Particle ratios at SPS, AGS and SIS.

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Abstract

Ratios of integrated particle yields provide the best method for determining the temperature and the chemical potential. The chemical freeze-out parameters obtained at CERN/SPS, BNL/AGS and GSI/SIS energies all correspond to a unique value of 1 GeV per hadron in the local rest frame of the system, independent of the beam energy and of the target and beam particles.

1 Particle Ratios

Ratios of integrated particle yields provide the best way to determine the freeze-out values of the temperature and baryon chemical potential of hadronic matter produced in relativistic heavy ion collisions. Various effects (e.g. flow) which can severely distort the momentum spectra of the particles produced cancel out in such ratios (see e.g. \cite{1}). The analysis of particle ratios is therefore the best method to obtain reliable information on the chemical freeze-out parameters of the hadronic final state. Below we briefly summarize arguments supporting this statement.

1. Excluded volume corrections cancel out in particle ratios. In most models one simply has the following relations (see e.g. \cite{4,3})

\begin{equation}
\frac{N_i}{N_j} = \frac{1 + \sum N_i^0 V_0}{1 + \sum N_j^0 V_0} = \frac{N_i^0 / N_j^0}{(1)}
\end{equation}

where $N_i^0$ refers to the distribution from a fireball at rest which in the Boltzmann approximation is given by

\begin{equation}
N_i^0 = g \int \frac{d^3p}{(2\pi)^3} e^{-E/T}e^\mu/T
= gm_i^2TK_2(m_i/T)e^\mu/T.
\end{equation}

Equation (1) shows that effects due to excluded volume corrections cancel out in particle ratios. As a word of caution: there exist models where these corrections do not cancel exactly as above, see e.g. \cite{4}.
2. If the hadronic gas is made up of a superposition of fireballs having different rapidities but the same temperature and chemical potential then the particle ratios are as if there were only one fireball:

\[
\frac{N_i}{N_j} = \frac{\int_{-\infty}^{\infty} dy \int_{-Y_{FB}}^{Y_{FB}} \rho(Y_{FB}) \frac{dN_i^0}{dy} (y - Y_{FB})}{\int_{-\infty}^{\infty} dy \int_{-Y_{FB}}^{Y_{FB}} \rho(Y_{FB}) \frac{dN_j^0}{dy} (y - Y_{FB})},
\]

\[
= \frac{N_i^0}{N_j^0} \frac{\int_{-Y_{FB}}^{Y_{FB}} \rho(Y_{FB})}{\int_{-Y_{FB}}^{Y_{FB}} \rho(Y_{FB})},
\]

\[
= \frac{N_i^0}{N_j^0}.
\]  

Hence effects due to a superposition of similar fireballs cancel out. It is of course a severe limitation that the fireballs must all have the same temperature, this has been lifted partially by Becattini recently [5].

3. Transverse flow (or random walk [6]) effects with instantaneous freeze-out cancel out if one considers particle yields that have been integrated over transverse momentum. The discussion about transverse flow needs the following two integrals:

\[
\int_{0}^{\infty} dp_{Tm} T m_i K_1 \left( \frac{m_i}{T} \cosh y_T \right) I_0 \left( \frac{p_T}{T} \sinh y_T \right) = m_i^2 K_2 \left( \frac{m_i}{T} \right) \cosh y_T,
\]

and

\[
\int_{0}^{\infty} dp_{Tm} T K_0 \left( \frac{m_i}{T} \cosh y_T \right) I_1 \left( \frac{p_T}{T} \sinh y_T \right) = m_i^2 K_2 \left( \frac{m_i}{T} \right) \sinh y_T.
\]  

For instantaneous freeze-out and constant transverse velocity, one has

\[
\frac{N_i}{N_j} = \frac{\int_{0}^{\infty} dp_{Tm} m_i^2 K_1 \left( \frac{m_i}{T} \cosh y_T \right) I_0 \left( \frac{p_T}{T} \sinh y_T \right)}{\int_{0}^{\infty} dp_{Tm} m_j^2 K_1 \left( \frac{m_j}{T} \cosh y_T \right) I_0 \left( \frac{p_T}{T} \sinh y_T \right)},
\]

\[
= \frac{m_i^2}{m_j^2} K_2 \left( \frac{m_i}{T} \right) \cosh y_T
\]

\[
= \frac{N_i^0}{N_j^0}.
\]  

i.e. effects due to transverse flow cancel out if the particle yields have been integrated over all transverse momenta.

4. For a longitudinal expansion à la Bjorken accompanied by a transverse expansion, the effects also cancel if one considers particle ratios integrated over transverse momenta as shown below. The distribution is, in this case, given by

\[
\left( \frac{dN_i}{dymTdmT} \right)_{y=0} = \frac{g}{\pi} \int_{\sigma} r dr \tau_F(r) \left\{ m_i^2 I_0 \left( \frac{p_T \sinh y_T}{T} \right) K_1 \left( \frac{m_i^2 \cosh y_T}{T} \right) - \left( \frac{\partial \tau_F}{\partial r} \right) p_T I_1 \left( \frac{p_T \sinh y_T}{T} \right) K_0 \left( \frac{m_i^2 \cosh y_T}{T} \right) \right\}
\]

\[
(7)
\]
where $\tau_F(r)$ is the proper freeze-out time which is a function of the radial distance, $r$. After integration over $m_T$ one is left with

$$ \left( \frac{dN_i}{dy} \right)_{y=0} = \frac{g}{\pi} \int_\sigma r \, dr \, \tau_F(r) \left\{ \cosh(y_T r) - \left( \frac{\partial \tau_F}{\partial r} \right) \sinh(y_T r) \right\} m_i^2 T K_2 \left( \frac{m_i}{T} \right) $$

(8)

Since the freeze-out parameters are the same for all particles it follows that

$$ \frac{dN_i/dy}{dN_j/dy} = \frac{N_i^0}{N_j^0} $$

(9)

This relation shows that in a model where the momentum distribution shows a plateau in rapidity space and where the flow pattern could be completely change the transverse momentum distribution, the ratio of particles is still as if one would have a single fireball at rest.

2 Results

We show in Fig. 1 the results obtained from an analysis [7] of $Au - Au$ collisions at an energy of 1 A-GeV. As one can see from an inspection of this figure there is a consistency with a freeze-out temperature
of about 50 MeV and a a freeze-out baryon chemical potential of about 850 MeV. A more complete analysis of the GSI/SIS data can be found in [7] and in [8, 10]. A notable exception is the $\eta/\pi^0$ ratio which does not fit the expectations of the thermal model. Combining the results from different accelerators (for a review and references see Sollfrank [4]) it can be seen that a unified description of the hadronic abundances produced in heavy ion collisions at the CERN/SPS, the BNL/AGS and the GSI/SIS accelerators is possible [12]. This description covers a range in beam energies from 200 A·GeV to below 1 A·GeV. As it turns out the same description can also be applied to the hadronic abundances in LEP and in $p - p$ and $\bar{p} - p$ collisions with slightly different treatment of strangeness sector which accounts for strangeness under-saturation. The result can be summarized in a surprisingly simple way: the hadronic composition of the final state is determined by an energy per hadron being approximately 1 GeV per hadron in the rest frame of the produced system. This generalizes an observation made a long time ago by Hagedorn [13] for $p - p$ collisions, namely, as one increases the beam energy, the available energy is used to produce more particles, but not to increase the temperature of the system. This led Hagedorn to the idea of a limiting temperature. For heavy ion collisions one has to take into account not only the temperature but also the finite baryon density of the system, which is described by the baryon chemical potential $\mu_B$. This, as it was first indicated by P. Braun-Munzinger and J. Stachel [4], leads to a freeze-out curve in the $T, \mu_B$ plane. In Fig. 2 the values of the freeze-out parameters are shown, as obtained by various groups (a summary can be found in [11]). The solid line corresponds to 1 GeV per hadron, the dashed line corresponds to 0.94 GeV per hadron. This energy corresponds to the chemical freeze-out stage, namely, before the hadrons decay into the stable hadrons. Such an analysis, relying as much as possible on fully integrated particle multiplicities was carried out for BNL/AGS and for CERN/SPS data. In Fig. 2 the SPS points are indicated by open squares [4, 14, 15] while the AGS points are indicated by open circles [4, 14, 15].

Data using $Ni$ and $Au$ beams at energies between 0.8 and 1.9 A·GeV have become available recently from the GSI/SIS accelerator. These data have attracted considerable interest due to the surprisingly large number of $K^-$ mesons being produced below threshold. A very detailed and extensive discussion of these results in the framework of thermal models has been presented in [8, 9, 7]. The results for the freeze-out parameters for $Ni - Ni$ at 1.9 A·GeV is shown as an open triangle in Fig. 2. The points with the lowest temperature correspond to $Au - Au$ collisions at 0.8 and 1.0 A·GeV and $Ni - Ni$ collisions at 1.0 and 1.8 A·GeV. and are also shown as open triangles.

A similar analysis has been performed in [5] for $e^+e^-$ annihilation into hadrons at LEP. Since no baryons are involved here this corresponds to zero baryon chemical potential, $\mu_B = 0$. An impressive fit has been obtained here since no less than 29 different hadronic abundances can be reproduced. It is our view that such a good agreement cannot simply be a coincidence. This analysis was subsequently extended [5] to $p - p$ and $\bar{p} - p$ reactions at CERN. In this case, one reproduces the Hagedorn temperature obtained many years ago.

In the underlying hadronic gas model all these points can be described by a single curve corresponding to a fixed energy per particle, $\epsilon/n$, which has approximately the value of 1 GeV per particle in the hadronic gas. This value characterizes all the final states produced by beams having 1 A·GeV all the way up to 200 A·GeV. Thus, the only modification one needs to make to the concept of Hagedorn’s limiting temperature it that there exist a "limiting" - freeze-out energy per particle of 1 GeV at which hadrons are formed in a collision.

This observation leads to a considerable unification in the description of the hadronic final states produced in high energy collisions.

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Figure 2: Freeze-out values obtained from hadronic abundances at CERN/SPS, BNL/AGS and GSI/SIS. Also indicated are the points obtained from observed hadronic abundances at LEP and in $p-p$ collisions at CERN. The smooth curves correspond to a fixed energy per hadron in the hadronic gas model (from [12]).

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