USM-TH-120

The \((\mu^-, \mu^+)\) conversion in nuclei as a probe of new physics

Fedor Šimkovic\(^*\), Amand Faessler

Institut für Theoretische Physik, Universität Tübingen, Auf der Morgenstelle 14, D-72076 Tübingen, Germany

Sergey Kovalenko\(^†\), Ivan Schmidt

Departamento de Física, Universidad Técnica Federico Santa María, Casilla 110-V, Valparaíso, Chile

(October 28, 2018)

Abstract

A detailed study of the muonic analogue of neutrinoless double beta decay, \((\mu^-, \mu^+)\) conversion, has been carried out for the \(A=44\) nuclear system. We studied several lepton number violating (LNV) mechanisms potentially triggering this process: exchange by light and heavy Majorana neutrinos as well as exchange by supersymmetric particles participating in R-parity violating interactions. The nuclear structure has been taken into account within the renormalized Quasiparticle Random Phase Approximation method. To our knowledge, this is the first realistic treatment of nuclear structure aspects of the \((\mu^-, \mu^+)\) conversion. We estimated the rate of this process utilizing the existing experimental constraints on the parameters of the underlying LNV

\(^*\)On leave of absence from Department of Nuclear Physics, Comenius University, Mlynská dolina F1, SK–842 15 Bratislava, Slovakia

\(^†\)On leave of absence from Joint Institut for Nuclear Research, Dubna, Russia
interactions and conclude that the \((\mu^-, \mu^+)\) conversion is hardly detectable in the near future experiments.

1. INTRODUCTION

Muonic atoms are known to be a suitable laboratory for studying lepton flavor and lepton number violation (LFV, LNV) \cite{1-9}. The main attention has been previously paid to the reactions of muon–electron \([(\mu^-, e^-)]\) and muon–positron \([(\mu^-, e^+)]\) conversions in nuclei. The new forthcoming experiments with stopped muon beams MECO \cite{10} at BNL and PRISM at KEK \cite{11} are going to substantially improve sensitivity to different LFV/LNV decay channels. The aimed intensity of muon beam from the proton machines is about four orders of magnitude higher than that available at present. This substantial experimental progress is raising the question on observability of other muonic LFV/LNV processes as yet not addressed experimentally.

In the present paper we are studying the \((\mu^-, \mu^+)\) conversion in nuclei

\[
\mu_b^- + (A, Z) \rightarrow (A, Z - 2) + \mu^+,
\]

the muonic analogue of neutrinoless double decay, first proposed in Ref. \cite{2}. The \(\mu_b^-\) is a bounded muon in a 1S atomic state. This process has never been searched for experimentally. The rough estimates of Ref. \cite{2} within some extensions of the standard model (SM) show that its branching ratio \(R_{\mu^+\mu^-} = \Gamma_{\mu^-\mu^+}/\Gamma_{\mu\nu}\) to ordinary muon capture does not exceed \(10^{-18}\). Although this is a very small number it does not look unreachable for future experiments. In fact, the expected sensitivity of the forthcoming experiments MECO and PRIME to another neutrinoless nuclear process, \((\mu^-, e^-)\) conversion, is of the same order of magnitude \(\sim 10^{-18}\). Despite significant differences in searching strategies for the \((\mu^-, e^-)\) and the \((\mu^-, \mu^+)\) conversion processes as well as in the technological treatment of the corresponding nuclear targets, the above observation might be encouraging for future searches of nuclear \((\mu^-, \mu^+)\) conversion. The results of our present study lead to a less optimistic conclusion.
We argue that the rate of this process is likely by at least 4 orders of magnitude less than that given in Ref. \cite{2}, and thus its possible observation is shifting to a quite distant future.

We study the \((\mu^-, \mu^+)\) conversion in nuclei within the most conventional LNV extensions of the SM with light and heavy Majorana neutrinos as well as in the minimal supersymmetric SM extension with R-parity violation. We carried out a detailed analysis of the nuclear structure effects in the \((\mu^-, \mu^+)\) conversion within the renormalized quasiparticle random phase approximation (RQRPA) \cite{12, 14}. The theoretical upper bounds on the branching ratios are evaluated using the current constraints on the relevant LNV parameters.

The paper is organized as follows. In Section 2 we discuss kinematical conditions and admissible nuclei for searching for the \((\mu^-, \mu^+)\) conversion. There we also give the \((\mu^-, \mu^+)\) conversion rate formulas and specify their parameters. Sections 3, 4, 5 deal with specific mechanisms of \((\mu^-, \mu^+)\) conversion. We determine the corresponding LNV parameters in terms of the fundamental parameters of the considered models, derive the \((\mu^-, \mu^+)\) transition operators and their nuclear matrix elements. In Section 6 we calculate these nuclear matrix elements for the long-living radioactive isotope \(^{44}\)Ti in the RQRPA. Then we derive upper bounds on the \((\mu^-, \mu^+)\) conversion in \(^{44}\)Ti using existing experimental constraints on the parameters of the studied LNV models and discuss prospects for its observability in future experiments. The summary is given in Section IV.

II. THEORETICAL PRELIMINARIES

The \((\mu^-, \mu^+)\) nuclear conversion violates total lepton number conservation by two units and, therefore, is forbidden in the standard model. Thus, its experimental observation, as well as the observation of any other LNV process, would be an unambiguous signal of physics beyond the SM. In general, a lepton number violating process may proceed if and only if neutrinos are Majorana particles with non-zero masses \cite{15} which can not be accommodated into the SM.

The underlying LNV mechanisms for different LNV processes are very similar and can
be adopted from the previously studied case of neutrinoless double beta decay (0νββ-decay) [10]. However, kinematically different LNV processes are rather different.

As observed in Ref. [2], the energy conservation requirement for the (µ⁻, µ⁺) conversion,

\[ T = E_i - E_f - \varepsilon_b > 0 \]  

(2.1)
is not fulfilled by any stable nucleus (A,Z). Here, T is the kinetic energy of the outgoing muon, \(E_i\) (\(E_f\)) is the ground state energy of initial (final) nucleus and \(\varepsilon_b\) is the muon binding energy in the muonic atom. Thus, experimental searches for the (µ⁻, µ⁺) conversion are possible only with radioactive isotopes. The only feasible target nucleus would be the isotope \(^{44}\text{Ti}\) with sufficiently long lifetime of 47 years [2].

Following Ref. [2] we write down the rate of the (µ⁻, µ⁺) conversion as

\[ \Gamma^{(i)}_{\mu \mu} = \frac{1}{\pi} E_{\mu^+} \ p_{\mu^+} \ F(Z-2, E_{\mu^+}) \ c_{\mu\mu} |\mathcal{M}^\Phi_{(i)}|^2 |\eta_{(i)}|^2, \]  

(2.2)

where \(E_{\mu^+} = m_\mu - \varepsilon_b + E_i - E_f\), \(p_{\mu^+} = \sqrt{E_{\mu^+}^2 - m_\mu^2}\) (\(m_\mu\) is the mass of muon), \(c_{\mu\mu} = 2G_F^4 \left( 4\pi m_\mu R \right)^2 g_A^4\). For the isotope \(^{44}\text{Ti}\) one has: \(\varepsilon_b = 1.28 \text{ MeV}\), \(p_{\mu^+} = 18.5 \text{ MeV/c}\). The index \((i)\) denotes the specific mechanism of the (µ⁻, µ⁺) conversion that we will study in the subsequent sections. In Eq. (2.2) the factors \(\eta_{(i)}\) and \(\mathcal{M}^\Phi_{(i)}\) are the LNV parameters and the nuclear matrix elements, respectively, associated with these mechanisms. The latter involve smearing over the muon wave function \(\Phi(r)\). Finally, the relativistic Coulomb factor \(F(Z-2, E_{\mu^+})\) is [17]

\[ F(Z-2, E_{\mu^+}) = \left( \frac{\Gamma(5)}{\Gamma(2\gamma + 1)} \right)^2 (2p_{\mu^+}R)^{2(\gamma-1)}|\Gamma(\gamma - iy)|^2 e^{-\pi y}, \]  

(2.3)

where \(\gamma = \sqrt{1 - \left(\alpha(Z-2)\right)^2}\), \(y = \alpha(Z-2)E_{\mu^+}/p_{\mu^+}\). R is the nuclear radius.

In the experiments with stopped muon beams one measures the branching ratio \(R^{(i)}_{\mu^+\mu^-}\) of the (µ⁻, µ⁺) conversion

\[ R^{(i)} = \frac{\Gamma^{(i)}_{\mu^+\mu^-}}{\Gamma_{\mu}} \]  

(2.4)

with respect to the ordinary muon capture \(\mu^- + (A, Z) \rightarrow (A, Z-1) + \nu_\mu\). The rate \(\Gamma_{\mu}\) of this SM allowed process can be calculated with the Primakoff formula [20].
\[
\Gamma_\mu = \frac{1}{2\pi} m_\mu^2 (G_F \cos \theta_c)^2 < \Phi_\mu >^2 Z [G_V^2 + 3G_A^2 + G_P^2 - 2G_A G_P] f(Z, A), \quad (2.5)
\]

The Pauli blocking factor for \(^{44}\text{Ti}\) takes the value \(f(22, 44) = 0.16\) [30]. The numerical value of the quadratic combination of the nucleon weak coupling constants is \([G_V^2 + 3G_A^2 + G_P^2 - 2G_A G_P] \approx 5.9\).

In the subsequent sections we derive the LNV parameters \(\eta_{(i)}\) and the nuclear matrix elements \(\mathcal{M}_\Phi^{(i)}\) for three examples of mechanisms of \((\mu^-, \mu^+)\) conversion based on the light and heavy Majorana neutrino exchange as well as on the LNV couplings of R-parity violating SUSY.

### III. Light Majorana Neutrino Exchange Mechanism

This mechanism is described by the diagrams in Fig. 1(a,b). In this case the LNV parameter for Eq. (2.2) is given by

\[
\eta_\nu = \left| \frac{< m_\nu >_{\mu\mu}}{m_e} \right|^2 \quad (3.1)
\]

where the effective muon neutrino mass is

\[
< m_\nu >_{\mu\mu} = \sum_k^{\text{light}} (U_{\mu k})^2 \xi_k m_k. \quad (3.2)
\]

Here, \(U\) is the unitary neutrino mixing matrix relating the weak and Majorana mass eigenstates of neutrinos. The mass eigenvalues and the factor of relative CP-phase are \(m_k\) and \(\xi_k\) respectively. The summation in Eq. (3.2) runs over the light neutrino mass eigenstates with the masses \(m_k\) much less than the typical energy scale of the \((\mu^-, \mu^+)\) conversion \((m_k \ll 100 \text{ MeV})\).

We calculate the nuclear matrix element \(\mathcal{M}_\nu^\Phi\) for this mechanism in the standard way, using the non-relativistic impulse approximation [17]. The final result can be written in terms of the Gamow-Teller \(M_{GT(\nu)}^\Phi\) and the Fermi \(M_{F(\nu)}^\Phi\) nuclear matrix elements as

\[
\mathcal{M}_\nu^\Phi = -\frac{M_{F(\nu)}^\Phi}{g_A^2} + M_{GT(\nu)}^\Phi \quad (3.3)
\]
with

\[ M_{F(n)}^\Phi = \frac{4\pi R}{(2\pi)^3} \int \frac{d\vec{q}}{2q} \times \]
\[ \sum_n \left( \frac{\langle 0^+_i | \sum \tau^+_l e^{-i\vec{q} \cdot \vec{r}_l} | n \rangle \langle n | \sum_m \tau^+_m e^{i\vec{q} \cdot \vec{r}_m} \Phi(r_m) | 0^+_f \rangle}{q - E_b + E_n - E_i + i\varepsilon_n} \right) + \]
\[ \frac{\langle 0^+_i | \sum \tau^+_m e^{-i\vec{q} \cdot \vec{r}_m} \Phi(r_m) | n \rangle \langle n | \sum_l \tau^+_l e^{-i\vec{q} \cdot \vec{r}_l} | 0^+_f \rangle}{q + E_{\mu^+} + E_n - E_i + i\varepsilon_n} \right), \quad (3.4) \]

\[ M_{GT(n)}^\Phi = \frac{4\pi R}{(2\pi)^3} \int \frac{d\vec{q}}{2q} \times \]
\[ \sum_n \left( \frac{\langle 0^+_i | \sum \tau^+_l \sigma_l e^{-i\vec{q} \cdot \vec{r}_l} | n \rangle \cdot \langle n | \sum_m \tau^+_m \sigma_m e^{i\vec{q} \cdot \vec{r}_m} \Phi(r_m) | 0^+_f \rangle}{q - E_b + E_n - E_i + i\varepsilon_n} \right) + \]
\[ \frac{\langle 0^+_i | \sum \tau^+_m \sigma_m e^{i\vec{q} \cdot \vec{r}_m} \Phi(r_m) | n \rangle \cdot \langle n | \sum_l \tau^+_l \sigma_l e^{-i\vec{q} \cdot \vec{r}_l} | 0^+_f \rangle}{q + E_{\mu^+} + E_n - E_i + i\varepsilon_n} \right). \quad (3.5) \]

Here, \( \Phi(r) \) is the radial part of bound muon wave function in the 1S-state, \( E_b \) and \( E_n \) are the energies of bound muon \( \mu^- \) and intermediate nuclear state, respectively. The width of the n-th intermediate nuclear state, denoted by \( \varepsilon_n \), is very small being determined only by the electromagnetic and weak decay channels. For low lying nuclear states it is much smaller than the corresponding intermediate state energy, \( \varepsilon_n \ll E_n \). This observation will allow us to significantly simplify the calculations presented in Section 6.

Deriving the \((\mu^-, \mu^+)\) conversion matrix elements we assumed that the outgoing muon \( \mu^+ \) is in \( s_{1/2} \) state and, therefore, \( p_{\mu^+} R/3 \approx 0.1 \). We also neglected the contributions from the weak-magnetism and induced pseudoscalar couplings of the nucleon current. These contributions correspond to higher order terms of non-relativistic expansion and are not important for our analysis. In fact, their inclusion may reduce the values of the nuclear matrix elements only up to \( 20 - 30\% \) [18].

The \((\mu^-, \mu^+)\) conversion matrix elements differ considerably from the nuclear matrix elements of 0ν\( \beta\beta \)-decay [18]. Note, first, that the value of \( -E_b + E_n - E_i \) entering the denominators of Eqs. (3.4), (3.3) is negative, i.e., the nuclear matrix elements exhibit a singular behavior. Therefore, the widths of the intermediate nuclear states are of great importance, and the imaginary part of the \((\mu^-, \mu^+)\) conversion matrix element can be large.
We note that this point was missed in Ref. [2]. A similar behavior of the amplitude of the muon to positron conversion process has been indicated only recently [3].

In order to simplify the numerical calculation of the \((\mu^-, \mu^+)\) conversion matrix element we adopt two additional approximations.

(i) We assume that the 1S muon wave function varies very little inside the nucleus \(^{44}\text{Ti}\).

Thus we apply the usual approximation [4]

\[
|M_\nu^2| = <\Phi_\mu^2> |M_\nu|^2, \tag{3.6}
\]

where the muon average probability density over the nucleus is

\[
<\Phi_\mu^2> = \frac{\int |\Phi_\mu(|\vec{x}|)|^2 \rho(|\vec{x}|) d^3x}{\int \rho(|\vec{x}|) d^3x}, \tag{3.7}
\]

Here, \(\rho(|\vec{x}|)\) is the nuclear density. To a good approximation it has been found that [4]

\[
<\Phi_\mu^2> = \frac{\alpha^3 m^3_\mu Z_{\text{eff}}^4}{Z}, \tag{3.8}
\]

i.e., the deviation from the behavior of the wave function at the origin has been taken into account by the effective proton number \(Z_{\text{eff}}\) (\(Z_{\text{eff}} = 17.5\) for \(Z = 22\) [4]).

(ii) We complete the sum over intermediate nuclear states by closure after replacing \(E_n, \varepsilon_n\) by some average values \(\bar{E}, \bar{\varepsilon}\), respectively.

Then we obtain

\[
M_\nu = -\frac{M_{F(\nu)}}{g_A^2} + M_{GT(\nu)} = M^D + M^C, \tag{3.9}
\]

with

\[
M_{F(\nu)} = M_F^D + M_F^C \tag{3.10}
\]

\[
= \langle 0^+_i | \sum_{kl} \tau_k^+ \tau_l^+ \frac{R}{\pi} \int_0^\infty \frac{j_0(qr_{kl})}{q - E_b + \bar{E} - E_i + i\bar{\varepsilon}} f_A^2(q^2) q dq | 0^+_f \rangle + \nonumber
\]

\[
\langle 0^+_i | \sum_{kl} \tau_k^+ \tau_l^+ \frac{R}{\pi} \int_0^\infty \frac{j_0(qr_{kl})}{q + \bar{E}_\mu + \bar{E} - E_i + i\bar{\varepsilon}} f_A^2(q^2) q dq | 0^+_f \rangle,
\]

\[
M_{GT(\nu)} = M_{GT}^D + M_{GT}^C \tag{3.11}
\]

\[
= \langle 0^+_i | \sum_{kl} \tau_k^+ \tau_l^+ \tilde{\sigma}_k \cdot \tilde{\sigma}_l \frac{R}{\pi} \int_0^\infty \frac{j_0(qr_{kl})}{q - E_b + \bar{E} - E_i + i\bar{\varepsilon}} f_A^2(q^2) q dq | 0^+_f \rangle + \nonumber
\]

\[
\langle 0^+_i | \sum_{kl} \tau_k^+ \tau_l^+ \tilde{\sigma}_k \cdot \tilde{\sigma}_l \frac{R}{\pi} \int_0^\infty \frac{j_0(qr_{kl})}{q + \bar{E}_\mu + \bar{E} - E_i + i\bar{\varepsilon}} f_A^2(q^2) q dq | 0^+_f \rangle.
\]
Here we take the nucleon form factors in the dipole form
\[ f_V(q^2) = \frac{1}{1 + q^2/\Lambda_V^2} \quad [\Lambda_V = 0.71 \ (GeV)^2], \]
\[ f_A(q^2) = \frac{1}{1 + q^2/\Lambda_A^2} \quad [\Lambda_A = 1.09 \ GeV]. \]
Note that the nuclear matrix elements \( M_{F(\nu)} \) and \( M_{GT(\nu)} \) are normalized in the same way as the matrix elements of the \( 0\nu\beta\beta \)-decay process, allowing for their direct comparison. We also separate the \((\mu^-, \mu^+)\) conversion nuclear matrix elements into two terms associated with the direct \( M^D_F, M^D_{GT} \) and \( M^D \) and the cross \( M^C_F, M^C_{GT} \) and \( M^C \) diagrams (see Fig. [1]). The direct terms contain the denominators with singular behavior.

**IV. HEAVY MAJORANA NEUTRINO EXCHANGE MECHANISM**

We assume that the neutrino mass spectrum include heavy Majorana states \( N \) with masses \( M_k \) much larger than the typical energy scale of the \( (\mu^-, \mu^+) \) conversion, \( M_k \gg 100 \) MeV. These heavy states can mediate this process according to the same diagrams in Fig. 1(a,b) as the previous light neutrino exchange mechanism. The difference is that the neutrino propagators in the present case can be contracted to points and, therefore, the corresponding effective transition operators are local unlike in the light neutrino exchange mechanism with long range internucleon interactions. As shown latter, the effect of nuclear structure in both cases is very different.

The corresponding LNV parameter \( \eta_N \) is given by
\[
\eta_N = \sum_k \left(U_{\mu k}\right)^2 \xi_k \frac{m_p}{M_k}. \quad (4.1)
\]
Here, \( m_p \) is the mass of proton. We assume that the mass of heavy neutrinos \( M_k \) is large in comparison with their average momenta \( (M_k >> 1 \ \text{GeV}) \) and that the muon wave function varies only slightly inside the nucleus. Then the resulting nuclear matrix element \( \mathcal{M}_N \) takes the same form as the corresponding nuclear matrix element for the heavy neutrino mass mechanism of \( 0\nu\beta\beta \)-decay [18]. Separating the Fermi (F), Gamow-Teler (GT) and the tensor (T) contributions we write down
\[
\mathcal{M}_N = -\frac{M_{F(N)}}{g_A^2} + M_{GT(N)} + M_{T(N)}
\]
\[ \langle 0^+_i \vert \sum_{kl} \tau^+_k \tau^+_l \frac{H_F(r_{kl})}{g^2} + H_{GT}(r_{kl})\sigma_{kl} - H_T(r_{kl})S_{kl} \vert 0^+_f \rangle, \quad (4.2) \]

where

\[ S_{kl} = 3(\vec{\sigma}_k \cdot \hat{r}_{kl})(\vec{\sigma}_l \cdot \hat{r}_{kl}) - \sigma_{kl}, \quad \sigma_{kl} = \vec{\sigma}_k \cdot \vec{\sigma}_l. \quad (4.3) \]

\( r_k \) and \( r_l \) are coordinates of nucleons undergoing weak interaction. \( r_{kl} = r_k - r_l, r_{kl} = |r_{kl}|, \hat{r}_{kl} = r_{kl}/r_{kl} \). The radial part of the exchange potential is

\[ H_I(r_{kl}) = \frac{2}{\pi} \frac{R}{m_p m_e} \int_0^{\infty} j_0(q r_{kl}) h_I(q^2) q^2 dq \quad (I = F, GT, T), \quad (4.4) \]

with

\[ h_F(q^2) = f_V^2(q^2) \]

\[ h_{GT}(q^2) = f_A^2(q^2) \left[ 1 - \frac{2}{3} \frac{q^2}{q^2 + m^2_\pi} + \frac{1}{3} \frac{q^2}{q^2 + m^2_\pi} \right] + \frac{2}{3} \frac{f_M^2(q^2)q^2}{4m^2_p}, \]

\[ h_T(q^2) = f_A^2(q^2) \left[ \frac{2}{3} \frac{q^2}{q^2 + m^2_\pi} - \frac{1}{3} \frac{q^2}{q^2 + m^2_\pi} \right] + \frac{1}{3} \frac{f_M^2(q^2)q^2}{4m^2_p}, \quad (4.5) \]

Here, \( m_\pi \) is the mass of pion, \( f_V(q^2), f_A(q^2) \) are the nucleon form factors introduced in Eqs. (3.11), (3.12) and \( g_M(q^2) = (\mu_p - \mu_n)f_V(q^2) \).

**V. Trilinear R-parity breaking contribution**

In the SUSY models with R-parity non-conservation there are present the LNV couplings which may trigger the \((\mu^-, \mu^+)\) conversion. Recall, that R-parity is a multiplicative quantum number defined by \( R = (-1)^{2S+3B+L} \) (S,B,L are spin, baryon and lepton number). Ordinary particles have \( R = +1 \) while their superpartners \( R = -1 \). The LNV couplings emerge in this class of SUSY models from the R-parity breaking part of the superpotential

\[ W_{R\mu} = \lambda_{ijk}L_iL_jE^c_k + \lambda'_{ijk}L_iQ_jD^c_k + \mu_iL_iH_2, \quad (5.1) \]

where \( L, Q \) stand for lepton and quark \( SU(2)_L \) doublet left-handed superfields, while \( E^c, D^c \) for lepton and down quark singlet superfields. Below we concentrate only on the trilinear
\(\lambda\)'-couplings. This is motivated by the fact that they give the dominant contribution in the case of the \(0\nu\beta\beta\)-decay \cite{19}.

The \((\mu^-, \mu^+)\) conversion mechanisms in this model can be derived in a close similarity with the \(0\nu\beta\beta\)-decay \cite{20,21}. An example of the diagram representing such a mechanism at the quark level is given in Fig. 2. This and all the other diagrams of this mechanism involve only the \(\lambda'_{211}\) coupling.

At the hadron level we assume dominance of the pion-exchange mode (see Fig. 3). The reasons are the same as in the case of \(0\nu\beta\beta\)-decay \cite{20,21}. Enhancement of the pion exchange mode with respect to the conventional two-nucleon mechanism is due to the long-range character of nuclear interaction and the details of the bosonization of the \(\mu^- + \pi^+ \rightarrow \pi^- + \mu^+\) vertex.

The \((\mu^-, \mu^+)\) conversion rate in the considered SUSY mechanism can be given in the form of Eq. (2.2) with the following LNV parameter

\[
\eta_{\lambda} = \frac{3}{8} (\eta_R + \frac{5}{8} \eta_{PS}).
\] (5.2)

The R-parity violating parameters associated both with gluino and neutralino exchange mechanisms are given as follows (see \cite{20,21}):

\[
\eta_{PS} = \eta_{\lambda e} + \eta_{\lambda f} + \eta_\chi + \eta_\tilde{g} + 7 \eta'_{\tilde{g}},
\] (5.3)

\[
\eta_R = \eta_{\lambda f} - \eta_{\lambda e} + \eta_\tilde{g} - \eta'_{\tilde{g}},
\] (5.4)

with

\[
\eta_{\tilde{g}} = \frac{\pi \alpha_s}{6} \frac{\lambda_{211}^2}{G_F^2 m_{\tilde{g}}^4} m_p \left[ 1 + \left( \frac{m_{\tilde{d}_R}}{m_{\tilde{u}_L}} \right)^4 \right],
\]

\[
\eta_\chi = \frac{\pi \alpha_2}{2} \frac{\lambda_{211}^2}{G_F^2 m_{\tilde{g}}^4} \sum_{i=1}^4 \frac{m_p}{m_{\chi_i}} \left[ \epsilon_{R_d}^2(d) + \epsilon_{L_d}^2(u) \left( \frac{m_{\tilde{d}_R}}{m_{\tilde{u}_L}} \right)^4 \right],
\]

\[
\eta_{\lambda e} = 2 \pi \alpha_2 \frac{\lambda_{211}^2}{G_F^2 m_{\tilde{g}}^4} \left( \frac{m_{\tilde{d}_R}}{m_{\tilde{e}_L}} \right)^4 \sum_{i=1}^4 \epsilon_{L_d}^2(e) \frac{m_p}{m_{\chi_i}},
\]

\[
\eta'_{\tilde{g}} = \frac{\pi \alpha_s}{12} \frac{\lambda_{211}^2}{G_F^2 m_{\tilde{g}}^4} \left( \frac{m_{\tilde{d}_R}}{m_{\tilde{u}_L}} \right)^2,
\]
\[ \eta_{ij} = \frac{\pi \alpha_2}{2} \frac{\lambda_{211}^2}{G_F^2 \bar{m}_{dR}^4} \left( \frac{\bar{m}_{\tilde{d}_R}}{m_{\tilde{e}_L}} \right)^2 \sum_{i=1}^{4} \frac{m_p}{m_{\chi_i}} \left[ \epsilon_{R_i}(d) \epsilon_{L_i}(e) + \epsilon_{L_i}(u) \epsilon_{R_i}(d) \right] \left( \frac{\bar{m}_{\tilde{d}_R}}{m_{\tilde{u}_L}} \right)^2. \]  

(5.5)

Here, \( G_F \) is the Fermi constant, \( \alpha_2 = g_2^2/(4\pi) \) and \( \alpha_s = g_3^2/(4\pi) \) are SU(2)_L and SU(3)_c gauge coupling constants respectively; \( m_{\tilde{u}_L}, m_{\tilde{d}_R}, m_{\tilde{g}} \) and \( m_{\chi_i} \) are masses of the u-squark, d-squark, gluino and neutralinos. We used the neutralino couplings in the form [22]:

\[ \epsilon_{L_i}(\phi) = -T_3(\phi)N_{i2} + \tan\theta_W[T_3(\phi) - Q(\phi)]N_{i1}, \]

(5.6)

\[ \epsilon_{R_i}(\phi) = Q(\phi)\tan\theta_W N_{i1}, \]

where \( N_{ij} \) is \( 4 \times 4 \) neutralino mixing matrix.

Assuming the dominance of gluino exchange, which is well motivated for \( 0\nu\beta\beta \)-decay [21], we obtain for the LNV parameter in Eq. (5.2) the following simplified expression

\[ \eta_{ij} = \frac{\pi \alpha_s}{6} \frac{\lambda_{211}^2}{G_F^2 \bar{m}_{dR}^4 \bar{m}_{\tilde{g}}} \left[ 1 + \left( \frac{\bar{m}_{\tilde{d}_R}}{m_{\tilde{u}_L}} \right)^2 \right]^2. \]

(5.7)

We denote the \((\mu^-, \mu^+)\) conversion nuclear matrix element to be substituted in Eq. (2.2) as \( \mathcal{M}_{\lambda} \). It can be written in the same form as for the \( 0\nu\beta\beta \)-decay [14, 21]. This is because for light nuclear systems the approximation used in Eq. (3.6) is reasonable. Also, relatively small momentum of the outgoing electron \((p_{\mu^+} \approx 0.1)\) allows considering only the \( s_{1/2} \) electron wave state, whose radial dependence is negligible. Thus we have

\[ \mathcal{M}_{\lambda} = c_A \left[ \frac{4}{3} \alpha_1^{1\pi} \left( M_{GT}^{1\pi} + M_{T}^{1\pi} \right) + \alpha_2^{2\pi} \left( M_{GT}^{2\pi} + M_{T}^{2\pi} \right) \right] \]

(5.8)

with \( c_A = m_A^2/(m_p m_e) \) \((m_A = 850 \text{ MeV})\). The structure coefficients of the one-pion \( \alpha_1^{1\pi} \) and two-pion mode \( \alpha_2^{2\pi} \) are [20, 21]: \( \alpha_1^{1\pi} = -0.044 \) and \( \alpha_2^{2\pi} = 0.20 \). The partial nuclear matrix elements of the \( \mathbb{R}_p \) SUSY mechanism for the \((\mu^-, \mu^+)\) process are:

\[ M_{GT}^{k\pi} = \langle 0^+_i | \sum_{i \neq j} \tau_i^+ \tau_j^+ R_0^{ij} F_{GT}^{k\pi}(x) \sigma_i \cdot \sigma_j, | 0^+_i \rangle, \]

\[ M_{T}^{k\pi} = \langle 0^+_i | \sum_{i \neq j} \tau_i^+ \tau_j^+ R_0^{ij} F_{GT}^{k\pi}(x) S_{ij}, | 0^+_i \rangle, \]

(5.9)
where \( k = 1, 2 \) and \( R_0 \) is the nuclear radius. The structure functions \( F_{I,J,K} \) can be expressed as:

\[
\begin{align*}
F^{1\pi}_{GT}(x) &= e^{-x}, \\
F^{1\pi}_T(x) &= (3 + 3x + x^2) e^{-x}, \\
F^{2\pi}_{GT}(x) &= (x - 2) e^{-x}, \\
F^{2\pi}_T(x) &= (x + 1) e^{-x}.
\end{align*}
\]

(5.10)

We used the notation: \( \hat{r}_{ij} = (r_i - r_j)/|r_i - r_j|, r_{ij} = |r_i - r_j|, x_A = m_A r_{ij} \) and \( x_\pi = m_\pi r_{ij} \).

Here \( r_i \) is the coordinate of the \( i \)-th nucleon.

VI. NUCLEAR STRUCTURE CALCULATIONS AND NUMERICAL RESULTS

Let us give a short account of our approach to calculate nuclear matrix elements of the \((\mu^-, \mu^+)\) conversion process. We used the proton-neutron renormalized Quasiparticle Random Phase Approximation \[12,18,14\] and analyzed the full \( 0 - 3\hbar\omega \) shells plus \( 2s_{1/2}, 0g_{7/2} \) and \( 0g_{9/2} \) levels as a single-particle model space both for protons and neutrons. The single particle energies have been obtained by using a Coulomb–corrected Woods–Saxon potential. Two-body G-matrix elements we derived from the Bonn one-boson exchange potential within the Brueckner theory. The pairing interactions have been adjusted to fit the empirical pairing gaps \[23\]. The particle-particle and particle-hole channels of the G-matrix interaction of the nuclear Hamiltonian \( H \) are renormalized by introducing the parameters \( g_{pp} \) and \( g_{ph} \), respectively. The calculations have been carried out for \( g_{ph} = 1.0 \) and \( g_{pp} = 0.8, 1.0, 1.2 \). The effect of two-nucleon correlation has been taken into account as in Refs. \[18,14\].

In Table I we present the nuclear matrix elements of the light Majorana neutrino exchange mechanism for the \((\mu^-, \mu^+)\) conversion in \( ^{44}Ti \). The adopted value of the average nuclear excitation energy \( \bar{E} - E_i \) is 10 MeV. We have found that our results depend weakly on this value within its physical range \( 2 \text{MeV} \leq (\bar{E} - E_i) \leq 15 \text{MeV} \). We used the advantage of that the widths of the low lying nuclear states are negligible in comparison with their energies (see comments after Eq. (3.5)) and carried out the calculation in the limit \( \varepsilon \to 0 \).
This allowed us to separate the real and imaginary of the nuclear matrix element with the formula

\[
\frac{1}{\alpha + i\varepsilon} = \mathcal{P}\frac{1}{\alpha} - i\pi\delta(\alpha).
\]  

(6.1)

As seen from Table I the \((\mu^-, \mu^+)\) conversion nuclear matrix element significantly depends on details of the nuclear model, in particular on the renormalization of the particle-particle channel of the nuclear Hamiltonian, and on the two-nucleon short-range correlation effect (s.r.c.). Especially, the real part of the nuclear matrix element \(M^D\) exhibits strong sensitivity to the s.r.c.. It is worthwhile to notice that the imaginary part of the nuclear matrix element \(\mathcal{M}_\nu\) dominates in \(|\mathcal{M}_\nu|\), in contrast to \(0\nu\beta\beta\)-decay where the imaginary part of the nuclear matrix element is zero. The latter is a direct consequence of special choice of nuclei for the \(0\nu\beta\beta\)-decay searches. These are nuclei with energetically forbidden ordinary beta decay channels, which, otherwise, would be the source of additional background.

The nuclear matrix elements of heavy Majorana neutrino exchange and \(R_p\) SUSY mechanisms of the \((\mu^-, \mu^+)\) conversion in \(^{44}\text{Ti}\) are listed in Table II. Notice that these nuclear matrix elements are strongly suppressed by the two-nucleon s.r.c. The trilinear R-parity breaking matrix elements exhibit weaker dependence on the two-nucleon s.r.c. due to the long range character of the two- and one-pion exchange mechanisms. The sensitivity of the obtained results to the renormalization of the particle–particle interaction strength is not strong. According to our numerical analysis, variation of the nuclear matrix elements presented in Table II do not exceed 20% within the physical region of the nuclear structure parameter \(g_{pp} (0.8 \leq g_{pp} \leq 1.2)\).

Latter on in our analysis we use the following values of the \((\mu^-, \mu^+)\) conversion matrix elements of \(^{44}\text{Ti}\)

\[
|\mathcal{M}_\nu| = 2.66, \quad \mathcal{M}_N = 14.6, \quad \mathcal{M}_\lambda = -426,
\]

(6.2)
calculated for \(g_{pp} = 1.0\) taking into account the two-nucleon s.r.c.

With these values of matrix elements we find from Eqs. (2.2)-(2.5) and (6.2) the \((\mu^-, \mu^+)\) conversion branching ratios.
\[
R^{(\nu)} = 1.0 \times 10^{-23} \left| \frac{<m_{\nu}^{\mu\mu}}{m_e} \right|^2, \quad R^{(N)} = 3.0 \times 10^{-22} |\eta_N|^2, \\
R^{(\lambda)} = 2.6 \times 10^{-19} |\eta_\lambda|^2.
\] (6.3)

for the above discussed light Majorana neutrino exchange \(R^{(\nu)}\), heavy Majorana neutrino exchange \(R^{(N)}\) and the SUSY R-parity violating \(R^{(\lambda)}\) mechanisms.

Eqs. \((6.3)\) allow us to derive the theoretical upper bound on the \((\mu^-, \mu^+)\) conversion branching ratio and estimate the prospects for experimental observation of this process. Towards this end we first find the experimental upper bounds for the LNV parameters \(\eta_i\) from other processes.

Atmospheric and solar neutrino oscillation data, combined with the tritium beta decay endpoint, set upper bounds on the masses of the three light neutrinos \(24\) \(m_{e, \mu, \tau} \leq 3\text{eV}\). Thus, we have conservatively

\[
\langle m_{\nu} \rangle_{\mu\mu} \leq 9 \text{ eV}.
\] (6.4)

More detailed analysis of neutrino oscillation data in combination with the data on \(0\nu\beta\beta\)-decay leads to more stringent constraints on this LNV parameter \(25\):

\[
\langle m_{\nu} \rangle_{\mu\mu} \leq 0.76 \text{ eV}.
\] (6.5)

Assuming the existence of heavy neutrinos \(N\), we may estimate an upper bound on \(\eta_N\) using the LEP limit on heavy stable neutral leptons \(M_N \geq 39.5 \text{ GeV} \) \(26\), which leads to

\[
\eta_N \leq 2.4 \times 10^{-2}. 
\] (6.6)

Adopting the current upper bound \(\lambda'_{211} \leq 0.059 \) \(28\) we obtain from Eq. \((5.7)\) an upper bound on the LNV parameter \(\eta_\lambda\) of the SUSY R-parity violating mechanism

\[
\eta_\lambda \leq 6.8 \times 10^{-4}, 
\] (6.7)

for superparticle masses \(m_{\tilde{g}} \sim m_{\tilde{q}} \sim 100 \text{ GeV}\).

Inserting the upper bounds on the LNV parameters from Eqs. \((6.4)-(6.7)\) to Eqs. \((6.3)\) we obtain the following constraints
\[ R^{(\nu)} \leq 2.3 \times 10^{-35}(3.2 \times 10^{-33}), \quad R^{(N)} \leq 1.7 \times 10^{-25}, \] \[ R^{(A)} \leq 1.2 \times 10^{-25}. \tag{6.8} \]

The value in the brackets correspond to the conservative bound in Eq. (6.4). The weakest constraint can be viewed as an expected upper bound on the \((\mu^-, \mu^+)\) conversion branching ratio \(R \leq 1.0 \times 10^{-25}\). Note that this is about 7 orders of magnitude lower than the corresponding limit in Ref. [2]. One of the main reasons of this difference is strong overestimation of the corresponding nuclear matrix element in Ref. [2] due to simplifying assumptions valid for the \((\mu^-, e^-)\) but not for the \((\mu^-, \mu^+)\) conversion. A particular example of such assumptions is the closure approximation inapplicable in the latter case. In fact, studying the \((\mu^-, \mu^+)\) conversion we deal with a two vacua problem since the initial and final nuclear states are different.

**VII. SUMMARY**

In the present work we developed the theory for the muonic analogue of neutrinoless double beta decay, paying special attention to nuclear structure effects. We studied the three presently most reliable mechanisms for \((\mu^-, \mu^+)\) conversion: light and heavy Majorana neutrino exchange mechanisms as well as the SUSY R-parity violating mechanism. Detailed analysis of the corresponding nuclear matrix elements has been performed. We uncovered the singular behavior of the nuclear matrix element of the light Majorana neutrino exchange mechanism. These sort of singularities are absent in the case of the \(0\nu\beta\beta\)-decay. We also have shown that the imaginary part of the matrix element of this mechanism, which was neglected before, plays a dominant role in the numerical analysis.

We derived the \((\mu^-, \mu^+)\) conversion matrix elements for \(^{44}\text{Ti}\) within the pn-RQRPA approach, and shown that their values existing in the literature [2] were significantly overestimated.

We computed the upper bounds on the branching ratio of \((\mu^-, \mu^+)\) conversion from the current constraints on the LNV parameters. The largest bound corresponds to the SUSY R-
parity violating mechanism $R \leq 1.0 \times 10^{-25}$ which we treat as an expected upper bound on the branching ratio of $(\mu^-, \mu^+)$ conversion. This upper bound is about 7 order of magnitude more stringent than those existing in the literature [2]. With this result we conclude that the nuclear $(\mu^-, \mu^+)$ conversion is probably out of reach of the present and future generation experiments.

**Acknowledgments**

This work was supported in part by the Deutsche Forschungsgemeinschaft grant 436 SLK 17/298, by Fondecyt (Chile) under grant 8000017 and by RFBR (Russia) under grant 00-02-17587.
REFERENCES

[1] W.J. Marciano, Lepton flavor violation, summary and perspectives, Summary talk in the conference on “New initiatives in lepton flavor violation and neutrino oscillations with very intense muon and neutrino beams”, Honolulu-Hawai, USA, October 2-6, 2000, [http://meco.ps.uci.edu/lepton_workshop](http://meco.ps.uci.edu/lepton_workshop).

[2] J.H. Missimer, R.N. Mohapatra, N.C. Mukhopadhyay, Phys. Rev. D 50, 2067 (1994).

[3] F. Šimkovic, P. Domin, S. Kovalenko, A. Faessler, hep-ph/0103029.

[4] T.S. Kosmas, G.K. Leontaris, and J.D. Vergados, Prog. Part. Nucl. Phys. 33, 397 (1994).

[5] T.S. Kosmas, Amand Faessler, and J.D. Vergados, J. Phys. G 23, 693 (1997).

[6] A. Faessler, T.S. Kosmas, S.G. Kovalenko, and J.D. Vergados, Nucl. Phys. B 587, 25 (2000).

[7] T.S. Kosmas, S. Kovalenko, and I. Schmidt, Phys. Lett. B 511, 203 (2001).

[8] T.S. Kosmas, A. Faessler, F. Šimkovic, and J.D. Vergados, Phys. Rev. C 56, 526 (1997).

[9] SINDRUM II Coll., C. Dohmen et al, Phys. Lett. B 317, 631 (1993); W. Honecker et al, Phys. Rev. Lett. 76, 200 (1996); P. Wintz, Status of Muon Electron Conversion at PSI, Invited talk at [1].

[10] MECO Collaboration, J.L. Popp hep-ex/0101017, J. Sculli, The MECO experiment, Invited talk at [1].

[11] The homepage of the PRISM project: [http://www-prism.kek.jp](http://www-prism.kek.jp). Y. Kuno, Lepton Flavor Violation Experiments at KEK/JAERI Joint Project of High Intensity Proton Machine, in Proceedings of Workshop of ”LOWNU/NOON 2000”, Tokyo, December 4-8, 2000.

[12] J. Schwieger, F. Šimkovic, A. Faessler, Nucl. Phys. A 600, 179 (1996).
[13] A. Faessler and F. Šimkovic, J. Phys. G 24, 2139 (1998).

[14] A. Wodecki, W. A. Kamiński and F. Šimkovic, Phys. Rev. D 60, 115007 (1999).

[15] J. Schechter and J.W.F. Valle, Phys.Rev. D 25 (1982) 2951; M. Hirsch, H.V. Klapdor-Kleingrothaus and S.G. Kovalenko, Phys.Lett. B 398 (1997) 311; Phys. Lett. B 403 (1997) 291.

[16] Add the most recent review paper on neutrinoless double beta decay and new physics.

I remember you wrote some paper of this type with Faessler.

[17] M. Doi, T. Kotani, and E. Takasugi, Prog. Theor. Phys., Suppl., 83, 1 (1985).

[18] F. Šimkovic, G. Pantis, J.D. Vergados, and A. Faessler, Phys. Rev. C 60, 055502 (1999).

[19] A. Faessler, S. Kovalenko, F. Šimkovic, Phys.Rev. D57 (1998) 055004.

[20] A. Faessler, S. Kovalenko, F. Šimkovic, and J. Schwieger, Phys. Rev. Lett. 78, 183 (1997); ibid. Phys. Atom. Nucl. 61, 1229 (1998).

[21] A. Faessler, S. Kovalenko, and F. Šimkovic, Phys. Rev. D 58, 115004 (1998).

[22] H. E. Haber and G. L. Kane, Phys. Rep. 117, 75 (1985); J. F. Gunion, H. E. Haber, and G. L. Kane, Nucl. Phys. B 272, 1 (1986).

[23] M.K. Cheoun, A. Bobyk, A. Faessler, F. Šimkovic and G. Teneva, Nucl. Phys. A 561, 74 (1993).

[24] V. Barger, T.J. Weiler, and K. Whisnant, Phys.Lett. B442 (1998) 255.

[25] O. Haug, J.D. Vergados, A. Faessler, S.G. Kovalenko, Nucl. Phys. B 565, 38 (2000).

[26] P. Abreu et al., DELPHI Collab. Z.Phys. C74 (1997) 57.

[27] A. Faessler, F. Šimkovic, Phys. Atom. Nucl. 63, 1165 (2000).

[28] V. Barger, G. F. Giudice, and T. Han, Phys. Rev. D 40, 2987 (1989); K. Enquist,
A. Masiero, and A. Riotto, Nucl. Phys. B 373, 95 (1992); The updated bounds by F. Ledroit and G. Sajot, GDR-S-008(ISN, Grenoble, 1998).

[29] H. Primakoff, Rev. Mod. Phys. 31, 802 (1959); B. Goulard and H. Primakoff, Phys. Rev. C 11, 1984 (1975).

[30] G.K. Leontaris and J.D. Vergados, Nucl. Phys. B 224, 137 (1983).
TABLE I. Nuclear matrix elements of the light Majorana neutrino exchange mechanism of the \((\mu^-, \mu^+)\) conversion in \(^{44}\text{Ti}\). The upper indices \(D\) and \(C\) denote the contributions associated with the direct (Fig. 1a) and cross (Fig. 1b) Feynman diagrams, respectively. The calculations have been performed within pn-RQRPA without and with taking into account the two-nucleon short-range correlations (s.r.c.).

| \(g_{pp}\) | Direct terms | Cross terms | \(|\langle \bar{\nu}_{\mu}\rangle_{\mu\mu}|\) |
|------|--------------|-------------|----------------|
|      | \(Re(M^D_F)\) | \(Re(M^D_{GT})\) | \(Im(M^D_F)\) | \(Im(M^D_{GT})\) | \(Re(M^D)\) | \(Im(M^D)\) | \(M^C_F\) | \(M^C_{GT}\) | \(M^C\) |       |
|      | \(Re(M^D_F)\) | \(Re(M^D_{GT})\) | \(Im(M^D_F)\) | \(Im(M^D_{GT})\) | \(Re(M^D)\) | \(Im(M^D)\) | \(M^C_F\) | \(M^C_{GT}\) | \(M^C\) |       |
| 0.80 | -0.30        | 1.23        | 1.38          | -3.10         | 1.43          | -3.99         | -0.53       | 1.33       | 1.67       | 5.05   |
| 1.00 | -0.29        | 1.12        | 1.25          | -2.61         | 1.31          | -3.41         | -0.49       | 1.16       | 1.47       | 4.40   |
| 1.20 | -0.28        | 1.06        | 1.12          | -2.14         | 1.24          | -2.85         | -0.44       | 1.00       | 1.28       | 3.81   |
|      |              |             |               |               |               |               |             |            |            |        |
|      |              |             |               |               |               |               |             |            |            |        |
| 0.80 | 0.06         | 0.17        | 1.10          | -2.25         | 0.13          | -2.95         | -0.35       | 0.79       | 1.01       | 3.16   |
| 1.00 | 0.04         | 0.17        | 0.98          | -1.83         | 0.14          | -2.46         | -0.32       | 0.67       | 0.87       | 2.66   |
| 1.20 | 0.03         | 0.20        | 0.88          | -1.43         | 0.18          | -1.99         | -0.29       | 0.56       | 0.74       | 2.20   |

without s.r.c

with s.r.c
TABLE II. Nuclear matrix elements of heavy Majorana neutrino exchange and the SUSY R-parity violating mechanisms of the \( (\mu^-, \mu^+) \) conversion in \(^{44}\text{Ti}\). The shorthand s.r.c. denotes two nucleon short range correlations.

| \( g_{pp} \) | \( M_{F}^{\text{heavy}} \) | \( M_{GT}^{\text{heavy}} \) | \( M_{T}^{\text{heavy}} \) | \( M_{s}^{\mu\mu} \) | \( R_{pp} \) SUSY mech. | \( M_{1\pi}^{\text{GT}} \) | \( M_{1\pi}^{\text{T}} \) | \( M_{2\pi}^{\text{GT}} \) | \( M_{2\pi}^{\text{T}} \) | \( M_{\chi_{211}}^{\mu\mu} \) |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.80 | -102. | 380. | -25.3 | 421. | 2.68 | -1.68 | -3.48 | -0.696 | -1346. |
| 1.00 | -94.8 | 344. | -24.9 | 380. | 2.39 | -1.65 | -3.13 | -0.690 | -1215. |
| 1.20 | -87.6 | 311. | -24.4 | 342. | 2.11 | -1.61 | -2.81 | -0.676 | -1094. |
| 0.80 | -28.2 | 20.9 | -21.2 | 17.8 | 1.03 | -1.13 | -0.930 | -0.661 | -471. |
| 1.00 | -26.0 | 18.9 | -21.0 | 14.6 | 0.889 | -1.11 | -0.824 | -0.655 | -426. |
| 1.20 | -23.9 | 17.1 | -20.5 | 11.8 | 0.760 | -1.09 | -0.729 | -0.643 | -385. |
FIG. 1. The direct (a) and cross (b) Feynman diagrams of the \((\mu^-, \mu^+)\) conversion in nuclei mediated by Majorana neutrinos.
FIG. 2. An example of the supersymmetric contribution to $(\mu^-, \mu^+)$ conversion at the quark level.

FIG. 3. The pion-exchange mechanism of $(\mu^-, \mu^+)$ conversion in nuclei. An example of an R-parity violating SUSY contribution to the elementary vertex $\mu^- + \pi^+ \rightarrow \pi^- + \mu^+$ is presented in Fig. 2.