Measurements of $CP$-Violating Amplitudes in the Decay $B^0 \to K^+K^-K^0$

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We analyze the decay $B^0 \to K^+ K^- K^0$ using 383 million $B \bar{B}$ events collected by the 
$\bar{B}$ABAR detector at SLAC to extract CP violation parameter values over the Dalitz plot. 
Combining all $K^+ K^- K^0$ events, we find $A_{CP} = -0.015 \pm 0.077 \pm 0.053$ and $\beta_{eff} = 0.352 \pm 0.076 \pm 0.026$ rad, 
corresponding to a CP violation significance of $4.8\sigma$. A second solution near $\pi/2 - \beta_{eff}$ is disfavored 
with a significance of $4.5\sigma$. We also report $A_{CP}$ and $\beta_{eff}$ separately for decays to $\phi(1020)K^0$, 
$f_0(980)K^0$, and $K^+ K^- K^0$ with $m_{K^+ K^-} > 1.1$ GeV/c$^2$.

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In the Standard Model (SM), the phase in the 
Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing ma-
trix is the sole source of CP violation in the quark sec-
tor. Due to interference between decays with and without 
mixing, this phase yields observable time-dependent CP 
asymmetries in $B^0$ meson decays. In particular, signif-
ificant CP asymmetries in $\to s\bar{s}s$ decays, such as $B^0 \to 
K^+ K^- K^0$ [2], are expected [2, 1]. Deviations from the 
predicted CP asymmetry behavior for $B^0 \to K^+ K^- K^0$ 
are expected to depend weakly on Dalitz plot (DP) po-
sition [2, 3]. Since the $\to s\bar{s}s$ amplitude is dominated 
by loop contributions, heavy virtual particles beyond the 
SM might contribute significantly [6, 7]. This sensitivity 
motivates measurements of CP asymmetries in multiple 
$\to s\bar{s}s$ decays [3, 4, 11].

Previous measurements of CP asymmetries in $B^0 \to 
K^+ K^- K^0$ have been performed separately for events 
with $K^+ K^-$ invariant mass $(m_{K^+ K^-})$ in the $B$ mass 
region, and for events excluding the $B$ region, neglecting 
interference effects among intermediate states [3, 4, 10]. 
In this Letter we describe a time-dependent DP analysis 
of $B^0 \to K^+ K^- K^0$ decay from which we extract the 
values of the CP violation parameters $A_{CP}$ and $\beta_{eff}$ by 
taking into account the complex amplitudes describing 
the entire $B^0$ Dalitz plots. We first extract the 
values of the parameters of the amplitude model, and 
measure the average CP asymmetry in $B^0 \to K^+ K^- K^0$ 
over the entire DP. Using this model, we then mea-
Sure the CP asymmetries for the $\phi K^0$ and $f_0 K^0$ decays, from a “low-mass” analysis of events with 
m_{K^+ K^-} < 1.1$ GeV/c$^2$. Finally, we perform a “high-
mass” analysis to determine the average CP asymmetry 
for events with $m_{K^+ K^-} > 1.1$ GeV/c$^2$.

The data sample for this analysis was collected with 
the 
$\bar{B}$ABAR detector [12] at the PEP-II asymmetry-energy 
e$^+ e^-$ collider at SLAC. Approximately $383 \times 10^6 B\bar{B}$ 
pairs recorded at the $T(4S)$ resonance were used.

We reconstruct $B^0 \to K^+ K^- K^0$ decays by combing 
two oppositely-charged kaon candidates with a $K^0$ 
reconstructed as $K^0_S \to \pi^+ \pi^- (B^0_{0(\pm)})$ [13], $K^0_S \to \pi^0 \phi$ 
($B^0_{0(0)}$), or $K^0_L (B^0_{L})$. Each $K^0_L \to \pi^0 \pi^0$ candidate is 
formed from two $\pi^0 \to \gamma \gamma$ candidates. Each photon 
have $H_\gamma > 50$ MeV and transverse shower shape consistent 
with an electromagnetic shower. Both $\pi^0$ candidates satisfy 
$100 < m_{\gamma \gamma} < 155$ MeV/c$^2$ and yield an invariant mass 
m_{\pi^0 + \pi^0} in the range $-20 < m_{\pi^0 + \pi^0} - m_{K^0} < 30$ MeV/c$^2$. 
A $K^0_L$ candidate is defined by an unassociated energy de-
posit in the electromagnetic calorimeter or an isolated 
signal in the Instrumented Flux Return [8].

For each fully reconstructed $B^0$ meson ($B_{CP}$), we use 
the remaining tracks in the event to reconstruct the decay 
vertex of the other $B$ meson ($B_{tag}$), and to identify its 
flavor $q_{tag}$ [3]. For each event we calculate the difference 
$\Delta t = t_{CP} - t_{tag}$ between the proper decay times of the 
$B_{CP}$ and $B_{tag}$ mesons, and its uncertainty $\sigma_{\Delta t}$. 

We characterize $B^0_{(\pm)}$ and $B^0_{(00)}$ candidates using two 
kinematic variables: the beam-energy-substituted mass 
m_{ES} and the energy difference $\Delta E$ [8]. The signal region 
(SR) is defined as $m_{ES} > 5.26$ GeV/c$^2$, and $|\Delta E| < 0.06$ 
GeV for $B^0_{(\pm)}$, or $-0.12 < \Delta E < 0.06$ GeV for 
$B^0_{(00)}$. For $B^0_{(L)}$, the SR is defined by $-0.01 < \Delta E < 
0.03$ GeV [8], and the missing momentum for the entire 
event is required to be consistent with the calculated $K_L^0$ 
laboratory momentum.

The main source of background is continuum $e^+ e^- \to 
q\bar{q}$ ($q = u, d, s, c$) events. We use event-shape variables to 
nlack the jet-like structure of these events in order to 
rmout much of this background [8].

We perform an unbinned maximum likelihood fit to the 
selected $K^+ K^- K^0$ events using the likelihood 
function defined in Ref. [8]. The probability density function 
(PDF), $P_i$, is given by

$$
P_i = \frac{P(m_{ES}) \cdot P(|\Delta E|) \cdot P_{\text{Low}}}{P_{\text{DP}}(m_{K^+K^-}, \cos \theta_H, \Delta t, q_{tag}) \cdot R(\Delta t, \sigma_{\Delta t})} \quad (1)$$

where $i = (\text{signal, continuum, } B\bar{B} \text{ background})$, and $R$ 
is the $\Delta t$ resolution function [4]. For $B^0_{(i)}$, $P(m_{ES})$ is not 
used. $P_{\text{Low}}$ is a PDF used only in the low-mass fit, which 
depends on the event-shape variables and, for $B^0_{(L)}$ only, 
the missing momentum in the event [8]. We characterize 
$B^0_{(i)}$ events on the DP in terms of $m_{K^+K^-}$ and $\cos \theta_H$, 
the cosine of the helicity angle between the $K^+$ and the 
$K^0$ in the rest frame of the $K^+ K^-$ system. The DP PDF for signal events is

$$
P_{DP} = d\Gamma \cdot \varepsilon(m_{K^+K^-}, \cos \theta_H) \cdot |J|, \quad (2)$$

where $d\Gamma$ is the time- and flavor-dependent decay rate 
over the DP, $\varepsilon$ is the efficiency, and $J$ is the Jacobian 
of the transformation to our choice of DP coordinates.
The time- and flavor-dependent decay rate is
\[
\frac{d\Gamma}{d\Delta \tau} \propto e^{-|\Delta t|/\tau} \times \left[ |A|^2 + |\bar{A}|^2 \right] + g_{ag} 2\text{Im} \left( \xi A A^* \right) \sin \Delta m_d \Delta t - g_{ag} \left( |A|^2 - |\bar{A}|^2 \right) \cos \Delta m_d \Delta t, \tag{3}
\]
where $\tau$ and $\Delta m_d$ are the lifetime and mixing frequency of the $B^0$ meson, respectively.\(^{14}\) The parameter $\xi = \eta_{CP} e^{-2i\beta}$, where $\beta = \arg(-V_{cb} V_{cb}^*/V_{td} V_{td}^*)$ and $V_{td}^*$ are CKM matrix elements.\(^{11}\) The CP eigenvalue $\eta_{CP}$ is 1 ($-1$) for the $K^0_d$ ($K^0_s$) mode. We define the amplitude $\overrightarrow{A}$ for $B^0$ decay as a sum of isobar amplitudes\(^{14}\),
\[
\overrightarrow{A}(m_{K^+ K^-}, \cos \theta_H) = \sum_r \overrightarrow{A}_r, \tag{4}
\]
where the minus signs are associated with the $\overrightarrow{A}$, the parameters $c_r$ and $\varphi_r$ are the magnitude and phase of the amplitude of component $r$, and we allow for different isobar coefficients for $B^0$ and $\overline{B}^0$ decays through the asymmetry parameters $b_r$ and $\delta_r$.

Our isobar model includes resonant amplitudes $\phi$, $f_0$, $\chi_{00}(1P)$, and $X_0(1550)$\(^{15,16}\); non-resonant terms; and incoherent terms for $B^0$ decay to $D^{*+} K^+$ and $D_s^- K^+$. For each resonant term, the function $f_r$ is $F_r \times T_r \times Z_r$ describes the dynamical properties, where $F_r$ is the Blatt-Weisskopf centrifugal barrier factor for the resonance decay vertex\(^{17}\), $T_r$ is the resonant mass-line shape, and $Z_r$ describes the angular distribution in the decay\(^{18}\). The barrier factor $F_r = 1/\sqrt{1+(R\bar{q})^2}$ for the $\phi$, where $\bar{q}$ is the $K^+$ momentum in the $\phi$ rest frame and $R = 1.5$ GeV\(^{-1}\); $F_r = 1$ for the scalar resonances. For $\phi$ decay $Z_r \sim \bar{q} \cdot \bar{p}$, where $\bar{p}$ is the momentum of the $K^0$ in the $\phi$ rest frame, while $Z_r = 1$ for scalar decays. We describe the $\phi$, $X_0(1550)$, and $\chi_{00}(1P)$ with relativistic Breit-Wigner line shapes\(^{14}\). For the $\phi$ and $\chi_{00}(1P)$ parameters we use average measurements\(^{14}\). For the $X_0(1550)$ resonance, we use parameters from our analysis of the $B^+ \to K^+ K^- K^+\bar{K}^-$ decay\(^{15}\). The $f_0$ resonance is described by a coupled-channel amplitude\(^{19}\), with the parameter values of Ref.\(^{20}\).

We include three non-resonant (NR) amplitudes parameterized as $f_{NR,k} = \exp(-\alpha c^2)$, where the parameter $\alpha = 0.14 \pm 0.01$ GeV\(^2\) is taken from measurements of $B^+ \to K^+ K^- K^+$ decays with larger signal samples\(^{15,16}\). We include a complex isobar coefficient for each component $k = (K^+ K^+, K^+ K^0, K^- K^0)$. PDFs for $q\bar{q}$ background in $B^0 \to K^+ K^- K^0$ are modeled using events in the region $5.2 < m_{ES} < 5.26$ GeV/c\(^2\). The region $0.02 < \Delta E < 0.04$ GeV is used for $B_{(s)}^{0(L)}$. Simulated $B \bar{B}$ events are used to define $B \bar{B}$ background PDFs. We use two-dimensional histogram PDFs to model the DP distributions for $q\bar{q}$ and $B \bar{B}$ backgrounds.

We compute the $CP$ asymmetry parameters for component $r$ from the asymmetries in amplitude ($b_r$) and phase ($\delta_r$) given in Eq.\(^{11}\). The rate asymmetry is
\[
A_{CP,r} = \frac{|A_r|^2 - |\bar{A}_r|^2}{|A_r|^2 + |\bar{A}_r|^2} = \frac{-2b_r}{1 + b_r^2}, \tag{5}
\]
and $\beta_{eff,r} = \beta + \delta_r$ is the phase asymmetry.

The selection criteria yield 3266 $B^{0}_{(s)}(\to \phi)$, 1611 $B_{(s)}^{0}$, and 27513 $B_{(s)}^{0}$ candidates which we fit to obtain the event yields, the isobar coefficients of the DP model, and the $CP$ asymmetry parameters averaged over the DP. The parameters $b_r$ and $\delta_r$ are constrained to be the same for all model components, so in this case $A_{CP,r} = A_{CP}$ and $\beta_{eff,r} = \beta_{eff}$. We find $947 \pm 37$ $B^{0}_{(s)}$, $144 \pm 17$ $B_{(s)}^{0}$, and $770 \pm 71$ $B_{(s)}^{L}$ signal events. Isobar coefficients and fractions are reported in Table \(^{11}\) and $CP$ asymmetry results are summarized in Table \(^{11}\). The fraction $F_r$ for resonance $r$ is computed as in Ref.\(^{15}\). Note that there is a $\pm \pi$ rad ambiguity in the $\chi_{00}(1P) K^+$ phase.

### Table I: The isobar amplitudes $c_r$, phases $\varphi_r$, and fractions $F_r$ from the fit to the full $K^+ K^- K^0$ DP. The three NR components are combined for the fraction calculation. Errors are statistical only. Because of interference, $\sum F_r \neq 100%$.

| Isobar Mode | Amplitude $c_r$ | Phase $\varphi_r$ (rad) | $F_r$ (%) |
|-------------|-----------------|------------------------|----------|
| $X_0(1550) K^+$ | 0.0885 $\pm$ 0.0010 | $-0.016 \pm 0.234$ | 12.5 $\pm$ 1.3 |
| $f_0 K^+$ | 0.622 $\pm$ 0.046 | $-0.14 \pm 0.14$ | 40.2 $\pm$ 9.6 |
| $X_0(1550) K^+$ | 0.114 $\pm$ 0.018 | $-0.47 \pm 0.20$ | 4.1 $\pm$ 1.3 |
| $(K^+ K^-)^{NR} K^+$ | 1 (fixed) | 0 (fixed) | |
| $(K^+ K^-)^{(NR)} K^-$ | 0.33 $\pm$ 0.07 | 1.95 $\pm$ 0.27 | 112.0 $\pm$ 14.9 |
| $(K^- K^0)^{(NR)} K^+$ | 0.31 $\pm$ 0.08 | 1.34 $\pm$ 0.37 | |
| $\chi_{00}(1P) K^0$ | 0.0306 $\pm$ 0.0049 | $-0.34 \pm 0.54$ | 3.0 $\pm$ 1.2 |
| $D^+ K^+$ | 1.11 $\pm$ 0.17 | | 3.6 $\pm$ 1.5 |
| $D^0 K^+$ | 0.76 $\pm$ 0.14 | | 1.8 $\pm$ 0.6 |

In Fig.\(^{11}\) we plot twice the change in the negative logarithm of the likelihood as a function of $\beta_{eff}$. We find that the $CP$-conserving case of $\beta_{eff} = 0$ is excluded at 4.8$\sigma$ (5.1$\sigma$), including statistical and systematic errors (statistical errors only). Also, the interference between $CP$-even and $CP$-odd amplitudes leads to the exclusion of the $\beta_{eff}$ solution near $\pi/2 - \beta$ at 4.5$\sigma$ (4.6$\sigma$).

![FIG. 1: The change in twice the negative log likelihood as a function of $\beta_{eff}$ for the fit to the whole DP.](image.png)
We also measure $CP$ asymmetry parameters for events with $m_{K^+K^-} < 1.1 \text{ GeV}/c^2$. In this region, we find $1359 B_{(s)}^0, 348 B_{(0)}^0, \text{and } 7481 B_{(s)}^0$ candidates. The fit yields $282 \pm 20, 37 \pm 9$ and $266 \pm 36$ signal events, respectively. The most significant contributions in this region are from $\phi K^0$ and $f_0 K^0$ decays, with a smaller contribution from the low-mass tail from non-resonant decays. In this fit we vary the amplitude asymmetries $b_\gamma$ and $d_\gamma$ for the $\phi$ and $f_0$, while the other components are fixed to the SM expectations of $\beta_{\text{eff}} = 0.370 \text{ rad}$ and $A_{CP} = 0$ [21]. We also vary the isobar coefficient for the $\phi$, while fixing the others to the results from the whole DP fit. There are two solutions with likelihood difference of only $\Delta \log L = 0.1$. Solution (1) is consistent with the SM, while in Solution (2) $\beta_{\text{eff}}$ for the $f_0$ differs significantly from the SM value (Table III). The solutions also differ significantly in the values of the $\phi$ isobar coefficient. There is also a mathematical ambiguity of $\pm \pi \text{ rad}$ in the solution for $\varphi_\phi$. This ambiguity is present for both solutions. The fit correlation between the $\phi$ and $f_0$ in $d_\gamma$ is 0.71 [22].

Finally, we perform a fit to extract the average $CP$ asymmetry parameters in the high-mass region. In the 2384 $B_{(s)}^0, 1406 B_{(0)}^0, \text{and } 20032 B_{(s)}$ selected events with $m_{K^+K^-} > 1.1 \text{ GeV}/c^2$, we find signal yields of $673 \pm 31, 87 \pm 14$ and $462 \pm 56$ events, respectively; the $CP$ asymmetry results are shown in Table III. We find that for this fit the $CP$-conserving case of $\beta_{\text{eff}} = 0$ is excluded at $5 \sigma$, including statistical and systematic errors.

Figure 2 shows distributions of the DP variables $m_{K^+K^-}$ and $\cos \theta_H$ obtained using the method described in [23]. Figure 3 shows the $\Delta t$-dependent asymmetry between $B^0$ and $\bar{B}^0$-tagged events.

Systematic errors on the $CP$-asymmetry parameters are listed in Table III. The fit bias uncertainty includes effects of detector resolution and possible correlations among the fit variables determined from full-detector simulations. We also account for uncertainties due to the isobar model: experimental precision of resonance parameter values; alternate $X_0(1550)$ parameter values [16]; and, in the low- and high-mass fits, the statistical uncertainties on the isobar coefficients determined in the fit to the whole DP. Other uncertainties common to many $B\bar{B}$ time-dependent analyses, including those due to fixed PDF parameters, and possible $CP$ asymmetries in the $B\bar{B}$ background are also taken into account [24].

Uncertainties due to fixed PDF parameters are evaluated by shifting the fixed parameters and refitting the data. As a cross-check, we perform the analysis using $B_{(s)}^0$ alone and find results consistent with those in Table III. The summary of the systematic errors on the $CP$ asymmetry parameter values are shown in Table III. We find that for this fit the $CP$-conserving case of $\beta_{\text{eff}} = 0$ is excluded at $5 \sigma$, including statistical and systematic errors.

### Table II: The $CP$-asymmetries for $B^0 \rightarrow K^+K^-K^0$ for the entire DP, in the high-mass region, and for $\phi K^0$ and $f_0 K^0$ in the low-mass region. The first errors are statistical and the second are systematic. The solutions (1) and (2) from the low-mass fit are discussed in the text.

| Source | Whole DP | High-mass | $\phi K^0$ | $f_0 K^0$ |
|--------|----------|-----------|------------|-----------|
| $A_{CP}$ | $-0.015 \pm 0.077 \pm 0.053$ | $0.352 \pm 0.076 \pm 0.026$ | $-0.08 \pm 0.18 \pm 0.04$ | $11 \pm 0.14 \pm 0.06$ |
| $\beta_{\gamma}$ | $0.436 \pm 0.087 \pm 0.032$ | $0.41 \pm 0.23 \pm 0.07$ | $0.14 \pm 0.15 \pm 0.05$ | $0.10 \pm 0.13$ |
| $\beta_{\text{eff}}$ | $-0.11 \pm 0.18$ | $0.10 \pm 0.13$ | $-0.20 \pm 0.31$ | $3.09 \pm 0.19$ |
In summary, in a sample of $383 \times 10^6$ $B\bar{B}$ meson pairs we simultaneously analyze the DP distribution and measure the time-dependent CP asymmetries for $B^0 \rightarrow K^+K^-K^0$ decays. The values of $\beta_{\mathrm{eff}}$ and $A_{\mathrm{CP}}$ are consistent with the SM expectations of $\beta \simeq 0.370$ rad, $A_{\mathrm{CP}} \simeq 0$ [21]. The significance of CP violation is 4.8$\sigma$, and we reject the solution near $\pi/2 - \beta$ at 4.5$\sigma$. We also measure CP asymmetries for the decays $B^0 \rightarrow \phi K^0$ and $B^0 \rightarrow f_0 K^0$, where we find $\beta_{\mathrm{eff}}$ lower than the SM expectation by about $2\sigma$. The CP parameters in the high-mass region are compatible with SM expectations, and we observe CP violation at the level of 5.1$\sigma$.

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