Data-Driven Distributed Intersection Management for Connected and Automated Vehicles

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Abstract—In this paper, we seek a scalable method for safe and efficient coordination of a continual stream of connected and automated vehicles at an intersection without signal lights. To handle a continual stream of vehicles, we propose trajectory computation in two phases - in the first phase, vehicles are constrained to not enter the intersection; and in the second phase multiple vehicles’ trajectories are planned for coordinated use of the intersection. For computational scalability, we propose a data-driven method to obtain the intersection usage sequence through an online “classification” and obtain the vehicles’ trajectories sequentially. We show that the proposed algorithm is provably safe and can be implemented in a distributed manner. We compare the proposed algorithm against traditional methods of intersection management and against some existing literature through simulations. We also demonstrate through simulations that for the proposed algorithm, the computation time per vehicle remains constant over a wide range of traffic arrival rates.

I. INTRODUCTION

The advent of connected and automated vehicles (CAVs) presents an opportunity to rethink the problem of intersection management. Further, in recent years, relevance of intersection management has grown in non-traditional domains such as robot traffic control in warehouses. The onboard sensing, computation and communication capabilities that are available on CAVs or robots allow us to do real time coordination of the CAVs or robots to achieve a more efficient and un-signalized intersection management. In this work, we propose a computationally efficient distributed algorithm and a framework for offline data-driven tuning for the management of an isolated intersection in the context of CAVs or robots.

Literature review: Un-signalized intersection management has been studied extensively in recent years using a variety of tools. The survey papers [1], [2] highlight the different methods and tools employed for un-signalized or autonomous intersection management.

Some early works [3]–[6] focused on reservation and multi-agent simulation based algorithms. Some disadvantages of such solutions is that they are computationally demanding, centralized and do not easily provide insights into the system. Since then a major trend in the literature has been to design model based, provably safe algorithms. For example, [7] uses reservations for scheduling intersection usage times, and [8], [9] (see also the references therein) propose a supervisory control method where a supervisor takes over only when a collision is imminent. Another major trend in the field of autonomous intersection management is the use of an optimal control framework for determining the schedules and trajectories of the vehicles. [10] formulates an optimal control problem to optimize the trajectory of each vehicle individually. However, intersection management problem in general requires coordination of multiple vehicles through a combined optimization. Such a combined optimization problem is combinatorial. Thus, the overall problem of trajectory optimization becomes a mixed integer program [11]–[14]. In particular, the complexity of such formulations scales exponentially with the number of vehicles. This limits the practical utility of the exhaustive mixed integer program formulations as intersection management is a time and safety critical application.

Given the computational complexity of the problem, along with the motivation of designing distributed algorithms, several works have sought to decompose the overall autonomous intersection management problem into simpler sub-problems. [15], [16] together propose a high level intersection access management by treating the vehicles on different lanes as queues and use the idea of platooning for local vehicular control. Given intersection usage schedule for the vehicles, [17], [18] (and the references therein) seek to solve the trajectory optimization problem in a decentralized manner by relaxing the rear-end collision avoidance constraints and guarantee existence of initial conditions under which the safety constraints are satisfied. Further, these works also propose a method to drive the vehicles to good “initial conditions” under which safety can be guaranteed subsequently. [19]–[25] also decompose the problem into scheduling and trajectory optimization. [24] proposes an algorithm, in which a central intersection manager groups vehicles into bubbles and schedules the bubbles as a whole to use the intersection. Given the schedule, the vehicles compute provably safe trajectories using a distributed switched controller. [26] proposes to achieve coordination of vehicles by optimizing a notion of joint rewards for the vehicles. For this, it employs Q-learning where the joint actions are found using the $\epsilon$-greedy approach. The learnt actions are stored in Q-tables which can be used by the vehicles when the system is deployed. Since the Q-tables are learnt from episodic data with a focus on minimizing the intersection delay, the resulting trajectories can turn out to be non-smooth. Although the vehicles can obtain near-optimal joint actions, the paper does not provide safety guarantees.

Comfort of passengers and generation of smooth trajectories for vehicles is another area of interest in autonomous intersection management. [27] surveys driver comfort in autonomous vehicles and highlights the inadequate research on passenger comfort in path and motion planning of autonomous vehicles.
[28] studies the problem of vehicles merging into highways and uses a model predictive control architecture that optimizes comfort by minimizing the squares of both acceleration and jerk. [29] introduces a metric of comfort which is a combination of vehicle-jitter, jerk and deviation from a desired velocity. [18] and [30] focus on vehicles turning at the intersection and impose a curvature-based acceleration constraint to capture the comfort of the passengers.

**Contributions:** In this paper, we propose a computationally scalable algorithm for coordinating and optimizing the trajectories of a continual stream of CAVs at and near an isolated, un-signalized autonomous intersection. Most of the papers in the literature consider only the problem of coordinating a fixed set of vehicles. Applying such solutions to a continual stream of vehicles can result in inefficiency or feasibility/safety itself may be violated in the long run. Further, the optimal coordination of vehicles at an intersection is a mixed integer problem, which scales badly with the number of vehicles and lanes. This work is the only paper, to the best of our knowledge, that addresses these issues systematically.

The key insight behind the proposed data-driven framework is that the computation of the optimal intersection usage sequence can be thought of as an online classification problem from a space of features that encode the “demand” of a vehicle and the traffic following it to the vehicle’s precedence for using the intersection. With this insight, we design a computationally very efficient and scalable data-driven algorithm for the problem of autonomous intersection management. The proposed framework also has the ability to incorporate both “micro” information about the individual vehicles’ state as well as “macro” information such as traffic arrival rates. Such a combination again has not been explored in the literature. This element of our framework is particularly useful under high traffic arrival rates.

The second set of major contributions of this paper relates to how we handle a continual stream of vehicles. Most papers in the area essentially propose a one-shot algorithm with the implicit suggestion that the one-shot algorithm should be run repeatedly. However, not explicitly considering the continual stream of vehicles could in general lead to loss of feasibility of safe trajectories. In the proposed framework of this paper, we split the trajectory of each vehicle into two phases (1) *provisional phase* and (2) *coordinated phase*. Every vehicle operates in the provisional phase as soon as it enters the system and it is restricted from entering the intersection. Periodically, the vehicles in the provisional phase obtain a trajectory for their coordinated phase and start executing them. This framework thus offers a complete for a continual stream of vehicles while ensuring safety, feasibility and near optimality of the solutions.

Lastly, we evaluate the performance of our algorithm through an extensive collection of simulations. In particular, we compare our algorithm with that of an “optimal” algorithm, signalized intersection management, first-in first-out based intersection management as well as the algorithm proposed in [24]. We also demonstrate the computational efficiency of the proposed algorithm through some coarse metrics. In particular, we observe from the simulations that for the proposed algorithm the computation time per vehicle essentially remains constant for a wide range of traffic arrival rates. This is in contrast to an exhaustive optimization algorithm for which the computation time per vehicle scales exponentially with the traffic arrival rate.

**Notation:** We use \( \mathbb{R} \) and \( \mathbb{N}_0 \) for the set of real and whole numbers, respectively. For a discrete set \( V \), we let \( |V| \) be the cardinality of the set \( V \).

II. Model and Problem Formulation

A. Model

**Geometry of the Region of Interest:** We consider an isolated intersection with a set of lanes \( \mathcal{L} \). We refer to the lanes together with the intersection itself as the region of interest. Every lane \( l \in \mathcal{L} \) has a unique fixed path \( P_l \subset \mathbb{R}^2 \) associated with it. We denote by \( s_l \) the length of the portion of the path \( P_l \) that lies within the intersection. The length of all the paths leading to the intersection is \( d \). Along the path on lane \( l \), we let the positions at the beginning of the region of interest, the beginning of the intersection and the end of the intersection be \( -d, 0 \) and \( s_l \), respectively. Figure 1 presents the basic geometry of an example region of interest, where the set of lanes is \( \mathcal{L} = \{1, 2, 3, \ldots, 12\} \) with 3 lanes (for going left, straight and right) on each branch. Figure 1 labels lanes 1, 8 and 12 and skips the rest for clarity. The translucent box shaded in red represents the conflict region or the intersection, which is the region of potential inter-lane collisions.

![Fig. 1: Region of interest and the geometry of the intersection. Here only 3 lanes with numbers 1, 8 and 12 have been labeled.](image)

As it is apparent from Figure 1, the paths of some lanes do not intersect while others do. For each pair of lanes \( l, m \in \mathcal{L} \), we define a notion of compatibility \( c(l, m) \) as

\[
c(l, m) := \begin{cases} 
1, & P_l \cap P_m = \emptyset \\
0, & P_l \cap P_m \neq \emptyset, \ P_l \neq P_m \\
-1, & P_l = P_m
\end{cases}
\]

We say that a pair of lanes \( l \) and \( m \) are compatible if \( c(l, m) = 1 \) and incompatible if \( c(l, m) = 0 \). In the sequel, we require that vehicles on incompatible lanes not be in the intersection at the same time. Note that our framework can handle other configurations of incoming branches and intersections by appropriately defining the compatibility pairs \( c(l, m) \).
choose the standard configuration, shown in Figure 1, purely for ease of exposition. We assume that vehicles do not change lanes within the region of interest.

We assume that all vehicles are CAVs - they can communicate with each other and the infrastructure, and are automated. We also assume that the vehicles do not change lanes within the region of interest. The CAVs can enter the region of interest on any lane in \( L \). We denote the lane that vehicle \( i \) traverses on by \( l_i \in L \) and the vehicle’s length by \( L_i \). The state of the vehicle at time \( t \) is \((x_i(t), v_i(t))\), where \( x_i \) and \( v_i \) are the position of the front bumper of the vehicle and the vehicle’s velocity respectively on the path \( P_i \). The dynamics of the vehicle \( i \) is

\[
\dot{x}_i(t) = v_i(t), \quad \dot{v}_i(t) = u_i(t), \quad (1)
\]

where \( u_i \) is the acceleration input to vehicle \( i \). The vehicles are in the region of interest for different time durations. In particular, vehicle \( i \) enters the region of interest at the arrival time, \( t_{i,A}^A \), enters the intersection at the entry time, \( t_{i,E}^A \), and leaves the intersection at the exit time, \( t_{i,E}^X \). Thus, \( x_i(t_{i,A}^A) = -d \), \( x_i(t_{i,E}^A) = 0 \), and \( x_i(t_{i,E}^X) = s_i \).

**B. Problem**

The aim of the autonomous intersection management problem is to compute safe trajectories for the CAVs while maximizing the following objective function

\[
J := \sum_{i \in V} \int_{t_{i,A}^A}^{t_{i,A}^X + T_h} \left[ W_i v_i(t) - (W_a u_i^2(t) + W_j u_j^2(t)) \right] dt, \quad (2)
\]

where \( V \) is the set of all vehicles that arrive in the region of interest during a time interval of interest, \( \dot{u}_i \) is the jerk of vehicle \( i \) and \( W_i, W_a \) and \( W_j \) are non-negative weights. We model the instantaneous discomfort of the passengers in vehicle \( i \) by the linear combination of the squares of acceleration and jerk. This metric penalizes sporadic high-magnitude disturbances caused by braking and acceleration manoeuvres performed by a vehicle [27]. Thus, each vehicle’s contribution to the objective function is a linear combination of the distance it travels and the comfort of passengers in a time horizon \( T_h \), starting from the vehicle’s arrival time \( t_{i,A}^A \).

**Remark 1** (Objective function). In the objective function (2), the contribution of vehicle \( i \) is a linear combination of the distance traversed (integral of the velocity) and the comfort (negative of discomfort) experienced by the passengers of the vehicles. For comfort, we consider the acceleration and jerk only in the longitudinal direction along a vehicle’s path. For vehicles turning left or right, we ignore the fact that forces also act in the lateral direction. However, the principles we illustrate in this paper could easily be extended to also consider ‘lateral’ comfort. We seek to maximize the social (over all vehicles) objective function of traversal distance and comfort. Maximizing traversal distance over a fixed horizon in each term of (2) is a proxy for minimizing traversal time for crossing the intersection. We choose this indirect metric because directly minimizing the traversal time results in a problem with a variable and unknown horizon for each vehicle. On the other hand, the fixed horizon formulation (2) provides a computational advantage in the online construction of constraints for a stream of vehicles.

**Constraints:** The first set of constraints on the CAVs are bounds on their acceleration \( u_i(t) \) and velocity \( v_i(t) \), i.e.,

\[
u_i(t) \in [\underline{u}, \bar{u}], \quad v_i(t) \in [\underline{v}, \bar{v}], \quad (3)
\]

for all \( \forall i \in V \) over an appropriate time interval. We assume that \( \underline{u} < 0 \), and \( \bar{u} = 0 \).

The second set of constraints ensure safety between the vehicles. Two types of collisions can occur in the region of interest: (1) rear-ended collision between successive vehicles on the same lane, and (2) collision between vehicles on incompatible lanes, within the intersection. To ensure in-lane safety we impose a safe-following distance between any two successive vehicles travelling on the same lane. Consider two vehicles \( i \) and \( j \) on the same lane \( (l_i = l_j) \) such that \( i \) is the vehicle immediately following \( j \). This arrangement is formally denoted using the follower indicator function as,

\[
q(i, j) := \begin{cases} 1, & \text{if } l_i = l_j, \quad x_i < x_j, \\ \bar{q}_k \text{ s.t. } l_k = l_i, \quad x_i < x_k < x_j \\ 0, & \text{otherwise}. \end{cases}
\]

The minimum safe-following distance \( D \) between vehicles \( i \) and \( j \), when \( q(i, j) = 1 \), is a function of the velocities of the two vehicles and is given by [24], [31].

\[
D(v_i, v_j) = L_j + r + \max\left\{0, \frac{1}{-2\bar{u}} \left( v_i^2(t) - v_j^2(t) \right) \right\}. \quad (4)
\]

Here, \( r \) is a robustness parameter which is a constant distance to account for measurement and communication errors and delays. Then the rear-end safety constraint is

\[
x_j(t) - x_i(t) \geq D(v_i, v_j), \quad j \text{ s.t. } q(i, j) = 1 \quad (5)
\]

for the time interval of interest. Note that the rear-end safety constraint (5) is more robust to loss of coordination, either due to breakdown in communication, control or due to malicious vehicles, than rear-end non-collision constraints [24], [31]. To ensure safety within the intersection, we additionally impose the constraint that vehicles on incompatible lanes cannot be within the intersection simultaneously. Thus, the intersection safety constraint for a pair of vehicles \( i \) and \( k \) is

\[
t^X_k \geq t^X_i \quad \text{OR} \quad t^X_i \geq t^X_k, \quad \text{if } c(l_i, l_k) = 0. \quad (6)
\]

Then, the proposed optimal control problem for intersection management is as follows,

\[
\max_{u_i(\cdot), \forall i \in V} J \quad (7a)
\]

s.t. \( (1), (3), (5) \quad \forall t \in [t_{i,A}^A, t_{i,A}^X + T_h], \forall i \in V \quad (7b)\)

\[
\forall i, \forall i \in V \text{ s.t. } c(l_i, l_k) = 0. \quad (7c)
\]

**Remark 2** (Challenges in solving (7) and problem statement). There are several challenges in solving Problem (7). First, vehicles arrive randomly in a stream into the system and
the information about their arrival and state is revealed only incrementally. Thus, Problem (7) cannot be "solved" in the usual sense. Hence, we seek an algorithm that satisfies the constraints in the problem and we utilize (2) as a metric for evaluating the performance of an algorithm after it makes all the decisions. Further, although the exact arrival times of the vehicles are not known a priori, we allow for the knowledge of the statistical data such as the mean arrival rate of vehicles. We seek to leverage this information for more efficient traffic management. Second, Problem (7) is a mix of large scale optimal control and combinatorial optimization. In particular, the number of optimal control sub-problems that constraints (6) generate scales exponentially with the number of vehicles and lanes. This is a serious issue since intersection management is a time and safety critical problem. Hence, we seek algorithms that are computationally scalable and yet provide near optimal performance.

III. OVERVIEW OF THE ALGORITHM

Considering the complexities and time-criticality associated with Problem (7), we propose a computationally efficient algorithm to compute a sequence for intersection usage as well as the trajectories for the vehicles. To overcome the randomness in the arrival of traffic, and the challenges associated with incremental revelation of information, we split the trajectory of each vehicle into two phases: provisional and coordinated. The provisional phase begins as soon as a vehicle arrives into the region of interest. The vehicle seeks to maximize its objective under the constraint of a safe approach towards the intersection. At a prescribed time, the vehicle switches to its coordinated phase from its provisional phase. The vehicles in their coordinated phase use the information safely while aiming to optimize the overall objective.

In this section, we give an overview of the proposed algorithm to solve Problem (7). For ease of exposition, we initially assume the presence of a central intersection manager (IM) that has communication and computation capabilities with which it carries out the coordination of the traffic. At the end of Section IV, we discuss how essentially all the functions of the IM can be carried out in a distributed manner. We present the overview of the algorithm in two parts: from the perspectives of an arbitrary vehicle $i$ and the IM in Algorithm 1, and in Algorithm 2, respectively.

A. Vehicle $i$’s Perspective

A vehicle $i$ starts execution of Algorithm 1 at $t_i^A$, its time of arrival into the region of interest. Vehicle $i$ communicates with the IM as soon as it arrives at $t_i^A$. The IM prescribes $t_i^C$, the start time of coordination phase for vehicle $i$, and also informs about the planned trajectory of the vehicle (if any) that precedes vehicle $i$ on its lane. This is sufficient for vehicle $i$ to plan its trajectory for the provisional phase, which ends at $t_i^C$. In particular, vehicle $i$ computes its trajectory for the provisional phase by solving optimal control Problem (9), which we refer to in Algorithm 1 as prov_phase($i$). Vehicle $i$ communicates its provisional phase trajectory back to the IM and starts executing it at $t_i^A$. At $t_i^C$, vehicle $i$ receives a new trajectory from the IM for the coordinated phase.

Algorithm 1: Algorithm from a vehicle $i$’s perspective

| Line | Description |
|------|-------------|
| 1    | if $t = t_i^A$ then |
| 2    | receive $t_i^C$ and trajectory of vehicle preceding $i$ in its lane |
| 3    | prov_phase($i$) |
| 4    | send provisional trajectory to IM |
| 5    | start provisional phase |
| 6    | end |
| 7    | if $t = t_i^C$ then |
| 8    | receive new trajectory from IM for coordinated phase |
| 9    | start coordinated phase |
| 10   | end |

B. Intersection Manager’s Perspective

Now, we describe Algorithm 2, which is from the IM’s perspective. As soon as a vehicle $i$ enters the region of interest,

Algorithm 2: Algorithm from IM’s perspective

| Line | Description |
|------|-------------|
| 1    | if $t = t_i^A$ then |
| 2    | $t_i^C \leftarrow kT_c$, with $k = \min\{k \in \mathbb{N}_0 : kT_c \geq t_i^A\}$ |
| 3    | send to vehicle $i$, $t_i^C$ and trajectory of vehicle preceding $i$ in its lane |
| 4    | receive vehicle $i$’s provisional trajectory |
| 5    | end |
| 6    | if $t = kT_c$ then |
| 7    | $V_c(k) \leftarrow \{i : t_i^A \in ( (k-1)T_c, kT_c ] \}$ |
| 8    | coord_phase($V_c(k)$) |
| 9    | send trajectories to vehicles $V_c(k)$ |
| 10   | $k \leftarrow k + 1$ |
| 11   | end |

the IM sends to vehicle $i$, the next instance of coordinated
trajectory planning as \( t_i^C \) and the trajectory of the vehicle preceding vehicle \( i \) on its lane \( l_i \) so that vehicle \( i \) can compute its provisional phase trajectory and communicate it back to the IM. In this paper, for simplicity, we assume that IM carries out coordinated planning periodically at the instances \( kT_c \), where \( k \in \mathbb{N}_0 \). And we let \( t_i^C = kT_c \), where \( k \) is the smallest integer such that \( kT_c \geq t_i^A \).

At each coordinated trajectory planning time instance \( kT_c \), the IM first considers \( V_c(k) \), the set all the vehicles that have arrived during the interval \( [(k-1)T_c, kT_c] \). Then, it computes a trajectory for each vehicle in \( V_c(k) \) seeking to achieve optimized coordination and ensures the vehicles cross the intersection safely. We denote the coordinated phase planning problem at the instance \( kT_c \) by \( \text{coord\_phase}(V_c(k)) \). The IM communicates the trajectories for the coordinated phase to the vehicles in \( V_c(k) \), which then execute them.

In Section IV, we present \( \text{coord\_phase}(V_c(k)) \), the algorithm for planning the trajectories in the coordinated phase.

Remark 3 (Computation instances). The coordination planning instances need not be periodic and may be adapted to the traffic. Also, in Algorithms 1 and 2, we have presented various functions to be executed at specific time instances. But, this is purely for ease of exposition and one could easily modify these algorithms to account for computational delays.

In Section IV, we present \( \text{coord\_phase}(V_c(k)) \), the algorithm for planning the trajectories in the coordinated phase.

IV. COORDINATED PHASE

This section presents the trajectory optimization for the coordinated phase. We first present combined optimization, which is a naive centralized method and is not computationally scalable. Building on this method, we present the data-driven sequential weighted algorithm, that is significantly superior in terms of computational requirements. Moreover, as we demonstrate through simulations in Section V, this algorithm performs almost as well as the combined optimization.

The planning for the coordinated phase is carried out periodically with period \( T_c \). In particular, at the instance \( kT_c \), trajectory planning is carried out for the set of vehicles \( V_c(k) \) that arrive into the region of interest during the interval \( [(k-1)T_c, kT_c] \). In this section, we discuss the methods for coordinated planning at an arbitrary but fixed instance \( kT_c \). We also introduce the set \( V_s(k) \), which then executes them.

A. Combined Optimization

In this method, the IM computes the trajectories for all the vehicles in \( V_c \) simultaneously. The optimal control problem for the combined optimization method is a variation of the problem (7). The only differences are: the time horizon for the coordinated phase is \( T_c \) and the set of participating vehicles is \( V_c \). Specifically, the objective function is

\[
J^c = \sum_{i \in V_c} \int_{t^C}^{T_c} \left( W_v v_i(t) - [W_a u_i^2(t) + W_j u_i^2(t)] \right) dt
\]

and the optimal control problem for combined optimization is

\[
\max_{u_i(\cdot), i \in V_c} J^c \\
\text{s.t.} (1), (3), (5) \forall t \in [t^C, t^C + T_c], \forall i \in V_c \tag{10a}
\]

\[
(6) \forall i, j \in V_c \cup V_s \text{ s.t. } c(l_i,l_j) = 0. \tag{10c}
\]

Subsequent to solving (10) and updating the trajectories for the vehicles, \( V_s \) is updated to \( V_s \cup V_c \).

Combined optimization (10) requires the IM to compute optimal trajectories for each feasible intersection usage sequence and then pick the best sequence and the corresponding optimal trajectories. However, the number of feasible sequences grows exponentially with the number of vehicles on incompatible lanes. Thus, this method is not scalable and is not well suited for the time and safety critical problem of autonomous intersection management. Hence, we next propose a computationally scalable and efficient method for computing near optimal sequences and trajectories for the coordinated phase.

B. Data Driven Sequential Weighted Algorithm (DD-SWA)

We now propose a scalable method for optimizing the intersection usage sequence and trajectories of vehicles in the coordinated phase. We call it data-driven sequential weighted algorithm (DD-SWA). The method scales linearly with the number of vehicles and as we discuss in the sequel, this method is very amenable to a distributed implementation.

We present an overview of DD-SWA in Algorithm 3. The algorithm begins with the set of unscheduled vehicles, \( V_c \). In Step 6, we identify \( F \), the set of vehicles in \( V_c \) that are closest to the intersection. In Step 8 of the algorithm, the

\[
\text{Algorithm 3: DD-SWA}
\]

| Line | Description |
|------|-------------|
| 1    | if \( t = 0 \) then |
| 2    | \( V_s \leftarrow \emptyset \) \{set of scheduled vehicles\} |
| 3    | end |
| 4    | if \( t = kT_c \), for \( k \in \mathbb{N}_0 \), then |
| 5    | while \( \left| V_c \right| > 0 \) do |
| 6    | \( F \leftarrow \{i \in V_c \mid x_i \geq x_j, \forall j \in V_c \text{ s.t. } l_j = l_i\} \) |
| 7    | for \( i \in F \) do |
| 8    | \( p_i \leftarrow \text{precedence}(i) \) |
| 9    | end |
| 10   | \( i^* \leftarrow \arg\max\{p_i \mid i \in F\} \) |
| 11   | \( \text{traj\_opti}(i^*) \) |
| 12   | \( V_c \leftarrow V_c \setminus i^* \) \{remove \( i^* \) from \( V_c \)\} |
| 13   | \( V_s \leftarrow V_s \cup i^* \) \{\( i^* \) is scheduled\} |
| 14   | end |
The vehicle \( i^* \in \mathcal{F} \) with the highest precedence index (after arbitrarily resolving any potential ties) is allowed to use the intersection before any other vehicle in \( \mathcal{F} \). A trajectory for the coordinated phase is then computed for \( i^* \) in Step 11, after which \( i^* \) is removed from \( V_c \) and added to \( V_s \). This process is repeated until \( V_c \) is empty. The vehicles optimize their trajectories sequentially so as to satisfy the intersection safety constraint (6). Next, we describe the computation of the precedence indices \( \text{precedence}(i) \) and the trajectory optimization.

1) Computation of the Precedence Index \( \text{precedence}(i) \): We let the precedence index \( p_i \) be a linear combination of certain scheduling features related to the vehicle \( i \in \mathcal{F} \)

\[
p_i := w_x(d + x_i(t_0^c)) + w_{v_i}v_i(t_0^c) + w_i(t_0^c - t_a^c) + w_n|Q_i| + \sum_{j \in Q_i}(x_j(t_j^c) - x_j(t_i^c)) + w_\sigma \sigma_i - w_\tau \tau_i.  \tag{11}
\]

Three of the scheduling features are based on the state at time \( t_0^c \) and history of the vehicle \( i \), namely, distance traveled since arrival \( d + x_i(t_0^c) \), velocity \( v_i(t_0^c) \) and time since arrival \( t_0^c - t_a^c \) of vehicle \( i \). Three features capture the “demand” on lane \( l_i \) that is “following” vehicle \( i \). First of these features is the number of vehicles \( |Q_i| \), where \( Q_i \) is the set of vehicles that follow vehicle \( i \) on lane \( l_i \) at time \( t_i^c \). The second feature is the average separation of vehicles in \( Q_i \) from vehicle \( i \). The third feature in this group is the average rate of arrival of vehicles \( \sigma_i \) on lane \( l_i \). The final feature is the minimum wait time to use the intersection, \( \tau_i \), for vehicle \( i \). Specifically,

\[
\tau_i := \max\{t_m^c - t_m^c \mid m \in V_s \text{ s.t. } c(l_i, l_m) = 0\}.
\]

All the weights in (11) are positive, which means minimum wait times effects the precedence index negatively. This is to prevent frequent switching of the right of way between incompatible lanes. The weighted linear combination of the features makes the computation of the precedence indices extremely simple. In this work, we propose tuning the weights based on offline simulations.

2) Trajectory Optimization: In Algorithm 3, the trajectories of the vehicles are computed sequentially. In each iteration, the vehicle \( i^* \) with the greatest precedence index is selected for trajectory optimization. However, notice from the combined optimization problem (10) that the optimization of the trajectory of vehicle \( i \) is coupled to the optimization of the other vehicles’ trajectories through the constraints. One of the purposes of the precedence indices is to set a precedence in the constraint (6). Even then, the coupling is not fully eliminated. In order to compute the trajectories sequentially, we seek to decouple (10) into several optimization problems - one per vehicle. We have to do this in a manner that ensures we still get near optimal solutions to (10). Such a method aids in developing a distributed algorithm.

A natural starting point for constructing such a decoupled problem is to consider only the term involving \( i^* \) in J of (10). However, this ignores the “demand” for the intersection usage. Thus, we seek to modify the “marginal” cost function of the vehicle \( i^* \) by incorporating a measure of the demand. We first introduce a notion of demand, \( D_i \), from vehicle \( i \) and those following it on the lane \( l_i \). Specifically,

\[
D_i := p_i + w_\tau \tau_i.
\]

Then, we let the objective function for generating a trajectory for vehicle \( i^* \) to be

\[
J_{i^*} = \int_{t_0^c}^{t_1^c} \left[ \mathcal{W}_v v_{i^*}(t) - [W_v u_{i^*}^2(t) + W_j u_j^2(t)] \right] dt,
\]

where \( \mathcal{W}_v := w_i \frac{\sum_{i \in \mathcal{F}} D_i}{|\mathcal{F}|} \mathcal{W}_v \) and \( w_i \) is a scaling factor. Then, \( \text{traj}_{\text{opt}}(i^*) \) in Step 11 of Algorithm 3 is

\[
\max_{u_{i^*}(\cdot)} J_{i^*} \text{ s.t. } (1), (3), (5) \forall t \in [t_0^c, t_1^c + T_c] \tag{13a}
\]

\[
t_0^c \geq \tau_i + t_1^c, \tag{13b}
\]

with \( i = i^* \) in (1), (3) and (5).

Remark 4 (Computational complexity of DD-SWA). In DD-SWA, we obtain the intersection usage order by computing the precedence indices for vehicles in \( \mathcal{F} \) as a weighted linear combination of the scheduling features and selecting the maximizer of the precedence indices. Also, note that |\( \mathcal{F} \)| \leq |\( \mathcal{L} \)|, the number of lanes. These aspects make the computation of the intersection usage order very simple. Further, the computation of the trajectory of the vehicles is also of significantly lesser complexity since for each vehicle we essentially need to solve an optimal control problem in which the only decision variables are those related to the vehicle itself and the constraints are significantly simplified. In the sequel, we use simulations to demonstrate that DD-SWA performs only marginally worse compared to combined optimization while the computation time per vehicle essentially stays constant for a wide range of traffic arrival rates. On the other hand for combined optimization, the computation time per vehicle increases exponentially with the traffic arrival rate.

We see that in all the three optimal control problems (9), (10) and (13) feasibility implies safety. In the following result we show that if the vehicles arrive into the region of interest in a safe configuration then they are always in a safe configuration (both in-lane and within the intersection) for all time during the provisional as well as the coordinated phases.

Theorem 1 (Sufficient condition for system wide inter-vehicle safety). If every vehicle \( i \) satisfies the rear-end safety constraint (5) at the time of its arrival, \( t_a^c \), and its initial velocity is such that \( v_i(t_a^c) \leq \min\{\bar{v}, |\mathcal{V}(-d)|\} \), feasibility of problems (9), (10) and (13) is guaranteed. Consequently, safety of all the vehicles is also guaranteed for all time.

Proof. The assumption that each vehicle \( i \) satisfies the rear-end safety constraint (5) at \( t_a^c \) ensures that there exists a control trajectory to ensure rear-end safety with the vehicle that precedes \( i \) on its lane \( l_i \). Further, the assumption that \( v_i(t_a^c) \leq \min\{\bar{v}, |\mathcal{V}(-d)|\} \) implies that there exists a control trajectory that ensures that the vehicle can come to a stop immediately if it is following.
before the beginning of the intersection. Thus, the optimization problem for the provisional phase (9) is feasible.

If problem (9) is feasible, the trajectory for the provisional phase guarantees that vehicle \( i \) satisfies the rear-end safety constraint (5) and the intersection entry prevention constraint (8) at \( t^*_l \), the start time of the coordinated phase of vehicle \( i \). This property ensures that the intersection safety constraints (10c) and (13b) for combined optimization and DD-SWA respectively are also feasible as the vehicle can come to a stop before the beginning of the intersection if necessary. Thus, feasibility of problem (9) guarantees the feasibility for problems (10) and (13). Since feasibility of the problems ensure rear-end safety and intersection safety, safety between every pair of vehicles is also guaranteed. \( \square \)

C. Distributed Implementation of the Algorithm

Note that each vehicle \( i \) can implement its provisional phase in a distributed manner by communicating only with the vehicle preceding it in its lane. The design of DD-SWA is also amenable to a distributed implementation. The information required to calculate a vehicle’s precedence index and to solve its trajectory optimization problem can be obtained with distributed communication.

Each vehicle can obtain information such as distance travelled, velocity and time since arrival locally. The other scheduling features, the safety constraints (5) and (6) and the weights for the scheduling features require communication. We make a distinction between the types of communication required for this purpose. The three types of communication required are: (1) intra-lane, (2) inter-lane and (3) central communication. In intra-lane communication, a vehicle \( i \in F \) needs to communicate with only a vehicle \( j \) such that \( q(i, j) = 1 \) or \( q(j, i) = 1 \), i.e., \( i \) needs to communicate with just the vehicles immediately preceding or following it on its lane \( l_i \). The number of vehicles following \( i \), \( |Q_i| \), can be counted in a distributed manner and can be communicated from one car to the next in \( Q_i \), the vehicles following \( i \) and ultimately to the vehicle \( i \) itself. Similarly, the vehicle immediately in front of \( i^* \) can communicate its position and velocity trajectory which are sufficient to compute (5). The intersection safety constraint and the minimum wait time feature require \( i^* \) to communicate and receive the exit time of the vehicle on an incompatible lane. We denote such communication as inter-lane communication. Lastly, intersection-specific information such as the weights for the scheduling features \( w_{x_1}, \ldots, w_{x_5} \) and the average arrival rate of traffic \( \sigma_l \) need to be communicated to the vehicles in \( F \) from a central infrastructure, such as an IM. Thus the central infrastructure’s or IM’s function is essentially restricted to communication.

V. Simulations

To evaluate the proposed algorithm, a simulation framework using Casadi [32] and Python was created. All the simulations were performed on an Intel i9-9900k 3.6GHz processor with 128GB of RAM. In the simulations, we assume that vehicles arrive according to a Poisson process with an average arrival rate of \( \sigma_l \) on lane \( l \in L \). To evaluate the proposed algorithm, we compare combined optimization and DD-SWA against a signalized intersection, the Hierarchical-Distributed algorithm [24] and the coordinated phase with a first-in first-out (FIFO) protocol for the sequence of intersection usage. The simulation results are for the particular case where vehicles only pass straight across the intersection. However, the proposed algorithms hold even when turning is allowed. We present the algorithms and the comparisons in greater detail below.

Arrival of Vehicles: Recall that the existence of feasible trajectories for the provisional and coordinated phases is guaranteed if the conditions mentioned in Theorem I are satisfied. Thus to ensure feasibility, we restrict the maximum velocity of the vehicles at the time of their arrival, to \( \min\{\bar{v}, V(\bar{d})\} \). In the simulations here, we assume that vehicles arrive according to a Poisson process with an average arrival rate of \( \sigma_l \) on lane \( l \in L \). However, a specific realization of arrival times of the vehicles may cause a violation of the rear-end safety constraint at the arrival time itself. To avoid this, we check the separation between the successive vehicles at the time of arrival. If the constraint (5) is violated, the arrival of the vehicle is delayed until the constraint is satisfied.

To evaluate the proposed algorithm, we compare combined optimization and DD-SWA against a signalized intersection, the Hierarchical-Distributed algorithm proposed in [24] and the coordinated phase with a first-in first-out (FIFO) protocol for the sequence of intersection usage. The simulation results that we present here are for the particular case where vehicles only pass straight across the intersection, i.e. vehicles arrive in lanes \( l \in L_s = \{2, 5, 8, 11\} \). However, the proposed algorithms hold even when turning is allowed. We present the algorithms and the comparisons in greater detail below.

Signalized Intersection: In this algorithm, every vehicle \( i \) that enters the region of interest performs \( \text{prov\_phase}(i) \) to approach the intersection. When a lane \( l \) receives a green signal, all the vehicles in lane \( l \) are considered to be a part of \( V_l \) and they are given a green trajectory to exit the intersection by solving problem (10). The cycle times and green times for the signals are obtained using Webster’s method [33] corresponding to the arrival rate \( \sigma_l \) in each lane.

Hierarchical-Distributed (HD) Algorithm: This is the algorithm presented in [24].

First-In First-Out (FIFO): We use a FIFO protocol for the coordinated phase in these simulations.

Simulation Parameters: Table I lists the parameters of the intersection and the vehicles that are common to all the algorithms. We conducted simulations using all the algorithms for several arrival rates of traffic. We chose the simulation time for each simulation to be equal to the time duration of 10 cycles of a signalized intersection corresponding to the particular arrival rate \( \sigma \) obtained from the Webster’s method [33]. In each of the simulations, we conducted 20 trials for each of the algorithms for each arrival rate \( \sigma \). Then, we compared the average time to cross and average objective function value per vehicle over the 20 trials for each value of \( \sigma \) across all the algorithms.

1 A video describing the main features of the proposed algorithm and simulations is available at: http://www.ee.iisc.ac.in/people/faculty/pavant/files/figs/Data-Driven-IM.mp4.
A. Results

We present 3 sets of comparisons between the various algorithms mentioned previously. Table II indicates the weights on the scheduling features used for computing the precedence index (11) in DD-SWA in each of the comparative simulations.

It also indicates the weights on the scheduling features used in DD-SWA in each of the comparative simulations. Next, we discuss in detail each of the four comparisons.

1) Comparison 1: In this comparison, there is no weight on comfort of the passengers in the objective functions and the vehicles aim to only maximize the distance travelled. In particular, in the “running cost”, the weights on acceleration and jerk terms (W_a and W_j respectively) are set to 0 and the weight on velocity term (W_v) is set to 1 for trajectory optimization problems for the provisional phase and the coordinated phase. In the HD algorithm, the fuel cost represented by \( F_i(\dot{v}_i) \) in Equation (5) of [24] is set to 0 so that the vehicles only aim to minimize the time spent within the intersection. This ensures a fair comparison between the HD algorithm and other algorithms. We compare the average time to cross (TTC) for the vehicles under the different algorithms. We show simulation results for arrival rates (\( \sigma \)) in the range of 0.01 to 0.09 vehicles/s per lane with an increment of 0.01 vehicles/s per lane. Figure 2(a) shows that the average TTC for vehicles with combined optimization and DD-SWA is comparable for all arrival rates in the considered range. FIFO performance is marginally poor compared to DD-SWA. We compare the computation time per vehicle for combined optimization and DD-SWA for

2) Comparison 2: In Comparison 2, the objective is again to maximize only the distance travelled by the vehicles, but the comparison is made for arrival rates (\( \sigma \)) from 0.1 to 0.9 vehicles/s per lane. As the computation time for combined optimization is significantly higher compared to the other algorithms, we choose to not include it for comparison 2. Figure 2(b) shows that DD-SWA continues to perform better than all the other algorithms. Although the time to cross initially increases for DD-SWA, it saturates at 0.4 vehicles/s per lane. FIFO is initially better than the signalized intersection until 0.2 vehicles/s but it’s performance rapidly deteriorates as the arrival rate increases. The signalized algorithm outperforms FIFO at 0.3 vehicles/s and outperforms the HD algorithm at all arrival rates in this range. However, it does not perform better than the DD-SWA. The HD algorithm performs significantly worse than the other algorithms due to its nature of creating bubbles with multiple vehicles.

3) Comparison 3: In Comparison 3, we compare between combined optimization, DD-SWA and FIFO when the arrival of traffic is inhomogeneous, i.e., when the arrival rates are not the same for all the lanes in consideration. In particular, we set \( \sigma_2 = \sigma_8 = \sigma \) and \( \sigma_3 = \sigma_{11} = \frac{\sigma}{3} \) for different values of \( \sigma \). We again set the weights on acceleration and jerk terms to 0 and the weight on the velocity term to 1 in this comparison. Figure 2(c) shows the average time to cross for the vehicles for combined optimization and DD-SWA. The performance of DD-SWA is only marginally poor compared to that of combined optimization, but FIFO performs significantly poorly compared to both the algorithms. Figure 2(d) shows the average objective value (2) over a period of \( t_A^i \) to \( t_A^i + T_h \) for every vehicle. To compute the objective value, only the vehicles that crossed the intersection by the end of the simulation time were considered to be in the set \( V \) in the objective function (2). The average objective value per vehicle is depicted in 2(d). Note that the weights on the scheduling features in DD-SWA were tuned to improve its performance.

4) Comparison 4: Combined optimization, DD-SWA and FIFO are compared against each other with weights on the velocity, acceleration and jerk terms are all set to 1. Figures 3(a) and (b) depict the average TTC and the average objective value for the two methods from \( t_A^i \) to \( t_A^i + T_h \). A decrease in the average objective value and an increase in the average TTC can be observed as there is an emphasis on both comfort and the distance travelled by the vehicles. It can be observed that combined optimization marginally outperforms DD-SWA both in terms of the average objective value and the average TTC. FIFO performs significantly poorly compared to the other two algorithms. Similar to the legend in Figure 2, the blue and red bars in Figure 3 correspond to DD-SWA and combined optimization respectively.

Computation time comparison: We compare the computation time per vehicle for combined optimization and DD-SWA to emphasize the computational advantage of DD-SWA. We initially compare the size of \( V_c \) for every round of trajectory optimization for the coordinated phase. We inspect the variation in \(|V_c|\) by using box plots. In the Figure 4. The lower and the upper edges of the boxes represent the first quartile (Q1, 25th percentile) and the third quartile (Q3, 75th percentile) respectively. The whiskers above and below the boxes represent the maximum and the minimum of the data. The maximum is calculated as \( Q3 + 1.5(Q3 – Q1) \) and the minimum as \( Q1 – 1.5(Q3 – Q1) \). Data beyond the maximum and the minimum are considered as outliers and they are represented by small circles. The mean of the data is represented by the bold black line. Figures 4(a) and (b) show the box plots of \(|V_c|\) for combined optimization and DD-SWA respectively and Figures 4 (c) and (d) compare the computation time per vehicle for combined optimization and DD-SWA for
TABLE II: Weights for comparisons

| Parameter Symbol | Comparisons 1 and 2 | Comparison 3 | Comparison 4 |
|------------------|---------------------|--------------|--------------|
| Weight on acceleration term \( W_a \) | 0 | 0 | 1 |
| Weight on jerk term \( W_j \) | 0 | 0 | 1 |
| Weight on velocity term \( W_v \) | 1 | 1 | 1 |

DD-SWA WEIGHTS

| Scheduling Features | Weight Symbol | Comparison 1 and 2 | Comparison 3 | Comparison 4 |
|---------------------|---------------|---------------------|--------------|--------------|
| Distance travelled since arrival \( w_x \) | 0 | 0.5 | 0.8 |
| Velocity \( w_v \) | 5 | 4 | 7 |
| No. of vehicles following \( i \) in \( l_i \) \( w_{n_i} \) | 4.5 | 6 | 5 |
| Time since arrival \( w_t \) | 3 | 3 | 5 |
| Average arrival rate in \( l_i \) \( w_{\sigma_i} \) | 40 | 65 | 40 |
| Average separation of vehicles from \( i \) in \( l_i \) \( w_s \) | 6 | 7 | 7 |
| Minimum wait time to use the intersection \( w_w \) | 0.5 | 1 | 5 |
| Average demand scaling factor \( w_l \) | 0.02 | 0.02 | 0.02 |

Fig. 2: Results of Comparisons 1, 2 and 3 for various arrival rates \( \sigma \). In Comparisons 1 and 2, the arrival rate is homogeneous across all lanes. In Comparison 3, the arrival rate is inhomogeneous. (a) Comparison 1 - the average time to cross (TTC) for low arrival rates. (b) Comparison 2 - the average time to cross (TTC) for high arrival rates. (c) Comparison 3 - the average time to cross (TTC) for the vehicles. (d) Comparison 3 - the average objective value, which is the average distance traversed by the vehicles from the time of their arrival.

Saturation of arrival rate: In Comparison 2, at high arrival rates of traffic, the true arrival rate of the vehicles is potentially reduced due to the feasibility conditions mentioned in Theorem 1. As mentioned earlier, if the rear-end safety constraint is violated at the time of arrival of a vehicle, the arrival time is delayed until the constraint is satisfied. At high arrival rates, the rear-end safety constraint is violated often. If necessary, the actual arrival of the vehicle is delayed until the constraint is satisfied. In Figure 5, we present this idea by plotting the true arrival rate per lane versus the set arrival rate per lane for DD-SWA. We make use of the box plot to capture the variation of the true arrival rate on the y-axis. This plot illustrates that the true arrival rate saturates beyond 0.4 vehicles/s per lane. In Figure 2(b), the average TTC of vehicles in DD-SWA also saturates at 0.4 vehicles/s per lane which is consistent with various arrival rates. Although the trend of \( |V_c| \) is similar for both the algorithms, the trend of computation time per vehicle is significantly different. The computation time for combined optimization increases exponentially as \( |V_c| \) increases with the arrival rate. Figure 4(d) shows that DD-SWA has a nearly constant value of computation time per vehicle. We attribute this to the low computational effort required to determine the sequence of intersection usage and for sequentially optimizing the trajectories of the vehicles in \( V_c \).
the saturation of the true arrival rate.

Fig. 5: The true arrival rate versus the desired $\sigma$ recorded for DD-SWA in Comparison 2. The upper and lower edges of the boxes represent the third and first quartile respectively. The whiskers of the boxes represent the maximum and minimum of the data. The circles beyond the whiskers are the outliers.

Fig. 4: Results of the computation time comparison. Figures (a) and (c) correspond to combined optimization and (b) and (d) correspond to DD-SWA. In (a) and (b), the box plots represent the number of vehicles that participate in the trajectory optimization problem for the coordinated phase, which is denoted by $|V_c|$. In (c) and (d), computation time per vehicle is compared for combined optimization and DD-SWA. The lower and upper edge of the boxes represent the first and third quartile of the data respectively. The minimum and maximum of the data is represented by whiskers beyond the edges of the boxes. The outliers of the data are represented by circles beyond the whiskers.

VI. CONCLUSION

In this work, we introduced a provably safe data-driven algorithm for intersection management. By decomposing into two phases, we ensured system wide safety and feasibility of vehicle trajectories. Simulations suggest that the proposed algorithm performs significantly better than traditional methods such as signalized intersections and first-in-first-out algorithms. We also demonstrated through simulations that DD-SWA takes significantly less computational effort compared to the centralized implementation with only marginal loss in the objective value. Future work can be focused on developing learning-based methodologies to automate the tuning of the parameters in DD-SWA for various traffic scenarios. Other promising directions include extension to traffic management parameters in DD-SWA for various traffic scenarios. Other learning-based methodologies to automate the tuning of the objective value. Future work can be focused on developing a centralized implementation with only marginal loss in the computational effort compared to the traditional methods.

for a network of intersections and hardware implementation on multi-robot systems in regulated environments.

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