Monte-Carlo simulation study for uncertainty evaluation in turbulent statistics obtained by constant-temperature anemometry

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Abstract. We evaluate the uncertainty of a recalibration scheme correcting for measurements of constant-temperature anemometry (CTA), where an observed velocity by CTA would be affected by variations in ambient temperature in an experiment. The present study elucidates the characteristics of the uncertainty quantitatively. A temporally evolving calibration curve is used to examine the uncertainty of the calibration curve. In the present study, we consider the temporally evolving curve to be a reference curve. The calibration curve predicted by the scheme is first examined. Here, the uncertainty of the gradient of the calibration curve is also considered. A Monte-Carlo simulation investigates the uncertainty in intensity and higher-order statistics of output-voltage fluctuation. Using the gradient of the calibration curve, this study derives a model for the statistics of the output voltage fluctuation, which is validated by the numerical simulation based on the Monte-Carlo simulation.

1. Introduction
Turbulent flow is often observed in fields such as mechanical engineering. Statistical tools are generally used to analyze turbulent flows. The governing equations for the turbulent statistics are not closed because these equations are highly nonlinear. Experimental measurements are performed in order to investigate the statistical properties of the flow. Constant-temperature anemometry (CTA) is a hot-wire anemometry technique (e.g., [1]) that is often used because, for velocity measurements, the S/N ratio is high and the dynamic range is wide. Constant-temperature anemometry measures the instantaneous velocity based on the change in resistance of a hot wire due to the fluid velocity. The calibration curve of CTA is generally derived in terms of the output voltage $E$ and the calibration velocity $U$: $E^2 = A_o + B_o U^n$, where $A_o$ and $B_o$ are coefficients which need to be calibrated, and $n_o$ is a coefficient, the value of which is assumed or calibrated and is constant in the moderate-velocity range [2].

Measurement results obtained by CTA are affected by variations in fluid temperature, because the calibration curve is affected by temperature variation. The magnitude of the temperature dependency of a measured velocity has been observed to be approximately 1% to 2% per degree Celsius. The uncertainty in the observed velocity due to the temperature variation could be rather large [3]. Correcting for the effects of fluid temperature variation is an important issue in CTA. Baerman's method is a classical method for temperature correction [4]. There are a number of methods of temperature correction based on the method (e.g., [5]). There are also temperature corrections for CTA that are not method based. For example, Abdel-Rahman et al. [6] used a temperature-compensation
technique based on Collis and Williams's law. The effects of fluid temperature on hot-wire anemometry have been investigated theoretically [3]. Dijk and Nieuwstadt [7] proposed a practical procedure that corrects for the effects of fluid temperature variation. Hultmark and Smits [8] proposed a calibration curve which accounts for the effects of relatively large temperature changes. Then, a correction method that predicts the temporal development of a calibration curve by measuring a series of output voltages at known fluid velocities, referred to as the recalibration method, has been proposed [9]. This recalibration method predicts a calibration function that evolves temporally without assuming an analytical equation for the effects of temperature variation, and is used for the measurement of grid-generated turbulence in the recent experiment [10]. Also there are still recent researches to study the measurement technique of CTA (e.g., [11, 12]).

The uncertainty in the velocity observed by CTA measurements should be sufficiently small. For instance, the uncertainty in the observed mean velocity of a turbulent boundary layer should be smaller than approximately 2% or 0.2% in order to reduce the uncertainty of the observed value of the Kármán constant to 1% or 0.1%. The uncertainty of CTA calibration is examined in the order of 1% to 0.1% of the relative uncertainty [2]. There are a number of issues to be solved with respect to this recalibration method. Since the uncertainty of the scheme depends on the relative difference between the pre- and post-calibration curves, the dependency of the uncertainty on the relative difference should be investigated. The contributions of the coefficient to the uncertainty should be determined independently because the calibration curve involves a number of coefficients [2]. The gradient of the calibration curve as well as the curve should be considered in the measurement of a turbulent flow, because the observed velocity fluctuation is related to the gradient rather than the curve. The uncertainty would be a function of time, because the proportional drift of the scheme [9] is a function of time. A model of the uncertainty would allow more practical CTA experiments. Also numerical simulation is an appropriate method of validation, because simulation involves no uncertainty of the measurement instruments.

The purpose of the present study is to evaluate the uncertainty of a predicted calibration curve. We apply a temporally varying calibration curve to the present examination as a reference. We first examine the uncertainty of the calibration curve predicted by this method. The statistics of the uncertainty in the output voltage predicted by the method are obtained and modeled. Monte-Carlo simulation (MCS) is used to investigate the uncertainty in the statistics of the output voltage fluctuation. This MCS could simulate the uncertainty in the statistics of the output voltage fluctuation, which is related to velocity fluctuation using a calibration curve. Here, the output voltage fluctuation is modeled by Gaussian-white random numbers. In the present MCS, the uncertainty in the statistics of the output voltage fluctuation is shown as a function of rms of the velocity fluctuation.

2. Methods

2.1. Recalibration scheme

We start by defining non-dimensional time \( t \): \( t = (t' - t'_\text{pre}) / (t'_\text{post} - t'_\text{pre}) \), where \( t' \) is dimensional time, and \( t'_\text{pre} \) and \( t'_\text{post} \) are the times at pre- and post-calibrations, respectively. The recalibration scheme predicts a calibration curve at any time \( E(U; t) \) using pre- and post-calibration curves, \( E_{\text{pre}}(U) \) and \( E_{\text{post}}(U) \). The scheme predicts the curve at the intermediate time \( E(U) \) in the following form:

\[
E(U; t) = R(t) \left[ E_{\text{post}}(U) - E_{\text{pre}}(U) \right] + E_{\text{pre}}(U). \tag{1}
\]

This relation includes the proportional drift \( R(t) \), which is defined as follows:

\[
R(t) = \left[ E(U_o; t) - E_{\text{pre}}(U_o) \right] / \left[ E_{\text{post}}(U_o) - E_{\text{pre}}(U_o) \right]. \tag{2}
\]

where \( U_o \) is the freestream velocity. The value of \( U_o \) must be known in order to be used in the scheme. The value of \( R(t) \) can be obtained through practical experiments. The value of the output voltage for a
known freestream velocity can be measured through a CTA measurement. By calculating the value of \( R_i(t) \), a calibration curve can be obtained at intermediate times. A calibration curve could vary with time. This temporally varying calibration curve is calculated by using the recalibration scheme. In the recalibration scheme, a temporal variation of \( E(U_o) \) is focused on, where \( E(U_o) \) is a value of a calibration curve at the free-stream velocity. By using the temporal variation of \( E(U_o) \), the proportional drift \( R_i(t) \) is calculated. By using the proportional drift \( R_i(t) \) (Equation (2)) and Equation (1), the temporally varying calibration curve could be calculated as \( E_i(U,t) \).

### 2.2. Temporally varying calibration curve

A reference calibration curve is needed in order to evaluate the uncertainty of a predicted calibration curve. We consider an experimental condition in which the variation of ambient temperature is not large. This is acceptable because most CTA experiments make a sufficient effort to reduce the uncertainty of the calibration curve. Using the non-dimensional time, we introduce the relation between the output voltage and the calibration velocity:

\[
E(U,t) = A_o(t) + B_o(t) \cdot U_o^\beta(t),
\]

where \( A_o(t) \) and \( B_o(t) \) are expressed as coefficients that are functions of time, because the temperature difference depends on the time. For the same reason, the coefficient \( n_o(t) \) is also defined as a function of time. The temporal variation of ambient temperature can be considered to be linear, because the variation of ambient temperature is small. Based on previous studies, we consider the calibration coefficients to be proportional to the temperature \([1, 3]\). Therefore, we introduce the following equations: \( A_o(t) = A(1 + \alpha t) \), \( B_o(t) = B(1 + \beta t) \), and \( n_o(t) = n(1 + \gamma t) \), where, respectively, \( \alpha = \partial(A_o(t)/A)/\partial t \), \( \beta = \partial(B_o(t)/B)/\partial t \), and \( \gamma = \partial(n_o(t)/n)/\partial t \), where \( A, B, \) and \( n \) are coefficients which are determined by a calibration procedure at the pre-calibration time, and \( A_o(t), B_o(t), \) and \( n_o(t) \) are assumed such that their temporal variations are linear.

Using the above variations of the coefficients, \( E_{pre}(U) \) and \( E_{post}(U) \) are obtained as follows: \( E_{pre}(U) = (A + B \cdot U^n)^{\beta(t)} \) and \( E_{post}(U) = (A(1 + \alpha t) + B(1 + \beta t) \cdot U^{n(1 + \gamma t)})^{\beta(t)} \). The temporally varying calibration curve \( E_{int}(U,t) \) is finally obtained as follows: \( E_{int}(U) = (A(1 + \alpha t) + B(1 + \beta t) \cdot U^{n(1 + \gamma t)})^{\beta(t)} \). We consider the above curve to be the reference of the curve and use these curves to examine the scheme.

### 2.3. Numerical simulation

In the present study, we consider calibration curves, the statistics for the output voltage fluctuation, and velocity statistics of a turbulent channel flow. Following a previous experiment on CTA calibration \([8]\), we set the values of these coefficients to be as follows: \((A,B,n) = (6.210,4.399,0.413)\) for \( U = 1 - 40 \) m/s and \((A,B,n) = (6.163,4,707,0.3995)\) for \( U = 1 - 6 \) m/s. In these cases, we set \( U_o = 40 \) m/s and \( U_o = 5 \) m/s, respectively. We define the relative deviation of output voltage \( \Delta E \) as follows: \( \Delta E = (E_{post}(U_o) - E_{pre}(U_o)) / E_{pre}(U_o) \). There could be a rather large difference in the temporally varying calibration curve between the variations of the coefficients. Thus, we set \( \Delta E = \pm 0.05 \). Using these derivations of the coefficients, the temporally varying calibration curve is set. The time is set to the intermediate time, \( t = 1/2 \). Here, we examine not only the uncertainty of the curve but also the gradient of the curve. The gradient of the curve is calculated using the fourth-order central difference scheme. Sufficient spatial resolution is verified using the analytical form of the curve.

In this study, a simplified calibration scheme is additionally examined to discuss the uncertainty of the present recalibration scheme. In this simplified scheme, the form of proportional drift in Equation (2) is given as follows: \( R_{i}(t) = \left[ E(U_o,t) - E_{pre}(U_o) \right] / \left[ E_{post}(U_o) - E_{pre}(U_o) \right] \), where \( E(U_o,t) = E_{post}(U_o) - E_{pre}(U_o) \) : \( t + E_{pre}(U_o) \). When \( t = 1/2, R_{i} = 1/2 \) in the simplified scheme. Therefore, the form of proportional drift yields the following form for \( t = 1/2: E_{avg}(U) = \left[ E_{post}(U_o) - E_{pre}(U_o) \right] (1/2) + E_{pre}(U_o) = (1/2) \left[ E_{pre}(U_o) + E_{post}(U_o) \right] \).
Figure 1. $100 \times \varepsilon_E [%]$, where $\varepsilon_E$ indicates the uncertainty of a calibration curve. When $100 \times \varepsilon_E [%] = 0.1$, it does means $\varepsilon_E = 0.001$. Results of $100 \times \varepsilon_E [%]$ due to (a) $\alpha$, (b) $\beta$, and (c) $\gamma$.

Figure 2. $100 \times \varepsilon_{dE/dU} [%]$, where $\varepsilon_{dE/dU}$ indicates the uncertainty of a calibration curve. When $100 \times \varepsilon_{dE/dU} [%] = 0.1$, it does means $\varepsilon_{dE/dU} = 0.001$. Results of $100 \times \varepsilon_{dE/dU} [%]$ due to (a) $\alpha$, (b) $\beta$, and (c) $\gamma$. 
The uncertainty in the statistics of the output voltage fluctuation is simulated through MCS. The output-voltage fluctuations of $E_i(U, t)$ and $E_{int}(U, t)$, $e_i(t)$ and $e(t)$, respectively, are calculated based on the velocity fluctuation. The velocity fluctuation is a series of Gaussian-white random numbers, which is generated by the Mersenne twister [13]. Box-Muller transform is used to transform series of uniform pseudo-random number to those of Gaussian pseudo-random number, which is used for the present MCS. In the present MCS, mean velocity is zero and a value of variance $\langle u^2 \rangle^{1/2} / U_o$ is 0.2, where $u$ is the velocity fluctuation, and $\langle \cdot \rangle$ denotes the ensemble average. The intensity and fourth-order statistics of the output voltage fluctuation are calculated.

3. Results and discussion

3.1. Uncertainty of calibration curve

We examine the uncertainty of both a calibration curve $\varepsilon_{Ec}$ defined as follows: $\varepsilon_{Ec} = (E(U) - E_{int}(U)) / E_{int}(U)$, where $E(U) = E(U)$ or $E_{avg}(U)$. Also the gradient of calibration curve $\varepsilon_{E_{avg}}$ defined as follows: $\varepsilon_{E_{avg}} = (dE(U)/dU - dE_{int}(U)/dU) / dE_{avg}(U)/dU$, where $dE(U)/dU = dE(U)/dU$ or $dE_{avg}(U)/dU$. Following previous studies (e.g., [7]), $\varepsilon_{E_{avg}}$ characterizes the uncertainty of the velocity fluctuation observed by CTA. $\varepsilon_{E}$ and $\varepsilon_{E_{avg}}$ characterize the uncertainty of mean velocity and that of velocity fluctuation, respectively. We start with examining the uncertainty of the curve $\varepsilon_{Ec}$. Figure 1 shows the uncertainty of the curves, where $\Delta E = 0.05$. The relative deviations are individually considered as shown in the figure. In all cases, the magnitude of the uncertainty is smaller than 1% and is considered to be small. The uncertainty due to this scheme is smaller than that for $E_{avg}(U)$ in all cases. The reduction of the uncertainty was especially noticeable around the free-stream velocity. The uncertainty due to $\alpha$ is larger in the lower-velocity range and could be larger than 0.1%. This magnitude is found when $U/U_o$ is small. Although the uncertainty due to $\beta$ also increases as $U/U_o$ decreases, the maximum magnitude is smaller than that due to $\alpha$.

Figure 2 shows the uncertainty in the gradient, where the value of $\Delta E$ is the same as that for the curve. In all cases, the absolute magnitude of the uncertainty in the gradient can be larger than that of the curves. The uncertainty due to $\alpha$ and $\gamma$ can be larger than 0.1%. The uncertainty due to $\alpha$ is large in the low-velocity range. The uncertainty due to $\gamma$ is large in this range and is also rather large at the free-stream velocity. The magnitude of these uncertainties depends on the value of $U_o$. The uncertainty due to $\alpha$ is large when $U/U_o$ is small. The value of $U/U_o$ could be small in practical CTA measurements when $U_o$ is large. The uncertainty due to $\gamma$ decreases as the value of $U_o$ increases, in contrast to the results of $\alpha$. The maximum magnitude of the uncertainty due to $\beta$ is smaller than that of $\alpha$ and $\gamma$. In addition, the uncertainty due to $\beta$ is not larger than 0.1%, which is in contrast to that due to $\alpha$ and $\gamma$.

The three coefficients $A$, $B$, and $n$ could be affected by the temperature variation. The dependency of these coefficients on the temperature variation is investigated by a previous study [3]. When a value of the exponent $n$ is fixed to 0.5, King's law, the temperature dependency of the coefficient could be analytically derived. Specifically, the following relation is yielded [1]: $E(U, t)^2 = A(t) + B(t) U^{1/2}$. Here $dA/A = 0.886 dT_o/T_o$ and $dB/B = 0.006 dT_o/T_o$. Here $T_o$ is ambient fluid temperature. As shown in the above relation, the coefficient $B$ may be less sensitive to the temperature variation. In Figure 3 (a) the uncertainty of the recalibration scheme is shown for three conditions of $dT_o/T_o$, where $A = 6.21$, $B = 4.399$, $U_o = 40$ [m/s]. Figure 3 (b) shows the uncertainty for $U = 1$ [m/s] as a function of $dT_o/T_o$. Thus, if King's law is assumed, the sensitivity to the ambient temperature change of the coefficients of the calibration curve could be derived [1]. The ambient temperature change can significantly affect the coefficient $A$. The coefficient $A$ corresponds to calibration drift of a curve. An error in the calibration curve could be caused by the ambient temperature change affecting calibration drift. The exponent $n$ of the calibration curve is also affected by the ambient temperature change [3]. The sensitivity of the exponent $n$ to the ambient temperature change is smaller than that of the coefficient $A$. 
Figure 3. The uncertainty of the recalibration scheme is shown for three conditions of $dT_a/T_a = 0.03$, 0.05, and 0.1 (a), where $A = 6.21$, $B = 4.399$, $U_o = 40$ [m/s]. (b): The uncertainty for $U = 1$ [m/s] as a function of $dT_a/T_o$.

Figure 4. Results for the uncertainty of the output voltage fluctuation obtained by Monte-Carlo simulation. As shown in the figure, observed values obtained by the present model agree with those of numerical results.

3.2. Monte-Carlo simulation

We evaluate the uncertainty of the statistics of the output voltage fluctuation due to the scheme, where we intend intensity, fourth-order statistics, and flatness factor of the fluctuation. Since, in this simulation, the magnitude of the third-order statistics is zero analytically, we do not intend this factor. The relative uncertainty of the intensity and the fourth-order statistics is defined as $(\langle e_{int}^m \rangle - \langle e_{int}^m \rangle) / (\langle e_{int}^m \rangle)$, where $m = 2$ and 4 for the intensity and the fourth-order statistics, respectively. Here, $e_{int}^m$ and $e_i^m$ are the output voltage fluctuation of the reference curve and the curve predicted by the scheme, respectively. The uncertainty in the statistics of the output voltage fluctuation, which depends on the rms of the velocity fluctuation, is shown in Figure 4. Although the uncertainty of the intensity is
The purpose of the present study is to address the uncertainty of the calibration curve given by the recalibration scheme for CTA measurement. By introducing a linear assumption for the temporally varying coefficients of the calibration curve, a calibration curve that evolves temporally is derived and is used as a reference for the examination. Velocity fluctuation is related to the gradient of the calibration curve. Therefore, we examine both the gradient and the value of the calibration curve. The uncertainty is more sensitive to negative deviation of the output voltage than to positive deviation. Monte-Carlo simulation examines the statistics of fluctuations of output voltage. Using the gradient of the calibration curve, the statistics of the output voltage fluctuation are modeled. As indicated by these results, in the present study, the uncertainty of the recalibration scheme is investigated. In the recalibration scheme, as described in the previous study, the relation between the coefficients in the calibration curve and a variation of ambient temperature. We believe that the recalibration scheme is an effective calibration scheme for CTA measurement. The knowledge of the uncertainty of the recalibration scheme, which is investigated in the present study, will be useful in the application of the recalibration scheme.
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