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Abstract

Functional Reactive Programming (FRP) has come to mean many things. Yet, scratch the surface of the multitude of realisations, and there is great commonality between them. This paper investigates this commonality, turning it into a mathematically coherent and practical FRP realisation that allows us to express the functionality of many existing FRP systems and beyond by providing a minimal FRP core parameterised on a monad. We give proofs for our theoretical claims and we have verified the practical side by benchmarking a set of existing, non-trivial Yampa applications running on top of our new system with very good results.

Categories and Subject Descriptors D.3.3 [Programming Languages]: Language Constructs and Features – Control Structures

Keywords functional reactive programming, reactive programming, stream programming, monadic streams, Haskell

1. Introduction

Functional Reactive Programming (FRP) [9, 10, 18] is a declarative approach to implementing reactive applications centred around programming with time-varying values (as opposed to operational descriptions of how to react to individual events). There are a very large number of different implementations of FRP frameworks catering for different settings in terms of platforms and languages, specific application needs, and ideas about how to structure such frameworks in the most appropriate way [5]. Particular differences include whether a discrete or hybrid (mixed discrete and continuous) notion of time is adopted, and how to handle I/O and other effects in a way that is both notationally convenient and scalable.

While this diversity makes for a rich and varied FRP landscape, it does raise practical obstacles and concerns for the FRP user in terms of which system to pick, especially if the needs are diverse or can be anticipated to change. Given the underlying conceptual similarity between the various implementations, this raises the question of whether, through appropriate generalisations, it might be possible to provide a single framework that covers a significantly broader set of applications and needs than what any one FRP implementation up until now has done, and that is easily extensible.

This paper proposes such a unifying framework by adopting stream functions parametrised over monads as the central reactive abstraction. We demonstrate that such a minimalist framework subsumes and exceeds existing FRP frameworks such as Yampa [8, 18] and Reactive Values [21]. Through composition of standard monads like reader, exception, and state, any desirable combination of time domain, dynamic system structure, flexible handling of I/O and more can be obtained in an open-ended manner.

Specifically, the contributions of this paper are:

- We define a minimal Domain-Specific Language of causal Monadic Stream Functions (MSFs), give them precise meaning and analyse the properties they fulfill.
- We systematically explore how MSFs with different standard monads give rise to known reactive notions like termination, switching and sinks, how to compose effects at a stream level using monad transformers, and how this addresses shortcomings of existing FRP frameworks.
- We implement three different FRP variants on top of our framework: 1) Arrowized FRP, 2) Classic FRP and 3) Signal/Sink-based reactivity.
- We demonstrate the practical feasibility of our approach by applying it to real-world games and applications.

2. Background

In the interest of making this paper sufficiently self-contained, we summarise the basics of FRP and Yampa in the following. For further details, see earlier papers on FRP and Arrowized FRP (AFRP) as embodied by Yampa [9, 10, 18]. This presentation draws heavily from the summary in [9].

2.1 Functional Reactive Programming

FRP is a programming paradigm to describe hybrid systems that operate on time-varying data. FRP is structured around the concept of signal, which conceptually can be seen as a function from time to values of some type:

\[ \text{Signal } \alpha \approx \text{Time } \rightarrow \alpha \]

Time is (notionally) continuous, and is represented as a non-negative real number. The type parameter \( \alpha \) specifies the type of values carried by the signal. For example, the type of an animation would be Signal Picture for some type Picture representing static pictures. Signals can also represent input data, like the position of the mouse on the screen.

Additional constraints are required to make this abstraction executable. First, it is necessary to limit how much of the history of a signal can be examined, to avoid memory leaks. Second, if we are interested in running signals in real time, we require them to be causal: they cannot depend on other signals at future times. FRP implementations address these concerns by limiting the ability to sample signals at arbitrary points in time.

The space of FRP frameworks can be subdivided into two main branches, namely Classic FRP [10] and Arrowized FRP [18]. Classic FRP programs are structured around signals or a similar notion representing internal and external time-varying data. In contrast,
2.2 Fundamental Concepts

Yampa is based on two concepts: signals and signal functions. A signal, as we have seen, is a function from time to values of some type, while a signal function is a function from Signal to Signal:

\[ \text{Signal } \alpha \approx \text{Time} \rightarrow \alpha \]
\[ \text{SF } \alpha \beta \approx \text{Signal } \alpha \rightarrow \text{Signal } \beta \]

When a value of type \( \text{SF } \alpha \beta \) is applied to an input signal of type Signal \( \alpha \), it produces an output signal of type Signal \( \beta \). Signal functions are first-class entities in Yampa. Signals, however, are not; they only exist indirectly through the notion of signal function.

In order to ensure that signal functions are executable, we require them to be causal: The output of a signal function at time \( t \) is uniquely determined by the input signal on the interval \([0, t]\).

2.3 Composing Signal Functions

Programming in Yampa consists of defining signal functions compositionally using Yampa's library of primitive signal functions and a set of combinators. Yampa's signal functions are an instance of the arrow framework proposed by Hughes [12]. Some central arrow combinators are \( \text{arr} \) that lifts an ordinary function to a stateless signal function, composition \( \gg \), parallel composition \( \& \& \& \& \), and the fixed point combinator \( \text{loop} \). In Yampa, they have the following types:

\[ \text{arr} \quad :: \quad (a \rightarrow b) \rightarrow \text{SF } a \quad b \]
\[ \gg \quad :: \quad \text{SF } a \quad b \rightarrow \text{SF } b \quad c \rightarrow \text{SF } a \quad c \]
\[ \& \& \& \& \quad :: \quad \text{SF } a \quad b \rightarrow \text{SF } a \quad c \rightarrow \text{SF } a \quad (b, c) \]
\[ \text{loop} \quad :: \quad \text{SF } (a, c) \quad (b, c) \rightarrow \text{SF } a \quad b \]

We can think of signals and signal functions using a simple flow chart analogy. Line segments (or "wires") represent signals, with arrowheads indicating the direction of flow. Boxes represent signal functions, with one signal flowing into the box's input port and another signal flowing out of the box's output port. Figure 1 illustrates the aforementioned combinators using this analogy. Through the use of these and related combinators, arbitrary signal function networks can be expressed.

2.4 Time-variant Signal Functions: Integrals and Derivatives

Signal Functions must remain causal and leak-free, and so Yampa introduces limited ways of depending on past values of other signals. Integrals and derivatives are important for application domains like games, multimedia and physical simulations, and they have well-defined continuous-time semantics. Their types in Yampa are as follows:

\[ \text{integral} \quad :: \quad \text{VectorSpace } v \quad s \Rightarrow \text{SF } v \quad v \]
\[ \text{derivative} \quad :: \quad \text{VectorSpace } v \quad s \Rightarrow \text{SF } v \quad v \]

Time-aware primitives like the above make Yampa specifications highly declarative. For example, the position of a falling mass starting from a position \( p_0 \) with initial velocity \( v_0 \) is calculated as:

\[ \text{fallingMass} \quad :: \quad \text{Double} \rightarrow \text{Double} \rightarrow \text{SF } () \quad \text{Double} \]
\[ \text{fallingMass} \quad p_0 \quad v_0 \quad = \quad \text{arr} \quad (\text{const } (-9.8)) \]
\[ \gg \quad \text{integral} \gg \quad \text{arr} \quad (+v_0) \]
\[ \gg \quad \text{integral} \gg \quad \text{arr} \quad (+p_0) \]

which resembles well-known physics equations (i.e. "the position is the integral of the velocity with respect to time") even more when expressed using Paterson's Arrow notation [20]:

\[ \text{fallingMass} \quad :: \quad \text{Double} \rightarrow \text{Double} \rightarrow \text{SF } () \quad \text{Double} \]
\[ \text{fallingMass} \quad p_0 \quad v_0 \quad = \quad \text{proc } () \rightarrow \text{do} \]
\[ v \leftarrow \text{arr} \quad (+v_0) \ll \text{integral} \ll (-9.8) \]
\[ p \leftarrow \text{arr} \quad (+p_0) \ll \text{integral} \ll -v \]
\[ \text{return } A \leftarrow p \]

2.5 Events and Event Sources

To model occurrences at discrete points in time, Yampa introduces the Event type [18]:

\[ \text{data} \quad \text{Event } a \quad = \quad \text{NoEvent} \mid \text{Event } a \]

A signal function whose output signal is of type Event \( T \) for some type \( T \) is called an event source. The value carried by an event occurrence may be used to convey information about the occurrence. The operator \( \text{tag} \) is often used to associate such a value with an occurrence:

\[ \text{tag} \quad :: \quad \text{Event } a \rightarrow b \rightarrow \text{Event } b \]

2.6 Switching

The structure of a Yampa system may evolve over time. These structural changes are known as mode switches. This is accomplished through a family of switching primitives that use events to trigger changes in the connectivity of a system. The simplest such primitive is \( \text{switch} \):

\[ \text{switch} \quad :: \quad \text{SF } a \quad (b, \text{Event } c) \rightarrow (c \rightarrow \text{SF } a \quad b) \rightarrow \text{SF } a \quad b \]

The switch combinator switches from one subordinate signal function into another when a switching event occurs. Its first argument is the signal function that is active initially. It outputs a pair of signals. The first defines the overall output while the initial signal function is active. The second signal carries the event that will cause the switch to take place. Once the switching event occurs, \( \text{switch} \) applies its second argument to the value tagged to the event and switches into the resulting signal function.

Yampa also includes parallel switching constructs that maintain dynamic collections of signal functions connected in parallel [18]. Signal functions can be added to or removed from such a collection at runtime in response to events, while preserving any internal state of all other signal functions in the collection (see Fig. 2). The first class status of signal functions in combination with switching over dynamic collections of signal functions makes Yampa an unusually flexible language for describing hybrid systems.
2.7 Limitations of Current AFRP Systems

Yampa has a number of limitations, described in the following. Most of these also apply to other current AFRP systems.

Fixed Time Domain and Clock Yampa has a global clock that progresses as driven by an external time-sensing monadic (IO) action. This is a serious limitation, as some games require the use of nested clocks (game clock versus application clock), and others require that time progresses at different speeds or with different precisions for different parts of the game. As a consequence, Continuous Collision Detection in Yampa is very complex.

The time domain in Yampa is fixed to Double, which is not always the most appropriate. Many games run on a discrete clock, while others require a rational clock with arbitrary precision or no clock at all. In such cases, keeping and passing an additional continuous clock becomes an unnecessary nuisance.

I/O Bottleneck Yampa’s input and output is connected to the external world once at the top level, in the invocation of the function that runs a signal function. This helps keep Signal Functions pure and referentially transparent across executions, but at the expense of having to poll all input data every time and handling more complex data structures in the output.

Explicit Wiring Pure implementations of AFRP do not allow communication between signal functions except through explicit input/output signals. All data that a signal function needs must be manually routed down, and outputs manually routed up. In practice, we often want to make part of that wiring implicit.

A manifestation of this problem is that it is not possible to debug from within signal functions except by adding explicit output signals carrying debugging information or by using functions like Debug.Trace.trace, which output to standard output directly (using unsafePerformIO under the hood). Code that uses trace is not portable, for instance, to platforms like Android, as debug messages must be printed to a special debug log facility.

In the following section we introduce a more fundamental abstraction to specify reactive programs that addresses these concerns, while remaining able to express all the definitions of Yampa.

3. Monadic Stream Functions

Monadic Stream Functions (MSFs) are a minimal abstraction to represent synchronous, effectful, causal functions between streams. MSFs are a generalisation of Yampa’s Signal Functions (SFs), with additional combinators to control and stack side effects. While the fundamentals of the abstraction are standard (continuations, parameterisation on a monad [13, 18]), the support for systematic effect extensions and how this subsumes existing FRP frameworks, and addresses common shortcomings of them, is novel.

In this section we introduce the definitions of MSFs and the basic combinators of our library. In the next sections we will extend our language with combinators to integrate effects using monad stacks, demonstrating with specific monads.

Notation The definitions that follow are simplified for clarity; performance and memory footprint of our implementation are discussed in Section 10. We use the shorter name MSF, instead of MStreamF, used in the implementation. The first argument of an MSF is always a monad, and consequently also a functor and an applicative. We usually omit the corresponding monad constraint from type signatures in the following to reduce clutter.

3.1 Definitions

Monadic Stream Functions are defined by a polymorphic type MSF and an evaluation function that applies an MSF to an input and returns, in a monadic context, an output and a continuation:

\[
\text{newtype } \text{MSF } m a b \\
\text{step } : \text{Monad } m \Rightarrow \text{MSF } m a b \rightarrow a \rightarrow m (b, \text{MSF } m a b)
\]

The type MSF and the step function alone do not represent causal functions on streams. It is only when we successively apply the function to a stream of inputs and consume the side effects that we get the unrolled, streamed version of the function. Causality is given by this progressive application, sample by sample.

For the purposes of exposition we provide functions to express the meaning of applying an MSF to an input and looking only at the output or only at the continuation:

\[
\text{head}_{M} : \text{Monad } m \Rightarrow \text{MSF } m a b \rightarrow a \rightarrow m b \\
\text{head}_{M} \text{msf } a = \text{fst } <\rangle \text{step } msf a
\]

We also provide a function to apply an MSF to a finite list of inputs, with effects and continuations chained sequentially. This is merely a debugging aid, not how MSFs are actually executed:

\[
\text{embed } : \text{Monad } m \Rightarrow \text{MSF } m a b \rightarrow [a] \rightarrow m [b]
\]

3.2 Lifting

The simplest kind of transformation we can apply to a stream is point-wise to every sample. We provide two functions for this purpose: \text{arr}, which produces an output and no side effects, and \text{liftS}, which applies an effectful function to every input sample.

\[
\text{arr } : (a \rightarrow b) \rightarrow \text{MSF } m a b \\
\text{liftS } : (a \rightarrow m b) \rightarrow \text{MSF } m a b
\]

We describe their meaning by showing that they act point-wise on the head, and that the continuation is the same MSF unchanged:

\[
\text{head}_{M}(\text{arr } f) a \equiv \text{return } (f a) \\
\text{head}_{M}(\text{liftS } f) a \equiv f a \\
\text{tail}_{M}(\text{arr } f) a \equiv \text{return } (\text{arr } f) \\
\text{tail}_{M}(\text{liftS } f) a \equiv f a \gg \text{return } (\text{liftS } f)
\]

Example One trivial way of using these combinators is the stream function that adds a constant number to the input:

\[
\text{add } : (\text{Num } n, \text{Monad } m) \Rightarrow n \rightarrow \text{MSF } m n n \\
\text{add } n0 = \text{arr } (\lambda n \rightarrow n + n0)
\]

which we test in a session (in GHC, monad-parametric computations are run in the IO monad and the results printed, if possible):

\[
> \text{embed } (\text{add } 7) \ [1, 2, 3] [8, 9, 10]
\]

3.3 Widening

We can define new MSFs that only affect the first or second components of pairs, passing the other component completely unmodified:

\[
\text{first } : \text{MSF } m a b \rightarrow \text{MSF } m (a, c) (b, c) \\
\text{second } : \text{MSF } m b c \rightarrow \text{MSF } m (a, b) (a, c)
\]
sequenced in the same order, which we detail using step $g$. Com-
posing $f \gg g$ first applies $f$ to the input producing an output, and
applies $g$ to that output, producing a final result. Side effects are
sequenced in the same order, which we detail using step $g$.

**Examples**

Extending the previous example, we can write:

We provide a way of keeping state by producing an extra value or

Example

These combinators are not primitive; their standard definitions
are based on combinators defined previously:

When composed in parallel like above, the effects of the $f$ are
produced before those $g$. This is of prime importance for non-
commutative monads. For a discussion, see Section 9.

**Example**

The following example of palindromes demonstrates monadic and compositional combinators:

When composed in parallel like above, the effects of the $f$ are
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effects in MSFs. In Section 7 we use these combinators to express FRP abstractions like Arrowized FRP.

4.1 Reader

We now want to make our example MSFs parametric on the player positions. One option is to pass the player position as an argument, as in ballToRight :: Monad m ⇒ Int → MSF m () Ball. However, this complicates the implementation of every MSF that uses ballToRight, which needs to manually pass those settings down the reactive network. We can define such a parametrisation in a modular way using a Reader monad with the game preferences as in

\[
\text{ReaderT in an environment (we use in a modular way using a Reader monad with the game preferences uses ballToRight)} \text{, which needs to manually pass those settings down the reactive network. We can define such a parametrisation in a modular way using a Reader monad with the game preferences in an environment (we use ReaderT for forward compatibility):}
\]

\[
\text{type GameEnv } = \text{ReaderT GameSettings}
\]

\[
\text{data GameSettings } = \text{GameSettings}
\]

\[
\begin{aligned}
\text{leftPlayerPos } &:: \text{Int} \\
\text{rightPlayerPos } &:: \text{Int}
\end{aligned}
\]

We rewrite the game to pass this environment in the context:

\[\text{ballToRight :: Monad m ⇒ MSF (GameEnv m) () Ball ballToRight = count } \gg \text{liftS addLeftPlayerPos } \gg \text{liftS checkHitR} \]

where checkHitR :: n → GameEnv m Int checkHitR n = do

rp ← asks rightPlayerPos
when (rp > n) $ tell ["Ball at " ++ show n]

Notice that we have changed the monad and ballToRight, but the rest of the game remains unchanged. Using the transformer ReaderT instead of Reader in the previous step now pays off in the form of added modularity.

Like with the reader monad, we may be interested in consuming the context (for instance, to print accumulated messages and empty the log). We provide the temporal execution function:

\[
\text{runWriterS :: Monad m} \Rightarrow \text{MSF (WriterT r m) a b} \\
\rightarrow \text{MSF m a (b, r)}
\]

We can test this combinator as follows:

\[
> \text{embed (runWriterS (runReaderS testMSF (GameSettings 0 3)))}
\]

\[
[((1, False), []), ((2, False), []), ((3, True), []), ((4, True), []), ((5, True), ["Ball at 4"]), ((5, True), ["Ball at 5"])]\]

Similarly we could have used a State monad to define configurable game settings (for instance, settings that can be adjusted using an options menu, but remain immutable during a game run).

4.2 Writer

We can use a similar approach to introduce monads like Writer or State, for instance, to log debug messages from MSFs. We first extend our environment with a WriterT wrapper:

\[
\text{type GameEnv m } = \\text{WriterT [String] (ReaderT GameSettings m)}
\]

We now modify ballToRight to print a message when the position is past the right player (indicating a goal):

\[
\text{ballToRight :: Monad m } \Rightarrow \text{MSF (GameEnv m) () Ball ballToRight = count } \gg \text{liftS addLeftPlayerPos } \gg \text{liftS checkHitR} \]

where checkHitR :: Int \rightarrow GameEnv m Int

\[
\text{checkHitR } \_ \rightarrow \text{do}
\]

rp ← asks rightPlayerPos
when (rp > n) $ tell ["Ball at " ++ show n]

We provide the temporal execution function:

\[
\text{runWriterS :: Monad m} \Rightarrow \text{MSF (WriterT r m) a b} \\
\rightarrow \text{MSF m a (b, r)}
\]

We can test this combinator as follows:

\[
> \text{embed (runWriterS (runReaderS testMSF (GameSettings 0 3)))}
\]

\[
[((1, False), []), ((2, False), []), ((3, True), []), ((4, True), []), ((5, True), ["Ball at 4"]), ((5, True), ["Ball at 5"])]\]

Similarly we could have used a State monad to define configurable game settings (for instance, settings that can be adjusted using an options menu, but remain immutable during a game run).

4.3 Exceptions and Control Flow

MSFs can use different monads to define control structures. One common construct is switching, that is, applying a transformation until a certain time, and then applying a different transformation.

We can implement an equivalent construct using monads like Either or Maybe. We could define a potentially-terminating MSF as an MSF in a MaybeT m monad. Following the same pattern as before, the associated execution function would have type:

\[
\text{runMaybeS :: Monad m} \Rightarrow \text{MSF (MaybeT m) a b} \\
\rightarrow \text{MSF m a (Maybe b)}
\]

Our evaluation function step, for this monad, would have type MSF Maybe a b → MSF Maybe (Maybe a b) indicating that it may produce no continuation. runMaybeS outputs Nothing continuously once the internal MSF produces no result.

“Recovering” from failure requires an additional continuation:

\[
\text{catchM :: Monad m} \Rightarrow \text{MSF (MaybeT m) a b} \\
\rightarrow \text{MSF m a b}
\]

We can now make the ball bounce when it hits the right player:

\[
\text{ballBounceOnce :: MSF (GameEnv m) () Ball ballBounceOnce } = \text{ballUntilHitRight } \gg \text{catchM } \gg \text{ballLeft} \gg \text{ballUntilRight}
\]

where

\[
\text{filterHit (b, c) = MaybeT$ return $ if c then Nothing else Just b}
\]

The utility function liftST is defined in Section 5. We define ballUntilLeft analogously and complete the game:
Let us interpret the game by inserting a list as input stream. The output shows the ball position going up and bouncing back between 10 (the right player’s position) and 0 (the left player’s position).

The implementation of switching in our library is based on a more general monad `ExceptT c m`, but the key idea is the same.

### 4.4 Creating and Destroying Objects with `ListT`  

In games it is often necessary to create and destroy “objects”. For example, a gun may fire a bullet or a target vanish when hit. Such dynamicity requires specific combinators in other reactive frameworks. In ours, the list monad provides the sought-after behaviour.

A stream function over the list monad can produce zero, one or several output values and continuations. How we continue depends on our interpretation of that list. If we explore all continuations simultaneously, we will be implementing parallel broadcasting.

To produce multiple outputs we provide `zeroA`, which produces no outputs or continuations, and `<|>`, which concatenates lists produced by two MSFs in a list monad. (In our library these are available for any instance of `Alternative` and `MonadPlus`.)

```haskell  
zeroA :: Monad m => MSF (ListT m) a b  
<|> :: Monad m => MSF (ListT m) a b  
         -> MSF (ListT m) a b  
         -> MSF (ListT m) a b
```

We now change the game logic such that, each time the ball starts moving left, it splits into two balls (note the use of `<|>`):

```haskell  
type GameEnv m = ReaderT GameSettings (ListT m)  
ballLeft :: Monad m => MSF (GameEnv m) () Ball  
between = singleBallLeft <|> singleBallLeft  
where  
singleBallLeft =  
  liftS (λn → (λp → p - n) <|> asks rightPlayerPos)
```

We can escape the list monad or the `ListT` transformer by collecting the outputs from all continuations into a list:

```haskell  
runListS :: Monad m => MSF (ListT m) a b  
         -> MSF m a [b]
```

Our approach proves to be very modular, and we only need to modify our top function slightly to extract the list effect:

```haskell  
mainMSF :: MSF IO () ()  
mainMSF =  
  runListS (runReaderS game (GameSettings 20 17))  
  &&& runReaderS game (GameSettings 10 4)
```

Note that the `ReaderT` layer is inside the `ListT` layer, and therefore both games are duplicated when either ball starts moving left. Running the above MSF prints the following output (presented in two columns for reasons of space):

```
[(18, 5)]  [(18, 9), (18, 9)]  
[(19, 6)]  [(17, 10), (17, 10)]  
[(20, 7)]  [(18, 9), (18, 9), (18, 9), (18, 9)]  
[(19, 8), (19, 8)] ...  
```

The standard implementation of `ListT` is only valid for commutative monads. Alternative implementations exist, but the discussion is beyond the scope of this article.

### 4.5 State  

Keeping type signatures parametric, like in the stream function `ballLeft :: Monad m => MSF (GameEnv m) () Ball`, renders more reusable definitions, since we can stack more monads on top. For example, it is easy to introduce a global state using the `State` monad, with a counter of the number of rounds played. The counter can be increased with:

```haskell  
incOneRound :: Monad m => StateT Integer m ()  
incOneRound = modify (+1)
```

which we use in the game, accounting for the side effect in the type:

```haskell  
game :: Monad m  
      ⇒ MSF (GameEnv (StateT Integer m)) () Ball  
game =  
      ballToRight 'untilM' hitRight  
        'catchM': ballToLeft 'untilM' hitLeft  
        'catchM' (lift incOneRound 'andThen' game)
```

The function `andThen :: Monad m => m () ⇒ MSF m a b  
⇒ MSF m a b` performs the monadic action in the first argument once and immediately carries on processing the MSF in the second argument. The function `lift :: (MonadTrans t, Monad m) ⇒ m a → t m a` from the transformers package lifts the state modification into the `GameEnv` monad.

To run this reactive program we have to pass the initial state, in a similar way to `runStateS :: StateT m s m a → s → m (a, s)`, which passes an initial state to a monadic state transformation and extracts the computed value and the final state. The corresponding function for streams has type:

```haskell  
runStateS :: Monad m => StateT (GameSettings) m a b  
          → s  
          → MSF m a (s, b)
```

Using this function in the main loop is a simple change:

```haskell  
mainMSF = runStateS parallelGame 0 >>= liftS print  
where  
parallelGame = runReaderS game (GameSettings 20 17)  
            &&& runReaderS game (GameSettings 10 4)
```

The output of running this MSF (presented in two columns) is:

```
(0, (18, 5))  (1, (19, 8))  
(0, (19, 6))  (1, (20, 7))  
(0, (20, 7))  (1, (19, 6))  
(0, (19, 8))  (1, (18, 5))  
(0, (18, 9))  (1, (17, 4))  
(0, (17, 10)) (3, (18, 5))  
(1, (18, 9)) ...  
```

The first value is the counter of total rounds, the other two values are the positions of the ball in the two games, respectively.

We have introduced this change without altering any code that did not use the state variable. In standard Arrowized FRP, we would have had to alter all signal functions and pass the state manually.

### 5. Monadic Lifting/Running Combinators  

In the last section we saw functions to combine MSFs on different monads, all of which conform to one of the following patterns:

- Lifting a purer MSF into an MSF in an impurer monad (e.g. `MSF Identity a b → MSF IO a b`).
- Running a more structured computation inside a less structured monad (e.g. `MSF (t m) a b → MSF m (I a) (F b)`).

This section explores these transformations and presents a way of thinking about MSFs over different monads and monad stacks.
5.1 Lifting MSFs
Whenever we use monad transformers or other effect stacking
mechanisms, we may be interested in embedding an MSF \( m \ a \ b \)
into a larger MSF \( (t m) \ a \ b \) (where \( t m \) denotes a monad that
encloses the effects of \( m \), such as a transformer \( t \) applied to \( m \)).

We can lift between any two arbitrary monads, provided we
have a monad morphism \((\text{Monad } m, \text{Monad } n) \Rightarrow m \rightarrow n\).

\[
\text{lifLM} \;::\; (\text{Monad } m, \text{Monad } n) \\
\Rightarrow (\forall a \cdot m \ a \rightarrow n \ a) \\
\Rightarrow MSF \; m \ a \ b \rightarrow MSF \; n \ a \ b
\]

Users are responsible for providing an actual monad morphism
that retains any information in \( m \) inside the context in \( n \). For
Monad Transformers, with a monad morphism \( \text{lift} :: m \rightarrow t \; m \),
we define the convenience lifting function \( \text{lifST} = \text{lifLM lift} \).

5.2 Temporal Execution Functions
Monad stacks are a common design pattern. If one introduces a
transformer on the stack, one frequently wants to remove it after
performing some effects in it, by executing it. An example is the
"running function" \( \text{runReaderT} :: ReaderT \; r \; m \ a \\rightarrow r \rightarrow m \).

In Section 4, we introduced temporal execution functions that
remove a transformer from the monad stack inside the MSF, such as
\( \text{runReaderS} :: MSF \; (ReaderT \; r \; m) \ a \ b \\rightarrow r \rightarrow MSF \; m \ a \ b \).
Similar examples were given for \( \text{WriterT}, \text{MaybeT}, \text{ListT} \) and
\( \text{StateT} \). Essentially, they are implemented by commuting the
transformer past MSF and then applying the running function of
the transformer. This will be explained in the following.

Monadic Streams and MSFs To simplify the implementation of
the temporal running functions, we are going to exploit an isomor-
phism between Monadic Stream Functions and certain Monadic
Streams. MSFs are defined as follows:

\[
\text{newtype MSF} \; m \; a \; b = MSF \\
\{ \text{step} :: a \rightarrow m \ (b, MSF \; m \; a \; b) \}
\]

Depending on the monad \( m \), we may have one, none or several
outputs and continuations. Moving from a monad like \( \text{Maybe} \) or
\( \text{Either} \) to another monad requires retaining the possibility
of providing no output, or recovering from a termination or exception.

In turn, Monadic Streams can be defined as:

\[
\text{Stream} \; m \ a = \{ m \ (a, \text{Stream} \; m \ a) \}
\]

It is easy to see that our abstraction MSF is isomorphic to a
Monadic Stream in a Reader context:

\[
\text{MSF} \; m \; a \; b = \text{Stream} \; (\text{ReaderT} \ a \ m) \ b
\]

Note that \( \text{Stream} \) is a transformer, with \( \text{lift} :: a \rightarrow \text{Stream} \; m \ a \) given by the infinite repetition of the same effect.
Because \( \text{MSF} \; m \; a \; b = \text{Stream} \; (\text{ReaderT} \ a \ m) \), this makes
\( \text{MSF} \) with the second argument \( a \) preapplied also a transformer.

Streams are functors and applicatives, but not necessarily mon-
ads. As transformers they can still be applied and, with some con-
straints, comma, so the lack of a general monad instance does
not invalidate our argument.

Implementing Temporal Execution Functions Commuting a
transformer \( t \) past MSF means commuting \( t \) past \( \text{ReaderT} \) and
subsequently past \( \text{Stream} \). We can capture all this in a type class:

\[
\text{class MonadTrans} \; t \Rightarrow \text{Commutation} \; t \; \text{where} \\
\text{commute} \;::\; (\text{Monad } m, \text{Monad } n) \\
\Rightarrow \text{ReaderT} \; r \; (t \; m) \; a \rightarrow r \rightarrow (t \; (\text{ReaderT} \; r \; m) \; a) \\
\text{commute} \;\text{Stream} \;::\; \text{Monad } m \\
\Rightarrow \text{Stream} \; (t \; m) \; a \rightarrow r \rightarrow (t \; (\text{Stream} \; m) \; a) \\
\text{preserveMH} \;::\; (\text{Monad } m1, \text{Monad } m2) \\
\Rightarrow (\forall a \cdot m1 \ a \rightarrow m2 \ a) \rightarrow t \; m1 \ b \rightarrow t \; m2 \ b
\]

As a technicality, we also have to assume that \( t \) preserves the
isomorphism \( \text{MSF} \; m \ a \ b \rightleftharpoons \text{Stream} \; (\text{ReaderT} \ a \ m) \), but
most transformers preserve monad homomorphisms. There is no
requirement that the commutations be isomorphisms, and in cases
like \( \text{ListT} \) and \( \text{MaybeT} \) a change of effects is actually desired.

The above type class gives rise to a commutation function:

\[
\text{commute} :: \text{Commutation} \; t \\
\Rightarrow \text{MSF} \; (t \; m) \ a \ b \rightarrow t \; (\text{MSF} \; m \ a \ b)
\]

Defining a temporal execution function is now as simple as defining
an instance of the \( \text{Commutation} \) type class and composing
\( \text{commute} \) with any running function. To give a few examples:

\[
\begin{align*}
\text{runReaderS} & \equiv \text{runReaderT} \circ \text{commute} \\
\text{runWriterS} & \equiv \text{runWriterT} \circ \text{commute} \\
\text{runStateS} & \equiv \text{runStateT} \circ \text{commute} \\
\text{catchM} \;\text{msf} \;\text{handler} & \equiv \\
& \text{fromMaybe } \langle\rangle \;\text{handler }\langle\rangle \;\text{runMaybeT} \;\text{commute} \;\text{msf}
\end{align*}
\]

Transformers commuting with \( \text{ReaderT} \) are abundant, and our
library defines \( \text{Commutation} \) instances for common transformers.

Running with a Changing Input Other execution functions are
also useful in practice. For example, we may be interested in ob-
aining an \( \text{Reader} \) context at every input sample, requiring:

\[
\begin{align*}
\text{runReaderS} & \equiv \text{runReaderT} \circ \text{commute} \\
\text{runWriterS} & \equiv \text{runWriterT} \circ \text{commute} \\
\text{runStateS} & \equiv \text{runStateT} \circ \text{commute} \\
\text{catchM} \;\text{msf} \;\text{handler} & \equiv \\
& \text{fromMaybe } \langle\rangle \;\text{handler }\langle\rangle \;\text{runMaybeT} \;\text{commute} \;\text{msf}
\end{align*}
\]

This kind of definition is trivial once we observe that Reader
commutes with itself, and that the following are isomorphic:

\[
\begin{align*}
\text{ReaderT} \; r1 \ (\text{ReaderT} \; r2 \ a) \ a & = \text{ReaderT} \; (r1, r2) \; m \ a \\
\text{and therefore:}
\text{MSF} \; (\text{ReaderT} \; r \ a \ b) \\
& \equiv \{ \text{MSFs as Streams} \} \\
& \equiv \text{Stream} \; (\text{ReaderT} \; a \ (\text{ReaderT} \; r \ a) \ b) \\
& \equiv \{ \text{Commutativity of reader} \} \\
& \equiv \text{Stream} \; (\text{ReaderT} \; r \ (\text{ReaderT} \; a \ a) \ b) \\
& \equiv \{ \text{Reader r1 (Reader r2 a)} =\text{Reader (r1,r2) a} \} \\
& \equiv \text{Stream} \; (\text{ReaderT} \; (r, a) \ b) \\
& \equiv \{ \text{Streams as MSFs} \} \\
& \equiv \text{MSF} \; m \; (r, a) \ b
\end{align*}
\]

6. Reactive Programming and Monadic Streams
Reactive Programming is a programming paradigm organised
around information producers and consumers, or \( \text{streams} \) and \( \text{sinks} \).
In this section we present definitions of Streams and Sinks based on
Monadic Stream Functions. Some properties of monadic streams
also apply to stream functions or are easier to prove in that set-
ing. The existing research on streams and causal stream functions
makes establishing this relation useful in its own right.

We present definitions of Streams and Sinks based on Monadic
Stream Functions, and demonstrate how to do Reactive Program-
ing using the ideas introduced in the previous section. We also
present an extension suitable for event-driven settings like GUIs.
In Section 7.2 we use similar concepts to implement Classic FRP.

6.1 Streams and Sinks
Monadic Stream Functions that do not depend on their input model
Monadic Streams. We can capture that idea with:

\[
\text{type Stream} \; m \; b = \text{MSF} \; m \; () \; b
\]

Disregarding bottoms and applying unit as the only possible arg-
ument, the above expands to \( \text{Stream} \; m \; b \rightleftharpoons \text{m} \; (b, \text{Stream} \; m \; b) \),
and \( \text{Stream Identity} \) is isomorphic to standard infinite streams.

If streams can be seen as MSF that do not depend on their inputs,
sinks can be seen as MSF do not produce any output:
6.2 Reactive Values

External mutable entities can be regarded as both sources and sinks. For instance, in programs with Graphical User Interfaces, text boxes can be seen as String-carrying Streams, and overload the standard numeric operators:

\[
\text{mirroredMouseX} :: \text{Stream} \, \text{IO} \, \text{Int}, \quad \text{mm} = 1024 - \text{mouseX}
\]

We can sometimes simplify code further. For example, we can give a `Num` instance for `Num`-carrying `Streams`, and overload the standard numeric operators:

\[
\text{mirroredMouseX} :: \text{Stream} \, \text{IO} \, \text{Int}, \quad \text{mm} = 1024 - \text{mouseX}
\]

Note that, in this new definition, 1024 has type `Stream IO Int`.

Streams and sinks are MSFs, so we can use MSF combinators to transform them and connect them. The following reactive program chains a stream and a sink to print the mouse position:

\[
\text{reactiveSink} = \text{Stream} \, \text{IO} \, \text{Int}
\]

\[
\text{printSink} = \text{Sink} \, \text{IO} \, \text{String}
\]

\[
\text{printSink} = \text{liftS} \, \text{putStrLn}
\]

With this interface, push-based connections need to be specified at monadic level. Both directional and bi-directional connections can be expressed declaratively using rule-binding combinators:

\[
\text{:=} :: \text{ReactiveValueRW} \, \text{IO} \, \text{a} \\
\text{:=} :: \text{ReactiveValueWO} \, \text{IO} \, \text{a} \\
\text{:=} :: \text{ReactiveValueRO} \, \text{IO} \, \text{a} \\
\text{:=} :: \text{ReactiveValueWO} \, \text{IO} \, \text{a} \\
\text{:=} :: \text{ReactiveValueRO} \, \text{IO} \, \text{a}
\]

Defining RVs in terms of MSFs simplifies the implementation, as we can use MSF combinators to implement RV transformations. At a conceptual level, this also shows that MSFs are a suitable abstraction to reason about bidirectional connections between reactive entities, and can aid in the design of combiners for compositional bidirectional reactive rules with well-defined semantics.

7. Extensible Functional Reactive Programming

Functional Reactive Programming (FRP) [9, 10, 18] is a paradigm to describe systems that change over time. Time in FRP is explicit and conceptually continuous. FRP is structured around a concept of signals, which represent time-varying values:

\[
\text{type} \, \text{Signal} \, a \\ 
\text{type} \, \text{Time} \, \cong \, \mathbb{R}^+
\]

While this conceptual definition enables giving FRP denotational semantics, execution is still carried out by sampling signals progressively. The ideal semantics are approximated more precisely as the sampling frequency increases [25].

Using Monadic Stream Functions we can add time information to the monadic context. In this section we use this approach to implement Arrowized FRP [9, 18] and Classic FRP [10]. We show that our alternatives remain flexible enough to address the concerns expressed in the introduction. We limit our discourse to the programming abstractions; performance and comparisons to other FRP variants are discussed in Sections 8 and 9.

7.1 Extensible Arrowized FRP

Arrowed FRP (AFRP) is an FRP formulation structured around the concept of signal functions. Signals, in AFRP, are not first-class citizens. Conceptually:

\[
\text{type} \, \text{SF} \, a \, b \\ 
\text{type} \, \text{AFRP} \, a \, b
\]

Multiple AFRP implementations exist. In the following we implement Yampa [8, 18], used to program multimedia and games. We demonstrate that our implementation can easily address some of the limitations of Yampa by implementing systems with multiple clocks and a form of Continuous Collision Detection.

7.1.1 Core Definitions

Signal Functions (SFs) are executed by successively feeding in a stream of input samples, tagged with their time of occurrence. The time always moves forward, and is expressed as the strictly positive time passed since the occurrence of the previous sample.

Leaving optimisations aside, Yampa’s running SFs are defined as type `SF a b = DTime i (a → b)`, which we can realise by passing time deltas in a Reader monad environment:
type SF a b = MSF ClockInfo a b

7.1.2 Reactimating Signal Functions

The second part of our Yampa replacement is a reactimation or simulation function. The signature of Yampa’s top-level simulation function demonstrates the bottleneck effect mentioned earlier:\footnote{Yampa’s reactimate has a more complex signature for reasons beyond the scope of this paper that do not invalidate our claims. Our library follows Yampa’s specification.}

reactimate :: ReaderT ReaderT deltas in a reader environment for the Yampa monad. Second, after

There are two notable aspects in

The first argument gathers inputs and delta times, the second consumes outputs, producing side effects. Our MSF reactimate function has a simpler type signature, consumes outputs, producing side effects. Our MSF

Effect are consumed and presented to the user progressively.

We have verified our implementation with multiple Yampa domains. For instance, the following runs MSFs at different speeds:

MSFs at different speeds:

Most of Yampa’s core primitives \cite{23}, like arr, (\Rightarrow) or

![Image](https://via.placeholder.com/150)

7.1.3 Time, Clocks and Continuous Collision Detection

The introduction of subsampling mechanisms like the latter needs to be addressed with care. Arbitrarily fine subsampling can lead to inherent memory leaks.

Continuous Collision Detection

7.2 Classic FRP

Classic FRP lets users define time-varying values, or signals, using combinators and applying functions to other signals. Signals may represent external information sources (e.g., mouse position).

With a Reader monad with time deltas in the environment, like in Arrowized FRP, Streams can model FRP Signals. To allow for external sources, we nest IO with the monad, obtaining:

\[
\text{type Signal } a = \text{Stream } (\text{ReaderT } DTime IO) a
\]

A simple simulation of a ball moving around the mouse position could be written in Classic FRP style as follows:
ballInCirclesAroundMouse :: Signal (Int, Int)
ballInCirclesAroundMouse =
  addPair <> mousePos <> ballInCircles

ballInCircles :: Signal (Double, Double)
ballInCircles = (λ x → (rad * cos x, rad * sin x)) <> time
  where rad = 45 -- radius in pixels

mousePos :: Signal (Int, Int)
mousePos = liftS (λ () → lift getMousePos)

-- Predefined
addPair :: Num a ⇒ (a, a) → (a, a) → (a, a)
getMousePos :: IO (Int, Int)
time :: Signal Time

With non-commutative monads like IO, additional measures
must be taken to ensure referential transparency at the same con-
ceptual time. If several signals depend on, for example, mousePos,
the mouse position might be sampled twice, with different results
at the same conceptual time. This can be addressed with a monad
that caches results and enables garbage collection [19].

7.3 Limiting the Impact of IO
MSFs over the IO monad limit out ability to apply equational rea-
soning and to debug programs. In contrast, the separation between
IO and Signal Functions in pure Arrowized FRP makes it possible
to record all inputs and replicate program behaviour precisely.

Monads are “contagious”, and MSFs that do IO, directly or
indirectly, must say so in their signature. We consider this a strength
of our framework, as it allows, but discourages, impurity.

Additionally, we can limit side effects with a monad without
support to lift arbitrary IO actions and a safer API. Monadic re-
quirements can be abstracted into a type class, as follows:

class Monad m ⇒ GameMonad m where
  getMousePos :: m (Int, Int)

Signatures require additional type constraints, but implementa-
tions remain unchanged. We can now use a pure instance for debug-
ging and the IO monad during normal execution. This makes our
framework more flexible than pure Arrowized FRP, while main-
taining its benefits in terms of testing and referential transparency.

8. Evaluation
We have implemented the ideas presented in this paper in two li-
braries: Dunai 2, a core implementation of Monadic Stream Func-
tions, and Bear River 3, a Yampa replacement built on top of Dunai
by using the ideas described in Section 7. Bear River defines Signal
Functions parameterised over a monad, making this implementa-
tion more versatile than Yampa. In order to remain backwards com-
patible, we provide an FRP. Yampa module that reexports Bear-
River and declares SF a b = BearRiver.SF Identity a b.

We have tested our library by compiling and executing existing
games implemented in Yampa. We discuss two games: the com-
mercial game Magic Cookies! and the open-source Haskanoid.

Magic Cookies! [3] is an FRP board game in which the user
needs to turn off all the lights on a board, following a rule: touching
any position toggles surrounding positions in a cross-like shape.
The game uses SDL2 for multimedia, and is compiled via the
unofficial GHC Android backend. This game is available from the
official store, Google Play for Android.

Haskanoid [2] is a clone of the game Arkanoid written in
Haskell using Yampa, SDL multimedia, and supporting input de-
vices like Kinect and Wii remotes. This game implements a simple
collision system with convex shapes like rectangles and circles.

Our Yampa replacement Bear River can execute both of these
programs in constant memory (Figure 5). In Haskanoid, memory
consumption decreases as the blocks are removed during gameplay.
Enabling compiler optimisations, Magic Cookies runs in 325KB of
constant memory with BearRiver, while with Yampa it consumes
300KB. Haskanoid consumes a maximum of 2.2MB of memory
with BearRiver, while with Yampa it requires approximately 2MB.

2http://hackage.haskell.org/package/dunai
3http://hackage.haskell.org/package/bearriver

Figure 3. Screenshot of Magic Cookies! running on Android, compiled with BearRiver.

Figure 4. Screenshot of the open-source Yampa game Haskanoid.

Figure 5. Heap profile of Magic Cookies! running with our library.
9. Related Work

Our abstraction is a generalisation of Yampa’s signal functions. One central difference is that Yampa has a fixed, notionally continuous time domain, with (broadly) all active signal functions sensing the same time flow. This is implemented by passing the time delta since the previous step to each signal function. In our framework, we can do the same using the Reader monad, but the time domain is no longer fixed, and it is easy to arrange for nested subsystems with different time domains and other variations as needed. Our proposal also eliminates some of the bottlenecks of Yampa thanks to the flexibility offered by the monadic parametrisation.

We have tested several Yampa games with our library, through a mediating layer as outlined above, including games like Haskanoid [2] and the commercial game Magic Cookies! for Android [3]. Our tests show that our replacement library runs these games in constant memory, consuming about ten per cent more memory than Yampa. Yampa is optimised exploiting algebraic identities like the arrow laws [12, 17], suggesting that we could deliver comparative performance with similar optimisations.

Netwire, inspired by Yampa, is an FRP framework parametric over the time domain and a monad, in which signals can be inhibited (producing no output) and with a switch that maintains the global clock (unlike Yampa’s switch, which resets time). We can implement equivalent behaviour using EitherT to inhibit signals, and adding the total global time to the context in a ReaderT layer.

Causal Commutative Arrows (CCA) [16] defines an abstraction of causal stream transformer, isomorphic to MSFs over the Identity monad. Programs written in terms of causal stream transformers, or any CCA, can be optimised by orders of magnitude in speed. MSFs are valid CCAs for commutative monads [6], making it possible in principle to apply similar optimisations.

Multiple FRP libraries implement push or push/pull evaluation, such as Elerea, Sodium and reactive banana, all structured in terms of signals and/or event streams. These libraries are optimised to run efficiently with minimal data propagation, often using weak references and an IO monad behind the scenes. Additional measures must be taken to guarantee referential transparency and efficient garbage collection in the presence of non-commutative monads like IO. These frameworks also include combinators to connect signals or event streams to external sources and sinks. This is a pragmatic choice, but the drawback is that the IO connection is not shown explicitly in the type. We consider the manifest reflection of all effects in the types a strength of our framework. For example, for pure MSFs (no general IO), we can perform reproducible tests and obtain more guarantees than in IO-based FRP implementations by connecting with automated testing tools like QuickCheck [7].

Monadic FRP [24] is, despite its name, most similar to Yampa’s Tasks (which we can model using MSFs using EitherT transformer). Monadic FRP is not parametric over the monad, which makes our framework more general in that respect.

UniTi [22] is a hybrid-system simulation framework with local clocks in which signal functions can output debugging information. Our work can accommodate local clocks using a reader monad, as well as debugging facilities. Additionally, our framework enforces causality and can enforce temporal consistency, whereas UniTi can provide inconsistent results for past values of signals.

There are also stream-based programming libraries that share notions with our framework. Iteratees [14] are stream transformers parameterised over a monad and oriented towards memory-efficient processing of data streams gathered from network and file sources. The application domain is thus rather different from ours, and this is reflected in an asynchronous API with a somewhat imperative feel, centred around reading and writing individual stream elements. Pipes [4] describe interconnected data processors and their coordination in an asynchronous setting. At a higher-level, pipes are expressed in terms of constructs like Producer and Pipe, conceptually similar to MSFs with an EitherT monad transformer. Internally, pipes are based on a Proxy type which is parameterised dually on the input and the output, making pipes bi-directional. Our framework can be used to describe bi-directional connections with an adapted reactimation function (Section 6).

Our approach can be used to implement discrete reactive programming like Reactive Values [21]. Reactive Values minimise forward propagation with push-based evaluation and change detection. This can be implemented in MSFs with an adapted push-based reactimation function. Expressing Reactive Values in terms of MSFs is also a way to define and study the semantics of the former.

Similarly, Wormholes [26] pairs whiteholes (monadic streams) with blackholes (sinks) to represent external resources. Reads and writes are sorted to guarantee referential transparency and commutativity. This could be achieved with custom monads in our setting. While they use a Monadic Stream Function representation to introduce IO, their types carry resource information, imposing additional constraints that require a customized Arrow type-class.

Hughes uses the same representation as our MSFs in a circuit simulator [13]. The approach is stream based, centred around data processors that actively wait for input events. Hughes does not explore the use of different pure monads or how they impact modularity. However, the work shows how to leverage active waits on the monad, something which is also applicable in our setting.

Finally, a representation similar to our MSFs was mentioned in [1], where the author briefly suggests using different monads to achieve different effects. To the best of our knowledge, that blog post did not spawn further work exploring such a possibility.

10. Conclusions and Future Work

We started this paper by observing how fractured the FRP landscape has become for a number of reasons, including domain-specific aspects, the inflexibility of current FRP frameworks, and different opinions on how to best structure FRP systems. Indeed, it is not easy to pinpoint what FRP is. While diversity brings many benefits, there are also significant costs in terms of duplication of effort and end-user uncertainty about what specific FRP system or approach to use, potentially hampering the use of FRP as such.

To address these concerns, this paper proposed to refactor FRP into a minimal core, capturing what arguably is the essence of FRP, but parameterised over a monad to make it open ended. Realisation of domain- and application-specific features is then just a matter of picking an appropriate monad. We evaluated our approach both from a practical and theoretical perspective. For the practical evaluation, we reimplemented an existing FRP system, Yampa, using our minimal core, demonstrating good time and space performance over a selection of medium-sized open-source and commercial games, including Haskanoid and Magic Cookies!, on both standard mobile and desktop platforms. Additionally, we showed how a range of other features from other systems and proposed extensions from the FRP literature can be expressed. For the theoretical side, we showed that our framework has an appropriate mathematical structure, such as satisfying the expected laws, preserving commutativity of the monad, and interoperating with monad morphisms.

So far, our implementation is unoptimised. We expect optimization techniques like those used in Yampa [8, 18] to carry over, bringing similar performance gains. We are further working on support for change propagation to avoid redundant computation when signals are unchanging. We anticipate that this will result in performance on par with push-based FRP implementations.

An advantage of making effects manifest, as when structuring code using arrows or monads, is that it becomes easier to reexecute code. This in turn enables sophisticated approaches to automated testing. At present, we are exploiting this to test commercial
Yampa games and debug traces gathered from users’ devices using QuickCheck [7] in combination with a DSL of temporal predicates. We are planning to provide similar capabilities for MSFs. Even when IO is needed, one can provide the needed IO capabilities through a wrapper that is responsible for carrying out the necessary logging and bookkeeping to also make such code re-executable.

Finally, we are currently exploring the addition of clock information to MSFs to express the speed at which asynchronous MSFs produce and consume data, and how to coordinate them precisely.

A. MSFs, Arrows and Arrow laws

The arrow laws and other properties hold for Monad Stream Functions [6]. To prove them, we model Haskell types as complete partial orders (CPOs), functions as continuous maps, and use fixpoint induction [11]. Each fundamental category and arrow combinator like id, >>=, arr and first is a fixpoint of a corresponding recursion function. Below we include a shortened proof of one arrow law.

Definitions

For conciseness, we will leave out the value constructors, so the type of MSF is simply MSF m a b = a → m (b, MSF m a b).

\[
\begin{align*}
  arr &= fix \text{arrRec} \\
  \text{arrRec} f a &= return (f \; a, \text{rec} f) \\
  \Rightarrow& = fix \text{compRec} \\
  \text{compRec} g a &= do \{\; f \leftarrow f a \\
  \quad \{\; g \leftarrow g b \\
  \quad \{\; \text{return} (c, \text{rec} f \; g') \}
  \}
\]

Proof: \(arr (f \Rightarrow g) \equiv arr f \Rightarrow arr g\)

The application of fixpoint induction means that if \(P\) is a predicate on values of some type \(a\), and \(f : a \rightarrow a\), and \(P\) holds for \(\bot\), and for every value \(x\) we can infer \(P \; x \Rightarrow P (f \; x)\), then \(P\) holds for \(\text{fix} \; f = \text{def} \; f \; (\text{fix} \; f)\). We define the predicate \(P\) as:

\[
P (x, y) = def \; \forall \; g. \; (x \; f) \; 'y' \; (g \; x) \equiv x \; (g \; o \; f)
\]

We proceed as follows:

\[
P (\bot, \bot) \iff \{\; \text{Definition of } P \;\}

\forall \; f. \; (\bot \; f) \; 'y' \; (\bot \; g) \equiv \bot \; (g \; o \; f)

\iff \{\; \text{Application to } \bot\;\}

\bot \equiv \bot

\iff \text{true}

P (arrRec x, compRec y)

\iff \{\; \text{Definition of } P \;\}

\forall \; f. \; (arrRec x \; f) \; 'y' \; (\text{compRec} \; y \; (\text{arrRec} \; x \; g)) \equiv \text{arrRec} x \; (g \; o \; f)

\iff \{\; \text{Definition of } \text{compRec} \; \text{and } \text{arrRec} \; \text{and } \beta\text{-Reduction}\;\}

\forall \; f. \; \text{lhs} \equiv \lambda a. \; \text{return} \; (g \; o \; f \; a, x \; (g \; o \; f)) \quad \text{where}

\text{lhs} = \lambda a \rightarrow \text{do}

\{\; \text{Monad laws, function composition}\;\}

\forall \; f. \; \lambda a. \; \text{return} \; (g \; f \; a, x \; 'y' \; x \; g) \equiv \lambda a. \; \text{return} \; (g \; f \; a, x \; (g \; o \; f))

\iff \{\; \text{Definition of } \text{compRec} \; \text{and } \text{arrRec} \; \text{and } \beta\text{-Reduction}\;\}

P (x, y)

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