1. Introduction

In tokamaks, crossing certain operational boundaries in plasma density ($n_e$) [1], current ($I_p$) [2, 3], or normalized pressure ($\beta$) [4] can lead to a disruption, a sudden and uncontrolled loss of thermal and magnetic energy in the plasma. Among these, high-$\beta$ disruptions are particularly challenging, not only because of the high thermal energy content of the plasma, but also because of their extremely fast time-scales in some cases. Here the plasma $\beta$ is defined as $\beta \equiv \frac{2\mu_0\langle p \rangle}{B^2}$, where $\langle p \rangle$ is the volume-averaged pressure.

Most high-$\beta$ disruptions are mediated by a (neoclassical) tearing mode (NTM), typically with the poloidal and toroidal mode numbers $m = 2, n = 1$, respectively, that for various reasons lock to the wall, grow in size and eventually cause a loss of confinement [5, 6]. Even when they do not lead to disruptions, NTM’s tend to degrade confinement significantly so that their avoidance or stabilization in the ITER ELMy H-mode baseline scenario has been a high-priority research item (see for example [7, 8]). When the plasma $\beta$ is pushed higher beyond the no-wall limit in ‘hybrid/advanced tokamak’ regimes, more dangerous $n = 1$ kink modes can become unstable. In the presence of a close-fitting wall, these are generally transformed into slow-growing resistive wall modes (RWM’s) [9]. Again, if they are allowed to lock to the wall, RWM’s can lead to disruptions. Fortunately, plasma rotation [10–13], kinetic effects [14–16], coupled with feedback-control methods [5, 17, 18], can stabilize RWM’s well above the no-wall $\beta$ limit.

Since both NTM’s and RWM’s grow on a slow, resistive time scale, disruptions caused by these modes are easily identified by their long precursors on various diagnostics. In fact, because of their relatively slow time scale, these are precisely the type of disruptions that are targeted by various
disruption mitigation schemes, which require at least a few 10’s of milliseconds of warning time [19, 20]. As stated earlier, however, tokamaks disrupt for a wide variety of reasons (see for example [6]), and not all disruptions follow this slow path where their arrival is well-advertised in advance; some in fact occur with little warning. Unfortunately, their very fast time scale apparently makes detailed studies difficult, and it is likely that their rare appearance in the literature does not accurately reflect their actual frequency in the experiments.

There do exist some documented high-$\beta$ disruptions with precursors of the order of a millisecond or less. For example: $\beta$-limit disruptions in TFTR due to toroidally localized ballooning modes in the presence of $n = 1$ magnetohydrodynamic (MHD) activity [21, 22], localized resistive interchange mode that couple to a global $n = 1$ mode and lead to a disruption in negative central shear (NCS) discharges in DIII-D [23, 24], and disruptions following an internal transport barrier (ITB) collapse in JET [25]. In these discharges, some of the important details were clearly different: in TFTR, at least initially, the $q = 1$ surface was involved, whereas DIII-D and JET presumably both had $q_{\text{min}} \simeq 2$. But generally, a large pressure gradient in regions of weak magnetic shear is believed to have played an essential role. Thus, one of the things we will do in this work will be a short review of the resistive and ideal stability of such configurations.

However, linear stability analysis alone cannot explain the fast timescale of these disruptions. The mode that is involved has to be growing near Alfvénic rates to account for the time scale, but it is not clear how a discharge evolving on the slow transport time scale can generate an unstable mode with a near-Alfvénic growth rate without producing a long series of precursor oscillations during its sub-Alfvénic period. Thus the mode that is involved in this work will be a short review of the resistive and ideal MHD dispersion relation for pressure-driven modes leads to

$$\gamma(t) = \gamma_{\text{mbd}}(t/\tau_{\text{H}})^{1/2}, \quad \gamma_{\text{mbd}} \propto 1/\tau_{\text{mbd}},$$

and $\gamma_{\text{mbd}}$ is a measure of the ideal MHD time scale. At this point, we can generalize to a wider class of modes and let

$$\gamma(t) = \gamma_{\text{mbd}}(t/\tau_{\text{H}})^{\alpha}, \quad \alpha = 1/2 \text{ for ideal modes, } \alpha = 2/3 \text{ for resistive interchange modes, etc.}$$

Then the time evolution of the displacement $\xi$ for a linear perturbation will be determined by $d\xi/dt = \gamma(t)\xi$, which leads to

$$\xi(t) = \xi_0 e^{(t/t_{\text{d}})^{1+\alpha}}, \quad \tau \equiv [(1+\alpha)/\gamma_{\text{mbd}}\tau_{\text{H}}]^{-1/\alpha}.$$  \hspace{1cm} (1)

Thus the displacement will grow at a faster-than-exponential rate $(1 + \alpha \geq 3/2)$, with the apparent potential to avoid long precursors. However, as Cowley has shown [28], because of the large separation in the MHD and transport time scales, this path to super-exponential growth requires an unrealistically small initial perturbation and can be ruled out. This point follows from the following simple argument: a fast disruption implies that the growth rate $\gamma \propto \gamma_{\text{mbd}}$. But $\gamma(t)$ will grow to this amplitude in a time $t \sim \tau_{\text{H}}$, as implied by the discussion above. Unfortunately, during this time, the initial perturbation $\xi_0$ will have grown by a factor of $\xi(\tau_{\text{H}})/\xi_0 = \exp(\tau_{\text{H}}/\gamma_{\text{mbd}})^{(1+\alpha)/\alpha} \approx \exp(\tau_{\text{H}}/\gamma_{\text{mbd}})$. Conservatively using $\tau_{\text{H}} \sim 10^{-1} \text{ s}$, $\gamma_{\text{mbd}}^{-1} = 10^2 \tau_{\text{mbd}}^{-1} \sim 10^{-5} \text{ s}$, we see that any reasonable initial perturbation will grow to macroscopic, machine-size length scales long before the growth rate becomes Alfvénic.

There are, however, nonlinear processes in plasmas that can generate explosive (faster-than-exponential) growth while the underlying mode is still not far from marginal stability. In a numerical study of the semi-collisional/collisionless $m = 1$ mode using a reduced two-fluid model, nonlinearities involving the parallel pressure gradient were shown to give a near-exponential increase in the growth rate of the mode [29], providing a possible explanation for precursor-less, fast sawtooth crashes. Similarly, Cowley and colleagues [28, 30–32] have shown that the nonlinear evolution of pressure-driven modes can generate finite-time singularities, again demonstrating how a long period of precursors can be avoided during a fast disruptive event.

Some previous numerical work has failed to confirm the existence of an explosive instability. Park et al [33], in their MHD studies of a high-$\beta$ disruption on TFTR, does not reproduce the apparently explosive growth in the ECE data (compare the computational result in their figure 6 with the experimental ECE signal in figure 7, which is very much like the ECE signals from JET [25] discussed elsewhere in this work). Lütjens and Luciani [34] find that $n = 1$ kink-driven ballooning modes can exhibit a ‘nonlinear destabilization’ without an explosive phase, but the calculation seems to have been terminated prematurely—a longer calculation made possible with more poloidal and toroidal Fourier modes might have given a different result. Similarly, Zhu et al [35], in an under-resolved nonlinear study of an $n = 15$ ballooning mode, see only an exponential growth well into the nonlinear regime. However, a work by Myers et al [31] studying ballooning modes near marginal stability does find an explosive phase that seems to terminate in a finite-time singularity.

In this work we extend our study of a specific example [36], a pressure-driven $n = 1$ kink-balloon mode that can continue to grow exponentially well into its nonlinear regime and become explosive with an apparent finite-time singularity at the end. We show that it can actually exhibit two very different types of nonlinear behavior depending on small differences in the assumed transport levels and linear perturbation amplitudes (bifurcated states in section 4). In addition to the explosive behavior, it can also display a more benign evolution and saturate in a ‘long-lived mode’ (LLM) with only minor confinement degradation [37]. The LLM itself is shown to be a metastable state (section 4). It can be pushed into the explosive regime with small changes in the transport coefficients or with a finite-size perturbation.
The experimental context for this computational study is KSTAR discharges with $q_{95} \simeq 7$, $q_{99} \simeq 2$, and a low inductive current fraction, similar to some hybrid/advanced scenarios [38]. With an axial electron cyclotron resonance heating (ECRH), the pressure profile peaks and drives an ideal $m/n = 2/1$ mode that saturates at a small amplitude. The resulting LLM survives many tens of seconds (as long as ECRH is maintained), with only a small effect on confinement [37]. Although there are no documented examples for the explosive version of this mode in KSTAR, in the absence of any detailed study of KSTAR disruptions, their existence cannot be ruled out. The computational tool used here is the CTD code, which solves the nonlinear MHD equations in toroidal geometry (see [39] and the references therein).

Before moving on to a discussion of the nonlinear results, in the next section we briefly review the salient features of the linear stability of pressure-driven modes.

2. Linear stability

A general understanding of the stability of pressure-driven modes can be obtained from a cursory examination of the ideal MHD energy integral, written here in its `intuitive form' (plasma contribution only) [9, 40]:

\[
\delta W_p = \frac{1}{2} \int \left\{ \frac{Q_\perp^2}{\mu_0} + B^2/\mu_0 |\nabla \cdot \xi_\perp + 2\xi_\perp \cdot \kappa|^2 + \gamma p |\nabla \cdot \xi_\perp|^2 \right\} dV + \frac{1}{2} \int \left\{ -2(\xi_\perp \cdot \nabla p)(\xi_\perp \cdot \kappa) - J/\mu_0 (\nabla \times B) \cdot \xi_\perp \right\} dV.
\]

The largest stabilizing contribution tends to be the $|Q_\perp|^2$ term in the first integral representing the line-bending energy, where $Q = \nabla \times \xi = B_\parallel$ is the perturbed field. The second and third terms in the first integral represent the energy required to compress the magnetic field and the pressure, respectively [9].

The destabilizing pressure-gradient and parallel current terms are grouped together in the second integral.

The pressure gradient makes a destabilizing contribution to $\delta W_p$ only in those regions where the field line curvature $\kappa \equiv \nabla \cdot B = (\mu_0/\mu_0) \nabla \cdot (p + B^2/2\mu_0)$ is `unfavorable, i.e. where $\kappa \cdot \nabla p > 0$. By having the displacement vanish where the curvature is favorable, $\kappa \cdot \nabla p < 0$, the net destabilizing contribution from the pressure forces can be maximized. This is the path the ballooning modes take, but they pay a price in excess line-bending energy since the perturbation is not constant along the field lines.

Another path for pressure-driven instabilities opens up if the magnetic shear is weak in a region of finite width. Simplifying and expanding $Q$ around a rational surface, we have $Q_\perp \simeq \xi_\perp (k \cdot B_0)$, and

\[
Q_\perp^2 \simeq \xi_\perp^2 (m - n)^2 \simeq \xi_\perp^2 (nq)^2 (r - r_c)^2,
\]

where $r_c$ is the radius of the rational surface. Thus, if the global shear $s \equiv r q/q$ is weak enough in regions with strong pressure gradients, interchange-like modes become possible even in ‘Mercier-stable’ equilibria with $q^2 > 1$, first recognized by Zakharov [41]. In fact, a rational surface is not necessary for instability. With $q \simeq (m + \epsilon)/n$, $0 < \epsilon \ll 1$, and $s \ll 1$, the line-bending energy can be minimized again since $Q_\perp^2 \simeq \xi_\perp^2 c^2$, which can be overcome by a strong-enough pressure gradient.

This simple piece of physics, strong pressure drive coupled with weak shear, is behind the quasi-interchange mode [42–45] (for $q_{\text{min}} \simeq 1$) and the ‘infernal’ modes [46–48] (for $q_{\text{min}} > 1$), both pressure-driven modes in low-shear equilibria. The former was studied in the context of fast sawtooth crashes caused by an internal $m/n = 1/1$ mode. The latter are particularly dangerous global modes that can be unstable much below the $n \to \infty$ ballooning limit and lead to major disruptions. They are typically thought of as ‘low-$n$’ modes, but of course the same physics can also make the $n = 1$ mode unstable, which will be the focus of this work.

For computational economy, our earlier work [36] focused on the nonlinear evolution of pressure-driven $n = 1$ modes in circular geometry, in both monotonic and weakly-reversed $q$ profiles. Here we will extend it to non-circular geometry and provide a brief review of the linear properties of the relevant modes. Partly because of KSTAR’s recent interest in advanced scenarios with internal transport barriers (ITB’s), we will mainly consider reversed-shear equilibria with $q_{\text{min}} > 2$.

Typical equilibrium profiles used in the linear and some of the nonlinear calculations are shown in figure 1. The shaped geometry has $\kappa = 1.5$ (elongation) and $\delta = 0.6$ (triangularity) within a perfectly conducting boundary; these geometric parameters are held fixed, except when we revisit circular-geometry. The CTD code uses a conformal transform from the poloidal plane to a unit circle in $(\rho, \omega)$ coordinates to deal with weakly-shaped equilibria [49]. The coordinate axis is shifted to approximately align the $\rho \equiv \text{const.}$ surfaces with flux surfaces, but $\rho$ is not a flux coordinate. For this reason, the plots as in figures 1(b) and (c) show both the $\omega = 0$ (outboard) and $\omega = \pi$ (inboard) sections of the mid-plane. Note that a simple pressure profile without an internal or edge transport barrier is used to simplify the discussion.

As expected, resistivity enlarges the instability domain for the $n = 1$ mode ($n > 1$ stability is not considered in this work) so that an unstable mode is observed well below the ideal MHD stability limits. However, we find that the nature of the unstable resistive mode can be confusing. The ‘infernal’ mode theory predicts a mode with a tearing scaling, $\gamma \tau \propto S^{-3/5}$, at low $\beta$. Close to the ideal stability boundary, the resistivity scaling is weaker, $\gamma \tau \propto S^{-3/13}$, becoming independent of $S$ beyond the ideal limit [47]. Here we define $\tau_r$ as the shear-Alfvén time, $\tau_A = R_0/v_A$, $v_A = B/\sqrt{\mu_0 \rho m_c}$. Then the magnetic Reynolds (Landquist) number is given by $S = \tau_r/\tau_A$, where $\tau_r = \mu_0 c^2/\eta$ is the resistive diffusion time, and $\alpha, R_0$ are the minor and major radii of the torus, respectively.

The well-known interchange theory also predicts a resistive mode with the usual interchange scaling, $\gamma \tau \propto S^{-1/3}$ [50], but only for reversed-shear equilibria. If we briefly recall the relevant theory, the Mercier (ideal interchange) modes are unstable for $D_l \equiv D_M - 1/4 > 0$, where $D_M = -(2\mu_0 \rho p' / B^2 \xi^2)^2 (1 - q^2)$ in circular geometry [51]. Although rare, Mercier modes have actually been observed experimentally [52]. Resistive instabilities require
weakly-reversed shear (assumes viscous effects. The classical viscous-tearing mode theory that assumes a resistive interchange mode scaling (S^-3/5) that is in competition with the ideal tearing scaling (S^-3/4). Here it is possible that there is a resistive infernal mode (with the S^-3/13 scaling [47]) that is in competition with the interchange, but it is not observed numerically. The weak reversed shear (not considered in the infernal mode theory) may be making the resistive interchange the dominant mode in this particular parameter regime.

At $\beta_N = 1.82$ (figure 2(b)), the mode is still resistive and has a clear resistive interchange scaling, $\gamma T_A \propto S^{-1/3}$. Here it is possible that there is a resistive infernal mode (with the $S^{-3/13}$ scaling [47]) that is in competition with the interchange, but it is not observed numerically. The weak reversed shear (not considered in the infernal mode theory) may be making the resistive interchange the dominant mode in this particular parameter regime.

Figure 1. Typical equilibrium profiles used in the linear and some of the nonlinear calculations. Here $q_0 = 2.15$, $q_{\text{min}} = 2.02$, $q_i = 9.21$, $\beta_N = 1.82$, $\delta = 0.6$, $\kappa = 1.5$. (a) Flux surfaces. (b) Current density (left axis) and pressure (right axis) profiles. The horizontal coordinate $\rho$ is that of the ($\rho, \omega$) conformal coordinates (see text). (c) Safety factor profile. The inset shows a magnified view of the central region.

Figure 2. Growth rate of the $n = 1$ mode as a function of the magnetic Reynolds number $S$ for various values $\beta_N$, for $q_0 = 2.15$, $q_{\text{min}} = 2.02$, $q_i = 9.21$ (see figure 1). (a) The blue dashed line represents the $S^{-3/5}$ tearing mode scaling, but $S^{-3/4}$ (red line) fits the computational data better (see text). (b) The data exhibits $S^{-3/4}$ dependence with $\gamma \tau_A \propto S^{-3/4} \bar{\kappa}$. Here $\bar{\kappa} = a/(1 + \kappa^2)/2^{1/2}$ is an equivalent minor radius defined for an equilibrium with elongation $\kappa$, and $I_p$ is the plasma current. The $S$-scans are performed at a constant magnetic Prandtl number, $P_M = \mu/\eta = 10$, where $\mu$ is the normalized viscosity coefficient. Although normalized viscosity tends to be higher than resistivity in fusion plasmas, this value of $P_M$ is chosen entirely for numerical reasons.

At $\beta_N = 1.82$ (figure 2(a)), there is a weakly unstable resistive mode; both the inertial mode and the resistive interchange theory seem to predict here a tearing-like scaling with $\gamma T_A \propto S^{-3/5}$ (the blue dashed line), but we find that a stronger dependence with $\gamma T_A \propto S^{-3/4}$ (the red line) is a better fit to the numerical data. Neither one of these theories takes into account viscous effects. The classical viscous-tearing mode theory that assumes $P_M < 1$ predicts a mode with the $S^{-2/3}$ scaling [53], which is somewhat weaker than the $S^{-3/4}$ scaling we observe. It is possible that for $P_M > 1$, the $S^{-2/3}$ scaling changes to $S^{-3/4}$, but that possibility has not been investigated.

Figure 3. Eigenfunctions for the radial velocity $u_r$ and magnetic field $B_p$ for $m/n = 2/1$ and $m/n = 3/1$ (in arbitrary units) at $S = 10^6$ for the equilibrium shown of figure 1. (a) and (b) $\beta_N = 1.82$. (c) and (d) $\beta_N = 2.62$. At $\beta_N = 2.62$ (figure 2(b)), the mode is still resistive and has a clear resistive interchange scaling, $\gamma T_A \propto S^{-1/3}$. Here it is possible that there is a resistive infernal mode (with the $S^{-3/13}$ scaling [47]) that is in competition with the interchange, but it is not observed numerically. The weak reversed shear (not considered in the infernal mode theory) may be making the resistive interchange the dominant mode in this particular parameter regime.
At an even higher $\beta$ ($\beta_N = 3.35$, panel (c)), the mode is ideally unstable (with wall). During this study, $\beta_N$ was increased in large steps so that the locations of the transition points between various regimes are not known, a task left for a future work.

Since there is no $q = 2$ rational surface in the plasma for the series of equilibria considered here ($q_{\text{min}} > 2$), and because of the wide region of weak magnetic shear around $q_{\text{min}}$ (see figure 1), the eigenfunctions do not exhibit a distinctive 'singular' behavior there. In fact, they have the features of a global kink mode, as seen in figures 3(a) and (b) for $\beta_N = 1.82$, and figures 3(c) and (d) for $\beta_N = 2.62$. Toroidal coupling to the 2/1 mode seems to drive the 3/1 unstable also, which appears to be a $\Delta'$-stable tearing mode. The mixing of the $m = 2, 3$ components is visible also in the pressure perturbation shown in figure 4(a). Note that although only two modes are shown, the linear calculations for $n = 1$ included all poloidal mode numbers in the range $m \in [-7, 57]$.

Above we discussed in some detail the linear stability for $q_0 = 2.15$, $q_{\text{min}} = 2.02$, mainly to place our nonlinear calculations below in some context. Summarizing our other linear results, for a more deeply-reversed equilibrium with $q_0 = 2.57$, $q_{\text{min}} = 2.02$ we find no ideal instability for $\beta_N \leq 3.08$, the limit of our numerical explorations for this q-profile. On the other hand, for an equilibrium with $q_0 = 2.03$, $q_{\text{min}} = 2.02$ (much weaker central shear), we find that $\beta_N \geq 2.72$ is ideally unstable, although the actual stability boundary has not been explored and is probably lower (but higher than $\beta_N = 1.90$, where we find a resistive mode).

Note that the theory of toroidal instabilities is usually not as clear cut and unambiguous as their simplified versions in cylindrical geometry. Here a non-resonant pressure-driven mode, essentially a 2/1, is toroidally coupled to a resonant 3/1 (and 4/1, etc) that is driven unstable, leading to reconnection at the $q = 3$ surface. At the same time, the 2/1 is close to an ideal instability boundary and becomes resistively unstable itself before the ideal mode is triggered. Because of weak shear and the absence of the $q = 2$ rational surface, this interchange-like mode fails to become localized and exhibits a more global eigenfunction (figure 3). Whether the mode displays a tearing, interchange, or ideal character is determined by the coupling among all these processes. Therefore, the numerical results tend to be ambiguous and confusing below the ideal limit.

3. The explosive instability and disruptions

Nonlinearly the pressure-driven $n = 1$ mode can turn into an explosive instability, as it was first demonstrated in circular geometry [36]. Here we discuss the nonlinear evolution of the mode in shaped geometries using the linear results of the previous section as starting points.

An ideally unstable $n = 1$ mode (e.g. one from figure 2(c)) with its large growth rate will naturally lead to a fast disruption. As discussed at some length in the Introduction, however, MHD modes are not ‘born’ in this robustly unstable state. They tend to come into existence as weak resistive instabilities as the equilibrium slowly (on transport time scale) passes through some marginal stability point due to the evolving discharge conditions. Hence our goal in this section is to demonstrate how a weak resistive instability can evolve into a robust mode that will result in a fast disruption with only a brief period of precursors.

Thus we start with a weakly unstable equilibrium similar to that of figure 1: some of the relevant parameters are: $q_0 = 2.145$, $q_{\text{min}} = 2.023$, $q_1 = 9.424$, $\beta_N = 1.67$, which results in an even weaker instability than that of figure 2(a).

The nonlinear calculations are typically performed at the Lundquist number $S = 10^6$ and the Prandtl number $P_M = \mu_0S = \mu_0/\eta_0 = 10$. The viscosity coefficient is fixed in time and space, $\mu = \mu_0$. Throughout this work, $S$ is defined in terms of the value of resistivity at the coordinate axis, but the resistivity itself is in general a function of the poloidal coordinates such that $\eta(\rho, \omega)J_{\rho\omega}(\rho, \omega) = E_0 = \text{const}$. The electric field $E_0$ is associated with the ‘loop voltage’. Numerically it is used to prevent the Ohmic diffusion of the equilibrium current during long nonlinear calculations. Of course other resistivity profiles, e.g. a ‘classical resistivity’ with a density and temperature dependence, can also be used, as long as its effect is small enough not to adversely affect the current and q profiles during the calculations. The mass density $\rho_m$ is not explicitly evolved, and the normalized density is set to

![Figure 4](image-url)
unity, \( \rho_0 = 1 \). Instead, the pressure is evolved, typically with \( \kappa_A = 10^{-6} \). Parallel thermal conductivity, normally treated using an algorithm similar to Park’s [54], is not used for reasons of computational economy. These simplifications were found not to qualitatively change the results in nonlinear tests. The nonlinear calculations use 21 toroidal Fourier modes. The poloidal Fourier expansions have \( m, n \in [0, 64] \) for \( n = 0 \) and typically \( m, n \in [-5, 64] \) for \( n \in [1, 20] \). The finite difference scheme in the radial direction uses 192 grid points. Some of the algorithmic details of the CTD code used here can be found in [39] and the references therein.

With weak shear and \( q_{\text{max}} > 2 \), an important feature of the linear eigenfunction is the dominance of the \( m = 2 \) poloidal component. This is clearly seen in figure 4(a), which shows the quadrupole geometry of the pressure perturbation (some coupling to an \( m = 3 \) on the outside is also visible). This perturbation leads to an elliptical deformation of the flux surfaces in the core plasma that eventually forms a ballooning finger, as seen in figures 4(b) and (c). The finger pushes through the flux surfaces on near-Alvénic time scales and brings the core plasma in contact with the boundary (figure 4(d)). Using typical parameters for modern tokamaks (\( B = 3 \, \text{T}, n_e = 10^{19} \, \text{cm}^{-3}, R_0 = 1.5 \, \text{m} \)), the state with no visible deformation in figure 4(b) and the final state in figure 4(d) are separated by less than 1 ms. Thus an actual disruption following this path would have a very short warning time.

The rapidity of the final disruptive phase is clearly due to a nonlinear increase in the growth rate rather than a slow, transport time scale change. This is seen in figure 5 where the kinetic energy in the \( n \geq 1 \) modes (excluding the \( n = 0 \) equilibrium flows) is plotted. Two points are immediately obvious: (i) the mode continues to exponentiate well into the nonlinear regime with a growth rate \( \gamma_A = 1.70 \times 10^{-3} \) (the dashed red line in panel (a)). Figure 4(b) is meant to be from the beginning of the nonlinear phase, here defined as the start of the finger formation. (ii) Instead of saturation, the late stages are characterized by a superexponential or explosive growth. In fact this phase has the appearance of a finite-time singularity (panel (b)) where the growth has the form

\[
W_K(t) = W_K(t_i)[(t_f - t_i) / (t_f - t)]^\nu,
\]

where \( t_i = 2773.9 \), \( t_f = 4277.6 \), and the exponent \( \nu = 2.05 \). In our earlier calculations in circular geometry the explosive phase was even faster, with \( \nu = 3.37 \) [36]. The slowdown seen here has both physical and numerical sources: the underlying \( n = 1 \) mode is inherently more stable in shaped geometry. And, because the poloidal spectrum for each toroidal mode is much wider in shaped geometry, fewer \( n_{\text{max}} = 20 \) toroidal modes were used here. In circular geometry we were able to use \( n_{\text{max}} = 30 \), which allowed us to continue the calculations further into the explosive phase.

A more intuitive picture of the instability can be obtained by examining figure 6 in conjunction with figure 4. The initial elliptical expansion of the plasma is facilitated by weak shear, which allows the pressure perturbation to push the field lines without deforming them much. At this stage, the mode is toroidally extended, as seen in figure 6(a). Beyond the low-shear region, further expansion requires that the reduction in the internal energy more than compensate the increase in the line-bending energy. Thus, the mode assumes an obvious ballooning character by toroidally localizing to the ‘bad curvature’ side of the torus. As it pushes through field lines with increasing safety factor \( q \), it becomes increasingly more localized in both parallel (along the field) and perpendicular

\[ \text{Figure 5. Time history of the total kinetic energy in the } n \geq 1 \text{ modes. (a) The dashed (red) line corresponds to a growth rate of } \gamma_A = 1.70 \times 10^{-3}. \text{ Note that the growth becomes superexponential beyond } t \approx 3000. \text{ (b) The finite-time singularity model of equation (4) is a good fit to the explosive phase (the dashed curve). The ballooning finger slows down beyond } t \approx 4100 \text{ as it starts coming into contact with the wall.} \]

\[ \text{Figure 6. Pressure contours in the } (\omega, \zeta) \text{ plane at } t = 4270.9. \text{ The outboard (inboard) midplane is at } \omega = 0 (\pi). \text{ Two constant-} \rho \text{ surfaces are shown: (a) At } \rho = 0.31, \text{ the finger shows a ballooning structure but is almost completely extended around the torus along the field lines. (b) Further out at } \rho = 0.84, \text{ the finger is more localized, in both parallel and perpendicular directions. Note that the } m/n = 2/1 \text{ helicity is preserved.} \]

\[ \text{Figure 6. Pressure contours in the } (\omega, \zeta) \text{ plane at } t = 4270.9. \text{ The outboard (inboard) midplane is at } \omega = 0 (\pi). \text{ Two constant-} \rho \text{ surfaces are shown: (a) At } \rho = 0.31, \text{ the finger shows a ballooning structure but is almost completely extended around the torus along the field lines. (b) Further out at } \rho = 0.84, \text{ the finger is more localized, in both parallel and perpendicular directions. Note that the } m/n = 2/1 \text{ helicity is preserved.} \]
Figure 7. Comparison of synthetic ECE diagnostics with the JET data from [25]. (a) Computational data with an assumed rigid toroidal rotation of period $425 \tau_R$. Note the narrowing of the finger in the time domain as it propagates radially outward, which of course corresponds to toroidal localization. (b) Experimental ECE contours from JET. Reprinted from [25], Copyright 2005, with permission from Elsevier.

Figure 8. Comparison of synthetic ECE diagnostics from our calculations with the JET data from [25] (their figure 4) shows good agreement, as seen in figure 7. In addition to disruptions, there are also detailed experimental studies of rapidly propagating finger-like structures in JET that cause serious confinement degradation without leading to a major disruption [56, 57]. The ballooning finger seen in this work is certainly a good candidate to explain the ‘black-out’ in ECE and other diagnostics seen during such events.

Although there are several observations in KSTAR that may be associated with fast, high-$\beta$ disruptions without significant precursors, the necessary analysis to link them formally to an explosive instability has not been carried out.

4. Bifurcated states

Generally, away from exact marginal points, we expect small changes in a relevant parameter to result in similarly small changes in the evolution of an unstable mode. Thus, small differences in the resistive dissipation level rarely have a significant impact on the saturation width of a tearing mode. However, there are counter examples where, for instance, an increase in the Prandtl number (ratio of viscous to resistive dissipation) beyond a threshold leads to a qualitatively different nonlinear regime [58].

Here we demonstrate an extreme case where a small change in a transport coefficient leads to a bifurcation between a benign, saturated state (the LLM) and an explosive instability for the $n = 1$ kink-ballooning mode. For computational economy, we expand upon our earlier results [36, 37] while staying in circular geometry. The bifurcation is summarized in figure 8 where we follow the nonlinear evolution of the mode starting with the same initial conditions and linear perturbation, but using slightly different transport coefficients. With $S = 10^6$, thermal conductivity $\kappa_\perp = 4 \times 10^{-6}$ and viscosity $\mu = 1 \times 10^{-5}$, the mode goes through an exponential growth phase but saturates at a small amplitude (curve (1) in figure 8(a)). This regime is identified with the LLM observed in KSTAR, where an $m/n = 2/1$ perturbation is seen in the electron cyclotron imaging (ECEI) data for many tens of seconds during the current flat-top period. The experimental conditions under which the LLM was observed is described in more detail in [37, 59].

In a slightly less dissipative system with $\kappa_\perp = 3.5 \times 10^{-6}$ but $\mu = 1 \times 10^{-5}$ still, the mode initially follows a similar path and goes through an exponential-growth phase (curve (2) in (a)). However, instead of saturating, it gradually enters a super-exponential regime where the growth rate increases rapidly, eventually becoming explosive. A similar effect is seen with reduced viscosity: with $\mu = 6 \times 10^{-6}$, $\kappa_\perp = 4 \times 10^{-6}$ the mode again transitions into the explosive regime (curve (3)).

This bifurcation can be understood qualitatively if we assume the explosive phase has a finite threshold in the perturbation amplitude. Dissipation affects both the linear growth rate and the nonlinear saturation amplitude of the unstable mode. The higher dissipation level clearly causes the mode to saturate below the apparent threshold. This point is confirmed in the next section where we show that the curve (1) of figure 8(a) represents a continuous set of metastable states.
5. Metastability

The \( n = 1 \) mode for the set of equilibria we consider here is lin-
early unstable, which implies that an infinitesimal perturbation
will grow in time exponentially, at least until the mode attains
a finite amplitude. The results of the previous section imply
that whether it turns into a LLM or becomes explosive is
determined by a critical perturbation amplitude that itself is a
function of the dissipation coefficients, \( \xi = \xi(\eta, \mu, \kappa_{\perp}) \). If
it nonlinearly saturates below the threshold, \( \xi < \xi_c \), the result
is a benign LLM. Above \( \xi_c \) it becomes explosive. Thus, the
initial equilibrium that is unstable to the pressure-driven
\( n = 1 \) mode is said to exhibit metastability. In this section this
theoretically predicted behavior [32, 55] is demonstrated numerically.

To show metastability, a LLM, identified by the dashed
curve (1a) in figure 9, is perturbed at various points along its
trajectory with \( n \) perturbations of varying amplitude. The relevant transport coefficients for this baseline, metastable
state were: \( S = 10^6 \), thermal conductivity \( \kappa_{\perp} = 4.75 \times 10^{-6} \),
viscosity \( \mu = 1.0 \times 10^{-5} \), and the initial equilibrium was
perturbed with the \( n = 1 \) linear eigenfunction using a per-
turbation amplitude of \( \epsilon = 1.0 \times 10^{-6} \) (The parameter \( \epsilon \)
is used to rescale the \( n = 1 \) component of the velocity and
other computational fields.). The same equilibrium, when
perturbed with \( \epsilon = 2 \times 10^{-6} \), evolves into an explosive mode
(curve 1b). Similar numerical experiments are performed at
later points in the nonlinear evolution of the baseline LLM:
at \( t = 1012 \), \( \epsilon = 4.0 \times 10^{-6} \) (curve 2a) is stable, however
\( \epsilon = 5.0 \times 10^{-6} \) becomes explosively unstable (curve 2b). At
a later time, \( t = 1256 \), the same perturbation (\( \epsilon = 5 \times 10^{-6} \)
leads to large-amplitude, damped oscillations but the mode
remains stable (curve 3a). A slightly larger perturbation
(\( \epsilon = 6 \times 10^{-6} \)) again becomes explosively unstable (curve 3b).

Clearly, the quasi-equilibrium states representing the early
nonlinear phase of the LLM are metastable with respect to
finite (as opposed to infinitesimal) perturbations. Transition to
the explosive regime requires a perturbation amplitude above

a threshold. The critical amplitude increases in time, which
is expected since the background pressure profile relaxes due
to dissipation, thus gradually reducing the free-energy source
for the mode.

Note that metastability is not uncommon in fusion plasmas.
For instance, neoclassical tearing modes are known to require
a seed-island larger than a critical width \( w_{\text{crit}} \) (see, for example,
[60, 61]). For an initial island width \( w < w_{\text{crit}} \), the perturbation
decays away, while an NTM is formed for \( w > w_{\text{crit}} \). Figure 9
describes a similar behavior, except below a critical amplitude
the mode saturates in a LLM rather than decaying away, and
the other branch of the bifurcation is an explosive mode.

6. One-dimensional model

In this section we present a simple system that exhibits linear
instability, metastability and explosive behavior, which should
make the nonlinear results of the previous sections more intu-
itive and easier to understand. Of course the results of this
one-dimensional model are not meant to be a quantitative
the particle is able to move past the saddle point s at oscillations. However, with only slightly lower dissipation, any positive velocity perturbation \( v > v_{\text{cr}} \) leads to a stable state. Slightly lower dissipation, the damped oscillations of figure 10(a) will not be able to climb out of the well and eventually settle at the metastable equilibrium point marked with m at \( \xi = 1.39 \). This behavior is shown in figure 10(b), where, after a period of exponential growth, the displacement \( \xi(t) \) exhibits damped oscillations. However, with only slightly lower dissipation, the particle is able to move past the saddle point s at \( \xi = 2.60 \) and become ‘explosively unstable’, as seen in panel (c). Of course the same result can be achieved by keeping the dissipation level constant but increasing the initial perturbation (panel (d)).

The damped oscillations of figure 10(b) correspond to the LLM described by curve (1) in figure 8(a). The explosive instability in figure 10(c) that develops at a lower dissipation level corresponds to the curves (2) and (3) of figure 8(a). Finally the explosive instability in figure 10(d) that results from a higher initial velocity corresponds to the explosive curve 1b in figure 9 that follows a large perturbation of the initial equilibrium. Curves 2b, 3b of figure 9 have not been simulated with the 1D model, but clearly they correspond to large-velocity perturbations of the damped oscillations in figure 10(b).

Of course the results of this 1D model are meant to be only suggestive of the 3D MHD observations. But some of the differences seen above, ironically, have to do with the computational expense of the 3D calculations, not the shortcomings of the 1D model. For instance, the plateaus seen in figures 10(b)–(d) are due to the extreme closeness of the relevant parameters to a bifurcation point–changes in \( v_0 \) or \( \mu \) occur in the third decimal place. We would expect a similar plateau-like region in, for example figure 8, curves (2), (3), if we could determine the bifurcation points in \( \kappa_{\perp} \) or \( \mu \) with such accuracy, a trial-and-error procedure that would have used too much of our computational resources.

7. Summary and discussion

In this work we demonstrate that the nonlinear evolution of a pressure-driven \( n = 1 \) kink-ballooning mode can exhibit a bifurcation between a benign final state with little confinement degradation—a long-lived mode (LLM), and an explosive instability that results in a fast disruption with very short precursors. The bifurcation depends sensitively on assumed transport levels and the initial perturbation amplitude. Large diffusive transport or too small a perturbation leads to a saturated \( n = 1 \) LLM. Equivalently, there is a transport-dependent critical perturbation amplitude, \( v_{\text{cr}} = \eta \kappa_{\perp} (\beta, \kappa_{\perp}, \ldots) \), such that \( v > v_{\text{cr}} \) leads to explosive behavior. The LLM itself is metastable and can be pushed into the explosive regime, again with a finite perturbation above a threshold. Thus it is possible that a LLM can abruptly terminate with a fast disruption.

Since a benign LLM is a possible end state, it is clear that the initial \( n = 1 \) instability has to be weak and not too far from an instability threshold; a robust and ideally unstable \( n = 1 \) is unlikely to saturate without serious deleterious effects on confinement. Thus we have concentrated on weak, resistive modes far from ideal instability boundaries. This choice follows also from the expectation that an MHD mode does not come into existence as a robustly unstable ideal mode with a large growth rate; the resistive thresholds tend to be much lower and the instability generally appears first as a weak, resistive mode. But this feature (weak instability) that makes a LLM possible would at the same time seem to make it difficult to explain a fast disruption with little or no precursors, the other possible end state of the bifurcation.

This difficulty is resolved by the numerical observation, with some theoretical support (e.g. [32]), that a weakly growing \( n = 1 \) mode can become explosive nonlinearly. Thus, a feeble resistive instability can transform into a robust ideal mode in a short period of time. Although a detailed understanding of the nonlinear process responsible for this transformation is lacking at this point, a simple one-dimensional model is presented that mimics essentially all its important features: a linear instability that can either saturate in a metastable state or lead to a nonlinear explosive instability. Of course the most important consequence of this explosive instability is that it obviates the need for a long period of mode
evolution on transport time scales where it gradually becomes stronger with the changing background equilibrium. Thus a long series of precursor oscillations are avoided.

One particular feature of the pressure-driven \( n = 1 \) mode that plays an important role in the transition to the explosive regime is the quadrupole geometry of the pressure perturbation due to the dominant \( n/m = 2/1 \) harmonic. This perturbation naturally adds an elliptical deformation to the flux surfaces, which can nonlinearly turn into a ballooning finger. Once formed, the finger rapidly moves outward, pushing through flux surfaces while essentially maintaining its original 2/1 symmetry. To minimize bending of the field lines as it moves into regions with \( q \gtrsim 2 \), it becomes localized both in parallel and perpendicular directions. Thus, although it is originally quite extended along the field lines, it turns into a highly local nonlinear structure as it moves to the edge, transporting a significant portion of the energy in the core to a small area on the wall. Although not shown here but discussed elsewhere [36], the rapid ejection of the finger, while the finger itself remains well-confined by regular flux surfaces. These features are consistent with jet-like flows in some high-\( \beta \) disruptions in TFTR [22] and JET [25]. It is likely that the global \( n = 1 \) mode responsible for the well-documented fast high-\( \beta \) disruption in DIII-D [23, 24] also has similar origins. Note that not all ballooning-finger-like events lead to disruptions; in JET sometimes they seem to only degrade the confinement without causing a disruption [56, 57]. And sometimes conditions that we would assume to be favorable do not seem to generate these fast events at all [62].

Finally, ITER disruption mitigation efforts seem to be based on the anticipation that a resistive wall mode or a tearing mode would slowly lock to the wall prior to the disruption, giving at least a few tens of milliseconds of warning time. Without a perfect pressure-profile control system, fast disruptions of the type discussed in this work would make the efficacy of this approach questionable in some of the advanced scenarios planned for ITER.

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