Primordial Black Holes and Quantum Effects

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Abstract Primordial black holes (PBHs) are of special interest because of the crucial role of quantum effects in their formation and evaporation. This means that they provide a unique probe of the early universe, high-energy physics and quantum gravity. We highlight some recent developments in the subject, including improved limits on the fraction of the Universe going into evaporating PBHs in the mass range \(10^9 - 10^{17}\) g and the possibility of using PBHs to probe a cosmological bounce.

1 Introduction

A comparison of the cosmological density at a time \(t\) after the big bang with the density associated with a black hole of mass \(M\) shows that PBHs should have of order the particle horizon mass, \(M_H(t) \approx 10^{15} (t/10^{-23}\text{s}) \text{g}\), at formation. They could thus span an enormous mass range: from \(10^{-5}\) g for those formed at \(10^{-43}\) s to \(10^5 M_\odot\) for those formed at 1 s. By contrast, black holes forming at the present epoch could never be smaller than about \(1 M_\odot\). However, the high density of the early Universe is not a sufficient condition for PBH formation. One either needs large-amplitude density fluctuations, possibly of inflationary origin, so that overdense regions can eventually stop expanding and recollapse, or some sort of cosmological phase transition at which PBHs can form spontaneously (eg. via the collapse of cosmic loops or the collisions of bubbles of broken symmetry). All these formation mechanisms depend in some sense on quantum effects and they are discussed in detail in Ref. [1] (henceforth CKSY).

The realization that PBHs might be small prompted Hawking to study their quantum consequences. This led to his famous discovery [2] that black holes radiate thermally with a temperature \(T \approx 10^{-7} (M/M_\odot)^{-1}\text{K}\) and evaporate on a timescale
\[ \tau(M) \approx 10^{64}(M/M_\odot)^3 \gamma. \] Only black holes smaller than 10^{15}g would have evaporated by the present epoch and 10^{15}g ones would be exploding today. Since the latter would be producing photons with energy of order 100 MeV, the observational limits on the \( \gamma \)-ray background intensity imply that their density could not exceed \( 10^{-8} \) times the critical density. Nevertheless, this does not preclude PBHs playing other important cosmological roles. Indeed, their study provides a unique probe of four areas of physics: gravitational collapse for \( M > 10^{15}g \), high energy physics for \( M \sim 10^{15}g \), the early Universe for \( M < 10^{15}g \) and quantum gravity for \( M \sim 10^{-5}g \).

Since both their formation and evaporation are a consequence of quantum effects, PBHs may offer the only astrophysical realization of what might be termed “quantum black holes” (i.e. holes for which quantum effects are important) [3]. This article will focus on their evaporation rather than their formation. In particular, it will discuss the upper limit on the fraction of the Universe going into PBHs as a function of mass because this provides important constraints on models (such as inflation) predicting their formation. The fraction of the Universe collapsing into PBHs at time \( t \) is related to their current density parameter \( \Omega_{\text{PBH}} \) by

\[
\beta \approx 10^{-6} \Omega_{\text{PBH}}(t/s)^{1/2} \approx 10^{-18} \Omega_{\text{PBH}}(M/10^{15}g)^{1/2} \tag{1}
\]

where the \( t \) dependence reflects the decreasing ratio of the PBH and radiation densities at early times [4]. Any limit on \( \Omega_{\text{PBH}} \) therefore places a constraint on \( \beta \) as a function of \( M \). The constraints on \( \beta(M) \) have been studied by numerous authors but the most recent and comprehensive discussion is that of Ref. [1]. The limits cover the mass range \( 10^9 - 10^{17}g \) and are shown in Fig. 1. The important point is that the value of \( \beta(M) \) must be tiny throughout this mass range, so any cosmological model which entails an appreciable fraction of the Universe going into PBHs is immediately excluded. The most stringent limits – associated with big bang nucleosynthesis (BBN), the extragalactic \( \gamma \)-ray background (EGB) and observations of anisotropies in the cosmic microwave background (CMB) – are discussed below. Positive evidence for PBHs might come from cosmic rays or short-period gamma-ray bursts but this is not covered below since the status of the observations is still ambiguous.

Fig. 1: Combined BBN and EGB limits (solid), compared to other constraints on evaporating PBHs from LSP relics and CMB distortions (short-dashed), extragalactic antiprotons and neutrinos (dotted), the Galactic \( \gamma \)-ray background (long-dashed), CMB anisotropies (dash-dotted) and the density limit from the smallest unevaporated black holes (dashed). From Ref. [1].
2 PBH Evaporations

A black hole with mass \( M = M_{10} \times 10^{10} \text{ g} \) emits thermal radiation with temperature

\[
T_{\text{BH}} = \frac{1}{8\pi GM} \approx 1.06M_{10}^{-1} \text{ TeV}.
\] (2)

The average energy of the emitted particles is \((4 - 6)kT_{\text{BH}}\), depending on their spin. Charge and angular momentum are neglected because these will be lost through quantum emission on a shorter timescale. The mass loss rate can be expressed as

\[
\frac{dM_{10}}{dt} = -5.34 \times 10^{-5} f(M) M_{10}^{-2} \text{ s}^{-1}.
\] (3)

Here \( f(M) \) is a measure of the number of emitted particle species, normalised to unity for a black hole with \( M \gg 10^{17} \text{ g} \), emitting only particles which are (effectively) massless: photons, neutrinos and gravitons. Holes with \( 10^{15} \text{ g} < M < 10^{17} \text{ g} \) emit electrons, while those with \( 10^{14} \text{ g} < M < 10^{15} \text{ g} \) also emit muons, which subsequently decay into electrons and neutrinos.

Once \( M \) falls to around \( 10^{14} \text{ g} \), a black hole can also begin to emit hadrons. However, hadrons are composite particles, made up of quarks held together by gluons, so for temperatures exceeding \( \Lambda_{\text{QCD}} = 250 - 300 \text{ MeV} \), one would expect the emission of quark and gluon jets rather than hadrons [5]. The jets would subsequently fragment into hadrons but only after travelling a distance \( \Lambda_{\text{QCD}} \sim 10^{-13} \text{ cm} \), which is much larger than the size of the hole. The QCD fragmentation has been calculated using the PYTHIA [1] and HERWIG [6] codes but with similar results. Since there are many quark and gluon degrees of freedom, the value of \( f \) should roughly quadruple once the QCD temperature is reached. If we sum up the contributions from all particles in the Standard Model up to 1 TeV, this gives \( f(M) = 15.35 \) and a lifetime

\[
\tau \approx 407 \left( \frac{f(M)}{15.35} \right)^{-1} M_{10}^3 \text{ s}.
\] (4)

The critical mass for which \( \tau \) equals the age of the Universe \( (t_0 \approx 13.7 \text{ Gyr}) \) is \( M_* \approx 5.1 \times 10^{14} \text{ g} \), corresponding to \( f_* = 1.9 \) and \( T_{\text{BH}}(M_*) = 21 \text{ MeV} \).

The direct Hawking emission is termed the primary component, while the jet fragmentation emission is termed the secondary component. The spectrum of secondary photons is dominated by the 2\( \gamma \)-decay of soft neutral pions and peaks around \( E_\gamma \approx m_{\pi^0}/2 \approx 68 \text{ MeV} \). The emission rates of primary and secondary photons for four typical temperatures are shown in Fig. 2. Although QCD effects are initially small for PBHs with \( M = M_* \), only contributing a few percent, they become important once \( M \) falls to \( M_q \approx 0.4M_* \approx 2 \times 10^{14} \text{ g} \) since the peak energy becomes comparable to \( \Lambda_{\text{QCD}} \) then. This means that an appreciable fraction of the time-integrated emission from the PBHs evaporating at the present epoch goes into quark and gluon jet products. However, a PBH with somewhat larger initial mass, \( M = (1 + \mu)M_* \) will today have a mass \( M(t_0) \approx (3\mu)^{1/3}M_* \) for \( \mu \ll 1 \). Since this falls below \( M_q \).
only for $\mu < 0.02$, the fraction of the black hole mass going into secondaries falls off sharply above $M_\ast$. The ratio of the secondary to primary peak energies and the ratio of the time-integrated fluxes are shown as functions of $M$ in Fig. 2.

There has been some dispute about the interactions between emitted particles beyond the QCD scale. The usual assumption that there is no interaction has been refuted by Heckler [7], who claims that QED interactions could produce an optically thick photosphere once the black hole temperature exceeds $T_{\text{BH}} = 45\text{GeV}$. He has proposed that a similar effect may operate at an even lower temperature, $T_{\text{BH}} \approx 200\text{MeV}$, due to QCD effects [8]. Variants of these models and their astrophysical implications have been studied by various authors. However, MacGibbon et al. [9] have reviewed all these models and identified a number of effects which invalidate them. They conclude that emitted particles do not interact sufficiently to form a QED photosphere and that the conditions for QCD photosphere formation could only be temporarily satisfied (if at all) when the black hole temperature is of order $\Lambda_{\text{QCD}}$.

### 3 Constraints on $\beta(M)$ Imposed by BBN, EGB and CMB

PBHs with $M \sim 10^{10}\text{g}$ and $T_{\text{BH}} \sim 1\text{TeV}$ have a lifetime $\tau \sim 10^3\text{s}$ and therefore evaporate at the epoch of big bang nucleosynthesis (BBN). The effect of these evaporation on BBN has been a subject of long-standing interest and jet-produced hadrons are particularly important. Long-lived hadrons remain in the ambient medium long enough to leave an observable signature on BBN. These effects were first discussed for the relatively low mass PBHs evaporating in the early stages of BBN [10] but the analysis has now been extended by CKSY to incorporate the effects of heavier PBHs evaporating after BBN.

High energy particles emitted by PBHs modify the standard BBN scenario in three different ways: (1) high energy mesons and antinucleons induce extra inter-

![Fig. 2](image-url)
conversion between background protons and neutrons even after the weak interaction has frozen out in the background Universe; (2) high energy hadrons dissociate light elements synthesised in BBN, thereby reducing $^4\text{He}$ and increasing $D$, $T$, $^3\text{He}$, $^6\text{Li}$ and $^7\text{Li}$; (3) high energy photons generated in the cascade further dissociate $^4\text{He}$. The PBH constraints depend on the initial baryon-to-photon ratio (allowing for PBH entropy production) and the ratio of the PBH number density to the entropy density, $Y_{\text{PBH}} \equiv n_{\text{PBH}}/s$, which is related to the initial mass fraction by $\beta = 5.4 \times 10^{21} (\tau/1\,\text{s})^{1/2} Y_{\text{PBH}}$.

The results of these calculations are summarized in Fig. 3. PBHs with $M < 10^9\,\text{g}$ ($\tau < 10^{-2}\,\text{s}$) are free from BBN constraints because they evaporate before weak freeze-out. PBHs with $M = 10^9 - 10^{10}\,\text{g}$ ($\tau = 10^{-2} - 10^2\,\text{s}$) are constrained by process (1), those with $M = 10^{10} - 10^{12}\,\text{g}$ ($\tau = 10^2 - 10^7\,\text{s}$) by process (2) and those with $M > 10^{12} - 10^{13}\,\text{g}$ ($\tau = 10^7 - 10^{12}\,\text{s}$) by process (3). We also show as a broken line the limits obtained earlier [10]. The helium limit is weaker because the primordial abundance is now known to be smaller, while the deuterium limit is stronger because of its extra production by hadrodissociation of helium.

It has been known for 40 years that observations of the diffuse extragalactic $\gamma$-ray background (EGB) constrain $\Omega_{\text{PBH}}(M_\ast)$ to be less than around $10^{-8}$ [11]. This limit has subsequently been refined by numerous authors and most recently by CKSY. In order to determine the present background spectrum of photons generated by PBH evaporations, one must integrate over the lifetime of the black holes, allowing for the fact that particles generated in earlier cosmological epochs will be redshifted in energy by now. The highest energy photons are associated with PBHs of mass $M_\ast$. Those from PBHs with $M > M_\ast$ are at lower energies because they are cooler, while those from PBHs with $M < M_\ast$ (although initially hotter) are at lower energies because they are redshifted.

Fig. 3 Left: Upper bounds on $\beta(M)$ from BBN, with broken line giving earlier limit. Right: Upper bounds on $\beta(M)$ from the extragalactic photon background, with no other contributors to the background having been subtracted. From Ref. [11].
The most recent X-ray and γ-ray observations are summarized by CKSY and correspond to an intensity $I_{\text{obs}} \propto E_{\gamma}^{-\epsilon} \gamma_0$ where $\epsilon$ lies between 0.1 and 0.4. The origin of these backgrounds is thought to be primarily distant astrophysical sources, such as blazars, and in principle one should remove these contributions before calculating the PBH constraints \[12\]. CKSY do not attempt such a subtraction, so their constraints may be overly conservative. The limits on $\beta(M)$ are shown in Fig. 3 and depend on the relative magnitude of the primary and secondary components. PBHs with $M > M_\ast$ can never emit secondary photons and one obtains $\beta(M) < 4 \times 10^{-29} (M/M_\ast)^{7/2+\epsilon}$. Those with $M \leq M_\ast$ will do so once $M$ falls below $M_\eta \approx 2 \times 10^{14} \text{g}$ and one obtains $\beta(M) < 3 \times 10^{-27} (M/M_\ast)^{-5/2-2\epsilon}$. These $M$-dependences explain the qualitative features of Fig. 3 and the associated limit on the density parameter is $\Omega_{\text{PBH}}(M_\ast) \leq 5 \times 10^{-10}$. Since photons emitted at sufficiently early times cannot propagate freely, there is a minimum mass $M_{\text{min}} \approx 3 \times 10^{13} \text{g}$ below which the above constraint is inapplicable.

If PBHs of mass $M_\ast$ are clustered inside our own Galactic halo, as expected, then there should also be a Galactic γ-ray background. Some time ago it was claimed that such a background had been detected by EGRET between 30MeV and 120GeV and that this could be attributed to PBHs \[13\]. A more recent analysis of EGRET data between 70MeV and 150GeV gives a limit $\Omega_{\text{PBH}}(M_\ast) < 2.6 \times 10^{-9}$ or $\beta(M_\ast) < 1.4 \times 10^{-26}$ \[14\], which is a factor of 5 above the EGB constraint. However, CKSY point out that the EGB constraint on $\beta(M)$ comes from the time-integrated contribution of the $M_\ast$ black holes, which peaks at 120 MeV, whereas the Galactic background is dominated by PBHs which are slightly larger than this. The emission from PBHs with initial mass $(1 + \mu)M_\ast$ currently peaks at an energy $E \approx 100 (3 \mu)^{-1/3} \text{MeV}$, which is in the range 70 MeV–150 GeV for $0.7 < \mu > 0.08$. The corrected limit is shown in Fig. 1.

The CMB anisotropy constraint arises because electrons and positrons from PBHs heat the matter content of the Universe after recombination, thereby damping small-scale anisotropies. CKSY find $\beta(M) < 3 \times 10^{-30} (M/10^{13} \text{g})^{3.1}$ for $2.5 \times 10^{13} \text{g} < M < 2.4 \times 10^{14} \text{g}$. The upper limit corresponds to evaporation at the epoch of reionization ($z = 6$), since the opacity is too low for emitted particles to heat the matter thereafter. This is stronger than all the other limits in this mass range.

### 4 PBHs and Dark Matter

Roughly 30% of the total density of the Universe is now thought to be in the form of “cold dark matter”. There has been a lot of interest in whether PBHs could provide this, since those larger than $10^{15} \text{g}$ would not have evaporated yet and would certainly be massive enough to be dynamically cold. One possibility is that PBHs with a mass of around $1M_\odot$ could have formed efficiently at the quark-hadron phase transition at $10^{-5}$s because of a temporary reduction in pressure \[15\]. At one stage there seemed to be evidence for this from microlensing observations. The data no longer support this but there are no constraints excluding PBHs in the sublunar range $10^{20} \text{g} < M < \ldots$
10^{26} \, \text{g} \quad [16] \text{or intermediate mass range } 10^2 M_\odot < M < 10^4 M_\odot \quad [17] \text{from having an appreciable density.}

Some people have speculated that black hole evaporation could cease once the hole gets close to the Planck mass (M_P) due to the influence of extra dimensions, higher order corrections to the gravitational Lagrangian, string effects, the Generalized Uncertainty Principle etc. The resulting stable relics would then be natural candidates for the dark matter [18]. In an inflationary scenario, if the relics have a mass $\kappa M_{\text{Pl}}$ and reheating occurs at a temperature $T_R$ (when the PBHs form), then the requirement that the relic density be less than the dark matter density implies $\beta(M) < 2 \times 10^{-28} \kappa^{-1} (M/M_{\text{Pl}})^{3/2}$ for $(T_R/T_{\text{Pl}})^{-2} < M/M_{\text{Pl}} < 10^{11} \kappa^{2/5}$ [19]. The lower mass limit arises because PBHs generated before reheating are diluted exponentially. (If there is no inflationary period, the constraint extends all the way down to the Planck mass.) The upper mass limit arises because PBHs larger than this dominate the total density before they evaporate, in which case the current cosmological photon-to-baryon ratio is determined by the baryon asymmetry associated with their emission.

5 PBHs as a Probe of a Cosmological Bounce

In some cosmological scenarios, the Universe is expected to eventually recollapse to a big crunch and then bounce into a new expansion phase. Such a bounce may arise through either classical or quantum gravitational effects. Even if the universe is destined to expand forever, it may have been preceded by an earlier collapsing phase. Both past and future bounces would arise in cyclic models, as reviewed in Ref. [20]. It is therefore interesting to ask whether black holes could either be generated by a big crunch or survive it if they were formed earlier [20]. We refer to these as “big-crunch black holes” (BCBHs) and “pre-crunch black holes” (PCBHs), respectively. If such black holes were detectable today, they would provide a unique probe of the last cosmological bounce, although this raises the question of whether one could differentiate between black holes formed just before and just after the last bounce.

Let us assume that the universe bounces at some density $\rho_B$. Since the density associated with a black hole of mass $M$ is $\rho_{BH} = (3M/4\pi R_S^3)$, this corresponds to a lower limit on the BCBH mass $M_{\text{min}} \sim (\rho_{\text{eq}}/\rho_B)^{1/2} M_P$. There is also a mass range in which pre-existing PCBHs lose their individual identity by merging with each other prior to the bounce. If the fraction of the cosmological density in these black holes at the bounce epoch is $f_B$, then the average separation between them is less than their size (i.e. the black holes merge) for $M > f_B^{-1/2} M_{\text{min}}$. The important point, as indicated in Fig. [4], is that there is a always range of masses in which BCBHs may form and PCBHs do not merge. However, one must distinguish between $f_B$ and the present fraction $f_0$ of the Universe’s mass in black holes. Since the ratio of the black hole to radiation density scales as the cosmic scale factor, the fraction of the universe in black holes at a radiation-dominated bounce is $f_B \approx f_0 (\rho_{\text{eq}}/\rho_B)^{1/4}$.
where $\rho_{eq} \sim 10^{12} \rho_0 \sim 10^{-17} \text{g cm}^{-3}$. The merger condition therefore becomes $f_0 > 10^{28} (\rho_B/\rho_P)^{-3/4} (M/M_P)^{-2}$, as indicated by the line on the right of Fig. 4.

There are various dynamical constraints on the form of the function $f_0(M)$ for non-evaporating PCBHs. They must have $f_0 < 1$ in order not to exceed the observed cosmological density and this gives a minimum value for the merger mass, $M_{\text{merge}} \sim 10^4 (t_B/t_P)^{3/4} \text{g}$, where $t_B$ is the time of the bounce as measured from the notional time of infinite density. This is around $10^{15} \text{g}$ for $t_B \sim 10^{-35} \text{s}$ but as large as $10^4 M_\odot$ for $t_B \sim 10^{-5} \text{s}$, so the observational consequences would be very significant. Another important constraint, deriving from Poisson fluctuations in the black hole number density, is associated with large-scale structure (LSS) formation [21]. This gives a limit $f_0 < (M/10^4 M_\odot)^{-1}$, as shown by the line at the top right of Fig. 4.

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Fig. 4 This shows the domain in which black holes of mass $M$ containing a fraction $f_0$ of the present density can form in a big crunch or avoid merging if they exist before it. From Ref. [20].