A coherent coupler is proposed to spin a Bose-Einstein condensate composed of ultracold alkali atoms into a vortex state (VS). The proposal is based on a Raman transition induced by two copropagating $\sigma^+$ and $\sigma^-$ polarized Laguerre-Gaussian laser beams with different frequencies. We show that the transfer of angular momentum of photons to the condensed atoms through a Raman transition leads to a coherent coupling of the ground-state condensate to a rotating condensate in a VS. The detection of such a VS is discussed.

It is well-known that VS's play a central role in characterizing the superfluid properties of large-size Bose systems such as superfluid helium (see, e.g., cf. [1]). Recently the experimental realizations of Bose-Einstein condensation in trapped ultracold alkali atomic gases [3] have generated great interest in studying the superfluid aspects of the small-scale trapped Bose gases [3]. To understand how the small-scale Bose-Einstein condensate is related to a superfluid, it is natural to study the rotational properties of the condensate and to examine VS's. Being different from superfluid helium, the trapped Bose gases are not in direct contact to an external container. How to rotate such gases and to create VS's is still an open question.

In this letter, we propose to employ a vortex coupler to realize such a goal. The principle for the vortex coupler is illustrated in Fig.1. Two copropagating $\sigma^+$ and $\sigma^-$ polarized laser beams along the $z$ direction (gravity’s direction) are used to induce a Raman transition between two hyperfine levels of ground-state alkali atoms. Initially we assume that a Bose-Einstein condensate is prepared in the trapping state $|\mp\rangle = |F=1, M_F=\mp1\rangle$. We consider the time evolution of the condensate after the trap is switched off and the Raman laser beams are applied. In addition, gravity effects are excluded here since we are only interested in the rotational motion of the condensates in the $x$-$y$ plane. To avoid the destructive incoherent heating of the condensate due to spontaneous decay of excited states $|j\rangle$, the two Raman beams are detuned by a frequency $\Delta$ from the optical transition between the ground state and the excited state manifolds. In this case adiabatic elimination of the excited states $|j\rangle$ results in a nonlinear Schrödinger equation describing the coherent Raman-type coupling [4] between the condensate wave functions $\phi_-$ and $\phi_+$ corresponding to the magnetic sublevel $|\mp\rangle$ and $|\mp\rangle = |F=1, M_F=\mp1\rangle$:

$$ih\dot{\phi}_- = (T-\mu)\phi_- + \frac{4\pi\hbar^2a_{sc}N}{m} \{2|\phi_-|^2 + |\phi_+|^2\}\phi_- + \frac{\hbar\Omega_1^{(-)}|^2}{4\Delta}\phi_- + \frac{\hbar\Omega_1^{(-)}\Omega_2^{(+)}(+)}{4\Delta} e^{-i\Delta t}\phi_+$$

$$ih\dot{\phi}_+ = (T-\mu)\phi_+ + \frac{4\pi\hbar^2a_{sc}N}{m} \{2|\phi_+|^2 + |\phi_-|^2\}\phi_+ + \frac{\hbar\Omega_2^{(-)}|^2}{4\Delta}\phi_+ + \frac{\hbar\Omega_1^{(-)}\Omega_2^{(+)}(-)}{4\Delta} e^{-i\Delta t}\phi_-$$

(1)

where $\Omega_1^{(+)}$ denote the positive frequency part of the corresponding Rabi frequencies of the two Raman beams. $\Delta\omega := \omega_2 - \omega_1$ is the frequency difference between the two Raman beams, $m$ the atomic mass, $a_{sc}$ the scattering length, and $N$ the total number of atoms in the initial condensate. $\mu$ denotes the chemical potential needed to fix the mean number of atoms. We have normalized the condensate wave functions $|+\rangle$ and $|-\rangle$ corresponding to the magnetic sublevel $|\pm\rangle$ and $|\pm\rangle = |F=1, M_F=\pm1\rangle$.

The last terms in Eqs. (1) represent the Raman coupling which coherently transfers the initial condensate in the level $|\mp\rangle$ to the initially empty level $|\pm\rangle$. Evidently, if the Raman laser beams are generated by ordinary laser sources with Hermite-Gaussian modes, Eqs. (1) only describe a normal coherent coupler which doesn’t cause a rotation of the condensate since no angular momentum is transferred by the coupling term. Thus, to realize a coherent rotational coupler, the Raman laser beams must carry an angular momentum. In view of the recent experimental progress in laser physics and atom optics [5], a natural choice is to employ Laguerre-Gaussian laser beams. For this kind of modes the Rabi frequency $\Omega_1^{(+)}$ for the $\sigma^+$ polarized beam given by

$$\Omega_1^{(+)}(\vec{x}) = \Omega_0 e^{-r^2/w^2} \left( \frac{\sqrt{2r}}{w} \right) i e^{i\varphi} e^{ikz}$$

(2)

($l \geq 1$), where $r^2 := x^2 + y^2$. A similar expression applies to the $\sigma^-$ polarized beam $\Omega_2^{(+)}$ with $\varphi$ replaced by $-\varphi$. By analogy between quantum mechanics and paraxial optics, the azimuthal angular dependence of the phase factor in Eq. (4) determines an orbital angular momentum of $\hbar l$ per photon in the Raman beams. Here we
consider the simple case that both Raman beams are in the first Laguerre-Gaussian mode \((l = 1)\) and their waist \(w\) is much larger than the size of the initial condensate. We then can approximate the above expression to \(\Omega_{\perp}^{(+)} = \sqrt{2}\Omega_{0}(x + iy)/w\) and likewise for \(\Omega_{\parallel}^{(+)}\) with \(x + iy\) replaced by \(x - iy\). Inserting these expressions into Eq. (3) we obtain
\[
i\hbar \phi_{-} = (T + V - \mu)\phi_{-} + V e^{-2i\varphi} e^{-i\Delta \omega t} \phi_{+} + \frac{4\pi \hbar^{2} a_{\omega} N}{m} [2|\phi_{-}|^{2} + |\phi_{+}|^{2}] \phi_{-}
\]

(3)
\[
i\hbar \phi_{+} = (T + V - \mu)\phi_{+} + V e^{2i\varphi} e^{i\Delta \omega t} \phi_{-} + \frac{4\pi \hbar^{2} a_{\omega} N}{m} [2|\phi_{+}|^{2} + |\phi_{-}|^{2}] \phi_{+} .
\]

(4)

We see that, because of the donught-shaped transverse profile of the Laguerre-Gaussian mode, the light-induced potentials and the Raman coupling terms have approximately the spatial structure of a harmonic oscillator potential \(V := (1/2)m\omega_{\text{eff}}^{2}r^{2}\), where the effective oscillator frequency is defined as \(\omega_{\text{eff}} := \sqrt{\hbar/(m\Delta)}\Omega_{0}/w\). In order to avoid excitations of the condensate after the original magnetic trap is switched off, we choose the appropriate laser intensities so that the effective oscillator frequency \(\omega_{\text{eff}}\) coincides with the transverse oscillating frequency of the original magnetic trap. In this sense, the donught-shaped Laguerre-Gaussian mode spatially acts as a waveguide for condensates where the transverse motion of the condensates is confined by the light-induced potentials. In addition, one more important feature of Eqs. (3, 4) is that the Raman coupling terms now contain phase factors \(\exp(\pm 2i\varphi)\).

The physics implicit in Eqs. (3, 4) is very obvious. As sketched in Fig.1, atoms in the condensate corresponding to level \(|-\rangle\) absorb photons from the \(\sigma^{+}\) beam and make a Raman-transition to the \(|+\rangle\) level by emitting photons into \(\sigma^{-}\) beam. Each cycle of photon absorption and emission in the Raman transition directly leads to the phase factors which describe the transfer of an orbital angular momentum \(2\hbar\), originating from two photons in the \(\sigma^{+}\) and \(\sigma^{-}\) laser beams, to the condensed atoms. As a result, the condensate fraction in the state \(|\text{+}\rangle\) is set into a VS with twice the elementary quantum of circulation by the coherent Raman transfer.

To quantitatively estimate the Raman transfer of the non-rotating condensate in level \(|-\rangle\) to the vortex-state condensate in level \(|+\rangle\), we further reduce Eqs. (3, 4) by assuming separately for each of the two states that the spatial part of the condensate varies little during the Raman coupling. For the present case this approximation is justified by two reasons: (1) phonon excitations of the condensate \(\phi_{-}\) are avoided by choosing the light-induced potential \(V\) to match the original magnetic trapping potential, and (2) the spatial dependence of the Raman coupling term in Eq. (3) is proportional to \(r^{2}\) which exactly leads to a transverse spatial structure matching the transverse spatial shape required by a VS with two quantum circulations.

Using this approximation we can make the following ansatz for the condensate wavefunctions, \(\psi_{-}(\bar{x}, t) = \alpha(t) \exp[i(\mu/\hbar - \kappa)t] \psi_{g}(\bar{x})\) and \(\psi_{+}(\bar{x}, t) = \beta(t) \exp[i (\Delta \omega + \mu/\hbar - \kappa)t] \psi_{v}(\bar{x})\), with \(\psi_{g}(\bar{x})\) denoting the spatial dependence of the nonrotating condensate and \(\psi_{v}\) representing the VS with two elementary quanta of circulation. Further, we approximate the spatial parts \(\psi_{g}(\bar{x})\) and \(\psi_{v}(\bar{x})\) of the wavefunctions by that of a trapped ideal Bose gas [3]. The spatial wavefunctions are then given by \(\psi_{g} = \exp[-(1/2)(r/L_{\perp})^{2} + (z/L_{z})^{2}]/(\pi^{3/4} L_{\perp} L_{z}^{1/2})\) for the ground state and \(\psi_{v} = (x + iy)^{2} \psi_{g}/(\sqrt{2} L_{\perp})\) for the VS. Here \(L_{\perp}\) and \(L_{z}\) denote the size parameters of the original trap in the x-y plane and the z-direction, respectively. The parameter \(\kappa\) determines the strength of the interatomic interaction and is defined as \(\kappa := \pi a_{\omega} N/(m(2\pi)^{1/2} L_{\perp}^{3/2} L_{z})\). Projecting Eqs. (3, 4) on these two states we arrive at a system of two ordinary differential equations,
\[
i\dot{\alpha} = \frac{\omega_{\text{eff}}^{2} \beta}{\sqrt{2}} + 7\kappa |\alpha|^{2} \alpha
\]
(5)
\[
i\dot{\beta} = (\Delta \omega + 2\omega_{\text{eff}}) \beta + \frac{\omega_{\text{eff}}}{\sqrt{2}} \alpha + 2\kappa |\beta|^{2} \beta ,
\]
(6)

where \(|\alpha|^{2}\) gives the occupying fraction of the condensate in level \(|-\rangle\) and \(|\beta|^{2}\) that of the VS in level \(|+\rangle\). Because of probability conservation, we have the constraint \(|\alpha|^{2} + |\beta|^{2} = 1\). The terms proportional to \(\omega_{\text{eff}}/\sqrt{2}\) describe the laser-induced Raman coupling between the non-rotating condensate and the VS. The nonlinear contributions proportional to \(\kappa\) have their origin in the interatomic interaction. As the strength of the nonlinear interaction depends on the density of the condensate, which is different for the non-rotating condensate and the VS, the nonlinear terms are of different magnitude for the two states. Finally, the expression linear in \(\beta\) of Eq. (5) includes the energy difference \(2\omega_{\text{eff}}\) between the VS and the ground state as well as the energy transfer \(\hbar \Delta \omega\) from the laser beams to the condensate.

To solve Eqs. (5, 6) it is convenient to consider the population difference \(f(t) := |\alpha|^{2} - |\beta|^{2}\) which also is the physical quantity we are interested in. By using the initial condition \(f(0) = 1, \dot{f}(0) = 0\) (all atoms are initially in the \(|-\rangle\) state) one can derive from the system of equations (5, 6) the first order equation
\[
j^{2} = (1 - f) \left\{ 2\omega_{\text{eff}}^{2}(1 + f) - \frac{\kappa^{2}}{16} (1 - f)(19 - \varepsilon + 9f)^{2} \right\} ,
\]
(7)

with \(\varepsilon := 4(\Delta \omega + 2\omega_{\text{eff}})/\kappa\). Eq. (5) indicates that \(f(t)\) can be expressed in term of elliptic functions [6]. The exact analytical solution is given by
\[
f(t) = \frac{a \, da(\nu t|m) + b}{da(\nu t|m) + a + b - 1} .
\]
(8)
dn(z|m) is one of the Jacobian elliptic functions. The parameter $a$ is determined by one of the real roots of a 6th order polynomial $P(ξ) = \sum_{i=0}^{6} g_i ξ^i$, where $g_i$ are 2nd order polynomials in the parameters $ε, κ$. The other parameters $b, m$, and $ν$ can be expressed as a rational polynomial of 5th order in the parameter $a$. The explicit expressions are rather lengthy and will therefore not be given here. Instead, we will rely on a numerical calculation to determine $a, b, m$, and $ν$.

As an example we use the experimental parameters of the $^{87}\text{Rb}$ experiment \cite{8}. The number $N$ of condensed atoms is 4500, and the laser intensity to be chosen so that the effective oscillator frequency $ω_{\text{eff}}$ is equal to the transversal trap frequency $ω_⊥ = 132$ Hz. The trap size parameters are given by $L_⊥ = 2.35\mu$m and $L_∥ = 1.4\mu$m.

The scattering length is taken to be $a = 5\text{nm}$ so that the nonlinear coupling parameter becomes $κ = 422$ s$^{-1}$. In Fig. 2 we show three solutions corresponding to different values of the frequency difference $Δω$. The dotted line corresponds to $Δω = 2700$ Hz for which the parameters in Eq. (8) take the values $a = 0.86$, $b = 0.39$, $m = 1.07$, and $ν = 514$ Hz. As will be explained below the dot-dashed line ($Δω = 1962$ Hz, $a = 0.72$, $b = 0.70$, $m = 656$, $ν = 16.4$ Hz) is a very peculiar case for which the maximal population transfer is reached (for fixed $Δω$). Within our model about one half of the $^{87}\text{Rb}$ atoms could be transferred to the $|+⟩$ state. The dashed curve shows that for a slightly smaller frequency difference ($Δω = 1850$ Hz, $a = 0.70$, $b = 0.74$, $m = −2.97$, $ν = 236$ Hz) the solution does only perform relatively small oscillations.

In order to better understand this behaviour we consider the equation which determines the extremal points in the time evolution of the population difference, $f = 0$. According to Eq. (6) these extremal points are determined by the (real) roots of a 4th order polynomial in $f$. This allows us to determine the maximum population transfer for a given $Δω$. In Fig. 3 we show the real roots of the polynomial given by the r.h.s. of Eq. (6) as a function of $Δω$. $f = 1$ is always a root, and since we start just with this value (all atoms in the $|−⟩$ state) the solution can only oscillate between $f = 1$ and the next root for a given $Δω$. It can be read off from Fig. 3 that the maximum population transfer is about 50% and appears around $Δω = 1962$ Hz (solid line in Fig. 2). The shape of the oscillations for frequency differences around this value is heavily affected by the fact that the polynomial becomes very small in the middle of the cycle, causing a large delay in the time evolution. For slightly smaller values of $Δω$ a new root appears relatively close to 1. This is responsible for the fact that the maximum population transfer is dramatically reduced if $Δω$ passes the critical value around 1950 Hz.

Though a population transfer of 50% is satisfactory it would be desirable to transfer almost the whole condensate into the VS. Fig. 3 indicates that this is only possible if we can reach the peak at $f = −1$ (corresponding to a complete population transfer). As is shown above this is impossible for a fixed $Δω$ if the system starts from $f = 1$. But this restriction may be circumvented by introducing a linearly time-dependent frequency difference between the two lasers. We have numerically solved Eqs. (6) by assuming that the frequency difference varies with $Δω(t) = 2900$ Hz $−(3336$ Hz $)^2t$. Obviously this leads to almost complete transfer of the condensate to the VS. This effect can be explained by noting that the nonlinear interaction between the atoms causes a time-dependent energy shift between the two states. This energy shift is due to the fact that the nonlinear interatomic interaction is proportional to the populations which change with time. Thus, after some time the transition will be out of resonance even if it was initially at resonance. A phase modulation of $Δω$ can compensate for this time-dependent energy shift.

So far we have shown how to create a VS by coherent Raman coupling. How to detect the VS remains to be discussed. A direct detection of the VS is to observe the off-resonance absorption image of the rotational cloud. For a vortex state one should expect a bright ”hole” in the image which accounts for the vortex ”core” in the density distribution of the VS. Another possibility to detect a VS is to observe the Doppler frequency shift due to the quantized circular motion of the atoms. Here we focus on discussing the absorption image for the circular motion.

To realize a non-destructive detection scheme we propose to employ two counterpropagating weak $σ^+$ and $σ^−$ probe beams in the $x$-direction and to observe the absorption image of the $σ^−$ probe beam. The principle for the detection is simply based on Raman absorption of the $σ^−$ beam (if the VS is prepared in the level $|+⟩$). The Raman absorption coefficient of the $σ^−$ probe beam is proportional to the probability of the Raman transition from $|+⟩$ to $|−⟩$ which has the form \cite{8}

$$
γ(y, z) \propto \int_{−∞}^{∞} dx \frac{Ω_2}{W} \sin(Wt_0)|ψ_{ψ}(\vec{x})|^2
$$

with $W := \sqrt{δ^2 + |Ω_+|^2|Ω_-|^2/(4Δ^2)}$. $Ω_±$ are the Rabi frequencies of the two probe-beams. $δ = Δω − 2(ℏk_L^2/m) − Δω_v$, is the effective frequency difference in the two probe-beams with $Δω_v = 4(ℏk_L/m)ξ_x' \nabla ϕ$ being the Doppler frequency shift induced by circular motion of the atoms in the VS. Eq. (6) clearly shows that the Raman absorption image directly displays the rotational properties of the VS. Fig. 4 shows the image of a VS discussed in this paper. We see that the absorption is asymmetric around the vortex axis. Such an asymmetry directly gives signature for the Doppler effect induced by the quantized circular motion of the atoms in the vortex state. Since the $σ^−$ probe beam propagates along the $x$-direction, the strong absorption (black spot in Fig. 4) corresponds to atoms with velocity antiparallel to the wavevector $k_Lξ_x'$ of the $σ^−$ beam. The weak absorption
on the opposite side of the vortex line appears because there the rotating atoms are moving parallel to the $\sigma^-$ beam.

Acknowledgement: The work has been supported by the Australian Research Council.

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Figure captions:

Fig. 1: Two Laguerre-Gaussian laser beams induce a Raman transition and generate a VS in the condensate.

Fig. 2: The population difference between the two ground states as a function of time for different constant values of the frequency difference between the lasers, and for a linearly varying frequency difference (solid line).

Fig. 3: The extremal points of the population difference as a function of $\Delta\omega$ (in Hz). The time evolution starts at $f = 1$ and moves downwards until it hits another extremal point.

Fig. 4: A typical Raman absorption image of a VS. The Doppler shift of the rotating atoms leads to a higher absorption if the atom move towards the $\sigma^-$ laser. $y$ and $z$ are given in units of the trap size parameter.
