Crash severity analysis of rear-end crashes in California using statistical and machine learning classification methods

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**ABSTRACT**

Investigating drivers’ injury level and detecting contributing factors that aggravate the damage level imposed on drivers and vehicles is a critical subject in the field of crash analysis. In this study, a comprehensive vehicle-by-vehicle crash data set is developed by integrating 5 years of data from California crash, vehicles involved, and road databases. The data set is used to model the severity of rear-end crashes for comparing three analytic techniques: multinomial logit, mixed multinomial logit, and support vector machine (SVM). The results of the crash severity models and the role of contributing factors to the severity outcome of rear-end crashes are extensively discussed. In terms of prediction performance, all three models yielded comparable results; although, the SVM performed slightly better than the other two methods. The results from this study will inform aspects of our driver safety education and design, either vehicle or roadway design, required to be improved to alleviate the probability of severe injuries.

**KEYWORDS**

Traffic safety; crash severity classification; machine learning; mixed multinomial logit; support vector machine

1. Background

According to crash statistics report presented annually by the Fatality Analysis Reporting System (FARS), in 2015, nearly 32,000 people were killed in vehicle crashes throughout the United States (Fatality Analysis Reporting System (FARS) Encyclopedia, National Highway Traffic Safety Administration (NHTSA), 2017). According to this report, since 1995, California, Texas, and Florida are among the states with the largest number of fatalities. The number of fatalities has decreased about 30% from the year the report was first initiated, from about 47,500 fatalities in 1994 to about 32,000 fatalities in 2015; however, the current number is still...
extremely high, which reflects the need to find remedies to decrease the rate more quickly. In this regard, the crash analysis in the context of traffic safety has become one of the main areas of focus among the traffic engineers.

Crash databases are usually built by using police reports and comprising information such as the status of the crash, driver’s information, road segment detail, environmental factors, and traffic condition. Understanding crash models and identifying contributing factors and their significance are crucial as the outcomes can be used in higher organizational- and management-level actions to define countermeasures that could prevent future crashes, improve the standards for the roadway and network design, improve public health policies, provide better emergency services, alleviate driver’s injury severity, and nurture safer driving experience.

Crashes are naturally randomly occurring incidents. Statistical models aid in better understanding the variability of these random events by examining the factors associated with them. There is a large body of research in the context of crash analysis. A review paper written by Lord and Mannering (2010) discusses the most common methodologies that have been used in studying crash frequency as well as the issues associated with them. Savolainen, Mannering, Lord, and Quddus (2011) published a similar paper discussing the most common methodologies for studying crash-severity analysis. A comprehensive study was carried out by Mannering and Bhat (2014) in which an updated list of the most recent methodologies since the two previous studies (Lord & Mannering, 2010; Savolainen et al., 2011) was presented. Their analysis also focused on demonstrating how the crash analysis approach has evolved over time and how issues identified in previous studies have been addressed in more recently introduced, advanced models (Mannering & Bhat, 2014). Most recently, Mannering, Shankar, and Bhat (2016) conducted a detailed discussion of how different statistical techniques can address the unobserved heterogeneity focusing on injury-severity analysis and analysis of accident likelihood. In the remainder of this section, the most recent and relative studies concerning the modeling of injury severity in crash databases are discussed.

Several studies that have developed injury-severity analysis models have focused on how the model’s classification performance would change in absence or presence of various factors. For example, Li, Wang, Liu, Bigham, and Ragland (2013) have concentrated on demographic attributes that are believed to vary from one city to other cities. They showed the impact of implementing spatial attributes in development of crash prediction models in the framework of a geographically weighted Poisson model (GWPM). Investigating the crash data and sociodemographic attributes of 58 counties in California, they compared the GWPM model’s performance to the popular generalized linear models (GLM) in predicting fatal crashes.
(Li et al., 2013). In another study, Wu et al. (2014) used mixed logit models to analyze the injury severity in single-vehicle crashes and crashes that involve two or more than two vehicles. In a similar approach, focusing on truck-involved crashes, Chen, Zhang, Tian, Bogus, and Yang (2015) employed a modified Hierarchical Bayesian Random Intercept model to predict the truck driver’s injury severities based on a 2-year database of truck-involved crashes on rural roadways from New Mexico. Ye and Lord (2014) investigated how the sample size can influence the performance of the multinomial logit, ordered probit, and mixed logit. Their results indicated that the sample size has a significant influence on the model performance. For example, a mixed logit model requires a much larger sample size compared to an ordered probit model, which can provide reasonable results even with small sample sizes (Ye & Lord, 2014).

The most basic crash severity modeling approaches, such as binary logit and probit models, have evolved into more advanced parametric and non-parametric models. These advanced models can address more of the unobserved characteristics of the data, which were not examined in earlier models. One such class of analytic approaches are mixed models (Milton, Shankar, & Mannering, 2008; Wu et al., 2014; Yasmin & Eluru, 2013; Ye & Lord, 2014). For instance, Milton et al. (2008) used the mixed logit model that is able to differentially account for the effect of independent variables on the level of severity over different road segments. Previous models had considered this effect to be the same on all road segments that results in biased results. The mixed feature of the model allows the coefficient of each variable affecting the injury severity (explanatory variables) to vary across all individuals in the crash database to consider the heterogeneous effect and correlations of unobserved factors (Savolainen et al., 2011; Train, 2009). In another study, when studying single-vehicle crashes in the state of California from 2003 to 2004, Kim, Ulfarsson, Kim, and Shankar (2013) employed mixed logit models in injury-severity analysis. Their results indicated the importance of considering population heterogeneity according to the notable differences in results for different age groups, especially when comparing the older age group to younger drivers. Developing separate models for two different age groups of older and younger drivers and for two gender groups on three different road surfaces, Morgan and Mannering (2011) utilized the mixed logit analysis to assess the effect of those factors on the severity of crashes in single-vehicle crashes. In a more recent study, Behnood and Mannering (2016) utilized mixed logit modeling and compared it to latent-class models using the 8-year pedestrian-injury database in Chicago. A similar modeling approach has shown promising application in other studies as well (for examples, see Anastasopoulos & Mannering, 2011; Aziz, Ukkusuri, & Hasan, 2013; Behnood & Mannering,
2015, 2017; Cerwick, Gkritza, Shaheed, & Hans, 2014; Malyschkina, Mannering, & Tarko, 2009; Manner & Wünsch-Ziegler, 2013; Moore, Schneider, Savolainen, & Farzaneh, 2011; Ye & Lord, 2011).

More recently, nonparametric modeling methods such as SVM, artificial neural networks, and decision table/Naïve Bayes (Chen, Zhang, Yang, Milton, & Alcantara, 2016) have become popular in the crash analysis studies. Nonparametric models are not built on the assumptions made based on the distribution properties of the data, which is usually the base for parametric models. An advantage of nonparametric models over parametric models is that they do not require the predefined relationship between the dependent and explanatory variables (Li, Liu, Wang, & Xu, 2012).

SVM, a supervised machine learning technique, is one of the methods of classification/regression used in many different transportation and traffic safety-related areas (for examples see (Balali & Golparvar-Fard, 2016; Jahangiri & Rakha, 2015; Jahangiri, Rakha, & Dingus, 2016; X. Li, Lord, Zhang, & Xie, 2008; C.-H. Wu, Ho, & Lee, 2004). Chen, Wang, and van Zuylen (2009) demonstrated the application of SVM as an incident detection tool to identify traffic incidents that reduce the capacity of the road. In the context of active traffic management, Yu and Abdel-Aty (2013) showed the application of SVM in real-time risk analysis to predict crash occurrences. Studying 326 freeway sections around the state of Florida, Li et al. (2012) employed SVM and ordered probit (OP) to predict the injury severity of individual crashes. Using the radial basic function (RBF), they indicated that the SVM model performed better than the OP in terms of the percent of correct predictions. The result was achieved by comparing the two models with multi-class response (five injury-severity levels). It has also been demonstrated that classification results for a two-level SVM also resulted in a significant improvement in the prediction accuracy of SVM model (Li et al., 2012).

Yu & Abdel-Aty (2013) also developed fixed parameter logit, SVM (with RBF), and random parameter logit models for 4-year data collected from a mountainous freeway section in Colorado. Comparing three models, they indicated that SVM and random parameter models provided a better fit than the fixed parameter logit models. In a more recent study, based on a 2-year crash data from New Mexico, Chen, Zhang, Qian, Tarefder, and Tian (2016) studied the application of SVM in mapping the injury severity in rollover crashes. Their result indicated that the SVM model provided reasonable performance in terms of predicting the injury severity (Chen et al., 2016). Unlike the study conducted by Li et al. (2012), where every single crash were taken as a single research unit, Chen et al. (2012) has taken each individual driver/vehicle as the research unit and taken into analysis a variable with the number of vehicles involved in the crash as an independent variable (Chen, Zhang, Qian, et al., 2016).
In this article, crash databases consisting of information on the crash, environment, vehicles, and occupants for five consecutive years (2007–2011) in the State of California were integrated. To the best of our knowledge, this vehicle-by-vehicle integrated database has not been used to develop crash models. Three approaches for modeling the severity of rear-end crashes, support vector machine (SVM), multinomial logit (MNL), and mixed multinomial logit (MMNL), were applied to this database and compared. The remainder of this article is structured as follows. The next section describes the methodology including data description and methods used to analyze the data. Then, results of the classification methods are presented and discussed for all three methods. The last section provides conclusions and future directions.

2. Methodology

2.1. Data description

The data used in this study was obtained from the Highway Safety Information System (HSIS) in the state of California for 5 consecutive years from 2007 to 2011. The database consists of three tables including the crash database, vehicle database, and road database. The rows in the crash database are built based on each case of a crash and includes information such as weather condition and lighting that is common among all vehicles involved in the crash. The vehicle database includes information specific to each vehicle such as driver’s age, sex, and vehicle type. Finally, the road database is built based on information of each road segment such as a number of lanes and terrain level. To perform a vehicle by vehicle analysis for the purpose of this study, it was required to attach the information of each crash to every single vehicle involved in that specific crash, and then add the road information to each vehicle. This task was performed using a Matlab script.

The observations for the rear-end crashes were extracted from the database. The severity-injury levels in the data set include five levels of (1) property damage only, (2) complaint of pain, (3) other visible injury, (4) severe injury, and (5) fatal. To select a subset of the independent variables for developing the models, all variables were individually examined to determine how well they classified severity levels using the area under the curve (AUC) measure to rank the variables. Top variables that provided the best results in predicting the severity level were used in the full modeling procedure. The selected variables were age, sex, terrain level, weather condition, lighting, vehicle type, number of lanes, and crash cause. The models were developed on 9,468 individual vehicles with 70% of the data randomly selected to build the training model and the remaining 30% left
as the test data to compare the performance of three models in terms of their prediction performance.

2.2. Multinomial logit (MNL) and mixed multinomial logit (MMNL) regression

Mixed multinomial logit regression, also known as random parameters logit model, is a generalized form of the multinomial logistic regression in which the coefficients of any of the variables are allowed to vary across the individuals and not be limited to a fixed value. Consequently, it allows the model to take into account the heterogeneity of the population. For the standard logistic regression (multinomial logit), the probability of individual $i$ experiencing the severity level of $l$ from the set of severity outcomes $J$ is (Croissant, 2012):

$$P_{il} = \frac{e^{\beta'x_{il}}}{\sum_{j=1}^{J} e^{\beta'x_{ij}}}$$

Here, $x$ is the factor and $\beta'$ is the fixed coefficient for all individuals. In the mixed multinomial logit, each individual has her or his own coefficient $\beta'_{i}$, and probabilities are described as probability of the individual $i$, conditional on the vector of individual-specific coefficient $\beta'_{i}$, experiencing severity level $l$ as:

$$P_{il} | \beta_{i} = \frac{e^{\beta'_{i}x_{il}}}{\sum_{j=1}^{J} e^{\beta'_{i}x_{ij}}}$$

Because there are many observations, and finding an individual coefficient for each observation may not be of interest, the coefficients are considered to be random variables and the probabilities of each individual $i$ is found conditional on the vector of random coefficients $\beta$. Later, the average of the probabilities for all values of $\beta$ is found to obtain the unconditional probability. Given that $\beta_{i}$ has the density of $f(\beta, \theta)$ ($\theta$ as distribution parameters of $\beta$), for one individual coefficient, the unconditional probability for individual $i$ experiencing injury level of $l$ is:

$$P_{il} = \int_{\beta} (P_{il}|\beta_{i}) f(\beta|\theta) d\beta$$

Here, the function $f$ indicates the density function for $\beta$ with $\theta$ defining the parameters of the density function. Solving this integral becomes more complicated when there is more than one parameter, which requires defining a separate $\beta$ for each of the random variables. This necessitates simulation techniques. In this study, 'mlogit' package was utilized through the R software to perform the mixed logit analysis. Using 200 draws was found
to be sufficient for results to converge. For more detail on how the simulation process is performed by the package and how to define simulation parameters, we refer the readers to (Croissant, 2012; Train & Croissant, 2012; Train, 2009).

2.3. Binary logit

Binary logit is the simple form of the multinomial logit (detailed above) with two outcomes instead of multiple outcomes. In this study, we build a binary model with the set of same variables for each of the severity levels. In each model, the first level is whether a specific severity occurs and the second level is whether that specific severity does not occur. For example, regarding the property damage only level, the first outcome is when the severity was property damage only, and the second outcome is when the severity is not property damage only. In other words, the second outcome is the combination of the four remaining levels in the dataset (e.g., all possible outcomes except property damage only). The procedure is repeated for each level which makes a total of 5 binary models.

It is expected that the binary logit does not yield results as efficiently as multinomial logit; however, comparing results for all five models, we can investigate how disparate each of severity levels is from the rest of levels.

2.4. Support vector machine (SVM)

In addition to multinomial logit and mixed multinomial logit, SVM, a supervised machine learning algorithm, was employed to classify severity levels. SVM is known as a powerful method of classification (and also regression) problems as it tries to find the best possible decision boundaries between different classes. In model development, SVM applies the function $\phi(.)$ to transform the data from $X$ space into some $Z$ space. This transformation rearranges the data in such a way that the classification becomes an easier task. SVM was first introduced in (Boser, Guyon, & Vapnik, 1992) for separable data and was further expanded in (Cortes & Vapnik, 1995) for nonseparable data. The SVM objective function maximizes the margin between different classes and at the same time accepts some errors that can be regulated through a penalty parameter. The formulation of SVM is as follows.

$$\min_{w, b, \xi} \left( \frac{1}{2} w^T w + C \sum_{n=1}^{N} \xi_n \right)$$ (1)
Subject to:

\[ y_n \left( w^T \phi(x_n) + b \right) \geq 1 - \xi_n, \quad n = 1, \ldots, N \]  
(2)

\[ \xi_n \geq 0, \quad n = 1, \ldots, N \]  
(3)

Where,

\( w \) = Parameters to define decision boundary between classes
\( C \) = Regularization (or penalty) parameter
\( \xi_n \) = Error parameter to denote margin violation
\( b \) = Intercept associated with decision boundaries
\( \phi(x_n) \) = Function to transform data from X space into some Z space
\( y_n \) = Target value for the \( n^{th} \) observation

To develop a multiclass classification model using SVM, one-versus-one approach based on a voting strategy has been employed; models were developed using only two classes (e.g., fatal vs. severe injury, fatal vs. other visible injury, etc.). To predict a new observation, all these models are used to produce votes for different classes. The class with the highest vote is identified as the predicted class. Because there are five classes for crash severity, combinations of two classes from five classes results in 10 different combinations, which led to the development of ten models.

3. Results

3.1. Multinomial logit and mixed multinomial logit regression

Tables 1 and 2, respectively, represent the estimation results for the coefficient of each variable in the MNL and MMNL models as well as overall performance. Property damage only severity level is selected as the base of comparison in the analysis and all the other severity levels are compared to it. Levels of each independent variable are shown below the name of each variable with the first level representing the base level.

Selecting random parameters

To find the variables in which the coefficients vary across the individuals (random parameters in MMNL), all independent variables were checked to determine how strong the hypothesis of coefficients varying across individuals is. In the first run, all variables were put in the model assuming that the distribution of their coefficients was normal. The existence of variability among the coefficient (having the random effect) was signaled by a statistically significant variable that also had a standard deviation that is statistically different from zero (\( p \) value lower than 0.05 in this study). For instance, according to Table 2, the coefficient associated to “4 lanes” level of the variable “Number of lanes” was found to be a significant factor for
severe injury crashes and have a high standard deviation that is statistically different from zero \((p\ \text{value} = .00)\). Examining the normal distribution of coefficient associated with this variable, it was found that almost 74% of the sample has a negative coefficient and the rest are positive. This, in fact, indicates that this factor decreases the probability of crash resulting in a severe injury for 74% of the population and increases the probability for the remaining 26% of the population. In a similar manner, significant variables with random effect were selected to remain in the model as variables with random effects and the rest of the variables that did not show the mixed effect were considered to have fixed coefficients.

**Variables significance**

Only a few factors including driving along a dark road with street light, driving on four-lane and six-lane roads, and speeding show the heterogeneity effect. Results from MNL and MMNL are very similar with many variables showing significance in MNL and MMNL. However, to discuss the random effects as well, we used results found from MMNL (Table 2) as the basis for variable significance analysis.

Several variables were found to have a significant impact on different levels of severity. The results for different age groups show that aging has an increasing effect on the probability of getting killed in a crash. This, in fact, is consistent with the results from Yasmin, Eluru, Pinjari, and Tay (2014) in which they maintain the increase in the risk of fatal crashes for drivers of age 65 or above (compared to other age groups) for different types of crashes. Similar results were also achieved by Wu et al. (2014) in a study on the contribution of a variety of factors on different levels of injury in multivehicle crashes, where the probability of fatality was increased for drivers of 65 years or older. Some other studies by (Kim et al., 2013; Xie, Zhang, & Liang, 2009; Yasmin & Eluru, 2013) have also confirmed the same finding.

Regarding the drivers’ gender, it was found that male drivers, compared to female drivers, are less likely to complain about pain after a crash. An explanation might be that men, in general, are physically stronger. There is also evidence that men are more likely to be killed compared to female drivers that raises the idea that male drivers might be more aggressive drivers than female drivers. This is in line with the finding by Shankar, Mannering, and Barfield (1996) where they maintain greater probability of fatal and disabling injury for crashes involved male drivers. Different results, however, have been concluded from the gender analysis. For instance, Wu et al. (2014) found a significant heterogeneity in effect of female drivers’ behavior on the severity they experience in multivehicle crashes on rural two-lane highways. Based on their results, 32.6% were
Table 1. Summary of results for multinomial logit model.

| Variable       | Complaint of pain | Other visible injuries | Severe injury | Fatal |
|----------------|-------------------|------------------------|---------------|-------|
|                | Parameter estimate| SE t-value Pr(>|t|) | Parameter estimate| SE t-value Pr(>|t|) | Parameter estimate| SE t-value Pr(>|t|) | Parameter estimate| SE t-value Pr(>|t|) |
| (intercept)    | 0.26              | 0.31                   | 0.84          | 0.40  | 1.39              | 0.27                   | 5.11          | 0.00*** | 1.94              | 0.26                   | 7.40          | 0.00*** | 1.22              | 0.28                   | 4.44          | 0.00*** |
| Age            |                   |                        |               |       |                   |                        |               |        |                   |                        |               |         |                   |                        |               |         |
| Young adult    | 0.06              | 0.10                   | 0.58          | 0.56  | 0.04              | 0.10                   | 0.45          | 0.65   | 0.02              | 0.10                   | 0.16          | 0.88   | 0.13              | 0.12                   | 1.09          | 0.27   |
| Adult          | 0.03              | 0.11                   | 0.31          | 0.76  | 0.11              | 0.11                   | 1.03          | 0.30   | 0.09              | 0.12                   | 0.78          | 0.44   | 0.27              | 0.13                   | 2.10          | 0.04***|
| Middle aged    | 0.03              | 0.18                   | 0.18          | 0.86  | 0.16              | 0.18                   | 0.86          | 0.39   | 0.20              | 0.19                   | 1.03          | 0.30   | 0.63              | 0.20                   | 3.09          | 0.00***|
| Old            | -0.30             | 0.08                   | -3.70         | 0.00***| -0.06             | 0.08                   | -0.66         | 0.51   | 0.10              | 0.09                   | 1.14          | 0.26   | 0.25              | 0.10                   | 2.47          | 0.01*   |
| Sex            |                   |                        |               |       |                   |                        |               |        |                   |                        |               |         |                   |                        |               |         |
| Female         |                   |                        |               |       |                   |                        |               |        |                   |                        |               |         |                   |                        |               |         |
| Male           | -0.06             | 0.11                   | 0.18          | 0.16  | -0.02             | 0.12                   | 0.05          | 0.29   | -0.13             | 0.14                   | 0.36          | 0.13   | 1.06              | 0.13                   | 8.29          | 0.00***|
| Terrain        |                   |                        |               |       |                   |                        |               |        |                   |                        |               |         |                   |                        |               |         |
| Flat           | 0.06              | 0.21                   | 0.26          | 0.79  | 0.07              | 0.10                   | 0.76          | 0.45   | 0.16              | 0.10                   | 1.64          | 0.10   | 0.34              | 0.11                   | 3.24          | 0.00***|
| Mountainous    | 0.01              | 0.11                   | 0.10          | 0.28  | -0.02             | 0.24                   | -0.10         | 0.92   | -0.03             | 0.14                   | -0.92         | 0.36   | -0.30             | 0.10                   | -1.29         | 0.20   |
| Rolling        |                   |                        |               |       |                   |                        |               |        |                   |                        |               |         |                   |                        |               |         |
| Weather        |                   |                        |               |       |                   |                        |               |        |                   |                        |               |         |                   |                        |               |         |
| Clear          | 0.11              | 0.11                   | 1.08          | 0.28  | 0.12              | 0.23                   | 0.53          | 0.60   | -0.27             | 0.26                   | -1.04         | 0.30   | -0.36             | 0.28                   | -1.28         | 0.20   |
| Cloudy         |                   |                        |               |       |                   |                        |               |        |                   |                        |               |         |                   |                        |               |         |
| Raining        | -0.02             | 0.24                   | -0.10         | 0.92  | 0.12              | 0.23                   | 0.53          | 0.60   | -0.27             | 0.26                   | -1.04         | 0.30   | -0.36             | 0.28                   | -1.28         | 0.20   |
| Fog            | 0.89              | 0.84                   | 1.06          | 0.29  | 0.62              | 0.87                   | 0.71          | 0.48   | 2.33              | 0.75                   | 3.10          | 0.00** | 2.22              | 0.77                   | 2.87          | 0.00**  |
| Daylight       |                   |                        |               |       |                   |                        |               |        |                   |                        |               |         |                   |                        |               |         |
| Dusk – Dawn    | -0.43             | 0.22                   | -1.93         | 0.05   | -0.31             | 0.22                   | -1.38         | 0.17   | 0.14              | 0.22                   | 0.66          | 0.51   | 0.87              | 0.22                   | 3.93          | 0.00***|
| Dark – Street lights | -0.01         | 0.12                   | -0.08         | 0.93  | -0.08             | 0.12                   | -0.65         | 0.52   | 0.57              | 0.12                   | 4.81          | 0.00***| 1.06              | 0.13                   | 8.29          | 0.00***|
| Dark – No street lights | -0.13      | 0.14                   | -0.92         | 0.36  | 0.29              | 0.13                   | 2.17          | 0.03*  | 1.00              | 0.13                   | 7.78          | 0.00***| 1.78              | 0.13                   | 13.56         | 0.00***|
| Vehicle type   |                   |                        |               |       |                   |                        |               |        |                   |                        |               |         |                   |                        |               |         |
| Passenger car  |                   |                        |               |       |                   |                        |               |        |                   |                        |               |         |                   |                        |               |         |
| Truck          | -0.17             | 0.24                   | -0.69         | 0.49  | 0.36              | 0.21                   | 1.72          | 0.09   | 1.13              | 0.19                   | 5.88          | 0.00***| 1.62              | 0.19                   | 8.40          | 0.00***|
| Motorcycle     | 1.72              | 0.78                   | 2.21          | 0.03*  | 3.47              | 0.72                   | 4.80          | 0.00***| 4.73              | 0.72                   | 6.62          | 0.00***| 4.74              | 0.72                   | 6.60          | 0.00***|
| Pickup truck   | -0.01             | 0.11                   | -0.05         | 0.96  | 0.02              | 0.11                   | 0.17          | 0.87   | 0.08              | 0.12                   | 0.71          | 0.48   | -0.04             | 0.13                   | -0.30         | 0.76   |
| Bus            | 1.04              | 1.16                   | 0.90          | 0.37  | 1.11              | 1.16                   | 0.96          | 0.34   | 2.39              | 1.07                   | 2.23          | 0.03*  | 2.06              | 1.14                   | 1.80          | 0.07   |
| Emergency vehicle | 0.22           | 0.55                   | 0.40          | 0.69  | -0.96             | 0.83                   | -1.17         | 0.24   | -0.44             | 0.73                   | -0.60         | 0.55   | -0.51             | 0.81                   | -0.63         | 0.53   |
Table 1. Continued.

| Variable            | Complaint of pain | Other visible injuries | Severe injury | Fatal |
|---------------------|-------------------|------------------------|---------------|-------|
|                     | Parameter estimate | SE | t-value | Pr(>|t|) | Parameter estimate | SE | t-value | Pr(>|t|) | Parameter estimate | SE | t-value | Pr(>|t|) | Parameter estimate | SE | t-value | Pr(>|t|) |
| Number of lanes     |                   |    |         |          |                   |    |         |          |                   |    |         |          |                   |    |         |          |
| :2                  | 0.46              | 0.71 | 0.64    | 0.52      | 0.38              | 0.72 | 0.53    | 0.59      | −1.00             | 0.85 | −1.18    | 0.24      | 0.60              | 0.70 | 0.86    | 0.39      |
| :4                  | −0.09             | 0.20 | −0.46   | 0.65      | −0.05             | 0.20 | −0.25   | 0.80      | −0.51             | 0.20 | −2.57    | 0.01*     | −0.79             | 0.21 | −3.82    | 0.00***   |
| :5                  | 0.33              | 0.35 | 0.92    | 0.36      | 0.28              | 0.37 | 0.76    | 0.45      | −0.18             | 0.36 | −0.49    | 0.63      | −0.52             | 0.38 | −1.39    | 0.16      |
| :6                  | −0.55             | 0.20 | −2.76   | 0.01**    | −0.36             | 0.20 | −1.78   | 0.07*     | −1.15             | 0.20 | −5.81    | 0.00***   | −1.34             | 0.21 | −6.44    | 0.00***   |
| :7                  | −0.76             | 0.30 | −2.53   | 0.01*     | −0.32             | 0.29 | −1.11   | 0.27      | −0.84             | 0.29 | −2.95    | 0.00***   | −1.31             | 0.32 | −4.13    | 0.00***   |
| :8                  | −0.26             | 0.19 | −1.39   | 0.16      | −0.34             | 0.20 | −1.73   | 0.08*     | −0.86             | 0.19 | −4.56    | 0.00***   | −1.22             | 0.20 | −6.14    | 0.00***   |
| :> 8                | −0.46             | 0.19 | −2.48   | 0.01*     | −0.27             | 0.19 | −1.44   | 0.15      | −1.01             | 0.18 | −5.46    | 0.00***   | −1.33             | 0.19 | −6.85    | 0.00***   |
| Cause               |                   |    |         |          |                   |    |         |          |                   |    |         |          |                   |    |         |          |
| :Under influence    |                   |    |         |          |                   |    |         |          |                   |    |         |          |                   |    |         |          |
| :Following too closely | 0.35             | 0.31 | 1.15    | 0.25      | −1.40             | 0.28 | −5.05   | 0.00***   | −2.51             | 0.32 | −7.94    | 0.00***   | −3.07             | 0.43 | −7.14    | 0.00***   |
| :Improper turn      | 0.34              | 0.40 | 0.86    | 0.39      | −0.81             | 0.35 | −2.31   | 0.02*     | −0.45             | 0.32 | −1.42    | 0.16      | 0.16              | 0.32 | 0.51    | 0.61      |
| :Speeding           | 0.26              | 0.24 | 1.04    | 0.30      | −1.31             | 0.19 | −6.91   | 0.00***   | −1.90             | 0.18 | −10.45   | 0.00***   | −1.80             | 0.19 | −9.67    | 0.00***   |
| :Other violations   | −0.01             | 0.29 | −0.05   | 0.96      | −1.30             | 0.24 | −5.50   | 0.00***   | −1.61             | 0.23 | −7.00    | 0.00***   | −2.00             | 0.26 | −7.84    | 0.00***   |
| :Other than driving | 0.90              | 0.90 | 1.00    | 0.32      | −0.75             | 0.93 | −0.80   | 0.43      | −0.32             | 0.84 | −0.38    | 0.70      | 1.69              | 0.77 | 2.20    | 0.03*     |
| :Unknown            | −0.77             | 0.64 | −1.19   | 0.23      | −2.46             | 0.63 | −3.87   | 0.00***   | −3.35             | 0.72 | −4.64    | 0.00***   | −1.84             | 0.55 | −3.33    | 0.00***   |

Log-Likelihood: −9693.1
McFadden $R^2$: 0.09
Likelihood ratio test: $\chi^2 = 1894.7$ (p-value = < 2.22e−16)

*** 0.000 < p-value ≤ 0.001, ** 0.001 < p-value ≤ 0.010, * 0.010 < p-value ≤ 0.050, / 0.050 < p-value ≤ 0.100.
Table 2. Summary of results for mixed multinomial logit model.

| Variable                  | Complaint of pain | Other visible injuries | Severe injury | Fatal |
|---------------------------|-------------------|------------------------|---------------|-------|
|                           | Parameter estimate| SE t-value Pr(>|t|)   | Parameter estimate| SE t-value Pr(>|t|)   | Parameter estimate| SE t-value Pr(>|t|)   | Parameter estimate| SE t-value Pr(>|t|)   |
| (intercept)               | 0.25             | 0.31 0.82 0.41         | 1.37          | 0.32 4.33 0.00*** | 1.97          | 0.27 7.34 0.00*** | 1.29          | 0.29 4.48 0.00*** |
| Age                       |                   |                       |               |                   |               |                   |               |                   |
| Young adult               | 0.06             | 0.10 0.61 0.54         | 0.07          | 0.13 0.55 0.58    | 0.02          | 0.11 0.18 0.86    | 0.17          | 0.13 1.30 0.19    |
| Adult                     | 0.04             | 0.11 0.33 0.74         | 0.16          | 0.14 1.11 0.27    | 0.09          | 0.12 0.73 0.46    | 0.33          | 0.14 2.31 0.02*   |
| Middle aged               | 0.03             | 0.19 0.17 0.86         | 0.10          | 0.24 0.40 0.69    | 0.18          | 0.21 0.89 0.37    | 0.75          | 0.22 3.34 0.00**  |
| Old                       |                   |                       |               |                   |               |                   |               |                   |
| Sex                       | Male             | -0.30 0.08 -3.68 0.00*** | -0.08        | 0.11 -0.75 0.46   | 0.13          | 0.10 1.35 0.18    | 0.28          | 0.11 2.48 0.01*   |
| Female                    |                   |                       |               |                   |               |                   |               |                   |
| Terrain                   | Flat             | 0.06 0.22 0.26 0.80    | 0.46          | 0.25 1.81 0.07*   | 0.47          | 0.22 2.15 0.03*   | 0.29          | 0.23 1.27 0.20    |
| Rolling                   | Mountainous      | 0.11 0.10 1.15 0.25    | 0.02          | 0.12 0.19 0.85    | 0.13          | 0.11 1.24 0.22    | 0.40          | 0.12 3.43 0.00*** |
| Weather                   | Flat             | 0.06 0.22 0.26 0.80    | 0.46          | 0.25 1.81 0.07*   | 0.47          | 0.22 2.15 0.03*   | 0.29          | 0.23 1.27 0.20    |
| Clear                     | Flat             | 0.11 0.11 1.04 0.30    | -0.26         | 0.15 -1.74 0.08*  | -0.19         | 0.13 -1.49 0.14   | -0.17         | 0.14 -1.17 0.24   |
| Cloudy                    | Flat             | 0.11 0.11 1.04 0.30    | -0.26         | 0.15 -1.74 0.08*  | -0.19         | 0.13 -1.49 0.14   | -0.17         | 0.14 -1.17 0.24   |
| Rolling                   | Fog              | 0.91 0.85 1.07 0.28    | 0.30          | 1.00 0.30 0.77    | 2.59          | 0.80 3.23 0.00**  | 2.17          | 0.85 2.55 0.01*   |
| Weather                   | Flat             | 0.06 0.22 0.26 0.80    | 0.46          | 0.25 1.81 0.07*   | 0.47          | 0.22 2.15 0.03*   | 0.29          | 0.23 1.27 0.20    |
| Daylight                  | Flat             | 0.11 0.11 1.04 0.30    | -0.26         | 0.15 -1.74 0.08*  | -0.19         | 0.13 -1.49 0.14   | -0.17         | 0.14 -1.17 0.24   |
| Dark - Street lights      | Flat             | 0.11 0.11 1.04 0.30    | -0.26         | 0.15 -1.74 0.08*  | -0.19         | 0.13 -1.49 0.14   | -0.17         | 0.14 -1.17 0.24   |
| Dark - No street lights   | Flat             | 0.11 0.11 1.04 0.30    | -0.26         | 0.15 -1.74 0.08*  | -0.19         | 0.13 -1.49 0.14   | -0.17         | 0.14 -1.17 0.24   |
| Vehicle type              | Flat             | 0.11 0.11 1.04 0.30    | -0.26         | 0.15 -1.74 0.08*  | -0.19         | 0.13 -1.49 0.14   | -0.17         | 0.14 -1.17 0.24   |
| Passenger car             | Flat             | 0.11 0.11 1.04 0.30    | -0.26         | 0.15 -1.74 0.08*  | -0.19         | 0.13 -1.49 0.14   | -0.17         | 0.14 -1.17 0.24   |
| Truck                     | Flat             | 0.11 0.11 1.04 0.30    | -0.26         | 0.15 -1.74 0.08*  | -0.19         | 0.13 -1.49 0.14   | -0.17         | 0.14 -1.17 0.24   |
| Motorcycle                | Flat             | 0.11 0.11 1.04 0.30    | -0.26         | 0.15 -1.74 0.08*  | -0.19         | 0.13 -1.49 0.14   | -0.17         | 0.14 -1.17 0.24   |
| Pickup truck              | Flat             | 0.11 0.11 1.04 0.30    | -0.26         | 0.15 -1.74 0.08*  | -0.19         | 0.13 -1.49 0.14   | -0.17         | 0.14 -1.17 0.24   |
| Bus                       | Flat             | 0.11 0.11 1.04 0.30    | -0.26         | 0.15 -1.74 0.08*  | -0.19         | 0.13 -1.49 0.14   | -0.17         | 0.14 -1.17 0.24   |
| Emergency vehicle         | Flat             | 0.11 0.11 1.04 0.30    | -0.26         | 0.15 -1.74 0.08*  | -0.19         | 0.13 -1.49 0.14   | -0.17         | 0.14 -1.17 0.24   |

(continued)
Table 2. Continued.

| Variable                  | Complaint of pain | Other visible injuries | Severe injury | Fatal |
|---------------------------|-------------------|------------------------|---------------|-------|
|                           | Parameter estimate | SE          | t-value | Pr(>|t|) | Parameter estimate | SE          | t-value | Pr(>|t|) | Parameter estimate | SE          | t-value | Pr(>|t|) |
| Number of lanes           |                   |             |         |       |                   |             |         |       |                   |             |         |       |
| :2                        | 0.45              | 0.72        | 0.63    | 0.53  | 0.63              | 0.81        | 0.77    | 0.44  | −1.01             | 0.88        | −1.15   | 0.25   |
| :3                        | −0.09             | 0.20        | −0.45   | 0.66  | 0.03              | 0.27        | 0.11    | 0.91  | −1.05             | 0.36        | −2.92   | 0.00** |
| :4                        | 0.33              | 0.36        | 0.92    | 0.36  | 0.35              | 0.45        | 0.77    | 0.44  | −0.19             | 0.37        | −0.52   | 0.61   |
| :5                        | −0.54             | 0.20        | −2.71   | 0.01**| −0.25             | 0.27        | −0.92   | 0.36  | −1.82             | 0.39        | −4.62   | 0.00***|
| :6                        | −0.75             | 0.30        | −2.51   | 0.01* | −0.14             | 0.38        | −0.37   | 0.71  | −0.87             | 0.30        | −2.88   | 0.00***|
| :7                        | −0.26             | 0.19        | −1.36   | 0.17  | −0.27             | 0.25        | −1.06   | 0.29  | −0.89             | 0.19        | −4.70   | 0.00***|
| :8                        | −0.46             | 0.19        | −2.44   | 0.01* | −0.12             | 0.26        | −0.47   | 0.64  | −1.04             | 0.18        | −5.64   | 0.00***|
| Cause                     |                   |             |         |       |                   |             |         |       |                   |             |         |       |
| :Under influence          |                   |             |         |       |                   |             |         |       |                   |             |         |       |
| :Following too closely    | 0.35              | 0.31        | 1.14    | 0.26  | −1.44             | 0.29        | −5.05   | 0.00***| −2.67             | 0.35        | −7.64   | 0.00***|
| :Improper turn            | 0.35              | 0.40        | 0.86    | 0.39  | −0.85             | 0.36        | −2.39   | 0.02* | −0.41             | 0.34        | −1.22   | 0.22   |
| :Speeding                 | 0.26              | 0.25        | 1.04    | 0.30  | −2.21             | 0.63        | −3.53   | 0.00***| −1.97             | 0.19        | −10.14  | 0.00***|
| :Other violations          | −0.01             | 0.29        | −0.04   | 0.97  | −1.35             | 0.24        | −5.55   | 0.00***| −1.63             | 0.24        | −6.73   | 0.00***|
| :Other than driving       | 0.88              | 0.95        | 0.92    | 0.36  | −0.81             | 0.94        | −0.86   | 0.39  | −0.57             | 0.91        | −0.63   | 0.53   |
| :Unknown                  | −0.77             | 0.64        | −1.21   | 0.23  | −2.52             | 0.66        | −3.84   | 0.00***| −3.53             | 0.78        | −4.56   | 0.00***|
| Random parameters         |                   |             |         |       |                   |             |         |       |                   |             |         |       |
| Light - Dark - Street lights |                 |             |         |       |                   |             |         |       |                   |             |         |       |
| Number of lanes :4        |                   |             |         |       |                   |             |         |       |                   |             |         |       |
| Number of lanes :6        |                   |             |         |       |                   |             |         |       |                   |             |         |       |
| Cause :Speeding           | −2.14             |                   | −2.34   | 0.02**|                   |             |         |       |                   |             |         |       |
| Model overall             |                   |             |         |       |                   |             |         |       |                   |             |         |       |
| Log-Likelihood: −9684.8   |                   |             |         |       |                   |             |         |       |                   |             |         |       |
| McFadden R²: 0.09         |                   |             |         |       |                   |             |         |       |                   |             |         |       |
| Likelihood ratio test: Χ² = 1911.3 (p.value = < 2.22e-16) |                  |             |         |       |                   |             |         |       |                   |             |         |       |

*** 0.000 < p-value ≤ 0.001, ** 0.001 < p-value ≤ 0.010, * 0.010 < p-value ≤ 0.050, ’ 0.050 < p-value ≤ 0.100.
more likely to experience a severe or fatal crash, and 67.4% were more likely to experience no injury. On the other hand, in some studies (Abdel-Aty, 2003; Xie et al., 2009), it was found that female drivers are more likely to get killed in a crash with the same circumstances.

Investigating driving on different terrain levels, the outcome of a crash is more likely to result in a severe injury or fatality when driving on mountainous and rolling terrains compared to flat terrains. This was expected because driving on mountainous or rolling terrains is usually more complex than driving on the flat terrains. This also confirms the results from Shankar et al. (1996), where they indicated that high proportion of horizontal curves was found to increase the likelihood of a possible injury crash. As for driving in different weather conditions, it was found that driving in foggy condition increases the probability of crashes leading to fatality or severe injuries compared to clear weather condition.

Results from the lighting condition of the street indicated that driving in a low light condition such as dusk or dark without street lights increases the probability of a fatal crash compared to driving in daylight. This is in line with the results of (Kim et al., 2013; Wu et al., 2014; Xie et al., 2009), where they also found that undesirable lighting increases the propensity of a crash to be more severe or fatal. In this study, however, driving at night when there is street light was found to have mixed effect. Evaluation of the distribution of the coefficient shows the existence of a statistically high Standard Deviation of 1.83 \( (p \text{ value} = .00) \) and a Mean of 0.55 for the coefficients of the “dark-street light” level of variable light. This might be associated to the fact that some drivers tend to drive more carefully when their range of view is limited due to the lack of proper lighting whereas some drivers may not feel the need to compensate in this situation.

Comparing different vehicle types, it was found that motorcycle riders and truck drivers are more prone to higher levels of severity and fatality and less likely to experience property damage and surface injuries compared to passenger cars.

Increase in the number of lanes for most of the levels found to have a negative effect on the log odds of the crash to become fatal or severe. This, in fact, raises the idea that crashes are more likely to have higher levels of severity on two-lane roads than roads and highways with a higher number of lanes.

Regarding crash causes, behavior such as following too closely and speeding are less likely to lead to fatality or severe injury compared to the situation where the driver is under influence of the alcohol. A similar result was achieved in other studies such as (Xie et al., 2009; Yasmin & Eluru, 2013).

### 3.2. Binary logit

Table 3 demonstrates the summary of the results for the binary logit model. Results from Tables 1 and 3 indicate the same effects from selected
factors in different ways. For example, looking at the motorcycle data, from Table 1 (multinomial approach), we see that motorcycle drivers are more prone to being killed in accidents compared to passenger car drivers. The same result could be ascertained by comparing the estimated coefficients/significances retrieved from each of the five binary logits. Starting from the first binary model (property damage only vs. all levels) there is a low negative coefficient that increases and eventually becomes positive as you move toward the fifth binary model (fatal vs. all levels). The same concept applies to other variables as well.

3.3. Support vector machine (SVM)

The same train and test data sets that were used for the MNL and MMNL models were applied to develop and validate the SVM model using Gaussian kernels. All attributes (variables) were scaled as SVM usually performs poorly without feature scaling. This is a simple preprocessing step that could significantly affect the SVM model performance. To find the optimal SVM model, two parameters—the regularization (cost) parameter, and the Gamma parameter—must be tuned. This parameter tuning was conducted for all 10 models, one of which is shown in Figure 1. Focusing on the fatal versus property damage only model, Figure 1 illustrates how different parameter settings affect the AUC value. This figure shows a grid search of the Gamma and cost parameters to achieve the best possible performance of one of the SVM models. The best AUC was achieved with cost and Gamma parameters being equal to 0.32 and 0.1, respectively. The darker regions in the figure represent higher values of AUC, which correspond to better performance. The optimal performance of the other SVM models was achieved in the same fashion.

4. Discussion of prediction results

In this section, the SVM, MNL (binary and multiclass), and MMNL are compared in terms of their predicting accuracy on the test data. Table 4 demonstrates the number of true and false predictions in each model as well as for each class.

Although binary and multinomial logit models indicate similar information regarding the effect of factors on the outcome of the accident, they do not maintain the same accuracy when used for prediction. Based on Table 4, the binary models built for the first three levels (property damage only, complaint of pain, and other visible injury) did not maintain good prediction results based on the test data, where every single prediction for the desired level (e.g., property damage only in the first model) is incorrect.
Table 3. Summary of results for binary logit model (each column belongs to one of the five binary logits).

| Variable          | Property damage only versus all levels | Complaint of pain versus all levels | Other visible injuries versus all levels | Severe injury versus all levels | Fatal versus all levels |
|-------------------|----------------------------------------|-------------------------------------|-----------------------------------------|-------------------------------|------------------------|
|                   | Parameter estimate | SE | t-value | Pr(>|t|) | Parameter estimate | SE | t-value | Pr(>|t|) | Parameter estimate | SE | t-value | Pr(>|t|) | Parameter estimate | SE | t-value | Pr(>|t|) |
| (intercept)       | −2.90 | 0.23 | −12.38 | 0.00*** | −2.51 | 0.23 | −10.71 | 0.00*** | −1.35 | 0.18 | −7.55 | 0.00*** | −0.62 | 0.16 | −3.88 | 0.00*** | −1.55 | 0.18 | −8.62 | 0.00*** |
| Age               |                          |    |         |        |                          |    |         |        |                          |    |         |        |                          |    |         |        |                          |    |         |        |                          |    |         |        |
| Young adult       | −0.06 | 0.08 | −0.73 | 0.46   | 0.02 | 0.08 | 0.22 | 0.82   | 0.00 | 0.08 | −0.05 | 0.96   | −0.05 | 0.08 | −0.65 | 0.52   | 0.09 | 0.10 | 0.95 | 0.34   |
| Middle aged       | −0.11 | 0.09 | −1.20 | 0.23   | −0.06 | 0.09 | −0.71 | 0.48   | 0.03 | 0.09 | 0.34 | 0.74   | −0.04 | 0.09 | −0.41 | 0.68   | 0.19 | 0.10 | 1.84 | 0.07*   |
| Old               | −0.20 | 0.15 | −1.35 | 0.18   | −0.16 | 0.15 | −1.09 | 0.27   | −0.01 | 0.15 | −0.10 | 0.92   | −0.02 | 0.15 | −0.16 | 0.88   | 0.51 | 0.16 | 3.10 | 0.00*** |
| Sex               |                          |    |         |        |                          |    |         |        |                          |    |         |        |                          |    |         |        |                          |    |         |        |
| Male              | 0.07 | 0.07 | 1.03 | 0.30   | −0.34 | 0.07 | −5.16 | 0.00*** | −0.02 | 0.07 | −0.27 | 0.79   | 0.15 | 0.07 | 2.09 | 0.04*   | 0.31 | 0.09 | 3.61 | 0.00*** |
| Terrain           |                          |    |         |        |                          |    |         |        |                          |    |         |        |                          |    |         |        |                          |    |         |        |
| Flat              | −0.26 | 0.17 | −1.53 | 0.13   | −0.20 | 0.16 | −1.21 | 0.23   | 0.16 | 0.15 | 1.13 | 0.26   | 0.25 | 0.14 | 1.76 | 0.08*   | 0.02 | 0.16 | 0.12 | 0.90   |
| Rolling           | −0.15 | 0.08 | −1.88 | 0.06*  | 0.00 | 0.08 | −0.06 | 0.95   | −0.06 | 0.07 | −0.83 | 0.41   | 0.02 | 0.07 | 0.29 | 0.77   | 0.24 | 0.08 | 2.96 | 0.00**  |
| Weather           |                          |    |         |        |                          |    |         |        |                          |    |         |        |                          |    |         |        |                          |    |         |        |
| Clear             | 0.08 | 0.09 | 0.92 | 0.36   | 0.24 | 0.09 | 2.77 | 0.01**  | −0.15 | 0.09 | −1.59 | 0.11   | −0.15 | 0.09 | −1.59 | 0.11   | −0.06 | 0.10 | −0.59 | 0.56   |
| Cloudy            | 0.09 | 0.20 | 0.48 | 0.63   | 0.06 | 0.20 | 0.29 | 0.77   | 0.28 | 0.18 | 1.53 | 0.13   | −0.17 | 0.21 | −0.82 | 0.41   | −0.28 | 0.23 | −1.23 | 0.22   |
| Raining           | −1.63 | 0.73 | −2.24 | 0.03*  | −0.64 | 0.49 | −1.32 | 0.19   | −1.01 | 0.53 | −1.92 | 0.05*  | 1.07 | 0.31 | 3.51 | 0.00*** | 0.74 | 0.35 | 2.12 | 0.03*   |
| Light             |                          |    |         |        |                          |    |         |        |                          |    |         |        |                          |    |         |        |                          |    |         |        |
| Daylight          |                         |    |         |        |                          |    |         |        |                          |    |         |        |                          |    |         |        |                          |    |         |        |
| Dusk – Dawn       | 0.05 | 0.17 | 0.27 | 0.78   | −0.53 | 0.19 | −2.76 | 0.01**  | −0.39 | 0.19 | −2.12 | 0.03*  | 0.08 | 0.18 | 0.46 | 0.64   | 0.97 | 0.18 | 5.51 | 0.00*** |
| Dark – street lights | −0.27 | 0.10 | −2.86 | 0.00*** | −0.29 | 0.10 | −3.00 | 0.00*** | −0.39 | 0.10 | −4.13 | 0.00*** | 0.35 | 0.09 | 4.00 | 0.00*** | 0.91 | 0.10 | 9.16 | 0.00*** |
| Dark – No street lights | −0.66 | 0.11 | −6.05 | 0.00*** | −0.83 | 0.11 | −7.31 | 0.00*** | −0.40 | 0.10 | −4.12 | 0.00*** | 0.37 | 0.09 | 4.17 | 0.00*** | 1.37 | 0.09 | 14.94 | 0.00*** |
| Vehicle type      |                          |    |         |        |                          |    |         |        |                          |    |         |        |                          |    |         |        |                          |    |         |        |
| Passenger car     |                          |    |         |        |                          |    |         |        |                          |    |         |        |                          |    |         |        |                          |    |         |        |
| Truck             | −0.81 | 0.17 | −4.75 | 0.00*** | −0.99 | 0.19 | −5.29 | 0.00*** | −0.45 | 0.15 | −3.04 | 0.00**  | 0.40 | 0.12 | 3.27 | 0.00**  | 1.09 | 0.12 | 8.99 | 0.00*** |
| Motorcycle        | −3.95 | 0.71 | −5.55 | 0.00*** | −2.18 | 0.33 | −6.70 | 0.00*** | −0.32 | 0.15 | −2.09 | 0.04*   | 1.32 | 0.12 | 10.86 | 0.00*** | 1.10 | 0.14 | 8.12 | 0.00*** |
| Pickup truck      | −0.02 | 0.09 | −0.18 | 0.86   | −0.02 | 0.09 | −0.25 | 0.80   | 0.01 | 0.09 | 0.11 | 0.91   | 0.10 | 0.09 | 1.06 | 0.29   | −0.06 | 0.11 | −0.58 | 0.56   |
| Bus               | −1.64 | 1.03 | −1.59 | 0.11   | −0.50 | 0.64 | −0.78 | 0.43   | −0.45 | 0.63 | −0.72 | 0.47   | 1.23 | 0.46 | 2.65 | 0.01**  | 0.54 | 0.58 | 0.93 | 0.35   |

(continued)
Table 3. Continued.

| Variable                  | Property damage only versus all levels | Complaint of pain versus all levels | Other visible injuries versus all levels | Severe injury versus all levels | Fatal versus all levels |
|---------------------------|----------------------------------------|------------------------------------|----------------------------------------|-------------------------------|-------------------------|
|                           | Parameter estimate | SE | t-value | Pr(>|t|) | Parameter estimate | SE | t-value | Pr(>|t|) | Parameter estimate | SE | t-value | Pr(>|t|) | Parameter estimate | SE | t-value | Pr(>|t|) |
| Emergency vehicle         | 0.26 | 0.60 | -0.43 | 0.66 | 0.03 | 0.53 | 0.60 | -0.89 | 0.75 | -1.19 | 0.24 | 0.63 | 0.60 | -0.25 | 0.63 | 0.60 | -0.35 | 0.72 | -0.49 | 0.63 |
| Number of lanes           | 2 | 0.24 | 0.63 | -0.38 | 0.70 | 0.33 | 0.46 | 0.71 | 0.48 | 0.26 | 0.45 | 0.57 | 0.57 | -1.44 | 0.62 | -2.31 | 0.02 | 0.73 | 0.40 | 1.84 | 0.07 |
|                           | 4 | 0.31 | 0.17 | 1.81 | 0.07 | 0.21 | 0.15 | 1.44 | 0.15 | 0.30 | 0.15 | 2.00 | 0.05 | -0.23 | 0.13 | -1.70 | 0.09 | -0.57 | 0.15 | -3.89 | 0.00 |
|                           | 5 | 0.05 | 0.31 | -0.17 | 0.86 | 0.38 | 0.25 | 1.52 | 0.13 | 0.31 | 0.25 | 1.25 | 0.21 | -0.21 | 0.23 | -0.88 | 0.38 | -0.62 | 0.26 | -2.40 | 0.02 |
|                           | 6 | 0.77 | 0.16 | 4.60 | 0.00 | 0.07 | 0.15 | 0.50 | 0.62 | 0.38 | 0.15 | 2.57 | 0.01 | -0.51 | 0.14 | -3.72 | 0.00 | -0.71 | 0.15 | -4.71 | 0.00 |
|                           | 7 | 0.75 | 0.23 | 3.19 | 0.00 | 0.21 | 0.24 | -0.88 | 0.38 | 0.39 | 0.22 | 1.75 | 0.08 | -0.18 | 0.21 | -0.85 | 0.40 | -0.77 | 0.25 | -3.06 | 0.00 |
|                           | 8 | 0.59 | 0.16 | 3.68 | 0.00 | 0.27 | 0.14 | 1.94 | 0.05 | -0.23 | 0.14 | 1.62 | 0.10 | -0.33 | 0.13 | -2.58 | 0.01 | -0.76 | 0.14 | -5.32 | 0.00 |
|                           | 8 | 0.69 | 0.16 | 4.40 | 0.00 | 0.10 | 0.14 | 0.74 | 0.46 | 0.41 | 0.14 | 2.93 | 0.00 | -0.42 | 0.13 | -3.34 | 0.00 | -0.79 | 0.14 | -5.70 | 0.00 |
| Cause                     | Under influence |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |
|                           | Following too closely | 1.58 | 0.24 | 6.68 | 0.00 | 2.01 | 0.24 | 8.30 | 0.00 | 0.00 | 0.20 | 0.00 | 1.00 | -1.30 | 0.24 | -5.36 | 0.00 | -1.76 | 0.38 | -4.64 | 0.00 |
|                           | Improper Turn       | 0.35 | 0.30 | 1.18 | 0.24 | 0.72 | 0.30 | 2.44 | 0.01 | -0.58 | 0.22 | -2.59 | 0.01 | -0.27 | 0.12 | -1.52 | 0.13 | 0.65 | 0.18 | 3.67 | 0.00 |
|                           | Speeding            | 1.38 | 0.17 | 8.04 | 0.00 | 1.65 | 0.19 | 8.78 | 0.00 | -0.11 | 0.11 | -0.99 | 0.32 | -0.85 | 0.09 | -9.10 | 0.00 | -0.62 | 0.10 | -6.20 | 0.00 |
|                           | Other violations    | 1.41 | 0.21 | 6.84 | 0.00 | 1.34 | 0.23 | 5.89 | 0.00 | -0.07 | 0.16 | -0.43 | 0.67 | -0.42 | 0.14 | -3.01 | 0.00 | -0.86 | 0.18 | -4.84 | 0.00 |
|                           | Other than driving  | -0.44 | 0.75 | -0.59 | 0.56 | 0.54 | 0.56 | 0.95 | 0.34 | -1.35 | 0.61 | -2.21 | 0.03 | -1.19 | 0.46 | -2.60 | 0.01 | 2.05 | 0.35 | 5.79 | 0.00 |
|                           | Unknown             | 2.27 | 0.45 | 5.03 | 0.00 | 1.14 | 0.58 | 1.96 | 0.05 | -0.68 | 0.55 | -1.24 | 0.21 | -1.74 | 0.63 | -2.78 | 0.01 | 0.28 | 0.43 | 0.64 | 0.52 |

Model Overall

Log-Likelihood: -3127.9  Log-Likelihood: -3171.6  Log-Likelihood: -3207.9  Log-Likelihood: -2570.5
McFadden $R^2$: 0.07  McFadden $R^2$: 0.07  McFadden $R^2$: 0.07  McFadden $R^2$: 0.07
Likelihood ratio test : Likelihood ratio test : Likelihood ratio test : Likelihood ratio test :
$\chi^2 = 482.72$ (p-value = < 2.22e-16)  $\chi^2 = 514.85$ (p-value = < 2.22e-16)  $\chi^2 = 386.92$ (p-value = < 2.22e-16)  $\chi^2 = 797.17$ (p-value = < 2.22e-16)

*** 0.000 < p-value ≤ 0.001, ** 0.001 < p-value ≤ 0.010, * 0.010 < p-value ≤ 0.050, / 0.050 < p-value ≤ 0.100.
Although binary models built for last two levels (severity and fatal) maintained better prediction results, the accuracy was still not very good. Although the prediction results from binary models display estimation inefficiency, it also demonstrates how discrete the last two levels (severity and fatal) are from the other levels.

Regression and support vector approaches with all five levels in the model led to more accurate predictions. The prediction results are also included in Table 4 for comparison. MNL and MMNL models maintained a comparable accuracy, which is expected because most parameters were not found to have random effects. Yet it is important that random parameters are included to better report the effect of these variables. It should be noted that most machine learning algorithms such as SVM are considered as black-box methods that are difficult to interpret. In general, they could perform better than classical statistical methods such as logit models but have limited explanatory power. In the present study, the SVM performed slightly better in specific cases as explained below.

According to Table 4, the SVM has maintained greater accuracy for other visible injury and fatal class of the database. On the other hand,
Table 4. Summary of number of true and false predictions for each model.

| Property | Binary Outcome Model | Multilevel Outcome Model |
|----------|----------------------|-------------------------|
|          | True | False | True | False | True | False | True | False | True | False | True | False | True | False | True | False | True | False |
| vs. all other levels |     |       |     |       |     |       |     |       |     |       |     |       |     |       |     |       |     |       |
| Damage only vs. all other levels | 0   | 620   | 216 | 404   | 215 | 405   | 208 | 412   |     |       |     |       |     |       |     |       |     |       |
| Complaint of Pain vs. all other Levels | 0   | 579   | 275 | 304   | 280 | 299   | 270 | 310   |     |       |     |       |     |       |     |       |     |       |
| Other Visible injuries vs. all other Levels | 0   | 609   | 17  | 592   | 17  | 592   | 30  | 579   |     |       |     |       |     |       |     |       |     |       |
| Severe Injury vs. all other levels | 29  | 563   | 225 | 367   | 222 | 370   | 213 | 379   |     |       |     |       |     |       |     |       |     |       |
| Fatal vs. all other levels | 48  | 392   | 133 | 307   | 132 | 308   | 158 | 282   |     |       |     |       |     |       |     |       |     |       |
| Total prediction | Total | 2219 | 621 | 2258 | 582 | 2231 | 609 | 2244 | 596 | 2391 | 449 | 866  | 1974 | 866  | 1974 | 879  | 1962 |
| AUC     | 0.65 | 0.66 | 0.54 | 0.67 | 0.74 |       |     |       |     |       |     |       |     |       |     |       |     |       |

Note. AUC = area under the curve.
Table 5. Confusion matrix for multinomial logit (1), mixed multinomial logit (2), and support vector machine (3).

(1) Multinomial Logit

| Prediction                    | Reference          | Property Damage Only | Complaint of Pain | Other Visible Injury | Severe Injury | Fatal |
|-------------------------------|--------------------|----------------------|-------------------|----------------------|---------------|-------|
| Property damage only          | 216                | 197                  | 168               | 115                  | 65            |       |
| Complaint of pain             | 275                | 275                  | 236               | 138                  | 68            |       |
| Other visible injury          | 24                 | 12                   | 17                | 19                   | 10            |       |
| Severe injury                 | 67                 | 53                   | 122               | 225                  | 164           |       |
| Fatal                         | 38                 | 42                   | 66                | 95                   | 133           |       |
| True predictions in %         | 35                 | 47                   | 3                 | 38                   | 30            |       |

(2) Mixed Multinomial Logit

| Prediction                    | Reference          | Property Damage Only | Complaint of Pain | Other Visible Injury | Severe Injury | Fatal |
|-------------------------------|--------------------|----------------------|-------------------|----------------------|---------------|-------|
| Property damage only          | 215                | 192                  | 166               | 115                  | 64            |       |
| Complaint of pain             | 278                | 280                  | 238               | 136                  | 68            |       |
| Other visible injury          | 24                 | 13                   | 17                | 25                   | 14            |       |
| Severe injury                 | 67                 | 50                   | 117               | 222                  | 162           |       |
| Fatal                         | 36                 | 44                   | 71                | 94                   | 132           |       |
| True predictions in %         | 35                 | 48                   | 3                 | 38                   | 30            |       |

(3) Support Vector Machine

| Prediction                    | Reference          | Property Damage Only | Complaint of Pain | Other Visible Injury | Severe Injury | Fatal |
|-------------------------------|--------------------|----------------------|-------------------|----------------------|---------------|-------|
| Property damage only          | 208                | 178                  | 147               | 108                  | 62            |       |
| Complaint of pain             | 266                | 270                  | 229               | 121                  | 66            |       |
| Other visible injury          | 20                 | 21                   | 30                | 19                   | 15            |       |
| Severe injury                 | 64                 | 53                   | 119               | 213                  | 139           |       |
| Fatal                         | 62                 | 58                   | 84                | 131                  | 158           |       |
| True predictions in %         | 34                 | 47                   | 5                 | 36                   | 36            |       |
MNL and MMNL had greater accuracy for property damage only, complaint of pain, and severe injury. Although, in total, the number of true predictions by the SVM is slightly higher than the total for MNL and MMNL.

More detailed results regarding the prediction of each model in the form of confusion matrix are shown in Table 5. According to Table 5, the largest number of true predictions belongs to complaint of pain level. On the other hand, the lowest number of true predictions belong to other visible injury level. This is a broad, and somewhat vague, a category that is overlapping with the complaint of pain category, so it is expected that it would be hard to predict. The remaining property damage only, severe injury, and fatal levels have similar accuracy levels. The last row under each confusion matrix indicates the percent of true prediction for each individual level of severity.

5. Conclusion

Considering the significant losses resulting from crashes, predicting the likelihood of crash severity has been a vital line of research. Studying crashes and their contributing factors allow traffic engineers and practitioners to determine and prioritize dangerous crash prone situations, which in turn aids efforts to implement suitable countermeasures and enhance the level of safety. In this study, crash severity models were developed using three statistical approaches of MNL (binary and multiclass), MMNL, and SVM. The database used for the model training is an integrated database of the California crash database from 2007 to 2011 including information on driver, vehicle, and environment.

Studying effects of different factors on severity of accidents, results from MNL and MMNL models showed that older and male drivers are more likely to have a fatal crash whereas complaint of pain is observed more among female drivers compared to males. As expected, driving in foggy and mountainous or rolling terrains is associated with more fatal and severe injury crashes. Similarly, low light conditions are associated to more fatal crashes. Regarding the vehicle type, motorcycle riders and truck drivers are more likely to experience a higher level of crash severity. Also, a fewer number of lanes was found to be associated with higher level of severity. Regarding the cause of crashes, improper turning and driving under influence were found to cause severe injuries and fatal crashes. In terms of random parameters, factors including driving on dark roads with a street light, driving on four- and six-lane roads, and speeding could have various impact on the severity outcome of accidents as for some people
they have been a contributing factor and for the others, they have acted as the opposite.

Studying the performance of the models based on their prediction accuracy, the MNL and MMNL maintained comparable results. This result was expected because only a few of the variables were found to have a mixed effect, and those that found to have mixed effect were not dominant enough to make a significant change in the overall outcome of the model. The SVM approach maintained a slightly better prediction accuracy using the test data.

In the future, the crash analysis will be conducted for other crash types than rear-end crashes. Also, other techniques such as neural network and discriminant analysis method will be considered on the same dataset and results will be compared in terms of model performance.

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