Addressing the high- $f$ problem in pseudo-Nambu–Goldstone boson dark energy models with dark matter–dark energy interaction

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Abstract We consider a dark energy scenario driven by a scalar field $\phi$ with a pseudo-Nambu–Goldstone boson (pNGB) type potential $V(\phi) = \mu^4 \left(1 + \cos(\phi/f) \right)$. The pNGB originates out of breaking of spontaneous symmetry at a scale $f$ close to Planck mass $M_{pl}$. We consider two cases namely the quintessence dark energy model with pNGB potential and the other, where the standard pNGB action is modified by the terms related to Slotheon cosmology. We demonstrate that for this pNGB potential, high- $f$ problem is better addressed when the interaction between dark matter and dark energy is taken into account and that Slotheon dark energy scenario works even better over quintessence in this respect. To this end, a mass limit for dark matter is also estimated.

1 Introduction

The observational data [1–4] reveal that our Universe is not only expanding with time but is undergoing an accelerated expansion. One of the most challenging problems of modern cosmology is to explain such a late time cosmic acceleration. The general conjecture is that a mysterious component of the Universe with negative pressure, broadly known as dark energy (DE) is responsible for the recent acceleration of the Universe. In theory, the commonly used candidate for dark energy is the cosmological constant $\Lambda$ introduced by Einstein. The popular and widely used dark energy model namely $\Lambda$CDM model that includes cosmological constant $\Lambda$ and the cold dark matter (CDM) has some unsolved theoretical problems like the fine-tuning problem [5] and cosmic coincidence problem [7].

Another concept to address the dark energy is to assume the existence of a scalar field $\phi$ with a slow rolling potential $V(\phi)$ as the source of dark energy. This scalar field $\phi$, named as quintessence field, provides the dynamical nature of the dark energy, in contrast to the cosmological constant explanation according to which the dark energy is constant throughout the evolution of the Universe. Extensive studies have been done to understand the nature of quintessence dark energy model [8,9]. A well motivated alternative description of dark energy is given by modified gravity models of dark energy, inspired by the theories of extra dimensions [10,11]. It is observed that to obey the observational results, the scalar field $\phi$ should possess a very flat potential $V(\phi)$ and very light mass.

The other dark sector component of the Universe is dark matter (DM) which contains about 26.5% of the total mass–energy content of the present Universe. The existence of the dark matter is confirmed by various observational results [12–14]. Various attempts have been made in the literature to explain the unknown dark matter in theories beyond Standard Model of particle physics. But in this work where we addressed dark matter–dark energy interactions, the dark matter is considered to be a scalar field.

Till date there are no theoretical considerations or experimental observations that seem to suggest that a possible dark matter–dark energy interaction can not exist. We consider here a nonminimal coupling between the two dark sector components namely, dark matter and dark energy instead of treating them independently. In literature various authors discussed the interacting dark energy (IDE) models [15] to address different phenomenological problems [16–30].

As mentioned, a popular dark energy model is scalar field dark energy model where one considers a slowly varying potential (slow roll) for the scalar field which the latter tracks
as the potential changes over time. Even though the idea of the
scalar field dark energy model is well motivated but in
Ref. [31] Kolda and Lyth pointed out a serious problem for
any slow rolling scalar field dark energy model. The problem
is to prevent any additional terms to the field potential $V(\phi)$,
which would spoil the flatness of the potential. In order to
avoid this problem, the dark energy models are considered
where the light mass of the scalar field $\phi$ is protected by a
symmetry. Such scenario can be realised if a pseudo-Nambu–
Goldston boson (pNGB) acts as a dynamical dark energy
field. This concept is studied in Refs. [32,33].

The spontaneous symmetry breaking of a global $U(1)$
symmetry results in producing a massless Goldstone boson.
Such spontaneous symmetry breaking leads to two modes,
one is the massive radial mode and the other is the mass-
less angular mode (named as NGB) at the symmetry break-
ing energy scale. A pseudo-Nambu–Goldston boson is pro-
duced when the NGB acquires mass at the soft explicit sym-
metry breaking scale which is lower than the spontaneous
symmetry breaking scale. A popular example of pNGB is
Axion [34–36]. As mentioned, pNGB could play the role of
the quintessence dark energy field with the following form
of the potential [32],

$$
V(\phi) = \mu^4 \left( 1 + \cos(\phi/f) \right).
$$

In the above, $f$ is the spontaneous global symmetry break-
ning scale which controls the steepness of the potential and $\mu$
is the explicit global symmetry breaking scale. The above
potential is periodic with period $2\pi f$ and the field value $\phi$
ranges from 0 to $2\pi f$. This periodic potential is special
because it stabilises the mass from quantum corrections [31]
and suppresses the fifth-force constraints [37]. In Refs. [38–
40] the authors found that generally for the quintessence
dark energy with pNGB potential, $f \geq M_{Pl}$, $M_{Pl}$ being the
reduced Planck mass and indicated that the larger value of $f$
corresponds to the flatter potential. But it is very difficult
to interpret such high value of $f$ since such values of $f$
are not compatible with the valid domain of field theory.
For this range, quantum gravity corrections can not be controlled
[40]. This problem is termed as high-$f$ problem of pNGB
quintessence dark energy model. Few authors have attempted
to solve this high-$f$ issue earlier [41,42]. For example in Ref.
[41] this has been suggested that $N$ number of pNGB fields
drive the late time acceleration and in Ref. [42] the authors
discussed the issue by adding extra phenomenological terms
to the quintessence lagrangian. Here in this work we explore
a new approach where we consider interacting dark energy
(IDE) model and the nonminimally coupled dark matter–
dark energy scenario to address the high-$f$ value problem
of pNGB quintessence.

Dark matter–dark energy (DM–DE) interactions are some-
times marked by considering both dark energy and dark
matter to be fluids or both of them to be scalar fields or
even one of them is a scalar field and the other is a fluid.
In this work we take both the dark energy and dark mat-
ter as real scalar fields and treat the interactions in a way
which is more fundamental in nature. Here, we consider two
dark energy models, the standard quintessence dark energy
model and the Slotheon field dark energy model [43–45]. The
Slotheon dark energy model is a modified gravity model,
inspired by extra-dimensional theories. This follows from
Dvali Gabadadze Porrati (DGP) model [46] (an extra dimen-
sional model) which in its decoupling limit can be described
by a scalar field named as Galileon field in Minkowski spac-
time and the field obeys Galileon shift symmetry [47,48].
The generalisation of the Galileon shift symmetry to curved
spacetime leads to the Slotheon field [45]. In literature there
are references where it is shown that in some cases such dark
energy models fit better with the observational or theoretical
constraints than the standard quintessence model [43,49–51].
Therefore in this work too we evaluate our results for each
of these two cases (namely quintessence dark energy and
Slotheon field dark energy) and compare them. For both the
cases we consider the nonminimal coupling between dark
matter and dark energy and furnish our results for both the
scenarios with and without DM–DE interactions.

This paper is organised as follows. In Sect. 2 we briefly
describe in general the dark matter–dark energy interactions.
Section 3 is devoted to explain the coupled (coupled to dark
matter) quintessence dark energy model with pNGB potential
while in Sect. 4, coupled Slotheon field dark energy model
with pNGB potential is described. We present our calcula-
tions and results in Sect. 5 and finally in Sect. 6 a summary
and discussions are given.

2 Dark energy–dark matter interaction

In this section we briefly furnish the formalism for the DM–
DE interactions that is followed in this work.

In order to describe the DM–DE interactions the energy-
momentum conservation equations are written with an intro-
duction of an exchange term $Q$ as,

$$
\dot{\rho}_{DE} + 3H (\rho_{DE} + p_{DE}) = Q, \quad (2)
$$

$$
\dot{\rho}_{DM} + 3H \rho_{DM} = -Q, \quad (3)
$$

where $\rho_{DE}$ and $\rho_{DM}$ are the energy densities of dark energy
and dark matter respectively, $H$ denotes the Hubble param-
eter while $\dot{A}$ represents time derivative of the variable $A$.
In the above, $Q$ refers to the energy transfer between dark
matter and dark energy and defines the DM–DE interaction
coupling. In literature there are several studies of this interac-
tion where the dark sector is considered to be a two compo-
ment fluid with some usual forms for coupling $Q$ are adopted.
φ signifies DM–DE interaction. The mass function energy is assumed to be a scalar field [59]. The action contains a kinetic term for dark energy field transformation on the metric have also shown that by rewriting (carrying out a conformal transformation on the metric $g_{\mu\nu}$) the lagrangian from Jordan frame to Einstein frame, one can obtain the same form of coupling $Q$ (Eq. (4)).

A more fundamental approach to describe the DM–DE interactions is to treat both the dark matter and dark energy as fields [57–59]. In the present work both the fields are chosen as real scalar fields similar to what is considered in Ref. [59]. The action contains a kinetic term for dark energy field $\phi$, a kinetic term for dark matter field $\chi$ and a total potential $V(\phi, \chi)$, where $V(\phi, \chi)$ includes the potentials for dark energy and dark matter. The action for such a system can be written as,

$$S = \int d^4x \sqrt{-g} \left[ -\frac{1}{2} g^{\mu\nu} \left( \partial_\mu \phi \partial_\nu \phi + \partial_\mu \chi \partial_\nu \chi \right) - V_{DE}(\phi) - \frac{1}{2} m^2 \phi^2 - \beta \phi^2 \chi^2 \right] + S_m + S_r, \quad (6)$$

where $M_{pl}$ and $R$ are reduced Planck mass and Ricci scalar respectively and $\beta$ denotes the coupling constant for DM–DE interaction. In the above $S_m$ and $S_r$ refer to the action of baryonic matter and action of radiation component respectively. We consider the quintessence field with pNGB potential for our dark energy model, therefore the expression for the potential $V_{DE}(\phi)$ is similar to that in Eq. (1),

$$V_{DE}(\phi) = \mu^4 \left( 1 + \cos(\phi/f) \right). \quad (7)$$

Hence in our case the effective potential is given by

$$V_{eff}(\phi, a) = \mu^4 \left( 1 + \cos(\phi/f) \right) + n_o a^{-3} M(\phi), \quad (8)$$

where symbols have similar meanings as described in the previous section and here $M^2(\phi)$ is given by

$$M^2(\phi) = m^2 + 2 \beta \phi^2. \quad (9)$$

By varying the action of Eq. (6) with respect to the metric, the Friedmann equations are obtained as $3M^4_{pl} H^2 = \rho_m + \rho_r + \frac{\phi^2}{2} + V_{eff}(\phi, a)$ ($\rho_m$ and $\rho_r$ are mass and radiation densities respectively), $M^2_{pl}(2 \dot{H} + 3 H^2) = -\rho_r - \frac{\phi^2}{2} + V_{eff}(\phi, a)$ ($H$ is the Hubble parameter) and $0 = \dot{\phi} + 3 H \phi + \frac{\partial V_{eff}(\phi, a)}{\partial \phi}$.

These are solved by constructing the autonomous set of equations in terms of a chosen set of dimensionless variables

$$x = \frac{\dot{\phi}}{\sqrt{6 H M_{pl}}}, \quad y = \sqrt{V_{DE}(\phi)/3 H M_{pl}}, \quad \lambda = -M_{pl} \frac{dV_{DE}(\phi)}{d\phi} \quad \text{and} \quad \xi = \frac{\phi}{m} \text{as}$$

$$\frac{dx}{dN} = \frac{P}{\sqrt{6}} - x \frac{\dot{H}}{H^2}, \quad \frac{dy}{dN} = -y \left( \sqrt{\frac{3}{2}} \lambda x + \frac{\dot{H}}{H^2} \right),$$

$$\frac{d\lambda}{dN} = -\sqrt{6} x \lambda^2 \left( \frac{V_{DE} d^2 V_{DE}}{d\phi^2} - 1 \right) - 1, \quad \frac{d\xi}{dN} = \sqrt{6} x \frac{P}{b}. \quad (10)$$

with $N = \ln a$ is the number of e-foldings, $b = \frac{m_{pl}}{n_o}$ and $\frac{\dot{H}}{H^2} = \frac{1}{2} \left( \frac{3 \Omega_{do}}{a^2} - 3 \lambda^2 + 3 \gamma^2 - 3 \Omega_r - 3 \right)$, $P = -\frac{6 \beta \Omega_{do}}{a^2 (2 \beta \phi^2 + b)} - 3 \sqrt{6} \lambda + 3 \gamma \lambda^2$, $\frac{d^2 V_{DE}}{d\phi^2} = \frac{1}{2} \left( 1 - \frac{1}{\lambda^2} \right)$, where the other notations carry usual significance and $\Omega_{do}$ is the dark matter density parameter at the present epoch.

3 Coupled quintessence dark energy model

The action for coupled quintessence scenario where the standard quintessence scalar $\phi$ is the dark energy candidate is written as
The effective equation of state parameter \( \omega_{\text{eff}} \) and equation of state parameter of dark energy \( \omega_{\text{DE}} \) are now obtained as

\[
\omega_{\text{eff}} = -1 - \frac{2 \dot{H}}{3H^2}, \quad \omega_{\text{DE}} = \frac{\omega_{\text{eff}} - \Omega_b}{\Omega_{\text{DE}}}. \quad (11)
\]

4 Coupled Slotheon dark energy model

The Slotheon field model [45] is a scalar field model which is classified as a modified gravity model of dark energy. The Slotheon field model is inspired by Dvali Gabadadze Porrati (DGP) model [46] – an extra dimensional model with one extra dimension. The DGP model in its decoupling limit \( r_c \rightarrow \infty \) [60,61] \( (r_c \) separates 4-D and 5-D regimes and defined as \( r_c = \frac{M_5^2}{2M_5^2} \), where \( M_5 \) and \( M_5 \) are bulk and brane Planck masses respectively) can be described by a scalar field (say \( \pi \)), dubbed as Galileon field [47,48]. Here the field \( \pi \) respects a shift symmetry known as Galileon shift and is given by \( \pi \rightarrow \pi + a + b \mu x^\mu \) [62]. The symbols \( a \) and \( b \) represent a constant and a constant vector respectively. The Slotheon field arises when this Galileon transformation is generalised to curved spacetime [45] and the Slotheon field obeys this curved Galileon transformation,

\[
\pi(x) \rightarrow \pi(x) + c + c_a \int_{\Gamma, x_0} \gamma^a, \quad (12)
\]

where \( \gamma^a \) is a set of Killing vectors, \( x_0 \) is a reference point connected to another point \( x \) through the curve \( \Gamma \) while \( c \) and \( c_a \) are respectively a constant and a constant vector. It is observed in Refs. [43,44] that if Slotheon field model of dark energy is considered, then the slow rolling criteria will be more favoured than the standard quintessence dark energy model. This is because the former induces an extra friction which favours the slow rolling nature of the field. Moreover in Ref. [49] it is demonstrated that the Swampland criteria are better satisfied with the Slotheon field model of dark energy over the quintessence model.

In this section we consider the Slotheon field \( \pi \) as dark energy and explore the behaviours of different cosmological parameters when it is coupled to the dark matter. The action of the coupled Slotheon dark energy field is given as

\[
S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} M_5^2 R - \frac{1}{2} \left( g^{\mu\nu} \partial_\mu \pi \partial_\nu \pi + \frac{G^{\mu\nu}}{M^2} \partial_\mu \pi \partial_\nu \pi \right) - V_{\text{DE}}(\pi) - \frac{1}{2} m^2 \pi^2 - \frac{\beta \pi^2}{\chi^2} \right] + S_m + S_r. \quad (13)
\]

In the above, \( G^{\mu\nu} \) is the Einstein tensor and \( M \) represents an energy scale and all the symbols are same as in Eq. (6). It can be noted here that without the term \( \frac{G^{\mu\nu}}{M^2} \partial_\mu \pi \partial_\nu \pi \) in the action of Eq. (13), both the actions of Eqs. (6) and (13) are identical. In the Slotheon case also we have similar form of dark energy potentials as in Eqs. (7)–(9) by replacing \( \phi \) by the Slotheon field \( \pi \) (e.g. \( V_{\text{DE}}(\pi) = \mu^4 (1 + \cos(\pi/f)) \) and similarly for \( V_{\text{eff}}(\pi, a) \) and \( M^2(\pi) \)).

The Friedmann equations for the action in Eq. (13) can now be derived as

\[
3M_5^2 H^2 = \rho_m + \rho_r + \frac{\dot{\pi}^2}{2} + \frac{9H^2 \dot{\pi}^2}{2M^2} + V_{\text{eff}}(\pi, a), \quad (14)
\]

\[
M_5^2(2 \dot{H} + 3H^2) = -\frac{\rho_r}{3} - \frac{\dot{\pi}^2}{2} + V_{\text{eff}}(\pi, a)
\]

\[
+ (2 \dot{H} + 3H^2) \frac{\dot{\pi}^2}{2M^2} + \frac{2H \dot{\pi}}{M^2}, \quad (15)
\]

\[
0 = \dot{\pi} + 3H \dot{\pi} + 3 \frac{M^2}{2} \left( \frac{\dot{\pi}}{M^2} + \frac{2 \dot{H} \dot{\pi}}{H^2} \right) + \frac{\partial V_{\text{eff}}(\pi, a)}{\partial \pi}. \quad (16)
\]

The Friedmann equations above for Slotheon field \( \pi \) are solved by constructing a set of differential equations in terms of a suitable set of dimensionless variables as described for quintessence field in Sect. 3. But here, we need to define one more dimensionless variable \( \epsilon = H^2/2M^2 \) and an additional differential equation \( \frac{d\epsilon}{dN} = 2\epsilon \frac{\dot{H}}{H^2} \). This additional variable \( \epsilon \) arises due to the term \( \frac{G^{\mu\nu}}{M^2} \partial_\mu \pi \partial_\nu \pi \) in the action of Eq. (13).

For the Slotheon field the quantities \( P \) and \( \frac{H}{H} \) (in the set of autonomous equations (see Sect. 3)) are

\[
\frac{H}{H^2} = \frac{\dot{\pi}^2(6\epsilon + 1)(18\epsilon + 1) + 4\sqrt{\Delta} \epsilon \left( \lambda \pi^2 - \frac{2\epsilon H G_{\mu\nu}}{M^2 (2\epsilon H^2)} \right)}{4\epsilon \left( 6\epsilon(1 + \epsilon + 1)(2\epsilon H^2) \right)} \quad (17)
\]

\[
P = \frac{6G_{\mu\nu}}{a^3 b (2\pi \epsilon + 1 + 1) (6\epsilon (x^2 (1 + 1 + 1) + 1))
\]

\[
+ 6\epsilon x^2 (1 + 1 + 1) + 2\epsilon x^2 (1 + 1 + 1))
\]

\[
+ \frac{3a^3 b (2\pi \epsilon + 1 + 1) \Delta \pi}{a^3 b (2\pi \epsilon + 1 + 1) (6\epsilon (x^2 (1 + 1 + 1) + 1))}
\]

\[
+ \frac{3a^3 b (2\pi \epsilon + 1 + 1) \Delta \pi}{a^3 b (2\pi \epsilon + 1 + 1) (6\epsilon (x^2 (1 + 1 + 1) + 1))}
\]

\[
+ \frac{3a^3 b (2\pi \epsilon + 1 + 1) \Delta \pi}{a^3 b (2\pi \epsilon + 1 + 1) (6\epsilon (x^2 (1 + 1 + 1) + 1))}
\]

\[
+ \frac{3a^3 b (2\pi \epsilon + 1 + 1) \Delta \pi}{a^3 b (2\pi \epsilon + 1 + 1) (6\epsilon (x^2 (1 + 1 + 1) + 1))}
\]

\[
+ \frac{3a^3 b (2\pi \epsilon + 1 + 1) \Delta \pi}{a^3 b (2\pi \epsilon + 1 + 1) (6\epsilon (x^2 (1 + 1 + 1) + 1))}
\]

\[
+ \frac{3a^3 b (2\pi \epsilon + 1 + 1) \Delta \pi}{a^3 b (2\pi \epsilon + 1 + 1) (6\epsilon (x^2 (1 + 1 + 1) + 1))}
\]

\[
+ \frac{3a^3 b (2\pi \epsilon + 1 + 1) \Delta \pi}{a^3 b (2\pi \epsilon + 1 + 1) (6\epsilon (x^2 (1 + 1 + 1) + 1))}
\]

\[
+ \frac{3a^3 b (2\pi \epsilon + 1 + 1) \Delta \pi}{a^3 b (2\pi \epsilon + 1 + 1) (6\epsilon (x^2 (1 + 1 + 1) + 1))}
\]

where all the symbols have their meaning as mentioned earlier. The effective equation of state parameter \( \omega_{\text{eff}} \) and the equation of state of dark energy \( \omega_{\text{DE}} \) for Slotheon field dark energy model can now be constructed using Eqs. (14)–(16) and they will be of the same forms as in Eq. (11). The variations of cosmological parameters \( \Delta \pi \) and \( \Delta \rho_{\text{DE}} \) are obtained from solving these equations with properly chosen initial conditions.

5 Calculations and results

In this section we furnish the results we obtain from solving the equations in Sects. 3 and 4.
In Fig. 1, the evolutions of the density parameters ($\Omega$) of different components of the Universe as functions of redshift $z$ are shown. In this figure, dynamics of the density parameters for radiation, matter and dark energy are plotted by considering the quintessence dark energy model coupled to the dark matter field. From Fig. 1 it can be observed that at a much later stage when $\ln(1+z) \simeq 0.4$, or $z \simeq 0.49$ the dark energy begins to dominate and the Universe enters into the phase of late time acceleration.

In Fig. 2 we show the variations of the dark energy equation of state parameter $\omega_{DE}$ with redshift $z$ for standard quintessence dark energy model with potential given in Eq. (7). Figure 2 also indicates the thawing nature [63] of the dark energy, i.e., $\omega_{DE}$ is equal to $-1$ at early epochs which gradually deviates from $-1$ with time, is clear. In Fig. 2 we compare variations for $\omega_{DE}$ with $z$ when the DM–DE interactions are considered (red line in the plot, $\beta \neq 0$) with the same with no such interactions included (the green line in the plot, $\beta = 0$). It is interesting to note that quintessence dark energy is more akin to $\Lambda$CDM model (where $\omega_{DE} = -1$, throughout the time of evolution) when the dark energy field is coupled to the dark matter field. In this case and in the following cases we take $\beta = 0.01$, $f = M_{pl}$, $m = 1$ keV if not otherwise stated. We may also add here that in this work we do not propose or adopt any particular particle dark matter theory or theories for the choice of a viable dark matter candidate. As the purpose of this work is to explore whether the introduction of a dark energy–dark matter interaction term would be helpful in addressing the high $f$ problem for the pNGB potential of periodic nature (axion type $\sim (1 + \cos(\phi/f))$), we simply adopt here a dark matter with mass $m$ assuming this to be a viable candidate.

In Fig. 3 we plot the evolutions of dark energy equation of state $\omega_{DE}$ as a function of redshift $z$ for different initial values of the field ($\phi_i$ or $\pi_i$). We consider in Fig. 3a the quintessence dark energy model and in Fig. 3b the Slotheon field dark energy model. For both the cases it can be noted that for smaller values of $\frac{\phi_i}{f}$ (or $\frac{\pi_i}{f}$) the evolutions of $\omega_{DE}$ appears to resemble more to that for $\Lambda$CDM. Thus when initially the field is nearer to the top of the potential (from $V_{DE}(\phi)$ and $V_{DE}(\pi)$) it can be clearly observed that the potentials have their maximum value for $\frac{\phi_i}{f} = 0$ or $\frac{\pi_i}{f} = 0$ respectively, it will feel the steepness of the potential less severely and heads towards a slower rolling. We observe that if $\phi_i \simeq 0$ (or $\pi_i \simeq 0$) then $\omega_{DE}$ is almost equal to $-1$ all through, i.e., the field may not experience the slope of the potential and the effective dynamics is independent of $f$.

In Fig. 4, we plot the variations of the dark energy equation of state parameter $\omega_{DE}$ with redshift for both the dark energy models namely quintessence (the solid lines) and the Slotheon (dashed lines). It can be easily observed from the graph that the behaviour of $\omega_{DE}$ for the Slotheon model is closer to the same for $\Lambda$CDM model than what is obtained for the quintessence model. This is expected as the Slotheon field favours the slow roll criteria more than the quintessence field, which we have discussed earlier. It is also interesting to note here that for both the cases when interactions between dark matter and dark energy are considered ($\beta \neq 0$), the behaviours of the fields resemble more to the behaviour of $\Lambda$CDM model. In other words the fields better satisfy the slow rolling criteria.

In order to understand how sensitive the dark energy equation of state $\omega_{DE}$ and the dark energy density parameter $\Omega_{DE}$ at the present epoch are, to the variation of $f$, we calculate these variations using the formalism given in Sects. 3 and 4 and the results are plotted in Fig. 5a, b and in Fig. 6a, b.

In Fig. 5a (left panel of Fig. 5) the variations of the present value of the equation of state parameters $\omega_{DE}^0$ (value of $\omega_{DE}$ at redshift $z = 0$) with the spontaneous symmetry breaking scale $f$ for both the models quintessence (marked with solid lines) and the Slotheon model (marked with dashed lines) are shown. We not only compare the variations for both the dark energy models considered here but also the results we obtain when DM–DE interaction ($\beta \neq 0$) is included in the calculations and the case when such interaction is not considered ($\beta = 0$). We adopt the range of the present value of $\omega_{DE}$ to be $-1.038 \leq \omega_{DE}^0 \leq -0.884$ as given by PLANCK.
2018 [64]. It is to be noted that in all the four cases shown in Fig. 5a the calculated values of $\omega_{\text{DE}}$ lie well within the range of PLANCK for higher values of $f$ while the former deviates from this range for smaller values of the scale $f$. This is due to the fact that since $f$ is associated with a steepness of the pNGB potentials, pNGB potential tends to be flat as $f$ increases. A discussion is in order. For the quintessence case with $\beta = 0$ the value of $\omega_{\text{DE}}^0$ goes beyond the PLANCK range at $f < 0.4M_{\text{pl}}$ while for $f \geq 0.4M_{\text{pl}}$, $\omega_{\text{DE}}^0$ remains barely within the PLANCK range. The situation is much improved when DM–DE interaction is switched on ($\beta \neq 0$). In this situation $\omega_{\text{DE}}^0$ lies well within the PLANCK range even upto $f \sim 0.3M_{\text{pl}}$. A similar situation is observed for the Slotheon case too. But as seen from Fig. 5a, Slotheon results are always better than those obtained from quintessence since for both $\beta = 0$ and $\beta \neq 0$, the range of $f$ for which the $\omega_{\text{DE}}^0$ agrees with the PLANCK limit are always larger for the latter case. Similar conclusions can be drawn from Fig. 5b (right panel of Fig. 5) too where present value of dark energy density parameter $\Omega_{\text{DE}}^0$ (value of $\Omega_{\text{DE}}$ at $z = 0$) is plotted for various $f$ values and compare with PLANCK range given by [64] $0.678 \leq \Omega_{\text{DE}}^0 \leq 0.692$. In addition one also note that in case of $\Omega_{\text{DE}}^0$ (Fig. 5b) the Slotheon field results for both $\beta = 0$ and $\beta \neq 0$ are in better agreement with the PLANCK limit than those for quintessence considerations. Therefore from Fig. 5a, b we may conclude that the Slotheon field dark energy model with DM–DE interactions address the higher-$f$ problem most effectively.

In order to study how the nature of variation of $\omega_{\text{DE}}^0$ with $f$ changes for different “bare” dark matter masses $m$ in presence of DM–DE interaction ($\beta \neq 0$) we repeat the analyses shown in Fig. 5 for different choices of $m$. We found that when $m < 1$ keV the values of $\omega_{\text{DE}}^0$ for the chosen range of $f/M_{\text{pl}}$ do not lie within the PLANCK limit for $\omega_{\text{DE}}^0$. Again for $m > 1$ TeV all $\omega_{\text{DE}}^0$ values for the same choice of $f/M_{\text{pl}}$ range lie beyond the PLANCK limit for $\omega_{\text{DE}}^0$. In Fig. 6a, b we plot $\omega_{\text{DE}}^0$ vs $f/M_{\text{pl}}$ (Fig. 6a) and $\Omega_{\text{DE}}^0$ vs $f/M_{\text{pl}}$ (Fig. 6b) for the cases of Slotheon field and quintessence field for two values of “bare” dark matter masses namely 1 TeV and 1 keV and compare the results.

It is obvious that similar trend as in Fig. 5 is reflected in Fig. 6 too. Although even for small $f$-values the PLANCK result is satisfied for $\omega_{\text{DE}}^0$ for both the quintessence and Slotheon dark energy case when $m = 1$ keV (Fig. 6a) but for Slotheon case the variation of $\omega_{\text{DE}}^0$ lies below the quintessence case indicating that the Slotheon case satisfies the PLANCK limit better even if $f$ values are further lowered. For $m = 1$ TeV (higher value) the PLANCK range is generally satisfied for both the Slotheon and quintessence cases when $f$ is high but in this case also more entered lower range of $f$ can be explored for Slotheon model results than those for quintessence. From Fig. 6b, similar conclusions can be drawn by observing how the variations of $\Omega_{\text{DE}}^0$ with $f$ obey the PLANCK range. Therefore from Fig. 6a, b it is observed that the Slotheon field dark energy when coupled
6 Summary and discussions

In this work we explore the dark energy equation of state and dark energy density parameter in case of a dark matter–dark energy interaction where the dark energy is considered to have driven by a pNGB scalar field $\alpha$ with potential having a form $V(\alpha) = \mu^4(1 + \cos(\alpha f))$. A pNGB is produced when the Nambu Goldstone boson that arises due to spontaneous breaking of a global symmetry acquires a mass at soft explicit symmetry breaking scale lower than the spontaneous symmetry breaking scale $f$. We address in this work the high-$f$ problem of such dark energy models. The high-$f$ problem arises from the consideration that although large value of $f$ leads to a flatter potential, but it is not compatible with the field theory limits and quantum gravity corrections. In this work we show that when dark matter–dark energy interaction is taken into account, the calculated cosmological parameters agree much better with the observational results of PLANCK 2018 experiment for lower values of $f$ and then the high-$f$ problem can be averted. We show this for both the quintessence dark energy model and for another model namely the Slotheon dark energy model. The latter is inspired by the theories of extra dimensions such as the DGP theory. We also find that Slotheon dark energy model with the DM–DE interaction addresses better the high-$f$ problem than the quintessence model.

We also explore in the present framework of dark matter–dark energy interaction, the mass limits of dark matter that would produce dark energy equation of state and dark energy density parameters for low values of $f$. We found this limit to lie between $\sim 1$ keV and $\sim 1$ TeV. The PLANCK limit is found to be better satisfied for lower values of the mass of the dark matter (the lower limit being $\sim 1$ keV). Here too the Slotheon dark energy consideration appears to be better than the quintessence dark energy.

Differences among the three dark energy models we mention in this work (coupled quintessence dark energy model, coupled Slotheon dark energy model and $\Lambda$CDM model) can also be demonstrated by considering the statefinder parameters [65]. The statefinder parameters $\{r, s\}$ serve as a geometrical diagnostic pair to probe dark energy and defined as [65]

$$ r = \frac{\ddot{a}}{aH^2}, \quad (19) $$

$$ s = \frac{r - 1}{3(q - 1/2)}, \quad (20) $$

where deceleration parameter $q = -\frac{\ddot{a}}{aH^2}$. In the late time ($z < 10^4$) when the Universe is dominated by its two components namely the dark energy component and the matter component, the pair $\{r, s\}$ can be expressed as [66]

$$ r = 1 + \frac{9}{2} \Omega_{DE} \omega_{DE}(1 + \omega_{DE}) - \frac{3}{2} a \Omega_{DE} \frac{d\omega_{DE}}{da}, \quad (21) $$

$$ s = 1 + \omega_{DE} - \frac{1}{3} \omega_{DE} \frac{d\omega_{DE}}{da}. \quad (22) $$

For $\Lambda$CDM model, as $\omega_{DE} = -1$ is constant throughout the evolution of the Universe the pair $\{r, s\}$ takes the value $r = 1$ and $s = 0$. Hence any deviation of $r$ from 1 and $s$ from 0 would show the deviation of the nature of dark energy from cosmological constant and would reveal the dynamical nature of dark energy. In Ref. [67] authors showed that the quintessence models of dark energy are characterized by the values in the region $r < 1$ and $s > 0$.
In Ref. [68] Sahni et al. proposed an alternative route to distinguished $\Lambda$CDM model from other dynamical dark energy models. They introduced a new parameter $Om(z)$ to distinguished different dark energy models and defined it as

$$Om(z) = \frac{h^2(z) - 1}{(1 + z)^3 - 1},$$

where $h(z) = H(z)/H_0$. For $\Lambda$CDM model $Om(z) = \Omega_{0m}$ (matter density parameter at $z = 0$) while for quintessence dark energy ($\omega > -1$) $Om(z) > \Omega_{0m}$ whereas $Om(z) < \Omega_{0m}$ for phantom dark energy model ($\omega < -1$) [68]. Moreover in Ref. [69] the authors described the two point $Om$ diagnostic and three point $Om$ diagnostic in detail and show that different $Om$ diagnostics are suitable for analysing different cosmological observations.

In Fig. 7 we plot the variations of the statefinder parameter $r$ and $s$ with redshift $z$ and the variations of $Om(z)$ with $z$ for four different cases, namely minimally coupled quintessence model ($\beta = 0$, solid green line), coupled quintessence model ($\beta \neq 0$, solid red line), minimally coupled Slotheon dark energy model ($\beta = 0$, dashed purple line) and coupled Slotheon dark energy model ($\beta \neq 0$, dashed blue line). From the above discussion we have for $\Lambda$CDM $r = 1$, $s = 0$ and $Om(z) = \Omega_{0m} \simeq 0.3$. Therefore the plots in Fig. 7 clearly reveal the dynamical nature of these dark energy models. The plots (Fig. 7) also indicate the thawing nature [63] of dark energy models i.e., the models are identical with $\Lambda$CDM at early time but gradually deviate from it with time. The characteristics of quintessence dark energy models also are evident in Fig. 7 since we obtain $r < 1$, $s > 0$ and $Om(z) > \Omega_{0m}$ for quintessence model. In Fig. 7 the quintessence dark energy models and Slotheon dark energy models can also be distinguished from each other. Moreover it is clear from the plots in Fig. 7 that statefinder pair $\{r, s\}$ and $Om(z)$ diagnostic can distinguish between minimally coupled dark energy models ($\beta = 0$) and non-minimally coupled dark energy models ($\beta \neq 0$).

It may also be mentioned that the Slotheon dark energy model can not be ruled out from the LIGO detections of Gravitational Waves. From the LIGO observation of Gravitational Wave (GW) arising out of the neutron star merger (GW 170817), the GW speed is constrained as [70]

$$-3 \times 10^{-15} \leq (c_T/c - 1) \leq 7 \times 10^{-16},$$

where $c_T$ denotes the GW phase velocity and $c$ the speed of light which implies luminal speed of GW. The operational frequency of LIGO for the above detection of GW is 10–100 Hz [70]. In the same reference [70] this is also summarised that generically speaking the GW speed $c_T$ is obtained at the frequency it is measured. Therefore the GW speed at the measured frequency could be the constraint on the GW speed obtained from an effective field theory (EFT) such as Horndeski theory [71,72] of dark energy at that frequency. The Slotheon model of dark energy which arises out of the decoupling limit of DGP model also belongs to the Horndeski EFT model. Such a DGP theory can have UV completion [73] and the sound speed remains luminal without being influenced by the background configuration. The theory can yield luminal GW speed at LIGO at the frequency of LIGO [70]. Thus Slotheon dark energy model also could produce luminal GW speed at LIGO frequency and therefore could not be ruled out at present.

From the present analyses and calculations, this may be concluded that the high- $f$ value problem for pNGB potential can be better addressed if dark matter–dark energy interaction is present. Also the Slotheon dark energy model is better suited than the quintessence model for the purpose. In addition dark matter mass limit for such a scenario is estimated.
Fig. 7 Variations of $r$ with $z$ (top left), $s$ with $z$ (top right) and $\Omega m(z)$ with $z$ (bottom) for four different cases, namely minimally coupled quintessence model ($\beta = 0$, solid green line), coupled quintessence model ($\beta \neq 0$, solid red line), minimally coupled Slotheon dark energy model ($\beta = 0$, dashed purple line) and coupled Slotheon dark energy model ($\beta \neq 0$, dashed blue line).

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Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors’ comment: In this work we explained a formalism and computed numerical solutions related to the formalism by using Mathematica. Here no datasets were taken to produce our results.].

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