Longitudinal electromagnetic waves in the framework of standard classical electrodynamics

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Abstract—The link between the longitudinal electromagnetic waves and the system of Maxwell equations is demonstrated. The longitudinal wave component of the electric field strength vector is found as the exact solution of the standard Maxwell equations with specific gradient-type case of electric current and charge densities.

I. INTRODUCTION

The theoretical and experimental description of the longitudinal electromagnetic waves is a subject of the different level discussions. There is a lot of indications of the existence of such waves in some kind of literature references (so called internet cafe messages). On the other hand, the Maxwell equations for the free electromagnetic field contain only the transverse solutions. The last fact is in all handbooks on electrodynamics. Moreover, the internet cafe messages contain some confusion in using the notions "longitudinal electric wave" and "longitudinal wave component of the electric field strength vector". It leads to the confusion in understanding.

The goal of this paper is to explain the situation by means of an exact solutions of well defined physical and mathematical systems of equations. Here we are trying to follow only the investigations presented in the well defined physical journals both on experimental [1, 2] and theoretical [3] levels.

The hypothesis about the longitudinal electromagnetic waves, which presence in the mathematical formalism follows from the massless Dirac equation (on the basis of a link between the massless Dirac and slightly generalized Maxwell equations), was suggested in [3]. The start in [3] was related to the results [4]. Therefore, in our little known publication [5], we presented our preliminary point of view on this problem.

However, the problem on the longitudinal electromagnetic waves can be considered independently (without any relation to the massless Dirac equation and the Maxwell equations in the Dirac-like form).

Below the longitudinal wave component of the electric field strength vector $\vec{E}$ is found as the exact solution of the standard Maxwell equations with specific partial case of electric current and charge densities.

Other approaches to the problem should be mentioned as well, see, e.g. [6].

II. EXACT SOLUTION OF THE MAXWELL SYSTEM WITH GRADIENT-TYPE ELECTRIC SOURCES

As ordinarily in field theory, the system of units $\hbar = c = 1$ is used. Further, in the Minkowski space-time $\mathbb{M}(1, 3) = \{x \equiv (x^\mu) = (x^0 = t, \vec{x}^\mu \equiv (x^j)) ; \mu = 0, 3, j = 1, 2, 3\}$, the variable $x^\mu$ denote the Cartesian (covariant) coordinates of the points of the physical space-time in the arbitrary-fixed inertial reference frame.

Consider the Maxwell equations in the form

$$\partial_0 \vec{E} - \text{curl}\vec{H} = -\text{grad}E^0, \quad \partial_0 \vec{H} + \text{curl}\vec{E} = 0,$$

$$\text{div}\vec{E} = -\partial_0 E^0, \quad \text{div}\vec{H} = 0.$$ 

Compare with the standard Maxwell equations for the free field

$$\partial_0 \vec{E} - \text{curl}\vec{H} = 0, \quad \partial_0 \vec{H} + \text{curl}\vec{E} = 0,$$

$$\text{div}\vec{E} = 0, \quad \text{div}\vec{H} = 0,$$

and with the standard Maxwell system with electric sources.

From the physical point of view, equations (1) are the partial case of the standard Maxwell equations (the partial case of a standard classical electrodynamics). In this case, the current $\vec{J}(x)$ and charge $\rho(x)$ densities have the form

$$\vec{J}(x) = -\text{grad}E^0(x), \quad \rho(x) = -\partial_0 E^0(x).$$

The mathematical point of view shows that objects (1) and (2) are the different partial differential equations. Indeed, the general solution of the system (2) is well known and is given by the transverse electromagnetic waves only

$$\vec{E}(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3k \sqrt{\frac{\omega}{2}} \left\{ \left[ c_k^1 e^1 + c_k^2 e^2 \right] e^{-ikx} + \left[ c_k^{*1} e^{*1} + c_k^{*2} e^{*2} \right] e^{ikx} \right\},$$

$$\vec{H}(x) = \frac{i}{(2\pi)^{\frac{3}{2}}} \int d^3k \sqrt{\frac{\omega}{2}} \left\{ \left[ c_k^1 e^1 - c_k^2 e^2 \right] e^{-ikx} - \left[ c_k^{*1} e^{*1} - c_k^{*2} e^{*2} \right] e^{ikx} \right\}.$$
Here \((\overrightarrow{E}(x), \overrightarrow{H}(x))\) are the real electric and magnetic field strengths, \(c_k^1, c_k^2\) are the complex quantum-mechanical momentum-helicity amplitudes of a photon (the amplitudes of the transverse electromagnetic waves),

\[
kx \equiv \omega t - \vec{k} \cdot \vec{x}, \quad \omega \equiv \sqrt{k^2},
\]

and the 3-component basis vectors \((\vec{e}_1, \vec{e}_2, \vec{e}_3)\), which, without any loss of generality, can be taken as

\[
\vec{e}_1 = \frac{1}{\omega \sqrt{2(k^1k^4 + k^2k^3)}} \begin{vmatrix} \omega k^2 - ik^1k^3 \\ -\omega k^1 - ik^2k^3 \\ i(k^1k^4 + k^2k^3) \end{vmatrix} ,
\]

\[
\vec{e}_2 = \vec{e}_1^*, \quad \vec{e}_3 = \frac{\vec{k}}{\omega}.
\]

are the eigenvectors of the quantum-mechanical helicity operator for the spin \(s = 1\).

The general solution (4) is ordinarily found by the Fourier method, and the normalization factor \(C \equiv \sqrt{\frac{2\pi}{\omega k^2}}\) in (4) is taken from the condition

\[
P_0 = \frac{1}{2} \int d^3x \left( \overrightarrow{E}^2 + \overrightarrow{H}^2 \right) = \int d^3k \omega \left( |c_k^1|^2 + |c_k^2|^2 \right).
\]

It is interesting to find the general solution of the system (1), which is expected to have another (maybe not only transverse) form. The Fourier method in the corresponding rigged Hilbert space leads to the general solution

\[
\overrightarrow{E}(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3k \sqrt{\frac{\omega}{2}} \left[ c_k^1 \vec{e}_1 + c_k^2 \vec{e}_2 + \alpha_k \vec{e}_3 \right] e^{-i\omega t + \vec{k} \cdot \vec{x}} + \left[ c_k^1 \vec{e}_1^* + c_k^2 \vec{e}_2^* + \alpha_k^* \vec{e}_3^* \right] e^{i\omega t - \vec{k} \cdot \vec{x}}.
\]

\[
\overrightarrow{H}(x) = \frac{i}{(2\pi)^{\frac{3}{2}}} \int d^3k \sqrt{\frac{\omega}{2}} \left[ c_k^1 \vec{e}_1 - c_k^2 \vec{e}_2 \right] e^{-i\omega t + \vec{k} \cdot \vec{x}} - \left[ c_k^1 \vec{e}_1^* - c_k^2 \vec{e}_2^* \right] e^{i\omega t - \vec{k} \cdot \vec{x}}.
\]

\[
E^0(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3k \sqrt{\frac{\omega}{2}} \left( \alpha_k e^{-i\omega t - \vec{k} \cdot \vec{x}} + \alpha_k^* e^{i\omega t + \vec{k} \cdot \vec{x}} \right).
\]

Here both the electric field strength \(\overrightarrow{E}(x)\) and the magnetic field strength \(\overrightarrow{H}(x)\) contain (together with ordinary transverse waves) the corresponding longitudinal waves as well. These longitudinal electric and longitudinal magnetic wave are given by the amplitudes \(c_k^3 + c_k^4\) and \(c_k^3 - c_k^4\) respectively. The scalar functions \((E^0(x), H^0(x))\) specify the electromagnetic currents and charges densities in the Maxwell-like system of equations (9). The scalar waves \((E^0(x), H^0(x))\) are longitudinal as well.

In our articles [7, 8], the system of equations (9) is called as the slightly generalized Maxwell equations with gradient-type sources. Indeed, at the first step, the system (9) is the generalization of the Maxwell equations because it contains the condition \(\text{div}\, \overrightarrow{H} \neq 0\) and the nonzero magnetic current density in the equation \(\partial_0 \overrightarrow{H} + \text{curl}\, \overrightarrow{E} = -\text{grad}\, H^0\). Nevertheless, at the second step the system (9) is the simplification (specification) of the Maxwell equations. Its electromagnetic currents and charge densities are the partial gradient-type forms of the general form of the electromagnetic sources.

Thus, the system of equations (9) due to the conditions \(\partial_0 \overrightarrow{H} + \text{curl}\, \overrightarrow{E} \neq 0, \text{div}\, \overrightarrow{H} \neq 0\) is not the standard Maxwell electrodynamics. Therefore, it is better to start the experimental detection of the longitudinal electromagnetic waves in the experimental modeling of the situation given by the system (1), which is inside the standard Maxwell electrodynamics. This system of equations predicts an existence of the longitudinal component of the vector of electric field strength \(\overrightarrow{E}\).
The arguments in the prospect of system (9) are not so evident. Nevertheless, let us mention (i) that system (9) has the maximally possible symmetry properties among the Maxwell and the Maxwell-like systems of equations. In [9] the 256 dimensional algebra of invariance of equations (9) considered in the terms of complex functions

$$\mathcal{E} \equiv E - iH = \begin{vmatrix} \vec{E} - i\vec{H} \\ E^0 - iH^0 \end{vmatrix}$$

(11)

was mentioned. Taking into account the symmetries found recently in [10, 11], the number 256 can be increased. Let us recall that the role of the symmetry principle in electrodynamics is known from the times of Maxwell and Heaviside.

(ii) The system of equation (9) has the property of the Fermi–Bose duality. It can describe both the massless spin 1/2 particle-antiparticle doublet of the fermions and the massless spin (1,0) doublet of bosons (photon and massless spinless boson). In the terms of complex 4-vector (11) equations (9) has the manifestly covariant form

$$\partial_{\mu}\mathcal{E}_{\nu} - \partial_{\nu}\mathcal{E}_{\mu} + i\varepsilon_{\mu\nu\rho\sigma}\partial^\rho\mathcal{E}^\sigma = 0, \quad \partial_{\mu}\mathcal{E}^\mu = 0,$$

(12)

and the form of massless Dirac equation

$$\gamma^\mu \partial_{\mu}\xi = 0,$$

(13)

with specific Clifford–Dirac algebra \(\gamma\) matrix representation (see, e. g., the formulae (14) in [8]).

(iii) The system of equations (9) considered in the specific medium [12, 13] can be applied to inneratomic phenomena and describe the hydrogen spectrum [7–9] (another approach to the hydrogen spectrum description on the basis of stationary Maxwell equations in this medium is given in [14, 15]).

Therefore, there is some sense in experimental modeling the situation given by the system (9) in the problem of the longitudinal electromagnetic waves investigation. Of course, the advantages of system (1), which is inside the standard electrodynamics, are evident.

Note that solution (8) can be found both by the direct application of the Fourier method and as a partial case of solution (10).

The validity of solution (10) can be verified by the direct substitution of (10) into equations (9).

IV. THE SUBSYSTEMS OF THE MAXWELL-LIKE EQUATIONS WITH GRADIENT-TYPE ELECTRIC AND MAGNETIC SOURCES

Due to the above mentioned properties, the system of equations (1) is the most interesting subsystem of the Maxwell-like equations (9) (in the problem of longitudinal electromagnetic waves consideration). Equations (1) follow from equations (9) after substitution \(H^0 = 0 \Rightarrow c_2^2 = c_1^2\). In this case, in solution (8) the amplitude of the longitudinal wave \(\alpha_\xi = 2c_1^2 k\). This partial case of (9) is presented in Sec. 2 in details.

Another interesting subsystem of (9) follows from (9) after the substitution \(E^0(x) = 0\). In this case, the vector \(\vec{H}(x)\) of the magnetic field strength contains the longitudinal wave component. Other properties of the general solution are similar to (8) and are to (8) in evident symmetry. Nevertheless, the interest to this case is lower then for \(H^0 = 0\) case. The conditions \(\text{div}\vec{H} \neq 0\) and \(\partial_0\vec{H} + \text{curl}\vec{E} \neq 0\) are not in the framework of the standard Maxwell electrodynamics.

V. CONCLUSION

The electric field strength vector \(\vec{E}(x)\) that contains the longitudinal wave component is found as the exact solution of the standard Maxwell equations with the partial gradient case of electric current and charge densities (3). The link between the longitudinal electromagnetic waves and the standard Maxwell electrodynamics is demonstrated. The author has a hope that corresponding experimental situation can be created and may be useful.

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