A Supplement: on the Quantum-vacuum Geometric Phases

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Three related topics on the quantum-vacuum geometric phases inside a noncoplanarly curved optical fiber is presented: (i) a brief review: the investigation of vacuum effect and its experimental realization; (ii) our sequence of ideas of geometric phases of photons in the fiber; (iii) three derivations of effective Hamiltonian that describes the wave propagation of photon field in a curved fiber.

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I. A BRIEF REVIEW: THE INVESTIGATION OF VACUUM EFFECT AND ITS EXPERIMENTAL REALIZATION

Geometric phases has received special attention of many researchers since Berry showed that there exists a topological phase (i.e., the adiabatic geometric phase) in quantum-mechanical wavefunction in adiabatic quantum process where the cyclic evolution of wavefunction yields the original state plus a phase shift, which is a sum of a dynamical phase and a geometric phase shift [1]. In the published papers [2,3] Gao and we considered a second-quantized spin model which can characterize the wave propagation of photon field in a noncoplanarly curved optical fiber [4,5]. In this work [2,3], we dealt only with the case where the photon spin operator is taken the normal-order form, which does not involve the vacuum zero-point electromagnetic fluctuation fields. So, the previous mathematical treatment [2,3] cannot predict the existence of the geometric phases resulting from the vacuum photon fluctuation in the curved fiber. Here we refer to such vacuum-induced topological phases as the quantum-vacuum geometric phases. In order to treat the so-called quantum-vacuum geometric phases, we should study the non-normal-order spin operators of photon field, where the zero-point electromagnetic fields is involved in the effective Hamiltonian (and hence in the time-evolution equation, i.e., the time-dependent Schrödinger equation). For the detailed discussions, readers may be referred to the paper [6], where we discussed the contribution of vacuum quantum fluctuation to geometric phases of photons moving inside a sufficiently perfect optical fiber and suggested an experimental scheme to detect them by using the noncoplanar fiber made of certain gyrotropic materials with suitable permittivity and permeability tensors [6].

Nearly four years has passed since we began to consider the quantum-vacuum geometric phases. The reason that drew our attention to such vacuum effects was as follows: in the quantum field theory the infinite zero-point vacuum energy is often cancelled (deleted) by using the normal-product procedure and a new vacuum background is thus re-defined [7]. It is believed that this formalism is valid for the time-independent quantum systems since in these cases the divergent background energies may have no observable effects (except for the Casimir’s effect [8], vacuum polarization leading to Lamb’s shift [9], atomic spontaneous radiation due to the interaction of the excited atom with the zero-point electromagnetic field, as well as the anomalous magnetic moment of electron) and hence do not influence the observed physical results of the interacting quantum fields. However, if the normal product technique is applied to the time-dependent systems of quantum field theory (such as field theory in the expanding universe and the time-dependent gravitational backgrounds [10–12]), and thus the vacuum background is so re-defined by removing different zero-point energies at different time, then some observable vacuum effects (e.g., the vacuum contribution to

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Berry’s phase that arises in time-dependent quantum systems) may be deleted theoretically and therefore the validity of this formalism may deserve incredulity\(^1\).

Unfortunately, the left-handed polarized light due to vacuum fluctuation always inescapably coexists with the zero-point right-handed polarized light and the total of quantum-vacuum geometric phases is therefore vanishing [6], which may follow from the following expressions for the cyclic adiabatic geometric phases [6]

\[
\phi_L = - \left( n_L + \frac{1}{2} \right) \cdot 2\pi (1 - \cos \lambda), \quad \phi_R = \left( n_R + \frac{1}{2} \right) \cdot 2\pi (1 - \cos \lambda),
\]

where \(n_L\) and \(n_R\) denote the occupation numbers of left- and right-handed (LRH) circularly polarized photons, respectively. Note that the geometric phases at quantum-vacuum level of left-handed polarized light,

\[
\phi_L^{(\text{vacuum})} = - \frac{1}{2} \cdot 2\pi (1 - \cos \lambda),
\]

acquires a minus sign, which may be cancelled (counteracted) by \(\phi_R^{(\text{vacuum})}\), i.e.,

\[
\phi_R^{(\text{vacuum})} = + \frac{1}{2} \cdot 2\pi (1 - \cos \lambda)
\]

that possesses a plus sign [6].

Since the total vacuum geometric phases \((\phi_L^{(\text{vacuum})} + \phi_R^{(\text{vacuum})})\) is vanishing and therefore trivial, it prevents physicists from investigating experimentally this nontrivial vacuum effect \((i.e.,\) vacuum contribution to the geometric phases). This, therefore, means that our above theoretical remarks as to whether the normal-product procedure is valid or not for the time-dependent quantum field theory (TDQFT) cannot be easily examined experimentally. It depressed me to be confronted with such difficulties.

Moreover, as was stated more recently by Fuentes-Guridi et al., in a strict sense, the Berry phase has been studied only in a semiclassical context until now [13]. Thus the effects of the vacuum field on the geometric evolution are still unknown [13]. So, we think that in addition to its physical significance in investigating the normal-product procedure in time-dependent field theory, the quantum-vacuum geometric phases itself is also physically interesting. These factors always encouraged me to seek a way to test this vacuum effect.

During the last four years, I tried my best but often unfortunately failed to suggest an excellent idea of experimental realization of this quantum-vacuum geometric phases of photons in the fiber. We conclude that it seems not quite satisfactory to test the quantum-vacuum geometric phases by using the optical fiber that is made of isotropic media, inhomogeneous media \((e.g.,\) photonic crystals\(^2\)), left-handed media (a kind of artificial composite metamaterial with the negative permittivity and permeability and the consequent negative refractive index [14,15]), uniaxial (biaxial) crystals or chiral materials. Is it truly extremely difficult to realize such a goal? It is found finally that perhaps in the fiber composed of some anisotropic media such as gyrotropic materials (gyroelectric or gyromagnetic media) the quantum-vacuum geometric phases may be achieved test experimentally.

Gyrotropic media is such electromagnetic materials where both the electric permittivity and the magnetic permeability are tensors, which can be respectively written as [14]

\[
(\epsilon)_{ik} = \begin{pmatrix} \epsilon_1 & i\epsilon_2 & 0 \\ -i\epsilon_2 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_3 \end{pmatrix}, \quad (\mu)_{ik} = \begin{pmatrix} \mu_1 & i\mu_2 & 0 \\ -i\mu_2 & \mu_1 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix}. \tag{4}
\]

\(^1\)If in experiments we find no information on the existence of this geometric phases at quantum-vacuum level \((i.e.,\) this vacuum effect does not exist), then it is believed that the normal-product procedure in second quantization is still valid for time-dependent quantum systems. But, once if the quantum-vacuum geometric phases is truly present experimentally, then we might argue that the normal-product procedure in second quantization may be no longer valid for time-dependent quantum systems, namely, these vacuum effects associated with the time-dependent evolution process in the time-dependent quantum field theory should not be removed by the normal-product procedure \((i.e.,\) it is not appropriate to re-define a vacuum background by removing different zero-point energies at different time).

\(^2\)Photonic crystals are artificial materials patterned with a periodicity in dielectric constant, which can create a range of forbidden frequencies called a photonic band gap. Such dielectric structure of crystals offers the possibility of molding the flow of light (including the zero-point electromagnetic fields of vacuum). It is believed that, in the similar fashion, this effect \((i.e.,\) modifying the mode structures of vacuum electromagnetic fields) may also take place in gyrotropic media. The suppression of spontaneous emission in certain gyrotropic media can be regarded as an illustrative example.

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Let us assume that the electromagnetic wave is propagating along the third component of the Cartesian coordinate system. By using the Maxwellian equations, it is verified that the optical refractive indices squared of gyrotrropic media corresponding to the two directions of polarization vectors (left- and right-handed polarized) are of the form $n_{\pm}^2 = (\epsilon_1 \pm \epsilon_2)(\mu_1 \pm \mu_2)$. According to this expression for $n_{\pm}^2$, it is possible for us to choose: $n_+^2 > 0$ and $n_-^2 < 0$ [14], and then in these gyrotrropic media only one of the LRH polarized lights can be propagated without being absorbed by media. This result holds also for the zero-point electromagnetic fields [16–19] (e.g., the inhibition and enhancement of spontaneous emission in photonic crystals and cavity resonator, where the vacuum mode density is modified under certain conditions). If, for example, in some certain gyrotrropic media one of the LRH polarized lights, say, the left-handed polarized light (including its vacuum fluctuation field), dissipates due to the medium absorption and only the right-handed polarized light is allowed to be propagated (in the meanwhile the vacuum mode structure in these anisotropic media also alters correspondingly), then only the quantum-vacuum geometric phase of right-handed polarized light is retained and can be easily tested in the fiber fabricated from these gyrotrropic media [6].

Since it is known that people can manipulate vacuum so as to alter the zero-point mode structures of vacuum, which has been illustrated in photonic crystals [16] and Casimir’s effect (additionally, the space between two parallel mirrors, cavity resonator in cavity QED [17–19], etc.)\(^3\), we will put forward another scheme to extract the nontrivial quantum-vacuum geometric phases in the noncoplanar fiber by using Casimir’s effect. It is well known that in a finitely large space (e.g., the space between two parallel metallic plates, which is the main equipment of Casimir’s effect experiment), the vacuum-fluctuation electromagnetic field alters its mode structures\(^4\), namely, the zero-point field with wave vector $k$ less than $\sim (\frac{\pi}{a})$ is expelled from this space with a finite scale length $a$. Likewise, in some certain gyrotrropic media, if the electromagnetic parameters $\epsilon_1$, $\epsilon_2$, and $\mu_1$, $\mu_2$ in expressions (4) for electric permittivity and magnetic permeability tensors can be chosen to be $\epsilon_1 \simeq \epsilon_2$ and $\mu_1 \simeq \mu_2$, then the optical refractive index $n_-$ of left-handed polarized light is very small (or approaching zero) and hence the wave vector $k_- \simeq 0$, since in these media $k_-$ is proportional to $n_-$. Thus the left-handed polarized zero-point field inside these gyrotrotropic media is absent if, for example, the media are placed in a finitely large space, and consequently the only retained quantum-vacuum geometric phase is that of right-handed circularly polarized light. In order to perform the detection of this geometric phase, we should make use of the optical fiber that is fabricated from the above gyrotrropic media, and the devices used in Tomita-Chiao fiber experiments [4,5] should be placed in a sealed metallic chamber or cell, where the zero-point circularly polarized field with lower wave vector does not exist. Hence the measurement of quantum-vacuum geometric phases of photons may be achievable by means of this scheme.

Although the infinite vacuum energy in conventional time-independent quantum field theories is harmless and easily removed theoretically by normal-order procedure, here for a time-dependent quantized-field system, we think that the existence of quantum-vacuum geometric phases indicates that zero-point fields of vacuum will also participate in the time evolution process and perhaps can no longer be regarded merely as an inactive onlooker in time-dependent quantum field theories such as field theory in curved space-time, e.g., in time-dependent gravitational backgrounds and expanding universe [10,11]. In order to investigate this fundamental problem of quantum field theory, we hope the vacuum effect presented here would be tested experimentally in the near future. We also hope to see anyone else putting forward some new more clever suggestions of detecting this interesting geometric phases at quantum-vacuum level.

**Note added:** Just after I finished the paper [6], S.L. Zhu of Hongkong University drew my attention to Fuentes-Guridi et al.’s recently published work [13], where they considered the interaction between a spin-1/2 charged particle (similar to the case of two-level atom) and the external quantized electromagnetic field, and studied a so-called vacuum-induced Berry’s phase which they regarded as the contribution of vacuum-field fluctuation. I think that Fuentes-Guridi et al.’s result can also be treated in more detail by the method presented in our paper [3], where we obtained the exact solutions of the time-dependent supersymmetric two-level multiphoton Jaynes-Cummings model.

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\(^3\)Spontaneous emission from an excited electronic state reflects the properties of the surrounding vacuum-field fluctuations. By placing the radiator near a metallic surface or in a cavity, one can modify the spectral density of these fluctuations and either enhance or inhibit spontaneous emission [18]. For instance, spontaneous radiation by an atom in a Rydberg state has been inhibited by use of parallel conducting planes to eliminate the vacuum modes at the transition frequency [17].

\(^4\)Because its “wavelength” is much larger than the length scale of the surrounding space, it cannot form a stationary wave.
II. OUR SEQUENCE OF IDEAS OF INVESTIGATION OF GEOMETRIC PHASES IN THE FIBER

During the past four years, we studied the noncyclic nonadiabatic time evolution of photon wavefunction in the noncoplanar fiber by using the Lewis-Riesenfeld invariant theory, second-quantized spin model and non-normal product procedure [2,3,6]. The outline of our sequence of ideas is given as follows:

Chiao-Wu’s model (photons moving inside a helically curved fiber)  
Berry’s phase formula
  
  cyclic adiabatic geometric phase \( \phi^{(\sigma)}_{\text{ad}}(T) = 2\pi \sigma(1 - \cos \lambda) \)  
  Lewis-Riesenfeld invariant theory
  
  non-cyclic non-adiabatic geometric phase \( \phi^{(\sigma)}(t) = \sigma \int_0^t \gamma(t')[1 - \cos \lambda(t')]dt' \)  
  second-quantized spin model

quantal geometric phases \( \phi^{(\sigma)}(t) = (nR - nL)\int_0^t \gamma(t')[1 - \cos \lambda(t')]dt' \)  
  non-normal order procedure

quantum-vacuum phases \( \phi^{(\text{vacuum})}_{\sigma = \pm 1}(t) = \pm \frac{i}{\hbar} \int_0^t \gamma(t'[1 - \cos \lambda(t')]dt' \)  
  validity problem of normal order in TDQFT

experimental test is required  
unfortunately, \( \phi^{(\text{vacuum})}_{L}(t) + \phi^{(\text{vacuum})}_{R}(t) = 0 \)  
by using gyrotropic-medium fiber

III. THREE DERIVATIONS OF EFFECTIVE HAMILTONIAN OF PHOTONS INSIDE A NONCOPLANARLY CURVED FIBER

The effective Hamiltonian describing the light wave propagation in a curved optical fiber is helpful in considering the nonadiabatic noncyclic time evolution process of photon wavefunction in the fiber. We have three methods to derive this effective Hamiltonian [2,3]

\[
H_{\text{eff}}(t) = \frac{k(t) \times \dot{k}(t)}{k^2} \cdot S, \tag{5}
\]

where dot denotes the derivative of wave vector \( k \) with respect to time and \( S \) stands for the photon spin operators.

**Method i** By using the infinitesimal rotation operator of wavefunction

The photon wavefunction \( |\sigma, k(t)\rangle \) varies as it rotates by an infinitesimal angle, say \( \vec{\theta} \), namely, it obeys the following transformation rule

\[
|\sigma, k(t + \Delta t)\rangle = \exp \left[ -i \vec{\theta} \cdot J \right] |\sigma, k(t)\rangle, \tag{6}
\]

where \( \exp \left[ -i \vec{\theta} \cdot J \right] \approx 1 - i \vec{\theta} \cdot J \) with \( J \) being the total angular momentum operator of photon and \( k(t + \Delta t) = k(t) + \Delta k(t) \) with \( \Delta k(t) = \dot{k} \Delta t \). Here \( |\vec{\theta}\rangle \) is the angle displacement vector between \( k(t) \) and \( k(t + \Delta t) \), and the direction of \( \vec{\theta} \) is parallel to that of \( k(t) \times k(t + \Delta t) \). One can therefore arrive at

\[
\vec{\theta} = \frac{k(t) \times k(t + \Delta t)}{k^2} \Delta t. \tag{7}
\]

Thus it follows from Eq.(6) and (7) that

\[
\frac{\partial |\sigma, k(t)\rangle}{\partial t} = \frac{k(t) \times \dot{k}(t)}{k^2} \cdot J |\sigma, k(t)\rangle \tag{8}
\]

by calculating the time derivative of \( |\sigma, k(t + \Delta t)\rangle \). The total angular momentum is \( J = L + S \), where the orbital angular momentum \( L \) is orthogonal to the linear momentum \( k \) for the photon. So, \( \frac{k(t) \times k(t)}{k^2} \cdot L = 0 \) and the only retained term in \( \frac{k(t) \times k(t)}{k^2} \cdot J \) is \( \frac{k(t) \times k(t)}{k^2} \cdot S \). This, therefore, means that if we think of Eq.(8) as the time-dependent Schrödinger equation governing the propagation of photons in the noncoplanar fiber, then we can obtain the effective Hamiltonian (5).
Method ii  By using the equation of motion of a photon in a “gravitomagnetic” field
If the momentum squared $k^2$ of a photon moving in a noncoplanarly curved and sufficiently perfect optical fiber is conserved, then we can derive the following identity
\[
\dot{k} + k \times \left( \frac{k \times \dot{k}}{k^2} \right) = 0,
\]  
(9)
which can be regarded as the equation of motion of a photon in the noncoplanarly curved fiber. Since Eq. (9) is exactly analogous to the equations of motion of a charged particle moving in a magnetic field or a spinning particle moving in a rotating frame of reference, $-k \times \left( \frac{k \times \dot{k}}{k^2} \right)$ can be considered a “magnetic field” or “gravitomagnetic field” (thus $-k \times \left( \frac{k \times \dot{k}}{k^2} \right)$ can be thought of as a “Lorentz magnetic force” or “Coriolis force”). Similar to the Mashhoon et al.’s work (i.e., the derivation of the interaction Hamiltonian of gravitomagnetic dipole moment in a gravitomagnetic field) [20,21], one can also readily write the Hamiltonian describing the coupling of the photon “gravitomagnetic moment” (i.e., photon spin $S$) [6] to the “gravitomagnetic field” as follows
\[
H = \frac{k \times \dot{k}}{k^2} \cdot S,
\]  
(10)
which is just the expression (5).

Method iii  By using the Liouville-Von Neumann equation
If a photon is moving inside a noncoplanarly curved optical fiber that is wound smoothly on a large enough diameter [5], then its helicity reversal does not easily take place [22] and the photon helicity $I(t) = k(t) \cdot S$ is therefore conserved [4] and can thus be considered a Lewis-Riesenfeld invariant $I(t)$ [23], which agrees with the Liouville-Von Neumann equation
\[
\frac{\partial I(t)}{\partial t} + \frac{1}{i} [I(t), H(t)] = 0.
\]  
(11)
With the help of the spin operator commuting relations $S \times S = iS$, one can solve the Liouville-Von Neumann equation (11), namely, if the effective Hamiltonian\(^5\) is written as $H(t) = \frac{k(\varphi(t))}{k} \cdot S$, then according to the Liouville-Von Neumann equation, one can arrive at $[I(t), H(t)] = \left[ \frac{k(\varphi(t))}{k} \times h(t) \right] \cdot iS$, and readily obtain the expression for the effective Hamiltonian (5) of photons in the curved optical fiber, i.e., the coefficients of the effective Hamiltonian is $h(t) = \frac{k(\varphi(t)) \times k(t)}{k^2}$.

It is apparently seen that substitution of $I(t) = \frac{k(\varphi(t))}{k} \cdot S$ and $H(t) = \frac{k(\varphi(t)) \times k(t)}{k^2} \cdot S$ into the Liouville-Von Neumann equation (11) yields the equation of motion of a photon in a “gravitomagnetic” field, i.e., Eq.(9).

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\(^5\)Since in the Liouville-Von Neumann equation, $I(t) = \frac{k(\varphi(t))}{k} \cdot S$, it is certain that $H(t)$ should also be the linear combination of photon spin operator $S_1, S_2, S_3$. 

[1] M.V. Berry, Proc. Roy. Soc. London, Ser. A 392 (1984) 45.
[2] X.C. Gao, Chin. Phys. Lett. 19 (2002) 613.
[3] J.Q. Shen and H.Y. Zhu, Ann. Phys. (Leipzig) 12 (2003) 131.
[4] R.Y. Chiao and Y.S. Wu, Phys. Rev. Lett. 57 (1986) 933.
[5] A. Tomita and R. Y. Chiao, Phys. Rev. Lett. 57 (1986) 937.
An experimental realization of quantum-vacuum geometric phases by using the gyrotropic-medium optical fiber (2003).

J.D. Bjorken and S.D. Drell, Relativistic Quantum Fields (New York: Mc Graw-Hill Company, 1965) Chap. 15.

H.B.G. Casimir, Proc.K.Ned.Acad.Wet. 51, 793(1948)

H.A. Bethe, Phys. Rev., 72 (1947) 339; H.A. Bethe, L.M. Brown, and J.R. Stehn, ibid, 77 (1950) 370.

S.A. Fulling, What have we learned from quantum field theory in curved space-time? Quantum Theory of Gravity (Essays in honor of the 60th birthday of Bryce S Dewitt) Eds. S.M. Christensen (Bristol: Adam Hilger Ltd, 1984) 42-52.

L.H. Ford, Aspects of interacting quantum field theory in curved space-time Quantum Theory of Gravity (Essays in honor of the 60th birthday of Bryce S Dewitt) Eds. S.M. Christensen (Bristol: Adam Hilger Ltd, 1984) 125-134.

X.C. Gao, J. Fu, and J.Q. Shen, Eur. Phys. J. C, 13 (2000) 527.

I. Fuentes-Guridi, A. Carollo, S. Bose, and V. Vedral, Phys. Rev. Lett. 89 (2002) 220404.

V.G. Veselago, Sov. Phys. Usp. 10 (1968) 509.

J.B. Pendry, A.J. Holden, D.J. Robbins, and W.J. Stewart, IEEE Trans. Microwave Theory Tech. 47 (1999) 2075.

E. Yablonovitch, Phys. Rev. Lett. 58 (1987) 2059.

R.G. Hulet, E.S. Hilfer, and D. Kleppner, Phys. Rev. Lett. 55 (1985) 2137.

W. Jhe, A. Anderson, E.A. Hinds, D. Meshede, L. Moi, and S. Haroche, Phys. Rev. Lett. 58 (1987) 666.

D. Meshede, H. Walther, and G. Müller, Phys. Rev. Lett. 54 (1985) 551.

J.Q. Shen, H.Y. Zhu, S.L. Shi, and J. Li, Phys. Scr. 65 (2002) 465; J.Q. Shen, H.Y. Zhu, and J. Li, Acta Phys. Sin. 50 (2001) 1884.

B. Mashhoon, Phys. Lett. A 173 (1993) 347.

K.H. Guo and X.D. Jiang, High Ener. Phys. Nucl. Phys.(China) 26 (2002) 543.

H.R. Lewis and W.B. Riesenfeld, J. Math. Phys. 10 (1969) 1458.