Bosonic Matrix Theory and Matrix Dbranes

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Abstract

We develop new tools for an in-depth study of our recent proposal for Matrix Theory. We construct the anomaly-free and finite planar continuum limit of the ground state with $SO(2^{13})$ symmetry matching with the tadpole and tachyon free IR stable high temperature ground state of the open and closed bosonic string. The correspondence between large $N$ limits and spacetime effective actions is demonstrated more generally for an arbitrary D25brane ground state which might include brane-antibrane pairs or NS-branes and which need not have an action formulation. Closure of the finite $N$ matrix Lorentz algebra nevertheless requires that such a ground state is simultaneously charged under all even rank antisymmetric matrix potentials. Additional invariance under the gauge symmetry mediated by the one-form matrix potential requires a ground state charged under the full spectrum of antisymmetric $(p+1)$-form matrix potentials with $p$ taking any integer value less than 26. Matrix Dbrane democracy has a beautiful large $N$ remnant in the form of mixed Chern-Simons couplings in the effective Lagrangian whenever the one-form gauge symmetry is nonabelian.

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1 Introduction

The pre-eminent task facing string theorists of our time is finding the answer to the deep, and puzzling, question: “What is String Theory?” [1]. We need an answer that is plausible, consistent with all of the known facts about weak-strong-dual effective field theory limits of nonperturbative String/M theory, and that is both mathematically, and aesthetically, satisfactory [2, 3, 4, 5, 6, 7, 8]. In recent work, we have proposed a new matrix framework for nonperturbative string/M theory [9]. In what follows, we explain the motivation underlying our proposal, illustrating many points within the context of the simpler bosonic matrix theory. In particular, we also include in this paper a worldsheet analysis of the open and closed unoriented bosonic string which is to be matched with the $1/N$ expansion about a planar limit of bosonic matrix theory. We will develop new tools necessary for an in-depth study of the proposed matrix framework for M theory given in [9].

For the most part, our discussion focuses in this paper on Bosonic Matrix Theory.2 The inequivalent multiple-scaled large $N$ states of the quantum matrix effective action are conjectured to be in one-to-one correspondence with the nonperturbative states of the quantum open and closed bosonic string. Recall that unoriented, open and closed, bosonic string theory is the unique bosonic string that is both tadpole-free, and with nonabelian gauge fields in the critical spacetime dimension [13, 14, 15]. There is a tachyonic mode in both the $SO(2^{13})$ and $SO(2^{12}) \times SO(2^{12})$ bosonic string ground states at zero temperature. The tachyon can, however, be removed at temperatures above a critical temperature of order the string scale [18]: $\beta < \beta_c$, where $\beta_c^2 = 4\pi^4 \alpha'/(\Delta w)^2$, in the presence of a time-like Wilson line. $T_c$ is the self-dual temperature of the bosonic closed string theory. The $\Delta w = \pm 1$ states may be interpreted in the $T$-dual coordinate as winding modes in Euclidean time. The stretched strings are massive at temperatures below the string scale, where the string vacuum has a tachyonic instability—a runaway direction in the tachyon potential.

At $T = T_c$, however, the stretched strings turn massless enhancing the gauge symmetry to $SO(2^{13})$. Above $T_c$ the scalar field has positive mass, and $T = T_c$ is therefore a metastable turning point of the tachyon potential. This is the prediction of classical string theory. It is this high temperature stable ground state of bosonic string theory, both tadpole and tachyon free, which will be the model of interest for us in this paper. We will conjecture that the quantum string vacuum with $SO(2^{13})$ enhanced gauge symmetry is the stable IR limit of the classically metastable open and closed bosonic string, at both zero and at finite temperatures. Some motivation for this identification will be given, based both on prior work, and on expectations from open string field theory [3]. We perform matching calculations between the anomaly-free and IR stable high temperature ground state of the bosonic string and the planar limit of the quantum matrix effective action, in a ground state with $SO(2^{13})$ symmetry arising from $2^{12}$ matrix D24branes. We emphasize that the matrix theory calculations will give an unambiguous prediction for the higher derivative interactions in the effective action of the quantum open and closed bosonic string.3

The worldsheet analysis of the classically metastable open and closed bosonic string at one-loop

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2We should clarify at the outset that our proposal has no relation to the intriguing conjecture put forth in [11].

3We note that if, instead, the metastable behavior of the classical bosonic string ground state is also found in the planar continuum limit of the bosonic matrix action, this would provide a nice example of a model with an inflationary potential. Modulo the issue of finding analogous phenomena in a realistic supersymmetric matrix ground state with Standard Model symmetries, and at suitable energy scales, this is potentially a phenomenon of great interest [18, 22].
order is introduced in section 2 of this paper, moving on to the high temperature stable quantum ground state of the bosonic string theory with $SO(2^{13})$ Yang-Mills gauge symmetry. Section 3 begins with a discussion of classical matrix actions, and the implementation of the Noether procedure for the bosonic finite $N$ matrix algebras. In particular, we discuss the construction of manifestly covariant matrix actions for $U(N) \times SL(d, R) \times G$, where $G$ is the group of extended Yang-Mills symmetry at finite $N$, and $d=26, 10$, respectively, in the bosonic, or super, matrix model cases. $G$ includes symmetries mediated by the higher rank antisymmetric $p$-form matrix potentials. These symmetries are described in some detail in section 3.2, moving on to a discussion of the matrix quantum effective action for an arbitrary Dbrane state in section 3.2.

Matrix Dbrane states demonstrate a remarkably simple and elegant phenomenon we will refer to as Dbrane democracy: closure of the finite $N$ matrix Lorentz algebra on any matrix theory ground state with D25brane charge requires that the ground state is simultaneously charged under all even rank antisymmetric matrix potentials. Additional invariance under the gauge symmetry mediated by the one-form matrix potential requires a ground state charged under the full spectrum of antisymmetric $(p+1)$-form matrix potentials with $p$ taking any integer value less than 26. Matrix Dbrane democracy has a beautiful large $N$ remnant in the form of mixed Chern-Simons couplings in the effective Lagrangian whenever the one-form gauge symmetry is nonabelian. This is described in section 3.3. In the conclusions, we comment on the match between higher derivative terms in the $1/N$ expansion of the quantum matrix effective action and the $\alpha'$ expansion of the quantum string effective action, clarifying also the physics intuition underlying large $N$ reduced models [10, 6, 9]. We explore future directions of interest in relation to our work.

In closing, we would like to apologize in advance for the absence in this paper of motivation and review of prior work, especially in the area of matrix theory techniques. This is due to circumstances which prevented the author from following developments in this field on a regular basis. We hope that this odd absence in the introduction is adequately made up for, in part, by our referencing of papers we have found stimulating, and of possible relevance to our work. We offer our sincere apologies to those authors whose work has been thus overlooked.

## 2 Quantum Open and Closed Bosonic String Theory

We begin with a review of the unoriented open and closed bosonic string theory [13]. The one-loop vacuum amplitude for the tadpole-free unoriented string with gauge group $SO(2^{13})$ is given in detail in Chap. 7 of Polchinski’s text [13]. In the modern language of Dbranes, this vacuum can be interpreted as that with $2^{12}$ coincident D25branes, whose worldvolume is the Minkowskian signature spacetime with $(1, 25)$ noncompact dimensions. In the tadpole-free vacuum, the one-loop vacuum amplitude may be written in the simple form [13]:

$$ A = iV_{26}(2\pi^2\alpha')^{-13} \int_0^\infty \frac{dt}{4t^4} t^{-13} \eta(it)^{-24} \left( 1 \mp \eta(it)^{12} \Theta_{00}(0; it)^{-12} \right). $$

For clarity, we have separated the contributions from Klein bottle (1) and annulus (2) inside the round brackets. It is obvious from the $q$ expansion of this expression, where large $t$ gives the lowest-lying states in the open string sector, that the spectrum will be tadpole free. But the leading

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4This is distinct, although not unrelated, to the use of the term $p$-brane democracy in [12].
contribution from Klein bottle and annulus contains a tachyonic state, contributing $3e^{2\pi t}$ to the $q$ expansion. This is the result for the, classically unstable, 26-dimensional open and closed bosonic string at zero temperature.\(^5\)

Consider compactifying one dimension on a circle of radius $R$. For a spatial compact dimension, we can include a spacelike Wilson line thus breaking the nonabelian gauge symmetry to $SO(2^{12}) \times SO(2^{12})$. The $T_{25}$-dual picture is a configuration of spatially separated stacks of $2^{11}$ D24branes, lying on orientifold planes at $X_{24}'=0$, and $X_{24}'=\pi R'$. The $T$-dual radius is given by $R'=\alpha' / R$. Consider the background with a stretched Dstring linking the well-separated D24brane stacks. The stretched strings contribute a term of classical origin in the formula for the open string mass spectrum, and the mass of the tachyonic mode will be shifted for spatial separations much larger than the string scale. The one-loop amplitude in this background is given by:

$$B = i V_{25} (2\pi^2 \alpha')^{-25/2} \int_0^{\infty} \frac{dt}{4t} \eta(it)^{-24} \left( 1 + 2 \mp \eta(it)^{12} \Theta_{00}(0; \eta(it))^{-12} \right) q^{R'^2 / \pi^2 \alpha'} ,$$

and where it is understood that the spatial radius is much larger than string scale distances. If $R'$ is reduced to string scale values, the stretched strings turn massless, contributing additional gauge bosons that enhance the symmetry to the full $SO(2^{13})$. For all separations above $R'=R_c$, the tachyon is massive. The divergence due to the tachyon is recovered below $R_c$, there is a runaway potential for the tachyonic state. We emphasize that these are the predictions from the classical string theory analysis we are able to perform at zero temperature. Our interest here is in results that hold at weak, not necessarily vanishing, open and closed string couplings.

Consider the finite temperature behavior of this theory in the limit of well-separated D24branes. A Wick rotation to Euclidean signature with compact “time” yields the finite temperature effective action functional, $W(\beta)$, for open and closed bosonic string theory at one-loop order for this string state. The reader is referred to the discussion given in [18], based on the earlier works cited in that paper. Besides the zero temperature modes described above, there are Matsubara thermal modes, all of which are tachyonic at low temperatures, turning massless at given critical temperature of order the string scale. The free energy, $F(\beta) \equiv -\frac{1}{\beta} W(\beta)$, at one-loop order in this background is given (in part) by an expression $F_1$ of the form [18]:

$$F_1 = \beta^{-1} \int_0^{\infty} \frac{dt}{4t} (2\pi t)^{-12} \eta(it)^{-24} \left( 1 + 2 \mp \frac{\eta(it)^{12}}{\Theta_{00}(0; \eta(it))^{12}} \right) \sum_{n \in \mathbb{Z}, \Delta w=0,1} q^{R'^2 / \pi^2 \alpha' + \alpha' \pi^2 n^2 / \beta^2} ,$$

where it is understood that the D24brane stacks are well-separated, and the spatial separation, $R'$, is much larger than string scale. Upon lowering the separation $R'$ to $R_c$, the stretched strings will turn massless, contributing additional gauge bosons which enhance the gauge symmetry to $SO(2^{13})$. At low temperatures, the tachyon will re-appear in the limit of coincident Dbranes. This would seem to be a bit contrived, since we are accustomed to thinking of the Dbrane separation, or the compactification radius of the T-dual theory, as a freely varying modulus. In particular, the simple-minded prescription for the finite temperature string state given here leads to the unphysical, and

\(^5\)Since the classical bosonic string at zero temperature is anyway unstable, one may ask why we are focused on a tadpole-free background. The reason is that we are interested in finding a mechanism with clear analog for the supersymmetric type I string. There, the dilaton tadpole must vanish in order to remove an accompanying unphysical tadpole for a Ramond-Ramond eleven form (see the discussion in [15, 13]). Moreover, as explained below, the classical bosonic string is actually metastable at finite temperature.
inescapable, conclusion that there are low temperature tachyons even in the case of supersymmetric string theories. Fortunately, the finite temperature prescription has since been refined in our recent work [18], and we will use these papers for guidance in the discussion below.

Notice that the expression in Eq. (3) is clearly incomplete, since it does not explained what happens to the finite temperature behavior of the $SO(2^{13})$ theory with coincident D24branes, the analog of the $SO(32)$ type I’ string at finite temperature. As explained by us in [18], the finite temperature theory in either case, i.e., with well-separated, or coincident, D24brane stacks, is well-defined upon incorporating a timelike Wilson line, $\beta A^0=(18,0^8)$. This step simply corresponds to quantization of the Yang-Mills theory in a modified axial gauge. The timelike component of the Yang-Mills potential, $A_0$, has been set to a temperature-dependent constant, rather than zero. It implies that the nonabelian gauge group at finite temperature is $SO(2^{12})\times SO(2^{12})$. The circle radius, $R_{24}$, or its $T$-dual D24brane separation, $R'_{24}$, is now a free modulus, as in the zero temperature string, precisely as one would expect on physical grounds.

In the presence of the time-like Wilson line, there are timelike winding modes stretched between the well-separated D24brane stacks, as in the analogous type I’ example described in [18]. Let us denote the stacks by $w=0$, and $w=1$, respectively. Then the free energy, $F(\beta)=-\frac{1}{\beta}W(\beta)$, at one-loop order in this background is given by the expression [18]:

$$F = \beta^{-1} \int_0^\infty \frac{dt}{4t} (2\pi t)^{-12} \eta(it)^{-24} \left( 1 + 2 \frac{\eta(it)^{12}}{\Theta_{00}(0; it)^{12}} \right) \sum_{n \in \mathbb{Z}, \Delta \omega = 0,1} q^{R^2/n^2} \frac{\alpha^4}{\beta^2 + (\Delta \omega)^2 + \beta^2}. $$

The classical term in the worldsheet action can now be removed without appearance of the tachyon. In other words, $R'$ can be taken smoothly to below string scale distances without a tachyonic instability. Approached from the high temperature end, the stretched strings turn massless at a critical temperature, $T_c$, given by $T^2_c=1/4\pi \alpha'$. Keeping $T=T_c$ fixed, we can perform a $T$-duality, $R=\alpha'/R'$. Working in the $T$-dual theory with coincident D25branes on a compactified circle, we can smoothly examine the distance regime far below the string scale in the primed coordinate: $R\rightarrow\infty$, $R'\ll\alpha'^{1/2}$. Conversely, this is the noncompact limit from the viewpoint of the original coordinate, $X_{25}$. The gauge symmetry is enhanced to $SO(2^{13})$ at $T_c$; this string state is free of both tachyon, and dilaton tadpoles, thereby stable at temperatures of order the string scale and beyond. To summarize, the critical temperature above which the bosonic open and closed string state, with as many as 25, arbitrarily small, compact spatial dimensions, is well-defined is $T^2_c=1/4\pi \alpha'$. It is this bosonic string state with enhanced gauge symmetry, and fixed critical temperature, $T=T_c$, we will be interested in while matching to a bosonic matrix theory analysis. We comment that $T_c$ is also the self-dual temperature of the closed bosonic string. Our conjecture is that the classically stable string state at $T_c$ represents the IR stable quantum ground state for the classically unstable, or metastable, bosonic string states. As predicted at weak coupling in open and closed string perturbation theory.

We comment that it may be possible to test this conjecture using string field theory methods [3, 22]. From the perspective of matrix theory, this is not very significant since our main focus is on the conjectured equivalence of the classically stable bosonic string ground state at $T=T_c$ with the planar limit of bosonic matrix theory. This equivalence can be tested by comparison of the higher derivative interactions in either case. If this conjecture turns out to be true, it would represent a
remarkable advance in our understanding of the matrix framework for M theory.

3 Classical Matrix Actions and Matrix Algebras

We now move on to a discussion of finite $N$ matrix algebras. We begin with the classical matrix action. We will construct it analogous to the well-understood example of a Lagrangian in classical field theory, where we begin by picking a (spacetime)$\times$(internal) symmetry group. Recall that the fields in the Lagrangian are required to transform in irreducible multiplets (irreps) of the spacetime Lorentz group. In addition, they may carry nontrivial charge under one, or more, internal symmetries. Translational and rotational symmetries, save for Lorentz boosts, could also be broken. This case implies the inclusion of certain fields in the Lagrangian with nontrivial momentum in one or more spatial direction. The procedure by which we arrived at our proposed action for Matrix Theory in [9] is simply the supersymmetric analog of this analysis, except that we work directly with $U(N)$ matrix variables. For clarity, we will only discuss the simpler bosonic matrix theory in what follows.

The fundamental variables in a bosonic matrix action are objects living in the $N^2$-dimensional adjoint representation of the unitary group $U(N)$, or any of the higher-dimensional irreducible multiplets obtained from the decomposition into irreps of the tensor product of an arbitrary number of adjoint multiplets. We begin with bosonic variables living in either the adjoint, antisymmetric traceless, symmetric traceless, or singlet, representations of $U(N)$. These irreducible representations appear already in the decomposition of the tensor product of two adjoints. Notice that although the individual components of a bosonic matrix are variables taking value in the field of ordinary real (complex) numbers, the matrix itself is a noncommuting object obeying the rules of $U(N)$ matrix multiplication. Thus, the ordering of matrices within a composite product is of crucial importance. An unambiguous prescription for matrix ordering is necessary prior to any meaningful analysis of matrix actions.

3.1 Manifest $SL(26, R)\times U(N)$ Covariance

In [9], we pointed out that an unambiguous prescription for the ordering of matrices in a composite operator is given by requiring each $U(N)$ multiplet to simultaneously transform covariantly under the $SL(26, R)$ subgroup of the finite $N$ Lorentz group. Thus, in analogy with spacetime Lagrangians, each term in the classical matrix action will be required to be an invariant of the finite $N$ Lorentz group. In addition, each matrix variable is required to live in some finite dimensional representation of the group $U(N)\times SL(26, R)$. The construction of invariant matrix actions then proceeds by the Noether procedure, familiar from classical field theory.

We work in the first order formalism for Einstein gravity. The basic objects are the vierbein, an array of $d^2 U(N)$ matrix variables, $E^a_\mu$, subject to the $d$ constraints, $E^{a\mu}E^{b_\mu}=\eta^{ab}$, and the nonabelian vector potential, $A_\mu$. The dimensionality, $d$, of the auxiliary (flat) tangent space is undetermined in the classical theory, allowing for the possibility of ground states with an arbitrary number of

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6 The failure to give a clear prescription for the ordering of $U(N)$ matrix variables at the outset resulted in considerable confusion in subsequent matching calculations for graviton scattering between the BFSS M(atrix) theory and eleven-dimensional supergravity [20].
noncompact dimensions <d. We will assume, however, the Minkowskian signature \((-,+,\cdots,+)\) in the tangent space, coordinatized by \(d\) real-valued parameters, \(\xi^a\), and with box-regulated volume \(V_d\). Thus, associated with each point in tangent space is a whole \(d(d-1)N^2\) unrestricted variables contained in the vierbein, encapsulating information about the background spacetime geometry of some large \(N\) ground state.

The individual \(E^a_\mu\) transform covariantly under \(U(N)\times SL(26, R)\). Recall that \(E^a_\mu, E^a_{\nu a}\), transform, respectively, as \((1,0)\) and \((0,1)\) bispinors under \(SL(26, R)\); each is in an adjoint multiplet of the \(U(N)\). These \(d(d-1)\) independent \(U(N)\) matrices may be assembled into composite matrix operators corresponding to the physical graviton, antisymmetric twoform, and scalar multiplets of the finite \(N\) Lorentz group. Thus, the metric tensor, \(G_{\mu\nu}\), is a \((1,1)\) tensor under \(SL(26, R)\), but is expressed as a bilinear composite of \(U(N)\) adjoints, \(E^a_\mu E^a_\nu\). Likewise, for the physical antisymmetric twoform potential, \(E^a_\mu E^b_\nu \epsilon_{ab}\), and the physical dilaton, \(E^{\mu a} E_{\mu a}\). The nonabelian vector potential also transforms covariantly under \(U(N)\) adjoints, \(A^i_\mu \tau^i\). Each element of the array is a Lorentz vector, transforming in the adjoint representation of the nonabelian gauge group \(G\). The choice of gauge group is, apriori, arbitrary. It is however uniquely determined to be \(SO(2^{13})\) once we require that the planar spacetime continuum limit of the Bosonic Matrix Theory is an anomaly-free low energy effective field theory.

Finally, we can give Lorentz invariant definitions for infinitesimal length and volume elements in a given vector space as follows. We define the length of a \(d\)-vector \(dV^\mu \in \mathcal{V}\), where each element of \(\mathcal{V}\) is an infinitesimal \(N\times N\) matrix, as follows:

\[
|dV|^2 = \text{Tr}_{U(N)} E^a_\mu dV^\mu E^a_\nu dV^\nu ,
\]

where the trace is over \(U(N)\) indices. Similar definitions can be given for the \(p\)-th volume form, \(p=2, \cdots, 26\), in any finite-dimensional vector space. The result in each case is an ordinary real number. It is evident that this definition corresponds to the usual definition of length and volume invariants of the \(d\)-dimensional Diffeomorphism group in the large \(N\) limit.

Notice that, thus far, all of the required matrix variables in the bosonic matrix action have been expressed in terms of \(U(N)\) adjoints, for e.g., \(A_\mu\), or appropriate composites of \(U(N)\) adjoints, namely, the \(E^a_\mu\). It is often helpful to express the classical action in terms of the composite matrix variables since these correspond directly to the fields appearing in the low energy spacetime Lagrangian: the scalar dilaton, \(\Phi\), symmetric two-form or metric, \(G_2\), and antisymmetric two-form, \(A_2\). When the large \(N\) ground state carries nontrivial Dbrane charges, additional higher dimensional \(U(N)\) irreps will be required in the matrix action. These new matrix variables cannot be expressed as composites of adjoints. Recall that, due to the presence of nonabelian gauge fields, the low energy spacetime Lagrangian of the bosonic string contains mixed Chern-Simons terms coupling \(A_1\) to antisymmetric \(p\)-form potentials, \(C_{[p]}\), with \(p = 0, \cdots, 25\). Such Chern-Simons terms have their counterparts in the matrix theory action. Given an understanding of the manifest symmetries desired in the classical matrix action, let us proceed with implementing the Noether procedure and with examining the detailed form of the action.
3.2 Extended Yang-Mills Symmetry

We begin with the definition of the gauge covariant derivative. At finite $N$, the gauge covariant derivative operator takes the matrix form, $D_a = \Omega_a + g A_a^i \tau^i$. The symbol “;” will be used to denote left-multiplication by $\Omega_a$. It acts as the general covariant derivative of the object to its immediate right in the planar limit: $\Omega_a \rightarrow \partial_a$, in a flat spacetime background. Recall that $\Omega_a$ transforms as a Lorentz vector, in the $(\frac{1}{2}, \frac{1}{2})$ representation of SL(26,R). The Riemann curvature scalar is expressed in the form:

$$R[E] = (\Omega_b E^{b\lambda})(\Omega_a E^a_{\lambda}) - (\Omega_a E^{b\lambda})(\Omega_b E^a_{\lambda}) + E^{a\lambda}(\Omega_a E^b_{\sigma})(\Omega_b E^c_{\sigma}) E^c_{\sigma} - E^{a\lambda}(\Omega_a E^c_{\sigma}) E^\sigma_c (\Omega_b E^b_{\sigma}) \ .$$

$R$ is quadratic in covariant derivatives of the $E^\mu_a$. Notice that operator ordering has been determined by requiring that each term in the matrix action is invariant under the finite $N$ matrix Lorentz transformations [9]:

$$\delta_L(U_a \Phi) = [L_a^c, U_c^b] \Phi, \quad \delta_L(\Phi U_a) = -\Phi [U^c, L_a^b]$$

$$\delta_L(U_a V_b) = [L_a^c, U_c^b] V_b + U_a [L_b^c, V_c]$$

$$\delta_L(U^a V_b) = U^a [L_b^c, V_c] - [U^c, L_a^b] V_b \ ,$$

for arbitrary Lorentz vectors $U$, $V$, and scalar $\Phi$. The parameter of Lorentz transformations, $L_{ab}$, is an array of infinitesimal $N \times N$ matrices, antisymmetric under the interchange of indices $a, b$.

The Yang-Mills and antisymmetric threeform field strength can be written in matrix form as follows:

$$F_{ab}^i = \Omega_a A_b^i - \Omega_b A_a^i + gf^{ijk} A_a^j A_b^k$$

$$H_{abc} = \Omega_{[a} A_{bc]} - X_{abc} \equiv \Omega_{[a} A_{bc]} - 2^{1/2} \mathrm{tr}_{ijk} (\delta_{ij} A_{[a}^i \Omega_b A_{bc]}^j - \frac{2}{3} A_{[a}^i A_b^j A_{bc]}^k) \ .$$

With this definition, the kinetic terms for both $F$ and $H$ take the standard form while incorporating the extended Yang-Mills symmetry. The mixed Chern-Simons terms are responsible for the coupling between matrix gauge potentials of different rank. Each of the matrix potentials transforms in an irrep of $U(N)$, respectively, adjoint, and antisymmetric tensor. This argument extends to all of the higher rank matrix potentials. The appropriate $U(N)$ irrep required is identified by consulting a good group theory table, requiring that the kinetic term for the $p$-form potential in the matrix action is manifestly invariant under $U(N) \times SL(26, R)$. In each case, we define the shifted field strength in order that the mixed Chern-Simons terms are incorporated naturally (see the discussion in Chap. 12 of [13]).

For example, consider a ground state of bosonic matrix theory which couples to an even rank $p$-form matrix gauge potential, $C_{[p]}$. Due to nontrivial $C_{[p]}$ charge, the bosonic matrix theory action contains a term from the series given below. Note that the $C_{[p]}$ live in independent, increasingly higher rank, irreps of $U(N)$:

$$S_0 = (F_a)^2, \quad F_a = \Omega_a C_0 - A_i$$

$$S_2 = \frac{1}{2} F_3 \wedge F_3, \quad F_{abc} = \Omega_{[a} C_{bc]} - X_{abc}$$

$$X_{abc} = 2^{1/2} \mathrm{tr}_{ijk} (\delta_{ij} A_{[a}^i \Omega_b A_{bc]}^j - \frac{2}{3} f_{ijk} A_{[a}^i A_b^j A_{bc]}^k) \ .$$
\[ S_4 = \frac{1}{2} F_5 \wedge F_5, \quad F_{abcdef} = \Omega_{[aC_{bcde]} - X_{abcdef}}, \]
\[ X_{abcdef} = \text{tr}_{ijk} (\delta_{ij} A_i^a \Omega_a A_j^b C_{de}] - f_{ijk} A_i^a A_j^b A_k^c C_{de}) + \cdots \]
\[ S_{26} = \frac{1}{2} F_{27} \wedge F_{27}, \quad \cdots \quad (9) \]

We should emphasize that the necessity for independent \( U(N) \) irrep for each of the matrix potentials in the action for Matrix Theory is forced upon us by the properties of the symmetry algebra. This is not unlike the case of open and closed string theory; while open strings do produce closed strings at the loop level, the renormalization of open and closed string coupling are independent results (see the detailed explanation in \[14, 16\]). Thus, at the quantum level, the open and closed string sectors contribute independent degrees of freedom in string theory. In particular, it is not true that “gauge theory contains gravity” in String Theory, an unfortunate misconception that appears in some of the recent literature.

The existence of Extended Yang-Mills symmetries in String/M theory simply implies the necessity for still further independent degrees of freedom associated with each of the higher rank gauge potentials. These are the Dbranes and, possibly, brane-antibranes, Mbranes, and other solitons. In Matrix Theory, we will find that all of the higher rank matrix potentials associated with Dbranes are represented democratically in the matrix effective action.

### 3.3 Quantum Effective Action and Dbrane Democracy

We will now make an important observation. Notice that the commutator of the first member of the series given in Eq. (9) with the generator of matrix Lorentz transformations, \( [C_o, L_{ab}] \), is nontrivial. This implies that finite \( N \) Lorentz invariance requires the ground state to couple to a two-form potential, \( C_{[2]} \), also an \( U(N) \) matrix. The argument can be iterated to conclude that coupling of the ground state to any one even rank potential implies, as a consequence of Lorentz invariance, the coupling to all even rank matrix potentials, with \( p<25 \). Conversely, if we begin with the ground state with nontrivial coupling to a 26 form matrix gauge potential, \( C_{[26]} \), the commutator with \( L_{ab} \) results in nontrivial coupling to the full spectrum of even \( p \)-form potentials, with \( p<25 \).

In addition, since we include a one-form Yang-Mills potential in the matrix action, the nontrivial \( U(N) \) matrix commutator, \( [A_a, C_{26}] \), has an expansion in terms of couplings to the odd rank \( p \)-form potentials, where \( p \) takes all odd integer values less than 26. Thus, the requirement of invariance under the gauge symmetry mediated by the oneform vector potential in addition to Lorentz invariance, implies a coupling to all of the remaining odd rank potentials. We emphasize that, as regards the finite \( N \) algebraic structure, matrix Dbrane democracy follows even when the one-form vector field, \( A_a \), is abelian. However, in the presence of a Yang-Mills symmetry, the finite \( N \) matrix algebra has a beautiful spacetime remnant, in the form of mixed Chern-Simon terms that survive in the planar continuum action. We have recovered a well-known result from the spacetime low energy effective field theory analyses of Dbranes. Namely, that Dbrane charge conservation must be carefully defined so as to correct for the effect of mixed Chern-Simons terms in the effective field theory \[21\]. The origin of this mixing lies in closure under the finite \( N \) Lorentz and extended gauge symmetry algebras.
We move on to a discussion of the quantum matrix effective action, exhibiting manifest invariance under the full $SL(26, R) \times U(N) \times G$. We have the following dimensionless free parameters available to us in the unoriented bosonic string theory: $g_0$, or the Yang-Mills coupling, and the constant parts of the background fields, $A_{ab}$, $F_{ab}$, and $\Phi$, assuming an expansion about flat Lorentzian spacetime. Notice that the gravitational, or closed string, coupling has been traded for the background vev of the dilaton scalar. In addition, we have the antisymmetric higher rank $p$-form matrix potentials described above.

We work in the matrix analog of the modified axial gauge for quantization of the one-form potential, preserving the residual gauge invariances of axial gauge and other symmetries of the effective action. This corresponds to a $1/N$ expansion about the timelike Wilson line background. Here, $\mathbf{1}$ denotes the identity $N \times N$ matrix in $U(N)$ space, and the $t^A, A=1, \ldots, 213$, are the hermitian generators of the Yang-Mills group $SO(213)$. Thus, the background potentials are mutually-commuting matrix variables. We consider expanding about the following classical background of bosonic matrix theory:

$$\bar{E}_\mu^a E_{\nu a} = (\eta_{\mu \nu} + \kappa h_{\mu \nu}) \mathbf{1}, \quad \bar{A}_{0 t}^A = \beta^{-1} \mathbf{1}, \quad \bar{A}_{i t}^A = 0, \quad i = 1, \ldots, 25 \quad \bar{C}_{[p]} = c\delta_{p25} \mathbf{1} \quad (10)$$

As explained earlier, the quantum effective action will contain induced charges for the full spectrum of $p$-form matrix potentials. Moreover, the charges and relative normalizations of all terms in the matrix effective action are entirely determined by symmetry given the classical input from above. This large $N$ ground state describes a flat 26-dimensional space with compact (Euclidean) time, and a timelike Wilson line. There are $2^{12}$ space-filling D25branes and, consequently, a constant 26-form antisymmetric tensor potential. The propagating degrees of freedom are 26-dimensional nonabelian gauge fields and gravitons, originating in the linearized matrix-valued perturbations: $\tilde{A}_a, \tilde{h}_{ab}, \tilde{\phi}, \tilde{A}_{ab}$, and $\tilde{C}_{[p]}$.

The quantum path integral for bosonic matrix theory is given by an expression of the form:

$$Z = \int \frac{[dE][dA][dC]}{Vol(SL(26, R) \times U(N) \times G)} e^{-S_{E,A,C}(g, A)} \equiv \exp \left(-S_{\text{eff}}[\bar{E}, \bar{A}, \bar{C}]\right) \quad (11)$$

where $\{g; \bar{A}\}$ denotes the set of free couplings and parameters describing the large $N$ background about which we expand using linearized perturbation theory. But as we will see in a moment, the matrix quantum effective action is fully determined by symmetry alone. Thus, it is possible to write down the result for $S_{\text{eff}}$ directly:

$$S_{\text{eff}} = e^{\Phi} \left(\frac{1}{2} g_2 F_{ab} F_{ab} + \frac{1}{2} \kappa \sum_{p=1}^{27} (\eta_{\mu \nu} \eta_{\rho \sigma} F_{\mu \nu} \wedge F_{\rho \sigma}) + \frac{1}{2} \kappa (\mathcal{R} - 4 \Omega^a \Phi \Omega_a \Phi + 3 e^{2\Phi} H^{abc} H_{abc}) \right) \quad (12)$$

We emphasize that there is no choice involved in either including, or excluding, the higher rank potentials: in their absence, the ground state would have no gauge fields, and the theory has no IR stable ground state. Thus, we must incorporate Yang-Mills fields. Due to finite $N$ Lorentz invariance, this automatically implies inclusion of all of the terms appearing in Eq. (12).

Notice also that there is no ambiguity in the relative normalizations of the kinetic terms of the higher rank $p$-form potentials, as a consequence of the finite $N$ Lorentz-extended-Yang-Mills invariance. Thus, the remnant mixed Chern-Simons terms obtained in the planar limit are unambiguously
determined by the extended $p$-form-Yang-Mills gauge symmetry. This is a clear prediction for the quantum string effective action to which it is matched. Notice that, unlike the unoriented bosonic string effective action, prior to taking the large $N$ limit, all of the terms required by the finite $N$ symmetries are present in the matrix action: there is both an $H_{[3]}$, and an $F_{[3]}$. This fact will be noteworthy in the supersymmetric case [9]. Notice that the matrix potential $A_{ab}$ arises in the vierbein (gravity) multiplet; $C_{ab}$ occurs as a consequence of Yang-Mills invariance. Owing to the fact that we begin with a ground state charged under $C_{[2]}$, and invariant under the gauge symmetry mediated by the one-form potential $A_a$.

In the fully democratic ground state with $SO(2^{13})$ Yang-Mills symmetry, it is possible to verify invariance of the quantum effective action, $S_{\text{eff}}$, under the finite $N$ matrix algebra, namely, under $U(N) \times SL(26, \mathbb{R}) \times G$. For completeness, we write these down explicitly as in [9]. We will introduce an infinitesimal hermitian matrix, $L_{ab}$, antisymmetric under the interchange of tangent space indices $a, b$. Keeping terms up to linear in $L_{ab}$, it is easy to verify that each term in $S$ is invariant under the matrix transformations:

\[
\delta A_a = [L^c_a, A_c] \\
\delta A_{ab} = [L^c_a, A_{eb}] + [A_{ae}, L^c_b] \\
\vdots = \\
\delta (\Omega_a \Phi) = [L^c_a, \Omega_j] \Phi - \Omega_a L^c_j \Phi, \\
\delta (\Phi \Omega^a) = -\Phi [\Omega^c, L^a_c] + \Phi L^a_c \Omega^c
\] (13)

Likewise, consider a $d_G$-plet of infinitesimal real matrices, $\{\alpha^j\}$, each of which takes diagonal $N \times N$ form. Here, $d_G$ is the dimension of the nonabelian gauge group with hermitian generators $\{\tau_j\}$. We can verify that every term in $S$ is invariant under the Yang-Mills transformations:

\[
\delta (g A^j_a \tau^j) = [\Omega_a, \tau^j \alpha^j] \\
\delta (\Omega_a \Phi) = i \tau^j \alpha^j \Omega_a \Phi, \\
\delta (\Phi \Omega^a) = -i \tau^j \Phi \Omega^a \alpha^j
\] (14)

Finally, it is easy to verify invariance under the higher rank symmetries, since the kinetic term for the field strength has been written in standard form by defining an appropriate shift.

4 Conclusions

Bosonic Matrix Theory has one dimensionless free parameter, $g$, and one intrinsic mass scale, $\alpha'$. We can, of course, trade these for the D25brane tension and Yang-Mills gauge coupling.\footnote{In a realistic ground state of the supersymmetric Matrix theory, these parameters are determined by matching with the physical coupling unification scale, and the strength of the unified Yang-Mills-gravity coupling.} In addition, we have available a broad choice of dimensionless, and dimensionful, scalings of the spectrum of background fields, namely, the spacetime metric, Yang-Mills and antisymmetric two-form potential, and higher rank $p$-form matrix potentials. As was pointed out by us in [9], such dimensionful, multiple-scaled, large $N$ limits of Matrix Theory are in one-to-one correspondence with the modified low energy effective field theory limits distinct from the original zero slope limit of string theory [7]. Such gauge-gravity effective duals arise precisely as a consequence of holding one, or more, additional mass scale fixed while taking the zero slope limit of a string theory. In matrix theory
language, such a modification of the large $N$ limit is simply an extension of the notion of the $(g, N)$ double-scaling limit familiar from the one-matrix model [19].

It should be noted that the $1/N$ corrections to the planar limit of the Matrix Theory quantum effective action contain higher derivative terms which may be compared with those obtained in the leading orders of the $\alpha'$ expansion of the string effective action. This should provide new insight into quantum corrections to the classical string theory predictions for the widely-studied gauge-gravity dualities of M Theory [7]. In connection to this subject, we should also mention the role of string loop corrections to the quantum effective action. In this paper, we have naturally focussed our attention on results which are meaningful at weak, but non-vanishing, string coupling, and in an IR stable ground state. In the fully supersymmetric case, studied by us earlier in [9], there is in addition, the possibility of incorporating weak-strong coupling duality. Specifically, we can match with the weak-strong dual heterotic-type I string theory limits but, also, the self-dual type IIB string theory limit. The matrix theory analog of the IIB string theory should be of great interest in the context of matrix Dbrane democracy. Direct comparison with some of the results of the Ishibashi-Kawai-Kitazawa-Tsuchiya matrix model [6] may be possible. It would be particularly interesting to develop analogous techniques to study genuinely nonperturbative phenomena, such as the matrix computation of Wilson Loop correlators or the dynamical selection of the quantum ground state [6], in our full-fledged proposal for Matrix Theory [9].

We emphasize that it is very important to study the full scope of the correspondence we have described above, since upon its validity hinges the success of our proposal for Matrix Theory [9]: does our matrix framework account for all of the known facts about weak-strong-dual effective field theory limits of M theory? Are there any known backgrounds which cannot be incorporated within the Matrix Theory framework? We remind that reader that it was an inability to incorporate certain backgrounds with fewer than five noncompact dimensions [4, 20]— at least in testable form, that led to the eventual demise of the BFSS M(atrix) theory. Although we have emphasized the action formulation at the outset in our work, because of its pedagogical value, it is obvious that there is nothing in our algebraic framework that requires it. In formulating the arbitrary matrix Dbrane state in section 3.2, we have described it in terms of a closed algebra. Likewise, although we are biased in this paper towards backgrounds of Matrix Theory whose planar limits contain propagating Yang-Mills and gravitational fields, neither is necessary in the arbitrary Dbrane state. Thus, little string theories [5] are expected to be compatible with our framework. It is well-known that type I states with additional Dbrane-antiDbrane pairs likewise have a simple description in terms of a closed algebra [8]. As is also true of states with NS fivebranes, and more complicated M-brane solitons. In summary, we see no insurmountable difficulties in accommodating the most general background of M theory in our proposed algebraic framework.

It remains a matter of great interest to develop the Hamiltonian formulation of Matrix Theory, analogous to the Banks-Fischler-Shenker-Susskind M(atrix) Theory framework [4]. In [9], we have emphasized that the notion that the “fundamental” degrees of freedom in Nature correspond to zero-dimensional matrix variables is a fact best appreciated from the representation-independent viewpoint of Dirac’s Matrix Quantum Mechanics. From this perspective, it is a perfectly natural assumption. Since we have a comparatively clear understanding of propagating degrees of freedom that fill both space and time, namely, quantum field theory, we can simplify our considerations, eliminating space by invoking translational invariance to “reduce” the dynamical degrees of freedom.
to those living on a single spatial point [10]. This leads to a Hamiltonian framework with a uniquely specified time. Or, we can eliminate both space and time, leading naturally to the pre-geometrical notions of noncommutative geometry [10]. Hence the close connection between noncommutative geometry and the reduced matrix models, exploited in the work of [10, 6].

This is the main idea underlying Eguchi-Kawai reduction [10]. It is the framework within which we have cast our formalism for Matrix Theory, and the Action Principle remains an important tool for its study. We emphasize that semi-classical investigations which will undoubtedly shed much light on this framework, can make full use of the related methodology of Hamilton-Jacobi theory. We should comment here that it would be very interesting to examine the matrix analog of the classical Dirac-Born-Infeld action for Dbranes. In particular, this could lead to a much simplified discussion of the vexing nonabelian DBI action describing the limit of coincident Dbranes [17].

Although we have emphasized the arbitrary Dbrane state above, we find it encouraging that the fully democratic quantum ground state of Bosonic Matrix Theory with a finite, and anomaly-free, planar limit corresponding to the zero slope limit of the unoriented open and closed bosonic string, has a simple and elegant description within the action formulation. Our investigations have their obvious parallel in an analogous state of the nonperturbative type I'-heterotic theory and its matrix analog [9]. Future analyses which explore the full nonperturbative dynamics of Matrix Theory will tell us whether these observations are prescient.

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NOTE ADDED (JULY 2005): I have corrected one inexplicable (silly) typo in the draft; $SL(2, C)$ in place of, the obvious, $SL(26, R)$. I wish somebody had brought this to my notice. I consider this a beautiful paper, with many ideas, whose impact has still to be realized in the full supersymmetric matrix proposal for nonperturbative String/M theory. The expression for the “free energy” in the discussion in section 2 is obviously only part of the answer, missing the unoriented and closed string contributions, clarified already above Eq. (3). The “inescapable” conclusion at the top of page 4 refers only to the type II closed superstrings in flat spacetime with trivial Ramond-Ramond sector. “Extended” Yang-Mills symmetries just means higher rank pform gauge potentials.

Appendix: Low Energy Spacetime Action of the Unoriented Bosonic String

For completeness, we list our conventions and recall the precise form of the Lorentz invariant spacetime Lagrangian describing the low energy limit of the unoriented open and closed bosonic string theory [13]. We use the first order vierbein formalism for Einstein gravity, and work in the weak field approximation, perturbing about the fixed background metric, $G_{\mu\nu}(x)=\eta_{\mu\nu}+2\kappa h_{\mu\nu}$, as in [23]. The dimensionful coupling $\kappa$ is the 26-dimensional Newton’s constant, $(8\pi G_N)^{1/2}$. For full generality, we could allow for a possible constant background electric field pointing in the spatial direction $X^{24}$, as well as a constant background $B$ field. Thus, the two-dimensional space ($X^{23}, X^{24}$) could be noncommutative. This range of backgrounds will permit us to make appropriate comparisons with a broad range of results from perturbative open string theory.
The physical fields of the gravity multiplet consist of the graviton, dilaton scalar, and antisymmetric twoform tensor field. The vierbein is denoted $e_\mu(x)$, where $\mu=0, \cdots, 26$, parameterizes the flat local tangent space, and $\mu$ is the spacetime index. The physical composite fields can be expressed as follows:

$$g_{\mu\nu}(x) = e_\mu^a(x)e_\nu^b(x)\eta_{ab}, \quad \phi(x) = e_\mu^a(x)e_\mu^a(x), \quad b_{\mu\nu}(x) = e_\mu^a(x)e_\mu^b(x)e_{ab} \quad .$$  \hfill (15)

The dilaton vev and gravitational coupling appear in the action only when multiplied together, $\kappa e^{-\phi}$. Thus, any finite rescaling, or renormalization, of the closed string coupling should be understood as a change in the vacuum expectation value for the dilaton field $[13]$. The Yang-Mills multiplet is composed of the vector potential, $f^{ijk}A^i_\mu(x)$, $i=1, \cdots, d_G$, transforming in the adjoint representation of the orthogonal group $SO(2\kappa)$. The choice of gauge group can be determined by requiring the absence of all worldsheet anomalies or, equivalently, all gauge and gravitational spacetime anomalies $[13]$. The Yang-Mills coupling is dimensionless and is to be identified with the open string coupling, $g = g_{\text{open}}$ $[13]$. We emphasize that the perturbative renormalizations of open and closed string couplings are known to have independent origin $[13, 16]$. This result implies the analogous independence of the perturbative renormalizations of the gauge and gravitational couplings appearing in the low energy spacetime effective Lagrangian.

In the weak field approximation, the Ricci curvature tensor takes the simple form:

$$R_{\mu\nu} = \kappa \left( \Delta h_{\mu\nu} - \partial_\lambda h_{\lambda\mu} + \partial_\mu \partial_\nu h^\lambda \right) \quad .$$ \hfill (16)

where the symbol “;” denotes the general covariant derivative, allowing for a nontrivial Christoffel connection. The nonabelian gauge covariant field strength takes the familiar form:

$$F_{\mu\nu}[A(x)] = \partial_\mu A_\nu^i(x) - \partial_\nu A_\mu^i(x) + g f^{ijk}A_\mu^j(x)A_\nu^k(x) \quad .$$ \hfill (17)

More generally, the scalar curvature, $R[e(x)]$, can be expressed as:

$$e^a_\mu(x)e^\nu_b(x) \left[ e^{b\lambda}_\nu(x)e^{a}_{\lambda\mu}(x) - e^{b\lambda}_\mu(x)e^{a}_{\lambda\nu}(x) + e^{a\lambda}(x)e^{b\nu}(x)e^\sigma_c(\partial_\mu e^{\sigma\nu}(x)e^b_\nu(\partial_\mu e^{\nu\mu}(x)) - e^{a\lambda}(x)e^{c\nu}(x)e^{b\sigma}(\partial_\mu e^{\nu\mu}(x)) \right] .$$ \hfill (18)

Thus, allowing up to two time derivatives, the manifestly local Lorentz invariant and diffeomorphism invariant spacetime Lagrangian coupled to both a Yang-Mills and possible higher $p$-form antisymmetric gauge potential takes the form:

$$S_B = -\int d^dx e^{d-7} \left[ \frac{1}{4}F_{\mu\nu}[A(x)]F^{\mu\nu}[A(x)] + \frac{1}{2\kappa^2} R[e(x)] + \frac{3}{2}H_{\mu\nu\rho}[e(x)]H^{\mu\nu\rho}[e(x)] - \phi \Delta \phi[e(x)] \right] .$$ \hfill (19)

Due to the presence of $\kappa$ dependent terms, the Yang-Mills symmetry is automatically extended to include transformations involving the two-form tensor field accompanied by ordinary Yang-Mills gauge transformations $[23, 13]$. Furthermore, upon inclusion of a nontrivial $C_{26}$ antisymmetric tensor potential, the presence of mixed Chern-Simons couplings induced by requiring closure of the extended gauge symmetry group implies induced $p$-form potentials for all of the antisymmetric gauge potentials with $p<26$. The kinetic terms for the gauge potentials can be restored to the canonical form by appropriate field redefinitions in the antisymmetric $(p+1)$-form field strength tensors. For the threeform field strength, the shift takes the form $[23]$

$$H_{\mu\nu\rho} \equiv \partial_\mu A_{\nu\rho} - X_{\mu\nu\rho} \equiv \partial_\mu A_\nu^i A_\rho^j - 2^{1/2} \text{tr} \left( A_{[\mu} \partial_\nu A_{\rho]} - \frac{2}{3} A_{[\mu} A_\nu A_\rho] \right) \quad .$$ \hfill (20)
We emphasize that in the absence of the gauge and gravitational couplings—or in the free field limit, the different possible \((p-1)\)-form charges should be understood as being independently conserved charges on the vacuum.

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