Novel similarity measures, entropy of intuitionistic fuzzy sets and their application in software quality evaluation

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Abstract

Intuitionistic fuzzy sets (IFSs), including member and nonmember functions, have many applications in managing uncertain information. The similarity measures of IFSs proposed to represent the similarity between different types of sensitive fuzzy information. However, some existing similarity measures do not meet the axioms of similarity. Moreover, in some cases, they could not be applied appropriately. In this study, we proposed some novel similarity measures of IFSs constructed by combining the exponential function of membership functions and the negative function of non-membership functions. In this paper, we also proposed a new entropy measure as a stepping-stone to calculate the weights of the criteria in the proposed multi-criteria decision-making (MCDM) model. The similarity measures used to rank alternatives in the model. Finally, we used this MCDM model to evaluate the quality of software projects.

Keywords

Intuitionistic fuzzy set · Similarity measure · Software quality model

1 Introduction

In 1986, Atanassov introduced intuitionistic fuzzy sets (IFSs) (Atanassov 1986), which are a generalization of a fuzzy set (Zadeh 1965). An IFS considers the information involving membership functions and non-membership functions. From their inception to the present day, IFSs have been proven to be a highly effective tool for processing uncertainties in real-world problems, including pattern recognition and decision-making. Later, Atanassov and Gargov introduced interval-valued IFSs (IVIFSs), in which membership functions and non-membership functions are subintervals of a unit interval [0, 1] (Atanassov and Gargov 1989). Similar to fuzzy sets, IFSs have wide applications in processing uncertain data for various purposes such as decision-making, medical diagnoses and agriculture (Xu 2010; Papakostas et al. 2013; Shidpour et al. 2013; Bharati and Singh 2014; Li and Zeng 2015; Liu et al. 2016; Xuan Thao 2018; Thao and Duong 2019; Thao et al. 2019a, b; Joshi 2020; Garg and Kumar 2020; Xue & Deng 2020; Thao 2021a). Along with distance and correlation measures, similarity measures of IFSs have been studied and widely used in many fields such as decision-making, machine learning and pattern recognition (Li and Cheng 2002; Szmidt and Kacprzyk 2004; Xu 2007; Ye 2011; Hwang et al. 2012; Park et al. 2013; Rajarajeswari and Uma 2013; Shi and Ye 2013; Tian 2013; Song et al. 2015; Ye 2016). Today, they still are a hot topic attracting research attention. Liang and Shi proposed a similarity measure of IFSs and applied the measure for pattern recognition (Liang and Shi 2003). Xu and Chen developed a similarity measure by using the distance measure in the multi-attribute decision-making process (Xu 2007; Xu and Chen 2008). Furthermore, Zhou proposed a similarity measure and applied it in multi-criteria decision-making (MCDM) process (Zhou 2016). The aforementioned methods used either polynomial or fractional functions. In 2013, similarity measures of IFSs based on the cotangent function investigated and applied to medical diagnoses

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(Rajarajeswari and Uma 2013; Tian 2013). Ye introduced vector cosine similarity measures for IFSs and presented their applications (Ye 2011). In 2016, Ye introduced similarity measures of IFSs based on the cosine function for decision-making in mechanical design schemes (Ye 2016). These similarity measures of Ye alleviated the limitations of other existing measures. These limitations are discussed in Sect. 3 (Examples 1 and 2). These suggest that we continue to search for new measures to overcome the limitations of existing measures.

The International Organization for Standardization (ISO 2017) declared that “The scope of the application of the quality models including supporting the specification and evaluation of software and software-intensive computer systems based on different perspectives by using the specifications associated with their acquisition, requirements, development, use, evaluation, support, maintenance, quality assurance and control, and audit. For example, the models can be used by developers, acquirers, quality assurance and control staff, and independent evaluators, particularly those responsible for specifying and evaluating software product quality”. Activities during product development that can benefit from the use of these quality models are as follows:

- Identifying software and system requirements
- Validating the comprehensiveness of requirement definitions
- Identifying software and system design objectives
- Identifying software and system testing objectives
- Identifying quality control criteria as part of quality assurance
- Identifying acceptance criteria for a software product and/or a software-intensive computer system
- Establishing measures of quality characteristics in support of these activities.

The quantification of these criteria to evaluate and rank software has practical significance. This will depend on human feelings and knowledge. Whereas IFS has proven to be an effective tool when dealing with this kind of knowledge. In this paper, we applied new measures on IFSs to the problem of pattern recognition and decision-making software quality assessment.

Contribution of this study as follows:

- We introduced a new similarity measure of IFSs by combining exponential and negative functions.
- Compare new proposed similarity measure to see that they are feasible.
- To propose new entropy of IFS by combining exponential and negative functions.
- Compare new proposed entropy measure to see that they are reasonable.
- Recommend a method to rank and evaluate product quality. It is useful to help us consider to select, evaluate software quality. This is done through illustrative numerical examples.
- Compare the new method to some other ranking method to demonstrate the performance of the new method.

The remainder of this paper is organized as follows: Sect. 2 reviews the concepts of IFSs and IVIFSs and their similarity measures. Section 3 describes how we constructed a new similarity measure between different IFSs and provides some examples in which the results obtained using our method are compared with those obtained using other methods. Section 4 presents the intuitionistic fuzzy software quality model and provides an example to illustrate the model. Moreover, the results of the model obtained using our similarity measure were compared with the results of a model obtained using the similarity measure proposed by (Ye 2011, 2016). Finally, in Sect. 5, we provide a conclusion about our method and present the future research.

2 Preliminary

Let X be a universal set. We reminder some concept related to IFS as follows.

Definition 1. (Atanassov 1986) An intuitionistic fuzzy set on X is defined by form.

\[ A = \{ (x, \mu_A(x), \nu_A(x)) | x \in X \} \]

in which \( \mu_A(x) \in [0, 1] \) and \( \nu_A(x) \in [0, 1] \) are the membership and non-membership degree of the element \( x \) in \( A \), respectively, such that.

\[ \mu_A(x) + \nu_A(x) \leq 1, \forall x \in X. \]

The degree of hesitant of element \( x \in X \) in \( A \) is

\[ \pi_A(x) = 1 - (\mu_A(x) + \nu_A(x)). \]

We denote IFS(\( X \)) is the collection of all IFSs on \( X \).

For two intuitionistic fuzzy sets \( A, B \in \text{IFS}(X) \), we have:

- Subset: \( A \subset B \) if only if \( \mu_A(x) < \mu_B(x) \) and \( \nu_A(x) > \nu_B(x) \) for all \( x \in X \).
- Equal: \( A = B \) if only if \( \mu_A(x) = \mu_B(x) \) and \( \nu_A(x) = \nu_B(x) \) for all \( x \in X \).
- For all \( \lambda > 0 \), we have

\[ A^\lambda = \left\{ x, \mu_A^\lambda(x), 1 - (1 - \mu_A(x))^\lambda \right\} | x \in X \]  \hspace{1cm} (1)

Now, we recall the similarity measure in literal.

Given a finite universal set \( X = \{ x_1, x_2, ..., x_n \} \). Let \( A = \{ (x_i, \mu_A(x_i), \nu_A(x_i)) | x_i \in X \} \), \( B = \{ (x_i, \mu_B(x_i), \nu_B(x_i)) | x_i \in X \} \) be two IFSs on \( X \).
Definition 2. (Li and Cheng 2002) A mapping $S$: IF$S(X) \times$IF$S(X) \rightarrow [0, 1]$ is the similarity measure of the intuitionistic fuzzy sets if it satisfies the following conditions:

(S1) $0 \leq S(A, B) \leq 1$ for all $A, B \in$ IF$S(X)$,
(S2) $S(A, B) = S(B, A)$ for all $A, B \in$ IF$S(X)$,
(S3) $S(A, A) = 1$ for all $A \in$ IF$S(X)$,
(S4) For all $A, B, C \in$ IF$S(X)$. If $A \subseteq B \subseteq C$ then $S(A, C) \leq \min\{S(A, B), S(B, C)\}$.

3 The new similarity measures of the IFSs

Let $X = \{x_1, x_2, ..., x_n\}$ be a finite universal set, and $A$ and $B$ are two arbitrary IFSs on $X$. We denote $S_i^\mu(A, B) = e^{-|\mu_{\mu}(x_i) - \nu_{\mu}(x_i)| - e^1}$, $S_i^\nu(A, B) = 1 - |\nu_{\mu}(x_i) - \nu_{\nu}(x_i)|$ for all $i = 1, 2, ..., n$.

Proposition 1. Let $A$ and $B$ be two arbitrary IFSs in $X$. Then $S_i^\mu(A, B)$ and $S_i^\nu(A, B)$ $(i = 1, 2, ..., n)$ satisfy the following conditions:

(s1) $0 \leq S_i^\mu(A, B), S_i^\nu(A, B) \leq 1$ for all $A, B \in$ IF$S(X)$,
(s2) $S_i^\mu(A, B) = S_i^\mu(B, A)$ and $S_i^\nu(A, B) = S_i^\nu(B, A)$ for all $A, B \in$ IF$S(X)$
(s3) $S_i^\mu(A, A) = S_i^\nu(A, A) = 1$ for all $A \in$ IF$S(X)$
(s4) For all $A, B, C \in$ IF$S(X)$. If $A \subseteq B \subseteq C$ then $S_i^\mu(A, C) \leq \min\{S_i^\mu(A, B), S_i^\nu(B, C)\}$

and

$$S_i^\nu(A, C) \leq \min\{S_i^\nu(A, B), S_i^\nu(B, C)\}.$$ 

Proof. (s1). For all $A \in$ IF$S(X)$ and $B \in$ IF$S(X)$, we have $-1 \leq -|\mu_{\mu}(x_i) - \nu_{\mu}(x_i)| \leq 0$. and $0 \leq |\nu_{\mu}(x_i) - \nu_{\nu}(x_i)| \leq 1$

so that

$$e^{-1} \leq S_i^\mu(A, B) = e^{-|\mu_{\mu}(x_i) - \mu_{\nu}(x_i)|} \leq 1$$

and

$$0 \leq S_i^\nu(A, B) = 1 - |\nu_{\mu}(x_i) - \nu_{\nu}(x_i)| \leq 1.$$ 

(s2). It is obvious.

(s3). Consider two intuitionistic fuzzy sets $A$ and $B$ on $X$. If $A = B$, then $\mu_{\mu}(x_i) = \mu_{B}(x_i)$, $\nu_{\mu}(x_i) = \nu_{B}(x_i)$. This implies that $S_i^\mu(A, B) = 1$, $S_i^\nu(A, B) = 1$.

(s4). For three intuitionistic fuzzy sets $A$, $B$ and $C$ such that $A \subseteq B \subseteq C$, then we have $\mu_{\mu}(x_i) \leq \mu_{B}(x_i) \leq \mu_{\nu}(x_i)$, and $\nu_{\nu}(x_i) \leq \nu_{B}(x_i) \leq \nu_{\mu}(x_i)$. So that, we have

$$\max\{|\mu_{\mu}(x_i) - \mu_{B}(x_i)|, |\mu_{B}(x_i) - \mu_{\nu}(x_i)|\} \leq |\mu_{\mu}(x_i) - \mu_{\nu}(x_i)|$$

and

$$\max\{|\nu_{\mu}(x_i) - \nu_{B}(x_i)|, |\nu_{B}(x_i) - \nu_{\nu}(x_i)|\} \leq |\nu_{\mu}(x_i) - \nu_{\nu}(x_i)|.$$ 

Then we have

$$\min\{-|\mu_{\mu}(x_i) - \mu_{B}(x_i)|, -|\mu_{B}(x_i) - \mu_{\nu}(x_i)|\} \geq -|\mu_{\mu}(x_i) - \mu_{\nu}(x_i)|,$$

and

$$\min\{-|\nu_{\mu}(x_i) - \nu_{B}(x_i)|, -|\nu_{B}(x_i) - \nu_{\nu}(x_i)|\} \geq -|\nu_{\mu}(x_i) - \nu_{\nu}(x_i)|.$$ 

Since, we obtain

$$S_i^\mu(A, C) \leq \min\{S_i^\mu(A, B), S_i^\nu(B, C)\}$$

and

$$S_i^\nu(A, C) \leq \min\{S_i^\nu(A, B), S_i^\nu(B, C)\}.$$ 

Proof. From Proposition 1, we have:

(S1) Since $0 \leq S_i^\mu(A, B), S_i^\nu(A, B) \leq 1$, we have

$$0 \leq S_i(A, B) = \sum_{i=1}^{n} \omega_i \frac{S_i^\mu(A, B) + S_i^\nu(A, B)}{2} \leq \sum_{i=1}^{n} \omega_i = 1.$$ 

Theorem 1. Let $A$ and $B$ be two arbitrary IFSs in $X$. The expression $S_1(A, B)$ in the Eq. (2) satisfies the following conditions:

(S1) $0 \leq S_1(A, B) \leq 1$ for all $A, B \in$ IF$S(X)$,
(S2) $S_1(A, B) = S_1(B, A)$ for all $A, B \in$ IF$S(X)$,
(S3) $S_1(A, A) = 1$ for all $A \in$ IF$S(X)$,
(S4) For all $A, B, C \in$ IF$S(X)$. If $A \subseteq B \subseteq C$ then, we have

$$S_1(A, C) \leq \min\{S_1(A, B), S_1(B, C)\}.$$ 

It means that

$$S_1(A, B) = \sum_{i=1}^{n} \omega_i \frac{S_i^\mu(A, B) + S_i^\nu(A, B)}{2}$$

is a similarity measure of two intuitionistic fuzzy sets.
The similarity measure based on the cotangent function introduced by (Tian 2013)
\[ \text{CT}_1(A, B) = \frac{\sum_{i=1}^{n} \cos \left( \left\{ \frac{\pi}{4} + \max \left( \left| \mu_A(x_i) - \mu_B(x_i) \right|, \left| v_A(x_i) - v_B(x_i) \right| \right) \right\} \right) }{n} \]

- The similarity measure based on the cotangent function proposed by (Rajarajeswari and Uma 2013)
\[ \text{CT}_2(A, B) = \frac{\sum_{i=1}^{n} \cos \left( \left\{ \frac{\pi}{4} + \max \left( \left| \mu_A(x_i) - \mu_B(x_i) \right|, \left| v_A(x_i) - v_B(x_i) \right| \right) \right\} \right) }{n} \]

- The similarity measure based on the cosine function defined by (Ye 2016).
\[ \text{CS}_1(A, B) = \frac{1}{n} \sum_{i=1}^{n} \cos \left( \left\{ \frac{\pi}{4} + \max \left( \left| \mu_A(x_i) - \mu_B(x_i) \right|, \left| v_A(x_i) - v_B(x_i) \right| \right) \right\} \right) \]

- The similarity measure proposed by (Xu and Chen 2008).
\[ \text{SX}_{C}(A, B) = 1 - \frac{1}{n} \sum_{i=1}^{n} \left[ \left| \mu_A(x_i) - \mu_B(x_i) \right| + \left| v_A(x_i) - v_B(x_i) \right| \right] + \frac{1}{2} \max \left( \left| \mu_A(x_i) - \mu_B(x_i) \right|, \left| v_A(x_i) - v_B(x_i) \right| \right) \]

- The similarity measures given by (Papakostas et al. 2013)
\[ \text{SP}_1(A, B) = 1 - \exp \left( -\frac{1}{n} \sum_{i=1}^{n} \left[ \left| \mu_A(x_i) - \mu_B(x_i) \right| + \left| v_A(x_i) - v_B(x_i) \right| \right] \right) \]

and
\[ \text{SP}_2(A, B) = 1 - \frac{1}{n} \sum_{i=1}^{n} \left[ \left| \sqrt{\mu_A(x_i)} - \sqrt{\mu_B(x_i)} \right| + \left| \sqrt{v_A(x_i)} - \sqrt{v_B(x_i)} \right| \right] \]

- The similarity measure introduced by of (Song et al. 2015)

(S2). It is obviously.

(S3). If \( A = B \), then \( \mu_A(x_i) = \mu_B(x_i), v_A(x_i) = v_B(x_i) \), so that we get.
\[ S_i^A(A, B) = 1, S_1(A, B) = \frac{\sum_{i=1}^{n} \omega_i S_i^A(A, B) + S_i^B(A, B)}{\sum_{i=1}^{n} \omega_i} = 1. \]

(S4). For all \( A, B, C \) in IFS(X) such that \( A \subseteq B \subseteq C \), then we have
\[ S_i^A(A, C) \leq \min \{ S_i^A(A, B), S_i^A(B, C) \} \]
and
\[ S_i^A(A, C) \leq \min \{ S_i^A(A, B), S_i^A(B, C) \} \]
so that, we get
\[ S_i^A(A, B) = \sum_{i=1}^{n} \omega_i S_i^A(A, B) + S_i^A(A, B) \]
\[ \leq \sum_{i=1}^{n} \omega_i \min \left\{ \frac{S_i^A(A, B) + S_i^A(A, B)}{2}, \frac{S_i^A(B, C) + S_i^A(B, C)}{2} \right\} \]
\[ = \min \{ S_i(A, B), S_i(B, C) \}. \]

More generally, with \( p \in N^* \), we have the similarity measures determined in Definition 5.

**Definition 4.** A mapping \( S : \text{IFS}(X) \times \text{IFS}(X) \rightarrow [0, 1] \)
defined by,
\[ S_p^A(A, B) = \sum_{i=1}^{n} \omega_i \left( \frac{S_i^A(A, B) + S_i^A(A, B)}{2} \right) \]
is the similarity measures of the intuitionistic fuzzy sets.

### 3.1 Comparison with some existing similarity measures of IFS.

Now, we remind some existing similarity measures on IFSs

- The cosine similarity measure proposed by (Ye 2011)
\[ C_Y(A, B) = \frac{1}{n} \sum_{i=1}^{n} \frac{\mu_A(x_i) \mu_B(x_i) + v_A(x_i) v_B(x_i)}{\sqrt{\mu_A^2(x_i) + v_A^2(x_i)}} \]

- The cosine similarity measure proposed by (Shi and Ye 2013)
\[ C_{XY}(A, B) = \frac{1}{n} \sum_{i=1}^{n} \frac{\mu_A(x_i) \mu_B(x_i) + v_A(x_i) v_B(x_i) + \pi_A(x_i) \pi_B(x_i)}{\sqrt{\mu_A^2(x_i) + v_A^2(x_i) + \mu^2_B(x_i) + v^2_B(x_i)}} \]

- The similarity measure based on the cotangent function introduced by (Tian 2013)
The classification results in Example 1

| Similarity measures                | $S(A, C)$ | $S(B, C)$ | The classification results |
|------------------------------------|-----------|-----------|---------------------------|
| $C_T$ (Ye 2011)                    | 0         | Null      | Null                      |
| $C_{SY}$ (Shi and Ye 2013)         | 0         | 0         | Null                      |
| $CT_1$ (Tian 2013)                 | 0         | 0         | Null                      |
| $CT_2$ (Rajarajeswari and Uma 2013)| 0         | 0         | Null                      |
| $CS_1$ (Ye 2016)                   | 0         | 0         | Null                      |
| $CS_2$ (Ye 2016)                   | 0         | 0         | Null                      |
| $SP_1$ (Papakostas et al. 2013)    | 0         | 0.3775    | $C$ belongs to class of $B$|
| $SP_2$ (Papakostas et al. 2013)    | 0         | 0.3775    | $C$ belongs to class of $B$|
| $SX_{C}$ (Xu and Chen 2008)        | 0         | 0.25      | $C$ belongs to class of $B$|
| $S_3$ (Song et al. 2015)           | 0         | 0.5       | $C$ belongs to class of $B$|
| $S_{31}$ (Song et al. 2019)        | 0         | 0.3333    | $C$ belongs to class of $B$|
| $S_4$ (proposed measure)           | 0         | 0.5       | $C$ belongs to class of $B$|
| $S_4'$ (proposed measure, in Eq. (2) with $p = 2$) | 0 | 0.7071 | $C$ belongs to class of $B$|

Null means that we cannot determine where class $C$ belong to.
4 New entropy of intuitionistic fuzzy sets

Entropy of intuitionistic fuzzy set used to measure the intuition of the intuitionistic fuzzy sets. In 2001, Szmidt E and Kacprzyk introduce the concept of entropy of intuitionistic fuzzy set.

Definition 5. (Szmidt and Kacprzyk 2001). An entropy on $IFS(X)$ is a function $E: IFS(X) \rightarrow [0,1]$, satisfying all following conditions.

(E1) $E(A) = 0$ if $A \in IFS(X)$ has $\mu_A(x_i), v_A(x_i) \in \{0,1\}$ for all $x_i \in X$.

(E2) $E(A) = 1$ if only if $\mu_A(x_i) = v_A(x_i)$ for all $x_i \in X$.

(E3) $E(A) = E(A^c)$, for all $A \in IFS(X)$.

(E4) $E(A) \leq E(B)$ for all $A, B \in IFS(X)$ satisfy either if $\mu_A(x_i) \leq \mu_B(x_i) \leq v_B(x_i) \leq v_A(x_i)$ or $\mu_A(x_i) \geq \mu_B(x_i) \geq v_B(x_i) \geq v_A(x_i)$ for all $x_i \in X$.

Definition 6. A mapping $E$: $IFS(X) \rightarrow [0,1]$ is defined by.

$$E_T(A) = \frac{1}{2n} \sum_{i=1}^{n} \left[ e^{-|\mu_A(x_i) - v_A(x_i)|} - e^{-1} \right] + \left( 1 - |\mu_A(x_i) - v_A(x_i)| \right)$$

(4)

Remark. We easy verify that $E_T(A) = S_1(A, A^c)$ for all $A \in IFS(X)$.

Theorem 2. Let $A$ is an arbitrary $IFS$ in $X$. The function $E_T(A)$ in Eq. (4) is an entropy on $IFS(X)$.

Proof.

|   | Similarity measures | $S(A, C)$ | $S(B, C)$ | The classification results |
|---|---------------------|-----------|-----------|----------------------------|
| C_T(Ye 2011) | 0.9258 | 0.9560 | C belongs to class of $B$ |
| C_{ST}(Shi and Ye 2013) | 0.8486 | 0.9567 | C belongs to class of $B$ |
| C_{T2}(Tian 2013) | 0.7903 | 0.7903 | Null |
| C_{T2}(Rajarajeswari and Uma 2013) | 0.7903 | 0.7903 | Null |
| C_S1(Ye 2016) | 0.9694 | 0.9694 | Null |
| C_S2(Ye 2016) | 0.9694 | 0.9694 | Null |
| S_T1(Papakostas et al. 2013) | 0.8145 | 0.8145 | Null |
| S_T2(Papakostas et al. 2013) | 0.7413 | 0.5841 | C belongs to class of $A$ |
| S_XT1(Xu and Chen 2008) | 0.8625 | 0.8625 | Null |
| S_X(Song et al. 2015) | 0.9893 | 0.9854 | C belongs to class of $A$ |
| S_Y(Song et al. 2019) | 0.9681 | 0.9714 | C belongs to class of $B$ |
| S_1(proposed measure) | 0.8407 | 0.8497 | C belongs to class of $B$ |
| S_2(proposed measure, in Eq. (2) with $p = 2$) | 0.8435 | 0.8501 | C belongs to class of $B$ |

Null means that we cannot determine where class $C$ belong to.

4.1 Compare to some existing other entropies

We consider the comparison of six entropies via examples as follows.

Example 3. Let $A = \{(x_6, 0.2, 0.8), (x_7, 0.3, 0.5), (x_8, 0.6, 0.4), (x_9, 0.9, 0.0), (x_{10}, 1.0)\}$ be a PFS on the universal set $X = \{x_6, x_7, x_8, x_9, x_{10}\}$, where $x_i$ is an apartment having $i$ rooms. In the characterizations of the linguistic variables, we regard $A$ as “Large”; then, according to the operator law in Eq. (1) we have

$A^0.5$ may be regarded as “Quite Large,” $A^2$ may be regarded as “Very Large,”

$A^3$ may be regarded as “Quite Very Large,” $A^4$ may be regarded as “Very Very Large,”

in which
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• Entropy of (Burillo and Bustince 1996)

\[ E_{BB}(A) = \frac{1}{n} \sum_{i=1}^{n} \pi_A(x_i) \]

• Entropy of (Szmidt and Kacprzyk 2001)

\[ E_{SK}(A) = \frac{1}{n} \sum_{i=1}^{n} \max \{ \text{count}(A(x_i) \cap A^c(x_i)) \} \]

where \( \max \{ \text{count}(A(x_i)) = \mu_A(x_i) + \pi_A(x_i) \) for all \( x_i \in X \).

• Entropy of (Hung and Yang 2006)

\[ E_{H}(A) = \frac{1}{n(1-\beta)} \sum_{i=1}^{n} \log \left( \mu_A(x_i) + \pi_A(x_i) \right) \]

where \( 0 < \beta < 1 \).

• Entropy of (Vlachos and Sergiagis 2007)

\[ E_{VS}(A) = -\frac{1}{n \ln 2} \sum_{i=1}^{n} \left[ \mu_A(x_i) \ln \mu_A(x_i) + \pi_A(x_i) \ln \pi_A(x_i) \right] \]

• Entropy of (Ye 2010)

\[ E_{Y}(A) = \frac{1}{n(\sqrt{2}-1)} \sum_{i=1}^{n} \left[ \cos(\pi(1-\mu_A(x_i)+\pi_A(x_i))) \right. \]

\[ + \cos(\pi(1+\mu_A(x_i)-\pi_A(x_i))) \left. \right] - 1 \]

• Entropy of (Zhang and Jiang 2008)

\[ E_{ZJ}(A) = \frac{1}{n} \sum_{i=1}^{n} \mu_A(x_i) \lor v_A(x_i) \]

\[ E_{ZL}(A) = \frac{1}{4n} \sum_{i=1}^{n} \left[ (1+\pi_A(x_i))(1-\mu_A(x_i)-\pi_A(x_i)) + \pi_A(x_i) \right] \]

5 Application of similarity measures and entropy in a software quality model

There are some models for evaluating software quality, such as the software quality evaluation model generated using the hesitant fuzzy uncertain linguistic information (Li et al. 2014) or the model based on the fuzzy analytic hierarchy process (Chang et al. 2008). These models are quite complex because they either require building linguistic variables (Li et al. 2014) or require using primitive ISO standards published in 2001 (Chang et al. 2008). In this section, we constructed a MCDM model based on the proposed similarity measures (in Sect. 3) and entropy (in Sect. 4) to evaluate the quality of software projects. Software is an important factor in the development of computers. Efficient orchestrated software enables applications solving real-world problems more effectively. Software engineers often focus on software production. Good-quality software that can keep up with budget constraints and time is considered to be effective software. In 2011, ISO defined a new standard ISO 25010 to evaluate system and software product quality. This standard confirmed in (IOS 2017) and included the following key quality identifiers.

- Functional Suitability \( C_1 \)
- Functional Correctness \( C_2 \)
- Testability \( C_3 \)
- Performance Efficiency \( C_4 \)
- Compatibility \( C_5 \)
- Usability \( C_6 \)
- Appropriateness Recognizability \( C_7 \)
- User Interface Aesthetics \( C_8 \)
- Reliability \( C_9 \)
- Security \( C_{10} \)
- Maintainability \( C_{11} \)
- Modifiability \( C_{12} \)
- Portability \( C_{13} \)

In the quality evaluation of software projects, the quality of software projects is compared with the best example based on a set of criteria (i.e., one that matches all criteria).
1. Determine the criteria for quality assessment
6. Calculate the similarity measure
7. Rank projects based on the similarity measure. The

For the criterion of benefits, being the best is often desirable. However, for the criterion of cost, having the lowest cost is usually desirable. Quality evaluation is an MCDM problem. Moreover, the digitization of qualitative criteria is highly dependent on human psychology. The theory of IFSSs is more useful for solving uncertainty problems. Therefore, we proposed an MCDM model for evaluating software quality by using an IFS similarity measure, namely the intuitionistic fuzzy software quality model.

The IFSQM has seven steps as follows:

1. Determine the criteria for quality assessment
2. Identify different projects for evaluation
3. Determine the intuitionistic fuzzy sets
4. Determine the weight of the criteria
5. Build the best project
6. Calculate the similarity measure
7. Rank projects based on the similarity measure. The project $P_i$ is better than the project $P_k$ (we denote $P_i \succ P_k$) if $S(P_i, P_b) > S(P_k, P_b)$, $i = 1, 2, ..., m$ and $k = 1, 2, ..., m$.

This algorithm consists of simple computation steps on fuzzy decision-making matrix IFM with $m$ rows and $n$ columns (corresponding to $m$ alternatives and $n$ criteria). So we can define its computational complexity as just $O(mn)$ polynomial time.

**Example 4.** We consider the IFSQM to evaluate the software quality of five software projects: $P_1, P_2, P_3, P_4$ and $P_5$. The software quality criteria are $U = \{C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9, C_{10}, C_{11}, C_{12}, C_{13}\}$. In this model, we consider $P_i (i = 1, 2, ..., m)$ are the intuitionistic fuzzy sets on $U$. Then, we have an intuitionistic fuzzy decision matrix of software based on software quality criteria (Table 4). These are also step 1, step 2 and step 3 of IFSQM.

**Step 4.** We compute the weight of each $C_j(j = 1, 2, ..., n)$ based on Eq. (5), then the weigh vector of criteria as follows:

$$\omega = (0.062, 0.1, 0.097, 0.12, 0.063, 0.07, 0.072, 0.097, 0.063, 0.072, 0.06, 0.068, 0.065)$$

**Step 5.** Building the best project: In this example, all the software quality criteria used for evaluation are the criteria of benefit. Therefore, we chose the best project is $P_b = \{(C_i, 1) | i = 1, 2, ..., 13\}$.

**Step 6.** The similarity measures of each project with the best project are shown in Tables 5 and 6.

**Step 7.** Ranking projects are shown in Tables 5 and 6.

We denote IFSQM 1 and IFSQM 1 as versions of IFSQM when using the similarity measure in step 6 by Eqs. (2) and (3) with $p = 2$, respectively.

Evaluation of the quality of the product based on similarity measures is carried out in accordance with the following principles. If $0 < S(P_i, P_b) < 0.5$, then the project $P_i$ belongs to the class Low (L); if $0.5 \leq S(P_i, P_b) < 0.75$, then the project $P_i$ belongs to the class Medium (M); if $0.75 \leq S(P_i, P_b) < 0.9$, then the project $P_i$ belongs to the class High (H); if $0.9 \leq S(P_i, P_b) < 1$, then the project $P_i$ belongs to the class Very High (VH).

Accordingly, we can see that the rank of project $P_3$ is the maximum and its quality is Very High. The second is

|   | $E_{1}$ | $E_{BB}$ | $E_{SK}$ | $E_{H}$ | $E_{VS}$ | $E_{Y}$ | $E_{ZI}$ | $E_{ZL}$ |
|---|--------|--------|--------|--------|--------|--------|--------|--------|
| $A^{0.1}$ | 0.3989 | 0.0421 | 0.3270 | 0.2346 | 0.4716 | 0.5066 | 0.4087 | 0.1237 |
| $A$ | 0.3874 | 0.0600 | 0.3033 | 0.2436 | 0.4601 | 0.5452 | 0.3792 | 0.1285 |
| $A^{2}$ | 0.2350 | 0.0700 | 0.1448 | 0.2154 | 0.2519 | 0.3896 | 0.1810 | 0.0885 |
| $A^{4}$ | 0.1472 | 0.0738 | 0.0629 | 0.1817 | 0.1446 | 0.2844 | 0.0786 | 0.0664 |
| $A^{14}$ | 0.1132 | 0.0797 | 0.0318 | 0.1538 | 0.1061 | 0.2262 | 0.0398 | 0.0599 |

"bold" means that the entropy values are sensible.
project $P_2$, which has its quality High. Three projects $P_1$, $P_4$ and $P_5$ have their quality being Medium, simultaneously ranked third, fourth and fifth, respectively.

Table 4 The intuitionistic fuzzy decision matrix of software based on software quality criteria

| Criteria | $P_1$       | $P_2$       | $P_3$       | $P_4$       | $P_5$       |
|----------|-------------|-------------|-------------|-------------|-------------|
| $C_1$    | (0.49,0.1)  | (0.6,0.04)  | (0.36,0.04) | (0.81,0.05) | (0.25,0.25) |
| $C_2$    | (0.7,0.16)  | (0.8,0.01)  | (0.73,0.03) | (0.6,0.11)  | (0.81,0.05) |
| $C_3$    | (0.8,0.1)   | (0.8,0.01)  | (1.0)       | (0.49,0.19) | (0.64,0.1)  |
| $C_4$    | (0.81,0.05) | (0.9,0.1)   | (1.0)       | (0.8,0.01)  | (0.81,0.05) |
| $C_5$    | (1.0)       | (0.25,0.1)  | (1.0)       | (0.25,0.3)  | (0.25,0.3)  |
| $C_6$    | (0.25,0.4)  | (1.0)       | (1.0)       | (0.25,0.16) | (0.25,0.4)  |
| $C_7$    | (0.25,0.4)  | (0.6,0.04)  | (1.0)       | (0.16,0.18) | (0.81,0.05) |
| $C_8$    | (0.8,0.1)   | (0.8,0)     | (1.0)       | (0.49,0.19) | (0.64,0.1)  |
| $C_9$    | (1.0)       | (0.25,0.1)  | (1.0)       | (0.25,0.3)  | (0.25,0.3)  |
| $C_{10}$ | (0.25,0.4)  | (0.6,0.04)  | (1.0)       | (0.16,0.176)| (0.81,0.05) |
| $C_{11}$ | (0.49,0.1)  | (0.6,0.04)  | (0.36,0.04) | (0.81,0.05) | (0.25,0.25) |
| $C_{12}$ | (0.25,0.4)  | (1.0)       | (1.0)       | (0.25,0.16) | (0.25,0.4)  |
| $C_{13}$ | (0.3,0.4)   | (1.0)       | (1.0)       | (0.25,0.16) | (0.25,0.4)  |

Table 5 The quality of software projects using IFSQM1

| Software projects | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ |
|-------------------|-------|-------|-------|-------|-------|
| Similarity values | 0.6988| 0.8541| 0.9510| 0.6954| 0.6680|
| Ranking           | 3     | 2     | 1     | 4     | 5     |
| Quality           | M     | H     | VH    | M     | M     |

Table 6 The quality of software projects using IFSQM2

| Software projects | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ |
|-------------------|-------|-------|-------|-------|-------|
| Similarity values | 0.6753| 0.8305| 0.9400| 0.6543| 0.6381|
| Ranking           | 3     | 2     | 1     | 4     | 5     |
| Quality           | M     | H     | VH    | M     | M     |

To further demonstrate the feasibility of the proposed method, we compare the ranking results with some existing ranking methods in (Song et al., 2015; Ye, 2016; Zhou, 2016; Thao and Duong, 2019; Song et al., 2019; Quynh et al., 2020;
Thao 2021a). The comparison results are shown in Fig. 1. According to Fig. 1, we easily see that almost method is shown that: $P_3$ is the highest, $P_2$ is the second, $P_3$ is the third, $P_5$ and $P_4$ are in the fourth and fifth place, respectively. Therefore, this is the final ranking option. It also coincides with the ranking results of the proposed new method. This explains the effectiveness of our proposed methods. We also calculate some scenarios having small changes in the weight of the criteria. According to that, we compare the ranking results of the options based on these different methods to evaluate the stability of the proposed method. Figures 2 and 3 illustrate the ranking results of the alternatives when using weights $\omega_2 = (13, 13, 13, 13, 13, 13, 13, 13, 13, 13)$ and $\omega_3 = (0.062, 0.1, 0.096, 0.119, 0.063, 0.07, 0.072, 0.096, 0.063, 0.072, 0.06, 0.062, 0.065)$, respectively. In these cases, we see that having a small change in the ranking results of $P_4$ and $P_5$ when using the ranking methods of (Ye 2016), (Zhou 2016) and (Quynh et al. 2020). The rankings of the other options still remain. Meanwhile, the proposed method along with the remaining methods (Song et al. 2015 2019; Thao and Duong 2019; Thao 2021a, b) all give stable ranking results.

6 Conclusion

The evaluation of software quality based on ISO standards (IOS 2017) associated with the difficulty is that the most of the standards are qualitative. The evaluation considerably depends on the opinion of the decision-maker. When assessing software quality, the evaluation result is affected by the understanding and subjective psychology of decision-makers. Therefore, using the intuitionistic fuzzy set is an appropriate selection for this problem. In this study, we provided some formulas that define similarity measures between different IFSs. Another advantage of this IFSQM model over some other MCDM models is that it combines the entropy induced from the similarity measure to calculate the weights of the criteria. This helps us to avoid emotional mistakes made by decision-makers when assigning weights to criteria. The proposed similarity measures were judged to be superior to existing measures. The evidence for this statement was provided by comparing the proposed measures with other similarity measures. Finally, a software quality model based on the similarity measures of IFSs was introduced. This model is simpler and easier to use for evaluating software quality than some previous models. Moreover, we presented a numerical example to illustrate the proposed model. The proposed method is also limited as it has not mentioned the handling of linguistic variables or the integration of many expert opinions to assess product quality more comprehensively. Those are very meaningful expansion directions, and we will continue to study them in the near future.

In future, we will continue to find other methods to evaluate software quality and will apply similarity, entropy measures to other applied mathematical problems, such as the labeled classification problem (Quynh et al. 2020), clustering problems (Zhou et al. 2016; Thao et al. 2019a 2019b), decision-making problems (Joshi 2020; Thao 2021b), applications to supply chain management (Xiao et al. 2020; Thao 2021a), using IFS and other extended fuzzy sets to “buy online and pick up in store mode” (Wang et al. 2020), and/or other measures as dissimilarity measure (Dinh et al. 2017; Duong and Thao, 2021; Chou et al. 2021) or other methods (Garg and Kumar 2020; Nguyen et al. 2014; Nguyen and Nguyen 2015; Zhou et al. 2016; Thao et al. 2019a, b; Thao 2021b).

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Declarations

Conflict of interest The author declare that they do not have any conflict of interests.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.
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