An application of super mean and magic graphs labeling on cryptography system

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Abstract. An edge-magic total (EMT) labeling on a simple graph $G$ with $p$ vertices and $q$ edges is a bijection $f: V(G) \to \{1, 2, \ldots, p + q\}$ such that for each edge $xy$ of $G$, $f(x) + f(xy) + f(y) = k$, for a fixed positive integer $k$. Moreover, an EMT labeling $f$ is a super edge-magic total labeling (SEMT) if $f(V) = \{1, 2, \ldots, p\}$. Furthermore, a graph $G$ is called a super mean (SM) graph if there exists an injective function $f$ from the vertices of $G$ to $\{0, 1, 2, \ldots, q\}$ such that when each edge $uv$ is labeled with $(f(u) + f(v))/2$ when $f(u) + f(v)$ is even, and $(f(u) + f(v) + 1)/2$ when $f(u) + f(v)$ is odd, then the resulting edge labels are distinct. There are two essentials results that will be proposed in this paper. In the first result, we show that a graph $(n, 1) - F$ Caterpillar has a SM labeling and a graph $H_{u, y} \odot K_n$ has a SEMT labeling. In the second result, we will give an application of these labeling to increase the security level of Affine Cipher in which to encrypt a text on social media.

1. Introduction

Let $G(V,E)$, in short $G$, be a graph with the vertex set $V$ and the edge set $E$, respectively. By a simple graph $G$ we mean a finite undirected, graph with neither loops nor multiple edges. The number of vertices of $G$ is called order of $G$ and it is denoted by $|V| = p$. The number of edges of $G$ is called size of $G$ and it is denoted by $|E| = q$. In further, we mention that a notation $G$ is a graph with $p$ vertices and $q$ edges. We follow that the complement of a graph $G$ is a graph $\overline{G}$ on the same vertices such that two distinct vertices of $G$ are adjacent if and only if they are not adjacent in $G$. The notation $K_n$ is a complete graph on $n$ vertices. In more, terms and notations not defined here are used in the sense of the book by Harary [5].

Nowadays, information technology is not only developing so fast but also playing important role in many aspect of human life. So that, the information security is the one important keyword to keep developed by the new method in mathematical aspect, in [8]. In fact by Rosen [7], a basic Affine Cipher is to easy attacked by frequency analyze. Therefore [9] in 2013, proposed increase one level security of the basic Affine Cipher by using super edge anti magic total (SEAMT) labeling $t$ copies of wheels. On the other hand, some families of graphs with another labeling have been deeply studied by researchers such as in [1], [2], [3], [6], [10], and [11].

In this paper, there are two essentials results that will be proposed. In the first result, we show that a graph $(n, 1) - F$ Caterpillar has a super mean (SM) labeling and a graph $H_{u, y} \odot K_n$ has a super edge
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magic total (SEMT) labeling. In the second one, we will give an application of these labeling to increase
the secure of Affine Cipher which is to encrypt a text on socials media in two levels of security.

2. Materials and methods
We construct the formula of an SM labeling for graph \((n, 1) - F Caterpillar\) and a SEMT labeling for
graph \(H_{u,v} \otimes K_n\) by proving that the graphs have a function with SM and SEMT properties, respectively.
And then, we give an application of these labeling to increase the secure of basic Affine Cipher in which
to encrypted a text on socials media in two levels of security. For building of the application (in Android
Apps), we use JavaScript NetBeans.

3. Results and discussion
3.1. A Super Mean (SM) labeling of Graph \((n, 1) - F Caterpillar\)
In this section, we give a basic definition of graph \(F\). The graph \(F\) in Figure 1, formed by the vertex set
\(V(F) = \{u, v, w, x, y, z\}\) and the edge set \(E(F) = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9\}\) with \(e_1 = uv, e_2 = vw, e_3 = vx, e_4 = vz, e_5 = wx, e_6 = xy, e_7 = yz, e_8 = zu, e_9 = ux\). The graph \((n, m) - F Caterpillar\) is the graph formed by \(m\) graphs \(F\) and joining the vertex \(u\) such that forms a path with
length \(n - 1\). In this study, we consider the case when \(m = 1\), denoted by \((n, 1) - F Caterpillar\) graph, Figure 2.

![Figure 1. The graph F](image)

Now we give a definition of super mean (SM) labeling on graph. An SM labeling on graph \(G\) with
\(|V| = p\) vertices dan \(|E| = q\) edges is an injective function \(f: V \to \{1, 2, 3, ..., p + q\}\) such that for each
edge \(e = uv\) induced function \(f^*(e) = \left\lceil \frac{f(u) + f(v)}{2} \right\rceil\) and forms the set \(f(V) \cup \{f^*(e): e \in E\} = \{1, 2, 3, ..., p + q\}\) by [4].

![Figure 2. The notation of vertex set and the edge set of \((n, 1) - F Caterpillar\) graph](image)
Theorem 1. The graph \((n, 1) - F \text{ Caterpillar}\) has an SM labeling.

Proof. Consider the vertex set and the edge set of graph \((n, 1) - F \text{ Caterpillar}\). Define the function on the vertex set of \((n, 1) - F \text{ Caterpillar}\) graph as follows:

\[
\begin{align*}
f(u_i) &= \begin{cases} 
16i - 1 & , 1 \leq i \leq n, \text{i even} \\
16i - 15 & , 1 \leq i \leq n, \text{i odd}
\end{cases}, \\
f(v_i) &= \begin{cases} 
16i - 3 & , 1 \leq i \leq n, \text{i even} \\
16i - 13 & , 1 \leq i \leq n, \text{i odd}
\end{cases}, \\
f(w_i) &= \begin{cases} 
16i - 5 & , 1 \leq i \leq n, \text{i even} \\
16i - 11 & , 1 \leq i \leq n, \text{i odd}
\end{cases}, \\
f(x_i) &= \begin{cases} 
16i - 15 & , 1 \leq i \leq n, \text{i even} \\
16i - 1 & , 1 \leq i \leq n, \text{i odd}
\end{cases}, \\
f(y_i) &= \begin{cases} 
16i - 13 & , 1 \leq i \leq n, \text{i even} \\
16i - 3 & , 1 \leq i \leq n, \text{i odd}
\end{cases}, \\
f(z_i) &= \begin{cases} 
16i - 11 & , 1 \leq i \leq n, \text{i even} \\
16i - 5 & , 1 \leq i \leq n, \text{i odd}
\end{cases}
\]

Therefore, we have the induced function on the edge set of graph \((n, 1) - F \text{ Caterpillar}\) in the following.

\[
f^*(e_{ij}) = \left[ \frac{f(u_i)+f(u_{i+1})}{2} \right] = \frac{\frac{16i + 16(i+1) - 15}{2}}{2} = \frac{32i}{2} = 16i , 1 \leq i \leq n - 1;
\]

\[
f^*(e_{1,2}) = \begin{cases} 
16i - 14 & , 1 \leq i \leq n, \text{i odd} \\
16i - 2 & , 1 \leq i \leq n, \text{i even}
\end{cases}
\]

\[
f^*(e_{1,3}) = \begin{cases} 
16i - 10 & , 1 \leq i \leq n, \text{i odd} \\
16i - 6 & , 1 \leq i \leq n, \text{i even}
\end{cases}
\]

\[
f^*(e_{2,3}) = \begin{cases} 
16i - 9 & , 1 \leq i \leq n, \text{i odd} \\
16i - 7 & , 1 \leq i \leq n, \text{i even}
\end{cases}
\]
\[ f^*(e_{i,4}) = \begin{cases} \frac{f(v_i) + f(w_i)}{2} & = 16i - 12, \quad 1 \leq i \leq n, \text{i odd} \\ \frac{f(v_i) + f(w_i)}{2} & = 16i - 4, \quad 1 \leq i \leq n, \text{i even} \end{cases} \]

\[ f^*(e_{i,5}) = \begin{cases} \frac{f(v_i) + f(x_i)}{2} & = 16i - 7, \quad 1 \leq i \leq n, \text{i odd} \\ \frac{f(v_i) + f(x_i)}{2} & = 16i - 9, \quad 1 \leq i \leq n, \text{i even} \end{cases} \]

\[ f^*(e_{i,6}) = \begin{cases} \frac{f(u_i) + f(x_i)}{2} & = 16i - 8, \quad 1 \leq i \leq n \end{cases} \]

\[ f^*(e_{i,7}) = \begin{cases} \frac{f(v_i) + f(y_j)}{2} & = 16i - 4, \quad 1 \leq i \leq n, \text{i odd} \\ \frac{f(z_i) + f(v_j)}{2} & = 16i - 12, \quad 1 \leq i \leq n, \text{i even} \end{cases} \]

\[ f^*(e_{i,8}) = \begin{cases} \frac{f(w_i) + f(x_i)}{2} & = 16i - 6, \quad 1 \leq i \leq n, \text{i odd} \\ \frac{f(w_i) + f(x_i)}{2} & = 16i - 10, \quad 1 \leq i \leq n, \text{i even} \end{cases} \]

\[ f^*(e_{i,9}) = \begin{cases} \frac{f(x_i) + f(y_i)}{2} & = 16i - 2, \quad 1 \leq i \leq n, \text{i odd} \\ \frac{f(x_i) + f(y_i)}{2} & = 16i - 14, \quad 1 \leq i \leq n, \text{i even} \end{cases} \]

Form the following sets:

\[ A_1 = \{ f(u_i) = 16i - 1, \quad 1 \leq i \leq n, \text{i even} \}; \quad A_2 = \{ f(u_i) = 16i - 15, \quad 1 \leq i \leq n, \text{i odd} \}; \]

\[ A_3 = \{ f(v_i) = 16i - 3, \quad 1 \leq i \leq n, \text{i odd} \}; \]
\[ A_4 = \{ f(v_i) = 16i - 13, \quad 1 \leq i \leq n, \text{i odd} \}; \]
\[ A_5 = \{ f(w_i) = 16i - 5, \quad 1 \leq i \leq n, \text{i even} \}; \]
\[ A_6 = \{ f(w_i) = 16i - 11, \quad 1 \leq i \leq n, \text{i even} \}; \]
\[ A_7 = \{ f(x_i) = 16i - 15, \quad 1 \leq i \leq n, \text{i even} \}; \]
\[ A_8 = \{ f(x_i) = 16i - 1, \quad 1 \leq i \leq n, \text{i odd} \}; \]
\[ A_9 = \{ f(y_i) = 16i - 13, \quad 1 \leq i \leq n, \text{i even} \}; \]
\[ A_{10} = \{ f(y_i) = 16i - 3, \quad 1 \leq i \leq n, \text{i odd} \}; \]
\[ A_{11} = \{ f(z_i) = 16i - 11, \quad 1 \leq i \leq n, \text{i even} \}; \]
\[ A_{12} = \{ f(z_i) = 16i - 5, \quad 1 \leq i \leq n, \text{i odd} \}; \]
\[ A_{13} = \{ f^*(e_i) = 16i, \quad 1 \leq i \leq n - 1 \}; \]
\[ A_{14} = \{ f^*(e_{i,1}) = 16i - 14, \quad 1 \leq i \leq n, \text{i odd} \}; \]
\[ A_{15} = \{ f^*(e_{i,1}) = 16i - 2, \quad 1 \leq i \leq n, \text{i even} \}; \]
\[ A_{16} = \{ f^*(e_{i,2}) = 16i - 10, \quad 1 \leq i \leq n, \text{i odd} \}; \]
\[ A_{17} = \{ f^*(e_{i,2}) = 16i - 6, \quad 1 \leq i \leq n, \text{i even} \}; \]
\[ A_{18} = \{ f^*(e_{i,3}) = 16i - 9, \quad 1 \leq i \leq n, \text{i odd} \}; \]
\[ A_{19} = \{ f^*(e_{i,3}) = 16i - 7, \quad 1 \leq i \leq n, \text{i even} \}; \]
\[ A_{20} = \{ f^*(e_{i,4}) = 16i - 12, \quad 1 \leq i \leq n, \text{i odd} \}; \]
\[ A_{21} = \{ f^*(e_{i,4}) = 16i - 4, \quad 1 \leq i \leq n, \text{i even} \}; \]
\[ A_{22} = \{ f^*(e_{i,5}) = 16i - 7, \quad 1 \leq i \leq n, \text{i odd} \}; \]
\[ A_{23} = \{ f^*(e_{i,5}) = 16i - 9, \quad 1 \leq i \leq n, \text{i even} \}; \]
\[ A_{24} = \{ f^*(e_{i,6}) = 16i - 8, \quad 1 \leq i \leq n \}; \]
\[ A_{25} = \{ f^*(e_{i,7}) = 16i - 4, \quad 1 \leq i \leq n, \text{i odd} \}; \]
$A_{26} = \{f^*(e_{i,7}) = 16i - 12, \quad 1 \leq i \leq n, i \text{ even}\};$

$A_{27} = \{f^*(e_{i,8}) = 16i - 6, \quad 1 \leq i \leq n, i \text{ odd}\};$

$A_{28} = \{f^*(e_{i,9}) = 16i - 10, \quad 1 \leq i \leq n, i \text{ even}\};$

$A_{29} = \{f^*(e_{i,9}) = 16i - 6, \quad 1 \leq i \leq n, i \text{ odd}\};$

$A_{30} = \{f^*(e_{i,9}) = 16i - 14, \quad 1 \leq i \leq n, i \text{ even}\}.$

Finally, we have the set $\bigcup_{i=1}^{n} A_i = f\left(V((n, 1) - F \textit{Caterpillar})\right) \cup \{f^*: \in E((n, 1) - F \textit{Caterpillar})\} = \{1, 2, 3, ..., 16n - 1\}. This concludes that the graph $(n, 1) - F \textit{Caterpillar}$ has an SM labeling and so $(n, 1) - F \textit{Caterpillar}$ is an SM graph.

3.2. A Super Edge Magic Total (SEMT) Labeling of Graph $H_{u,y} \ominus \bar{K}_n$

Firstly we start with the definition of a graph $H$. A graph $H$ is a graph formed from the vertex set $V(H) = \{t, u, v, w, x, y, z\}$ and the edge set $E(H) = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9\}$ with $e_1 = tu, e_2 = uv, e_3 = vw, e_4 = wx, e_5 = xy, e_6 = yz, e_7 = zt, e_8 = bt, e_9 = tx$.

**Figure 3.** The graph $H$

The graph $H_{u,y} \ominus \bar{K}_n$, in Figure 4, is a graph obtained from $H$ by joining all vertices of $\bar{K}_n$ (the complement of complete graph on $n$ vertices) to the vertices $u$ and $y$ in $H$, respectively.

**Figure 4.** The notation on the vertex set and the edge set of graph $H_{u,y} \ominus \bar{K}_n$

Next, we give a definition of magic labeling on graph. An edge magic total (EMT) labeling on a $(p, q)$-graph $G$ is a bijection $f: VUE \rightarrow \{1, 2, ..., p + q\}$ with the property that, for each edge $xy$ in $E$ of $G$, $f(x) + f(xy) + f(y) = k$, for a fixed positive integer $k$. Moreover, an EMT labeling $f$ is a super edge-magic total labeling (SEMT) if it has the property that $f(V) = \{1, 2, ..., p\}$. A graph $G$ is called EMT (SEMT), if graph $G$ admits EMT (SEMT) labeling [4].

Now based on Figure 4, we obtain that the vertex set and the edge set of graph $H_{u,y} \ominus \bar{K}_n$ are $V(H_{u,y} \ominus \bar{K}_n) = \{v'_1, v''_1, t, u, v, w, x, y, z; 1 \leq i \leq n\}$ the vertex set and $E(H_{u,y} \ominus \bar{K}_n) = \{e_i, e'_i, e''_i\}$ the edge set. Clearly that, the order of graph $H_{u,y} \ominus \bar{K}_n$ is $2n + 7$ and $2n + 9$ is the size of graph $H_{u,y} \ominus \bar{K}_n$.

In the following theorem stated that the graph $H_{u,y} \ominus \bar{K}_n$ has a SEM labeling.
Theorem 2. The graph $H_{u,v} \odot \overline{K}_n$ has a SEMT labeling with the magic constant $k = 6n + 19$.

Proof. Consider the vertex set and the edge set of graph $H_{u,v} \odot \overline{K}_n$. Define a bijection $f$ on graph $H_{u,v} \odot \overline{K}_n$ as follows:

\[
\begin{align*}
    f(t) & = n + 1 \\
    f(v) & = n + 2 \\
    f(x) & = n + 3 \\
    f(z) & = n + 4 \\
    f(u) & = 2n + 5 \\
    f(w) & = 2n + 6 \\
    f(y) & = 2n + 7 \\
    f(v_i) & = i; \ i = 1, 2, 3, ... , n; \ f(v_i') = i + n + 4; \ i = 1, 2, 3, ... , n. \\
    f(e_i) & = 14 - i + 4n; \ 1 \leq i \leq n \\
    f(e_i') & = 8 - i + 3n; \ 1 \leq i \leq n \\
    f(e_i'') & = 14 - i + 3n; \ 1 \leq i \leq 6 \\
    f(e_7) & = 14 + 4n \\
    f(e_8) & = 16 + 4n \\
    f(e_9) & = 15 + 4n.
\end{align*}
\]

Now, we can check that the magic constant $k$ is in the following:

\[
\begin{align*}
    k & = f(u) + f(e_1') + f(v_1') \\
    & = 2n + 5 + 14 - i + 4n + 4 + i \\
    & = 6n + 19 \\
    k & = f(u) + f(e_2) + f(t) \\
    & = 2n + 5 + 14 - 1 + 3n + n + 1 \\
    & = 6n + 19 \\
    k & = f(u) + f(e_2) + f(v) \\
    & = 2n + 5 + 14 - 2 + 3n + n + 2 \\
    & = 6n + 19 \\
    k & = f(v) + f(e_3) + f(w) \\
    & = n + 2 + 14 - 3 + 3n + 2n + 6 \\
    & = 6n + 19 \\
    k & = f(x) + f(e_4) + f(w) \\
    & = n + 3 + 14 - 4 + 3n + 2n + 6 \\
    & = 6n + 19 \\
    k & = f(x) + f(e_5) + f(y) \\
    & = n + 3 + 14 - 5 + 3n + 2n + 7 \\
    & = 6n + 19 \\
    k & = f(z) + f(e_6) + f(y) \\
    & = n + 4 + 14 - 6 + 3n + 2n + 7 \\
    & = 6n + 19 \\
    k & = f(t) + f(e_8) + f(v) \\
    & = n + 1 + 16 + 4n + n + 2 \\
    & = 6n + 19 \\
    k & = f(t) + f(e_9) + f(x) \\
    & = n + 1 + 15 + 4n + n + 3 \\
    & = 6n + 19 \\
    k & = f(t) + f(e_7) + f(z) \\
    & = n + 1 + 14 + 4n + n + 4 \\
    & = 6n + 19 \\
    k & = f(y) + f(e_9') + f(v_9') \\
    & = 2n + 7 + 8 - i + 3n + i + n + 4
\end{align*}
\]
This concludes that the graph $H_{u,y} \odot \overline{K}_n$ admits a SEMT labeling with the magic constant $k = 6n + 19$.

In the next section, we will use the Theorems 1 and 2 to modified Affine Cipher.

### 3.3. A modified of affine cipher

For any ciphertext $y$ and plaintext $x$, a basic form of Affine Cipher or encryption is $y = e(x) \equiv (ax + b) \mod m$ and decryption is $x = d(y) \equiv a^{-1}(y - b) \mod m$, where $a, b, m$ are natural number with $a$ co-prime respecte with $m$. In fact by Rosen [7], a basic Affine Cipher is easy attacked by frequency analyze, therefore we modified the Affine Cipher by using a SM labeling on path $(n, 1) - F$ Caterpillar and a SEMT labeling on graph $H_{u,y} \odot \overline{K}_n$ based on Theorems 1 and 2 such that having high security level.

Furthermore, we have Affine Cipher in new form as follows: $y = e(x) \equiv (ax + g) \mod m$ and $x = d(y) \equiv a^{-1}(y - g) \mod m$, where $g = f(v)$ or $g = f(e)$ is a label of vertex $v$ or edge $e$ in SM graph $(n, 1) - F$ Caterpillar and SEMT graph $H_{u,y} \odot \overline{K}_n$ reserved by Theorems 1 and 2.

#### Example:

Let $x = \text{NAMA} \text{space}$ be plaintext with length 5. Then, based on Theorem 1, we use $n = 3$ on $(3, 1) - F$ Caterpillar. Suppose that space $= 0, A = 1, B = 2, \ldots, Z = 26$ and take $a = 2$ is co-prime with 27 and so invers of 2 is 14 in modulo 27. By using Theorem 1, we have all labels on vertices and edges of path in graph $(3, 1) - F$ Caterpillar, are $f(u_1) = 1, f(u_2) = 31, f(u_3) = 33$ and $f(e_1) = 16, f(e_2) = 32$. Now, use modified Affine Cipher in encryption form $y = e(x) \equiv (ax + g_1) \mod m$ with $g_1 \in \{f(u_1) = 1, f(u_2) = 31, f(u_3) = 33, f(e_1) = 16, f(e_2) = 32\}, a = 2, a^{-1} = 14, \text{and } m = 27$.

We obtain

$$y_1 = e(N) \equiv (2.14 + \lambda(u_1)) \mod 27$$

$$\equiv (2.14 + 1) \mod 27$$

$$\equiv 29 \mod 27 \equiv 2 = B$$

$$y_2 = e(A) \equiv (2.1 + \lambda(e_1)) \mod 27$$

$$\equiv (2.1 + 16) \mod 27$$

$$\equiv 18 \mod 27 \equiv 18 = R$$

and so on, we have ciphertext $y = \text{BRTGF}$. In the second process on encryption, we use Theorem 2 with graph $H_{u,y} \odot \overline{K}_3$ and $g_2$ in $\{f(v_1') = 1, f(v_2') = 2, f(v_3') = 3, f(v_4') = 8, f(v_5') = 9\}$. We use again, $z = e(y) \equiv (ay + g_2) \mod m$ and we now obtained that:

$$z_1 = e(B) \equiv (2.2 + f(v_1')) \mod 27$$

$$\equiv (2.2 + 1) \mod 27$$

$$\equiv 5 \mod 27 \equiv 5 = E$$

$$z_2 = e(R) \equiv (2.18 + f(v_2')) \mod 27$$

$$\equiv (2.18 + 2) \mod 27$$

$$\equiv 38 \mod 27 \equiv 11 = K$$

and so on, we have the second levels of ciphertext $z = \text{EKPVU}$.

In the decryption process we need two step also, the first one is $z = \text{EKPVU}$ back to ciphertext level 1 by using $y = d(z) \equiv a^{-1}(z - g_2) \mod m$, and so:

$$d(E) \equiv a^{-1}(5 - f(v_1')) \mod 27$$

$$\equiv 14(5 - 1) \mod 27$$

$$\equiv 56 \mod 27 \equiv 2 = B$$

$$d(K) \equiv a^{-1}(11 - f(v_2')) \mod 27$$

$$\equiv 14(11 - 2) \mod 27$$

$$\equiv 126 \mod 27 \equiv 18 = R.$$
and so on, we obtain that $y = \text{BRTGF}$. Furthermore, by using $x = d(y) \equiv a^{-1}(y - g_1) \mod m$, and then we have:

\[
d(B) \equiv a^{-1}(2 - f(u_1)) \mod 27
\]

\[
\equiv 14(2 - 1) \mod 27
\]

\[
\equiv 14 \mod 27 \equiv 14 = N
\]

\[
d(R) \equiv a^{-1}(18 - f(e_1)) \mod 27
\]

\[
\equiv 14(18 - 16) \mod 27
\]

\[
\equiv 28 \mod 27 \equiv 1 = A.
\]

and finally, we obtain back that plaintext $x$ is NAMA.

### 3.4. An encryption-decription flowcharts and algorithms

In this section, we give the flowcharts and algorithms based on Java script.

![Flowchart](image-url)

*Figure 5. Flowchart encryption process*
Encryption Algorithms:
1. Begin
2. Check length of plaintext $x = x_1, x_2, x_3, \ldots, x_n$,
3. Build a graph $(n, 1) - F$ Caterpillar base on length of $x$,
4. If $n$ even use graph $\left(\frac{n+2}{2}, 1\right) - F$ Caterpillar, else use graph $\left(\frac{n+1}{2}, 1\right) - F$ Caterpillar,
5. Construct label of graph $(n, 1) - F$ Caterpillar based on Theorem 1,
6. Use ASCII on 256 character.
7. Encryption in the first step by using modified Affine Cipher $y_i = e(x_i) \equiv (ax_i + g_1) \mod m$
8. Write encryption result in the first process, the first Cipherertext $y = y_1, y_2, y_3, \ldots, y_n$,
9. Build a graph $H_{u,y} \odot \bar{K}_n$, if $n$ even use graph $H_{u,y} \odot \bar{K}_n / 2$, else use graph $H_{u,y} \odot \bar{K}_{n+1} / 2$,
10. Construct label of graph $H_{u,y} \odot \bar{K}_n$ based on Theorem 2,
11. Encryption in the second process by using modified Affine Cipher $z_i = e(y_i) \equiv (ay_i + g_2) \mod m$,
12. Write encryption result in the second process, the second Cipherertext $z = z_1, z_2, z_3, \ldots, z_n$,
13. End.

Decryption Algorithms:
1. Begin
2. Read length of the second Cipherertext, $z = z_1, z_2, z_3, \ldots, z_n$,
3. Build a graph $H_{u,y} \odot \bar{K}_n$, if $n$ even use graph $H_{u,y} \odot \bar{K}_n / 2$, else use graph $H_{u,y} \odot \bar{K}_{n+1} / 2$,
4. Read back label of graph $H_{u,y} \odot \bar{K}_n$ based on Theorem 2,
5. Use ASCII on 256 character,
6. Decryption in the first step by using modified Affine Cipher $y_i = d(z_i) \equiv (a^{-1}(z_i - g_2) \mod m$,
7. Write the second plaintext, $y = y_1, y_2, y_3, \ldots, y_n$.

Figure 6. Flowchart decryption process
8. Read back label of graph \((n, 1) - F \text{Caterpillar}\) based on Theorem 1.
9. Decryption in the second process by using modified Affine Cipher \(x_i = d(y_i) \equiv (a^{-1}(y_i - g_i)) \text{mod} \ m\)
10. Write plaintext, \(x = x_1x_2x_3 \ldots x_n\).
11. End.

*View on Android Apps:*

![Image](image1.jpg)

*Figure 7.* View of plaintext (a) and Cipher text (b) in Whatsapp platform

![Image](image2.jpg)

*Figure 8.* Decryption of Cipher text to plaintext

4. **Remark and conclusion**
In this section, we give the new results corresponding to SM labeling of graph \((n, 1) - F \text{Caterpillar}\) and SEMT labeling of graph \(H_{u, y} \bigodot \bar{K}_n\). This work is an effort to have a deep study of application of graph labeling on cryptography system. In future, it is not only possible obtaining the new technic to increase security level of information but also a wide application of graph labeling in general as well.
References

[1] Afzal H U 2015 *Util. Math.* 97 97-108

[2] Agustin I H, Susanto F, Dafik, Prihandini R M, Alfarisi R and Sudarsana I W 2019 *Int. J. Math. Math. Sci.* 7 1-7

[3] Ahmad Y, Ali U, Bilal M, Zafar S and Zahid Z 2017 *Appl. Math. Nonlinear Sci.* 2 (1) 61-72

[4] Gallian J A 2019 “A Dynamic Survey of Graph Labeling”, *The Elec. J. Combin.* 6 1-535

[5] Harary F 1972 Recent results on generalized Ramsey theory for graphs in *Graph Theory and Applications*: Lecture Notes in Mathematics (Berlin: Springer Berlin Heidelberg)

[6] Inayah N, Sudarsana I W, Musdalifah S and Mangesa N D 2018 *Int. J. Math. Math. Sci.* 6 1-5

[7] Rosen K H 2005 *Elementary Number Theory and Its Applications* (New York: Pearson- Addison Wesley)

[8] Satyaputra A and Aritonang M E 2014 *Beginning Android Programming with ADT Bundle.* (Tanggerang: Gramedia)

[9] Sudarsana I W, Laila R, Lutfi A and Farhamsa D 2013 *J. Math. and Sci.* 6 78-84

[10] Sudarsana I W, Hendra A, Adiwijaya and Setyawan D Y 2012 *Far East J. Math. Sci.* 69 (2) 275-283

[11] Sudarsana I W, Baskoro E T, Uttunggadewa S and Ismaimuza D 2009 *J. Combin. Math. Combin. Comput.* 71 189-199