What does the $W p_T$ distribution tell us about the $W + 1$ jet/$W + 0$ jet ratio at the Tevatron?

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We show that the $W p_T$ distribution measured by DØ at the Tevatron agrees well with the NLO QCD theoretical prediction for this quantity, whilst the $W + 1$ jet/$W + 0$ jet ratio, $R^{10}$, measured by DØ lies significantly above the NLO QCD theoretical prediction. We derive an approximate relationship between these two quantities, and show that this rules out the majority of theoretical explanations for the DØ excess in $R^{10}$. We discuss possible physics that could give rise to the $R^{10}$ excess, which have little effect on the $W p_T$.

For some time now DØ at the Tevatron has reported an excess in the preliminary measurement for the ratio of $W + 1$ jet events to $W + 0$ jet events,

$$R^{10}(E_{\text{min}}^T) = \frac{\sigma(W + 1 \text{ jet})}{\sigma(W + 0 \text{ jet})}, \quad (1)$$

over the theoretical NLO prediction, where the jets are defined with transverse energy above some $E_{\text{min}}^T$, and both the numerator and denominator are exclusive with respect to the number of jets. This is shown in Fig.1 where the preliminary DØ measurement lies about 30% above the theoretical prediction for all values of $E_{\text{min}}^T$.

For this measurement $W$ bosons are tagged (both theoretically and experimentally) in their leptonic decay to electrons by requiring there to be an isolated lepton and significant amounts of missing energy,

$$E_T^e > 25 \text{ GeV}, \quad |\eta^e| < 1.1, \quad E_T > 25 \text{ GeV},$$

$$\Delta R(e, \text{jet}) > 0.4 \quad \text{for} \quad E_T^{\text{jet}} > 10 \text{ GeV}. \quad (2)$$

Then jets are formed in the rapidity range $|\eta_{\text{jet}}| < 3.5$ using the standard cone algorithm where all particles are clustered within $\Delta R$ cones, with $\Delta R = 0.7$. For the theoretical predictions we
simulate the experimental jet algorithm by clustering all pairs of partons that lie within $R_{\text{sep}} \Delta R$ of each other to form a proto jet, then test that all clustered partons lie within $\Delta R$ of the proto jet. As a default parameter, we set $R_{\text{sep}} = 1.3$. The jet direction and transverse energy is constructed using the DØ recombination procedure.

Now the definition for $\mathcal{R}^{10}$ can be rearranged to read,

$$
\mathcal{R}^{10}(E_{T}^{\text{min}}) = \frac{\int_{E_{T}^{\text{min}}}^{\infty} dE_{T} \frac{1}{\sigma} \frac{d\sigma^{\text{incl}}}{dE_{T}}}{1 - \int_{E_{T}^{\text{min}}}^{\infty} dE_{T} \frac{1}{\sigma} \frac{d\sigma^{\text{excl}}}{dE_{T}}}.
$$

(3)

In this form the total $W$ cross-section, $\sigma$, has no dependence on the value of $E_{T}^{\text{min}}$ at which jets are defined, and this means that the theoretical calculation does not contain any large logarithms of $E_{T}^{\text{min}}$ and so should be accurately calculable. We calculate $\sigma$ at fixed NLO in QCD, that is $O(\alpha_S)$, using the program DYRAD. We choose to set the renormalization and factorization scale set equal at $\mu = \mu_R = \mu_F = m_W$. On the other hand $d\sigma^{\text{incl}, \text{excl}}/dE_{T}$, the $W + 1$ jet rate, inclusive or exclusive in the number of jets, which explicitly depends on $E_{T}^{\text{min}}$, we calculate at one higher order in perturbation theory, that is NLO (as for finite $p_T$ we are forced to have an additional parton), or $O(\alpha_S^2)$. We keep the renormalization and factorization scale equal, and by default use $\mu = \mu_R = \mu_F = m_W$, although the theoretical calculation changes by only a few percent if the scale is changed between $\mu = 2m_W, m_W/2, E_{T}^{\text{jet}},$ and $E_{T}^{W}$. Throughout we use the

Figure 1: DØ experimental measurement for $\mathcal{R}^{10}$, and the corresponding next-to-leading order QCD predictions. For the theoretical calculations we have chosen the scale $\mu = M_W$. 

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Figure 2: a) The DØ measured $p_T^W$ distribution, and the $O(\alpha_s^2)$ predictions for the three choices of scale $\mu = M_W$, $E_T^W$, $p_T^W$; with the cuts described in the text. b) The ratio of the theoretical predictions, and the preliminary DØ measurement, to the theoretical prediction with $\mu = M_W$. Note that in comparing with the DØ data, we have integrated over the appropriate range of $p_T^W$.

CTEQ4M parton densities\(^8\) with $\alpha_s(M_Z) = 0.116$.

This apparent difference between the experimental measurement and theoretical prediction has led to several suggested explanations. For example, Balázs and Yuan\(^9\) have considered the effect of soft gluon resummation on the related quantity $R_W$,

$$R_W(p_T^{W,\min}) = \frac{\int_{p_T^{W,\min}}^{\infty} dp_T^W \frac{1}{\sigma} \frac{d\sigma}{dp_T^W}}{1 - \int_{p_T^{W,\min}}^{\infty} dp_T^W \frac{1}{\sigma} \frac{d\sigma}{dp_T^W}}. \quad (4)$$

New physics effects are also possible and Choudhury et al.\(^10\) have considered the effect that a massive vector boson with the quantum numbers of both a $W$ boson and a gluon would have on the observed value of $R^{10}$. A more mundane explanation is that an increase in the gluon parton distribution at medium Bjorken $x$ values would boost the $W + 1$ jet rate, which receives contributions from $qg$ scattering, while having little effect on the zero jet rate\(^11\).

In order to consider possible origins for difference between DØ 's measurement of $R^{10}$ and the theoretical prediction of DYRAD it is worth considering variables that contain similar physical information. At lowest order the observed jet in $W + 1$ jet events is produced by a single parton that recoils against the $W$ boson. This means that,

$$E_{T^\text{jet}} = p_T^W. \quad (5)$$

Beyond lowest order this equality is not exactly satisfied, however we expect it to hold approximately. Hence we expect,

$$R^W(p_T^{W,\min}) \simeq R^{10}(E_{T^\text{min}} = p_T^{W,\min}). \quad (6)$$

Unfortunately $R^W$ is not directly measured by DØ, however they do measure the normalized $W_{p_T}$ distribution, $1/\sigma \frac{d\sigma}{dp_T^W}$. This is shown in Fig\(^2\) along with the NLO theoretical prediction for the quantity. Clearly the same theoretical description that provided a bad description of the $R^{10}$ data, gives a good description of the $W_{p_T}$ distribution.
The difference between the measurements of $W$ $p_T$ and $R^{10}$ can be made more directly, by transforming the $W$ $p_T$ into $R^W$ using Eqn.4. If we assume that the experimental errors are independent this gives the result shown in Fig.3. We also show the $R^{10}$ measurement from Fig.1.

We can see that the theoretical prediction and experimental measurement for $R^W$ agree within errors, recall that due to the definition of $R^W$ each data point is not independent of the others. That the theoretical prediction for $R^W$ is greater than $R^{10}$ is easily understood, this is because $R^{10}$ is defined in terms of the exclusive 1 jet rate, whereas $R^W$ receives contributions from all $W$ events. This means that if 2 or more jets are observed $R^W$ receives a contribution that $R^{10}$ does not. Experimentally the measurement for $R^{10}$ lies above $R^W$, which seems hard to understand. This rules out most explanations for the measured $R^{10}$ excess, as whatever explains the $R^{10}$ excess must leave $R^W$ unchanged.

So how can we understand the $R^{10}$ excess in light of the agreement between theory and experiment for $R^W$? The essential difference between $R^{10}$ and $R^W$ is that for the former measurement jets need to be formed, whereas for the latter they do not. At leading order the theoretical calculation in insensitive to how jets are formed, however at NLO we gain sensitivity to how jets are formed as two partons can be clustered into a single jet; however this is still very far from the experimental situation where typically many hadrons are clustered into each jet. Some feeling for the difference between experiment and theory can be obtained by varying the theoretical parameter $R_{sep}$ between its natural limits $1 < R_{sep} < 2$. $R_{sep} = 1$ corresponds experimentally to there being no hadrons (seed towers) between the two parton directions, while $R_{sep} = 1$ corresponds to their being a seed hadron precisely between the two partons. We show
Figure 4: The dependence of the Dyrad prediction for $R^{10}$ on the jet clustering algorithm. We show the predictions normalized to that for $R_{\text{sep}} = 1.3$ clustering. In all cases we have chosen $\mu = M_W$.

the dependence of the theoretical prediction on the theoretical parameter $R_{\text{sep}}$ in Fig.4. Clearly this only changes the calculation for $R^{10}$ by a few percent, and so we do not expect the difference between experimental and theoretical jets to be the source of the difference between the QCD theory and experimental measurement for $R^{10}$.

That $R^{10}_{\text{Exp}} > R^{10}_{\text{Th}}$ means that experimentally there must be more energy associated with the jet than theoretically. However the $W p_T$ is not increased by this additional energy clustered with the jet, this can only be if there is even more additional energy flowing in the direction of the $W p_T$ which balances the $p_T$ clustered with the jet. Hence to simultaneously explain the $R^{10}$ excess, while keeping $R^W$ unchanged, one needs significant additional transverse energy flowing in all directions.

What possible explanations can there be for this additional energy? Several ideas come to mind,

- soft gluons at higher orders in perturbation theory.
- the underlying event.
- multiple interactions.
- overlapping events.

Each of these give additional energy in the event that will increase any measured jets $E_T$, while the first two will only have a minor effect on the $W p_T$ and the last two have no effect on the
That theory and experiment are different for $R^{10}$ is not a sign that QCD is breaking down, as theory and experiment agree so well for $R^{W}$. Instead we should look at the differences between $R^{10}$ and $R^{W}$ (or $1/\sigma \, d\sigma /dE_T$ and $1/\sigma \, d\sigma /dp_T^{W}$) as a probe as to how jets are experimentally and theoretically formed. In this way $W$ events give a somewhat independent test of how we study jet physics. $W$ events through the production of the $W$ boson are known to have a hard scattering, and this gives a somewhat different environment from the usual environment in which jets are formed, and as such have somewhat orthogonal sensitive to physics that can affect jets, such as the underlying event and multiple interactions.

Finally we should note that the CDF collaboration at the Tevatron has recently made their own measurement for the variable $R^{11}$ which agrees well with the NLO QCD theoretical prediction. Although one may take this as a hint of an experimental problem in the DØ measurement, this is far from clear as the measurement made by CDF is of a slightly different quantity than DØ measurement. CDF calculate the ratio of the inclusive 1 or more jet rate to the inclusive zero or more jet rate, whereas DØ measure the exclusive jet rates. As the fraction of $W$ events that contain two or more jets is relatively small we do not expect the theory prediction to work well in one case, but not in the other. Perhaps more importantly CDF define their jets with a cone size of $\Delta R = 0.4$, and this can have less innocent effects. For example if the DØ excess is caused by a misunderstanding of the underlying event in $W$ events, then as the underlying event is approximately flat in rapidity and azimuthal angle, we would expect the larger DØ jet cones to show approximately 3 times the effect of the smaller CDF cones. Such an effect may cause the DØ measurement to be inconsistent with NLO QCD theory, while the CDF measurement remains consistent.

References

1. S. Abachi et al., DØ Collaboration, Submitted to 28th International Conference on High Energy Physics, Warsaw, July 1996, FERMILAB-Conf-96/172-E; G. Guglielmo for the DØ Collaboration, Proceedings of the XI Topical Workshop on $\bar{p}p$ Collider Physics, Padova, May 1996, FERMILAB-Conf-96/245-E; Jay R. Dittmann, Hadronic session, Proceedings of the XXXIIInd Rencontres de Moriond, Editions Frontieres (1997).
2. W. T. Giele, E. W. N. Glover and D. A. Kosower, Nucl. Phys. B403, 633 (1993); W. T. Giele and E. W. N. Glover, Phys. Rev. D46, 1980 (1992).
3. Harry Melanson, private communication.
4. Jaehoon Yu, Ph.D. Thesis, SUNY at Stony Brook, (1993).
5. S.D. Ellis, Z. Kunszt and D.E. Soper, Phys. Rev. Lett. 69, 3615 (1992).
6. S. Abachi et al., DØ Collaboration, Phys. Lett. B 357, 500 (1995).
7. E.W.N. Glover and D.J. Summers, Phys. Lett. B 419, 363 (1998), hep-ph/9704429.
8. H.L. Lai et al, CTEQ Collaboration, Phys. Rev. D55, 1280 (1997).
9. C. Balázs and C.P. Yuan, ‘Comment on the $W+1$ jet to $W+0$ jets ratio’, hep-ph/9704429.
10. D. Choudhury, S. Raychaudhuri, and K. Sridhar, Phys. Lett. B 413, 93 (1997), hep-ph/9706536.
11. Cecilia Gerber, these proceedings.