Hadley circulations
and large scale motions of moist convection
in the two dimensional numerical model

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Abstract

As a tool for understanding the meridional circulation of the atmosphere, a two-dimensional (latitude–height) numerical model is used to clarify the relationship between the Hadley circulation and large-scale motions associated with moist convection. The model is based on the primitive equations including the moist process, and two kinds of coordinates are used: the spherical coordinate and the Cartesian coordinate with a uniform rotation. The surface temperature is externally fixed and the troposphere is cooled by the radiation; unstable stratification generates large-scale convective motions.

Dependencies on the surface temperature difference from north to south $\Delta T_s$ are investigated. The numerical results show that a systematic multi-cell structure exists in every experiment. If the surface temperature is constant ($\Delta T_s = 0$), convective motions are organized in the scale of the Rossby deformation radius and their precipitation patterns have a periodicity of the advective time $\tau_D$. As $\Delta T_s$ becomes larger, the organized convective system tends to propagate toward warmer regions. The convective cells calculated in the Cartesian coordinate model is very similar to those of the mid-latitudes in the spherical coordinate model. In particular, the Hadley cell can be regarded as the limit of the convective cells in the equatorial latitudes.
1 Introduction

Although it is well understood that two-dimensional axisymmetric models are important for studying the Hadley circulation (Schneider, 1977; Held and Hou, 1980), this kind of model has not been thought useful for large-scale motions in the extratropics, since non-axisymmetric baroclinic waves are essential in the real atmosphere. Until the beginning of this century, however, meridional cellular structure in an axisymmetric situation had been a major problem of the general circulation of the atmosphere (e.g., Thomson, 1892; Lorenz, 1967): not only the Hadley circulation but also the cellular structure in the extratropics. Views of the general circulation has changed from that given by Hadley(1735) in which only a direct cell exists in each hemisphere (upward motion in the equatorial side and downward motion in the polar side) to that given by Ferrel(1856) and Thomson(1892) in which a direct cell co-exists with an indirect cell (downward motion in the equatorial side and upward motion in the polar side). Although various cellular structures had been presented, the problem of the cell shapes was almost settled by the requirement of the angular momentum balance; the direct cell must override the indirect cell in the axisymmetric circulations, and the cell boundary is inclined to the pole as in the upper layers. This cellular structure is presented by Bjerknes(1937) and Eliassen(1951). The numerical experiments by Schneider(1977) and Held and Hou(1980) have qualitatively reproduced this structure.

Satoh(1994a) has calculated the axisymmetric circulations of a moist atmosphere by using a two-dimensional model in the global domain (hereafter, referred to as S94). The surface temperature is externally fixed and the free atmosphere is cooled by the gray radiation model. Large-scale convective motions are naturally resolved in the model and the stratification is stabilized, even if no cumulus parameterization except for the large-scale condensation is introduced. The results show that, in the low-latitudes, there exists a large-scale direct cell which corresponds to the Hadley cell. The cell widths are broader as in the upper layers, so that the cell shape is very similar to that presented by Bjerknes(1937) and Eliassen(1951). In the extratropics, however, unsteady multiple cells exist instead of an indirect cell and they propagate toward the equator (S94, Fig. 3; reproduced as Fig.
2(c) of the present paper). Hereafter, the multiple cells calculated in S94 in the mid- and high-latitudes are referred as the symmetric cells.

The circulations in the mid-latitudes of S94 are different from those obtained by Schneider(1977) and Held and Hou(1980). The difference may be ascribed to the maintenance mechanism of the stratification of the models. In S94, the stratification tends to be unstable because of the radiative cooling in the troposphere and the heating from the surface. Convective motions must be generated in order to suppress the unstable stratification. In contrast to this, Schneider(1977) and Held and Hou(1980) used a stable temperature profile as a reference state of the Newtonian cooling, so that no local convection exist in their models.

In the extratropics of the real atmosphere, non-axisymmetric motions such as baroclinic waves are dominant, and the symmetric cells calculated in S94 is hardly observed. As will be shown in sections 2 and 3, however, the symmetric cells generally exist in all experiments of the two-dimensional model, and play an important role in the transport of the angular momentum. It is required that dynamical mechanisms of the symmetric cells should be clarified, in particular, for their propagation and horizontal scales, to gain a complete view of the axisymmetric systems. The symmetric cells are also valuable, since they will provide a further insight into the Hadley circulation. It will be shown that the Hadley cell can be regarded as one of the symmetric cells.

In this paper, the circulations in the mid-latitudes of the axisymmetric atmospheres are focused. A two-dimensional model in horizontal-vertical section is utilized to investigate the dynamics of the symmetric cells. Two kinds of coordinates are used; the model in the spherical coordinate is referred as the global model, and the model in the Cartesian coordinate is referred as the $f$–plane model. Formulation of the model is described in the appendix. In section 2, the angular momentum budget of the standard experiment of S94 is analyzed, and additional results for the global model are shown. In sections 3 and 4, the symmetric cells are discussed with the results for the $f$–plane model. Section 5 summarizes
the relationship between the symmetric cells and the Hadley circulations and discuss the generalization of these findings.
2 Global condition

2.1 Angular momentum budget

The angular momentum budget plays a key role in meridional circulations of the axisymmetric atmosphere. When steady direct and indirect cells co-exist as depicted by Bjerknes (1937) and Eliassen (1951), the cell shapes are determined by the requirements of the angular momentum balance in the following manner. The direct cell transports the angular momentum upward, and the indirect cell transports it downward, so that there must be an exchange of the angular momentum between both cells. In the axisymmetric system, the viscous stress is the only process of the momentum exchange between steady cells. If coefficients of the viscosity are isotropic, horizontal transport caused by the viscous stress is much smaller than vertical transport by the viscous stress, since the aspect ratio of the atmosphere is very small. Thus, the angular momentum is vertically transported by the viscous stress which is associated with the vertical shear of the zonal winds. In order for the zonal shear to be maintained, the direct cell must override the indirect cell, and the cell boundary is inclined to the pole as in the upper layers.

In the beginning, we examine the angular momentum budget of the results for the global model of S94, and compare it with the above consideration. In the standard experiment of S94, the surface temperature is given by

$$T_s(\varphi) = T_A - (T_B - T_A) \sin^2 \varphi,$$

where \( \varphi \) is the latitude, \( T_A = 300 \) K, and \( T_B = 260 \) K. The diffusion coefficient of momentum flux is 5 m\(^2\)/s, and that of heat and moisture fluxes is 1 m\(^2\)/s.

In the standard experiment, the Hadley cell exists in the low-latitudes, and a multiple cell structure which propagates equatorward exists in the mid- and high-latitudes. In order to isolate transports by the symmetric cells in the mid- and high-latitudes, we divide
convergence of the angular momentum into steady and unsteady parts:

\[- \nabla \cdot (lv) = - \nabla \cdot (\overline{lv}) - \nabla \cdot (lv').\]  

(2)

\(l\) is the absolute angular momentum, \(v\) is the meridional velocity, and \(\nabla \cdot\) is the divergence in the meridional plane. \((\overline{)}\) denotes time average and \((\cdot)'\) the deviation from the average.

Fig. 1 shows the respective terms in Eq. (2). The total component (a) shows a large divergence (gain of the angular momentum in the atmosphere) in the lower boundary layers of the Hadley cell. At the polar boundary of the Hadley cell (the latitudes 10°–20°), divergent and convergent regions are vertically adjacent to each other. Both regions are inclined to higher latitudes in the upper layers. The steady component (b) is uniformly divergent in the lower layers in every latitude, while it is convergent in the upper layers in the extratropics. Conversely, the unsteady component (c) is divergent in the upper layers, while it is convergent in the lower layers.

Fig. 1 indicates that it is the symmetric cells that transport the angular momentum downward in the mid-latitudes. The angular momentum is supplied to the atmosphere in the surface easterly regions in the low-latitudes, and is transported upward and poleward by the Hadley cell. It is transferred downward by the viscous stress at the polar boundary of the Hadley cell, where a large vertical shear of the westerlies exists. In the mid- and high-latitudes, westerly shear is maintained by the poleward transport of the angular momentum by a weak steady direct cell, which has a hemispheric scale as shown by Fig. 1(b). On the other hand, the unsteady symmetric cells transport the angular momentum downward in these regions. Although both easterlies and westerlies exist at the surface boundary of the symmetric cells, the westerly component is a little stronger, hence the angular momentum is transferred from the atmosphere to the ground due to the net effect of the surface friction.

The role of the Hadley cell in the angular momentum budget is the same as that depicted by Bjerknes(1937) and Eliassen(1951). In the extratropics, however, the symmetric cells play a role of the downward transport of the angular momentum instead of a steady indirect

\(^{1}\) In precisely, the convergence is defined by \(- \nabla \cdot (p_s lv)/p_s\), since the model is expressed by the \(\sigma\) coordinate. Fig. 1(a) is the total convergence \(- \nabla \cdot (p_s lv)/p_s\), (b) is the steady part \(- \nabla \cdot (\overline{p_s lv})/p_s\), and (c) is the difference between (a) and (b).


2.2 Additional experiments

Additional experiments of the global model are performed, so as to investigate the dependencies of the symmetric cells on the surface temperature differences $\Delta T_s = T_A - T_B$ and the viscosity coefficients $\nu$. Fig. 2 shows the time sequences of the latitudinal distribution of the precipitation for $\Delta T_s = 0\text{K}, 20\text{K}, 40\text{K}$ (the standard experiment), and 80K. The temperature at the equator is $T_A = 300\text{K}$ for every case. In the standard experiment, the symmetric cells are propagating equatorward at a speed of approximately 1 m/s, and have a period of about 20 days. The speed of the propagation becomes faster as $\Delta T_s$ becomes larger. In the case of $\Delta T_s = 0$, the precipitation patterns are periodic and they do not change their positions except for the equatorial latitudes. These results imply that the gradient of the surface temperature and the Coriolis parameter affect the propagation of the symmetric cells. It should be noted that, in the case of $\Delta T_s = 0$ (Fig. 2(a)), precipitation is not concentrated at the equator but randomly distributed in the low-latitudes. That is, the Hadley cell no longer exists in this model, when the surface temperature gradients are sufficiently small near the equator.

Fig. 3 displays the dependencies of the precipitation on the viscosity coefficients of the free atmosphere ($\nu$) and of the surface layer ($\nu_s$): $\nu = \nu_s = 1$ and 25 m$^2$/s. In the case of $\nu = \nu_s = 25$ m$^2$/s, the symmetric cells around the latitudes 30° do not propagate. This result indicates the propagation of the symmetric cells is related with the angular momentum transport, since the viscous diffusion controls the vertical momentum transport.
3 Mid-latitude condition

3.1 Model

In this section, an "f-plane model" is used to investigate the symmetric cells in the mid-latitudes; the model is two-dimensional (horizontal-vertical) with a uniform rotation rate in the Cartesian coordinate. As summarized in the appendix, the formulation of the model, i.e. the basic equations, the discretization, and the physical processes, is almost the same as that of the global model. Only the metrics and the domain are different.

The model is based on the primitive equations with a $\sigma$ coordinate ($\sigma = p/p_s$; $p$ the pressure and $p_s$ the surface pressure). The lateral widths of the domain is $y_0 = 10^7$ m. Here, the lateral coordinate is denoted by $y$, which corresponds to the latitude of the global model; larger $y$ indicates the northward direction. The lateral boundaries are assumed to be walls with slip and insulating conditions. The number of grid points is 100 in the horizontal direction and 50 in the vertical direction. The grid intervals are $\Delta y/y_0 = 0.02$ and $\Delta \sigma = 0.02$, respectively. The Coriolis parameter is prescribed as $f = \Omega_0 = 7.272 \times 10^{-5}$ rad/s, which is the value at the latitude 30° ($\Omega_0$ is the rotation rate of the earth).

As for the radiation process, a uniform cooling rate 2K/day is given below the level $\sigma = 0.1$ to the ground just for simplicity. The level $\sigma = 0.1$ can be regarded as the tropopause. The surface temperature is externally fixed, and the ground surface is assumed to be wet everywhere. Moisture is supplied from the surface to the atmosphere by the bulk formula. In the same way as section 3a of S94, only the “large-scale condensation” is employed as the moist process in the atmosphere, and any other cumulus parameterization is used; phase change of moisture is explicitly calculated at each of the grid point, and the liquid part of the air is removed as precipitation when it is saturated. Diffusion coefficients for the momentum, heat, and moisture are set to be constant as in S94.
3.2 Results

The surface temperature is linearly prescribed as

\[ T_s = T_A + (T_A - T_B) \frac{y}{y_0}. \]  

(3)

The temperature difference in the domain is denoted by \( \Delta T_s = T_A - T_B \). Fig. 4 shows time variations (0 – 100 days) of precipitation for (a) \( T_A = T_B = 300 \) K, (b) \( T_A = 305 \) K, \( T_B = 295 \) K, (c) \( T_A = 310 \) K, \( T_B = 290 \) K, and (d) \( T_A = 320 \) K, \( T_B = 280 \) K. In every case, there exists a systematic cellular structure similar to that in the mid-latitudes of the global model. The precipitation occurs in the upward motion regions of the symmetric cells. When there is no temperature difference (\( \Delta T_s = 0 \)), the precipitation patterns have a periodicity of about 20 days. When there is a temperature difference (\( \Delta T_s \neq 0 \)), the precipitation patterns propagate to the warmer regions (smaller \( y \)). As the temperature difference becomes larger, the velocity of the propagation becomes larger. Although the lateral intervals between the precipitating regions are not sensitive to \( \Delta T_s \), they are larger in the warmer regions. The periodicity or the propagation of the precipitation is associated with the circulations of the symmetric cells. Structure and mechanism of the symmetric cells will be examined in the following subsections.

3.3 Case of \( \Delta T_s = 0 \)

The symmetric cells in the case \( \Delta T_s = 0 \) (Fig. 4(a)) are examined here. Fig. 5 shows the time variation of the surface zonal winds for the days 0 – 100. \footnote{The winds perpendicular to the model domain refer to the zonal winds; positive winds are westerly, and negative winds are easterly.} Westerlies exist in the southern side of the precipitating regions, whereas easterlies exist in the northern side. Their phases correspond to those of the precipitation. Figs. 6(a) and (b) show the zonal winds in the meridional plane (\( y-\sigma \)) for the averages of the days 46–55 and the days 56–65, respectively. Figs. 7(a) and (b) are the temperature perturbation from the \( y \)-average for the same periods. The phase of the precipitation changes at about the day 55. For instance, precipitation occurs at \( y/y_0 = 0.52 \) during the days 46–55, and it jumps
to $y/y_0 = 0.41$ and $0.62$ at the day 55. At the precipitating regions, the circulations of the lower layers are cyclonic with convergence of lateral winds, while those of the upper layers are anticyclonic with divergence of lateral winds. The temperatures in the upward motion regions are warmer than those in the downward motion regions irrespective of height. The temperatures are almost in thermal wind balance with the zonal winds.

The periodicity of the symmetric cells can be explained as follows. Associated with the convective circulation, cyclonic winds develop in the lower layers, and anticyclonic winds develop in the upper layers. In the northern part of a precipitating region (upward motion region), easterlies develop in the lower layers while westerlies develop in the upper layers. As the convective motion continues, the westerlies in the upper layers gradually come down. By the time they reach the ground, the lateral gradient of pressure changes its sign due to the geostrophic adjustment to the westerlies. The inverted pressure gradient force causes lateral winds of the opposite direction near the ground. Finally, the circulations are reversed; the downward motions are generated in the previously upward motion region, and upward motions are generated in the previously downward motion region. The period is given by the overturning time $\tau_D$, which is defined by $H/w_d$ where $H$ is the height of the troposphere and $w_d$ is the speed of the downward motion. (This is also estimated by Eq. (29) of S94.)

The lateral scale of the symmetric cells $L$ can be estimated in the following manner. Note that there are two cells between adjacent precipitating regions, so that the lateral interval of the precipitating regions is expressed by $2L$. If one denotes the lateral temperature difference in the middle of the troposphere by $\Delta T$, the thermal wind balance is expressed by

$$\frac{g}{T_0} \frac{\Delta T}{L} = \frac{fU/2}{H},$$  \hspace{1cm} (4)

where $U$ is the maximum value of the zonal wind at the tropopause $H$, and $T_0$ is the mean temperature. By estimating $U = fL$ from the angular momentum conservation at the
height $H$, one obtains
\[ L = \left( \frac{2gH \Delta T}{f^2 T_0} \right)^{1/2} \]  
(5)
where $g$ is the acceleration of gravity.

The value of $\Delta T$ should be known in order to obtain $L$. As shown below, only an upper bound of $\Delta T$ can be given from the consideration of the stratification. However, it will be shown that $\Delta T$ is actually close to the upper bound. If one supposes a thermal wind balance of a large $L$, $\Delta T$ also becomes larger as $L$ is increased. In this case, the temperature profile in the upward motion regions does not change, since it is prescribed by a moist adiabat. Thus, the temperature in the downward motion regions must be cooler in this balance. Since the surface temperature is uniform, the stratification of the downward motion regions becomes unstable. This implies that another convective motion will occur from these regions. Although the downward motion regions are, in general, conditionally unstable, they would be absolutely unstable in the balance of a sufficiently large $L$. Therefore, the upper bound of $\Delta T$ is given by that in the case when temperature lapse rate in the downward motion regions is equal to the dry adiabat. Fig. 8 displays a time variation of lapse rate ($-dT/dz$) at $\sigma = 0.9$ which is just above the mixed layers. The values above 8 K/km are shown by hatching. Comparison with Fig. 4(a) shows that the lapse rate is about 9K/km at the downward motion regions when the circulation changes its direction. This lapse rate is close to the dry adiabat $\Gamma_d = g/C_p = 9.8$K/km ( $C_p$ is specific heat for constant pressure ). From this, one may conclude that the temperature difference $\Delta T$ is given by this upper bound.

The temperature difference is given by
\[ \Delta T = \left( T_s - \Gamma_m \frac{H}{2} \right) - \left( T_s - \Gamma_d \frac{H}{2} \right) = \left( \frac{g}{C_p} - \Gamma_m \right) \frac{H}{2}, \]  
(6)
where $\Gamma_m$ is the lapse rate of the moist adiabat. One obtains, therefore, by using Eq. (5) and (6),
\[ L = \left[ \frac{H^2 g}{f^2 T_0} \left( \frac{g}{C_p} - \Gamma_m \right) \right]^{1/2} = \frac{NH}{f} = R_N, \]  
(7)
3 The angular momentum in the $f$-plane is expressed by $u - fy$. 

where $N^2 = g/T_0(g/C_p-\Gamma_m)$ is the Brunt Väisälä frequency for the moist adiabatic profile. In conclusion, the lateral scale is given by the Rossby deformation radius $NH/f$. It is estimated as $L \approx 0.11y_0 = 1.1 \times 10^6$ for the values in the case of Fig. 4(a): $N^2 = 7 \times 10^{-5}$ [sec$^{-2}$], $H = 1 \times 10^4$ [m] and $f = 7.27 \times 10^{-5}$ [sec$^{-1}$]. Fig. 4(a) shows that the lateral interval of the precipitating regions is approximately 0.18 $y_0$; that is, $L \approx 0.09y_0$, which is very close to the above estimation.

One might suspect that the result that the scale of the convective cell is given by the Rossby deformation radius is only a trace of linear theories (e.g., Gill, 1982; Emanuel, 1983; Bretherton, 1987). However, the present case is considered in the nonlinear statistically equilibrium states, to which the linear theories are not directly applied. The nonlinear advection of the angular momentum and the asymmetry between the lower and the upper boundary conditions for the momentum (the angular momentum is exchangeable at the lower surface, but it is not at the upper boundary) are key points of the above derivation. If, instead, one applies the slip conditions in which momentum exchange is forbidden both at the lower and the upper boundaries, the angular momentum will be horizontally uniform in the whole layer around the cumulus region in the nonlinear equilibrium states. In this case, the vertical component of the absolute vorticity becomes zero, so that the circulation field associated with cumulus heating will not feel the basic rotation $f$ and the scale of the convective cell will be larger than the Rossby deformation radius. In the present study, however, the circulation field can feel the basic rotation through the angular momentum exchange at the lower surface, and, as a result, the Rossby deformation radius becomes a natural scale for the convective cell.

### 3.4 Case of $\Delta T_s \neq 0$

As an example for $\Delta T_s \neq 0$, the wind fields in the case of $T_A = 320$ K and $T_B = 300$ K are examined. Fig. 9 shows a time variation (the days 0 – 100) of the precipitation.

4 As for Figs. 4(c) and 9, $\Delta T_s = 20$ K is the same, but $T_A$ and $T_B$ are different. These figures show that the absolute values of temperature affect the lateral intervals of the precipitating regions; the intervals are larger as the temperature is higher.
wind fields are averaged for 50 days along the propagating precipitating region A → B of Fig. 9 (A : $y/y_0 = 0.68$ at the day 51, B : $y/y_0 = 0.45$ at the day 100 ). Figs. 10 (a), (b) show the zonal winds and the stream functions, respectively. The line A → B corresponds to $y/y_0 = 0.5$ of Fig. 10. In the lower layers, the northern side of the precipitating region ($y/y_0 = 0.5$) is westerly, while the southern side is easterly. In the upper layers, contrary to this, the northern side of $y/y_0 = 0.5$ is easterly, while the southern side is westerly. The region of the upper westerlies is connected with that of the surface westerlies ($y/y_0 = 0.6 - 0.66$), which are associated with the inflow to the next precipitating region ($y/y_0 = 0.66$). As shown by the stream functions, each precipitating region has both northern and southern cells. The northern cell is broader than the southern cell especially in the upper layers, and the northern cell spreads over the next southern cell which is associated with the next precipitating region ($y/y_0 = 0.66$). That is, the direct cell (positive sign of the stream functions) is in contact with the indirect cell (negative sign) by an inclined boundary (the contour of zero). The angular momentum is transported upward by the direct cell, and is transported downward by the indirect cell, hence the angular momentum should be transferred from the direct cell to the indirect cell. The inclination of the cell boundary makes possible the vertical transfer between the cells by the viscosity. This cell structure is similar to the relation between the Hadley cell and the Ferrel cell depicted by Eliassen (1951), and is essential for the angular momentums transport between the two cells.

Fig. 11 shows the temperature and the normalized angular momentum $(u - fy)/(fy_0)$ at the day 100. The upper angular momentum and temperature is uniform in the regions $y/y_0 = 0.22 - 0.4$ and $y/y_0 = 0.41 - 0.64$, for example. These uniform regions are associated with the precipitation at $y/y_0 = 0.24$ and 0.46, respectively (Fig. 9). The lateral gradient of the temperature is larger as in the lower layer, and the temperature is in the thermal wind balance. In this respect, the structure of the symmetric cells is very similar to that of the Hadley cell; the angular momentum is conserving in the upper layers and the temperature is balanced with the vertical shear (compare with Figs. 2 and 4 of S94).
As for the relationship with the Hadley cell, the symmetric cells in the $f$-plane model should be investigated in terms of the argument by Plumb and Hou (1992), in which the symmetric circulations are classified into two regimes: a non-meridional circulation regime in thermal wind balance with the prescribed heating (a thermal equilibrium (TE) solution) and a meridional circulation regime with an angular momentum conserving flow in the upper layer (an AMC solution). They argued that the response takes the form of the TE solution if the thermal forcing is weak or broad or it is far from the equator (Eq. (8) and Appendix of Plumb and Hou). Although the Newtonian cooling is used as the thermal forcing in Plumb and Hou, the similar argument is partly applied to the present study if their reference temperature $\hat{T}_s$ is replaced by the surface temperature $T_s$. In the $f$-plane model, Eq. (8) of Plumb and Hou corresponds to

$$\frac{-gH}{T_0} \frac{\partial^2 T_s}{\partial y^2} < f. \quad (8)$$

If this condition is satisfied, the angular momentum balanced with the surface temperature does not have an extremum in the interior of the atmosphere and the TE solution is regular provided. Plumb and Hou further argued that the AMC solution cannot exist when the TE solution is regular provided. If the surface temperature is linearly distributed (Eq. (3)), the left hand side of (8) vanishes and the inequality holds. However, it should be pointed out that the proof in their Appendix is insufficient for the $f$-plane model, since they implicitly assumed that two Hadley cells coexist: the summer cell and the winter cell. Their relation (A13), which is derived from this assumption, does not hold in the present situation. Thus, the possibility of the AMC solution is not precluded even if (8) is satisfied.

In contrast to Plumb and Hou, S94 derived the condition for the AMC solution and the latitudinal scale of the Hadley cell from the relation between the surface temperature $T_s$ and the temperature in the middle layer $T$ which is in the thermal wind balance with the angular momentum conserving flow (Figs. 11 and 13 of S94). From this, the AMC solution can exist when $T$ is higher than $T_s$ around the upward motion region. This condition is
rewritten in terms of the gradients:

\[ \left| \frac{\partial T_s}{\partial y} \right| > \left| \frac{\partial T}{\partial y} \right|. \]  (9)

The temperature deviation in the middle layer from that at the upward motion region of the AMC solution is given by \( \Delta T \) in Eq. (5). Replacing \( L \) by the distance from the upward motion region \( \delta y \) and \( \Delta T \) by \( -\delta T(\delta y) \), one obtains

\[ \delta T(\delta y) = -\frac{T_0 f^2 \delta y^2}{2gH}. \]  (10)

From this, the right hand side of (9) becomes zero as \( \delta y \to 0 \), so that the AMC solution can be possible if there is a non-zero surface temperature gradient. If one denotes the deviation of \( T_s \) from that at the upward motion region by \( \delta T_s(\delta y) \), the relation between \( \delta T_s \) and \( \delta T \) gives the scale of the cell \( L \) as shown by Fig. 12(b). By equating the change of \( \delta T \) in \( L \) with that of \( \delta T_s \):

\[ \delta T(L) = \delta T_s(L) = -\Delta T_s \frac{L}{y_0}, \]  (11)

one obtains with Eq. (10),

\[ L = \frac{2gH \Delta T_s}{f^2 y_0 T_0} \equiv R_T. \]  (12)

This indicates that \( L \) increases with \( \Delta T_s \).

The scale of the cell \( L \) corresponds to the cell width of the northern side of the precipitating region; the cell width is defined by the upper spread of the stream functions of the cell. (Remind that \( L \) is not the interval of the precipitating regions. In the case of \( \Delta T_s = 0 \), the cell width is \( L \) whereas the interval of the precipitating regions is \( 2L \).) For \( \Delta T_s = 20 \) K and \( T_0 = 270\)K, Eq. (12) estimates \( L = 0.027y_0 \), which is much smaller than the cell width shown by Fig. 10. Furthermore, Eq. (12) approaches zero as \( \Delta T_s \to 0 \) and contradicts Eq. (9). One of the reasons for this underestimation is that the effect of static stability, which is important in the case of \( \Delta T_s = 0 \), is not taken into account in the derivation of Eq. (12). As shown by Fig. 11, the temperatures in the middle layers are maximum around \( y/y_0 = 0.46 \) where the precipitation occurs, and the temperatures are minimum near \( y/y_0 = 0.40 \) and 0.64. The northern and the southern cells exist in the regions between these minimum and
maximum temperatures; the northern cell spreads in the region \( y/y_0 = 0.46 - 0.64 \), and
the southern cell spreads in the region \( y/y_0 = 0.40 - 0.46 \). As in the case of \( \Delta T_s = 0 \), the
static stability is smaller around the minimum temperatures. One can take the effect of
the static stability on the cell scale into account by simply adding Eq. (11) to Eq. (11) (Fig.
12(c)):

\[
\delta T(L) = -\Delta T_s \frac{L}{y_0} - \left( \frac{g}{C_p} - \Gamma_m \right) \frac{H}{2},
\]

which yields with Eq. (10),

\[
L^2 - R_T L - R_N^2 = 0.
\]

From this, one obtains

\[
L_\pm = \frac{R_T}{2} \pm \sqrt{\frac{R_T^2}{4} + R_N^2},
\]

where \( L_+ \) is the width of the northern cell and \( -L_- (> 0) \) is the width of the southern cell.
If one notes

\[
\frac{R_T}{R_N} = \frac{2g\Delta T_s}{f N y_0 T_0} \approx 0.24 \frac{\Delta T_s}{20K} \ll 1,
\]

then the interval of the precipitating regions is given by

\[
L_+ - L_- = 2 \sqrt{\frac{R_T^2}{4} + R_N^2} \\
\approx 2R_N \left( 1 + \frac{R_T^2}{8R_N^2} \right) \approx 2R_N \left[ 1 + 0.007 \left( \frac{\Delta T_s}{20K} \right)^2 \right],
\]

which is not sensitive to \( \Delta T_s \). This result is consistent with Fig. 4, in which the intervals
of the precipitating regions are almost the same regardless of \( \Delta T_s \): 0.18 \( y_0 \) in the case of
\( \Delta T_s = 0 \) and 0.2 \( y_0 \) in the case of \( \Delta T_s = 40K \).

The reason for the propagation of the symmetric cells is suggested as follows. In the case
of \( \Delta T_s = 0 \), as shown in the previous subsection, the symmetric cells are periodic rather
than propagating. It has been argued that, if the upper angular momentum descends in the
northern side, zonal winds become westerlies, and that these westerlies lead to the change
of direction of the circulation and hence the periodicity. In order for the symmetric cells
to preserve their circulation structure, the surface winds must be easterly in the northern
side of the precipitating region. The zonal winds can be easterly only if the symmetric cells
propagate southward during the overturning period, since the upper angular momentum descends to a more southern place than the place where it is supplied (this is schematically shown by Fig. 13). The above consideration suggests that, as a convenient estimation, the upper bound of the speed of the propagation of the symmetric cells is given by $L/\tau_D$, where $\tau_D$ is the overturning time and $L$ is the cell length. This corresponds to the ultimate case in which the angular momentum is conserved (Fig. 13). The value of the angular momentum is reduced by the viscous transfer with the adjacent indirect cell in the northern side, and it is also preferable for the easterly. The viscosity also affects the speed as shown by Fig. 3, though this effect is not easy to be taken into account in the simple scaling argument.

It should be noted, however, that the above consideration does not tell which is the preferable direction for the propagation. The experiments show that the symmetric cells always propagate southward (toward the warmer regions). The direction of the propagation seems to be controlled by the requirement of the surface easterlies. One can suppose a basic state that has a westerly shear in thermal wind balance with the prescribed surface temperature gradient. The upper westerly does not change its direction if it descends at the same $y$. Only if it descends southward, it becomes easterly. There must be both easterlies and westerlies in the surface zonal winds on the constraint of the angular momentum conservation. The symmetric cells should move southward as a whole in order to cancel the basic westerly component and to produce surface easterlies by the advection. If they move northward, westerlies would be stronger at the surface.

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5 Even in the case of $\Delta T_s = 0$, the symmetric cells occasionally propagate. For instance, Fig. 4(a) shows that there is a precipitating region propagating toward larger $y$ near $y \approx 0.9$ during the days 20 – 35. This propagation seems to occur by chance.

6 In fact, the westerly shear is produced by the symmetric cells themselves. However, the basic state is formally defined by the steady component of the zonal winds.
4 Relation with the symmetric instability

In this section, the symmetric cells in the case of $\Delta T_s \neq 0$ are examined in terms of the symmetric instability (SI) (e.g., Stone, 1966; Emanuel, 1979, 1982). It should be noted that it is impossible in the strict sense to say which instability is the origin of the symmetric cells, since the symmetric cells of the present study are disturbances in the statistically equilibrium states of a long time integration. Stability of a prescribed initial field and time evolution of the field are mainly concerned in the instability problems. When one relates disturbances in the equilibrium states to some kind of instability, one should appropriately choose a reference state that is thought to be an initial state for the instability. A reference state is sometimes defined by an idealistic balanced state of forcings prescribed to the systems under consideration. An alternative definition of a reference state is given by a simple average (in time or in space) of the equilibrium states. In the following, relation between the symmetric cells and instabilities is considered according to the two kind of the reference states.

Forcings imposed on the present system are the radiative cooling below the tropopause ($\sigma = 0.1$) and the sensible and the latent heats from the surface. If any motion is suppressed, a balanced state of the forcings is in principle statically unstable. Convective motions will be produced to release the unstable stratification. In this respect, it is natural to consider the symmetric cells to be the circulations derived from the “convective instability” (CI) rather than SI.

As for the reference state as an average of the equilibrium states, a time-mean or space-mean state has, in general, no relation with the initial state for the instability; the initial state will be lost when instability occurs, and may be no longer similar to the state of the full development of the instability. It is well known, however, that the equilibrium state is sometimes very close to the critical state for the instability, or becomes a slightly

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7 Radiative equilibrium will be established in each latitude if the radiation transfer models are used. In the present study, however, only the body cooling is simply used so that there may be a balance between the radiative cooling and the thermal diffusion.
unstable state, as represented by “convective adjustment” and “baroclinic adjustment”. If this idea is also true for the present study, one may infer the origin of the symmetric cells by examining a reference state as a time-average of the equilibrium states.

Fig. 14 shows the time-mean fields over the days 50 – 100 in the case of Fig. 9 (\(T_A = 320\) K, \(T_B = 300\) K): relationships between the normalized angular momentum \(l = (u - fy)/y_0\) and (a) potential temperature \(\theta\), (b) equivalent potential temperature \(\theta_e\) and (c) saturation equivalent potential temperature \(\theta^*_e\). Since the contours of \(l\) are inclined more vertically than those of \(\theta\) (i.e., the Richardson number is much greater than one), it is stable for the “dry” SI (Stone, 1966). As for the “moist” SI, Bennetts and Hoskins (1979) discussed that moist conditional SI is preferable if the contours of \(\theta_e\) are more inclined than those of \(l\). Fig. 14(b) shows that it is stable in the upper layers. In the lower layers, however, it is conditionally “convectively” unstable rather than conditionally symmetrically unstable. In this respect, therefore, the symmetric cells seem to be ascribable to CI.

As seen above, the reference states of the present study is preferable for CI rather than SI, then it seems difficult to relate the properties of the symmetric cells to SI. In the moist situation, however, a view of the moist SI may still be applied to the symmetric cells, if one isolates circulations associated with cumulus heating from the effect of moist process on stratification. Emanuel (1982) considered moist SI as CISK in a conditionally statically unstable state with a baroclinic flow. He pointed out that a symmetric mode which propagates toward warmer regions exists. This result is consistent with the present study, in which the symmetric cells are always propagating toward the warmer regions. It should be reminded that, in the present case, it is not clear whether moist circulations can be separately considered from its effect on the stratification as in Emanuel, since the stratification of the system is maintained by the symmetric cells themselves.

As for the scale of the symmetric cells, it does not seem to be relevant to the scale predicted by the theories of SI. According to the dry SI theory, there are two horizontal scales: the horizontal wavelength and the horizontal projection of the isentropes based on the layer.
depth (Emanuel, 1979; Thorpe and Rotunno, 1989). The wavelength of the dry SI is controlled by frictional process particularly in the inviscid limit, but the viscosity is not important for the cell scale of the present study, although it may control the scale of the upward branch. The second scale, the horizontal projection of the isentropes, is given by

\[ L_\theta = 2H \frac{\partial \theta}{\partial z}, \quad (18) \]

where \( H \) is the layer depth. This scale seems to correspond to the wavelength of the moist SI (Emanuel, 1982; Bennetts and Hoskins, 1979). Emanuel (1982) normalized horizontal length by \( L_\theta \), and showed the wavelength of the most growing mode is proportional to \( L_\theta \) (Eq. (8) of Emanuel (1982)). In the numerical experiments by Bennetts and Hoskins (1979), the cell scale is also approximately \( L_\theta \), although they did not clearly argue about the scale. However, the scale of the present symmetric cells is irrelevant to \( L_\theta \); the scale is rather related to the slope of the angular momentum:

\[ L = \frac{2gH \Delta T_s}{f^2 y_0 T_0} = 2H \frac{\partial U}{\partial z} = 2H \frac{\partial l}{\partial y}, \quad (19) \]

where the effect of stability is neglected as in Eq. (12). Fig. 14(a) shows that this scale is much smaller than \( L_\theta \).
5 Summary and further comments

The symmetric cells which exist in the two-dimensional model are examined through the dependency on the surface temperature gradient. In the case of no surface temperature gradient ($\Delta T_s = 0$), the circulations of the symmetric cells oscillate. The oscillation period is given by the convective time $\tau_D$, and the cell scale by the Rossby deformation radius. Advection of the angular momentum by the convective motion of the cells is essential for the oscillation. In the case of a surface temperature gradient ($\Delta T_s \neq 0$), the symmetric cells propagate toward warmer regions. Each symmetric cell consists of a cell in the warmer side of the precipitating region and a cell in the colder side. The cell shapes are different between the two types of the cells; the width of the cells in the colder side is broader as in the upper layers, whereas the width in the warmer side is smaller as in the upper layers. Successive cells are sloping and vertically in contact with each other, so that the angular momentum is vertically transferred by diffusive process from the upper cell (in the warmer region) to the lower cell (in the colder region).

The Hadley cell in the axisymmetric model can be viewed as an extreme case of the symmetric cells; it is the only symmetric cell that does not move. It is because a pair of the Hadley cells of both hemisphere is symmetric about the equator and the upward motion of the Hadley cells is located at the warmest region (the equator). The structure of the symmetric cells is very similar to that of the symmetric Hadley cell; the cell width of colder regions (higher latitudes) is broader as in the upper layers. The scale of the symmetric cells is also determined by a similar mechanism to that of the Hadley cell. It is determined by the relation between the surface temperature and the vertical mean temperature which is in the thermal wind balance with the angular momentum flow in the upper layers. In the Hadley case, the latitudinal dependency of the Coriolis parameter (the equatorial $\beta$ effect) must be considered in the estimation of the cell width (S94, Fig. 13).

To show how the Hadley cell and the symmetric cells are related with each other, we calculate the motion of the position of the symmetric cells in the global condition. If the
surface temperature is given by Eq. \((1)\), the substitution of a local Coriolis parameter and a surface temperature gradient into Eq. \((12)\) yields

\[
L = \frac{gH}{2 \Omega_0^2 \sin^2 \varphi T_0} \frac{1}{a} \frac{\partial T_s}{\partial \varphi} = \frac{gH \Delta T_s}{\Omega_0^2 a T_0} \cot \varphi
\]

\[
\equiv Ra \cot \varphi,
\]

where \(\Delta T_s \equiv T_B - T_A\) is the temperature difference from the pole to the equator, \(a\) is the radius of the earth, and \(R \equiv gH \Delta T_s / (\Omega_0^2 a^2 T_0)\). If the latitude of a precipitating region is denoted by \(\varphi(t)\), the propagation speed is estimated by

\[
\frac{d\varphi}{dt} = - \frac{L}{a \tau_D} = - \frac{R}{\tau_D} \cot \varphi.
\]

Integration for time under the initial condition \(\varphi(0) = \varphi_0\) yields

\[
\varphi(t) = \cos^{-1}\left( \cos \varphi_0 \cdot e^{\frac{R t}{\tau_D}} \right).
\]

Fig. 15 shows the calculated paths of the cells, where \(\tau_D = 20\) days and \(R = 0.068\) are used. In this figure, the paths in the higher latitudes than 60° are omitted. Fig. 15 shows that Eq. \((22)\) reproduces a similar pattern to that of Fig. 2(c). According to Eq. \((46)\) of S94, the Hadley scale is given by \(\varphi_H = \sin^{-1}(\sqrt{2R / (1 + 2R)}) = 20.2^\circ\). It is comparable to the scale of the symmetric cells at the moment they reach the equator.

As for the results for \(\Delta T_s = 0\) as shown by Fig. 4, the concentration of the precipitation to the equator becomes less clear as the surface temperature gradient becomes smaller, while a cellar structure still exists in the mid-latitudes. This difference may be attributed to the effect of rotation; moist convective motions are organized into a cellar structure if there is sufficient rotation. The discussion by Schneider\((1977)\) should be reconsidered in terms of these results. Schneider estimated the Hadley width on the assumption that a reference temperature is horizontally uniform. This assumption corresponds to the case \(\Delta T_s = 0\) of the present study. However, the Hadley cell no longer exists in this case and Schneider’s estimation is not applicable. Of course, it is too early to say that the Hadley cell in a three-dimensional model will show the same behavior as in the present two-dimensional models in the case of \(\Delta T_s = 0\). It has been reported that the concentration of the precipitation...
to the equator is sensitive to cumulus parameterization, in particular, when the surface
temperature gradient is small near the equator (Hess et al., 1993; Numaguti, 1993).

It is not clear how the symmetric cells are relevant to the phenomena in the observed
atmospheres. The latitudinal motion of the cumulus bands such as ITCZ may have a common
mechanism of the movement of the symmetric cells. Goswami and Shukla (1984) simulated
a northward propagation of the Indian monsoon by using an axisymmetric GCM. In their
calculation, there exists a disturbance propagating toward the warmer region with period
around 15 days, and this disturbance has many similarities to the observed fluctuation of
the Indian monsoon. The structure of their disturbance is confined to the lower atmosphere
and they pointed out the interaction with the land surface through the evaporation plays
a key role. These properties are not seen in the present study, so it is not clear their
disturbance and the symmetric cells of the present study have the same origin.

Although the symmetric cells are convective modes, they should be re-examined in terms
of the symmetric instability (e.g., Stone, 1966; Emanuel, 1979, 1982). In particular,
Emanuel (1982) pointed out that a symmetric mode which propagates toward warmer re-
gions exists in a shear flow of moist atmosphere. His results are consistent with the present
study. It will be also interesting to study the relationship with the motion of the spiral
bands of typhoon or with the phenomena in the planetary atmospheres such as the striped
pattern of Jupiter.

The symmetric cells are also expected to exist in the dry atmosphere or in the Boussinesq
fluid, although all the experiments in the present study are for the moist atmosphere. They
will oscillate under the uniform surface temperature condition, and will propagate toward
warmer regions if there is a surface temperature gradient, since these are the consequences of
the conservation of the angular momentum. The difference will be in their horizontal scale;
horizontal viscosity will play a major role in the scale of the cells in the dry atmosphere or
in the Boussinesq fluid, as in the case of the Bénard convection. The symmetric cells should
be discussed in a more general context, in which they are viewed as a convective motion
on a rotating plane with a rigid boundary below and a slip boundary above. It should be also examined how the symmetric cells are modified in a large-scale three-dimensional field, particularly in the situation where both vertical convection and baroclinic waves co-exist in a vertically unstable basic stratification.
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APPENDIX

Model formulation

The model is two-dimensional with a lateral direction and a vertical direction. The primitive equations with a $\sigma$-coordinate are used. The global model, that is, a two-dimensional axisymmetric model with a spherical coordinate is described in Sec 2 of S94. Formulation of the $f$-plane model (with the Cartesian coordinate) is almost similar to that of the global model. The symbols are summarized in Table. 1.

Let the direction perpendicular to the two-dimensional plane be denoted by $x$ (longitude, in the spherical case), the lateral direction by $y$, the vertical direction by $z$, and the corresponding metrics by $h_1$, $h_2$, and $h_3$, respectively. We also define $\sqrt{g} = h_1 h_2 h_3$. The lateral coordinate is bounded by $Y_0 \leq y \leq Y_1$, where $Y_0 = 0$ and $Y_1 = 1$ if $y$ is non-dimensionalized by the length of the lateral domain $y_0$ in the Cartesian coordinate system (Only in this appendix, $y$ is non-dimensional). In the spherical coordinate system, $y = \sin \varphi$ ($\varphi$ is the latitude) is used, and hence $Y_0 = -1$ and $Y_1 = 1$. The metrics and the angular momentum $l$ are expressed, in the Cartesian coordinate, by

\[
(h_1, h_2, h_3) = (1, y_0, 1),
\]
\[
l = u - f y y_0.
\]

whereas, in the spherical coordinate, by

\[
(h_1, h_2, h_3) = \left( a \sqrt{1 - y^2}, \frac{a}{\sqrt{1 - y^2}}, 1 \right),
\]
\[
l = u h_1 + \Omega h_1^2,
\]

where $f = 2\Omega$ is the Coriolis parameter, $\Omega$ is the rotation velocity, and $a$ is the radius of the earth.

The surface pressure $p_s$, the angular momentum $l$, the lateral velocity $v$, the moist enthalpy $h$, and the specific humidity $q$ are predicted variables in this system. The choice of $l$ and $h$ as the predicted variables automatically guarantees the conservations of the angular
momentum and the moist enthalpy. The basic equations are expressed as follows:

\[ \partial_t p_s = -\frac{1}{\sqrt{g}} \partial_y (h_1 p_s v) - \partial_\sigma (p_s \dot{\sigma}), \quad (23) \]

\[ \partial_t (p_s l) = -\frac{1}{\sqrt{g}} \partial_y (h_1 p_s vl) - \partial_\sigma (p_s \dot{\sigma} l) + h_1 p_s f_x, \quad (24) \]

\[ \partial_t (p_s v) = -\frac{1}{\sqrt{g}} \partial_y (h_1 p_s v^2) - \partial_\sigma (p_s \dot{\sigma} v) - \frac{1}{h_2} \left[ \frac{p_s}{\rho} \partial_\gamma p_s + p_s \partial_y \Phi \right] + p_s F_C + p_s f_y, \quad (25) \]

\[ \partial_t (p_s h) = -\frac{1}{\sqrt{g}} \partial_y (h_1 p_s v h) - \partial_\sigma (p_s \dot{\sigma} h) + \frac{p_s}{\rho} \frac{d}{dt} p + \frac{p_s}{\rho} Q^{df} + \frac{p_s}{\rho} Q^{rad} + \frac{p_s}{\rho} Q^{fric}, \quad (27) \]

\[ \partial_t (p_s q) = -\frac{1}{\sqrt{g}} \partial_y (h_1 p_s v q) - \partial_\sigma (p_s \dot{\sigma} q) + \frac{p_s}{\rho} S^{df} - \frac{p_s}{\rho} S^{prc}, \quad (28) \]

where \( \Phi = g z \) is the gravitational potential and \( F_C \) is the centrifugal force, which is given, in the spherical coordinate, by

\[ F_C = -\frac{1}{h_2} \partial_y \Phi_C - \frac{l}{h_2} \partial_\gamma \frac{1}{2 h_1^2}, \quad (29) \]

where \( \Phi_C = -\Omega^2 h_1^2 / 2 \) is the centrifugal potential. In the Cartesian coordinate, it corresponds to the Coriolis force \( F_C = fu \). The vertical \( \sigma \)-velocity \( \dot{\sigma} \) is given by the integration from \( \sigma \) to \( \sigma = 1 \) of Eq. (23):

\[ \dot{\sigma} = \frac{1}{p_s} \left[ (1 - \sigma) \partial_\gamma p_s + \frac{1}{\sqrt{g}} \partial_y \left( h_1 p_s \int_\sigma^1 v \, d\sigma \right) \right]. \quad (30) \]

The diffusion terms \( f_x, f_y, Q^{fric}, Q^{dif} \), and \( S^{dif} \) in the basic equations are given by the same form as in section 2 of S94. \( S^{prc} \) of the humidity equation is a term representing the precipitation, which is calculated as a removal of a liquid part of the humidity after each of time integration (this process is referred as the “large-scale condensation”). The radiative cooling \( Q^{rad}/\rho \) is specified as a constant value in the \( f \)-plane model. Boundary conditions are \( \dot{\sigma} = 0 \) at \( \sigma = 0, 1 \), and \( v = 0 \) at \( y = Y_0, Y_1 \). In the spherical coordinate, although \( l = 0 \) at the poles \( y = Y_0, Y_1 \), the angular velocity \( l/h_1^2 \) does not generally vanish. The fluxes
at the boundaries at \( y = Y_0, Y_1 \), and \( \sigma = 0 \) are zero, while those at \( \sigma = 1 \) are given by bulk formulae.

The grid points are defined as follows. As shown by Fig. 16, the lateral boundary \( y = Y_0 \) is denoted by \( j = 1/2 \), and \( y = Y_1 \) by \( j = J + 1/2 \), while the vertical boundary \( \sigma = 1 \) is denoted by \( k = 1/2 \) and \( \sigma = 0 \) by \( k = K + 1/2 \). The grid points are defined at both half-integer \( j + 1/2 \) ( \( k + 1/2 \) ) and integer \( j \) ( \( k \) ). The half-integer point which is placed between \( j \) and \( j + 1 \) ( \( k \) and \( k + 1 \) ) is denoted by \( j + 1/2 \) ( \( k + 1/2 \) ). The number of the half-integer points is \( J + 1 \) in the lateral direction, while it is \( K + 1 \) in the vertical direction. \( J \) is even in the case of the spherical coordinate system. The grid intervals of the lateral direction are equal both for integer and half-integer points. As for the vertical direction, the half-integer points are equally placed, while the integer points are based on Arakawa and Suarez(1983):

\[
y_{j-\frac{1}{2}} = Y_0 + (Y_1 - Y_0) \frac{j-1}{J}; \quad (j = 1, ..., J + 1), \quad \text{(31)}
\]

\[
y_j = \frac{1}{2} \left( y_{j+\frac{1}{2}} + y_{j-\frac{1}{2}} \right); \quad (j = 1, ..., J). \quad \text{(32)}
\]

\[
\sigma_{k-\frac{1}{2}} = 1 - \frac{k-1}{K}; \quad (k = 1, ..., K + 1). \quad \text{(33)}
\]

\[
\sigma_k = \frac{1}{\kappa} \frac{\sigma_{k+\frac{1}{2}} - \sigma_{k-\frac{1}{2}}}{\sigma_{k+\frac{1}{2}} - \sigma_{k-\frac{1}{2}}}; \quad (k = 1, ..., K). \quad \text{(34)}
\]

where \( \kappa = R_d/C_p \).

The variables are defined at the following points:

\[
p_s \quad p_{s,j-\frac{1}{2}}; \quad (j = 1, ..., J + 1)
\]

\[
l \quad l_{j-\frac{1}{2},k}; \quad (j = 1, ..., J + 1; k = 1, ..., K)
\]

\[
v \quad u_{j,k}; \quad (j = 1, ..., J; k = 1, ..., K)
\]

\[
h \quad h_{j-\frac{1}{2},k}; \quad (j = 1, ..., J + 1; k = 1, ..., K)
\]

\[
q \quad q_{j-\frac{1}{2},k}; \quad (j = 1, ..., J + 1; k = 1, ..., K)
\]

\[
\dot{\sigma} \quad \dot{\sigma}_{j-\frac{1}{2},k}; \quad (j = 1, ..., J + 1; k = 1, ..., K + 1)
\]

Vertical discretization by Arakawa and Lamb(1977) and Arakawa and Suarez(1983) is used so as to conserve total kinetic energy. The angular momentum and the equations of motion are discretized so as to conserve total kinetic energy(Satoh, 1994b).
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Figure Captions

**Fig. 1.** Convergence of the angular momentum. The standard experiment of the global model. (a) total convergence $-\nabla \cdot (\langle l v \rangle)$, (b) steady component $-\nabla \cdot (\langle l \bar{v} \rangle)$, and (c) unsteady component $-\nabla \cdot (\langle l \bar{v}' \rangle)$. The contour intervals are (a) 100 m$^2$/s$^2$, and (b), (c) 60 m$^2$/s$^2$.

**Fig. 2.** Time variation of the meridional distribution of the precipitation. Dependencies on the pole-to-equator temperature difference: (a) $\Delta T_s = 0$ K, (b) $\Delta T_s = 20$ K, (c) $\Delta T_s = 40$ K (the standard experiment), and (d) $\Delta T_s = 80$ K. The contour intervals are $5 \times 10^{-5}$ kg/m$^2$ s.

**Fig. 3.** Time variation of the meridional distribution of the precipitation. Dependencies on the coefficient of the viscosity: (a) $\nu = \nu_s = 1$ m$^2$/s, (b) $\nu = \nu_s = 25$ m$^2$/s, where $\nu$ is the viscosity in the free atmosphere and $\nu_s$ is the viscosity at the surface. The contour intervals are $5 \times 10^{-5}$ kg/m$^2$ s.

**Fig. 4.** Time variation of the lateral distribution of the precipitation in the $f$-plane model. The surface temperature is linearly distributed from $T_A$ at $y/y_0 = 0$ to $T_B$ at $y/y_0 = 1$: (a) $T_A = T_B = 300$ K, (b) $T_A = 305$ K, $T_B = 295$ K, (c) $T_A = 310$ K, $T_B = 290$ K, and (d) $T_A = 320$ K, $T_B = 280$ K. The contour intervals are $2 \times 10^{-4}$ kg/m$^2$ s.

**Fig. 5.** Time variation of the zonal winds (winds perpendicular to the model domain) at the surface. Surface temperature is uniform: $T_A = T_B = 300$ K. The contour intervals are 2 m/s.

**Fig. 6.** Distribution of the zonal winds (winds perpendicular to the figures). Surface temperature is uniform: $T_A = T_B = 300$ K. (a) 10 days average for the days 46 – 55, and (b) 10 days average for the days 56 – 65. The contour intervals are 5 m/s.

**Fig. 7.** As in Fig. 6 but for the temperature difference from the horizontal average. The contour intervals are 1 K.
**Fig. 8.** Time variation of the temperature lapse rate \((-dT/dz)\) at \(\sigma = 0.9\). Surface temperature is uniform: \(T_A = T_B = 300\) K. The contour intervals are 2 K/km. The hatched area indicates the values larger than 8 K/km.

**Fig. 9.** Time variation of the lateral distribution of the precipitation in the \(f\)-plane model for \(T_A = 320\) K and \(T_B = 300\) K. The contour intervals are \(2 \times 10^{-4}\) kg/m\(^2\) s. \(A \rightarrow B\) denotes a reference line for the composite maps.

**Fig. 10.** Composite maps averaged with the propagation of the symmetric cells from \(y/y_0 = 0.68\) at the day 51 to \(y/y_0 = 0.45\) at the day 100 (shown by \(A \rightarrow B\) in Fig. 9). The center of the symmetric cell is placed at \(y/y_0 = 0.5\). (a) zonal winds (winds perpendicular to the figure) with the contour interval of 5 m/s and (b) stream functions with the contour interval of 1,000 kg/m s. (Stream functions are those per unit length perpendicular to the figure.)

**Fig. 11.** Temperature (solid lines) and normalized angular momentum \((u - fy)/fy_0\) (broken lines) at the day 100 in the case of \(T_A = 320\) K and \(T_B = 300\) K. The contour intervals are 5K for the temperature and 0.02 for the normalized angular momentum.

**Fig. 12.** The scales of the symmetric cells. (a) scale in the case of \(\Delta T_s = 0\): \(R_N\), (b) scale in the case of \(\Delta T_s \neq 0\) when the effect of static stability is not included : \(R_T\), and (c) scale in the case of \(\Delta T_s \neq 0\) when the effect of static stability is included: the warmer side \(-L_+\) and the colder side \(L_+\). Thick allows indicate the precipitating region. The curves are the temperature difference from the precipitating region in the middle of the troposphere \(\delta T\), and the sloping lines are that of the surface temperature \(\delta T_s\). The three vertical segments of lines shown by \(dT_0\) in (a) and (c) are the same temperature difference \((g/C_p - \Gamma_m)H/2\). In (b), \(dT_1 = \Delta T_s R_T/y_0\), and, in (c), \(dT_2 = \Delta T_s L_+/y_0\).

**Fig. 13.** Relationship between surface zonal winds and motion of a parcel which starts from the origin \(y = 0\) shown by a thick arrow. \(y = 0\) is the position of precipitation at the time when the parcel starts. If the angular momentum is conserved along the motion of the parcel, zonal wind becomes easterly if the parcel comes down to the
negative $y$, whereas it becomes westerly if the parcel comes down to the positive $y$. Zonal wind is zero only if the parcel returns to the starting position $y = 0$. In order for the surface wind to be easterly, the precipitation must move toward the negative direction during the cycle of the parcel.

**Fig. 14.** Comparison of time averaged distributions of (a) potential temperature $\theta$, (b) equivalent potential temperature $\theta_e$ and (c) saturation equivalent potential temperature $\theta^*_e$ with distribution of normalized angular momentum $(u - fy)/fy_0$. The normalized angular momentum is the same in (a)-(c), and is shown by broken lines with the contour interval of 0.02. The potential temperatures, $\theta$, $\theta_e$ and $\theta^*_e$, are shown by solid lines with the contour interval of 5K. These are 50 days average for the days 51-100 in the case of $T_A = 320$ K and $T_B = 300$ K.

**Fig. 15.** Calculated patterns of the symmetric cells propagation by Eq. (22).

**Fig. 16.** Arrangement of the grid points.
Table. 1 Symbols.

\begin{align*}
x & \quad \text{coordinate perpendicular to the two-dimensional plain} \\
y & \quad \text{lateral coordinate} \\
z & \quad \text{altitude} \\
h_1, h_2, h_3, \sqrt{g} = h_1 h_2 h_3 & \quad \text{metrics} \\
p_s & \quad \text{surface pressure} \\
p & \quad \text{pressure} \\
\sigma = p/p_s & \quad \text{vertical coordinate} \\
u, v & \quad \text{velocity component} \\
l & \quad \text{angular momentum} \\
\dot{\sigma} & \quad \sigma\text{-velocity} \\
T & \quad \text{temperature} \\
q & \quad \text{specific humidity} \\
C_p & \quad \text{specific heat for constant pressure} \\
R_d & \quad \text{gas constant for dry air} \\
L & \quad \text{latent heat} \\
h = C_p T + L q & \quad \text{moist enthalpy} \\
\rho & \quad \text{density} \\
T_s & \quad \text{surface temperature} \\
\Phi = g z & \quad \text{gravitational potential} \\
\Phi_c & \quad \text{centrifugal potential} \\
a & \quad \text{radius of the earth} \\
\Omega & \quad \text{rotation velocity} \\
\Omega_0 & \quad \text{rotation velocity of the earth} \\
f_x, f_y & \quad \text{viscous forces} \\
Q^{\text{dif}} & \quad \text{energy convergence due to thermal and moisture diffusions} \\
Q^{\text{rad}} & \quad \text{energy convergence due to radiation} \\
Q^{\text{fric}} & \quad \text{energy convergence due to dissipation} \\
S^{\text{dif}} & \quad \text{moisture convergence due to diffusion} \\
S^{\text{prec}} & \quad \text{precipitation}
\end{align*}