CREDIBLE IDENTIFICATION IN ECONOMIES WITH FRICTIONS
THE ROLE OF SURVEY DATA.

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ABSTRACT. The paper demonstrates that identification in heterogeneous agent economies can be robust to alternative assumptions regarding the underlying structure using identifying restrictions that are consistent with a variety of mechanisms. These restrictions generate moment inequalities and therefore set identification, which can lead to wide confidence sets for the structural parameters. The paper shows that employing aggregated survey data that are informative about the extensive margin of adjustment i.e. the proportion of agents whose behavior is distorted over time due to financial frictions can provide additional information that can tighten the corresponding bounds. The paper provides identification analysis both in an extended partial equilibrium analytical example and in a general equilibrium setting. I apply this approach to the Spanish economy, where the proportion of constrained consumers is identified by combining information from different surveys. The results suggest that the extensive margin is empirically informative.

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1. Introduction

Frictions in dynamic macroeconomic models are undoubtedly a source of heterogeneity, and have appealing theoretical and empirical implications. For example, borrowing constraints may inhibit the ability of individuals to insure fluctuations in income and this generates heterogeneous consumption outcomes, while this can have interesting implications for the persistence of macroeconomic time series.

Nevertheless, the specification of the corresponding mechanism is a complicated process which may involve some degree of arbitrariness which can distort parameter identification and policy conclusions in general.

The paper demonstrates that identification in heterogeneous agent (HA) economies can be robust to alternative assumptions regarding the underlying structure by employing identifying restrictions that are consistent with a variety of mechanisms. These restrictions generate moment inequalities and therefore set identification.

Moment inequality restrictions have been used to characterize frictions in specific markets, see for example Luttmer (1996) and Chetty (2012). To our knowledge, this is the first paper that characterizes such restrictions in dynamic stochastic macroeconomic models with heterogeneity and thus contributes to the literature that deals with partial identification in structural macroeconomic models (e.g. Lubik and Schorfheide (2004); Coroneo, Corradi, and Santos Monteiro (2011)) and the literature on applications of moment inequality models1.

Due to set identification, many models with frictions are likely to be consistent with the robust identifying restrictions. Thus, additional data other than macroeconomic time series can be potentially useful in order to further constrain the set of admissible models.

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1See Pakes, Porter, Ho, and Ishii (2015) and references therein.
Employing aggregated survey data that are informative about the extensive margin of adjustment e.g. the proportion of agents whose behavior is distorted over time due to financial frictions can provide additional information that can tighten the corresponding bounds.

The paper provides identification analysis both in an extended partial equilibrium analytical example and in a general equilibrium setting. I apply this approach to a benchmark heterogeneous agent model of the Spanish economy, where the proportion of constrained consumers is identified by combining information from different surveys. Empirical results suggest that controlling for variations in the extensive margin is indeed informative.

The paper also contributes to the strand of the literature that is related to the intricacies of calibration in representative agents models when the data is generated by heterogeneous agent economies. Browning, Hansen, and Heckman (1999) cast doubt on using microeconomic studies as the mapping to aggregate models is affected by aggregation and sample selection issues. Guvenen (2006) finds that estimates of the elasticity of intertemporal substitution using aggregate data reflect the behavior of poorer agents while Constantinides and Duffie (1996) find that cross sectional heterogeneity is another source of bias. Chang, Kim, and Schorfheide (2013) show that structural parameters are not invariant to policy when heterogeneity is ignored.

This paper’s empirical strategy delivers heterogeneity-consistent estimates that account for the lack of policy invariance. Survey data control both for the extensive margin and for consumption variance, whose variation would otherwise be absorbed by the structural parameters, bridging the gap between micro and macro elasticities.

The rest of the paper is organized as follows. Section 2 provides an extended analysis of the methodology in a partial equilibrium context and the informativeness of the
extensive margin. Section 3 presents the second order approximation of the HA model and the corresponding aggregate identifying restrictions. Section 4 provides identification analysis based limited information. Section 5 presents the empirical study and Section 6 concludes. Appendix A contains the general identification results based on moment inequalities, the proofs and the mixed frequency model used to measure the borrowing constrained and the prior distributions and estimates. Appendix B contains additional derivations and data explanations.

Finally, a word on notation. $T$ signifies the length of aggregate data, and $N$ the number of agents. $\theta \in \Theta$ signifies the parameters of interest with $\Theta_I$ the identified set and $\Theta_{CS}^{\alpha}$ the corresponding $1 - \alpha$ level confidence set. Bold capital letters e.g. $\mathbf{Y}$ denote a matrix containing observations of vector $Y_t$ up to time $\tau$, $\{Y_j\}_{j \leq \tau}$. $\perp$ signifies the orthogonal complement.

2. CREDIBLE IDENTIFICATION - A MOTIVATING EXAMPLE

This section provides an illustrating analytical example on what can be learned by observing household consumption behavior. This will be relevant when we next consider aggregate fluctuations as the arguments are similar.

Assuming an instantaneous utility function $U(c_{i,t}; \omega)$, where $\omega$ signifies preference related parameters, each household receives total income $y_{i,t}(l_{i,t})$ which might depend on labor hours supplied ($l_{i,t}$) and makes consumption, labor supply and savings decisions ($c_{i,t}, l_{i,t}, s_{i,t}$ respectively) taking prices as given. Wealth can be stored in $J$ assets $\{a_{j,t}\}_{j=1,J}$ earning returns $R_{i,t}^j = 1 + r_t^j(a_{t}^j)$, and there is a general constraint technology $g(.)$ that restricts trades between these assets which is increasing in wealth\footnote{This includes popular restrictions e.g. on short selling and non-collateralized borrowing ($a_{t+1}^j \geq -\delta$) and collateralized borrowing ($a_{t+1}^j \geq -\sum_{k \neq j} w_k a_{t+1}^k$) where $w_k$ summarize restrictions on quantities across assets.} while $\nu_{i,t}^j$ is the cost of accessing asset $j$. 


The household problem is as follows:

$$\max \sum_{t=0}^{\infty} \beta^t U(c_{i,t}, l_{i,t}; \omega)$$

s.t. $c_{i,t} - y_{i,t}(l_{i,t}) = \sum_{j=1}^{J} (1 + r_j(a_{i,t}^j))a_{i,t}^j - \sum_{j=1}^{J} a_{i,t+1}^{j} u_{i,t+1}^j$

$c_{i,t} > 0$, $g_i \left( \{a_{i,t}^j; a_{i,t+1}^j\}_{j=1}^{J} \right) > 0$

where we allow for asset level specific rate of return. Denoting the marginal utility of consumption by $U_1(c_{i,t}; l_{i,t}, \omega)$, the Euler equation for any asset $j$ is distorted by the non-negative Lagrange multipliers on the occasionally binding liquidity constraint, denoted by $\mu_{i,t}$, which are positive when the borrowing constraint is binding:

$$U'(c_{i,t}; l_{i,t}, \omega) = \beta \mathbb{E}_t \frac{1}{u_{i,t+1}^j} \left( 1 + r_j(a_{i,t+1}^j) + \frac{\partial r_j(a_{i,t+1}^j)}{\partial a_{i,t+1}^j} a_{i,t+1}^j \right) U'(c_{i,t+1}; l_{i,t+1}, \omega)$$

$$\mu_{i,t} \frac{\partial g_i \left( \{a_{i,t}^j; a_{i,t+1}^j\}_{j=1}^{J} \right)}{\partial a_{i,t+1}^j} + \mathbb{E}_t \mu_{i,t+1} \frac{\partial g_i \left( \{a_{i,t+1}^j; a_{i,t+2}^j\}_{j=1}^{J} \right)}{\partial a_{i,t+1}^j}$$

$$:= \beta \mathbb{E}_t \frac{1}{u_{i,t+1}^j} \left( 1 + r_j(a_{i,t+1}^j) + \frac{\partial r_j(a_{i,t+1}^j)}{\partial a_{i,t+1}^j} \right) U'(c_{i,t+1}; l_{i,t+1}, \omega) + \lambda_{i,t}$$

Given this setup, I next focus on what can be learned from the data in terms of the preference parameters $(\beta, \omega)$ and the unobserved endogenous variable $\lambda_{i,t}$. Since (1) holds for all assets, it suffices to look at the condition for a savings account (or a riskless government bond):

$$U'(c_{i,t}; l_{i,t}, \omega) = \beta (1 + r) \mathbb{E}_t U'(c_{i,t+1}; l_{i,t+1}, \omega) + \lambda_{i,t}$$

There are alternative ways of achieving identification. For example, if cross section or panel data is available, one can split the sample into unconstrained and constrained households using some criterion. In his seminal paper, Zeldes (1989) splits the sample using a cut-off rule on the end of period nonhuman wealth and discusses different
rules that could have been used, as well as liquidity constraints arising from credit market imperfections and illiquid assets as e.g. in Pissarides (1978). The importance of illiquid assets has been recently revived by the heterogeneous agent literature, see for example Kaplan, Violante, and Weidner (2014) and references therein, where they also identify groups of wealthy and poor hand to mouth consumers using different rules. However, this approach cannot be readily applied to time series data, as is the case for aggregate data which is the focus in the rest of the paper. More importantly, alternative choices can lead to sample selection and thus whimsical conclusions on whether liquidity constraints are present. An alternative way is a structural treatment which amounts to employing an estimated income process and compute optimal consumption. One would therefore place much more faith on the technology of the borrowing constraint and the nature of idiosyncratic income risk.

A more credible way to identify \((\beta, \omega)\) is to use the more general implication of borrowing constraints, which would be true across models with different types of heterogeneity and insurance mechanisms that would give rise to different borrowing limit technologies (specifications for \(g_i\)) and therefore \(\lambda_{i,t}^3\). The implication is that liquidity constraints generate a positive discrepancy between current and next period marginal utility of consumption, as the household cannot smooth consumption as much as it desires\(^4\). Conditions in (1) therefore generate conditional moment inequalities, one of which is the case of the riskless asset:

\[
U'(c_{i,t}; l_{i,t}, \omega) = \beta(1 + r)E_t U'(c_{i,t+1}; l_{i,t+1}, \omega)
\]

Using income \(y_{i,t}(\geq 0)\) as an instrument, the following unconditional moment inequality holds:

\(^3\)Notice also that we did not specify a specific process for labor income risk. It can therefore include any kind of individual specific risk to income as well as any kind of transfers.

\(^4\)Similar restrictions also arise in the context of limited commitment and endogenous solvency constraints as in Alvarez and Jermann (2015).
The inequality does not pin down a unique vector of values for $(\omega, \beta)$.

In order to derive explicit identification regions I simplify the model by assuming exogenous income (so labour supply is not relevant) and I adopt the approximation of Hall (1978) using CRRA utility $U(c_{i,t};\omega) := \frac{c_{i,t}^{1-\omega}-1}{1-\omega}$, which implies the following law of motion for consumption:

\begin{equation}
   c_{i,t+1} = \rho c_{i,t} + \epsilon_{i,t+1} + \tilde{\lambda}_{i,t} + \phi_{i,t}
\end{equation}

where $\tilde{\lambda}_{i,t} \equiv -(U''(c_{i,t};\omega))^{-1} \lambda_{i,t}$, $U''(c_{i,t};\omega)$ is the second derivative of the utility function, $\rho = (\beta(1+r)) \frac{U''(c_{i,t};\omega)}{U'(c_{i,t};\omega)} = (\beta(1+r))^\frac{1}{\omega}$, $\epsilon_{i,t+1}$ is the rational forecast error such that $\mathbb{E}_t \epsilon_{i,t+1} = 0$ and $\phi_{i,t}$ is the residual of this approximation which is set to zero for simplicity.\(^5\) Equivalently, consumption growth is equal to

\[ \Delta c_{i,t+1} = (\rho - 1)c_{i,t} + \epsilon_{i,t+1} + \tilde{\lambda}_{i,t} \]

Since higher income relaxes the budget constraint ($\text{Cov}(\tilde{\lambda}_{i,t}, y_{i,t}) < 0$), the identified set for $\rho$ is as follows:

\begin{equation}
   \rho_{ID,1} := \left[ 0, 1 + \frac{\text{Cov}(y_{i,t}, \Delta c_{i,t+1})}{\text{Cov}(y_{i,t}c_{i,t})} \right] = (0, 1 + \rho_{IV})
\end{equation}

Avoiding to take a stance on the exact nature of $\lambda_{i,t}$, which can involve a vast amount of unobserved information, results in set identification. Nevertheless, we are still able to infer certain facts about the household’s preferences and economic behavior and conclusions are valid across alternative environments.

2.1. Bounds on Risk Aversion. Given the set of admissible values for the reduced form coefficient $\rho$, we can recover the implied bounds for the risk aversion parameter,$^5$

\(^5\)In a recent paper Commault (2019) challenged Hall (1978)'s random walk result for consumption in the case of isoelastic utility. In the Appendix I clarify why the identification analysis is unaffected by this simplification; in a nutshell, $\phi_{i,t}$ has very similar properties to $\tilde{\lambda}_{i,t}$.  

\[ \mathbb{E}[(U'(c_{i,t};l_{i,t},\omega) - \beta(1+r)U'(c_{i,t+1};l_{i,t+1},\omega))y_{i,t}] \geq 0 \]
Treating \((r, \beta)\) as known, if \(\beta(1 + r) = 1\), risk aversion is unidentified \((\omega_{ID} = \mathbb{R}^+)\) as consumption follows a random walk. If \(\beta(1 + r) \neq 1\), then risk aversion is set identified and has a very intuitive interpretation, as it reflects restrictions on preferences implied by the presence of non-diversifiable income risk. In particular, for \(\beta(1 + r) < 1\) the household is impatient and does not accumulate wealth indefinitely. Using (5), since income and consumption growth are negatively correlated see e.g. (Deaton, 1991), risk aversion is bounded by above and the set of values consistent with the data is:

\[
\omega_{ID, 1} = \left\{ \omega \in \mathbb{R}^+ : \omega < \frac{\| \log(\beta(1 + r)) \|}{\log \left( \frac{\text{Cov}(y_{i,t}c_{i,t})}{\text{Cov}(y_{i,t}c_{i,t}) - |\text{Cov}(y_{i,t}\Delta c_{i,t+1})|} \right)} \right\}
\]

The stronger the negative correlation, the lower the upper bound on risk aversion, indicating that the less risk averse household is not accumulating enough wealth to fully insure against income risk\(^6\).

The next subsection illustrates the additional identifying information qualitative data can provide.

**The extensive margin as additional information.** Suppose that we observe the dichotomous response of the household over time to a survey question that asks whether the household is (or expects to be in the near future) financially constrained. An honest household will answer positively whenever \(\lambda_{i,t} > 0\). In order to analyze the implications of observing \(\tilde{\chi}_{i,t} = \mathbf{1}(\lambda_{i,t} > 0)\), I next characterize the form of \(\lambda_{i,t}\) had it been generated by the model in (2) e.g. a restriction on borrowing.

For analytical tractability, consumption is approximated by a piece-wise function. When the household is constrained, consumption is equal to cash on hand, \(c_{i,t}^{\text{con}} = \)

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\(^6\)In the language of Arellano, Hansen, and Sentana (2012) under-identification depends on this correlation, which depends on \((\omega, \beta, r)\).
\( x_{i,t} := y_{i,t} + (1 + r)a_{i,t} \), while when income is not autocorrelated, unconstrained consumption is equal to \( c_{i,t}^{unc} = \frac{(1+r-\rho)}{1+r}x_{i,t} \).

Given the approximation, it can be shown that consumption growth under incomplete markets becomes as follows:

\[
\Delta c_{i,t+1} = (\rho - 1)c_{i,t} + \epsilon_{i,t+1} + 1(\lambda_{i,t} > 0) \left( -\rho c_{i,t} + \bar{y}_i \left( 1 - \frac{\rho}{1+r} (1 - F_{y_i}(x_i^*)) \right) \right)
\]

where \( F_{y_i}(x_i^*) \) is the the probability of hitting the constraint. The higher this probability is, the higher the savings rate out of income, increasing thereby consumption growth.

For the analysis that follows, this is treated as the true model. Had we known the mechanism that generates this distortion we could control for the occasionally binding constraint using the interaction term \( 1(\lambda_{i,t} > 0) (\lambda_1 c_{i,t} + \lambda_{0,i}) \). This would restore point identification of \( \rho, \omega \) and \( \mathbb{E}\lambda_{i,t} \). I instead examine what can be learned from the data without explicit knowledge of the mechanism. First, responses \( \{x_{i,t}\}_{t < N} \) can be used to estimate \( \mathbb{P}_t(\lambda_{i,t} > 0) \). Therefore, for \( v \in \mathbb{R} \) and \( \epsilon_{i,t+1} \) with cumulative density \( \Phi_{i,t}(\cdot) \), the distribution function of \( \Delta c_{i,t+1} \) is:

\[
\mathbb{P}_t(\Delta c_{i,t+1} < v) = \Phi_{i,t}(v - (\rho - 1)c_{i,t}) \mathbb{P}_t(\lambda_{i,t} = 0) + \Phi_{i,t}(v - (\rho - 1)c_{i,t} - \lambda_{i,t}) \mathbb{P}_t(\lambda_{i,t} > 0) \\
\leq \Phi_{i,t}(v - (\rho - 1)c_{i,t}) \mathbb{P}_t(\lambda_{i,t} = 0) + \Phi_{i,t}(v - \tilde{\rho}_{IV} c_{i,t}) \mathbb{P}_t(\lambda_{i,t} > 0)
\]

where we use the only information available: that is \( \lambda_{i,t} > 0 \), and that the IV regression estimate of \( \rho - 1 \) in (6), \( \tilde{\rho}_{IV} \), is a lower bound.

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7Note that the assumption that income is not autocorrelated is done to simplify the argument and its exposition and has absolutely no bearing on its substance.

8See the Appendix for the derivation.

9To see this, notice that \( \tilde{\rho}_{IV} = \rho - 1 + (\text{Cov}(c_{i,t}, y_{i,t})^{-1} \text{Cov}(y_{i,t}, \lambda_{i,t} | \lambda_{i,t} > 0) \mathbb{P}(\lambda_{i,t} > 0)) < \rho - 1 \). The bias term is negative as higher income relaxes the constraint and can be verified by the solution of the model, as \( \lambda_1 = -\rho \).
How does this additional restriction refine the set of admissible estimates for the risk aversion parameter implied by (5)? The simplest way to show such a refinement is to show that there exists parameter value in $\rho \in \mu_{ID,1}$ that is not consistent with (7). In Appendix A, it is shown that using $\tilde{\rho}_{IV}$ as the "test point", (7) is not satisfied when

$$P_t(\lambda_{i,t} > 0) \in (g_{i,t}, 1)$$

where

$$g_{i,t} := \frac{\Phi_{i,t}(v - (\rho - 1)c_{i,t}) - \Phi_{i,t}(v - \tilde{\rho}_{IV}c_{i,t})}{\Phi_{i,t}(v - (\rho - 1)c_{i,t}) - \Phi_{i,t}(v - \lambda_{0,t} - (\rho - 1 + \lambda_1)c_{i,t})}$$

The upper bound in (5) is no longer admissible. To gain intuition, consider the extreme cases where $P_t(\lambda_{i,t} > 0)$ is zero or one. Condition (8) becomes invalid as (0, 1) and (1, 1) do not contain $P_t(\lambda_{i,t} > 0)$. In either case, additional survey information is redundant. When liquidity constraints are occasionally binding, survey information matters as long as (8) is a proper interval. Figure 1 below provides an example when $\Phi_{i,t}$ is the cumulative Normal distribution.

**Figure 1.** Range for $v = 0, c_{i,t} = 1, r = 0.05, \sigma_e = 0.1, \bar{y} = 0$
For higher values of the intensive distortion $\lambda_1$, e.g. $2\lambda_1$, concavity increases, and the admissible range (grey shade) becomes larger. The more severe the distortions, the more likely is that additional information matters. Conversely, in the frictionless limit, $g_{i,t} \xrightarrow{\lambda_i \to 0} P_t(\lambda_{i,t} > 0)$, contradicting (8). Survey data can provide additional information that can narrow down the set of plausible values for $\rho, \omega$.

2.2. Discussion.

2.2.1. Soft Constraints. The model setup employed in this section is consistent with many environments, including those in which the household can face a different interest rate for a different level of assets, given by $\hat{\beta}_j(a_{i,t+1}^j)/a_{i,t+1}^j$ in the smooth case. If we examined the condition for a different asset e.g. credit line which also involved higher interest rates at higher levels of unsecured borrowing, then identification is restored only if we observe the policy that relates interest rates to the level of borrowing. Otherwise we are still in the world of set identification, as $\hat{\beta}_j(a_{i,t+1}^j)/a_{i,t+1}^j$ becomes an unobservable variable as well, which has the same properties as $\lambda_{i,t}$. In this case, the distortion to the Euler equation does not have to do with the insurance mechanism itself, but rather with the fact that part of the marginal increase in the cost is not observed. Nevertheless, the same moment inequality is valid, while survey data will be informative as long as the household is constrained with positive probability.

2.2.2. Alternative Survey Questions. Surveys that report households that have applied for a loan and have been rejected are robust to alternative assumptions about the economic environment. In fact, such questions also ask for the reason for which these households have been rejected, and these reasons vary depending on individual characteristics, employment, guarantees, changes in the institution’s credit policy, excessive debt etcetera\textsuperscript{10}. We do not claim that this is the only survey question that

\textsuperscript{10}This is the case for example in the Spanish Survey of Household Finances.
matters, but the information that is key here is the one that identifies when the individual is likely to be constrained or not, as this data provides more information than the moment inequality that only captures average behavior and is consistent with being constrained for e.g. 99% or 1% of the time. Such data will also be informative when we consider aggregate fluctuations, as this probability will describe the mass of financially constrained agents over time and will be related to variations in consumption heterogeneity and precautionary effects.

3. The second order approximation and Aggregate Restrictions

The above example investigated identification from the perspective of observing a single household. This section characterizes the corresponding restrictions using macroeconomic data. Since identification is based on moment conditions, we do not need to specify the rest of the economy to identify the structural parameters of interest. In fact, one can focus on a few moment conditions that are the aggregate counterparts of e.g. condition (2).

Researchers have recently employed a second order approximation to the heterogeneous agent model around the representative agent allocation as a tractable way of analyzing its aggregate implications (Debortoli and Gali, 2018; Tryphonides, 2020)\textsuperscript{11}. This paper will also follow this route. In particular, looking at marginal utility of consumption in the CRRA case:

\[ c_{i,t}^{-\omega} \approx C_t^{-\omega} - \omega C_t^{-\omega-1}(c_{i,t} - C_t) + \frac{\omega(\omega + 1)}{2} C_t^{-\omega-2}(c_{i,t} - C_t)^2 \]

\textsuperscript{11}For example Debortoli and Gali (2018) show that a simple two-agent New Keynesian (TANK) model which abstracts completely from heterogeneity within unconstrained agents, captures reasonably well the implications of a baseline HANK model regarding the effects of aggregate shocks on aggregate variables. Tryphonides (2020) evaluates the quality of this approximation and utilizes it to identify how heterogeneity and the extensive and intensive margins of adjustment depend on aggregate shocks.
In order to aggregate (2), let \( s_{i,t} \equiv (\nu_{i,t}, y_{i,t}, \{ a_{i,t,1}^{j}, a_{i,t,1+1}^{j} \}_{j=1}^{J}) \) and \( S_t \) be the vector that includes all aggregate states, including the distribution of \( s_{i,t} \). Aggregating the bond Euler equation using \( p(s_{i,t}|S_t) \), and using that agent expectations are formed using \( p(s_{i,t+1}, S_{t+1}|s_{i,t}, S_t) \), the approximate aggregated first order condition for bonds is equal to:

\[
\Xi_t C_{t}^{-\omega} = \beta E_t C_{t+1}^{-\omega} \Xi_{t,t+1} R_{t+1} + \int \lambda_{i,t} p(s_{i,t}|S_t) ds_{i,t}
\]

\[
= \beta E_t C_{t+1}^{-\omega} \Xi_{t,t+1} R_{t+1} + \mu_t
\]

where

\[
C_{t+1} = \int c_{i,t+1} p(s_{i,t+1}, s_{i,t}|S_{t+1}, S_t) d(s_{i,t+1}, s_{i,t}), \quad C_t = \int c_{i,t} p(s_{i,t}|S_t) ds_{i,t}
\]

\[
\Xi_{t} \equiv 1 + \frac{\omega(\omega + 1)}{2} C_t^{-2} Var_t(c_{i,t}), \quad \Xi_{t,t+1} \equiv 1 + \frac{\omega(\omega + 1)}{2} C_{t+1}^{-2} Var_t(c_{i,t+1})
\]

Since \( \lambda_{i,t} \) is weakly positive by construction, the aggregate distortion to the bond Euler equation will be a weakly positive random variable as well.

If data on the cross sectional variance of consumption is also available to the econometrician, it immediately follows that -using any instrument that is positive e.g. an aggregate random variable \( X_{t-1} \)- gives rise to the following moment inequality, which is also consistent with a general equilibrium response in \( R_t \)^12:

\[
E \left( C_{t}^{-\omega} - \beta C_{t+1}^{-\omega} \left( \frac{\Xi_{t,t+1}}{\Xi_t} \right) R_{t+1} \right) X_{t-1} \geq 0
\]

From the above condition it is clear that there are two ways in which heterogeneity matters for aggregate fluctuations to first order. The first is fluctuations in the cross sectional variance, \( \Xi_t \). The relevance of fluctuations in the variance of consumption is best described by ignoring momentarily the presence of constrained consumers. In

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^12In a closed economy, this will put downward pressure on the interest rate which then relaxes the borrowing constraint for some of the agents. Even if all the agents become unconstrained then the weak inequality is still valid.
this case, the (log-linearized) bond equation reads as follows:

$$\tilde{C}_t = \mathbb{E}_t \tilde{C}_{t+1} - \frac{1}{\omega} \mathbb{E}_t \left( \tilde{R}_{t+1} + (\tilde{\Xi}_{t+1} - \tilde{\Xi}_t) \right)$$

Growth in consumption risk increases aggregate savings and equilibrium allocations are different even if all agents are "on their Euler equations". Measures of cross sectional consumption variance are therefore important to account for fluctuations in heterogeneity even if no household reports financial constraints.

Since partial insurance is the most likely result a priori, controlling only for consumption risk might not be enough to capture distortions to aggregate consumption. The next section analyzes what we learn about the aggregate distortions from combining aggregated survey data, quantitative and qualitative.

4. IDENTIFICATION WITH LIMITED INFORMATION

As illustrated in the previous section, the HA model can be approximated around the RA allocation, which results in distortions that are partially observable. Denote by $Y^f_t$ the representative agent (frictionless) model prediction,

$$Y_t = Y^f_t + \lambda^o_t + \lambda^{un}_t$$

where $\lambda^o_t$ is the observable component (up to the parameter vector $\theta$) of the aggregate distortion and $\lambda^{un}_t$ the unobservable component. For example, in the previous section, $Y_t = C_t^{-\omega}, Y^f_t \equiv \beta \mathbb{E}_t C_{t+1}^{-\omega} R_{t+1}, \lambda^o_t \equiv \mu_t$ and $\lambda^{un}_t \equiv \beta \mathbb{E}_t \left( \frac{\tilde{X}_{t+1} - \tilde{X}_t}{\tilde{X}_t} - 1 \right) R_{t+1}$.

The only theoretical restriction on $\mu_t$ is its positive sign. In the absence of any additional information, theoretical or empirical, and assuming additive separability in $\lambda^o_t$, we could utilize the following moment inequality to estimate the structural parameters, where $X_{t-1}$ is any positive random variable which is $Y_{t-1}$ measurable:

$$\text{sign} \left( \mathbb{E} \left( g(Y_t, Y_{t+1}, \theta) - \lambda^o_t(Y_t, Y_{t+1}, \theta) \right) X_{t-1} \right) = \text{sign} \left( \mathbb{E}(\mu_t X_{t-1}) \right)$$
where \( g(Y_t, Y_{t+1}, \theta) := Y_t - Y_{t+1} \). This restriction is unlikely to point identify \( \theta \), and could result in a quite wide confidence set. Yet, \( \mu_t \) is a quite complicated object; abstaining from imposing further restrictions results in more credible inference.

The question is however whether we could do better using additional information that does not undermine the credibility of the assumptions we have already done. One way of achieving this is to notice that the latent \( \mu_t \), while it depends on a lot of information, its evolution over time can be explained by either changes in the proportion of agents whose behavior is distorted, the extensive margin, and by how much they distort their behavior, on average. More particularly, we can decompose \( \mu_t \) into two multiplicative components:

\[
\mu_t = \mathbb{E}_t(\mu_{i,t} | \mu_{i,t} \neq 0)\mathbb{P}_t(\mu_{i,t} \neq 0) \equiv \kappa_t B_t
\]

where \( \kappa_t \) is the intensive margin and \( B_t \) the extensive margin. These margins can always be defined for a broad range of models and are consistent with a lot of heterogeneity. If we could find a way to measure either margin, e.g. \( B_t \), then this could be utilized to sharpen inference as it generates a new set of moment inequalities, where we utilize that \( B_t \) and \( \kappa_t \) have the same sign:

\[
(12) \quad \text{sign} \left( \mathbb{E}(g(Y_t, Y_{t+1}, \theta) - \lambda_t^i(Y_t, Y_{t+1}, \theta)) X_{t-1} B_t^{-1} \right) = \text{sign} \left( \mathbb{E}(\kappa_t X_{t-1}) \right)
\]

Obviously, when \( B_t \) is zero for all \( t \), then condition (12) collapses to the standard moment equality restriction. The question is nevertheless whether (12) yields more identifying power than (11), which is not guaranteed a priori, since as illustrated in the motivating example in Section 2, it depends on the data generating process.

4.0.1. Identification Analysis. While I briefly lay out the identification framework based on moment inequalities, in Appendix A I establish the general conditions under

\[\text{As I will illustrate in the empirical application, combining information from different surveys is one fruitful way.}\]

\[\text{Appendix A includes a more formal proof.}\]
which additional moment inequalities are informative for the parameters of interest. First, conditions like (11) and (12) can be re-expressed as moment equalities:

\[
E \mathbf{g}_r(Y_t, Y_{t+1}, \theta) - \lambda^o_r(Y_t, Y_{t+1}, \theta) = E \mu_t \phi(Y_t - 1) = U \in [0, \infty)
\]

where \( \phi(.) \) is an inequality preserving function of \( Y_t - 1 \), \( r \) is the number of conditions and \( U \) is a vector of nuisance parameters that restore the moment equality. From an economic point of view, the set of model completions is simply the set of all possible mechanisms that generate the wedge \( \mu_t \) such that \( U \geq 0 \). The identified set is thus defined as follows:

\[
\Theta_I := \left\{ \theta \in \Theta : \exists U \in \mathbb{R}^+ : E \left( m(Y_t, Y_{t+1}, \theta) \phi(Y_t - 1) \right) - U = 0 \right\}
\]

where \( m(Y_t, Y_{t+1}, \theta) := (g(Y_t, Y_{t+1}, \theta) - \lambda^o_t(Y_t, Y_{t+1}, \theta)) \).

The next proposition characterizes how informative \( B_t \) can be within the context of limited information moment restrictions. If it constrains the stochastic properties of \( \mu_t \) as implied by (11), then the size of \( \Theta_I \) is likely to be refined.

**Proposition 1. Identification with \( B_t \).** Given Theorem 2 in Appendix A,

1. If \( \hat{B}_t \xrightarrow{N \rightarrow \infty} B_t \in (0, 1) \), \( \Theta'_I \subset \Theta_I \).
2. If \( \text{Var}(\hat{B}_t) \xrightarrow{N, T \rightarrow \infty} \text{Var}(B_t) = 0 \), then \( \Theta'_I = \Theta_I \).
3. Impossibility of point identification: When \( B_t \neq 0 \), \( \Theta_I \) is not a singleton.

*Proof. See Appendix* □

\( B_t \in (0, 1) \) is essential for shrinking the admissible set of predictions. Moreover, absence of time variation in the proportion of agents that are constrained i.e. if the economy reaches the stationary state, implies that \( B_t \) has no additional information for the parameters of the model.
Other micro moments. The advantage of the second order approximation to the HA model is that it makes clear which moments are likely to be important. In this particular case, the micro moment that is necessary for identifying the structural parameters of interest is the cross sectional variance of consumption. Since consumption is a nonlinear function of wealth, this is equivalent to using the corresponding moment from the wealth distribution. On the other hand, sharpening inference without compromising credibility requires information on $B_t$ and $\kappa_t$. In the application that follows, we will combine information from different sources to identify $B_t$.

5. The case of Spain

This section applies the above analysis to the case of Spain, as it is a country in which financial frictions are a priori expected to have played a significant role post 1999, and especially during 2008-2014.

I will employ moment conditions that correspond to the model in (1) where I also allow for external habit formation e.g. "catching up with the Joneses" effects and disutility in hours worked which is separable from utility in consumption. The employed individual utility function is therefore as follows:

$$u(c_{i,t}, l_{i,t}, C_t) := \frac{(c_{i,t} - hC_{t-1})^{1-\omega} - 1}{1 - \omega} - \frac{l_{i,t}^{1+\eta}}{1 + \eta}$$

The two individual first order conditions that are relevant for identification are the Euler equation and the intratemporal labor supply condition:

$$(c_{i,t} - hC_{t-1})^{-\omega} = \beta E_t (c_{i,t+1} - hC_t)^{-\omega} R_{t+1} + \lambda_{i,t}$$

$$p_{i,t}^l = \frac{W_t}{P_t} c_{i,t}^{-\omega}$$

where we have used that $\frac{\partial y_{i,t}(l_{i,t})}{\partial l_{i,t}} = \frac{W_t}{P_t}$, that is, the derivative of total individual income with respect to hours worked is the real wage. The parameters of interest are
therefore \((\omega, \beta, \eta, h)\), the coefficient of risk aversion, the discount factor, the (inverse) Frisch elasticity of labor supply and the habit parameter respectively. The corresponding aggregated moment conditions (without \(B_t\)) are as follows\(^\text{15}\):

\[
\mathbb{E} \left( (C_t - hC_{t-1})^{-\omega} - \beta(C_{t+1} - hC_t)^{-\omega} \left( \frac{\Xi_{t+1}^{1}}{\Xi_{t}^{1}} \right) \pi_{t+1}^{-1}R_{t+1} \right) Y_{t-1} \geq 0
\]

(15)

\[
\mathbb{E} \left( L_t - w_t^{1/\eta} (C_t - hC_{t-1})^{-\eta} \Xi_{lab}^{1/\eta} \right) Y_{t-1} = 0
\]

(16)

where

\[
\Xi_t = 1 + \frac{\omega(1 + \omega)}{2} \left( 1 - h \left( \frac{C_{t-1}}{C_t} \right) \right)^{-2} V_t^c
\]

and

\[
\Xi_{lab}^{1/\eta} = 1 + \frac{\omega(\eta + \omega)}{2\eta^2} \left( 1 - h \left( \frac{C_{t-1}}{C_t} \right) \right)^{-2} V_t^c
\]

where \(V_t^c := \text{Var}_t \left( \frac{C_{t-1}}{C_t} \right) \).

5.0.1. Measurement issues. As evident from the analysis in the previous section, being able to measure \(B_t\) improves identification and it is therefore useful to first discuss pertinent measurement issues and how they can be addressed. The exact measure of \(B_t\) is computed from the Survey of Household Finances (SHF) and its corresponding 95% confidence bands are plotted in Figure 2. This measure asks households whether they have been denied a loan application during this period\(^\text{16}\) but is nevertheless available every three years. In order to obtain a quarterly measure, I utilize monthly data from the Business and Consumer Survey (BCS) of the European Commission\(^\text{17}\).

In particular, the survey asks the households about their financial situation, and the possible answers range from "We are saving a lot" to "We are running into debt". In

---

\(^{\text{15}}\)See Appendix B for the derivations.

\(^{\text{16}}\)Accounting for consumers that have not applied for loans because they expect to be rejected (also coined as "discouraged borrowers" in Cox and Jappelli (1993) ) or consumers that were granted a fraction of the requested loan does not affect the estimate of this proportion.

\(^{\text{17}}\)BCS data can be downloaded from here.
Figure 2, I plot the proportion that corresponds to the latter category (those that claim to be getting indebted), which increased from 5% in 2008 to 10% by the end of 2016. This is an upper bound to the true fraction of households that are likely to face borrowing constraints within the quarter, as some of those (already into) getting into debt may (have been) be denied a loan application, partially or in total.

To infer the true proportion of constrained households for the periods in which it is not observed, so all periods between the triennial measure from SHF, I employ a mixed frequency model, where the quarterly observations on consumers running into debt are linked to the triennial exact measure.

Letting $B_t$ be the exact measure and $\Pi_t$ the proportion of consumers running into debt, then $B_t = \zeta_t \Pi_t$ where $\zeta_t \in (0, 1)$, as the agents that hit the minimum level of permissible debt are a fraction of the total measure of indebted households. As shown in the appendix, this serves as a measurement equation in a mixed frequency state space model. The extracted measure is plotted in Figure 2.

Since this measure is primarily informed from the SHF survey, one might still be concerned that the latter might under-state the true measure of constrained households.
As I have argued in 2.2.2, the SHF measure is consistent with a variety of underlying reasons for which credit was denied and in the state-space formulation we used, we control for measurement error. More importantly, notice that even if \( \hat{\mathbf{B}}_t \rightarrow \hat{B}_t \leq B_t \), the moment inequality restrictions are still valid as:

\[
\begin{align*}
    m_t \hat{B}_t^{-1} & \geq m_t B_t^{-1} \\
\end{align*}
\]

\[
\begin{align*}
\mathbb{E}(m_t \hat{B}_t^{-1} X_{t-1}) & \geq \mathbb{E}(m_t B_t^{-1} X_{t-1}) \geq 0
\end{align*}
\]

where \( m_t := \left( (C_t - hC_{t-1})^{-\omega} - \beta \mathbb{E}_t(C_{t+1} - hC_t)^{-\omega} \left( \frac{\Xi_{t+1}}{\xi_t} \right) \pi_{t+1}^{-1} R_{t+1} \right) \).

Finally, another variable that needs to be measured from micro data is consumption dispersion \( V_{c,t} \). I construct two different measures, one from the Survey of Household Finances (SHF) and one from the Household Budget Survey (HBS)\(^{18}\). In order to impute the value of \( V_{c,t} \) when it is latent, I make the simplifying assumption that it is constant. This is not likely to be a detrimental assumption for this kind of exercise as consumption is smoother than income, and its distribution is a slowly moving variable\(^{19}\).

**Estimation.** In the estimation results that follow, I examine whether additional moment conditions generated by utilizing the measure of \( B_t \) are informative for structural parameter estimates obtained using limited information methods, providing therefore empirical support to Proposition 1. I compare the confidence sets for all the structural parameters when using the extensive margin as additional information and when not using it, for the two measures of cross sectional consumption variance.

\(^{18}\)HBS Data can downloaded from [here](#) and SHF data from [here](#). Please consult the appendix for a detailed explanation of how the measure is constructed.

\(^{19}\)See for example in [Anghel (2018)](#) (Figure 13).
The table below presents the (projections) of the 95% joint confidence set obtained using quantiles from the posterior draws of the objective function as in Chen, Christensen, and Tamer (2018) while in Appendix B I specify the prior distributions.

|                | with B - SHF | without B - SHF | with B - HBS | without B - HBS |
|----------------|--------------|-----------------|--------------|-----------------|
| \( q_{2.5\%} \) | 0.3421       | 0.2514          | 0.5312       | 0.2232          |
| \( q_{97.5\%} \) | 15.6193      | 18.5916         | 15.2901      | 18.5544         |
| \( q_{2.5\%} \) | 0.0139       | 0.0044          | 0.0266       | 0.0077          |
| \( q_{97.5\%} \) | 2.3734       | 4.1882          | 1.9627       | 4.2942          |
| \( \omega \)    | 0.9800       | 0.9800          | 0.9800       | 0.9800          |
| \( \eta \)      | 0.0001       | 0.0001          | 0.0000       | 0.0000          |
| \( \beta \)     | 0.1336       | 1.0000          | 0.9850       | 1.0000          |
| \( h \)         | 0.9113       | 0.9585          | 0.8862       | 0.9510          |

As expected, the moment inequality approach delivers wide confidence sets, that is, credibility comes at a cost. Nevertheless, what is really important here is that information from micro-survey data does matter. Accounting for the constrained agents shrinks the confidence sets across all cases. As in the introductory example, the set of admissible values becomes smaller. Had we neglected the extensive margin of constrained consumers smaller values of risk aversion would be admissible. Impatient agents can drive the equilibrium rate upwards, which validates a lower upper bound on \( \beta \) since \( \beta < R^{-1} \). Moreover, the upper bound on \( \eta \) becomes smaller. The smallest admissible value for the Frisch elasticity becomes higher when we control for the extensive margin. If all agents are constrained, then aggregate consumption is very responsive to fluctuations in the wage, which implies that fluctuations in employment are driven primarily by the wealth effect. If only a fraction of consumers are constrained, the substitution effect is more likely to dominate, leading to larger admissible values for the Frisch elasticity.

6. Conclusion

This paper has shown that within the context of economies with financial frictions, the identification of structural parameters can be robust to alternative mechanisms.
that can generate these frictions and the corresponding heterogeneity in individual outcomes. While set identification can result in wide confidence sets, the paper shows that additional distributional information can be useful in shrinking the identified set. Nevertheless, identification of the micro-moments that are required to make inference more precise is not void of challenges as micro data is usually available at much lower frequencies than macroeconomic data. The paper has explored one way of dealing with this by exploiting information from different surveys in a mixed frequency setting to identify the borrowing constrained consumers. Empirically, this information seems to matter for all structural parameters.
7. APPENDIX A

7.1. Constrained versus Unconstrained Consumption. Notice that for large values of \( x_t \), \( C_t^{inc} \) and \( C_t^{per,for} \) become parallel, as expected.

![Figure 3. Example of Optimal Consumption in Incomplete Markets](image)

**Figure 3.** Example of Optimal Consumption in Incomplete Markets

7.2. Deriving Consumption Growth. When the household is constrained, \( c_{i,t}^{con} = x_{i,t} \) while unconstrained consumption is equal to \( c_{i,t}^{unc} = \frac{(1+r-\rho)}{1+r} x_{i,t} \). The latter is parallel to the perfect foresight solution, \( c_{i,t}^{pf} = (1 + r - \rho) a_{i,t} + \frac{1 + r - \rho}{1 + r} (y_{i,t} + \frac{\bar{y}_t}{r}) = \frac{(1+r-\rho)}{1+r} (x_{i,t} + \frac{\bar{y}_t}{r}) \) where \( \frac{(1+r-\rho)}{1+r} \bar{y}_t \) is interpreted as the share of permanent income \( \bar{y}_t \equiv \mathbb{E}y_{i,t} \) that is channeled to precautionary savings\(^{20}\). Using the constrained agent’s Euler equation, it has to be the case that \( \rho x_{i,t} = \mathbb{E}(c_{i,t+1} | \lambda_{i,t} > 0) - \lambda_{i,t} \).

Expectations include the possibilities of being constrained or not in \( t + 1 \), thus:

\[
\rho x_{i,t} = \mathbb{E}(c_{i,t+1} | \lambda_{i,t} > 0) - \lambda_{i,t} \\
= \mathbb{E}(x_{i,t+1} | \lambda_{i,t} > 0) \mathbb{P}(\lambda_{i,t+1} > 0 | \lambda_{i,t} > 0)
\]

\(^{20}\)See Carroll (2001) and Figure 3 for an example of a numerical solution that verifies this.
\[
+ \left(1 - \frac{\rho}{1 + r}\right) \mathbb{E}(x_{i,t+1}|\lambda_{i,t} > 0) \mathbb{P}(\lambda_{i,t+1} = 0|\lambda_{i,t} > 0) - \lambda_{i,t}
\]

\[
= \mathbb{E}(x_{i,t+1}|\lambda_{i,t} > 0) \left[1 - \frac{\rho}{1 + r} \mathbb{P}(\lambda_{i,t+1} = 0|\lambda_{i,t} > 0)\right] - \lambda_{i,t}
\]

\[
= \bar{y}_i \left[1 - \frac{\rho}{1 + r} \mathbb{P}(\lambda_{i,t+1} = 0|\lambda_{i,t} > 0)\right] - \lambda_{i,t}
\]

\[
= \bar{y}_i \left[1 - \frac{\rho}{1 + r} (1 - F_{y_i}(x_i^*))\right] - \lambda_{i,t}
\]

where in the last two lines we use that cash on hand in \(t + 1\) if constrained at \(t\) is equal to \(y_{i,t+1}\) whose expected value is \(\bar{y}_i\), while the probability of being constrained in \(t + 1\) is \(F_{y_i}(x_i^*)\), \(x_i^*\) being the cash on hand threshold. Thus, \(\lambda_{i,t} = -\rho x_{i,t} + \bar{y}_i \left[1 - \frac{\rho}{1 + r} (1 - F_{y_i}(x_i^*))\right]\) and

\[
\Delta e_{i,t+1} = \bar{c}_{i,t} + \epsilon_{t+1} + 1(\lambda_{i,t} > 0) \left(-\rho c_{i,t} + \bar{y}_i \left(1 - \frac{\rho}{1 + r} (1 - F_{y_i}(x_i^*))\right)\right)
\]

\[
:= \bar{c}_{i,t} + \epsilon_{t+1} + 1(\lambda_{i,t} > 0) (\lambda_1 c_{i,t} + \lambda_0, i)
\]

7.3. Alternative Derivation to Hall (1978). Following Commault (2019), consider the Euler equation:

\[
U'((\beta(1 + r))^{\frac{1}{\beta}} c_{i,t}) = \mathbb{E}_t U'(c_{i,t+1}) + (\beta(1 + r))^{-1} \lambda_{i,t}
\]

\[
(\beta(1 + r))^{\frac{1}{\beta}} c_{i,t} = U^{-1} \left(U'(\mathbb{E}_t c_{i,t+1} - \phi_{i,t}) + (\beta(1 + r))^{-1} \lambda_{i,t}\right)
\]

where \(\phi_{i,t} > 0\) if marginal utility is convex. A first order approximation of the RHS around \(\lambda_{i,t} = 0\) yields:

\[
(\beta(1 + r))^{\frac{1}{\beta}} c_{i,t} \approx \mathbb{E}_t c_{i,t+1} - \phi_{i,t} - \tilde{\lambda}_{i,t}
\]

where \(\tilde{\lambda}_{i,t} = -\frac{(\beta(1 + r))^{-1} \lambda_{i,t}}{U''((\beta(1 + r))^{\frac{1}{\beta}} c_{i,t})} > 0\). The distortion to random walk comes from both the precautionary savings effect and occasionally binding constraint. Repeating the analysis in 7.2 yields that:

\[
\lambda_{i,t} + \phi_{i,t} = \bar{y}_i \left[1 - \frac{\rho}{1 + r} (1 - F_{i,y}(x_i^*))\right] - \rho x_{i,t}, \quad \text{and}
\]
\[
\Delta c_{i,t+1}^{inc} = \tilde{\rho} c_{i,t} + \epsilon_{t+1} + 1(\lambda_{i,t} > 0) \left(-\rho c_{i,t} + \bar{y}_t \left(1 - \frac{\rho}{1 + r} (1 - F_{y_i}(x^*))\right)\right) + 1(\lambda_{i,t} = 0) \phi_{i,t}
\]

Finally, the bound derived in Section 2.1 is identical because \(\dot{\phi}_{i,t}\) and \(\text{Cov}(\phi_{i,t}, y_{i,t})\) have the same sign as \(\lambda_{i,t}\) and \(\text{Cov}(\lambda_{i,t}, y_{i,t})\) respectively.

7.4. Proofs for results in main text.

Proof. of Result (8)

Let \(p_{i,t} := \mathbb{P}(\lambda_{i,t} > 0)\). If \(\tilde{\rho}_{IV}\) is admissible, evaluating the LHS of (7) at the true \(\tilde{\rho} = \rho - 1\) and the RHS at \(\tilde{\rho}_{IV}\), and solving for \(p_{i,t}\):

\[
p_{i,t} \leq \frac{\Phi_{i,t}(v - \tilde{\rho} c_{i,t}) - \Phi_{i,t}(v - \tilde{\rho}_{IV} c_{i,t})}{\Phi_{i,t}(v - \tilde{\rho} c_{i,t}) - \Phi_{i,t}(v - \lambda_0 - (\tilde{\rho} + \lambda_1) c_{i,t})} := g(p_{i,t})
\]

The function \(g(p_{i,t})\) satisfies \(g(0) = 0\) (and \(g(1) = 1\) when \(\bar{y} = 1\)). \(\square\)

Proof. of Results (11) and (12)

(1) Starting from the moment function which is separable in \(\lambda_t^0\),

\[
g(Y_t, Y_{t+1}, \theta) - \lambda_t^0(Y_t, Y_{t+1}, \theta) = \mu_t
\]

where \(\mu_t\) is positive or negative, then multiplying by \(X_{t-1} > 0\) maintains the same sign. Taking unconditional expectations we conclude.

(2) Since \(\mu_t = \kappa_t B_t\), dividing both sides by \(B_t\) and taking unconditional expectations with \(X_{t-1}\) we conclude. \(\square\)
7.5. Identification: Limited Information Framework. I first define two notions that will be helpful for the proposition that follows:

**Definition 1.** Denote \( m(Y_t, Y_{t+1}, \theta) \) as correctly specified if there exists \( \theta_0 \in \Theta \) such that if \( U = 0 \), \( \mathbb{E}m(Y_t, Y_{t+1}, \theta_0) = 0 \).

**Definition 2.** For some \( U \in \times \mathbb{R}^+ \), \( \theta_1(U) \) is conditionally identified if there does not exist any other \( \theta \in \Theta \), \( \theta_2(U) \) such that \( \mathbb{E}m(Y_t, Y_{t+1}, \theta_1(U)) = \mathbb{E}m(Y_t, Y_{t+1}, \theta_2(U)) \).

The first ensures that the identified set is not empty, while the second guarantees that more information on the wedge \( U \) leads to a smaller identified set for \( \theta \). Given these definitions, Theorem 2 establishes that fixing a value for the wedge \( U \), there exists a unique value for \( \theta \), defined as \( \theta(U) \), that satisfies (13). Any additional information is likely to make the identified set smaller.

**Theorem 2.** \( \Theta_I \) when \( r = n_\theta \)

Under correct specification of \( m(\cdot) \) and conditional identification,

\[ \exists ! \text{ mapping } \mathcal{G} : \Theta_I = \mathcal{G}^{-1}(U) \cap \Theta \]

**Proof.** Fix \( U_{n_\theta \times 1} \in (\underline{U}, \bar{U}] \). Assuming conditional identification, there is a unique value of \( \theta^* \) such that \( \mathbb{E}m(Y_t, Y_{t+1}, \theta^*) = \mathcal{G}(\theta^*) = U_{n_\theta \times 1} \). Thus, \( \Theta_I = \Theta \cap \mathcal{G}^{-1}(U) \). \( \square \)

7.5.1. The use of additional conditions. I distinguish between moments \( m_{\alpha,t}(\theta) \), the necessary moments functions for Theorem 2 being true, and \( m_{\beta,t}(\theta) \) the additional moment functions where for notational brevity, I have dropped the dependence on \( Y_0^{-1} \). In addition, let \( \hat{m}_{\alpha,t}(\theta) := m_{\alpha,t}(\theta)\phi_t \) and \( \hat{m}_{\beta,t}(\theta) := m_{\beta,t}(\theta)\phi_t \), \( \hat{m}_\alpha(\theta) \) and \( \hat{m}_\beta(\theta) \) the corresponding vectors, and \( \hat{m}_\alpha(\theta) \) and \( \hat{m}_\beta(\theta) \) the vector means. Let \( W \) be a real valued, possibly data dependent matrix, diagonal in \( (W_\alpha, W_\beta) \). Furthermore, denote by \( Q_\alpha \) the conditional expectation operator when conditioning on \( W_\alpha^2 \hat{m}_\alpha(\theta) \) and by \( Q_\alpha^T \) the residual operator.
The following proposition specifies that additional moments have to be less than perfectly correlated with the necessary conditions for the identified set to be smaller.

**Theorem 3.** For $\mathcal{U}_t = \{U_t \in [\underline{U}, \bar{U}]\}$ define

$$\mathcal{U}_t^c = \left\{ U_t \in [\underline{U}, \bar{U}] : \mathbb{E} Q_\alpha^1 \left( W_\beta^{1T} \mathbf{m}_\beta(\theta(U)) - U_t \right) = 0 \right\}$$

Then, $\Theta_t' \subset \Theta_I$ iff $\mathcal{U}_t^c \subset \mathcal{U}_t$

**Proof.** Denote the moment conditions using $n_\theta$ macroeconomic variables by $q_1(\theta, Y^{r-1})$ and moment conditions using $k = r - n_\theta$ survey variables by $q_2(\theta, Y^{r-1})$:

$$q_1(\theta, Y^{r-1}) = \mathcal{V}_1(Y^{r-1}) \in [\bar{\mathcal{V}}(Y_t)_1, \bar{\mathcal{V}}(Y_t)_1]$$

$$q_2(\theta, Y^{r-1}) = \mathcal{V}_2(Y^{r-1}) \in [\bar{\mathcal{V}}(Y_t)_2, \bar{\mathcal{V}}(Y_t)_2]$$

which imply the following unconditional moment equalities:

$$\mathbb{E}(q_1(\theta, Y^{r-1}) - \mathcal{V}_1(Y^{r-1}) \phi_t) = 0$$

$$\mathbb{E}(q_2(\theta, Y^{r-1}) - \mathcal{V}_2(Y^{r-1}) \phi_t) = 0$$

Denote this vector of moment conditions by $m$. Partition the vector $m \equiv (m_\alpha^T, m_\beta^T)^T$ where $m_\alpha$ contains the first $n_\theta$ moments. Since $r > n_\theta$, and given a weighting matrix $W$, the first order conditions are as follows:

$$\mathbb{E} J(\theta)^T(W_\alpha^{1T} m_\alpha + W_\beta^{1T} m_\beta - \mathcal{V}_1) \phi_t = 0$$

where the Jacobian $J(\theta)$ has full rank. To economize on notation, redefine the weights after pre-multiplication with the Jacobian, which implies that:

$$\mathbb{E}(W_\alpha^{1T} m_\alpha + W_\beta^{1T} m_\beta) - \bar{U} = 0$$

This is a projection of $m$ on a lower dimensional subspace. Since $W$ is an arbitrary matrix, and $(m_\alpha, m_\beta)$ are possibly correlated, we reproject the sum onto the space spanned by $W_\alpha^{1T} m_\alpha$. Denote the projection $Q_\alpha := W_\alpha^{1T} m_\alpha (m_\alpha^T W_\alpha^T m_\alpha)^{-1} m_\alpha^T W_\alpha^{1T}$ and
$Q^\perp$ the orthogonal projection. Since the original sum satisfies the moment condition, then the two orthogonal complements will also satisfy it:

$$Q_{\alpha} \left( W_\alpha^T m_\alpha + W_\beta^T m_\beta - U \right) = W_\alpha^T m_\alpha + Q_{\alpha} \left( W_\beta^T m_\beta - U \right) = 0$$

$$Q^\perp_{\alpha} \left( W_\alpha^T m_\alpha + W_\beta^T m_\beta - U \right) = Q^\perp_{\alpha} \left( W_\beta^T m_\beta - U \right) = 0$$

where $U \in \left[ W_\alpha^T \mathcal{V}(Y_t)_\alpha + W_\beta^T \mathcal{V}(Y_t)_\beta, \quad W_\alpha^T \mathcal{V}(Y_t)_\alpha + W_\beta^T \mathcal{V}(Y_t)_\beta \right] \otimes \phi_t$

As in Theorem 2 the first set of restrictions identifies a one to one mapping from $U$ to $\Theta_I$, and therefore $\theta^*(U) \equiv G^{-1}(U)$. Plugging this in the second set of restrictions eliminates dependence on $m_\alpha$ and imposes further restrictions on the domain of variation of $U$. The admissible set for $U$ is now

$$\left\{ U \in (-\infty, \bar{U}) : Q^\perp_{\alpha} \left( W_\beta^T m_\beta(U) - U \right) = 0 \right\}$$

Therefore, $\exists \theta \in \theta(U) : \theta \notin \Theta_I(U')$ and consequently $\Theta'_I \subset \Theta_I$. The same result carries through if we replace the linear projection with conditional expectations. Letting $Q_{\alpha}$ be the conditional expectation operator implies that any integrable moment function $m$ can be decomposed as $m = Q_{\alpha}m + Q^\perp_{\alpha}m$ such that $Q_{\alpha}(m - Q_{\alpha}m) = 0$. □

**Proof. of Proposition 1**

(1): When $B_t = 0$, $\mathbb{E} (g(Y_t, Y_{t+1}, \theta) - \lambda^\alpha_t(Y_t, Y_{t+1}, \theta)) X_{t-1} = 0$ which trivially restores point identification. When $B_t = 1$, (13) collapses to (12).

$$\text{sign} \left( \mathbb{E} (g(Y_t, Y_{t+1}, \theta) - \lambda^\alpha_t(Y_t, Y_{t+1}, \theta)) X_{t-1} \right) = \text{sign} \left( \mathbb{E}(\kappa_t X_{t-1}) \right) = \text{sign} \left( \mathbb{E}(\mu_t X_{t-1}) \right)$$

For $B_t \in (0,1)$, the moment conditions in (12) and (13) are not perfectly correlated as $\text{Corr}(\kappa_t, \mu_t) = \text{Corr}(\kappa_t, B_t \kappa_t) \neq 1$. By Theorem 3 we conclude.

(2): If $\text{Var}(B_t) = 0$, then $\text{Corr}(\kappa_t, B_t \kappa_t) = 1$.

(3): Suppose that $\Theta_I$ is a singleton. Then it must be that the RHS of (13) is zero: Either $B_t = 0$ for all $t$, or $p(s_{i,t}|S_t)$ has unit mass on one agent who is unconstrained, and thus $B_t = 0$ as well. □
7.6. **Mixed Frequency Model for Quarterly** $B_t$. Using $\tilde{b}_t = \tilde{\pi}_t + \tilde{\zeta}_t$ as a measurement equation, the true (log) quarterly proportion of liquidity constrained consumers is extracted using the following mixed frequency Gaussian linear state space model, where $t = 4(j - 1) + q$, $t$ is the quarterly observation at year $j$ and $q = \{1, 2, 3, 4\}$ the within year quarter index:

State Equation (ignoring identities):

$$
\begin{pmatrix}
\tilde{b}_{4(j)} \\
\tilde{\zeta}_{4(j)}
\end{pmatrix} =
\begin{pmatrix}
\rho_b & 0 \\
0 & \rho_\zeta
\end{pmatrix}
\begin{pmatrix}
\tilde{b}_{4(j-1)+3} \\
\tilde{\zeta}_{4(j-1)+3}
\end{pmatrix} +
\begin{pmatrix}
\nu_{b,4(j)} \\
\nu_{\zeta,4(j)}
\end{pmatrix}
$$

Observation equation:

$$
\begin{pmatrix}
\tilde{\pi}_{4(j)} \\
\tilde{b}_{4(j)}^o
\end{pmatrix} =
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \ldots & 0 \\
\frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & 0 & 0 & \ldots & 0
\end{pmatrix}
\begin{pmatrix}
\tilde{b}_{4(j)} \\
\tilde{b}_{4(j-1)+3} \\
\tilde{b}_{4(j-1)+2} \\
\tilde{b}_{4(j-1)+1} \\
\tilde{b}_{4(j-1)} \\
\tilde{b}_{4(j-2)+3} \\
\tilde{b}_{4(j-2)+2} \\
\tilde{b}_{4(j-2)+1} \\
\tilde{\zeta}_{4(j-1)+3} \\
\tilde{\zeta}_{4(j-1)+2} \\
\tilde{\zeta}_{4(j-1)+1} \\
\tilde{\zeta}_{4(j-2)} \\
\tilde{\zeta}_{4(j-2)+3} \\
\tilde{\zeta}_{4(j-2)+2} \\
\tilde{\zeta}_{4(j-2)+1} \\
\tilde{\zeta}_{4(j-2)}
\end{pmatrix} +
\nu_{4(j)}
$$
where $(\nu_{b,t}, \nu_{\zeta,t}) \sim N(0, \text{diag}(\Sigma_{\nu}))$ and $v_t \sim N(0, \text{diag}(\Sigma_{v}))$. The last diagonal component of $\Sigma_v$ is calibrated to the standard error from the estimation of $B_t$ in the SHF survey. The rest of the components of the diagonal are calibrated to 1% of the variance of the BCS measure ($\tilde{\pi}_t$).

8. Appendix B

8.1. Derivations with external Habits in Consumption. With external habit formation, the utility of the household becomes as follows:

$$u(c_{i,t}, l_{i,t}, C_t) := \left( \frac{(c_{i,t} - hC_{t-1})^{1-\omega} - 1}{1 - \omega} - \frac{\eta^{1+\eta}}{1 + \eta} \right)$$

and the corresponding Euler equation is:

$$(c_{i,t} - hC_{t-1})^{-\omega} = \beta \mathbb{E}_t (c_{i,t+1} - hC_t)^{-\omega} \pi^{-1}_{t+1} R_{t+1} + \mu_{i,t}$$

Expanding individual consumption around aggregate consumption, we have that

$$\int (c_{i,t+1} - hC_t)^{-\omega} p(s_{i,t}|S_t) ds_{i,t} \approx (C_{t+1} - hC_t)^{-\omega} \left( 1 + \frac{\omega(\omega + 1)}{2}(C_t - hC_{t-1})^{-2} \text{Var}_t(c_{i,t}) \right)$$

$$\quad := (C_{t+1} - hC_t)^{-\omega} \Xi_t(h)$$

In particular, the bond Euler equation becomes:

$$(C_t - hC_{t-1})^{-\omega} \Xi_t(h) = \beta \mathbb{E}_t (C_{t+1} - hC_t)^{-\omega} \Xi_{t+1}(h) \pi^{-1}_{t+1} R_{t+1} + \int \mu_{i,t} p(s_{i,t}|S_t) ds_{i,t}$$

Correspondingly, the aggregate intratemporal condition becomes:

$$\int l_{i,t} p(s_{i,t}|S_t) ds_{i,t} = w_i^{\frac{1}{\eta}} (c_{i,t} - hC_{t-1})^{-\frac{\omega}{\eta}}$$

$$\quad = w_i^{\frac{1}{\eta}} (C_t - hC_{t-1})^{-\frac{\omega}{\eta}} \left( 1 + \frac{\omega(\omega + \eta)}{2\eta^2}(C_t - hC_{t-1})^{-2} \text{Var}_t(c_{i,t}) \right)$$

$$\quad \equiv w_i^{\frac{1}{\eta}} (C_t - hC_{t-1})^{-\frac{\omega}{\eta}} \Xi_{t}^{lab}$$
8.2. Priors.

Table 2. Priors for $\Theta_{FI}$

| $\Theta$ | Prior                  | $\Theta$ | Prior                  |
|----------|------------------------|----------|------------------------|
| $\omega$ | $Inv - Gamma(3, 2)$    | $\eta$  | $Inv - Gamma(2, 0.6)$  |
| $\beta$  | $U(0.98, 1)$           | $h$     | $Beta(1.2, 1.2)$       |

8.3. Consumption Dispersion: SHF versus HBS. Two measures of durable and non-durable consumption dispersion are used using data from two different surveys. More specifically, for the SHF, we compute the variance of annual nominal consumption, using survey weights, computed as follows:

$$c_{i,t}^{nom, annual} = 12 \cdot (Imputed \ Monthly \ Rent[code : 2.31]$$

$$+ Car \ Purchases/12[code : 2.74] + House \ Durables/12[code : p2.70]$$

$$+ monthly \ non \ durable \ consumption[code : p9.1])$$

To convert it to the real consumption ratio, I divide by aggregate consumption. For the Household Budget Survey, I compute the variance of total consumption expenditure ($code: GASTOT$, for those who do not report zero total income), which is itself a weighted measure of consumption expenditure for the representative household of a particular classification. To convert it to the real consumption ratio, I divide by aggregate consumption. Notice also the survey switched from the COICOP to the ECOICOP classification in 2016.

Figure 4 plots the demeaned measures of consumption dispersion computed from the triennial Spanish Survey of Household Finances (SHF), and the annual Household Budget Survey (HBS). The two series have a similar pattern over time while the SHF
measure exhibits sharper falls in 2009 and in 2015. As evident in Figure 4 this can be attributed to durable consumption.

![Figure 4](image)

**Figure 4**
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