Chiral gauge leptoquark mass limits and branching ratios of $K_L^0$, $B^0$, $B_s \to l_i^+ l_j^-$ decays with account of the general fermion mixing in leptoquark currents

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Abstract

The contributions of the chiral gauge leptoquarks $V_{L,R}^{L,R}$ induced by the chiral four color quark-lepton symmetry to the branching ratios of $K_L^0, B^0, B_s \to l_1 l_2$ decays are calculated and analysed using the general parametrizations of the fermion mixing matrices in the leptoquark currents. From the current experimental data on these decays under assumption $m_{V,L} \ll m_{V,R}$ the lower mass limit $m_{V,L} \cos \gamma_L > 5.68$ TeV is found, which in particular case of equal gauge coupling constants gives $m_{V,L} > 8.03$ TeV. The branching ratios of the decays under consideration predicted by the chiral gauge leptoquarks are calculated and analysed in dependence on the leptoquark masses and the mixing parameters. It is shown that in consistency with the current experimental data these branching ratios for $B_s, B^0 \to \mu e$ decays can be close to their experimental limits and those for $B_s, B^0 \to \tau e, \tau \mu$ decays can be of order of $10^{-7}$. The calculated branching ratios will be useful in the further experimental searches for these decays.

Keywords: Beyond the SM; four-color symmetry; Pati–Salam; leptoquarks; B physics; rare decays.

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The search for a new physics beyond the Standard Model (SM) is one of the directions of the modern studies in the high energy physics. There is a lot of possible variants of new physics (such as supersymmetry, left-right symmetry, two Higgs model, extended dimension models, etc.) which are now under theoretical discussions and experimental searches (including the experimental searches at the LHC).

One such variant of new physics can be induced by the possible four color symmetry between quarks and leptons regarding leptons as the quarks of the fourth color. Proposed firstly [1] on the basis of the gauge group $G_{PS} = SU_V(4) \times SU_L(2) \times SU_R(2)$ this symmetry can be unified with the SM in the minimal way in frame of the model based on the gauge group [2–4] $G_{MQLS} = SU_V(4) \times SU_L(2) \times U_R(1)$, where the last two factors correspond to the usual electroweak symmetry of the SM. These groups can be embedded into the

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GUT group $SO(10)$ $[5, 7]$ as the intermediate stages in appropriate schemes $[8]$ of the $SO(10)$ symmetry breaking. In both cases the four color symmetry is described by the group $SU_V(4)$ of the vector-like type and predicts in particular the new gauge particles – the vector leptoquarks which belong to the 15-plet of the group $SU_V(4)$ and form the color triplet $(3, 1)_{2/3}$ of the SM group $G_{SM} = SU_c(3) \times SU_L(2) \times U(1)$.

The vector-like group $SU_V(4)$ can be extended to the gauge group of the chiral four color symmetry for example in frame of group $[9] SU_L(4) \times SU_R(4) \times SU_L(2) \times SU_R(2)$ $[9] \times U(1)'$. In this case the chiral four color symmetry predicts two types of the chiral gauge leptoquarks which are the color triplets $(3, 1)_{2/3}$ of the SM group and each of them interacts with left-handed and right-handed leptoquark currents separately. The possibility of the chiral four color symmetry has been also considered in Refs. $[10, 11]$ in context of the mass limits for the leptoquarks. In general case the leptoquarks are the gauge or scalar particles carrying both the baryon and lepton numbers. They appear in many models and can led to varied new physics effects, the comprehensive review of the physics of leptoquarks can be found in Ref. $[12]$.

In the last years the leptoquarks are intensively used for the possible explanations of the known anomalies in the semileptonic $B$ meson decays. In this case to obviate the existing high limits on the masses of the vector leptoquarks the conventional Pati-Salam four color symmetry is subjected to modifications by the appropriate choice of couplings $[13, 14]$ or mixings $[15]$ in fermion sector, by introducing the additional factor $SU(3)'$ $[16, 17]$ with the third family quark-lepton unification $[18]$, by PS triplication $[19]$, by using also the scalar leptoquarks $[20, 23]$, by extending the fermion sector of the Pati-Salam model $[24]$ with introducing the nonunitary mixings due to additional vector-like heavy leptons $[25]$.

The mass limits for the chiral gauge leptoquarks are more mild and by this reason such leptoquarks look as the more natural ones for explanations of the $B$ anomalies. The gauge leptoquarks which couple to quarks and leptons in a predominately left-handed manner were considered in the model with the gauge symmetry $SU(4)_C \times SU(2)_L \times U(1)_Y$ $[26]$, and with accounting also the new scalar leptoquarks in frame of the Pati-Salam-like group $SU_C(4) \times SU_L(2) \times SU_R(2)$ with unification of the left-handed quarks and leptons into a fundamental representation of $SU_C(4)$ and with a separate treatment of right-handed quarks and leptons $[29]$. The chiral leptoquarks as the gauge bosons of the chiral four color symmetry were considered in the model with the gauge symmetry $SU_L(4) \times SU_R(4) \times SU_L(2) \times SU_R(2)$ with the subgroup $SU_R(4)$ assumed to be broken at a much higher scale than $SU_L(4)$, leading to a suppression of right-handed leptoquark currents.

The possible effects of leptoquarks in experiments depend on the masses of leptoquarks. The lower mass limits for leptoquarks from their direct searches are of about or less 1 TeV. The essentially more stringent lower mass limits are resulting from the rare decays of pseudoscalar mesons of the type

$$K_L^0, B^0, B_s \to l_i^+ l_j^-.$$  

(1)

The most stringent of them are resulting from the $K_L^0 \rightarrow e^\mp \mu^\pm$ decay and with neglect of fermion mixing in leptoquark currents are of order of 2000 TeV for the vector leptoquark $[30, 32, 10, 11]$ and of order of 260 TeV for the chiral one $[10, 11]$. These mass limits can be essentially lowered by account of fermion mixing in leptoquark currents $[33–36]$ and instead of 2000 TeV the current lower mass limit for the vector leptoquark with

A possibility to lower the mass scale of the $SU_C(4) \times SU_L(2) \times SU_R(2)$ symmetry by the leptoquark interaction only with the right-handed currents and introducing additional exotic leptons was considered some time ago in Refs. $[27, 28]$. 
account of the fermion mixing in the leptoquark currents of the general form is of order of 90 TeV \cite{35}. It is interesting now to know what can be the lower mass limits for the chiral gauge leptoquarks resulting from the decays (1) with account of the fermion mixing in the leptoquark currents.

In this paper the new lower mass limit for the chiral gauge leptoquarks which results from the current experimental data on the decays (1) is obtained with account of the general parametrizations of the fermion mixing matrices in the leptoquark currents. The contributions of the chiral gauge leptoquarks to the branching ratios of these decays are calculated and analysed in dependence on the leptoquark masses and mixing angles and phases in comparison with the corresponding current data on these decays. The leptoquarks under consideration are regarded as the gauge bosons of the chiral four color symmetry group

\[ G_c = SU^L_c(4) \times SU^R_c(4), \]

where the left( right )-handed quarks and leptons of each generation (after fermion mixing) are unified into a fundamental representation of \( SU^L_c(4) \times SU^R_c(4) \) group.

The interaction of the chiral gauge leptoquarks \( V^L_\alpha, V^R_\alpha \) with down quarks \( d_{p\alpha} \) and leptons \( l_i \) which are responsible for the decays (1) and induced by the gauge group \( (2) \) can be written in general case as \( \text{Ref.} \ [10,11] \)

\[ L^{V,LR}_{d\ell} = \frac{g^L_d}{\sqrt{2}} (\bar{d}_{p\alpha}[(K^L_2)_{p\alpha}\gamma^\mu P_L]l_i)V^L_{\alpha\mu} + \frac{g^R_i}{\sqrt{2}} (\bar{d}_{p\alpha}[(K^R_2)_{p\alpha}\gamma^\mu P_R]l_i)V^R_{\alpha\mu} + h.c., \]

where \( g^L_d, g^R_i \) are the gauge coupling constants of the group \( (2) \), \( p,i=1,2,3 \) are the quark and lepton generation indexes, \( \alpha = 1,2,3 \) is the \( SU_c(3) \) colour index, \( d_p = (d,s,b) \), \( l_i = (e,\mu,\tau) \) are down quarks and leptons, \( P_{L,R} = (1+\gamma_5)/2 \) are the left and right operators of fermions and \( K^L_{2R} \) are the unitary matrices which describe the mixing of down fermion in the leptoquark currents. In general case the interaction of the leptoquarks with fermions contains four mixing matrices \( K^L_{2R} \) for up(\( a=1 \)) and down(\( a=2 \)) quarks and leptons. These matrices are specific for the models with the four color quark-lepton symmetry. Some details concerning the mixing matrices \( K^L_{a2R} \) can be found in Refs. \[2,3,10,11,35\].

The group \( (2) \) after breaking down to the \( SU_c(3) \) colour group reproduces in particular the usual QCD interaction and as a result the gauge coupling constants \( g^L_4, g^R_4 \) should satisfy the relation

\[ g^L_4 g^R_4 / \sqrt{(g^L_4)^2 + (g^R_4)^2} = g_{st}(M_c), \]

where \( g_{st}(M_c) \) is the strong coupling constant at the mass scale \( M_c \) of the chiral four color symmetry \( (2) \).

The branching ratios of the decays of pseudoscalar mesons \( P = (K^0_L, B^0, B_s) \) into lepton-antilepton pairs of type (1) induced by the chiral gauge leptoquarks \( V^L, V^R \) can be presented in the form

\[ Br_{VLR}(P \rightarrow l_i^+ l_i^-) = B_{Pl} \sqrt{1-4m_i^2/m_p^2} \beta^2_{P,ii}, \quad \text{for } l_i = e,\mu,\tau, \]

\[ Br_{VLR}(P \rightarrow ll') = B_{Pl} (1-m_i^2/m_p^2)^2 \beta^2_{P,ll'}, \quad \text{for } lll' = \mu e, \tau e, \tau \mu, \]

where \( B_{Pl} \) are the typical branching ratios of these decays and \( \beta^2_{P,ii}, \beta^2_{P,ll'} \) are the mixing factors depending on the mixing matrices \( K^L_2, K^R_2 \) and on the masses and coupling constants of leptoquarks \( V^L, V^R \).
The branching ratio $B_{P^l}$ in eqs. (5), (6) can be written as

$$B_{P^l} = \frac{m_P \pi \alpha_s^2(M_c) f_P^2 m_l^2}{2 (\bar{m}_V)^4 \Gamma_P^{tot}},$$

where $m_P$, $m_l$ are the masses of $P$ meson and lepton, $f_P$ is the form factor parametrizing the matrix elements of the axial and pseudoscalar quark currents of $P$ meson in the standard way, $\bar{m}_V$ is some typical leptoquark mass (conveniently the mass of the lightest leptoquark $m_{V_L}$ or $m_{V_R}$) and $\Gamma_P^{tot}$ is the total width of $P$ meson.

The branching ratios $Br_{V,L,R}(P \to LL')$ in eq.(11) denote the sums of the branching ratios of the charge conjugated final states

$$Br_{V,L,R}(P \to LL') = Br_{V,L,R}(P \to \mu^+e^-) + Br_{V,L,R}(P \to e^+\mu^-),$$

$$Br_{V,L,R}(P \to \tau e) = Br_{V,L,R}(P \to \tau^+e^-) + Br_{V,L,R}(P \to e^+\tau^-),$$

$$Br_{V,L,R}(P \to \tau\mu) = Br_{V,L,R}(P \to \tau^+\mu^-) + Br_{V,L,R}(P \to \mu^+\tau^-)$$

with $m_l$ being the mass of heaviest lepton ($m_l > m_{\nu}$) and with account of the relations

$$m_{\tau} \gg m_{\mu} \gg m_{e}.$$  

According to the definitions (9)–(11) the mixing factors $\beta_{P,LL'}^2$ in eq.(6) can be written as

$$\beta_{P,LL'}^2 = \beta_{P,21}^2 + \beta_{P,12}^2, \quad \beta_{P,LL'}^2 = \beta_{P,31}^2 + \beta_{P,13}^2, \quad \beta_{P,LL'}^2 = \beta_{P,32}^2 + \beta_{P,23}^2.$$  

With branching ratios (5)–(7) the mixing factors $\beta_{P,ij}^2$ in eqs.(11), (12) with account of the quark content of $P$ meson depend on the matrix elements $(K_{2L,R}^{L,R})_{pi}$, $(K_{2L,R}^{L,R})_{qj}$ of the mixing matrices $K_{2L,R}^{L,R}$ and on the relations between the masses and coupling constants of the leptoquarks $V_L, V_R$.

Each of the unitary $3 \times 3$ mixing matrices $K_{2L,R}^{L,R}$ in general case can be parametrized by three angles $\theta_{L,R}^{12}, \theta_{L,R}^{13}, \theta_{L,R}^{12}$ and six phases $\delta_{L,R}, \varepsilon_{L,R}, \delta_0, \varphi_1, \varphi_{2L}, \varphi_{3L}$ as

$$K_{2L,R}^{L,R} = e^{i\varphi_{0L,R}} \begin{pmatrix} c_{12}c_{13} e^{i \varphi_{21}} & s_{12}c_{13} e^{i \varphi_{22}} & s_{13} e^{i \varphi_{23}} \\ -s_{12}c_{23} + c_{12}s_{23}s_{13}i\delta & c_{12}c_{23} - s_{12}s_{23}s_{13}i\delta & c_{12}s_{23} e^{i \varphi_{23}} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}i\delta & -s_{12}s_{23} c_{13} e^{i \varphi_{23}} & e^{i \varphi_{23}} c_{23}c_{13} e^{i \varphi_{23}} \end{pmatrix},$$

where

$$\varphi_{21} = (\varphi_1 + \varepsilon)^{L,R}, \quad \varphi_{22} = (\varphi_2 + \varepsilon)^{L,R}, \quad \varphi_{23} = (\varphi_3 + \delta + \varepsilon)^{L,R},$$

$$\varphi_{31} = -(\varphi_2 + \varphi_3 + \delta + \varepsilon)^{L,R}, \quad \varphi_{32} = -(\varphi_1 + \varphi_3 + \delta + \varepsilon)^{L,R}, \quad \varphi_{33} = -(\varphi_1 + \varphi_2 + \varepsilon)^{L,R},$$

$$s_{ij}^{L,R} = \sin \theta_{ij}^{L,R}, \quad c_{ij}^{L,R} = \cos \theta_{ij}^{L,R}.$$  

The use of the mixing matrices $K_{2L,R}^{L,R}$ in the form (13) gives for the mixing factors $\beta_{P,ij}^2$ the next expressions for $P = K_L$

$$\beta_{K_{2L,11}}^2 = \frac{1}{8} \left[ \frac{\mu_L^2}{\cos^2 \gamma_L} \right] c_{12} c_{13} (e^{i \varepsilon_L} (s_{12} c_{23} + e^{i \delta_L} c_{12} s_{13} c_{23}) + c.c.) + L \leftrightarrow R \right]^2,$$

$$\beta_{K_{2L,22}}^2 = \frac{1}{8} \left[ \frac{\mu_L^2}{\cos^2 \gamma_L} \right] s_{12} c_{13} (e^{i \varepsilon_L} (c_{12} c_{23} - e^{i \delta_L} s_{12} s_{13} c_{23}) + c.c.) + L \leftrightarrow R \right]^2,$$

$$\beta_{K_{2L,21}}^2 = \beta_{K_{2L,12}}^2 = \frac{1}{16} \left[ \frac{\mu_L^4}{\cos^4 \gamma_L} \right] c_{12}^2 (c_{12} c_{23} - e^{i \delta_L} s_{12} s_{13} c_{23}) - s_{12} e^{-i \varepsilon_L} (c_{23} c_{12} + e^{-i \delta_L} c_{13} s_{13} c_{23}) \right]^2 + L \leftrightarrow R,$$

$$\beta_{K_{2L,33}}^2 = \beta_{K_{2L,31}}^2 = \frac{1}{16} \left[ \frac{\mu_L^4}{\cos^4 \gamma_L} \right] c_{12}^2 (c_{12} c_{23} - e^{i \delta_L} s_{12} s_{13} c_{23}) - s_{12} e^{-i \varepsilon_L} (c_{23} c_{12} + e^{-i \delta_L} c_{13} s_{13} c_{23}) \right]^2 + L \leftrightarrow R,$$

$$\beta_{K_{2L,22}}^2 = \beta_{K_{2L,21}}^2 = \frac{1}{16} \left[ \frac{\mu_L^4}{\cos^4 \gamma_L} \right] c_{12}^2 (c_{12} c_{23} - e^{i \delta_L} s_{12} s_{13} c_{23}) - s_{12} e^{-i \varepsilon_L} (c_{23} c_{12} + e^{-i \delta_L} c_{13} s_{13} c_{23}) \right]^2 + L \leftrightarrow R,$$
for $P = B^0$

\begin{align}
\beta_{B^0,11}^2 &= \frac{1}{4} \frac{\mu_L^2}{\cos^2 \gamma L} c_{12} L c_{13} L e^{i\chi L + i\epsilon L} \left( s_{12} L s_{13} L e^{i\delta L} - c_{12} s_{13} L c_{23} L \right) + L \leftrightarrow R \bigg|, \\
\beta_{B^0,22}^2 &= \frac{1}{4} \frac{\mu_L^2}{\cos^2 \gamma L} s_{12} L c_{13} L e^{i\chi L + i\epsilon L} \left( c_{12} s_{23} L e^{i\delta L} + s_{12} s_{13} L c_{23} L \right) + L \leftrightarrow R \bigg|, \\
\beta_{B^0,33}^2 &= \frac{1}{4} \frac{\mu_L^2}{\cos^2 \gamma L} c_{13} L s_{13} L c_{23} L e^{i\chi L + i\epsilon L} + L \leftrightarrow R \bigg|, \\
\beta_{B^0,21}^2 &= \frac{1}{8} \frac{\mu_L^4}{\cos^3 \gamma L} \left( c_{13} L \right)^2 \left( c_{13} L \right)^2 \left| s_{12} L s_{13} L c_{23} L \right|^2 \left( s_{12} s_{13} c_{23} L + c_{12} s_{13} L c_{23} L \right) + L \leftrightarrow R \bigg|, \\
\beta_{B^0,12}^2 &= \frac{1}{8} \frac{\mu_L^4}{\cos^3 \gamma L} \left( s_{13} L \right)^2 \left( s_{13} L \right)^2 \left| s_{12} L s_{13} L c_{23} L \right|^2 \left( s_{12} s_{13} c_{23} L + c_{12} s_{13} L c_{23} L \right) + L \leftrightarrow R \bigg|, \\
\beta_{B^0,31}^2 &= \frac{1}{8} \frac{\mu_L^4}{\cos^3 \gamma L} \left( c_{13} L \right)^2 \left( c_{13} L \right)^2 \left| s_{12} L s_{13} L c_{23} L \right|^2 \left( s_{12} c_{13} L c_{23} L + c_{12} s_{13} c_{23} L \right) + L \leftrightarrow R \bigg|, \\
\beta_{B^0,32}^2 &= \frac{1}{8} \frac{\mu_L^4}{\cos^3 \gamma L} \left( c_{13} L \right)^2 \left( c_{13} L \right)^2 \left| c_{12} L L c_{13} L \right|^2 \left( c_{12} s_{13} c_{23} L + s_{12} s_{13} c_{23} L \right) + L \leftrightarrow R \bigg|, \\
\beta_{B^0,23}^2 &= \frac{1}{8} \frac{\mu_L^4}{\cos^3 \gamma L} \left( s_{13} L \right)^2 \left( s_{13} L \right)^2 \left| s_{12} L s_{13} L c_{23} L \right|^2 + L \leftrightarrow R \bigg|,
\end{align}

and for $P = B_s$

\begin{align}
\beta_{B_s,11}^2 &= \frac{1}{4} \frac{\mu_L^2}{\cos^2 \gamma L} e^{i\chi L + 2i\epsilon L} \left( -c_{12} s_{13} L c_{23} L + s_{12} s_{13} L c_{23} L \right) \left( s_{12} c_{23} L + c_{12} s_{13} c_{23} L \right) + L \leftrightarrow R \bigg|, \\
\beta_{B_s,22}^2 &= \frac{1}{4} \frac{\mu_L^2}{\cos^2 \gamma L} e^{i\chi L + 2i\epsilon L} \left( s_{12} s_{13} L c_{23} L + c_{12} s_{13} L c_{23} L \right) \left( c_{12} s_{23} L - s_{12} s_{13} s_{23} L \right) + L \leftrightarrow R \bigg|, \\
\beta_{B_s,33}^2 &= \frac{1}{4} \frac{\mu_L^2}{\cos^2 \gamma L} \left( c_{13} L \right)^2 \left( c_{13} L \right)^2 \left| c_{12} L L c_{13} L \right|^2 \left( c_{12} c_{13} L c_{23} L + c_{12} s_{13} c_{23} L \right) + L \leftrightarrow R \bigg|, \\
\beta_{B_s,21}^2 &= \frac{1}{8} \frac{\mu_L^4}{\cos^3 \gamma L} \left( c_{13} L \right)^2 \left( c_{13} L \right)^2 \left| s_{12} s_{13} L c_{23} L \right|^2 \left( s_{12} s_{13} c_{23} L + c_{12} s_{13} L c_{23} L \right) + L \leftrightarrow R \bigg|, \\
\beta_{B_s,12}^2 &= \frac{1}{8} \frac{\mu_L^4}{\cos^3 \gamma L} \left( s_{13} L \right)^2 \left( s_{13} L \right)^2 \left| s_{12} s_{13} L c_{23} L \right|^2 \left( s_{12} s_{13} c_{23} L + c_{12} s_{13} L c_{23} L \right) + L \leftrightarrow R \bigg|, \\
\beta_{B_s,31}^2 &= \frac{1}{8} \frac{\mu_L^4}{\cos^3 \gamma L} \left( c_{13} L \right)^2 \left( c_{13} L \right)^2 \left| s_{12} s_{13} L c_{23} L \right|^2 \left( s_{12} c_{13} L c_{23} L + c_{12} s_{13} c_{23} L \right) + L \leftrightarrow R \bigg|, \\
\beta_{B_s,32}^2 &= \frac{1}{8} \frac{\mu_L^4}{\cos^3 \gamma L} \left( c_{13} L \right)^2 \left( c_{13} L \right)^2 \left| c_{12} L L c_{13} L \right|^2 \left( c_{12} s_{13} c_{23} L + s_{12} s_{13} c_{23} L \right) + L \leftrightarrow R \bigg|, \\
\beta_{B_s,23}^2 &= \frac{1}{8} \frac{\mu_L^4}{\cos^3 \gamma L} \left( s_{13} L \right)^2 \left( s_{13} L \right)^2 \left| s_{12} s_{13} L c_{23} L \right|^2 + L \leftrightarrow R \bigg|,
\end{align}

where $\mu_{L,R} = \tilde{m}_V/m_{V,L,R}$, $\gamma_{L,R}$ are defined through $\sin^2 \gamma_{L,R} = (g_4^L R)^2/[(g_4^L)^2 + (g_4^R)^2]$ so that

\[
\cos^2 \gamma_{L,R} = g_{\ell s}^2(M_\ell)/(g_4^L R)^2 = \alpha_{\ell s}(M_\ell)/\alpha_4^L R, \quad \cos^2 \gamma_L + \cos^2 \gamma_R = 1
\]
with $g_{st}(M_s)$ being defined by the relation (1) and $\chi^{L,R} = \varphi_1^{L,R} + \varphi_2^{L,R} + \varphi_3^{L,R}$.

The mixing factors $\beta^2_{P,ii}$, (12), (14)–(34) describe in the general form the effect of the fermion mixing in leptoquark currents in the case (11) on the branching ratios (5), (6) of the decays of pseudoscalar mesons $P = (K_s^0, B^0, B_s)$ into lepton-antilepton pairs induced by the chiral gauge leptoquarks. These mixing factors in general case depend on six angles $\theta_{12}^{L,R}, \theta_{23}^{L,R}, \theta_{13}^{L,R}$ and six phases $\delta^{L,R}, \varepsilon^{L,R}, \chi^{L,R}$ and can be used for the analysis of the branching ratios of these decays in dependence on the mixing angles and phases of the mixing matrices (13). With account of the expressions (17)–(34) one can see that for $P = (B^0, B_s)$ the mixing factors $\beta^2_{P,ii}$ and (12) in the case of $\mu_{R(L)} = 0$ satisfy the relations

$$\sum_{i=1}^3 \beta^2_{P,ii} + 2(\beta^2_{P,\mu e} + \beta^2_{P,\tau e} + \beta^2_{P,\tau \mu}) = \frac{1}{4} \frac{\mu_{L(R)}^4}{\cos^4 \gamma_{L(R)}},$$

which is a result of the relations (11) and of unitarity of the matrices (13).

Further we will consider the special case that one of two leptoquarks is much more heavy that the another so that its contribution to decays under consideration can be neglected. For definiteness we assume that

$$m_{V_R} \gg m_{V_L}$$

and for account of the contribution only of the lightest leptoquark $V^L$ it is sufficient in the expressions (7), (14)–(34) to set $\bar{m}_V = m_{V_L}, \mu_L = 1, \mu_R = 0$. In this case the mixing factors (14)–(16) for $K^0_L$ meson still depend on three angles $\theta_{12}^L, \theta_{23}^L, \theta_{13}^L$ and on two phases $\delta^L, \varepsilon^L$ whereas the mixing factors (17)–(34) for $B^0, B_s$ mesons depend on these three angles and on the only phase $\delta^L$. The opposite case of $m_{V_L} \gg m_{V_R}$ can be obtained from the formulas given below by the simple exchange $L \leftrightarrow R$.

We have numerically analysed the branching ratios (5)–(7) with mixing factors $\beta^2_{P,ii}$ and $\beta^2_{P,ii'}$ defined by the equations (12), (13)–(34) with account of experimental data on the decays (1). The experimental data on the branching ratios $Br(P \to l^+ l^-)^{exp}$, $Br(P \to ll')^{exp}$ are taken from the Ref. [37] except the branching ratios of the decays $B^0, B_s \to \tau \mu$ for which we use the recent data [38]

$$Br(B^0 \to \tau \mu)^{exp} < 1.4 \cdot 10^{-5},$$
$$Br(B_s \to \tau \mu)^{exp} < 4.2 \cdot 10^{-5}.$$ (37)

The experimental data under consideration are presented in the second column of the Table 1. In the third column of this table we present for comparison the SM predictions $Br_{SM}(P \to l_1 l_2)$ for the branching ratios of the diagonal leptonic decays $K^0_L \to e^+ e^-$ [39, 40], $K^0_L \to \mu^+\mu^-$ [41] and $B_s, B^0 \to l^+_i l^-_j$ [42]. We vary the mixing angles $\theta_{12}^L, \theta_{23}^L, \theta_{13}^L$ and phases $\delta^L, \varepsilon^L$ to find the minimal chiral gauge leptoquark mass $m_{V_L}$ satisfying these experimental data with account that the SM contributions to some of the diagonal leptonic decays are dominant.

As seen from the Table 1 the experimental branching ratios of the diagonal $K^0_L \to e^+ e^-, \mu^+\mu^-$ decays are almost completely saturated by the (long-distance) SM contributions so that the new physics contributions to these decays should be small. With account also of the very stringent experimental limit on the branching ratio of the non-diagonal $K^0_L \to \mu e$ decay the possible leptoquark contributions to the branching ratios of $K^0_L \to e^+ e^-, \mu^+\mu^-, \mu e$ decays should be small. It means that for the small leptoquark
mass the mixing factors $\beta_{K^0_{L},ij}^2$ must be very small (close to zero). Below we believe for definiteness that

$$\beta_{K^0_{L},ij}^2 = 0. \quad (39)$$

The measured values $Br(B_s, B^0 \to \mu^+ \mu^-)^{\text{exp}}$ are consistent with the corresponding SM predictions within the experimental and theoretical uncertainties with the sufficiently good degree of this consistency in the case of $B_s \to \mu^+ \mu^-$ decay. We assume below that the experimental branching ratios $Br(B_s, B^0 \to \mu^+ \mu^-)^{\text{exp}}$ of these decays are saturated mainly by the SM contributions so that the contributions $Br_{V-L}(B_s, B^0 \to \mu^+ \mu^-)$ of the chiral gauge leptoquark $V^L$ to these branching ratios can be assumed to be small. In this case the mixing factors $\beta_{B_s,22}^2, \beta_{B^0,22}^2$ for the small leptoquark mass should be sufficiently small. Below we assume for definiteness that

$$\beta_{B_s,22}^2 = 0, \quad \beta_{B^0,22}^2 = 0, \quad (40)$$

that is $Br_{V-L}(B_s, B^0 \to \mu^+ \mu^-) = 0$, and find the minimal chiral gauge leptoquark mass $m_{V^L}$ satisfying the experimental data with account of the conditions $\beta_{K^0_{L},ij}^2 = 0$. Below we use it in the further analysis.

We have found three solutions of the equations $\beta_{K^0_{L},ij}^2 = 0$:

solution 1: \quad $\theta_{23}^L = \pi/2, \quad \delta^L + \epsilon^L = 3\pi/2, \quad (41)$

solution 2: \quad $\theta_{13}^L = \pi/2, \quad (42)$

solution 3: \quad $\theta_{23}^L = \pi/2, \quad \theta_{13}^L = 0. \quad (43)$

The solution $\theta_{23}^L$ contains two free independent angles $\theta_{12}^L, \theta_{13}^L$ whereas each of the other ones contains only one angle (in the case $\theta_{23}^L$ two angles $\theta_{23}^L, \theta_{13}^L$ form one effective angle $\tilde{\theta}^L$ defined as $\sin \tilde{\theta}^L = |c_{23}^L c_{12}^L - s_{23}^L s_{12}^L e^{i\delta^L}|$). The solution $\theta_{23}^L$ results in the more lower mass limits for the chiral gauge leptoquarks in comparision with the solutions $\theta_{12}^L, \theta_{13}^L$ and we use it in the further analysis.

In the case $\theta_{23}^L$ the mixing factors $\beta_{P,ii}^2, \beta_{P,ii}^2$ defined by the equations $\beta_{P,ii}^2, \beta_{P,ii}^2$ (44)–(54) are simpliyed and the nonzero mixing factors take the form for $P = B^0$

$$\beta_{B^0_{ii},11}^2 = \beta_{B^0_{ii},22}^2 = \frac{1}{4 \cos^4 \gamma_L} (c_{12}^L s_{12}^L c_{13}^L)^2, \quad (44)$$

$$\beta_{B^0_{ii},\mu e}^2 = \frac{1}{8 \cos^4 \gamma_L} \left[ (c_{12}^L)^4 + (s_{12}^L)^4 \right] (c_{13}^L)^2, \quad (45)$$

$$\beta_{B^0_{ii},\tau e}^2 = \frac{1}{8 \cos^4 \gamma_L} \left( s_{12}^L \right)^2 \left( s_{13}^L \right)^2, \quad (46)$$

$$\beta_{B^0_{ii},\tau \mu}^2 = \frac{1}{8 \cos^4 \gamma_L} \left( c_{12}^L \right)^2 \left( s_{13}^L \right)^2, \quad (47)$$

and for $P = B_s$

$$\beta_{B_s,11}^2 = \beta_{B_s,22}^2 = \frac{1}{4 \cos^4 \gamma_L} (c_{12}^L s_{12}^L s_{13}^L)^2, \quad (48)$$

$$\beta_{B_s,\mu e}^2 = \frac{1}{8 \cos^4 \gamma_L} \left[ (c_{12}^L)^4 + (s_{12}^L)^4 \right] (s_{13}^L)^2, \quad (49)$$

$$\beta_{B_s,\tau e}^2 = \frac{1}{8 \cos^4 \gamma_L} \left( s_{12}^L \right)^2 \left( c_{13}^L \right)^2, \quad (50)$$

$$\beta_{B_s,\tau \mu}^2 = \frac{1}{8 \cos^4 \gamma_L} \left( c_{12}^L \right)^2 \left( c_{13}^L \right)^2, \quad (51)$$
With account of expressions (44), (48) the equations (40) give

$$\theta_{12}^L = 0 \left(\pi/2\right)$$

(52)

and the nonzero mixing factors in (44)-(51) take the form

$$\beta_{B^0,\mu e}^2 = \frac{1}{8 \cos^4 \gamma_L} \left( c_{13}^L \right)^2,$$

(53)

$$\beta_{B_s,\mu e}^2 = \frac{1}{8 \cos^4 \gamma_L} \left( s_{13}^L \right)^2,$$

(54)

$$\beta_{B_s,\tau e}^2 \left( \beta_{B^0,\tau e}^2 \right) = \frac{1}{8 \cos^4 \gamma_L} \left( s_{13}^L \right)^2 \text{ for } \theta_{12}^L = 0, \left(\pi/2\right),$$

(55)

$$\beta_{B_s,\tau e}^2 \left( \beta_{B^0,\tau e}^2 \right) = \frac{1}{8 \cos^4 \gamma_L} \left( c_{13}^L \right)^2 \text{ for } \theta_{12}^L = 0, \left(\pi/2\right).$$

(56)

In the numerical analysis we use the data on the masses of leptons and quarks and the data on the masses and life times

$$\tau_P \left( \tau_P \rightarrow \Gamma_{P_{tot}} \right) \text{ of mesons } P = (K_0^L, B_0^0, B_s) \text{ from Ref. [37]. For the form factors } f_P \text{ we use the values [41]}

$$f_{K_0^L} = 155.72 \text{ MeV}, \ f_{B_0^0} = 190.9 \text{ MeV}, \ f_{B_s} = 227.2 \text{ MeV.}$$

For the mass scale $M_c$ of the chiral four color symmetry breaking we choose the value $M_c = 10 \text{ TeV}$ and for the strong coupling constant at this mass scale we use the value $\alpha_{st}(10 \text{ TeV}) = 0.07134.$

Using (5)–(7) and varying the angle $\theta_{13}^L$ in (53)–(56) we have found the lower mass limit for the chiral gauge leptoquark $V^L$ in the form

$$m_{V^L \cos \gamma_L} > 5.68 \text{ TeV}$$

(57)

where $\cos \gamma_L$ is defined by the formulas (35). The lightest allowed in (57) mass $m_{V^L}$ satisfying the relation $m_{V^L \cos \gamma_L} = 5.68 \text{ TeV}$ in the case of (11), (52) is ensured by the value

$$\theta_{13}^L = 1.096$$

(58)

of the mixing angle $\theta_{13}^L$.

Numerically the mass limit (57) depends on the relation between the coupling constants $g_1^L$ and $g_4^R$. In particular case of $\alpha_{1}^L = \alpha_{4}^R (= 2 \alpha_{st}(M_c) = 0.143)$ from (35) we have $\cos \gamma_L = \cos \gamma_R = 1/\sqrt{2}$ and the eq. (57) for $g_4^L = g_4^R$ gives the mass limit

$$m_{V^L} > 8.03 \text{ TeV}.$$ 

(59)

Variation of $\cos \gamma_L$ in (57) according to (35) by varying the coupling constants $\alpha_{1}^{L,R}$ in the region ensuring the validity of the pertubation theory leads to some region for the lower mass limits. For example with assumption $\alpha_{1}^L, \alpha_{4}^R < 1$ for $0.08 \lesssim \alpha_{1}^L \lesssim 0.5(1.0)$ the relation (57) gives for the values of the lower mass limit the region

$$m_{V^L} \gtrsim 6.1 - 15.0(21.3) \text{ TeV.}$$

(60)

The solutions (42) and (43) with account of (40) instead of the mass limit (57) result in the more high mass limits $m_{V^L \cos \gamma_L} > 6.02 \text{ TeV}$ and $m_{V^L \cos \gamma_L} > 8.39 \text{ TeV}$ respectively.
It is worthy to note that in the case with neglect of fermion mixing (\(K_2^V = I\)) the chiral gauge leptoquark \(V^L\) contributes only to the decays \(K_L^0 \rightarrow \mu e, B^0 \rightarrow \tau e, B_s \rightarrow \tau \mu\) and instead of (57) we obtain in this case the mass limits

\[
m_{V^L} \cos \gamma_L > 212.28 \text{ TeV}, \quad 2.50 \text{ TeV}, \quad 2.48 \text{ TeV}
\]

from the current data on the branching ratios \(\text{Br}(K_L^0 \rightarrow \mu e)^\text{exp}, \text{Br}(B^0 \rightarrow \tau e)^\text{exp}, \text{Br}(B_s \rightarrow \tau \mu)^\text{exp}\) respectively. As seen the account of the fermion mixing in leptoquark currents essentially reduces the mass limit for the chiral gauge leptoquark from the largest value in (61) to the value (57).

It is worthy to note also that the mass limits (57), (59), (60) for the chiral gauge leptoquarks are essentially lower then corresponding lower mass limit of order of 90 TeV [35] for the vector leptoquark. Such light chiral gauge leptoquarks can manifest themselves to the leptonic decays of pseudoscalar mesons of type (1) and in other decays with the lepton flavour violation.

The mass limits (57), (59), (60) are obtained with neglecting the possible small contributions to the diagonal decays \(K_L^0 \rightarrow e^+e^-, \mu^+\mu^-\) and \(B_s, B^0 \rightarrow \mu^+\mu^-\) from the leptoquark \(V^L\). The account of these contributions can result in the small lowering of these mass limits. These possible deviations can be approximately estimated keeping in mind that the amplitude \(M\) and the branching ratio \(\text{Br}\) of the diagonal decay \(P \rightarrow l_i^+ l_i^-\) with simultaneous account of SM and \(V^L\) contributions can be written as \(M = M_{SM} + \xi M'_{V^L}\) and \(\text{Br} = \text{Br}_{SM}(1 + \xi k + \xi^2)\) where \(\xi\) is a (small) parameter defined as \(\xi^2 = \text{Br}_{V^L}/\text{Br}_{SM}\) with \(\text{Br}_{V^L}\) defined by expression (5) and \(\xi k\) parametrizes the interference term with \(k \sim 1\). The limiting values of parameter \(\xi\) are defined by the equation \(\text{Br}_{SM}(1 + \xi k + \xi^2) = \text{Br}_0^\text{exp} \pm \Delta\text{Br}_0^\text{exp}\) where \(\text{Br}_0^\text{exp}\) and \(\Delta\text{Br}_0^\text{exp}\) are the central value and the experimental errors of the measured branching ratio. These limiting values can be found from this equation for each decay using the experimental data and the SM predictions for \(K_L^0 \rightarrow e^+e^-, \mu^+\mu^-\) and \(B_s, B^0 \rightarrow \mu^+\mu^-\) decays. The numerical analysis of the part \(\text{Br}_{V^L} = \xi^2 \text{Br}_{SM}\) of the total branching ratio of each of these decays in dependence on the leptoquark mass \(m_{V^L}\) leads to the deviation of the right side of the relation (57) by the value \(\Delta m_{V^L} \cos \gamma_L \sim -0.1\) TeV, which induces the corresponding deviations \(\Delta m_{V^L}\) in the mass limits (59), (60). In the case of agreement of the future improved measurement of the branching ratio of the \(B^0 \rightarrow \mu^+\mu^-\) decay with the SM prediction within the 10% experimental error the deviation \(\Delta m_{V^L} \cos \gamma_L\) can be reduced to the value \(\Delta m_{V^L} \cos \gamma_L \sim -0.01\) TeV.

We have calculated the contributions to the branching ratios (5) - (11) in the case (36) from the chiral gauge leptoquark \(V^L\) with the lightest allowed mass \(m_{V^L}\) satisfying the relation \(m_{V^L} \cos \gamma_L = 5.68\) TeV with mixing parameters (11), (52), (58). These contributions \(\text{Br}_{V^L}(P \rightarrow l_1 l_2)\) are presented in the fourth column of the Table I.

As seen the contributions of the chiral gauge leptoquark \(V^L\) to all the diagonal leptonic decays \(B_s, B^0 \rightarrow l_i^+ l_i^-\) of \(B_s, B^0\) mesons (including the decays \(B_s, B^0 \rightarrow e^+e^-\) and \(B_s, B^0 \rightarrow \tau^+\tau^-\) in the case (39), (11)) occur to be equal to zero whereas such contributions to \(B_s, B^0 \rightarrow \mu e\) decays are close to the corresponding experimental limits. Just the decays \(B_s, B^0 \rightarrow \mu e\) define in this case the lower mass limit (57) and the further search for these decays will give the new mass limits for the chiral gauge leptoquarks. The analogous contributions to the branching ratios of the decays \(B_s, B^0 \rightarrow \tau \mu\) (or \(B_s, B^0 \rightarrow \tau e\)) are predicted to be of order of \(10^{-7}\) and are approximately by order of 2 less than their current experimental limits (the experimental data on the decays \(B_s \rightarrow \tau e\) are still absent) and the search for these decays will need the more high statistics.
Table 1: Contributions to the branching ratios of the decays $P \to l_1 l_2$ from the chiral gauge leptoquarks with the lightest allowed mass with account of the general fermion mixing a) in the case of $Br_{VL}(K^0_L \to l_1 l_2) = 0$, $Br_{VL}(B_s, B^0 \to \mu^+ \mu^-) = 0$, $m_{VL} \cos \gamma_L = 5.68$ TeV and $\theta^L_{12} = 0 (\pi/2)$, b) in the symmetrical case of $g^R_4 = g^L_4$, $m_{VL} = m_{VR} = 8.03$ TeV with $\chi^R = \chi^L + \pi$.

| $P \to l_1 l_2$ | $Br(P \to l_1 l_2)$ | $Br_{SM}(P \to l_1 l_2)$ | $Br_{VL}(P \to l_1 l_2)$ | $Br_{VL,R}(P \to l_1 l_2)$ |
|-----------------|---------------------|--------------------------|----------------------------|--------------------------|
| $K^0_L \to e^+ e^-$ | $(9 \pm 5) \times 10^{-12}$ | $9 \times 10^{-12}$ | 0 | 0 |
| $K^0_L \to \mu^+ \mu^-$ | $(6.84 \pm 0.11) \times 10^{-9}$ | $(6.64 \pm 0.07) \times 10^{-9}$ | 0 | 0 |
| $K^0_L \to \mu e$ | $< 4.7 \times 10^{-12}$ | $< 4.7 \times 10^{-12}$ | 0 | 0 |
| $B^0 \to e^+ e^-$ | $< 8.3 \times 10^{-8}$ | $(2.48 \pm 0.21) \times 10^{-13}$ | 0 | 0 |
| $B^0 \to \mu^+ \mu^-$ | $(1.4 \pm 1.6) \times 10^{-10}$ | $(1.06 \pm 0.09) \times 10^{-10}$ | 0 | 0 |
| $B^0 \to \tau^+ \tau^-$ | $< 2.1 \times 10^{-3}$ | $(2.22 \pm 0.19) \times 10^{-8}$ | 0 | 0 |
| $B^0 \to \mu e$ | $< 1.0 \times 10^{-9}$ | $0.99 \times 10^{-9}$ | 0 | 0 |
| $B^0 \to \tau e$ | $< 2.8 \times 10^{-5}$ | $8.37 \times 10^{-7}$ | 8.47 $\times 10^{-7}$ | 0 |
| $B^0 \to \tau \mu$ | $< 1.4 \times 10^{-5}$ | $8.37 \times 10^{-7} (0)$ | 8.47 $\times 10^{-7}$ | 0 |
| $B_s \to e^+ e^-$ | $< 2.8 \times 10^{-7}$ | $(8.54 \pm 0.53) \times 10^{-14}$ | 0 | 0 |
| $B_s \to \mu^+ \mu^-$ | $(3.0 \pm 0.4) \times 10^{-9}$ | $(3.65 \pm 0.23) \times 10^{-9}$ | 0 | 0 |
| $B_s \to \tau^+ \tau^-$ | $< 6.8 \times 10^{-3}$ | $(7.73 \pm 0.49) \times 10^{-7}$ | 0 | 0 |
| $B_s \to \mu e$ | $< 5.4 \times 10^{-9}$ | $5.38 \times 10^{-9}$ | 5.39 $\times 10^{-9}$ | 0 |
| $B_s \to \tau e$ | $< 4.2 \times 10^{-5}$ | $3.19 \times 10^{-7}$ | 3.16 $\times 10^{-7}$ | 0 |
| $B_s \to \tau \mu$ | $< 4.2 \times 10^{-5}$ | $3.19 \times 10^{-7} (0)$ | 3.23 $\times 10^{-7}$ | 0 |

It is interesting to note that in the case of $m_{VR} \sim m_{VL}$ there is the possibility of the mutual cancellation of the contributions to the branching ratios of the diagonal leptonic decays $B_s, B^0 \to l_1^+ l_1^-$ from the chiral gauge leptoquarks $V^L$ and $V^R$, as it can be seen from the structure of the mixing factors [17]–[19] and [26]–[28]. In particular, in the symmetrical case with

$$m_{VR} = m_{VL}, \quad g^R_4 = g^L_4, \quad (\theta_{ij}, \delta, \varepsilon)^R = (\theta_{ij}, \delta, \varepsilon)^L, \quad \chi^R = \chi^L + \pi$$

and with account of [44] we obtain the mass limit

$$m_{VL} = m_{VR} > 8.03 \text{ TeV},$$

where the lightest mass is ensured in the case [44] by the angles

$$\theta^L_{12} = \theta^R_{12} = 0.78, \quad \theta^L_{13} = \theta^R_{13} = 1.096.\quad (64)$$

The contributions $Br_{VL,R}(P \to l_1 l_2)$ to the branching ratios of $P \to l_1 l_2$ decays from the chiral gauge leptoquarks $V^L$ and $V^R$ with their lightest mass in [63] and mixing angles [11], [64] are presented in the fifth column of the Table I. As seen these contributions to the branching ratios for $B_s, B^0 \to \mu e$ decays are also close to the corresponding experimental limits as well as those for $B_s, B^0 \to \tau e, \tau \mu$ decays are predicted to be of order of $10^{-7}$.

The possible contributions of the chiral gauge leptoquarks to the branching ratios of the decays $P \to l_1 l_2$ presented in the Table I are consistent with current data on these decays and will be useful in designing of the current and future experimental searches [43] for these decays. The nondiagonal leptonic decays $B_s, B^0 \to \mu e, \tau e, \tau \mu$ look as the perspective ones for search for new physics effects such as the chiral gauge leptoquark contributions to these decays.

In conclusion we resume the results of the work.
The contributions of the chiral gauge leptoquarks $V^L, V^R$ induced by the chiral four
color symmetry between quarks and leptons to the branching ratios of $K^0_L, B^0, B_s \rightarrow l_1 l_2$
decays are calculated and analysed using the general parametrizations of the fermion
mixing matrices in the leptoquark currents.

From the current experimental data on the branching ratios of these decays the lower
mass limit $m_{V^L} \cos \gamma_L > 5.68$ TeV is found (assuming that $m_{V^L} \ll m_{V^R}$), which gives
$m_{V^L} > 8.03$ TeV for $\alpha_4^L = \alpha_4^R$ and $m_{V^L} \gtrsim 6.1 - 15.0(21.3)$ TeV for $0.08 \leq \alpha_4^L \leq 0.5(1.0)$.

The contributions of the chiral gauge leptoquarks to the branching ratios of $K^0_L, B^0, B_s \rightarrow l_1 l_2$
decays are analysed in dependence on the leptoquark masses and the mixing param-
eters and the predictions for these contributions to the branching ratios of $B^0, B_s \rightarrow l_1 l_2$
decays are obtained in consistency with the current experimental data.

In particular, it is shown that the contributions the chiral gauge leptoquarks to the
diagonal leptonic decays $B_s, B^0 \rightarrow l^+_i l^-_i$ can be suppressed by the appropriate choice of
the mixing angles and phases (in the case of $m_{V^L} \ll m_{V^R}$) or by the possible mutual
cancellation of the contributions of the chiral gauge leptoquarks to these decays (in the
case of $m_{V^L} \sim m_{V^R}$). In this case with the contributions of the chiral gauge leptoquarks to
the branching ratios of $B_s, B^0 \rightarrow \mu e$ decays being close to the corresponding experimental
limits such contributions to the branching ratios of $B_s, B^0 \rightarrow \tau e, \tau \mu$ decays are predicted
to be of order of $10^{-7}$. The nondiagonal leptonic decays $B_s, B^0 \rightarrow \mu e, \tau e, \tau \mu$ look as the
perspective ones for search for the chiral gauge leptoquark contributions to these decays.

The estimations of the possible contributions of the chiral gauge leptoquarks to the
branching ratios of the decays $K^0_L, B^0, B_s \rightarrow l_1 l_2$ calculated and discussed in this paper
with account of the fermion mixing in leptoquark currents of the general form and in
consistency with the current experimental data will be useful in designing of the further
experimental searches for these decays.

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