Nonlinear dimensionality reduction for the acoustic field measured by a linear sensor array

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Abstract. Dimensionality reduction is one of the central problems in machine learning and pattern recognition, which aims to develop a compact representation for complex data from high-dimensional observations. Here, we apply a nonlinear manifold learning algorithm, called local tangent space alignment (LTSA) algorithm, to high-dimensional acoustic observations and achieve nonlinear dimensionality reduction for the acoustic field measured by a linear sensor array. By dimensionality reduction, the underlying physical degrees of freedom of acoustic field, such as the variations of sound source location and sound speed profiles, can be discovered. Two simulations are presented to verify the validity of the approach.

1 Introduction

With the development of sensor technique and materials science, the cost of acoustic sensors becomes constantly decreasing. In order to obtain higher spatial resolution and signal-to-noise ratio, more receiver arrays with large aperture and densely distributed sensors are applied to acoustic observations\textsuperscript{[1,2]}, which often makes acousticians confronted with large volumes of high-dimensional data. Consequently, dimensionality reduction for acoustic fields becomes much more important than ever before, which aims to discover meaningful low-dimensional structures that underlie high-dimensional observations. However, because the dependence of acoustic fields on physical parameters, like sound source location and sound speed profile, is generally nonlinear, traditional dimensionality reduction methods, such as principal component analysis (PCA)\textsuperscript{[3]} and multidimensional scaling (MDS)\textsuperscript{[4]}, are not capable of finding nonlinear degrees of freedom.

Some new dimensionality reduction methods have been proposed since the 21th century. Among these new approaches, manifold learning algorithms have attracted extensive attention because of their nonlinear nature, geometric intuition, and computational feasibility. The crucial assumption of manifold learning algorithms is that the input data lie on or close to a smooth low-dimensional manifold\textsuperscript{[5]}. Each manifold learning algorithm attempts to preserve a certain geometric property of underlying manifold. Isometric feature mapping (ISOMAP)\textsuperscript{[6]} try to preserve the geodesic distances of all pairs of data points. Because ISOMAP requires computing the shortest path between all pairs of points on the manifold, it is computationally expensive and can only be applied to intrinsically flat manifolds. In order to overcome the weaknesses of ISOMAP, some local approaches, such as locally linear embedding (LLE)\textsuperscript{[7]} and Laplacian eigenmaps (LE)\textsuperscript{[8]}, have been presented. LLE and LE aim to preserve the neighbourhood relationship among the data. Yan\textsuperscript{[9]} presents a general formulation known as graph embedding to unify them within a common framework. Inspired by LLE, Zhang and Zha\textsuperscript{[10]} propose a new algorithm, called local tangent space alignment (LTSA), which describes local properties of the high-dimensional data using the local tangent space of each data point. LTSA algorithm simultaneously searches for the coordinates of the low-dimensional data representations, and for the linear mappings of the low-dimensional data points to the local tangent space of the high-dimensional data\textsuperscript{[11]}. Compared to LLE, LTSA algorithm shows some improved performance\textsuperscript{[10]}.

In this article, we apply LTSA algorithm to high-dimensional acoustic observations and achieve nonlinear dimensionality reduction for the acoustic field measured by a linear sensor array. It turns out that LTSA algorithm can discover the intrinsically physical degrees of freedom of acoustic fields, such as the variations of sound source location and sound speed profile, which underlies in high-dimensional observation space. The validation of the approach is verified by two simulations in a three-dimensional axial symmetry shallow water waveguide.

The remainder of the paper is organized as follows. In sec.2, the mathematic frameworks of dimensionality reduction and LTSA algorithm are introduced. Sec.3 presents two stimulations to verify the validity of the approach. Finally, a conclusion is given in sec.4.

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2 Dimensionality reduction and LTSA algorithm

2.1 Dimensionality reduction problem for the acoustic field measured by a linear sensor array

Dimensionality reduction problem for the acoustic field measured by a linear sensor array is introduced in this section. Without loss of generality, we take the acoustic field measured by a vertical linear array as an example.

Given a set of acoustic field vectors \(x_i, x_2, \ldots, x_n\) in \(\mathbb{R}^r\) observed by a vertical linear array,

\[ x_i = [p(z_i; \gamma_i), p(z_i; \gamma_2), \ldots, p(z_i; \gamma_n)], \quad i = 1, 2, \ldots, m \] (1)

where \(p(z_i; \gamma_j), j = 1, 2, \ldots, n\) denotes the acoustic field recorded by the \(j\)th sensor of vertical linear array. \(z_i\) denotes the depth of the \(i\)th sensor and \(\gamma_j \in \mathbb{R}^r\) represents the physical parameter vector that controls the \(j\)th observed acoustic fields, which may consist of sound source location, medium parameter and so on, the goal of dimensionality reduction is to map these points to a set of points \(y_1, y_2, \ldots, y_n\) in \(\mathbb{R}^l (l \ll n)\). Manifold learning algorithms assume that \(x_1, x_2, \ldots, x_n \in M\) and \(M\) is a nonlinear manifold embedded in \(\mathbb{R}^r\).

2.2 LTSA algorithm

In this section, we will briefly review the mathematic framework of LTSA algorithm. If readers would like to know much more details about the algorithm, please refer to [10].

Given the data set \(X = [x_1, x_2, \ldots, x_n]\) in \(\mathbb{R}^r\), for each point \(x_i\), we denote the set of its \(i\) nearest neighbours by a matrix \(X_i = [x_1, x_2, \ldots, x_i]\). To preserve the local structure of each \(X_i\), we should compute the local linear approximation for the data points in \(X_i\) using tangent space. We have

\[
\text{arg min}_{x \in X_i} \sum_{x_j \in X_i} \left\| x - (x + Q \theta) \right\|_2 = \text{arg min}_{x \in X_i} \left\| X_i H_i - Q \theta \right\|_2
\] (2)

where \(H_i = I - \epsilon e e' / k \), \(e\) represents the vector of all 1’s, \(Q\) is an orthonormal basis matrix of the tangent space and has columns, and \(\theta = [0, 0, \ldots, 0]\), where \(\theta\) is the local coordinate corresponding to the basis \(Q\). The optimal \(x\) in the above optimization is given by \(X_i\), the mean of all \(x_i\’,s\) and the optimal \(Q\) is given by \(\theta\), the matrix of \(l\) left singular vectors of \(X_i H_i\) corresponding to its \(l\) largest singular values, and \(\theta\) is given by \(\theta\) defined as

\[
\theta_i = Q' X_i H_i = \left[\theta^{(1)}_i, \theta^{(2)}_i, \ldots, \theta^{(m)}_i\right], \quad \theta^{(i)}_i = Q' (x_i - \bar{x}_i)
\] (3)

Note that, the algorithm just mentioned essentially performs a local principal component analysis; the \(\theta_i\)s are the projections of the points in a local neighbourhood on the local PCA. Then, we can construct the global coordinate \(y_i, y_2, \ldots, y_n\) in \(\mathbb{R}^l\) based on the local coordinate \(\theta_i\), which represents the local geometry,

\[
y_j = y_i + L \theta^{(j)}_i + e^{(j)}_i, \quad j = 1, 2, \ldots, k, \quad i = 1, 2, \ldots, m
\] (4)

where \(y_j\) is the mean of \(y_j\), \(L_i\) is a local affine transformation matrix that need to be determined, and \(e^{(j)}_i\) is the local reconstruction error. Let

\[
Y_i = [y_1, y_2, \ldots, y_n] \quad \text{and} \quad E_i = [e^{(1)}_1, e^{(2)}_1, \ldots, e^{(m)}_1],
\]

we have

\[
Y_i H_i = L_i \Theta_i + E_i
\] (5)

To preserve as much of the local geometric properties in the low-dimensional feature space as possible, we intend to find \(Y_i\) and \(L_i\) to minimize the reconstruction errors \(e^{(j)}_i\), i.e.

\[
\text{arg min}_{i,j} \sum_{i,j} \left\| E_i \right\|_2 = \text{arg min}_{i,j} \sum_{i,j} \left\| Y_i H_i - L_i \Theta_i \right\|_2
\] (6)

Therefore, the optimal affine transformation matrix has the form \(L_i = Y_i H_i \Theta_i\), and \(E_i = Y_i H_i (1 - \Theta_i \Theta_i)\), where \(\Theta_i\) is the Moore-Penrose generalized inverse of \(\Theta_i\).

Let \(Y = [y_1, y_2, \ldots, y_n]\) and \(S_i\) be the 0-1 section matrix such that \(Y S_i = y_i\). The objective function is converted to this form

\[
\text{arg min}_{i} \sum_{i} \left\| E_i \right\|_2 = \text{arg min}_{i} \left\| S W W^T \right\|_F
\] (7)

where \(S = [S_1, S_2, \ldots, S_n]\) and \(W = \text{diag}(W_1, W_2, \ldots, W_n)\) with \(W_i = H_i (1 - \Theta_i \Theta_i)\). In order to uniquely determine \(Y\), we will impose the constraint \(Y^T = I\). It turns out that the vector \(e\) of all 1’s is an eigenvector of

\[
B = S W W^T S^T
\] (8)

corresponding to a zero eigenvalue. Therefore the optimal \(Y\) is given by the \(l\) eigenvectors of \(B\) corresponding to the 2nd to \(l + 1\) smallest eigenvalues.

3 Simulations

In this section, we will give two simulations to verify the validity of the approach. Both of them are carried out in a three-dimensional axial symmetry shallow water waveguide. The intrinsically physical parameters that control the observed acoustic fields are sound source location and sound speed profile, respectively. All of acoustic data used in the two simulations are computed by KRAKEN programming[12] and the source frequency is fixed at 500Hz.

3.1 The variations of sound source location

This simulation is conducted to verify that LTSA algorithm can discover the variations of sound source location which actually controls the observed acoustic fields. The diagram of the simulated environment is illustrated in Fig.1. The sound speed profile of water
column and the parameters of fluid halfspace bottom (density 1.8g/cm$^3$, sound speed 1668m/s and attenuation coefficient 0.5dB/λ) are shown in Fig.1(a). The source-receiver configuration is depicted in Fig.1(b). The vertical linear array consists of 101 hydrophones spanning the whole water column with uniform intersensor spacing of 1m. The location of sound source is variable in the rectangular shadow region as indicated by ‘I’ in Fig.1(b).

Let $\mathbf{p}_i$ be the complex pressure vector generated by simulation for the vertical linear array when sound source locates at $\gamma_i$, 

$$
\mathbf{p}_i = \left[ p(z_1; \gamma_i), p(z_2; \gamma_i), \ldots, p(z_{101}; \gamma_i) \right]^T, \quad i = 1, 2, \ldots, m
$$

where $\gamma_i = [r_i, z_i]^T$, $r_i \in [1, 600]$ m and $z_i \in [50,100]$ m denotes the range and depth of sound source corresponding to the $i$th observation respectively. The rectangular shadow region ‘I’ at which sound source locates is sampled with the fixed step size of $\Delta r = 3$ m and $\Delta z = 1$ m so that we can get $m = 10200$ complex pressure vectors. We use the real parts of normalized complex pressure vectors as the input to LTSA algorithm, i.e.

$$
\mathbf{x}_i = \text{Re} \left( \frac{\mathbf{p}_i}{\sqrt{\mathbf{p}_i^H \mathbf{p}_i}} \right), \quad i = 1, 2, \ldots, m
$$

where the symbol $^H$ denotes the conjugate transpose.

Note that the observed acoustic field denoted by $\mathbf{x}_i$ is a 101-dimensional vector, while the intrinsically physical parameter vector $\gamma_i$ which controls the acoustic field is in fact two-dimensional. The goal of LTSA algorithm is to find the underlying low-dimensional nonlinear structure. The two-dimensional result of dimensionality reduction using LTSA algorithm with 12 nearest neighbours is given in Fig.2(b). As a comparison, the result of PCA is also given in Fig.2(a). The corresponding variations of sound source location are given in Fig.2(c). The red and blue lines denote the variations along the depth-direction at a certain range and the range-direction at a certain depth, respectively. It can be seen that LTSA algorithm can successfully discover the low-dimensional structure hidden in the high-dimensional observations. The result of LTSA shows clearly the variations along two directions and the low-dimensional structure is relatively smooth. In contrast, the result of PCA is more distorted because of the intrinsically nonlinear dependence of acoustic fields on physical parameters.

3.2 The variations of sound speed profile

In this section, we will use LTSA algorithms to discover the low-dimensional variations of sound speed profile from the high-dimensional acoustic observations generated by simulation. Fig.3(a) shows the average sound speed profile of water column. The parameters of waveguide, fluid halfspace bottom and source-receiver configuration are described in Fig.3(b). The depth of
sound source is fixed at 35m as indicated by the red star in Fig.3(b) and the vertical linear array consists of 36 hydrophones spanning the whole water column with uniform inter-sensor spacing of 1m. The distance between source and array is 5km.

Fig. 3. (a) Sound speed profile of water column (b) The parameters of fluid halfspace bottom and source-receiver configuration.

In order to depict the variations of sound speed profile quantitatively, we introduce the empirical orthonormal functions (EOFs) derived from an experimental dataset. The first two order EOFs are given in Fig.4. We model the sound speed profile \( c_i(z) \) corresponding to the \( i \)th observation as follows

\[
c_i(z) = c_0(z) + \sum_{j=1}^{2} \beta_j \Psi_j(z) \tag{11}
\]

where \( c_0(z) \) denotes the average sound speed profile, \( \Psi_j(z) \) represents the \( j \)th order EOF and the corresponding coefficient is given by \( \beta_j \). So, the intrinsic parameters which control the variations of sound speed profile as well as observed acoustic fields are \( \beta_1 \) and \( \beta_2 \) in this simulation, i.e. \( \gamma = [\beta_1, \beta_2]^T \).

Let \( \beta_1 \in [-10, 10] \) and \( \beta_2 \in [-5, 5] \). We sample the two intervals with fixed step size of \( \Delta \beta_1 = 0.2 \) and \( \Delta \beta_2 = 0.1 \), so that 10201 observed acoustic fields are simulated. We use the same input \( x \) to LTSA algorithm as those used in section 3.1.

Fig. 4. The first two order EOFs derived from an experimental dataset.

Fig. 5. (a) The result of PCA. (b) The result of LTSA. Both of them are dimensionless. (c) The variations of \( \beta_1 \) and \( \beta_2 \).

The two-dimensional results of dimensionality reduction using PCA and LTSA with 12 nearest neighbours are shown in Fig.5(a) (b). The corresponding variations of \( \beta_1 \) and \( \beta_2 \) are given in Fig.5(c). The red and blue lines indicate the variations along the \( \beta_2 \)-direction at \( \beta_1 = 0 \) and the \( \beta_1 \)-direction at \( \beta_2 = 0 \), respectively. Similar
to the results shown in Fig.2, LTSA algorithm successfully discovers the two-dimensional structure whose geometric property is obviously smoother than that found by PCA.

4 Conclusions

In this paper, we have applied a manifold learning algorithm, called LTSA, to the acoustic fields measured by a linear sensor array and achieved nonlinear dimensionality reduction. Compared with the traditional dimensionality reduction methods, such as PCA, LTSA algorithm can successfully discover the underlying physical degrees of freedom which actually control observed acoustic fields. Two simulations are conducted to verify the validation of the method. The approach presented here is likely to be more promising when combined with other machine learning algorithms that focus on source localization and geoacoustic inversion.

References

1. G. F. Edelmann, J. S. Rogers, S. L. Means, J. Acoust. Soc. Am., 136(4), 2214-2214 (2014)
2. J. E. Quijano, L. M. Zurk, J. Acoust. Soc. Am., 138(3), 1840-1840 (2015)
3. I. T. Jolliffe, Principal Component Analysis (Springer, New York, 1986).
4. T. Cox, M. Cox, Multidimensional Scaling (Chapman & Hall, London, 1994)
5. H. S. Seung, D. D. Lee, Science, 290,2268-2269 (2000)
6. J. Tenenbaum, V. De Silva, J. Langford, Science, 290, 2319-2323 (2000)
7. S. Roweis, L. Saul, Science, 290, 2323-2326 (2000)
8. M. Belkin, P. Niyogi, Neural. Comput., 15(6), 1373-1396 (2003)
9. S. Yan, D. Xu, B. Zhang, H. J. Zhang, Q. Yang, S. Lin, IEEE Trans. Pattern Anal. Mach. Intell., 29(1), 40-51 (2007)
10. Z. Zhang, H. Zha, SIAM J. Sci. Comput., 26 (1), 313-338 (2004)
11. L.V. der Maaten, E. Postma, J. V. den Herik, J. Mach. Learn. Res., 10, 66-71 (2009)
12. M.B. Porter, The KRAKEN Normal Mode Program, http://oalib.hlsresearch.com/Modes