PAQMAN: A Principled Approach to Active Queue Management

Sounak Kar*, Bastian Alt†, Heinz Koepl† and Amr Rizk‡
* EPFL, sounak.kar@epfl.ch
† Technical University of Darmstadt, firstname.lastname@bcs.tu-darmstadt.de
‡ University of Duisburg-Essen, amr.rizk@uni-due.de

Abstract—Active Queue Management (AQM) aims to prevent bufferbloat and serial drops in router and switch FIFO packet buffers that usually employ drop-tail queueing. AQM describes methods to send proactive feedback to TCP flow sources to regulate their rate using selective packet drops or markings. Traditionally, AQM policies relied on heuristics to approximately provide Quality of Service (QoS) such as a target delay for a given flow. These heuristics are usually based on simple network and TCP control models together with the monitored buffer filling. A primary drawback of these heuristics is that their way of accounting flow characteristics into the feedback mechanism and the corresponding effect on the state of congestion are not well understood. In this work, we show that taking a probabilistic model for the flow rates and the queuing pattern, a Semi-Markov Decision Process (SMDP) can be formulated to obtain an optimal packet dropping policy. This policy-based AQM, denoted PAQMAN, takes into account a steady-state model of TCP and a target delay for the flows. Additionally, we present an inference algorithm that builds on TCP congestion control in order to calibrate the model parameters governing underlying network conditions. Finally, we evaluate the performance of our approach using simulation compared to state-of-the-art AQM algorithms.

Keywords: Active Queue Management, Markov Decision Processes.

I. INTRODUCTION

Striking a balance between the two most common metrics of performance in IP networks, i.e., throughput (or utilization) and delay is a fundamental problem for routing devices. Active queue management (AQM) has evolved as a mechanism to augment prevalent end-system protocols such as TCP to tune the performance with regards to the said metrics and is usually implemented in connecting network devices such as routers and switches. Most modern network switches have built-in buffers, which accumulate incoming data packets while the switch is busy processing and transmitting processed packets to the output ports [1]. A large buffer has the advantage that it can potentially minimize droptail packet loss, i.e., the dropping of incoming data packets waiting to be processed when the buffer is full. Understandably, this impacts network utilization positively while causing excess buffering of packets and thus longer delays. This phenomenon is commonly referred to as bufferbloat [2]. Although a shallow buffer alleviates this problem, it can also lead to frequent packet drops, which harm the TCP flows1 throughput [4]. Evidently, the trade-off between higher utilization and lower queuing delay is a matter of policy [5], which is delineated by the implemented AQM algorithm at the switch. The algorithm usually decides whether to drop an incoming packet depending upon inputs such as current buffer-filling, packet delay history, or recent packet drop pattern.

In the last three decades, a range of algorithms have been proposed [6], [7], [8] to address the AQM problem, which aims to achieve an acceptable trade-off between link utilization and packet delay. Occasionally, these algorithms focus on additional aspects such as scalability, robustness, or fairness; for an extensive survey, see [9] and the references therein. The seminal AQM algorithm Random Early Detection (RED), proposed by Floyd and Jacobson in 1993, calculates an exponentially weighted average queue length and, as a linear function of this average, computes an initial packet drop probability in $[0, p_{\text{max}}]$, $p_{\text{max}} \in (0,1)$. To avoid serial drops, the initial probability is further transformed into a final drop probability that takes into the account the number of packets admitted since the last packet drop. Clearly, the resulting policy depends critically on the two threshold parameters for the queue length and $p_{\text{max}}$. Although RED is shown to be fairer to bursty traffic than the classic drop tail [6], the main challenge lies in identifying the model parameters and there has been no consensus on the corresponding parameter engineering process. To get around this problem, a range of variants and extensions of RED have been proposed, which address this issue with fairly limited success. For a thorough discussion of these algorithms, we refer to [9] and the references therein.

A popular AQM algorithm that claims to have successfully circumvented the problem of parameter engineering is CoDel [7]. Although CoDel is driven by two input parameters, one that signifies a target delay (at the AQM enabled node) and the other specifying how often a packet should be dropped, their default value is hardly changed in practice. Thus, CoDel is often considered to be knob-free. Starting with the default value of the window parameter that signifies the frequency of packet drops, CoDel successively adapts its value until it meets the target delay. Although claimed to be knob-free due to the prescribed default values of the parameters under universal traffic conditions, the authors of [10] demonstrate that these values can be tuned to yield superior performance for a given environment. However, the analysis for the choice of the default values under varied conditions has been limited to minimal empirical investigations [10].
Different implementations exist for CoDel, e.g., while originally devised to drop a packet after it is already enqueued, it can also be implemented to obtain the current queuing delay and drop packets at the ingress depending on the information exposed on the data plane. Similarly, the AQM algorithm denoted PIE [8] drops packets directly at the input port. PIE calculates the drop probability of an incoming packet by looking at the current queue-filling and the departure rate from the queue. This further improves the processing overhead compared to CoDel, which requires calculation of delay per packet. Additionally, the consideration of the current queue length to calculate the drop probability implies that congestion is directly controlled. Although claimed to be knob-free like CoDel, PIE actually requires default values for target delay, drop frequency and parameters for drop probability adaptation [10]. Further, the drop probability parameters are adapted according to a rule that is based on judgements. This phenomenon extends to most, if not all, popular AQM algorithms of today [10].

In light of the above, we propose a principled approach to address the AQM problem, called PAQMAN, which only requires the target delay as an input. To detect potential congestion, PAQMAN uses the current queue-filling and an estimate of the arrival rate as well as estimates of the flow RTTs. We make the case that to optimally decide on packet drops, an AQM algorithm requires this information, or estimates thereof, to predict congestion.

PAQMAN is derived based on a model of the state evolution of the switch that uses the Markov Decision Processes (MDP) framework. In this model, the state description includes the congestion indicators mentioned above. We further encode the optimization goal through a reward function that combines delay and throughput objectives and reflects the immediate gain following a packet drop/admit decision. Consequently, we derive an optimal policy, i.e., the AQM algorithm, using tools from the MDP framework that use the state transition probabilities calculated from the model and the reward function as inputs. The policy provides the optimal decision, i.e., whether it is best to drop or admit an incoming packet for every switch state, comprising of current queue length, flow rate, and RTT. PAQMAN treats the simpler case where the switch deals with a single flow having negligible RTT in greater generality, whereas a more tractable model is chosen for the more involved case with multiple flows. Finally, we compare the performance of PAQMAN with the state-of-the-art algorithm CoDel and the classic droptail queues. Our findings show that PAQMAN yields equivalent utilization/throughput to CoDel while minimizing delays considerably.

The remainder of the paper is structured as follows: We first provide a background of AQM research in the last decades in Sect. II and subsequently formulate the AQM decision problem in Sect. III. While Sect. III also contains the details of the algorithm when the switch performs AQM on a single flow with negligible RTT, we discuss the case with non-negligible RTT with single or multiple flows in Sect. V. Finally, we present numerical simulation results in Sect. VI and conclude the paper in Sect. VII.

II. RELATED WORK

In this section, we review AQM algorithms that are most relevant to our work and highlight the aspects where they differ from PAQMAN. AQM was developed as an additional module on top of congestion control functions and aimed to keep the congestion levels lower than traditional droptail queues by sending early congestion signals. To detect early congestion, earlier AQM algorithms relied on queue length based congestion indicator which was later generalized to incorporate other factors such as packet arrival/departure rate. The congestion signal was traditionally in form of a packet drop but ECN markings also came into use to enhance throughput. While the packet drop action was deterministic for droptail queues, it was randomized in the first AQM algorithm RED [6]. To be specific, RED assigned a drop probability for each packet, calculated based on certain thresholds on the queue length. The lack of unanimity on the process of determining these thresholds led to newer algorithms, which generalized the congestion indicator even further. Apart from RED and droptail, we take the example of PIE [8] and CoDel [7] here. While CoDel uses the minimum delay of dequeued packets over an observation window as its congestion indicator, PIE looks at the queue length and departure rate from the queue to calculate a drop probability.

In general, the congestion indicator(s) for an AQM may include queue length, packet arrival/departure rate, flow round-trip time, link capacity, and number of flows. Under PAQMAN, the current packet arrival rate and the queue length are used to detect early congestion in the simplified canonical case of considering one flow and negligible RTT. For the general case with non-negligible RTT and multiple flows, the RTTs, the flow index of the incoming packet, and the time since the last decision event are also included in the set of congestion indicators. Thus, we have a more comprehensive description of the state of the switch under PAQMAN, which allows us to model the state evolution and to derive the optimal policy for dropping packets.

The core of the AQM algorithm can be viewed as a rule which translates the congestion indicator into the drop/admit action or, more generally, the likelihood of the same. That is, by looking at the current values of the congestion indicators, it determines if there is indeed an indication of potential congestion and accordingly decides whether to send a congestion signal. There are primarily three approaches [9] to formulate this rule: heuristic-based, control theoretic, and deterministic optimization-based. While the early algorithms such as RED, its variants [5], [12] and the popular algorithm CoDel follow heuristic approaches, the later approaches adopted control theoretic framework to circumvent the judgemental aspects of the former approach such as parameter tuning. For simplicity and tractability, most of the control
unknown flow properties: Sect. III

Fig. 1: Part (a) shows how PAQMAN distinguishes between different AQM scenarios: the time until the receipt of the feedback of drop/admit action is the transmission time between the switch and the sender via the receiver, which is approximated as one RTT. The case with negligible feedback time (RTT) is discussed in Sect. III and the other case is taken up in Sect. V. When traffic properties are unknown, we estimate corresponding parameters according to Sect. IV. Part (b) highlights how AQM acts in tandem with the underlying congestion control by sending early congestion signals and thereby influencing the sending rate to achieve a desired level of congestion.

Theoretic algorithms build on the Additive Increase Multiplicative Decrease (AIMD) principle of TCP ignoring other aspects like slow start and retransmission timeouts. Further, these works assume a linearized fluid model of TCP proposed in [13], which analyzes RED from a control theoretic perspective. Some prominent examples under the control theoretic approach include: Proportional-Integral [13], Proportional-Derivative [14], Proportional-Integral-Derivative [15], and Proportional-Integral-Enhanced [8] controller. For a thorough comparison of the algorithms under this approach, see Table VI-IX of [9]. Recently, an information compression approach has been adopted in [16], where starting from the TCP fluid model, a simpler relation between queueing delay and drop probability is formulated. The control law here specifies the change in drop probability to drive the delay towards a reference value. The authors finally show that for their choice of parameters and experiment setup, the prescribed approach yields similar performance to CoDel and PIE for different RTTs and a range of number of flows. We note that the control theoretic approaches in general require certain input parameters whose default values are set according to judgement. In comparison, PAQMAN employs an MDP-based approach and only requires the target delay as its input parameter. A closely related yet different approach has been discussed in the works [17], [18] where the authors first assume that the system dynamics is given by a deterministic fluid model for which they set out to perform a non-equilibrium analysis. To that end, they first identify a Markov chain that closely approximates the behaviour of the deterministic system, i.e., estimate the transition matrix of the chain for a given level of state-space discretization. Subsequently, they adopt an MDP-based approach to find out the best policy, where the set of possible actions in a given state comprises of drop-tail, RED, or a interpolated version of these two policies. Further, the reward function is chosen based on judgement and it is shown via ns-2 simulation that the optimal policy suggested by the MDP yields stabler queue lengths compared to RED. While our approach uses the MDP formulation to derive an AQM policy, this work utilizes the MDP framework to pick the best possible AQM from a set of candidate AQMs given the current state of the system.

Compared to control theoretic approaches, which analyze the transient behaviour of the system, deterministic optimization approaches [19], [20] primarily focus on steady-state measures. Also, many AQMs under this approach primarily rely on arrival/departure rate of packets as congestion indicator. In contrast to the above, we take an MDP-based approach to solving the AQM problem. PAQMAN builds on a stochastic framework where following each drop/admit decision a reward is calculated. The reward combines delay and throughput goals and reflects the immediate gain in terms of these objectives. To find the optimal packet dropping policy, PAQMAN uses a standard value iteration algorithm for MDPs in the simpler case and a function approximator for more involved scenarios to maximize the long-term accumulated reward.

To send a congestion signal to the underlying congestion control, an AQM algorithm uses packet drop or packet marking [9]. For example, packets are dropped under RED whereas they are ECN-marked in a switch deploying REM. While we use packet drop for PAQMAN throughout this paper, it can also be used in conjunction with ECN marking and the corresponding changes in the derivations are straightforward. We highlight this in part (b) of Fig. 1 where the AQM is seen as an intermediate module between the sender and the congestion control mechanism of the network.

Needless to say, the objective of an AQM algorithm shapes its functioning, i.e., the inputs indicating potential congestion and the subsequent rule to send a congestion signal to the underlying congestion control mechanism. While the initial goal of AQM algorithms was to strike an acceptable trade-off between utilization and delay, the aspects of drop rate, jitter and fairness also gained focus over the years. However, there is no standardized evaluation criteria for AQM schemes and the authors in [21] attribute the slow progress of AQM research to this fact. In addition to the performance measures mentioned above, the authors in [9] consider scalability, stability, responsiveness, and robustness to be crucial metrics.
of performance. Scalability focuses on the feasibility of implementation of an AQM algorithm as the number of flows increases, whereas stability measures, for example, the change in queue length as the number of flows varies. Further, the speed of convergence is referred to as responsiveness [22] and robustness signifies the ability of an AQM algorithm to work under diverse network conditions. Thus, robustness necessitates dynamic parameter tuning to suit changing traffic loads, i.e., variation of network parameters. Following the inclusion of varied performance measures, RED was modified to meet the new objectives. For example, authors in [12] introduced SRED to enhance stability, ARED [5] used auto-tuning to increase robustness, and FRED [23] focused on fairness. We refer the reader to Table XIV-XXIV of [9] for a detailed comparison of the heuristic AQM schemes that address certain aspects of performance. Coming back to the issue of dynamic parameter tuning, the paper [24] introduces a general parameter tuning method that can be used in conjunction with any given AQM algorithm. Using the proportion of ECN-Echo (ECE) marked packets in a fixed time interval as congestion indicator, the authors aim to predict this proportion over the next time interval using an Long Short-Term Memory (LSTM) architecture which takes the past values of the proportion as inputs. Independently of the underlying AQM algorithm, the problem of parameter tuning is then formulated as an MDP, where the (discretized) predicted value of the congestion indicator denotes the state of the system and the action set consists of all possible values of the AQM parameter. Taking the measured throughput-to-RTT ratio as the immediate reward, the authors adopt a Q-learning approach [25] to find the optimal AQM parameter value for a given estimate of the congestion predictor in the next time interval.

While delay and throughput are the primary objectives of any AQM algorithm, most of the control theoretic approaches additionally focus on stability, responsiveness, and robustness by analyzing the transient response, the oscillation around the target queue length, and the steady-state error, respectively. However, fairness is usually not measured under control theoretic as well as deterministic optimization approaches. In comparison, we empirically evaluate PAQMAN in terms of delay, throughput, and the rate of convergence to steady-state behaviour. However, we do not evaluate its performance in terms of jitter, responsiveness, scalability, or robustness.

The algorithms that dominate the present-day AQM landscape are PIE and CoDel. They have been compared extensively to each other and to some variants of RED [26], [10]. The former paper considers both performance and scalability under multiplexing. It concludes that CoDel visibly leads to better performance in terms of delays, while the performance of PIE scales well for multiplexed flows. Similarly, the authors in [10] recommend CoDel over PIE after their extensive evaluation over the range of respective default parameters, as the empirical delay distribution under PIE was seen to have a longer tail. Hence, in this work, we evaluate PAQMAN against CoDel and droptail queue as they can be justly considered as AQM and non-AQM benchmarks, respectively.

III. AQM AS AN OPTIMAL DECISION PROBLEM

In the following, we formulate the AQM problem as finding an optimal policy of a Semi Markov Decision Process where the underlying system is essentially an AQM-capable router or switch that carries IP traffic. We recognize that the problem can be framed in various ways depending upon the flow RTT and flow properties. Such distinctions, e.g., with respect to flow RTT can naturally arise during intra-datacenter or inter-datacenter communications as depicted in Fig. 1. Under the scenario of negligible RTT, we allow the arrival process to have a more general form whereas, keeping tractability in sight, a simpler model is adopted for the case with non-negligible RTT. In this section, the former is described in greater detail and we take up the latter in Sect. V.

In our framework where negligible RTT is assumed, the buffer is observed at arrival instants3 and we aim to find out the optimal policy, i.e., whether it is ideal to drop or admit a packet given the state of the system. Apart from the action, the state evolution of the system is influenced by the packet interarrival time and the service time distributions. We formulate the problem using a model of the packet data arrivals that is given by gamma distributed interarrival times where the parameters of the distribution depend on the history. Our choice of distribution for the interarrival times allows us to fit a wide class of observed traffic flows to the model. Further, the service times are assumed to be exponentially distributed. Before formalizing the system description and the framework in general, we emphasize that optimality here is defined in terms of expected long term average reward. The expected long term average reward can be thought of as the long-term accumulation rate of instant rewards where the instant reward can be specified precisely depending upon the intended objective such as a function of throughput and/or delay. Specifically, we formulate the reward function to capture the immediate gain in relative throughput while the given delay threshold is adhered to. Next, we introduce the required notations to formally define the AQM problem:

Notations:

\[ Q_t: \] queue length immediately before \( t \)-th packet arrival,
\[ \alpha: \] common Gamma shape parameter of packet interarrival time distributions,
\[ \beta_t: \] gamma rate parameter of the packet interarrival time immediately before \( t \)-th arrival,
\[ s_t: \] state of the system observed by the packet corresponding to \( t \)-th arrival, given by \((Q_t, \beta_t)\),
\[ \mu: \] service rate of the packets,
\[ A_t: \] action taken upon \( t \)-th arrival, i.e. admit or drop,
\[ P(S_{t+1}|S_t, A_t): \] transition probability to state \( S_{t+1} \) from \( S_t \) given \( A_t \) action was taken,
\[ R(S_t, A_t): \] reward when action \( A_t \) is taken in state \( S_t \),
\[ \tau(S_t, A_t): \] expected transition time when action \( A_t \) is taken in state \( S_t \),
\[ X_t: \] time of \( t \)-th packet arrival.

3We use instant and epoch interchangeably
Further, we denote the state space by $\mathcal{S}$ and the action space by $A = \{0, 1\}$ where 0 and 1 denote admitting and dropping of a packet, respectively. These notations are used across sections with minor variation which is mentioned in respective contexts. 

Looking at the consequence of available actions, we see that admitting a packet causes the queue length to increase by one and the effective arrival rate of the TCP flow that is given as $\beta_t/\alpha$ increases to $f_u(\beta_t/\alpha)$ immediately. Here, the function $f_u$ is dependent upon the exact TCP congestion control algorithm. For example, for an additive increase multiplicative decrease (AIMD) algorithm [4], $f_u(\beta_t/\alpha) = \beta_t/\alpha + a$, for some $a > 0$. For tractability, we assume the shape parameter $\alpha$ remains fixed and vary the rate parameter $\beta_t$ appropriately to reflect this change. This implies that following an admission action, $\beta_{t+1}$ assumes the value $\alpha f_u(\beta_t/\alpha)$ in the next period.

In contrast, packet drops evidently do not change the queue length although the effective arrival rate $\beta_t/\alpha$ drops to $f_d(\beta_t/\alpha)$ immediately where $f_d$ is dependent upon the exact TCP congestion control algorithm. Again, for an AIMD algorithm, $f_d(\beta_t/\alpha) = b\beta_t/\alpha$, for some $0 < b < 1$. As before, we assume the shape parameter remains fixed and take $\beta_{t+1} = b\beta_t$. For the sake of simplicity, we derive our results with the simplest version of AIMD algorithm which uses $a = 1$ and $b = 1/2$. Our results remain valid under a wide class of elementary congestion control functions and can be obtained by replacing the increment and decrement of flow arrival rate with corresponding $f_u$ and $f_d$.

To formalize the state evolution under the given AIMD algorithm, the action $A_t = 0$ causes the queue length to increase to $(Q_t + 1)$ instantaneously and the possible states in the next arrival epoch could be any of the elements of the set: $Q_0 = \{(q, \beta_t + \alpha) : 0 \leq q \leq Q_t + 1\}$. That is, $S_{t+1} \in Q_0$ and $P((q, \beta_t + \alpha)|S_t, 0)$ is the probability that exactly $Q_t + 1 - q$ many packets are served until next packet arrival. Similarly, for the action $A_t = 1$, we have $Q_1 = \{(q, \beta_t/2) : 0 \leq q \leq Q_t\}$ and $P((q, \beta_t/2)|S_t, 1)$ is the probability of serving $Q_t - q$ many packets until the next packet arrival. The transition probabilities to any state outside these designated sets are zero.

Next, we derive the expression for state transition probabilities which dictate the pattern of transition between the system states. These transition probabilities are crucial inputs to the optimal policy derivation process.

A. State Transition Probabilities

Recall that the state system is described as the vector consisting of the current queue length and the arrival rate, i.e., $S_t = (Q_t, \beta_t)$. We are interested in deriving an expression for the transition probabilities $P(S_{t+1}|S_t, A_t)$ as these are inputs to the Bellman operator which iteratively determines the value of each state. The value of each system state in turn determines the policy; see Chap. 7 of [27] for details. To that end, we state the following lemma.

Lemma III.1. For two independent Gamma random variables, $Y_{u,v} \sim \text{Gamma}(u, v)$ and $X_{w,z} \sim \text{Gamma}(w, z)$, \[ P(Y_{u,v} > X_{w,z}) = \frac{\Gamma(u+w-1)}{\Gamma(u)} \left(\frac{v}{v+z}\right)^{u-1} \left(\frac{z}{v+z}\right)^w, \] where $u > 1$. This implies \[ P(Y_{u,v} > X_{w,z}) = \sum_{k=0}^{u-1} \frac{\Gamma(k+w)}{\Gamma(k+1)} \left(\frac{v}{v+z}\right)^k \left(\frac{z}{v+z}\right)^w. \]

Proof. The proof is given in Sect. VIII.

Now, we can derive the transition probabilities as follows.

Theorem III.2. Given the state $S_t = (Q_t, \beta_t)$ at $t$-th arrival epoch, $Q_t$ and $\beta_t$ being the queue length and arrival rate, respectively, the transition probabilities to the state $S_{t+1} = (Q_{t+1}, \beta_{t+1})$ in the next epoch under the action $A_t = 0$ are given by

\[ P((Q_{t+1}, \beta_{t+1})|S_t, 0) = 1_{\beta_t+1 = \beta_t + \alpha} \frac{\Gamma(Q_t + 1 - Q_{t+1} + \alpha)}{\Gamma(Q_t + 2 - Q_{t+1})} \left(\frac{\mu}{\mu + \beta_{t+1}}\right)^{\alpha}, \]

for $1 \leq Q_{t+1} \leq Q_t$ and

\[ P((0, \beta_{t+1})|S_t, 0) = 1_{\beta_t+1 = \beta_t/2} \left(1 - \sum_{k=0}^{Q_t - Q_{t+1}} \frac{\Gamma(k + \alpha)}{\Gamma(k+1)\Gamma(\alpha)} \left(\frac{\mu}{\mu + \beta_{t+1}}\right)^k \left(\frac{\beta_{t+1}}{\mu + \beta_{t+1}}\right)^{\alpha}\right). \]

Proof. We start by observing that $P((q, \beta_{t+1})|S_t, 0)$ denotes the probability of the event that exactly $Q_t + 1 - q$ packets are served between two packet arrivals for $1 \leq q \leq Q_t + 1$. Now, at least $n$ packets are served between two arrivals if and only if the corresponding service time $Y_{n,\mu}$ is less than the respective interarrival time $X_{\alpha,\beta_{t+1}}$. The change in the arrival rate parameter is given by the fact that following an action on the $t$-th packet arrival, the arrival rate parameter is instantaneously changed to $\beta_{t+1} = \beta_t + \alpha$ which determines the distribution of the next interarrival time. Thus, given $\beta_{t+1} = \beta_t + \alpha$, for $1 \leq q \leq Q_t + 1$,

\[ P((q, \beta_{t+1})|S_t, 0) = P(Y_{Q_t+2-q,\mu} > X_{\alpha,\beta_{t+1}}) - P(Y_{Q_t+1-q,\mu} > X_{\alpha,\beta_{t+1}}) \]

\[ = \frac{\Gamma(Q_t + 1 - q + \alpha)}{\Gamma(Q_t + 2 - q)} \left(\frac{\mu}{\mu + \beta_{t+1}}\right)^{\alpha} \left(\frac{\beta_{t+1}}{\mu + \beta_{t+1}}\right)^{\alpha}. \]

For $q = 0$, we observe that no more than $Q_t + 1$ packets can be served between two arrivals. Hence, given $\beta_{t+1} = \beta_t + \alpha$, $P((0, \beta_{t+1})|S_t, 0) = P(Y_{Q_t+1,\mu} > X_{\alpha,\beta_{t+1}})$

\[ = 1 - P(Y_{Q_t+1,\mu} > X_{\alpha,\beta_{t+1}}) = 1 - \sum_{k=0}^{Q_t - 1} \frac{\Gamma(k + \alpha)}{\Gamma(k+1)\Gamma(\alpha)} \left(\frac{\mu}{\mu + \beta_{t+1}}\right)^k \left(\frac{\beta_{t+1}}{\mu + \beta_{t+1}}\right)^{\alpha}, \]

which completes the proof.

\(\square\)
The transition probabilities for the case when a packet is dropped are derived similarly. Note that only the rate parameter of the interarrival time variable changes to \( \beta_t/2 \) due to the drop and the possible queue length in the next decision epoch is at most \( Q_t \). Thus,

\[
P((Q_{t+1}, \beta_{t+1})|S_t, 1) = \mathbb{I}_{\beta_{t+1} = \beta_t/2} \frac{\Gamma(Q_t - Q_{t+1} + \alpha)}{\Gamma(Q_t + 1 - Q_{t+1}) \Gamma(\alpha)} \left( \frac{\mu}{\mu + \beta_t} \right)^{Q_t - Q_{t+1}} \left( \frac{\beta_t}{\mu + \beta_t} \right)^\alpha,
\]

for \( 1 \leq Q_{t+1} \leq Q_t \) and

\[
P((0, \beta_{t+1})|S_t, 1) = \mathbb{I}_{\beta_{t+1} = \beta_t/2} \left( 1 - \sum_{k=0}^{Q_t-1} \frac{\Gamma(k + \alpha)}{\Gamma(k + 1) \Gamma(\alpha)} \left( \frac{\mu}{\mu + \beta_t} \right)^Q \left( \frac{\beta_t}{\mu + \beta_t} \right)^\alpha \right).
\]

Unlike the MDP framework, the expected time between two decision epochs plays a significant role in the SMDP framework that forms the basis of our analysis. Since drop/admit action affects the packet sending rate of TCP, the expected time between two arrival events varies. We see that the expected transition times are: \( \tau(S_t, 0) = \alpha/(\beta_t + \alpha) \) and \( \tau(S_t, 1) = 2\alpha/\beta_t \), following the AIMD principle of TCP. We occasionally abbreviate \( \tau(S_t, A_t) \) as \( \tau_t \). A brief sketch of state transitions in this framework is provided in Fig. 2.

**B. Reward function**

Equipped with the transition probabilities, we now focus on choosing the reward function which shapes the objective of the policy learning process. The reward function represents the immediate incentive and the policy learning process aims to maximize the average accumulated reward in the long run. To align the learning process with our objective of maximizing throughput given a user-defined delay constraint, we use the reward function described in the following. The reward function penalizes heavily whenever the delay constraint is breached and provides immediate incentive that equals the change in the square rate of throughput. The rationale here is to reflect the relative change rather than its absolute value. Formally,

\[
R(S_t, 0) = -M \mathbb{I}_{(Q_{t+1})/\mu > \eta} \left( \sqrt{\frac{\beta_t + \alpha}{\alpha}} - \sqrt{\frac{\beta_t}{\alpha}} \right),
\]

\[
R(S_t, 1) = -M \mathbb{I}_{(Q_{t+1})/\mu > \eta} \left( \sqrt{\frac{\beta_t}{2\alpha}} - \sqrt{\frac{\beta_t}{\alpha}} \right),
\]

where \( \eta \) denotes a target delay threshold and \( M \) is a large penalty value for breaching it. Thus, the reward function adopts the goal of state-of-the-art heuristic AQM algorithms that take a target delay as a single input variable [28].

Equipped with the framework above and the reward function, we focus on finding the AQM policy using the value-iteration algorithm for SMDP’s. The AQM policy runs on the buffer and takes the system state \((Q_t, \beta_t)\), essentially, the current buffer filling and the current arrival rate parameter as input and provides the action, i.e., whether to drop or admit to maximize accumulated predefined reward.

Formally, a policy \( \pi \) is defined as a mapping \( \pi : S \mapsto A \), i.e., given the system state in terms of buffer filling and arrival rate, the policy returns one of the possible actions, i.e., admission or dropping. Let the accumulated reward up to time \( x \) be denoted by \( Z(x) \), i.e., \( Z(x) = \sum_{t=1}^{x} R(S_t, A_t) \), where \( V(x) = \max \{ t : X_t \leq x \} \), and \( X_t \) is the arrival time of the \( t \)-th packet. The expected average long-term reward is then given by:

\[
g_t(\pi) = \lim_{x \to +\infty} \frac{1}{x} E_{t, \pi}[Z(x)],
\]

where \( i \) denotes the fact that the initial state was \( S_t \) and \( \pi \) is the used policy.

As mentioned, our objective is to find a policy \( \pi \) which maximizes \( g \) for each state \( S_t \). To that end, we convert the SMDP problem to a discrete-time MDP using the data transformation method [27] by appropriately scaling the rewards and transforming the transition probabilities. This is due to the fact that an SMDP with average reward criterion can be converted for the purpose of solving it to a discrete time MDP where the rewards are modified to reward accumulation rate and the transition probabilities are adjusted to reflect the changes from continuous time to discrete time transformation. Subsequently, we use the value iteration method for discrete time MDP’s to determine the optimal policy. The transformed reward function \( \bar{R} \) and transition probability \( \bar{P} \) are respectively given by:

\[
\bar{R}(S_t, A_t) = \frac{R(S_t, A_t)}{\tau(S_t, A_t)},
\]

\[
\bar{P}(S_{t+1}|S_t, A_t) = P(S_{t+1}|S_t, A_t) \frac{\tau}{\tau(S_t, A_t)}, \quad S_{t+1} \neq S_t,
\]

\[
\bar{P}(S_t|S_t, A_t) = P(S_t|S_t, A_t) \frac{\tau}{\tau(S_t, A_t)} + 1 - \frac{\tau}{\tau(S_t, A_t)},
\]

where \( \tau \) is chosen such that \( 0 < \tau \leq \min_{s,a} \tau(s, a) \).
IV. AQM FOR UNKNOWN TRAFFIC FLOW PROPERTIES

In this section, we consider the problem of finding the optimal AQM policy for unknown traffic flow properties. As established in the Sect. III, the optimal AQM policy requires accurate knowledge of the current traffic arrival rate which is determined by the initial arrival rate and the series of drop or admit actions. Taking the current arrival rate and the queue filling as inputs, the AQM policy then prescribes whether it is optimal to drop or admit an incoming packet. Therefore, to propose a policy under unknown arrival characteristics, we first need to infer the respective parameters of the arrival flows. However, estimating the model parameters while learning the optimal policy introduces the classical problem of dual control [29]. However, this can be alleviated by an exploration-exploitation Bandit-heuristic [30], which is computationally tractable.

A. Estimation of Traffic Flow Parameters

Next, we infer the parameters governing the arrival process at data packet arrivals. To that end, we derive Maximum Likelihood Estimates (MLE) of arrival shape \(\alpha\) and arrival rates \(\beta_i\) as used by the parametric models in Sect. III. Let \(\{X_n\}\) and \(\{A_n\}\) denote the sequence of arrival times and the actions respectively. Further, the corresponding packet interarrival times are denoted by \(W_t\), i.e., \(W_t = X_{t+1} - X_t\). Recall that \(A_t = 1\) signifies a packet drop and \(A_t = 0\) denotes packet admission. We take a tractable model of Gamma interarrival times, i.e.

\[
W_t | \alpha, \beta_t \sim \text{Gamma}(\alpha, \beta_t),
\]

following the AIMD congestion control principle of TCP. Given the sequence of interarrival times \(W_t = \{W_n\}_{n=1}^k\) and actions \(A = \{A_n\}_{n=1}^k\), we aim to estimate the common shape parameter \(\alpha\) and the initial rate parameter \(\beta_t\) to derive the transition probabilities \(P(S_{t+1} | S_t, A_t)\) described in Sect. III to find the optimal policy. The likelihood of the parameters for the observed sequence \((W_t, A)\) is given by

\[
L(\alpha, \beta_t | W, A) = \frac{\prod_{n=1}^k \beta_n^\alpha (\prod_{n=1}^k W_n)^{\alpha-1} e^{-\sum_{n=1}^k \beta_n W_n}}{(\Gamma\alpha)^k}.
\]

Note that the RHS is simply product of Gamma densities and the dependence on \(A\) is implicit. Now, taking logarithm we obtain the log-likelihood \(l(\alpha, \beta_t)\) as

\[
l(\alpha, \beta_t) = \alpha \sum \log \beta_n + (\alpha - 1) \sum \log W_n - \sum \beta_n W_n - k \log(\Gamma\alpha).
\]

To optimize the likelihood we obtain

\[
\frac{\partial l}{\partial \alpha} = \sum \log \beta_n + \sum \left( \frac{\alpha}{\beta_n} - W_n \right) \frac{\partial \beta_n}{\partial \alpha} \quad \text{and} \quad \frac{\partial l}{\partial \beta_1} = \sum \left( \frac{\alpha}{\beta_n} - W_n \right) \frac{\partial \beta_n}{\partial \beta_1},
\]

where

\[
\frac{\partial \beta_n}{\partial \alpha} = \left( 1 + \frac{\partial \beta_{n-1}}{\partial \alpha} \right) ^{-1} \left( \frac{1}{2} \left( \frac{\partial \beta_{n-1}}{\partial \alpha} \right) ^{-1} \right) ^{n-1},
\]

\[
\text{with} \quad \frac{\partial \beta_n^2}{\partial \alpha} = 1_{\{A_1 = 0\}} \quad \text{and} \quad 0^0 = 1, \text{ by convention.}
\]

Further,

\[
\frac{\partial \beta_n}{\partial \beta_1} = \left( \frac{1}{2} \right) ^{-1} \frac{\partial \beta_{n-1}}{\partial \beta_1}, \quad n = 2, 4, \ldots, k.
\]

To get the maximum likelihood estimates \(\hat{\alpha}\) and \(\hat{\beta}_1\), we numerically find the zero point of (6) that maximizes (5).

B. Inference under unknown TCP Congestion Control

In case the exact congestion control algorithm of TCP is not known beforehand, we assume the effective arrival rate \(\beta_{t+1}/\alpha\) in the next epoch is changed according to a polynomial function (e.g., TCP CUBIC [31]) of the present effective arrival rate \(\beta_t/\alpha\). We can then rewrite (4) as

\[
W_t | \alpha, \beta_t \sim \text{Gamma}(\alpha, \beta_t),
\]

\[
\frac{\beta_{t+1}}{\alpha} = \left( f_u \left( \frac{\beta_t}{\alpha} \right) \right)^{1-A_t} \left( f_d \left( \frac{\beta_t}{\alpha} \right) \right)^{A_t}, \quad (7)
\]

where \(f_u\) and \(f_d\) are polynomials governing the change of the arrival rate following an admit and drop, respectively. The log-likelihood has a similar form as shown above, although estimates of the coefficients of the polynomials, together with \(\alpha\) and \(\beta_1\), are required. This can either be done numerically or by calculating partial derivatives explicitly, similar to (6). Subsequently, one can find their zeros corresponding to the maxima, albeit with additional equations for \(\partial f / \partial \beta_j\), for each coefficient \(\beta_j\). For example, if \(f_u\) is a cubic polynomial and \(f_d\) is a linear function, this method will lead to calculation of six additional partial derivatives.

Using the parameter estimates along with the observed series of drop or admit actions, we infer the current arrival rate \(\beta_t\) which together with the current queue filling \(Q_t\) describes the system state. We thus use the AQM policy similar to Sect. III to find the optimal action.

V. AQM UNDER NON-NEGligible RTT

In this section, we focus on the case where the RTT between the sender and the receiver of a flow is non-negligible. As mentioned in Sect. III, we trade model flexibility for tractability for this case. This tractability is essential given the case where the switch acts on multiple concurrent flows. We derive our policy for the single flow case in Sect. V-A whereas the multiple flow case is described in Sect. V-B. As before, our objective is to maximize the total throughput subject to a user-defined delay constraint.
Fig. 3: Arrival times $X_i$ and decision times $T_i$ are shown on the time axis. The first job arrives at time $X_1$ with rate $\beta_1$. After $X_1$, $X_2$ is the only arrival in $(X_1, X_1 + r]$ and is assumed to have no bearing on the arrival rate. We see $X_3 = T_2$ and the arrival rate is changed to $\beta_2$ at $T_2$ according to the action taken at $X_1$. The rate again changes to $\beta_3$ at $T_3 = X_6$.

A. Single Flow

In the following, we assume that both packet interarrival times and the packet service times are exponentially distributed. Further, the RTT is denoted as $r$ and we have $r > 0$. We assume that an estimate of the exact flow RTT is known to the receiver, which approximately equals $r/2$. Thus an action takes effect at the switch only after a time period $r$, and all arrivals meanwhile are admitted given there is enough room in the output buffer. For tractability, we further assume that the arrivals and the corresponding admit actions in this period have no direct consequence on the way arrival rate is changed. Although an approximation, we expect this assumption to have no major consequence and take advantage of the corresponding simpler state representation.

Recall that we denote the arrival times as $\{X_i\}_{i \in \mathbb{N}}$. We take a decision whether to admit or drop on the first packet arrival at time $X_1$. Thereafter all arrivals in the interval $[X_1, X_1 + r)$ are admitted subject to enough room in the buffer and the first arrival after $(X_1 + r)$ can be potentially dropped. The arrival rate changes at this new arrival after $(X_1 + r)$ according to the action taken at time $X_1$. Subsequently, the decision making process goes dormant for another time period of length $r$ and the drop/admit decision is taken again at the first arrival after this period. The change in arrival rate also takes place at this time. An illustrative example is given in Fig. 3. We call the arrival epochs where a drop decision can be possibly made as decision times and denote them as $\{T_i\}_{i \in \mathbb{N}}$. As explained, $T_i$’s ($i \geq 2$) are also the time points where the arrival rate changes according to the action taken at $T_{i-1}$.

The service times are assumed independently and identically distributed (iid) as $\text{Exp}(\mu)$ and given the relevant arrival rate parameter $\beta_k$, we have $X_{i+1} - X_i \sim \text{Exp}(\beta_k)$, $i \in \mathbb{N}$, for some $k \in \mathbb{N}$. Note that, unlike Sect. III, the arrival rates do not change at every arrival. Rather the change happens at the decision times owing to the delay factor of $r$ and the fact that intermediate arrivals do not influence the arrival rate directly. Focusing on the inter-decision time, we observe that the residual time to the next decision time after one RTT $r$ is also distributed as $\text{Exp}(\beta_j)$, for some $j \in \mathbb{N}$, following the memoryless property of exponential distribution. Further, the change in the arrival rate $\beta$ happens only at the decision times. Therefore, we know for the inter-decision times

$$T_{i+1} - T_i | \beta_i \sim r + \text{Exp}(\beta_i), \ i \in \mathbb{N} , \quad (8)$$

as $\beta_i$ is the arrival rate during the interval $[T_i, T_{i+1})$.

To derive the optimal decision at times points $T_i$, we adopt the SMDP framework similar to the derivations of Sect. III. We retain much of our notations from Sect. III with the exception of $\beta_i$ which denotes the rate parameter of packet interarrival times at $t$-th decision epoch. In contrast to Sect. III, $\beta_i$ in this case is already known at $(t - 1)$-th arrival epoch and, hence, is a valid input to the policy.

Note that in the time interval $(T_i, T_i + r]$, the queue length grows as the population of a birth-death process, where the transition matrix $P(s)$ over a time interval of length $s$ is given by $e^{sG}$. Here, $G_j$ is the $(L + 1) \times (L + 1)$ intensity matrix of the process in the relevant interval where $L$ is the buffer size. It is given by

$$G_j = \begin{bmatrix} -\beta_j & \beta_j & 0 & \cdots & \cdots \\ \mu & -\mu - \beta_j & \beta_j & 0 & \cdots \\ 0 & \mu & -\mu - \beta_j & \beta_j & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & \cdots & \mu & -\mu \end{bmatrix} , \quad (9)$$

for some $j$. Further, in the interval $(T_i + r, T_{i+1})$ there can only be packet departures and hence the transition matrix for any interval of length $s$ is given by $e^{sG}$ where

$$G = \begin{bmatrix} 0 & 0 & 0 & \cdots & \cdots \\ \mu & -\mu & 0 & \cdots & \cdots \\ 0 & \mu & -\mu & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & \cdots & \mu & -\mu \end{bmatrix} . \quad (9)$$

The following lemma helps us derive the transition matrix between two consecutive decision epochs.

**Lemma V1.** If all eigenvalues of the matrix $A$ are positive,

$$\int_0^\infty e^{-uA} du = A^{-1} .$$

**Proof.** This well-known result follows from the fact that $\lim_{u \to \infty} e^{-tA} = 0$ for a positive definite matrix $A$. \qed

We now derive the transition matrix for the queue length over the interval between two consecutive decision epochs. The first step towards this is to find the transition probabilities for the queue length as the change of arrival rate following a transition is deterministic.

**Theorem V2.** The transition probability matrix $P_i$ for queue length $Q$ over the interval $(T_i, T_{i+1})$ is given by

$$P_i = \beta_i e^{sG} (\beta_i I - G)^{-1} .$$
We have used the facts that $\forall (P_H, P_i)$ implying
\[
P_t = e^{G \beta_t} e^{-\beta_t t} dt = \beta_i e^{G \beta_t} e^{-\beta_t t} dt.
\]
We have used the facts that $e^{-t} I = e^{-c t}$ and $cI$ commutes with any matrix of similar dimension $\forall c \in \mathbb{R}$. Now,
\[
\beta_i I - G = \begin{bmatrix} \beta_i & 0 & 0 & \cdots & 0 \\ -\mu & \mu + \beta_i & 0 & \cdots & 0 \\ 0 & -\mu & \mu + \beta_i & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & \mu + \beta_i \end{bmatrix}.
\]
Therefore, by Gershgorin’s theorem [32], all eigenvalues of $\beta_i I - G$ lie in the following closed balls: $B(\beta_i, 0), B(\mu + \beta_i, \mu)$. Hence, by lemma V.1
\[
P_t = \beta_i e^{G \beta_t} (\beta_i I - G)^{-1}.
\]
Observe that $P(Q_{t+1}|Q_t, 1)$ corresponds to the $Q_t$-th row of the transition matrix over the interval $[T_i, T_{i+1})$. Under $A_t = 1$, the transition matrices over intervals $[T_i, T_{i+1})$ and $(T_{i+1}, T_{i+2})$ are identical as the incoming packet is dropped. Hence, $P(Q_{t+1}|Q_t, 1)$ is given by the $(Q_t, Q_{t+1})$-th element of $P_t$. However, under $A_t = 0$, the queue length jumps to $\max(Q_{t+1}, 1)$ at time $T_t$ and the $(Q_t, \max(Q_{t+1}, 1), L)$-th element represents $P(Q_{t+1}|Q_t, 0)$. Finally, the state transition probabilities can be expressed as follows:
\[
P(S_{t+1}|S_t, 0) = P(Q_{t+1}|Q_t, 0) \mathbb{I}_{\beta_{t+1} = \beta_t + 1},
\]
\[
P(S_{t+1}|S_t, 1) = P(Q_{t+1}|Q_t, 1) \mathbb{I}_{\beta_{t+1} = \beta_t / 2}.
\]
Further, similar to (1), we define the reward function as:
\[
R(S_t, 0) = -M \mathbb{I}_{\{Q_t < \mu\}} + \sqrt{\beta_t + 1 - \sqrt{\beta_t}}.
\]
\[
R(S_t, 1) = -M \mathbb{I}_{\{Q_t \geq \mu\}} + \sqrt{\beta_t / 2 - \sqrt{\beta_t}}.
\]
Also, it is immediate from (8) that the expected inter-decision time $\tau(S_t, A_t) = r + 1/\beta_t$. Equipped with the transition probabilities, reward function and expected transition times, we can now transform the SMDP problem into a discrete time MDP problem like Sect. III and perform value iteration on the transformed MDP to find out the best policy.

B. Multiple Flows

In this section, we extend our SMDP formulation for the single flow case to obtain an optimal AQM policy in presence of concurrent flows with different RTT. We assume that the flow RTT’s are known or estimated at the switch running AQM.

We define the decision times for a flow of interest as the instances of the first packet arrivals that are at least an RTT apart. Simply put, when an AQM decision is carried out for one packet the subsequent packets within on RTT window are admitted. The next AQM decision is then taken for the first packet after the RTT window has passed. We will see in the following that, unlike the single flow case, the time since the last decision epoch involving a packet from each flow plays an important role in formalizing the framework for multiple flows.

Let us denote the packet arrival times for the $j$-th flow as $\{X_{js}\}$, with packet index $s \in \mathbb{N}$ and, similar to the single flow case, we assume for the $j$-th flow
\[
X_{j(t+1)} - X_{j(t)} | \beta_{jk} \sim \text{Exp}(\beta_{jk}), \ i \in \mathbb{N},
\]
for some $k \in \mathbb{N}$, $j \in \{1, 2, \ldots, n\}$ where $n$ denotes the number of concurrent flows. The decision times are denoted as $\{T_{js}\}_{s \in \mathbb{N}}$ and we see that the mean arrival rate for each flow remains constant between two decision epochs. This is because we assume that the AQM signal reaches the sender after an RTT.

Let $\beta_t = (\beta_{t1}, \beta_{t2}, \ldots, \beta_{tn})^T$ be the parameter vector of packet interarrival times at the $t$-th decision epoch and $r = (r_1, r_2, \ldots, r_n)^T$ denote the vector of flow RTTs. Further, we express the age vector at $t$-th decision epoch by $u_t = (u_{t1}, u_{t2}, \ldots, u_{tn})^T$ where $u_{jt}$ denotes the time since the last decision epoch concerning a packet from the $j$-th flow. In absence of a last decision time concerning flow $j$, i.e., before the arrival of the first packet from flow $j$, we define $u_{jt} = 0$. Observe that if the $t$-th decision epoch involves a packet from flow $j$, we have $u_{jt} = 0$. An illustration of the decision epochs for the case of three concurrent flows is provided in Fig. 4.

At the $t$-th decision epoch, let $Y_{jt}$ denote the time until the next AQM decision on flow $j$. In other words, $Y_{jt}$ indicates the time until the arrival of a packet from flow $j$ that is at least one RTT after the last decision epoch from flow $j$. Given the RTT $r_j$ of flow $j$, we can write
\[
Y_{jt} = W_j + \max(0, r_j - u_{jt}),
\]
where $W_j$ denotes the residual time to the packet arrival from $j$-th flow after $r_j$ amount of time has passed since the last decision epoch involving flow $j$. By the memoryless property of exponential distribution we know that $W_j \sim \text{Exp}(\beta_{jt})$, for $j \in \{1, 2, \ldots, n\}$. Thus, the time until the $(t + 1)$-st decision epoch across all flows is given by
\[
Y_t = \min_{j \in \{1, 2, \ldots, n\}} Y_{jt}.
\]

Therefore, we can write the inter-decision time $T_{t+1} - T_t = Y_t$ as
\[
Y_t | \beta_t, u_t \sim \min_{j \in \{1, 2, \ldots, n\}} \{W_j + \max(0, r_j - u_{jt})\}, \ t \in \mathbb{N}.
\]
Equipped with the description above we now focus on formalizing the SMDP to derive the optimal AQM action corresponding to any given switch state. Apart from the individual flow arrival rates and the queue length, the state description in this case includes the age vector $u_t$ and the flow index of the incoming packet. To formulate the AQM problem as an SMDP, we use the notations introduced in Sect. III with the following exceptions:

$\beta_t$: flow-wise rate parameter vector of packet interarrival times at the $t$-th decision epoch,
Fig. 4: Illustration of decision times when three flows pass through the switch buffered link. The first decision time is the first arrival, which takes place at \( T_1 = X_{21} \). Next decision time concerning flow 2 cannot be earlier than \( X_{21} + r_2 \). The first arrival from flow 1 or 3 can potentially be the next decision time, which turns out to be \( T_2 = X_{11} \). Likewise, the decision process on flow 1 goes dormant in \( (X_1, X_1 + r) \). Note that decision times are the times where the arrival rates change for the associated flow. The interarrival time \( W_{ij} \) is exponentially distributed with a parameter that was fixed at the last decision time concerning flow \( i \), according to the action taken at the second last decision time on that flow.

\( u_t \): flow-wise age vector, i.e., time since last decision epoch involving a packet from the concerned flow at the \( t \)-th decision epoch,

\( S_t \): state of the system observed by the packet corresponding to the \( t \)-th decision epoch, given by \((j, Q_t, \beta_t, u_t)\) where \( j \) denotes the flow index of the packet for which an AQM decision is to be taken.

With the flow-wise age \( u_t \) being a continuous variable we derive next the corresponding transition kernels. Let \( B = \{j, Q_{t+1}, \beta_{t+1}, U\} \) denote the set that the system state at \( t + 1 \), i.e., \( S_{t+1} \), can possibly belong to. As usual, we restrict our focus on \( U \in B(\mathbb{R}^n) \), the Borel sigma algebra over \( \mathbb{R}^n \). Due to the known fact that right semi-closed rectangles generate \( B(\mathbb{R}^n) \), it is sufficient to consider the sets \( U \) having the form \( U = I_1 \times I_2 \times \ldots \times I_n \) where each \( I_j \) is a right semi-closed interval.

Going back to (12), we see that the value of the age vector at the decision epoch \((t+1)\) is given by \( u_t + Y_t \). This motivates us to define the translates

\[ I_j' = \{x : x + u_{jt} \in I_j\}, \quad j \in [n], \]

which enable expressing the transition kernel as follows:

\[
\begin{align*}
\mathbb{P}(S_{t+1} \in B | S_t, 0) &= 1_{\beta_{t+1} = \beta_t + c_t}, \\
\mathbb{P} \left( Y_t \in \bigcap_{k \in [n]} I_k', Y_t = Y_{jt}, V_t = Q_{t+1} - Q_t | A_t = 0 \right), \\
\mathbb{P}(S_{t+1} \in B | S_t, 1) &= 1_{\beta_{t+1} = h(\beta_t, i)}, \\
\mathbb{P} \left( Y_t \in \bigcap_{k \in [n]} I_k', Y_t = Y_{kt}, V_t = Q_{t+1} - Q_t | A_t = 1 \right),
\end{align*}
\]

where \( V_t \) denotes the change in queue length in the interval \([T_t, T_{t+1})\) and \( h(x, i) = (x_1, x_2, \ldots, x_i/2, \ldots, x_n) \) and \( c_t \) denotes the unit vector whose \( i \)-th coordinate equals 1. Essentially, the indicator function involving \( \beta_t \) in the RHS of (13) denotes the fact that the arrival rate can only change to specific values which are determined by the drop or admit action and the AIMD principle of TCP. Further, \((t+1)\)-th decision epoch involves flow \( j \) iff the corresponding inter-decision time is \( Y_t = Y_{jt} \). Finally, the age vector \( u_{t+1} \) in the next epoch belongs to the set \( U \) iff \( Y_t \in I_k - u_{kt}, 1 \leq k \leq n \). We further introduce the following shorthand notations:

\[
\bigcap_{j \in [n]} I_j' = \{a, b\}, \quad \bigcap_{j \in [n]} I_j \neq \emptyset, \\
\max(0, r_j - u_{jt}) := c_j, \quad j \in [n].
\]

Given a decision at time zero we use \( c_j \) to denote the time span during which there cannot be a decision concerning flow \( j \). In particular, this time span equals the RTT as \( c_t = r_t \), since \( S_t = \{i, Q_t, \beta_t, u_t\} \). Now let's order the time spans as \( c_{i_1} \leq c_{i_2} \leq \cdots \leq c_{i_n} \). Evidently, for \((a, b) \subset (-\infty, c_{i_j})\), we have for the time to the next decision of the flow \( i_j \) that \( \mathbb{P}(Y_t \in (a, b)) = 0 \). Further, for \((a, b) \subset (c_{i_j}, c_{i_{j+1}})\) we can express the probability of the time to the next decision falling in this time
span as

\[
P\left(Y_i \in (a, b], Y_i = Y_{jt}, V_i = Q_{t+1} - Q_t\right)
= \int_a^b P\left(Y_i = y, V_i = Q_{t+1} - Q_t\right) dF_{Y_i}(y)
= \sum_{j \in \{i_1, \ldots, i_l\}} \int_a^b \prod_{k \leq j, \Delta_k \neq j} P\left(Y_{ik} > y\right) P\left(V_i = Q_{t+1} - Q_t\right) dF_{Y_i}(y)
= \sum_{j \in \{i_1, \ldots, i_l\}} \int_a^b \prod_{k \leq j} P\left(Y_{ik} > y\right) P\left(V_i = Q_{t+1} - Q_t\right) dF_{Y_i}(y).
\]

(14)

Note that the indicator function \( \mathbb{1}_{j \in \{i_1, \ldots, i_l\}} \) indicates the fact that in order to have the flow index of the packet involving the next decision epoch as \( j \), \( c_j \leq c_i \) should hold. Further, the first term in the integrand corresponds to the fact that given \( Y_{jt} = y \), the events \( \{Y_{ik} = y\} \) and \( \{Y_{ik} > y\ \forall k \leq l, \Delta_k \neq j\} \) are equivalent. Now, to evaluate the expression in (14), we notice that the queue length of the underlying process grows like the population of a birth-death process where the death rate is always \( \mu \) subject to a positive queue length. The birth rate, however, depends upon the time interval. Observe that there cannot be any packet arrival from \( i_k \)-th flow once the time since the last decision epoch exceeds \( c_{ik} \). Thus, the \((L+1) \times (L+1)\) intensity matrix in the interval \( \{c_{ik-1}, c_{ik}\}, 1 \leq k \leq n \) is given by

\[
H_{ik} = \begin{bmatrix}
-\mu & b_k & 0 & \ldots & \ldots \\
\mu & -\mu - b_k & b_k & 0 & \ldots \\
0 & \mu & -\mu - b_k & b_k & \ldots \\
\ldots & \ldots & \ldots & \ldots & \mu - \mu \\
\end{bmatrix},
\]

with the sum of flow rates \( b_k = \sum_{m \geq k} \beta_{im} \) and \( c_{i0} = 0 \) by definition. Further, the intensity matrix in the interval \( \{c_{in}, \infty\} \) is given by \( G \) from (9). The different intensity matrices for the arrivals in the different time spans after the last AQM decision is illustrated in Fig. 5.

Replacing \( P(V_i = Q_{t+1} - Q_t\|Y_i = y) \) in (14) by the corresponding transition matrix and denoting the respective transition kernel for \( P(Y_i \in (a, b], Y_i = Y_{jt}, V_i = Q_{t+1} - Q_t) \) as \( G_j(a, b) \), we obtain the formulation (*)

Here, the corresponding probability under the drop action \( A_t = 1 \) is given by the \((Q_t, Q_{t+1})\)-th element of the kernel whereas the \((Q_t, \max(Q_t + 1, L))\)-th gives the required probability under the admit action \( A_t = 0 \). Finally, similar to the single flow case, we define the reward function \( R(S_t, A_t) \) as

\[
R(S_t, 0) = -M \mathbb{1}_{\{Q_t + 1 > \mu > \eta\}} + \left(\sqrt{\beta_j + 1} - \sqrt{\beta_j}\right),
\]

\[
R(S_t, 1) = -M \mathbb{1}_{\{Q_t > \mu > \eta\}} + \left(\sqrt{\beta_j/2} - \sqrt{\beta_j}\right).
\]

(15)

To derive the optimal policy, we can now simulate the trajectory of the system according to (14) and use a function approximator, e.g., DQN [33] to learn the Q-values for each state-action pair. Subsequently, the optimal policy is given by the action with higher Q-value for each state. The advantage of our approach over model-free learning is that it requires a lot less data to reliably predict the optimal action.

VI. EVALUATION

In this section, we evaluate PAQMAN under different network scenarios using simulations. We first focus on the case where the switch deals with a single flow having negligible RTT. The evaluation under non-negligible RTT is subsequently taken up in Sect. VI-B. Throughout our simulations, PAQMAN aims to achieve a delay shorter than the target delay \( \eta = 50 \) ms while optimizing the throughput. Recall from (1) and (11) that the reward function combines individual delay and throughput objectives. The penalty amount in the reward function for breaching this delay is fixed at \( M = 10^6 \). To make comparisons fair, we set the target delay parameter of CoDel to the same target delay, wherever applicable. All other internal CoDel parameters are left unchanged.

While comparing the performance of different policies, we look at the empirical stationary behaviour of the corresponding system. Here, we consider the behavior of long-lived flows. To derive the stationary behaviour, we conduct 200 simulation runs. Each run spans across \( 5 \times 10^4 \) packet arrivals and the run starts with an arrival rate equal to the link service rate. To generate comparison plots, we subsequently time-average the
We first plot the derived policies using the inputs from (3) for two different average service rates. The simulation setup for the first scenario corresponds to a service rate \( \mu = 5 \text{ Mbit/s} \), while the second is given by \( \mu = 10 \text{ Mbit/s} \). Further, we calculate the policy for arrival rates in the range \([0.01, 12]\) Mbit/s, simulated using a Gamma distributed packet interarrival times with fixed shape parameter \( \alpha = 1.5 \). For our simulations, we only change the rate parameter \( \beta \) of the Gamma random variable to reflect the impact of the drop/admit action on the effective arrival rate \( \beta/\alpha \). The resulting policies are shown in Fig. 6, where an incoming packet should be dropped if the current state of the switch, i.e., buffer filling and the packet arrival rate belong to the dark region. As expected, a lower service rate at the switch entails more aggressive packet drops reflected by the difference in Fig. 6a and 6b. The non-trivial effect of flow arrival rate on the policy is also noteworthy.

Next, we compare PAQMAN to CoDel and droptail queues. As mentioned earlier, the target delay parameter of CoDel is set to the delay threshold for PAQMAN. The evaluation is shown in Fig. 8, where the policy from Fig. 6b is used to generate the left subplot. We plot the system state, given by the (i) current buffer filling and (ii) flow arrival rate, time-averaged over multiple runs. Our plots suggest that, in stationarity, PAQMAN results in an arrival rate that is comparable to CoDel, although the queue length appears to be much shorter. This immediately translates to the fact that PAQMAN yields equivalent stationary throughput while keeping the delay much shorter. As already known, the droptail policy in Fig. 6c generates near-perfect utilization at the cost of a longer delays.

### B. Non-negligible RTT

Next, we consider the case when the flow RTT is non-negligible. We assume throughout this subsection that the RTT is estimated by (or known to) the switch. We follow the method from Sect. V to derive PAQMAN under each setting and accordingly simulate the system. As before, we compare the resulting performance with CoDel and droptail queues.

In Fig. 7, we illustrate PAQMAN under low and high RTTs. We fix the packet service rate at 10 Mbit/s and restrict our attention to arrival rate \( \in [0.01, 12] \) Mbit/s. Recall that in this case, PAQMAN admitted all arrivals between two decision epochs subject to sufficient room at the buffer of the switch. This leads to increased packet buffering for higher RTTs, especially in high arrival regimes as the inter-decision times are RTT-based. To alleviate this problem, the policy is more aggressive and starts dropping earlier as seen via a comparison of Fig. 7a and 7b.

In Fig. 9 and 10, we compare PAQMAN with CoDel and Droptail when there is only one flow passing through the buffered switch port. The state evolution under PAQMAN (left subplot) in these figures is generated using the corresponding policy from Fig. 7, which follows the workflow described in Sect. V-A. Similar to Fig. 8, we see that PAQMAN achieves shorter delay than CoDel while yielding comparable stationary throughput. Further, a quick comparison of Fig. 8-10 reveals

---

**A. Negligible RTT**

In this section, we analyze PAQMAN for the case of a single flow having negligible RTT. We follow the method described in Sect. III to compute the packet drop policy.
Fig. 8: State evolution under different policies for a switch port with service rate = 10 Mbit/s. As already known, the droptail queue achieves higher stationary utilization than the AQMs at the cost of higher delay. PAQMAN leads to much shorter delay than CoDel, while achieving comparable stationary throughput and faster convergence to steady-state.

Fig. 9: State evolution under different policies for a flow with known RTT = 2 ms and switch service rate = 10 Mbit/s. Similar to Fig. 8, PAQMAN converges faster to the steady-state characterized by shorter delay and equivalent throughput.

Fig. 10: State evolution under different policies for a setup same as Fig. 9 except that the flow RTT is much higher (10 ms). We observe similar phenomena as Fig. 9 although both the stationary delay and the throughput decrease across AQM policies.

that a longer flow RTT leads to a reduction in both throughput and delay, which we investigate further in Fig. 12. Here, we first derive PAQMAN and subsequently simulate the system for long-lived flows (5 × 10^4 packet arrivals) for every given RTT. The box plots per RTT in Fig. 12 are generated using 200 simulation runs. Consistent with Fig. 8-10, we see that the stationary delay and the stationary throughput tend to diminish as the RTT increases.

Finally, in Fig. 11, we compare PAQMAN’s performance in the multi-flow case. Here, we consider three flows with RTT [2, 4, 6] ms and the service is assumed to be μ = 10 Mbit/s. We start the system in a phase where the total arrival rate equals the switch port service rate and the individual arrival rates are equal, i.e., each flow has equal share of the bandwidth. Similar to the experiments in the single flow case, we simulate the system for 200 runs, where each run spans across 5 × 10^4 packet arrivals. To learn the policy, we pre-simulate the system and use a DQN to approximate the Q-values for each system state, which in turn decides the optimal action. The time-averaged plot of the aggregated system state expressed in terms of queue length and total arrival rate is shown in Fig. 11. We see that that PAQMAN achieves similar throughput to CoDel, while keeping the delay considerably low here as well.


\[ \gamma(x + 1, y) = \int_{y}^{\infty} t^x e^{-t} dt = x \gamma(x, y) + y^{x-1} e^{-y}, \]

for \( R(x) > 0 \). We see that

\[
P(Y_{u,v} > X_{w,z}) = \int_{0}^{\infty} P(Y_{u,v} > X_{w,z} | X_{w,z} = t) dP_X(t)
\]

\[
= \int_{0}^{\infty} P(Y_{u,v} > t) dP_X(t), \quad Y_{u,v} \perp X_{w,z}
\]

\[
= \frac{\gamma(u, vt)}{\Gamma(u)} \frac{\gamma(v, z)}{\Gamma(v)}
\]

\[
= \int_{0}^{\gamma(u-1, vt)} \gamma(u-1, vt) + (vt)^{u-1} e^{-vt} z^w t^{w-1} e^{-zt} dt
\]

\[
= \frac{\gamma(u-1, v) \Gamma(u) \Gamma(w)}{\Gamma(u+w-1) v + z}
\]

\[
= \frac{\gamma(u-1, v) \Gamma(u) \Gamma(w)}{\Gamma(u) \Gamma(w) (v + z)}
\]

\[
= \frac{\gamma(u-1, v) \Gamma(u) \Gamma(w) (v + z)}{\Gamma(u) \Gamma(w) (v + z + w)}
\]

\[
= \frac{\gamma(u-1, v) \Gamma(u) \Gamma(w) (v + z)}{\Gamma(u) \Gamma(w) (v + z + w)}
\]

which proves the first part of the lemma. By induction on \( u \),

\[
P(Y_{u,v} > X_{w,z}) = P(Y_{1,v} > X_{w,z}) + \sum_{k=2}^{u} \frac{\Gamma(k+w-1)}{\Gamma(k) \Gamma(w)} \left( \frac{v}{v+z} \right)^{k-1} \left( \frac{z}{v+z} \right)^w.
\]

Since \( Y_{1,v} \) is exponentially distributed with parameter \( v \),

\[
P(Y_{1,v} > X_{w,z}) = \int_{0}^{\infty} e^{-vt} z^w t^{w-1} e^{-zt} dt
\]

\[
= \left( \frac{z}{v+z} \right)^w \int_{0}^{\infty} (v+z)^w t^{w-1} e^{-(v+z)t} dt
\]

\[
= \left( \frac{z}{v+z} \right)^w \int_{0}^{\infty} \frac{(v+z)^w t^{w-1} e^{-(v+z)t}}{\Gamma(w)} dt
\]

Hence from (16),

\[
P(Y_{u,v} > X_{w,z}) = \sum_{k=0}^{w} \frac{\Gamma(k+w)}{\Gamma(k+1) \Gamma(w)} \left( \frac{v}{v+z} \right)^k \left( \frac{z}{v+z} \right)^w.
\]
REFERENCES

[1] S. Iyer, R. Zhang, and N. McKeown, “Routers with a single stage of buffering,” in Proceedings of the 2002 conference on Applications, technologies, architectures, and protocols for computer communications, 2002, pp. 251–264.

[2] J. Gettys, “Bufferbloat: Dark buffers in the internet,” IEEE Internet Computing, vol. 15, no. 3, pp. 96–99, 2011.

[3] D. Lee, B. E. Carpenter, and N. Brownlee, “Observations of udp to tcp ratio and port numbers,” in 2010 Fifth International Conference on Internet Monitoring and Protection, 2010, pp. 99–104.

[4] D.-M. Chiu and R. Jain, “Analysis of the increase and decrease algorithms for congestion avoidance in computer networks,” Computer Networks and ISDN systems, vol. 17, no. 1, pp. 1–14, 1989.

[5] S. Floyd, R. Gummadi, S. Shenker et al., “Adaptive red: An algorithm for increasing the robustness of red’s active queue management,” 2001.

[6] S. Floyd and V. Jacobson, “Random early detection gateways for congestion avoidance,” IEEE/ACM Transactions on networking, vol. 1, no. 4, pp. 397–413, 1993.

[7] K. Nichols and V. Jacobson, “Controlling queue delay,” Communications of the ACM, vol. 55, no. 7, pp. 42–50, 2012.

[8] R. Pan, N. Natarajan, C. Figliolino, M. S. Prabhhu, V. Subramanian, F. Baker, and B. VerSteeg, “Pic: A lightweight control scheme to address the bufferbloat problem,” in 2013 IEEE 14th International Conference on High Performance Switching and Routing (HPSR), 2013, pp. 148–155.

[9] R. Adams, “Active queue management: A survey,” IEEE Communications Surveys Tutorials, vol. 15, no. 3, pp. 1425–1476, 2013.

[10] N. Khademi, D. Ros, and M. Welzl, “The new aqm kids on the block: An experimental evaluation of codel and pic,” in 2014 IEEE Conference on Computer Communications Workshops (INFOCOM WKSHPS), 2014, pp. 85–90.

[11] R. Kundel, A. Rizk, J. Blendin, B. Koldehofe, R. Hark, and R. Steinmetz, “P4-codel: Experiences on programmable data plane hardware,” in ICC 2014, 2014, pp. 159–167.

[12] T. J. Ott, T. Lakshman, and L. H. Wong, “Sred: stabilized red,” in IEEE INFOCOM’99. Conference on Computer Communications. Proceedings. Eighteenth Annual Joint Conference of the IEEE Computer and Communications Societies. The Future is Now (Cat. No. 99CH36320), vol. 3. IEEE, 1999, pp. 1346–1355.

[13] C. V. Hollot, V. Misra, D. Towsley, and W.-B. Gong, “A control theoretic analysis of red,” in Proceedings IEEE INFOCOM 2001. Conference on Computer Communications. Twentieth Annual Joint Conference of the IEEE Computer and Communications Society (Cat. No. 01CH37213), vol. 3. IEEE, 2001, pp. 1510–1519.

[14] K. B. Kim, A. Tang, and S. H. Low, “Design of aqm in supporting tcp based on the well-known aimd model,” in GLOBECOM’03. IEEE Global Telecommunications Conference (IEEE Cat. No. 03CH37489), vol. 6. IEEE, 2003, pp. 3226–3230.

[15] F. Yanfei, R. Fenyuan, and L. Chuang, “Design a pid controller for active queue management,” in Proceedings of the Eighth IEEE Symposium on Computers and Communications. ISCC 2003. IEEE, 2003, pp. 985–990.

[16] Z. Liu, J. Sun, S. Hu, and X. Hu, “An adaptive aqm algorithm based on a novel information compression model,” IEEE Access, vol. 6, pp. 31 180–31 190, 2018.

[17] T. Alpcan, P. G. Mehta, T. Basar, and U. Vaidya, “Control of non-equilibrium dynamics in communication networks,” in Proceedings of the 45th IEEE Conference on Decision and Control. IEEE, 2006, pp. 5216–5221.

[18] T. Alpcan, P. Wang, P. G. Mehta, and T. Basar, “A non-equilibrium analysis and control framework for active queue management,” Automatica, vol. 44, no. 10, pp. 2474–2486, 2008.

[19] S. Kunniyur and V. Srikant, “Analysis and design of an adaptive virtual queue (avq) algorithm for active queue management,” ACM SIGCOMM Computer Communication Review, vol. 31, no. 4, pp. 123–134, 2001.

[20] S. Athuraliya, Y. H. Li, S. H. Low, and A. V. Vin, “Rem: Active queue management,” in Teletraffic Science and Engineering. Elsevier, 2001, vol. 4, pp. 817–828.

[21] A. Bitorika, M. Robin, and M. Huggard, “An evaluation framework for active queue management schemes,” in 11th IEEE/ACM International Symposium on Modeling, Analysis and Simulation of Computer Telecommunication Systems, 2003. MASCOTS 2003. IEEE, 2003, pp. 200–206.

[22] C. Hollot, Y. Liu, V. Misra, and D. Towsley, “Unresponsive flows and aqm performance,” in IEEE INFOCOM 2003. Twenty-second Annual Joint Conference of the IEEE Computer and Communications Societies (IEEE Cat. No. 03CH37428), vol. 1. IEEE, 2003, pp. 85–95.

[23] D. Lin and R. Morris, “Dynamics of random early detection,” in Proceedings of the ACM SIGCOMM’97 conference on Applications, technologies, architectures, and protocols for computer communication, 1997, pp. 127–137.

[24] C. A. Gomez, X. Wang, and A. Shami, “Intelligent active queue management using explicit congestion notification,” in 2019 IEEE Global Communications Conference (GLOBECOM). IEEE, 2019, pp. 1–6.

[25] C. J. Watkins and P. Dayan, “Q-learning,” Machine learning, vol. 8, no. 3–4, pp. 279–292, 1992.

[26] I. Järvinen and M. Kojo, “Evaluating codel, pic, and hred aqm techniques with load transients,” in 39th Annual IEEE Conference on Local Computer Networks. IEEE, 2014, pp. 159–167.

[27] H. C. Tijms, A first course in stochastic models. John Wiley and sons, 2003.

[28] K. Nichols, V. Jacobson, and A. McGregor, “and j. iyengar, ed,” controlled delay active queue management,” RFC 8289, DOI 10.17487/RFC8289, January 2018, < https://www. rfc-editor. org . . . >, Tech. Rep.

[29] A. Feldbaum, “Dual control theory. i,” Avtomatika i Telemekhanika, vol. 21, no. 9, pp. 1240–1249, 1960.

[30] R. S. Sutton and A. G. Barto, Reinforcement Learning: An Introduction, 2nd ed. The MIT Press, 2018. [Online]. Available: http://incompleteideas.net/book/the-book-2nd.html

[31] S. Ha, I. Rhee, and L. Xu, “Cubic: a new tcp-friendly high-speed tcp variant,” ACM SIGOPS operating systems review, vol. 42, no. 5, pp. 64–74, 2008.

[32] S. A. Gershgorin, “Über die Abgrenzung der Eigenwerte einer Matrix,” Bull. Acad. Sci. URSS, vol. 1931, no. 6, pp. 749–754, 1931.

[33] V. Mnih, K. Kavukcuoglu, D. Silver, A. A. Rusu, J. Veness, M. G. Bellemare, A. Graves, M. Riedmiller, A. K. Fidjeland, G. Ostrovski et al., “Human-level control through deep reinforcement learning,” nature, vol. 518, no. 7540, pp. 529–533, 2015.