A New Numerical Force Analysis Method of CBR Reducer with Tooth Modification

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Abstract. A new one stage cycloidal reducer called China Bearing Reducer (CBR) which has large transmission ratio, high payload, high torsional stiffness, high tilting stiffness and compact size is designed. In this paper, a new force analysis algorithm is proposed to compute contact force of each tooth with modification. The cycloid drive theory and structure of CBR reducer is introduced firstly. Then the steps of force analysis algorithm are described and a numerical example of CBR25 reducer is analysed based on the algorithm. This method of force computing is more accurate than conventional force computing and can help better designing of CBR reducer.

1. Introduction

The cycloid drive has the advantages of large reducer ratio, high torque, high precision, high stiffness and compact structure. It has been widely used in precision reducer, such as RV reducer. In this paper, the CBR reducer is designed and a new numerical force analysis method is proposed.

Many researches had been done by scholars on cycloid-pin gear reducer. Yang and Blanche [1,2] discussed the formulas of cycloid drive and investigated the effect of machining tolerances on backlash and torque ripple. Then they presented an analytical and computer-aided analysis and synthesis of cycloid drives. Teruaki Hidaka et al. [3] proposed a method by using equivalent dynamic model with equivalent error to analyze rotational transmission error. Subsequently, the effects of machining and assembly errors of elements on the rotational transmission error were investigated [4]. Yunhong Meng et al. [5] proposed a mathematical model of 2K-H cycloid-pin reducer with one tooth difference and analyzed the transmission performance of clockwise and counterclockwise. Carlo Gorla et al. [6] proposed an innovative cycloidal speed reducer whose profile is the external offset of an epitrochoid and investigated the structural characteristics and the kinematic principles. Mirko Blagojevic et al. [7] designed a two-stage cycloid gear reducer and analyzed the loading sharing and stress by FEM. Bingkui Chen et al. [8] proposed a new cycloid drive by applying double-enveloping gear theory, establishing the meshing equation, deriving the equation of tooth profile and meshing line and the formula of induced normal curvature. And the superior characteristics of the new conjugated tooth profile is represented by comparison of induced normal curvature with conventional cycloid drives.

The aim of the paper is to investigate the contact force of cycloidal gear reducer. It will be expected to analyze the load distribution among the multiple contact tooth pairs.

2. Cycloid drive and design of CBR reducer

2.1 Cycloid profile and its modification
As shown in Fig. 3, an overall fixed coordinate system $S_f(X_f - O_f - Y_f)$ and a moving coordinate system $S_p(X_p - O_p - Y_p)$ fixed to the pin gear are established in the center of the pin gear. And a moving coordinate system $S_c(X_c - O_c - Y_c)$ is established in the center of the cycloid gear. In the initial position, the $Y_p$ axis coincides with the $Y_f$ axis, and the $Y_c$ axis is parallel to the $Y_f$ axis. In the IGTS, the cycloid gear rotates $\varphi_c$ at the angular speed $w_c$ counterclockwise around the point $O_c$ (that is, $S_c$ rotates $\varphi_c$ around $O_c$). According to the relative motion relation, the pin gear will rotate $\varphi_p$ at the angular speed $w_p$ counterclockwise around the point $O_p$ (that is, $S_p$ rotates $\varphi_p$ around $O_p$). So we can get

$$i^H = \frac{w_c}{w_p} = \frac{\varphi_c}{\varphi_p} = \frac{z_p}{z_c} \quad (1)$$

Where $i^H$ is the transmission ratio of the cycloid gear and the pin gear. $z_c$ is the number of teeth of the cycloidal gear and $z_p$ is the number of teeth of pins.

Based on the theory of gear meshing and differential geometry, the cycloid disc profile in $S_c$ coordinate system can be expressed as

$$\begin{align*}
x_c &= \left(R_p - R_{rp}S_r^{-\frac{1}{2}}\right)\cos\left[\left(1 - i^H\right)\varphi_p\right] - \left(a - K_1R_{rp}S_r^{-\frac{1}{2}}\right)\cos\left(i^H\varphi_p\right) \\
y_c &= \left(R_p - R_{rp}S_r^{-\frac{1}{2}}\right)\sin\left[\left(1 - i^H\right)\varphi_p\right] + \left(a - K_1R_{rp}S_r^{-\frac{1}{2}}\right)\cos\left(i^H\varphi_p\right)
\end{align*} \quad (2)$$

Fig. 1 Profile generation for a cycloidal disc.

To compensate for the errors caused by assembly and manufacturing, it is necessary to modify the cycloidal gear profile. Three ways of tooth modification are commonly used: isometric modification (modifying the pin radius $R_{rp}$), offset modification (modifying the pin gear radius $R_p$) and angle rotation modification (modifying the rotation angle $\varphi_c$ of cycloid gear). The general cycloid gear profile equation with three modification methods can be established as
$$\begin{align*}
x'_{c} &= \left( (R_p + \Delta R_p) - (R_{rp} + \Delta R_{rp})S_{r}^{-\frac{1}{2}} \right) \cos[(1 - i^H)\varphi_p - \delta] - \\
&\quad \frac{a}{R_p + \Delta R_p} \left( (R_p + \Delta R_p) - z_p \left( (R_{rp} + \Delta R_{rp})S_{r}^{-\frac{1}{2}} \right) \right) \cos(i^H\varphi_p + \delta) + \\
y'_{c} &= \left( (R_p + \Delta R_p) - (R_{rp} + \Delta R_{rp})S_{r}^{-\frac{1}{2}} \right) \sin[(1 - i^H)\varphi_p - \delta] + \\
&\quad \frac{a}{R_p + \Delta R_p} \left( (R_p + \Delta R_p) - z_p \left( (R_{rp} + \Delta R_{rp})S_{r}^{-\frac{1}{2}} \right) \right) \sin(i^H\varphi_p + \delta)
\end{align*}$$

(3)

### 2.2 Structure of CBR reducer

CBR reducer is a new type of cycloid gear reducer, which adopts a symmetrical transmission structure. As shown in Fig.2, it is mainly composed of bearing end cover, outputting flange, cross roller bearing, crank bearing, disc connector, turning bearing, crank shaft, pins, cycloid gear, case, outputting bearing and input support. Compared with RV reducer, it has the advantages of simple structure, small number of parts, simple assembly process and low manufacturing cost.

Because of its compact structure, it can achieve the installation dimension of the harmonic drive reducer. So, it can be used in robot joints with space restricted instead of harmonic reducer, and its payload, torsional stiffness and instantaneous impact resistance are several times higher than that of the harmonic drive reducer.

![Fig.2 Exploded view of CBR Reducer.](image)

### 3. Force analysis algorithm with tooth modification

In this section, a new numerical algorithm is proposed to compute contact force. Not only can it analyze initial gap, but also can compute the contact force and simultaneous contact teeth. Below are the detail steps of the algorithm.

1. First, calculating the initial gap.

   The initial gap has been derived by the method of geometric analysis and conventional TCA [9,10]. Here, the initial gap is solved by the new force analysis method, and the main flow is summarized as follows:

   Step 1. As shown in Fig.3, the cycloid gear is discretized and divided into $z_p$ equal parts, each corresponding to a pin. Each part is discretized into $n$ points, the coordinates of each point are $(x_{ci}, y_{ci})$. $\varphi_c$ is the rotational angle of cycloid gear. According to the modified cycloid equation Eqn. (3), the coordinates $(x_{ci}, y_{ci})$ of every point on the cycloidal profile are obtained.
Step 2. Update the value of the rotational angle of the cycloid gear, \( \varphi_c = \varphi_c + d\varphi_c \), and get the coordinates \((x_{ci}', y_{ci}')\) of the discrete points.

Step 3. Calculating the coordinates \((x_{pi}, y_{pi})\) of the center of the pin.

Step 4. Calculating the distance between the center of the pins and the points of the corresponding part of the cycloid gear, \( l_i = \sqrt{(x_{pi} - x_{ci}')^2 + (y_{pi} - y_{ci}')^2} \).

Step 5. Judgement condition. If \( l_i - R_{rp} < 0 \), then stop calculating and extract the value of \( \varphi_c \). Otherwise, turn to step 2.

Step 6. Outputting the initial gap. The distance between center of other pins and normal direction of cycloid profile minus the radius of the pin is the initial gap \( \Delta \varphi_i \).

The new force analysis method uses the numerical method to solve the initial gap. It can avoid the error solved by the approximate geometric method. The result can be obtained accurately by reducing the iteration value \( d\varphi_c \) and increasing the numbers \( n \) of discretized points in the tooth of cycloid gear.

(2) Then, calculating the contact force.

After determining the first contact tooth of the cycloid gear, the cycloid gear continues to rotate, and the pins and the cycloid gear will produce contact force. According to the magnitude of the loading torque, the number of gear teeth that produces contact force will be different. Assuming the loading torque is rated load \( T_r \), the contact force and the number of simultaneous meshing teeth are solved as follows:

Step 1. Calculating the contact stiffness \( K_m \).

The cycloid disc tooth as a non-uniform cantilever beam, according to the beam theory and Hertzian theory [11], the mesh stiffness of one tooth pair of cycloid gear in mesh can be expressed as:

\[
K_m = \frac{1}{K_h + \frac{1}{K_b} + \frac{1}{K_s} + \frac{1}{K_a} + \frac{1}{K_c}}
\]

\[
\frac{1}{K_h} = \frac{4(1 - \mu^2)}{\pi E W}, \quad K_b = \int_0^d \frac{(x \cos \alpha - h \sin \alpha)^2}{E I_x} dx, \quad K_s = \int_0^d \frac{1.2 \cos^2 \alpha}{G A_x} dx, \quad K_a = \int_0^d \frac{1.2 \cos^2 \alpha}{E A_x} dx, \quad K_c = \int_0^d \frac{\sin^2 \alpha}{E A_x} dx
\]

The cycloid-pin gear mesh stiffness of CBR25 reducer is calculated according to Eqn. (4) and is shown in Fig.4.
Step 2. Calculating the maximum nominal contact depth $h_{imax}$. After obtaining the $q_c$ value of the initial gap distribution, increasing the $q_c$ value continually, $q_c = q_c + dq_c$, the pins which meet the condition $l_i - R_{rp} < 0$ engage contact. As shown in Fig. 5, the maximum normal contact depth can be obtained as follow

$$h_{imax} = |l_i - R_{rp}|$$

(5)

Step 3. Calculating the contact force $F_i$ for each tooth,

$$F_i = K_m \cdot h_{imax}.$$  \hspace{1cm} (6)

Step 4. Calculating the total torque $T_c$, as shown in Fig. 6, the pins are enumerated, $n$ is the number of simultaneously contact teeth, $L_i$ is the arm length of contact force,

$$T_c = \sum_{i=1}^{n} F_i \cdot L_i = \sum_{i=1}^{n} F_i \cdot (a \cdot z_c \cdot \sin \phi \cdot S_r \cdot \frac{1}{2})$$ \hspace{1cm} (7)

Step 5. Comparing the torque $T_c$ and $T_N$, if $T_c \leq T_N$, then stop calculating, else go to step 2.

Step 6. Outputting the contact force and simultaneously contact number.

The flowchart of the new force analysis method is shown in Fig. 7.
4. Numerical examples
Taking the CBR25 reducer as an example, its design parameters are shown in Table 1. The isometric and offset modification are used in cycloidal profile. The output flange of the reducer is applied 100Nm torque. The input crank shaft rotates $2\pi*49$, so the cycloid gear will rotate $2\pi$. We use the new force analysis method described above to calculate the initial gap and contact force of CBR25. The initial gap with respect to input crank angle is shown in Fig. 8 and each tooth contact force in one state is shown in Fig. 9.

Calculating the distance between the center of the pins and the points of the corresponding part of the cycloid gear,
\[ l_i = \sqrt{(x_{pi} - x'_{ci})^2 + (y_{pi} - y'_{ci})^2} \]

Increasing the rotational angle of the cycloid gear $\phi_c = \phi_c + d\phi_c$ and get the coordinates $(x'_{ci}, y'_{ci})$ of the discrete points.

Calculating the coordinates $(x_{pi}, y_{pi})$ of the center of the pin.

Calculating the contact force $F_i$ for each contact tooth
\[ F_i = K_m * h_{max} \]

Outputting the contact force and simultaneously contact number.

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5. Conclusion
This article presents the design and a new force analysis method of CBR reducer. CBR is a one-stage reducer with a compact structure and a wide range of installation size. The modified profile equation of cycloid gear was derived and the algorithm process of force analysis was described. The numerical example of CBR25 reducer was simulated and the initial gap and contact force of each teeth were obtained. With the help of new force analysis method, the designer can obtain the initial gap and contact force accurately, and hence to improve the design of the CBR reducer in the product development phase.

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