Abstract. Based on the principle of relativity, we find that the sufficient and necessary condition for the general covariance of a field theory actually requires more than the invariance of its local Lagrangian density. If the spacetime is not a flat one, its derivative requirement from the analysis of the parallel transportation of tensor fields over spacetime restricts the generally supposed covariance group, the group of diffeomorphisms, to the group of linear coordinate transformations. Moreover, for any of a field theory with linear equations of motion, it stipulates for a universal physical propagation speed of interaction over the spacetime manifold.

1 Introduction

General covariance, as the technical realization of the principle of general relativity, is regarded as one of the most powerful symmetries of nature. Its conceptual simplicity leads many to believe that it should be a prominent attribute of all fundamental physical laws. A physical theory is said to be covariant between two systems of coordinates, say K and K’, if all the concerned physical quantities (including the observables) expressed or measured in them are in one-to-one correspondence according to the respective laws of tensor and spinor transformations induced by the spacetime coordinate transformation connecting K and K’ and, as a result, the described physical processes take place in the similar manner in both of them if the transformation also keeps the metric field form-invariant. Under this view, there is general covariance if this property holds good to an arbitrary (differentiable) spacetime coordinate transformation

\[ x'^\mu = f^\mu(x^\nu). \]  

(1)

For a field action on a general spacetime manifold M with metric field \( g_{\mu \nu}(x) \), e.g. the action

\[ S = \int_M \sqrt{|g(x)|} d^4x L(x) = \int_M \sqrt{|g(x)|} d^4x \left[ -\frac{1}{4} (\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) - \frac{1}{c^2} J_{\mu A}(x) \right] \]  

(2)

that describes an Abelian gauge field with a linear coupling to the external source field \( J^\mu(x) \), the following transformations

\[ A'^\mu(x') = \frac{\partial x'^\mu}{\partial x^\mu} A^\mu(x), \]  

(3)
Constraints on Covariance

\[ J^\mu(x') = \frac{\partial x'^\mu}{\partial x^\nu} J^\nu(x) \]  

\[ g_{\mu'\nu'}(x') = \frac{\partial x^\mu}{\partial x'^\mu} \frac{\partial x^\nu}{\partial x'^\nu} g_{\mu\nu}(x) \]

induced by the arbitrary spacetime transformation (1) leave the field action \( S \) invariant and therefore, through the variation with respect to \( A^\mu(x) \), the local equations of motion take on the same form in all systems of coordinates. In this way the related physical processes involving \( A^\mu(x) \) and \( J^\mu(x) \) are thought to take place similarly in different systems of coordinates, given that the initial, boundary data for the field equations should be the same and the metric field should be form-invariant in these systems. The invariance of the field action or the invariance of Lagrangian density under the spacetime coordinate transformation is presently used as the sufficient condition for the covariance of any field theory, be its equations of motion linear or nonlinear.

It has been long recognized that general covariance for a physical theory is a mathematical rather than a physical requirement (E. Kretschmann 1917, V. Fock 1959, C. W. Misner et al. 1975, M. Friedman 1983); nevertheless, it stipulates for the way in which the local quantities should be mathematically correlated, such as those in Eqs. (3)-(5). In the more recent literature general covariance group is identified with the group of dffermorphisms of spacetime manifolds on which the fundamental physical laws can be reduced to equations involving only local differential operators of finite order. Various theories involving gravitational effect have this group as the largest one preserving the form-invariance of local field equations, while specify some subgroup of it as the group of automorphisms to preserve certain geometrical structures, called absolute elements in (A. Trautman 1966, J. Anderson 1967, A. Trautman 1973), invariant. The local quantities in a physical theory must be in one-to-one correspondence according to the laws of tensor and spinor transformations, as long as the theory is a covariant one; once the measures or expressions of them are determined in one coordinate system, those in all other physically admissible coordinates are also determined if the concerned spacetime transformations are specified. Obviously it is the consequence of the validity of the principle of general relativity or the principle of general invariance, which requires that all the admissible systems of coordinate be equal in the description of nature.

With a closer look at the original meaning of the principle of relativity, however, we find that there is a stronger condition on the covariance of a field theory established on general spacetime manifold, i.e. that for the consistency of description of the physical processes in different coordinate systems, and it limits the arbitrary spacetime transformation group (1) to its much smaller subgroup, which is picked out to meet the requirement for consistency. Hereafter, for a distinction from ‘physically covariant’ or ‘covariant’, we will call the field equations of a field action ‘form covariant’ if its Lagrangian density satisfies the invariance under spacetime coordinate transformation. In this letter we first present a brief discussion on our sufficient and necessary condition for the covariance of a field theory, then apply it to the study of the covariance of the field described by Eq. (2) under arbitrary spacetime transformation (1), and finally we use it to study the covariance of the field theory under linear transformations in homogeneous spacetime. The advantage of choosing action (2) as the example is that the linearity of its field equations allows us to have a better understanding of their solution structure.
2 Sufficient and necessary condition for covariance

The equivalence of systems of coordinate in describing a covariant field theory is the essential requirement raised by the principle of relativity. Here the term ‘equivalence’ has a two-fold meaning: 1) The definite correlation of the local physical quantities in terms of the transformations of tensors (spinors) between any couple of systems of coordinate in the equivalent class; 2) The arbitrariness in selecting a coordinate among the equivalent ones for the construction of solution to these field equations. Only after the two points are met, can system K with the geometrical structure in it described by $g_{\mu\nu}(x)$, $\Gamma^\tau_{\mu\nu}(x)$, etc. of M, be exactly equivalent to system K’, where the geometrical structure is characterized by $g'_{\mu'\nu'}(x')$, $\Gamma'^\tau'_{\mu'\nu'}(x')$, etc. of M’ (the differmorphic image of M), and the field theory under study respectively by the observers in K and K’ be truly covariant. Of course the realization of the completely equivalent description of the fields (at the classical level) among different systems of coordinate also requires that the boundary data and initial data for the field equations should be preserved in the proper way under the concerned transformations, but here we only consider the boundary and time origin at infinity and therefore their influence are negligible.

For clarification we construct the following diagram to illustrate the equivalence of system K and K’ in the study of a covariant field theory (here the action (2) is taken as example).

\[
\begin{align*}
g_{\mu\nu}(x), J^\mu(x) & \rightarrow g'_{\mu'\nu'}(x'), J'^\mu(x') \\
\downarrow & \downarrow \\
A^\mu(x) & \rightarrow A'^\mu(x')
\end{align*}
\]

In this diagram the horizontal arrows represent the transformations of the related fields from system K to K’, and the perpendicular arrows the construction of the solution to field equations with the source field and geometrical structure in K and K’, respectively. Thus the equivalence of K and K’ leads us to the sufficient and necessary condition for the covariance of a field theory: a field theory is covariant between two systems of coordinate, if and only if such kind of diagram for it commutes. The ‘only if’ part of the condition is more nontrivial; it guarantees the compatibility of the transformations for the geometrical structure of the spacetime manifold with those for the physical processes taking place on the spacetime manifold.

For a better understanding of the condition we imagine such a picture: In system K established on M, the physical fields $A^\mu(x), J^\mu(x)$ in action (2) are determined by an observer through measurement (and solving the field equations) and, after the specification of the exact form of spacetime coordinates transformation between K and K’, they are transformed according to Eqs. (3) and (4) to $A'^1_\mu(x')$, $J'^\mu(x')$ in K’, where another observer solves the form covariant field equations only with the transformed $J'^\mu(x')$ and $g'_{\mu'\nu'}(x')$. Unless the diagram is commutative, the solution $A'^2_\mu(x')$ thus constructed in K’, which is compatible with the geometrical structure described by $g'_{\mu'\nu'}(x')$ there, will not be equal to $A'^1_\mu(x')$, and therefore an awkward situation will arise from the ambiguity of $A'^\mu(x')$. At first sight this requirement seems redundant since any form covariant equation is expressed in the form: a tensor (spinor) = 0, which is true to any coordinate system, and indicates that $A'^1_\mu(x')$ transformed from $A^\mu(x)$ according to Eq. (3) should satisfy the field equations in K’ too. However, we will see later that the undesired non-commutation of the above diagram does arise from the non-local geometrical structure we have to take into account in the construction of a solution involving the propagation of physical fields
from one point on spacetime manifold to another, which is governed by the natural geometry of the spacetime. (A. Logunov 1990, p.173).

3 Application to an arbitrary spacetime coordinate transformation

For simplicity we study $A^\mu(x)$ produced by a point particle moving in the region of an arbitrary spacetime manifold without the presence of other matter and radiation. If the radiation of its own is week enough, it can be regarded as moving over the spacetime background with almost identically vanishing Ricci tensor field $R_{\mu\nu}$ (from field equations $R_{\mu\nu} = \frac{8\pi G_N}{c^4} (T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T)$). The source field in this case can be expressed as (A. Barut 1980, p.165) (N. Straumann 1984, p.252)

$$J^\mu(x) = \frac{ec}{\sqrt{|g(x)|}} \int_{-\infty}^{+\infty} ds \frac{dx_0(s)^\mu}{ds} \delta(x - x_0(s)),$$

where $dx_0(s)^\mu/ds$ is the four-velocity of the particle, $s$ the proper time, and $ec$ the coupling constant. Throughout our discussion Lorentz gauge (in the sense of covariant derivative) may be used. The linearity of the equations of motion derived from action (2) determines definitely a light cone structure (a hypersurface on spacetime manifold) at each point on M. The point particle in Eq. (6) contributes only to field $A^\mu(x)$ over and inside its forward light cone (the propagation of $A^\mu(x)$ along the surface of light cone and its scattering by the interaction with curvature of spacetime to the inside of the light cone (H. Stephani 1982, p.72)), because of the retardation effect caused by the limited propagation speed of interaction. Although the exact equations of light cone on a general spacetime manifold is hard to know, we are sure of their existence by constructing the local light cones with locally constant $g_{\mu\nu}(x)$ (in the form of local world line element) and gluing them together to form a global one. More generally, if the source field $J^\mu(x)$ is distributed on the whole spacetime manifold, the contribution to $A^\mu(x)$ comes from the integral inside and over the backward light cone of point $x$.

On the subset of a spacetime manifold where $R_{\mu\nu} \approx 0$, the physical considerations require that the propagation of $A^\mu(x)$ from the source particle to another spacetime point should be along the null geodesics connecting them. In a specific system of reference, say K, an observer sees that the field $A^\mu(x)$ at the intersecting point, $x^\mu$, of his/her world line with the forward light cone of an arbitrary point on the world line of the source particle ($x_0^\mu(s)$) is proportional to vector $n^\mu(x)$ that is produced by the parallel translation of the four-velocity vector

$$n^\mu(x) = (dx_0^\mu/ds) / \left( \sqrt{g_{\mu\nu}(x_0)(dx_0^\mu/ds)(dx_0^\nu/ds)} \right) = dx_0^\mu/ds$$

along the null geodesics connecting the two points:

$$A^\mu(x) = C(x, x_0)n^\mu(x) = C(x, x_0) \left[ n^\mu(x_0) - \int_{x_0}^{x} \Gamma^\mu_{\nu\tau}(z)n^\nu(z)dz^\tau \right],$$
where the integral is along the null geodesic, and \( C(x, x_0) \) is a scalar function which reduces to

\[
\frac{e \sqrt{\eta_{\mu \nu} (dx^\mu_0/ds) \eta_{\nu \tau} (dx^\tau_0/ds)}}{4\pi (x - x_0)^\tau (dx_0^\tau/ds)} = \frac{e}{4\pi (x - x_0)^\tau (dx_0^\tau/ds)}
\]

in Minkowski spacetime. One component of the four-velocity of the source particle contributes to the others of \( A^\mu(x) \) after the parallel displacement along the null geodesics, since the field equations on a general spacetime manifold cannot be split into the uncoupled ones with respect to the components of the vector fields. Particularly, if there is more than one geodesic connecting the two points (e.g., they are conjugate points of each other), \( n^\mu(x) \) is a sum of the parallel translations of \( n^\mu(x_0) \) along all geodesics.

In the light of the above result, the covariance of the produced field \( A^\mu(x) \) on the forward light cone of its source particle can be studied through the covariance of the field \( n^\mu(x) \) produced by the parallel transportation of \( n^\mu(x_0) \) along the null geodesics on the forward light cone of \( x_0(s) \); the covariance of \( n^\mu(x) \) is the necessary condition for the covariance of \( A^\mu(x) \) on the forward light cone of its source particle. The necessary condition for the covariance of the field theory described by Eq. (2) is then specified to the commutation of the following diagram:

\[
\begin{array}{c}
n^\mu(x_0) \rightarrow n^\mu(x'_0) \\
\downarrow \quad \downarrow \\
n^\mu(x) \rightarrow n^\mu(x')
\end{array}
\]

(10)

Here the perpendicular arrows represent the parallel displacement of vectors along the null geodesics to the forward lightcone of \( x'_0 \) (resp. \( x_0 \)) in \( K \) (resp. \( K' \)). If we require that \( g_{\mu \nu}(x) \) transform as what is given in the first diagram, the null geodesics connecting \( x^\mu \) and \( x'_0 \) are always transformed piecewise through the transformation of the local light cone to those connecting \( x'^\mu \) and \( x'_0 \), i.e., a null geodesic is generally covariant. The commutation of the diagram means that the field \( n^\mu(x) \) produced by the parallel displacement of \( n^\mu(x_0) \) along the above-mentioned null geodesics will be transformed to the field \( n^\mu(x') \) by the parallel displacement of \( n^\mu(x'_0) \) along the corresponding null geodesics in \( K' \), so the two following results

\[
n^\mu_1(x') = \left( \frac{\partial x'^\mu}{\partial x^\mu} \right) n^\mu(x) = \left( \frac{\partial x'^\mu}{\partial x^\mu} \right) (n^\mu(x_0) - \int_{x_0}^x \Gamma^\mu_{\nu \tau}(z)n^\nu(z)dz^\tau), \quad (11)
\]

and

\[
n^\mu_2(x') = n^\mu(x'_0) - \int_{x'_0}^{x'} \Gamma^\mu_{\nu \tau}(z') n^\nu(z')dz'^\tau
\]

\[
= \left( \frac{\partial x'^\mu_0}{\partial x^\mu_0} \right) n^\mu(x_0) - \int_{x_0}^x \left( \frac{\partial z'^\mu}{\partial z^\mu_0} \right) \Gamma^\mu_{\nu \tau}(z)n^\nu(z)dz^\tau + \int_{x_0}^x \left( \frac{\partial^2 z'^\mu}{\partial z^\nu \partial z^\tau} \right) n^\nu(z)dz^\tau \quad (13)
\]

must be equal.

Two different cases we should consider: 1) The curvature tensor field \( R^\mu_{\nu \sigma \tau}(x) \) is non-vanishing in \( K \). Obviously, the two results are not equal for a nonlinear transformation, with the different transformation Jacobians at different points and an additional summand in Eq. (11)
Thus, on a general spacetime manifold, the only candidates that preserve the covariance of the field theory described by Eq. (2) are linear transformation groups. 2) The curvature tensor field $R_{\mu\nu\rho\sigma}(x)$ vanishes identically in $K$. Because $n^\mu$ in this case remains constant under parallel displacement along any of a curve in $K$, i.e. the solution of the partial differential equation,

$$\partial_\mu n^\nu(x) = -\Gamma^\nu_{\mu\tau}(x)n^\tau(x),$$

exists [P. Bergmann 1947, p.167], $n^\mu_1(x')$ and $n^\mu_2(x')$ will be absolutely equal under an arbitrary spacetime transformation (1).

For completeness we can add the gravitational field term

$$L_G = \frac{c^4}{16\pi G_N} g_{\mu\nu} R^{\mu\nu}$$

with

$$R_{\mu\nu} = \partial_\alpha \Gamma^\alpha_{\mu\nu} - \partial_\nu \Gamma^\alpha_{\mu\alpha} + \Gamma^\beta_{\alpha\nu} \Gamma^\alpha_{\mu\beta} - \Gamma^\beta_{\alpha\nu} \Gamma^\alpha_{\mu\beta}$$

to the Lagrangian density in Eq. (2). Because of the presence of the nonlinear self-interaction term the structure of the solution to the field equations is beyond our knowledge, so, except for a few special cases (e.g. the propagation of gravitation wave discussed in terms of its wave front equation in a harmonic coordinate system given in [V. Fock 1959, p.175]), we are not sure whether or not gravitational wave propagates along the null geodesics on the forward light cone of its source. A linear approximation of the theory can however be realized if we consider only the gravitational perturbation $\epsilon^{\mu\nu}(x)$ over the spacetime background $h^{\mu\nu}(x)$:

$$g^{\mu\nu}(x) = h^{\mu\nu}(x) + \epsilon^{\mu\nu}(x),$$

then the covariance of $\epsilon^{\mu\nu}(x)$ can be discussed in the similar way. Together with the our previous discussion on the covariance of $A^{\mu}(x)$ we conclude that the form covariance of the field equations is not sufficient to guarantee the physical covariance of the fields. In other words, it doesn’t generally enable the physical fields (classical solutions to the actions of the fundamental interactions) expressed or measured in different coordinate systems to be in one-to-one correspondence through the tensor transformations induced by the arbitrary spacetime transformation (1) connecting them, since the parallel transportation of tensor fields should be an absolute element in a field theory.

The fact that the physically admissible spacetime coordinate transformations in a curved spacetime (in the sense of maintenance of physical covariance) are only those of linear transformation groups implies the existence of some preferred systems for the study of the physical phenomena in large spacetime scale. For a possible realization of physical covariance under the arbitrary spacetime coordinate transformation (1), the structure of spacetime in which all physical processes take place must be that of an absolutely flat one—the curvature tensor field $R^{\mu\nu\rho\sigma}(x)$ degenerates identically over the spacetime manifold. So physical covariance under the

1 The transformation of $\Gamma^\mu_{\nu\rho}$ in Eq. (11) is defined to guarantee the commutation of the second diagram under arbitrary spacetime transformation (1) in a small neighborhood of $x_0$ (see [C. Møller 1952, p.276]). But it depends on considering the Taylor expansion of the transformation Jacobian (at $x$) around $x_0$ only to the first order and the fact $dx^{\mu'} = \frac{\partial x^{\mu'}}{\partial x^{\mu}} dx^{\mu}$, which are invalid for a finite distance from $x_0$. 
arbitrary spacetime coordinate transformation (1) is connected with the flatness of spacetime (in curved spacetime the covariance of the field theories under arbitrary spacetime coordinate transformation (1) in a small neighborhood of every spacetime point can be regarded as the result of the local Minkowskian property from the principle of equivalence), and it is necessary, as it was originally pointed out by V. A. Fock (V. Fock 1959, p.370), to distinguish relativity in the sense of uniformity of spacetime from relativity in the sense of the possibility of using arbitrary reference systems. In the theory of relativity with the latter sense, the term ‘motion’ is defined by the Jacobian matrix \( \left( \frac{\partial x'_{\mu}}{\partial x^\mu} \right) \) between two systems of reference (M. Sachs 1982, p.7). As a matter of fact, however, the mathematical expression for a spacetime coordinate transformation between two arbitrary systems of reference established on general spacetime manifolds can never be determined. In one of the systems an observer only knows (through measurement) the trajectory of the other expressed by \( x^i(t) \) (i=1,2,3), where \( t \) is the coordinate time. The additional symmetry, the existence of which entails certain degree of homogeneity for spacetime structure, is indispensible for the specification of the exact form of spacetime transformation between two reference systems. For example, the establishment of Lorentz transformation between inertial reference systems actually needs the invariance of Minkowski metric or the form invariance of spacetime interval (an absolute element in the theory) as such additional symmetry for the spacetime structure. In (C. Møller 1952, p.121), a procedure is given to determine the form of the spacetime coordinate transformation between two systems in an arbitrary rectilinear motion relative to each other in Minkowski spacetime, but it doesn’t hold good on a more general spacetime manifold for the lack of the additional symmetry. Again, from practical point of view, it is concluded that only homogeneous spacetimes are relevant to the physical covariance under the arbitrary spacetime coordinate transformation (1).

4 Application to the linear transformation \( \Lambda^\mu_{\mu'} \) in homogeneous spacetime (constant metric fields)

This is a situation where such important tool as Green’s function method is applicable in dealing with the linear field equations. The field \( A^\mu(x) \) and its source field \( J^\mu(x) \) are then connected by the Green’s function (propagator):

\[
A^\mu(x) = \int_M d^4y \, G(x,y)J^\mu(y).
\]

(18)

Hence our sufficient and necessary condition can be expressed by the first diagram, with the perpendicular arrows representing the combination of the construction of Green’s function and the performance of the integral (in Eq. (18)) in system K and K’, respectively. The requirement for the commutation of the diagram is therefore translated to the following relation:

\[
det(\Lambda^\mu_{\mu'})G^\mu(x',y') = G(x,y).
\]

(19)

With the help of the verbiage fields \( e^\mu_a \), which connect \( g_{\mu\nu} \) and Minkowski metric \( \eta_{ab} \) as

\[
\eta_{ab} = e^\mu_a e^\nu_b g_{\mu\nu},
\]

(20)
we can construct through Fourier transformation the Green’s function (propagator) in system K (resp. K’):

\[ G(x, y) = \det(e^a_\mu) \int \frac{(dk_a)}{(2\pi)^4} \frac{1}{\eta^{ab}k ak b} e^{ik_a e^a_\mu(x^\mu - y^\mu)} \]

\[ = \det(e^a_\mu) \frac{1}{4\pi R} \theta(x^0 - y^0)[\delta(e^0_\mu(x^\mu - y^\mu) + R) + \delta(e^0_\mu(x^\mu - y^\mu) - R)], \]

where \( R^2 = \sum_{i=1}^{3} (e_i^0(x^\mu - y^\mu))^2 \), and only the second term contributes because we take \( R > 0 \).

From the singularity of the \( \delta \) function in Eq. (22) we directly obtain the light cone equation

\[ g_{00}(wt)^2 + 2g_{0i}(wt)x^i + g_{ij}x^i x^j = 0, \]

with which the physical propagation speed of interaction \( w \) (in the sense of the isotropic one in [A. Logunov 1990, p.82] in the specific system of coordinate can be obtained. The covariance of Green’s function requires that \( w \) should remain invariant under linear spacetime coordinate transformations. The same argument applies to the situation on Riemannian manifold too, if we consider the covariance of the equation of local lightcone derived from the local limit of Green’s function. Going back to the real physical world, Minkowski spacetime, we have the universal propagation speed \( c \) for all kinds of long-range interactions because of the uniqueness of energy-momentum relation (the conjugate of Eq. (23) in momentum space) for the massive particles; otherwise in system K’ one of them will be

\[ w' = \left( \begin{array}{c}
\frac{w_1 - v}{1 - w_1 v/c^2}, \\
\frac{w_2 \sqrt{1 - v^2/c^2}}{1 - w_1 v/c^2}, \\
\frac{w_3 \sqrt{1 - v^2/c^2}}{1 - w_1 v/c^2}
\end{array} \right), \]

and it breaks the covariance of the Green’s function required by Eq. (19). Thus the invariance of light speed is indispensable as one of the two postulates for the special theory of relativity, since it starts with an attempt at a covariant electromagnetic field theory between inertial systems of reference. It should be specially emphasized that, because the integral range is over the whole spacetime manifold in Eq. (18), the invariant propagation speed must exist at every point in space and keep invariant with the progress of time. This requirement imposes the other constraint on the covariance of the field theory described by Eq. (2).

Finally, we obtain from the above discussion three physical results related to the propagation speed of interaction in Minkowski spacetime:

1) The invariance of propagation speed of interaction under spacetime transformation will further limit the form of admissible linear transformation groups preserving the covariance of...
the field theory described by Eq. (2). In the case of 1+1 dimension, for example, the form of such linear transformations:

\[ ct' = a_{00}ct + a_{01}x^1, \]
\[ x^1' = a_{10}ct + a_{11}x^1, \]

should be given the following constraint:

\[ a_{01} = a_{10}, \]
\[ a_{00} = a_{11}, \]
\[ a_{00}^2 - a_{01}^2 = 1. \]

A proper Lorentz transformation obviously falls into the class.

2) To make the situation more complicated we put some refractive media, which gives rise to a refractive index \( n \neq 1 \), into the spacetime manifold. Certainly the density of it is low enough \( (T^{\mu\nu} \ll 1) \) not to influence the spacetime metric too much, but the electromagnetic propagation speed \( c/n \) is no longer invariant under coordinate transformation in where the media is distributed. The above discussion tells us that any small patch of such media can destroy the covariance of electromagnetic field, even if the field equations are still form covariant on the whole rest of the spacetime manifold. Thus the covariance of electromagnetic field is only an approximate symmetry in reality.

3) If the mass term \( -\frac{1}{2}m^2A^\mu A_\mu \) is added to the Lagrangian density in Eq. (2), then the contribution to the field \( A^\mu(x) \) at point \( x \) in Minkowski spacetime comes from the source field within the backward light cone of the point. With the time reversal symmetry of light cone, the covariance of the field requires that the velocity of the massive quanta of \( A^\mu(x) \) emitted by its source range from \( -c \) to \( c \), so the four-momentum of them will diverge. Independently of the requirement for gauge symmetry, therefore, a massive long-range intermediate Boson field is forbidden in nature by the requirement for covariance.

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References

J. L. Anderson (1967). *Principle of Relativity Physics*, Academic Press, New York.

A. O. Barut (1980) *Electrodynamics and Classical Theory of Fields, Particles*, Dover.

P. G. Bergmann (1947) *An Introduction to The Theory of Relativity*, Prentice-Hall, New York.

V. A. Fock (1959) *The Theory of Space, Time and Gravitation*, Pergamon, London.

M. Friedman (1983) *Foundations of Space-time Theories*, Princeton University Press, Princeton.

E. Kretschmann (1917) Ann. Physik 53.
A. Logunov (1990) *Lectures in Relativity and Gravitation—A modern look*, Nauka Publishers, Moscow; Pergamon, London.

C. W. Misner, K. S. Thorne and J. A. Wheeler (1975) *Gravitation*, Freeman, San Francisco.

C. Møller (1952) *The Theory of Relativity*, Oxford, London.

M. Sachs (1982), *General Relativity and Matter*, D. Reidal, Dordrecht, Holland.

H. Stephani (1982), *General Relativity: An introduction to the theory of gravitational field*, Cambridge University Press, Cambridge.

N. Straumann (1984), *General Relativity and Relativistic Astrophysics*, Springer-Verlag, New York.

A. Trautman (1966), Uspekhi Fiz. Nauk 89.

A. Trautman (1973), *Theory of Gravitation* In *The Physicist’s Conception of Nature*, (J. Mehra, Ed.), p. 179, Dordrecht, Boston.