Computational model of coil placement in cerebral aneurysm with using realistic coil properties

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Abstract
This study presents a computational mechanical model of coil placement in cerebral aneurysms with using realistic coil properties. The mechanical behavior of the coil is expressed by solving a variational problem including the kinetic and elastic energies of the coil and geometric constraints due to the multiple contacts. The coil is discretized as a set of beam elements obeying Timoshenko beam theory, in which these elastic properties are determined based on the actual shape and material property of the coil. Numerical examples of the coil placement are presented for four cases with different coil properties in terms of the elastic property (hard and soft) and reference configurations (helical loop and complex loop) within the realistic range. These results exhibit the effects of the coil properties on not only the transition of the coil configuration during the placement process but also the contact force of the coil against the aneurysm wall which has a fatal risk of bleeding in the operation. The proposed computational model will assist surgeons to select the appropriate coil and coil deployment protocol to achieve the sufficient treatment effectiveness with minimizing the risk of bleeding in the view of the mechanical sense.

Key words: Computational simulation, Beam element, Coil placement, Cerebral aneurysm

1. Introduction

The cerebral aneurysm is a cerebrovascular disease, in which a balloon-like structure is locally formed on cerebral arteries. Rupture of the aneurysm is a main cause of subarachnoid hemorrhages and often poses a serious risk of death. It is commonly considered that growth and rupture of aneurysms are induced by an abnormal hemodynamic stimulus, and then several surgical methods have been proposed to occlude the aneurysm. A coil embolization is one of the popular methods in clinical practice to achieve the aneurysm occlusion. In this procedure, single helical coils consisting of a platinum wire are manually placed inside the aneurysm by a micro-catheter to fill in the aneurysm. This coil placement induces blood flow stagnation, which enhances blood clot formation, and finally the aneurysm occlusion is accomplished.

In clinical practice, the treatment effectiveness by the coil embolization is not sustained by the inadequate conditions, such as a low packing density (PD) (Sluzewski et al., 2004; Tamatan et al., 2002), which is the volume ratio of the coils to the aneurysm, and inhomogeneous coil distribution (Raymond et al., 2003; Roy et al., 2001). To aim for the high PD and homogeneous distribution, various kinds of coils have been developed by changing two factors (Khan et al., 2012; Mehra et al., 2011; White et al., 2008). One is the reference configuration of the coil, which is normally set to the helical loop or spherical complex loop structures with various kinds of loop size. The other is the elastic property of the coil, which is determined by the coil shape (e.g., pitch, diameter of coil and wire constituting the coil) (Den Hartog, 1952; Wahl, 1963). The experimental observations exhibited that hard coils are more stable after placement in the aneurysm (Marks et al., 1996; Schloesser et al., 2007), whereas it is likely that the hard coils strongly
press the aneurysm wall, which may cause the bleeding during the operation.

In a framework of the placement of multiple coils, the appropriate selection of the first coil is the most important part of the treatment procedure because of the difficulty of the placement into aneurysms comparing to that of subsequent coils (Cloft et al., 2000). The appropriate coil is selected based on aneurysm morphology leading to create a stable flame like a basket-like structure, for delivering subsequent coils inside to achieve the aneurysm occlusion (Eboli et al., 2014). However, the aneurysm morphology includes individual difference (normally described by the existence of blebs and several shape indices) (Raghavan et al., 2005). Therefore, most of coils with spherical loop structure may be undersized or oversized comparing to the aneurysm size, and then this mismatch leads to increase the aneurysm wall stress and induce the failure of creating the stable flame (Osanai et al., 2013). In the clinical practice, the appropriate coil is empirically selected based on the experience of surgeons and manually placed inside the aneurysm through a trial and error process. The evidence-based guideline to determine the most appropriate coil would be a powerful tool to support surgeons and reduce the risk of the aneurysm recurrence after coil embolization.

The numerical experiment of the coil placement inside the aneurysm is expected to contribute the consideration of the appropriate coil for each patient. Several computational-mechanical simulations associated with the coil placement have proposed (Babiker et al., 2013; Dequidt et al., 2009; Dequidt et al., 2008; Stoop et al., 2011; Vetter et al., 2014; Vetter et al., 2013), aiming for the clinical support simulator (Dequidt et al., 2009; Dequidt et al., 2008) or the construction of the realistic coil configuration used for the blood flow analysis in the coil-embolized aneurysm (Babiker et al., 2013). However, to our knowledge, the coil kinematics was normally expressed to be that of a wire consisting of a normal linear elastic material, and then the realistic elastic property of the coil was not fully represented. The mechanical simulation with realistic elastic property of the coils enables not only to represent more accurate coil configuration, but also to deduce the force of the coil against the aneurysm wall and assist the coil placement to reduce the risk of bleeding.

In this study, we present a computational model of the coil placement in the cerebral aneurysm with realistic coil properties. The mechanical behavior of the coil is represented by solving the variational problem including the kinetic and elastic energies of the coil and geometric constraints originated from the multiple contacts between coil-aneurysm, coil-catheter and coil-coil. Elastic property of the coil (i.e., tensile, shear and torsion) is assigned based on the realistic shape and material property of the coil. The effectiveness of the present computational model is illustrated through numerical examples of the coil placement simulation by changing the elastic property and reference configuration of the coil.

2. Computational methods

In this section, a methodology of the computational simulation of the aneurysm coil placement is presented. In section 2.1, the variational problem including the kinetic, elastic and contact energies is formulated to express the coil dynamics. The kinetic description of the coil is shown in section 2.2 and the treatment of the multiple contacts is explained in section 2.3. The discretization of the stationary conditions is introduced in section 2.4 and the time integration scheme is shown in section 2.5. Finally the determination of the material parameters to represent the realistic coil property is introduced in section 2.6.

2.1 Stationary conditions

The computational simulation of the coil placement is conducted by solving the variational problem based on the minimum energy principle with satisfying the geometric constraints due to the multiple contacts between coil-aneurysm, coil-catheter and coil-coil. The geometrical constraints by the contact between two materials can be expressed by the Signorini (KKT) condition given by

\[ g\lambda = 0, \]

where \( g \) is the minimum distance between two materials and \( \lambda \) is the magnitude of the contact force being understood as the Lagrange multiplier (Wriggers, 2006). Thus the functional of the system, \( \mathcal{I} \) is defined by

\[ \mathcal{I} = W + g\lambda, \]

where \( W \) is the mechanical energy including both the kinetic and elastic energies. The stationary conditions of eq. (2)
are given by
\[
\frac{\partial \Pi}{\partial \mathbf{u}} + \frac{\partial W}{\partial \mathbf{u}} + g \frac{\partial g}{\partial \mathbf{u}} = 0,
\]
\[
\frac{\partial \Pi}{\partial g} = g = 0,
\]
where \( \mathbf{u} \) is the displacement vector. The principle of virtual work and total Lagrange formulation are used to solve the derivative of the \( W \) with respect to the \( \mathbf{u} \) in eq. (3) such that
\[
\frac{\partial W}{\partial \mathbf{u}} \cdot \delta \mathbf{u} = \rho \int_{\Omega} \mathbf{u} \cdot \delta \mathbf{u} dV + \int_{\Omega} \mathbf{S} : \delta \mathbf{E} dV,
\]
where \( \rho \) is the density of the coil, \( V \) is the volume of the coil at the reference state, \( \mathbf{S} \) is the second Piola-Kirchhoff stress tensor, \( \delta \mathbf{u} \) is the virtual displacement vector and \( \delta \mathbf{E} \) is the virtual variation of the Green-Lagrange strain tensor. The first term in right-hand side of (5) represents the inertial forces based on d’Alembert’s principle.

2. 2 Kinematics of the coil
2. 2. 1 Description of the coil configuration
The Galerkin finite element method is used to solve the coil kinematics expressed in eq. (5). The coil is discretized by a set of degenerated beam elements (e.g., Bathe, 1996; Dvorkin et al., 1988) based on Timoshenko beam theory, in which the deformation in the cross-section of the beam is assumed to be negligible. Schematic description of the beam element is shown in Fig. 1. We assume the coil shape as the tube with the outer radius of \( D \) and thickness of \( d \), which corresponds to the diameter of the coil wire for the numerical simplicity. We adopt the second order beam element consisting of three nodes. Local coordinate system \((r_1, r_2, r_3)\) is defined in each element: \( r_1 \) axis is set along the centroid; \( r_2 \) and \( r_3 \) axis are set in the cross-section of the beam element. Shape functions of the second-order beam element, \( N_1, N_2, N_3 \) are given by
\[
N_1 = \frac{1}{2} r_1 (1 - r_1), \quad N_2 = \frac{1}{2} r_1 (1 + r_1), \quad N_3 = (1 - r_1^2),
\]
s.t. \(-1 \leq r_1 \leq 1\).

The orthonormal basis \((\mathbf{V}_1, \mathbf{V}_2, \mathbf{V}_3)\) is assigned to each node corresponding to the local coordinate axes at the reference state. Hence, arbitrary position vector in the beam element at the reference state, \( \mathbf{X} \) is given by
\[
\mathbf{X}(r_1, r_2, r_3) = N^p(r_1)\left[ \mathbf{X}^p + \frac{D}{2} r_2 \mathbf{V}_2^p + \frac{D}{2} r_3 \mathbf{V}_3^p \right],
\]
s.t., \(\left( \frac{D - 2d}{D} \right)^2 \leq r_2^2 + r_3^2 \leq 1\),
where \( p \) (\( p = 1-3 \)) is a dummy index of summation. \( \mathbf{X}^p \) and \( \mathbf{V}_i^p \) are the position vector and orthogonal basis assigned the node \( p \), respectively. Suppose the rotation of the orthogonal basis from \( t \) to \( t + \Delta t \) is enough small, the \({}^{t+\Delta t}\mathbf{V}\) can be given by
\[
{}^{t+\Delta t}\mathbf{V} \equiv \mathbf{V} + \mathbf{\Theta} \times \mathbf{V} = \mathbf{V} + \begin{pmatrix} 0 & -\theta_2 & \theta_1 \\ \theta_2 & 0 & -\theta_1 \\ -\theta_1 & \theta_2 & 0 \end{pmatrix} \mathbf{V} = \mathbf{V} + \begin{pmatrix} 0 & V_3 & -V_2 \\ -V_3 & 0 & V_1 \\ V_2 & -V_1 & 0 \end{pmatrix} \mathbf{0} = \mathbf{V} + \mathbf{R} \mathbf{0}
\]
where \( \mathbf{\Theta} \) (\( \theta_1, \theta_2, \theta_3 \)) is the axial vector and \( \mathbf{R} \) is the skew-symmetric tensor consisted of the components of \( \mathbf{V}(V_1, V_2, V_3)\). According to eq. (7) and (8), incremental displacement vector of the beam element \( {}^{t+\Delta t}\mathbf{u} \) from time \( t \) to \( t + \Delta t \) is calculated by
where $U^p$ is the incremental displacement vector and $\Theta^p$ is the incremental rotation vector of node $p$. $R_2$ and $R_3$ are skew-symmetric tensors consisted of the components of $V_2$ and $V_3$, respectively. For detailed explanation about this beam element, one can find in (Bathe, 1996; Dvorkin et al., 1988; Hisada and Noguchi, 1995).

2.2.2 Constitutive law

In this study, we assume the coil deformation can be represented by the finite deformation but small strain elasticity theory. According to the Bathe (1996), the linear elastic constitutive law is given by

$$S = C : E,$$  \hspace{1cm} (10)

where $S$ is the second Piola-Kirchhoff stress tensor, $C$ is fourth-order constitutive tensor and $E$ is the Green-Lagrange strain tensor such that

$$E = \frac{1}{2} \left( \frac{\partial u}{\partial X} + \frac{\partial u}{\partial X}^T + \left( \frac{\partial u}{\partial X} \right)^T \frac{\partial u}{\partial X} \right).$$  \hspace{1cm} (11)

The elastic property of the beam elements is independently determined with respect to each axis of the polar coordinate system (i.e., longitudinal, radial and circumferential directions), which is assigned in each beam element. The unit basis vectors for the polar coordinate system are given by

$$\mathbf{e}_1 = N^p V_1^p,$$ \hspace{1cm} (12)

$$\mathbf{e}_2 = N^p V_2^p \cos \theta + N^p V_3^p \sin \theta,$$ \hspace{1cm} (13)

$$\mathbf{e}_3 = -N^p V_2^p \sin \theta + N^p V_3^p \cos \theta,$$ \hspace{1cm} (14)

where $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$ are the unit basis vectors of the longitudinal, radial and circumferential directions, respectively. $\theta$ is the angle from the basis $V_2$ in the cross-section. According to Timoshenko beam theory, the relationship between the second Piola-Kirchhoff stress tensor $\mathbf{S}$ and Green-Lagrange strain tensor $\mathbf{E}$ can be simply defined by

$$\begin{pmatrix} \bar{S}_{11} \\ \bar{S}_{12} \\ \bar{S}_{13} \end{pmatrix} = \begin{pmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{pmatrix} \begin{pmatrix} \bar{E}_{11} \\ 2\bar{E}_{12} \\ 2\bar{E}_{13} \end{pmatrix},$$  \hspace{1cm} (15)

where $k_1$, $k_2$ and $k_3$ are the elastic coefficients of the longitudinal, radial and circumferential directions. Thus the components of the constitutive tensor in the polar coordinate system, $C_{ijkl}$ is given by

$$\bar{C}_{i111} = k_1.$$  \hspace{1cm} (16)
\[
\begin{align*}
\bar{C}_{1212} &= \bar{C}_{2121} = \bar{C}_{1212} = k_2, \\
\bar{C}_{1313} &= \bar{C}_{3131} = \bar{C}_{1313} = k_3.
\end{align*}
\]

Other components of the \( \bar{C} \) are set to be zero. The coordinate transform of the constitutive tensor from the polar to the global (Cartesian) coordinates is given by
\[
C_{ijkl} = C_{xyz} (\hat{e}_i \cdot \hat{e}_x) (\hat{e}_j \cdot \hat{e}_y) (\hat{e}_k \cdot \hat{e}_z),
\]
where \( \hat{e}_i \) \((i = 1, 2, 3)\) is the normal basis vector of the global coordinate system.

### 2.3 Treatment of the contact

#### 2.3.1 Contact search

The magnitude of the contact force \( \lambda \) is treated by the Lagrange multiplier method, e.g., (Wriggers 2006) and the occurrence of the contact is detected when the minimum distance between two objects \( g \) is lower than zero. The contact search is done following two continuous phases: (I) global search to detect the candidate space where the contact occurs and (II) local search to calculate \( g \) (Wriggers, 2006) and judge the contact occurrence. In the phase (I), we adopt the grid cell algorithm, e.g., (Munjiza and Andrews, 1998), and in the phase (II), the contact detection algorithm (Munjiza and Andrews, 1998; Wriggers, 2006) is used with the little modification to fit our specific situation. In the following sections, we briefly explain the process of phase (II), regarding to the contact between coil-plane (surface of the aneurysm or catheter) and coil-coil.

#### 2.3.1.1 Minimum distance between coil and plane

In order to calculate the minimum distance between the coil and the surface of the aneurysm or the catheter \( g_a \), we consider calculating the minimum distance between the node of the coil and closest surface of the aneurysm or the catheter (Fig. 2(a)). This tangential plane is discretized by a set of first-order triangle elements. The closest point on the triangle element from the node of the coil, \( x_a \), is described by
\[
x_a = N^p_a x^p_a ,
\]
where \( x^p_a \) \((p = 1 \sim 3)\) is the position vector of the nodes constituting the triangle element and \( N^p_a \) is the shape function of the triangle element and identified by the parameters \( s_a \) and \( t_a \) given by
\[
N^1_a = 1 - s_a - t_a , \quad N^2_a = s_a , \quad N^3_a = t_a ,
\]
s.t. \( 0 \leq s_a , t_a , s_a + t_a \leq 1 \).

Here, the wall of the aneurysm and catheter is assumed to be rigid, i.e., \( x^p_a \) is fixed even if any interaction force is exerted on the plane from the node of the coil. Finally, \( s_a \) and \( t_a \) are given by solving a minimization problem for \( g_a \) such that
\[
g_a = \min \| x_a - x_{ca} \| - \frac{D}{2} , \quad \text{w.r.t} \quad s_a \text{ and } t_a ,
\]
where \( x_a \) is the current position vector of the node of the coil. The contact between the coil and the plane are detected when \( g_a \leq 0 \).

#### 2.3.1.2 Minimum distance between adjacent coils

The minimum distance between two coils (beam elements), \( g_c \), is calculated by the method of (Wriggers, 2006). To reduce the computational cost, each second-order beam element introduced in section 2.2 is separated into two first-order beam elements in this section. Schematic description of the coil (beam)-coil (beam) contact is shown in Fig. 2 (b). Suppose the contact between two first-order beam elements \( c1 \) and \( c2 \) occurs, the contact point on each beam element, \( x_{c1} \) and \( x_{c2} \), is defined by
\[
\begin{align*}
x_{c1} &= N^p_{c1} x^p_{c1} , \quad x_{c2} = N^p_{c2} x^p_{c2} ,
\end{align*}
\]
where \( x^p_{c1} \) and \( x^p_{c2} \) \((p = 1, 2)\) is the position vector of each node of the beam element. \( N^p_{c1} \) and \( N^p_{c2} \) are shape
functions of each beam element and identified by the parameter $s_{c1}$ and $s_{c2}$ given by

$$N_{c1}^i = \frac{1}{2}(1 - s_{c1}), \quad N_{c2}^i = \frac{1}{2}(1 + s_{c1}),$$

$$N_{c1}^e = \frac{1}{2}(1 - s_{c2}), \quad N_{c2}^e = \frac{1}{2}(1 + s_{c2}),$$

s.t. $-1 \leq s_{c1}, s_{c2} \leq 1$.

Taking into account of the outer diameter of the contacted coils, $D$, $s_{c1}$ and $s_{c2}$ are given by solving a minimization problem for $g_c$ such that

$$g_c = \min \left| \left| \mathbf{x}_{c2} - \mathbf{x}_{c1} \right| \right| - D, \text{ w.r.t } s_{c1} \text{ and } s_{c2}. \quad (26)$$

Fig. 2 Schematic drawing of the contact between the vertical point of the coil and triangle element (a) and two beam elements (b). Red arrows show the basis vectors for local coordinates defined in each element.

2.3.2 Friction force on the contact plane

Friction force is treated by the penalty method to satisfy coulomb friction law. The magnitudes of the maximum static friction force $T_{\text{stick}}$ and kinetic friction force $T_{\text{slip}}$ are represented by the absolute value of the contact force $\lambda$ such that

$$T_{\text{stick}} = \kappa_1 |\lambda|, \quad T_{\text{slip}} = \kappa_2 |\lambda|,$$ \quad (27)

where $\kappa_1$ and $\kappa_2$ are the static and kinetic friction constant, respectively. In this study, we use same parameter to treat the friction force for both the coil-aneurysm and coil-coil contacts.

The admittance-type friction model proposed by (Kikuuwe et al., 2006) is adopted to avoid numerical oscillation due to the friction force. A brief review of this model is provided as follows. Suppose a mass point moves on the frictional plane, equation of motion can be given by

$$m \frac{d\mathbf{U}_{\text{tan}}}{dt} = \mathbf{T},$$ \quad (28)

where $\mathbf{T}$ is the friction force vector and $\mathbf{U}_{\text{tan}}$ is the tangential velocity vector. Using backward Euler scheme, eq. (28) is discretized by

$$m \frac{\mathbf{U}_{\text{tan}}^{i+\Delta t} - \mathbf{U}_{\text{tan}}^i}{\Delta t} = \frac{\mathbf{T}}{\Delta t},$$ \quad (29)

When the $\mathbf{U}_{\text{tan}}^{i+\Delta t} = 0$ (static friction), the friction force vector is given by

$$\frac{\mathbf{T}}{\Delta t} = -m \frac{\mathbf{U}_{\text{tan}}^i}{\Delta t}.$$ \quad (30)

Hence, the generalized friction force vector $\frac{\mathbf{T}}{\Delta t}$ is given by
where \( \mathbf{u} \) is the velocity vector of the node, \( \mathbf{I} \) is the identity matrix, and \( \mathbf{n} \) is the unit normal vector from the plane to the node perpendicularly. Thereby, the friction force on the coil-plane contact is calculated by eqs. (30) and (31) with the tangential velocity (32).

### 2.3.2.2 Friction of the coil-coil contact

Each friction force of contacted beam elements is rewritten from eq. (30) by

\[
\begin{align*}
\frac{m_1}{\Delta t} \left( \mathbf{U}_{\text{tan},1}^{n} - \mathbf{U}_{\text{tan},1}^{n-1} \right) &= \mathbf{T}_{\text{slip}}^{n}, \\
\frac{m_2}{\Delta t} \left( \mathbf{U}_{\text{tan},2}^{n} - \mathbf{U}_{\text{tan},2}^{n-1} \right) &= \mathbf{T}_{\text{slip}}^{n},
\end{align*}
\]

(33)

where \( m_1 \) and \( m_2 \) is the mass of each beam element. These values are easily obtained by the linear interpolation using the mass of each node as well as eq. (23). The mass assigned each node is calculated in the next section. \( \mathbf{U}_{\text{tan},1} \) and \( \mathbf{U}_{\text{tan},2} \) is the tangential velocity vector of each beam element, which corresponds to the velocity vector along the direction of each beam element calculated by that of each node as well as (23). In the case of the coil-coil contacts, the friction force works to reduce the relative tangential velocity vector of the coils. Suppose the relative velocity \( \mathbf{U}_{\text{tan},1}^{n} - \mathbf{U}_{\text{tan},2}^{n} \) is zero, eq. (33) can be also rewritten by

\[
\frac{m_1 m_2}{m_1 + m_2} \left( \mathbf{U}_{\text{tan},1}^{n} - \mathbf{U}_{\text{tan},2}^{n} \right) = \frac{\Delta t}{m_1 + m_2} \mathbf{T}_{\text{slip}}^{n}.
\]

(34)

Using eq. (31) and (34), the friction forces between two coils (beam elements) are derived.

### 2.4 Finite element discretization

In the finite element discretization, numerical integration obeying the Gauss-Legendre quadrature is implemented to calculate the volume integral of the eq. (3). Reduced integration technique is adopted to calculate the second term of eq. (3) in the longitudinal direction to avoid the shear locking phenomena. Taking into account of the damping and friction effects represented in the form of external forces, linear algebraic equation derived from eq. (3)-(5) are described by

\[
\left( \begin{array}{cc}
\mathbf{M}_{uu} & \mathbf{M}_{u\theta} \\
\mathbf{M}_{\theta u} & \mathbf{M}_{\theta\theta}
\end{array} \right) \left( \begin{array}{c}
\dot{\mathbf{U}}_u \\
\dot{\mathbf{U}}_\theta
\end{array} \right) + \left( \begin{array}{cc}
\mathbf{C}_{uu} & \mathbf{C}_{u\theta} \\
\mathbf{C}_{\theta u} & \mathbf{C}_{\theta\theta}
\end{array} \right) \left( \begin{array}{c}
\mathbf{U}_u \\
\mathbf{U}_\theta
\end{array} \right) + \left( \begin{array}{cc}
\mathbf{Q}_u \\
\mathbf{Q}_\theta
\end{array} \right) + \left( \begin{array}{c}
\lambda \mathbf{G} \\
0
\end{array} \right) + \left( \begin{array}{c}
\mathbf{T}
\end{array} \right) = 0,
\]

(35)

\[
\mathbf{G} \mathbf{x} = \mathbf{r},
\]

(36)

where \( \mathbf{M}_{uu}, \mathbf{M}_{u\theta}, \mathbf{M}_{\theta u}, \mathbf{M}_{\theta\theta} \) are the mass matrices, \( \mathbf{C}_{uu}, \mathbf{C}_{u\theta}, \mathbf{C}_{\theta u}, \mathbf{C}_{\theta\theta} \) are the damping matrices treated as Rayleigh damping given by the product of mass matrices and artificial viscosity coefficient \( \omega \), \( \mathbf{Q}_u \) and \( \mathbf{Q}_\theta \) are the equivalent nodal force vectors, \( \mathbf{G} \) is the contact boundary condition matrix introduced by the partial derivative of \( g \) with respect to \( \mathbf{u} \) (Wriggers, 2006), \( \mathbf{x} \) is the position vector of all nodes of the coil and \( \mathbf{r} \) indicates the distance between the contact nodes in each contact point, and then the component of \( \mathbf{r} \).
\[ r^x = \begin{cases} \frac{D}{2} g_u & \leq 0 \\ D g_e & \leq 0 \end{cases} . \] (37)

2.5 Time integration

Time integration is treated by the method used in (Carpenter, 1991), based on the multi-step central differential scheme. For numerical simplicity, \( M_{u\theta} \) and \( M_{\theta \theta} \) are treated as lumped matrixes and \( M_{u} \) and \( M_{\theta} \) are ignored to avoid solving the huge linear simultaneous equation. Eq. (35) is separated into 4 steps described by

\[ M_{uu} \frac{t^{1/2} U - 2 \dot{U} + t^{1/2} U}{\Delta t^2} + C_{uu} t^{1/2} \dot{U} + \dot{Q}_u = 0 , \] (38)

\[ M_{u\theta} \frac{t^{1/2} \theta - 2 \dot{\theta} + t^{1/2} \theta}{\Delta t^2} + C_{u\theta} t^{1/2} \dot{\theta} + \dot{Q}_\theta = 0 , \] (39)

\[ M_{u} \frac{t^{1/2} U - \dot{U}}{\Delta t^2} + \dot{\lambda} G = 0 , \] (40)

\[ M_{\theta} \frac{t^{1/2} \theta - \dot{\theta}}{\Delta t^2} + \lambda \dot{G} T = 0 , \] (41)

where \( \Delta t \) is time increment, \( t^x \) and \( t^u \) are the temporal time. When the time discretization form of the eq. (36) is given by

\[ G^r x = \dot{r} , \] (42)

where the \( \lambda \) is explicitly calculated by the eq. (40) and (42) such that

\[ \lambda = \left( \Delta t^2 GM_{uu} G^u \right)^{-1} \left( G^r x - \dot{r} \right) . \] (43)

The velocity and angular velocity vector are given by

\[ t^{1/2} \dot{U} = \frac{t^{1/2} U - \dot{U}}{\Delta t} , \] (44)

\[ t^{1/2} \dot{\theta} = \frac{t^{1/2} \theta - \dot{\theta}}{\Delta t} . \] (45)

2.6 Elastic properties of the coil

The elastic coefficients of the coil, \( k_1, k_2 \) and \( k_3 \) defined in eq. (15) are determined based on the material property and shape of the coil. It is common that the equivalent spring constant of the coil spring \( k_s \) and the equivalent torsional spring constant \( k_\theta \) are given by

\[ k_s = \frac{G_s d^4}{8 n D^3} , \] (46)

\[ k_\theta = \frac{E_s d^4}{64 n D} , \] (47)

where \( E_s \) and \( G_s \) are the Young’s modulus and the shear modulus of the wire constituting the coil, respectively. The \( n \) is the number of turns of the coil. As well, when considering the tubular beam (Fig. 1), the equivalent spring constant and torsional spring constant are also commonly known by

\[ k = \frac{E A}{l} , \] (48)
\[ GJ_k l = \frac{\theta}{I}, \]  

(49)

where \( l \) is the beam length, \( E \) and \( G \) are the Young’s modulus and the shear modulus of the tubular beam, respectively. \( A \) and \( J \) are the cross-sectional area and polar moment of inertia given by

\[ A = \pi \left( Dd - d^2 \right), \]  

(50)

\[ J = \frac{\pi \left( D^4 - (D-2d)^4 \right)}{32}. \]  

(51)

With the assumption of the close-coiled helical spring \((l=nd)\), the equivalent Young’s modulus \( E \) and shear modulus \( G \) are determined to satisfy \( k_1 = \bar{k}_1 \) and \( k_\theta = \bar{k}_\theta \), respectively.

The elastic coefficient of the longitudinal resistance, \( k_1 \) is set to the equivalent Young’s modulus \( E \). The elastic coefficient of the shear resistance, \( k_2 \) is set to be equal to \( k_1 \) since the strength of the shear resistance is close to that of the longitudinal resistance when the longitudinal strain is enough small (Wahl, 1963). The elastic parameter of the torsional resistance, \( k_3 \) is set to the equivalent shear modulus \( G \).

The physical parameters of the coil placement simulation are shown in Table 1. Density of the coil is set corresponding to the density of Pt/W (8%) reported in the PGM Database (http://www.pgmdatabase.com/) hosted by Johnson Matthey. Artificial viscosity coefficient \( \omega \) is empirically defined to produce the stability of the simulation. The friction coefficients of the coil-plane and coil-coil contact are also empirically set to the small value not to determine the coil behavior dominantly since the effects of the wet friction are likely to be significantly small. The time increment \( \Delta t \) is set with the Courant number < 1 for the elastic wave speed. Young’s modulus and shear modulus of the wire, \( E_w \) and \( G_w \) are set corresponding to the value of Pt/W (8%). The diameter of the coil is set to \( 2.54 \times 10^{-4} \) m by referring to (White et al. 2008).

| Physical parameters used in the coil placement simulation |
|---------------------------------------------------------|
| Density \( \rho \) [kg/m\(^3\)] \quad 2.1 \times 10^4 |
| Artificial viscosity coefficient \( \omega \) [kg/s] \quad 2.0 \times 10^2 |
| Static friction coefficient \( k_1 \) [-] \quad 1.0 \times 10^{-3} |
| Kinetic friction coefficient \( k_2 \) [-] \quad 1.0 \times 10^{-3} |
| Time increment \( \Delta t \) [s] \quad 1 \times 10^{-5} |
| Young’s modulus \( E_w \) [GPa] \quad 230 |
| Shear modulus \( G_w \) [GPa] \quad 82 |
| Diameter of the coil \( D \) [m] \quad 2.54 \times 10^{-4} |

3. Numerical examples

This section presents the numerical examples of the coil placement simulation for an idealized geometry of the aneurysms. The idealized geometry of the aneurysm with the diameter of \( 5 \times 10^{-3} \) m is constructed and the micro catheter is set in the aneurysm (Fig. 3). The configuration of the micro catheter is assumed to be a straight tube with the diameter of \( 3 \times 10^{-4} \) m. In order to avoid moving the coil outside the aneurysm, the rigid plane is set in the entrance of the aneurysm by assuming the placement of the balloon or the stent, which are usually placed due to the same reason in the clinical practice. The base size of the triangle element constituting the wall of the aneurysm and catheter is \( 2.5 \times 10^{-4} \) m (minimum proximity is 10% of the base size).
The initial configuration of the coil in the catheter is set to be straight and each orthogonal basis is set to be identical direction. In order to drive the coil motion, the insertion velocity being $1 \times 10^{-3}$ m/s (Matsubara et al., 2011) is imposed to the longitudinal direction of the catheter at the bottom node of the coil, in which the rotation of this node is prohibited until entering into the aneurysm. In the catheter, the coil is allowed only to move along the longitudinal direction of the catheter and the friction between the coil and inner wall of the catheter is ignored for simplicity.

We conduct four cases of the coil placement simulations, in which the different reference configurations and elastic properties of the coil are adopted to each case (Table 2). Two patterns of the reference coil configuration are prepared (Fig. 4): the helical loop structure, in which the pitch of the helical loop is set to be the same length as the coil diameter $D$; complex loop structure, in which the geometrical parameters are set by the methods of (Babiker et al., 2013). For considering the effects of the coil elastic property, two kinds of wire diameter $d$ are assigned as the hard coil ($d = 7.62 \times 10^{-5}$ m) and the soft coil ($d = 4.45 \times 10^{-5}$ m) based on (White et al., 2008). The relationship between the equivalent elastic constants (Young’s modulus $E$ and shear modulus $G$) and the wire diameter are shown in Fig. 5. It is obvious that $E$ and $G$ are dramatically increased with increasing the $d$. The coil length is set to be $1 \times 10^{-1}$ m in all case.

![Fig. 4 Reference configuration of the coil with helical loop (a) and complex loop (b).](image)

Table 2 Geometric parameters of the coil in each case

| Reference configuration | Wire diameter (m) | $E$ (MPa) | $G$ (MPa) |
|-------------------------|------------------|----------|----------|
| Helical-hard            | Helical loop     | $7.62 \times 10^{-5}$ | 37.8 | 91.3 |
| Helical-soft            | Helical loop     | $4.45 \times 10^{-5}$ | 0.687 | 1.21 |
| Complex-hard            | Complex loop     | $7.62 \times 10^{-5}$ | 37.8 | 91.3 |
| Complex-soft            | Complex loop     | $4.45 \times 10^{-5}$ | 0.687 | 1.21 |
Fig. 5 Relationships between the elastic coefficients and the diameter of the coil wire. Solid and Dashed lines show the values of the equivalent Young's modulus and shear modulus, respectively.

Figure 6 illustrates the temporal coil configurations during placement process in the case of complex and helical hard coils. Color map shows the magnitude of the contact forces against the aneurysm wall. Both coils were located along the aneurysm wall being curved and tangled regardless of the reference configuration. The complex-hard coil formed spatially uniform distribution through the placement process (Fig. 6 (a-1)-(a-5)), whereas the helical-hard coil initially formed the helical configuration (Fig. 6 (b-2)) and gradually distributed uniformly (Fig. 6 (b-3)-(b-5)). These temporal configurations in each coil show in good agreements to the clinical observations. Figure 7 also shows the temporal configurations of the complex- and helical- soft coil. Both coils initially distributed locally in the aneurysm and gradually expanded to a whole region with inserting the coil inside.
Fig. 7 Temporal coil configurations with its contact force against the wall in case of complex-soft (a) and helical-soft (b) at $t = 20$ s (1), 40 s (2), 60 s (3), 80 s (4) and 100 s (5).

One of our primary motivations of the coil placement simulation is to estimate the contact force of the coil against the aneurysm wall, which may cause the bleeding during operation. The temporal changes of the contact force (ave.$\pm$S.D.) pressing on the aneurysm wall in the case of complex-hard and soft coils are shown in Fig. 8. In the results of complex-hard coil, force magnitude of approximately $1\pm1 \mu$N consistently pressed on the aneurysm wall through the whole process of the coil placement. Its magnitude is approximately one order higher than that of the complex-soft coil. This one order difference is also shown in the results of the helical coil (Fig. 9).

Fig. 8 Temporal change of the average (a) and standard derivation (b) of contact forces by the coil to the aneurysm wall in cases of complex-hard and complex-soft coils.
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Fig. 9 Temporal change of the average (a) and standard derivation (b) of contact forces by the coil to the aneurysm wall in cases of helical-hard and helical-soft coils.

The total contact forces loading against the aneurysm wall during the placement process (0 s – 100 s with time increments of 0.5 s) is counted and its histogram is generated as shown in Figure 10 in the case of hard coils (Fig. 10(a)) and soft coils (Fig. 10(b)). The horizontal axis is set to assess the general tendencies of the distribution and it should be noted that less than 1% of the contact forces over the range of this histogram can be observed in all cases. Most of contact forces distribute from 0 to 2 μN in the case of the complex and helical hard coils (Fig. 10(a)), and that of helical-hard coils mainly distributes under 0.5 μN and its variation is relatively small comparing to that of complex-hard coil. These tendencies are also shown in the case of complex and helical soft coil, but these values distributes from 0 to 0.2 μN and the difference between the complex and helical coil is slight comparing to that of complex and helical hard coils. These results give an insight into the effects of coil elastic properties: the hard coil can provide the uniform coil distribution around the aneurysm wall but loads high magnitude of the contact force, whereas the soft coil can placed the aneurysm inside with small contact force regardless of the coil reference configuration, but this uniformity is less than that of hard coil.

Fig. 10 Histograms of contact force of the coil against the aneurysm wall counted during the coil placement (0 s-100 s) in the case of hard coils (a) and soft coils (b).

4. Conclusions

The present study developed the computational model for the aneurysm coil placement with the realistic coil properties. The mechanical behavior of the coil was successfully expressed based on the minimum energy principle and geometric constraints due to the multiple contacts. Realistic coil properties used in the clinical practice were assigned considering the reference configurations and elastic properties. In the present numerical examples of the coil placement, the difference of the coil elastic properties demonstrated the obvious effects on both the coil configuration and the contact force on the aneurysm wall. The reliable estimation of these contact forces is in great demand in order
to select the appropriate coils for each clinical case, and thus the present model has a great potential to contribute the clinical practice in terms of the mechanical sense.

It should be noted that the proposed model of the coil placement includes further considerations about several physical parameters since the mechanical interaction between the coils and blood flow is not accurately expressed in the model. Thus, the damping coefficient and friction coefficients are empirically determined to maintain the stability of the simulation. For the consideration of the more accurate values of these parameters, the experimental measurements (e.g., (Takashima et al., 2007) for the measurement of the friction force) or the numerical experiment focusing on the interactions between the coils and blood flow are hopeful.

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