Low-rank representation of head impact kinematics: 
A data-driven emulator

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Abstract

Head motion induced by impacts has been deemed as one of the most important measures in brain injury prediction, given that the vast majority of brain injury metrics use head kinematics as input. Recently, researchers have focused on using fast approaches, such as machine learning, to approximate brain deformation in real time for early brain injury diagnosis. However, training such models requires large number of kinematic measurements, and therefore data augmentation is required given the limited on-field measured data available. In this study we present a principal component analysis-based method that emulates an empirical low-rank substitution for head impact kinematics, while requiring low computational cost. In characterizing our existing data set of 537 head impacts, each consisting of 6 degrees of freedom measurements, we found that only a few modes, e.g. 15 in the case of angular velocity, is sufficient for accurate reconstruction of the entire data set. Furthermore, these modes are predominantly low frequency since over 70% to 90% of the angular velocity response can be captured by modes that have frequencies under 40Hz. We compared our proposed method against existing impact parametrization methods and showed significantly better performance in injury prediction using a range of kinematic-based metrics – such as head injury criterion (HIC) and rotational injury criterion (RIC) – and brain tissue deformation-based metrics – such as brain angle metric (BAM), maximum principal strain (MPS) and axonal fiber strains (FS). In all cases, our approach reproduced injury metrics similar to the ground truth measurements with no significant difference, whereas the existing methods obtained significantly different ($p < 0.01$) values as well as substantial differences in injury classification sensitivity and specificity. This emulator will enable us to provide the necessary data augmentation to build a head impact kinematic data set of any size.

Introduction

Traumatic brain injury (TBI) is one of the most debilitating health problems in our society today, with nearly two million new cases in the US every year [1]. The majority of these cases are considered mild, also known as concussion [2]. The substantial increase in reported concussions in
contact sports [3], together with the recent findings of increased long-term pathological changes [4], has sparked a public discussion and raised awareness about TBI. An important requirement is an accurate and objective diagnosis of concussions, which in turn could inform better protective equipment design and safer activities [5–9].

Head motion kinematics, including the rate, frequency and direction of head’s movement during collision, has been deemed as one of the most consequential metric in predicting brain injury. Historically, kinematic-based metrics such as head injury criterion (HIC) [10], rotational injury criterion (RIC) [11], and brain injury criterion (BrIC) [12] have been used to detect injury. These metrics are still widely used ones among researchers and are endorsed by safety regulating organizations such as the National Highway Traffic Safety Administration (NHTSA) and the National Operating Committee on Standards for Athletic Equipment (NOCSAE) [13]. More recently, brain tissue deformation-based metrics have been introduced that use head kinematics as input to computational models that can approximate the effect of head motion on brain displacement and deformation. These metrics either use simple discrete mechanical elements in lumped-parameter models, i.e. mass-spring-damper combinations, to give a rigid-body estimate of brain’s relative motion with respect to the skull [14–17], or more complex finite element (FE) models with detailed geometry of the brain anatomy, which can simulate the local brain deformation and interaction with the stiff bony or membranous structures [18–20]. In the case of lumped models, brain angle metric (BAM), developed based on a data set of concussive and sub-concussive head impacts [21], and in the case of FE models, maximum principal strain (MPS) and axonal fiber strain (FS) along the white matter axon fibers have been proposed as an effective injury diagnosis metrics [22].

Evidently, both for the kinematic-based and the brain deformation-based metrics, head impact kinematics play a major role. With the advent of wearable sensor technology, several groups have been collecting on-field head kinematic measurements during contact sports events [23–27]. However, despite these pioneering efforts, on-field head kinematic measurements are not widely available. As a result, researchers have resorted to simplifying and parametrizing head collisions as idealized biphasic acceleration impulses (with only two parameters) represented either by a triangle or half-sine [28–30], whose height and width constitute the magnitude and duration of a head impact impulse. The simplification of kinematic impulses serves the objective of emulating real kinematic data of a head impact with a few and manageable number of parameters to populate an otherwise infinite-dimensional loading space to investigate and establish a relation between head motion and brain injury. However, a potential disadvantage of these simplifications is overlooking valuable information that could prove detrimental in developing injury metrics. Therefore, it is paramount to understand the characteristics of real-world head impacts and whether we can accurately capture them through simplified approximations. Furthermore, advances in computational methods, including machine learning algorithms, have provided new and exciting avenues for fast and reliable prediction and diagnosis of brain injury. As a result, given the prohibitively high computational cost of current FE models, the biomechanics community has been trying to utilize such interpolative and machine learning techniques [22, 28, 31, 32]. However, a limitation of those techniques is the large number of kinematic data required to train these algorithms (in the order of thousands of head impacts [32]). Currently such a data set is not widely available. Thus, artificial augmentation of kinematic samples has been utilized as an alternative to satisfy that training data set requirements of such algorithms.

In this study we present an alternative method of empirically characterizing head impact kine-
matics that could reduce the loading space using a low-rank version of a set of kinematic data. We extract the most dominant characteristics of head impact kinematics by modal decomposition on an existing data set in the context of contact sports. Our main goal is to represent a low dimensional characteristic for these head impacts by accurately capturing their behavior by utilizing only a small number of modes. Our hypothesis is that by comparing their accuracy in predicting brain displacement and deformation as well as injury metrics, we would show that our proposed model is more accurate than the current biphasic reconstruction of head impact impulses, while conserving a low computational cost. Furthermore, we provide a formal approach to emulating new and expanded head impact kinematics data sets based on modal reconstruction and expansion.

Materials and Methods

In order to study the characteristics of head impact kinematics and the efficacy of simplified approximations, we use a previously-collected data set of 537 head impact kinematics measured during contact sports: American football, boxing, and mixed martial arts [23,24]. For each impact, measurements with 6 degrees of freedom (DoF) – linear acceleration and angular velocity in the three anatomical directions – were collected at 1,000Hz for 100ms using a mouthguard instrumented with a triaxial accelerometer and a triaxial gyroscope [23]. We construct reduced kinematics data sets to approximate the measured kinematics. First, using principal component analysis (PCA), we decrease the dimensionality of the measured head impact kinematics to construct a low-rank kinematics data set. Subsequently, using previously proposed biphasic assumptions for acceleration impulses with only two variables (acceleration magnitude and duration), we fit triangle (tri) and half-sine (hs) approximations to derive a biphasic data set. We investigate the efficacy of each approximation through comparing its performance in detecting brain motion/deformation and injury prediction using three types of metrics: 1) kinematics-based injury metrics, including HIC, RIC and BrIC, 2) brain angle metric (BAM), and 3) tissue deformation-based finite element injury metrics, including maximum principal strain (MPS) and axonal fiber strain (FS) in the whole brain and corpus callosum (CC). We compare the performance of each approximation against the ground truth (GT) data described above.

Dimension reduction through principal component analysis

Our goal is to exploit the correlations between different measurements and find a reduced representation for three quantities of interest (QoIs), including: linear acceleration, angular velocity and angular acceleration in each anatomical direction. In the case of linear kinematics, anterior-posterior, inferior-superior, and lateral directions are considered and in the case of angular kinematics, axial, coronal and sagittal directions are considered as separate DoF. To this end, we apply principal component analysis (PCA) to our data set. For each QoI, we form a data matrix $X_{m \times n}$, where $n = 537$ is the number of measured head impacts and $m = 100$ is the number of time steps, and $X = [x_1 | x_2 | \ldots | x_n]$, where each column represents the measured QoI for a particular head impact and each row represents the time instance of the measurement. To perform PCA, we compute the singular value decomposition (SVD) of the data matrix: $X = U \Sigma Y^T$, where $U = [u_1 | u_2 | \ldots | u_n]$ are a set of orthonormal modes, i.e. $u_i^T u_j = \delta_{ij}$, $\Sigma = \text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_n)$ is a diagonal matrix, where $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_n$ are the singular values, and $Y = [y_1 | y_2 | \ldots | y_n]$ are the uncorrelated linear components, i.e. $y_i^T y_j = \delta_{ij}$ with the joint probability distribution function.
A reduced representation of the impact data is obtained by: \( \mathbf{X} \simeq \sum_{i=1}^{k} \sigma_i \mathbf{u}_i \mathbf{y}_i^T \). To quantify the performance of the reduction, we introduce:

\[
\eta(k) = \frac{\left( \sum_{i=1}^{k} \sigma_i \right)}{\left( \sum_{i=1}^{n} \sigma_i \right)},
\]

Once we extract the modal characteristics of the head impact data set, a reduced emulator of the head impact kinematics is obtained by truncating the PCA expansion at \( k \) modes:

\[
\mathbf{x}^* = \sum_{i=1}^{k} \sigma_i \mathbf{y}_i^* \mathbf{u}_i,
\]

where \( (\mathbf{y}_1^*, \mathbf{y}_2^*, \ldots, \mathbf{y}_k^*) \) is a random point with \( k \) components drawn from the marginal PDF of \( p(y_1, y_2, \ldots, y_k) \). Equation 2 can be used as an emulator for producing new time series \( (\mathbf{x}^*) \) for each of the QoIs that are nearly indistinguishable from the ground truth head impact kinematics measurements. We provide this emulator as a MATLAB code that can easily be used to construct new head impact data sets.

### Parametrizing biphasic impulse profiles

In order to compare the performance of previously proposed biphasic models for angular and linear acceleration impulses [28,29] against our low-rank PCA approximation, we construct triangle (tri) and half-sine (hs) representations of the 537 head impacts described above. First, the absolute maximum of the angular acceleration profile \( (\alpha_M) \) – computed by differentiating angular velocity measurements – is identified, including the time of peak \( (t_M) \). The impact duration \( (\Delta t) \) is defined as the time interval on both sides of \( t_M \), where either the sign or the convexity of the acceleration profile (whichever comes first) changes. This process is described in Figure 1 where the impact duration \( (\Delta t = t_1 - t_0) \) is the time elapsed between the initiation \( (t_0 < t_M) \) and the completion
of impact \( t_1 > t_M \). In the cases where \( t_M \rightarrow 0ms \) or \( t_M \rightarrow 100ms \), since it is not possible to define \( t_0 \) or \( t_1 \) correctly, only half of the simplified pulse is created. Finally, the change in velocity is computed as the area under the acceleration impulse through \( \Delta \omega = \int_{t_0}^{t_1} \alpha(t)dt \), and the corresponding acceleration magnitudes for the triangle \((\alpha_{tri})\) and half-sine \((\alpha_{hs})\) approximations are calculated through:

\[
\alpha_{tri} = 2\frac{\Delta \omega}{\Delta t}, \quad \alpha_{hs} = \frac{\pi \Delta \omega}{2\Delta t}.
\]

Angular velocity pulses are computed through direct temporal integration from low-rank angular acceleration pulses. Thus, this computed angular velocity will also be low dimensional. Consequently, we derive an ordered array of the modal contributions inherent to head impacts response. Finally, we perform power spectral density (PSD) analysis on the derived temporal modes to obtain their predominant frequencies. These values are given by the maximum PSD values of each mode.

### Accuracy for injury prediction metrics

**Kinematic-based injury metrics:**

We use HIC_{15}, RIC_{36} and BrIC to compare the performance of our proposed PCA reduction against the current simplified biphasic signals, i.e. triangle and half-sine approximations. To this end, we use previously published injury threshold values: 1) for HIC_{15}, values of 240 and 667 have been reported as a 50% risk of concussion [34] and skull fracture [35], respectively; 2) for RIC_{36}, a value of \( 10.3 \times 10^6 \) there is a 50% risk of concussion [11]; and 3) for BrIC, a value of 0.5 constitutes a 50% [12]. We use these thresholds to assess the performance of each reduction approach in providing injury predictions in terms of sensitivity and specificity. Injury thresholds are used as indicators, defining true positives (above) and negatives (below), while the predictive value of each impulse approximation was compared against the GT.
Brain angle injury metric:

We further compared the performance of each approximation using injury criterion based on simplified lumped-model systems. These models generally consider simplifying assumptions: skull and brain are considered rigid bodies and relative motion between the two represents a form of deformation and injury, and the compliance of the brain-skull interface such as the effect of bridging veins, dura and pia mater is represented by linear spring and damper elements [15, 36, 37]. Given the head kinematics as the base excitation input to this system, provided an estimate of the relative motion of brain and skull, particularly the angular motion since that has been seen as the more consequential type of motion [38]. Recently, brain angle metric (BAM) was developed based on the characteristics of human brain and skull in finite element simulations, and validated against observed concussive and sub-concussive head impacts [21]. We compute BAM for each kinematic approximation (PCA, triangle and half-sine).

Tissue deformation-based injury metrics:

As a final step in studying the efficacy of the different kinematic approximations, we compare the performance of each approximation in predicting the tissue-level deformation metrics using finite element simulations, including maximum principal strain (MPS) in the whole brain (WB) and in the corpus callosum (CC) region, as well as axonal fiber strains (FS) in the corpus callosum region, which have all been proposed as reliable tissue-level metrics for injury prediction [20, 24, 39] (Figure 2). Recently a convolutional neural network (CNN) was developed based on pre-trained FE simulations based on the Worcester Head Injury Model (WHIM) [40]. This CNN method uses angular velocity data as input to approximate the regional brain deformations, i.e. maximum principal and axonal fiber strain [32].

Injury metric error analysis:

Having simulated the injury metrics for each head impact measurement, we calculate the corresponding injury metric (metric\(_{GT}\)) and the injury metric estimated by the kinematics approximation (metric\(_{approx}\)) using the equation below:

\[
error = \frac{|metric_{approx} - metric_{GT}|}{metric_{GT}} \times 100. \tag{4}
\]

Subsequently, we performed statistical analysis of variance (ANOVA) with a \(p\) value of 0.01 (MATLAB, \texttt{anova1}) to show significant differences between each approximated impulse and the ground truth. We also performed sensitivity and specificity analysis to provide an estimate for the efficacy of approximating the metrics for injury diagnosis. For this purpose, we used previously published values for 50% risk of concussion, including MPS\(_{WB}\) = 0.2 [41], MPS\(_{CC}\) = 0.2 [39], and FS\(_{CC}\) = 0.074 [42].
Results

Dimension reduction through PCA

We perform PCA decomposition of the measured kinematic data for the QoIs, i.e. linear acceleration, angular velocity and angular acceleration in each anatomical direction. We propose the reduction criterion of $\eta = 0.90$, as defined in Equation (1), for all these cases. In the case of angular velocity, the minimum number of modes that satisfy this reduction criterion are $k = 13$, 15 and 10 modes for coronal, sagittal and axial directions, respectively. In Figure 3 (top row), the PCA reconstruction of angular velocity in three anatomical directions for a sample case is showed. The sample case was chosen randomly from the 537 cases and it is represented by a column of the data matrix $X$. The ground truth measurement for the sample case as well as the reconstructed impulses with different levels of reduction are showed. It is clear that the 15-mode reduction yields a satisfactory reconstruction. In Figure 3 (bottom row) the individual and cumulative contribution of PCA modes are showed for the entire kinematics data set. These results demonstrate that with a relatively small number of PCA modes (i.e. 15) an accurate approximation of the head kinematic measurements can be achieved.

To further study the distribution of these modal approximations, we show the orthonormal projection of the first five PCA modes ($y_2, \ldots, y_5$) against the first and most energetic mode ($y_1$) (Figure 5). It is clear that the modes follow a Gaussian distribution, which would be an important consideration for emulating more data points. Furthermore, the first three temporal modes show a classic modal behavior with 1, 2, and 3 peaks in all three anatomical directions (Figure 7).
Together, these first three modes capture nearly a half of the total angular velocity response, and with each additional mode, we can reconstruct a closer approximation with the ground truth. For more analysis of the modes, see Supplementary Materials Figures SM.2 and SM.4. We will provide a MATLAB code that allow users to build low-rank head kinematics data sets with various approximation levels (η) and emulate a pre-defined number of impacts.

Figure 4: Distribution graphs for second to fifth principal components $y_2, \ldots, y_5$ projected to the first principal component $y_1$ for angular velocity in the sagittal direction, showing Gaussian distribution.

Figure 5: The three most energetic temporal modes for angular velocity for the entire data set.

The contribution of the most dominant frequencies, obtained through PSD criterion, are displayed in Figure 6. In general, low frequencies interval such from 10 to 40Hz have the highest predominance for each parameter. Notably, cumulated contribution for rotational velocity is progressively decreasing with increasing frequency.

**Parametrizing biphasic impulse profiles**

Using the criteria described above, we fitted triangle and half-sine analog pulses to the ground truth kinematics measurements in order to parametrize the rotational acceleration magnitude and duration for each head impact. As a result, we derived 537 analog impulses in the three anatomical directions for both triangle and half-sine approximations. The results are presented in Table 1.
where the rotational acceleration magnitude and duration average and standard errors of the mean are given for the ground truth and the impulses approximations.

Table 1: Means and standard errors of acceleration magnitude and durations for ground truth and the biphasic impulses. The PCA magnitudes were obtained by decomposing the ground truth angular accelerations for the criterion $\eta = 0.9$, which constituted of 21 modes for coronal and sagittal directions and 20 modes for axial direction (See the Supplementary Materials Section for more information).

|                     | Coronal direction | Sagittal direction | Axial direction |
|---------------------|-------------------|--------------------|-----------------|
| **Ground truth magnitude** ($rad/s^2$) | 818.89 ± 937.12  | 1,498.10 ± 1,753.40 | 655.16 ± 535.03 |
| **PCA magnitude** ($rad/s^2$)             | 801.41 ± 922.08  | 1,460.50 ± 1,736.10 | 641.52 ± 523.06 |
| **Triangle magnitude** ($rad/s^2$)         | 871.53 ± 1,011.8 | 1,625.40 ± 1,923.20 | 697.04 ± 553.95 |
| **Half-sine magnitude** ($rad/s^2$)        | 681.25 ± 790.38  | 1,269.30 ± 1,500.40 | 545.64 ± 433.40 |
| **Duration** ($ms$)                         | 15.20 ± 6.75     | 15.00 ± 8.03        | 17.90 ± 8.45     |

**Accuracy for injury prediction metrics**

**Kinematic-based injury metrics**

The injury metrics HIC$_{15}$, RIC$_{36}$ and BrIC are computed for every model and the ground truth. Figure 7 shows the distribution of all samples and the corresponding concussion and skull fracture thresholds. The PCA predictions show similar mean and standard deviations for HIC and RIC (38.20 ± 139.55 and 1.89 · 10$^6$ ± 7.66 · 10$^6$ respectively) as the ground truth (38.49 ± 140.13 and 2.26 · 10$^6$ ± 9.04 · 10$^6$); however, there is a significant difference between the ground truth predictions and the triangle (15.74 ± 41.71 and 3.98 · 10$^5$ ± 1.06 · 10$^7$) and half-sine (14.80 ± 39.31 and 3.86 · 10$^5$ ± 1.02 · 10$^6$) impulses. Whereas the PCA-based impulses show accurate predictions compared to the ground truth, we see the biphasic approximations either under-predict injury (higher number of false negatives) in terms of HIC and RIC, or over-predict injury (higher number of false positives) in terms of BrIC (Figure 7). To better illustrate this, we perform sensitivity and specificity analysis for injury classification, where the PCA-based signals showed high predictive performance compared to the biphasic impulses (Table 2).
Figure 7: Computed HIC\textsubscript{15}, RIC\textsubscript{36} and BrIC for each head impact case and impulses model. Circles represent each sample, and blue and red regions describe the standard deviation and standard error. The solid blue and dashed red lines represent 50% risk of concussion and skull fracture, respectively. Significant differences are indicated for $p < 0.01$.

Table 2: Sensitivity and specificity of kinematic injury metrics for HIC\textsubscript{15}, RIC\textsubscript{36} and BrIC of PCA-based method, triangles (Tri.) and half-sines (H.S.) compared to the ground truth, considering thresholds of 50% risk of injuries.

|                | HIC (50% risk – skull fracture) | HIC (50% risk – concussion) | RIC (50% risk – concussion) | BrIC (50% risk – concussion) |
|----------------|---------------------------------|-----------------------------|----------------------------|----------------------------|
|                | PCA | Tri. | H.S. | PCA | Tri. | H.S. | PCA | Tri. | H.S. | PCA | Tri. | H.S. |
| Sensitivity    | 1.00 | 0.00 | 0.00 | 0.91 | 0.27 | 0.27 | 0.79 | 0.04 | 0.04 | 0.9 | 0.77 | 0.77 |
| Specificity    | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 0.97 | 0.97 |

**Brain angle injury metric:**

We compute 3DoF relative brain angles using the lumped model proposed in [21] to obtain the maximum relative brain angle as a result of each head impact based on the ground truth kinematics and each of the three approximations. In coronal and axial directions, triangular and half-sine approximations give significantly lower predictions for the brain angle metric, whereas the PCA modes show no statistically significant difference from the ground truth (Figure 8). Next, we calculate approximation errors using Equation 4 and observe substantially smaller errors for the PCA-based approach compared to the two biphasic approximations (Figure 8). Indeed, error of those approximations are more than eightfold that for PCA-based method (with $\sim 3\%$ of error).

Figure 8: Computed brain angle metric (BAM) values for each impact case based on ground truth and the corresponding PCA and biphasic approximations. Error comparison between models with means and SEM of each data set. The significant differences are indicated for $p < 0.01$. 


Tissue deformation-based injury metrics:

Similar to brain angle metric, we calculate the errors between the ground truth strain metrics and those of low-rank approximations (Figure 9). As can be seen, the PCA-based approach closely follows the ground truth simulation results in all three strain metrics: maximum principal strain (MPS) in the whole brain (WB) and corpus callosum (CC), as well as axonal fiber strain (FS) in the corpus callosum. The strains computed for the biphasic models significantly differ from the ground truth values and successively provide higher approximation errors for more region-specific, from fivefold the PCA-method error (of ∼ 3%) and more morphologically-relevant (including axonal fibers) comparisons. Furthermore, based on the 50% concussion risk thresholds, the PCA-based approach provides substantially higher sensitivity and specificity (Table 3).

Figure 9: Simulated strain metrics for ground truth head impact measurements as well as the low-rank PCA approximation and triangle (Tri.) and half-sine (H.S.) biphasic approximations. The blue line shows the 50% risk of mTBI for each metric. Also average and standard error of the mean are showed for strain estimation errors. The significant differences are indicated for p < 0.01.

Table 3: Sensitivity and specificity of strain metrics for maximum principal strain (MPS) for whole brain (WB) and corpus callosum (CC), as well as fiber strain (FS) for CC compared to the ground truth, with respect the threshold of 50% risk of concussion.

|          | MPS WB  |          | MPS CC  |          | FS CC  |          |
|----------|---------|----------|---------|----------|--------|----------|
|          | PCA     | Tri.     | H.S.    | PCA      | Tri.   | H.S.     | PCA      | Tri.   | H.S.   |
| Sensitivity | 0.93   | 0.74     | 0.63    | 0.98     | 0.55   | 0.53     | 0.97     | 0.63   | 0.62   |
| Specificity | 0.99   | 0.99     | 0.99    | 0.99     | 0.99   | 0.99     | 0.99     | 0.94   | 0.95   |

Discussion

In this study we provide a formal approach for reducing the dimensionality of head impact kinematics in contact sports settings. We first derive the most important modes contributing to the head kinematics through principal component analysis and then compare those to existing methods that approximate head kinematics with simple biphasic impulses. We show that the modal decomposition approach can capture the kinematic behavior of the head with better accuracy and provide better approximations of brain deformation and injury. This analysis confirms that although head kinematics during head collisions span a wide range of magnitudes and frequencies [24], using only a relatively small number of modes can capture accurately the impact head kinematics. The low-rank database constructed based on PCA analysis require only 15 modes to
build the ground truth angular velocity kinematics with over 90% accuracy as well as accurately capture the predictive value of head impact kinematics using a variety of injury metrics. A major advantage of our approach is that with the acquired modes above, we are able to emulate a head impact data set with any given number of cases without needing access to the ground truth measurements. This emulated data set would closely replicate head impacts measured by on-field wearable sensors that constitute current state of the art. The advantage of this low-rank emulator, in addition to its computational efficiency, is that it avoids simplifying assumptions for the shape of acceleration impulses.

In contrast, the conventional biphasic assumption for head impacts as simple impulses with only two variables, i.e. acceleration magnitude and duration, falls short in providing accurate estimates. This effect is more pronounced for acceleration impulses that are more variable due to the time derivative but is true for velocity profiles as well. This apparent lack of accuracy in injury prediction in the biphasic approximations might be due to the fact that the biphasic triangle and half-sine signals are built using acceleration signals and then integrated to give the corresponding velocity profiles. Since there is no restitutive deceleration for these impulses, angular velocity eventually becomes constant after the acceleration return to zero, contrary to the actual measured impulses [30]. In the case of kinematics-based injury metrics, differences about false negatives and positives between HIC, RIC and BrIC could be explained by HIC and RIC’s pulse shape dependence. In contrast, BrIC is a function of velocity peaks only and therefore shows a different behavior (Figure 7). In the case of brain angle metrics, the biphasic impulses approximations show five-to-six times larger errors compared to PCA-based impulses. This difference could be attributed to the simplification of the biphasic models that influences the solution of the mechanical lumped-parameter models. This discrepancy seems to affect the coronal direction the most and the sagittal direction the least for the biphasic approximations. Similar to the brain angle results, brain finite element strains showed superior performance by our PCA-based approach. In previous publications, it has been showed that the closer the model is to the correct anatomical and morphological attributes of the brain, the more accurate the injury predictions become [43, 44]. Similarly, we see a worse performance for the biphasic models as we go from maximum principal strains from whole brain to corpus callosum and fiber strains (Table 3).

There are a few other points worth noting based on our analysis. Relative with time domain, we observe differences in kinematics profiles in the three anatomical directions. It seems that the head experiences higher linear accelerations in the anterior-posterior direction (3.54±2.97 m/s²) compared to lateral (2.62±2.25 m/s²) and inferior-superior (3.01±3.44 m/s²) direction, and higher angular accelerations in the sagittal direction (190.53±177.34 rad/s²) compared to coronal (314.50±401.53 rad/s²) and axial (177.34±128.16 rad/s²) directions. This observation might be attributed to the type and direction of loading in the specific activity, e.g. direction of tackling in football, as well as anatomical features such as the neck constraint in those directions [45]. In the frequency domain, there is a low-frequency dominance (10 to 40Hz) in the kinematic response, expressed by ~ 90% of the total kinematic response of angular velocity in the axial plane, ~ 83% for the coronal plane and ~ 74% for the sagittal plane, confirming previous findings on frequency dependence of head impacts [24, 46]. This results could be useful for designing better helmet and other safety devices to avoid brain injury by targeting specific low-frequency range of motion.

In summary, our proposed PCA decomposition approach not only provides a deeper understanding of the head’s response during impacts, it also provides a formal basis for reconstructing
and augmenting head impact kinematics data. Our current emulator is built on the 537 head impacts described; however, another advantage of this approach is that with additional head impact measurements, we can improve the representative PCA modal bases and provide better accuracy. This type of approach might prove necessary given the increased need for larger training data sets in modern machine learning algorithms.

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Supplementary Materials

Figure SM.1: Top: Representative reconstruction of the linear acceleration data for a single head impact. Bottom: Individual and cumulative contribution of the temporal modes derived through PCA for angular velocity in each anatomical direction, constituting over 90% of the ground truth measurements.
Figure SM.2: The three most relevant temporal modes for linear acceleration for the entire dataset.

Table SM.1: $p$-values for the different injury metrics.

| Parameter       | PCA  | Tri. | H.S  |
|-----------------|------|------|------|
| $HIC_{15}$      | 0.972| < 0.001| < 0.001 |
| $RIC_{36}$      | 0.478| < 0.001| < 0.001 |
| $BrIC$          | 0.625| 0.523| 0.607 |
| Coronal BAM     | 0.966| 0.005| 0.001 |
| Sagittal BAM    | 0.968| 0.524| 0.307 |
| Axial BAM       | 0.894| < 0.001| < 0.001 |
| MPS WB          | 0.772| 0.005| < 0.001 |
| MPS CC          | 0.738| < 0.001| < 0.001 |
| FS CC           | 0.790| < 0.001| < 0.001 |
Figure SM.3: Top: Representative reconstruction of the angular acceleration data for a single head impact. Bottom: Individual and cumulative contribution of the temporal modes derived through PCA for angular velocity in each anatomical direction, constituting over 90% of the ground truth measurements.

Figure SM.4: The three most relevant temporal modes for angular acceleration for the entire dataset.