Dynamic Model for Determining Disaster Evacuation Locations with Game Theory

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Abstract. Disaster management can be grouped into three stages with several activities that
can be carried out starting from pre-disaster, during emergency response and post-disaster. In
the pre-disaster stage one of which is mitigation, which is a series of efforts to reduce disaster
risk, both through physical development and awareness raising and capacity to face the threat of
disaster. In mitigation one important factor is about evacuation. Evacuation locations in disaster
transportation management are urgently needed for an effective evacuation process. Evacuation
location determination is a benchmark in handling evacuation that can minimize the impact of
losses from the disaster. To determine the evacuation location dynamically using game theory
can solve the problem by providing several alternative locations by showing the dominant factor
in the variables in the selection of the evacuation site. Game theory will use pure and mixed
strategies to provide the most optimum choice of criteria. With this method will produce a
decision support system to determine the optimal evacuation location or safe area in disaster
mitigation

1. Introduction
Disaster is an event or series of events that threaten and disrupt people’s lives and livelihoods
casted, both by natural factors and / or non-natural factors as well as human factors resulting
in human casualties, environmental damage, property losses and psychological impacts (Law
No. 24 of 2007 concerning Disaster Management). Evacuation is the process by which people
are placed from dangerous places to safer places to reduce health problems and people’s lives
that are vulnerable to being affected (Saadatseresht et al., 2009) (Mei et al., 2016) (Coutinho-
Rodrigues et al., 2012). Disaster management as a process of evacuation of victims and an early
warning system for disaster arrivals is slow due to difficulties in determining the location of the
evacuation. The selection of evacuation sites in mitigation must consider the places that are
available and based on geographical data from the disaster area. Each disaster affected area
has different characteristics and this needs to be considered during evacuation. Considerations
in the evacuation process are calculations of the number of people who can be mobilized, the
capacity that can be carried, and the route chosen to go to safety(Campos et al., 2012). Bringing
and directing people to safe areas is a major challenge in managing evacuation areas (Kendra
et al., 2008). Evacuation is characterized by an immediate and rapid movement of people who
are in unsafe locations or disaster locations to safer locations (Hamidazada et al., 2019). The important thing to note is that there is no congestion and chaos during the evacuation process (LIM et al., 2013).

In (Rogers & Luna, 2004) One of the anticipatory actions that can be taken before a disaster comes is to determine the location of the evacuation or a place to stop for disaster relief and disseminated to the wider community to accelerate the process of evacuation of disaster victims so as to minimize losses from the disaster, Geographical Information Systems (GIS) will be utilized with use the Overlay and Buffer functions and proceed with calculations using the Simple Additive Weighting (SAW) method which is expected to determine the best evacuation location so that it can be taken into consideration and help related agencies such as BPBD in order to improve mitigation. Game Theory is a mathematical model used in situations of conflict or competition between various interests that face each other as competitors (Kondratev & Mazalov, 2020). This theory was developed to analyze the decision making process of different competitive situations, and involves two or more interests (Bennett, 1995). Applying game theory will produce an optimal evacuation location in the decision support system in disaster mitigation (Li et al., 2014).

2. Game Theory with Linear Programs

To solve mixed strategy games with dimensions or larger, one can use a linear program by transforming the maximal and minimax equations into a linear programming form (Singh & Tech, 2010). The method that will be used in this research is a linear program method which can be formulated generally as follows:

\[ X = X_1, X_2, \ldots, X_m \text{ dan } Y = Y_1, Y_2, \ldots, Y_n \]

which optimizes the mathematical expectation value

\[ E = (X, Y) = \sum_{i=1}^{m} \sum_{j=1}^{n} \alpha_{ij} X_i Y_j \]

with the provision of

\[ \sum_{i=1}^{m} X_i = \sum_{j=1}^{n} Y_j = 1 \]

\[ X_i, Y_j \geq 0; \text{ for all } i \text{ and } j. \]

In solving the problem of game theory using a linear program using the simplex method, namely forming a linear program and finding the optimum solution (Kirtiwant; P. Ghadle, 2014). The steps in linear programming game theory are as follows:

2.1. For P1 Players (Line Players)

Player I is a line player (maximizing player), so it can be expressed hope of winning player I in a sign of greater inequality. This means that player I might get more wins than V if player II
uses a weak strategy. So the expected value of winning player I is:

\[
\sum_{i=1}^{m} \alpha_1 X_i \geq V, \\
\sum_{i=1}^{m} \alpha_2 X_i \geq V, \\
\sum_{i=1}^{m} \alpha_n X_i \geq V \text{ and} \\
\sum_{i=1}^{m} X_i = 1 \\
X_i \geq 0 \text{ for all } i = 1, 2, ..., m
\]

By dividing the above inequality with V, we get:

\[
\sum_{i=1}^{m} = \frac{\alpha_1 X_i}{V} \geq 1, \\
\sum_{i=1}^{m} = \frac{\alpha_2 X_i}{V} \geq 1, \\
\sum_{i=1}^{m} = \frac{\alpha_n X_i}{V} \geq 1, \\
\sum_{i=1}^{m} X_i = 1 \frac{1}{V}, X_i \geq 0, \text{ for all } i = 1, 2, ..., m
\]

From Example:

\[
\sum_{i=1}^{m} = \alpha_1 X_i \geq 1, \\
\sum_{i=1}^{m} = \alpha_2 X_i \geq 1, \\
\sum_{i=1}^{m} = \alpha_n X_i \geq 1, \\
\sum_{i=1}^{m} X_i = 1 \frac{1}{V}, X_i \geq 0, \text{ for all } i = 1, 2, ..., m
\]

Because player I is the maximizing player, the objective function is to maximize the value of V or equal to minimize $\frac{1}{V}$, then the linear program for player I can be formulated as follows:

\[
Maks V = Min \frac{1}{V} = Min \sum_{i=1}^{m} \frac{X_i}{V} = Min \sum_{i=1}^{m} X_i
\]

From equation $X_1 + X_2 + ... + X_m = \frac{1}{V}$, The linear player program for player I is: Min $0 = X_1 + X_2 + ... + X_m$, With restrictions:
\[ a_{11}X_1 + a_{21}X_2 + \ldots + a_{m1}X_m \geq 1 \\
\]
\[ a_{12}X_1 + a_{22}X_2 + \ldots + a_{m2}X_m \geq 1 \\
\]
\[ a_{1n}X_1 + a_{2n}X_2 + \ldots + a_{mn}X_m \geq 1 \]
\[ X_i \geq 0, i = 1, 2, \ldots, m \]

Where \( f_0 = \frac{1}{v} \) dan \( X_i = \frac{X_i}{v} \)

2.2. For \( P'^2 \) Players (Column Players)
Player II is a column player (minimizing player), so it can be expressed hope of winning Player II in a smaller sign of inequality. This means that Player II may suffer less than V if Player I uses a weak strategy. So the expected value of winning player II is:

\[ \sum_{j=1}^{n} \alpha_{1j}X_j \leq V, \]
\[ \sum_{j=1}^{n} \alpha_{2j}X_j \leq V, \]
\[ \sum_{j=1}^{n} \alpha_{mj}X_j \leq V \text{ and} \]
\[ \sum_{j=1}^{n} Y_j = 1 \]
\[ Y_j \leq 0 \text{ for all } j = 1, 2, \ldots, n \]

By dividing the above inequality with V, we get:

\[ \sum_{i=1}^{n} \frac{\alpha_{1i}Y_i}{V} \leq 1, \]
\[ \sum_{i=1}^{n} \frac{\alpha_{2i}Y_i}{V} \leq 1, \]
\[ \sum_{i=1}^{n} \frac{\alpha_{mi}Y_i}{V} \leq 1, \]
\[ \sum_{i=1}^{n} \frac{Y_i}{V} = \frac{1}{V}, Y_i \geq 0, \text{ for all } i = 1, 2, \ldots, n \]
Example: \( Y_i = \frac{Y_j}{v} \); \( j = 1, 2, \ldots, n \)

Then

\[
\sum_{j=1}^{n} \alpha_i Y_j \leq 1, \\
\sum_{j=1}^{n} \alpha_i^2 Y_j \leq 1, \\
\sum_{j=1}^{n} \alpha_i^m Y_j \leq 1,
\]

\[
\sum_{j=1}^{n} Y_j = \frac{1}{v}, Y_j \geq 0, \text{forall} j = 1, 2, \ldots, n
\]

Because Player II is a minimizing player, the objective function is to minimize the value of \( V \) or equal to maximize \( \frac{1}{v} \), then the linear program can be formulated for player II as follows:

\[
Min V = Maks \frac{1}{v} = Maks \sum_{j=1}^{n} \frac{Y_j}{v} = Maks Y_j
\]

From equation \( Y_1 + Y_2 + \ldots + Y_n = \frac{1}{v} \), The linear player program for player II is: \( \text{Min} g_0 = Y_1 + Y_2 + \ldots + Y_n \)

With restrictions:

\[
a_{11} Y_1 + a_{21} Y_2 + \ldots + a_{n1} Y_n \leq 1 \\
a_{12} Y_1 + a_{22} Y_2 + \ldots + a_{n2} Y_n \leq 1 \\
a_{1m} Y_1 + a_{2m} Y_2 + \ldots + a_{nm} Y_m \leq 1 \\
Y_j \geq 0, J = 1, 2, \ldots, n
\]

Where \( g_0 = \frac{1}{v} \) dan \( Y_j = \frac{V_j}{v} \)

The linear program formulation in player I is dual from player II and vice versa. Obtained an optimal strategy for player I to be the optimal strategy for player II. Furthermore, to solve this linear program problem can be solved by using the simplex method.

3. Results and Discussion

Analysis of game theory with pure strategy explains the results of calculating the meeting point between the maximum and minimum values at the value of pay off called the saddle point. The pay off value is obtained from the reduction between the first player and the second player, in this case obtained based on the results of the questionnaire. The strategies used are as follows:

- **L1** = Evacuation Location 1
- **L2** = Evacuation Location 2
- **L3** = Evacuation Location 3
- **Ln** ... = Evacuation Location n
- **x1** = Altitude (The higher the evacuation site the better)
- **x2** = Number of Population (The less number of population the better)
- **x3** = Length of evacuation route (The shorter the evacuation route the better)
- **x4** = Hierarchy of roads (The fewer bends, the better)
- **x5** = type of transportation (the more modes used the better)
x6 = Evacuation Place Area (The wider the evacuation area the better)

Players and strategies are used to determine the results of the calculation of pay-offs that show the optimal solution for the strategies used in evacuation to achieve benefits in determining the location of the evacuation. The count of each player is explained by comparing the evacuation location 1 (L1) with evacuation location 3 (L3), evacuation location 2 (L2) with evacuation location 3 (L3), and evacuation location 1 (L1) with evacuation location 2 (L2). At each evacuation site get a strategy that can be used to achieve profits in evacuation, with the acquisition value of the player comparison game as follows:

3.1. Evacuation Location 1 (L1) with Evacuation Location 3 (L3)

|   | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 |
|---|----|----|----|----|----|----|
| X1 | 10 | 10 | 33 | 28 | 24 | 39 |
| X2 | 0  | 39 | 11 | 27 | 9  | 19 |
| X3 | 4  | 36 | 19 | 40 | 22 | 18 |
| X4 | 55 | 17 | 15 | 20 | 34 | 8  |
| X5 | 20 | 33 | 25 | 10 | 32 | 11 |
| X6 | 31 | 17 | 40 | 21 | 28 | 26 |

In the table above, it can be seen that the smallest value (maximin) in the row is -20 while in the column player the greatest value (minimaks) is 35. The maximin value is not the same, so the game does not have a saddle point so the next step will be to find the balance point with rules of domination. For Location 1 (L1) can be formulated into a linear program for line players as follows:

Minimize $Z = \frac{1}{v}
\sum_{i=1}^{6} X_i = X_1 + X_2 + X_3 + X_4 + X_5 + X_6$

With restrictions:

$10X_1 + 10X_2 + 33X_3 + 28X_4 + 24X_5 + 39X_6 \geq 1$
$0X_1 + 39X_2 + 11X_3 + 27X_4 + 9X_5 + 19X_6 \geq 1$
$4X_1 + 36X_2 + 19X_3 + 40X_4 + 22X_5 + 18X_6 \geq 1$
$55X_1 + 17X_2 + 15X_3 + 20X_4 + 34X_5 + 8X_6 \geq 1$
$20X_1 + 33X_2 + 25X_3 + 10X_4 + 32X_5 + 11X_6 \geq 1$
$31X_1 + 17X_2 + 40X_3 + 21X_4 + 28X_5 + 26X_6 \geq 1$

The following optimal solution is obtained:
Table 2. Optimal Solution for Matrix Games L1 and L3.

| X1 | X2 | X3 | X4 | X5 | X6 | Z   |
|----|----|----|----|----|----|-----|
| 0  | 0,01 | 0 | 0 | 0 | 0,03 | 0,04 |

Because $Z = \frac{1}{V}$ and $X_i = \frac{X_i}{V}$

Maka:

$$V = \frac{1}{0.04} = 25$$

$$X1 = X1xV = 0x25 = 0$$

$$X2 = X2xV = 0,01x25 = 0.25$$

$$X3 = X3xV = 0x25 = 0$$

$$X4 = X4xV = 0x25 = 0$$

$$X5 = X5xV = 0x25 = 0$$

$$X6 = X6xV = 0,03x25 = 0.75$$

Because the acquisition matrix elements above have been added with $K = 20$, the game value becomes $V = 25 - 20 = 5$, so the optimal solution is 5. Obtained optimal strategy for Location 3, namely strategy (Height of Place), strategy (Number of Population), strategy (Length of Evacuation Path), strategy (Hierarchy of Roads), Strategy (Type of Transportation) and strategy (Area of Evacuation Place) with large game value (value of games) of 5. For Location 3 (L3) can be formulated into a linear program for line players as follows:

Maximize:

$$Z = \frac{1}{V} \sum_{i=1}^{6} Y_i = Y1 + Y2 + Y3 + Y4 + Y5 + Y6$$

With restrictions:

$$10Y1 + 10Y2 + 33Y3 + 28Y4 + 24Y5 + 39Y6 \geq 1$$

$$0Y1 + 39Y2 + 11Y3 + 27Y4 + 9Y5 + 19Y6 \geq 1$$

$$4Y1 + 36Y2 + 19Y3 + 40Y4 + 22Y5 + 18Y6 \geq 1$$

$$55Y1 + 17Y2 + 15Y3 + 20Y4 + 34Y5 + 8Y6 \geq 1$$

$$20Y1 + 33Y2 + 25Y3 + 10Y4 + 32Y5 + 11Y6 \geq 1$$

$$31Y1 + 17Y2 + 40Y3 + 21Y4 + 28Y5 + 26Y6 \geq 1$$

After calculating using the QM POM program, the following matrix is obtained:
### Table 3. Solusi Optimal Matriks Permainan L1 dan L3

| X1   | X2  | X3   | X4   | X5   | X6   | Z     |
|------|-----|------|------|------|------|-------|
| 0.01 | 0.02| 0    | 0    | 0    | 0.01 | 0.04  |

Because $Z = \frac{1}{V}$ and $X_i = \frac{X_i}{V}$

Maka:

$$V = \frac{1}{0.04} = 25$$

- $Y_1 = 1xV = 0.01x25 = 0.25$
- $Y_2 = 2xV = 0.01x25 = 0.25$
- $Y_3 = 3xV = 0x25 = 0$
- $Y_4 = 4xV = 0.01x25 = 0.25$
- $Y_5 = 5xV = 0.01x25 = 0.25$
- $Y_6 = 6xV = 0x25 = 0$

Because the acquisition matrix elements above have been added with $K = 20$, the game value becomes $V = 25 - 20 = 5$, so the optimal solution is 5. Obtained optimal strategy for Location 3, namely strategy (Height of Place), strategy (Number of Population), strategy (Length of Evacuation Path), strategy (Hierarchy of Roads), Strategy (Type of Transportation) and strategy (Area of Evacuation Place) with large game value (value of games) of 5.

#### 3.2. Evacuation location 1 (L1) with evacuation location 2 (L2)

### Table 4. Matrix Payoff L1 and L2

|     | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 | Min |
|-----|----|----|----|----|----|----|-----|
| X1  | 5  | 7  | 13 | 19 | -5 | 39 | -5  |
| X2  | -15| 45 | -31| 47 | -9 | 9  | -31 |
| X3  | 3  | 19 | 5  | 27 | 17 | -7 | -7  |
| X4  | 63 | 7  | 15 | 3  | 25 | 1  | 1   |
| X5  | 17 | 13 | 13 | 3  | 45 | -31| -31 |
| X6  | 15 | 7  | 1  | 13 | 25 | 39 | 1   |
| Max | 63 | 45 | 15 | 47 | 45 | 39 |     |

In the table above, it can be seen that the smallest value (maxima) in the row is -31 while in the column player, the biggest value (minimax) is 63. The maximal value with the minimax value is not the same, so the game does not have a saddle point, so the next step will be to find the balance point with rules of domination.
Table 5. Matrix Payoff L1 and L2

|      | Y2 | Y4 | Minimum |
|------|----|----|---------|
| X4   | 7  | 3  | 3       |
| X6   | 7  | 13 | 7       |
| Maximum | 7  | 13 |         |

In the table above it can be seen that there is a similar value of 7, then the game is optimal because it has saddle points.

3.3. Evacuation location 2 (L2) with evacuation location 3 (L3)

Table 6. Matriks Payoff L2 dan L3

|      | Y1 | Y2 | Y3 | Y4 | Y5 | Y6 | Min |
|------|----|----|----|----|----|----|-----|
| X1   | -19| -17| -10| -11| 9  | -20| -20 |
| X2   | 19 | -26| 22 | -40| -2 | -10| -40 |
| X3   | -19| -3 | -6 | -7 | -15| 5  | -19 |
| X4   | -28| -10| 20 | -3 | -11| -13| -28 |
| X5   | -17| 0  | -13| -33| 22 | -33|     |
| X6   | -11| -10| 3  | -12| -17| -33| -33 |
| Max  | 19 | 0  | 22 | -3 | 9  | 22 |     |

In the table above, it can be seen that the smallest value (maximum) in the row is -40 while in the column player the biggest value (minimax) is 22. The maximum value with the minimum value is not the same so the game does not have a saddle point so the next step will be to find the balance point with rules of domination. For Location 2 (L2) Can be formulated into a linear program for line players as follows:

$$\sum_{i=1}^{m} X_i = X_1 + X_2 + X_3 + X_4 + X_5 + X_6$$

Minimize $Z=\frac{1}{x}$

With restrictions:

$$21X_1 + 23X_2 + 30X_3 + 29X_4 + 49X_5 + 20X_6 \geq 1$$
$$21X_1 + 14X_2 + 62X_3 + 0X_4 + 38X_5 + 30X_6 \geq 1$$
$$21X_1 + 37X_2 + 34X_3 + 33X_4 + 25X_5 + 45X_6 \geq 1$$
$$12X_1 + 30X_2 + 20X_3 + 37X_4 + 29X_5 + 27X_6 \geq 1$$
$$23X_1 + 40X_2 + 32X_3 + 27X_4 + 7X_5 + 62X_6 \geq 1$$
$$29X_1 + 30X_2 + 43X_3 + 28X_4 + 23X_5 + 7X_6 \geq 1$$
The following optimal solution is obtained:

| X1  | X2  | X3  | X4  | X5  | X6  | Z     |
|-----|-----|-----|-----|-----|-----|-------|
| 0.01| 0   | 0   | 0   | 0.01| 0.02| 0.04  |

Because $Z = \frac{1}{v}$ and $X_i = X_i$

Then: $V = \frac{1}{0.04} = 25$

$$X1 = X1xV = 0x25 = 0$$
$$X2 = X2xV = 0, 01x25 = 0, 25$$
$$X3 = X3xV = 0x25 = 0$$
$$X4 = X4xV = 0x25 = 0$$
$$X5 = X5xV = 0x25 = 0$$
$$X6 = X6xV = 0, 03x25 = 0, 75$$

Since the acquisition matrix elements above have been added $K = 40$, the game value is $V = 25-40 = -15$. So the optimal strategy is -15. Obtained optimal strategy for Location 2, namely strategy (Height of Place), strategy (Number of Population), strategy (Length of Evacuation Path), strategy (Road Hierarchy), Strategy (Type of Transportation) and strategy (Area of Evacuation Place) with large game value (value of games) of -15.

4. Conclusion

This research presents dynamically determining disaster evacuation location techniques to overcome the problem of determining the location that has been determined by the natural state when a disaster occurs. This research proposes the Game Theory approach to the Linear Program in problem solving. The rationale of this research is to analyze the weaknesses of previous studies that only discuss the problem of route and vehicle selection during evacuation.

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