A deep neural network (NN) is used to simultaneously detect laser beams in images and measure their center coordinates, radii and angular orientations. A dataset of images containing simulated laser beams and a dataset of images with experimental laser beams—generated using a spatial light modulator—are used to train and evaluate the NN. After training on the simulated dataset the NN achieves beam parameter root-mean-square-errors (RMSEs) of less than 3.4% on the experimental dataset. Subsequent training on the experimental dataset causes the RMSEs to fall below 1.1%. The NN method can be used as a stand-alone measurement of the beam parameters or can compliment other beam profiling methods by providing an accurate region-of-interest.

1. INTRODUCTION

Profiling multiple laser beams on a single image sensor has become increasingly important due to the growing number of multi-beam applications. Spatial light modulators [1], for example, can create multiple, dynamically controlled laser beams—used for optical tweezer arrays in cold atom experiments [2–4] and multi-site neuron activation in two-photon microscopy [5]—while diffractive optical elements allow multiple beams to be created for machining applications [6, 7] and can also form laser beam arrays used in medical skin treatment procedures [8, 9].

In recent years deep neural networks (NNs) have been applied with great success to the analysis of scientific image data. Convolutional neural networks (CNNs) [10, 11] often form the basis for image analysis NNs and have been used within optics for tasks such as laser beam mode classification [12, 13], modal decomposition [14–16] and determination of a beam’s center coordinates [17]. Object detection neural networks (ODNN) [18, 19], which are based on CNNs, can detect objects in images, classify the objects [20] and determine regions-of-interest (ROIs) which bound the objects. In this work, an ODNN [21] which returns rotated regions-of-interest (RROIs) is used to identify multiple TEM\(_{00}\) Gaussian laser beams in images and simultaneously measure all their spatial parameters.

The intensity distribution \(I(x, y)\) for a TEM\(_{00}\) Gaussian beam (see Fig. 1) in a plane orthogonal to the beam’s axis of propagation is given by

\[
I(x, y) = I_0 e^{-\frac{[(x-x_0)\cos\theta+(y-y_0)\sin\theta]^2}{w_x^2} + \frac{[(y-y_0)\cos\theta-(x-x_0)\sin\theta]^2}{w_y^2}},
\]

where \(I_0\) is the peak intensity of the beam, \(x_0\) and \(y_0\) are the beam’s center coordinates, \(w_x\) and \(w_y\) are the major and minor radii and \(\theta\) is the angular orientation. Although higher-order modes (e.g. Hermite-Gaussian) have experimental applications [22], we exclusively focus on the TEM\(_{00}\) Gaussian beams—henceforth referred to simply as Gaussian beams. The majority of laser beams used in both research and industrial applications have a mode content composed primarily of the TEM\(_{00}\) mode thus making it a good approximation for a beam’s intensity distribution.

There are several standardized ways to measure Gaussian beams including scanning slit [23], knife edge [24] and camera based methods [25]. Within the camera based methods, the second moment measurement [26] is the industry standard [27] as it allows for fast calculation of multi-modal beams. However, the second moment method is prone to statistical error from
image noise [28] and several standardized methods are used to combat this including thresholding low-intensity pixels and performing calculations within a ROI centered on the beam [27].

When profiling Gaussian beams, a two-dimensional (2D) fit of the Gaussian beam to Eq. 1 can also be used due to a priori knowledge of the beam mode. A properly chosen ROI increases the 2D fit accuracy by removing portions of the image without relevant data and also decreases the calculation time by fitting a smaller area. Even with an appropriate ROI, the 2D fit is significantly slower than the second moment, but allows for higher accuracy—particularly in noisy images.

For both the second moment and 2D fit methods the ROI is generally found via an iterative method such as calculating the beam width within a ROI, recalculating the ROI using this value and then repeating the process until the beam width converges [27]. However, iterative methods are computationally expensive, have difficulty converging if the image noise is too high and are generally applicable when only a single laser beam is present in the image.

We present a deep neural network based method that allows for an arbitrary number of beams on a single image to be detected and their spatial parameters \([x_0, y_0, w_x, w_y, \theta]\) determined simultaneously, which significantly simplifies the laser beam analysis pipeline. If either the second moment or 2D fit of the beam is still required (e.g. for an ISO 11146 [27] compliant beam measurement), the spatial parameters returned by the NN can be used to determine, ROIs, RROIs or elliptical RROIs in which these calculations can be performed. Furthermore, this method allows for measurement of Gaussian beams with overlapping edges—which cannot be done with the second moment method and would require prior knowledge of the number of beams for the 2D fit method.

This paper is organized as follows: section 2 describes the NN model used to detect the laser beams and measure their spatial parameters, section 3 and section 4 explain how the simulated and experimental datasets are created. Finally, section 5 discusses training the NN and the accuracy achieved for both detection and determination of the beams’ spatial parameters.

2. ROTATED REGION PROPOSAL NEURAL NETWORK

Although the Gaussian equation includes an intensity parameter (amplitude \(I_0\)), beam profiling is generally only concerned with the shape and location of the laser beam which can be described by the spatial parameters \([x_0, y_0, w_x, w_y, \theta]\). The goal is therefore to first detect each laser beam (object) in the image and then measure (regress) their geometric parameters.

Object detection neural networks have been heavily researched in the last decade with several different architectures developed. Region-CNN (RCNN) [29] class NNs are extremely popular and utilize a convolutional neural network base (CNN) followed by a region-proposal-network (RPN) which returns rough ROIs where objects are likely located. The CNN’s output is cropped and pooled using a ROI pooling/alignment stage and passed into one or more classification/regression branches—one of which regresses the ROI coordinates to yield a more accurate value. Although this could be useful for detecting beams, the ROI is aligned along the image axes and only yields information about the center coordinates \([x_0, y_0]\) and the projection of the beam radii on the image axes.

To regress all the beam’s geometric parameters we use the Rotated Region Proposal Network (RRPN) [21] which was initially developed for detecting rotated text in images, but is well suited for detecting laser beams. RRPN is similar to other RCNNs [18], but returns rotated regions-of-interest (RROIs) rather than ROIs. RROIs are rotated rectangles which are defined via the center coordinates \([x_0r, y_0r]\), widths \([dx_r, dy_r]\) and angular orientation \(\theta_r\) of the rectangle. For a RROI centered and aligned on a laser beam in an image, the RROI parameters \([x_0r, y_0r, dx_r, dy_r, \theta_r]\) directly correspond to the laser beam’s geometric parameters and can thus be rewritten \([x_0, y_0, \alpha w_r, \alpha w_y, \theta]\) where \(\alpha\) is a scale factor relating the RROI widths and beam radii. Thus, RRPN can be used to simultaneously detect and measure laser beams.

RRPN (see Fig. 2) begins with a CNN base (we use ResNet50 [11]) which outputs a feature map [30] for a given input. The feature map is then fed into the rotated region proposal network (single node within RRPN) which is similar to the RPN in Faster-RCNN [18], but returns rough RROIs—rather than ROIs—where objects are likely to be located. The RROIs and feature map are both passed into the RROI alignment stage [31] which returns fixed size feature maps via bi-linear interpolation as successive layers require a fixed input. The fixed-size feature maps are then fed into two separate branches: the first classifies the object within the RROI and assigns a score to its prediction, whereas the second does further regression of the RROI parameters.

To train RRPN two datasets are created; the first dataset is comprised of images with simulated Gaussian beams, whereas the second is composed of experimental images with the beams generated using a spatial light modulator. For both the simulated and experimental datasets, the images contain between one and five laser beams, although RRPN could easily be trained to detect a larger number of beams on a single image.

3. SIMULATED DATASET

CNN’s require diverse image training data to allow them to generalize to new data during inference. However, supervised-learning dataset sizes are often limited due to practical considerations such as the time it takes to manually annotate images. Simulated data allows the annotation bottleneck to be circumvented [32–34] as the annotations are calculated directly from the simulation parameters. Since Gaussian beams are relatively easy to simulate, we can create an arbitrarily large dataset filled with unique images by randomizing each beam’s Gaussian parameters (see Fig. 1).

When randomizing a beam’s parameters, an initial beam radius is first drawn from a uniform distribution with a minimum
bound of 5 pixels and a maximum value of 1/6 the 512 pixel image width. An ellipticity value is then drawn from a normal distribution and multiplied by the initial radius to create the second radius value. The larger of the two radii is the major radius \( w_x \), the smaller is the minor radius \( w_y \), and the angular orientation \( \theta \) defines the angle between \( w_x \) and the \( x \) axis. The angular orientations are randomly chosen between \(-\frac{\pi}{2}\) and \(\frac{\pi}{2}\)—which gives a strict definition the NN can learn to regress while still covering the full range of possible orientations. The beam’s center coordinates are randomly drawn from a uniform distribution, but subject to the constraint that the beam’s entire RROI must lie on the simulated sensor surface. Additionally, when more than one beam is present the overlap between beams is restricted to the edges of the distributions. Beam amplitudes are randomly chosen between 0.1 and 1 for all beams and simulated Gaussian noise—with a standard deviation \( \sigma_n \) randomly chosen—is added to the image. The background intensity is set to \( b_0 = 2.5c_0 \) to prevent the Gaussian noise from being substantially clipped.

Using the process above, a simulated dataset with 5000 simulated images (512×512 pixels, see Fig. 1) is generated—1000 for each beam class (number of beams on the image). The initially monochrome images are normalized and mapped to RGB using the Viridis colormap as the pre-trained NN we use (see Section 5) expects RGB input. The ground truth RROI annotations are calculated from the simulated beam parameters and defined as \([x_0, y_0, aw_x, aw_y, \theta]\) where we choose \( a = 3 \). Finally, the dataset is randomly split into a training set with 4000 images and validation dataset with 1000 images.

4. EXPERIMENTAL DATASET

The experimental dataset is created using a spatial light modulator (SLM) [1] which allows structured light [35, 36] to be created using holograms (see Fig. 3a). Our setup begins with a MSquared frequency doubled Ti-Sapphire laser which produces 370 nm light and is coupled into a single-mode optical fiber. The beam exits the fiber and is collimated and reflected off the SLM’s surface—after which the beam passes through a \( f=400 \) mm lens placed a distance \( f \) away from the SLM. An aperture is used to select the first-order diffracted light which is subsequently imaged by a camera placed at the lens’ focus (Fourier plane).

Complex amplitude modulation holograms (CAM) [37] allow both the amplitude and phase of the electric field to be modulated using a phase-only SLM (see Fig. 3b). The amplitude and phase of the laser beam are encoded into a single hologram [38] along with a blazed grating so that the beam parameters \([I_0, x_0, y_0, w_x, w_y, \theta]\) can be dynamically set in the Fourier plane. Adding multiple CAM holograms together—with different blazed grating frequencies—creates multiple beams in the Fourier plane (see Fig. 3c) which can have different sizes, orientations and positions.

The experimental images contain between one and five beams with the CAM parameters for each beam randomly drawn from a uniform distribution—within physically realizable bounds. The ground truth beam parameters \([x_0, y_0, w_x, w_y, \theta]\) at the Fourier plane were found for each beam by performing a 2D fit within a ROI 2× the \( 1/e^2 \) radii; however, for overlapping beams a multi-Gaussian fit was performed. After fitting all the beams in the experimental dataset, the fits were manually inspected\(^1\)

\[ f=100 \text{ mm} \]
\[ f=400 \text{ mm} \]

![Experimental setup](image)

**Fig. 3.** Experimental setup. (a) A 370 nm laser beam exits a single-mode fiber and is collimated with a 100 mm lens before being reflected from a Hamamatsu X13267-05 spatial light modulator (SLM)—which features an 800×600 grid with a pixel pitch of 12.5 \( \mu \)m. A \( 1/2 \) wave plate between the fiber coupler and collimating lens sets the beam’s polarization parallel to the SLM’s vertical axis as the SLM is polarization sensitive. The reflected beam is focused using a \( f=400 \) mm lens placed a distance \( f \) from the SLM and passes through an aperture placed directly (~1 cm) before the lens’ focus—where the different diffraction orders can be resolved. The aperture allows through only the first-order diffracted light which is then imaged by a camera at the Fourier plane. (b) A complex amplitude modulation hologram used to generate Gaussian beams in the Fourier plane for the experimental dataset. (c) Image generated using the hologram in (b).

and subsequently used to calculate the RROIs as \([x_0, y_0, aw_x, aw_y, \theta]\) where again \( a = 3 \).

The 1050 images in the experimental dataset—210 images for each number of beams—are split into a training dataset with 800 images and a validation dataset with 250 images. Although the camera has a sensor size of 1280×1024 pixels, the beams in the Fourier plane are incident on a small area of the sensor and the images are cropped to 256×256 pixels. As with the simulated dataset, the image intensities are normalized and mapped to RGB.

5. TRAINING AND EVALUATION

Rather than building the NN model from scratch, Facebook Artificial Intelligence Research’s (FAIR) Detectron2 [39] framework is utilized which implements common machine vision models and is written for speedy training and inference. Since Detectron2 only has pre-trained weights for the CNN base, transfer learning [40] can only be partially implemented and significantly more images are needed to train the NN. Our strategy is therefore to first pre-train RRFN on the larger, more diverse simulated dataset before (optionally) doing final training on the experimental dataset.

RRFN is trained on the simulation dataset for 120 epochs
using a stochastic gradient descent optimizer with an initial learning rate which is decayed four times. The initial learning rate, learning rate decay scalar and the epochs at which the learning rate is decayed are all used as hyperparameters—along with the momentum and batch size. Nominally either random search [41] or Bayesian optimization would be used to tune the hyperparameters; however, due to the large size of the simulated dataset and limited computational resources (we train our NN in a Google Colab [42] notebook) we manually set the hyperparameters to sensible values: batch size of 4, learning rate of 0.01, momentum of 0.9, learning rate decay of 0.1 and learning rate decay at epochs [80, 100, 110, 115].

After each training epoch, the NN is evaluated on both the simulated and experimental validation datasets using the mean average precision (mAP) metric [43]. The mAP is a standard object detection metric where intersection-over-union scores (IoU) [44] between the ground truth and NN predicted RROIs are used to form precision-recall curves [45] for different IoU thresholds—which are integrated and averaged to give the mAP value [43].

At the beginning of training, the loss quickly decays while the mAPs climb, whereas towards the end of training both the loss and mAP values become asymptotic (see Fig. 4a). Maximum mAPs of 96.8% and 93.9% are achieved on the simulation and experimental validation datasets respectively which correspond to the NN correctly finding 2996/3000 beams [2] on the simulated validation dataset and 750/750 beams on the experimental validation dataset (correct prediction threshold is set as an IoU>0.5 between the ground truth and predicted RROIs).

Along with the mAP, the beam parameter errors are calculated after each training epoch. For both the ground truth and NN, the parameters are normalized before calculating the error. The center coordinates \((x_0, y_0)\) are normalized by dividing by the ground truth beam radii along the \(x\) and \(y\) laboratory axes, whereas the major and minor radii \((w_x, w_y)\) are divided by the ground truth major and minor radii. For the angular orientation error, \(\theta\) is normalized by dividing by the range of angles, \(\pi\). Beams with ellipticities below \(w_x/ w_y=1.15\) are considered radially symmetric [27] and removed from the further angular error calculations as the angle for radially symmetric beams is arbitrary.

### Table 1.

| Parameters | Lower Bound | Upper Bound | Log Scale | Best Value |
|------------|-------------|-------------|-----------|------------|
| Learning Rate | 0.001 | 0.01 | Yes | 0.0056 |
| Momentum | 0.8 | 0.925 | No | 0.85 |
| Decay Epoch | 1 | 30 | No | 21 |
| LR Decay | 0.01 | 0.1 | No | 0.01 |
| Batch Size | 2 | 8 | No | 5 |

2For 35 beams, the NN predicts two RROIs, but the extra predictions are easily filtered using the IoU between predictions and the NN’s prediction score.

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**Fig. 4.** Training on the simulated dataset. (a) The loss on the simulated training dataset and the mean average precision (mAP) on the simulated and experimental validation datasets plotted against the training epoch. (b) The root-mean-square errors (RMSE) for the simulation validation dataset beam parameters \(\{x_0, y_0, w_x, w_y, \theta\}\) vs. the training epoch. The spatial parameters are normalized by the beam size (see text), and the angular orientation is normalized by \(\pi\).
allows high accuracies to be reached due to pre-training on the simulated dataset. Similar to the simulated training run, both the mAP and RMSEs are calculated after every training epoch on the experimental validation dataset. The NN trained with the best set of hyperparameters achieves a mAP of 97.7% and successfully detects all 750 beams. Furthermore, the Gaussian parameter RMSEs are all below 1.1% (see Table 1), which are lower than the NN’s experimental validation dataset RMSEs when trained on the simulated dataset alone; however, the accuracy gain is not substantial.

6. CONCLUSION

The method developed uses a deep neural network to detect an arbitrary number of Gaussian laser beams in an image and simultaneously measure their spatial parameters. The NN requires a single pass on an image which significantly simplifies the beam analysis pipeline compared to other methods. Since training the NN on simulated data alone results in high accuracies on experimental data, this method can be applied in a wide range of experimental settings.

The NN can be used alone or can compliment other beam measurement methods by detecting laser beams in an image and determining ROIs for further calculations, such as a 2D fit or second moment measurement. This removes the need for iterative ROI algorithms which can generally only find a single beam. Furthermore, if 2D fitting is implemented, the beam parameters extracted with the NN can be used to seed the fit which increases fitting speed and the likelihood of fit convergence.

Although this method is applied to TEM_{00} beams, it can be extended to higher-order Gaussian modes since RPFN natively handles multiple object types. However, this would require the training datasets to include higher-order beams with a label for each beam mode. Furthermore, for higher-order and multi-modal beams the radius is determined numerically rather than analytically—which would need to be accounted for when generating the datasets’ RROI annotations.

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DISCLOSURES

L.H. was previously employed at DataRay Inc.

DATA AVAILABILITY

The data that support the findings of this study are openly available at the following URL: DOI: https://doi.org/10.5287/bodleian:JbDXrNQ1. We additionally make code available at https://github.com/Dipolar-Quantum-Gases/nn-beam-profiling.

REFERENCES

1. Konforti N, Marom E and Wu S T 1988 Optics Letters 13 251–253 URL https://doi.org/10.1364/OL.13.000251
2. Barreto D, de Léséleuc S, Lienhard V, Lahaye T and Browaeys A 2016 Science 354 1021–1023 URL https://doi.org/10.1126/science.aah3778
3. Ohi de Mello D, Schäffner D, Werkmann J, Preuschoff T, Kohlhahl L, Schlosser M and Birkl G 2019 Physical Review Letters 122(20) 203601 URL https://doi.org/10.1103/PhysRevLett.122.203601
4. Endres M, Bernien H, Keeling A, Levine H, Anschuetz E R, Kranjcnbrink A, Senko C, Vuletic V, Greiner M and Luhn M D 2016 Science 354 1024–1027 URL https://doi.org/10.1126/science.aah3752
5. Nikolenko V, Watson B, Araya R, Woodruff A, Peterka D and Yuste R 2008 Frontiers in Neural Circuits 2 ISSN 1662-5110 URL: https://doi.org/10.3389/neuro.04.005.2008
6. Hauschitz P, Stoklasa B, Kuchař J, Turčiová H, Plašický M, Brajer J, Rostohar D, Moczek T, Duda M and Lucianetti A 2020 Materials 13 ISSN 1996-1944 URL https://doi.org/10.3390/ma13123962
7. Katz S, Kaplan N and Gasssinger I 2018 Optik & Photonik 13 83–86 URL https://doi.org/10.1016/j.ijoi.20180021
8. Tanghetti E A 2016 Lasers in Surgery and Medicine 48 646–652 URL http://doi.org/10.1002/lsm.22540
9. Lee H C, Childs J, Chung H J, Park J, Hong J and Cho S B 2019 Scientific Reports 9 1–10 URL https://doi.org/10.1038/s41598-019-41021-7
10. Krizhevsky A, Sutskever I and Hinton G E 2012 Advances in Neural Information Processing Systems 25 1097–1105 URL https://doi.org/10.1145/3065386
11. He K, Zhang X, Ren S and Sun J 2016 IEEE Conference on Computer Vision and Pattern Recognition 770–778 URL https://doi.org/10.1109/CVPR.2016.90
12. Doster T and Wadnir A T 2017 Applied Optics 56 3386–3396 URL https://doi.org/10.1364/AO.56.003386
13. Hofer L R, Jones L W, Goedert J L and Dragone R V 2019 Journal of the Optical Society of America A 36 936–943 URL https://doi.org/10.1117/1.JOSAA.36.060036
14. Lohani S, Knutson E M, O’Donnell M, Huver S D and Glasser R T 2018 Applied Optics 57 4180–4190 URL https://doi.org/10.1364/AO.57.004180
15. An Y, Hou T, Li J, Huang L, Leng J, Yang L and Zhou P 2020 Applied Optics 59 1954–1959 URL https://doi.org/10.1364/AO.377189
16. Schiowski M G, Brown D D and Ottaway D J 2021 Journal of the Optical Society of America A 38 1603–1611 URL http://dx.doi.org/10.1364/JOSAA.428214
17. Lin C S, Huang Y C, Chen S H, Hsu Y L and Lin Y C 2018 Applied Optics 57 1542 URL https://doi.org/10.1364/AO.2018.1542
18. Ren S, He K, Girshick R and Sun J 2015 Advances in Neural Information Processing Systems 28 91–99 URL https://proceedings.neurips.cc/paper/2015/file/14b1a5bb1d0176a435e05b2a08a252ed39046-Paper.pdf
19. Redmon J and Farhadi A 2018 arXiv preprint arXiv:1804.02767 URL https://arxiv.org/abs/1804.02767
20. Hofer L R, Krstajic M, Juhász P, Marchant A L and Smith R P 2021 Machine Learning: Science and Technology 2 045008 URL: https://doi.org/10.1088/2632-2153/abf5ee
21. Ma J, Shao W, Ye H, Wang L, Wang H, Zheng Y and Xue X 2018 IEEE Transactions on Multimedia 20 3111–3122 URL https://doi.org/10.1109/TMM.2018.2818020
22. Gaunt A L, Schmidutz T F, Götlibovych I, Smith R P and Hadzibabic Z 2013 Physical Review Letters 110 URL https://doi.org/10.1103/PhysRevLett.110.200406
23. McCally R L 1984 Applied Optics 23 2227–2227 URL https://doi.org/10.1364/AO.23.002227
24. Siegman A E, Sasnell M and Johnston T 1991 IEEE Journal of Quantum Electronics 27 1099–1104 URL https://doi.org/10.1109/3.33946
25. Siegman A E 1998 DPSS (Diode Pumped Solid State) Lasers: Applications and Issues M1 URL https://doi.org/10.1364/DAI.1998.M1
26. Ross T S 2013 Laser beam quality metrics (SPIE Press) URL https: //doi.org/10.1117/3.1000595
27. 2005 ISO 11146-1: Lasers and laser-related equipment—test methods for laser beam widths, divergence angles and beam propagation ratios—part 1: Stigmatic and simple astigmatic beams
28. Hofer L R, Dragone R V and MacGregor A D 2017 Optical Engineering 56 043110 URL https://doi.org/10.1117/1.OE.56.4.043110
29. Girshick R, Donahue J, Darrell T and Malik J 2014 IEEE Conference on Computer Vision and Pattern Recognition 580–587 URL https://doi.org/10.1109/CVPR.2014.81
30. Zeiler M D and Fergus R 2014 Computer Vision – ECCV 818–833 URL https://doi.org/10.1007/978-3-319-10590-1
31. Huang J, Sivakumar V, Mnatsakanyan M and Pang G 2018 arXiv preprint arXiv:1811.07031 URL https://arxiv.org/abs/1811.07031
32. Wood E, Baltrusaitis T, Morency L P, Robinson P and Bulling A 2016 Proceedings of the Ninth Biennial ACM Symposium on Eye Tracking Research & Applications 131–138 URL https://doi.org/10.1145/2857491.2857492
33. Yoo D, Kim N, Park S, Paek A S and Kweon I S 2016 European Conference on Computer Vision 517–532 URL http://doi.org/10.1007/978-3-319-46484-8_31
34. Zhang X, Fu Y, Zang A, Sigal L and Agam G 2015 arXiv preprint arXiv:1503.03163 URL https://arxiv.org/abs/1503.03163
35. Forbes A 2019 Laser & Photonics Reviews 13 1900140 URL https://doi.org/10.1002/lpor.201970043
36. Forbes A, Dudley A and McLaren M 2016 Advances in Optics and Photonics 8 200–227 URL https://doi.org/10.1364/AOP.8.000200
37. Arrizón V, Ruiz U, Carrada R and González L A 2007 Journal of the Optical Society of America A 24 3500–3507 URL https://doi.org/10.1364/JOSAA.24.003500
38. Rosales-Guzmán C and Forbes A 2017 How to shape light with spatial light modulators (SPIE Press) URL https://doi.org/10.1117/3.2281295
39. Wu Y, Kirillov A, Massa F, Lo W Y and Girshick R 2019 Detectron2 https://github.com/facebookresearch/detectron2
40. Yosinski J, Clune J, Bengio Y and Lipson H 2014 Advances in Neural Information Processing Systems 27 3320–3328 URL https://proceedings.neurips.cc/paper/2014/file/375c71349b295f6be2ddcda9206f20a06-Paper.pdf
41. Bergstra J and Bengio Y 2012 The Journal of Machine Learning Research 13 281–305 URL https://dl.acm.org/doi/abs/10.5555/2188385.2188395
42. Bisong E 2019 Google Colaboratory (Springer) pp 59–64 URL https://link.springer.com/book/10.1007/978-1-4842-4470-8
43. Lin T Y, Maire M, Belongie S, Hays J, Perona P, Ramanan D, Dollár P and Zitnick C L 2014 Computer Vision – ECCV 740–755 URL https://doi.org/10.1007/978-3-319-10602-1
44. Everingham M, Van Gool L, Williams C K, Winn J and Zisserman A 2010 International Journal of Computer Vision 88 303–338 URL https://doi.org/10.1007/s11263-009-0275-4
45. Boyd K, Eng K H and Page C D 2013 Joint European Conference on Machine Learning and Knowledge Discovery in Databases 451–466 URL http://doi.org/10.1007/978-3-642-40994-3
46. Snoek J, Larochelle H and Adams R P 2012 Advances in Neural Information Processing Systems 25 2951–2959 URL https://proceedings.neurips.cc/paper/2012/file/05311655a15b75fab869566663e1819cd-Paper.pdf
47. Frazier P I 2018 arXiv preprint arXiv:1807.02811 URL https://arxiv.org/abs/1807.02811
48. Balandat M, Karrer B, Jiang D R, Daulton S, Letham B, Wilson A G and Bakshy E 2020 arXiv preprint arXiv:1910.06403 URL https://arxiv.org/abs/1910.06403
49. Sobol’ I M 1967 Zhurnal Vychislitel’noi Matematiki i Matematicheskoi Fiziki 7 784–802 URL http://mi.mathnet.ru/eng/vzvmmf7334