Comparative Research on Characteristics of the Isolation Systems with Dry Friction Damping and with Vicious Damping under Base Excitation

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Abstract: As for the isolation problem of electronic equipments on vehicle, the vibration response characteristics of dry friction damping isolation system under base displacement excitation was analyzed in theory by harmonic balance method, and the displacement response was compared between the isolation systems with dry friction damping and vicious damping separately. The results show that the isolation system with small dry friction damping can’t meet the demands of displacement reduction close to the natural frequency, and it can realize full-frequency vibration isolation by improving dry friction damping when the lock frequency passes beyond the resonance frequency band. The results imply that the damping mechanism of dry friction isolator can’t be described only by dry friction damping, and the composite damping with dry friction and vicious damping is more appropriate.

1. Introduction
The electronic equipments on vehicle will subject to large vibration and shock environment when the vehicle running on off-road pavement. Under large vibration and shock, electronic devices may malfunction such as electric loose contact, wire distortion or dislocation, loose weld of electronic component and so on, which can reduce severely the reliability of the electronic devices. To ensure reliable work of the electronic devices on vehicle in rough environment, effective measures should be taken to isolate the vibration and absorb the shock except for strengthening of the electronic devices.

The main vibration isolators used for vibration isolation of electronic devices in armored vehicles include damping rubber type of vibration isolator and dry friction type of vibration isolator. Due to its vulnerability to high and low temperature, the damping capacity and elastic behavior of the damping rubber type of vibration isolator would get worse in extreme temperature used in armored vehicles. In addition, the rubber material is prone to aging gradually. Therefore, the damping rubber type of vibration isolator is recently replaced by dry friction type of vibration isolators, such as wire-cable vibration isolator [1], wire mesh vibration isolator [2,3] and structural dry friction vibration isolator. The wire-cable and wire mesh vibration isolators damp the vibration by the dry friction damping force between sliding wires. The structural dry friction vibration isolator [4] damps the vibration by the dry friction damping force between friction pairs constituted by metal structural parts. The theory analysis [5] and experimental research [6] was conducted by researchers.

The vibration response characteristics of dry friction damping isolation system under base displacement excitation was analyzed in this paper, and the displacement response was compared between the isolation systems with dry friction damping and vicious damping separately.
2. Forced Vibration of Single DOF System with Dry Friction Damping Induced by Base Excitation

The typical mechanical model of wire mesh isolator and wire-cable isolator is shown in Figure 1, in which the mechanical components include cubic nonlinear elastic force, vicious damping force, and dry friction force with hysteresis character. The mechanical model of the structural dry friction vibration isolator constituted by metal structural parts is shown in Figure 2, in which the elastic component is linear spring and the damping component is metal friction pairs.

\[ m\ddot{x} + k(x - y) + F\text{sgn}(\dot{x} - \dot{y}) = 0 \]  \hspace{1cm} (1)

And the base excitation is

\[ y = y_0 \sin(\omega t) \]  \hspace{1cm} (2)

Given \( x - y = z \), then

\[ m\ddot{z} + k\dot{z} + F\text{sgn}(\dot{z}) = m\omega^2 y_0 \sin(\omega t) \]  \hspace{1cm} (3)

The motion vibration isolation of the base is transformed into the forced vibration shown in formula (3), and the exciting force \( m\omega^2 y_0 \sin(\omega t) \) is the inertia force of the isolated mass induced by the displacement of the base.

The free vibration attenuates rapidly for the dry friction damping. The steady-state forced vibration of the system expressed by formula (3) is

\[ z = A\sin(\omega t - \varphi) \]  \hspace{1cm} (4)

The dry friction force \( F\text{sgn}(\dot{z}) \) can be expanded into Fourier series, and the primary harmonic component is taken into consideration. Then the dry friction force is

\[ F\text{sgn}(\dot{z}) \approx \frac{4F}{\pi} \cos(\omega t - \varphi) \]  \hspace{1cm} (5)

Substitute formulas (4) and (5) into (3), then

\[ (kA - m\omega^2 A)\sin(\omega t - \varphi) + \frac{4F}{\pi} \cos(\omega t - \varphi) = m\omega^2 y_0 \sin(\omega t) \]  \hspace{1cm} (6)
Considering the trigonometric function transformation, then
\[
\sin(\omega t) = \sin(\omega t - \varphi) = \sin(\omega t - \varphi) \cos \varphi + \cos(\omega t - \varphi) \sin \varphi
\]  
(7)

Substitute formula (7) into (6), then
\[
(k - m\omega^2)A = m\omega^2 y_0 \cos \varphi
\]
(8)
\[
\frac{4F}{\pi} = m\omega^2 y_0 \sin \varphi
\]
(9)

According to formulas (8) and (9), the amplitude and phase of the forced vibration is
\[
A = \sqrt{(m\omega^2 y_0)^2 - (\frac{4F}{\pi})^2}
\]
(10)
\[
\varphi = \arcsin\left(\frac{4F/m\omega^2 y_0}{\pi}\right)
\]
(11)

It’s seen from (9) that the steady-state vibration solution \( z \) of the system can be expressed by formula (4) on the condition that the inertia exciting force amplitude is not less than the dry friction force amplitude, that is \( m\omega^2 y_0 \geq \frac{4F}{\pi} \). As for the un-simplified dry friction force model \( F \) \( \text{sgn}(\dot{z}) \), the isolation system is locked when \( m\omega^2 y_0 \leq F \), and there is no relative displacement between the base and the isolated mass. The isolation system is unlocked when \( m\omega^2 y_0 > F \). From the above analysis, the unlock frequency \( \omega_u \) of the isolation system is
\[
\omega_u = \sqrt{\frac{F}{my_0}}
\]
(12)

According to formulas (2) and (4), the displacement of the isolated mass can be expressed as
\[
x = y_0 \sin(\omega t) + A \sin(\omega t - \varphi)
\]
(13)
\[
= x_0 \sin(\omega t + \psi)
\]
In which
\[
x_0 = \sqrt{A^2 + y_0^2 + 2Ay_0 \cos \varphi}
\]
(14)
\[
\psi = \arctan(-A \sin \varphi/(A \cos \varphi + y_0))
\]
(15)

And the displacement transmissibility \( T \) of the isolation system is
\[
T = \frac{x_0}{y_0} = \sqrt{1 + (A/y_0)^2 + 2A \cos \varphi / y_0}
\]
(16)

3. Comparison of the systems with dry friction damping and with viscous damping
The displacement transmissibility curves of the isolation system with dry friction damping is shown in Figure3, in which, the horizontal ordinate is frequency ratio \( \gamma = \omega / \omega_b \), and \( \omega_b = \sqrt{k/m} \) is the natural frequency. The following conclusions can be summarized based on Figure3:

1) The unlock frequency of the isolation system increases with the dry friction damping.

2) When the unlock frequency is less than the natural frequency of the system, the displacement transmissibility increases significantly while the exciting frequency close to the natural frequency. And
$T \to \infty$ while $\gamma \to 1$.

(3) All the displacement transmissibility curves pass through the point $(\sqrt{2}, 1)$ with no matter how much the dry friction damping is.

(4) The non-resonant vibration isolation at all frequency band can be achieved by increasing the dry friction damping in order that the displacement transmissibility curve passes beyond the displacement enlarging frequency band when the unlock frequency larger than $\sqrt{2}$ times of natural frequency of the system. Meanwhile, the displacement isolation effectiveness at high frequency band would be reduced when continuing increasing the dry friction damping. And the lower dry friction damping is better to the displacement isolation when the unlock frequency larger than $\sqrt{2}$ times of natural frequency of the system.

Figure 3. Displacement transmissibility curves

Figure 4. Transition of the displacement transmissibility curves
Compared with the vibration isolation with viscous damping, there exists locking phenomenon in the isolation system with dry friction damping, and relative displacement between the isolated mass and base exists only when the exciting frequency larger than unlock frequency. Furthermore, the displacement transmissibility increases significantly near the natural frequency of the system when the dry friction is small, which is similar to the undamped vibration isolation system.

The dry friction damping force in formula (5) is the primary harmonic component, which can be equivalent to viscous damping:

\[ F \text{ sgn}(\dot{z}) \approx \frac{4F}{\pi} \cos(\omega t - \varphi) = \frac{4F}{A\omega \pi} \dot{z} \]  

(17)

The equivalent viscous damping coefficient is

\[ \bar{c} = \frac{4F}{A\omega \pi} \]  

(18)

The exhausted energy of the equivalent viscous damping in one vibration period is

\[ \Delta E_c = \oint \frac{4F}{\pi} \cos(\omega t - \varphi) dz = 4FA \]  

(19)

It’s shown from the above analysis that the exhausted vibration energy by the equivalent viscous damping is the same as the dry friction damping.

The formula (18) shows that the equivalent viscous damping coefficient is relevant not only to the dry friction force, but also to the relative displacement amplitude and exciting frequency of the isolation system. On the condition that the exciting frequency \( \omega \) is close to \( \omega_0 \) and \( \omega < \omega_0 \), the relative displacement amplitude \( A \) increases to \( A + \Delta A \) with \( \omega \) increasing to \( \omega + \Delta \omega \). And the equivalent viscous damping coefficient is

\[ \bar{c}_i = \frac{4F}{(A + \Delta A)(\omega + \Delta \omega)\pi} = \bar{c} - \Delta \bar{c} \]  

(20)

Due to \( \bar{c}_i < \bar{c} \), the equivalent viscous damping coefficient decreases and the displacement transmissibility transits from the value corresponding to the viscous damping coefficient \( \bar{c} \) (relevant to frequency \( \omega \)) to the value corresponding to \( \bar{c}_i \) (relevant to frequency \( \omega + \Delta \omega \)), as shown in Figure 4. The displacement transmissibility trends up to infinite with \( \omega \) close to \( \omega_0 \).

**4. Conclusion**

Vibration response characteristics of the isolation system with dry friction damping under base displacement perturbation was analyzed using harmonic balance method, and the displacement response was compared between the isolation systems with dry friction damping and viscous damping separately. The results show that the isolation system with small dry friction damping can’t meet the demands of displacement reduction near the natural frequency, and it can realize full-frequency vibration isolation when the lock frequency passes beyond the resonance frequency band by improving dry friction.

The damping mechanism of the dry friction type of vibration isolator used in engineering is more complex, and the dissipative mechanism cannot be described only by dry friction damping. The composite damping with dry friction and viscous damping is more appropriate for the dry friction type of isolator, and the displacement transmissibility at the frequency close to the natural frequency of the system is similar to the system with viscous damping.
5. References

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