Heavy quarkonium production provides an ideal laboratory to understand quantum chromodynamics. In contrast to the unpolarized cross section, the quarkonium polarization measurement may provide more complete information for the production mechanism of heavy quarkonium.\(^1\)

A distinct example is the \(J/\psi\) polarization at hadron colliders. The polar asymmetry coefficient \(\lambda_0\) in the angular distribution of the leptons from the \(J/\psi\) decay is an important observable that encodes the \(J/\psi\) polarization information. At the Tevatron, the CDF Collaboration measured the quantity many years ago\(^2\). Their measurements show that \(\lambda_0\) for prompt \(J/\psi\) production in its helicity frame is around zero up to \(p_T = 30\text{GeV}\), indicating that the \(J/\psi\) are produced unpolarized at the Tevatron. The state-of-the-art theory that describes the heavy quarkonium dynamics, non-relativistic QCD (NRQCD)\(^3\), predicts that the heavy quark pair is allowed to be created in a color-octet(CO) intermediate state at short distances and then evolves nonperturbatively into a color-singlet(CS) quarkonium at long distances. Although this CO mechanism provides an opportunity to account for the CDF yield data\(^4\) that cannot be resolved in the CS model (CSM) even by including the higher-order QCD corrections\(^5\), the leading-order (LO) in \(\alpha_S\) NRQCD prediction gives a completely transverse polarization result at high \(p_T\) due to the gluon fragmentation contribution to the CO \(S_1^{[8]}\) intermediate state\(^6\). Recently, three groups have reported their next-to-leading order QCD corrections to the \(J/\psi\) polarization\(^7\). Remind that the \(J/\psi\) polarization is strongly dependent on the specific choice of the nonperturbative CO long-distance matrix elements (LDMEs), which can only be determined from the experimental data. Choosing different \(p_T\) regions of the input experimental data may result in very different predictions. Therefore, the precise measurement of polarization, especially at high \(p_T\), may provide a smoking-gun signature to distinguish between various production mechanisms of heavy quarkonium. Moreover, it was pointed out in Ref.\(^1\) that there is still a CO LDMEs parameter space left to make both the unpolarized yields and \(\lambda_0\) quite satisfactory compared to the hadroproduction data.

However, the prompt \(J/\psi\) production at the Tevatron and LHC is affected substantially by the higher charmonia (e.g. \(\chi_c\) and \(\psi'\)) transitions to \(J/\psi\). Furthermore, even for direct \(J/\psi\) production there are three leading CO LDMEs, which makes the precise determination of CO LDMEs uneasy. In contrast to the \(J/\psi\), the feed-down contribution only comes from \(\psi'\) to \(\chi_c\) transition but is not significant, and there is only one leading CO state \(S_1^{[8]}\) involving \(\chi_c\) direct production, which can make the determination of the nonperturbative LDMEs more easily and precisely. Moreover, the higher-order QCD corrections to the conventional P-wave CS state suffer from severe infrared divergences, while in NRQCD these divergences can be absorbed by the CO state and thus make the P-wave observables well defined beyond LO. Given these reasons, the investigation of \(\chi_c\) production at the LHC is an important way to test the validity of NRQCD factorization and the CO mechanism.

The first investigation for the unpolarized \(\chi_c\) hadroproduction at NLO level was performed in Ref.\(^8\). In this work, we extend our calculation to the polarized case, with the method described in Refs.\(^9,10\). Similar to the case of \(J/\psi\) polarization, the measurement of \(\chi_{c1}\) and \(\chi_{c2}\) polarizations at the LHC may provide important information for the production mechanism of heavy quarkonium. The polarization observables of \(\chi_{c1}\) and \(\chi_{c2}\) have been proposed in Refs.\(^11,12\). Experimentally, one may have two ways to measure the polarization of the \(\chi_{c1}\) and \(\chi_{c2}\) through the angular distributions of their decay products. One is to measure the \(J/\psi\) angular distribution from \(\chi_c \rightarrow J/\psi\gamma\). The angular distribution with respect to the \(J/\psi\) polar angle \(\theta\) in the rest frame of \(\chi_c\) can be formulated as\(^13\):

\[
\frac{dN_{\chi_{c1}}}{d \cos \theta} \propto 1 + \sum_{k=1}^{j} \lambda_{k\theta} \cos^{2k} \theta, \tag{1}
\]

where the polar asymmetry coefficients \(\lambda_{k\theta}\) can be ex-
pressed as the rational functions of the $\chi_{cJ}$'s production spin density matrix $\rho^{\chi_{cJ}}$. More specifically, for $\chi_{c1}$ it is

$$\lambda_{\theta} = \frac{(1 - 3\delta)}{(1 + \delta)N_{\chi_{c1}} + (1 - 3\delta)\rho_{0,0}^{\chi_{c1}}},$$

with $N_{\chi_{c1}} \equiv \rho_{1,1}^{\chi_{c1}} + \rho_{0,0}^{\chi_{c1}} + \rho_{-1,-1}^{\chi_{c1}}$, while for $\chi_{c2}$, the coefficients are

$$\lambda_{\theta} = 6[(1 - 3\delta_{0} - \delta_{1})N_{\chi_{c2}} - (1 - 7\delta_{0} + \delta_{1})(\rho_{1,1}^{\chi_{c2}} + \rho_{-1,-1}^{\chi_{c2}}) - (3 - \delta_{0} - 7\delta_{1})\rho_{0,0}^{\chi_{c2}}]/R,$$

$$\lambda_{2g} = (1 + 5\delta_{0} - 5\delta_{1})[N_{\chi_{c2}} - 5(\rho_{1,1}^{\chi_{c2}} + \rho_{-1,-1}^{\chi_{c2}}) + 5\rho_{0,0}^{\chi_{c2}}]/R,$$

with

$$N_{\chi_{c2}} = \rho_{1,1}^{\chi_{c2}} + \rho_{0,0}^{\chi_{c2}} + \rho_{-1,-1}^{\chi_{c2}} + \rho_{-2,-2}^{\chi_{c2}},$$

$$R = (1 + 5\delta_{0} + 3\delta_{1})N_{\chi_{c2}} + 3(1 - 3\delta_{0} - \delta_{1})(\rho_{1,1}^{\chi_{c2}} + \rho_{-1,-1}^{\chi_{c2}}) + (5 - 7\delta_{0} - 9\delta_{1})\rho_{0,0}^{\chi_{c2}}.$$

The parameters $\delta$, $\delta_{0}$ and $\delta_{1}$ can be determined by the normalized multipole amplitudes. Following the notations in Ref. [18], we denote the normalized electric dipole (E1) transition amplitudes by $a_{1,J}^{0,1}$ and $a_{2,J}^{0,2}$ for $\chi_{c1}$ and $\chi_{c2}$ respectively, while $a_{1,1}^{J=1}$ and $a_{2,2}^{J=2}$ are the $\chi_{c1}$ and $\chi_{c2}$'s normalized magnetic quadrupole (M2) amplitudes and $\chi_{c2}$'s electric octupole amplitude (E3). The explicit expressions for $\delta$, $\delta_{0}$ and $\delta_{1}$ are

$$\delta = \frac{(1 + 2a_{1,J}^{J=1}a_{2,J}^{J=1})}{2},$$

$$\delta_{0} = \frac{(1 + 2a_{1,J}^{J=2}(\sqrt{5}a_{2,J}^{J=2} + 2a_{2,J}^{J=2}) + 4a_{2,J}^{J=2}(a_{1,J}^{J=2} + \sqrt{7}a_{2,J}^{J=2}) + 3(3a_{3,J}^{J=2})^{2}}{10},$$

$$\delta_{1} = \frac{(9 + 6a_{1,J}^{J=2}(\sqrt{5}a_{2,J}^{J=2} - 2a_{2,J}^{J=2}) - 4a_{2,J}^{J=2}(\sqrt{2}a_{2,J}^{J=2} + 2\sqrt{5}a_{2,J}^{J=2}) + 7(3a_{3,J}^{J=2})^{2}}{30}.$$

An alternative way to study the polarizations of $\chi_{c1}$ and $\chi_{c2}$ is to measure the dilepton angular distributions from $\chi_{c,J} \rightarrow J/\psi \gamma \rightarrow l^{+}l^{-}\gamma$. There are two scenarios to describe the dilepton angular distributions [17]. Here, we only choose the second scenario presented in Ref. [17], where the $z$ axis in the rest frame of $J/\psi$ coincides with the direction of the spin quantization axis in the $\chi_{c}$ rest frame. The generic lepton polar angle $\theta'$ dependence is

$$\frac{dN_{\chi_{cJ}}}{d\cos\theta'} \propto 1 + \lambda_{\theta'} \cos^{2}\theta',$$

where

$$\lambda_{\theta'}^{\chi_{c1}} = \frac{-N_{\chi_{c1}} + 3\rho_{0,0}^{\chi_{c1}}}{R_{1}},$$

$$\lambda_{\theta'}^{\chi_{c2}} = \frac{6N_{\chi_{c2}} - 9(\rho_{1,1}^{\chi_{c2}} + \rho_{-1,-1}^{\chi_{c2}})}{R_{2}} - 12\rho_{0,0}^{\chi_{c2}},$$

with

$$R_{1} = [(15 - 2(a_{2,J}^{J=1})^{2})N_{\chi_{c1}} - (5 - 6(a_{2,J}^{J=1})^{2})\rho_{0,0}^{\chi_{c1}}]/(5 - 6(a_{2,J}^{J=1})^{2}),$$

$$R_{2} = [2(21 + 14(a_{2,J}^{J=2})^{2}) + 5(a_{2,J}^{J=2})^{2})N_{\chi_{c2}} + 3(7 - 14(a_{2,J}^{J=2})^{2}) - 5(a_{2,J}^{J=2})^{2})/(\rho_{1,1}^{\chi_{c2}} + \rho_{-1,-1}^{\chi_{c2}}) + 4(7 - 14(a_{2,J}^{J=2})^{2} - 5(a_{2,J}^{J=2})^{2})\rho_{0,0}^{\chi_{c2}}]/(7 - 14(a_{2,J}^{J=2})^{2} - 5(a_{2,J}^{J=2})^{2}).$$

In this case, the angular distribution observable is just the component of the $\chi_{c}$ feeddown to the $J/\psi$ polarization when $p_{T} \gg m_{c} - m_{J/\psi}$. Note that $\lambda_{2g}$ for $\chi_{c2}$ is suppressed by the higher-order multipole amplitudes, i.e. $a_{2,J}^{J=2}, a_{3,J}^{J=3}$. The observable is expected to be near zero. Hence, we refrain from establishing the $p_{T}$ distribution of $\lambda_{2g}$ here.

In our numerical computation, we choose the same input parameters as those presented in Ref. [11]. The renormalization scale $\mu_{r}$, factorization scales $\mu_{f}$ and NRQCD scale $\mu_{A}$ are chosen as $\mu_{r} = m_{c} = \sqrt{4m_{c}^{2} + p_{T}^{2}}$ and $\mu_{A} = m_{c}$. The CO LDMEs are chosen as $\langle O^{(5)}(\chi_{c1}) \rangle = (2J + 1) \times (2.2^{+0.48}_{-0.32}) \times 10^{-3} \text{GeV}^{5}$, which are obtained by fitting the ratio $\sigma_{c2}/\sigma_{c1}$ at NLO level to the CDF data [18], while the CS LDMEs are estimated using the B-T potential model [20] as $\langle O^{(5)}(\chi_{c2}) \rangle = (2J + 1) \times (2.54^{+0.33}_{-0.29}) \times 0.075 \text{GeV}^{5}$. The uncertainties from the scale dependence, which is estimated by varying $\mu_{r}, \mu_{f}$ by a factor of $\frac{1}{2}$ to 2 with respect to their central values, the charm quark mass $m_{c} = 1.5 \pm 0.1 \text{GeV}$ and the error in the CDF data [18] are all encoded in the error estimations of the CO LDMEs. The normalized multipole amplitudes used here are taken from the CLEO measurement [21], i.e. $a_{2,J}^{J=2} = (-6.26 \pm 0.68) \times 10^{-2}$, $a_{3,J}^{J=3} = (-0.3 \pm 0.1) \times 10^{-2}$, $a_{3,J}^{J=3} = 0$. We keep the E3 amplitude $a_{3,J}^{J=3}$ vanishing, which is the consequence of the single quark radiation hypothesis [21, 22].

As performed in Ref. [17], we have tried to improve the extraction of CO LDMEs ($\langle O^{(5)}(\chi_{c1}) \rangle$) by including the LHcb data [24]. However, the magnitudes and accuracies of these parameters are not changed significantly. Measurements with higher resolution in the high $p_{T}$ region will be useful to improve the theoretical predictions. In Fig. 4, the cross section ratios $\sigma_{\chi_{c2}}/\sigma_{\chi_{c1}}$ at the Tevatron Run II and LHC are shown. For comparison, besides the NLO NRQCD predictions, we also plot the LO NRQCD results and the LO CSM results. We see the NLO NRQCD result is consistent with the CDF data [18] at the Tevatron in the whole $p_{T}^{J/\psi}$ region, while in the forward rapidity region the NLO NRQCD prediction is in agreement with the LHcb data [23] at the LHC only when $p_{T}^{J/\psi} > 8 \text{GeV}$, which may be attributed to the fact that the non-perturbative effects make our fixed-order results unreliable when $p_{T}^{J/\psi}$ is lower. Note that $p_{T}^{J/\psi}$ is obtained from $p_{T}$ of $\chi_{c}$ by the mass rescaling $p_{T}^{J/\psi} = \frac{m_{J/\psi}}{m_{\chi_{c}}}p_{T}$, which is proven to be a good approxi-
FIG. 1: (color online) The unpolarized cross-section ratio \( \sigma_{c,1}/\sigma_{c,2} \) vs. the transverse momentum \( p_T^{J/\psi} \) at the Tevatron Run II (left panel) and LHC with \( \sqrt{S} = 7 \text{TeV} \) (right panel). The rapidity cuts are the same as in the experiments [24, 25]. Results for LO NRQCD (solid line), NLO NRQCD (dashed line) and LO CSM (dotted line) are shown.

FIG. 2: (color online) Predictions of \( p_T \) spectra for the unpolarized \( \chi_{c1} \) (left column) and \( \chi_{c2} \) (right column) at the LHC with \( \sqrt{S} = 7 \text{TeV} \). Cross sections in the central rapidity region \((|y| < 2.4)\) and forward rapidity region \((2 < y < 4.5)\) for \( \chi_c \) are plotted. Results for LO NRQCD (solid line), NLO NRQCD (dashed line) and LO CSM (dotted line) are shown.

FIG. 3: (color online) The \( p_T \) dependence of \( \lambda_0 \) with \( J/\psi \) angular distributions from radiative decays \( \chi_{c1} \to J/\psi \gamma \) (left column) and \( \chi_{c2} \to J/\psi \gamma \) (right column) in the helicity frame at the LHC with \( \sqrt{S} = 7 \text{TeV} \). Results in central and forward rapidity regions are plotted. The LO NRQCD (solid line), NLO NRQCD (dashed line) and LO CSM (dotted line) predictions are shown.

The polar observables for \( \chi_{c1} \) approach to their maximal values, while the minimal values are obtained when \( \rho_{1,1}^{\chi_{c1}} = \rho_{1,1}^{\chi_{c2}} \gg \rho_{0,0}^{\chi_{c1}} \). For \( \chi_{c2} \), the polar asymmetry coefficients \( \lambda_0 \) and \( \lambda_\rho \) are maximum when \( \rho_{2,2}^{\chi_{c1}} = \rho_{0,0}^{\chi_{c1}} \gg \rho_{0,0}^{\chi_{c2}} \) and minimum when \( \rho_{2,2}^{\chi_{c2}} = \rho_{0,0}^{\chi_{c2}} \ll \rho_{0,0}^{\chi_{c1}} \). The \( p_T \) distributions of \( \lambda_0 \) and \( \lambda_{\rho} \) are shown in Fig. 3 and Fig. 4, respectively. It is worth noting that the transformation relation between the spin density matrices of \( S_1^{\chi_{c}} \) and those of \( T_2^{[1]} \)

\[
\rho_{J_s^c,J_s^c}\propto \sum_{l_z,s_z,s_z'} \langle 1, l_z; 1, s_z | J_s | 2, s_z' \rangle \langle 2, s_z' | J_s | 1, l_z; 1, s_z \rangle^{3/8}
\]

are used in our numerical results. The error bars are due to uncertainties of the CO LDMEs \( \langle S_1^{\chi_{c}} \rangle \) and errors in the normalized multipole amplitudes. From Figs. 3 and 4, we see that the measurements of these polarization observables may provide another important way to test the CO mechanism in the hadroproduction of heavy quarkonium. Moreover, our polarization predictions may also substantially reduce the systematic errors of experimental data at the LHC.

In summary, we have performed an analysis of the polarized \( \chi_{c1} \) and \( \chi_{c2} \) production at the LHC in NRQCD...
and in the color-singlet model. The complete NLO NRQCD predictions are given for the first time. These observables may provide important information, which is not available in the unpolarized $p_T$ spectra, in testing the validity of NRQCD factorization. In addition, the predictions of the $\chi_c$ polarizations can be used to reduce the experimental systematic errors in the measurement of $\chi_c$ production. Compared with $J/\psi$ production, the prompt $\chi_c$ production may play a unique role in understanding the heavy quarkonium production mechanism. Therefore we propose to measure these polarization observables at the LHC.

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