Abstract

We provide a detailed, model-independent, study for $CP$ violation effects due to the $T$-odd top-quark electric dipole moment (EDM) and weak dipole moment (WDM) in the top-quark pair production via $e^+e^-$ and two-photon annihilation at a next $e^+e^-$ linear collider (NLC). There are two methods in detecting $CP$ violation effects in these processes. One method makes use of measurements of various spin correlations in the final decay products of the produced top-quark pair, while the other is to measure various $CP$-odd polarization asymmetry effects of the initial states. In the $e^+e^-$ case only the first method can be used, and in the $\gamma\gamma$ case both methods can be employed. We provide a complete classification of angular correlations of the $t$ and $\bar{t}$ decay products under $CP$ and $CPT$ which greatly facilitate $CP$ tests in the $e^+e^-$ mode. Concentrating on the second method with the Compton back-scattered high-energetic laser light off the electron or positron beam in the two-photon mode, we construct two $CP$-odd and $CPT$-even initial polarization configurations and apply them to investigating $CP$-violating effects due to the top-quark EDM. With a typical set of experimental parameters at the NLC, we compare the 1-\(\sigma\) sensitivities to the top-quark EDM and WDM in the $e^+e^-$ mode and the two-photon mode. Some model expectation values of the $T$-odd parameters are compared with the results.
1 Introduction

Precision measurements of various production and decay modes of the top quark are expected to provide useful information on physics beyond the SM. Testing new physics in observables which are sensitive to $CP$ violation seems especially promising. As the top quark hardly mixes with other generations, the GIM mechanism of unitarity constraints leads to negligibly small effects of $CP$ violation in the SM. Thus, observation of $CP$ non-invariance in top-quark physics would definitely be a signal for physics beyond the SM.

An important property of heavy top ($m_t \sim 175$ GeV)\([1]\) is that hadronization of the top quark can be neglected to a good approximation because on average it decays before it can form hadronic bound states\([2]\). This implies in particular that spin effects, for instance spin correlations between the produced $t$ and $\bar{t}$ quarks are not be severely distorted by hadronization. These spin effects can be analyzed through the distributions and angular correlations of the weak decay products of the $t$ and $\bar{t}$ quarks. Moreover, these effects can be calculated in perturbation theory. Hence they provide an additional means of testing the SM predictions and of searching for possible new physics effects in top quark production and decay.

The $\gamma t\bar{t}$ coupling consists of the SM tree-level and the magnetic dipole moment (MDM) couplings as well as the EDM coupling. Likewise, in addition to the tree-level SM $Zt\bar{t}$ coupling, we have the analogous $Z$ MDM and $Z$ EDM couplings, the latter of which is called the top-quark WDM. In both cases these couplings may have imaginary parts. The MDM-like couplings are present in the SM at the one-loop level. On the other hand, the EDM-like couplings violate $CP$ and, due to the structure of the SM, they are only present perturbatively in the SM at the three loop level. In some extensions to the SM, however, EDM couplings may be present at lower order in perturbation theory. Some models\([3]\) which can give relatively large fermion EDM's include left-right symmetric theories, additional Higgs multiplets, supersymmetry, and composite models. Neglecting the MDM-like couplings, we consider both the $T$-odd top-quark EDM and WDM in the reactions $e^+e^- \to t\bar{t}$ and $\gamma\gamma \to t\bar{t}$ at NLC\([4]\).

Previously $CP$ violation in the process $e^+e^- \to t\bar{t}$ has been extensively investigated. Those previous works can be classified in two categories according to their own emphasized aspects: (i) the classification\([5, 6, 7, 8]\) of spin correlations of the $t$ and $\bar{t}$ decay products without electron beam polarization, and (ii) the use of a few typical $CP$-odd observables with electron beam polarization\([1, 10]\). In the first class they have constructed all the $CP$-odd observables according to their ranks. However, since the $t$ and $\bar{t}$ are spin-1/2 particles, the $CP$-odd spin correlation only up to rank-two can appear in the process. Therefore, all the constructed $CP$-odd observables previously investigated are not linearly independent. In the present work we completely define all the linearly-independent $CP$-odd correlations by which all other $CP$-odd correlations can be expressed. In the second class, it has been shown that electron beam polarization is very crucial for a few specialized $CP$-odd correlations. We extend their works to investigate which $CP$-odd correlations depend crucially on electron polarization and which ones do not. After expressing all the strongly-dependent $CP$-odd correlations in terms of the linearly-independent correlations we can provide simple explanations for why those specialized observables depend crucially on electron beam polarization.

Detailed studies have been performed mainly in the processes $e^+e^- \to t\bar{t}$ including general studies of $tt\gamma$, $ttZ$ and $tbW$ couplings\([1, 2, 4, 8]\) previously. A photon linear collider (PLC),
employing polarized photons by the Compton back-scattering of polarized laser light on electron/positron beams of NLC, enables us to measure the $tt\gamma$ and $tbW$ couplings and to investigate the possibility of extracting the effective couplings of the top quark to the photon.

We can employ two methods to extract the top-quark effective couplings at a PLC. One method makes use of the quasi-freely decaying property$^2$ of the top quark by measuring various spin correlations in the $tt$ final system, $(bW^+)(\bar{b}W^-)$ or $(bf_1\bar{f}_2)(\bar{bf}_3f_4)$. The other method is to employ linearly-polarized photon beams to measure various polarization asymmetries of the initial states. It is, of course, possible to combine the two methods. The former technique is essentially same as that employed in $e^+e^-$ collisions$^3, 4, 5$ with one difference; in $e^+e^-$ collisions the spin of the $tt$ system is restricted to $J = 1$, while in photon fusion $J = 0$ or $J \geq 2$ is allowed.

Section 2 is devoted to the introduction of the top-quark EDM and WDM, and to some model expectations for the $CP$-odd parameters. In Section 3 we classify all angular dependencies and angular correlations of $t$ and $\bar{t}$ decay products in the $e^+e^-$ mode under $CP$ and $CPT$ transformations, where $T$ is the “naive” time reversal operation which flips particle momenta and spins but does not interchange initial and final states. Then we apply the $CP$-odd angular correlations to probing $CP$ violation due to the top-quark EDM and WDM in the $e^+e^-$ mode by considering the polarized electron case as well as the unpolarized electron case.

In Section 4 we give a detailed description of the energy spectrum and polarization of the high-energy Compton backscattered light. The study of $CP$ violation in the two-photon mode$^1$ is extended in Section 5, where we construct two $CP$-odd and $CPT$-even initial photon polarization configurations, and apply them to obtain the 1-$\sigma$ sensitivities of the top-quark EDM without the detailed information on the complicated $t$ and $\bar{t}$ decay patterns.

After comparing the 1-$\sigma$ sensitivities to the top-quark EDM in the $e^+e^-$ mode and the two-photon mode, we close Section 6 with some prospects for further studies related with $CP$ violation.

The Appendices are devoted to the definition and explicit form of the angular distributions $P_{\alpha X}$ and the definition of the angular correlations $D_\alpha$ and $D'_\beta$.

2 Top-quark EDM

One of the most commonly studied $CP$-violating operators is the EDM of a fermion and its generalizations to weak and strong couplings. The most general matrix element of the electromagnetic current between two top-quark spinors contain $T$-odd term:

$$\langle t|j^\mu_m^e|t\rangle = iF_3(q^2)\bar{u}(p_2)\sigma_{\mu\nu}q^\nu\gamma_5u(p_1).$$

The value of this form factor at zero-momentum transfer:

$$F_3(q^2 = 0) \equiv d_1^t,$$

is called the EDM. This induces a local interaction that can be derived from the effective Lagrangian:

$$L_{eff} = \frac{1}{2}d_1^t \bar{u}\sigma_{\mu\nu}\gamma_5uF_{\mu\nu} + \frac{1}{2}d_2^Z \bar{u}\sigma_{\mu\nu}\gamma_5uZ^\mu\nu + \frac{1}{2}d_3^g \bar{u}\sigma_{\mu\nu}\gamma_5uG_{\mu\nu}^a \frac{\lambda_a}{2},$$

3
where we have also added the generalizations to fermion couplings to the $Z$ boson and gluons.

The quark EDM in the SM vanishes at the one-loop order, because of the unitarity of CKM matrix. Diagrams at two-loop order can have a $CP$-violating phase, but it has been shown by Shabalin\cite{12} that the sum of two-loop contributions to $F_3(q^2 = 0)$ vanishes. It is thus thought that the lowest-order SM contribution to the quark EDM occurs at least at the three-loop level.

There are models which generate a non-zero quark EDM at the one-loop level. Typical examples are the models of $CP$ violation with extra scalars\cite{13}. When $CP$ violation comes from the exchange of a neutral Higgs boson, the EDM for up-type quarks, down-type quarks, or charged leptons is given in the $M_H \gg m_f$ limit by:

$$d_f^\gamma = e Q_f \sqrt{2} G_F m_f^3 \frac{\text{Re}(A) \text{Im}(A)}{M_H^2} \log \left( \frac{m_H^2}{M_f^2} \right),$$

where $A$ is a dimensionless parameter for the Higgs coupling with the left-handed fermion. This is largest for the top-quark although in the case of the top-quark it may be a poor approximation to take $q^2 = 0$. In the case where the $CP$ violation arises in the charged scalar sector, the EDM for down-type quarks is given by

$$d_d^\gamma = e \sqrt{2} G_F m_d \text{Im}(\alpha_1 \beta_1^* |V_{td}|^2 \left( x_t - \frac{3}{2} x_t \log x_t \right),$$

where $\alpha_1$ and $\beta_1$ are dimensionless parameters for the charged Higgs coupling with fermions, and $x_t = m_t^2/M_H^2$. This result follows from the dominance of the top-quark in the loop and assumes that the dominant contribution comes from the lightest charged scalar $H^+$. For the case of an up-type quark the result is:

$$d_u^\gamma = e \sqrt{2} G_F m_u \text{Im}(\alpha_1 \beta_1^*) \left( x_t - \frac{3}{2} x_t \log x_t \right),$$

where $\text{Im}(\alpha_1 \beta_1^*)$ is a dimensionless parameter for the charged Higgs coupling with fermions, and $x_t = m_t^2/M_H^2$.

We denote the $\gamma tt$ and $Z tt$ couplings by the vertex factor $ie \Gamma_V^{\mu}$ (See Figure 1), where

$$\Gamma_V^{\mu} = v_{V\gamma\mu} + a_{V\gamma\mu} \gamma_5 + \frac{c_{V\gamma\mu}}{2m_t} \sigma_{\mu\nu} \gamma_5 q^\nu, \quad V = \gamma, Z,$$

with the vector and axial-vector couplings of the $t$ quark given in the SM by

$$v_\gamma = \frac{2}{3}, \quad a_\gamma = 0, \quad v_Z = \frac{(1 - \frac{2}{3} x_W)}{\sqrt{x_W (1 - x_W)}}, \quad a_Z = -\frac{1}{4 \sqrt{x_W (1 - x_W)}},$$

and $x_w = \sin^2 \theta_W$, $\theta_W$ being the weak mixing angle. Here, $q$ is the four-momentum of the vector boson, $V(=\gamma, Z)$. For $x_w = 0.23$, we find that $v_Z = 0.23$ and $a_Z = -0.59$. We assume that the only additional couplings to the SM ones are the $CP$-violating EDM and WDM factors,

$$d_t^{\gamma; Z} = \frac{e}{m_t} c_{\gamma,Z} \approx 1.13 \times 10^{-16} c_{\gamma,Z} (ecm),$$

for $m_t = 175$ GeV, and that the $CP$-violating form factors, $c_{\gamma,Z}$ are small.
3 Electron-positron mode

3.1 Helicity amplitudes for top-quark pair production

We define the helicities of the $t$ and $\bar{t}$ in the $e^+e^-$ c.m. frame. Let us define the coordinate system $F_0$ for the $t\bar{t}$ production process, $e^+e^- \rightarrow t\bar{t}$. The scattering is in the $x$-$z$ plane and the $z$-axis is along the top-quark momentum direction. The $y$-axis is along $\vec{p}_e - \vec{p}_\ell$ and the $x$-axis is given by the right-handed rule.

We calculate the polarization amplitudes $M_{\sigma,\bar{\sigma};\lambda,\bar{\lambda}} = e^2 M_{\sigma,\bar{\sigma};\lambda,\bar{\lambda}}^0$ for the production process $e^+e^- \rightarrow t\bar{t}$ by using a very straightforward and general method based on two-component spinors. The helicity amplitudes $M_{\sigma,\bar{\sigma};\lambda,\bar{\lambda}}$ are presented in the $e^+e^-$ c.m. frame where the positive $z$-axis is chosen along the top quark momentum direction:

\[
p_e = \frac{\sqrt{s}}{2}(1, -\sin \Theta, 0, \cos \Theta), \quad p_{\ell} = \frac{\sqrt{s}}{2}(1, \sin \Theta, 0, -\cos \Theta),
\]

\[
p_t = \frac{\sqrt{s}}{2}(1, 0, 0, \beta), \quad p_{\bar{t}} = \frac{\sqrt{s}}{2}(1, 0, 0, -\beta).
\]

Then the helicity amplitudes of the process $e^+e^- \rightarrow t\bar{t}$ can be expressed in a compact fashion as follows

\[
M_{\sigma,\bar{\sigma};\lambda,\bar{\lambda}} = \frac{1}{i} \sum_{\alpha=L,R} \sum_{\alpha' = \pm} (v_\alpha + \alpha' a_\alpha) \left[ J^e_\alpha(\sigma, \bar{\sigma}) \cdot J^{\bar{\lambda}}_{\alpha'}(\lambda, \bar{\lambda}) \right] + \frac{i}{2m_{t,s}} \sum_{\alpha=L,R} c_\alpha \left[ S^e_\alpha(\sigma, \bar{\sigma}) \cdot P^t(\lambda, \bar{\lambda}) \right],
\]

where

\[
J^{e\mu}_\alpha(\sigma, \bar{\sigma}) = \bar{v}(p_e, \bar{\sigma})\gamma^\mu P_\alpha u(p_e, \sigma), \quad J^{\bar{\lambda}_{\alpha'}}(\lambda, \bar{\lambda}) = \bar{u}(p_t, \lambda)\gamma^\mu P_{\alpha'} v(p_{\bar{t}}, \bar{\lambda}),
\]

\[
S^e_\alpha(\sigma, \bar{\sigma}) = 2\bar{v}(p_e, \bar{\sigma}) P_\alpha u(p_e, \sigma), \quad P^t(\lambda, \bar{\lambda}) = \bar{u}(p_t, \lambda)\gamma_5 v(p_{\bar{t}}, \bar{\lambda}),
\]

with $\sigma, \bar{\sigma} = L, R$, $\lambda, \bar{\lambda} = \pm$, and $P_{R,L} = P_{\pm} = \frac{1}{2}(1 \pm \gamma_5)$.

In the vanishing electron mass limit, the positron helicity should be opposite to the electron helicity, that is to say, $M_{L,L;\lambda,\bar{\lambda}} = M_{R,R;\lambda,\bar{\lambda}} = 0$. Therefore, it is convenient to rewrite $M_{L,R;\lambda,\bar{\lambda}} = M^L_{\lambda,\bar{\lambda}}$ and $M_{R,L;\lambda,\bar{\lambda}} = M^R_{\lambda,\bar{\lambda}}$, which are given by

\[
M^L_{\pm \pm} = \mp (v_L \mp \beta a_L)(1 \pm \cos \Theta), \quad M^L_{\mp \mp} = \pm \frac{\sqrt{s}}{2m_t} \left[ \mp \frac{m_t^2}{s} v_L \mp i\beta c_L \right] \sin \Theta,
\]

for the initial $e^- e^+$ configuration, and

\[
M^R_{\pm \pm} = \pm (v_R \mp \beta a_R)(1 \mp \cos \Theta), \quad M^R_{\mp \mp} = \pm \frac{\sqrt{s}}{2m_t} \left[ \pm \frac{m_t^2}{s} v_R \mp i\beta c_R \right] \sin \Theta,
\]

for the initial $e^- e^+$ configuration with the scattering angle $\Theta$ and the dimensionless variables defined as

\[
v_L = v_\gamma + r_L v_z, \quad v_R = v_\gamma + r_R v_z,
\]

\[
a_L = a_\gamma + r_L a_z, \quad a_R = a_\gamma + r_R a_z,
\]

\[
c_L = c_\gamma + r_L c_z, \quad c_R = c_\gamma + r_R c_z.
\]
Here the two $\sqrt{s}$-dependent parameters $r_L$ and $r_R$ in the SM are defined as

$$r_L = \frac{\left(\frac{1}{2} - x_w\right)}{\left(1 - \frac{m_Z^2}{s}\right) \sqrt{x_w(1 - x_w)}} \approx +0.64 \left(1 - \frac{m_Z^2}{s}\right)^{-1},$$

$$r_R = \frac{-x_w}{\left(1 - \frac{m_Z^2}{s}\right) \sqrt{x_w(1 - x_w)}} \approx -0.55 \left(1 - \frac{m_Z^2}{s}\right)^{-1},$$

where we have inserted $x_w = 0.23$ and have neglected the $Z$ boson width $\Gamma_Z$, which is easily incorporated but its numerical effect is minute for $\sqrt{s} \geq 2m_t$ since $\Gamma_Z / \sqrt{s} \leq 7 \times 10^{-3}$.

### 3.2 Top and anti-top quark decays

We calculate the helicity amplitudes of $t \to W^+ b$ and $\bar{t} \to W^- \bar{b}$ for on-shell $W^\pm$ bosons. For the process $t \to W^+ b$, the top quark is taken to decay in its rest frame where the top quark momentum is $p_t = (m_t, 0, 0, 0)$. Spherical coordinates are used to describe the outgoing particles; the polar angle $\theta$ is taken from the positive $z$ axis and the azimuthal angle $\phi$ is taken from the positive $x$ axis in the $x$-$y$ plane. The bottom quark and $W$ boson are taken on their mass shells with the four-momenta $p_b$ ($p_{\bar{b}}$) for the bottom (anti-)quark and the four-momenta $p_W$ ($p_{\bar{W}}$) for the $W^+ (W^-)$ bosons taken as

$$p_b = E_b^* (1, - \sin \theta \cos \phi, - \sin \theta \sin \phi, - \cos \theta),$$

$$p_W = E_W^* (1, \beta_W \sin \phi, \beta_W \sin \theta \sin \phi, \beta_W \cos \theta),$$

$$p_{\bar{b}} = E_{\bar{b}}^* (1, - \sin \bar{\theta} \cos \bar{\phi}, - \sin \bar{\theta} \sin \bar{\phi}, - \cos \bar{\theta}),$$

$$p_{\bar{W}} = E_{\bar{W}}^* (1, \beta_{\bar{W}} \sin \bar{\phi}, \beta_{\bar{W}} \sin \bar{\theta} \sin \bar{\phi}, \beta_{\bar{W}} \cos \bar{\theta}),$$

where we neglect the bottom quark mass, which is about 5 GeV, and then

$$E_b^* = \frac{m_b^2 - m_{W^+}^2}{2m_t}, \quad E_W^* = \frac{m_{W^+}^2 + m_{W^-}^2}{2m_t}, \quad \beta_{W^+}^* \equiv \frac{E_b}{E_W}.$$

The angles $\theta$ ($\bar{\theta}$) and $\phi$ ($\bar{\phi}$) in the $t$ ($\bar{t}$) decay refer to the direction of the $W^+$ ($W^-$) boson. We denote the helicity amplitudes as $M_{h_t; \lambda_W; h_b}$ and as $\tilde{M}_{h_{\bar{t}}; \lambda_{W^+}; h_{\bar{b}}}$ after extracting a common factor as

$$M_{h_t; \lambda_W; h_b} = -\frac{e}{\sqrt{2} \sin \theta_W} \sqrt{m_t^2 - m_{W^+}^2} \langle h_t; \lambda_W, h_b \rangle_t,$$

$$\tilde{M}_{h_{\bar{t}}; \lambda_{W^+}; h_{\bar{b}}} = -\frac{e}{\sqrt{2} \sin \theta_W} \sqrt{m_t^2 - m_{W^+}^2} \langle h_{\bar{t}}; \lambda_{W^+}, h_{\bar{b}} \rangle_{\bar{t}}.$$

There are four non-vanishing helicity amplitudes for each decay mode in the rest frame of the top quark and the top anti-quark for $m_b = m_{\bar{b}} = 0$:

$$\langle -; 0- \rangle_t = \frac{m_t}{m_W} \sin \frac{\theta}{2}, \quad \langle -; -\rangle_t = \sqrt{2} \cos \frac{\theta}{2}.$$
\begin{align}
\langle +;0- \rangle_t &= \frac{m_t}{m_W} \cos \frac{\theta}{2} e^{i\phi}, \quad \langle +;-- \rangle_t = -\sqrt{2} \sin \frac{\theta}{2} e^{i\phi}, \\
\langle +;0+ \rangle_t &= -\frac{m_t}{m_W} \cos \frac{\theta}{2} e^{-i\phi}, \quad \langle +;++ \rangle_t = -\sqrt{2} \sin \frac{\theta}{2} e^{-i\phi}, \\
\langle -;0+ \rangle_t &= -\frac{m_t}{m_W} \sin \frac{\theta}{2} e^{i\phi}, \quad \langle -;++ \rangle_t = \sqrt{2} \cos \frac{\theta}{2}.
\end{align}

The helicity amplitudes can be used to derive the density matrix of the top quark. When the \( W \) polarization is not measured, the \( t \) and \( \bar{t} \) decay density matrices are given by

\begin{align}
D_t &= \frac{1}{2} \begin{pmatrix} 1 + \kappa_w \cos \theta & \kappa_w \sin \theta e^{i\phi} \\
\kappa_w \sin \theta e^{-i\phi} & 1 - \kappa_w \cos \theta \end{pmatrix}, \\
\bar{D}_t &= \frac{1}{2} \begin{pmatrix} 1 + \kappa_w \cos \bar{\theta} & \kappa_w \sin \bar{\theta} e^{-i\phi} \\
\kappa_w \sin \bar{\theta} e^{i\phi} & 1 - \kappa_w \cos \bar{\theta} \end{pmatrix},
\end{align}

respectively, where the polarization efficiency \( \kappa_w \) is given by

\begin{equation}
\kappa_w = \frac{m_t^2 - 2m_W^2}{m_t^2 + 2m_W^2} \approx 0.41,
\end{equation}

for \( m_t = 175 \text{ GeV} \) and \( m_W = 80 \text{ GeV} \).

We now discuss the angular distributions of the leptons \( t^\pm \) arising from semileptonic decays

\begin{equation}
t \to W^+(p_W) b \to l^+(p_l) \nu b(p_\nu), \\
\bar{t} \to W^-(p_W) \bar{b} \to l^-(p_\bar{\nu}) \bar{\nu} b(p_\bar{\nu}),
\end{equation}

for the polarized \( t \) and \( \bar{t} \) quarks, where the momenta in the parentheses refer to the rest systems of \( t \) and \( \bar{t} \) and serve to analyze the spin polarization of \( t \) and \( \bar{t} \). Neglecting lepton masses we write the lepton momenta as

\begin{align}
p_l &= E_l^+ (1, \sin \theta_l \cos \phi_l, \sin \theta_l \sin \phi_l, \cos \theta_l), \\
p_\nu &= E_\nu^+ (1, \sin \theta_\nu \cos \phi_\nu, \sin \theta_\nu \sin \phi_\nu, \cos \theta_\nu), \\
p_\bar{\nu} &= E_\bar{\nu}^- (1, \sin \bar{\theta}_\nu \cos \bar{\phi}_\nu, \sin \bar{\theta}_\nu \sin \bar{\phi}_\nu, \cos \bar{\theta}_\nu),
\end{align}

where \( E_l^+ \) and \( E_\nu^+ \) are the lepton energies and \( E_\nu^- \) and \( E_\bar{\nu}^- \) are the neutrino energies in the \( t \) and \( \bar{t} \) rest frames, respectively. We denote the helicity amplitudes as \( M_{h_t}^l \) and as \( M_{h_t}^l \) after extracting a common factor as follows

\begin{align}
M_{h_t}^l &= 2g^2 \frac{\sqrt{(m_t^2 - q^2) E_l E_\ell}}{q^2 - m_W^2 + im_W \Gamma_W} \langle h_t, h_\ell \rangle_l, \\
M_{h_t}^l &= 2g^2 \frac{\sqrt{(m_t^2 - q^2) E_\nu E_\ell}}{q^2 - m_W^2 + im_W \Gamma_W} \langle h_\ell, h_\nu \rangle_l.
\end{align}
In the semileptonic decays, there are two non-vanishing helicity amplitudes for each decay mode in the rest frames of the top quark and the top anti-quark for $m_b = m_{\bar{b}} = 0$:

\[
\langle +, - \rangle^t = \cos \frac{\theta_t}{2} \left[ \cos \frac{\theta_b}{2} \sin \frac{\theta_b}{2} e^{i \phi_b} - \sin \frac{\theta_b}{2} \cos \frac{\theta_b}{2} e^{i \phi_b} \right],
\]

\[
\langle -, + \rangle^t = -\cos \frac{\theta_b}{2} \left[ \cos \frac{\theta_b}{2} \sin \frac{\theta_b}{2} e^{i \phi_b} - \sin \frac{\theta_b}{2} \cos \frac{\theta_b}{2} e^{i \phi_b} \right],
\]

\[
\langle +, + \rangle^t = -\cos \frac{\theta_t}{2} \left[ \cos \frac{\theta_t}{2} \sin \frac{\theta_t}{2} e^{-i \phi_t} - \sin \frac{\theta_t}{2} \cos \frac{\theta_t}{2} e^{-i \phi_t} \right],
\]

\[
\langle -, - \rangle^t = -\cos \frac{\theta_t}{2} \left[ \cos \frac{\theta_t}{2} \sin \frac{\theta_t}{2} e^{-i \phi_t} - \sin \frac{\theta_t}{2} \cos \frac{\theta_t}{2} e^{-i \phi_t} \right],
\]

(27)

It is well known that within the SM the angular distribution of the charged lepton is a much better spin analyzer of the top quark than that of the $b$ quark or the $W$-boson arising from semi- or non-leptonic $t$ decays. As a matter of fact, the decay matrices of the semileptonic decays of polarized $t$ and $\bar{t}$ are given in the $t$ and $\bar{t}$ helicity bases by

\[
D^t = \frac{1}{2} \begin{pmatrix}
1 + \cos \theta_t & \sin \theta_t e^{i \phi_t} \\
\sin \theta_t e^{-i \phi_t} & 1 - \cos \theta_t
\end{pmatrix},
\]

\[
D^\bar{t} = \frac{1}{2} \begin{pmatrix}
1 + \cos \bar{\theta}_t & \sin \bar{\theta}_t e^{-i \phi_t} \\
\sin \bar{\theta}_t e^{i \phi_t} & 1 - \cos \bar{\theta}_t
\end{pmatrix},
\]

(28)

(29)

respectively. $\theta_t$ and $\phi_t$ are the polar and azimuthal angles of $l^+$ from the $t$ decay, which are defined in the $t$ rest frame $F_t$ constructed by boosting the $t\bar{t}$ c.m. frame $F_0$ along the top quark momentum direction. Similarly, the polar angle $\bar{\theta}_t$ and the azimuthal angle $\phi_t$ of $l^-$ from the $\bar{t}$ decay are defined in the $\bar{t}$ rest frame $F_{\bar{t}}$ constructed by boosting the $t\bar{t}$ center of mass frame $F_0$ along the anti-top quark momentum direction. Through the present work, it is important to keep in mind that the three coordinate systems $F_0$, $F_t$, and $F_{\bar{t}}$ have parallel directions of coordinate axes. Note that the polarization efficiency is unity in the semileptonic decays, implying that the charged lepton analyzes the spin of the top quark much more efficiently than the corresponding $b$ quark.

To lowest order in the SM and in the narrow-width approximation we obtain the following normalized distribution of the semileptonic $t$ decay:

\[
N(t \to b\bar{\ell}\nu)_{\lambda\lambda'} = \frac{12x(1 - x)}{(1 + 2w)(1 - w)^2} \left[ D^t_\lambda(\lambda') \right] d\Omega^t, \quad (30)
\]

\[
\bar{N}(\bar{t} \to b\bar{\ell}\nu)_{\bar{\lambda}\bar{\lambda'}} = \frac{12\bar{x}(1 - \bar{x})}{(1 + 2\bar{w})(1 - \bar{w})^2} \left[ D^{\bar{t}}_{\bar{\lambda}}(\bar{\lambda}') \right] d\Omega^{\bar{t}}, \quad (31)
\]

where $\lambda(\ell)$ and $\bar{\lambda}(\bar{\ell})$ refer to the helicities of the $t$ and $\bar{t}$, respectively, and

\[
x = \frac{2E_t}{m_t}, \quad \bar{x} = \frac{2E_{\bar{t}}}{m_{\bar{t}}}, \quad w = \frac{m_{\bar{\nu}}}{m_t}, \quad \bar{w} \leq x(\bar{x}) \leq 1,
\]

\[
d\Omega^t = d \cos \theta_t d\phi_t, \quad d\Omega^{\bar{t}} = d \cos \bar{\theta}_{\bar{t}} d\bar{\phi}_{\bar{t}}. \quad (32)
\]
The factorization of the lepton distribution into an energy and angular dependent part holds and this property is irrespective of whether the $W$ boson in on-shell or not. It was shown in Ref. 16 that even the order $\alpha_s$ QCD corrections respect this factorization property to a high degree of accuracy.

### 3.3 CP-odd observables

The differential cross section of the process $e^+e^- \to t\bar{t}$, followed by the decays $t \to bX^+$ and $\bar{t} \to \bar{b}X^-$ is given by

$$
d\sigma\left(e^+e^- \to t\bar{t} \to bX^+\bar{b}X^-\right)_{L,R} = \frac{6\pi\alpha^2\beta}{s} \left[ B_{X^+\bar{B}_{X^-}} \right] \times \Sigma_{L,R}(\Theta; \xi_1, \bar{\xi}_1; \xi_2, \bar{\xi}_2; \xi_3, \bar{\xi}_3) \left[ d\cos \Theta \left[ \frac{d\cos \theta d\phi}{4\pi} \right] \right],$$

where for notational convenience the abbreviations

$$\xi_1 = \sin \theta \cos \phi, \quad \xi_2 = \sin \theta \sin \phi, \quad \xi_3 = \cos \theta,$$

$$\bar{\xi}_1 = \sin \bar{\theta} \cos \bar{\phi}, \quad \bar{\xi}_2 = \sin \bar{\theta} \sin \bar{\phi}, \quad \bar{\xi}_3 = \cos \bar{\theta},$$

are used, $\Theta$ is the scattering angle between the electron and top-quark momenta, and $B_{X^+}$ and $B_{X^-}$ are the branching fractions of $t \to bX^+$ and $\bar{t} \to \bar{b}X^-$. Here, the angular dependence $\Sigma_{L,R}$ is given by

$$\Sigma_{L,R}(\Theta; \xi_1, \bar{\xi}_1; \xi_2, \bar{\xi}_2; \xi_3, \bar{\xi}_3) \equiv \sum_{\lambda\lambda'\lambda''} M^{L,R}_{\lambda\lambda'} M^*_{\lambda'\lambda''} \phi_{\lambda\lambda'} \phi_{\lambda''}.$$ \hspace{1cm} (35)

In the $e^+e^-$ c.m frame the angular dependence $\Sigma_{L,R}$ for the process $e^+e^- \to t\bar{t} \to (X^+b)(X^-\bar{b})$ can be written as

$$\Sigma_{L,R}(\Theta; \xi_1, \bar{\xi}_1; \xi_2, \bar{\xi}_2; \xi_3, \bar{\xi}_3) = \mathcal{P}_{1L,R} \mathcal{D}_1 + \kappa\bar{\kappa}\mathcal{P}_{2L,R} \mathcal{D}_2$$

$$+ \frac{(k + \bar{k})}{2} \mathcal{P}_{3L,R} + \frac{(k - \bar{k})}{2} \mathcal{P}_{4L,R} \mathcal{D}_3 + \left[ \frac{(k + \bar{k})}{2} \mathcal{P}_{4L,R} + \frac{(k - \bar{k})}{2} \mathcal{P}_{3L,R} \right] \mathcal{D}_4$$

$$+ \frac{(k + \bar{k})}{2} \mathcal{P}_{5L,R} + \frac{(k - \bar{k})}{2} \mathcal{P}_{6L,R} \mathcal{D}_5 + \left[ \frac{(k + \bar{k})}{2} \mathcal{P}_{6L,R} + \frac{(k - \bar{k})}{2} \mathcal{P}_{5L,R} \right] \mathcal{D}_6$$

$$+ \frac{(k + \bar{k})}{2} \mathcal{P}_{7L,R} + \frac{(k - \bar{k})}{2} \mathcal{P}_{8L,R} \mathcal{D}_7 + \left[ \frac{(k + \bar{k})}{2} \mathcal{P}_{8L,R} + \frac{(k - \bar{k})}{2} \mathcal{P}_{7L,R} \right] \mathcal{D}_8$$

$$+ \kappa\bar{\kappa} \sum_{\alpha=9}^{16} \mathcal{P}_{\alpha L,R} \mathcal{D}_{\alpha} \right],$$ \hspace{1cm} (36)

where the definition of the sixteen (16) functions $\mathcal{P}_{\alpha X}$ ($X = L, R$) and the sixteen correlation functions $\mathcal{D}_{\alpha}$ ($\alpha = 1$ to 16) is given in Appendices A and B, respectively, and $\kappa(k) = \kappa_w$ for the inclusive $t(\bar{t})$ decay and unity for the semileptonic $t(\bar{t})$ decay.

The terms $\mathcal{P}_{\alpha}$ and $\mathcal{D}_{\alpha}$ can thus be divided into four categories under $CP$ and $CP\bar{T}$: even-even, even-odd, odd-even, and odd-odd terms. $CP$-odd coefficients directly measure $CP$
violation and \(CP\tilde{T}\)-odd terms indicate rescattering effects. Table 1 shows that there exist six (6) independent \(CP\)-odd terms among which \(P_5\), \(P_{12}\), \(P_{14}\) and \(P_3\) are \(CP\tilde{T}\)-even, and \(P_3\), \(P_7\) and \(P_{16}\) \(CP\tilde{T}\)-odd.

From now on we make a detailed investigation of the production process \(e^+e^-\rightarrow t\bar{t}\). Recently, \(CP\) violation in the production process has been extensively investigated. Here, we study the \(CP\)-violating effects of the process \(e^+e^-\rightarrow t\bar{t}\) in a rather unified manner. As shown previously, we can consider three \(CP\)-odd and \(CP\tilde{T}\)-even and three \(CP\)-odd and \(CP\tilde{T}\)-odd terms.

Including electron beam polarization, we can obtain thirty-two (32) observables in total:

\[
32 = 2 \times (2 \times 2) \uparrow \uparrow \quad \text{Electron Pol.} \quad t \text{ and } \bar{t} \text{ Pol.}
\]

Here, the first 2 is for the electron helicity, and two 2’s in the parentheses for the degrees of top and anti-top polarizations. It is therefore clear that the classification according to the \(CP\) and \(CP\tilde{T}\) transformation properties gives us a complete set of observables that can be measured in the process \(e^+e^-\rightarrow t\bar{t}\) with left- and right-handed polarized electron beams.

The \(CP\)-odd part of the angular dependence (36) can be separated into two parts

\[
\Sigma_{CP}^{L,R}(\Theta; \xi_1, \bar{\xi}_1; \xi_2, \bar{\xi}_2; \xi_3, \bar{\xi}_3) = \Sigma_{CP}^{E,R} + \Sigma_{CP}^{O,R},
\]

where \(\Sigma_{CP}^{E,R}\) and \(\Sigma_{CP}^{O,R}\) terms are \(CP\tilde{T}\)-even and \(CP\tilde{T}\)-odd, respectively, and given by

\[
\Sigma_{CP}^{E,R} = P_{5L,R} \left[ \frac{2}{2} D_5 + \frac{2}{2} D_6 \right] + \kappa\bar{\kappa} \left[ P_{12L,R} D_{12} + P_{14L,R} D_{14} \right],
\]

\[
\Sigma_{CP}^{O,R} = P_{3L,R} \left[ \frac{2}{2} D_3 + \frac{2}{2} D_4 \right] + P_{7L,R} \left[ \frac{2}{2} D_7 + \frac{2}{2} D_8 \right] + \kappa\bar{\kappa} P_{16} D_{16}.
\]

The explicit form of all the \(CP\)-odd terms is listed in Appendix [A]. The upper three \(CP\)-odd \(P_{\alpha X}\) terms are \(CP\tilde{T}\)-even and the lower three \(CP\)-odd terms are \(CP\tilde{T}\)-odd.

Including electron beam polarization, Poulose and Rindani [10] recently have considered two new \(CP\)-odd and \(CP\tilde{T}\)-even asymmetries, which are essentially equivalent to the so-called triple vector products, and two new \(CP\)-odd and \(CP\tilde{T}\)-odd asymmetries in addition to the two conventional lepton energy asymmetries. It is clear that we can use six more asymmetries among which four asymmetries are \(CP\)-odd and \(CP\tilde{T}\)-even and the other two terms are \(CP\)-odd and \(CP\tilde{T}\)-odd.

### 3.4 Top-quark momentum reconstruction

Purely semileptonic decay modes of a \(t\bar{t}\) pair also give the cleanest signal for the top-pair production process in \(e^+e^-\) collisions:

\[
e^-(p_e) + e^+(p_{\bar{e}}) \rightarrow t(p_t) + \bar{t}(p_{\bar{t}}),
\]

\[
t(p_t) \rightarrow b(p_b) + W^+(p_{W^+}),
\]
\[\bar{t}(p_t) \rightarrow \bar{b}(p_b) + W^-(p_W),\]
\[W^+(p_W) \rightarrow \bar{l}(p_l) + \nu(p_{\nu}),\]
\[W^-(p_W) \rightarrow l(p_l) + \bar{\nu}(p_{\bar{\nu}}).\]  (40)

The process is observed experimentally as shown in Figure 2;
\[e^+ + e^- \rightarrow b + \bar{b} + l + \bar{l} + \text{missing momentum},\]  (41)
where the final lepton pair can be either one of \(e^-e^+, e^-\mu^+, e^-\tau^+, \mu^-e^+, \) and \(\mu^-\mu^+.\) The four-momenta of the particles are given in parentheses.

A simple kinematical analysis, presented below, shows that the two unobserved neutrino momenta can be determined from the observed \(b, \bar{b},\) and lepton momenta with no ambiguity, in the limit where the \(t\) and \(W\) widths and photon (or gluon) radiation are neglected.

The kinematics of the process (40) is determined by ten angles, two for the scattering, four each for the semileptonic \(t\) decays. Since we observe the four three-momenta of the final particles, generally we have superfluous observables to fix the whole configuration. Here we present an explicit solution for the two momenta \(p_W\) and \(p_{\bar{W}}\) in terms of the observed \(b, \bar{b},\) and lepton momenta so that the \(t\) and \(\bar{t}\) momenta can be reconstructed.

It suffices to solve for the three-momentum \(\vec{p}_W\) and then \(p_W\) is given by momentum conservation. As the \(t\) energy is equal to the beam energy \(E\), we have
\[p_W^0 = E - p_b^0, \quad \vec{p}_W^2 = (E - p_b^0)^2 - m_W^2.\]  (42)

A similar equation holds for the \(\bar{t} \rightarrow \bar{b}W^-\) decay:
\[\vec{p}_{\bar{W}}^2 = (E - p_{\bar{b}}^0)^2 - m_{\bar{W}}^2.\]  (43)

Using momentum conservation \(\vec{p}_W = -\vec{p}_{\bar{W}} + \vec{p}_b + \vec{p}_{\bar{b}}\) and Eq. (42), this last equation can be written in terms of \(\vec{p}_{\bar{W}}:\)
\[(\vec{p}_b + \vec{p}_{\bar{b}}) \cdot \vec{p}_W = E(p_b^0 - p_{\bar{b}}^0) - \vec{p}_{\bar{b}}^0 - \vec{p}_b + m_b^2.\]  (44)

The third constraint comes from the condition that the \(b-W^+\) system should have the mass of the \(t\) quark:
\[(p_b + p_W)^2 = m_t^2,\]  (45)
which gives
\[\vec{p}_b \cdot \vec{p}_W = E p_b^0 - p_{\bar{b}}^0 + \frac{1}{2}(m_W^2 + m_b^2 - m_t^2).\]  (46)

Eqs. (44) and (46) lead to
\[\vec{p}_{\bar{b}} \cdot \vec{p}_{\bar{W}} = -E p_{\bar{b}}^0 - \vec{p}_{\bar{b}} \cdot \vec{p}_{\bar{b}} + \frac{1}{2}(m_t^2 + m_b^2 - m_{\bar{W}}^2).\]  (47)
The sequential $W^+ \rightarrow l\nu$ decay yields another condition:

$$(p_W - p_l)^2 = 0,$$  \hspace{1cm} (48)

which gives

$$p_l \cdot p_W = E p_l^0 - p_b^0 p_l^0 - \frac{1}{2}(m_W^2 + m_l^2).$$  \hspace{1cm} (49)

The four conditions (42), (46), (47), and (49) provide the solution for $p_W$. We rewrite the right-hand sides of these equations for the sake of clarity:

$$p_W^2 = K, \hspace{1cm} p_b \cdot p_W = L, \hspace{1cm} p_b^0 \cdot p_W = M, \hspace{1cm} p_l \cdot p_W = N.$$  \hspace{1cm} (50)

Let us assume, for the moment, that the three three-momenta $p_b$, $p_b^0$ and $p_l$ are not parallel. Then we can expand $p_W$ in terms of any combination of two momenta among the three momenta. Here, we choose $p_b$ and $p_b^0$, for which $p_W$ is expressed as

$$p_W = a p_b + b p_b^0 + c p_b \times p_b^0.$$  \hspace{1cm} (51)

The second and third expressions in Eq. (50) constrain $p_W$ to lie on a line in three-dimensional space. They give

$$a p_b^2 + b p_b \cdot p_b^0 = L,$$
$$a p_b \cdot p_b^0 + b p_b^2 = M,$$  \hspace{1cm} (52)

which can be explicitly solved:

$$\binom{a}{b} = \frac{1}{|p_b \times p_b^0|^2} \begin{pmatrix} p_b^2 & -p_b \cdot p_b^0 \\ -p_b \cdot p_b^0 & p_b^2 \end{pmatrix} \begin{pmatrix} L \\ M \end{pmatrix}.$$  \hspace{1cm} (53)

The remaining variable $c$ is determined using the final two conditions of Eq. (50):

$$c^2 = \frac{1}{|p_b \times p_b|^2} \left[ K - a^2 p_b^2 - b^2 p_b^0 - 2ab p_b \cdot p_b^0 \right],$$  \hspace{1cm} (54)

$$c = \frac{1}{p_l \cdot (p_b \times p_b)} \left[ N - a p_l \cdot p_b + b p_l \cdot p_b^0 \right].$$  \hspace{1cm} (55)

The sign of $c$ can not be determined by the first equation, but this twofold discrete ambiguity is cleared out through the second constraint which stems from the extra information on the antilepton momentum.

There are two exceptional cases where the $t$ and $\bar{t}$ momenta can not be determined. (i) In the exceptional case that two momenta are parallel, one has a twofold discrete ambiguity, and (ii) in the more exceptional case that three momenta are parallel, one obtains an one-parameter family of solution for which the azimuthal angle of $p_W$ with respect to $p_b$ is left undetermined. Even from experimental point of view such two cases are so exceptional that the reconstruction of the $t$ and $\bar{t}$ momenta can be almost always possible.
3.5 *CP*-odd observables in the laboratory frame

Experimentally it is, however, difficult to perform a 16-parameter fit (corresponding to the 16 angular coefficients) for each of several cos Θ bins. Rather one would like to obtain from the experimental data the moments of those angular distributions that are most sensitive to new physics, i.e. the anomalous *CP*-violating form factors, $c_\gamma$ and $c_Z$ at hand. However, a sufficiently precise reconstruction of the top quark direction is required to measure all the angular variables. The reconstruction is easy if either the top quark or the top anti-quark decays into a b quark and W boson that decays hadronically. We have shown in Section 3.4 that in the process $e^+e^\to tt$ even the purely semileptonic decays of the $t$ and $\bar{t}$ quarks allow the full reconstruction of the particle momenta, especially the top and anti-top momenta. In practice, the use of the directly measurable momenta of the charged leptons and/or b-jets might be easier. When the top quark and anti-top quark directions are not determined, the cross section should be rewritten in the laboratory frame and variables independent of the top quark and anti-quark directions should be used. These transformations can be straightforwardly performed and several useful angular variables can be introduced.

The previous works have concentrated on the *CP*-odd observables expressed in terms of the directly measurable particle momenta. However, the analytic expressions of those observables are very much involved even if the *CP*-odd terms for the specific $t$ and $\bar{t}$ helicity values are very simple.

First of all we investigate the kinematics of the production-decay sequence

$$e^-e^+ \to tt \to X^+bX^-\bar{b}. \quad (56)$$

The $t$ and $\bar{t}$ momenta are, of course, back to back. The $b$ and $\bar{b}$ momenta can be measured. In the laboratory frame, the momenta of $X^+(q_X, \vartheta, \varphi)$ and $X^-(q_{\bar{X}}, \bar{\vartheta}, \bar{\varphi})$ are referred with respect to the direction of the top quark. The boosts between the laboratory frame and each of the top and anti-top rest frames are defined by the parameters $\gamma = \sqrt{s}/(2m_t)$ and $\beta = \sqrt{1 - \gamma^{-2}}$. The momentum variables between the laboratory frame and the top rest frame are related by

$$E_X = \gamma(E^*_X + \beta q^*_X \cos \theta), \quad \varphi = \phi, \quad q_X \cos \vartheta = \gamma(q^*_X \cos \theta + \beta E^*_X), \quad q_X \sin \vartheta = q^*_X \sin \theta, \quad (57)$$

and those between the laboratory frame and the anti-top rest frame are related by

$$E_X = \gamma(E^*_X - \beta q^*_X \cos \bar{\theta}), \quad \bar{\varphi} = \bar{\phi}, \quad q_X \cos \bar{\vartheta} = \gamma(q^*_X \cos \bar{\theta} - \beta E^*_X), \quad q_X \sin \bar{\vartheta} = q^*_X \sin \bar{\theta}. \quad (58)$$

Observables which are constructed from the (unit) momenta of the charged leptons and/or $b$ jets originating from $t$ and $\bar{t}$ decay are directly measurable in future experiments. Both the nonleptonic and semileptonic decay channels

$$t \to bX_{\text{had}}, \quad (59)$$

$$t \to bl^+\nu; \quad l = e, \mu, \tau, \quad (60)$$

together with the corresponding charge-conjugated ones are used in the following. The first set of observables which we consider involves the momentum of a lepton or $b$ jet from $t$ decay
correlated with the momentum of a lepton or $b$ jet from $\ell$ decay. These correlations apply to the reactions

$$e^+(p_e) + e^-(p_{\ell}) \rightarrow t + \bar{t} \rightarrow a(q_+^a) + \bar{c}(q_-^c) + X,$$

(61)

where we use the notation $a, c = e^+, \mu^+, \tau^+, b$ jet, and $\bar{a}, \bar{c}$ will denote the corresponding charge conjugate particles. The momenta $p_{e,\ell}$ and $q_{\pm}$ are defined in the overall c.m. frame. Light quark jets resulting from the hadronic decays are difficult to identify and are therefore not used for constructing observables in the following. We shall assume that the $\tau$ momentum is measurable with a suitable vertex chamber. The subsequent analysis holds for all reactions of the form irrespectively of the intermediate $tt$ state and of the unobserved part $X$ of the final state.

Let us start with the $CP$-odd energy asymmetries

$$A_E^b = E_b - E_\ell, \quad A_E^\ell = E_\ell - E_t.$$

(62)

The $CP$-odd asymmetries are proportional to the $CP\bar{T}$-odd correlation function $D_7$:

$$A_E^b = -\sqrt{\frac{2}{3}}\gamma E_b^* D_7, \quad \langle A_E^\ell \rangle_E = \sqrt{\frac{2}{3}}\gamma E_\ell^* E D_7,$$

(63)

where $E_b^* = (m_t^2 - m_W^2)/2m_t$ and the notation $X)_E$ denotes the average of the observable $X$ over the lepton energy distribution. Explicitly we obtain for the average of the lepton energy

$$\langle E_\ell^* \rangle_E = \frac{m_t}{2} \int \frac{6x^2(1-x)}{(1+2w)(1-w)^2} \ dx = \frac{m_t}{4} \left[ \frac{1 + 2w + 3w^2}{1 + 2w} \right],$$

(64)

with $w$ defined in Equation (32), and for the expectations of the $CP$-odd energy asymmetries

$$\langle A_E^b \rangle_{L,R} = \frac{4}{9} E_b^* \gamma^2 v_{L,R} \text{Im}(c_{L,R}),$$

$$\langle A_E^\ell \rangle_{L,R} = -\frac{4}{9} \langle E_\ell^* \rangle_E \gamma^2 v_{L,R} \text{Im}(c_{L,R}) \text{Im}(c_{L,R}).$$

(65)

Secondly, let us investigate the $CP$-odd vector observables

$$A_1^b = \bar{p}_e \cdot (\bar{p}_b \times \bar{p}_\ell), \quad A_2^b = \bar{p}_e \cdot (\bar{p}_b + \bar{p}_\ell),$$

$$A_1^\ell = \bar{p}_e \cdot (\bar{p}_\ell \times \bar{p}_t), \quad A_2^\ell = \bar{p}_e \cdot (\bar{p}_\ell + \bar{p}_t).$$

(66)

(67)

The four $CP$-odd vector observables can be expressed in the $t$ and $\bar{t}$ rest frames in terms of the angular correlations $D_\alpha$ ($\alpha = 1$ to 16) defined in Appendix E, as

$$A_1^b = \sqrt{\frac{2}{3}} E_b^* \left[ \cos \Theta D_{12} - \gamma \sin \Theta D_{14} - \sqrt{3}\gamma \sin \Theta D_3 \right],$$

$$A_2^b = -\sqrt{\frac{2}{3}} E_b^* \left[ \gamma \cos \Theta D_7 - \sin \Theta D_3 \right],$$

$$\langle A_1^b \rangle_E = \sqrt{\frac{2}{3}} E_b^* \left[ \cos \Theta D_{12} - \gamma \sin \Theta D_{14} + \sqrt{3}\gamma \sin \Theta D_5 \right],$$

$$\langle A_2^b \rangle_E = \sqrt{\frac{2}{3}} E_b^* \left[ \gamma \cos \Theta D_7 - \sin \Theta D_3 \right].$$

(68)

(69)
It is now straightforward to obtain the analytic expressions for the expectation of the observables $O_i^X (= A_i^b, (A_i^b)_E) \ (X = b, l$ and $i = 1, 2$) defined by

$$\langle O_i^X \rangle = \frac{1}{2} \frac{1}{(4\pi)^2} \int_{-1}^{1} d\cos \Theta \int d\Omega \int d\bar{\Omega} \left[ O_i^X \Sigma(\Theta; \xi_1; \xi_2; \xi_3) \right],$$

where $d\Omega = d\cos \theta d\phi$ and $d\bar{\Omega} = d\cos \bar{\theta} d\bar{\phi}$. We obtain for the expectations of the $CP$-odd vector observables

$$\langle A_1^b \rangle_{L,R} = \pm \frac{4}{27} E_b^2 \beta \gamma^2 \kappa_w \left( 3v_{L,R} - \kappa_w a_{L,R} \right) \text{Re}(c_{L,R}),$$

$$\langle A_2^b \rangle_{L,R} = \pm \frac{4}{9} E_b^2 \beta \gamma \kappa_w a_{L,R} \text{Im}(c_{L,R}),$$

$$\langle \langle A_1^b \rangle_{L,R} \rangle = \pm \frac{4}{27} (E_1^b)^2 \beta \gamma^2 \left( 3v_{L,R} + a_{L,R} \right) \text{Re}(c_{L,R}),$$

$$\langle \langle A_2^b \rangle_{L,R} \rangle = \pm \frac{4}{9} (E_1^b)^2 \beta \gamma a_{L,R} \text{Im}(c_{L,R}).$$

Thirdly, we turn to the $CP$-odd tensor observables and take $i, j = 3$ to consider the $(3, 3)$ components with respect to the electron momentum direction

$$T_{33}^b = 2(\vec{p}_b - \vec{p}_b) \times (\vec{p}_b \times \vec{p}_b),$$

$$Q_{33}^b = 2(\vec{p}_b + \vec{p}_b) \times (\vec{p}_b - \vec{p}_b) - \frac{2}{3}(\vec{p}_b^2 - \vec{p}_b^2).$$

The four $CP$-odd tensor observables can be expressed in the $t$ and $l$ rest frames in terms of the angular correlations $\mathcal{D}_a \ (\alpha = 1$ to $16$) and some extra angular correlations $\mathcal{D}'_b \ (\beta = 1$ to $12$), which are defined in Appendix $B$, as

$$T_{33}^b = \frac{2\sqrt{3}}{9} E_b^{\ast 3} \left[ -\{7\sqrt{3}(\gamma^2 - 1)\mathcal{D}_5 - 9\gamma^2 \beta \mathcal{D}_4\} \sin \Theta \cos \Theta \right.$$

$$\left. + 3\gamma \beta (3 \cos^2 \Theta - 1) \mathcal{D}_{12} \right]$$

$$+ \frac{2\sqrt{10}}{15} E_b^{\ast 3} \left[ (\sqrt{3} \gamma^2 \beta \mathcal{D}_4' + \mathcal{D}_6' + \frac{\sqrt{3}}{3}(2\gamma^2 + 1)\mathcal{D}_8' \right.$$  

$$\left. - \mathcal{D}_7' + \gamma^2 \mathcal{D}_9' \sin \Theta \cos \Theta - \gamma(\mathcal{D}_{11}' + \sqrt{3} \beta \mathcal{D}_3') \sin^2 \Theta \right.$$  

$$\left. + \gamma \cos^2 \Theta \mathcal{D}_{10}' - \gamma(2 \cos^2 \Theta - 1) \mathcal{D}_{11}' \right],$$

$$Q_{33}^b = \frac{4\sqrt{6}}{9} E_b^{\ast 2} \beta \left[ 3 \sin \Theta \cos \Theta \mathcal{D}_3 - \gamma(3 \cos^2 \Theta - 1) \mathcal{D}_7 \right]$$

$$- \frac{2\sqrt{10}}{15} E_b^{\ast 2} \frac{1}{3}(2\gamma^2 + 1)(3 \cos^2 \Theta - 1) \mathcal{D}_1' - \sqrt{3} \sin^2 \Theta \mathcal{D}_2'$$

15
\[ T_{33}^l \] = \frac{2\sqrt{2}}{9} \langle E_l^2 \rangle_2 \langle E_l^* \rangle_E \left[ \{7\sqrt{3}(\gamma^2 - 1)D_5 - 9\gamma^2 \beta D_{14}\} \sin \Theta \cos \Theta \\
+ 3\gamma \beta (3 \cos^2 \Theta - 1)D_{12} \right] \\
+ \frac{2\sqrt{10}}{15} \langle E_l^2 \rangle_2 \langle E_l^* \rangle_E \left[ \left(\sqrt{3}\gamma^2 \beta D_4' - D_6' - \frac{\sqrt{3}}{3}(2\gamma^2 + 1)D_8' \right) \sin \Theta \cos \Theta \\
+ \gamma \cos^2 \Theta D_{10}' + \gamma (2 \cos^2 \Theta - 1)D_{11}' \right], \\
\langle Q_{33}^l \rangle = \frac{4\sqrt{6}}{9} \langle E_l^2 \rangle_2 (E_l^* \rangle_E \left[ 3 \sin \Theta \cos \Theta D_3 - \gamma (3 \cos^2 \Theta - 1)D_7 \right] \\
- \frac{2\sqrt{10}}{15} \langle E_l^2 \rangle_2 \left[ \frac{1}{3} (2\gamma^2 + 1)(3 \cos^2 \Theta - 1)D_1 - \sqrt{3} \sin^2 \Theta D_2' \right] \\
+ 2\sqrt{3}\gamma \sin \Theta \cos \Theta D_5', \\
(74) \\
(75)
\]

where the average of the lepton energy squared \( \langle E_l^2 \rangle \) is given by
\[
\langle E_l^2 \rangle_3 = \frac{m_l^2}{4} \int_0^1 \frac{dx}{w} \left( \frac{6x^3(1-x)}{(1+2w)(1-w)^2} \right) = \frac{3m_l^2}{40} \left[ \frac{1+2w+3w^2+4w^3}{1+2w} \right], \\
(76)
\]

with \( w = m_W^2/m_t^2 \). We do not present the analytic expressions for the expectation values of the \( CP \)-odd tensor observables although they are straightforward to obtain.

### 3.6 Observable consequences of the top-quark EDM

For the sake of numerical analysis we insert the values of the SM vector and axial-vector couplings and then we obtain

\[
v_L = 0.67 + 0.15 \delta_Z, \quad v_R = 0.67 - 0.13 \delta_Z, \\
a_L = -0.38 \delta_Z, \quad a_R = 0.32 \delta_Z, \\
c_L = c_\gamma + 0.64 \delta_Z c_Z, \quad c_R = c_\gamma - 0.55 \delta_Z c_Z, \\
(77)
\]

where \( \delta_Z = (1 - m_Z^2/s)^{-1} \). The contribution from the Z-boson exchange diagram decreases as the c.m. energy \( \sqrt{s} \) increases. For \( m_t = 175 \) GeV and \( m_Z = 91.2 \) GeV, \( 1 \leq \delta_Z \leq 1.073 \). Note that the \( c_Z \) contribution to \( c_L \) and \( c_R \) is similar in size but different in sign. Naturally, the electron polarization is expected to play a crucial role in discriminating \( c_\gamma \) and \( c_Z \), as pointed out earlier by Cuypers and Rindani\[9\].

If a non-vanishing expectation value \( \langle O_X \rangle \) for a given observable \( O_X \) is observed, it has a statistical significance as far as it is compared with the expectation variance \( \langle O_X^2 \rangle \). For instance, to observe a deviation from the SM expectation with better than one-standard deviations one
\[ \langle O_X \rangle \geq \sqrt{\frac{\langle O_X^2 \rangle}{N_{\bar{t}t}}} \quad \text{where} \quad N_{\bar{t}t} = \varepsilon [B_{X^+} - B_{X^-}] L_{ee} \sigma(e^+e^- \rightarrow t\bar{t}), \]  

(78)

where \( N_{\bar{t}t} \) is the number of events, \( L_{ee} \) is the \( e^+e^- \) collider luminosity, and \( \varepsilon \) is the detection efficiency.

Implementing Eq. (78) we can determine the areas in the \((c_\gamma, c_z)\) plane which can not be explored with a given confidence level. Clearly, because of the linear dependence of the expectation values on the \( CP \)-odd electroweak dipole form factors, these areas are delimited by straight lines which are equidistant from the SM expectation \( c_\gamma = c_z = 0 \). The slopes of these straight lines vary with the polarization degree of the initial \( e^+e^- \) beams. The use of more than two \( CP \)-odd distributions can help to determine independently the real and imaginary parts of the electric as well as weak dipole couplings. Of course, longitudinal beam polarization, if present, obviates the need for the simultaneous measurement of more than one distribution and it can enhance the sensitivity to the \( CP \)-odd parameters. Our numerical results are presented for the assumed detection efficiency \( \varepsilon = 10\% \) and for the following set of experimental parameters:

\[ \sqrt{s} = 0.5 \text{ TeV}, \quad L_{ee} = \begin{cases} 10 \text{ fb}^{-1} & \text{for polarized electrons}, \\ 20 \text{ fb}^{-1} & \text{for unpolarized electrons}, \end{cases} \]  

(79)

The shadowed parts in Figure 3 show the 1-\( \sigma \) allowed regions of the \( CP \)-odd parameters \( \text{Re}(c_\gamma) \) and \( \text{Re}(c_z) \) through (a) \( A_1^b \) and \( T_{33}^b \) and (b) \( A_1^l \) and \( T_{33}^l \) with left- and right-handed polarized electron beams, respectively. The solid lines with a positive (negative) slope are for \( A_1^b \) and \( A_1^l \) with right-handed (left-handed) electrons while the long-dashed lines with a positive (negative) slope are for \( T_{33}^b \) and \( T_{33}^l \) with right-handed (left-handed) electrons. On the other hand, the shadowed parts in Figure 4 show the 1-\( \sigma \) allowed regions of the parameters \( \text{Re}(c_\gamma) \) and \( \text{Re}(c_z) \) through (a) \( A_1^b \) and \( T_{33}^b \) and (b) \( A_1^l \) and \( T_{33}^l \) with unpolarized electron beams, respectively. The solid lines are for \( A_1^l \) and \( A_1^b \) while the long-dashed lines are for \( T_{33}^l \) and \( T_{33}^b \). Two figures present us with several interesting results:

- The allowed regions strongly depend on electron polarization. Combining the bounds obtained with left-handed and right-handed electron beams, we obtain very tightly constrained 1-\( \sigma \) regions for \( \text{Re}(c_\gamma) \) and \( \text{Re}(c_z) \).

- Even with unpolarized electrons and positrons, it is possible to obtain a closed region for the \( CP \)-odd parameters by using two or more \( CP \)-odd asymmetries. The 1-\( \sigma \) regions become very loose for the parameter \( \text{Re}(c_\gamma) \), but the 1-\( \sigma \) regions for \( \text{Re}(c_z) \) remain rather intact.

- With polarized electrons, the tightest bound is obtained through the \( CP \)-odd vector asymmetry \( A_1^b \) in the inclusive top-quark decay mode.

Numerically, the 1-\( \sigma \) allowed region of \( \text{Re}(c_\gamma) \) and \( \text{Re}(c_z) \) at the c.m. energy \( \sqrt{s} = 500 \text{ GeV} \) with the total \( e^+e^- \) integrated luminosity 20 \( \text{ fb}^{-1} \), which is the sum of the integrated luminosities for left- and right-handed electrons, is

\[ |\text{Re}(c_\gamma)| \leq 0.12, \quad |\text{Re}(c_z)| \leq 0.20. \]  

(80)
The shadowed parts in Figure 5 show the 1-σ allowed regions of the CP-odd parameters \( \text{Im}(c_\gamma) \) and \( \text{Im}(c_\varepsilon) \) through the CP-odd and CP\( \tilde{T} \)-odd asymmetries (a) \( A_E^b, A_L^b \) and \( Q_{33}^b \) and (b) \( A_E^l, A_L^l \) and \( Q_{33}^l \) with polarized electron beams, respectively. The solid lines with a positive (negative) slope are for \( A_E^b \) and \( A_L^b \) with right-handed (left-handed) electrons while the long-dashed lines with a positive (negative) slope are for \( A_E^l \) and \( A_L^l \) with right-handed (left-handed) electrons. And, the dot-dashed lines with a positive (negative) slope are for \( Q_{33}^b \) and \( Q_{33}^l \) with right-handed (left-handed) electrons. On the other hand, the shadowed parts in Figure 6 show the 1-σ allowed regions of the parameters \( \text{Im}(c_\gamma) \) and \( \text{Im}(c_\varepsilon) \) through (a) \( A_E^b, A_L^b \) and \( Q_{33}^b \) and (b) \( A_E^l, A_L^l \) and \( Q_{33}^l \) with unpolarized electron beams, respectively. The solid lines are for \( A_E^b \) and \( A_L^b \) while the long-dashed lines are for \( A_E^l \) and \( A_L^l \). And, the dot-dashed lines are for \( Q_{33}^b \) and \( Q_{33}^l \). Two figures present us with several interesting results:

- The allowed regions strongly depend on electron polarization. Combining the bounds obtained with left-handed and right-handed electron beams, we obtain very tightly constrained 1-σ regions for \( \text{Im}(c_\gamma) \) and \( \text{Im}(c_\varepsilon) \).
- Even with unpolarized electrons and positrons, it is possible to obtain a bounded region for the CP-odd parameters by using two or more CP-odd asymmetries. We find that the 1-σ regions become very loose for the parameter \( \text{Im}(c_\varepsilon) \), but the 1-σ regions for \( \text{Re}(c_\gamma) \) remain rather intact.
- With polarized electrons, the tightest bound is obtained through the CP-odd energy asymmetry \( A_E^b \) in the inclusive top-quark decay mode.

Numerically, the 1-σ allowed region of the parameters \( \text{Im}(c_\gamma) \) and \( \text{Im}(c_\varepsilon) \) with the total \( e^+e^- \) integrated luminosity 20 fb\(^{-1} \) at the c.m. energy \( \sqrt{s} = 500 \text{ GeV} \) is

\[
\text{Im}(c_\gamma) \leq 0.16, \quad \text{Im}(c_\varepsilon) \leq 0.27. \quad (81)
\]

4 Compton backscattered laser light

Let us describe in a general framework how photon polarization can provide us with an efficient mechanism\(^{17} \) to probe CP invariance in the two-photon mode. With purely linearly-polarized photon beams, we classify all the distributions according to their CP and CP\( \tilde{T} \)-properties. Then, we show explicitly how linearly polarized photon beams allow us to construct two CP-odd and CP\( \tilde{T} \)-even asymmetries which do not require detailed information on the momenta and polarizations of the final-state particles.

4.1 Formalism

Generally, a purely polarized photon beam state is a linear combination of two helicity states and the photon polarization vector can be expressed in terms of two angles \( \alpha \) and \( \phi \) in a given coordinate system as

\[
|\alpha, \phi \rangle = -\cos(\alpha)e^{-i\phi}|+\rangle + \sin(\alpha)e^{i\phi}|-\rangle, \quad (82)
\]
where \(0 \leq \alpha \leq \pi/2\) and \(0 \leq \phi \leq 2\pi\). The photon polarization vector \(\mathbf{n}_\alpha\) implies that the degrees of circular and linear polarization are determined by

\[
\xi = \cos(2\alpha), \quad \eta = \sin(2\alpha),
\]

respectively, and the direction of maximal linear polarization is denoted by the azimuthal angle \(\phi\) in the given coordinate system. Note that \(\xi^2 + \eta^2 = 1\) as expected for a purely polarized photon. For a partially polarized photon beam it is necessary to rescale \(\xi\) and \(\eta\) by its degree of polarization \(P\) \(0 \leq P \leq 1\) as

\[
\xi = P \cos(2\alpha), \quad \eta = P \sin(2\alpha),
\]

such that \(\xi^2 + \eta^2 = P^2\).

Let us now consider the two-photon system in the c.m. frame where two photon momenta are opposing along the \(z\)-axis. The two-photon state vector is

\[
|\alpha_1, \phi_1; \alpha_2, \phi_2\rangle = |\alpha_1, \phi_1\rangle |\alpha_2, -\phi_2\rangle = \cos(\alpha_1) \cos(\alpha_2) e^{-i(\phi_1 - \phi_2)}|++\rangle - \cos(\alpha_1) \sin(\alpha_2) e^{-i(\phi_1 + \phi_2)}|+\rangle
\]

\[- \sin(\alpha_1) \cos(\alpha_2) e^{i(\phi_1 + \phi_2)}|-\rangle + \sin(\alpha_1) \sin(\alpha_2) e^{i(\phi_1 - \phi_2)}|\rangle,
\]

and then the transition amplitude from the polarized two-photon state to a final state \(X\) is simply given by

\[
\langle X|M|\alpha_1, \phi_1; \alpha_2, \phi_2\rangle.
\]

The azimuthal angles \(\phi_1\) and \(\phi_2\) are the directions of maximal linear polarization of the two photons, respectively, in a common coordinate system (For instance, see Figure 7.). In the process \(\gamma\gamma \rightarrow t\bar{t}\), the scattering plane is taken to be the \(x-z\) plane in the actual calculation of the helicity amplitudes. The maximal linear polarization angles are then chosen as follows. The angle \(\phi_1\) (\(\phi_2\)) is the azimuthal angle of the maximal linear polarization of the photon beam, whose momentum is in the positive (negative) \(z\) direction, with respect to the direction of the \(t\) momentum in the process \(\gamma\gamma \rightarrow t\bar{t}\). Note that we have used \(|\alpha_2, -\phi_2\rangle\) in Eq. (85) for the photon whose momentum is along the negative \(z\) direction in order to employ a common coordinate system for the two-photon system.

For later convenience we introduce the abbreviation

\[
M_{\lambda_1\lambda_2} = \langle X|M|\lambda_1\lambda_2\rangle,
\]

and two angular variables:

\[
\chi = \phi_1 - \phi_2, \quad \phi = \phi_1 + \phi_2,
\]

where \(-2\pi \leq \chi \leq 2\pi\) and \(0 \leq \phi \leq 4\pi\) for a fixed \(\chi\). It should be noted that (i) the azimuthal angle difference, \(\chi\), is independent of the final state, while the azimuthal angle sum, \(\phi\), depends on the scattering plane, and (ii) both angles are invariant with respect to the Lorentz boost along the two-photon beam direction.
It is straightforward to obtain the angular dependence of the $\gamma\gamma \rightarrow X$ cross section on the initial beam polarizations in terms of the Stokes parameters $(\xi, \bar{\xi})$ for the degrees of circular polarization and $(\eta, \bar{\eta})$ for those of linear polarization of the two initial photon beams, respectively, as

$$\Sigma(\xi, \bar{\xi}; \eta, \bar{\eta}; \chi, \phi) \equiv \sum_X |\langle X|M|\xi, \bar{\xi}; \eta, \bar{\eta}; \chi, \phi \rangle|^2,$$

(89)

where the summation over $X$ is for the polarizations of the final states. Incidentally, the Stokes parameters are expressed in terms of two parameters $\alpha_1$ and $\alpha_2$ by

$$\xi = P \cos(2\alpha_1), \quad \bar{\xi} = \bar{P} \cos(2\alpha_2),$$
$$\eta = P \sin(2\alpha_1), \quad \bar{\eta} = \bar{P} \sin(2\alpha_2),$$

(90)

where $P$ and $\bar{P}$ ($0 \leq P, \bar{P} \leq 1$) are the polarization degrees of the two colliding photons. There exist sixteen independent terms, all of which are all measurable in polarized two-photon collisions. Purely linearly polarized photon beams allow us to determine nine terms among all the sixteen terms, while purely circularly polarized photon beams allow us to determine only four terms. The unpolarized cross section is determined in both cases. However, both circular and linear polarizations are needed to determine the remaining four terms.

Even though we obtain more information with both circularly and linearly polarized beams, we study mainly the case where two photons are linearly polarized but not circularly polarized. The expression of the angular dependence then greatly simplifies to

$$D(\eta, \bar{\eta}; \chi, \phi) = \Sigma_{\text{unpol}} - \frac{1}{2} \left[ \eta \cos(\phi + \chi) + \bar{\eta} \cos(\phi - \chi) \right] \Re(\Sigma_{02})$$
$$+ \frac{1}{2} \left[ \eta \cos(\phi + \chi) - \bar{\eta} \cos(\phi - \chi) \right] \Im(\Sigma_{02}) - \frac{1}{2} \left[ \eta \cos(\phi + \chi) - \bar{\eta} \cos(\phi - \chi) \right] \Re(\Delta_{02})$$
$$+ \frac{1}{2} \left[ \eta \sin(\phi + \chi) + \bar{\eta} \sin(\phi - \chi) \right] \Im(\Delta_{02}) + \eta \bar{\eta} \cos(2\phi) \Re(\Sigma_{22}) + \eta \bar{\eta} \sin(2\phi) \Im(\Sigma_{22})$$
$$+ \eta \bar{\eta} \cos(2\chi) \Re(\Sigma_{00}) + \eta \bar{\eta} \sin(2\chi) \Im(\Sigma_{00}),$$

(91)

where the invariant functions are defined as

$$\Sigma_{\text{unpol}} = \frac{1}{4} \sum_X \left[ |M_{++}|^2 + |M_{+-}|^2 + |M_{-+}|^2 + |M_{--}|^2 \right]$$
$$\Sigma_{02} = \frac{1}{2} \sum_X \left[ M_{++}^* (M_{+-}^* + M_{+}^* M_{-+}^*) + (M_{+-} + M_{-+}) M_{+}^* M_{-+} \right]$$
$$\Delta_{02} = \frac{1}{2} \sum_X \left[ M_{++}^* (M_{+-}^* - M_{+}^* M_{-+}^*) - (M_{+-} + M_{-+}) M_{+}^* M_{-+} \right]$$
$$\Sigma_{22} = \frac{1}{2} \sum_X (M_{++}^* M_{++})^*,$$
$$\Sigma_{00} = \frac{1}{2} \sum_X (M_{++}^* M_{++})^*,$$

(92)

with the subscripts, 0 and 2, representing the magnitude of the sum of two photon helicities of the initial two-photon system.
4.2 Symmetry properties

It is useful to classify the invariant functions according to their transformation properties under the discrete symmetries, $CP$ and $CP\overline{T}$. We find that $CP$ invariance leads to the relations

$$\sum_{X} \left(M_{\lambda_{1}\lambda_{2}} M_{\lambda_{1}'\lambda_{2}'}^{*}\right) = \sum_{X} \left(M_{-\lambda_{2},-\lambda_{1}} M_{-\lambda_{2}',-\lambda_{1}'}^{*}\right),$$

$$d\sigma(\phi, \chi; \eta, \bar{\eta}) = d\sigma(-\phi, -\chi; \bar{\eta}, \eta), \quad (93)$$

and, if there are no absorptive parts in the amplitudes, $CP\overline{T}$ invariance leads to the relations

$$\sum_{X} \left(M_{\lambda_{1}\lambda_{2}} M_{\lambda_{1}'\lambda_{2}'}^{*}\right) = \sum_{X} \left(M_{-\lambda_{2},-\lambda_{1}} M_{-\lambda_{2}',-\lambda_{1}'}^{*}\right),$$

$$d\sigma(\phi, \chi; \eta, \bar{\eta}) = d\sigma(-\phi, \chi; \bar{\eta}, \eta). \quad (94)$$

The nine invariant functions in Eq. (91) can then be divided into four categories under $CP$ and $CP\overline{T}$: even-even, even-odd, odd-even, and odd-odd terms as in Table 2. $CP$-odd coefficients directly measure $CP$ violation and $CP\overline{T}$-odd terms indicate rescattering effects (absorptive parts in the scattering amplitudes). Table 2 shows that there exist three $CP$-odd functions; $\Im(\Sigma_{02})$, $\Re(\Sigma_{00})$ and $\Re(\Delta_{02})$. Here, $\Re$ and $\Im$ are for real and imaginary parts, respectively. While the first two terms are $CP\overline{T}$-even, the last term $\Re(\Delta_{02})$ is $CP\overline{T}$-odd. Since the $CP\overline{T}$-odd term $\Re(\Delta_{02})$ requires the absorptive part in the amplitude, it is generally expected to be smaller in magnitude than the $CP\overline{T}$-even terms. We therefore study the two $CP$-odd and $CP\overline{T}$-even distributions; $\Im(\Sigma_{02})$ and $\Im(\Sigma_{00})$.

We can define two $CP$-odd asymmetries from the two distributions, $\Im(\Sigma_{02})$ and $\Im(\Sigma_{00})$. First, we note that the $\Sigma_{00}$ term does not depend on the azimuthal angle $\phi$ whereas the $\Sigma_{02}$ does. In order to improve the observability we may integrate the $\Im(\Sigma_{02})$ term over the azimuthal angle $\phi$ with an appropriate weight function. Without any loss of generality we can take $\eta = \bar{\eta}$. Then, the quantity $\Im(\Sigma_{00})$ in Eq. (91) can be separated by taking the difference of the distributions at $\chi = \pm \pi/4$ and the $\Im(\Sigma_{02})$ by taking the difference of the distributions at $\chi = \pm \pi/2$. As a result we obtain the following two integrated $CP$-odd asymmetries:

$$\hat{A}_{02} = \left(\frac{2}{\pi}\right) \frac{\Im(\Sigma_{02})}{\Sigma_{\text{unpol}}}, \quad \hat{A}_{00} = \frac{\Im(\Sigma_{00})}{\Sigma_{\text{unpol}}}. \quad (95)$$

where the factor $(2/\pi)$ in the $\hat{A}_{02}$ stems from taking the average over the azimuthal angle $\phi$ with the weight function sign($\cos \phi$):

$$\hat{A}_{02} = \frac{\int_{0}^{4\pi} d\phi \text{sign}(\cos \phi) \left[ \left(\frac{d\sigma}{d\phi}\right)_{\chi=\frac{\pi}{2}} - \left(\frac{d\sigma}{d\phi}\right)_{\chi=-\frac{\pi}{2}} \right]}{\int_{0}^{4\pi} d\phi \left[ \left(\frac{d\sigma}{d\phi}\right)_{\chi=\frac{\pi}{2}} + \left(\frac{d\sigma}{d\phi}\right)_{\chi=-\frac{\pi}{2}} \right]}, \quad (96)$$

$$\hat{A}_{00} = \frac{\int_{0}^{4\pi} d\phi \left[ \left(\frac{d\sigma}{d\phi}\right)_{\chi=\frac{\pi}{4}} - \left(\frac{d\sigma}{d\phi}\right)_{\chi=-\frac{\pi}{4}} \right]}{\int_{0}^{4\pi} d\phi \left[ \left(\frac{d\sigma}{d\phi}\right)_{\chi=\frac{\pi}{4}} + \left(\frac{d\sigma}{d\phi}\right)_{\chi=-\frac{\pi}{4}} \right]}, \quad (97)$$
In pair production processes such as $\gamma \gamma \rightarrow t \bar{t}$, all the distributions, $\Sigma_i$, can be integrated over the scattering angle $\theta$ with a CP-even angular cut so as to test CP violation.

### 4.3 Photon spectrum

Recently, the well-known Compton backscattering has drawn a lot of interest because it can be utilized as a powerful high-energy photon source at NLC experiments. In this section we give a detailed description of the energy spectrum and polarization of the Compton backscattered laser lights off high energy electrons or positrons.

We are interested in the situation where a purely linearly polarized laser beam of frequency $\omega_0$ is focused upon an unpolarized electron or positron beam of energy $E$. In the collision of a laser photon and a linac electron, a high energy photon of energy $\omega$, which is partially linearly polarized, is emitted at a very small angle, along with the scattered electron of energy $E' = E - \omega$. The kinematics of the Compton backscattering process is then characterized by the dimensionless parameters $x$ and $y$:

$$x = \frac{4E\omega_0}{m_e^2} \approx 15.3 \left( \frac{E}{\text{TeV}} \right) \left( \frac{\omega_0}{\text{eV}} \right), \quad y = \frac{\omega}{E}. \quad (98)$$

In general, the backscattered photon energies increase with $x$; the maximum photon energy fraction is given by $y_m = x/(1 + x)$. Operation below the threshold for $e^+e^-$ pair production in collisions between the laser beam and the Compton-backscattered photon beam requires $x \leq 2(1 + \sqrt{2}) \approx 4.83$; the lower bound on $x$ depends on the lowest available laser frequency and the production threshold of a given final state.

Figure 8(a) shows the photon energy spectrum for various values of $x$. Clearly large $x$ values are favored to produce highly energetic photons. On the other hand, the degree $\eta(y)$ of linear polarization of the backscattered photon beam reaches the maximum value at $y = y_m$ (See Figure 8(b)),

$$\eta_{\text{max}} = \eta(y_m) = \frac{2(1 + x)}{1 + (1 + x)^2}, \quad (99)$$

and approaches unity for small values of $x$. In order to retain large linear polarization we should keep the $x$ value as small as possible.

### 4.4 Linear polarization transfers

In the two-photon collision case only part of linear polarization of each incident laser beam is transferred to the high-energy photon beam. We introduce two functions, $A_\eta$ and $A_{\eta\eta}$, to denote the degrees of linear polarization transfer as

$$A_\eta(\tau) = \frac{\langle \phi_0\phi_3 \rangle_\tau}{\langle \phi_0\phi_0 \rangle_\tau}, \quad A_{\eta\eta}(\tau) = \frac{\langle \phi_3\phi_3 \rangle_\tau}{\langle \phi_0\phi_0 \rangle_\tau}, \quad (100)$$

where $\phi_0(y)$ is the photon energy spectrum function and $\phi_3(y) = 2y^2/(x(1 - y))^2$ and $\tau$ is the ratio of the $\gamma\gamma$ c.m. energy squared $\hat{s}$ to the $e^+e^-$ collider energy squared $s$. The function $A_\eta$ is for the collision of an unpolarized photon beam and a linearly polarized photon beam, and the
function $A_{\eta\eta}$ for the collision of two linearly polarized photon beams. The convolution integrals
\[
\langle \phi_i \phi_j \rangle_\tau \quad (i, j = 0, 3)
\]
for a fixed value of $\tau$ are defined as
\[
\langle \phi_i \phi_j \rangle_{\tau} = \frac{1}{N^2(x)} \int_{\frac{\tau}{\gamma_m}}^{\gamma_m} \frac{dy}{y} \phi_i(y) \phi_j(\frac{\tau}{y}),
\]
(101)
where the normalization factor $N(x)$ is by the integral of the photon energy spectrum $\phi_0$ over $y$.

The event rates of the $\gamma\gamma \rightarrow X$ reaction with polarized photons can be obtained by folding a photon luminosity spectral function with the $\gamma\gamma \rightarrow X$ production cross section as (for $\eta = \bar{\eta}$)
\[
dN_{\gamma\gamma \rightarrow X} = dL_{\gamma\gamma} d\hat{\sigma}(\gamma\gamma \rightarrow X),
\]
(102)
where
\[
dL_{\gamma\gamma} = \kappa^2 L_{ee}(\phi_0 \phi_0)_{\tau} d\tau,
\]
(103)
\[
d\hat{\sigma}(\gamma\gamma \rightarrow X) = \frac{1}{2s} d\Phi_X \left[ \sum_{\text{unpol}} - \eta A_{\eta} \cos \phi \Im \left(e^{-i\chi \Sigma_{02}}\right) 
\right.
\]
\[
+ \eta A_{\eta} \sin \phi \Re \left(e^{-i\chi \Delta_{02}}\right) + \eta^2 A_{\eta\eta} \Re \left(e^{-2i\phi \Sigma_{22}} + e^{-2i\chi \Sigma_{00}}\right) \right].
\]
(104)
Here, $\kappa$ is the $e-\gamma$ conversion coefficient in the Compton backscattering and $d\Phi_X$ is the phase space factor of the final state. The distribution (104) of event rates enables us to construct two $CP$-odd asymmetries;
\[
A_{02} = \left(\frac{2}{\pi}\right) \frac{N_{02}}{N_{\text{unpol}}}, \quad A_{00} = \frac{N_{00}}{N_{\text{unpol}}},
\]
(105)
where with $\tau_{\text{max}} = \frac{y^2_m}{s}$ and $\tau_{\text{min}} = \frac{M^2_X}{s}$ we have for the event distributions
\[
\begin{pmatrix}
N_{\text{unpol}} \\
N_{02} \\
N_{00}
\end{pmatrix}
= \kappa^2 L_{ee} \frac{1}{2s} \int_{\tau_{\text{min}}}^{\tau_{\text{max}}} \frac{d\tau}{\tau} \int d\Phi_X (\phi_0 \phi_0)_{\tau} \left[ \begin{array}{c}
\sum_{\text{unpol}} \\
\eta A_{\eta} \Im (\Sigma_{02}) \\
\eta^2 A_{\eta\eta} \Im (\Sigma_{00})
\end{array} \right].
\]
(106)

The asymmetries depend crucially on the two-photon spectrum and the two linear polarization transfers.

We first investigate the $\sqrt{\tau}$ dependence of the two-photon spectrum and the two linear polarization transfers, $A_{\eta}$ and $A_{\eta\eta}$ by varying the value of the dimensionless parameter $x$. Three values of $x$ are chosen: $x = 0.5, 1, 4.83$. Two figures in Figure 9 clearly show that the energy of two photons reaches higher ends for larger $x$ values but the maximum linear polarization transfers are larger for smaller $x$ values. We also note that $A_{\eta}$ (solid lines) is larger than $A_{\eta\eta}$ (dashed lines) in the whole range of $\sqrt{\tau}$. We should keep the parameter $x$ as large as possible to reach higher energies. However, larger $CP$-odd asymmetries can be obtained for smaller $x$ values. Therefore, there should exist a compromised value of $x$, i.e. the incident laser beam frequency $\omega_0$ for the optimal observability of $CP$ violation. The energy dependence of the subprocess cross section and that of the $CP$-odd asymmetries are both essential to find the optimal $x$ value.
5 Two-photon mode

In this section we reinvestigate $CP$ violation due to the top-quark EDM in the two-photon mode by extending the previous work\cite{11} and revising its numerical errors.

5.1 Helicity Amplitudes

The process $\gamma \gamma \rightarrow t\bar{t}$ consists of two Feynman diagrams and its helicity amplitudes in the $\gamma \gamma$ c.m. frame are given by

$$M_{\lambda_1 \lambda_2; \sigma \bar{\sigma}} = \frac{4\pi \alpha Q_t^2 N_c}{(1 - \beta^2 \cos^2 \Theta)} \left[ A_{\lambda_1 \lambda_2; \sigma \bar{\sigma}} + (i\delta_t)B_{\lambda_1 \lambda_2; \sigma \bar{\sigma}} + (i\delta_t)^2 C_{\lambda_1 \lambda_2; \sigma \bar{\sigma}} \right],$$

where $\Theta$ is the scattering angle between $t$ and a photon, and the top-quark EDM factor $\delta_t$ is given by

$$\delta_t = \frac{3}{4} \frac{c_G}{m_t} = \frac{3}{4} d_t^i.$$  (108)

The SM contributions $A_{\lambda_1 \lambda_2; \sigma \bar{\sigma}}$ are given by

$$A_{\lambda \lambda; \sigma \sigma} = -\frac{4m_t}{\sqrt{s}} (\lambda + \sigma \hat{\beta}), \quad A_{\lambda \lambda, -\sigma} = 0,$$

$$A_{\lambda, -\lambda; \sigma \sigma} = \frac{4m_t \hat{\beta}}{\sqrt{s}} \sigma \sin^2 \Theta, \quad A_{\lambda, -\lambda, -\sigma} = 2 \hat{\beta} (\lambda \sigma + \cos \Theta) \sin \Theta,$$

the terms $B_{\lambda_1 \lambda_2; \sigma \bar{\sigma}}$, which are linear in $\delta_t$, and the terms $C_{\lambda_1 \lambda_2; \sigma \bar{\sigma}}$, which are quadratic in $\delta_t$, are given by

$$B_{\lambda \lambda; \sigma \sigma} = 2\sqrt{s} \left[ \frac{8m_t^2}{s} + \hat{\beta} (\hat{\beta} - \sigma \lambda) \sin^2 \Theta \right],$$

$$B_{\lambda \lambda, -\sigma} = -4m_t \lambda \hat{\beta} \sin \Theta \cos \Theta,$$

$$B_{\lambda, -\lambda; \sigma \sigma} = 2\sqrt{s} \hat{\beta}^2 \sin^2 \Theta,$$

$$B_{\lambda, -\lambda, -\sigma} = 0,$$  (109)

and

$$C_{\lambda \lambda; \sigma \sigma} = -2m_t \sqrt{s} \lambda \left[ \frac{4m_t^2}{s} + \hat{\beta} (\hat{\beta} - \sigma \lambda) \sin^2 \Theta \right],$$

$$C_{\lambda \lambda, -\sigma} = 4m_t^2 \hat{\beta} \sin \Theta \cos \Theta,$$

$$C_{\lambda, -\lambda; \sigma \sigma} = -2m_t \sqrt{s} \lambda \hat{\beta} \sin^2 \Theta,$$

$$C_{\lambda, -\lambda, -\sigma} = -\hat{s} \hat{\beta} \sin \Theta \left[ \frac{4m_t^2}{s} \cos \Theta + \lambda \sigma (1 - \hat{\beta}^2 \cos^2 \Theta) \right],$$  (111)

where $\lambda, \bar{\lambda}$ and $\sigma/2, \bar{\sigma}/2$ are the two-photon and $t, \bar{t}$ helicities, respectively, $\hat{s}$ is the $\gamma \gamma$ c.m. energy squared, and $\hat{\beta} = \sqrt{1 - 4m_t^2/\hat{s}}$. 


5.2 Differential cross section

In counting experiments where the final $t$ polarizations are not analyzed, we measure only the following combinations:

$$\sum_{X} M_{\lambda_1 \lambda_2} M_{\lambda'_1 \lambda'_2}^* = (eQ_t)^4 \sum_{\sigma} \sum_{\sigma'} \tilde{M}_{\lambda_1 \lambda_2; \sigma \sigma'} \tilde{M}_{\lambda'_1 \lambda'_2; \sigma' \sigma}^*. \tag{112}$$

We then find $\Sigma_{\text{unpol}}$, $\Sigma_{02}$, $\Delta_{02}$, $\Sigma_{22}$, and $\Sigma_{00}$ from Equation (92). The differential cross section for a fixed angle $\chi$ is

$$\frac{d^2\sigma}{d\cos \Theta d\phi}(\chi) = \frac{\alpha^2 Q_t^4 N_c}{8\hat{s}(1 - \beta^2 \cos^2 \Theta)^2} \left\{ \hat{\Sigma}_{\text{unpol}} - \frac{1}{2} \Re\left[ (\eta e^{-i(\chi + \phi)} + \bar{\eta} e^{-i(\chi - \phi)}) \hat{\Sigma}_{02} \right] \right. $$

$$\left. + \frac{1}{2} \Re\left[ (\eta e^{-i(\chi + \phi)} - \bar{\eta} e^{-i(\chi - \phi)}) \hat{\Delta}_{02} \right] + \eta \bar{\eta} \Re\left[ e^{-2i\phi} \hat{\Sigma}_{22} + e^{-2i\chi} \hat{\Sigma}_{00} \right] \right\}, \tag{113}$$

$$\hat{\Sigma}_i = \frac{e^4 Q_t^4 \hat{\Sigma}_i}{(1 - \beta^2 \cos^2 \Theta)^2}, \quad \hat{\Delta}_{02} = \frac{e^4 Q_t^4 \hat{\Delta}_{02}}{(1 - \beta^2 \cos^2 \Theta)^2}, \tag{114}$$

for $i = \text{unpol}, 02, 22, \text{and } 00$.

We first note that all the real parts of the distributions (112) are independent of the anomalous $CP$-odd form factors $c_\gamma$ up to linear order

$$\hat{\Sigma}_{\text{unpol}} = 4 \left[ 1 + 2\beta^2 \sin^2 \Theta - \beta^4 (1 + \sin^4 \Theta) \right],$$

$$\Re(\hat{\Sigma}_{02}) = \frac{16}{15} \beta^2 \sin^2 \Theta, \quad \Re(\hat{\Delta}_{02}) = 0, \quad \Re(\hat{\Sigma}_{22}) = -4\beta^4 \sin^4 \Theta, \quad \Re(\hat{\Sigma}_{00}) = -\frac{4}{15}. \tag{115}$$

Two $CP$-odd distributions $\Im(\hat{\Sigma}_{02})$ and $\Im(\hat{\Sigma}_{00})$ have contributions from the $CP$-odd form factor $c_\gamma$ and they are given by

$$\Im(\hat{\Sigma}_{02}) = 0, \quad \Im(\hat{\Sigma}_{00}) = \frac{24}{m_t} (1 - \beta^2 \cos^2 \Theta) \Re(c_\gamma). \tag{116}$$

A few comments on the $CP$-odd distributions are in order.

- $\Im(\hat{\Sigma}_{02})$ is zero so that it can not be used to probe $CP$-violating effects from the real part of the top quark EDM.

- $\Im(\hat{\Sigma}_{00})$ is not suppressed at threshold.

- The $CP$-odd distribution $\Im(\hat{\Sigma}_{00})$ has the angular dependence $(1 - \beta^2 \cos^2 \Theta)$ which becomes largest at the scattering angle $\Theta = \pi/2$, where the SM contribution is generally small. We, therefore, expect a large $CP$-odd asymmetry at $\Theta \approx \pi/2$. 

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5.3 Observable consequences of the top-quark EDM

The CP-odd distribution \( \Im(\Sigma_{02}) \) is useless in determining the top EDM parameter \( \Re(c_\gamma) \), because it vanishes in the top-pair production via two-photon fusion as shown in Eq. (116). In case of \( \Im(\Sigma_{00}) \), no spin analysis for the decaying top quarks is required and furthermore even the scattering plane does not need to be identified. Even if one excludes the \( \tau^+\tau^- \) modes of 1%, the remaining 99% of the events can be used to measure \( \Im(\Sigma_{00}) \).

We present our numerical results for the experimental parameters \( \sqrt{s} = 0.5 \) and 1.0 TeV, \( \kappa^2 L_{ee} = 20 \) fb\(^{-1} \).

The dimensionless parameter \( x \), which depends on the laser frequency \( \omega_0 \), is treated as an adjustable parameter. We note that \( \kappa = 1 \) is the maximally allowed value for the e-\( \gamma \) conversion coefficient \( \kappa \) and it may be as small as \( \kappa = 0.1 \) if the collider is optimized for the e\(^+\)e\(^-\) model\(^{[19]} \).

All one should note is that the significance of the signal scales as \( (\epsilon \cdot \kappa^2 \cdot L_{ee}) \), where \( \epsilon \) denotes the overall detection efficiency that is different for \( A_{00} \) and \( A_{02} \).

Folding the photon luminosity spectrum and integrating the distributions over the polar angle \( \Theta \), we obtain the \( x \)-dependence of available event rates:

\[
\left( \frac{N_{\text{unpol}}}{N_{00}} \right) = \kappa^2 L_{ee} \frac{\pi \alpha^2 Q_t^4 N_C}{2s} \int_{\tau_{\text{min}}}^{\tau_{\text{max}}} \frac{1}{\tau} \int_{-1}^{1} \frac{d\tilde{\beta}(\phi_0) \tau d\cos \Theta}{(1 - \tilde{\beta}^2 \cos^2 \Theta)^2} \left( \frac{\tilde{\Sigma}_{\text{unpol}}}{A_{\eta\eta} \Im(\hat{\Sigma}_{00})} \right),
\]

where \( \tau_{\text{max}} = (\tau/(1+x))^2 \) and \( \tau_{\text{min}} = 4m_t^2/s \). After extracting the top EDM form factor \( \Re(c_\gamma) \) from the asymmetry \( A_{00} \) as

\[
A_{00} = \Re(c_\gamma) \tilde{A}_{00},
\]

we obtain the 1-\( \sigma \) allowed sensitivity of the form factor \( \Re(c_\gamma) \)

\[
\text{Max}(|\Re(c_\gamma)|) = \frac{\sqrt{2}}{|A_{00}\sqrt{\epsilon N_{\text{unpol}}}|},
\]

if no asymmetry is found. Here, \( \epsilon \) denotes the multiplication of the branching fraction times the experimental detection efficiency. The \( N_{SD}-\sigma \) upper bound is determined simply by multiplying Max(|\Re(c_\gamma)|) by \( N_{SD} \).

A crucial issue is to find an optimal means for maximizing the denominator in Eq. (120) experimentally. It requires obtaining the smallest possible value of \( x \) to make the linear polarization transfer as large as possible. However, the large top-quark mass does not allow \( x \) to be very small. For a given c.m. energy squared, \( \sqrt{s} \), the allowed range of \( x \) is given by

\[
\frac{2m_t}{\sqrt{s} - 2m_t} \leq x \leq 2(1 + \sqrt{2}).
\]
Experimentally, the process $\gamma\gamma \rightarrow W^+W^-$ is the most severe background process against the process $\gamma\gamma \rightarrow t\bar{t}$. Without a detailed background estimation, we simply take the detection efficiency $\varepsilon$ to be

$$\varepsilon = 10\%,$$  \hspace{1cm} (122)

even though more experimental analyses are required to estimate the efficiency precisely. It would be, however, rather straightforward to include the effects from any experimental cuts and efficiencies in addition to the branching factors discussed above.

Figure 10 shows a very strong $x$ dependence of the Re($c_\gamma$) upper bound, $\text{Max}(|\text{Re}(c_\gamma)|)$, at $\sqrt{s} = 0.5$ and 1 TeV, from the asymmetry $A_{00}$. The solid line is for $\sqrt{s} = 0.5$ TeV and the long-dashed line for $\sqrt{s} = 1$ TeV. The doubling of $e^+e^-$ c.m. energy improves the sensitivity so much and renders the optimal $x$ value smaller than that at $\sqrt{s} = 0.5$ TeV. The $x$ values for the optimal sensitivities and the optimal 1-$\sigma$ sensitivities to the $CP$-odd parameter Re($c_\gamma$) for $\sqrt{s} = 0.5$ and 1 TeV are listed in Table 3.

6 Conclusions

Large top-quark mass implies that a top quark can serve as an excellent tool to probe $CP$ violation from new interactions at NLC.

In the production process $e^+e^- \rightarrow t\bar{t}$, followed by the $t$ and $\bar{t}$ decays, $CP$ violation from the $T$-odd top-quark EDM and WDM can be investigated through the angular correlations of the $t$ and $\bar{t}$ decay products.

We have completely defined all the available $CP$-odd correlations and have established the relations between a lot of previously suggested $CP$-odd correlations and the linearly-independent $CP$-odd correlations. We have fully analyzed the dependence of all the $CP$-odd observables on the electron beam polarization.

Most $CP$-odd asymmetries in the process $e^+e^- \rightarrow t\bar{t}$ depend on both the top EDM and the top WDM. Therefore, the separation of two contributions requires introducing electron beam polarization and/or using at least two independent $CP$-odd observables. We found that electron polarization is quite effective in separating the top-quark EDM and WDM effects.

In the polarized $\gamma\gamma$ mode, initial $CP$-odd two-photon polarization configurations allow us to measure the top-quark EDM by counting $t\bar{t}$ pair production events in a straightforward way. Without any direct information on the momenta of the top-quark decay products linearly-polarized laser beams with an adjustable beam energy provide us with a very efficient way of probing the top-quark EDM at a PLC.

The strongest 1-$\sigma$ sensitivity on the top EDM factor Re($c_\gamma$) for $\sqrt{s} = 500$ GeV in the polarized $e^+e^-$ mode is obtained through the vector asymmetry $A_1^\parallel$ and, numerically, it is $|\text{Re}(c_\gamma)| \leq 0.13$ for the total $e^+e^-$ integrated luminosity 20 fb$^{-1}$. On the other hand, the optimal 1-$\sigma$ sensitivity on Re($c_\gamma$) through the asymmetry $A_{00}$ for $\sqrt{s} = 0.5$ TeV in the polarized two-photon mode is $|\text{Re}(c_\gamma)| \leq 0.16$. Consequently, the polarized $e^+e^-$ mode and the polarized two-photon mode are competitive in probing $CP$ violation in the top-quark pair production processes. Certainly, for more rigorous comparison, we should take the momentum-dependent top EDM and WDM into account.
Soni and Xu\[21\] have estimated the top EDM factor Re($c_6$) in Higgs-boson-exchange models of CP nonconservation to be typically of the order of $10^{-3}$-$10^{-4}$, which is still much smaller than the experimental sensitivities in the processes $e^+e^-(\gamma\gamma) \to t\bar{t}$ for the total integrated luminosity 20 fb$^{-1}$ and the c.m. energy $\sqrt{s} = 500$ GeV. However, as indicated in Table 3 and Figure 10, the two-photon mode is expected to greatly improve the experimental constraints on the $T$-odd top-quark EDM by increasing the c.m. energy and by adjusting the laser beam frequency.

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**A  The definition and explicit analytic form of $P_{\alpha X}$**

The definition of $P_{\alpha X}$ ($\alpha = 1$ to 16 and $X = L, R$) in terms of the helicity amplitudes $M_{\lambda\bar{\lambda}}^X$ ($X = L, R$ and $\lambda, \bar{\lambda} = \pm$) is as follows

\[
\begin{align*}
\mathcal{P}_{1X} &= \frac{1}{4} \left[ |M_{++}^X|^2 + |M_{--}^X|^2 + |M_{+-}^X|^2 + |M_{-+}^X|^2 \right], \\
\mathcal{P}_{2X} &= \frac{1}{3\sqrt{3}} \left[ \text{Re}(M_{++}^X M_{++}^* X_{++}) + \frac{1}{4} (|M_{++}^X|^2 + |M_{--}^X|^2 - |M_{+-}^X|^2 - |M_{-+}^X|^2) \right], \\
\mathcal{P}_{3X} &= \frac{1}{2\sqrt{6}} \left[ \text{Re} \left( (M_{++}^X + M_{--}^X)(M_{++}^X + M_{--}^X)^* \right) \right], \\
\mathcal{P}_{4X} &= -\frac{1}{2\sqrt{6}} \left[ \text{Re} \left( (M_{++}^X - M_{--}^X)(M_{+-}^X - M_{-+}^X)^* \right) \right], \\
\mathcal{P}_{5X} &= \frac{1}{2\sqrt{6}} \left[ \text{Im} \left( (M_{++}^X + M_{--}^X)(M_{+-}^X - M_{-+}^X)^* \right) \right], \\
\mathcal{P}_{6X} &= -\frac{1}{2\sqrt{6}} \left[ \text{Im} \left( (M_{++}^X - M_{--}^X)(M_{+-}^X + M_{-+}^X)^* \right) \right], \\
\mathcal{P}_{7X} &= \frac{1}{2\sqrt{6}} \left[ |M_{++}^X|^2 - |M_{--}^X|^2 \right], \\
\mathcal{P}_{8X} &= \frac{1}{2\sqrt{6}} \left[ |M_{+-}^X|^2 - |M_{-+}^X|^2 \right], \\
\mathcal{P}_{9X} &= \frac{1}{3\sqrt{6}} \left[ \text{Re}(M_{--}^X M_{++}^*) - \frac{1}{2} (|M_{++}^X|^2 + |M_{--}^X|^2 - |M_{+-}^X|^2 - |M_{-+}^X|^2) \right].
\end{align*}
\]
\[ \mathcal{P}_{10X} = \frac{1}{3\sqrt{2}} \text{Re} \left( M_{++}^X M_{++}^{X*} \right), \]
\[ \mathcal{P}_{11X} = \frac{1}{\sqrt{2}} \text{Im} \left( M_{++}^X M_{++}^{X*} \right), \]
\[ \mathcal{P}_{12X} = \frac{1}{3\sqrt{2}} \text{Im} \left( M_{++}^X M_{++}^{X*} \right), \]
\[ \mathcal{P}_{13X} = \frac{1}{\sqrt{2}} \text{Im} \left[ (M_{++}^X M_{++}^{-}) (M_{++}^{X} M_{++}^{-})^* \right], \]
\[ \mathcal{P}_{14X} = -\frac{1}{6\sqrt{2}} \text{Im} \left[ (M_{++}^X + M_{++}^{-}) (M_{++}^{X} + M_{++}^{-})^* \right], \]
\[ \mathcal{P}_{15X} = \frac{1}{6\sqrt{2}} \text{Re} \left[ (M_{++}^X - M_{++}^{-}) (M_{++}^{X} - M_{++}^{-})^* \right], \]
\[ \mathcal{P}_{16X} = \frac{1}{6\sqrt{2}} \text{Re} \left[ (M_{++}^X + M_{++}^{-}) (M_{++}^{X} + M_{++}^{-})^* \right]. \]  

(123)

It is simple to derive all the \( \mathcal{P}_{\alpha X} \) terms up to linear in \( c_\gamma \) and \( c_Z \) from the helicity amplitudes of \( e^+e^- \rightarrow t\bar{t} \), neglecting higher-order terms in the form factors. In the linear approximation, the \( \text{CP-even} \) and \( \text{CPT-even} \) terms independent of \( c_\gamma \) and \( c_Z \) are given by

\[ \mathcal{P}_{1L,R} = \frac{1}{2} (v_{L,R}^2 + \beta^2 a_{L,R}^2)(1 + \cos^2 \Theta) \mp 2\beta v_{L,R} a_{L,R} \cos \Theta \]
\[ + \frac{1}{2} (1 - \beta^2) v_{L,R}^2 \sin^2 \Theta, \]
\[ \mathcal{P}_{2L,R} = -\frac{1}{3\sqrt{3}} \left[ \frac{1}{2} (v_{L,R}^2 + \beta^2 a_{L,R}^2)(1 + \cos^2 \Theta) \mp 2\beta v_{L,R} a_{L,R} \cos \Theta \right. \]
\[ + \frac{1}{2} (1 - \beta^2) v_{L,R}^2 \sin^2 \Theta \],
\[ \mathcal{P}_{4L,R} = -\frac{2}{\sqrt{6}} \frac{1}{v_{L,R}} (\beta a_{L,R} \cos \Theta \mp v_{L,R}) \sin \Theta, \]
\[ \mathcal{P}_{8L,R} = \frac{2}{\sqrt{6}} \left[ \beta v_{L,R} a_{L,R} (1 + \cos^2 \Theta) \mp (v_{L,R}^2 + \beta^2 a_{L,R}^2) \cos \Theta \right], \]
\[ \mathcal{P}_{9L,R} = \frac{1}{3\sqrt{6}} \left[ (v_{L,R}^2 + \beta^2 a_{L,R}^2)(1 + \cos^2 \Theta) \mp 4\beta v_{L,R} a_{L,R} \cos \Theta \right. \]
\[ -2(1 - \beta^2) v_{L,R}^2 \sin^2 \Theta \],
\[ \mathcal{P}_{10L,R} = -\frac{1}{3\sqrt{2}} (v_{L,R}^2 - \beta^2 a_{L,R}^2) \sin^2 \Theta, \]
\[ \mathcal{P}_{15L,R} = \frac{\sqrt{2}}{2\sqrt{3}} v_{L,R} (v_{L,R} \cos \Theta \mp \beta a_{L,R}) \sin \Theta, \]  

(124)

where \( v_{L,R}, a_{L,R} \) and \( c_{L,R} \) are defined in Eqs. (15) and (16).

Every \( \text{CP-even} \) and \( \text{CPT-odd} \) term vanishes at the tree level:

\[ \mathcal{P}_{6L} = \mathcal{P}_{6R} = \mathcal{P}_{11L} = \mathcal{P}_{11R} = \mathcal{P}_{13L} = \mathcal{P}_{13R} = 0. \]  

(125)
These $\bar{T}$-odd terms can have finite contributions from QCD or QED loop corrections through the absorptive parts in the amplitude so that the terms can provide an important QCD test since the dominant contributions are from one-loop QCD contributions.

Every $CP$-odd and $CPT$-even term $P_{\alpha X}$, which depends on the real parts of $c_\gamma$ and $c_z$ is given by

$$P_{5L,R} = \sqrt{6} \frac{3\gamma}{2} (\beta a_{L,R} \cos \Theta \mp v_{L,R}) \sin \Theta \mathrm{Re}(c_{L,R}),$$

$$P_{12L,R} = -\frac{\sqrt{2}}{6} \beta v_{L,R} \sin^2 \Theta \mathrm{Re}(c_{L,R}),$$

$$P_{14L,R} = -\frac{\sqrt{2}}{6} \gamma (v_{L,R} \cos \Theta \mp \beta a_{L,R}) \sin \Theta \mathrm{Re}(c_{L,R}),$$

(126)

whereas every $CP$-odd and $CPT$-odd term $P_{\alpha X}$, which depends on the imaginary parts of the form factors, $c_\gamma$ and $c_z$, is given by

$$P_{3L,R} = -\frac{\sqrt{6}}{3} \gamma \beta (v_{L,R} \cos \Theta \mp \beta a_{L,R}) \sin \Theta \mathrm{Im}(c_{L,R}),$$

$$P_{7L,R} = -\frac{\sqrt{6}}{3} \beta v_{L,R} \sin^2 \Theta \mathrm{Im}(c_{L,R}),$$

$$P_{16L,R} = -\frac{\sqrt{2}}{3} \gamma \beta (\beta a_{L,R} \cos \Theta \mp v_{L,R}) \sin \Theta \mathrm{Im}(c_{L,R}).$$

(127)

B The definition of angular correlations $D_\alpha$ and $D'_\beta$

For notational convenience we use the following abbreviations

$$\xi_1 = \sin \theta \cos \phi, \quad \xi_2 = \sin \theta \sin \phi, \quad \xi_3 = \cos \theta,$$

$$\bar{\xi}_1 = \sin \bar{\theta} \cos \bar{\phi}, \quad \bar{\xi}_2 = \sin \bar{\theta} \sin \bar{\phi}, \quad \bar{\xi}_3 = \cos \bar{\theta}.$$ 

(128)

Then the orthonormal decay angular correlations $D_\alpha (\alpha = 1 \text{ to } 16)$ and $D'_\beta (\beta = 1 \text{ to } 12)$ are defined in terms of $\xi_i$ and $\bar{\xi}_i (i = 1, 2, 3)$ as

$$D_1 = 1,$$

$$D_3 = \sqrt{3} \xi_1 + \xi_2 \bar{\xi}_2 + \xi_3 \bar{\xi}_3),$$

$$D_4 = \frac{\sqrt{3}}{\sqrt{2}} \xi_1 - \bar{\xi}_1,$$

$$D_5 = \frac{\sqrt{3}}{\sqrt{2}} \xi_2,$$

$$D_6 = \frac{\sqrt{3}}{\sqrt{2}} \xi_3 - \bar{\xi}_3,$$

$$D_7 = \frac{\sqrt{3}}{\sqrt{2}} \xi_3,$$

$$D_8 = \frac{\sqrt{3}}{\sqrt{2}} (\xi_3 - \bar{\xi}_3),$$

$$D_9 = \frac{3}{\sqrt{6}} (\xi_1 \xi_1 + \xi_2 \xi_2 - 2 \xi_3 \bar{\xi}_3),$$

$$D_{10} = \frac{3}{\sqrt{2}} (\xi_1 \bar{\xi}_1 - \xi_2 \bar{\xi}_2),$$

$$D_{11} = \frac{3}{\sqrt{2}} (\xi_1 \bar{\xi}_2 + \xi_2 \bar{\xi}_1),$$

$$D_{12} = \frac{3}{\sqrt{2}} (\xi_1 \bar{\xi}_2 - \xi_2 \bar{\xi}_1).$$

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\[ D_{13} = \frac{3}{\sqrt{2}} (\xi_2 \xi_3 + \xi_3 \xi_2), \]
\[ D_{15} = \frac{3}{\sqrt{2}} (\xi_3 \xi_1 + \xi_1 \xi_3), \]
\[ D_{14} = \frac{3}{\sqrt{2}} (\xi_2 \xi_3 - \xi_3 \xi_2), \]
\[ D_{16} = \frac{3}{\sqrt{2}} (\xi_3 \xi_1 - \xi_1 \xi_3), \]

and
\[ D'_{1} = \frac{3 \sqrt{5}}{2 \sqrt{2}} (-\xi_3^2 + \xi_3^2), \]
\[ D'_{3} = \frac{\sqrt{15}}{\sqrt{2}} (\xi_1 \xi_2 - \xi_1 \xi_2), \]
\[ D'_{5} = \frac{\sqrt{15}}{\sqrt{2}} (\xi_3 \xi_1 - \xi_3 \xi_1), \]
\[ D'_{7} = \frac{3 \sqrt{5}}{\sqrt{2}} \xi_1 \xi_2 \xi_2, \]
\[ D'_{9} = \frac{3 \sqrt{5}}{\sqrt{2}} \xi_3 \xi_2 \xi_2, \]
\[ D'_{11} = \frac{3 \sqrt{5}}{\sqrt{2}} \xi_2 \xi_3 \xi_1 + \xi_2 \xi_3 \xi_1), \]

The correlation functions \( D \) and \( D' \) are normalized to satisfy the orthonormality conditions;
\[ \langle D_\alpha D_{\alpha'} \rangle \equiv \frac{1}{(4\pi)^2} \int d\Omega d\bar{\Omega} D_\alpha D_{\alpha'} = \delta_{\alpha\alpha'}, \]
\[ \langle D_\alpha D'_{\beta'} \rangle \equiv \frac{1}{(4\pi)^2} \int d\Omega d\bar{\Omega} D_{\alpha} D'_{\beta'} = 0, \]
\[ \langle D'_{\beta} D'_{\beta'} \rangle \equiv \frac{1}{(4\pi)^2} \int d\Omega d\bar{\Omega} D'_{\beta} D'_{\beta'} = \delta_{\beta\beta'}, \]

where \( \alpha(\alpha') = 1 \) to 16 and \( \beta(\beta') = 1 \) to 12.

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Tables

Table 1: \(CP\) and \(CPT\) properties of \(P_{\alpha X}\)’s and \(D_{\alpha}\)’s \((X = L, R\) and \(\alpha = 1\) to \(16)\).

Table 2: \(CP\) and \(CPT\) properties of the invariant functions and the angular distributions.

Table 3: The optimal 1-\(\sigma\) sensitivities to the \(CP\)-odd top EDM form factor \(\text{Re}(c_\gamma)\) and their corresponding \(x\) values for \(\sqrt{s} = 0.5\) and 1 TeV.

Figures

Figure 1: Feynman diagram for the \(Vtt\) \((V = \gamma, Z)\) vertex.

Figure 2: Schematic view of the sequential processes \(e^+e^- \rightarrow t\bar{t} \rightarrow (bW^+)(\bar{b}W^-) \rightarrow (bl^+\nu_l)(\bar{b}l^-\bar{\nu}_l)\). The dashed lines are for invisible particle trajectories in a particle detector.

Figure 3: The 1-\(\sigma\) allowed region of the \(CP\)-odd parameters \(\text{Re}(c_\gamma)\) and \(\text{Re}(c_\varphi)\) through the \(CP\)-odd and \(CPT\)-even asymmetries (a) \(A_b^1\) and \(T_{33}^b\) and (b) \(A_l^1\) and \(T_{33}^l\) with polarized electron beams, respectively, for the \(e^+e^-\) integrated luminosity 10 fb\(^{-1}\) and for the c.m. energy \(\sqrt{s} = 500\) GeV. The solid lines with a positive (negative) slope are for \(A_b^1\) and \(A_l^1\) with right-handed (left-handed) electrons while the long-dashed lines with a positive (negative) slope are for \(T_{33}^b\) and \(T_{33}^l\) with right-handed (left-handed) electrons.

Figure 4: The 1-\(\sigma\) allowed region of the \(CP\)-odd parameters \(\text{Re}(c_\gamma)\) and \(\text{Re}(c_\varphi)\) through the \(CP\)-odd and \(CPT\)-even asymmetries (a) \(A_b^1\) and \(T_{33}^b\) and (b) \(A_l^1\) and \(T_{33}^l\) with unpolarized electron beams, respectively, for the \(e^+e^-\) integrated luminosity 20 fb\(^{-1}\) and for the c.m. energy \(\sqrt{s} = 500\) GeV. The solid lines are for \(A_b^1\) and \(A_l^1\) while the long-dashed lines are for \(T_{33}^b\) and \(T_{33}^l\).

Figure 5: The 1-\(\sigma\) allowed region of the \(CP\)-odd parameters \(\text{Im}(c_\gamma)\) and \(\text{Im}(c_\varphi)\) through the \(CP\)-odd and \(CPT\)-odd asymmetries (a) \(A_{bE}^1\), \(A_{b2}^1\) and \(Q_{33}^b\) and (b) \(A_{lE}^1\), \(A_{l2}^1\) and \(Q_{33}^l\) with polarized electron beams, respectively, for the \(e^+e^-\) integrated luminosity 10 fb\(^{-1}\) and for the c.m. energy \(\sqrt{s} = 500 GeV\). The solid lines with a positive (negative) slope are for \(A_{bE}^1\) and \(A_{lE}^1\) with right-handed (left-handed) electrons while the long-dashed lines with a positive (negative) slope are for \(A_{b2}^1\) and \(A_{l2}^1\) with right-handed (left-handed) electrons. And, the dot-dashed lines with a positive (negative) slope are for \(Q_{33}^b\) and \(Q_{33}^l\) with right-handed (left-handed) electrons.

Figure 6: The 1-\(\sigma\) allowed region of the \(CP\)-odd parameters \(\text{Im}(c_\gamma)\) and \(\text{Im}(c_\varphi)\) through the \(CP\)-odd and \(CPT\)-odd asymmetries (a) \(A_{bE}^1\), \(A_{b2}^1\) and \(Q_{33}^b\) and (b) \(A_{lE}^1\), \(A_{l2}^1\) and \(Q_{33}^l\) with unpolarized
electron beams, respectively, for the $e^+e^-$ integrated luminosity 20 fb$^{-1}$ and for the c.m. energy $\sqrt{s} = 500$ GeV. The solid lines are for $A_E^b$ and $A_E^l$ while the long-dashed lines are for $A_2^b$ and $A_2^l$. And, the dashed lines are for $Q_{33}^b$ and $Q_{33}^l$.

**Figure 7:** The coordinate system in the colliding $\gamma\gamma$ c.m. frame. The scattering angle, $\Theta$, and the azimuthal angles, $\phi_1$ and $\phi_2$, for the linear polarization directions measured from the scattering plane are described.

**Figure 8:** (a) the photon energy spectrum and (b) the degree of linear polarization of the Compton backscattered photon beam for $x = 4E\omega_0/m_e^2 = 0.5, 1$ and 4.83.

**Figure 9:** (a) the $\gamma\gamma$ luminosity spectrum and (b) the two linear polarization transfers, $A_\eta$ (solid lines) and $A_{\eta\eta}$ (dashed lines), for $x = 4E\omega_0/m_e^2 = 0.5, 1$ and 4.83.

**Figure 10:** The $x$ dependence of the Re($c_\gamma$) upper bound, Max(|Re($c_\gamma$)|), at $\sqrt{s} = 0.5$ and 1 TeV, from the asymmetry $A_{00}$. The solid line is for $\sqrt{s} = 0.5$ TeV and the long-dashed line for $\sqrt{s} = 1$ TeV.
| CP | CPT | $\mathcal{P}_{\alpha X}$ | $\mathcal{D}_\alpha$ | Number |
|----|-----|----------------|----------------|--------|
| even | even | $\mathcal{P}_{1X}, \mathcal{P}_{2X}, \mathcal{P}_{4X}, \mathcal{P}_{8X}$ | $D_1, D_2, D_4, D_8$ | 7 |
| | | $\mathcal{P}_{9X}, \mathcal{P}_{10X}, \mathcal{P}_{15X}$ | $D_9, D_{10}, D_{15}$ | |
| even | odd | $\mathcal{P}_{6X}, \mathcal{P}_{11X}, \mathcal{P}_{13X}$ | $D_6, D_{11}, D_{13}$ | 3 |
| odd | even | $\mathcal{P}_{5X}, \mathcal{P}_{12X}, \mathcal{P}_{14X}$ | $D_5, D_{12}, D_{14}$ | 3 |
| odd | odd | $\mathcal{P}_{3X}, \mathcal{P}_{7X}, \mathcal{P}_{16X}$ | $D_3, D_7, D_{16}$ | 3 |

Table 2

| CP | CPT | Invariant functions | Angular dependences |
|-----|-----|-------------------|-------------------|
| even | even | $\Sigma_{\text{unpol}}$ | $\eta \cos(\phi + \chi) + \bar{\eta} \cos(\phi - \chi)$ |
| | | $\Re(\Sigma_{02})$ | $\eta \eta \cos(2\phi)$ |
| | | $\Re(\Sigma_{22})$ | $\eta \eta \cos(2\chi)$ |
| even | odd | $\Im(\Delta_{02})$ | $\eta \sin(\phi + \chi) + \bar{\eta} \sin(\phi - \chi)$ |
| | | $\Im(\Sigma_{22})$ | $\eta \eta \sin(2\phi)$ |
| odd | even | $\Im(\Sigma_{02})$ | $\eta \sin(\phi + \chi) - \bar{\eta} \sin(\phi - \chi)$ |
| | | $\Im(\Sigma_{00})$ | $\eta \eta \sin(2\chi)$ |
| odd | odd | $\Re(\Delta_{02})$ | $\eta \cos(\phi + \chi) - \bar{\eta} \cos(\phi - \chi)$ |

Table 3

| $\sqrt{s}$ | 0.5 | 1.0 |
|-----------|-----|-----|
| $x$       | 3.43 | 0.85 |
| $\Re(c_\gamma)$ | 0.16 | 0.02 |
Figure 1
Figure 2
Figure 3
Figure 4
Figure 5
Figure 6
Photon Spectrum

Linear Polarization

Figure 8
Figure 9
Figure 10