Two-loop contributions to electroweak precision observables in the MSSM

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The evaluation of the two-loop MSSM-contributions to the electroweak precision observables $M_W$ and $\sin^2 \theta_{\text{eff}}$ at $\mathcal{O}(\alpha^2 t)$, $\mathcal{O}(\alpha t \alpha b)$, $\mathcal{O}(\alpha^2 b)$ is presented. These contributions enter via $\Delta \rho$, and it is explained in detail how one can retain the true, non-vanishing value of the MSSM Higgs boson mass $M_h$ in spite of using the gauge-less limit in the calculation. The numerical results can be sizeable, in particular for strong squark mixing. By comparing the results in the on-shell and $\overline{\text{DR}}$ renormalization schemes, the remaining theoretical uncertainty is found to be small.

1. INTRODUCTION

Electroweak precision observables (EWPO) like $M_W$ or the effective leptonic weak mixing angle $\sin^2 \theta_{\text{eff}}$ are intimately related to the quantum structure of the electroweak interactions. The experimental determination of these quantities, obtained at LEP and Tevatron, has a precision of better than one per-mille: $\delta M_W = 34 \text{ MeV (0.04\%)}$ and $\delta \sin^2 \theta_{\text{eff}} = 16 \times 10^{-5}$ (0.07\%) [1]. In the future, these accuracies will improve, and at the GigaZ option of a linear $e^+e^-$ collider a precision of $\delta M_W = 7 \text{ MeV}$ [2, 3] and $\delta \sin^2 \theta_{\text{eff}} = 1.3 \times 10^{-5}$ [3, 4] can be achieved.

These precise measurements constitute tests of the quantum level of the Standard Model (SM) that are sensitive even to two-loop effects. The corresponding theoretical evaluation of the SM predictions up to the two-loop level is quite advanced, see in particular Ref. [5] for the most recent developments. On the other hand, the measurements of the EWPO can be used to discriminate between different models of electroweak interactions and to derive constraints on unknown parameters. The comparison of the SM and its minimal supersymmetric extension, the MSSM, is particularly interesting since the MSSM agrees with all precision data at least as well as the SM, in some cases even better.

It is therefore highly desirable to know the MSSM predictions for the EWPO with a precision that matches the one of the SM and the experiments. The comparison of the SM and the MSSM predictions with the data could then lead to precise constraints e.g. on masses of supersymmetric particles. As a step towards this goal we have evaluated [6] the two-loop MSSM-corrections to the EWPO that enter via $\Delta \rho$ at $\mathcal{O}(\alpha^2 t)$, $\mathcal{O}(\alpha t \alpha b)$, $\mathcal{O}(\alpha^2 b)$, where

$$\Delta \rho = \frac{\Sigma_Z(0)}{M_Z^2} - \frac{\Sigma_W(0)}{M_W^2}$$

in terms of the $Z$ and $W$ self energies $\Sigma_{Z,W}$. These are leading two-loop contributions involving the top and bottom Yukawa couplings and come from three classes of diagrams as shown in Fig. 1. These contributions to $\Delta \rho$ induce universal two-loop corrections to the EWPO as follows (with $1 - s_W^2 = c_W^2 = M_W^2/M_Z^2$):

$$\delta M_W \approx \frac{M_W}{2} \frac{c_W^2}{c_W^2 - s_W^2} \Delta \rho, \quad \delta \sin^2 \theta_{\text{eff}} \approx - \frac{c_W^2 \Delta \rho}{c_W^2 - s_W^2} \Delta \rho.$$  

The previously known two-loop contributions to EWPO in the MSSM comprise only QCD and SUSY-QCD corrections [7] and the $\mathcal{O}(\alpha^2 t)$, $\mathcal{O}(\alpha t \alpha b)$, $\mathcal{O}(\alpha^2 b)$ corrections for the class $(q)$ in Fig. 1 [8]. Obviously, the diagrams of class $(q)$ contain no supersymmetric particles. The contributions from classes $(\tilde{q})$, $(\tilde{H})$ considered here in addition can be expected to be important for squark/Higgsino masses in the electroweak range and to have a more pronounced dependence on the MSSM parameters. A similar situation was found for the case of $(g - 2)_\mu$, where the two-loop contributions from squark-Higgs diagrams are more important than the ones from quark-Higgs diagrams [9].
Figure 1: Sample diagrams for the three classes of contributions to $\Delta \rho$ considered here: (q) $t/b$-quark loop with Higgs exchange, ($\tilde{q}$) $\tilde{t}/\tilde{b}$-squark loop with Higgs exchange, ($\tilde{H}$) mixed quark/squark loop with Higgsino exchange.

In the following we will first discuss in detail the renormalization and the restrictions imposed by approximating the EWPO by $\Delta \rho$ as in eq. (2). It turns out that a strict calculation of $\Delta \rho$ at $O(\alpha_2^t), O(\alpha_1 \alpha_b), O(\alpha_2^b)$ would imply a vanishing MSSM Higgs boson mass, $M_h = 0$, which would lead to a bad approximation for the EWPO. We will show that it is possible to improve the approximation by taking into account the true value of $M_h$ essentially everywhere. Finally we will discuss the numerical results and give an account of the remaining theoretical uncertainty based on the dependence on the renormalization scheme.

2. THE ROLE OF THE GAUGE-LESS LIMIT AND THE LIGHT HIGGS BOSON MASS $M_h$

2.1. Gauge-less limit and $M_h = 0$

$\Delta \rho$ as defined in eq. (1) constitutes a part of the loop corrections to the EWPO but is no observable. By itself, $\Delta \rho$ is only UV-finite and gauge independent if corrections are considered that enter the EWPO only through vector boson self energies in the form (2) (as opposed to e.g. vertex or box diagrams). This is the case for the $O(\alpha_1^t), O(\alpha_1 \alpha_b), O(\alpha_2^b)$ corrections.

Taking into account strictly only terms of $O(\alpha_1^t), O(\alpha_1 \alpha_b), O(\alpha_2^b)$ corresponds in particular to neglecting the gauge coupling $\alpha$ and thus to taking the gauge-less limit $\alpha \to 0$. In this limit, $M_{Z,W} \to 0$ while the ratio $c_W = M_W/M_Z$ is fixed.

In the SM the gauge-less limit is a reasonable approximation since $\alpha \ll \alpha_t$. In the MSSM, however, the gauge-less limit has side-effects in the Higgs sector since supersymmetry relates the Higgs self couplings to gauge couplings. At tree-level, the gauge-less limit implies the relations

$$M_h = 0, \quad M_{H^\pm}^2 = M_H^2 = M_A^2, \quad \sin \alpha = -\cos \beta, \quad \cos \alpha = \sin \beta$$

for the light and heavy CP-even, the CP-odd and the charged Higgs boson masses $M_{h,H}, M_A, M_{H^\pm}$ and the angles $\alpha, \beta$ where $\tan \beta = v_2/v_1$, the ratio of the two vacuum expectation values.

The particularly troublesome of these relations is the masslessness of the light Higgs boson, $M_h = 0$. In the SM, where the Higgs boson mass is a free parameter even in the gauge-less limit, one knows that the result for $\Delta \rho$ at $O(\alpha_1^t), O(\alpha_1 \alpha_b), O(\alpha_2^b)$ is proportional to a factor $[10, 11]$

$$19 - 2\pi^2 \text{ for } M_H = 0, \quad 19 - 2\pi^2 + f(M_H) \text{ for } M_H \neq 0.$$  

(4)

For typical values of $M_H = O(100 \text{ GeV})$, this factor is about an order of magnitude larger than for $M_H = 0$, and the result for $M_H = 0$ leads to a bad approximation for the EWPO.

Due to the similar structure of the SM and MSSM diagrams it can be expected that taking the gauge-less limit relation $M_h = 0$ literally would also lead to a bad approximation for the EWPO and should therefore be avoided.
Indeed, in Ref. [8] it was observed that the class \((q)\) contributions to \(\Delta \rho\) in the MSSM are already UV-finite if all relations in eq. (3b) but not \(M_h = 0\) are employed. In the following, we give an explanation of this result and show how it extends to the contributions of classes \((\tilde{q}), (\tilde{H})\).

### 2.2. Comparison of the MSSM and a general two-Higgs doublet model

In order to study these questions it is very useful to regard the MSSM as a special case of a more general two-Higgs-doublet model \(2HDM\) without supersymmetry relations for the couplings. \(\Delta \rho\) can be calculated at \(O(\alpha_s^2), O(\alpha_s\alpha_t), O(\alpha_t^2)\) in the gauge-less limit in both models, but in a general \(2HDM\), the gauge-less limit does not enforce any of the relations in eq. (3).

Comparing first the contributions from class \((q)\) in the MSSM and the \(2HDM\), we find that the corresponding two-loop diagrams and the counterterm contributions from the top/bottom sector are identical. The only difference concerns the Higgs sector counterterm contributions. In a general \(2HDM\), they can be derived from the one-loop expression \((F_0\) is a symmetric function satisfying \(F_0(x,x) = 0, \partial_x F_0(x,y)|_{y=x} = 0\)

\[
\Delta \rho_{2HDM}^\tilde{H} \propto \left[ F_0(M_{H^+}^2, M_{A^0}^2) + s^2_{\beta-\alpha} (F_0(M_{H^+}^2, M_{H}^2) - F_0(M_{A^0}^2, M_{H}^2)) + c_{\beta-\alpha}^2 (F_0(M_{H^+}^2, M_{H}^2) - F_0(M_{A^0}^2, M_{H}^2)) \right] \tag{5}
\]

by performing the renormalization transformation \(M_h \to M_h + \delta M_h\) etc. In the MSSM, eq. (5) and the Higgs sector counterterms vanish because of the gauge-less limit relations (3). Thus the class \((q)\) contributions can be decomposed as \(\Delta \rho_{2HDM}^{\tilde{q}, \tilde{H}} = \Delta \rho_{2HDM}^{\tilde{q}, H} + \Delta \rho_{2HDM}^{\tilde{q}, \tilde{H}}\) in the \(2HDM\) and as \(\Delta \rho_{MSSM}^{\tilde{q}, \tilde{H}} = \Delta \rho_{MSSM}^{\tilde{q}, H} + \Delta \rho_{MSSM}^{\tilde{q}, \tilde{H}}\) in the MSSM. The \(2HDM\) result is UV-finite for all values of the Higgs sector parameters.

From this comparison we find that the MSSM result is not only UV-finite if \((3)\) is used but more generally if \(\Delta \rho_{\tilde{H}}^{\tilde{q}, \tilde{H}}\) as derived from \((5)\) is finite. From eq. (5) we can explicitly read off that \(\Delta \rho_{\tilde{H}}^{\tilde{q}, \tilde{H}} = 0\) already if only the relations \((3b)\) are used. This explains the observation made in Ref. [8]. The MSSM result \(\Delta \rho_{\tilde{q}, \tilde{H}}^{\tilde{q}, \tilde{H}} + \Delta \rho_{\tilde{q}, H}^{\tilde{q}, \tilde{H}}\) with the relations \((3b)\) corresponds to a certain special case of the general \(2HDM\) calculation. Therefore it is finite even for the true, non-vanishing value of \(M_h\).

For the class \((\tilde{q}, \tilde{H})\) contributions we obtain similar decompositions

\[
\Delta \rho_{2HDM}^{\tilde{q}, \tilde{H}} = \Delta \rho_{2HDM}^{\tilde{q}, \tilde{H}} + \Delta \rho_{\tilde{H}}^{\tilde{q}, \tilde{H}} + \Delta \rho_{\tilde{q}, \tilde{H}}^{\tilde{q}, \tilde{H}, OS3} + \Delta \rho_{\tilde{q}, \tilde{H}}^{\tilde{q}, \tilde{H}, OS4}, \tag{6}
\]

\[
\Delta \rho_{MSSM}^{\tilde{q}, \tilde{H}} = \Delta \rho_{MSSM}^{\tilde{q}, \tilde{H}} + \Delta \rho_{\tilde{H}}^{\tilde{q}, \tilde{H}} + \Delta \rho_{\tilde{q}, \tilde{H}}^{\tilde{q}, \tilde{H}, OS3} + \Delta \rho_{\tilde{q}, \tilde{H}}^{\tilde{q}, \tilde{H}, OS4}. \tag{7}
\]

In this case, even if the relations \((3b)\) are used such that \(\Delta \rho_{\tilde{H}}^{\tilde{q}, \tilde{H}}\) vanishes, there is still a difference between the \(2HDM\) and the MSSM result because the \(t/\bar{b}\) sector counterterms differ as indicated by the superscripts \(OS4\) and \(OS3\).

In the MSSM, supersymmetry in conjunction with \(SU(2)\) gauge invariance correlates the four sfermion masses \(m_{\tilde{t}_1, \tilde{t}_2}, m_{\tilde{b}_1, \tilde{b}_2}\). Therefore only three of them can be renormalized independently. We choose to renormalize \(m_{\tilde{t}_1, \tilde{t}_2}\) and \(m_{\tilde{b}_2}\) independently by on-shell conditions \((OS3)\). Then the fourth renormalization constant \(\delta m_{\tilde{t}_1}\) is determined as a function of the other three in the MSSM, while in the \(2HDM\) all four sfermion masses can be defined independently by on-shell conditions \((OS4)\). In the simple case of vanishing mixing between left- and right-handed sfermions, \(m_{\tilde{t}_1} = m_{\tilde{t}_2}\) and we obtain the following different results for \(\delta m^2_{\tilde{b}_2}\)

\[
\delta m^2_{\tilde{b}_2} |_{OS3} = \delta m^2_{\tilde{b}_2} + \delta m^2_{\tilde{h}} - \delta m^2_{\tilde{t}_1}, \quad \delta m^2_{\tilde{b}_2} |_{OS4} = \Sigma_{\tilde{b}_2} (m^2_{\tilde{b}_2}). \tag{8}
\]

The difference between the sfermion sector counterterms in the \(2HDM\) and the MSSM is thus contained in the mass shift

\[
\Delta m^2_{\tilde{b}_2} = \Sigma_{\tilde{b}_2} (m^2_{\tilde{b}_2}) - \left( \delta m^2_{\tilde{b}_2} + \delta m^2_{\tilde{h}} - \delta m^2_{\tilde{t}_1} \right). \tag{9}
\]

It turns out that this mass shift is only UV-finite if all gauge-less limit relations are used, including \(M_h = 0\). Accordingly, the MSSM result \((7)\) is only finite if all relations in \((3)\) are employed. The \(2HDM\) result with on-shell renormalization of all four sfermion masses is of course UV-finite for all choices of \(M_h\).
The best way to take into account the non-vanishing value of $M_h$ as much as possible is to consider the combination

$$\Delta \rho_{\text{MSSM}}(M_h = 0) + \left( \Delta \rho_{\text{2HDM}}^{(\tilde q, \tilde H)}(M_h, \tilde \rho) - \Delta \rho_{\text{2HDM}}^{(\tilde q, \tilde H)}(M_h = 0) \right),$$

where the relations (3b) are used everywhere. As explained above, all three terms are individually UV-finite. Since the difference between $\Delta \rho_{\text{MSSM}}$ and $\Delta \rho_{\text{2HDM}}^{(\tilde q, \tilde H)}$ is confined to the $\tilde t/\tilde b$ counterterms, the combination (10) can be written as

$$\Delta \rho_{\text{2-Loop}}^{(\tilde q, \tilde H)}(M_h) + \Delta \rho_{\text{t/b-cts}}^{(\tilde q, \tilde H)}(M_h) + \Delta \rho_{\text{t/b-cts}}^{(\tilde q, \tilde H, \text{OS4})}(M_h) + \left[ \Delta \rho_{\text{t/b-cts}}^{(\tilde q, \tilde H, \text{OS3})}(M_h = 0) - \Delta \rho_{\text{t/b-cts}}^{(\tilde q, \tilde H, \text{OS4})}(M_h = 0) \right].$$

The first three terms correspond to the MSSM calculation where $M_h$ is set to its true, non-vanishing value but where all sfermion masses are renormalized by on-shell conditions instead of using the mass relation imposed by supersymmetry. The term in the square brackets is proportional to the mass shift $\Delta m_t^2(M_h = 0)$ that restores the necessary supersymmetry mass relation. It is only here that $M_h = 0$ has to be employed.

### 3. Numerical Results

#### 3.1. Results for different values of the supersymmetry parameters

Fig. 2 demonstrates that it is in general important to take into account the true value of the Higgs boson mass $M_h$. In the left and right panels of Fig. 2, $\Delta \rho^{(q)}$ and $\Delta \rho^{(\tilde q, \text{2HDM})}$, which is the only $M_h$-dependent term in eq. (10), are shown as functions of $M_h$ in a scenario with a light stop (see caption). For both the fermion and the sfermion loop contributions the difference of setting $M_h = 0$ or $M_h = \mathcal{O}(100 \text{ GeV})$ amounts to more than $10^{-4}$.

In the remainder we focus on the numerical results from the classes $(\tilde q, \tilde H)$ (for class $(q)$ see Ref. [8]). In Figs. 3, 4 the results for $\Delta \rho$ as defined in eqs. (10,11) are shown, split up into the contributions for the individual classes. We choose several representative values for the supersymmetry parameters as described in the captions, basically always starting from the SPS1a scenario [12] and varying one or two of the parameters. In the class $(\tilde q)$ contributions the Higgs boson mass $M_h$ is set to either 120 GeV, which is a good approximation for the true, loop-corrected value of $M_h$, or to zero; the class $(\tilde H)$ contributions are $M_h$-independent.

According to eq. (2) a contribution of $\Delta \rho = 10^{-4}$ leads to shifts

$$\delta M_W = 6 \text{ MeV}, \quad \delta \sin^2 \theta_{\text{eff}} = -3 \times 10^{-5}.$$

We find that contributions of this order of magnitude are possible, in particular for strong sfermion mixing (large values of $\mu$ or $A_t$ in Fig. 4) but also for a light common sfermion mass parameter $M_{\text{SUSY}}$ as in Fig. 3.
Figure 3: Results for $\Delta \rho^{(q)} (M_{h} = 120 \text{ GeV})$ (1, brown), $\Delta \rho^{(h)}$ (2, green), and, for comparison, $\Delta \rho^{(q)} (M_{h} = 0)$ (3, red), shown as functions of the common sfermion mass $M_{\text{SUSY}}$. In the left panel, the $A_{t,b} = 0$; in the right panel, $A_{t,b}$ are chosen such that the ratios $M_{\text{SUSY}} : A_{t} : A_{b}$ are as in the SPS1a scenario. The remaining supersymmetry parameters are always set to the values of the SPS1a scenario.

Figure 4: Results for $\Delta \rho^{(q)} (M_{h} = 120 \text{ GeV})$ (1, brown), $\Delta \rho^{(h)}$ (2, green), and, for comparison, $\Delta \rho^{(q)} (M_{h} = 0)$ (3, red). In the left panel, the results are shown as functions of the $\mu$-parameter for $\tan \beta = 40$, such that for large $\mu$ one sbottom becomes light; in the right panel, the results are shown as functions of $A_{t}$. The remaining supersymmetry parameters are always set to the values of the SPS1a scenario.

3.2. Different renormalization schemes and estimate of remaining theoretical uncertainty

So far we have chosen the on-shell renormalization scheme for the independent sfermion masses $m_{\tilde{t}_{1,2}}$ and $m_{\tilde{b}_{2}}$. In Fig. 5 the on-shell results are compared with the results in the $\overline{\text{DR}}$ scheme, where the counterterms for the soft supersymmetry breaking parameters in the sfermion mass matrices are defined as pure divergences. This $\overline{\text{DR}}$ scheme implies non-vanishing finite parts of the sfermion mass counterterms, which read, in the case of vanishing left-right mixing, $\delta m_{j}^{2}_{\text{fin-part}} = \delta m_{\tilde{j}}^{2}_{\text{fin-part}}$. The comparison of the two renormalization schemes is interesting in order to assess the numerical stability of the result and the intrinsic theoretical uncertainty of the two-loop contributions.

We find that the difference between the on-shell and $\overline{\text{DR}}$ results for the sfermion loop contributions at the one-loop level is of the order $10^{-5}$ to $10^{-4}$ for large sfermion mixing. At the two-loop level this renormalization-scheme dependence is significantly reduced to well below $10^{-5}$. We have checked that this result is general and not restricted to the particular parameter choice in Fig. 5.
Figure 5: The sfermion loop one-loop result $\Delta \rho^{(1L)}$ (dashed) and the sum $\Delta \rho^{(1L)} + \Delta \rho^{(\tilde{q}, \tilde{H})}$ (full lines) in the on-shell (blue) and the DR (red) renormalization scheme. The parameters are as in the right panel of Fig. 4. The left panel shows the full results, where the differences between the curves are hardly visible; the right panel shows the differences of the results to $\Delta \rho^{(1L)}|_{\text{OS}}$. For simplicity, $M_h = 0$ is used here, but the conclusions do not change for non-vanishing $M_h$.

In conclusion, we have evaluated the $O(\alpha_s^2)$, $O(\alpha_s\alpha_t)$, $O(\alpha^2_t)$ corrections to $\Delta \rho$ and thus to the EWPO $M_W$ and $\sin^2 \theta_{\text{eff}}$ in the MSSM. Although the gauge-less limit is necessary and leads to the tree-level relation $M_h = 0$, we have shown that the true MSSM Higgs boson mass can be taken into account. The numerical values of the class $(\tilde{q}, \tilde{H})$ contributions (10), (11) to $\Delta \rho$ can amount to $10^{-4}$, corresponding to shifts $\delta M_W = 6$ MeV, $\delta \sin^2 \theta_{\text{eff}} = -3 \times 10^{-5}$.

The comparison of the two renormalization schemes shows that the inclusion of the two-loop result in the MSSM-prediction for $\Delta \rho$ and the EWPO leads to a significantly improved accuracy. The residual theoretical uncertainty due to unknown three-loop corrections of $O(\alpha_t^3)$ is well below the foreseen experimental resolution achievable at the GigaZ option of a linear $e^+e^-$ collider.

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