Double stratification effects on boundary layer over a stretching cylinder with chemical reaction and heat generation

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Abstract: An analysis of double stratification effects on boundary layer flow along a stretching cylinder with chemical reaction and heat generation is presented in this study. The governing non-linear partial differential equations are transformed into a system of non-linear ordinary differential equations using similarity transformations and solved by Runge-Kutta forth-fifth order (RKF45) with shooting technique. The effects of various parameters on the velocity, temperature and concentration distributions are analyzed graphically. The present analysis is validated by comparing with previously published work and found to be in good agreement.

1. Introduction

The boundary layer flow and heat transfer due to the stretching cylinder is important in industrial manufacturing processes such as the aerodynamic extrusion of plastic sheets, glass blowing and polymer extrusion [1]. Authors such as Hayat et al. [2], Mukhopadhyay [3] and Elbashbeshy et al. [4] have analyzed the effects of boundary layer flow along a stretching cylinder with various physical situations.

Heat and mass transfer study on fluid with chemical reaction effect over a stretching sheet have important role in metallurgy and chemical engineering industries, such as food processing and polymer production [5]. The effects of chemical reaction have been investigated by Tripathy et al. [6] and Mukhopadhyay [7].

Heat generation or absorption is also important during chemical reaction where heat may be generated or absorbed due to the reactions. According to Cheng [8], a lot of physical phenomenon involve convection is driven by heat generation. Boundary layer flow in presence of heat generation was considered by authors such as Elbashbeshy et al. [9] and Makinde and Sibanda [10].

The phenomenon of mixed convection occurs in many technical and industrial problems such as electronic devices cooled by fans, nuclear reactors cooled during an emergency shutdown and solar collectors [11]. Mukhopadhyay and Ishak [12] and Poornima [13] considered the mixed convection flow along a stretching cylinder with different aspects of the problems.

Stratification occurs due to the variations in temperature and concentration or present of different fluids of different densities. According to Ibrahim and Makinde [14] the analysis of mixed convection in a doubly stratified medium is a vital problem because of its occurrence in geophysical flows. Mukhopadhyay et al. [15] analyzed the effects of thermal stratification on flow and heat transfer past a porous vertical stretching surface using Lie group transformation method. Rehman et al. [16] studied dual stratified mixed convection flow of Eyring-Powell fluid over an inclined stretching cylinder.
Motivated by the above applications, the aim of present study is to investigate the effects of double stratification on boundary layer over a stretching cylinder with chemical reaction and heat generation.

2. Mathematical modeling

We consider a steady boundary layer flow of a viscous and incompressible Newtonian fluid along a stretching horizontal cylinder. \((x,r)\) are coordinates measured in axial and radial direction of the cylinder respectively. The governing boundary layer equations following \([12, 16]\) and \([7]\) are:

\[
\frac{\partial (u)}{\partial x} + \frac{\partial (v)}{\partial r} = 0
\]

\[
u \frac{\partial u}{\partial x} + \frac{\partial u}{\partial r} = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} - \frac{\partial^2 u}{\partial \theta \partial r} \right)
\]

\[
u \frac{\partial v}{\partial x} + \frac{\partial v}{\partial r} = -k^* \left( C - C_\infty \right)
\]

where \(u\) and \(v\) are velocity components along \(x\) and \(r\) directions, respectively, \(\nu = \mu/\rho\) is the kinematic viscosity, \(\rho\) is the fluid density, \(\mu\) is the coefficient of fluid viscosity, \(g\) is the acceleration due to gravity, \(\beta\) is coefficient of thermal expansion, \(T\) is the fluid temperature, \(\kappa\) is the thermal diffusivity of the fluid, \(c_p\) denotes specific heat at constant pressure, \(Q_o\) is the heat generation coefficient, \(D\) be the mass diffusivity coefficient, \(k^*\) is the rate of chemical reaction and \(C\) is the fluid concentration.

The boundary conditions:

\[
u \left( \frac{\partial C}{\partial r} + \frac{1}{r} \frac{\partial C}{\partial \theta} \right) = D \left( \frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} \right)
\]

where \(\eta\) is the similarity variable, \(f, \theta\) and \(\phi\) are dimensionless stream function, temperature and concentration, respectively.

The transformed ordinary differential equations are:

\[
(1 + 2M\eta) f'' + 2Mf' - f^2 + \eta f\theta = 0
\]

\[
(1 + 2M\eta) \theta'' + 2M\theta' + \Phi f (f\theta' - f'\theta - f\varepsilon_1 + \varepsilon_1 \theta) = 0
\]

\[
(1 + 2M\eta) \phi'' + 2M\phi' + \Phi (f\phi' - f'\phi - f\varepsilon_2 + \varepsilon_2 \phi) = 0
\]

The new boundary conditions are:

\[
\eta = 0: \quad f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1 - \varepsilon_1, \quad \phi(0) = 1 - \varepsilon_2
\]

\[
\eta \to \infty: \quad f'(\infty) \to 0, \quad \theta(\infty) \to 0, \quad \phi(\infty) \to 0
\]
where \( M = \left( \frac{\nu L}{U_0 R} \right)^{1/2} \) is the curvature parameter, \( \lambda = g \beta L b U_0^2 \) is the mixed convection parameter, \( \text{Pr} = \mu C_p / \kappa \) is a Prandtl number, \( \varepsilon_1 = c/b \) is the thermal stratification parameter, \( \delta = L Q_0 / U_0 \rho C_p \) is heat generation parameter, \( S_c = \nu / D \) is a Schmidt’s number, \( \varepsilon_2 = e / d \) is the solutal stratification parameter and \( \alpha = k^2 L / U_0 \) is reaction rate parameter. 

The physical significances of the flow are the skin friction coefficient \( C_f \), local Nusselt number \( N_u \), and local Sherwood number \( S_h \), defined as:

\[
C_f = \frac{\tau_w}{\rho U^2 / 2}, \quad N_u = \frac{x q_w}{k(T_w - T_0)}, \quad S_h = \frac{x j_w}{D(C_w - C_0)}
\]

where the wall shear stress \( \tau_w \), the surface heat flux \( q_w \), and the surface mass flux are given by:

\[
\tau_w = \mu \left( \frac{\partial u}{\partial r} \right)_{r=R}, \quad q_w = -k \left( \frac{\partial T}{\partial r} \right)_{r=R}, \quad J_w = -D \left( \frac{\partial C}{\partial r} \right)_{r=R}
\]

Using non-dimensional variables in (7) we obtain:

\[
C_f \text{Re}^{1/2} = 2(i/R)f'(0), \quad N_u \text{Re}^{1/2}(1-\varepsilon_1) = -\theta'(0), \quad S_h \text{Re}^{1/2}(1-\varepsilon_2) = -\phi'(0)
\]

where \( \text{Re} = U_0 \sqrt{\nu L} \) as a local Reynolds number, \( \varepsilon_1 = c/b \) is the thermal stratification and \( \varepsilon_2 = e / d \) is the solutal stratification.

3. Results and Discussion

Table 1 shows the comparison of numerical values of Nusselt number \([ -\theta'(0) \] for several values of Prandtl number, \( \text{Pr} \) where \( M = 0, \lambda = 0, \varepsilon_1 = 0, \varepsilon_2 = 0, S_c = 0, \delta = 0 \) and \( \alpha = 0 \). The table shows the results of the present study are in excellent agreement with that reported by Rehman et al. [16].

| \( \text{Pr} \) | Rehman et al. [16] | Present study |
|---------------|-------------------|---------------|
| 0.72          | 0.8089            | 0.8088        |
| 1.00          | 1.0000            | 1.0000        |
| 3.00          | 1.9239            | 1.9237        |
| 10.00         | 3.7208            | 3.7207        |

Many numerical results were obtained in this study. Some of the results were shown from figure 1 through figure 12 to illustrate the effects of curvature \( M \), mixed convection \( \lambda \), thermal stratification \( \varepsilon_1 \), solutal stratification \( \varepsilon_2 \), heat generation \( \delta \) and reaction rate \( \alpha \) parameters on the characteristics of the flow field. Figure 1 shows that as curvature parameter \( M \) increases, the velocity profile initially decrease and then it increases after a distance \( \eta \) from the surface. As \( M \) increases, the radius of curvature decreases, thus the contact area between fluid and cylinder decreases which cause less resistance to fluid flow. Figure 2 elucidates that increase in curvature parameter \( M \), the temperature profile initially decreases and then it increases. Larger value of \( M \) reduces the heat transport rate, thus the temperature increases. Figure 3 shows the effect of mixed convection parameter \( \lambda \) on velocity profile. When mixed convection parameter \( \lambda \) increases, the velocity profile increases. This is due to the increase in thermal buoyancy force that leads to increase in velocity within a boundary layer. Figure 4 depicts that when mixed convection \( \lambda \) increases, thermal buoyancy force also increases which is responsible for high rate of heat transfer which cause the temperature profile to decrease.

Figure 5 shows that velocity profile decreases when thermal stratification parameter \( \varepsilon_1 \) increases. Higher thermal stratification \( \varepsilon_1 \) lowers the effective convective potential between ambient temperature and surface of cylinder, thus the reduction in the velocity of the fluid. Figure 6 demonstrates that temperature decreases when thermal stratification parameter \( \varepsilon_1 \) increases. This is because of the
decrease in temperature difference between ambient fluid and surface of cylinder. Figure 7 shows when thermal stratification parameter \( \varepsilon_2 \) increases, the concentration profile increases.

Figure 8 shows that when solutal stratification parameter \( \varepsilon_2 \) increases, the concentration profile and mass boundary layer thickness decreases. Figure 9 elucidates that velocity profile increases when heat generation \( \delta \) increases. Higher heat generation reduces the shear stress along the wall and hence it increases the rate of transport. Figure 10 illustrates that temperature profile increases when heat generation parameter \( \delta \) increases. An increase in heat generation parameter \( \delta \) also increases the thermal boundary layer thickness. It is noticed from figure 11 that when heat generation increases, concentration profile decreases accompanied by the decrease in the concentration boundary layer thickness. Figure 12 shows the effects of reaction rate parameter \( \alpha \) on concentration profile. When reaction rate parameter \( \alpha \) increases, concentration profile decreases. The reaction rate parameter \( \alpha \) is the decelerating agent to this profile. The concentration boundary layer thickness reduces near the wall due to the conversion of species that occurs as a result of chemical reaction and hence decreases the concentration in the boundary layer.

**Figure 1.** Velocity profile for various values of \( M \).

**Figure 2.** Temperature profile for various values of \( M \).

**Figure 3.** Velocity profile for various values of \( \lambda \).

**Figure 4.** Temperature profile for various values of \( \lambda \).

**Figure 5.** Velocity profile for various values of \( \varepsilon_1 \).

**Figure 6.** Temperature profile for various values of \( \varepsilon_1 \).
4. Conclusion
The effects of double stratification on boundary layer flow over a stretching cylinder with chemical reaction and heat generation has been investigated. It can be concluded that the velocity profile increases with the increasing values of curvature parameter, mixed convection parameter and heat generation parameter. Meanwhile, it shows a decline for thermal stratification parameter. The temperature profile increases for larger values of curvature parameter and heat generation parameter while it decreases for mixed convection parameter and thermal stratification parameter. The concentration profile increases for increasing values of curvature parameter and thermal stratification parameter whereas, concentration profile shows the opposite effects on mixed convection parameter, solutal stratification parameter, heat generation parameter and reaction rate parameter. The increase in the thermal stratification parameter resulted in lower velocity and temperature profiles, but increase in the concentration of the fluid.
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