Reconstruction of vertical and L-shaped ancient Egyptian sundials and methods for measuring time

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Abstract

The article presents the results of the study of design features of vertical and L-shaped ancient Egyptian sundials. With the help of astronomical methods were developed their models, based on which the reconstruction of a sundial was held. Also, the original scheme is a simple way to fairly precise of measurement of time with them has been developed. Large urgency of the task due to the lack of similar models and schemes to date.

Model offered by us, which describes the vertical sundial, is a vertical sundial, with a sloping gnomon, which takes into account latitude of area. It is based on the assumption of the existence in ancient Egypt representations about an hour (and a half hour) of equal duration throughout the day, does not depend on the time of year. Offered by us model is characterized by marking hour lines from 6 to 12 hours after each hour. From 12 to 12.5 hours produced displacement in the markup of hour lines on half an hour, then the markup is repeated every hour.

As a consequence, the reconstruction of the vertical sundials, we have developed and proposed a model that describes the design features and operation of the L-shaped sundials of two types. They had to work together with the inclined gnomon, like vertical sundials or directly with vertical sundials. In this case, L-shaped sundials can complement vertical sundials, providing an opportunity to read the caption to hour markers and interpret the indications of vertical sundials because vertical sundials inscription missing.

The article also describes explanation of the inscriptions from the tomb of Seti I, long intriguing researchers. It is proved that the inscription contains the length of the intervals between adjacent markers L-shaped sundial of the second type, where the first interval corresponds to a half of hour.

Keywords: sundial, model, astronomical methods, archaeoastronomy, ancient Egypt.

Introduction

Technologies of ancient Egypt, including measurement of time, reached a high level. In ancient Egypt there is a water clocks and sundials, which had a different design: L-shaped, stepped, vertical. Examples of such tools are stored in the Cairo and Berlin museums.

The earliest written evidence of the existence of the Egyptian sundials was associated with a description of the battle of Pharaoh Thutmose III and belongs to the XVI century BC [1]. In the tomb of Seti I (1300 BC), there is an image of sundial in the form of an L-shaped bar. [2] Similar green schist sundial dating from the reign of Thutmose III, and a sundial dating from Fayoum 1000-600 BC stored in the Egyptian Museum of Berlin [3]. Ancient Egyptian vertical sundial dating to the reign of Pharaoh Mernefta (XIII century BC) are also known (Fig. 1) [4].
The sundial in the form of a disk, also dating from the XIII century BC, were found in 2013 during archaeological excavations in the Valley of the Kings, expedition University of Basel (Switzerland) under the leadership of S. Bickel and E. Pauline-Grothe. Found sundial are limestone disk with marked sectors (Fig. 2).

To prove that the find indeed a sundial, were carried out interdisciplinary research with the help of natural science methods. In recent decades, advances in archaeological researches is largely 

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\[1\] http://aegyptologie.unibas.ch/forschung/projekte/university-of-basel-kings-valley-project/report-2013/
associated with integrating interdisciplinary research, through which appeared and develop new research areas such as archaeoastronomy, arheozoologiya, historical informatics, etc. [5-10].

Swiss researchers using astronomical methods conducted study of the limestone disk found in the Valley of the Kings and have concluded that it was a vertical sundial really, oriented to the South [11]. It is believed that the division on the hours first appeared in ancient Egypt. Already from 2100 BC egyptian priests shared the day on 24 hours [12]. On the basis of the design features of the L-shaped sundials (unequal distance between the marks on the scale), the German egyptologist Ludwig Borchardt in 1910 put forward the hypothesis of the existence in ancient Egypt unequal hours, the duration of which depended on the season. As part of his hypothesis, the duration of hour was constantly changing throughout the year and was equal to 1/12 of the time from sunrise to sunset or from sunset to sunrise. Hour duration, hence varied depending on the latitude and time of year [13]. However, analyzing the markings on the sundial from the Valley of the Kings, S. Bickel and R. Gautschy make assumptions about the use of hours equal to the duration [14]. During the study, they pay attention that the longest match in the range of 6 to 12 hours provides a model of vertical sundial with a sloping gnomon. However, poor agreement between the calculated values with the real in the range of 12 to 18 hours has forced researchers to give up this model and adopt the model of vertical sundial with a horizontal gnomon and approximate marking with a sufficiently high average error - 12 minutes (Fig. 3).

Figure 3. The sundial from the Valley of the Kings with marked existing hour lines (black solid lines), with lines calculated for the equinox (gray solid lines), with lines calculated for the summer solstice (gray dashed lines) and winter solstice (gray line of dots) [15].

The aim of our study was to reconstruct and develop a model of a sundial, which describes a sundial from the Valley of the Kings, and reconstruct the ways of measuring time by means of vertical and L-shaped sundials in ancient Egypt.
Relevance of this topic is not reduced, since the discovery of the first fragments of a sundial in ancient Egypt, because still has not been proposed models adequately describe their design, and allowing determining the time by a simple method and with high accuracy.

As a result of our study was reconstructed and developed new models of ancient Egyptian vertical and the L-shaped sundials, allowing determining the time in a simple manner with sufficient accuracy.

**Vertical sundial from the Valley of the Kings**

During the study, we analyzed the markup of a sundial from the Valley of the Kings. After the calculations for horizontal and vertical sundials we concluded on the greatest similarity of markup found sundial with markup vertical sundial with a sloping gnomon, especially in the range from 6 to 12 hours.

Because on the sundial, except time lines, were plotted point marks of about dividing watch sector in half, we have calculated the hour angles of every 0.5 hours. Calculations hour angles we produced according to formulas 1 and 2 [16]:

\[
H' = \arctg(\cos \varphi \cdot \tan H),
\]

\[
H = 15^\circ \cdot (t - 12)
\]

где H – hour angle of the Sun (for noon H = 0\(^\circ\)), t – time (time format - from 0 to 24 hours), H' - angle between the meridian line and hour line on the sundial, \(\varphi\) – latitude of location (Fig. 4).

![Diagram illustrating the operation of the vertical sundial with a sloping gnomon; \(\varphi\) – latitude [17]](image-url)
The results of our calculations for the latitude of the Valley of the Kings (Egypt) $\phi = 25^\circ 44'\ N$ in the range of from 6 to 18 hours are presented in Tables 1 and 2.

**Table 1.** Hour angles of vertical sundial for latitude $25^\circ 44'\ N$ (before noon). $H$ - hour angle of the Sun, $H'$ - calculated angle between the meridian line and hour line, $t$ - time.

| $t$, (hour) | 6.0 | 6.5 | 7.0 | 7.5 | 8.0 | 8.5 | 9.0 | 9.5 | 10.0 | 10.5 | 11.0 | 11.5 | 12.0 |
|------------|-----|-----|-----|-----|-----|-----|-----|-----|------|------|------|------|------|
| $H$, (°)  | -90.0 | -82.5 | -75.0 | -67.5 | -60.0 | -52.5 | -45.0 | -37.5 | -30.0 | -22.5 | -15.0 | -7.5 | 0.0 |
| $H'$, (°) | -90.0 | -81.7 | -73.4 | -65.3 | -57.3 | -49.6 | -42.0 | -34.7 | -27.5 | -20.5 | -13.6 | -6.8 | 0.0 |

**Table 2.** Hour angles of vertical sundial for latitude $25^\circ 44'\ N$ (afternoon). $H$ - hour angle of the Sun, $H'$ - calculated angle between the meridian line and hour line, $t$ - time.

| $t$, (hour) | 12.0 | 12.5 | 13.0 | 13.5 | 14.0 | 14.5 | 15.0 | 15.5 | 16.0 | 16.5 | 17.0 | 17.5 | 18.0 |
|------------|-----|-----|-----|-----|-----|-----|-----|-----|------|------|------|------|------|
| $H$, (°)  | 0.0 | 7.5 | 15.0 | 22.5 | 30.0 | 37.5 | 45.0 | 52.5 | 60.0 | 67.5 | 75.0 | 82.5 | 90.0 |
| $H'$, (°) | 0.0 | 6.8 | 13.6 | 20.5 | 27.5 | 34.7 | 42.0 | 49.6 | 57.3 | 65.3 | 73.4 | 81.7 | 90.0 |

Hour lines were marked on the photo of sundial found in the Valley of the Kings (Fig. 5).

**Figure 5.** Photo of a sundial found in the Valley of Kings, with marked existing hour lines (gray solid lines), with lines calculated for the integers hours (black lines), and calculated half-hour lines (blue lines). Figures noted the time corresponding to hour angles.

For comparison of the accuracy of the coincidence hour lines of the model, proposed by Swiss researchers, and our model, we used the values of measured hour angles $H'$, and the results of calculations hour angles for vertical sundial with a horizontal gnomon from Swiss researchers.
article $H''_{\text{AQU}}$, $H''_{\text{WSW}}$, $H''_{\text{SSW}}$ (Tab. 3, Tab. 4) [18]. To compare the results, we used the hour angles $H''=H'+90^\circ$ (Tab. 1, Tab. 2).

**Table 3.** Hour angles of vertical sundial (before noon). $H$ - hour angle of the Sun, $H''$ - calculated hour angle, $H''_t$ - measured hour angle, $H''_{\text{AQU}}$ - hour angle, calculated for the sundial with a horizontal gnomon at the equinox, $H''_{\text{WSW}}$ - at the winter solstice, $H''_{\text{SSW}}$ - at the summer solstice, $t$ - time.

| t, (hour) | 6 | 6.5 | 7 | 7.5 | 8 | 8.5 | 9 | 9.5 | 10 | 10.5 | 11 | 11.5 | 12 |
|-----------|---|-----|---|-----|---|-----|---|-----|----|------|---|------|----|
| $H''_t$, ($^\circ$) | 0.0 | - | 17.0 | - | 33.0 | - | 47.0 | - | 60.0 | - | 76.0 | - | 90.0 |
| $H''_{\text{AQU}}$, ($^\circ$) | 0.0 | - | 14.0 | - | 28.0 | - | 42.5 | - | 57.5 | - | 73.5 | - | 90.0 |
| $H''_{\text{WSW}}$, ($^\circ$) | 0.0 | - | 12.0 | - | 25.0 | - | 39.0 | - | 54.5 | - | 72.0 | - | 90.0 |
| $H''_{\text{SSW}}$, ($^\circ$) | 0.0 | - | 15.0 | - | 29.5 | - | 44.0 | - | 59.0 | - | 74.5 | - | 90.0 |
| $H''$, ($^\circ$) | 0.0 | 8.3 | 16.6 | 24.7 | 32.7 | 40.4 | 48.0 | 55.3 | 62.5 | 69.5 | 76.4 | 83.2 | 90.0 |
| $H''_F - H''_{\text{AQU}}$, ($^\circ$) | 0.0 | - | 3.0 | - | 5.0 | - | 4.5 | - | 2.5 | - | 2.5 | - | 0.0 |
| $H''_F - H''_{\text{WSW}}$, ($^\circ$) | 0.0 | - | 5.0 | - | 8.0 | - | 8.0 | - | 5.5 | - | 4.0 | - | 0.0 |
| $H''_F - H''_{\text{SSW}}$, ($^\circ$) | 0.0 | - | 2.0 | - | 3.5 | - | 3.0 | - | 1.0 | - | 1.5 | - | 0.0 |
| $H''_F - H''$, ($^\circ$) | 0.0 | - | 0.4 | - | 0.3 | - | -1.0 | - | -2.5 | - | -0.4 | - | 0.0 |

**Table 4.** Hour angles of vertical sundial (afternoon). $H$ - hour angle of the Sun, $H''$ - calculated hour angle, $H''_t$ - measured hour angle, $H''_{\text{AQU}}$ - hour angle, calculated for the sundial with a horizontal gnomon at the equinox, $H''_{\text{WSW}}$ - at the winter solstice, $H''_{\text{SSW}}$ - at the summer solstice, $t$ - time.

| t, (hour) | 12.5 | 13 | 13.5 | 14 | 14.5 | 15 | 15.5 | 16 | 16.5 | 17 | 18 |
|-----------|------|----|------|---|------|---|------|---|------|---|----|
| $H''_t$, ($^\circ$) | 100.0 | 100.0 | 111.0 | 111.0 | 124.0 | 124.0 | 139.0 | 139.0 | 156.0 | 156.0 | 180.0 |
| $H''_{\text{AQU}}$, ($^\circ$) | - | 106.5 | - | 122.5 | - | 137.5 | - | 152.0 | - | 166.0 | 180.0 |
| $H''_{\text{WSW}}$, ($^\circ$) | - | 108.0 | - | 125.5 | - | 141.0 | - | 155.0 | - | 168.0 | 180.0 |
| $H''_{\text{SSW}}$, ($^\circ$) | - | 105.5 | - | 121.5 | - | 136.0 | - | 150.5 | - | 165.0 | 180.0 |
| $H''$, ($^\circ$) | 96.8 | 103.6 | 110.5 | 117.5 | 124.7 | 132.0 | 139.6 | 147.3 | 155.3 | 163.4 | 180.0 |
| $H''_F - H''_{\text{AQU}}$, ($^\circ$) | - | 6.5 | - | 11.5 | - | 13.5 | - | 13.0 | - | 10.0 | 0.0 |
| $H''_F - H''_{\text{WSW}}$, ($^\circ$) | - | 8.0 | - | 14.5 | - | 17.0 | - | 16.0 | - | 12.0 | 0.0 |
| $H''_F - H''_{\text{SSW}}$, ($^\circ$) | - | 5.5 | - | 10.5 | - | 12.0 | - | 11.5 | - | 9.0 | 0.0 |
| $H''_F - H''$, ($^\circ$) | 3.2 | - | 0.5 | - | -0.7 | - | -0.6 | - | 0.7 | - | 0.0 |

The average error of the markup of vertical sundial with a horizontal gnomon model relative markup of Valley of the Kings sundial: for the equinox is $6^\circ$, the winter solstice is $8.2^\circ$, for the summer solstice is $5^\circ$. Average error of our model markup is only $0.9^\circ$. Thus, all the lines on the limestone disk, found in the Valley of the Kings, agree sufficiently well with the calculated hour lines for vertical sundial with a sloping gnomon and a half-hour shift in the afternoons.

The most striking exception is the hour line, corresponding to 12.5 hours. It painted carelessly and most deviates from the calculated lines. Line, painted after it, marks the 13.5 hours. Each subsequent hour line is also drawn through the space of one hour. We assume, that the line of 12.5 hours was applied formally had of little importance compared with the other hour lines. It is
possible that the time from 12 (noon) to 13.5 hours was merged and corresponded midday rest of workers, near the stone dwellings that sundial had been discovered.

After midday rest time working time, also as before noon, divided on integer hours. The first working hour began at 13.5 hours, and each subsequent - in exactly one hour. In 16.5 hours working day most likely ended, because hour lines, point marks is not on the disk after 16.5 to 18 hours. It should be noted that the edge of the disc chipped, so the point mark could well be on the lost fragments of the sundial. Line of 18 hours, most likely, was applied for the symmetry with the line of 6 hours.

Noon time has been allocated and is associated with rest and / or taking food in many traditions. For example, it is known that in ancient Greece working day began at sunrise and lasted until noon, which marked the end of the working time (Anth. PA1., X, 43). Dinner time in ancient Rome (Mart., IV, 8) began around noon [19]. In many countries with a hot climate is still widespread midday rest - siesta. Perhaps, marking sundial found in the Valley of Kings, is one of the oldest evidence of the existence of this tradition in the era of ancient Egypt.

What hour lines of our model of vertical sundial for hours equal and constant duration, with such high precision coincide with hour lines of Valley of the Kings sundial, testify to the reality of existence in ancient Egypt as a vertical sundial with a sloping gnomon, well as division of the day into 24 equal hour. As a consequence, we have attempted to apply this system for the reconstruction of a method of measuring time by L-shaped sundials.

**L-shaped sundial**

It is well known image of a sundial in the form of an L-shaped bar from the tomb of Seti I (Fig. 6) [20].

![Figure 6. The image of a sundial from the tomb of Seti I at Abydos [2].](image-url)
Sundial with L-shaped structures made of green schist dating from epoch of board Thutmose III is stored in the Berlin Egyptian Museum under accession number 19744 (Fig. 7a), and the sundial of Fayoum dating from 1000-600 BC- under number 19743 (Fig. 7b) [3].

Since their discovery, there have been several hypotheses about both have to work such a sundial. All hypotheses are based on the assumption that it was necessary to measure the time in the division of day on the clock variable duration [21-26]. None of the hypotheses presented, could not be described sufficiently accurate and simple method of measuring time by a sundial of L-shaped structure.

Is quite well known in the scientific world a fragment of papyrus from Tanis of Roman period, which in the upper part of drawing shows the lines that resemble hour lines of vertical sundial, and under them, presumably, is a fragment of the L-shaped sundial [27]. Papyrus fragment from the upper part of the L-shaped sundial and the center of the fan-like divergent lines, unfortunately, is missing.

We offer the following simple reconstruction of the image (Fig. 8). We believe that in the lower part of drawing can be shown L-shaped sundial of standard form, without any additional crossbar or bar, to increase their height. In the upper part of drawing, in the form of a point, to which converge time lines, was depicted gnomon perpendicular to the plane of the drawing.

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2 http://members.aon.at/sundials/berlin-egypte.htm
3 http://www.aegyptisches-museum-berlin-verein.de/c31.php
Figure 8. Fragment of papyrus, which depicts a vertical sundial presumably (partial reconstruction Flanders Petrie (Flinders Petrie)) [28]: a – drawing of papyrus fragment, b - our image of reconstruction L-shaped sundial (selected in gray) on the same fragment of papyrus.

On the basis of this interpretation of images on papyrus from Tanis, vertical sundial can function in conjunction with the L-shaped sundial, complementing each other. To clarify this possibility, we calculated according to the formula 3 the relative lengths of the segments between the projections of neighboring $a_i$ hour lines at an arbitrary horizontal line (Fig. 9):

$$a_i = \frac{\tan H_i - \tan H_{i-1}}{\tan H_i}$$

where $H_i$ - calculated angle between meridian line and i-th hour line ($H_1 = 0^\circ$), $H_1$ - calculated angle between meridian line and first hour line (11 hours); $i$ - number of hour line, $t_i$ - time
corresponding to the hour line, $a_i$ - the relative length of the segment corresponding to the $i$-th hour line ($a_1 = 1$).

Figure 9. Diagram illustrating the arrangement of the segments formed by the projections of adjacent hour lines of the vertical sundial in the time range from 8 to 12 hours, $c$ - the distance to the fixing of the gnomon.

The results of calculations for ideal vertical sundial at latitude $25^\circ 44'$ N (the Valley of the Kings, Egypt) are presented in Table 5.

Table 5. Calculated relative length of segments $a_i$ for ideal vertical sundial in the time range from 8 to 12 hours. $H_i$ - calculated angle between the meridian line and the $i$-th hour line, $t_i$ - time corresponding to the $i$-th hour line, $a_i$ - the relative length of the segment on the horizontal line corresponding to the $i$-th hour line.

| $i$ | 1   | 2   | 3   | 4   |
|-----|-----|-----|-----|-----|
| $t_i$ (hour) | 11  | 10  | 9   | 8   |
| $H_i$ ($^\circ$) | 13.6 | 27.5 | 42  | 57.3 |
| $\tan(H_i)$ | 0.24 | 0.52 | 0.90 | 1.56 |
| $a_i$ | 1.0 | 1.2 | 1.6 | 2.7 |

To compare with the real L-shaped sundial, we also made a similar calculation for the vertical sundial of the Valley of the Kings (Tab. 6).

Table 6. Calculated relative length of segments $a_i$ for vertical sundial of the Valley of the Kings in the time range from 8 to 12 hours. $H_i$ - calculated angle between the meridian line and the $i$-th hour line, $t_i$ - time corresponding to the $i$-th hour line, $a_i$ - the relative length of the segment on the horizontal line corresponding to the $i$-th hour line.

| $i$ | 1   | 2   | 3   | 4   |
|-----|-----|-----|-----|-----|
| $t_i$ (hour) | 11  | 10  | 9   | 8   |
| $H_i$ ($^\circ$) | 14.0 | 30.0 | 43.0 | 57.0 |
| $\tan(H_i)$ | 0.25 | 0.58 | 0.93 | 1.54 |
| $a_i$ | 1.0 | 1.3 | 1.4 | 2.4 |
Having the linear dimensions of the L-shaped sundial from the Berlin Museum (Tab. 7), we calculated the relative lengths of the intervals between adjacent marks (Tab. 8). From the obtained results it can be concluded that the relative lengths of the segments between adjacent marks are sufficiently close.

Table 7. Linear parameters of L-shaped sundial. \( l_i \) - the distance between adjacent marks (between "i" and "i + 1" mark).

| Museum number of sundial | Distance from the beginning of the bar to the first mark, (cm) | \( l_{1} \) (cm) | \( l_{2} \) (cm) | \( l_{3} \) (cm) | Distance from the last mark to the edge of the bar, (cm) | total length of the bar \(^{4,5} \) (cm) |
|-------------------------|---------------------------------------------------------------|----------------|----------------|----------------|---------------------------------------------------------------|-------------------------------|
| №19743                 | 3.2                                                           | 2.5            | 3.8            | 5.1            | 6.3                                                           | 9.4                           | 30.3                         |
| №19744                 | 2.9                                                           | 2.4            | 3.4            | 4.3            | 5.7                                                           | 4.5                           | 23.2                         |

Table 8. Summary table of the lengths of sundial segments. \( a_i \) - the relative length of the segment on the horizontal line corresponding to the \( i \)-th hour line for vertical sundial or the relative length of the segment between adjacent marks on L-shaped sundial. I - ideal vertical sundial for latitude 25\(^{0} \) 44\(^{4} \) N (Valley of the Kings, Egypt), II - vertical sundial found in the Valley of the Kings, III - L-shaped sundial № 19743, IV - L-shaped sundial № 19744.

|     | \( a_1 \) | \( a_2 \) | \( a_3 \) | \( a_4 \) |
|-----|---------|---------|---------|---------|
| I   | 1.0     | 1.2     | 1.6     | 2.7     |
| II  | 1.0     | 1.3     | 1.4     | 2.4     |
| III | 1.0     | 1.5     | 2.0     | 2.5     |
| IV  | 1.0     | 1.4     | 1.8     | 2.4     |

For comparison, we also calculated the absolute length of the first segment for the vertical sundial at the level of the lower edge by the formula 4:

\[
l_i = c \cdot tg\left(H'_1\right)
\]  

(4)

where \( c = 11.8 \text{ (cm)} \) - the height of the gnomon fixing, relatively the lower edge of the vertical sundial of the Valley of the Kings, \( H'_1 \) - angle between the 12 and 11 hour lines of the vertical sundial.

As a result calculations based on formula 4 we obtain for ideal sundial \( l_1=2.85 \) (cm), and for the sundial from the Valley of the Kings \( l_1=2.94 \) (cm). When account is taken the height of the L-shaped bar sundial, which, in the case sundial from the Berlin Museum, is approximately equal to one inch - the ancient Egyptian unit of length one "djeba", equal to 18.75 mm [29], for ideal sundial obtain \( l_1=2.4 \) (cm) (similar to sundial №19744), and for the sundial from the Valley of the Kings \( l_1=2.5 \) (cm) (similar to sundial №19743).

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\(^4\) http://members.aon.at/sundials/berlin-egypte.htm\#1.2 Measuring of the height of the shadow respectively the length of the shadow with the help of an instrument

\(^5\) http://www.aegyptisches-museum-berlin-verein.de/c31.php
These results support the possibility of working together L-shaped sundial and vertical sundial. Figure 10 illustrates such joint work. L-shaped sundial complement vertical sundial, providing an opportunity to read the caption to hour marks and interpret indications of the vertical sundial, because no inscriptions on the vertical sundial. Perhaps this was done on purpose in order to limit circle of people who can interpret the indications of vertical sundial. It appears that determine the time could only specially prepared and authorized people are likely representatives of the priesthood.

Figure 10. The complex of the vertical sundial found in the Valley of the Kings and the L-shaped sundial №19744. Dotted lines illustrate directions of the gnomon shadow, corresponding hour lines of the vertical sundial.

In the tomb of Seti I at Abydos over the image of L-shaped sundial next with marks plotted the signature in the form of Egyptian numerals: 3, 6, 9, 12 (Fig. 6). Until now there is no satisfactory hypothesis allows to explain how a sundial with such markings worked and that actually stands for these numerals. We assumed that these numerals may be associated with relatively long intervals between adjacent marks $b_i$ when the countdown begins with 12 hours, but the first hour of the line corresponds to 12.5 hours (Fig. 11). We did calculation using the formula 5:

$$b_i = \frac{\left(\operatorname{tg} \left( H'_i \right) - \operatorname{tg} \left( H'_{i-1} \right) \right)}{\operatorname{tg} \left( H'_i \right)}$$

где $t_i \in [12.5, 13.5, 14.5, 15.5]$

where $H'_i$ - calculated angle between meridian line and $i$-th hour line ($H'_0 = 0^\circ$), $H'_i$ - calculated angle between meridian line and first hour line; $i$ - number of hour line, $t_i$ - time corresponding to the hour line, $b_i$ - the relative length of the segment corresponding to the $i$-th hour line ($b_1 = 1$).
Figure 11. Diagram illustrating the arrangement of the segments formed by the projections of adjacent hour lines of the vertical sundial in the time range from 12 to 15.5 hours, $c$ - the distance to the fixing of the gnomon.

The results of calculations by the formula 5 for ideal vertical sundial at latitude $25^\circ \text{44'} \text{N}$ (Valley of the Kings, Egypt), in the time range from 12.0 to 15.5 hours are presented in Table 7.

Table 7. Calculated relative length of segments $b_i$ for vertical sundial of the Valley of the Kings in the time range from 12 to 15.5 hours. $H_i$ - calculated angle between the meridian line and the $i$-th hour line, $t_i$ - time corresponding to the $i$-th hour line, $b_i$ - the relative length of the segment on the horizontal line corresponding to the $i$-th hour line, $\approx b_i$ - rounding to integers.

| $i$ | $t_i$, (hour) | $H_i,(^\circ)$ | $\text{tg}(H_i)$ | $b_i$ | $\approx b_i$ | $\approx b_i \times 3$ |
|-----|--------------|----------------|-------------------|------|-------------|------------------|
| 1   | 12.5         | 6.8            | 0.12              | 1.0  | 1           | 3                |
| 2   | 13.5         | 20.5           | 0.37              | 2.1  | 2           | 6                |
| 3   | 14.5         | 34.7           | 0.69              | 2.7  | 3           | 9                |
| 4   | 15.5         | 49.6           | 1.17              | 4.0  | 4           | 12               |

Calculated the relative lengths of the $b_i$ when rounding to integers and multiplying by three give the same series of numbers, as in a fresco in the tomb of Seti I: 3, 6, 9, 12. Because these numbers have a common factor - the "three", it is logical to assume that on the fresco were depicted not relative lengths of the segments, but their absolute values, eg length in the ancient Egyptian inches (djeba).

Offered by us method of measuring time by L-shaped sundial does not contradict the instructions for their use on the ancient Egyptian language, placed on the fresco in gorbnitse Seti I under the figure of the sundial, translation of which was performed only partially [30].

Figure 12 illustrates the joint work of vertical sundial (for example, a sundial from the Valley of the Kings) and the L-shaped sundial with the first half hour interval.
Figure 12. The complex of vertical sundial found in the Valley of the Kings and the L-shaped sundial with the first half hour period. Dotted lines illustrate the direction of the shadow from the gnomon, the corresponding hour marks on the L-shaped sundial.

Having L-shaped sundial both types - with hour and half-hour first time interval, can be more subtle and difficult to mark the vertical sundial and organize working time.

Conclusion

Thus, as a result of our study was reconstructed and proposed new models of functioning of Egyptian sundial. Our model describing the vertical sundial is a vertical sundial with a sloping gnomon, which takes into account latitude of the area. It is based on the assumption about the existence in ancient Egypt of representations about an hour (and a half) of equal duration throughout the day, does not depend on the time of year.

Offered by us model is characterized by marking hour lines from 6 to 12 hours after each hour. From 12 to 12.5 hours shift is made in the markup hour lines on half an hour. We associate a half-hour shift in the markup with the need for marking hour and a half of time interval of midday rest for workers - the traditional siesta, which is characteristic for countries with hot climates.

A similar selection one and a half hours in afternoon already detected on the sundial XIII - XII century BC [31, 32]. In the framework of our model the average error of markup of Valley of the Kings sundial is 0.90, which corresponds to an average accuracy of the measurement of time about 3.6 minutes.

Such accuracy nearly four times higher than the accuracy of the model proposed by S. Bickel and R. Gautschy and evidence in favor of our proposed model allowing reconstruct and design features of the Valley of the Kings sundial and a way of measuring time with their using with high degree of reliability.
As a consequence of the reconstruction of the vertical sundials, we have developed and proposed a model that describes the design features and functioning of L-shaped sundials. They had to work together with the inclined gnomon, as that of vertical sundials or directly with vertical sundials.

In such a case, L-shaped sundials can complement vertical sundials, providing an opportunity to read captions to hour marks and interpret indications of vertical sundials, because inscriptions absent from vertical sundials.

During the study we found that existed minimum of two types of L-shaped sundials. The first kind of L-shaped sundial marks on the bar marks the place of the shadow of the gnomon every hour, starting at 12 hours. The second type - the first interval between the marks correspond to half an hour, and the rest for one hour.

Presumably, L-shaped sundial can be used for marking of vertical sundial. The combined use of the L-shaped sundials both types allowed to do more complex the markup of vertical sundials and, thus, for example, more optimal to organize working time and its control.

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