Modeling of PV Systems Based on Inflection Points Technique Considering Reverse Mode

Modelado de Sistemas Fotovoltaicos Basado en la Técnica de Puntos de Inflexión Considerando Modo Inverso

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Abstract
This paper proposes a methodology for photovoltaic (PV) systems modeling, considering their behavior in both direct and reverse operating mode and considering mismatching conditions. The proposed methodology is based on the inflection points technique with a linear approximation to model the bypass diode and a simplified PV model. The proposed mathematical model allows to evaluate the energetic performance of a PV system, exhibiting short simulation times in large PV systems. In addition, this methodology allows to estimate the condition of the modules affected by the partial shading since it is possible to know the power dissipated due to its operation at the second quadrant.

Keywords
Photovoltaic systems; modeling; Reverse Mode Biased; Shaded PV Cells; Mismatched conditions; bypass diode.

Resumen
Este artículo propone una metodología para el modelado de sistemas fotovoltaicos, considerando su comportamiento tanto en el modo de operación directo como en modo inverso bajo condiciones no uniformes de irradiación. La metodología propuesta se basa en la técnica de puntos de inflexión con una aproximación lineal del modelo del diodo de bypass y un modelo simplificado del módulo fotovoltaico. El modelo matemático planteado permite evaluar el rendimiento energético de un sistema fotovoltaico, con tiempos cortos de simulación para arreglos de gran tamaño. Adicionalmente, esta metodología permite estimar el estado de los módulos afectados por el sombreado parcial ya que es posible conocer la potencia disipada debido a la operación en el segundo cuadrante.

Palabras clave
Sistemas fotovoltaicos; modelado; modo inverso; condiciones no uniformes; diodo de bypass.
1. INTRODUCTION

Due to the interest of worldwide governments to incentive renewal energy projects, photovoltaic (PV) systems have become in a wide research area. For urban networks where is required to connect a PV system to an existing grid, it is necessary to realize a design that points to optimal energy efficiency based on: load consumption requirements, environmental conditions and the behavior of the distribution system in which it will be connected.

The assessment of energy efficiency in a photovoltaic system depends on the study of the individual behavior of each module that composes it. Thus, if a module in the system is affected by shadows, either partial or total, the module will not deliver power to the system, instead the module will consume power degrading the system performance (Bishop, 1988; Wang & Hsu, 2010). This behavior is due to the nature of the photovoltaic cell, since the current flowing through it \( I_{PV} \) is higher than the short-circuit current \( I_{SC} \), the cell is polarized inversely, i.e. the cell dissipates energy as heat since the cell voltage becomes negatively (Alonso-Garcia et al., 2006). In such an operating condition the cell exhibits hot spots which lead to degradation of the module decreasing the life cycle (premature aging). Moreover, if the temperature reaches the thermal breakdown, the cell will suffer irreversible damages (Zegaoui et al., 2011). Commercial modules are made of series-connected PV cells in parallel with a bypass diode, which avoids large negative voltage to the cells. In any case on the shadowing conditions, the module is subjected to negative voltages that force the cell to operate at the second quadrant (negative voltage, positive current).

Simulation of large PV systems require huge efforts in design circuitual schematics required for classical electrical simulators like PSIM or PSpice (Patel & Agarwal, 2008). To avoid the requirement of such simulation environments it is necessary to develop mathematical models that take into account the modules behavior in both: direct (first quadrant) and reverse (second quadrant) conditions. Such models are required to estimate the energy production of PV installations in order to provide an accurate viability analysis and technical design.
The proposed methodology consists of a mathematical model based on the inflection points technique proposed in (Petrone & Ramos-Paja, 2011), but considering also the operation at the second quadrant. To minimize the complexity of the model, a simplified model of a single diode is used to represent the module, while a linear approximation is used to represent the bypass. The proposed model requires to solve a nonlinear equation system which is addressed using the Newton-Raphson algorithm. Finally the proposed model considers individual irradiance, temperature and parameters tolerance conditions for each module. This paper is organized as follows: Section 2 presents the modeling of the PV system and the description of the inflection points technique with the new approach considering the operation of the PV cell at the second quadrant. Section 3 provides simulation results of the proposed approach and its comparison with the Fast method. Finally, Section 4 gives the conclusions of the work.

2. MODULE MODELING

PV systems are commonly represented using the single diode model (Fig. 1). Unfortunately, for this model is not possible to obtain an explicit relationship between voltage and current (Petrone et al., 2007). Moreover, it requires to solve a complex system, which increases significantly the computational cost.

In Fig. 1, \( I_{SC} \) corresponds to the short-circuit current, \( D_j \) is the diode which represents the PN junction, \( I_d \) is the diode current, \( R_s \) and \( R_H \) are the series and parallel resistances, \( D_B \) is the bypass diode and \( I_{PV} \) is the module current. A simplified version of this
model is obtained by neglecting $R_S$ and $R_H$ resistance (Chao et al., 2008), which leads to an explicit equation between the voltage and current $I$ in the module as in (1). In such an expression, $I_{SC}$, $A$ and $B$ parameters are from the datasheet data (Eicker, 2003).

$$I_{PV} = I_{SC} - A e^{BV_{PV}} \tag{1}$$

Concerning the bypass diode, a linear approximation of the exponential model (Fig. 2(a)) is adopted. Both the cells and bypass diode models lead to the module model depicted in Fig. 2(b). The activation voltage of the bypass diode $V_{ad}$ is represented by a voltage source and the activation resistance $R_{ad}$, which define the slope of the voltage - current characteristic of the diode, is also observed. Both parameters can be extracted from the diode datasheet or estimated using a computational tool (Boylestad & Nashelsky, 2009).

![Fig. 2. (a) Bypass Diode Models. (b) Circuital diagram of the module with a linear approximation of the bypass diode. Source: Authors](image)

### 2.1 String Modeling

In the design of a photovoltaic system, the number of modules connected in series depends on the required system voltage. The series connection of PV modules is named PV strings (Petrone et al., 2007). Similarly the number of parallel connected strings depends on the required PV power. Fig. 3 shows a PV system
composed by two PV strings, each one of them formed by N series-connected modules. In addition each string exhibits a blocking diode to protect the string from reverse current. Thus, the string voltage is given by the sum of the voltage in each module $V_{k}$ and the blocking diode voltage as given in (2). Moreover the modules and blocking diodes in a string share the same current (3).

$$V_{\text{String}} = \sum_{k=1}^{N} V_{PV,k} + V_{\text{Diode}}$$ (2)

$$I_{PV,1} = I_{PV,k} \forall k \in [2, N]$$ (3)

The blocking diode is modeled using the Schottky equation (4), where $I_{0D}=0$, $B_{D}<0$ and $A_{D} \ll A$. In this way, the equations system based on (2) and (3) may include the blocking diode, forming an $N+1$ equations system to calculate the string current.

$$I_{\text{Diode}} = I_{0D} - A_{D}e^{B_{D}V_{D}}$$ (4)

$$I_{PV,1} = I_{\text{Diode}}$$ (5)

![Fig. 3. Typical connection of PV modules. Source: Authors](image-url)

### 2.2 Inflection Points Calculation

The inflection points define the voltage and current conditions in which the bypass diode become active. Considering that each
cell may have variation in their parameters, (1) can be rewritten as follows:

$$I_{Str} = I_{PV,k} = I_{SC,k} - A_k e^{B_k V_{PV,k}}$$  \hspace{1cm} (6)$$

If $I_{Str}$ is higher than the short-circuit current of module $k$ ($I_{SC,k}$), the voltage $V_{PV,k}$ of such a module is given by negative bypass diode voltage which forces the module to dissipate power as is shown in Fig. 4, where $V_{ad}$ and $V_{br}$ are the activation voltage and the breakdown voltage respectively and $V_{op}$ is the voltage at which the module operates depending on the string current $I_{Str}$, the gray area represents the dissipated power.

![Fig. 4. Direct-reverse operation mode of a PV cell. Source: Authors](image)

Therefore it is important to predict the conditions in which the bypass diode becomes active, since such conditions imply the PV power degradation due to the power consumption of the module operating at the second quadrant. Then the bypass diode $k_{th}$ associate to the $k_{th}$ module becomes active at the voltage $V_{0j,k}$ given in (7), which correspond to the voltage of module $j$. Finally the inflection voltage, which represents the minimum string voltage at which the bypass diode becomes active, is given by (8).

$$V_{0j,k} = \frac{1}{B_j} \ln \left( \frac{I_{SC,j} - I_{SC,k} + A_k e^{B_k V_{ad}}}{A_j} \right)$$  \hspace{1cm} (7)$$

$$V_{0k} = \sum_{m=1}^{k-1} V_{0m,k}$$  \hspace{1cm} (8)$$
2.3 PV System Modeling

The nonlinear equations system representing the PV systems obtained from (2) – (5) as:

\[
\begin{align*}
    f_1(V) &= \sum_{k=1}^{N+1} V_k - V_{\text{String}} = 0 \\
    f_2(V) &= -A_1 e^{B_1 V_{PV,1}} + A_2 e^{B_2 V_{PV,2}} + I_{SC_1} - I_{SC_2} = 0 \\
    \vdots \\
    f_{N+1}(V) &= -A_1 e^{B_1 V_{PV,1}} + A_{N+1} e^{B_{N+1} V_{PV,N+1}} + I_{SC_1} - I_{SC_k} = 0
\end{align*}
\]

The system solution \( V \) is obtained using the Newton-Raphson iterative algorithm (Petrone et al., 2007), from the Jacobian matrix of \((N + 1) \times (N + 1)\) elements. The \(N + 1\) terms in the first row of the matrix are equal to 1. The values associated with the first column are \( J_{k,1} = \partial f_k / \partial V_{PV,1} = -A_1 B_1 e^{B_1 V_{PV,1}} \) and the diagonal elements are given by \( J_{k,k} = \partial f_k / \partial V_{PV,k} = A_k B_k e^{B_k V_{PV,k}} - \frac{1}{R_{ad,k}} \), with \( k \in [2, N + 1] \). When the bypass diode is active the associated module is represented by (10), when the bypass diode is not active it is represented by (6). Finally those equations compose the equations system in (9).

\[
I_{PV,k} = I_{SC,k} - A_k e^{B_k V_{PV,k}} + \frac{1}{R_{ad,k}} (V_{PV,k} - V_{ad,k})
\]

3. RESULTS AND DISCUSSION

The methods Fast and the new approach named Second Quadrant Approach (SQ approach), were implemented in Matlab®. The PV system used for the simulation consists of a string of three modules. The parameters of the PV modules used in the simulation are: \( V_{OC} = 21.78 \) [V], \( I_{SC} = 5.13 \) [A], \( I_{ph1} = 0.9 \times I_{SC} (90\%) \), \( I_{ph2} = 0.6 \times I_{SC} (60\%) \), \( I_{ph3} = 0.3 \times I_{SC} (30\%) \), \( A = 2.281 \times 10^{-7} \) [A] and \( B = 0.7782 \) [V^{-1}]. The parameters of the linear approximation for bypass diode are \( V_{ad} = 0.2166 \) [V] and \( R_{ad} = 0.003 \) [\Omega] (Bastidas et al., 2013). Fig. 5 shows the comparison between the models. Com-
Computation time for the SQ approach was 5.25 [s] and for the Fast method was 0.84 [s]; however, some imprecisions related with the overestimation of the Fast method are observed.

The average overestimation of the power provides by the Fast method with respect to the one provides by the SQ approach was 0.4 [W]. In large PV fields this difference could be significant and can affect seriously the planning and performance of the system. For the same PV string, the I-V curves for each module are presented in Fig. 6. Due to the mismatching condition, the bypass diodes of the modules 2 and 3 become actives giving place to negative operation voltages. This situation is detailed in Fig. 7 where the operation of the module 2 in the second quadrant is shown. The white lines section represents the power consumed by the module due to its operation at the currents imposed by the string which are higher than its own short-circuit current. In the same way the module 3 consumes power due to the activation of its bypass diode, in this case the power consumed was 1.0427 [W]. These results represent an advantage of the SQ approach, since it is possible to know the behavior of the modules at the second quadrant which allows to predict how many modules are consuming power and consequently presenting hot spots. That information cannot be obtained by using the Fast method.
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Fig. 6. I-V characteristic for each module of the PV string. Source: Authors

Fig. 7. Detail of the operation in the second quadrant in the Module 2. Source: Authors

The upper part of Fig. 8 shows an irradiance profile of the center of Colombia for 7 days with samples taken hour by hour, while the lower part shows the predicted power provides by the two methods for a 5x2 PV field with a shading pattern of 100%, 95%, 80%, 50% and 40% for the first string and 100%, 90%, 70%, 50% and 20% for the second one. The average difference between the prediction provided by the Fast method and the SQ approach was 0.56 [W] per sample.
4. **CONCLUSIONS**

In this paper a new methodology based on the inflection points technique for PV systems modeling was proposed. The system was modeled using a simplified representation of a PV cell and a linear approximation of the bypass diode. In this way, it is possible to obtain a nonlinear equations system which can be solved using an iterative algorithm like Newton-Raphson to reconstruct I-V and P-V curves for the whole system. The SQ approach provides an improvement in the accuracy of the power prediction compared to the Fast method without increasing significantly the simulation time. In addition, the proposed approach allows to estimate the condition of the modules affected by the partial shading since it is possible to know the power dissipated due to its operation at the second quadrant.

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