Cavity Optomechanical Bistability with an Ultrahigh Reflectivity Photonic Crystal Membrane

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Photonic crystal (PhC) membranes patterned with sub-wavelength periods offer a unique combination of high reflectivity, low mass, and high mechanical quality factor. Using a PhC membrane as one mirror of a Fabry–Perot cavity, a finesse as high as $F = 35\,000(500)$ is demonstrated, corresponding to a record high PhC reflectivity of $R = 0.999835(6)$ and an optical quality factor of $Q_{\text{opt}} \approx 10^7$. The fundamental mechanical frequency is 426 kHz, more than twice the optical linewidth, placing it firmly in the resolved-sideband regime required for ground-state optical cooling. The mechanical quality factor in vacuum is $Q = 1.1(1) \times 10^6$, allowing values of the single-photon cooperativity as high as $C_\gamma = 6.6 \times 10^{-3}$. Optomechanical bistability is easily observed as hysteresis in the cavity transmission. As the input power is raised well beyond the bistability threshold, dynamical backaction induces strong mechanical oscillation above 1 MHz, even in the presence of air damping. This platform will facilitate advances in optomechanics, precision sensing, and applications of optomechanically-induced bistability.

1. Introduction

It has long been recognized that patterning a dielectric membrane with a 1D array of lines or a 2D array of holes can dramatically increase its optical reflectivity. In the 1D case, such devices are often called “high contrast gratings,”[1] and their remarkable optical and mechanical properties have been exploited to realize high-finesse optical cavities,[2,3] high quality factor monolithic optical resonators,[4] second harmonic emission,[5] tunable vertically-coupled surface-emitting lasers (VCSELs),[6] algorithms for optical computing,[7] optical cooling,[8] and phonon lasing.[9] In the 2D case, such “photonic crystal slabs”[10] or “PhC membranes” have found numerous applications in both optics[11–13] and optomechanics.[16–22] For cavity optomechanical applications, a key property of these devices is the reflectivity that is achieved, which determines the finesse of the optical cavity in which it is employed. Early work with 1D structures demonstrated cavities with finesse $F = 1\,200$ to $F = 2\,800$, and subsequent “membrane-in-the-middle”[21] experiments showed that even higher values of finesse were possible. More recently, a finesse of $F = 6\,390(150)$ was demonstrated in a SiN structure patterned with a 2D square lattice of holes.[18] In this work, we demonstrate a record high finesse of $F = 35\,000(500)$, corresponding to a PhC reflectivity of $R = 0.999835(6)$ and an optical quality factor of $Q_{\text{opt}} \approx 10^7$, using a PhC membrane with a hexagonal lattice. We find that the wavelength dependence of the finesse is Lorentzian, and elucidate the functional form of the global transmission spectrum.

From the beginning of this study, the phenomenon of optomechanical bistability has played a key role. When light is coupled into an optical cavity, the amount of circulating power in the cavity depends on the detuning of the incident light from a cavity resonance frequency. If the cavity is mechanically compliant, it elongates in response to the radiation pressure of the stored light, finding a new equilibrium length where the radiation pressure balances the mechanical restoring force. The cavity resonant frequencies shift downward, in turn modifying the detuning. For certain initial detunings and sufficiently large circulating power, the system becomes bistable, with two different cavity lengths for which the mechanical force exactly balances the optical force. Bistability resulting from this Kerr-type nonlinearity was first observed via hysteresis in the power transmitted by a mechanically compliant cavity as the drive power was swept.[24] This pioneering experiment was very sensitive to seismic noise, and the rare successful experiments took place only at night. Following the initial demonstration, a great deal of interest in optomechanical bistability was driven by the recognition that the underlying Kerr-like nonlinearity does not suffer from the material losses limiting traditional Kerr media.[25,26] These
ideas have motivated proposals for nonreciprocal devices,[27] all-optical switches and memories based on spontaneous symmetry breaking,[28] and devices with negative effective mass.[29] For sensing, it has been shown theoretically that bistable optomechanical systems provide force sensitivity superior to that of their linear counterparts,[29] and in the quantum domain, ponderomotive squeezing[30] and entanglement[31] are enhanced near the bistable critical points. Very recent work in a magnetic system described by the same Kerr nonlinearity has shown enhanced optomechanical cooling by exploiting the asymmetric lineshape characteristic of the system below the bistability threshold.[32] The backside of the wafer was then patterned with square openings to form silicon nitride membranes by fully etching through the silicon wafer. Deep reactive ion etching (DRIE) was used to etch the back side of the silicon wafer until \( \approx 50 \) \( \mu \)m of silicon was left. After dicing into 1 cm square chips, the membranes were fully released using KOH with a concentration of 30% at 60 °C. In this fabrication process, the lattice constant can be controlled precisely, whereas the hole radius is much more prone to process variations. Therefore, we fabricated multiple PhCs with the same lattice constant (\( a \approx 1.51 \) \( \mu \)m) but different target radii (varying from \( r = 0.475 \) \( \mu \)m to \( r = 0.515 \) \( \mu \)m; simulations gave an optimum radius of \( r = 0.515 \) \( \mu \)m). This approach has been used previously,[33] an alternative method involving iterative etching to approach a target wavelength has also been demonstrated.[34] Despite the numerous proposals for experiments and applications, very few other optomechanical systems exhibiting radiation-pressure induced bistability or multistability have been reported.[34–38] In some cases, reports of bistability have involved photothermal mechanisms including expansion of the mirror coatings[39,40] and circulating optical powers of several kilowatts.[41] Here, the low mass and high reflectivity of our PhC membrane enable an optomechanical cavity with a fundamental mechanical frequency of 426 kHz in which hysteresis induced by bistability is easily seen with incident optical powers below 500 \( \mu \)W. We present a quantitative discussion of our observations, which include the appearance of oscillations (instability of the upper branch[31]) as the power is raised, despite the damping provided by air. While we focus on static bistability in the present study, the mechanical quality factor of our device in vacuum is \( Q = 1.11 \times 10^8 \), which, in combination with its low mass and ultrahigh reflectivity, allows us to achieve a single-photon cooperativity[42] as high as \( C_\text{S} = 6.6 \times 10^{-3} \). This renders our device attractive for dynamic applications as well, especially where the simplicity of a two-mirror cavity is desired.

2. Experimental Setup

2.1. Photonic Crystal Membrane

Our photonic crystal membrane is illustrated in Figure 1a. It is a square silicon nitride (SiN) membrane, 220 \( \mathrm{nm} \) in thickness and 800 \( \mu \)m on a side, suspended in a silicon frame. A 2D PhC of diameter 300 \( \mu \)m is patterned in the center. Figure 1b shows an optical image of the PhC membrane, and Figure 1c is a scanning electron micrograph of the center. The PhC is made of a hexagonal lattice of circular holes etched into the SiN device layer. Its geometry was chosen to maximize the reflection of a normally incident Gaussian beam at 1550 nm using \( S^0[43] \) a freely available software package employing the rigorous coupled wave analysis (RCWA) algorithm. For membranes of 220 nm thickness, simulations of both square and hexagonal lattices show comparable reflectivity, but the hexagonal lattice is somewhat less susceptible to fabrication imperfections. A detailed study of these matters will appear in a subsequent publication.

Fabrication started by coating a 525 \( \mu \)m thick silicon wafer with a 220 nm layer of low-stress SiN (chosen for ease of fabrication) using low pressure chemical vapor deposition (LPCVD). The photonic crystals were patterned with electron-beam lithography and the silicon nitride was etched with reactive ion etching (RIE).

The experimental apparatus is shown in Figure 1d. A Fabry–Perot cavity is created by using the PhC membrane as the input coupler, and a concave dielectric mirror (M) as the output coupler. The output coupler has a nominal radius of curvature of 25 mm and a transmission (as documented by the manufacturer) of \( T_\text{r} = 10^{-5} \). The PhC membrane is mounted on a translation stage allowing alignments in 3D, as well as tip and tilt (not shown in the figure). The output coupler is mounted by means of a piezoelectric transducer (PZT) on a 1D translation stage (not shown), enabling motion along the axis of the cavity. Laser light from a narrow-linewidth tunable laser is collimated and mode-matched with a lens (L1, \( f = 200 \) mm) into the cavity. A half-wave plate (HWPI) and a polarizing beamsplitter (PBS) are used to adjust the input power, and the polarization of the incident light is optimized by means of another half-wave plate (HWPI). Transmitted light is collimated by a lens (L2, \( f = 200 \) mm) and captured by a photodetector (PD1), while the reflection is detected by another photodetector (PD2). An electro-optic intensity modulator (EOM1) allows the incident light to be rapidly extinguished for cavity ringdown measurements, and an electro-optic phase modulator (EOM2) is used to introduce sidebands to the laser frequency spectrum. These sidebands can either be driven by a synthesizer to establish a precise frequency calibration, or driven by a voltage-controlled oscillator in order to rapidly sweep the optical frequency.

3. Optical Characterization

3.1. Photonic Crystal Membrane Transmission Spectrum

Initial measurements of the photonic crystal properties were made in the absence of the cavity output coupler by simply focusing light onto the sample and monitoring the transmission as a function of wavelength. The position and tilt of the PhC were optimized to minimize the transmission as the wavelength was swept. Transmission spectra are shown in Figure 2a for two of the eight devices on a sample. Both spectra exhibit minima in transmission along with resonant features that are well known to occur in such devices.[35] The Fano resonances underlying these features have been explored theoretically[44] and studied...
Figure 1. Photonic crystal design and experimental setup for characterization. a) A silicon nitride (SiN) membrane is suspended by a Si frame, which has a patterned PhC disk at the center. A focused laser beam is normally incident. b) An optical image of the photonic crystal chip with an 800 \( \mu \)m square SiN membrane (light blue), a 300 \( \mu \)m diameter photonic crystal, and a Si frame (black). c) Scanning electron micrograph of the photonic crystal, showing a hexagonal lattice of circular holes. d) Experimental setup used to characterize properties of the photonic crystal. EOM1: electro-optic intensity modulator; EOM2: electro-optic phase modulator; HWP1, HWP2: half-waveplates; PBS: polarizing beam splitter; BS: beam splitter; L1, L2: lenses (\( f = 200 \) mm); PhC: photonic crystal membrane; M: high-reflectivity concave mirror; PD1, PD2: photodetectors.

experimentally.

While the spectrum of the device with design radius \( r = 475 \) nm (blue) has its minimum very close to the target wavelength of 1550 nm, the device with \( r = 505 \) nm (green) has a slightly lower minimum transmission, and is the one studied in the remainder of this work. The inset shows the transmission spectra as simulated with \( S^4 \) with slightly perturbed thickness and radii values to account for nanofabrication process variations, showing reasonable agreement with the measurements.

3.2. Cavity Transmission Spectrum

We then installed the concave dielectric output coupler to form a Fabry–Perot cavity, and measured the transmission and reflection of the cavity near the wavelength that minimizes the transmission of the photonic crystal. The cavity mode profile was verified to be Gaussian by use of a camera in place of photodetector PD1 to monitor the cavity transmission. We optimized the cavity finesse by working with a cavity length \( L \) such that the cavity waist is \( \psi_0 \approx 60 \mu \)m, obtained by taking either \( L \approx 2.4 \) mm (“short” cavity) or \( L \approx 22.8 \) mm (“long” cavity). The optimal cavity waist reflects a compromise between the limited size of the PhC (leading to “clipping” loss at radii beyond that of the PhC) and the lower reflectivity of a more tightly focused optical mode, due to its broader angular spectrum, as addressed in previous work.[8,48] As we will show, both choices of cavity length give the same finesse, but the free spectral range and linewidth differ by about a factor of ten in the two configurations. A transmission spectrum made with a “long” cavity, driven with incident power \( P_{\text{in}} = 700 \) \( \mu \)W, is shown in Figure 2b. It exhibits \( \approx 100 \) modes over the range of 1533 to 1540 nm. The cavity transmission varies dramatically over this range, exhibiting maxima at 1536.5 and 1538.2 nm, and dropping to nearly zero in between; we will discuss the transmission envelope in Section 4.2. Profiles of both transmission and reflection for three of the modes are shown in Figure 2c–e. It is clear that the resonance at 1537.297 nm (Figure 2d) is considerably...
Figure 2. Optical characterization of the photonic crystal membrane. a) Transmission spectra of two different devices for light focused to a spot size of 60 μm. Transmission minima are exhibited at around 1537 (green) and 1548 nm (blue), corresponding to devices with designed hole radii of 505 and 475 nm respectively. The inset shows the calculated transmission spectra for two PhCs using $S_4$, where the membrane thickness is 203 nm and $r = 515$ nm (green) and $r = 485$ nm (blue). b) Transmission spectrum for a “long” cavity made using an input optical power of 700 μW. The green line is a theoretical transmission envelope calculated from the measured wavelength dependence of the cavity finesse. c–e) Zoomed-in views of modes at 1535.210, 1537.297, and 1539.021 nm, respectively, with both transmission (red) and reflection (blue) signals displayed. The resonance at 1537.297 nm has considerably narrower linewidth. The lineshapes of the modes are asymmetric, with steeper slopes on the red-detuned sides, due to optomechanical nonlinearity.

narrower than the others; in addition, all three modes are asymmetric, with a steeper slope on the long-wavelength (red-detuned) side of the resonance. As we will discuss in Section 5, the lineshape asymmetry is due to optomechanical nonlinearity.

In general the asymmetric lineshapes preclude fitting to a Lorentzian functional form. A brute-force approach to determining the finesse from the linewidth is to reduce the circulating optical power to the point that the optomechanical distortion in the resonances is negligible. This approach is demonstrated in Figure 3a, which shows the transmission at a wavelength of $\lambda = 1537.435$ nm of a “long” cavity with a free spectral range of FSR = 6.83(5) GHz. The cavity is driven such that the circulating power is $P_{\text{circ}} \approx 70$ mW, well below the threshold for bistability (as discussed in Section 5) of $P_{\text{th}} = 750$ mW. An electro-optic phase modulator is used to generate sidebands at a separation of 2 MHz for frequency calibration. The linewidth (FWHM) of the central resonance is found to be 215(16) kHz, corresponding to a finesse $F = 31900(2500)$, where the uncertainties correspond to the standard deviation of a series of 14 measurements. This approach clearly introduces the laser linewidth as a broadening mechanism, as the resulting profile is a convolution of the resonance linewidth and the laser linewidth. In addition, it requires one to sweep the laser frequency relatively slowly in order to obtain an adequate signal-to-noise ratio, so that the measurement is also susceptible to cavity length fluctuations. We thus employed alternative means to determine the cavity finesse.

3.3. Frequency-Swept Measurements

A more desirable method to determine the cavity finesse in this case is to sweep the laser wavelength rapidly through the resonance. The transmission of a cavity whose length is rapidly changed, or that is driven by an input field whose frequency is rapidly swept, exhibits secondary oscillations (ringing), whose functional form can be used to infer the cavity finesse. As discussed previously, this approach is particularly well suited to high-finesse cavities. In the present work, the method presents two advantages. First, the light is swept through the resonance so quickly that very little circulating power builds up, reducing the associated nonlinearity induced by radiation pressure. Second, the approach mitigates the effect of technical noise related to environmental disturbances of the cavity length, once again because the measurement is done so quickly.

We perform the measurements by sweeping the drive frequency of the electro-optic phase modulator at a rate $\beta$.
Figure 3. Cavity finesse measurements. a) An Airy transmission resonance normalized to the input power at 1537.435 nm with 2 MHz sidebands for calibration, along with a Lorentzian fit (red). The measurement is made with very low circulating power to minimize optomechanical nonlinearities. b) Measured transmission with a phase modulator sideband swept rapidly across a resonance peak centered at 1537.296 nm. A theoretical profile (red) is fit to the data. c) Cavity ringdown measurement. The decay signal is displayed on a natural logarithmic scale (blue) with a linear fit (red). The black curve shows extinction of the excitation light. d) The measured finesse as a function of wavelength using the cavity ringdown method (blue diamonds) and fast sweeping method (red squares) for a “long” cavity. The ringdown data are fit to a Lorentzian lineshape (black line). Error bars are given by the standard deviation of five individual measurements. e) Same as (d), but for a “short” cavity.

\[ (8 \times 10^6 \text{ MHz s}^{-1} < \beta < 10^8 \text{ MHz s}^{-1} ) \text{ such that one sideband scans through a cavity resonance, and monitoring the cavity transmission. The transmitted power } P_{\text{trans}}(t) \text{ is proportional to (see Supporting Information)} \]

\[ P_{\text{trans}}(t) \propto e^{-2t} \left| \text{erfc} \left( \frac{1}{2} \left( 1 + i \frac{\gamma - i \beta t}{\sqrt{-\beta}} \right) \right) \right|^2 \]  

(1)

where \( \gamma \) is the cavity half-width at half maximum (HWHM) in units of angular frequency. Figure 3b shows a measured transmitted signal (blue) for \( \lambda = 1537.296 \text{ nm} \) and \( \beta = 8.8(3) \times 10^6 \text{ MHz s}^{-1} \), along with a fit (red) to Equation (1). The linewidth inferred is \( 2\gamma = 2\pi \times 208(20) \text{ kHz} \), corresponding to a finesse \( F = 32800(3000) \), where the uncertainties correspond to the standard deviation of five successive measurements. Larger sweep rates \( \beta \) are used at wavelengths where the cavity decay rate \( 2\gamma \) is larger, in order to maintain the “fast sweep” condition \( 2\gamma^2 / \beta < 1 \) mentioned in Supporting Information.

3.4. Ringdown Measurements

A complementary approach to measuring optical cavity losses is to work in the time domain and study cavity ringdown. Here light is coupled into the cavity and then abruptly extinguished, and the light stored in the cavity leaks out with a time constant given by \( \tau = 1/(2\gamma) \). This approach is immune to broadening due to the laser linewidth, as well as optomechanical effects and environmental perturbations in the cavity length over the course of the measurement. Incoupled light was intensity modulated with a square wave, and the power exiting the output coupler was measured with a fast photodetector (3 ns risetime). Ringdown data for a cavity with a free spectral range of 6.58 GHz at a wavelength of \( \lambda = 1537.4 \text{ nm} \) is shown in Figure 3c on a logarithmic scale. From a linear fit we infer \( \tau = 874(36) \text{ ns} \), corresponding to a linewidth of \( 2\gamma = 2\pi \times 183(8) \text{ kHz} \) and a finesse of \( F = 35900(1500) \); once again the uncertainty is given by the standard deviation of five consecutive measurements.

4. Wavelength Dependence of Finesse and Transmission

4.1. Wavelength Dependence of Finesse

Figures 3d and 3e show the cavity finesse inferred by the fast sweep and ringdown approaches for both the “long” and “short” cavity configurations, respectively. The range of wavelengths covered in these graphs is very close to the minimum of the transmission curve shown in Figure 2a, for which one can expand the
transmission of the photonic crystal in a Taylor series as a function of wavelength as

\[ T_1(\lambda) = T_1^0 + \alpha(\lambda - \lambda_0)^2 + \cdots \]  

(2)

The finesse of a low-loss cavity is given by

\[ F = \frac{2\pi}{\sum \text{round trip losses}} \]  

(3)

where the “roundtrip losses” in the denominator refer to mirror transmission as well as undesired losses, in particular scattering S, absorption A, and “clipping” \( \Gamma_{\text{clip}} \) due to the finite size of the PhC. Denoting the undesired losses by \( \Gamma_{\text{cav}} = S_i + A_i + \Gamma_{\text{clip}} \), where \( i = 1, 2 \) refers to the PhC and dielectric mirror, respectively, the wavelength dependence of the finesse is, in the vicinity of \( \lambda_0 \)

\[ F(\lambda) = \frac{2\pi}{\Gamma_1 + \Gamma_2 + \Gamma_{\text{cav}} + \alpha(\lambda - \lambda_0)^2} \]  

(4)

Here it is implicit (as documented in the spectrophotometer data provided by the manufacturer) that the transmission of the output coupler \( T_2 \) is independent of wavelength for the wavelengths of interest. Equation (4) has the functional form of a Lorentzian; fitting Lorentzians to the ringdown data shown in Figure 3d,e yields a peak finesse \( F_{\text{peak}} = 35\,000(500) \), and a corresponding determination of the total cavity losses as

\[ \Gamma_1 + \Gamma_2 + \Gamma_{\text{cav}} + T_1^0 = 1.80(3) \times 10^{-4} \]  

(5)

Taking the known \( T_2 = 10^{-5} \), we make the reasonable assumption that the absorption and scattering losses of the dielectric mirror are comparable to, but somewhat smaller than, its transmission, so that \( \Gamma_2 + T_2 = 1.5(5) \times 10^{-5} \). We can then infer for the photonic crystal

\[ \Gamma_1 + T_1^0 = 1.65(6) \times 10^{-4} \]  

(6)

or, equivalently, a peak reflectivity \( R_1 = 1 - \Gamma_1 - T_1^0 = 0.999835(6) \).

### 4.2. Wavelength Dependence of Transmission

We now address the envelope of the transmission spectrum shown in Figure 2b. The resonant transmission of a Fabry–Perot cavity is given by

\[ T_{\text{cav}} = \eta \left( \frac{F}{\pi} \right)^2 T_1 T_2 \]  

(7)

where \( \eta \) is a numerical factor (\( 0 \leq \eta \leq 1 \)) representing the degree of mode-matching. Combining Equations (2), (4), and (7), we obtain

\[ T_{\text{cav}} = 4\eta T_1 T_2 \left[ T_1^0 + \alpha(\lambda - \lambda_0)^2 \right] \left[ \Gamma_1 + \Gamma_2 + T_1^0 + \alpha(\lambda - \lambda_0)^2 \right] \]  

(8)

Noting that \( \Gamma_1 + \Gamma_2 + T_1^0 \) and \( \alpha \) have already been determined from the fits in Figure 3d,e, the only free parameters are \( T_1^0 \) and \( \eta \). The value of \( T_1^0 \) plays little role here; tentatively taking \( T_1^0 = 0 \) and \( \eta = 0.64 \), we find the envelope shown in green in Figure 2b. The qualitative agreement is quite good, and shows that the shape of the global transmission spectrum is set by competition between the finesse (favoring a small \( T_1(\lambda) \)) and the input coupling (favoring a large \( T_1(\lambda) \)).

A more refined value of \( T_1^0 \) can be found by using transmission data such as that shown in Figure 3a in conjunction with Equation (7). Indeed, at \( \lambda = \lambda_0 \), we can infer from Equation (7)

\[ T_1^0 = \frac{T_{\mu0}(\lambda_0)}{\eta T_2} \left( \frac{\pi}{F_{\text{max}}} \right)^2 \]  

(9)

From a series of 14 measurements like the one in Figure 3a, we infer a cavity transmission \( T_{\mu0}(\lambda_0) = 3.7(4) \times 10^{-4} \), where the uncertainty is given by the standard deviation of the measurements, and a corresponding value \( T_1^0 = 5(1) \times 10^{-7} \). It is clear from comparison to Equation (6) that the undesired absorption, scattering, and “clipping” losses comprising \( \Gamma_{\text{cav}} \) dominate the transmission at the wavelength \( \lambda_0 \). In fact, the transmission \( T_1(\lambda) \) of the PhC is lower than that of the dielectric output coupler \( T_2 \) over a wavelength range of \( \approx 0.4 \) nm centered at \( \lambda_0 \), or about nine modes of the “long” cavity. When operating at these wavelengths, where the finesse is highest, it may thus be advantageous to couple light in through the curved dielectric mirror rather than the photonic crystal.

At this point we pause to elucidate the dominant loss mechanism limiting the PhC reflectivity. In the absence of losses, RCWA simulations show that the reflectivity achievable with an infinitely large PhC with our geometry for light with a spot size of \( \omega_0 = 60 \mu m \) would be \( R = 0.9999822 \). A simple estimate of beam clipping loss with a PhC of diameter 300 \( \mu m \), based on a Gaussian beam profile, gives \( \Gamma_{\text{clip}} = 4.5 \times 10^{-5} \), reducing the reflectivity to 0.9999772. The largest finesse that could be achieved with such a PhC would then be \( F_{\text{max}} = 2\pi/|1 - R| \approx 100\,000 \). Our measurements give total PhC losses \( \Gamma_{\text{cav}} \approx 1.65 \times 10^{-4} \), from which we can infer \( \Gamma_1 = \Gamma_2 = \Gamma_{\text{cav}} \approx 1.2 \times 10^{-4} \). While the experiment does not allow us to distinguish absorption and scattering losses, it is straightforward to simulate the absorption of our PhC for a given material extinction coefficient (imaginary part of the index of refraction), and one finds \( A = 1.2 \times 10^{-4} \) for an extinction coefficient of \( n_1 = 1.14 \times 10^{-5} \). This is slightly lower than measured extinction coefficients of low-stress SiN membranes fabricated commercially\(^{[50]} \) and in our facility\(^{[51]} \) in 2014, which we expect reflects improvements in the SiN fabrication techniques over those in use at that time. It seems likely, then, that absorption, rather than scattering, is presently the limiting loss mechanism.

In the future, we plan to use stoichiometric (high stress) SiN to fabricate these devices, as it is known to have lower absorption.\(^{[52]} \)

Waveguide loss measurements\(^{[53]} \) suggest that an extinction coefficient of \( n_1 < 10^{-4} \) is possible, but as absorption drops, the role of scattering will doubtless become more of an issue.

### 4.3. Optimal Power Buildup and Impedance Matching

The dependence of the finesse and transmission on wavelength, governed by the reflection spectrum of the photonic crystal, allows the properties of the cavity resonances to be chosen for the...
task at hand. In some cases a very narrow resonance might be desired, and it would be advantageous to choose a wavelength close to the one maximizing the finesse. In other cases, however, it might be preferable to work at a wavelength that maximizes the circulating optical power. Applications of this flexibility include optical cooling with a “pump” field coupled to a mode with high circulating power while probing with a “probe” field tuned to a weaker mode with high finesse,[19] or using two “pump” fields coupled to modes with different characteristics to simultaneously implement optical cooling and an optical spring.[14] Differentiating Equation (8), one finds that the value of $T_1(\lambda)$ maximizing the transmission is

$$T_1(\lambda) = \Gamma_1 + \Gamma_2 + T_2$$

This condition, in which the transmission of the input coupler equals the sum of all other round-trip losses, is well known to define an impedance-matched cavity,[51] which, in the case of perfect mode-matching, has a vanishing resonant reflection. Indeed, the reflection dip shown in Figure 2e, while not at the exact wavelength of impedance matching, is substantially deeper than those of Figure 2c,d. In the limit $T_1 \to 0$, the wavelength detuning yielding this impedance-matched situation is equal to the half-width at half maximum of the Lorentzian function (Figure 3d,e) describing the variation of the finesse with wavelength.

5. Optomechanical Bistability and Dynamics at Atmospheric Pressure

We return now to the asymmetric lineshapes shown in Figure 2c–e. When optical power is stored in a Fabry–Perot cavity, the associated radiation pressure elongates the cavity and shifts its resonance frequencies down. While this effective Kerr nonlinearity is in theory always present, it is only apparent if the cavity is sufficiently mechanically compliant. By balancing the mechanical restoring force against the radiation pressure, the steady-state circulating power is found to obey a cubic equation (derived in Supporting Information)

$$P_{\text{circ}} = \frac{P_{\text{max}}}{\left(\frac{\delta}{\gamma} + \frac{P_{\text{nl}}}{P}\right)^2 + 1}$$

where

$$\bar{P} = \frac{k_{\text{static}} c \lambda}{8F}$$

is the circulating power needed to shift the resonant frequency of the cavity to the red by one half-linewidth $\gamma$, and $P_{\text{max}}$ is the maximum possible circulating power, dependent on the detuning $\delta$ and mode-matched input power (see Supporting Information). Here $\delta$ is defined relative to an undriven cavity, where there is no radiation pressure, and $k_{\text{static}}$ is the mechanical spring constant of the optical cavity. In addition to the static nonlinearity discussed here, it is well known that the optomechanical interaction modifies the system dynamics,[41] and the individual mechanical modes have their own effective masses and corresponding spring constants.

The fundamental mechanical frequency of our photonic crystal membrane is $v_m \approx 426$ kHz, as illustrated by the power spectral density shown in Figure 4. This measurement of the thermal Brownian motion was made using a Michelson interferometer at a pressure of $1.3 \times 10^{-4}$ Pa ($10^{-6}$ Torr). The inset shows a ringdown measurement from which we infer a mechanical quality factor $Q = 1.1(1) \times 10^6$, where the uncertainty corresponds to the standard deviation of three measurements. For radiation pressure from a Gaussian beam with spot size $\omega_0 = 60 \mu$m, the static spring constant of our membrane can be expressed in terms of the fundamental frequency $v_m = 2\pi v_0$, of the membrane as $k_{\text{static}} = 0.089 m_0\omega_0^2$. (see Supporting Information), enabling one to calculate $\bar{P}$ for any value of the finesse.

For $\bar{P} \to \infty$, corresponding to a perfectly rigid cavity, Equation (11) gives the usual Lorentzian response associated with a driven Fabry–Perot cavity. With a finite spring constant, however, the Lorentzian becomes distorted, and when $P_{\text{max}}$ exceeds a threshold power $P_{\text{th}} = 8\sqrt{3}/9\bar{P}$, the system becomes bistable and hysteretic (see Supporting Information for details). Figure 5a shows the “shark fin” solution to Equation (11) for $P_{\text{max}} = 2.25 P_{\text{th}} = 2\sqrt{3}\bar{P}$, in which the maximum circulating power for a detuning sweep from blue to red is 17% larger than the maximum when the detuning is swept from red to blue. For values of the detuning $-3.54 \gamma < \delta < -2.57 \gamma$, Figure (11) admits three real solutions, but only the two depicted in green are physically stable. Sweeping the detuning from red to blue or from blue to red gives different responses (see Supporting Information for details), as illustrated in the inset.

Figure 5b shows bidirectional spectra of the power transmitted by the cavity when driven with an input power of $P_{\text{in}} = 500 \mu$W at a wavelength of $\lambda = 1536.576$ nm, where the finesse is $F \approx 18,500$. This measurement and those that follow were performed at atmospheric pressure. The known sweep rate of the laser is used to calibrate the frequency axis. The spectra are in good qualitative agreement with those shown in Figure 5a; in particular, the transmitted power is ≈17% larger when sweeping from blue to red detuning than when sweeping from red to blue. It
Figure 5. Hysteresis related to optomechanical bistability. All detunings are relative to an undriven cavity; the detuning relative to the optomechanically elongated cavity is blue-shifted by $P_\gamma$. b–d) Circulating power is related to transmitted power by $P_{\text{circ}} = P_{\text{trans}}/T_2 = 10^5 P_{\text{trans}}$. a) “Shark fin” solution to Equation (11) for the circulating power in units of $P_\gamma$ as a function of detuning, for $P_{\text{max}} = 2\sqrt{3} P_\gamma$. The solution shown in black is physically unstable. Inset: Associated hysteretic behavior for sweeps from red to blue detuning (red curve), and from blue to red detuning (blue curve). The circulating power is expressed in Watts by means of Equation (12), for a value of $F \approx 18500$. b) Cavity transmission for a bidirectional laser sweep with an input power of $P_\text{in} = 500 \mu$W at a wavelength of $\lambda = 1536.576$ nm, where the finesse is $F \approx 18500$. The color convention is the same as that taken for the inset to (a). c) Bidirectional sweep when the laser input power is raised to 1 mW; the hysteretic behavior is still clearly observed, but it is accompanied by significant oscillation on the blue-detuned side of the resonance. d) At still higher input powers, the dynamical behavior dominates. Here the cavity is driven with 4 mW at a wavelength of $\lambda = 1539.594$ nm and the detuning is swept from red to blue; the inset shows that the optically-sprung oscillation takes place at a frequency slightly in excess of 1 MHz.

is also notable that there is a small amount of oscillation on the blue-detuned side of the resonance, regardless of the direction of the detuning sweep. By referring to the original time series, its frequency is found to be $\nu = 427$ kHz. The oscillation is the usual consequence of dynamical backaction,\(^{[34]}\) and related to the fact that the upper branch of the optomechanical system can be unstable, unlike the case for a true Kerr medium, as discussed theoretically.\(^{[33]}\)

More quantitatively, the physical mass of our membrane is $m = 4.2 \times 10^{-10}$ kg, yielding $k_{\text{static}} \approx 269$ Nm\(^{-1}\) and a power $P \approx 860$ mW as calculated from Equation (12) with $F \approx 18500$. The choice $P_{\text{max}} = 2\sqrt{3} P_\gamma$ made in Figure 5a, corresponding to the amount of asymmetry observed in the peak heights when the laser is swept in the two directions, is then equal to $P_{\text{max}} \approx 3$ W, as shown in the inset. The maximum circulating power $P_{\text{circ}} = P_{\text{trans}}/T_2$ that we infer from the transmitted power in Figure 5b, on the other hand, is in the vicinity of $P_{\text{circ}} \approx 1.5$ W. The reasons for the discrepancy are not yet understood, but may include a value $T_2$ of the output coupler transmission lower than specified, modification of the transition threshold due to dynamical back-action, or photothermal effects. Figure 5c shows spectra when the input power is doubled to $P_{\text{in}} = 1$ mW; in this case, the oscillations severely perturb the lineshapes predicted by the static theory alone, and are chirped from $\nu \approx 434$ kHz to $\nu \approx 474$ kHz. It is noteworthy that the mechanical frequency is shifted upward, consistent with the expectation for an optical spring induced by blue-detuned radiation pressure;\(^{[34]}\) the chirp is related to the fact that the circulating
optical power and detuning governing the optical spring are both varying during the sweep. As the input power is raised further, the oscillations become so prominent as to largely obscure the static bistability. Figure 5d shows a sweep from red to blue detuning made with an input power of $P_{\text{in}} = 4 \text{ mW}$, but at a wavelength of $\lambda = 1539.594$ nm, where the finesse is $F \approx 5600$. In this case, the mechanical oscillation, a segment of which is shown in the inset, is chirped in frequency from $\nu \approx 958 \text{ kHz}$ to $\nu \approx 1.04 \text{ MHz}$. We believe that the oscillation corresponds in this case not to the fundamental mechanical mode with frequency $\omega_0$, (using standard notation for the eigenfrequencies of a square membrane as used in the Supporting Information), but rather the mode with frequency $\omega_{13} = \sqrt{5} \omega_0$, $\approx 953 \text{ kHz}$. The issue of which mode of an optomechanical system oscillates in the presence of gain is a complicated one, and has been discussed previously.[9]

6. Conclusion

In conclusion, we have demonstrated a Fabry–Perot cavity employing a mechanically compliant PhC membrane as one end mirror with a record high finesse of $F = 35 000 (500)$ and an optical quality factor of $Q_\text{opt} \approx 10^7$. We measured the finesse in three different ways, and the results of the various measurements are consistent, accounting for a small contribution to the measured cavity linewidth from that of the probe laser. The finesse is found to have a Lorentzian dependence on wavelength, and the global transmission spectrum reflects a tradeoff between the role of the PhC as gatekeeper for photons seeking to enter the cavity, and its role as a loss mechanism limiting the finesse. With a cavity linewidth less than half the $\nu_m \approx 426 \text{ kHz}$ frequency of the fundamental mechanical mode, the “long” cavity is firmly in the resolved-sideband regime, making it suitable for ground-state optical cooling. With a mechanical quality factor in excess of one million, the “long” and “short” cavities that we construct have single-photon cooperativity values of $C_0 = 6 \times 10^{-4}$ and $C_s = 6.6 \times 10^{-3}$, respectively. By way of comparison, the systems noted in the classic review paper,[42] with similar mechanical frequencies exhibit cooperativities in the range of $C_0 = 10^{-7}$ to $C_s = 10^{-4}$. Very recent work with a PhC device similar to that reported here, embedded in a phononic shield,[38] has demonstrated an even higher cooperativity of $C_0 \approx 3.6 \times 10^{-2}$. We have found the regime of optomechanical bistability to be easily accessible, as demonstrated by hysteresis in the cavity transmission as the detuning of the injected light is swept. Experimental results for drive powers even modestly above the bistable threshold are complicated, however, by the oscillation arising from dynamical backaction. In fact, we have found strong oscillation at an optically-sprung mechanical frequency in excess of 1 MHz, despite the damping provided by the air environment. We believe these devices have an exciting future in traditional applications of optomechanics, such as cooling, and can be improved still more. The finesse we achieve seems to be limited by material absorption, so we are optimistic that future devices fabricated with stoichiometric silicon nitride will exhibit even higher reflectivity. In parallel, we are attempting to fabricate membranes incorporating these PhC mirrors within a phononic shield,[38,39] in an effort to improve the mechanical quality factor still further as well. Finally, we are hopeful that this work will stimulate activity in some of the many fascinating applications of optomechanical bistability that have been proposed since its original demonstration.

Supporting Information

Supporting Information is available from the Wiley Online Library or from the author.

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Conflict of Interest

The authors declare no conflict of interest.

Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Keywords

cavity optomechanics, Fabry–Perot, finesse, optomechanical bistability, photonic crystal membrane

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