Strong and radiative decays of the $D_{s0}^*(2317)$ and $D_{s1}(2460)$

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Abstract. Since their discovery in 2003, the open charm states $D_{s0}^*(2317)$ and $D_{s1}(2460)$ provide a challenge to the conventional quark model. In recent years, theoretical evidence has been accumulated for both states in favor of a predominantly $DK$ and $D^*K$ molecular nature, respectively. However, a direct experimental proof of this hypothesis still needs to be found. Since radiative decays are generally believed to be sensitive to the inner structure of the decaying particles, we study in this work the radiative and strong decays of both the $D_{s0}^*(2317)$ and $D_{s1}(2460)$, as well as of their counterparts in the bottom sector. While the strong decays are indeed strongly enhanced for molecular states, the radiative decays are of similar order of magnitude in different pictures. Thus, the experimental observable that allows one to conclusively quantify the molecular components of the $D_{s0}^*(2317)$ and $D_{s1}(2460)$ is the hadronic width, and not the radiative one, in contradistinction to common belief. We also find that radiative decays of the sibling states in the bottom sector are significantly more frequent than the hadronic ones. Based on this, we identify their most promising discovery channels.

1 Introduction

Since the beginning of this millennium, mounting experimental evidence in hadronic spectroscopy puts into question quark models like the Godfrey-Isgur model [1] that successfully described the ground and some low excited states of mesons with open charm or bottom. This picture was challenged when two narrow resonances with open charm or bottom were discovered by the BaBar [2] and CLEO Collaborations [3], respectively. These states are now named $D_{s0}^*(2317)$ and $D_{s1}(2460)$ and referred to in the following as $D_{s0}^*$ and $D_{s1}$, respectively. Their respective masses were about 160 MeV and 70 MeV below the predictions of the Godfrey-Isgur quark model. On the other hand, the states are located by almost the same amount of about 45 MeV below the $DK$ and $D^*K$ thresholds, respectively.

This appears a mere numerical coincidence in quark models, and is a consequence of the parity doubling assumption in refs. [4–6]. However, as stressed in ref. [7], this can be explained naturally if the systems are bound states of the $DK$ and $D^*K$ meson pairs, respectively [8–16]. Another possible explanation is the mixing of $D_{s0}^*$ with another axial-vector state, $D_{s1}(2536)$ [17].

Weinberg introduced a model-independent way to quantify the molecular admixture of the most prominent continuum component in the wave function of a physical state [18]. The relation between the coupling constant $g$ of a hadronic molecule with a mass $M$ and a binding energy $\epsilon = m_1 + m_2 - M$ to its constituents with masses $m_1$ and $m_2$ and the reduced mass $\mu = m_1 m_2 / (m_1 + m_2)$ is found to be

$$g^2 = 16\pi\lambda^2 \frac{(m_1 + m_2)^2}{\mu} \frac{1}{2\mu} + O\left(R\sqrt{2\mu}\right),$$

where $1/R$ denotes the momentum scale related to dynamics not included explicitly, such as effective range corrections or other channels. The parameter $\lambda^2$ is the probability of finding a two-body continuum state in the physical state. It is thus zero for an elementary particle and one for a pure two-body molecule. For a shallow bound state,
whose binding energy is small so that $R \sqrt{2 \mu e} \ll 1$, the pole contribution dominates the $S$-matrix elements in the near-threshold region, in particular the scattering length. This makes $g$ in principle accessible to experiment, albeit $DK$ scattering is not likely to be directly observed experimentally in the near future. However, the scattering properties can be calculated using lattice quantum chromodynamics (QCD). Indeed, there have been lattice calculations of the $S$-wave isoscalar $DK$ scattering length. It was calculated directly in refs. [19,20] at two pion masses, and the obtained values agree with the ones from the indirect calculation in ref. [21]. In lattice QCD, the isoscalar $DK$ scattering is relatively difficult because of the presence of the disconnected Wick contractions which are of leading order at both the $1/N_c$ expansion and chiral expansion [22]. In ref. [21], the charmed meson-light meson scattering lengths for the channels which are free of disconnected contractions are calculated, and then the $DK$ scattering length was extracted [23] using unitarized chiral perturbation theory, the parameters of which were determined from fitting to the lattice results. In this sense, we refer to the calculation of the $DK$ scattering length as “indirect”. These lattice results agree perfectly with the prediction of eq. (1) for $\lambda = 1$ taking into account the uncertainties. This provides strong evidence from the theoretical point of view that the $D_{s0}$ and $D_{s1}$ are $D^{(*)}K$ molecular states. However, a clear experimental proof is still missing.

It was stressed in refs. [21,24–26] that the leading loop contributions to the hadronic widths of $D_{s0}$ and $D_{s1}$ are quite sensitive to $g^2$ and thus allow one to quantify their molecular admixtures experimentally. Especially, no additional counterterm is present at leading order (LO). The situation for the radiative decays is less clear. While refs. [25,27,28] provide predictions, a LO counterterm obstructs a prediction in ref. [29] and hampers the sensitivity to the coupling constant $g^2$.

In this work, we reinvestigate the decays of the $D_{s0}$ and $D_{s1}$ in an effective field theory description appropriate for these systems. Our key finding is that the radiative decays of the $D_{s0}$ and $D_{s1}$ are insensitive to their precise nature, contrary to the strong decays. In particular, the electromagnetic transition rates are not enhanced by the nature, contrary to the strong decays. In particular, the electromagnetic transition rates are not enhanced by the nature, contrary to the strong decays. In particular, the electromagnetic transition rates are not enhanced by the nature, contrary to the strong decays.

Our findings are in conflict with those of ref. [30], where it is argued that, if $D_{s0}$ and $D_{s1}$ were molecules, their radiative decays necessarily would be more frequent than their strong ones, which conflicts with experiment (see discussion in sect. 3.2.2). To come to this conclusion, the authors factorize the transition matrix elements into the wave function at the origin times the on-shell transition matrix elements of the molecular constituents to the final state. However, our formalism is different from that used for molecules in ref. [30] in three important aspects: first, the factorization used neglects the cut that occurs after the photon emission (cf. the detailed discussion in ref. [31]); second, the short ranged contributions significant for the radiative decays were neglected and third, the non-analyticities due to the mass splittings of the constituents, which are crucial to the hadronic decays of the molecules, were omitted [21,24–26].

Unfortunately, the experimental information available at present is rather limited. At best, ratios between hadronic and radiative decays are published, with only upper limits for most transitions. There is hope that with the advent of high precision and high intensity experiments like PANDA, this situation will be improved significantly.

Since heavy quark flavor symmetry connects the open charm and bottom sectors, states similar to the $D_{s0}$ and $D_{s1}$ are expected in the open bottom sector. Such predictions have been made for conventional mesons with parity doubling [5], using heavy quark effective theory [32,33] or for $B^{(*)}K$ molecules in a variety of publications [11–14,34–38]. Here, we update the latter class of works, and especially identify radiative decays as the probably most promising discovery modes of the bottom partners of the $D_{s0}$ and $D_{s1}$.

The paper is organized as follows. The theoretical framework and the interaction Lagrangians are presented in sect. 2. Both the isospin breaking hadronic decays and radiative decays of the $D_{s0}$ and $D_{s1}$ and their bottom partners are calculated in sect. 3. The last section contains a brief summary.

### 2 Framework

Various earlier works demonstrated that both the $D_{s0}$ and $D_{s1}$ can straightforwardly be produced by unitarizing $D^{(*)}K$ scattering amplitudes which are derived, for example, from chiral perturbation theory (CHPT) at LO [11,12,14,39,40] or next-to-leading order (NLO) [21,23,24,26,29]. These amplitudes were also used to calculate their strong decays. In principle, the electromagnetic decays could also be addressed with the full set of equations by gauging the integral equation [41]. However, since we are interested in an observable close to the resonance pole only, we can take a simpler route. First, we extract the pole residues from the full calculation, and then use these as input of a one-loop evaluation of the actual decays. For a proper field theoretical derivation of the connection between the two approaches in a different context, see sect. 3.3 of ref. [42].

Our approach is based on the Lagrangian describing the coupling of the molecules to a heavy-light meson pair in an $S$-wave (all other Lagrangians needed in this work...
Table 1. Comparison of our predictions of the masses of the $\bar{B}K$ and $\bar{B}^*K$ bound states with those in refs. [5,11,12,14,32]. All masses are given in MeV.

|          | This calculation | Ref. [11] | Refs. [12,14] | Ref. [32] | Ref. [34] | Ref. [5] |
|----------|-----------------|-----------|---------------|-----------|-----------|---------|
| $M_{\bar{B}^*0}$ | 5625 ± 45     | 5643      | 5725 ± 39     | 5667      | 5696 ± 40 | 5718 ± 35 |
| $M_{\bar{B}^*1}$ | 5671 ± 45     | 5690      | 5778 ± 7      | 5714      | 5742 ± 40 | 5765 ± 35 |

are listed in the appendix):

$$
\mathcal{L}_{\text{Mol}}^D = g_{DK} D_{\eta}^{0*} (D^+ K^0 + D^0 K^+) \\
+ g_{D^*K} D_{\eta}^{1*} (D^+ K^0 + D^0 K^+) \\
+ g_{D^+\eta} D_{\eta}^{0*} D_{1*}^{1*} + g_{D^+\eta} D_{\eta}^{0*} D_{s*}^{1*} + \text{h.c.},
$$

(2)

where $g_i$ denote the corresponding coupling constants. As pointed out already in ref. [30], kinematics allows one to treat the kaon as heavy and not to exploit its nature as pseudo-Goldstone-boson. In particular this justifies the use of a coupling without derivatives. This treatment simplifies the actual calculations considerably. In ref. [21], the $D_{\eta}^{0*}$ pole was generated dynamically using unitarized NLO heavy meson-Goldstone boson scattering amplitudes. The low-energy constants (LECs) were fit to lattice calculations for various scattering lengths. The same values of the LECs are used in this work. We here mainly present the extension of the earlier formalism necessary for this work. For more details, we refer to refs. [21,23,43]. Each of the isoscalar heavy meson-kaon scattering amplitudes has a pole below threshold which corresponds to the particle of interest. The coupling constants defined in the Lagrangian in eq. (2) are then determined from the residues of these poles:

$$
g_{DK} = (9.0 \pm 0.5) \text{ GeV}, \quad g_{D^*K} = (10.0 \pm 0.3) \text{ GeV} \\
g_{D^+\eta} = (8.0 \pm 0.2) \text{ GeV}, \quad g_{D^+\eta} = (7.5 \pm 0.5) \text{ GeV},
$$

(3)

where the uncertainties are propagated from the errors of the LECs with correlations taken into account. The couplings of the $D_{\eta}^{0*}$ ($D_{\eta}^{1*}$) to the $D$ ($D^*$) turn out to be larger than those to the $D_s$ ($D_s^*$). Applying eq. (1) to the couplings in eq. (3), we find values of $\lambda^2$ for both $D_{\eta}^{0*}$ and $D_{\eta}^{1*}$ of about 0.8. It should be stressed that this formula can only be applied to the principal components, i.e. $D^{(*)} K$. The possible impact of the $D^{(*)\eta}$ channel is included in the uncertainty, which turns out to be of the order of 50% when using for the range of forces $R \sim 1/\sqrt{2(m_\eta + m_D - M)}$ (cf. eq. (1)). This provides additional evidence for the interpretation of $D_{\eta}^{0*}$ and $D_{\eta}^{1*}$ as predominantly molecular states. Note that the mentioned large uncertainty refers to quantifying the molecular component of the scalar and axial-vector states; the residues themselves are known to much higher accuracy, see eq. (3), and it is their uncertainty that matters for the calculations below.

Besides the value of the couplings there is an additional reason, why the $D^*$ channel indeed dominates over the $D\eta$ channel in the key observables studied here, thanks to an enhancement by a factor of

$$
\sqrt{(m_\eta + m_{D^*} - M)/(m_D + m_K - M)} \simeq 2,
$$

(4)

which emerges since the triangle diagrams driving the radiative decays scale as the inverse of the meson momenta.

Since heavy quark flavor symmetry allows us to use the same parameters and predict the heavy-flavor partners, we can extend these calculations to the open bottom sector. In our previous study [34], we took the same subtraction constant which is used to regularize the divergent two-meson loop integrals in dimensional regularization for both the bottom and charm systems. Now, we use a different method which makes the transmission of the scale-dependence of the loop integrals more transparent/physical: we first use a three-momentum sharp cutoff to regularize the loop integral and fix it to reproduce the dimensional-regularized loop in the charm sector, and use the same cutoff to determine the value of the subtraction constant in the bottom sector. Then the masses of the generated states with positive parity can be calculated by searching for poles of the scattering amplitudes. The results are presented in the first column of table 1. The uncertainty contains both that of the LECs and of the heavy-flavor symmetry breaking, added in quadrature. We estimated the latter as $(A_{QCD}/m_c)\sim 40$ MeV. Within uncertainties, the masses obey the relation

$$
M_{B_{s1}} - M_{B_{s0}} \simeq M_{B^*} - M_B.
$$

(5)

In table 1, we also compare our results with previous studies. Our values agree within errors with refs. [11,32,34], while there is some discrepancy to the results of refs. [5,12,14].

The Lagrangian for coupling the bottom molecules to heavy-light meson pairs is analogous to eq. (2). The corresponding residues for the $B_{s0}^*$ and $B_{s1}$ read

$$
g_{BBK} = (30 \pm 1) \text{ GeV}, \quad g_{BB^*K} = (30 \pm 1) \text{ GeV} \\
g_{BB^+\eta} = (12 \pm 6) \text{ GeV}, \quad g_{BB^+\eta} = (10 \pm 7) \text{ GeV}.
$$

(6)

The larger couplings reflect the fact that the bottom states are more deeply bound, as expected from eq. (1). Indeed, the large binding energy renders useless any estimate of the probability as $x^2$ via eq. (1).

To calculate the radiative decays, we need the magnetic moments of the heavy mesons in addition to the electric photon-meson coupling which comes from gauging the kinetic term of the heavy mesons. The Lagrangian
where \( \text{cancel the loops at the poles, cf. fig. 1 and ref. [27]}. \) The single lines charmed mesons, and dashed lines kaons.

**Fig. 1.** The mass renormalization mechanism that ensures that the \( D_0^* \) and \( D_1^* \) do not mix for the physical particles. The loops are evaluated at \( \mu^2 = M_{D_0^*}^2 \). Double lines denote molecules, single lines charmed mesons, and dashed lines kaons.

reads \([44,45]^2\)

\[
\mathcal{L}_{\text{mag. mom.}} = \frac{i}{2} e F_{\mu \nu} M_H \\
\times \left[ \varepsilon^{\mu \nu a \beta} v_\alpha \left( \frac{Q}{m_Q} + \frac{Q'}{m_Q} \right)_{ab} P_a P_{b+}^* - P_{a}^* P_b^* \right] \\
+ P_{a}^* P_b^* \beta^\mu \left( \frac{Q}{m_Q} - \frac{Q'}{m_Q} \right)_{ab}, \tag{7}
\]

where \( M_H \) is the mass of the heavy meson, and the pseudoscalar (vector) mesons with open charm are collected in \( P_a (P_{a})^* \) with \( a \) labeling the light flavors.

\[
P = (D^0, D^+, D^*_+), \quad P^*_a = (D^0_{a}, D^*_{a}, D^*_{a}) \tag{8}
\]

The \( \beta Q \) term, where \( Q = \text{diag}(2/3,-1/3, -1/3) \) is the light quark charge matrix, comes from the magnetic moment of the light degrees of freedom, and the \( Q'/m_Q \) term is the magnetic moment coupling of the heavy quark with \( Q' \) and \( m_Q \) being the charge and mass of the heavy quark, respectively. The quantities \( \beta \) and \( m_Q \) can be fixed from experimental data for \( \Gamma(D^0 \rightarrow D^0 \gamma) \) and \( \Gamma(D^{*+} \rightarrow D^{+} \gamma) \). We will use one set of values determined in ref. [45], which are \( 1/\beta = 379 \) MeV and \( m_c = 1863 \) MeV. The transition to the bottom sector is made by using \( m_b = 4650 \) MeV and the same value for \( \beta \).

Since the longitudinal components of the vector fields, \( \partial_\mu P_{a}^{*} \), have scalar quantum numbers, hadronic loops couple them to the scalar fields. In this way, the longitudinal components of the vector fields contribute to the self-energy of the scalar field. Analogously, the longitudinal components of the axial vector couplings to the pseudoscalar fields. Thus, for the purpose of renormalization, we have to add the counterterms

\[
\mathcal{L}_{\text{long.}} = -C_{D^* K} \left( \partial_\mu D_{a}^{* \mu} \right) D_{a}^{*} + \frac{1}{2} \epsilon^{\mu \nu a \beta} \left( D_\mu D_\nu^{a} \right) v^\alpha D_{a}^{* \alpha}, \tag{9}
\]

with the coupling constants \( C_{D K} \) and \( C_{D^* K} \) adjusted to cancel the loops at the poles, cf. fig. 1 and ref. [27]. The gauge covariant derivative is defined by

\[
D_\mu = \partial_\mu + ieQ_F A_\mu, \tag{10}
\]

where \( A_\mu \) is the electromagnetic vector potential and \( Q_F = 1 \) for \( D_{a1} \) and \( -1 \) for \( D_{a1}^{*} \).

\[2\] Notice that in our notation, the presence of the factor \( M_H \) renders the fields \( P_a \) and \( P_{a}^* \) to have an energy dimension 1. This is different from refs. [44,45] where the dimension of these fields is 3/2.

Furthermore, the Lagrangian for the leading contact interactions for the radiative decays reads

\[
\mathcal{L}_{\text{Contact}} = \kappa F_{\mu \nu} \left( v_{a}^{\mu} D_{a}^{* \nu} + D_{a}^{* \nu} v_{a}^{\mu} + \epsilon^{\mu \nu a \beta} D_{a1}^{*} D_{a}^{* \beta} \right) + \tilde{\kappa} \epsilon^{\mu \nu a \beta} F_{\mu \nu} v_{a} D_{a1} D_{a}^{* \beta} + \text{h.c.} \tag{11}
\]

We will discuss the relative importance of contact interactions and loop diagrams in a CHPT power counting scheme in sect. 3.2.

For numerical calculations, we will take the following values for the meson masses [46]:

\[
\begin{align*}
M_{D^0} &= 1864.86 \text{ MeV}, \quad M_{D^+} = 1869.62 \text{ MeV}, \\
M_{D^0_s} &= 1968.49 \text{ MeV}, \quad M_{D^{*+}} = 2006.98 \text{ MeV}, \\
M_{D^{*+}} &= 2100.28 \text{ MeV}, \quad M_{D^{*+}} = 2112.3 \text{ MeV}, \\
M_{B^+} &= 5279.25 \text{ MeV}, \quad M_{B^0} = 5279.58 \text{ MeV}, \\
M_{B^0} &= 5366.77 \text{ MeV}, \quad M_{B^{*+}} = 5325.2 \text{ MeV}, \\
M_{B^{*+}} &= 5325.2 \text{ MeV}, \quad M_{B^{*+}} = 5415.4 \text{ MeV}, \\
M_{\pi^+} &= 134.98 \text{ MeV}, \quad M_{\eta} = 139.57 \text{ MeV}, \\
M_{K^+} &= 493.677 \text{ MeV}, \quad M_{K^0} = 497.614 \text{ MeV}, \\
M_{\eta} &= 547.85 \text{ MeV}.
\end{align*}
\]

It is important to also specify the uncertainties of the mass differences used in our approach [46]:

\[
\begin{align*}
M_{D^+} - M_{D^0} &= (4.8 \pm 0.2) \text{ MeV}, \\
M_{D^{*+}} - M_{D^{0*}} &= (3.3 \pm 0.2) \text{ MeV}, \\
M_{B^{*+}} - M_{B^{0*}} &= (-0.33 \pm 0.24) \text{ MeV}.
\end{align*}
\tag{12}
\]

The mass splittings in the charm and bottom sectors have different patterns because the interference between the \( m_d - m_u \) contribution and the electromagnetic contribution is different [47].

### 3 Two-body decays

#### 3.1 Hadronic decays

In this section, we calculate the hadronic decay widths \( D_0^* \rightarrow D_s^* \pi^0 \) and \( D_0^* \rightarrow D_s^* \pi^0 \) and their corresponding bottom partners. The narrow widths of the charmed states can only be understood, if they are isoscalar states for then the pionic decays violate isospin. One natural decay mechanism, which is present irrespective of the assumed nature of the states, is the strong decay of the scalar (axial vector) state into a \( D_s^* \) (\( D_s^* \)) and a virtual \( \eta \)-meson, followed by the isospin violating transition to a pion via the \( \pi^0, \eta \) mixing amplitude \( \epsilon_{\pi \eta} = 0.013 \pm 0.001 \). This amplitude is analytic in the quark masses and scales as \( (m_u - m_d)/m_s \). Different groups using different underlying models for the \( D_s^* \) states report hadronic widths due to the \( \pi^0, \eta \) mixing ranging between 3 and 25 keV [5, 12, 14, 25, 26, 29].
The column last of table 2 lists the full result, showing that interference effects play an important role. The differences between the bottom and charm sectors are even larger for the full result since interference between the two mechanisms is vastly different. The uncertainties for the full results are obtained by adding the uncertainties for the individual contributions linearly. This is done to incorporate the fact that the residues, $g_{DK}$ and $g_{D_{s}π}$ in the case of $D^{0}_{s}$, are not necessarily independent quantities while they contribute largely to the uncertainties in the individual channels, respectively. The fact that our results are consistent with those of ref. [21] is an a posteriori justification to use eq. (2).

Note that our results differ significantly from the phenomenological studies of ref. [48]. There, the predicted hadronic widths for the $B^{(*)}K$ molecules are much larger, in the range from 50 to 90 MeV. The reason is that therein the masses of the molecules predicted in refs. [12,14] were used, which are around 100 MeV larger than those calculated in this paper, see table 1. We checked that, if we keep all the LECs to the best fit values of ref. [21], and only change the subtraction constant to produce a mass of $B^{0}_{s}$ at 5725 MeV used in ref. [48], then we obtain a larger width of 73 keV, which is consistent with the result in ref. [48].

### 3.2 Radiative decays

#### 3.2.1 Power counting

We first address the relative size of the different contributions. We employ here the standard power counting scheme of CHPT coupled to heavy fields [49–51]. The relevant momentum scale is $p \sim \sqrt{2m_{K}} \sim m_{K}$. The integration measure counts as $\mathcal{O}(p^{4})$, the light meson propagator as $\mathcal{O}(p^{-2})$ and the heavy one as $\mathcal{O}(p^{-4})$. Similarly, the coupling of a photon to the electric charge gives $\mathcal{O}(p^{2})$ for light and $\mathcal{O}(p)$ for heavy mesons. The field strength tensor of the photon, relevant for the coupling to the magnetic moments and the contact interaction, enters as $\mathcal{O}(p^{2})$. The hard scale in CHPT is given by $A_{\chi} \sim 1$ GeV. Thus, higher orders are suppressed by positive powers in $p/A_{\chi}$.

Consider first the one-loop diagrams where the photon couples to the electric charge of the involved mesons. For a photon emission inside the loop from a light meson, fig. 3 EC(d), we find a factor of $\mathcal{O}(p^{-5})$ from two light and one heavy propagators, plus the axial vector coupling as $\mathcal{O}(p)$, and the coupling of a photon to the electric charge of a light meson as $\mathcal{O}(p^{2})$, so that this diagram counts in total as

$$\mathcal{O}\left(p^{4} \frac{1}{p^{0}} \frac{1}{p^{2}}\right) = \mathcal{O}(p^{2}) .$$

(13)

Here, we have counted the $S$-wave coupling of the generated state to $D^{(*)}K$ as $\mathcal{O}(p^{0})$. The same process with a charged intermediate heavy meson, fig. 3 EC(c), gives

$$\mathcal{O}\left(p^{4} \frac{1}{p^{0}} \frac{1}{p^{2}}\right) = \mathcal{O}(p^{2}) .$$

(14)
Fig. 3. Different contributions to the radiative decays. In this case, the dashed lines denote kaons only. A photon coupling to the magnetic moment is denoted by a filled box.

All other diagrams in the same set are of the same order, as required by gauge invariance. Since the contact interactions are proportional to the photon field strength tensor, they also contribute at order $p^2$. This means that there is no enhancement of the loop diagrams compared with the contact term and, contrary to the hadronic decays, we expect different models for the $D^*_s$ structure to lead to similar results. Below we will use the available data to fix the contact interaction in one channel in order to predict the others.

We also consider the contributions from the magnetic couplings of the heavy mesons. The size of the diagrams in fig. 3 MM(a) are estimated as $O\left(1/p^4\right) \sim O\left(p^3\right)$.

Typical further higher order diagrams include an additional pion exchange in the loop. This leads to additional factors of order $(p/\Lambda)^2$, which imply that they can be safely neglected. The largest subleading contribution stems from the NLO term for the axial vector coupling. It is suppressed by one order, $p/\Lambda$, and we will use it to estimate the theoretical uncertainty for the amplitudes.

We close with some technical remarks. The diagrams in the first line of fig. 3 show the full gauge invariant set of diagrams for which the photon couples to the electric charge. These are obtained by gauging the kinetic terms and the axial vector coupling. In ref. [43] the explicit expressions for the amplitudes of all possible transitions are given. An explicit calculation confirms that the two subsets ($D^0 K^+$ and $D^+ K^0$ for $D^*_{s0}$) are gauge invariant separately, but there is still a remaining divergence. However, once the mass renormalization diagrams shown in the last line of fig. 3 are included, all divergences are cancelled leaving a renormalized divergence-free amplitude.

In ref. [29] vector and axial vector states are treated in the tensor formulation [52, 53]. In our approach, we modify the standard treatment of the vector formulation by employing a trick introduced by Stückelberg (see, for instance, ref. [54]). The standard Lagrangian for an arbitrary vector particle $V$ with mass $m_V$ reads

$$\mathcal{L}_V = -\frac{1}{2} V^{\mu\nu} V_{\mu\nu} + m_V^2 V^{\mu\nu} V^\dagger_{\mu\nu},$$

with the field strength tensor $V^{\mu\nu} = D_\mu V_\nu - D_\nu V_\mu$ and the covariant derivative $D_\mu = \partial_\mu - ie A_\mu + \ldots$, where the
Table 3. The decay widths (in keV) calculated only from the coupling to the electric charge (EC), from the magnetic moments (MM) and from the contact term (CT), respectively, compared to the total (including interference). The CT strength for the transitions to odd parity mesons is fixed to data, while that to even parity states, marked as \( \gamma^0 \), is undetermined and part of the uncertainty.

| Decay channel                  | EC    | MM     | CT     | Sum   |
|--------------------------------|-------|--------|--------|-------|
| \( D_{s0} \to D_s^\ast \gamma \) | 2.0   | 0.03   | 3.3    | 9.4   |
| \( D_{s1} \to D_s \gamma \)    | 4.2   | 0.2    | 11.3   | 24.2  |
| \( D_{s1} \to D_s^\ast \gamma \) | 9.4   | 0.5    | 10.3   | 25.2  |
| \( B_{s0} \to B_s^\ast \gamma \) | 22.4  | 0.6    | 5.2    | 32.6  |
| \( B_{s1} \to B_s^\ast \gamma \) | 39.4  | 25.8   | 5.1    | 4.1   |
| \( B_{s1} \to B_s \gamma \)    | 46.5  | 0.1    | 6.4    | 46.9  |
| \( B_{s1} \to B_{s0} \gamma \)  | 0.02  | ?      | 0.02   | 0.02  |

Ellipses indicate the presence of additional terms not relevant for the following discussion and \( A_\mu \) denotes the photon field. The propagator resulting from this Lagrangian is

\[
\frac{i}{l^2 - m_V^2 + i\epsilon} \left( \frac{|\mu|\nu}{m_V^2} - g^{\mu\nu} \right),
\]

with \( l \) the vector meson momentum. This may make calculations rather extensive. Therefore, one may add another term to the Lagrangian that vanishes for on-shell vector particles:

\[
\mathcal{L}_V = -\frac{1}{2} V^{\mu\nu} V^\dagger_{\mu\nu} + m_V^2 V^{\mu\nu} V^\dagger_{\mu\nu} - \lambda (D_\mu V^\mu)(D^\nu V^\dagger_{\mu}),
\]

and thus the propagator changes to

\[
\frac{i}{l^2 - m_V^2 + i\epsilon} \left( \frac{1}{\lambda} \left( \frac{|\mu|\nu}{l^2 - m_V^2 + i\epsilon} \right) + g^{\mu\nu} \right),
\]

where we are free to choose \( \lambda = 1 \). Since we are dealing with radiative decays it is important to notice that also after gauging the coupling to a photon changes. For a more detailed discussion of vector meson Lagrangians and the St"uckelberg construction, we refer the reader to the comprehensive review [55].

3.2.2 Results

Our amplitudes consist of three different contributions. For the decays via heavy meson-kaon loops, we considered the coupling of the photon to the electric charges and to the magnetic moment of the heavy mesons, as well as the contact interaction. In table 3, we show the central values for the width calculated using one of the three contributions exclusively while discarding the remaining ones. With the contact interaction fixed to data as described below, the largest loop contributions come from the loop diagrams where the photon couples to the electric charges. Coupling the photon to magnetic moments, which is of one order higher, gives in general small contributions. Therefore, the chiral expansion converges well. The magnetic contribution to the transition \( D_{s1} \to D_{s0}^\ast \gamma \) is larger than the others in the charm sector, since it scales, as expected, with the product of two sizable resonance couplings \( g_{D0}K_{D0}K \), while all others scale with the product of one of them and the axial coupling constant \( g_A \) for the \( D^\ast D_s K \) vertex.

Two results stand out here and need to be explained. In the decay \( B_{s1} \to B_s \gamma \) the contribution from the magnetic moment is particularly large because of a constructive interference that does not appear in any of the other channels. Individually, the interfering diagrams do not contribute more than expected from the power counting. However, this large contribution interferes destructively with the electric charge contribution. This leads, in turn, to a width about one order of magnitude smaller than those for the \( B_{s1} \to B_s^\ast \gamma \) and \( B_{s0} \to B_s^\ast \gamma \), despite of having the largest phase space. The same mechanism is not present in the other channels for different reasons. In the case of the corresponding open charm decay, \( D_{s1} \to D_s \gamma \), the charge of the heavy quark, 2/3 instead of −1/3, prevents a similar effect. For the other open bottom channels \( B_{s0} \to B_s^\ast \gamma \) and \( B_{s1} \to B_s^\ast \gamma \) the relevant loops give too small contributions. The second interesting result is the small decay width for \( B_{s1} \to B_{s0} \gamma \). Here we notice that the decay width scales with \( E_2^3/M_{B_{s1}}^2 \). When we compare this to the same factor in the charm equivalent, we find a suppression of \( \sim 1/150 \), explaining the tiny decay width.

The currently available data are rather limited. Only upper limits exist for some decay widths, while others are not yet measured at all. The only available ratios are:

\[
R_1 := \frac{\Gamma(D_{s0} \to D_s \gamma)}{\Gamma(D_{s0} \to D_s \pi^0)}, \quad R_2 := \frac{\Gamma(D_{s1} \to D_s \gamma)}{\Gamma(D_{s1} \to D_s \pi^0)},
\]

\[
R_3 := \frac{\Gamma(D_{s1} \to D_s^\ast \gamma)}{\Gamma(D_{s1} \to D_s^\ast \pi^0)}, \quad R_4 := \frac{\Gamma(D_{s1} \to D_{s0} \gamma)}{\Gamma(D_{s1} \to D_{s0}^\ast \pi^0)},
\]

\[
R_5 := \frac{\Gamma(D_{s1} \to D_s^\ast \pi^0)}{\Gamma(D_{s1} \to D_s^\ast \gamma) + \Gamma(D_{s1} \to D_{s0}^\ast \gamma)}, \quad R_6 := \frac{\Gamma(D_{s1} \to D_s \gamma)}{\Gamma(D_{s1} \to D_s^\ast \gamma) + \Gamma(D_{s1} \to D_{s0} \gamma)},
\]

\[
R_7 := \frac{\Gamma(D_{s1} \to D_s \pi^0)}{\Gamma(D_{s1} \to D_s^\ast \gamma) + \Gamma(D_{s1} \to D_{s0} \gamma)}, \quad R_8 := \frac{\Gamma(D_{s1} \to D_{s0} \gamma)}{\Gamma(D_{s1} \to D_s^\ast \gamma) + \Gamma(D_{s1} \to D_{s0} \gamma)}.
\]

In table 4, we compare our results to the experimental values. We have chosen the ratio \( R_2 \) to fix the free parameter, namely the strength of the contact interaction \( \kappa \) in eq. (11). The same term contributes to the decays \( D_{s0} \to D_s^\ast \gamma \), \( D_{s1} \to D_s \gamma \) and \( D_{s1} \to D_s^\ast \gamma \). However, an independent contact term \( \kappa \) in eq. (11) contributes to \( D_{s1} \to D_{s0} \gamma \). We thus assign an uncertainty of 100% to this transition amplitude. The uncertainties include those from neglecting higher order contributions and of the coupling constants. Within the theoretical uncertainties, all
results agree with the measured ratios or upper limits. The only possible exception is \( R_1 \), which is consistent only within two standard deviations. In our approach, almost identical values are found for each of the pairs of ratios \( R_2 \) and \( R_6 \), \( R_3 \) and \( R_7 \) since \( \Gamma(D_{s1} \rightarrow D_s^{*}\pi^0) \gg \Gamma(D_{s1} \rightarrow D_s^{*}\gamma) \), in line with the observed proximity of \( R_2 \) and \( R_6 \).

Table 5 contains the results for the individual radiative decay widths. The theoretical uncertainties given there contain various contributions, we only show the sum of all added in quadrature. The largest uncertainty stems from the chiral expansion, which is estimated by multiplying the amplitudes by \((1 \pm \sqrt{2}\mu \lambda / A_\gamma)\), followed by the uncertainty of the contact term propagated from the error of the data used to fix it. Smaller uncertainties come from the residues and the axial vector coupling \( g_\gamma \), determined from the pionic decay of the \( D^* \). In the case of the latter improved experimental data on the \( D^* \) width would be helpful.

In absence of experimental information, we can compare our results only to the results of previous calculations which are given in the table as well, where the values in the last two columns were obtained on the hadronic molecular picture of the \( D_{s0}^* \) and \( D_{s1}^* \). Another result in the molecular picture was performed by Gamermann et al. [27], using an flavor-SU(4) Lagrangian, with a width of \( 0.475_{-0.290}^{+0.831} \text{keV} \) for the \( D_{s0}^* \rightarrow D_{s1}^* \).

The results by Lutz and Soyeur [29] are the closest to ours, while other calculations generally find smaller numbers. However, the differences are not large: all results, even those from the parity-doubling model for the \( c\bar{s} \) mesons [5], agree with ours within two standard deviations. Similarly, the values from different models in the bottom sector agree within three standard deviations. This is however only based on our uncertainties. Once other models quote their residual error as well, the statistical significance of any deviation will decrease.

In contrast to the radiative width, the hadronic width is enhanced significantly for molecular states due to an additional loop contribution, but here the leading contact interaction is proportional to \((m_{u} - m_{d})E_{\gamma}\) and thus suppressed. Consequently, calculations performed for compact \( c\bar{s} \) states predict significantly smaller values. In contradistinction, the origins of larger contributions for molecular states are the two-particle cuts in meson loops, resulting in total widths of the order of 100 keV, and a larger coupling constant in eq. (3). As can be seen in eq. (1), the pure molecule sets an upper bound for \( \lambda^2 \), with an uncertainty of \( \mathcal{O}(R\sqrt{2}\mu \lambda) \). In principle, the \( D^{(*)}K \) meson loops can also contribute to the width of the \( c\bar{s} \) mesons. However, the coupling constant would be much smaller since \( \lambda^2 \ll 1 \) for such states.

| Decay channel | Our result | Exp. | [5] | [56] | [29] | [25, 28, 48] |
|---------------|------------|------|-----|------|-----|--------------|
| \( D_{s0}^* \rightarrow D_{s1}^* \gamma \) | (9.4 ± 3.8) keV | 1.74 | 4.6 | 1.94(6.47) keV | 0.55–1.41 |
| \( D_{s1} \rightarrow D_s \gamma \) | (24.2 ± 10.7) keV | 5.08 | 19–29 | 44.50(45.14) keV | 2.37–3.73 |
| \( D_{s1} \rightarrow D_{s1}^* \gamma \) | (25.2 ± 9.7) keV | 4.66 | 0.6–1.1 | 21.8(12.47) keV | – |
| \( D_{s1} \rightarrow D_{s0} \gamma \) | (1.3 ± 1.3) keV | 2.74 | 0.5–0.8 | 0.13(0.59) keV | – |
| \( B_{s0} \rightarrow B_{s} \gamma \) | (32.6 ± 20.8) keV | 58.3 | – | – | 3.07–4.06 |
| \( B_{s1} \rightarrow B_{s} \gamma \) | (4.1 ± 10.9) keV | 39.1 | – | – | 2.01–2.67 |
| \( B_{s1} \rightarrow B_{s}^* \gamma \) | (46.9 ± 33.6) keV | 56.9 | – | – | – |
| \( B_{s1} \rightarrow B_{s0} \gamma \) | (0.02 ± 0.02) keV | 0.0061 | – | – | – |

Table 4. Results for the relevant decay channels, compared to PDG 2012 [46]. The * denotes an input quantity.

Table 5. Results for the radiative decay widths in keV. The first column gives our result with all uncertainties from higher orders and coupling constants added in quadrature. The numbers in the second column are from a parity-doubling model by Bardeen et al. [5]; in the third from light-cone sum rules by Colangelo et al. [56]; and in the fourth from Lutz and Soyeur [29], who provide two values with reasonable estimates for their remaining free parameter. The fifth column reports model calculations by Faessler et al. [25, 28, 48].
4 Summary and conclusion

We presented hadronic and radiative decay widths of the charm-strange resonances \( D_{s0}^* (2317) \) and \( D_{s1} (2460) \) under the assumption that they are \( D^{(*)} K \) bound states. Our results are in fair agreement with available data. In detail, the decay widths are larger by more than one order of magnitude for the isospin violating hadronic decays than for the radiative decays: the hadronic widths are around 100 keV while the radiative ones are of the order of a few keV.

Our analysis revealed that only the hadronic decays are sensitive to a possible molecular component of both the \( D_{s0}^* \) and \( D_{s1} \)—a hadronic width of 100 keV or larger can be regarded as a unique feature for molecular states. This experimental accuracy could possibly be reached with PANDA at the future accelerator facility FAIR. The origin of this enhanced width is the presence of two-meson cuts and the large coupling constant of the molecules to their constituents—those should be much smaller in for non-molecular states. In contrast to this, the radiative decays turn out to be similar in size for all models for the non-molecular states. In this section we give a more comprehensive overview of the Lagrangians and fields used in this work.

Appendix A. Lagrangians

In this section we give a more comprehensive overview of the Lagrangians and fields used in this work.

The light fields are defined as

\[
\phi = \begin{pmatrix}
\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta \\
\pi^- - \frac{1}{\sqrt{6}} \pi^0 + \frac{1}{\sqrt{3}} \eta \\
K^-
\end{pmatrix}
\]

where

\[
\phi = \begin{pmatrix}
\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta \\
\pi^- - \frac{1}{\sqrt{6}} \pi^0 + \frac{1}{\sqrt{3}} \eta \\
K^-
\end{pmatrix}
\]

and \( F_\pi = 92.21 \text{ MeV} \) [46]; From this one can construct the (axial-)vector current

\[
\Gamma_\mu = \frac{1}{2} (u^T \partial_\mu u + u_\mu u^T),
\]

\[
u_\mu = i (u^T \partial_\mu u - u_\mu u^T).
\]

(A.3)

The pseudoscalar (vector) mesons with open charm are given by

\[
P = (D^0, D^+, D_s^+) \), \quad P^* = (D_{s0}^0, D_{s1}^{*+}, D_{s1}^{*+})
\]

(A.4)

To switch to the bottom sector just replace the pseudoscalar and vector mesons with

\[
P = (B^-, \bar{B}^0, B_s^0) \), \quad P^* = (B_{s0}^-, \bar{B}_{s0}^0, B_{s1}^{*0})
\]

(A.5)

and the heavy quark charge with \( q_B = -1/3 \). The chiral and gauge covariant derivative on the heavy pseudoscalar field is (for vector fields simply replace \( P \) by \( V^i \))

\[
\mathcal{D}_\mu P^i = \partial_\mu P^i + i e A_\mu (P^i Q_B - Q_i P^i)
\]

(A.6)

where \( A_\mu \) is the electromagnetic vector potential, \( Q_B \) is the heavy quark’s charge, \( Q_2 = 2/3 \) or \( Q_3 = -1/3 \), respectively, and \( Q_1 = \text{Diag}(2, -1/3, -1/3) \) is the light quark charge matrix. \( Q_B \) is the heavy meson charge matrix with \( Q_D = \text{Diag}(0, 1, 1) \) and \( Q_D = \text{Diag}(-1, 0, 0) \).

The interaction Lagrangians are derived from the spin symmetric multiplets. We need the axialvector coupling, \( i.e. \) the emission of a light field from a heavy field:

\[
\mathcal{L}_{AV} = g_\pi \sqrt{M_D M_D} \times \frac{e}{M_D} \pi^0 \pi^0 \mathcal{P}^i - \frac{e}{M_D} \partial_\mu P^i + \frac{e}{M_D} \mathcal{P}_{\mu

(A.7)

Since partial widths of the decays \( D^{\mu} \to D^{0} \pi^+ \) and \( D^{\mu} \to D^{+} \pi^0 \) have been measured we can deduce \( g_\pi \) from data [46]:

\[
g_\pi = 0.61 \pm 0.07
\]

(A.8)

To fully describe the systems at hand in addition to eqs. (9) and (11) we use the leading-order chiral covariant Lagrangian for pseudoscalar charmed mesons

\[
\mathcal{L}_{LQ}^{Rel} = (D_\mu P) \left( \mathcal{D}^\mu P \right) - M^2 P P^\dagger
\]

as well as the next-to-leading-order one

\[
\mathcal{L}_{NLQ}^{Rel} = P (h_0 (\chi_+ - h_1 \chi_+) + h_2 (u_\mu u^\mu - h_5 u_\mu u^\mu)) P^\dagger
\]

(A.10)

For the vector mesons simply replace \( P \to P^{\*a} \). The coefficients \( h_1 \) can be found in ref. [21].
