Neutral top-pion and lepton flavor violating processes

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Abstract

In the context of topcolor-assisted techicolor (TC2) models, we study the contributions of the neutral top-pion $\pi^0_t$ to the lepton flavor violating (LFV) processes $l_i \rightarrow l_j \gamma$ and $l_i \rightarrow l_j l_k l_l$. We find that the present experimental bound on $\mu \rightarrow e \gamma$ gives severe constraints on the free parameters of TC2 models. Taking into account these constraints, we consider the processes $l_i \rightarrow l_j l_k l_l$ generated by top-pion exchange at the tree-level and the one loop level, and obtain $Br(\mu \rightarrow 3e) \approx 2.87 \times 10^{-14}$, $1.1 \times 10^{-15} \leq Br(\tau \rightarrow 3e) \approx Br(\tau \rightarrow 2e\mu) \leq 4.4 \times 10^{-15}$, $3.1 \times 10^{-15} \leq Br(\tau \rightarrow 2\mu e) \approx Br(\tau \rightarrow 3\mu) \leq 1.5 \times 10^{-14}$ in most of the parameter space.

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1. Introduction

The standard model (SM) accommodates fermion and weak gauge boson masses by including a fundamental scalar $Higgs$. However, the SM can not explain the dynamics responsible for the generation of mass. Furthermore, the scalar sector suffers from the problems of triviality and unnaturalness. Thus, the SM can only be an effective field theory below some high-energy scale. New physics should exist at energy scales around $TeV$.

The cause of electroweak symmetry breaking (EWSB) and the origin of fermion masses are important problems of current particle physics. Given the large value of the top quark mass and the sizable splitting between the masses of the top and bottom quarks, it is natural to wonder whether $m_t$ has a different origin from the masses of other quarks and leptons. There may be a common origin for EWSB and top quark mass generation. Much theoretical work has been carried out in connection to the top quark and EWSB. Topcolor-assisted technicolor (TC2) models [1], flavor-universal TC2 models [2], top see-saw models [3], and top flavor see-saw models [4] are four of such examples. The common feature of these kinds of models is that topcolor interactions are assumed to be chiral critically strong at the scale about $1 TeV$, and it is coupled preferentially to the third generation. In TC2 models, EWSB is mainly generated by TC interactions or other strong interactions. The topcolor interactions also make small contributions to EWSB and give rise to the main part of the top quark mass. Then, the presence of the physical top-pions in the low-energy spectrum is an inevitable feature of these kinds of models. Thus, studying the possible signatures of the top-pions at present and future high- or low- energy colliders can help the collider experiments to search for top-pions, test topcolor scenario and further to probe EWSB mechanism.

It is well known that the individual lepton numbers $L_e, L_\mu, \text{and } L_\tau$ are automatically conserved and the tree-level lepton flavor violating (LFV) processes are absent in SM, due to unitary of the leptonic analog of CKM mixing matrix and the masslessness of the three neutrinos. However, the solar neutrino experiments [5] and the atmospheric neutrino experiments [6] confirmed by reactor and accelerator experiments [7] provide very strong
evidence for mixing and oscillation of the flavor neutrinos, which presently provide the
only direct observation of physics that can not be accommodated within the SM and
imply that the separated lepton number are not conserved. Thus, the SM requires some
modification to account for the pattern of neutrino mixing, in which the LFV processes
like \( l_i \to l_j \gamma \) and \( l_i \to l_j k l_i \) are allowed. The observation of these LFV processes would be
a clear signature of new physics beyond the SM. The fact and the improvement of their
experimental measurements force one to make more elaborate theoretical calculation in
the framework of some specific models beyond the SM and see whether the LFV effects
can be tested in the future experiments. For instance, these LFV processes have been
widely studied in a model independent way in Ref.[8], in the SM with extended right-
headed and left-handed neutrino sectors [9], in supersymmetric models [10], in the general
two Higgs doublet model(2HDM) type III [11], in the Zee model [12], and in the topcolor
models [13].

The aim of this paper is to study the contributions of the neutral top-pion \( \pi_t^0 \) predicted
by TC2 models to the LFV processes \( l_i \to l_j \gamma \) and \( l_i \to l_j k l_i \) and see whether \( \pi_t^0 \) can
give significant effects on these processes. The paper is organized as follows: in section
2 we give the flavor-diagonal(FD) and flavor-changing(FC) couplings of \( \pi_t^0 \) to the three
family leptons and calculate its contributions to the LFV process \( l_i \to l_j \gamma \). Using the
experimental upper limit of the LFV process \( \mu \to e \gamma \), we give the constraints on the flavor
mixing factors. The tree-level and one loop-level contributions of \( \pi_t^0 \) to the branching
ratios \( Br(l_i \to l_j k l_i) \) are calculated in section 3. The conclusions are given in section 4.

2. The flavor mixing factors of \( \pi_t^0 \) and the LFV processes \( l_i \to l_j \gamma \)

2.1 The couplings of the neutral top-pion \( \pi_t^0 \) to leptons.

For TC2 models [1], \( TC \) interactions play a main role in breaking the electroweak
symmetry. Topcolor interactions make small contributions to EWSB, and give rise to
the main part of the top quark mass, \( (1 - \varepsilon) m_t \), with the parameter \( \varepsilon \ll 1 \). Thus, there
is the following relation:

\[
\nu_\pi^2 + F_t^2 = \nu_W^2,
\]
where $\nu_\pi$ represents the contributions of TC interactions to $EWSB$, $\nu_W = \nu/\sqrt{2} \simeq 174 GeV$. Here $F_t \simeq 50 GeV$ is the physical top-pion decay constant, which can be estimated from the Pagels-Stokar formula. This means that the masses of the gauge bosons $W$ and $Z$ are given by absorbing the linear combination of the top-pions and technipions. The orthogonal combination of the top-pions and technipions remains unabsorbed and physical [14]. However, the absorbed Goldstone linear combination is mostly the technipions while the physical linear combination is mostly the top-pions, which are usually called physical top-pions ($\pi^\pm_t, \pi^0_t$). The existence of the physical top-pions in the low-energy spectrum can be seen as characteristic of topcolor scenario, regardless of the dynamics responsible for $EWSB$ and other quark masses.

The $FD$ couplings of the neutral top-pion $\pi^0_t$ to leptons can be written as:

$$\frac{m_l l_5^l}{\nu} \nu_l \gamma^5 l \pi^0_t$$

(2)

where $l = \tau, \mu$ or $e$. For TC2 models, the underlying interactions, topcolor interactions, are non-universal and therefore do not possess GIM mechanism. The non-universal gauge interactions result in the new $FC$ coupling vertices when one writes the interactions in the mass eigen-basis. Thus, the top-pions can induce the new $FC$ scalar coupling vertices [15]. The $FC$ couplings of $\pi^0_t$ to leptons can be written as:

$$\frac{m_\tau k_{\tau_i} l_5^i}{\nu} \nu_\tau \gamma^5 l_i \pi^0_t$$

(3)

where $l_i (i = 1, 2)$ is the first(second) lepton $e(\mu)$, $k_{\tau_i}$ is the flavor mixing factor, which is the free parameter.

Using the $FD$ and $FC$ couplings of the neutral top-pion $\pi^0_t$ to fermions, we have studied the $FC$ process $\mu^+ \mu^- \rightarrow 7c$ mediated by $\pi^0_t$ exchange [16]. We find that $\pi^0_t$ can generate several hundred $7c$ events and the signals of $\pi^0_t$ might be detected via the process $\mu^+ \mu^- \rightarrow 7c$ in the first muon collider. In the next subsection, we will study the contributions of $\pi^0_t$ to the $LFV$ processes $l_i \rightarrow l_j \gamma$ and see whether the experimental upper limits of these processes can give significant constraints on the flavor mixing factor $k_{\tau_i}$.
2.2 The LFV processes $l_i \rightarrow l_j \gamma$

The observation of neutrino oscillations [5,6] implies that individual lepton numbers $L_{e,\mu,\tau}$ are violated, suggesting the appearance of the LFV processes, such as $l_i \rightarrow l_j \gamma$ and $l_i \rightarrow l_i l_j l_l$. The branching ratios of these processes are extremely small in the SM with right-handed neutrinos. For example, reference [17] has showed $Br(\mu \rightarrow e\gamma) < 10^{-47}$. Such small branching ratios are unobservable.

![Feynman diagrams](image)

Figure 1: Feynman diagrams contribute to the LFV processes $l_i \rightarrow l_j \gamma$ due to the neutral top-pion $\pi^0_t$ exchange in TC2 models.

The present experimental upper limits are [18]:

$$Br(\tau \rightarrow \mu\gamma) < 1.1 \times 10^{-6},$$

$$Br(\tau \rightarrow e\gamma) < 2.7 \times 10^{-6},$$

$$Br(\mu \rightarrow e\gamma) < 1.1 \times 10^{-11}.$$  \hspace{1cm} (4)

These bounds are expected to be improved by a few orders of magnitude in the future. For example, in an experiment under preparation at PSI [19], it is planned to reach a sensitivity

$$Br(\mu \rightarrow e\gamma) < 1 \times 10^{-14}.$$  \hspace{1cm} (5)

Thus, these processes are ideal tools to search for new physics. The observation of any rate for one of these processes would be a signal of new physics.

The relevant Feynman diagrams for the contributions of the neutral top-pion $\pi^0_t$ to the LFV processes $l_i \rightarrow l_j \gamma$ are shown in Fig.1. The internal fermion lines may be $\tau, \mu$, or $e$. However, the internal fermion propagator provides a term proportional to $m_f^2$ in
the numerator, which is not cancelled by the $m_\tau^2$ in the denominator since the heavy $\pi^0_t$ mass dominates the denominator. Thus, we only take the internal fermion line as the $\tau$ fermion line.

Using Eq.(2), Eq.(3) and other relevant Feynman rules, the decay widths of the $LFV$ processes $l_i \rightarrow l_j \gamma$ can be written as:

$$\Gamma(\tau \rightarrow m \gamma) = \frac{n_r^2 k_\tau^2 k_\pi^2}{2048 \alpha_e^4 \pi^4} [F_1 - \frac{1}{2} m_\tau^2 (F_2^2 + F_2 F_3)] - m_\tau F_1 F_2,$$

where $m = \mu$, or $e$, $F_i$ are

$$F_1 = B_0 + m_{\pi_t}^2 C_0 - 2C_{24} + m_\tau^2 (C_{11} - C_{12}) - B_0^* - B_1^*,$$
$$F_2 = 2m_\tau (-C_{21} - C_{22} + 2C_{23}),$$
$$F_3 = 2m_\tau (C_{22} - C_{23}).$$

with

$$\Gamma(\mu \rightarrow e \gamma) = \frac{n_\mu^2 m_\mu^2 k_\tau^2 k_\pi^2 \alpha_e}{2048 \alpha_e^4 \pi^4} [F_1' - m_\mu F_1 F_2' - \frac{1}{2} m_\mu^2 (F_2^2 + F_2 F_3')].$$

The standard Feynman integrals $B_n$, $C_0$, and $C_{ij}$ can be written as:

$$C_{ij} = C_{ij}(-p_l, p_\gamma, m_{\pi_t}, m_\tau, m_\tau), \quad C_0 = C_0(-p_l, p_\gamma, m_{\pi_t}, m_\tau, m_\tau),$$

$$B_0 = B_0(p_\gamma, m_\tau, m_\tau), \quad B_0^* = B_0(-p_m, m_{\pi_t}, m_\tau), \quad B_1^* = B_1^*(-p_l, m_{\pi_t}, m_\tau),$$

where $m_{\pi_t}$ is the mass of the top-pion, the variable $P_m(m = \mu$ or $e)$ is the momentum of the final state lepton, $P_l$ is the momentum of the initial state lepton, and $l = \tau$ or $\mu$, which corresponds the lepton $\tau$ decay $\tau \rightarrow e(\mu)\gamma$ and the lepton $\mu$ decay $\mu \rightarrow e\gamma$, respectively. In above equations, we have assumed that the masses of the final state lepton equal to zero.
For TC2 models, the topcolor interactions only contact with the third generation. The new particles, such as gauge boson $Z'$ and top-pions $\pi_0^\pm_t$, treat the fermions in the third generation differently from those in the first and second generation and treat the fermions in the first generation same as those in the second generation. So, in our calculation, we will assume that the mixing factor $k_{\tau\mu}$ is equal to the mixing factor $k_{\tau e}$. In this case, we have $\Gamma(\tau \to \mu\gamma) \simeq \Gamma(\tau \to e\gamma)$ for $m_\mu \simeq 0, m_e \simeq 0$. The corresponding branching ratios $Br(l_i \to \ell_j\gamma)$ can be written as:

\begin{align}
Br(\tau \to \mu\gamma) & \simeq Br(\tau \to e\gamma) = Br^{\text{exp}}(\tau \to e\nu_\tau\bar{\nu}_\tau) \frac{\Gamma(\tau \to e\gamma)}{\Gamma(\tau \to e\nu_\tau\bar{\nu}_\tau)}, \quad (16) \\
Br(\mu \to e\gamma) & = \frac{\Gamma(\mu \to e\gamma)}{\Gamma(\mu \to e\nu_\tau\bar{\nu}_\mu)} \quad (17)
\end{align}

with

\begin{align}
\Gamma(\tau \to e\nu_\tau\bar{\nu}_\tau) = \frac{m_\tau^5 G_F^2}{192\pi^3}, & \quad \Gamma(\mu \to e\nu_\tau\bar{\nu}_\mu) = \frac{m_\mu^5 G_F^2}{192\pi^3}. \quad (18)
\end{align}
Where the Fermi coupling constant $G_F = 1.16637 \times 10^{-5} GeV^{-2}$ and the precision measured branching ratio $Br^{exp}(\tau \rightarrow e\nu e\bar{\nu}) = (17.83 \pm 0.06)\%$ [18].

Figure 3: The branching ratio $Br(\mu \rightarrow e\gamma)$ as a function of the flavor mixing factor $k$ for three values of the top-pion mass $m_{\pi_t}$.

To obtain numerical results, we take the SM parameters as $\alpha_e(m_Z) = \frac{1}{128.8}$, $m_\tau = 1.777 GeV$, $m_\mu = 0.105 GeV$ [18]. The limits on the mass $m_{\pi_t}$ of the top-pion may be obtained via studying its effects on various experimental observables [20]. It has been shown that $m_{\pi_t}$ is allowed to be in the range of a few hundred $GeV$ depending on the models. As numerical estimation, we take the top-pion mass $m_{\pi_t}$ and the mixing factor $k = k_{\tau\mu} = k_{\tau e}$ as free parameters.

We plot the branching ratios $Br(\tau \rightarrow e\gamma)$ and $Br(\mu \rightarrow e\gamma)$ as function of the mixing factor $k$ for three values of the top-pion mass in Fig.2 and Fig.3, respectively. To compare the value of $Br(\mu \rightarrow e\gamma)$ given by $\pi^0_t$ exchange with its current experimental limit, we
have used the horizontal solid line to denote $Br(\mu \to e\gamma) = 1.1 \times 10^{-11}$ in Fig.3. One can see from Fig.2 and Fig.3 that the branching ratios of the LFV processes $l_i \to l_j \gamma$ are $Br(\tau \to \mu \gamma) \simeq Br(\tau \to e\gamma) < 2 \times 10^{-10}$ and $Br(\mu \to e\gamma) < 1.4 \times 10^{-7}$ in most of the parameter space of TC2 models. The branching ratio $Br(\tau \to l\gamma)$ is at least four orders of magnitude below the present experimental bound on $\tau \to l\gamma$, which is far from the reach of present or next generation experiments. The branching ratio $Br(\mu \to e\gamma)$ can be above or below the present experimental bound on $\mu \to e\gamma$, which depends on the value of the top-pion mass $m_{\pi t}$ and the mixing factor $k$. In most of the parameter space, the value of the $Br(\mu \to e\gamma)$ is in the range of $2.5 \times 10^{-12} \sim 1.4 \times 10^{-7}$.

![Figure 4: The flavor mixing factors $k$ as a function of top-pion mass $m_{\pi t}$ for $Br(\mu \to e\gamma) = 1.2 \times 10^{-11}$.

Using the present experimental bound on the LFV process $\mu \to e\gamma$, we can give the constraints on the mixing factor $k$ for $150GeV \leq m_{\pi t} \leq 400GeV$. The numerical results are showed in Fig.4. From Fig.4 we can see that the mixing factor $k$ increases as $m_{\pi t}$ increasing. If we demand that the top-pion mass is smaller than $400GeV$, then there must
be \( k \leq 0.21 \). Thus, the present experimental upper bound of the LFV process \( \mu \to e\gamma \) gives severe constraints on the free parameters \( m_{\pi_t} \) and \( k \) of TC2 models. In the next section, we will take the \( \mu \to e\gamma \) constraints into account and calculate the branching ratios of the LFV processes \( l_i \to l_j l_k l_l \).

3. The LFV processes \( l_i \to l_j l_k l_l \)

In TC2 models, the LFV processes \( l_i \to l_j l_k l_l \) can be generated at the tree level and also can be induced via photon penguin diagrams at the one-loop level, as shown in Fig.5. For the diagrams Fig.5 (b),(c),(d), we have taken \( k = l \).

\[
\begin{align*}
\textbf{Figure 5: The tree-level and one-loop Feynman diagrams contribute to the LFV processes} \\
l_i \to l_j l_k l_l \text{ induced by } \pi_t^0 \text{ exchange.}
\end{align*}
\]

Let us first consider the contributions of the neutral top-pion \( \pi_t^0 \) to the LFV processes \( l_i \to l_j l_k l_l \) via Fig.5(a). For the decay \( \mu \to 3e \), it is induced by the FC scalar coupling \( \pi_t^0 \mu e \). However, the topcolor interactions only contact with the third generation fermions. The flavor mixing between the first and second generation fermions is very small [15].
In numerical estimation, we will assume $k_{\mu e} \simeq 0$. So, the branching ratio $Br(\mu \to 3e)$ induced by $\pi_i^0$ exchange at the tree-level is zero. The LFV processes $\tau \to 2\mu e$ and $\tau \to 2e\mu$ can only be generated via the FC couplings $\pi_i^0\tau e$ and $\pi_i^0\tau \mu$. The decay widths of the processes $\tau \to l_il_jl_k$ are given by:

\begin{align*}
\Gamma(\tau \to 3e) &= \frac{m_\tau^7 m_e^2}{1042\pi^3 m_\mu^4 \nu^4} k^2, \\
\Gamma(\tau \to 3\mu) &= \frac{m_\tau^7 m_\mu^2}{1042\pi^3 \nu^4 m_\pi^4} k^2, \\
\Gamma(\tau \to 2\mu e) &= \frac{m_\tau^7 m_\mu^2}{3072\pi^3 m_\pi^4 \nu^4} k^2, \\
\Gamma(\tau \to 2e\mu) &= \frac{m_\tau^7 m_e^2}{3072\pi^3 m_\pi^4 \nu^4} k^2,
\end{align*}

where $m_l$ ($l = \mu, e,$ or $\tau$) represents the lepton mass and $k = k_{\tau \mu} = k_{\tau e}$.

Now we consider the one-loop contributions of the neutral top-pion $\pi_i^0$ to the LFV processes $\tau \to l_il_jl_k$ via the photonic penguin diagrams shown in Fig. 5(b)(c)(d). Same as Fig.1, the internal fermion line of the photonic penguin diagrams only is the $\tau$ fermion line. Comparing Fig.5 with Fig.1, one can use the branching ratios $Br(l_i \to l_j \gamma)$ to express the branching ratios $Br(l_i \to l_jl_kl_l)[10]$. The one-loop expressions of the branching ratios $Br(l_i \to l_jl_kl_l)$ can be written as:

\begin{align*}
Br^{1\text{-loop}}(\tau \to 3\mu) &= Br^{1\text{-loop}}(\tau \to 3e) = Br^{1\text{-loop}}(\tau \to 2\mu e) = Br^{1\text{-loop}}(\tau \to 2e\mu) \\
&\simeq \frac{\alpha_e}{3\pi} (\ln \frac{m_e^2}{m_\mu^2} - \frac{11}{4}) Br(\tau \to e\gamma), \\
Br^{1\text{-loop}}(\mu \to 3e) &\simeq \frac{\alpha_e}{3\pi} (\ln \frac{m_e^2}{m_\mu^2} - \frac{11}{4}) Br(\mu \to e\gamma).
\end{align*}

For the LFV processes $\tau \to l_il_jl_k$, we have assumed $m_\mu \simeq 0$, $m_e \simeq 0$ and taken $Br(\tau \to e\gamma) \simeq Br(\tau \to \mu\gamma)$. The expressions of the $Br(\tau \to e\gamma)$ and $Br(\mu \to e\gamma)$ have been given in Eq.(16) and Eq.(17), respectively.

Comparing the one-loop contributions of $\pi_i^0$ to the $l_i \to l_jl_kl_l$ with the tree-level contributions, we find that the branching ratios $Br(\tau \to 2e\mu)$, $Br(\tau \to 3e)$, and $Br(\mu \to 3e)$ given by the one-loop diagrams mediated by $\pi_i^0$ exchange are larger than those generated by the tree-level diagrams at least by four orders of magnitude. This is because the
FD coupling $\pi^0 ee$ is proportional to $\frac{m_{\tau}}{\nu}$, which can strongly suppress the values of these branching ratios. However, for the processes $\tau \to 3\mu$ and $\tau \to 2\mu e$, two kinds of contributions are comparable. Thus, we will ignore the tree-level contributions of $\pi^0 t$ exchange to the $\tau \to 3e$, $\tau \to 2e\mu$ and $\mu \to 3e$ in the following numerical estimation.

![Figure 6: The branching ratios $Br(l_i \rightarrow l_j l_k l_l)$ as functions of the top-pion mass $m_{\pi_t}$.](image)

We have assumed $Br_1 = Br(\tau \rightarrow 3e)$ and $Br_2 = Br(\tau \rightarrow 3\mu)$.

Taking into account the constraints of the current experimental upper bound $Br(\mu \rightarrow e\gamma) \leq 1.1 \times 10^{-11}$ on the free parameters $m_{\pi_t}$ and $k$, we find that the branching ratio $Br(\mu \rightarrow 3e)$ is approximately equal to $2.87 \times 10^{-14}$, which might be observable in the planned experiments of the next generation. Combining the tree-level and one-loop contributions, we have $Br(\tau \rightarrow 3e) \simeq Br(\tau \rightarrow 2e\mu)$ and $Br(\tau \rightarrow 3\mu) \simeq Br(\tau \rightarrow 2\mu e)$ in TC2 models, which are shown in Fig.6 as functions of the top-pion mass $m_{\pi_t}$. In Fig.6, we have used $Br_1$ and $Br_2$ represent $Br(\tau \rightarrow 3e)$ and $Br(\tau \rightarrow 3\mu)$, respectively. One can see from Fig.6 that the branching ratios slowly decrease as $m_{\pi_t}$ increasing. As long as $m_{\pi_t} < 400 GeV$, we have $Br(\tau \to 3e) \simeq Br(\tau \to 2e\mu) \geq 1.1 \times 10^{-15}$ and
\( Br(\tau \to 3\mu) \simeq Br(\tau \to 2\mu e) \geq 3.1 \times 10^{-15} \). Even we take \( m_{\pi_t} = 200 GeV \), the branching ratio \( Br(\tau \to 3\mu) \) can only reach \( 1.54 \times 10^{-14} \), which is far below the experimental bound on \( \tau \to l_i l_j l_k \) (\( 10^{-6} \) or \( 10^{-7} \)) [18,21].

4. Conclusions

The presence of physics top-pions in the low-energy spectrum is a common feature of topcolor models. The physics top-pions have large Yukawa couplings to the third family fermions and can induce the FC scalar couplings, which might give significant contributions to the FC processes. The effects of top-pion on these processes are governed by its mass \( m_{\pi_t} \) and the relevant flavor mixing factors.

In this paper we study the contributions of the neutral top-pion \( \pi_t^0 \) predicted by TC2 models on the LFV processes \( l_i \to l_j \gamma \) and \( l_i \to l_j l_k l_l \). We find that the branching ratio \( Br(\tau \to e\gamma) \) is approximately equal to the branching ratio \( Br(\tau \to \mu\gamma) \) and is smaller than \( 2 \times 10^{-10} \) in all parameter space of TC2 models. The present experimental bound on \( \mu \to e\gamma \) produces severe constraints on the top-pion mass \( m_{\pi_t} \) and the mixing factor \( k \).

Based on these constraints on the free parameters of TC2 models, we further calculate the contributions of \( \pi_t^0 \) to the LFV processes \( l_i \to l_j l_k l_l \) at the tree-level and one-loop level. For the LFV processes \( \mu \to 3e, \tau \to 3e, \) and \( \tau \to 2e\mu \), the contributions coming from photonic penguin diagrams are larger than those from the tree-level top-pion exchange at least by four orders of magnitude. While two kinds of contributions are comparable for the processes \( \tau \to 3\mu \) and \( \tau \to 2\mu e \). If we take the top-pion mass \( m_{\pi_t} \leq 400 GeV \), we have \( Br(\tau \to 3e) \simeq Br(\tau \to 2e\mu) \geq 1.1 \times 10^{-15} \) and \( Br(\tau \to 2\mu e) \simeq Br(\tau \to 3\mu) \geq 3.1 \times 10^{-15} \), which can not be observable in the near future experiments. However, the branching ratio \( Br(\mu \to 3e) \) is approximately equal to \( 2.8 \times 10^{-14} \), which may be observable in the planned experiments of the next generation.

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