Article

A New Analysis Method Based on the Coupling Effect of Saturation and Expansion for the Shallow Stability of Expansive Soil Slopes

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Abstract: Expansive soil is a kind of unsaturated soil that is rich in hydrophilic clay minerals. The shallow slope stability of expansive soil is one of the important research topics in geotechnical engineering. However, there are no suitable methods for analyzing the shallow slope stability of expansive soil. Hence, this paper proposes a new method based on a coupling effect of saturation and expansion for analyzing the shallow slope stability. Especially, the coupling effect of saturation and expansion is introduced in detail, and used to further study the shallow slope stability. With the described coupling effect and the infinite slope, a formula calculating the overlying load of the shallow soil is established by the symmetrical limited expansion along the slope and perpendicular to the plane. Moreover, a calculation model for the factor of safety is presented according to the limit equilibrium method. The experiments are designed to demonstrate the feasibility and effectiveness of the proposed analysis method for the shallow stability of newly excavated and newly filled expansive soil slopes by rainfall. In the present study, the moisture content and shear strength of the shallow expansive soil slope are investigated, and the factor of safety is calculated. The results also show that the initial moisture content has an important influence on the shallow stability in terms of the two expansive slopes previously mentioned.

Keywords: slope engineering; expansive soil slope; rainfall; shallow stability; saturation–expansion coupling effect

1. Introduction

Expansive soil is a type of unsaturated soil rich in hydrolphilic clay minerals, and creates significant challenges for serious problems in the fields of the development of civil engineering, water conservation and transportation. In particular, the shallow slope instability of expansive soil slopes induced by rainfall is the most common engineering problem [1,2], such as the instability of the river bank slope in the middle route of China’s south-to-north water transfer project.

Since the thickness of the sliding mass of expansive soil slopes is not more than 6 m, the instability of expansive soil slopes belongs to shallow landslides. The shallow stability of expansive soil slopes is a key research topic in geotechnical engineering. Due to the dry shrinkage and wet expansion of expansive soil, drying–wetting cycles have an important influence on shallow stability. Drying–wetting cycles can result in a change in shear...
strength [3–5] and the development of cracks [6–8], facilitating the infiltration of rainwater into expansive soil slopes [9–14]. The reduction in shear strength is mainly caused by the first three drying–wetting cycles [3–5], indicating that the effect of drying–wetting cycles on shear strength is limited. The shallow layer of expansive soil slopes often cracks under atmospheric conditions [6–8]. The shallow slope in expansive soil has been studied from the cracks, including the following two aspects: the existence of cracks reduces the shear strength of expansive soil [12–14]; and, Under high rainfall conditions, the existence of cracks also causes a seepage force and hydrostatic pressure [13–17], leading to difficulties for the shallow slope’s stability [13,14,17].

Combined with the results obtained from the above-mentioned literature [3–17], the effects of drying–wetting cycles and fissures on the shallow stability of expansive soil slopes have been extensively studied. With low rainfall levels, cracks in the shallow layer of expansive soil slopes tend to heal because of water absorption and expansion. Moreover, rainfall also induces the shallow instability of newly excavated and newly filled expansive soil slopes, which are not affected by drying–wetting cycles and cracks. Hence, the rainfall-induced shallow slope instability of expansive soil cannot be completely solved by only analyzing the influences of drying–wetting cycles and cracks. It can be further explained that most of the current analysis methods for the shallow stability of expansive soil slopes mainly consider the influences of drying–wetting cycles and cracks, and are not suitable for newly excavated and newly filled expansive soil slopes.

In the above research, expansive soil is studied as a special soil. Expansive soil should be studied under the framework of unsaturated soils [18], although expansive soil slopes are unstable in shallow layers due to expansive action [19]. Based on unsaturated soil mechanics, researchers have studied the stability of expansive soil slopes in shallow layers in [20], but the presence of suction and the change in volume needs to be considered. This causes many difficulties for the shallow stability analysis of expansive soil slopes. In order to simplify the shallow stability analysis of expansive soil slopes under rainfall conditions, it is necessary to analyze the saturation process of expansive soil in detail.

The suction theory is the basis of unsaturated soil mechanics [21]. The matrix suction of expansive soil consists of capillary suction and crystal layer suction [21,22]. Matrix suction mainly occurs during the water absorption process of expansive soil, and significantly influences the behavior and properties of expansive soil. A change in the saturation of expansive soil causes a change in capillary suction. Therefore, the comprehensive action of capillary suction and surface tension can change the binding action between soil particles [23], and a change in crystal layer suction leads to an expansion deformation of expansive soil [21].

Saturation is used to describe water–gas morphology [24], and swelling deformation reflects the change in the void ratio. According to the saturation change and swelling deformation, the water absorption process of expansive soil can be divided into two stages [21,25]. In the first stage, the saturation of expansive soil changes and gas particles exist as isolated and dispersed bubbles in the soil pores [24]. In the second stage, water absorption leads to an expansion deformation of expansive soil.

To clarify the coupling effect of saturation and expansion, the saturation process of swelling soil must be analyzed, based on the aforesaid suction theory and the water absorption process of expansive soil. In the first stage of water absorption, mainly a change in capillary suction occurs, and saturation rapidly increases to a greater value. The water–gas phase of expansive soil exists in a gas-tight state, and the pore volume remains unchanged. In the air-tight state, the coalescence between soil particles formed by the combined action of capillary suction and surface tension disappears and capillary suction is negligible. The means that the mechanical properties of expansive soil are basically not affected by capillary suction. As the first stage of the water absorption is similar to the saturation process of sand, this stage can be regarded as its initial saturation stage. In the second stage of water absorption, a change in the crystal layer suction leads to an expansion deformation of expansive soil. Consequently, the pore volume increases,
and soil pores are filled with water. The second stage is unique to expansive soil and can be regarded as the second saturation stage. After the second stage of water absorption, expansive soil reaches the final saturation state.

The water potential theory is mainly used to study the characteristics of unsaturated soils. Under the action of long-term low-intensity rainfalls, the soil-water potential for the shallow soil of an expansive soil slope is mainly composed of a matrix potential, osmotic (internal) potential, and pressure (external) potential [21,22]. During the water absorption and saturation processes of shallow soil in an expansive soil slope, the permeability potential of the internal potential part remains unchanged, and the matrix potential changes. The matrix potential consists of the capillary potential and crystalline potential, where the crystal potential is closely related to the external pressure potential [21]. At the wetting peak of an expansive soil slope, the capillary potential of the soil first changes during the saturation process. After the soil reaches its initial saturation, (see Equation (2)) the pressure potential changes, and, subsequently, the changes in the left and right crystal layer potentials affect the expansion of the soil. During the second saturation stage of expansive soil, a coupling effect exists between saturation and expansion. Therefore, the effect of saturation–expansion coupling must be considered for investigating the shallow stability of expansive soil slopes under rainfall conditions.

In light of the above, in order to thoroughly study the shallow stability of expansive soil slopes under the action of rainfall, it is necessary to consider the coupling effect of saturation and expansion. In order to develop a method based on the coupling action of saturation and expansion to analyze the shallow stability of expansive soil slopes, two calculation models are proposed for the overlying load and the factor of safety affecting the final saturation of the shallow soil of the expansive soil slopes. In order to verify the feasibility and effectiveness of the proposed analysis method, focusing on the newly excavated and newly filled expansive soil slope, the moisture content and shear strength tests of the shallow soil of expansive soil slopes under the coupling action of saturation and expansion are carried out, and the factors of safety are calculated. The research results can provide a scientific basis for the treatment of expansive soil slopes.

2. Analysis of the Shallow Stability of Expansive Soil Slopes under the Coupling Effect of Saturation and Expansion

The shallow slope stability of expansive soil is one of the important research topics in geotechnical engineering. However, there are no suitable methods for analyzing the shallow slope stability of expansive soils. Therefore, it is necessary to develop an effective method for studying the shallow slope stability of expansive soil from the coupling effect of saturation and expansion.

2.1. Infinite Slope

The length and thickness of the sliding body in the shallow layers of expansive soil slopes become relatively large, and the sliding surface is almost parallel to the slope surface [20]. Therefore, an infinite slope can be used for the study of shallow slope stability in expansive soil [20]. The infinite slope formulation has been widely used to study the shallow slope stability in unsaturated soils under rainfall conditions [26–32]. Since only one vertical force balance needs to be considered (see Figure 1), the assumption of an infinite slope can simplify the shallow slope stability analysis of expansive soil.
is the weight of water, and $\gamma_{\alpha}$ is the saturation weight of the soil strip, $\alpha = -\frac{1}{2}$.

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Figure 1. Schematic diagram of the mechanical analysis.

2.2. Calculation Model of the Overlying Load of Shallow Soil on an Expansive Soil Slope

The overlying load has a significant impact on the expansive deformation of expansive soil and the final saturation of the shallow soil of expansive soil slopes. Therefore, to study the shallow stability of expansive soil slopes under the coupling effect of saturation and expansion, a suitable calculation model for overlying load needs to be developed.

To analyze the shallow stability, the assumption of the infinite slope is adopted. Moreover, the following assumptions are made under the condition of rainfall infiltration: (1) a long-term low-intensity rainfall occurs, and no ponding is created on the slope; (2) the wetting front is parallel to the slope surface, and the wetting peak continues to advance into the slope after the soil becomes completely saturated at the wetting peak; and (3) the effect of expansion deformation on slope geometry is negligible.

The sliding surface for the rainfall-induced shallow slope stability of the expansive soil is almost parallel to the slope [20]. For the expansive soil slope, under the condition of rainfall infiltration, the matric suction in the wet area decreases, and a dangerous surface appears at the wet front. Figure 1 describes the stresses acting on the shallow of the expansive soil slope, which shows the shear and normal stresses at the wetting front. The overlying load, affecting the final saturation of the soil at the wetting front, is calculated according to the equilibrium stress state. The above wet peak is considered as a unit soil strip. The length of the unit soil strip along the slope surface is $l$, and its height and thickness are $z$ and $h$, respectively. The schematic diagram of the soil strip is displayed in Figure 1.

According to the saturation process of expansive soil, it is assumed that hydrostatic pressure was formed after the initial saturation of the soil at the moist front. The normal stress on the bottom slope of the soil strip $\sigma_n$ can be expressed as:

$$\sigma_n = (\gamma_{\text{sat}} - \gamma_w)z \cos^2 \alpha$$  \hspace{1cm} (1)

where $\gamma_{\text{sat}}$ is the saturation weight of the soil strip, $\gamma_w$ is the weight of water, and $\alpha$ is the slope angle.

If the above assumptions are not considered, the normal stress of the bottom slope of the soil strip can be written as Equation (2).

$$\sigma_n = \gamma_{\text{sat}}z \cos^2 \alpha$$  \hspace{1cm} (2)

Due to the symmetrical limited expansion along the slope and perpendicular to the plane, the expansion deformation of the expansive soil slope is along the normal direction of the vertical slope. Therefore, the saturation process of the shallow soil on the expansive soil slope can be regarded as a one-dimensional expansion. The procedure is similar to a confined one-dimensional expansion chamber experiment. Based on the above analysis, it is noticeable from Equations (1) and (2) that the normal stress acting on the bottom slope of the soil strip is the smallest, implying that the overlying load limiting the expansion of the
soil at the wet front is the smallest, implying that the smaller the overlying load, the smaller the cohesion and internal friction angle of expansive soil after the final saturation [33,34]. Moreover, the smaller the overlying load, the smaller the restriction on the swelling capacity of expansive soil [24]. Hence, it can be inferred that the overlying load hindering the final saturation of the soil at the wetting front is the smallest, the water absorption capacity of the soil at the wetting front is the largest after its complete saturation, and the cohesion and internal friction angle of the soil are the smallest. Therefore, the shallow layer of the infinite slope could be dangerous for engineering applications. Considering the stability of the shallow layer of a non-expansive soil slope, the overlying load of the soil in the wet area can be calculated according to Equation (1).

In Equation (1), \( \gamma_{sat} - \gamma_w \) is the floating weight \( \gamma' \), and \( \gamma' \) can be expressed as:

\[
\gamma' = \frac{m_s - v_s \rho_w}{z l \cos \alpha} g
\]  

(3)

where \( v_s \) is the volume of the soil strip, given by

\[
v_s = \frac{m_s}{G_s}
\]  

(4)

with the specific gravity \( G_s \) of soil particles and the solid particle mass

\[
m_s = \rho_d z l \cos \alpha
\]  

(5)

and \( \rho_d \) is the initial dry density of the soil strip.

The vertical height and thickness of the soil strip maintains the following relationship.

\[
h = z \cos \alpha
\]  

(6)

Now, substituting Equations (3)–(6) into Equation (1), the overlying load affecting the final saturation of the soil at the wetting front \( \sigma_{ol} \) can be expressed as Equation (7).

\[
\sigma_{ol} = \left( \rho_d h - \frac{\rho_d h \rho_w}{G_s} \right) g \cos \alpha
\]  

(7)

2.3. Calculation Model of the Factor of Safety

The limit equilibrium method has been widely applied to slope stability analysis. Thus, it is employed to calculate the factor of safety.

After the final saturation of the soil at the wetting front of the expansive soil slope, the shallow slope is the most prone to landslides. Now, taking the above unit soil strip as the research object, the factor of safety \( F_s \) at the wetting peak \( H \) can be calculated as:

\[
F_s = \frac{\tau_f}{\tau_m}
\]  

(8)

where \( \tau_f \) is the undrained shear strength and \( \tau_m \) is the shear stress on the sliding surface.

According to Equation (1), the shear strength \( \tau_f \) can be formulated as:

\[
\tau_f = (\gamma_{sat} - \gamma_w) z \cos^2 \alpha \tan \varphi + c
\]  

(9)

where \( \varphi \) and \( c \) are the undrained internal friction angle and cohesive force of the soil at the wetting front after the final saturation, respectively. When the initial saturation of expansive soil is completed, the stress state is based on the principle of effective stress.

The shear stress \( \tau_m \) can be calculated as Equation (10).

\[
\tau_m = \gamma_{sat} z \cos \alpha \sin \alpha
\]  

(10)
Now, substituting Equations (6), (9), and (10) into Equation (8), the factor of safety \( F_s \) can be formulated as Equation (11).

\[
F_s = \left( \frac{\gamma_{sat} - \gamma_w}{\gamma_{sat} \tan \alpha} \right) \tan \varphi + \frac{c}{\gamma_{sat} h \sin \alpha}
\]  

(11)

3. Experimental Study on the Water Content and Shear Strength of Shallow Soil in Expansive Soil Slopes under the Coupling Action of Saturation and Expansion

In this section, the experiments are presented to investigate the moisture content and shear strength for shallow soil in newly excavated and newly filled expansive soil slopes by the described coupling effect. The initial saturation of shallow soil on expansive soil slope affects the overlying load, while the overlying load affects the final saturation of shallow soil by affecting the expansion. Hence, it can be observed that the saturation–expansion coupling effect of shallow soil on expansive soil slopes is reflected by the overlying load. For newly excavated and newly filled expansive soil slopes, the corresponding moisture content and shear strength are experimentally calculated by using the developed overlying load calculation model.

Nanning is a typical expansive soil distribution area in China. The expansive soil of Nanning is a kind of clay rich in hydrophilic clay minerals. In order to produce expansive soil slope samples, according to the T0101-2007 of the Highway Geotechnical Test Standard [35], expansive soil was collected from a project site in Nanning, with a sampling depth of about 2 m; the natural moisture content of the soil sample was 19% and the natural dry density of the soil sample was 1.54 g/cm\(^3\). In order to make the initial water content of the samples representative, four different initial water contents were selected. One initial water content was below the plastic limit, another initial water content was close to the natural water content, and the other two initial water contents were above the natural water content, but not above the liquid limit. According to Article 3.1 of the Standard Geotechnical Test Methods (GB/T 50123-1999) [36], the soil sample with the prepared target moisture content was sealed for 24 h to make the moisture of the sample profile uniform, and then the prepared soil sample was made into a ring-knife specimen with a diameter of 618 mm and a height of 20 mm by the static pressing method. Therefore, the ring-knife samples with initial moisture contents of 15%, 18%, 22%, 25% and a dry density of 1.5 g/cm\(^3\) (close to the natural dry density of 1.54 g/cm\(^3\)) were prepared. The physical indexes of the soil samples are listed in Table 1.

| Plastic Limit (%) | Liquid Limit (%) | Free Swelling Ratio (%) | Soil Specific Gravity |
|------------------|------------------|-------------------------|----------------------|
| 18               | 36.8             | 51                      | 2.7                  |

Table 1. Physical indexes of the soil samples.

For an expansive soil slope with undeveloped fissures, due to the small permeability coefficient of the expansive soil, rainfall mainly affect the soil in the shallow layer of the expansive soil slope that is tens of centimeters thick. Therefore, in this study, it is considered that rainfall mainly affected the shallow 60 cm thick soil of the slope.

Rainfall can induce the shallow instability of the expansive soil slopes when the slope is low. The slope angle and height are 10° and 6 m, respectively. For calculation convenience, the thickness of the soil strip is defined as the depth. Moreover, the locations at 0 cm, 20 cm, 40 cm, and 60 cm away from the slope are taken as research objects. The density of the water intake is 1 g/cm\(^3\), and the gravity acceleration is 10 m/s\(^2\). According to the aforesaid overlying load calculation model and Equation (7), overlying loads affecting the final saturation of the soil at the expansive soil depths of 0 cm, 20 cm, 40 cm, and 60 cm are calculated (Table 2). The overlying load is only related to the expansive soil depth under the same initial dry density.
Table 2. Overlying loads at different expansive soil depths.

| Initial Moisture Content (%) | Depth (cm) | Overlying Load (kPa) |
|-----------------------------|------------|----------------------|
| 15                          | 0          | 0                    |
|                             | 20         | 1.86                 |
|                             | 40         | 3.72                 |
|                             | 60         | 5.58                 |
| 18                          | 0          | 0                    |
|                             | 20         | 1.86                 |
|                             | 40         | 3.72                 |
|                             | 60         | 5.58                 |
| 22                          | 0          | 0                    |
|                             | 20         | 1.86                 |
|                             | 40         | 3.72                 |
|                             | 60         | 5.58                 |
| 25                          | 0          | 0                    |
|                             | 20         | 1.86                 |
|                             | 40         | 3.72                 |
|                             | 60         | 5.58                 |

3.1. Test Scheme

Method

The swelling forces of the samples were first completed. To complete the final saturation of the samples, the applied overlying load should be smaller than the expansion forces of the samples. According to Article 22 of the Standard for Geotechnical Engineering Test Methods (GB/T 50123-1999) [36], the expansion force test of the samples was carried out in a low-pressure consolidation instrument by the loading balance method, and the volume of the samples was constant during the test. The initial moisture contents of the samples were 15%, 18%, 22%, and 25%, and the dry density was 1.5 g/cm$^3$. The measured expansion forces of these four samples were 60 kPa, 52 kPa, 45 kPa, and 40 kPa, respectively. Comparing the expansion force test results with the calculated overlying loads presented in Table 2, it is clear that the samples are completely saturated.

Furthermore, a final saturation test was carried out on the samples in a low-pressure consolidation instrument by the Standard for Geotechnical Test Methods (GB/T 50123-1999) [36]. The schematic diagram of the saturation test is presented in Figure 2. To prevent the compression of the samples under the application of the overlying loads, the diameter of the upper permeable stones was set slightly larger than that of the ring cutter. The final saturation was considered complete when the expansion deformation was constantly less than 0.01 mm for 2 h, and the saturation was not less than 95%. The average value of the swelling deformations of the four samples after the final saturation was taken as the swelling deformation of the samples.

Finally, the samples were sheared after the final saturation. As the self-weight stress of the shallow soil was small, the four samples, after the final saturation, were subjected to a rapid shear test under the vertical pressures of 12.5 kPa, 18.75 kPa, 25 kPa, and 31.25 kPa. The rapid shear test was carried out according to Article 18.3 of the Standard for Soil Test Methods (GB/T 50123-1999) [36]. The drying method was used to measure the moisture contents of the four samples after shearing, and the average value of the moisture contents was taken as the moisture content of the samples after the final saturation.

3.2. Results

3.2.1. Saturation Test Results

The expansion deformation test results of the samples with the initial water contents of 15%, 18%, 22%, and 25% after the final saturation under the overlying loads of 0 kPa, 1.86 kPa, 3.72 kPa, and 5.58 kPa are presented in Figure 3. It is noticeable that the expansion
deformation of the samples decreases with the increase in the overlying load. Since the overlying load is larger, the expansion limitation is larger.

![Diagram of saturation test](image)

**Figure 2.** Schematic diagram of the saturation test. (a) Without the application of the overlying loads. (b) Under the application of the overlying loads.

![Graph of expansion deformation](image)

**Figure 3.** Expansion deformation test results.
For the samples with initial moisture contents of 15%, 18%, 22%, and 25%, the overlying loads were 0 kPa, 1.86 kPa, 3.72 kPa, and 5.58 kPa. The moisture contents of the samples after saturation are presented in Figure 4. Due to the limitation of the overlying load on the expansion, it is clear that the moisture content of the samples decreases with the increase in the overlying load.

![Figure 4. Water content test results.](image)

### 3.2. Results

#### 3.2.1. Saturation Test Results

The expansion deformation test results of the samples with the initial water contents of 15%, 18%, 22%, and 25% after the final saturation under the overlying loads of 0 kPa, 1.86 kPa, 3.72 kPa, and 5.58 kPa are presented in Figure 3. It is noticeable that the expansion deformation of the samples decreases with the increase in the overlying load. Since the overlying load is larger, the expansion limitation is larger.

![Figure 3. Expansion deformation test results.](image)

### 3.2.2. Shear Strength Test Results

The least-square method was adopted to fit the shear strength test results of the four samples (Figure 5). Due to the limitation of the overlying load on the expansion, it can be seen from Figure 5 that the shear strength of the four samples increases with the increase in the overburden load. It is noticeable from Figure 5a,b that the undrained cohesion of the samples with the initial water contents of 15% and 18% after the final saturation is 0 kPa. However, when the initial water contents are 22% and 25%, the undrained cohesion of the samples after the final saturation is greater (Figure 5c,d). Moreover, the undrained internal friction angles of the samples after the final saturation are small. Combined with the saturation process of expansive soil, the reason for the above test results may be caused by expansion.

### 3.3. Influence of the Overlying Load on the Water Content

Figure 6 is obtained by Figures 3 and 4. Since the larger the expansion deformation, the greater the change in the crystalline suction; Figure 6 shows that the final moisture content of the sample increases with the increase in the expansion deformation, and it is evident that the final water content of the samples increases with the increase in the swelling deformation. Figure 4 shows that the larger the overlying load, the smaller the expansion deformation. According to the saturation process of expansive soil, the larger the overlying load, the smaller the change in the crystal layer suction; therefore, the larger the overlying load, the smaller the final moisture content of the samples. It is clear from Equation (7) that the water content of the shallow soil of the expansive soil slope decreased with the increase in the soil depth after saturation.

### 3.4. Influence of the Overlying Load on the Shear Strength Parameters

Figure 7, manifesting the change in the undrained cohesion with the overlying load, is based on Figure 5. The overlying load has an important effect on the undrained cohesion, due to its influence on the expansion.
internal friction angles of the samples after the final saturation are small. Combined with the saturation process of expansive soil, the reason for the above test results may be caused by expansion.

\[
\begin{align*}
\phi = 5.2^\circ & \quad c = 0 \text{kPa} \\
\phi = 5.4^\circ & \quad c = 0 \text{kPa} \\
\phi = 5.1^\circ & \quad c = 0 \text{kPa} \\
\phi = 4.9^\circ & \quad c = 0 \text{kPa}
\end{align*}
\]

\[
\begin{align*}
\phi = 5.8^\circ & \quad c = 7.951 \text{kPa} \\
\phi = 6.1^\circ & \quad c = 8.715 \text{kPa} \\
\phi = 5.7^\circ & \quad c = 7.339 \text{kPa} \\
\phi = 6.3^\circ & \quad c = 6.88 \text{kPa}
\end{align*}
\]

\[
\begin{align*}
\phi = 7^\circ & \quad c = 11.001 \text{kPa} \\
\phi = 6.6^\circ & \quad c = 7.051 \text{kPa} \\
\phi = 6.4^\circ & \quad c = 7.339 \text{kPa} \\
\phi = 6.3^\circ & \quad c = 6.88 \text{kPa}
\end{align*}
\]

\[
\begin{align*}
\phi = 7.8^\circ & \quad c = 12.232 \text{kPa} \\
\phi = 7.4^\circ & \quad c = 11.001 \text{kPa} \\
\phi = 7.2^\circ & \quad c = 9.939 \text{kPa} \\
\phi = 7^\circ & \quad c = 9.17 \text{kPa}
\end{align*}
\]

**Figure 5.** Shear strength test results. (a) Initial water content: 15%. (b) Initial water content: 18%. (c) Initial water content: 22%. (d) Initial water content: 25%.
After the final saturation, the coalescence between the soil particles under the combined undrained cohesion of the two samples after saturation is completely lost. The undrained cohesive force of the shallow soil mass of the expansive soil slopes with the initial water greater the damage to the soil skeleton, and the larger the overlying load, the smaller the contents for cases of 15% and 18% after saturation are greater than the liquid limit (Figure 5). The combined action of capillary suction and surface tension disappeared. The final water contents for cases of 15% and 18% were completely lost after saturation, whereas the undrained cohesive force of the shallow soil mass of the expansive soil slopes with the initial water contents of 22% and 25% increased with the increase in the soil depth.

Initial water content: 22%.

The undrained cohesion of the samples with the initial water contents of 15% and 18% was 0 kPa, but for cases of 22% and 25%, it increased with the increase in the overlying load. After the final saturation, the coalescence between the soil particles under the combined action of capillary suction and surface tension disappeared. The final water contents for cases of 15% and 18% after saturation are greater than the liquid limit (Figure 4), indicating that there is almost no coupling force between the soil particles, and the undrained cohesion of the two samples after saturation is completely lost. The undrained cohesion of the samples with the initial moisture contents of 22% and 25% increases with the increase in the overlying load (Figure 7). The larger the expansion deformation, the greater the damage to the soil skeleton, and the larger the overlying load, the smaller the expansion deformation. Hence, under the condition of rainfall infiltration, the undrained cohesive force of the shallow soil mass of the expansive soil slopes with the initial water contents of 15% and 18% were completely lost after saturation, whereas the undrained cohesive force of the shallow soil mass of the expansive soil slopes with the initial water contents of 22% and 25% increased with the increase in the soil depth.

It is noticeable that the undrained internal friction angle of the samples increased with the increase in the overlying load. The undrained internal friction angle is determined by the surface friction of the soil particles and the occlusal friction caused by the embedding and interlocking of the soil particles. Due to the effect of crystal layer suction, the surface of soil particles became smoother with the increase in the expansion deformation, meaning that the embedding and interlocking failure between the soil particles was intensified.

**Figure 6.** Influence of expansion deformation on the final water content.

**Figure 7.** Influence of the overlying load on cohesion.

| Overlying load (kPa) | Cohesion (kPa) | ϕ |
|----------------------|--------------|----|
| 0                    | 0            | 7.4° |
| 2                    | 2            | 7.8° |
| 4                    | 4            | 8.0° |
| 6                    | 6            | 8.2° |
| 8                    | 8            | 8.4° |
| 10                   | 10           | 8.6° |
| 12                   | 12           | 8.8° |
| 14                   | 14           | 9.0° |
| 16                   | 16           | 9.2° |
| 18                   | 18           | 9.4° |

**Figure 8.** Change in the undrained internal friction angle with the overlying load.
As the expansion deformation increased, the undrained internal friction angle decreased. It is observable from Figure 3 that, with the increase in the overlying load, the expansion deformation decreases. This implies that the undrained internal friction angle of the samples increases with the increase in the overlying load. Therefore, for the expansive soil slopes with the initial water contents of 15%, 18%, 22%, and 25%, the undrained internal friction angle increased with the increase in the soil depth after saturation.

![Figure 8. Influence of the overlying load on the internal friction angle.](image)

### 4. Calculation and Analysis of the Factor of Safety

According to the above experimental results, the factors of safety for the shallow expansive soil slopes with different initial water contents are calculated by Equation (11).

For the expansive soil slopes with the initial water contents of 15% and 18% under the condition of rainfall infiltration, the undrained cohesive force at the soil depths of 0 cm, 20 cm, 40 cm, and 60 cm was completely lost. The calculation of the factor of safety of the expansive soil slopes with the initial water contents of 15% and 18% do not need to consider the second term on the right-hand side of Equation (11). To further simplify the calculation of the shallow safety factor, for the molecular part of the first term on the right-hand side of Equation (11), the water weight term is not considered. Hence, the factors of safety at the wetting peak of the expansive soil slopes with the above two contents are calculated by the following formula:

$$
F_s = \tan \varphi \tan \alpha
$$

(12)

When the undrained cohesion of the shallow soil on the expansive soil slope is significant after saturation is completed, the cohesion has an important influence on the safety factor, and Equation (12) does not need to be applied to this situation. It can be seen that Equation (12) is not suitable for calculating the safety factor of expansive soil slopes with an initial moisture contents of 22% and 25% under the condition of rainfall infiltration.

In order to understand the variation law of shear stress during the advancing process of the wetting front, the saturation weight of the soil strip is determined. The saturation weight of the expansive soil is presented as follows:

$$
\gamma_{sat} = \frac{m_s + m_w}{z l \cos \alpha} g
$$

(13)

where \( w \) is the moisture content of the soil strip.

Now, substitute Equations (5) and (13) into Equation (10), and the shear stress is calculated by Equation (14).

$$
\tau_m = (\rho_d h + \rho_d h w) g \sin \alpha
$$

(14)
For the expansive soil slopes with the initial water contents of 22% and 25% under the condition of rainfall infiltration, the factor of safety at the wetting peak has the following relationship, as is expressed in Equation (15):

\[
F_s > \frac{c}{(\rho_d h + \rho_w h w)g \sin \alpha}
\]  

(15)

Based on the variation law of undrained cohesion and moisture content with soil depth during the advancement of the wetting front, the undrained cohesion of the expansive soil at the slope surface is the smallest, and the final moisture content of the expansive soil at the slope surface is the greatest. Therefore, for the expansive soil slopes with the initial moisture contents of 22% and 25%, the factor of safety on the slopes has the following relationship:

\[
F_s > \frac{c_0}{(\rho_d h_{\text{max}} + \rho_w h_{\text{max}} w_0)g \sin \alpha}
\]

(16)

where \( h_{\text{max}} \) is the maximum infiltration depth of rainwater after the final saturation (0.6 m), \( c_0 \) and \( w_0 \) are the undrained cohesive force and final moisture content of the expansive soil on the surface of the slopes, respectively. In particular, \( c_0 \) and \( w_0 \) are obtained by performing the above experiments.

The factors of safety of the expansive soil slopes with the initial water contents of 15% and 18% at different soil depths (e.g., 0, 20, 40, and 60 cm) are calculated by Equation (12) (Figure 9). Due to the small undrained internal friction angle and the complete loss of undrained cohesion after the final saturation of the expansive soil slopes with the initial moisture contents of 15% and 18%, it is noticeable that the factors of safety of the expansive soil slopes with initial water contents of 15% and 18% at these depths are less than 1, indicating that the slopes have lost stability under rainfall infiltration. Therefore, the slope instability caused by rainfall is minor and gentle.

![Figure 9. Factors of safety for the expansive soil slopes with the initial water contents of 15% and 18% at different soil depths.](image_url)

The factors of safety of the expansive soil slopes with the initial water contents of 22% and 25% under the condition of rainfall infiltration are calculated by Equation (16), and the corresponding results are presented in Table 3. It can be observed from Table 3 that rainfall does not induce the shallow instability of expansive soil slopes with the two water contents. As the undrained cohesion of the shallow soil on these slopes is large, it can be observed from Table 3 that the factors of safety for the two slopes are great, indicating that rainfall will not induce the shallow instability of the two slopes.
### Table 3. Factors of safety for the expansive soil slopes with the initial water contents of 22% and 25%.

| Initial Water Content (%) | Factor of Safety |
|--------------------------|------------------|
| 22                       | >3.23            |
| 25                       | >4.41            |

Combined with the above calculation results, with respect to the factor of safety, the factors of safety for the initial moisture contents of 15% and 18% at different depths were less than 1, and they were much greater than 1 for 22% and 25%. This means that rainfall easily induces the shallow instability of the expansive soil slopes with the initial moisture contents of 15% and 18%, while rainfall cannot easily induce shallow stability for the cases of 22% and 25%. Therefore, it is shown that the initial moisture content has an important influence on the shallow stability of the newly excavated and newly filled expansive soil slopes.

### 5. Discussion

In this study, a novel shallow stability analysis method of expansive soil slopes under the coupling effect of saturation and expansion was presented. According to this method, the shallow stability of four representative initial moisture content expansive soil slopes was analyzed, and the results show that rainfall easily induces the shallow instability of expansive soil slopes with initial moisture contents of 15% and 18%. It can be observed that the shallow instability of expansive soil slopes has been verified, indicating that the proposed analysis method is reasonable.

The proposed shallow stability analysis method for expansive soil slopes consisted of two parts: the overlying load calculation model, and the factor of safety calculation model. It is noticeable from Equations (7) and (11) that the parameters of these calculation models are commonly used in geotechnical engineering. Moreover, when the shallow stability of the expansive soil slopes was analyzed, the suction and volume did not need to be tested, implying that the proposed shallow stability analysis method for expansive soil slopes was easy to implement.

In this study, for the newly excavated and newly filled expansive soil slopes, the calculation results of the factors of safety show that, when the slope of the expansive soil slope is 10 degrees, the rainfall also induces the shallow instability of the expansive soil slopes, indicating that the shallow instability of this kind of expansive soil slope cannot be effectively solved by reducing the slope.

### 6. Conclusions

This paper elucidated the coupling effect of saturation and expansion for expansive soil, and proposed a new method based on the coupling effect for analyzing the corresponding shallow stability. Especially, an infinite slope was used for the shallow slope stability analysis in expansive soil. Based on the symmetrical limited expansion along the slope and perpendicular to the plane, the calculation of the overlying load of the shallow soil in expansive soil slopes was established. Furthermore, according to the limit equilibrium method, the calculation model of the factor of safety was presented. From the experiment results in newly excavated and newly filled expansive soil slopes, the parameters for analyzing the shallow stability of expansive soil slopes were obtained, and the shallow stability of the expansive soil slopes with different initial water contents was analyzed. The main observations of this work are presented below:

1. A new method for analyzing the shallow stability of expansive soil slopes was proposed, and it was shown to be reasonable and easy to implement.
2. Rainfall could easily induce instability in the shallow layer of the expansive soil slopes with initial water contents of 15% and 18%, but this was not applied for the cases of 22% and 25%. Therefore, for the newly excavated and newly filled expansive soil slopes, the initial water content had an important influence on the slope stability.
(3) For newly excavated and newly filled expansive soil slopes, reducing the slope could not effectively solve the shallow instability of such expansive soil slopes.

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