Partial-wave analysis of $J/\psi \rightarrow K^+K^-\pi^0$
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A partial-wave analysis of the decay $J/\psi \rightarrow K^+K^-\pi^0$ has been made using $(223.7 \pm 1.4) \times 10^6$ $J/\psi$ events collected with the BESIII detector in 2009. The analysis, which is performed within the isobar-model approach, reveals contributions from $K_1(1430)\pm, K_2(1890)\pm$ and $K_3^*(2045)\pm$ decaying to $K^\pm \pi^0$. The two latter states are observed in $J/\psi$ decays for the first time. Two resonance signals decaying to $K^+K^-$ are also observed. These contributions can not be reliably identified and their possible interpretations are discussed. The measured branching fraction $B(J/\psi \rightarrow K^+K^-\pi^0)$ of $(2.88 \pm 0.01 \pm 0.12) \times 10^{-4}$ is more precise than previous results. Branching fractions for the reported contributions are presented as well. The results of the partial-wave analysis differ significantly from those previously obtained by BESII and BABAR.

I. INTRODUCTION

A good knowledge of the spectrum and properties of hadrons is one of the key issues for understanding the strong interaction at low and intermediate energies. The conventional quark model implies that quark-antiquark states are produced as nonets, which consist of mesons with strange and non-strange quarks. Therefore, an accurate identification of mesons with one strange quark can help to establish nonet members in the isoscalar sector, where the situation is more complicated. This is due to a potential mixing between octet and singlet states as well as possible mixing with glueball states.

The identification of meson radial excitations also helps in the understanding of quark-antiquark interaction at intermediate energies. Quark potential models [1] predict that the squared masses of radial excitations depend on the excitation number quadratically. However, in the analysis of proton-antiproton annihilation in flight, it was found that this dependence is close to the linear one similar to the Regge trajectories [2]. If correct, this behavior has the potential to reveal a new symmetry of the quark-antiquark interaction [3] [4]. Therefore, the experimental confirmation (or disproof) of this behavior is an important task in experimental hadron physics.

$J/\psi$ decays are ideal for the study of meson spectra and the determination of meson properties. They can provide important information about meson states with masses up to 3 GeV/$c^2$ and partial-wave analysis is facilitated due to the well-known quantum numbers of the initial state. Moreover, the $J/\psi$ radiative decay is favored for the production of glueball states which makes it a perfect tool to search for and study such exotics [5].

In this paper we report the results of a partial-wave analysis (PWA) of the decay $J/\psi \rightarrow K^+K^-\pi^0$. This decay channel has been previously studied by the MARK [6], MARK-II [7], MARK-III [8], DM2 [9], BESII [10], and BABAR [11][12] Collaborations, but only two recent publications report PWA results. In the first of these [10], BESII analyzes 58 million $J/\psi$ decays and observes a very broad exotic resonance $X(1575)$ with pole position $(1576^{+49}_{-55} + 98)MeV/c^2$ and branching fraction $B(J/\psi \rightarrow X(1575)p^0 \rightarrow K^+K^-\pi^0) = (8.5 \pm 0.6^{+32}_{-25}) \times 10^{-4}$. In the second analysis [12], BABAR reports a PWA solution based on a smaller data set of 2102 events, which consists of $K^+(892)\pm$, $K^+(1410)\pm$ and $K_2^*(1430)\pm$ states in the $K^+\pi^0$ channels, while the enhancement at low $K^+K^-$ invariant masses is attributed to the $\rho(1450)$. The analysis presented in this paper is based on a data set of 182,972 event candidates selected from $(223.7 \pm 1.4) \times 10^6 J/\psi$ decays [13] collected by the BESIII experiment in 2009. The high statistics and good data quality allow us to reveal signals from states that have not been observed before and precisely determine properties of intermediate states. Moreover, the obtained PWA solution can be used for the simulation of the irreducible background from this channel to the $J/\psi \rightarrow \gamma K^+K^-$ decay, which is one of the key channels to be studied in the search for a low-mass glueball.

II. BESIII EXPERIMENTAL FACILITY

The BESIII detector is a magnetic spectrometer [14] located at the Beijing Electron Positron Collider (BEPCCII) [15]. The cylindrical core of the BESIII detector consists of a helium-based multilayer drift chamber (MDC), a plastic scintillator time-of-flight sys-
system (TOF), and a CsI(Tl) electromagnetic calorimeter (EMC), which are all enclosed in a superconducting solenoidal magnet providing a 1.0 T magnetic field. The solenoid is supported by an octagonal flux-return yoke with resistive plate counter muon identifier modules interleaved with steel. The geometrical acceptance of charged particles and photons is 93% over the 4π solid angle. The charged-particle momentum resolution at 1 GeV/c is 0.5%, and the dE/dx resolution is 6% for electrons from Bhabha scattering. The EMC measures photon energies with a resolution of 2.5% (5%) at 1 GeV in the barrel (end cap) region. The time resolution of the TOF barrel part is 68 ps, while that of the end-cap part is 110 ps.

The GEANT4-based simulation software BOOST [10] is used to simulate the detector response. An inclusive J/ψ Monte Carlo (MC) sample is used to estimate the background. In this sample the production of the J/ψ resonance is simulated by the MC event generator KKMC[17,18] and decays are generated by EVGEN[19,20]. The branching fractions of known decay modes are set to the Particle Data Group (PDG) [21] world-average values and the remaining unknown decays are generated according to the Lund-Charm model [22].

III. EVENT SELECTION

The K^+K^-π^0 candidate events are required to have two charged tracks with zero net charge and at least two good photons. Charged tracks must be reconstructed within the geometrical acceptance of the detector (|cosθ| < 0.93, where θ is the angle with respect to the beam axis) and originate from the interaction point (|z| < 10 cm and R < 1 cm, where z and R are minimal distances from a track to the run-averaged interaction point along the beam direction and in the transverse plane, respectively). An event is rejected if the transverse momentum of at least one charged track is too low (p_T < 120 MeV/c). Particle identification (PID) is performed using TOF and MDC dE/dx information. Their measurements are combined to form particle identification confidence levels (C.L.) for π, K, and p hypotheses, and the particle type with the highest C.L. is assigned to the track. Both tracks are required to be identified as kaons.

Signal clusters in the EMC within the acceptance region, which are not associated with charged tracks and possess energy E > 25 MeV in the barrel part of the detector and E > 50 MeV in the end caps, are treated as photon candidates. To exclude showers from association with charged particles, the angle between the shower direction and the charged tracks extrapolated to the EMC must be greater than 10 degrees. The requirement on the EMC cluster time with respect to the start of the event (0 ns ≤ t ≤ 700 ns) is used to reject electronic noise and energy deposits not related to the analyzed event.

Consistency between the detector response and a final state hypothesis (for the signal and specific background decays) is evaluated by a four-momentum constrained (4C) kinematic fit. Firstly, the accepted pair of charged tracks and each pair of the selected photon candidates with invariant mass M_γγ < 300 MeV/c^2 are fitted under the γγK^+K^- hypothesis. A combination with the lowest value of χ^2_4C(γγK^+K^-) is selected and an event is retained if χ^2_4C(γγK^+K^-) < 60. Secondly, the χ^2_4C(γγK^+K^-) is compared to the corresponding value obtained in the best fits under the main background hypotheses: γγπ^+π^−, γK^+K^−, and, in the cases more than two good photon candidates are selected, γγγK^+K^-. If any of the background hypotheses results in a lower χ^2 value, the event is rejected. Finally, the π^0 candidates are reconstructed requiring the two-photon mass of the selected pair to be within a 110 MeV/c^2 < M_γγ < 150 MeV/c^2 interval. For the partial-wave analysis, we use particle momenta after the five-constrained (5C) kinematic fit, which also constrains the invariant mass of the selected photon pair to the nominal π^0 mass.

A total of 182,972 candidates satisfy the selection criteria. The corresponding number of background events is estimated from the inclusive MC: N_bg = 565±24 (or 0.3%). The largest background contributions come from the decay channels J/ψ → γη_c, η_c → K^+K^−π^0 and J/ψ → γK^+K^−. The continuum background, i.e. that due to the e^+e^- → γ∗ → K^+K^−π^0 process, is estimated from the analysis of a data sample of approximately 280 nb^{-1} collected from e^+e^- collisions at 3.08 GeV. It gives N_continuum = 855 ± 499, where the uncertainty is statistical. The background treatment in the PWA will be described in the next section.

The Dalitz plot for the selected data is shown in Fig. 1(a). Its most striking feature is a clear K*(892)^± signal. In the internal region of the plot a clear signal from K_2^0(1430)^± is seen as well as structures at M^2(K^±π^0) ≈ 4 GeV^2/c^4. These structures are likely to be the result of positive interference of resonances in the K^±π^0 channels. In the K^+K^- channel there are indications for a resonance signal at 1.6 − 1.7 GeV/c^2 and a signal at higher masses.

IV. PARTIAL-WAVE ANALYSIS

We use the isobar model to describe the J/ψ decay into K^+K^-π^0. The amplitude is parameterized as a sum of sequential quasi two-body decay processes in this approach. The subprocess described by intermediate state production and the subsequent decay to a specific pair of the final state mesons is referred to as the decay kinematic channel. The angular-dependent parts of the partial-wave amplitudes are calculated in the framework of the covariant tensor approach as described in detail in Ref. [23]. Note that in our case the conservation of P- and C-parities restricts the number of allowed partial waves for production and decay of any resonance to one. To account for the finite size of a hadron each decay vertex
also includes Blatt-Weisskopf form factors, which depend on the Blatt-Weisskopf radius $r$. The Breit-Wigner term for the resonance $a$ in the kinematic channel $m$ (labeled by the number of the spectator particle) is

$$A_{m,a}^{BW} = \frac{1}{M_a^2 - s_m - iM_a \Gamma(s_m, J_a)}.$$ 

Here $M_a$, $J_a$ and $s_m$ are the resonance mass, spin and the invariant mass squared of its daughter particles, respectively. The width of the $K^*(892)^\pm$ state is defined by its decay to $K\pi$ and is parameterized as:

$$\Gamma(s_m, J_a) = \frac{\rho_f(s_m)}{\rho_f(M_a^2)} \Gamma_a,$$

$$\rho_f(s_m) = \frac{2q}{\sqrt{s_m}} F^2(q^2, r, J).$$

Here, $\Gamma_a$ is the resonance width, $q$ is the relative momentum of the daughter particles calculated in the resonance rest frame and $F(q^2, r, J)$ is the above-mentioned Blatt-Weisskopf form factor. The same parameterization is used for the width of the $K^*_2(1430)^\pm$ resonance, whose decay branching fraction to $K\pi$ is about 0.5. For other states we use a constant width $\Gamma(s_m, J_a) = \Gamma_a$ due to the absence of reliable information about their branching fractions.

The masses, widths, decay radii (for the $J/\psi$, $K^*(892)^\pm$ and $K^*_2(1430)^\pm$) of resonances as well as the product of their production and decay couplings (complex numbers in general case) are initially free parameters of our fit. We find fit results weakly sensitive to the $J/\psi$ decay radius. Hence, we set this parameter to be 0.7 fm, as is obtained in Ref. [24].

The analysis is performed within the framework of the event-by-event maximum likelihood method, which allows us to take into account all correlations in the multidimensional phase space. The negative log-likelihood function $\text{NLL}$ is expressed as

$$\text{NLL} = - \sum_i \ln \frac{\omega_i \epsilon_i}{\int \epsilon \omega d\Phi} = - \sum_i \ln \frac{\omega_i}{\epsilon_i \omega d\Phi} + \text{const}$$

and is minimized. Here index $i$ runs over the selected data events, $\omega_i$ is the decay-amplitude squared, summed over transverse $J/\psi$ polarizations and evaluated from the four-momenta of final particles in the event $i$. The detector and event selection efficiency for the measured four-momenta is denoted by $\epsilon_i$, the denominator is a normalization integral over the phase space ($\Phi$), and the $\text{const}$ term is independent of the fit parameters. The normalization integral is calculated using phase-space distributed MC events that pass the detector simulation and the event reconstruction. To take the background into account we estimate its contribution to the NLL function and subtract it. This is done by the evaluation of the NLL function over properly normalized data samples that have a kinematic distribution similar to that of the background. We consider two types of background channels: those producing a peak at the $\pi^0$ mass in the

\[ \begin{align*}
    \text{NLL}_{\text{background}} &= - \sum_i \ln \frac{\omega_i \epsilon_i}{\int \epsilon \omega d\Phi} \\
    &= - \sum_i \ln \frac{\omega_i}{\epsilon_i \omega d\Phi} + \text{const}
\end{align*} \]

\[ \text{Figure 1. Dalitz plots for the selected data (a), the PWA solution I (b) and the PWA solution II (c).} \]
two-photon invariant-mass distribution ("peaking" background) and those exhibiting a smooth shape below the peak ("non-peaking" background). The former is estimated from $J/\psi \to \gamma \eta_c, \eta_c \to \gamma K^+ K^- \pi^0$ events selected under criteria similar to ones of the main event selection, and the latter is estimated from the $\pi^0$ mass cut sideband: $190 \text{ MeV}/c^2 < M_{\gamma\gamma} < 230 \text{ MeV}/c^2$.

This approach neglects the detector resolution, which is a good approximation for all resonances except for the $K^*(892)^\pm$. The MC simulation shows that estimated bias to the measured width of $K^*(892)^\pm$ is much larger than the corresponding systematic uncertainty estimated from other sources. At the same time, this bias is much smaller than the $K^*(892)^\pm$ width, which allows us to use the approximation proposed in Ref. [25] to take into account the detector resolution. Due to the significant computation time, this method is used only to correct the final PWA results.

The quality and consistency of the obtained solution is evaluated by the comparison of the mass and angular distributions of the experimental data and reconstructed phase-space generated MC events weighted according to the PWA solution.

The conservation of $P$- and $C$-parities strictly restricts the allowed quantum numbers of intermediate states. In the $K^{\pm\pi^0}$ channels only resonances with quantum numbers $I = 1/2$, $J^P = 1^-, 2^+, 3^-, 4^+ \ldots$ can be produced. The reaction is dominated by $K^*(892)^\pm$ production. There are two other established vector states which are in the accessible mass region: $K^*(1410)$ and $K^*(1680)$. In the $2^+, 3^-$ and $4^+$ partial waves three states are well established: $K_2^+(1430)$, $K_2^*(1780)$ and $K_3^+(2045)$. Possible contributions must also be considered from two observations reported by the LASS Collaboration: a $2^+$ state at $1980 \text{ MeV}/c^2$ (also claimed to be seen by SPEC [28]) and a $5^-$ state at $2380 \text{ MeV}/c^2$ [29], which needs confirmation. As for the $K^+ K^-$ channel, the produced resonances are restricted to quantum numbers $J^{PC} = J^{--}$, where $J = 1, 3, 5\ldots$. For the strong decays of the $J/\psi$ isospin and $G$-parity conservation requires $I^G = 1^+$. There are two well known isovector resonances in the $J^{PC} = 1^{--}$ sector: the $\rho(1450)$ and $\rho(1700)$, and a set of observations that needs confirmation: the $\rho(1570)$, $\rho(1900)$ and $\rho(2150)$ (see Ref. [26]). In the isovector $J^{PC} = 3^{--}$ sector one can expect the production of the well known and relatively narrow $\rho_3(1690)$ state. At higher energies there have been observations of two $J^{PC} = 3^{--}$ states: the $\rho_3(1990)$ and $\rho_3(2250)$. The first isovector $J^{PC} = 5^{--}$ state is expected to have a mass of around 2350 $\text{ MeV}/c^2$. Such a resonance is observed in the analysis of the GAMS2 data for the reaction $\pi^- p \to \omega n$ [30] and in the analyses of proton-antiproton annihilation in flight into different meson final states (e.g. see Ref. [31]). The decay of the $J/\psi$ through a virtual photon does not forbid but even favors the production of $I^G = 0^-$ resonances. The $J/\psi \to \phi \pi^0$ decay is strongly suppressed [32], hence the production of excited $\phi$ mesons is expected to be negligible assuming the absence of strong mixing of excited $\phi$ and $\omega$ states. However, the production of excited $\omega$ resonances is possible. The isovector and isoscalar states can be distinguished in a combined analysis of the decay under consideration and the $J/\psi$ decay to $K^\pm K^0\pi^\mp$.

### A. Fit to the data

The masses and widths of all states included in the solution (with the sole exception of the $\rho(770)$) are initially free fit parameters. For the well-established $K\pi$ resonances we use results of the LASS fits to the elastic $K\pi$ scattering amplitudes [33] as reference values. The masses and widths of these states are allowed to vary within $\pm\sigma$ of the LASS measurements (here $\sigma$ stands for the LASS uncertainty). If no NLL minimum is found for the mass or width within this range or the minimum is unstable (with respect to variations of the PWA solution used for estimation of systematic errors), the parameter is set to the central value of the LASS results. Motivated by the claim of an observation of the $K_2^*(1890)^\pm$ by LASS [27] and by Regge trajectories predicting a state at approximately $1.8 \text{ GeV}/c^2$ we introduce a second $J^P = 2^+$ contribution with a mass allowed to vary within the $1.75 \text{ GeV}/c^2 - 2.1 \text{ GeV}/c^2$ interval. Two clear resonance-like $K^+ K^-$ signals are found to significantly contribute to the data description in all fits. The first contribution has a mass of around $1.65 \text{ GeV}/c^2$ and is likely a manifestation of the $\rho(1700)$ or $\omega(1650)$, or interference between the two. Note that the parameters of both these states remain highly uncertain. For the $\rho(1700)$, the PDG quotes the results with the mass varying roughly from $1540 \text{ MeV}/c^2$ to $1860 \text{ MeV}/c^2$, which may indicate the presence of two states. Quark potential models [4] suggest two resonances close to this mass range: $1^3D_1$ and $3^3S_1$. This possibility is implied in the interpretation of the fit results. The second contribution has a mass of around $2.0 \text{ GeV}/c^2 - 2.1 \text{ GeV}/c^2$, close to the mass of the $\rho(2150)$. No limitations on their parameters are imposed in the fits. For the $\rho(1450)$ the mass range from $1.3 \text{ GeV}/c^2$ up to $1.5 \text{ GeV}/c^2$ is studied, but no NLL minima are found, and so its mass and width are fixed to the PDG estimates [26].

In the analysis we find that the PWA solution can not be saturated with well-known states included as Breit-Wigner resonances and constant contributions in the lowest partial waves. At the same time, the “missing part” of the PWA solution can not be reliably attributed to a single resonance and mainly manifests itself as a slow changing background in the $J^P = 3^-$ partial wave of the $K^\pm \pi^0$ pairs at high $K^\pm \pi^0$ masses. Below we provide two solutions constructed with and without the smooth contribution in this partial wave to demonstrate that the conclusions of this analysis are not strongly affected by assumptions on the “missing part” of the PWA solution.
B. Solution I

The results for the best fit based on the well-established resonances and constant contributions in the lowest partial waves are given in Table I. Only contributions improving the NLL by more than 17 are included to the fit (corresponding to a statistical significance of 5σ for 4 degrees of freedom). The data description as a Dalitz plot are shown in Fig. 1(b). Fig. 2 and Fig. 3 show the corresponding invariant mass spectra and angular distributions. The kinematic distributions in Fig. 3 are restricted to the inner part of the Dalitz plot $(M(K^\pm\pi^0) > 1.05 \text{ GeV}/c^2)$ to exclude the huge peaks from the $K^*(892)^\pm$.

The dominant contribution stems from the $K^*(892)^\pm$ and $K_2^*(1430)^\pm$ resonances in the $K^\pm\pi^0$ kinematic channels. The first decay is well-known and contributes about 90% to the total decay rate. The interference term between the $K^*(892)^+K^-$ and $K^*(892)^-K^+$ intermediate states contributes about 10%. The mass and the width of the $K^*(892)^\pm$ are determined with high statistical precision. The Blatt-Weisskopf radius of the resonance is reliably determined from the fit and is set to 0.4 fm, which is the meson-interaction radius used in Ref. [20]. The contribution of the $K_2^*(1430)^\pm K^\mp$ channel to the reaction is approximately 10 times smaller than the contribution from the $K^*(892)^\pm K^\mp$ channel. Taking into account this result and using a branching fraction of 49.9% for the $K_2^*(1430)^\pm$ decay to $K\pi$ [20], we find that the $J/\psi$ decay to $K_2^*(1430)^\pm K^\mp$ is suppressed by an approximate factor of 5 compared to the decay to $K^*(892)^\pm K^\mp$. For $J^P = 1^-$, the inclusion of the $K^*(1680)^\pm$ provides a significant improvement in the data description, but no NLL minima consistent with its mass and width are found. The $J^P = 2^+$ partial wave requires another $2^+$ state with a relative contribution of approximately 0.4%. Its mass and width are found to be $1817 \pm 11 \text{ MeV}/c^2$ and $312 \pm 28 \text{ MeV}/c^2$, respectively. This mass is much lower than the mass of the $K_2^*(1800)^\pm$ observed by LASS. The $K_2^*(1780)^\pm$ state provides a significant improvement in the log-likelihood, but no NLL minima consistent with its measured parameters are found. Finally, there is a small, but very distinct and stable contribution of $(0.18 \pm 0.02)\%$ from the $K_2^*(2045)^\pm$. Its fitted mass is lower than that obtained in other measurements [26], which can be attributed to the uncertainties of the PWA solution (see solution II).

In the $K^+K^-$ kinematic channel, the first stable contribution has $J^{PC} = 1^{-+}$, a mass of $1643 \pm 3 \text{ MeV}/c^2$, a width of $167 \pm 12 \text{ MeV}/c^2$ and a decay fraction of 1%. It can also be clearly seen in the Dalitz plot. As mentioned above, this contribution can be attributed to the $\rho(1700)$. The structure is also reasonably consistent with the $\omega(1650)$ (the mass is consistent with the PDG estimate, and the width is well within the spread of the results quoted by PDG) or an interference between these states. The second contribution that can be reliably determined from the data is a $J^{PC} = 1^{--}$ resonance with a mass of $2078 \pm 6 \text{ MeV}/c^2$ and width of $149 \pm 21 \text{ MeV}/c^2$.

The largest relative contribution of $(1.8 \pm 0.2)\%$ comes from the tail of the $\rho(770)$. Since the mass of this state is significantly below the $K^+K^-$ production threshold, no reliable claim can be made about its observation. The $\rho_3(1690)$ and $\rho(1450)$ provide NLL improvement by 14 and 27, but no NLL minimum consistent with the parameters of each state is found. The smooth contribution in the $J^{PC} = 1^{--}$ partial wave is also found to be significant.

Additionally, we try to set the mass and the width of the $J^{PC} = 1^{--}$ contribution at 1.65 GeV/$c^2$ to the PDG mean values for the $\rho(1700)$ averaged from $\eta\pi(770)$ and $\pi^+\pi^-$ modes. In this case, the NLL worsens by 42, and so one may consider including the $\omega(1420)$ and $\omega(1650)$ in the fit. In these fits we set their masses and width to the mean values of the PDG estimates. If the $\omega(1420)$ ($\omega(1650)$) is included, the NLL is still worse by 14 (7) compared to the result of solution I. If the $\rho(1450)$ is substituted by the $X(1575)$, instead of adding a resonance, the NLL improves by 28, but remains worse by 14 than the result for solution I.

Adding further well-established resonances with the nominal PDG parameters does not improve log-likelihood by more than 17 units. Despite this, the solution is not saturated: if additional contributions (parametrized as Breit-Wigner resonances with parameters not required to correspond to a physical state) are added, they can improve NLL by up to 95 in a single partial wave, which is much larger than the contribution of other resonances included to the solution. The only notable additional contribution indicating resonance behavior is in the $J^{PC} = 1^{--}$ $K\pi$ partial wave with a mass of around 2.4 GeV/$c^2$, but there is lack of qualitative evidence to report a new state. The largest improvement in the NLL function comes from contributions that tend to be broad and cannot be interpreted as resonances. These conclusions are not surprising if one considers the measured two-particle $K\pi$ scattering amplitudes obtained by the LASS Collaboration [23]. Here the $F$-wave intensity, apart from the $K_2^*(1780)$ peak, has a strong contribution from nontrivial structures, which are not resolved in the LASS analysis. The inability to provide a consistent data description for this solution prevents us from making a reliable estimation of systematic uncertainties.

C. Solution II

We find that the largest improvement to the NLL of the solution I comes from the inclusion of a smooth contribution in the $J^{PC} = 3^{--}$ partial wave, which we parametrize with a broad Breit-Wigner shape. Its mass is found to
be close the maximal allowed invariant mass of the $K^\pm \pi^0$ system. The width can vary in the approximate interval of 0.5 GeV/$c^2$ – 1.2 GeV/$c^2$, depending on small variations of the PWA solution, and its value only slightly affects other components in the fit. Such a mass and width does not allow an interpretation of this contribution as a single resonance. The solution where this broad component is added and the significance of the resonance is reevaluated is shown in Table I. For this solution, we use the more conservative resonance significance criteria: the minimum NLL improvement is required to be 40. We ensure that no other allowed resonance contributions improve the NLL value above this number, considering possibilities with spins up to $J = 5$, which is the maximum spin of previously reported states allowed in this decay. Those contributions which give the most significant NLL improvement are used to estimate systematic uncertainties. The NLL value for this solution is better by 116 than that of solution I. The systematic uncertainties listed in Table I will be discussed later. The Dalitz plot for the solution II is shown in Fig. [c]. Mass and angular distributions are given in Fig. [d] and Fig. [e] for the data and for the two models. The two descriptions are very similar, but solution II is superior in specific kinematic regions.

Solution II has the same set of well-defined contributions as solution I. The fitted mass and width for the $K^*(892)\pm$ and $K^*_2(1430)\pm$ are almost the same. The mass, width and Blatt-Weisskopf radius of the $K^*(892)$ are found to be $M = 893.6 \pm 0.1^{+0.2}_{-0.3}$ MeV/$c^2$, $\Gamma = 46.7\pm 0.2^{+0.1}_{-0.2}$ MeV/$c^2$ and $r = 0.20 \pm 0.02^{+0.14}_{-0.04}$ fm, respectively, where here and subsequently the first uncertainty is statistical, and the second systematic. The mass lies between the PDG averages for measurements performed where the $K^*(892)\pm$ is produced in hadronic collisions and those were it is produced in $\tau$ decays 25. The fitted width is consistent with the $\tau$-decay results 31. For the $K^*_2(1430)$ we fix the Blatt-Weisskopf radius to 0.4 fm. The $2^+$ partial amplitude in the $K^\pm \pi^0$ kinematic channels also requires a second contribution with a mass higher than that of the previous solution with large systematic uncertainties for both the mass and width: $M = 1868 \pm 8^{+40}_{-57}$ MeV/$c^2$ and $\Gamma = 272 \pm 24^{+50}_{-15}$ MeV/$c^2$. The mass is approximately 100 MeV/$c^2$ below the LASS measurement for the $K^*_2(1980)$ 27, but both the mass and the width are compatible with the PDG averages within 2.2 standard deviations. As in solution I, there is a very clear contribution to the $J^P = 4^+$ partial wave with $M = 2090 \pm 9^{+11}_{-10}$ MeV/$c^2$ and $\Gamma = 201 \pm 19^{+57}_{-17}$ MeV/$c^2$, which is consistent with the parameters of the $K^*_3(2045)$ 26. For the $K^*(1410)$, which is required in this solution, the $K^*(1680)\pm$ and $K^*_2(1780)\pm$, no NLL minima consistent with parameters of these resonances are found. In the $K^+K^-$ kinematic channel we see again two stable contributions at 1.65 GeV/$c^2$ and 2.05 GeV/$c^2$. The contributions from the $\rho(1450)$, $\rho_3(1690)$ and $\rho(770)$ are marginal.
A striking feature of solution II is the presence of a non-resonance component in the $J^P = 3^- K^\pm \pi^0$ partial waves, which can not be clearly interpreted as an interference between Breit-Wigner states. A possible interpretation is that this component is the manifestation of non-resolved contributions present in the $F$-wave $K\pi$ scattering amplitude [33]. This may include the presence of several resonances, non-resonant production and final-state particle rescattering effects.

The stability of the found NLL minimum with respect to the parameters of the reported resonances is demonstrated in Fig. 4.

The systematic errors due to the uncertainty of the PWA solution are assigned to be the largest deviations for the following variations of the solution:

- variation of the masses and widths for the $K^\pm \pi^0$ resonances with the parameters fixed in the fit, and varied by one standard deviation of the LASS results [33];
- variation of the Blatt-Weisskopf radius of the $K_2^*(1430)^\pm$ by $\pm 0.2$ fm;
- inclusion of contributions that strongly improve the log-likelihood below the acceptance criteria ($J^P = 1^- (K\pi)$ at approximately 2.5 GeV/$c^2$ and $J^{PC} = 1^- (K^+K^-)$ at $M(K^+K^-) \approx 2.3$ GeV/$c^2$);
reparametrization of the broad background part of partial waves.

To evaluate the latter variation, broad contributions in the $1^-, 2^+ (K\pi)$ amplitudes and $1^{--} (K^+K^-)$ partial wave parametrized with $\rho(770)^0$ and $\rho(1450)^0$ are studied. In all these fits the states $K^+(892)^\pm$, $K^*_2(1430)^\pm$, $K^*_2(2045)^\pm$ and the structures at 1.65 GeV/c$^2$ and 2.05 GeV/c$^2$ in the $K^+K^-$ channels remain stable. The high-mass broad $K^\pm\pi^0$ $3^-$ contribution always remains significant, but its relative fraction varies to much smaller values in some fits. The $1^-$ additional contribution mostly manifests resonant behavior. No stable contribution can be associated with the $\rho(1450)^0$, but its relative decay fraction at the level of 1% does not contradict the data.

The total systematic uncertainties for the masses, widths and decay fraction given in Table II are calculated as a quadratic sum of:

- the variation in results due to the uncertainty of the PWA solution;
- the bias introduced by imperfections of the detector simulation and the event reconstruction;
- the uncertainties due to the differences in kaon tracking and PID efficiencies between data and the MC simulation.

The differences in kaon tracking and PID efficiencies between data and the MC simulation are studied with a high-purity control sample of $J/\psi \rightarrow K\bar{K}K^\pm\pi^\mp$ decays as a function of kaon transverse momentum $p_T$ and are found to be within 1% per track both for the tracking and the PID. The effect on the PWA result is estimated by varying the selection efficiency difference for data and MC in $p_T$ bins within these errors. Uncertainties on the fit parameters due to the efficiency variation in each bin are summed quadratically.

The background uncertainty, estimated by varying the subtracted NLL contribution by 50%, is found to be negligible.

D. Summary on PWA

Our analysis shows that there is a set of states in the PWA solutions that remains stable for both considered cases: when contributions corresponding to well-known resonances are considered or when broad contributions are introduced to parameterize the missing part of the partial amplitudes. In the $K^\pm\pi^0$ channels this set of resonances includes the $K^+(892)^\pm$, $K^*_2(1430)^\pm$, and $K^*_2(2045)^\pm$. The second $J^P = 2^+$ state, labeled here as $K^*_2(1430)^0$, has a mass much lower than that observed by the LASS Collaboration [27]. However, given the large systematic uncertainties on this quantity, our result is compatible within 2.2 standard deviations. The first stable structure in the $K^+K^-$ channel has a mass of about 1.65 GeV/c$^2$ and a decay fraction of 1.0% – 1.5%. The absence of a distinct contribution from the first radial excitation of the $\rho(770)$ favors its interpretation as a $3D_1$ $\omega$ resonance. At the same time such a small decay fraction is consistent with $\omega(1650)$ production in $J/\psi$ decay through a virtual photon. Its mass is consistent with the PDG estimate for the $\omega(1650)$ and its width is well within the spread of experimental results quoted by the PDG. It could also be the result of interference between these isovector and isoscalar states. The second stable contribution has a mass of about 2.05 GeV/c$^2$ – 2.10 GeV/c$^2$ and decay fraction of 0.1% – 0.2%. Given the large systematic uncertainties it could be interpreted as either the $\rho(2150)$ or as another isovector-vector state observed in proton-antiproton annihilation in flight [35]. Clarification of the nature of these excited vector mesons requires further investigation.

V. BRANCHING FRACTIONS

The $J/\psi \rightarrow K^+K^-\pi^0$ branching fraction is determined as $B(J/\psi \rightarrow K^+K^-\pi^0) = \frac{N_{sel} - N_{bg} - N_{continuum}}{cN_{J/\psi}(\epsilon_{J/\psi})}$. Here $N_{sel}$, $N_{bg}$ and $N_{continuum}$ are the number of selected events, the estimated background from the $J/\psi$ decays, and the continuum production, respectively. The number of $J/\psi$ events $N_{J/\psi} = (223.7 \pm 1.4(syst.)) \times 10^6$ is taken
even if one of the particles is badly reconstructed. This is consistent with the recoil mass calculated using the other and the reported signals in the $K^+K^-$ channel ($J^{PC} = 1^{--}$ signals with masses around 1650 MeV/$c^2$ and 2050 MeV/$c^2$) the first uncertainty is statistical and the second is systematic. In the $K^+$ channel the decay fraction is given for both charged conjugated modes ($b^+$) and for the contribution of one charged mode ($b^{+(-)}$), so that their interference can be determined. As the $K^*(1410)^\pm$, $K^*(1680)^\pm$ and $K_0^*(1780)^\pm$ contributions are not reliably identified (see main text), their masses and widths are fixed (marked with *) and only statistical uncertainties are given for their decay fractions.

Table II. List of components for solution II. For the reported states in the $K\pi$ channel ($K^*(892)^\pm$, $K_2^*(1430)^\pm$, $K_2^*(1780)^\pm$ and $K_2^*(2045)^\pm$) and the reported signals in the $K^+K^-$ channel ($J^{PC} = 1^{--}$ signals with masses around 1650 MeV/$c^2$ and 2050 MeV/$c^2$) the first uncertainty is statistical and the second is systematic. In the $K^+$ channel the decay fraction is given for both charged conjugated modes ($b^+$) and for the contribution of one charged mode ($b^{+(-)}$), so that their interference can be determined. As the $K^*(1410)^\pm$, $K^*(1680)^\pm$ and $K_0^*(1780)^\pm$ contributions are not reliably identified (see main text), their masses and widths are fixed (marked with *) and only statistical uncertainties are given for their decay fractions.

| $J^{PC}$ | PDG | $M$(MeV/$c^2$) | $\Gamma$(MeV/$c^2$) | $b$(%) | $b^{+(-)}$(%) | $\Delta NLL$ |
|--------|-----|----------------|------------------|--------|----------------|------------|
| 1$^-$  | $K^*(892)^\pm$ | 893.6$^{+0.2}_{-0.3}$ | 46.7$^{+0.1}_{-0.2}$ | 93.4$^{+1.8}_{-5.8}$ | 42.5$^{+0.5}_{-1.7}$ | - |
| 1$^-$  | $K^*(1410)^\pm$ | 1380$^*$ | 176$^*$ | 0.26$\pm$0.04 | 0.11$\pm$0.02 | 80 |
| 1$^-$  | $K^*(1680)^\pm$ | 1677$^*$ | 205$^*$ | 0.20$\pm$0.03 | 0.08$\pm$0.01 | 56 |
| 2$^+$  | $K_2^*(1430)^\pm$ | 1432.7$^{+2.2}_{-2.3}$ | 102.5$^{+3.1}_{-2.8}$ | 9.4$^{+0.8}_{-0.5}$ | 4.2$^{+0.3}_{-0.2}$ | - |
| 2$^+$  | $K_2^*(1780)^\pm$ | 1868$^{+50}_{-57}$ | 272$^{+50}_{-15}$ | 0.38$^{+0.22}_{-0.05}$ | 0.15$^{+0.08}_{-0.02}$ | 192 |
| 3$^+$  | $K_0^*(1840)^\pm$ | 1781$^*$ | 203$^*$ | 0.16$\pm$0.02 | 0.07$\pm$0.01 | 105 |
| 4$^+$  | $K_0^*(2045)^\pm$ | 2090$^{+11}_{-29}$ | 201$^{+57}_{-17}$ | 0.21$^{+0.10}_{-0.05}$ | 0.09$^{+0.04}_{-0.02}$ | 212 |
| 3$^-$  | non-resonant | -- | -- | -- | -- | -- |

Table III. Summary of systematic uncertainties for $B(J/\psi \rightarrow K^+K^-\pi^0)$.

| Source                  | Uncertainty (%) |
|-------------------------|-----------------|
| $N_{bg}$                | 0.2             |
| $N_{continuum}$         | 0.3             |
| Track reconstruction efficiency | 2.0           |
| PID efficiency          | 2.0             |
| Photon reconstruction efficiency | 2.0          |
| Kinematic fit cut efficiency | 2.4           |
| $N_{J/\psi}$            | 0.6             |
| Total                   | 4.3             |

from Ref. [13], and $B(\pi^0 \rightarrow \gamma\gamma) = (98.823\pm0.034) \times 10^{-2}$ is taken from the PDG [26]. The selection efficiency $\epsilon$ is obtained using the PWA solution II and the detector performance simulation. The dominant contribution to the statistical uncertainty comes from $N_{sel}$. The systematic uncertainty on the branching fraction is estimated from the sources listed in Table III. The background uncertainty is estimated by varying $N_{bg}$ by $\pm 50\%$. The uncertainty associated with the subtraction of the continuum background is assigned to be the statistical error on $N_{continuum}$. The charged track reconstruction efficiency and the PID efficiency uncertainties are $1\%$ each per track as is discussed above. The photon detection efficiency is studied with the decays $\psi(3686) \rightarrow \pi^+\pi^-J/\psi$, $J/\psi \rightarrow \rho^0\pi^0$ and photon conversion control samples [30, 37]. In this analysis, an uncertainty of $1\%$ per photon is assigned. The uncertainty introduced by the cut on $N_{K^+K^-\gamma\gamma}$ is estimated using a control sample. This is selected using similar selection criteria, with the kinematic-fit cut replaced by the requirement that at least one particle out of three ($K^+, K^-, \pi^0$) has a mass hypothesis consistent with the recoil mass calculated using the other two particles. Such a procedure accepts a signal event even if one of the particles is badly reconstructed. This gives $B(J/\psi \rightarrow K^+K^-\pi^0) = (2.88 \pm 0.01 \pm 0.12) \times 10^{-3}$.

Knowing the $J/\psi \rightarrow K^+K^-\pi^0$ branching fraction and the decay fractions for the individual components from the PWA, we determine branching fractions for the decay via individual resonances. Results for solution II are summarized in Table IV. The branching fraction $B(J/\psi \rightarrow K^+K^-\pi^0)$ and the branching fractions for the decay via the $K^*(892)^\pm$ that are obtained in solution II are compared to the results from previous experiments in Table IV. Our result for $B(J/\psi \rightarrow K^+K^-\pi^0)$ is up to now the most precise measurement. It differs from the PDG value [26], obtained indirectly from Ref. [11], by about 2.8 standard deviation. The systematic uncertainty of our results for decays through the $K^*(892)^\pm$ is somewhat larger than that of Ref. [11], which can be attributed to the uncertainties present in the PWA model.
VI. CONCLUSION

A partial-wave analysis of the decay $J/\psi \rightarrow K^+K^−π^0$ using a data sample of $(223.7 \pm 1.4) \times 10^6 J/\psi$ events collected by the BESIII reveals a set of resonances that have not been observed by previous experiments. In the $K^+π^0$ channels our analysis reveals signals from $K_1^*(892)^{±}$ and $K_1^*(1430)^{±}$ resonances. This is the first observation of these states in $J/\psi$ decays. The mass of the former state is determined with a central value around 100 MeV/c² lower than that reported by the LASS Collaboration [21]. This lower value is in better agreement with the expectation from the linear Regge trajectory of radial excitations with the standard slope [35]. As for the known decays through $Kπ$ resonances, we determine the parameters, decay ratios, and branching fractions for the $K^*(892)^{±}$ and $K_1^*(1430)^{±}$ with improved precision compared to previous measurements. In the $K^+K^−$ channel we observe a clear $J^{PC}=1^{−−}$ resonance structure with a mass of 1.65 GeV/c² and another $J^{PC}=1^{−−}$ contribution at 2.05 GeV/c². The first structure may be interpreted as the ground $^3D_1$ isovector state. At the same time its mass, width and small relative contribution to the decay are reasonably consistent with the production of the $\omega(1650)$ in $J/\psi$ decays through a virtual photon. The second state can be interpreted as the $^3P_1$ isovector state. At $1.45$ GeV/c², the mass of the former state is $1.65$ GeV/c², and the latter is $2.05$ GeV/c².

The precise identification of these two states requires further analysis of more channels, such as $J/\psi\rightarrow K^+K^-\pi^0$. Our PWA solutions have notable differences from those presented in Ref. [10] and more recently in Ref. [12]. We also report the most precise measurement of the branching fraction $B(J/\psi \rightarrow K^+K^-π^0)$.

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Table IV. Branching fractions for decays via reliably identified intermediate states (solution II). \( R_{K\pi} \) and \( R_{KK} \) denotes \( K^\pm \pi^0 \) and \( K^+ K^- \) resonances, respectively, and \( R_{KK}^{\pi\pi} \) denotes for one possible charged combination. The first uncertainty is statistical and the second one is systematic.

| Intermediate resonance in the \( K\pi \) system | \( R_{KK} \) | \( B(J/\psi \to R_{KK}^{\pi^0} K^+ K^- \pi^0) \) |
|-----------------------------------------------|----------------|----------------------------------|
| \( K^*(892) \) | \( 1.22\pm0.01^{+0.05}_{-0.07} \times 10^{-3} \) & \( 2.69\pm0.04^{+0.13}_{-0.10} \times 10^{-4} \) |
| \( K_2^*(1430) \) | \( 4.3\pm0.5^{+2.3}_{-0.6} \times 10^{-6} \) & \( 1.1\pm0.1^{+0.6}_{-0.7} \times 10^{-5} \) |
| \( K_2^*(1800) \) | \( 2.6\pm0.3^{+1.1}_{-0.6} \times 10^{-6} \) & \( 6.2\pm0.7^{+2.8}_{-1.4} \times 10^{-6} \) |

Table V. Comparison between this work and previous measurements. For \( B(J/\psi \to K^{*+} K^- + c.c. \to K^+ K^- \pi^0) \) and \( B(J/\psi \to K^{*+} K^- + c.c. \to K^{*+} K^- \pi^0) \) we give two numbers for solution II: the first one is a sum of branching fractions through \( K\pi \) and \( K^{*+} \) and the second number (in parenthesis) accounts for their interference. Results marked with \(*^\dagger\pi\) are obtained by averaging the \( K_0 K^\pm \pi^\mp \) and \( K^* \pi^\mp \) final states. Results recalculated by us using numbers from this work are marked with \( *^{\dagger\dagger}\pi\).

| Channel | This work | BABAR [11] | DM2 [2] | MARK-III [5] | MARK-II [1] |
|---------|-----------|------------|--------|---------------|-------------|
| \( B(J/\psi \to K^+ K^- \pi^0) \) | \( 2.88\pm0.01\pm0.12 \) | — | — | — | \( 2.8\pm0.8 \) |
| \( B(J/\psi \to K^{*+} K^- + c.c. \to K^+ K^- \pi^0) \) | \( 2.45\pm0.01^{+0.10}_{-0.14}(2.69\pm0.01^{+0.12}_{-0.20}) \) & \( 1.97\pm0.16\pm0.13 \) & \( 1.50\pm0.23\pm0.27^{\dagger} \) & \( 1.87\pm0.04\pm0.28^{\dagger} \) & \( 2.6\pm0.8 \) |
| \( B(J/\psi \to K^{*+} K^- + c.c.) \) | \( 7.34\pm0.03^{+0.33}_{-0.43}(8.07\pm0.04^{+0.30}_{-0.41}) \) & \( 5.2\pm0.3\pm0.2^{\dagger} \) & \( 4.57\pm0.17\pm0.70^{\dagger} \) & \( 5.26\pm0.13\pm0.53^{\dagger} \) & \( 7.8\pm2.4^{\dagger} \) |