Flat directions and gravitino production in SUSY models

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Flat directions in supersymmetric models can get large vacuum expectation values in the early Universe which leads to a large mass for gauge bosons and gauginos. We point out that this can then result in enhanced gravitino production because the cross-section for the production of the $\pm 1/2$ helicity states of the gravitino is proportional to the square of the gaugino masses. We consider gravitino production after inflation in such a scenario and find that the abundance in some cases can be much larger than the upper bound on the gravitino abundance from cosmological constraints unless the flat direction field has a very small vacuum expectation value when it commences oscillating.

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I. FLAT DIRECTIONS IN SUSY MODELS

Generic supersymmetric (SUSY) models have a large number of flat directions i.e. directions in the field space of scalars that have a flat potential $\phi$. However SUSY breaking and non-renormalizable terms lift these flat directions and can give rise to a non-zero minimum for the potential of the field $\phi$ that parametrises a flat direction. During inflation the minimum of the potential which varies with the Hubble parameter $H$. When $H \approx m_0$, where $m_0$ is of the order of SUSY breaking scale $\sim 100$ GeV, the minimum shifts to the origin with $\phi$ displaced from the origin. In certain other cases, the minimum of the potential during inflation is at the origin yet the flat direction vev is non-zero due to quantum fluctuations of $\phi$ during inflation.

The flat direction vevs give mass to gauge bosons through terms like $g^2 A_\mu \tilde{f}^* A^{\mu} \tilde{f}$ where $\tilde{f}$ represents a squark or a slepton associated with $\phi$. The gauge bosons get a SUSY conserving mass $m_g^2 \sim g^2 \varphi^2$ where $\varphi^2 = \langle \phi^* \phi \rangle$. In this process one or more Standard Model (SM) gauge symmetries can get broken (at late times $\phi$ decays and gauge symmetry is restored.) Gauginos get an equivalent mass, and we take $m_{\tilde{g}} = m_g = \sqrt{\alpha} \varphi$.

There can be two possible scenarios in the context of flat directions with large vevs.

1. If all the gauge bosons get large masses, then thermalization of the inflaton decay products is delayed. This is because gauge bosons mediate the scattering processes relevant for thermalisation and the scattering cross-section is suppressed due to their large masses. Lack of thermalization then affects the cosmology of the Universe. In particular, one needs to recalculate the gravitino production for a non-thermal Universe, and for a lower final reheat temperature. This idea was first proposed in Refs. 1, 8. A more detailed calculation is done in Ref. 11.

2. If not all gauge bosons get mass then thermalization happens as usual. However, the large vev of a flat direction gives a large mass to one or more gauginos which can still affect gravitino production.

The gravitino has $\pm 1/2$ and $\pm 3/2$ helicity states and the total gravitino production cross section is proportional to $\left(1 + m_{\tilde{g}}^2/(3m_G^2)\right)$, where the factor “1” is due to the $\pm 3/2$ helicity states and the factor $m_{\tilde{g}}^2/(3m_G^2)$ is due to the $\pm 1/2$ helicity states 10. A large gaugino mass can lead to a large gravitino abundance in both the above scenarios. This has been overlooked in the literature so far. In this article we consider the second scenario in which only gluons and gluinos get mass and the Universe does thermalize quickly after inflation. The first scenario of a non-thermal Universe after inflation and enhanced gravitino production has been studied in Ref. 8.

II. GRAVITINO PRODUCTION

Gravitinos are produced by the scattering of the decay products of the inflaton. Refs. 11, 12 provide a list of processes for gravitino production. The number density of gravitinos generated is then obtained using the Boltzmann equation

$$\frac{dn_{\tilde{G}}}{dt} + 3Hn_{\tilde{G}} = \langle \Sigma_{tot}|v|\rangle n^2,$$

where $n = \langle \zeta(3)/\pi^2\rangle T^3$ is the number density of inflaton decay products ($\zeta(3)$ = 1.20206, is the Riemann zeta function of 3), $\Sigma_{tot}$ is the total scattering cross section for gravitino production, $v$ is the relative velocity of the incoming particles, and $\langle ... \rangle$ refers to thermal averaging. We ignore the gravitino decay term above as the gravitino lifetime is $10^{-8}(100 \text{ GeV}/m_{\tilde{G}})$ [11] and is not relevant during the gravitino production era for gravitinos of mass.
\(10^{2-3}\text{ GeV}\) that we consider. \(\langle \Sigma_{\text{tot}} | v| \rangle\) is given by

\[
\langle \Sigma_{\text{tot}} | v| \rangle = \frac{\eta(T)}{M^2} = \frac{1}{M^2} \frac{3\pi}{16 \zeta(3)} \sum_{i=1}^{3} \left[ 1 + \frac{m_{3i}^2(T)}{3m_G^2} \right] c_i g_i^2 \ln \left( \frac{k_i}{g_i} \right). \tag{2}
\]

The \(i = 1, 2, 3\) in \(\eta(T)\) refer to the three gauge groups \(U(1)_Y, SU(2)_L, SU(3)_c\), respectively. \(g_i(T)\) are the gauge coupling constants. The parameters \(c_i\) and \(k_i\) are constants associated with the gauge groups and are given by \(c_{1,2,3} = 11, 27, 72\) and \(k_{1,2,3} = 1.266, 1.312, 1.271\) respectively (see Table 1 of Ref. [13]). \(M = M_{Pl}/\sqrt{8\pi} \approx 2.4 \times 10^{18}\) GeV is the reduced Planck mass. The crosssection above includes corrections to the expressions obtained earlier in Refs. [10, 14]. We presume that the inflaton decays perturbatively and the products thermalise quickly as discussed in Appendix A of Ref. [13]. We shall only consider \(i = 3\) because we are studying the enhancement in the gravitino production rate due to the large gluino mass and we take \(\alpha_3 = 6 \times 10^{-2}\) for the relevant energy scales below. Then \(\eta(T)\) is given by

\[
\eta(T) = \frac{3\pi}{16 \zeta(3)} c_3 g_3^2 \ln \left( \frac{k_3}{g_3} \right) \left( 1 + \frac{m_{33}^2}{3m_G^2} \right). \tag{3}
\]

where \(m_{33}\) is now the gluino mass.

To account for the expansion of the Universe one should consider the abundance of a species \(i\) in a comoving volume. This is achieved by considering the ratio \(Y_i = n_i/s\), where \(n_i\) is the number density of particles of the species \(i\) in a physical volume and \(s\) is the entropy density given by \(s = (2\pi^2/45) g_* T^3\), where \(g_* = 228.75\) in the MSSM. Thus Eq. (1) is rewritten as

\[
T \frac{dY_{G3}}{dT} = \langle \Sigma_{\text{tot}} | v| \rangle Y_n. \tag{4}
\]

In the radiation dominated era, the temperature \(T\) is given by

\[
T = T_R \left( \frac{1}{2H_R(t-t_R)+1} \right)^{1/2}, \tag{5}
\]

where \(T_R\) is the reheat temperature and

\[
H_R = \sqrt{\frac{8\pi^3 g_* T_R^2}{90}} \frac{T_R^2}{M_{Pl}}. \tag{6}
\]

This implies that \(\dot{T}\) is

\[
\dot{T} = \frac{H_R}{T_R^3} T^3 = - \left( \frac{g_* R^2}{90} \right)^{1/2} \frac{T^3}{M}. \tag{7}
\]

Thus

\[
\frac{dY_{G3}}{dT} = - \left( \frac{90}{g_* R^2} \right)^{1/2} \left( \frac{1}{(2\pi^2/45) g_*} \right) \left( \frac{\eta}{M} \right) \left( \frac{\zeta(3)}{\pi^2} \right)^2. \tag{8}
\]

To obtain the gravitino abundance we integrate the above equation from \(T_R\) to \(T_f\) where \(f\) corresponds to the time when the flat direction condensate decays. Here we have ignored gravitino production during the period of reheating, as studied in Refs. [14, 16, 20]. Regarding \(t_f\), the perturbative condensate decay rate is given by \(\Gamma_\phi = m_{\phi}^3/\varphi^2\) [1, 21] and as argued in Ref. [21] the condensate decay products do not dominate the Universe for \(\varphi < 10^{-2} M_{Pl}\) for gravitational decay of the inflaton. (We discuss alternate mechanisms for condensate decay below.) For gravitino production after \(t_f\) there will be no enhancement due to a large gluino mass and the abundance generated after \(t_f\) will be proportional to \(T_f\) as similar to the standard scenario.

We take the inflaton mass \(m_i\) to be \(10^{13}\) GeV and the inflaton decay rate \(\Gamma_d \sim m_{\phi}^3/M_{Pl}^2 \sim 10\text{ GeV}\). Below we are primarily concerned with the evolution of \(\phi\) after \(t_0 \sim m_{\phi}^{-1}\). In the both the cases mentioned at the beginning of Sec. 4 \(\phi\) effectively has a quadratic potential after \(t_0\) with a minimum at the origin with a positive curvature of \(m_{\phi}^2\) (ignoring thermal effects for now). The gluino mass is given by \(m_{33}^2 = \alpha_3 \varphi^2\), and \(\varphi\) is \(\psi_0\) at \(t_0\) and then falls as \(1/\alpha_3^{3/2}\) once the condensate starts oscillating at \(t_0\). Then for \(t > t_d\), where \(t_d = \Gamma_d^{-1}\) is the inflaton decay time, the gluino mass is

\[
m_{33}^2 = \alpha \varphi_0^2 \left( \frac{a_0}{a} \right)^3 = \alpha \varphi_0^2 \left( \frac{a_d}{a_0} \right)^3 = \alpha \varphi_0^2 \left( \frac{\Gamma_d}{m_0} \right)^2 \left( \frac{T}{T_R} \right)^3, \tag{9}
\]

where we have used \(a \sim t^{3/2}\) for \(t_0 < t < t_d\) for an inflaton oscillating in a quadratic potential during reheating and \(a \sim 1/T\) for \(t > t_d\).

Integrating the Boltzmann equation in Eq. (5) from \(T_R\) to \(T_f\) gives the gravitino abundance generated between \(t_d\)
and $t_f$ as

$$Y_{\tilde{G}} = \left( \frac{90}{g_\ast \pi^2} \right)^{1/2} \frac{45}{2\pi^2 g_\ast R} \left( \frac{\sqrt{8\pi}}{M_{Pl}} \right) \left( \frac{\zeta(3)}{\pi^2} \right)^2 \left[ \frac{3\pi c_3}{16 \zeta(3)} \right] \times g_\ast^3(T_R) \ln \left( \frac{k_3}{g_3(T_R)} \right) \left( T_R - T_f \right) + \frac{1}{3} \frac{m_{\tilde{G}}^2(T_R)}{3m_{\tilde{G}}^2} T_R \left( 1 - \frac{T_f^4}{T_R^4} \right).$$  \hspace{1cm} (10)

We have ignored the variation of $g_\ast$ and $g_i$ with temperature and have used the value at $T_R$, since for $T_R \gg T_f$ most gravitino production occurs close to $T_R$. The term proportional to $T_R - T_f$ is associated with the production of $\pm 3/2$ helicity gravitinos and the other term is associated with the production of the $\pm 1/2$ helicity states.

The reheat temperature is given by $T_R = 0.55 g_\ast^{-1/4}(M_{Pl}\Gamma_d)^{1/2} = 2 \times 10^9 \text{GeV}^{22}$. The condensate decays when its decay rate $\Gamma = m_0^3/\varphi^2$ equals $H$, i.e., when $t_f = \varphi_f^2/m_0^3$, where

$$\varphi_f = \varphi_0 \left( \frac{a_0}{a_f} \right)^{3/2} = \varphi_0 \left( \frac{a_0}{a_d} \right)^{3/2} \left( \frac{a_d}{a_f} \right)^{3/2} = \varphi_0 \left( \frac{\Gamma_d}{m_0} \right) \left( \frac{m_0^3}{\Gamma_d \varphi_f^2} \right)^{3/4}. \hspace{1cm} (11)$$

Here we have used $a \sim t^{2/3}$ for $t_0 < t < t_d$ and $a \sim t^{1/2}$ for $t > t_d$. This implies

$$\varphi_f = \varphi_0^{2/5} m_0^{1/2} \Gamma_d^{1/10}. \hspace{1cm} (12)$$

The temperature at $t_f$, using $a \sim t^{1/2}$ for $t > t_d$, is

$$T_f = T_R \left( \frac{a_d}{a_f} \right) = T_R \left( \frac{m_0^3}{\Gamma_d \varphi_f^2} \right)^{1/2} = T_R \left( \frac{m_0}{2/5 \varphi_f^2 \Gamma_d} \right) = 0.55 g_\ast^{-1/4} \frac{m_0}{\varphi_0^{2/5}} \frac{M_{Pl}^{1/2}}{\Gamma_d^{1/10}}. \hspace{1cm} (13)$$

For the condensate to decay after inflaton decay, or $T_f < T_R$, we need $\varphi_0 > 30 m_0$ or $\varphi_f > 3 m_0$, for $\Gamma_d = 10 \text{GeV}$.  

III. RESULTS AND CONCLUSION

A large gravitino abundance can be in conflict with cosmological observations. If the gravitino is stable its energy density can overclose the Universe. If the gravitino is unstable its decay products can either overclose the Universe or dissociate the light nuclei generated during primordial nucleosynthesis. We use an upper bound of $Y_{\tilde{G}} < 10^{-12}$ for consistency with cosmological observations. For the abundance obtained in Eq. (10) to be consistent with observations then requires

$$\frac{\varphi_0}{m_{\tilde{G}}} < 300. \hspace{1cm} (14)$$

For $m_{\tilde{G}} = 100 \text{GeV}$, the constraint is

$$\varphi_0 < 3 \times 10^4 \text{GeV}. \hspace{1cm} (15)$$

Note that if we fix $m_{\tilde{G}} = 100 \text{GeV}$ and allow the condensate mass, so far taken to be $m_0$, to vary one gets the same constraint as in Eq. (14) with $m_{\tilde{G}}$ replaced by the condensate mass $m_\varphi$ (since in Eq. (14) $m_\varphi^2/m_{\tilde{G}}^2 \sim \varphi_0^2/(m_0^2 m_{\tilde{G}}^2)$). This is relevant if one includes thermal corrections to the condensate potential.

We obtain this bound assuming $T_f \ll T_R$ in Eq. (10). From Eq. (13) $\varphi_0 = 3 \times 10^4 \text{GeV}$ implies $T_f = 6 \times 10^9 \text{GeV}$. For this value of $T_f$ and $T_R = 2 \times 10^9 \text{GeV}$, our assumption only slightly affects the contribution of the first term in Eq. (10), and does not alter the contribution of the second term. Since the contribution of the second term is larger than that of the first term our assumption is therefore justified. Smaller values of $\varphi_0$ will correspond to larger $T_f$ and therefore greater compatibility of $Y_{\tilde{G}}$ with cosmological constraints.

We now consider plausible values of $\varphi_0$. During inflation the non-zero vacuum energy breaks SUSY and can give large positive masses to the flat direction of order $H_I$, where $H_I$ is the Hubble parameter during inflation $\phi$. This may be true in supergravity models with minimal or non-minimal Kahler potentials $23$. The flat direction potential has a minimum at the origin and the field is driven to this minimum. Quantum fluctuations during inflation then give a vev of order $H_I$ $\phi$. Assuming that the field does not vary much till $t_0$, then $\varphi_0 \sim H_I$ and the upper bound on $\varphi_0$ implies that the inflationary scale $V_\phi^{1/4} < 4 \times 10^{11} \text{GeV}$.

Alternatively, in no scale supergravity or more generally in any supergravity theory with a Heisenberg symmetry of the kinetic function in the Kahler potential, one gets contributions during inflation to the flat direction potential from supersymmetry breaking (due to $H_I$) only at the one loop level and these are negative at the origin for flat directions which do not involve a
stop and if the inflaton is not the standard string dilaton \cite{23}. This correction to the potential, along with non-renormalizable terms, leads to a shifted minimum of the potential and subsequently a large vev of order $M_{Pl}$ \cite{23} or $10^{12-14}$ GeV on including GUT interactions \cite{24} as the field rolls to the minimum of its potential and oscillates about it. (Negative corrections at the origin were also discussed in Ref. \cite{2}.)

We consider the shifted minimum of the potential for $\phi$ to be $M(H/M)^{1/(n+1)}$ where non-renormalizable terms in the potential are of the form $\phi^{2n+1}/M^{2n}$, $n \geq 1$ \cite{6,22}. $\phi$ oscillates about this time dependent minimum which decreases as $H$ decreases. When $H \sim m_0$, the potential minimum goes to zero and the field oscillates about the origin in a quadratic potential with curvature $m_0^2$. Then \cite{2}

$$\varphi_0 \sim M(H(t_0)/M)^{1/(n+1)}, \quad (16)$$

where $H(t_0) = 100$ GeV. For $n = 1$, $\varphi_0 < 3 \times 10^4$ GeV is satisfied for

$$M < 10^7 \text{ GeV}. \quad (17)$$

For larger values of $n$, the upper bound on $M$ reduces further and one requires $M < 10^{4-5}$ GeV for $n > 1$. However we are considering scales up to at least $T_R \sim 10^{9}$ GeV, and setting $M = 10^9$ GeV gives a value of $\varphi_0$ in conflict with the upper bound on $\varphi_0$. If we choose $\varphi_0 \sim 10^{12}$ GeV as mentioned above then we get the gravitino abundance $Y_G$ to be $8 \times 10^5$ which is orders of magnitude higher than the cosmological bound of $10^{-12}$. The abundance will increase as $\varphi_0^2$ for larger values of $\varphi_0$.

Thermal corrections to the condensate potential can increase the mass of the condensate field \cite{25,27}. Ref. \cite{27} obtains masses of order $10^{10}$ GeV, $10^3$ GeV and $10^4$ GeV for non-renormalizable terms with $n = 1, 2, 3$ and $M = 10^{18}$ GeV. We find that the condition $\varphi_0 < 300 m_0$ discussed above is not satisfied for $\varphi_0$ obtained from Eq. (16) with $H(t_0)$ replaced by $m_0$.

Our results imply that either one has a scenario of supergravity with a positive contribution to the $\phi$ potential from $H_I$ that gives $H_I < 3 \times 10^4$ GeV and $V_I^{1/4} < 4 \times 10^{11}$ GeV, or the flat directions must decay quickly in a way so as to not affect the cosmology of the Universe.

With regards to the quick decay of the flat directions, the longevity of flat directions has been debated in Refs. \cite{21,23,36}. However it has been argued in Refs. \cite{33,36} that even if non-perturbative rapid decay via parametric resonance occurs for scenarios with multiple flat directions it leads to a redistribution of energy of the condensate amongst the fields in the $D$ flat superspace and hence to practically the same cosmological consequences, including at least as large gauge boson and gaugino masses as in the scenario with only perturbative decay.

Scattering of particles of the thermal bath off the flat direction condensate can lead to the decay of the condensate \cite{23,25,26}, though thermal effects are less important for larger values of $n$. For example, for $n = 3$ the condensate decays much after the decay of the inflaton \cite{26}. Decay via fragmentation into solitonic states called Q-balls \cite{32,42} or Q-axitons \cite{40} due to inhomogeneities in the condensate may also be relevant. However the time scales for the formation of Q-balls and Q-axitons can be larger than $t_f$. $t_f$ is less than $60 m_0^{-1}$, for $\varphi_f < 800$ GeV obtained from Eq. (12).

In conclusion, the presence of flat directions in SUSY models and associated large vevs seems to have very important consequences for gravitino production in the early Universe. These large vevs give a large mass to gauginos which enhances gravitino production and can violate cosmological constraints on the gravitino abundance. This scenario can be avoided if the flat direction has a vev smaller than $3 \times 10^4$ GeV at $t \sim m_0^{-1}$, or if the flat direction decays early.

\begin{thebibliography}{99}
\bibitem{1} I. Affleck and M. Dine, Nucl. Phys. B\textbf{249}, 361 (1985).
\bibitem{2} M. Dine, L. Randall, and S. D. Thomas, Phys. Rev. Lett. \textbf{75}, 309 (1995), hep-ph/9503305.
\bibitem{3} M. Dine, L. Randall, and S. D. Thomas, Nucl. Phys. B\textbf{458}, 291 (1996), hep-ph/9507453.
\bibitem{4} T. Gherghetta, C. F. Kolda, and S. P. Martin, Nucl. Phys. B\textbf{468}, 37 (1996), hep-ph/9510370.
\bibitem{5} K. Enqvist and A. Mazumdar, Phys. Rept. \textbf{380}, 99 (2003), hep-ph/0209244.
\bibitem{6} M. Dine and A. Kusenko, Rev. Mod. Phys. \textbf{76}, 1 (2003), hep-ph/0303065.
\bibitem{7} R. Allahverdi and A. Mazumdar (2005), hep-ph/0505050.
\bibitem{8} R. Allahverdi and A. Mazumdar, JCAP \textbf{0610}, 008 (2006), hep-ph/0512227.
\bibitem{9} R. Rangarajan and A. Sarkar (2012), arXiv: May 2012.
\bibitem{10} M. Bolz, A. Brandenburg, and W. Buchmuller, Nucl. Phys. B\textbf{606}, 518 (2001), Erratum-ibid. B\textbf{790} (2008) 336, hep-ph/0012052.
\bibitem{11} J. R. Ellis, J. E. Kim, and D. V. Nanopoulos, Phys. Lett. B\textbf{145}, 181 (1984).
\bibitem{12} M. Kawasaki and T. Moroi, Prog. Theor. Phys. \textbf{93}, 879 (1995), hep-ph/9403364.
\bibitem{13} J. Pradler and F. D. Steffen, Phys. Rev. D\textbf{75}, 023509 (2007), hep-ph/0608344.
\bibitem{14} M. Kawasaki, K. Kohri, and T. Moroi, Phys. Rev. D\textbf{71}, 083502 (2005), astro-ph/0408426.
\bibitem{15} D. J. Chung, E. W. Kolb, and A. Riotto, Phys. Rev. D\textbf{60}, 063504 (1999), hep-ph/9809453.
\bibitem{16} G. Giudice, A. Riotto, and I. Tkachev, JHEP \textbf{11}, 036 (1999), hep-ph/9911302.
\bibitem{17} J. Pradler and F. D. Steffen, Phys. Lett. B\textbf{648}, 224 (2007), hep-ph/0612291.
\end{thebibliography}
[18] R. Rangarajan and N. Sahu, Mod. Phys. Lett. A23, 427 (2008), hep-ph/0606228.
[19] R. Rangarajan and N. Sahu, Phys. Rev. D79, 103534 (2009), hep-ph/0608096.
[20] V. S. Rychkov and A. Strumia, Phys. Rev. D75, 075011 (2007), hep-ph/0701104.
[21] K. A. Olive and M. Peloso, Phys. Rev. D74, 103514 (2006), hep-ph/0608096.
[22] E. W. Kolb and M. S. Turner, The Early Universe (Addison-Wesley, RedWood City, 1990).
[23] M. K. Gaillard, H. Murayama, and K. A. Olive, Phys. Lett. B355, 71 (1995), hep-ph/9504307.
[24] B. A. Campbell, M. K. Gaillard, H. Murayama, and K. A. Olive, Nucl. Phys. B538, 351 (1999), hep-ph/9805300.
[25] R. Allahverdi, B. A. Campbell, and J. R. Ellis, Nucl. Phys. B579, 355 (2000), hep-ph/0001122.
[26] A. Anisimov and M. Dine, Nucl. Phys. B619, 729 (2001), hep-ph/0008058.
[27] A. Anisimov, Phys. Atom. Nucl. 67, 640 (2004), hep-ph/0111233.
[28] R. Allahverdi, R. D. Shaw, and B. A. Campbell, Phys. Lett. B473, 246 (2000), hep-ph/9909256.
[29] M. Postma and A. Mazumdar, JCAP 0401, 005 (2004), hep-ph/0304246.
[30] R. Allahverdi and A. Mazumdar, JCAP 0708, 023 (2007), hep-ph/0608296.
[31] A. Basboll, D. Maybury, F. Riva, and S. M. West, Phys. Rev. D76, 055005 (2007), hep-ph/0703015.
[32] A. Basboll, Phys. Rev. D78, 023528 (2008), 0801.0745.
[33] R. Allahverdi and A. Mazumdar, Phys. Rev. D78, 043511 (2008), 0802.4430.
[34] A. E. Gumrukcuoglu, K. A. Olive, M. Peloso, and M. Sexton, Phys. Rev. D78, 063512 (2008), 0805.0273.
[35] A. E. Gumrukcuoglu, Phys. Rev. D80, 123520 (2009), 0910.0854.
[36] R. Allahverdi, R. Brandenberger, F.-Y. Cyr-Racine, and A. Mazumdar, Ann. Rev. Nucl. Part. Sci. 60, 27 (2010), 1001.2600.
[37] A. Kusenko and M. E. Shaposhnikov, Phys. Lett. B418, 46 (1998), hep-ph/9709492.
[38] K. Enqvist and J. McDonald, Phys. Lett. B425, 309 (1998), hep-ph/9711514.
[39] S. Kasuya and M. Kawasaki, Phys. Rev. D61, 041301 (2000), hep-ph/9909509.
[40] K. Enqvist, A. Jokinen, and J. McDonald, Phys. Lett. B483, 191 (2000), hep-ph/0004050.
[41] S. Kasuya and M. Kawasaki, Phys. Rev. Lett. 85, 2677 (2000), hep-ph/0006128.
[42] S. Kasuya and M. Kawasaki, Phys. Rev. D62, 023512 (2000), hep-ph/0002285.
[43] S. Kasuya and M. Kawasaki, Phys. Rev. D64, 123515 (2001), hep-ph/0106119.
[44] K. Enqvist, A. Jokinen, T. Multamaki, and I. Vilja, Phys. Rev. D63, 083501 (2001), hep-ph/0011134.
[45] T. Multamaki and I. Vilja, Phys. Lett. B535, 170 (2002), hep-ph/0203195.
[46] K. Enqvist and J. McDonald, Nucl. Phys. B570, 407 (2000), hep-ph/9908316.