On decay of bubble of disoriented chiral condensate

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Abstract

We discuss the space-time structure for the process of decay of a bubble of hypothetic phase – disoriented chiral condensate (DCC). The evolution of the initial classical field configuration corresponding to the bubble of DCC is studied both numerically and analytically. The process of decay of this initial configuration depends crucially on selfinteraction of the pionic fields. It is shown that in some cases this selfinteraction leads to the formation of sort of breather solution, formed from pionic fields situated in the center of the initial bubble of DCC. This breather looks like a long-lived source of pionic fields.

I. INTRODUCTION

Disoriented chiral condensate (DCC) is nothing more than a piece of vacuum, chirally rotated from its usual orientation in internal symmetry space. The history of the subject is rather old [1–4]. Formation of a large domain with DCC has been proposed by Anselm [5], by Blaizot and Krzywicki [6] and by Bjorken, Kowalski and Taylor [7]. The more detailed lists of references devoted to this subject one may find in the more recent publications, see, e.g. Refs. [8,9].

Usually people discuss the radiation of pionic fields from initially formed bubble of DCC. The most important feature of this radiation is semiclassical coherent nature of the initial pionic configuration. As usual, the four fields $\sigma$ and $\vec{\pi}$ are constrained by the relation

$$\sigma^2 + \vec{\pi}^2 = f_\pi^2. \quad (1)$$
Usually one supposes that in ordinary vacuum \( \langle \sigma \rangle = f_\pi \). The DCC-hypothesis means that inside the DCC bubble the chiral vacuum does not point in the \( \sigma \) direction but rather in one deflected toward one of the \( \pi \) directions.

If the event-by-event deviation of the chiral orientation from the \( \sigma \) direction is random, it follows that the distribution of neutral fraction

\[
f = \frac{N_{\pi^0}}{N_{\pi^+} + N_{\pi^-} + N_{\pi^0}} = \frac{N_{\pi^0}}{N}
\]

is inverse square root, as \( N \) becomes large:

\[
\frac{dP}{df} = \frac{1}{2\sqrt{f}}.
\]

At the same time the "standard" non-coherent mechanisms (like we get at high energies) predict the probability

\[
\frac{dP}{df} = \delta(f - 1/3).
\]

The distribution (3) is the most bright and evident signal of DCC-phase.

It would be interesting to find out what are other consequences in terms of observables for decay of the DCC bubble. In other words, what is the space-time picture of the process of a DCC bubble decay? To answer this question, let us consider the simplified model with only a pair of fields \( \sigma \) and \( \pi \) under constraint (1). In this case the choice \( \langle \sigma \rangle = f_\pi \) means "ordinary" vacuum state. It is more suitable to introduce new angular variable \( \phi \): \( \pi = f_\pi \cos \phi, \sigma = f_\pi \sin \phi \). In terms of this new field \( \phi \in [0, 2\pi] \) vacuum is degenerate. So we have \( U(1) \) symmetry in the vacuum sector.

It is evident, that if we are limiting ourselves by noninteractive fields only, the evolution of any initial field configuration in terms of \( \phi \) field is to satisfy the equation

\[
\phi_{tt} - \Delta \phi = 0.
\]

At the same time one may introduce some violation of the chiral symmetry, by adding mass of "pion" in this equation, namely

\[
\phi_{tt} - \Delta \phi + m^2 \phi = 0.
\]

This assumption is equivalent to the condition that the theory has only one real vacuum, \( \langle \sigma \rangle = f_\pi, \langle \pi \rangle = 0 \) and any other position on the circle \( \sigma^2 + \pi^2 = f_\pi^2 \) is not the real vacuum state. Nevertheless, the bubbles with nonzero expectation value of \( \phi \) could be initially formed, and we may discuss properties of decay of such states. One may consider also a more complicated case of the structure of vacuum, considering the situation with more than one vacuum state. This situation may be approximated by the following equation of motion:

\[
\phi_{tt} - \Delta \phi + \frac{1}{2}m^2 \sin 2\phi = 0.
\]

There are two vacuum states in this theory with \( \phi = 0 \) and \( \phi = \pi \) \((0 \leq \phi \leq 2\pi)\). This form of potential in terms of the fields \( \sigma \) and \( \pi \) resembles a Mexican folk hat though arranged in the Texas style with back and front brims curved down.
As we shall demonstrate, the decay of the DCC bubble looks quite different in these three cases (4), (5) and (6). That is why it is possible to get information about real interaction in the system studying the decay of DCC bubbles. Evidently, cases (4)-(6) give us only some examples of selfinteraction and the physical picture may be more complicated. Nevertheless, limiting ourselves by these examples, we get some general features of the decay of DCC.

II. ONE-DIMENSIONAL CASE

Let us consider first for pedagogical reason the evolution of the initially formed bubble of DCC in (1+1)-dimensions.

a) **Massless case.** The solution of the equation of motion (4) is trivial in this case

\[ \phi(x, t) = \frac{1}{2} [\phi_0(x - t) + \phi_0(x + t)] , \]

where \( \phi_0(x) \) is the initial field configuration. Taking the initial field configuration in the form

\[ \phi(x, 0) \equiv \phi_0(x) = \frac{\phi_0}{1 + (x/a)^2}, \phi_t(x, 0) = 0 , \]

\( \alpha \gg 1 \) is positive integer,

we just immediately get from Eq. (7) that a detector being placed at the distance \( X \gg a \) from the position of the domain of DCC, will fix the flux outgoing from the region of DCC after the time interval \( t \sim X/c \). The signal lasts the period \( \tau \sim 2a/c \) and consists of two separate peaks corresponding to the forward and backward walls of the initial bubble passing across the detector. Concentration of energy in the boundaries reflects only the fact, that vacuum is exactly degenerate, so all the excess of energy of the DCC bubble is concentrated on its surface due to the gradient terms in the expression for energy density of the system.

b) **Massive case.** The situation is getting only slightly more complicated in the massive case (5). In this case the solution of the problem can also be obtained analytically as Eq. (5) is linear. Time evolution of the initial configuration \( \phi_0(x) \) (8) depends on the value of the dimensionless parameter \( \xi = ma \).

The case \( \xi \ll 1 \) looks very similar to the massless case with the only exception. The interior region of the bubble carries also some part of energy (volume energy). This is the consequence of the chiral symmetry violation in the vacuum sector. The solution in the interior region oscillates in time. Also it is worth mentioning, that practically all the flux of energy is transferred with near-to-light velocity. There present also some more slowly moving tail, but its energy is negligibly small under condition \( \xi \ll 1 \). The situation changes with increasing \( \xi \). Part of energy, carried with near-to-light velocities, is decreasing. At the same time the process of emitting of slowly moving particles dominates. The flux of energy through the point \( X = 20 \) as a function of time is represented in Fig. 1 for several values of \( \xi \). Obviously, that \( \xi = 0 \) corresponds to the massless case (4).

c) **sine-Gordon case.** The situation is getting more complicated in the case of dynamics with degenerate vacua, described by Eq. (6). In this case the process of bubble evolution depends on the initial mean value of \( \phi \) in the interior region of the bubble.

i) Consider first the case of small \( \phi_0 \): \( |\phi_0| \ll 1 \). In this case the decay of the bubble is similar to one in massive case b). The only difference is in formation in the center of the
bubble a sort of breather solution of small amplitude. This breather solution is a well-known breather of the sine-Gordon equation \[^{10}\]. It is stable and it keeps part of the initial energy of the bubble in the center. So not all the initial energy of the DCC is emitted.

ii) In the case $|\phi_0 - \pi| \ll 1$ the field oscillations take place already not around $\phi = 0$, as for $|\phi_0| \ll 1$, but around $\phi = \pi$ (second local minimum in the system). If we take initial field configuration in the form of (8) with $\phi_0 = \pi$, $\xi = ma \gg 1$, we get that for large times the field configuration will look like a far situated kink-antikink pair

$$\phi_{KK} \approx 2 \arctan \exp[m(x + a)] - 2 \arctan \exp[m(x - a)]$$

with some small oscillations localized in the center near $x = 0$. These oscillations are due to the initial condition (8) for field configuration and they leave the region of interaction relatively fast. If $|\phi_0 - \pi| \neq 0$, we observed the formation of a breather-like solution of small amplitude in the central region. We would like to emphasize, that in the selfinteraction case considered above part of energy, concentrated initially in the DCC bubble, is not emitted at all but forms special stable localized solution called breather. Conservation of part of energy of the DCC inside the bubble is indeed the direct consequence of integrability of Eq. (6) in one-dimensional case.

### III. THREE-DIMENSIONAL CASE

Solution of Eq. (4) in spherically symmetric three-dimensional case is also trivial. Really, making substitution $\phi(r) = w(r)/r$, we get

$$w_{tt} - w_{rr} = 0 , \quad (9)$$

so the solution is known analytically, see Eq. (7). The situation looks analogously in the massive case (5), and we get for $w(r, t)$ in this case:

$$w_{tt} - w_{rr} + m^2 w = 0 . \quad (10)$$

The situation with the sine-Gordon equation looks more complicated in three-dimensional case, namely we get for $w(r, t)$ the following equation:

$$w_{tt} - w_{rr} + \frac{1}{2} m^2 r \sin(2w/r) = 0 . \quad (11)$$

Later on we shall consider the bubbles according to Eqs. (9)-(11) with initial conditions, chosen in the form

$$\phi(r, 0) = \frac{\phi_0}{1 + (r/a)^{2\alpha}} , \quad \phi_t(r, 0) = 0 , \quad (12)$$

where $\alpha$ is positive integer, as in (8). The behavior of the energy flux in cases (9)-(11) is shown in Fig. 2. The general picture looks similar to that of one-dimensional case. Nevertheless, some comments are needed when comparing results for the sine-Gordon equation in one-dimensional and three-dimensional cases. In one-dimensional case the breather solution is stable. In three-dimensional case with $|\phi_0| \ll 1$ numerically we also observed formation of sort of breather in the center of the initial bubble. This three-dimensional breather was
already found long ago [11]. It is known, that this solution is quasi-stable and it decays slowly, emitting radial waves. That is why the process of decay of the DCC with dynamics corresponding to Eq. (11) has two stages. During the first stage the main part of energy is emitted. This first stage is finished by the formation of breather in the center of the initial bubble. Afterwards the emission of waves becomes a slow process. We solved Eq. (11) also with the following initial condition:

$$\phi(r,0) = 2 \arctan \exp[-m(r-a)], \quad \phi_t(r,0) = 0.$$  (13)

Initial condition (13) corresponds to the minimum of energy of the field which links two vacua of the theory. Initial condition (12) does not correspond to this minimization procedure. It has additional excess of energy, especially if $\alpha \gg 1$. This excess of energy may be considered as excitation over soliton (13). In the sine-Gordon case this excitation belongs to the continuum and may be emitted from the soliton region. Thus, the evolution of the bubble is similar to the case of initial condition of the type of (13) with the only difference. The initially formed bubble with $\phi = \pi$ vacuum inside ($\phi_0 = \pi$ in (12)) is not only collapsing, but emitting radial waves from its boundary too.

The time dependence of the field $\phi(r,t)$ at the origin and the flux of energy from this solution under initial condition (13) are represented in Fig. 3. The time dependence of $\phi(0,t)$ may be conventionally separated into two periods. The first one $0 < t \leq T_0$ is characterized by oscillations of large amplitude and large damping. The second period $t \geq T_0$ ($T_0 \sim 200$ in our units) may be characterized by oscillations with much smaller amplitudes (breather). The damping is also small at this stage of evolution.

The ratio of energies being emitted during the first fast (gross-structure) and the second slow (breather) evolution stages of the decay depends on the initial configuration of the bubble. It is worth mentioning that the evolution of a spontaneously formed spherical bubble was studied long ago in papers [12] for $\lambda \phi^4$-field theory. The main conclusion of those papers was that the bubble of “wrong” vacuum with the initial radius $R_0 \gg 1$ starts to collapse. The collapse lasted the time $T \simeq 1.3R_0$, which means that the walls of the bubble are moving toward the center at nearly the speed of light.

The pulsations of large amplitude (gross-structure) were found for $\lambda \phi^4$-theory in a narrow vicinity of the initial radius $R_0 = 3.875$ [3,12], see also review [13]. In contrast to the case of $\lambda \phi^4$-theory for the sine-Gordon model we observed the gross-structure shown in Fig. 3 in the wide range of the initial size of the bubble: $5 \leq a \leq 50$ for $m = 1$. We have not performed calculations for larger $a$, but there seems to be the same gross-structure for all $a \gg 1$. For smaller $a \leq 5$ we observed the formation of a breather solution just after shrinking of the initial configuration.

So if we consider the picture of decay of the DCC bubble of small amplitude, with $|\phi_0| \ll 1$, it expands and nothing drastical happens. But in the case of decay of bubbles with large amplitudes $|\phi_0 - \pi| \ll 1$, the picture of decay looks quite different and it is similar to the $\lambda \phi^4$-theory case, studied long ago.

IV. CONCLUSION

We studied the space-time structure for the decay of DCC bubble. The main point is that characteristics of this decay depend crucially on selfinteraction in the system of pionic
fields. Namely, presence of selfinteraction (nonlinear effects) leads to the formation of sort of breather solution in the center of the initially formed bubble of DCC. This breather solution decays slowly and keeps part of energy in the central region. That is why the decay of the bubble is not instant \((\tau \sim a/c)\), but rather long process. The detailed picture of this decay depends on details of selfinteraction in the system as well as on the initial conditions (amplitude and size of the bubble.) That is why studying experimentally the space-time structure of the signal followed the decay of the DCC bubble, one may get limitations on the form of selfinteraction potential in terms of chiral fields.

As the decay of the DCC bubble may be experimentally observed by fixing the flux of outgoing pions, it is worth to stress once again, that this flux is to follow the time dependence drawn in Fig. 3b. So we may find not only the prompt signal from decaying DCC, but also some delay pions emitted via the breather decay.

Notice, that in spite of the fact that we studied the most trivial case of the two-component \(\sigma\)-theory, our result is indeed more general. Really, one may consider theory with three pions and one sigma. In this case vacuum sector is \(S^3\)-sphere. But the evolution of the system will take place along the ”large circle” of this \(S^3\)-sphere. That is why the time evolution of the initial field configuration looks like in \(S^1\)-case, studied in this our paper.

In this paper we didn’t study the influence of selfinteraction on charge distribution. We think, that this distribution is to be independent on this interaction and is given by Eq. (3).

It is also worth mentioning, that the space-time structure of signal from decay of the DCC bubble was the subject of discussion in papers \([16,17]\). The authors of paper \([16]\) consider the evolution picture for DCC phase in terms of linear sigma model. In paper \([17]\) decay of the DCC bubble with both linear and nonlinear \(\sigma\)-models was studied. In contrary to papers \([16,17]\) in this our paper we concentrated on the formation of a long-lived breather-like solution in the center of the bubble. The existence of this solution is a consequence of selfinteraction of the fields in vacuum sector.

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FIGURE CAPTIONS

Fig. 1. Flux of energy (in units of total energy) from DCC bubble in one-dimensional case for different \(\xi = ma\) as a function of time \((\alpha = 3)\).

Fig. 2. Flux of energy as a function of time for three-dimensional case:
a) Eq. (10) with initial conditions (12), \(\alpha = 5, a = 1\), flux through the sphere \(r = 4\);
b) Eq. (11) with initial conditions (12), \(\alpha = 25, a = 10, \phi_0 = 0.1\), flux through the sphere \(r = 20\);
c) Eq. (11) with initial conditions (12), \(\alpha = 25, a = 10, \phi_0 = \pi\), flux through the sphere \(r = 20\).
Fig. 3. Three-dimensional sine-Gordon case (11) with initial conditions (13), $a = 20$, $m = 1$:
a) time dependence of the field $\phi(r, t)$ at the origin $r = 0$;
b) flux of energy as a function of time in this case.
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Fig. 1

Flux vs. time for different values of $\xi$: $\xi=0$, $\xi=0.5$, $\xi=2$, $\xi=5$.
Fig. 2a
Fig. 2b
Fig. 2c

Flux vs. time for different values of $\xi$: $\xi = 0.1$, $\xi = 3$, and $\xi = 5$. The graph shows the evolution of flux with time for these values.
Fig. 3a
Fig. 3b