Problems about Causality in Fermi’s Two-Atom Model and Possible Resolutions

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Abstract

In order to check finite propagation speed Fermi, in 1932, had considered two atoms $A$ and $B$ separated by some distance $R$. At time $t = 0$, $A$ is in an excited state, $B$ in its ground state, and no photons are present. Fermi’s idea was to calculate the excitation probability of $B$. In a model-independent way and with minimal assumptions – Hilbert space and positive energy only – it is proved, not just for atoms but for any systems $A$ and $B$, that the excitation probability of $B$ is nonzero immediately after $t = 0$. Possible ways out to avoid a contradiction to finite propagation speed are discussed. The notions of strong and weak Einstein causality are introduced.

1 Introduction

One of the pillars of special as well as general relativity is the assumption that no signals can be transmitted faster than the speed of light. If there were arbitrarily high signal velocities in nature then either

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This article has appeared in: NONLINEAR, DEFORMED AND IRREVERSIBLE QUANTUM SYSTEMS, Proceedings of an International Symposium on Mathematical Physics at the Arnold Sommerfeld Institute 15–19 August 1994, Clausthal, Germany. Editors: H.-D. Doebner, V.K. Dobrev, P. Nattermann. WORLD SCIENTIFIC, Singapore (1995), p. 253 – 264
those superfast signals could be used to synchronized clocks to yield absolute simultaneity and thus a breakdown of relativity theory, or

there would exist “tachyons”, and the sequence of cause and effect could be reversed.

The second alternative has captured imaginative minds and prompted them to create science-fiction like scenarios. The concept of finite signal velocity or, more precisely, the speed of light as highest signal velocity, is therefore often called “Einstein causality”. In my opinion, though, if Einstein causality were to fail most physicist would adopt the first alternative and reformulate or abandon relativity theory.

For this reason the question of finite signal velocity in quantum theory attracted the interest of Heisenberg and Fermi in the early thirties, in particular whether photons traveled with the speed of light.

In 1932, Fermi [1] consider for this purpose a simple model. Two atoms, A and B, are separated by a distance $R$. At time $t = 0$, A is assumed to be in excited state and B in its ground state, with no photons present. Atom A will decay into its ground state under the emission of a photon. This photon can then, with a small probability, be absorbed by atom B. Fermi asked the question at what earliest time atom B will “notice” the decay of atom A with its accompanying photon. He expected that B moves out of its ground state only after a time $t = R/c$, in accordance with the speed of light. And indeed, this was what he found by his calculations.

Fermi’s calculations were based on second-order perturbation theory – a technique still quite common today – and on the approximation of an integral over positive frequencies by an integral over positive and negative frequencies ranging from $-\infty$ to $\infty$ instead of 0 to $\infty$. More than thirty years later Shirokov [2] pointed out that without this replacement the calculations would not yield the desired result [3]. It remained unclear, however, what would happen if one went to higher orders in perturbation theory.

The setup of Fermi’s model will be further discussed in the next section. Fermi had calculated the probability for the following transition: A nonexcited, B excited and no photons. As will be discussed this is an exchange probability [2] and it does not directly, without further assumptions, refer to Einstein causality but to what one nowadays calls local and nonlocal correlations.

Fermi’s problem was investigated by many authors in this or in a related form, e.g. by Heitler and Ma [4], Hamilton [5], Fierz [6], Ferretti [7], Milonni and Knight [8], Shirokov [2] and his review [9], Rubin [10], Biswas et al. [11], and Valentini [12]. The older papers confirmed Fermi’s conclusion, while the results of the later papers depend on the model and the approximations used. At present there seems to be agreement that Fermi’s ‘local’ result is not correct, but that this nonlocality cannot be used for superluminal signal transmission since measurements on A and B as well as on photons are involved.
The present contribution, which is partially based on Ref. [13], is mathematically very simple. It analyzes Fermi’s model under quite general and simple assumptions—essentially just positivity of the energy. No perturbation theory, no specific form of the Hamiltonian, nor further assumptions of quantum field theories like the locality postulate, are used, and the conclusions hold for relativistic and nonrelativistic theories. Moreover, the atoms can be replaced by more general “sources” and “detectors”.

Specifically, it is shown for the model considered by Fermi that the excitation probability of atom $B$ would be immediately nonzero if the experiment could really be performed. At first sight this result might seem to indicate serious difficulties with causality for Fermi’s two-atom model. However, already in Ref. [13] I pointed out several ways to avoid this disastrous consequence, and in the last section I will discuss additional ones. The message is that finite signal velocity is a delicate question.

Somewhat surprisingly, the results of my paper [13] received great publicity and were discussed not only in science journals like Nature [3] or New Scientist [14], but also made it to the daily press and weekly magazines [15]. While some discussions were reasonably serious, smaller tabloids tended to sensationalize by picking on acausality and omitting the ways out [16]. The moral to draw from this is that one should not rely on second or third hand accounts, in particular not on sensational ones.

2 Fermi’s model: Correlations, Excitation Probabilities, and Bare States

Fermi supposed in his model that by some means one had prepared, at time $t = 0$, atom $A$ in an excited state, $|e_A\rangle$, and atom $B$ in its ground state, $|g_B\rangle$, with no photons present. The state of the complete system then developed in time and Fermi calculated the “exchange” probability to find the state $|g_A\rangle|e_B\rangle|0_{ph}\rangle$ at time $t$. He probably had in mind that this could occur only by deexcitation of $A$, emission of a photon by $A$, absorption of it by $B$ and excitation of $B$. However, actually to check that there are no photons requires, at least in principle, photon measurement over all space, not only measurements of the states of $A$ and $B$. Hence such an exchange probability cannot be used for signals, it just refers to statistical correlations. Really needed in this model approach to finite signal velocity is the probability of finding $B$ excited, irrespective of the state of $A$ and possible photons; if there turn out to be no photons, all the better. This excitation probability could then, in Fermi’s approach, be determined by a measurement on $B$ alone.

Using “bare” states, as Fermi did, the Hilbert space is simply the tensor product

$$\mathcal{H}_{\text{bare}} = \mathcal{H}_A \times \mathcal{H}_B \times \mathcal{H}_F$$

(1)
and the Hamiltonian is of the form

\[ H_{\text{bare}} = H_A + H_B + H_F + H_{AF} + H_{BF}. \]  

(2)

At time \( t = 0 \), the initial state is

\[ |\psi_0\rangle = |e_A\rangle|g_B\rangle|0_{ph}\rangle. \]  

(3)

At time \( t \), the probability of finding \( B \) in some excited state is then a sum overall excited states \( |e_B\rangle \), over all states \( |i_A\rangle \) of \( A \) and over all photon states \( |\{n\}\rangle \), i.e.

\[
\sum_{e_B} \sum_{i_A} \sum_{\{n\}} |\langle \{n\} | e_B | i_A \rangle |^2 \\
= \langle \psi_t^{\text{bare}} | \{ \sum_{i_A,e_B,\{n\}} |i_A\rangle|e_B\rangle|\{n\}\rangle \langle \{n\} | e_B | i_A \rangle | \psi_t^{\text{bare}} \rangle \\
= \langle \psi_t^{\text{bare}} | 1_A \times \sum_{e_B} |e_B\rangle \langle e_B | \times 1_F | \psi_t^{\text{bare}} \rangle.
\]  

(4)

The r.h.s. is the expectation of the operator

\[ O_{e_B}^{\text{bare}} \equiv 1_A \times \sum_{e_B} |e_B\rangle \langle e_B | \times 1_F. \]  

(5)

This operator represents the observable “\( B \) is in a bare excited state”, and here it is a projection operator.

### 3 Renormalized States

Bare states are widely used in quantum optics and are usually quite adequate. However, for subtle questions of principle of a physical theory great care is needed. Approximations and perturbation theory may give misleading results. In one order an effect might show up, but not in the next order, and so on. Unrenormalized bare theories are, without cut-off, mathematically not well-defined. They are plagued by infinities whose cancellation has not been investigated for signal velocities.

For the present purpose it suffices to use only rudiments of a renormalized theory. We just need the following two simple properties,

(i) existence, with a Hilbert space \( \mathcal{H}_{\text{ren}} \),

(ii) a Hamiltonian, \( H_{\text{ren}} \), which is bounded from below and self-adjoint (“positive energy”).
Then, in general, $\mathcal{H}_{\text{ren}}$ is no longer the tensor product in Eq. (1), and the initial state, denoted by $|\psi_0\rangle$, will not be a simple product state,

$$|\psi_0\rangle \neq |e_A\rangle|g_B\rangle|0_{\text{ph}}\rangle.$$  

Similarly, if the observable

"B is in an excited state" \hspace{1cm} (6)

makes sense and is represented by an operator $O_{e_B}$ then in general

$$O_{e_B} \neq O_{e_B}^{\text{bare}}.$$  

However, its expectation values must lie between 0 and 1 to represent a probability, i.e.,

$$0 \leq O_{e_B} \leq 1.$$ \hspace{1cm} (7)

For example, $O_{e_B}$ might be a projector, as for bare states, but this would not be the most general case. Thus

$$P_{e_B}^B(t) \equiv \langle \psi_t | O_{e_B} | \psi_t \rangle$$

would be the excitation probability of $B$ at time $t$, and it would involve a measurement on $B$ only. It should be noted that the explicit form of the operator $O_{e_B}$ is not required in the following, only Eq. (7) will be used.

Also no point-like localization of $A$ and $B$ are required. Generalizing Fermi’s model, $A$ and $B$ may be systems initially localized in two regions separated by a distance $R$, with no photons present. The ground state of $B$ may be degenerate. Again, with Fermi, one would suppose that one had somehow managed to prepare this initial state at $t = 0$. The analog of Fermi’s original result would then be that the excitation probability $P_{e_B}^B(t)$ of $B$ would vanish for $t \leq R/c$.

In the next section I will prove a simple mathematical theorem which applies to this situation and which yields that either

(i) $P_{e_B}^B(t) \neq 0$ for almost all $t$,  

or

(ii) $P_{e_B}^B(t) \equiv 0$ for all $t$.

This result does not agree with Fermi’s original expectation. In the last section I will show how one can by-pass potential difficulties for finite signal velocity by modifying and clarifying the physical assumptions employed. There is also a mathematical loophole which could be used, although the theorem, of course, remains true.
4 The Theorem

To more clearly separate what is Physics and what Mathematics, I will phrase the theorem in purely mathematical terms although its main and possibly sole interest lies in its applications to the physical situation described above. So what was previously $H_{\text{ren}}$ now becomes any self-adjoint operator $H$ bounded from below, and the initial state $|\psi_0\rangle$ can now be any state, while in the application it represents a physical situation in which $A$ is supposed to be in an excited state, $B$ in a ground state and with no photons.

**Theorem.** Let $H$ be self-adjoint and bounded from below and let $\mathcal{O}$ be any operator satisfying

$$0 \leq \mathcal{O} \leq 1 .$$  \hspace{1cm} (8)

Let $\psi_0$ be any vector and define

$$\psi_t \equiv e^{-iHt/\hbar} \psi_0 .$$

Then one of the following two alternatives hold.

(i) $\langle \psi_t, \mathcal{O} \psi_t \rangle \neq 0$ for almost all $t$, and the set of such $t$’s is dense and open.

(ii) $\langle \psi_t, \mathcal{O} \psi_t \rangle \equiv 0$ for all $t$.

**Proof.** Let us define

$$P(t) = \langle \psi_t, \mathcal{O} \psi_t \rangle .$$  \hspace{1cm} (9)

Since $\psi_t$ is continuous in $t$, so is $P(t)$. From this it follows immediately that the set $\mathcal{N}_0 := \{t; P(t) = 0\}$ is closed and its complement $\mathcal{N}_0^c$ is open. Since $\mathcal{O}$ is a positive operator, its positive square-root $\mathcal{O}^{1/2}$ exists, and one has

$$\langle \psi_t, \mathcal{O} \psi_t \rangle = \langle \mathcal{O}^{1/2} \psi_t, \mathcal{O}^{1/2} \psi_t \rangle .$$

For $t \in \mathcal{N}_0$ this vanishes, and thus

$$\mathcal{O}^{1/2} \psi_t = 0 \quad \text{for } t \in \mathcal{N}_0 .$$ \hspace{1cm} (10)

Now let $\phi$ be any fixed vector and define the auxiliary function $F_\phi(t)$ by

$$F_\phi(t) = \langle \phi, \mathcal{O} e^{-iHt/\hbar} \psi_0 \rangle .$$  \hspace{1cm} (11)

Hence, by Eq. (10),

$$F_\phi(t) = 0 \quad \text{for } t \in \mathcal{N}_0 .$$ \hspace{1cm} (12)

Since $H \geq - \text{const}$, one has that the operator

$$e^{-iH(t+iy)/\hbar}$$
is well-defined for \( y \leq 0 \). Putting
\[
z = t + iy
\]
one sees that \( F_\phi(z) \) can be defined as a continuous function for \( \text{Im} \, z \leq 0 \), and, moreover, \( F_\phi(z) \) is analytic for \( \text{Im} \, z < 0 \).

Let us now assume that (i) does not hold, i.e. that either \( N_0 \) is not a null set or that its complement \( N_0^c \) is not dense. It would suffice to consider the former, but the latter can be treated in an almost elementary way, so I consider it first. If \( N_0^c \) is not dense, \( N_0 \) contains some nontrivial interval, \( I \) say. Hence \( F_\phi(z) \) vanishes on \( I \), by Eq. (12), and one can directly use the Schwarz reflexion principle \[17\] or proceed as follows. One defines an extension of \( F_\phi \) to the upper half plane by putting
\[
F_\phi(z) = F_\phi(z^*)^* \quad \text{for } \text{Im} \, z > 0 . \tag{13}
\]
Since \( F_\phi(t) \) is real for \( t \in I \) it follows that the extension is continuous on \( I \), and from this one can show that it is analytic for \( z \not\in \mathbb{R} \setminus I \). Hence \( I \) is contained in the analyticity domain. Since \( F_\phi(z) = 0 \) for \( z \in I \), it therefore vanishes identically in its domain of analyticity. However, since it is continuous when approaching the real axis, it follows that \( F_\phi(t) = 0 \) for all \( t \). Since \( \phi \) was arbitrary this implies
\[
\mathcal{O}\psi_t = 0 \quad \text{for all } t ,
\]
and this gives case (ii).

This proves the interesting part of the theorem, namely that \( P(t) \) is either nonzero on a dense open set or that it vanishes identically. Since a dense open set need not have full Lebesgue measure this does not yet prove the full theorem. However, as a boundary value of a bounded analytic function, \( F_\phi(t) \) satisfies the inequality \[15\]
\[
\int_{-\infty}^{\infty} dt \ln |F_\phi(t)|/(1 + t^2) > -\infty \tag{14}
\]
unless it vanishes identically. If \( N_0 \) had positive measure the integral would be \(-\infty\), and thus \( F_\phi(t) \) would vanish identically, for each \( \phi \). This would again imply case (ii). Incidentally, this last argument also covers the previous case since, if \( N_0 \) is a null set, its complement is dense. Since the argument based on the Schwarz reflexion principle is very transparent it has been included. This completes the proof of the theorem.

There is a similarity of this result with the Reeh-Schlieder theorem \[19\] which also exploits analyticity but uses stronger assumptions of field theory, in particular locality. It is therefore not directly applicable to the general situation considered here.

Taking for \( \mathcal{O} \) in the theorem the previously considered observable \( \mathcal{O}_{e_B} \) and for \( \psi_0 \) the state \( |\psi_0\rangle \) representing the initial state with \( A \) excited, \( B \) in its ground state and no photons – provided they exist – one obtains from the theorem that the excitation
probability of $B$ is immediately nonzero after $t = 0$ – unless it vanishes for all times, a case one might exclude on physical grounds.

Another application can be made to the correlations mentioned in the Introduction. Let $|\psi_{\text{ex}}\rangle$ denote the state representing $A$ in a ground state, $B$ in an excited state, and no photons, either in $\mathcal{H}_{\text{ren}}$ or $\mathcal{H}_{\text{bare}}$, provided again the notion makes sense in the case of $\mathcal{H}_{\text{ren}}$. In the bare case one just has

$$|\psi_{\text{ex}}\rangle_{\text{bare}} = |g_A\rangle|e_B\rangle|0_{\text{ph}}\rangle.$$ 

We define

$$\mathcal{O}_{\text{ex}} \equiv |\psi_{\text{ex}}\rangle\langle\psi_{\text{ex}}|.$$ 

The expectation value of $\mathcal{O}_{\text{ex}}$,

$$\langle\psi_t|\mathcal{O}_{\text{ex}}|\psi_t\rangle = |\langle\psi_{\text{ex}}|\psi_t\rangle|^2$$

is just the transition probability to $|\psi_{\text{ex}}\rangle$. Since $\mathcal{O}_{\text{ex}}$ is a projector the theorem yields that the transition probability is immediately nonzero, unless it vanishes identically. But since this is just a correlation function of measurements at different positions this result has no bearing on signal velocities.

Further, more general, applications are possible. $A$ and $B$ can be any quantum systems, e.g. a “source” and detector; they may be moving. One may also envisage other particles and interactions. One may apply it also to a problem of Heisenberg who had suggested to consider an excited atom $A$ with no photons and to calculate the probability to find a photon at time $t$ in a region a distance $R$ away. At that time this probability was found to vanish for $t < R/c$ [20].

5 Discussion and Ways out

As already stressed in Ref. [13], the theorem is a mathematically rigorous result, and its applications in Physics depend on the physical assumptions, leading to statements of the form, “If ..., then ...”. For example, I had been careful, when introducing the excitation observable $\mathcal{O}_{\text{ex}}$, to say, “if this makes sense”. If it does exist, then indeed the excitation probability of $B$ is immediately nonzero, or identically zero which one would exclude. Does this mean that atom $B$ has been excited by a superluminal photon emitted by $A$? Not necessarily. Before discussing this we discuss another, more mathematical, way out.

Possible mathematical resolution

An explicit calculation of transition probabilities or other quantities in quantum optics, or quantum electrodynamics or theories involving fields, will in general not
start from a renormalized theory with Hilbert space $\mathcal{H}_{\text{ren}}$ and Hamiltonian $H_{\text{ren}}$ – the form is not known, not even the existence. Instead one will introduce cutoffs in the bare theory to make it well-defined, then calculate transition probabilities, and finally one will remove the cutoffs, taking care of divergent expressions by renormalization. For each cutoff the theorem may be applicable and may yield a nonzero probability for almost all $t$ in $t < R/c$. However, as the cutoffs are tending to infinity, the nonzero probability in this time interval may in principle become smaller and smaller, and in the limit one might conceivably have 0 for $t < R/c$. If this were so, then the mathematical assumptions – existence of $\mathcal{H}_{\text{ren}}$, $H_{\text{ren}}$, and $\mathcal{O}_{\epsilon_B}$ – could not be fulfilled. Such a possibility, nonexistence of a Hilbert space after renormalization, has indeed been discussed in the literature [21].

*Physical ways out*

We now discuss the more or less implicit physical assumptions that have been made, and how to by-pass them.

(a) Systems localized in disjoint regions might not exist as a matter of principle, i.e. systems might always “overlap”.

In ordinary quantum mechanics the wave-function associated with an energy level of a hydrogen atom extends to infinity, and this has been proposed before as a reason for overlapping [22]. But since this happens in a nonrelativistic theory, this particular argument probably does not go to the heart of the matter. Moreover, in nonrelativistic quantum mechanics, there do exist wave-functions of the hydrogen atom which vanish outside some finite volume at a given fixed time. By completeness these wave-function can be obtained by suitable superpositions, but they spread out to infinitely instantaneously [23].

A better argument for overlapping may seem that one might conceivably create particle-antiparticle pairs or other particles whenever one tries to localize a system too well. This has been advanced as an explanation for the difficulties one has in obtaining good localization or position operators [24, 25, 26, 27, 28]. This is connected to the next point.

(b) Renormalization may introduce a sort of photon cloud around each system, e.g. due to ”vacuum fluctuations”. This essentially means an overlapping of the systems with their clouds and leads back to (a). More specifically, photons of one cloud may excite one or the other system, and this might even happen with only one system present.

(c) The notion of a ground state of $B$, either with or without $A$ present, might not
make sense, due to renormalization.

**Strong and weak Einstein causality**

How then to check finite propagation speed? To clarify matters it is useful to differentiate between two notions of Einstein causality.

(i) **Strong Causality:** For each individual process or experiment there is no excitation or disturbance of the second system for $t < R/c$.

This notion is similar to energy – momentum and Baryon conservation in each individual scattering process in particle physics. Strong causality would hold if the transition probabilities considered above were strictly zero for $t < R/c$. It seems to me that both Fermi [1] and Heisenberg - Kikuchi [20] had this in mind when they set out to prove that certain probabilities vanished for $t < R/c$. The above theorem shows that strong causality cannot be checked, unless the way out via cut-off theories holds, or it may fail, a possibility I do not advocate.

(ii) **Weak Causality:** This notion was introduced in Ref. [29], and loosely speaking it means that Einstein causality holds for expectation values only, i.e. for ensembles, not for individual processes. For weak causality to hold, expectation values, i.e. ensemble averages, need not vanish for $t < R/c$, but it takes a time at least $t = R/c$ to produce an effect on them. To exhibit this effect one may suitably subtract possible fluctuations of system $B$ alone, e.g. vacuum fluctuations.

Without additional assumptions to those of the theorem (Hilbert space, positive energy) probably nothing can be said about weak causality in Fermi’s setup. Model calculations [11, 12] point in the right direction, although the renormalization problem remains unsolved. “Bare” theories can sometimes give useful indications on the problem of weak causality, but cannot provide definitive answers. This is all the more true for bare theories with nonrelativistic atoms.

Buchholz and Yngvason [30] use the assumptions of the theory of local observables (“algebraic quantum field theory”), which are stronger than those employed for the above theorem. They point out that in this theory transition probabilities and the above observable $O_{e_B}$ are no legitimate quantities and thus not allowed. The idea is therefore to consider only observables which are allowed in the theory of local observables. These are observables associated with bounded space-time regions; in particular, sharp-time observables are excluded. To give the essential ideas, let $|\psi_{AB}(t)\rangle$ denote the state with systems $A$ and $B$ present, $|\psi_B(t)\rangle$ the state with only $B$ present, and let $\mathcal{B}$ be an observable associated with a space-time region of system $B$. Then weak causality requires

$$
\langle \psi_{AB}(t)|\mathcal{B}|\psi_{AB}(t)\rangle = \langle \psi_B(t)|\mathcal{B}|\psi_B(t)\rangle \quad \text{for } t < R/c ;
$$

i.e. only the difference of both sides is zero for $t < R/c$. With the assumptions inherent in the theory of local observables this does hold [30].
For simplicity let us assume that this could be applied to Fermi’s question about the excitation probability of $B$ (as remarked before, this is not a good observable in algebraic quantum field theory). This would then mean that the excitation probability of $B$ could indeed be nonzero, but until $t = R/c$ it would not depend on the presence of $A$.

What does this mean physically? Expectation values are ensemble averages, and to check this nondependence on $A$ experimentally one could not use a single pair of systems $A$ and $B$. Instead one would need an ensemble of such pairs – either by repetition of the experiment or by simultaneous realization of many, $N$ say, of such pairs at $t = 0$. At time $t$ one would then measure how many of the $B$ systems are excited. Their fraction is

$$P^e_B(\text{with } A)(t) = P^e_B(t)$$

if $N \to \infty$; for finite $N$ this holds only approximately, due to statistical fluctuations. Now one would calculate the excitation probability of $B$ without system $A$ present, which is denoted by $P^e_B(\text{w/o } A)(t)$, and subtract it. Weak causality would then assert

$$P^e_B(\text{with } A)(t) - P^e_B(\text{w/o } A)(t) = 0 \quad \text{for } t < R/c .$$

(15)

Only for $N \to \infty$ would this be strictly true experimentally since for finite $N$ there would always be statistical fluctuations.

Hence finite propagation velocity or speed of light in the sense of weak causality cannot be checked experimentally in a strict sense for finite ensembles since in this case there are always deviations from the exact zero in Eq. (15). What is needed are rigorous bounds on the $N$ dependence of the statistical fluctuations. The theory of local observables does not provide these, and rigorous model studies may be more promising for the question of bounds.

In a strict sense, finite propagation speed as expressed by weak Einstein causality can only be checked experimentally for infinite ensembles, and this may suggest that this notion somehow belongs to a macroscopic context.

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[15] L’Unità, Frankfurter Allgemeine Zeitung, Der Spiegel, to mention a few with a somewhat more reliable coverage.

[16] At least one local paper even had a front page headline: “There are things faster than light”. One can imagine the weird calls I got.

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