Several classes of neutrino seesaw mass models, which can naturally account for hierarchical neutrino masses and the bi-large pattern of neutrino mixing, are constructed from a bottom-up perspective based on the idea of decoupling of one right-chiral neutrino from the seesaw mechanism. The interplay between the predictions for the lepton-flavour violation in the MSSM with universal soft masses and for leptogenesis is studied. In particular, classes of neutrino mass models in which successful low-temperature leptogenesis implies $\text{BR}(\mu \rightarrow e\gamma)$ potentially observable in upcoming experiments are identified.

1 Introduction

The discovery of neutrino flavour transitions, whose only explanation are neutrino oscillations, necessarily implies that neutrinos have masses, much smaller than the masses of other elementary particles, and a bi-large pattern of mixing, very different from that of quarks. These results cannot be accommodated in the Standard Model of particle physics, in which, by construction, the neutrinos are massless and each of the partial lepton numbers $L_e$, $L_\mu$ and $L_\tau$ is separately conserved. Hence, neutrino experiments provide unquestionable empirical evidence for the necessity of extending the Standard Model. This necessity is supported by a serious theoretical argument that the explanation of the huge hierarchy between the energy scale of weak interactions and the Planck scale (or, in general, a high scale of new physics) requires an enormous fine-tuning between the tree-level parameters of the Standard Model and quantum corrections. This hierarchy problem can be solved by introducing supersymmetry, softly broken by masses of new particles of order of 1 TeV.

The smallness of neutrino masses can, in turn, be elegantly explained by the seesaw mechanism, postulating that they are generated in Yukawa interactions, involving an exchange of very heavy Majorana fermions, the right-chiral neutrinos. Their interactions can be described by the fol-
lowing Lagrangian:

$$\Delta \mathcal{L} = -\epsilon_{ij} H_i N_K \ell_{A} \ell_{A^j} - \frac{1}{2} M_{KL} N_K N_L + \text{H.c.},$$

where $H$ and $\ell_A$ are Higgs and lepton doublets, $N_K, K = 1, 2, 3$ denote the right-chiral neutrinos, $M_{KL}$ is their Majorana mass matrix and $Y^K_{\nu A}$ is the matrix of the neutrino Yukawa couplings. Integrating out the heavy fields $N_K$ provides (after the electroweak symmetry breaking) the remaining neutrinos with small Majorana masses:

$$m_{\nu} = -\langle H \rangle^2 Y^T_{\nu} M^{-1} Y_{\nu},$$

where $m_{\nu}$ has eigenvalues $m_{\nu_1}, m_{\nu_2}$ and $m_{\nu_3}$, and $\langle H \rangle$ denotes the vacuum expectation value of the relevant Higgs field.

The seesaw mechanism introduces new parameters to the Standard Model (or to its supersymmetric extension, the MSSM), which cannot be fully determined even with infinitely accurate measurements of neutrino masses, mixing angles and $CP$ properties. One can attempt to reduce the number of unconstrained parameters by building appropriate neutrino mass models, e.g. by postulating that the hierarchical neutrino masses and the bi-large pattern of neutrino mixing is generated without fine-tuning in the seesaw mechanism. This can be achieved by assuming that one right-chiral neutrino gives very small contributions to the mass matrix of the light neutrinos, i.e. it decouples from the seesaw mechanism. In our analysis, we have used the Casas-Ibarra parametrization of the neutrino Yukawa matrix, which directly links the neutrino observables to the parameters of the seesaw mechanism, thereby allowing to use the observables as input parameters for building neutrino mass models.

$$Y^A_{\nu B} = \frac{M_A^{1/2}}{\langle H \rangle} \sum_D \Omega_{AD} m_{\nu_D}^{1/2} U_{BD}^* \Omega^T \Omega = 1,$$

where $M_A$ is the mass of $N_A$ and $U_{\nu}$ is the neutrino mixing matrix. This parametrization takes a particularly simple form in neutrino seesaw mass models with one right-chiral neutrino decoupled:

$$\Omega \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & z & p \\ 0 & \mp p & \pm z \end{pmatrix}, \quad \begin{pmatrix} 0 & z & p \\ 0 & 1 & 0 \\ \mp p & \pm z \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 & z & p \\ 0 & 1 & 0 \\ \mp p & \pm z \end{pmatrix},$$

for $N_1, N_2$ and $N_3$ decoupled, respectively.

Due to the Majorana nature of the right-chiral neutrinos, the total lepton number $L = L_e + L_\mu + L_\tau$ is violated in their interactions. At presently accessible energies, the only experimental sign of $L$ violation would be the observation of neutrinoless double beta decay, since the right-chiral neutrinos are probably too heavy to be produced in any terrestrial experiments. They could have been, however, fairly abundant in the early Universe, if it was sufficiently hot, and their $CP$ violating decays could have produced a lepton asymmetry, subsequently transformed, thanks to sphaleron transitions, into the baryon asymmetry observed today (leptogenesis).

In the MSSM embedded in supergravity, there is, however, an upper bound on the temperature of the Universe after inflation, the so-called reheating temperature $T_{RH}$. If $T_{RH}$ is too high, too many gravitinos are produced and they make up too much dark matter or their late decays destroy the observationally confirmed predictions of the primordial nucleosynthesis. Depending on the mass spectra of supersymmetric particles, the bound on the reheating temperature can be strong ($T_{RH} \lesssim 10^{7} - 8$ GeV) or weak ($T_{RH} \lesssim 10^{9}$ GeV).
Table 1: Current and expected sensitivities (90% CF) of experiments searching for LFV.

| process                  | experimental constraints | current experiment | planned sensitivity | planned year | planned experiment |
|--------------------------|--------------------------|-------------------|---------------------|--------------|-------------------|
| $\mu^+ \rightarrow e^+ \gamma$ |                           | $1.2 \times 10^{-11}$ | MEGA-LAMPF[11]     | $10^{-13}$    | 2006 MEG[12]      |
| $\mu^- Ti \rightarrow e^- Ti$ |                           | $6.1 \times 10^{-13}$ | SINDRUM II[13]    | $10^{-13}$    | 2009 MECO[12]     |
| $\mu^- Pb \rightarrow e^- Pb$ |                           | $4.6 \times 10^{-11}$ | SINDRUM II[14]    | $10^{-13}$    | 2009 MECO[12]     |
| $\mu^- Au \rightarrow e^- Au$ |                           | $8 \times 10^{-13}$   | SINDRUM II[15]    | $2 \times 10^{-17}$ | 2009 MECO[12]     |
| $\tau \rightarrow e \gamma$ |                           | $3.7 \times 10^{-7}$   | Belle[16]         | $10^{-9}$    | 2007 LHC[17]      |
| $\tau \rightarrow \mu \gamma$ |                           | $6.8 \times 10^{-8}$   | BABAR[18]         | $10^{-9}$    | 2007 LHC[17]      |

It is interesting to study a possible interplay between lepton-flavour violating (LFV) decays of the charged leptons in the MSSM and the violation of the total lepton number in leptogenesis. As we shall show, in some neutrino mass models, the requirement of successful leptogenesis consistent with the gravitino bounds on the reheating temperature can give some predictions for the rates of the LFV decays of charged leptons and reduce the ambiguity in determining the neutrino couplings from the low-energy experiments.

2 LFV radiative decays of the charged leptons

In the MSSM extended with the seesaw mechanism, even if the mechanism of supersymmetry breaking does not introduce by itself any flavour violation (e.g. as in mSUGRA with sfermion mass matrices $\tilde{m}_f^2 = m_0^2 1$, gaugino masses $M_{1/2}$ and the three-scalar couplings proportional to the relevant Yukawa matrices, $A_f = A_0 Y_f$, at the scale of Grand Unification), there are flavour-violating quantum corrections to the mass matrix of sleptons, the scalar partners of leptons. These corrections,

$$\left(\tilde{m}_L^2\right)_{AB} \approx -\frac{3m_0^2 + A_0^2}{8\pi^2} \sum_{K=1}^3 Y^K_{\nu A} Y^K_{\nu B} \ln \left(\frac{M_{GUT}}{M_K}\right),$$

(5)

can lead to lepton-flavour violating (LFV) radiative decays of charged leptons, e.g. $\mu \rightarrow e \gamma$, $\tau \rightarrow e \gamma$ and $\tau \rightarrow \mu \gamma$. Assuming, for simplicity, that there is a common mass scale $M$ of the right-chiral neutrinos and that there are no large cancellations in the seesaw formula, Eq. 2, one can estimate the LFV rates. Neglecting $O(1)$ factors, as well as the logarithmic factor in Eq. 5, one obtains:

$$\frac{\text{BR}(\ell_A \rightarrow \ell_B \gamma)}{\text{BR}(\ell_A \rightarrow \ell_B \nu_B \nu_A)} \approx 10^{-7} \times \left(\frac{100 \text{ GeV}}{m_{\text{SUSY}}}\right)^4 \left(\frac{\tan \beta}{10}\right)^2 \left(\frac{m_{\nu_3}}{5 \times 10^{-2} \text{ eV}}\right)^2 \left(\frac{M}{10^{14} \text{ GeV}}\right)^2 .$$

(6)

No such decays have been observed so far, but the upcoming experiments aim at searching for these processes with high accuracy. The current experimental bounds for the LFV radiative decays and related $\mu \rightarrow e$ conversion in nuclei, as well as the sensitivities of planned experiments are presented in Table 1.

The specific predictions for the LFV decays (as well as for leptogenesis) depend crucially on the parameters of the seesaw model which are not constrained by neutrino experiments.

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*a*For some values of $m_0$, $M_{1/2}$, the approximation in Eq. 5 does not work well and corrections beyond the leading logarithm have to be included.
There is a very wide range of predictions for the rates of the LFV decays, which, as follows from Eq. 6, depend on the masses of the right-chiral neutrinos, on the textures of the neutrino Yukawa couplings and on the parameters of the MSSM. In principle, more specific predictions can be obtained under an additional assumption of successful leptogenesis, possibly consistent with the gravitino bounds.

3 Leptogenesis and LFV decays

If the right-chiral neutrinos have hierarchical masses, then the lepton asymmetry produced in leptogenesis is directly proportional to the $CP$ asymmetry $\varepsilon_1$ in the decays of the lightest right-chiral neutrino. Under reasonable assumptions about the neutrino Yukawa couplings, this $CP$ asymmetry must satisfy the Davidson-Ibarra bound

$$|\varepsilon_1| \leq \varepsilon_1^{DI} = \frac{3}{8\pi} \frac{M_1(m_{\nu_3} - m_{\nu_1})}{\langle H \rangle^2}$$

Successful leptogenesis requires sufficiently large $\varepsilon_1$, which, as follows from Eq. 7, is possible if the mass $M_1$ of the lightest right-chiral neutrino is sufficiently large. Interestingly, the case of maximally efficient leptogenesis corresponds to the decoupling of the lightest right-chiral neutrino and $T_{RH} \gtrsim M_1 \gtrsim 10^9$ GeV. As regards models with one of the heavier right-chiral neutrino decoupled, successful leptogenesis generically requires $5T_{RH} > M_1 \gtrsim 10^{11}$ GeV, due to stronger washout of the generated lepton asymmetry and a slight suppression of the maximal $CP$ asymmetry. The first case is marginally consistent with the weak gravitino bound, but there are no additional constraints on the predictions for LFV decays. In the second case, the requirement of successful thermal leptogenesis implies that $M_1$ is so large that $\mu \rightarrow e\gamma$ should be, in principle, observable in the upcoming experiments; this case is, however, inconsistent with both gravitino bounds.

Various alterations of generic neutrino mass models have been proposed to allow for successful leptogenesis with a low reheating temperature. In particular, for the lightest right-chiral neutrino decoupled, it has been shown that the simplest version of nonthermal leptogenesis embedded in the scenario of sneutrino-driven inflation can be successful for $T_{RH} \sim 10^6$ GeV. The right-chiral (s)neutrinos are rather heavy in this scenario, as the temperature fluctuations of the cosmic microwave background require $M_1 = 2 \times 10^{13}$ GeV, and, according to Eq. 7, the model predicts a large rate of $\mu \rightarrow e\gamma$ possibly observable in the upcoming experiments, as shown in Figure 1. One can also construct a model, in which the $CP$ asymmetry in the decays of the lightest right-chiral neutrino is enhanced due to large Yukawa couplings of the heavy right-chiral neutrinos. In this model, successful thermal leptogenesis can be achieved for arbitrarily low $T_{RH}$ and the model actually implies that $\mu \rightarrow e\gamma$ will be observed in the forthcoming experiments, unless the supersymmetric particles are very heavy. For the heaviest right-chiral neutrino decoupled, successful leptogenesis is possible for $T_{RH}$ consistent with the strong gravitino bound, if the non-decoupled right-chiral neutrinos are tightly degenerate in mass. In particular, it has been proposed that this quasi-degeneracy results, via renormalization group corrections, from an exactly degenerate mass spectrum of the right-chiral neutrinos at the scale of Grand Unification. Such model is, however, subject to certain consistency conditions, partially compensating this enhancement of the $CP$ asymmetry, but successful leptogenesis is, nevertheless, possible for $T_{RH}$ consistent with the strong gravitino bound, if $\tan \beta \gtrsim 10$.

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Throughout this work, we do not mention the predictions for the rates of LFV $\tau$ decays, since they are similar to those of $\mu \rightarrow e\gamma$, but the relevant experiments are less sensitive.
Figure 1: Extremal values of $\text{BR}(\mu \rightarrow e\gamma)/f(m_0, M_{1/2})$ as a function of $|z|$ for $\sin \theta_{13} = 0$ (panel a) and $\sin \theta = 0.1, \delta = -\pi/2$ (panel b), $\tan \beta = 10$ and $A_0 = 0$. We have chosen $f(m_0, M_{1/2}) = 1$ for values $m_0 = 100 \text{ GeV}$, $M_{1/2} = 500 \text{ GeV}$, consistent with the dark matter relic density. Dotted, dashed and solid curves correspond to $\text{arg} \ z = 0, \pi/4$ and $\pi/2$, respectively. The masses of the right-chiral neutrinos are $M_{1,2,3} = (2, 10, 50) \times 10^{13} \text{ GeV}$. The values of the remaining mixing angles are set to their central values and the neutrino Majorana phases vary freely. The horizontal line represents the current experimental bound.

4 Summary

Let us conclude this presentation by an observation that, if sparticle mass spectrum consistent with the hypothesis of universal soft masses and dark matter constraints is measured at the LHC, and if $\mu \rightarrow e\gamma$ is observed, this would be a strong argument that the seesaw mechanism is correct and the strong gravitino bound on the reheating temperature holds. Analogous conclusions can also be drawn from a confirmed observation of neutrinoless double beta decay. Then, if the baryon asymmetry of the Universe is to be generated in leptogenesis, it will be interesting to look for a symmetry principle, which could justify special features of the seesaw neutrino mass models with a right-chiral neutrino decoupled, presented in this work.

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