Thermal-mechanical coupling analysis of functionally graded cylindrical shells

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Abstract. Functionally graded materials is a new material for thermal protection in the aerospace field. It is extremely important to study the temperature and stress changes in the thermal environment to ensure the normal operation of the spacecraft and the safety of crews. In this paper, ABAQUS is used to perform sequential thermal coupling and full thermal coupling analysis. The temperature variation and mechanical properties of functionally graded cylindrical shells in thermal environment under temperature and force field and temperature field interacts with force field are studied. Based on the first law of thermoelasticity and thermodynamics, the three-dimensional stress and energy equations of functionally graded cylindrical shells in thermal environment are derived. The example shows that the temperature variation law and mechanical characteristics of the two types of thermo-mechanical coupling analysis are almost the same. The temperature value results of the sequential thermal-mechanical coupling analysis are the same as the results of the full thermal coupling, but the stress value results are lower than the results of the full thermal coupling.

1. Introduction
Functionally graded materials, as a new type of thermal protection material, were first proposed by Japanese scholar Niino and Hirai, etc., which usually consist of two or more materials. The functionally graded materials used in the aerospace field are mainly made of metal and ceramics. The metal has good strength to resist deformation or fracture. The ceramic has good heat resistance, heat insulation and high temperature oxidation resistance. The continuous change in composition between the two eliminates the macroscopic interface in the material. The overall material behaves good thermal stress relaxation characteristics, enabling it can be used under harsh environmental conditions such as large temperature difference, ultra-high temperature, and high-speed heat flow impact. It is expected to be used as the fuselage of the new-generation space shuttle, the inner wall of the combustion chamber, and the ultra-high temperature heat-resistant materials for turbine engines and high-efficiency gas turbines.

In recent years, with the continuous exploration of human beings in the field of extraterrestrial, experts have increasingly explored the application of functionally graded materials in the aerospace field. Domestic and foreign scholars have gradually deepened their research on thermal analysis. Loy and Lam studied the mechanical problems of FGMs and obtained the mechanical properties of metal-based ceramic FGMs[1]. Cho and Oden used the Crank-Nicolson-Galerkin scheme to establish a thermal stress analysis model for FGMs cylindrical shells[2]. Na and others considered the stress and
critical temperature, and studied the material fraction by the power-distribution of the functional gradient material volume fraction optimization[3]. Huang and Rao used the classical Donnell shell theory to derive the buckling controlling equations and the analytical expressions of the buckling critical temperature of the FGMs elliptical cylindrical shells, and analyzed the effects of uniform, linear and nonlinear temperature rise environments on the physical properties of FGMs. And the variation of the critical temperature of buckling with the eccentricity of the section and the parameters of the material composition[4]. Xu and Zhang et al. prepared HATi axisymmetric biological FGMs by powder metallurgy method, and determined the mechanical properties and thermal expansion coefficient of HATi composite materials[5]. Lin and Bai developed the axisymmetric WC/Co heavy metal FGMs by microwave sintering method, and observed the microstructure and hardness of the material[6]. Hu and Zhang developed a numerical manifold research method for two-dimensional steady-state heat conduction problems of FGMs. The governing equations and boundary conditions are given, and the quadrature method of correlation matrix is discussed[7]. Lan and Fu proposed the Layered Exponential Precise Method (LPEM) for the steady-state heat conduction equation of FGMs. The effectiveness of the method is illustrated by an example[8]. Amini et al. considered the temperature field, studied the nonlinear free vibration and forced vibration of thick annular FGM plates based on the first-order shear deformation plate theory and von Karman equation, and studied two kinds of constituent materials in the thermal environment. The effect of functional gradient plate material composition and thermal load on vibration characteristics and stress in a hot environment were investigated[9]. Noda and Jin used a finite inhomogeneous elastic solid containing cracks to study the problem of stable thermal stress, and calculated the thermal stress intensity factor of FGMs[10]. Zhao et al. used the asymptotic solution method to derive the transient temperature field and transient thermal stress field formula of the double-sided functionally graded coating infinite sandwich plate under convective boundary conditions[11]. Zhang et al. used the Laplace transform and the power series method to obtain the dynamic temperature field of the FGMs shell. The differential quadrature method was developed to obtain the transient thermal stress of the FGMs cylindrical shell under thermal shock. And when the volume of the component was changed, it can reduce the results of thermal stress[12]. Ozturk et al. studied the elastoplastic deformation of a functionally graded solid cylinder with fixed and internal heat sources according to Tresca's yield criterion and its associated flow law. The unknown interface radius values were determined by Mathematica 5.2 to analyze the expression of the stress, strain and radial displacement distribution. And the time history of temperature field and thermal stress are obtained by using the residual theorem and the fast Laplace inverse transform method (FLIT) respectively. The transient stress field and the thermoelastic stress wave propagation of the functionally graded thick hollow cylinder under any thermomechanical impact load are studied[13]. Nie and Batra analyzed the axially symmetric deformation of the isotropic FGMs rotating disk and physical properties change in the radial direction. The Airy stress function is used to analyze the hoop stress and radial stress in the constitutive relation of the material[14].

Studies have shown that domestic and foreign scholars have carried out a lot of research on the design optimization and strength of FGMs structure, they mainly focus on the research methods of heat conduction and thermal stress of FGMs, and most of the research objects are plate and shell structures. Rarely involved in finite element software to compare the results of different thermodynamic coupling analysis types. And many structures in the aerospace field, such as the space shuttle’s tail nozzle, can be simplified into a cylindrical shell axisymmetric structure. In this paper, through the thermal analysis module of finite element software ABAQUS, sequential thermal coupling and full thermal coupling analysis of functionally graded cylindrical shells are carried out respectively. The law of temperature and stress variation of the functional gradient materials with different thermal couplings analysis type under the thermal environment temperature field and force field are revealed. It will provide a reference for the temperature and stress of the functionally graded material when it is used in actual working conditions.
2. Functional gradient cylindrical shell modeling

Figure 1 is the FGMs cylindrical shell model. The outer surface is pure ceramic ZrO₂, the inner surface is pure metal Ti-6Al-4V, and the material distribution is a power exponential function distribution in the thickness direction. In the middle surface, the composition of the two components is the same. Material properties such as modulus of elasticity, density, coefficient of thermal expansion, thermal conductivity, etc., all show continuous change in the thickness direction and are influenced by the external environment and volume fraction index.

![Figure 1. FGMs cylindrical shell model.](image)

The coordinate system shown in figure 1 is established in the middle surface of the FGMs cylindrical shell. The length \( L = 0.8 \text{m} \), the middle surface radius \( R_m = 0.405 \text{m} \), and the shell thickness \( h = 0.01 \text{m} \). The material property parameter expression is:

\[
\Gamma = (\Gamma_c - \Gamma_m)(z/h + 0.5)^N + \Gamma_m
\]

(1)

Where \( \Gamma \) can be expressed as elastic modulus \( E \), poisson's ratio \( \nu \), density \( \rho \), thermal expansion coefficient \( \alpha \), thermal conductivity \( \kappa \), etc., and \( \Gamma_c \) and \( \Gamma_m \) are material properties of ceramics and metals, respectively. \( N \) is the volume fraction index and takes a value of \( 0 < N < \infty \).

Since the material properties are affected by temperature, the specific expression of the material property parameter in the thermal environment is:

\[
\Gamma_{(i)} = P_i(P_c T^{-4} + 1 + PT + PT^2 + PT^3)
\]

(2)

Where \( i \) is c or m. \( P \) is the temperature sensitivity coefficient of the material, using international units, as shown in table 1. \( T \) is the ambient temperature.

| Material   | \( P_c \) | \( P_0 \) | \( P_1 \) | \( P_2 \) | \( P_3 \) |
|------------|----------|----------|----------|----------|----------|
| ZrO₂       | 0        | 132.20   | 3.805e-4 | -6.127e-8| 0        |
| Ti-6Al-4V  | 0        | 122.70   | -4.605e-4| 0        | 0        |
| ZrO₂       | 0        | 0.3330   | 0        | 0        | 0        |
| Ti-6Al-4V  | 0        | 0.2888   | 1.108e-4 | 0        | 0        |
| ZrO₂       | 0        | 3657     | 0        | 0        | 0        |
| Ti-6Al-4V  | 0        | 4420     | 0        | 0        | 0        |
| ZrO₂       | 0        | 13.300e-6| -1.421e-3| 9.549e-7 | 0        |
| Ti-6Al-4V  | 0        | 7.4300e-6| 7.483e-4 | -3.621e-7| 0        |
| ZrO₂       | 0        | 1.78     | 0        | 0        | 0        |
| Ti-6Al-4V  | 0        | 6.10     | 0        | 0        | 0        |

3. Basic principles of thermal coupling analysis

3.1. Thermal stress theory

Determining the temperature field, since the FGMs cylindrical shell is an axisymmetric model, the stress, strain and displacement components throughout the interior are symmetrical about the axis and are only related to the parameters \( R \) and \( z \), and the shear stress is zero.
The shear stress along the z-axis on the cylindrical surface and the shear stress in the R direction perpendicular to the z-axis are ignored. Assuming that the temperature $T$ is only a function of $R$, a cylindrical coordinate system as shown in Figure 2 is established, and $M$ is a point of the cylindrical shell space. In this coordinate system, the stress-temperature difference relationship is obtained according to the generalized Hooke's law in thermoelasticity[15].

$$\sigma_k = 2G[I_1 \frac{\partial u}{\partial R} + I_2(\frac{u}{R} + \frac{\partial \omega}{\partial z})] - \beta T$$

$$\sigma_\theta = 2G[I_1 \frac{u}{R} + I_2(\frac{\partial \omega}{\partial z} + \frac{\partial u}{\partial R})] - \beta T$$

$$\sigma_z = 2G[I_1 \frac{\partial \omega}{\partial z} + I_2(\frac{\partial u}{\partial R} + \frac{u}{R})] - \beta T$$

Where: $u$, $\omega$ are the radial displacement in the $R$ direction and the axial displacement along the $z$ axis, respectively. $\sigma_R$, $\sigma_\theta$ and $\sigma_z$ are radial normal stress, hoop normal stress, and axial normal stress, respectively. Among them:

$$I_1 = \frac{1-v}{1-2v}, \quad I_2 = \frac{v}{1-2v}, \quad G = \frac{E}{2(1+v)}, \quad \beta = \frac{\alpha E}{1-2v}$$

### 3.2. Thermal stress calculation

Due to the temperature field and force field of the FGMs cylindrical shell in the thermal environment, the applied load and temperature are axisymmetric. If the inner diameter is $R_i$ and the outer diameter is $R_e$, it is assumed that the temperature change $T$ is only a function of $R$, and the temperature difference between the inner and outer surfaces is $\Delta T$, and is independent of $z$. Then the equilibrium equation of the cylindrical shell is simplified to

$$\frac{\partial \sigma_k}{\partial R} + \frac{\sigma_R - \sigma_\theta}{R} = 0$$

$$\frac{\partial \sigma_\theta}{\partial z} = 0$$

According to the boundary conditions that have been set on the inner and outer surfaces, the equation (3) is substituted into equation (5), then the parameters of the temperature $T$ are calculated to obtain the thermal stress expression:

$$\sigma_R = \frac{\alpha E \Delta T}{2(1-v)\ln k_i} \left[- \ln k_i - k_i (1-k_i^2) \ln k_i \right]$$

$$\sigma_\theta = \frac{\alpha E \Delta T}{2(1-v)\ln k_i} \left[1 - \ln k_i - k_i (1+k_i^2) \ln k_i \right]$$

$$\sigma_z = \frac{\alpha E \Delta T}{2(1-v)\ln k_i} \left[1 - 2\ln k_i - 2k_i \ln k_i \right]$$

Among them:
3.3. Energy equation

When the FGMs cylindrical shell acts under external force and thermal load, the external force is converted into deformation energy and stored in the body. In addition, the internal particle motion generates kinetic energy. Due to the input and output of heat accompanying the process, according to the first law of thermodynamics, the deformation energy and heat caused by external load are equal to the sum of the kinetic energy and the internal energy increment of the FGMs cylindrical shell per unit time. Its expression is

\[ dK + dU = \delta W + \delta Q \]  

Where: \( dK \), \( dU \), \( \delta W \), \( \delta Q \) are per unit volume, per unit time the kinetic energy increment, the internal energy increment, the work done by the surrounding materials, and the external heating amount.

According to the second theorem of thermodynamics, if the process is reversible, then

\[ \delta Q = Tds \]  

Where: \( T \) and \( s \) are the absolute temperature and the entropy per unit volume.

Assume that the volumetric forces of the FGMs cylindrical shell are \( X, Y, Z \), the area forces are \( X, Y, Z \), and the displacements of the mass points are \( \delta u, \delta v \), and \( \delta w \). According to the Green’s theorem, the integral operation is used to integrate the area into volume integral, the \( \delta W_i \) which is external force do work on the cylindrical shell is determined by the boundary conditions:

\[
\delta W_i = \iiint_V \left[ \left( \frac{\partial \sigma_{x}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + X \right) du + \left( \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{y}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + Y \right) dv + \left( \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{z}}{\partial z} + Z \right) dw \right] \]

\[
\frac{\partial \sigma_{x}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + X = 0 \\
\frac{\partial \sigma_{y}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + \frac{\partial \tau_{yz}}{\partial x} + Y = 0 \\
\frac{\partial \sigma_{z}}{\partial z} + \frac{\partial \tau_{zz}}{\partial x} + \frac{\partial \tau_{zz}}{\partial y} + Z = 0
\]

Where: \( \sigma_{x}, \sigma_{y}, \sigma_{z}, \tau_{xy}, \tau_{xz}, \tau_{yz} \) are the six stress components of the cylindrical shell, respectively. \( \varepsilon_{x}, \varepsilon_{y}, \varepsilon_{z}, \gamma_{xy}, \gamma_{yz}, \gamma_{xz} \) are the six strain components of the cylindrical shell, respectively.

Then, according to the equilibrium differential equation (11), the simplified equation (12) is obtained.

\[
\delta W_i = \iiint_V (\sigma_{x} \delta \varepsilon_{x} + \sigma_{y} \delta \varepsilon_{y} + \sigma_{z} \delta \varepsilon_{z} + \tau_{xy} \delta \gamma_{xy} + \tau_{xz} \delta \gamma_{xz} + \tau_{yz} \delta \gamma_{yz}) dV
\]

Then, according to \( W_i = \int W dV \), equation (13) is obtained.

\[
\delta W = \sigma_{x} \delta \varepsilon_{x} + \sigma_{y} \delta \varepsilon_{y} + \sigma_{z} \delta \varepsilon_{z} + \tau_{xy} \delta \gamma_{xy} + \tau_{xz} \delta \gamma_{xz} + \tau_{yz} \delta \gamma_{yz}
\]

For kinetic energy, assuming that the process is reversible, the force and temperature change slowly with time, which can be treated as a quasi-static problem, ignoring kinetic energy, therefore

\[
dK = 0
\]

Substituting the equations (9), (13), and (14) into (8), so the energy equation of the FGMs cylindrical shell under thermal coupling is
4. Thermal coupled finite element analysis

The FGMs cylindrical shell was studied at room temperature of 300 K. The inner surface was subjected to a stable temperature field of 300 K, and the outer surface was subjected to a uniform load of 2000 MPa and a temperature field varying with time \( T = 300 + t \), \( t \) was 600 s. The density \( \rho \), the thermal conductivity \( \kappa \), and the temperature-related parameters \( E \), \( \alpha \) and specific heat \( C \) are set in the ABAQUS material attribute unit, and the temperature gradient when calculating the parameters is 100 K. The Poisson's ratio \( \nu \) is 0.3. At room temperature of 300 K, the specific heat of ZrO\(_2\) is 467 J/kg•K, and the specific heat of Ti-6Al-4V is 690 J/kg•K. It is assumed that the specific heat of the material changes linearly and uniformly in the thickness direction with the change of temperature gradient.

In the analysis step module, the set time length is 600 s, the maximum incremental step number is 100, and the incremental step size is 10, and other options are default. In the boundary condition module, constrain the displacement in the \( z \) direction and the rotation in the \( x \) and \( y \) directions. In the load module, set the amplitude table of temperature field. The input time is 0 K at time 0 s and 800 K at 600 s. Figure 3 shows the FGMs cylindrical shell finite element model with a grid number of 1540.

![FGMs cylindrical shell finite element model](image.png)

**Figure 3.** FGMs cylindrical shell finite element model.

4.1. Sequential thermal coupling

The stress-strain field in such an analysis depends on the temperature field, but the temperature field is unaffected by stress strain. In the finite element software, the heat conduction analysis is first performed, and then the obtained temperature field is used as a known condition, and thermal stress analysis is performed to obtain a stress strain field. The grid used to analyze heat transfer problems and the grid of thermal stress analysis can be different. The heat conduction grid unit type is DC3D8, and the thermal stress grid unit type is C3D8R.

4.2. Full thermal coupling

There is a strong interaction between the stress-strain field and the temperature field in this type of analysis, which needs to be solved simultaneously in the finite element software. The grid unit type is C3D8T.
Figure 4. Temperature and stress cloud diagram of sequential thermal coupling (Figure a is the temperature cloud diagram. Figure b is the stress cloud diagram only under the temperature field. Figure c is the stress cloud diagram under the action of temperature field and force field.).

Figure 4 shows that the temperature of the FGMs cylindrical shell gradually increases from the inner surface to the outer surface under the action of the temperature field in the sequential thermal-mechanical coupling analysis. And owing to the temperature field tends to be stable over time, the inner and outer layer temperatures will also tend to stabilize. Eventually it tends to be stable. Only under the action of temperature field, the stress on the outer surface is larger than the stress on the inner surface, which also shows that the outer surface generates thermal stress due to the temperature field. Due to the external surface force loading, the stress after the sequential thermal coupling is higher, and the metal layer is stronger than the ceramic layer, which embodies the advantages of the functionally graded material.

Figure 5. Temperature and stress clouds diagram of full thermal coupling (Figure d is the temperature cloud diagram. Figure e is the stress cloud diagram.).
Figure 5 shows that in the full thermal-mechanical coupling analysis, the temperature of the FGMs cylindrical shell gradually increases from the inner surface to the outer surface under the action of temperature field and force field. It is also shown that the temperature of the inner and outer layers eventually tends to be stable as the temperature field tends to stabilize over time. Under the action of the temperature field and the force field, the stress of the complete thermal coupling is also behaved that the metal layer is larger than the ceramic layer.

![Figure 5](image1)

**Figure 5.** Temperature map that changes with time and space (Figure f is the sequential thermal-mechanical coupling. Figure g is the full thermal coupling.).

![Figure 6](image2)

**Figure 6.** Temperature map that changes with time and space (Figure f is the sequential thermal-mechanical coupling. Figure g is the full thermal coupling.).

![Figure 7](image3)

**Figure 7.** Stress map that changes with time and space (Figure h is the stress map only under the temperature field of sequential coupling. Figure i is the stress map under the temperature field and force field of sequential coupling. Figure j is the stress map under the temperature field and force field of full thermal coupling.).

Figure 6 shows that the temperature variation laws of the two types of thermal coupling analysis are basically the same, and the temperature rises gradually from the inner surface to the outer surface with time increasing, and roughly linear increase in the same section. Figure 7 shows that when only under the temperature field of the sequential thermal-mechanical coupling analysis, the stress is the
largest at the outer surface ceramic (z/h=0.5), and the internal stress is not much different. When temperature field and force field simultaneously, although the stress values of the two types of analysis are the largest at the inner surface metal, and the stress increases with time, but the rate of increase is different. Moreover, the stress value of full thermal coupling is larger than the sequential thermal coupling, it gradually decreases from the middle to the top at 102 s.

5. Conclusion
For the functionally graded cylindrical shell, the temperature change law and stress distributions of the two types of thermo-mechanical coupling analysis are approximately the same. The temperature values of the sequential thermo-mechanical coupling analysis are the same as the full thermal coupling, but the stress values are lower than the full thermal coupling.

6. References
[1] Loy C T, Lam K Y and Reddy 1999 INT J. Mech. Sci. 41, 3
[2] Cho J R and Oden J T 2000 Comput. Method. Appl. M. 188, 1
[3] Na K S and Kim J H 2009 Compos. Struct. 89, 4
[4] Huang H W and Rao D H 2017 J. South. China. Univ Techno; Nat. Sci. Ed. 45, 5
[5] Xu Q, Zhang X H, Han J C and He X D 2003 Acta. Mater. Compos. Sin. 423-425, 5
[6] Lin W, Bai X D, Ling Y H and Jiang Z H 2003 Mater. Sci. Forum. 423-425
[7] Hu G D, Zhang H H and Tan Y X 2017 Chin. J. Appl. Mech. 34, 2
[8] Lan L H, Fu M H and Gao W L 2011 Acta. Scientiarum. Nat. U. Sunyatseni: Nat. Sci. 50 4
[9] Amini M H, Rastgoo A and Soleimani M 2010 J. Therm. Stresses. 33, 11
[10] Noda N, Jin Z H 1994 Arch. Appl. Mech. 64, 2
[11] Zhao J, Li Y and Ai X 2008 Thin. Solid. Films. 516, 21
[12] Zhang J H, Li G Z, Li S R and Ma Y B 2015 J. Therm. Stresses. 38, 9
[13] Ozturk A, Gulgec M 2011 Int. J. Eng. Sci. 49, 10
[14] Nie G J, Batra R C 2010 Compos. Struct. 92, 3
[15] Li W T, Huang B H and Bi Z B 2004 Theoretical Analysis and Application of Thermal Stress (Beijing: China Electric Power Press) p 90

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