Charmed Baryons Circa 2015

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Abstract
This is basically the update of [1], a review on charmed baryon physics around 2007. Topics of this review include the spectroscopy, strong decays, lifetimes, nonleptonic and semileptonic weak decays, and electromagnetic decays of charmed baryons.
I. INTRODUCTION

Charm baryon spectroscopy provides an excellent ground to study the dynamics of light quarks in the environment of a heavy quark. In the past decade, many new excited charmed baryon states have been discovered by BaBar, Belle, CLEO and LHCb. A very rich source of charmed baryons comes both from $B$ decays and from the $e^+e^-\rightarrow c\bar{c}$ continuum. A lot of efforts have been devoted to identifying the quantum numbers of these new states and understand their properties.

Consider the strong decays $\Sigma_Q\rightarrow \Lambda_Q\pi$ and $\Xi_Q^*\rightarrow \Xi_Q\pi$ with $Q = c, b$. It turns out that the mass differences between $\Sigma_c$ and $\Lambda_c$ and between $\Xi_c^*$ and $\Xi_c$ in the charmed baryon sector are large enough to render the strong decays of $\Sigma_c$ and $\Xi_c^*$ kinematically allowed. As a consequence, the charmed baryon system offers a unique and excellent laboratory for testing the ideas and predictions of heavy quark symmetry of the heavy quark and chiral symmetry of the light quarks. This will have interesting implications for the low-energy dynamics of heavy baryons interacting with the Goldstone bosons.

Theoretical interest in the charmed baryon hadronic weak decays was peaked around early nineties and then faded away. Until today, we still don’t have a good phenomenological model, not mentioning the QCD-inspired approach as in $B$ meson decays, to describe the complicated physics of baryon decays. We do need cooperative efforts from both experimentalists and theorists to make progress in this arena.

This review is basically the update of [1] around 2007. The outline of the content is the same as that of [1] except we add discussions on the spectroscopy and lifetimes of doubly charmed baryons.

Several excellent review articles on charmed baryons can be found in [2–7].

II. SPECTROSCOPY

A. Singly charmed baryons

The singly charmed baryon is composed of a charmed quark and two light quarks. Each light quark is a triplet of the flavor SU(3). There are two different SU(3) multiplets of charmed baryons: a symmetric sextet $6$ and an antisymmetric antitriplet $\bar{3}$. The $\Lambda_c^+$, $\Xi_c^+$ and $\Xi_c$ form an $\bar{3}$ representation and they all decay weakly. The $\Omega_c^0$, $\Xi_c^*$, $\Xi_c^0$ and $\Sigma_c^{++}$, $\Sigma_c^+$, $\Sigma_c^0$ form a $6$ representation; among them, only $\Omega_c^0$ decays weakly. We follow the Particle Data Group’s convention [8] to use a prime to distinguish the $\Xi_c$ in the $6$ from the one in the $\bar{3}$.

In the quark model, the orbital angular momentum of the light diquark can be decomposed into $L_\ell=L_\rho+L_\lambda$, where $L_\rho$ is the orbital angular momentum between the two light quarks and $L_\lambda$ the orbital angular momentum between the diquark and the charmed quark. Although the separate spin angular momentum $S_\ell$ and orbital angular momentum $L_\ell$ of the light degrees of freedom are not well defined, they are included for guidance from the quark model. In the heavy quark limit, the spin of the charmed quark $S_c$ and the total angular momentum of the two light quarks $J_\ell = S_\ell + L_\ell$ are separately conserved. The total angular momentum is given by $J = S_c + J_\ell$. It is convenient to use $S_\ell$, $L_\ell$ and $J_\ell$ to enumerate the spectrum of states. Moreover, one can define two independent relative momenta $p_\rho = \frac{1}{\sqrt{2}} (p_1 - p_2)$ and $p_\lambda = \frac{1}{\sqrt{6}} (p_1 + p_2 - 2p_c)$ from the two light quark momenta $p_1$, $p_2$ and the heavy quark momentum $p_c$. Denoting the quantum numbers $L_\rho$ and $L_\lambda$ as the eigenvalues of $L_\rho^2$ and $L_\lambda^2$, the $\rho$-orbital momentum $L_\rho$ describes relative orbital excitations of the
two light quarks, and the $\lambda$-orbital momentum $L_\lambda$ describes orbital excitations of the center of the mass of the two light quarks relative to the heavy quark (see Fig. 1). The $p$-wave heavy baryon can be either in the $(L_\rho = 0, L_\lambda = 1)$ $\lambda$-state or the $(L_\rho = 1, L_\lambda = 0)$ $p$-state. It is obvious that the orbital $\lambda$-state ($p$-state) is symmetric (antisymmetric) under the interchange of $p_1$ and $p_2$. In the following, we shall use the notation $B_{cJ_p}(J^P)$ ($\tilde{B}_{cJ_p}(J^P)$) to denote the states symmetric (antisymmetric) in the orbital wave functions under the exchange of two light quarks. The lowest-lying orbitally excited baryon states are the $p$-wave charmed baryons with their quantum numbers listed in Table I.

The next orbitally excited states are the positive-parity excitations with $L_\rho + L_\lambda = 2$. There exist multiplets (e.g. $\Lambda_{c2}$ and $\bar{\Lambda}_{c2}$) with the symmetric orbital wave function, corresponding to $L_\lambda = 2, L_\rho = 0$ and $L_\lambda = 0, L_\rho = 2$ (see Table II). We use a hat to distinguish them. Since the orbital $L_\lambda = L_\rho = 1$ states are antisymmetric under the interchange of two light quarks, we shall use a tilde to denote them. Moreover, the notation $\tilde{B}_{cJ_\rho}^{L_\rho}(J^P)$ is reserved for tilde states in the $3$ as the quantum number $L_\rho$ is needed to distinguish different states.

The observed mass spectra and decay widths of charmed baryons are summarized in Table III (see also Fig. 2). Notice that except for the parity of the lightest $\Lambda_c^+$ and the heavier one $\Lambda_c(2880)^+$,

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**FIG. 1**: Singly charmed baryon where $L_\rho$ describes relative orbital excitation of the two light quarks and $L_\lambda$ the orbital excitation of the center of the mass of the two light quarks relative to the charmed quark.

**TABLE I**: The $p$-wave charmed baryons denoted by $B_{cJ_p}(J^P)$ and $\tilde{B}_{cJ_p}(J^P)$ where $J_\ell$ is the total angular momentum of the two light quarks. The orbital $\rho$-states with $L_\rho = 1$ and $L_\lambda = 0$ have odd orbital wave functions under the permutation of the two light quarks and are denoted by a tilde.

| State      | SU(3) | $S_\ell$ | $L_\ell(L_\rho, L_\lambda)$ | $J_\ell^{P_\ell}$ | State      | SU(3) | $S_\ell$ | $L_\ell(L_\rho, L_\lambda)$ | $J_\ell^{P_\ell}$ |
|------------|-------|---------|-------------------------------|-------------------|------------|-------|---------|-------------------------------|-------------------|
| $\Lambda_{c1}(\frac{1}{2}^+, \frac{3}{2}^-)$ | 3     | 0       | $(0,1)$                       | $1^-$             | $\Sigma_{c0}^{'}(\frac{1}{2}^-)$ | 6     | 1       | $(0,1)$                       | $0^-$             |
| $\Lambda_{c0}(\frac{1}{2}^-)$             | 3     | 1       | $(1,0)$                       | $0^-$             | $\Sigma_{c1}(\frac{1}{2}^-, \frac{3}{2}^-)$ | 6     | 1       | $(0,1)$                       | $1^-$             |
| $\tilde{\Lambda}_{c1}(\frac{1}{2}^-, \frac{3}{2}^-)$ | 3     | 1       | $(1,0)$                       | $1^-$             | $\Sigma_{c2}(\frac{1}{2}^-, \frac{5}{2}^-)$ | 6     | 1       | $(0,1)$                       | $2^-$             |
| $\tilde{\Lambda}_{c2}(\frac{3}{2}^-, \frac{5}{2}^-)$ | 3     | 1       | $(1,0)$                       | $2^-$             | $\Sigma_{c1}(\frac{3}{2}^-, \frac{3}{2}^-)$ | 6     | 0       | $(1,0)$                       | $1^-$             |
| $\Xi_{c1}(\frac{1}{2}^-, \frac{3}{2}^-)$      | 3     | 1       | $(0,1)$                       | $1^-$             | $\Xi_{c0}^{'}(\frac{1}{2}^-)$ | 6     | 1       | $(0,1)$                       | $0^-$             |
| $\Xi_{c0}(\frac{1}{2}^-)$                  | 3     | 1       | $(1,0)$                       | $0^-$             | $\Xi_{c1}^{'}(\frac{1}{2}^-, \frac{3}{2}^-)$ | 6     | 1       | $(0,1)$                       | $1^-$             |
| $\Xi_{c1}(\frac{1}{2}^-, \frac{3}{2}^-)$      | 3     | 1       | $(1,0)$                       | $1^-$             | $\Xi_{c2}^{'}(\frac{1}{2}^-, \frac{5}{2}^-)$ | 6     | 1       | $(0,1)$                       | $2^-$             |
| $\Xi_{c2}(\frac{3}{2}^-, \frac{5}{2}^-)$      | 3     | 1       | $(1,0)$                       | $2^-$             | $\Xi_{c1}^{'}(\frac{3}{2}^-, \frac{3}{2}^-)$ | 6     | 0       | $(1,0)$                       | $1^-$             |
none of the other $J^P$ quantum numbers given in Table III has been measured. One has to rely on the quark model to determine the spin-parity assignments.

In the following we discuss some of the excited charmed baryon states:

1. $\Lambda_c$ states

The lowest-lying $p$-wave $\Lambda_c$ states are $\tilde{\Lambda}_c(1720), \Lambda_c(1750), \tilde{\Lambda}_c(1750), \lambda_c(2595)$ and $\lambda_c(2595)$, and $\tilde{\Lambda}_c(2625)$, and $\lambda_c(2625)$, and $\tilde{\Lambda}_c(2625)$, and $\lambda_c(2625)$, and $\tilde{\Lambda}_c(2625)$, and $\lambda_c(2625)$. A doublet $\Lambda_c(1/2^−)$ and $\Lambda_c(3/2^−)$ is formed by $\Lambda_c(2595)$ and $\lambda_c(2625)$, $\lambda_c(2595)$, $\lambda_c(2625)$, and $\lambda_c(2625)$. The allowed strong decays are $\Lambda_c(1/2^−) \rightarrow [\Sigma_c\pi]_S, [\Sigma_c^*\pi]_D$ and $\Lambda_c(3/2^−) \rightarrow [\Sigma_c\pi]_D, [\Sigma_c^*\pi]_S, [\Lambda_c\pi\pi]_P$. This explains why the width of $\Lambda_c(2625)$ is narrower than that of $\Lambda_c(2595)$. Because of isospin conservation in strong decays, $\Lambda_c^+(1385)$ is not allowed to decay into $\Lambda_c^+\pi^0$.

$\Lambda_c(2765)$ is a broad state first seen in $\Lambda_c^+(1385)$ by CLEO [16]. However, whether it is a $\Lambda_c^+$ or a $\Sigma_c^+$ and whether the width might be due to overlapping states are still not known. The Skyrme model [16] and the quark model [17] suggest a $J^P = 1^+$ $\Lambda_c$ state with a mass 2742 and 2775 MeV, respectively. Therefore, $\Lambda_c(2765)^+$ could be a first positive-parity excitation of $\Lambda_c$. It has also

| State | SU(3)$_F$ | $S_L$ | $L_{\ell}(L_\rho, L_\lambda)$ | $J^P_{\ell}$ | State | SU(3)$_F$ | $S_L$ | $L_{\ell}(L_\rho, L_\lambda)$ | $J^P_{\ell}$ |
|-------|----------|------|-----------------|--------|-------|----------|------|-----------------|--------|
| $\Lambda_c(1/2^−)$ | 3 | 0 | 2(0.2) | 2$^+$ | $\Sigma_c^0(\frac{1}{2}^−, \frac{3}{2}^−)$ | 6 | 1 | 2(0.2) | 1$^+$ |
| $\Lambda_c(3/2^−)$ | 3 | 0 | 2(0.2) | 2$^+$ | $\Sigma_c^2(\frac{3}{2}^−, \frac{5}{2}^−)$ | 6 | 1 | 2(0.2) | 2$^+$ |
| $\Lambda_c^+(1/2^−)$ | 3 | 1 | 0(1.1) | 1$^+$ | $\Sigma_c^0(\frac{1}{2}^−, \frac{3}{2}^−)$ | 6 | 1 | 2(0.2) | 3$^+$ |
| $\Lambda_c^+(3/2^−)$ | 3 | 1 | 0(1.1) | 1$^+$ | $\Sigma_c^2(\frac{3}{2}^−, \frac{5}{2}^−)$ | 6 | 1 | 2(0.2) | 3$^+$ |
| $\Lambda_c^+(1/2^−)$ | 3 | 1 | 1(1,1) | 0$^+$ | $\Sigma_c^0(\frac{1}{2}^−, \frac{3}{2}^−)$ | 6 | 1 | 2(0.2) | 1$^+$ |
| $\Lambda_c^+(3/2^−)$ | 3 | 1 | 1(1,1) | 1$^+$ | $\Sigma_c^2(\frac{3}{2}^−, \frac{5}{2}^−)$ | 6 | 1 | 2(0.2) | 2$^+$ |
| $\Lambda_c^+(5/2^−)$ | 3 | 1 | 1(1,1) | 2$^+$ | $\Sigma_c^0(\frac{1}{2}^−, \frac{3}{2}^−)$ | 6 | 1 | 2(0.2) | 3$^+$ |
| $\Lambda_c^+(3/2^−)$ | 3 | 1 | 2(1,1) | 1$^+$ | $\Sigma_c^2(\frac{3}{2}^−, \frac{5}{2}^−)$ | 6 | 1 | 2(0.2) | 1$^+$ |
| $\Lambda_c^+(5/2^−)$ | 3 | 1 | 2(1,1) | 1$^+$ | $\Sigma_c^0(\frac{1}{2}^−, \frac{3}{2}^−)$ | 6 | 1 | 2(0.2) | 2$^+$ |
| $\Lambda_c^+(3/2^−)$ | 3 | 1 | 2(1,1) | 2$^+$ | $\Sigma_c^2(\frac{3}{2}^−, \frac{5}{2}^−)$ | 6 | 1 | 2(0.2) | 2$^+$ |
| $\Lambda_c^+(5/2^−)$ | 3 | 1 | 2(1,1) | 2$^+$ | $\Sigma_c^0(\frac{1}{2}^−, \frac{3}{2}^−)$ | 6 | 1 | 2(0.2) | 2$^+$ |

TABLE II: The first positive-parity excitations of charmed baryons denoted by $B_{cJ}(J^P)$, $\hat{B}_{cJ}(J^P)$ and $\hat{B}^L_{cJ}(J^P)$. Orbital $L_\rho = L_\lambda = 1$ states with antisymmetric orbital wave functions are denoted by a tilde. States with the symmetric orbital wave functions $L_\rho = 2$ and $L_\lambda = 0$ are denoted by a hat. For convenience, we drop the superscript $L_\ell$ for tilde states in the sextet.
TABLE III: Mass spectra and widths (in units of MeV) of charmed baryons. Experimental values are taken from the Particle Data Group [8]. For the widths of the $\Sigma_c(2455)^{0/++}$ and $\Sigma_c(2520)^{0/++}$ baryons, we have taken into account the recent Belle measurement [11] for average. The width of $\Xi_c(2645)^+$ is taken from [12]. For $\Xi_c(3055)^0$, we quote the preliminary result from Belle [13].

| State   | $J^P$ | $S_\ell$ | $L_\ell$ | $I_F^G$ | Mass      | Width      | Decay modes                        |
|---------|-------|----------|----------|---------|-----------|-----------|------------------------------------|
| $\Lambda_c(2595)^+$ | $\frac{1}{2}^+$ | 0     | 0       | 0+$^+$ | 2286.46 ± 0.14 | weak          |
| $\Sigma_c(2455)^0$ | $\frac{1}{2}^+$ | 1     | 0       | 1+$^+$ | 2453.98 ± 0.16 | 1.94±0.08−0.16 | $\Lambda_c\pi$          |
| $\Sigma_c(2455)^+$ | $\frac{1}{2}^+$ | 1     | 0       | 1+$^+$ | 2452.9 ± 0.4 | 4.6       | $\Lambda_c\pi$          |
| $\Sigma_c(2455)^0$ | $\frac{1}{2}^+$ | 1     | 0       | 1+$^+$ | 2453.74 ± 0.16 | 1.87±0.17−0.17 | $\Lambda_c\pi$          |
| $\Sigma_c(2520)^+$ | $\frac{3}{2}^+$ | 1     | 0       | 1+$^+$ | 2517.9 ± 0.6 | 14.8±0.3−0.4 | $\Lambda_c\pi$          |
| $\Sigma_c(2520)^0$ | $\frac{3}{2}^+$ | 1     | 0       | 1+$^+$ | 2517.5 ± 2.3 | 17        | $\Lambda_c\pi$          |
| $\Xi_c(2645)^+$ | $\frac{1}{2}^+$ | 1     | 0       | 1+$^+$ | 2645.9±0.5−0.6 | 2.6±0.5       | $\Xi_c\pi$          |
| $\Xi_c(2645)^0$ | $\frac{1}{2}^+$ | 1     | 0       | 1+$^+$ | 2645.9 ± 0.9 | 5.5       | $\Xi_c\pi$          |
| $\Xi_c(2790)^+$ | $\frac{3}{2}^+$ | 0     | 1       | 1−     | 2789.9 ± 3.2 | 19        | $\Xi_c^\prime\pi$          |
| $\Xi_c(2790)^0$ | $\frac{1}{2}^+$ | 0     | 1       | 1−     | 2791.8 ± 3.3 | 12        | $\Xi_c^\prime\pi$          |
| $\Xi_c(2815)^+$ | $\frac{3}{2}^+$ | 0     | 1       | 1−     | 2816.6 ± 0.9 | 3.5       | $\Xi_c^\prime\pi, \Xi_c\pi, \Xi_c^\prime\pi$ |
| $\Xi_c(2815)^0$ | $\frac{3}{2}^+$ | 0     | 1       | 1−     | 2819.6 ± 1.2 | 6.5       | $\Xi_c^\prime\pi, \Xi_c\pi, \Xi_c^\prime\pi$ |
| $\Xi_c(2930)^0$ | $\frac{3}{2}^+$ | 0     | 1       | 1−     | 2931 ± 6     | 36±13      | $\Lambda_c\overline{K}$          |
| $\Xi_c(2980)^+$ | $\frac{3}{2}^+$ | 0     | 1       | 1−     | 2971.4 ± 3.3 | 26±7       | $\Sigma_c\overline{K}, \Lambda_c\overline{K}, \Xi_c\pi, \Xi_c\pi$ |
| $\Xi_c(2980)^0$ | $\frac{3}{2}^+$ | 0     | 1       | 1−     | 2968±2.6     | 20±7       | $\Sigma_c\overline{K}, \Lambda_c\overline{K}, \Xi_c\pi, \Xi_c\pi$ |
| $\Xi_c(3055)^+$ | $\frac{3}{2}^+$ | 0     | 1       | 1−     | 3054.2±1.3   | 17±13      | $\Sigma_c\overline{K}, \Lambda_c\overline{K}, \Xi_c\pi, \Xi_c\pi$ |
| $\Xi_c(3055)^0$ | $\frac{3}{2}^+$ | 0     | 1       | 1−     | 3059.7±0.8   | 7.4±3.9    | $\Sigma_c\overline{K}, \Lambda_c\overline{K}, \Xi_c\pi, \Xi_c\pi$ |
| $\Xi_c(3080)^+$ | $\frac{3}{2}^+$ | 0     | 1       | 1−     | 3077.0±0.4   | 5.8±1.0    | $\Sigma_c\overline{K}, \Lambda_c\overline{K}, \Xi_c\pi, \Xi_c\pi$ |
| $\Xi_c(3080)^0$ | $\frac{3}{2}^+$ | 0     | 1       | 1−     | 3079.0±1.4   | 5.6±2.2    | $\Sigma_c\overline{K}, \Lambda_c\overline{K}, \Xi_c\pi, \Xi_c\pi$ |
| $\Xi_c(3123)^+$ | $\frac{3}{2}^+$ | 0     | 1       | 1−     | 3122.9±1.3   | 4.4±3.8    | $\Sigma_c\overline{K}, \Lambda_c\overline{K}, \Xi_c\pi, \Xi_c\pi$ |
| $\Omega_c(2770)^0$ | $\frac{1}{2}^+$ | 1     | 0       | 1+$^+$ | 2695.2±1.7   | weak         |

6
been proposed in the diquark model [18] to be either the first radial \((2S)\) excitation of the \(\Lambda_c\) with \(J^P = \frac{1}{2}^-\) containing the light scalar diquark or the first orbital excitation \((1P)\) of the \(\Sigma_c\) with \(J^P = \frac{3}{2}^-\) containing the light axial vector diquark.

The state \(\Lambda_c(2880)^+\) first observed by CLEO [15] in \(\Lambda_c^+\pi^+\pi^-\) was also seen by BaBar in the \(D^0\bar{p}\) spectrum [19]. Belle has studied the experimental constraint on the \(J^P\) quantum numbers of \(\Lambda_c(2880)^+\) [20] and found that \(J^P = \frac{5}{2}^+\) is favored by the angular analysis of \(\Lambda_c(2880)^+ \rightarrow \Sigma_c^{0,+}\pi^\pm\) together with the ratio of \(\Sigma^c\pi/\Sigma\pi\) measured to be

\[
R \equiv \frac{\Gamma(\Lambda_c(2880) \rightarrow \Sigma_c^+\pi^0)}{\Gamma(\Lambda_c(2880) \rightarrow \Sigma_c^+\pi^-)} = (24.1 \pm 6.4^{+1.1}_{-4.5})\%.
\]

Given the quark model, the candidates for the parity-even spin-\(\frac{5}{2}\) state are \(\Lambda_{c2}(\frac{5}{2}^+), \hat{\Lambda}_{c2}(\frac{5}{2}^+), \hat{\Lambda}_{c2}(\frac{5}{2}^+)\), \(\hat{\Lambda}_{c3}(\frac{5}{2}^+), \hat{\Lambda}_{c3}(\frac{5}{2}^+), \hat{\Lambda}_{c3}(\frac{5}{2}^+)\) (see Table 11). The first four candidates with \(I_\ell = 2\) decay to \(\Sigma_c\pi\) in a \(F\) wave and \(\Sigma_c\pi\) in \(F\) and \(P\) waves. Neglecting the \(P\)-wave contribution for the moment,

\[
\frac{\Gamma(\Lambda_{c2}(5/2^+) \rightarrow [\Sigma_c^\ast\pi]_F)}{\Gamma(\Lambda_{c2}(5/2^+) \rightarrow [\Sigma_c\pi]_F)} = \frac{4}{5} \frac{p_\pi^2(\Lambda_c(2880) \rightarrow \Sigma_c\pi)}{p_\pi^2(\Lambda_c(2880) \rightarrow \Sigma_c\pi)} = \frac{4}{5} \times 0.29 = 0.23,
\]

where the factor of \(4/5\) follows from heavy quark symmetry. At first glance, it appears that this is in good agreement with experiment. However, the \(\Sigma_c^\ast\pi\) channel is available via a \(P\)-wave and is enhanced by a factor of \(1/p_\pi^4\) relative to the \(F\)-wave one. However, heavy quark symmetry cannot be applied to calculate the contribution of the \([\Sigma_c^\ast\pi]_F\) channel to the ratio \(R\) as the reduced matrix elements are different for \(P\)-wave and \(F\)-wave modes. In this case, one has to rely on a phenomenological model to compute the ratio \(R\). As for \(\hat{\Lambda}_{c3}(\frac{5}{2}^+)\), it decays to \(\Sigma_c^\ast\pi, \Sigma_c\pi\), and \(\Lambda_c\pi\) all in \(F\) waves. It turns out that

\[
\frac{\Gamma(\hat{\Lambda}_{c3}(5/2^+) \rightarrow [\Sigma_c^\ast\pi]_F)}{\Gamma(\hat{\Lambda}_{c3}(5/2^+) \rightarrow [\Sigma_c\pi]_F)} = \frac{5}{4} \frac{p_\pi^2(\Lambda_c(2880) \rightarrow \Sigma_c\pi)}{p_\pi^2(\Lambda_c(2880) \rightarrow \Sigma_c\pi)}
\]
Although this deviates from the experimental measurement (2.1) by 1σ, it is a robust prediction. This has motivated us to conjecture that the first positive-parity excited charmed baryon \( \Lambda_c(2880)^+ \) could be an admixture of \( \Lambda_{c2}(\frac{5}{2}^+), \Lambda_{c3}^0(\frac{5}{2}^+) \) and \( \Lambda_{c3}^2(\frac{5}{2}^+) \) [10].

It is worth mentioning that the Peking group [21] has studied the strong decays of charmed baryons based on the so-called \( ^3P_0 \) recombination model. For the \( \Lambda_c(2880) \), Peking group found that (i) the possibility of \( \Lambda_c(2880) \) being a radial excitation is ruled out as its decay into \( D^0p \) is prohibited in the \( ^3P_0 \) model if \( \Lambda_c(2880) \) is a first radial excitation of \( \Lambda_c \), and (ii) the states \( \Lambda_{c2}(\frac{5}{2}^+), \Lambda_{c3}^0(\frac{5}{2}^+) \) and \( \Lambda_{c3}^2(\frac{5}{2}^+) \) are excluded as they do not decay to \( D^0p \) according to the \( ^3P_0 \) model. Moreover, the predicted ratios of \( \Sigma_c^+\pi/\Sigma_c\pi \) are either too large or too small compared to experiment, for example,

\[
\frac{\Gamma(\Lambda_{c2}(5/2^+) \to \Sigma_c^+\pi)}{\Gamma(\Lambda_{c2}(5/2^+) \to \Sigma_c\pi)} = 89, \quad \frac{\Gamma(\Lambda_{c2}(5/2^+) \to \Sigma_c^+\pi)}{\Gamma(\Lambda_{c2}(5/2^+) \to \Sigma_c\pi)} = 0.75.
\]

Both symmetric states \( \Lambda_{c2} \) and \( \Lambda_{c3} \) are thus ruled out. Hence, it appears that \( \Lambda_{c3}^2(\frac{5}{2}^+) \) dictates the inner structure of \( \Lambda_c(2880) \). However, there are several issues with this assignment: (i) the quark model indicates a \( \Lambda_{c2}(\frac{5}{2}^+) \) state around 2910 MeV which is close to the mass of \( \Lambda_c(2880) \), while the mass of \( \Lambda_{c3}^2(\frac{5}{2}^+) \) is even higher [17], (ii) \( \Lambda_{c3}^2(\frac{5}{2}^+) \) can decay to a \( F \)-wave \( \Lambda_c\pi \) and this has not been seen by BaBar and Belle, and (iii) the calculated width 28.8 MeV is too large compared to the measured one 5.8 ± 1.1 MeV. One may argue that the \( ^3P_0 \) model’s prediction can be easily off by a factor of 2 ~ 3 from the experimental measurement due to its inherent uncertainties [21].

It is interesting to notice that, based on the diquark idea, the quantum numbers \( J^P = \frac{5}{2}^+ \) have been correctly predicted in [23] for the \( \Lambda_c(2880) \) before the Belle experiment.

The highest \( \Lambda_c(2940)^+ \) was first discovered by BaBar in the \( D^0p \) decay mode [10] and confirmed by Belle in the decays \( \Sigma_c^0\pi^+, \Sigma_c^{*+}\pi^- \) which subsequently decay into \( \Lambda_c^+\pi^0\pi^- \) [20]. Its spin-parity assignment is quite diversified. For example, it has argued that \( \Lambda_c(2940)^+ \) is the radial excitation of \( \Lambda_c(2959) \) with \( J^P = \frac{1}{2}^- \), but the predicted mass is too large by of order 40 MeV or it could be the first radial excitation of \( \Sigma_c \) (not \( \Lambda_c \!) with \( J^P = 3/2^+ \) [24]. The latter assignment has the advantage that the predicted mass is in better agreement with experiment. Since the mass of \( \Lambda_c(2940)^+ \) is barely below the threshold of \( D^*0p \), this observation has motivated the authors of [25] to suggest an exotic molecular state of \( D^*0 \) and \( p \) with a binding energy of order 6 MeV and \( J^P = \frac{1}{2}^- \) for \( \Lambda_c(2940)^+ \). The quark potential model predicts a \( \frac{5}{2}^- \) \( \Lambda_c \) state at 2900 MeV and a \( \frac{3}{2}^+ \) \( \Lambda_c \) state at 2910 MeV [17]. A similar result of 2906 MeV for \( \frac{3}{2}^+ \) \( \Lambda_c \) is also obtained in the relativistic quark model [26].

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1 It has been argued in [22] that in the chiral quark model \( \Lambda_c(2880) \) favors to be the state \( |\Lambda_c^2S+1L_sJ^P\rangle = |\Lambda_c^2D_{\lambda\lambda}\frac{3}{2}^+\rangle \) with \( L_\rho = 0 \) and \( L_\lambda = 2 \) rather than \( |\Lambda_c^2D_{\lambda\lambda}\frac{5}{2}^+\rangle \) with \( L_\rho = L_\lambda = 1 \) as the latter cannot decay into \( D^0p \). However, this is not our case as \( \Lambda_{c3}^2(\frac{5}{2}^+) \) does decay to \( D^0p \) and can reproduce the measured value of R.
2. $\Sigma_c$ states

The highest isoscalar charmed baryons $\Sigma_c(2800)^{++,+}0$ decaying to $\Lambda_c^+\pi$ were first measured by Belle [27] with widths of order 70 MeV. The possible quark states are $\Sigma_{c0}(1^-), \Sigma_{c1}(5^-, 3^-), \Sigma_{c2}(5^-, 5^-)$. The states $\Sigma_{c1}$ and $\Sigma_{c3}$ are ruled out because their decays to $\Lambda_c^+\pi$ are prohibited in the heavy quark limit. Now the $\Sigma_{c2}(5^-, \frac{3}{2}^-)$ baryon decays principally into the $\Lambda_c\pi$ system in a D-wave, while $\Sigma_{c0}(1^-)$ decays into $\Lambda_c\pi$ in an S-wave. Since HHChPT implies a very broad $\Sigma_{c0}$ with width of order 885 MeV (see Sec.III.B below), this p-wave state is also excluded. Therefore, $\Sigma_c(2800)^{++,+}0$ are likely to be either $\Sigma_{c2}(\frac{3}{2}^-)$ or $\Sigma_{c2}(\frac{5}{2}^-)$ or their mixing. In the quark-diquark model [24], both of them have very close masses compatible with experiment. Given that for light strange baryons, the first orbital excitation of the $\Sigma$ has also the quantum numbers $J^P = 3/2^-$ (see Fig. 2), we will advocate a $\Sigma_{c2}(3/2^-)$ state for $\Sigma_c(2800)$.

3. $\Xi_c$ states

The states $\Xi_c(2790)$ and $\Xi_c(2815)$ form a doublet $\Xi_{c1}(1^-, \frac{3}{2}^-)$. Since the diquark transition $1^- \to 0^+ + \pi$ is prohibited, $\Xi_{c1}(\frac{1}{2}^-, \frac{3}{2}^-)$ cannot decay to $\Xi_c\pi$. The dominant decay mode is $[\Xi_c^0\pi]_S$ for $\Xi_{c1}(\frac{1}{2}^-)$ and $[\Xi_c^1\pi]_S$ for $\Xi_{c1}(\frac{3}{2}^-)$.

Many excited charmed baryon states $\Xi_c(2980), \Xi_c(3055), \Xi_c(3080)$ and $\Xi_c(3123)$ have been seen at B factories [12, 28, 29]. Another state $\Xi_c(2930)^0$ omitted from the PDG summary table has been only seen by BaBar in the $\Lambda_c\pi$ system in a $P$-wave state. According to the Regge phenomenology is very useful for the $J^P$ assignment of charmed baryons [24, 32]. The Regge analysis suggests $J^P = 3/2^+$ for $\Xi_c(3055)$ and $5/2^+$ for $\Xi_c(3080)$ [24]. From Table V below we shall see that $\Xi_c(3080)$ and $\Lambda_c(2880)$ form nicely a $J^P = 5/2^+$ antitriplet.

In the relativistic quark-diquark model [24], $\Xi_c(2980)$ is a sextet $J^P = \frac{1}{2}^+$ state. According to Table I possible candidates are $\Xi_{c1}(1^+, \frac{1}{2}^+), \Xi_{c1}(\frac{1}{2}^+, \frac{3}{2}^+), \Xi_{c0}(1^+)$ and $\Xi_{c1}(\frac{1}{2}^+)$. As pointed out in [30], strong decays of these four states studied in [21] using the $3P_0$ model show that $\Xi_{c1}(1^+)$ does not decay to $\Xi_c\pi$ and $\Lambda_cK$ and has a width of 28 MeV consistent with experiment. Therefore, the favored candidate for $\Xi_c(2980)$ is $\Xi_{c1}(\frac{1}{2}^+)$ which has $J_L = L = 1$.

The possible quark states for $J^P = \frac{5}{2}^+$ $\Xi_c(3080)$ baryon in an antitriplet are $\Xi_{c2}(5^+, \frac{1}{2}^+), \Xi_{c2}(5^+, \frac{3}{2}^+), \Xi_{c3}(5^+, \frac{5}{2}^+)$ and $\Xi_{c3}(5^+, \frac{5}{2}^-)$ (see Table I). Since $\Xi_c(3080)$ is above the $D\Lambda$ threshold, the two-body mode $D\Lambda$ should exist though it has not been searched for in the $D\Lambda$ spectrum. Recall that the neutral $\Xi_c(3055)^0$ was observed recently by Belle in the $D^0\Lambda$ spectrum [13]. According to the $3P_0$ model, the first four states are excluded as they do not decay into $D\Lambda$ [21]. The only possibility left is $\Xi_{c3}(\frac{5}{2}^+)$. This is the analog of $\Lambda_{c3}(\frac{5}{2}^+)$ for $\Lambda_c(2880)$. Nevertheless, the identification of $\Xi_{c3}(\frac{5}{2}^+)$ with $\Xi_c(3080)$ encounters two potential problems: (i) its width is dominated by $\Xi_c\pi$ and $\Lambda_cK$ modes which have not been seen experimentally, and (ii) the predicted width of order 47 MeV [21] is too large compared to the measured one of order 5.7 MeV.
The possible spin-parity quantum numbers for excited charmed baryon resonances are partially summarized in Table IV. Some of the models that have been suggested in the literature are:

| Model          | \( \Lambda_c(2765) \) | \( \Lambda_c(2880) \) | \( \Lambda_c(2940) \) | \( \Sigma_c(2800) \) | \( \Xi_c(2930) \) | \( \Xi_c(2980) \) | \( \Xi_c(3055) \) | \( \Xi_c(3080) \) | \( \Xi_c(3123) \) |
|----------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| Capstick et al. [17] | \( \frac{1}{2}^+ \) | \( \frac{1}{2}^+ (2S) \) | \( \frac{1}{2}^+ (1D) \) | \( \frac{1}{2}^+ (2P) \) | \( \frac{1}{2}^+ (1D) \) | \( \frac{1}{2}^+ (1D) \) | \( \frac{1}{2}^+ (1D) \) | \( \frac{1}{2}^- (2P) \) |
| B. Chen et al. [33]  | \( \frac{1}{2}^- (2S) \) | \( \frac{1}{2}^- (1D) \) | \( \frac{1}{2}^- (2P) \) | \( \frac{1}{2}^- (1P) \) | \( \frac{1}{2}^- (1P) \) | \( \frac{1}{2}^- (1P) \) | \( \frac{1}{2}^- (1P) \) | \( \frac{1}{2}^- (1P) \) |
| H. Chen et al. [34]  | \( \frac{1}{2}^- (2S) \) | \( \frac{1}{2}^- (1D) \) | \( \frac{1}{2}^- (2P) \) | \( \frac{1}{2}^- (1P) \) | \( \frac{1}{2}^- (1P) \) | \( \frac{1}{2}^- (1P) \) | \( \frac{1}{2}^- (1P) \) | \( \frac{1}{2}^- (1P) \) |
| Cheng et al. [10]   | \( \frac{1}{2}^- (2S) \) | \( \frac{1}{2}^- (1D) \) | \( \frac{1}{2}^- (2P) \) | \( \frac{1}{2}^- (1P) \) | \( \frac{1}{2}^- (1P) \) | \( \frac{1}{2}^- (1P) \) | \( \frac{1}{2}^- (1P) \) | \( \frac{1}{2}^- (1P) \) |
| Ebert et al. [24]    | \( \frac{1}{2}^- (2S) \) | \( \frac{1}{2}^- (1D) \) | \( \frac{1}{2}^- (2P) \) | \( \frac{1}{2}^- (1P) \) | \( \frac{1}{2}^- (1P) \) | \( \frac{1}{2}^- (1P) \) | \( \frac{1}{2}^- (1P) \) | \( \frac{1}{2}^- (1P) \) |
| Garcilazo et al. [26] | \( \frac{1}{2}^- (2S) \) | \( \frac{1}{2}^- (1D) \) | \( \frac{1}{2}^- (2P) \) | \( \frac{1}{2}^- (1P) \) | \( \frac{1}{2}^- (1P) \) | \( \frac{1}{2}^- (1P) \) | \( \frac{1}{2}^- (1P) \) | \( \frac{1}{2}^- (1P) \) |
| Gerasyuata et al. [35]| \( \frac{1}{2}^- (2S) \) | \( \frac{1}{2}^- (1D) \) | \( \frac{1}{2}^- (2P) \) | \( \frac{1}{2}^- (1P) \) | \( \frac{1}{2}^- (1P) \) | \( \frac{1}{2}^- (1P) \) | \( \frac{1}{2}^- (1P) \) | \( \frac{1}{2}^- (1P) \) |
| Liu et al. [36]      | \( \frac{1}{2}^- (2S) \) | \( \frac{1}{2}^- (1D) \) | \( \frac{1}{2}^- (2P) \) | \( \frac{1}{2}^- (1P) \) | \( \frac{1}{2}^- (1P) \) | \( \frac{1}{2}^- (1P) \) | \( \frac{1}{2}^- (1P) \) | \( \frac{1}{2}^- (1P) \) |
| Wilczek et al. [23]  | \( \frac{1}{2}^- (2S) \) | \( \frac{1}{2}^- (1D) \) | \( \frac{1}{2}^- (2P) \) | \( \frac{1}{2}^- (1P) \) | \( \frac{1}{2}^- (1P) \) | \( \frac{1}{2}^- (1P) \) | \( \frac{1}{2}^- (1P) \) | \( \frac{1}{2}^- (1P) \) |
| Zhong et al. [22]    | \( \frac{1}{2}^- (2S) \) | \( \frac{1}{2}^- (1D) \) | \( \frac{1}{2}^- (2P) \) | \( \frac{1}{2}^- (1P) \) | \( \frac{1}{2}^- (1P) \) | \( \frac{1}{2}^- (1P) \) | \( \frac{1}{2}^- (1P) \) | \( \frac{1}{2}^- (1P) \) |

4. **\( \Omega_c \) states**

Only two ground states have been observed thus far: \( 1/2^+ \, \Omega_c^0 \) and \( 3/2^+ \, \Omega_c(2770)^0 \). The latter was seen by BaBar in the electromagnetic decay \( \Omega_c(2770) \to \Omega_c \gamma \). [37]

**Molecule picture**

Since \( \Lambda_c(2940)^+ \) and \( \Sigma_c(2800) \) are barely below the \( D^* \) and \( DN \) thresholds, respectively, it is tempting to conjecture an exotic molecular structure of \( D^* \) and \( p \) for the former and a molecule state of \( DN \) for the latter. [25, 38, 42]. Likewise, \( \Xi_c(2980) \) could be a molecule state of \( DA \).

The coupled-channel calculation of the baryon-meson \( ND \) system has been performed to look for the isospin-spin channel which is attractive enough to form a molecule state [40, 43]. It turns out that \( (I)J^P = (0)\frac{1}{2}^+ \) is the most attractive one followed by \( (I)J^P = (1)\frac{3}{2}^- \). This suggests the possibility of \( \Sigma_c(2800) \) being an \( s \)-wave \( DN \) molecule state with \( (I)J^P = (0)\frac{1}{2}^- \) and \( \Lambda_c(2940) \) an \( s \)-wave \( D^*N \) molecular state with \( (I)J^P = (1)\frac{3}{2}^- \) (see Fig. 3 of [43]). Another possibility is a \( DN \) molecular state with \( (I)J^P = (1)\frac{3}{2}^- \) for \( \Sigma_c(2800) \) and a \( D^*N \) one with \( (I)J^P = (0)\frac{1}{2}^- \) for \( \Lambda_c(2940) \). Since \( \Sigma_c(2800) \) has isospin 1 and moreover we have noted in passing that \( \Sigma_c(2800) \) will be too broad if it is assigned to \( J^P = 1/2^- \), we conclude that the second possibility is more preferable (see also [42]).

The possible spin-parity quantum numbers of the higher excited charmed baryon resonances that have been suggested in the literature are partially summarized in Table IV. Some of the predictions are already ruled out by experiment. For example, \( \Lambda_c(2880) \) has \( J^P = \frac{5}{2}^+ \) as seen by Belle. Certainly, more experimental studies are needed in order to pin down the quantum numbers.

Charmed baryon spectroscopy has been studied extensively in various models. It appears that the spectroscopy is well described by the heavy quark-light diquark picture elaborated by Ebert, Faustov and Galkin (EFG) [24] (see also [33]). As noted in passing, the quantum numbers \( J^P = \frac{5}{2}^+ \) of \( \Lambda_c(2880) \) have been correctly predicted in the model based on the diquark idea before the Belle experiment [23]. Moreover, EFG have shown that all available experimental data on heavy baryons...
Charmed baryon spectra

\begin{align*}
\Lambda_c(2940)^{7^+} & \quad \Xi_c(3123)^{7/2^+} (1D) \\
\Lambda_c(2880)^{5/2^+} (1D) & \quad \Xi_c(3080)^{5/2^+} (1D) \\
\Lambda_c(3055)^{3/2^+} (1D) & \quad \Omega_c(3055)^{3/2^+} (1D) \\
\Xi_c(2980)^{1/2^+} (2S) & \quad \Xi_c(2930)^{3/2} (1P) \\
\end{align*}

FIG. 3: Singly charmed baryon states where the spin-parity quantum numbers in red are taken from [24].

fit nicely to the linear Regge trajectories, namely, the trajectories in the \((J, M^2)\) and \((n_r, M^2)\) planes for orbitally and radially excited heavy baryons, respectively:

\[ J = \alpha M^2 + \alpha_0, \quad n_r = \beta M^2 + \beta_0, \]

where \(n_r\) is the radial excitation quantum number, \(\alpha, \beta\) are the slopes and \(\alpha_0, \beta_0\) are intercepts. The linearity, parallelism and equidistance of the Regge trajectories were verified. The predictions of the spin-parity quantum numbers of charmed baryons and their masses in [24] can be regarded as a theoretical benchmark (see Fig. 3).

Antitriplet and sextet states

The antitriplet and sextet states of charmed baryons are listed in Table V. By now, the \(J^P = 1/2^+, 1/2^-\) and \(3^-\) 3 states: \((\Lambda_c^+, \Xi_c^+, \Xi_c^0)\), \((\Lambda_c(2595)^+, \Xi_c(2790)^+, \Xi_c(2790)^0)\), \((\Lambda_c(2625)^+, \Xi_c(2815)^+, \Xi_c(2815)^0)\) respectively and \(J^P = 1/2^+\) and \(3^+\) 6 states: \((\Omega_c, \Sigma_c, \Xi_c^0)\), \((\Omega_c^*, \Sigma_c^*, \Xi_c^*)\) respectively are established. It is clear that the mass difference \(m_{\Xi_c} - m_{\Lambda_c}\) in the antitriplet states lies in the range of 180-200 MeV. We note in passing that \(\Xi_c(3080)\) should carry the quantum numbers \(J^P = 5/2^+\). From Table V we see that \(\Xi_c(3080)\) and \(\Lambda_c(2880)\) form nicely a \(J^P = 5/2^+\) antitriplet as the mass difference between \(\Xi_c(3080)\) and \(\Lambda_c(2880)\) is consistent with that observed in other antitriplets. Likewise, the mass differences in \(J^P = 3/2^-\) sextet \((\Omega_c(3050), \Xi_c(2930), \Sigma_c(2800))\) predicted by the quark-diquark model are consistent with that measured in \(J^P = 1/2^+\) and \(3/2^+\) sextets. Note that there is no \(J^P = 1/2^-\) sextet as the \(\Sigma_c(2800)\) with these spin-parity quantum numbers will be too broad to be observed.
TABLE V: Antitriplet and sextet states of charmed baryons. The $J^P$ quantum numbers of $\Xi_c(3080), \Xi'_c(2930), \Sigma_c(2800)$ are not yet established and the $\Omega_c(3/2^-)$ state has not been observed. Mass differences $\Delta m_{\Xi_c, \Lambda_c} = m_{\Xi_c} - m_{\Lambda_c}$, $\Delta m_{\Xi_c, \Sigma_c} = m_{\Xi_c} - m_{\Sigma_c}$, $\Delta m_{\Omega_c, \Xi_c} = m_{\Omega_c} - m_{\Xi_c}$ are in units of MeV.

| $J^P$ | States | Mass difference | status |
|-------|--------|-----------------|--------|
| 3     | $\Lambda_c(2287)^+, \Xi_c(2470)^+, \Xi_c(2470)^0$ | $\Delta m_{\Xi_c, \Lambda_c} = 183$ | estab  |
|       | $\Lambda_c(2595)^+, \Xi_c(2790)^+, \Xi_c(2790)^0$ | $\Delta m_{\Xi_c, \Lambda_c} = 198$ | estab  |
|       | $\Lambda_c(2625)^+, \Xi_c(2815)^+, \Xi_c(2815)^0$ | $\Delta m_{\Xi_c, \Lambda_c} = 190$ | estab  |
|       | $\Lambda_c(2880)^+, \Xi_c(3080)^+, \Xi_c(3080)^0$ | $\Delta m_{\Xi_c, \Lambda_c} = 196$ |        |
| 6     | $\Omega_c(2695)^0, \Xi'_c(2575)^+, \Xi_c(2455)^{++,+0}$ | $\Delta m_{\Xi', \Sigma_c} = 124$, $\Delta m_{\Omega_c, \Xi_c} = 119$ | estab  |
|       | $\Omega_c(2770)^0, \Xi'_c(2645)^++, \Xi_c(2520)^{++,+0}$ | $\Delta m_{\Xi', \Sigma_c} = 128$, $\Delta m_{\Omega_c, \Xi_c} = 120$ | estab  |
|       | $\Omega_c(3050)^0, \Xi'_c(2930)^++, \Sigma_c(2800)^{++,+0}$ | $\Delta m_{\Xi', \Sigma_c} = 131$, $\Delta m_{\Omega_c, \Xi_c} = 119$ |        |

On the basis of QCD sum rules, many charmed baryon multiplets classified according to $[6_F, \rho \rho \ell, S_{\ell}, \rho / \ell]$ were recently studied in [34]. Three sextets were proposed in this work: $(\Omega_c(3250), \Xi'_c(2980), \Sigma_c(2800))$ for $J^P = 1/2^-, 3/2^-$ and $(\Omega_c(3320), \Xi'_c(3080), \Sigma_c(2890))$ for $J^P = 5/2^-$. Notice that $\Xi'_c(2980)$ and $\Xi'_c(3080)$ were treated as $p$-wave baryons rather than the first positive-parity excitations as we have discussed before. The results on the multiplet $[6_F, 1, 0, \rho]$ led the authors of [34] to suggest that there are two $\Sigma_c(2800), \Xi'_c(2980)$ and $\Omega_c(3250)$ states with $J^P = 1/2^-$ and $J^P = 3/2^-$. The mass splittings are $14 \pm 7, 12 \pm 7$ and $10 \pm 6$ MeV, respectively. The predicted mass of $\Omega_c(1/2^-, 3/2^-)$ is around $3250 \pm 200$ MeV. Using the central value of the predicted masses to label the states in the multiplet $[6_F, 1, 0, \rho]$ (see Table I of [34]), one will have $(\Omega_c(3250), \Xi'_c(2980), \Sigma_c(2730))$ for $J^P = 1/2^-$ and $(\Omega_c(3260), \Xi'_c(2980), \Sigma_c(2750))$ for $J^P = 3/2^-$. One can check that $\Delta m_{\Xi', \Sigma_c} = 230 \pm 234$ MeV, and $\Delta m_{\Omega_c, \Xi_c}$ is of order $285 \pm 250$ MeV. Due to the large theoretical uncertainties in masses, it is not clear if the QCD sum-rule calculations are compatible with the mass differences measured in $J^P = 1/2^+$ and $3/2^+$ sextets. At any rate, it will be interesting to test these two different model predictions for $J^P = 3/2^-$ and $1/2^-$ sextets in the future.

B. Doubly charmed baryons

Evidence of doubly charmed baryon states has been reported by SELEX in $\Xi_{cc}(3520)^+ \to \Lambda_c^+ K^- \pi^+$ [44]. Further observation of $\Xi_{cc} \to pD^+ K^-$ was also announced by SELEX [45]. However, none of the doubly charmed states discovered by SELEX has been confirmed by FOCUS [46], BaBar [47], Belle [12] and LHCb [48] in spite of the $10^6 \Lambda_c$ events produced in $B$ factories, for example, versus 1630 $\Lambda_c$ events observed at SELEX.

The doubly charmed baryons $\Xi_{cc}^{(++)}, \Xi_{cc}^{(+0)}, \Omega_{cc}^{(++)}$ with the quark contents $ccu, ccd, ccs$ form an SU(3) triplet. They have been studied extensively in many different approaches: quark model, light quark-heavy diquark model, QCD sum rules and lattice simulation. Tabulation of the predicted doubly charmed baryon masses calculated in various models can be found in [49, 50]. For recent QCD sum rule calculations, see e.g. [51]. Chiral corrections to the masses of doubly heavy baryons
FIG. 4: Doubly charmed low-lying baryon spectra taken from [53].

up to N^3LO were presented in [52].

There have been a number of recent lattice studies of doubly and triply charmed baryon spectra displayed in Fig. 4 by different groups: RQCD [53], HSC [54], Brown et al. [55], ETMC [56], ILGTI [57], PACS-CS [58], Durr et al. [59], Briceno et al. [60], Liu et al. [61] and Na et al. [62]. A new lattice calculation of Ω^{(*)}_{cc} and Ω^{(*)}_{ccc} was available in [63]. Various lattice results are consistent with each other and they fall into the ranges

\[ M(\Xi_{cc}) = 3.54 \sim 3.68 \text{ GeV}, \quad M(\Xi^{*}_{cc}) = 3.61 \sim 3.72 \text{ GeV}, \]
\[ M(\Omega_{cc}) = 3.57 \sim 3.76 \text{ GeV}, \quad M(\Omega^{*}_{cc}) = 3.68 \sim 3.85 \text{ GeV}, \]

and

\[ M(\Omega^{(*)}_{ccc}) = 4.70 \sim 4.84 \text{ MeV}. \] (2.7)

Although lattice study suggests that the mass of the low-lying Ξ_{cc} is larger than 3519 MeV, it is interesting to notice that the authors of [49] have calculated the masses of doubly and triply charmed baryons based on the Regge phenomenology and found \( M(\Xi^{(*)}_{cc}) = 3520.2^{+40.6}_{-39.8} \text{ MeV}, \) in good agreement with SELEX.

III. STRONG DECAYS

Due to the rich mass spectrum and the relatively narrow widths of the excited states, the charmed baryon system offers an excellent ground for testing the ideas and predictions of heavy quark symmetry and light flavor SU(3) symmetry. The pseudoscalar mesons involved in the strong decays of charmed baryons such as \( \Sigma_c \to \Lambda_c \pi \) are soft. Therefore, heavy quark symmetry of the heavy quark and chiral symmetry of the light quarks will have interesting implications for the low-energy dynamics of heavy baryons interacting with the Goldstone bosons.
The strong decays of charmed baryons are most conveniently described by the heavy hadron
chiral perturbation theory (HHChPT) in which heavy quark symmetry and chiral symmetry are
incorporated\textsuperscript{[64, 65]. Heavy baryon chiral Lagrangians were first constructed in \textsuperscript{[64]}
for strong decays of s-wave charmed baryons and in \textsuperscript{[9, 14]} for p-wave ones. Previous phenomenological studies
of the strong decays of p-wave charmed baryons based on HHChPT can be found in \textsuperscript{[9, 10, 14, 66, 67].}
The chiral Lagrangian involves two coupling constants $g_1$ and $g_2$ for $P$-wave transitions between
s-wave and s-wave baryons \textsuperscript{[64]}, six couplings $h_2 - h_7$ for the S-wave transitions between s-wave
and p-wave baryons, and eight couplings $h_8 - h_{15}$ for the D-wave transitions between s-wave and
p-wave baryons \textsuperscript{[64].} The general chiral Lagrangian for heavy baryons coupling to the pseudoscalar
mesons can be expressed compactly in terms of superfields. We will not write down the relevant
Lagrangians here; instead the reader is referred to Eqs. (3.1) and (3.3) of \textsuperscript{[9]. The partial widths
relevant for our purposes are \textsuperscript{[9]}:

\[
\begin{align*}
\Gamma(\Sigma_c^* \rightarrow \Sigma_c \pi) &= \frac{g_1^2}{2\pi f_\pi^2} \frac{m_{\Sigma_c}}{m_{\Sigma_c^*}} p_\pi^3, \quad \Gamma(\Sigma_c \rightarrow \Lambda_c \pi) = \frac{g_2^2}{2\pi f_\pi^2} \frac{m_{\Lambda_c}}{m_{\Sigma_c}} p_\pi^3, \\
\Gamma(\Lambda_c(1/2^-) \rightarrow \Sigma_c \pi) &= \frac{h_2^2}{2\pi f_\pi^2} \frac{m_{\Sigma_c}}{m_{\Lambda_c}} E_\pi^2 p_\pi, \quad \Gamma(\Sigma_c(0/1^-) \rightarrow \Lambda_c \pi) = \frac{h_3^2}{2\pi f_\pi^2} \frac{m_{\Lambda_c}}{m_{\Sigma_c}} E_\pi^2 p_\pi, \\
\Gamma(\Lambda_c(3/2^-) \rightarrow \Sigma_c \pi) &= \frac{2h_8^2}{9\pi f_\pi^2} \frac{m_{\Sigma_c}}{m_{\Lambda_c}} p_\pi^5, \quad \Gamma\left(\Sigma_c(3/2^-) \rightarrow \Sigma_c^{(*)} \pi\right) = \frac{h_9^2}{2\pi f_\pi^2} \frac{m_{\Sigma_c^{(*)}}}{m_{\Sigma_c}} p_\pi^5, \\
\Gamma\left(\Sigma_c(3/2^-) \rightarrow \Lambda_c \pi\right) &= \frac{4h_9^2}{15\pi f_\pi^2} \frac{m_{\Lambda_c}}{m_{\Sigma_c}} p_\pi^5, \quad \Gamma\left(\Sigma_c(3/2^-) \rightarrow \Sigma_c^{(*)} \pi\right) = \frac{h_9^2}{10\pi f_\pi^2} \frac{m_{\Sigma_c^{(*)}}}{m_{\Sigma_c}} p_\pi^5, \\
\Gamma\left(\Sigma_c(5/2^-) \rightarrow \Sigma_c \pi\right) &= \frac{2h_{12}^2}{45\pi f_\pi^2} \frac{m_{\Sigma_c}}{m_{\Sigma_c}} p_\pi^5, \quad \Gamma\left(\Sigma_c(5/2^-) \rightarrow \Sigma_c^{(*)} \pi\right) = \frac{7h_{12}^2}{45\pi f_\pi^2} \frac{m_{\Sigma_c^{(*)}}}{m_{\Sigma_c}} p_\pi^5,
\end{align*}
\]

where $f_\pi = 132$ MeV. The dependence on the pion momentum is proportional to $p_\pi^3$, $p_\pi^5$ and $p_\pi^5$ for S-wave,
$P$-wave and $D$-wave transitions, respectively. It is obvious that the couplings $g_1, g_2, h_2, \ldots, h_7$
are dimensionless, while $h_8, \ldots, h_{15}$ have canonical dimension $E^{-1}$.

A. Strong decays of s-wave charmed baryons

Since the strong decay $\Sigma_c^* \rightarrow \Sigma_c \pi$ is kinematically prohibited, the coupling $g_1$ cannot be extracted
directly from the strong decays of heavy baryons. In the framework of HHChPT, one can use some
measurements as input to fix the coupling $g_2$ which, in turn, can be used to predict the rates
of other strong decays. Among the strong decays $\Sigma_c^{(*)} \rightarrow \Lambda_c \pi, \Sigma_c^{+} \rightarrow \Lambda_c^+ \pi^+$ is the most well
measured. Hence, we shall use this mode to extract the coupling $g_2$. Based on the 2006 data \textsuperscript{[68]}
of $\Gamma(\Sigma_c^{+}) = \Gamma(\Sigma_c^{+} \rightarrow \Lambda_c^{+} \pi^+) = 2.23 \pm 0.30$ MeV, the coupling $g_2$ is extracted to be

\[
|g_2|_{2006} = 0.605^{+0.039}_{-0.043}.
\]

The predicted rates of other modes are shown in Table\textsuperscript{[VI]. It is clear that the agreement between
theory and experiment is excellent except the predicted width for $\Sigma_c^{+} \rightarrow \Lambda_c^{+} \pi^+$ is a bit too large.

Using the new data from 2014 Particle Data Group \textsuperscript{[8] in conjunction with the new measurements
of $\Sigma_c$ and $\Sigma_c^*$ widths by Belle \textsuperscript{[11]}, we obtain the new average $\Gamma(\Sigma_c^{+} \rightarrow \Lambda_c^{+} \pi^+) = 1.94^{+0.08}_{-0.16}$ MeV
(see Table\textsuperscript{[III]. Therefore, the coupling $g_2$ is reduced to

\[
|g_2|_{2015} = 0.565^{+0.011}_{-0.024}.
\]
TABLE VI: Decay widths (in units of MeV) of s-wave charmed baryons where the measured rates are taken from 2006 PDG [68].

| Decay               | Expt. | HHChPT       |
|---------------------|-------|---------------|
| $\Sigma_c^{++} \rightarrow \Lambda_c^+ \pi^+$ | 1.94 ± 0.2 | 2.23 ± 0.30   |
| $\Sigma_c^+ \rightarrow \Lambda_c^0 \pi^0$   | < 4.6  | 2.6 ± 0.4     |
| $\Sigma_c^0 \rightarrow \Lambda_c^+ \pi^-$ | 2.2 ± 0.4  | 2.2 ± 0.3     |
| $\Sigma_c(2520)^{++} \rightarrow \Lambda_c^+ \pi^+$ | 14.9 ± 1.9  | 16.7 ± 2.3    |
| $\Sigma_c(2520)^+ \rightarrow \Lambda_c^0 \pi^0$ | < 17     | 17.4 ± 2.3    |
| $\Sigma_c(2520)^0 \rightarrow \Lambda_c^+ \pi^-$ | 16.1 ± 2.1  | 16.6 ± 2.2    |
| $\Xi_c(2645)^+ \rightarrow \Xi_c^{0+,+0}$     | < 3.1    | 2.8 ± 0.4     |
| $\Xi_c(2645)^0 \rightarrow \Xi_c^{+,0-,0}$    | < 5.5    | 2.9 ± 0.4     |

TABLE VII: Decay widths (in units of MeV) of s-wave charmed baryons. Data are taken from 2014 PDG [8] together with the new measurements of $\Sigma_c$, $\Sigma_c^*$ [11] and $\Xi_c(2645)^+$ widths [12]. Theoretical predictions of $\Sigma_c$ are taken from Table IV of [70].

| Decay               | Expt. | HHChPT       | Tawfiq et al. [69] | Ivanov et al. [70] | Huang et al. [71] | Albertus et al. [72] |
|---------------------|-------|---------------|--------------------|--------------------|--------------------|----------------------|
| $\Sigma_c^{++} \rightarrow \Lambda_c^+ \pi^+$ | 1.94±0.2 | 1.51±0.17     | 2.85±0.19          | 2.5                | 2.41±0.07           |
| $\Sigma_c^+ \rightarrow \Lambda_c^0 \pi^0$   | < 4.6  | 2.3±0.1       | 3.63±0.27          | 3.2                | 2.79±0.08           |
| $\Sigma_c^0 \rightarrow \Lambda_c^+ \pi^-$ | 1.9±0.1  | 1.9±0.1       | 2.65±0.19          | 2.4                | 2.37±0.07           |
| $\Sigma_c(2520)^{++} \rightarrow \Lambda_c^+ \pi^+$ | 14.8±0.3  | 14.5±0.5      | 21.99±0.87         | 8.2                | 17.52±0.75          |
| $\Sigma_c(2520)^+ \rightarrow \Lambda_c^0 \pi^0$ | < 17     | 15.2±0.6      | 16.7±0.74          | 8.6                | 16.90±0.72          |
| $\Sigma_c(2520)^0 \rightarrow \Lambda_c^+ \pi^-$ | 15.3±0.1  | 14.7±0.6      | 21.2±0.81          | 8.2                | 16.90±0.72          |
| $\Xi_c(2645)^+ \rightarrow \Xi_c^{0+,+0}$     | 2.6±0.5  | 2.4±0.1       | 3.04±0.37          | 3.18±0.10          |
| $\Xi_c(2645)^0 \rightarrow \Xi_c^{+,0-,0}$    | < 5.5    | 2.5±0.1       | 3.12±0.33          | 3.03±0.10          |

From Table VII we see that the agreement between theory and experiment is further improved: The predicted $\Xi_c(2645)^+$ width is consistent with the first new measurement by Belle [12] and the new calculated width for $\Sigma_c^{++} \rightarrow \Lambda_c^+ \pi^+$ is now in agreement with experiment. It is also clear that the $\Sigma_c$ width is smaller than that of $\Sigma_c^*$ by a factor of $\sim 7$, although they will become the same in the limit of heavy quark symmetry.

### B. Strong decays of p-wave charmed baryons

Since $\Lambda_c(2595)^+$ and $\Lambda_c(2625)^+$ form a doublet $\Lambda_c((\frac{2}{3}^-, \frac{3}{2}^-)$, it appears from Eq. (3.1) that the couplings $h_2$ and $h_3$ in principle can be extracted from $\Lambda_c(2595) \rightarrow \Sigma_c \pi$ and from $\Lambda_c(2625) \rightarrow \Sigma_c \pi$, respectively. Likewise, the information on the couplings $h_{10}$ and $h_{11}$ can be inferred from the strong...
TABLE VIII: Decay widths (in units of MeV) of p-wave charmed baryons where the measured rates are taken from 2006 PDG [68].

| Decay                          | Expt. [68] | HHChPT [10] |
|-------------------------------|------------|-------------|
| \(\Lambda_c(2595)^+ \to (\Lambda_c^+ \pi^0)_R\) | 2.63^{+1.56}_{-1.09} | input       |
| \(\Lambda_c(2595)^+ \to \Sigma_c^{++}\pi^-\) | 0.65^{+0.41}_{-0.31} | 0.75^{+0.43}_{-0.30} |
| \(\Lambda_c(2593)^+ \to \Sigma_c^0\pi^+\) | 0.67^{+0.41}_{-0.31} | 0.77^{+0.46}_{-0.32} |
| \(\Lambda_c(2593)^+ \to \Sigma_c^0\pi^0\) | 1.57^{+0.93}_{-0.65} |              |
| \(\Lambda_c(2595)^+ \to \Sigma_c^{++}\pi^-\) | < 0.10 | 0.029       |
| \(\Lambda_c(2625)^+ \to \Sigma_c^0\pi^+\) | < 0.09 | 0.029       |
| \(\Lambda_c(2625)^+ \to \Sigma_c^0\pi^0\) | 0.041 |              |
| \(\Lambda_c(2625)^+ \to \Lambda_c^+\pi\pi\) | < 1.9 | 0.21        |
| \(\Sigma_c(2800)^{++} \to \Lambda_c\pi, \Sigma_c^{(*)}\pi\) | 75^{+22}_{-17} | input       |
| \(\Sigma_c(2800)^0 \to \Lambda_c\pi, \Sigma_c^{(*)}\pi\) | 62^{+60}_{-40} | input       |
| \(\Xi_c(2790)^+ \to \Xi_c^{0,+}\pi^+,0\) | < 15 | 8.0^{+4.7}_{-3.3} |
| \(\Xi_c(2790)^0 \to \Xi_c^{0,+}\pi^-,0\) | < 12 | 8.5^{+5.7}_{-3.5} |
| \(\Xi_c(2815)^+ \to \Xi_c^{0,+}\pi^+,0\) | < 3.5 | 3.4^{+2.0}_{-1.4} |
| \(\Xi_c(2815)^0 \to \Xi_c^{0,+}\pi^-,0\) | < 6.5 | 3.6^{+2.1}_{-1.5} |

decays of \(\Sigma_c(2800)\) identified with \(\Sigma_c(3/2^-)\). Couplings other than \(h_2\), \(h_8\) and \(h_{10}\) can be related to each other via the quark model [9].

Although the coupling \(h_2\) can be inferred from the two-body decay \(\Lambda_c(2595) \to \Sigma_c\pi\), this method is less accurate because this decay is kinematically barely allowed or even prohibited depending on the mass of \(\Lambda_c(2595)^+\). For the old mass measurement \(m(\Lambda_c(2595)) = 2595.4 \pm 0.6\) MeV [68], \(\Lambda_c(2595)^+ \to \Sigma_c^{++}\pi^-\), \(\Sigma_c^0\pi^+\) and \(\Lambda_c(2595)^+ \to \Sigma_c^0\pi^0\) are kinematically barely allowed. But for the new measurement \(m(\Lambda_c(2595)) = 2592.25 \pm 0.28\) MeV by CDF [73], only the last mode is allowed. Moreover, the finite width effect of the intermediate resonant states could become important [69].

We next turn to the three-body decays \(\Lambda_c^+\pi\pi\) of \(\Lambda_c(2595)^+\) and \(\Lambda_c(2625)^+\) to extract \(h_2\) and \(h_8\). As shown in [10], the 2006 data for \(\Gamma(\Lambda_c(2595)) = 3.6^{+2.0}_{-1.4}\) MeV [68] and for the \(\Lambda_c(2595)\) mass lead to the resonant rate [10]

\[
\Gamma(\Lambda_c(2593)^+ \to \Lambda_c^+\pi\pi)_R = (2.63^{+1.56}_{-1.09})\text{ MeV},
\]

as shown in Table IX. Assuming the pole contributions to \(\Lambda_c(2595)^+ \to \Lambda_c^+\pi\pi\) due to the intermediate states \(\Sigma_c\) and \(\Sigma_c^*\), the resonant rate for the process \(\Lambda_c^+(2595) \to \Lambda_c^+\pi^+\pi^-\) can be calculated in the framework of heavy hadron chiral perturbation theory [9]. Numerically, we found

\[
\Gamma(\Lambda_c(2595)^+ \to \Lambda_c^+\pi\pi)_R = 13.82h_2^2 + 26.28h_8^2 - 2.97h_2h_8,
\]

\[
\Gamma(\Lambda_c(2625)^+ \to \Lambda_c^+\pi\pi)_R = 0.617h_2^2 + 0.136 \times 10^6h_8^2 - 27h_2h_8,
\]

where \(\Lambda_c^+\pi\pi = \Lambda_c^+\pi^+\pi^- + \Lambda_c^+\pi^0\pi^0\). It is clear that the limit on \(\Gamma(\Lambda_c(2625))\) gives an upper bound on \(h_8\) of order \(10^{-3}\) (in units of MeV\(^{-1}\)), whereas the decay width of \(\Lambda_c(2595)\) is entirely governed
FIG. 5: Calculated dependence of $\Gamma(\Lambda_c^+\pi^0\pi^0)/h_2^2$ (full curve) and $\Gamma(\Lambda_c^+\pi^+\pi^-)/h_2^2$ (dashed curve) on $m(\Lambda_c(2595)^+) - m(\Lambda_c^+)$, where we have used the parameters $g_2 = 0.565$, $h_2 = 0.63$ and $h_8 = 0.85 \times 10^{-3}$ MeV$^{-1}$.

by the coupling $h_2$. Specifically, we have

$$\vert h_2 \vert_{2006} = 0.437^{+0.114}_{-0.102}, \quad \vert h_8 \vert_{2006} < 3.65 \times 10^{-3} \text{ MeV}^{-1}. \quad (3.6)$$

It was pointed out in [67] that the proximity of the $\Lambda_c(2595)^+$ mass to the sum of the masses of its decay products will lead to an important threshold effect which will lower the $\Lambda_c(2595)^+$ mass by 2–3 MeV than the one observed. A more sophisticated treatment of the mass lineshape of $\Lambda_c(2595)^+ \rightarrow \Lambda^+_c\pi^+\pi^-$ by CDF yields $m(\Lambda_c(2595)) = 2592.25 \pm 0.28$ MeV [73], which is 3.1 MeV smaller than the 2006 world average. Therefore, the strong decay $\Lambda_c(2595) \rightarrow \Lambda_c\pi\pi$ is very close to the threshold. With the new measurement of $m(\Lambda_c(2595))$, we have (in units of MeV) [30]

$$\Gamma(\Lambda_c(2595)^+ \rightarrow \Lambda^+_c\pi\pi)_R = g_2^2(20.45h_2^2 + 43.92h_8^2 - 8.95h_2h_8),$$

$$\Gamma(\Lambda_c(2625)^+ \rightarrow \Lambda^+_c\pi\pi)_R = g_2^2(1.78h_2^2 + 4.557 \times 10^6h_8^2 - 79.75h_2h_8). \quad (3.7)$$

By performing a fit to the measured $M(pK^-\pi^+\pi^+) - M(pK^-\pi^+)$ mass difference distributions and using $g_2^2 = 0.365$, CDF obtained $h_2^2 = 0.36 \pm 0.08$ or $\vert h_2 \vert = 0.60 \pm 0.07$ [73]. This corresponds to a decay width $\Gamma(\Lambda_c(2595)^+) = 2.59 \pm 0.30 \pm 0.47$ MeV [73]. For the width of $\Lambda_c(2625)^+$, CDF observed a value consistent with zero and therefore calculated an upper limit 0.97 MeV using a Bayesian approach. From the CDF measurements $\Gamma(\Lambda_c(2595)^+) = 2.59\pm 0.56$ MeV and $\Gamma(\Lambda_c(2625)^+) < 0.97$ MeV, we obtain

$$\vert h_2 \vert_{2015} = 0.63 \pm 0.07, \quad \vert h_8 \vert_{2015} < 2.32 \times 10^{-3} \text{ MeV}^{-1}. \quad (3.8)$$

Hence, the magnitude of the coupling $h_2$ is greatly enhanced from 0.437 to 0.63. Our $h_2$ is slightly different from the value of 0.60 obtained by CDF. This is because CDF used $\vert g_2 \vert = 0.604$ to calculate the mass dependence of $\Gamma(\Lambda^+_c\pi\pi)$, while we used $\vert g_2 \vert = 0.565$.

The fact that the coupling $h_2$ obtained in 2006 and 2015 is so different is ascribed to the fact that the mass of $\Lambda_c(2595)^+$ is 3.1 MeV lower than the previous world average due to the threshold effect. To illustrate this, we consider the dependence of $\Gamma(\Lambda^+_c\pi^+\pi^-)/h_2^2$ and $\Gamma(\Lambda^+_c\pi^0\pi^0)/h_2^2$ on $\Lambda M(\Lambda_c(2595)) \equiv M(\Lambda_c(2595)^+) - M(\Lambda_c^+)$ as depicted in Fig. [5]. It is evident that $\Gamma(\Lambda^+_c\pi\pi)/h_2^2$
at $\Delta M(\Lambda_c(2595)) = 305.79$ MeV is smaller than that at 308.9 MeV. This explains why $h_2$ should become larger when $\Delta M(\Lambda_c(2595))$ becomes smaller.

The $\Xi_c(2790)$ and $\Xi_c(2815)$ baryons form a doublet $\Xi_c(1^-|1\frac{3}{2}^-)$. Using the coupling $h_2$ obtained from (3.8) and assuming SU(3) flavor symmetry, the predicted $\Xi_c(2790)$ and $\Xi_c(2815)$ widths are shown in Table IX. It is evident that the predicted two-body decay rates of $\Xi_c(2790)^0$ and $\Xi_c(2815)^+$ exceed the current experimental limits because of the enhancement of $h_2$ (see Table IX). Hence, there is a tension for the coupling $h_2$ as its value extracted from from $\Lambda_c(2595)^+ \to \Lambda_c^+\pi\pi$ will imply $\Xi_c(2790)^0 \to \Xi_c^0\pi$ and $\Xi_c(2815)^+ \to \Xi_c^0\pi$ rates slightly above current limits. It is conceivable that SU(3) flavor symmetry breaking could help account for the discrepancy.

Some information on the coupling $h_{10}$ can be inferred from the strong decays of $\Sigma_c(2800)$. From Eq. (3.11) and the quark model relation $|h_3| = \sqrt{3}|h_2|$ from (9), we obtain, for example, $\Gamma(\Sigma_{c0}^+ \to \Lambda_c^+\pi^+) \approx 885$ MeV. Hence, $\Sigma_c(2800)$ cannot be identified with $\Sigma_{c0}(1/2^-)$. Using the quark model relation $h_{10}^2 = 2h_{10}^2$ and the measured widths of $\Sigma_c(2800)^{++,+0}$ (Table III), we obtain

$$|h_{10}| = (0.85^{+0.11}_{-0.08}) \times 10^{-3} \text{ MeV}^{-1}. \quad (3.9)$$

The quark model relation $|h_8| = |h_{10}|$ then leads to

$$|h_8| \approx (0.85^{+0.11}_{-0.08}) \times 10^{-3} \text{ MeV}^{-1}. \quad (3.10)$$

| Decay | Expt. | HHChPT | Tawfiq et al. | Ivanov et al. | Huang et al. | Zhu et al. |
|-------|-------|---------|---------------|---------------|--------------|-----------|
| $\Lambda_c(2595)^+ \to (\Lambda_c^+\pi\pi)_R$ | $2.59 \pm 0.56$ | input | 2.5 | | | |
| $\Lambda_c(2595)^+ \to \Sigma_c^+\pi^-$ | $1.47 \pm 0.57$ | | $0.79 \pm 0.09$ | $0.55^{+1.3}_{-0.55}$ | $0.64$ | |
| $\Lambda_c(2595)^+ \to \Sigma_c^0\pi^+$ | | | $1.78 \pm 0.70$ | $0.83 \pm 0.09$ | $0.89 \pm 0.86$ | $0.86$ |
| $\Lambda_c(2595)^+ \to \Sigma_c^0\pi^0$ | $2.74^{+0.67}_{-0.60}$ | | $1.18 \pm 0.46$ | $0.98 \pm 0.12$ | $1.7 \pm 0.49$ | $1.2$ |
| $\Lambda_c(2625)^+ \to \Sigma_c^+\pi^-$ | $< 0.10$ | $\leq 0.028$ | $0.44 \pm 0.23$ | $0.076 \pm 0.009$ | $0.013$ | $0.011$ |
| $\Lambda_c(2625)^+ \to \Sigma_c^0\pi^+$ | $< 0.09$ | $\leq 0.040$ | $0.47 \pm 0.25$ | $0.080 \pm 0.009$ | $0.013$ | $0.011$ |
| $\Lambda_c(2625)^+ \to \Sigma_c^0\pi^0$ | | $\leq 0.029$ | $0.42 \pm 0.22$ | $0.095 \pm 0.012$ | $0.013$ | $0.011$ |
| $\Lambda_c(2625)^+ \to \Lambda_c^+\pi^-$ | $< 0.97$ | $\leq 0.35$ | | | $0.11$ | |
| $\Sigma_c(2800)^{++} \to \Lambda_c\pi, \Sigma_c^{(*)}\pi$ | $75^{+22}_{-17}$ | input | | | | |
| $\Sigma_c(2800)^+ \to \Lambda_c\pi, \Sigma_c^{(*)}\pi$ | $62^{+60}_{-40}$ | input | | | | |
| $\Sigma_c(2800)^0 \to \Lambda_c\pi, \Sigma_c^{(*)}\pi$ | $72^{+22}_{-15}$ | input | | | | |
| $\Xi_c(2790)^+ \to \Xi_c^{0,+0}\pi^0, +$ | $< 15$ | $16.7^{+4.6}_{-3.6}$ | | | | |
| $\Xi_c(2790)^0 \to \Xi_c^{0,+0}\pi^0, -$ | $< 12$ | $17.7^{+2.9}_{-3.8}$ | | | | |
| $\Xi_c(2815)^+ \to \Xi_c^{0,+0}\pi^0, +$ | $< 3.5$ | $7.1^{+1.3}_{-1.5}$ | $2.35 \pm 0.93$ | $0.70 \pm 0.04$ | | |
| $\Xi_c(2815)^0 \to \Xi_c^{0,+0}\pi^0, -$ | $< 6.5$ | $7.7^{+1.7}_{-1.7}$ | | | | |

TABLE IX: Decay widths (in units of MeV) of $p$-wave charmed baryons. Data are taken from 2014 PDG [8] together with the new measurements of $\Sigma_c, \Sigma_c^*$ [11] and $\Xi_c(2645)^+$ widths [12]. Theoretical predictions of [69] are taken from Table IV of [70].
which improves the previous limit (3.8) by a factor of 3. The calculated partial widths of $\Lambda_c(2625)^+$ shown in Table IX are consistent with experimental limits.

IV. LIFETIMES

A. Singly charmed baryons

Among singly charmed baryons, the antitriplet states $\Lambda_c^+$, $\Xi_c^+$, $\Xi_c^0$, and the $\Omega_c^0$ baryon in the sextet decay weakly. The world averages of their lifetimes in 2006 were

$$\tau(\Lambda_c^+) = (200 \pm 6) \times 10^{-15} \text{s}, \quad \tau(\Xi_c^+) = (442 \pm 26) \times 10^{-15} \text{s},$$

$$\tau(\Xi_c^0) = (112^{+13}_{-10}) \times 10^{-15} \text{s}, \quad \tau(\Omega_c^0) = (69 \pm 12) \times 10^{-15} \text{s}.$$  (4.1)

These results remain the same even in 2014 [8]. As we shall see below, the lifetime hierarchy $\tau(\Xi_c^+) > \tau(\Lambda_c^+) > \tau(\Xi_c^0) > \tau(\Omega_c^0)$ is qualitatively understood in the OPE (operator product expansion) approach but not quantitatively.

Based on the OPE approach for the analysis of inclusive weak decays, the inclusive rate of the charmed baryon is schematically represented by

$$\Gamma(B_c \to f) = \frac{G_F^2 m_c^5}{192\pi^3} V_{\text{CKM}} \left( A_0 + \frac{A_2}{m_c^2} + \frac{A_3}{m_c^3} + \mathcal{O}(1/m_c^4) \right),$$  (4.2)

with $V_{\text{CKM}}$ being the relevant CKM matrix element. The $A_0$ term comes from the $c$ quark decay and is common to all charmed hadrons. There is no linear $1/m_Q$ corrections to the inclusive decay rate due to the lack of gauge-invariant dimension-four operators [75, 76], a consequence known as Luke’s theorem [77]. Nonperturbative corrections start at order $1/m_Q^2$ and they are model independent. Spectator effects in inclusive decays due to the Pauli interference and $W$-exchange contributions account for $1/m_Q^3$ corrections and they have two eminent features: First, the estimate of spectator effects is model dependent; the hadronic four-quark matrix elements are usually evaluated by assuming the factorization approximation for mesons and the quark model for baryons. Second, there is a two-body phase-space enhancement factor of $16\pi^2$ for spectator effects relative to the three-body phase space for heavy quark decay. This implies that spectator effects, being of order $1/m_Q^3$, are comparable to and even exceed the $1/m_Q^2$ terms.

In general, the total width of the charmed baryon $B_c$ receives contributions from inclusive nonleptonic and semileptonic decays: $\Gamma(B_c) = \Gamma_{\text{NL}}(B_c) + \Gamma_{\text{SL}}(B_c)$. The nonleptonic contribution can be decomposed into

$$\Gamma_{\text{NL}}(B_c) = \Gamma_{\text{dec}}(B_c) + \Gamma_{\text{ann}}(B_c) + \Gamma_{\text{int}}^-(B_c) + \Gamma_{\text{int}}^+(B_c),$$  (4.3)

corresponding to the $c$-quark decay, the $W$-exchange contribution, destructive and constructive Pauli interferences. It is known that the inclusive decay rate is governed by the imaginary part of an effective nonlocal forward transition operator $T$. Therefore, $\Gamma_{\text{dec}}$ corresponds to the imaginary part of Fig. 6(a) sandwiched between the same $B_c$ states. At the Cabibbo-allowed level, $\Gamma_{\text{dec}}$ represents the decay rate of $c \to su\bar{d}$, and $\Gamma_{\text{ann}}$ denotes the contribution due to the $W$-exchange diagram $cd \to us$. The interference $\Gamma_{\text{int}}^- (\Gamma_{\text{int}}^+)$ arises from the destructive (constructive) interference between the $u$ ($s$) quark produced in the $c$-quark decay and the spectator $u$ ($s$) quark in the charmed baryon $B_c$. Notice that the constructive Pauli interference is unique to the charmed baryon sector as it does not occur in the bottom sector. From the quark content of the charmed baryons, it is clear
FIG. 6: Contributions to nonleptonic decay rates of charmed baryons from four-quark operators: (a) $c$-quark decay, (b) $W$-exchange, (c) destructive Pauli interference and (d) constructive interference.

TABLE X: Various contributions to the decay rates (in units of $10^{-12}$ GeV) of singly charmed baryons [78]. Experimental values are taken from [8].

|          | $\Gamma_{\text{dec}}$ | $\Gamma_{\text{ann}}$ | $\Gamma_{\text{int}}^-$ | $\Gamma_{\text{int}}^+$ | $\Gamma_{\text{SL}}$ | $\Gamma_{\text{tot}}$ | $\tau(10^{-13}s)$ | $\tau_{\text{expt}}(10^{-13}s)$ |
|----------|------------------------|------------------------|--------------------------|--------------------------|----------------------|----------------------|------------------|---------------------|
| $\Lambda_c^+$ | 1.006                  | 1.342                  | -0.196                   | 0.323                    | 2.492                | 2.64                 | 2.00 ± 0.06      |
| $\Xi_c^+$   | 1.006                  | 0.071                  | -0.203                   | 0.364                    | 0.547                | 1.785                | 3.68 ± 0.26       |
| $\Xi_c^0$   | 1.006                  | 1.466                  | 0.385                    | 0.547                    | 3.404                | 1.93                 | 1.12 ± 0.10       |
| $\Omega_c^0$| 1.132                  | 0.439                  | 1.241                    | 1.039                    | 3.851                | 1.71                 | 0.69 ± 0.12       |

that at the Cabibbo-allowed level, the destructive interference occurs in $\Lambda_c^+$ and $\Xi_c^+$ decays (Fig. 6(c)), while $\Xi_c^+, \Xi_c^0$ and $\Omega_c^0$ can have constructive interference $\Gamma_{\text{int}}^+$ (Fig. 6(d)). Since $\Omega_c^0$ contains two $s$ quarks, it is natural to expect that $\Gamma_{\text{int}}^+(\Omega_c^0) \gg \Gamma_{\text{int}}^+(\Xi_c)$. The $W$-exchange contribution (Fig. 6(b)) occurs only for $\Xi_c^0$ and $\Lambda_c^+$ at the same Cabibbo-allowed level. In the heavy quark expansion approach, the above-mentioned spectator effects can be described in terms of the matrix elements of local four-quark operators.

The inclusive nonleptonic rates of charmed baryons in the valence quark approximation and in the limit of $m_s/m_c = 0$ have the expressions [78]:

\[
\begin{align*}
\Gamma_{\text{NL}}(\Lambda_c^+) &= \Gamma_{\text{dec}}(\Lambda_c^+) + \cos \theta_C^2 \Gamma_{\text{ann}} + \Gamma_{\text{int}}^- + \sin \theta_C^2 \Gamma_{\text{int}}^+, \\
\Gamma_{\text{NL}}(\Xi_c^+) &= \Gamma_{\text{dec}}(\Xi_c^+) + \sin \theta_C^2 \Gamma_{\text{ann}} + \Gamma_{\text{int}}^- + \cos \theta_C^2 \Gamma_{\text{int}}^+, \\
\Gamma_{\text{NL}}(\Xi_c^0) &= \Gamma_{\text{dec}}(\Xi_c^0) + \Gamma_{\text{ann}} + \Gamma_{\text{int}}^+, \\
\Gamma_{\text{NL}}(\Omega_c^0) &= \Gamma_{\text{dec}}(\Omega_c^0) + 6 \sin \theta_C^2 \Gamma_{\text{ann}} + \frac{10}{3} \cos \theta_C^2 \Gamma_{\text{int}}^+, \quad (4.4)
\end{align*}
\]

with $\theta_C$ being the Cabibbo angle.

The results of a model calculation in [78] are shown in Table X. It is clear that the lifetime pattern

\[
\tau(\Xi_c^+) > \tau(\Lambda_c^+) > \tau(\Xi_c^0) > \tau(\Omega_c^0) \quad (4.5)
\]

is in accordance with experiment. This lifetime hierarchy is qualitatively understandable. The $\Xi_c^+$ baryon is longest-lived among charmed baryons because of the smallness of $W$-exchange and partial
TABLE XI: Predicted lifetimes of doubly charmed baryons in units of $10^{-13}s$.

|       | Kiselev et al. | Guberina et al. | Chang et al. | Karliner et al. |
|-------|----------------|-----------------|--------------|-----------------|
|       | [79]           | [80]            | [81]         | [82]            |
| $\Xi_{cc}^{++}$ | 4.6 ± 0.5     | 10.5            | 6.7          | 1.85            |
| $\Xi_{cc}^+$     | 1.6 ± 0.5     | 2.0             | 2.5          | 0.53            |
| $\Omega_{cc}^+$  | 2.7 ± 0.6     | 3.0             | 2.1          |                 |

cancellation between constructive and destructive Pauli interferences, while $\Omega_c$ is shortest-lived due to the presence of two s quarks in the $\Omega_c$ that renders the contribution of $\Gamma_{_{\text{int}}}^+$ largely enhanced. Since $\Gamma_{_{\text{int}}}^+$ is always positive, $\Gamma_{_{\text{int}}}^-$ is negative and that the constructive interference is larger than the magnitude of the destructive one, this explains why $\tau(\Xi_{cc}^{++}) > \tau(\Lambda_{cc}^{++})$. It is also clear from Table X that, although the qualitative feature of the lifetime pattern is comprehensive, the quantitative estimates of charmed baryon lifetimes and their ratios are still rather poor.

B. Doubly charmed baryons

The inclusive nonleptonic rates of doubly charmed baryons in the valence quark approximation and in the limit of $m_s/m_c = 0$ have the expressions:

$$
\Gamma_{\text{NL}}(\Xi_{cc}^{++}) = \Gamma_{\text{dec}}(\Xi_{cc}^{++}) + \cos \theta_C^2 \Gamma_{\text{ann}} + \sin \theta_C^2 \Gamma_{_{\text{int}}}^+, \\
\Gamma_{\text{NL}}(\Xi_{cc}^+) = \Gamma_{\text{dec}}(\Xi_{cc}^+) + \Gamma_{_{\text{int}}}^-, \\
\Gamma_{\text{NL}}(\Omega_{cc}^+) = \Gamma_{\text{dec}}(\Omega_{cc}^+) + \sin \theta_C^2 \Gamma_{\text{ann}} + \cos \theta_C^2 \Gamma_{_{\text{int}}}^+.
$$

(4.6)

Since $\Gamma_{_{\text{int}}}^+$ is positive and $\Gamma_{_{\text{int}}}^-$ is negative, it is obvious that $\Xi_{cc}^{++}$ is longest-lived, while $\Xi_{cc}^+$ ($\Omega_{cc}^+$) is shortest-lived if $\Gamma_{_{\text{int}}}^+ > \Gamma_{\text{ann}}$ ($\Gamma_{_{\text{int}}}^- < \Gamma_{\text{ann}}$). In general, we have

$$
\tau(\Xi_{cc}^{++}) \gg \tau(\Omega_{cc}^+) \sim \tau(\Xi_{cc}^+).
$$

(4.7)

The predictions available in the literature are summarized in Table XI. Note that the lifetime of $\Xi_{cc}^+$ was measured by SELEX to be $\tau(\Xi_{cc}^+) < 0.33 \times 10^{-13}s$ [44].

Since the mass splitting between $\Xi_{cc}^*$ and $\Xi_{cc}^+$ and between $\Omega_{cc}^*$ and $\Omega_{cc}$ is less than 100 MeV (see also Eq. (2.6) for lattice calculations)

$$
m_{\Xi_{cc}^*} - m_{\Xi_{cc}} = m_{\Sigma_c^*} - m_{\Sigma_c} \approx 65 \text{ MeV}, \quad m_{\Omega_{cc}^*} - m_{\Omega_{cc}} = m_{\Omega_c^*} - m_{\Omega_c} \approx 71 \text{ MeV},
$$

(4.8)

it is clear that only electromagnetic decays are allowed for $\Omega_{cc}^*$ and $\Xi_{cc}^*$.

V. HADRONIC WEAK DECAYS

A. Nonleptonic decays

Contrary to the significant progress made over the last 10 years or so in the studies of hadronic weak decays in the bottom baryon sector, advancement in the arena of charmed baryons, both theoretical and experimental, has been very slow.
In the naive factorization approach, the coefficients \( a_1 \) for the external \( W \)-emission amplitude and \( a_2 \) for internal \( W \)-emission are given by \( (c_1 + \frac{c_2}{N_c}) \) and \( (c_2 + \frac{c_1}{N_c}) \), respectively. However, we have learned from charmed meson decays that the naive factorization approach never works for the decay rate of color-suppressed decay modes, though it usually operates for color-allowed decays. Empirically, it was learned in the 1980s that if the Fierz-transformed terms characterized by \( \frac{1}{N_c} \) are dropped, the discrepancy between theory and experiment is greatly improved [83]. This leads to the so-called large-\( N_c \) approach for describing hadronic \( D \) decays [84]. Theoretically, explicit calculations based on the QCD sum-rule analysis [85] indicate that the Fierz terms are indeed largely compensated by the nonfactorizable corrections.

As the discrepancy between theory and experiment for charmed meson decays gets much improved in the \( \frac{1}{N_c} \) expansion method, it is natural to ask if this scenario also works in the baryon sector? This issue can be settled down by the experimental measurement of the Cabibbo-suppressed mode \( \Lambda_c^+ \rightarrow p\phi \), which receives contributions only from the factorizable diagrams. As pointed out in [86], the large-\( N_c \) predicted rate is in good agreement with the measured value. By contrast, its decay rate predicted by the naive factorization approximation will be too small by a factor of 15. Therefore, the \( 1/N_c \) approach also works for the factorizable amplitude of charmed baryon decays. This also implies that the inclusion of nonfactorizable contributions is inevitable and necessary. If nonfactorizable effects amount to a redefinition of the effective parameters \( a_1 \), \( a_2 \) and are universal (i.e., channel-independent) in charm decays, then we still have a new factorization scheme with the universal parameters \( a_1 \), \( a_2 \) to be determined from experiment.

It is known in the heavy meson case that nonfactorizable contributions will render the color suppression of internal \( W \)-emission ineffective. However, the \( W \)-exchange in baryon decays is not subject to color suppression even in the absence of nonfactorizable terms. A simple way to see this is to consider the large-\( N_c \) limit. Although the \( W \)-exchange diagram is down by a factor of \( 1/N_c \) relative to the external \( W \)-emission one, it is compensated by the fact that the baryon contains \( N_c \) quarks in the limit of large \( N_c \), thus allowing \( N_c \) different possibilities for \( W \) exchange between heavy and light quarks [87]. That is, the pole contribution can be as important as the factorizable one. The experimental measurement of the decay modes \( \Lambda_c^+ \rightarrow \Xi^0 K^+, \Delta^{++} K^- \), which proceed only through the \( W \)-exchange contributions, indicates that \( W \)-exchange indeed plays an essential role in charmed baryon decays.

On the theoretical side, various approaches had been made to investigate weak decays of heavy baryons, including the current algebra approach [88, 89], the factorization scheme, the pole model technique [86, 90–94], the relativistic quark model [87, 95] and the quark-diagram scheme [96, 97]. Various model predictions of the branching fractions and decay asymmetries can be found in Tables VI-VII of [1] for \( B_c \rightarrow B + P \) decays, Table VIII for \( B_c \rightarrow B + V \) decays and Table IX for \( B_c \rightarrow B(\frac{3}{2}^+) + P(V) \) decays.

**B. Discussions**

1. \( \Lambda_c^+ \) decays

Experimentally, nearly all the branching fractions of the \( \Lambda_c^+ \) are measured relative to the \( pK^-\pi^+ \) mode. Based on ARGUS and CLEO data, PDG has made a model-dependent determination of the absolute branching fraction \( B(\Lambda_c^+ \rightarrow pK^-\pi^+) = (5.0 \pm 1.3)\% \) [3]. Recently, Belle has
TABLE XII: Branching fractions of the Cabibbo-allowed two-body decays of $\Lambda_c^+$ in units of %. Data are taken from PDG [8] except that the absolute branching fraction $\mathcal{B}(\Lambda_c^+ \to pK^-\pi^+)$ = (5.0±1.3)% is replaced by the new measurement of (6.84±0.24±0.21)% by Belle [98].

| Decays | B       | Decays | B       | Decays | B       |
|--------|---------|--------|---------|--------|---------|
| $\Lambda_c^+ \to \Lambda\pi^+$ | 1.46±0.13 | $\Lambda_c^+ \to \Lambda\rho^+$ | < 6.5 | $\Lambda_c^+ \to \Delta^{++}K^-$ | 1.16±0.07 |
| $\Lambda_c^+ \to \Sigma^0\pi^+$ | 1.44±0.14 | $\Lambda_c^+ \to \Sigma^0\rho^+$ | < 1.9 | $\Lambda_c^+ \to \Sigma^{+}\pi^0$ |            |
| $\Lambda_c^+ \to \Sigma^{+}\pi^0$ | 1.37±0.30 | $\Lambda_c^+ \to \Sigma^{+}\rho^0$ | < 1.9 | $\Lambda_c^+ \to \Sigma^{+}\pi^0$ |            |
| $\Lambda_c^+ \to \Sigma^{+}\eta$ | 0.75±0.11 | $\Lambda_c^+ \to \Sigma^{+}\omega$ | 3.7±1.0 | $\Lambda_c^+ \to \Sigma^{+}\eta$ | 1.16±0.35 |
| $\Lambda_c^+ \to \Sigma^{+}\eta'$ |            | $\Lambda_c^+ \to \Sigma^{+}\phi$ | 0.42±0.07 | $\Lambda_c^+ \to \Sigma^{+}\eta'$ |            |
| $\Lambda_c^+ \to \Xi^0K^+$ | 0.53±0.13 | $\Lambda_c^+ \to \Xi^0K^+$ | 0.53±0.19 | $\Lambda_c^+ \to \Xi^{0}K^+$ | 0.36±0.10 |
| $\Lambda_c^+ \to pK^0$ | 3.2±0.3 | $\Lambda_c^+ \to pK^0$ | 2.1±0.3 | $\Lambda_c^+ \to \Delta^{++}K^0$ | 1.36±0.44 |

reported a value of (6.84±0.24±0.21)% [98] from the reconstruction of $D^+p\pi$ recoiling against the $\Lambda_c^+$ production in $e^+e^-$ annihilation. Hence, uncertainties are much reduced and, most importantly, this measurement is model independent! More recently, BESIII has also measured this mode directly with the preliminary result $\mathcal{B}(\Lambda_c^+ \to pK^-\pi^+) = (5.77\pm 0.27)%$ (statistical error only) [98]. Its precision is comparable to the Belle’s result. Another approach is to exploit a particular $B^+ \to p\pi^+\pi^+\pi^-$ and its charge conjugate to measure $\mathcal{B}(\Lambda_c^+ \to pK^-\pi^+)$ also in a model independent manner [100]

Branching fractions of the Cabibbo-allowed two-body decays of $\Lambda_c^+$ are displayed in Table XII. Data taken from PDG [8] are scaled up by a factor of 1.37 for the central values due to the new measurement of $\mathcal{B}(\Lambda_c^+ \to pK^-\pi^+)$ by Belle [98]. BESIII has recently measured 2-body, 3-body and 4-body decay modes of $\Lambda_c^+$ with precision significantly improved [99]. For example, $\mathcal{B}(\Lambda_c^+ \to \Lambda\pi^+)(1.20\pm 0.07)%$ obtained by BESIII has a much better precision than the value of (1.07± 0.28)% quoted by PDG [8].

Many of the $\Lambda_c^+$ decay modes such as $\Sigma^+K^-\pi^+, \Sigma^+\phi, \Xi^{(*)}K^{(*)}$ and $\Delta^{++}K^-$ can only proceed through $W$-exchange. The experimental measurement of them implies the importance of $W$-exchange, which is not subject to color suppression in charmed baryon decays.

Some Cabibbo-suppressed modes such as $\Lambda_c^+ \to \Lambda K^+$ and $\Lambda_c^+ \to \Sigma^0K^+$ have been measured by Belle [101] and BaBar [102], respectively. Their branching fractions are of order $10^{-3} - 10^{-4}$. The first measured Cabibbo-suppressed mode $\Lambda_c^+ \to p\phi$ is of particular interest because it receives contributions only from the factorizable diagram and is expected to be color suppressed in the naive factorization approach. A calculation in [103] yields

$$\mathcal{B}(\Lambda_c^+ \to p\phi) = 2.26 \times 10^{-3}a_2^2, \quad \alpha(\Lambda_c^+ \to p\phi) = -0.10.$$  \hspace{1cm} (5.1)

From the experimental measurement $\mathcal{B}(\Lambda_c^+ \to p\phi) = (11.2 \pm 2.3) \times 10^{-4}$ [8], it follows that

$$|a_2|_{\text{expt}} = 0.70 \pm 0.07.$$ \hspace{1cm} (5.2)

This is consistent with the $1/N_c$ approach where $a_2 = c_2(m_c) \approx -0.59$.

---

2 We have scaled up the PDG number $(8.7 \pm 2.7) \times 10^{-4}$ [8] by a factor of 1.37 for its central value.
All the models except the model of \[93\] predict a positive decay asymmetry \(\alpha\) for the decay \(\Lambda_c^+ \to \Sigma^+\pi^0\) (see Table VII of \[1\]). Therefore, the measurement of \(\alpha = -0.45 \pm 0.31 \pm 0.06\) by CLEO \[104\] is a big surprise. If the negative sign of \(\alpha\) is confirmed in the future, this will imply an opposite sign between s-wave and p-wave amplitudes for this decay, contrary to the model expectation. The implication of this has been discussed in detail in \[86\]. Since the error of the previous CLEO measurement is very large, it is crucial to carry out more accurate measurements of the decay asymmetry for \(\Lambda_c^+ \to \Sigma^+\pi^0\).

2. \(\Xi_c^{+}\) decays

No absolute branching fractions have been measured. The branching ratios listed in Tables VI and VIII of \[1\] are the ones relative to \(\Xi_c^+ \to \Xi^-\pi^+\). Several Cabibbo-suppressed decay modes such as \(pK^{*0}\), \(\Sigma^+\phi\), \(\Sigma^+\pi^+\pi^-\), \(\Sigma^-\pi^+\pi^+\) and \(\Xi(1690)K^+\) have been observed \[8\].

The Cabibbo-allowed decays \(\Xi_c^+ \to B(3/2^+) + P\) have been studied and they are believed to be forbidden as they do not receive factorizable and \(1/2^\pm\) pole contributions \[87, 92\]. However, the \(\Sigma^+K^0\) mode was seen by FOCUS before \[105\] and this may indicate the importance of pole contributions beyond low-lying \(1/2^\pm\) intermediate states.

3. \(\Xi_c^0\) decays

No absolute branching fractions have been measured so far. However, there are several measurements of the ratios of branching fractions, for example \[8\],

\[
R_1 = \frac{\Gamma(\Xi_c^0 \to \Lambda K_0^0)}{\Gamma(\Xi_c^0 \to \Xi^-\pi^+)} = 0.21 \pm 0.02 \pm 0.02, \quad R_2 = \frac{\Gamma(\Xi_c^0 \to \Omega^-K^+)}{\Gamma(\Xi_c^0 \to \Xi^-\pi^+)} = 0.297 \pm 0.024. \tag{5.3}
\]

The decay modes \(\Xi_c^0 \to \Omega^-K^+\) and \(\Sigma^+K^-\) and \(\Sigma^+\pi^-\) proceed only through \(W\)-exchange. The measured branching ratio of \(\Omega^-K^+\) relative to \(\Xi^-\pi^+\) implies the significant role played by the \(W\)-exchange mechanism. The model of Körner and Krämer \[87\] predicts \(R_2 = 0.33\) (see Table IX of \[1\]), in agreement with experiment, but its prediction \(R_1 = 0.06\) is too small compared to the data.

4. \(\Omega_c^0\) decays

One of the unique features of the \(\Omega_c^0\) decay is that the decay \(\Omega_c^0 \to \Omega^-\pi^+\) proceeds only via external \(W\)-emission, while \(\Omega_c^0 \to \Xi^{*0}\bar{K}^0\) via the factorizable internal \(W\)-emission diagram. Various model predictions of Cabibbo-allowed \(\Omega_c^0 \to B(3/2^+) + P(V)\) are displayed in Table IX of \[1\] with the unknown parameters \(a_1\) and \(a_2\). From the decay \(\Lambda_c^+ \to p\phi\) we learn that \(|a_2| = 0.70 \pm 0.07\). Recently, the hadronic weak decays of the \(\Omega_c^0\) have been studied in \[106\] in great details with the finding that most of the decay channels in \(\Omega_c^0\) decays proceed only through the \(W\)-exchange diagram; moreover, the \(W\)-exchange contributions dominate in the rest of processes with some exception. Observation of such decays would shed some light on the mechanism of \(W\)-exchange effects in these decay modes.
C. Charm-flavor-conserving nonleptonic decays

There is a special class of weak decays of charmed baryons which can be studied in a reliable way, namely, heavy-flavor-conserving nonleptonic decays. Some examples are the singly Cabibbo-suppressed decays $\Xi_c \to \Lambda_c \pi$ and $\Omega_c \to \Xi'_c \pi$. The idea is simple: In these decays only the light quarks inside the heavy baryon will participate in weak interactions; that is, while the two light quarks undergo weak transitions, the heavy quark behaves as a “spectator”. As the emitted light mesons are soft, the $\Delta S = 1$ weak interactions among light quarks can be handled by the well known short-distance effective Hamiltonian. This special class of weak decays usually can be calculated more reliably than the conventional charmed baryon weak decays. The synthesis of heavy-quark and chiral symmetries provides a natural setting for investigating these reactions \[107\]. The weak decays $\Xi \to \Lambda Q \pi$ with $Q = c, b$ were also studied in \[108, 109\].

The combined symmetries of heavy and light quarks severely restricts the weak interactions allowed. In the symmetry limit, it is found that there cannot be $B_3 - B_6$ and $B_6^* - B_6$ nonleptonic weak transitions \[107\]. Symmetries alone permit three types of transitions: $B_3 - B_3$, $B_6 - B_6$ and $B_6^* - B_6$ transitions. However, in both the MIT bag and diquark models, only $B_3 - B_3$ transitions have nonzero amplitudes. The general amplitude for $B_i \to B_f + P$ reads

$$M(B_i \to B_f + P) = i u_f (A - B \gamma_5) u_i, \quad \text{(5.4)}$$

where $A$ and $B$ are the $S$- and $P$-wave amplitudes, respectively. The $S$-wave amplitude can be evaluated using current algebra in terms of the parity-violating commutator term. For example, the $S$-wave amplitude of $\Xi_c^+ \to \Lambda_c^+ \pi^0$ is given by

$$A(\Xi_c^+ \to \Lambda_c^+ \pi^0) = \frac{1}{\sqrt{2 f_\pi}} \langle \Lambda_c^+ \uparrow | H^{PC} | \Xi_c^+ \uparrow \rangle, \quad \text{(5.5)}$$

while the $P$-wave amplitude arises from the ground-state baryon poles \[107\]

$$B(\Xi_c^+ \to \Lambda_c^+ \pi^0) = \frac{g_2 m_{\Xi_c}}{2 f_\pi} \langle \Lambda_c^+ \uparrow | H^{PC} | \Xi_c^+ \uparrow \rangle \sin \phi, \quad \text{(5.6)}$$

where $\phi$ is mixing angle of $\Xi_c$ with $\Xi'_c$ and $\Xi_{c3}$, $\Xi_{c2}$ being their mass eigenstates. The matrix element $\langle \Lambda_c^+ \uparrow | H^{PC} | \Xi_c^+ \uparrow \rangle$ was evaluated in \[107\] using two different models: the MIT bag model \[110\] and the diquark model.

The predicted rates are \[107\]

$$\Gamma(\Xi_c^0 \to \Lambda_c^+ \pi^-) = 1.7 \times 10^{-15} \text{ GeV}, \quad \Gamma(\Xi_c^+ \to \Lambda_c^+ \pi^0) = 1.0 \times 10^{-15} \text{ GeV},$$
$$\Gamma(\Omega_c^0 \to \Xi_c^{*+} \pi^-) = 4.3 \times 10^{-17} \text{ GeV}, \quad \text{(5.7)}$$

and the corresponding branching fractions are

$$B(\Xi_c^0 \to \Lambda_c^+ \pi^-) = 2.9 \times 10^{-4}, \quad B(\Xi_c^+ \to \Lambda_c^+ \pi^0) = 6.7 \times 10^{-4},$$
$$B(\Omega_c^0 \to \Xi_c^{*+} \pi^-) = 4.5 \times 10^{-6}. \quad \text{(5.8)}$$

As stated above, the $B_6 - B_6$ transition $\Omega_c^0 \to \Xi_c^{*+} \pi^-$ vanishes in the chiral limit. It receives a finite factorizable contribution as a result of symmetry-breaking effect. At any rate, the predicted branching fractions for the charm-flavor-conserving decays $\Xi_c^0 \to \Lambda_c^+ \pi^-$ and $\Xi_c^+ \to \Lambda_c^+ \pi^0$ are of order $10^{-3} \sim 10^{-4}$ and should be readily accessible in the near future.
TABLE XIII: Predicted semileptonic decay rates (in units of $10^{10} s^{-1}$) and decay asymmetries (second entry) in various models. The absolute branching fraction $B(\Lambda_c^+ \to pK^-\pi^+)=(5.0 \pm 1.3)\%$ is replaced by the new measurement of $(6.84 \pm 0.24^{+0.21}_{-0.22})\%$ by Belle [98] for the data of $\Gamma(\Lambda_c^+ \to \Lambda^0 e^+\nu_e)$ taken from PDG [8]. Predictions of [111] are obtained in the non-relativistic quark model and the MIT bag model (in parentheses).

| Process          | [103] | [111] | [112] | [113] | [114] | [115] | [116] | [117] | [118] | [119] | Expt. [8] |
|------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-----------|
| $\Lambda_c^+ \to \Lambda^0 e^+\nu_e$ | 7.1   | 11.2 (7.7) | 9.8   | 7.22  | 7.0   | 13.2 \pm 1.8 | 10.9 \pm 3.0 | 14.4 \pm 2.6 |       |       |           |
|                  |       |       |       | -0.812|       | -1     | -0.88 \pm 0.03 | -0.86 \pm 0.04 |       |       |           |
| $\Lambda_c^+ \to \Lambda^0 \mu^+\nu_e$ | 7.1   | 11.2 (7.7) | 9.8   | 7.22  | 7.0   | 13.2 \pm 1.8 | 10.9 \pm 3.0 | 13.3 \pm 2.8 |       |       |           |
|                  |       |       |       | 1.32  |       | 1.01   | 0.96, 1.37 |       |       |           |
| $\Xi_c \to \Xi e^+\nu_e$       | 7.4   | 18.1 (12.5) | 8.5   | 8.16  | 9.7   |       | 64.8 \pm 22.6 |       |       | seen     |
| $\Xi_c \to \Xi^+ e^+\nu_e$      |       |       |       |       |       | 3.3 \pm 1.7 | |       |       |           |

D. Semileptonic decays

The exclusive semileptonic decays of charmed baryons: $\Lambda_c^+ \to \Lambda c^+(\mu^+\nu_e)$, $\Xi_c^+ \to \Xi^0 c^+(\mu^+\nu_e)$ and $\Xi_c^0 \to \Xi^- e^+\nu_e$ have been observed experimentally. Their rates depend on the $B_c \to B$ form factors $f_i(q^2)$ and $g_i(q^2)$ ($i=1,2,3$) defined by

$$ \langle B_f(p_f)|V_\mu|A_\mu|B_c(p_i)\rangle = \bar{u}_f(p_f)[f_1(q^2)\gamma_\mu + if_2(q^2)\sigma_{\mu\nu}q^\nu + f_3(q^2)q_\mu]
- (g_1(q^2)\gamma_\mu + ig_2(q^2)\sigma_{\mu\nu}q^\nu + g_3(q^2)q_\nu)\gamma_5\gamma_5 u_c(p_i). $$

These form factors have been evaluated in the non-relativistic quark model [103, 111, 112, 119], the MIT bag model [111], the relativistic quark model [113, 117], the light-front quark model [114] and QCD sum rules [115, 116, 118]. Experimentally, the only information available so far is the form-factor ratio measured in the semileptonic decay $\Lambda_c \to \Lambda e\bar{\nu}$. In the heavy quark limit, the six $\Lambda_c \to \Lambda$ form factors are reduced to two:

$$ \langle \Lambda(p)|\bar{s}_c(1-\gamma_5)c|\Lambda_c(v)\rangle = \bar{u}_\Lambda \left(F_1^{\Lambda_c}(v \cdot p) + \gamma_5 F_2^{\Lambda_c}(v \cdot p)\right)\gamma_\mu(1-\gamma_5)u_{\Lambda_c}. $$

Assuming a dipole $q^2$ behavior for form factors, the ratio $R = \hat{F}_2^{\Lambda_c}/\hat{F}_1^{\Lambda_c}$ is measured by CLEO to be \[120\]

$$ R = -0.31 \pm 0.05 \pm 0.04. $$

Various model predictions of the charmed baryon semileptonic decay rates and decay asymmetries are shown in Table XIII. Dipole $q^2$ dependence for form factors is assumed whenever the form factor momentum dependence is not available in the model. Four different sets of predictions for $\Lambda_c^+ \to n e^+\nu_e$ not listed in Table XIII were presented in the sum rule calculations of [121]. The semileptonic decays of $\Omega_c$ have been treated in [122] within the framework of a constituent quark model. From Table XIII we see that the computed branching fractions of $\Lambda_c^+ \to \Lambda e^+\nu$ falling in the range 1.4% ~ 2.6% are slightly smaller than experiment, $\langle 2.9 \pm 0.5 \% \rangle [2.1 \pm 0.6 \%]$ in PDG [8]. Branching fractions of $\Xi_c^0 \to \Xi^- e^+\nu$ and $\Xi_c^+ \to \Xi^0 e^+\nu$ are predicted to lie in the ranges $(0.8 \sim 2.0)\%$ and $(3.3 \sim 8.1)\%$, respectively, except that the QCD sum rule calculation in [118] predicts a much large rate for $\Xi_c \to \Xi^+ e^+\nu_e$. Experimentally, only the ratios of the branching fractions are available.
so far \[8\]

\[
\frac{\Gamma(\Xi_c^+ \to \Xi^0e^+\nu)}{\Gamma(\Xi_c^+ \to \Xi^-\pi^+\pi^+)} = 2.3 \pm 0.6^{+0.3}_{-0.6}, \quad \frac{\Gamma(\Xi_c^0 \to \Xi^-e^+\nu)}{\Gamma(\Xi_c^+ \to \Xi^-\pi^+)} = 3.1 \pm 1.0^{+0.3}_{-0.5}. \tag{5.12}
\]

There has been active studies in semileptonic decays of doubly charmed baryons. The interested reader can consult to [123–126] for further references.

Just as the hadronic decays discussed in the last subsection, there are also heavy-flavor-conserving semileptonic processes, for example, \(\Xi_c^0 \to \Lambda_c^+(\Sigma_c^+e^-\bar{\nu}_e)\) and \(\Omega_c^0 \to \Xi_c^+e^-\bar{\nu}_e\). In these decays only the light quarks inside the heavy baryon will participate in weak interactions, while the heavy quark behaves as a spectator. This topic has been recently investigated in [109]. Due to the severe phase-space suppression, the branching fractions are of order \(10^{-6}\) in the best cases, typically \(10^{-7}\) to \(10^{-8}\).

VI. ELECTROMAGNETIC AND WEAK RADIATIVE DECAYS

Although radiative decays are well measured in the charmed meson sector, e.g. \(D^* \to D\gamma\) and \(D^{*+}_s \to D^{*+}_s\gamma\), only three of the radiative modes in the charmed baryon sector have been seen, namely, \(\Xi_c^0 \to \Xi_c^0\gamma\), \(\Xi'_c \to \Xi'_c\gamma\) and \(\Omega_c^0 \to \Omega_c^0\gamma\). This is understandable because \(m_{\Xi_c^0} - m_{\Xi_c} \approx 108\ MeV\) and \(m_{\Omega_c^0} - m_{\Omega_c} \approx 71\ MeV\). Hence, \(\Xi'_c\) and \(\Omega_c^*\) are governed by the electromagnetic decays. However, it will be difficult to measure the rates of these decays because these states are too narrow to be experimentally resolvable. Nevertheless, we shall systematically study the two-body electromagnetic decays of charmed baryons and also weak radiative decays.

A. Electromagnetic decays

In the charmed baryon sector, the following two-body electromagnetic decays are of interest:

\[
\begin{align*}
B_6 & \to \mathcal{B}_\gamma + \gamma : \Sigma_c \to \Lambda_c + \gamma, \quad \Xi'_c \to \Xi_c + \gamma, \\
B_6^* & \to \mathcal{B}_\gamma + \gamma : \Sigma_c^* \to \Lambda_c + \gamma, \quad \Xi'_c^* \to \Xi_c + \gamma, \\
B_6^0 & \to \mathcal{B}_0 + \gamma : \Sigma_c^0 \to \Sigma_c + \gamma, \quad \Xi'_c^0 \to \Xi'_c + \gamma, \quad \Omega_c^* \to \Omega_c + \gamma,
\end{align*}
\tag{6.1}
\]

where we have denoted the spin \(\frac{1}{2}\) baryons as \(B_6\) and \(\mathcal{B}_\gamma\) for a symmetric sextet \(6\) and antisymmetric antitriplet \(\bar{3}\), respectively, and the spin \(\frac{3}{2}\) baryon by \(B_6^*\).

An ideal theoretical framework for studying the above-mentioned electromagnetic decays is provided by the formalism in which the heavy quark symmetry and the chiral symmetry of light quarks are combined [64, 65]. When supplemented by the nonrelativistic quark model, the formalism determines completely the low energy dynamics of heavy hadrons. The electromagnetic interactions of heavy hadrons consist of two distinct contributions: one from gauging electromagnetically the chirally invariant strong interaction Lagrangians for heavy mesons and baryons given in [64, 65], and the other from the anomalous magnetic moment couplings of the heavy particles. The heavy quark symmetry reduces the number of free parameters needed to describe the magnetic couplings to the photon. There are two undetermined parameters for the ground-state heavy baryons. All these parameters are related simply to the magnetic moments of the light quarks in the nonrelativistic quark model. However, the charmed quark is not particularly heavy \((m_c \simeq 1.6\ GeV)\), and it carries a charge of \(\frac{2}{3}e\). Consequently, the contribution from its magnetic moment cannot be
neglected. The chiral and electromagnetic gauge-invariant Lagrangian for heavy baryons can be found in Eqs. (3.8) and (3.9) of [127], denoted by \( \mathcal{L}_B^{(1)} \) and \( \mathcal{L}_B^{(2)} \), respectively.

The most general gauge-invariant Lagrangian Eq. (3.9) of [127] for magnetic transitions of heavy baryons can be recast in terms of superfields [128]

\[
\mathcal{L}_B^{(2)} = -i3a_1 \text{tr}(\bar{S}^\mu Q F_{\mu\nu} S^{\nu}) + \sqrt{3}a_2\epsilon_{\mu\nu\alpha\beta}\text{tr}(\bar{S}^\mu Q v^\nu F^{\alpha\beta}) + h.c.
\]

\[
+3a'_1 \text{tr}(\bar{S}^\mu Q' \sigma \cdot F S_\mu) - \frac{3}{2}a'_1 \text{tr}(TQ' \sigma \cdot FT),
\]

(6.2)

where \( \sigma \cdot F \equiv \sigma_{\mu\nu} F^{\mu\nu}, \) \( Q = \text{diag}(2/3, -1/3, -1/3) \) is the charge matrix for the light \( u, d, \) and \( s \) quarks, \( Q' = e_Q \) is the charge of the heavy quark. In the above equation,

\[
T = B_3, \quad S^\mu = B_6^\mu - \frac{1}{\sqrt{3}}(\gamma^\mu + v^\mu)\gamma_5 B_6.
\]

(6.3)

It follows that [128]

\[
A[S^\mu_B(v) \to S^\nu_B + \gamma(\epsilon, k)] = i \frac{2}{3}a_1 \bar{U'}(Q_{ii} + Q_{jj})(k_\mu \epsilon_\nu - k_\mu \epsilon_\nu)U^\mu - 6a'_1 Q\bar{T}^\mu U^\mu,
\]

\[
A[S^\mu_B(v) \to T_{ij} + \gamma(\epsilon, k)] = -2\sqrt{3/2} a_2 \epsilon_{\mu\nu\alpha\beta} \bar{u}_3 \gamma^\nu k^\alpha \epsilon^\beta (Q_{ii} - Q_{jj})U^\mu \quad (i < j),
\]

(6.4)

where \( k_\mu \) is the photon 4-momentum and \( \epsilon_\mu \) is the polarization 4-vector. As stressed in [127], SU(3) breaking effects due to light-quark mass differences can be incorporated by replacing the charge matrix \( Q \) by

\[
Q \to \tilde{Q} = \text{diag} \left( \frac{2}{3}, -\frac{\alpha}{3}, -\frac{\beta}{3} \right)
\]

(6.5)

with \( \alpha = M_u/M_d \) and \( \beta = M_u/M_s \). To avoid any confusion with the current quark mass \( m_q \), we have used capital letters to denote the constituent quark masses. In the quark model, the coefficients \( a_1 \) and \( a_2 \) are simply related to the Dirac magnetic moments of the light quarks

\[
a_1 = \frac{e}{3} \frac{1}{M_u}, \quad a_2 = \frac{e}{2\sqrt{6}} \frac{1}{M_u},
\]

(6.6)

whereas \( a'_1 \) is connected to those of heavy quarks. Explicitly, \( a'_1 \) is fixed by heavy quark symmetry to be

\[
a'_1 = \frac{e}{12} \frac{1}{M_Q}.
\]

(6.7)

Within the framework of HHChPT, the authors of [129] proceeded to construct chiral Lagrangians at the level \( \mathcal{O}(p^2) \) and \( \mathcal{O}(p^3) \) and then calculated the electromagnetic decay amplitudes of charmed baryons up to \( \mathcal{O}(p^3) \). It is not clear if their \( \mathcal{O}(p^2) \) Lagrangian (see Eq. (12) of [129]) characterized by the four couplings \( f_2, f_3, \tilde{f}_3 \) and \( f_4 \) are equivalent to the first two terms of the \( \mathcal{O}(p) \) Lagrangian given by Eq. (6.2). The unknown couplings there were also estimated using the quark model.

The general amplitudes of electromagnetic decays are given by [127]

\[
A(B_6 \to B_3 + \gamma) = i\eta_1 \bar{u}_3 \sigma_{\mu\nu} k^\mu \epsilon^\nu u_6,
\]

\[
A(B_6^* \to B_3 + \gamma) = i\eta_2 \epsilon_{\mu\nu\alpha\beta} \bar{u}_3 \gamma^\nu k^\alpha \epsilon^\beta u_6,
\]

\[
A(B_6^* \to B_6 + \gamma) = i\eta_3 \epsilon_{\mu\nu\alpha\beta} \bar{u}_6 \gamma^\nu k^\alpha \epsilon^\beta u_6.
\]

(6.8)
The corresponding decay rates are [127]

\[ \Gamma(B_0 \rightarrow B_3 + \gamma) = \eta_1 \frac{k_3}{\pi}, \]
\[ \Gamma(B_0^* \rightarrow B_3 + \gamma) = \eta_2 \frac{k_3}{3\pi} \frac{3m_i^2 + m_f^2}{4m_i^2}, \]
\[ \Gamma(B_0^* \rightarrow B_6 + \gamma) = \eta_3 \frac{k_3}{3\pi} \frac{3m_i^2 + m_f^2}{4m_i^2}, \]

where \(m_i\) (\(m_f\)) is the mass of the parent (daughter) baryon. The coupling constants \(\eta_i\) can be calculated using the quark model for \(a_1, a_2\) and \(a'_1\) [127, 130]:

\[ \eta_1(\Sigma^+_c \rightarrow \Lambda^+_c) = \frac{e}{6\sqrt{3}} \left( \frac{2}{M_u} + \frac{1}{M_d} \right), \]
\[ \eta_1(\Xi^{0}_c \rightarrow \Xi^{0}_c) = \frac{e}{3\sqrt{6}} \left( \frac{2}{M_u} + \frac{1}{M_d} \right), \]
\[ \eta_2(\Xi^{+}_c \rightarrow \Xi^{+}_c) = \frac{e}{3\sqrt{6}} \left( \frac{2}{M_u} + \frac{1}{M_d} \right), \]
\[ \eta_3(\Sigma^{++}_c \rightarrow \Sigma^{++}_c) = \frac{2\sqrt{2}e}{9} \left( \frac{1}{M_u} - \frac{1}{2M_d} - \frac{2}{M_c} \right), \]
\[ \eta_3(\Xi^{+}_c \rightarrow \Xi^{+}_c) = \frac{2\sqrt{2}e}{9} \left( \frac{1}{M_u} - \frac{1}{2M_d} - \frac{2}{M_c} \right), \]

where \(M_u, M_d, M_s\) and \(M_c\) are the constituent quark masses, \(M_u = 338\) MeV, \(M_d = 322\) MeV, \(M_s = 510\) MeV [8], and \(M_c = 1.6\) GeV. The calculated results are summarized in the second column of Table XIV. Some other model predictions are also listed there for comparison.

Radiative decays of \(s\)-wave charmed baryons are considered in [131] in the quark model with predictions similar to ours. A similar procedure is followed in [132] where the heavy quark symmetry is supplemented with light-diquark symmetries to calculate the widths of \(\Sigma^+_c \rightarrow \Lambda^+_c \gamma\) and \(\Sigma^{*+}_c \rightarrow \Sigma^{*+}_c \gamma\). The authors of [70] apply the relativistic quark model to predict various electromagnetic decays of charmed baryons. Besides the magnetic dipole (M1) transition, the author of [133] also considered and estimated the electric quadrupole (E2) amplitude for \(\Sigma^{*+}_c \rightarrow \Lambda^+_c \gamma\) arising from the chiral loop correction. A detailed analysis of the E2 contributions was presented in [134]. The E2 amplitudes appear at different higher orders for the three kinds of decays: \(O(1/\Lambda^2_\chi)\) for \(B_0^* \rightarrow B_0 + \gamma\), \(O(1/m_Q \Lambda^2_\chi)\) for \(B_0^* \rightarrow B_3 + \gamma\), and \(O(1/m_Q^3 \Lambda^2_\chi)\) for \(B_0^* \rightarrow B_3 + \gamma\). Therefore, the E2 contribution to \(B_0 \rightarrow B_3 + \gamma\) is completely negligible. The electromagnetic decays were calculated in [135, 136] using the QCD sum rule method, while they were studied within the framework of the modified bag model in [137].

It is evident from Table XIV that the predictions in [127, 130] and [129] all based on HHChPT are quite different for the following three modes: \(\Sigma^{++}_c \rightarrow \Sigma^{++}_c \gamma\), \(\Sigma^{*+}_c \rightarrow \Lambda^+_c \gamma\), and \(\Xi^{*+}_c \rightarrow \Xi^{+}_c \gamma\). Indeed, the results for the last two modes in [129] are larger than all other existing predictions by one order of magnitude! It is naively expected that all HHChPT approaches should agree with each other to the lowest order of chiral expansion provided that the coefficients are inferred from the nonrelativistic quark model. The lowest order predictions \(\Gamma(\Sigma^{*+}_c \rightarrow \Lambda^+_c \gamma) = 756\) keV and \(\Gamma(\Xi^{*+}_c \rightarrow \Xi^{+}_c \gamma) = 403\) keV obtained in [129] are still very large. Note that a recent lattice calculation in [139] yields \(\Gamma(\Omega^*_c \rightarrow \Omega_c \gamma) = 0.074 \pm 0.008\) keV which is much smaller than \(\Gamma(\Omega^*_c) = 4.82\) keV predicted in [129].
TABLE XIV: Electromagnetic decay rates (in units of keV) of s-wave charmed baryons. Among the four different results listed in [131] and [138], we quote those denoted by $\Gamma^{(0)}_{\gamma}$ and "Present (ecqm)", respectively.

| Decay | HHChPT | HHChPT | Dey et al. [131] | Ivanov et al. [70] | Tawfiq et al. [132] | Baanuls et al. [134] | Aliev et al. [135] | Wang et al. [136] | Bernotas et al. [137] | Majethiya et al. [138] |
|-------|--------|--------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| $\Sigma_c^+ \rightarrow \Lambda_c^+ \gamma$ | 91.5 | 164.16 | 120 | 60.7 ± 1.5 | 87 | 130 ± 4 | 46.1 | 60.55 | 126 | 154.48 |
| $\Sigma_c^+ \rightarrow \Lambda_c^{++} \gamma$ | 150.3 | 892.97 | 310 | 151 ± 4 | 1.3 | 11.60 | 1.6 | 3.04 | 2.65 ± 1.20 | 6.36 ± 0.21 |
| $\Sigma_c^+ \rightarrow \Sigma_c^+ \gamma$ | 0.002 | 0.85 | 0.001 | 0.14 ± 0.004 | 0.19 | 0.40 ± 0.16 | 0.40 ± 0.21 | 0.004 | < 10^{-4} |
| $\Sigma_c^+ \rightarrow \Sigma_c^0 \gamma$ | 1.2 | 2.92 | 1.2 | 0.76 | 0.08 ± 0.03 | 1.25 ± 0.06 |
| $\Xi_c^+ \rightarrow \Xi_c^+ \gamma$ | 19.7 | 54.31 | 14 | 12.7 ± 1.5 | 63.5 | 502.11 | 71 | 54 ± 3 | 52 ± 0.6 | 0.011 |
| $\Xi_c^+ \rightarrow \Xi_c^0 \gamma$ | 0.06 | 1.10 | 0.10 | 1.2 ± 0.7 | 0.96 ± 0.62 |
| $\Xi_c^{*0} \rightarrow \Xi_c^{*0} \gamma$ | 0.4 | 0.02 | 0.33 | 0.17 ± 0.02 | 1.26 ± 0.80 |
| $\Xi_c^{*0} \rightarrow \Xi_c^{*0} \gamma$ | 1.1 | 0.36 | 1.7 | 0.68 ± 0.04 | 5.1 ± 2.7 |
| $\Omega_c^{*0} \rightarrow \Omega_c^{*0} \gamma$ | 1.0 | 3.83 | 1.6 | 1.66 ± 0.32 | 0.908 | 0.30 |

TABLE XV: Electromagnetic decay rates (in units of keV) of p-wave charmed baryons.

| Decay | Ivanov et al. [70] | Tawfiq et al. [132] | Aziza Baccouche et al. [141] | Zhu et al. [142] | Chow et al. [143] | Gamermann et al. [144] |
|-------|----------------|----------------|----------------|----------------|----------------|----------------|
| $1/2^{-} \rightarrow 1/2^{+}(3/2^{+})\gamma$ | 115 ± 1 | 25 | 36 | 16 | 274 ± 52 |
| $\Lambda_c(2595)^+ \rightarrow \Lambda_c^+ \gamma$ | 77 ± 1 | 71 | 11 | 2.1 ± 0.4 |
| $\Lambda_c(2595)^+ \rightarrow \Sigma_c^+ \gamma$ | 6 ± 0.1 | 11 | 1 | 0.3 ± 0.6 |
| $\Lambda_c(2595)^+ \rightarrow \Sigma_c^{*+} \gamma$ | 151 ± 2 | 48 | 21 | 0.3 ± 0.6 |
| $\Lambda_c(2625)^+ \rightarrow \Lambda_c^+ \gamma$ | 35 ± 0.5 | 130 | 5 | 0.3 ± 0.6 |
| $\Lambda_c(2625)^+ \rightarrow \Sigma_c^+ \gamma$ | 46 ± 0.6 | 32 | 6 | 0.3 ± 0.6 |
| $\Xi_c(2815)^+ \rightarrow \Xi_c^+ \gamma$ | 190 ± 5 | 0.3 ± 0.6 |
| $\Xi_c(2815)^0 \rightarrow \Xi_c^0 \gamma$ | 497 ± 14 | 0.3 ± 0.6 |

Chiral-loop corrections to the M1 electromagnetic decays and to the strong decays of heavy baryons have been computed at the one loop order in [128]. The leading chiral-loop effects we found are nonanalytic in the forms of $m/\Lambda_X$ and $(m_c^2/\Lambda_X^2)\ln(\Lambda^2/m_c^2)$ (or $m_q^{1/2}$ and $m_q\ln m_q$, with $m_q$ being the light quark mass). Some results are [128]

$$\Gamma(\Sigma_c^+ \rightarrow \Lambda_c^+ \gamma) = 112 \text{ keV}, \quad \Gamma(\Xi_c^0 \rightarrow \Xi_c^0 \gamma) = 29 \text{ keV}, \quad \Gamma(\Xi_c^0 \rightarrow \Xi_c^0 \gamma) = 0.15 \text{ keV}, \quad (6.11)$$

which should be compared with the corresponding quark-model results: 92 keV, 20 keV and 0.4 keV (Table XIV).
The electromagnetic decays $\Xi_c^{∗0} \to \Xi_c^0 \gamma$ and $\Xi_c^{0} \to \Xi_c^0 \gamma$ are of special interest. It has been advocated in [140] that a measurement of their branching fractions will allow us to determine one of the coupling constants in HHChPT, namely, $g_1$. They are forbidden at tree level in SU(3) limit [see Eq. (6.10)]. In heavy baryon chiral perturbation theory, this radiative decay is induced via chiral loops where SU(3) symmetry is broken by the light current quark masses. By identifying the chiral loop contribution to $\Xi_c^{∗0} \to \Xi_c^0 \gamma$ with the quark model prediction given in Eq. (6.10), it was found in [130] that one of the two possible solutions is in accord with the quark model expectation for $g_1$.

For the electromagnetic decays of $p$-wave charmed baryons, the search for $\Lambda_c(2593)^+ \to \Lambda_c^+ \pi^\gamma$ and $\Lambda_c(2625)^+ \to \Lambda_c^+ \pi^\gamma$ has been failed so far. On the theoretical side, the interested reader is referred to [14, 70, 132, 140–145] for more details. Some predictions are collected in Table XV and they are more diversified than the $s$-wave case. For the electromagnetic decays of doubly charmed baryons, see e.g. [137, 146].

The electromagnetic decays considered so far do not test critically the heavy quark symmetry nor the chiral symmetry. The results follow simply from the quark model. There are examples in which both the heavy quark symmetry and the chiral symmetry enter in a crucial way. These are the radiative decays of heavy baryons involving an emitted pion. Some examples which are kinematically allowed are

$$\Sigma_c \to \Lambda_c \pi \gamma, \quad \Sigma_c^* \to \Lambda_c \pi \gamma, \quad \Sigma_c' \to \Sigma_c \pi \gamma, \quad \Xi_c^* \to \Xi_c \pi \gamma.$$  \hfill (6.12)

It turns out that the contact interaction dictated by the Lagrangian $L_B^{(1)}$ can be nicely tested by the decay $\Sigma_c^0 \to \Lambda_c^+ \pi^- \gamma$, whereas a test on the chiral structure of $L_B^{(2)}$ is provided by the process $\Sigma_c^+ \to \Lambda_c^+ \pi^0 \gamma$; see [127] for the analysis.

**B. Weak radiative decays**

At the quark level, there are three different types of processes which can contribute to the weak radiative decays of heavy hadrons, namely, single-, two- and three-quark transitions [148]. The single-quark transition mechanism comes from the so-called electromagnetic penguin diagram. Unfortunately, the penguin process $c \to u \gamma$ is very suppressed and hence it plays no role in charmed hadron radiative decays. There are two contributions from the two-quark transitions: one from the $W$-exchange diagram accompanied by a photon emission from the external quark, and the other from the same $W$-exchange diagram but with a photon radiated from the $W$ boson. The latter is typically suppressed by a factor of $m_q k/M_W^2$ ($k$ being the photon energy) as compared to the former bremsstrahlung process [147]. For charmed baryons, the Cabibbo-allowed decay modes via $c\bar{u} \to s\bar{d} \gamma$ (Fig. 7) or $cd \to us \gamma$ are

$$\Lambda_c^+ \to \Sigma^+ \gamma, \quad \Xi_c^0 \to \Xi_c^0 \gamma.$$  \hfill (6.13)

Finally, the three-quark transition involving $W$-exchange between two quarks and a photon emission by the third quark is quite suppressed because of very small probability of finding three quarks in adequate kinematic matching with the baryons [148, 149].

The general amplitude of the weak radiative baryon decay reads

$$A(B_i \to B_f \gamma) = i\bar{u}_f(a + b\gamma_5)\sigma_{\mu\nu}e^\mu k^\nu u_i,$$  \hfill (6.14)
where $a$ and $b$ are parity-conserving and -violating amplitudes, respectively. The corresponding decay rate is

$$\Gamma(B_i \rightarrow B_f \gamma) = \frac{1}{8\pi} \left( \frac{m_i^2 - m_f^2}{m_i^2} \right)^3 (|a|^2 + |b|^2).$$  \hfill (6.15)

Nonpenguin weak radiative decays of charmed baryons such as those in (6.13) are characterized by emission of a hard photon and the presence of a highly virtual intermediate quark between the electromagnetic and weak vertices. It has been shown in [150] that these features should make possible to analyze these processes by perturbative QCD; that is, these processes are describable by an effective local and gauge invariant Lagrangian:

$$H_{\text{eff}}(c \bar{u} \rightarrow s \bar{d} \gamma) = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* (c \gamma_{\mu} O_{\mu}^+ + c \gamma_{\mu} O_{\mu}^-),$$  \hfill (6.16)

with

$$O_{\pm}^\mu(c \bar{u} \rightarrow s \bar{d} \gamma) = \frac{e}{m_i^2 - m_f^2} \left\{ \left( e_s \frac{m_f}{m_s} + e_u \frac{m_i}{m_u} \right) \left( \tilde{F}_{\mu\nu} + iF_{\mu\nu} \right) O_{\pm}^{\mu\nu} + \left( e_s \frac{m_f}{m_d} + e_c \frac{m_i}{m_c} \right) \left( \tilde{F}_{\mu\nu} - iF_{\mu\nu} \right) O_{\mp}^{\mu\nu} \right\},$$  \hfill (6.17)

where $m_i = m_c + m_u$, $m_f = m_s + m_d$, $\tilde{F}_{\mu\nu} \equiv \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} F^{\alpha\beta}$ and

$$O_{\pm}^{\mu\nu} = \bar{s} \gamma^\rho (1 - \gamma_5) c \bar{u} \gamma^\nu (1 - \gamma_5) d \pm \bar{s} \gamma^\rho (1 - \gamma_5) d \bar{u} \gamma^\nu (1 - \gamma_5) c.$$  \hfill (6.18)

For the charmed baryon radiative decays, one needs to evaluate the matrix element $\langle B_f | O_{\pm}^{\mu\nu} | B_i \rangle$. Since the quark-model wave functions best resemble the hadronic states in the frame where both baryons are static, the static MIT bag model was thus adopted in [150] for the calculation. The predictions are

$$B(\Lambda_c^+ \rightarrow \Sigma^+ \gamma) = 4.9 \times 10^{-5}, \quad \alpha(\Lambda_c^+ \rightarrow \Sigma^+ \gamma) = -0.86,$$

$$B(\Xi_c^0 \rightarrow \Xi^0 \gamma) = 3.5 \times 10^{-5}, \quad \alpha(\Xi_c^0 \rightarrow \Xi^0 \gamma) = -0.86.$$ \hfill (6.20)

\footnote{The branching fraction of $\Xi_c^0 \rightarrow \Xi^0 \gamma$ has been updated using the current lifetime of $\Xi_c^0$.}
A different analysis of the same decays was carried out in [151] with the results
\[
\mathcal{B}(\Lambda_c^+ \to \Sigma^+ \gamma) = 2.8 \times 10^{-4}, \quad \alpha(\Lambda_c^+ \to \Sigma^+ \gamma) = 0.02, \\
\mathcal{B}(\Xi_c^0 \to \Xi^0 \gamma) = 1.5 \times 10^{-4}, \quad \alpha(\Xi_c^0 \to \Xi^0 \gamma) = -0.01. 
\]
(6.21)

Evidently, these predictions (especially the decay asymmetry) are very different from the ones obtained in [150].

Finally, it is worth remarking that, in analog to the heavy-flavor-conserving nonleptonic weak decays as discussed in Sec. VI.C, there is a special class of weak radiative decays in which heavy flavor is conserved, for example, \(\Xi_c \to \Lambda_c \gamma\) and \(\Omega_c \to \Xi_c \gamma\). In these decays, weak radiative transitions arise from the light quark sector of the heavy baryon whereas the heavy quark behaves as a spectator. However, the dynamics of these radiative decays is more complicated than their counterpart in nonleptonic weak decays, e.g., \(\Xi_c \to \Lambda_c \pi\). In any event, it deserves an investigation.

VII. CONCLUSIONS

In this report we began with a brief overview of the spectroscopy of charmed baryons and discussed their possible structure and spin-parity assignments in the quark model. For the \(p\)-wave baryons, We have assigned \(\Sigma_{c2}(\frac{3}{2}^-)\) to \(\Sigma_c(2800)\). As for first positive-parity excitations, with the help of the relativistic quark-diquark model and the \(3^P_0\) model, we have identified \(\tilde{\Lambda}_{c3}(\frac{5}{2}^+)\) with \(\Lambda_c(2800)\), \(\tilde{\Xi}_c(\frac{1}{2}^+)\) with \(\Xi_c(2980)\), and \(\tilde{\Xi}_{c3}(\frac{3}{2}^+)\) with \(\Xi_c(3080)\), though the first and last assignments may encounter some potential problems.

It should be stressed that the mass analysis alone is usually not adequate to pin down the spin-parity quantum numbers of higher excited charmed baryon states, a study of their strong decays is necessary. For example, \(\Sigma_{c0}(\frac{1}{2}^-)\), \(\Sigma_{c1}(\frac{1}{2}^-, \frac{3}{2}^-)\) and \(\Sigma_{c2}(\frac{3}{2}^-, \frac{5}{2}^-)\) for \(\Sigma_c(2800)\) all have similar masses. The analysis of strong decays allows us to exclude the first two possibilities. It should be stressed that the Regge phenomenology and the mass relations for antitriplet and sextet multiplets also provide very useful guidance for the spin-parity quantum numbers.

Based on various theoretical tools such as lattice QCD and the QCD sum rule method, there are a lot of theoretical activities on charmed baryon spectroscopy, especially for doubly and triply charmed baryons. However, progress in the hadronic decays, radiative decays and lifetimes has been very slow. Experimentally, nearly all the branching fractions of the \(\Lambda_c^+\) are measured relative to the \(pK^-\pi^+\) mode. The recent measurements \(\mathcal{B}(\Lambda_c^+ \to pK^-\pi^+) = (6.84 \pm 0.24^{+0.21}_{-0.25})\%\) by Belle and \((5.77 \pm 0.27)\%\) (statistical error only) by BESIII are very encouraging. Moreover, BESIII has recently measured 2-body, 3-body and 4-body nonleptonic decay modes of \(\Lambda_c^+\) with precision significantly improved. It is conceivable that many new data emerged from LHCb and BESIII in the immediate future and from the experiments at J-PARC and PANDA in the future can be used to test the underlying mechanism for hadronic weak decays.

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