Magnetic Susceptibility of the Orbitally Degenerate \( (J = 5/2) \) Periodic Anderson Model

- Analysis on the Basis of the Fermi Liquid Theory -

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Abstract

In the orbitally degenerate \( (J = 5/2) \) Periodic Anderson Model, the magnetic susceptibility is composed of both the Pauli term and the Van Vleck term, as is well known. The former is strongly enhanced by the strong correlation between \( f \)-electrons. But, for the latter, the influence of the strong correlation has been obscure for years. In this paper we give the solution of the longstanding problem. With the aid of the \( d = \infty \) approximation, we study this problem on the basis of the Fermi liquid theory with degenerate orbitals, taking account of all the vertex corrections in a consistent way. As a result, we obtain the simple expression for the magnetic susceptibility, and show unambiguously that the Van Vleck term is also highly enhanced in the strong correlation regime. This fact explains naturally the enhanced magnetic susceptibility observed in many insulating systems (i.e., Kondo insulator). Moreover, we show that the Wilson ratio takes a value around 1 in the metallic system, in good agreement with experiments.

KEYWORDS : magnetic susceptibility, Van Vleck susceptibility, orbitally degenerate \( (J = 5/2) \) Periodic Anderson Model, Heavy Fermion, Kondo insulator, Fermi liquid theory, \( d = \infty \) approximation
I. INTRODUCTION

In this paper, we investigate the uniform magnetic susceptibility, \( \chi \), for the orbitally degenerate \(( J = 5/2 \) ) Periodic Anderson Model \(( J = 5/2 \text{ PAM} )\). This model is much closer to real heavy Fermion systems because it includes both the \( f \)-electron orbital degeneracy and the \( l-s \) coupling. Based on this model, we can explain several characteristic behaviors observed experimentally, which are unable to be derived from the SU(N)-PAM. \(( \text{SU(N)-PAM is familiar for theorists but less realistic model than } J = 5/2 \text{ PAM in that both the } f \)-electron bands and the conduction electron bands possess the same \( N \)-fold degeneracy. )\)

One of the characteristic behaviors of the model is the presence of the anomalous Hall effect (AHE). Reference \([3]\) clearly explains that the anomalous Hall coefficients of heavy Fermion systems are proportional to the square of the resistivity at low temperatures, and points out that its mechanism is similar to that of AHE on the ferromagnetic metals proposed by Karplus and Luttinger. \([4,5]\)

The other characteristic property is the presence of the Van Vleck magnetic susceptibility. \([6,7]\) As is well known, the magnetic susceptibility of this model contains the Van Vleck susceptibility \( \chi_V \) in addition to the usual Pauli susceptibility \( \chi_P \), so the total magnetic susceptibility is given by \( \chi = \chi_P + \chi_V \). \( \chi_P \) comes from the intra-band contribution, which vanishes in the absence of the Fermi surface, and \( \chi_V \) comes from the inter-band contribution. Of course, \( \chi \) is one of the most important and characteristic physical quantities in the heavy Fermion systems. But, unfortunately, the property of the Van Vleck susceptibility \( \chi_V \) remains quite obscure in the presence of the large Coulomb interaction.

Furthermore, its general properties should be studied for understanding the origin of the strongly enhanced magnetic susceptibility for so-called Kondo insulators, \([8]\) where \( \chi_P = 0 \) and only the Van Vleck susceptibility remains. We also have to understand the property of \( \chi \) in order to interpret the Knight shift data for super-conductive heavy Fermion systems. \([9]\)

Historically, this problem was first studied in ref. \([1]\). The authors paid attention only to \( \chi_P \), and claimed that \( \chi_V \) could be neglected. On the contrary, mean-field like treatments give the enhanced \( \chi_V \). \([3,7,10]\) But, such approximations seem to be insufficient to settle this problem because they assume the frequency independent renormalization factor, which is a less-realistic assumption. The settlement of this problem will bring us the useful understanding for the electronic properties for the orbitally degenerate model.

The existence of \( \chi_V \) in \( J = 5/2 \text{ PAM} \) is closely related with the fact that the magne-
tization operator \( \hat{M} \) is not conserved in this model, i.e., \([\hat{H}, \hat{M}] \neq 0\). Whenever \( \hat{M} \) is conserved, \( \chi_V \) vanishes identically and the susceptibility has been obtained without ambiguity in the framework of the Fermi liquid theory (FLT). For instance, the susceptibility for SU(2)-PAM or \( J = 5/2 \) single-site Anderson model is made of \( \chi_P \) alone, and obtained by the Luttinger’s method.\[13\]–\[15\] On the other hand, in order to analyze the magnetic susceptibility of our model, we have to find another useful method. (By the RPA-approximation, which is an insufficient analysis for our problem, the enhancement for \( \chi_V \) is less than factor of two even in the strong correlation regime where \( \chi_P \lesssim \infty \).\[16\])

In this paper, we analyze the properties of \( \chi \) and \( \chi_V \) by investigating all the vertex correction for them by use of a kind of the Bethe-Salpeter equation. Then we employ the \( d = \infty \) approximation,\[17\] which is considerably effective for our problem on \( J = 5/2 \) PAM.\[18\] We also discuss the Wilson ratio \( R \) in the strong correlation regime. The definition of \( R \) is

\[
R \equiv \frac{\chi/\chi_0}{\gamma/\gamma_0} \cdot R^0,
\]

where \( R^0 \) is the unrenormalized Wilson ratio. \( \gamma \) (\( \gamma_0 \)) is the (unrenormalized) \( T \)-linear coefficient of the specific heat, \( \chi \) (\( \chi_0 \)) is the (unrenormalized) uniform magnetic susceptibility. \( R^0 \) is given by

\[
R^0 = \frac{3\chi_0}{2g^2\mu_B^2 J_{\text{eff}}^2 \rho^0(0)},
\]

where \( \rho^0(0) \), \( g \) and \( \mu_B \) are the density of states for \( f \)-electrons, the Lande’s \( g \)-factor, and the Bohr magneton respectively, and \( J_{\text{eff}}^2 = J(J + 1) \) in the spherical system, i.e., without any electronic crystal field (ECF). For instance, for an isolated Kondo atom \( R \) is universally given by \( R = (2J + 1)/2J \) and \( R^0 = 1 \).

The composition of this paper is as follows: in §2, we explain main properties of \( J = 5/2 \) PAM and its Green’s functions. In §3, we express the magnetic susceptibility in terms of the dynamical susceptibility on the basis of the multicomponent FLT. In the insulating systems, the susceptibility is given by only the Van Vleck susceptibility defined in this section. In §4, we calculate \( \chi \) and \( \chi_V \) explicitly in case \( U = 0 \). The obtained results are the same as those obtained previously. In §5, we investigate \( \chi \) and \( \chi_V(P) \) in \( d = \infty \) in the strong correlation regimes, by taking all the vertex corrections into account. Then, in the case of no ECF, we find that both \( \chi \) and \( \chi_V \) are proportional to the same enhancement factor. This implies that the Van Vleck susceptibility \( \chi_V \) is enhanced, which is the main result of this paper. In §6, we also discuss the Wilson ratio in our system and find out that \( R \sim 1 \). We also point out that
the results for $d = \infty$ systems are qualitatively correct for the three dimensional system. In §7, the obtained results are summarised and the future problems are pointed out.

In Appendix A, we study the magnetic susceptibility for SU(N)-PAM, and show that the Van Vleck susceptibility is small in this model, which arises from the difference between the magnetic momentum of $f$-electrons and that of conduction electrons. In Appendix D, we confirm our conclusions numerically, by use of the ( self-consistent ) second order perturbation treatment ( ( SC- ) SOPT ). In Appendix E, we show in detail that $\chi''_V$, given by (78), vanishes identically.

II. MODEL AND GREEN’S FUNCTION

In this paper, we study $J = 3/2$ PAM, which is a realistic model for the Ce-compound heavy Fermion systems in that it represents the $f$-orbital degeneracy. In the absence of the magnetic field $H$, our Hamiltonian is given by

$$H = H_0 + H_1$$

$$H_0 = \sum_{k\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + \sum_{kM} E_f f_{kM}^\dagger f_{kM} + \sum_{M\sigma} (V_{kM\sigma}^* f_{kM}^\dagger c_{k\sigma} + V_{kM\sigma} c_{k\sigma}^\dagger f_{kM}),$$

$$H_1 = \frac{U}{2} \sum_{kk'qM \neq M'} f_{k-qM}^\dagger f_{k'qM'}^\dagger f_{k'qM'} f_{kM},$$

where $c_{k\sigma}^\dagger$ is the creation operator of a conduction electron with momentum $k$ and spin $\sigma$, and $f_{kM}^\dagger$ is the creation operator of an $f$-electron with momentum $k$ and angular momentum $M$, which is the eigenvalue of $J_z = l_z + s_z$. As $J = 7/2$ states are ignored in our model as is usually done, the $f$-orbital has six-fold degeneracy, i.e., $M = \{5/2, 3/2, \cdots, -5/2\}$. $V_{kM\sigma}$ in (4) is the mixing potential between the f-electrons and the conduction electrons, which is given by

$$V_{kM\sigma} = (4\pi)^{1/2} \sum_{m} a_{m\sigma}^M Y_{l=3}^m(\theta_k, \varphi_k) \cdot V,$$

$$a_{m\sigma}^M = -\sigma \{ (7/2 - M\sigma) / 7 \}^{1/2} \delta_{m,M-\sigma/2}$$

for $J = 5/2$,

where $a_{m\sigma}^M$ is the Clebsh-Goldan ( C-G ) coefficient and $Y_{l=3}^m(\theta_k, \varphi_k)$ is the spherical harmonic function. In the Coulomb interaction term (4), we take account of the Pauli’s principle, i.e., $M \neq M'$.

In the presence of the magnetic field $H$ along $z$-axis, both $E^f$ and $\epsilon_k$ in (4) are shifted by the Zeeman energies as
\begin{align}
E^f_M &= E^f + g\mu_B M \cdot H, \\
\epsilon_{k\sigma} &= \epsilon_k + \mu_B \sigma \cdot H,
\end{align}

(5)

where \(\mu_B\) is the Bohr magneton and \(\sigma = 1\) for up-spin and \(\sigma = -1\) for down-spin. \(g\) is the Lande’s \(g\)-factor (\(g = 6/7\) for \(J = 5/2\)). Hereafter, we put \(\mu_B = 1\).

The general forms of the Green’s functions for \(J = 5/2\) PAM in the absence of the magnetic field are given by (3.11) \(\sim\) (3.13) of ref. [3]. In the presence of the magnetic field \(H\), the \(f\)-electron Green’s functions can be expressed as

\[
G_{kMM'}(\omega) = G^f_{kMM'}(\omega) + \sum_{\sigma\sigma'} \alpha_{kM\sigma}(\omega) G_{k\sigma\sigma'}(\omega) \alpha_{kM'\sigma'}(\omega),
\]

(6)

\[
G^f_{kMM'}(\omega) \equiv \left(\omega \hat{1} + \mu \hat{1} - \hat{E}^f - \hat{\Sigma}(\omega)\right)^{-1}_{M'M},
\]

(7)

\[
\alpha_{kM\sigma} \equiv \sum_{M'} V_{kM'\sigma} G^f_{kM'M}(\omega),
\]

(8)

where \(\mu\) denotes the Fermi energy of this system and \(G_{k\sigma\sigma'}(\omega)\) is the conduction electron Green’s function. Considering the Zeeman terms for only the \(f\)-electrons, we can derive the expression for the conduction electron Green’s function in the presence of the magnetic field:

\[
G_{k\sigma\sigma'}(\omega) = \frac{\delta_{\sigma\sigma'} \cdot (\omega + \mu - \epsilon_k) + (1 - 2\delta_{\sigma\sigma'}) S_{k\sigma\sigma'}(\omega)}{(\omega + \mu - \epsilon_k - S_{k\uparrow\uparrow}(\omega))(\omega + \mu - \epsilon_k - S_{k\downarrow\downarrow}(\omega)) - S_{k\uparrow\downarrow}(\omega) S_{k\downarrow\uparrow}(\omega)},
\]

(9)

where

\[
S_{k\sigma\sigma'}(\omega) \equiv \sum_{M'M'} V_{kM\sigma} G^f_{kMM'}(\omega) V^*_{kM'\sigma'}.\]

(10)

Note that \(S_{k\sigma,\sigma'}(\omega) = 0\) in case \(H = 0\). The pole of \(G_{k\sigma}(\omega)\) represents the spectrum for the quasiparticle (i.e., heavy electron), which we denote as \(E^*_k\). The pole of \(G^f(\omega)\) represents the local \(f\)-electron spectrum, \(E^f\), and never represent the quasiparticle spectrum. \(G_{kM}(\omega)\) contains both poles.

In Fig. 1, the electronic structure of \(J = 5/2\) PAM in case \(U = 0\) is sketched in terms of the one-body picture. There remains four-fold degenerate local \(f\)-electron spectrum, which reflects the difference of the degeneracy between the conduction electrons and the \(f\) electrons. (They do not exist in SU(N)-PAM.) In Fig. 1(a), the Fermi energy \(\mu\) lies below the lower edge of the \(f\)-\(c\) hybridization gap, so the system is metallic. \(k_F\) is the Fermi momentum and \(D\) (\(-D\)) is the upper (lower) edge of the conduction band. We can show that \(V^2/D^2\) is proportional to \(m_{f\text{-band}}/m_c\), where \(m_{f\text{-band}}\) and \(m_c\) represent the unperturbed mass of \(f\)-electrons and the conduction electrons, respectively. In usual heavy
fermion systems, \( V^2/D^2 \ll 1 \) is satisfied. On the other hand, in Fig. 1(b), the Fermi energy \( \mu \) lies in the hybridization gap and below \( E_f \), so the system is insulating. \( \Delta_\text{-} ( \Delta_+ ) \) represents the lower (higher) edge of the hybridization gap measured from the Fermi energy. This situation is a prototype for the so-called Kondo insulator. More detailed electronic structure is studied beyond the one-body picture by use of the numerical perturbation calculation.

In the absence of the magnetic field, \( G_{k\sigma\sigma'}(\omega) \) is diagonal with respect to the spin \( \sigma \) even if \( U \neq 0 \). The density of states (DOS) of the \( f \)-electrons, \( \rho^0(\omega) \), and that of the conduction electrons, \( \rho_c^0(\omega) \), are given by the retarded Green’s functions as

\[
\rho_c^0(\omega) = -\frac{1}{\pi N} \sum_k \text{Im} G_{k\sigma}^{0R}(\omega),
\]

\[
\rho^0(\omega) = -\frac{1}{\pi N} \sum_k \text{Im} G_{kM}^{0R}(\omega) = \frac{3V^2}{(\omega + \mu - E_f)^2} \cdot \rho_c^0(\omega),
\]

where \( N \) represents the total number of the \( f \)-sites. In case \( U \neq 0 \), these relations (11) and (12) are also satisfied with \( E_f \) replaced by \( E_f + \Sigma(\omega) \) for \( |\omega| \lesssim T^* \), where \( T^*(>0) \) represents the characteristic energy within which the quasiparticles are well defined. In usual heavy Fermion systems, \( T^* \) is much smaller than \( D \).

As is shown in ref. [3], \( G_{kM\sigma}(\omega) \propto e^{i(M' - M)\varphi_k} \) even in case \( U \neq 0 \), which is an important relation throughout this paper.

### III. DEFINITION OF THE PAULI AND VAN-VLECK SUSCEPTIBILITY

In this section, we consider both the Pauli susceptibility \( \chi_P \) and the Van Vleck susceptibility \( \chi_V \), and derive their general expressions on the basis of the FLT for \( J = 5/2 \) PAM. Though our discussion in this section is restricted to \( J = 5/2 \) PAM (in the presence of any ECF) for simplicity, it is more general and is valid for many kinds of multicomponent Fermi liquid systems. Now, we consider the situation where the system is in a finite magnetic field \( H \) along \( z \)-axis. The magnetization of this system, \( \langle M \rangle \), is expressed by use of the Green’s functions as

\[
\langle M \rangle = \frac{1}{N} \sum_k \int_\infty^{i\infty} \frac{d\omega}{2\pi} e^{+i\omega \cdot \varphi_k} \left\{ \sum_M gMG_{kMM}(\omega) + \sum_\sigma \sigma G_{k\sigma\sigma}(\omega) \right\}.
\]

The second term of the above equation is the contribution from the spin of the conduction electrons. In this paper we neglect this contribution because it is very small. The total susceptibility \( \chi \) is given by
\[ \chi = \frac{\partial \langle M \rangle}{\partial H} \bigg|_{H=0}. \]  

(14)

As is well known, the Green’s function \( G_{k\sigma}(\omega) \) is expressed by the retarded and the advanced Green’s functions, \( G^R_{k\sigma}(\omega) \) and \( G^A_{k\sigma}(\omega) \) as

\[ G_{k\sigma}(\omega) = G^R_{k\sigma}(\omega) \cdot \theta(E^*_k - \mu) + G^A_{k\sigma}(\omega) \cdot \theta(\mu - E^*_k) \]  

(15)

for \(|k| \sim k_F\) and \(\omega \sim 0\). \(E^*_k\) is the quasiparticle spectrum obtained by

\[ \{G_{k\sigma}(E^*_k)\}^{-1} = 0. \]  

(16)

From the definition of \(E^*_k\) above, we can show for \(|k| = k_F\) that

\[ \frac{\partial}{\partial H} E^*_{k\sigma} = a_k(0) \cdot \sum_{M,M'} \alpha_{kM\sigma}(0) \left( M \delta_{MM'} + \frac{\partial}{\partial H} \Sigma_{kMM'}(0) \right) \alpha^*_{kM'M\sigma}(0), \]  

(17)

where \(\alpha_{kM\sigma}(\omega)\) is introduced by (8), and \(a_k(\omega)\) is the renormalization factor for the conduction electrons, given by

\[ 1/a_k(\omega) = \sum_{MM'} \alpha_{kM\sigma}(\omega) \cdot \left( \delta_{MM'} - \frac{\partial}{\partial \omega} \Sigma_{kMM'}(\omega) \right) \alpha^*_{kM'M\sigma}(\omega). \]  

(18)

Note that \(a_k(\omega) \ll 1\) for \(|\omega| \lesssim T^*\). By use of (15) and (17), we can derive that

\[ -\frac{\partial}{\partial \omega} G_{kMM'}(\omega) = \sum_{M'',M'''} \varphi_{kMM',M''M'''}(\omega) \cdot \left( \delta_{M'M''} - \frac{\partial}{\partial \omega} \Sigma_{kM'M''}(\omega) \right), \]  

(19)

\[ \frac{\partial}{\partial H} G_{kMM'}(\omega) = \sum_{M'',M'''} \left\{ \varphi_{kMM',M''M'''}(\omega) + Q_{kMM',M''M'''}(\omega) \right\} \times \left( M'' \delta_{M'M''} + \frac{\partial}{\partial H} \Sigma_{kM'M''}(\omega) \right), \]  

(20)

where \(\varphi\) and \(Q\) are defined by

\[ \varphi^{R(A)}_{kMM',M''M'''}(\omega) = \{G_{kMM''}(\omega)G_{kMM'}(\omega)\}^{R(A)}, \]  

(21)

\[ Q_{kMM',M''M'''}(\omega) = -2\pi i a_k(0)^2 \delta(\omega) \delta(E^*_k) \sum_{\sigma\sigma'} \alpha_{kM\sigma}(0) \alpha_{kM'\sigma'}(0) \alpha^*_{kM''\sigma'}(0) \alpha^*_{kM''\sigma}(0), \]  

(22)

where the superscript \(R(A)\) represents retarded (advanced) function. On the other hand, we can show that

\[ \lim_{k \to k'} \lim_{\omega \to \omega'} G_{kMM''}(\omega)G_{k'M'M'''}(\omega') = Q_{kMM',M''M'''}(\omega) + \varphi_{kMM',M''M'''}(\omega), \]  

(23)

\[ \lim_{\omega \to \omega'} \lim_{k \to k'} G_{kMM''}(\omega)G_{k'M'M'''}(\omega') = \varphi_{kMM',M''M'''}(\omega), \]  

(24)
on the basis of the well-known FLT. \cite{12,20,21} By use of \cite{13}, \cite{14}, \cite{20} and \cite{23}, we can prove that

\[
\chi = \lim_{|k| \rightarrow 0} \lim_{\omega \rightarrow 0} \chi_k(\omega). \tag{25}
\]

This relation is trivial whenever the magnetization operator is conserved \cite{22,21} but should be proved for our model. \cite{12} Anyway, by use of (25), we can discuss our problem in terms of the more familiar FLT. \cite{12,20,21} So, \(\chi\) can be expressed more explicitly as

\[
\chi = \frac{1}{N} \sum_p \int \frac{d\epsilon}{2\pi i} \text{Tr} \left\{ \hat{M} \left( \hat{\phi}_p(\epsilon) + \hat{Q}_p(\epsilon) \right) \hat{\Lambda}_p^k(\epsilon) \right\}, \tag{26}
\]

\[
\hat{\Lambda}_p^k(\epsilon) \equiv \hat{M} + \frac{1}{N} \sum_q \int \frac{d\epsilon'}{2\pi i} \left\{ \hat{\Gamma}_{pq}^k(\epsilon, \epsilon') \left( \hat{\phi}_q(\epsilon') + \hat{Q}_q(\epsilon') \right) \hat{M} \right\}
\]

\[
= \hat{M} + \frac{\partial}{\partial H} \hat{\Sigma}_p^k(\epsilon)
\]

where \(\text{Tr}\) represents the trace with respect to the angular momentum, and \(\{\hat{M}\}_{M_1,M_2} = M_1 \cdot \delta_{M_1,M_2}\). Here, we have introduced the \(k\)-limit and \(\omega\)-limit of the four-point vertex \(\hat{\Gamma}_{pp',q}(\epsilon, \epsilon', \omega)\), and represents them as \(\hat{\Gamma}_{pp'}^{k}(\epsilon, \epsilon')\) and \(\hat{\Gamma}_{pp'}^{\omega}(\epsilon, \epsilon')\), respectively:

\[
\hat{\Gamma}_{pp'}^{k}(\epsilon, \epsilon') \equiv \lim_{|q| \rightarrow 0} \lim_{\omega \rightarrow 0} \hat{\Gamma}_{pp',q}(\epsilon, \epsilon', \omega), \tag{27}
\]

\[
\hat{\Gamma}_{pp'}^{\omega}(\epsilon, \epsilon') \equiv \lim_{\omega \rightarrow 0} \lim_{|q| \rightarrow 0} \hat{\Gamma}_{pp',q}(\epsilon, \epsilon', \omega), \tag{28}
\]

where \(\hat{\Gamma}_{pp'}^{k(\omega)}(\epsilon, \epsilon')\) expresses the matrix whose \((MM', M''M''')\)-component is \(\Gamma_{pp',MM',M''M'''}(\epsilon, \epsilon')\). They are expressed by Fig. 2.

As is well known, \(\hat{\Gamma}_{pp'}^{k}(\epsilon, \epsilon')\) and \(\hat{\Gamma}_{pp'}^{\omega}(\epsilon, \epsilon')\) are related to each other by the following Bethe-Salpeter equation: \cite{20,12}

\[
\hat{\Gamma}_{pp'}^{k}(\epsilon, \epsilon') = \hat{\Gamma}_{pp'}^{\omega}(\epsilon, \epsilon') + \frac{1}{N} \sum_q \int \frac{d\omega}{2\pi i} \left\{ \hat{\Gamma}_{pq}^{k}(\epsilon, \omega) \hat{Q}_q(\omega) \hat{\Gamma}_{qp'}^{\omega}(\omega, \epsilon') \right\}. \tag{29}
\]

For instance, we can write the \(H\)-derivative and the \(\omega\)-derivative of the selfenergy as follows:

\[
-\frac{\partial}{\partial \omega} \hat{\Sigma}_p(\epsilon) = \frac{1}{N} \sum_q \int \frac{d\omega}{2\pi i} \left\{ \hat{\Gamma}_{pq}^{\omega}(\epsilon, \omega) \hat{\phi}_q(\omega) \right\}, \tag{30}
\]

\[
\frac{\partial}{\partial H} \hat{\Sigma}_p(\epsilon) = \frac{1}{N} \sum_q \int \frac{d\omega}{2\pi i} \left\{ \hat{\Gamma}_{pq}^{k}(\epsilon, \omega) \left( \hat{Q}_q(\omega) + \hat{\phi}_q(\omega) \right) \hat{M} \right\}. \tag{31}
\]

In this paper, we define the Van Vleck susceptibility \(\chi_V\) and the Pauli susceptibility \(\chi_P\) as
\[ \chi_V \equiv \lim_{\omega \to 0} \lim_{|k| \to 0} \chi_k(\omega), \quad (32) \]
\[ \chi_P \equiv \chi - \chi_V. \quad (33) \]

According to (32) and (33), we can divide \( \chi \) into \( \chi_P \) and \( \chi_V \) in a unique way. The standard FLT tells us that \( \chi_V = 0 \) and \( \chi = \chi_P \) if the magnetization operator \( \hat{M} \) is conserved, i.e., \([\hat{H}, \hat{M}] = 0 : [21, 22]\) we briefly show this in Appendix A. Equations (32) and (33) can be expressed as

\[ \chi_V = \frac{1}{N} \sum_p \int \frac{d\epsilon}{2\pi i} \text{Tr} \left\{ \hat{M} \hat{\phi}_p(\epsilon) \hat{\Lambda}_p^\omega(\epsilon) \right\}, \quad (34) \]
\[ \chi_P = \frac{1}{N} \sum_p \int \frac{d\epsilon}{2\pi i} \text{Tr} \left\{ \hat{\Lambda}_p^\omega(\epsilon) \hat{Q}_p(\epsilon) \hat{\Lambda}_p^k(\epsilon) \right\}; \quad (35) \]
\[ \hat{\Lambda}_p^\omega(\epsilon) \equiv \hat{M} + \frac{1}{N} \sum_q \int \frac{d\epsilon'}{2\pi i} \left\{ \hat{\Gamma}_{pq}^\omega(\epsilon, \epsilon') \hat{\phi}_q(\epsilon') \hat{M} \right\}, \quad (36) \]
\[ \left( = \hat{M} + \frac{\partial}{\partial H} \hat{\Sigma}_p(\epsilon) \bigg|_{\omega \to 0} \neq \hat{\Lambda}_p^k(\epsilon). \right) \]

Equation (34) is depicted by Fig. 3. In the following sections, we investigate the expressions of \( \chi \) and \( \chi_V \), given by (26) and (34), further.

Here we stress that the principle of division of the total susceptibility into (32) and (33) is very clear; the former does not contain any \( \hat{Q}_p(\omega) \) defined by (22) while the latter contains at least one \( \hat{Q}_p(\omega) \) in itself. Equation (33) indicates that \( \chi_P \) vanishes and the total susceptibility is given by \( \chi_V \) in the case where \( \hat{Q}_p(\epsilon) \equiv 0 \), i.e., where the Fermi surface disappears for some reasons or other. This is true for any multicomponent Fermi liquid systems.

**IV. MAGNETIC SUSCEPTIBILITY \( \chi^0 \): IN CASE \( U = 0 \)**

In this section, we calculate both \( \chi_V^0 \) and \( \chi_P^0 \) defined by (34) and (35) in case \( U = 0 \), and show that our results are identical to those obtained previously by several authors. [6, 7]

\[ \chi^0 = -\frac{1}{N} \sum_{kMM'} \int \frac{d\omega}{2\pi i} M \left\{ \varphi_{k,MM,M'M'}^0(\omega) + Q_{k,MM,M'M'}^0(\omega) \right\} M', \quad (37) \]
\[ \chi_V^0 = -\frac{1}{N} \sum_{kMM'} \int \frac{d\omega}{2\pi i} M \varphi_{k,MM,M'M'}^0(\omega) M'. \quad (38) \]

a) Metallic case: For simplicity, we consider the case where the DOS for the conduction electrons is constant, i.e., \( \rho_c(\omega) = \rho_c \). We also neglect the contribution from the spin of the conduction electrons, which is of order \( \sim \rho_c / \rho \sim O(V^2 / D^2) \) compared with the total
magnetic susceptibility. (see §2.) Moreover, we assume the six-fold degenerate $f$-electron spectrum and the spherical Fermi surface. The calculated results of (37) and (38) are

$$\chi_0 = 2g^2J(J+1) \cdot A\rho^0(0),$$  \hspace{1cm} (39)  
$$\chi_0^V = 2g^2J(J+1) \cdot C\rho^0(0),$$  \hspace{1cm} (40)  
$$A = 0.333, \ C = 0.152.$$  

Note that $\rho^0(\omega)$, given by (12), is the DOS for $f$-electrons. Moreover, the ratio $\chi_0^V/\chi_0^P = (A - C)/C = 1.19$ is derived. The Wilson ratio $R^0$ of this system, whose definition is given by (1), is given by

$$R^0 = 1.$$  \hspace{1cm} (41)  

The obtained results (39) ~ (41) are identical to those shown previously by several authors. \cite{6,7} Note that these results are not universal for $J = 5/2$ PAM in that it depends on the $\omega$-dependence of $\rho_c(\omega)$.

b) Insulating Case: Next, we consider the insulating case where the Fermi energy $\mu$ lies in the $f$-$c$ hybridization gap. In this case, apparently $\chi_0^P = 0$ because $\rho^0(0) = 0$. So, the total magnetic susceptibility $\chi^0$ is equal to the Van Vleck susceptibility, which is given by

$$\chi_{\text{ins}}^0 = 2g^2J(J+1) \cdot C\rho^0(\Delta_-),$$  \hspace{1cm} (42)  
$$\rho^0(\Delta_-) = \rho_c \cdot \frac{3V^2}{(\Delta_- + \mu - Ef)^2} > 0,$$

where $\Delta_-$ is the lower edge of the hybridization gap. In this case, $\chi_{\text{ins}}^0$ remains finite and will take a large value. This result makes highly contrast to that of the insulating SU(6)-PAM, whose magnetic susceptibility is about $V^2/D^2$ times smaller than (42). \cite{12} (see Appendix A.)

V. MAGNETIC SUSCEPTIBILITY $\chi$ IN THE $D = \infty$ LIMIT: IN CASE $U \neq 0$

In this section, we analyze the magnetic susceptibility $\chi$ in the strong coupling regime, taking account of all the vertex corrections due to the strong Coulomb interaction $U$ in a consistent way. In §5 and §6, we assume the six-fold degeneracy of $f$-electron spectrum and spherical Fermi surface in the extended zone scheme. Such a assumption will be allowed for our aim to elucidate the essential properties of the susceptibility under the influence of the strong Coulomb interaction.

We study this problem concerning only the leading term with respect to $1/d$-expansion; we call such an approximation the $d = \infty$ approximation or the $d = \infty$ limit. In the $d = \infty$
limit, the selfenergy becomes momentum-independent, as is well known. \[17\] For $J = 5/2$

PAM, the electronic properties obtained by this approximation will be quite similar to that

in the realistic $d = 3$ system. \[18\] So, the conclusions obtained by the $d = \infty$ limit analysis

will be valid also for the $d = 3$ system.

A. Brief review of the $d = \infty$ approximation

In the $d = \infty$ approximation, the irreducible four-point vertex is considered as local,
as well as the selfenergy. So, the reducible four-point vertex $\Gamma$ is composed of the local

irreducible four-point vertex $\Gamma'$ and the non-local particle-hole Green's functions $\hat{\phi}(\omega)$, as is

shown in Fig. 4. We explain the reasoning briefly:

Fig. 5 shows the two examples of $U^2$-order contributions for the susceptibility. We

assume the $d$-dimensional hyper-cubic $f$-electron lattice for instance. Each site has $z = 2d$

nearest neighbor sites. $j, l, m$ and $n$ represent the $f$-electron sites, and we have to take

summation over them. Note that an $f$-electron on one site cannot hop to other sites without

mixing with the conduction electrons. In Fig. 5 (a), $(j, m)$ and $(l, n)$ are connected by the

Green's functions, while they are not in Fig. 5 (b). Here, we fix $l$ and $m$ as the $\xi$-th nearest

neighbor sites. Such selections exist about $\sim z^\xi \sim d^\xi$ different ways in total. In the spirit of

$1/d$-expansion, an itinerant Green’s function in the real space connecting the $\xi$-th nearest

neighbor sites is scaled as $\sim O(d^{-\xi/2})$. After the summation over $j, n, l$ and $m$, while $l$ and

$m$ are limited to the $\xi$-th nearest neighbor pairs, Fig. 5 (a) and (b) becomes of order $\sim d^\xi$

and $\sim d^0$, respectively. Thus in the $d = \infty$ limit, the irreducible four-point vertices, included

in Fig 5 (a) for instance, are local and momentum-independent, as well as the selfenergy.

On the other hand, the particle-hole pair should be regarded as non-local. Below, we take

account of $O(d^0)$ terms at most, i.e., the $d = \infty$ approximation.

In this approximation, \(k\)-summation becomes

$$
\frac{1}{N} \sum_k \rightarrow \int \rho_c^0(\epsilon_k)d\epsilon_k,
$$

(43)

where $\rho_c^0(\epsilon_k)$ is the unperturbed DOS for the conduction electrons. We see below that

the results of this paper are sensitive to the value of $\rho_c^0(0)$, but insensitive to the energy-
dependence of $\rho_c^0(\epsilon_k)$ in heavy Fermion systems. So, we do not pay attention to the functional form of $\rho_c^0(\epsilon_k)$.
B. The local Green’s function, the selfenergy and the irreducible four point vertex in 

the \( d = \infty \) case 

As is shown in the previous subsection, in the \( d = \infty \) limit both the selfenergy and the irreducible four-point vertex are composed of the local Green’s functions, \( \hat{g}(\omega) \). When we assume the spherical Fermi surface, \( g(\omega) \) is given by 

\[
\frac{1}{N} \sum_k G_{kMM'}(\omega) \equiv g(\omega) \cdot \delta_{MM'}.
\] 

(44)

Thus, the local Green’s function \( g(\omega) \) is diagonal with respect to \( M \), and it is independent of \( M \) in case \( H = 0 \). ( Notice that \( G_{kMM'}(\omega) \) is proportional to the phase factor \( \propto e^{i(M' - M)\varphi_k}, \) which vanishes after the \( k \)-summation in case \( M \neq M' \). ) This is a remarkable simplification occurring in the \( d = \infty \) approximation. Thus, in the \( d = \infty \) limit, both the selfenergy and the four-point vertex become diagonal with respect to \( M \) even in a finite magnetic field \( H \), and independent of \( M \) in case \( H = 0 \).

For simplicity, we consider the case in the absence of ECF, i.e., \( E_M^f = E_M^f \) for all \( M \). In the absence of the magnetic field \( H \), we can rewrite the \( f \)-electron Green’s function given by (9), into a simpler form as follows :

\[
G_{kMM'}(\omega) = h_{kMM'}G_k(\omega) + d_{kMM'}G_f^f(\omega),
\] 

(45)

where

\[
\begin{align*}
G_k(\omega) &= \left( \omega + \mu - E^f - \Sigma(\omega) - 3V^2/(\omega + \mu - \epsilon_k) \right)^{-1}, \\
G_f^f(\omega) &= \left( \omega + \mu - E^f - \Sigma(\omega) \right)^{-1}, \\
h_{kMM'} &= \sum_{\sigma} V_{kM\sigma}^* V_{kM'\sigma} / 3V^2, \\
d_{kMM'} &= \delta_{MM'} - h_{kMM'},
\end{align*}
\]

(46)

where \( \Sigma(\omega) \) represents the selfenergy in the \( d = \infty \) limit, which depends on neither \( k \) nor \( M \).

Here, we consider the quasiparticle representation of \( G_k(\omega) \) given by (46). For \( |\omega| \lesssim T^* \), the retarded Green’s function is

\[
G^R_k(\omega) = \frac{z_1(\omega)}{\omega - E_k^* + i\delta},
\] 

(47)

\[
\left( \text{i.e., } (-1/\pi) \text{Im} G^R_k(\omega) = z_1(\omega) \cdot \delta(\omega - E_k^*) \right),
\]

where \( T^* \) represents the characteristic energy, within which the quasiparticle can be well defined. ( \( T^* \) corresponds to the renormalized Fermi energy. ) Here, \( z_1(\omega) \) is the momentum-independent renormalization factor of \( f \)-electrons, given by
Both \( z(\omega) \) and \( z_1(\omega) \) are the renormalization factors of \( f \)-electrons. Apparently, \( 1/z_1(0) \approx 1/z(0) \gg 1 \) in the heavy Fermion system.

Finally, in the \( d = \infty \) limit, we can show that the irreducible parallel four-point vertex \( \Gamma_{MM,M'M'}^I(\omega, \omega') \) satisfies the following property:

\[
\Gamma_{MM,M'M'}^I(\omega, \omega') = \Gamma^I(\omega, \omega') \cdot \delta_{MM'},
\]

i.e., \( \Gamma_{MM,M'M'}^I(\omega, \omega') \) is diagonal with respect to \( M \) and independent of \( M \) ( see Fig. 6. ) This property is explained in Appendix. B, which is no more true for reducible vertices.

Here, we summarise the results of this subsection : in our \( J = 5/2 \) PAM, in the \( d = \infty \) limit, the angular momentum \( M \) is conserved in the local Green’s function, the selfenergy and the irreducible four point vertex, because they are estimated as local processes in the \( d = \infty \) limit.

\[ \]

C. Vertex Corrections for the magnetic susceptibility

Contrary to \( \Sigma(\epsilon) \) and \( \Gamma^I(\epsilon, \epsilon') \), the particle-hole Green’s functions are never restricted to the local process, so \( \hat{\varphi}(\omega) \) and \( \hat{Q}(\omega) \) are not diagonal with respect to \( M \). In \( d = \infty \), \( \hat{\varphi}(\omega) \) is given by

\[
\hat{\varphi}(\omega) = \frac{1}{N} \sum_j \hat{G}_{i,j}(\omega) \cdot \hat{G}_{j,i}(\omega) = \frac{1}{N} \sum_k \hat{G}_k(\omega) \cdot \hat{G}_k(\omega),
\]

which is a matrix with respect to \( M \). In \( \{11\} \), \( \hat{G}_{i,j}(\omega) \) \( (\hat{G}_k(\omega)) \) is the Green’s function in the real- (momentum-) space representation.

Here, we introduce a \( M \)-independent particle-hole Green’s function in \( d = \infty \), \( \varphi_a(\omega) \),

\[
\varphi_a(\omega) \equiv \frac{1}{N} \sum_k \varphi_{k,MM,M'M'}(\omega) \cdot 1
= (1 - A) \frac{1}{N} \sum_k G_{k}^2(\omega) + A \frac{1}{N} \sum_k G_k(\omega),
\]

where
\[ A \equiv 1/3 = \int \frac{d\Omega_k}{4\pi} \cdot h_{kMM}. \] (53)

\( \varphi_a(\omega) \) is related with frequency-derivative of the selfenergy, which is depicted by Fig. 7 (a). Note that the summation on \( M' \) is taken in (52).

Next, we also introduce another \( M \)-independent particle-hole Green’s functions, \( \varphi_b(\omega) \),

\[ \varphi_b(\omega) \equiv \frac{1}{M} \left\{ \frac{1}{N} \sum_k \sum_{M'} \varphi_{k,MM,M'M'}(\omega) \cdot M' \right\} \]

\[ = (1 - 2A + B) \frac{1}{N} \sum_k G_k^{f2}(\omega) + B \frac{1}{N} \sum_k G_k^2(\omega) \]

\[ + 2(-A + B) \frac{1}{N} \sum_k \left( \frac{\omega + \mu - \epsilon_k}{3V^2} \right) G_k^f(\omega) + 2(A - B) \frac{1}{N} \sum_k \left( \frac{\omega + \mu - \epsilon_k}{3V^2} \right) G_k(\omega), \] (54)

where

\[ B \equiv 0.181 = \int \frac{d\Omega_k}{4\pi} \cdot j_{kMM}/M. \] (55)

\[ j_{kMM'} = \sum_{\sigma\sigma'M''} V_{k\sigma\sigma'}^{M''} V_{k\sigma'\sigma''}^{M''} V_{k\sigma''\sigma'}^{M'M''}/(3V^2)^2. \] (56)

\( \varphi_b(\omega) \) is related with the magnetic field-derivative of the selfenergy, which is depicted by Fig. 7 (b).

We also define the following functions, which are \( M \)-independent :

\[ Q_\alpha(\omega) \equiv \frac{1}{N} \sum_k \sum_{M'} Q_{k,MM,M'M'}(\omega) = -A \cdot 2\pi i z_1(0)\rho(0)\delta(\omega) \] (57)

\[ Q_\beta(\omega) \equiv \frac{1}{M} \left\{ \frac{1}{N} \sum_k \sum_{M'} Q_{k,MM,M'M'}(\omega) \cdot M' \right\} = -B \cdot 2\pi i z_1(0)\rho(0)\delta(\omega) \] (58)

\[ Q_\gamma(\omega) \equiv Q_\alpha(\omega) - Q_\beta(\omega) = -C \cdot 2\pi i z_1(0)\rho(0)\delta(\omega), \] (59)

(i.e., \( C = A - B = 0.152 \)),

where we have used the relation \( dE_k^*/d\epsilon_k = a_k(0) \) and \( z_1(0) = a_k(0) \cdot 3V^2/(\mu - E^f - \Sigma(0))^2 \) for \( |k| = k_F \), which is rigorous because our selfenergy is \( k \)-independent. Note that \( \rho(0) \) is not an enhanced quantity. Especially, in the \( d = \infty \) approximation, \( \rho(0) = \rho^0(0) \) is satisfied rigorously on condition that \( E^f \) is modified so as to fix the value of \( \mu \), because of the momentum-independence of the selfenergy.

Here, we notice that \( \varphi_a(\omega) \neq \varphi_b(\omega) \) and \( Q_\alpha(\omega) \neq Q_\beta(\omega) \) in our model, which is the consequence of the fact that the magnetization \( M \) is not conserved. i.e., \( [\hat{H}, \hat{M}] \neq 0 \). This situation has been prevented us from treating this problem on the basis of the FLT.

Then, we introduce the reducible four-point vertices \( \Gamma^r(\epsilon, \epsilon') \) and \( \Gamma^r\xi(\epsilon, \epsilon') \) so as to satisfy the following Bethe-Salpeter equations :

14
\[
\Gamma'(\epsilon, \epsilon') = \Gamma'(\epsilon, \epsilon') + \int \frac{d\omega}{2\pi i} \Gamma'(\epsilon, \omega) \varphi_r(\omega) \Gamma'(\omega, \epsilon'), \quad (60)
\]

\[
\Gamma^{\xi}(\epsilon, \epsilon') = \Gamma'(\epsilon, \epsilon') + \int \frac{d\omega}{2\pi i} \Gamma'(\epsilon, \omega) (\varphi_r(\omega) + Q(\omega)) \Gamma^{\xi}(\omega, \epsilon'), \quad (61)
\]

where \( r = \{a, a', b\} \) and \( \xi = \{\alpha, \beta, \gamma\} \). For example, we can represent the mass-enhancement factor as follows:

\[
-\frac{1}{2\pi i} \sum \varphi_a(\omega, \epsilon) \left\{ \varphi_a(\epsilon) + \frac{1}{N} \sum_k G_k^2(\epsilon) \frac{V^2}{(\epsilon + \mu - \epsilon_k)^2} \right\}
\]

\[
\approx \int \frac{d\epsilon}{2\pi i} \varphi_a(\epsilon) \varphi_a(\epsilon), \quad (62)
\]

which is shown in Fig. 8. Here, we notice that \( \frac{1}{N} \sum_{k,M} \varphi_{k,M,M',M''} = 0 \) for \( M \neq M' \) because of the phase factor, \( e^{i(M'-M)\varphi_k} \). In the same way, the Van Vleck susceptibility \( \chi_V \) and the total susceptibility \( \chi \), given by (34) and (20) respectively, are expressed as

\[
\chi_V \approx -2g^2 J(J+1) \int \frac{d\epsilon}{2\pi i} \varphi_a(\epsilon) \left\{ 1 + \int \frac{d\epsilon'}{2\pi i} \Gamma_a(\epsilon, \epsilon') \varphi_a(\epsilon') \right\}, \quad (63)
\]

\[
\chi \approx -2g^2 J(J+1) \int \frac{d\epsilon}{2\pi i} (\varphi_a(\epsilon) + Q(\epsilon)) \left\{ 1 + \int \frac{d\epsilon'}{2\pi i} \Gamma_{a,\beta}(\epsilon, \epsilon') (\varphi_a(\epsilon') + Q(\epsilon')) \right\}. \quad (64)
\]

Notice that \( \sum_{M,M'} \int \Gamma(\epsilon) \varphi_{M,M',M'} = \sum_{M',M''} \int \Gamma(\epsilon) \varphi_{M,M',M',M''} \) and \( \sum_{M} M^2 = 2J(J+1) \) for \( J = \frac{5}{2} \). In (32) ( (33) and (34)), we have omitted the contribution from the (spins of the) conduction electrons. We neglect it hereafter because its value is at most of order \( \rho_c(0)/\rho(0) \sim O(V^2/D^2) \ll 1 \) in the real heavy Fermion systems, \( [23] \) where \( \rho(0) (\rho_c(0)) \) are the density of states of \( f \)-electrons (conduction electrons) at the Fermi energy, introduced by (12) (by (11)).

In heavy Fermion systems, the behavior of \( \text{Im} \varphi_a(\epsilon) \) or \( \text{Im} \varphi_b(\epsilon) \) only for \( 0 \leq -\epsilon \lesssim T^* \) plays a predominant role for the enhancement factor, as is discussed in Appendix C. Below, we show the relation between \( \text{Im} \varphi_a(\epsilon) \) and \( \text{Im} \varphi_b(\epsilon) \), which is derived in Appendix C.

(i) In the metallic case: Here, we introduce the new particle-hole Green’s functions, \( \varphi_a'(\epsilon) \), as

\[
\varphi_a'(\epsilon) \equiv \varphi_b(\epsilon) - Q(\epsilon). \quad (65)
\]

Then, the following important relation is obtained:

\[
\text{Im} \varphi_a'(\epsilon) \approx \text{Im} \varphi_a(\epsilon) \quad \text{for} \quad 0 \leq -\epsilon \lesssim T^*. \quad (66)
\]

(ii) In the insulating case (i.e., \( \rho(0) = 0 \)) : Here, we introduce \( \varphi_a'(\epsilon) \) as
\[ \varphi_{a'}(\epsilon) \equiv \varphi_b(\epsilon) - Q^i(\epsilon), \]  
\[ Q^i(\epsilon) \equiv -C \cdot 2\pi iz(\Delta^*) \rho(\Delta^*) \cdot \delta(\epsilon - \Delta^*), \]  
where \( \Delta^* \) denotes the renormalized lower edge of the hybridization gap measured from the fermi energy, and \( \rho(\Delta^*) = \rho_0(\Delta^*) + \mu - \epsilon_{k_F})^2/3V^2 \). Apparently, the relation \( \rho(\Delta^*) \approx \rho_0(\Delta^*) \) is satisfied very well. Because the relation \( 1/z(\Delta^*) \approx 1/z(0) \) is naturally expected by the numerical calculation in Appendix D, the relation \( \Delta^* \approx z(0) \cdot \Delta^- \) is also expected.

We can show that

\[ \text{Im}\varphi_{a'}(\epsilon) \approx \text{Im}\varphi_a(\epsilon), \quad \text{for} \quad 0 \leq -\epsilon - \Delta^* + T^* \].

In the next subsection, we investigate the magnetic susceptibility \( \chi \) and \( \chi_V \) by use of the relation (66) and (69), respectively.

**D. The expression for the magnetic susceptibility in the \( d = \infty \) case.**

In this subsection, we study the (63) and (64) further and obtain the simple expressions for \( \chi, \chi_V \) and \( \chi_{\text{ins}} \). By use of the relations \( \varphi_b = \varphi_{a'} + Q_\gamma \) and \( \varphi_b + Q_\beta = \varphi_{a'} + Q_\alpha \) given in the previous section, we can check the following Bethe-Salpeter equations in a straightforward way:

\[ \Gamma^b(\epsilon, \epsilon') = \Gamma^{a'}(\epsilon, \epsilon') + \int \frac{d\omega}{2\pi i} \Gamma^{a'}(\epsilon, \omega) Q_\gamma(\omega) \Gamma^b(\omega, \epsilon'), \]  
\[ \Gamma^{b,\beta}(\epsilon, \epsilon') = \Gamma^{a'}(\epsilon, \epsilon') + \int \frac{d\omega}{2\pi i} \Gamma^{a'}(\epsilon, \omega) Q_\alpha(\omega) \Gamma^{b,\beta}(\omega, \epsilon'). \]

The first line is depicted in Fig. 9. Taking account of the relations (66) and (54) obtained in §5-3, we can estimate as

\[ \left\{ \begin{array}{ll}
\Gamma^a(\epsilon, \epsilon') \approx \Gamma^{a'}(\epsilon, \epsilon') & \text{for} \quad 0 < \{-\epsilon, -\epsilon'\} \approx T^*, \\
\Gamma^{a,\alpha}(\epsilon, \epsilon') \approx \Gamma^{b,\beta}(\epsilon, \epsilon') & \text{for} \quad 0 < \{-\epsilon, -\epsilon'\} \approx T^*,
\end{array} \right. \]

because the contributions to them from the quasiparticle excitations are dominant in heavy Fermion systems. These results play important roles in the following discussions.

Then, we introduce the new enhancement factor, \( 1/z'(\epsilon) \), as

\[ \frac{1}{z'(\epsilon)} = 1 + \int \frac{d\epsilon'}{2\pi i} \Gamma^{a'}(\epsilon, \epsilon') \varphi_{a'}(\epsilon'). \]
Considering the relation (72),

$$\frac{1}{z(\epsilon)} \cong \frac{1}{z'(\epsilon)} \quad \text{for} \quad 0 < -\epsilon < \lesssim T^*$$  \hspace{1cm} (74)

is satisfied in heavy Fermion systems. In Appendix D, we show that the relation (74) is well supported by the numerical calculation by SOPT and SC-SOPT.

Here, we introduce the enhancement factor for the magnetic susceptibility $\frac{1}{z_H(\epsilon)}$ as

$$\frac{1}{z_H(\epsilon)} \equiv 1 + \frac{1}{gM} \left. \frac{\partial}{\partial H} \Sigma_M(\epsilon) \right|_{H=0}$$  \hspace{1cm} (75)

$$= 1 + \int \frac{d\epsilon'}{2\pi i} \Gamma^{b,\beta}(\epsilon, \epsilon') (\phi_b(\epsilon') + Q_{\beta}(\epsilon')) \frac{1}{z'(\epsilon')},$$  \hspace{1cm} (76)

where (76) is given for the metallic case. In the insulating case, $\Gamma^{b,\beta}, Q_{\alpha}$ in (76) is replaced by $\Gamma^b, Q^\gamma$, respectively. Apparently, $1/z_H(0) \sim 1/z(0)$ is expected.

Then, we can rewrite the total susceptibility $\chi$, given by (64), and obtain a simple expression as

$$\chi = -2g^2J(J+1) \int \int \frac{d\epsilon d\epsilon'}{(2\pi i)^2} \frac{1}{z'(\epsilon)} Q_{\alpha}(\epsilon) \left(2\pi i \delta(\epsilon - \epsilon') + \Gamma^{b,\beta}(\epsilon, \epsilon') Q_{\alpha}(\epsilon') \right) \frac{1}{z'(\epsilon')} + \chi''_V,$$

$$= \chi^0 \cdot \frac{1}{z_H(0)} \cdot \frac{z(0)}{z'(0)} + \chi''_V,$$  \hspace{1cm} (77)

where $\chi^0 = 2g^2J(J+1) \cdot A\rho(0)$, which is similar to the unperturbed value given by (69). And $\chi''_V$ is given by

$$\chi''_V = -2g^2J(J+1) \int \frac{d\epsilon}{2\pi i} \phi_{a'}(\epsilon) \frac{1}{z'(\epsilon')}.$$  \hspace{1cm} (78)

We can estimate that $\chi''_V = 0$ by use of (74), because

$$\chi''_V \cong -\int_{-\infty}^{\infty} \frac{d\epsilon'}{2\pi i} \phi_{a'}(\epsilon)/z_1(\epsilon) = \frac{1}{N} \sum_k \int \frac{d\epsilon}{2\pi i} \cdot \frac{\partial}{\partial \epsilon} G_k(\epsilon) = 0.$$  \hspace{1cm} (79)

More accurate derivation of (79) is given in Appendix E. Equation (77) (and (64)) is depicted in Fig. 10. Thus, by use of (74) and (78), we get the simple expression,

$$\chi \cong \chi^0 \cdot \frac{1}{z_H(0)}.$$  \hspace{1cm} (80)

This is the main result of this paper, and is proved to be rigorous when $1/z(\omega) \equiv 1/z'(\omega)$ is satisfied identically. (see Appendix D.)

Next, we also rewrite the Van Vleck susceptibility $\chi_V$, given by (63), as follows,
\[ \chi_V = -2g^2J(J + 1) \int \int \frac{d\epsilon d\epsilon'}{(2\pi i)^2} \frac{1}{z'(\epsilon)} Q_\gamma(\epsilon) \left( 2\pi i \delta(\epsilon - \epsilon') + \Gamma^b(\epsilon, \epsilon') Q_\gamma(\epsilon') \right) \frac{1}{z'(\epsilon')} + \chi''_V, \tag{81} \]

where \( \chi''_V \) is given by (78).

Before concluding this subsection, we consider the insulating case. In this case, apparently \( \chi_P = 0 \) and \( \chi_{\text{ins}} = \chi_V \) at zero temperature. By use of the results obtained in the previous sections, we can show that

\[ \chi_{\text{ins}} = -2g^2J(J + 1) \int \int \frac{d\epsilon d\epsilon'}{(2\pi i)^2} \frac{1}{z'(\epsilon)} Q_\gamma(\epsilon) \left( 2\pi i \delta(\epsilon - \epsilon') + \Gamma^b(\epsilon, \epsilon') Q_\gamma(\epsilon') \right) \frac{1}{z'(\epsilon')} + \chi''_V \]

\[ = \chi_{\text{ins}}^0 \cdot \frac{1}{z_H(\Delta^\ast)} \cdot \frac{z(\Delta^\ast)}{z'(\Delta^\ast)} + \chi''_V, \tag{82} \]

where \( \chi_{\text{ins}}^0 = 2g^2J(J + 1) \cdot C\rho(\Delta^\ast) \), which is similar to the unperturbed value given by (12). \( \chi''_V \) is given by (78). Thus, we get the simple expression

\[ \chi_{\text{ins}} \simeq \chi_{\text{ins}}^0 \cdot \frac{1}{z_H(\Delta^\ast)}. \tag{83} \]

From the mathematical point of view, the obtained expression for \( \chi \), given by (77), is identical to (26). In fact, the energy-integration appearing in the definition of \( \chi \), given by (26), has been included in the enhancement factor at the Fermi energy, \( 1/z_H(0) \). Also, the expression (81) and (82) are identical to (34) mathematically. However, the expressions obtained in this section show clear physical information: We can see that \( \chi \) and \( \chi_V \) (or \( \chi_{\text{ins}} \)) are apparently enhanced by the interaction because they are proportional to the enhancement factor, \( 1/z(0) \) (or \( 1/z(\Delta^\ast) \)).

VI. FURTHER ANALYSIS FOR \( \chi, \chi_V \) AND \( \chi_{\text{ins}} \)

A. Estimation for the Wilson ratio in the \( d = \infty \) limit: in case \( U \neq 0 \)

In the previous section, we get the general expression for \( \chi \) and \( \chi_V \), and find that they are enhanced by \( 1/z \). Furthermore, we estimate the Wilson ratio in this section by examining the magnetic enhancement factor, \( 1/z_H \). For this purpose, we consider the \( T \)-linear coefficient of the specific heat and the charge susceptibility at first.

(i) \( T \)-linear Coefficient of the Specific Heat: \( \gamma \)

In the metallic case, the \( \gamma \) is given by (2)

\[ \gamma = \gamma^0/z_1(0) \cong \gamma^0/z(0), \tag{84} \]
where $\gamma^0$ is the value in case $U = 0$. $\gamma$ is proportional to the mass-enhancement factor $1/z_1(0) \cong 1/z(0)$.

(ii) Charge Susceptibility ; $\chi_{ch}$

By use of the relation

$$\frac{\partial}{\partial \mu} \Sigma_M(\omega) \bigg|_{Q_{\alpha} = 0} = \frac{\partial}{\partial \omega} \Sigma_M(\omega),$$

the charge susceptibility in the metallic case is given by

$$\chi_{ch} = \frac{\partial}{\partial \mu} \langle n_f \rangle = \int d\epsilon \frac{1}{2\pi i} (\varphi_a(\epsilon) + Q_\alpha(\epsilon)) \left( 1 - \frac{\partial}{\partial \mu} \Sigma_M(\epsilon) \right)$$

$$= \int d\epsilon \frac{\varphi_a(\epsilon)}{z_1(\epsilon)} + \int \int d\epsilon d\epsilon' \frac{1}{(2\pi i)^2} z(\epsilon) Q_\alpha(\epsilon) \left\{ 2\pi i \delta(\epsilon - \epsilon') + (\Gamma^{a,\alpha}(\epsilon, \epsilon') + (2J + 1) T^{a,\alpha}(\epsilon, \epsilon')) Q_\alpha(\epsilon) \right\} \frac{1}{z(\epsilon')},$$

where $T^{a,\alpha}(\epsilon, \epsilon')$ is the antiparallel four-point (reducible) vertex. (see Fig. B·1.) The first term of (86) turns out to vanish identically, as proved by (79).

Usually, in heavy Fermion systems, $\chi_{ch}$ is considerably suppressed by the strong Coulomb repulsion between $f$-electrons. [15] Here, we assume the following relation approximately:

$$\Gamma^{a,\alpha}(0, 0) = -T^{a,\alpha}(0, 0),$$

which is rigorous for $J = 5/2$ single-site Anderson model, and expected to be valid approximately even for the lattice problem. (In reality in the periodic system, the relation (86) may be modified by the magnetic correlation between different sites. But, we do not consider such an effect now.) If we put $\chi_{ch} \ll 1$ in the strong coupling limit region, we get from (86) that

$$-\Gamma^{a,\alpha}(0, 0) \cdot A z(0) \rho(0) = \frac{1}{2J} = \frac{1}{5} \quad \text{for } J = 5/2.$$  

(iii) Magnetic Susceptibility ; $\chi$

Here, we consider $\chi$ of $J = 5/2$ PAM, in case there is no ECF and $\rho(\epsilon) \cong \text{constant}$ for $0 \leq -\epsilon \lesssim T^*$. At first, we consider the metallic case. By use of (76), (88) and $\Gamma^{b,\beta}(0, 0) \cong \Gamma^{b,\beta}(0, 0)$, the relation $1/z_H(0) \cong 1.2 \cdot 1/z(0)$ is obtained. Thus, $\chi$ is given by

$$\chi \sim 1.2 \cdot \chi^0 \cdot \frac{1}{z(0)},$$

So, the obtained Wilson ratio is $R \sim 1.2 \cdot R^0$. ($R^0$ is given by (41).) In fact, the result (89) is changed if the assumption (87) is incorrect. Nonetheless, $R \sim 1$ is expected because the
It should be stressed that the Wilson ratio for $J = 5/2$ PAM becomes equal to that for $J = 5/2$ impurity Anderson model, $R = 1.2$, if the magnetic correlation between different sites can be neglected. This result is nontrivial in the previous works, and is contrastive to that for SU(2) PAM, $R \sim 2$. 

On the other hand, because the relation $0 < -\Gamma^0(0, 0) \cdot Cz(0)\rho(0) \lesssim 1/5$ is expected, the Van Vleck susceptibility is written by

$$\chi_V \sim \chi^0_V \cdot \frac{1}{z(0)},$$

where $\chi^0_V = 2g^2J(J+1) \cdot C\rho(0)$, which is similar to the unperturbed value. Thus, $\chi_V$ is enhanced also by $1/z$.

We stress that the RPA-type diagrams are included only in the polarization factor (i.e., in $R/R^0$), and never included in the enhancement factor $1/z$ or $1/z'$ which brings the highly enhanced magnetic susceptibility observed in the heavy Fermion systems. This situation also holds in the SU(N)-PAM.

In conclusion, the Van Vleck susceptibility for the $J = 5/2$ PAM is strongly enhanced by $1/z$, both in the metallic case and in the insulating case, and $\chi_{\text{ins}}$ for the $J = 5/2$ PAM is much larger than that in the insulating SU(N)-PAM without ECF.

### B. Consideration on the $d = 3$ system: in case $U \neq 0$

Here, we briefly consider the magnetic susceptibility in the three dimensional system beyond the $d = \infty$ approximation. The relation $\chi = \chi^0 \cdot (1/z_H)(z/z')$ is also derived in $d = 3$, because the equation is also satisfied in $d = 3$ by the replacement of $Q_\gamma(\omega)$ with $Q_\gamma(\omega) \cdot z(0)\delta(E_k^*)$ in (33). On the contrary, the relation $1/z \sim 1/z'$ is not guaranteed in $d = 3$, because the relation is rigorous only for the $d = \infty$ case.

In the $d = 3$ system, there exist the correction from the non-local part of the irreducible four-point vertex, $\Gamma_{\text{nl}}^J$, which violates the relation. Fortunately, SOPT for $J = 5/2$ PAM by means of the $1/d$-expansion shows that the correction term $\Gamma_{\text{nl}}^J$ is approximately negligible in $d = 3$, so will be satisfied very well. As a result, the relation $\chi_V/\chi^0_V \sim \chi_P/\chi^0_P \sim 1/z$ holds also in the three dimensional $J = 5/2$ PAM.

At last, we make a more general but qualitative consideration on the original definition for $\chi_V$ given by (34). Approximately, we can transform to $\chi_V \sim -\int_{-\infty}^0 d\epsilon \rho(\epsilon) \frac{\partial}{\partial \epsilon}(\Lambda^\epsilon(\epsilon)z(\epsilon))$, where $\rho(\epsilon)$ is the DOS for the $f$-electrons. Here, we consider the quasi-particle contribution to $\chi_V$, i.e., the contribution from the integration range, $0 < -\epsilon \lesssim T^*$. 

20
It is not enhanced only when the relation \(|\hat{A}^\omega(\epsilon) \cdot z(\epsilon)| \cong 1\) is satisfied. The enhancement of \(\chi_V\) is not determined by the value of the enhancement factor at the Fermi energy, \(\hat{A}^\omega(0)\), but by the \(\epsilon\)-dependence of \(\hat{A}^\omega(\epsilon)\). ( From the definition of \(T^*\), \(|z(0)/z(-T^*)| \ll 1\) should be satisfied, and \(z(\pm \infty) = 0\). ) From the result of this paper, \(\chi_V/\chi^0_V \sim 1/z\) in \(d = \infty\) \(J = 5/2\) PAM, \(|\hat{A}^\omega(-T^*)|/|\hat{A}^\omega(0)| \lesssim O(1)\) should be realized. Such a property of \(\hat{A}^\omega(\epsilon)\) will not depend so much on the dimension of the system. Thus, \(\chi_V\) should be enhanced by the strong correlation in three-dimensional systems.

VII. DISCUSSIONS

Here, we summarise the conclusions of this paper. At first, we obtain the general but abstract expression for the magnetic susceptibility on the basis of the orbitally degenerate FLT. In the next stage, we employ two simplifications: One of them is to assume no ECF and the spherical Fermi surface, and another is the \(d = \infty\) approximation. Since our aim of this paper is to elucidate unambiguously the essential properties of the susceptibility under the influence of the strong Coulomb interaction, these over-simplifications will be allowed. Needless to say, it is significant to confirm the property for the zero ECF limit case. The opposite limit case, where only the lowest Kramers doublet contributes to the ground state, has already been discussed elsewhere. [28] ( see Appendix A. )

After the two simplifications, we make further analysis on the expressions both for \(\chi\) and for \(\chi_V\), taking account of all the vertex corrections in a consistent way. Below, we summarize the results of this paper, which will hold qualitatively even in the three-dimensional case.

(i) Metallic Case
In this case the Fermi energy lies below the hybridization gap, which is a prototype of the ( Ce-compound ) heavy Fermion systems. [8] The total susceptibility is given by \(\chi = \lim_{k \to 0} \chi_k(0) = \chi_V + \chi_P\). In \(d = \infty\) approximation, we get the simple expression, \(\chi = \chi^0/z_H(0)\). ( \(1/z_H(0)\) is the magnetic enhancement factor at the Fermi energy. ) We can also express \(\chi\) by use of the mass enhancement factor \(1/z(0)\) as \(\chi \sim 1.2 \cdot \chi^0/z(0)\), which means that the Wilson ratio is \(R \sim 1.2\). In the same way, the Van-Vleck susceptibility is also expressed as \(\chi_V \sim 1.2 \cdot \chi^0_V/z(0)\). In conclusion, both \(\chi\) and \(\chi_V\) are proportional to the mass-enhancement factor. Our conclusion contradicts to the conclusion in ref. [1].

(ii) Insulating Case
In this case the Fermi energy lies in the hybridization gap. This is a prototype of the so called Kondo insulators, some of which exhibit the large magnetic susceptibility experimentally at
In this case, $\chi_P = 0$ and the magnetic susceptibility $\chi_{\text{ins}}$ is given only by the Van Vleck susceptibility at zero temperature. We get the simple expression as $\chi_{\text{ins}} = \chi_{\text{ins}}^0 / z_H(\Delta^*)$, where $\Delta^*$ is the renormalized lower hybridization edge. Because $1 / z_H(\Delta^*) \approx 1 / z_H(0)$ is expected, the magnetic susceptibility for the orbitally degenerate model is strongly enhanced. This result is consistent with the result (i) because the Van Vleck susceptibility will be insensitive to the state of the Fermi surface. So, $\chi_V$ will be little affected by the superconducting transition.

As our calculations are very lengthy and involved, we briefly summarise the mathematical analysis for the Van Vleck susceptibility, $\chi_V$: As is shown by (32), $\chi_V$ is given by the $\omega$-limit of the dynamical magnetic susceptibility. After the $d = \infty$ approximation, it is written as $\chi_V \sim \varphi_b (1 + \Gamma^b \varphi_b)$, where $\varphi_b$ and $\Gamma^b$ are given by (52) and (60), respectively. (Here, the symbols for the energy-integration are implicit.) On the other hand, $1 / z(\omega) \sim (1 + \Gamma^a \varphi_a)$. $\Gamma^a$ and $\Gamma^b$ are related by the Bethe-Salpeter equation given by (70), $\Gamma^b = \Gamma^a + \Gamma^a Q_\gamma \Gamma^b$. (Here, we have identified $\Gamma^a$ with $\Gamma^a'$ for simplicity.) By use of this equation, we can finally show that $\chi_V \sim \int d\omega \{1 / z(\omega) \cdot Q_\gamma(\omega) \cdot 1 / z(\omega)\} \sim 1 / z(0)$. In the same way, $\chi \sim 1 / z(0)$ is derived.

Here, we make the physical consideration on $\chi_P$ and $\chi_V$ in terms of the one-body picture, which was done previously by Anderson and Zou. In the one-body picture, $\chi_P / \chi_P^0 \sim 1 / z(0)$ and $\chi_V / \chi_V^0 \sim (E_f - \mu) / (E_f^* - \mu)$, where $z(0)$ is the renormalization factor at the Fermi energy and $E_f^*$ is the renormalized $f$-electron spectrum. In the mean-field approximation, where the frequency-independent renormalization factor $z_{\text{const}}$ is assumed, then $E_f^*$ is strongly renormalized towards the Fermi energy so that $\chi_V / \chi_V^0 \sim 1 / z_{\text{const}}$. On the other hand, from the viewpoint of the FLT, renormalization is caused by the strong energy-dependence of the selfenergy. The renormalized value of $E_f^*$ critically depends on the energy range of the coherent region (around the Fermi energy), in where the energy dependence of the selfenergy is large. Thus, the question 'to what extent $\chi_V$ is enhanced or not by the strong correlation' is never trivial within the one-body picture. The simple mean-field approximation never answer this question.

Our analysis shows that the excitation of the quasiparticles, which are well defined only within $T^*$, bring the enhancement of $\chi_V$, $\chi_V / \chi_V^0 \sim 1 / z_H(0)$. Our work also suggests that $E_f^*$ is strongly renormalized to the Fermi energy.

Finally, we point out some future problems. At first, it is interesting to estimate the influence of the electronic crystal field, or the shape of the Fermi surface (i.e., the shape of the lattice). These effects are ignored in this paper. Secondly, the effect of the antiferromagnetic fluctuation should be taken into account correctly. The antiferromagnetic fluctuations will
make the magnetic susceptibility $\chi$ smaller than our prediction, although they cannot change the value of $\chi$ and $\chi_V$ drastically. In fact, some heavy Fermion compounds are under the influence of the prominent antiferromagnetic fluctuations, which may be the driving force to the superconducting state. Such a study will give us much information on the electronic properties of the heavy Fermion systems.

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APPENDIX A: THEORY OF THE MAGNETIC SUSCEPTIBILITY FOR SU(N)-PAM

In this Appendix, we briefly consider the magnetic susceptibility for SU(N)-PAM. In case $U = 0$, it is easy to show that $\chi^0_V$ is much smaller than $\chi^0_P$. Here, we study the case $U \neq 0$, making no use of the $d = \infty$ approximation. The Hamiltonian without ECF is given by

\begin{align}
H_0 &= \sum_{kM} \epsilon_k c_{kM}^\dagger c_{kM} + \sum_{kM} E_f f_{kM}^\dagger f_{kM} + \sum_{Mk} (V f_{kM}^\dagger c_{kM} + V c_{kM}^\dagger f_{kM}), \\
H_1 &= \frac{U}{2} \sum_{kk'qM \neq M'} f_{k-qM}^\dagger f_{k'+qM'}^\dagger f_{k'M'} f_{kM}.
\end{align}

We note that the conduction electrons have the six-fold degeneracy. This model is analyzed usually by the slave boson technique. [26]

Here, we assume the existence of the magnetic field $H$ along $z$-axis. At first, we assume that both $E_f$ and $\epsilon_k$ appearing in (A1) are shifted by the same Zeeman energy as

\begin{align}
\begin{cases}
E_f^f = E_f + g_f \mu_B M \cdot H, \\
\epsilon_{kM} = \epsilon_k + g_c \mu_B M \cdot H.
\end{cases}
\end{align}
Only in the special case $g_f = g_c$, the magnetization operator $\hat{M}$ is conserved and $\chi$ is derived after the Luttinger’s manner. [13,15] But, we consider the case $g_f \neq g_c$ here. The Green’s functions for $f$-electrons $G_{kM}(\omega)$ and that for the conduction electrons $G_{kM}^c(\omega)$ are given by

$$G_{kM}(\omega) = \left(\omega + \mu - E^f - \Sigma_{kM}(\omega) - \frac{V^2}{(\omega + \mu - \epsilon_k)^2}\right)^{-1}, \quad (A4)$$

$$G_{kM}^c(\omega) = G_{kM}(\omega) \cdot \frac{V^2}{(\omega - \mu - \epsilon_k)^2}, \quad (A5)$$

where $\Sigma_{kM}(\omega)$ is the selfenergy.

Here, we define the three types of the enhancement factors.

$$\frac{1}{z(\omega)} = 1 - \frac{\partial}{\partial \omega} \Sigma_{kM}(\omega), \quad (A6)$$

$$\frac{1}{z_H(\omega)} = 1 + \frac{1}{g_f M} \frac{\partial}{\partial H} \Sigma_{kM}(\omega), \quad (A7)$$

$$\frac{1}{z'(\omega)} = \frac{1}{z_H(\omega)} \bigg|_{\omega\text{-limit}}, \quad (A8)$$

where (A8) is defined by (36). From the definition of $\chi_P$, given by (33), we get

$$\chi_P = \chi_P^0 \cdot \left(\frac{1}{z'(0)} + \frac{g_c}{g_f (\mu - \epsilon_{k_f})^2} \right) \cdot \left(\frac{1}{z(0)} + \frac{V^2}{(\mu - \epsilon_{k_f})^2}\right)^{-1} \cdot \left(\frac{1}{z_H(0)} + \frac{g_c}{g_f (\mu - \epsilon_{k_f})^2}\right), \quad (A9)$$

$$\chi_P^0 = \sum_M g_f^2 M^2 \rho(0),$$

where $\rho(0)$ is the DOS of the $f$-electrons at the Fermi energy. On the other hand, by use of (34), we get

$$\chi_V = -\frac{1}{N} \sum_{kM} g_f^2 M^2 \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} \cdot \left(\frac{1}{z'(\omega)} + \frac{g_c}{g_f (\omega + \mu - \epsilon_k)^2}\right) \cdot \left(\frac{1}{z(\omega)} + \frac{V^2}{(\omega + \mu - \epsilon_k)^2}\right) \times \frac{\partial}{\partial \omega} \left(G_{kM}(\omega) + \frac{g_c}{g_f} G_{kM}^c(\omega)\right). \quad (A10)$$

In case $g_f = g_c$, $1/z(\omega) = 1/z'(\omega)$ is satisfied rigorously. Then, $\chi_P = \chi_P^0 / z_H(0)$ and $\chi_V = 0$ is derived. Although $1/z(\omega) = 1/z'(\omega)$ is not satisfied rigorously in case $g_f \neq g_c$, its discrepancy will be of order $\sim (V/D)^2 / z$ at most because the relation $\frac{\partial}{\partial (g_f, H)} \Sigma_M(0) \sim \frac{\partial}{\partial (g_f, H)} \Sigma_M(0) \cdot D^2 / V^2$ is expected from (34) and ref. [23].

Thus, $\chi_V$ is negligible compared with $\chi_P$. So, the susceptibility for the insulating case, $\chi_{ins}$, cannot become large. This reasoning has been already applied to the system with the Kramers doublet ground state in the strong ECF limit. [2,28] In the Gutzwiller approximation, $\chi_V / \chi_V^0$ in this model is not enhanced because it is proportional to the conduction electron DOS, which is not renormalized. But the more detailed analysis should be required. Finally, we point out that $\chi_V = 0$ for any model where $\hat{M}$ is conserved, because $1/z(\omega) = 1/z_H(\omega)$ is satisfied then.
APPENDIX B: SOME PROPERTIES OF THE IRREDUCIBLE VERTEX

At first, we show that we can ignore the Pauli’s principle in (4). The term excluded in (4) due to the Pauli’s principle is, in the real-space representation, given by

\[ U \sum_{\{i\},M} f_{iM}^\dagger f_{iM} f_{iM}^\dagger f_{iM}, \]  

(B1)

where \( \{i\} \) represents the set of the \( f \)-electron sites. Apparently, (B1) represents a constant energy shift and is independent of \( M \) in the paramagnetic state. Therefore, we can ignore the Pauli’s principle in the following discussion if all the diagrams are taken into consideration. In other words, contributions from the diagrams violating the Pauli’s principle cancel out in each order of the perturbation in the paramagnetic state. [27]

Then, we investigate the general properties of four-point vertices. We classify four-point vertices into the parallel vertex \( \Gamma \) and the anti-parallel vertex \( T \), as is shown by Fig. B.1. Needless to say, \( T \) never contributes to \( \frac{\partial}{\partial \omega} \Sigma_M(\omega) \). Moreover, \( \frac{\partial}{\partial H} \Sigma_M(\omega) \) also have no contribution from \( T \) because Pauli’s principle is ignored now.

In \( d = \infty \) limit, by neglecting the Pauli’s principle, we can regard that the irreducible parallel four-point vertex \( \Gamma \) satisfy the relation

\[ \Gamma_{MM,M'M'}^I(\epsilon, \epsilon') = \Gamma^I(\epsilon, \epsilon') \cdot \delta_{MM'}, \]  

(B2)

because the local Green’s function is independent of \( M \) and diagonal with respect to \( M \).

On the other hand, when we investigate \( \frac{\partial}{\partial \mu} \Sigma_M(\omega) \), we have to take account of the contribution from both \( T \) and \( \Gamma \). The irreducible antiparallel four-point vertex \( T \) satisfy the relation

\[ T_{MM,M'M'}^I(\epsilon, \epsilon') = T^I(\epsilon, \epsilon') \quad \text{for any} \quad M, M', \]  

(B3)

by neglecting the Pauli’s principle.

APPENDIX C: THE CONSIDERATION ON \( \varphi_{A(B)}(\omega) \) AND THE PROOF OF (66) AND (69)

At first, we consider the following calculation as a preparation of calculating the enhancement factor.

\[ \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} \Gamma'(0,\omega) \varphi_{a(0)}(\omega) \Gamma'(\omega,0), \]  

(C1)
where $\Gamma'(\epsilon, \epsilon')$ is an irreducible four-point vertex with respect to the Coulomb interaction $U$. Then,

$$\{ \text{eq. (C1)} \} = \int_C \frac{d\omega}{2\pi i} \Gamma'(0, \omega) \varphi_{a(b)}(\omega) \Gamma'(\omega, 0) = \int_{-\infty}^0 \frac{d\zeta}{\pi} \left\{ \text{Im}\{\varphi_{a(b)}(\zeta)\} \text{Re}\{\Gamma'(\zeta, 0)\} + \text{Re}\{\varphi_{a(b)}(\zeta)\} \text{Im}\{\Gamma'(\zeta, 0)\} \right\}. \quad (C2)$$

In the first line of the r.h.s. of (C2), we have changed the path of $\omega$-integration as shown in Fig. C.1. Even if $\Gamma'$ is replaced with reducible vertex $\Gamma$, this procedure is correct because $\varphi_a \sim \varphi_b \sim \omega^{-2}$ for $\omega \gtrsim D$. The last term in (C2) can be obtained by substituting the following spectrum representations (which is possible because $\varphi_{a(b)}(\pm \infty) = \Gamma'(\pm \infty, 0) = 0$),

$$\varphi_{a(b)}(\omega) = -\int \frac{d\zeta}{\pi} \left\{ \frac{\theta(\zeta)}{\omega - \zeta + i\eta} + \frac{\theta(-\zeta)}{\omega - \zeta - i\eta} \right\} \cdot \text{Im}\varphi_{a(b)}^R(\zeta), \quad (C3)$$

$$\Gamma'(\omega, 0) = -\int \frac{d\zeta}{\pi} \left\{ \frac{\theta(\zeta)}{\omega - \zeta + i\eta} + \frac{\theta(-\zeta)}{\omega - \zeta - i\eta} \right\} \cdot \text{Im}\Gamma'^R(\zeta, 0), \quad (C4)$$

and by performing $\omega$-integration at first. In the last line of (C2), only $\text{Re}\{\Gamma'(\zeta, 0)^2\}$ for $0 < -\zeta < T^*$ contributes predominantly to the value of the $\zeta$-integration, which is largely enhanced. Thus, in heavy Fermion systems, the behavior of $\text{Im}\varphi_{a(b)}(\epsilon)$ only for $0 \leq -\epsilon < T^*$ is dominant in determining the enhancement factor.

In the next stage, we prove the important relation given by (46). The spectral representations for the local Green’s functions, given by (44), are

$$g^{(f)}(\omega) = \int d\zeta \left\{ \frac{\theta(\zeta)}{\omega - \zeta + i\eta} + \frac{\theta(-\zeta)}{\omega - \zeta - i\eta} \right\} \cdot \rho^{(f)}(\zeta), \quad (C5)$$

$$\rho(\zeta) = -\frac{1}{\pi} \text{Im} \frac{1}{N} \sum_k G_{kMM}^R(\zeta) = -\frac{1}{\pi} \text{Im} g^R(\zeta),$$

$$\rho^f(\zeta) = -\frac{1}{\pi} \text{Im} \frac{1}{N} \sum_k d_{kMM} G_k^{Rf}(\zeta),$$

where $\rho^f(\zeta)$ represents the DOS of the localized $f$-electrons, and $d_{kMM}$ is given by (46). Apparently, $\rho(\zeta) \geq \rho^f(\zeta)$ for any $\zeta$ and $\rho^f(0) = 0$. On the other hand, we can also express the particle-hole Green’s functions in the spectral representation as follows (see (C4)):

$$\frac{1}{N} \sum_k G_k(\omega)^2 = -z(\omega) \cdot \frac{1}{N} \sum_k \partial_{\omega} G_k(\omega)$$

$$= -z(\omega) \int d\zeta \left\{ \frac{\theta(\zeta)}{\omega - \zeta + i\eta} + \frac{\theta(-\zeta)}{\omega - \zeta - i\eta} \right\} \cdot \frac{\partial}{\partial_k} \rho(\zeta) + 2\pi i z(0) \rho(0) \cdot \delta(\omega), \quad (C6)$$

where in deriving the last line above, we have done the partial integration.
At first, we investigate the case where the system is metallic at $T = 0$, i.e., $\rho(0) \neq 0$. (see Fig. 1(a).) For $0 < -\omega \lesssim T^*$,

$$\rho(\omega) \cong \frac{1}{N} \sum_{k} a_k(\omega)\delta(\omega - E^*_k) \cdot \frac{3V^2}{(\omega + \mu - E^f - \Sigma(\omega))^2}$$

$$= \rho_c(\omega) \cdot \frac{3V^2}{(\omega + \mu - E^f - \Sigma(\omega))^2}. \quad (C7)$$

So, we obtain

$$\frac{\partial}{\partial \omega} \rho(\omega) \cong -2\rho(\omega) \cdot \frac{1}{\omega + \mu - E^f - \Sigma(\omega)} \cdot \frac{1}{z(\omega)} + \frac{\partial}{\partial \omega} \rho_c(\omega) \cdot \frac{3V^2}{(\omega + \mu - E^f - \Sigma(\omega))^2}. \quad (C8)$$

Considering that the first term of the r.h.s. of (C8) has the enhancement factor $1/z$ contrary to the last term, we can neglect the last term of (C8) for $0 \leq -\omega < T^*$ in metallic heavy Fermion systems. On the other hand, $\rho^f(\omega)$ is non-zero only for $\omega \sim |E^f^* - \mu| > 0$, corresponding to the renormalized $E^f$ spectrum, and $\omega \sim \pm U/2$, ($U \gg T^*$), corresponding to the broad satellite on both sides of the Fermi energy [18]. Especially, $\rho^f(0) = 0$. Thus, both $\text{Im} G^f_k(\omega)$ and $\text{Im} G^2_k(\omega)$ make little contribution to the behavior of (66) or (69), so they are negligible. By use of the relation for $0 < -\omega \lesssim T^*$,

$$\text{Im} \frac{1}{N} \sum_{k} (-1/\pi) G^R_k(\omega) \cdot \frac{\omega + \mu - E_k}{3V^2} \cong \frac{1}{\omega + \mu - E^f - \Sigma(\omega)} \cdot \rho(\omega), \quad (C9)$$

we can show from (52) that

$$\text{Im} \varphi_\theta(\omega) \cong \text{Im} \left\{ B \frac{1}{N} \sum_k G^2_k(\omega) + (A - B) \left( \frac{1}{N} \sum_k G^2_k(\omega) - 2\pi i z(0) \rho(0) \delta(\omega) \right) \right\}$$

$$\cong \text{Im} \{ \varphi_a(\omega) + Q_\gamma(\omega) \},$$

for $0 < -\omega \lesssim T^*$. Here we have used (54) and (C8). We stress that when $U = 0$ and $\rho^0_c(\omega)$ is constant with respect to $\omega$, the relation (53) is rigorous for $\omega \leq 0$.

In the same way, we consider the case where the system is insulating, i.e., $\rho(0) = 0$. (see Fig. 1(b).) We prove the important relation given by (69). Note that $\rho(\omega) = 0$ for $\Delta^*_e < \omega < E^f^* - \mu$, where $\Delta^*_e$ and $E^f^*$ are the renormalized lower edge of the hybridization gap and the renormalized local $f$-electron level, respectively. They are given by $\Delta^*_e \cong \Delta_+ \cdot z(0)$ and $E^f^* \cong (E^f - \mu) \cdot z(0) + \mu$. [18] In this insulating case, taking account of the fact $\text{Im} \Sigma(\omega) = 0$ for $\Delta^*_e \leq \omega < E^f^* - \mu$, we can show that

$$\rho(\omega) \cong \rho_c(\omega) \frac{3V^2}{(\omega + \mu - E^f - \Sigma(\omega))^2} \cdot \theta(\Delta^*_e - \omega), \quad (C10)$$

for $\Delta^*_e - T^* < \omega < E^f^* - \mu$. So, we get for $\omega \leq 0$,
\[ \frac{\partial}{\partial \omega} \rho(\omega) \approx \{ \text{eq.(C8)} \} - \rho(\Delta^*) \delta(\omega - \Delta^*). \]  

Thus, for \(0 \lesssim -\omega \lesssim -\Delta^* + T^*\), we can show the relation (63) in the insulating case. Here, we comment that if the anisotropy of the Brillouin zone is taken into account in \(d = 3\) system, the step function in (C10) becomes a continuous function because of the van Hove singularity. So, the delta function in (C11) has finite (but narrow) width. In this sense, the relation (63) is less universal than the relation (63).

**APPENDIX D: NUMERICAL CALCULATIONS FOR \(1/Z(\omega)\) AND \(1/Z'(\omega)\)**

In this Appendix, we explain the method and the results of the numerical calculation by SOPT and SC-SOPT with respect to \(U\) in the \(d = \infty\) limit. We calculate both \(1/z(\omega)\) and \(1/z'(\omega)\) and check the relation (74), which is derived in the analytical way. The relation (74) is very significant because our main results (80) and (83) are based on it.

At first, we calculate \(1/z(\omega)\) and \(1/z'(\omega)\) by SOPT, which are depicted in Fig. D-1(a) and D-1(b), respectively. Here, we use the constant DOS for the conduction electrons, i.e., \(\rho_c^0(\omega) = \rho_c^0\). The numerical results of their real part are shown in Fig. D-2. The obtained \(T^*\) in SOPT is about \(|E_f - \mu|\). Surprisingly, the resultant \(1/z(\omega)\) and \(1/z'(\omega)\) are almost the same not only for \(|\omega| \lesssim T^*\) but also for \(|\omega| \gg T^*\). By use of SOPT, we also calculate \(1/z(\omega)\) and \(1/z'(\omega)\) for insulating case. We put Fermi energy \(\mu\) in the middle of \(\Delta_+\) and \(E_f\). The numerical results of their real parts are shown in Fig. D-3. In the framework of the SOPT, \(\Delta_+\) is renormalized to be \(\Delta^* \approx z(0) \cdot \Delta_+\).

In the second stage, we calculate \(1/z(\omega)\) and \(1/z'(\omega)\) by SC-SOPT. The irreducible four-point vertex \(\Gamma^I\) is composed of at most two local Green’s functions, and is shown by Fig. D-4. In SC-SOPT, we construct \(1/z(\omega)\) and \(1/z'(\omega)\) with \(\Gamma^I\) and \(g(\omega)\) by referring to their definitions, (62) and (73), respectively. (Then, \(U\)-linear terms of \(\Gamma^I\) turn out to give no contribution, as is shown below.) The numerical results of their real parts are shown in Fig. D-5. Here, we assume \(\rho_c^0(\omega) \propto (\omega + D)^2\). The obtained \(T^*\) is renormalized to be \(\sim z(0) \cdot |E_f + \Sigma(0) - \mu|\).

Surprisingly, in spite of \(\rho_c^0(\omega) \neq \rho_c^0\), the resultant \(1/z(\omega)\) and \(1/z'(\omega)\) are almost the same not only for \(|\omega| \lesssim T^*\) but also for \(|\omega| \gg T^*\). The discrepancy becomes almost invisible in case \(\rho_c^0(\omega) = \rho_c^0\). It becomes large only when \(\rho_c^0(\omega)\) has drastic energy-dependence for \(0 \leq -\omega \lesssim T^*\). In real heavy Fermion systems, \(T^*\) is renormalized as is pointed out by
SC-SOPT, so the susceptibility will be quite insensitive to the energy dependence of the \( \rho_c^0(\omega) \).

In calculating for \( 1/z'(\omega) \) in case \( \rho_c^0(\omega) \neq \rho_c^0 \), we do not fix the value of \( C \) to (59), but adjust it so as to the equation (79) is satisfied. Then, the obtained \( 1/z'(\omega) \) has no RPA-type enhancement, as well as \( 1/z(\omega) \). The resultant values of \( C_{\text{cal}} \) are given by Table D · I. The larger the \( 1/z(0) \) is, the smaller \( |C - C_{\text{cal}}| \) becomes, as is expected in Appendix C. ( see (C8). ) Taking account of the smallness of \( 1/z \) in our calculation, we can say that our numerical result supports (74) quite well even in case \( \rho_c^0(\omega) \neq \rho_c^0 \).

| \( U \) | \( C_{\text{cal}} \) | \( 1/z(0) - 1 \) |
|---|---|---|
| 0.5 | 0.143 | 4.67 |
| 0.3 | 0.140 | 3.64 |

Table D-I : The values of \( C_{\text{cal}} \) which satisfies by eq. (79) by SC-SOPT. All the parameters except for \( U \) are the same as that used in Fig. D-5.

Here, we do not calculate the magnetic enhancement factor \( 1/z_H(\omega) = 1 + \frac{1}{g\mu B M} \Sigma_M(\omega) \) because \( 1/z_H(\omega) \) calculated by SC-SOPT will contain much contribution from RPA-type diagrams. We know that their contribution is limited only to the polarization factor, \( R/R^0 \sim 1.2. \) ( see §6 and Appendix E. ) On the calculation of \( 1/z_H(\omega) \) by SC-SOPT, we have to take caution against the unphysical enhancement of the Wilson ratio caused by the RPA-type diagrams. In the rigorous perturbation calculation, most of them turn out to be canceled with other diagrams not contained in SC-SOPT, and amount to a little contributions totally.

**APPENDIX E: PROOF OF (79)**

In this appendix, we examine the relation \( \chi''_{vl} = 0 \), shown by (79), in detail. To do this, we study the following integrals at first :

\[
\frac{1}{N} \sum_k \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi i} G_k^2(\epsilon)/z'(\epsilon) = \frac{1}{N} \sum_k (-2i) \text{Im} \int_{-\infty}^{0} \frac{d\epsilon}{2\pi i} G_k(\epsilon) \cdot \frac{\partial}{\partial \epsilon} \{z_1(\epsilon)/z'(\epsilon)\}; \quad (E1)
\]

\[
\frac{1}{N} \sum_k \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi i} G_k^2(\epsilon)/z'(\epsilon) = \frac{1}{N} \sum_k (-2i) \text{Im} \int_{-\infty}^{0} \frac{d\epsilon}{2\pi i} G_k(\epsilon) \cdot \frac{\partial}{\partial \epsilon} \{z(\epsilon)/z'(\epsilon)\}, \quad (E2)
\]
where we have done partial integration. Because \( \frac{\partial}{\partial \epsilon}(z^{(1)}(\epsilon)/z' (\epsilon)) \ll 1 \) for \( 0 < - \epsilon \lesssim T^* \) is expected, both (E1) and (E2) are not enhanced. As \( \chi_V'' \) is expressed by both (E1) and (E2) (see (66) and (78)), we conclude that \( \chi_V'' \) is not enhanced, and small quantity. Even if \( \chi_V'' \neq 0 \), a slight modification of the value of \( C \) in the definition of \( Q_\gamma (\omega) \) can make \( \chi_V'' = 0 \). (As is shown in Appendix D, this is confirmed well within SC-SOPT.) This reason is as follows: the behavior of \( z'(\epsilon) \) for \( 0 < - \epsilon \lesssim T^* \) is sensitive to the value of \( C \), and the absolute value of (E1) is largely enhanced when (74) is not satisfied. Thus, the relation (79) is justified.

\[
1/z'(\omega) \text{ can be rewritten as}
1/z'(\omega) = \left(1 + \int \frac{d\epsilon}{2\pi i} \Gamma_\text{non-tad}^{\alpha}(\omega, \epsilon) \varphi_{a'}(\epsilon) \right) \cdot \left(1 + U \chi_V''(3/2g^2J(J+1))\right),
\]
where \( \Gamma_\text{non-tad}^{\alpha}(\omega, \epsilon) \) is derived from \( \Gamma^{\alpha}(\omega, \epsilon) \) by dropping the reducible terms with respect to \( U \). When we choose the value of \( C \) so that \( \chi_V'' = 0 \) is satisfied, (which will be close to the value given by (59),) the enhancement factor \( 1/z'(0) \) has no contributions from any irreducible vertices with respect to \( U \), i.e., from any tadpole diagrams. (see Fig. E-1.) Moreover, the mass-enhancement factor \( 1/z(0) \) also has nothing to do with any tadpole diagrams, that is,

\[
1/z(\omega) = 1 + \int \frac{d\epsilon}{2\pi i} \Gamma_\text{non-tad}^{\alpha}(\omega, \epsilon) \varphi_{a}(\epsilon).
\]

Then, we can conclude that both \( 1/z(\omega) \) and \( 1/z'(\omega) \) has no contributions from RPA-type enhancement. This results suggests that the relation \( 1/z(\omega) \cong 1/z'(\omega) \) holds for a wider range of \( \omega \).
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\[ 6 \times \text{constant}] \]
2.19)) are obtained straightforwardly. $\chi^0$ and $\chi^0_V$ is given by §4. When $U \lesssim 6/\rho(0)$, $\chi_{RPA}^0 \lesssim \infty$ and $\chi_{RPA}^V = 1.84$.

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Figure

- Fig. 1: Effective band structure for $J = 5/2$ PAM. $E^f$ is the four-fold degenerate local $f$-electron spectrum, and $E^*_k$ is the two-fold degenerate quasiparticle spectrum, respectively. (a) Fermi energy $\mu$ lies below $\Delta_- + \mu$. The system is metallic. (b) $\mu$ lies between $\Delta_- + \mu$ and $E^f$. The system is insulating.

- Fig. 2: (a) Four point vertex $\Gamma_{pp',qq'}^M,M',M''(\epsilon, \epsilon'; \omega)$. (b) $k$-limit (or $\omega$-limit) four point vertex $\hat{\Gamma}_{pp}^k(\epsilon, \epsilon')$.

- Fig. 3: The schematic structure for $\chi_V$.

- Fig. 4: The structure of the vertex correction for the (effective mass or magnetic) enhancement factor in the $d = \infty$ approximation. Here, $I$ represents the local irreducible four-point vertex, $\Gamma^I$, and $\hat{\varphi}$ represents the particle-hole Green’s functions, connecting between two sites. $l$, $m$, $n$ and $j$ represent the $f$-electron sites, on which we have to take summation.

- Fig. 5: Two examples of $U^2$-order contributions for the susceptibility. $j$, $l$, $m$ and $n$ represent the $f$-electron sites, on which we have to take summation.

- Fig. 6: $\Gamma_{MM',M'M''}^I(\epsilon, \epsilon')$ represents the irreducible parallel four-point vertex with respect to the particle-hole pair. In the $d = \infty$ limit, it is diagonal with respect to $M$ and independent of $M'$.

- Fig. 7: The definition for two kinds of the particle-hole Green’s functions, $\varphi_a(\omega)$ and $\varphi_b(\omega)$. Both of them are independent of $M$. $l$ represents the $f$-electron sites, on which we have to take summation. In (b), factor $M'$ comes from the Zeeman term.

- Fig. 8: The schematic structure for the energy-derivative of the selfenergy. This is composed of $\{ \Gamma^I, \varphi_a \}$.
• Fig. 9: The first line represents the Bethe-Salpeter equation relating between $\Gamma^a$ and $\Gamma^b$. The second line is derived by using the first line twice.

• Fig. 10: The schematic structure for $\chi_V$. This is composed of $\{ \Gamma^I, \varphi^a \}$. We stress that two enhancement factors appear in total on both sides of $Q^r(\omega)$ in the last line. $\chi''_V$ turns out to vanish identically.

• Fig. B·1: $\Gamma(\epsilon, \epsilon')$ represents the parallel four-point vertex and $T(\epsilon, \epsilon')$ represents the anti-parallel four-point vertex, respectively.

• Fig. C·1: Complex integration path $C$ in eq. (C2).

• Fig. D·1: The diagrams for the enhancement factors in SOPT. The broken lines represent the Coulomb potential $U$. (a) $\{ 1/z(\omega) - 1 \}$ in SOPT. This is constructed by the unperturbed $\varphi^0_a(\omega)$. (b) $\{ 1/z'(\omega) - 1 \}$ in SOPT. This is constructed by the unperturbed $\varphi^0_a'(\omega)$.

• Fig. D·2: Numerical results for the frequency dependence of the real parts of enhancement factors by SOPT, for the metallic case. The line and the broken line represent $\{ 1/z(\omega) - 1 \}$ and $\{ 1/z'(\omega) - 1 \}$, respectively. The energy region for large enhancement in SOPT is about $\lesssim |\mu - E^f|$. We put $U^2 = 1, 3V^2 = 0.16, E^f = -0.3, k_F = 0.885\pi$ and $\epsilon_k = 2(|k|/\pi)^3 - 1$, respectively.

• Fig. D·3: Numerical results for the frequency dependence of the real parts of enhancement factors by SOPT, for the insulating case. The line and the broken line represent $\{ 1/z(\omega) - 1 \}$ and $\{ 1/z'(\omega) - 1 \}$, respectively. We put $U^2 = 1, 3V^2 = 0.25, E^f = -0.5, \mu = (\Delta_e + E^f)/2$ and $\epsilon_k = 2(|k|/\pi)^3 - 1$, respectively.

• Fig. D·4: The diagrams for the irreducible parallel four point vertex $\Gamma^I(\epsilon, \epsilon')$ used in SC-SOPT.

• Fig. D·5: Numerical results for the frequency dependence of the real parts of the enhancement factors by SC-SOPT, for the metallic case. The line and the broken line represent $\{ 1/z(\omega) - 1 \}$ and $\{ 1/z'(\omega) - 1 \}$, respectively. The energy region for large enhancement in SC-SOPT is about $\lesssim z(0) \cdot |\mu - E^f|$, which is renormalized compared with that by SOPT. (see Fig. D3.) We put $U^2 = 0.5, 3V^2 = 0.16, E^f = -0.3, k_F = 0.885\pi$ and $\epsilon_k = 2(|k|/\pi)^3 - 1$, respectively.

• Fig. E·1: The diagram for $1/z'(\omega)$. The first term is irreducible and the second term is reducible with respect to $U$. As the second term includes $\chi''_V$, it vanishes identically.
Figure 1.

(a) Energy vs. Momentum

\[ \Delta^+ + \mu \]

\[ \Delta^- + \mu \]

\[ E_f \]

\[ E_k^* \]

(b) Energy vs. Momentum

\[ \Delta^+ + \mu \]

\[ \Delta^- + \mu \]

\[ E_f \]

\[ E_k^* \]

Figure 2.

(a) \( p' - q \) vs. \( \epsilon' - \omega \)

(b) \( p' \) vs. \( \epsilon' \)
Figure 3.

$$\chi_V = \phi_k + \phi_{k'} \Gamma_{kk'}$$

Figure 4.

Figure 5.

(a) (b)
Figure 6.

\[ \Gamma^{I}_{MM',M'M'} = \Gamma^{I} \cdot \delta_{MM'} \]

Figure 7.

(a) \[ \varphi_{a} = \sum_{M'} \sum_{l}^{j} \]

(b) \[ M \cdot \varphi_{b} = \sum_{M'} \sum_{l}^{j} \]

Figure 8.

\[ \Gamma^{r} \Gamma^{l} = \begin{bmatrix} I \end{bmatrix} + \begin{bmatrix} I & I \end{bmatrix} \begin{bmatrix} \Gamma^{r} \varphi_{r} \Gamma^{l} \end{bmatrix} + \begin{bmatrix} I & I & I & I \end{bmatrix} \begin{bmatrix} \Gamma^{r} \varphi_{r} \Gamma^{l} \varphi_{r} \Gamma^{l} \varphi_{r} \Gamma^{l} \end{bmatrix} + \cdots \]

\( (r = a, a', b) \)

\[ \frac{d}{d\omega} \sum_{M} = \begin{bmatrix} \Gamma^{a} \varphi_{a} \end{bmatrix} \]
Figure 9.

\[
\Gamma_b = \Gamma_{a'} + \Gamma_{a'} Q_\gamma \Gamma_b
\]

\[
= \epsilon + Q_\gamma + Q_\gamma \Gamma_b Q_\gamma
\]

Figure 10.

\[
\chi_V = \frac{\phi_b(\varepsilon) + Q_\beta(\varepsilon)}{z_H(\varepsilon)} \frac{1}{z_H(\varepsilon)}
\]

\[
\chi_V = \frac{1}{z_H'(0)} + \frac{1}{z_H'(0)} Q_{a'} + \phi_{a'} \frac{1}{z_H'(\varepsilon)}
\]
Figure B1.

\[ \Gamma(\epsilon, \epsilon') \quad , \quad T(\epsilon, \epsilon') \]

Figure C. 1

\[ \text{Im} \omega \quad \text{Re} \omega \]

Figure D1.

\[
\begin{pmatrix}
1/z(\omega) \\
1/z'(\omega)
\end{pmatrix} = 1 + \phi_0^a + \phi_0^{a'}
\]
Figure D.4

\[ \Gamma^I(\epsilon, \epsilon') = \begin{array}{c}
\text{\includegraphics{diagram1.jpg}}
\end{array} \]

Figure D.5

![Graph showing Re(1/z(ω)-1) and Re(1/z'(ω)-1) vs Energy](image.png)
Figure E1.

\[
1/z'(\omega) = \omega + \frac{1}{\omega} \left( \frac{\Gamma^a_{\text{non-tad}} \cdot \varphi^a + 1}{\chi''} \right)
\]