Future Dependent Initial Conditions from Imaginary Part in Lagrangian

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**Abstract**

We want to unify usual equation of motion laws of nature with “laws” about initial conditions, second law of thermodynamics, cosmology. By introducing an imaginary part – of a similar form but different parameters as the usual real part – for the action to be used in the Feynmann path way integral we obtain a model determining (not only equations of motion but) also the initial conditions, for say a quantum field theory. We set up the formalism for e.g. expectation values, classical approximation in such a model and show that provided the imaginary part gets unimportant except in the Big Bang era the model can match the usual theory. Speculatively requiring that there be place for Dirac strings and thus in principle monopoles in the model we can push away the effects of the imaginary part to be involved only with particles not yet found. Most promising for seeing the initial condition determining effects from the imaginary part is thus the Higgs particle. We predict that the width of the Higgs particle shall likely turn out to be (appreciably perhaps) broader than calculated by summing usual decay rates. Higgs machines will be hit by bad luck.

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1 Introduction

Usually when we talk about “theory of everything” as superstring theory is hoped to be, it is not really meant that the initial state of the universe is included in the model immediately. Rather one needs to make extra assumptions – cosmology, second law of thermodynamics\cite{1, 2, 3, 4}, etc. – about the initial conditions or one simply leaves it for the applicator of the theory to somehow himself manage to find out what the initial conditions are for the experiment he wants to describe with the theory. It is, however, the intention of the series of articles\cite{5, 6, 7, 8, 9} to which this article belongs to set up assumptions telling the initial conditions in a way that can be called that these initial condition assumptions are unified\cite{10} with the part of the theory describing the equations of motion and the particle content (the usually T.O.E.). Our unification may though be mainly a bit formal in as far as our main point is to use in the Feynmann path integral an action which has both a real $S_R$ and an imaginary part $S_I$. Usually of course the action is real and the imaginary part $S_I = 0$ (Total $S = S_R + iS_I$). We may quickly see that the imaginary part gives a typically hugely different extra factor in the probability for different paths obeying equations of motion. Thus such an imaginary part essentially fix the path obeying equations of motion which should almost certainly be the realized one. In this way we can claim that to a good approximation an imaginary part of the action will choose/settle the initial conditions.

In the present article it is not the point to settle on any choice of the in usual sense “theory of everything”. Rather we shall present our idea of introducing an imaginary part in the Lagrangian and thereby also in the action as a modification that can be made on any theory as represented by the real action $S$.

We have already published a few articles on essentially a classical formulation of the present model. We sought in these articles to be a little more general by simply defining a probability weight called $P$(path) defined for all possible paths. In classical theory it is really only the paths which obey the classical equations of motion for which we need to define $P$. We already in the earlier articles suggested that this probability $P$(path) for a certain track, path, to be the one realized in nature should be given as the exponential of an expression depending on the track, path, of the form of a space-time integral over a locally defined quantity $\mathcal{L}_I$ depending on the fields in the development, path. Really this quantity $\mathcal{L}_I$ (really $-\mathcal{L}_I$) comes into determining
the probability as if it were the imaginary part of the Lagrangian density.

A major point of the present article is to set up the quantum formulation of our already published model, now really settling on taking the suggestive idea of just making the action complex, but with a priori a different set of coupling constants and $m^2$ for real and imaginary part separately.

A genuine problem with our kind of model is that very likely it predicts that special simple configurations leading to big probability may be arranged at a priori any time. That is to say, with our type of model it needs an explanation that one in particle almost never see any great arrangements being organized to occur later on. Really such arrangements might seem to us to be something like a hand of God, but they seem very seldom. Thus at first it looks that our type of model is already falsified by the non-appearance of arrangements. Really such a problem is almost obviously expected to occur in a model that like ours does not a priori make any time reversal asymmetric assumption at the fundamental level. Unless in the Hartle-Hawking no boundary postulate [10] we add some time reversal asymmetry spontaneously other otherwise that theory will be up to similar problems [11, 12].

A model-language describing how final states can be imposed by a density matrix $\rho_f$ is put forward by Hartle and Gell-Mann [14].

In the present work we hope for that a certain moment in the ‘middle of times’ will turn out to become dominant w.r.t. fixing the special solution selected as the realized one, and that this time can then be interpreted as a close to Big Bang time (there may not really be a true big bang but just an inflation era coming out of a deflation era continuously). Then since we live in the time after this decisive Big Bang simulating era there is for us a time reversal asymmetry, nevertheless it is a problem that like ours is even timetranslational invariant w.r.t. the law that finally settle the ‘initial conditions’ to explain that there are not more prearranged events than one seemingly see.

However, we believe to have found some explanations able to suppress so many of these prearrangements that our model can be made compatible with present experience of essentially no prearrangements.

For really avoiding it we shall assume consistency of Dirac strings, but let us postpone that discussion to section 13 below.

Our model is really inspired from the considerations of time machines [13] and the
troubles of needs for prearrangements in order to avoid the so called grand mother paradoxes, meaning the inconsistencies occurring when one seeks to go back in time and changes the events there.

We shall present the work by making two attempts to assumption about how to interpret the Feynmann path integrals with the imaginary part of the action non-zero. In the first part of the paper we start out from letting the average of a dynamical variable $\mathcal{O}$ be given by equation (2.9) below, but that this is a priori not so good is seen by it not being (safely) real even if the dynamical variable $\mathcal{O}$ is real. Therefore in section 7 we restart the discussion so now from the side of the interpretation of the Feynmann path way integrals in our model.

**First trial of interpretation**

In the next section 2, we shall put forward the basic formula for expectation values with our complex action model and the philosophy that this model even deliver the initial condition, or better the solution of equation of motion to be the one realized.

In section 3 we review our earlier reasons for that future should have only little influence on what happens.

In section 4 we then shall argue for some approximate treatment of the functional integral in the late times $t$, the future.

In section 5 we shall make use of the approximation of the future to obtain the usual quantum mechanics expressions at least in the case where our imaginary part $S_I$ of the action can be ignored. (It should be stressed that we actually have used already a philosophy based on this $S_I$ being non zero, so it is not fully zero.)

In section 6 it turns out that we – perhaps not completely convincing though – can make the effect be that we return to probability in practical scattering experiments say get conserved.

**Second trial of interpretation:**

In section 7 we restart the discussion of making the interpretation formula for the Feynmann path way integral, which after some talk takes the way of using the classical approximation weighted with the exponential of minus 2 times the imaginary part of the action. In a subsection 7.5 we formally connect our model to our earlier one based on the probability weight $P(path)$.

In section 8 we develop a rather general formula for the correlated probability
for a series of dynamical quantities or operators $O_i$ at different moments of time take values inside small ranges specified.

In section 9 we go a bit further in making the expressions like the ones one uses in practice in usual theories. Most importantly we again consider how to approximate the future when the effects of the imaginary part of the action is very small.

In section 10 we put the simplest example of the more general formula, namely a formula for the probability of just one operator at one time being in a given range. (This question would be impossible to predict even in principle in other theories, but we in principle can, but in practice not usually). But the resulting formula has what we call “squared form” in the sense that the projector comes in twice as a factor in it. The finding of a reduction to an unsquared form is left to section 12, while we in section 11 then give an example of application of very interesting physical significance. In fact section 11 predicts a broadening of the width of the Higgs particle due to the imaginary action.

In section 12 we then bring about a connection between the postulated interpretation formulas for probabilities put forward in part I and part II. In fact we find that they coincide under rather suggestive assumptions.

In section 13 we bring the promised argument for removing the effects of the imaginary action $S_I$ from the domain of older accelerators, since otherwise our model would have been falsified. The argument is based on assuming monopoles.

In section 14 we conclude and give a bit of outlook.

**Part I, First Trial of Interpretation**

**2 Philosophy and formula**

Our basic modification of introducing an imaginary part in the actions leads to that integrand $e^{iS}$ or $e^{i\hbar S}$ of the Feynmann path way integrand $\int e^{iS} \mathcal{D}\phi$ or $\int e^{i\hbar S} \mathcal{D}\phi$ (if the Planck constant is written explicitly) varies a lot in magnitude, and not only in phase as usual. This effect is likely to make some regions in the space of paths – or we could restrict to the space paths with $\delta S = 0$, i.e. the space classical solutions – get a very much bigger weight in the integral than others. Actually it can likely happen that only a very narrow range of paths or better solutions (= paths obeying
\[ \delta S = 0 \) will quite dominate the integral
\[ \int e^{iS} \mathcal{D}\phi. \] (2.1)

That should naturally be taken to mean that the presumed narrow range of dominating paths represent the paths being actually realized in nature. It is in this way that we hope our model to essentially predict the initial state for the realized solutions. It is important to have in mind that such an effect of the imaginary action \( S_I \) of selecting narrow bunches of solutions can make the boundary conditions at an initial and a final time for a period to be studied say, superfluous. A bit optimistically we might imagine that the imaginary part of action makes the functional integral converge even without boundary condition specifications. Note that being allowed to throw boundary conditions away – having them replaced by effects of \( S_I \) – is a great/nice simplification. We consider this achievement as an aesthetically very nice feature of our model! Supposing that this works to deliver a meaningful Feynmann-path integral (2.1) even without boundary conditions this way we must now decide how one is supposed to extract information now in principle for the true expectation value as it should occur even without further input. Note here that we are – but only in principle – proposing an exceedingly ambitious model compared to usual quantum field theories:

We want to predict expectation values without any further input than the mere complex action! This of course corresponds to that our level of ambition is to in addition to the usual time-development laws of nature also predict the initial conditions, i.e. what really happens!

To write down the formula for some physical quantity let us first exercise by a quantity \( \mathcal{O}(\varphi|_t) \) which is a function of the fields \( \varphi|_t \) restricted to some time \( t \), where \( \varphi \) is a general symbol for all the fields in the model.

If we for instance use the Standard Model as the starting model, then providing it with an imaginary part of the Lagrangian density, then the symbol \( \varphi(x) (x \in \mathbb{R}^4) \) is really a set
\[ \varphi = (A^a_{\mu}, \psi^b, H) \] (2.2)
where the indices on the Fermion fields runs through the combination of flavor and color and/or \( W \)-spin components, while the index on the gauge fields run through
the 12 gauge fields – 8 gluon color combination plus \((B_\mu)\) the \(U(1)\)-component and 3 \(W\)’s –. Finally \(H\) is the two complex component Higgs field.

The quantity \(\mathcal{O}(\varphi|t)\) can of course be considered a functional of the whole field development \(\mathcal{O}(\varphi)\) also, i.e. it could be consider a functional of the path of one wants.

The simplest proposal for what the average quantity \(\mathcal{O}(\varphi)\) would be

\[
\langle \mathcal{O}(\varphi) \rangle = \frac{\int e^{iS[\varphi]} \mathcal{O}(\varphi) \mathcal{D}[\varphi]}{\int e^{iS[\varphi]} \mathcal{D}[\varphi]}.
\]

This would mean that we have a “sort of probability” given by

\[
\text{“Probability of } \mathcal{O} \text{ being } \mathcal{O}_0 = \frac{\int \delta(\mathcal{O}(\varphi) - \mathcal{O}_0) e^{iS[\varphi]} \mathcal{D}[\varphi]}{\int e^{iS[\varphi]} \mathcal{D}[\varphi]}
\]

Now, however, we must admit that conceiving of this expression as a probability is upset by the severe problem that it will typically be a complex number. There is no guarantee that it is positive or zero.

Thus a priori one would say that this simple expression for the probability density is quite untenable.

Nevertheless it is our intention to claim that we should – and that is then part of our model – use the simple expression \([2.4]\) and the corresponding \([2.3]\) and the expression to be given below for more general operators \(\mathcal{O}\) corresponding also to \([2.3]\) and \([2.4]\).

First let us again stress that it is our a priori philosophy that somehow the imaginary part \(S_I\) managed to fix both a state in future and in past. Thereby asking the average of some quantity \(\mathcal{O}\) becomes much like in an already finished double slit experiment (Bohr-Einstein) in which a particle already have been measured on the photographic plate (presumable on an interference line) what were the average position of the particle when it past the double slit screen. Really asking such a question concerning a quantity \(\mathcal{O}\) that were not measured and could not have been measured without having disturbed the outcome of something later is one of the forbidden questions in quantum mechanics. Indeed it is by asking this sort of questions which are not answerable by measurement that Einstein can find ammunition against quantum mechanics. In other words our proposal \([2.4]\) for “probability distribution” is a priori – with our present philosophy of a future essentially determined by \(S_I\) – an answer to a quantum mechanically forbidden question. Niels Bohr would say we should not ask it.
In that light it may of course not be so serious that our formula gives a rather stupid or crazy answer, a complex probability!

But now we have the problem of justifying that if we made a true measurement the answer would turn out to give positive (or zero) probability.

Let us take as the important feature of a measurement of some quantity \( \mathcal{O} \) that there is an apparatus which makes a lot of degrees of freedom, \( \xi \) say (really macroscopic systems) develop in a way depending on value of \( \mathcal{O} \). Such an amplification of the effect of the actual value of \( \mathcal{O} \) is characteristic for a measurement. Unless somehow there are special reasons for that \( S_I \) be insensitive to \( \xi \) (as we shall actually later seek to show but do not assume to be the case) we expect that \( S_I \) typically will depend on the macroscopically many d.o.f. \( \xi \) being influenced by \( \mathcal{O} \)-value measured. Now we argue like this: Since there is a huge (macroscopical) number of variables \( \xi \) depending on the value of \( \mathcal{O} \) “measured”, the imaginary part \( S_I \) of the action is likely to depend very strongly on this measured value – very rapidly varying.

We here think of \( S_I \) as the integral over the imaginary part of the Lagrangian \( L_I \) over all times \( t \in ]-\infty, \infty[ \)

\[
\left( S_I = \int_{-\infty}^{\infty} L_I dt \right)
\] (2.5)

Because of the great complications in an actual measuring apparatus, let alone the further developments depending the measured value, publications and so on, the imaginary action \( S_I \) can easily be a very complicated function of the measured \( \mathcal{O} \) value. Even if \( S_I \) as function of the measured \( \mathcal{O} \) value should in principle be continuous it may in practice vary so much up and down – caused by accidents influenced by the broadcasted measuring value – that very likely the smallest value of \( S_I \) occurs for a seemingly accidental value of the measured \( \mathcal{O} \). If the \( S_I \)-variation with the “measured \( \mathcal{O} \)” is indeed very strong so that the \( S_I \) variations are big the exponential weight \( e^{-S_I} \) contained in (2.3) and (2.4) will have a completely dominant value for only one measured \( \mathcal{O} \)-value.

In this way our model has the integral in the numerator of (2.4) be much bigger for one single value of \( \mathcal{O}_0 \). If so, then the ratio (2.4) is actually \( \propto 1 \) for this \( \mathcal{O}_0 \)-value and negligible for all other \( \mathcal{O}_0 \)-values. This means that our model much like usual measurement theory (in Copenhagen interpretation) predicts that crudely only one value of a measured quantity is realized. In principle it is even so that, bearing
a very special situation, the result of the measurement is calculable by essentially minimizing the imaginary action \( S_I \). In practice, however, such calculation will only be doable in extremely rare cases. (If we impress a special result by threatening with a Higgs-producing machine).

We postpone the argumentation for that the probability distribution to be obtained in practice shall be the one of usual quantum mechanical measurement theory partly to the later sections and partly to a subsequent paper.

At the end of this section let us extend slightly our formula (2.3) and thereby also (2.4), to the case where the quantity \( \mathcal{O} \) corresponds in usual quantum mechanics to an operator that do not commute with the fields \( \varphi \).

An operator corresponding to a quantity measurable at a moment of time \( t \) will in general in the quantum field theory considered be given by a matrix with a columns and rows in correspondence with field functions \( \varphi|_t \) restricted to the time \( t \). I.e. \( \mathcal{O} \) is given by a “matrix”

\[
(\varphi'|_t |\mathcal{O}| \varphi|_t) = \mathcal{O}(\varphi'|_t, \varphi|_t). \tag{2.6}
\]

What should be the formulas replacing (2.3) and (2.4) in this more general case?

Well, our main starting point were that we assumed our imaginary part \( S_I \) to (essentially) fix both a further \( |B\rangle \) and a past state \( |A\rangle \). A natural notation to introduce is in fact – for the past –

\[
\langle \varphi|_t |A\rangle = A[\varphi|_t] = \int_{\text{ending at } \varphi|_t} e^{iS_{-\infty}^t} \mathcal{O} \varphi \tag{2.7}
\]

and analogously

\[
\langle B|\varphi|_t\rangle = \langle \varphi|_t |B\rangle^* = B[\varphi|_t]^* = \int_{\text{beginning at } \varphi|_t} e^{iS_{t}^{\infty}} \mathcal{O} \varphi \tag{2.8}
\]

In this notation our previous formulas (2.3) and (2.4) are for the in \( \varphi|_t \) diagonal operators \( \mathcal{O}(\varphi|_t) \) become

\[
\langle \mathcal{O} \rangle = \frac{\int e^{iS} \mathcal{O}(\varphi|_t) \mathcal{O} \varphi}{\int e^{iS} \mathcal{O} \varphi} = \frac{\int_{\varphi|_t} \langle B|\varphi|_t\rangle \mathcal{O}(\varphi|_t) \langle \varphi|_t |A\rangle}{\int_{\varphi|_t} \langle B|\varphi|_t\rangle \langle \varphi|_t |A\rangle} = \frac{\langle B|\mathcal{O}|A\rangle}{\langle B|A\rangle}. \tag{2.9}
\]
and (2.4) becomes

\[
\text{“Probability of } \mathcal{O} \text{ being } \mathcal{O}_0 \text{” } = \frac{\langle B|\delta(\mathcal{O} - \mathcal{O}_0)|A \rangle}{\langle B|A \rangle} \quad (2.10)
\]

Now really we want to suggest that formula (2.9) and (2.10) can also be used for operators that are not simply functions of the fields \( \varphi|_t \) at time \( t \), used in the functional integral.

In order to justify that extension of our interpretation formulas we want to remark:

1. Provided a Hermitean operator \( \mathcal{O} \) has either \( |A\rangle \) or \( |B\rangle \) as eigenstate then the eigenvalue \( \mathcal{O}' \) in question can of course be extracted as

\[
\mathcal{O}' = \frac{\langle B|\mathcal{O}|A \rangle}{\langle B|A \rangle} \quad (2.11)
\]

2. One can quite generally – by Fourier transformations at every step in a time lattice – rewrite a functional integral of the Feynmann path way integral form from some set of variables \( \varphi \) to a conjugate set:

\[
\int e^{iS} \mathcal{D} \varphi \overset{\text{latticitation}}{=} \int \prod_{t \in \{t-\text{lattice}\}} \mathcal{D}^{(3)} \varphi|_t e^{i \sum_{t \in \{\text{lattice}\}} L_{\text{discr}} (\varphi|_t, \varphi|_t + \Delta t - \varphi|_t)} \Delta t
\]

\[
= \int \prod_{t \in \{t-\text{lattice}\}} \mathcal{U} (\varphi|_t + \Delta t, \varphi|_t) \mathcal{D}^{(3)} \varphi|_t \quad (2.12)
\]

where

\[
\mathcal{U} (\varphi|_t + \Delta t, \varphi|_t) = e^{i L (\varphi|_t + \Delta t - \varphi|_t)},
\]

can be rewritten into \( \mathcal{W} (\Pi|_t + \Delta t, \Pi|_t) \) matrices obtained from the \( \mathcal{U} (\varphi|_t + \Delta t, \varphi|_t) \) by Fourier functional transformations

\[
\mathcal{W} (\Pi|_t + \Delta t, \Pi|_t) \overset{\text{def}}{=} \int \mathcal{D}^{(3)} \varphi|_t + \Delta t e^{i \varphi|_t + \Delta t \Pi|_t + \Delta t} \mathcal{U} (\varphi|_t + \Delta t, \varphi|_t) e^{-i \varphi|_t \Pi|_t} \mathcal{D}^{(3)} \varphi|_t \quad (2.14)
\]

Now of course for long chains of \( \mathcal{W} \)-matrices (ignoring end problems) you have

\[
\int \prod_{t \in \{t-\text{lattice}\}} \mathcal{U} (\varphi|_t + \Delta t, \varphi|_t) \mathcal{D}^{(3)} \varphi|_t
\]

except for end problems

\[
\int \prod_{t \in \{t-\text{lattice}\}} \mathcal{W} (\Pi|_t + \Delta t, \Pi|_t) \mathcal{D}^{(3)} \Pi|_t \quad (2.15)
\]
Supposedly you can put the right hand side into a form

\[ \int e^{iS(\text{in} \Pi)}[\Pi, \Delta \Pi - \text{defferences}] \mathcal{D}\Pi \]  

(2.16)

Now you may argue with the same intuitive suggestion for getting

\[ \mathcal{O}(\Pi|_{t}) = \frac{\int e^{iS(\text{in} \Pi)} \mathcal{O}(\Pi|_{t}) \mathcal{D}\Pi}{\int e^{iS(\text{in} \Pi)} \mathcal{D}\Pi} \]  

(2.17)

as we did for (2.3). By thinking of doing the just presented Fourier transformation partly we might argue for a similar average formula for any operator

\[ \langle \mathcal{O}(\varphi|_{t}, \Pi \mathcal{J}) \rangle = \frac{\int e^{iS} \mathcal{O}(\varphi|_{t}, \Pi|_{t}) \mathcal{D}\varphi}{\int e^{iS} \mathcal{D}\varphi} = \frac{\langle B_{t}|\mathcal{O}(\varphi|_{t}, \Pi|_{t})|A_{t} \rangle}{\langle B_{t}|A_{t} \rangle}. \]  

(2.18)

Really this proposal looks very bad because of several lacks of good correspondence with usual quantum mechanics a priori:

a) Obviously \( |A_{t} \rangle \) is here (a sort of) wave function of the universe at time \( t \), but our probability density \( 2.4 \) or

\[ \text{“Probability for } \mathcal{O} \text{ being } \mathcal{O}_{0} \text{”} = \frac{\langle B_{t}|\delta(\mathcal{O} - \mathcal{O}_{0})|A_{t} \rangle}{\langle B_{t}|A_{t} \rangle} \]  

(2.19)

is not quadratic in \( |A_{t} \rangle \) as we expect from the usual corresponding formula

\[ \text{“Probability for } \mathcal{O} \text{ being } \mathcal{O}_{0} \text{ usual”} = \frac{\langle A_{t}|\delta(\mathcal{O} - \mathcal{O}_{0})|A_{t} \rangle}{\langle A_{t}|A_{t} \rangle}. \]  

(2.20)

b) As already stated the “probability density” \( 2.19 \) is even usual complex and needs the above measurement special case to become just positive.

We shall below argue for an approximate treatment of the future part \( |B_{t} \rangle \) of the integral thereby achieving indeed a rewriting into an expression which is of the form with \( |A_{t} \rangle \) coming squared. Indeed we shall rewrite \( 2.19 \) into \( 2.20 \) below.

2.1 Justification of philosophy from semiclassical approximation

In semiclassical approximation one simply evaluates different contributions to the functional integral (1) by seeking the different extrema for \( e^{iS} \) or equivalent \( S = S_{R} + \)
$iS_I$. Around such an extremum it is extremely well known that one can approximate $S$ by the Taylor expansion up to second order

$$S = S(\text{extremum}) + \frac{1}{2} \int \frac{\partial^2 S}{\partial \varphi_1(x_1) \partial \varphi_2(x_2)} \cdot (\varphi_1(x_1) - \varphi_1^{\text{extr}}(x_1)) (\varphi_2(x_2) - \varphi_2^{\text{extr}}(x_2)) + \cdots \ d^4x_1 d^4x_2 \tag{2.21}$$

where then the linear terms

$$\int \frac{\partial S}{\partial \varphi_1(x_1)} (\varphi_1(x_1) - \varphi_1^{\text{extr}}(x_1)) \ d^4(x_1) \tag{2.22}$$

vanish because of the extremiticity condition. Here $\varphi_1^{\text{extr}}(x_1)$ and $\varphi_2^{\text{extr}}(x_2)$ denote the fields at the extremum field configuration development. Such an extremum as is well known corresponds to a solution to

$$\delta S = 0 \tag{2.23}$$

i.e. solving the variational principle leading to classical equations of motion.

The main term in the exponent $iS(\text{extremum})$ is in the usual real action case purely imaginary and thus only gives rise to a phase factor so that in this approximation the contribution has the same size for all the classical solutions, provided they can go on for real field configurations. With our $S_I$ included, however, we tend to get even to the approximation of the first term in the Taylor expansion (2.21) a real term $-S_I$ into the exponent and thus the order of magnitude for one classical solution compared to another can easily become tremendous

$$|e^{iS(\text{extremum})}| = e^{-S_I(\text{extremum})}. \tag{2.24}$$

It is our philosophy that only relatively very few classical solution have $e^{-S_I(\text{extremum})}$'s dominating violently the rest. In this sense we expect and assumed that such one or a very few classical solutions could be considered the only one realized. With very big size of $S_I$ – and that can easily come about for a couple of reasons – it gets relatively only exceedingly few classical solutions that are competitive in the sense that for most classical solutions (of (2.23)) you have exceedingly small $e^{-S_I}$ compared to the few dominant ones. As the reasons for $S_I$ being big when it is not forbidden by gauge invariance and the condition that Dirac strings shall be unobservable we can give:
1. There is in analogy to the $S_R$-term a $\frac{1}{\hbar}$-factor in front of $S_I$. For practical purposes we know that we shall consider the Planck constant $\hbar$ to be very small.

2. We could easily get Avogadro's number come in as a factor in the $S_I$ because it would get such a factor a priori since there are typically in the world of macroscopic bodies of that order magnitude molecules.

3. Approximate treatment of future part of functional integral (treatment of $|B_t\rangle$)

In our earlier works[8] – in which we mainly worked in the classical approximation – we presented some arguments that in the era which have been going on since short time of after some effective (or real) Big Bang the imaginary Lagrangian or action $L_I$ or $S_I$ effectively became very trivial. That should mean that under the times starting after some early Big Bang and extending into the future we could approximately take $L_I$ and the part of $S_I$ coming from this era as independent of what are the practical possibilities for what can go on. Thus we should in this present era supposed to extend into even the infinite future be allowed to ignore in first approximation the imaginary parts $L_I$ or $S_I$.

The reasons, which we presented for that were that this present era including supposedly all future is dominated by two types of particles:

1. Massless particles (really the entropy of the universe is today dominated by the massless microwave background radiation of photons).

2. Non-relativistic particles carrying practically conserved quantum numbers (the nucleons and the electrons are characterized by their charges and baryon or lepton number so as to make their decays into lighter particles impossible).

The argument then went that we could write the action – actually both real $S_R$ and imaginary $S_I$ – for these particles, treated as particles, as a sum having each giving a contribution proportional to the eigentimes for them:

$$S_R, S_I = \sum_{\text{particles}} K_P \cdot \tau_P. \quad (3.1)$$
That is to say that each of the particles contribute to $S_I$ say a contribution proportional to the eigentime

$$S_{I\text{ from }P} \propto \tau_P. \quad (3.2)$$

Now for massless particles any step in eigentime

$$\Delta \tau_P = 0 \quad \text{(for massless)} \quad (3.3)$$

and for nonrelativistic ($\simeq$ slow) particles, such a step is

$$\Delta \tau_P = \Delta t \quad (3.4)$$

equal to the usual time. Since the number of the conserved quantum numbers protected particles are all the time the same the whole contribution to the $S_I$ from the present era becomes very trivial:

Zero from the massless, and just a constant integrated over coordinate time for the conserved particles.

In addition there are terms from interactions contributing a priori to say $S_I$ also. Since, however, in the era since a little after Big Bang the density of particles were low in fundamental units presumably also the interaction contributions would be much suppressed in this after Big Bang era.

So all together we estimate that it is only the very early Big Bang times that will dominate $S_I$. Thus the solution to the equations of motion being in a model with an imaginary action $S_I$ selected to be the realized one will mainly depend on what happened in that solution in the early Big Bang era. This means that it will be in our era as if it were the initial state that were a rather special one determined by having an especially small contribution to $S_I$ from Big Bang times. This would mean a rather well determined starting state roughly which interpreted as a macrostate would be one with low entropy. That is at least a good beginning for obtaining the second law of thermodynamics, since then there are supposedly no strong effects of $S_I$ any more to enforce the universe to go to any special macrostate. Rather it will go into bigger and bigger macrostates meaning that they have higher and higher entropy.

Although we have now argued for approximately seeing no effects of $S_I$ in the era after Big Bang implying that our model should have no effects in this era, this is however, presumably not being quite sufficiently accurate.
We shall, however, below in section 9 invent or find arguments that will allow us to get completely rid of the $L_I$ or $S_I$ from the in the Standard Model already found particles. Only for the Higgs involving processes our arguments in section 9 based on gauge symmetry and the assumption of unobservability of Dirac strings associated with monopoles will not quite function. Thus we still expect that an $S_I$-contribution pops up with Higgs-particles. But since Higgs-particles are so far not well studied such an effect of $S_I$ might well have been overlooked so far.

4 Treatment of $|B_t\rangle$ or Treatment of the future factor in the functional integral

In equation (2.8) above we defined what one could call “the future part” of the functional integral relative to the time $t$. It should however be kept in mind that it is a part in the sense that the full integral is a contraction (a sort of product) of the past part and this future part,

$$\int e^{iS} \mathcal{D}\varphi = \langle B_t|A_t\rangle.$$  \hspace{1cm} (4.1)

Now we must remember that according to the second law of thermodynamics the state of the universe if at all obtainable (calculable) should be so by considering the development in the past having lead to it. The future, however, should be rather shaped after what happened earlier. This suggests that we should mainly have the possibility to guess or know $|A_t\rangle$ but determined from the fundamental Lagrangian as our model suggests. Really in order not to disagree drastically with the second law of thermodynamics the future should be shaped from the past and reflect the latter. However, there should not be – at least not much – adjustment of the happenings at say time $t$ in order to arrange something special simple happening in future. This means in or formalism that the by the $S_I$ future contributions determined $|B_t\rangle$ should according to second law better disappear quite from our formula for predicting probabilities for operator values, i.e. from (2.4) or more generally (2.19).

Now, however, as we argued in foregoing section – section 3 – reviewing previous articles working in the classical approximation it should be the state of a solution to the equations of motion in the early Big Bang time that dominates the selection of such a solution to be the realized one. The future on the other hand has only a
small effect, if any, on choosing the true or realized solution. With the arguments to be given in section 9 we argue for the effects of $S_I$ being even smaller in the future. Nevertheless we have if we talk exactly also effects of $S_I$ even in the future. Otherwise the hypothesis that the integral (2.8) defining $|B_t\rangle$ would be senseless since the $e^{-S_I}$-weighting is needed to suppress the integrand $e^{-S_I}$ enough to make hope of a sensible practical convergence.

However, we have in section 3 and will in section 9 argue for that $S_I$ varies much less in the future than in Big Bang era.

It is now the purpose of the present section to use this only weak $S_I$ variation with the fields in the future to argue for an approximation in density matrix terminology for the future part $|B_t\rangle$ of the functional integral.

Let us indeed perform the following considerations for estimating the crude treatment of $|B_t\rangle$ which we shall use:

a) Since $S_I$ has in practice only small non-trivial contributions in the future it is needed to involve contributions in the integral

$$S_{I t' \to +\infty} = \int_{t'}^{\infty} dt \int d\vec{x} L_I$$

from very large $t \geq t'$.

b) At these enormous $t$ regions then at the end we get finally a rather restricted range of solutions. – we can think of classical solutions here, if we like –

c) Now the solutions from the enormously late times under a) have to be developed backward in time to the time $t'$ say to deliver the state $|B_{t'}\rangle$ (really we first get $\langle B_{t'}|\phi\rangle$ from equation (2.8)).

d) Now we make the assumption that the system/world is sufficiently “ergodic” and the large times so large and so smeared out (also because of the smallness of the $L_I$-effects) that we can take it that there is almost the same probability for finding the system in state $|B_{t'}\rangle$ at any place in phase space allowed by the conserved quantum numbers of the theory practically valid in the future era.

e) Ignoring for simplicity the conserved quantities we thus argued that with equal probability; equally distributed in phase space, we have that $|B_{t'}\rangle$ will be any state.
We can especially imagine that we have chosen a basis of wave packet states \(|w\rangle\) in the field configuration space so that they fill smoothly the phase space – accessible without violating the conservation laws relevant –. Taking these to be – approximately – orthonormal \(\langle w|w'\rangle \approx \delta_{ww'}\) we clearly get for the average expectation of the projection operator

\[ P_{B'} = |B'\rangle\langle B'| \quad (4.3) \]

the estimate

\[ \text{av}(|B'\rangle\langle B'|) = \frac{1}{N} \sum_{w} |w\rangle \langle w| \approx \frac{1}{N} \frac{1}{N} \quad (4.4) \]

where \(N\) is the number of states in the basis

\[ |w\rangle, \ w = 1, 2, \cdots, N. \quad (4.5) \]

That is to say we have argued for that our weak \(S_I\)-influence in future combined with an assumed approximate ergodicity leads to that we can approximate

\[ |B'\rangle\langle B'| \approx \frac{1}{N} \frac{1}{N} \quad (4.6) \]

in practice for all \(t'\) at least a bit later than the earliest Big Bang.

The crude estimate that we could replace \(|B_t\rangle\langle B_t|\) by \(\frac{1}{N} \frac{1}{N}\) derived as formula \((|B_t\rangle\langle B_t| \approx \frac{1}{N} \frac{1}{N})\) were based on that \(L_I\) were in practice small.

### 5 Deriving a more usual probability formula

We shall now make use of approximation (4.6) for the “future factor” in the functional integral in order to obtain an expression rewriting the formulas like (2.3), (2.4) and (2.18) and (2.19) into expressions analogous to (2.20).

The calculation is in fact rather trivial, starting say from the most general of our
postulated expressions (2.18):

\[
\langle O(\varphi|t, \Pi|t) \rangle = \frac{\int e^{iS} O(\varphi|t, \Pi|t) \mathcal{D}\varphi}{\int e^{iS} \mathcal{D}\varphi} = \frac{\langle B_t|O(\varphi|t, \Pi|t)|A_t \rangle}{\langle B_t|A_t \rangle}
\]

trivial step \(\Rightarrow\)

\[
\frac{\langle A_t|B_t\rangle \langle B_t|O(\varphi|t, \Pi|t)|A_t \rangle}{\langle A_t|B_t \rangle \langle B_t|A_t \rangle} = \frac{\langle A_t|\frac{1}{N} O(\varphi|t, \Pi|t)\rangle}{\langle A_t|\frac{1}{N}|A_t \rangle}
\]

using (4.6)

\[
= \frac{\langle A_t|O(\varphi|t, \Pi|t)|A_t \rangle}{\langle A_t|A_t \rangle} (5.1)
\]

which is the completely usual quantum mechanical expression for the expectation value of the operator \(O(\varphi|t, \Pi|t)\) in the wave functional state \(|A_t\rangle\).

With this expression we see that we should be allowed, as we anyway would expect, to use \(|A_t\rangle\) as the quantum state of the universe.

It should be noted though that our \(|A_t\rangle\) is in principle calculable from the “theory” when as we shall of course, count also the \(S_I\)-expression as part of the theory. In this way our model is widely more ambitious than usual quantum mechanics:

We have – much like the Hartle-Hawking no boundary proposal – a functional integral (2.7) delivering in principle the wave functional \(|A_t\rangle\). In usual quantum mechanics the wave function is left for the experimental physicist to find out from his somewhat difficult job of preparing the state. In practice we would presumably have to let him be so helped by observation and arrangements under the preparation that we almost leave to him the usual job. We should, however, have in mind that in preparing a state one will usually need to trust that some material is a rather pure chemical substance or that no disturbing cosmic radiation spoils the preparation. These kinds of trusts are usually based on some empirical experience which in turn makes use of that big assembles of pure substances are easily/likely available and that generally cosmic ray has low intensity. Such trusts however, are at the very root connected with the starting state – the cosmology – of our world. But this starting state for practical purposes is in our model based on the activity of our \(L_I\) in early Big Bang times of the initial state of the universe.

Thus it is even in the practical way of preparing a quantum state a lot of reference to our \(S_I\).
If, however, somehow the universe develops into states where \( L_I \) is no longer negligible we should expect corrections to such an approximation (\(|B_t\rangle\langle B_t| \approx \frac{1}{N}1\)).

## 6 Time development and \( S_I \) corrections to \(|B_t\rangle\)

From the definitions (2.7) and (2.8) of \(|A_t\rangle\) and \(|B_t\rangle\) it is trivial to derive the time development formulas for these Hilbert space vectors (say for \( t' > t \))

\[
|A_{t'}\rangle = \int_{\text{time } -\infty \text{ to } t'} e^{iS_{-\infty \text{ to } t'}D\varphi} \\
= \int_{t \text{ to } t'} e^{iS_{t \text{ to } t'}A_t[\varphi]|_t} \cdot D\varphi \\
= \mathcal{U}(t', t)|A_t\rangle
\]

(6.1)

where \( \mathcal{U}(t', t) \) is the operator corresponding to the matrix (with columns and rows marked by \( \varphi |_t \) configurations)

\[
\mathcal{U}(\hat{\varphi}|_{t'}, \hat{\varphi}|_t) = \int_{\text{over } t \text{ to } t'} \begin{cases} \mathcal{U}(\varphi|_{t'}, \varphi|_t) \end{cases} e^{iS_{t \text{ to } t'}D\varphi}. \]

(6.2)

Similarly we have from (2.8) for \( t' > t \) again, first taking the complex conjugate of (2.8)

\[
\langle \varphi|_t |B_t\rangle = \int_{\text{beginning at } \varphi|_t} e^{-iS_{t+\infty \text{ to } t'}D\varphi} \tag{6.3}
\]

and thus

\[
\langle \varphi|_t |B_t\rangle = \int_{\text{over } t \text{ to } t'} e^{-iS_{t+\infty \text{ to } t'}\langle \varphi|_t |B_t|_{t'}\rangle} D\varphi \tag{6.4}
\]

which can be written

\[
|B_t\rangle = \mathcal{U}_{\text{with } L_I \rightarrow -L_I}(t', t)\dagger |B_{t'}\rangle. \tag{6.5}
\]

Here we used that e.g.

\[
S_{t+\infty} = \int_t^\infty dt \int \text{d}^3 \vec{x} (\mathcal{L}_R + i\mathcal{L}_I) \tag{6.6}
\]

where \( \mathcal{L}_R \) and \( \mathcal{L}_I \) are respectively the real and the imaginary parts of the Lagrangian densities. So

\[
S_{t+\infty}^* = \int_t^\infty dt \int \text{d}^3 \vec{x} (\mathcal{L}_R - i\mathcal{L}_I), \tag{6.7}
\]

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and now restricting ourselves for \{pedagogics/simplicity\} at first to boson fields we have (usually) that for them $L_R$ and $L_I$ are even order in the time derivatives which are under latticification

$$\partial_t \varphi(t, \vec{x}) \approx \frac{\varphi(t + \Delta t, \vec{x}) - \varphi(t, \vec{x})}{\Delta t}.$$  \hfill (6.8)

Thus conceived as operators between the configuration at the two close by times $t$ and $t + \Delta t$, i.e. with columns and rows marked by $\varphi|_{t+\Delta t}$ and $\varphi|_t$ we have e.g.

$$(L_R + iL_I)^+ = L_R - iL_I$$ \hfill (6.9)

because

$$L_R^T = L_R \text{ and } L_I^T = L_I$$ \hfill (6.10)

and

$$L_R^* = L_R \text{ and } L_I^* = L_I.$$  \hfill (6.11)

In formula (6.5) of course the meaning of the under symbol text in the expression $U_{\text{with } L_I \rightarrow -L_I}(t', t)^+$ is that in addition to taking the Hermitian conjugation of $U(t', t)$ as defined by the matrix representation (6.2) one shall shift the sign for all occurrences of the $L_I$-part of the Lagrangian or of the $L_I$-part of the Lagrangian density. One should have in mind that it is easily seen that

$$U(t', t)^{-1} = U_{\text{with } L_I \rightarrow -L_I}(t', t)^+.$$  \hfill (6.12)

Especially the “usual” case of $L_I = 0$ means that $U(t', t)$ becomes unitary. This relation (6.12) together with (6.5) and (6.11) ensures that

$$\langle B_t | A_t \rangle = \int e^{iS_{\text{to } +\infty}} D\varphi$$  \hfill (6.13)

can be true independent of the time $t$ chosen on the left hand side.

Since (6.1) represents a completely usual time development of the ‘wave function’ $|A_t\rangle$ we have of course analogously to the usual theory

$$i \frac{d|A_t\rangle}{dt} = H|A_t\rangle$$ \hfill (6.14)

where then $H$ is the to the action

$$S = S_R + S_I$$ \hfill (6.15)
corresponding Hamiltonian. As we saw under point a) in section 2 formula (2.20) we can consider

\[ |A_t\rangle \quad (6.16) \]

the wave function for the universe essentially. But really because of the normalizing denominator in (2.20) it is rather the normalized \(|A_t\rangle\), namely

\[ |A_t\rangle_{\text{norm}} = |A_t\rangle / \sqrt{\langle A_t | A_t \rangle} \quad (6.17) \]

which is the true wave function.

It is important to remark that precisely because we now find that we shall use the normalized wave function rather than \(|A_t\rangle\) itself we do not get as could be feared a lack of conservation of probability due to the non-unitarity of the time development. Have in mind that the to a non-real action corresponding Hamiltonian \(H\) will not be Hermitean! But with the normalization coming from the \(\langle A_t | A_t \rangle\) in the denominator in (2.20) the total probability will anyway remain unity. This result matches nicely with the from the slightly different start evaluated (9.22) below.

**Part II, Second Trial of Interpretation**

### 7 Second Interpretation of the functional integral

Usually one only uses the functional integral over a time interval to evaluate a transition matrix element from an initial time \(t_i\) to a final time \(t_f\)

\[ U (\psi_f(\phi|f), \psi_i(\phi|i)) = \int \mathcal{D}\text{fixed time }\phi|f \int \mathcal{D}\text{fixed time }\phi|i \mathcal{D}\phi \ e^{iS_{t_i \to t_f}[\phi]} \quad (7.1) \]

where

\[ S_{t_i \to t_f} = \int_{t_i}^{t_f} \int \mathcal{L}(x)d^3\vec{x}dt \quad (7.2) \]

and the functional integral over \(\mathcal{D}\phi\) is restricted to \(\phi\)-functions (field developments, or paths) which at times \(t_i\) and \(t_f\) respectively coincides with \(\phi|i\) and \(\phi|f\) respectively.

In the present article we, however, have the ambition of having the functional integral determine a priori not only the development with time but also say something about the initial conditions so that we a priori might ask for the probability
of some dynamical variable $\mathcal{O}$ say having certain value $\mathcal{O}$ at a certain time without imposing any initial conditions. In order to obtain a formula or procedure or how to obtain such probabilities for what shall happen we have to assume such a formula.

We therefore need some intuitive and phenomenological guess leading to such a formula/prescription.

In order to propose such a formula in a sensible way we shall first consider a semiclassical approximation for our functional integral supposed to be connected with and describing the development of the Universe

$$\int \mathcal{D}\phi \, e^{iS[\phi]}$$

(7.3)

where we remember that in our model the $S[\phi]$ is not as usual real but is allowed to be complex.

### 7.1 Semiclassical approach

For first orientation let us imagine that the imaginary part of the action $S[\phi]$ is effectively small in the sense that we can obtain the most significant contributions to the functional integral by asking for saddle points for the real part $S_R$. That is we ask for field development solutions to the variational principle

$$\delta S_R = 0.$$  \hspace{1cm} (7.4)

Without specifying the boundary conditions at $t \to \pm \infty$ in our functional integral there should be (essentially) one solution for any point in the (classical) phase space of the field theory described. For the enumeration of the various development solutions $\phi$ we could use the field and conjugate field configuration at any chosen moment of time, to say. However, now our hope and speculation is that the imaginary part should give a probability weight distribution over the set ($\simeq$ phase space) of these classical solutions.

### 7.2 A first but wrong thinking

It is clear that we must make a definition of an expectation value for function(al) $\mathcal{O}$ say of the field development $\phi$ so that if a single (semi) classical solution $\phi_{sol}$ comes to be highly weighted then this expectation value should be $\mathcal{O}[\phi_{sol}]$. 

We might therefore at first think of
\[
\langle O \rangle = \frac{\int \mathcal{D}\phi O[\phi] e^{iS[\phi]}}{\int \mathcal{D}\phi e^{iS[\phi]}}. \quad (7.5)
\]

If really a single classical path contributed completely dominantly to both numerator and denominator, then indeed we would obtain that this proposal would obey
\[
\langle O \rangle = O[\phi_{\text{sol dom}}] \quad (7.6)
\]
where \(\phi_{\text{sol dom}}\) is this single dominant solution.

It is however likely that if will be more realistic to imagine that there is a huge number of significant classical solutions \(\phi_{\text{sol}}\). But then appears the “problem” that in the expansion of the numerator functional integral into contributions from the various (semi) classical solutions \(\phi_{\text{sol} i}\):
\[
\int \mathcal{D}\phi O[\phi] e^{iS[\phi]} = \sum_{\phi_{\text{sol} i \text{ all the classical solutions}}} e^{iS[\phi_{\text{sol} i}]} O[\phi_{\text{sol} i}] \sqrt{\det i^{-1}} \quad (7.7)
\]
the various contributions contribute with quite different signs or rather phases due to the appearance of the phase factor \(e^{iS_R[\phi_{\text{sol} i}]}\). The proposal just put forward thus is not as it stands a usual average, it lacks the usual requirement of an average of being performed with a positive weight. Rather the summation over the contribution becomes a summation with random phases to a good approximation. That means that if we classify in some ways the different solutions \(\phi_{\text{sol} i}\) into classes, then what would sum up when such classes are combined would be the squared contributions rather than the contributions themselves. In other words, if we define a contribution to
\[
\int \mathcal{D}\phi e^{iS[\phi]} O[\phi] = \sum_{\phi_{\text{sol} i}} \sqrt{\det i^{-1}} e^{iS[\phi_{\text{sol} i}]} O[\phi_{\text{sol} i}] \quad (7.8)
\]
where
\[
\det i = \det \left( \frac{\delta^2}{\delta \phi_1(x_1) \delta \phi_2(x_2)} \right) \quad (7.9)
\]
from a certain class of semi classical solution \(\mathcal{C}_k\) then the quantities such as
\[
\int O e^{iS} \mathcal{D}\phi \bigg|_{\text{from class } \mathcal{C}_k} = \sum_{\phi_{\text{sol} i \in \mathcal{C}_k}} \sqrt{\det i^{-1}} e^{iS[\phi_{\text{sol} i}]} O[\phi_{\text{sol} i}] \quad (7.10)
\]
obey approximately
\[
\left| \int \mathcal{O} e^{iS \mathcal{D} \phi} \right|_{\text{from class } \mathcal{C}_1}^2 + \left| \int \mathcal{O} e^{iS \mathcal{D} \phi} \right|_{\text{from class } \mathcal{C}_2}^2 \\
\approx \left| \int \mathcal{O} e^{iS \mathcal{D} \phi} \right|_{\text{from class } \mathcal{C}_1 \cup \mathcal{C}_2}^2.
\] (7.11)

However we do not have a similar addition formula for numerical values as
\[
\left| \int \mathcal{O} e^{iS \mathcal{D} \phi} \right|_{\text{from class } \mathcal{C}_1}^2,
\] (7.12)
when they are not squared. However, of course, we do have
\[
\left| \int \mathcal{O} e^{iS \mathcal{D} \phi} \right|_{\text{from class } \mathcal{C}_1}^2 + \left| \int \mathcal{O} e^{iS \mathcal{D} \phi} \right|_{\text{from class } \mathcal{C}_2}^2 \\
\approx \left| \int \mathcal{O} e^{iS \mathcal{D} \phi} \right|_{\text{from class } \mathcal{C}_1 \cup \mathcal{C}_2}^2
\] (7.13)
but this relation has terms of typically rather random phases.

### 7.3 Approaching a probability assumption

If we take \( \mathcal{O} \) to be a “projection operator” in the sense of being a functional of \( \phi \) only taking the values 0 and 1 then \( \left| \int \mathcal{O} e^{iS \mathcal{D} \phi} \right|_{\text{from class } \mathcal{C}_k} \) should give the chance for solutions in the class \( \mathcal{C}_k \) to pass through the configuration-class for which \( \mathcal{O}[\phi] = 1 \). Because of the (random) phase and the lack of simple numerical additivity mentioned if the foregoing subsection we are driven to assume that the probability for \( \phi \) being in the \( \mathcal{O}[\phi] = 1 \) region must be given by the squared contributions
\[
\left| \int \mathcal{O} e^{iS \mathcal{D} \phi} \right|_{\text{from class } \mathcal{C}_k}^2.
\] (7.14)

Calling the region in the space of \( \phi \)'s consisting of the \( \phi \)'s obeying \( \mathcal{O}[\phi] = 1 \) with our “project \( \mathcal{O} \)”, for region \( M \), we get
\[
\text{Prob}(M) \propto \left| \int M e^{iS \mathcal{D} \phi} \right|_{\text{from class } \mathcal{C}_k}^2
\] (7.15)
for restriction to the class \( \mathcal{C}_k \).

This means using the probability of the complementary set \( \mathbb{C} M \) of \( M \)
\[
\text{Prob}(\mathbb{C} M) \propto \left| \int \mathbb{C} M e^{iS \mathcal{D} \phi} \right|_{\text{from class } \mathcal{C}_k}^2
\] (7.16)
and the additivity (7.11)

\[
\left| \int e^{iS} \mathcal{D}\phi \right|_{\text{from class } \mathcal{C}_k}^2 = \left| \int_M e^{iS} \mathcal{D}\phi \right|_{\text{from class } \mathcal{C}_k}^2 + \left| \int_{\mathcal{C}_M} e^{iS} \mathcal{D}\phi \right|_{\text{from class } \mathcal{C}_k}^2
\]

we derive

\[
\text{Prob}(M) = \frac{\left| \int_M e^{iS} \mathcal{D}\phi \right|_{\text{from class } \mathcal{C}_k}^2}{\left| \int e^{iS} \mathcal{D}\phi \right|_{\text{from class } \mathcal{C}_k}^2} = \frac{\sum_{\phi_{\text{sol}} i \in M} e^{iS} \sqrt{\det_i^{-1}} \left| \text{from class } \mathcal{C}_k \right|^2}{\sum_{\phi_{\text{sol}} i \in \mathcal{M} \cap \mathcal{C}_k} e^{-2S_I \det_i^{-1}}} \quad \text{using random phases} \quad \approx \frac{\sum_{\phi_{\text{sol}} i \in \mathcal{M} \cap \mathcal{C}_k} e^{-2S_I \det_i^{-1}}}{\sum_{\phi_{\text{sol}} i \in \mathcal{C}_k} e^{-2S_I \det_i^{-1}}}.
\]

Here in principle of a classical approximation the $e^{-2S_I}$ factor is much more important than the “quantum correction” $\det^{-1}$. Thus we would ignore the determinant $\det^{-1}$ factor in first approximation.

Then we arrived to the picture here that the probability distribution over phase space - at some chosen time, that due to Liouville’s theorem does not matter - is given by $e^{-2S_I[\phi_{\text{sol}}]}$ where $\phi_{\text{sol}}$ is the classical field solution associated with the point in phase space for which $e^{-2S_I[\phi_{\text{sol}}]}$ shall be the probability density.

### 7.4 About the effect of $S_I$ in the classical approximation

To appreciate the just given probability density $e^{-2S_I[\phi_{\text{sol}}]}$ over phase space

\[
P(\phi|_{t_0}, \Pi|_{t_0}) \mathcal{D}\phi|_{t_0} \mathcal{D}\Pi|_{t_0} \propto e^{-2S_I[\phi_{\text{sol}}]} \mathcal{D}\phi|_{t_0}, \mathcal{D}\Pi|_{t_0}
\]

one should have in mind that in the classical approximation of the universe developing along a solution $\phi_{\text{sol}}$ to the equations of motion

\[
\delta S_R = 0,
\]

(7.20)
the development is given quite uniquely by the equations of motion. The only place in which the imaginary part then comes in is in weighting with various probability densities the various “initial state data” \((\phi|t_0, \Pi|t_0)\) – i.e. the phase space point –. Once you know the initial state of the (sub)system considered the equation of motion determines everything in the classical approximation determined by \(S_R\) just described, the \(S_I\) gets totally irrelevant. In other words it is only to know something about the “initial state” that \(S_I\) has relevance. Here the usual terminology of “initial state” shall especially in our model not be taken too seriously as far as it with the word “initial” refers to a beginning moment, the Big Bang start say. No, as we just mentioned one can use any moment of time \(t_0\) for the description of the phase space describing the set of classical solutions \(\phi_{sol}\). This \(t_0\) time does not have to be the first moment – even if such one should exist –. Rather we can use any moment of time as \(t_0\). In the usual theory we would tend to use \(t_0\) being the initial moment and the state at this moment should then be one of very low entropy describing our start of universe state. However, in our model there is the rather unusual feature that the probability weight \(e^{-2S_I[\phi_{sol}]}\) is given via a functional \(S_I[\phi_{sol}]\) depending on how the solution \(\phi_{sol}\) behaves at all different times \(t\) and not only at \(t_0\). Since we by the classical equations of motion can calculate the whole time development \(\phi_{sol}\) from the fields and their conjugate \((\phi|t_0, \Pi|t_0)\) at some chosen time \(t_0\), we can of course also consider \(e^{-2S_I[\phi_{sol}]}\) as a function of only the data at \(t_0\), \((\phi|t_0, \Pi|t_0)\).

So it is only by the fact that in our model \(e^{-2S_R[\phi_{sol}]}\) is a rather simple function of \(\phi_{sol}\) and thus because of the often chaotic development of the fields by the classical equations of motion typically a complicated function(al) of the time \(t_0\) data. With a more usual model one might think of the initial state in a “first moment” \(t_0\) would be specified by some sort of cosmological model or no boundary condition. In this case the probability density should be rather simple in terms of the first moment data. The simplicity of \(e^{-2S_R[\phi_{sol}]}\) as functional of the \(\phi_{sol}\)-behavior even at late times to some extend is extremely dangerous for our model showing observable effects not observed experimentally. Indeed an especially high probability for initial states leading to a special sort of happenings today say would look as a hand of God effect seeking to arrange just this type of happenings to occur. In practise we never know the state of the universe totally at a moment of time. So there would usually be possibilities to adjust a bit the initial conditions. That could then in our
model have happened in such a way that events or things to happen in the future gets arranged, if it can be done so as to organize especially big $e^{-2S_{R[\phi_{\text{sol}}]}}$ i.e. an especially low $S_{I}$. So a priori there would in our model be such “hand of God effects”. In a later section we shall, however, invent or propose a possible explanation that could make the era of today be of very little significance for the value of $S_{I}$ so that in the first approximation it mainly the early time features of a solution that counts for its probability density

$$e^{-2S_{I[\phi_{\text{sol}}]}} \sim f(\phi_{\text{sol}}|\text{“early times”}) = f(\text{early time part of } \phi_{\text{sol}})$$  (7.21)

### 7.5 Relation to earlier publications

We have earlier published articles working in the classical approximation seeking to produce a model behind the second law of thermodynamics by assigning a probability $P$ over the phase space of the Universe. It were also there the point that this probability density $P$ in our model should depend in the same way on the state at all times. We already proposed that this $P$ were obtained by imposing an imaginary part for the action $S_{I}$. According to the above we clearly have

$$P \propto e^{-2S_{I}}.$$  (7.22)

### 8 Suggestion of the quantum formula

We already suggested above that if $M$ denotes a subset of paths, e.g. those taking values in certain subset of $\phi|_{t}$-configuration space in a moment of time $t$, then the probability for the true path being in $M$ would be

$$\text{Prob}(M) = \frac{|\int_{M} e^{iS\phi}|^{2}}{\int |e^{iS\phi}|^{2}}.$$  (8.1)

We imagine the paths to be described by the field $\phi$ as function over $\mathbb{R}^{4}$, the Minkowski space. Thus we could use such an $M$ to describe e.g. the project of the possible development $\phi$ to some subspace of configuration space $M_{i}$ for a series of moments $t_{i}, i = 1, 2, \cdots, n$. In fact then we would have

$$M = \left\{ \phi \in \{\text{paths}\} | \phi|_{t_{i}} \in M_{i} \text{ for all } i \right\}.$$  (8.2)
It would in this case be natural to think of the functional integral
\[ \int_M e^{iS} D\phi \]  
(8.3)
as a product of a series of functional integral associated with the various time intervals in the series of times \(-\infty < t_1 < t_2 < \cdots < t_n < \infty\). In fact let us define
\[ \mathcal{U}_{t_i \to t_{i+1}}(\phi|_{t_{i+1}}, \phi|_{t_i}) \equiv \int_{\text{BOUNDARY}}^{t_{i+1} \to t_i} e^{iS} D\phi. \]
(8.4)
Here
\[ S_{t_i \to t_{i+1}}[\phi] = \int_{t_i}^{t_{i+1}} \int L(x) d^3\vec{x} dt \]
(8.5)
and remember that we here have the complex \(d(x)\),
\[ \mathcal{L}(x) = \mathcal{L}_R(x) + i\mathcal{L}_I(x). \]
(8.6)
We then can write
\[ \int_M e^{iS} D\phi = \mathcal{U}_{t_n \to \infty} \mathcal{U}_{t_{n-1} \to t_n} \cdots \mathcal{U}_{t_1 \to t_1}. \]
(8.7)
where \(\theta_i[\phi|_{t_i}]\) is the function
\[ \theta_i[\phi|_{t_i}] = \begin{cases} 1 & \text{for } \phi|_{t_i} \in \mathcal{U}_i \\ 0 & \text{for } \phi|_{t_i} \notin \mathcal{U}_i \end{cases} \]
(8.8)
We can also write this expression in language of a genuine operator product
\[ \int_M e^{iS} D\phi = \mathcal{U}_{t_n \to \infty} \mathcal{U}_{t_{n-1} \to t_n} \cdots \mathcal{U}_{t_1 \to t_1}. \]
(8.9)
where the \(\theta_i\)'s are now conceived of as projection operators on the space of wave functionals characterized by being zero outside \(M_i\),
\[ \theta_i \psi(\phi|_{t_i}) = \theta_i(\phi|_{t_i}) \psi(\phi|_{t_i}) = \begin{cases} 0 & \text{for } \phi|_{t_i} \notin M_i \\ \psi(\phi|_{t_i}) & \text{for } \phi|_{t_i} \in M_i \end{cases} \]
(8.10)
Here \(\psi\) is a possible/general wave functional for the state of the universe, in the formula presented at the moment \(t_i\).
In this operator formalism our probability formula takes the form

\[
\text{Prob}(M) = \text{Prob}(M_1, M_2, \cdots, M_n) = \frac{|U_{t_n}^{t_\infty} \theta_n U_{t_{n-1}}^{t_n} \theta_{n-1} \cdots \theta_1 U_{-\infty}^{t_1}|^2}{|U_{t_n}^{t_\infty} U_{t_{n-1}}^{t_n} \cdots U_{-\infty}^{t_1}|^2} \tag{8.11}
\]

Since we are anyway in the process of arguing along to just make an assumption about how to interpret in terms of probabilities for physical quantities of our complex action functional integral, we might immediately see that it would be suggestive to extend the validity of this formula for probabilities for field variables to also be valid for distributions in the conjugate fields \(\Pi|_{t_i}\) or in combinations,

\[
\text{Prob}(O_1 \in \tilde{M}_1, O_2 \in \tilde{M}_2, \cdots, O_n \in \tilde{M}_n) = \frac{|U_{t_n}^{t_\infty} P_{O_n \in \tilde{M}_n} U_{t_{n-1}}^{t_n} P_{O_{n-1} \in \tilde{M}_{n-1}} \cdots P_{O_1 \in \tilde{M}_1} U_{-\infty}^{t_1}|^2}{|U_{t_n}^{t_\infty} U_{t_{n-1}}^{t_n} \cdots U_{-\infty}^{t_1}|^2} \tag{8.12}
\]

Provided this proposal is not inconsistent to assume, we will assume it because it would be quite reasonable to assume that the analogous formula to (8.11) should be valid for any change for variables between \(\phi\) and \(\Pi\) in the formulation of our functional integral.

### 8.1 Consistency and no need for boundary conditions

It should be kept in mind that we expect that due to the presence of the imaginary part \(S_I\) in the action \(S\) it is not needed to require any boundary conditions at \(t \to \pm \infty\) so that we basically can remove as not relevant the \(\phi|_\infty\) and \(\phi|_-\infty\) boundaries which one would at first have considered to be needed in the expressions \(U_{-\infty}^{t_1}(\phi|_{t_1}, \phi|_-\infty)\) and \(U_{t_n}^{t_\infty}(\phi|_\infty, \phi|_{t_n})\). The imaginary part \(S_I\) is in fact expected to weight various contributions so strongly different that whenever the by this weighting flavored component in \(\phi|_\infty\) say is at all allowed by a potential choice of boundary condition then that contribution will dominate so much that all over contributions will be relatively negligible. So after taking the ratio for normalization such as (8.12) the choice of the boundary conditions for \(\phi|_\infty\) and \(\phi|_-\infty\) becomes irrelevant. This irrelevance of the boundary conditions would indeed allow us to formally put in according to our convenience of calculation whatever boundaries we might like provided it does not precisely kill the boundary wave function component.
flavored by the $S_I$. For instance we could put in at the infinity density matrices taken to be unity, since it does not matter anyway what we put and a unit matrix $\rho_1 = 1$ and $\rho_k = 1$ would not suppress severely any state such as the flavored one(s).

By this trick we could write our formula for probability

$$\text{Prob}(O_1 \in \tilde{M}_1, O_2 \in \tilde{M}_2, \ldots, O_n \in \tilde{M}_n) = \frac{\text{Tr}(\mathcal{U}_{t_n \to \infty} P_{O_n \in \tilde{M}_n} \mathcal{U}_{t_{n-1} \to t_n} P_{O_{n-1} \in \tilde{M}_{n-1}} \cdots P_{O_1 \in \tilde{M}_1} \mathcal{U}_{-\infty \to t_1} P_{O_1 \in \tilde{M}_1} U^\dagger_{t_{n-1} \to t_n} P_{O_{n-1} \in \tilde{M}_{n-1}} \mathcal{U}_{t_{n-2} \to t_{n-1}} P_{O_{n-2} \in \tilde{M}_{n-2}} \cdots P_{O_1 \in \tilde{M}_1} U^\dagger_{t_1 \to t_2} P_{O_1 \in \tilde{M}_1} U^\dagger_{-\infty \to t_1} P_{O_1 \in \tilde{M}_1} U^\dagger_{-\infty \to t_1})}{\text{Tr}(\mathcal{U}_{t_n \to \infty} U_{t_n \to \infty} P_{O_n \in \tilde{M}_n} \mathcal{U}_{t_{n-1} \to t_n} P_{O_{n-1} \in \tilde{M}_{n-1}} \cdots P_{O_1 \in \tilde{M}_1} U^\dagger_{t_{n-1} \to t_n} P_{O_{n-1} \in \tilde{M}_{n-1}} \mathcal{U}_{t_{n-2} \to t_{n-1}} P_{O_{n-2} \in \tilde{M}_{n-2}} \cdots P_{O_1 \in \tilde{M}_1} U^\dagger_{t_1 \to t_2} P_{O_1 \in \tilde{M}_1} U^\dagger_{t_1 \to \infty})}. \quad (8.13)$$

Here the reader should have in mind that because of the imaginary part in the action $S = S_R + iS_I$ the different development operators

$$\mathcal{U}_{t_i \to t_{i+1}}(\phi|_{t_{i+1}}, \phi|_{t_i}) = \int e^{iS[\phi]} \mathcal{D}\phi \quad (8.14)$$

are in general not as usual unitary. Therefore it is quite important to distinguish,

$$\mathcal{U}_{t_i \to t_{i+1}}^\dagger \text{ in general } \neq \mathcal{U}_{t_i \to t_{i+1}}^{-1}. \quad (8.15)$$

9 Practical Application Formulas

9.1 Practical application philosophy

Although in principle our theory is so much a theory of everything that it should even tell what really happens and not only what is allowed by the equations of motion, we must of course admit that even we knew the parameters of both $S_I$ and $S_R$ it would be so exceedingly hard to calculate what really happens that cannot do that.

We are thus first of all interested in using some reasonable approximations to derive (in a spirit of a correspondence principle) some rules coinciding under practical conditions with the quantum mechanics (or quantum field theory rather) rules we usually use.

Now as is to be explained in this section we can by means of requirements of monopoles and using the Standard Model gauge symmetries and homogeneity of the Lagrangian in the fermion fields argue that there will in the present era where Higgs
particles are seldom and Standard model applicable be only very small effects of $S_I$. We also seem to have justified to make the same assumption for very huge time spans in the future so that also until very far out in future the influence of $S_I$ is small. Even if we imagine that in the very long run $S_I$ selects almost uniquely the state or rather development - as we used above to argue that the boundary conditions $\phi|_{-\infty}$ and $\phi|_{+\infty}$ were unimportant - then in the practical (i.e. rather near) future we would expect the far future determination to deliver under an ergodicity assumption an effective density matrix proportional to the unit matrix.

### 9.2 Insertion of practical future treatment into interpretation formula

The above suggestion for the treatment of the practical future to be equally likely in “all” (practical) states is implemented by replacing what is basically taking the place of a future density matrix $\rho_f$ in our interpretation formula (8.13) namely

$$\rho_f \approx \mathcal{U}_{t_n \to \infty} \mathcal{U}_{t_1 \to \infty}$$  \hspace{1cm} (9.1)

by a normalized unit density matrix

$$\rho_f \approx \frac{1}{N} \mathbb{1}.$$  \hspace{1cm} (9.2)

where $N$ is the dimension of the Hilbert space. (In practice $N$ is infinite) So the interpretation formula becomes

$$\text{Prob}(O_1 \in \tilde{M}_1, O_2 \in \tilde{M}_2, \ldots, O_n \in \tilde{M}_n)$$

$$= \text{Tr}(P_{O_n \in \tilde{M}_n} \mathcal{U}_{t_{n-1} \to t_n} \mathcal{U}_{t_{n-1} \to \infty} \cdots P_{O_1 \in \tilde{M}_1} \mathcal{U}_{-\infty \to t_1})$$

$$\bigg/ \text{Tr}(\mathcal{U}_{t_n \to \infty} \cdots \mathcal{U}_{-\infty \to t_1} \cdots \mathcal{U}_{t_{n-1} \to t_n}) \bigg).$$  \hspace{1cm} (9.3)

where we used that

$$P_{O_n \in \tilde{M}_n} = P_{O_n \in \tilde{M}_n}^2.$$  \hspace{1cm} (9.4)
9.3 Conditional Probability

With formulas like (8.13) or (9.3) we can easily form also conditional probabilities such as

\[
\begin{align*}
\text{Prob}(O_{p+1} \in \bar{M}_{p+1}, \cdots, O_n \in \bar{M}_n | O_1 \in \bar{M}_1, \cdots, O_p \in \bar{M}_p) \\
= \text{Prob}(O_1 \in \bar{M}_1, \cdots, O_p \in \bar{M}_p, \cdots, O_n \in \bar{M}_n) \\
/ \text{Prob}(O_1 \in \bar{M}_1, \cdots, O_p \in \bar{M}_p).
\end{align*}
\]

(9.5)

In order to determine what happens if we know the wave function in some moment. Let us as an example consider the idealized situation of a case in which we know – by preparation set up – the whole state of the universe of one moment of time. This we could imagine being described by taking a series of \( P_{O_i \in \bar{M}_i} \) projection of the same moment of time, the moment in which we suppose that we know the wave function. For consistency and for being able to take the limit of them being at same time – and therefore with an ill-determined algebraic order in (8.13) we must assume these same time projectors to commute. If we consider the situation that we already know that the system has all these \( O_i \) in the small regions. \( \bar{M}_i \) because we know the wave function at their common time \( t_{\text{com}} \) then we are after that discussing only the conditional probabilities with the set of relations \( O_i \in \bar{M}_i, i = 1, \cdots, p \), taken as fixed.

For simplicity let us consider the simple case that we just ask for if a variable \( O_n \) at a later time being in \( \bar{M}_n \) or not then the conditional probability is in (8.13) form

\[
\begin{align*}
\text{Prob}(O_n \in \bar{M}_n | \psi) \\
= \text{Tr}(\mathcal{U}_{t_n}^{\dagger} P_{O_n \in \bar{M}_n} \mathcal{U}_{t_{\text{com}}}^{\dagger} \cdots P_{O_1 \in \bar{M}_1} P_{O_2 \in \bar{M}_2} \cdots P_{O_p \in \bar{M}_p} \mathcal{U}_{t_{\text{com}}} \cdots P_{O_n \in \bar{M}_n} \mathcal{U}_{t_n}^{\dagger}) \\
/ \text{Tr}(\mathcal{U}_{t_{\text{com}}} \cdots P_{O_1 \in \bar{M}_1} P_{O_2 \in \bar{M}_2} \cdots P_{O_p \in \bar{M}_p} \mathcal{U}_{t_{\text{com}}} \cdots P_{O_n \in \bar{M}_n} \mathcal{U}_{t_n}^{\dagger})
\end{align*}
\]

(9.6)

Herein we can substitute

\[
|\psi\rangle \langle \psi| = P_{O_1 \in \bar{M}_1} P_{O_2 \in \bar{M}_2} \cdots P_{O_p \in \bar{M}_p}
\]

(9.7)
and obtain using as usual for traces $\text{Tr}(AB) = \text{Tr}(BA)$

$$
\text{Prob}(O_n \in \tilde{M}_n \mid \psi) = \frac{\langle \psi \mid U_{\text{com}} \circ t_n P_{O_n \in \tilde{M}_n} U_{\text{com}} \circ t_n \mid \psi \rangle}{\langle \psi \mid U_{\text{com}} \circ t_n \mid \psi \rangle} \cdot \frac{\langle \psi \mid P_{O_n \in \tilde{M}_n} U_{\text{com}} \circ t_n \mid \psi \rangle}{\langle \psi \mid U_{\text{com}} \circ t_n \mid \psi \rangle}
$$

(9.8)

We may rewrite this expression in a suggestive way of the how it is modified relative to usual quantum mechanics by defining the final state density matrix from time $t_n$

$$
\rho_f \circ t_n \equiv \frac{U_{\text{com}} \circ t_n}{U_{\text{com}} \circ t_n} \frac{U_{\text{com}} \circ t_n}{U_{\text{com}} \circ t_n}
$$

(9.9)

in the following way

$$
\text{Prob}(O_n \in \tilde{M}_n \mid \psi) = \frac{\langle \psi \mid U_{\text{com}} \circ t_n P_{O_n \in \tilde{M}_n} \rho_f \circ t_n P_{O_n \in \tilde{M}_n} U_{\text{com}} \circ t_n \mid \psi \rangle}{\langle \psi \mid U_{\text{com}} \circ t_n \rho_f \circ t_n U_{\text{com}} \circ t_n \mid \psi \rangle}
$$

(9.10)

Here $U_{\text{com}} \circ t_n$ is really a non unitary $S$-matrix or development matrix for the time interval $t_{\text{com}}$ at which we have $\psi$ to $t_n$ at which we look for $O_n$. It is easy to see that we could have replaced $P_{O_n \in \tilde{M}_n}$ by a large set of commuting projections at a second common time $t_f$ destined to the single wave function $\mid \psi_f \rangle$ and thus allowing to replace the series of projections put in place of $P_{O_n \in \tilde{M}_n}$ by $\mid \psi_f \rangle \langle \psi_f \mid$. Then we get for the probability of the transition from $\mid \psi \rangle$ to $\mid \psi_f \rangle$

$$
\text{Prob}(\mid \psi_f \rangle \mid \psi) = \frac{\langle \psi \mid U_{\text{com}} \circ t_n \mid \psi_f \rangle \langle \psi_f \mid \rho_f \circ t_n \mid \psi_f \rangle}{\langle \psi \mid U_{\text{com}} \circ t_n \rho_f \circ t_n U_{\text{com}} \circ t_n \mid \psi \rangle}
$$

(9.11)

Now we can compare this expression with the usual transition probability expression when $S$ is only real $= S_R$

$$
\text{Prob}_{\text{usual}}(\mid \psi_f \rangle \mid \psi) = \frac{\langle \psi \mid U_{\text{com}} \circ t_n \mid \psi_f \rangle \cdot \langle \psi_f \mid U_{\text{com}} \circ t_n \mid \psi \rangle}{\langle \psi \mid U_{\text{com}} \circ t_n \mid \psi \rangle}
$$

(9.12)

Denoting the transition operator

$$
S \equiv U_{\text{com}} \circ t_n
$$

(9.13)
this means we have
\[
\text{Prob} \left( |\psi_f \rangle |\psi \rangle \right) = |\langle \psi_f | S |\psi \rangle|^2 \cdot \frac{\langle \psi_f | \rho_{f\text{ from } t_n} |\psi \rangle}{\langle \psi | \rho_{f\text{ from } t_n} |\psi \rangle} \tag{9.14}
\]
compared to the usual expression
\[
\text{Prob}_{\text{usual}} \left( |\psi_f \rangle |\psi \rangle \right) = |\langle \psi_f | S |\psi \rangle|^2 \tag{9.15}
\]
The deviations are thus the following:

1. With our imaginary part in \( S \) there is no longer unitality, i.e.
\[
S^\dagger \neq S^{-1}. \tag{9.16}
\]
The transition \( S \) is calculated by the Feynmann path integral with the full
\[
S = S_R + iS_I. \tag{9.17}
\]
2. There is the extra wright factor \( \langle \psi_f | \rho_{f\text{ from } t_n} \) describing the effect of the happenings and the \( S_I \) in the future of the “final measurement” \( |\psi_f \rangle \).
3. There is the only on the initial state \( |\psi \rangle \) dependent “normalization factor” in the denominator
\[
\langle \psi | S^\dagger \rho_{f\text{ from } t_n} S |\psi \rangle \tag{9.18}
\]
This denominator is indeed a normalization factor normalizing the total probability for reaching a complete set – an orthonormal basis – of final states \( |\psi_{f k}\rangle, k = 1, 2, \cdots \) which we for simplicity choose as eigenstate of \( \rho_{f\text{ from } t_n} \) so that \( \langle |\psi_{f k} \rangle \rho_{f\text{ from } t_n} |\psi_{f k} \rangle \) gets a diagonal matrix. Then namely
\[
\sum_{k=1,2,\cdots} \text{Prob} \left( |\psi_{f,k} \rangle |\psi \rangle \right)
= \frac{1}{\langle \psi | S^\dagger \rho_{f\text{ from } t_n} S |\psi \rangle} \sum_k |\langle \psi_{f,k} | S |\psi \rangle|^2 \langle \psi_{f,k} | \rho_{f\text{ from } t_n} |\psi_{f,k} \rangle
= \frac{1}{\langle \psi | S^\dagger \rho_{f\text{ from } t_n} S |\psi \rangle} \sum_k \langle \psi | S^\dagger \langle \psi_{f,k} | \rho_{f\text{ from } t_n} |\psi_{f,k} \rangle \langle \psi_{f,k} | S |\psi \rangle \tag{9.19}
\]
which by using that the off diagonal elements of \( \rho_{f\text{ from } t_n} \) were chosen to be zero can be rewritten as a double sum – i.e. over both \( k \) and \( k' \) –
\[
\sum_k \text{Prob} \left( |\psi_{f,k} \rangle |\psi \rangle \right) \langle \psi | S^\dagger \rho_{f\text{ from } t_n} S |\psi \rangle
= \sum_{k,k'} \langle \psi | S^\dagger |\psi_{f,k} \rangle \langle \psi_{f,k} | \rho_{f\text{ from } t_n} |\psi_{f,k'} \rangle \langle \psi_{f,k'} | S |\psi \rangle
= \langle \psi | S^\dagger \rho_{f\text{ from } t_n} S |\psi \rangle \tag{9.20}
\]
Thus we see that this denominator just ensures that the total probability for all that can happen at time $t_n$ starting from $|\psi\rangle$ at $t_{com}$ becomes just one.

### 9.4 Simplifying formula for conditional probability by approximating future

We have already suggested that we should approximate

$$\rho_{f\text{ from }t_n} \approx \frac{1}{N^1}$$

provided the future after $t_n$ is so that we can practically consider the $S_I$-effects small or so much delayed into the extremely far future that our above ergodicity argument can be used. By such an approximation we remove the deviation number 2 above given by $\langle \psi_f^{\prime} | \rho_{f\text{ from }t_n} | \psi_f^{\prime}\rangle$ because we approximate this matrix element $\langle \psi_f^{\prime} | \rho_{f\text{ from }t_n} | \psi_f^{\prime}\rangle$ by a constant as a function of $|\psi_f\rangle$. After this approximation we get

$$\text{Prob} (|\psi_f\rangle | |\psi\rangle) = \frac{|\langle \psi_f^{\prime} | S | \psi_f^{\prime}\rangle|^2}{\langle \psi | S \dagger S | \psi \rangle}$$

(9.22)

We should have in mind that $S^+ S \neq 1$ in general since with the imaginary part of the action the Hamiltonian will be non-hermitean and $S$ non unitary. Thus the usual $|\langle \psi_f^{\prime} | S | \psi_f^{\prime}\rangle|^2$ would by itself not deliver total probability for what comes out of $|\psi\rangle$ to be unity. Only after the division by the normalization $\langle \psi | S \dagger S | \psi \rangle$ would it become normalized to unity.

### 10 Can we make an unsquared form?

The formulas for the extraction of probabilities from our Feynmann path integral with imaginary part of action $S_I$ also were derived by considerations of statistical addition with essentially random phases of various classical path. But our crucial formula, say (8.13), for probabilities is seemingly surprisingly complicated in as far as each projection operator occurs twice in the trace in the numerator. Even the simplest example of asking if some variable $O$ at time $t$ falls into the range $\bar{M}$ gets
the expression

\[
\text{Prob}(O \in \bar{M}) = \frac{\text{Tr}(U_{t\to\infty} P_{O \in \bar{M}} U_{-\infty\to t} U_{-\infty\to t} P_{O \in \bar{M}} U_{t\to\infty})}{\text{Tr}(U_{-\infty\to\infty} U_{-\infty\to\infty})}
\]

(10.1)

containing the projection operator \(P_{O \in \bar{M}}\) twice as factor in the expression. If we make the approximation of no \(S_I\)-effects in the \(t\) to \(\infty\) time range by taking

\[
\rho_{f\text{ from } t_n} = U_{t\to\infty} U_{-\infty\to t}
\]

(10.2)

and approximating it by being proportional to the unit matrix then, however, the two projection operators come together and we could formally replace their product by just one of them. So in the case of the in this way approximated future we could write

\[
\text{Prob}(O \in \bar{M}) = \frac{\text{Tr}(P_{O \in \bar{M}} U_{-\infty\to t} U_{-\infty\to t} P_{O \in \bar{M}} U_{t\to\infty})}{\text{Tr}(U_{-\infty\to\infty} U_{-\infty\to\infty})}.
\]

(10.3)

If \(O\) were a variable among the variables used as the path-description in the Feynmann path integral the formula (10.1) would by functional integral be written

\[
\text{Prob}(O \in \bar{M}) = \frac{\left| \int_{\text{final}} P_{O \in \bar{M}} e^{iS} \mathcal{D}\phi \right|^2}{\left| \int e^{iS} \mathcal{D}\phi \right|^2}.
\]

(10.4)

Strictly speaking these Feynmann integrals should be summed over all end of time configurations, but with significant \(S_I\) presumably this summation would be dominated by a few “true” initial and states at \(\pm\infty\) and the summation would not be so important.

So strictly speaking we have

\[
\text{Prob}(O \in \bar{M}) = \frac{\sum_{\text{init},\text{final}} \left| \int_{\text{initial}} P_{O \in \bar{M}} e^{iS} \mathcal{D}\phi \right|^2}{\sum_{\text{init},\text{final}} \left| \int e^{iS} \mathcal{D}\phi \right|^2}.
\]

(10.5)

### 11 The Higgs width broadening

As an example of application of the \(S_I\)-caused modification of the usual transition matrices we may consider the decay of a particle – which we for reasons to be explained below take to be the Weinberg-Salam Higgs particle – which has from \(S_I\)
induced an imaginary term in the mass (or energy). Let us say take this term to have the effect of delivering a term being a positive constant number multiplied by $-i$ in the Hamiltonian. In the Schrodinger equation

$$i \frac{d\psi}{dt} = H\psi \tag{11.1}$$

such a term will cause the wave function $\psi$ to decrease with time so that it will decay exponentially with time $t$. If the particle in addition decays “normally” into decay products, say $b\bar{b}$ as the Higgs particles do the exponential decay rate will be the sum $\Gamma_{\text{normal}} + \Gamma_{S_I}$ of the $S_I$-induced width $\Gamma_{S_I}$ and the “normal” decay width $\Gamma_{\text{normal}}$. Let us for simplicity take as an approximation that the real part of the mass is very large compared to both the “normal” and the $S_I$-induced widths so that we can work effectively non relativistically with a resting Higgs particle. We can let it be produced in a short moment of time which is short compared to the inverse widths $\frac{1}{\Gamma_{S_I}}$ and $\frac{1}{\Gamma_{\text{normal}}}$ while still allowing the particle may be considered at rest approximately.

If we at first used the “usual” formula $|\langle \psi_f | S | \psi \rangle|^2$ for the decay process and calculate the total probability for the particle to decay into anything one will find that this probability is only $\frac{\Gamma_{\text{normal}}}{\Gamma_{\text{normal}} + \Gamma_{S_I}}$ because the average lifetime has been reduced by this factor, namely from $\frac{1}{\Gamma_{\text{normal}}}$ to $\frac{1}{\Gamma_{\text{normal}} + \Gamma_{S_I}}$. Since of course the usual particle with $\Gamma_{S_I} = 0$ will decay into something with just probability unity, we thus need a normalization factor $\langle \psi | S^\dagger S | \psi \rangle$ to rescale the total probability to be (again) unity in our imaginary action theory.

By Fourier transforming from time $t$ to energy the Higgs decay time distribution we obtain in our model again a Breit-Wigner energy distribution

$$P(E) = \frac{\Gamma_{\text{normal}} + \Gamma_{S_I}}{2\pi \left[ (E - m_{\text{Higgs}})^2 + \left( \frac{\Gamma_{\text{normal}} + \Gamma_{S_I}}{2} \right)^2 \right]} \tag{11.2}$$

If indeed we effectively should have such an $S_I$-induced imaginary part in the mass of the Higgs, then the Higgs-width could be made bigger than calculated in the usual width $\Gamma_{\text{normal}}$. This is an effect that might have been already seen in the LEP-collider provided one has indeed seen some Higgses in this accelerator. Indeed there has been found an excess of Higgs-like events with masses slightly below the established lower bound for the Higgs mass of 114 GeV/c.
11.1 The effect of the $\langle \psi_f | \rho_{f \text{ from } t_n} | \psi_f \rangle$ suppression factor

As an example of a (perhaps realistic) case of an effect of the factor $\langle \psi_f | \rho_{f \text{ from } t_n} | \psi_f \rangle$ we could imagine that two particles are coming together organized to hit head on – say in a relative s-wave - able to potentially form two different resonances of which say one is a Higgs which as above is assumed to have an imaginary term in its mass. Now it is easy to see that the $\rho_{f \text{ from } t_n}$ (here $t_n$ is the moment the collision just formed one of the two resonances thought upon as physical objects) will have $\langle \psi_f \text{Higgs} | \rho_{f \text{ from } t_n} | \psi_f \text{Higgs} \rangle < \langle \psi_f \text{all} | \rho_{f \text{ from } t_n} | \psi_f \text{all} \rangle$ (11.3) where $|\psi_f \text{Higgs}\rangle$ and $|\psi_f \text{all}\rangle$ represent respectively the two possible resonances the Higgs and the alternative resonance. Compared to the usual calculation of the transition to one of the resonances – essentially the square of the coupling constant – the Higgs-resonance will occur with suppressed probability because of the $\langle \psi | \rho_{f \text{ from } t_n} | \psi \rangle$-factor in the formula [9.11]. Really if the collision were safely organized that the collision occurs because of s-wave impact preensured the total probability for one or the other of the two resonances to be formed would be with properly normalized probability 1 because of the $\langle \psi | S^\dagger S | \psi \rangle$ normalization factor. However, the effect of $\langle \psi_f | \rho_{f \text{ from } t_n} | \psi_f \rangle$ would be to increase the probability to form the alternative resonance while decreasing the formation of the Higgs.

12 Approaching a more beautiful formulation

Taking the regions in which $O$ may lie or not $\bar{M}$ as infinitesimally extended we would the formula for the probability density in the form

$$\text{Prob}(O = O_0) = \frac{\sum_{i,f} \left| \int_{\text{BOUNDARY} : i,f} e^{iS[\phi]} P_{\bar{M}} D\phi \right|^2}{\sum_{i,f} \left| \int_{\text{BOUNDARY} : i,f} e^{iS[\phi]} D\phi \right|^2} \propto \frac{\sum_{i,f} \left| \int_{\text{BOUNDARY} : i,f} e^{iS[\phi]} \delta(O - O_0) D\phi \right|^2}{\sum_{i,f} \left| \int_{\text{BOUNDARY} : i,f} e^{iS[\phi]} D\phi \right|^2}$$ (12.1)

We may claim that this kind of formula the probability density for finding $O$ taking a value infinitesimally close to $O_0$ is a bit unaestetic because of having the projection operator $P_{\bar{M}}$ or the equivalent $\delta(O - O_0)$ occurring twice while one
might have said it would be simpler to have just one $\delta(O - O_0)$ or $P_{O \in \bar{M}}$ factor in the expression.

We should now seek to reformulate our expression with these factors occurring twice into a simpler one with only single occurrence of $P_{O \in \bar{M}}$ or $\delta(O - O_0)$. To perform this hoped for derivation we first argue that for nonoverlapping $O$-value regions $\bar{M}_1$ and $\bar{M}_2$

$$\sum_{i,f} \left( \int_{\text{BOUNDARY};i,f} \delta(O - O_1) e^{iS[\phi]} \mathcal{P}_\phi \right)^* \cdot \int_{\text{BOUNDARY};i,f} P_{O \in \bar{M}_2} e^{iS[\phi]} \mathcal{P}_\phi$$

$$\approx 0 \quad \text{for} \quad \bar{M}_1 \cap \bar{M}_2 = \emptyset. \quad (12.2)$$

This is argued for by maintaining that giving $O$ a different value at time $t$ very typically by “butterfly effect” – Lyapunov exponent – will cause very different states $f$ and $i$ at $\mp \infty$ respectively. If the two factors in (12.2) have very different final $f$ and initial $i$ states dominate at the boundaries and even random phases the total sum is indeed much smaller than what one would obtain if $\bar{M}_1$ and $\bar{M}_2$ were taken to be the same region $\bar{M}_1 = \bar{M}_2 = \bar{M}$. If we now use the zero expression (12.2) by adding such terms into the numerator and analogously in the denominator of (12.1) we can formulate this expression for the probability of $O$ being in $\bar{M}$ into an expression involving a summation over the value or region for $O$ in one of the occurrences

$$\text{Prob}(O = O_0^{(2)}) = \left( \sum_{i,f} \int dO_0^{(1)} \left( \int_{\text{BOUNDARY};i,f} \delta(O - O_0^{(1)}) e^{iS[\phi]} \mathcal{P}_\phi \right)^* \cdot \int_{\text{BOUNDARY};i,f} \delta(O - O_0^{(2)}) e^{iS[\phi]} \mathcal{P}_\phi \right)$$

$$/ \left( \sum_{i',f'} \left( \int_{\text{BOUNDARY};i',f'} e^{iS[\phi]} \mathcal{P}_\phi \right)^* \int_{\text{BOUNDARY};i',f'} e^{iS[\phi]} \mathcal{P}_\phi \right). \quad (12.3)$$

But now obviously we have

$$\int dO_0^{(1)} \delta(O - O_0) = 1 \quad (12.4)$$

and thus we get

$$\text{Prob}(O - O_0^{(2)}) = \frac{\sum_{i,f} \left( \int_{\text{BOUNDARY};i,f} e^{iS[\phi]} \mathcal{P}_\phi \right)^* \int_{\text{BOUNDARY};i,f} \delta(O - O_0^{(2)}) e^{iS[\phi]} \mathcal{P}_\phi}{\sum_{i',f'} \left( \int_{\text{BOUNDARY};i',f'} e^{iS[\phi]} \mathcal{P}_\phi \right)^* \int_{\text{BOUNDARY};i',f'} e^{iS[\phi]} \mathcal{P}_\phi} \quad (12.5)$$
In this expression we have achieved to have $\delta(O - O_0^{(2)})$ only occurring once as factor. We could therefore trivially extract from it an expression for the average of the $O$-variable

$$
\langle O \rangle = \int O_0^{(2)} \text{Prob}(O - O_0^{(2)}) dO_0^{(2)}
$$

$$
= \frac{\sum_{i,f} \left( \int_{\text{BOUNDARY}:i,f} e^{iS[\phi]} \partial \phi \right)^*}{\sum_{i',f'} \left( \int_{\text{BOUNDARY}:i',f'} e^{iS[\phi]} \partial \phi \right)^*} \frac{\int_{\text{BOUNDARY}:i,f} e^{iS[\phi]} \partial \phi}{\int_{\text{BOUNDARY}:i',f'} e^{iS[\phi]} \partial \phi}
$$

(12.6)

If we could somehow remove the after all identical complex conjugate functional integrals

$$
\left( \int_{\text{BOUNDARY}:i,f} e^{iS[\phi]} \partial \phi \right)^* 
$$

(12.7)

and

$$
\left( \int_{\text{BOUNDARY}:i',f'} e^{iS[\phi]} \partial \phi \right)^* 
$$

(12.8)

only deviating by the dummy initial and final state designations respectively ($i, f$) and ($i', f'$), then we could achieve the simple expression (7.5). But in order to argue for such removal being possible we would have to speculate say that some – we could say the true – boundary condition combination for the functional integrals (12.7, 12.8) completely dominates. This is actually not at all unrealistic since indeed the $S_I$ will tend to very few paths dominate. In such a case of dominance we would have a set of dominant ($f, i$). Presumably to make the chance that there should be such dominance we should allow ourselves to be satisfied with a linear combinations of $i$-state and of $f$-states to dominate. But now if indeed we could do that and call these linear combinations ($f_{\text{dom}}, i_{\text{dom}}$), then we could approximate

$$
\langle O \rangle \approx \frac{\int_{\text{BOUNDARY} f_{\text{dom}}, i_{\text{dom}}} O e^{iS[\phi]} \partial \phi}{\int_{\text{BOUNDARY} f_{\text{dom}}, i_{\text{dom}}} e^{iS[\phi]} \partial \phi}.
$$

(12.9)

Now we would like not to have the occurrence in this expression of the rather special states ($f_{\text{dom}}, i_{\text{dom}}$). However, these dominant boundary conditions are precisely the dominant boundary conditions for the denominator integral, because it were really
just the complex conjugate for the latter for which we looked for the dominant boundaries.

So if we let the boundaries free then at least the denominator should become dominantly just as if we had used the boundaries \((f_{\text{dom}}, i_{\text{dom}})\). It even seems that because of the smoothness and boundedness of the variable \(\mathcal{O}\) as functional of \(\phi\) the dominant boundaries \((i, f)\) should not be much changed by inserting an extra factor \(\mathcal{O}\) so that also by letting the boundaries free in the numerator functional integral \(\int \mathcal{O} e^{iS[\phi]} \mathcal{D}\phi\) would not change much the dominant boundaries from those of the same integral with the \(\mathcal{O}\)-factor removed. But the removal of this \(\mathcal{O}\) leads to the denominator functional integral, for which we already saw that the dominating boundary behavior were \((f_{\text{dom}}, i_{\text{dom}})\). Thus we have argued that we can rewrite (12.9) into

\[
\langle \mathcal{O} \rangle = \frac{\int \mathcal{O} e^{iS[\phi]} \mathcal{D}\phi}{\int e^{iS[\phi]} \mathcal{D}\phi} \quad (12.10)
\]

where it is understood that the boundaries for \(t \to \pm \infty\) are “free”. Then we suggested they would automatically go to be dominated by \((f_{\text{dom}}, i_{\text{dom}})\) thus fitting on to the formulas with double occurrence of \(\delta(\mathcal{O} - \mathcal{O}_0)\)’s.

The argumentation that the factor \(\mathcal{O}\) does not matter for the dominant behavior at \(\pm \infty\) may sound almost contradictory to our assumption using the “butterfly effect” to derive the rapid variation of (2.4) which meant that an insertion of \(\delta(\mathcal{O} - \mathcal{O}_0)\) would drastically change behavior, including that of the boundary.

It is, however, not totally unreasonable that a sharp function \(\delta(\mathcal{O} - \mathcal{O}_0)\) which is zero in most places could modify the boundary conditions, while a smooth one \(\mathcal{O}\), almost never zero would not modify them. Basically we hope indeed for that the \(S_I\)-caused weighting is so severely restricting the set of significant paths, that it practically means that a single path, “the realized path” is selected. In this case the insertion of the factor \(\mathcal{O}\) into the functional integral would just multiply it by the value of \(\mathcal{O}\) on “realized path”. If you however insert \(\delta(\mathcal{O} - \mathcal{O}_0)\) and it as most likely the case \(\mathcal{O}_0\) is not the value of \(\mathcal{O}\) on the realized path then we kill by the zero-value of \(\delta(\mathcal{O} - \mathcal{O}_0)\) at the realized path would totally kill the dominant contribution. Then of course the possibility for a completely different path is opened and the orthogonality used in (12.2) gets realistic.

As conclusion of the just delivered derivation of (12.10) we see that the starting
point in the beginning the articles is indeed consistent under the suggested approximations with the forms derived from the semiclassical start.

13 The monopole argument for suppressing the $S_I$ in the Standard Model

We have already above in section 3 argued that due to the material in the present era, and the future too, being either massless or protected from decay by in practice conserved quantum numbers and due to weakness of the interactions the contribution to $S_I$ from these eras must be rather trivial.

It were also for the above argument important that the non-zero mass particles were non-relativistic in these eras. That above argument may, however, not be sufficient for explaining that no effect of our $L_I$ or $S_I$ would have been seen so far. We have indeed had several high energy accelerators such as ISR (=intersecting storage ring at CERN) in which massive particles – such as protons – have been brought to run for days with relativistic speeds. That means that they would during this running in the storage rings say have had eigentime contributions significantly lower than the coordinate time or rather the time on earth. This would presumably easily have given significant contributions to $S_I$ which going to the exponent would suppress – or priori perhaps enhance – the probability of developments, solutions, to equations of motion, leading to the running of such storage rings. Since the protons have not already been made to run around dominantly relativistically we should deduce that most likely the storage rings would lead to increasing $S_I$ and thus lowering of the probability weight. Thus one would expect that the initial conditions should have been so adjusted as to prevent funding for this kind of accelerators, at least for them running long time. Contrary to Higgs producing accelerators which have so far not been able to work on big scale (may be L.E.P. produced a few Higgses for a short time) the accelerators with relativistic massive particles have seemingly worked without especially bad luck. In order to rescue our model it seems therefore needed to invent a crutch for it of the type that there are actually no $L_I$-contributions involving the particles for which the massive relativistically running accelerators have been realized. We have actually two mechanisms to offer which at the end can argue away our $L_I$ or $S_I$ effects for all the hitherto humanly produced
or found particles, leaving the hopes for finding observable effects – bad luck for accelerators, mysterious broadening of resonance peaks – to experiments involving the Higgs particle or particles outside the Standard Model. The point is indeed that we shall argue away the effects of $S_I$ for gauge particles and for Fermions (coupled to them).

The suppression rules to be argue for are:

1) Supposing the existence of monopoles we deduce that the corresponding full gauge coupling constants must be real, basically as a consequence of the Dirac relation.

2) For fields which like the Fermion fields in renormalizable theories occur homogeneously in the Lagrangian density $\mathcal{L}_R + i\mathcal{L}_I$ this Lagrangian density can be shown to be zero by inserting the equations of motion.

13.1 Spelling out the suppression rules

Spelling out a bit the suppression rules let us for the monopole based argument remind the reader that although we consider a complex Lagrangian density $\mathcal{L}_R + i\mathcal{L}_I$ for instance the electric and magnetic fields and the four potential $A_\mu(x)$ for electrodynamics are still real as usual. Now if we have fundamental monopoles there must exist corresponding Dirac strings which, however, as is well known must be unphysical. The explicit flux in the Dirac string must to have the Dirac string unobservable – to be unphysical – be compensated by an at the string singular behavior of the four potential $A_\mu$ around the Dirac string. The singular flux to compensate the extra flux in the Dirac string can, however, only be real since the $A_\mu$-field is real and it is given by a curve integral $\oint A_\mu \, dx^\mu$ around the Dirac string. Now as is well known the fluxes mentioned equal the monopole charges. Thus the monopole charge $g$ must be real. But then the Dirac relation

$$\tag{13.1} \ e \ g = 2\pi n, \quad n \in \mathbb{Z} $$

tells that also the electric charge $e$ must be real. Now, however, in the formalism with the electric charge absorbed into the four potential $\hat{A}_\mu = eA_\mu$ the coefficient on the $F_{\mu\nu}^2$-term in the Lagrangian density is $-\frac{1}{4\pi^2}$ so that the pure electromagnetic,
kinetic, Lagrangian density

\[ (L_R + iL_L) \bigg|_{\text{pure.m.}} = -\frac{1}{4e^2} F_{\mu\nu}^2 \]  

becomes totally real. I.e.

\[ L_L |_{\text{pure.m.}} = 0. \]  

We may skip or postpone a similar argument for non-abelian, Yang Mills, theories to another paper, but really you may just think of some abelian subgroup and make use of gauge invariance.

Concerning the rule 2) for homogeneously occurring fields, such as the Fermion fields in renormalizable theories the trick is to use the equations of motion. For example the part of the Lagrangian density \( L_R + iL_I \) involving a Fermion field \( \psi \) is of the form \( L_F = Z \cdot \bar{\psi}(i\slashed{D} - m)\psi \) where \( Z \) is a constant and \( D_\mu \) the covariant derivative and of course \( \slashed{D} = \gamma^\mu D_\mu \). This Fermionic Lagrangian density is homogeneous of rank two in the Fermion field \( \psi \). The Euler-Lagrange equations, the equations of motion for the Fermion fields are derived from functional differentiation w.r.t. the field \( \psi \)

\[ \frac{\delta S}{\delta \psi(x)} = 0 \]  

and end up giving equations of motion of the form

\[ \frac{\partial L_F}{\partial \psi} = 0 \]  

or

\[ \frac{\partial L_F}{\partial \bar{\psi}} = 0 \]  

(really these forms are only trustable modulo total divergences but that is enough) leading as is well known to

\[ \bar{\psi}(i\slashed{D} - m) = 0 \]  

or

\[ (i\slashed{D} - m)\psi = 0. \]  

But now it is a general rule that a homogeneous expression, \( L_F \) say, can be recovered from its partial derivatives

\[ \sum \frac{\partial L_F}{\partial \psi} \psi + \sum \bar{\psi} \frac{\partial L_F}{\partial \bar{\psi}} = \text{rank} \cdot L_F \]
where rank is in the present case rank= 2. Such a recovering for homogeneous Lagrangian densities, however, means that the Lagrangian density – at least modulo total divergences – can be expressed by the equation of motions, which are zero, if obeyed. But then at least in the classical approximation the Lagrangian density is zero at least modulo total divergences. This means that the total $S_R + iS_I$ contribution from the just discussed homogeneous terms end up zero. Especially the imaginary part also ends up zero, although its form does not have to be zero. It is only insertion of equations of motion that makes it zero.

14 Conclusion

We have put up a formalism for a non-unitary model based on extending the Lagrangian and thereby the action to be complex by allowing complex coefficients in the Lagrangian density $L_R + iL_I$.

We used two starting points for how to extract probabilities and expectation values from the Feynmann path way integral in our ambitious model that shall even be able to tell what really happens rather than just the equations of motion. The first were the interpretation that an operator $O(t)$ should have the expectation value

$$\langle O \rangle = \frac{\int O(t)e^{iS[\phi]} \mathcal{D}\phi}{\int e^{iS[\phi]} \mathcal{D}\phi}$$ (14.1)

but this expression is a bit dangerous in as far as it is a priori not guaranteed to be real even though the quantity $O(t)$ is real. The second approach would rather have a series of projections onto small regions $\bar{M}_i$ for operator $O_i(t_i)$ denoted $P_{O_i \in \bar{M}_i}$ inserted into the functional integral but then this integral is numerically squared for any combination $(i, f)$ of boundary behaviors at respectively $-\infty$ and $+\infty$ times.

That is to say that the insertions are to be performed into the integral $\int e^{iS[\phi]} \mathcal{D}\phi$ so as to replace the latter by $\int \prod_i P_{O_i \in \bar{M}_i} e^{iS[\phi]} \mathcal{D}\phi$ just as in the first approach, but then one forms the numerical square summed over the initial $i$ and final $f$ behaviors

$$\sum_{i,f} \left( \int_{\text{BOUNDARY}:i,f} e^{iS[\phi]} \mathcal{D}\phi \right) \ast \int_{\text{BOUNDARY}:i,f} e^{iS[\phi]} \mathcal{D}\phi.$$ (14.2)

The probability distribution is then obtained by inserting the projection operators into both factors in (14.2) and then as normalization divide the (14.2) having these insertion with (14.2) not having the insertions.
Under some suggestive assumptions we argued that the two approaches approximately will agree with each other. The most important formula derived is presumably the formula to replace usual unitary $S$-matrix or $U$-matrix transition between two moments in time in our model. This formula turns out for transition an initial state $|\psi\rangle$ to a final $|\psi_f\rangle$ to be

$$\text{Prod}(|\psi_f\rangle, |\psi\rangle) = \frac{|\langle \psi_f | S |\psi\rangle|^2 \langle \psi_f | \rho_{\text{from} t_f} |\psi_f\rangle}{\langle \psi | S^\dagger S |\psi\rangle}$$

(14.3)

We used that to derive the broadening in our model of the Higgs-width.

As an outlook we may mention some of the expectations of our model used in a more classical language in our earlier publications: If the Higgs – especially freely running Higgses — decrease significantly the probability (7.21) then the initial state should be organized so that Higgs production be largely avoided. This would actually make the prediction that some how or the other an accident will happen and the LHC-accelerator will be prevented from coming to full energy and luminosity.

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