Studies of the Shapes of Heavy Nuclei at ISOLDE

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In this talk I will describe how recent experiments carried out at REX-ISOLDE have furthered our understanding of shape co-existence and reflection-asymmetric shapes in atomic nuclei. I will discuss how the experimental results have constrained nuclear theory and how the latter measurements might contribute to tests of extensions of the Standard Model. I will also discuss future prospects for measuring nuclear shapes from inelastic scattering.

1. Quadrupole shape coexistence

The evolution and microscopic origin of quadrupole collectivity and shape coexistence at low excitation energies in neutron mid-shell nuclei near the $Z = 82$ shell closure are still not fully understood [1,2]. The shape coexistence phenomenon in neutron-deficient nuclei close to $Z = 82$ was first observed when isotope shift measurements at ISOLDE for Hg nuclides revealed a sharp transition between $^{187}$Hg and $^{185}$Hg [3]. This change was interpreted as a transition from a weakly oblate shape to a more deformed prolate structure [4, 5]. Nowadays there exists a large body of experimental information supporting the coexistence of different shapes at low excitation energies in mercury isotopes. Similarly, in neutron deficient even-mass Pb isotopes intruder states were first identified in $\alpha$- and $\beta$-decay studies using on-line isotope separators [6]. The low-lying excited $0^+$ states associated with weakly deformed oblate proton $2p$–$2h$ intruder structures have been observed in $\alpha$-decay in several Pb isotopes with $N \geq 106$ [7, 8], and prolate deformed bands built upon $4p$-$4h$ excitations have also been identified in separate experiments (e.g. [9]). The lowest excited states in $^{186}$Pb were found to have spin and parity $0^+$ and are associated with oblate and (predicted) more deformed prolate shape respectively [10].

2. Recent experiments on shape coexistence

In order to determine the structure of the coexisting states in this mass region, Coulomb excitation experiments have been carried out [11] at REX-ISOLDE, CERN. Here, beams of neutron deficient mercury isotopes were accelerated to energies of $\sim 3$MeV/u and allowed to impinge upon various targets in the MINIBALL [12] $\gamma$-ray spectrometer. In order to determine $E2$ matrix elements in $^{182–188}$Hg, the Coulomb excitation least-squares fitting code GOSIA [13] was used. The code fits a set of reduced matrix elements to reproduce the measured yield of $\gamma$-ray transitions depopulating the Coulomb-excited states of $^{182–188}$Hg, taking into account known spectroscopic data related to electromagnetic matrix elements: branching ratios and conversion coefficients [e.g. 14],...
and lifetimes of the yrast and non-yrast states \[15,16\]. Crucial for this analysis was the knowledge of the conversion coefficient of the \( 2_1^+ \rightarrow 2_1^+ \) transitions, as they contain a large \( E_0 \) component \[14\].

The extracted matrix elements can be analyzed in terms of the quadrupole deformation parameters, \( Q \) and \( \delta \), that describe the charge distribution of a nucleus in a given nuclear state. Using the quadrupole sum rules approach \[17\] the quadrupole invariants, \( \langle Q^2 \rangle \) and \( \langle Q^2 \cos(3\delta) \rangle \), can then be obtained. The sums of products of the relevant matrix elements between \( 0^+ \) and \( 2^+ \) states are shown in Fig. 1: the sum of squared E2 matrix elements (SSM) related to \( \langle Q^2 \rangle \) and the sum of triple products of E2 matrix elements (STM) related to \( \langle Q^2 \cos(3\delta) \rangle \). The quadrupole invariants can be further related to the collective model variables \( \beta \) (overall deformation parameter) and \( \gamma \) (triaxiality parameter). It can be concluded that the ground states of the even-even mercury isotopes are weakly deformed with a \( \beta \) value close to 0.15 and are consistent with an oblate like deformation \( \langle \cos(3\delta) \rangle \simeq -1 \), while the excited \( 0^+ \) states are more deformed.

These data are compared to the equivalent sums calculated from beyond-mean field (BMF) \[18\] and interacting-boson based models (IBM) \[19\] (Fig. 1). According to the BMF calculation, for \( N > 104 \) the Hg ground states are predicted to be predominantly oblate and the first excited \( 0^+ \) state to be prolate, whereas for \( 100 \leq N \leq 104 \), the ground state is predominantly prolate and the excited \( 0^+ \) state is an almost equal mixture of prolate and oblate configurations \[18\]. For the latter, this is inverted with respect to experiment. In the IBM approach \[19\], whereby particle-hole pair excitations across the \( Z=82 \) closed shell are explicitly included, only partial agreement between experiment and theory is observed. The disagreements for the two models are not understood and point to missing ingredients in the calculations.

### 3. Octupole shapes and EDMs

Strong octupole correlations leading to pear shapes can arise when nucleons near the Fermi surface occupy states of opposite parity with orbital and total angular momentum differing by 3. This condition is met for proton number \( Z \sim 34, 56 \) and 88 and neutron number \( N \sim 34, 56, 88 \) and 134. The largest array of evidence for reflection asymmetry is seen at the values of \( Z \sim 88 \) and \( N \sim 134 \), where phenomena such as interleaved positive-
and negative-parity rotational bands in even-even nuclei, parity doublets in odd-mass nuclei, and enhanced $E1$ transition moments have been observed [20]. Many theoretical approaches have been developed to describe the observed experimental features: shell-corrected liquid-drop models, mean-field approaches using various interactions, models that assume $\alpha$-particle clustering in the nucleus, algebraic models and other semi-phenomenological approaches.

Atoms with octupole-deformed nuclei are very important in the search for permanent atomic electric-dipole moments (EDMs). The observation of a non-zero EDM at the level of contemporary experimental sensitivity would indicate time-reversal ($T$) or equivalently charge–parity ($CP$) violation due to physics beyond the standard model. Experimental limits on EDMs provide important constraints on many proposed extensions to the standard model. For a neutral atom in its ground state, the Schiff moment (the electric-dipole distribution weighted by radius squared) is the lowest-order observable nuclear moment. Octupole-deformed nuclei with odd nucleon number $A$ will have enhanced nuclear Schiff moments owing to the presence of the large octupole collectivity and the occurrence of nearly degenerate parity doublets that naturally arise if the deformation is static [21,22].

4. Electric octupole moments and recent experiments

In order to determine the shape of nuclei, the rotational model can be used to connect the intrinsic deformation, which is not directly observable, to the electric charge moments that arise from the non-spherical charge distribution. The $E3$ transition moment is collective in behaviour ($> 10$ single particle units) and is largely insensitive to single-particle effects, as it is generated by coherent contributions arising from the quadrupole-octupole shape. The $E3$ moment is therefore an observable that should provide direct evidence for enhanced octupole correlations and, for deformed nuclei, can be related to the intrinsic octupole deformation parameters. Twenty years ago Coulomb excitation was applied to the detailed measurements of the octupole shape of $^{148}$Nd in the $Z \sim 56$, $N \sim 88$ region [23] and of $^{226}$Ra in the $Z \sim 88$ and $N \sim 134$ region [24], but measurements of other nuclei in the latter mass region have had to wait for the development of accelerated radioactive beams.

Experiments have been carried out that used REX-ISOLDE to accelerate beams of radon and radium to bombard various targets in the MINIBALL $\gamma$-ray spectrometer. The measured values [25] of the $E2$ and $E3$ matrix elements in $^{220}$Rn and $^{224}$Ra are all consistent with the geometric predictions expected from a rotating, deformed distribution of electric charge, although these data do not distinguish whether the negative-parity states arise from the projection of a quadrupole-octupole deformed shape or from an octupole oscillation of a quadrupole shape. Figure 2 compares the experimental values of $Q_3$ derived from the matrix elements connecting the lowest states for nuclei near $Z = 88$ and $N = 134$ measured by Coulomb excitation. It is striking that while the $E2$ moment increases by a factor of 6 between $^{208}$Pb and $^{234}$U, the $E3$ moment changes by only 50% in the entire mass region. Nevertheless, the larger $Q_3$ values for $^{224}$Ra and $^{226}$Ra indicate an enhancement in octupole collectivity that is consistent with an onset of octupole deformation in this mass region. On the other hand, $^{220}$Rn has similar octupole strength to $^{208}$Pb, $^{230,232}$Th and $^{234}$U, consistent with it being an octupole vibrator. In the case of a vibrator, the coupling of an octupole phonon to the ground state rotational band will give
zero values for matrix elements such as $<1^+|E3|4^+>$, because an aligned octupole phonon would couple the $4^+$ state to a $7^-$ state. Although the radioactive beam experiments do not have sensitivity to this quantity, this effect has been observed for $^{148}$Nd [23], consistent with behaviour expected for an octupole vibrator. In contrast the intrinsic moment derived from the measured $<1^+|E3|4^+>$ in $^{226}$Ra [24] is similar to that derived from the value of $<0^+|E3|3^+>$, as expected for a static deformed system.

The values of $Q_3$, deduced from the measured transition matrix elements, are plotted in figure 3 as a function of $N$ and compared to various theoretical calculations. The trend of the experimental data is that the values decrease from a peak near $^{226}$Ra with decreasing $N$ (or $A$). This is in marked contrast to the predictions of the cluster model calculations [26]. It is also at variance with the Gogny HFB [27], Woods-Saxon macroscopic-microscopic [28] and relativistic Hartree-Bogoliubov (mapped onto an IBM Hamiltonian) [29] mean-field predictions of a maximum for $^{224}$Ra, although the agreement with the measurements for $^{220}$Rn and $^{222}$Ra by themselves is quite good. As can be seen, the relativistic mean field calculations [30] predict that the maximum value of $Q_3$ occurs for radium isotopes between $A = 226$ and 230, depending on the parameterization, and Skyrme Hartree-Fock calculations [31] predict that $^{226}$Ra has the largest octupole deformation, consistent with the data.

The recent measurements of $Q_3$ values in $^{220}$Rn and $^{224}$Ra are also consistent with suggestions from the systematic studies of energy levels [32] that the even-even isotopes $^{218,220}$Rn and $^{220}$Ra have vibrational behaviour while $^{222,228}$Ra have octupole-deformed character. The parity doubling condition that leads to enhancement of the Schiff moment is therefore unlikely to be met in $^{219,221}$Rn. On the other hand $^{223,225}$Ra, having parity doublets separated by ~50 keV, will have large enhancement of their Schiff moments, provided that the states of opposite parity have the same intrinsic structure.

5. Future developments

Future experiments at the HIE-ISOLDE facility [33] at CERN will study $^{182,184}$Hg at higher energies, in order to carry out stringent tests of beyond-mean field and algebraic-model based calculations, and carry out new measurements of $B(E3)$s in $^{222,228}$Ra and $^{222,228}$Rn, which should greatly enhance our knowledge of the systematic behaviour of E3 moments in this mass region and help us to assess the likelihood of parity doubling in the odd-A nuclei. There are also plans to measure directly the low-lying structure of odd-mass Rn isotopes, from in-beam conversion-electron and $\gamma$-ray measurements following Coulomb excitation or from the decay of the astatine parent. These measurements are vital in order to identify the best candidates for the radon EDM programme. More challenging is the measurement of the inter-band $E3$ matrix elements.
between the low-lying states in odd-mass nuclei, required for both radon and radium EDM programmes. This will require advances in precision spectroscopy. An approach advocated here is to employ (d,d') scattering \cite{34} to populate low-lying states in odd-A nuclei via single-step excitation. We have carried out simulations that indicate that \( \sim 20 \) keV energy resolution for the final states can be reached by using a helical orbit spectrometer (HELIOS) such as that developed at the Argonne National Laboratory \cite{35}, provided that the radioactive heavy beams are cooled in a storage ring. The cooling should reduce the transverse emittance of the beam to 0.01 mm mrad or better and the FWHM energy spread should be 0.025\% or better. Such beam characteristics can be achieved using the Heidelberg Test Storage Ring (TSR) \cite{36} that can be coupled to HIE-ISOLDE. In order to reduce the contribution to the final energy spread from multiple scattering and energy loss in the target, several targets of thickness \( 20 \mu \text{g/cm}^2 \) will be employed with separations along the beam axis sufficient to be identified by time-of-flight. The other requirement is that the position resolution of the linear silicon array should be at least 1 mm along and perpendicular to the beam direction, and the intrinsic energy resolution should be 10 keV or better. The required homogeneity in the magnetic field in HELIOS \( (10^{-3}) \) is easily achieved in MRI-type magnets.

6. Conclusion

We have demonstrated that radioactive beams of heavy nuclei with \( A \approx 200 \) can be successfully accelerated with sufficient intensity to extract precise values of electric-quadrupole and electric-octupole moments. Comparing our data with predictions of \( E_2 \) and \( E_3 \) strengths from recent models, we find that there are detailed differences with most calculations. We have confirmed the suitability of Ra isotopes with \( N \sim 136 \) for EDM searches. Further studies of Hg, Rn and Ra isotopes will be extended at the HIE-ISOLDE facility.

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References
[1] Julin R Helariutta K and Muikku M 2001 J. Phys. G 27 R109
[2] Heyde K and Wood JL 2001 Rev. Mod. Phys. 83 1467
[3] Bonn J et al. 1972 Phys. Lett. B 38 308
[4] Frauendorf S and Pashkevich V V 1975 Phys. Lett. B 55 365
[5] Bonn J, Huber G, Kluge H-J and Otten E W 1976 Z. Phys. A 276 203
[6] Van Duppen P et al. 1984 Phys. Rev. Lett. 52 1974
[7] Wood J L et al. 1992 Phys. Rep. 215 101
[8] Bijnens N et al. 1996 Z. Phys. A 356 3
[9] Jenkins D G et al. 2000 Phys. Rev. C 62 021302(R)
[10] Andreyev A N et al. 2000 Nature 405 430
[11] Bree N et al. 2014 Phys. Rev. Lett. 112 162701
[12] Warr N et al. 2013 Eur. Phys. J. A 49 40
[13] Czosnyka T Cline D and Wu CY 1982 Bull. Am. Phys. Soc. 28 745
[14] Rapisarda E et al. (to be published)
[15] Scheck M et al. 2010 Phys. Rev. C 81 014310
[16] Gaffney LP et al. 2014 Phys. Rev. C 89 024307
[17] Kumar K 1972 Phys. Rev. Lett. 28 249
[18] Yao JM Bender M and Heenen PH 2013 Phys. Rev. C 87 034322
[19] García-Ramos JE and Heyde K 2014 Phys. Rev. C 89 014306
[20] Butler PA and Nazarewicz W 1996 Rev. Mod. Phys. 68 349
[21] Auerbach N Flambaum VV and Spevak V 1996 Phys. Lett. Lett. 76 4316
[22] Dobaczewski J and Engel J 2005 Phys. Rev. Lett. 94 232502
[23] Ibotson RW et al. 1997 Nucl. Phys. A 619 213
[24] Wollersheim HJ et al. 1993 Nucl. Phys. A 556 261
[25] Gaffney LP et al. 2013 Nature 497 199
[26] Shneidman TM et al. 2003 Phys. Rev. C 67 014313
[27] Robledo LM and Butler PA 2013 Phys. Rev. C 88 051302(R)
[28] Nazarewicz W et al. 1984 Nucl. Phys. A 429 269
[29] Nomura K et al. 2014 Phys. Rev. C 89 024312
[30] Rutz K et al. 1995 Nucl. Phys. A 590 680
[31] Engel J et al. 2003 Phys. Rev. C 68 025501
[32] Cocks JFC et al. 1999 Nucl. Phys. A 645 61
[33] Lindroos M Butler PA Huyse M and Riisager K 2008 Nucl. Instrum. Meth. B 266 4687
[34] Thorsteinsen TF Nybo K and Lovhøiden G 1990 Physica Scripta 42 141
[35] Lighthall JC et al. 2010 Nucl. Instrum Meth. A 622 97
[36] M Gieser M et al. 2012 Eur. Phys. J. Special Topics 207 1