Solving Wave Flow Energy Propagated by Twin-Hull Ships (Catamarans)

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Abstract. The main objective of this paper is to put forward a method of solution of wave flow energy propagated by twin-hull ships utilizing ship wave resistance integral equation by Tuck-Lazauskas [8]. The underlying principle of the governing equation is based on the linear thin ship theory pioneered by Michell. The ship wave resistance integral equation by Tuck-Lazauskas is improvised in which the upper limit of the mathematical integration of the governing equation is set to the final root instead of $\pi/2$ as in the original equation to eliminate the problem of solving such a divergent improper integral. The method of solution and the theoretical results are validated against works of others experimentally and theoretically. The results obtained by the proposed method are found to be very consistent and more closely fit to the experimental results in comparison with those obtained by other theoretical method as presented in this paper.

1. Introduction

Twin-hull ships or catamarans are more advantageous than mono-hull ships of equal displacement for their higher operational speed, significantly larger deck area, more stable platform, shallower draft, less susceptible to rolling, less wave resistance to a certain extent of Froude numbers due to their hull slenderness and invariably less total resistance. Nevertheless, the disadvantages of catamarans over mono-hull ships amongst others are more vulnerable to pitch and heave responses, structurally more complex and complicated waves flow between the hulls. Continuous studies had been made on predicting ship wave resistance of multi-hill ships for more accurate solutions since the past centuries until today and this reflects for its importance.

Many fundamentals and modern methods evolved through these periods of times. The search for better methods and more accurate solutions has driven for deeper studies and motivates to bring together the fundamentals of the past and current works and the relevant theories. The objective of the study is to search for an effective and reliable method of solution of the wave resistance or wave energy utilizing the wave resistance integral by Tuck and Lazauskas (1998) based on the underlying linear thin ship theory by Michell (1898) and the subsequent related works by Wigley (1926 - 1942) and Newman (1977). The ship wave resistance in this study is in fact equivalent to the wave energy propagated or generated by the catamaran while underway or in motion cutting through the water.
2. Governing Equation
The study made to a specific reference to the governing equation of the ship wave resistance of multi-hull ships by Tuck and Lazauskas suited for twin-hull configuration. This equation is connected to the Michell-Wigley’s and Newman’s wave amplitude functions of the demi hull ships. The waves system in a global coordinates \((x,y,z)\) and origin local coordinates \((x_{j0}, y_{j0}, z_{j0})\) of a multi-hull ships is expressed as;

\[
\zeta(x,y) = \text{Re} \int A_j(\theta) \exp\left[(-ig/U^2)\cos^2\theta(x_{j0}\cos\theta + y_{j0}\sin\theta)\right] \exp\left[(-ig/U^2)\cos^2\theta(x\cos\theta + y\sin\theta)\right] d\theta;
\]

\(-\pi/2 < \theta < \pi/2\) (1)

The above expression is the real part of the complex equation defining the waves profile or system generated by a multi-hull ship with \(j = 1, 2 \ldots n\) and suited for a twin-hull ship or catamaran with \(j = 1, 2\). Where \(A_j(\theta)\) is the wave amplitude function for \(j\) numbers of ship hull, \(U\) is the speed, \(g\) is the gravitational acceleration, \(x_{j0}\) and \(y_{j0}\) are the axial and transverse positions referred to the origin of the local coordinate system for \(j\) numbers of ship hull. Further explanation on the above expression are given below.

The wave amplitude function is generalized as;

\[
A(\theta) = \sum A_j(\theta) \cos\left[(-g/U^2)\cos\theta(x_{j0}\cos\theta + y_{j0}\sin\theta)\right]; \quad j = 1, 2 \ldots n
\]

(2)

Taking the real part;

\[
A(\theta) = \sum A_j(\theta) \cos\left[-g/U^2\cos\theta(x_{j0}\cos\theta + y_{j0}\sin\theta)\right]; \quad j = 1, 2 \ldots n
\]

(3)

The centre plane wave amplitude function would be for \(y = 0\);

\[
A(\theta)_{CP} = \sum A_j(\theta) \cos\left[-g/U^2\cos\theta x\right]; \quad j = 1, 2 \ldots n
\]

(4)

The wave resistance of a twin-hull ship \(R_w\) is much dependent on the square of the speed \(U^2\), square of the wave amplitude function \(|A(\theta)|^2\), the transverse hull interference factor \(HIF_T(\theta)\) and the cube of the cosine of the wave propagation angle \(\theta\) (i.e. \(\cos^3\theta\)) associated with the waves system. The mathematical expression of the ship wave resistance for a twin-hull ship due to Tuck-Lazauskas can be derived from the ship wave resistance expression of a multi-hull ship for \(j = 1\) to \(2\), non-staggered hull configuration with \(x = 0\) and wave amplitude function of the demi hull by Newman may finally be written in the following forms;

\[
R_w = \left(\pi \rho U^2/2\right) \int |A(\theta)_{CP}|^2 HIF_T(\theta) \cos^2\theta d\theta; \quad -\pi/2 < \theta < \pi/2
\]

(5)

As for the demi hull wave amplitude function due to Michell-Wigley, the mathematical expression of the ship wave resistance for a twin-hull ship due to Tuck-Lazauskas is of the form;

\[
R_w = \left(4\rho g/\pi U^2\right) \int (I^2 + J^2) HIF_T(\theta) \sec^2\theta d\theta; \quad 0 \leq \theta \leq \pi/2
\]

(6)

\[
J = bd \int \left[\frac{\partial I}{\partial \zeta^2}\right] \exp(-dg\sec^2\theta/U^2) \sin(tg\zeta\sec\theta/U^2) \delta \zeta^2\delta \zeta; \quad -1 \leq \zeta \leq +1, \quad 0 \leq \xi \leq 1
\]

(7)

\[
I = bd \int \left[\frac{\partial J}{\partial \zeta^2}\right] \exp(-dg\sec^2\theta/U^2) \cos(tg\zeta\sec\theta/U^2) \delta \zeta^2\delta \zeta; \quad -1 \leq \zeta \leq +1, \quad 0 \leq \xi \leq 1
\]

(8)
The transverse hull interference factor $HIF_T(\theta)$ is expressed as:

$$HIF_T(\theta) = 4 \cos^2 \left[ 0.5 \left( \frac{2p}{L} \right) \sin \theta (F_n \cos^2 \theta) \right]$$

With reference to Michell-Wigley for wave amplitude function of a thin ship is given as:

$$\int \int (\partial \eta / \partial x) \exp \left[ \upsilon \sec^2 \theta (z + ix \cos \theta) \right] dz dx$$

Where:

Subscript $CP$ means the central plane, $\upsilon = g/U^2$, $\zeta = z/d$, $\xi = x/\ell$, $\ell = L/2$, $\eta_x = \partial y / \partial x$, $L$ – Length of ship, $d$ – Draft of ship

3. Problem Formulation

Hypothetically the magnitude of wave resistance or the wave energy propagated by a moving ship in water is also dependent on a precise maximum resultant propagation angle of the combined divergent and transverse waves. In this study the ship wave resistance equation by Tuck and Lazauskas is improvised to eliminate the problem of solving such a divergent improper integral that the denominator tends to zero (i.e. improper integral of type 2). The ship wave resistance is solved by first determining the final root of solution of the ship wave resistance integral which sets the upper limit of the mathematical integration. This unique angle is termed as the maximum wave propagation angle abbreviated by $\theta_{max}$ replaces the maximum limit of integration $\pi/2$ of the original expressions. $\theta_{max}$ dictates the absolute solution of the ship wave resistance problem and the method is herein called Final Root Method. The theoretical results are validated utilizing a twin-hull ship model of National Physical Laboratory (NPL) round bilge hull form.

Mathematically, the integral of the ship wave resistance $R_w$ contains infinite roots of solution. This complex integral is not easily differentiable directly to determine the roots of solution. Nonetheless, practically one may solve for the root iteratively from the wave amplitude function or the ship wave integral for zero values. The study zoomed on the final root of minima which exists in the range of $\theta$ approaching the value of $\pi/2$. The so-called precise maximum resultant wave propagation angle $\theta_{max}$ would be the final root of the ship wave resistance integral function and that the values of the integral $I$, $J$ or $R_w$ are equal to zero on the x-axis, beyond this point there should distinctively no other root exists.

The improvised ship wave resistance integral or equation for a twin-hull ship is hence written in the form as shown below in which the upper limit of the mathematical integration is reassigned with the precise final root of solution $\theta_{max}$;

$$R_w = (\pi p U^2) \int [A(\theta)_{CP}]^2 HIF_T(\theta) \cos^2 \theta d\theta; \ 0 < \theta < \theta_{max}$$

$$A(\theta)_{CP} = (2 \upsilon \pi \sec^2 \theta) \int (\partial \eta / \partial x) \exp \left[ \upsilon \sec^2 \theta (z + ix \cos \theta) \right] dz dx$$

$$= 2 \upsilon \pi \sec^2 \theta \int \eta \exp \left[ (g \zeta / U^2) \sec^2 \theta \cos \left( (g \xi / U) \sec \theta \right) \right] dz dx; \ 0 < z < d, -\ell < x < +\ell$$

4. Computational Technique

In implementing the calculation technique the model of principal particulars as given below are divided into frames or stations of equal interval or spacing longitudinally along the length of the ship and divided into waterlines of equal intervals vertically down from the free surface to the keel bottom. The mathematical calculation technique makes use of Simpson’s Rules of Integration as appropriate.

The integration performed in strip-wise sequence, first in the $\eta$-$\zeta$-direction at each cross-sectional plane of the hull followed by the integration in the $\xi$-direction along the length of the ship hull to compute the integral 1 with the origin at amidships. The coordinate systems for the twin-hull models are shown in Figures 1 and 2. The integration is then continued in the angular $\theta$-direction from zero up to the limiting resultant propagation angle i.e. $\theta_{max}$ to finally determine $R_w$. The integration in the angular
θ-direction is done at every interval and finer intervals near the final wave propagation angle $\theta_{\text{max}}$. The values of integrals $I$ and $J$ are read from calculation of separate subroutines for the demi hull. The effect of transverse hull interference factor $HIF_T(\theta)$ or the interacted wave amplitude function is necessary to be calculated following the given expression.

5. Theoretical Analysis
The theoretical analysis was performed according to the improvised ship wave resistance equation described in Paragraph 3. The values of integrals $I$ and $J$ for the selected interval of angular spacing $\theta$ are first calculated separately. Next are to calculate the transverse hull interference factor $HIF_T(\theta)$ for each interval of $\theta$ and the ship wave resistance $R_w$ for a chosen speed. The calculation of $R_w$ is then necessary to be iterated until the final root of solution $\theta_{\text{max}}$ is found. The calculations for $R_w$ are then continued for different speeds and varying ratios of transverse hull spacing to the ship length $2p/L$. Several validations of method and results are also made against works done by M. Insel, A.F. Molland and J.F. Wellicome [13] using an NPL hull form model with the model principal particulars are give in Table 1. The coordinate system is given Figure 1 and Figure 2 below.

![Figure 1. Twin-Hull Configuration and the Coordinate System (Cross-sectional View)](image1)

![Figure 2. Twin-Hull Configuration and the Coordinate System (Plan View)](image2)
6. The Results

The validation for the twin-hull ships are carried out utilizing the NPL Hull for the ratio of transverse hull spacing to the ship length of 2p/L = 0.3 and 0.5 and for Froude numbers, F_n between 0.3 to 1.0. The coefficients of ship wave resistance C_w were evaluated (C_w = R_w/0.5ρSV^2, R_w – ship wave resistance, ρ – water density, S – hull wetted surface area, V – ship speed), for the range of the Froude numbers. The calculation results are shown in Tables 2 and 3 and plotted as shown in Figures 2, 3, 4 and 5.

Table 1 - Model Principal Particulars (NPL Hull):

| Particulars                  | Value         |
|------------------------------|---------------|
| Length Between Perpendiculars, L<sub>BP</sub> | 1.600 m       |
| Length Waterline, L<sub>WL</sub>          | 1.600 m       |
| Draft, T                     | 0.089 m       |
| Beam Waterline, B<sub>WL</sub>       | 0.178 m       |
| Wetted Surface Area, S at draft T | 0.338 m^2     |

Table 2. Theoretical and Experimental Coefficient of Ship Wave Resistance C_w, 2p/L = 0.3

| V (m/s) | F_n  | C<sub>W</sub> x 10<sup>3</sup> (Experimental) | C<sub>W</sub> x 10<sup>3</sup> (Theoretical) | Newman (final root) |
|--------|------|---------------------------------|---------------------------------|-------------------|
| 1.19   | 0.30 | 2.200                           | 3.000                           | 2.132             |
| 1.39   | 0.35 | 3.000                           | 1.000                           | 2.909             |
| 1.59   | 0.40 | 4.000                           | 3.000                           | 3.875             |
| 1.82   | 0.46 | 6.600                           | 5.900                           | 6.398             |
| 1.98   | 0.50 | 6.000                           | 5.600                           | 5.812             |
| 2.38   | 0.60 | 4.000                           | 3.600                           | 3.873             |
| 2.77   | 0.70 | 2.700                           | 2.600                           | 2.616             |
| 3.17   | 0.80 | 1.800                           | 1.700                           | 1.744             |
| 3.57   | 0.90 | 1.500                           | 1.400                           | 1.453             |
| 3.96   | 1.00 | 1.400                           | 1.300                           | 1.356             |

Table 3. Theoretical and Experimental Coefficient of Ship Wave Resistance C_w, 2p/L = 0.5

| V (m/s) | F_n  | C<sub>W</sub> x 10<sup>3</sup> (Experimental) | C<sub>W</sub> x 10<sup>3</sup> (Theoretical) | Michell-Wigley (final root) |
|--------|------|---------------------------------|---------------------------------|--------------------------|
| 1.19   | 0.30 | 2.200                           | 2.400                           | 2.132                    |
| 1.39   | 0.35 | 2.400                           | 1.200                           | 2.325                    |
| 1.58   | 0.40 | 3.800                           | 2.400                           | 3.875                    |
| 1.82   | 0.46 | 5.200                           | 4.300                           | 5.038                    |
| 1.98   | 0.50 | 4.800                           | 4.200                           | 4.651                    |
| 2.38   | 0.60 | 3.400                           | 3.200                           | 3.294                    |
| 2.77   | 0.70 | 2.700                           | 2.600                           | 2.617                    |
| 3.17   | 0.80 | 2.100                           | 2.000                           | 2.035                    |
| 3.57   | 0.90 | 1.700                           | 1.600                           | 1.647                    |
| 3.96   | 1.00 | 1.500                           | 1.400                           | 1.453                    |
Figure 2. Graphs of Experimental and Theoretical (Newman – Final Root) 
$C_w$ Versus $F_n$, $2p/L = 0.3$

Figure 3. Graphs of Experimental and Theoretical (Newman - Final Root) 
$C_w$ Versus $F_n$, $2p/L = 0.5$
7. Conclusion
The results calculated theoretically in comparison with those experimentally indicated that the method of solution employed in this study is indeed useful, consistent and worthwhile. The accuracy of the calculated results are well within 3% to 5%. Obviously, this new method of solution could provide an option as alternative method for solving such a divergent improper integral equation of ship wave resistance visa-vis the flows of wave energy propagated by a moving twin-hull ship (catamaran). The
method could also be used for the study of hull optimization and hydrodynamics performances of twin-hull ships particularly in terms of the wave resistance component during the ship design activities with the ultimate objective is for predicting the powering requirement and the selection of the appropriate propulsion plant.

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