Limit states for the damage assessment of bridges supported on LRB bearings

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Abstract. There is now a common awareness that excessive strength is neither essential nor a synonymous of good performance in strong earthquakes. It is widely recognized that the design process should put more emphasis on predicting structural damage, according to the performance-based design philosophy. This paper presents an approach for assessing the damage level of lead rubber isolation devices and reinforced concrete piers, according to the performance-based design framework. The proposed performance limit states are based on the control of the materials strain, by means of the shear displacement of the bearing and the hinge rotation of the pier. The performance of the piers of different types of bridges is evaluated with the proposed methodology considering four different limit states.

1. Introduction

The emphasis in seismic design of bridge structures has shifted from resistance of large seismic forces to the control of deformations, rotations or displacements. There is now a common awareness that excessive strength is neither essential nor a synonymous of good performance in strong earthquakes. There are numerous cases of structures that were designed for a life safety performance according to current codes, but they were not operational after moderate earthquakes [1]. It means, that present code approach of designing only for the life safety performance, is not satisfactory.

Today, it is widely recognized that the design process should put more emphasis on predicting structural performance, and several methodologies have been proposed for performance-based design. A discussion about the characteristics and capabilities of each method can be found in [1].

Given the many sources of uncertainty that are inherent in earthquake design, a true performance-based design procedure must be based on the reliability-design principles. An important task in performance-based design is the association of earthquake probability of exceedance with suitable measures of structural performance. In this context, limit states relating the seismic intensity with the damage level and structural performance are needed. This paper presents an approach for assessing the performance levels of lead rubber isolation devices and reinforced concrete piers, according to the performance-based design framework by means of four limit states. The proposed performance limit states are based on the control of the materials strain, by means of the shear displacement of the bearing and the hinge rotation of the pier.
2. Limit states for lead rubber bearings

Common earthquake protection systems for bridges are seismic isolators in association with passive energy dissipaters. In particular, lead rubber bearing (LRB) is probably one of the systems more cited in literature as an example of a single isolation device that combines flexibility and damping, providing also an economic and simple solution for protecting bridges. Rubber bearings provide the vertical stiffness needed for supporting gravitational loads and lead bars supply horizontal stiffness to control the lateral bridge displacements under service loads while giving energy dissipating capability under strong motion movements. If the bearing displacements are large enough the lead yields in shear as it is illustrated in figure 1. When coupled with the linear properties of the elastomeric bearing, the combined unit showed a bilinear force – shear deformation relationship. Typically, for computer modelling purposes, a bilinear hysteresis loops are assumed.

![Figure 1. Hysteresis loops for lead elements](image)

Since the shear deformation is the principal displacement in a lead rubber bearing, the shear strain \( \gamma_b \) is selected as the damage index for this type of isolators. Shear strain is defined as the lateral displacement \( x_b \) divided by the total thickness of rubber, excluding the total thickness of internal steel plates (figure 2).

\[
\gamma_b = \frac{x_b}{h_t} \quad \text{and} \quad h_t = h_r - \sum_{i=1}^{n} t_i
\]

![Figure 2. Shear strain as damage index for lead rubber bearings](image)

2.1 Limit state for service earthquakes

For frequent or service earthquakes it is desirable that the bridge continues fully operational without damage; residual displacements are not allowed for the isolator bearings in order to avoid the necessity of application of restoring forces, or the substitution of one or more devices. Thanks to the recovery of the lead mechanical properties after shear inelastic deformations, the yield strength of lead can be exceeded during frequent earthquakes without residual displacements.

As suggested by Chen [2], the response of the idealized bilinear model for the lead rubber bearing shown in figure 3 can be obtained by solving a sequence of linear problems. Then, the solution of the equation of motion can be divided in the steps: 0-1, 1-2, 2-3, 3-4 and 4-0 shown in figure 3. The necessary conditions for obtaining a non-residual displacement are twofold: a) the displacement at the beginning of the last branch \( t = tx4 \) must be: \( xt4 = -xy \), and b) the velocity at \( t = tx4 \) must be: 0. Then, the maximum displacement of the bearing for non-residual displacements occurs when \( xt4/xy = -1.0 \). Figure 4 shows the relationship between the bearing displacement ductility (\( \mu_b = x_{\text{max}}/x_y \)) and the...
displacement ratio $\frac{x_t4}{x_y}$, for the inelastic to elastic stiffness ratio $k_2 / k_1= 0.1$, typical ratio for lead rubber bearings. The lower curve corresponds to $\xi = 0\%$; the middle curve is for a $5\%$ equivalent damping; and the upper curve corresponds to a variable damping for the LRB isolators, obtained as a function of the displacement ductility developed by the bearing, according to the following equation [3]:

$$\xi_{eq} = 0.05 + \eta \ln(\mu_b) \leq 0.30$$  \hspace{1cm} (1)$$

where the coefficient $\eta$ depended on the characteristics of the record: $\eta = 0.065$ for earthquakes recorded on firm soils and $0.085$ for earthquakes recorded on soft soils.

![Figure 3. Idealized bilinear model for the lead rubber bearing](image)

![Figure 4. Displacement at time xt4 for different displacement ductility of the bearing](image)

As the maximum displacement of the bearing for non-residual displacements occurs when $\frac{x_t4}{x_y} = -1.0$, the corresponding allowable displacement ductility of the bearing (figure 4) is:

$$(\mu_b)_{max} = 17 \text{ for } \xi = 0\% \quad (\mu_b)_{max} = 20 \text{ for } \xi = 5\% \quad (\mu_b)_{max} = 24 \text{ for } \xi_{eq} = \phi(\mu_b)$$

Based on the mechanical properties of the LRB bearings, the relationship between the shear deformation and displacement ductility is [3]:

$$\gamma_{x,t=0} = 17 \frac{k_2}{k_1} \frac{A_y}{A_t} \frac{r_y}{G_r} \left[ \frac{1}{1+10 \frac{A_y}{A_t}} \right]$$  \hspace{1cm} (2)$$
where, $\tau_y$ is the lead shear yield stress, $G_r$ the shear modulus of the rubber, $A_l$ the lead core area and $A_r$ the plan area of the rubber. After substituting typical values for the stiffness ratio, $A_l/A_r$ ratio, shear yield stress and shear modulus of the bearing, the maximum distortion expression may be simplified [3] as:

$$\gamma_{x, r=0} = 17 (0.1)(0.05)(10) \left( \frac{1}{1 + 10(0.05)} \right) = 0.57$$  \hspace{1cm} (3)

An initial shear deformation of the bearing can be present due to tolerances and imperfections during construction; then, a reduction coefficient of 0.7 is considered for accounting the potential initial distortion of the device. The proposed maximum shear deformation for non-residual displacement of the LRB bearing (service limit state) is:

$$\gamma_{\text{max}} \leq 0.7(0.57) = 0.4$$  \hspace{1cm} (4)

### 2.2 Limit state for service earthquakes

It is expected that the bridge remains in almost total operation condition after the earthquake, and only minor limitations to vehicle circulation, if any, can be admitted. Therefore the maximum shear deformation for non-residual displacement of the bearing is also desirable for this limit state, without any consideration of possible initial distortion of the bearing, and the proposed maximum deformation is:

$$\gamma_{\text{max}} \leq 0.6$$  \hspace{1cm} (5)

### 2.3 Limit state for rare earthquakes

The life safety is the principal objective for this limit state and the structural damage and rehabilitation works are limited to easy access zones. Restricted traffic on the bridge can be allowed after the earthquake. For this condition it is expected that significant amount of energy is dissipated by the bearing without excessive displacements. To achieve this objective it is proposed that the maximum force transmitted by the bearing to the piers is limited to four times the yield strength of the device [3]. Then, the maximum shear distortion of the bearing for a maximum force $F_{\text{max}} = 4F_y$ is equal to:

$$\gamma_{\text{max}} \leq \mu_0 \frac{k_1}{k_1 A_r G_r} \frac{\tau_y}{1 + 10 \lambda} = \left[ 1 + \frac{k_1}{k_2} \left( \frac{F_{\text{max}}}{F_y} - 1 \right) \right] \frac{k_2}{k_1 A_r G_r} \frac{\tau_y}{1 + 10 \lambda}$$  \hspace{1cm} (6)

Based on the LRB’s properties the previous expression is simplified to:

$$\gamma_{\text{max}} \leq 1 + 10 \left( \frac{F_{\text{max}}}{F_y} - 1 \right) \frac{0.05}{1.5} = 1.03$$  \hspace{1cm} (7)

### 2.4 Limit state for rare earthquakes
The purpose is to maintain the stability of the bridge after the maximum credible earthquake in the region. The allowable deformation of the bearing must be below the rupture strain of the rubber (Ramberger [4] reports a rupture strain $\gamma_u$ between 3.0 and 5.0). The maximum deformation of the bearing must consider the seismic action and the additional deformations produced by rotation of the bearing, compressive stresses, creep, shrinkage, posttension and temperature. The aspects of stability, overturning, and resistance of connection should also be carefully checked. Taking into account all these aspects, the proposed maximum deformation of the isolator is:

$$\gamma_{\text{max}} \leq 2.0$$ (8)

3. Limit states for concrete piers

Based on the experience of past earthquakes and laboratory tests, damage level of reinforced concrete elements can be associated for practical purposes to maximum strain. Then, the proposed performance limit states derived in this study are based on the control of the materials strain, by means of the hinge rotation of the pier. The following expressions for estimating the allowable rotations for each limit state were obtained from the mechanical properties and usual hypothesis adopted in the design of reinforced concrete structures and were favorable compared to the experimental results reported on [5-6]. Initially, the yield rotation and ultimate compression strain of the concrete should be defined.

3.1 Yield Rotation $\theta_y$

Rotation of the plastic hinge is due to the contribution of three effects: flexural deformations, shear deformations and bond slip of the steel reinforcement. The yield rotation caused by these factors can be assessed with the following equation proposed by [7]:

$$\theta_y = \frac{\phi_y L}{2} \left( 1 - \frac{H}{3L} \right) + 0.0025 + \frac{0.25 \epsilon_{sy} d_b f_y}{(d - d') \sqrt{f_c}}$$ (9)

In the above expression L is the free length of the pier and H is the distance between the pier top and the center of the plastic hinge, usually $H \approx L$. The second term is the shear deformation contribution to the plastic rotation and the last term is the effect of bond slip. According to the analyses of the experimental results obtained by [7], inclined cracks and shear deformations can be estimated by a constant factor of 0.0025. The third term in the above expression is proportional to the yield force of the steel in tension ($\pi d^2 f_{ys}/4$), and inverse to the perimeter of the bar ($\pi d_b$) and the bond strength ($\sqrt{f_c}$). If bond slip can be neglected, the last term in the equation can be dropped.

3.2 Ultimate Compression Strain for Concrete $\epsilon_{cu}$

According to the strain energy balance equation [8], the total stored energy equals the strain energy in the stirrups before the rupture plus the strain energy for the unconfined concrete and the required strain energy for maintaining the plastic strain in the longitudinal steel until the stirrups failure. After solving this energy balance equation, the ultimate compression strain for concrete is [3]:
\[ \varepsilon_{cu} = \frac{110 \rho_s + 3.4 \rho_{sl} + 0.017 \sqrt{f'_{co}} - 0.07}{0.94 f_{yl} + 302} - 0.015 f'_{c} + 1.1 f'_{c} + 8 \]  

(10)

### 3.3 Limit State for Service Earthquakes

The bridge must be fully operational after the earthquake for this limit state, then, if some minor cracks are formed during the earthquake, they have to be closed after the event. For typical pier elements, the first cracks are horizontal as a consequence of flexural strains, and shear deformations and bond slip can be neglected. According to experimental data, the strains before the initiation of cracking are \( \varepsilon_c = 0.0015 \) and \( \varepsilon_s = 0.7 \varepsilon_{sy} \), for the concrete and longitudinal steel respectively, and the allowable rotation for this limit state is:

\[ \theta_{ser} = \frac{0.7 f_{sl}}{E_s} + 0.0015 \frac{1}{3d} L \]  

(11)

### 3.4 Limit State for Occasional Earthquakes

Traffic on the bridge after the earthquake is not disrupted and only unimportant limitations to vehicle circulation can be admitted during the repairing works. Minimum repairing works such as epoxy injections in minor cracks are allowed, width cracks should be less than 2 mm without loss of cover concrete, but yield strain can be exceeded. Inclined cracks might be present and the second term of Eqn. 3.1 should be included in the estimation of \( \theta_y \). Bond slip has a small contribution at this level of strain and can be neglected. Two non-linear conditions may occur: one is the result of steel yielding, and the other is present when the concrete strain is clearly in the non linear range (\( \varepsilon_{cu} \geq 1.2 f_c / E_c \)). Applying the usual hypothesis of reinforced concrete design, the rotations for these non-linear conditions are:

\[ \theta_{ed,x} = \frac{L f_{sl}}{3E_s (d - c_y)} + 0.0025 \]  

\[ \theta_{ed,c} = \frac{1.2 L f_c}{3E_c c_y} + 0.0025 \]  

(12)

In the above expressions \( c_y \) is the distance of the outer compression fiber at the beginning of yield.

### 3.5 Limit State for Rare Earthquakes

For this limit state the bridge stability must be assured and restricted traffic over the bridge is admitted after the seismic event. Loss of the concrete cover is allowed but damage of the concrete core is limited to minor cracks. As spalling of concrete cover is admitted, the rotation is the result of \( \theta_y \) and some additional plastic rotation:

\[ \theta_{pe} = \theta_y + (\phi_{pe} - \phi_y) L_p \left(1 - \frac{L_p}{2L} \right) \]  

(13)

where the limit curvature \( \phi_{pe} \) depends on the maximum strain in concrete \( \varepsilon_{ps,c} \) or steel \( \varepsilon_{ps,s} \) for the expected damage level. The maximum values of strain can be estimated by [5]:

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\[ \varepsilon_{cu} = \frac{110 \rho_s + 3.4 \rho_{sl} + 0.017 \sqrt{f'_{co}} - 0.07}{0.94 f_{yl} + 302} - 0.015 f'_{c} + 1.1 f'_{c} + 8 \]  

(10)
\[ e_{pv,s} = \frac{\varepsilon_{cc} f_{cc} b}{1.25 N} \left( d - \frac{1.25 N}{f_{cc} b} \right) \]
\[ e_{pv,c} = \frac{1.25 N (0.5 \varepsilon_{cm})}{f_{c} b_{co} d_{co} - 1.25 N} \] (14)

In the above expressions \( N \) is the axial force on the pier, \( f_{cc} \) is the confined compression resistance of the concrete, \( \varepsilon_{cc} \) the strain corresponding to the confined resistance and \( b_{co} \) and \( d_{co} \) are the width an effective depth of the confined or core section respectively.

3.6 Limit State for Very Rare Earthquakes
The bridge stability for the maximum earthquake expected in the region must be assured and emergency traffic is allowed over the bridge after the shock. Moderate damage of the core concrete is permitted but the stability of the element is not compromised neither its vertical load capacity. Rupture of stirrups must not happen and vertical rebars do not buckle for this condition. Ultimate rotation \( \theta_u \) is obtained using the ultimate curvature \( \phi_{uc} \) instead of \( \phi_{pv} \). For assessing \( \theta_u \), bond slip should be added. \( \phi_u \) can be originated in the rupture of the transverse bars, rupture of longitudinal steel or the loss of axial load carrying capacity of the pier. Eqn. 13 gives the limit state for very rare earthquakes, after the substitution of Eqn. 15.

\[ \phi_{uc} = \frac{\varepsilon_{cu}}{e_{c}} = \frac{0.68 b_{co} f_{cu} e_{cu}}{N} \] (15)

4. Displacement demand
The nonlinear static analysis procedures proposed in some current seismic design recommendations are a reasonable tool for the evaluation of the displacement capacity of new and existing bridges. According to these procedures the seismic demand is represented in an acceleration-displacement response spectra space (ADRS). Considering the limit states proposed in this study, the seismic demand can be represented in an acceleration-rotation response spectra space as illustrated in figure 5. Using the capacity spectrum method for a reinforced concrete bridge, the rotation demand and the capacity for the four limit states proposed in this study are summarised in table 1. According to the capacity/demand ratio, the bridge is adequate for the very rare earthquake with a rotation capacity twice the rotation demand; however, for the other limit states, the bridge has not adequate capacity for the seismic demand imposed by the earthquake.

| Limit state | Rotation capacity (rad) | Equivalent damping (%) | Effective period (s) | Rotation demand | Capacity/demand |
|-------------|------------------------|------------------------|----------------------|----------------|-----------------|
| Service     | 0.0076                 | 9                      | 1.58                 | 0.0143         | 0.53            |
| Occasional  | 0.0106                 | 16                     | 1.87                 | 0.0182         | 0.58            |
| Rare        | 0.0227                 | 18                     | 2.22                 | 0.0292         | 0.78            |
| Very rare   | 0.0629                 | 22                     | 2.41                 | 0.0316         | 2.00            |
5. Conclusions
This paper presents expressions for determining limit states for concrete piers and LRB isolators according to the performance-based design framework. The rotations of piers and shear deformations are selected as damage indexes and the expressions proposed were favorable compared to experimental data bases.

An expression for estimating the ultimate compression strain in concrete is also proposed. The maximum deformation predicted with this equation, improves the results obtained with other expressions proposed previously.

Considering the four limit states proposed in this study, the seismic demand can be represented in an acceleration-rotation response spectra space, and the capacity/demand ratio can be obtained for isolated bridges, with some of the nonlinear static procedures proposed in the latest version of seismic design codes.

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