Study of low-lying orbital excited $B_c$ mesons

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In this paper, we study the mass spectra and the decays of low-lying orbital excited $B_c$ mesons. We predict the mass spectra of the high excited $B_c$ states using Cornell potential model taking into account the screening effect. We adopt a wave function which is expanded with a set of complete Simple Harmonic Oscillator (SHO) bases to calculate the radiative decay width of the $B_c$ mesons. Furthermore, the widths of the two-body strong decays of $B_c$ mesons are calculated by the quark pair creation(QPC) model, and the branching ratio are given accordingly. We expect that our research have reference value for searching for the other $B_c$ mesons that have not been observed and establishing the $B_c$ meson family in the future.

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I. INTRODUCTION

$B_c$ mesons are mainly produce by strong interaction and decay by weak interaction. Their are composed of $c$-quarks and $b$-quarks. $B_c$ mesons cannot annihilate into gluons or photons, they are very stable [1]. Therefore, the decay behaviors and mass spectra of $B_c$ mesons predicted in the present work are helpful to understand perturbative or non-perturbative QCD and electroweak interaction, etc.

In the past few decades, significant progress has been achieved in exploring $B_c$ mesons experimentally and theoretically [2–6]. In 1998, the ground state of $B_c$ meson was first observed by the CDF Collaboration at Fermilab [3], with its mass $M_{B_c} = 6.40 \pm 0.39 \pm 0.13$ GeV. There was no reported evidence of the excited $B_c$ state until 2014, the ATLAS Collaboration reported a structure with mass of 6842 ± 9 MeV [4], which is consistent with the value predicted for $B_c(2S)$. Recently, the excited $B_c(2^3S_0)$ and $B_c(2^3S_1)$ states have been observed in the $B_c^+\pi^+\pi^-$ invariant mass spectrum by the CMS and LHCB Collaboration, with their masses determined to be 6872.1 ± 2.2 MeV and 6841.2 ± 1.5 MeV [5, 6], respectively. So far, different experimental groups have successively observed $B_c$ mesons as the $S$-wave, the experimental information about the $B_c$ mesons are still scarce. The Particle Data Group(PDG) has only included two $B_c$ mesons [7]. There is a lack of the information of the $B_c$ mesons which needed to be urgently studied.

Cornell potential model have achieved great success in describing mass spectra of mesons [8–10]. When using it to predict mass spectra of mesons, the masses of the ground and low excited states agree well with the experimental values. However, for the mass of the high excited states, the prediction of theoretical value is significantly higher than the experimental value [11, 12]. Theorists believe that this deviation is partly caused by the coupled channel effect [13, 14].

The structure of this paper is as follows. In Sec. II, we adopt the Cornell potential model by including the screening effect to study the mass spectra of $B_c$ mesons. In Sec. III, we present the detailed study of the OZI-allowed two-body strong decays of the discussed $B_c$ mesons and predict their decay widths by using the QPC model. We calculate the decay widths of the electric dipole(E1) and magnetic dipole(M1) radiative transition of $B_c$ mesons in Sec. V. The paper ends with a conclusion.

II. MASS SPECTRUM

$B_c$ mesons are composed of two heavy quarks. The heavier mass of the heavy quarks make that the velocity of the quark in the $B_c$ meson system is relatively small. So we can regard the $B_c$ meson system as a non-relativistic system. The mass spectra of the system can be obtained by solving the Schrödinger equation [15, 16], and the Hamiltonian is

$$H|\psi\rangle = (H_0 + V)|\psi\rangle = E|\psi\rangle. \quad (2.1)$$

For a non-relativistic system, $H_0$ is

$$H_0 = \sum_{i=1}^{2} \left( m_i + \frac{p_i^2}{2m_i} \right). \quad (2.2)$$

$m_1, m_2$ are the masses of $b$-quarks and $c$-quarks. we use the simple Cornell potential as a starting point [17],

$$s(r) = br + c, \quad (2.3)$$

$$G(r) = -\frac{4\alpha_s}{3r}, \quad (2.4)$$

where $s(r)$ is the linear potential, $G(r)$ is the Coulomb potential, and the parameter $c$ is the scaling parameter [18]. When we add the screening potential [19], the linear potential becomes

$$s(r)' = \frac{b(1 - e^{-\mu r})}{\mu} + c. \quad (2.5)$$

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For the spin-dependent term, we refer to it given in the GI model [20–24]. The addition of the screening potential make that the spin correlation term related to the linear potential will also have a corresponding transformation. We obtain that

\[ V = H_{\text{conf}} + H_{\text{cont}} + H^{\text{m}} + H^{\text{ten}}, \tag{2.6} \]

where

\[ H_{\text{conf}} = s(r)' + G(r) \tag{2.7} \]

includes the screening potential and Coulomb-like interaction.

\[ H_{\text{cont}} = \frac{32\pi\alpha_s}{9m_1m_2} \left( \frac{\sigma}{\pi} \right)^3 e^{-\alpha r^2} S_1 \cdot S_2 \tag{2.8} \]

is the colour contact interaction.

\[ H^{\text{m}} = H^{\text{soc(cm)}} + H^{\text{m}(p)} \tag{2.9} \]

is the spin-orbit interaction.

\[ H^{\text{soc(cm)}} = \frac{4\alpha_s}{3} \frac{1}{r^3} \left( \frac{1}{m_1} + \frac{1}{m_2} \right)^2 L \cdot S_{1(2)}, \tag{2.10} \]

its colour magnetic piece arising from one-gluon exchange.

\[ H^{\text{m}(p)} = -\frac{1}{2r} \frac{\partial H_{\text{conf}}}{\partial r} \left( \frac{S_1}{m_1^2} + \frac{S_2}{m_2^2} \right) \cdot L \tag{2.11} \]

\[ = -\frac{1}{2r} \left( \frac{4\alpha_s}{3} \frac{1}{r^2} + be^{-\alpha r} \right) \left( \frac{1}{m_1^2} + \frac{1}{m_2^2} \right) L \cdot S_{1(2)} \tag{2.12} \]

is the Thomas precession term with screening effect.

\[ H^{\text{ten}} = \frac{4}{3} \frac{\alpha_s}{m_1m_2} \frac{1}{r^3} \left( \frac{3S_1 \cdot r S_2 \cdot r}{r^2} - S_1 \cdot S_2 \right) \tag{2.13} \]

is the colour tensor interaction.

\[ T = \frac{3S_1 \cdot r S_2 \cdot r}{r^2} - S_1 \cdot S_2, \tag{2.14} \]

\[ \langle T \rangle = \begin{cases} \frac{L}{6(L+1)} & J = L + 1 \\ \frac{L}{2} & J = L \\ \frac{L}{2(L-1)} & J = L - 1 \end{cases} \tag{2.15} \]

\[ T \] is the tensor operator, \( S_1 \) and \( S_2 \) are the spins of the quarks which constitute the mesons. \( L \) is the orbital angular momentum [25].

\( S \cdot L \) coupling causes the mixing of spin singlet and spin triplet states. The Hamiltonian corresponding to the \( S \cdot L \) term can be written as symmetric and anti-symmetric parts. The result of the symmetric part is zero. We only need to consider the anti-symmetric part,

\[ H_{\text{anti}} = \frac{1}{4} \left( \frac{4\alpha_s}{3} \frac{1}{r^2} - be^{-\alpha r} \right) \left( \frac{1}{m_1^2} - \frac{1}{m_2^2} \right) (S_1 - S_2) \cdot L, \tag{2.16} \]

so that the quality of the mixture can be obtained

\[ L' = L_J \cos \theta + 3L_J \sin \theta, \tag{2.17} \]

To solve the Schrödinger equation, we use the simple harmonic oscillator(SHO) wave functions as a set of complete bases,

\[ \Psi_{nLM_\ell}(r) = R_{n\ell}(r)Y_{LM_\ell}(\Omega_\ell), \tag{2.19} \]

\[ \Psi_{nLM_\ell}(p) = R_{n\ell}(p)Y_{LM_\ell}(\Omega_\ell), \tag{2.20} \]

where

\[ R_{n\ell}(r,\beta) = \beta^2 \sqrt{\frac{2n!}{\Gamma(n + L + \frac{3}{2})}} e^{-\frac{2r^2}{\beta^2}} L_n^{L+\frac{1}{2}}(\beta^2 r^2), \tag{2.21} \]

\[ R_{n\ell}(p,\beta) = (\frac{-1}{\beta^2}) e^{-\frac{p^2}{\beta^2}} \sqrt{\frac{2n!}{\Gamma(n + L + \frac{3}{2})}} (\frac{p}{\beta})^L L_n^{L+\frac{1}{2}}(\frac{p^2}{\beta^2}), \tag{2.22} \]

\( Y_{LM_\ell}(\Omega_\ell) \) is a spherical harmonic function, \( R_{n\ell} \) is a radial wave function, \( L_n^{L+\frac{1}{2}}(x) \) is a Laguerre polynomial.

In order to study the spectra of \( cc \) mesons, we should determine the properties of the potential model by fitting the experimental data. \( B_c \) mesons come between \( bb \) and \( cc \) mesons in the the properties. The decay of \( B_c \) mesons in the two-body strong decay contain \( B_c \), \( B_c^* \), \( D \) and \( D_s \) mesons. We need to consider their experimental values when fitting the parameters of the potential model. The experimental value of \( B_c \) mesons are taken from LHCb, and the experimental value of other mesons are taken from PDG. We define the error \( \chi^2 \) of all mesons from the experimental values:

\[ \chi^2 = \sum \frac{(\text{Th} - \text{Exp})^2}{\text{Error}^2}, \tag{2.23} \]

where \( \text{Th} \), \( \text{Exp} \), and \( \text{Error} \) represent the theoretical, experimental data, and fitting error, respectively.

The parameter which we need is obtained when \( \chi^2 \) takes the minimum value. When \( \chi^2/d.o.f. \) is 90, we get the parameters of the potential model as shown in Table II. \( d.o.f. \) is the degree of freedom and it is 32.

Using the parameters in Table II, we obtain the mass spectra of \( B_c \) mesons. In Table III, we give the masses of \( B_c \) mesons from \( S \)-wave to \( P \)-wave, and give the predicted value of other Refs. [1, 26–28]. Comparing our calculated results with other literatures, we can find that the theoretical values are in good agreement with the experimental values.

The ground state is in good agreement with experimental value and we predict the high excited states considering screening effect. We calculate the theoretical values of \( B_c \) mesons on the high excited states (\( n = 3, 4, 5, 6 \)) which are given in Table III. We find that the mass of the \( 3S_0 \) and \( 3S_1 \) states are about 7216 and 7232 MeV respectively. The mass of the \( 4S_1 \) state is about 7513 MeV higher than the \( 4S_0 \) state, which mass is 7500 MeV.

\( P \)-waves are mixed, where the \( 1P_1 \) and \( 3P_1 \) states are mixed into \( P_1' \) and \( P_1 \) states. The masses of \( 1P_1' \) and \( 1P_1 \) states are 6760 and 6745 MeV, and the mixing angle \( \theta_{1P} = -25.2^\circ \). The
masses of $1P_1$ and $1P_1$ states are very close to those predicted by the GI model, they are 10 MeV larger and 4 MeV smaller than the GI model, respectively. The values of other states are shown in Table III.
The QPC model was first proposed by Micu and has been widely used to calculate the strong two-body decay allowed by OZI after further development [29–34]. In the QPC model, the transition matrix of the decay process \( A \rightarrow B+C \) is defined by [35–39]

\[
\langle BC|\mathcal{T}|A \rangle = \delta^3(P_B + P_C)\mathcal{M}^{M_A, M_B, M_C},
\]

(3.1)

where \( \mathcal{M}^{M_A, M_B, M_C} \) is the amplitude of \( A \rightarrow B + C \). \( \mathcal{T} \) is the transition operator, which can describe the creation of a quark-antiquark pair from vacuum. It can be expressed as

\[
\mathcal{T} = -3\gamma \sum_m \langle 1m; 1 - m|00 \rangle \int dp_3 dp_4 \delta^3(p_3 + p_4)
\times Y_{1m} \left( \frac{P_3 - P_4}{2} \right) \chi^\dagger \phi \omega \chi \phi \omega
\]

(3.2)

where \( \chi, \phi \) and \( \omega \) denote the quark and antiquark, respectively. \( \gamma \) is a dimensionless constant, which describes the generation rate of a quark-antiquark pair from vacuum, and it is determined by fitting experimental data. \( Y_{1m}(p) = |p|^4 Y_{1m}(p) \) is the solid harmonic. According to the Jacobi-Wick formula, the amplitude is converted into the partial wave amplitude, and the amplitude is expressed as

\[
\mathcal{M}^{JL}(P) = \frac{\sqrt{4\pi(2L + 1)}}{2J + 1} \sum_{M_A, M_B, M_C} \langle L0; JM_{J1} | JM_{J2} \rangle \times \langle J_B M_{J1}; M_{J2} | J_A M_J \rangle \mathcal{M}^{M_A, M_B, M_C}.
\]

(3.3)

The decay width of \( A \rightarrow BC \) is read as

\[
\Gamma = \frac{\pi |P_E|}{m_A^2} \sum_{JL} |\mathcal{M}^{JL}(P)|^2,
\]

(3.4)

where \( m_A \) is the mass of the initial state \( A \)-meson.

In addition, the meson wave function is defined as a mock state, i.e.

\[
|A(2S + 1LJM_J)\rangle(p_A) = \sqrt{2E} \sum_{M_S, M_L} \langle LMS M_S | JM_J \rangle \chi^A M_S \times \phi^A \omega^A \int dp_1 dp_2 \delta^3(p_A - p_1 - p_2) \times \Psi_{nLM_L} \langle p_1, p_2 | q_1(1) \bar{q}_2(2) \rangle,
\]

(3.5)

here, the spatial wave function \( \Psi_{nLM_L}(p) \) of the meson is obtained by solving Eq. 2.1.

We can filter out the decay channels according to OZI-allowed two-body strong decay of hadrons. For strong decays that produce a pair of strange quarks from vacuum, \( \gamma_s = \gamma/\sqrt{3} \). The experimental information of \( B_c \) mesons is not integrated so that we take \( \gamma = 0.4 \) in this paper [40]. The value of \( \gamma \) we calculated is \( \sqrt{96\pi} \) times the value of \( \gamma \) in Ref. [41] and it is 0.4 \( \sqrt{96\pi} \). The decay width of the two-body strong decay are calculated and the results are shown in Table IV. The branching ratios of the decay widths of \( B_c \) mesons are also given.

| Initial state | Final state | \( \Gamma \) (MeV) | \( B_c \) |
|--------------|-------------|-----------------|--------|
| \( 3^1S_0 \)  | \( B^D \)    | 193             | 100\%  |
| \( 4^1S_0 \)  | \( B^D \)    | 35.0            | 17.2\% |
| \( B^D^* \)   |             | 75.0            | 36.9\% |
| \( B^D^{*+} \)|             | 90.5            | 44.5\% |
| \( B^D_1 \)   |             | 0.0151          | 0.00744\% |
| \( B^D_2 \)   |             | 3.03            | 1.49\% |
| \( 3^1S_1 \)  | \( B^D \)    | 58.1            | 27.7\% |
| \( B^D' \)    |             | 152             | 72.3\% |
| \( 4^1S_1 \)  | \( B^D \)    | 0.624           | 30.6\% |
| \( B^D'^* \)  |             | 14.1            | 6.91\% |
| \( B^D'^{*+} \)|             | 48.1            | 23.6\% |
| \( B^D'' \)   |             | 139             | 68.1\% |
| \( B^D_1' \)  |             | 0.0518          | 0.0254% |
| \( B^D_2' \)  |             | 2.12            | 1.04\% |
| \( 3^3P_0 \)  | \( B^D \)    | 93.5            | 32.1\% |
| \( B^D' \)    |             | 195             | 66.9\% |
| \( B^D' \)    |             | 2.69            | 0.926\% |
| \( 3^3P_1 \)  | \( B^D \)    | 8.94            | 3.40\% |
| \( B^D'^* \)  |             | 0.416           | 0.158\% |
| \( B^D'^{*+} \)|             | 38.2            | 14.5\% |
| \( B^D'' \)   |             | 213             | 81.0\% |
| \( B^D_1' \)  |             | 1.36            | 0.516\% |
| \( B^D_2' \)  |             | 1.09            | 0.416\% |
| \( 3^3P_1' \)|             | 39.5            | 11.1\% |
| \( B^D'^* \)  |             | 91.0            | 25.6\% |
| \( B^D'^{*+} \)|             | 224             | 62.9\% |
| \( B^D'' \)   |             | 1.56            | 0.438\% |
| \( 3P_1 \)    | \( B^D \)    | 17.6            | 5.14\% |
| \( B^D'^* \)  |             | 92.6            | 27.0\% |
| \( B^D'^{*+} \)|             | 231             | 67.5\% |
| \( B^D'' \)   |             | 1.27            | 0.371\% |

From Table IV, we can observe that the total width of each decay channel is between 175 MeV and 365 MeV. In addition, the branching ratio of the decay channel whose final state is \( B^D'^{*+} \) is very large, especially when the initial state is \( 3^3P_2 \), the branching ratio of this channel is 81.0%. The branching ratios of the decay channels whose final states are \( B^D_P \) and \( B^D_1' \) make small contribution, less than 5% in all results. The largest differences are for decays involving the \( 3P_1' \) and \( 3P_1 \) states which are mixtures of the initial states. We give the decay widths when the mixing angle is \( \theta_{13} = -28.4^\circ \). When \( B^D'^{*+} \) channel is their final state, the widths are 224 MeV and
231 MeV, respectively, and the branching ratio is larger than other channels.

Considering the dependence of the decay width on the mixing angle, we give the decay widths as a function whose initial states are $3P_1$ and $3P_1$ states versus $\theta$ in Fig 1 and Fig 2. The mixing angle varies between $-90^\circ$ and $90^\circ$. The total width of $3P_1$ varies between 295 MeV and 395 MeV. The total width of $3P_1$ varies between 245 MeV and 445 MeV. We can see that the decay width of the $B^*D^*$ channel is the largest, the minimum channel is $B^*_sD_s$. The results are consistent with our table above. The curves show that $BD^*$ and $B^*_sD_s$ channel are insensitive to mixing angle, while $B^*D^*$ is sensitive to mixing angle. $B^*D^*$ is the main decay channel, while $B^*_sD_s$ and $B^*_cD_s$ contribute little in the two-body strong decay.

![FIG. 1: Decay widths of $B_c(3P)$ versus $\theta$. The dotted line here represents the mixing angle $\theta_{3P} = -28.4^\circ$.](image1)

![FIG. 2: Decay widths of $B_c(3P)$ versus $\theta$. The dotted line here represents the mixing angle $\theta_{3P} = -28.4^\circ$.](image2)

IV. RADIATION DECAY

Radiative transitions play an important role in the discovery and identification of $B_c$ mesons [1, 43–47]. In this section, we calculate the E1 and M1 radiative widths of $B_c$ mesons.

A. E1 Transitions

The partial width for an E1 radiative transition between states in the nonrelativistic quark model is given by [1]

$$\Gamma(i \rightarrow f + \gamma) = \frac{4}{3} (\epsilon_\gamma)^2 \alpha \omega^3 C_{fi} \delta_{SS'} |\langle f | r | i \rangle|^2,$$  \hspace{1cm} (4.1)

where

$$C_{fi} = \text{Max} (L, L') (2J' + 1) \left\{ \begin{array}{ccc} L' & J' & S \\ L & J & 1 \end{array} \right\}^2,$$  \hspace{1cm} (4.2)

and

$$\langle e_\gamma \rangle = \frac{m_b e_c - m_c e_b}{m_b + m_c},$$  \hspace{1cm} (4.3)

$m_c$ and $m_b$ are the masses of the quarks. $e$ represents the charge, the charge of the quark is $e_b = -1/3, e_c = 2/3$. $\omega$ is the energy of the photon. The following equation can be obtained by the conservation of energy and momentum

$$M_1 = \sqrt{M_2^2 + \omega^2 + \omega}.$$  \hspace{1cm} (4.4)

In Table V, we give the width of the electric dipole decay and the values are all less than 0.1 MeV. Although the experimental information of $B_c$ mesons decay behaviors is still lacking, the widths we calculate used potential model are closed to Ref. [1]. There is still a significant difference for the E1 radiation transition widths calculated from the present work and the GI model [1], which is mainly due to the difference of the potential modes. This situation can be happen when comparing the result from the decay of $1F^+_3 \rightarrow 1D'_3 \gamma, 1D_2$ and $1F^+_3 \rightarrow 1D'_2 \gamma, 1D_2$, which maybe be caused by the mixing angle. $B_c(1S)\gamma$ is important for $1P$ states. Especially, the process of $B_c(1P_2) \rightarrow 1S_1 \gamma$ has a predicted width of 91 keV. If $B_c(1P_2)$ is confirmed in the future, we suggest experiments to search for the missing $1P_2$ state by studying this radiative process of $B_c(1P_2)$. Likewise, compared to other radiative transition processes, the decay process to the $2S$ -wave $B_c$ state is very significant for $B_c(2P)$ state.

B. M1 Transitions

Radiative transitions which flip spin are described by magnetic dipole (M1) transitions. The rates for magnetic dipole transitions in quarkonium bound states are given in the nonrelativistic approximation by [1]

$$\Gamma(i \rightarrow f + \gamma) = \frac{\alpha}{3} \mu^2 \omega^3 (2J_f + 1) \left| \left\langle f \left| \frac{k}{2} \right| i \right\rangle \right|^2,$$  \hspace{1cm} (4.5)
where

$$\mu = \frac{e_c}{m_c} - \frac{e_b}{m_b}, \quad (4.6)$$

Our predictions for the widths of the magnetic dipole decays we calculated of $S$ states are listed in Table VI. By comparing our results with those of the GI model [1], we find that although most of decay modes have not changed much, the screening effects have demonstrated the power in the M1 radiation transitions. The measurements of M1 radiative decays are much lower than theoretical results as shown in Table V.

As the low spin states, the $S$-wave $B_c$ mesons have no experimental signals at present. If the states can be observed in experiment, it could be a good confirmation for the theoretical calculation of the potential model. The predicted decay properties of $S$-wave $B_c$ mesons for partial widths of magnetic dipole transition are listed in Table VI, where we can find a very interesting common decay feature. That is, the width of low radial excited states are higher than the width of high radial excited states when the initial state is the same. The phenomenon can also be found in Ref. [1]. We hope that these predicted results will be helpful for the future experimental studies on the high excited states of $B_c$ mesons. Different from the case of strong decay modes, the decay width of radiative decay is very small, but it can be compared with experimental values to test the potential model.

### V. CONCLUSION

Mass spectra of $B_c$ mesons are studied in this paper using Cornell potential with the screening effect. The parameters of the potential model by fitting $B_c$, $b\bar{b}$, $c\bar{c}$, $B$, $B_s$, $D$ and $D_s$ mesons are given, we also predict the masses of the excited states of $B_c$ mesons.

For the radiative decays of $B_c$ mesons, we obtain the decay widths of the E1 and M1 dipoles of their $S$ – $F$ waves. The decay widths are all small, within 1 MeV. Comparing with the Ref. [1] and analysis the data that we calculated, the potential model we used in this paper is reasonable.

### TABLE V: Widths of the E1 transitions for the $S_1, P_1, D_1$ and $F_1$ waves $B_c$ states compared with the other model predictions. The width results are in units of keV.

| Initial state | Final state | This work | GI [1] | EFG [48] | GKLT [49] |
|---------------|-------------|-----------|--------|----------|----------|
| $1^1P_2$      | $1^3S_1^+$  | 91        | 83     | 107      | 103      |
| $1^1P_1$      | $1^3S_1$    | 16        | 11     | 14       | 8.1      |
| $1^3S_1$      |             | 84        | 80     | 132      | 11       |
| $1^3P_0$      | $1^3S_1$    | 52        | 55     | 67       | 65       |
| $2^3S_1$      | $1^3P_2$    | 5.6       | 5.7    | 5.2      | 15       |
| $1^3P_1$      |              | 0.74      | 0.7    | 0.63     | 1.0      |
| $1^3P_0$      | $1^3P_2$    | 4.7       | 4.7    | 5.1      | 13       |
| $2^3P_0$      | $1^3P_2$    | 3.7       | 3.9    | 3.8      | 7.7      |
| $1^3D_3$      | $1^3P_2$    | 6.1       | 6.1    | 3.7      | 16       |
| $1^3D_2$      | $1^3P_2$    | 2.0       | 1.3    | 1.0      | 1.9      |
| $1^3D_1$      | $1^3P_2$    | 7.7       | 8.8    | 13       | 6.8      |
| $1^3P_1$      |              | 56        | 63     | 116      | 46       |
| $1^3P_0$      | $1^3P_2$    | 9.2       | 7      | 7.3      | 25       |
| $2^3P_2$      | $2^3S_1$    | 49        | 55     | 57       | 49       |
| $1^3S_1$      |              | 32        | 14     | 14       | 19       |
| $1^3D_3$      | $1^3S_1$    | 9.4       | 6.8    | 1.6      | 11       |
| $1^3D_2$      |              | 0.86      | 0.70   | 0.11     | 0.5      |
| $1^3D_1$      | $1^3D_2$    | 0.73      | 0.60   | 0.27     | 1.5      |
| $2^3P_0$      | $2^3S_1$    | 53        | 55     | 128      | 80       |
| $1^3S_1$      |              | 0.79      | 1.0    | 0.79     | 16.1     |
| $1^3D_3$      | $1^3D_2$    | 0.081     | 4.2    | 0.036    | 3.2      |
| $1^3F_4$      | $1^3D_2$    | 61        | 81     |          |          |
| $1^3P_1$      |              | 4.2       | 3.7    |          |          |
| $1^3D_3$      |              | 0.34      | 78     |          |          |
| $1^3D_1$      | $1^3D_2$    | 49        | 49     |          |          |
| $1^3F_3$      | $1^3D_2$    | 2.8       | 5.4    |          |          |
| $1^3D_3$      |              | 73        | 0.04   |          |          |
| $1^3D_1$      |              | 0.32      | 82     |          |          |
| $1^3F_2$      | $1^3D_2$    | 0.29      | 0.4    |          |          |
| $1^3D_3$      |              | 5.5       | 6.3    |          |          |
| $1^3D_1$      |              | 4.8       | 6.5    |          |          |
| $1^3D_1$      |              | 58        | 75     |          |          |

### TABLE VI: Partial widths of the M1 transitions for the low-lying $S$-, $P$-, $D$-, and $F$-wave $B_c$ states compared with the other model predictions. The width results are in units of eV.

| Initial state | Final state | This work | GI [1] | EQ [26] | EFG [48] | FU [50] |
|---------------|-------------|-----------|--------|---------|----------|---------|
| $2^1S_0$      | $1^3S_1^+$  | 144       | 300    | 93      | 488      | 59      |
| $3^3S_0$      | $2^3S_1^+$  | 12        | 60     |         |          |         |
| $1^3S_1$      | $1^3S_1^+$  | 48        | 80     | 135     | 33       | 59      |
| $2^3S_1$      | $2^3S_1^+$  | 3        | 10     | 29      | 17       | 12      |
| $1^3S_1$      | $1^3S_1^+$  | 350       | 600    | 123     | 428      | 122     |
| $1^3S_1^+$    |              | 408       | 600    |          |          |         |

$j_0$ is a spherical Bessel function

$$j_0(x) = \frac{\sin x}{x}, \quad (4.7)$$
We also study the decay width of the S-wave $B_s$ meson state to P-wave $B_c$ meson state using the $^3P_0$ model and give the branching ratios. Analysis the strong two-body decay of the $B_c$ mesons, the important decay channel is $B_c^*D_s^*$. In addition, $BD^*$ also has certain contribution. $B_s^*D_s$ and $B_cD_c^*$ make small contribution. We expect that our research can provide some helpful information for searching for $B_c$ mesons in future experiments.

VI. ACKNOWLEDGMENTS

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