Determining the optimal window length of the time-varying copula parameter: a simulation study

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Abstract. A time-varying copula is a multivariate cumulative distribution function that can accommodate the temporal dependence of random variables. To capture the temporal dependence of random variables using a copula, we can form a time variation on copula parameters. Some studies reported that a function of a latent variable following an autoregressive process defines time-varying copula parameters. The process consists of an autoregressive coefficient describing the influence of the parameters in previous times and a forcing variable representing the relationship between the marginal variables which utilize previous information over a certain number of window lengths of the observations. Previous studies reported that ARMA(1,10) defines time-varying copula parameters. However, the optimal parameters of time-varying copulas do not always require ten last observations. Therefore, we determine the optimal window length of the time-varying copula parameter by conducting a numerical simulation on four types of copula with different strengths and directions of dependence. The window length is defined as a new parameter and the optimal parameters can be obtained by finding the values producing the smallest AIC. We also provide a numerical simulation on how to improve the accuracy of the predicted series by changing the mean and standard deviation of the residual pairs.

1. Introduction

There are many methods in dependence modeling, one that is quite popular and continues to be developed is copula model. Copula couple the marginal distribution functions into a multivariate distribution function without any normality assumption. The dependence measures, such as rank correlation and tail dependence, can be represented as a parameter of copula. In recent years, temporal dependence is considered to better describe the structure of dependence between variables, particularly for the joint historical data such as in financial, actuarial, and climatological field. Therefore, many researches are conducted to analyze the temporal parameter of the time-varying copula, i.e. the copula which varies over time.

In previous studies, the temporal parameter of time-varying copula is often defined by the autoregressive process [1, 2]. Specifically, the process follow an ARMA(1,10) [3, 1, 2], where the order of autoregressive $p = 1$ defines the influence of the parameter at time $t-1$. While the order of moving average $q = 10$ represents the involvement of the last 10 observations toward the interdependence of the marginal variables. The use of first order in the autoregressive process simplifies computation because the larger the order used, the more parameters that must be estimated. In addition, the parameters of the previous time are considered to have the greatest
role in the dependence of the present time. Furthermore, there is no specific judgment toward the use of the last 10 observations to the moving average process. In other similar study, the temporal parameter of the time-varying copula is defined by the Fisher transformation, called by Fisher dynamic, to maintain the parameter of the Gaussian and \( t \) copula at the range of \([-1, 1]\) [4]. Unlike the studies previously mentioned, Fisher dynamic only use one lag of the past observations.

According to the previous studies, there is a difference in determining the window length from past observations that determine the calculation of the temporal parameters. This paper aims to determine the optimal window length of the temporal parameter of time-varying copulas by considering the window length of the past observations as a new parameter that must be estimated. We conduct a numerical simulation to study the behavior of the optimal window length of dependence with different strengths and directions, i.e. strong positive, weak positive, and strong negative. We use four kinds of copulas, that are time-varying Normal, Clayton, Gumbel, and Symmetry Joe-Clayton (SJC) copulas, each represents the copula having zero tail dependence, lower tail, upper tail, and both lower and upper tail dependence. We also provide the procedures to obtain the optimal window length of the time-varying copula parameter. In order to obtain a better understanding related to the modeling using time-varying copula with optimal window length, we employ the procedures to the agricultural data. We identify the dynamic dependency between the price index received and paid by farmer which determine the farmer exchange rate index as an indicator of the farmer’s welfare. Similar to the previous studies, we model the marginal variables using a time-series model. The most frequently used time-series models are \( ARCH/GARCH \) or the combination of \( AR – GARCH \) [5, 6, 4, 7, 8].

The rest of the paper is organized as follows. Section 2 presents a brief overview of the time-varying copula models and the model specification including the temporal parameter. Section 3 provides the numerical simulation to determine the optimal window length of the time-varying copula parameter. Section 4 delivers the application of the modeling using the agricultural data and a numerical simulation on how to improve the accuracy of the predicted series. Section 5 gives the conclusions of the studies.

### 2. Time-Varying Copulas

Suppose that \( \{X_{1,t}, X_{2,t}\} \) is a bivariate time-series data and assume that each univariate process follows a particular time-series model. Suppose that each model has a residual notated by \( \varepsilon_{i,t}, i = 1, 2 \), where the two residuals are correlated with each other. The cumulative distribution function of the residuals are \( F(\varepsilon_{1,t}) \) and \( G(\varepsilon_{2,t}) \) and the bivariate distribution function of the joint vector of residuals is \( H(\varepsilon_{1,t}, \varepsilon_{2,t}) \). By Sklar’s Theorem [9], a copula function \( C(\cdot) : [0, 1]^2 \rightarrow [0, 1] \) exists such that

\[
H(\varepsilon_{1,t}, \varepsilon_{2,t}) = C(F(\varepsilon_{1,t}), G(\varepsilon_{2,t}))
\]

(1)

The functions and specifications of the time-varying Normal, Clayton, Gumbel, and SJC copulas are provided in Table 1 [10, 2]

| Time-Varying Copula | Function | Parameter |
|---------------------|----------|-----------|
| Normal              | \[
\int_{-\infty}^{\Phi^{-1}(u_t)} \int_{-\infty}^{\Phi^{-1}(v_t)} \frac{1}{2\pi \sqrt{1-\rho_t^2}} \exp \left\{ \frac{-r^2-2\rho_r s}{2(1-\rho_t^2)} \right\} ds \right] dr \\
\] | \( \rho_t \in (-1, 1) \) |
| Clayton             | \[
(u_t^{-\theta_t^C} + v_t^{-\theta_t^C} - 1)^{-\frac{1}{\theta_t^C}}
\] | \( \theta_t^C \in (0, +\infty) \) |
| Gumbel              | \[
\exp \left\{ \frac{(-\ln u_t)^{\theta_t^G} + (-\ln v_t)^{\theta_t^G}}{\theta_t^G} \right\}
\] | \( \theta_t^G \in [1, +\infty) \) |
| SJC                 | \[
0.5(C_{JC}(u_t, v_t) + C_{JC}(1-u_t, 1-v_t) + u_t + v_t - 1)
\] | \( \tau_t^U, \tau_t^L \in (0, 1) \) |

Table 1: Copula Specification
where \( u_t = F(\varepsilon_{1,t}) \), \( v_t = G(\varepsilon_{2,t}) \), and

\[
C_{JC}(u_t, v_t) = 1 - \left( 1 - \{1 - (1 - u_t)^{\alpha t}\}^{-\gamma t} + \{1 - (1 - v_t)^{\alpha t}\}^{-\gamma t} - 1 \right) \frac{1}{\gamma t}
\]

with \( \kappa_t = \frac{1}{\log_2(2 - \tau_t^2)} \), \( \gamma_t = \frac{1}{\log_2(\tau_t^2)} \).

Let \( \Theta_t = \{\rho_t, \theta_t^C, \theta_t^G, \{\tau_t^U, \tau_t^L\}\} \) be the dynamic parameter of time-varying Normal, Clayton, Gumbel, and SJC copulas, respectively. The temporal parameters of the time-varying copulas follow an ARMA(1,10) process and are given by [1, 2]

\[
\hat{\Theta}_t = \begin{cases} 
\tilde{\lambda} \left( \omega + \beta \Theta_{t-1} + \alpha \frac{1}{10} \sum_{j=1}^{10} \Phi^{-1}(u_{t-j}) \cdot \Phi^{-1}(v_{t-j}) \right), & \text{time-varying Normal} \\
\lambda \left( \omega + \beta \Theta_{t-1} + \alpha \frac{1}{10} \sum_{j=1}^{10} |u_{t-j} - v_{t-j}| \right), & \text{others} 
\end{cases}
\]

(2a)

where \( \Phi^{-1}(u_{i,t}) \) is the inverse cdf transform of standard Normal distribution, \( \tilde{\lambda}(x) = (1 - e^{-x})(1 + e^{-x})^{-1} \) for the dynamic parameter of time-varying Normal copula, and \( \lambda(x) = \{e^x, e^x + 1, (1 + e^{-x})^{-1}\} \) for time-varying Clayton, Gumbel, and SJC copulas, respectively.

In this study, we define the order of the MA process on the function of the temporal parameter to be a new parameter that must be estimated. Therefore, Eqs. 2a and 2b are rewritten by

\[
\hat{\Theta}_t = \begin{cases} 
\tilde{\lambda} \left( \omega + \beta \Theta_{t-1} + \alpha \frac{1}{m} \sum_{j=1}^{m} \Phi^{-1}(u_{t-j}) \cdot \Phi^{-1}(v_{t-j}) \right), & \text{time-varying Normal} \\
\lambda \left( \omega + \beta \Theta_{t-1} + \alpha \frac{1}{m} \sum_{j=1}^{m} |u_{t-j} - v_{t-j}| \right), & \text{others} 
\end{cases}
\]

(3a)

The parameters \( \Omega_t = \{\omega, \beta, \alpha, m\} \) are estimated by maximizing the log-likelihood function as follows

\[
\Omega_t = \arg \max_{\Omega_t} \left( \sum_{t=1}^{T} \log c(F(\varepsilon_{1,t}; \lambda_1), G(\varepsilon_{2,t}; \lambda_2)) \right)
\]

(4)

where \( c(\cdot) \) is the copula density function and \( \lambda_1 \) and \( \lambda_2 \) are the parameter of the marginal variables \( \varepsilon_{1,t} \) and \( \varepsilon_{2,t} \), respectively. The best copula is selected from those has the smallest AIC value.

3. Numerical Simulation to Determine the Optimal Window Length

In this section, we conduct a numerical simulation using the generalized function of the temporal parameter presented in Eqs. 3a and 3b. The marginal distribution used for joint vector \( \varepsilon_{i,t} \) is Gamma distribution with parameters \( \{\alpha, \beta\} = \{0.5, 4.5\} \). This type of distribution shows that copulas can handle the structure of dependence of non-elliptical distribution. Table 2 shows the real parameters of \( \Theta = \{\rho, \theta^C, \theta^G, \{\tau^U, \tau^L\}\} \) for each copula, the sample sizes \( n \), and the number of iteration \( r \) used in this simulation.

We conduct 9 simulations consist of 3 simulations for time-varying Normal and 2 simulations, each for time-varying Clayton, Gumbel, and SJC. The three simulations for time-varying Normal
Table 2: Characteristic of Data Generation

| Time-Varying Copula | Simulation | Parameter(s) | n     | r     |
|---------------------|------------|--------------|-------|-------|
| Normal              | 1          | $\rho_1 = 0.95$ | 50,100,500,1000 |       |
|                     | 2          | $\rho_2 = 0.1$  |       |       |
|                     | 3          | $\rho_3 = -0.95$ |       |       |
| Clayton             | 1          | $\theta^C_1 = 15$ | 50,100,500,1000 |       |
|                     | 2          | $\theta^C_2 = 0.25$ |       |       |
| Gumbel              | 1          | $\theta^G_1 = 10$ | 50,100,500,1000 |       |
|                     | 2          | $\theta^G_2 = 1.11$ |       |       |
| SJC                 | 1          | $\{\tau^U, \tau^L\} = \{0.86, 0.78\}$ | 50,100,500,1000 |       |
|                     | 2          | $\{\tau^U, \tau^L\} = \{0.2, 0.1\}$ |       |       |

consist of simulations with strong positive, weak, and strong negative dependencies. Whereas the two simulations for the other copulas consist of simulations with strong and weak positive dependencies. There is no simulation with strong negative dependence for simulation other than time-varying Normal because the range of parameters of the other three copulas is at intervals of greater than zero value (see Table 1). For each copula, we consider various sample sizes $n$, i.e., 50, 100, and 500, to study the different behaviors of the parameter estimation, and perform $r = 1000$ iterations. In other words, we conduct 9 simulations $\times$ 3 sample sizes $\times$ 1000 iterations. We set the window length $m$ from 1 to 15 with the assumption that the larger the window length, the less significant the effect of the previous observations to the dynamic parameter. Moreover, the selection of the window length reduces the complexity of the computation.

Considering the various outcomes of the parameter estimates of $\hat{\omega}$, $\hat{\beta}$, and $\hat{\alpha}$, we focus on the estimation result of the window length $\hat{m}$ to study the effect of $m$ past observations to the temporal parameter. Although the first three parameter estimates vary, the result of the temporal parameter $\Theta_t$ estimate has fluctuations that are not much different after the four estimated parameters are combined following Eqs. 3a and 3b. Thus, by focusing on the percentage of the appearance intensities of window length $\hat{m}$, we study whether the short or the long period of the past observation has a stronger impact on the dynamic parameter of time-varying copulas.

Previous studies used the last 10 observations for the window length $m$ in the moving average process of the temporal parameter (Eqs. 2a–2b) [1, 2]. However, information regarding the window length selection is lacking. In our preliminary study, we tried to generate two pairs of random data using a copula and vary the number of $m$. Our results show that the best window length does not always come from the last 10 observations. For this reason, we assume that the best window length can be obtained by considering $m$ as a parameter that can be estimated so that the estimation results are optimal. Therefore, we consider four parameters ($\omega$, $\beta$, $\alpha$, and $m$) to be estimated for time-varying copulas. However, obtaining the optimal value of $m$ directly using the optimization method is complicated because the optimization needs to be performed for continuous and integer parameter values. Thus, we set the value of $m$ from 1 to 15, considering that more than 15 recent observations are insufficient to exert a significant effect. Then select $m$ that optimizes the log likelihood function of the time-varying copulas by selecting the smallest $AIC$ value. Figures 1 and 2 present the percentage of appearance of the window length $m$ in 1000 iterations for simulation 1 and 2 (strong positive and weak dependency) of all time-varying copulas.

The simulation with sample sizes of 50 and 100 in Figures 1 and 2 show that the last one observation greatly influences the temporal parameter of the time-varying copula and has a small likelihood of being affected by a long period of observation in the past. This result is evidenced...
Figure 1: Percentage of Appearance of the Window Length in Simulation 1 (Strong Positive Dependency)

by the decreasing percentage of window length appearances for the longer period from previous observations. Moreover, no fluctuation can be observed in the distribution of the window length percentages of appearance except a small jump in the order of 15 (m = 15). On the basis of the value of the parameter in simulation 1, \( \Theta \) = \{\( \rho_1, \theta^{C}_1, \theta^{G}_1, \tau^{U}_1, \tau^{L}_1 \)\} presented in Table 1, each of

Figure 2: Percentage of Appearance of the Window Length in Simulation 2 (Weak Dependency)
the parameter has a strong value in terms of dependence measures. Therefore, the smaller the window length, the stronger the dependence between the variables. Similar to the estimation results of simulations with a sample sizes of 50 and 100, the window length that often appears in the simulation with sample size of 500 is the last one to two observations. However, the larger the sample size, the more the window length fluctuates. It means that for the data with the longer period, the temporal parameter can be influenced by the last few observations, which are longer than the data with the shorter period. However, the percentage of a small \( m \) is generally more significant than that of a large \( m \). Furthermore, Figure 3 presents the percentage of appearance of the window length \( m \) in 1000 iterations for simulation 3 (strong negative dependency).

Figure 3: Percentage of Appearance of the Window Length for Simulation 3 (Strong Negative Dependency)

Similar to simulation 1 and 2, the window length that often appear in 1000 iterations for all the sample sizes \( n \) in simulation 3 is the last one observation. However, unlike the simulation 1 and 2, there are some fluctuations of the percentage of appearance of window length in simulation 3. It may happen because negative value of the parameter \( \rho_{t-1} \) affect the value of the \( \rho_t \) in a different way compare to the positive ones. Generally, the simulations for all sample sizes show similar behavior, that is the percentage of appearance of the window length decreases when the window length increases. Basing from all of the simulations, we cannot determine the optimal window length \( m \) by picking any positive integer number. Otherwise, \( m \) can be estimated by calculating \( AIC \) from various amounts of window length and selecting the value of \( m \) that minimizes the value of \( AIC \). Moreover, we cannot guarantee a specific \( m \) value that will produce the optimal parameter estimates. However, we can ensure that the smaller the amount of \( m \), the more significant the effect of \( m \) past observations to the value of the temporal parameter.

4. Application

In this section, we apply the time-varying copula modeling to real data in the agricultural field. We use two variables involved in the calculation of farmer exchange rate, i.e., price index received and paid by farmers in Indonesia. The price index received by farmers is an indicator of farmer’s welfare in terms of income, whereas the price index paid by farmers is an indicator of farmer’s expenditure for daily needs and production cost. The farmer exchange rate is the ratio between the two variables. Hence, the farmer exchange rate is calculated by follows [11]

\[
FER_t = \frac{X_{t}^{PR}}{X_{t}^{PP}} \cdot 100
\]
where $FER_t$ is the farmer exchange rate at time $t$, $X_t^{PR}$ and $X_t^{PP}$ are the price index received and paid by farmers at time $t$, respectively.

The data used are monthly data from January 2008 to December 2018, reported by the Indonesian Bureau of Statistics (Badan Pusat Statistik/BPS). The original data of the price index received and paid by farmers are calculated using two base years. That is, the data from January 2008 to November 2013 use 2007 as the base year, whereas the data from December 2013 to December 2018 use 2012 as the base year. Therefore, we transform the two marginal data series into the same base year of 2012 without changing the farmer exchange rate data. Figure 4 presents the original and transformed data.

We identify the dependence between the price index received and paid by farmers using time-varying copula models. The ultimate purpose is to model the farmer exchange rate using time-varying copula and compare the result with the original farmer exchange rate data. First, we model the marginal variables using the time-series $ARIMA$ model. After some modeling, we obtain the best $ARIMA$ models for the two marginal variables; $ARIMA(2,1,2)$ and $ARIMA(1,1,2)$, respectively. The parameter estimates of the two marginal models is presented in Table 3.
Table 3: Parameter Estimates of Marginal Models

| Parameters | Price Index Received by Farmers | Price Index Paid by Farmers |
|------------|---------------------------------|----------------------------|
|            | Value   | Std | Value   | Std |
| \( \hat{\alpha}_1 \) | -0.5939 | 0.1228 | 1 | 2e-04 |
| \( \hat{\alpha}_2 \) | 0.3898 | 0.1203 | - | - |
| \( \hat{\theta}_1 \) | 1.3468 | 0.1275 | -0.6308 | 0.0862 |
| \( \hat{\theta}_2 \) | 0.3845 | 0.1226 | -0.3618 | 0.0857 |

Then, we estimate the temporal parameter of time-varying Normal, Clayton, Gumbel, and SJC copulas which couple the residuals of the marginal variables \( \varepsilon_{1,t} \) and \( \varepsilon_{2,t} \). The optimal parameter \( \hat{\Omega} = \{ \hat{\omega}, \hat{\beta}, \hat{\alpha}, \hat{m} \} \) are selected based on the smallest AIC value. Table 4 presents the temporal parameter estimates of time-varying copulas.

Table 4: Parameter Estimates of Time-Varying Copulas

| Time-Varying Copula | \( \hat{\omega} \) | \( \hat{\beta} \) | \( \hat{\alpha} \) | \( \hat{m} \) | AIC |
|---------------------|-----------------|-----------------|-----------------|-----------------|-----|
| Normal              | 1.105           | -1.151          | -1.248          | 1               | -9.882 |
| Clayton             | 1.189           | -3.013          | -5.000          | 8               | -8.439 |
| Gumbel              | 2.647           | -2.686          | -5.000          | 1               | -16.998 |
| SJC                 | -4.125          | 3.943           | 6.132           | 5               | -38.606 |

The temporal parameter estimates in Table 4 show that the optimal parameter of time-varying copula used in this case is from the time-varying SJC copula. The results also show that 5 previous observations give the most significant effect on the temporal parameter for this case. Furthermore, the optimal parameter of time-varying copula obtained compared to the parameter of the static copula is presented in Figure 5.

Figure 5: Optimal Parameter Estimates of the Time-Varying SJC Copula for the Price Index Received and Paid by Farmer
Figure 5 shows that there are positive upper and lower tail dependence between two residuals of the marginal variables. It also shows the fluctuation in the temporal parameter $\hat{\tau}_U$ of time-varying SJC copula, which is not owned by the parameter of the static copula. This result means that temporal upper tail dependence exists between the residuals of the price index received and paid by farmers. The lower tail dependence (see second graph in Figure 5) has no single fluctuation in its temporal parameter after the second observation. However, variations occur on a reasonably small scale starting from the second observation until the last observation. This result indicates that temporal lower tail dependence also exists between the residuals of the two marginal variables. The optimal window length shows that the most significant period of the previous observations is from the 5 last observations. Therefore, the joint residuals between the price index received and paid by farmers have a temporal upper and lower tail dependence.

Finally, the optimal parameter obtained is used to generate a new joint vector of residuals $\{\hat{\varepsilon}_{1,t}, \hat{\varepsilon}_{2,t}\}$ to replace the residuals of ARIMA model of the marginal variables and determine the new series of the price index received and paid by farmers. Figure 6 presents the time-varying copula-based new set of the two marginal variables and the related farmer exchange rate. We use mean squared error ($MSE$) and mean absolute percentage error ($MAPE$) to evaluate the performance of the time-varying copula models in predicting the data with dependence, which are presented in Table 5.

![Figure 6](image)

**Figure 6: Time-Varying Copula-Based Estimated Data of the Price Index Received and Paid by Farmer and the Farmer Exchange Rate**

**Table 5: Performance Measurement of the Time-Varying Copula-Based Estimated Data**

| Variable                     | $MSE$  | $MAPE$   |
|------------------------------|--------|----------|
| Price Index Received by Farmers | 0.7529 | 0.7141%  |
| Price Index Paid by Farmers  | 1.0942 | 0.8165%  |
| Farmer Exchange Rate         | 1.8553 | 1.0702%  |
Figure 6 shows that the estimated marginal variables and farmer exchange rate are close to the original data. Furthermore, the amounts of $MSE$ and $MAPE$ show that the time-varying SJC copula with the window length of 5 provides a good estimation result to the data.

Based on the best copula estimation results for the price index received and paid by farmers, the upper tail dependence between the two variables has a stronger and more dynamic value than the lower tail dependence. It shows that there is a strong enough relationship between the two variables in their upper values. Based on these results, farmers’ welfare can be improved by increasing the price index received by farmers. Although intuitively, the greater the price index received by farmers, the greater the price index that must be paid by farmers, but by paying attention and increasing the price index received by farmers, farmers will tend to be in a stable condition so that the farmer’s welfare can be improved. In practice, the price index received by farmers can be increased by making efforts to increase the quality and quantity of agriculture because the index is obtained by calculating the amount of income obtained from agricultural products. Meanwhile, the price index paid by farmers can be lowered by reducing the cost of daily living and production capital without reducing the value of products and services needed for agricultural household life.

Furthermore, we conduct a numerical simulation on how to improve the accuracy measure of the farmer exchange rate prediction. We change the value of the mean and standard deviation of the residual pair generated from the selected time-varying copula with optimal window length without changing the dependency measure obtained from the modeling. The sequences of the mean and standard deviation are \{(\mu_1, \sigma_1), (\mu_2, \sigma_2), (\mu_3, \sigma_3), (\mu_4, \sigma_4)\} = \{(0, 1), (0.1, 0.1), (0.01, 0.01), (0, 0.001)\}. The result is presented in Figure 7.

Figure 7 shows that the smaller the mean and standard deviation of the residual pairs, the more accurately the model in predicting the FER series. It is indicated by the smaller $MSE$ and $MAPE$ values obtained when the mean and standard deviation of the residual pairs are reduced.
5. Conclusion
We generalized the temporal parameters of time-varying copula models to obtain the optimal window length. We assume four parameters to be estimated, where the three previous parameters introduced by [2] are added with a parameter related to the effect of the past observations to the relationship between two marginal variables in the temporal parameter, called window length $m$. Numerical simulation shows that the optimal parameter, specifically the window length, does not always come from a specific number. Otherwise, the optimal value of window length can vary depending on the data and the degree of dependence between marginal variables. However, the optimal value of window length that often appears from 1000 iterations in each type of copula is a short order from the last observation. Therefore, the longer the period of the previous observations, the smaller the impact on the dynamic parameter.

Application to real data shows that the optimal parameter obtained is from the time-varying SJC copula with a window length of order 5. Positive dependence values of $\hat{\tau}^U_t$ and $\hat{\tau}^L_t$ indicate that there are positive upper and lower tail dependence between the joint residuals of the price index received and paid by farmer. Fluctuations in the value of $\hat{\tau}^U_t$ which are sometimes strong and weak are also sufficient evidence that the required window length in the optimal parameter estimation is 5. Thus, based on the numerical simulation and the application, the window length must be treated as a parameter to obtain the order of previous observations which give a significant effect on the temporal parameter. We also found that the accuracy of the predicted data can be obtained by minimizing the mean and standard deviation of the residual pairs which are generated from the selected time-varying copula models.

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