A New Approach to the Green-Schwarz Superstring

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Abstract

By replacing two of the bosonic scalar superfields of the N=2 string with fermionic scalar superfields (which shifts $d_{critical}$ from (2,2) to (9,1)), a quadratic action for the ten-dimensional Green-Schwarz superstring is obtained. Using the usual N=2 super-Virasoro ghosts, one can construct a BRST operator, picture-changing operators, and covariant vertex operators for the Green-Schwarz superstring. Superstring scattering amplitudes with an arbitrary number of loops and external massless states are then calculated by evaluating correlation functions of these vertex operators on N=2 super-Riemann surfaces, and integrating over the N=2 super-moduli.

These multiloop superstring amplitudes have been proven to be SO(9,1) super-Poincaré invariant (by constructing the super-Poincaré generators and writing the amplitudes in manifest SO(9,1) notation), unitary (by showing agreement with amplitudes obtained using the light-cone gauge Green-Schwarz formalism), and finite (by explicitly checking for divergences in the amplitudes when the Riemann surface degenerates). There is no multiloop ambiguity in these Green-Schwarz scattering amplitudes since spacetime-supersymmetry is manifest (there is no sum over spin structures), and therefore the moduli space can be compactified.
There are two main reasons for studying the Green-Schwarz superstring. The first reason is to obtain a more efficient method for calculating superstring amplitudes than is possible using the NSR formalism. Because of manifest spacetime-supersymmetry in the Green-Schwarz formalism, there is no need to perform GSO projections or sum over spin structures. As will be described later in this talk, this simplifies calculations involving external fermions (there are no square-root cuts and fermionic vertex operators do not require ghosts), removes the multiloop ambiguity (the moduli space can be compactified since there is no need for a cutoff associated with summing over spin structures), and allows a direct proof of finiteness (the amplitudes can be explicitly checked to be free of divergences). Although these amplitude calculations are perturbative in the string coupling constant, they may be useful for studying possible quantum corrections to general relativity, or for finding new symmetries in superstring theory.

The second reason for studying the Green-Schwarz superstring is to get a better understanding of super-Yang-Mills and supergravity. Since two-dimensional non-linear sigma models provide a natural framework for studying the massless fields of the string, one would expect that by constructing the appropriate sigma model for the Green-Schwarz superstring, one could learn something about off-shell super-Yang-Mills and supergravity. For the case of the four-dimensional Green-Schwarz superstring, this expectation has been confirmed (four-dimensional super-Yang-Mills and supergravity fields are scalar potentials and Kahler vectors of N=2 non-linear sigma models), while for the ten-dimensional case, work is still in progress. Hopefully, a better understanding of these massless supersymmetric field theories will be useful in unraveling how superstring theory produces a consistent quantum theory of gravity.

Until recently, the only method available for calculating Green-Schwarz superstring amplitudes was the light-cone gauge method in which all world-sheet symmetries, including conformal invariance, are non-manifest. Amplitudes are calculated in this method by evaluating correlation functions of light-cone vertex operators and interaction-point operators on a two-dimensional surface, and integrating over the positions of the vertex-operator punctures and the moduli of the surface. These interaction-point operators are required for Lorentz invariance and can be understood as the light-cone analog of picture-changing operators, which come from integrating out the world-sheet gravitini. However unlike picture-changing operators, their locations on the surface are completely fixed, and in fact are extremely complicated functions of the puncture positions, the surface moduli, and the \( P^+ \) momenta of the external states. Because of this complication, the light-cone Green-Schwarz method has not yet produced manifestly Lorentz-invariant expressions for any amplitude with more than one loop or more than four external states. An additional problem of the light-cone Green-Schwarz method is that in order to remove non-physical divergences when interaction-points coincide, one needs to introduce contact-terms whose precise form has not yet been determined.

Recently, a new method has been developed for calculating Green-Schwarz superstring amplitudes which starts from the manifestly N=(2,0) worldsheet supersymmetric action:

\[
S = \int d^2 z d^2 \kappa [X^\alpha \partial_\bar{\alpha} X^{\bar{\alpha}} + W^- \partial_\bar{\alpha} \Theta^+ - W^+ \partial_\alpha \Theta^-],
\] (1)
subject to the chirality constraints, \( D_+ X^\alpha = D_+ \Theta^- = D_- X^\alpha = D_- \Theta^+ = 0 \) \((a, \bar{a} \text{ range from 1 to } A \text{ and } D_\pm = \partial_\pm + \kappa^\mp \partial_z)\), the N=2 superconformal constraint, \( D_+ X^a D_- X^{\bar{a}} + D_- W^- D_+ \Theta^+ + D_+ W^+ D_- \Theta^- = 0 \), and the global constraint, \( D_+ \Theta^+ D_- \Theta^- - \frac{1}{2} (\Theta^+ \partial_z \Theta^- + \Theta^- \partial_z \Theta^+) = \partial_z X^+ \) for some real superfield \( X^+ \). This action is manifestly invariant under an \( SU(A) \times U(1) \) subset of the super-Poincaré group, which includes the \( 2A + 2 \) spacetime-supersymmetry transformations, \( \delta X^a = \epsilon^a \Theta^+, \delta X^{\bar{a}} = \epsilon^{\bar{a}} \Theta^-, \delta \Theta^+ = \epsilon^+, \delta \Theta^- = \epsilon^- \), \( \delta W^- = \epsilon^a X^a, \delta W^+ = \epsilon^{\bar{a}} X^{\bar{a}} \). Although only the heterotic superstring (ignoring lattice degrees of freedom) will be discussed in this talk, the new method easily generalizes to non-heterotic Green-Schwarz superstrings.

However up to now, it is possible only for the heterotic case to obtain this quadratic action by partially gauge-fixing a manifestly Lorentz-covariant action.\(^{1,11}\)

The action of equation 1 is just the usual N=2 string action,\(^{12,13}\) except that the two “longitudinal” pairs of bosonic scalar superfields, \((X^0, X^\bar{0})\) and \((X^d, X^{\bar{d}})\), have been exchanged for two pairs of fermionic scalar superfields, \((\Theta^+, W^-)\) and \((\Theta^-, W^+)\). Note that although the \( W^\pm \) superfields are not chiral, only \( D_+ W^+ \) and \( D_- W^- \) contribute to the action. Since the central charge contribution of the longitudinal superfields is thereby flipped from +6 to −6, the critical N=2 string now contains four pairs of transverse superfields, \( X^a = x^a + \kappa^\dagger \Gamma^a \) and \( X^{\bar{a}} = x^{\bar{a}} + \kappa^{-1} \Gamma^{\bar{a}} \) for \( a, \bar{a} = 1 \) to 4, which describe the usual light-cone Green-Schwarz content of eight scalar bosons and eight spin-\( \frac{3}{2} \) fermions (because of spectral flow, these eight spin-\( \frac{3}{2} \) fermions could alternatively be treated as four spin-0 and four spin-1 fermions).

The longtitudinal degrees of freedom of the Green-Schwarz superstring are described by the four bosonic and four fermionic components of the superfields, \( \Theta^\pm = \theta^\pm + \kappa^\pm \lambda^\pm \) and \( D_\pm W^\pm = w^\pm + \kappa^\pm \epsilon^\pm \), which are subject to the global constraint, \( \lambda^+ \lambda^- - \frac{1}{2} (\Theta^+ \partial_\pm \Theta^- + \Theta^- \partial_\pm \Theta^+) = \partial_z x^+ \) for some real field \( x^+ \). Since this global constraint on \( \lambda^\pm \) commutes with the N=2 stress-energy tensor, it does not affect the conformal anomaly calculation. However in order to construct vertex operators and calculate scattering amplitudes, it is necessary to solve the constraint in the following way:

\[
\lambda^+ = (\partial_z x^+ + \frac{1}{2} (\Theta^+ \partial_\pm \Theta^- + \Theta^- \partial_\pm \Theta^+)) e^{h^+} + e^{-h^-}, \quad \lambda^- = -e^{-h^+}, \quad (2)
\]

\[
w^+ = e^{h^+} \partial_z (h^+ + h^-) + x^- (\partial_z x^+ + \frac{1}{2} (\Theta^+ \partial_\pm \Theta^- + \Theta^- \partial_\pm \Theta^+)) + x^- e^{-h^-}, \quad w^- = x^- e^{-h^+},
\]

where \( x^+ \) and \( x^- \) are the usual longtitudinal bosonic scalars, and \( h^\pm \) are chiral bosons of screening charge \(-1\) with the operator-product \( \partial_\pm h^+ \partial_\pm h^- = z^{-2} \) (note that the relationship between \( w^\pm, \lambda^\pm \) and \( x^\pm \) closely resembles the twistor condition of Penrose.\(^{14,15}\) \( w = x \lambda \)). Because this field redefinition preserves the operator-product relations of \( \lambda^\pm \) and \( w^\pm \), the free-field action of equation 1 is still a free-field action when \( \lambda^\pm \) and \( w^\pm \) are replaced by \( x^\pm \) and \( h^\pm \). As will be shown later, equation 2 has the effect of transforming a global constraint on the fields into a constraint on the U(1) moduli of the surface.\(^2\)

Using the full SO(9,1) vector, \( x^\mu \), one can now construct N=2 superconformally invariant Lorentz generators out of the free fields.\(^3\) This is done by combining the free fields \([x^\mu, \Gamma^a, \Theta^\pm, \epsilon^\pm, h^\pm] \) into a pair of
The only correlation function that is not straightforward to evaluate is that of the $h^\pm$ fields. Since $e^{h^+ - h^-}$ has negative conformal weight, it is not possible on a general surface to define the holomorphic correlation function $< \exp(\sum_k (c_k h^-(z_k) + d_k h^+(z_k))) >$ without allowing unphysical poles (this situation also arises with the fields, $\phi$, that come from bosonizing the bosonic super-reparameterization ghosts, but in that case, the residues of the unphysical poles are BRST trivial$^{17}$). However on a surface with the special values of the U(1) moduli, $m_j = \sum_k c_k \int \omega_j$ where $\omega_j$ is the $j^{th}$ canonical holomorphic one-form, the unphysical poles are not present. Therefore, it is necessary for BRST invariance to define the correlation function $< \exp(\sum \mu (c_k h^-(z_k) + d_k h^+(z_k)) >$ to be proportional to $\prod_{j=1}^9 \delta(m_j - \sum_k c_k \int \omega_j)$, where the proportionality factor is completely fixed by its conformal properties. In this way, the global constraint on the $\lambda^\pm$ fields has transformed into a restriction on the U(1) moduli of the surface.$^2$

After performing the functional integral over the free fields, one is left with an integrand which depends on the $2(2g - 2)$ arbitrary points where the picture-changing operators have been inserted, and which must be integrated over the usual $(6g - 6 + 2N)$ Teichmuller parameters and puncture locations. Although the integrand is not manifestly Lorentz-invariant, it is possible to use knowledge of the Lorentz invariance (recall that Lorentz generators have been constructed which commute with the BRST operator and transform the vertex operators covariantly) to rewrite the integrand in a manifestly SO(9,1) invariant form.$^3$

To prove that these Lorentz-invariant expressions are unitary, one must show agreement with amplitudes obtained using the manifestly unitary light-cone Green-Schwarz formalism.$^7$–$^9$ This has been done by choosing the moduli (and corresponding Beltrami differentials) for the surface to be the light-cone interaction
points, twists, and internal $P^+$ momenta, and choosing the locations of the picture-changing operators to be precisely the interaction points. With this choice for the surface moduli, it is straightforward to show that the path integrals over the longitudinal matter fields $[x^\pm, h^\pm, \theta^\pm, \varepsilon^\pm]$ precisely cancel the path integrals over the ghost fields $[c, b, u, v, \beta^\pm, \gamma^\pm]$, since each bosonic/fermionic longitudinal matter field couples in the same way as a corresponding fermionic/bosonic ghost field. The remaining path integrals over the transverse matter fields give precisely the light-cone gauge prescription for calculating amplitudes, with the transverse part of the picture-changing operators becoming the light-cone Green-Schwarz interaction-point operators.\(^4\)

Finally, it has been shown\(^4\) that these superstring amplitudes (for external massless boson-boson states in the Type II superstring) are finite by explicitly checking for divergences when the Riemann surface degenerates either into one surface with a pinched handle, or into two surfaces connected by a thin tube (all other possibly divergent regions in moduli space are related to these by modular transformations). Note that a similar analysis has not yet been done in the NSR formalism since before summing over spin structures, the NSR amplitudes are not divergence-free.\(^{18,19}\) When the surface degeneracy corresponds to a pinched handle of radius $R$, the amplitude behaves like $(\log R)^5 R^{-1} dR$ as $R \to 0$, and is therefore divergence-free. When the surface degenerates into two surfaces connected by a tube of radius $R$, the amplitude factorizes into $A_1 A_2 R^{k^\mu - 1} dR$ as $R \to 0$, where $k^\mu$ is the momentum flowing through the tube. If there are vertex operators on both surfaces, this divergence corresponds to the physical massless pole that is present in all field theories containing massless particles. In the case when all vertex operators are located on one of the surfaces, $A_1$ vanishes since there are no zero modes for the $\theta^\pm$ path integrals (these are two of the manifest spacetime supersymmetries), and the amplitude is divergence-free.

It should be noted that under a change in the locations of the picture-changing operators, the integrand of the scattering amplitude changes by a total derivative in the Teichmuller parameters which can lead to surface term contributions if the moduli space has a boundary. For example, in the NSR formalism the need to sum over spin structures before obtaining a finite amplitude means that a cutoff in the moduli space has to be introduced, which causes a multiloop ambiguity in the amplitude.\(^{20}\) However such an ambiguity does not occur in the above Green-Schwarz amplitudes since the integrand is well-behaved when the surface degenerates, and therefore there is no need to introduce a cutoff and the moduli space contains no boundary.

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