The Effect of Primordial Anti-Biasing on the Local Measurement of the Key Cosmological Parameters

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ABSTRACT
The best-fit values of the density parameter and the amplitude of the linear density power spectrum obtained from the Cosmic Microwave Background (CMB) temperature field scanned by the Planck satellite are found to notably disagree with those estimated from the abundance of galaxy clusters observed in the local universe. Basically, the observed cluster counts are significantly lower than the prediction of the standard flat ΛCDM model with the key cosmological parameters set at the Planck best-fit values. We show that this inconsistency between the local and the early universe can be well resolved without failing the currently favored flat ΛCDM cosmology if the local universe corresponds to a region embedded in a crest of the primordial gravitational potential field. Incorporating the condition of positive primordial potential into the theoretical prediction for the mass function of cluster halos, we show that the observed lower number densities of the galaxy clusters are in fact fully consistent with the Planck universe.

Key words: cosmology:theory — large-scale structure of universe

1 INTRODUCTION
The currently prevalent flat ΛCDM (Euclidean geometry, cosmological constant Λ and cold dark matter) cosmology can be completed only if the values of its key parameters are determined as precisely and accurately as possible. Among the various probes that have so far been developed for the accurate measurements of the key cosmological parameters, the best one is undoubtedly the Cosmic Microwave Background (CMB) temperature spectrum since the CMB sky provides us the least evolved version of our universe whose physics we believe are well understood. Although using the CMB spectrum as a cosmological probe suffers from the parameter degeneracy, this downside has been overcome by combining the CMB measurements with the results from the other complimentary probes such as the local cluster counts, the weak gravitational lensing, the baryonic acoustic oscillation (BAO) features and etc.

The recently reported tensions between the parameter values determined from the local universe and from the CMB sky scanned by the Planck satellite (Planck Collaboration XVI 2013) gave a great anxiety to the community, since the last (and perhaps the most crucial) puzzle to the standard flat ΛCDM picture is the consistency between the local and the distant scales (or the late and the early universe) (Verde et al. 2013). For instance, the Planck constraint on the Hubble constant, \(H_0 = 67.3 \pm 1.2\), is lower than the locally determined value, \(H_0 = 73.8 \pm 2.4\), by the Hubble Space Telescope (HST) observations of the Cepheid variable stars (Riess et al. 2011).

The Planck constraints on the density parameter, \(\Omega_m = 0.314 \pm 0.020\), and the amplitude of the linear power spectrum, \(\sigma_8 = 0.834 \pm 0.027\), are also in significant tension with the local measurements, \(\Omega_m = 0.255 \pm 0.043\) and \(\sigma_8 = 0.805 \pm 0.011\), from the low-z cluster counts (Vikhlinin et al. 2007). To make matters worse, very recently, the Planck team traced the abundance evolution of the 189 massive clusters detected via the Sunyaev-Zel’dovich (SZ) effect and determined the best constraints as \(\Omega_m = 0.29 \pm 0.02\) and \(\sigma_8 = 0.77 \pm 0.02\), which are even more serious 3σ deviations from the Planck best-fit ranges (Planck Collaboration XX 2013). These best-fit values of \(\Omega_m\) and \(\sigma_8\) determined from the late universe indicate that the observed number densities of the galaxy clusters in the local universe are much lower than predicted in the Planck cosmology.

These tensions have caught immediate attentions, provoking a burst of research to find solutions as well as their origins. Although some unknown systematics could have biased the local measurements of the cosmological parameters, the recent hot trend is to suspect the model-dependent values of the Planck experiments and to suggest possible solutions based on such non-standard models as a ΛCDM with massive neutrinos (Wyman et al. 2013), a ΛCDM with scale dependent non-Gaussian initial conditions...
In the current work, we explore a possibility to explain away the tensions within the standard flat ΛCDM cosmology. Our exploration will start from a core assumption that the local universe has formed in a crest of the primordial gravitational potential field and then will proceed in the direction of investigating if the observed lower amplitudes of the cluster mass functions are consistent with the Planck cosmology under this assumption.

2 CLUSTER COUNTS IN A PRIMORDIAL POTENTIAL CREST

In the standard theory of structure formation, the primordial gravitational potential field, ψ, is regarded as a Gaussian random field, being related to the linear density contrast field δ as δ = ∇ψ. Lee & Shandarin (1998) demonstrated that since the primordial potential field is much smoother than the linear density field, it is possible to determine a characteristic comoving scale, Rψ, of the "raw" unsmoothed primordial potential fluctuations as \( R_{\psi} = (3\sigma_{\psi}^2/\sigma_\delta^2)^{1/2} \)

where the rms fluctuations of \( \psi \) and \( \nabla \psi \) can be written in terms of the linear density power spectrum, \( P_\delta(k) \), as

\[
\sigma_\psi^2 = \int_{k_{\min}}^{\infty} dk \, k^{-2} P_\delta(k), \quad \sigma_\delta^2 = \int_{0}^{\infty} dk \, \delta P_\delta(k),
\]

where \( k_{\min} \) is the wave number corresponding to the comoving cosmic horizon introduced to prevent a divergence of \( \sigma_\psi^2 \) in practical calculation. Note that there is no filtering by any kernel in Equation (1), as stated in Lee & Shandarin (1998). Since the probability density of \( \psi \) is Gaussian distributed, finding a region with \( \psi > 0 \) (crest) in the primordial potential field is as equally probable as that with \( \psi < 0 \) (trough), i.e., \( P(\psi > 0) = P(\psi < 0) = 1/2 \).

Here, we set up a hypothesis that the present local universe have formed in a primordial potential crest with \( \psi > 0 \) rather than in a trough with \( \psi < 0 \) in the early universe. This hypothesis naturally leads to an expectation that the cluster number densities measured in the local universe would be lower than the global counterparts for a given background cosmology. The intriguing question is whether or not this hypothesis can explain away the tension between the local and the Planck measurements of the key cosmological parameters, especially, \( \Omega_m \) and \( \sigma_8 \), on which the cluster number densities are most strongly dependent.

It is of importance to understand that the characteristic comoving scale \( R_{\psi} \) of the primordial potential field represents its scale of coherence. According to Lee & Shandarin (1998), even though the formation of massive clusters are strongly biased toward the primordial potential troughs with \( \psi < 0 \) (see also Sahni et al. 1994, Buriak et al. 1992, Madsen et al. 1998, Demianski & Doroshkevich 1999), it is not totally impossible for the massive clusters to form in a primordial potential crest. Now, imagine a halo formed in a primordial potential crest with \( \psi > 0 \). Since the peculiar velocity of this halo is always in the direction from a crest to a trough of the primordial potential field, it is expected that the halo would be displaced during the evolution by gravitational effect from the formation site. If its maximum displacement distance from the formation site is smaller than \( R_{\psi} \), then this halo can be regarded as having effectively stayed in the primordial potential crest during the whole evolution. On the other hand, if its maximum displacement distance exceeds \( R_{\psi} \), then the halo is regarded as having been displaced from the crest region in the subsequent evolution. As for the galaxy clusters, their maximum displacement distances are usually much smaller than (see eq.[11] in Lee & Shandarin 1998). Therefore, if formed in a primordial potential crest, the galaxy clusters are expected to have stayed in the crest during the whole evolution. Unlike the smoothing scale, the characteristic scale \( R_{\psi} \) is not an extrinsic scale but an intrinsic one determined only by the background cosmology. For the Planck cosmology with \( \Omega_m = 0.318, \Omega_\Lambda = 0.683, n_s = 0.962, \Omega_b = 0.049, \sigma_8 = 0.834, h = 0.671 \) (Planck Collaboration XVI 2013), it is found to be \( R_{\psi} \sim 100 \, h^{-1} \text{Mpc} \).

We adopt the analytic prescription laid out in the work of Lee & Shandarin (1998) which basically incorporated the effect of primordial potential on the number densities of the dark halos into the classical mass function formalism of Press & Schechter (1974, hereafter, PS). Although the PS mass function has been well known to be inaccurate when tested against the numerical results (e.g., Sheth & Tormen 1999, Reed et al. 2003), it is the only "purely" analytic mass function theory into which it is rather straightforward to incorporate the condition of \( \psi > 0 \) without resorting to any empirical adjustment from N-body simulations. Since our goal here is not to model as accurately as possible the number densities of cluster halos in a primordial potential crest but to see how much the condition of \( \psi > 0 \) decreases the cluster number densities relative to the unconditional one, we believe that a modified version of the PS formalism should suffice to achieve this goal.

The PS formalism relates the differential mass function, \( dN/dM \), of dark halos to the fraction of the volumes, \( F(\delta_c; M) \), occupied by those regions whose linear density contrasts, \( \delta \), exceed a unique threshold value \( \delta_c = 1.686 \) (Gunn & Gott 1972, Peebles 1980, Eke et al. 1996), when the linear density field is smoothed on a given mass scale \( M \):

\[
\frac{dN}{dM} = 2 \frac{\bar{\rho}}{M} \left| \frac{dF(\delta_c; M)}{dM} \right|,
\]

\[
F(\delta_c; M) = \int_{\delta_c}^{\infty} d\delta \, p(\delta; \sigma_\delta),
\]

where \( p(\delta; \sigma_\delta) \) is a Gaussian probability density of \( \delta \), with the standard deviation \( \sigma_\delta \), \( \bar{\rho} \) is the mean mass density of the universe and the factor of 2 before represents the normalization factor of the mass function (e.g., Bond et al. 1991, Jedamzik 1993). The standard deviation, \( \sigma_\delta \), depends on the mass scale \( M \) as

\[
\sigma_\delta^2(M) = \int_{0}^{\infty} dk \, k^2 \sigma_{\psi}^2(2\pi)^2 W^2(k; M),
\]

where \( W(k; M) \) is the Fourier transform of a kernel by which
Equation (6) can be straightforwardly calculated by using the linear density contrast field is smoothed on the mass scale of $M$.  

Incorporating the condition of $\psi > 0$ into the PS formalism amounts to modifying the volume fraction $F(\delta_c; M)$ in Equation (5) into  

$$F(\delta_c; M|\psi > 0) = \int_{\delta_c}^{\infty} d\delta \ p(\delta; \delta_c, \sigma_c, \sigma_\psi|\psi > 0) ,$$  

with  

$$p(\delta, \delta_c, \sigma_c, \sigma_\psi|\psi > 0) = \frac{\int_0^\infty d\psi \ p(\delta, \psi; \delta_c, \sigma_c, \sigma_\psi)}{\int_0^\infty d\psi \ p(\psi; \sigma_\psi)} = 2 \int_0^\infty d\psi p(\delta, \psi; \delta_c, \sigma_c, \sigma_\psi) ,$$  

where $\sigma_c$ is the square root of the cross correlation between the unsmoothed primordial potential field, $\psi$, and the smoothed density field on the mass scale $M$, $\delta$. Since the Fourier transform of the unsmoothed primordial potential field $\psi$ is related to the Fourier transform of the smoothed linear density field $\tilde{\delta}$ as $\psi = k^{-2} \tilde{\delta} W^{-1}(k; M)$, the cross-correlation $\sigma^2_c(M)$ can be evaluated as  

$$\sigma^2_c(M) = \langle \psi \delta \rangle = \int_0^\infty dk \ P_\delta(k) W(k; M) .$$  

The joint probability density, $p(\delta, \delta_c, \sigma_c, \sigma_\psi|\psi > 0)$, in Equation (6) can be straightforwardly calculated by using the statistics of a Gaussian random field (Bardeen et al. 1986, and references therein) as  

$$p(\delta, \psi; \delta_c, \sigma_c, \sigma_\psi) = \frac{1}{[(2\pi)^2 \sigma_\psi^2 (\sigma_c^2 - \sigma_\psi^2)/(\delta_c^2)]^{1/2}} \times$$  

$$\exp \left[-\frac{\delta^2}{2\sigma^2} \ - \frac{(\psi + \sigma_\psi^2 \delta/\sigma_c^2)^2}{2(\sigma_c^2 - \sigma_\psi^2/\sigma_c^2)} \right].$$  

Now, let us perform a change of variable as  

$$\nu = \frac{\delta}{\sigma_c}, \quad \mu = \frac{(\psi + \sigma_\psi^2 \delta/\sigma_c^2)^2}{2(\sigma_c^2 - \sigma_\psi^2/\sigma_c^2)} .$$  

By applying the probability conservation relation of $p(\delta, \psi)d\delta d\psi = p(\nu, \mu)d\nu d\mu$ to $p(\delta, \psi)$, we derive the joint probability density distribution, $p(\mu, \nu)$, as  

$$p(\nu, \mu) d\nu d\mu = \frac{1}{(2\pi)^{1/2}} \exp \left(-\frac{\nu^2}{2}\right) \frac{1}{(2\pi)^{1/2}} \exp \left(-\frac{\mu^2}{2}\right),$$  

As can be seen, the two variables, $\mu$ and $\nu$ are mutually uncorrelated and thus the joint distribution, $p(\mu, \nu)$, is expressed as a product of two one-point distributions, $p(\mu)$ and $p(\nu)$.  

Now, the volume fraction in Equation (6) can be readily calculated in terms of $\mu$ and $\nu$ as  

$$F(M; \nu_c, \mu_c) = 2 \int_{\nu_c}^{\infty} d\nu \ \frac{1}{(2\pi)^{1/2}} \exp \left(-\frac{\nu^2}{2}\right) \times$$  

$$\int_{\mu_c}^{\infty} d\mu \ \frac{1}{(2\pi)^{1/2}} \exp \left(-\frac{\mu^2}{2}\right) ,$$  

where $\nu_c = \nu(\delta = \delta_c)$ and $\mu_c = \mu(\delta = \delta_c, \psi = 0)$.  

Finally, the mass function of the cluster halos formed in a primordial potential crest is evaluated as  

$$\frac{dN}{dM} = 2 \frac{\bar{\rho}}{M} \frac{dF}{dM} = 2 \frac{\bar{\rho}}{M} \left[ \frac{d\nu_c}{dM} dF_c + \frac{d\mu_c}{dM} dF_\mu \right] ,$$  

where the four differentials can be computed with the help of the chain rule as  

$$\frac{dF}{d\nu_c} = \frac{1}{(2\pi)^{1/2}} \exp \left(-\frac{\nu_c^2}{2}\right) \left[ \frac{1}{2} \text{erfc} \left( \frac{\nu_c}{2} \right) \right] ,$$  

$$\frac{dF}{d\mu_c} = \frac{1}{(2\pi)^{1/2}} \exp \left(-\frac{\mu_c^2}{2}\right) \left[ \frac{1}{2} \text{erfc} \left( \frac{\mu_c}{2} \right) \right] ,$$  

$$\frac{d\nu_c}{dM} = \frac{dv_c}{dM} \frac{d\sigma_c}{dM} ,$$  

$$\frac{d\mu_c}{dM} = \frac{d\mu_c}{d\sigma_c} \frac{d\sigma_c}{dM} + \frac{d\mu_c}{d\sigma_\psi} \frac{d\sigma_\psi}{dM} .$$  

The top panel of Figure 1 plots the conditional mass function, $dN/dM$, of the cluster halos per unit volume with $M \geq 10^{15} h^{-1} M_\odot$ at $z = 0$ for two different cases. The solid and dashed lines correspond to the cases that the cluster halos formed in a primordial potential crest and trough, respectively. The latter can be straightforwardly evaluated by repeating the same steps described in Equations (5) - (19) but with changing the crest condition of $\psi > 0$ into the trough condition of $\psi < 0$ as in Lee & Shandarin (1998). The dotted line in Figure 1 corresponds to the case of no condition, i.e., the original unconditional PS mass function of the cluster halos. The bottom panel shows the ratio of the two conditional mass functions to the unconditional one as solid and dashed lines, respectively, while the horizontal
dashed line corresponds to the low-$z$ cosmology with $\Omega_m = 0.318$, $\Omega_\Lambda = 0.683$, $h = 0.671$, $\sigma_8 = 0.834$ (Planck Collaboration XVI 2013), while the dotted line corresponds to the case of the Planck cosmology with $\Omega_m = 0.255$, $\Omega_\Lambda = 0.745$, $h = 0.722$, $\sigma_8 = 0.805$ (Vikhlinin et al. 2009). The conditional mass function of the clusters formed in a primordial potential crest ($\psi > 0$) for the Planck cosmology is plotted as solid line.

Since what has been always assumed in the theoretical modeling of the cluster mass function is that there is no difference between the local and the global average number densities of the clusters, our result implies that the comparison between the observed number densities of the local clusters and the analytic model of the unconditional mass function would yield different best-fit values of $\Omega_m$ and $\sigma_8$ from the Planck constraints, even when the background is truly the Planck universe.

We evaluate the unconditional mass functions of the cluster halos by Equations (2, 3) for two different cosmologies and plot them in Figure 2 as dotted and dashed lines, respectively: The dotted line corresponds to the case of the Planck cosmology with $\Omega_m = 0.3175$, $\Omega_\Lambda = 0.6825$, $h = 0.6711$, $\sigma_8 = 0.8344$ (Planck Collaboration XVI 2013). As can be seen, the number densities of the cluster halos formed in a primordial potential crest (trough) are indeed lower (higher) than the unconditional counterpart.

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Evaluating the conditional mass function of the cluster halos formed in a primordial potential crest by Equations (4, 10) for the Planck cosmology, we show it as solid line in Figure 3. Surprisingly, the solid line is in an excellent agreement with the dashed line. A crucial implication of this result is that the observed lower number densities of the local clusters are in fact fully consistent with the Planck constraints as far as the condition of $\psi > 0$ for the local universe is properly taken into account when the theoretical prediction for the cluster mass function is made.

It is worth mentioning here that our analytic prescription of evaluating the conditional mass function of galaxy clusters in the local universe suffers from one ambiguity. Since we deal with the scale-free unsmoothed primordial potential field, the scale of its crest region in which our local universe is assumed to reside is unknown and thus has to be determined empirically. Given that the effective redshift of the clusters considered by Vikhlinin et al. (2009) is approximately 0.15, the scale of the local universe should extend at least up to the same redshift, $z = 0.15$, which amounts to $430 \, h^{-1} \text{Mpc}$ for the Planck cosmology. In other words, to
reconcile the Planck cosmology with the mass function of the low-z clusters in the sample of Vikhlinin et al. (2009), it has to be assumed that the local universe corresponds to a primordial potential crest of comoving size as large as 430 h⁻¹ Mpc.

Let us examine if incorporating the condition of ψ > 0 can also resolve the other tension between the constraints of Ω_m and σ_8 from the Planck SZ catalogs and the Planck cosmology. As mentioned in section 1, the abundance evolution of the SZ clusters measured by Planck Collaboration XX (2013) has yielded the best-fit constraints of Ω_m = 0.28, σ_8 = 0.78 provided that the dimensionless Hubble parameter is given as h = 0.722 (say, the SZ cluster cosmology). Here, the abundance evolution of the galaxy clusters, dN/dz, is proportional to the number counts, N(> M_{min}, z), of the cluster halos as a function of redshift defined as N(> M_{min}, z) = ∫_{M_{min}}^{∞} dM dN/dM, where M_{min} is the mass threshold set at 10⁻⁴ h⁻¹ M_⊙ for our analysis.

We evaluate the unconditional number counts of the galaxy clusters as a function of redshift, N(> M_{min}, z), by integrating Equation (2) over mass for the SZ cluster cosmology and show it as dashed line in Figure 3. Comparing the dashed line with the dotted line in Figure 3 which is nothing but the unconditional number counts, N(> M_{min}, z), for the Planck cosmology, one can see that the difference in the prediction for N(> M_{min}, z) between the two cosmologies is quite large. Now, evaluating the conditional number counts, N(> M_{min}, z|ψ > 0), by integrating Equation (10) over M for the Planck cosmology, we show it as solid line in Figure 3. It is exciting to see that the solid line exhibits a wonderful match (especially in amplitude) to the dashed line.

It is worth mentioning here that although the redshifts of the galaxy clusters in the Planck SZ catalog are distributed in a wide range of 0 ≤ z ≤ 1, the constraints from the Planck SZ clusters on the values of σ_8 and Ω_m were decisively determined in the low-z section (0 ≤ z < 0.4). From Figure 7 in Planck Collaboration XX (2013) which plots the number counts of the SZ clusters as a function of redshift, one can see that the data points in the high-z section (0.4 < z ≤ 1) suffer from large uncertainties and that the amplitude of dN/dz is determined almost decisively by those data points with small errors in the low-z section (0 ≤ z < 0.2). Thus, the anti-biasing effect of the primordial potential fluctuation can also reconcile the redshift distribution of the local SZ clusters with the Planck cosmology.

3 DISCUSSION AND CONCLUSION

To solve the problem that the constraints on Ω_m and σ_8 from the observed cluster mass functions in the local universe (Vikhlinin et al. 2009) and from the redshift evolution of the SZ cluster counts (Planck Collaboration XXI 2013) are both in tension with the constraints from the Planck experiments (Planck Collaboration XVI 2013), we have put forward a new hypothesis that the local universe formed in a crest of the primordial gravitational potential and has stayed in the potential crest after the formation. Under this hypothesis, we have evaluated analytically the number densities of the cluster halos formed in a primordial potential crest at present epoch with the help of the analytic prescriptions suggested by Lee & Shandarin (1998) who had modified the original PS mass function theory to incorporate the effect of primordial potential.

When the condition of being in a primordial potential crest is imposed, the resulting conditional mass function of cluster halos has been found to exhibit significantly lower amplitude than the unconditional one. We have finally explained away the tension between the Planck and the local measurements of Ω_m and σ_8 by revealing the following two results: (i) The conditional mass function of cluster halos for the case of the Planck cosmology with Ω_m = 0.318 and σ_8 = 0.834 agree almost perfectly with the unconditional mass function for the low-z cluster cosmology with Ω_m = 0.255 and σ_8 = 0.805; (ii) The redshift evolution of the conditional cluster counts for the case of the Planck cosmology matches that of the unconditional ones for the case of the cosmology from the Planck SZ clusters with Ω_m = 0.28 and σ_8 = 0.78.

It should be worth discussing the apparent similarity and the essential difference between our analytic prescription of evaluating the cluster mass function and the one proposed by Alonso et al. (2012) in the context of the Lemaître-Tolman-Bondi cosmology (Lemaître 1933; Tolman 1934; Bondi 1947). Although both of the models have evaluated the halo mass function with an assumption that the local universe corresponds to a low-density region, the latter is based on the radical hypothesis that there is no dark energy in the universe (see also Nadathur & Sarkar 2011, and references therein) in a direct contrast to the former which is well accommodated by the standard ΛCDM cosmology.

Here we have followed the purely theoretical approach based on the analytic PS formalism of the halo mass function. However, it would be definitely desirable to make a direct comparison with the observed cluster counts, which requires to construct a more improved formalism for the conditional mass function, given the inaccuracy of the PS formalism (e.g., Sheth & Tormen 1999; Reed et al. 2003). As for the unconditional mass function of cluster halos, there have been plenty of literatures which developed more improved analytic formalism by making more complicated and realistic assumptions about the halo formation process such as ellipsoidal collapse, diffusive collapse threshold, and non-Markovian random walks and etc (e.g., Sheth et al. 2001; Maggiore & Riotto 2010; Corasaniti & Achitouv 2011; Musso & Sheth 2012; Paranjape et al. 2012). The formalism for the conditional mass function has to be improved and refined similarly before testing directly our hypothesis against observations.

Although we have focused mainly on the parameters of Ω_m and σ_8 in the current work, it will be intriguing to investigate if our hypothesis can also lead to a reconciliation between the Planck and the local measurements of the Hubble constant H_0 (e.g., Riess et al. 2011). In fact, it can be logically expected that if the local universe corresponds to a primordial potential crest, then the density contrast averaged over the local universe would be lower than the global average, which would be reflected by a higher value of H_0 when measured locally than its global value determined from the CMB analysis. To quantitatively investigate this effect of primordial potential on the Hubble constant, however, it will be first required to study rigorously the evolution of the linear overdense region embedded in a primordial potential crest.
The other interesting issue that our hypothesis might be useful to address is the lower growth rate, $\frac{d \ln D}{d \ln a}$, inferred from the recently available deep galaxy surveys than predicted by the Planck cosmology (Macaulay et al. 2013, and references therein). Although the change of the linear growth factor, $D(z)$, by the primordial anti-biasing has not been accounted for in our analysis under the assumption that it would be small (Lee & Shandarin 1998), the growth rate in a primordial potential crest that has a locally negative curvature must be lower than the average value. We believe that incorporating the condition of $\psi > 0$ into the calculation of the linear growth rate might also provide a clue to resolving the apparent inconsistency found in Macaulay et al. (2013). Our future work is in the direction of conducting these works.

ACKNOWLEDGMENTS

I thank the anonymous referee for helping me improve the original manuscript. This research was supported by Basic Science Research Program through the National Research Foundation of Korea(NRF) funded by the Ministry of Education (NO. 2013004372) and partially by the research grant from the National Research Foundation of Korea to the Center for Galaxy Evolution Research (NO. 2010-0027910).

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