DEFAULTABLE BONDS VIA HKA

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ABSTRACT. To construct a no-arbitrage defaultable bond market under the physical measure \( P \), we work in the state price density framework. Using the heat kernel approach (HKA for short) and the killing of a Markov process, we construct a single defaultable bond market that enables an explicit expression of the price of a defaultable bond and the credit spread. In the quadratic Gaussian settings, we show that the model is not only tractable but realistic via some simulation results.

Keywords: (Non-)Systematic risk, State price density, Killed HKA, Markov functional model, Quadratic Gaussian

1. Introduction. Considering the (non-)systematic risk in credit markets, it is reported in [1] that the risk neutral default probability for BBB-rated bonds is eight times greater than the physical one. It should be noted that under the risk neutral measure \( Q \) we leave behind the systematic risk since the market price is something that has already been eliminated. The difference between \( P \) and \( Q \), however, is sometimes very critical. Therefore, it is reasonable, or should we say, desirable, to construct a credit market model under the physical measure \( P \) and explore modeling the market risk.

In this paper, we rely on the heat kernel approach introduced by one of the author and his collaborators in [2]. The HKA, an abbreviation of “Heat Kernel Approach”, was introduced by one of the authors and his collaborators in [2]. Briefly speaking, HKA is a systematic method to produce a tractable interest rate model which is “Markov functional” in the sense of Hunt-Kennedy-Pelsser [3]. In the fundamental paper [2], four different types of implementation methods are introduced, namely, 1) Eigenfunction models, 2) Weighted HKA, 3) Killed HKA and 4) the Trace Approach. As is pointed out in [2], the eigenfunction models are tailor-made for swaption pricing, and a deeper understanding for its mathematical structure leads to the trace approach, which is mathematically the most involved. The weighted HKA is extended to a time-inhomogeneous setting and applied to information-based models by J. Akahori and A. Macrina [4].

In the present paper, we will demonstrate how the Killed HKA is applied to the modeling of defaultable bonds by constructing a market where the market price of risk and the default probability are “built in the same block” (the precise meaning will be given later). We stress that the HKA is basically a state-price density approach where everything is written under the physical = statistical measure. Since the HKA furthermore gives an analytically tractable model, the framework proposed in this paper is promising in respect of modelling defaultable markets.

The organization of the present paper is as follows. After recalling the plain-vanilla HKA in Section 2.1 and the killed HKA in Section 2.2, we shall give the main result, a