Mechanism of Coulomb breakup reactions of two-neutron halo nuclei $^6$He and $^{11}$Li

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Abstract. We investigate the three-body Coulomb breakup reactions of two-neutron halo nuclei and discuss the correlations of every binary subsystem such as of core-$n$ and $n$-$n$ by showing the invariant mass spectra. It is found that the final-state interactions of core-$n$ and $n$-$n$ binary subsystems dominantly determine the observed energy distributions of the breakup cross sections, such as the low-lying enhancements. Furthermore, we investigate the effects of the $^9$Li core excitations on the Coulomb breakup cross section of $^{11}$Li. It is shown that the integrated $E_1$ strength at low energy region is reduced by $\sim 15\%$ by the inclusion of the $^9$Li core excitations.

1. Introduction

Since the discovery of neutron halo nuclei such as $^{11}$Be, $^6$He, and $^{11}$Li, extensive studies have been performed to understand their exotic structure caused by the weakly-bound nature of valence neutrons [1]. Theoretically, using the models based on the core + valence neutrons picture, the halo structure of the ground states and the excitations of halo nuclei have been investigated [2–5]. In two-neutron halo nuclei, it has been pointed out the correlation between two halo neutrons is important to explain the observed small two-neutron separation energies and large matter radii [3].

Experimentally, the Coulomb breakup reactions have been performed to expose the role of the $n$-$n$ correlation in the two-neutron halo nucle. The observed Coulomb breakup cross section provides with interesting information on the properties of weak binding and the excitations of halo nuclei. It is also expected to gain a deeper understanding of the $n$-$n$ correlation in the two-neutron halo nuclei through the Coulomb breakup reactions [6]. To understand the $n$-$n$ correlation in the two-neutron halo nuclei, there are at least two problems to be solved: (i) clarifying the dominant breakup process such as the direct breakup to a noninteracting three-body continuum or sequential decay via the resonance of the binary subsystem, (ii) evaluating the influence of the final-state interactions (FSI) on the cross section.

It has been observed in $^{11}$Li that the amount of the $(1s_{1/2})^2$ component ($45\pm10\%$) of the halo neutrons is comparable to that of $(0p_{1/2})^2$ [7]. This fact indicates the breaking of the $N = 8$ magic number in the ground state of $^{11}$Li. To reproduce this situation in $^{11}$Li, most of
the theoretical studies based on the $^9\text{Li} + n + n$ three-body models assume the $1s_{1/2}$ orbit is degenerated with the $0p_{1/2}$ one energetically by using the different $^9\text{Li}$-$n$ interactions for even and odd parity states [6; 8; 9]. However, Esbensen et al. [8] have mentioned that the $^9\text{Li} + n + n$ three-body model assuming the inert $^9\text{Li}$ core fails to explain the observed charge radius and the $E1$ strength in $^{11}\text{Li}$ consistently.

In Ref. [10], Myo et al. have shown that various physical quantities such as the matter and charge radii and the $E1$ strength of halo nuclei. The Coulomb breakup cross section as function of the total energy $E$ as

$$
\sigma(E) = \frac{16\pi^3}{9\hbar c} N_{E1}(E_\gamma) \frac{d^3B(E1)}{dKdk},
$$

where $N_{E1}(E_\gamma)$ is the virtual photon number with the photon energy $E_\gamma$. Two momenta, $k$ and $K$, represent relative momenta in Jacobi coordinates for the three-body system. The $E1$ strength distribution is given as

$$
\frac{d^3B(E1)}{dKdk} = \frac{1}{2J_{gs} + 1} \left| \langle \Phi^{(-)}(k, K) | \hat{O}(E1) | \Phi_{gs} \rangle \right|^2,
$$

where $\Phi_{gs}$ and $\Psi^{(-)}(k, K)$ are the wave functions for the ground and excited states, respectively, and the ground-state wave function for $^6\text{He}$ and $^{11}\text{Li}$ are the same ones as in Ref. [4] and [5], respectively. The total spin of the ground state is given as $J_{gs}$. Using Eq. (2), we obtain the Coulomb breakup cross section as function of the total energy $E$ as

$$
\frac{d\sigma}{dE} = \int dKdk \frac{d^3\sigma}{dKdk} \delta \left( E - \frac{\hbar^2k^2}{2\mu} - \frac{\hbar^2K^2}{2M} \right),
$$

where $\mu$ and $M$ are reduced masses corresponding to $k$ and $K$, respectively. Similarly, we can calculate the invariant mass spectra as

$$
\frac{d\sigma}{d\varepsilon_1} = \int dKdk \frac{d^3\sigma}{dKdk} \delta \left( \varepsilon_1 - \frac{\hbar^2k^2}{2\mu} \right),
$$

where $\varepsilon_1$ is the energy of the binary subsystem. Using Eq. (4), we can discuss the effects of binary subsystem correlations, such as core-$n$ and $n-n$, on the Coulomb breakup reaction.
To evaluate the distributions given in Eqs. (3) and (4), it is necessary to obtain the scattering states, $\Psi^-(k, K)$. Here, we obtain $\Psi^-(k, K)$ using CSLS[11; 12]. The incoming scattering state $\Psi^-$ in the bra-representation is described as

$$\langle \Psi^-(k, K) \rangle = \langle \Phi_0(k, K) \rangle + \langle \Phi_0(k, K) \rangle \hat{V} \lim_{\varepsilon \to 0} \frac{1}{E - \hat{H} + i \varepsilon},$$

where $\Phi_0$ is a solution of an asymptotic Hamiltonian $\hat{H}_0$. The interaction $\hat{V}$ is defined by subtracting $\hat{H}_0$ from the total hamiltonian $\hat{H}$ and becomes the source of FSI.

The Green’s function in Eq. (5) is replaced with the complex-scaled Green’s function in CSLS, and the relation between the Green’s function in Eq. (5) and the complex-scaled one is given as

$$\lim_{\varepsilon \to 0} \frac{1}{E - \hat{H} + i \varepsilon} = \hat{U}^{-1}(\theta) \frac{1}{E - \hat{H}^\theta} \hat{U}(\theta) = \sum_{n} \hat{U}^{-1}(\theta) |\chi_n^\theta\rangle \frac{1}{E - E_n^\theta} \langle \tilde{\chi}_n^\theta | \hat{U}(\theta) \rangle.$$  

In derivation of the right-hand-side of Eq. (6), we insert the complete set constructed with $\{\chi_n^\theta\}$, being the eigenstates of the complex-scaled Hamiltonian $\hat{H}^\theta$. We here calculate the eigenstates and eigenvalues of $\hat{H}^\theta$, $\{|\chi_n^\theta\rangle\}$ and $\{E_n^\theta\}$ by using the core + $n + n$ three-body models [4; 5]. We obtain $\Psi^-(k, K)$ as

$$\langle \Psi^-(k, K) \rangle = \langle \Phi_0(k, K) \rangle + \sum_{n} \langle \Phi_0(k, K) \rangle \hat{V} \hat{U}^{-1}(\theta) |\chi_n^\theta\rangle \frac{1}{E - E_n^\theta} \langle \tilde{\chi}_n^\theta | \hat{U}(\theta) \rangle.$$  

The scattering state in CSLS consists of two terms: The first term represents the noninteracting continuum states, and the second is the effects of FSI. This description of the scattering states is useful to investigate the effects of correlations in the final states after the breakup. The correlations in the ground state can also be seen in the transition to the first term.

### 3. Coulomb breakup cross sections of $^6$He and $^{11}$Li

We first show the Coulomb breakup cross section with respect to the total energy of $^6$He and $^{11}$Li, in Fig. 1, in comparison with the observed ones [13; 14]. In both cases, the calculated cross sections show the low-lying enhancements, and well reproduces the observed data. We also estimate the effect of FSI on the cross sections by taking only the first term in Eq. (7) in

**Figure 1.** Obtained Coulomb breakup cross sections of $^6$He and $^{11}$Li, measured from the threshold energy of core + $n + n$. The left and right panels represent the cross sections for $^6$He and $^{11}$Li, respectively. The observed data, shown as solid squares, are taken from Ref. [13] for $^6$He and from Ref. [14] for $^{11}$Li.
Figure 2. Invariant mass spectra of the Coulomb breakup reaction of $^6$He. The left and right panels present the spectra for $\alpha$-$n$ and $n$-$n$, respectively. The observed data, shown as solid squares, are taken from Ref. [13]. An arrow in the left panel indicates the position of the $^5$He$(3/2^-)$ resonances in our calculation.

Figure 3. Invariant mass spectra of the Coulomb breakup reaction of $^{11}$Li. The left and right panels present the spectra for $^9$Li-$n$ and $n$-$n$, respectively. In the left panel, three types of spectra are presented: The total distribution (solid), $s$-wave component (dotted), and $p$-wave component (dashed). Two arrows in the left panel indicate the position of the $p$-wave resonances in our calculation.

the calculation, and it is found that correlations in final states play decisive roles in reproducing the Coulomb breakup cross sections.

To discuss what kinds of FSI are important in the Coulomb breakup reactions, we calculate the invariant mass spectra for binary subsystems. We show the spectra for $^6$He in Fig. 2. From the results, the invariant mass spectra for the $\alpha$-$n$ subsystem shows the peak at around 0.7 MeV corresponding to the $^5$He$(3/2^-)$ resonance. The $\alpha$-$n$ correlation is clearly confirmed in the invariant mass spectra of the Coulomb breakup cross section. For the $n$-$n$ subsystem, the low-lying enhancement is seen near the zero-energy region, which indicates the importance of the $n$-$n$ virtual state in the final states.

In Fig. 3, we also show the calculated invariant mass spectra for $^{11}$Li. From the results, we confirm that both invariant mass spectra for $^9$Li-$n$ and $n$-$n$ show the low-lying enhancements. For $^9$Li-$n$, it is seen that the peak comes from the $s$-wave component as shown in the left panel in Fig. 3. This result indicates the virtual-state correlation of the $^9$Li-$n$ subsystem. For $n$-$n$, we find the similar behavior to that for $^9$Li-$n$, and the virtual-state correlation of the $n$-$n$ subsystem is confirmed in the spectra. One the other hand, from Fig. 3, the $p$-wave resonances in $^{10}$Li does not contribute to the invariant mass spectra for the $^9$Li-$n$ subsystem since the energies of the
Figure 4. Comparison between the $E1$ strength distributions with different $(s_{1/2})^2$ components. The solid line represent the result of using the ground-state wave function with $(s_{1/2})^2 = 44\%$, which is used in the right panel of Fig. 1. The dashed line is the result with $(s_{1/2})^2 = 21\%$. The dotted line is taken from Ref. [9].

$p$-wave resonances, which are obtained at 275 keV and 506 keV in our calculation, are higher than the peak position of the total energy of the breakup cross section shown in the right panel of Fig. 1.

From the results for $^6$He and $^{11}$Li, it is found that the invariant mass spectra for core-$n$ subsystems reflect the characters of the core-$n$ correlations in two-neutron halo nuclei, while the behaviors of the spectra for $n$-$n$ subsystem are commonly dominated by the $n$-$n$ virtual-state correlation in two nuclei.

4. Effects of $^9$Li core excitations on the Coulomb breakup reaction of $^{11}$Li

To investigate the effects of the $^9$Li core excitations on the Coulomb breakup reaction of $^{11}$Li, we compare the $E1$ transition strength calculated in our coupled-channel $^9$Li + $n + n$ model [5] with the result in Ref. [9], in which the three-body model assuming the inert $^9$Li core is employed.

We here calculate the $E1$ strength distribution using the different $^{11}$Li wave function, in which the coupling to the 2p-2h configuration involved only by the pairing correlation in $^9$Li core is taken into account. This restriction leads to the small $(s_{1/2})^2$ component as 21.0% in the ground state, which gives a similar value to that in the three-body model assuming the inert $^9$Li core. The calculated distribution is shown in Fig.4 as dashed line in comparison with the result in the model assuming the inert core (dotted line). The wave function used in the results shown as dashed and dotted lines contain almost the same amount of the $s$-wave components in the $^{11}$Li ground state. In two kinds of results, the $E1$ strength distributions commonly have peaks at around 0.5 MeV; however, there exists a large difference of strength around the peak energy. This is due to the fact that about 15% of the integrated strength in our calculation escapes to the higher excited states having the excited components of the $^9$Li core.

We confirm that the observed Coulomb breakup cross section and the $E1$ strength distribution cannot be reproduced only by taking into account the coupling to the 2p-2h excitation due to the pairing correlation in $^9$Li. To reproduce the observed cross section, it is essential to include the 2p-2h excitation coming from not only the pairing correlation but also the tensor one in the $^9$Li core. By taking into account both correlations in $^9$Li, we obtain the result shown as solid line in Fig. 4, which is used in the calculation of the cross section in the right panel in Fig. 2.
5. Summary

In this study, we calculate the three-body Coulomb breakup cross sections and invariant mass spectra for binary subsystems of $^6$He and $^{11}$Li. Our calculations reproduce the Coulomb breakup cross sections well. From the result of cross sections, it is found that the FSI has a dominant contribution to explain the low-lying enhancement seen in the cross sections. It is confirmed that the subsystem correlations of the $^5$He($3/2^-$) resonance and the $^{10}$Li virtual state dominantly determine the distributions of the invariant mass spectra of the breakup reactions of $^6$He and $^{11}$Li, respectively, while the n-n virtual-state correlations are found in both cases. Furthermore, we investigated the effects of the $^9$Li core excitations on the Coulomb breakup reaction of $^{11}$Li. It is shown that the integrated $E1$ strength is reduced by $\sim 15\%$ by inclusion of the $^9$Li core excitations. It is essential to take into account 2p-2h excitations due to the tensor and pairing correlations in the $^9$Li core in reproducing the observed Coulomb breakup cross section of $^{11}$Li.

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