On the $^{21}\text{Na}(p,\gamma)^{22}\text{Mg}$ thermonuclear rate for $^{22}\text{Na}$ production in novae

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Classical novae are potential sources of $\gamma$-rays, like the 1.275 MeV gamma emission following $^{22}\text{Na}$ beta decay, that could be detected by appropriate instruments on board of future satellites like INTEGRAL. It has been shown that the production of $^{22}\text{Na}$ by novae is affected by the uncertainty on the $^{21}\text{Na}(p,\gamma)^{22}\text{Mg}$ rate and in particular by the unknown partial widths of the $E_x=5.714, J^\pi=2^+$, $^{22}\text{Mg}$ level. To reduce these uncertainties, we performed shell model calculations with the OXBASH code, compared the results with available spectroscopic data and calculated the missing partial widths. Finally, we discuss the influence of these results on the $^{21}\text{Na}(p,\gamma)^{22}\text{Mg}$ reaction rate and $^{22}\text{Na}$ synthesis.

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I. INTRODUCTION

Classical novae are potential sources of $^{22}\text{Na}$ which beta decays ($\tau=3.75$ years) towards the first excited state of $^{22}\text{Ne}$ followed by a prompt gamma ray emission ($E_\gamma=1.275$ MeV) that could be observed. Even though it has not been detected yet, this could happen with higher sensitivity instruments on board of future space missions like INTEGRAL (including a Ge based gamma-ray telescope) or gamma-ray focusing telescope projects. If conditions are favorable (i.e. an oxygen–neon nova at a distance of less than $\approx2$ kpc) the $^{22}\text{Na}$ line could be detected with the INTEGRAL spectrometer. Nova outbursts occur at the surface of an accreting white dwarf within a binary system. The accreted H–rich matter enriched with the C–O or O–Ne matter from the white dwarf undergo a thermonuclear runaway that synthesizes new isotopes. The formation of $^{22}\text{Na}$ (in O–Ne novae) proceeds from initial $^{20}\text{Ne}$ present in large quantities through the two possible paths: $^{20}\text{Ne}(p,\gamma)^{21}\text{Na}(\beta^+)^{21}\text{Ne}(p,\gamma)^{22}\text{Na}$ and $^{20}\text{Ne}(p,\gamma)^{21}\text{Na}(\gamma)^{22}\text{Mg}(\beta^+)^{22}\text{Na}$. The first path has been found to be more favorable to $^{22}\text{Na}$ formation because of its longer time scale. The preferred path is governed by the competition between the $^{21}\text{Na} \beta^+$ decay and the $^{21}\text{Na}(p,\gamma)^{22}\text{Mg}$ reaction whose rate remains uncertain mainly because of the unknown resonance strength associated with the $E_x=5.714, J^\pi=2^+$, $^{22}\text{Mg}$ level.

Estimates of the $^{21}\text{Na}(p,\gamma)^{22}\text{Mg}$ reaction rate have been provided considering the first three levels ($E_x; J^\pi=5.714; 2^+, 5.837; (0–5)$ and 5.965 MeV; $0^+$) above the proton threshold (5.501 MeV). With the exception of the first level, the total widths can be identified with the proton widths, so that $\omega \gamma \approx \omega \Gamma_\gamma$. The resonance strength associated with the first $E_x=5.714$ MeV, $J^\pi=2^+$ level suffers from a significant uncertainty that affects the $^{21}\text{Na}(p,\gamma)^{22}\text{Mg}$ rate in the temperature domain of nova nucleosynthesis. For this level, only the total width is known experimentally ($\Gamma=16.5\pm4.4$ MeV) and to calculate the corresponding resonance strength, estimated proton widths have been used together with the relation $\Gamma_\gamma=\Gamma_\beta \gamma$. The two estimates ($l_\beta=0$ and $\theta_\beta^2=0.01$ or $l_\beta=2$ and $\theta_\beta^2=0.5$) lead to similar values ($\gamma=3.4$ or 3.8 MeV) very close to the maximum value ($\Gamma/4$), obtained when $\Gamma_\gamma=\Gamma_\beta \gamma=\Gamma/2$. However, based on the data available for the $^{22}\text{Ne} E_x=6.120$ MeV analog level, it was argued that the proton width could be much smaller because the radiative width estimated from the analog level is such that $\Gamma_\gamma \approx \Gamma$ and the neutron spectroscopic factor in the analog level should be very small according to experimental data.

Accordingly, values of $\omega \gamma$ = 2.5, 0.25, 0.0 MeV have been adopted for upper ($\Gamma_\gamma=\Gamma_\beta \gamma=\Gamma/2$), recommended (with the usual 0.1 reduction factor) and lower limit for the calculation of $^{22}\text{Na}$ production in novae. This induces a factor of $10^5$ uncertainty on the rate around a temperature of $\approx 10^8$ K, typical of novae, and a factor of up to 3 in the $^{22}\text{Na}$ yields. Another, much less important, source of uncertainty comes from the assumed value for the radiative width of the third ($E_x; J^\pi=5.965$ MeV; $0^+$) level. It is important to reduce this uncertainty on the $^{22}\text{Na}$ yield that directly affects the detectability distance of the 1.275 MeV gamma emission in order to interpret future nova observations. In consequence, we performed shell model calculations of spectroscopic factors and radiative strengths for $^{22}\text{Ne}$ and $^{22}\text{Mg}$ nuclei. In this paper, we first compare calculated values with existing experimental spectroscopic data in order to validate the calculations. We re–analyze existing experimental data to extract supplementary information on missing spectroscopic factors. From this analysis, we derive better estimates for the spectroscopic factor of the 5.714; $2^+$ level and the radiative width of the 5.965 MeV; $0^+$ level. Finally, we discuss the influence of these new values on the $^{21}\text{Na}(p,\gamma)^{22}\text{Mg}$ rate and $^{22}\text{Na}$ production in novae.

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II. SHELL MODEL CALCULATIONS

In order to estimate spectroscopic factors and radiative widths, we have performed shell model calculations using the OXBASH code [4]. Since we are interested only in positive parity states, we used the well-known USD interaction of Wildenthal [3] for the sd shell model space. The results of the calculations in comparison with the experimental spectra of $^{22}\text{Ne}$ and $^{22}\text{Mg}$ are shown in Figs. 1 and Table I. The correspondence of the experimental and theoretical results is remarkably good.

In Table I the calculated neutron spectroscopic factors are compared with the experimental values obtained [13] through the neutron stripping reaction $^{21}\text{Ne}(d,p)^{22}\text{Ne}$. The agreement between calculated and experimental values is very good except for the levels labeled (in ref. [13]) 6.120, 6.350; 6$^+$ and 7.341; 0$^+$. However, these discrepancies can be explained when new spectroscopic data [2] are included (see the following section).

Assuming the equality between spectroscopic factor of conjugate reactions and according to the good agreement between our calculations and experiment we can confidently use them to obtain the $^{22}\text{Mg}$ proton width of the $E_x$; $J^+$= 5.714; 2$^+$ level. The fourth calculated 2$^+$ state at 6.179 MeV corresponds to the fourth 2$^+$ state of $^{22}\text{Ne}$ at 6.120 MeV and to the 2$^+$ state of $^{22}\text{Mg}$ at 5.714 MeV (Fig. 1) which are of main interest here. To check the correctness of the assignment and the proximity of the calculated and physical 2$^+$ state, we compared calculated radiative strength with those available experimentally [13] for the $E_x$=6.120 MeV level in $^{22}\text{Ne}$. There is a fair agreement between the experimental and theoretical values as shown in Table I increasing the confidence in the assignment. The calculated spectroscopic factors for the 2$^+$ state are small as expected from experiments on $^{22}\text{Ne}$ (see Table I). The corresponding proton reduced widths are obtained, using the relation $\Gamma_p = C^2 S \Gamma_{s.p.}$, where the single-particle width $\Gamma_{s.p.}$ has been estimated from the scattering phase shifts in the Woods-Saxon potential with the depth required to reproduce the experimentally known energy of the resonance. The contribution of $l = 2$ transfer to the 5.714 MeV state in $^{22}\text{Mg}$ is negligible as compared to $l = 0$ transfer and we obtain the values of $\Gamma_{0d_{3/2}} = 2 \times 10^{-6}$ eV, $\Gamma_{0d_{5/2}} = 2 \times 10^{-7}$ eV, $\Gamma_{0s_{1/2}} = 4.5 \times 10^{-3}$ eV, that leads to $\Gamma_p = 4.5$ meV.

We also obtained the radiative width of the third level above threshold ($E_x$; $J^+$ = 5.965 MeV; 0$^+$) by calculating $B(E2)$ or $B(M1)$ for the transition to the lower lying 2$^+$ and 1$^+$ levels (Table III). The calculated value, $\Gamma_{\gamma} = 33.6$ meV leads to a resonance strength of $\omega \gamma = 4.2$ meV, only slightly higher than the estimated value [11] of 2.5 meV.
III. REANALYSIS OF EXPERIMENTAL DATA

We performed a DWBA analysis of some of Neogy et al. \[13\] data (6.120, 6.350 and 7.341 levels) to extend the comparison between calculated and experimental values and to put more constraint on the 6.120 MeV $^{22}\text{Ne}$ (or $^{5.714}\text{MeV}\; ^{22}\text{Mg}$) level spectroscopic factor. For this purpose, we used the ECIS code \[17\] with the same optical potential parameters as Neogy et al.

The 6.350 MeV level (Fig. 5 of Neogy et al. \[13\]) was assumed to have a $6^+$ spin and parity and accordingly ($l = 4$) no DWBA analysis was performed at that time. However, since then \[12\], a $6^+$ level has been located at $6.311\;\text{MeV}$ and a $4^+$ level at $6.345\;\text{MeV}$. Hence, it is almost certain that the 6.350 MeV level in ref. \[13\] was unduly identified with the $6^+$ instead of the $4^+$ level. Accordingly, we made a DWBA analysis of the Neogy et al. data assuming that the reported level is a $4^+$ and extracted a spectroscopic factor. This new value is close (within a factor of two) to the calculated, $4^+_3$ level, value (see Table I). Hence, shell model calculations reinforce the idea that the 6.35 MeV level seen by Neogy et al. \[13\] is the 6.345; $4^+$ one instead of the 6.311; $6^+$.

Our shell model calculations lead to a very small spectroscopic factor for the 7.341; $0^+$ level in complete contradiction with the value reported by Neogy et al. However, less than three keV above ($E_x = 7.344\;\text{MeV}$) lies a $J^\pi = (3,4)^+$ level whose calculated spectroscopic factors (for the $3^+_1$ and $4^+_1$ states) agree much better with those extracted from experimental data (Table I). Hence, the experimentally determined spectroscopic factor could be attributed to the 7.344 MeV; $(3,4)^+$ level rather than to the 7.341; $0^+$ one.

In order to put more constraint on the 6.120 MeV $^{22}\text{Ne}$ level we also performed a DWBA analysis of the Neogy et al. data for this level. (This analysis was not performed in the original work \[13\] because “the angular distribution does not exhibit characteristics of direct reactions\[8\].) The experimental data and DWBA cross sections are represented in Fig. \[2\]. As expected, the calculated transfer cross sections are more forward peaked than the experimental angular distribution suggesting a strong contribution from fusion reactions. When the theoretical spectroscopic factors are used, the $d_{5/2}$ contribution is negligible while the $s_{1/2}$ contribution is compatible with experimental data except for the most forward angle. Requesting that the DWBA cross sections remain below all experimental data points lead to upper limits for the spectroscopic factors of $C^2S \lesssim 0.0025$ or 0.015 for $l = 0$ or 2 respectively. While the $l = 2$ upper limit is fully compatible with shell model calculations, the $l = 0$ one is a factor of four below the calculated spectroscopic factor. From this upper limit, we obtain $\Gamma_p \lesssim 1\;\text{MeV}$. One can note however that this limit should be taken with caution as it comes from a single data point at the smallest angle. As it can be seen in Fig. 1 of Neogy et al., the 6.120 MeV, $^{22}\text{Ne}$ peak is close to an other one from $^{23}\text{Ne}$ (arising from a (d,p) reaction on $^{22}\text{Ne}$ in the target.) Resolving these two peaks at a lower angle should be more difficult because of the unfavorable evolutions of both their energy separation and relative heights. Hence one cannot exclude that the experimental error bars were underestimated in this case.

IV. IMPLICATIONS ON THE $^{21}\text{Na(p,}\gamma)^{22}\text{Mg}$ RATE

The contribution of the $E_x$; $J^\pi = 5.714; 2^+$ level to the $^{21}\text{Na(p,}\gamma)^{22}\text{Mg}$ rate depends directly from its adopted proton width. From experimental data \[8\] we deduced an approximate upper limit (see comment above) for this width ($\Gamma_p \approx 1\;\text{MeV}$). Our shell model calculations give a slightly higher value ($\Gamma_p = 4.5\;\text{meV}$). Using the experimental total width ($\Gamma = 16.5\;\text{meV}$), one obtains $\gamma \approx 3.2$ or 1 meV, and $\omega\gamma \approx 2.$ or 0.6 meV, when using the calculated or experimental upper limit for the proton widths. These values are close to the first $(3.4\;\text{meV})$ and $3.8\;\text{meV}$ \[10\] and more recent estimated strengths \(2.5\;\text{upper and} 0.25\;\text{meV nominal values} [11]\). One important conclusion resulting from the shell model calculations is that is it most unlikely that the spectroscopic factor is much smaller than one meV. This would exclude the lower limit for the rate \[9\] obtained with a null spectroscopic factor. The contribution of the 5.965 MeV; $0^+$ level is obtained through our shell model calculation of its radiative width ($\Gamma_{\gamma} = 33.6\;\text{meV}$ and $\omega\gamma = 4.2\;\text{meV}$). As in previous works \[8\] we assume that the 5.837 MeV; $(0-5)$ level, is the analog of the 5.910; $3^-$ level in $^{22}\text{Ne}$. This assignment made by \[1\], is not present anymore in \[13\] but is likely
from the examination of Fig. 2 and because of their similar gamma decay modes [16]. Accordingly, we take \( \Gamma = 13 \text{ meV} \) from the analog \(^{22}\text{Ne}\) level and \( \omega_\gamma = 11.4 \text{ meV} \). The direct capture term [9, 8] is also left unchanged as it is based on experimental spectroscopic factors.

Following the good general agreement between calculated and experimental quantities, we derive the \(^{21}\text{Na}(p, \gamma)^{22}\text{Mg}\) rate using the shell model calculated values presented above. The resonant part of the rate is approximated as usual by \( \sum A_i \exp (-B_i/T) \) with \( A_i = 334., 1862., 686. \) for \( B_i = 2.52, 3.95, 5.49 \) respectively. The resulting rate is very close to the previous rates [9, 10, 8] and of the recent upper rate limit [7] but it is now put on a safer ground, in the domain of nova nucleosynthesis, as it now relies on shell model calculations rather than estimates. Using the experimental upper limit instead of the calculated one for the first resonance strength would lead to a rate lying within a factor of three of the rates from refs. [9, 10, 8] and from the nominal and upper rate limit from ref. [7]. (The proximity of these different rates is due to the strong experimental constraint introduced by \( \Gamma = \Gamma_p + \Gamma_\gamma = 16.5 \pm 4.4 \text{ meV} \) [11].)

In conclusion, the lower limit for the \(^{21}\text{Na}(p, \gamma)^{22}\text{Mg}\) rate, used for calculating \(^{22}\text{Na}\) yields in novae seems now excluded. Unfortunately it was also the more efficient for \(^{22}\text{Na}\) production through the \(^{21}\text{Na}(\beta^+)^{21}\text{Ne}(p, \gamma)^{22}\text{Na}\) chain. Hence the highest \(^{22}\text{Na}\) yields reported [8] are now chain. A precise conclusion on gamma emission detectability will require further hydrodynamical calculations of nova outbursts. However, the \(^{21}\text{Na}(p, \gamma)^{22}\text{Mg}\) rate resulting from this analysis is not too far from the nominal rate used in previous calculations [8] so that the nominal detectability distance of \(^{22}\text{Na}\) gamma emission [8] should not be too much affected.

V. ACKNOWLEDGMENTS

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*[1] D.D. Clayton, & F. Hoyle, Astrophys. J., 187, L101 (1974).
[2] A.F. Iyudin, et al., Astron. Astrophys. 300, 422 (1995).
[3] G. Vedrenne et al., Proceedings of the 3rd INTEGRAL Workshop, Astrophysical Letters and Communications, 39, 325 (1999).
TABLE I. Experimental, positive parity, energy levels of $^{22}\text{Mg}$ and $^{22}\text{Ne}$ and spectroscopic factors deduced from the $^{21}\text{Ne}(d,p)^{22}\text{Ne}$ reaction compared with calculated ($T = 1$, $A = 22$) values.

| $\text{J}^\pi$ | $E_x$ (MeV) | $\text{J}^\pi$ | $E_x$ (MeV) | $(2J + 1)S$ | $l$ | $\text{J}^\pi$ | $E_x$ (MeV) | $(2J + 1)S$ |
|----------------|-------------|----------------|-------------|-------------|-----|----------------|-------------|-------------|
| $0^+$          | 0.0         | $0^+$          | 0.0         | $\leq 0.20$ | 2   | $0^+_1$        | 0.0         | $0.13$      |
| $2^+$          | 1.246       | $2^+$          | 1.275       | 3.25        | 2   | $2^+_1$        | 1.368       | 4.9         |
| $4^+(2^+)$     | 3.308       | $4^+$          | 3.358       | 0.44        | 2   | $4^+_2$        | 3.378       | 0.29        |
| $2^+(1^+)$     | 4.401       | $2^+$          | 4.456       | 0.27        | 0   | $2^+_2$        | 4.455       | 0.30        |
|                |             |                | 0.72        |             | 2   |                |             | 0.95        |
| $(0 - 4)^+$    | 5.006       |                |             |             |     |                |             |             |
| $(1,2)$        | 5.317       | $1^+$          | 5.329       | 0.15        | 0   | $1^+_1$        | 5.437       | 0.15        |
| $(2,3,4)$      | 5.464       | $2^+$          | 5.363       | 1.56        | 0   | $2^+_1$        | 5.032       | 1.22        |
| $(2^+,3)$      | 5.293       | $3^+$          | 5.641       | 0.49        | 2   | $3^+_1$        | 5.635       | 1.16        |
| $(0-5)$        | 5.837       |                |             |             |     |                |             |             |
| $2^+$          | 5.714       | $2^+$          | 6.120       | $\leq 0.012$ | 2   | $2^+_1$        | 6.179       | 0.05        |
|                |             |                |             | $\leq 0.07$ |     |                |             | 0.06        |
| $0^+$          | 5.965       | $0^+$          | 6.235       |             | 2   | $0^+_2$        | 6.344       | 0.05        |
| $4^+$          | 6.267       | $6^+$          | 6.311       |             | 6   | $6^+_1$        | 6.396       |             |
| $(2,3)^+$      | 6.336       | $2^+$          | 6.345       | $\approx 0.5\,^d,e$ | 2   | $2^+_1$        | 6.430       | 1.0         |
| $2^+$          | 6.819       | $1^+$          | 6.854       | 1.65        | 0   | $1^+_1$        | 6.663       | 1.65        |
| $0^+$          | 7.341       | $(0.35)\,^f$  | 7.344       | $\approx 0.48\,^{d,f}$ | 2   | $0^+_2$        | 7.264       | 0.007       |
| $(3,4)^+$      | 7.344       |                |             | $\approx 0.46\,^{d,f}$ |     |                |             |             |
| $(3,5)^+$      | 7.423       |                |             |             |     |                |             |             |
| $2^+$          | 7.644       |                |             |             |     |                |             |             |
|                |             |                |             |             |     |                |             |             |

$^a$ $E_x$ and $J^\pi$ from Ref. $^1$

$^b$ $E_x$ and $J^\pi$ from Ref. $^2$

$^c$ $(2J + 1)S$ from Ref. $^3$, unless otherwise stated.

$^d$ Our analysis of Neogy et al. data $^3$.

$^e$ Assuming that the 6.35; $6^+$ $^{22}\text{Ne}$ data in Neogy et al. $^3$ corresponds to the 6.345; $4^+$ level.

$^f$ Assuming that the 7.341; $0^+$ $^{22}\text{Ne}$ data in Neogy et al. $^3$ corresponds to the 7.344; $(3,4)^+$ level.

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TABLE II. Experimental and theoretical reduced transition probabilities from 6.120 MeV $2^+$ state $^{22}\text{Ne}$.

| Transition | Experimental $^a$ | Theoretical |
|------------|-------------------|-------------|
| $2^+ \rightarrow 0^+_1$ | $B(E2) = (1.91 \pm 1.11) \, e^2\text{fm}^4$ | $B(E2) = 3.1 \, e^2\text{fm}^4$ |
| $2^+ \rightarrow 2^+_1$ | $B(E2) = (1.15 \pm 1.15) \, e^2\text{fm}^4$ | $B(E2) = 1.8 \, e^2\text{fm}^4$ |
| $2^+ \rightarrow 2^+_2$ | $B(M1) = (0.046 \pm 0.023) \, \mu_N^2$ | $B(M1) = 0.138 \, \mu_N^2$ |

$^a$ From Ref. $^4$
TABLE III. Theoretical reduced transition probabilites from $0^+_2$ state of which is assumed to correspond to the 5.965 MeV $0^+$ level in $^{22}$Mg.

| Transition          | Transition rates |
|---------------------|------------------|
| $0^+_2 \rightarrow 2^+_1$ | $B(E2) = 16.92$ e²fm² |
| $0^+_2 \rightarrow 2^+_2$ | $B(E2) = 8.0$ e²fm² |
| $0^+_2 \rightarrow 2^+_3$ | $B(E2) = 6.2$ e²fm² |
| $0^+_2 \rightarrow 2^+_4$ | $B(E2) = 0.01$ e²fm² |
| $0^+_2 \rightarrow 1^+_1$ | $B(M1) = 0.57 \, \mu^2_N$ |