Some new theorems of α-admissible mappings on c-distance in cone metric spaces over Banach algebras

Han Yan\textsuperscript{1,a}, Bian Yi-duo\textsuperscript{2b}, Shan Chang-ji\textsuperscript{3c}

\textsuperscript{1} School of Mathematics and Statistics, Zhaotong University, Zhaotong, 657000, Yunnan, PRC
\textsuperscript{2} School of Foreign Languages, Zhaotong University, Zhaotong, 657000, Yunnan, PRC
\textsuperscript{3} Institute of Physics and Information Engineering, Zhaotong University, Zhaotong, 657000, Yunnan, PRC
\textsuperscript{a}553752771@126.com, \textsuperscript{b}2098373248@qq.com, \textsuperscript{c}157458629@qq.com

Abstract. In this work, we introduce some new fixed point results of α-admissible mappings on c-distance in cone metric spaces over Banach algebras, which are significant extensions and generalizations of many important known results. Furthermore, we give an example to illustrate our main results.

1. Introduction and preliminaries

Cone metric space was introduced in 2007 by Huang and Zhang[1], which is one of the generalizations of metric space. Subsequently, there are many related fixed point theorems in [2-4]. In view of the arguments given in [5, 6], Liu and Xu [7] defined the concept of cone metric space over Banach algebra and proved some fixed point results in this space. Moreover, they gave some examples to elucidate that fixed point results in cone metric spaces over Banach algebras are not equivalent to metric spaces (in usual sense). In 2011, c-distance was defined by Cho et al.[8] and Wang et al.[9]. This is a cone version of w-distance of Kada et al.[10]. On the other hand, a new type of mapping called α-admissible was introduced in metric space by [11]. Recently, some fixed point theorems of α-admissible mappings in cone metric spaces or cone metric spaces over Banach algebras were investigated by [12-15].

In our work, we introduce some fixed point results of α-admissible mappings on c-distance in cone metric spaces over Banach algebras, which extend some meaningful known results in the literature [7, 12-15]. In addition, an example is also provided to show that the main results improve and generalize the corresponding conclusions in the literature.

First, we review some basic concepts about Banach algebras and cone metric spaces. A Banach algebra $A$ is a real Banach space in which an operation of multiplication is defined subject to the following properties: for all $x, y, z \in A, \lambda \in \mathbb{R}$ : (1) $(xy)z = x(yz)$; (2) $x(y + z) = xy + xz$ and $(x + y)z = xz + yz$; (3) $\lambda(xy) = (\lambda x)y = x(\lambda y)$; (4) $\|xy\| = \|x\|\|y\|$\textsuperscript{[17]}.
In this paper, we shall assume that the Banach algebra $A$ has a unit (i.e., a multiplicative identity) $e$ such that $xe = ex = x$ for all $x \in A$. An element $x \in A$ is said to be invertible if there is an inverse element $y \in A$ such that $xy = yx = e$. The inverse of $x$ is denoted by $x^{-1}$ (see [17]).

**Proposition 1.1.** ([17]) Let $A$ be a real Banach algebra with a unit $e$ and $x \in A$. If the spectral radius $r(x)$ of $x$ is less than 1, i.e.,

$$r(x) = \lim_{n \to \infty} \|x^n\|^\frac{1}{n} < 1,$$

then $e - x$ is invertible. Actually,

$$(e - x)^{-1} = \sum_{n=0}^{\infty} x^n.$$  

A subset $P$ of $A$ is called a cone if: (1) $P$ is nonempty, closed and $\{0, e\} \in P$ where $0$ denotes the zero element of $A$; (2) $a, b \in \mathbb{R}^+ \Rightarrow aP + bP \subseteq P$; (3) $P^2 = PP \subseteq P$; (4) $P \cap (-P) = \{0\}$.

For a given cone $P \subseteq A$, we can denote a partial ordering $\leq$ with respect to $P$ by $x \leq y$ if and only if $y - x \in P$. We shall write $x < y$ if $x \leq y$ and $x \neq y$, while $x \ll y$ will stand for $y - x \in int P$, where $int P$ denotes the interior of $P$.

A cone $P$ is called normal if there is a number $K > 0$ such that for all $x, y \in A$,

$$\theta \leq x \leq y \Rightarrow \|x\| \leq K \|y\|,$$

The least positive number satisfying the above inequality is called the normal constant of $P$.

**Definition 1.2.** ([1], [7]) Let $X$ be a nonempty set. Suppose that the mapping $d : X \times X \to A$ satisfies:

1. $\theta \leq d(x, y)$ for all $x, y \in X$ and $d(x, y) = \theta$ if and only if $x = y$;
2. $d(x, y) = d(y, x)$ for all $x, y \in X$;
3. $d(x, y) \leq d(x, z) + d(z, y)$, for all $x, y, z \in X$.

Then $d$ is called a cone metric on $X$ and $(X, d)$ is called a cone metric space over a Banach algebra $A$.

**Definition 1.3.** ([1], [7]) Let $(X, d)$ be a cone metric space over a Banach algebra $A$, $x \in X$ and $\{x_n\}$ a sequence in $X$. Then we say that is:

1. $(x_n)$ converges to $x$ if for every $c \in int P$, there is a natural number $N$ such that for all $n > N, d(x_n, x) \ll c$;
2. $(x_n)$ is a Cauchy sequence if for every $c \in int P$, there is a natural number $N$ such that for all $n, m > N, d(x_n, x_m) \ll c$;
3. $(X, d)$ is a complete cone metric space over Banach algebra $A$ if every Cauchy sequence is convergent in $X$.

**Definition 1.4.** ([8], [9], [16]) Let $(X, d)$ be a cone metric space over a Banach algebra $A$. Then the mapping $q : X \times X \to A$ is called a $c$-distance on $X$ if the following are satisfied:

1. For all $x, y \in X$, $\theta \leq q(x, y)$;
2. For all $x, y, z \in X$, $q(x, z) \leq q(x, y) + q(y, z)$;
3. For each $x \in X$ and $n \geq 1$, if $q(x, y_n) \leq u$ for some $u = u_x \in P$, then $q(x, y) \leq u$,
4. For all $c \in A$ with $c \gg \theta$, there exists $w \in A$ with $w \gg \theta$ such that $q(z, x) \ll w$ and
\[ q(z, y) \leq w \text{ imply } d(x, y) \leq c. \]

**Remark 1.5.** ([9]) The following facts are well-known.
(1) \( q(x, y) = q(y, x) \) does not necessarily hold for all \( x, y \in X \).
(2) \( q(x, y) = \Theta \) is not necessarily equivalent to \( x = y \) for all \( x, y \in X \).
(3) The \( c \)-distance is a great generalization of the \( w \)-distance.
(4) If \( q(x, y) = d(x, y) \), then \( q \) is a \( c \)-distance. That is, \( c \)-distance is also a generalization of cone metric.

**Definition 1.6.** ([11]) A mapping \( f : X \rightarrow X \) is said to be continuous at \( x \in X \), if for every sequence \( \{x_n\} \) in \( X \) such that \( x_n \rightarrow x (i \rightarrow \infty) \), we have \( f(x_n) \rightarrow f(x) (i \rightarrow \infty) \).

**Lemma 1.7.** ([18]) Let \( (X, d) \) be a cone metric space over Banach algebra and \( q \) be a \( c \)-distance on \( X \). Let \( \{x_n\} \) and \( \{y_n\} \) be sequences in \( X \) and \( x, y, z \in X \). Suppose that \( \{u_n\}, \{v_n\} \) are two sequences in \( P \) converging to \( \Theta \). Then the following hold:
1. If \( q(x_n, y) \leq u_n \) and \( q(x_n, z) \leq v_n \), then \( y = z \).
2. If \( q(x_n, y_n) \leq u_n \) and \( q(x_n, z) \leq v_n \), then \( \{y_n\} \) converges to \( z \in X \).
3. If \( q(x_n, x_m) \leq u_n \) for \( m > n \), then \( \{x_n\} \) is a Cauchy sequence in \( X \).

**Lemma 1.8.** ([17]) Let \( A \) be a Banach algebra with a unit \( e \). If \( x, y \in A \) and \( x \) commutes with \( y \), then
\[ r(x + y) \leq r(x) + r(y) \; ; \; r(xy) \leq r(x)r(y). \]

**Lemma 1.9.** ([15]) Let \( A \) be a Banach algebra with a unit \( e \) and let \( k \) be a vector in \( A \). If \( 0 \leq r(k) < 1 \), then we have \( r((e - k)^{-1}) \leq (1 - r(k))^{-1} \).

## 2. Main Results

Inspired by Definition 2.2 and Theorem 2.2(iii) in [11], we introduced the concept of \( \alpha \)-admissible mapping and \( \alpha \)-regularity for \( c \)-distance in cone metric spaces over Banach algebras \( A \).

**Definition 2.1.** Let \( (X, d) \) be a cone metric space over Banach algebra \( A \) and \( q \) be a \( c \)-distance on \( X \), \( \alpha : X \times X \rightarrow A \) and \( f : X \rightarrow X \) be mappings. Then
(1) \( f \) is \( \alpha \)-admissible mapping if \( \alpha(x, y) \geq e \Rightarrow \alpha(fx, fy) \geq e, \forall x, y \in X \).
(2) \( (X, d) \) is \( \alpha \)-regular if for any sequence \( \{x_i\} \) in \( (X, d) \) with \( \alpha(x_i, x_{i+1}) \geq e \) for all \( i \in \mathbb{N} \) and \( x_i \rightarrow x^* \in X (i \rightarrow \infty) \), then \( \alpha(x_i, x^*) \geq e \) for all \( i \in \mathbb{N} \).

**Theorem 2.2.** Let \( (X, d) \) be a complete cone metric space over Banach algebra \( A \) and \( P \) be the underlying solid cone. Let \( q \) be a \( c \)-distance on \( X \), \( f : X \rightarrow X \) and \( \alpha : X \times X \rightarrow A \) be two functions. Suppose the following conditions are satisfied:
1. \( f \) is \( \alpha \)-admissible mapping;
2. there exists \( x_0 \in X \) such that \( \alpha(x_0, fx_0) \geq e \);
3. \( f \) is continuous;
4. for all \( x, y \in X \) with \( \alpha(x, y) \geq e \)
\[ q(fx, fy) \leq a_1 q(x, y) + a_2 q(x, fx) + a_3 q(x, fy), \] (2.1)
where, \( a_i \in P(i = 1, 2, 3) \) such that \( r(a_1) + r(a_1 + a_2 + a_3) < 1 \). If \( a_i \) commutes with \( a_1 + a_2 + a_3 \), then \( f \) has a fixed point.
Proof. Let $x_0$ be an element of $X$ such that $\alpha(x_0, f x_0) \geq e$. Set $x_n = f x_{n-1} = f^n x_0$, $n = 0, 1, 2 \ldots$ Suppose $x_n = x_{n+1}$ for some $n \in \mathbb{N}$, then $x = x_n$ is a fixed point of $f$. Hence, we assume that $x_n \neq x_{n+1}$ for all $n \in \mathbb{N}$. As $f$ is $\alpha$-admissible mapping, we have

$$\alpha(x_0, x_1) = \alpha(x_0, f x_0) \geq e \Rightarrow \alpha(f x_0, f^2 x_0) = \alpha(x_1, x_2) \geq e.$$ 

We obtain by induction that

$$\alpha(x_n, x_{n+1}) \geq e, n \in \mathbb{N}. \quad (2.2)$$

According to (2.1), we get

$$q(x_n, x_{n+1}) = q(f x_{n-1}, f x_n) \leq a_1 q(x_{n-1}, x_n) + a_2 q(x_{n-1}, f x_{n-1}) + a_3 q(x_{n-1}, f x_n) = a_1 q(x_{n-1}, x_n) + a_2 q(x_{n-1}, x_n) + a_3 q(x_{n-1}, x_{n+1}) \leq a_1 q(x_{n-1}, x_n) + a_2 q(x_{n-1}, x_n) + a_3 [q(x_{n-1}, x_n) + q(x_n, x_{n+1})].$$

It means that

$$(e - a_3) q(x_n, x_{n+1}) \leq (a_1 + a_2 + a_3) q(x_{n-1}, x_n). \quad (2.3)$$

Since $r(a_3) < r(a_3) + r(a_1 + a_2 + a_3) < 1$, then by Proposition 1.1 it is obvious that $e - a_3$ is invertible. Moreover

$$(e - a_3)^{-1} = \sum_{i=0}^{\infty} a_i^i.$$ 

Let

$$h = (e - a_3)^{-1} (a_1 + a_2 + a_3).$$

As $a_3$ commutes with $a_1 + a_2 + a_3$, it follows that

$$(e - a_3)^{-1} (a_1 + a_2 + a_3) = \left( \sum_{i=0}^{\infty} a_i^i \right) (a_1 + a_2 + a_3) = \left( \sum_{i=0}^{\infty} a_i^i \right) (a_1 + a_2 + a_3) (e - a_3)^{-1},$$

that is to say $(e - a_3)^{-1}$ commutes with $a_1 + a_2 + a_3$. By Lemma 1.8 and Lemma 1.9, we get

$$r(h) = r((e - a_3)^{-1} (a_1 + a_2 + a_3)) \leq r((e - a_3)^{-1}) r(a_1 + a_2 + a_3) \leq (1 - r(a_3))^{-1} r(a_1 + a_2 + a_3) = \frac{1}{1 - r(a_3)} r(a_1 + a_2 + a_3) < 1,$$

that is

$$(e - h)^{-1} = \sum_{i=0}^{\infty} h_i^i \quad \text{and} \quad \|h_i^i\| \to 0 \quad \text{as} \quad n \to \infty.$$ 

Multiplying by $(e - a_3)^{-1}$ in both sides of (2.3), we have

$$q(x_n, x_{n+1}) \leq (e - a_3)^{-1} (a_1 + a_2 + a_3) q(x_{n-1}, x_n) = h q(x_{n-1}, x_n) = \cdots \leq h^n q(x_0, x_1). \quad (2.4)$$

Let $m > n \geq 1$, we infer

$$q(x_n, x_m) \leq q(x_n, x_{n+1}) + q(x_{n+1}, x_{n+2}) + \cdots + q(x_{m-1}, x_m) \leq (h^n + h^{n+1} + \cdots + h^{m-1}) q(x_0, x_1) = (e + h + \cdots + h^{m-1}) h^n q(x_0, x_1) \leq \left( \sum_{i=0}^{m} h_i^i \right) h^n q(x_0, x_1)$$

$$\leq \left( \sum_{i=0}^{\infty} h_i^i \right) h^n q(x_0, x_1).$$
\[ (e - h)^{-1} h^n q(x_n, x_i). \]

Owing to \( \| (e - h)^{-1} h^n q(x_n, x_i) \| \leq \| (e - h)^{-1} h^n \| \| q(x_0, x_i) \| \to 0 (n \to \infty) \), it leads to \( (e - h)^{-1} h^n q(x_n, x_i) \to \theta (n \to \infty) \). By Lemma 1.7(3), \( \{ x_n \} \) is a Cauchy sequence in \((X, d)\). As \((X, d)\) is complete, there is a point \( x^* \in X \) such that \( x_n = f x_{n-1} \to x^* \) as \( n \to \infty \). If \( f \) is continuous, it follows that \( x_{n+1} = f x_n \to f x^* \) as \( n \to \infty \). By the uniqueness of limit, we get \( x^* = f x^* \). So, \( x^* \) is the fixed point in \( X \).

**Theorem 2.3.** Let \((X, d)\) be a complete cone metric space over Banach algebra \( A \) and \( P \) be the underlying solid cone \( P \). Let \( q \) be a \( c \)-distance on \( X \), \( f : X \to X \) and \( \alpha : X \times X \to A \) be two functions. Suppose the following conditions are satisfied:

1. \( f \) is \( \alpha \)-admissible mapping;
2. there exists \( x_0 \in X \) such that \( \alpha(x_0, f x_0) \geq e \);
3. \((X, d)\) is \( \alpha \)-regular;
4. for all \( x, y \in X \) with \( \alpha(x, y) \geq e \)

\[ q(f x, f y) \leq a_1 q(x, y) + a_2 q(x, f x) + a_3 q(x, f y), \]

where \( a_i \in P (i = 1, 2, 3) \) such that \( r(a_i) + r(a_1 + a_2 + a_3) < 1 \). If \( a_i \) commutes with \( a_1 + a_2 + a_3 \), then \( f \) has a fixed point.

Proof. From the proof of Theorem 2.2, we obtain that \( \{ x_n \} \) is a Cauchy sequence in \((X, d)\). By the completeness of \((X, d)\), there exists \( x^* \in X \) such that \( x_n = f x_{n-1} \to x^* \) as \( n \to \infty \). By Definition 1.4(q3) and (2.4), we know

\[ q(x_n, x^*) \leq (e - h)^{-1} h^n q(x_0, x_i). \]

Then, from (2.2) and hypothesis (3), we get

\[ \alpha(x_i, x^*) \geq e, i \in \mathbb{N}. \]

Now, we show that \( f x^* = x^* \). By (2.5), we obtain

\[ q(x_n, f x^*) = q(f x_{n-1}, f x^*) \leq a_1 q(x_{n-1}, x^*) + a_2 q(x_{n-1}, f x_{n-1}) + a_3 q(x_{n-1}, f x^*) \]

\[ = a_1 q(x_{n-1}, x^*) + a_2 q(x_{n-1}, x_i) + a_3 q(x_{n-1}, x_i) + q(x_n, f x^*). \]

It implies that \( (e - a_3) q(x_n, f x^*) \leq a_1 q(x_{n-1}, x^*) + (a_2 + a_3) q(x_{n-1}, x_i). \) Since \( r(a_3) < 1 \), \( e - a_3 \) is invertible. So, by (2.4) and (2.6), we know

\[ q(x_n, f x^*) = (e - a_3)^{-1} [a_1 q(x_{n-1}, x^*) + (a_2 + a_3) q(x_{n-1}, x_i)] \]

\[ \leq (e - a_3)^{-1} [a_1 (e - h)^{-1} h^{n-1} q(x_0, x_i) + (a_2 + a_3) h^{n-1} q(x_0, x_i)] \]

\[ = (e - a_3)^{-1} [a_1 (e - h)^{-1} + (a_2 + a_3) h^{n-1}] q(x_0, x_i). \]

Set \( u_n = (e - h)^{-1} h^n q(x_0, x_i) \) and \( v_n = (e - a_3)^{-1} [a_1 (e - h)^{-1} + (a_2 + a_3)] h^{n-1} q(x_0, x_i). \) As \( r(h) < 1 \) and \( \| h^n \| \to 0 (n \to \infty) \), we know \( u_n, v_n \to \theta (n \to \infty) \). Thus, by (2.6), (2.7) and Lemma 1.7(1), we get \( f x^* = x^*. \)

**Remark 2.4.** Theorem 2.2, 2.3 improve and extend lots of meaningful theorems in the literature from many aspects. Firstly, our results dismiss the condition of normality of the cone. Secondly, \( c \)-distance greatly generalize cone metric (see [8]). Thus, we improve and generalized Theorem 2.1 in [9], Theorem 3.1, 3.2 in [12]. Thirdly, there is selectivity to the preimage of contractive mappings, not for all \( x, y \in X \), which improve Theorem 1 and Corollary 1, 2 in [15]. Since the non-equivalence between
cone metric spaces over Banach algebras and metric spaces, the results in this paper are mainly about fixed point theorems in cone metric spaces over Banach algebras instead of theorems in cone metric spaces (see [1-4, 10, 17-18]), which is more meaningful. If \( E = \mathbb{R} \) in our results, the corresponding fixed point theorems in metric spaces are obtained, which extend many conclusions in [17].

**Example 2.5.** Let \( A = \mathbb{R} \times \mathbb{R} \) and define a norm on \( A \) by \[ ||x|| = |x_1| + |x_2|, \quad x = (x_1, x_2) \in A. \]

Define multiplication as follows \[ xy = (x_1, x_2) (y_1, y_2) = (x_1 y_1 + x_2 y_2, x_1 y_1 + x_2 y_2), \]

where \( x = (x_1, x_2), y = (y_1, y_2) \in A. \) Thus, \( A \) is a Banach algebra with a unit \( e = (1, 0) \). Let \( P = \{ x = (x_1, x_2) \in A : x_1 \geq 0, x_2 \geq 0 \} \). It is clear that \( P \) is a solid cone in \( A \). Let \( X = [0, \infty) \times [0, \infty) \) and define a mapping \( d : X \times X \to A \) by \[ d(x, y) = d((x_1, x_2), (y_1, y_2)) = (|x_1 - y_1|, |x_2 - y_2|) \in P. \]

Then \( (X, d) \) is a complete cone metric space over \( A \). Suppose the mapping \( q : X \times X \to A \) by \[ q(x, y) = y, \]

then \( q \) is a \( c \)-distance. Next, let \( f : X \to X \) and \( \alpha : X \times X \to A \) defined by \[ f(x_1, x_2) = \begin{cases} \left( \frac{x_1}{3}, \frac{x_2}{3} \right), & (x_1, x_2) \in [0, 1] \times [0, 1]; \\ (x_1, x_2), & \text{otherwise}, \end{cases} \]

and \[ \alpha((x_1, x_2), (y_1, y_2)) = \begin{cases} \left( \frac{1}{|x_1 - y_1|}, \frac{1}{|x_2 - y_2|} \right), & (x_1, x_2), (y_1, y_2) \in [0, 1] \times [0, 1] \text{ and } x_1 \neq y_1, x_2 \neq y_2; \\ (0, 0), & \text{otherwise}. \end{cases} \]

Then, the mapping \( f \) is an \( \alpha \)-admissible. Indeed, let \( x, y \in X \) such that \( \alpha(x, y) \geq e \). By the definition of mapping \( \alpha \), it implies that \( x, y \in [0, 1] \times [0, 1] \). Thus,

\[ \alpha(fx, fy) = \alpha((\frac{x_1}{3}, \frac{x_2}{3}), (\frac{y_1}{3}, \frac{y_2}{3})) = (\frac{3}{|x_1 - y_1|}, \frac{3}{|x_2 - y_2|}) \geq e. \]

And there existing \( x_0 \in X \) such that \( \alpha(x_0, fx_0) \geq e \). In fact, for \( x_0 = (1, 1) \), we have \[ \alpha(x_0, fx_0) = \alpha((1, 1), (\frac{3}{2}, \frac{3}{2})) = (\frac{3}{2}, \frac{3}{2}) \geq e. \]

Since \([0, 1] \times [0, 1]\) is complete, then \((X, d)\) is \( \alpha \)-regular. Furthermore, denote \( k = (k_1, k_2) \), where \( k_1, k_2 > 0 \), we obtain \[ r(k) = \lim_{n \to \infty} \left\| k^n \right\|_{\infty} = \lim_{n \to \infty} \left\| \left(nk_1k_2^{n-1}, k_2^n\right) \right\|_{\infty}^{\frac{1}{2}} = k_2. \]

Take \( a_1 = (\frac{1}{2}, \frac{1}{3}), a_2 = (\frac{1}{4}, \frac{1}{6}), a_3 = (\frac{1}{5}, \frac{1}{12}) \). We observe that \( a_1 \) commutes with \( a_1 + a_2 + a_3 \) and \[ r(a_1) + r(a_1 + a_2 + a_3) = \frac{2}{3} < 1. \]

Obviously, \( \forall x, y \in [0, 1] \times [0, 1] \),

\[ q(fx, fy) = fy = (\frac{y_1}{3}, \frac{y_2}{3}) \]

\[ \leq \left( \frac{17}{30} y_2 + \frac{13}{36} y_1, \frac{13}{36} y_2 \right). \]
\[ \leq \left( \frac{1}{2}, \frac{1}{3} \right) (y_1, y_2) + \left( \frac{1}{4}, \frac{1}{6} \right) \left( \frac{x_1}{3}, \frac{x_2}{3} \right) + \left( \frac{1}{5}, \frac{1}{12} \right) \left( \frac{y_1}{3}, \frac{y_2}{3} \right) \]

\[ = a, q(x, y) + a, q(x, f x) + a, q(x, f y) \].

So, we can apply Theorem 2.3 and come to a conclusion that \( f \) has a fixed point in \( X \). In this example, the mapping \( f \) is not continuous and the fixed point is not unique. Thus, the mapping \( f \) is different from the contraction in[7]. In conclusion, our results in this paper are genuine generalization of some known results.

3. Conclusion

Our theorems in this paper are new, which are genuine improvements and generalizations of many recent known results in the literature. In addition, the research was partially supported by Yunnan Applied Basic Research Projects, Yunnan Province, China (No. 2016FD082).

References

[1] Huang L G, Zhang X, Conemetricspaceandfixedpointtheoremsofcontractive mappings[J]. J. Math. Anal. Appl. 2007, 332, 1468–1476.

[2] Rezapour Sh, Hambarani R, Some notes on the paper"Cone metric space and fixed point theorems of contractive mappings"[J]. J. Math. Anal. Appl., 2008, 345: 719-724.

[3] Abbas M, Rhoades B E. Fixed and periodic point results in cone metric spaces[J]. Appl. Math. Lett., 2009, 22: 511-515.

[4] Nashie H K, Rohen Y, Chhatrajit T. Common coupled fixed point theorems of two mappings satisfying generalized contractive condition in cone metric space[J]. International Journal of Pure and Applied Mathematics, 2016, 106(3), 791-799.

[5] Du Wei-Shih. A note on cone metric fixed point theory and its equivalence[J]. Nonlinear Anal., 2010, 72(5), 2259-2261.

[6] Feng Y, Mao W. The equivalence of cone metric spaces and metric spaces[J]. Fixed Point Theory, 2010, 11(2), 259-264.

[7] Liu H and Xu S. Cone metric spaces with Banach algebras and fixed point theorems of generalized Lipschitz mappings[J]. Fixed Point Theory Appl., 2013, 2013: 320.

[8] Cho Y J, Saadati R, Wang S. Common fixed point theorems on generalized distance in ordered cone metric spaces[J]. Computers and Mathematics with Applications, 2011, 61: 1254-1260.

[9] Wang S and Guo B. Distance in cone metric spaces and common fixed point theorems[J]. Appl. Math. Lett., 2011, 24, 1735-1739.

[10] Kada O, Suzuki T and Takahashi W. Nonconvex minimization theorems and fixed Point theorems in complete metric spaces[J]. Math. Japonica, 1996, 44: 381-391.

[11] Samet B, Calogero V, Pasquale V. Fixed point theorems for \( \alpha-\psi \)-contractive type mappings[J]. Non-linear Anal., 2012, 75, 2154-2165.

[12] Malhotra S K, Sharma J B, Shukla S. Fixed points of \( \alpha \)-admissible mappings in cone metric spaces with Banach algebra[J]. Int. J. Anal. Appl. 2015, 9, 9-18.

[13] Malhotra S K, Sharma J B, Shukla S. Fixed Points of Generalized Kannan Type \( \alpha \)-admissible Mappings in Cone Metric Spaces with Banach Algebra[J], Theory Appl. Math. Comp. Sci., 2017, 7(1), 1-13.

[14] Huang H, Deng G. Fixed point theorems under c-distance in cone metric spaces over Banach algebras[J]. Acta Scientiarum Naturalium Universitatis Pekinensis. 2018, 4, 693-698.

[15] Xu S and Radenovic S. Fixed point theorems of generalized Lipschitz mappings on cone metric spaces over Banach algebras without assumption of normality[J]. Fixed Point Theory Appl. 2014, 2014: 102.

[16] Huang H, Radenovic S, Dosenovic T. Some common fixed point theorems on c-distance in cone metric spaces over Banach algebras[J]. Comput. Math. 2015, 14(2), 180–193.

[17] Rudin W. Functional Analysis[M], 2nd edn. McGraw-Hill, New York, 1991.
[18] Dorevi M, Doric D, Kadelburg Z, Radenovic S, Spasic D. Fixed point results under c-distance in tvs-cone metric spaces[J]. Fixed Point Theory and Applications. doi: 10.1186/1687-1812-2011-29.