Is there a d.c. Josephson effect in bilayer quantum Hall systems?

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We argue on the basis of phenomenological and microscopic considerations that there is no d.c. Josephson effect in ordered bilayer quantum Hall systems, even at $T = 0$. Instead the tunnel conductance is strongly enhanced, approaching a finite value proportional to the square of the order parameter as the interlayer tunneling amplitude vanishes.

Introduction: Weakly disordered bilayer quantum Hall systems with small inter-layer separation $d$ have broken symmetry ground states that can be regarded either as easy-plane ferromagnets or as excitonic superfluids. Experimental evidence for this state, in which phase coherence between different layers occurs spontaneously, was first uncovered in quantum Hall effect activation energy studies that showed anomalous in-plane field dependence indirectly related to its collective properties. Recently, however, Spielman et al. discovered a spectacular feature in the zero-bias conductance $G_T$ that appears when bilayer quantum Hall system parameters are tuned into the regime where order occurs. Although it appears clear that the dramatic conductance peak they observe is due to collective inter-layer tunneling in the ordered state, it has not yet been possible to explain its height and width or its dependence on proximity to the ordered state phase boundary. In particular, the close similarity between the phenomenological effective theory of an ordered bilayer quantum Hall system and that of a Josephson junction suggests that bilayers should exhibit a d.c. Josephson effect, i.e., that persistent currents can flow between the layers without any bias potential. Recent theoretical work has addressed the tunneling characteristics of clean and disordered bilayer quantum Hall systems at finite bias voltages, finite in-plane fields, and finite temperatures, demonstrating among other things that in the ordered state the inter-layer tunneling amplitude $\Delta_t$ cannot be treated perturbatively, i.e. that $G_T/\Delta_t^2$ diverges. The central question concerning these experiments, whether or not a d.c. Josephson effect occurs in principle, has not yet been addressed directly, although divergent views have been expressed by different authors. Experimentally there is no evidence for a d.c. Josephson effect, i.e. the $T \rightarrow 0$ zero-bias conductance peak appears to be finite. The explanation for this finding need not be fundamental, however; for example the $T \rightarrow 0$ order-parameter could vanish in current samples due to quantum fluctuations that are expected to be enhanced by disorder, or the highest measured conductance values could be limited by extrinsic experimental effects. In this Letter we argue that in bilayer quantum Hall systems $G_T$ always remains finite at $T = 0$. Nevertheless, $G_T/\Delta_t^2$ does diverge for $\Delta_t \rightarrow 0$ and inter-layer tunneling cannot be treated perturbatively. In the following paragraphs, we first discuss the physical picture that underlies our theory and then present an approximate but fully microscopic calculation that we believe captures all essentials of the effect.

Phenomenological Theory: We use a pseudospin language to describe bilayer quantum Hall systems.

$$S_\alpha = \frac{1}{2} \sum_{k,\sigma',\sigma} c_{k,\sigma'}^{\dagger} \tau_{\sigma',\sigma}^{(\alpha)} c_{k,\sigma}$$

(1)

is the total pseudospin component in direction $\alpha$, $k$ is a Landau level orbital labels, $\sigma, \sigma'$ are the pseudospin labels, and $\tau_{\sigma',\sigma}^{(\alpha)}$ are Pauli spin matrices with pseudospin up/down representing electrons in top/bottom layers. In this language the ordered state is an $XY$ easy-plane pseudospin ferromagnet. Its long-wavelength, low-frequency, small-amplitude dynamics should be described by the linearized Landau-Lifshitz-Gilbert equation of magnetism

$$\frac{dM_z}{dt} = \frac{\Delta_t}{\hbar} M_y,$$

$$\frac{dM_y}{dt} = \frac{1}{\hbar} \left[ eV - (\Delta_t + 4\pi l^2 \beta) M_z \right] - \frac{M_y}{\tau}.$$  

(2)

In Eq.(2) $\hat{M} \approx (1 - [M_y^2 + M_z^2]/2, M_y, M_z)$ is a unit vector which specifies the pseudospin ordered moment orientation, and $\beta$ is the pseudospin anisotropy energy per unit area which will be discussed at greater length below. For quantum Hall ferromagnets there is no relaxation term in the first of Eqs.(2) because only inter-layer tunneling, which we take to be constant, violates the separate conservation of charge in each layer. The factor in square brackets in the second of Eqs.(2) is the effective field in the $z$-direction which includes both external and induced
contributes to the electrochemical potential difference $\delta \mu$ between the two layers. It follows from the first of the Eqs.(3) that the inter-layer current is $I = (NeM_0/2)(dM_z/dt) = (NeM_0\Delta_i/2\hbar)M_y$, where $N$ is the total number of electrons, $\lim_{\Delta_i \to 0} \langle S_y \rangle = N M_0/2$ is the pseudospin ordered moment. Here $M_0$ is a dimensionless order parameter that approaches 1 for layer separation $d \to 0$ in the absence of disorder [13]. In the steady state, the driving and relaxation terms in the second of Eqs.(2) cancel, from which it follows that $M_y = \delta \mu \tau/\hbar$ and that the tunnel conductance is [13]

$$G_T = \frac{e^2}{\hbar} \frac{NM_0 \Delta_i \tau}{2\hbar}.$$  

The relaxation time in bilayer quantum Hall systems can be evaluated microscopically by evaluating the dynamic pseudospin response function. It follows from Eq.(3) that at low frequencies

$$\chi_{yz}(\omega) = \frac{-M_0 i\hbar \omega}{[(\Delta_i + 4\pi^2 \hbar^2 \omega^2)\Delta_i - \hbar^2 \omega^2] - i\hbar^2 \omega/\tau}.$$  

We derive a response function of precisely this form from a microscopic theory below.

**Is there a d.c. Josephson Effect?** In using Eq.(3) we are asserting that the d.c. conductance is equal to the $\omega \to 0$ limit of the a.c. conductance and thus denying the possibility of persistent d.c. tunnel currents. In a conventional Josephson junction geometry, persistent currents are enabled by order parameter phase rigidity across the entire system. From the point of view of microscopic mean-field-theory, the anomalous self-energy couples electrons and holes, forming an equilibrated quasiparticle system which self-consistently establishes different order parameter phase values on opposite sides of the junction. In other words, the Dyson equation has a set of self-consistent solutions with continuously variably order parameter phase difference across the junction. In a quantum Hall system, on the other hand, there is no analog of overall phase rigidity, only phase difference rigidity supported by non-local interlayer interactions. From the microscopic point of view, the exchange self-energy associated with order couples electrons in balanced layers through a pseudospin effective field that points in the same direction as the pseudospin order parameter. Since inter-layer tunneling adds a pseudospin effective field in the $\hat{x}$ direction, a self-consistent equilibrated quasiparticle system is possible only when the pseudospin order points in the $\hat{x}$ direction, in which case no current flows. A collective interlayer current can always decay by making particle-hole transitions in the non-equilibrium quasiparticle system. The microscopic calculations presented below describe this effect. Of course, persistent currents can also decay in standard Josephson junctions, even at $T = 0$, because of collective quantum tunneling of the order parameter field. This decay mechanism is, however, qualitatively weaker and easily distinguished.

**Microscopic Theory:** To describe a quantum-Hall bilayer, we use a self-consistent Born approximation for the disorder self-energy, a self-consistent Hartree-Fock approximation for the interaction self-energy, and include consistent ladder and bubble vertices in the two-particle Greens function of interest. It is essential for our analysis that this approximation, summarized diagrammatically in Fig.1, captures the physics associated with the separate conservation of charge in each layer when $\Delta_i \to 0$ and that all necessary diagram sums can be evaluated accurately. The disorder averaged Greens function depends on frequency only and satisfies the Dyson equation

$$G^{-1}(i\nu_n) = \begin{pmatrix} -i\nu_n + \rho_{TT}\Gamma_A + v^2_G \rho_{TT}(i\nu_n) + \Delta_i/2 + \rho_{TB}\Gamma_E + v^2_G \rho_{TB}(i\nu_n) & -i\nu_n + \rho_{BB}\Gamma_A + v^2_G \rho_{BB}(i\nu_n) \\ +\Delta_i/2 + \rho_{BT}\Gamma_E + v^2_G \rho_{BT}(i\nu_n) \\ -i\nu_n + \rho_{BB}\Gamma_A + v^2_G \rho_{BB}(i\nu_n) \end{pmatrix},$$

where $\Gamma_A$ and $\Gamma_E$ are intralayer and interlayer exchange [16] integrals, $v^2_s$ and $v^2_d$ are the disorder correlation functions in same and different layers, and the labels $T$ and $B$ refer to top and bottom layers. In Eq.(3) $\rho_{\sigma\sigma'}$ is the density matrix which is determined self-consistently by integrating Greens function spectral weights up to the Fermi energy. We concentrate here on balanced bilayer systems at total filling factor $\nu = 1$ so that $\rho_{TT} = \rho_{BB} = 1/2$, and take $v^2_d = 0$ since the disorder potentials in different layers are not expected to be correlated. $\rho_{TB} = \rho_{BT}$ is non-zero when order is established. Unlike the analogous Josephson-junction-system Dyson equation, when $\Delta_i \neq 0$, Eq.(3) has a solution only for purely real $\rho_{TB}$. Persistent currents would occur if complex solutions existed.

Inversion symmetry in balanced bilayers separates the $y$ and $z$ pseudospin response function components, related to correlations between many-particle states with opposite parity, from the charge and $x$ pseudospin response functions, related to correlations between states with the same parity. A somewhat lengthy but elementary calculation that follows line similar to earlier work [16,17] leads to the following expressions:

$$\begin{pmatrix} \chi_{yy}(\omega) & \chi_{yz}(\omega) \\ \chi_{zy}(\omega) & \chi_{zz}(\omega) \end{pmatrix}^{-1} = \begin{pmatrix} -\Gamma_E & 0 \\ 0 & 2V_x - \Gamma_A \end{pmatrix} + \begin{pmatrix} \Pi_{yy}(\omega) & \Pi_{yz}(\omega) \\ \Pi_{zy}(\omega) & \Pi_{zz}(\omega) \end{pmatrix}^{-1},$$

where $2V_x$ is the difference between intra-layer and inter-layer Coulomb interactions,
where we have used that $\Delta_{qp}$

The relaxation rate $\tau_{\alpha\beta}$ compare the SCBA response functions with Eq.( 4). The inset to Fig.( 2) shows the dependence of the quasiparticle tunneling amplitude. It follows from Eqs.( 7) and ( 8) that at low frequencies

For small $\Delta_{qp}$, it follows from the standard Golden rule argument that $G_T \propto \Delta_{qp}^2$ which, since $M_{qp}$ is proportional to $\Delta_{qp}$, implies that $\tau_{qp}$ approaches a constant as $\Delta_{qp} \to 0$. This result is expected since in this limit, the two quasiparticle systems are independent and $\Pi_{zz}(\omega) = (\Delta_t/i\hbar \omega)\Pi_{yz}(\omega)$ is the standard single-layer diffusive response.

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Fig. 1. Diagrammatic summary of SCBA and GRPA. It is essential to include disorder vertex corrections along with the disorder broadening of the single-particle bands. The direct and the exchange channels included in the GRPA capture the competing Hartree and exchange interactions in these systems.

Fig. 2. Typical plots of the $\chi_{yz}$ response function. Our microscopic results for this response function are consistent with phenomenological expression [3]. The inset shows the dependence of quasiparticle relaxation time on $\Delta_{qp}$, obtained by fitting the $\Pi_{yz}$ response function to Eq. (10).

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