Data Detection in Massive MIMO Systems Based on a Novel Total Least Squares Algorithm

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Abstract. In this paper, we propose a novel total least squares (NTLS) -based data detection algorithm for TDD massive MIMO uplink systems, where the impact of the channel estimation errors and the characteristics of channel hardening are both taken into consideration. Firstly, aiming at the problems caused by the channel estimation error, the NTLS method is adopted to minimize the impact. Secondly, QR decomposition and channel hardening properties are employed respectively to simplify the computational complexity. Finally, experimental results show that the algorithm not only has lower computational complexity, but also performs better than conventional linear data detection algorithms without requiring channel statistics.

1. Introduction

In recent years, massive MIMO as one of the key technologies in the next - generation 5G mobile cellular network, has attracted the attention of researchers with its potential in terms of capacity and performance. The technology uses hundreds of antennas to further increase channel capacity and energy efficiency without increasing the spectrum resources [1]. However, with the increase of the number of antennas, the spatial channel environment will become more complex, which poses a challenge for the efficient recovery of the transmitted signal.

The traditional Maximum Likelihood (ML) [2] and Sphere Decoding (SD) detection algorithms [3], have near-optimal detection performance, but the complexity of the algorithms will increase exponentially with the number of antennas, and their application in massive MIMO becomes less practical. Common low-complexity linear detection algorithms, such as Zero-Forcing (ZF) [4], Minimum Mean Square Error (MMSE) [5], and other optimization algorithms, such as MMSE-SIC [6], require channel statistics and matrix inversion which are difficult to implement in hardware, so they are also thorny issues in massive MIMO systems. Recently, a number of nonlinear detection algorithms have been proposed, such as Genetic Soft-heuristic Algorithm (GSA) [7]. Although the performance is better than SD, it can only be applied in the case of lower modulation order, such as BPSK and QPSK. However, these methods not only do not fully utilize the characteristics of the massive MIMO channel hardening [8] to reduce the complexity of the algorithm, but also do not consider the influence of the channel estimation errors on the data detection.

In this paper, a data detection method based on the novel total least squares (NTLS) is proposed. Firstly, in order to minimize the impact of the channel estimation errors on the data detection process, the NTLS method is used to reconstruct the original transmitting signal. Next, for solving the matrix inversion operations occurring in the algorithm, a layer processing approach is adopted. That is, when the number of system antennas is on the normal or super large scale, the QR decomposition method or
the characteristics of channel hardening is employed skillfully to simplify the complexity, respectively. The experimental results show that the algorithm has lower computational complexity and better performance than conventional linear data detection algorithms in the massive MIMO system, which is a reliable solution.

2. System Model
In this paper, a Multi-cell multi-user and Time Division Duplexing (TDD) massive MIMO system is considered. In Fig.1, assume that the system has $L$ hexagonal cells, each cell has a base station (BS) with $M$ antennas and serves $K$ ($M>>K$) single-antenna user terminals (UT). For uplink, the signal vector in the time domain received by the BS in one cell $j$ is $y_j \in \mathbb{C}^{M \times \Gamma}$, which can be written as

$$
y_j = \sqrt{p_d} \sum_{i=1}^{L} H_{ji} s_i + n_j = \sqrt{p_d} \sum_{i=1}^{L} F_{ji} D_{ji}^{1/2} s_i + n_j,$$

where $\sqrt{p_d}$ represents the average transmission power of each UT, $H_{ji}$ represents the channel matrix between the BS in cell $i$ to all the users in cell $j$, $F_{ji}$ and $D_{ji}$ denote small and large scale fading matrix respectively. $s_i = [s_{i1}, s_{i2}, \cdots, s_{ik}] \in \mathbb{C}^{K \times \Gamma}$ denotes the signal vector sent by all terminals in cell $i$ at a certain time, $\Gamma$ is the length of the signal sequence, and $n_j \in \mathbb{C}^{M \times \Gamma}$ is the additive complex Gaussian white noise vector with a mean of zero and a variance of $\sigma^2$.

For simplicity, (1) can be rewritten as

$$
y_j = Hs + n_j,$$

where $s = [\sqrt{p_d}s_1, \sqrt{p_d}s_2, \cdots, \sqrt{p_d}s_L]^T \in \mathbb{C}^{KL \times \Gamma}$, and $H = [H_{j1}, H_{j2}, \cdots, H_{jL}] \in \mathbb{C}^{M \times KL}$.

Since when $M \to \infty$, the column vectors of $H_{ji}$ are asymptotically orthogonal, that is, the characteristics of channel hardening [1], which can be stated as follows

$$
H_{ji}^H H_{ji} = D_{ji}^{1/2} F_{ji}^H F_{ji} D_{ji}^{1/2} \approx M D_{ji}^{1/2} I_K D_{ji}^{1/2} = MD_{ji},
$$

where $D_{ji} = \text{diag}(\beta_{ji1}, \beta_{ji2}, \cdots, \beta_{jiK}) \in \mathbb{C}^{K \times K}$ is a diagonal matrix, which contains path loss and shadow fading. When the BS antenna number $M$ tends to be infinite, according to (3), it can be seen that the channel model only has large-scale fading, and the effect of small-scale fading could be completely eliminated, which is very favorable for the analysis of massive MIMO systems.

3. Data Detection Scheme
On the one hand, due to the environment noise and the imperfect channel estimation accuracy, the channel matrix $H$ obtained at the base station has an error relative to the truth value, which will further affect the numerical detection of the transmitted signal $s$. On the other hand, the traditional linear detection algorithms need to inverse the channel correlation matrix, but in large-scale MIMO systems, the inversion of the larger dimensional matrix is difficult to implement in hardware. This article will
focus on the impact of the error on the detection system and propose the following solution referred to as the novel total least squares (NTLS) data detection algorithm.

Assuming that the channel matrix $H$ has an error $\mathbf{E}_H$, (2) can be further expressed as

$$\mathbf{y}_j = (\mathbf{H} + \mathbf{E}_H)\mathbf{s} + \mathbf{n}_j,$$

(4)

In the recovery process of the signal $\mathbf{s}$, we hope to make $\mathbf{E}_H$ and $\mathbf{n}_j$ both as small as possible. The problem can be described as the following constraint optimization problem:

$$\min_{\mathbf{E}_H, \mathbf{n}_j, \mathbf{s}} ||\mathbf{E}_H||_2^2 + ||\mathbf{n}_j||_2^2, \text{ subject to } (\mathbf{H} + \mathbf{E}_H)\mathbf{s} + \mathbf{n}_j = \mathbf{y}_j,$$

(5)

To solve the problem (5), firstly, the singular value decomposition of channel $\mathbf{H}$ and received signal $\mathbf{y}_j$ is performed

$$\mathbf{H} \mathbf{y}_j = \mathbf{U} \mathbf{U}^T \mathbf{P}_1^T \mathbf{P}_2 \Lambda,$$

(6)

where $\mathbf{U} = [\mathbf{U}_1 \mathbf{U}_2] \in \mathbb{C}^{M \times M}$ is an orthogonal unitary matrix, $\mathbf{U}_1 = [u_1 \cdots u_n]$, $\mathbf{U}_2 = [q_{n+1} \cdots q_M]$, $q_k \in \mathbb{C}^{M \times 1}$, $\mathbf{U}^T \mathbf{U} = \mathbf{I}_M$; $\mathbf{P} = [\mathbf{P}_1 \mathbf{P}_2] \in \mathbb{C}^{(KL+T) \times (KL+T)}$ is an orthogonal matrix, and $\mathbf{P}_1 = [p_1 \cdots p_n]$, $\mathbf{P}_2 = [p_{n+1} \cdots p_{KL+L}]$, $p_k \in \mathbb{C}^{(KL+T) \times 1}$, $\mathbf{P}_2^T \mathbf{P} = \mathbf{I}_{KL+L}$; $\Lambda = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix}$

is a diagonal matrix, $\Lambda_1 = \text{diag}(\sigma_1, \ldots, \sigma_{KL+L})$, $\Lambda_2 = \text{diag}(\sigma_{KL+1}, \ldots, \sigma_{KL+L}, 0, \ldots, 0)$, where, $\sigma_1 \geq \cdots \geq \sigma_{KL+L}$ is a singular value.

**Lemma 1.** According to Theorem 5 in [9], when $\sigma_{KL} \neq \sigma_{KL+1}$, the detection value of $\mathbf{s}$ can be determined by

$$\hat{s} = (\mathbf{H}^H \mathbf{H} - \sigma^2 I)^{-1} \mathbf{H}^H \mathbf{y}_j,$$

(7)

where, $\sigma^2 = \sigma^2_{KL+1} + \cdots + \sigma^2_{KL+L}$. In the massive MIMO system, the size of the channel matrix $\mathbf{H}$ will be increased multiplied by the number of antennas $M$, and the inverse of the $\mathbf{H}$ in the solution will become very complex. Therefore, in order to simplify the calculation, two sets of schemes are proposed in this paper, corresponding to the two conditions of infinite number of antennas $M$ and normal conditions.

**Theorem 1.** For sufficiently large $M$ ($M \rightarrow \infty$), the sending signal $\mathbf{s}$ can be obtained as

$$\hat{s} = \text{diag}(\frac{1}{d_1}, \ldots, \frac{1}{d_{KL}}) \mathbf{H}^H \mathbf{y}_j,$$

(8)

where $d_k = M \cdot \beta_{jk} - \sigma^2$.

Proof: According to the (3), we have

$$\mathbf{H}^H \mathbf{H} - \sigma^2 I = \begin{bmatrix} \mathbf{H}_{j1}^H \mathbf{H}_{j1} & \cdots & \mathbf{H}_{j1}^H \mathbf{H}_{jL} \\ \vdots & \ddots & \vdots \\ \mathbf{H}_{jL}^H \mathbf{H}_{j1} & \cdots & \mathbf{H}_{jL}^H \mathbf{H}_{jL} \end{bmatrix} - \sigma^2 I$$

$$= \begin{bmatrix} \mathbf{M} \mathbf{D}_{j1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \mathbf{M} \mathbf{D}_{jL} \end{bmatrix} - \sigma^2 I$$

$$= \text{diag}(d_1, \ldots, d_k, \ldots, d_{KL})$$

where $d_k = M \cdot \beta_{jk} - \sigma^2$.

Then (7) can be further reduced to

$$\hat{s} = \text{diag}(d_1, \ldots, d_{KL})^{-1} \mathbf{H}^H \mathbf{y}_j$$
\[
\mathbf{H}_H \mathbf{y}_j = \operatorname{diag}(1/d_1, \ldots, 1/d_{KL}) \mathbf{H}^H \mathbf{y}_j
\]

**Theorem 2.** For normal conditions, the sending signal \( s \) can be stated as follows

\[
\hat{s} = \frac{Q_2 Q_H^H \mathbf{y}_j}{i \sigma}
\]  

(9)

Proof: Suppose \( \mathbf{W} = (\mathbf{H}^H \mathbf{H} - \sigma^2 \mathbf{I})^{-1} \mathbf{H}^H = (\mathbf{B}^H \mathbf{B})^{-1} \mathbf{H}^H \), where \( \mathbf{B} = [\mathbf{H} \sigma \mathbf{I}] \in \mathbb{C}^{(M + KL) \times KL} \). Then the matrix \( \mathbf{B} \) is decomposed by QR, which can be given as

\[
\mathbf{B} = [\mathbf{H} \sigma \mathbf{I}] = \mathbf{Q} \mathbf{R} = \begin{bmatrix} \mathbf{Q}_1 \\ \mathbf{Q}_2 \end{bmatrix} \mathbf{R},
\]

where \( \mathbf{Q} \in \mathbb{C}^{(M + KL) \times KL} \) is an orthogonal matrix, \( \mathbf{Q}_1 \in \mathbb{C}^{M \times KL} \), and \( \mathbf{Q}_2 \in \mathbb{C}^{KL \times KL} \); \( \mathbf{R} \in \mathbb{C}^{KL \times KL} \) is a diagonal matrix; \( \mathbf{H} = \mathbf{Q}_1 \mathbf{R} \). Then (7) is described by

\[
\hat{s} = (B^H B)^{-1} \mathbf{H}^H \mathbf{y}_j
\]

Algorithm 1. NTLS Based Data Detection

**Input:** Estimated channel \( \mathbf{H} \), received signal \( \mathbf{y}_j \), the length \( \Gamma \) of the signal sequence, the system has \( L \) hexagonal cells, each cell has a BS with \( M \) antennas and serves \( K \) (\( M \gg K \)) single-antenna UT, the large-scale fading diagonal matrix \( \mathbf{D} = \operatorname{diag}(\mathbf{D}_{j_1}, \mathbf{D}_{j_2}, \ldots, \mathbf{D}_{j_L}) \in \mathbb{C}^{KL \times KL} \), and a threshold parameter \( Y \)

**Initialize:** \( k \leftarrow 1, \quad \mathbf{y}_k \leftarrow \mathbf{0}_{M \times 1}, \quad \hat{s} \leftarrow \mathbf{0}_{K \times L} \quad Y = 500 \)

For \( a = 1: \Gamma \)

Calculate: \( \mathbf{y}_a = \mathbf{Y}(:, a) \)

Singular value decomposition of channel \( \mathbf{H} \) and received signal \( \mathbf{y}_a \): \( [\mathbf{U}, \Lambda, \mathbf{P}^T] = \operatorname{SVD}([\mathbf{H}, \mathbf{y}_a]) \)

Extract singular value: \( \sigma = \sigma_{KL+1} \)

If \( M \geq Y \)

Calculate: \( \mathbf{H}^H \mathbf{H} - \sigma^2 \mathbf{I} = M \cdot \mathbf{D} - \sigma^2 \)

\[
\mathbf{s}' = (\mathbf{H}^H \mathbf{H} - \sigma^2 \mathbf{I}) \mathbf{H}^H \mathbf{y}_a
\]

\[
\hat{s}(;, a) = \mathbf{s}'
\]

Else

\[
r = i \times \sigma^2 \times \text{eye}(K \times L)
\]

QR decomposition: \( [\mathbf{Q}_{\text{nts}}, \mathbf{R}_{\text{nts}}] = \text{QR}([\mathbf{H}; \ r]) \)

Extract value: \( \mathbf{Q}_1 = \mathbf{Q}_{\text{nts}}(1:M, 1:KL) \)

Extract value: \( \mathbf{Q}_2 = \mathbf{Q}_{\text{nts}}((M + 1):(M + KL), 1:KL) \)

\[
\hat{s}(;, a) = \mathbf{s}'
\]

**Output:** \( \hat{s} \)
As described in Algorithm 1, the traditional linear detection algorithm is applied to the massive MIMO system which can ensure certain algorithm performance. However, one of the major challenges for the linear detection algorithms is the inverse of the channel correlation matrix, which is mainly because the inversion of the matrix of larger dimensions is difficult to implement in hardware. In this paper, the NTLS detection algorithm is proposed, which could avoid the need for channel statistics or matrix inversion. And utilizing the channel hardening characteristics in massive MIMO, two different simplified calculation methods are adopted for antenna scales of different magnitudes, which are able to help the iterative algorithm converge to the optimal solution vector at a faster rate.

4. Simulation Results

In the simulation, the massive MIMO uplink model of multi cell and multiuser is considered, in which the system has 5 hexagonal cellular cells. Each cell has a base station containing multiple antennas and serves 8 single antenna users, which sends a data sequence of length 60. And it is assumed that the small-scale fading between cells obeys Rayleigh distribution, and the large-scale fading coefficients are generated by random numbers which can be detected in advance. The channel state information (CSI) of the base station is accomplished by the channel estimation technology. Each user uses binary phase shift keying (BPSK) to modulate and transmit data. In order to show the performance of the proposed algorithm, it is compared with a variety of classical data detection algorithms. In general, the channel parameters used in the data detection process of this experiment are provided by the new adaptive matching pursuit (NAMP) algorithm [10].

![Figure 2. BER performance of the ZF, ZF-SIIC, MMSE, MMSE-SIC, and NTLS algorithms with diverse SNR.](image)

In fig.2, it shows the bit error rate (BER) performance at different signal to noise ratio (SNR). It can be found that the higher the SNR, the better the performance of each algorithm; at the same SNR, the channel parameters provided by the channel estimation have errors, resulting in that the lower the SNR, the error of data detection results is larger. As the proposed NTLS algorithm takes into account the influence of the channel estimation error and can minimize the impact, it is better than other algorithms, and when the SNR is low, the effect is more obvious. At the same time, the influence of channel on detection performance can be seen. Obviously, the transmission signal obtained by using the original channel is more accurate.

![Figure 3. BER performance of the ZF, ZF-SIIC, MMSE, MMSE-SIC, and NTLS algorithms with 16QAM and BPSK modulation methods.](image)
Fig. 3 presents the BER performance comparison among different algorithms under two diverse modulation methods, such as BPSK and quadrature amplitude modulation (16QAM). It is experimentally shown that the performance of the BPSK is better than that of the 16QAM at different SNR. And with the increase of SNR, the data detection performance gap under the two modulation modes will also increase. However, at low SNR, the two modulation methods do not affect the performance of the algorithms.

Fig. 4 shows the BER performance of the NTLS algorithm under different receiving antenna numbers. Although the advantage of massive MIMO is that it can greatly improve the system throughput as the number of antennas increases [1], simulation results confirm that increasing the number of receiving antennas will also optimize the detection quality, which is because the diversity gain of the system is proportional to the number of receiving antennas when the number of transmitting antennas is constant. Since the algorithm considers the characteristics of channel hardening, the performance is better when the number of antennas is larger.

**Table 1.** Running time of different SNR for diverse algorithms, $M=100$.  

|       | -5dB  | 0dB   | 5dB   | 10dB  | 15dB  | 20dB  | 25dB  |
|-------|-------|-------|-------|-------|-------|-------|-------|
| MMSE-SIC | 0.225 | 0.183 | 0.204 | 0.171 | 0.179 | 0.177 | 0.171 |
| MMSE   | 0.179 | 0.175 | 0.161 | 0.157 | 0.159 | 0.149 | 0.153 |
| ZF-SIC  | 0.191 | 0.147 | 0.154 | 0.160 | 0.149 | 0.146 | 0.151 |
| ZF      | 0.179 | 0.166 | 0.145 | 0.142 | 0.139 | 0.138 | 0.145 |
| NTLS    | **0.159** | **0.153** | **0.136** | **0.129** | **0.127** | **0.121** | **0.130** |

**Table 2.** Running time of different antenna numbers, SNR=10dB.  

|       | $M=10$ | $M=20$ | $M=30$ | $M=40$ | $M=50$ | $M=60$ | $M=70$ |
|-------|--------|--------|--------|--------|--------|--------|--------|
| MMSE-SIC | 0.643  | 0.233  | 0.238  | 0.252  | 0.241  | 0.271  | 0.332  |
| MMSE   | 0.163  | 0.205  | 0.212  | 0.244  | 0.238  | 0.254  | 0.283  |
| ZF-SIC  | 0.162  | 0.176  | 0.182  | 0.217  | 0.236  | 0.250  | 0.271  |
| ZF      | 0.159  | 0.171  | 0.176  | 0.207  | 0.231  | 0.235  | 0.265  |
| NTLS    | **0.148** | **0.156** | **0.167** | **0.179** | **0.189** | **0.217** | **0.228** |

Table I intuitively illustrates the running time of each algorithm under different SNR, which indirectly reflects the computational complexity of each algorithm. One can see clearly that the lower the SNR, the longer the running time of each algorithm; under the same SNR, NTLS algorithm is superior to several other algorithms, and MMSE-SIC runs the longest, that is, the highest complexity.
Similarly, table II describes the influence of the number of different receiving antennas on the running time of various algorithms. On the one hand, increasing the number of receiving antennas will cost more running time of each algorithm, reflecting the more complex of the system. On the other hand, LTS algorithm is also superior to other data detection algorithms, which further validates the effectiveness of the algorithm.

5. Conclusion
In this paper, a NTLS-based data detection algorithm which does not require channel statistics is proposed for TDD massive MIMO uplink systems. In the first place, the NTLS method is adopted to minimize the impact of the channel estimation errors. In the next place, QR decomposition and channel hardening properties are applied respectively to simplify the algorithm complexity which can be propitious to implement on hardware. Numerical results verify the superiority of the proposed algorithm over the existing linear data detection algorithms, regardless of the complexity or detection performance.

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