Maxwell-Kostelecký Electromagnetism and Cosmic Magnetization

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Abstract. The Lorentz violating term in the photon sector of Standard Model Extension, $\mathcal{L}_K = -\frac{1}{4}(k_F)_{\alpha\beta\mu\nu}F^\alpha\beta F^{\mu\nu}$ (here referred to as the Kostelecký term), breaks conformal invariance of electromagnetism and enables a superadiabatic amplification of magnetic vacuum fluctuations during inflation. For a wide range of values of parameters defining Lorentz symmetry violation and inflation, the present-day magnetic field can have an intensity of order of nanogauss on megaparsec scales and then could explain the large-scale magnetization of the universe.

The methodical investigation of possible effects of violations of Lorentz symmetry has became stronger in recent years\textsuperscript{1} (for recent papers concerning Lorentz violation see, e.g., Refs.\textsuperscript{2,3}). The interest in Lorentz violation (LV) resides primarily in the fact that theories that attempt to unify Gravity to the other fundamental interactions, such as String Theory or Quantum Gravity, incorporate it in a natural way. Terrestrial experiments and astrophysical observations, however, have established that Lorentz violation effects (if ever they exist) have to be very tiny. Nevertheless, due to the increased sensitivity of experiments, the detection of LV signals could be a real possibility in a not-so-far future.

If on the one hand Lorentz symmetry violation is being searched for, though no compelling evidence for it exists, on the other hand astrophysical observations undoubtedly indicate that all large-scale, gravitationally bound systems (i.e. galaxies and clusters of galaxies) are pervaded by microgauss magnetic fields whose origin is still not well understood (for reviews on cosmic magnetic fields see Ref.\textsuperscript{4}). The fact that large-scale magnetic fields exist everywhere in the universe and possess approximatively the same intensity seem to indicate that they have a common and primordial origin. If one takes into account that the collapse of primordial large-scale structures enhances the intensity of any preexisting magnetic field of about a factor $10^3$\textsuperscript{4}, a primeval field with comoving intensity of order of nanogauss and correlated on megaparsec scales could explain the magnetization of the universe.
It is worth noting that, due to the large correlation of the presently observed fields (ranging from \( \sim 10 \text{kpc} \) for magnetic fields in galaxies to \( \sim 1 \text{Mpc} \) for those found in clusters), it is quite natural to suppose that they have been generated during an inflationary epoch of the universe. Indeed, during inflation all fields are quantum mechanically exited. Because the wavelength \( \lambda \) associated to a given fluctuation grows faster than the horizon, there will be a time, say \( t_1 \), when this mode crosses outside the horizon itself. After that, this fluctuation cannot collapse back into the vacuum being not causally self-correlated, and then “survives” as a classical real object. The energy associated to a given fluctuation is subjected to the uncertainty relation, \( \Delta E \Delta t \gtrsim 1 \). Therefore, the energy density in the volume \( \Delta V \), \( \mathcal{E} = \Delta E/\Delta V \), is approximatively given by \( \mathcal{E} \sim H^4 \), where \( H \) is the Hubble parameter. Here, we used the fact that at the horizon crossing \( \Delta t \sim H^{-1} \) and \( \Delta V \sim H^{-3} \). Taking into account the expression for the electromagnetic energy in standard Maxwell electromagnetism, one arrives to the result that the spectrum of magnetic fluctuations at the time of horizon crossing is given by \( B_1 \sim H^2 \sim M^4/m_{\text{Pl}}^2 \), where in the last equality we used the Friedmann equation

\[
H^2 = \left( \frac{8\pi}{3} \right) M^4/m_{\text{Pl}}^2.
\]

Here, \( M^4 \) is the (constant) total energy density during (de Sitter) inflation and \( m_{\text{Pl}} \sim 10^{19}\text{GeV} \) is the Planck mass.

Due to conformal invariance of Maxwell electromagnetism one finds, however, that the present magnitude of the inflation-produced field at the scale of, say, \( 10 \text{kpc} \) is vanishingly small, \( B_0 \sim 10^{-52} \text{G} \). Strictly speaking, this is true only if the background metric is spatially-flat, which indeed is the case discussed in this paper.

Since the seminal work of Turner and Widrow, a plethora of mechanisms have been proposed for generating cosmic magnetic fields in the early universe. Most of them repose on the breaking of conformal invariance attained by adding non-conformal invariant terms in the standard electromagnetic Lagrangian, e.g., non-minimal couplings between photon and gravity, interaction terms between photon and inflaton, dilaton, or axion, and so on.

Long time ago, it was noted by Kostelecký, Potting and Samuel that the breaking of conformal invariance is a natural consequence of LV. Indeed, they argued that the appearance of an effective photon mass owing to spontaneous breaking of Lorentz invariance could enable the generation of large-scale magnetic fields within inflationary scenarios. In a subsequent paper, that idea was developed by Bertolami and Mota. The connection between Lorentz symmetry violation and cosmic magnetic fields has been studied only in a few papers, namely within the
framework of noncommutative spacetimes \[12, 13\], quantum theory with noncommutative fields \[14\] and, recently, by considering the introduction of a Lorentz-violating Chern-Simons term in the standard electromagnetic Lagrangian \[15\].

In this paper, we study the generation of magnetic fields within the so-called Standard Model Extension (SME) \[16\], which is an effective field theory that includes all admissible Lorentz-violating terms in the $SU(3) \times SU(2) \times U(1)$ standard model of particle physics. In the photon sector, and in curved spacetimes, the SME action, here referred to as the Maxwell-Kostelecký action, reads \[17\]:

$$S_{\text{MK}} = \int d^4x e \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} (k_F)_{\alpha\beta\mu\nu} F^{\alpha\beta} F^{\mu\nu} \right], \quad (2)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field strength tensor and $e$ the determinant of the vierbein. The presence of the external tensor $(k_F)_{\alpha\beta\mu\nu}$ breaks, explicitly, (particle) Lorentz invariance \[17\] (and in general conformal invariance) and parameterizes then Lorentz violation. Note that $(k_F)_{\alpha\beta\mu\nu}$ is antisymmetric on the first two and on the last two indices while it is symmetric under interchange of the first and last pair of indices.

The external tensor $(k_F)_{\alpha\beta\mu\nu}$ is fixed in a given system of coordinates. Going in different system of coordinates will, generally, induces a change of the form of $(k_F)_{\alpha\beta\mu\nu}$. We stress that, in this paper, we assume that the form of $(k_F)_{\alpha\beta\mu\nu}$ refers to a system of coordinates at rest with respect to the Cosmic Microwave Background (CMB), the so-called “CMB frame”.

Since taking $(k_F)_{\alpha\beta\mu\nu}$ as a fixed tensor corresponds to have an explicit violation of Lorentz symmetry, this could introduce in the theory an instability associated to non-positivity of the energy. This difficulty could be overcome, in principle, by considering models in which LV is spontaneously broken, such as those which naturally emerge in string theory. In this case, however, action \[2\] loses its character of generality, owing to the fact that the tensor $(k_F)_{\alpha\beta\mu\nu}$ is now regarded as a vacuum expectation value of some tensor field with its own dynamics. For this reason, we will take $(k_F)_{\alpha\beta\mu\nu}$ to be a fixed tensor, that is we will assume that the effects of spontaneous breaking can be approximated by terms in the action with explicit symmetry breaking. This assumption is not completely satisfactory from a pure theoretical viewpoint but, nevertheless, we believe that the “simplified” model with explicit symmetry breaking catches the main characteristics of Lorentz violation in the photon sector.

It should be noted that the photon part of the action for the Standard Model Extension also contains, in principle, two more CPT-odd terms: a Chern-Simons term,

$$S_{\text{CS}} = \int d^4x \frac{1}{4} e (k_{AF})^\kappa\epsilon_{\kappa\lambda\mu\nu} A^\lambda F^{\mu\nu}, \quad (3)$$
and a term linear in the electromagnetic field,
\[ S_A = - \int d^4 x \, e (k_A)_{\kappa} A^\kappa, \]  
(4)

where \( \epsilon_{\mu\nu\rho\sigma} \) is the Levi-Civita tensor density, while \( (k_A F)_\kappa \) and \( (k_A)_{\kappa} \) are coefficients for Lorentz violation. The first term was analyzed in Ref. \[15\] while the second one, as it is easy to show, is not able to break conformal invariance in spatially-flat models of universe, so there is no production of astrophysically interesting magnetic fields. For these reasons, we neglect those two CPT-odd terms in the following analysis.

The equations of motion follow from action (2):
\[ D^\mu F_{\mu\nu} + D^\mu[(k_F)_{\mu}^{\alpha\beta} F^{\alpha\beta}] = 0, \]
(5)
while the Bianchi identities are:
\[ D^\mu \tilde{F}_{\mu\nu} = 0, \]
(6)
where \( D_\mu \) is the spacetime covariant derivative and
\[ \tilde{F}_{\mu\nu} = \frac{1}{2e} \epsilon_{\mu\nu\alpha\beta} F^{\alpha\beta} \]
(7)
is the dual electromagnetic field strength tensor.

We assume that the universe is described by a Robertson-Walker metric
\[ ds^2 = a^2(d\eta^2 - dx^2), \]
(8)
where \( a(\eta) \) is the expansion parameter, \( \eta \) the conformal time (related to the cosmic time \( t \) through \( d\eta = dt/a \)), and \( H \) the Hubble parameter. Introducing, in the usual way, the electric and magnetic fields as \( F_{0i} = -a^2 E_i \) and \( F_{ij} = \epsilon_{ijk}a^2 B_k \) (Latin indices run from 1 to 3, while Greek ones from 0 to 3), the equations of motion read:
\[ \partial_\eta(a^2 E_i) - \epsilon_{ijk}\partial_j(a^2 B_k) + \]
\[ \partial_\eta[2(k_F)_{i00}a^{-2}E_j - (k_F)_{0ijk}\epsilon_{jkl}a^{-2}B_l] + \]
\[ \partial_j[2(k_F)_{ijkl}a^{-2}E_k - (k_F)_{ij}k\epsilon_{klm}a^{-2}B_m] = 0, \]
(9)
\[ \partial_i(a^2 E_i) + \partial_i[2(k_F)_{i00}a^{-2}E_j - (k_F)_{0ijk}\epsilon_{jkl}a^{-2}B_l] = 0, \]
(10)
while the Bianchi identities become:
\[ \partial_\eta(a^2 \mathbf{B}) + \nabla \times (a^2 \mathbf{E}) = 0, \quad \nabla \cdot \mathbf{B} = 0. \]
(11)
Moreover, following the standard procedure, we find the electromagnetic energy-momentum tensor:

$$T_{\mu \nu} = \frac{1}{4} g_{\mu \nu} F_{\alpha \beta} F^{\alpha \beta} - F^{\mu \alpha \nu} F_{\nu \alpha} + \frac{1}{4} g_{\mu \nu} (k_F)_{\alpha \beta \gamma \delta} F^{\alpha \beta} F^{\gamma \delta} - (k_F)_{\mu \alpha \beta \gamma} F^{\gamma \delta} F_{\nu} \cdot F^{\alpha \beta}.$$  \hspace{1cm} (12)

The electromagnetic energy density is then given by

$$\mathcal{E} = T^0_0 = \frac{1}{2} (E^2 + B^2) + (k_F)_{000} a^{-4} E_i E_j + \frac{1}{4} (k_F)_{ijkl} a^{-4} \epsilon_{ijm} \epsilon_{klm} B^m B^n.$$  \hspace{1cm} (13)

In any sensible Lorentz-violating theory, we require positivity of the energy. In the Maxwell-Kostelecký theory, the energy density is not-positive defined in general. If in a particular system of coordinates the energy in not-positive defined, this simply means that this effective theory becomes meaningless and one needs to consider the full theory with spontaneous Lorentz symmetry breaking in order to get physically acceptable results.

We are interested in the evolution of electromagnetic fields well outside the horizon, that is to modes whose physical wavelength is much greater than the Hubble radius $H^{-1}$, $\lambda_{\text{phys}} \gg H^{-1}$, where $\lambda_{\text{phys}} = a \lambda$ and $\lambda$ is the comoving wavelength. Since $a \eta \sim H^{-1}$, introducing the comoving wavenumber $k = 2\pi/\lambda$, the above condition reads $|k \eta| \ll 1$. Observing that the first Bianchi identity gives $B \sim k \eta E$, where $B$ and $E$ stand for the average magnitude of the magnetic and electric field intensities, and assuming that all (non-null) components of $(k_F)_{\alpha \beta \mu \nu}$ have approximately the same magnitude, it easy to see that, at large scales, Eq. (9) reduces to

$$\partial_\eta (a^2 E_i) + \partial_\eta [2(k_F)_{00} a^{-2} E_j] = 0.$$  \hspace{1cm} (14)

Assuming that $||(k_F)_{000}|| \gg a^4$ we then have $(k_F)_{000} a^{-2} E_j = c_i$, where the $c_i$'s are constants of integration. For the sake of simplicity, we assume that $(k_F)_{000}$ is a constant isotropic tensor, so that we can write

$$(k_F)_{000} = k_F \delta_{ij},$$  \hspace{1cm} (15)

where $k_F$ is a constant which gives the magnitude of Lorentz violation effect, and $\delta_{ij}$ is the Kronecker delta. In this case, the average intensity of the electric field scales as $E \propto a^2$. Observing that $\eta \propto a^{-1}$ during de Sitter inflation and taking into account the first Bianchi identity, we get that the average magnetic intensity grows “superadiabatically”, $B \propto a$.

Before proceeding further, we estimate the spectrum of magnetic vacuum fluctuation generated during de Sitter inflation in Maxwell-Kostelecký electromagnetism. If, for the sake of simplicity, we assume that the nonzero components of $(k_F)_{\alpha \beta \mu \nu}$ are just given by Eq. (15, the electromagnetic energy density turns to be

$$\mathcal{E} = \frac{1}{2} (E^2 + B^2) + k_F a^{-4} E^2.$$  \hspace{1cm} (16)
As a consequence, at the time of crossing, where $|k\eta| \sim 1$, and in the limit $k_F \gg a^4$, we get $E \sim k_F a_1^{-4} B_1^2$, where $a_1 = a(t_1)$ and we used the first Bianchi identity. Remembering that $E \sim H^4$, we obtain the spectrum of magnetic fluctuations at the time of horizon crossing:

$$B_1 \sim k_F^{-1/2} a_1^2 H^2.$$  \hfill (17)

It is worth noting that, in order to have positivity of the energy, we are forced to assume $k_F > 0$.

After inflation, the universe enters in the so-called reheating phase, during which the energy of the inflaton is converted into ordinary matter. The reheating phase ends at the temperature $T_{RH}$ which is less than $M$ and constrained as \hfill [18]

$$T_{RH} \lesssim 10^8 \text{GeV}. \hfill (18)$$

Moreover, CMB analysis requires $M \lesssim 10^{-2} m_{Pl}$ \hfill [7], otherwise it would be too much of a gravitational waves relic abundance, and also one must impose that $T_{RH} \gtrsim 1 \text{GeV}$, so that the predictions of Big Bang Nucleosynthesis (BBN) are not spoiled \hfill [7].

It is worth noting that the condition $||(k_F)_{i00j}|| \gg a^4$ or, equivalently, $k_F \gg a^4$ is certainly fulfill during inflation and reheating if $k_F \gg a_{RH}^4$, where $a_{RH} = a(T_{RH})$. Since $a_{RH} \sim T_0/T_{RH}$, where $T_0 \sim 10^{-13} \text{GeV}$ is the actual temperature \hfill [19], we have $10^{-84} \lesssim a_{RH}^4 \lesssim 10^{-52}$ for $1 \text{GeV} \lesssim T_{RH} \lesssim 10^8 \text{GeV}$.

The most stringent upper bounds on $||(k_F)_{\alpha\beta\mu\nu}||$ come from the analysis of CMB polarization and polarized light of radiogalaxies and gamma-ray bursts, and are respectively: $10^{-30}$ \hfill [20], $10^{-32}$ \hfill [21], and $10^{-37}$ \hfill [22], although, in deriving these constraints, some additional assumptions have to be taken. A direct constraint on the quantity $k_F$ [defined through Eq. (15)] can be obtained if one takes into account that the maximal attained experimental sensitivity on the coefficient $\tilde{\kappa}_{tr} \equiv 2k_F$ introduced in Ref. \hfill [2] is about $10^{-11}$ and that there is no compelling evidence for nonzero values of $\tilde{\kappa}_{tr}$ (see the 2009 version of the Data Tables for Lorentz and CPT Violation \hfill [2] for more details).

In any case, for a wide range of allowed values of $k_F$, we have a superadiabatic amplification of magnetic vacuum fluctuations during inflation and reheating. In this latter phase, in particular, taking into account that $\eta \propto a^{1/2}$, we have $B \propto a^{5/2}$.

\hfill [1] It should be noted that, though the CMB bound is less stringent than the radiogalaxies and gamma-ray bursts bounds, it covers the whole portion of coefficient space for Lorentz violation. The point-source nature of radiogalaxies and gamma-ray bursts, instead, allows us to put constraints only on limited portions of coefficient space \hfill [2].
After reheating, the universe enters the radiation dominated era. In this era, as well as in the subsequent matter era, the effects of the conducting primordial plasma are important when studying the evolution of a magnetic field. They are taken into account by adding to the electromagnetic Lagrangian the source term \( j^\mu A_\mu \). Here, the external current \( j^\mu \), expressed in terms of the electric field, has the form \( j^\mu = (0, \sigma c E) \), where \( \sigma c \) is the conductivity. Plasma effects introduce, in the right-hand-side of Eq. (9), the extra term \(-a\sigma c(a^2E_i)\). In this case, it easy to see that modes well outside the horizon (assuming that \( k_F \gg a^4 \)) evolve as

\[
E \propto a^2 \exp\left(-\int d\eta a^5\sigma_c/2k_F\right).
\]  

(19)

Approximating \( \int d\eta a^5\sigma_c \) with \( \eta a^5\sigma_c \) and using \( a\eta \sim H^{-1} \), we get from Eq. (19):

\[
E \propto a^2 \exp(-a^4\sigma_c/2Hk_F).
\]  

(20)

In the radiation era \( H \sim T^2/m_{Pl} \) and, for temperature much greater than the electron mass, the conductivity is approximatively given by \( \sigma_c \sim T/\alpha \), where \( \alpha \) is the fine structure constant and \( T \) the temperature. Then, from Eq. (20) we obtain:

\[
E \propto a^2 \exp(- (T_*/T)^5)\],
\]  

(21)

where \( T_*/10\text{GeV} \sim (10^{-37}/k_F)^{1/5} \). This means that for \( T \gtrsim T_* \) we have \( E \propto a^2 \) (which in turn gives \( B \propto a^3 \) since \( \eta \propto a \) in radiation era), while for \( T \lesssim T_* \) the electric field is dissipated, so the magnetic field evolves adiabatically, \( B \propto a^{-2} \). [We have assumed that \( k_F \gg a^4 \) from the end of reheating until \( T_* \). This assumption is satisfied if \( k_F \gtrsim a_*^4 \), where \( a_* = a(T_*) \) or, taking into account the above expression for \( T_* \), if \( k_F \gtrsim 10^{-95} \). This is certainly our case since we have already assumed that \( k_F \gtrsim 10^{-52} \) if, for example, \( T_{RH} \sim 1\text{GeV} \) or \( k_F \gtrsim 10^{-84} \) if \( T_{RH} \sim 10^8\text{GeV} \).]

Finally, evolving along the lines discussed above the inflation-produced magnetic field from the time of horizon crossing until today, we get

\[
\frac{B_0}{\text{nG}} \sim \left( \frac{M}{10^{14}\text{GeV}} \right)^4 \left( \frac{T_{RH}}{10\text{GeV}} \right)^{5(n-1)/2} \left( \frac{k_F}{10^{-37}} \right)^{n/2} \lambda_{\text{Mpc}}^{-1},
\]  

(22)

where \( n \) takes the values \( \pm 1 \) according to \( T_* \lesssim T_{RH} \) or \( T_* \gtrsim T_{RH} \). The latter condition means, indeed, that the magnetic field evolves adiabatically from the end of reheating until today and is equivalent to have \( T_{RH}/10\text{GeV} \lesssim (10^{-37}/k_F)^{1/5} \).

It is clear from Eq. (22) and Fig. 1 that, for a wide range of values of parameters defining inflation (\( M \) and \( T_{RH} \)) and Lorentz symmetry violation \( (10^{-52} \lesssim k_F \lesssim 10^{-11}) \), the present-day magnetic field can be as strong as \( B_0 \sim 1.0\text{nG} \) on megaparsec scales.
FIG. 1: The inflation-produced magnetic field has an actual intensity of order of nanogauss on megaparsec scales if the values of parameter defining inflation, i.e. energy scale of inflation $M$ and the reheat temperature $T_{RH}$, stay on the curves. $k_F \sim \| (k_F)_{\alpha\beta\mu\nu} \|$ estimates the magnitude of the (constant) external tensor which parameterizes Lorentz violation [see Eq. (2)].

In conclusion, we have shown that the presence of large-scale magnetic fields in the present universe is a general prediction of Standard Model Extension. The presence of the “Kostelecký”, Lorentz violating term $L_K = -\frac{1}{4} (k_F)_{\alpha\beta\mu\nu} F^{\alpha\beta} F^{\mu\nu}$ in the photon sector of the theory allows electromagnetic vacuum fluctuations to be superadiabatically amplified during inflation. The resulting magnetic field has, today, a magnitude of order of nanogauss on megaparsec scales for a wide range of parameter space of inflation and Lorentz violation and then could explain why our universe is magnetized.

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