THE STRUCTURE, DYNAMICS, AND STAR FORMATION RATE OF THE ORION NEBULA CLUSTER

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Received 2014 June 23; accepted 2014 September 6; published 2014 October 13

ABSTRACT

The spatial morphology and dynamical status of a young, still-forming stellar cluster provide valuable clues to the conditions during the star formation event and the processes that regulated it. We analyze the Orion Nebula Cluster (ONC), utilizing the latest censuses of its stellar content and membership estimates over a large wavelength range. We determine the center of mass of the ONC and study the radial dependence of angular substructure. The core appears rounder and smoother than the outskirts, which is consistent with a higher degree of dynamical processing. At larger distances, the departure from circular symmetry is mostly driven by the elongation of the system, with very little additional substructure, indicating a somewhat evolved spatial morphology or an expanding halo. We determine the mass density profile of the cluster, which is well fitted by a power law that is slightly steeper than a singular isothermal sphere. Together with the interstellar medium density, which is estimated from average stellar extinction, the mass content of the ONC is insufficient by a factor \( \sim 1.8 \) to reproduce the observed velocity dispersion from virialized motions, in agreement with previous assessments that the ONC is moderately supervirial. This may indicate recent gas dispersal. Based on the latest estimates for the age spread in the system and our density profiles, we find that at the half-mass radius, 90% of the stellar population formed within \( \sim 5–8 \) free-fall times (\( t_{\text{ff}} \)). This implies a star formation efficiency per \( t_{\text{ff}} \) of \( \epsilon_{\text{ff}} \sim 0.04–0.07 \) (i.e., relatively slow and inefficient star formation rates during star cluster formation).

Key words: open clusters and associations: individual (Orion Nebula Cluster) – stars: formation – stars: kinematics and dynamics – stars: pre-main sequence

Online-only material: color figures

1. INTRODUCTION

The majority of stars, perhaps including our Sun, have their origin in clusters (Lada & Lada 2003; Gutermuth et al. 2009). Thus understanding the formation of star clusters is important both for their role as the basic building blocks of galactic stellar populations and for being the birth environments of most planetary systems.

In spite of this importance, some basic questions about star cluster formation are still debated, including whether it is a dynamically fast (Elmegreen 2000, 2007; Hartmann & Burkert 2007) or slow (Tan et al. 2006, hereafter TKM06; Nakamura & Li 2007, 2014) process—in essence, is the duration of star cluster formation similar to a dynamical time (similar to the free-fall time) of the natal gas clump or is it much longer? The latter scenario would be consistent with some theoretical expectations of relatively low star formation efficiency per free-fall time in turbulent and/or magnetized gas (Krumholz & McKee 2005; Padoan & Nordlund 2011) and with turbulence being maintained by self-regulated protostellar outflow feedback (Nakamura & Li 2007, 2014).

As discussed by TKM06, there are several ways to try to distinguish between these scenarios, including considering the morphologies of gas clumps or embedded stars, assessing the momentum flux of protostellar outflows, and looking at the age spreads of pre-main-sequence stars and the ages of dynamical ejection events. By considering these factors, especially in the context of the relatively nearby (414 pc; Menten et al. 2007) and massive (approximately few thousand \( M_\odot \); Hillenbrand & Hartmann 1998) Orion Nebula Cluster (ONC), TKM06 concluded that the duration of star cluster formation—as defined by \( t_{\text{form,90}} \), the time to form 90% of a cluster’s stars—was \( \geq 4 t_{\text{dyn}} \sim 8 t_{\text{ff}} \), where \( t_{\text{dyn}} = R/\sigma \), \( R \) being the local radius, \( \sigma \) being the one-dimensional velocity dispersion, and the bar indicating that this is the mass-weighted average of \( t_{\text{dyn}} \) over the region containing the stars that count toward \( t_{\text{form,90}} \). The free-fall time is defined via \( t_{\text{ff}} = \left[ \frac{3\pi}{32G\rho} \right]^{1/2} \), where \( \rho \) is the volume density, and for a virialized cloud with \( \alpha_{\text{vir}} \equiv 5\sigma^2 R/(GM) \sim 1 \) (Bertoldi & McKee 1992), we have \( t_{\text{ff}} \sim 0.5 t_{\text{dyn}} \). For the ONC, TKM06 adopted \( M = 4600 M_\odot \), \( \Sigma = M/(\pi R^2) = 0.12 \, \text{g cm}^{-2} \), \( R = 1.60 \, \text{pc} \), so that \( t_{\text{dyn}} \sim 7 \times 10^5 \, \text{yr} \) and \( t_{\text{ff}} \sim 3.5 \times 10^5 \, \text{yr} \). They assumed that star formation, which is still ongoing, has a duration \( t_{\text{form,90}} \geq 3 \, \text{Myr} \).

The estimate of \( t_{\text{form,90}} \) in the ONC is measured most directly via age spreads of pre-main-sequence stars, as revealed by spreads of luminosity in the HR diagram in comparison with stellar evolutionary models. However, other factors can also lead to this luminosity spread, including the difficulty in assigning stellar parameters to individual stars (Da Rio et al. 2010a; Reggiani et al. 2011) and episodic protostellar accretion (Baraffe et al. 2009, 2012; Hosokawa et al. 2011). Da Rio et al. (2014) examined the problem of age spread in the ONC and concluded from independent constraints that there is an intrinsic age spread of \( \sim 1.34 \, \text{Myr} \) as defined by \( 1\sigma \) dispersion in ages. Assuming a log-normal age distribution or a constant star formation rate in time, this leads to \( t_{\text{form,90}} \geq 4.1, 4.2 \, \text{Myr} \), respectively.

In this paper, we revisit questions of the timescale of star cluster formation as exemplified by the ONC. First, we consider the spatial structure of stars in the cluster to investigate the TKM06 assertion of progressively smoother stellar distributions (smaller amounts of substructure) as a cluster ages. We do this by examining angular substructure in annuli as a function of radius (adapting methods used by Gutermuth et al. 2005). The theoretical expectation is that the center of the cluster, being...
dynamically older because of its shorter local dynamical time but similar age spread (Da Rio et al. 2012), has a smoother distribution of stars because any initial substructure has had more dynamical timescales to be erased. This analysis first requires a careful assessment of the location of the center of mass of the ONC, and then an analysis of radial variation of angular substructure. This is presented in Section 3.

Then, in Section 4, utilizing the latest mass and age estimates of the stars in the ONC and allowing for contributions of gas to the total mass, we examine the mass density profile of the ONC. Using literature measurements for the velocity dispersion we refine previous assessments concerning the ONC dynamical equilibrium. In Section 5, we compute the ratio of \( t_{\text{ff}} \) with \( t_{\text{ff}} \) as a function of radius in the cluster. Together with the latest estimate of the age spread in the system, this allows us to measure the star formation efficiency per free-fall time, \( \epsilon_{\text{ff}} = 0.9 \epsilon_{\text{eff}} t_{\text{ff}} / t_{\text{form,90}} \), where \( \epsilon_{\text{eff}} \equiv M_*/M_{\text{tot}} = \rho_*/\rho_{\text{tot}} \). We are able to measure \( \epsilon_{\text{ff}} \) both locally as a function of projected radius (making the simplifying approximation that the projected radius is three-dimensional radius) and as an average interior to a given radius.

2. THE STELLAR CATALOGS

We assemble catalogs of stellar positions and properties in the ONC from the literature.

We first compiled a sample of all sources with available stellar parameters, including spectroscopically determined \( T_{\text{eff}} \) and \( \log L \), as well as (model dependent) ages and masses. In this context, the H-R diagram of Da Rio et al. (2012) represents the latest update; it is obtained by combining spectral types from either spectroscopy or narrow-band photometry with optical multi-band photometry to measure the reddening toward each source and calculate bolometric luminosities. This sample covers a field of view of about \( \sim 40' \times 40' \) on the Orion Nebula centered southwest of the Trapezium, and is nearly complete for \( A_V < 5 \) mag down to the H-burning limit, while also extending into the substellar regime. Sources flagged as probable non-member contaminants in Da Rio et al. (2012) have been excluded from the catalog. We further extended this catalog by adding new spectral types from Hillenbrand et al. (2013); for these stars, the extinction, \( A_V \), and thus \( \log L \), has been assigned using \( BVI \) photometry from Da Rio et al. (2010b) and adopting the same analysis technique as in that work. Also, because Da Rio et al. (2012) was incomplete toward the massive end of the population due to saturation of their photometry, we added all missing sources by adopting the stellar parameters from the Hillenbrand (1997) catalog. Finally, the masses of the Trapezium stars have been adjusted to account for their multiplicity, using estimates of masses for each multiple system from Grellmann et al. (2013).

The final catalog of optically determined stellar parameters contains 1597 sources.

We also constructed a catalog of near-infrared (NIR) photometry. We based this on the \( JHK \) catalog from Robberto et al. (2010), which covers a region slightly larger than that of our optical photometry and has a very deep detection limit (3\( \sigma \) detection at \( J = 19.5 \) mag, \( H = 18 \) mag), reaching down to planetary masses, and nearly complete for \( A_V < 20 \) mag for stellar objects. We complement this sample with the saturated bright end using data from the Two Micron All Sky Survey (2MASS; Skrutskie et al. 2006). We also collected the mid-infrared Spitzer survey in Orion from Megeath et al. (2012). This catalog provides stellar fluxes from the \( J \) band to 24 \( \mu \)m, and includes the classification of sources showing infrared excess from dusty stellar surroundings (either disks or protostellar objects). This was based on the multi-wavelength, near- and mid-infrared criteria described in Gutermuth et al. (2009).

Finally, we used the X-ray source catalog of Getman et al. (2005b) from the Chandra Orion Ultradeep Project (COUP), and rejected sources flagged by Getman et al. (2005a) as non-members of the Orion population (nebular shocks, extragalactic sources, or unconfirmed members).

All these catalogs have been cross-matched and the resulting data set limited to the sky area covered by the optical data. The X-ray sample has a smaller coverage than the other data sets, and is complete up to \( \sim 0.11 \) from the Trapezium; all the other catalogs extend to a radius of \( \sim 0.2 \) at any position angle. This corresponds to a projected distance of \( \sim 1.4 \) pc from the center, assuming a distance for the ONC of 414 pc (Menten et al. 2007).

Despite the richness of this data set, it is difficult to precisely isolate the ONC population, as these samples suffer from some combination of incompleteness or contamination from nonmembers. The optical catalog of stellar parameters is naturally limited by dust extinction, which is also not spatially uniform. The NIR photometry is nearly complete at high extinctions, lacking only a minor fraction of young members in the heavily embedded OMC-1 cloud, as well as protostellar objects in the vicinity of the Kleinmann-Low (KL) nebula; however, this sample suffers significant contamination from Galactic field populations, which increase toward low stellar luminosities. The X-ray sample is not limited significantly by extinction, but suffers incompleteness at low stellar masses (\( M < 0.2 M_\odot \)) and in the Trapezium cluster due confusion from the broad tails of the point-spread function of the bright objects. Last, the Spitzer survey also suffers from incompleteness in the detection at low masses, and confusion in the core due to the lower angular resolution compared to the other samples.

In the following sections, we assume the combination of the optical parameter and the X-ray sample as representative of the spatial structure of the ONC. This joint sample, although somewhat incomplete, is virtually immune from field contamination and not biased by patchy extinction. The remaining sources in the IR samples will be considered when assessing the total stellar mass and its radial density profile.

3. THE STRUCTURE OF THE ONC

3.1. The Center of the ONC

It is well established that the massive stars of the Trapezium cluster lie roughly at the center of the stellar population of the ONC; here, we aim to better characterize the actual position of the center of mass of the population. Hillenbrand & Hartmann (1998), based on ellipse fitting on isophotes on an optical-NIR stellar density map, found the center of the stellar positions to be located \( \sim 25'' \) north of \( \theta^1\text{C} \) and slightly to the west, which is just outside the Trapezium cluster. Feigelson et al. (2005), based on the COUP X-ray sample, noted that the heavily embedded sources (log \( N_\text{H} > 22 \text{ cm}^{-2} \), corresponding to \( A_V \gtrsim 6 \text{ mag} \)) are systematically offset to the northwest of the Trapezium, compared to the lightly obscured members. This, however, may be in part due to the spatial variations of extinction, which are present in the ONC at all column densities.

We have considered the merged catalog of available optical stellar parameters with X-ray members to evaluate the position of the center of mass of the ONC. The collected census counts 2228 members in total, or 1901 sources within the square area of size 0.4 centered on \( \theta^1\text{C} \) common to all catalogs, to which
we restrict our analysis. About 2/3 of the stars in this sample have a mass estimate from optical studies; the remaining are X-ray members with no mass estimate. We assign to these sources a mass $M = 0.5\, M_\odot$ (i.e., the mean mass of the Kroupa (2001) initial mass function). In fact, as mentioned, the X-ray sample is incomplete below $0.2\, M_\odot$, so its mean stellar mass should be higher; on the other hand all the massive stars in the ONC have optical parameters, which lowers the mean mass of the X-ray sample with no available mass estimate. The stellar positions included in our final catalog of optical and X-ray sources are shown in Figure 1, left and center panels; the circle of radius 0.11 in the center panel delimits the maximum aperture fully contained in the X-ray field of view.

The center of the ONC has been computed in an iterative way, from large to small scales. First, we considered the largest circular aperture contained in the whole area shown in Figure 1 and determined its center of mass. This was then used as a center of a slightly smaller circular aperture, where the sample was limited to, in order to re-derive its center of mass. The procedure was iterated to progressively smaller circles, down to $\sim 0.6$, each time using as the aperture center the center of mass of the previous one. Figure 1, right panel, shows the displacement of the center of mass from largest area to the X-ray complete aperture and then to the smallest aperture. We find that at large apertures the center of mass is displaced $\sim 30''$ (0.05 pc) north of the $\theta^1$C and outside the Trapezium, which is in agreement with previous works. When we reduce the aperture to sample the center at smaller scales, this progressively moves inside the Trapezium, indicating some degree of asymmetry at different scales. We consider the latter center, located at $\alpha_{2000} = 05^h 35^m 16^s 26; \delta_{2000} = -05^\circ 23' 16'' 4$, as our bona-fide center of the ONC.

We tested how this result is sensitive to the assumed value of stellar mass for uncharacterized X-ray sources, and found no displacement (less than 1$'$) if this value is lowered to $M_{X\text{-ray}} = 0.3\, M_\odot$.

Finally, we note that the local ONC center of mass lies close to the point where, tracking proper motions in reverse as proposed by Chatterjee & Tan (2012), $\theta^1$C and the Becklin-Neugebauer (BN) object were co-located $\sim 4500$ yr ago. It is proposed that a strong gravitational slingshot interaction ejected BN into the molecular cloud at $\sim 30$ km s$^{-1}$ with $\theta^1$C recoiling in the opposite direction to its current location. Thus, we have recalculated the ONC center of mass, both removing $\theta^1$C from the sample and displacing its position to the point of the past interaction. In both cases, as shown in Figure 1, our calculated center of mass moves even closer to the interaction point. This qualitative argument could strengthen the hypothesis of Chatterjee & Tan (2012), because the system of $\theta^1$C, which is the most massive star in the cluster, would tend to settle in the very center of the cluster via gentle interactions with other ONC members. Its current displacement from the ONC center of mass is then explained as a result of its strong interaction with BN. If true, this scenario has the potential to place constraints on the formation time and radial displacement from the cluster center of the formation site of $\theta^1$C.

3.2. Displacement of the ONC Center

We now study how the center of the ONC, and its variations upon aperture scale, depends on the sample of sources we have adopted. First, we aim to test whether the incompleteness of our optical and X-ray sample could bias our derived ONC center; second, we look for systematic displacements of the center of the population as a function of dust extinction, which provides some indication of the distribution of stars and gas along the line of sight. Hillenbrand & Hartmann (1998) found no significant variations of the spatial distribution of sources observed in the optical and near infrared. Using X-ray derived extinctions, which reach higher column densities than near infrared photometry, Feigelson et al. (2005), however, found the embedded population to be more concentrated to the east of the Trapezium cluster, with over-concentrations around the BN/KL region and the OMC-1S region. Feigelson et al. (2005), however, set the division between the lightly and heavily obscured samples at log $N_{H_1} = 22$ cm$^{-2}$. This corresponds to $A_V \sim 6$ mag, a value low enough to be sensitive to the non-uniformity of the average extinction of the ONC (see, e.g., the extinction maps from Scandariato et al. 2011), rather than a value separating highly embedded members, which cannot be detected in the optical or NIR.
We first considered the X-ray sample alone and converted the column densities log \( N_H \) derived from the X-ray spectral analysis of Getman et al. (2005b) into dust extinction \( A_V \), assuming the relation \( N_H/A_V = 1.58 \times 10^{21} \text{ cm}^{-2} \) (Vuong et al. 2003). We then separated the sample into two subsamples with \( A_V < 20 \text{ mag} \) and \( >20 \text{ mag} \), and computed the center. This was performed as in Section 3.1. Figure 2 (left panel) shows that the center of the ONC remains confined inside the Trapezium when considering the sample with up to 20 mag of extinction, which corresponds to \( \sim 85\% \) of the population. The remaining small fraction of heavily embedded sources remains spatially centered \( \lesssim 0.1 \text{ pc} \) to the northwest of the Trapezium in the direction of the BN/KL region. This is not surprising as the KL region is associated to the densest molecular core within the OMC-1 filament (Johnstone & Bally 1999; Grosso et al. 2005).

Next, we computed the center of the ONC adopting the NIR photometric sample. We separately considered the entire sample and a sample restricted to sources brighter than the reddening vector in the \( J \) versus \( J-H \) diagram corresponding to a mass of \( 0.075 \, M_\odot \). Robberto et al. (2010) has shown that below this locus in the NIR CMD, roughly corresponding to the substellar mass range, the vast majority of sources are faint field contaminants rather than young brown dwarfs. Figure 2 (middle panel) shows that removing this faint population of probable contaminants has little effect on the derived center of the ONC. For the X-ray sample, we divide the NIR catalog according to extinction. This was roughly computed by deredding the \( J \) versus \( J-H \) CMD on a 2.5 Myr Siess et al. (2000) isochrone. Again, the center of the lightly embedded population is confined within the Trapezium cluster, but this moves north at higher \( A_V \) at a higher declination than for the X-ray embedded population. This result has two origins: first, the NIR sample is severely limited by dust extinction in the densest BN-KL and OMC-1S regions whose combined embedded populations (dominated by OMC-1S) would thus shift the center of the obscured population to the southwest; second, at high extinction, the NIR catalog from Robberto et al. (2010) has a spatially variable completeness, and all sources with \( A_V > 15 \) are located on a stripe around \( \delta = -5^\circ18' \), which coincides with the overlapping area between adjacent exposures of the imaging mosaic, and thus has higher effective photometric depth. Such a feature in the spatial completeness therefore biases the center toward the north at high \( A_V \).

Finally, we considered the \( Spitzer \) sample. Figure 2 (right panel) shows that its center remains compatible with that of the other catalogs, and does not vary when limiting to sources showing infrared excess emission. This suggests that there are no significant spatial offsets between young members with disks and diskless sources, although their radial profile might not be identical.

### 3.3. Ellipticity

The ONC population is known to be elongated in a direction close to north–south, which follows the local filamentary structure of the Orion A molecular cloud (Johnstone & Bally 1999; Muench et al. 2008). Hillenbrand & Hartmann (1998) fitted ellipses to isophotes on a stellar spatial density map in the ONC, obtaining average ellipticity \( e = 1 - b/a = 0.30 \), identical for their optical and NIR sample, with a tilt of the major axis at about \( 10^\circ \) counterclockwise from the north–south direction.

We remeasure the ellipticity of the ONC, considering our catalog of optical parameters and X-ray members to be representative of the structure of the cluster. We evaluate both a local and a scale dependent overall ellipticity. We compute the first in a similar way as in Hillenbrand & Hartmann (1998), generate a stellar density map, and smooth it with a Gaussian kernel of \( 36'' \) (\( \sim 0.08 \text{ pc} \)). We recompute the tilt of the major axis of the cluster, which from our catalog is only \( 7^\circ \), and choose to view it as negligible throughout our analysis. We also constrain the center of each fitted ellipse to be our reference ONC center, computed on the same catalog, derived in Section 3.1.

We also compute an overall ellipticity of the entire ONC population within varying distances from the center. To this end, we proceed in an iterative way: we consider square boxes centered on our ONC center, and compute the ratio between the standard deviations in R.A. and decl. of the sources contained. If the ratio departs from one, we change the size of the box in R.A. (generally diminishing it given the north–south elongation of the ONC), and recompute the ratio of the dispersions of the sample contained in this rectangular area. We iterate until the
Figure 3. Axes ratio, b/a, of ONC stars. The green squares are the best elliptical fits to isophotes on the ONC map, and trace the local ellipticity. The green dashed line is a second-order polynomial fit to these results. Black solid line: cumulative flattening of the population in the north–south direction as a function of maximum distance in declination from the center, estimated from the ratio of the positional dispersions in R.A. and decl. (see text).

(A color version of this figure is available in the online journal.)

measured ratio of sample standard deviation converges to the ratio of the sides of the box.

The result, in terms of axis ratio $b/a = 1 - e$ as a function of distance in declination from the center, is shown in Figure 3. Our isophote fits show that the core of the cluster is relatively round and the distribution becomes more elliptical at increasing radii, reaching a value $b/a \sim 0.5$; this is compatible with Hillenbrand & Hartmann’s (1998) results, which were obtained up to a distance (semi-major axis) of 0.14 from the center. At larger distances, however, the cluster again becomes rounder, with a semi-major axes ratio of $\sim 0.8$. The black line in Figure 3 shows the cumulative $b/a$ as a function of radius: its increase at distances from the center (≥0.12) is more modest because the enclosed stellar content remains dominated by stars in the more elliptical region.

3.4. Angular Substructure

3.4.1. The Angular Dispersion Parameter

We estimate the degree of angular sub-structure in the ONC, or conversely, the smoothness of the stellar distribution, using a technique analogous to the azimuthal asymmetry parameter (AAP) defined by Gutermuth et al. (2005). This is based on dividing the spatial stellar distribution in equally sized circular sectors, and comparing the dispersion of the number counts among different sectors with the hypothesis of being drawn from a uniformly random distribution of position angles. Varying the width (or number) of the sectors allows one to probe positional substructure at different azimuthal multipole moments. The radial dependence of the degree of angular substructure can be investigated by further isolating the population within concentric annuli before counting sample numbers within azimuthal sectors.

In addition to radial variation, we also generalize the AAP of Gutermuth et al. (2005) to account for cluster ellipticity. We thus define a new angular dispersion parameter (ADP), $\delta_{\text{ADP},N}(R)$. For each annulus, the number $n_i$ of stars in each $i$th sector is counted over a total of $N$ sectors; the quantity $\delta_{\text{ADP},N}$ is then defined as follows:

$$
\delta_{\text{ADP},N} = \sqrt{\frac{1}{(N-1)\bar{n}} \sum_{i=1}^{N} (n_i - \bar{n})^2} = \sqrt{\frac{\sigma^2}{\sigma_{\text{Poisson}}^2}}.
$$

where $\sigma$ is the standard deviation of the $n_i$ values, $\bar{n}$ is the average of the number of stars per sector in the considered annulus, and $\sigma_{\text{Poisson}}$ is the expected standard deviation due to Poisson statistics. When the annuli follow the local or mean elliptical shape of the cluster, we indicate this via $\delta_{\text{ADP},e,N}$ and $\delta_{\text{ADP},e,L,N}$, respectively. The ADP simply corresponds to the measured sample standard deviation of counts in sectors normalized on that expected, assuming Poisson statistics. Practically, an azimuthal random distribution of stars would produce a measured $\delta_{\text{ADP},N} \sim 1$; in the presence of intrinsic cluster sub-structuring, the measured dispersion increases to values $>1$.

Strictly speaking, because the sample variance $\sigma^2$ follows a scaled $\chi^2$ distribution, as $(N - 1)\sigma^2/\sigma_{\text{Poisson}}^2$ follows a $\chi^2_{N-1}$ distribution with $N - 1$ degrees of freedom, we have that the expected value of $\delta_{\text{ADP},N}^2$ is 1 if the stellar distribution is azimuthally random, but the non-linearity of the square root in Equation (1) lowers the mean of $\delta_{\text{ADP},N}^2$ to $\sim 0.93$ for $N = 4$, to $\sim 0.95$ for $N = 6$ and to 1 for $N \to \infty$.

Since $\delta_{\text{ADP},N}$ is a random variable subject to a statistical error, deviations from 1 are expected even for a random distribution of stars. From the relations mentioned above, the standard error in $\delta_{\text{ADP},N}$ will be

$$
\sigma_{\delta N} = \sqrt{\text{Var}[\delta_{\text{ADP},N}]} = \sqrt{\text{Var} \left[ \frac{1}{N-1} \chi^2_{N-1} \right]}. \tag{2}
$$

which does not depend on the number of stars, but on the number of sectors. For a small number of sectors, this error is relatively large: for example $\sigma_{\delta N}$ $\simeq$ 0.25 for four sectors and $\simeq$0.17 for 10 sectors.

A way to lower this error is to decrease the probability of the measured $\delta_{\text{ADP},N}$ deviates from the expected value because of the particular orientation of the sector pattern (e.g., the edge between two contiguous sectors oriented in the north–south direction as shown in Figure 4). Instead, the value of $\delta_{\text{ADP},N}$ can be computed for multiple orientations of the sector pattern and the results averaged. The total number of unique redistributions of $n_{\text{tot}} = N\bar{n}$ sources within an annulus among the different sectors obtained by rigid rotations of the sector pattern is $n_{\text{rot}}$, so we compute $\delta_{\text{ADP},N}$ for each of these cases and average the $n_{\text{rot}}$ results.

We characterize the decrease of $\sigma_{\delta N}$ due to our averaging process through Monte Carlo simulations. We generate a large number of simulated stellar distributions with random positions within a circular aperture, and estimate the error $\sigma_{\delta N}$ as the standard deviation of the dispersions measured for each realization. We repeat the experiment changing the number of stars within the aperture (from 20 to 5000) and the number of sectors. In addition, in each case, we separately test the cases of measuring $\delta_{\text{ADP},N}$ assuming a fixed pattern of sectors, or performing an average over the $n_{\text{rot}}$ possible orientations of the patterns for each simulated distribution. Results show that $\sigma_{\delta N}$ is independent of $n_{\text{rot}}$ in the latter case. Instead (see Figure 5), the value of $\sigma_{\delta N}$ when the rotational averaging is performed is lower than that for a fixed sector pattern by an amount that depends on the number of sectors—from $\sim$30% smaller for
Figure 4. Example of the subdivision of the ONC population in sectors and annuli, assuming circular symmetry (left panel, blue lines), constant ellipticity (center panel, red lines), or radially variable ellipticity (right panel, green lines). The dashed lines show another possible orientation of the sector pattern. (A color version of this figure is available in the online journal.)

Figure 5. Measured error, \( \sigma_{\delta,N} \), in the angular dispersion parameter, \( \delta_{\text{ADP},N} \), vs. number of sectors, \( N \), from our Monte Carlo simulation, assuming a fixed pattern of sectors for each simulated stellar distribution (black histogram) or averaging \( \delta_{\text{ADP},N} \) for the \( n_{\text{rot}} \) possible rotations of the pattern (red histogram). The black line is the analytic prediction for the first from Equation (2). The red line is scaled to match the red histogram. (A color version of this figure is available in the online journal.)

We have established that the statistical uncertainty in \( \delta_{\text{ADP},N} \) does not depend on the number of stars, \( n_{\text{tot}} \), in an annulus or circular aperture. However, if a population of stars is not azimuthally uniform, a given degree of substructure will lead to different values \( \delta_{\text{ADP},N} \) \( > 1 \) depending on \( n_{\text{tot}} \). This is caused by the normalization of \( \delta_{\text{ADP},N} \) over the expected dispersion for a Poisson distribution: increasing \( n_{\text{tot}} \) causes the relative expected standard deviation of the star counts in sectors (over the total counts) to decrease. Thus, for a given relative increase in the measured standard deviation of counts produced by substructure, the higher the number of stars, the higher the value of \( \delta_{\text{ADP},N} \). Thus, when comparing the measured \( \delta_{\text{ADP},N} \) between different star clusters, different radial bins for the same cluster, or different assumed samples for the same cluster, the number of sources within an aperture or annulus must be fixed.

Before we analyze the properties of the ADP, \( \delta_{\text{ADP},N} \), in the ONC, we briefly characterize the typical ranges of the variation of this parameter between very smooth and highly substructured stellar populations, and in particular consider globular clusters (GCs) and the pre-main-sequence (PMS) stars in Taurus-Auriga. For the GCs, we adopt the catalogs from the ACS Survey of Galactic Globular Clusters (Sarajedini et al. 2007; Anderson et al. 2008), which include Hubble Space Telescope photometry of about 50 GCs. For the Taurus association we use the census of PMS stars from Kenyon et al. (2008) that includes 383 members. For each GC and Taurus we derive the center of the cluster as in Section 3.1, divide the population in circular annuli and sectors, forcing a fixed number of sources within each annulus, and compute \( \delta_{\text{ADP},N} \). Then, we average the results for multiple annuli in Taurus and the annuli of all GCs. Table 1 shows the results, compared with that obtained in the ONC assuming the optical parameters + X-ray sample, and imposing either 50 or 100 stars per annulus. Such a low number, which is very small compared to the number of sources in the GCs and the ONC, is required to allow a meaningful comparison with the small sample of the Taurus region.

Table 1 shows a clear trend in the ADP from the smooth, dynamically old GCs, where the \( \delta_{\text{ADP},N} \approx 1 \) indicates a near random azimuthal distribution of sources (indeed such low values are expected due to the regularization imposed by the global potential of the cluster, while increased values may...
result from a spread in stellar masses) to the ONC where departures from Poisson smoothness are detected \((d \simeq 1.4\) for 50 stars per annulus, \(\delta_{\text{ADP,N}} \simeq 1.8\) for 100 stars per annulus) to the substructured distribution in Taurus leading to an azimuthal dispersion that is up to twice as large as in the ONC. These results highlight the ability of the azimuthal dispersion parameter \(\delta_{\text{ADP,N}}\) to trace small departures from angular spatial smoothness: the minimum spanning tree \(Q\) parameter (Cartwright & Whitworth 2004) is another technique that is a powerful tracer of substructure for clumpy spatial distributions, but in the ONC \((Q \sim 0.8)\) would merely indicate central concentration.

3.4.2. Radial Dependence of the Angular Dispersion
Parameter in the ONC

We now look for radial variations in the ADP of the ONC (i.e., \(\delta_{\text{ADP,N}}(r)\)). In addition, we account for the ellipticity of the cluster and consider separately three assumptions.

**Circular symmetry.** We simply divide the stellar sample in concentric circular annuli in R.A. and decl. to derive \(\delta_{\text{ADP,N}}(r)\).

**Constant ellipticity.** We assume elliptical annuli, with an axes ratio \(b/a = 0.55\) corresponding to the overall ellipticity we determined within large apertures (\(a > 0:1\)) from the ONC center (see Figure 3) to derive \(\delta_{\text{ADP,e,N}}(r)\). The position angles of the segments separating neighboring sectors are corrected to maintain equal areas within each sector.

**Variable ellipticity.** We assume the polynomial fit to the best-fit isophotes shown in Figure 3, allowing the flattening of subsequent annuli to vary with the distance from the center to derive \(\delta_{\text{ADP,e,N}}(r)\). As in the previous case, the edge between neighboring sectors is defined to force the area of all sectors in each annulus to be constant, which produces curved lines separating sectors.

An example of the three configurations we explore is shown in Figure 4. As before, we assume the combined sample of optical and X-ray sources.

Figure 6 reports the radial dependence of \(\delta_{\text{ADP,N}}\) from our ONC stellar sample for multiple configurations of the number of sectors and stars per annulus, as reported in the legend. As anticipated, increasing the number of stars per annulus leads, on average, to a larger measured value of \(\delta_{\text{ADP,N}}\), whereas we do not detect significant differences in the results changing the number of sectors (i.e., the angular mode of substructure) from \(N = 4\) to 9. We assume that circular symmetry leads to the highest dispersion, with a broad peak at \(\sim 0.9\) from the center. This corresponds to the distance of maximum ellipticity of the cluster (see Figure 3); such a feature in the dispersion profile disappears when accounting for ellipticity (lower panels of Figure 6). This shows that the radial increase in the angular dispersion assuming circular symmetry is not indicative of clumpy substructure, but is largely due to the elongation of the ONC. Allowing for elliptical sectors with varying ellipticity leads to the lowest dispersion, which is almost radially constant at an average value \(\delta_{\text{ADP,N}} \sim 1.2\--1.6\).

As mentioned in Section 2, since our representative sample of sources with optical parameters or X-ray membership remains somewhat incomplete at the very low-mass end of the IMF, and at projected distances from the center >0.11 where part of the region has not been observed in X-rays, we tested the radial behavior of \(\delta_{\text{ADP,N}}\) measured from different assumptions for the stellar catalog. In particular, we compared the cases of assuming the sample with either optical parameters or X-ray membership separately; we considered the entire \(JHK\) photometry and that restricted to the stellar luminosity range, which is largely immune to contamination that is predominant at fainter luminosities. We also considered the catalog of Spitzer detections from Megeath et al. (2012), and separately its subsample of IR excess sources. These results are shown in the left column of Figure 7, for the three assumptions regarding the cluster ellipticity. We find that the qualitative behavior of \(\delta_{\text{ADP,N}}\) is largely unaffected by the choice of the sample, with typical differences between the measurements that are comparable with the statistical errors affecting each value. This indicates that the degree of substructure that we detect in the ONC is not being set by residual contamination, incompleteness of the adopted stellar sample, or patchy extinction.

In Section 3.1, we showed that the center of mass of the ONC shifts when computed within apertures of different radii, and at large scales is \(\sim 20^\prime\) north of the Trapezium (Figure 1), so we have also assessed whether the radial trend of \(\delta_{\text{ADP,N}}\) and its absolute values are driven by this effect. Figure 7 (right column) reports the radial dependence of \(\delta_{\text{ADP,N}}\) when computed, centering the pattern of annuli to the center of mass of the ONC within an aperture of 0.11 from the Trapezium (red square symbol in Figure 1). Also in this case, we find little or no change compared to the radial trend of \(\delta_{\text{ADP,N}}\) from our final selected center of the cluster.
Figure 7. Dispersion vs. distance from the center for different selections of the stellar samples, and the method to account for ellipticity as indicated in the legend. Left panels denote values of \( \delta \) for annuli centered on our ONC bona-fide center of mass, right panels assume the center of mass within an aperture of \( r = 0.1 \) pc to the north of the Trapezium. The value of \( \delta \) is computed by defining annuli to contain 200 stars and assuming six sectors.

(A color version of this figure is available in the online journal.)

The very center of the ONC appears to have a significantly smaller angular dispersion parameter, \(< 1\), with the values rising by factors of a few by \( R = 0.05 \). This behavior is independent of whether or not ellipticity is allowed for.

Moreover, as we have shown in Section 3.3, the inner part of the ONC is rounder than at larger distances. Such behavior is expected, considering that core has a shorter dynamical timescale than the halo, thus stellar interactions can smooth out the spatial distributions faster.

For the outer regions, once ellipticity is allowed for, the level angular substructure is relatively constant with radius. This indicates that the peak in \( \delta_{\text{ADP,N}} \) measured at \( R \approx 0.6 \) pc assuming circular symmetry is mainly driven by the elongation of the system, which is highest at this distance from the center (Figure 3). On close inspection of our catalogs, we also noted that at intermediate distances increases in \( \delta_{\text{ADP,N}} \) are also influenced by the relatively under abundance of sources to the east of the Trapezium compared to the west, which is an asymmetry already noted by Feigelson et al. (2005). Thus the value of \( \delta_{\text{ADP,N}} \) after correcting for ellipticity is larger than the mean value in the dynamically old OCs, but smaller than in the more dispersed Taurus region. This may indicate that there has been some dynamical processing if the stars in the ONC formed with the same initial substructuring as Taurus. Alternatively, if the stars in this extended region are part of an expanding halo of weakly bound or unbound cluster members that formed in a more central location, then this could also explain the observed flattening of \( \delta_{\text{ADP,N}} \) beyond \( \sim 0.4 \) pc.

\( N \)-body simulations utilizing varying initial conditions have investigated the temporal evolution of the \( Q \) parameter, stellar surface density distribution, and mass segregation (e.g., Allison et al. 2009, 2010; Parker et al. 2014). Analysis needs to be extended to the \( \delta_{\text{ADP,N}} \) parameter, which could further constrain the initial conditions and dynamical evolution of the ONC.

Further observational data is also needed. In a forthcoming paper, we will study \( \delta_{\text{ADP,N}} \) and its radial dependence in a large sample of young clusters, spanning a range of masses, densities, and ages.

4. DYNAMICS OF THE ONC

In this section, we use our collected ONC data sets to reevaluate its dynamical status. In particular, we constrain the overall mass profile, both due to stellar and gas potential, and compare it with kinematic studies from the literature to assess the virial equilibrium of the system.

4.1. Stellar Density Profile

Following the work of Hillenbrand & Hartmann (1998), we study the radial density profile of the stellar population in ONC. For simplicity, we neglect the ellipticity of the cluster and derive an average radial profile, adopting circular symmetry in the plane of sky. We consider all the stellar catalogs described in Section 2; as anticipated, these have been restricted to a square area with a size of \( 0.4 \times 2.9 \) pc centered on our bona-fide ONC center of mass (Section 1), where we have full coverage for the optical, near infrared, and \textit{Spitzer} catalogs. The maximum distance from the center to the corners of this area is thus \( 0.28 \) or \( 0.2 \) pc. We divided the samples in radial annuli, each containing 10 stars, up to the maximum distance from the center, and
measured the projected stellar mass density summing the stellar masses in each annulus and dividing the result by the area of each annulus. As in Section 3.4, stars with no available mass estimate were assigned a mass of 0.5 \( M_\odot \). The results for the outer annuli, which are in part outside our square field of view, have been corrected to account for this incompleteness.

The mass surface density profile of the ONC was computed for different combinations of the stellar samples. In particular, we considered the full photometric samples (optical, near infrared, and Spitzer), subsamples of sources with optically derived parameters, youth tracers (IR excess, X-ray emission), and several combinations of these selection criteria. An example of the surface stellar density profiles is shown in Figure 8 for the sample of sources with either optically derived parameters or X-ray membership.

Unlike in Hillenbrand & Hartmann (1998), our mass surface density profiles \( \Sigma(R) \) tend to not show a flattening in the core regions, but appear to follow roughly a straight line in logarithmic axes in the entire radial range. Thus, instead of adopting King models, we assume that the three-dimensional stellar density \( \rho(r) \) follows a power-law profile up to a maximum radius \( r_{\text{max}} \), which sets the boundary of the cluster:

\[
\rho(r) = \begin{cases} 
\rho_0 \left( \frac{r}{1 \text{ pc}} \right)^{-\alpha} & \text{if } r \leq r_{\text{max}} \\
0 & \text{if } r > r_{\text{max}}.
\end{cases}
\]

We assume \( r_{\text{max}} = 3 \text{ pc} \), and varying \( \alpha \) we project the three-dimensional \( \rho(r) \) in two-dimensional for \( \Sigma(R) \); the latter are then fit to the data using a \( \chi^2 \) minimization, thus determining the best-fit \( \alpha \) and \( \rho_0 \).

Table 2 reports the best-fit density profile parameters, as well as the extrapolated number of sources and total stellar mass within 2 pc from the center. Overall, the power-law exponent \( \alpha \) is found to be close to that of a single isothermal sphere (\( \alpha_{\text{SIS}} = 2 \)), and only weakly affected by the criteria for selecting the stellar sample, despite a factor of several difference in the number of stars. The values \( \alpha > 2.3 \) obtained for samples that include the X-ray sources is in part biased toward steeper slopes by the incomplete X-ray coverage at large distances from the center.

![Figure 8](image-url) Measured mass surface density profile of the ONC, adopting the sample of sources with available stellar parameters or X-ray membership (black line). The red line shows the best fit of a truncated three-dimensional power-law profile.

(A color version of this figure is available in the online journal.)

| Sample | \( \rho_0 \) (\( M_\odot \text{ pc}^{-2} \)) | \( \alpha \) | \( N_\ast \) (<2 pc) | \( M_\ast \) (<2 pc) |
|--------|------------------|-------|----------------|----------------|
| (a) All sources | 105 | 2.05 | 4323 | 2676 |
| (b) \( HHR \) photometry | 95 | 1.98 | 4129 | 2386 |
| (c) \( HHS_{\text{stellar}} \) | 55 | 2.25 | 2745 | 1539 |
| (d) Optical photometry | 52 | 1.90 | 2969 | 1285 |
| (e) Optical parameters | 33 | 2.05 | 1927 | 846 |
| (f) Young | 17 | 2.24 | 1099 | 465 |
| (g) Old | 17 | 1.88 | 939 | 405 |
| (h) Optical param + X-ray | 50 | 2.40 | 2595 | 1591 |
| (i) Optical param + X-ray + IR excess | 55 | 2.35 | 2741 | 1667 |
| (j) Optical param + IR excess | 46 | 2.02 | 2238 | 1156 |
| (k) IR excess | 23 | 2.01 | 1094 | 578 |
| (l) Optical param + X-ray + \( HHS_{\text{stellar}} \) | 18 | 1.96 | 915 | 452 |
| \( A_V < 1 \) mag | 32 | 2.29 | 1621 | 916 |
| \( A_V < 3 \) mag | 50 | 2.26 | 2733 | 1421 |
| \( A_V < 10 \) mag | 66 | 2.24 | 3132 | 1850 |
| \( A_V < 100 \) mag | 66 | 2.27 | 3230 | 1886 |

**Notes.** Samples are defined as follows. (a) any individual source detected in the photometry of Da Rio et al. (2010b), the NIR photometry of Robberto et al. (2010) complemented with 2MASS, Spitzer, and X-ray members from Getman et al. (2005a); (b) NIR photometry catalog; (c) as (b) but excluding sources in the CMD zone populated by brown dwarfs and contaminants (see Robberto et al. 2010); (d) optical photometry from Da Rio et al. (2010b); (e) sample of optically derived stellar parameters from Da Rio et al. (2012); (f) and (g) sources with available age estimate from the HRD, divided as younger or older than the mean cluster age. (k) Spitzer detection showing evidence of IR excess from circumstellar material (Megeath et al. 2012). From (h) to (l) combination of the above criteria.

Conversely, the optical photometric catalog shows a flatter slope than other samples, possibly due to lower completeness in the central regions of the ONC because of the bright nebular background in the vicinity the Trapezium. Similarly, stars with isochronal ages older than the mean cluster age are more likely to be missed in the central regions, as they are fainter than younger sources for the same mass. Finally, the exponent from the fit to the entire \( HJK \) photometry, which includes prominent background contamination at substellar luminosities, turns out to be flatter than that obtained by restricting to the stellar mass range where contamination is minimal. This is expected as the contamination from Galactic field sources is not as centrally concentrated as the ONC members. From all these comparisons, we estimate a bona-fide value \( \alpha \simeq 2.2 \) for the ONC population. We emphasize that, in principle, a power-law density profile is unphysical, in that it has infinite density at \( r = 0 \). However, for \( \alpha < 3 \) the mass contribution of the core does not diverge, and for the isothermal case each radial bin in linear units contributes the same amount of mass. Because our measurements find no deviation from a single power law down to \( r \sim 10^{-2} \text{ pc} \), changing the model to remove the singularity at smaller radii would have no effect on our analysis.

The assessment of the real value of \( \rho_0 \) from the values listed in Table 2 is critical to constrain the actual stellar mass of the ONC, and requires some considerations. First, the value \( \rho_0 \simeq 100 \text{ } M_\odot \text{ pc}^{-2} \) determined for the entire sample of unique detections summed from each catalog, as well as the whole \( HJK \) sample, is significantly overestimated due to the large contamination at faint luminosities. The normalization
value and the cluster mass nearly halve when restricting to near infrared sources above the stellar mass threshold of Robberto et al. (2010), below which almost every source is not a member. Even this sample, however, suffers from some contamination and incompleteness. For example, within the same field of view and luminosity range (M ≳ 0.2 M_☉), ∼15% of NIR sources are not X-ray members and this increases to ∼25% in the whole stellar luminosity range; this is both due to increasing incompleteness of the X-ray survey, and increasing contamination toward lower luminosities. However, the X-ray sample reaches deeper extinctions than our JHK photometry. Within an aperture of 0:11 from the ONC center, we find that 25% of X-ray sources have no counterpart in the JHK stellar sample. Thus, incompleteness and contamination should roughly cancel out, in stellar number, in the stellar luminosity range of the JHK sample. Yet, this sample will be affected by further incompleteness at faint luminosities under the stellar threshold in the JH color-magnitude diagram (CMD), and somewhat in the core of the region, where confusion limits the X-ray sample. This can be noted from Table 2 considering the sample of optically derived parameters (which extends somewhat in the substellar regime), X-rays, and IR excess sources: this sample is virtually immune from contamination but likely incomplete outside the FOV of the X-ray sample and at low luminosities near the core. Its measured normalization constant ρ_☉ = 55 M_☉ pc^-3 is identical to that of the JHK sample at stellar luminosities.

Based on this data, it is not clear what the degree of residual incompleteness in these samples is caused by substellar objects, unresolved binaries, and confusion in the center. With some uncertainty, we thus assume an additional 25% of total stellar mass. Thus we infer that the ONC stellar population is well represented by a density distribution:

\[ \rho_{\text{stellar}}(r) \simeq 70 M_\odot \text{pc}^{-3} \left( \frac{r}{1 \text{pc}} \right)^{-2.2}, \quad r < 3 \text{ pc}. \]  

We emphasize that this simple model is intended to be representative of the overall dynamical contribution from stellar mass in the ONC; on smaller scales, some degree of substructure, as well as elongation, remain present (see Section 3). Other studies (e.g., Rivilla et al. 2013; Kuhn et al. 2014) have also analyzed the spatial structure of the ONC population within the inner pc of the region, finding that the stellar distribution is well matched by a superposition of a denser core, which is basically coincident with the Trapezium, surrounded by a shallower halo.

Last, as we have shown in Section 3, the heavily embedded population, although it accounts for a small fraction of the population, appears slightly offset from the main cluster, following the densest cores in the region and the integral shape filament within the OMC-1. In this study, we do not consider this part of the population to be a separate population to be removed from the sample, as we infer that it will eventually be one with the rest of the system during the upcoming early evolution of the ONC. However, we check if the spatial distribution of the heavily embedded population affects the surface density profiles we derived. The last rows of Table 2 report the fitted profile parameters for the sample of sources with available A_V (either optical spectroscopy, NIR CMD, or X rays), limited below varying upper limits in extinction. Except for the lowest extinctions, which appear slightly less centrally concentrated, which is most likely because they belong to the very external shell of the system toward our direction, the power-law exponent remains largely unaffected by the chosen cut in extinction.

![Figure 9. Average extinction affecting the ONC members as a function of the projected radius from the center, obtained from the average ONC members A_V map from Scandariato et al. (2011).](image)

(A color version of this figure is available in the online journal.)

### 4.2. ISM Density

The distribution, density, and total mass of the interstellar medium (ISM) in the ONC is not well constrained. Some of the densest regions of the OMC cloud (the KL region and the OMC-1 South cores) can reach column densities of up to A_V ∼ 100 mag (Bergin et al. 1996; Scandariato et al. 2011; Lombardi et al. 2011), but the total mass estimates of these clumps and cores can be uncertain by at least a factor of two, given uncertainties in temperatures, dust emissivity, and gas to dust mass ratios. In addition, much of the gas in the region lies somewhat behind the ONC. This is demonstrated by the large difference between the total ISM column densities integrated along the line of sight and the relatively small extinction affecting ONC stellar members, which peaks at A_V = 1–2 mag and presents a tail extending to A_V ≳ 5–10 mag (Hillenbrand 1997; Da Rio et al. 2010b). As anticipated, only a minor fraction of young stellar members appear highly obscured.

Scandariato et al. (2011) used near-infrared data, together with optical parameters where available, to derive both the total extinction map—from statistics of background stars—and a map of the average A_V affecting the ONC members. We adopted the latter and computed its mean value as a function of angular distance from the center of the ONC. The result is shown in Figure 9. The mean stellar A_V is nearly constant at all distances from the centers, at a value ⟨A_V⟩ ≃ 2.5–3 mag. The peak extinction at R ≃ 0.5 pc is due to the Dark Bay (O’Dell 2001), an obscuring cloud in slight foreground with respect to the ONC population and H II region, which is located northeast of the Trapezium.

If we assume that the ISM is also uniformly distributed along the line of sight, we can translate this column density of dust into a total volume density of ISM. Since the A_V distribution is skewed, this is probably not the case, and the ISM density on average increases moving into the cluster along the line of sight, reaching high values for the few very embedded objects. However, this approximation is fair at the midplane of the system. If we thus assume that the ISM is uniformly distributed in either a cubic box or a sphere around the cluster center with the radius of 3 pc, which is the same truncation we assumed in Figure 8 for the stellar distribution, this translates...
into an optical depth $A_V = 1$ mag pc$^{-1}$ along the line of sight. Assuming the dust to gas relation from Vuong et al. (2003) $N_H / A_V = 1.58 \times 10^{21}$ cm$^{-2}$ and solar abundance of He, this corresponds to a constant gas density:

$$\rho_{\text{gas}} \simeq 22 M_\odot \text{ pc}^{-3}. \quad (4)$$

It could be argued that part of the extinction toward ONC sources may originate from foreground galactic ISM that is unrelated to the cluster. For example, O’Dell et al. (2008) finds that the Orion Nebula H II region is obscured by $A_V \sim 2$ mag of neutral material. However, the vast majority of such veil remains located within the stellar system, meaning that part of the ONC population is well in front of the H II region. This is confirmed by the fact that spectroscopic measurements of extinctions toward individual ONC members (Hillenbrand 1997; Da Rio et al. 2010a, 2012) measure $A_V$ values as low as $\sim 0$ with no evidence, within the uncertainties, of a positive minimum threshold value. Therefore the foreground extinction from Galactic ISM must be of a negligible amount, up to no more than a few tenths of magnitude, compared to the mean ONC extinction shown in Figure 9. Given the relatively large uncertainties already present in our mass estimates for both stars and gas, we thus do not attempt to constrain and remove the small foreground extinction.

Figure 9 also shows, together with the radial trend of the average $A_V$, the trend expected assuming this uniform amount of ISM either in a cubic or spherical geometry; in both cases, the simple model is fairly adequate to reproduce the data.

The $A_V$ map from Scandariato et al. (2011) was derived from NIR data, thus lacking potentially heavily embedded sources that could, despite being a small fraction of the population, shift the mean stellar $A_V$ to higher values. We thus also considered the extinctions derived from log $N_H$ from the COUP X-ray sample. Indeed, the X-ray mean $A_V$ is $\sim 15$ mag; this value however is strongly biased by the asymmetry of the distribution, with a few sources with derived values exceeding 100 mag, and large relative errors in the measurements. The median $A_V$ is 3.8 mag, which is in line with the mean from Scandariato et al. (2011).

In addition, the errors in $A_V$ from the X-ray analysis correlate with $A_V$, and the values of $N_H$ appear nearly symmetric around a mean value of $10^{21.7}$ cm$^{-2}$, which corresponds to $A_V \simeq 3$. Thus we conclude that our estimate for the value $\rho_{\text{gas}} \simeq 22 M_\odot \text{ pc}^{-3}$ adequately represents the present average ISM content within the ONC.

The above ISM density, when compared to the stellar mass profile (Equation (3)) is quite small: It is smaller than the stellar density for $r < 1.37$ pc, radius containing 73% of the stellar mass within 2 pc, or 53% within 3 pc, and negligible in the cluster core. If we approximate the contribution of stars and gas to the radial profile with a power law, the results depend on the considered range in radii. Limiting to the range of distances spanned by our stellar density profiles (e.g., Figure 8), the approximate total density follows

$$\rho_{\text{total}} \sim 100 \left( \frac{r}{\text{1 pc}} \right)^{-2.07} M_\odot \text{ pc}^{-3}. \quad (5)$$

We will use this as an estimate for the total observed mass for comparison with the dynamical mass (Section 4.3). Figure 10 summarizes the contribution to the density profile from Equations (3) and (4), and the approximation to the total of Equation (5).

**Figure 10.** Top panel: the estimated volume density profile of the ONC due to stars, gas, and total (stars + gas), together with the best power-law fit to the latter as reported in the legend. The red line represents the density profile of a singular isothermal sphere needed under the assumption of dynamical equilibrium given the observed velocity dispersion in the system (Section 4.3). Bottom panel: the predicted one-dimensional velocity dispersion, $\sigma_v$, as a function of radius for the these models. The horizontal lines on the left indicate the overall cluster $\sigma_v$, computed by averaging the curves with a weighting proportional to the fractional stellar mass at each radius. (A color version of this figure is available in the online journal.)

### 4.3. Dynamical Equilibrium

It has been pointed out in several works (e.g., Hillenbrand & Hartmann 1998; Scally et al. 2005) that the ONC may not be in dynamical equilibrium, as the dynamical mass determined from the kinematic properties of the cluster is twice or more the stellar mass. Here we follow up on these findings based on our updated estimates of stellar and gas content in the ONC (Sections 4.1 and 4.2).

Proper motion surveys in the ONC date back to the work of (Jones & Walker 1988); they measured a one-dimensional velocity dispersion $\sigma_v \simeq 2.3$ km s$^{-1}$. Radial velocity surveys (Sicilia-Aguilar et al. 2005; F{"e}r{é}sz et al. 2008; Tobin et al. 2009) derived a nearly identical velocity dispersion within the ONC region, except for evidence for lower velocities ($<1.8$ km s$^{-1}$) for bright members, and systematic variations with position at scales larger than that considered in this study along the north–south filament.

If we consider a singular isothermal profile $\rho(r) = \rho_{\text{SIS}} r^{-2}$, which is a fair approximation for the ONC given the power-law exponents 2.2 or 2.07 from Equations (3) and (5), an average $\sigma_v = 2.3$ km s$^{-1}$ under dynamical equilibrium would imply a normalization constant for the density at $r = 1$ pc of

$$\rho_{\text{SIS}} = \frac{\sigma_v^2}{2\pi G} = 37 \left( \frac{\sigma_v}{1 \text{ km s}^{-1}} \right)^2 M_\odot \text{ pc}^{-3} \rightarrow 195 M_\odot \text{ pc}^{-3}. \quad (6)$$

This is about twice the overall value we estimated from the contribution of stars and gas in the ONC (Equation (5)). Alternatively, if we consider that the best power-law fit of the estimated stellar plus gas density (Equation (5)) has an exponent close to that of an isothermal sphere, its normalization $\rho_0 = 100 M_\odot \text{ pc}^{-3}$ would lead to a velocity dispersion $\sigma_v \simeq 1.64$ km s$^{-1}$ if in virial equilibrium.

In Section 5, we find evidence for relatively prolonged star formation history and thus gradual build-up of the ONC,
in which case one expects a virialized star cluster to be established before gas removal (e.g., Fellhauer et al. 2009). However, subvirial initial conditions for stellar motions are also a possibility, as suggested by studies of dense gas cores (e.g., Kirk et al. 2007), with subsequent dynamical evolution investigated by a number of works (e.g., Proszkow & Adams 2009; Allison et al. 2009; Parker & Meyer 2012). In this case, the initial density structure would have an even higher normalization than that implied by Equation (6), (but within the context of a static density structure that does not account for cluster expansion or contraction).

We better characterize the radial dependence of the predicted \( \sigma_0 \) from the actual measured density profile of stars alone and with gas from Equations (3)–(5). For simplicity we assume isotropic velocities and thus \( \sigma_0 = v_{\text{rot}}/\sqrt{2} \), which would strictly hold for a model in equilibrium, and where \( v_{\text{rot}} \) is the Keplerian rotational velocity for circular orbits in the potential described by our volume density profile. The result is shown in the bottom panel of Figure 10; we find that given the actual volume density of stars and gas in the ONC, virial equilibrium would require \( \sigma_0 \approx 1.73 \), which is 75% of the measured velocity dispersion. This indicates that the ONC may be slightly supervirial, with a virial ratio (kinetic over potential energy) \( q \approx 0.9 \).

In this case, the ONC cannot be in perfect dynamical equilibrium, and should be expanded. This result, however, is affected by some degree of uncertainty, because it relies on our estimate of total stellar and gas mass—which remains a challenging estimate (Section 4.1)—as well as velocity dispersion measurements from proper motions and/or radial velocity surveys, which in turn are very sensitive to membership estimates, measurement accuracy, and binary properties of the ONC members.

A current supervirial state of the ONC would be consistent with general theoretical expectations of dynamical evolution, either quickly or slowly compared to the dynamical time, from an initially virialized state as gas is expelled by feedback during the star cluster formation process. Alternatively, dynamically fast star cluster formation scenarios can be imagined in which the natal, transient gas cloud was always in a supervirial state (e.g., if it was formed or affected by large-scale gas flows; Hartmann et al. 2012; Bonnell et al. 2006)

A supervirial state leading to cluster expansion and dissolution is also consistent with observed populations of young clusters that exhibit high infant mortality, with most clusters of a given mass not surviving at that mass during their first 10 Myr of evolution, most likely because of the relatively low overall star formation efficiencies from their natal gas clumps (e.g., Lada & Lada 2003).

5. STAR FORMATION EFFICIENCY PER FREE-FALL TIME

A long-standing debate in the star formation community concerns the timescales over which a molecular clump sustains star formation (i.e., the duration of star cluster formation). "Fast" scenarios (Elmegreen 2000; Hartmann et al. 2001; Elmegreen 2007; Hartmann & Burkert 2007) predict that star-forming clumps are relatively transient dynamical entities, with star cluster formation extending over just one or a few free-fall or dynamical times. The star formation efficiency per (local) free-fall time, \( \epsilon_{\text{ff}} \), would then be relatively high, \( \gtrsim 0.1 \), depending on the overall star formation efficiency that is achieved in the forming the cluster from the clump.

Alternatively, in the "slow" mode, the process is sustained in quasi-equilibrium for at least several crossing times (e.g., Tan et al. 2006; Nakamura & Li 2007), with star formation regulated by turbulence that is maintained by protostellar outflows or by support from relatively strong magnetic fields. In these models, \( \epsilon_{\text{ff}} \) is relatively low, \( \lesssim 0.1 \). There is also more time available for continued accretion of gas to the star-forming clump from its surroundings.
Figure 12. Radial variation (in projected two-dimensional radius, $R$) of the mean age of the ONC expressed in free-fall times for the four models shown in Figure 11 (thick line). The shaded contours delimit the 1σ, 2σ, and 3σ widths of the log-normal age spread as constrained in the literature (see text). The vertical dashed black line shows the half-mass radius of the observed stellar density profile, and the vertical dashed red lines show the half-mass radius of the particular combination of total mass (gas + stars) for each model.

(A color version of this figure is available in the online journal.)

The extent of the age spread in the ONC (as well as in other clusters) has recently been constrained by a number of works. The luminosity spread of its PMS stars, if interpreted as a distribution in radii for a given mass from a true age spread, leads to a large apparent width ($\sigma_{\log t} = 0.4$ dex; Hillenbrand 1997; Da Rio et al. 2010b) around a (model dependent) mean age of $\sim 2.5$ Myr. However, observational uncertainties, variability, and unresolved binarity cause this age spread to be overestimated. Reggiani et al. (2011) showed that these have a small effect on the overall luminosity broadening. Jeffries et al. (2011), on the other hand, posed an upper limit to the real $\sigma_{\log t}$ of 0.2 dex from the lack of correlation between the abundance of circumstellar disks around members and isochronal ages, suggesting that the apparent luminosity spread is in large part—if not all—due to protostellar accretion induced changes in the stellar structure evolution (Baraffe et al. 2009, 2012). However, Hosokawa et al. (2011) found that reasonable levels of episodic accretion were insufficient to explain the observed luminosity spread, suggesting that significant intrinsic age spreads are present. Finally, Da Rio et al. (2014) excluded a very short age spread from an analysis of the bias in the inferred temporal decay of mass accretion rates induced by uncertain ages of PMS stars, suggesting as a bona-fide compromise of all these independent constraints that there is a real age spread $\sigma_{\log t} = 0.2$ dex around a mean age $t = 2.5$ Myr, corresponding to 95% of the ONC population with ages between 1 and 6.3 Myr assuming a Gaussian distribution in $\log t$. If a uniform distribution in $\log t$ is assumed instead, this 95% of the stars lie in the interval between 1.1 and 5.5 Myr, and for a Gaussian distribution in linear age between 0.7 and 4.7 Myr. We stress that given the amplitude of the apparent age spread compared to the real one, the actual shape of the age distribution is largely unknown; this is also particularly true at very young ages, where the age of individual sources is largely uncertain. Hereafter, we will assume a lognormal distribution.

Here we use our constrained estimate of the stellar and gas content of the ONC to translate the age and age distribution of the ONC in terms of free-fall timescales. We consider different
models for the mass content of the region: first, the present-day estimates, separately for the radial distribution of mass volume density of stars alone (Equation (3)) and the sum of stars and gas (Equation (4)). Second, assuming that the supervirial state of the ONC we found in Section 4.3 is due to recent gas expulsion, we consider two simple assumptions for the total mass profile before this event: the singular isothermal sphere that reproduces the observed velocity dispersion (Equation (6)) and a model obtained by adding to the measured present-day stellar density profile a gas profile $\rho_{\text{gas}} \propto r^{-1}$ normalized so that the total mass contained within $r < 3$ pc coincides with that of the latter isothermal sphere.

Figure 11 shows the radial dependence of both the cumulative mass, and the free-fall time $t_{\text{ff}}$ for each model. The solid lines for $t_{\text{ff}}$ represent the exact $t_{\text{ff}}$ calculated by numerical integration of the motion of a test particle from rest within the modeled potential. For comparison, we also show the resulting $t_{\text{ff}}$ derived by adopting the common approximation valid for a uniform sphere, $t_{\text{ff}} = (3\pi \rho / 32G)^{1/2}$, where we assume for the volume density $\rho$ either the local one at any given $r$ (dotted lines in Figure 11) or the average density within the sphere of radius $r$ (dashed line). The first approximation leads to an overestimation of $t_{\text{ff}}$, since in reality the density increases moving toward the center. On the other hand, the second approximation leads to results closer to the exact solution, although in this case $t_{\text{ff}}$ are slightly underestimated. In Figure 11, we also show the mass profile obtained by simply multiplying the stellar mass by two and three. This shows that the two models that reproduce the dynamical state in equilibrium are also compatible with a simple assumption that the ONC initially had a similar density profile as the present-day stellar distribution, and stars have formed with an efficiency between $\sim 30\%$ and $50\%$. In this case, most of the remaining gas was expelled by the system during the star cluster formation process.

Using these four models, Figure 12 shows the mean age of the ONC—together with its age spreads in units of 1, 2, and $3\sigma$ from the mean age—expressed in terms of the number of free-fall times, $N_{\text{ff}}$, in the past at different distances from the cluster center. To this end we have assumed, as mentioned, a log-normal age distribution with a width of 0.2 dex around a mean age of 2.5 Myr. The system becomes increasingly dynamically older toward the core, even under the assumption of constant average age and age spread, due to the shorter free-fall time at higher densities.

Figure 12 also highlights that the age distribution of the ONC spans, on average, several $t_{\text{ff}}$ depending on the model. This is clarified in Figure 13, left panel; this shows, again as a function of projected radius from the center, the number $N_{\text{ff},90}$ of free fall times needed form $90\%$ of the stellar population in a symmetric interval with respect to the mean cluster age for the four models. When considering the present-day mass distribution, either from stars alone or from stars and present gas (blue and green) at the half-mass radius, $90\%$ of the stellar population has been forming within five to six free-fall times. Including the contribution to the missing mass needed for the ONC to be in dynamical equilibrium (red and magenta) increases the number of $t_{\text{ff}}$ from six to eight at the same radii, due to the shorter $t_{\text{ff}}$ for these models. In any case, we find these results to be compatible with a slow star formation scenario.

If star cluster formation takes place over a relatively extended period of time, a natural question to ask is whether the characteristics of star formation, such as the initial mass function, change systematically during this evolution. Or even more simply, do massive stars tend to form preferentially near the beginning or end of star cluster formation? If feedback from massive stars is the primary agent terminating star cluster formation, then one may expect that they will tend to form near the end of the process. In the ONC, Da Rio et al. (2012) did not find any evidence for a stellar mass versus age correlation, although such analyses are subject to inherent systematic uncertainties arising from pre-main-sequence stellar evolutionary models. On the other hand, Getman et al. (2014) found the ONC core, where most of the massive stars are located, to be younger than the outskirts. While massive stars are still forming today in the ONC, such as “source I” in the KL region (see Tan et al. 2014 for a review) Hoogerwerf et al. (2001) have claimed that the four massive ($\sim 20 M_\odot$) stars $\mu$ Col, AE Aur and the $\iota$ Ori binary formed in the ONC and were dynamically ejected about 2.5 Myr ago. This timescale is compatible with the age spread we adopted from the analysis of Da Rio et al. (2012) and would indicate that massive star formation, at least in the case of the ONC, has occurred throughout the star cluster formation process. It also suggests
that the destructive feedback from massive star formation can be mitigated by dynamical (self-)ejection of the massive stars—a process likely enhanced by their migration to the cluster center, as perhaps exemplified today by the case of θ1C and its recent interaction with BN (Tan 2004; Chatterjee & Tan 2012).

The right panel of Figure 13 shows the star formation efficiency per free-fall time, $\epsilon_{\text{ff}}$. This is simply estimated as $\epsilon_{\text{ff}} = 0.9\epsilon_{k,\text{ff}}/t_{\text{form}},0 = 0.9\epsilon_{*}/N_{\text{ff},0}$, where $\epsilon_{*}$ is the fraction of total mass converted into stars. In Figure 13 we adopt the projected two-dimensional radius, so here we assume $\epsilon_{*} = \Sigma_{\text{gas}}/\Sigma_{\text{tot}}$ instead of $\rho_{*}/\rho_{\text{tot}}$, as well as the projected $t_{\text{ff}}$ as shown in the bottom-right panel of Figure 11. As in the previous figures, the red and magenta lines are for the two models we adopt for the mass content before gas removal, respectively, the isothermal sphere reproducing the observed $\sigma_{*}$, and the stellar profile with gas $\sim r^{-1}$. The value of $\epsilon_{\text{ff}}$ decreases toward the core as a consequence of the smaller $t_{\text{ff}}$ compared to the age spread that we assumed was radially constant (consistent with the results of Da Rio et al. 2012).

On the other hand, the slow decrease in $\epsilon_{\text{ff}}$ at larger radii for the model with a shallower gas profile is due to the radial decrease of $\epsilon_{*}$ since the stellar profile falls more steeply than the gas in this model. The circles in this figure denote the value at the half-mass radius, where we find $\epsilon_{\text{ff}} \approx 0.05$. For comparison, in Figure 13 we also show the same quantity derived from the two approximations of $t_{\text{ff}}$ as computed from the local density at each radius, or the mean density enclosed within each radius, as in Figure 11.

Our derived values of $\epsilon_{\text{ff}}$ are very similar to the value adopted in the study of Krumholz & Tan (2007). It is comparable to the values seen in the simulations of Nakamura & Li (2007), in which star formation activity is regulated by protostellar outflow driven turbulence.

6. CONCLUSIONS

In this work, we reanalyzed the structural and dynamical properties of the ONC. We based our analysis on a collection of stellar catalogs, membership estimates, and stellar properties from the latest studies: optically derived stellar parameters, near-infrared photometry, Spitzer photometry, and X-ray data. We used previous studies to assess the level of contamination from the Galactic field in order to constrain the actual stellar population of the system. Finally, we use stellar $A_V$ properties to estimate the ISM density within the cluster as an additional component adding to the gravitational potential. Here we briefly summarize our findings.

1. We determine the center of mass of the ONC from a subsample of sources of known membership. The center is located within the Trapezium region and its position is only weakly sensitive on the assumptions for sources without available stellar parameters. We also note that the center roughly coincides with the location of the point where, according to Chatterjee & Tan (2012), the dynamical ejection of the BN object from θ1C took place. θ1C, being the most massive star in the cluster, is expected to migrate to this location via dynamical interactions with other cluster stars, which thus places a joint constraint on the age of the star and the distance of its formation site from the cluster center.

2. We analyze the degree of angular substructure of the spatial distribution of stars via the angular dispersion parameter, $\delta_{\text{ADP,N}}$, including its radial dependence. A random azimuthal distribution leads to $\delta_{\text{ADP,N}} \approx 1$, whereas a degree of additional intrinsic substructure, perhaps imparted from initial turbulence in the star-forming gas, increases its value. The measured $\delta_{\text{ADP,N}}$ in the ONC lies between that measured in GCs—as dynamically old stellar systems with no azimuthal substructure—and Taurus, chosen as an example of a very young, dynamically un-evolved, clumpy stellar system. The dispersion is found to be lower in the core of the ONC compared to the outskirts, indicating higher dynamical processing that has erased any initial substructure. However, we also find that the elongation of the system along the north–south direction, which is higher at increasing distances from the center, is the major contributor to the measured increase of $\delta_{\text{ADP,N}}$ with radius, if ellipticity is not accounted for. We test the dependence of the radial trend of the dispersion on the selection of stellar samples, finding no significant variations when different combinations of the stellar catalogs are used.

3. We derive the stellar mass surface density and volume density profiles of the ONC for different combinations of the catalogs affected by varying degrees of incompleteness and contamination. This allows us to accurately extrapolate the bona-fide profile of the ONC members, which is well reproduced by a power-law profile (Equation (3)). We use measured stellar $A_V$ to derive the average gas density within the cluster, which appears to be nearly constant at $\rho_{\text{gas}} \sim 22 M_\odot pc^{-3}$ (i.e., relatively small compared to the stellar density, except at large distances from the center).

4. We compare the total estimated mass density profile of the ONC with literature measurements of the velocity dispersion $\sigma_{*}$ in the region, confirming previous claims that the cluster is slightly supervirial, which is indicative that the ONC should be expanding. We expect that this supervirial state has most likely been caused by relatively recent gas expulsion, given that the duration of star formation appears to have been relatively long compared to the free-fall or dynamical time (below).

5. We derive the radial dependence of free-fall time, $t_{\text{ff}}$, assuming either the present-day measured mass density and different simple models for that required by dynamical equilibrium, as descriptive of possible configurations before gas dispersal. We compare $t_{\text{ff}}$ with recent constraints on the age and intrinsic age spread in the ONC: the cluster appears to be at least several $t_{\text{ff}}$ old, and 90% of the population has been forming over 5–8 $t_{\text{ff}}$, depending on the assumptions, which is consistent with slow star formation scenarios. From these results we infer a star formation efficiency per free-fall time for the cluster-forming gas of $\epsilon_{\text{ff}} \approx 0.05$.

We thank S. Chatterjee and A. Ordonez for their help on simulating clusters for testing the $\delta_{\text{ADP}}$ parameter (part of this research will appear in a forthcoming paper). N.D.R. is funded by the Theory Group Fellowship program at UF Astronomy Department.

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