1
The Gregory-Laflamme instability
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In this chapter we introduce the notion of higher dimensional gravity in the context of one extra spatial dimension. We focus on the black string: a simple extension of the Schwarzschild solution into five dimensions. Here, we will show that this solution is unstable to long wavelength perturbations, and discuss its implications and extensions.

1.1 Overview
Kaluza [1] and Klein [2], very shortly after Einstein’s general relativity had been verified by Eddington’s 1919 expedition, suggested that adding an extra dimension to our space could have an amazing consequence: ‘Gravity’ in five dimensions with the extra dimension stabilized has the appearance of Einstein-Maxwell theory in a four dimensional slice. Kaluza-Klein theory, as it is now known, is a construction adding extra dimensions to space which are much smaller than scales we can directly physically probe, and thus contribute only a few long range, or massless, additional forces to nature. Kaluza-Klein theory is reviewed in full in Chapter 4, but for the purposes of this chapter, we will only require some very basic intuition. We will consider only solutions to vacuum gravity in five dimensions, and focus on a particularly simple solution: the black string. We will describe the solution, discuss its properties, then demonstrate explicitly that it is unstable to linear perturbations. We conclude with a brief discussion of the more general situation.

1 Chapter of the book Black Holes in Higher Dimensions to be published by Cambridge University Press (editor: G. Horowitz).
1.2 Black holes in higher dimensions

The Schwarzschild solution in four dimensions is found by solving the vacuum Einstein equations

\[ R_{\mu\nu} = 0, \]  

subject to a physically motivated spherical symmetry restriction. It is known that in four dimensions, the only possible static black hole solution must be spherically symmetric – but what happens if we live in four, or more, spatial dimensions? In particular, what happens if that final spatial dimension has finite extent and is very small?

In five dimensions, we obviously have to solve the same equation

\[ R_{ab} = 0. \]  

However, we now have to choose an appropriate symmetry for the spacetime metric. An obvious generalisation of the Schwarzschild solution is to take a hyperspherically symmetric solution:

\[ ds^2 = -V_5(r) dt^2 + V_5(r)^{-1} dr^2 + r^2 d\Omega_3^2, \]

where \( V_5 \) is an appropriate generalisation of the four dimensional Schwarzschild potential. This is indeed the case (see Chapter 5 or [3] for a discussion of general solutions), if the extra dimension is infinite, when \( V_5 \) is given explicitly by

\[ V_5(r) = 1 - \frac{r_5^2}{r^2}, \]

where \( r_5 \) is the horizon radius, and is related to the mass of the black hole via [3],

\[ r_5^2 = \frac{8G_5 M_5}{3\pi}. \]

However, there is another simple solution we can easily guess, based on the properties of the Riemann tensor. If we assume that nothing depends on the extra dimension, we can consider a spacetime of the form

\[ ds^2 = g_{\mu\nu}(x^\mu) dx^\mu dx^\nu + dz^2, \]

for which the Riemann tensor has only four dimensional components. In this case, a solution of the four-dimensional Einstein equations will automatically

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2 We reserve the labels \( \mu, \nu \ldots \) for purely four dimensional indices, and use \( a, b \ldots \) for the full range of dimensions.
be a solution of the five-dimensional Einstein equations, as $R_{5\alpha} = 0$ by construction. Thus, we can extend the four-dimensional Schwarzschild solution uniformly into the extra dimension to obtain a black string:

$$ds^2 = -V(r)dt^2 + V(r)^{-1}dr^2 + r^2d\Omega_2^2 + dz^2,$$  \hspace{1cm} (1.7)

where

$$V(r) = \left(1 - \frac{r_+}{r}\right)$$ \hspace{1cm} (1.8)

is the Schwarzschild potential introduced in the previous chapter.

This may seem a rather unremarkable observation, as it is a straightforward solution to write down. However, it is the first sign that gravity in higher dimensions may have distinctively new phenomena to offer. In four dimensions, black holes are essentially unique, classified by very few parameters: mass, charge and angular momentum. Once these parameters are specified, the solution is known, and the horizon topology is always spherical. Here, by adding an extra dimension, with very little effort we have constructed two distinct black objects: one with a spherical and one with a cylindrical event horizon. Clearly, these black holes have different masses: the black string has (strictly) infinite mass and is not asymptotically flat, however, our simple construction shows that event horizons in higher dimensions need no longer be spherical \cite{5}; Chapter 7 discusses the possible topologies of black objects. As we will see in Chapters 6 and 8, this first clue of non-uniqueness of ‘black’ solutions is the tip of the iceberg: many distinct black solutions with the same charges exist, see e.g. \cite{6, 7}.

Now let us consider what happens if our extra dimension is compact, i.e., finite and of length $L$. This will give the black string a finite length, hence mass, and in fact corresponds to the traditional Kaluza-Klein picture. The black string therefore corresponds to a basic Kaluza-Klein black hole; there is no dependence of the geometry on the extra dimension, which remains a Killing symmetry of the full solution. From the four dimensional viewpoint, it looks just like a Schwarzschild black hole. At energies of order $L^{-1}$, new physics is expected to come into play – physics corresponding to the additional degrees of freedom of the extra dimension. In our case, as we are considering only vacuum Einstein gravity, our new degrees of freedom correspond to a dependence of the geometry on the extra dimension. Alternatively, from a four dimensional perspective, these can be interpreted as a tower of massive gravitons. We can therefore ask if there are any alternative solutions to the black string which excite these extra degrees of freedom. An obvious starting point is the five dimensional ‘Schwarzschild’ solution, \cite{11,58}. Once we have a finite spatial direction, we do not expect exact hyper-
spherical symmetry, since the finite size of the extra dimension introduces an effective periodicity in one direction, and hence the black hole will interact with its mirror image black holes (see Fig. 1.1), altering the gravitational potential along the extra dimension. For $r_5 \ll L$, the five dimensional potential (1.4) will be a good approximation to the solution, but for larger black holes, the nonlinearity of gravity does not allow for an analytic solution, and the geometry must be found numerically. As the mass of the black hole increases further, it eventually can no longer fit inside the extra dimension, and must become a string-like solution. These solutions are known as caged black holes, and nonuniform black strings, and will be discussed further in Chapter 4 (see also \[8\]).

Therefore, within the context of five dimensional Kaluza-Klein theory, there seem to be various options for a simple uncharged black hole. It can be a straight or nonuniform black string, or a caged black hole, but which is the physically relevant one, or which will form in a collapse process? Are all of these solutions possible, or is there a selection process which rules out one or the other?

The answer is provided by the black string instability: dependent on the size of the extra dimension and the mass of the black hole, there is a unique stable solution, and hence a unique preferred end state for gravitational collapse. The rest of this chapter is devoted to explaining why the instability is natural, and proving that it exists.

1.3 A thermodynamic argument for instability

Typically in nature, we decide which is the most likely state by determining which has the lowest energy, however, in the case of black holes and
black strings, both solutions can have the same energy, therefore a different physical principle must be used. In the last chapter, we saw how black holes could be assigned thermodynamic properties, and in thermodynamics it is the state with largest entropy which is preferred. Thus, if there is an entropy difference between the two states, then we might expect that one is preferred over the other.

In the previous chapter, the entropy of a black hole was shown to be proportional to its area: \( S = A/4 \) in Planck units. However, there is a subtlety if we are dealing with Kaluza-Klein theory; this formula actually contains a hidden Newton’s constant, and the Planck mass is dimension dependent in a compactification:

\[
M_\text{p}^2 = V_{D-4} M_D^{D-2},
\]

where \( V_{D-4} \) is the volume of the internal space on which we are compactifying, and \( D \) is the total number of dimensions. For the five dimensional case we are considering here, this gives

\[
G_5 = L G_4 = L,
\]

(1.10)

(where we have set \( G_4 = 1 \) in the last step) and hence what we mean by the Planck scale is renormalized by a factor of \( L \). We therefore obtain for the entropies of the black hole and black string (assuming, for simplicity, that the black hole is approximated by its exact analytic form (1.3)):

\[
S_{BH} = \frac{\pi}{2} r_5^3 5/2, S_{BS} = \pi r_+^2,
\]

(1.11)

where \( r_5 \) and \( r_+ \) are the horizon radii of the black hole and string respectively. We now need to compare these entropies for the same total mass of hole or string. The black string has a mass of \( r_+ / 2 \), and using (1.5) and (1.10), the black hole mass is

\[
M_5 = \frac{3 \pi r_5^3}{8L}.
\]

(1.12)

Hence, setting the masses equal, the entropies can be re-expressed as:

\[
S_{BH} = 4 \pi M^2 \sqrt{\frac{8L}{27 \pi M}}, S_{BS} = 4 \pi M^2.
\]

(1.13)

Clearly, if \( L \) becomes sufficiently large, the black hole will be thermodynamically preferred over the black string, and hence the black string solution should have a long wavelength instability. In order to prove this we have to look at perturbations around the black string solution.
1.4 Perturbing the black string

Now we will show explicitly that the black string is unstable. In order to do this, we perturb the metric (1.7), and solve the linearized Einstein equations to show that there is a growing mode.

There are several issues to bear in mind when considering perturbation theory in general relativity. First of all, a perturbation must be “small”, this may seem to be a statement of the obvious, but when a change of coordinates can make the components of a tensor large, we must be careful to interpret correctly what “small” means. Secondly, we need to specify an initial data surface for our perturbation problem, which as we will see ties in with regularity of the perturbation, and is easily resolved by choosing an appropriate Cauchy surface. Finally, gravity has an infinite gauge group, i.e., there are an infinite set of different coordinate transformations we can perform on any particular geometry, thus there will be many perturbations which are pure gauge – in other words, the act of changing coordinates gives a perturbation to the metric, but one that is not physical. We must therefore be careful to determine whether our perturbation is physical.

1.4.1 Perturbation theory

We begin by defining the perturbation. In Einstein gravity, a small perturbation of a spacetime is represented by a change in the metric:

\[ g_{ab} \rightarrow g_{ab} + h_{ab} \]  \hspace{1cm} (1.14)

under which the Ricci tensor acquires a perturbation

\[ R_{ab} \rightarrow R_{ab} - \frac{1}{2} \Delta_L h_{ab} \]  \hspace{1cm} (1.15)

where \( \Delta_L \) is the Lichnerowicz operator,

\[ \Delta_L h_{ab} = \Box h_{ab} + 2R_{acbd}h^{cd} - 2R_{[a}^{\quad c}h_{b)c} - 2\nabla_{(a}\nabla^{c}h_{b)c} + \nabla_{a}\nabla_{b}h \]  \hspace{1cm} (1.16)

the curved space wave operator for a spin two massless particle. Clearly, a perturbation of a vacuum spacetime must obey \( \Delta_L h_{ab} = 0 \).

For the black string, the facts that the Ricci tensor is zero (as the string is a solution of the vacuum Einstein equations), and that there are no \( z \) components of the Riemann tensor, will simplify the equations considerably. In addition, since we are in vacuum we can also choose the “transverse
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which further simplifies (1.16) to

\[ \Delta_{\ell} h_{ab} \rightarrow \Box h_{ab} + 2R_{acbd}h^{cd} = 0. \] (1.18)

It is now a matter of some algebraic computation and manipulation to compute the perturbation equations component by component, using the gauge choice to simplify equations where relevant.

As is standard practice, we use a separation of variables method, and decompose the perturbation in terms of the symmetries, or Killing vectors, of the background geometry. The black string has both time and \( z \)-translation invariances, as well as an \( SO(3) \) isometry corresponding to the four dimensional spherical symmetry of the Schwarzschild solution. For simplicity (and with the benefit of hindsight!) we consider spherically symmetric perturbations, since the entropy argument indicates that the instability should manifest at this level. This means that \( h_{ab} \) has no cross terms with an angular coordinate, and has the form:

\[
h_{ab} = \begin{bmatrix}
h_{\ell \ell} & h_{\ell r} & 0 & 0 & h_{\ell z} \\
h_{r \ell} & h_{rr} & 0 & 0 & h_{rz} \\
0 & 0 & h_{\theta \theta} & 0 & 0 \\
h_{t \ell} & h_{r \ell} & 0 & 0 & h_{zz}
\end{bmatrix}
\] (1.19)

In addition, the \( t \)- and \( z \)-translation symmetries allow us to factor out an oscillatory \( e^{i \omega t} \) behaviour, and a growing \( e^{\Omega t} \) mode, corresponding to an unstable perturbation.

### 1.4.2 Finding the perturbation

The full set of equations is rather lengthy, and not particularly illuminating, so we refer the reader to the original literature for the details, [4]. For the purposes of this review, we note that the perturbations \( h_{zz} \) and \( h_{\ell \mu} \) must vanish for any unstable mode. To see this is particularly straightforward for the \( h_{zz} \) perturbation as the lack of Riemann components in the \( z \)-direction means that the equation for this component decouples. Writing

\[ \nabla_{a} h_{b}^{a} - \frac{1}{2} \nabla_{b} h = 0, \]

just not the individual parts separately.
\[
\frac{d}{dr} \left( \frac{2r - r_+}{r - r_+} \frac{dh}{dr} \right) - \left( m^2 r (r - r_+) + \Omega^2 r^2 \right) \frac{h}{(r - r_+)^2} = 0. 
\tag{1.20}
\]

This equation has asymptotic solutions

\[
h \sim e^{\pm \sqrt{\Omega^2 + m^2} r} \quad \text{as} \quad r \to \infty,
\]

\[
h \sim (r - r_+)^{\pm \Omega r_+} \quad \text{as} \quad r \to r_+. 
\]

Clearly therefore, any regular solution must vanish at the horizon and infinity, with a turning point at some finite \( r \) at which \( h''/h < 0 \). However, examination of (1.20) shows that \( h''/h > 0 \) at any turning point, hence no such solution exists. A similar argument shows that \( h_{\mu} \) must also vanish.

We are now left with a perturbation which has only four dimensional components, \( h_{\mu \nu} = e^{imz} e^{i\Omega t} H_{\mu \nu}(r) \). After imposing the gauge constraints, the equations of motion reduce to a pair of first order ODE’s plus one constraint:

\[
H_+ = H_- \left( \frac{2r^2 \Omega^2 + r^2 m^2 V - (1 - V^2)/2}{(r^2 m^2 + 1 - V)} \right) - rH \left( \frac{4\Omega^2 + m^2(1 - 3V)}{r^2 (r^2 m^2 + 1 - V)} \right)
\tag{1.21}
\]

\[
H' = \frac{\Omega(H_+ + H_-)}{2V} - \frac{(1 + V)H}{rV} 
\tag{1.22}
\]

\[
H_-' = \frac{m^2 H}{\Omega} + \frac{H_+}{r} + \frac{(1 - 5V)H_-}{2rV} 
\tag{1.23}
\]

in the variables

\[
H_\pm = \frac{H_{tt}}{V} \pm VH_{rr} 
\tag{1.24}
\]

\[
H = H_{tr}. 
\tag{1.25}
\]

Once again, reading off the asymptotic behaviour gives

\[
r \to \infty : \begin{cases} 
H & \sim \pm \sqrt{\Omega^2 + m^2} e^{\pm \sqrt{\Omega^2 + m^2} r} \\
H_- & \sim \frac{m^2}{\Omega} e^{\pm \sqrt{\Omega^2 + m^2} r}
\end{cases}
\tag{1.26}
\]

\[
r \to r_+ : \begin{cases} 
H & \sim (\pm \Omega r_+ - \frac{1}{2}) (r - r_+)^{\pm \Omega r_+ - 1} \\
H_- & \sim \left( \frac{m^2}{\Omega} \pm \frac{2}{r_+} \right) (r - r_+)^{\pm \Omega r_+}
\end{cases}
\tag{1.27}
\]

An instability therefore corresponds to a solution of (1.22), (1.23) which is regular at both the horizon and infinity, as determined by the asymptotic forms (1.26), (1.27).
1.4.3 Regularity conditions

In order to determine the regularity of the perturbation, we clearly need $h_{ab}$ to tend to zero at large $r$, picking out the exponentially decaying branch of (1.26), and also to be regular at the horizon. Surprisingly, this latter constraint is not equivalent to the perturbation being regular in a local orthonormal system, as it is easy to see from (1.27) that \( h_{\hat{t}\hat{r}} \) blows up as \( r \to r_+ \) for \( \Omega r_+ < 1 \). Instead, we have to check regularity in a locally regular coordinate system at the horizon. A convenient choice of coordinates is based on the Kruskal system:

\[
T = 2e^{r_+/2r_+} \sinh\left(\frac{t}{2r_+}\right) , \quad R = 2e^{r_+/2r_+} \cosh\left(\frac{t}{2r_+}\right) ,
\]

where

\[
r_+ = r - r_+ + r_+ \log(r - r_+) \quad (1.29)
\]

is the standard tortoise coordinate in the Schwarzschild metric (see Fig. 1.2).

Transforming to this new coordinate system, we see that

\[
h_{TT} \sim \mathcal{U}(R,T) \frac{R^2 h_{\hat{t}\hat{t}} - 2RT h_{\hat{t}\hat{r}} + T^2 h_{\hat{r}\hat{r}}}{(R^2 - T^2)} \quad (1.30)
\]

\[
h_{TR} \sim \mathcal{U}(R,T) \frac{R(T h_{\hat{t}\hat{t}} + h_{\hat{r}\hat{r}}) - (T^2 + R^2) h_{\hat{t}\hat{r}}}{(R^2 - T^2)} \quad (1.31)
\]

\[
h_{RR} \sim \mathcal{U}(R,T) \frac{T^2 h_{\hat{t}\hat{t}} - 2RT h_{\hat{t}\hat{r}} + R^2 h_{\hat{r}\hat{r}}}{(R^2 - T^2)} \quad (1.32)
\]

must be regular at the horizon, where $h_{\hat{t}\hat{t}} = h_{tt}/V$ refers to the components in a local orthonormal system, and

\[
\mathcal{U}(R,T) = \mathcal{U}(R^2 - T^2) = \frac{r_+^2}{e^{(r-r_+)/r_+}} \quad (1.33)
\]

is the new (non-singular) Kruskal gravitational potential.

Substituting in the near horizon behaviour gives

\[
h_{TT} \simeq h_{RR} \propto \pm (R \pm T)^{\Omega r_+ - 2} \quad (1.34)
\]

\[
h_{TR} \propto (R \pm T)^{\Omega r_+ - 2} \quad (1.35)
\]

and we see that the upper branch of (1.27) is always regular on the future event horizon (\( R = T \)), and the lower branch on the past event horizon. At the bifurcation point, where \( \mathcal{H}^+ \) and \( \mathcal{H}^- \) meet, corresponding to \( r = r_+ \) at finite \( t \), neither branch is strictly regular, and to exclude both would render the Lichnerowicz operator non self-adjoint. For simplicity, it is easiest simply
1.4.4 The instability

To determine the existence of the instability we must numerically integrate the perturbation equations (1.22) and (1.23) between the horizon and infinity, looking for a solution which approaches the regular horizon branch, and is exponentially decaying at infinity. We do not expect solutions for all $\Omega$ and $m$, since the thermodynamic argument indicates that an instability can only set in for $r_+ m < 32/27$. We expect a single characteristic frequency, $\Omega_m$, for any wavelength, thus we must scan through the values of $\Omega$ for each
m to check if a solution exists. Fig. 1.3 shows a plot of the frequency pairs \((m, \Omega)\) for which a regular solution, and hence an instability, exists, and Fig. 1.4 shows the behaviour of the perturbation.

Having found an unstable solution to the perturbation equations, the final step of the argument is to demonstrate that this is a physical instability of the black string, and not just some odd gauge mode. In fact, this is easy to demonstrate by looking at \((1.18)\). Since both the perturbation and the Riemann tensor vanish in the extra dimension \((h_{za} = 0 = R_{zabc})\), the five dimensional Lichnerowicz operator reduces to the four dimensional Lich-
nerowicz operator with a mass term:

\[ \Delta^{(5)}_{L} h_{\mu\nu} = \Delta^{(4)}_{L} h_{\mu\nu} + \frac{\partial^2}{\partial z^2} h_{\mu\nu} = \left[ \Delta^{(4)}_{L} - m^2 \right] h_{\mu\nu}. \]  

(1.36)

However, if \( h_{\mu\nu} \) is a gauge mode, it must correspond to a purely four dimensional change of coordinates, in other words, it can have no dependence on \( z \). Thus any solution of the massive four dimensional Lichnerowicz operator must be a physical Kaluza-Klein instability.

1.4.5 More general instabilities

So far, the discussion has been strictly in terms of five dimensional vacuum Einstein gravity. This approach was chosen so that the mathematics and physics of the instability would be clearer, but of course if an instability exists with one extra dimension, then it will exist more generally. In [4], it was shown how black branes, objects with arbitrary numbers of extra dimensions, would be unstable, with \( 1 - 6 \) extra dimensions focussed on for the purposes of applying to the string theoretic solutions found by Horowitz and Strominger [9].

These instabilities look very similar to the five dimensional case detailed here: the instability is once more restricted to a four dimensional s-wave, where now the effective mass term in (1.36) is a general eigenvalue of the symmetries in the extra dimensions, \( e^{m_{i}z_{i}} \). The details of the \((m, \Omega)\) plot vary, but the qualitative shape and features are the same (see [4]). Instabilities of charged solutions, analogous to the four dimensional Reissner-Nordstrom family of black holes (see Chapter 11) can also be found.

Interestingly, the instability does not require a translation invariance along the string or brane, it also applies to more general higher dimensional spacetimes. Essentially, all that is required is some sort of factorizability of the metric and wave operator [10], so that we can decompose the perturbation in terms of effective four dimensional quantities with suitable eigenfunctions of the extra dimensions:

\[ h_{ab} \rightarrow h_{\mu\nu} = u_{m}(z^{i})e^{\Omega} H_{\mu\nu}(r) \]  

(1.37)

where the Riemann tensor and wave operator also factorize so that a massive wave equation in the form of (1.36) is obtained for \( H \).
1.5 Consequences of the instability

In the previous section, we proved that an instability of the black string exists, in that there is a linear perturbation of the black string solution which is exponentially growing in our coordinate time, $t$. However, it is not clear what the effect of this growing mode will be on the event horizon, which is a coordinate singularity, and in fact corresponds to $t \to \infty$, $r \to r_+$ in Schwarzschild coordinates.

To explore the effect of the instability, we return to the Kruskal coordinates and check what happens to outgoing light rays near the original event horizon. In the unperturbed spacetime, null geodesics satisfy $R = \pm T + R_0$, with $R = T$ being the future event horizon, as indicated in Fig. 1.2. In the perturbed spacetime, the geodesic equation becomes

\[
\left( \frac{dR}{dT} \right)^2 = 1 + \frac{1}{\mathcal{U}} \left( h_{TT} + 2h_{TR}\dot{R} + h_{RR}\dot{R}^2 \right)
\]

\[
= 1 + \epsilon \cos mz (R + T)^{2r_+ \Omega - 2} \left( 1 + \frac{dR}{dT} \right)^2
\]

where $\epsilon$ is an (arbitrary) small parameter representing the size of the initial perturbation. From this, we see that the event horizon is schematically shifted to

\[
R = T + \epsilon \cos mz T^{2r_+ \Omega - 1}
\]

or, in Schwarzschild coordinates

\[
r = r_+ + \epsilon T^{2\Omega} \cos mz
\]

in other words, the ‘horizon’ begins to ripple (see Fig. 1.5).

This now has very interesting consequences. In four dimensional gravity, the event horizon cannot shrink in any classical process without violating positivity of energy. This black string instability is a classical process, so what is happening? Clearly, although the horizon is shrinking in some places along the black string, in other places it is growing, hence overall this classical process is growing the total area, as we would expect from the thermodynamic argument of entropic instability. However, increasing the area of an event horizon is not the only classical relativistic constraint. If we follow the instability to its logical endpoint, dictated by the entropy argument, we might expect that the black string will eventually fragment, forming a black hole caged within the larger fifth dimension. A simple conclusion perhaps, but at the moment when the horizon pinches off, the curvature at the horizon diverges, forming a naked singularity.
In the catalogue of four dimensional relativity, a pattern emerges where, apart from a few very well known examples such as the Big Bang, singularities tend to be clothed by an event horizon, and certainly singularities which form during a physical collapse. This led Penrose to conjecture \[11\] that a ‘censorship’ applies in gravitational collapse, which prevents any singularity forming which could be visible from infinity. Although a full proof of this conjecture remains elusive, any counter-examples that have been constructed are either unphysical in some way, or highly non-generic. The Cosmic Censor therefore has been assumed to be an omnipotent authority in classical gravity. Yet here, within the bounds of classical gravity, a physical, generic process has been shown to exist which strongly suggests a violation of cosmic censorship at the moment the string fragments into the black hole. For this reason, for many years after the discovery of the black string instability, the final fate of the black string was viewed as an open question \[12\], and cosmic censorship was an unknown factor in higher dimensional gravity. The story of what happens to the black string, and how the instability proceeds
requires a tour de force numerical simulation [13], a description of which forms the core of the next chapter.
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