Energy-momentum in gauge gravitation theory

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Abstract

Building on the first variational formula of the calculus of variations, one can derive the energy-momentum conservation laws from the condition of the Lie derivative of gravitation Lagrangians along vector fields corresponding to generators of general covariant transformations to be equal to zero. The goal is to construct these vector fields. In gauge gravitation theory, the difficulty arises because of fermion fields. General covariant transformations fail to preserve the Dirac spin structure $S_h \rightarrow X$ on a world manifold $X$ which is associated with a certain tetrad field $h$. We introduce the universal Dirac spin structure $S \rightarrow \Sigma \rightarrow X$ such that, given a tetrad field $h : X \rightarrow \Sigma$, the restriction of $S$ to $h(X)$ is isomorphic to $S_h$. The canonical lift of vector fields on $X$ onto $S$ is constructed. We discover the corresponding stress-energy-momentum conservation law. The gravitational model in the presence of a background spin structure also is examined.

1 Introduction

There are several approaches to discovering energy-momentum conservation laws in gravitation theory. Here we are concerned with the gauge gravitation model of classical fields represented conventionally by sections of bundles $Y \rightarrow X$ over a world manifold $X$. In the framework of this model, the first variational formula of the calculus of variations can be utilized in order to discover differential conservation laws. This formula provides the canonical decomposition of the Lie derivative of Lagrangians along vector fields of infinitesimal gauge transformations into the two terms. The first one is expressed into the variational derivatives and, therefore, vanishes on shell. Other is the divergence $d_\lambda T^\lambda$ of the corresponding symmetry flow $T$. If a Lagrangian is gauge-invariant, its Lie derivative is equal to zero and the weak conservation law

$$0 \approx d_\lambda T^\lambda$$  (1)
takes place. When gauge transformations involve diffeomorphisms of a world manifold, i.e., their generators are the lift of vector fields \( \tau \) on \( X \) onto \( Y \), we have the stress-energy-momentum (SEM) conservation law \([1]\) \([2]\) \([3]\) \([4]\) \([5]\) \([6]\) \([7]\).

In the model of metric gravity without matter \([13]\), the SEM conservation law \([1]\) occurs due to invariance of the Hilbert–Einstein Lagrangian under general covariant transformations. It takes the form

\[
0 \approx d_\lambda \mathcal{T}^\lambda, \quad \mathcal{T}^\lambda \approx d_\mu U^\lambda_\mu, \tag{2}
\]

where

\[
U^\lambda_\mu = \frac{\sqrt{-g}}{2\kappa} (g^{\lambda\nu} \nabla_\nu \tau^\mu - g^{\mu\nu} \nabla_\nu \tau^\lambda)
\]

is the well-known Komar superpotential. By \( \nabla \) are meant the covariant derivatives with respect to the Levi-Civita connection of a metric \( g \).

The same conservation law has been discovered in the Palatini model whose Lagrangian is an arbitrary function of the scalar curvature of a torsionless connection on \( X \) \([2]\).

The affine-metric gravitation theory with tensor matter fields also has been considered \([3]\) \([4]\) \([10]\). In case of a Lagrangian expressed into the curvature tensor \( R^\alpha_\nu\lambda_\mu \) of a general linear connection \( K \) on \( X \), the SEM conservation law is brought into the form (2) where

\[
U^\lambda_\mu = 2 \frac{\partial L_{\text{AM}}}{\partial R^\alpha_\nu\lambda_\mu} (\partial_\nu \tau^\alpha + K^\alpha_\sigma_\nu \tau^\sigma)
\]

is the generalized Komar superpotential.

In gauge gravitation theory, the difficulty arises because of Dirac fermion fields. These fields admit the Lorentz gauge transformations only and, therefore, they are described in a pair with a certain tetrad field \( h \) on \( X \) by sections of the corresponding spinor bundle \( S_h \rightarrow X \). Recall that tetrad fields constitute the 2-fold covering of pseudo-Riemannian metrics on \( X \). Since active general covariant transformations alter metrics on \( X \), they do not preserve spinor bundles \( S_h \rightarrow X \). We thus face the problem how to construct the appropriate lift of vector fields on \( X \) onto spinor fields in order to discover the SEM conservation laws. The solutions may be the following.

(i) The "world" spinors are considered. They carry representation of the universal covering \( SL(4, \mathbb{R}) \) of the group \( SL(4, \mathbb{R}) \) \([12]\).

(ii) The total system of fermion-gravitation pairs is described \([3]\) \([4]\) \([21]\). In the present work, these pairs are represented by sections of the composite spinor bundle \( S \rightarrow \Sigma \rightarrow X \) where \( \Sigma \rightarrow X \) is the bundle of tetrad fields on \( X \) \([22]\) \([24]\). We show that gauge theory on this bundle is reduced to the affine metric-gravitation theory in the presence of fermion fields. We construct the canonical lift of vector fields \( \tau \) on \( X \) onto the spinor bundle \( S \) and, as a result, come to the energy-momentum conservation law \([4]\) where \( U^\mu_\lambda \) is the generalized
Komar superpotential (3) with accuracy to the standard term which is related to particular choice of the Lepagean equivalent of a Lagrangian (see (11)).

(iii) Given a tetrad field \( h \), nonvertical gauge transformations which keep \( h \) are considered. As a result, we come to the affine-metric modification of the relativistic theory of gravity by A. Logunov [13, 14] where independent dynamic variables are non-metric gravitational fields and world connections on \( X \) in the presence of a background pseudo-Riemannian metric \( g \). The Lagrangian of this model, by construction, is invariant under the above-mentioned gauge transformations, but not the general covariant transformations. As a consequence, the total energy-momentum flow is the sum of the superpotential term and the metric energy-momentum tensor. It is conserved if \( g \) is the Minkowski metric.

Throughout, \( X \) is a locally compact paracompact oriented connected 4-manifold. It is assumed to be noncompact and parallelizable in order that a pseudo-Riemannian metric, a spinor structure and a causal space-time structure can exist on \( X \).

2 Conservation laws

Differential operators and the Lagrangian formalism on sections of bundles are phrased conventionally in the terms of jet manifolds [15 23 24 26].

As a shorthand, one can say that the \( k \)-order jet manifold \( J^kY \) of a bundle \( Y \to X \) comprises the equivalence classes \( j^k_s \)'s, \( x \in X \), of sections \( s \) of \( Y \) identified by the first \( k+1 \) terms of their Taylor series at a point \( x \). Given bundle coordinates \( (x^{\lambda}, y^i) \) of \( Y \to X \), the \( k \)-jet manifold \( J^kY \) is provided with the coordinates \( (x^{\lambda}, y^i, y^i_{\lambda}, y^i_{\lambda\mu}, \ldots, y^i_{\lambda_1\ldots\lambda_k}) \) where

\[(y^i, y^i_{\lambda}, \ldots)(j^k_x s) = (s^i(x), \partial_{\lambda}s^i(x), \ldots).\]

We are concerned with the first order Lagrangian formalism on the configuration space \( J^1Y \) coordinatized by \( (x^{\lambda}, y^i, y^i_{\lambda}) \) together with the affine transition functions

\[ y^i_{\lambda} = \frac{\partial x^\mu}{\partial x'^{\lambda}}(\partial_{\mu} + y^j_{\mu} \partial_j)y^i. \]

A first order Lagrangian on \( J^1Y \) is defined to be a horizontal density

\[ L = \mathcal{L}(x^\mu, y^i, y^i_{\mu})\omega, \quad \omega = dx^0 \wedge \cdots \wedge dx^3. \tag{4} \]

Differential conservation laws in classical field theory are derived from the condition of Lagrangians to be invariant under 1-parameter groups of gauge transformations.

By a gauge transformation is meant an isomorphism \( \Phi \) of the bundle \( \pi : Y \to X \) over a diffeomorphism \( f \) of \( X \) which sends the fibres \( \pi^{-1}(x) \) onto the fibres \( \pi^{-1}(f(x)) \). Every
1-parameter group $G[\alpha]$ of isomorphisms of $Y \to X$ over diffeomorphisms $f[\alpha]$ of $X$ yields the complete vector field

$$u = u^\lambda(x^\mu)\partial_\lambda + u^i(x^\mu, y^j)\partial_i$$

(5)

which plays the role of the generator of $G[\alpha]$. This vector field is projected onto the vector field $\tau_u = u^\mu\partial_\mu$ on $X$ which is the generator of $f[\alpha]$. For instance, if $G[\alpha]$ is the group of vertical isomorphisms over Id$_X$ (or simply over $X$), the vector field (5) is vertical one $u = u^i(y)\partial_i$. Conversely, one can think on an arbitrary projective vector field $u$ (5) on a bundle $Y$ as being the generator of a local 1-parameter group of local isomorphisms of $Y$.

Given such a group $G[\alpha]$, the Lie derivative $L_u L$ of a Lagrangian $L$ along its generator $u$ is equal to zero iff $L$ is invariant under $G[\alpha]$.

To calculate the Lie derivative $L_u L$ of the Lagrangian (4) along the vector field $u$ (5), one utilizes the canonical lift $j^1u = u^\lambda\partial_\lambda + u^i\partial_i + (d_\lambda u^i - y^i_{\mu}\partial_\lambda u^\mu)\partial^\lambda_i$ (6)
of $u$ onto $J^1Y$ where $d_\lambda = \partial_\lambda + y^i_{\lambda}\partial_i + y^i_{\mu\lambda}\partial^\mu_i$ denote the total derivatives. We have

$$L_u L = [\partial_\lambda u^\lambda L + (u^\lambda \partial_\lambda + u^i \partial_i + (d_\lambda u^i - y^i_{\mu}\partial_\lambda u^\mu)\partial^\lambda_i)L]|\omega.$$ (7)

The first variational formula provides the canonical decomposition of the Lie derivative $L_u L$ in accordance with the variational task. This decomposition takes the coordinate form

$$\partial_\lambda u^\lambda L [\partial_\lambda + u^i \partial_i + (d_\lambda u^i - y^i_{\mu}\partial_\lambda u^\mu)\partial^\lambda_i]L \equiv \quad (8)
$$

$$\quad (u^i - y^i_{\mu}u^\mu)(\partial_i - d_\lambda \partial^\lambda_i)L - d_\lambda[\pi^\lambda_i(u^\mu y^i_{\mu} - u^i) - u^\lambda L]$$

where

$$\delta_i L = (\partial_i - d_\lambda \partial^\lambda_i)L$$ (9)

are the components of the Euler-Lagrange operator and

$$\mathcal{T} = \mathcal{T}^\lambda \omega_\lambda = [\pi^\lambda_i(u^\mu y^i_{\mu} - u^i) - u^\lambda L]|\omega_\lambda, \quad \pi^\lambda_i = \partial^\lambda_i L, \quad \omega_\lambda = \partial_\lambda |\omega,$$ (10)

is the symmetry flow along the vector field $u$.

It should be emphasized that the symmetry flow $\mathcal{T}$ in the first variational formula (8) is not defined uniquely. It has the general form

$$\mathcal{T}^\lambda = [\pi^\lambda_i(u^\mu y^i_{\mu} - u^i) - u^\lambda L] + d_\mu[c^\mu_i(u^i_{\nu} u^\nu - u^i)]$$ (11)

where $c^\mu_i = -c^\mu_i$ are arbitrary skew-symmetric functions on $Y$ which correspond to different Lepagean equivalents of the Lagrangian $L$. The flow (11) corresponds to choice of the Poincaré–Cartan form as the Lepagean equivalent of $L$. 

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The first variational formula (8) on shell

\[(\partial_i - d_\lambda \partial_i^\lambda)\mathcal{L} = 0\]  \hspace{1cm} (12)

comes to the weak transformation law

\[\partial_\lambda u^\lambda \mathcal{L} + [u^\lambda \partial_\lambda + u^i \partial_i + (d_\lambda u^i - y^i_\mu \partial_\lambda u^\mu)\partial_i^\lambda] \mathcal{L} \approx -d_\lambda [\pi^\lambda_i (u^\mu y^i_\mu - u^i) - u^\lambda \mathcal{L}].\]  \hspace{1cm} (13)

If the Lie derivative \(L_u \mathcal{L}\) vanishes, we obtain the conservation law

\[0 \approx d_\lambda [\pi^\lambda_i (u^\mu y^i_\mu - u^i) - u^\lambda \mathcal{L}].\]  \hspace{1cm} (14)

On solutions \(s(x)\) of the differential Euler-Lagrange equations

\[\partial_i \mathcal{L} - (\partial_\lambda + \partial_\lambda s^i \partial_j + \partial_\lambda \partial_\mu s^i \partial_j^\mu)\partial_i^\lambda \mathcal{L} = 0,\]  \hspace{1cm} (15)

the weak identity (14) comes to the differential conservation law

\[0 \approx \frac{d}{dx^\lambda} (\pi^\lambda_i (u^\mu \partial_\mu s^i - u^i) - u^\lambda \mathcal{L}).\]

3 Conservation laws and background fields

Gauge symmetries of Lagrangians are broken in the presence of background fields.

We restrict ourselves to the direct product \(Y_{tot} = Y \times Y'\) of a bundle \(Y\) whose sections are dynamic fields and a bundle \(Y'\) whose sections play the role of background fields. Let \(Y\) and \(Y'\) be coordinatized by \((x^\lambda, y^i)\) and \((x^\lambda, y^A)\) respectively. A total Lagrangian \(L\) of dynamic and background fields is set on the configuration space \(J^1 Y_{tot}\). Dynamic fields are assumed to live on shell (12), whereas the background fields take the background values

\[y^B = \phi^B(x), \quad y^B_\lambda = \partial_\lambda \phi^B(x).\]  \hspace{1cm} (16)

Let us consider projectable vector fields

\[u = u^\lambda(x) \partial_\lambda + u^A(x^\mu, y^B) \partial_A + u^i(x^\mu, y^B, y^j) \partial_i\]  \hspace{1cm} (17)

on \(Y_{tot} \to X\) which are projectable on \(Y \times Y' \to Y'\), for gauge transformations of background fields do not depend on dynamic fields. Substituting (17) into (8), we find the first variational formula in the presence of background fields. This formula on shell (12) of dynamic fields results in the weak identity

\[\partial_\lambda u^\lambda \mathcal{L} + [u^\lambda \partial_\lambda + u^A \partial_A + u^i \partial_i + (d_\lambda u^i - y^i_\mu \partial_\lambda u^\mu)\partial_i^\lambda + (d_\lambda u^A - y^A_\mu \partial_\lambda u^\mu)\partial_A^\lambda] \approx -d_\lambda [\pi^\lambda_i (u^\mu y^i_\mu - u^i) - u^\lambda \mathcal{L}].\]  \hspace{1cm} (18)

\[d_\lambda \pi^\lambda_A (u^A - y^A_\lambda \partial_\lambda \mathcal{L} + \pi^\lambda_A \partial_\lambda \mathcal{L}) + \pi^\lambda_A d_\lambda (u^A - y^A_\lambda \partial_\lambda \mathcal{L}).\]
In practice, total Lagrangians, by construction, are invariant under gauge transformations, and their Lie derivatives along the corresponding vector fields (17) are equal to zero. In this case, the identity (18) is reduced to the transformation law

\[ 0 \approx -d\lambda [\pi^\lambda_i (u^\mu y^i_\mu - u^i) - u^\lambda L] + (u^A - y^A_\lambda u^\lambda) \partial_A L + \pi^\lambda_A d\lambda (u^A - y^A_\mu u^\mu) \] (19)

where the two last terms characterize disturbance of the conservation law (14) in the presence of background fields (16).

### 4 SEM conservation laws

It is readily observed that the transformation law (13) is linear in a vector field \( u \). Therefore, one can consider superposition of the transformation laws along different vector fields.

For instance, every vector field on \( Y \) projected onto a vector field \( \tau \) on \( X \) is the sum of the lift of \( \tau \) onto \( Y \) and of a vertical vector field on \( Y \). It follows that every transformation law (13) derived from the first variational formula (8) appears to be superposition of (i) the Noether type transformation law

\[ [u^i \partial_i + d\lambda u^i \partial^\lambda_i] L \approx d\lambda (\pi^\lambda_i u^i) \] (20)

for the Noether flow

\[ T^\lambda = -\pi^\lambda_i u^i \] (21)

along a vertical vector field and of (ii) the SEM transformation law along the lift of a vector field \( \tau \) on \( X \) onto \( Y \) [7, 9, 25].

A vector field \( \tau \) on \( X \) gives rise to a vector field on a bundle \( Y \to X \) only by means of a connection on \( Y \).

We follow the definition of connections on a bundle as sections

\[ \Gamma = dx^\lambda \otimes (\partial_\lambda + \Gamma^i_\lambda \partial_i) \] (22)

of the affine jet bundle \( J^1 Y \to Y \). For instance, a linear connection \( K \) on the tangent bundle \( TX \) of \( X \) and the dual connection \( K^* \) on the cotangent bundle \( T^* X \) read

\[ K^\alpha_\lambda = -K^\alpha_\nu (x) \dot{x}^\nu, \quad K^*_{\alpha\lambda} = K^*_{\nu\alpha}(x) \dot{x}^\nu \] (23)

where \( \dot{x}^\nu \) and \( \dot{x}_\nu \) are induced coordinates with respect to the holonomic frames \( \{ \partial_\nu \} \) and \( \{ dx^\nu \} \) in \( TX \) and \( T^* X \) respectively. We shall call (23) the world connections. Difference of two connections \( \Gamma \) and \( \Gamma' \) (24) is a soldering 1-form

\[ \Gamma - \Gamma' = (\Gamma^i_\lambda - \Gamma'^i_\lambda) dx^\lambda \otimes \partial_i \]
on $Y$ which takes its values into the vertical tangent bundle $VY$ of $Y$. Every connection $\Gamma$ yields the first order differential operator
\[
D : J^1Y \to T^*X \otimes VY, \quad D = (y^i_\lambda - \Gamma^i_\lambda)dx^\lambda \otimes \partial_i,
\]
on $Y$ called the covariant differential with respect to $\Gamma$.

Let $\tau = \tau^\mu \partial_\mu$ be a vector field on $X$ and $\tau_\Gamma = \tau \circ \Gamma = \tau^\mu (\partial_\mu + \Gamma^i_\mu \partial_i)$ (25)
it its horizontal lift onto $Y$ by a connection $\Gamma$ (22) on $Y$.

The weak identity (13) along the vector field (25) takes the form
\[
\mathcal{L}_\tau_\Gamma \approx -d \lambda (\tau^\mu \mathcal{T}^\lambda_\mu) \omega
\]
along with the system of weak equalities
\[
(\partial_\mu + \Gamma^i_\mu \partial_i + d_\lambda \Gamma^i_\mu \partial_\lambda)\mathcal{L} \approx -d_\lambda \mathcal{T}^\lambda_\mu.
\]
\[
\mathcal{L}_\tau \approx -d_\lambda \mathcal{T}^\lambda_\mu
\]
where
\[
\mathcal{T}^\lambda_\mu = \pi^\lambda_i (y^i_\mu - \Gamma^i_\mu) - \delta^\lambda_\mu \mathcal{L}
\]
is termed the SEM tensor relative to the connection $\Gamma$ [6, 7, 9, 11].

To obtain SEM conservation laws, one may choose different connections $\Gamma$ for different vector fields $\tau$ on $X$ and for different solutions of the Euler–Lagrange equations. The SEM flows $\mathcal{T}_\Gamma$ and $\mathcal{T}_{\Gamma'}$ relative to different connections $\Gamma$ and $\Gamma'$ differ from each other in the Noether flow (21) along the vertical vector field $u = \tau^\mu (\Gamma^i_\mu - \Gamma'^i_\mu)$.

If all vector fields $\tau$ on $X$ gives rise onto $Y$ by means of the same connection $\Gamma$, the transformation law (26) is equivalent to the system of weak equalities
\[
(\partial_\mu + \Gamma^i_\mu \partial_i + d_\lambda \Gamma^i_\mu \partial_\lambda)\mathcal{L} \approx -d_\lambda \mathcal{T}^\lambda_\mu.
\]
\[
\mathcal{L}_\tau \approx -d_\lambda \mathcal{T}^\lambda_\mu
\]
For instance, let us choose the trivial local connection $\Gamma^i_\mu = 0$. In this case, the identity (26) recovers the well-known transformation law
\[
\frac{\partial \mathcal{L}}{\partial x^\mu} + \frac{d}{dx^\lambda} \mathcal{T}_0^\lambda_\mu(s) \approx 0
\]
of the canonical energy-momentum tensor
\[
\mathcal{T}_0^\lambda_\mu(s) = \pi^\lambda_i \partial_\mu s^i - \delta^\lambda_\mu \mathcal{L}
\]
which however fails to be a true tensor. At the same time, the transformation law (29) on solutions $s$ of the differential Euler–Lagrange equations (13) is well-defined. It issues from the identity (29) when, for every solution $s$, we choose the proper connection $\Gamma$ such that $s$ is
an integral section of $\Gamma$, i.e., $\Gamma^i \circ s = \partial_{\mu} s^i$. The transformation law (\ref{29}) does not contain the Noether flows associated with vertical gauge transformations. Each transformation law (\ref{28}) on solutions $s$ of the differential Euler–Lagrange equations (\ref{15}) represents a superposition of (\ref{29}) and the transformation law

$$\left(\Gamma^i_{\mu} \partial_i + d\lambda^i_{\mu} \partial^i_{\lambda}\right) L \approx \frac{d\lambda}{\pi^\lambda_{\mu}} \left(\pi^\lambda_{\mu} \Gamma^i_{\lambda}\right). \tag{30}$$

The latter looks locally like the Noether transformation law (\ref{24}) where $u^i = \tau^\mu \Gamma^i_{\mu}$, but the entity $\tau^\mu \pi^\lambda_{\mu} \Gamma^i_{\lambda}$ is not well-behaved. At the same time, if the Lie derivatives of a Lagrangian $L$ along all local vector field $\tau^\mu \Gamma^i_{\mu}$ vanishes, one can equate the left-hand side of (\ref{30}) to zero.

Building on this fact, one can use invariance of Lagrangians under vertical gauge transformations, e.g., under internal symmetry transformations in order to simplify SEM conservation laws. If a Lagrangian is not invariant under internal gauge transformations, the total Lie derivative does not vanish and the SEM flow fails to be conserved in general.

## 5 General covariant transformations

Generators of general covariant transformations exemplify the bundle lift of a vector field $\tau$ on $X$ by means of the proper connection.

Let $LX \to X$ be the principal bundle of oriented linear frames in the tangent spaces to a world manifold $X$. Its structure group is $GL_4 = GL_4^+(4, \mathbb{R})$. Let $T$ denote the bundles associated with $LX$. They are exemplified by tensor bundles

$$T = (\otimes^m TX) \otimes (\otimes^k T^*X). \tag{31}$$

Recall that a structure group $G$ of a principal bundle $\pi_P : P \to X$ acts canonically on $P$ on the right

$$r_g : p \mapsto pg, \quad \pi_P(p) = \pi_P(pg), \quad p \in P, \quad g \in G. \tag{32}$$

Accordingly, every $P$-associated bundle $Y \to X$ with a standard fibre $V$ is isomorphic to the quotient $Y = (P \times V)/G$ with respect to identification of elements $(p, v)$ and $(pg, g^{-1}v)$ for all $g \in G$. Isomorphisms $\Phi$ of a principal bundle $P$, by definition, are equivariant under the canonical action (\ref{32}), that is, $r_g \circ \Phi = \Phi \circ r_g$. They generate the corresponding isomorphisms

$$\Phi_Y : (P \times V)/G \to (\Phi(P) \times V)/G \tag{33}$$

of bundles $Y$ associated with $P$.

Turn to the principal frame bundle $LX$. With respect to holonomic frames $\{\partial_{\mu}\}$, every element $\{H_a\}$ of $LX$ takes the form $H_a = S^\mu_a \partial_{\mu}$ where $S^\mu_a$ are matrices of representation of the group $GL_4$ in $\mathbb{R}^4$. They constitute the bundle coordinates

$$(x^\lambda, S^\mu_a), \quad S^\mu_a = \frac{\partial x^\mu}{\partial x^\lambda} S^\lambda_a.$$
of \( LX \). Relative to these coordinates, the canonical action (32) of \( GL_4 \) on \( LX \) reads \( r_g : S^\mu_a \mapsto S^{\mu}_b g^b_a \).

The peculiarity of the frame bundle \( LX \to X \) lies in the fact that every diffeomorphism \( f \) of \( X \) gives rise canonically to the isomorphism

\[
\bar{f} : (x^\lambda, S_{a}^{\lambda}) \mapsto (f^\lambda(x), \partial^\lambda f S^\mu_a)
\]  

(34)
of \( LX \) and to the corresponding isomorphisms (33) of associated bundles \( T \). These isomorphisms are called the general covariant transformations or the holonomic isomorphisms because they send holonomic frames onto holonomic frames.

The lift (34) implies the canonical lift \( \tilde{\tau} \) of every vector field \( \tau \) on \( X \) onto the principal bundle \( LX \) and the associated bundles. This lift takes the form

\[
\tilde{\tau} = \tau^\mu \partial_\mu + \partial_\nu \tau^{\alpha_1 \cdots \alpha_m} x^{\alpha_1 \cdots \alpha_m} + \cdots - \partial_\beta \tau^{\nu} x_\beta \partial_\nu - \cdots \frac{\partial}{\partial \dot{x}_\alpha}
\]  

(35)
on the tensor bundle (31) and, in particular,

\[
\tilde{\tau} = \tau^\mu \partial_\mu + \partial_\nu \tau^{\alpha} x^{\nu} \frac{\partial}{\partial \dot{x}_\alpha}, \quad \tilde{\tau} = \tau^\mu \partial_\mu - \partial_\beta \tau^{\nu} \dot{x}_\nu \frac{\partial}{\partial \dot{x}_\beta}
\]  

(36)
on the tangent bundle \( TX \) and the cotangent bundle \( T^*X \) respectively.

In fact, the canonical lift \( \tilde{\tau} \) (35) is the horizontal lift (25) by means of the world connection (23) which meets \( \tau \) as a horizontal vector field \( \partial_\mu \tau^\beta = -K^\beta_\lambda \tau^\lambda \).

One can construct the horizontal lift \( \tau_K \) of a vector field \( \tau \) on \( X \) onto \( LX \) by means of any world connection \( K \). Such a lift is the generator of nonholonomic isomorphisms of \( LX \). Transformations of this type are called into play in the framework of gauge theories of the general linear group \( GL_4 \) \( [12] \). At the same time, the term \( -y_i^\mu \partial_\lambda u^\nu \partial_\nu^\lambda \) in the jet lift (3) of a vector field on a bundle \( Y \) is exactly the canonical lift (36). This is the reason why we should consider the canonical lift \( \tilde{\tau} \) of vector fields on \( X \) onto the \( LX \)-associated bundles \( T \), in particular, onto the bundle \( \Sigma_g \subset \hat{2}TX \) of pseudo-Riemannian metrics on \( X \). Indeed, gravitation Lagrangians are not invariant under nonholonomic gauge transformations in general.

6 Superpotential

The main peculiarity of SEM conservation laws along generators of general covariant transformations consists in the phenomenon that the corresponding SEM flow is reduced to the sum of a superpotential term and a terms which displays itself only in the presence of a background metric on \( X \).
To illustrate clearly this phenomenon, we consider tensor fields. Let $T$ be a tensor bundle coordinatized by $(x^\lambda, y^A)$ where the collective index $A$ is employed. Given a vector field $\tau$ on $X$, its canonical lift $\tilde{\tau}$ on $T$ reads

$$\tilde{\tau} = \tau^\lambda \partial_\lambda + u^A_\alpha \partial_\lambda \tau^\alpha \partial_A.$$  

In this case, the strong equality $L\tilde{\tau}L = 0$ and the weak identity (14) take the coordinate form

$$0 \approx d\lambda [g^\lambda_A(y^A_\alpha \tau^\alpha - u^A_\alpha \partial_\lambda \tau^\alpha - \tau^\lambda \mathcal{L}].$$  

Due to the arbitrariness of the functions $\tau^\alpha$, the equality (38) is equivalent to the system of equalities

$$\partial_\lambda \mathcal{L} = 0,$$

$$\delta_\beta^\alpha \mathcal{L} + u^A_\alpha \partial_\lambda \tau^\alpha \partial_A \mathcal{L} + d_\mu(u^A_\alpha \partial_\beta \tau^\alpha)\pi_\mu^A - y^A_\alpha \partial_\lambda \tau^\alpha \pi^\beta_A = 0,$$

$$u^A_\alpha \pi^\beta_A + u^A_\alpha \pi^\beta_A = 0.$$  

Substituting the relations (40b) and (40c) into the weak identity (39), we get the conservation law

$$0 \approx d\lambda[u^A_\alpha \delta_A \mathcal{L} \tau^\alpha + d_\mu(u^A_\alpha \pi^\mu_A \tau^\alpha)]$$

where $\delta_A \mathcal{L}$ are the variational derivatives (9). A glance at the expression (41) shows that the conserved flow on shell takes the form

$$\mathcal{T} = W + d_H U,$$

$$\mathcal{T}^\lambda = (W^\lambda + d_\mu U^\mu), \quad U^\lambda = -U^\lambda, \quad (42)$$

where $W \approx 0$ and

$$U^\lambda = u^A_\alpha \pi^\mu_A \tau^\alpha.$$  

is a superpotential. It is readily observed that the superpotential (13) arises since the lift $\tilde{\tau}$ (37) depends on the derivatives of the components of the vector field $\tau$. At the same time, dependence of the superpotential (13) on the vector field $\tau$ reflects the fact that the SEM conservation law (42) is maintained under general covariant transformations.

Also in gauge theory, if vector fields (5) depend on derivatives of the parameters of gauge transformations, the conserved flow $\mathcal{T}$ is brought into the form (12). For instance, the conserved flow $-\mathcal{T}^\lambda$ (12) in electromagnetic theory is the familiar electric current, the superpotential $4\pi U^{\mu\lambda}$ consists with the electromagnetic strength, and the relations (12) are exactly the Maxwell equations.

Let us consider now tensor fields in the presence of a background pseudo-Riemannian metric on a world manifold $X$. The total configuration space of this model is the jet manifold.
\( J^1Y \) of the product \( Y = T \times \Sigma_g \) coordinatized by \((x^\lambda, y^A, \sigma^{\mu\nu})\). Given a vector field \( \tau \) on \( X \), its canonical lift onto \( Y \) reads
\[
\tilde{\tau} = \tau^\lambda \partial_\lambda + u^{A\beta}_\alpha \partial_\beta \partial_A + (\partial_\nu \tau^\alpha \sigma^{\nu\beta} + \partial_\nu \tau^\beta \sigma^{\nu\alpha}) \partial_\alpha \beta. \tag{44}
\]

Let the total Lagrangian \( L \) of tensor fields and a metric field be invariant under general covariant transformations. Its Lie derivative along the vector field \( \tilde{\tau} \) reads
\[
(\partial_\nu \tau^\alpha \lambda + \partial_\nu \tau^\alpha \sigma^{\nu\beta} + \partial_\nu \tau^\beta \sigma^{\nu\alpha}) \partial_\alpha \beta + (\partial_\nu \tau^\alpha \lambda + \partial_\nu \tau^\beta \sigma^{\nu\alpha}) \partial_\alpha \beta = 0, \tag{45a}
\]
and the weak transformation law \( (19) \) which restricted to \( \sigma^{\mu\nu} = g^{\mu\nu}(x) \) takes the form
\[
0 \approx -d_\lambda\left[ \pi^\lambda_\mu(y^A_\alpha \tau^\alpha - u^{A\beta}_\alpha \partial_\beta \tau^\alpha) - \tau^\lambda \partial_\mu \right] + (\partial_\nu \tau^\alpha \lambda + \partial_\nu \tau^\beta \sigma^{\nu\alpha} - \partial_\lambda \sigma^{\nu\beta} \tau^\lambda) \partial_\alpha \beta. \tag{46}
\]
Substituting \( (45a) \) and \( (45b) \) into \( (46) \), we get the transformation law
\[
0 \approx -d_\lambda\left[ \tau^\mu_\lambda \sqrt{|g|} + u^{A\lambda}_\alpha \sigma^{\mu\alpha} \partial_\lambda \partial_\mu \right] + \partial_\lambda \tau^\mu_\lambda \sqrt{|g|} + \tau^\mu_\beta \sqrt{|g|} \{ \beta^{\mu\alpha} \}
\]
where
\[
t^\mu_\beta \sqrt{|g|} = 2g^{\mu\nu} \partial_\alpha \beta \tag{47}
\]
is the metric energy-momentum tensor of tensor fields. A glance at this transformation law expression shows that the SEM flow of tensor fields in the presence of a background metric is the sum
\[
T^\lambda = \tau^\mu_\lambda t^\mu_\beta \sqrt{|g|} + d_\mu(u^{A\lambda}_\alpha \pi^{\mu\alpha} \tau^\alpha)
\]
of the metric energy-momentum tensor \( t^\lambda_\mu \) and the superpotential \( (13) \). The latter does not make any contribution into the differential transformation law which takes the familiar form
\[
\nabla_\lambda t^\lambda_\mu \approx 0.
\]
At the same time, if a metric field is dynamic, the metric energy-momentum tensor \( (17) \) on shell comes to zero. Thus, we observe that the local part of the SEM flow in gravitation models displays itself only if symmetry under general covariant transformations is broken. This is the phenomenon of "hidden" energy.

7 Energy-momentum of affine-metric gravity

As we show below, gauge gravitation theory is reduced to the affine-metric gravitation model in the presence of fermion fields. Therefore, let us consider briefly the SEM conservation laws
in the affine-metric theory of gravity without matter where dynamic variables are a pseudo-Riemannian metric and a world connection on $X$. Being principal connections on $L_X$, the world connections are represented by sections $K$ of the quotient bundle $C := J^1L_X/GL_A → X$. (48)

This bundle is coordinatized by $(x^\lambda, k^{\alpha\beta\lambda})$ so that $k^{\alpha\beta\lambda} = k^{\alpha\beta\lambda\nu}$. The bundle $C$ (48) is not associated with $L_X$, but it is an affine bundle modelled on the vector bundle associated with $L_X$. There exists the canonical lift

$$\tilde{\tau} = \tau^\mu \partial_\mu + [\partial_\nu \tau^\alpha k_{\beta\mu} - \partial_\beta \tau^\nu k_{\alpha\nu\mu} - \partial_\mu \tau^\nu k_{\alpha\beta\nu} - \partial_\beta \tau^\nu] \frac{\partial}{\partial k^{\alpha\beta\mu}}$$

of a vector field $\tau$ on $X$ onto $C$.

The total configuration space of affine-metric gravity is the jet manifold $J^1Y$ of the product $Y = \Sigma_g \times C$ coordinatized by $(x^\lambda, \sigma^{\alpha\beta}, k^{\alpha\beta\lambda})$. Given a vector field $\tau$ on $X$, its canonical lift onto this product reads

$$\tilde{\tau} = \tau^\lambda \partial_\lambda + (u_A^{\beta\alpha} \partial_\beta \tau^\alpha - u_A^{\alpha\beta} \partial_\epsilon \tau^\alpha) \partial_A.$$

For the sake of simplicity, let us utilize the compact notation

$$\tilde{\tau} = \tau^\lambda \partial_\lambda + (u_A^{\beta\alpha} \partial_\beta \tau^\alpha - u_A^{\alpha\beta} \partial_\epsilon \tau^\alpha) \partial_A.$$

We assume that a Lagrangian density $L_{AM}$ of affine-metric gravity depends on the curvature tensor

$$R^{\alpha\beta\nu\lambda} = k^{\alpha\beta\lambda\nu} - k^{\beta\alpha\nu\lambda} + k^{\alpha\epsilon\nu k^{\beta\lambda} - k^{\alpha\epsilon\lambda k^{\epsilon\beta\nu}}.$$

Then, there are the corresponding relations

$$\frac{\partial L_{AM}}{\partial k^{\alpha\beta\nu}} = \pi_\sigma^{\beta\nu} k_{\alpha\lambda} - \pi_\sigma^{\alpha\nu} k_{\beta\lambda}, \quad \pi_\sigma^{\beta\nu} = -\pi_\alpha^{\beta\lambda\nu}. (51)$$

Let $L_{\tilde{\tau}}L_{AM} = 0$. We get the weak conservation law

$$0 \approx d_\lambda [\pi_A^{\lambda}(u_A^{\beta\alpha} \partial_\beta \tau^\alpha - u_A^{\alpha\beta} \partial_\epsilon \tau^\alpha - y_A^{\alpha} \tau^\alpha) + \tau^\lambda L]$$

where

$$\pi_A^{\lambda}(u_A^{\beta\alpha} = \pi_\alpha^{\epsilon\beta\lambda},$$

$$\pi_\sigma^{\epsilon\beta\alpha} = \pi_\alpha^{\gamma\mu \epsilon \beta k_{\gamma\epsilon}} - \pi_\sigma^{\beta\mu \epsilon k_{\alpha\mu}} - \pi_\sigma^{\gamma\beta\epsilon k_{\sigma\gamma}} = \partial_\alpha^{\beta\epsilon} L_{AM} - \pi_\sigma^{\gamma\beta\epsilon} k_{\sigma\gamma\alpha}.$$
and the strong equality

$$\delta^\alpha_\beta L_{AM} + \sqrt{-g} T^\beta_\alpha + u^A_\alpha \partial_A L_{AM} + d_\mu (u^A_\alpha) \pi^\mu_A = y^A_\alpha \pi^\beta_A$$

(53)

where one can think of

$$\sqrt{-g} T^\beta_\alpha = 2 \sigma^\beta_\nu \partial_\nu A$$

as being the metric energy-momentum tensor of world connections.

Substituting \(y^A_\alpha \pi^\beta_A\) from the expression (53) into the conservation law (52), we bring the latter into the form

$$0 \approx -d_\lambda [\sqrt{-g} T^\lambda_\alpha \tau^\alpha - \pi^\lambda_A (u^A_\alpha \partial_\beta \tau^\alpha + u^A_\alpha \partial_\alpha \tau^\beta) + u^A_\alpha \tau^\alpha \partial_A L_{AM} + \pi^\mu_A d_\mu (u^A_\alpha) \tau^\alpha].$$

After separating the variational derivatives, we find that the SEM conservation law (52) of affine-metric gravity comes to the superpotential form

$$0 \approx -d_\lambda [2 g^\lambda_\mu \tau^\alpha \delta^\lambda_\alpha \partial_\alpha \tau^\alpha + (k^\lambda_\gamma \mu \delta^\gamma_\alpha \partial_\alpha \tau^\mu \tau^\lambda - k^\sigma_\gamma \delta^\lambda_\alpha \partial_\alpha \tau^\gamma \tau^\sigma - k^\sigma_\alpha \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma - k^\sigma_\gamma \delta^\lambda_\gamma \partial_\gamma \tau^\lambda \tau^\sigma$$

(54)

where \(U\) is the generalized Komar superpotential (3).

Let us now consider the total system of the affine-metric gravity and, e.g., a covector Proca field coordinatized by \(y_\mu = \dot{x}_\mu\). Proca fields can model the gauge potentials when internal symmetries are ignored. In virtue of the relation (40c), the Lagrangian \(L_P\) of Proca fields depends on the strength \(F_{\mu\nu} = y_{\nu\mu} - y_{\mu\nu} - \Omega^\sigma_{\nu\mu} y_{\sigma}\)

where \(\Omega^\sigma_{\nu\mu} = k^\sigma_{\nu\mu} - k^\sigma_{\mu\nu}\) is the torsion tensor. In this case, the superpotential term in the energy-momentum flow of the Proca fields (see (43)) is eliminated due to the additional contribution \(-d_\mu (\partial_\lambda \partial^\mu L_p \tau^\alpha)\). The total SEM conservation law of affine-metric gravity and Proca fields takes the form (54) where \(L_{AM}\) is replaced with \(L_{AM} + L_P\), but \(U\) remains the generalized Komar superpotential (3). We observe below that also fermion fields do not contribute into the total SEM flow because of interaction with a torsion.

### 8 Gauge gravitation theory

In gauge gravitation theory, a gravitational field appears as a Higgs field corresponding to spontaneous symmetry breaking because of the Dirac fermion fields [21].

We describe Dirac fermions as follows [3, 16, 20]. Given a Minkowski space \(M\), let \(\mathbb{C}_{1,3}\) be the complex Clifford algebra generated by elements of \(M\). A spinor space \(V\) is defined to be a minimal left ideal of \(\mathbb{C}_{1,3}\) on which this algebra acts on the left. We have representation
\( \gamma : M \otimes V \to V \) of elements of the Minkowski space \( M \subset C_{1,3} \) by the Dirac's matrices \( \gamma \) on \( V \). Let us consider the pairs \((l, l_s)\) of the Lorentz transformations \( l \) of the Minkowski space \( M \) and the invertible elements \( l_s \) of \( C_{1,3} \) such that
\[
\gamma(lM \otimes l_s V) = l_s \gamma(M \otimes V).
\]

Elements \( l_s \) constitute the Clifford group whose action on \( M \) however is not effective. We take its spin subgroup \( L_s = SL(2, C) \) with the generators
\[
I_{ab} = \frac{1}{4}[\gamma_a, \gamma_b].
\]

Let us consider some bundle of Clifford algebras \( C_{3,1} \) over a world manifold \( X \). Its subbundles are both a spinor bundle \( S_M \to X \) and the bundle \( Y_M \to X \) of Minkowski spaces of generating elements of \( C_{3,1} \). To describe Dirac fermion fields on a world manifold \( X \), the bundle \( Y_M \) must be isomorphic to the cotangent bundle \( T^*X \) of \( X \). It takes place if the structure group of \( LX \) is reducible to the connected Lorentz group \( L \) and \( LX \) contains a reduced \( L \)-principal subbundle \( L^hX \) such that
\[
Y_M = (L^hX \times M)/L.
\]

In this case, the spinor bundle
\[
S_M = S_h = (P_h \times V)/L_s \tag{55}
\]
is associated with the \( L_s \)-lift \( P_h \overset{\gamma}{\to} L^hX \) of \( L^hX \).

In accordance with the well-known theorem, there is the 1:1 correspondence between the reduced \( L \) subbundles \( L^hX \) of \( LX \) and the global sections \( h \) of the quotient bundle
\[
\Sigma := LX/L \to X \tag{56}
\]
with the standard fibre \( GL_4/L \). This bundle is the 2-fold covering of the bundle \( \Sigma_g \) of pseudo-Riemannian metrics. Its sections are called the tetrad fields.

Every tetrad field \( h \) defines an associated Lorentz atlas \( \Psi^h \) of \( LX \) such that the corresponding local sections \( z^h_\xi \) of \( LX \) take their values into \( L^hX \). Given \( \Psi^h \) and a holonomic atlas \( \Psi = \{\psi_\xi\} \) of \( LX \), the tetrad field \( h \) can be represented by a family of local \( GL_4 \)-valued tetrad functions
\[
h_\xi = \psi_\xi \circ z^h_\xi, \quad dx^\lambda = h^\lambda_a(x)h^a, \quad g_{\mu\nu} = h^{\mu}_a h^{\nu}_b \eta^{ab}. \tag{57}
\]

For every tetrad field \( h \), there exists representation
\[
\gamma_h : T^*X \otimes S_h = (P_h \times (M \otimes V))/L_s \to (P_h \times \gamma(M \otimes V))/L_s = S_h \tag{58}
\]
of covectors to a world manifold $X^4$ by the Dirac’s $\gamma$-matrices on elements of the spinor bundle $S_h$. Relative to an atlas $\{}z_\xi\{}$ of $P_h$ and the associated Lorentz atlas $\{}z^h_\xi = z \circ z_\xi\{}$ of $LX$, the representation (58) reads

$$\gamma_h(h^a \otimes y^A v_A(x)) = \gamma^{aA}_B y^B v_A(x)$$

where $\{v_A(x)\}$ are the corresponding fibre bases for $S_h$. As a shorthand, we write

$$\hat{dx}^\lambda = \gamma_h(dx^\lambda) = h^\lambda_a(x)\gamma^a.$$

One may say that the representation (58) sets a Dirac spin structure on a world manifold. Sections $\psi_h$ of the spinor bundle $S_h$ describe Dirac fermion fields in the presence of the tetrad field $h$. Indeed, let $A_h$ be a principal connection on $S_h$ and

$$D : J^1 S_h \to T^* X \otimes S_h,$$

$$D = (y^A - A^{ab}_\lambda I_{ab}^A y^B) dx^\lambda \otimes \partial_A,$$

the corresponding covariant differential (24). Then, the first order differential Dirac operator

$$D_h = \gamma_h \circ D : J^1 S_h \to T^* X \otimes S_h \to S_h,$$

$$y^A \circ D_h = h^\lambda_a(y^B - A^{ab}_\lambda I_{ab}^A y^B),$$

is defined on $S_h$.

The crucial point lies in the fact that, for different tetrad fields $h$ and $h'$, the representations $\gamma_h$ and $\gamma_{h'}$ (58) are not equivalent. Given two reduced Lorentz subbundles $L^h X$ and $L^{h'} X$, there exists the vertical isomorphism $\Phi$ of the bundle $LX$ which sends $L^h X$ onto $L^{h'} X$. However, it does not give rise to a morphism $\Phi_s : P_h \to P_{h'}$ in general. Moreover, the bundles $L^h X$ and $L^{h'} X$ (accordingly, $S_h$ and $S_{h'}$) are not the equivalent locally trivial bundles since the union of their atlases fails to be an atlas with the Lorentz transition functions. This is the fact that characterizes the symmetry breaking picture in gravitation theory and the physical nature of gravity as a Higgs field.

We may follow the general procedure of describing spontaneous symmetry breaking in gauge theory [22, 24]. Since different tetrad fields $h$ and $h'$ define nonequivalent representations $\gamma_h$ and $\gamma_{h'}$, each Dirac fermion field is regarded only in a pair with a certain tetrad field $h$. There is the 1:1 correspondence between these pairs $(\psi_h, h)$ and the sections of the composite spinor bundle

$$S \to \Sigma \to X$$

where $\Sigma$ is the quotient bundle (60). This spinor bundle is defined as follows [21, 24].

Given the trivial $L$-principal bundle $LX \to \Sigma$, let us consider the $L_s$-principal bundle $P_\Sigma \to \Sigma$ which is the 2-fold covering $P_\Sigma \to LX$ of $LX$. In particular, there is imbedding of
the $L_{\ast}$-lift $P_{\ast}$ of $L^hX$ onto the restriction of $P_{\Sigma}$ to $h(X)$. Remark that the bundle $P_{\Sigma} \to X$ is not diffeomorphic to the universal covering of $LX$ with the structure group $\overline{GL}(4, \mathbb{R})$.

The composite spinor bundle $S \to \Sigma$ is defined to be the $P_{\Sigma}$ associated bundle

$$S = (P_{\Sigma} \times V)/L_s.$$ 

One may say that the bundle (60) introduces the universal Dirac spin structure on a world manifold since, given a global section $h$ of $\Sigma$, the restriction of $S_{\Sigma}$ to $h(X)$ is isomorphic to the spinor bundle $S_h(55)$.

Let us provide the principal bundle $LX$ with a holonomic atlas $\{\psi_\xi, U_\xi\}$ and the principal bundles $P_{\Sigma}$ and $LX \to \Sigma$ with associated atlases $\{z^s, U_s\}$ and $\{z, U\} \subset U_\xi$. Relative to these atlases, the composite spinor bundle $S$ is endowed with the bundle coordinates $(x^\lambda, \sigma^a_\mu, \psi_A)$ where $(x^\lambda, \sigma^a_\mu)$ are coordinates of $\Sigma$ such that $\sigma^a_\mu$ are the matrix components of the group element $(\psi_\xi \circ z_\epsilon)(\sigma), \sigma \in U_\epsilon, \pi_{X\Sigma}(\sigma) \in U_\xi$. Given a section $h$ of $\Sigma$, we have $(\sigma^a_\mu \circ h)(x) = h^a_\mu(x)$ where $h^a_\mu(x)$ are the tetrad functions (57).

Let us consider the bundle of Minkowski spaces $(LX \times M)/L \to \Sigma$. Since it is isomorphic to the pullback $\Sigma \times T^*X$, there exists the representation

$$\gamma : T^*X \otimes S = (P_{\Sigma} \times (M \otimes V))/L_s \to (P_{\Sigma} \times \gamma(M \otimes V))/L_s = S,$$ 

$$\hat{dx}^\lambda = \gamma_{\Sigma}(dx^\lambda) = \sigma^\lambda_a \gamma^a.$$ 

This representation restricted to $h(X) \subset \Sigma$ recovers the morphism $\gamma_h$ (58). We are based on this representation in order to construct the total Dirac operator on the composite spinor bundle $S$ as follows.

Every principal connection

$$\tilde{A} = dx^\lambda \otimes (\partial_\lambda + A^B_\lambda \partial_B) + d\sigma^a_\mu \otimes (\partial_\mu + A^{B_a}_\mu \partial_B)$$ 

(62)

on the bundle $S \to \Sigma$ yields the first order differential operator

$$\tilde{D} : J^1Y \to T^*X \otimes S,$$

$$\tilde{D} = dx^\lambda \otimes (\psi_B^\lambda - \tilde{A}^B_\lambda - A^{B_a}_\mu \sigma^{a}_\mu \partial_B)$$ 

(63)

on $S$. Let $h$ be a global section of $\Sigma$ and $S_h$ the restriction of the bundle $S \to \Sigma$ to $h(X)$. The corresponding restriction of $\tilde{D}$ to $J^1S_h \subset S^1Y$ recovers to the familiar covariant differential on $S_h$ with respect to the connection

$$A_h = dx^\lambda \otimes [\partial_\lambda + (\tilde{A}^B_\lambda + A^{B_a}_\mu \partial_\lambda h^a_\mu) \partial_B].$$ 

(64)
Building on the representation (61) and the differential (63), we construct the first order differential operator

\[ D = \gamma_\Sigma \circ \bar{D} : J^1S \to T^*_\Sigma S \to S, \]

\[ \psi^A \circ D = \sigma^\lambda_A \gamma^A_B (\psi^B_\lambda - \bar{A}^B_\lambda - A^B_\mu \sigma^\mu_{a\lambda}), \]

on \( S \). One can think of \( D \) as being the total Dirac operator since, for every tetrad field \( h \), the restriction of \( D \) to \( J^1S_h \subset J^1S \) comes to the Dirac operator \( D_h \) relative to the connection (64) on the bundle \( S_h \).

Let us choose the principal connection \( \bar{A} \) (62) on \( P_\Sigma \) given by the following coefficients of the local connection form:

\[ \bar{A} = (\bar{A}^a_{\mu} dx^\mu + A^{ab}_{\mu} d\sigma^\mu_a) \otimes I_{ab}, \]

\[ \bar{A}^{ab}_{\mu} = \frac{1}{2} \left( \eta^{ca} \sigma^b_{\mu} - \eta^{cb} \sigma^a_{\mu} \right), \]

where \( K \) is a world connection on \( X \) and (66) corresponds to the canonical left-invariant free-curvature connection on the bundle \( GL_4 \to GL_4/L \). The corresponding differential \( \bar{D} \) (63) reads

\[ \bar{D} = dx^\lambda \otimes [\partial_\lambda \left( \frac{1}{2} A^{abc}_{\mu} (\sigma^\mu_{c\lambda} + K^\mu_{\nu\lambda} \sigma^\nu_{c}) I_{ab} \psi^B B \partial_A \right). \]

Given a tetrad field \( h \), the connection \( \bar{A} \) (65) is reduced to the principal connection

\[ \bar{K}^{ab}_{\lambda \mu} = A^{abc}_{\mu} (\partial_\lambda h^c_\mu + K^\mu_{\nu\lambda} h^\nu_c) \]

on \( S_h \), and the differential (67) comes to the covariant differential of Dirac fermion fields in the presence of the tetrad field \( h \) and the world connection \( K \). As a result, gauge gravitation theory is reduced to the model of affine-metric gravity and fermion fields.

The total configuration space of this model is the jet manifold \( J^1Y \) of the bundle

\[ Y = C \times S \oplus S^+ \]

coordinatized by \( (x^\mu, \sigma^\mu_a, k^\mu_{\nu\lambda}, \psi^A, \psi^+_A) \). The total Lagrangian on this configuration space is the sum

\[ L = L_{AM} + L_\psi \]

of (i) the affine-metric Lagrangian \( L_{AM}(R^\alpha_{\beta\nu\lambda}, \sigma^\mu\nu) \) expressed into the curvature tensor \( R^\alpha_{\beta\nu\lambda} \) (50) and the metric tensor \( \sigma^\mu\nu = \sigma^\mu_a \sigma^\nu_b \eta^{ab} \) and (ii) the Lagrangian of fermion fields

\[ L_\psi = \left( \frac{i}{2} \bar{\psi}^+_A (\gamma^0 \gamma^\lambda)_A^B \psi^B - \frac{1}{2} A^{abc}_{\mu} (\sigma^\mu_{c\lambda} + k^\mu_{\nu\lambda} \sigma^\nu_{c}) I_{ab} \psi^B C \psi^C \right) - \]

\[ (\psi^+_A \lambda - \frac{1}{2} A^{abc}_{\mu} (\sigma^\mu_{c\lambda} + k^\mu_{\nu\lambda} \sigma^\nu_{c}) \psi^+_C I_{ab} A \psi^B A_B \psi^B) (\gamma^0 \gamma^\lambda)_A^B \psi^B \right) - \]

\[ - m \psi^+_A (\gamma^0)_A^B \psi^B \sqrt{\left| \sigma \right| \omega} \]
where $\gamma^\mu = \sigma^a \gamma^a$ and $\sigma = \det(\sigma_{\mu\nu})$. One can easily verify that

$$\frac{\partial L_\psi}{\partial k_{\nu\lambda}^\mu} + \frac{\partial L_\psi}{\partial k_{\nu\lambda}^\mu} = 0,$$

(71)

that is, the Lagrangian (70) depends on the torsion of world connections $k$.

9 Energy-momentum of gauge gravity

Recall: (i) that there is the 1:1 correspondence between the invariant vector fields on the groups $L$ and $L_s$ and (ii) that, for every connection form on $LX$, its Lorentz-valued component restricted to $L^hX$ is a connection on $L^hX$. Building on these facts, we can define the canonical lift

$$\tilde{\tau} = \tau^\mu \partial_\mu + (\partial_\nu \tau^\alpha k^\nu_\beta \mu - \partial_\beta \tau^\nu k^\alpha_\nu \mu - \partial_\mu \tau^\nu k^\alpha_\beta \nu - \partial_\beta \tau^\alpha) \frac{\partial}{\partial k^\alpha_\beta \mu} +$$

$$\partial_\nu \tau^\mu \sigma_c^\nu \frac{\partial}{\partial \sigma_c^\mu} + \frac{1}{2} \partial_\tau \tau^\beta (\eta^{\alpha B} \sigma_c^\alpha \sigma_f^\beta - \eta^{\beta B} \sigma_c^\alpha \sigma_f^\beta) \left( (\eta_{ac} \delta_d^\alpha - \eta_{ac} \delta_b^\alpha) \sigma_c^d \frac{\partial}{\partial \sigma_c^d} + I_{ab}^A A_B \psi A \partial_A + I_{ab}^A A_B \psi A \partial_B \right),$$

(72)

of a vector field $\tau$ on $X$ onto the bundle (68). For the sake of simplicity, let us employ again the compact notation

$$\tilde{\tau} = \tau^\mu \partial_\mu + \partial_\nu \tau^\nu \sigma_c^\mu \frac{\partial}{\partial \sigma_c^\mu} + (u^A \partial_\beta \tau^\alpha - u^A \partial_\beta \tau^\alpha) \partial_A.$$ 

The Lagrangian (69), by construction, is invariant under transformations of the holonomic atlases of the principal frame bundle $LX$ (passive general covariant transformations acting on the Greek indices) and under transformations of the atlases of the principal bundle $P_\Sigma$ and $LX \rightarrow \Sigma$ (passive Lorentz gauge transformations acting on the Latin indices). It follows that this Lagrangian is invariant under infinitesimal active gauge transformations whose generator is the canonical lift (72) and

$$L_{\tilde{\tau}}L = 0.$$ 

(73)

Then, the weak conservation law

$$0 \approx d_\Lambda [\partial_\Lambda L_{AM}(u^A \partial_\beta \tau^\alpha - u^A \partial_\beta \tau^\alpha - y^A \tau^\alpha)$$

$$+ \frac{\partial L_\psi}{\partial \sigma_c^\alpha} (\partial_\beta \tau^\nu \sigma_c^\nu - \sigma_c^\alpha \mu \tau^\nu) - \frac{\partial L_\psi}{\partial \psi^A \psi^A} \psi^A \psi^A \tau^\alpha + \tau^\Lambda L]$$

(74)

takes place. We have also the relations (51) and the relation

$$\frac{\partial L_\psi}{\partial k_{\nu\lambda}^\mu} = \frac{\partial L_\psi}{\partial \sigma_c^\alpha} \sigma_c^\alpha.$$

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Due to arbitrariness of the functions $\tau^\alpha$, the equality (73) implies the strong equality (53) where $\sqrt{|g|}$ is replaced by $\sqrt{|\sigma|}$ and the strong equality

$$\delta^\beta_\alpha L\psi + \sqrt{|\sigma|} t^\beta_\alpha + \partial L\psi \sigma^\beta_{\sigma\alpha} - \partial L\psi \sigma^\beta_{\sigma\epsilon} \sigma^\epsilon_{\alpha\beta} + \partial A L\psi u^{\beta}_{\alpha\beta} - \partial L\psi \frac{\partial}{\partial \psi^A_{\alpha\beta}} \psi^A_{\alpha\beta} = 0$$

(75)

where

$$\sqrt{|\sigma|} t^\beta_\alpha = \sigma^\beta_\alpha \frac{\partial L\psi}{\partial \psi^A_{\alpha\beta}}.$$

Substituting the term $y^{\beta}_A \partial^{\beta}_A L_{AM}$ from the expression (53) and the term

$$\frac{\partial L\psi}{\partial \sigma^\mu_{\sigma\alpha}} \sigma^\mu_{\epsilon\alpha} + \frac{\partial L\psi}{\partial \psi^A_{\beta\alpha}} \psi^A_{\beta\alpha} + \frac{\partial L\psi}{\partial \psi^A_{\alpha\beta}} \psi^A_{\alpha\beta}$$

from the expression (74) into the conservation law (74), we bring the latter into the form

$$0 \approx d_\lambda \left[ -\sigma^\lambda_\alpha \tau^\alpha \delta^\alpha_\lambda L - (k^\lambda \gamma^\mu_\alpha \delta^\mu_\lambda A_\alpha L - k^\sigma_\alpha \partial_\sigma \delta^\lambda_\mu A_\alpha L - k^\gamma_\alpha \delta^\lambda_\sigma A_\alpha L) \tau^\alpha + \delta^\gamma_\alpha A_\lambda \partial_\epsilon \tau^\alpha - d_\mu(\delta_\lambda^\alpha \epsilon^\mu L_{AM}) \tau^\alpha \right] - d_\lambda \left[ d_\mu(\pi^\mu_\alpha \lambda^\nu \epsilon^\lambda_\nu \partial_\epsilon \tau^\alpha) \right]

+ d_\lambda \left[ \frac{\partial L\psi}{\partial \sigma^\alpha_{\sigma\mu}} \sigma^\alpha_{\sigma\lambda} + \frac{\partial L\psi}{\partial \sigma^\alpha_{\sigma\lambda}} \sigma^\alpha_{\sigma\mu} \right] \partial_\mu \tau^\alpha].$$

(76)

In accordance with the relation (71), the last term in the expression (76) is equal to zero, i.e., fermion fields do not contribute into the superpotential. It follows that the SEM conservation law (74) comes to the form (8) where $U$ is the generalized Komar superpotential (8).

We thus may conclude that the generalized Komar superpotential (8) appears to be universal for several gravitation models.

10 Relativistic theory of gravity

There is another lift $\Phi$ of diffeomorphisms $f$ of the world manifold $X$ onto $LX$ which preserve a given tetrad field $h$. In this case, we have

$$\gamma_h(dx^\lambda) \rightarrow \gamma_h(\Phi^\lambda_\mu dx^\mu) = \Phi^\lambda_\mu h^\mu_\gamma \gamma^\alpha.$$

(77)

It is readily observed that the Dirac operator (59) is not invariant under these transformations. To overcome this difficulty, one may introduce an additional geometric field $q^\lambda_\mu$ and then rewrite the Dirac operator into the form

$$D^{qh} = q^\lambda_\mu h^\mu_\gamma \gamma^\alpha D^h_{\lambda}$$

(78)

which is invariant

$$D^{qh} \rightarrow \Phi^\lambda_\nu q^\lambda_\mu h^\mu_\gamma \gamma^\alpha D^h_{\lambda} = q^\lambda_\mu h^\mu_\gamma \gamma^\alpha D^h_{\lambda}$$
under the transformations (77). As a result, we get the affine-metric modification of the
relativistic theory of gravity (RTG) [13, 14] where independent dynamic variables are non-
metric gravitational fields $q$ and world connections $\Gamma$ on $X$ in the presence of a background
tetrad (or metric) field, e. g. the Minkowski metric. Since the above-mentioned gauge
transformations do not act on the indices of the background Minkowski metric $\eta^{\mu\nu}$ and the
low index $\mu$ of a gravitational field $q^{\lambda\mu}$, it is the tensor field

$$q^{\mu\nu} = q^{\mu}_{\alpha} q^{\nu}_{\beta} g^{\alpha\beta} = H_{a}^{\mu} H_{b}^{\nu} \eta^{ab}$$ (79)

which perform contraction of indices in the Lagrangians of matter fields and world connec-
tions. Thus, the tensor field (79) plays the role of an effective metric. Obviously, it is not a
true metric, for the coframes $H^{a} = H_{\mu}^{a} dx^{\mu}$ are carried by the $\gamma$-matrices

$$\gamma_{h}(H^{a}) = h_{\mu\nu} \eta^{ab} q^{\nu}_{\alpha} h^{\alpha}_{a} h^{\mu}_{c} \gamma^{c}$$

with respect to the same representation as the coframes $h^{a} = h_{\mu}^{a} dx^{\mu}$ are done. At the same
time, there always exists a pseudo-Riemannian metric $g'$ such that $g'^{\mu\nu} = q^{\mu\nu}$.

Note that additional tensor fields $q$ and generalized tetrad fields $H_{a}^{\mu} = q^{\mu}_{\nu} h_{a}^{\nu}$ have been
considered in the framework of both the $GL_{4}$-gauge models [17, 12] and the gauge model of
the translation group where $q$ describe deformations of a world manifold [21].

Now let us describe our construction in details. Given a tetrad field $h$, let $\Gamma_{h}$ be a principal
connection on $L^{h}X$ extended to $LX$. Any 1-parameter group $f[\alpha]$ of diffeomorphisms of $X$
whose generator is a vector field $\tau$ on $X$ gives rise to the local group of nonholonomic
isomorphisms $f_{\Gamma}[\alpha]$ of $LX$ whose generator is the vector field

$$\tau_{\Gamma} = \tau^{\lambda} (\partial_{\lambda} - \Gamma_{h}^{\mu\nu\lambda} S_{\nu}^{\mu a} \frac{\partial}{\partial S_{\alpha}^{a}})$$
on $LX$. Isomorphisms $f_{\Gamma}[\alpha]$ keep the Lorentz subbundle $L^{h}X$ and the tetrad field $h$. Given
a 1-parameter group $\tilde{f}[\alpha]$ of general covariant transformations over diffeomorphisms $f[\alpha]$, its elements are represented by composition

$$\tilde{f}[\alpha] = \Phi[\alpha] \circ f_{\Gamma}[\alpha]$$ (80)
of $f_{\Gamma}[\alpha]$ and the vertical isomorphisms $\Phi[\alpha]$ of $LX$ with the generator

$$u = D_{\nu}^{\mu} S_{\alpha}^{\mu} \frac{\partial}{\partial S_{\alpha}^{a}}$$

where by $D_{\nu}$ are meant the covariant derivatives with respect to the connection $\Gamma_{h}$.

Recall the following. Let $P \to X$ be a principal bundle with a structure group $G$ and
$\tilde{P} \to X$ the associated group bundle with the standard fibre $G$ on which the structure
group acts by the adjoint representation. Given a bundle $Y$ associated with $P$, there is the canonical fibre-to-fibre morphism $\tilde{P} \times Y \to Y$. As a consequence, every vertical isomorphism $\Phi_Y$ of an associated bundle $Y$ is brought into the form

$$\Phi_Y(y) = q_\Phi(\pi(y)) \cdot y, \quad y \in Y,$$

where $q_\Phi(x)$ is a global section of the group bundle $\tilde{P}$. In particular, we can show that the matrix functions $\Phi^\lambda_\mu(x)$ in the expression (77) are local functions of a section $q_\Phi[\alpha]$ of the group bundle $LX \subset TX \otimes T^*X$ which corresponds to the vertical isomorphism $\Phi[\alpha]$ in the expression (80).

Therefore, let us introduce additional dynamic fields $q$ represented by sections of the group bundle $LX$ and consider the compositions $q \cdot h$. Then, every vertical gauge transformation of $q \cdot h$ reads

$$\Phi(q \cdot h) = q_\Phi \cdot q \cdot h = q' \cdot h.$$  

The field $q$ in the expression (78) exemplifies such a section of $LX$.

Thus, we come to RTG in case of a background tetrad field $h$, the dynamic gravitational fields $q$ and the effective tetrad fields $H = q \cdot h$. The total configuration space of RTG in the presence of fermion fields is the jet manifold of the bundle

$$Y = LX \times X \times S_h \oplus S^+_h$$  

(81)

coordinatized by $(x^\mu, q^\mu, k^{\mu\nu}, \psi^A, \psi^\pm_A)$. Let us consider the gauge transformations of the bundle (81) whose generator is the vector field

$$\tau = \tau^\mu \partial_\mu + \left( \partial_\nu \tau^\alpha k^{\mu\nu}_\beta - \partial_\beta \tau^\nu k^{\alpha\mu}_\nu - \partial_\mu \tau^\nu k^{\alpha\beta}_\nu - \partial_\beta \tau^\alpha k^{\mu\nu}_\alpha \right) \frac{\partial}{\partial q_{\alpha\beta}^{\mu\nu}} +$$

$$+ (\partial_\nu \tau^\sigma q^{\sigma\nu} + \tau^\lambda \Gamma^\nu_{\alpha\beta} q^{\mu\nu}_{\alpha\beta}) \frac{\partial}{\partial q^{\mu\nu}_{\alpha\beta}} + \frac{1}{2} \partial_\alpha \tau^\beta (\eta^{ap} h^{a\mu}_p h^{p\beta}_\beta - \eta^{bp} h^{a\mu}_p h^{a\beta}_\beta) [I_{ab} A^B \psi^B \partial_A + I_{ab} A^B \psi^A \partial_B].$$

These transformations act on the metric functions $g^{\mu\nu}$ and the tetrad functions $h^\mu_a$ of the background field $h$ by the law

$$g^{\mu\nu} \to \tau^\lambda (\partial_\lambda g^{\mu\nu} - \Gamma^\mu_{\alpha\lambda} g^{\alpha\nu} - \Gamma^\nu_{\alpha\lambda} g^{\mu\alpha}),$$

$$h^\mu_a \to \tau^\lambda (\partial_\lambda h^\mu_a - \Gamma^\mu_{\alpha\lambda} h^\alpha_a) + \partial_{\alpha \tau^\beta} (\eta^{ap} h^{a\mu}_p h^{p\beta}_\beta - h^a_{\beta} h^{\alpha\mu}_a) h^\mu_d.$$  

At the same time, they act on the effective metric field $q$ in accordance with the standard law of general covariant transformations

$$q^{\mu\nu} \to \tau^\lambda \partial_\lambda q^{\mu\nu} + \partial_\alpha \tau^\mu q^{\alpha\nu} + \partial_\alpha \tau^\nu q^{\mu\alpha},$$

$$H^\mu_a \to \tau^\lambda \partial_\lambda H^\mu_a + \partial_\alpha \tau^\mu H^\alpha_a + \partial_{\alpha \tau^\beta} (\eta^{ap} h^{a\mu}_p h^{p\beta}_\beta - h^a_{\beta} h^{\alpha\mu}_a) H^\mu_d.$$
The total gauge-invariant Lagrangian of RTG can be given by the sum

$$L_{RG} = L_{AM}(q, \Gamma) + L_{\psi} + L_{q}(q, g)$$  \hspace{1cm} (83)$$

of the Lagrangian $L_{AM}$ of the affine-metric gravity and the Lagrangian $L_{\psi}$ of fermion fields where the metric field $\sigma$ is replaced by the effective metric $q$ and of a Lagrangian $L_{q}$ of fields $q$ where contraction is performed by means of the background metric $g$. Therefore, $L_{q}$ is not invariant under familiar general covariant transformations.

In particular, put

$$L_{AM} = (-\lambda_{1}R + \lambda_{2})\sqrt{|g|}, \quad L_{\psi} = 0, \quad L_{q} = \lambda_{3}\eta_{\mu\nu}g^{\mu\nu}\sqrt{|\eta|}$$ \hspace{1cm} (84)$$

where $R = g^{\mu\nu}R^{\alpha}_{\mu\nu}$ is the effective scalar curvature of a connection $\Gamma$. Then, the conventional RTG is recovered.

Let us examine the SEM conservation laws in the RTG model. Building on the formula (19), we get

$$0 \approx d_{\lambda}(2\tau^{\mu}\partial^{\lambda}_{\mu}\partial L_{RG}/\partial g^{\alpha\mu} + d_{\mu}U^{\lambda\mu}) + (\tau^{\lambda}\partial_{\lambda}g^{\mu\nu} - \partial_{\alpha}\tau^{\mu}g^{\alpha\nu} + \partial_{\alpha}\tau^{\nu}g^{\mu\alpha})\partial L_{RG}/\partial g^{\mu\nu}$$

where $U$ is the generalized Komar superpotential. A glance at this relation shows that, if the Lagrangian $L_{RG}$ contains the Higgs term $L_{q}$, the energy-momentum flow does not reduce to the superpotential, and we have the standard covariant conservation law

$$\nabla_{\alpha}t_{\lambda}^{\alpha} \approx 0, \quad \sqrt{|g|}t_{\lambda}^{\alpha} = 2g^{\alpha\mu}\partial L_{q}/\partial q^{\mu\nu}.$$  \hspace{1cm} (85)$$

In particular, it is brought into the conservation law if $g$ is the Minkowski metric.

Note that one can express the energy-momentum tensor $t_{\lambda}^{\alpha}$ into the tensor

$$\sqrt{|g|}T_{\lambda}^{\alpha} = 2g^{\alpha\mu}\partial L_{q}/\partial q^{\mu\nu}$$

owing to the field equations

$$\partial L_{RG}/\partial q^{\alpha\mu} = \partial L/\partial q^{\alpha\mu} + \partial L_{q}/\partial g^{\alpha\mu} = 0.$$ 

In particular, if $L_{q}$ is equal to the Higgs massive term (84), we have the relation

$$\sqrt{|g|}t_{\lambda}^{\alpha} = \sqrt{|q|}(T_{\lambda}^{\alpha} + \frac{1}{2}\delta^{\lambda}_{\alpha}T^{\mu}).$$

Let us emphasize that this relation does not depend on the constant $\lambda_{3}$ which therefore can be as small as will.
References

1. Aringazin A and Mikhailov A 1991 *Class. Quant. Grav.* **8** 1685

2. Borowiec A, Ferraris M, Francaviglia M and I.Volovich 1994 *Gen. Rel. Grav.* **26** 637

3. Crawford J 1991 *J. Math. Phys.* **32** 576

4. Fatibene F, Ferraris M and Francaviglia M 1994 *J. Math. Phys.* **35** 1644

5. Fatibene F, Ferraris M, Francaviglia M and Godina M 1996 gr-qc/9609042

6. Ferraris M and Francaviglia M 1985 *J. Math. Phys.* **26** 1243

7. Giachetta G and Sardanashvily G 1995 gr-qc/9510067

8. Giachetta G and Sardanashvily G 1995 gr-qc/9511040

9. Giachetta G and Sardanashvily G 1996 *Gravitation, Particles and Space-Time* ed P Pronin and G Sardanashvily (Singapore: World Scientific) p 471

10. Giachetta G and Sardanashvily G 1996 *Class. Quant. Grav.* **13** L67

11. Gotay M and Marsden J 1992 *Contemp. Math.* **132** 367

12. Hehl F, McCrea J, Mielke E and Ne’eman Y 1995 *Phys. Reports* **258** 1

13. Logunov A and Mestvirishvili M 1986 *Int. J. Theor. Phys.* **16** 1

14. Logunov A, Loscutov Yu and Mestvirishvili M 1988 *Int. J. Mod. Phys.* **43** 2067

15. Novotný J 1984 *Geometrical Methods in Physics* Proceedings of the Conference on Differential Geometry and its Application, Czechoslovakia, 1983 ed D Krupka (Brno: University of J.E.Purkyně) p 207

16. Obukhov Yu and Solodukhin S 1994 *Int. J. Theor. Phys.* **33** 225

17. Percacci R 1991 *Nucl. Phys.* **B353** 271

18. Pommaret J 1978 *Systems of Partial Differential Equations and Lie Pseudogroups* (Glasgow: Gordon and Breach)

19. Ponomarev V and Obukhov Yu 1982 *GRG* **14** 309

20. Rodrigues W and de Souza Q 1993 *Found. Phys.* **23** 1465

23
21. Sardanashvily G and Zakharov O 1992 *Gauge Gravitation Theory* (Singapore: World Scientific)

22. Sardanashvily G 1992 *J. Math. Phys.* **33** 1546

23. Sardanashvily G 1994 [hep-th/9411089](https://arxiv.org/abs/hep-th/9411089)

24. Sardanashvily G 1995 *Generalized Hamiltonian Formalism for Field Theory. Constraint Systems* (Singapore: World Scientific)

25. Sardanashvily G 1997 *J. Math. Phys.* (accepted)

26. Saunders D 1989 *The Geometry of Jet Bundles* (Cambridge: Cambridge Univ. Press)

27. Tucker R and Wang C 1995 *Class. Quant. Grav.* **12** 2587