Complex amplitude phase motion in Dalitz plot heavy meson three body decay.

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Abstract

We propose a method to determine the phase motion of a complex amplitude in three body heavy meson decays. We show that the phase variation of a complex amplitude can be directly revealed through the interference observed in the Dalitz Plot region where it crosses with a well established resonant state. This method could be applied to the decays $D^+ \to \pi^- \pi^+ \pi^+$ and $D^+ \to K^- \pi^+ \pi^+$, to determine whether the low mass states $\sigma$ and $\kappa$, suggested by E791, have phase motions compatible with resonances.

Keywords: Heavy Meson Decay; Dalitz plot; Scalar Mesons; Resonances.

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Recently the Fermilab experiment E791 with Dalitz plot analysis showed strong evidence for the existence of two light and broad scalar resonances. One resonance is compatible with the isoscalar meson $\sigma$, and is observed in Cabbibo suppressed decay $D^+ \to \pi^- \pi^+ \pi^+$. The other, its strange partner, the meson $\kappa$ seen in the $D^+ \to K^- \pi^+ \pi^+$ decay. These results opened a new experimental window for understanding scalar meson spectroscopy.

In these analyses, each possible resonance amplitude is represented by Breit-Wigner functions multiplied by angular distributions associated with the spin of the resonance. The various contributions are combined in a coherent sum with complex coefficients that are extracted from maximum likelihood fits to the data. The absolute value of the coefficients are related to the relative fraction of each contribution and the phases takes into account final state interaction (FSI) between the resonance and the bachelor pion. To fit E791 data, it was necessary to include extra, not previously observed scalar resonances. For the new states, modeled by Breit-Wigner amplitudes, they measure mass and width. The $D^+ \to \pi^- \pi^+ \pi^+$ decay required a scalar with mass $478^{+24}_{-23} \pm 17$ MeV/$c^2$ and width $324^{+42}_{-40} \pm 21$ MeV/$c^2$. The high statistics sample of $D^+ \to K^- \pi^+ \pi^+$ needed a scalar with mass of $797 \pm 19 \pm 43$ MeV/$c^2$ and width $410 \pm 43 \pm 87$ MeV/$c^2$ for a good confidence level fit. In both cases, the extra contribution is dominant, accounting for approximately half of the decay. Due the importance of these mesons in many areas of particle and nuclear physics, it is desirable to be able to have confirmation with a direct observation of their phase motion, without having to assume a priori the Breit-Wigner phase variation.

Partial wave analyses (PWA) have been used successfully in hadron-hadron scattering to observe the phase motion of complex amplitudes. For example, the LASS experiment did a partial wave analysis of the $K^- \pi^+$ spectrum in $K^- p \to K^- \pi^+ n$ interaction with transverse momentum less than 0.2 (GeV/$c^2$). The low momentum transfer assures first that one $\pi^+$ meson exchange between the incoming $K^-$ and the proton is dominant, and second that there is no important contribution to final state interactions between the $K^- \pi^+$ system and the outgoing neutron. This is a physical situation completely different from charm three body decay, where all phase space is taken into account, including low and high momentum transfer between the resonance state and the bachelor particle. The strength of the resonance-bachelor particle interaction in charm decay is given by the FSI phase. In fact, experimental results show a tendency of large values for the strong phase, thus non negligible FSI.

The method proposed here regards the Dalitz plot just in the region where a well established resonance, represented by a Breit-Wigner, for example, in $s_{12}$ spectrum.

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1. This phase is considered constant because it depends only on the total energy of the system, i.e. the heavy meson mass.
2. In the three body decay $D \to ijk$, $s_{ij}$ and $s_{jk}$ are two of the three possible Dalitz plot variables, that is the square of the invariant mass of the particles $i$ plus $j$ and $j$ plus $k$ respectively.
crosses the complex amplitude under study in the $s_{13}$ variable. In Figure 1 we show a Monte Carlo simulation of a Dalitz plot of two scalar resonances, one the $f_0(980)$ in the variable $s_{12}$ and the other a broad and low mass resonance compatible with the $\sigma$ in $s_{13}$. We show that the interference created in this particular region of the phase space, can reveal the unknown phase variation of the amplitude as a function of $s_{13}$. The method can be applied to retrieve slow variation similar to the one observed in $K\pi$ elastic scattering at low invariant mass $[5]$, as well as rapidly moving ones expected for resonant amplitudes.

For simplicity we suppose that the only contributions we have in the Dalitz plot are the amplitude we want to study in $s_{13}$ and a scalar state represented by a constant width Breit-Wigner in $s_{12}$. This is not a limitation of the method, it being straightforward to include known effects like extra states populating the region as well as angular momentum distributions and varying width Breit-Wigner functions. In this simple case, the total amplitude is given by:

$$A(s_{12}, s_{13}) = a_R BW(s_{12}) + a_s \sin \delta(s_{13}) e^{i(\delta(s_{13}) + \gamma)}$$

(1)

$\gamma$ represents the constant FSI phase difference between the two body scattering amplitude and the bachelor particle. $a_R$ contains all constant or almost constant parameters, like form factors and the magnitude. $a_s e^{i\delta(s_{13})} \sin \delta(s_{13})$ is the most general form to represent a complex amplitude, $a_s$ can be considered constant in the range

Figure 1: Dalitz plot distribution for two crossing scalar mesons, $f_0(980)$ in the variable $s_{12}$ and $\sigma$ in $s_{13}$. The left and right hashed regions correspond to $|A(m_0^2 - \epsilon, s_{13})|^2$ and to $|A(m_0^2 + \epsilon, s_{13})|^2$ respectively.
of the invariant mass region of interest. The relativistic Breit-Wigner function used in many Dalitz plot analyses is given by:

\[ \mathcal{B}W(s_{12}) = \frac{m_0 \Gamma_0}{s_{12} - m_0^2 + im_0 \Gamma_0} \]  

(2)

\( m_0 \) and \( \Gamma_0 \) are mass and the width of the resonance. The square modulus of the above amplitude can be written as:

\[ |\mathcal{A}(s_{12}, s_{13})|^2 = a_R^2 |\mathcal{B}W(s_{12})|^2 + a_s^2 \sin^2 \delta(s_{13}) + \frac{2a_R a_s m_0 \Gamma_0 \sin \delta(s_{13})}{(s_{12} - m_0^2)^2 + m_0^2 \Gamma_0^2} \left( (s_{12} - m_0^2) \cos(\delta(s_{13}) + \gamma) + m_0 \Gamma_0 \sin(\delta(s_{13}) + \gamma) \right) \]  

(3)

In this case, \( |\mathcal{B}W(s_{12})|^2 \) is a symmetric function and thus \( \mathcal{B}W(s_{12} = m_0^2 + \epsilon) \) equals \( \mathcal{B}W(s_{12} = m_0^2 - \epsilon) \), where \( \epsilon \) is a small \( s_{12} \) interval. Consequently the difference of the amplitudes square takes the simple form:

\[ |\mathcal{A}(m_0^2 + \epsilon, s_{13})|^2 - |\mathcal{A}(m_0^2 - \epsilon, s_{13})|^2 = \frac{4a_R a_s \epsilon m_0 \Gamma_0}{\epsilon^2 + m_0^2 \Gamma_0^2} \sin \delta(s_{13}) \cos(\delta(s_{13}) + \gamma). \]  

(4)

\[ |\mathcal{A}(m_0^2 - \epsilon, s_{13})|^2 \] and \( |\mathcal{A}(m_0^2 + \epsilon, s_{13})|^2 \) are taken from data as becomes clear from Figure [1]. We can rewrite Eq. 4 as:

\[ \Delta |\mathcal{A}|^2 = |\mathcal{A}(m_0^2 + \epsilon, s_{13})|^2 - |\mathcal{A}(m_0^2 - \epsilon, s_{13})|^2 = \mathcal{C} \sin(2\delta(s_{13}) + \gamma) - \sin \gamma. \]  

(5)

\( \mathcal{C} \) is equal to \( 4a_R a_s \epsilon m_0 \Gamma_0 / (\epsilon^2 + m_0^2 \Gamma_0^2) \). \( \gamma \) being a constant, \( \Delta |\mathcal{A}|^2 \) directly reflects the behavior of \( \delta(s_{13}) \). A constant \( \Delta |\mathcal{A}|^2 \) would imply constant \( \delta(s_{13}) \), this would be the case of non-resonant contribution. The same way a slow phase motion will produce a slowly varying \( \Delta |\mathcal{A}|^2 \) and a full resonance phase motion produces a clear signature in \( \Delta |\mathcal{A}|^2 \) with the presence of zero, maxim and minin values. Next we discuss how to extract the constant phase \( \gamma \) and \( \delta(s_{13}) \) for two situations. We start with the case of a rapidly moving phase like a resonance where we extract direct information on \( \delta(s_{13}) \). Then we deal with slow phase variation, for which we use as the example the LASS \( K \pi \) low mass phase behavior.

To study a resonant amplitude in the \( \Delta |\mathcal{A}|^2 \) distribution, we produced a fast simulation of 10K events with a hypothetical decay \( D^+ \rightarrow \pi^- \pi^+ \pi^+ \) with only a broad and low mass scalar resonance \( \sigma \) in \( s_{13} \) and the well known resonance \( f_0(980) \) in \( s_{12} \) and strong phase difference between the two of 2.8 rad, Figure [1] shows the relevant portion of the Dalitz plot. To generate the \( f_0(980) \) we used the constant width Breit-Wigner (Eq. 2). For the low mass wide \( \sigma \), the Breit-Wigner parameterization produces undesirable threshold features. We could, in principle, introduce form-factors to avoid this problem, but this would bring unnecessary complexity to the
example. Instead, we parameterized the low mass resonance as $\sin \delta(s_{13}) e^{i \delta(s_{13})}$, like the amplitude we used in Eq. 1, with $\delta(s_{13}) = \pi / (1 + \exp(a(s_{13} - m_0^2)))$, the parameter $a$ is associated with the $\sigma$ width. We used $a = -15$ GeV$^{-2}$. With this choice, we have a complete $\pi$ phase variation around the $\sigma$ mass, as would behave a Breit-Wigner far from the threshold, and the mass plot given by $|\sin \delta(s_{13}) e^{i \delta(s_{13})}|^2$ reproduces the overall characteristics of the $\sigma$ resonance.

The $\Delta |A|^2$ distribution for the fast-MC produced with the above parameters is presented in Figure 2a. We can see a strong variation of this distribution with the presence of three zeros: one at the threshold implicit from Eq. 5, the second at $s_{13} \approx 0.3$ associated with $\delta = 3\pi/2 - \gamma$ and the third at $s_{13} \approx 0.55$ corresponding to $\delta = \pi$. Thus, it is possible to observe in a simple way, a clear signature of the presence of large phase variation associated with the number of zeros in $\Delta |A|^2$.

We can extract directly the phase motion $\delta(s_{13})$ noticing that the maximum and minimum values of $\Delta |A|^2$ corresponds respectively to the $\sin(2\delta(s_{13}) + \gamma) = 1$ and $\sin(2\delta(s_{13}) + \gamma) = -1$ conditions. These and the $\Delta |A|^2$ value, contents of bins 3 and 6 of the plot Figure 2a provide us two equations that can be used to get $\gamma$ and $\mathcal{C}$. Then we invert Equation 5 and get from each value of $\Delta |A|^2$ one value of $\delta(s_{13})$. Figure 2b compares $\delta(s_{13})$ obtained in this way with the functional form (dashed)
used to generate the fast-MC sample. To evaluate error in $\delta(s_{13})$ (Figure 2b), and the ability of this procedure to measure the phase $\gamma$ we repeated the fast-MC experiment 1000 times. The result is shown in Figure 3. We are able to recover the input value (2.8 rad) and the rms of the distribution is consistent with the individual experiment errors.

We have presented an ideal situation in our example, with constant width, and isolated scalar resonances. In a realistic case, the width dependence introduces an asymmetry in $s_{13}$, the same occurs with respect to resonant amplitudes with spin 1 or 2 and even form factors [10]. These effects can be included in Equation 3 in a complete way, since we know very well the functional form of a Breit-Wigner, the width dependence with $s_{12}$, and the spin amplitude. However, the simplified amplitude for $\Delta |\mathcal{A}|^2$ still works if we are willing to sacrifice the measurement of $\gamma$. In fact, we observed that many of these realistic effects are absorbed by the parameter $\gamma$, keeping the $\delta(s_{13})$ distribution almost unchanged if we include or not the realistic effects. Since with this method, one is primarily interested on the direct measurement of the phase motion, and since the usual Dalitz plot method has been the best technique to get the FSI phase, one can use $\Delta |\mathcal{A}|^2$ without much elaboration in most cases.

The other case that we wish to discuss is when the complex amplitude is not
a resonance, but has only a small phase variation. A good example is the scalar smooth phase variation at low mass obtained by the LASS collaboration in $K\pi$ scattering. The LASS phase variation is given by $\cot\delta = 1/d.q + e.q/2$, where $q$ is the momentum in the $K^-\pi^+$ center of mass, $d = 4.03\pm1.72$ GeV$^{-1}$ and $e = 1.29\pm0.89$ GeV$^{-1}$ are fit parameters. We included this amplitude as a function of $s_{13}$ in a hypothetical three body charm decay $D^+ \rightarrow K^-\pi^+\pi^+$ with the well known resonance $K_0(1430)$ in the crossing channel, $s_{12}$. Figure 4a shows the $\Delta |A|^2$ behavior, given by Eq. 5, for several values of $\gamma$ between $0^0$ to $300^0$. While all curves show slow variation of $\Delta |A|^2$, we notice the clear distinction among the $\gamma$ values.

In this case we can not extract directly the phase motion since we have not maxima and minima, as we have for the resonance example. Instead we have to use a functional form to fit data and get the shape. The fit function should be monotonic with a constant asymptotic behavior. We used for the $\delta(s_{13})$ form the same function we used to generated the resonance in the preceding example, that is $\delta(s_{13}) = ph / (1 + \exp(a(s_{13} - t)))$, where $ph$, $a$ and $t$ are free fit parameters. With this simple function we are able to retrieve the $\delta(s_{13})$ behavior and $\gamma$ values. In Figure 4b we show the LASS phase behavior, input to the exercise and the dashed line the functional form we get with our fit function.

![Figure 4](image-url)

Figure 4: (a) $\Delta |A|^2$ distributions for different $\gamma$ values. (b) LASS low mass phase motion used as input (dotted line) and obtained fitting $\Delta |A|^2$ distribution using the functional form of $\delta(s_{13})$ given in text (dashed line).

We have presented here a method to observe phase motion of a complex amplitude in a Dalitz plot of three body decays. We showed that the difference of amplitudes,
around the central mass squared of a well known resonance is a good way to observe this variation. We also showed that one can determine rapid or even slow phase variation with this method. We used as example two scalar resonances, the $\sigma$ and the $f_0(980)$ in $D^+ \rightarrow \pi^- \pi^+ \pi^+$ decay. This is the simplest possible situation, however one could use also the crossing between the $\sigma$ and the $f_2(1270)$ resonance including the well known angular distribution. The advantage of using $f_2(1270)$ in real data is the large statistics in the $D^+ \rightarrow \pi^- \pi^+ \pi^+$ decay, associated with the size of the branching fraction and the effective area of the crossing region. Nevertheless, we believe that both $f_0(980)$ and $f_2(1270)$ could be used to measure the phase motion of the low $\pi^+ \pi^-$ invariant mass region. The analysis of the $\kappa$ resonance in $D^+ \rightarrow K^- \pi^+ \pi^+$ decay, should be slightly more complicated because of the proximity of the vector resonance $K^*(890)$ to the new scalar state. However the $K^*(890)$ would have to be incorporated in Eq. 1.

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References

[1] E791 Collaboration, E.M. Aitala et al., Phys. Rev. Lett. 86, 770 (2001).
[2] E791 Collaboration, E.M. Aitala et al. Phys. Rev. Lett. 89, 121801-1 (2001).
[3] F.E. Close and N. Törnqvist, hep-ph/0204203. J. Phys. : Nucl. Part. Phys. 28G, 249 (2002)
[4] Peter Minkowski and Wolfong Ochs, hep-ph/0209225. To be appear in the proceeding of QCD 2002 Euroconference, Montpellier 2-9 July 2002.
[5] LASS Collaboration, D. Aston et al., Nucl. Phys. 296B , 493 (1988).
[6] E791 Collaboration, E.M. Aitala et al., Phys. Rev. Lett. 86, 765 (2001).
[7] CLEO Collaboration, H. Muramatsu et al., hep-ex/0207067.
[8] E687 Collaboration, P.L. Frabetti et al., Phys. Lett. 331B, 217 (1994).
[9] E687 Collaboration, P.L. Frabetti et al., Phys. Lett. 407B, 79 (1997).
[10] V.V. Anisovich, D.V. Bugg, A.V. Sarantsev, B.S. Zou, Phys. Rev. D50, 1972 (1994).