Causal Limit of Neutron Star Maximal Mass in $f(R)$ Gravity in View of GW190814

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We investigate the causal limit of maximum mass for stars in the framework of $f(R)$ gravity. We choose a causal equation of state, with variable speed of sound, and with the transition density and pressure corresponding to the SLy equation of state. The transition density is chosen to be equal to twice the saturation density $\rho_t \sim 2\rho_0$, and also the analysis is performed for the transition density, chosen to be equal to the saturation density $\rho_t \sim \rho_0$. We examine numerically the combined effect of the stiff causal equation of state and of the sound speed on the maximum mass of static neutron stars, in the context of Jordan frame of $f(R)$ gravity. This yields the most extreme upper bound for neutron star masses in the context of extended gravity. As we will evince for the case of $R^2$ model, the upper causal mass limit lies within, but not deeply in, the mass-gap region, and is marginally the same with the general relativistic causal maximum mass, indicating that the $\sim 3M_\odot$ general relativistic limit is respected. In view of the modified gravity perspective for the secondary component of the GW190814 event, we also discuss the strange star possibility. Using several well established facts for neutron star physics and the Occam’s razor approach, although the strange star is exciting, for the moment, it remains a possibility for describing the secondary component of GW190814. We underpin the fact that the secondary component of the compact binary GW190814 is probably a neutron star, a black hole or even a rapidly rotating neutron star, but not a strange star. We also discuss, in general, the potential role of the extended gravity description for the binary merging.

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I. INTRODUCTION

Neutron stars (NSs) are currently in the epicenter of scientific interest, since they are truly laboratories in the sky, for many scientific purposes, like nuclear physics [1–11], particle physics [12–15] and theoretical astrophysics [16–23]. NSs have been thoroughly studied in the last 40 years, and the latest LIGO-Virgo astronomical observations of gravitational waves emitted from merging of NSs, makes the once but long ago theoretical dream of understanding how NSs are composed, a scientifically strong and overwhelming reality. For a mainstream of textbooks and reviews on NSs, we refer the reader, for example, to Refs. [24–28].

Even after so much time since the first pulsar was observed far back in 1967 by Jocelyn Bell, to date the crust and core of NSs are still mysteries for scientists. The reason is that NSs consist mainly of nuclear matter, the density of which nearly after the crust-core interface becomes supranuclear, and at the outer core bottom is nearly twice the nuclear saturation density. The behavior of nuclear matter at supranuclear densities is to date one of the greatest mysteries of theoretical astrophysics and nuclear physics of course. The main problem is that such high densities cannot be reached by experiments conducted on Earth, thus, for the moment, in most cases, theoretical astrophysicists rely on non-relativistic (mostly) results obtained by nucleon-nucleon scattering, or by strongly coupled Coulomb systems experiments. Moreover, to date, one can have strong hints about the main characteristics of the equation of state (EoS) for the outer core, which ends at densities of the order $\rho \sim 2\rho_0$, where $\rho_0$ is the nucleus energy density $\rho_0 \sim 2.8 \times 10^{14} g/cm^3$, but, at higher densities, there are only speculations for the state of matter, with the possibilities being that the EoS of matter is determined by ordinary symmetric nuclear matter, hyperons, strange quarks, or the EoS is determined by the condensation of Kaons or even of Pions. Novel approaches exist, based on firm phenomenological data, that may provide a piecewise polytropic EoS, valid up to large central densities [29–30].

Apart from nuclear matter, all the other possibilities we mentioned above, belong to exotic classes of NS models. Besides the standard General Relativity (GR) description of NS, there is also the possibility of Extended Gravity [31–38] that may also describe NSs. In the context of Extended Gravity, large mass neutron stars can be harbored

\[ R^2 = g_{\mu\nu}R^\mu^\nu - \frac{4}{3}g_{\alpha\beta}R^\alpha^\beta - \frac{4}{3}g^\mu^\nu R_{\mu\nu} + \lambda_{\alpha\beta}R_{\alpha\beta} + \lambda_{\mu\nu}R_{\mu\nu} + \lambda R^2, \]

where $R$ is the Ricci scalar, $\lambda$, $\lambda_{\alpha\beta}$, and $\lambda_{\mu\nu}$ are the parameters of the gravitational theory, and $(\alpha, \beta, \mu, \nu)$ are dummy indices. The most popular extended gravity theories are Horndeski’s theory [39], the f(R) gravity [40], and the f(R, T) gravity [41], where $T$ is the trace of the energy-momentum tensor. In this work, we consider the $f(R)$ gravity as a representative example of extended gravity theories, and we examine the causal limit of maximum mass for stars in the $f(R)$ gravity framework.
In this context, we shall investigate the causal limit of NS maximum mass in the Jordan frame. We shall adopt the stiffest EoS one can use, the causal limit EoS, with variable speed of sound. In the latter case, the big puzzle of dark side, not yet solved at experimental level, can be encompassed in the ultraviolet and infrared phenomena which escape the standard description. In the first case, the lack of a self-consistent limit in its curvature based formulation, can be seen as the natural extension of GR as soon as one wants to address phenomena generated for smaller central densities or even equal, compared to GR. On the other hand, Extended Gravity, at least in its curvature based formulation, can be seen as the natural extension of GR as soon as one wants to address ultraviolet and infrared phenomena which escape the standard description. In the first case, the lack of a self-consistent theory of Quantum Gravity needs the semiclassical approach of quantum field theory formulated on curved space-time \[10\]. In the latter case, the big puzzle of dark side, not yet solved at experimental level, can be encompassed in the framework of gravitational phenomena \[11,12\].

In this paper, we shall investigate the causal limit maximum mass of NSs in the context of $f(R)$ gravity formulated in the Jordan frame. We shall adopt the stiffest EoS one can use, the causal limit EoS, with variable speed of sound. In this context, we shall investigate the causal limit of NS maximum mass in the $f(R)$ frame, and how this limit is affected by the sound speed. The causal limit EoS will be linked at densities of the order $\sim 2\rho_0$ to the SLy EoS \[13\], with $\rho_0$ being the saturation density, which is standard in the literature. However, we shall also include the case where the transition density is $\rho_t \sim \rho_0$, mainly motivated by the fact that the exact nuclear matter behavior is known for densities of the order of saturation density.

The results of our investigation will be critically examined discussing the pros and cons of Extended Gravity description of NSs with masses in the mass gap region. Finally, we shall critically take into account the possibility that the secondary compact stellar object of GW190814 binary event \[14\] is a strange star. In this perspective, we shall discuss why a NS, or even a black hole (BH) possibility are more compatible with an Occam's razor approach. The probability that a NS or a small mass BH are candidates for the secondary component of GW190814, is also stressed in the literature \[15,16\], where a thorough analysis of data is performed.

The layout of the paper is the following. Sec II is devoted to a discussion of causal limit of maximum NS mass in the Jordan frame of $f(R)$ gravity. We consider also the role of Extended Gravity with respect to the gravitational wave event GW190814. The strange star perspective, according to the data of GW190814 event, is discussed in Sec. III. Conclusions and perspectives are drawn in Sec. IV.

II. CAUSAL LIMIT OF MAXIMUM NEUTRON STAR MASS IN JORDAN FRAME $f(R)$ GRAVITY AND THE ROLE OF EXTENDED GRAVITY FOR GW190814 EVENT

The maximum mass allowed in a NS is one of the most difficult issues in NS physics, and it is strongly related with the EoS and the adopted theory of gravity. In other words, gravity and the EoS determine the maximum NS mass. For standard GR, the maximum mass obtained for the softest BPAL12 EoS is $M_{\text{max}} \sim 1.4M_\odot$ while for the stiffest, BGN2 EoS, the maximum mass is $M \sim 2.5M_\odot$ \[24\]. Actually, for $Ne\mu$ cores, $M_{\text{max}}$ varies in the range $1.8 \pm 2.2M_\odot$, and, in presence of hyperons (assuming two body interactions), the above limit becomes roughly $1.5 \pm 1.8M_\odot$. The existence of hyperons softens significantly the EoS, unless a three body interaction is assumed for hyperons. In this case, the EoS may result significantly stiffer. If the secondary component of GW190814 proves to be a NS, such a massive NS, in the context of GR, indicates that the core should be composed by nuclear matter solely. This issue however may not be necessarily true in the context of modified gravity, since hyperon based EoS also yield large masses, with the EoS not being too stiff \[39\], thus the hyperon puzzle can be solved in the context of modified gravity. The very own existence of a maximum NS mass is due to the combined effect of GR (or Extended Gravity) and of EoS of supranuclear densities cold nuclear matter. GR calculations indicate that the maximum NS mass is mainly determined by the EoS at supranuclear densities of at least $\rho \geq \rho_0$ \[24\], with $\rho_0$ being the saturation density. Causality is a severe constraint for most EoSs, so a reliable EoS must respect causality. The EoS can become superluminal when $dP/d\rho > 1$ in natural units, and also can become ultrabaric when $P > \rho$.

The question is whether an EoS which predicts superluminality in the speed of sound, is inconsistent with Lorenz invariance and causality. The answer is no, however for the most known and frequently used EoSs, like the SLy, superluminality occurs for energy densities where the NS becomes hydrodynamically unstable \[24\]. The same applies even for the stiffest BGN2 EoS.

Taking into account the stability of nuclear matter at high densities ($\frac{dP}{d\rho} > 0$), and the subluminality condition...
FIG. 1: Mass-radius relation for equation of state SLy+(5) with \( \rho_u = 2\rho_0 \) at \( v^2 = c^2/3 \) (upper right panel), \( v_s^2 = 3c^2/5 \) (upper left panel), \( v_s^2 = c^2 \) (down panel).

\( \left( \frac{dP}{d\rho} \leq 1 \right) \) in natural units, the widely accepted causal upper bound limit on the maximum mass for the static NS case is \([17, 18]\).

\[
M_{\text{max}}^{CL} = 3M_\odot \sqrt{\frac{5 \times 10^{14} \text{g/cm}^3}{\rho_u}},
\]

(1)

where \( \rho_u \) is the maximum density for which the EoS is well known, with corresponding pressure being \( P_u(\rho_u) \). Also the causal limit maximum mass of Eq. (1) is obtained by assuming the causal limit EoS,

\[
P_{sn}(\rho) = P_u(\rho_u) + (\rho - \rho_u)c^2.
\]

(2)

A safe statement, also widely known in astrophysics, is that the actual maximum mass of a static NS \( (P \geq 3\text{ms}) \), built from baryon mass, is

\[
M_{\text{max}} \leq 3M_\odot.
\]

(3)
Rotation increases the causal upper limit of the maximum NS mass to,

$$M_{\text{max}}^{\text{CL,rot}} = 3.89M_\odot \sqrt{\frac{5 \times 10^{14} \text{g/cm}^3}{\rho_u}}.$$  \hspace{1cm} (4)$$

The maximum mass of a NS in Extended Gravity is the most serious issue to be addressed. Some useful insights may be provided by examining the causal limit maximum mass for NSs in the context of Extended Gravity. This limit will provide us with an upper limit on the maximum mass, thus it is an indication of what is the most extreme upper bound on NS masses in the context of Extended Gravity.

Here we shall calculate numerically the causal limit of the maximum mass for NSs in the context of $f(R)$ gravity considering the Jordan frame. More importantly: Is there a gap between maximum NS mass and lowest BH mass, or can they be continuously found even in the mass gap region $M \sim 2.5 \div 5 M_\odot$? In the literature, some ways for discriminating NS and BH are given \[50\]. These criteria are based on gravitational waves tidal deformability measurements.

However, even if one is able to discriminate a BH and a NS in binaries, the question is, what is the theoretical upper limit for NS masses and the lower limit of BH masses (if any)? The true question towards understanding the mass gap region is, what is the maximum allowed mass of a NS.

Hence, this upper limit on NS mass for Extended Gravity is the one obtained by using a causal EoS, which we shall calculate numerically. The questions posed above are important and closely related to future LIGO-Virgo observations and the upcoming LISA project. Thus NSs are truly modern laboratories that may put to test several theoretical physics frameworks, such as modified gravity in its various forms, both in the Jordan \[51\]-\[55\] or Einstein frame \[56\]-\[58\], or even string-motivated gravity \[59\]-\[61\], theoretical astrophysics, nuclear physics \[11\] and even particle physics \[13\]-\[15\], \[62\]-\[64\]. The calculation of the maximum mass causality limit or subclasses of scalar-tensor gravity has been performed in \[65\] and the results are rather interesting. In this paper, we shall perform the same calculation in the context of the Jordan frame of $f(R)$ gravity, by using the causal limit EoS of Eq. (2), with a variable speed of sound $v_s$. We shall adopt the notation of \[65\] for our study. The causal limit EoS we shall use in this paper is,

$$P_{\text{sn}}(\rho) = P_u(\rho_u) + (\rho - \rho_u)v_s^2,$$  \hspace{1cm} (5)$$

and although there exist a conjecture that the sound speed should be less than $c/\sqrt{3}$ \[65\], we shall assume that the sound speed is in the range $c^2/3 \leq v_s^2 \leq c^2$, as in Ref. \[65\]. Also $P_u$ and $\rho_u$ in Eq. (5) correspond to the pressure and density of the well known segment of a low-density EoS, with $\rho_u \simeq 2 \rho_0$. We shall refer to these quantities as transition pressure and transition density hereafter, and we shall assume that the low-density EoS is the SLy \[43\] EoS. Thus, $\rho_u = 2.7223 \times 10^{14} \text{ g/cm}^3$, and the corresponding pressure is $P_u = 3.5927 \times 10^{53} \text{ erg/cm}^3$. But motivated by the fact that ground based nuclear experiments truly describe well nuclear matter up to the saturation density, we shall perform the same numerical analysis for the SLy EoS, by taking the transition density to be of the order of the saturation density, namely $\rho_u \sim \rho_0$.

In the following, we shall solve the $f(R)$ gravity Tolman-Oppenheimer-Volkoff (TOV) equations numerically for the EoS given in Eq. (5) assuming various values of the sound speed in the range $c^2/3 \leq v_s^2 \leq c^2$.

In order to be self-contained, let us briefly recall the $f(R)$ gravity framework for a spherically symmetric compact object, and the corresponding TOV equations. The $f(R)$ action is given by,

$$A = \frac{\epsilon^4}{16\pi G} \int d^4x \sqrt{-g} [f(R) + \mathcal{L}_{\text{matter}}],$$  \hspace{1cm} (6)$$

where $g$ denotes the determinant of the metric tensor $g_{\mu\nu}$ and $\mathcal{L}_{\text{matter}}$ is the perfect fluid matter Lagrangian. Upon varying \[6\] with respect to the metric $g_{\mu\nu}$, we obtain the field equations \[34\],

$$\frac{df(R)}{dR}R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - \left[\nabla_{\mu}\nabla_{\nu} - g_{\mu\nu}\Box\right] \frac{df(R)}{dR} = \frac{8\pi G}{\epsilon^4}T_{\mu\nu},$$  \hspace{1cm} (7)$$

where $T_{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta g^{\mu\nu}}$ denotes the energy-momentum tensor of the matter perfect fluids. The metric describing spherically symmetric spacetimes is,

$$ds^2 = e^{2\psi}c^2dt^2 - e^{2\lambda}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2),$$  \hspace{1cm} (8)$$

where $\psi$ and $\lambda$ are $r$-dependent functions, with $r$ denoting the radial coordinate. For the perfect matter fluid describing the compact stellar interior, the energy-momentum tensor is $T_{\mu\nu} = \text{diag}(e^{2\psi}\rho c^2, e^{2\lambda}\rho, r^2p, r^2p \sin^2\theta)$ where $\rho$ is the
FIG. 2: Mass-radius relation for equation of state SLy+(5) with \( \rho_u = \rho_0 \) at \( v_s^2 = c^3/3 \) (upper right panel), \( v_s^2 = 3c^2/5 \) (upper left panel), \( v_s^2 = c^2 \) (down panel).

matter density and \( p \) denotes the pressure [66]. The equations for the stellar compact object are obtained by adding the hydrostatic equilibrium condition from the contracted Bianchi identities,

\[ \nabla^\mu T_{\mu\nu} = 0, \]

which yield the Euler conservation equation

\[ \frac{dp}{dr} = - (\rho + p) \frac{d\psi}{dr}. \]

Combining the metric [8] and the field Eqs. [7], the equations describing the functions \( \lambda \) and \( \psi \) are [67],

\[ \frac{d\lambda}{dr} = \frac{e^{2\lambda}[r^2(16\pi p + f(R)) - f'(R)(r^2R + 2)] + 2R^2 f''(R)r^2 + 2rf''(R)[Rr_{rr} + 2R_{rr}] + 2f'(R)}{2r [2f'(R) + rf''(R)]}, \]

\[ \frac{d\psi}{dr} = \frac{e^{2\lambda}[r^2(16\pi p - f(R)) + f'(R)(r^2R + 2)] - 2(2rf''(R)R_r + f'(R))}{2r [2f'(R) + rf''(R)]}, \]
where the prime in both Eqs. \([11]\) and \([12]\) denotes differentiation with respect to the Ricci scalar \(R(r)\).

The above equations constitute the modified TOV equations, which, for \(f(R) = R\), reduce to the standard GR TOV equations \([68, 69]\). In the context of \(f(R)\) gravity, the Ricci scalar is dynamically evolving with respect to the variable \(r\), thus the TOV equations should be solved together with the following equation

\[
\frac{d^2 R}{dr^2} = R_r \left( \lambda_r + \frac{1}{r} \right) + \frac{f'(R)}{f''(R)} \frac{1}{r} \left( 3\psi_r - \lambda_r + \frac{2}{r} \right) - e^{2\lambda} \left( \frac{R}{2} + \frac{2}{r^2} \right) \right] - \frac{R^2 f'''(R)}{f''(R)},
\]

which is derived from the trace of Eqs. (7) by using the metric (8).

### Table I: Parameters of compact stars (maximal mass for given EoS and corresponding radius) for various equation of states

| EoS          | \(\rho_u = 2\rho_0\) | \(\rho_u = \rho_0\) |
|--------------|----------------------|----------------------|
|              | \(r_g^2\) | \(M_{\text{max}}\) | \(R_c\) | \(\Delta M_{\text{max}}\) | \(M_\odot\) | \(r_g^2\) | \(M_{\text{max}}\) | \(R_c\) | \(\Delta M_{\text{max}}\) | \(M_\odot\) |
| SLy+(5) with \(v_s^2 = e^2/3\) | 2.50 | 1.92 | 11.28 | 0 | 10 | 2.11 | 11.54 | 0.12 |
| SLy+(5) with \(v_s^2 = 3e^2/5\) | 2.50 | 0.25 | 2.49 | 12.09 | 10 | 2.53 | 12.45 | 0.04 |
| SLy+(5) with \(v_s^2 = e^2\) | 2.50 | 0.25 | 2.97 | 12.85 | 10 | 2.93 | 13.28 | -0.04 |
| SLy+(5) with \(v_s^2 = 3e^2/5\) | 2.50 | 0.25 | 2.98 | 13.57 | 10 | 3.10 | 13.71 | 0.13 |

Thus, in the following we shall solve the TOV Eqs. \([10]\), \([11]\) and \([12]\) together with \([13]\), for

\[
f(R) = R + \alpha R^2,
\]

where the parameter \(\alpha\) is expressed in units of \(r_g^2 = 4G^2M_\odot^2/c^4\) where \(r_g\) is the gravitational radius of Sun. The nuclear matter inside the stellar object will be assumed to satisfy the EoS \([3]\). The sound speed is assumed to be a free parameter that may vary, and also the transition density in Eq. \([5]\) is be chosen to be that of the SLy EoS at \(\rho \sim 2\rho_0\), which is \(\rho_u = 2.7223 \times 10^{14} \text{ g/cm}^3\). \(P_u\) is the corresponding pressure which is \(P_u = 3.5927 \times 10^{33} \text{ erg/cm}^3\) \([13]\). The same analysis can be performed by assuming \(\rho_u \sim \rho_0\).

The results of numerical analysis are presented in Table I and in Figs. 1 and 2. In Fig. 1, we present the mass-radius relation for the causal EoS of Eq. \([6]\) with the low density EoS at \(\rho_u = 2\rho_0\) being the SLy. The upper left plot corresponds to \(v_s^2 = e^2/3\), the upper right plot to \(v_s^2 = 3e^2/5\) and the bottom plot to \(v_s^2 = e^2\). Also the same numerical analysis for the transition density \(\rho_u = \rho_0\) is presented in Fig. 2. The resulting picture is rather interesting.
and rich in qualitative conclusions that can be made. Specifically, it seems that, for all the values of the parameter \( \alpha \), when the sound speed is less than that of light, the causal limit maximum NS mass is larger in comparison to the causal limit maximum mass of GR case, for both the cases \( \rho_u = 2\rho_0 \) and \( \rho_u = \rho_0 \). However, when \( v_s = c \), for small values of the parameter \( \alpha \), the GR causal maximum mass is larger if compared to the one of \( R^2 \) model, but, for larger values of the parameter \( \alpha \), the \( R^2 \) model overwhelms over the GR values. For the standard values in the literature, for the transition density \( \rho_u = 2\rho_0 \), the results on causal limit maximum mass indicate that NSs for the specific \( f(R) \) model and the low density SLy EoS, the masses belong to the mass gap region, but not as deeply as we expected. In fact the GR results are quite close, so even in the context of the \( R^2 \) gravity, the \( \sim 3M_\odot \) upper mass barrier \([3]\) seems to remain the same. Finally, as a last comment, it is worth noticing that when the transition density of the causal EoS is assumed to be equal to the nuclear saturation density, the maximum masses are larger if compared to the case when the transition density is twice the saturation density.

### III. THE STRANGE STAR PERSPECTIVE: IS IT POSSIBLE OR PROBABLE ACCORDING TO GW190814 EVENT?

Using the Occam’s razor approach, the answer to the question posed in the title is possible for the moment, but not probable. Although nature will continue to surprise us, and the strange star perspective is very exciting for theoretical physics, for the moment we evince that the strange star is just a possibility, but not a probability according to the description of the GW190814 event.

The most important argument that makes the strange star possibility small for the GW190814 event is the very well accepted procedure of how stars, undergoing thermonuclear evolution, remain hydrodynamically stable. In ordinary stars, and NSs in particular, gravity keeps the star in hydrodynamical equilibrium, it keeps the star compact. Gravity counteracts the Fermi pressure of the degenerate fermionic material in ordinary stars. In addition, for NSs, gravity compensates even the repulsive neutron-neutron interaction occurring well beyond the superfluid crust and outer core, at supranuclear densities. Thus gravity and Fermi pressure are the forces that keep an ordinary star and NS, in particular, in hydrodynamical equilibrium.

On the other hand, strange stars rely on the synergy of gravity and Quantum Chromodynamics (QCD), to compensate the Fermi pressure of the hadronic matter. In the strange stars, the ground state of hadronic matter is, of course, self-bound strange quark matter. Thus, this argument is the strongest argument against strange quark stars.

In order for a strange star to be the secondary object of the GW190814 event, one must change the very own well established fact that gravity is the only opposing force that compensates the Fermi pressure of nuclear matter in stars, while in the case of strange stars, QCD and gravity compensate the Fermi pressure. Thus NSs, in contrast to atomc nuclei and strange stars, are solely bound by gravity, hence they serve as the Occam’s razor description for the secondary component of the binary merging GW190814.

To our knowledge, a smoking gun evidence for strange stars would be either the observation of an apparent radius \( R_\infty = \frac{R}{\sqrt{1 - \frac{M}{M_\odot}}} \) below 9Km, that is \( R_\infty < 9km \), or the observation of half-millisecond pulsars. In principle, the sub-millisecond periods cannot be sustained by ordinary nuclear matter NSs, since this high spin rotation would severely affect its hydrodynamic stability. Strange stars however can sustain such spin periods since they are self-bound by QCD and gravity.

Apart from the above smoking gun evidence, there is no other possible and scientifically consistent way to confirm the existence of strange stars. Strange stars may have been formed primordially, during the early Universe, and of course during the so-called Asymptopia era. There were two possibilities, that may work in favor or against the strange star hypothesis, either that the primordial strangelets \([70]\) survived after the Asymptopia. If they did so, no NS presence would be justified in the Universe, and this argument would work against the strange stars. If on the other hand some primordial strangelets \([70]\) survived, then these could co-exist with NSs and would not pollute the Universe with their presence \([24]\).

To our personal opinion, the primordial strangelets fall into the same category where primordial BHs belong, so both strangelets and primordial BHs are primordial hypothetic objects, and the question is how did these survive and why did NSs appear in the presence of primordial strange stars in the first place? These questions cannot be answered for the time being in a self-consistent way, a fact that casts doubt on the same existence of strange stars. Still, no one can exclude strange stars as possible compact stellar objects, but these are not probable for the moment.

Another puzzle is the very own EoS of strange stars. One could claim that it is QCD-derivable, where one assumes a \( O(1) GeV \) energy scale (or a few GeV) one gluon exchange interaction among quarks (tree-order process), and thus the soft coupling of QCD applies, and the EoS is easily found. However, the Asymptopia state we just described, cannot be easily reached in strange stars. In order for the soft QCD coupling physical state to occur even at tree order (one gluon exchange), one must have high quark densities, many orders above \( \rho \sim 10^{15} g/cm^3 \), and energies \( \sim 1 GeV \). These conditions are not realized in strange stars, and are only realized during the pre-inflationary epoch.
where SU(N) GUTs are unbroken or are about to be broken. On the other hand, for NSs the compressibility modulus of symmetric nuclear matter is higher or smaller than $K_0 = 220$ MeV, and this can be achieved by using stiff or soft EoSs. In all cases though, the density is of the order of a few $\rho \sim 10^{15} g/cm^3$, the two compact stellar objects are quite significantly different.

Another argument which works to date against strange stars, and makes them a hypothetical and not the Occam’s razor choice, is the existence of Pulsar glitches. They are associated with the undoubted presence of NS crust, and they occur due to the sudden unpinning of superfluid crust vortices, situated near the neutron dripping density $\rho_{ND} \sim 0.001 \rho_0$. According to this interpretation of glitches, the superfluid component of the crust must have at least 1% of the total momentum of inertia of the NSs. Evidence for the presence of a crust in NS is mainly supported by the observed free precession of radio pulsars. Strange stars, on the other hand, cannot describe glitches, so this observation serves, at the moment, against the presence of strange stars.

An interesting argument, but still questionable, is the absence of r-process electromagnetic radiation during the GW190814 merging process, which could act against the BH-NS candidates. As it is known, BH mergers are thought to produce no electromagnetic radiation, thus one claim that the GW190814 is a BH merger due to the fact that no kilonova observation was observed and secondly, it cannot be a NS-BH merger because we have never observed such a binary. These claims however are not strongly supported by all the theoretical and observational facts. Firstly, the GW190814 event was six times farther in comparison to the GW170817 event, where a kilonova was also observed. Moreover, if the secondary object of GW190814 was a NS, it would have been swallowed by the $\sim 23 M_\odot$ BH. Again, if the small mass component of the GW190814 event was a NS, no electromagnetic or r-process ejecta would have be observed because the tidal forces would be small $\sim 0$. In the same scenario, simulations indicate that the disk mass would be small, so the electromagnetic processes would have been difficult to detect $\sim 0$. Moreover, the observation or not of a kilonova depends on the line of sight, and on the large uncertainties of the source, as these are determined by LIGO and Virgo.

Furthermore, there is the argument that strong interactions play a significant role both in strange star and in NS physics, so both NSs and strange stars are likely to be found, since the same physics of strong interactions control them. This is not correct of course, since if we recall that the maximum mass of a NS for a free Fermi neutron gas is $\sim 0.72 M_\odot$, the famous Oppenheimer-Volkoff limit, which by simply assuming beta equilibrium becomes $\sim 0.7 M_\odot$. However, the presence of a strongly interacting fluid is verified by the existence of NSs with masses quite larger than the Oppenheimer-Volkoff limit. Thus strong interactions make their presence apparent in NSs too, however in the most part of NSs, in the outer core and inner core, and always at supranuclear densities, where the repulsive component of the neutron-neutron interaction takes place, and thus NSs are not held together by strong interactions, but by gravity itself.

**IV. CONCLUDING REMARKS**

In this work, we investigated the causal maximum mass limit of NSs in $f(R)$ gravity focusing on the $R^2$ model. Particularly, we used the stiffest EoS one can use, the causal EoS, and we numerically solved the TOV equations for the $R^2$ model in order to find the maximum NS mass for the causal EoS. With regard to the causal EoS, it consists of two parts, the low density and the high density, and, for the low-density part, we assumed it consists of the SLy EoS. The density where the low and high density parts meet, is called the transition density, denoted by $\rho_u$ and, for the transition density, we assumed that it is equal to either twice the saturation density $\rho_0$ or it is just equal to the transition density. Accordingly, we solved numerically the TOV equations and extracted the maximum causal masses for various values of the free parameter of the model, which is the coupling of the $R^2$ term denoted $\alpha$. The results are interesting since the conclusion is that, for the choice $\rho_u = 2 \rho_0$, the maximum causal limit for the $f(R)$ gravity is quite close to the GR maximum causal mass and more importantly, the three solar masses causal limit is also marginally respected by the $R^2$ model, for the SLy EoS at least. In addition, a notable feature is that when the speed of sound is equal to the speed of light, and simultaneously for small values of the coupling parameter $\alpha$, the GR causal maximum mass is larger compared to the $R^2$ model, and this feature was also observed in a different context though in $\sim 0$. However, this behavior does not hold true for larger values of the coupling parameter $\alpha$. Moreover, it is also notable that the maximum mass limits, for $\rho_u \sim \rho_0$, are larger compared to the case $\rho_u \sim 2 \rho_0$.

In view of the fact that the upper causal mass limit lies within the mass-gap region (but not deeply as we verified, at least for the $R^2$ model), and in relation with our previous work $\sim 0$, the GW190814 event may be well described by Extended Gravity. However another candidate description for the secondary component of GW190814 is a strange star, so we critically discussed the possibility that the large mass component of the merging event GW190814 is a strange star. As we evinced, by using the Occam’s razor approach, this possibility is deemed unlikely, at least for the moment. As it is demonstrated in Ref. $\sim 0$, the presence of a large mass NS or a small mass BH is probable, not just possible. Of course, Nature has a unique way to utterly change the scientific perspective, but, for the moment, the
most logical way is to stick to the simplest solution that seems to describe all compact astrophysical objects. That is, gravity and Fermi pressure keep the stellar objects hydrodynamically stable, not the synergy of QCD and gravity against Fermi pressure of nuclear matter.

However, even if we stick with the NS and BH solutions, there are still many issues that need to be appropriately addressed. The most important is, what is the maximum mass allowed for NSs, and what is the minimum mass for astrophysical BHs. If NSs are found, with masses larger than $3M_\odot$, this observation will clearly show that GR is unable to describe large mass NSs, and thus Extended Gravity might be the optimal physical description for NSs, combined with an appropriate EoS. Which EoS though could be the optimal? The GW170817 event clearly excluded several stiff EoSs, like the WFF1 [23], however the optimal, to our opinion, EoSs are the SLy [43] the FPS EoS and piecewise polytropic EoSs [29, 30]. SLy and FPS EoSs are the only EoSs which provide a unified description of the crust core, since the EoS is continuous at the crust-core interface. To our knowledge, only these two EoSs have this appealing property. But to our opinion, the piecewise polytropic EoSs, is a particularly appealing description of matter at supranuclear densities, based on optimization of data and simulation of polytropic EoSs [29, 30].

One may claim that any modified gravity it is not the standard gravity, especially the Jordan frame forms of modified gravity as higher-order gravity. But this claim is rather naive due to the fact that the same claim could be done for the Newtonian description of stars, and the general relativistic one. If one sticks on the foundations of Newtonian gravity, then the data indicate that NSs cannot be appropriately described by Newtonian gravity. On the other hand, GR, for which the Newtonian theory is the weak field limit, perfectly describes medium mass NSs. Thus a generalization of Newtonian gravity, namely GR, is what actually perfectly describes medium mass neutron stars. Therefore, a generalization of GR, namely Extended Gravity, may be a more appropriate theory accurately describing large mass NSs, and accurately predicting their maximum mass limit for a wide range of EoSs. In the case of modified gravity description of NSs, gravity still describes the hydrostatic equilibrium of NS, in contrast to the strange star hypothesis, where the synergy of QCD and gravity keeps the star in hydrodynamic equilibrium.

Now let us critically discuss what is the role of modified gravity in the GW190814 description, adopting again the Occam’s razor approach. In the context of modified gravity, neutron stars are kept in hydrodynamic equilibrium by gravity and Fermi pressure of the supranuclear density matter. In the context of modified gravity, and even scalar-tensor gravity, one may obtain larger maximum neutron star masses for the same EoSs used in GR [65], as we also evinced in a previous work [51] and we also demonstrated in this work for the causal limit EoS. Of course, the causal EoS cannot describe a physical NS, but it is an indication of the upper limits of the maximum masses for a specific theory of gravity. Also modified gravity descriptions of NS may solve the hyperon problem for NS masses larger than $2M_\odot$, due to the fact that, in the context of GR, only stiff nuclear matter, without hyperons, can achieve such large maximum NS masses [39]. In modified gravity though, this is not an issue, since large maximum NS masses can be achieved even for hyperon related EoSs [39]. A future perspective task, that may shed some light on the question of what is the maximum NS mass and which is the smallest BH mass, is the study of the maximum baryon mass for static NSs. Specifically, the maximum static baryon mass could serve as a lower limit for BHs, since supermassive NSs with masses larger than the maximum baryon masses will collapse to BHs. Thus studying the baryon masses for Extended Theories of Gravity could be proven very useful for NS physics, and we hope to address this issue in the near future.

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