PAPER

Bouncing dynamics of electrostatically actuated NEM switches

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Abstract

The aim of the present research is to understand the bouncing dynamic behavior of nanoelectromechanical (NEM) switches in order to improve switch performance and reliability. It is well known that bouncing can dramatically degrade the switch performance and life; hence, in the present study, the bouncing dynamics of a cantilever-based NEM switch has been studied in detail. To this end, the repulsive van der Waals force is incorporated into a nano-switch model to capture the contact dynamics. Intermolecular forces, surface effects, and gas rarefaction effects were also included in the proposed model. The Euler–Bernoulli beam theory and an approximate approach based on Galerkin’s method have been employed to predict transient dynamic responses. In the present study, performance parameters such as initial contact time, permanent contact time, major bounce height, and the number of bounces, were quantified in the presence of interactive system nonlinearities. The performance parameters were used to investigate the influence of surface effects and rarefaction effects on the performance of an electrostatically actuated switch. Recommended operating conditions are suggested to avoid excessive bouncing for these types of NEM switches.

1. Introduction

Nanoelectromechanical systems (NEMS) combine different engineering disciplines such as material, mechanical, electrical, chemical, optics, and fluids. This technology integrates interrelated device components into comprehensive systems at very small scales that complete functions such as sensing and actuation, and has advantages of NEMS technology such as low cost, low power consumption, high functionality, and small size and lightweight of the devices. It also opens enormous opportunities in many engineering applications such as automotive, aerospace, electronics instrumentation, industrial process control, biotechnology, and telecommunications. In many of these applications, switches are fundamental building blocks in the design of NEMS. Changing the state of the switch can be utilized via electrostatic, magnetostatic, piezoelectric, or thermal designs or by utilizing intermolecular forces. Electrostatic actuation is often regarded as the main driving source of NEM switches.

Cantilever-based nanomechanical switches are one of the most frequently used configurations in NEM switch applications, but their instability behavior and transient response are not adequately understood. In the present research, emphasis will be placed on NEM devices, in particular, the bouncing dynamic response. Typically, upon closing, the nano-beam (switch) bounces several times before making permanent contact with the adjoining structure. Bouncing due to switching can dramatically degrade the performance owing to an increase in the transition time and electrical breakdown. As the characteristic dimensions of NEM devices are reduced to the nanometers scale, intermolecular forces such as Casimir and van der Waals forces have considerable magnitudes, and surface effects of nanostructured materials, such as surface stress and surface elasticity, and rarefaction effects have significant influences on the dynamic behavior of the NEM device. The bouncing dynamic behavior of NEM switches has not been studied adequately, particularly with the presence of these effects and forces. Hence, the present study is concerned with the dynamic behavior of a NEM-based...
cantilever switch with consideration of intermolecular forces, surface effects, and gas rarefaction effects under electrostatic actuation. The effects of some relevant parameters on the bouncing behavior have been investigated when this class of NEM switches is actuated electrostatically. To this end, a switch subjected to these effects and forces is modeled by a nonlinear partial differential equation that governs the beam motion. Since a closed-form solution for such a system is not achievable, Galerkin’s method is employed to develop a discretized system which then is solved numerically to obtain the response.

NEMS technology has been an attractive field for many researchers, and the literature is extensive in these fields. In general, dynamic behavior analysis of nanoscale devices is challenging due to the nonlinear coupling of the different physics and the influences that are accompanied by small-size effects. When the size of a device is reduced from micro to nanoscale, models based on the classical theories of mechanics that are valid at the microscale may not provide accurate predictions for nanostructures. In particular, when the characteristic sizes of these devices shrink to nanometers, intermolecular forces such as van der Waals force and Casimir force, surface effects of nanostructured materials such as surface stress, surface elasticity, and rarefaction effects may play an important role in predicting the dynamic behavior of NEM switches. The performance and reliability of switches can be improved by understanding the bouncing behavior owing to the influence of these effects. The above-mentioned forces and effects exist, by nature, in nanostructures and hence incorporating them in the model is expected to provide a more accurate understanding of the bouncing dynamics. Therefore, if a switch is designed using the obtained response, it is envisaged that its performance can be optimized. Although research in this field is still considered significant, the bouncing associated with switching has not been considered thus far in the literature.

Intermolecular forces such as van der Waals forces and Casimir forces have been considered in the modeling of nanostructures [1, 2]. Although these intermolecular forces describe the same physical phenomena at different nano levels [3], the authors modeled the intermolecular forces in nanosystems separately or simultaneously. In NEM switches, actuation is typically performed by applying a voltage difference between two electrodes [4–6]. The effect of the van der Waals force on the pull-in voltage was investigated for electrostatically actuated nanodevices in a study performed by Rotkin [4]. In this study, an analytical model for calculating pull-in parameters was presented, and a single degree of freedom model was used to investigate the van der Waals effects on the pull-in behavior. Dequesnes et al studied the pull-in voltage characteristics of several nanotube electromechanical switches [5]. The influence of van der Waals forces in the design of nanoelectromechanical switches has been investigated using a simple continuum model. A molecular dynamic simulation was employed to confirm the results of the continuum model. It is worth noting that damping, surface effects, and the Casimir force were not considered in these studies.

Lin and Zhao [7] investigated the pull-in instability of electrostatically actuated nanostructures considering the Casimir force. Approximate analytical expressions for a one-degree-of-freedom model were used to describe the nonlinearity of the system; however, surface effects were not considered in this investigation. Liu [6] investigated the nonlinear pull-in behavior of a NEM actuator considering the influence of surface effects, the fringing field effect, and the Casimir force effect. It should be mentioned that the van der Waals force was not included in these models because typically, the gap for this force to be effective is not reached. However, for a switch, the gaps between the contacting elements are small upon contact, and the van der Waals force should be considered.

The effects of both Casimir and van der Waals forces on the pull-in stability of nano-actuators were considered in a study performed by Sorosh et al [8]. Adomian decomposition is used to obtain an analytical solution to the distributed parameter model; however, this study ignored both the surface effects of nanostructured materials and damping in the system. The static behavior and dynamic responses of a beam subjected to an electrostatic force in the presence of intermolecular forces were investigated in a study performed by Vakili-Tahami et al [9]. In this study, the Euler–Bernoulli beam theory was employed to model the switch, and Galerkin’s approach was used to discretize the system for solving the response numerically. However, only simple constant damping without the rarefaction effect was considered, and the surface effects were ignored in this investigation.

Owing to the inherently large surface area to volume ratio that is exhibited by typical nanoscale structures, the surface energy becomes a significant part of the total elastic energy, and surface energy may play an important role in the elastic deformation of these nanostructures and their material properties. The dependence of mechanical properties on their size is considered in many related studies [10–14]. Hence, much effort has been devoted to understanding the effect of the size of the structures on the material properties.

Using contact atomic force microscopy, Jing et al [11] measured the Young’s modulus of silver nanowires with different diameters. The dependence of Young’s modulus on the size of the wires due to surface effects has been experimentally demonstrated. Results show that the Young modulus increases as the size of the wires decreases. Moreover, in this study, a theoretical analysis of the elastic property of the nanowires is carried out using the classical continuum model with consideration of the surface effects. The obtained results show that the model with surface effects fits reasonably well with the experimental results. The dependence of the properties
on the material size was also measured by C Q Chen et al for [0001]-oriented ZnO nanowires [12]. In their experiments, a dramatic increase in Young’s modulus for smaller diameters (less than 120 nm) was measured, whereas, for larger diameters (up to 550 nm), the modulus approaches the bulk value. In addition to the above-mentioned experimental investigations, atomistic models have also been used to characterize the size dependency of materials for nanostructures. Miller and Shenoy [10] developed a theoretical model for the effective Young’s modulus of nanowires (NWs) with surface effects and explained the dependency of Young’s modulus on the NW size. Rudd and Lee [14] studied the size dependence of the Young’s modulus of NWs based on first-principles density functional theory and compared the results with classical molecular dynamics. Park [15] also studied the size-dependent resonant frequencies of silicon nanowires with finite deformation. In this study, it was concluded that the residual surface stress affects the resonance frequencies of the nanowires.

Continuum modeling has also been considered in many investigations related to the surface effects of nanostructures based on the linear surface elastic theory developed by Gurtin and Murdoch and the generalized Young–Laplace equations [16, 17]. This approach was used by Wang and Feng [18], in which the effects of both surface elasticity and surface stress on the natural frequencies of a microbeam have been investigated. He and Lilley [19] investigated the elastic behavior of the static bending of NWs for different support configurations. The Euler–Bernoulli beam theory is used with modified Young–Laplace equations to introduce the surface effects on nanowires. Ma et al [20] investigated the influence of surface effects on the pull-in instability of a nano-electrostatically actuated switch using an Euler–Bernoulli beam model considering the Casimir force. The results demonstrate that surface effects play a significant role in the selection of basic design parameters of NEMS switches, such as static deflection, pull-in voltage and detachment length. It is worth noting that this study focused only on static deflection and pull-in instabilities. Liu [5] studied the effects of surface effects on the pull-in behavior of a cantilever-type nanomechanical electrostatic actuator using a continuum model via the Euler–Bernoulli beam. The influence of surface effects, fringing field effect, and Casimir force effect are considered in this study.

In the dynamics of NEM devices, squeeze-film damping has a significant influence on device behavior. Several models have been developed for these devices; however, when the dimensions become very small, certain macro- and micro-scale models do not accurately predict nanoscale behavior. In nanostructures, where the characteristic length of the gaps is very small and the surrounding pressure is low, rarefied gas effects need to be considered in the damping. Guo and Alexeenko [21] proposed a compact model of squeeze-film damping based on rarefied flow simulations. The model provides a simple expression for the damping coefficient valid for free-molecular regimes. Gallis and Torczynski [22] developed a molecular-dynamics-based model employing the direct simulation Monte Carlo method incorporating squeeze-film damping for a rigid beam. Their model takes advantage of both the Reynolds equation and molecular dynamics. Sumali [23] experimentally validated some existing models, including the Gallis and Torczynski models, of squeeze-film damping in the free molecular regime for a wide range of Knudsen numbers. Further improvements to the Gallis and Torczynski models were made by Parkos et al and led to a near-contact gas damping model [24]. This model has also been validated experimentally and shown to be applicable for nano switches because the parameters are comparable.

It is worth noting that the bouncing phenomenon is not exclusive to MEM switches since contact also exists in nano-switches. In NEM switches, bouncing was reported in a study carried out by Loh and Espinosa [25], and it is expected to have a performance degradation similar to that of MEM devices. Liao et al [26] fabricated electrostatically actuated suspended single-crystal diamond nanowires for high-performance nanoelectromechanical switches. The current-voltage characteristics of the switch were measured experimentally. An abrupt behavior was observed in the ON state as the applied voltage increased. This abrupt behavior may be attributed to the bouncing of the nanoswitch. Yahiaoui et al [27] fabricated and tested a nanogap radio frequency microelectromechanical system metal-contact switch. Bouncing of the switch under vacuum conditions was also reported in this study.

To the best of the author’s knowledge, the dynamics of nano-switches with consideration of bouncing have not been addressed in the literature so far. Therefore, in the present study, a comprehensive model of a NEM switch that incorporates a realistic contact force along with consideration of nanostructure forces and effects such intermolecular forces and surface effects, as well as damping, is essential for an in-depth understanding of the switch bouncing dynamics, which are beneficial for further development in this class of switches.

2. Modelling of a NEM switch

In this study, a mathematical model of a cantilever-based NEM switch under electrostatic actuation was employed to study the switch dynamics. Other forces and effects related to nanostructures, such as intermolecular forces, surface effects, and rarefaction effects, are included in the model. As discussed in the literature review section, modeling of NEM devices that use conventional theories available in mechanics may not accurately predict the behavior when the characteristic sizes shrink to nanometers; therefore, continuum
models for MEMS need to be modified to accommodate these effects at the nanoscale. The following sections describe the mathematical modeling of these forces and effects, followed by a comprehensive mathematical model of the switch under electrostatic actuation.

2.1. Intermolecular forces

The origin of both the van der Waals and Casimir forces is known to be related to the quantum fluctuations of the electromagnetic fields [28]. When the gap between two surfaces is much smaller than the plasma (for metals) or characteristic absorption (for dielectrics) wavelengths, the retardation effects are not negligible. At this separation level, such a force is usually called the van der Waals force. At relatively large atomic separations, the forces are usually called Casimir–Polder for atom-atom and atom-wall interactions or Casimir for interaction between two macroscopic body forces [29]. In the following subsections, detailed mathematical modeling of intermolecular forces is introduced.

2.1.1. Casimir force

In the literature, the parallel plate concept is employed for beams if the gap is very small, as in NEM switches. In this case, the Casimir force per unit length of the beam is given in a study by Farrokhabadi et al. [30] as

$$f_{CA} = \frac{1}{240} \frac{\pi^2 h c a}{(d - y(x))^4},$$

where \(h = 1.055 \times 10^{-34}\) J represents Planck’s constant divided by \(2\pi\) and \(c = 2.998 \times 10^8\) ms\(^{-1}\) denotes the speed of light. It should be noted that the Casimir force is not affected by material properties.

2.1.2. Van der Waals force

The well-known Lennard–Jones potential, which describes the interaction between two atoms, is usually used to formulate a continuum model of the van der Waals force. The attractive and repulsive parts of the Lennard-Jones potential should be considered when the gap between two surfaces is smaller than a few nanometers; however, the repulsive part of the potential decays extremely fast with separation and plays an important role only after the contact of surfaces takes place. The van der Waals force described by the Lennard-Jones potential is determined per unit length of the beam as follows:

$$f_{VDW} = \frac{A_h a}{6\pi} \left[ \frac{1}{(d - y(x))^3} - \frac{\varepsilon^6}{(d - y(x))^6} \right],$$

where \(A_h\) is the Hamaker constant, and \(\varepsilon\) is the equilibrium distance at which \(f_{VDW}\) becomes zero (see, e.g., [31, 32]). The Hamaker constant depends on the material properties, and its typical values are in the range of \((0.4 - 4) \times 10^{-19}\) J for most polar molecules [20], while the equilibrium distance varies around 2 Å for most materials. The cubic term in equation (2) represents the long-range van der Waals attraction forces, and the ninth-order term describes the short-range Born repulsion [32]. It is worth noting that the Casimir force and the van der Waals force cannot be considered to act simultaneously because they describe the same physical phenomenon and both act at different gap separations. In the present study, separation of 20 nm as suggested by Ramezani et al. [33], was employed in the simulations to distinguish between the ranges of the dominance of each force. Hence, in this study, the Casimir force is considered to be dominant for gaps greater than 20 nm. It should also be noted that the repulsive part of the van der Waals force participates during the contact phase.

2.2. Force due to surface effects and beam elasticity

In nanomaterials, surface effects arise because atoms at the material-free surfaces have a different bonding configuration than bulk atoms. Atoms at a surface are not at an energy minimal state, and surface stresses try to cause these atoms to deform to minimize their energy. The different bonding environments of atoms at a surface and in bulk make the elastic properties of surfaces differ from those of bulk material, and the effect of the difference increases with an increase in the surface-to-volume ratio [34]. Thus, for accurate prediction of dynamic behavior, consideration of surface effects in the modeling of nanostructures, such as nano-switches in the present study, is essential. As mentioned previously in section 2, experimental confirmation of the existence of surface effects in nanostructures reveals that surface effects play a significant role in shaping the switch response. Hence inclusion of these effects in the dynamic modeling of nanostructures is expected to result in more accurate response predictions. The continuum modeling of nanostructures considering surface effects is based on the linear surface elastic theory developed by Gurtin and Murdoch and the generalized Young–Laplace equations [16, 17].

In the surface elasticity model, the surface consists of a thin layer with negligible thickness perfectly bonded to the bulk material, and slipping between the surface and bulk materials is not allowed. The properties of the surface are different from those of the bulk materials. A surface-layer-based model constitutes the foundation
for continuum modeling of nanostructures with consideration of surface effects. In this model, the nanostructure is assumed to be composed of a bulk and a surface, that is, a nanostructure = bulk + surface. A thin layer with an elastic modulus $E_1$ and negligible thickness, $t_s$, beneath the surface, represents the surface layer of a cantilever nanobeam with a rectangular cross-section, as shown in figure 1. The constitutive equation for the surface is different from that of the bulk because of the different local environments around the atoms. For a bending beam, the relation between the surface stress, $\tau$, and longitudinal strain $\epsilon_s$, as given in the study by Miller and Shenoy [10], is

$$\tau = \tau^0 + E'\epsilon_s,$$

(3)

where $\tau^0$ is the residual surface stress along the longitudinal direction of the beam, and $E'$ represents the surface elastic modulus. The relationship between $E^s$ and $E'$ is expressed as $E' = Et'$. By using the composite beam theory under the assumption that the thickness of the surface layer $t_s$ is much smaller than the beam thickness $b$, the effective bending rigidity $(EI)_{eff}$, for a beam with a rectangular cross-section is derived as

$$(EI)_{eff} = EI + \frac{1}{2}E'ab^2 + \frac{1}{6}E'b^3,$$

(4)

where the last two terms in equation (4) contribute to the surface elasticity. The generalized Young-Laplace equation provides a mathematical description of the out-of-plane stresses induced from in-plane stresses on curved interface surfaces [19]. According to the generalized Young-Laplace equation, the residual surface stress results in a jump in the normal stress $\sigma_{ij}$ across the interface surface between the surface layer and the bulk volume, that is,

$$\langle \sigma_{ij}^+ - \sigma_{ij}^- \rangle n_in_j = \tau^0 \kappa,$$

(5)

where $\sigma_{ij}^+$ and $\sigma_{ij}^-$ represent the stresses above and below the surface, respectively. The parameter $n_i$ is the unit normal vector to the surface and $\kappa$ is the curvature of the beam. For a beam with small deformation, the curvature is approximated by the second derivative $y''(x)$ of the beam deflection. For a deformed beam, equation (5) predicts that the stress jump due to the residual surface stress will generate a distributed transverse load $f_x(x)$ along the longitudinal direction of the beam, which is given in the study by He and Lilley [19] as follows:

$$f_x(x) = 2\tau^0 a d^2y(x) dx^2.$$  

(6)

Equation (6) indicates that surface stress comes into effect once the beam is bent with a non-zero curvature, as shown in figure 1. This distributed transverse force may stiffen or soften NEM switches during its bending process depending on the sign of the surface stress. In the present study, the surface effects are incorporated into the nonlinear model of the NEM switch to investigate the dynamic bouncing behavior.

2.3. Damping force

NEM devices usually work in a dry and clean environment and require vacuum packaging for these purposes. In the dynamics of NEM devices, gas damping has a significant influence on the device behavior since the surface is large compared to the volume. However, under these conditions of low pressure and small characteristic gaps, the rarefied damping effect needs to be considered in the modeling. If the dimensions of the structure containing the gas are comparable to the mean free path of the gas molecules, the fluid can no longer be regarded as a continuum flow and rarefaction will occur. The ratio between the mean free path and the characteristic dimension of the flow geometry, $d$, is commonly referred to as the Knudsen number, $K_n$, and is given by
In the beam case, \( d \) represents the gap between the beam and the substrate. For gases, the mean free path of the molecule, \( \lambda \), can be related to temperature, \( T \), and pressure, \( P \), via

\[
\lambda = \frac{\mu}{P} \sqrt{\frac{\pi K_B T}{m_m}},
\]

where \( K_B = 1.38066 \times 10^{-23} \text{J K}^{-1} \) is the Boltzmann’s constant, \( T \) represents the temperature, \( P \) denotes the pressure, and \( m_m \) denotes the mass of the molecules \([23]\). The value of the Knudsen number determines the degree of gas rarefaction and the validity of the continuum flow assumption. For \( K_n < 0.001 \), continuum flow is applicable, and the Navier–Stokes equations can be used. When \( 0.001 \leq K_n < 0.1 \) rarefaction effects start, and the Navier–Stokes equations can be used along with slip-velocity boundary conditions. Beyond \( K_n = 0.1 \), the continuum assumptions of the Navier–Stokes equations are not appropriate. In this case, alternative simulation techniques, such as particle-based direct simulation Monte Carlo are used. For \( K_n \geq 10 \) the continuum approach breaks down completely, and the flow refers to free molecular flow \([35]\). Several models have been developed that are valid for high Knudsen numbers. Gallis and Torczyński \([22]\) developed a molecular-dynamics-based model using the direct simulation Monte Carlo method for squeeze-film damping on a rigid beam. Parkos et al \([24]\) further improved Gallis and Torczyński’s model and validated it experimentally. In Parkos’ model, the variable damping coefficient \( C_f \) is defined as

\[
C_f = \frac{f_D}{(dy/dt)},
\]

where \( f_D \) represents the damping force per unit length. The velocity of the beam, \( dy/dt \), also varies with the lengthwise location. Based on this formulation, the near-contact damping coefficient is assumed to be in the following form:

\[
C_f = A_1 \left( \frac{a}{d - y(x)} \right)^\gamma + A_2
\]

for \( [a/(d - y(x))] \) values less than a certain cut-off value \( z_c \). Beyond this cut-off value, the damping coefficient takes the following form:

\[
C_f = \frac{\mu}{(1 + 6\chi K_n)} \left[ 1 + 6\eta \frac{a}{d - y(x)} + 12\zeta \frac{a}{d - y(x)} \right].
\]

The cut-off value \( z_c \) is used to extend the near-contact model to a model that is not applicable near the contact developed by Gallis and Torczyński \([22]\). Suggestions for the typical parameter value for \( \gamma \) were determined via simulations, while the constants \( A_1 \) and \( A_2 \) in equation (10) were shown to depend on the Knudsen number. The suggested values of the coefficients \( A_1 \) and \( A_2 \) are

\[
A_1 = \frac{-3\mu_G}{(1 + 6\chi K_n)} \left( \frac{1}{z_c^2} + \frac{4\varphi}{z_c^2} + \frac{4\zeta}{z_c^2} \right) d^{-1} z_c^{-a},
\]

\[
A_2 = \frac{\mu_G}{(1 + 6\chi K_n)} \left( \frac{1}{z_c^2} + \frac{6\varphi}{z_c^2} + \frac{12\zeta}{z_c^2} \right) - z_c^a A_1,
\]

where \( \mu_G = 0.84\mu \) and \( \mu \) is the air viscosity. The coefficients \( \chi \), \( \varphi \), and \( \zeta \) are calculated as functions of the Knudsen number by Gallis and Torczyński \([22]\). Their values can be obtained from:
\[ \chi = \frac{1 + 8.834 K_n}{1 + 5.118 K_n} \]  
\[ \varphi = \frac{0.634 + 1.572 K_n}{1 + 0.537 K_n} \]  
\[ \zeta = \frac{0.445 + 11.20 K_n}{1 + 5.510 K_n} \]

2.4. Electrostatic force
Considering the fringing effect, the electrostatic force per unit length of the beam is denoted by \( f_E \) and takes the following form [36]:
\[ f_E = \varepsilon_0 V^2 a \frac{1 + \beta}{2h^2} \left[ H(x - L_1) - H(x - L_2) \right]. \]
The parameter \( \varepsilon_0 \) denotes the permittivity of air, \( V \) represents the applied voltage between the electrode and the beam, \( h = d - y \), and \( \beta \) represents the fringe effect correction coefficient. The function, \( H(x) \), denotes the Heaviside step function, and the electrostatic force is considered to act on the beam segment between lengths \( L_1 \) and \( L_2 \). The fringe effect correction coefficient takes the following form [36]:
\[ \beta = \frac{h}{\pi a} \ln \left( \frac{\pi a}{h} \right) + \frac{h}{\pi a} \ln \left( 1 + \frac{2b}{h} + 2 \sqrt{\frac{b}{h} + \frac{b^2}{h^2}} \right). \]

2.5. Equations of motion for a switch under electrostatic force
Figure 2 shows a typical cantilever switch with length \( L \), beam tip gap \( d_T \), beam initial gap \( d \), beam thickness \( b \), beam tip length \( L_T \), and beam width \( a \). The electrostatic force is considered to act on the beam segment between lengths \( L_1 \) and \( L_2 \). The repulsive part of the van der Waals force is incorporated to capture the bouncing dynamics. In addition, rarefied damping forces, surface effects due to surface stress, and surface elasticity were incorporated into the model. The Euler–Bernoulli beam theory is used to develop the equation of motion that governs the flexural dynamics of the switch. When these forces and effects are incorporated into the modeling, the equation of motion takes the form [6]:
\[ \frac{\partial^2}{\partial x^2} \left( (EI)_{eff} \frac{\partial^2 y(x, t)}{\partial x^2} \right) + \rho a \frac{\partial^2 y(x, t)}{\partial t^2} = f_E + f_{CA} + f_{VDW} + f_s + f_D, \]
where \( y(x, t) \) is the transverse displacement of the beam, \((EI)_{eff}\) represents the effective bending rigidity of the beam considering the surface elasticity, and \( \rho \) is the beam material mass density per unit volume, while \( A \) denotes the area of the beam’s cross-section. \( f_E \) represents the actuation electrostatic force, while \( f_{CA} \) and \( f_{VDW} \) denote, respectively, the Casimir force and van der Waals force. \( f_s \) represents the load force due to the surface stresses. \( f_D \) denotes the dissipating damping force due to the squeezed air film damping effect between the beam and the substrate. It should be noted that all applied forces were expressed per unit length of the beam. Detailed expressions for the forces are provided above.

It should be noted that the NEM switches exhibit inherent nonlinear behavior owing to the nonlinearity in the considered forces and effects. The exact solution for the resulting nonlinear beam equation is difficult to obtain, and an approximate numerical approach based on Galerkin’s method was employed in the present study to predict the responses.

To the best of the author’s knowledge, it appears that the influence of the surface effects as well as the incorporation of van der Waals forces, as described in section 2.1.2, as a contact model on the behavior of NEM switches, has not been investigated thus far. Hence, the objective of the current work is to present a general mathematical model for NEM switches by incorporating the electrostatic force, intermolecular forces, surface effects, and ratification effects under electrostatic actuation to characterize the bouncing dynamic behavior. The presented comprehensive model that incorporates these forces and effects is expected to accurately predict the dynamic behavior, which is considered necessary for developing new NEM devices and for controlling/improving their performances.

2.6. Discretized model for response predictions
Galerkin’s approach is used to develop an approximate solution since closed-form solutions of the governing equations are not achievable. For this purpose, the natural frequencies and mode shapes of the cantilever beam were evaluated first. By employing Galerkin’s method, the continuous system can be discretized to form a system of ordinary differential equations (ODEs), which is solved numerically. The solutions of these ODEs were then used to predict the response. To this end, the transverse displacement of the beam is written in the following...
In equation (20), $Y_r(x)$ and $q_t(t)$, respectively, represent the normalized normal mode shapes and temporal coordinates that govern the beam motion. The cantilever beam was subjected to the following boundary conditions:

$$y(0, t) = 0; \quad \frac{\partial y(0, t)}{\partial x} = 0;$$
$$EI \frac{\partial^2 y(L, t)}{\partial x^2} = 0; \quad EI \frac{\partial^3 y(L, t)}{\partial x^3} = 0. \tag{21}$$

For the fixed end, both the displacement and slope are zero, and for the free end, both the shear force and bending moment are zero.

Substituting equation (20) into the beam equations of motion and multiplying by $Y_r(x)$ on both sides and integrating over the length of the beam, the following ODEs representing the motion are obtained:

$$Mq + Kq = N, \tag{22}$$

The elements of the mass matrix, $M$, and stiffness matrix $K$, are given as follows:

$$m_{rs} = \int_0^L \rho A Y_r(x) Y_s(x) dx, \quad r, s = 1, 2, \ldots, n, \tag{23}$$
$$k_{rs} = \int_0^L EI \frac{d^2 Y_r(x)}{dx^2} \frac{d^2 Y_s(x)}{dx^2} dx, \quad r, s = 1, 2, \ldots, n. \tag{24}$$

and $N$ represents the vector of modal forces given by the expression

$$N_r = \int_0^L Y_r(x) f(x) dx, \quad r = 1, 2, \ldots, n. \tag{25}$$

where $n$ represents the number of normal modes used in the simulation, and $f$ denotes the resultant external applied forces on the beam.

The equations of motion in the normal coordinates, as given by equation (22), represent a system of nonlinear equations since the force term includes forces that depend on the system output, such as displacements and velocities. The system is subject to the initial displacement and velocity, $x(0) = x_0$ and $\dot{x}(0) = v_0$, respectively. If an assumption is made that during a sufficiently small time step, only a finite small displacement is obtained, the forces $N_r$ can be considered to take a constant value during this finite small time step. Then, the equations can be solved, and the temporal coordinates in equation (22) can be used to calculate the system response during each time step.
3. Results and discussion

The dynamic behavior of the NEM switches actuated by an electrostatic force was investigated. Performance parameters such as initial contact time, permanent contact time, major bounce height, and the number of significant bounces, were quantified to investigate the influences of some effects on the switch performance. For electrostatically actuated NEM switches, in the presence of the Casimir force, the influence of surface effects on the dynamic behavior was studied by comparing the results for cases with and without surface effects under different low-pressure vacuum conditions. The dynamic pull-in voltage is determined for such switches and verified with that presented in a previous work [20]. It is worth noting that in the modeling of MEM switches, a realistic asperity-based contact model that has been used previously in many studies (see, e.g., [37]) is adopted here for the NEM switch model with the consideration of surface effects to represent the contact force along with the repulsive van der Waals force. However, it was demonstrated that the elastic forces due to asperities had negligible or no influence on the bouncing behavior. Hence, the van der Waals force is instrumental in governing the tip contact dynamic behavior. Therefore, the van der Waals force is considered sufficient for understanding the bouncing dynamics of NEM switches. The present study is expected to yield greater insight into the design and performance predictions of nano-switches and help improve the performance predictions of such switches.

The switch employed in this analysis is depicted in figure 2, and dynamically, the switch is governed by equation (22). Transient response simulation of the nano-switch was performed to quantify important switch design parameters based on the predicted response considering the contact bouncing phenomenon. To validate the model, a silver cantilever NEM switch, as considered in the study by Ma et al [20], has been chosen in the present study. The dimensional and material properties of the switch are listed in table1. It worth noting that for all predictions, five normal modes were used, that is, \(n = 5\). The nano-switch beam employed in the above study was used to verify the proposed model. Although the current study focuses on the dynamic behavior of a nano-switch, the static deflections of a similar beam under an electrostatic force are used to validate the current model via static deflection. For this purpose, a slow application of an actuation voltage of 1 V is imposed on the nano-beam to mimic the static behavior demonstrated in their study. A dimensionless deflection of 0.0076 at the free end of the beam is obtained which agrees with the deflection of 0.0079 demonstrated in their study. Further, it is worth noting that the deflection curvature obtained in both studies are similar. This good agreement provides confidence in using the current model for response predictions. It may be noted that the simulations carried out to investigate the bouncing behavior considered all forces, including the Casimir and asperity contact forces. However, it was found that at the nanoscale, the asperity-contact forces made negligible or no contribution to the bouncing dynamics; hence, it was ignored in all of the simulations associated with tip substrate contact. Hence, bouncing due to the repulsive part of the van der Waals force is important in the contact phase because it governs the bouncing behavior. Figure 3 shows a typical switch response when all the relevant forces are included in the model. When a sufficient voltage is applied between the electrodes, the beam switches on and its tip touches the substrate, then bounces several times before it makes permanent contact. The figure shows some of the important parameters that characterize the switch performance. The initial contact time, \(t_i\), is the time taken for the first contact, while the permanent contact time, \(t_p\), is the time taken for the switch to maintain permanent contact. Bouncing time, \(t_b\), is the time from the instant of making first contact to the instant when the
single significant bounce ends, whereas the major bounce height and number of significant bounces are designated by $BH$ and $NB$, respectively.

For the set of parameters used in this study, a dynamic pull-in voltage, $V_{th}$, was determined using the NEM switch model. For the present switch configuration, value of 7.3 V for the dynamic pull-in voltage has been obtained. At this voltage, the NEM switch collapses as the electrostatic force and other attractive forces overcome the elastic restoring force by considering the inertia effect and other relevant effects. An increase in the dynamic pull-in voltage to a value of 7.5 V is observed when the surface effects are ignored. An oscillatory damped motion is predicted for voltages less than the dynamic pull-in voltage associated with cases with and without the consideration of surface effects. This oscillatory motion eventually settles to an equilibrium position, and the beam deflects but does not touch the substrate. For comparison, the dynamic behavior of the switch is predicted under different vacuum pressures namely at 0.1 mTorr, 1 mTorr, and 100 mTorr. It is worth noting that the Casimir force has been measured experimentally in a study by Chan et al at 1 mTorr [38]. Having determined the dynamic pull-in voltage $V_{th}$, the influence of surface effects on the switch performance parameters at different actuation voltages and pressures were quantified and are shown in figures 4–8. Figure 4 shows the
influence of surface effects on the initial contact time $t_i$ for increasing actuation voltages, while figure 5 depicts the influence of surface effects on the permanent contact time $t_p$ at different pressures and voltage values. An overall reduction in the switching time with increasing actuation voltages can be observed from figures 4 and 5, which indicates a performance improvement with a higher actuation voltage. It is also obvious from the figures that the reduction in the initial contact time, $t_i$, is greater than the reduction in the permeant contact time owing to the bouncing. Increasing the pressure improves $t_p$ but it has no significant effects on $t_i$ and hence is not provided. This may be attributed to the low damping values associated with the rarefication. An underestimation of the initial contact time $t_i$ is obvious when the influence of the surface effects is ignored. In addition, the trend of the variation of $t_i$ as seen in figure 4 may be attributed to the absence of any curvature sign.

Figure 6. Influence of surface effects on bounce time $t_b$ at various actuation voltages and pressures.

Figure 7. Influence of surface effects on major bounce height $BH$ at various actuation voltages and pressures.
change during the beam flight stage, that is, prior to actual contact. However, when the beam touches the substrate and starts bouncing, the beam curvature sign changes along different regions of the beam length, and these curvature sign changes lead to stiffening and softening effects along different segments of the beam. As a result, in general, a non-consistent variation in the permanent contact time $t_p$ at different actuation voltages are evident in figure 5. This may be attributed to the effects of bounce-induced curvature changes that influence surface effects as well as other bounce-related forces. Further the model used in the present study is completely deterministic and there are no random effects introduced to either forces or effects. The resulting fluctuation may also be attributed to structural dynamics (modal truncation using the first five modes) and nonlinear interactions between the forces and effects considered in the present study. In general, there is an underestimation of $t_p$ at lower voltages when the surface effects are not considered at low and high pressures, but they overestimate the performance parameters at medium pressures for high voltages. Figure 5 also shows a reduction in $t_p$ with increasing pressure due to an increase in damping, which depresses the bouncing.

It is worth noting that the improvement in the switching time by increasing the actuation voltage is seen to be gained at the expense (increase) of other bouncing parameters, as illustrated in figures 6–8. In addition to the drawback of actuating the switch at higher voltages, the figures show the influence of surface effects at different pressures on the pertinent performance parameters. Figure 6 shows the influence of surface effects at different pressures on the bouncing time for different actuation voltages. These figures illustrate a general increasing trend of switch bouncing time $t_b$ with the increase in actuation voltage at lower pressures. However, at higher pressures, the bouncing time improved significantly compared with the performance at lower pressures. As in the contact time behavior, surface effects lead to a non-consistent trend since the bouncing time is considered during the beam bouncing stage with various stiffening/softening behavior. Further, in general, an overestimation of bounce time can be observed at higher voltages when the surface effects are ignored. Similar performance degradation behavior can be observed from figures 7 and 8 with increasing actuation voltage for other performance parameters such as major bounce height, $BH$, and number of significant bounces $NB$. Figure 7 shows a general increasing trend of major bounce height with increasing actuation voltages at all pressures, while a relatively smaller increase can be seen in the major bounce height at higher pressures. An overestimation of the major bounce height $BH$ at higher voltages is evident in these figures when the surface effects are ignored. The disadvantage of using a higher actuation voltage to improve the switching time is also limited by the increase in the number of bounces. Figure 8 shows such behavior where an increase in the number of bounces is associated with increasing actuation voltages. In addition, the observation that a slight reduction in $NB$ with increasing pressure is evident. Further, ignoring the surface effects appears to overestimate $NB$ at higher voltages. It may be noted that relatively larger rates of increase in the major bounce height $BH$ and the bounce time $t_p$ with increasing actuation voltages are evident when the actuation voltage exceeds $1.3 V_{th}$. Applying voltages higher than $1.3 V_{th}$ causes multiple bounces with larger heights compared to the voltages...
V_{th}, The increase in these performance parameters limits the use of high voltages to improve the switching time. Thus, more favorable bounce behavior may be obtained when the voltage is less than this voltage. In these cases, a significant improvement in the switching time is attained prior to reaching the severe bouncing stage, which will certainly have a detrimental effect on the switch performance. It is worth noting that increasing the pressure is shown to improve the above-mentioned performance parameters owing to the increased damping levels. However, NEM devices are usually operated at low pressures, and it is known that the Casimir force cannot be considered at higher pressures since this force is reported in the literature only at low pressures. Thus, situations where the pressure is higher as discussed above are not realistic in NEM applications, and they are reported here for comparison purposes only.

4. Conclusions

The dynamic behavior of NEM switches has been investigated under electrostatic forces. A nano-switch model that includes the relevant forces and effects and a suitable numerical scheme for providing insight into the switch nonlinear dynamics is employed. The Euler–Bernoulli beam theory and an approximate approach based on Galerkin’s method have been employed for predicting dynamic responses. The predicted simulation responses show that the beam tip bounces, then makes a final contact with the substrate. Performance parameters such as initial contact time, permanent contact time, major bounce height, and the number of significant bounces, have been quantified to evaluate the switch performance and to investigate the influences of some effects and parameters on the switch performance.

The dynamic pull-in voltage was determined for an electrostatically actuated switch. Simulations show that consideration of the van der Waals force is sufficient for understanding the bouncing dynamics of NEM switches. The influence of surface effects on the performance of a class of electrostatically actuated nano switches has been investigated to assist in the design and performance of switchable devices under rarefication. The switch performance parameters were determined in cases of actual contact. Significant improvements are achieved when the applied voltage is approximately 1.3 times the dynamic pull-in voltage; however, beyond this voltage, bouncing deteriorates the performance and limits the advantage of using higher voltages. Surface effects appear to have inconsistent effects on various bounce parameters during the bouncing stage owing to the curvature changes of the beam deflection. However, in general, an overestimation of performance parameters at a high actuation voltage is observed when the surface effects are ignored. The present study is envisaged to help in the design and improve the performance predictions of electrostatically actuated nano switches.

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Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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References

[1] Israelachvili J N 1992 Intermolecular and Surface Forces (London: Academic)
[2] Casimir H B G 1948 On the attraction between two perfectly conducting plates Proc. K. Ned. Acad. Wet. 51 793
[3] Palasantzas G, Zvod P J and Hosson J T 2008 Transition from Casimir to van der Waals force between macroscopic bodies Appl. Phys. Lett. 93 121912
[4] Rotkin S V 2002 Analytical calculations for nanoscale electromechanical systems Electromech. Soc. Proc. 6 90–7
[5] Dequesnes M, Rotkin S V and Aluru N R 2002 Calculation of pull-in voltages for carbon-nanotube-based nanoelectromechanical switches Nanotechnology 13 120–31
[6] Liu C 2016 Dynamic behavior analysis of cantilever–type nano-mechanical electrostatic actuator Int. J. Non-Linear Mech. 82 124–30
[7] Lin W and Zhao Y 2009 Casimir effect on the pull-in parameters of nanometer switches Microm. Technol. 11 80–5
[8] Soroush R, Koochi A, Kazemi A S, Noghrehabadi A, Hakladpour H and Abadyan M 2010 Investigating the effect of Casimir and van der Waals attractions on the electrostatic pull-in instability of nano-actuators Phys. Scr. 82 045801
[9] Vakili-Tahami F, Mobiki H, Keyvani-Janbahan A and Rezaadeh G 2009 Pull-in phenomena and dynamic response of a capacitive nano-beam switch Sensors & Transducers Journal 110 26–37
[10] Miller R E and Shenoy V B 2000 Size-dependent elastic properties of nanosized structural elements Nanotechnology 11 139–47
[11] Jing Y G, Duan H L, Sun X M, Zhang Z S, Xu J, Li Y D and Yu D F 2006 Surface effects on elastic properties of silver nanowires: contact atomic–force microscopy Phys. Rev. B 73
[12] Chen C Q, Shi Y, Zhang Y S, Zhu J and Yan Y J 2006 Size dependence of Young’s modulus in ZnO nanowires Phys. Rev. Lett. 96
[13] Guo J and Zhao Y 2007 The size-dependent bending elastic properties of nanowires with surface effects Nanotechnology 18 295701
[14] Rudd R and Lee B 2008 Mechanics of silicon nanowires: size–dependent elasticity from first principles Mol. Simul. 34 1–8
[15] Park H S 2009 Quantifying the size–dependent effect of the residual surface stress on the resonant frequencies of silicon nanowires if finite deformation kinematics are considered Nanotechnology 20 115701
[16] Kurtin M E and Murdoch A J 1975 Addenda to our paper A continuum theory of elastic material surfaces Arch. Ration. Mech. Anal. 59 389–90
[17] Chen T, Chiu M and Weng C 2006 Derivation of the generalized Young-Laplace equation of curved interfaces in nanoscaled solids J. Appl. Phys. 100 074308
[18] Wang G and Feng X 2007 Effects of surface elasticity and residual surface tension on the natural frequency of microbeams Appl. Phys. Lett. 90 231904
[19] He J and Lilley C M 2008 Surface effect on the elastic behavior of static bending nanowires Nano Lett. 8 1798–802
[20] Ma J B, Jiang L and Asokanthan S F 2010 Influence of surface effects on the pull-in instability of NEMS electrostatic switches Nanotechnology 21 505708
[21] Guo X and Alexeenko A 2009 Compact model of squeeze–film damping based on rarefied flow simulations J. Micromech. Microeng. 19 045026
[22] Gallis M and Torczynski J 2004 An improved Reynolds–equation model for gas damping of microbeam motion J. Microelectromech. Syst. 13 653–9
[23] Sumali H 2007 Squeeze–film damping in the free molecular regime: model validation and measurement on a MEMS J. Micromech. Microeng. 17 2231–40
[24] Parkos D, Raghunathan N, Venkatraman A, Sanborn B, Chen W, Peroulis D and Alexeenko A 2013 Near-contact gas damping and dynamic response of high–g MEMS accelerometers J. Microelectromech. Syst. 22 1089–99
[25] Loh O Y and Espinosa H D 2012 Nanoelectromechanical contact switches Nat. Nanotechnol. 7 283–95
[26] Liao M, Hishita S, Watanabe E, Koizumi S and Koide Y 2010 Suspended single–crystal diamond nanowires for high–performance nanoelectromechanical switches Adv. Mater. 22 5393–7
[27] Yahiaoui A, Lemoine E, Poither A and Blondy P 2014 Mechanical nanogap switch for low–power onboard electronics Int. J. Microwave Wireless Technology 7 515–20
[28] Lamoreaux S K 2004 The Casimir force: background, experiments, and applications Rep. Prog. Phys. 68 201–36
[29] Klimchitskaya G L, Mohideen U and Mostepanenko V M 2009 The Casimir force between real materials: experiment and theory Rev. Mod. Phys. 81 1827–85
[30] Farrokhlabadi A, Moheshbahan A, Rach R and Duan J 2016 An improved model for the cantilever NEMS actuator including the surface energy, fringing field, and Casimir effects Physica E 75 02–209
[31] Hariri A, Zu J and Mrad R 2004 Modeling of surface forces between micron–sized objects in dry condition 2004 Int. Conf. on MEMS, NANO, and Smart Systems (ICMENS04)
[32] Yu N and Polycarpou A 2004 Adhesive contact based on the Lennard–Jones potential: a correction to the value of the equilibrium distance as used in the potential J. Colloid Interface Sci. 278 428–35
[33] Ramezani A, Alasty A and Akbari J 2007 Closed–form approximation and numerical validation of the influence of van der Waals force on electrostatic cantilevers at nano–scale separations Nanotechnology 19 015501
[34] Park H S 2008 Surface stress effects on the resonant properties of silicon nanowires J. Appl. Phys. 103 123504
[35] Barber B W and Emerson D R 2002 The influence of Knudsen number on the hydrodynamic development length within parallel plate micro–channels WIT Trans. Eng. Sci. 36 12
[36] Bao M 2001 Analysis and Design Principles of MEMS Devices (Amsterdam: Elsevier)
[37] Bognash M and Asokanthan S F 2018 Uncertainty considerations for nonlinear dynamics of a class of MEMS switches undergoing tip contact bouncing Int. J. Comput. Nonlinear Dyn. 14 021014–8
[38] Chan H B, Akysuk V A, Kleiman R N, Bishop D J and Capasso F 2001 Quantum mechanical actuation of microelectromechanical systems by the Casimir force Science 291 1941–4