Rainfall prediction climatological station of Banjarbaru using arima kalman filter

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Abstract. Time series analysis is a method built in a particular time sequence for prediction. One of the models in time series analysis used for prediction is the ARIMA model introduced by Box and Jenkins. As time goes by, the ARIMA model was developed by applying algorithms, one of which was the Kalman Filter algorithm. This study aims to estimate the parameters of the ARIMA model used as the Kalman Filter’s initial value to forecast rainfall using ARIMA and ARIMA Kalman Filter. Determination of the ARIMA model is done by dividing the data into training and testing. The results obtained from the three training data have the same model, namely ARIMA (0,0,0) × (0,1,1)12 models but with different parameter values than those used as initial values for the Kalman Filter. The results obtained using the ARIMA model with Kalman Filter significantly affect the initial data of 90% training data model parameters with an RMSE value of 155,13. Then predictions are made, the results obtained by ARIMA Kalman Filter can follow the actual data, but from June to October, the prediction results cannot approach the actual data. According to events in the field, June to October is the dry season, where rainfall is deficient.

1. Introduction
The Meteorology, Climatology, and Geophysics Agency (BMKG) has a function, one of which is to process climate data. One of the processing of climate data and information is to make accurate rainfall predictions. One of the fields of study in Statistics that focuses on researching past data and aims to predict future conditions is time series analysis. The ARIMA model is one of the general time series analysis models used to predict rainfall. Over time, ARIMA models were developed by applying algorithms to optimize predictions. One of them is the application of the Kalman Filter to optimize the prediction of the ARIMA model by using the ARIMA model parameters as the initial value. Kalman filter is the statistically optimal sequential estimation procedure for dynamic systems [1]. The main advantage of Kalman Filter is the easy adaption to any alteration of the observation and the fact that it needs short series of background information [2].

According to research [3], Kalman Filter is used for parameter estimation of the ARIMA model; the conclusion obtained by parameter estimation using the Kalman Filter algorithm for Covid-19 pandemic dataset of Pakistan has better results with a MAPE value of 0,0058% compared to the SuttleARIMA model which has a MAPE value of 0,0088% and Holt-winters method which has a MAPE value 0,030%. Research from [4] Kalman Filter was used to estimating the parameters of the ARIMA model, the results obtained by applying the Kalman Filter had an increased accuracy ARIMA model compared to those of the ARIMA model alone. In the study [5] the Kalman Filter was compared with applying other models...
besides the ARIMA model, including Neural Networks, Hybrid of LSTM and Genetic Algorithm and Kalman-Savitzky-Golay Filter; it was concluded that the ARIMA model with Kalman Filter has the smallest error value compared to other models. Kalman filter according research [6] was used for prediction wind speed, the result shows that Kalman Filter method is more accurate than pure time series analysis method. Kalman filter algorithm [7] can obtain optimal prediction results, and root-mean-square error decreases by 68.3% on average. For Seasonal ARIMA model, the study [8] shows that the Kalman Filter is efficient for forecasting by input to (S)ARIMA process.

The purpose of this study is to estimate the ARIMA parameter model of rainfall data at the Banjarbaru Climatology Station to be applied to the Kalman Filter and to predict the onset of rain at the Banjarbaru Climatology Station using the ARIMA Kalman Filter model with training for distribution data that has an error value.

2. Theoretical Review

2.1. Preprocessing
The preprocessing process aims to so that the data used does not contain noise (outliers and errors), has smaller dimensions, and is more structured to be processed further. The preprocessing is important, because proper preprocessing of imbalanced data can enable researchers to reduce defects as much as possible, which, in turn, may lead to the elimination of defects in existing data sets [9].

2.2. Stationarity
Stationarity is divided into 2 namely, mean stationarity and variance stationarity [10]. Mean stationarity Data stationary in the mean means that there is no trend pattern; in other words, changes in the data are in the vicinity of the average value; if it is not stationary, differencing is performed. Variance stationarity Stationarity invariance means that the data has a constant or constant change. To see stationarity invariance, see if the rounded value ($\lambda$) = 1. If it is not stationary in the variance, a Box-Cox transformation can be performed [11].

2.3. Box Jenkins Method
This method was introduced in 1970, aiming to form a time series model for prediction. This model is known as the ARIMA Model ($p$, $d$, $q$), expansion of ARIMA Model ($p$, $d$, $q$) is known as the ARIMA Seasonal ($p$, $d$, $q$) × ($P$, $D$, $Q$)S

The ARIMA process of time series analysis can be expressed by

$$\phi_p(B)(1 - B)^dY_t = \theta_q(B)a_t$$

with

$$\phi_p(B) : 1 - \phi_1(B) - \cdots - \phi_pB^p$$

$$\theta_q(B) : 1 - \theta_1(B) - \cdots - \theta_qB^p$$

The Seasonal ARIMA process of time series analysis can be expressed by

$$\phi_p(B)\Phi_p(B^S)(1 - B)^d(1 - B^S)^D Y_t = \theta_q(B)\Theta_q(B^S)a_t$$

with

$$\phi_p(B) : 1 - \phi_1(B) - \cdots - \phi_pB^p$$

$$\Phi_p(B^S) : 1 - \Phi_1(B^S) - \cdots - \Phi_pB^{ps}$$

$$\theta_q(B) : 1 - \theta_1(B) - \cdots - \theta_qB^p$$

$$\Theta_q(B^S) : 1 - \Theta_1(B) - \cdots - \Theta_qB^{ps}$$
2.4. Model Identification, Estimation, Testing of ARIMA Model, and White Noise
The selection of several AR orders (p) and MA (q) orders can be done by observing the pattern of the autocorrelation function (ACF) and the partial autocorrelation function (PACF). The estimation of ARIMA model parameters \((p, d, q)\) was performed using the Maximum likelihood method. The Ljung Box-Pierce test was carried out to test the assumption of white noise with the hypothesis \([12]\).

2.5. Kalman Filter
Kalman Filter is a recursive updating procedure that consists of forming an initial estimate of the state space, then revising the estimate by adding corrections to the initial estimate. The magnitude of the correction is determined by how well the initial guess predicts the new observation \([13]\). In general, the Kalman Filter can be expressed as follows \([14]\):

\[
x_{t+1} = A_t \hat{x}_t + w_t \tag{3}
\]

\[
z_t = H x_t + v_t \tag{4}
\]

\[x_0 \sim (\bar{x}_0, P_{x0}), w_t \sim (0, Q_t), v_t \sim (0, R_t)\]

Initialization Stage:

\[P_0 = P_{x0}, \hat{x}_0 = \bar{x}_0 \tag{5}\]

Prediction Stage:

Estimation : \[\hat{x}_{t+1} = A_t \hat{x}_t \tag{6}\]

Covariance Error : \[P_{t+1} = A_t P_t A_t^T + Q_t \tag{7}\]

Correction Stage:

Kalman Gain : \[K_{t+1} = P_{t+1} H_t^T (H_{t+1} P_{t+1} + R_{t+1})^{-1} \tag{8}\]

Estimation : \[\hat{x}_{t+1} = \hat{x}_{t+1} + K_{t+1} (z_{t+1} - H_{t+1} \hat{x}_{t+1}) \tag{9}\]

Covariance error : \[P_{t+1} = (I - K_{t+1} H_{t+1}) P_{t+1} \tag{10}\]

2.6. Application Kalman Filter To ARIMA Model
After obtaining the ARIMA or Seasonal ARIMA model, the Kalman Filter is then applied. With coefficients \(\phi_p, \Phi_p, \theta_q, \Phi_Q\) as initial values. It is assumed that the state vector is formed from the coefficients \(\phi_p, \Phi_p, \theta_q, \Phi_Q\) namely \(x_t = [\phi_p, \Phi_p, \theta_q, \Phi_Q]^T\).

2.7. Best Model Selection
RMSE is used as the calculation of the error value. RMSE is the root of the difference between the actual data and the predicted data squared. The results are added up and divided by the number of observations. RMSE is defined using the formula \([15]\):

\[
RMSE = \sqrt{\frac{\sum(y_t - \hat{y}_t)^2}{n}} \tag{11}
\]
3. Method
The data comes from BMKG Climatological Station of Banjarbaru. The research variable is the monthly rainfall for the first 10 years from 2010 to the end of 2019; as many as 120 pieces of data are available in the Banjarbaru Class I Climatology Station database. The unit of research variables is expressed in millimeters (mm).

**The procedures in this research are as follows:**

a. Perform data preprocessing.

b. Analyzing the shape of the rainfall data pattern descriptively.

c. Conduct a data stationarity test.

d. Divide the data into 2 (two), called training and testing. The training data aims to create a model, then the best model is selected from the training data, and testing data is used to see the prediction accuracy of the best model.

e. Forming the ARIMA model and choosing the best ARIMA model.

f. Changing the best ARIMA model obtained into state space, including determining the system model and measurement, initialization, prediction stage, and Kalman Gain, then iteration is carried out.

g. Calculate the error value of the ARIMA Kalman Filter model to see the accuracy.

4. Result dan Discussion

4.1. Preprocessing

Outlier Detection

Rainfall data is considered extreme above normal if it has an intensity of more than 500 mm and an extreme value below normal if it has an intensity of 0 mm. The extreme value with an intensity of 0 mm in the rainfall data is that there is no rain (blank) and there is rain but in very low intensity. To detect outliers, a boxplot is used by looking at the distribution of data based on a five-digit summary ("minimum", first quartile (Q1), median, third quartile (Q3), and "maximum").

![Boxplot Rainfall Data Climatological Station of Banjarbaru](image)

*Figure 1. Boxplot Rainfall Data Climatological Station of Banjarbaru*

Based boxplot in Figure 1, extreme values (outliers) are more than the upper limit, namely in December 2011 data with 856.40 mm of rainfall. Extreme value data is due to actual natural events in the field, so extreme values are still included in the analysis. However, extreme values will be handled in the stationary test for further research, which is carried out by transforming the data.
4.2. Descriptive Analysis of Rainfall Data

Table 1. Descriptive Analysis of Rainfall Data

| Characteristic                  | Value       |
|--------------------------------|-------------|
| The number of data (N)         | 120         |
| Mean (mm/12 month)             | 228.2       |
| Minimum (mm/12 month)          | 0.0         |
| Maximum (mm/12 month)          | 856.4       |
| Standard Deviation             | 143.6       |

The table shows that the average value of rainfall data for Banjarbaru Climatology Station as much as 120 data is 228.2 mm with a standard deviation of 143.6 mm, the highest value of rainfall is 856.4, the lowest value is 0.0 mm.

Figure 2. Time Series Plot Rainfall Data Climatological Station of Banjarbaru

Figure 2 shows that the Climatological Station of Banjarbaru has a seasonal pattern of 12 periods. However, the data is identified as not stationary invariance, it can be seen from the data pattern that is almost flat but spread out to build a widened or narrowed pattern.

4.3. Determination of ARIMA Model (p, d, q)

a. Stationary Test

Stationary test data in the variance seen with the Box-Cox transformation plot is considered stationary if the value of $\lambda$ is 1.

Figure 3. Plot Box-Cox Rainfall Data

Figure 4. Plot Box-Cox Rainfall Data Transformation

Figure 3 shows the $\lambda$ of value 0.50 value of 0.50, which indicates that the data is not stationary. The data is transformed using Box-Cox with $\sqrt{Y_t}$, $Y_t$ is the observation data. The plot of the transformation
results can be seen in Figure 4, which shows the $\lambda = 1$ which means it is stationary in the variance. The following is a plot of rainfall data for Banjarbaru Climatology Station after the first transformation.

Figure 5. Plot Time Series Rainfall Data After Transformation

Figure 5 shows that the data is already moving in Mean Area, which means it the already stationary in the mean.

b. ARIMA Model (p,d,q) Determination

Furthermore, the distribution of the training data and data testing with a ratio of 90% (108 data), 80% (96 data), and 75% (90 data). Here are the ACF and PACF plots for 90%, 80%, and 75% training data:

Figure 6. Plot ACF and PACF Data Training 90%
Figure 7. Plot ACF and PACF Data Training 80%
Figure 8. Plot ACF and PACF Data Training 75%

Figure 6, Figure 7, and Figure 8 show the ACF pattern for truncated data in lags 1 and 2. The ACF plot is truncated in lag 1. Figure 5 shows the data has a seasonal pattern of 12 periods, so differencing is carried out according to the seasonal period. As an illustration, the following is a plot of rainfall data for Banjarbaru Climatological Station after differencing on lag 12 training:

Figure 9. Plot Data Training 90% Seasonal Differencing 12
Figure 10. Plot Data Training 80% Seasonal Differencing 12
Figure 11. Plot Data Training 75% Seasonal Differencing 12

The figure above shows the training data plot of 90%, 80%, and 75% after differencing at lag 12; it appears that the data is not stationary on average; it can be seen from the changes in the movement of the data that are not around the average. Then do differencing again to obtain stationary data. Then do the determination of the Seasonal Order.
Next is parameter estimation, parameter significance test, and finally, white noise assumption test for the best model. Here is the summary table:

**Table 2. Estimation, Test of Signifikansi, and Test Assumption of White Noise Best Model**

| Ratio of Training Data | Seasonal ARIMA Model | Parameter | Coefficient | Ljung Box | P-value |
|------------------------|----------------------|-----------|-------------|-----------|---------|
| 90%                    | $(0,0,0) \times (0,1,1)_{12}$ | Seasonal MA(1) = $\theta_1$ | 0.8798 | 0.603 | 0.000 |
| 80%                    | $(0,0,0) \times (0,1,1)_{12}$ | Seasonal MA(1) = $\theta_1$ | 0.8623 | 0.589 | 0.000 |
The equation model from the general model equation (2) is obtained:

$$Y_t = Y_{t-12} - \Theta_1 a_{t-12} + a_t$$  \hspace{1cm} (12)

4.4. Application of Kalman Filter For ARIMA Seasonal Model \((p, d, q) \times (P, D, Q)_s\)

Kalman filter for the Seasonal ARIMA model used in equation (12) is converted to state-space based on equation (3) for 90% training data as follows:

$$\begin{bmatrix} \Phi_0 \\ \Theta_1 \\ Y_t \end{bmatrix}_{t+1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ Y_{t-12} & -a_{t-12} & 0 \end{bmatrix} \begin{bmatrix} \Phi_0 \\ \Theta_1 \\ Y_t \end{bmatrix}_t + w_t$$

with

$$x = \begin{bmatrix} \Phi_0 \\ \Theta_1 \\ Y_t \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ Y_{t-12} & -a_{t-12} & 0 \end{bmatrix}$$

Measurement model using equation (4) as follows:

$$z = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Phi_0 \\ \Theta_1 \\ Y_t \end{bmatrix} + v_t$$

with

$$H = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

$$x = \begin{bmatrix} \Phi_0 \\ \Theta_1 \\ Y_t \end{bmatrix}$$

Initialization Stage

Initial value \(Y_t\) is first of rainfall data which already stationary in variants and mean:

Covariance error system model: \(R = 10^{-6}\)

Covariance error measurement model:

$$Q_0 = \begin{bmatrix} 10^{-6} & 0 & 0 \\ 0 & 10^{-6} & 0 \\ 0 & 0 & 10^{-6} \end{bmatrix}$$

Initial value of estimation:

$$\hat{x}_0 = \begin{bmatrix} 1 \\ 0.87 \\ 18.00 \end{bmatrix}$$  \hspace{1cm} (13)
Initial value of covariance error:

\[ P_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \]

**Prediction Stage**

Value of \( w \) randomly generated so that the following results are obtained:

\[
\hat{x}_1^- = A_0 \hat{x}_0 + w_0
\]

\[
= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0.87 \\ 18.00 \\ 187.960000 \end{bmatrix} + \begin{bmatrix} 123.89 \times 10^{-4} \\ 162.39 \times 10^{-4} \end{bmatrix}
\]

\[
= \begin{bmatrix} 1,0000000 \\ 87,960000 \\ 0,180000 \end{bmatrix}
\]

(14)

\[
P_1^- = A_0 P_0 A_0^T + Q_0
\]

\[
= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 10^{-6} & 0 & 0 \\ 0 & 10^{-6} & 0 \\ 0 & 0 & 10^{-6} \end{bmatrix}
\]

\[
= \begin{bmatrix} 1,000001 & 0 & 0 \\ 0 & 1,000001 & 0 \\ 0 & 0 & 1,000001 \end{bmatrix}
\]

(15)

**Correction Stage:**

\[
K_1 = P_1^- H_1^T (H_1 P_1 H_1^T + R_1)^{-1}
\]

\[
= \begin{bmatrix} 1,000001 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1,000001 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1,000001 \end{bmatrix}\times
\]

\[
\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + 10^{-6}
\]

\[
= \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\]

(16)

\[
P_1 = (I - K_1 H_1) P_1^-
\]

\[
= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1,000001 & 0 & 0 \\ 0 & 1,000001 & 0 \\ 0 & 0 & 1,000001 \end{bmatrix}
\]

\[
= \begin{bmatrix} 1,000001 & 0 \\ 0 & 1,000001 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}
\]

(17)
$$\hat{\mathbf{x}}_1 = \mathbf{H}_1 (z_1 - \mathbf{H}_1 \hat{\mathbf{x}}_1) + K_1 (\mathbf{x}_1 - \mathbf{H}_1 \hat{\mathbf{x}}_1)$$

$$= \begin{bmatrix} 1,0000000 \\ 87,960000 \\ 0,180000 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0,5 \end{bmatrix} \left( \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1,0000000 \\ 87,960000 \\ 0,180000 \end{bmatrix} + \begin{bmatrix} 0 \\ 275,90 \times 10^{-4} \\ 933,54 \times 10^{-4} \end{bmatrix} \right) -$$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1,0000000 \\ 87,960000 \\ 0,180000 \end{bmatrix} \begin{bmatrix} 1,0000000 \\ 87,960000 \\ 3,2430000 \end{bmatrix}$$

$$= \begin{bmatrix} 1,0000000 \\ 87,960000 \\ 0,180000 \end{bmatrix}$$


(18)

Iterations were carried out as much as observation data, namely 120 of data. The error value is obtained from the above steps by reducing the training data to testing data; at 90% training data, the RMSE value is obtained as follows:

$$RMSE = \sqrt{\frac{(141)^2 + (-185)^2 + \cdots + (-98)^2}{12}}$$

$$= 155,1344$$

Furthermore, the value of equation (13) is changed according to the parameter values of the Seasonal ARIMA model in table 2 before it produces the RMSE value as follows:

**Table 3. RMSE Value For Each Initial Value using Kalman Filter**

| No. | Training Data and Testing Data | Initial Value | RMSE’s Value |
|-----|--------------------------------|---------------|--------------|
| 1   | Data training 90%              | \begin{bmatrix} 1 \\ 0,87 \\ 18,00 \end{bmatrix} | 155,13       |
| 2   | Data training 80%              | \begin{bmatrix} 1 \\ 0,86 \\ 18,00 \end{bmatrix} | 176,96       |
| 3   | Data training 75%              | \begin{bmatrix} 1 \\ 0,83 \\ 18,00 \end{bmatrix} | 165,49       |

From table 3, it can be seen that the smallest RMSE value is at the initial value of 90% of training data, namely the Seasonal ARIMA model(0,0,0) × (0,1,1)_{12}. Thus the Seasonal ARIMA model with Kalman Filter that will be used to predict rainfall in the next stage is the Seasonal ARIMA model with Kalman Filter using 90% training data.

The following are the prediction results for the Seasonal ARIMA model with Kalman Filter using 90% training data model parameters.
From Figure 18, it can be seen from June to October that predictions using the Kalman Filter cannot follow the actual data, according to events in the field, that month is the dry season. During the dry season, the rainfall is very low; some even do not rain. Therefore, further research can examine separately between the rainy season and the dry season.

5. Conclusion
The Seasonal ARIMA model used for the initial value of the Kalman Filter is the Seasonal ARIMA model \( (0,0,0) \times (0,1,1)_12 \) with the mathematical model \( Y_t = Y_{t-12} - 0.87a_{t-12} + a_t \) and \( \Theta_1 \) which is \(-0.87\) as parameters have a smaller RMSE value than the Seasonal ARIMA model with Kalman Filter on 80% and 75% training data, while the RMSE value for Seasonal ARIMA Kalman Filter on 90% training data is 155, 13 for 90% training data, RMSE value of 176.96 for 80% training data and RMSE value of 165.49 for 75% training data. So to predict rainfall, we use the Seasonal ARIMA Kalman Filter model with 90% training data.

Rainfall prediction using Seasonal ARIMA Model with Kalman Filter ARIMA Seasonal \( (0,0,0) \times (0,1,1)_12 \) with mathematical model \( Y_t = Y_{t-12} - 0.87a_{t-12} + a_t \) and \( \Theta_1 \) which is \(-0.87\) as parameters, it seems that they can still follow the actual data but do not follow the actual data in June to October. According to field observations, from June to October, is the dry season; during the dry season, the rainfall is very low, and there is even no rain. Therefore, further research can examine the rainy season and dry season separately.

References
[1] Gelb A 1974 Optimal linear filtering. In Applied Optimal Estimation vol 8
[2] Song D, Yang J, Liu Y, Su M, Liu A and Joo Y H 2017 Int. J. Control. Autom. Syst. 15 1720–8
[3] Aslam M 2020 Data Br. 31 105854
[4] Hage R-M S and Mghames S J 2020 Eur. J. Sci. Res. 155 440–54
[5] Sivagami, Vaishali A, Ramakrishnan R and Subasini A 2019 Procedia Comput. Sci. 165 449–55
[6] Yuan S and Shen Y 2020 J. Phys. Conf. Ser. 1650 032095
[7] Lai X, Yang T, Wang Z and Chen P 2019 Appl. Sci. 9
[8] Merabet F and Zeghdoudi H 2020 WSEAS Trans. Syst. Control 15 235–46
[9] Felix E A and Lee S P 2019 IET Softw. 13 479–96
[10] Wei W W S 1991 *Technometrics* **33** 108
[11] Boukharouba K and Kettab A 2016 *Desalin. Water Treat.* **57** 17095–103
[12] Makridakis S, Wheelwright S. C and Hybdman, R J 2008 Forecasting Methods and Applications (John Wiley & Sons).
[13] Meinhold R J and Singpurwalla N D 1983 *Am. Stat.* **37** 123
[14] Lewis F L, Xie L and Pop 2007 *Optimal and Robust Estimation With an Introduction to Stochastic Control Theory* (CRC Press)
[15] Makridakis S, Wheelwright SC H R 1997 *Third Ed.* 1–632