A wind power plant site selection algorithm based on $q$-rung orthopair hesitant fuzzy rough Einstein aggregation information

Attaullah¹, Shahzaib Ashraf², Noor Rehman³, Asghar Khan¹, Muhammad Naeem⁴ & Choonkil Park⁵

Wind power is often recognized as one of the best clean energy solutions due to its widespread availability, low environmental impact, and great cost-effectiveness. The successful design of optimal wind power sites to create power is one of the most vital concerns in the exploitation of wind farms. Wind energy site selection is determined by the rules and standards of environmentally sustainable development, leading to a low, renewable energy source that is cost effective and contributes to global advancement. The major contribution of this research is a comprehensive analysis of information for the multi-attribute decision-making (MADM) approach and evaluation of ideal site selection for wind power plants employing $q$-rung orthopair hesitant fuzzy rough Einstein aggregation operators. A MADM technique is then developed using $q$-rung orthopair hesitant fuzzy rough aggregation operators. For further validation of the potential of the suggested method, a real case study on wind power plant site has been given. A comparison analysis based on the unique extended TOPSIS approach is presented to illustrate the offered method’s capability. The results show that this method has a larger space for presenting information, is more flexible in its use, and produces more consistent evaluation results. This research is a comprehensive collection of information that should be considered when choosing the optimum site for wind projects.

Providing sustainable and widely accessible energy to human populations became one of the most challenging problems over the last several decades. Between 2000 and 2030 AD, global energy consumption is expected to expand by an average of 8% each year¹. Fossil fuels supply the majority of the energy needed and have the largest effect. To reduce their reliance on fossil fuels, several developed nations have enacted laws promoting the use of renewable energy sources such as wind and solar power. Wind power is one of the most reliable and long-term renewable energy sources accessible. Wind energy has developed into a large, environmentally beneficial, and financially feasible resource. It has become more desirable as a renewable resource attributable to technological improvements and productivity improvements. Wind energy has grown in popularity, and governments have implemented several successful policies that encourage its installation. The expense of wind energy generation has become comparable with the cost of fossil fuel generation. As a result, wind energy is a surprisingly safe and risk-free source of renewable energy that is economically feasible, ecologically safe, and contributed substantially to the reduction of hazardous substances. The main goal of the study is to assess the requirement for excess power resources as a consequence of growing populations. Prior to initiating the technical project, it is essential to choose the suitable location for the wind power plant. The enhancement of facility location provides relevant information of analyzing challenges associated with installations in specific locations based on specified criteria²³. Wind farm site selection is complicated, with many factors to consider, including the finance, environment, infrastructure, ecological, geographical features, hydrogeological conditions, ground hydrological conditions, industry, and practicality of wind power⁴⁵.

Owing to the overwhelming ambiguity and complexity of local and global surroundings, as well as the capacity of human intelligence to apprehend reality, it is not always possible for decision makers to express assessment

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values or ideas in simple quantitative measures. To alleviate this challenge, Zadeh established the fuzzy set theory, which is characterized by a membership function with a degree of membership of [0,1]. In recent decades, fuzzy set theories have significantly been incorporated with MCDM as a beneficial tool for resolving imprecision and ambiguity, resulting in a profusion of fuzzy MCDM techniques. However, in dealing with erroneous information caused by several sources of uncertainty in reality, the traditional fuzzy set has certain limitations.

Therefore, various expansions of fuzzy sets have already been suggested over the last few decades. Torra introduced hesitant fuzzy sets (HFS) in 2007, which enabled the analysis of the membership degree of an element to various sets to have equal weights or significance. Obviously, in real-world MCDM challenges, it may not be appropriate to use these sets as they provide premature results. Moreover, in some cases, the degree of membership is uncertain, while in others, it is precise. One of the critical challenges in MCDM is to define the membership degree of an element to a fuzzy set. Therefore, the use of hesitant fuzzy set theory can be beneficial in this regard.

Hesitant fuzzy set theory is a method for capturing the uncertainty in a more flexible and comprehensive way than traditional fuzzy set theory. In this theory, the membership degree of an element to a fuzzy set is represented by a set of possible values, rather than a single value. This allows for a more accurate representation of the uncertainty associated with the data.

In recent years, various innovative generalised versions of HFS have been presented to effectively address ambiguity in real problems. Qian developed the generalised HFS, which is used to describe a range of attributes or categories, and potentially implement it to practical MCDM. Zhu introduced dual hesitant fuzzy sets (DHFS) and analysed the basic operations and features while proposing a DHFS expansion concept. Rodriguez et al. addressed the linguistic term set (HFTS) and its applicability in group decision challenges. Chen initiated hesitant intuitionistic fuzzy sets (HIFS) and provided the composite operators for MCDM with HIFS information. Mahmoudi et al. expanded the PROMETHEE to an HF environment that was irrelevant to aggregation and distance operators. Alcantud et al. established HFS decomposition theorems employing newly specified families of cuts and also presented two HFS extension principles that broadened crisp maps.

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discovered in the q-ROF information, according to current understanding. As a consequence, we specify a collection of operators based on rough data, such as q-rung orthopair hesitant fuzzy rough geometric, ordered weighted geometric, and hybrid weighted geometric aggregation operators.

This manuscript contributes to the literature of FRSs theory by introducing some innovative ideas, which are as follows:

1. To compile a list of Aops based on Einstein’s t-norm and t-conorm, namely q-ROHFR weighted geometric, q-ROFRS ordered weighted geometric, and q-ROHFR hybrid weighted geometric operators, and explore their essential operational laws. Also, discuss the related properties thoroughly.
2. To develop a DM approach for synthesising uncertain information utilising suggested aggregation operators.
3. A numerical case study of a real-world problem concerning wind power project site selection evaluation is developed using the established operators.
4. Furthermore, the findings are interpreted through comparisons to the q-ROHFR-TOPSIS technique. The acquired outcomes are displayed graphically.

The concluding section of the article is described as follows: “Fundamental concepts” summarises key concepts in q-ROFSs, HFSs, rough set theory and q-rung orthopair hesitant fuzzy sets. “The q-rung orthopair hesitant fuzzy rough aggregation operators” summarises a list of Einstein aggregation operators that are used to aggregate uncertain data based on Einstein operational laws. “The multi-attribute decision making methodology” explains a decision-making approach based on developed AOps. “The application of proposed decision-making approach” illustrates how to formulate a strategy for wind farm site selection numerically. Additionally, this section explores the application of the technique. “Comparison analysis” details the proposed q-ROHFR-TOPSIS technique for analysing the MADM method based on AOps. “Conclusion” concludes the manuscript.

**Fundamental concepts**
This section introduces some fundamental concepts, particularly q-ROFSs, RS, and q-ROFRSs.

**Definition 2.1** (Ref.15) Let \( \mathbb{X} \) be a universe. A q-ROFS \( F \) over \( \mathbb{X} \) is define as follows:

\[
F = \{(v, \delta_F(v), \psi_F(v)) | v \in \mathbb{X}\}
\]

for each \( v \in \mathbb{X} \) the functions \( \delta_F : \mathbb{X} \rightarrow [0, 1] \) and \( \psi_F : \mathbb{X} \rightarrow [0, 1] \) denotes the positive and negative membership functions respectively with constraint that \((\psi_F(v))^q + (\delta_F(v))^q \leq 1, (q > 2 \in \mathbb{Z})\).

**Definition 2.2** (Ref.15) Let \( \mathbb{X} \) be the universal set and \( \mathcal{Y} \in q - \text{ROFS}(\mathbb{X} \times \mathbb{X}) \) be an IF relation. Then

1. \( \mathcal{Y} \) is reflexive if \( \delta_{\mathcal{Y}}(\mu, \mu) = 1 \) and \( \delta_{\mathcal{Y}}(\mu, \mu) = 0, \forall \mu \in \mathbb{X} \);
2. \( \mathcal{Y} \) is symmetric if \( \forall (\mu, a) \in \mathbb{X} \times \mathbb{X}, \delta_{\mathcal{Y}}(\mu, a) = \delta_{\mathcal{Y}}(a, \mu) \) and \( \delta_{\mathcal{Y}}(\mu, a) = \delta_{\mathcal{Y}}(a, \mu) \);
3. \( \mathcal{Y} \) is transitive if \( \forall (\mu, b) \in \mathbb{X} \times \mathbb{X}, \delta_{\mathcal{Y}}(\mu, b) \geq \bigvee_{a \in \mathbb{X}} \{ \delta_{\mathcal{Y}}(\mu, a) \wedge \delta_{\mathcal{Y}}(a, b) \} \); and

\[
\delta_{\mathcal{Y}}(\mu, b) = \bigwedge_{a \in \mathbb{X}} \{ \delta_{\mathcal{Y}}(\mu, a) \wedge \delta_{\mathcal{Y}}(a, b) \}.
\]

**Definition 2.3** Let \( \mathbb{X} \) be the universal set. Then any \( \mathcal{Y} \in q - \text{RFS}(\mathbb{X} \times \mathbb{X}) \) is called q-rung relation. The pair \((\mathbb{X}, \mathcal{Y})\) is said to be a q-rung approximation space. Now for any \( \mathcal{B} \subseteq q - \text{RFS}(\mathbb{X}) \), the upper and lower approximations of \( \mathcal{B} \) with respect to q-rung fuzzy approximation space \((\mathbb{X}, \mathcal{Y})\) are two q-RFSs, which are denoted by \( \overline{\mathcal{Y}}(\mathcal{B}) \) and \( \underline{\mathcal{Y}}(\mathcal{B}) \) and is defined as:

\[
\overline{\mathcal{Y}}(\mathcal{B}) = \{ \langle \mu, \delta_{\overline{\mathcal{Y}}(\mathcal{B})}(\mu), \delta_{\overline{\mathcal{Y}}(\mathcal{B})}(\mu) \rangle | \mu \in \mathbb{X} \} ;
\]

\[
\underline{\mathcal{Y}}(\mathcal{B}) = \{ \langle \mu, \delta_{\underline{\mathcal{Y}}(\mathcal{B})}(\mu), \delta_{\underline{\mathcal{Y}}(\mathcal{B})}(\mu) \rangle | \mu \in \mathbb{X} \} ;
\]

where
\[ \theta_{qY}(\mu) = \bigvee_{g \in \mathcal{Y}} [\theta_Y(\mu, g) \bigvee \delta_B(g)]; \]
\[ \delta_{qY}(\mu) = \bigwedge_{g \in \mathcal{Y}} [\delta_Y(\mu, c) \bigwedge \delta_B(g)]; \]
\[ \theta_{qY}(\mu) = \bigvee_{g \in \mathcal{Y}} [\theta_Y(\mu, c) \bigvee \delta_B(g)]; \]
\[ \delta_{qY}(\mu) = \bigwedge_{g \in \mathcal{Y}} [\delta_Y(\mu, c) \bigwedge \delta_B(g)]; \]

such that \(0 \leq ((\theta_{qY}(\mu))^q + (\delta_{qY}(\mu))^q) \leq 1\) and \(0 \leq ((\theta_{qY}(\mu))^q + (\delta_{qY}(\mu))^q) \leq 1\).

As \((\mathcal{Y}(B), \overline{\mathcal{Y}}(B))\) are \(q\)-RFSs, so \(\overline{\mathcal{Y}}(B), \overline{\mathcal{Y}}(B) : q - \text{RFS}(\mathcal{Y}) \rightarrow q - \text{RFS}(\mathcal{Y})\) are upper and lower approximation operators. The pair \((\mathcal{Y}(B), \overline{\mathcal{Y}}(B)) = \{ (\mu, (\theta_{qY}(\mu), \delta_{qY}(\mu), (\theta_{qY}(\mu), \delta_{qY}(\mu))) \} | \mu \in \mathcal{Y} \}\) is known as \(q\)-RFR value. For simplicity \((\mathcal{Y}(B) = \{ (\mu, (\theta_{qY}(\mu), \delta_{qY}(\mu), (\theta_{qY}(\mu), \delta_{qY}(\mu))) \} | \mu \in \mathcal{Y} \}\) is represented as \((\mathcal{Y}(B) = ((\theta_{qY}, \delta_{qY}) , (\overline{\mathcal{Y}}(B)))\) and is known as \(q\)-RFRV.

**Definition 2.4** Let \(\mathcal{X}\) be a nonempty finite set and for any subset \(\mathcal{Y} \in q - ROHFS(\mathcal{X} \times \mathcal{X})\) is said to be a \(q\)-RFR relation. The pair \((\mathcal{X}, \mathcal{Y})\) is said to be \(q\)-ROHFS approximation space. If for any \(\mathcal{Y} \subseteq q - ROHFS(\mathcal{X})\), then the upper and lower approximations of \(\mathcal{Y}\) with respect to \(q\)-ROHFS approximation space \((\mathcal{X}, \mathcal{Y})\) are two \(q\)-ROHFSS, which are denoted by \(\mathcal{Y}(B)\) and \(\overline{\mathcal{Y}}(B)\) and defined as:

\[
\mathcal{Y}(B) = \left\{ (\mu, (\theta_{qY}(\mu), \psi_{\overline{\mathcal{Y}}(\mu)}) | \mu \in \mathcal{Y} \right\} ;
\]
\[
\overline{\mathcal{Y}}(B) = \left\{ (\mu, (\theta_{qY}(\mu), \psi_{\overline{\mathcal{Y}}(\mu)}) | \mu \in \mathcal{Y} \right\} ;
\]

where

\[
\theta_{qY}(\mu) = \bigvee_{k \in \mathcal{X}} \left[ \theta_{\mathcal{Y}}(\mu, k) \bigvee \delta_{\mathcal{Y}}(k) \right] ;
\]
\[
\psi_{\overline{\mathcal{Y}}}(\mu) = \bigwedge_{k \in \mathcal{X}} \left[ \psi_{\mathcal{Y}}(\mu, k) \bigwedge \psi_{\mathcal{Y}}(k) \right] ;
\]
\[
\theta_{\overline{\mathcal{Y}}}(\mu) = \bigvee_{k \in \mathcal{X}} \left[ \theta_{\mathcal{Y}}(\mu, k) \bigvee \delta_{\mathcal{Y}}(k) \right] ;
\]
\[
\psi_{\overline{\mathcal{Y}}}(\mu) = \bigwedge_{k \in \mathcal{X}} \left[ \psi_{\mathcal{Y}}(\mu, k) \bigwedge \psi_{\mathcal{Y}}(k) \right] ;
\]

such that \(0 \leq \left( \max(\theta_{qY}(\mu)) \right)^q + \left( \min(\theta_{qY}(\mu)) \right)^q \leq 1\) and \(0 \leq \left( \max(\theta_{qY}(\mu)) \right)^q + \left( \min(\theta_{qY}(\mu)) \right)^q \leq 1\).

As \((\mathcal{Y}(B), \overline{\mathcal{Y}}(B))\) are \(q\)-RHSFs, so \(\mathcal{Y}(B), \overline{\mathcal{Y}}(B) : q - \text{ROHFS}(\mathcal{X}) \rightarrow q - \text{ROHFS}(\mathcal{X})\) are upper and lower approximation operators. The pair \((\mathcal{Y}(B), \overline{\mathcal{Y}}(B)) = \{ (\mu, (\theta_{qY}(\mu), \delta_{qY}(\mu), (\theta_{qY}(\mu), \delta_{qY}(\mu))) \} | \mu \in \mathcal{Y} \}\) will be called \(q\)-ROHFRSs. For simplicity

\[
\mathcal{Y}(B) = \left\{ (\mu, (\theta_{qY}(\mu), \delta_{qY}(\mu), (\theta_{qY}(\mu), \delta_{qY}(\mu))) | \mu \in \mathcal{Y} \right\} ;
\]

is represented as \((\mathcal{Y}(B) = ((\theta_{qY}, \delta_{qY}) , (\overline{\mathcal{Y}}(B)))\) and is known as \(q\)-ROFR value. We provide the following example to demonstrate the above notion of \(q\)-ROHFRS.

**Example 2.5** (Ref. 35) Suppose \(\mathcal{X} = \{ \mu_1, \mu_2, \mu_3, \mu_4 \}\) be any arbitrary set and \((\mathcal{X}, \mathcal{Y})\) is \(q\)-ROHFS approximation space with \(\mathcal{Y} \in q - \text{ROHFS}(\mathcal{X} \times \mathcal{X})\) be the \(q\)-ROHFR relation as given in Table 1. Now an expert in decision-making presents the ideal normal decision object mathcalR, which is a \(q\)-ROHFS.

and

\[
B = \left\{ (\mu_1, [0,2,0.3,0.4]), (\mu_2, [0.2,0.3,0.7,0.8]), (\mu_3, [0.5,0.7,0.8,0.9]), (\mu_4, [0.6,0.8,0.9,0.2,0.6,0.7]) \right\}.
\]

Afterwards, it follows that
By routine calculations, we get

Further

Similarly,

By routine calculations, we get

Now,
ψ_{2\cup R}(μ_1) = \bigvee_{k \in \mathcal{Y}} [ψ_{2\cup R}(μ, c) \bigvee ψ_{2\cup R}(k)]

= \left\{ \begin{array}{l}
[0.2 \lor 0.5, 0.5 \lor 0.7, 0.7 \lor 0]\lor
[0.7 \lor 0.2, 0.9 \lor 0.3, 0.3 \lor 0.7]\lor
[0.2 \lor 0.1, 0.3 \lor 0.5, 0.5 \lor 0.7]\lor
[0.8 \lor 0.2, 0.0 \lor 0.6, 0.0 \lor 0.7]
\end{array} \right.

= \left\{ \begin{array}{l}
[0.5, 0.7, 0.7] \lor [0.7, 0.9, 0.7]\lor
[0.2, 0.5, 0.7] \lor [0.8, 0.6, 0.7]
\end{array} \right.

= [0.8, 0.9, 0.7].

Continuing in the same way, we find the other values,

ψ_{2\cup R}(μ_2) = [0.7, 0.9], ψ_{2\cup R}(μ_3) = [0.7, 0.9, 0.8], ψ_{2\cup R}(μ_4) = [0.6, 0.9, 0.9].

Thus the lower and upper q-ROHFR approximation operators are

\mathcal{Y}(\mathcal{B}) = \left\{ (μ_1, [0.1, 0.3], [0.8, 0.9, 0.7]), (μ_2, [0.1, 0.3], [0.7, 0.9, 0.9]), (μ_4, [0.2, 0.3], [0.6, 0.9, 0.9]) \right\},

\mathcal{Y}(\mathcal{B}) = \left\{ (μ_1, [0.6, 0.8, 0.9], [0.2]), (μ_2, [0.6, 0.8, 0.9], [0.1]), (μ_4, [0.6, 0.8, 0.9], [0.1, 0.5]) \right\}.

Hence

\mathcal{Y}(\mathcal{B}) = (\mathcal{Y}(\mathcal{B}), \mathcal{Y}(\mathcal{B}))

\left\{ \begin{array}{l}
(μ_1, [0.1, 0.3], [0.8, 0.9, 0.7]), [0.6, 0.8, 0.9], [0.2]),
(μ_2, [0.1, 0.3], [0.7, 0.9, 0.9]), [0.6, 0.8, 0.9], [0.1]),
(μ_4, [0.2, 0.3], [0.6, 0.9, 0.9]), [0.6, 0.8, 0.9], [0.1, 0.5])
\end{array} \right\}.

Definition 2.6 (Ref.28) Let \mathcal{Y}(\mathcal{B}_1) = (\mathcal{Y}(\mathcal{B}_1), \mathcal{Y}(\mathcal{B}_1)) and \mathcal{Y}(\mathcal{B}_2) = (\mathcal{Y}(\mathcal{B}_2), \mathcal{Y}(\mathcal{B}_2)) be two q-ROHFRSs. Then

1. \mathcal{Y}(\mathcal{B}_1) \cup \mathcal{Y}(\mathcal{B}_2) = (\mathcal{Y}(\mathcal{B}_1) \cup \mathcal{Y}(\mathcal{B}_2), \mathcal{Y}(\mathcal{B}_1) \cup \mathcal{Y}(\mathcal{B}_2))
2. \mathcal{Y}(\mathcal{B}_1) \cap \mathcal{Y}(\mathcal{B}_2) = (\mathcal{Y}(\mathcal{B}_1) \cap \mathcal{Y}(\mathcal{B}_2), \mathcal{Y}(\mathcal{B}_1) \cap \mathcal{Y}(\mathcal{B}_2)).

Definition 2.7 Let \mathcal{Y}(\mathcal{B}_1) = (\mathcal{Y}(\mathcal{B}_1), \mathcal{Y}(\mathcal{B}_1)) and \mathcal{Y}(\mathcal{B}_2) = (\mathcal{Y}(\mathcal{B}_2), \mathcal{Y}(\mathcal{B}_2)) be two q-ROHFRSs. Then

1. \mathcal{Y}(\mathcal{B}_1) \oplus \mathcal{Y}(\mathcal{B}_2) = (\mathcal{Y}(\mathcal{B}_1) \oplus \mathcal{Y}(\mathcal{B}_2), \mathcal{Y}(\mathcal{B}_1) \oplus \mathcal{Y}(\mathcal{B}_2))
2. \mathcal{Y}(\mathcal{B}_1) \otimes \mathcal{Y}(\mathcal{B}_2) = (\mathcal{Y}(\mathcal{B}_1) \otimes \mathcal{Y}(\mathcal{B}_2), \mathcal{Y}(\mathcal{B}_1) \otimes \mathcal{Y}(\mathcal{B}_2))
3. \mathcal{Y}(\mathcal{B}_1) \leq \mathcal{Y}(\mathcal{B}_2) = (\mathcal{Y}(\mathcal{B}_1) \leq \mathcal{Y}(\mathcal{B}_2), \mathcal{Y}(\mathcal{B}_1) \leq \mathcal{Y}(\mathcal{B}_2))
4. σ \mathcal{Y}(\mathcal{B}_1) = (σ \mathcal{Y}(\mathcal{B}_1), σ \mathcal{Y}(\mathcal{B}_1)) for σ ≥ 1
5. (\mathcal{Y}(\mathcal{B}_1))^m = (\mathcal{Y}(\mathcal{B}_1))^m, (\mathcal{Y}(\mathcal{B}_1))^m) for σ ≥ 1
6. \mathcal{Y}(\mathcal{B}_1)^c = (\mathcal{Y}(\mathcal{B}_1)^c, \mathcal{Y}(\mathcal{B}_1)^c) where \mathcal{Y}(\mathcal{B}_1)^c and \mathcal{Y}(\mathcal{B}_1)^c represents the complement of q-RFR approximation operators \mathcal{Y}(\mathcal{B}_1) and \mathcal{Y}(\mathcal{B}_1), that is \mathcal{Y}(\mathcal{B}_1)^c = (\mathcal{Y}(\mathcal{B}_1)^c, \mathcal{Y}(\mathcal{B}_1)^c).
7. \mathcal{Y}(\mathcal{B}_1) = \mathcal{Y}(\mathcal{B}_2) if \mathcal{Y}(\mathcal{B}_1) = \mathcal{Y}(\mathcal{B}_2) and \mathcal{Y}(\mathcal{B}_1) = \mathcal{Y}(\mathcal{B}_2).

The score function will be used to compare/rank two or more q-ROHFR values. The q-ROHFRV with higher score value will be considered greater, and the q-ROHFRV values with smaller score will be considered smaller. If the score values are same, we will employ the accuracy function. The q-ROHFRV with higher accuracy will be considered greater, and the q-ROHFRV values with smaller accuracy will be considered smaller.

Definition 2.8 (Ref.26) The score function for q-ROHFRV \mathcal{Y}(\mathcal{B}) = (\mathcal{Y}(\mathcal{B}), \mathcal{Y}(\mathcal{B})) = ((\mathcal{Y}_1, \mathcal{Y}_2), \mathcal{Y}_1, \mathcal{Y}_2)) is given as:

SR(\mathcal{Y}(\mathcal{B})) = \frac{1}{4} \left( 2 + \frac{1}{\mathcal{Y}_1} \sum_{x \in \mathcal{Y}_1} \frac{1}{\mathcal{Y}_2} \sum_{y \in \mathcal{Y}_2} (\mathcal{Y}_1 - \mathcal{Y}_2) + \frac{1}{\mathcal{Y}_1} \sum_{x \in \mathcal{Y}_1} \frac{1}{\mathcal{Y}_2} \sum_{y \in \mathcal{Y}_2} (\mathcal{Y}_1 - \mathcal{Y}_2) \right).

The accuracy function for q-ROHFRV \mathcal{Y}(\mathcal{B}) = (\mathcal{Y}(\mathcal{B}), \mathcal{Y}(\mathcal{B})) = ((\mathcal{Y}_1, \mathcal{Y}_2), \mathcal{Y}_1, \mathcal{Y}_2)) is given as;
\[ \text{ACY}(B) = \frac{1}{4} \left( \frac{1}{M_F} \sum_{\delta_t \in \psi_{\delta_t}(\phi)} (\delta_t) + \frac{1}{M_F} \sum_{\delta_t \in \psi_{\delta_t}(\phi)} (\delta_t) + \frac{1}{N_F} \sum_{\delta_t \in \psi_{\delta_t}(\phi)} (\delta_t) + \frac{1}{N_F} \sum_{\delta_t \in \psi_{\delta_t}(\phi)} (\delta_t) \right), \]

where \( M_F \) and \( N_F \) represent the number of elements in \( \psi_{\delta_t} \) and \( \psi_{\delta_t} \), respectively.

**Definition 2.9** (Ref.\(^{34}\)) Suppose \( Y(B_1) = (Y(B_1), Y(B_1)) \) and \( Y(B_2) = (Y(B_2), Y(B_2)) \) are two \( q \)-ROHFR values. Then

1. If \( \text{SR}(Y(B_1)) > \text{SR}(Y(B_2)) \), then \( Y(B_1) > Y(B_2) \),
2. If \( \text{SR}(Y(B_1)) < \text{SR}(Y(B_2)) \), then \( Y(B_1) < Y(B_2) \),
3. If \( \text{SR}(Y(B_1)) = \text{SR}(Y(B_2)) \), then

(a) If \( \text{ACY}(Y(B_1)) > \text{ACY}(Y(B_2)) \), then \( Y(B_1) > Y(B_2) \),
(b) If \( \text{ACY}(Y(B_1)) < \text{ACY}(Y(B_2)) \), then \( Y(B_1) < Y(B_2) \),
(c) If \( \text{ACY}(Y(B_1)) = \text{ACY}(Y(B_2)) \), then \( Y(B_1) = Y(B_2) \).

**The \( q \)-rung orthopair hesitant fuzzy rough aggregation operators**

In this part, we provide a novel concept of \( q \)-ROHF rough AOPs by incorporating RS and \( q \)-ROHF aggregation operators to get aggregation concepts of \( q \)-ROHFREWG, \( q \)-ROHFREWG, and \( q \)-ROHFREHWG operators. In addition, some of the essential features of the concepts are examined.

**\( q \)-rung orthopair hesitant fuzzy rough Einstein weighted geometric aggregation operators**

This section describes the \( q \)-ROHFREWG aggregation operator and highlights its important features.

**Definition 3.1** Let \( \mathcal{Y}(\varphi_t) = (\mathcal{Y}(\varphi_t), \mathcal{Y}(\varphi_t)) \) \((t = 1, 2, 3, \ldots, n)\) be the collection of \( q \)-ROHFR values. Then \( q \)-ROHFREWG operator is follows as:

\[ q - \text{ROHFREWG}(\mathcal{Y}(\varphi_1), \mathcal{Y}(\varphi_2), \ldots, \mathcal{Y}(\varphi_n)) = \left( \bigoplus_{t=1}^{n} (\mathcal{Y}(\varphi_t))^m \right) \left( \bigoplus_{t=1}^{n} (\mathcal{Y}(\varphi_t))^m \right). \]

where \( \sigma = (\sigma_1, \sigma_2, \ldots, \sigma_n)^T \) is the weight vector such that \( \bigoplus_{t=1}^{n} \sigma_t = 1 \) and \( 0 \leq \sigma_t \leq 1 \).

**Theorem 3.2** Let \( \mathcal{Y}(\varphi_t) = (\mathcal{Y}(\varphi_t), \mathcal{Y}(\varphi_t)) \) \((t = 1, 2, 3, \ldots, n)\) be the collection of \( q \)-ROHFR values with weight vectors \( \sigma = (\sigma_1, \sigma_2, \ldots, \sigma_n)^T \) such that \( \bigoplus_{t=1}^{n} \sigma_t = 1 \) and \( 0 \leq \sigma_t \leq 1 \). Then \( q \)-ROHFREWG operator is described as:

\[ q - \text{ROHFREWG}(\mathcal{Y}(\varphi_1), \mathcal{Y}(\varphi_2), \ldots, \mathcal{Y}(\varphi_n)) = \left( \bigoplus_{t=1}^{n} (\mathcal{Y}(\varphi_t))^m \right) \left( \bigoplus_{t=1}^{n} (\mathcal{Y}(\varphi_t))^m \right), \]

where \( \sigma = (\sigma_1, \sigma_2, \ldots, \sigma_n)^T \) is weight vector such that \( \bigoplus_{t=1}^{n} \sigma_t = 1 \) and \( 0 \leq \sigma_t \leq 1 \).

**Proof** We \( \frac{1}{4} \) induction. If \( n = 2 \), then...
Further, we show that the result holds for \( n = 2 \). Assume it is valid for \( n = k \):

\[
q - \text{ROHFREWG}(\mathcal{Y}(\mathcal{E}_1), \mathcal{Y}(\mathcal{E}_2) \ldots \mathcal{Y}(\mathcal{E}_k)) = \left( k \sum_{t=1}^{k} (\mathcal{Y}(\mathcal{E}_t))^{m_t} \otimes (\mathcal{Y}(\mathcal{E}_{k+1}))^{m_{k+1}} \right)
\]

Further, we show that the result hold for \( n = k + 1 \). Consider

\[
q - \text{ROHFREWG}(\mathcal{Y}(\mathcal{E}_1), \mathcal{Y}(\mathcal{E}_2), \ldots, \mathcal{Y}(\mathcal{E}_k), \mathcal{Y}(\mathcal{E}_{k+1}))
\]

Hence the result hold for \( n = k + 1 \). Therefore, the result is true for all \( n \geq 1 \).

**Theorem 3.3** Let \( \mathcal{Y}(\mathcal{E}_t) = (\mathcal{Y}(\mathcal{E}_t), \mathcal{Y}(\mathcal{E}_t)) \) (\( t = 1, 2, 3, \ldots, n \)) be the collection of \( q \)-ROHF RVs and \( \mathcal{V} = (\sigma_1, \sigma_2, \ldots, \sigma_n) \) is weight vector such that \( \sigma_t \in [0, 1] \) and \( \sum_{t=1}^{n} \sigma_t = 1 \). Then \( q \)-ROHFREWG operator satisfy the following properties:

1. **Idempotency**: If \( \mathcal{Y}(\mathcal{E}_t) = \mathcal{Y}(\mathcal{E}_t) \) for \( t = 1, 2, 3, \ldots, n \) where \( \mathcal{Y}(\mathcal{E}_t) = \left( \mathcal{Y}(\mathcal{E}_t), \mathcal{Y}(\mathcal{E}_t) \right) \). Then
   \[
   q - \text{ROHFREWG}(\mathcal{Y}(\mathcal{E}_1), \mathcal{Y}(\mathcal{E}_2), \ldots, \mathcal{Y}(\mathcal{E}_n)) = \mathcal{Y}(\mathcal{E}_t).
   \]

2. **Boundedness**: Let \( (\mathcal{Y}(\mathcal{E}))_{\min} = \min_{t} \mathcal{Y}(\mathcal{E}_t) \) and \( (\mathcal{Y}(\mathcal{E}))_{\max} = \max_{t} \mathcal{Y}(\mathcal{E}_t) \). Then
   \[
   (\mathcal{Y}(\mathcal{E}))_{\min} \leq q - \text{ROHFREWG}(\mathcal{Y}(\mathcal{E}_1), \mathcal{Y}(\mathcal{E}_2), \ldots, \mathcal{Y}(\mathcal{E}_n)) \leq (\mathcal{Y}(\mathcal{E}))_{\max}.
   \]

3. **Monotonicity**: Suppose \( \mathcal{Y}(\mathcal{E}) = (\mathcal{Y}(\mathcal{E}_1), \mathcal{Y}(\mathcal{E}_2)) \) (\( t = 1, 2, 3, \ldots, n \)) is another collection of \( q \)-ROHF RV values such that \( \mathcal{Y}(\mathcal{E}_t) \leq \mathcal{Y}(\mathcal{E}_t) \) and \( \mathcal{Y}(\mathcal{E}_t) \leq \mathcal{Y}(\mathcal{E}_t) \). Then
   \[
   q - \text{ROHFREWG}(\mathcal{Y}(\mathcal{E}_1), \mathcal{Y}(\mathcal{E}_2), \ldots, \mathcal{Y}(\mathcal{E}_n)) \leq q - \text{ROHFREWG}(\mathcal{Y}(\mathcal{E}_1), \mathcal{Y}(\mathcal{E}_2), \ldots, \mathcal{Y}(\mathcal{E}_n)).
   \]

**Proof**

1. **Idempotency**: As \( \mathcal{Y}(\mathcal{E}_t) = \mathcal{Y}(\mathcal{E}_t) \) (for all \( t = 1, 2, 3, \ldots, n \)) where \( \mathcal{Y}(\mathcal{E}_t) = \left( \mathcal{Y}(\mathcal{E}_t), \mathcal{Y}(\mathcal{E}_t) \right) \). It follows that
\[ q - \text{ROHFREW}(\mathcal{Y}(q_1), \mathcal{Y}(q_2), \ldots, \mathcal{Y}(q_n)) = \left( \bigotimes_{t=1}^{n} \left( \mathcal{Y}(q_t) \right)^{m_t} \bigotimes_{t=1}^{n} \left( \overline{\mathcal{Y}}(q_t) \right)^{m_t} \right) \]

for all \( t, \mathcal{Y}(q_t) = \mathfrak{g}(q_t) = (\mathfrak{g}(q_t), \overline{\mathfrak{g}}(q_t)) = (b_{h(t)}, d_{h(t)}, b_{\overline{h}(t)}, d_{\overline{h}(t)}) \). Therefore,

\[ \mathcal{Y}(q_t) = \mathfrak{g}(q_t) \]

Hence \( q - \text{ROHFREW}(\mathcal{Y}(q_1), \mathcal{Y}(q_2), \ldots, \mathcal{Y}(q_n)) = \mathfrak{g}(q) \).

(2) **Boundedness:** As

\[ \mathcal{Y}(q^-) = \left( \begin{array}{c} \min_t \{ \vartheta_{h(t)} \}, \max_t \{ \vartheta_{h(t)} \} \\ \min_t \{ \overline{\vartheta}_{h(t)} \}, \max_t \{ \overline{\vartheta}_{h(t)} \} \end{array} \right) \]

\[ \mathcal{Y}(q^+) = \left( \begin{array}{c} \max_t \{ \vartheta_{h(t)} \}, \min_t \{ \vartheta_{h(t)} \} \\ \max_t \{ \overline{\vartheta}_{h(t)} \}, \min_t \{ \overline{\vartheta}_{h(t)} \} \end{array} \right) \]

and \( \mathcal{Y}(q_t) = \left( (\vartheta_{h(t)}, \overline{\vartheta}_{h(t)}), (\vartheta_{\overline{h}(t)}, \overline{\vartheta}_{\overline{h}(t)}) \right) \). To prove that

\[ \mathcal{Y}(q^-) \leq q - \text{ROHFREW}(\mathcal{Y}(q_1), \mathcal{Y}(q_2), \ldots, \mathcal{Y}(q_n)) \leq \mathcal{Y}(q^+) \]

Let \( g(x) = \sqrt{\frac{2-x^2}{x^2}}, x \in (0, 1) \), then \( g'(x) = \frac{3}{2} \sqrt{\frac{(2-x^2)^2}{x^4}} < 0 \). So \( g(x) \) is decreasing on \( (0, 1) \). Since

\[ \vartheta_{h(t)} < \vartheta_{h(t)} \leq \vartheta_{h(t)} \]

for all \( t \). Then \( g \left( \vartheta_{h(t)} \right) \leq g \left( \vartheta_{h(t)} \right) \leq g \left( \vartheta_{h(t)} \right) \) i.e.,

\[ \frac{2 - \left( \vartheta_{h(t)} \right)^3}{\left( \vartheta_{h(t)} \right)^3} \leq \frac{2 - \left( \vartheta_{h(t)} \right)^3}{\left( \vartheta_{h(t)} \right)^3} \leq \frac{2 - \left( \vartheta_{h(t)} \right)^3}{\left( \vartheta_{h(t)} \right)^3} \]

and let \( \sigma = (\sigma_1, \sigma_2, \ldots, \sigma_n)^T \) is weight vectors such that \( \sigma \in [0, 1] \) and \( \sum_{t=1}^{n} \sigma_t = 1 \). We have
Similarly, we can show that

\[ f(y) = 3\sqrt{1 - y^3 + y^3} = 3\sqrt{1 - y}, \quad y \in [0, 1]. \]

Then

\[ f'(y) = -\frac{2y}{(1+y^3)^{3/2}} < 0. \]

Thus \( f(y) \) is a decreasing function over \([0, 1]\). Since \( \delta_{h_{\text{max}}} \leq \delta_{h_t} \leq \delta_{h_{\text{min}}} \) for all \( t \). Then \( \frac{\delta_{h_{\text{min}}}}{\delta_{h_t}} \leq \frac{\delta_{h_{\text{max}}}}{\delta_{h_t}} \) \( r = 1, 2, 3, \ldots, n \) i.e.,

\[ \frac{1 - \delta_{h_{\text{min}}}}{1 + \delta_{h_{\text{min}}}} \leq \frac{1 - \delta_{h_t}}{1 + \delta_{h_t}} \]

\[ \leq \frac{1 - \delta_{h_{\text{min}}}}{1 + \delta_{h_{\text{max}}}} \]

and let \( \sigma = (\sigma_1, \sigma_2, \ldots, \sigma_n)^T \) is weight vector such that \( \sigma_t \in [0, 1] \) and \( \odot_{t=1}^n \sigma_t = 1 \), we have
Let \( q - \text{ROHFREOWG} \) operator is determined as:

\[
q - \text{ROHFREOWG}(\mathcal{Y}(q_1), \mathcal{Y}(q_2), \ldots, \mathcal{Y}(q_n)) = \left( \bigoplus_{t=1}^{n} \left( \mathcal{Y}_{\rho}(q_t) \right)^{\sigma_t} \right)^{\frac{1}{m_t}} \bigoplus_{t=1}^{n} \left( \mathcal{Y}_{\rho}(q_t) \right)^{\sigma_t},
\]

where \( \sigma = (\sigma_1, \sigma_2, \ldots, \sigma_n)^T \) is the weights vector such that \( \bigoplus_{t=1}^{n} \sigma_t = 1 \) and \( 0 \leq \sigma_t \leq 1 \).

**Theorem 3.5** Let \( \mathcal{Y}(q_t) = (\mathcal{Y}(q_1), \mathcal{Y}(q_2), \ldots, \mathcal{Y}(q_n)) \) be the collection of \( q \)-ROHFRE values with weights vector \( \sigma = (\sigma_1, \sigma_2, \ldots, \sigma_n)^T \) such that \( \bigoplus_{t=1}^{n} \sigma_t = 1 \) and \( 0 \leq \sigma_t \leq 1 \). Then \( q \)-ROHFREOWG operator is described as:
Let \( q - ROHFR E OwG(\mathcal{Y}(e_1), \mathcal{Y}(e_2), \ldots, \mathcal{Y}(e_n)) \)

\[
= \left\{ \begin{array}{l}
\bigcup_{\theta_{h_1} \in \Phi_{\mathcal{Y}(e_1)}} \left( \frac{2 \bigoplus_{i=1}^{n} (\theta_{h_1})_{q}^{\theta}}{2 \bigoplus_{i=1}^{n} (\theta_{h_1})_{q}^{\theta}} \right) \bigoplus \bigoplus_{i=1}^{n} (\theta_{h_1})_{q}^{\theta}
\end{array} \right.
\]

where \( \sigma = (\sigma_1, \sigma_2, \ldots, \sigma_n)^T \) is the weight vector such that \( \sum_{i=1}^{n} \sigma_i = 1 \) and \( 0 \leq \sigma_i \leq 1 \).

**Proof** The proof is straightforward and is similar to Theorem 3.2.

**Theorem 3.6** Let \( \mathcal{Y}(e_t) = (\mathcal{Y}(q_1), \mathcal{Y}(q_t)) \) \( (t = 1, 2, 3, \ldots, n) \) be the collection of \( q \)-ROHFR values and \( \sigma = (\sigma_1, \sigma_2, \ldots, \sigma_n)^T \) is the weight vector such that \( \sum_{i=1}^{n} \sigma_i = 1 \) and \( 0 \leq \sigma_i \leq 1 \). Then \( q - ROHFR E OwG \) operator satisfy the following properties:

1. **Idempotency:** If \( \mathcal{Y}(e_t) = \mathcal{F}(q_t) \) for \( t = 1, 2, 3, \ldots, n \) where \( \mathcal{F}(q_t) = (\mathcal{F}(q_t), \mathcal{F}(q_t)) \). Then
   \( q - ROHFR E OwG(\mathcal{Y}(e_1), \mathcal{Y}(e_2), \ldots, \mathcal{Y}(e_n)) = \mathcal{F}(q_t) \).

2. **Boundedness:** Let \( (\mathcal{Y}(q)_t)_{\min} = (\min_{t} \mathcal{Y}(q_t), \max_{t} \mathcal{Y}(q_t)) \) and \( (\mathcal{Y}(q)_t)_{\max} = (\max_{t} \mathcal{Y}(q_t), \min_{t} \mathcal{Y}(q_t)) \). Then
   \( (\mathcal{Y}(q)_t)_{\min} \leq q - ROHFR E OwG(\mathcal{Y}(e_1), \mathcal{Y}(e_2), \ldots, \mathcal{Y}(e_n)) \leq (\mathcal{Y}(q)_t)_{\max} \).

3. **Monotonicity:** Suppose \( \mathcal{F}(q_t) = (\mathcal{F}(q_1), \mathcal{F}(q_2), \ldots, \mathcal{F}(q_n)) \) \( (t = 1, 2, \ldots, n) \) is another collection of \( q \)-ROHFR values such that \( \mathcal{F}(q_t) \leq \mathcal{Y}(e_t) \) and \( \mathcal{F}(q_t) \leq \mathcal{Y}(e_t) \). Then
   \( q - ROHFR E OwG(\mathcal{F}(q_1), \mathcal{F}(q_2), \ldots, \mathcal{F}(q_n)) \leq q - ROHFR E OwG(\mathcal{Y}(e_1), \mathcal{Y}(e_2), \ldots, \mathcal{Y}(e_n)) \).

**Proof** The proof is straightforward and is similar to Theorem 3.3.

The \( q \)-runge orthopair hesitant fuzzy rough Einstein hybrid geometric aggregation operator. In this part a \( q \)-ROHFR EWGW aggregation operator is introduce, as well as the essential properties of the suggested operators are addressed.

**Definition 3.7** Let \( \mathcal{Y}(e_t) = (\mathcal{Y}(q_1), \mathcal{Y}(q_t)) \) \( (t = 1, 2, 3, 4, \ldots, n) \) be the collection of \( q \)-ROHFR values and let \( \sigma = (\sigma_1, \sigma_2, \ldots, \sigma_n)^T \) be the weights vector of the given collection of \( q \)-ROHFR values such that \( \sum_{i=1}^{n} \sigma_i = 1 \) and \( 0 \leq \sigma_i \leq 1 \). Let \( (w_1, w_2, \ldots, w_n)^T \) be the associated weights such that \( \sum_{i=1}^{n} w_i = 1 \) and \( 0 \leq w_i \leq 1 \). Then the \( q \)-ROHFR EWGW operator is determined as:

\[
q - ROHFR E OwG(\mathcal{Y}(e_1), \mathcal{Y}(e_2), \ldots, \mathcal{Y}(e_n)) = \left( \bigoplus_{t=1}^{n} (\mathcal{Y}_t(q_t))^{\sigma} \right) ^{\sum_{t=1}^{n} (\mathcal{Y}_t(q_t))^{\sigma}}
\]

where \( \left( \bigoplus_{t=1}^{n} (\mathcal{Y}_t(q_t))^{\sigma}, \sum_{t=1}^{n} (\mathcal{Y}_t(q_t))^{\sigma} \right) \)

**Theorem 3.8** Let \( \mathcal{Y}(e_t) = (\mathcal{Y}(q_1), \mathcal{Y}(q_t)) \) \( (t = 1, 2, 3, 4, \ldots, n) \) be the collection of \( q \)-ROHFR values and let \( \sigma = (\sigma_1, \sigma_2, \ldots, \sigma_n)^T \) be the weights vector such that \( \sum_{i=1}^{n} \sigma_i = 1 \) and \( 0 \leq \sigma_i \leq 1 \). Let \( (w_1, w_2, \ldots, w_n)^T \) be the associated weights of the given collection of \( q \)-ROHFR values such that \( \sum_{i=1}^{n} w_i = 1 \) and \( 0 \leq w_i \leq 1 \). Then the \( q \)-ROHFR EWGW operator is described as:
\[ q - ROHFREHWG(\mathcal{Y}(q_1), \mathcal{Y}(q_2), \ldots, \mathcal{Y}(q_n)) \]

\[ = \left( \bigotimes_{t=1}^{n} \left( \sum_{\rho_t}^{'} \mathcal{Y}(q_1) \right) \right)^{w_t} \left( \bigotimes_{t=1}^{n} \left( \sum_{\rho_t}^{''} \mathcal{Y}(q_1) \right) \right)^{w_t} \]

\[ = \left( \bigcup_{\delta_{q_t} \in \delta_{\mathcal{Y}(q_t)}} \left( \left( \bigotimes_{t=1}^{n} \left( \delta_{\rho_t}^{q_t} \right) \right)^{w_t} \right) \right) \left( \bigcup_{\delta_{q_t} \in \delta_{\mathcal{Y}(q_t)}} \left( \left( \bigotimes_{t=1}^{n} \left( \delta_{\rho_t}^{q_t} \right) \right)^{w_t} \right) \right) \]

**Proof** The proof is straightforward and is similar to Theorem 3.2.

**Theorem 3.9** Let \( \mathcal{Y}(q_t) = (\mathcal{Y}(q_1), \mathcal{Y}(q_2), \ldots, \mathcal{Y}(q_n)) \) \((t = 1, 2, 3, 4, \ldots, n)\) be the collection of \( q \)-ROHFR values and let \((w_1, w_2, \ldots, w_n)^T\) be the associated weights such that \( \bigoplus_{t=1}^{n} w_t = 1 \) and \( 0 \leq w_t \leq 1 \). Let \( \sigma = (\sigma_1, \sigma_2, \ldots, \sigma_n)^T \) be the weight vector of the given collection of \( q \)-ROHFR values such that \( \bigoplus_{t=1}^{n} \sigma_t = 1 \) and \( 0 \leq \sigma_t \leq 1 \). Then \( q \)-ROHFREHWG operator satisfy the following properties:

1. **Idempotency**: If \( \mathcal{Y}(q_t) = \mathcal{F}(q) \) for \( t = 1, 2, 3, \ldots, n \) where \( \mathcal{F}(q) = (\mathcal{F}(q_1), \mathcal{F}(q_2), \ldots, \mathcal{F}(q_n)) \). Then \( q - ROHFREHWG(\mathcal{Y}(q_1), \mathcal{Y}(q_2), \ldots, \mathcal{Y}(q_n)) = \mathcal{F}(q) \).

2. **Boundedness**: Let \( (\mathcal{Y}(q_t))_{\min} = \left( \min_{t} \mathcal{Y}(q_t), \max_{t} \mathcal{Y}(q_t) \right) \) and \( (\mathcal{Y}(q_t))_{\max} = \left( \max_{t} \mathcal{Y}(q_t), \min_{t} \mathcal{Y}(q_t) \right) \). Then \( (\mathcal{Y}(q_t))_{\min} \leq q - ROHFREHWG(\mathcal{Y}(q_1), \mathcal{Y}(q_2), \ldots, \mathcal{Y}(q_n)) \leq (\mathcal{Y}(q_t))_{\max} \).

3. **Monotonicity**: Suppose \( \mathcal{F}(q) = (\mathcal{F}(q_1), \mathcal{F}(q_2), \ldots, \mathcal{F}(q_n)) \) \((t = 1, 2, \ldots, n)\) is another collection of \( q \)-ROHFR values such that \( \mathcal{F}(q_t) \leq \mathcal{Y}(q_t) \) and \( \mathcal{F}(q_t) \leq \mathcal{Y}(q_t) \). Then \( q - ROHFREHWG(\mathcal{F}(q_1), \mathcal{F}(q_2), \ldots, \mathcal{F}(q_n)) \leq q - ROHFREHWG(\mathcal{Y}(q_1), \mathcal{Y}(q_2), \ldots, \mathcal{Y}(q_n)) \).

**Proof** The proof is easy and is similar to the proof of Theorem 3.3.

**The multi-attribute decision making methodology**

In this section, we developed an approach to dealing with uncertainty in MAGDM using \( q \)-ROHFR information. Consider a DM problem with a set \([A_1, A_2, \ldots, A_n]\) of \( n \) alternatives and a set of \( n \) attributes \([\chi_1, \chi_2, \ldots, \chi_n]\) with \((\sigma_1, \sigma_2, \ldots, \sigma_n)^T\) the weights, that is, \( \sigma_t \in [0, 1], \bigoplus_{t=1}^{n} \sigma_t = 1 \). To test the reliability of kth alternative \( A_i \) under the attribute \( \chi_i \), let \([D_1, D_2, \ldots, D_j]\) be a set of decision makers (DMs). The expert evaluation matrix is defined as follows:

\[ M = \left[ \mathcal{Y}(q_t) \right]_{m \times n} \]

\[ = \left[ \begin{array}{cccc}
\mathcal{Y}(q_1), & \mathcal{Y}(q_2), & \ldots, & \mathcal{Y}(q_n) \\
\mathcal{Y}(q_1), & \mathcal{Y}(q_2), & \ldots, & \mathcal{Y}(q_n) \\
\vdots, & \vdots, & \ddots, & \vdots \\
\mathcal{Y}(q_1), & \mathcal{Y}(q_2), & \ldots, & \mathcal{Y}(q_n)
\end{array} \right] \]

where

\[ \mathcal{Y}(q) = \{ \langle \mathcal{Y}(q_t), \mathcal{Y}(q_2), \ldots, \mathcal{Y}(q_n) \rangle \mid \mathcal{Y}(q_t) \in \mathcal{Y}(q) \} \]

and

\[ \mathcal{Y}(q_t) = \{ \langle \mathcal{Y}(q_t), \mathcal{Y}(q_2), \ldots, \mathcal{Y}(q_n) \rangle \mid \mathcal{Y}(q_t) \in \mathcal{Y}(q) \} \]

\[ 0 \leq \left( \max_{\mathcal{Y}(q_t)} (\mathcal{Y}(q_t)) \right)^q + \left( \min_{\mathcal{Y}(q_t)} (\mathcal{Y}(q_t)) \right)^q \leq 1 \]

and

\[ 0 \leq \left( \min_{\mathcal{Y}(q_t)} (\mathcal{Y}(q_t)) \right)^q + \left( \max_{\mathcal{Y}(q_t)} (\mathcal{Y}(q_t)) \right)^q \leq 1, \]

are the \( q \)-ROHFR values. The following are the main steps for MAGDM:
Step-1 Construct the experts evaluation matrices as

\[
(E)\hat{j} = \begin{bmatrix}
(\mathcal{Y}(e_{11}), \mathcal{Y}(e_{11})), (\mathcal{Y}(e_{12}), \mathcal{Y}(e_{12})), \ldots, (\mathcal{Y}(e_{1j}), \mathcal{Y}(e_{1j})) \\
\cdot & \cdot & \cdot \\
(\mathcal{Y}(e_{n1}), \mathcal{Y}(e_{n1})), (\mathcal{Y}(e_{n2}), \mathcal{Y}(e_{n2})), \ldots, (\mathcal{Y}(e_{nj}), \mathcal{Y}(e_{nj}))
\end{bmatrix}
\]

where \( \hat{j} \) shows the number of experts.

Step-2 Explore the expert matrices that were normalised \((N)\hat{j}\), as

\[
(N)\hat{j} = \begin{bmatrix}
(\mathcal{Y}(e_{ij}), \mathcal{Y}(e_{ij})), (\mathcal{Y}(e_{ij}), \mathcal{Y}(e_{ij})), \ldots, (\mathcal{Y}(e_{ij}), \mathcal{Y}(e_{ij}))
\end{bmatrix}
\]

Step-3 Using the suggested aggregation information, compute the \( q \)-ROHFR values for each considered alternative with respect to the given list of criteria/attributes.

Step-4 Determine the ranking of alternatives based on the score function as follows:

\[
SR(\mathcal{Y}(e)) = \frac{1}{4} \left( 2 + \frac{1}{n_T} \sum_{\delta_h \in \partial e \cap \psi(h)} (\theta_h) + \frac{1}{n_T} \sum_{\delta_h \in \partial e \cap \psi(h)} (\gamma_h) \right)
\]

Step-5 All alternative scores must be ranked in descending order. The superior/best alternative will be the one with a higher value.

The application of proposed decision-making approach

To demonstrate the validity of the established operators, we present a numerical MCGDM example that use the suggested aggregations technique in combination with \( q \)-ROHFR information to identify the optimum location for a wind power plant.

Case study (the evaluation of wind power station site selection). Currently, the civilization is facing threat because of several environmental issues caused by the fossil fuel consumption. As a result, several renewable energy power generation projects have gained a prominent development. Renewable energy is the most cost-effective and environmentally safe energy source which is never going to exhaust. Renewable energy generation is a burgeoning field, with more and more renewable energy sources being investigated and it has a bright future. Therefore, nations have made significant investments in renewable energy power generation.

More comprehensive review techniques are needed to choose the appropriate initiatives, so that we can identify their strengths and weaknesses and put forward some new suggestions to accomplish the objectives. The site selection is usually a crucial challenge in dealing with all renewable energy projects for professional and decision-makers because several factors were evaluated while deciding on a location for a large-scale renewable energy installation. The aim is to optimize the location in which the power will produce in more efficient and cost-effective systems and fulfill the demand while maintaining a minimal impact on the environment and society.

Wind energy stations are among the most efficient and environmentally energy sources, making a significant contribution to existing energy supply. It is essential to mention that the installation of a wind power plant requires wind energy potential, distance from the electricity grid, distance to roads and urban areas.

1. Wind energy potential: The most significant aspect is the wind energy potential criteria. The average wind speed in the region is a fundamental requirement for the economic efficiency of wind generators. Winds are highly influenced and reformed by plant, water bodies, spatial patterns, local topography, weather changes, and a variety of other factors.
2. Slope of topography: The slope of a site is a significant economic and transportation factor. Wind energy power plant should ideally be erected on completely flat land. However, if this is not accessible, the slope must be developed, which will take effort and time, raising installation costs.
3. Range from the energy grid: It is intrinsically connected to the efficiency of energy transmission to power grids or transformers, because the shorter the range and the less energy spent, the closer to the energy stations. Additionally, shorter ranges lead to lower network connection costs.
4. Distance from roads and urban areas: Wind farms particularly in the urban and high-consumption regions provide financial benefits. When wind turbines are located near areas with significant energy use, the energy produced by the plantation will require minimum transmission lines to transfer the power, minimising the cost of transmitting the energy to consumers.

The evaluation procedure of a site selection for wind power station. Assume an organization intends to evaluate the selection process of a location for a wind power project. They will appoint a team of specialists to choose the best location for a wind power plant. In this problem, we analyse a case study for
-ROHFR values $q$.

Table 2. Decision making information.

| $X_1$ | $X_2$ |
|-------|-------|
| $A_1$ | $(0.1, 0.2, 0.5), $ $(0.3, 0.4), $ $(0.8), $ $(0.4, 0.6)$ | $(0.5, 0.7), $ $(0.5, 0.6), $ $(0.4, 0.5), $ $(0.7, 0.9)$ |
| $A_2$ | $(0.6, 0.7), $ $(0.7, 0.9), $ $(0.3, 0.5), $ $(0.6)$ | $(0.2, 0.4, 0.5), $ $(0.5), $ $(0.6, 0.7), $ $(0.3)$ |

Table 3. Decision making information.

| $X_3$ | $X_4$ |
|-------|-------|
| $A_1$ | $(0.4), $ $(0.3, 0.7), $ $(0.5), $ $(0.9)$ | $(0.6), $ $(0.7), $ $(0.6, 0.8, 0.9), $ $(0.7, 0.9)$ |
| $A_2$ | $(0.8), $ $(0.4, 0.5, 0.7), $ $(0.2, 0.5), $ $(0.4, 0.5)$ | $(0.8), $ $(0.5), $ $(0.7), $ $(0.1, 0.3, 0.4)$ |

Table 4. Decision making information.

| $X_1$ | $X_2$ |
|-------|-------|
| $A_3$ | $(0.4, 0.5, 0.6), $ $(0.6, 0.7), $ $(0.9), $ $(0.5)$ | $(0.1), $ $(0.5, 0.6), $ $(0.4, 0.6, 0.7), $ $(0.5, 0.7)$ |
| $A_4$ | $(0.4), $ $(0.5, 0.6), $ $(0.3, 0.4), $ $(0.8)$ | $(0.4, 0.5), $ $(0.4), $ $(0.1, 0.2), $ $(0.2, 0.3)$ |

selecting site in which four alternative locations, say, $A_1, A_2, A_3, A_4, A_5$ are evaluated in addressing the problem and we must select the ideal one. Let $\{X_1, X_2, X_3, X_4\}$ be the attributes of each alternative based on the influencing factors determined as follows: wind energy potential $(X_1)$, slope of topography $(X_2)$, distance from the electricity grid $(X_3)$ and distance from roads and urban areas $(X_4)$ of wind power site. Because of the uncertainty, the DMs' selection information is presented as $q$-ROHFR information. The weights vector for criteria is $\alpha = (0.13, 0.27, 0.29, 0.31)$.

To solve the DM problem using the developed methodology for evaluating alternatives, the following calculations are performed:

- **Step-1:** Tables 2, 3, 4 and 5 present professional expert information in the form of $q$-ROHFRS ($q=3$).

- **Step-2** The expert information is of benefit type. Therefore, we need not to normalise the $q$-ROHFR values in this case.

- **Step-3** Only one expert is considered in this problem for the collection of uncertain information. Therefore, we are not required to find the collected information.

- **Step-4** The following information is used to assess the aggregated information for the alternative under the specified set of attributes:

Case-1: Table 6 displays aggregation information using $q$-ROHFREW operator.

Case-2: Aggregation information using Einstein ordered weighted averaging operator is shown in Table 7.

Case-3: Tables 8 and 9 present the aggregation information using the $q$-ROHFRWG operator.
Table 5. Decision making information.

| $X_3$ | $X_4$ |
|-------|-------|
| A1    | (0.3, 0.7, 0.8) |
| A2    | (0.3, 0.6, 0.8) |
| A3    | (0.3, 0.6, 0.7) |
| A4    | (0.3, 0.7, 0.6) |

Table 6. Aggregated information using $q$-ROHFWRG.

| $X_3$ | $X_4$ |
|-------|-------|
| A1    | (0.0018, 0.0025, 0.0036, 0.0051, 0.0090, 0.0128, 0.5370, 0.6272, 0.9732, 0.6466, 0.5424, 0.6310, 0.5689, 0.6502) |
| A2    | (0.0128, 0.0244, 0.0302, 0.0222, 0.0302, 0.0386, 0.5156, 0.5386, 0.9045, 0.5930, 0.6900, 0.6008) |
| A3    | (0.0096, 0.0045, 0.0113, 0.0046, 0.0160, 0.0075, 0.0188, 0.0275, 0.0317, 0.0415, 0.0479, 0.0563, 0.3975, 0.3973, 0.9836, 0.4169, 0.4317, 0.4512) |
| A4    | (0.0006, 0.0022, 0.0013, 0.0045, 0.0008, 0.0030, 0.0017, 0.0060, 0.6258, 0.6728, 0.9843, 0.6298, 0.6763, 0.7409) |

Table 7. Aggregated information using $q$-ROHFWOG.

| $X_3$ | $X_4$ |
|-------|-------|
| A1    | (0.4012, 0.4458, 0.4376, 0.4856, 0.4913, 0.5440, 0.5240, 0.6235, 0.5544, 0.6446, 0.5296, 0.6274, 0.5594, 0.6481, 0.5612, 0.5273, 0.6107, 0.5747, 0.6359, 0.5990, 0.7713, 0.8334, 0.8298, 0.8759, 0.7800, 0.8397, 0.8362, 0.8806) |
| A2    | (0.5183, 0.5456, 0.6159, 0.6464, 0.6508, 0.6822, 0.5623, 0.6992, 0.5799, 0.7097, 0.6372, 0.7457, 0.3634, 0.4259, 0.4721, 0.5500, 0.3899, 0.4461, 0.4941, 0.5748, 0.6543, 0.4812, 0.4596, 0.4860, 0.4671, 0.4926) |
| A3    | (0.6220, 0.3775, 0.2288, 0.3889, 0.2348, 0.3987, 0.6789, 0.6972, 0.7155, 0.7314, 0.6994, 0.7079, 0.7254, 0.7406, 0.6180, 0.6935, 0.7258, 0.6451, 0.7221, 0.7548, 0.3975, 0.5116, 0.4327, 0.5335, 0.5482, 0.6160) |
| A4    | (0.3889, 0.4134, 0.3978, 0.4228, 0.4151, 0.4411, 0.5451, 0.5751, 0.6038, 0.6279, 0.5503, 0.5797, 0.6079, 0.6317, 0.2749, 0.3008, 0.3307, 0.3616, 0.3254, 0.3558, 0.3908, 0.4268, 0.6541, 0.6578, 0.6728, 0.6763, 0.7024, 0.7055) |

The score values of Tables 8 and 9 are presented in Table 10. On the basis of score values ordered the information shown in Tables 11 and 12. Aggregation information using $q$-ROHFWRG are presented in Table 13.

Step-5 Table 14 shows the score values for all alternatives under established aggregation operators.
Step-6 Table 15 illustrates the ranking of the alternatives $A_k (k = 1, 2, \ldots, 4)$. We determined that alternative $A_2$ is the best choice among the others based on the findings of the prior computational technique and so strongly recommend it. The graphical representation of alternatives are depicted in Fig. 1.

Comparison analysis
To demonstrate the characteristics of the developed technique clearly, we shall perform a comparison with TOPSIS approach.
The expert's decision matrix is as follows: ideal solution and furthest from the negative ideal solution. Through the following steps, we will develop an approach for evaluating ideal solutions, which enables policymakers to evaluate ideal positive and negative solutions. TOPSIS is predicated on the idea that the optimal alternative is the one that is closest to the positive ideal solution and furthest from the negative ideal solution. Through the following steps, we will develop an approach for ranking all of the alternatives using improved TOPSIS technique:

Firstly, let \( A = \{A_1, A_2, A_3, \ldots, A_m\} \) be the set of alternatives and \( C = \{x_1, x_2, x_3, \ldots, x_n\} \) be a set of criteria. The expert's decision matrix is as follows:

| \( x_1 \) | \( x_2 \) |
|---|---|
| \( A_1 \) | \( (0.8456, 0.8902, 0.9481), (0.1438, 0.1917) \) | \( (0.8609, 0.9217), (0.3276, 0.3944) \) |
| \( A_2 \) | \( (0.9559, 0.9703), (0.3400, 0.4666) \) | \( (0.7058, 0.8222, 0.8609), (0.3276) \) |
| \( A_3 \) | \( (0.9339, 0.9481, 0.9599), (0.2890, 0.3400) \) | \( (0.5970, (0.3276, 0.3944), (0.8222, 0.8933, 0.9217), (0.32760, 0.4638) \) |
| \( A_4 \) | \( (0.9339, 0.2400, 0.2890), (0.9159, 0.9339) \) | \( (0.8222, 0.8609), (0.2618) \) |

Table 8. Weighted information (EWG).

| \( x_3 \) | \( x_4 \) |
|---|---|
| \( A_1 \) | \( (0.8087), (0.2008, 0.4745), (0.8503), (0.6472) \) | \( (0.8811), (0.4797) \) |
| \( A_2 \) | \( (0.9440), (0.2679, 0.3352, 0.4745), (0.6846, 0.8503), (0.2679, 0.3352) \) | \( (0.9421), (0.3389) \) |
| \( A_3 \) | \( (0.7563), (0.4745, 0.5518), (0.91580, 0.9440), (0.0669, 0.2679, 0.4745) \) | \( (0.74780, 0.8811), (0.5578) \) |
| \( A_4 \) | \( (0.7563), (0.4745, 0.5518), (0.9158, 0.4036) \) | \( (0.8811, 0.9128, 0.9705), (0.2031, 0.2708) \) |

Table 9. Weighted information (EWG).

| \( x_5 \) | \( x_6 \) | \( x_7 \) | \( x_8 \) |
|---|---|---|---|
| \( A_1 \) | 0.8667 | 0.7014 | 0.6685 | 0.6915 |
| \( A_2 \) | 0.8012 | 0.7884 | 0.7627 | 0.8339 |
| \( A_3 \) | 0.8456 | 0.6798 | 0.7258 | 0.7416 |
| \( A_4 \) | 0.7996 | 0.7669 | 0.6888 | 0.7285 |

Table 10. Score value of weighted (EWG) matrix.

The TOPSIS approach utilizing q-ROHFR information. Hwang and Yoon introduced the TOPSIS approach for evaluating ideal solutions, which enables policymakers to evaluate ideal positive and negative solutions. TOPSIS is predicated on the idea that the optimal alternative is the one that is closest to the positive ideal solution and furthest from the negative ideal solution. Through the following steps, we will develop an approach for ranking all of the alternatives using improved TOPSIS technique:

Firstly, let \( A = \{A_1, A_2, A_3, \ldots, A_m\} \) be the set of alternatives and \( C = \{x_1, x_2, x_3, \ldots, x_n\} \) be a set of criteria. The expert's decision matrix is as follows:
|   | $X_1$                                      | $X_2$                                      |
|---|-------------------------------------------|-------------------------------------------|
| $A_1$ | $\begin{pmatrix} 0.8456, 0.8902, 0.9481, \\ 0.4350, 0.9179, 0.8290, \\ 0.3840, 0.9172, 0.8290 \end{pmatrix}$ |
| $A_2$ | $\begin{pmatrix} 0.9421, 0.9389, \\ 0.9627, 0.9203, 0.8290 \end{pmatrix}$ |
| $A_3$ | $\begin{pmatrix} 0.8139, 0.9481, 0.9599, \\ 0.4350, 0.9481, 0.2890 \end{pmatrix}$ |
| $A_4$ | $\begin{pmatrix} 0.9159, 0.9339, \\ 0.3960 \end{pmatrix}$ |

Table 11. Ordered weighted information (EWG).

|   | $X_3$                                      | $X_4$                                      |
|---|-------------------------------------------|-------------------------------------------|
| $A_1$ | $\begin{pmatrix} 0.8881, 0.4797, \\ 0.8811, 0.9421, 0.9705, \\ 0.4797, 0.6359 \end{pmatrix}$ |
| $A_2$ | $\begin{pmatrix} 0.70580, 0.8220, 0.8609, \\ 0.3276, 0.8637, 0.8989, \\ 0.1963 \end{pmatrix}$ |
| $A_3$ | $\begin{pmatrix} 0.7563, 0.4745, 0.5518, \\ 0.0669, 0.2679, 0.4745 \end{pmatrix}$ |
| $A_4$ | $\begin{pmatrix} 0.8811, 0.9128, 0.9705, \\ 0.2031, 0.2708, \\ 0.6740, 0.9128, 0.4797, 0.5578, 0.6359 \end{pmatrix}$ |

Table 12. Ordered weighted information (EWG).

|   | $X_5$                                      |
|---|-------------------------------------------|
| $A_1$ | $\begin{pmatrix} 0.8487, 0.8655, 0.8546, 0.8713, 0.8624, 0.8791, \\ 0.3544, 0.4220, 0.3722, 0.4348, 0.3558, 0.4229, 0.3735, 0.4357, 0.3558, 0.4229, 0.3735, 0.4357 \end{pmatrix}$ |
| $A_2$ | $\begin{pmatrix} 0.8814, 0.9140, 0.9247, 0.8844, 0.9170, 0.9277, \\ 0.3169, 0.3348, 0.3894, 0.3652, 0.3789, 0.4234 \end{pmatrix}$ |
| $A_3$ | $\begin{pmatrix} 0.8292, 0.8803, 0.8905, 0.8334, 0.8894, 0.8951, \\ 0.2450, 0.2734, 0.2507, 0.2780, 0.2584, 0.2843 \end{pmatrix}$ |
| $A_4$ | $\begin{pmatrix} 0.7261, 0.7620, 0.7280, 0.7640, 0.7297, 0.7675, \\ 0.4518, 0.6466, 0.4793, 0.4907, 0.4550, 0.4676, 0.4824, 0.4934 \end{pmatrix}$ |

Table 13. Aggregated information using $q$-ROHFREHWG.
where

\[ M = \left[ \Phi(\mu_{ij}^f), \Phi(\mu_{ij}^h) \right]_{m \times n} \]

\[ = \left[ \left( \Phi(e_{11}), \Phi(e_{12}) \right), \left( \Phi(e_{21}), \Phi(e_{22}) \right), \ldots, \left( \Phi(e_{ij}), \Phi(e_{ij}) \right) \right] \]

\[ \vdots \]

\[ = \left[ \left( \Phi(e_{11}), \Phi(e_{12}) \right), \left( \Phi(e_{21}), \Phi(e_{22}) \right), \ldots, \left( \Phi(e_{ij}), \Phi(e_{ij}) \right) \right] \]

such that

\[ \Phi(e_{ij}) = \left\{ \left( \mu, \delta_{\Phi_{e_{ij}}}(\mu), \psi_{\Phi_{e_{ij}}}(\mu) \right) \mid \mu \in \mathcal{E} \right\} \]

and

\[ \Phi(e) = \left\{ \left( \mu, \delta_{\Phi_{e_{ij}}}(\mu), \psi_{\Phi_{e_{ij}}}(\mu) \right) \mid \mu \in \mathcal{E} \right\} \]
Likewise, the geometric distance between all possible alternatives and the negative ideal

\[ 0 \leq \left( \max(\psi_{h_{Y_{10}}}(\mu)) \right)^q + \left( \min(\psi_{h_{Y_{10}}}(\mu)) \right)^q \leq 1 \]

and

\[ 0 \leq \left( \min(\psi_{h_{Y_{10}}}(\mu)) \right)^q + \left( \max(\psi_{h_{Y_{10}}}(\mu)) \right)^q \leq 1 \]

are the \(q\)-ROHF rough values. Secondly, we collect information from DMs in the form of \(q\)-ROHFRNs.

Thirdly, normalise the data defined by DMs, since the decision matrix may include both benefits and cost

to

Fourthly, assess the normalised matrices of experts \( \hat{\psi} \),

\[
(H)^{j} = \begin{bmatrix}
(\varphi(\varepsilon_{11}), \varphi(\varepsilon_{11})) & (\varphi(\varepsilon_{12}), \varphi(\varepsilon_{12})) & \cdots & (\varphi(\varepsilon_{1i}), \varphi(\varepsilon_{1i})) \\
(\varphi(\varepsilon_{21}), \varphi(\varepsilon_{21})) & (\varphi(\varepsilon_{22}), \varphi(\varepsilon_{22})) & \cdots & (\varphi(\varepsilon_{2i}), \varphi(\varepsilon_{2i})) \\
(\varphi(\varepsilon_{31}), \varphi(\varepsilon_{31})) & (\varphi(\varepsilon_{32}), \varphi(\varepsilon_{32})) & \cdots & (\varphi(\varepsilon_{3i}), \varphi(\varepsilon_{3i})) \\
\vdots & \vdots & \ddots & \vdots \\
(\varphi(\varepsilon_{n1}), \varphi(\varepsilon_{n1})) & (\varphi(\varepsilon_{n2}), \varphi(\varepsilon_{n2})) & \cdots & (\varphi(\varepsilon_{ni}), \varphi(\varepsilon_{ni})) 
\end{bmatrix}
\]

where \( j \) identifies the number of experts.

Fourthly, assess the normalised matrices of experts \( (N)^{j} \), as

\[
(N)^{j} = \begin{cases} 
\varphi(\varepsilon_{ij}) = (\varphi(\varepsilon_{ij}))^c & \text{if For benefit} \\
(\varphi(\varepsilon_{ij}))^c = (\varphi(\varepsilon_{ij}))^c & \text{if For cost}
\end{cases}
\]

Fifthly, determine the positive ideal solution and the negative ideal solution based on the score value. Positive ideal solutions and negative ideal solutions are represented as: \( \Upsilon^+ = (\Gamma_1^+, \Gamma_2^+, \Gamma_3^+, \ldots, \Gamma_n^+) \) and \( \Upsilon^- = (\Gamma_1^-, \Gamma_2^-, \Gamma_3^-, \ldots, \Gamma_n^-) \) respectively. For positive ideal solution \( \Upsilon^+ \), it can be calculated as follows:

\[
\Upsilon^+ = (\Gamma_1^+, \Gamma_2^+, \Gamma_3^+, \ldots, \Gamma_n^+) = \left( \max_{i} \text{score}(\Gamma_{i1}), \max_{i} \text{score}(\Gamma_{i2}), \max_{i} \text{score}(\Gamma_{i3}), \ldots, \max_{i} \text{score}(\Gamma_{in}) \right)
\]

Similarly, the following formula may be used to find the negative ideal solution:

\[
\Upsilon^- = (\Gamma_1^-, \Gamma_2^-, \Gamma_3^-, \ldots, \Gamma_n^-) = \left( \min_{i} \text{score}(\Gamma_{i1}), \min_{i} \text{score}(\Gamma_{i2}), \min_{i} \text{score}(\Gamma_{i3}), \ldots, \min_{i} \text{score}(\Gamma_{in}) \right)
\]

Afterwards, determine the geometric distance between all possible options and the positive ideal \( \Upsilon^+ \) as follows:

\[
d(\alpha_{ij}, \Upsilon^+) = \frac{1}{8} \left\{ \frac{1}{r^h} \sum_{i=1}^{\text{the}} \left( \frac{1}{2} \left( \frac{\mu_{ij(h)}}{\mu_{ij(h)}^+} \right)^2 - \left( \frac{\mu_{ij(h)}}{\mu_{ij(h)}^+} \right)^2 \right) \right\},
\]

where \( i = 1, 2, 3, \ldots, n \), and \( j = 1, 2, 3, \ldots, m \).

Likewise, the geometric distance between all possible alternatives and the negative ideal \( \Upsilon^- \) can be found as follows:

\[
d(\alpha_{ij}, \Upsilon^-) = \frac{1}{8} \left\{ \frac{1}{r^h} \sum_{i=1}^{\text{the}} \left( \frac{1}{2} \left( \frac{\mu_{ij(h)}}{\mu_{ij(h)}^-} \right)^2 - \left( \frac{\mu_{ij(h)}}{\mu_{ij(h)}^-} \right)^2 \right) \right\},
\]

where \( i = 1, 2, 3, \ldots, n \), and \( j = 1, 2, 3, \ldots, m \).

Sixthly, the following formula is used to determine the relative closeness indices for all decision makers of the alternatives:
Finally, determine the ranking order of alternatives, and then choose the most desirable alternative that is the smallest distance.

**Numerical example.** Through a numerical example, this section will describe the features and validity of the suggested approach for selecting a wind power plant location.

**Step-1** Tables 2, 3, 4 and 5 contain information regarding decision makers in the form of $q$-ROHFRNs.

**Step-2** Table 16 computes both positive and negative ideal solutions as follows:

**Step-3** Determine the distance between the positive and negative ideal solutions.

| Criteria | $T^+$ | $T^-$ |
|----------|-------|-------|
| $X_1$    | $(0.4, 0.5, 0.6, 0.0, 0.1, 0.2, 0.3, 0.4)$ | $(0.5, 0.6, 0.7, 0.8, 0.9)$ |
| $X_2$    | $(0.2, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9)$ | $(0.5, 0.6, 0.7, 0.8, 0.9)$ |
| $X_3$    | $(0.8, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9)$ | $(0.4, 0.5, 0.6, 0.7, 0.8, 0.9)$ |
| $X_4$    | $(0.6, 0.7, 0.8, 0.9, 0.0, 0.1, 0.2)$ | $(0.7, 0.8, 0.9, 0.0, 0.1, 0.2)$ |

Table 16. Ideal solutions.

\[
RC(\alpha_{ij}) = \frac{d(\alpha_{ij}, T^+)}{d(\alpha_{ij}, T^-) + d(\alpha_{ij}, T^+)}
\]

Finally, determine the ranking order of alternatives, and then choose the most desirable alternative that is the smallest distance.

**Numerical example.** Through a numerical example, this section will describe the features and validity of the suggested approach for selecting a wind power plant location.

**Step-1** Tables 2, 3, 4 and 5 contain information regarding decision makers in the form of $q$-ROHFRNs.

**Step-2** Table 16 computes both positive and negative ideal solutions as follows:

| Criteria | $T^+$ | $T^-$ |
|----------|-------|-------|
| $X_1$    | $(0.4, 0.5, 0.6, 0.0, 0.1, 0.2, 0.3, 0.4)$ | $(0.5, 0.6, 0.7, 0.8, 0.9)$ |
| $X_2$    | $(0.2, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9)$ | $(0.5, 0.6, 0.7, 0.8, 0.9)$ |
| $X_3$    | $(0.8, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9)$ | $(0.4, 0.5, 0.6, 0.7, 0.8, 0.9)$ |
| $X_4$    | $(0.6, 0.7, 0.8, 0.9, 0.0, 0.1, 0.2)$ | $(0.7, 0.8, 0.9, 0.0, 0.1, 0.2)$ |

**Step-3** Determine the distance between the positive and negative ideal solutions.

\[
0.6019
\]
\[
0.2140
\]
\[
0.3849
\]
\[
0.5427
\]

and

\[
0.3265
\]
\[
0.5313
\]
\[
0.5035
\]
\[
0.4266
\]

**Step-4** The following are the relative closeness indices for DMs of the alternatives:

| Criteria | $A_1$ | $A_2$ | $A_3$ | $A_4$ |
|----------|-------|-------|-------|-------|
| $X_1$    | 0.6483 | 0.2871 | 0.4333 | 0.5599 |

**Step-5** According to the aforementioned findings and Fig. 2, $A_2$ has the shortest distance. As a consequence, $A_3$ is the most appropriate option. By synthesizing the above concepts, we can conclude that proposed solution based on $q$-ROHFRSs is effective and reasonable for handling MCDM problems.

**Conclusion**

Choosing the best site for establishing the projects is a crucial stage in wind energy power stations. There are several aspects to consider while deciding the best location for the plants to be installed, which is a significant stage in wind energy projects. Therefore, a novel approach based on $q$-ROHFRS is suggested for assessment in order to overcome the restrictions and support the researcher in selecting an appropriate site for installing a wind power station. The knowledge of the concepts presented in this study provide a broad space for analyzing information, enabling decision-makers to incorporate the features of uncertain data and having a high computing capabilities for uncertain information. A list of geometric aggregation operators is presented based on the proposed approach, employing Einstein’s t-norm and t-conorm. The aforementioned methodology can handle the complication of the MADM approach based on the $q$-ROHFRS, and the evaluation information is very
reasonable. Furthermore, a case study in the evaluation of wind power plant site selection schemes together with comparative analysis using the improved q-ROHFR-TOPSIS approach demonstrates the approach’s validity and reasonability. In the future, the established approach can be extended to other fuzzy and uncertain situations such as language and probability sets to broaden the space for representation of analysing information, adapt to a wider range of evaluation environments, and improve the method’s flexibility. Additionally, within the context of three-way notions, it is worthwhile to investigate consensus procedures based on q-ROHFRS.

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All authors contributed equally to the manuscript.

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