Analysis of time in establishing synchronization radio communication system with expanded spectrum conditions for communication with mobile robots

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Abstract. This paper analyzes the influence of the Doppler Effect on the length of time to establish synchronization pseudorandom sequences in radio communication systems with an expanded spectrum. Also, this paper explores the possibility of using secure wireless communication for modular robots. Wireless communication could be used for local and global communication. We analyzed a radio communication system integrator, including the two effects of the Doppler signal on the duration of establishing synchronization of the received and locally generated pseudorandom sequence. The effects of the impact of the variability of the phase were analyzed between the said sequences and correspondence of the phases of these signals with the interval of time of acquisition of received sequences. An analysis of these impacts is essential in the transmission of signal and protection of the transfer of information in the communication systems with an expanded range (telecommunications, mobile telephony, Global Navigation Satellite System GNSS, and wireless communication). Results show that wireless communication can provide a safety approach for communication with mobile robots.

1. Introduction

Radio communication signal with expanding spectrum was introduced to provide an essential and effective method of protection of transfer signals from interfering and interception. This method of protecting information is more reliable than the previous methods of protection by coding the radio communication signal content. Expansion of the spectrum can be realized by modulating the useful signal with a relative speed pseudorandom sequence, whose generation must be only known to certain users in order to protect information from interference and interception. To make it possible to extract the contents of NF useful signal (speech or some information) from expanded spectrum, it is necessary to generate an identical sequence in a receiver needed for modulation of the useful signal in the transmitter. It is necessary to perform receiver synchronization and locally generated PSS in the receiver. It should be noted that the sequence is pseudorandom and it is very complicated to execute their synchronization thus to enable the compression of the spectrum of the received signal and further extraction of useful information or digitized voice signals by detection which in certain circumstances must not be accessible to unauthorized persons.

These systems are used in certain communications, satellite and aviation systems, different platforms and systems for remote sensing and reconnaissance of facilities, terrain, space research, diplomacy and statesmanship of communication, in order to protect the content of that information which is important to users of communication systems with expanded spectrum. So, to be able to use these communication systems special attention must be paid to the choice of method for synchronizing of pseudorandom transmitting and received sequences, because only due to this synchronization the compression of the spectrum of expanded signal can be achieved from which useful signal can be detected. In literature, scientific and professional papers and publications in IEEE journals various
methods of synchronization of PSS for radio transmission were analyzed. Systems for obtaining expanding spectrum are the following:

- Systems with direct sequence – "Direct -Sequence", whose code is DS-SS and modulation performed with fast pseudo-random sequence,
- Systems with the frequency - ("Frequency-Hopping"), whose code is FH-SS, and transfer of information is based on the elements of a standard system for digital transmission of information with incoherent demodulation signal,
- Systems with a change period ("Time-Hopping"), whose code is TH-SS in which expanding the spectrum of the information signal is performed by changing its parameters in the time domain.
- Hybrid systems, whose code is FH / DS-SS, which are obtained by combining certain elements of the three systems explained above [1].

The first radio communication system for transmitting information in the extended range was manufactured in the early 50s of the last century in Lincoln's laboratories for the purposes of the United States' Army, in communications with satellites for research NASA spacecraft, the GPS systems, asteroid belt, and then in NATO in general. By applying the techniques of expanded spectrum a high degree of resistance to impact interference signals and electronic surveillance is achieved thus enabling the system to be used by pre-planned users. The first radio communication systems operated in the band 30-88 MHz, and newer systems in the 1710-1850 MHz and higher frequencies and the width of the frequency band for signal transmission order of 22.5 MHz and data transfer speed of order 32 kb/s to 216 kb/s. Radio communication systems with the PSS are a special class of communication system, characterized by the following characteristics [2]:

- Transmitted signal (an expanded signal) occupies a much wider frequency range than the minimum required to transfer a message,
- Expansion of the frequency spectrum in the transmitter is realized using auxiliary, pseudo-random signal (PSS), which is independent of the message and is known to the receiver. PSS is in principle expressed in digital form, and in the literature it is referred to as PSS sequence or pseudorandom code sequence.

Graphic spectra of the information signal,

\[ S_{ui}(f) \equiv S_{ul}(f) \equiv S_i(f) \]

and spectrum of PSS sequences \( S_{PSS}(f) \) is shown in Figure 1.

![Figure 1. Graphical representation of spectra of the information signal](image)

By applying expanding spectrum the following advantages over the standard signal transmission are achieved:

- Increased electronic reconnaissance of expanded signal,
- Disabled interception,
- The high degree of immunity to electronic interference,
• Improved quality of transmission in radio channels with multiple transfers,
• Multiplex is formed where all channels simultaneously work on the same frequency,
• Selectively addressing individual users of the system,
• The precise location of individual cells in the system and so on.
These solutions can be achieved in small communication systems in a limited space, as well as large systems involving tens of satellites.

2. The merits of signal expansion and compression
Expanding the spectrum is usually achieved by using binary pseudorandom code sequences, which can be reproduced by an authorized user [3]. This code allows the user of the communications system the identification and selective calling. With conventional radio transmitting spectral density of the useful signal \( S_i(f) \) is significantly higher than the spectral density of the noise power \( N_0 \) [2].

Nonexpanded signal occupies the spectrum width \( B_b \equiv n \). In systems with an expanded range \( S_{PSS}(f) \), the signal occupies quite a wider range \( B_{PSS} \equiv N \), wherein \( B_{PSS} \gg B_b \). The width ratio of the expanded and nonexpanded spectra, generates a factor of expansion or amplification process:

\[
\eta = \frac{B_{PSS}}{B_b}
\]  

Since \( B_{PSS} \gg B_b \), this process can gain great value, and it is thus appropriate to expressed this factor in decibels:

\[
\eta = 10 \log \left( \frac{B_{PSS}}{B_b} \right) = 10 \log \eta [dB] \equiv PG
\]

In practice, it is usually \( \eta = (20-60)[dB] \).

In practice, there is a number of different configurations systems with an expanded spectrum, and their principle of operation is based on a common concept. In fact, the signal spectrum \( S_i(f) \), which carries the information in the transmitter where the spectrum is expanded, is transformed by a certain procedure so expansion (enlargement) of its spectrum is executed. The spectrum of the expanded signal can be represented by the relation [4]:

\[
S_{PSS}(f) = e_{PSS} \left[ S_i(f) \right]
\]  

where \( e_{PSS} \left[ S_i(f) \right] \equiv e_{PSS} [\bullet] \) marks an operation for the expansion of the spectrum signal \( S_i(f) \). In the radio receiver system with PSS a reverse process occurs, i.e. the compression of the spectrum of the received signal \( S_{PSS}(f) \), using the same procedure \( e_{PSS} \left[ S_i(f) \right] \equiv e_{PSS} [\bullet] \), to give a spectrum signal \( \hat{S}_i(f) \) from which NF or useful signal can be extracted by detection. Compression can occur only if the PSS generation law in the receiver is exactly the same as in the transmitter. Otherwise, there will be no compression of signal and the useful signal will not possible to detect. The compression of the spectrum of the received signal in the receiver can be displayed in the following form:

\[
\hat{S}_i(f) = e_{PSS} \left[ S_{PSS}(f) \right] = e_{PSS} \left[ \left[ S_i(f) \right] \right] = S_i(f)
\]
From relation (4) we see that operation $\varepsilon_{\text{PSS}} \left[ S_i(f) \right] \equiv \varepsilon_{\text{PSS}} \left[ S_i(\bullet) \right]$ is inverse of itself, that is $\varepsilon_{\text{PSS}}^2 \left[ S_i(f) \right] \equiv \varepsilon_{\text{PSS}}^2 \left[ S_i(\bullet) \right] = 1$.

So, after a double application of the operation for the expansion of the signal $\varepsilon_{\text{PSS}} \left[ S_i(f) \right] \equiv \varepsilon_{\text{PSS}} \left[ S_i(\bullet) \right]$ to any signal, a signal obtains the original form, i.e. the compression of the spectrum of the received signal $S_{\text{PSS}}(f)$ from which NF useful signal can be detected only on this way. Since in the process of expanding the spectrum total energy $S_{\text{f}}$ of the expanded signal remains unchanged, a condition must be satisfied:

$$S = S_i(f) \cdot B_h = S_{\text{PSS}}(f) \cdot B_{\text{PSS}} = \hat{S}_i(f) \cdot B_h$$

(5)

Both modulator and demodulator must be of the same kind, wherein they may be of any type: AM, FM, SSB, PSK, FSK, etc. PSK is usually used.

By using standard procedure of modulation of RF carrier signal with NF useful signal (e.g. a speech signal) in the transmitter a modulated signal $S(t)$ is obtained, whose spectrum is $S_i(f)$.

Multiplying the resulting modulated signal $S(t)$ with a pseudorandom sequence $C(t)$, whose range is $S(f)$ we get an expanded signal whose spectrum is $S_{\text{PSS}}(f)$. In practice, this process is implemented with a balancing modulator. The output level of the transmitter and the receiver input stage include broadband amplifiers, filter, networks for adaptation and so on. The expanded signal $S(t) \cdot C(t)$, during the current propagation can interfered by a signal of narrowband interference $J(t)$ and noise signal $N(t)$ which is time shifted during propagation $\tau$.

In order to extract useful information from the received signal, it is necessary for the generator of PSS sequences in the receiver to generate the exact same sequence of PSS as in the transmitter and the time shifted during time propagation $\tau$, so that by multiplying the received expanded signal, superimposed on the noise and narrow-band disturbance yields:

$$\left[ S(t-\tau) \cdot C(t-\tau) + j(t) + N(t) \right] \cdot C(t-\tau) =$$

$$= S(t-\tau) \cdot C^2(t-\tau) + j(t) \cdot C(t-\tau) +$$

$$+ N(t) \cdot C(t-\tau)$$

Considering that:

$$C^2(t-\tau) = 1$$

This previous expression can be written as:

$$\left[ S(t-\tau) \cdot C(t-\tau) + j(t) + N(t) \right] \cdot C(t-\tau) =$$

$$= S(t-\tau) \cdot j(t) \cdot C(t-\tau) + + N(t) \cdot C(t-\tau)$$

This process of expanding and compressing signal is illustrated in Figure 2 [4], [5].
The total energy of the signal before and after expanding remains unchanged. These energies are defined with terms:

\[ \int S^2(t) \, dt \] - energy signal before expanding the spectrum,

\[ \int \left( S(t)C(t) \right)^2 \, dt = \left( \text{for} \ C^2(t) = 1 \right) = \int S^2(t) \, dt \] (6)

energy signals after expanding the spectrum.

From the expression for the energy signal before and after expanding the spectrum, we see that this is the same energy. Therefore, we have the case that the level of energy of expanded signal is lower than the level of noise signal (Figure 3).

**Figure 2.** Block diagram of the sending and receiving part of the RK system with expanded spectrum

**Figure 3.** Spectrum signal in the process of expanding a) before, b) after and c) after compression
In Figure 4, spectra of the modulation signal, carrier signal, PSS, narrowband interference, noise and expanded signal are respectively labeled with:

\[ X(f), S_i(f) = S(f), J(f), C(f), S(t)*C(t), \]

where convolution is marked with *.

Figure 3.c shows the spectra of waveform signal after the compression of the spectrum in the receiver, from which we see that the range of interference \( J(t) \) is extended so that within the bandwidth of MF filter receiver only a small part of the disturbance energy enters. The same happens with noise \( N(t) \), to which the operation of expanding spectrum is applied after compression, causing a reduction in the amount and its energy covered by narrowband MF filter.

The process of enlargement of the spectrum of the useful signal is illustrated in Figure 4 which is used in the form of a bipolar signal information (information string) and the pseudorandom sequence [6].

![Figure 4](image)

**Figure 4.** The process of enlargement of the spectrum of the useful signal.

Usually, the minimum duration of the interval of the information signal \( m(t) \) is referred to as bit, which will be denoted by \( T_b \). The duration of the minimum interval of PSS sequences \( c(t) \) will be denoted by \( T_c \), wherein \( T_c < T_b \). Usually, within an interval of the bit, the useful signal can be 1000 and more natural PSS of code intervals. Thus, it is typical that \( T_b \geq 10^3 T_c \). The common name for one PSS code interval is a chip. The duration of the smallest interval duration of those signals defines bandwidth information and code PSS signals respectively:
\[ B_b = \frac{1}{T_b} \quad B_c = \frac{1}{T_c} \] (7)

The analysis shows that the bandwidth of spectrum signals \( B_{\text{PSS}} \) is significantly higher than the bandwidth of the signal prior to enlargement (expanding) spectrum \( B_b \), and it is defined by the formula:

\[ B_{\text{PSS}} = B_b + B_c \] (8)

The factor of expanding or extending the range is defined by the expression:

\[ \mu = \frac{B_{\text{PSS}}}{B_b} = \frac{B_b + B_c}{B_b} \] (9)

Bearing in mind that \( B_c >> B_b \), this expression (9) can be written as:

\[ \mu \approx \frac{B_c}{B_b} \] (10)

Based on the definition of bandwidth of coding and information signal, the expression for a factor of expanding the spectrum can be written in the form:

\[ \mu \approx \frac{B_c}{B_b} = \frac{T_b}{T_c} \] (11)

which is approximately equal to the process gain defined with expressions (1) and (2).

Two effects occur in the receiver, namely:

- Compression of the received expanded signal \( S(t-\tau) \cdot C(t-\tau) \) by re-application operations \( \varepsilon_{\text{PSS}} \left[ S_i(f) \right] \equiv \varepsilon_{\text{PSS}} \left[ \bullet \right] \), which is practically implemented by ordinary multiplication of the modulated signal with \( C(t-\tau) \), which can be seen in Figure 2.

- Signal interference and noise by using operation \( \varepsilon_{\text{PSS}} \left[ S_i(f) \right] \equiv \varepsilon_{\text{PSS}} \left[ \bullet \right] \), i.e., with multiplication with signal \( C(t-\tau) \), the range of the width of the spectrum extending gets equal to \( B_{\text{J,N}} = B_c \).

Narrowband filter, whose bandwidth \( B_b \), passes through the energy of the useful signal \( S(t-\tau) \) and only part of the energy interference and noise. The strength of this total signal interference at the output of the narrow-band filter is reduced by a factor:

\[ \mu \approx \frac{B_c}{B_b} = \frac{T_b}{T_c} \] (12)

The question is how factor of expanding the spectrum is determined by the degree of protection from interference and electronic discovery of content information. For example, when the input of the receiver next to the useful signal \( S_{\text{PSS}}(f) \), comes also a narrowband disturbance \( J(f) \), as shown in Figure 2, then the total signal at the receiver input in the spectral domain can be written as:

\[ R(f) = S_{\text{PSS}}(f) + J(f) \] (13)

After the application of the operation \( \varepsilon_{\text{PSS}} \left[ R(f) \right] \equiv \varepsilon_{\text{PSS}} \left[ \bullet \right] \) of the received signal obtains:
\[
\hat{S}(f) = \varepsilon_{PSS}\{R(f)\} = \varepsilon_{PSS}\{S_{PSS}(f)\} + 
\]

\[
+ \varepsilon\{J(f)\} = S(f) + J_c(f)
\]

where \(J_c(f)\) are spectrum disorders, whose range by applying operations expanding the spectrum \(\varepsilon_{PSS}[R(f)\equiv \varepsilon_{PSS}[\bullet]]\) is extended to the level of the band \(B_{PSS}\). With this operation affecting the disturbance the power of spectral density interference for expanding is reduced in the amount of the coefficient of the (enlargement) spectra. If \(J(f)\equiv J\) denoted overall strength of interference, then the ratio of signal/fault within the bandwidth of the useful signal \(B_b\) after the operation of range compression is defined by the following expression:

\[
\left(\frac{S}{J}\right)_{yl} = \frac{\left(S\right)_{ul}}{B_b} = \mu \left(\frac{S}{J}\right)_{ul}
\]

From expression (15) we see that the ratio of the signal and the interference in the process of expanding the spectrum improves amounting to the factor of expanding (expansion) \(\eta\) of the spectrum.

### 3. Time of the establishment of synchronization with a single integrator in the presence of Doppler signal

When the Doppler signal is present, then two aspects that influence of interference signals occur in the system for establishing synchronization between the locally generated and received PSS sequence. The first is when the phase difference between the received and locally generated PSS sequence is variable with time. This phase difference will lead to the decrease or increase of the probability of detection \(P_D\), depending on whether the Doppler signal increases or decreases the phase difference between said sequences. The second dominant influence of the Doppler signal is on the length of the average time to establish synchronization. If the phase shift is caused by the Doppler signal in time \(\tau_d\), which is approximately the same step time of acquisition, i.e. the establishment of synchronization, then the average or median duration of time to establish synchronization decreases. An analysis of these impacts is quite a complex process, so the process of derivation of the equation for the time of establishing synchronization is quite extensive. If Doppler signal in the chip / second is characterized by \(\Delta f_c\), then the average length of time steps for search (acquisition) of correspondence between the received and locally generated PSS sequences is defined with the following expression [1]:

\[
\mu = \frac{N_u}{q} + \Delta f_c \cdot \tau_d + \Delta f_c \cdot \tau_d \cdot K \cdot P_{FA} =
\]

\[
= \frac{N_u}{q} + \Delta f_c \cdot \tau_d \left(1 + K \cdot P_{FA}\right)
\]

where is:

\(\mu\) - step search (acquisition) cells – parts of the searching signal

\(\frac{N_u}{q}\) - the length of step for acquisitions in the areas of chip signal when there is no activity of the
Doppler signal, 
\[ \Delta f_c \cdot \tau_d \] - phase shift code (sample of a sampled signal), originating due to the effects of the Doppler signal at a time, 
\[ \Delta f_c \cdot \tau_d \cdot K \] - phase shift code that is created during verification of wrong signal.

If in the equation for the mean time to establish synchronization (acquisition time) when the Doppler signal is not present (execution of this term can be seen in [6] and equation (16a)).

\[ \bar{T}_{ACQ} = \frac{(2 - P_D)(1 + P_{FA})}{2P_D} \cdot q_\tau_d \] (16a)

We change \( q = \frac{N_u}{\mu} \) and if the \( \mu \) classified with expression (16), we get:

\[ \bar{T}_{ACQ} = \frac{(2 - P_D)(1 + P_{FA})}{2P_D} \cdot \frac{1}{\mu} \cdot N_u \cdot \tau_d \]

\[ = \frac{(2 - P_D)(1 + P_{FA})}{2P_D} \cdot \frac{1}{\mu} \cdot N_u \cdot \tau_d \]

\[ = \frac{N_u}{q} \cdot \frac{(2 - P_D)(1 + K \cdot P_{FA})}{q} \cdot \frac{1}{\mu} \cdot N_u \cdot \tau_d \]

\[ = \frac{(2 - P_D)(1 + K \cdot P_{FA})}{q} \cdot \frac{1}{\mu} \cdot N_u \cdot \tau_d \]

\[ = \frac{\bar{T}_{ACQ}}{1 + \frac{q}{N_u} \Delta f_c \cdot \tau_d (1 + K \cdot P_{FA}) \cdot N_u, \tau_d} \]

Where \( \bar{T}_{ACQ} \) - acquisition time without the Doppler signal.

The same procedure is carried out in the expression for the variance of the time of establishing the synchronization, shown in [6] based on the definition of the time of acquisition, and application of the term to \( \bar{T}_{ACQ} \) and the condition that the number of searched phases \( K << q \), for which the variance of the acquisition of the total time, can be written in the form:
\[ \sigma_{ACQ}^2 = \tau_d^2 (1 + KP_{FA})^2 \cdot \left[ \frac{1}{12} + \frac{1}{P_D^2} - \frac{1}{P_D} \right] \cdot q^2 \] (17)

Introducing expression \( q = \frac{N_u}{\mu} \) and (16) for \( \mu \) we get the following expression for the variance of the total time of acquisition:

\[ \sigma_{ACQ}^2 = \tau_d^2 (1 + KP_{FA})^2 \cdot \left[ \frac{1}{12} + \frac{1}{P_D^2} - \frac{1}{P_D} \right] \cdot \left[ \frac{N_u}{\mu} \right]^2 = \]

\[ \tau_d^2 (1 + KP_{FA})^2 \cdot \left[ \frac{1}{12} + \frac{1}{P_D^2} - \frac{1}{P_D} \right] \cdot N_u^2 = \]

\[ \frac{N_u + \Delta_{fc} \cdot \tau_d (1 + K \cdot P_{FA})}{q}^2 \]

\[ \sigma_{ACQ}^2 \text{ without Doppler signal} \]

\[ 1 + \frac{q}{N_u} \Delta_{fc} \cdot \tau_d (1 + K \cdot P_{FA}) \]

\[ \sigma_{ACQ}^2 = \frac{\sigma_{ACQ}^2}{\left[ 1 + \frac{q}{N_u} \Delta_{fc} \cdot \tau_d (1 + K \cdot P_{FA}) \right]^2} \] (18)

Considering that \( \Delta_{fc} \) may be greater or less than zero, the Doppler signal can speed up or slow down the search (acquisition) of correspondence of the received locally generated pseudorandom sequence. Usually it is in the expressions (17) and (18).

\[ \Delta_{fc} \cdot \tau_d (1 + K \cdot P_{FA}) \ll \frac{N_u}{q} \]

So, these equations are reduced to equations in terms of the absence of the Doppler signal. When the Doppler signal \( \Delta_{fc} \left[ \text{chip/sec} \right] = 0 \), then the correspondence of these equations is hundred percent and \( q' = q \).

Mean value of the search of correspondence is calculated by means of analogy with equation (16) by replacing \( q \) with \( \frac{N_u}{\mu} \) and then the equations for the mean and variance of time are used to establish synchronization.
Based on the expression (16) we can conclude that if Doppler signal is not present, then the mean value of steps for search \( \mu_{sred} \) correspondence of the received and locally generated pseudorandom sequence is time-invariant. If in addition to the useful signal of information the Doppler signal is present, then the mean value of steps for search \( \mu_{sred} \) changes with time and this is a function of examined cells. Average values of searched steps \( \mu_{sred} \) of any cell (segment signal) being searched is equal to the nominal step results \( \frac{N_u}{q} \) (typical \( \frac{N_u}{q} = \frac{1}{2} \) for half a chip or half-cells results) of local PN generator plus the mean change in phase of the received signal \( \Delta_{fc} \) [chip/sec^2] denotes the value of the Doppler signal, then the Markov’s model set for the time interval search of correspondence of the received and locally generated pseudorandom sequence obtains:

\[
\mu_{n+1} = \frac{N_u}{q} + \Delta_{fc} \cdot \tau_d \left( 1 + K \cdot P_{FA} \right) + \\
\frac{1}{2} \Delta_{fc} \cdot \tau_d^2 \left[ 1 + P_{FA} (K^2 + 2K) \right] + (19)
\]

where \( n \gg 0 \) is a number of examined cells.

From the expression (19) we conclude that the mean of the search step is a linear function of the number of cell search. Because of this dependence on \( n \gg 0 \), we cannot directly replace \( q \) with \( \frac{N_u}{\mu} \) in the equation for the mean time to establish synchronization (acquisition time), when the Doppler signal is not present (execution of this term can be seen in [6] expression (16a.) in order to get the expression for median time to establish synchronization \( T_{ACQ} \) in the presence of the Doppler signal and the Doppler impact. It is more suitable to find the average mean of search steps \( \mu_{sred} \), which can be obtained by searching the average value \( \mu_{n+1} \), from equation (19) for all search cells \( q \), i.e. [7]

\[
\mu = \frac{1}{q} \sum_{n=0}^{q-1} \mu_{n+1}
\]

(20)

and then to introduce the previously proposed changes in the expression for the mean time to establish synchronization (acquisition time) when the Doppler signal is not present. From equations (19) and (20), the following expression can be reached:

\[
\mu_{n} = \frac{N_u}{q} + \Delta_{fc} \cdot \tau_d \left( 1 + K \cdot P_{FA} \right) + \\
\frac{1}{2} \Delta_{fc} \cdot \tau_d^2 \cdot q \cdot (1 + K \cdot P_{FA})^2 + \\
K^2 \cdot P_{FA} (1 - P_{FA})
\]

(21)

Since the overall mean of search steps must match the total number of searched chips (cells), it obtains that:
\[ q\mu = \sum_{n=0}^{q-1} \mu_{n+1} = N_u \]  

(22)

Here it is assumed that the amounts \( \Delta f_c \) and \( \Delta f_c \) are sufficiently small sizes so that the \( \mu_{n+1} > 0 \) for each \( n = 1,2, \ldots, q - 1 \) and so that the search is performed only in one direction, which is defined by changing the phase of the local PSS signal replacing \( q \) with \( \frac{N_u}{\mu} \) in equation (21) and after settling of the term, we get a quadratic equation with a variable \( \mu \):

\[
\mu^2 - \mu \left[ \frac{N_u}{q} + \Delta f_c \cdot \tau_d (1 + K \cdot P_{FA}) + \frac{1}{2} \Delta f_c \cdot \tau_d^2 \cdot K^2 \cdot P_{FA} (1 - P_{FA}) \right] - \frac{1}{2} \Delta f_c \cdot \tau_d^2 \cdot N_u \cdot (1 + K \cdot P_{FA})^2 = 0
\]

(23)

If the following changes are introduced to the equation (23)

\[
A = 1.
\]

\[
B = \left[ \frac{N_u}{q} + \Delta f_c \cdot \tau_d (1 + K \cdot P_{FA}) + \frac{1}{2} \Delta f_c \cdot \tau_d^2 \cdot K^2 \cdot P_{FA} (1 - P_{FA}) \right]
\]

(24)

\[
C = \frac{1}{2} \Delta f_c \cdot \tau_d^2 \cdot N_u \cdot (1 + K \cdot P_{FA})^2
\]

Then equation (23) can be written in the following form:

\[ A\mu^2 - B\mu - C = 0 \]

(25)

The solution of the quadratic equation is

\[
\mu = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(B) \pm B \sqrt{1 - \frac{4A(-C)}{(-B)^2}}}{2A} =
\]

\[
B + B \sqrt{1 + \frac{4AC}{B^2}} = \frac{2A}{2A} \Rightarrow
\]

respectively

\[
\mu = \frac{B}{2A} \left[ 1 + \sqrt{1 + \frac{4AC}{B^2}} \right]
\]

(26)
If in the expression for the mean time to establish synchronization \( T_{ACQ} \) the expression (26) is added, and if we replace \( q \) with \( \frac{N_u}{\mu} \), we obtain an expression for the time of acquisition:

\[
T_{ACQ} = \frac{(2 - P_D)(1 + KP_{FA})}{2P_D \cdot \mu} \cdot N_u \cdot \tau_d = \\
= \frac{(2 - P_D)(1 + KP_{FA})}{2P_D \cdot \frac{B}{2A} \left[ 1 + \sqrt{1 + \frac{4AC}{B^2}} \right]} \cdot N_u \cdot \tau_d
\]  

(27)

From this formula can be seen if \( P_D \) is larger, time acquisition \( T_{ACQ} \) will be less, but the amount of size \( P_D \) is affected by the Doppler signal. Analogly and for a variance of time for searching correspondence of the received and locally generated PSS sequences, we get the following expression:

\[
\sigma_{ACQ}^2 = \tau_d^2 (1 + KP_{FA})^2 \cdot \left[ \frac{1}{12} + \frac{1}{P_D^2} - \frac{1}{P_D} \right] \cdot q^2 = \\
= \tau_d^2 (1 + KP_{FA})^2 \cdot \left[ \frac{1}{12} + \frac{1}{P_D^2} - \frac{1}{P_D} \right] \cdot \left[ \frac{N_u}{\mu} \right]^2 = \\
= \tau_d^2 (1 + KP_{FA})^2 \cdot \left[ \frac{1}{12} + \frac{1}{P_D^2} - \frac{1}{P_D} \right] \cdot \left[ \frac{B}{2A} \left[ 1 + \sqrt{1 + \frac{4AC}{B^2}} \right] \right]^{-2}
\]  

(28)

If \( \Delta_{fc} = \Delta_{fc} = 0 \) it obtains that the coefficients are:

\[
A = \frac{N}{q}, \quad B = \frac{N}{q} \quad \text{and} \quad C = 0.
\]

Based on which we can conclude that the expressions (27) and (28) are reduced the expressions for acquisition time and time variance of search of correspondence of the received and locally generated PSS sequences without the presence of the Doppler signal, respectively. The basic idea of the systems with an expanded spectrum is to reduce the time of acquisition of correspondence and variance of time of search of correspondence of the received and locally generated PSS sequences, which can be achieved by reducing the integration time within the time interval \( \tau_d \). This is achieved by reducing the probability of false alarm, increasing the possibility of correct detection of \( P_D \), which is mathematically proven. It is shown that a lot of time of integration increases the possibility of correct detection of \( P_D \), but if in the process of establishing synchronization false alarms appear, the average time increases to establish synchronization for repeating the acquisition process. In order to reduce this time, it is convenient to introduce a system in which the time of the first integration is very short. After the expiry of the first time of integration at the output of the assembly decision, with one integrator (Figure 5) [8].

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Figure 5. The circuit for establishing synchronization of PSS sequences with one integrator or more integrators obtains a decision whether there was an establishment of synchronization or not. If synchronization is established, further search of sequence phase is terminated and the process is focused on maintaining synchronization of PSS sequences. The method of sequence detection is suitable for practical application for the establishment of synchronization, especially in the case of long sequences, when the integration time is longer, and when the probability of correct detection of $P_D$ increases and time to establish synchronization and establishing communication between the participants in a communication system [9].

4. Radio Frequency Communication for Mobile Robots
We used the suitability of Wireless Radio Frequency (RF) inter-module communication for mobile robots. Our hypothesis is that, instead of using Infrared (IR) and wired links, RF could be used for local and global communication. Given the need to protect the contents of important information and data in the field of communications, the most reliable systems are with an expanded spectrum, which is why developing these communication systems is specially oriented for communication with mobile robots. Wi-fi Hops about every 0.1 sec, Bluetooth is hopping every 625 microseconds and as fast as 3200 hops a second when initially connecting. Hopping Benefits are:

- Bandwidth can be utilized more efficiently,
- Security level (difficult to intercept)

In order to avoid collisions with other Bluetooth transmitters in the same area, Bluetooth uses pseudo-random generation algorithm to identify the frequency hopping pattern between two devices. [10].

The hopping pattern is based on the Bluetooth Device Address information, a unique identifier that is assigned to every Bluetooth transmitter.

**Bluetooth Physical Layer (RF)**
Utilizes the free license 2.4GHz ISM radio band
Pseudo-random frequency-hopping scheme with 1600 frequency hops per second (FHSS) 79 carriers ($f=2402+k$ MHz, $k=0,\ldots,78$)
Performing Gaussian Frequency Shift Keying modulation
Power classes:
- I: Max output power: 100mW (20dBm) => 100m
- II: Max output power: 2.5mW (4dBm) => 20m
- III: Max Output power: 1mW(0dBm) => 10m
5. Conclusions

Given the need to protect the contents of important information and data in the field of communications in the area of dedicated communications, as well as in the field of satellite systems, military systems for communications systems for remote sensing of the universe, diplomatic and statesman communications, the most reliable systems are with an expanded spectrum, which is why special attention has been paid to the development of these communication systems for more than half a century. An important aspect of these communication systems is acquisition and the establishment of synchronization between the received and locally generated pseudorandom sequence. If specified synchronization is not achieved, communication cannot be established and the content of information cannot go through nor there can be a possibility for communication between users of these radio communication systems. Therefore, due attention is paid to the development of a system for establishing synchronization using the sliding correlator with one or more integrators. Simulation of synchronization of PN sequences in the process of establishing synchronization is performed in C++. Comparing RF communication with the other technologies we found that the main difference between RF, Infrared or Bluetooth is that RF can provide reliable local and global communication without regard to the inter-module docking orientation, mechanism and number of modules connected. Infrared can only provide neighbor-to-neighbor communication and suffers from module misalignment issues, while Bluetooth can communicate locally and globally but requires a central module that constrains the size of the network. With Radio Frequency we did not have the above problems to communicate with mobile robots [11].

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