Dynamics of Glue-Balls in $N = 1$ SYM Theory

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Abstract

The extension of the Veneziano-Yankielowicz effective Lagrangian with terms including covariant derivatives is discussed. This extension is important to understand glue-ball dynamics of the theory. Though the superpotential remains unchanged, the physical spectrum exhibits completely new properties.

1 Introduction

The low energy effective action of $N = 1$ SYM theory is written in terms of a chiral effective field $S = \varphi + \theta \psi + \theta^2 F$, which may be defined from the local source extension of the SYM action \cite{1,2,3,4}

$$S \propto \delta \delta J W[J,J], \quad e^{iW[J,J]} = \int D V e^{i \int d^4 x d^2 \theta (J + \tau_0) Tr W^\alpha W_\alpha + h.c.}. \quad (1)$$

With appropriate normalization $S$ is equivalent to the anomaly multiplet $\bar{D}^\alpha V_{\alpha \dot{\alpha}} = D_\alpha S$. $J(x)$ is the chiral source multiplet, with respect to which a Legendre transformation can be defined \cite{3,4}. The resulting effective action is formulated in terms of the gluino condensate $\varphi \propto \text{Tr} \lambda \lambda$, the glue-ball operators $F \propto \text{Tr} F_{\mu \nu} F^{\mu \nu} + i \text{Tr} F_{\mu \nu} \tilde{F}^{\mu \nu}$ and a spinor $\psi \propto (\sigma^{\mu \nu} \lambda)_{\alpha} F_{\mu \nu}$. An effective Lagrangian in terms of this effective field $S$ has the form \cite{1,2}

$$\mathcal{L}_{\text{eff}} = \int d^4 \theta K(S, \bar{S}) - \left( \int d^2 \theta S (\log \frac{S}{\Lambda^3} - 1) + h.c. \right). \quad (2)$$

The correct anomaly structure is realized by the superpotential and thus $K(S, \bar{S})$ is invariant under all symmetries. In ref. \cite{1} the explicit ansatz $K = k (SS)^{1/3}$ had been made, which leads to chiral symmetry breaking due to $\langle S \rangle = \Lambda^3$, but supersymmetry is not broken as $\varphi$ and $\psi$ acquire the same mass $m = \Lambda/k$. 

1
2 Glue-balls and constraint Kähler geometry

Though the spectrum found in ref. [1] does not include any glue-balls, such fields do appear in $F$. However, they drop out in the analysis of [1], as $F$ is treated as an auxiliary field. Indeed, the highest component of a chiral superfield is auxiliary in standard SUSY non-linear $\sigma$-models, i.e. there appear no derivatives acting onto this field and moreover its potential is not bounded from below, but from above. In case of the Veneziano-Yankielowicz Lagrangian the part depending on the auxiliary field reads

$$L_{aux} = k(\bar{\phi}\phi)^{\frac{2}{3}}F + \left(\frac{1}{3}\phi - \frac{2}{3}\bar{\phi}\frac{1}{3}F\bar{\psi}\psi - F\log\frac{\phi}{\Lambda^3} + h.c.\right),$$

and the supersymmetric spectrum is obtained, if and only if $F$ is eliminated by the algebraic equations of motion that follow from (3). This leads to the unsatisfactory result that glue-balls cannot be introduced in a straightforward way (cf. also [5]) which, in addition, contradicts available lattice-data [6].

However, in the special case of $N = 1$ SYM the elimination of $F$ is not consistent: If $F$ is eliminated from (3), this implies that the theory must be ultra-local in the field $F$ exactly, i.e. even corrections to the effective Lagrangian which are not included in (2) are not allowed to change the non-dynamical character of $F$. If this field would be related to the fundamental auxiliary field, this restriction would be obvious. But in $N = 1$ SYM the situation is different: $S$ is the effective field from a composite operator and $F$ is not at all related to the fundamental auxiliary field $D$. As a consequence, the restriction of ultra-locality on $F$ leads to an untenable constraint on the physical glue-ball operators (for details we refer to [4, 7, 8]).

As shown in ref. [2], the effective Lagrangian of [1] is not the most general expression compatible with all the symmetries, but the constant $k$ may be generalized to a function $k(\frac{\partial^{3/2}}{\partial S^{1/2}}, \frac{\partial^{3/2}}{\partial \bar{S}^{1/2}})$. This non-holomorphic part automatically produces space-time derivatives onto the field $F$, which is most easily seen when $K(S, \bar{S})$ is rewritten in terms of two chiral fields [5]:

$$K(S, \bar{S}) \rightarrow K(\Psi_0, \Psi_1; \bar{\Psi}_0, \bar{\Psi}_1)$$

$$\Psi_0 = S^{\frac{1}{2}} = \phi^{\frac{2}{3}} + \frac{1}{3}\phi^{\frac{2}{3}}\theta\psi + \frac{1}{3}\theta^2(\phi^{\frac{2}{3}}F + \frac{1}{3}\phi^{\frac{2}{3}}\psi\psi),$$

$$\Psi_1 = \bar{D}^2\bar{\Psi}_0 = \frac{1}{3}(\bar{\phi}^{\frac{2}{3}}\bar{F} + \frac{1}{3}\bar{\phi}^{\frac{2}{3}}\bar{\psi}\bar{\psi}) - \frac{i}{3}\theta\sigma^\mu\partial_\mu(\bar{\phi}^{\frac{2}{3}}\bar{\psi}) - \theta^2\square\bar{\phi}^{\frac{1}{3}}.$$
As $F$ appears as lowest component of $\bar{\Psi}_1$, the Lagrangian includes a kinetic term for that field. In contrast to the situation in \cite{1}, this is not inconsistent as the potential in $F$ may include arbitrary powers in that field (instead of a quadratic term only) and can be chosen to be bounded from below (instead of above). This way the field $F$ is promoted to a usual physical field. It has been shown in \cite{7} that there exist consistent models of this type. In \cite{8} these ideas have been applied to $N = 1$ SYM, leading to an effective action of that theory with dynamical glue-balls as part of the low-energy spectrum. Formally, the effective potential looks the same as in the case of Veneziano and Yankielowicz:

$$V_{\text{eff}} = -\tilde{g}_{\phi\bar{\phi}} F \bar{F} + \frac{1}{2} \tilde{g}_{\phi\bar{\phi}\bar{\phi}} F (\bar{\psi} \psi) + \frac{1}{2} \tilde{g}_{\phi\bar{\phi}\psi} F (\psi \bar{\psi}) + \frac{1}{4} \tilde{g}_{\phi\bar{\phi}\psi\bar{\phi}} (\bar{\psi} \psi)(\bar{\psi} \psi)$$

(7)

However, in contrast to \cite{1} the Kähler “metric”\footnote{This quantity is not equivalent to the true Kähler metric of the manifold spanned by $\Psi_0$ and $\Psi_1$, cf. \cite{8}.} is a function of $\phi$ and $F$, $\tilde{g}_{\phi\bar{\phi}} (\phi, F; \bar{\phi}, \bar{F})$. From eq. (7) the consistent vacua can be derived, for explicit expressions we refer to \cite{8}. The most important properties of the Lagrangian (2) with (4) are:

The effective potential is minimized with respect to all fields $\phi$, $\psi$ and $F$. Consequently, the dominant contributions that stabilize the potential must stem from the Kähler part, not from the superpotential: The superpotential is a holomorphic function in its fields and therefore its scalar part must have unstable directions. In the present context there exists no mechanism to transform these instabilities into stable but non-holomorphic terms.

Though the model has the same superpotential as the Lagrangian of ref. \cite{1} its spectrum is completely different: Chiral symmetry breaks by a vacuum expectation value (vev) of $\phi \propto \Lambda^3$, but this mechanism is more complicated than in \cite{1}. Any stable ground-state must have non-vanishing vev of $F$. But $\langle F \rangle$ is the order parameter of supersymmetry breaking and thus this symmetry is broken as well\footnote{The author of ref. \cite{2} concluded that this model cannot have a stable supersymmetric ground-state. This is in agreement with our results, as the model breaks down as $F \to 0$.}. $\psi$ is a massless spinor, the Goldstino.

The supersymmetry breaking scenario is of essentially non-perturbative nature\footnote{The importance of such a breaking mechanism has been pointed out in \cite{4} already, but a concrete description was not yet found therein.}: it is not compatible with perturbative non-renormalization theorems, as the value of $V_{\text{eff}}$ in its minimum and the vev of $T^\mu_\nu$ are no longer

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equivalent. In particular, the former can be negative, while the latter is positively semi-definite due to the underlying current-algebra relations. To our knowledge this is the first model, where this type of supersymmetry breaking has found a concrete description (cf. [7, 8] for details).

Any ground state with \( \langle \tilde{g}_{\varphi \bar{\varphi}} \rangle \neq 0 \) can be equipped with stable dynamics for \( p^2 < |\Lambda|^2 \). In the construction of concrete kinetic terms it is important to realize that \( \Xi \) may include expressions with explicit space-time derivatives. Again this is possible as \( F \) is not interpreted as an auxiliary field.

In summary, the Lagrangian of ref. [8] is the most general one, which can be formulated in terms of the effective field \( S \). Consistent ground-states can be found together with broken supersymmetry only. It would be interesting to compare these results with a different action, which has supersymmetric ground-states. But the "pièce de résistance" for such an action is the fact, that it cannot start from the effective field \( S \).

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