Radiative corrections to the
Reggeized quark – Reggeized quark – gluon
effective vertex

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Abstract

This paper is devoted to the calculation of the Reggeized quark – Reggeized quark –
gluon effective vertex in perturbative QCD in the next–to–leading order. The case of
QCD with massless quarks is considered and the correction is obtained in the $D \to 4$
limit. This vertex appears in the quark Reggeization theory, which next–to–leading
order extension is now under construction.

1 Introduction

The construction of the quark Reggeization theory in the next–to–leading approximation (NLA)
involves both the calculation of the next–to–leading order (NLO) corrections to the quark effective
vertices and trajectory and the proof of the Reggeization hypothesis. Some of these tasks have
already been done. Firstly, all multiparticle Reggeon vertices required in the NLA were obtained
[1]. It enabled one to prove the quark Reggeization hypothesis in the quasi–multi–Regge kinematics
(QMRK) [2], important in the NLA. Next, NLO corrections to the effective particle–particle–Reggeon
(PPR) vertices, appearing in the leading logarithmic approximation (LLA) were found [3]. It allowed
one to obtain the NLO correction to the quark Regge trajectory [4], which was also a test of the
hypothesis in the NLO but only for a particular elastic process.

In this paper the NLO correction to the coupling of two Reggeized quarks with external gluon is
calculated. The case of QCD with massless quarks is considered and the correction is obtained in the
$(D = 4 - 2\epsilon) \to 4$ limit.

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The paper is organized as follows. In Section 2 we introduce the Reggeized form of the gluon production amplitude required by analyticity, unitarity and crossing symmetry and calculate the part of the effective vertex contributing to the discontinuities of the production amplitude in $s_1$ and $s_2$ channels. In Section 3 we obtain the correction to the gluon production amplitude in the framework of dispersive approach in $t_1$ channel. In Section 4 we calculate one–loop corrections to quark-photon–Reggeized quark vertex. In Section 5 we present the result for the Reggeized quark – Reggeized quark – gluon effective vertex.

We perform calculations for massless quarks, but for generality we give the main formulae with $m$ kept.

## 2 Production amplitude in the multi–Regge kinematics

![Diagram of gluon production](image)

Fig. 1. Schematic gluon production diagram.

Let us consider the high energy process $A + B \rightarrow A' + G + B'$ of a gluon $G$ production in the multi–Regge kinematics (MRK), which means that all participating particles are well separated in the rapidity space and have limited transverse momenta (see fig. 1):

\[
(p_A + p_B)^2 = s \gg s_i \gg |t_j|, \quad s_1 = (p_{A'} + k)^2, \quad s_2 = (p_{B'} + k)^2, \quad t_j = q_j^2, \quad q_1 = p_A - p_{A'}, \quad q_2 = p_{B'} - p_B, \quad k = q_1 - q_2, \quad s_1 s_2 \simeq -s k^2_\perp.
\]

(1)

Here $k_\perp$ is the gluon momentum component transverse to the plane $p_A, p_B$:

\[
k = \alpha p_B + \beta p_A + k_\perp, \quad k^2 = -k^2_\perp
\]

(2)

and the last relation in eq.(1) for the product $s_1 s_2$ is a consequence of the reality of the massless gluon:

\[
k^2 = s \alpha \beta + k^2_\perp = 0.
\]

(3)

The behavior of the amplitude in the MRK is determined by the exchanges in $q_i$ channels. For the case of quark quantum numbers in $q_1$ and $q_2$ channels, the multi–Regge form of the amplitude proved for the LLA in [5] is

\[
A_{2\rightarrow 3} = \tilde{\Gamma}_{B'B} \frac{s_2}{m - q_2} \gamma^G_{\text{Born}} \frac{s_1}{m - q_1} \Gamma_{A'A}.
\]

(4)
where \( \Gamma_{PP'} \) and \( \Gamma_{PRP} \) are the particle-particle-Reggeon (PPR) effective vertices, describing \( P \to P' \) transitions due to the interaction with Reggeons and taken in tree approximation, \( \delta_i \equiv \delta(q_i) \) is the quark Regge trajectory [6]

\[
\delta(q) = g^2 C_F \int \frac{d^{D-2}r_{\perp}}{(2\pi)^{D-1}} \frac{(m - \hat{q}_1)}{(m - \hat{r}_{\perp})(q - r)^2_{\perp}} + O(g^4).
\]

(5)

Here, the space–time dimension \( D = 4 - 2\epsilon \), \( C_F = N/2 - 1/(2N) \), with the number of colors \( N = 3 \) for QCD. \( \gamma_{\text{Born}} = \gamma_{\text{Born}}(q_1, q_2) \) is the Reggeon-Reggeon-particle (RRP) effective vertex, describing the production of the gluon \( G \) at Reggeized quark transitions [6]

\[
\gamma^G_{\text{Born}} = -gt^G e_G^\mu \left( \gamma^\mu + (m - \hat{q}_1) \frac{2p_A^\mu}{s_1} - (m - \hat{q}_2) \frac{2p_B^\mu}{s_2} \right),
\]

(6)

with \( t^G \) being the color group generator. Amplitude (4) coincides with its real part because its imaginary part is subleading in the LLA, but it is not quite so for amplitudes in the NLA.

If the Reggeization hypothesis in the NLA is correct then, assuming Regge behavior in two subchannels \( s_1 \) and \( s_2 \) (see Fig.1) with the Reggeized quarks in the corresponding crossing channels \( t_1 \) and \( t_2 \), from the general requirements of analyticity, unitarity and crossing symmetry one obtains [7] the multi–Regge form

\[
A_{2\to3} = \frac{1}{4} \bar{\Gamma}_{BB'} \frac{1}{m - \hat{q}_2} \left\{ \left[ \left( \frac{s}{m - \hat{q}_2} \right)^{\delta_2} + \left( \frac{-s}{m - \hat{t}_2} \right)^{\delta_2} \right] \right.

\times \left[ \left( \frac{s_1}{\sqrt{(-t_2)k^2}} \right)^{\delta_2} \mathcal{R} \left( \frac{s_1}{\sqrt{k^2(-t_1)}} \right) \right. + \left. \left( -\frac{s_1}{\sqrt{(-t_2)k^2}} \right)^{-\delta_2} \mathcal{R} \left( -\frac{s_1}{\sqrt{k^2(-t_1)}} \right) \right]

\left. + \left[ \left( \frac{s_2}{\sqrt{(-t_2)k^2}} \right)^{\delta_2} \mathcal{L} \left( \frac{s_2}{\sqrt{k^2(-t_1)}} \right) \right. + \left. \left( -\frac{s_2}{\sqrt{(-t_2)k^2}} \right)^{-\delta_2} \mathcal{L} \left( -\frac{s_2}{\sqrt{k^2(-t_1)}} \right) \right] \right\}

\times \left[ \left( \frac{s}{m - \hat{q}_1} \right)^{\delta_1} + \left( \frac{-s}{-\hat{t}_1} \right)^{\delta_1} \right] \frac{1}{m - \hat{q}_1} \Gamma_{A'A},
\]

(7)

where PPR vertices \( \Gamma_{PP'} = \Gamma_{PRP}(t) \) depend on the polarization of the particles \( P, P' \) and the squared momentum transfer \( t \). They are real for \( t < 0 \). Hereafter, such expressions as \((-s_i)^\delta\) should be read as \((-s_i - i\delta\) so that for \( s_i > 0 \), \( \ln(-s_i) = \ln(s_i) - i\pi \). The RRP vertices \( \mathcal{R} = \mathcal{R}(q_1, q_2) \) and \( \mathcal{L} = \mathcal{L}(q_1, q_2) \) depend on the polarization of the gluon and momenta \( q_1, q_2 \). These vertices are real in all channels where \( t_i < 0 \) and \( k^2 > 0 \). Moreover, in NLO both PPR and RRP vertices become dependent on the energy normalization scale. Since the amplitude \( A_{2\to3} \) is physical, it does not depend on this scale. Therefore, the energy normalization points in eq.[7] may be chosen in an arbitrary way supposing that the corresponding PPR and RRP vertices are calculated at the same points. Our choice \(-t_j \) and \( \sqrt{k^2(-t_j)} \) yields a particularly symmetric real part of the amplitude \( A_{2\to3} \) in the NLA:

\[
\text{Re} \, A_{2\to3} = \bar{\Gamma}_{BB'} \frac{1}{m - \hat{q}_2} \frac{s_2}{\sqrt{(-t_2)k^2}} \delta_2 \left( \mathcal{R} + \mathcal{L} \right) \left( \frac{s_1}{\sqrt{k^2(-t_1)}} \right)^{\delta_1} \frac{1}{m - \hat{q}_1} \Gamma_{A'A}.
\]

(8)
Note, that because of freedom in choosing the energy normalization, one has to make sure that all PPR and RRP vertices in the amplitude are calculated at the proper normalization points.

It is clear from eq. (7) that the contribution of the sum $\mathcal{R} + \mathcal{L}$ is leading, while that of the difference $\mathcal{R} - \mathcal{L}$ is subleading. Indeed, in the LLA the imaginary part of the amplitude is negligible, therefore, only the sum $\mathcal{R} + \mathcal{L}$ contributes to the effective vertex in Born approximation:

$$\mathcal{R}^{(0)} + \mathcal{L}^{(0)} = \gamma_{\text{Born}}^G.$$  \hfill (9)

On the contrary, if the Reggeization is valid, the difference $\mathcal{R}^{(0)} - \mathcal{L}^{(0)}$ at the same order $g$ contributes to the amplitude only as a radiative correction. It can be obtained together with the order $g^3$ corrections to the sum $\mathcal{R}^{(1)} + \mathcal{L}^{(1)}$ from the analysis of the NLO gluon production amplitude. We compare projection of this amplitude on the color triplet state in $t_i$ channels taken with the positive signature, with its multi–Regge form, assuming that the one loop corrections $\Gamma_{P'P}^{(1)}$ are known. In fact, with such accuracy we get from eq. (7)

$$A_{2\rightarrow 3}(\text{one-loop}) = \tilde{\Gamma}_{B'B}^{(0)} \frac{1}{m - \hat{q}_2} \gamma_{\text{Born}}^G \frac{1}{m - \hat{q}_1} \Gamma_{A'A}^{(1)} + \tilde{\Gamma}_{B'B}^{(1)} \frac{1}{m - \hat{q}_2} \gamma_{\text{Born}}^G \frac{1}{m - \hat{q}_1} \Gamma_{A'A}^{(0)}$$

$$+ \frac{1}{4} \Gamma_{B'B}^{(0)} \frac{1}{m - \hat{q}_2}$$

$$\times \left\{ \frac{G_{\text{Born}}}{\hat{q}_1} \ln \left( \frac{s(-s)}{(t_1^2)^2} \right) + \delta_2 \frac{G_{\text{Born}}}{\hat{q}_2} \ln \left( \frac{s(-s)}{(t_2^2)^2} \right) + \left( \Gamma_{\text{Born}}^{(1)} \delta_1 - \delta_2 \Gamma_{\text{Born}}^{(1)} \right) \ln \left( \frac{s_1(-s_1)s_2(-s_2)}{s(-s)(k^2)^2} \right) \right\}$$

$$+ \left[ \left( \mathcal{R}^{(0)} - \mathcal{L}^{(0)} \right) \delta_1 - \delta_2 \left( \mathcal{R}^{(0)} - \mathcal{L}^{(0)} \right) \right] \ln \left( \frac{s_1(-s_1)s_2(-s_2)}{s(-s)(k^2)^2} \right) + 4 \left( \mathcal{R}^{(1)} + \mathcal{L}^{(1)} \right) \right\} \frac{1}{m - \hat{q}_1} \Gamma_{A'A}^{(0)}.$$  \hfill (10)

For massless QCD $\delta_i$ is a scalar and (10) simplifies to

$$A_{2\rightarrow 3}(\text{one-loop}) = \tilde{\Gamma}_{B'B}^{(0)} \frac{1}{\hat{q}_2} \gamma_{\text{Born}}^G \frac{1}{\hat{q}_1} \Gamma_{A'A}^{(1)} - \tilde{\Gamma}_{B'B}^{(1)} \frac{1}{\hat{q}_2} \gamma_{\text{Born}}^G \frac{1}{\hat{q}_1} \Gamma_{A'A}^{(0)}$$

$$- \frac{1}{4} \Gamma_{B'B}^{(0)} \frac{1}{\hat{q}_2} \gamma_{\text{Born}}^G \frac{1}{\hat{q}_1} \Gamma_{A'A}^{(0)} \left[ \delta_1 \ln \left( \frac{s(-s)}{(t_1^2)^2} \right) + \delta_2 \ln \left( \frac{s(-s)}{(t_2^2)^2} \right) + (\delta_1 - \delta_2) \ln \left( \frac{s_1(-s_1)s_2(-s_2)}{s(-s)(k^2)^2} \right) \right]$$

$$= \Gamma_{B'B}^{(0)} \frac{1}{\hat{q}_2} \left[ \left( \mathcal{R}^{(0)} - \mathcal{L}^{(0)} \right) \frac{\delta_1 - \delta_2}{4} \ln \left( \frac{s_1(-s_1)s_2(-s_2)}{s(-s)(k^2)^2} \right) + \mathcal{R}^{(1)} + \mathcal{L}^{(1)} \right] \frac{1}{\hat{q}_1} \Gamma_{A'A}^{(0)}.$$  \hfill (11)

Eq. (10) states that the difference $\mathcal{R}^{(0)} - \mathcal{L}^{(0)}$ contributes to the discontinuities of $A_{2\rightarrow 3}$ in $s_1$ and $s_2$ channels. For small $g$ we obtain

$$(\text{disc}_{s_1} + \text{disc}_{s_2})A_{2\rightarrow 3} = -\pi i \Gamma_{B'B}^{(0)} \frac{1}{m - \hat{q}_2} \left[ \left( \mathcal{R}^{(0)} - \mathcal{L}^{(0)} \right) \frac{\delta_1 - \delta_2}{4} \ln \left( \frac{s_1(-s_1)s_2(-s_2)}{s(-s)(k^2)^2} \right) + \mathcal{R}^{(1)} + \mathcal{L}^{(1)} \right] \frac{1}{m - \hat{q}_1} \Gamma_{A'A}^{(0)}.$$  \hfill (12)

On the other hand, the same expression can be found in the one–loop approximation using the
s-channel unitarity conditions [5,6]. Comparing these two approaches one obtains

\[
\begin{align*}
\mathcal{R}^{(0)} - \mathcal{L}^{(0)} &= \gamma_{\text{Born}}^G(q_1, q_2) \frac{\delta_1 + \delta_2}{\delta_1 - \delta_2} \\
&+ \frac{g^2 N}{\delta_1 - \delta_2} \int \frac{d^{D-2}r_{\perp}}{(2\pi)^{D-1}} \ g t G e_G^{*} C^\mu(q_1 + r, q_2 + r) \frac{\hat{q}_{\perp} \cdot \hat{r}_{\perp}}{(r + q_1)^2 (r + q_2)^2} \\
&+ \frac{1}{N} \int \frac{d^{D-2}r_{\perp}}{(2\pi)^{D-1}} \left[ (\hat{q}_{\perp} + \hat{r}_{\perp}) + m \right] \gamma_{\text{Born}}(q_1 + r, q_2 + r) \left[ (\hat{q}_{\perp} + \hat{r}_{\perp}) + m \right] (\hat{q}_{\perp} - m),
\end{align*}
\]

which gives for the massless case

\[
\begin{align*}
\mathcal{R}^{(0)} - \mathcal{L}^{(0)} &= \gamma_{\text{Born}}^G(q_1, q_2) \frac{\delta_1 + \delta_2}{\delta_1 - \delta_2} \\
&+ \frac{g^2 N}{\delta_1 - \delta_2} \int \frac{d^{D-2}r_{\perp}}{(2\pi)^{D-1}} \ g t G e_G^{*} C^\mu(q_1 + r, q_2 + r) \frac{\hat{q}_{\perp} \cdot \hat{r}_{\perp}}{(r + q_1)^2 (r + q_2)^2} \\
&+ \frac{1}{\delta_1 - \delta_2} \int \frac{d^{D-2}r_{\perp}}{(2\pi)^{D-1}} \left[ (\hat{q}_{\perp} + \hat{r}_{\perp}) + m \right] \gamma_{\text{Born}}(q_1 + r, q_2 + r) \left[ (\hat{q}_{\perp} + \hat{r}_{\perp}) + m \right],
\end{align*}
\]

where

\[
C^\mu(q_1, q_2) = -(q_1 + q_2)_{\perp} + p_A^\mu \left( \frac{s_2}{s} + \frac{2q_{\perp}}{s_1} \right) - p_B^\mu \left( \frac{s_1}{s} + \frac{2q_{\perp}}{s_2} \right)
= -(q_1 + q_2)^\mu + 2p_A^\mu \left( \frac{s_1}{s} \right) - 2p_B^\mu \left( \frac{s_2}{s} \right).
\]

The integral over the transverse components of the virtual gluon momentum \( r^\mu \) is calculated in Appendix A. Then, substituting the orthogonal momenta

\[
\begin{align*}
q_{\perp}^\mu &= q_1^\mu - \frac{s_2 - t_1}{s} p_A^\mu - \frac{t_1}{s} p_B^\mu \simeq q_1^\mu - \frac{s_2}{s} p_A^\mu, \\
q_2^\mu &= q_2^\mu + \frac{s_1 - t_2}{s} p_A^\mu + \frac{t_2}{s} p_B^\mu \simeq q_2^\mu + \frac{s_1}{s} p_B^\mu
\end{align*}
\]

we obtain

\[
\begin{align*}
\mathcal{R}^{(0)} - \mathcal{L}^{(0)} &= \frac{\delta_1 + \delta_2 - g_F^2}{\delta_1 - \delta_2} \gamma_{\text{Born}}^G + \frac{1}{N} \frac{\gamma_{\text{Born}}^G}{\delta_1 - \delta_2} \\
&- 4 (N - C_F) \frac{g^2 N}{\delta_1 - \delta_2} \left[ \hat{q}_{\perp} \gamma_{\text{Born}}^G \hat{q}_{\perp} \frac{q_{\perp}^2}{t_1} \ln \left( \frac{q_{\perp}^2}{k^2} \right) \right] \\
&- g t G e_G^{*} C^\mu(q_1, q_2) \left[ \hat{q}_{\perp} \frac{q_{\perp}^2}{t_1} \ln \left( \frac{q_{\perp}^2}{k^2} \right) \right] \\
&+ g t G \left( \frac{p_A e_G}{s_1} - \frac{p_B e_G}{s_2} \right) \left[ \frac{q_{\perp}^2}{k^2} \ln \left( \frac{q_{\perp}^2}{k^2} \right) \right].
\end{align*}
\]
As for the sum $\mathcal{R}(1) + \mathcal{L}(1)$, it is clear that, contrary to the difference $\mathcal{R}(0) - \mathcal{L}(0)$, it can not be defined by means of $s_t$–channel unitarity in the multi–Regge region. To calculate it we use a more suitable here $t$–channel dispersive approach based on analyticity and $t$–channel unitarity, developed in \[8,9,10\].

3 t–channel discontinuity

It is possible to calculate the corrections to the RRP vertex by considering gluon production in various annihilation processes: quark–antiquark, gluon–gluon, quark–gluon or antiquark–gluon. Moreover, we can consider photons instead of gluons as the scattered particles, which noticeably reduces the number of contributing diagrams. Of course, if the Reggeization hypothesis is valid, we must obtain the same vertex. In the approach based on $t_i$–channel unitarity it looks very natural.

We choose to consider gluon production in the process of quark–antiquark annihilation into two photons, therefore in Figs.1, 2 the particles $A', B'$ are photons and $G$ is a gluon. Let us take into account the amplitude discontinuity in the $t_1$ channel. The discontinuity in the $t_2$ channel could be considered analogously. The contribution of $t_1$ channel is represented schematically in Fig.2. It is given by an ordinary Cutkosky rule from cut of the corresponding diagrams. After this cut we come to consider the amplitudes being the left part ($A$) and the right part ($B$) of Fig.2. Since the external gluon is physical and the cut lines in Fig.2 are on the mass shell, both $A$ and $B$ are invariant under gauge transformations of the intermediate gluon polarization. Therefore, one can use an arbitrary gauge. We choose Feynman gauge with the polarization tensor $-g^{\mu\nu}$.

After the convolution we substitute $i(p^2 - m^2 + i\alpha)^{-1}$ for the on–mass–shell $\delta$–functions $2\pi \delta_+(p^2 - m^2)$ and perform the loop integration thus restoring the amplitude $A_{2\to3}^{t_1}$. This amplitude has the same $t_1$–channel singularities as the complete one. That is enough to restore the correct Regge asymptotic. Indeed, obtained amplitude has the same $t_1$ singularity as the real one and an arbitrary polynomial in $t_1$ could be added to the final result in the framework of this procedure but, such term would have a wrong asymptotic behavior incompatible with the renormalizability of the theory [14], and, contrary to the case with massive quarks, where the expression which is equal to the Born amplitude with some constant coefficient can be added, for the massless limit considered in this paper in detail the correct analytical properties together with the unitarity requirement in $t_i$–channels determine the amplitude in an unambiguous way.
In order to construct the amplitude with true Regge behavior in MRK, we have to ensure that it has correct quark quantum numbers, i.e. color triplet and positive signature in $t_i$ channels. While the former is done by applying the projection operator (which is not actually necessary when $A'$ and $B'$ are photons)

$$\langle c_1 | \hat{P}_3 | c_2 \rangle = \frac{t^{c_1} t^{c_2}}{C_F},$$  \hspace{1cm} (18)

the latter can be achieved via the "signaturization" procedure, which is naturally formulated for truncated amplitudes — the amplitudes with omitted wave functions (polarization vectors and Dirac spinors). The procedure is most simple for the massless case — to obtain positive signature states in $t_1$ and $t_2$ channels we have to symmetrize the truncated amplitude $A^t_{2 \rightarrow 3}$ with respect to the substitutions

$$p_A \rightarrow -p_{A'}, \hspace{1cm} s_1 \rightarrow -s_1, \hspace{1cm} s \rightarrow -s,$$

and

$$p_B \rightarrow -p_{B'}, \hspace{1cm} s_2 \rightarrow -s_2, \hspace{1cm} s \rightarrow -s.$$ \hspace{1cm} (19)

So, we have to consider

$$A^t_{2 \rightarrow 3} = \hat{S}_+ \int \frac{d^D r}{(2\pi)^D} \frac{\sum B A}{(p^2 - m^2 + i\alpha)(r^2 + i\alpha)}, \hspace{1cm} p = q_1 + r,$$

where the sum is taken over all polarizations of the intermediate particles, the convolution is performed on–mass–shell $(r^2 = 0, p^2 = m^2)$, and $\hat{S}_+$ stands for the "signaturization" operator returning the amplitude with positive signature.

On the one hand, we must use the exact expression for the amplitude $A$, as we integrate over the momenta $r$ and $p$ of the intermediate particles. On the other hand, since $s_2 \approx (p_B + k)^2$ is fixed and large, we take the amplitude $B$ in the quasi–multi–Regge kinematics, which means that gluon $G$ with momentum $k$ is produced in the fragmentation region of the intermediate quark and gluon:

$$B = \tilde{\Gamma}^{(0)}_{B'B} \frac{1}{m - q_2} \tilde{\Gamma}_{\{r,k\} p}.$$ \hspace{1cm} (22)

Here $\tilde{\Gamma}^{(0)}_{B'B} = -e \overline{v}_{B'} \hat{e}_{B'}$ and $e$ is a quark–photon coupling constant. We use the light–cone gauge for the external photon $(e_{B'} p_{B'}) = (e_B p_A) = 0$. One can show that the gauge invariant vertex $\tilde{\Gamma}_{\{r,k\} p}$, initially calculated from an inelastic quark–gluon process \[\Pi\], does not depend on the nature of the particles $B$ and $B'$, so $B'$ can be a photon. This vertex has the form

$$\tilde{\Gamma}(k_1, k_2)_p = -g^2 \left\{ \frac{t^2 t^1}{(k_1 + k_2)} \gamma^\mu(k_2) \frac{1}{\hat{p} - k_1 - m} \gamma^\nu + \frac{t^1 t^2}{(k_1 + k_2)} \gamma^\mu(k_1) \frac{1}{\hat{p} - k_2 - m} \gamma^\nu ight\} u_{\nu} e^\mu_1 e^\nu_2,$$

\hspace{1cm} (23)
where
\[ \gamma^\mu(r) = \gamma^\mu + (\tilde{q}_2 - m) \frac{p_B^\mu}{(rp_B)}, \quad (k_i e_i^*) = 0, \]
\[ \gamma^{\mu\rho}(k_1, k_2) = (k_1 - k_2)^\rho g^{\mu\nu} - 2k_1^\rho g^{\mu\mu} + 2k_2^\rho g^{\nu\nu}. \] (24)

So, projecting the color state of \( B \) on quark quantum numbers \(^{(18)}\), we get for massless quarks
\[
\Gamma_{\{r,k\}p} = -g^2 t^G t^r \left\{ C_F \left( \tilde{e}_G^* + \tilde{q}_2 \frac{2(e_G^p B)}{s_2} \right) \frac{1}{\tilde{q}_1} \tilde{e}_r^* - \frac{1}{2N} \left( \tilde{e}_r^* - \tilde{q}_2 \frac{2(e_r^p B)}{s_B} \right) \right\} \frac{1}{\hat{p} - \hat{k}} \tilde{e}_G^* \\
+ \frac{N}{2} \frac{1}{l} \left[ \tilde{r} - \tilde{k} + \tilde{q}_2 \left( 1 - 2 \frac{s_2}{u_B} \right) (e_r^* e_G^*) + 2 \left[ \tilde{e}_G^* + \tilde{q}_2 \frac{2(e_r^p B)}{u_B} \right] (ke_r^*) \right] - 2 \left[ \tilde{e}_r^* + \tilde{q}_2 \frac{2(e_r^p B)}{u_B} \right] (re_G^*) \right\} \frac{2(e_r^p B)(e_r^p B)}{s_2} \left( \frac{N}{u_B} + \frac{1}{N} \frac{1}{s_B} \right) \right\} u_p. \] (25)

Here we use some of the next denotations
\[
(p_A + r)^2 = s_A, \quad (p_A - p)^2 = u_A, \\
(p_B - r)^2 = s_B, \quad (p_B + p)^2 = u_B, \\
(r + k)^2 = l, \quad (p - k)^2 = n, \] (26)

and the properties of our kinematics, obtained for the mass shell momenta \( r, p, \) and \( k \):
\[
s_A + u_A = -t_1, \quad s_B + u_B \simeq s_2, \quad l + n = t_2 - t_1, \\
2(rp_A) = s_A, \quad 2(pp_A) = -u_A, \quad 2(rp_B) = -s_B, \\
2(pp_B) = u_B, \quad 2(q_1 p_B) \simeq s_2, \quad 2(q_2 p_A) \simeq -s_1. \] (27)

As for the amplitude \( \mathcal{A} \), it is very profitable here to decompose it into an "asymptotic" ("as") part, giving the asymptotic of the amplitude \( \mathcal{A} \) in the Regge limit \(|s_A| \approx |u_A| \gg |t_1|\), and a "non–asymptotic" ("na") part:
\[
\mathcal{A} = \mathcal{A}^{(as)} + \mathcal{A}^{(na)}, \\
\mathcal{A}^{(as)} = \Gamma_{pr}^{(as)} \frac{1}{m - \tilde{q}_1} \Gamma_{A'A}^{(0)} \mathcal{A}^{(na)} = -eg t^r \bar{u}_p \frac{\tilde{e}_A^* \tilde{e}_r}{s_A - m^2} u_A, \] (28)

where
\[
\Gamma_{A'A}^{(0)} = -e \tilde{e}_A^* u_A, \quad \Gamma_{pr}^{(as)} = -gt^r \bar{u}_p \left( \tilde{e}_r + (\tilde{q}_1 - m) \frac{2(p_A e_r)}{s_A - m^2} \right). \] (29)

These parts are invariant under gauge transformations of the intermediate gluon polarization as well as the complete amplitude. The "asymptotic" contribution to the full amplitude for massless quarks takes the form:
\[
A_{2\to3}^{l_1(\text{as})} = \hat{S}_+ \bar{\Gamma}_{B'p}^{(0)} \frac{1}{q_2} \int \frac{d^D r}{(2\pi)^D} \frac{\left( \Gamma_{r,k} \right)}{r^2 p^2} \frac{1}{q_1} \Gamma_{A'A}^{(0)}. \] (30)
The convolution of the vertices yields:

\[
\sum \gamma \Gamma_{r,k}^{(as)}(n) = g^2 C_F \left[ \gamma^G_{\text{Born}} - g tG \hat{q}_1 \frac{2(p_A e_G^*)}{s_1} \right] \frac{1}{\hat{q}_1} \left( \gamma^\mu \hat{p}^\mu + 2 \hat{p}_A \hat{p} \hat{q}_1 \right) \\
+ g^3 tG \left[ \frac{1}{N} (V_1 + V_2) + N (V_3 + V_4) \right],
\]

where

\[
V_1 = \frac{\gamma^\mu (\hat{p} - \hat{k}) \hat{e}_G^* \hat{p}^\mu}{2 n} - \frac{\hat{q}_2 (\hat{p} - \hat{k}) \hat{e}_G^* \hat{p} \hat{q}_1 s}{ns_A s_B} \\
+ \left( \frac{\hat{p}_A (\hat{p} - \hat{k}) \hat{e}_G^* \hat{p} \hat{q}_1}{ns_A} - \frac{\hat{q}_2 (\hat{p} - \hat{k}) \hat{e}_G^* \hat{p} \hat{q}_1}{ns_B} \right),
\]

\[
V_2 = -\frac{2(p_B e_G^*)}{s_2 s_B} \left( \hat{q}_2 \hat{p} \hat{p} + \frac{\hat{q}_2 \hat{p} \hat{q}_1 s}{s_A} \right), \\
V_3 = V_2(s_B \rightarrow u_B),
\]

\[
V_4 = \frac{\hat{k} \hat{p} \hat{e}_G^* - \hat{e}_G^* \hat{p} \hat{k}}{l} + \frac{\hat{k} \hat{p} \hat{q}_1 2(p_A e_G^*)}{l s_A} - \frac{\hat{q}_2 \hat{p} \hat{k} 2(p_B e_G^*)}{l u_B} \\
+ 2 \frac{\hat{q}_2 \hat{p} \hat{q}_1 (p_A e_G^*) s_2 - (p_B e_G^*) s_1}{l s_A u_B} + \frac{\hat{q}_2 \hat{p} \hat{e}_G^* s_2}{l u_B} + \frac{\hat{q}_2 \hat{p} \hat{q}_1 s_1}{l s_A} \\
+ 2 \left( \hat{p}_A \hat{p} \hat{q}_1 (r e_G^*) \frac{2(p_B e_G^*) s}{l s_A u_B} \right) + \frac{\hat{q}_2 \hat{p} \hat{p} \hat{p} (r e_G^*)}{u_B} \frac{\gamma^\mu \hat{p}^\mu (r e_G^*)}{l} \frac{\hat{q}_2 \hat{p} \hat{q}_1 (r e_G^*) s}{l s_A u_B}.
\]

The integrals appearing in eq. (30) from the convolution (31) are calculated in Appendix B of this paper and Appendix C of [8]. Due to its length we do not present the integrated expression for (30) here and will use it for calculating the answer for \( \mathcal{R} + \mathcal{L} \) in Section 5. It is important to stress that we integrate in the fixed order of limit taking, as it is done systematically in Regge approach: first \( s_i \rightarrow \infty \), and only after it \( D \rightarrow 4 \). Of course, these two limits must commute in final infrared stable results for observables, but at intermediate steps one must adhere to the initially set order everywhere. The consequence of our choice is the prohibition to expand such terms as \( s^{-\epsilon} \) to series with respect to \( \epsilon \rightarrow 0 \).

The integrated expression for (30) has a correct singularity (discontinuity) in the \( t_1 \) channel. It also has a \( t_2 \) channel singularity, but the latter is correct only for the terms with both \( t_1 \) and \( t_2 \) channel singularity. The amplitude with correct analytic properties in both channels can be easily yielded by symmetry consideration. Using the uncertainty of the dispersion formula (30) with respect to adding to the convolution (31) some terms proportional to \( r^2 \) or \( p^2 \), we can restore the expression which is symmetric (with respect to change left and right parts of the diagram (see. fig. 1)), and vanishes after \( e_G \rightarrow k \) substitution. We find such additional terms after analyzing the integrated form of (30). They depend on how we treat hidden \( r^2 \) and \( p^2 \) in \( V \)’s. In all cases we commute terms similar
Therefore, we obtain the simple multi-Regge form (4) with the corresponding substitution of momenta and color indices.

In the multi-Regge asymptotic region instead of complicated expression (25). In that region one may use the "non-asymptotic" part of the amplitude invariant, the essential region of integration over $r$. It implies that, in order to calculate the contribution of the amplitude discontinuity in the $t_2$ channel discontinuity.

Expression (35) is applicable for arbitrary $D$. After adding eq.(35) to (31) we do not need to consider the contribution of the amplitude discontinuity in the $t_2$ channel.

It is clear that regardless of energy normalization, $R^{(1)} + L^{(1)}$ contributes only to the real part of the l.h.s. of eq.(11) and $R^{(0)} - L^{(0)}$ is unambiguously determined by its imaginary part. Since both $\gamma^{G}_{\text{Born}}$ and $A_{2\to 3}$ vanish after the substitution of the gluon momentum $k$ for the polarization vector $e_{G}$, we can conclude that both $R^{(1)} + L^{(1)}$ and $R^{(0)} - L^{(0)}$ vanish as well.

Now, let us turn to the "non-asymptotic" contribution $A^{t(t_{1}(na))}_{2\to 3}$ given by the product $B A_{(na)}$. Since the "non-asymptotic" part $A_{na}$ (see (23)) does not contain terms of order $s_{A}$ for large values of this invariant, the essential region of integration over $r$ in this case is

$$s_{A} \sim u_{A} \sim r^{2} \sim p^{2} \sim t_{i},
\quad l \sim n \sim s_{1}, \quad s_{B} \sim u_{B} \gg |t_{i}|. \quad (36)$$

It implies that, in order to calculate the contribution of $A^{t(t_{1}(na))}_{2\to 3}$, one may take the amplitude $B$ in the multi-Regge asymptotic region instead of complicated expression (25). In that region one may use the simple multi-Regge form (4) with the corresponding substitution of momenta and color indices. Therefore, we obtain

$$B(\text{"na"-region}) = \tilde{\Gamma}^{(0)}_{B'B} \frac{1}{m - \tilde{q}_{2}} \gamma^{\text{Born}}_{G} \frac{1}{m - q_{1}} \Gamma^{k(\text{as})}_{rp},$$

$$\Gamma^{k(\text{as})}_{rp} = -gt^{r} \left( \hat{e}_{r}^{*} + (q_{1} - m) \frac{(ke_{G}^{*})}{(kr)} \right) u_{p}. \quad (37)$$

Thus, the problem of calculating the $A^{t(t_{1}(na))}_{2\to 3}$ contribution reduces to a simpler problem for the elastic case. Moreover, it is not necessary to calculate this contribution at all, since it is totally absorbed in the correction to the PPR vertices $\Gamma^{(1)}_{p'p}$. We stress that one-loop corrections to the gluon production amplitude $A_{2\to 3}$ (one-loop) include corrections to these vertices, as one can see from eq.(11). We will clarify this point in the next Section.
4 Asymptotic correction to PPR vertex

The calculation of $\mathcal{R}^{(1)} + \mathcal{L}^{(1)}$ includes the one loop correction to the PPR vertex $\Gamma^{(1)}_{P'p}$ (see eq. (11)). Assuming the validity of the Reggeization hypothesis, one may find $\Gamma^{(1)}_{P'p}$ from consideration of the amplitude of quark–antiquark annihilation into photons in Regge limit. We compare the projection of this amplitude on the positive signature and its Regge form:

$$A_{2\to2} = \frac{1}{2} \Gamma_{B'B}(q, s_0) \frac{1}{m-q} \left[ \left( \frac{s}{s_0} \right)^{\delta(q)} + \left( \frac{-s}{s_0} \right)^{\delta(q)} \right] \Gamma_{A'A}(q, s_0),$$

where $s_0$ is an energy normalization point. Eq. (7) is written for PPR vertex $\Gamma_{P'p}$ normalized at $s_0 = -t$, so we calculate the correction to this vertex at the same point. The one–loop approximation gives us

$$A_{2\to2}(\text{one–loop}) = \Gamma^{(1)}_{B'B} \frac{1}{m-q} \Gamma^{(0)}_{A'A} + \Gamma^{(0)}_{B'B} \frac{1}{m-q} \Gamma^{(1)}_{A'A} + \frac{1}{2} \Gamma^{(0)}_{B'B} \frac{\delta(q)}{m-q} \Gamma^{(0)}_{A'A} \ln \left( \frac{s(-s)}{(-t)^2} \right).$$

(39)

Let us note that formula (39), i.e. the Reggeization hypothesis at $\alpha_s^2$ order, is proved [4].

Similarly to the previous Section we use $t$ channel unitarity technique and, performing the Cutkosky cut, deal with the left ($A$) and the right ($B$) parts of Fig 3. After $q_1 \to q$ substitution the amplitude $A$ coincides with the ones presented in (28) and the amplitude $B$ can be easily obtained by its Hermitian conjugation and $p_A \leftrightarrow -p_B$ exchange. We mark its parts with letter $B$ (e.g. $\Gamma^{B(\text{as})}_{p'r}$).

The contribution of the product $B^{(\text{as})}A^{(na)}$ represents a part of the corrections to the amplitude connected with the piece of $\Gamma^{(1)}_{A'A}$. Because of the factorization property of $B^{(\text{as})}$ we have for the massless case

$$\Gamma^{(1)(\text{as}−\text{na})}_{A'A} = \hat{S}_+ \int \frac{d^D p}{(2\pi)^D} \sum_{r,p} \Gamma^{B^{(\text{as})}}_{r,p} A^{(na)} \Gamma^{(1)}_{A'A},$$

(40)

Here, as in the previous Section we use the light–cone gauges for external photons

$$\langle e_A p_A \rangle = \langle e_B p_B \rangle = \langle e_A p_B \rangle = \langle e_B p_A \rangle = 0,$$

(41)
which means, in other words, that the final result in general gauges will be given by the replacements

\[ e_{A'} \to e_{A'} - p_{A'} \left( \frac{e_{A'pB}}{p_{A'pB}} \right), \quad e_{B'} \to e_{B'} - p_{B'} \left( \frac{e_{B'pA}}{p_{A'pB}} \right). \]

The integrand of (40) is

\[ \sum \Gamma_{rp}^{B(\alpha s)} A^{(\alpha s)} = -e g^2 C_F \left( \frac{\gamma^\mu \hat{\gamma}^\nu e_{A'} \gamma^\mu}{s_A} - 2 \hat{q} \hat{\hat{p}} e_{A'} \hat{\hat{p}} B \right) u_A. \]

Necessary integrals can be found in Appendix C of this paper and in Appendix A of [8]. We obtain

\[ \Gamma_{A'A}^{(1)(\alpha s - \alpha s)} = -e \delta(q) \left( \hat{e}_{A'} \frac{-3 - 2\epsilon}{2(1 - 2\epsilon)} + \frac{\hat{q}(e_{A'q})}{t} \frac{2 + \epsilon}{1 - 2\epsilon} \right) u_A, \]

where \( \delta(q) \) is the quark trajectory (3) for \( m = 0 \).

While inserting \( \Gamma_{A'A}^{(1)(\alpha s - \alpha s)} \) (40) into (11) we substitute \( p_B \to k \), which leads to \( \Gamma_{rp}^{B(\alpha s)} \to \Gamma_{rp}^{k(\alpha s)} \) (cf. eq.(37)). Therefore, due to the analogous factorization property of \( B^{(\alpha s)} \)-region represented in eq.(37), the contribution of the product \( BA^{(\alpha s)} \) to the gluon production amplitude \( A_{3 \to 2}(\text{one-loop}) \) cancels the piece of the second term of (11) given by \( \Gamma_{A'A}^{(1)(\alpha s - \alpha s)} \). So, actually, we need to calculate neither the contribution of \( BA^{(\alpha s)} \) to \( A_{3 \to 2} \), nor the contribution of \( B^{(\alpha s)} A^{(\alpha s)} \) to \( A_{2 \to 2} \). The symmetric situation takes place for the part of \( B^{(1)}_{B'B} \).

The totally "non-asymptotic" contribution to the \( A_{2 \to 2} \) for massless quarks is given by the product \( B^{(\alpha s)} A^{(\alpha s)} \). For it, the essential values of \( (rp_A) \) and \( (rp_B) \) in the integration region are small: \( r^2 \sim p^2 \sim s_A \sim s_B \sim t \). Therefore, in Sudakov decomposition (2) parameters \( \alpha \) and \( \beta \) are suppressed \( \alpha \sim \beta \sim t/s \) and can be omitted in the propagators:

\[ r^2 \to r_1^2, \quad p^2 \to (r + q)^2, \quad s_A \to r_1^2 + s\alpha, \quad s_B \to r_1^2 - s\beta. \]

It leads to the factorization of the corresponding integral (see (21)) into two blob integrals with respect to \( \alpha \) and \( \beta \), which can be taken by residues. The following contribution to the total amplitude is pure imaginary at high energies and corresponds to the lowest-order term of the negative signature. Here we are interested in radiative corrections to the positive signedature amplitude and therefore contribution of \( B^{(\alpha s)} A^{(\alpha s)} \) is not important for us.

Therefore, we have to consider only the piece of \( \Gamma_{A'A}^{(1)} \) (and \( \Gamma_{B'B}^{(1)} \)) defined by \( B^{(\alpha s)} A^{(\alpha s)} \) and subtract from the corrections to the gluon production amplitude \( A_{3 \to 2}(\text{one-loop}) \) in eq.(11) only the part connected with this piece (\( \Gamma_{A'A}^{(1)(\alpha s - \alpha s)} \)).

\[ A_{2 \to 2}^{(\alpha s - \alpha s)} = \hat{S} + \bar{\Gamma}_{B'B}^{(0)} \frac{1}{-\hat{q}} \int \frac{d^D r}{(2\pi)^D} \sum \frac{\Gamma_{rp}^{B(\alpha s)} \bar{\Gamma}_{rp}^{(\alpha s)}}{r^2 p^2} \frac{1}{-\hat{q}} \Gamma_{A'A}^{(0)}. \]

The integrand here has the form:

\[ \sum \Gamma_{rp}^{B(\alpha s)} \bar{\Gamma}_{rp}^{(\alpha s)} = -g^2 C_F \left( \gamma^\mu \hat{\gamma}_{A}^\mu + \frac{2 \hat{p}_A \hat{p}_B q}{s_A} - \frac{2 \hat{q} \hat{p}_B s_B}{s_B} - 2 s \frac{\hat{q} \hat{p}_B q}{s_A s_B} \right). \]
After the integration we obtain
\[
\Gamma_{A'\Delta}^{(1)(as-as)} = -e \delta(q) \left( \hat{e}_{A'} \frac{1}{2} \left[ -\frac{1}{\epsilon} + \frac{3 + \epsilon}{2(1 - 2\epsilon)} + \psi(1) + \psi(1 + \epsilon) - 2\psi(1 - \epsilon) \right] - \frac{\hat{q}(e_A q)}{t} \frac{2}{1 - 2\epsilon} \right) u_A. \tag{47}
\]

Finally, we present the full photon–quark–Reggeized quark vertex with photon polarization satisfying light–cone gauge conditions (41):
\[
\Gamma_{A'\Delta} = -e \left( \hat{e}_{A'} (1 + \delta_e) + \frac{\hat{q}(e_A q)}{t} \delta_q \right) u_A, \tag{48}
\]
where
\[
\delta_e = \delta(q) \frac{1}{2} \left[ -\frac{1}{\epsilon} - \frac{3(1 + \epsilon)}{2(1 - 2\epsilon)} + \psi(1) + \psi(1 + \epsilon) - 2\psi(1 - \epsilon) \right] + O(g^4),
\]
\[
\delta_e = \delta(q) \frac{\epsilon}{1 - 2\epsilon} + O(g^4). \tag{49}
\]

This result coincides with the one, which can be easily obtained from the calculations of the gluon–quark–Reggeized quark vertex, presented in V.S. Fadin and R. Fiore’s paper [3]. Indeed, taking the coefficient only before \(-1/2N\) in their result and multiplying it by \(C_F\) corresponds to omitting the triple gluon vertex and corrected color algebra.

5 Radiative correction to RRP vertex

Finally, using the table of integrals presented in the Appendices and gathering results from previous sections we find the one–loop correction to the RRP vertex. The answer contains terms proportional to three gamma–matrix structure \(\hat{q}_2 \hat{e}_G^* \hat{q}_1\) (e.g. term \(\hat{q}_2 \gamma^G_{\text{Born}} \hat{q}_1\) in (17)), which in our kinematics (do not forget about surrounding spinors in the amplitude, where the RRG vertex has to be inserted) can be replaced by \(\hat{q}_{2\perp} \hat{e}_{G\perp}^* \hat{q}_{1\perp}\), so we have
\[
\hat{q}_2 \hat{e}_G^* \hat{q}_1 = \hat{q}_1 (e_{G\perp}^* q_2)_{\perp} + \hat{q}_2 (e_{G\perp}^* q_1)_{\perp} - (q_1 q_2)_{\perp} \hat{e}_{G\perp}^* + i \gamma^5 \gamma'^{a\perp b\perp} q_{1\perp} e_{G\perp}^* q_{2\perp} \hat{e}_{G\perp}^*, \tag{50}
\]
with the last term equal to zero. There is also a linear relation between the parts of the RRP vertex resulting from the fact that \(e_{G\perp}\) can be decomposed into the basis vectors: \(q_{1\perp}\), \(q_{2\perp}\).

\[
\gamma_{\text{Born}}^G = g t^G e_{G\perp}^* C^\mu(q_1, q_2) \frac{1}{\Delta} \left[ (\hat{q}_1 + \hat{q}_2) k^2 + (\hat{q}_1 - \hat{q}_2) (t_1 - t_2) \right] \nonumber
\]

\[
+ g t^G \left( \frac{(p_A e^G_A)}{s_1} - \frac{(p_B e^G_B)}{s_2} \right) \frac{2 t_1 t_2}{\Delta} \left[ \left( \frac{\hat{q}_1}{t_1} + \frac{\hat{q}_2}{t_2} \right) k^2 - \left( \frac{\hat{q}_1}{t_1} - \frac{\hat{q}_2}{t_2} \right) (t_1 - t_2) \right], \tag{51}
\]

13
where $\Delta = 4(q_1 q_2)^2 - 4t_1 t_2$. We used this relation to simplify our answer for $\mathcal{R}^{(1)} + \mathcal{L}^{(1)}$.

$$
(\mathcal{R}^{(0)} - \mathcal{L}^{(0)}) \frac{\delta_1 - \delta_2}{c_r g^2} = 2N \gamma_{\text{Born}} \left( \frac{1}{\epsilon} - \ln \left( k^2 \right) \right) 
+ (N - C_F) \left\{ \gamma_{\text{Born}}^G \left[ \frac{(k^2 + t_1 + t_2)^2 - 2t_1 (t_1 + t_2)}{k^2 t_1} \ln \left( \frac{k^2}{-t_2} \right) + \frac{(k^2 + t_1 + t_2)^2 - 2t_2 (t_1 + t_2)}{k^2 t_2} \ln \left( \frac{k^2}{-t_1} \right) \right] 
+ g t^G e^\mu G^\mu (q_1, q_2) \left[ \frac{q_1 + q_2}{t_1 t_2} V^+ + \frac{q_1 - q_2}{t_1 t_2} V^- \right] 
+ 2 g t^G \left( \frac{p_A e^\mu A}{s_1} - \frac{p_B e^\mu B}{s_2} \right) \left[ \left( \frac{q_1}{t_1} + \frac{q_2}{t_2} \right) V^+ - \left( \frac{q_1}{t_1} - \frac{q_2}{t_2} \right) \left( t_1 + t_2 \right) V^- \right] \right\} ,
$$

where

$$
V^+ = \ln \left( -\frac{t_1}{k^2} \right) t_1 + \ln \left( -\frac{t_2}{k^2} \right) t_2 , \quad V^- = \ln \left( -\frac{t_1}{k^2} \right) t_1 - \ln \left( -\frac{t_2}{k^2} \right) t_2 ,
$$

and the expressions for $\gamma_{\text{Born}}^G$ and $C^\mu$ are defined in eqs. $[50], [51]$.
Note that new spin structures appear in eqs. (52, 55) in comparison with the Born effective vertex and that the corrections obtained are obviously gauge invariant. It is easy to verify that poles at \( t_1 = t_2 \) in eq. (55) cancel. Moreover, one can check that when \(|q_{1\perp}| \) or \(|q_{2\perp}| \) approaches zero the coefficients at each spin structure remain finite.

Let us stress that the calculated amplitude \( A_{2\rightarrow 3} \) (one-loop) has the very same nontrivial structure in energy variables \( s_i \) as predicted by Regge ansatz, which leads to cancellation of all energy variables in eq. (11) and finally gives us eq. (55).

In all the above formulae we used the unrenormalized coupling constant \( g \). Therefore, expressions (52) and (55) for \( D \rightarrow 4 \) contain singularities from ultraviolet as well as from infrared divergences of Feynman integrals. We can remove the ultraviolet divergences from \( R - L \) and \( R + L \) expressing \( g \) in terms of the renormalized coupling constant, for example in the \( \overline{\text{MS}} \) renormalization scheme

\[
g = g_\mu \mu^{2-D/2} \left\{ 1 - \frac{1}{2} \left( \frac{11}{3} N - \frac{2}{3} n_f \right) \frac{g_\mu^2 \Gamma(2 - D/2)}{(4\pi)^{D/2}} + \cdots \right\},
\]

where \( n_f \) is the number of flavors and \( g_\mu \) is the renormalized coupling constant at the renormalization point \( \mu \). At first sight, the absence of the singularity proportional to \( n_f \) at \( D = 4 \) in our answer for \( R + L \) may look suspicious, but one should realize that such a term with the ultraviolet singularity cancels the infrared singularity which arises in the fermion part of gluon self–energy \( P_f(0) \) at \( m = 0 \).

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A Appendix

Here we calculate the transverse momentum tensor integral appearing in (14) for \( D \rightarrow 4 \). Expressions for its vector \((I_\Delta^{(D-2)})\) and scalar \((I_\Delta^{(D-2)})\) variants one can find in Appendix B of [8].

\[
I_\Delta^{\mu(D-2)} = \int \frac{r_\mu r'_\nu d^{D-2}\tau_\perp}{r_\perp^2 (r + q_1)^2 (r + q_2)^2} \\
= \Gamma(1 + \epsilon) \pi^{D/2 - 1} \left\{ \frac{g_\mu^{\mu'}}{2} \left( \frac{\ln(q_1^2) \ln(q_2^2)}{k^2 t_{12}} - \frac{\ln(q_1^2) \ln(q_2^2)}{k^2} - \frac{\ln(q_1^2) \ln(k^2)}{t_{12}} \right) - \frac{q_1^\mu q_2^{\mu'}}{k^2 t_{12}} \left( \frac{1}{\epsilon} - \ln(q_1^2 k^2) \right) \right\} \\
- \frac{(q_1^\mu q_2^{\mu'} + q_2^\mu q_1^{\mu'})}{2} \left( \frac{\ln(k^2)}{t_{12}} + \frac{\ln(q_1^2)}{k^2 t_{12}} + \frac{\ln(q_2^2)}{k^2 t_{12}} \right) + \mathcal{O}(\epsilon),
\]
Let us explain how one can obtain this short formula. The method used is well known, and we describe it shortly here to refer to it in the next Section. First, we introduce our notation for coefficients in the tensor decomposition of the $I^{\mu\nu(D-2)}_\Delta$:

$$I^{\mu\nu(D-2)}_\Delta = \Delta^{(D-2)}_\Delta [g] g^{\mu\nu}_{\perp[D-2]} + \sum_{i,j=1}^{2} I^{(D-2)}_\Delta [q_i q_j] q^{\mu}_i q^{\nu}_j, \quad (A.2)$$

where $g^{\mu\nu}_{\perp[D-2]}$ is the metric tensor in $D-2$ dimensions. These coefficients can be expressed through the vector $I^{(D-2)}_\Delta$ and scalar $I^{(D)}_\Delta$ integrals:

$$I^{(D-2)}_\Delta [q_1 q_2] = \frac{1}{t_1 t_2 - (q_1 q_2)^2} \left\{ \left[ I^{(D-2)}_\Delta [g] - P^\mu_1 \left( I^{\nu\mu(D-2)}_\Delta q^{\nu}_1 \right) \right] (q_1 q_2) \perp + t_1 P^\mu_1 \left( I^{\mu\nu(D-2)}_\Delta q^{\nu}_2 \right) \right\},$$

$$I^{(D-2)}_\Delta [q_1 q_1] = \frac{1}{(q_1 q_2)^2} \left\{ \left[ I^{(D-2)}_\Delta [g] - P^\mu_1 \left( I^{\nu\mu(D-2)}_\Delta q^{\nu}_1 \right) \right] t_2 + (q_1 q_2) \perp P^\mu_1 \left( I^{\mu\nu(D-2)}_\Delta q^{\nu}_2 \right) \right\}, \quad (A.3)$$

$$I^{(D-2)}_\Delta [q_2 q_2] = I^{(D-2)}_\Delta [q_1 q_1] (q_1 \leftrightarrow q_2), \quad I^{(D-2)}_\Delta [g] = \frac{1}{2\pi} I^{(D)}_\Delta,$$

where $P^\mu_1$ is the operator returning the coefficient of $q^{\mu}_{1\perp}$ in the vector integral $I^{\mu\nu(D-2)}_\Delta q^{\nu}_{i\perp}$, i.e.

$$P^\mu_1 = \frac{1}{(q_1 q_2)^2} \perp - \frac{1}{(q_1 q_2)^2} \perp (q_1 q_2)^2 \perp. \quad (A.4)$$

To find (A.3) we multiply (A.2) by $q^{\mu}_{i\perp}$ and solve the equations obtained w.r.t. $I^{(D-2)}_\Delta [q_i q_j]$. As one can see, relations (A.3) contain unpleasant $I^{(D-2)}_\Delta [g] = I^{(D)}_\Delta / (2\pi)$ expressed in radicals and polylogarithms [11]. But we notice that $I^{(D-2)}_\Delta [g]$ is finite for $D \to 4$. Therefore, in the decomposition of the $D-2$ dimensional metric tensor $g^{\mu\nu}_{\perp[D-2]}$ in eq. (A.2) into the two–dimensional metric tensor $g^{\mu\nu}_{\perp[2]}$ and the metric tensor in $D-4$ dimensions

$$g^{\mu\nu}_{\perp[D-2]} = g^{\mu\nu}_{\perp[2]} + g^{\mu\nu}_{\perp[D-4]}, \quad (A.5)$$

the last item can be neglected because it gives $O(\epsilon)$ contribution. Then, the relation

$$((q_1 q_2)^2 - t_1 t_2) g^{\mu\nu}_{\perp[2]} = (q^\mu_1 q^\nu_2 + q^\mu_2 q^\nu_1) \perp (q_1 q_2) \perp - q^\mu_1 q^\nu_1 t_2 - q^\mu_2 q^\nu_2 t_1 \quad (A.6)$$

helps us totally cancel $I^{(D-2)}_\Delta [g]$ in (A.2). Finally, we eliminate all Gram determinants $(q_1 q_2)^2 - t_1 t_2$ from denominators of $I^{(D-2)}_\Delta [q_i q_j]$ by the appropriate choice of a new $I^{(D-2)}_\Delta [g]$, see (A.1).

### B Appendix

Here we calculate in the Regge limit some of the integrals appearing in eqs. (30) and (35). Other necessary integrals can be found in Appendix C of [8] and Appendices A and B of [9]. We use the
The following denotations:

\[ c_{\Gamma} = \frac{\Gamma(1 + \epsilon) \Gamma(1 - \epsilon)^2}{\Gamma(1 - 2\epsilon)(4\pi)^{D/2}} \equiv \frac{(-s_1)(-s_2)}{(-s)} , \quad I(t) = \frac{c_{\Gamma}(-t)^{-\epsilon}}{\epsilon(1 - 2\epsilon)} , \quad (B.1) \]

\[ I_4^\mu = \int \frac{d^D r}{(2\pi)^D} \frac{r^\mu r^\nu}{r^2 p^2 s_{A} s_{B}} = p_A^\mu (2(D - 3)) I(t_1) - I(t_2) s_{t_1} + k^\mu \frac{2(D - 3)}{(D - 4) s_{t_1}} t_1 - t_2 - q_1^\mu I_{4A} , \quad (B.2) \]

where

\[ I_4 = \int \frac{d^D r}{(2\pi)^D} \frac{1}{r^2 p^2 s_{A} s_{B}} = \frac{2 c_{\Gamma}}{\epsilon s(-t_1)^{1+\epsilon}} \left( \ln \left( \frac{-s}{-s_2} \right) + \psi(1) - \psi(-\epsilon) \right) \quad (B.3) \]

may be found in [9] and \( \psi(z) = \Gamma'(z)/\Gamma(z) \).

\[ I_{4A} = \int \frac{d^D r}{(2\pi)^D} \frac{r^\mu}{r^2 p^2 n s_{A}} = p_A^\mu (2(D - 3)) I(t_1) - I(t_2) s_{t_1} + k^\mu \frac{2(D - 3)}{(D - 4) s_{t_1}} t_1 - t_2 - q_1^\mu I_{4A} , \quad (B.4) \]

\[ I_{4A}^{\mu \nu} = \int \frac{d^D r}{(2\pi)^D} \frac{r^\mu r^\nu}{r^2 p^2 n s_{A}} = (k^\mu q_1^\nu + k^\nu q_1^\mu) \frac{(D - 2)}{(D - 4) s_{t_1}} (I(t_1) - I(t_2)) + q_1^\mu q_1^\nu \frac{(D - 2) I_{4A}}{4(D - 3)} \]

\[ + \frac{k^\mu k^\nu}{s_{t_1}} \left( \frac{I(t_1) - I(t_2)}{t_1 - t_2} - 2 t_1 (I(t_1) - I(t_2)) \right) - p_A^\mu p_A^\nu \frac{2 I(t_1)}{(D - 4) s_{t_1}} \]

\[ + g^{\mu \nu} \left( \frac{I(t_1) - I(t_2)}{(D - 4) s_{t_1}} + \frac{t_1 I_{4A}}{4(3 - D)} \right) - \frac{(p_A^\mu q_1^\nu + p_A^\nu q_1^\mu) I(t_1)}{s_{t_1}} \]

\[ + (p_A^\mu k^\nu + p_A^\nu k^\mu) \frac{(D - 2) I(t_1) - 2 I(t_2)}{(D - 4) s_{t_1}^2} + \frac{t_1 I_{4A}}{2(D - 3) s_{t_1}} - \frac{(D - 6) I(s_1)}{(D - 4) s_{t_1}^2} \quad (B.5) \]

where \( I_{4A} \) may be found in [8].

\[ I_{4A} = \int \frac{d^D r}{(2\pi)^D} \frac{1}{r^2 p^2 n s_{A}} \]

\[ = \frac{c_{\Gamma} \Gamma(1 - 2\epsilon) (2 - 2 \epsilon \ln \left( \frac{(-s_1)(-t_1)}{(-s_2)}\right))}{s_{t_1} \Gamma(1 - \epsilon)^2} - \ln^2 (-t_2) + 2 \ln (-s_1) \ln (-t_1) - 2 Li_2 \left( 1 - \frac{t_2}{t_1} \right) + \mathcal{O}(\epsilon) \quad (B.6) \]

The most complicated of appearing integrals is a tensor pentabox, which we present in \( D \rightarrow 4 \).
decomposition:

\[ c^{-1}_r I^\mu_5^{\mu
u} = c^{-1}_r \int \frac{d^D r}{(2\pi)^D r^2 p^2 n A s B} \]

\[ = \frac{g^{\mu\nu}}{8 s_1 s_2} \left\{ \frac{(t_1 + t_2 + k^2)^2 - 4t_1 t_2}{t_1 t_2} \ln^2 \left( \frac{-k_{12}}{t_1 t_2} \right) + \frac{t_2 - t_1 - k^2}{t_2} \ln^2 \left( \frac{-k_{12}}{t_1 t_2} \right) \right. \]

\[ + \frac{t_1 - t_2 - k^2}{t_1} \left( \ln^2 \left( \frac{-k_{12}}{t_1 t_2} \right) - \ln^2 \left( \frac{-t_1}{t_2} \right) \right) - \frac{(t_1 - t_2)(t_1 + t_2 + k^2)}{t_1 t_2} 2 \text{Li}_2 \left( 1 - \frac{t_2}{t_1} \right) \]

\[ + \frac{\pi^2 (k^2)^2 - (t_1 - t_2)^2}{t_1 t_2} \right\} \]

\[ \frac{p_A^\mu p_B^\nu + p_B^\mu p_A^\nu}{4 s_1 s_2} \left\{ \frac{-(t_1 - t_2)^2 + k^2 (t_1 + t_2)}{t_1 t_2} \ln^2 \left( \frac{-k_{12}}{k^2} \right) \right. \]

\[ + (t_1 - t_2) \left( \frac{1}{t_2} \ln^2 \left( \frac{-k_{12}}{-t_1} \right) - \frac{1}{t_1} \ln^2 \left( \frac{-k_{12}}{-t_2} \right) + \frac{1}{t_1} \ln^2 \left( \frac{-t_1}{-t_2} \right) \right) + \]

\[ + \frac{t_1 - t_2}{t_1 t_2} 2 \text{Li}_2 \left( 1 - \frac{t_2}{t_1} \right) + \frac{\pi^2 (t_1 - t_2)^2 - k^2 (t_1 + t_2)}{3 t_1 t_2} \right\} \]

\[ + \frac{q_1^\mu q_2^\nu + q_2^\mu q_1^\nu}{4 s_1 s_2} \left\{ \frac{-4}{e (t_1 - t_2)} \ln \left( \frac{-t_1}{-t_2} \right) - \frac{1}{t_1} \ln^2 \left( \frac{-t_1}{-t_2} \right) \right. \]

\[ + \frac{2}{t_1 - t_2} \left( \ln^2 \left( t_1 - t_2 \right) - \ln^2 \left( t_2 - t_2 \right) \right) + \frac{t_1 - t_2}{t_1 t_2} 2 \text{Li}_2 \left( 1 - \frac{t_2}{t_1} \right) + \frac{\pi^2 t_1 + t_2 - k^2}{3 t_1 t_2} \right\} \]

\[ \left[ \frac{p_A^\mu q_1^\nu + p_A^\nu q_1^\mu}{s_1 t_1} \left\{ \frac{1}{e} \ln \left( \frac{-k_{12}}{s_1} \right) - \frac{1}{2} \ln^2 \left( \frac{-k_{12}}{s_1} \right) - \frac{1}{2} \ln^2 \left( \frac{-k_{12}}{k^2} \right) + \frac{1}{2} \ln^2 \left( \frac{-k_{12}}{t_1} \right) \right. \right. \]

\[ + \frac{1}{2} \ln^2 \left( s_1 - t_1 \right) - \frac{1}{2} \ln^2 \left( s_1 - t_2 \right) - \frac{1}{2} \ln^2 \left( s_1 - t_1 \right) - \frac{1}{2} \ln^2 \left( s_1 - t_2 \right) \right\} \]

\[ \left[ \frac{p_A^\mu q_2^\nu + p_A^\nu q_2^\mu}{4 s_1 s_2} \left\{ \frac{t_1 + t_2 + k^2}{t_1 t_2} \ln^2 \left( \frac{-k_{12}}{k^2} \right) + \frac{1}{t_1} \ln^2 \left( \frac{-t_1}{t_2} \right) - \frac{1}{t_1} \ln^2 \left( \frac{-k_{12}}{t_2} \right) \right. \right. \]

\[ - \frac{1}{t_2} \ln^2 \left( \frac{-k_{12}}{t_2} \right) - \frac{t_1 - t_2}{t_1 t_2} 2 \text{Li}_2 \left( 1 - \frac{t_2}{t_1} \right) - \frac{\pi^2 t_1 + t_2 - k^2}{3 t_1 t_2} \right\} \]

\[ \frac{q_1^\mu q_1^\nu}{s_1 s_2 t_1} \left\{ \frac{1}{e^2} + \frac{1}{e} \ln \left( \frac{-t_1}{t_2} \right) - \ln \left( \frac{-k_{12}}{t_1 t_2} \right) \right. \]

\[ + \frac{t_2}{2 (t_1 - t_2)} \left( \ln^2 \left( t_1 - t_2 \right) - \ln^2 \left( t_2 - t_2 \right) \right) \right\} \]

\[ (B.7) \]

where \((1 \leftrightarrow 2)\) means \(p_A \leftrightarrow -p_B\), and \(q_1 \leftrightarrow q_2\) substitutions.

To find \((B.7)\) we perform a procedure similar to the one used for calculating \(I_\Delta^{\mu\nu(D-2)}\) in Appendix [A]. Again, the coefficient of metric tensor is expressed through scalar pentabox in \(D + 2\) dimensions and,
therefore, is finite, which allows one totally cancel it using the relations analogous to \((A.3)\) and \((A.6)\). This cancellation looks much simpler in Sudakov basis: \(p_A, p_B, q_{i\perp}\), there corresponding formulae are very similar to eqs.\((A.3)\) and

\[
\gamma_{[4]}^\mu = \frac{p_A^\mu p_B^\mu + p_B^\mu p_A^\mu}{p_A p_B} + g_{[2]}^\mu, \tag{B.8}
\]

with \(g_{[2]}^\mu\) from \((A.6)\). Then, most of remaining vector integrals are known \([8,9]\), and the others may be calculated by Sudakov decomposition technique exploited in \([9]\). Fortunately, the analysis of \(I_{[4]}^\mu\) in Feynman parametrization shows that it does not contain terms proportional to \(s_i\) in fractional power, such as \(I(s_1)\) in \(I_{[4]}^\mu\). That is why our answer coincides with the one obtained in \([12]\) where the limit \(D \to 4\) was taken first. Then, we use freedom in choice of the coefficient of \(g^\mu\nu\) and choose it so (see eq.\((B.7)\)) that on the one side it eliminates all Gram determinants \((q_1 q_2)_{\perp}^2 - t_1 t_2\) in denominators, and on the other side, contains energy dependence compatible with Reggeization after its insertion in \((11)\). Moreover, in calculating \((17)\) the analogous freedom was exploited (see Appendix \(A\)). So, to perform the necessary cancellations of terms proportional to \(R\) in \((11)\) we have to choose in \((B.7)\) the coefficient of \(g^\mu\nu\) in a way consistent with corresponding coefficient of \(I_{[4]}^{\mu(D-2)}\). Because of this freedom, formula \((B.7)\) does not straightforwardly coincide with the corresponding one presented in \([13]\), there \(ln^2(s_i)\) may be found. Such \(ln^2(s_i)\) are in contradiction with Regge asymptotic \((11)\) but may be eliminated via discussed trick. At last, we present one additional vector integral useful for the calculation of \(I_{[4]}^\mu\)

\[
I_{[4]}^\mu = \int \frac{d^D r}{1(2\pi)^D} \frac{r^\mu}{r^2 s A s B} - p_A^\mu \left( \frac{s_1}{2s} I_{[4]} + \frac{2(D-3)}{(D-4)s s_1} I(s_1) \right) - p_B^\mu \left( \frac{s_1}{2s} I_{[4]} + \frac{2(D-3)}{(D-4)s s_1} I(s_2) \right) - \frac{1}{2} (q_1^\mu + q_2^\mu) I_{[4]}, \tag{B.9}
\]

where its scalar variant \(I_{[4]}\) also appears in \([35]\) and may be found in \([9]\)

\[
I_{[4]} = \int \frac{d^D r}{1(2\pi)^D} \frac{1}{r^2 s A s B} = \frac{c_T}{s (k^2)^{1+\epsilon}} \frac{2}{\epsilon} \left[ ln \left( \frac{(-s)k^2}{(-s_1)(-s_2)} \right) + \psi(-\epsilon) - \psi(1+\epsilon) \right].
\]

C Appendix

Here we present vector and tensor integrals necessary to calculate \(\Gamma^{(1)}_{A'A}\) (see \((16)\)):

\[
I_{\Box}^{\mu\nu} = \int \frac{d^D r}{1(2\pi)^D} \frac{r^\mu r^\nu}{r^2 s A s B} = - \left( p_A^\mu p_B^\nu + p_B^\mu p_A^\nu \right) \frac{2I(q)}{(D-4)s t} - \eta^\mu\nu \left( \frac{I(t) + I(s)}{(D-4)s} + \frac{t I_{\Box}}{4(D-3)} \right)
\]

\[
+ q^\mu q^\nu \left( \frac{2(D-3)}{(D-4)s t} I(t) + \frac{(D-2)}{4(D-3)} I_{\Box} \right) - \left( (p_A - p_B)^\mu q^\nu + (p_A - p_B)^\nu q^\mu \right) \frac{I(q)}{s t}
\]

\[
+ \frac{p_A^\mu p_B^\nu + p_B^\mu p_A^\nu}{s} \left( \frac{(D-2)I(t) - (D-6)I(s)}{(D-4)s} + \frac{t I_{\Box}}{2(D-3)} \right),
\]

\(19\)
\[ I^{\mu} = \int \frac{d^{D}r}{(2\pi)^D} \frac{r^{\mu}}{r^2 p_A s A s B} = -\frac{q^\mu}{2} I^\Box + \frac{2(D-3)}{D-4} \left( p_A - q - p_B \right)^\mu \frac{s t}{s t} I(t), \]  

(C.1)

where

\[ I^\Box = \int \frac{d^{D}r}{(2\pi)^D} \frac{1}{r^2 (r + q)^2 s A s B} = \frac{2 c_\Gamma}{s (-t)^{1+\epsilon}} \left\{ \ln \left( \frac{-s}{-t} \right) - 2\psi(-\epsilon) + \psi(1+\epsilon) + \psi(1) \right\}, \]  

(C.2)

was calculated in \([8]\), and for \(c_\Gamma\) see eq.(B.1).

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