Aharonov-Bohm effect, local field interaction, and Lorentz invariance

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Abstract

A field-interaction scheme is introduced for describing the Aharonov-Bohm effect, fully consistent with the principle of relativity. Our theory is based on the fact that local field interactions are present even when a particle moves only in a field-free region. The interaction Lagrangian between a charge and a flux is uniquely constructed from three obvious principles: Lorentz covariance, linearity in the interaction strength, and a correct stationary limit of charge. Our result resolves fundamental questions raised on the standard interpretation of the Aharonov-Bohm effect, concerning the principle of relativity and the equivalence between the potential and the field-interaction pictures for describing the electromagnetic interaction. Most of all, potential is eliminated in our theory, and all kind of the force-free Aharonov-Bohm effect is understood in a unified picture of Lorentz-invariant local interaction of electric/magnetic fields.

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I. INTRODUCTION

Aharonov-Bohm (AB) effect [1, 2] has changed our notion of electromagnetic field and potential. It is known as a milestone in our understanding of electromagnetic interactions, which describes a quantum interference of a charged particle moving in a region free of electric and magnetic fields. According to the standard interpretation, AB effect demonstrates that the classical picture of electromagnetism based on the local action of fields is invalid in quantum theory, because the interference is affected by a potential even when a charged particle moves in a field-free region. Aharonov-Casher (AC) effect [3], dual to the AB phenomenon, shows a phase shift of a fluxon moving around a charged rod. It has been shown that AC effect is also free of force [4, 5], but standard view draws a clear distinction between the two phenomena in that the fluxon moves under a nonvanishing field generated by the charge in the case of AC effect [6]. Despite the fact that the observable phenomena depend only on the relative motion of a charge and a fluxon in two dimension, a unified picture, fully consistent with the principle of relativity, is lacking.

In this paper, we provide a unified theory which resolves the question of relativity, based on a Lorentz-invariant field-interaction between a charge and a localized flux. The AB effect can be understood in this fully relativistic viewpoint based on the local action of fields. The AB phase shift is derived from the Lorentz-covariant interaction Lagrangian, and the force-free nature of the effect is also derived. In addition, we show that the AB effect should vanish under the condition of a perfect shielding of the field interactions, which has been widely overlooked. The present study suggests a fundamental change in our viewpoint of the potential: The AB effect can be described purely in terms of local interaction of fields, and we can unify the role of fields and potentials both in classical and quantum theories. We expect that our study initiates further research in this context. The role of shielding local interactions will be a key issue for understanding the underlying nature of the AB effect.

II. QUESTIONS CONCERNING THE STANDARD VIEWPOINT OF THE AHARONOV-BOHM EFFECT

Take, for instance, the interaction of a charge, \( e \), and a localized fluxon with its flux value \( \Phi \), in two dimension [3, 5] (Figure 1(a)). The observable phenomena here depend only on the
relative motion of the two entities, in that (1) a phase shift is acquired for one-loop rotation of one particle moving around the other, and, (2) the interaction between the particles is free of force in both cases. In spite of this equivalence, the standard view draws a clear distinction between the two cases, namely “Type I” and “Type II” (Figure 1(b)) [6]. In Type I, the charge is moving in a field-free region. The observed AB phase is interpreted as a pure topological effect where the charge is interacting with the flux only in a nonlocal way. In the case of Type II, the fluxon moves under the influence of the electric field generated by the charge [3]. Although the net force applied to the fluxon vanishes [4], the observed phase shift can be fully understood in terms of local interaction of the fluxon with the external electric field [4]. This distinction of the types depending on the reference frame should not be inherent, because the AB and the AC effect in two dimension is in fact the same problem in that only a relative motion of the two objects matters. A unified view, fully consistent with the principle of relativity, is desirable for understanding both types of the AB effect.

We also clarify a critical issue concerning a possible shielding of local field interaction. It is a common notion that the AB effect survives in a condition of shielding any kind of local field interactions. This viewpoint has been strengthened due to the seminal experiment by Tonomura et al. [8, 9]. However, this notion is misleading, as we show in the following analysis.

Let us consider a moving electron and a stationary flux confined in an ideal Faraday cage made of a hollow superconductor (Figure 2). Here the superconductor is introduced only as an ideal Faraday cage, without taking into account its material properties. For simplicity, the problem is analysed for a two dimension with a circular hollow cage, with the condition that no electric/magnetic field penetrates inside the hollow. In this condition, the electromagnetic interaction has three different contributions, namely, \( L_{\text{int}} = L_{e\Phi} + L_{s\Phi} + L_{es} \), where \( L_{e\Phi} \), \( L_{s\Phi} \), and \( L_{es} \) are Lagrangians representing electron-flux, superconductor-flux, and electron-superconductor interactions, respectively. \( L_{es} \) is independent of \( \Phi \), and therefore irrelevant to the AB effect. The electron-flux interaction is

\[
L_{e\Phi} = \frac{e}{c} \hat{r} \cdot A = \frac{e\omega}{2\pi c} \Phi, \tag{1}
\]

where \( \omega \) is the angular velocity of the electron and the vector potential \( A \) is given by a symmetric gauge. The moving electron induces a charge density, \( \delta\sigma(\phi) \), on the outer surface
FIG. 1. (a) A charge, $e$, and a fluxon, $\Phi$, in two dimension, moving with velocities $\dot{\mathbf{r}}$ and $\dot{\mathbf{R}}$, respectively. (b) Classification of the Aharonov-Bohm effect. In “Type I”, a charge is moving in a field-free region, while in “Type II”, a fluxon moves under the influence of an electric field produced by a charge.

of the superconductor with radius $R$, which depends on the angle $\phi$ on the conductor,

$$
\delta \sigma(\phi) = -\frac{e}{2\pi R} \left( \frac{a^2 - R^2}{a^2 + R^2 - 2aR \cos \phi} \right),
$$

where $a (> R)$ is the distance of the electron from the center. Accordingly a counter-flowing current

$$
\mathbf{j}_c(\phi) = R\omega \delta \sigma(\phi) \hat{\phi}.
$$

is generated, where $\hat{\phi}$ denotes the azimuthal unit vector. It is straightforward to find that this induced current gives the interaction term

$$
L_{s\phi} = \frac{1}{c} \int \mathbf{j}_c \cdot \mathbf{A} \, dr = -\frac{e\omega}{2\pi c} \Phi,
$$

(3)
thereby the $\Phi$-dependent interaction Lagrangian vanishes: $L_{e\Phi} + L_{s\Phi} = 0$. The charge-flux interaction is completely shielded by the counter-flowing current in the superconductor. Therefore, the flux $\Phi$ does not contribute any phase shift for an arbitrary evolution of the electron-conductor composite system. This implies that, although the vector potential is nonzero in the electron’s path, the AB effect vanishes under the condition of a perfect shielding of field interaction. This effect has been widely ignored, and is crucial in our understanding the role of field and potential.

Note that the quantization of charge (by $2e \times \text{integer } n$) in the superconductor does not affect the analysis given above. By taking the charge quantization into account, the ideal shielding condition of the induced charge density (Eq. (2)) is uniquely determined by

$$\int \sigma(\phi) R d\phi = 2ne,$$

$$\sigma(\phi) = \sigma_0 + \delta \sigma(\phi),$$

(4)

where $\delta \sigma(\phi)$, given by Eq. (2), is responsible for the image current: $j_c(\phi) = R \omega \delta \sigma(\phi) \hat{\phi}$, while the background charge $\sigma_0 = (2n + 1)e/2\pi R$ does not contribute to the electric/magnetic field inside the cage.

The result of our analysis invokes another important question: Is the local field interaction picture really irrelevant to describe the Aharonov-Bohm effect? In fact, equivalence of two different approaches for describing the electromagnetic interaction energy is well established in classical electrodynamics [10]. For a current distribution $(j_1)$ in the presence of a magnetic field $B_2 (= \nabla \times A_2)$ from another source, the interaction energy of the two independent sources is given by $\frac{1}{c} \int j_1 \cdot A_2 d\mathbf{x}$, coupling of the current and the vector potential $(A_2)$. Alternatively, it can be expressed as $\frac{1}{4\pi} \int B_1 \cdot B_2 d\mathbf{x}$, where $B_1$ is the magnetic field generated by the current $j_1$. When it is applied to the AB effect, the general consensus is that only the potential picture is valid. It is hard to see why and how the equivalence is broken in a quantum mechanical treatment of the interaction. If one tries to maintain the standard viewpoint that only the potential-based model is relevant, it should be explicitly shown how the equivalence of the two model is broken, which has never been done. In the following, we argue that this is not the case, and the equivalence of the two approaches is maintained also in quantum theory.
FIG. 2. Aharonov-Bohm effect under a perfect shielding of field interaction? A flux (Φ) is localized inside a Faraday cage (constructed by a hollow superconductor), and the magnetic field of the moving electron (e) is compensated by the counter-flowing current \( j_c \). This current is generated by the motion of the induced surface charges (marked by red plus signs) of the superconductor.

III. LORENTZ-INvariant FIELD-INTERACTION APPROACH

In order to resolve the questions raised above, we develop a field-interaction scheme which is fully consistent with the special theory of relativity. We treat the local field interaction of two particles (charge and fluxon) in two dimension, on an equal footing (Figure 1(a)). For moving charge and fluxon (with their velocities \( \dot{r} \) and \( \dot{R} \), respectively), there are two sources of field interactions, that is, (1) between the magnetic field of the fluxon and that of the moving charge, and, (2) between the electric field of the charge and that of the moving fluxon. Each interaction term was treated for a classical force explanation of the AB [11–13], and of the Aharonov-Casher (AC) [14] phase shifts, respectively. Although the classical force based on the field interactions have been refuted both theoretically [4, 5] and
experimentally [15], we point out that the field-interaction framework itself is not erroneous. The main drawback of the previous field-interaction approaches is that the interaction term depends on the reference frame, violating the principle of relativity. Below, we provide a Lorentz-covariant field interaction scheme to get rid of this problem.

A. Construction of the Lagrangian

The Lagrangian, $L$, of the system is given by $L = L_e + L_\Phi + L_{\text{int}}$, where $L_e$, $L_\Phi$, and $L_{\text{int}}$ represent a free charge ($e$), a fluxon ($\Phi$), and their interaction, respectively. The self field energies diverge for point particles and are therefore neglected. In any case, the self field energy does not affect our analysis. The interaction term, $L_{\text{int}} = \int L_{\text{int}} d\mathbf{x}$, is constructed from the following three obvious principles: (1) The Lagrangian density $L_{\text{int}}$ is invariant under Lorentz transformation and space inversion, (2) linear in field strengths, and (3) $L_{\text{int}}$ is reduced to $-eV$, the correct non-relativistic limit for a stationary charge, with the electric scalar potential $V$ generated by the moving fluxon. The interaction term is uniquely determined from these constraints as

$$L_{\text{int}} = \frac{1}{8\pi} \int F^{(e)}_{\mu\nu} F^{\mu\nu(\Phi)} d\mathbf{x}, \quad (5)$$

where $F^{(e)}_{\mu\nu}$ and $F^{\mu\nu(\Phi)}$ are the electromagnetic field tensors generated by the charge and the fluxon, respectively. Eq. (5) can also be written in terms of more familiar electric and magnetic fields as

$$L_{\text{int}} = \frac{1}{4\pi} \int \left( \mathbf{B}^{(e)} \cdot \mathbf{B}^{(\Phi)} - \mathbf{E}^{(e)} \cdot \mathbf{E}^{(\Phi)} \right) d\mathbf{x}, \quad (6)$$

where $\mathbf{B}^{(e)}$ ($\mathbf{E}^{(e)}$) and $\mathbf{B}^{(\Phi)}$ ($\mathbf{E}^{(\Phi)}$) represent the magnetic (electric) fields of the charge and the fluxon, respectively. $L_{\text{int}}$ can be simplified by adopting the relations $\mathbf{B}^{(e)} = \frac{1}{c} \mathbf{\dot{r}} \times \mathbf{E}^{(e)}$ and $\mathbf{E}^{(\Phi)} = -\frac{1}{c} \mathbf{\dot{R}} \times \mathbf{B}^{(\Phi)}$, as,

$$L_{\text{int}} = (\mathbf{\dot{r}} - \mathbf{\dot{R}}) \cdot \mathbf{\overline{\Pi}}, \quad (7)$$

where

$$\mathbf{\overline{\Pi}} = \frac{1}{4\pi c} \int \mathbf{E}^{(e)} \times \mathbf{B}^{(\Phi)} d\mathbf{x} \quad (8)$$

is the field momentum produced by the two particles. Note that Eq. (7) is equivalent to the interaction Lagrangian of [3, 5]

$$L_{\text{int}} = \frac{e}{c} (\mathbf{\dot{r}} - \mathbf{\dot{R}}) \cdot \mathbf{A}, \quad (9)$$
based on the vector potential $A$, except that in the former case (Eq. (7)) gauge dependence is absent.

From now on we focus on the nonrelativistic limit ($|\dot{r}|, |\dot{R}| \ll c$), although the Lorentz-covariant interaction Lagrangian of Eq. (7) should generally be valid. In this limit, the Lagrangian of the system is reduced to

$$L = \frac{1}{2} m \dot{r} \cdot \dot{r} + \frac{1}{2} M \dot{R} \cdot \dot{R} + (\dot{r} - \dot{R}) \cdot \vec{\Pi},$$

(10)

where the field momentum is

$$\vec{\Pi} = \frac{e\Phi}{2\pi c|\mathbf{r} - \mathbf{R}|} \hat{\phi},$$

with $\hat{\phi}$ the azimuthal unit vector of $\mathbf{r} - \mathbf{R}$. $m$ and $M$ denote the masses of the charge and the fluxon, respectively. It is instructive to note that this Lagrangian is transformed to a Hamiltonian

$$H = \left(\frac{p - \vec{\Pi}}{2m}\right)^2 + \left(\frac{P + \vec{\Pi}}{2M}\right)^2,$$

(11)

where $p$ and $P$ are the canonical momenta of the variables $\mathbf{r}$ and $\mathbf{R}$, respectively.

**B. Understanding the force-free Aharonov-Bohm effect**

Several noticeable features can be found from the Lagrangian of Eq. (10) (or from the Hamiltonian of Eq. (11)). First, this Lagrangian is equivalent to the one represented by a vector potential [3, 5], except that there is no freedom to choose a gauge in Eq. (10). The equivalence of the “potential” and the “field-interaction” approaches is restored. Second, the Euler-Lagrange equation for the classical dynamics leads to an equation of motion of

$$m\ddot{r} = -M\ddot{R} = q(\dot{r} - \dot{R}) \times B^{(\Phi)} = 0.$$

(12)

There is no mutual classical force between the two particles, contrary to the previous classical explanation of the AB effect based on the field interactions [11–14]. This indicates that the classical force claimed in the previous schemes is an artefact which arises from neglect of Lorentz invariance. For instance, if $-\dot{R} \cdot \vec{\Pi}$ term (which corresponds to the electric field interaction) is neglected in Eq. (10), we find that $M\ddot{R} \neq 0$, which is the basis of the claim in Ref. [12]. Apparently, this force is absent if the Lorentz invariance is taken into account in the Lagrangian. Third, on encircling around the fluxon, the charge acquires an AB phase

$$\phi_{AB} = \frac{1}{\hbar} \oint \vec{\Pi} \cdot d\mathbf{r} = \frac{e\Phi}{\hbar c}.$$  

(13)
The same is true for the fluxon encircling around the charge. Recall that, in obtaining this, we have not relied on the vector potential. Force-free AB phase can be explained in a general way without the notion of potential, in contrast to the widely accepted viewpoint.

Our findings are not limited to the two point particles in two dimension, but can be applied to more realistic case with distributed flux (charge). Consider, for example, a charge \((e)\) and distributed magnetic flux schematically drawn in Figure 3. The interaction Lagrangian in this case is given in the same form of Eq. (10) with the field momentum replaced by

\[
\vec{\Pi} = \frac{e}{2\pi c} \int S \hat{\phi} B d^2R |\mathbf{r} - \mathbf{R}|.
\]

The integral is over the cross sectional area \(S\) of the flux lines, and \(\hat{\phi}\) is the azimuthal unit vector of \(\mathbf{r} - \mathbf{R}\). The force-free nature of the interaction is maintained. Also, it is straightforward to show that the one-loop integral of \(\vec{\Pi}\) gives the AB phase of \(\phi_{AB} = e\Phi/(\hbar c)\), where \(\Phi\) is the net magnetic flux.

### IV. QUESTIONS ABOUT SHIELDING

An interesting question has been raised by Erlichson \[16\] already in 1970 concerning a shielding of local field interactions. In his suggestion, a superconducting barrier is placed between a charge and a magnetic flux in order to prevent the penetration of the charge’s magnetic field into the flux. Whether the local action of fields play a major role or not would be revealed with this kind of experiment. It is widely believed that this matter has been settled in a seminal experiment by Tonomura et al. \[8, 9\]. In this experiment, a phase shift is reported even when the flux is confined in a superconducting shield, and their result is interpreted in terms of the potential (gauge field). However, as we have shown in Eqs. (13), the AB effect should vanish in a condition of perfect shielding of field interaction. This point should be clarified. Absence of the AB effect can be more directly understood in our field interaction framework. For a moving electron and a stationary flux confined in an ideal Faraday cage (Figure 2), the magnetic field \((\mathbf{B}_e)\) generated by the moving electron does not penetrate into the hollow. Therefore, the local field interactions vanish inside the cage. To be precise, the moving electron’s magnetic field is compensated by a field generated in the superconductor. Counter-flowing current \((\mathbf{j}_c)\) of the induced surface charge generates a magnetic field \((\mathbf{B}_c)\), which compensates \(\mathbf{B}_e\) inside superconducting shield. The only non-
vanishing field interaction is the one between $B_e$ and $B_c$. This interaction is independent of the flux, and thus, the AB effect does not appear in this system.

The result of our analysis looks, at first glance, contradictory to the experimental result of Tonomura et al. They observed a phase shift of an electron beam, where the magnetic flux is shielded by a superconductor from the electron path. Their experiment demonstrates clear phase shift of $\pi$ in the electron interference, in the absence of a leakage of the magnetic flux. The observed phase shift was attributed to the odd number of flux quantum confined in a superconducting shield. The result is also interpreted as a demonstration of the AB effect with complete shielding of field interaction. However, this interpretation of Ref. 8 and 9 is invalid, as we show in the following.

In the AB interference experiment of Ref. 8, an electron beam of its wavelength about $3 \times 10^{-12}$m was used. The corresponding electron speed and the kinetic energy are $v_e \sim 2.4 \times 10^8$ m/s and $E_e \sim 150$ keV, respectively. In fact, no superconducting material can
shield the magnetic field produced by such a fast electron. As discussed in the introduction (See Eqs. 13), for a complete shielding of the field generated by the electron beam, the induced charge in the superconducting surface should adiabatically follow the motion of the electrons. This “adiabaticity” condition is satisfied if the transient time, $\Delta t$, a measure of the time interval over which the fields are appreciable, is larger than the characteristic time scale in the superconducting condensate, $\hbar/\Delta$, with $\Delta$ denoting the superconducting gap parameter. The transient time is given by \[ \Delta t = \frac{d}{\gamma v_e}, \] where $d$ is the distance of the superconductor from the moving charge, and $\gamma = 1/\sqrt{1 - (v_e/c)^2}$. So, our condition for an ideal shielding is given by $\Delta t > \hbar/\Delta$, or,

$$\gamma v_e < \frac{d\Delta}{\hbar}. \quad (15)$$

By putting a typical value of $d = 1\mu m$ and the gap energy of Niobium, $\Delta = 1.5\text{meV}$, (used in the experiment of Ref. 8), the adiabaticity condition of Eq. (15) gives

$$v_e < 3.6 \times 10^5\text{m/s}. \quad (16)$$

Therefore, the electron speed in the experiment of Ref. 8 was too high for a shielding of the field produced by that electron. The magnetic field generated by the moving electron penetrates freely into the superconducting barrier, and interacts with the localized flux in the situation of Ref. 8. The AB phase shift of $\pi$ observed in the experiment can be attributed to this local field interaction.

Our analysis also implies that no experiments so far have been performed under the condition of shielding the field interactions. The validity of local field interaction can, in principle, be verified through an electron interferometry based on the geometry of Figure 4. An ideal superconducting barrier shields both leakage of the localized flux and the moving electron’s field. For this purpose, the barrier should be thicker than the penetration depth without any flux trap inside the superconducting material. Also, the speed of incident electrons should be low enough that the superconductor can shield the magnetic field generated by the moving electrons. This condition could be achieved, for example, with a low-density 2-dimensional electron gas, since the Fermi velocity can be made smaller than the estimated value (Eqs. 15 and 16). In this ideal case, the moving electron’s field does not interact...
FIG. 4. Schematic of testing the Aharonov-Bohm effect with an electronic interferometer and a superconducting shield. The flux ($\Phi$) is localized inside the hollow superconductor.

with the localized flux, due to a complete shielding. The widely accepted notion - that the AB effect survives even though the field interaction is absent - is misleading, as we have shown in our analysis (both in terms of the potential-based and of the field-interaction framework). Instead, the interference pattern will be independent of the localized flux if the field interactions are perfectly shielded: That is, the AB effect vanishes under the condition of perfect shielding of local actions.

V. CONCLUSION

Since the discovery of the AB effect, it has become a common notion that a framework based on the local action of fields is impossible in quantum mechanics. Here we have provided an alternative, unified framework based on the Lorentz-covariant field-interaction, which shows that the AB effect can be universally described in terms of local interaction of
fields. Our result does not reduce the significance of the AB effect, nor that of its various applications, such as the concept of gauge field. It suggests, however, the following crucial change in our understanding of the electromagnetic interaction in quantum theory. First, vector potential is eliminated in our scheme, and thus, the force-free AB effect can be explained purely in terms of local field interactions. This possibility was recently addressed with some specific examples by Vaidman [18]. Our result shows, in a general way, that there is no physical effect in the absence of a local overlap of fields. Second, our study restores the equivalence of the potential-based and the field-interaction-based schemes in quantum theory of electromagnetic interactions. The equivalence is already present in the Lagrangian of the system, and therefore, there is no reason to discard the field-interaction approach for a quantum mechanical treatment of the interaction. Third, the AB effect can be described in a unified framework independent of the reference frame, without a need for distinction of its types [6]. With this unified viewpoint, the principle of relativity is fully restored. Fourth, there is no freedom to choose a gauge for a potential in our scheme, simply because the Lagrangian is uniquely determined by the field strengths. In fact, this is the underlying reason why the local phase can be uniquely determined in an Aharonov-Bohm loop, as shown recently by the present author [19]. Finally, it is straightforward to apply the Lorentz-covariant field-interaction framework for the electric AB effect. This implies that all kind of the AB effect can be described in a unified approach. Together with the issue of shielding, further experimental efforts are necessary to verify the validity of the local action of fields.

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