Application of a novel grey model with dynamic background value to forecast nuclear generation in China

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Abstract. Nuclear generation is an important energy construction task in China, which is regarded as the work focus of the next stage by the government. Achieving accurate predictions of nuclear generation behavior will effectively improve energy dilemma and further promote energy economics in China. In our work, the error of the classical grey model is analyzed. And a novel grey model optimized background value with series parameters is proposed, which enables more accurate calculation of background values. The application case results show the proposed model outperforms other grey models in forecasting nuclear generation in China.

1. Introduction
Faced with the shortage of energy resources and low carbon demand in the future, nuclear energy is one of the important pillars to meet energy supply and ensure national security, as well as an important way to achieve zero pollution emission. [1] At present, nuclear generation accounts for 10.4% of the total global electricity generation. Efficient development of nuclear generation on the basis of ensuring safety is an important policy of energy construction in China, which is of great significance to ensuring energy supply and safety, protecting the environment and realizing sustainable development. Therefore, it is urgent to propose a prediction model that can accurately predict the nuclear generation in China.

Grey system theory was first proposed in 1980s [2], which was known as the ability in solving the uncertain problems with little samples and poor information. For decades, the issue about background value of the grey model has always been a hot topic to research. Tan et.al [3] [4] [5] firstly improved the model according to the geometric meaning of the background value, which not only kept the characteristics of simple and easy operation of the grey model but also expanded the application scope of the model. Luo et.al [6] utilized homogeneous exponential function to conduct curve fitting of the background value. Tang et.al [7] designed a grey model based on quadratic interpolation to construct background values. Later, many researchers expected to employ numerical integration theory to improve the calculation accuracy of background values, which containing Gauss formula [8], Simpson formula [9] and Newton-Cotes formula [10]. Meanwhile, Xie et.al [11] proposed a new method about the calculation of background values with a tuneable parameter, which provided a new idea for the improvement of background values. The above research work improved the calculation accuracy of the background value and the prediction effect of the model to some extent.

All in all, how to calculate the background value accurately is considered to be the key to improve the prediction performance of the model. In this paper, a more general method for calculating background values is proposed, which effectively improves the prediction accuracy of the model.
Moreover, the real world application case results illustrate the model our proposed is reliable and effective.

2. Classical grey GM (1,1) model

The accumulated generation operation is a key premise to construct grey model, which has effectively reduced or even eliminated the randomness of the original data sequences. In general, for an original data sequence \( X^{(0)} = \left( x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n) \right) \), we define the accumulated generation operator (AGO) of \( X^{(0)} \) as follows.

**Definition 2.1** Assume the sequence \( X^{(i)} = \left( x^{(i)}(1), x^{(i)}(2), \ldots, x^{(i)}(n) \right) \) be the accumulated generating operator (AGO) of \( X^{(0)} \), where

\[
x^{(i)}(k) = \sum_{i=1}^{k} x^{(0)}(i), k = 1, 2, \ldots, n.
\]

Conversely, we can define \( X^{(0)} \) as the inverse accumulated generation operator (IAGO) of \( X^{(i)} \), where

\[
x^{(0)}(k) = x^{(i)}(k) - x^{(i)}(k-1), k = 1, 2, \ldots, n
\]

**Definition 2.2** Assume the sequence \( X^{(0)} \) and \( X^{(i)} \) be the original data sequence and its accumulated generation operator, the background value sequence can be defined as \( Z^{(i)} = \left( z^{(i)}(2), z^{(i)}(3), \ldots, z^{(i)}(n) \right) \), where

\[
z^{(i)}(k) = \frac{1}{2} \left( x^{(i)}(k) + x^{(i)}(k-1) \right), k = 2, 3, \ldots, n
\]

The equation

\[
x^{(0)}(k) + a z^{(i)}(k) = b
\]

is called the discrete formulation of the GM (1,1) model, and the parameters set \( \Theta = [a, b]^T \) can be estimated by least-square method and satisfies

\[
\Theta = \left( B^T B \right)^{-1} B^T Y
\]

Where

\[
Y = \begin{bmatrix}
x^{(0)}(2) \\
x^{(0)}(3) \\
\vdots \\
x^{(0)}(n)
\end{bmatrix},
B = \begin{bmatrix}
-\frac{1}{2} \left( x^{(i)}(2) + x^{(i)}(1) \right) & 1 \\
-\frac{1}{2} \left( x^{(i)}(3) + x^{(i)}(2) \right) & 1 \\
\vdots & \vdots \\
-\frac{1}{2} \left( x^{(i)}(n) + x^{(i)}(n-1) \right) & 1
\end{bmatrix}
\]
Definition 2.3 Assume all the parameters have the same definition as defined previously, the whitening differential equation of the GM (1,1) model is defined as follows:

\[
\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = b
\]  

(7)

3. The GM (1,1, λ(k)) model

In this section, the error about classical grey GM (1,1) model is discussed in subsection 1. After that, an ingenious construction method of the background value is presented and constructs a novel GM (1,1, λ(k)) model, more details about the proposed model is presented in this section.

3.1. Error analysis of the model

As our previous knowledge, the precision of modelling and prediction depend largely on the parameters \(a\) and \(b\). At the same time, the size of background value sequence \(z^{(1)}(k)\) is directly related to parameters \(a\) and \(b\). By integrating equation \(dx^{(1)}(t)/dt + ax^{(1)}(t) = b\) on interval \([k-1, k]\), one obtains

\[
\int_{k-1}^{k} dx^{(1)}(t) + a\int_{k-1}^{k} x^{(1)}(t)dt = \int_{k-1}^{k} bdt
\]  

(8)

Simplify the equation (8),

\[
x^{(1)}(k) - x^{(1)}(k-1) + a\int_{k-1}^{k} x^{(1)}dt = b
\]  

(9)

Compare the above equation and \(x^{(0)}(k) + az^{(1)}(k) = b\) one has

\[
z^{(1)}(k) = \int_{k-1}^{k} x^{(1)}(t)dt
\]  

(10)

In order to illustrate more intuitively, the geometric meaning about background value of grey model is drawn in figure 1. Obviously, we noticed that the area of curved trapezoid is background value of classical GM (1,1) model, which consists of curve \(x^{(1)}(t)\) on interval \([k-1, k]\) and horizontal axis \(t\). In fact, the trapezoid formula is applied to approximate the area of curved trapezoid in traditional grey GM (1,1) model, which also means that the traditional model ignores the errors caused by the shaded parts in figure 1.

Figure 1. Geometric representation of the background value
3.2. The representation of the grey GM (1, 1, λ(k)) model

To receive a satisfactory method to approach the exact value about the background value of grey model, a lot of mathematical methods and ideas are utilized to remedy this problem including the curve fitting, the interpolation theory as well as some tedious numerical integral formulas. These works have improved the calculation accuracy of the background value to some extent.

Unfortunately, they cannot fundamentally achieve the accurate calculation of the background value, and the improvement effect is not universal. Literature [11] designed a new GM (1, 1, λ) model with improved background value, and proved its improvement is effective using practical numerical cases. In this literature, the background value is designed as follows:

\[ z^{(1)}(k) = \lambda x^{(1)}(k) + (1 - \lambda) x^{(1)}(k - 1) \]  

where λ is an adjustable parameter and belongs to [0, 1]. Particularly, the GM (1, 1, λ) model would degenerate to GM (1, 1) model when λ=0.5. The mean value theorem inspires that this design is persuasive. Moreover, we have also taken noticed that the parameter of the background value in GM (1, 1, λ) is regarded as a constant. In general, it’s difficult to seek an optimal parameter to solve most problems. Therefore, a general background value improvement scheme is proposed in this paper.

**Definition 3.1** The background value sequence \( Z^{(i)} = \{z^{(1)}(2), z^{(1)}(3), \ldots, z^{(1)}(n)\} \) generated by consecutive neighbors of \( X^{(i)} \) is defined as:

\[ z^{(1)}(k) = \lambda(k-1) x^{(1)}(k) + (1 - \lambda(k-1)) x^{(1)}(k - 1) \]  

where \( \lambda(k) \) is adjustable parameters of the background value and \( \lambda(k) \in [0, 1], k = 1, 2, \ldots, n - 1. \)

Similarly, the equation

\[ x^{(0)}(k) + az^{(1)}(k) = b \]  

is defined as basic form of the GM (1, 1, λ(k)) model, in which the parameter set \( \Theta = [a, b] \) are estimated by least-square method and satisfies

\[ \Theta = [a, b]^T = (B^T B)^{-1} B^T Y \]  

Where

\[ Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}, B = \begin{bmatrix} \lambda(1) x^{(1)}(1) + (1 - \lambda(1)) x^{(1)}(2) & 1 \\ \lambda(2) x^{(1)}(2) + (1 - \lambda(2)) x^{(1)}(3) & 1 \\ \vdots & \vdots \\ \lambda(n-1) x^{(1)}(n-1) + (1 - \lambda(n-1)) x^{(1)}(n) & 1 \end{bmatrix} \]  

**Definition 3.2** Assume that the parameters of GM (1, 1, λ(k)) have the same definition as those in Definition 3.1, we definite

\[ \frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = b \]  

as the whitening differential of the GM (1, 1, λ(k)) model.

Similarly, we can obtain the time response sequence as follows:
\[
\hat{x}^{(1)}(k) = \left( x^{(0)}(1) - \frac{b}{a} \right) e^{-a(k-1)} + \frac{b}{a}, k = 1, 2, \ldots, n
\]

then the inverse accumulated generation operator was employed to obtain the

\[
\hat{x}^{(0)}(k) = (1-e^{a})(x^{(0)}(1) - \frac{b}{a}) e^{-a(k-1)}, k = 2, 3, \ldots, n.
\]

### 3.3. Optimization of the parameters \( \lambda(k) \) based on Grey Wolf Optimization algorithm

From the previous modelling process, we can realize the parameters \( \lambda(k) \) play a critical role in the proposed model. And the traditional solution method is not applicable to solve the parameters \( \lambda(k) \), thus an optimization problem with nonlinear constraints is formulated as follows:

\[
\min_{\lambda(k)} f(\lambda(k)) = \frac{1}{n} \sum_{i=2}^{n} \left| \frac{x^{(0)}(1) - \hat{x}^{(0)}(k)}{x^{(0)}(k)} \right| \times 100\%
\]

\[
Y = \begin{bmatrix} y^{(0)}(2) \\ y^{(0)}(3) \\ \vdots \\ y^{(0)}(n) \end{bmatrix}, \quad B = \begin{bmatrix} \lambda(1)y^{(0)}(1)+(1-\lambda(1))y^{(0)}(2) & 1 \\ \lambda(2)y^{(0)}(2)+(1-\lambda(2))y^{(0)}(3) & 1 \\ \vdots & \vdots \\ \lambda(n-1)y^{(0)}(n-1)+(1-\lambda(n-1))y^{(0)}(n) & 1 \end{bmatrix}
\]

Subject to,

\[
\Theta = [a, b]^T = (B^T B)^{-1} B^T Y
\]

\[
\hat{x}^{(0)}(k) = (1-e^{a})(x^{(0)}(1) - \frac{b}{a}) e^{-a(k-1)}
\]

Solving equation 19 directly is extremely challenging, while grey wolf optimization algorithm as an emerging intelligent optimization algorithm, who has excellent performance in searching the optimal solution. And it has been used by many researchers to solve similar problems. [12]. In this paper, the fitness function and constraints are expressed in equation 19. The maximum number of iterations in the grey wolf optimization algorithm is set to 300, and initialize the number of grey wolf population is 100. The upper and lower boundaries of the parameters are set to 0 and 1.

### 3.4. Properties of the GM (1,1, \( \lambda(k) \))

According to the previous modelling analysis of the GM (1,1, \( \lambda(k) \)) model, we take noticed that the proposed GM (1,1, \( \lambda(k) \)) model not only approximates the background value accurately, but also is a more general grey prediction model. The GM (1,1, \( \lambda(k) \)) model could reduce to the existing models when the parameters \( \lambda(k) \) of the background satisfy specified conditions, such as when \( \lambda(1) = \lambda(2) = \ldots = \lambda(n-1) \), the GM (1,1, \( \lambda(k) \)) model is reduced to the GM (1,1, \( \lambda(1) \)) model.

when \( \lambda(1) = \lambda(2) = \ldots = \lambda(n-1) = 1/2 \), the GM (1,1, \( \lambda(k) \)) model is reduced to the GM (1,1) model.

### 4. Applications

4.1. Metrics for evaluating the performance of models

To accurately evaluate the performance of the model, it is necessary to establish a unified metric standard. In our work, the absolute percentage error (ape) is employed to evaluate the modelling ability and prediction effect of prediction model. The calculation about it is as follows:
\[
ape(k) = \left( \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)} \right) \times 100\%
\]

where \( x^{(0)}(k) \) is the predicted value of the model and \( x^{(0)}(k) \) is the actual value. Thus \( ape \) can be interpreted as the relative error between the predicted value and the real value of the model, while it is not suitable for describing the overall error of the data sequence. Then mean absolute percentage error (MAPE) is employed to evaluate to the errors between data sequences in order to without losing its simplicity, its calculation formula is expressed as:

\[
MAPE = \frac{1}{n} \sum_{k=1}^{n} ape(k)
\]

where \( n \) represents the size of the data set being tested. Naturally, the meaning of MAPE be changed as \( n \) changes, such as when \( n \) represents the size of modelling data, it evolved into the simulation mean percentage (SMAPE), when \( n \) represents the size of prediction data, it evolved into the modelling mean percentage (PMAPE), when \( n \) represents the size of total sample data, it evolved into the modelling mean percentage (TMAPE).

4.2. Forecasting the nuclear generation in China

In order to effectively verify the modelling ability and prediction precision of the model our proposed, the nuclear generation is utilized as the real world application case. All raw data about the nuclear generation (Twh) are collected from BP Statistical Review of World Energy 2019 and are tabulated in table 1.

Table 1. Nuclear generation in China from 2009 to 2018

| Year | Nuclear generation | Year | Nuclear generation |
|------|--------------------|------|--------------------|
| 2009 | 70.1               | 2014 | 132.5              |
| 2010 | 73.9               | 2015 | 170.8              |
| 2011 | 86.4               | 2016 | 213.3              |
| 2012 | 97.4               | 2017 | 248.1              |
| 2013 | 111.6              | 2018 | 294.4              |

The sample data from 2009 to 2014 are used to construct the grey prediction models, the remaining sample data from 2015 to 2018 are used to examine the prediction accuracy of the model. Then the predicted values are calculated based on the time response functions of GM (1,1) model, GM (1,1, \( \lambda \)) model and GM (1,1, \( \lambda(k) \)) model, and the parameters of GM (1,1, \( \lambda \)) and GM (1,1, \( \lambda(k) \)) are 1 and (0.3377,0.3152,0.5455,0.8429,0.9853) respectively. Additionally, both them are solved using Grey Wolf Optimization algorithm. Those data are listed in table 2, also there are listed the relative error between the actual value and the predicted value of different grey models.

In addition, the modelling results and prediction effect of the grey model are intuitively drawn in figure 2, where directly illustrates the results of GM (1,1, \( \lambda(k) \)) compared with GM (1,1) model and GM (1,1, \( \lambda \) model.

It can be seen from figure 2, the simulation effect of GM (1,1, \( \lambda(k) \)) model is not the best and it not is the worst either in comparison with other grey models, but it much outperforms others during the prediction phase.
Table 2. Simulation and prediction results from grey models

| Year | Raw data | $\text{ape}(k)$ | $\text{ape}(k)$ | $\text{ape}(k)$ | $\text{ape}(k)$ |
|------|----------|----------------|----------------|----------------|---------------|
| 2009 | 70.1     | 70.1           | 70.1           | 70.1           | 70.1          |
| 2010 | 73.9     | 73.5391        | 0.4884         | 79.6967        | 7.8440        |
| 2011 | 86.4     | 93.0468        | 1.7233         | 82.6459        | 4.3450        |
| 2012 | 97.4     | 108.6332       | 0.6588         | 99.0919        | 1.7371        |
| 2013 | 111.6    | 126.8304       | 1.4361         | 118.8104       | 6.4609        |
| 2014 | 132.5    | 148.0760       | 1.3522         | 142.4529       | 7.5116        |
| 2015 | 170.8    | 172.8803       | 11.6389        | 170.8000       | 0.0000        |
| 2016 | 213.3    | 201.8397       | 18.3033        | 204.7880       | 3.9906        |
| 2017 | 248.1    | 235.6502       | 18.9011        | 245.5394       | 1.0321        |
| 2018 | 294.4    | 275.1242       | 21.0868        | 294.4000       | 0.0000        |

Furthermore, the error comparison results of the GM (1, 1, $\lambda(k)$) model and others are quantitatively displayed in figure 3. As we can see, the PMAPE of the proposed model is 1.2557%, those about GM (1, 1) and GM (1, 1, $\lambda$) are 17.4825% and 4.5391% respectively. The MTAPE of the proposed model is
3.5537%, those about GM (1,1) and GM (1,1, λ) are 8.3988% and 7.8477% respectively. These results
demonstrate that GM (1,1, λ(k)) model our proposed is more suitable for predicting the nuclear
generation in China.

5. Conclusion
Based on the grey system theory, the error sources of the classical grey model are analyzed in our
work. And a novel grey model named as GM (1,1, λ(k)) model with optimized background values is
proposed in this paper, which can effectively improve the model error and boost the prediction
precision of the model. At the end of this paper, the proposed model is applied to predicted nuclear
generation and shows satisfactory predictive performance.

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