Calibration of standard radiation thermometers using two fixed points

A Manoi\textsuperscript{1,4}, P Wongnut\textsuperscript{1}, X Lu\textsuperscript{2}, P Bloembergen\textsuperscript{2} and P Saunders\textsuperscript{3,}\textsuperscript{4}

\textsuperscript{1} National Institute of Metrology (Thailand), Pathumthani 12120, Thailand
\textsuperscript{2} National Institute of Metrology (NIM), Beijing 100029, People’s Republic of China
\textsuperscript{3} Measurement Standards Laboratory of New Zealand, Lower Hutt, New Zealand

E-mail: athikom@nimt.or.th

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Abstract
Radiation thermometry based on the measurement of two fixed points, the so-called \( n = 2 \) scheme, is studied in this work. Verification of the scheme was undertaken for various measurement conditions using seven radiation thermometers with nominal centre wavelengths of 650 nm, 890 nm, 900 nm, and 1600 nm. Comparison between the signals calculated using the \( n = 2 \) scheme and the signals measured directly with fixed-point cells was studied. For temperatures between the two reference fixed points, the equivalent temperature difference was less than 0.14 K in the temperature range from the In to Cu fixed points, and less than 0.3 K in the temperature range from the Cu to Re–C fixed points. The maximum normalised error was 0.35, confirming the validity of the \( n = 2 \) scheme; the consistency was observed not only in the interpolation temperature range, but also in the extrapolation range. The results also indicate that temperature realisation in the range from 150 °C to 2500 °C can be achieved by maintaining only the Zn, Cu, and Re–C fixed-point facilities. The \( n = 2 \) scheme would be of great benefit to developing national metrology institutes in terms of time consumed for scale realisation and cost for maintenance of the measurement system.

Keywords: radiation thermometry, realisation of ITS-90, two fixed point scheme

(Some figures may appear in colour only in the online journal)

1. Introduction

The 2018 redefinition of the International System of Units (SI), including the kelvin, was officially adopted on 20 May 2019. There are multiple methods for realisation of the kelvin either by absolute primary thermometry or relative primary thermometry [1]. For absolute primary radiometric thermometry, the absolute optical power of a blackbody must be determined by a radiometer with the measurement system fully characterised, including the absolute spectral responsivity, solid angle of the measurement, target emissivity, and refractive index of the medium along the optical path. Using this approach, there is no requirement to measure any reference point, and this is known as the \( n = 0 \) method, where \( n \) is the number of fixed points (FPs). For relative primary radiometric thermometry, the optical power is determined by measurements relative to at least one FP (\( n \geq 1 \); e.g. measurement of one FP in conjunction with the measurement of the relative spectral responsivity (\( n = 1 \) scheme). The full range of possible schemes is discussed in [2].

The signal, \( S \), measured by a radiation thermometer can be expressed by the Planck version of the well-known Sakuma–Hattori equation [3] as

\[
S = C \left[ \exp \left( \frac{c_2}{AT + B} \right) - 1 \right]^{-1},
\]

where \( A, B, \) and \( C \) are constants pertaining to the radiation thermometer, \( T \) is the temperature in kelvin (K), and \( c_2 \) is the second radiation constant (equal to 0.014 388 m · K). If measurements are made with at least 3 FPs (\( n \geq 3 \) schemes), the constants \( A, B, \) and \( C \) can be obtained by non-linear least squares fitting without any additional spectral responsivity.
measurements. With the development of metal–carbon and metal-carbide–carbon eutectic FPs over the past two decades, calibration of radiation thermometers using \( n \geq 3 \) schemes is widely used [4].

In 2003, temperature scale realisation using two fixed points (\( n = 2 \) scheme) was studied by Bloembergen et al [5]. The concept is that, based on the measured shape of spectral responsivity curve, the centre wavelength can be determined by fitting the integral of the shifted spectral responsivity over Planck's law to the measured signals at the two FPs. The study by Saunders et al [6] revealed the relationship between the parameters in interpolation equations for radiation thermometry, e.g. equation (1), and the radiometric characteristics of the thermometer itself, and described how to apply this relationship to the \( n = 2 \) scheme. The study showed that, for radiation thermometers with a sufficiently narrow bandwidth, the interpolation equation parameters are independent of the shape of the spectral responsivity function and can be expressed solely in terms of the mean wavelength and bandwidth as determined from the variance. This is discussed in more detail in section 2.

These studies also demonstrated that the \( n = 2 \) scheme results in minimum uncertainty. The uncertainty evaluation for this scheme has been intensively studied in [2, 7, 8]. Another publication that studied the \( n = 2 \) scheme [9] indicated that the temperature difference between using the \( n = 2 \) and \( n = 3 \) schemes can be as much as 1 °C for interpolation in the temperature range from the Co–C FP (1324.24 °C) to the Re–C FP (2474.38 °C), and the difference can rise up to 5.5 °C at 2500 °C for extrapolation above the Pt–C FP (1738.28 °C) if the Co–C and Pt–C FPs are used for the \( n = 2 \) scheme. The temperature differences between these two schemes were about the level of the measurement uncertainties. These differences are larger than the theoretical limits given in [6] (assuming no uncertainties in the fitted parameters), where the differences are of the order of a few mK.

Most of the studies of the \( n = 2 \) scheme have been performed using 650nm radiation thermometers only, and no additional work has been found in the literature to confirm the agreement between the theoretical and practical results. In this work, verification of the \( n = 2 \) scheme was undertaken for various measurement conditions, and the results show good agreement for all conditions within the measurement uncertainties. Thus, the \( n = 2 \) scheme will be particularly beneficial for calibration laboratories with limited resources wanting to efficiently maintain a radiation thermometry temperature scale.

2. Equations for the \( n = 2 \) scheme

The principle of radiation thermometry using the \( n = 2 \) scheme is described in [6]. By expanding the Planck function as a Taylor series about the mean wavelength of the spectral responsivity, for sufficiently narrow bandwidths, where \( c_2 \sigma_2^2 / 2 \lambda_0^3 T \ll 1 \), the signal, \( S \), of a radiation thermometer can be expressed as

\[
S = c' \left[ \exp \left( \frac{c_2}{\lambda_0^3} \left( 1 - 6 \frac{\sigma_2^2}{\lambda_0^3} T + \frac{\sigma_2^4}{2 \lambda_0^6} \right) \right) - 1 \right]^{-1},
\]

where \( c' \) is a constant related to the optical, geometrical, and electrical components of the radiation thermometer, \( \lambda_0 \) is the mean wavelength of the spectral responsivity, and \( \sigma \) is the standard deviation of the spectral responsivity. This approximation typically creates an error smaller than 3 mK over the temperature range 600 °C to 3000 °C when the relative bandwidth, \( \sigma / \lambda_0 \), is less than 0.01 (e.g. for a 650nm thermometer with a full width at half maximum (FWHM) less than about 20nm). Equation (2) is identical to equation (1), with the parameters in equation (1) defined by

\[
C = c',
\]

\[
A = \lambda_0 \left( 1 - 6 \frac{\sigma_2^2}{\lambda_0^3} \right),
\]

and

\[
B = \frac{c_2 \sigma_2^4}{2 \lambda_0^6}.
\]

When the spectral responsivity is rectangular in shape, as in the example shown in figure 1, the standard deviation \( \sigma \) can be determined from the FWHM as \( \sigma = \text{FWHM} / \sqrt{2} \). Conversion factors between \( \sigma \) and the FWHM for other spectral responsivity shapes are given in [6]. In equation (2), there are three unknown parameters; \( c', \sigma, \) and \( \lambda_0 \). The least sensitive parameter is \( \sigma \), and this can be estimated with sufficient accuracy from the FWHM of the spectral band, even by eye [8], so that the FWHM specified by thermometer manufacturers is good enough for this estimation in this work. By fixing the value of \( \sigma \), the other two parameters, \( \lambda_0 \) and \( c' \), can be determined by measuring the thermometer signals at two FPs with known temperatures, giving two temperature–signal pairs \( (T_1, S_1) \) and \( (T_2, S_2) \) that can be fitted using equation (2).

The uncertainty for the \( n = 2 \) scheme can be determined as described in [2, 8]. The total uncertainty depends on the
uncertainties in \( T_1, T_2, S_1, \) and \( S_2, \) respectively. i.e. \( u(T_1), u(T_2), u(S_1), \) and 
\( u(S_2), \) respectively, and the uncertainty in the estimation of \( \sigma, u(\sigma) \). The sensitivity coefficients for these uncertainties, \( \partial T / \partial T_1, \partial T / \partial T_2, \partial T / \partial S_1, \partial T / \partial S_2, \) and \( \partial T / \partial \sigma, \) can be derived by differentiating equation (2), applying the Wien approximation, and neglecting terms of order \( r^2 \) and higher, where

\[
\sigma = \frac{T}{\lambda_0}
\]

is known as the relative bandwidth. These sensitivity coefficients are given by

\[
\frac{\partial T}{\partial T_1} = \frac{(T - T_2)T}{(T_1 - T_2)T_1},
\]

(7)

\[
\frac{\partial T}{\partial T_2} = \frac{(T - T_1)T}{(T_2 - T_1)T_2},
\]

(8)

\[
\frac{\partial T}{\partial S_1} = \frac{(T - T_2)T\lambda_0 T_1}{(T_1 - T_2)c_2 S_1},
\]

(9)

\[
\frac{\partial T}{\partial S_2} = \frac{(T - T_1)T\lambda_0 T_2}{(T_2 - T_1)c_2 S_2},
\]

(10)

and

\[
\frac{\partial T}{\partial \sigma} = -\frac{(T - T_1)(T - T_2)c_2 \sigma}{T_1 T_2 \lambda_0^2}.
\]

(11)

The combined uncertainty in a scale realisation using the \( n = 2 \) scheme is given by

\[
u^2 = u_1^2 + u_2^2 + u_3^2,
\]

(12)

where

\[
u_1^2 = \left( \frac{\partial T}{\partial T_1} \right)^2 u^2(T_1) + \left( \frac{\partial T}{\partial S_1} \right)^2 u^2(S_1),
\]

(13)

\[
u_2^2 = \left( \frac{\partial T}{\partial T_2} \right)^2 u^2(T_2) + \left( \frac{\partial T}{\partial S_2} \right)^2 u^2(S_2),
\]

(14)

and

\[
u_3^2 = \left( \frac{\partial T}{\partial \sigma} \right)^2 u^2(\sigma).
\]

(15)

When using the thermometer, an additional uncertainty, \( u_s, \) due to the uncertainty, \( u(S), \) in the signal measurement at the unknown temperature must be considered. This is given by

\[
u_s = \frac{T^2 \lambda_0 u(S)}{c_2},
\]

(16)

where the components of \( u(S) \) include uncertainties due to non-linearity, gain ratio, size-of-source effect, and noise. For components that are uncorrelated with the signal measurements at the reference FPs, such as noise, equation (16) should be added in quadrature to equation (12). However, for correlated components, such as non-linearity and the size-of-source effect, the covariance needs to be included when propagating the uncertainty. A method for doing this for the uncertainty due to non-linearity is given in [10], where it is shown that the effect of correlation causes the contribution of this uncertainty component to be zero at each of the reference FPs and to vary systematically between these FPs.

Initially, it will be assumed that there are no correlated components of uncertainty in equation (12); that is, it will be assumed that \( T_1 \) and \( S_1 \) are not related to \( T_2 \) and \( S_2, \) respectively. Then in section 4.3, the effects of correlations between \( u(T_1) \) and \( u(T_2), \) and between \( u(S_1) \) and \( u(S_2), \) will be considered.

3. Measurements

Data from seven narrow-band radiation thermometers with nominal wavelengths of 650 nm, 890 nm, 900 nm, and 1600 nm, as detailed in table 1, were analysed in this work. The thermometers were calibrated by means of FP blackbody cells, including In, Sn, Zn, Al, Ag, Cu, Co-C, Pt-C, Ru-C, and Re-C, depending on the temperature range of the thermometers. Table 2 indicates the temperatures of each FP and the FPs used for each thermometer. For RT3, the thermometer was also calibrated by measurement of the relative spectral responsivity in conjunction with a measurement of the Cu FP (\( n = 1 \) scheme) for the range 1000 °C to 2000 °C. The measurements were performed at National Institute of Metrology, Thailand (NIMT) for RT1 to RT3, and at National Institute of Metrology, China (NIM) for RT4 to RT7. The measurement facilities and details of uncertainties in the measurements can be found in [11–13].

Different pairs of FPs were selected for the \( n = 2 \) scheme in order to study the effects of both interpolation and extrapolation. For interpolation, the output signals and the related FP temperatures of the lowest and highest measuring points were used to determine \( \lambda_0 \) and \( \epsilon \) from equation (2) for each thermometer. The value of \( \sigma \) was estimated from the FWHM as stated by the manufacturer, with the assumption that the spectral responsivity shapes of all thermometers are rectangular, as for the example of RT3 shown in figure 1: i.e. \( \sigma = \text{FWHM} / \sqrt{2} \) [6]. Then, the calculated output signal for the \( n = 2 \) scheme, \( S_{n=2}, \) as determined by equation (2), was directly compared to the measured signal, \( S_{\text{FP}} \) for any intermediate FP cells. The equivalent temperature difference, \( \Delta T, \) is simply determined as

| Code | RT1 | RT2 | RT3 | RT4 | RT5 | RT6 | RT7 |
|------|-----|-----|-----|-----|-----|-----|-----|
| Model | Chino IR-RST16H | Chino IR-RST16H | Chino IR-RST90H | KE-LP4 | KE-LP4 | KE-LP4 | KE-LP4 |
| Nominal centre wavelength / nm | 1600 | 1600 | 900 | 890 | 650 | 650 | 650 |
| FWHM / nm | 150 | 150 | 80 | 30 | 14 | 13 | 13 |
\[ \Delta T = \frac{\lambda_0 T^2 (S_{FP} - S_{n=2})}{c_2 S_{FP}}. \]  

(17)

The normalised error, \( E_n \), is used to indicate the equivalence of the \( n = 2 \) scheme and the direct FP measurement, and is defined as

\[ E_n = \frac{\left| \Delta T \right|}{\sqrt{U_{FP}^2 + U_{n=2}^2}}. \]  

(18)

where \( U_{n=2} \) and \( U_{FP} \) are the expanded uncertainties, with a coverage factor \( k = 2 \), associated with the \( n = 2 \) scheme and with the direct FP measurement, respectively. The equations for evaluating \( U_{n=2} \) were given in section 2, where the total standard uncertainty is determined using equations (12) and (16). The relative uncertainty in the FWHM has been assumed to be 10% with a coverage factor \( k = 2 \), and this translates to a 5% relative standard uncertainty in \( \sigma \); i.e. \( u(\sigma) / \sigma = 0.05 \). For the spectral responsivity of RT3, shown in figure 1, the actual value of \( \sigma \) obtained from integrating the whole curve, including the out-of-band peak at short wavelengths, is 25.3 nm. This differs by about 4.5% from the estimated value of 24.2 nm obtained from the FWHM. Example uncertainty budgets for \( U_{FP} \) and \( U_{n=2} \) for RT3 are given in tables 3 and 4, respectively.

For extrapolation, RT1, RT3, and RT6 were selected to represent the 1600 nm, 900 nm, and 650 nm radiation thermometers, respectively. The Ag and Cu FPs, defined in the realisation of the International Temperature Scale of 1990 (ITS-90), were used to determine \( \lambda_0 \) and \( c' \) for the \( n = 2 \) scheme. The output signals calculated by equation (2) at the other FP temperatures, i.e. those outside of the Ag to Cu temperature range, were compared with the signals directly measured at these FPs. For RT1 and RT3, the \( n = 2 \) scheme temperature scale realisation using the Zn and Cu FPs was also studied.

4. Results and discussions

4.1 Interpolation

The temperature differences (\( \Delta T \)) of the 1600 nm, 890 nm, and 900 nm radiation thermometers, i.e. RT1 to RT4, and the 650 nm radiation thermometers, i.e. RT5 to RT7, are illustrated in figures 2 and 3, respectively. In the legend of the figures, the fixed points used in solving equation (2) are indicated in the parentheses after the thermometer code. The uncertainties shown in the figures are only the uncertainties determined by the \( n = 2 \) scheme and with the direct FP measurement, respectively. The results indicate that \( \Delta T \) is less than 0.14 K in the temperature range between the In and Cu FPs for the 1600 nm radiation thermometer. The uncertainties shown in the figures are only the uncertainties determined by the \( n = 2 \) scheme and the direct FP measurement, respectively. The results indicate that \( \Delta T \) is less than 0.14 K in the temperature range between the In and Cu FPs for the 1600 nm radiation thermometer.
Table 4. Measurement uncertainties of RT3 using Zn and Cu FPs as the references.

| Uncertainty component | Standard uncertainty (K) |
|-----------------------|--------------------------|
| FP 1, \(u_1\)        | 0.095 0.082 0.031 0.000 |
| FP 2, \(u_2\)        | 0.000 0.031 0.093 0.125 |
| Estimation of \(\sigma, u(\sigma)\) | 0.000 0.058 0.038 0.000 |
| Non-linearity        | 0.000 0.019 0.013 0.000 |
| Combined standard uncertainty | 0.10 0.11 0.11 0.13 |
| Expanded uncertainty  | 0.19 0.21 0.21 0.25 |

Figure 2. Temperature differences for the 1600 nm radiation thermometers (RT1 and RT2) and the 900 nm and 890 nm radiation thermometers (RT3 and RT4). The FPs used for the \(n = 2\) scheme are shown in parentheses. The uncertainties shown are the expanded uncertainties \((k=2)\), \(U_{\text{expanded}}\), for the \(n = 2\) scheme.

Thermometers (RT1 and RT2), less than 0.10 K in the temperature range between the Zn and Ru–C FPs for the 890 nm (RT4) and 900 nm (RT3) radiation thermometers (see figure 2), and less than 0.31 K in the temperature range between the Ag and Re–C FPs for the 650 nm radiation thermometers (RT5, RT6, and RT7) (see figure 3). Table 5 indicates \(\Delta T, U/\text{FP}, U_{n=2}\), and \(E_n\) for RT1, RT3, and RT5, which give the maximum normalised errors for each type of thermometer. The maximum normalised error, \(E_n\), is 0.35. This indicates excellent agreement between the temperatures determined on the basis of the \(n = 2\) scheme and the temperatures of the FPs measured directly.

For RT1, the \(n = 2\) scheme uncertainty, \(U_{n=2}\), in the middle of the temperature range is higher than the uncertainties at the lowest and the highest points (table 5). In contrast, \(U_{n=2}\) for RT3 and RT5 increases as the temperature increases. This is because the uncertainty due to the estimate of \(\sigma\) dominates the combined uncertainty for RT1, but not for RT3 or RT5. In equation (15), the value of \(u_\sigma\) depends (via equation (11)) on the product of \(T - T_1, T - T_3\), and \(u(\sigma)\). The product of \(T - T_1\) and \(T - T_3\) is zero at the two ends of the range and peaks midway between \(T_1\) and \(T_3\). As the value of \(\sigma\) is proportional to the FWHM of the thermometer’s spectral responsivity, and a 10% relative uncertainty in the FWHM \((k=2)\) has been assumed, the value of \(u(\sigma)\) increases in proportion to the FWHM, which is 2.17 nm, 1.15 nm, and 0.20 nm for RT1, RT3, and RT5, respectively. For thermometers with a narrower spectral responsivity, such as RT5, the value of \(u_\sigma\) is much lower than \(u_1\) and \(u_2\) and would still be much lower even if the uncertainty of the FWHM were increased by a factor of 2 (i.e. 20% of the FWHM). Thus, the combined uncertainty is less sensitive to \(u_\sigma\) for narrow-bandwidth radiation thermometers.

The measurement results in the interpolation temperature range demonstrate the consistency of signals calculated using the \(n = 2\) scheme and direct signal measurements of the FPs. The technique can be used not only for a 650 nm radiation thermometer, as studied in [5, 8], but also for 890 nm, 900 nm, and 1600 nm radiation thermometers. The calibration uncertainty of radiation thermometers with two FPs in the interpolation region is generally comparable in size to the uncertainty of the \(n = 1\) and \(n = 3\) schemes [2, 5, 8], depending on the measurement capabilities of the individual laboratory. However, temperature scale realisation using the \(n = 2\) scheme has the advantages of ease of implementation and robustness, especially over the \(n = 1\) scheme. The \(n = 2\) scheme also extrapolates better than the \(n > 2\) schemes.

4.2. Extrapolation

In this section, scale realisation by extrapolation beyond the two reference FPs in the \(n = 2\) scheme is investigated. The thermometers RT1, RT3, and RT6 were selected for the study, and the Ag (961.78 °C) and Cu (1084.62 °C) FPs were chosen as the two reference FPs as they are the FPs used for the ITS-90 realisation (\(n = 1\) scheme) and most national metrology institutes (NMIs) have these facilities. The thermometers’ signals in the extrapolation temperature range were simulated by using the values of \(\lambda_0\) and \(c'\) obtained by solving equation (2) using the measured signals and the FP temperatures of the Ag and Cu FPs. The simulated signals were compared with the signals measured at a range of other FPs and, for RT3, with the signals determined by the \(n = 1\) scheme in the range from the Cu FP (1084.62 °C) to the Ru–C FP (1953 °C) based on RT3’s measured relative spectral responsivity (see figure 1).

The values of \(\Delta T\) and their uncertainties are shown in figure 4. The uncertainties for the \(n = 2\) scheme, \(U_{n=2}\),...


Table 5. Temperature differences, uncertainties \((k = 2)\), and normalised errors for the interpolation study of RT1, RT3, and RT5.

| Type of FP | \(\Delta T/K\) | \(U_{1p}/K\) | \(U_{u-2}/K\) | \(E_n\) | \(\Delta T/K\) | \(U_{1p}/K\) | \(U_{u-2}/K\) | \(E_n\) | \(\Delta T/K\) | \(U_{1p}/K\) | \(U_{u-2}/K\) | \(E_n\) |
|------------|----------------|------------|--------------|--------|----------------|------------|--------------|--------|----------------|------------|--------------|--------|
| In         | 0.00           | 0.21       | 0.21         | 0.00   |                |             |              |        |                |             |              |        |
| Sn         | −0.09          | 0.14       | 0.24         | 0.33   |                |             |              |        |                |             |              |        |
| Zn         | −0.14          | 0.22       | 0.32         | 0.35   | 0.00           | 0.19       | 0.19         | 0.00   |                |             |              |        |
| Al         | −0.01          | 0.22       | 0.35         | 0.04   | −0.10          | 0.19       | 0.21         | 0.35   |                |             |              |        |
| Ag         | −0.03          | 0.25       | 0.26         | 0.07   | −0.08          | 0.23       | 0.21         | 0.25   | 0.00           | 0.20       | 0.20         | 0.00   |
| Cu         | 0.00           | 0.28       | 0.28         | 0.00   | 0.00           | 0.25       | 0.25         | 0.00   | 0.02           | 0.33       | 0.22         | 0.06   |
| Co–C       | 0.16           | 0.53       | 0.39         | 0.24   |                |             |              |        |                |             |              |        |
| Pt–C       | 0.31           | 0.71       | 0.54         | 0.34   |                |             |              |        |                |             |              |        |
| Ru–C       | 0.00           | 1.04       | 1.04         | 0.00   |                |             |              |        |                |             |              |        |

Figure 4. Temperature differences for RT1, RT3, and RT6, with Ag and Cu as the reference FPs for the \(n = 2\) scheme. The uncertainties shown in the inset are the expanded uncertainties \((k = 2)\), \(U_{u-2}\), for the \(n = 2\) scheme.

are presented in the inset of the figure. These uncertainties were evaluated from the uncertainties determined at the Ag and Cu FPs, which are in the range 0.2 K to 0.3 K \((k = 2)\).

The maximum value of \(\Delta T\) and its uncertainty \((k = 2)\) for RT1, RT3, and RT6 are 0.26 K ± 1.0 K, 1.33 K ± 5.4 K, and 0.16 K ± 10.6 K, respectively. This indicates the consistency of the scale realisation by the \(n = 2\) scheme and the other methods. However, compared to the values of \(\Delta T\), the uncertainties are quite high, especially when the measurement temperature deviates significantly from the temperature of the reference FPs. These small values of \(\Delta T\) are not expected to be typical, particularly considering the relatively small temperature interval of the two reference fixed points compared to the extrapolation range. This unexpected result is discussed in section 4.3 below.

4.3. The effect of correlation on the uncertainties

The large uncertainties reported above compared to the \(\Delta T\) values, also indicated by the low \(E_n\) values, suggests that either the uncertainty components have been over-estimated or that correlations between some of the components can significantly reduce the total uncertainty. We consider first the effect of correlations on the uncertainties \(u(T_1)\) and \(u(T_2)\). Neglecting all other uncertainties for now, the total uncertainty, including correlations, is given by the propagation of uncertainty formula

\[
u_T^2 = \left( \frac{\partial T}{\partial T_1} \right)^2 u_1^2(T_1) + \left( \frac{\partial T}{\partial T_2} \right)^2 u_2^2(T_2) + 2r_{T_1T_2} \frac{\partial T}{\partial T_1} \frac{\partial T}{\partial T_2} u(T_1)u(T_2),
\]

which can be written as

\[
u_T^2 = (1 - r_{T_1T_2}) \left[ \left( \frac{\partial T}{\partial T_1} \right)^2 u_1^2(T_1) + \left( \frac{\partial T}{\partial T_2} \right)^2 u_2^2(T_2) \right] + r_{T_1T_2} \left[ \frac{\partial T}{\partial T_1} u(T_1) + \frac{\partial T}{\partial T_2} u(T_2) \right]^2,
\]

where \(r_{T_1T_2}\) is the correlation coefficient between the uncertainties \(u(T_1)\) and \(u(T_2)\). Thus, for uncorrelated uncertainties \((r_{T_1T_2} = 0)\), the square of total uncertainty is the sum of the squares of the individual components, and for fully correlated uncertainties \((r_{T_1T_2} = 1)\), it is the square of the linear sum of the individual components. In general, for partial correlation, the square of the total uncertainty is a linear combination of these two contributions.

It is interesting to examine the case when \(r_{T_1T_2} = 1\) and \(u(T_1) = u(T_2) = u\). In this case, equation (20) becomes

\[
u_T = \left| \frac{\partial T}{\partial T_1} + \frac{\partial T}{\partial T_2} \right| u.
\]

Substituting the sensitivity coefficients from equations (7) and (8) gives

\[
u_T = \frac{T}{T_1T_2} |T_1 + T_2 - T| u.
\]

When \(T \leq T_1 + T_2\), this is a parabola with a maximum occurring at \(T = (T_1 + T_2)/2\) and zeros at \(T = 0 K\) and \(T = T_1 + T_2\). Thus, the total uncertainty is a maximum at the centre of the interpolation region and reduces to zero on both sides of the extrapolation region. Figure 5 plots individually the correlated and uncorrelated terms of equation (20) for \(u(T_1) = u(T_2) = 0.15^\circ C\), with \(T_1\) and \(T_2\) corresponding to the Ag and Cu FPs, respectively. The square of the total uncertainty (equation (20)) is given by \((1 - r_{T_1T_2})\) multiplied by the square of the uncorrelated curve plus \(r_{T_1T_2}\) multiplied...
by the square of the correlated curve. Thus, any correlation between \( u(T_1) \) and \( u(T_2) \) will lead to a reduction in the total uncertainty.

If \( u(T_1) = u(T_2) \), then the conclusions differ somewhat, but the correlation still acts to reduce the total uncertainty as compared to the uncorrelated case.

Next, we consider only uncertainties in \( u(S_1) \) and \( u(S_2) \) and assume that the relative uncertainties are the same; i.e. \( u(S_1)/S_1 = u(S_2)/S_2 = u_{rel} \). In this case, the propagation of uncertainty formula can be written:

\[
\begin{align*}
\sigma_T^2 &= \left(1 - r_{S_1,S_2}\right) \left[ \left( \frac{\partial T}{\partial S_1} \right)^2 S_1^2 + \left( \frac{\partial T}{\partial S_2} \right)^2 S_2^2 \right] \\
&\quad + r_{S_1,S_2} \left[ \left( \frac{\partial T}{\partial S_1} S_1 + \frac{\partial T}{\partial S_2} S_2 \right)^2 \right] u_{rel}^2,
\end{align*}
\]

where \( r_{S_1,S_2} \) is the correlation coefficient between the relative uncertainties \( u(S_1)/S_1 \) and \( u(S_2)/S_2 \). When \( r_{S_1,S_2} = 1 \), using the sensitivity coefficients of equations (9) and (10), equation (23) reduces to

\[
\sigma_T = \frac{\lambda T^2}{c_2} u_{rel}.
\]

Thus, fully correlated relative signal uncertainties for the \( n = 2 \) scheme propagate in the same way as the relative signal uncertainty at the reference point for the \( n = 1 \) scheme [2].

Figure 6 plots the correlated and uncorrelated components for equation (23) with \( u_{rel} = 1.4 \times 10^{-4} \). As is the case for the temperature uncertainties, in the extrapolation region, correlations between the relative signal uncertainties can significantly reduce the total uncertainty.

The last two columns of table 3 give the uncertainty budget for the RT3 measurements at the Ag and Cu FPs, where all the uncertainties have been expressed as temperature uncertainties. Some of the uncertainty components, such as the FPBB calibration and FPBB emissivity, have significant contributions in common arising from systematic errors. By splitting the uncertainties into random and systematic components, the correlation coefficient \( r_{T_1,T_2} \) can be calculated according to

\[
r_{T_1,T_2} = \frac{u_{sys}^2}{u(T_1)u(T_2)},
\]

where \( u_{sys} \) is the total of the systematic components of uncertainty common to the measurements at the two FPs, and \( u(T_1) \) and \( u(T_2) \) are the total uncertainties (random and systematic) in the measurements of the Ag FP and Cu FP, respectively. For the uncertainty budget in table 3, equation (25) gives an estimated correlation coefficient of about 0.7. Using this value of \( r_{T_1,T_2} \) in equation (20) reduces the uncertainty for RT3 shown in the inset in figure 4 down to about the size of the errors, \( \Delta T \). However, for RT6, the extremely small values of \( \Delta T \) remain
a puzzle, and cannot be accounted for by a reasonable level of correlation.

4.4. Combination of interpolation and extrapolation for temperature scale realisation in the range 150 °C to 2500 °C

To maximise the accuracy of the \( n = 2 \) scheme for the extrapolation range (particularly when the uncertainties are uncorrelated), the temperature interval between the two reference fixed points should be as wide as possible. This reduces the sensitivity coefficients given by equations (7) to (10). In this section, the Zn FP (419.527 °C) was selected instead of the Ag FP (961.78 °C), and the thermometer signals at the Zn and Cu (1084.62 °C) FPs for RT1 and RT3 were used for the temperature scale realisation. The results of \( \Delta T \) are shown in figure 7.

For RT1, the maximum value of \( \Delta T \) is 0.12 K ± 0.22 K at the temperature of the Al FP (660.323 °C) and the value of \( U_{n=2} \) at the In FP (156.5985 °C) is 0.28 K, which is slightly higher than the uncertainty of the FP measurement. For RT3, the temperature differences in the range from the Zn to the Cu FP are the same as the data in table 5 because the same reference FPs were used. For the extrapolation range of RT3, from the Cu FP to the Ru–C FP (1953 °C), the maximum temperature difference is 0.33 K ± 1.43 K at the temperature of the Pt–C FP (1738 °C). The maximum uncertainty is 2.5 K at the temperature of the Ru–C FP (1953 °C). This is due to the significant increase of the non-linearity component of uncertainty to 0.7 K (\( k = 1 \)).

Both sets of results for RT1 and RT3 illustrate that by increasing the temperature interval between the two FPs, the accuracy of the measurement improves; i.e. the temperature differences and the uncertainties in the extrapolation range reduce. The \( U_{n=2} \) uncertainty values in the extrapolation range of RT3 are of the same magnitude as the uncertainty realisation by the \( n = 1 \) scheme for temperatures up to 1800 °C [12]. By combining these results with the results of a 650 nm radiation thermometer calibrated in the range of the Cu FP to the Re–C FP, as shown in table 1, the \( n = 2 \) scheme temperature scale realisation in the range 150 °C to 2500 °C can be achieved by maintaining just the Zn, Cu, and Re–C FPs. This illustrates the advantage of the technique, especially for developing NMIs, who can reduce the cost for the facilities and time consumption.

5. Conclusion

Temperature scale realisation by radiation thermometry using two fixed points was studied in this work. Seven radiation thermometers with different centre wavelengths of 650 nm, 890 nm, 900 nm, and 1600 nm were calibrated using fixed point blackbody cells within their measurement range. The results indicate excellent agreement between the signal calculated by the \( n = 2 \) scheme and the signal measured directly at the fixed points, not only in the interpolation temperature range, but also in the extrapolation temperature range. The uncertainty of the calibration using the \( n = 2 \) scheme increases with the temperature difference from the reference fixed points, especially for the extrapolation range, unless there are significant correlations between the uncertainties at the two defining fixed points. The study indicates that temperature realisation by means of the \( n = 2 \) scheme in the temperature range from say 150 °C to far beyond the Re–C point can be achieved by maintaining just the Zn (419.53 °C), Cu (1084.62 °C), and Re–C (2474 °C) FPs. It should be noted that the selection of the FPs used can be varied depending on the measuring range of a RT, and the overall temperature range required to be maintained. The equations presented in this paper can be used to determine whether the resulting uncertainty is satisfactory.

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ORCID iDs

A Manoi  @ https://orcid.org/0000-0002-6113-0592
X Lu  @ https://orcid.org/0000-0002-1090-2046
P Saunders  @ https://orcid.org/0000-0003-3928-8147

References

[1] Fellmeth B, Fischer J, Machin G, Picard S, Steur P P M, Tamura O, White D R and Yoon H 2016 The kelvin redefinition and its mise en pratique Phil. Trans. R. Soc. A 374 20150037
[2] Saunders P 2011 Uncertainties in the realization of thermodynamic temperature above the silver point Int. J. Thermophys. 32 26–44
[3] Sakuma F and Kobayashi M 1996 Interpolation equations of scales of radiation thermometers Proceedings of TEMPMEKO ’96, Sixth International Symposium on Temperature and Thermal Measurements in Industry and Science, edited by P Marcarino, Levrotto and B Torino pp 305–10
[4] Woolliams E R, Machin G, Lowe D H and Winkler R 2006 Metal (carbine)–carbon eutectics for thermometry and radiometry: a review of the first seven years Metrologia 43 R11
[5] Bloembergen P, Yamada Y, Yamamoto N and Hartmann J 2003 Realizing the High-Temperature Part of a Future ITS with the Aid of Eutectic Metal-Carbon Fixed Points Temperature: Its Measurement and Control in Science and Industry, Vol. 7, ed Riplie D C et al. (New York: Melville) AIP Conf. Proc. 684 291–6
[6] Saunders P and White R D 2003 Physical basis of interpolation equations for radiation thermometry Metrologia 40 195
[7] Bloembergen P and Yamada Y 2011 Measurement of thermodynamic temperature above the silver point on the basis of the scheme \( n = 2 \) Int. J. Thermophys. 32 45–67

[8] Saunders P, Bloembergen P and White D R 2004 Uncertainty in temperatures realised by radiation thermometry using two fixed points 9th Int. Symp. on Temperature and Thermal Measurements in Industry and Science (Tempmeko) ed D Zvizdic (Dubrovnik: Laboratory for Process Measurement, Faculty of Mechanical Engineering and Naval Architecture Zagreb, Croatia) pp 1149–54

[9] Willmott J R, Lowe D, Broughton M, White B S and Machin G 2016 Potential for improved radiation thermometry measurement uncertainty through implementing a primary scale in an industrial laboratory Meas. Sci. Technol. 27 044002

[10] Saunders P and White D R 2007 Propagation of uncertainty due to non-linearity in radiation thermometers Int. J. Thermophys. 28 2098–2110

[11] Lu X, Yuan Z, Wang J, Bai C, Wang T and Dong W 2017 Calibration of pyrometers by using extrapolation and interpolation methods at NIM Int. J. Thermophys. 39 12

[12] Keawprasert T, Yamada Y and Ishii J 2015 Pilot comparison of radiance temperature scale realization between NIMT and NMIJ Int. J. Thermophys. 36 315–26

[13] Yuan Z, Lu X, Wang J, Bai C, Dong W, Hao X and Wang T 2011 Realization of ITS-90 above the silver point at the NIM Int. J. Thermophys. 32 1611