspace{2em} Update $e_j[t] \leftarrow b_j[t] - q_j[t]/\delta$;
4. \hspace{1em} end for
5. \hspace{1em} // Phase 1: Select winners and Compute the payment
6. \hspace{2em} $(\zeta, \tau', R', J', \rho^*, J[t]) \leftarrow (1, 2^{\log_2 t}, \emptyset, \emptyset, \emptyset)$;
7. \hspace{2em} for $\zeta \leq \tau'$ do
8. \hspace{3em} if there is a user $n$ arriving at time step $\zeta$ then
9. \hspace{4em} if $e_n \leq V_n(J[t]/\rho^*$ and $V(J[t]) < R'$ then
10. \hspace{5em} $p_n \leftarrow V_n(J[t])/\rho^*$,
11. \hspace{5em} $p_n[t] \leftarrow p_n[t] + q_n[t]/\delta$;
12. \hspace{3em} else
13. \hspace{4em} $p_n \leftarrow 0$;
14. \hspace{4em} end if
15. \hspace{4em} $J[t] \leftarrow J'[t] \cup \{n\}$;
16. \hspace{3em} end if
17. \hspace{3em} // Phase 1: Compute Threshold
18. \hspace{4em} if $\zeta = \lceil \tau' \rceil$ then
19. \hspace{5em} Calculate $\rho^* \leftarrow \text{getDensityThreshold}(R', J')$;
20. \hspace{5em} set $R' \leftarrow 2R'$,
21. \hspace{5em} $\tau' \leftarrow 2\tau'$;
22. \hspace{3em} end if
23. \hspace{3em} $t \leftarrow t + 1$;
24. \hspace{3em} end for
25. \hspace{3em} // Phase 3: Update Rule
26. \hspace{4em} for $i = 1$ to $N$
27. \hspace{5em} $q_n[t] \leftarrow [q_n[t] - \hat{\delta}(t-1)]^+ + A_n$;
28. \hspace{4em} end for
29. end for

Fig. 2. Illustration of a multi-stage sample process with deadlines $\tau$ of each period $t$. (a)Budget constraints over quantiles $\zeta$; (b)Quantiles over time slots $t$. 
is the adjusted bid of user $n$ to satisfy the long-term user participation. Meanwhile, we give user $n$ a payment $V_n(\mathcal{J}[t]/\rho^n[t]$, and add this user to the set of selected users $\mathcal{J}[t]$. 

In the computation of the density threshold for the mechanism, we first find the maximal density for fulfilling $\eta R'[t]$ services from the sample set $\mathcal{J}'[t]$. Then the process is repeated by using a simple greedy manner until all of $\eta R'[t]$ services are allocated. The greedy manner sorts users according to their density, preferentially allocates services to users with higher density. Here, we set $\eta$ to blow up the required stage services so that the constant blowup services can be allocated at the next stage. Furthermore, we compute the total payment for fulfilling the constant blowup services. Furthermore, the algorithm calls the following the budget feasible mechanism for submodular function and then sets the density threshold to be $\rho[t]/\nu[t]$. $\nu[t]$ is introduced to guarantee enough users selected and avoid the waste of payment.

The above budget feasible mechanism for submodular function is an offline mechanism proposed in [9]. It adopts a proportional share allocation rule [9] to compute the density threshold from the sample set $\mathcal{J}'[t]$ and the budget $B'[t]$. First of all, users are sorted according to their increasing marginal densities. In this sorting the $(n+1)$-th user is the user $j$ such that $V_j(\mathcal{J}_n[t]/e_j[t]$ is maximized over $\mathcal{J}'[t] \setminus \mathcal{J}_n[t]$, where $\mathcal{J}_0[t] = \{1, 2, \ldots, n\}$ and $\mathcal{J}_n[t] = \emptyset$. Considering the submodularity of $V[t]$, this sorting implies that $V_j(\mathcal{J}_n[t]/e_j[t]) \geq V_{j+1}(\mathcal{J}_n[t]/e_{j+1}[t]) \geq V_{j+2}(\mathcal{J}_n[t]/e_{j+2}[t]) \geq \cdots \geq V(\mathcal{J}_n[t]/e_{n}[t])$.

Then, the computation process adopts a greedy strategy. That is, according to increasing marginal contributions relative to their bids from the sample set to find the largest $k[t]$ satisfying $e_k[t] \leq \frac{B'[t]}{V(\mathcal{J}_n[t]/e_{k}[t])}$. Furthermore, we can obtain the payment threshold estimated based on every sample set $\mathcal{J}'[t]$ with the privacy profile of users and the allocated stage-budget $R'[t]$. Finally, we set the density threshold to be $\frac{V(\mathcal{J}_n[t]/e_{k}[t])}{R'[t]}$. The detailed computation of the threshold density is illustrated in Algorithm 3 and Fig. 2.

Algorithm 3 getDensityThreshold

**Input:** Sample user set $\mathcal{J}'$, the stage-service $R'$.  
**Output:** The threshold density $\rho$.  
1: Initialize: $\mathcal{J} \leftarrow \emptyset$; $j^* \leftarrow \arg \max_{j \in \mathcal{J}'} \frac{V_j(\mathcal{J}[t])}{e_j}$;  
2: while $V(\mathcal{J}[t]) < \eta R'[t]$ do  
3: \hspace{1cm} $\mathcal{J} \leftarrow \mathcal{J} \cup \{j^*\}$;  
4: \hspace{1cm} Compute $j^* \leftarrow \arg \max_{j \in \mathcal{J}' \setminus \mathcal{J}} \frac{V_j(\mathcal{J}[t])}{e_j}$;  
5: end while  
6: $B' \leftarrow \sum_{j \in \mathcal{J}} b_j$;  
7: $\rho \leftarrow \text{getFeasibleDensity}(B', \mathcal{J})$;  
8: return $\rho/\nu$;

**Algorithm 4 getFeasibleDensity**

**Input:** Sample user set $\mathcal{J}'$, the budget $B'$.  
**Output:** The threshold density $\rho$.  
1: Initialize: $\mathcal{J} \leftarrow \emptyset$; $j^* \leftarrow \arg \max_{j \in \mathcal{J}'} \frac{V_j(\mathcal{J}[t])}{e_j}$;  
2: while $e_j^* \leq \frac{B'}{V_j(\mathcal{J}[t])}$ and $V(\mathcal{J}[t]) \leq B'$ do  
3: \hspace{1cm} $\mathcal{J} \leftarrow \mathcal{J} \cup \{j^*\}$;  
4: \hspace{1cm} Compute $j^* \leftarrow \arg \max_{j \in \mathcal{J}' \setminus \mathcal{J}} \frac{V_j(\mathcal{J}[t])}{e_j}$;  
5: end while  
6: $\rho \leftarrow \frac{V(\mathcal{J}[t])}{B'}$;  
7: return $\rho$;

### 5 MECHANISM ANALYSIS

We now prove that SFP mechanism and OFP mechanism satisfy the desirable properties.

#### 5.1 SFP Mechanism Analysis

**Lemma 1:** The SFP mechanism satisfies the long-term participation constraint.

**Proof:** In the SFP mechanism, the update rule in (53) of Algorithm 1 can be viewed as a regulation factor for lowering the sensing bid of each user, and thereby increasing the probability that user becomes a winner. By iteratively applying the update rule, we further obtain the following participation queue backlog of user $n$, i.e., $q_n[t] = q_n[0] - \sum_{k=0}^{t-1} \phi_n[k] + t \cdot A_n \quad (\forall n \in N, q_n[0]$ is a non-negative real valued random variable), which is also approximately derived by simply omitting the operation $\lceil \cdot \rceil^+$. This means that according to the Lyapunov drift theorem 4.1 in [20], if a mechanism greedily minimizes the Lyapunov drift $\Delta(q[t])$ in each period $t$, then all backlogs are consistently pushed towards a low level, which implies that $\lim_{t \rightarrow \infty} \frac{q_n[t]}{t} = 0$ with the probability 1. Since the SFP mechanism satisfy the condition, we have $\lim_{t \rightarrow \infty} \frac{q_n[t]}{t} = 0$ with probability 1. That is, each user $n$'s participation queue is stable.

Moreover, according to Lemma 2.1 in [20], for all periods $t > 0$ of each user $n$, the following inequality holds:

$$\frac{q_n[t]}{t} - \frac{q_n[0]}{t} \geq A_n - d_n(\phi_n),$$

where $d_n(\phi_n) = \frac{1}{T} \sum_{t=1}^{T} \phi_n[t] \geq A_n$ denotes the average allocation probability of user $n$. We can take limits in the above inequality as $t \rightarrow \infty$. Combining the previous result, we conclude that $0 \geq A_n - d_n(\phi_n)$, and hence $d_n(\phi_n) \geq A_n$, i.e., satisfying the long-term participation constraint. Thus, Lemma 1 holds.

**Lemma 2:** The SFP mechanism’s solution is approximately optimal.

**Proof:** According to Lyapunov optimization equation 3.31 in [20], we have

$$\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^{T} E(p[t]) \leq p^* + \frac{G}{\delta},$$
Besides, \( p^* \leq \tilde{p} \), where \( \tilde{p} \) denotes the payment of SFP mechanism when excluding the update rule supporting the long-term participation. Thus, we have

\[
\lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} E(p[t]) \leq \tilde{p} + \frac{G}{\delta}.
\]

As such, we can easily find that the SFP mechanism converges to the value \( \tilde{p} \) asymptotically, with a controllable approximation error bound \( O(1/\delta) \). Moreover, \( \tilde{p} \) is an approximate optimal solution according to [9, 10]. Thus, the SFP mechanism’s solution is approximately optimal. \( \square \)

Since the SFP mechanism is very similar with MSensing in [10], only with three differences. The one is that the services allocated to the winners is a constraint instead of a factor in the objective function. The next one is that SFP scenario applies for the periodical MCS instead of the independent MCS, thereby introducing lines 4-29 and 33 of Algorithm 1. The third one is that SFP is a frugal mechanism instead of a budget constraint mechanism, hence introducing line 11 of Algorithm 1. But these lines’ introduction has no impact on the following desirable properties. Thus, putting Lemma 1 and 2 together, we have the following theorem.

**Theorem 2**: The SFP mechanism satisfies individual rationality, computational efficiency, service feasibility, truthfulness, long-term participation constraint and approximately optimal under a semi-online scenario.

### 5.2 OFP Mechanism Analysis

**Lemma 3**: The OFP mechanism is incentive compatible or truthful.

*Proof*: To see that bid-independent auctions of the OFP mechanism are truthful, here consider a user \( n \) with cost of \( c_n \) that arrives at some stage during period \( t \), for which the threshold density is set to \( \rho^* \). If by the time the user arrives there are no remaining required stage services for the period, then the user’s cost declaration will not affect the allocation of the mechanism and thus cannot improve his utility by reporting a false cost. Otherwise, assume there are remaining required stage services for the period by the time the user arrives. In case \( e_n = (c_n[t] - q_n[t]/\delta) \leq V_n(J)/\rho^* \), reporting any cost that makes the adjusted cost \( e_n = c_n[t] - q_n[t]/\delta \) below \( V_n(J)/\rho^* \) wouldn’t make a difference in the user’s allocation and payment and his utility for each assignment would be \( V_n(J)/\rho^* + q_n[t]/\delta - c_n \geq 0 \), where \( V_n(J)/\rho^* + q_n[t]/\delta \) is his payment from the platform. Declaring a cost that makes the adjusted cost \( e_n = c_n[t] - q_n[t]/\delta \) above \( V_n(J)/\rho^* \) would make the user lose the auction, and his utility would be 0. In case \( e_n > V_n(J)/\rho^* \), declaring any cost above \( V_n(J)/\rho^* \) would leave the user unallocated with utility 0. If the user declares a cost that makes the adjusted cost \( e_n = c_n[t] - q_n[t]/\delta \) lower than \( V_n(J)/\rho^* \) he will be allocated. In such a case, however, his utility will be negative. Thus the user’s utility is always maximized by reporting his true cost: \( b_n = c_n \). Putting these discussions together, the OFP mechanism satisfies bid-independence.

**Proof**: At each stage \( \varsigma \in \{0, 1, \cdots, \lfloor \log_2 \tau \rfloor, \lfloor \log_2 \tau \rfloor + 1\} \), the mechanism uses a stage-service of \( R' = \frac{\varsigma - \varsigma'}{2^{\varsigma - \varsigma' + 1}} \). From the lines 9-10 of Algorithm 2 we can see that it is guaranteed that the current total allocated services do not exceed the stage-service \( R' \) for each period. Specially, the service constraint of the last stage of the period \( t \) is \( R[t] \). Therefore, every stage of the period is service feasible, and when the deadline \( \tau \) of the period arrives, the total allocated services do not exceed \( R[t] \). It is possible that the total required services can not be fulfilled. To the end, we compute the minimal cost for fulfilling a constant blowup of the required services by a frugal ratio \( \eta \) (see Algorithm 3). As such, \( R/2 \) required services could be completed at the last stage of the period \( t \) while the total payment of the period is no more than the budget \( B \). Thereby, the mechanism can guarantee that each stage of the period uses minimal payments to achieving the required stage services by blowing up to \( \eta R' \) until the total required services are fulfilled. Similarly, the above discussion also applies for other periods. Thus, Lemma 3 holds. \( \square \)

**Lemma 4**: The OFP mechanism is service feasible.

**Proof**: As such, \( R/2 \) required services could be completed at the last stage of the period \( t \) while the total payment of the period is no more than the budget \( B \). Thereby, the mechanism can guarantee that each stage of the period uses minimal payments to achieving the required stage services by blowing up to \( \eta R' \) until the total required services are fulfilled. Similarly, the above discussion also applies for other periods. Thus, Lemma 3 holds. \( \square \)

**Lemma 5**: The OFP mechanism is computational efficient.

*Proof*: Since the mechanism runs online, we only need to focus on the computation complexity at each time step \( \varsigma = \{1, 2, \cdots, \tau\} \) of each period \( t \). Computing the marginal value of user \( n \) takes \( O(r_n) \) time, which is at most \( O(I) \). Thus, the running time of computing the allocation and payment of user \( n \) (lines 8-16 of Algorithm 2) is bounded by \( O(I) \). Next, we analyze the complexity of computing the density threshold, namely Algorithm 3. Finding the user with maximum marginal density takes \( O(I,J) \) time. Since there are \( I \) PoIs and each selected user should contribute at least one new PoI, the number of winners is at most \( \min(I,|J|) \). Thus, the running time of lines 16 of Algorithm 3 is bounded by \( O(I,J) \min(I,|J|) \). The running time of line 7 of Algorithm 3 is the same as of lines 16 of Algorithm 3. Thus, the computation complexity at each time step (lines 8-22 in each period is bounded by \( O(I,J) \min(I,|J|) \)). At the last stage of the period, the sample set \( J' \) has the maximum number of samples, being \( N/2 \) with high probability.
Thus, the computation complexity at each time step of each period $t$ is bounded by $O(IN \min \{I, N\})$. Thus, Lemma 5 holds.

**Lemma 6:** The OFP mechanism is individually rational.

**Proof:** From the lines 9-13 of Algorithm 2, we can see that $p_n = V_n(J)[t] + q_n(t)/\rho \geq c_n + q_n(t)/\delta = b_n - q_n(t)/\delta + q_n(t)/\delta = b_n$ if $n \in J$, otherwise $p_n = 0$. Therefore, we have individual gain $u_n \geq 0$. Similarly, the above discussion also applies for other periods. Thus, Lemma 6 holds.

**Lemma 7:** The OFP mechanism satisfies the consumer sovereignty in economics.

**Proof:** Each stage for each period $n$ is an accepting process as well as a sampling process ready for the next stage. As a result, users are not automatically rejected during the sampling process for each period, and are allocated as long as their marginal densities are not less than the current threshold density, and the allocated stage services has not been exhausted. Similarly, the above discussion also applies for other periods. Thus, Lemma 7 holds.

**Lemma 8:** The OFP mechanism satisfies the long-term constraint.

The proof of this lemma is the same as Lemma 11’s proof.

If the stage services for each period $t$ could be achieved at each stage of each period $t$, then $R[t]$ required services of each period $t$ would be allocated finally. Since each period of the OFP mechanism consists of multiple stages, and dynamically increases the stage services of the period, it only needs to prove that $R[t]/2$ required services could be allocated at the last stage of the period while the total payment is no more than the budget $B$. Thereby, the mechanism can guarantee that each stage of the period uses minimal payments to achieving the required stage services of the period by blowing up to $\eta R'$ until the total required services of each period are fulfilled. The frugality ratio for achieving the required services of each period would be $\eta$, since at the last stage of each period $t$ the budget $B$ is the minimal cost for fulfilling the required stage services $\eta R' = \eta R[t]/2$ according to Algorithm 3. The OFP mechanism for minimizing payments is originated from the observations that the stage-service constraint at each stage of each period can be changed into the budget constraint at the correspondent stage of each period. If we show that at least $R[t]/2$ required services could be allocated at the last stage of each period under the budget constraint $B$, then it is equivalent to that $R[t]/2$ required services of each period could be allocated while the total payment of each period is no more than $B$. This means that the frugality ratio for achieving the required services of each period is $\eta$.

**Lemma 9:** The OFP mechanism satisfies $O(1)$-competitive in each period, i.e., constant frugal ratio. Specifically, under i.i.d. model, we can achieve the announced services from the platform when the frugal ratio $\eta = 8$ in each period. Under the secretary model, we can achieve the announced services from the platform when the frugal ratio $\eta = 24$ in each period. The OFP mechanism’s solution in all periods is constant frugal ratio $\eta$ with a controllable approximation error bound $O(1/\delta)$ under an online scenario.

### 6 Performance Evaluation

To evaluate the performance of our service-constraint mechanisms, we implemented the mechanisms SFP and OFP, and compared them against a full-offline benchmark (The platform can decide the explicit allocation of each user in current period as well as each period in future) with the same participation constraint of each user, the RADP-VPC policy that considers the participation incentive [14], and a random mechanism (A simple greedy algorithm that adopts a naive strategy for rewarding users based on an uninformed fixed bid threshold). To make our results more convincing, we also compare our policy with those not considering the long-term participation incentive based on the data set RollerNet [27]. The performance metrics include the participation ratio (the ratio between the number of participation users and total users), time average payments, control parameter $\delta$, and participation constraint parameter $A_n$ impact on minimum payments.

#### 6.1 Simulation Setup

We consider four two different simulation scenarios, depending on the different sensing value distributions in different areas. The first one is that sensing value distributions are uniform in different areas. Next one is that the Pols near to the center of the AoI have much larger than those far from the center of the AoI. We set the deadline (T) to 100s, and vary the required services $(R[t])$ of different periods from 200 to 2000 with the increment of 200. Users arrive according to a Poisson process in time with arrival rate $\lambda$. We vary $\lambda$ from 0.2 to 1 with the increment of 0.2. The sensing range $(r_n)$ of each user $n$ is set to 7 meters. The cost of each user is uniformly distributed over $[1, 10]$. The initial density threshold $(\epsilon)$ of Algorithm 1 and 4 is set to 1. Note that this threshold could be an empirical value for real applications. All the simulations were run on a PC with 1.7 GHz CPU and 8 GB memory. Each measurement is averaged over 100 instances.
Comparison on participation ratio: SFP and OFP evaluate the participation ratio on the mechanisms worse performance due to lower participation ratio. The mechanisms RADP-VPC and Random achieve a is illustrated in Fig. 5(a) in details. Fig. 4(b) shows that the parameter value errors, which is controlled by choosing different parameter. From Fig. 5(b), we can see that the minimum payment achieved by the mechanisms SFP and OFP decreases with the increase of the value of the control parameter $\delta$. Finally it will converge to the full-offline benchmark (an optimal solution) asymptotically with small approximation errors. This also validates our previous theory results. In Fig. 5(b) the lowest line denotes the lower-bound of the minimum payment with the participation constraint, which can be obtained by choosing an optimal control parameter. From Fig. 5(b) we can see that the minimum payment achieved by the mechanisms SFP and OFP increases with the increase of the value of the participation constraint value $A_n$ of each user, which means that more incentive cost is needed to retain users in the Periodical MCS system, hence the long-term participation incentive cost is necessary for Periodical MCS applications for achieving an optimal frugal payment.

Control parameter and participation constraint: Fig. 4 shows control parameter and participation constraint’s impact on the minimum payments. Fig. 5(a) shows that the minimum payment achieved by the mechanisms SFP and OFP increases with the increase of the value of the control parameter $\delta$. Finally it will converge to the full-offline benchmark (an optimal solution) asymptotically with small approximation errors. This also validates our previous theory results. In Fig. 5(b) the lowest line denotes the lower-bound of the minimum payment with the participation constraint, which can be obtained by choosing an optimal control parameter. From Fig. 5(b) we can see that the minimum payment achieved by the mechanisms SFP and OFP increases with the increase of the value of the participation constraint value $A_n$ of each user, which means that more incentive cost is needed to retain users in the Periodical MCS system, hence the long-term participation incentive cost is necessary for Periodical MCS applications for achieving an optimal frugal payment.

7 Conclusions

In this paper, we have designed two incentive mechanisms to motivate smartphone users to participate
in periodical MCS applications with the service constraint of each period. We first propose a SFP mechanism for the semi-online scenario. Furthermore, we design an OFP mechanism for a sequential arrival model, where users arrive one by one online. We also prove that the two mechanisms satisfy the above desirable properties.

APPENDIX A

Proof of Theorem 1:

**Proof:** From the sensing value function’s definition, we have

\[ V_{\phi[t]}(J) = \sum_{i=1}^{l} \min \left\{ 1, \sum_{n=1}^{N} \phi_{n}[t] \cdot r_{n,i}[t] \right\} \cdot \omega_{i}[t]. \]

For convenience, let \( \phi^J[t] \) and \( \phi^H[t] \) correspond to the allocated set \( J \) and \( H \) respectively. As such, \( V(J) = \sum_{i=1}^{l} \min \left\{ 1, \sum_{n \in J} r_{n,i}[t] \right\} \cdot \omega_{i}[t] \). According to Definition 1, we have

\[ V(J \cup \{ n \}) - V(J) = \sum_{i=1}^{l} \min \left\{ 0, 1 - \sum_{n \in J} r_{n,i}[t] \right\} \cdot \omega_{i}[t] \]

\[ \geq \sum_{i=1}^{l} \min \left\{ 0, 1 - \sum_{n \in J} r_{n,i}[t] \right\} \cdot \omega_{i}[t] \]

\[ = V(J \cup \{ n \}) - V(J). \]

Moreover, \( \forall J \subseteq \Omega \) and \( n \in \Omega \setminus J \), we have

\[ V(J \cup \{ n \}) - V(J) \geq 0. \]

Thus, \( V(J) \) is a monotone submodular function, i.e., \( V_{\phi[t]}[t] \) is a monotone submodular function in terms of Definition 1.

APPENDIX B

In order to make Lemma 9 hold, we first provide the following proposition.

**Proposition 1:** Given a sample set \( J' \), the total value of selected users computed by Algorithm 4 in our paper with the budget \( B'/2 \) is at least a half of that computed with the budget \( B' \) in each period \( t \).

**Proof:** Assume that the set of selected users computed with the budget \( B'/2 \) is \( J_1 = \{ 1, 2, \ldots, l \} \), and the set of selected users computed with the budget \( B' \) is \( J_2 = \{ 1, 2, \ldots, k \} \). Then, users can be sorted according to their increasing marginal densities as follows: \( V_1(J_0) / e_1 = V_2(J_0) / e_2 = \cdots = V_l(J_{l-1}) / e_l \geq 2V_l(J_l) / B' = V_{l+1}(J_{l+1}) / e_{l+1} = \cdots = V_k(J_{k-1}) / e_k \).

Thus, it can be easily derived that: \( V(J_1) \geq V(J_2) / 2 \). Thus, Lemma 9 holds.

**Proof of Lemma 9:**

**Proof:** Firstly, we make the following assumptions: Let \( J^* \) be the set of users selected by the offline Algorithm 4 in our paper before the time \( H \) and the budget \( 2B \), the value of \( J^* \) is \( V(J^*) \). The value density threshold of \( J^* \) is \( \rho = V(J^*) / B \). \( J^* \) is the sample set obtained at the time \( \tau / 2 \), \( J^*_1 = J^* \cap J^* \) and \( J^*_2 = J^* \setminus \{ V \setminus J^* \} \). \( J_1 \) is the set of users selected from the sample set \( J^* \) by Algorithm 4 before the time \( \tau \) of each period \( t \) and the budget \( B_t \), and \( J_2 \) is the set of users selected by Algorithm 2 in each period \( t \) at the last stage. Let \( \rho_1 = V(J_1') / B \) be the density computed using Algorithm 3 over \( J^* \) and \( \rho^* = \rho_1 / \nu \) is the density threshold of the last stage of each period \( t \). Assume that the value of each user \( n \) is at most \( \max_n V_n \leq V(J^*) / \omega \).

Then, we consider that the mechanism is constant frugal from the two class model: I.I.D. and the Secretary Model.

**I.I.D. Model’s Frugal Ratio of Each Period:** Since the costs and values of all users in \( U \) are i.i.d., they can be selected in the set \( J^* \) with the same probability. Thus, we have \( E[V(J_1')] = E[V(J_2')] = \frac{1}{2} \). Considering the submodularity of function \( V(J) \), it can be derived that: \( E[V(J_1')] \geq E[V(J_2')] \geq V(J^* / 2) = R / 2 \). Since \( V(J_1') \) is computed with the stage-budget \( B' / 2 \), it can be derived that: \( E[V(J_1')] \geq E[V(J_2')] \geq \frac{V(J^*)}{2} = R / 2 \) and \( E[\rho_1] \geq \rho \). where the first inequality follows from the fact that \( V(J_1') \) is the optimal solution computed by Algorithm 4. Therefore, we only need to prove that the ratio of \( E[V(J_2')] \) to \( E[V(J_1')] \) is at least a constant, then the OFP mechanism have a constant frugal ratio. Only two cases can exist according to the total payment to the selected users at the last stage.

First, consider the case that the total payment to the selected users at the last stage is at least \( \alpha B, \alpha \in (0,1/2] \), according to \( \alpha \in [0,1/2] \). In this case, since each selected user has marginal density at least \( \rho^* \), so we have that \( V(J_2') \geq \alpha B \cdot \rho^* = \alpha B \cdot \rho_1 / \nu = 2 \alpha \cdot V(J_1') / \nu \).

Second, consider the case that the total payment to the selected users at the last stage is less than \( \alpha B \). Consider the first case that the marginal densities of some users from \( J_2' \) are less than \( \rho^* \). In this case, these users are not allocated by the OFP mechanism. Even if these users are all in \( J^*_2 \), their expected total payment is at most \( B / 2 \). Because of submodularity, the expected total loss due to these missed users is at most \( \rho^* \cdot B / 2 = V(J_1') / \nu \). Considering the second case that the stage’s budget is exhausted before all users in \( J_2' \) arrives, it means that the payment for such a current user is larger than \( (1/2 - \alpha) B \) i.e., \( V_n(J) / \rho^* > (1/2 - \alpha) B \). From the above discussion, we know \( E[\rho_{1}] \geq \rho \). Thus, we have \( E[V_n(J)] > E[\rho^*](1/2 - \alpha) B \geq (1 - 2\alpha) B / 2 \nu \). Since the value of each user is at most \( V(J^*) / \omega \), and their expected total payment is at most \( B / 2 \), the expected total loss under the case is at most \( (\frac{\omega}{2 - 2\alpha} - 1) V(J^*) / \omega \). Thus, we have \( E[V(J_2')] \geq E[V(J_1')] \geq \frac{V(J^*)}{2} - (\frac{\omega}{2 - 2\alpha} - 1) V(J^*) / \omega \).
\[ \mathbb{E}[V(J_t')]/\nu \geq [1/2 - (1/2 - \alpha)/\nu - 1/\nu] \mathbb{E}[V(J_1')]. \]

Putting the two cases together, we have \(1/2 - (1/2 - \alpha)/\nu = 2\alpha/\nu.\) Thus, when \(\omega\) is sufficiently large (at least 12), we can obtain a constant ratio of \(\mathbb{E}[V(J_t')] / \mathbb{E}[V(J_1')].\) More importantly, the optimal ratio increases to 1/4 (i.e., \(2\alpha/\nu \to 1/4\)) as \(\omega\) increases.

According to Proposition \[\|\], we have \(\mathbb{E}[V(J_t')] \geq \eta R\). Furthermore, \(\mathbb{E}[V(J_t')] \geq 2\alpha R \mathbb{E}[V(J_1')] \geq \frac{\omega R}{\nu} \cdot \frac{\eta R}{4}.\) According to the previous discussions, to achieve the required services, the inequality \(\mathbb{E}[V(J_t')] \geq R/2\) holds by setting \(\frac{\omega R}{\nu} \geq \frac{R}{2}.\) As such, we have \(\eta \geq 2 \cdot \nu/2\alpha \geq 2 \times 4 = 8.\) Thus, we can set the frugal ratio \(\eta = 8\) to achieve the required services.

**Secretary Model’s Frugal Ratio of Each Period:**

In this model, let \(J^*\) be the set of users selected by the offline Algorithm 4 before the time \(H\) and the budget \(B\) other than the budget \(2B\) in the i.i.d. model. According to Lemma 15 in \[22\], for sufficiently large \(\omega\), the random variable \(|V(J^*) - V(J_2^*)|\) is bounded by \(V(J^*)/2\) with a constant probability. Because of the submodularity of \(V\), we have \(V(J_2^*) + V(J_2^*) \geq V(J^*)\). Thus, we easily obtain the result: For sufficiently large \(\omega\), both \(V(J_1^*)\) and \(V(J_2^*)\) are at least \(V(J^*)/4\) with a constant probability. Putting the result and Lemma \[21\] together, Putting the result and Proposition 1 together, we have \(\geq V(J_1)/2 \geq V(J^*)/8\) only two cases can exist according to the total payment to the selected users at the last stage.

First, consider the case that the total payment to the selected users at the last stage is at least \(\alpha B, \alpha \in (0, 1/2]\), according to \(\alpha \in [0, 1/2]\). In this case, since each selected user has marginal density at least \(\rho\), so we have that \(V(J_2^*) \geq \alpha B \cdot \rho^* = \alpha B \cdot \rho_1/\nu = 2\alpha \cdot V(J_1)/\nu.\)

Second, consider the case that the total payment to the selected users at the last stage is less than \(\alpha B.\) Consider the the first case that the marginal densities of some users in \(J_2^*\) are less than \(\rho^*\). In this case, these users are not allocated by the OFP mechanism. Even if these users are all in \(J_1^*\), their total payment is at most \(B.\) Because of submodularity, the expected total loss due to these missed users is at most \(\rho^* \cdot B = 2V(J_1)/\nu.\) Considering the second case that the stage’s budget is exhausted before all users in \(J_1^*\) arrives, it means that the payment for such a current user is larger than \((1/2 - \alpha)B, i.e., V_n(J)/\rho^* > (1/2 - \alpha)B).\) From the above discussion, we know \(2V(J_1)/B \geq V(J^*)/4B = \rho/4.\) Thus, we have \(V_n(J) > \rho^*(1/2 - \alpha)B = (1 - 2\alpha)\rho_1/2 B/\nu \geq (1 - 2\alpha)\rho B/8\nu.\) Since the value of each user is at most \(V(J^*)/\omega, \) and their total payment is at most \(B,\) the total loss under the case is at most \((\frac{\alpha R}{\nu} - 1)V(J^*)/\omega.\) Thus, we have \(V(J_2^*) \geq V(J_2^*) - \frac{\alpha R}{\nu} - 1)V(J^*)/\omega - 2V(J_1)/\nu \geq V(J^*)/4 - (\frac{\alpha R}{\nu} - 1)V(J^*)/\omega - 2V(J_1)/\nu \geq [1/4 - \frac{\alpha R}{\nu} - 1]/\omega + 2\nu) V(J_1).\)

**Frugal Ratio of All Periods:** Putting the two cases together, we have \(1/4 - (\frac{\alpha R}{\nu} - 1)/\omega - 2\nu = 2\alpha/\nu.\)

Thus, when \(\omega\) is sufficiently large (at least 12), we can obtain a constant ratio of \(V(J_2^*)\) to \(V(J_1^*).\) More importantly, the optimal ratio increases to 1/12 (i.e., \(2\alpha/\nu \to 1/12\)) as \(\omega\) increases.

According to Proposition \[1\] we have \(V(J_1^*) \geq \frac{\alpha R}{\nu}.\) Furthermore, \(V(J_2^*) \geq \frac{\alpha R}{\nu} V(J_1^*) \geq \frac{\alpha R}{\nu} \cdot \frac{\eta R}{4}.\) According to the previous discussions, to achieve the required services, the inequality \(V(J_2) \geq R/2\) holds by setting \(\frac{\alpha R}{\nu} \geq \frac{R}{2}.\) As such, we have \(\eta \geq 2 \cdot \nu/2\alpha \geq 2 \times 12 = 24.\) Thus, we can set the frugal ratio \(\eta = 24\) to achieve the required services.

Putting these together, we conclude that the first two results in Lemma 9 holds. Furthermore, according to the result to the Lemma 9, we have

\[
\lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} E[p(t)] \leq \bar{p} + \frac{G}{\delta},
\]

where for the OFP mechanism, \(\bar{p}\) should be represented as the payment of OFP mechanism when excluding the update rule supporting the long-term participation. Similarly, we can easily find that the OFP mechanism converges to the value \(\bar{p}\) asymptotically, with a controllable approximation error bound \(O(1/\delta).\) Moreover, \(\bar{p}\) is an approximate optimal solution with the above derived frugal ratio \(\eta \to \rho^*.\)

Thus, the SFP mechanism’s solution have frugal ratio \(\eta\) with a controllable approximation error bound \(O(1/\delta).\) Thus, the Lemma 9 holds.

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