On the impossibility of measuring a galvano–gravitomagnetic effect with current carrying semiconductors in a space–based experiment

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Abstract

In this paper we investigate the feasibility of a recently proposed space–based experiment aimed to detect the effect of the Earth’s gravitomagnetic field in spaceborne semiconductors carrying radial electric currents and following identical circular equatorial orbits along opposite directions. Identical voltages of gravitomagnetic origin would be induced across the orbiting semiconductors, while equal and opposite gravitoelectric larger voltages would be generated. It turns out that the deviations from the proposed idealized orbital configuration due to the unavoidable orbital injection errors would make impossible the measurement of the gravitomagnetic voltage of interest.
1 Introduction

Up to now, the only proposed or performed attempts to measure some general relativistic
gravitomagnetic effects (Ciufolini and Wheeler 1995) in the space environment of Earth by
means of artificial satellites are

- The well known Stanford GP–B mission (Everitt et al 2001), which is aimed to detect the
  gravitomagnetic precession (Schiff 1960) of the spins of four superconductor gyroscopes
  and which is scheduled to fly in the second quarter of 2004

- The LAGEOS–LAGEOS II Lense–Thirring experiment (Ciufolini et al 1998; Ciufolini
  2002), which is currently performed by analysing the orbital data of the existing LAGEOS
  and LAGEOS II geodetic laser–tracked satellites and which measures the Lense–Thirring
  gravitomagnetic precessions (Lense and Thirring 1918) of the longitude of the ascending
  node $\Omega$ and the argument of perigee $\omega$ of the orbits of the satellites

- The proposed LAGEOS–LARES experiment, which is based on the launch of a third
  LAGEOS–type satellite and which would measure the sum of the nodes of LAGEOS and
  LARES (Ciufolini 1986), or a suitable combination of the nodes and the perigees of the
  three LAGEOS satellites (Iorio et al 2002a). At present, funding is the major obstacle to
  the implementation of such mission

- The HYPER project, which is aimed to the detection of the decoherence on microscopic
  probes induced by the Earth gravitomagnetic field by means of a space–based atom in-
  terferometer. (http://www.esa.int/export/esaSC/SEM056WO4HD_index_0_m.html) At
  present, such mission is under feasibility assessment

Recently, in (Ahmedov 1999; 2003) another interesting experiment aimed to the detection of
a gravitomagnetic effect by means of a space based mission has been proposed. The analogy
between the General Theory of Relativity, in its linearized weak–field and slow–motion approxi-
imation, and electromagnetism suggests that there is a galvano-gravitomagnetic effect, which
is the gravitational analogue of the Hall effect. This new effect takes place when a current
carrying conductor is placed in a gravitomagnetic field and the conduction electrons moving
inside the conductor are deflected transversally with respect to the current flow. In connection with this galvano-gravitomagnetic effect, in (Ahmedov 1999) the possibility of using current carrying semiconductors for detecting the gravitomagnetic field of Earth by means of a pair of counter–orbiting satellites has been outlined.

In this paper we wish to realistically analyze if such an experiment could be effectively feasible. In Table 1 we quote the physical quantities of interest in the present case.

Table 1: Physical quantities of interest in MKSA system.

| Physical quantity                              | value       | unit          |
|-----------------------------------------------|-------------|---------------|
| $G$ Newtonian gravitational constant           | $6.67259 \times 10^{-11}$ | m$^3$ Kg$^{-1}$ s$^{-2}$ |
| $J$ Earth’s spin                              | $5.9 \times 10^{33}$ | Kg m$^2$ s$^{-1}$ |
| $c$ speed of light in vacuum                  | $2.9978 \times 10^8$ | m s$^{-1}$ |
| $GM$ Earth $GM$                               | $3.986 \times 10^{14}$ | m$^3$ s$^{-2}$ |
| $\delta(GM)$ error in Earth $GM$ (McCarthy 1996) | $8 \times 10^5$ | m$^3$ s$^{-2}$ |
| $r_0$ geostationary satellite orbit radius    | $4.2160 \times 10^7$ | m |
| $h$ difference of the satellites’ orbit radius | $5 \times 10^3$ | m |
| $\delta r_0$ error in $r_0$                  | $10^{-2}$ | m |
| $\delta h$ error in $h$                      | $10^{-2}$ | m |
| $m_e$ electron rest mass                      | $9.11 \times 10^{-31}$ | Kg |
| $e$ electron charge                           | $1.6 \times 10^{-19}$ | C |
| $\nu_e$ semiconductor electronic concentration | $10^{18}$ | m$^{-3}$ |
| $i_r$ radial current carried by the spaceborne semiconductor | $10^3$ | A |
| $d$ thickness of the spaceborne semiconductor | $10^{-4}$ | m |

2 The galvano–gravitomagnetic effect

According to (Ahmedov 1999; 2003), restricting ourselves to circular equatorial orbits, the gravitationally induced voltage across a spaceborne semiconductor carrying a radial current $i_r$ and thickness $d$ is given by the sum of a gravitoelectric contribution, which is sensitive to the direction of motion along the satellite orbit, and a gravitomagnetic one, which, instead, is insensitive to the direction of motion along the orbit$^1$

$$V = V_{GE} + V_{GM} = \pm \frac{nR_{gg}i_r}{d} + \frac{B_gR_{gg}i_r}{2cd}, \quad (1)$$

$^1$This situation is exactly reversed with respect to the so called gravitomagnetic clock effect (Mashhoon et al 1999; Iorio et al 2002b).
where the sign + and - refer to the opposite directions of motion of the satellites, \( n \) is the Keplerian mean motion
\[
n = (GM)^{\frac{1}{2}} r_0^{\frac{3}{2}},
\]
(2)
the gravitomagnetic field is given, in general, by
\[
B_g = \frac{2GJ}{cr^3} [\hat{J} - 3(\hat{J} \cdot \hat{r})\hat{r}],
\]
(3)
and the galvano–gravitomagnetic constant is
\[
R_{gg} = \frac{2m_e}{\nu_e e^2}.
\]
(4)
For an equatorial geostationary orbit (see \( r_0 \) in Table II) the gravitomagnetic field, which is directed along the \( z \) axis, amounts to
\[
B_g = \frac{2GJ}{cr_0^3} = 3.5 \times 10^{-8} \text{ m s}^{-2},
\]
(5)
while the galvano–gravitomagnetic constant, for a typical semiconductor (see Table II), amounts to
\[
R_{gg} = 7.117 \times 10^{-11} \text{ Kg m}^3 \text{ C}^{-2}.
\]
(6)
This implies that we have, with the values of Table II
\[
\frac{R_{gg} i_r}{2cd} \equiv H = 1.19 \times 10^{-12} \text{ Kg m} \text{ C}^{-1},
\]
(7)
\[
\frac{R_{gg} i_r}{d} \equiv K = 7.117 \times 10^{-4} \text{ Kg m}^2 \text{ s}^{-1} \text{ C}^{-1},
\]
(8)
so that, for a geostationary orbit, the gravitoelectric and gravitomagnetic voltages amount to
\[
V_{GE} \equiv Kn = 5.2 \times 10^{-8} \text{ V},
\]
(9)
\[
V_{GM} \equiv HB_g = 4.16 \times 10^{-20} \text{ V},
\]
(10)
respectively. It must be noted that an experimental sensitivity of \( 10^{-12} \text{ V} \) can be reached with great effort only by the today’s SQUID technology; a \( 10^{-15} \text{ V} \) level, or even better, is still a challenge (Vodel et al 1995). This limits could be reached, e.g., by increasing the intensity of the gravitomagnetic field \( B_g \) felt by the satellite by reducing its altitude. For a LAGEOS–type orbit, i.e. a semimajor axis \( a \) of \( 1.2270 \times 10^7 \text{ m} \), the gravitomagnetic field amounts to \( 1.4 \times 10^{-4} \text{ m s}^{-2} \).
3 The impact of the orbital injection errors

Here we wish to consider in detail the effects of deviations of the spaceborne semiconductors’ orbits from the idealized case; we will put aside the problem of shielding the Earth magnetic field which could induce much larger voltage due to the Hall effect.

Let us consider a couple of Earth artificial satellites following, in principle, identical orbits around opposite directions and denote them as (+) and (−): if this condition had been exactly fulfilled, it turns out from eq. (1) that in the sum of the gravitationally induced voltages the gravitoelectric contributions would cancel out while the gravitomagnetic ones would add up

\[ \Sigma V \equiv V^{(+)} + V^{(-)} = \Sigma V_{GE} + \Sigma V_{GM} = 0 + 2V_{GM} = 2HB_g. \] (11)

Of course, rocketry is not an exact science and the unavoidable orbital injection errors in the orbit radius would make the paths followed by the counter–orbiting satellites slightly different. This would induce an uncancelled gravitoelectric component in the sum of the voltages

\[ \Sigma V_{GE} = K[n^{(+)} - n^{(-)}], \] (12)

so that

\[ \Sigma V_{GM} = \Sigma V - \Sigma V_{GE}. \] (13)

Let us pose \( r_0^{(-)} = r_0^{(+)} + h \) with \( h/r_0 \ll 1 \); then, from eq. (2) it follows

\[ \Sigma V_{GE} = K\frac{3(GM)^{\frac{1}{2}}hr_0^{\frac{3}{2}}}{2}. \] (14)

Would the error in the gravitoelectric component \( \delta(\Sigma V_{GE}) \) be larger than the gravitomagnetic voltage of interest \( \Sigma V_{GM} \)? The error in \( \Sigma V_{GE} \) is induced by the uncertainty in the Earth \( GM \) and by the errors in the Keplerian mean motions. In turn, they are induced by the indirect effects due to \( \delta r_0 \) and \( \delta h \) and by the direct orbital perturbations on \( n \) of gravitational and non–gravitational origin. Let us consider the errors induced by \( \delta(GM) \), \( \delta r_0 \) and \( \delta h \). We have

\[
\delta(\Sigma V_{GE}) \leq K\frac{3hr_0^{-\frac{3}{2}}}{4(GM)^{\frac{1}{2}}} \delta(GM) + K\frac{15(GM)^{\frac{3}{2}}hr_0^{-\frac{3}{2}}}{4} \delta r_0 + K\frac{3(GM)^{\frac{1}{2}}r_0^{-\frac{3}{2}}}{2} \delta h, \tag{15}
\]

from which it follows

\[
\frac{\delta(\Sigma V_{GE})}{2V_{GM}} \leq \left| \frac{3c^2hr_0^{-\frac{3}{2}}}{8G^2M^{\frac{5}{2}}J} \right| \delta(GM) + \left| \frac{15M^{\frac{1}{2}}c^2hr_0^{-\frac{3}{2}}}{8G^\frac{5}{2}J} \right| \delta r_0 + \left| \frac{3M^{\frac{1}{2}}c^2r_0^{-\frac{3}{2}}}{4G^\frac{5}{2}J} \right| \delta h. \tag{16}
\]
It is interesting to note that eq. (16) depends on the Earth and satellite orbital parameters only.

According to the values of Table I, the normalized error of eq. (16) amounts to

\[
\frac{\delta (\Sigma V_{GE})}{2V_{GM}} \leq 1 \times 10^{-1} + 6.6 \times 10^{-2} + 2.21 \times 10^{2}.
\]  

(17)

It can be noted that, while the errors due to \(GM\) and \(r_0\) could be further reduced by increasing the accuracy of the orbital injection process, i.e. with a smaller \(h\), the major limiting factor is given by the uncertainty in the knowledge of the separation between the orbits of the two satellites. The value \(\delta h \sim 10^{-2}\) m is a conservative and realistic estimate; for such a value it turns out that, unfortunately, there is no hope of getting

\[
\frac{\delta (\Sigma V_{GE})}{2V_{GM}} \bigg|_{\delta h} \leq 1
\]  

(18)

by reducing the orbital radius \(r_0\). The situation does not change neither for \(\delta h \sim 10^{-3}\) m, which is a very stringent constraint, satisfied, for example, in the GRACE mission.

4 Conclusions

In this paper we have investigated the feasibility of a recently proposed space–based mission aimed to the measurement of a gravitomagnetic effect on spaceborne semiconductors carrying radial currents and following identical circular equatorial orbits along opposite directions.

It has been shown that the orbital injection errors in the orbits of the proposed counter–rotating satellites would induce a residual gravitoelectric voltage, which, instead, in an idealized situation involving exactly equal orbits would be cancelled. The associated uncertainty would be larger by one–two orders of magnitude than the gravitomagnetic voltage of interest. The main source of error would be the separation between the two orbits.

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