Effect of Model, Observer and their Interaction on State and Disturbance Estimation in Artificial Pancreas: an In-Silico Study

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ABSTRACT The state and disturbance estimations are an indispensable part of the state-of-the-art model-based controllers related to the artificial pancreas, supporting the decision-making and self-tuning of the algorithms. They are not just important when state-feedback kind of controller structure is applied, but also play a crucial role in the estimation of, for example, the amount of acting drug (insulin) in blood or meal intake estimation which has determining role in the short and long term effectivity of the given therapy. This information is also important for physicians to support them in knowledge-based decision-making to be sure a given therapy or device works well. This article compares three observers – a linear-parameter-varying (LPV) dual Kalman filter (KF), a LPV joint KF, and a nonlinear sliding mode observer (NSMO) – designed with two individualized models – Hovorka and Identifiable Virtual Patient model (IVP). The article also statistically quantifies the effect of the observer algorithm and model structure on the accuracy of the estimation of plasma insulin, rate of glucose appearance, and glucose. Data for the analysis was generated by the UVa-Padova simulator. Results indicated that, for the rate of glucose appearance and the plasma insulin, the type of model and the observer structure explain less than 10% of the variability in the error, while the inter-patient variability contributes to the error more than 50%. This reveals a limiting factor in the estimation accuracy that might be improved by model parameter adaptation.

INDEX TERMS Kalman filters, sliding mode observers, artificial pancreas, model identification, linear parameter varying model, estimation

I. INTRODUCTION

A major hurdle in artificial pancreas (AP) applications is data scarcity. In practice, the typically utilized information is the Continuous Glucose Monitoring Sensor (CGMS) measurements and demographic data such as body weight. As the physiology and processes of a human organism is a complex system – a network of multitudinous interconnected subsystems – it is cumbersome to describe them with a limited number of measurable variables. Observers are applied to provide information about the inner state variables.
Application for observers ranges from meal detection [1] to meal-annunciation free control [2]–[4] or insulin limitation for hypoglycaemia avoidance [5].

The most well-known of the observers in the field are the different realizations of the Kalman filter (KF) [6]; their performance is evaluated on different terms (general, fault detection, insulin estimation) in [7]–[9], respectively. Despite the prevalence of KFs, other observers have been applied, such as moving-horizon estimators [10], [11], extended-state observers [4], [12]–[14] or sliding mode observers [3], [15]. Among them, sliding mode observers are appealing since they are robust, have a simple structure, and can reconstruct disturbances [3].

A crucial element in the observer design is the model. The medium-complexity Hovorka model [16] has been widely used to design observers for glucose control [2], [17]. However, its large number of parameters could complicate model identification and observer tuning. For this reason, the use of simpler models might be desirable. Extensions of the Bergman’s minimal model [18], such as the Identifiable Virtual Patient Model (IVP) [19] might be an appealing option in this regard, as shown in [3] and [8]. Necessarily, a price to pay for model simplification is a loss of accuracy in modelling certain physiological behaviours. For example, while renal glucose clearance is considered in the Hovorka model, the IVP does not. Nevertheless, to the best of the authors’ knowledge, no study has yet evaluated whether the larger flexibility offered by the Hovorka model in modelling complex behaviours has a notable impact on the observer performance, compared to other simpler models.

This article extends the results of a preliminary work presented in [20]. It aims to fill the two above-mentioned gaps through an in-silico analysis with the UVa-Padova simulator [21]. The contributions of the article are the following:

- to evaluate the effect of the observer structure on the estimation performance by comparing three observers: two KFs and a sliding mode observer, which is simpler to tune.
- to evaluate the impact of model complexity by designing the above observers with two models of different complexity: a model with medium complexity (the Hovorka model) and a simpler one (IVP model).

The parameters of the models have been identified for the adult cohort in the simulator, and a genetic algorithm-based approach has been employed to tune the KFs. Finally, the individual factors (observer type and model structure) were evaluated with statistical methods such as analysis of variance (ANOVA) and multiple comparisons.

The paper is constructed in the following way. First, Section II describes the Methods. The applied models are introduced in Section II-A, theoretical background of the identification in Section II-B and sensitivity analysis in Section II-B2, followed by the explanation of the observers in Section II-C and II-D. Results about identification and the statistical analysis are provided in Section III. Finally, Section IV closes the paper with the conclusions.

II. METHODS

A. MODELS DESCRIPTION

One goal of this research is to determine how the model complexity affects the performance of the observer. To this end, the well-accepted Hovorka model in [16] was compared to IVP presented in [19]. For convenience, a brief description of the models is provided in this section.

1) Identifiable Virtual Patient Model

The IVP model [19] is a compromise between Bergman’s model [18] and the Hovorka model regarding structural complexity and accuracy. Its equations are described below:

\[
\dot{I}_{SC}(t) = -\frac{1}{\tau_1} \cdot I_{SC}(t) + \frac{1}{\tau_1 C_I} \cdot u(t) \tag{1}
\]

\[
\dot{I}_P(t) = -\frac{1}{\tau_2} \cdot I_P(t) + \frac{1}{\tau_2} \cdot I_{SC}(t) \tag{2}
\]

\[
\dot{I}_{EFF}(t) = -p_2 \cdot I_{EFF}(t) + p_2 \cdot S_I \cdot I_P(t) \tag{3}
\]

\[
\dot{G}(t) = -(GEZI + I_{EFF}(t)) \cdot G(t) + EPG + R_A(t) \tag{4}
\]

where \( I_{SC}(t) \) and \( I_P(t) \) represent the subcutaneous and plasma insulin concentrations (mU/L), respectively. \( I_{EFF}(t) \) is the insulin effect (min\(^{-1}\)) and \( G(t) \) is the plasma glucose concentration (mg/dL). The inputs of the model are the subcutaneous insulin infusion \( u(t) \) (mU/min), and the rate of glucose appearance \( R_A(t) \) (mg/dL/min) from the meal absorption. Note that the \( R_A(t) \) is generally unknown due to the complexity of meal absorption dynamics, influencing factors like nutritional composition and alcohol intake. Hence, the \( R_A(t) \) will be estimated in this work. Lastly, \( \tau_1 \) and \( \tau_2 \) (min) refer to the insulin absorption time constants, \( p_2 \) is the kinetic rate for insulin action (min\(^{-1}\)), \( S_I \) is the insulin sensitivity (mL/mU/min), \( EPG \) is the hepatic glucose production, \( GEZI \) is the glucose effectiveness at zero insulin (min\(^{-1}\)) and \( C_I \) denotes the insulin clearance (mL/min).

2) Hovorka model

The Hovorka or Cambridge model is considered a medium complexity model by the AP scientific community. The model consists of 8 dynamic states and a couple of additional algebraic equations if the carbohydrate absorption sub-model is lumped, and replaced with the \( R_A(t) \) term, as in the IVP model previously described. The dynamic equations are given by [16]:

\[
\dot{I}_{SC}(t) = -\frac{1}{\tau_1} \cdot I_{SC}(t) + \frac{1}{\tau_1 C_I} \cdot u(t) \tag{1}
\]

\[
\dot{I}_P(t) = -\frac{1}{\tau_2} \cdot I_P(t) + \frac{1}{\tau_2} \cdot I_{SC}(t) \tag{2}
\]

\[
\dot{I}_{EFF}(t) = -p_2 \cdot I_{EFF}(t) + p_2 \cdot S_I \cdot I_P(t) \tag{3}
\]

\[
\dot{G}(t) = -(GEZI + I_{EFF}(t)) \cdot G(t) + EPG + R_A(t) \tag{4}
\]

where \( I_{SC}(t) \) and \( I_P(t) \) represent the subcutaneous and plasma insulin concentrations (mU/L), respectively. \( I_{EFF}(t) \) is the insulin effect (min\(^{-1}\)) and \( G(t) \) is the plasma glucose concentration (mg/dL). The inputs of the model are the subcutaneous insulin infusion \( u(t) \) (mU/min), and the rate of glucose appearance \( R_A(t) \) (mg/dL/min) from the meal absorption. Note that the \( R_A(t) \) is generally unknown due to the complexity of meal absorption dynamics, influencing factors like nutritional composition and alcohol intake. Hence, the \( R_A(t) \) will be estimated in this work. Lastly, \( \tau_1 \) and \( \tau_2 \) (min) refer to the insulin absorption time constants, \( p_2 \) is the kinetic rate for insulin action (min\(^{-1}\)), \( S_I \) is the insulin sensitivity (mL/mU/min), \( EPG \) is the hepatic glucose production, \( GEZI \) is the glucose effectiveness at zero insulin (min\(^{-1}\)) and \( C_I \) denotes the insulin clearance (mL/min).
where \( Q_1(t) \) (mmol/kg) is the glucose content in the accessible compartment, \( Q_2(t) \) (mmol/kg) is the glucose content in the non-accessible compartment, \( R_A(t) \) (mmol/kg/min) is the rate of glucose appearance, \( I(t) \) (mU/L) is the plasma insulin concentration. \( x_1(t) \) (min\(^{-1}\)) is the remote effect of insulin on glucose distribution, \( x_2(t) \) (min\(^{-1}\)) is the remote effect of insulin on endogenous glucose production in the liver. \( S_1(t) \) (mU/kg) is the insulin in the accessible compartment and \( S_2(t) \) (mU/kg) is the insulin in the non-accessible compartment. Input of the system \( u(t) \) (mU/kg/min) is the insulin infusion. The parameters: \( k_{12} \) (min\(^{-1}\)) is the transfer rate from the non-accessible compartment to the accessible, \( EGP_0 \) (mmol/kg/min) is the endogenous glucose production in case of zero insulin level, \( k_c \) (min\(^{-1}\)) is the insulin clearance. The patient’s insulin sensitivity on transport, disposal and, glucose production is determined by the \( k_{a1,2,3} \) (min\(^{-1}\)) and \( k_{b1,2} \) (L/mU/min/min), \( k_{b3} \) (L/mU/min) parameters. The insulin absorption time constant is denoted by \( \tau_S \) (min) [16].

Supplementary algebraic equations are defined by:

\[
F_{01c}(t) = \begin{cases} 
F_{01}, & \text{if } G(t) \geq 4.5 \text{ mmol/L} \\
F_{01}G(t)/4.5, & \text{otherwise} 
\end{cases}, \tag{13}
\]

\[
F_R(t) = \begin{cases}
0.003 V_G (G(t) - 9), & \text{if } G(t) \geq 9 \text{ mmol/L} \\
0, & \text{otherwise} 
\end{cases}, \tag{14}
\]

\[
G(t) = \frac{Q_1(t)}{V_G}, \tag{15}
\]

where \( F_{01c} \) (mmol/kg/min) is the glucose consumption of the central nervous system, while \( F_{01} \) represents the consumption at ambient glucose concentration, \( F_R \) (mmol/kg/min) is the renal excretion of glucose in the kidneys, \( G(t) \) (mmol/L) is the actual output of the system: blood glucose concentration, \( V_G \) (L/kg) is the glucose distribution volume [16].

The units of the models can be converted in the following way: \( 1 \text{mmol/L} = 18 \text{mg/dL} \) for the blood glucose level, \( 1 \text{mmol/kg/min} = 180/V_G \text{mg/dL/min} \) for the rate of appearance, where \( V_g \) (dL/kg) is the glucose distribution volume. The conversion between the insulin infusion is \( 1 \text{mU/kg/min} = 1000/BW \text{(uU/min)} \), where \( BW \) (kg) is the body weight. Note that the IVP model expresses the variables relative to the volumes, while the Hovorka model, relative to the body weight.

### B. IDENTIFICATION OF THE MODELS

The UVa-Padova simulator was selected to identify the models in Section II-A. The simulator received the approval of the Food and Drug Administration to be used as a substitute for preclinical trials with animals [22]. Also, the simulator model could fit glucose profiles from clinical data [23], and its meal model reconstructed the \( R_A \) with more accuracy than other models in the literature [24]. Finally, it includes a realistic cohort of virtual subjects [25].

We identified two parameter sets: a population value parameter set (or average model) and individualized parameter set. The population value parameter set was determined from the average subject in the simulator. The individualized parameter sets were obtained by identifying two of the most sensitive parameters for each of the 10 individuals in the adult cohort (of the academic version of the simulator), while the remaining parameters were set to their corresponding values in the average model. The identification procedure followed the pathway of [26]: checking the structural identifiability, ranking the sensitive parameters, and identifying the parameters.

1) Identifiability analysis

Structural identifiability defines the possibility of determining under ideal conditions (e.g. absence of noise) a unique value (structurally globally identifiable) or a finite set of values (structurally locally identifiable) for model unknown parameters, with the only information of the inputs and outputs [27]. To analyse the structural identifiability, the generating series (GS) approach was used since it is a trade-off method between computational cost and provided information regarding other techniques in the literature [27]. The analysis was performed with the GenSSI 2.0 software described in [28].

The inputs considered in the identifiability analysis were the insulin infusion and \( R_A \). Since the \( R_A \) was available for the identification, parameters related to the carbohydrate absorption dynamics – the glucose distribution volume in the IVP model [19] and the parameters \( D_G \) (amount of carbohydrates digested), \( A_G \) (carbohydrate bioavailability), \( t_{max} \) (maximum time for the rate of glucose appearance) in the Hovorka model [16] – were excluded from the analysis. In addition, the weight \( BW \) in the Hovorka model is known, thus it was neither considered in the identification.

2) Sensitivity analysis

A global sensitivity analysis was performed with a twofold purpose: 1) to reduce the number of parameters to be identified in the Hovorka average model, and 2) to select the two...
parameters to be individualized in the personalized IVP and Hovorka models. The same parameters included in the identifiability analysis were considered in the sensitivity analysis.

For the sensitivity analysis, the AMIGO2 Matlab toolbox was utilized [29]. The sensitivity is calculated by changing the parameters which result in various trajectories. Different measures can be applied to these trajectories, the most common ones are the root-mean-square deviation and the mean absolute deviation. In order to get a more robust result, several steps are taken. The simulations of the trajectories are repeated with different initial conditions or input schemes. The sensitivity values are normalized at each time instance for all the parameters. The toolbox utilized the Latin Hypercube Sampling (LHS). The LHS method provides an even distribution of all the parameters in the given range, with a relatively low amount of necessary samples. The parameter bounds were calculated by applying to the nominal values in [30] and [19] a deviation of ±σ, where σ denotes the standard deviation provided in the referred articles, but saturating to 0 if negative values appeared. The simulation of a given parameter set is only considered in the calculation of the sensitivity if the BG remained in the following range: 1.8 < BG < 25 mmol/L in order to avoid entirely unrealistic trajectories.

If a general nonlinear system is defined in the form of $0 = f(\dot{x}, x, p, u, t)$, where $p$ are the parameters, then the sensitivities $s_p = \frac{\partial f}{\partial p}$ can be given with the Jacobians:

$$\frac{\partial s_p}{\partial p} = \frac{\partial f}{\partial x} s_p + \frac{\partial f}{\partial u} s_u + \frac{\partial f}{\partial t} s_t |_{t=0} = 0.$$  

The toolbox normalizes with the value of the parameter and with the value of the output in the given discrete step:

$$S_{p,k} = \frac{p_i}{y(k)} s_{p,k}$$  

where $S_{p,k}$ is the normalized or relative sensitivity of the parameter $p$ in the $k$-th discrete step, $y(k)$ is the output and $p_i$ is the $i$-th sample from the LHS. The ranking of the parameters are based on the final measure defined by:

$$\delta_p^{msqr} = \frac{1}{n_e n_{ths} n_k} \sqrt{\sum_{i=1}^{n_e} \sum_{t=1}^{n_{ths}} \sum_{k=1}^{n_k} (S_{p,k})^2}$$  

where $n_e$ is the number of different setups for experiments (initial conditions or input schemes), $n_{ths}$ is the number of sampled values of the parameters, $n_k$ is the number of discrete points at which the system is evaluated.

3) Identification

On the one hand, in the identification of the average models, all the sensitive and mildly sensitive parameters are considered, as suggested in [26]. The insensitive parameters were set to the nominal parameters in [16] for the Hovorka model, and the means of the parameters listed in [19] were considered for the IVP model. On the other hand, two parameters in the sensitive group were individualized for the personalized models. The remaining parameters were fixed to the corresponding values in the average model. To identify the Hovorka model, the parametrization of the model in terms of insulin sensitivities ($S_{i1} = k_{i1}/k_{a1}$, $S_{i2} = k_{i2}/k_{a2}$ and $S_{i3} = k_{i3}/k_{a3}$) was utilized as in [16].

The identification consisted of two optimization problems: one that identifies insulin-related states by minimizing the normalized root-mean-square error (NRMSE) of the plasma insulin; and another one that identifies the insulin effect and glucose-related parameters by minimizing the NRMSE of the glucose measurement. The NRMSE is defined by

$$NRMSE(x) = \sqrt{\frac{\sum_{i=1}^{N} (x_{model} - x_{UVa})^2}{N}} \cdot \frac{1}{x_{UVa} - x_{min}}$$  

where $x_{model}$ represents the glucose or plasma insulin obtained by the IVP or Hovorka models. $x_{UVa}$ denotes the “real” measurements, whereas the superscripts $max$ and $min$ refers to the maximum and minimum values of these measurements, respectively. This two-step optimization procedure targets a better identification of the insulin pharmacokinetics, as discussed in [19]. The inputs to the optimization problem were the glucose, the plasma insulin, the insulin infusion, and the meal disturbance. We generated these inputs from a 3-meal simulation of the average adult patient (for average models) or the adult cohort (for personalized models). Of note, the full knowledge of these inputs is impractical for real applications, but they were used in this study to ensure accurate identification of the parameters.

The genetic algorithm (GA) in the Global Optimization Toolbox of Matlab 2018b (ga function) was utilized to solve the above-described optimization problem. The algorithm was configured with its default settings [31].

Finally, the identification of the models was assessed in terms of the NRMSE using, as in the identification, a 3-meal scenario, but with different instances of variability.

C. KALMAN FILTERS

Besides the estimation of the state variables, our goal was to estimate the $R_A$ as well. To do so in the case of KFs, the engineer has two main options. The most straightforward and largely applied technique is the augmentation of the state vector with the parameter or disturbance term, giving rise to the joint KF (JKF) [32]. Another possibility is to use separate KF for the state and parameter/disturbance estimation problem; this observer is called the dual KF (DKF) [32]. Both observers hypothesize that the parameters are static: in the prediction phase, the parameter in the next step holds the value of the previous step. For the application of the observers, the models are either discretized by the Euler or Complete method. In addition, the benefits of linear parameter varying (LPV) formulation were exploited in this article to handle non-linearities in KFs design. Consequently, more complex KF-based algorithms (such as Extended KF or Unscented KF) were avoided [7].


1) Quasi LPV representations

LPV modeling technique is an approach to handle the nonlinearities of the given system. If one the parameters is not a free signal such as an inner state variable, then it is called quasi-LPV or qLPV. The continuous time qLPV state-space representation (qLPV-SS) is defined as follows [33]:

\[ x(t) = A(p(t))x(t) + B(p(t))u(t), \]
\[ y(t) = C(p(t))x(t) + D(p(t))u(t), \]

where the \( p(t) = [p_1(t) \ldots p_R(t)] \) parameter vector consists of the so-called scheduling parameters \( p_i(t), \ p_i(t) \in \Omega^R \subset \mathbb{R}^R \) is an \( R \)-dimensional real vector within the set \( \Omega = [p_{1,\min}, p_{1,\max}] \times [p_{2,\min}, p_{2,\max}] \times \ldots \times [p_{R,\min}, p_{R,\max}] \).

Either the IVP or the Hovorka model accept a qLPV-SS representation:

- **IVP model.** Selecting \( p(t) = G(t) \) as scheduling parameter leads to the following qLPV representation:

\[
A(p(t)) = \begin{bmatrix} -GEZI + \frac{EGP}{p(t)} & -p(t) & 0 & 0 & 0 \\ 0 & -p_2 & p_2S_1 & 0 & 0 \\ 0 & 0 & -\frac{1}{\tau_1} & \frac{1}{\tau_1} & 0 \\ 0 & 0 & 0 & -\frac{1}{\tau_2} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \end{bmatrix},
\]

\[
B = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{1}{\tau_G} \end{bmatrix}^T, \]

\[
C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ \end{bmatrix},
\]

\[
D = [0],
\]

where the additional 5th state variable is the estimated disturbance, namely the \( R_A \):

- **Hovorka model.** Taking

\[
p(t) = \frac{[EGP_0 - F_{011}(t) - F_R(t)]}{Q_1(t)}, \quad Q_1(t), Q_2(t)
\]

the qLPV representation is as follows:

\[
A(p(t)) = \begin{bmatrix} p_{11}(t) k_{12} & 0 & -p_2(t) & 0 & -EGP_0 & 0 & 0 & 0 & 0 \\ 0 & -k_{12} & 0 & p_{11}(t) & -p_2(t) & 0 & 0 & 0 & 0 \\ 0 & 0 & -k_{21} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & k_{31} & -k_{21} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & k_{32} & -k_{22} & 0 & -\frac{1}{\tau_1} & \frac{1}{\tau_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{\tau_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \end{bmatrix},
\]

\[
B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}^T, \]

\[
C = \begin{bmatrix} \frac{1}{\tau_G} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},
\]

\[
D = [0],
\]

Since the selected parameters are not directly measurable, they have to be estimated by the observer and can introduce additional inaccuracies compared to an LPV model where the parameters are measurable. The additional 9th state variable is the estimated disturbance, namely the \( R_A(t) \).

2) Applied Kalman filters

Utilizing the benefits of the discretized LPV representation, a linear discrete Kalman filter can be applied to the nonlinear system [34], [35]. In the case of the JKF, the dimension of the covariance matrix is extended by the number of parameters. In this case, because of the supposition of the parameters, the exact discretization method cannot be applied. This is due to the arising singularity issue, thus explicit Euler method is applied:

\[
A[p[k]] = I + T_s A(p(kT_s)),
\]

\[
B[p[k]] = T_s B(p(kT_s)),
\]

\[
C[p[k]] = C(p(kT_s)),
\]

\[
D[p[k]] = D(p(kT_s)),
\]

where the parentheses indicate the continuous LPV representations of state-space matrices, while the brackets the discretized ones of the corresponding model. Sample time is denoted by \( T_s \), while the discrete step by \( k \).

On the contrary, the DKF can be implemented using the exact discretization given by:

\[
A[p[k]] = e^{A(p(kT_s))T_s},
\]

\[
B[p[k]] = A^{-1}(p(kT_s))(e^{A(p(kT_s))T_s} - I) \cdot B(p(kT_s)),
\]

\[
C[p[k]] = C(p(kT_s)),
\]

\[
D[p[k]] = D(p(kT_s)),
\]

As a result of the LPV discretization, the discrete propagation can be expressed in the following state-space form:

\[
x^-[k] = A[p[k]]x[k - 1] + B[p[k]]u[k - 1],
\]

\[
P^-[k] = A[p[k]]P[k - 1]A^T[p[k]] + Q,
\]

\[
P_p^{-}[k] = P_p[k - 1]R^{-1},
\]

where \( u[k - 1] \) is the integral of the continuous input from \( (k-1)T_s \) to \( kT_s \). The \( x^- \) and \( P^- \) are the predicted state variables and error covariance, respectively, while \( \hat{x} \) and \( P \) are the posteriori state estimate and posteriori error covariance. The Q process noise covariance matrix is the tuning parameter and the \( \Lambda \) is the forgetting factor.

In the update phase, the predicted values are corrected with the new measurements:

\[
K[k] = P^-^{-}[k]C^T \cdot [p[k]](C[p[k]]P^-^{-}[k]C^T[p[k]] + R)^{-1},
\]

\[
\hat{x}[k] = \hat{x}^-[k] + K[k](y[k] - C[p[k]]\hat{x}^-[k]),
\]

\[
P[k] = (I - K[k]C[p[k]])P^-^{-}[k].
\]

The main differences between the filters are summarized in Table 1, where \( n \) is the number of state variables and \( p \) is the number of parameters.
3) Kalman filter tunings
The study included 4 different KFs. The filters can be categorized by type and utilized model. Henceforth, a specific filter is identified by a consequent notation: the model name (IVP or Hovorka), followed by the abbreviation of the filter type (DKF or JKF). During the tuning process, the virtual patient and the observer utilized the same model, either the IVP or the Hovorka; dissimilarity between them was caused by a fixed +30% variability in all the model parameters. This amount of variability is in physiologically relevant ranges according to [19], [30]. The Q covariance matrix had non-zero elements only in the main diagonal.

To lessen the human factor, and make the comparison between observers fairer, we tuned the Q matrix with Matlab GA. The optimization is done on a 24-hour-long BG trajectory of a virtual patient with parameter variability. The optimization is driven by a cost function where only the transients were considered, as we observed in previous simulations that the filters cannot compensate for the steady-state offset. If the whole trajectory would be taken into account, the optimizer would adjust the transient to reduce the cost function; however, it can result in distorted, unfavourable responses, as shown in Fig. 1. The result of a tuning considering only the transient is denoted by “A”, while a tuning taking into account the steady-state also is denoted by “B”. Although “B” yields a lower NRMSE compared to “A”, the latter one is preferred, since it provides a more realistic waveform.

The elements lying in the main diagonal of the Q covariance matrix were the tuning parameters, also known as genes of an individual. The lower bounds of the parameters were set to zero. The upper bounds were set to a value at least as large, as if this value would be the only non-zero element in the Q matrix, the filter would still converge to the measurements. The numerical results are shown in Table 4.

Since this tuning method utilizes all the state variables, it can be applied only in-silico, due to the non-measurable variables in clinical practice. This was a reasonable compromise to make since our main goal was to provide uniform settings for each virtual patient model: $J_{IVP} = k(e_G + e_{I_P} + e_{I_{eff}} + e_{R_A})$ (44) $J_{Hovorka} = k\frac{9}{5}(e_G + e_{I_P} + e_{R_A}) + \frac{3}{5}(e_Q + e_{I_{eff}} + e_{R_A}) + e_{x_1} + e_{x_2} + e_{x_3} + e_{S_1} + e_{S_2})$ (45)

where $e$ denotes the NRMSE of the given state variable (see the corresponding notations in [16] and [19] or in Section II-A) and $k$ is the penalizing coefficient.
D. Sliding Mode Observer

In this research, the performance of two KFs (see Section II-C) were compared to an NSMO, which has a simpler structure and tuning. The NSMO has several appealing features such as finite-time convergence of the measurable states despite matching disturbances or the ability to reconstruct disturbances without assuming that they are constant [36]. These features have motivated the design of sliding mode observers for a wide range of applications in the literature, including anti-disturbance control [37], [38], and fault detection and isolation [39], [40].

The NSMO used in this work was proposed by [36, Section 3.5], which extends the results of [41] or [42] for non-linear models. Shrestha et al. [36] considers a nonlinear system given in the unmeasurable-measurable form:

\[
\begin{aligned}
\dot{x}_1(t) &= A_{11} x_1(t) + A_{12} x_2(t) + \phi_1(x, u, t) \\
\dot{x}_2(t) &= A_{21} x_1(t) + A_{22} x_2(t) + \phi_2(x, u, t) + D_d \dot{d}(t), \\
y(t) &= C_2 x_2(t)
\end{aligned}
\]  

(46)

where \( y(t) \in \mathbb{R}^{ny} \) is the measurable output and \( x(t) := \text{col}(x_1(t), x_2(t)) \in \mathbb{R}^{n_x} \) denotes the system state, where \( x_2(t) \) represents the last \( ny \) elements of \( x(t) \). Plus, \( u(t) \in \mathbb{R}^{nu} \) is the known input; whereas, \( d(t) \in \mathbb{R}^{nu} \) is the unknown input appearing only in the measurable states \( x_2(t) \). Finally, \( \phi_1(x, u, t) \) and \( \phi_2(x, u, t) \) denote the nonlinear terms; and \( A_{11}, A_{12}, A_{22}, \text{and} \ C_2 \) refer to the system matrices with the proper dimension. Notice that the IVP model and the Hovorka model accept the form of (46) as shown below:

- **IVP model.** Taking \( d(t) := R_A(t), y(t) := G(t) \) and \( x(t) := [I_{SC}(t), I_F(t), I_{EFF}(t), G(t)]^T \), the equations (2–4) can be converted into (46) with the following matrices:

\[
A_{11} := \begin{bmatrix}
\frac{-1}{\tau_1} & 0 & 0 \\
\frac{1}{\tau_2} & -\frac{1}{\tau_2} & 0 \\
0 & p_2 S_i & -p_2
\end{bmatrix},
A_{12} = A_{21} := 0_{3x1}, A_{22} := -GEZI,
\]

\[
\phi_1(t) := \begin{bmatrix}
\frac{u(t)}{V_i} \\
0_{nx1}
\end{bmatrix}, \phi_2(t) := EGP - G(t) I_{EFF}(t),
\]

\[
C_2 = D_2 = 1
\]

- **Hovorka model.** For \( x(t) := [S_1(t), S_2(t), x_1(t), x_2(t), x_3(t), Q_2(t), Q_1(t)]^T, d(t) := R_A(t) \) and \( y(t) := G(t) \), the equations (5–12) can be transformed into (46) with:

\[
A_{11} = \begin{bmatrix}
\frac{-1}{\tau_1} & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{\tau_2} & -\frac{1}{\tau_2} & -k_c & 0 & 0 & 0 & 0 \\
0 & \frac{\tau V_i}{r} & -k_e & 0 & 0 & 0 & 0 \\
0 & 0 & k_b_1 & -k_s_1 & 0 & 0 & 0 \\
0 & 0 & k_b_2 & -k_s_2 & 0 & 0 & 0 \\
0 & 0 & k_b_3 & 0 & -k_s_3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -k_{12}
\end{bmatrix},
\]

\[
A_{21} := \begin{bmatrix} 0_{1x5} & -EGP_0 & k_{12} \end{bmatrix}, A_{12} := 0_{7x1}, A_{22} := 0
\]

\[
\phi_1(t) := \begin{bmatrix} u(t) \\
0_{5x1}
\end{bmatrix}, C_2 := 1/V_G, D_2 := 1
\]

\[
\phi_2(t) := -F_{01}(t) - x_1(t) Q_1(t) - F_R(t) + EGP_0
\]

The equations for the NSMO read as [3], [36]:

\[
\begin{aligned}
\dot{x}_1(t) &= A_{11} x_1(t) + A_{12} x_2(t) + \phi_1(x, u, t) \\
\dot{x}_2(t) &= A_{21} x_1(t) + A_{22} x_2(t) + \phi_2(x, u, t) + D_d \dot{d}(t) \\
\phi_2(t) &= -F_{01}(t) - x_1(t) Q_1(t) - F_R(t) + EGP_0
\end{aligned}
\]

(47)

with \( \nu(t) \) the discontinuous term defined by

\[
\nu(t) := \text{sign}(y(t) - \hat{y}(t))
\]

(48)

If the positive gain \( \kappa \) in (48) is chosen sufficiently larger than \( \dot{d}(t) \), then the output error \((e_y(t) := y(t) - \hat{y}(t))\) will converge in finite time to 0 [36, Chapter 3]. In this work, the maximum value of the disturbance \( d(t) \) was set to \( 50 \text{mg}/\text{dl}/\text{min} \) after exhaustive simulations under the same conditions as in the KFs tuning. Moreover, the estimation error of the unmeasurable states \((e_1(t) := x_1(t) - \hat{x}_1(t))\) will converge to 0 asymptotically. However, no gains applied to the unmeasurable states, which might be affected by modelling errors.

In addition, an estimation of \( d(t) \) can be realized by using the discontinuous term [36, Chapter 3]. Once the sliding motion is attained, i.e. \( e_y(t) = \hat{e}_y(t) = 0 \), the output error dynamics is described by

\[
0 = A_{21} e_1(t) + \phi_2(x, u) - \phi_2(\hat{x}, u) + D_2 d(t) - v_{eq}(t)
\]

(49)

where \( v_{eq} \) is the so-called equivalent output error injection [36, Chapter 3]. Because of the asymptotic convergence of \( e_1(t) \), \( v_{eq} \) asymptotically tends to \( D_2 d(t) \). As a result, an estimation of \( d(t) \) is given by:

\[
\hat{d}(t) = D_2^* \cdot v_{eq}(t)
\]

(50)

where \( D_2^* = (D_2 D_2)^{-1} D_2 \).

One of the main drawbacks of the NSMO is the chattering, that is, a zigzag behaviour of the estimated states variables that occurs when the continuous observer in (47–48) is discretized with explicit methods [43]. To reduce the chattering, Acary et al. [43] presented an implicit Euler discretization technique for control purposes. In [3], that technique was applied to sliding mode observers. However, it required the estimation of the disturbance in the discretization, which reduces the performance. To overcome this problem, the approach of [44] was applied in this article.

The discretized equations of the NSMO, using the implicit Euler method, are given by
leads to

\[ \hat{x}_1[k] = (I_{nx-n_y} + T_s \cdot A_{11})\hat{x}_1[k-1] + 
+ T_s \cdot A_{12} \hat{x}_2[k-1] + 
+ T_s \cdot \phi_1(\hat{x}[k-1], u[k-1]) \]

\[ \hat{x}_2[k] = T_s \cdot A_{21} \hat{x}_1[k-1] + 
+ (I_{n_y} + T_s \cdot A_{22})\hat{x}_2[k-1] + 
+ T_s \cdot \phi_2(x[k-1], u[k-1]) + 
+ T_s \cdot D_2 \cdot C_2^{-1} \cdot \cdot v[k] \] (51)

where \( v[k] \in \text{sign}(e_y[k]) \) and \( \text{sign}(\cdot) \) is the multivalued sign function used in [43]. Although the term \( v[k] = \text{sign}(e_y[k]) \) appears on the right hand of (51), the system is causal. To get an explicit expression of \( v[k] \), Kikuwe et al. [44] used \( \hat{y}[k] = C_2^{-1}(y[k] - e_y[k]) \) to express the output error \( e_y[k] \) as \( e_y[k] = e^*[k-1] - \beta[k-1]v[k] \). As an example, consider the implicit discretization equations of the observer (51) for the IVF model in (2 - 4) given by:

\[ I_{SC}[k] - I_{SC}[k-1] \]

\[ = \frac{I_D[k]}{\tau_1 C_1} - \frac{I_{SC}[k]}{\tau_1} \] (52)

\[ I_P[k] - I_P[k-1] \]

\[ = \frac{I_P[k]}{\tau_2} + \frac{I_{SC}[k]}{\tau_2} \] (53)

\[ I_{EFF}[k] - I_{EFF}[k-1] \]

\[ = -p_2 \cdot I_{EFF}[k] + \]

\[ + S_I p_2 I_P[k] \] (54)

\[ G[k] - G[k-1] \]

\[ = EGP + \kappa C_2^{-1} v[k] - 
- (GEZI + I_{EFF}[k]) G[k] \] (55)

By solving the above system of equations together with the expression \( \hat{y}[k] = C_2^{-1}(y[k] - e_y[k]) \), one can obtain \( e_y[k] = e^*[k-1] - \beta[k-1]v[k] \). Once \( e^*[k-1] \) and \( \beta[k-1] \) are calculated – they are not given for lack of space – the observer injection term becomes

\[ v[k] \in \text{sign}(e^*[k-1] - \beta[k-1]v[k]) \] (56)

To obtain a causal expression for \( v[k] \) two properties of convex sets were applied [43]:

- The inverse function of \( b \in \text{sign}(a) \) is determined by

\[ b \in \text{sign}(a) \iff a \in N_{[-1,1]}(b) \] (57)

with \( N_{[-1,1]} \) the normal cone onto the convex set \([-1,1]\) at point \( b \).

- Given a positive scalar \( M \in \mathbb{R} \) and a closed convex non-empty set, \( C \subseteq \mathbb{R} \) it follows that

\[ M(b - a) \in N_C(b) \iff b = \text{proj}(C; a) \] (58)

where \( \text{proj}(C;a) \) is the Euclidean projection of the point \( a \) into \( C \).

Finally, the application of the above properties to (56) leads to

\[ v[k] = \text{proj}([-1,1]; \beta[k]^{-1}e^*[k]) \] (59)

Note that, unlike in [3], the above explicit expression for \( v[k] \) does not depend on the disturbance since neither \( \beta[k] \) nor \( e^*[k] \) do it.

E. IN-SILICO COMPARISON

1) General description

The purpose of the comparison is to study the impact on the estimation performance of the following factors:

- “Model”: the structure of the model was considered by comparing the IVP model (Section II-A1) with the Hovorka model (Section II-A2).
- “Observer”: the observers were assessed with the comparison of the NSM (Section II-D) and the KFs (Section II-C3).

We performed 6 simulations – one for each combination of models and observers – with the academic version of the UVA-Padova simulator [21], which was extended with added features for intra-patient variability generation. The scenario consisted of a 24-h simulation including three meals (45 g at 7 h, 70 g at 14 h, and 60 g at 21 h) and multiple sources of variability, namely, CGMS error according to the default model in the simulator; sinusoidal-based circadian insulin variation with uniformly-distributed amplitude of ±30%; and, variability of subcutaneous insulin absorption according to a uniform distribution of ±30%. For each of the 10 virtual adults, we repeated the simulation three times with different variability instances.

Remark that when comparing different types of observers, tuning may bias the results. For the fairness of the comparison, as indicated in Section II-C3 and Section II-D, we tuned the three observers for them to perform similarly under the same tuning scenario. To this end, we set the Q matrix of the KFs by optimization, and the disturbance bound of the NSM by exhaustive simulations.

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Note that, unlike in [3], the above explicit expression for \( v[k] \) does not depend on the disturbance since neither \( \beta[k] \) nor \( e^*[k] \) do it.
was applied to control the false discovery rate in the case of multiple comparisons.

The ANOVA and sign test only inform whether the observed differences among the levels of a factor are non-random (statistically significant difference) or they originated from the randomness of the simulations. However, these tests do not inform about the magnitude of these differences or their practical relevance. To analyse this information, we calculate two metrics: the eta-squared ($\eta^2$) – which measures the proportion of variance associated with each factor [48] – and the 95% confidence interval around the median difference (with the correction in [49] to control the false coverage rate).

### III. RESULTS AND DISCUSSION

#### A. MODELS IDENTIFICATION

1) Virtual Patient Model

The analysis of the structural identifiability of the IVP model determines that all the parameters are structurally locally identifiable when the plasma insulin is measured, which is coherent with the results in [19]. The sensitive parameters are $S_I$, $EGP$, $C_I$, and $GEZI$; whereas, parameters $\tau_1$, and $p_2$ are mildly sensitive. The only insensitive parameter is $\tau_2$; its nominal value was used instead of being identified.

Among the sensitive parameters, we individualize $S_I$ and $C_I$ because they are related to the steady state conditions of all the state variables. Table 2 shows the identified parameters for the 10 virtual adults in the UVa-Padova simulator.

### TABLE 2. Virtual Patient model parameters. The parameter $\tau_2$ was obtained as the mean of the values included in [19].

| Parameter | Average patient | Individual |
|-----------|-----------------|------------|
| $S_I$ (mL/µU/min) | 7.03 $\cdot 10^{-4}$ | - |
| $C_I$ (mL/min) | 1.11 $\cdot 10^3$ | - |
| $GEZI$ (min$^{-1}$) | 3.03 $\cdot 10^{-6}$ | - |
| $\tau_1$ (min) | 52.71 | - |
| $\tau_2$ (min) | 45.41 | - |
| $p_2$ (min$^{-1}$) | 2.85 $\cdot 10^{-4}$ | - |

#### 2) Hovorka model

Likewise the IVP model, the Hovorka model parameters are structurally locally identifiable if the plasma insulin is known. The sensitive parameters of this model are $V_i$, $k_e$, $S_{i1}$, $k_{12}$, $S_{i2}$, $F_01$, $S_{i3}$; the mildly sensitive are $EGP$, $\tau_8$ and $V_g$ and finally, the insensitive parameters are the remaining ones.

As stated in Section II-B, the sensitive and mildly sensitive parameters are included in the identification of the average model, while the insensitive ones are fixed to the nominal values in [16]. Parameters $V_i$ and $S_{i1}$ are selected to be individualized, since they are the most sensitive gains of the insulin subsystem and glucose subsystem, respectively. Table 3 includes the identified parameters.

### TABLE 3. Hovorka’s model parameters

| Parameter | Average patient | Individual |
|-----------|-----------------|------------|
| $\tau_8$ (min) | 43.00 | - |
| $k_e$ (min$^{-1}$) | 1.36 $\cdot 10^{-1}$ | - |
| $V_i$ (L/kg) | 1.18 $\cdot 10^{-1}$ | - |
| $k_{12}$ (min$^{-1}$) | 2.22 $\cdot 10^{-2}$ | - |
| $V_g$ (L/kg) | 9.11 $\cdot 10^{-2}$ | - |
| $S_{i1}$ (min$^{-1}$ per mLUL$^{-1}$) | 2.13 $\cdot 10^{-3}$ | - |
| $S_{i2}$ (min$^{-1}$ per mLUL$^{-1}$) | 1.92 $\cdot 10^{-3}$ | - |
| $S_{i3}$ (L/mU) | 3.92 $\cdot 10^{-3}$ | - |
| $k_{a1}$ (min$^{-1}$) | 2.35 $\cdot 10^{-3}$ | - |
| $F_01$ (mmol·kg$^{-1}$ min$^{-1}$) | 1.28 $\cdot 10^{-4}$ | - |
| $EGP$ (mmol·kg$^{-1}$ min$^{-1}$) | 2.37 $\cdot 10^{-2}$ | - |

![FIGURE 2. Evaluation of the NRMSE for the identified IVP and Hovorka models](image-url)
3) Identification assessment

The identified parameters of the average models (Tables 2 and 3) accurately fit the glucose; they achieved an NRMSE of 9.87% for the IVP model and 6.32% for the Hovorka model.

Both models achieve a low NRMSE for the plasma insulin. However, the IVP model fits the plasma insulin better (3.69% vs. 6.38%). The main difference between the models is that the Hovorka model has a slower response due to a higher order in the insulin pharmacokinetics model.

Moreover, fit degrades when the average models are used for the different adult subjects in the cohort (see Fig. 2), specially for the BG fit provided by the Hovorka model. The larger degradation of the fit in the Hovorka model might be related to an over-parametrization of the insulin effect sub-system: whereas the simulator model uses two compartments to describe the insulin effect [21], the Hovorka model adds an additional compartment to describe the insulin effect on the glucose transport, i.e., the state variable $x_1(t)$. Also, it turns out that $S_{11}$ is a sensitive parameter which was chosen to individualize the model. This denotes a structural deficiency of the Hovorka model concerning the Dalla Man model. This over-parametrization could cause some overfitting problems, which make the model sensitive to different patients.

Model individualization improves the fit and reduces the difference in terms of NRMSE between both models. No statistically significant evidence exists to consider that the median difference in the NRMSE of the glucose between both models is different from 0, according to the sign test, at a confidence level of 0.05. Model individualization also reduces the variability of the plasma insulin, but the IVP model improves the accuracy of the Hovorka model.

Since the individualized models fit the plasma insulin and the plasma glucose better than the average model, we used the individualized models to design the observers.

### TABLE 4. KF tuning table. The notations indicate the element of the Q matrix. As an example 2.2 is the element in the second row and in the second column

| Not. | IVP | Hovorka | IVP | Hovorka |
|------|-----|---------|-----|---------|
| 1,1  | 18000 | 0.2418 | 17990 | 0.3814 |
| 2,2  | 2.993·10^{-6} | 2.498 | 2.909·10^{-6} | 1.831 |
| 3,3  | 5 | 8.377·10^{-5} | 4.986 | 1.252·10^{-5} |
| 4,4  | 48.13 | 1.867·10^{-4} | 49.99 | 0.005 |
| 5,5  | 18000 | 1.974·10^{-5} | 18010 | 0 |
| 6,6  | 0.75 | 2.199·10^{-3} | - | 0.0186 |
| 7,7  | - | 4.861·10^{-5} | - | 9.967·10^{-4} |
| 8,8  | - | 4.986·10^{-5} | - | 9.150·10^{-4} |
| 9,9  | - | - | - | 9.951 |
| $Q_p$ | - | 1000 | - | - |
| $\lambda$ | - | 0.8742 | - | - |

### B. IN-SILICO COMPARISON

1) Rate of glucose appearance

Overall, observers imprecisely estimate the $R_A$, since the CGMS noise and model uncertainties are coupled with the estimation. For example, in Fig. 3, the estimated $R_A$ has negative values: an unphysiological behaviour for a scenario without exercise.

The type of model significantly contributes to the variability of RMSE and MAE (Table 4) in the statistical sense (P-value<0.05). Observers designed with the IVP model underperform the observers designed with the Hovorka model (Fig. 4) in terms of RMSE and MAE because the former overestimates the $R_A$ after the postprandial peak and underestimate it in the steady-state (Fig. 3). The pairwise comparisons also confirm the superiority of Hovorka-based observers (Table 6 and Table 4). However, the median difference is 0.04 mg/dL/min for both RMSE and MAE, which might be negligible from the application side.

The observer structure also has a statistically significant effect on MAE (the influence on RMSE is not statistically significant, but it is closed to be it). Specifically, DKF observers overperform NSMO and JKF in 0.04 mg/dL/min in the median for the MAE (Fig. 7) and in 0.05 mg/dL for the RMSE (Fig. 6). Conversely, NSMO and JKF behave similarly, although the NSMO observers have a higher peak
and the maximum absolute error (MaxAE). Terms “metrics: the root-mean-square error (RMSE), the mean absolute error (MAE)
F-statistics, respectively. The asterisk (*) indicates a P-Value lower than 0.05. TABLE 8.
unveils the limitations of the personalized models used to
patient variability of the simulation. In addition, this result
observer technique managed to cope with the large inter-
explain more than 90% of the variability, evincing that no
interaction between the model and the observer (Table 5). The
DFK observer designed with the Hovorka model illustrates
the importance of this interaction since it has the largest
MaxAE (Fig. 4); it even underperforms the observers de-
that statistically differ; they lower, in the median, the RMSE
appeared.
2) Plasma insulin estimation
Model structure and observer type cause remarkable differ-
ences in the RMSE, MAE, and MaxAE (Fig. 5). Indeed,
ANOVA supports for the three metrics significant differences
between the IVP and Hovorka models on the one side and
between the DFK, JFK, and NSMO on the other side (Table 8).
The post-hoc analyses in Tables 9 - 11 determine that
no significant differences between the NSMO and the JFK
exist, which agrees with Fig. 6, where the responses of both
observers mostly overlap. DFK observers are the only ones
that statistically differ; they lower, in the median, the RMSE
in the 0.5 mU/L, the MAE in 0.3 mU/L, and the MaxAE in
2.92 mU/L. Since the gain related to the $I_p$ in the KFs is
near 0 and, it is exactly 0 for the NSMO, the only difference
between observers is the discretization method: whereas the
NSMO and the JFK are based, respectively, on an implicit
and explicit Euler method, the DFK employs a zero-order
hold discretization that leads to more accurate results.
The IVP model significantly improves the performance of the
estimation compared to the observers designed with the
Hovorka model – with median reductions of 0.92, 0.72, and 5
mU/L in the RMSE, the MAE, and the MaxAE, respectively.
In Section III-A3, we already observed these differences be-
tween models when analysing the fit of the identified models,
indicating that the use of the observer could not cope with
the deficiencies of the models. The superiority of the IVP is
related to a quicker response in the $I_p$ dynamics than the
Hovorka model(Fig. 6). The slower dynamics of the Hovorka
model might be explained by the 2-compartment model that
describes the subcutaneous absorption with an equal transfer
rate between compartments.

### TABLE 6.
Pairwise comparison of the median difference of the root-mean-square error for the rate of glucose appearance (in mg/dL/min) between the levels of the significant factors. Values are expressed as median (mean absolute deviation). The P-values and confidence intervals (CI) corresponded to the sign test with corrections of false discovery rate and false coverage rate. An asterisk indicates a significant result at 0.05 level.

|       | Difference | CI           |
|-------|------------|--------------|
| IVP-Hovorka | 0.04 (0.09) | [0.02, 0.06] * |
| JFK-DKF    | 0.04 (0.05) | [0.03, 0.05] * |
| NSMO-DKF   | 0.04 (0.02) | [0.03, 0.05] * |
| NSMO-JKF   | 0 (0.02)    | [-0.01, 0.01] * |

### TABLE 7.
Pairwise comparison of the median difference of the mean absolute error for the rate of glucose appearance (in mg/dL/min) between the levels of the significant factors. Values are expressed as median (mean absolute deviation). The P-values and confidence intervals (CI) corresponded to the sign test with corrections of false discovery rate and false coverage rate. An asterisk indicates a significant result at 0.05 level.

|       | Difference | CI           |
|-------|------------|--------------|
| IVP-Hovorka | 0.04 (0.09) | [0.02, 0.06] * |
| JFK-DKF    | 0.04 (0.05) | [0.03, 0.05] * |
| NSMO-DKF   | 0.04 (0.02) | [0.03, 0.05] * |
| NSMO-JKF   | 0 (0.02)    | [-0.01, 0.01] * |

### TABLE 8.
Summary of ANOVA results of plasma insulin. It summarizes three metrics: the root-mean-square error (RMSE), the mean absolute error (MAE) and the maximum absolute error (MaxAE). Terms “$\eta^2$” and “P-value” denote the eta squared effect size measurement and the P-value of the ANOVA F-statistics, respectively. The asterisk (*) indicates a P-Value lower than 0.05.

|        | RMSE | MAE | MaxAE |
|--------|------|-----|-------|
| Model  | $\eta^2$ | P   | $\eta^2$ | P   | $\eta^2$ | P   |
| Observer | 2.40 | 0.00* | 1.60 | 0.00* | 3.60 | 0.00* |
| Subject | 66.60 | 0.00* | 65.60 | 0.00* | 60.80 | 0.00* |
| Model:Observer | 0.20 | 0.54 | 0.10 | 0.63 | 0.30 | 0.35 |
| Model:Subject | 1.20 | 0.53 | 2.00 | 0.13 | 1.40 | 0.46 |
| Observer:Subject | 0.40 | 1.00 | 0.40 | 1.00 | 0.50 | 1.00 |

### TABLE 9.
Pairwise comparison of the median difference of the root-mean-square error for the plasma insulin (in mU/L) between the levels of the significant factors. Values are expressed as median (mean absolute deviation). The P-values and confidence intervals (CI) corresponded to the sign test with corrections of false discovery rate and false coverage rate. An asterisk indicates a significant result at 0.05 level.

|       | Difference | CI           |
|-------|------------|--------------|
| IVP-Hovorka | -0.74 (0.32) | [-0.82, -0.62] * |
| JFK-DKF    | 0.3 (0.21) | [0.24, 0.36] * |
| NSMO-DKF   | 0.3 (0.21) | [0.24, 0.36] * |
| NSMO-JKF   | 0 (0)    | [0, 0]        |

### TABLE 10.
Pairwise comparison of the median difference of the mean absolute error for the plasma insulin (in mU/L) between the levels of the significant factors. Values are expressed as median (mean absolute deviation). The P-values and confidence intervals (CI) corresponded to the sign test with corrections of false discovery rate and false coverage rate. An asterisk indicates a significant result at 0.05 level.

|       | Difference | CI           |
|-------|------------|--------------|
| IVP-Hovorka | -0.74 (0.32) | [-0.82, -0.62] * |
| JFK-DKF    | 0.3 (0.21) | [0.24, 0.36] * |
| NSMO-DKF   | 0.3 (0.21) | [0.24, 0.36] * |
| NSMO-JKF   | 0 (0)    | [0, 0]        |
explained by “Subject” and the interactions “Model:Subject” and “Observer:Subject” – 67.6% of the RMSE, 68.0% of the MAE, and 62.7% of the MaxAE – is much larger than the sum of type of model, observer structure and interaction – 11.1% of the RMSE, 12.2% of the MAE, and 15.7% of the MaxAE. Again, this reveals the limitation of the observers to overcome the inter-patient variability.

The interaction between the type of model and the observer structure is not significant. This result agrees with Fig. 5, where the DKF always reduces the RMSE compared to the JKF and NSMO, regardless of the utilized model.

The type of model contributes more to the performance of the observer than the observer structure does (Table 8). How-}

3) Glucose measurement

The ANOVA in Table 12 determines that the type of model, the observer structure, and its interaction significantly explain the variance of the RMSE, the MAE, and the MaxAE. Although all the levels in the interaction between the type of model and the observer structure are statistically different, the median differences are, at most, 0.10 mg/dL for the RMSE, 0.07 mg/dL for the MAE, and 0.4 mg/dL for the MaxAE. Such minor differences are negligible from the clinical point of view.

Unlike the analysis of the $R_A$ and the $I_p$, the type of model, the observer structure, and its interaction fully explain the
variability of the performance metrics – 98.7% of the RMSE, 98.9% of the MAE, and 97.6% of the MaxAE. Since glucose is the only signal available online, the observers handle the inter-patient variability, which contributes less than 1% to the variability.

IV. CONCLUSION

We designed three observers (dual Kalman filter, Joint Kalman filter, and non-linear sliding mode observer) based on two glucose-insulin models (Hovorka model and IVP model). The most sensitive parameters of these models were individualized for the 10-adult cohort in the UVa-Padova simulator.

A statistical analysis quantified the impact of the observer structure and model type on the performance of the observers in terms of root-mean-square error, mean absolute error, and maximum absolute error in the $I_p$, glucose, and $R_A$. It was an in-silico analysis; hence simulator accuracy might compromise the results. UVa-Padova simulator includes a realistic cohort of subjects [25], accurately fits the glucose profiles of clinical trials [23], and improves the estimation of the meal absorption compared to other models in the literature [24]. However, like any other simulator, it cannot fully represent all the complex mechanisms involved in the glucose-insulin system. Repeating this study with clinical data would be desirable, but considering the rate of glucose appearance would be unfeasible since its measurement requires elaborate and expensive procedures.

For the $I_p$ and $R_A$, the low values of $\eta^2$ indicate that the observer structure negligibly impacts the global performance. Although statistically significant differences between observer structures exist, these differences are either too marginal or due to causes beyond the observer structure. For example, the method used to discretize the plant model caused the differences between observers in plasma insulin; the observer type barely contributed to these differences.

The type of model influenced the performance of $I_p$ and $R_A$ estimations more than the observer one. This result is evident in the estimation of the $I_p$ where the same advantages of the IVP model, shown in the identification procedure, appeared after designing the observers, regardless of the observer structure.

Finally, the large values of the $\eta^2$ for the “Subjects” and “Model:Subjects” factors evince the lack of robustness of the observers against the inter-patient variability. Observers with a more complex structure – for example, moving horizon estimators [10] or observers based on neuro-fuzzy systems [50] – and new strategies to improve the personalization of the models must be explored and compared in the future to study if they increase the robustness.

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