Supersymmetric SU(3) × U(1) Gauge Model:
Higgs Structure at the Electroweak Energy Scale

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Abstract

We consider a supersymmetric version of the recently proposed SU(3) × U(1) extended gauge model. We show that it is possible to have only two Higgs doublets at the SU(2) × U(1) energy scale but they are not those of the minimal supersymmetric standard model. In particular, the upper bound on the lightest scalar boson of this model is $4M_Z \sin \theta_W$ at tree level and goes up to about 189 GeV after radiative corrections.
The quartic scalar couplings of a given supersymmetric gauge theory are determined by the gauge couplings as well as other possible couplings appearing in the superpotential. In the minimal supersymmetric standard model (MSSM), there are only two Higgs-doublet superfields from which a cubic invariant cannot be formed. Hence the quartic scalar couplings of the Higgs potential $V$ depend only on $g_1$ and $g_2$, i.e. the U(1) and SU(2) gauge couplings respectively. Let

$$V = \mu_1^2 \Phi_1^\dagger \Phi_1 + \mu_2^2 \Phi_2^\dagger \Phi_2 + \mu_{12}^2 (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1) + \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \frac{1}{2} \lambda_5 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_5^* (\Phi_2^\dagger \Phi_2)^2,$$

then in the MSSM,

$$\lambda_1 = \lambda_2 = \frac{1}{4}(g_1^2 + g_2^2), \quad \lambda_3 = -\frac{1}{4}g_1^2 + \frac{1}{4}g_2^2, \quad \lambda_4 = -\frac{1}{2}g_1^2, \quad \lambda_5 = 0. \quad (2)$$

However, if the standard model is really the remnant\[1\] of a larger theory such as SU(3) $\times$ U(1) as proposed recently\[2, 3\] and it is supersymmetric, then at the electroweak energy scale, after the heavier particles have been integrated out, the reduced Higgs sector may contain only two doublets but Eq. (2) is no longer valid. The reason is the appearance of possible cubic invariants in the superpotential according to the larger theory which have no analog in the MSSM. A first example based on an E$_6$-inspired left-right model has already been discovered.\[4\]

In this paper, we propose a supersymmetric version of SU(3) $\times$ U(1) which is interesting in its own right. We discuss how quark and lepton masses can be generated. We then focus on a specific scenario where only two Higgs doublets are relevant at the SU(2) $\times$ U(1) scale. In particular, we show that the upper bound on the lightest scalar boson of this model is $4M_Z \sin \theta_W$ at tree level and goes up to about 189 GeV after radiative corrections. This
bound is substantially higher than the 115 GeV of the MSSM or the 120 GeV of the left-right model mentioned above.\[4\]

The salient feature of the new SU(3) \(\times\) U(1) model\[2, 3\] is in the choice of the electric-charge operator within SU(3). Instead of the usual \(Q = I_3 + Y/2\), it is assumed here that \(Q = I_3 + 3Y/2\). Hence for SU(3) \(\times\) U(1), we have

\[
Q = I_3 + \frac{3}{2}Y + Y',
\]

where \(Y'\) is the U(1) hypercharge. Consider now the fermionic content of this model. The three families of leptons transform identically as \((3^*, 0)\). Specifically, \((\ell^c, \nu_\ell, \ell)_L\) form an antitriplet with \(I_3 = (0, 1/2, -1/2)\) and \(Y = (2/3, -1/3, -1/3)\). The quarks are different: the third family \((T, t, b)_L\) is also an antitriplet \((3^*, 2/3)\), but the first two, \((u, d, D)_L\) and \((c, s, S)_L\), are triplets \((3, -1/3)\) with \(I_3 = (1/2, -1/2, 0)\) and \(Y = (1/3, 1/3, -2/3)\). All the charge-conjugate quark states are singlets. As shown in Refs. [2] and [3], this structure ensures the absence of all axial-vector anomalies.

The Higgs sector of this model must consist of at least three complex triplets \((\eta^+, \eta^0, \eta^-)\), \((\rho^0, \rho^-, \rho^{--})\), and \((\chi^{++}, \chi^+, \chi^0)\), transforming as \((3, 0)\), \((3, -1)\), and \((3, 1)\) respectively. At the first step of symmetry breaking, \(\chi^0\) acquires a large vacuum expectation value, so that SU(3) \(\times\) U(1) breaks down to the standard SU(2) \(\times\) U(1) and the exotic quarks \(D, S\) (of electric charge \(-4/3\)) and \(T\) (of electric charge \(5/3\)) become massive. The subsequent breaking of SU(2) \(\times\) U(1) is accomplished with nonzero values of \(\langle \eta^0 \rangle\) and \(\langle \rho^0 \rangle\), such that \(t, s, \text{ and } d\) acquire masses proportional to the former and \(b, c, \text{ and } u\) acquire masses proportional to the latter. To obtain lepton masses, a Higgs sextet was proposed.\[3, 5\] However, we would like to adopt a simple alternative. Let \(E_L\) and \(E^c_L\) be singlets \((1, -1)\) and \((1, 1)\), then \(E_L E^c_L\) is an allowed mass term and the mass matrix linking \((\ell^c, E^c_L)\) to \((\ell_L, E_L)\) is of the see-saw form with \(\langle \chi^0 \rangle\) contributing to \(E_L \ell^c_L\) and \(\langle \rho^0 \rangle\) to \(\ell_L E^c_L\) respectively.

We now impose supersymmetry. In addition to changing all fields into superfields, we
need to add three complex scalar superfields \((\eta^+, \eta^0, \eta^-)\), \((\rho'^+, \rho'^0, \rho'^0)\), and \((\chi'^0, \chi'^-, \chi'^--)\), transforming as \((3^*, 0)\), \((3^*, 1)\), and \((3^*, -1)\) respectively. These are required for the cancellation of anomalies generated by the \(\rho\), \(\eta\), and \(\chi\) superfields. They also have invariant couplings to the quark superfields, so that \(m_T\) comes from \(\langle \chi^0 \rangle\), but \(m_S\) and \(m_D\) come from \(\langle \chi^0 \rangle\); \(m_t\) comes from \(\langle \eta^0 \rangle\), but \(m_c\) and \(m_u\) come from \(\langle \rho'^0 \rangle\); \(m_b\) comes from \(\langle \eta^0 \rangle\), but \(m_s\) and \(m_d\) come from \(\langle \eta^0 \rangle\). Furthermore, the superpotential now contains two cubic invariants \(f \epsilon_{ijk} \eta_i \rho_j \chi_k\) and \(f' \epsilon_{ijk} \eta_i' \rho_j' \chi_k\) which contribute to the Higgs potential.

The Higgs sector of our supersymmetric version of the SU(3) \(\times\) U(1) model now has 3 triplets and 3 antitriplets. The part of the Higgs potential related to the gauge interactions through supersymmetry is given by

\[
V_D = \frac{1}{2} G_1^2 [-\rho_i^* \rho_i + \chi_i^* \chi_i + \rho_i^* \rho_i - \chi_i^* \chi_i]^2
+ \frac{1}{8} G_3^2 \sum_a [\eta_i^a \chi_{ij}^* \eta_j + \rho_i^a \chi_{ij} \rho_j + \chi_i^* \chi_{ij} \chi_j - \eta_i^a \chi_{ij}^* \eta_j' - \rho_i^a \chi_{ij} \rho_j' - \chi_i^* \chi_{ij} \chi_j']^2,
\]

where \(G_1\) and \(G_3\) are the U(1) and SU(3) gauge couplings respectively and \(\chi_{ij}^a\) are the 8 conventional \(3 \times 3\) SU(3) representation matrices. Similarly, the part of the Higgs potential related to the superpotential is given by

\[
V_F = f^2 \sum_k [|\epsilon_{ijk} \eta_i \rho_j|^2 + |\epsilon_{ijk} \rho_i \chi_j|^2 + |\epsilon_{ijk} \chi_i \eta_j|^2]
+ f'^2 \sum_k [|\epsilon_{ijk} \eta_i' \rho_j'|^2 + |\epsilon_{ijk} \rho_i' \chi_j'|^2 + |\epsilon_{ijk} \chi_i' \eta_j'|^2].
\]

Let \(\langle \chi^0 \rangle = u \neq 0\) and \(\langle \chi'^0 \rangle = u' \neq 0\), then the SU(3) \(\times\) U(1) gauge symmetry is broken down to the standard SU(2) \(\times\) U(1). Five of the twelve degrees of freedom contained in \(\chi\) and \(\chi'\) are absorbed into the five vector gauge bosons which become massive. The remaining seven are heavy physical states. As for \(\eta, \rho, \eta',\) and \(\rho'\), if their doublet components are to be light, then their singlet components are necessarily heavy because their mass terms depend differently on \(u^2\) and \(u'^2\). In general, the reduced Higgs potential will contain four doublets if \(\langle \eta^0 \rangle, \langle \rho'^0 \rangle, \langle \eta'^0 \rangle,\) and \(\langle \rho'^0 \rangle\) are all nonzero. However, an interesting alternative exists if we
assume an extra discrete $Z_2$ symmetry under which $u_L^c, d_L^c, s_L^c$, and $c_L^c$ are odd and all other superfields are even. It has been shown that the breaking of this $Z_2$ by soft terms which also break the supersymmetry will allow $u, d, s$, and $c$ to acquire radiative masses in one-loop order through gluino exchange. Hence it is possible for $\langle \eta^0 \rangle$ and $\langle \rho^0 \rangle$ to be zero so that $\eta'$ and $\rho'$ may be assumed heavy and will not appear in the reduced Higgs potential at the electroweak energy scale.

Redefine $(-\rho^-, \rho^0)$ as $\Phi_1$ and $(\eta^+, \eta^0)$ as $\Phi_2$, then the parts of $V_D$ and $V_F$ which contain $\Phi_1, \Phi_2, \chi^0$, and $\chi'^0$ are given by

$$V' = \frac{1}{2} G_1^2 [(\Phi_1^\dagger \Phi_1)^2 - 2(\Phi_1^\dagger \Phi_1)(|\chi^0|^2 - |\chi'^0|^2) + (|\chi^0|^2 - |\chi'^0|^2)^2] + \frac{1}{6} G_3^2 (\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2)^2 - 3(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) - (\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2)(|\chi^0|^2 - |\chi'^0|^2) + (|\chi^0|^2 - |\chi'^0|^2)^2] + f^2 [(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + (\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2)|\chi^0|^2].$$

(6)

Since $\langle \chi^0 \rangle = u$ and $\langle \chi'^0 \rangle = u'$, there are cubic interactions in $V'$ involving $\chi^0$ and $\Phi_{1,2}$ as well as $\chi'^0$ and $\Phi_{1,2}$. These have to be taken into account in obtaining the effective quartic scalar couplings $\lambda_i$ of Eq. (1). However, because $\sqrt{2} R e \chi^0$ and $\sqrt{2} R e \chi'^0$ are not mass eigenstates, we need to consider their $2 \times 2$ mass-squared matrix given by

$$\mathcal{M}^2 = \begin{pmatrix} M^2 \cos^2 \gamma + M'^2 \sin^2 \gamma & -(M^2 + M'^2) \sin \gamma \cos \gamma \\ -(M^2 + M'^2) \sin \gamma \cos \gamma & M^2 \sin^2 \gamma + M'^2 \cos^2 \gamma \end{pmatrix},$$

(7)

where $M^2 = 2(G_1^2 + G_3^2/3)(u^2 + u'^2)$, $\tan \gamma \equiv u'/u$, and $M'$ is the mass of the heavy pseudoscalar boson $\sqrt{2}(\sin \gamma \text{Im} \chi^0 - \cos \gamma \text{Im} \chi'^0)$ which has no cubic coupling to $\Phi_{1,2}$. The determinant of $\mathcal{M}^2$ is equal to $M^2 M'^2 \cos^2 2\gamma$. Hence

$$\lambda_1 = \frac{1}{3} G_3^2 + G_1^2 - \frac{2(u^2 + u'^2)}{M^2 M'^2 \cos^2 2\gamma} [(f^2 - G_1^2 - G_3^2/6)^2 \cos^2 \gamma (\mathcal{M}^2)_{22} - 2(f^2 - G_1^2 - G_3^2/6)(G_1^2 + G_3^2/6) \sin \gamma \cos \gamma (\mathcal{M}^2)_{12}$$

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In the limit $f = 0$, 
\[
\lambda_1 = \lambda_2 = \frac{G_2^2(G_3^2 + 4G_1^2)}{4(G_3^2 + 3G_1^2)}, \quad \lambda_3 = \frac{G_3^2(G_3^2 + 2G_1^2)}{4(G_3^2 + 3G_1^2)}, \quad \lambda_4 = -\frac{1}{2}G_3^2, \quad \lambda_5 = 0. 
\] (12)

Assuming the tree-level relations $g_2 = G_3$ and $g_1^2 = G_1^2 + 3G_3^2$, we then have $G_1^2 = g_1^2g_2^2/(g_2^2 - 3g_1^2)$ and the MSSM conditions, i.e. Eq. (2), are obtained as expected. [4]

Since $f \neq 0$ in the general case, the Higgs potential of this model differs from that of the MSSM even though there are only two Higgs doublets at the electroweak energy scale. It also differs from that of the left-right model mentioned previously. [4] The $f^2$ and $f^4$ terms in $\lambda_{1,2,3}$ depend on $\gamma$ and the $f^4$ terms on $M^2/M^2$ as well. For illustration, let us take the special case $\cos \gamma = 1$, then
\[
\lambda_1 = \frac{1}{4}(g_1^2 + g_2^2) + f^2 \left(1 + \frac{3g_1^2}{g_2^2}\right) - \frac{3f^4}{g_2^2} \left(1 - \frac{3g_1^2}{g_2^2}\right), \quad (13)
\]
\[
\lambda_2 = \frac{1}{4}(g_1^2 + g_2^2) + f^2 \left(1 - \frac{3g_1^2}{g_2^2}\right) - \frac{3f^4}{g_2^2} \left(1 - \frac{3g_1^2}{g_2^2}\right), \quad (14)
\]
\[
\lambda_3 = -\frac{1}{4}g_1^2 + \frac{1}{4}g_2^2 + f^2 - \frac{3f^4}{g_2^2} \left(1 - \frac{3g_1^2}{g_2^2}\right), \quad (15)
\]
\[
\lambda_4 = -\frac{1}{2}g_2^2 + f^2, \quad \lambda_5 = 0. \quad (16)
\]

The requirement that $V$ be bounded from below puts an upper bound on $f^2$ so that
\[
0 \leq f^2 \leq \frac{1}{2}g_2^2. \quad (17)
\]
Let us now specialize further to the case $f = f_{\text{max}}$, we then find

$$
\lambda_1 = 4g_1^2, \quad \lambda_2 = g_1^2, \quad \lambda_3 = 2g_1^2, \quad \lambda_4 = \lambda_5 = 0. \tag{18}
$$

The equality of $\lambda_4$ and $\lambda_5$ means that an accidental custodial SU(2) symmetry exists so that the charged Higgs boson $H^\pm$ and the pseudoscalar Higgs boson $A$ form a triplet with a common mass given by

$$
m_A^2 = \frac{-2\mu_{12}^2}{\sin 2\beta}, \tag{19}
$$

where $\tan \beta \equiv \langle \phi_0^2 \rangle / \langle \phi_0^1 \rangle$. The $2 \times 2$ mass-squared matrix spanning $\sqrt{2} \Re \phi_0^1$ and $\sqrt{2} \Re \phi_0^2$ is now

$$
\mathcal{M}^2 = \begin{pmatrix}
16M_Z^2 \sin^2 \theta_W \cos^2 \beta + m_A^2 \sin^2 \beta & (8M_Z^2 \sin^2 \theta_W - m_A^2) \sin \beta \cos \beta \\
(8M_Z^2 \sin^2 \theta_W - m_A^2) \sin \beta \cos \beta & 4M_Z^2 \sin^2 \theta_W \sin^2 \beta + m_A^2 \cos^2 \beta + \epsilon / \sin^2 \beta
\end{pmatrix}, \tag{20}
$$

where

$$
\epsilon = \frac{3g_2^2 m_t^2}{8\pi^2 M_W^2} \ln \left(1 + \frac{\tilde{m}^2}{m_t^2}\right), \tag{21}
$$

comes from radiative corrections due to the $t$ quark and its two scalar supersymmetric partners of effective mass $\tilde{m}$. This implies

$$
m_h^2 \leq 4M_Z^2 \sin^2 \theta_W (1 + \cos^2 \beta)^2 + \epsilon \tag{22}
$$

as well as

$$
m_h^2 \leq \frac{m_A^2 (1 + \cos^2 \beta)^2 + 4\epsilon \cot^2 \beta}{1 + 3 \cos^2 \beta}, \tag{23}
$$

where $h$ is the lighter of the two mass eigenstates. Hence $m_h$ has an upper bound of $4M_Z \sin \theta_W$ at tree level and it goes up to about 189 GeV after radiative corrections assuming $m_t = 150$ GeV and $\tilde{m} = 1$ TeV.

In conclusion, we have presented in this paper a supersymmetric SU(3) × U(1) model which has a possible reduction to the standard SU(2) × U(1) model with two Higgs doublets at the electroweak energy scale. Because of the existence of cubic invariants in the
superpotential of the larger theory, the reduced Higgs potential is not that of the minimal supersymmetric standard model (MSSM). The quartic scalar couplings are given by Eqs. (8) to (11), instead of Eq. (2). For illustration, we have taken the special case $\cos \gamma = 1$ (i.e. neglecting $\langle \chi'^0 \rangle$) and then $f = g_2/\sqrt{2}$ (i.e. $f = f_{max}$), resulting in the very simple conditions of Eq. (18). We then show that instead of 115 GeV in the MSSM or 120 GeV in a left-right model discussed elsewhere[4], the upper bound of the lightest scalar boson in this model is $4M_Z \sin \theta_W$ and goes up to 189 GeV after radiative corrections assuming $m_t = 150$ GeV and $\tilde{m} = 1$ TeV. In the future, as it becomes possible experimentally to explore the Higgs sector at the electroweak energy scale, it is important to realize that even if supersymmetry exists, the MSSM is not the only possibility for two Higgs doublets.

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References

[1] E. Ma and D. Ng, Univ. of Calif., Riverside Report No. UCRHEP-T103 (1993).

[2] F. Pisano and V. Pleitez, Phys. Rev. D46, 410 (1992).

[3] P. H. Frampton, Phys. Rev. Lett. 69, 2889 (1992).

[4] E. Ma and D. Ng, Univ. of Calif., Riverside Report No. UCRHEP-T107 (1993).

[5] R. Foot, O. F. Hernandez, F. Pisano, and V. Pleitez, Phys. Rev. D47, 4158 (1993).

[6] T. Banks, Nucl. Phys. B303, 172 (1988); E. Ma, Phys. Rev. D39, 1922 (1989).