Diffeomorphism Symmetry in Two Dimensions and Celestial Holography

John H. Schwarz\textsuperscript{1}

\textit{Walter Burke Institute for Theoretical Physics}
\textit{California Institute of Technology 452-48}
\textit{Pasadena, CA 91125, USA}

Abstract

Two-dimensional diffeomorphism symmetry can be described by an operator algebra extension of the well-known Virasoro algebra description of conformal symmetry. Utilizing this extension, this note explains why the conformal symmetry that appears in celestial holography should not be extended to diffeomorphism symmetry, a possibility that several authors have proposed. The description of the two-dimensional diffeomorphism algebra presented here might be useful for other purposes.

\textsuperscript{1}jhs@theory.caltech.edu
1 Introduction

Unlike in higher dimensions, the conformal group in two dimensions is infinite dimensional. In the case of Euclidean signature, which is the main concern in this work, the generators consist of the product of a holomorphic Virasoro algebra and its complex conjugate anti-holomorphic algebra. An enormous amount of work has taken place over the past 50 years studying 2d conformal field theories. It is a rich subject with many important applications.

Two-dimensional Euclidean space can be parametrized by a complex coordinate $z$ and its complex conjugate $\bar{z}$. By means of stereographic projection, these coordinates can also be used to parametrize a sphere. By definition, a conformal operator $\Phi(z, \bar{z})$ has dimensions $(h, \bar{h})$ provided that $\Phi(z, \bar{z})(dz)^h(d\bar{z})^{\bar{h}}$ is invariant under conformal transformations. It is customary to define dimension $\Delta = h + \bar{h}$ and spin $s = h - \bar{h}$. The mode expansion of such an operator is

$$\Phi(z, \bar{z}) = \sum_{m,n} \frac{\Phi_{m,n}}{z^{m+h}\bar{z}^{n+\bar{h}}}.$$  \hfill (1)

The range of $m$ and $n$ are such that $m + h$ and $n + \bar{h}$ take all integer values.

Conformal transformations are generated by the operators

$$T(z) = \sum_{m\in\mathbb{Z}} \frac{L_m}{z^{m+2}} \quad \text{and} \quad \overline{T}(\bar{z}) = \sum_{m\in\mathbb{Z}} \frac{\overline{L}_m}{\bar{z}^{m+2}}.$$  \hfill (2)

The modes satisfy the Virasoro algebra

$$[L_m, L_n] = (m-n)L_{m+n}, \quad [\overline{L}_m, \overline{L}_n] = (m-n)\overline{L}_{m+n}, \quad [L_m, \overline{L}_n] = 0.$$  \hfill (3)

No central extension is required.\(^2\) Conformal transformations of an operator $\Phi(z, \bar{z})$ having dimensions $(h, \bar{h})$ are generated by the Virasoro operators

$$[L_k, \Phi_{m,n}] = ((h-1)k - m)\Phi_{m+k,n},$$  \hfill (4)

$$[\overline{L}_l, \Phi_{m,n}] = ((\bar{h}-1)l - n)\Phi_{m,n+l}.$$  \hfill (5)

These commutation relations correspond to the operator-product formulas (as $z \to w$)

$$T(z)\Phi(w, \bar{w}) \sim \frac{\Phi(w, \bar{w})}{(z-w)^2} + \frac{\partial \Phi(w, \bar{w})}{z-w},$$  \hfill (6)

$$\overline{T}(\bar{z})\Phi(w, \bar{w}) \sim \frac{\Phi(w, \bar{w})}{(\bar{z}-\bar{w})^2} + \frac{\overline{\partial} \Phi(w, \bar{w})}{\bar{z}-\bar{w}}.$$  \hfill (7)

\(^2\)See [1] for the possible role of a central extension in the context of celestial holography.
where $\partial = \partial/\partial w$ and $\bar{\partial} = \partial/\partial \bar{w}$.

Conformal algebras can be described either by commutation relations for modes of operators and or by the identification of the singular terms in operator product expansions (OPEs), as we have just demonstrated. The latter is more succinct and elegant, and therefore it is usually preferred. One advantage of considering the commutation relations of modes is that this makes it easier to check consistency with Jacobi identities of the form $[[A, B], C] + [[B, C], A] + [[C, A], B] = 0$. There are cases where checking this consistency is important, as will become clear later. These Jacobi identities correspond to associativity of the operator algebra, which can be investigated by introducing appropriate contour integrals.

One of the important precursors of celestial holography was the famous BMS analysis in the early 1960s [2][3][4]. These authors analyzed the symmetries of general relativity in asymptotically flat spacetime (AFS) geometries. They concluded that the usual translation generators, i.e., momenta, are supplemented by an infinite set of additional ones, which they called supertranslations. Similarly, the Lorentz generators acquire an infinite extension, which are called superrotations. These conclusions were reached by analyzing the asymptotic behavior of the spacetime metric in appropriately chosen coordinates, subject to assumptions about the asymptotic falloff of various functions that appear in the analysis.

Eventually it was realized that the BMS algebra should be extended to include the entire infinite-dimensional Virasoro algebra [5][6][7]. This implies that the Lorentz symmetry together with the superrotations can be interpreted as the infinite-dimensional conformal symmetry of the two-dimensional celestial sphere. Furthermore, there is compelling evidence for the existence of a dual description of gravitational physics in asymptotically flat four-dimensional spacetime in terms of a 2d conformal field theory on the celestial sphere. Much of the pioneering work has been done by Strominger, who has written a book on the subject [11]. There are also a number of more recent reviews including [12]–[16].

A basic premise of celestial holography is that bulk fields in four-dimensional asymptotically flat spacetime map to operators on the celestial sphere. Furthermore, spacetime scattering amplitudes map to correlation functions of these operators on the celestial sphere. Operator product expansions of these operators on the celestial sphere play a central role in the current research. They correspond to collinear limits of the bulk fields.

In recent years, various authors have revisited the BMS analysis. By making slight changes to the assumptions some of them have been led to conclude that the asymptotic

\footnote{Other possibilities considered in the literature include various Carrollian groups. See [8] – [10], for example.
symmetry is larger than the conformal group. Some of these studies are [17]–[23]. In most cases they appear to conclude that the conformal symmetry of celestial holography, Conf($S^2$), should be extended to diffeomorphism symmetry, Diff($S^2$). It is difficult to decide which conclusion is correct within the general relativity framework that is used in these analyses. Some works, such as [24][25], are careful to distinguish between asymptotic spacetime symmetries and symmetries of the celestial CFT, even though they must be closely related. For previous studies of Diff($S^2$) and related algebras see [26].

The goal of this paper is to answer this question by using considerations that do not refer explicitly to a spacetime metric. Specifically, the plan is to explore what is involved in extending the conformal symmetry algebra of a 2d Euclidean conformal field theory to diffeomorphism symmetry and what its physical implications are in the context of celestial holography. We will conclude that the conformal symmetry in celestial holography should not be extended to diffeomorphism symmetry.

2 The diffeomorphism algebra

The key fact is that the conformal symmetry generator $T(z)$ describes a subset of the modes of a diffeomorphism generator $J(z, \bar{z})$. The complex conjugate statement is that $\overline{T}(\bar{z})$ describes a subset of the modes of $\overline{J}(z, \bar{z})$. The mode expansions of these operators have the form given previously for operators of dimension $(2, 1)$ and $(1, 2)$, respectively. Thus,

$$J(z, \bar{z}) = \sum_{mn} \frac{J_{m,n}}{z^{m+2} \bar{z}^{n+1}} \quad \text{and} \quad \overline{J}(z, \bar{z}) = \sum_{mn} \frac{\overline{J}_{m,n}}{z^{m+1} \bar{z}^{n+2}} \quad m, n \in \mathbb{Z}. \quad (8)$$

However, as we will see, this statement does not adequately describe their algebra. The embedding of $T$ and $\overline{T}$ is given by

$$J_{m,0} = L_m \quad \text{and} \quad \overline{J}_{0,n} = \overline{L}_n. \quad (9)$$

Aside from a possible central extension, the natural extension of the conformal symmetry algebra to the diffeomorphism algebra is

$$[J_{k,l}, J_{m,n}] = (k - m)J_{k+m,l+n} \quad (10)$$

$$[\overline{J}_{k,l}, \overline{J}_{m,n}] = (l - n)\overline{J}_{k+m,l+n} \quad (11)$$

$$[J_{k,l}, \overline{J}_{m,n}] = lJ_{k+m,l+n} - m\overline{J}_{k+m,l+n}. \quad (12)$$
The coefficients have been determined by setting various indices to zero and implementing the correct conformal subalgebra. As we will show, these terms are correct, but another term involving a new operator needs to be added.

The fact that this is essentially the correct algebra for the generators of diffeomorphisms, also known as general coordinate transformations, is shown by representing them by differential operators

\[ J_{k,l} \sim z^{1-k} \bar{z}^{-l} \frac{\partial}{\partial z} \quad \text{and} \quad \mathcal{J}_{m,n} \sim z^{-m} \bar{z}^{1-n} \frac{\partial}{\partial \bar{z}}. \] (13)

Operators of dimension \((\hbar, \bar{\hbar})\) satisfy the same algebra as conjugate operators of dimensions \((1 - \hbar, 1 - \bar{\hbar})\). (The conjugate operator is called the “shadow transform.”) That is why the \((2, 1)\) and \((1, 2)\) operators \(J\) and \(\mathcal{J}\) satisfy the same algebra as the \((-1, 0)\) and \((0, -1)\) differential operators that transform the coordinates \(z\) and \(\bar{z}\). It is also why we used the \(\sim\) symbol here. These differential operators will not be used in the subsequent analysis.

If \(\Phi\) is an operator with dimensions \((\hbar, \bar{\hbar})\), but not \(J\) or \(\mathcal{J}\), diffeomorphisms are encoded in the commutation relations of its modes

\[
[J_{k,l}, \Phi_{m,n}] = ((\hbar - 1)k - m)\Phi_{k+m,l+n}
\] (14)
\[
[\mathcal{J}_{k,l}, \Phi_{m,n}] = ((\bar{\hbar} - 1)l - n)\Phi_{k+m,l+n}.
\] (15)

These formulas play a crucial role in the subsequent analysis, so we should say more about them. The first formula is certainly correct for \(l = 0\), since \(J_{k,0} = L_k\), so any modification should be proportional to \(l\). Similarly, any modification of the second formula should be proportional to \(k\). So let us be safe and (temporarily) write

\[
[J_{k,l}, \Phi_{m,n}] = ((\hbar - 1)k - m + \alpha l)\Phi_{k+m,l+n}
\] (16)
\[
[\mathcal{J}_{k,l}, \Phi_{m,n}] = ((\bar{\hbar} - 1)l - n + \beta k)\Phi_{k+m,l+n},
\] (17)

where \(\alpha\) and \(\beta\) are allowed to depend on \(\hbar\) and \(\bar{\hbar}\).

Diffeomorphism invariance is not sufficient by itself to describe theories containing operators with nonzero conformal spin. To demonstrate the issue let us examine the Jacobi identity involving \(J\), \(\mathcal{J}\), and an operator \(\Phi\) that has dimensions \((\hbar, \bar{\hbar})\). (There is no problem with the \(JJ\Phi\) and \(\mathcal{J}\mathcal{J}\Phi\) Jacobi identities.) All of the requisite commutators have been presented. Using them we find that

\[
[[J_{k,l}, \mathcal{J}_{m,n}], \Phi_{p,q}] + [[\mathcal{J}_{m,n}, \Phi_{p,q}], J_{k,l}] + [[\Phi_{p,q}, J_{k,l}], \mathcal{J}_{m,n}]
\]
\[
= [(h - \bar{\hbar})lm + \alpha ln - \beta km] \Phi_{k+m+p,l+n+q},
\] (18)
which shows that the algebra is inconsistent as it stands. The only way to rectify it is to set $\alpha = \beta = 0$ and to add an additional term to the $[J, \bar{J}]$ equation.

The appearance of the spin of $\Phi$, $h - \bar{h}$, suggests the need to incorporate a spin operator in the formulas. For this purpose let us introduce a dimension $(1, 1)$ spin operator $S(z, \bar{z})$, with the commutation relation

$$[S_{k,l}, \Phi_{m,n}] = (h - \bar{h})\Phi_{k+m,l+n}. \quad (19)$$

Then we add another term to eq. (12) giving

$$\bar{J}_{k,l} J_{m,n} = l J_{k+m,l+n} - m \bar{J}_{k+m,l+n} - lm S_{k+m,l+n}, \quad (20)$$

which repairs the Jacobi identity. Additional Jacobi identities require that

$$[J_{k,l}, S_{m,n}] = -m S_{k+m,l+n}, \quad (21)$$

$$[\bar{J}_{k,l}, S_{m,n}] = -n S_{k+m,l+n}, \quad (22)$$

$$[S_{k,l}, S_{m,n}] = 0. \quad (23)$$

Note that $J$ and $\bar{J}$ are special and do not follow the general rule for operators of dimension $(2, 1)$ and $(1, 2)$. In particular, eq. (19) does not apply to them. Instead, they transform $S$ in accordance with their general rule in eqs. (14) and (15). This conclusion was reached by studying Jacobi identities.

We can now convert the various commutation relations of modes into equivalent operator product expansions. Typically what happens when one evaluates a commutator of two operators, such as $[A(z, \bar{z}), B(w, \bar{w})]$ by inserting mode expansions for each of them is that one encounters ill-defined series of the form $\sum_m w^m/z^{m+1}$ times some operator $C(w)$. The ill-defined series, as well as its derivatives and complex conjugate, need to be interpreted. These series determine the poles in the corresponding OPE, which carry all of the relevant information. A convenient mnemonic that allows one to convert between commutators of modes and singular OPEs in either direction uses the correspondence

$$\sum_m w^m/z^{m+1} \leftrightarrow \frac{1}{z - w}, \quad (24)$$

as well as derivatives and complex conjugates of this correspondence. This rule reproduces the results obtained by the contour integral methods mentioned earlier.

We are now ready to re-express the complete algebra of the $J$, $\bar{J}$, and $S$ symmetry operators, combining diffeomorphism symmetry with the spin operator, as well as their
action on an arbitrary conformal operator $\Phi$ of dimensions $(h, \bar{h})$. The OPEs of various $\Phi$ operators among themselves addresses the dynamics of specific theories and (with one exception that will appear later) is beyond the scope of this note.

Using the procedure sketched above to deduce the OPEs from commutation relations, we find the following\(^4\)

\[
S(z, \bar{z})S(w, \bar{w}) \sim 0. \tag{25}
\]

\[
J(z, \bar{z})S(w, \bar{w}) \sim \frac{1}{|z - w|^2} \left( \frac{S}{z - w} + \partial S \right) \tag{26}
\]

\[
\overline{J}(z, \bar{z})S(w, \bar{w}) \sim \frac{1}{|z - w|^2} \left( \frac{S}{\bar{z} - \bar{w}} + \bar{\partial} S \right) \tag{27}
\]

\[
J(z, \bar{z})J(w, \bar{w}) \sim \frac{1}{|z - w|^2} \left( \frac{2J}{z - w} + \partial J \right) \tag{28}
\]

\[
\overline{J}(z, \bar{z})\overline{J}(w, \bar{w}) \sim \frac{1}{|z - w|^2} \left( \frac{2\overline{J}}{\bar{z} - \bar{w}} + \bar{\partial} \overline{J} \right) \tag{29}
\]

\[
J(z, \bar{z})\overline{J}(w, \bar{w}) \sim \frac{1}{|z - w|^2} \left( \frac{S}{z - w} + \frac{\partial S + J}{z - w} + \frac{\overline{J}}{\bar{z} - \bar{w}} + \bar{\partial} \overline{J} \right). \tag{30}
\]

\[
\overline{J}(z, \bar{z})J(w, \bar{w}) \sim \frac{1}{|z - w|^2} \left( \frac{S}{|z - w|^2} + \frac{\partial S + \overline{J}}{z - w} + \frac{J}{\bar{z} - \bar{w}} + \overline{\partial} J \right). \tag{31}
\]

Turning now to the action of these operators on $\Phi$, we find

\[
S(z, \bar{z})\Phi(w, \bar{w}) \sim \frac{(h - \bar{h})\Phi}{|z - w|^2} \tag{32}
\]

\[
J(z, \bar{z})\Phi(w, \bar{w}) \sim \frac{1}{|z - w|^2} \left( \frac{h\Phi}{z - w} + \partial \Phi \right) \tag{33}
\]

\[
\overline{J}(z, \bar{z})\Phi(w, \bar{w}) \sim \frac{1}{|z - w|^2} \left( \frac{\bar{h}\Phi}{\bar{z} - \bar{w}} + \bar{\partial} \Phi \right). \tag{34}
\]

### 3 Implications for Celestial Holography

The preceding analysis of diffeomorphism symmetry has an important implication for celestial holography. In celestial holography the helicity of a massless particle in four-dimensional spacetime is represented by the two-dimensional conformal spin of the corresponding operator on the celestial sphere. Thus, since the assumption of two-dimensional diffeomorphism

---

\(^4\)In order to save space, we do not display the $w$ and $\bar{w}$ dependence of the operators appearing on the right-hand side of the OPEs.
symmetry implies conservation of conformal spin, the holographic implication is conservation of four-dimensional helicity in the interactions of massless particles such as gravitons and photons. This is definitely not what we want. Thus, we are forced to conclude that the conformal symmetry of the holographic theory on the celestial sphere must not be extended to diffeomorphism symmetry.

To make perfectly clear what we are talking about, let us consider a specific example – the OPE of two positive-helicity graviton operators derived in [27]:

\[ G^{+}_{\Delta_1}(z_1, \bar{z}_1)G^{+}_{\Delta_2}(z_2, \bar{z}_2) \sim E_{+}(\Delta_1, \Delta_2) \frac{\bar{z}_{12}}{z_{12}} G^{+}_{\Delta_1+\Delta_2}(z_2, \bar{z}_2), \]  

(35)

where \( z_{12} = z_1 - z_2 \) and

\[ E_{+}(\Delta_1, \Delta_2) = -\frac{\kappa}{2} B(\Delta_1 - 1, \Delta_2 - 1). \]  

(36)

The left-hand side of the OPE has conformal spin \( s = 2 + 2 = 4 \) and the right-hand side has conformal spin \( s = 2 \). Thus, conformal spin is not conserved. As we have explained, this implies that diffeomorphism symmetry is violated as well.

Celestial holography may require all sorts of symmetries in addition to conformal symmetry, such as supertranslations and a \( w_{1+\infty} \) loop algebra [28], but \( \text{Diff}(S^2) \) is not one of them. There are classical field theories that do conserve helicity, such as theories of scalars only or Born–Infeld theory [29], not coupled to gravity, but they are unlikely to be fully consistent as quantum theories. Even if there were consistent helicity-conserving quantum theories, they would not be expected to have a dual celestial realization with infinite-dimensional conformal symmetry, let alone diffeomorphism symmetry, since they do not contain gravity.

It may be interesting to make precise the implications of this conclusion for the asymptotic metric analysis of GR in asymptotically flat spacetime. Although diffeomorphism symmetry is not appropriate for celestial holography, the description of the two-dimensional diffeomorphism algebra presented here might be useful for other purposes.

Acknowledgments

I am grateful to Clifford Cheung for reading a draft of this manuscript and making helpful comments. This work was supported in part by the Walter Burke Institute for Theoretical Physics at Caltech and by U.S. DOE grant DE-SC0011632. It was performed in part at the Aspen Center for Physics, which is supported by NSF grant PHY-1607611.
References

[1] G. Barnich, “Centrally extended BMS4 Lie algebroid,” JHEP 06, 007 (2017) [arXiv:1703.08704 [hep-th]].

[2] H. Bondi, M. G. J. van der Burg and A. W. K. Metzner, “Gravitational waves in general relativity. 7. Waves from axisymmetric isolated systems,” Proc. Roy. Soc. Lond. A 269, 21-52 (1962)

[3] R. K. Sachs, “Gravitational waves in general relativity. 8. Waves in asymptotically flat space-times,” Proc. Roy. Soc. Lond. A 270, 103-126 (1962)

[4] R. Sachs, “Asymptotic symmetries in gravitational theory,” Phys. Rev. 128, 2851-2864 (1962)

[5] J. de Boer and S. N. Solodukhin, “A Holographic reduction of Minkowski space-time,” Nucl. Phys. B 665, 545-593 (2003) [arXiv:hep-th/0303006 [hep-th]].

[6] G. Barnich and C. Troessaert, “Symmetries of asymptotically flat 4 dimensional space-times at null infinity revisited,” Phys. Rev. Lett. 105, 111103 (2010) [arXiv:0909.2617 [gr-qc]].

[7] G. Barnich and C. Troessaert, “Aspects of the BMS/CFT correspondence,” JHEP 05, 062 (2010) [arXiv:1001.1541 [hep-th]].

[8] C. Duval, G. W. Gibbons and P. A. Horvathy, “Conformal Carroll groups,” J. Phys. A 47, no.33, 335204 (2014) [arXiv:1403.4213 [hep-th]].

[9] L. Donnay, A. Fiorucci, Y. Herfray and R. Ruzziconi, “Carrollian Perspective on Celestial Holography,” Phys. Rev. Lett. 129, no.7, 071602 (2022) [arXiv:2202.04702 [hep-th]].

[10] A. Bagchi, S. Banerjee, R. Basu and S. Dutta, “Scattering Amplitudes: Celestial and Carrollian,” Phys. Rev. Lett. 128, no.24, 241601 (2022) [arXiv:2202.08438 [hep-th]].

[11] A. Strominger, “Lectures on the Infrared Structure of Gravity and Gauge Theory,” Princeton University Press, 2018 [arXiv:1703.05448 [hep-th]].

[12] A. M. Raclariu, “Lectures on Celestial Holography,” [arXiv:2107.02075 [hep-th]].

[13] S. Pasterski, “Lectures on Celestial Amplitudes,” [arXiv:2108.04801 [hep-th]].

[14] P. B. Aneesh, G. Compère, L. P. de Gioia, I. Mol and B. Swidler, “Celestial Holography: Lectures on Asymptotic Symmetries,” [arXiv:2109.00997 [hep-th]].
[15] S. Pasterski, M. Pate and A. M. Raclariu, “Celestial Holography,” [arXiv:2111.11392 [hep-th]].

[16] T. McLoughlin, A. Puhm and A. M. Raclariu, “The SAGEX Review on Scattering Amplitudes, Chapter 11: Soft Theorems and Celestial Amplitudes,” [arXiv:2203.13022 [hep-th]].

[17] M. Campiglia and A. Laddha, “Asymptotic symmetries and subleading soft graviton theorem,” Phys. Rev. D 90, no.12, 124028 (2014) [arXiv:1408.2228 [hep-th]].

[18] M. Campiglia and A. Laddha, “New symmetries for the Gravitational S-matrix,” JHEP 04, 076 (2015) [arXiv:1502.02318 [hep-th]].

[19] G. Compèrè, A. Fiorucci and R. Ruzziconi, “Superboost transitions, refraction memory and super-Lorentz charge algebra,” JHEP 11, 200 (2018) [erratum: JHEP 04, 172 (2020)] [arXiv:1810.00377 [hep-th]].

[20] M. Campiglia and J. Peraza, “Generalized BMS charge algebra,” Phys. Rev. D 101, no.10, 104039 (2020) [arXiv:2002.06691 [gr-qc]].

[21] D. Colferai and S. Lionetti, “Asymptotic symmetries and the subleading soft graviton theorem in higher dimensions,” Phys. Rev. D 104, no.6, 064010 (2021) [arXiv:2005.03439 [hep-th]].

[22] L. Donnay and R. Ruzziconi, “BMS Flux Algebra in Celestial Holography,” [arXiv:2108.11969 [hep-th]].

[23] V. Chandrasekaran, E. E. Flanagan, I. Shehzad and A. J. Speranza, “A general framework for gravitational charges and holographic renormalization,” Int. J. Mod. Phys. A 37, no.17, 2250105 (2022) [arXiv:2111.11974 [gr-qc]].

[24] É. É. Flanagan, K. Prabhu and I. Shehzad, “Extensions of the asymptotic symmetry algebra of general relativity,” JHEP 01, 002 (2020) [arXiv:1910.04557 [gr-qc]].

[25] C. Krishnan and J. Pereira, “Hypertranslations and Hyperrotations,” [arXiv:2205.01422 [hep-th]].

[26] M. Enriquez-Rojo, T. Procházka and I. Sachs, “On deformations and extensions of \textit{Diff(S}\textsuperscript{2})\textit{),” JHEP 10, 133 (2021) [arXiv:2105.13375 [hep-th]].
[27] M. Pate, A. M. Raclaru, A. Strominger and E. Y. Yuan, “Celestial operator products of gluons and gravitons,” Rev. Math. Phys. 33, no.09, 2140003 (2021) [arXiv:1910.07424 [hep-th]].

[28] A. Strominger, “w(1+infinity) and the Celestial Sphere,” [arXiv:2105.14346 [hep-th]].

[29] C. Cheung and J. Mangan, “Covariant color-kinematics duality,” JHEP 11, 069 (2021) [arXiv:2108.02276 [hep-th]].