Searching for signals of minimal length in extra dimensional models using dilepton production at hadron colliders

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Abstract

Theories of quantum gravity suggest the existence of a minimal length scale. It is interesting to speculate what the consequences of the existence of such a length scale would be for models of large extra dimensions, in particular, the ADD model. When ADD model is conflated with the minimal length scale scenario, processes involving virtual exchange of gravitons cease to be ultraviolet divergent. We study the production of dileptons at hadron colliders as an example of a process mediated by virtual gravitons. We find that the bounds we derive on the effective string scale are significantly different (in fact, less stringent) from those derived in the conventional ADD model, both at the upgraded Tevatron and the Large Hadron Collider.

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I Introduction

A reinterpretation of the gauge hierarchy problem by Arkani-Hamed, Dimopolous and Dvali (ADD) [1] involves an alteration in the behaviour of gravity at small distances owing to the existence of large extra compactified space dimensions. In the ADD scenario, the gauge interactions are confined to a 3-brane while gravity propagates in all the dimensions. The effective Planck scale \( M_S \) in this model gets related to the usual Planck scale by \( M^2_{Pl} = R^d M^d_S + 2 \), where \( R \) is the radius of compactification and \( d \) is the number of extra dimensions. It is possible to choose \( R \) for a given \( d \) such that the fundamental string scale comes down to \( M_S \sim 1 \text{ TeV} \). Such a low value of \( M_S \) is also exciting from the point of view of studying quantum gravity at present and future colliders. Various signals of graviton production and virtual graviton exchange have been proposed in several papers in the last few years [2, 3]. These are also reviewed in Refs. [4, 5].

Another important general feature of quantum gravity theories, apart from the requirement of extra dimensions, is the existence of a minimal length scale (MLS). In string theory, such a minimal length is suggested since strings cannot probe distances smaller than the string scale. If the energy of a string reaches the Planck scale, string excitations can occur causing its extension [6]. Even though we are inspired by this assertion of quantum gravity theories, nevertheless we implement the notion of a minimal length scale phenomenologically. Note that in some fundamental theory, like string theory, such a notion may have a different meaning and interpretation. In the context of large extra dimensional models exhibiting a low fundamental scale of \( M_S \) of order 1 TeV, the existence of a minimal length becomes phenomenologically important if we take it to be around an inverse TeV, viz. \( l_p \sim 1/M_S \). In the ADD model, for an amplitude involving virtual gravitons, one has to sum over
an infinite tower of graviton Kaluza-Klein (KK) states. The result is divergent and to cure the divergence an ad-hoc cutoff of the order of $M_S$ is used, see Ref. [2] for details. A very attractive feature of the MLS scenario is that one can sum over the entire KK graviton tower because the contribution of higher energy KK states is smoothly cut off, as we shall see later, rendering the amplitude finite. This constitutes the main motivation for incorporating the effects of a minimal length scale in the ADD model. This also leads to a significant deviation of the bounds on the conventional ADD model parameters, e.g. $M_S$, derived from collider searches, as we shall see later.

II The MLS scenario and $q\bar{q} \rightarrow l^+l^-$

We define the MLS scenario to mean the ADD model with the idea of a minimal length $l_p$ incorporated in it with $l_p$ to be of the order of $\text{TeV}^{-1}$. The uncertainty in position measurement now cannot be smaller than $l_p$, which means that the standard commutation relation between position and momentum needs to be modified [7, 8]. In the minimal length scheme, since distances less than $l_p$ do not exist, the Compton wavelength ($\lambda = 2\pi/k$) of a particle cannot be arbitrarily small. However, we suppose that even though the wave vector $k$ is bounded from above, the momentum $p$ can be made as large as possible. Similarly, to maintain the same dispersion relation the frequency $\omega$ is restricted from above while the energy $E$ can go up arbitrarily. This immediately requires that the standard relations $p = \hbar k$ and $E = \hbar \omega$ have to be modified. This can be realised by introducing the following ansatz for the modified relations, known as the Unruh relations [9]

\begin{equation}
\begin{aligned}
l_p k(p) &= \tanh^{1/\gamma} \left( \frac{p}{M_S} \right)^{\gamma}, \\
l_p \omega(E) &= \tanh^{1/\gamma} \left( \frac{E}{M_S} \right)^{\gamma},
\end{aligned}
\end{equation}

where $\gamma$ is a positive constant. Hereafter, we take $\gamma = 1$ for simplicity and set $l_p M_S = \hbar = 1$. The above simple relations capture the essence of a minimal length scale: as $p$ (or $E$) becomes very large, $k$ (or $\omega$) approaches the upper bound $1/l_p$ and hence one cannot probe arbitrarily small length and time scales. The relations (1) lead to a generalized position-momentum and energy-time uncertainty principle which can be written in a Lorentz covariant form as

\begin{equation}
[x^\nu, p_\mu] = i \frac{\partial p_\mu}{\partial k^\nu}.
\end{equation}

As explained in [7, 10], the effect of the modified relations (1) can conveniently be accounted for by a simple redefinition of the momentum measure, which in one dimension is

\begin{equation}
d p \rightarrow dp \frac{\partial k}{\partial p}.
\end{equation}

The Lorentz invariant 4-momentum measure scales as

\begin{equation}
d^4 p \rightarrow d^4 p \det \left( \frac{\partial k_\nu}{\partial p_\nu} \right) = d^4 p \prod\limits_\nu \frac{\partial k_\nu}{\partial p_\nu},
\end{equation}

where the Jacobian matrix $\frac{\partial k_\nu}{\partial p_\nu}$ can be kept diagonal. Some applications of the minimal length scenario have been discussed in Refs. [11, 12]. In this paper, we study dilepton production at hadron colliders as a probe of extra dimensional models with a minimal length scale. To this end, we compute the dilepton production cross section at hadron colliders (i) in the SM (the standard Drell-Yan process), (ii) in the conventional ADD model (i.e., with no minimal length), and (iii) in the MLS scenario. In the ADD model and in the MLS scenario, apart from the SM diagrams, the dilepton production cross section receives contributions from diagrams involving virtual gravitons coupled to quarks and gluons. As mentioned previously, for diagrams involving virtual gravitons the amplitude is a sum over an infinite tower of
KK states having masses \( m_n = n/R \), where \( n \) is an integer. Each state couples to fermions with a strength \( 1/M_{Pl} \) and a summation over the KK states enhances the effective strength to \( 1/M_S \). The derivation of the Feynman rules for the ADD model can be found in [2] and the expression for the Drell-Yan differential cross section is displayed in [13]. In the expression for the squared matrix-element the interference term between the SM diagram and the diagram with graviton exchange goes like \( F/M \). The section is displayed in [13]. In the expression for the squared matrix-element the interference term between the Feynman rules for the ADD model can be found in [2] and the expression for the Drell-Yan differential cross section given in [13] according to Eqs. (6) and (7).

\[
\frac{F}{M_S^2} = \frac{-2}{M_S^{d+2}} (\sqrt{s})^{d-2} I(M_S/\sqrt{s}), \quad \text{where} \quad I(\Lambda) = P \int_0^\Lambda dy \frac{y^{d-1}}{1-y^2},
\]

where \( \sqrt{s} \) is the partonic center-of-mass energy. The integral \( I(\Lambda) \) is ultraviolet divergent, showing that the summation over the graviton KK states is infinite, as mentioned before. But in the MLS scenario, because of the Unruh relations, the integral is modulated by a factor \( \partial \omega/\partial E \) which smoothly cuts off the contribution of the higher energy KK states. Hence we can perform the integration over all the KK states:

\[
\frac{F}{M_S^2} \to \frac{F'}{M_S^2} = \frac{-2}{M_S^d} \left( \frac{\sqrt{s}}{M_S} \right)^{d-2} I', \quad \text{where} \quad I' = P \int_0^\infty dy \frac{y^{d-1}}{1-y^2} \text{sech}^2 \left( \frac{\sqrt{s}}{M_S} \right).
\]

The sech-square function for large \( y \) goes like \( \exp(-2y) \) which damps the power law growth of \( y \), thus avoiding the need for an ad-hoc cutoff as in \( I \). The divergence is thus dynamically remedied by the requirement of minimal length.

Another important modification due to the MLS is the change in the cross section due to the rescaling of the momentum measure given in Eq. (4). It is straightforward to check, as derived in [10], that the phase space integration in the total cross section picks up the following modification factor:

\[
d\sigma(\text{modified}) = d\sigma \prod_n \frac{E_n}{\omega_n} \prod_\nu \frac{\partial k_\nu}{\partial p_\nu} \big|_{p_i = p_f}, \tag{7}
\]

where \( n \) runs over the four initial and final states in a \( 2 \to 2 \) process, and \( p_i \) and \( p_f \) are the total initial and final four momenta in a \( 2 \to 2 \) process. We work out this modification factor for the process we are studying viz. \( ab \to l^+l^- \), where \( a, b \) are the initial state partons. Using Eqs. (1) and (4), we can easily show that

\[
\prod_n \frac{E_n}{\omega_n} = \frac{sp^2}{4M_S^4} \prod_{i=1,2} x_i \cosh(y_i) \coth \left( \frac{x_i \sqrt{s}}{2M_S} \right) \coth \left( \frac{p_T \cosh(y_i)}{M_S} \right), \tag{8}
\]

\[
\prod_\nu \frac{\partial k_\nu}{\partial p_\nu} = \text{sech}^2 \left( \frac{(x_1 + x_2) \sqrt{s}}{2M_S} \right) \text{sech}^2 \left( \frac{(x_1 - x_2) \sqrt{s}}{2M_S} \right), \tag{9}
\]

where \( x_1 \) and \( x_2 \) are the momentum fractions of the hadrons carried by the initial state partons, while \( y_1 \) and \( y_2 \) are the pseudorapidities and \( p_T \) is the common transverse momentum of the final state leptons. It can be easily checked that in the decoupling limit \( M_S \gg \sqrt{s} \), the phase space correction factor goes to unity.

### III Discussion of results

In [13], the effect of virtual graviton exchange has been studied for dilepton production in the conventional ADD model. A lower bound of \( M_S \) in the 1 to 3.5 TeV range (Tevatron Run I and II) and 6.5 to 12.8 TeV range (LHC) has been obtained at 95% C.L. by varying \( d \) in the range 2 to 7. To incorporate the effects of the minimal length, we modify the expression for the differential cross section for the Drell-Yan process given in [13] according to Eqs. (6) and (7).

\[1\]While computing the pure graviton term in the cross section, we also include the resonant contribution by actually taking \( \sqrt{t^2 + \pi^2/4} \) in place of \( t' \) in Eq. (6).
Figure 1: The dilepton cross section integrated over invariant mass $M$ of the dilepton pair as a function of the fundamental scale $M_S$ at the (a) Tevatron Run II with $M > 250$ GeV and, (b) LHC with $M > 600$ GeV. The two curves are for the conventional ADD model and the MLS scenario, assuming the number of extra dimensions $d$ to be 3. Also shown are the 95% C.L. upper and lower bounds (‘SM2’ and ‘SM1’, respectively) on the SM cross section (assuming only statistical errors).
Fig. 1 contains the results of our numerical computation. Fig (1a) shows the dilepton production cross section, integrated over dilepton invariant masses greater than 250 GeV and over the pseudorapidity range $|y_1|, |y_2| \leq 1.1$ and $1.5 < y_1, y_2 < 2.5$, as a function of the scale $M_S$ for $p\bar{p}$ collisions at the upgraded Tevatron (Run II) operating at an energy of $\sqrt{s} = 2$ TeV and an integrated luminosity of 2 fb$^{-1}$. We have used the MRS2001 LO parton densities$^2$. The two curves shown in Fig. (1a) are for the conventional ADD model$^3$ and MLS scenario respectively. We have taken $d = 3$ for the curves shown in the figure$^4$. We have also shown the 2-$\sigma$ upper and lower limits of the SM, assuming only statistical errors. We observe that the cross section for the MLS is smaller than that for ADD. This is mainly because of the exponential suppression of the phase space factor. For the ADD case, a bound of about 1.63 TeV results at the 95% C.L., but this bound is diluted in the MLS scenario where we get a 95% bound of about 1.45 TeV. Fig. (1b) displays the case of LHC ($pp$ collisions at $\sqrt{s} = 14$ TeV), where we assume a luminosity of 100 fb$^{-1}$ and integrate over dilepton invariant masses greater than 600 GeV and over the pseudorapidity range $|y_1|, |y_2| \leq 2.5$. Again, we have taken $d = 3$. We obtain the lower bound on $M_S$ to be about 7 TeV for the conventional ADD model which goes down to about 6 TeV for the MLS scenario.

IV Conclusions

A minimal length scale is a generic prediction of quantum gravity theories. In brane world models, like the ADD model, the implications of such a minimal length can be probed in collider experiments in the TeV range. We study dilepton production at hadron colliders mediated by virtual graviton exchange in an ADD model having a minimal length of the order of the inverse of the effective string scale. The technical benefit of introducing the minimal length into the ADD model is that the sum over the graviton tower is regulated which does not any more require the introduction of an ad-hoc cutoff. We find that the bounds are substantially lowered in such a scenario as compared to the conventional ADD model without the minimal length scale. Our choice of studying the dilepton production cross section in hadron colliders is an illustrative one; similar analyses can be carried out for other processes. In all cases, the suppression factors associated with minimal length would relax the existing constraints.

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$^2$In principle, the MLS hypothesis should also modify the parton densities. But the effects are numerically insignificant: the parton densities are calculated by employing the QCD input data obtained at a few GeV scale, while the inverse minimal length is a TeV. The $Q^2$ evolution of the parton densities is also not numerically sensitive to this issue. A similar conclusion has been drawn in [14] in the context of black hole production in hadron colliders. We thank S. Hossenfelder for bringing this point and Ref. [14] to our attention.

$^3$Our treatment of the ADD model is somewhat different from that presented in Ref. [13]. While the authors of Ref. [13] have approximated the expression for $F$ in Eq. (6) by the leading terms yielding $F = \ln(M_S^2/s)$ for $d = 2$ and $F = 2/(d - 2)$ for $d > 2$, we have used the full expressions for the integral appearing in Eq. (6).

$^4$Since our primary aim is to compare the bounds obtained in the conventional ADD model and the MLS scenario, we have restricted ourselves to a fixed value of $d = 3$. We note that the $d$ dependence appears in the graviton summation and in the MLS scenario the sech-square modulation of the integral in Eq. (6) renders that dependence rather weak.
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