Research Article

Ancient Mesopotamian Stone Bridge: Numerical Modeling and Structural Assessment

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This study aimed to investigate the stress-strain and strain energy density (SED) states of Dalal stone arch bridge in Mesopotamia. Structural modeling of ancient bridge made of natural stone has been proven reliable, and accurate results have been obtained using 3D finite elements. Based on the more applicable theories of failure, a general methodology is presented for evaluating the ringstone of the largest ellipse-shaped arch of the Dalal Bridge. The elliptical arch was built in the COMSOL Multiphysics complex using 70 3D elements to represent the number of stones used along the length of the arch in the Dalal Bridge. Therefore, to create an accurate model, the coordinates of the four nodes of each stone were entered. Then, all domains were extruded for 0.8 m in the y-axis direction, i.e., 0.8 m of the bridge width was selected for investigation. That is, tapered fields were used to represent the stones of the arch ring. Using Rankine’s, St. Venant’s, and Haigh’s theories, the qualitative and quantitative characteristics of all components of the stresses and SED states are investigated. The maximum positive values of the principal stresses, \( \sigma_1 \), \( \sigma_2 \), and \( \sigma_3 \), in the 3D model reach 1.4, 0.51, and 0.09 MPa, respectively, and their maximum negative values were 13, 6.8, and 3.4 MPa, respectively. The equivalent principal stresses determined via a 2D investigation did not exceed these values. Evaluating the ringstone against the maximum principal strain theory (i.e., St. Venant’s theory) reveals a safety factor of four in the existing state. Also, application of Haigh’s theory confirms the results of the previously applied approaches. Even though the safety of the arch, according to the total strain energy theory (i.e., Haigh’s approach), has been verified, a significant variation in the nonuniformity of the distribution of the SED (0.0011 J/m\(^3\) –4416 J/m\(^3\)) confirmed that the geometry of the investigated arch is not optimal for applied loading. The maximum value of the vertical component of the displacement is 3.4 mm, significantly lower than the allowable deflection for such an arch span.

1. Introduction

A review of the literature showed that truly little scientific work had been conducted on historical buildings, bridges, and monuments in Mesopotamia. Most of the publications are devoted to the purely historical, artistic, or architectural aspects of the Mesopotamian civilization or the assessment of the practical experience of their construction and restoration [1–4]. In ancient Mesopotamia, the natural stone arch, one of the oldest structural elements, is widely used in buildings and bridges. Stone arches have been used for thousands of years owing to their excellent load-bearing capacity and ability to transmit loads to the foundation and soil through the help of compressive stresses. The structural analysis approaches used for stone arches until the second half of the last century were essentially based on the classical methods of statics and were likely the methods by which many arch bridges were designed [5, 6]. The advantages of taking this case study over others is the historical value of Dalal Bridge, as well as the unique design solution, which makes it unmatched in our country.

It is well known that current structural analysis methods, particularly the approaches based on elasticity theory, have focused the attention of structural engineers on the structural possibilities brought about by the development of steel and reinforced concrete. In contrast, stone has lost its principal role as an essential building material. In the past several decades, very few masonry arch bridges have been
built, and knowledge of the related design methods has ceased to form part of the civil engineers’ stock-in-trade. The lack of reliable methods for assessing the existing state of masonry arch bridges is a problem for structural engineers. It makes it exceedingly difficult to maintain and conserve ancient and historic bridges. Fanning and Boothby [7] recalled two standard methods for assessing the stress state of existing historic masonry bridges. The first method, called MEXE, is semi-empirical, and the second commonly used method is based on a computer model such as Archie-M. Practical static analysis of historical masonry structures requires advanced computational methods, modern software programs, and the cooperation of scientists from different disciplines. In the study of stone masonry bridges, in particular, the use of 3D finite element modeling (FEM) allows for the possibility of achieving the exact results of the old bridge without an excess of computational time due to the complexity of the model [8].

The study of the condition of roads and their components, including bridges, due to their hereditary value, is a good trend and truly relevant. Investigation of the state of stone bridges can be studied from different points of view. Lallam et al. [9] proposed the fuzzy analytic hierarchy process based on the French bridge evaluation method IQOA scoring system. Also, there is a proposed model based on the IQOA, which allows to collect simplified data, about bridge state, according to the visual condition [10]. At the same time, there are several works devoted to strengthening or sustainability of roads and bridges [11, 12].

The uniqueness of this work lies in the simultaneous simulation and application of capable failure theories to analyze the existing stone bridge. This study presents an advantageous analytical model for the Dalal stone arch bridge as the basis for the subsequent stress, strain, and energy investigations. For this purpose, the largest arch of Dalal Bridge was chosen for simulation, as it is one of the most significant symbols in the city of Zakho, which is in the Kurdistan Region of Iraq and Mesopotamia. The FEM in the COMSOL package was utilized. Then, the relevant theories of strength and failure were applied, and a qualitative and quantitative assessment of the arch’s stress-strain-energy state was performed. Pavelka [13] reported that the Dalal stone bridge consists of five turns, including a wide, high arch in the middle and smaller arches on the sides. The exact measurement data are presented in Figure 1. Pavelka warned that the bridge’s state was poor, and it had been repaired many times without regard for its historical value. In this work, a structural assessment of the bridge is presented, which will give an impetus to the fact that does this bridge needs strengthening. Indeed, incorrect or uncertain inputs during analytical modeling will lead to unavoidably erroneous outputs. The most difficult step of such studies is to perform a quantitative assessment of the stones used in the bridge. In this regard, one must be as vigilant as possible. A comprehensive search for accurate, quantitative data led to the use of visual inspections and photogrammetric data as the basis for the input data. The type of stones used in the bridge, as well as the exact dimensions of the bridge and even each stone, were easily obtained from the photogrammetric work of the Czech Technical University in Prague. During a symposium in Kyoto, Japan, in 2009, Pavelka [13] presented a detailed work on the history and current state of Dalal Bridge in Zakho. The photogrammetric and geodetic measurements from that work, which was the first step in the documentation of the bridge as a historical monument, became the basis for modeling the largest arch span of Dalal Bridge (Figure 1).

2. 3D Modeling and Simulation

There are many ways to model arches using various software programs. However, it is difficult to model the behavior of masonry structures with high accuracy, several programs convenient for such types of modeling, such as ANSYS, ABAQUS, DIANA, and most suitable COMSOL Multiphysics. One can simulate an arch using multiple 1D, 2D, or 3D domains generated along an arc to represent the geometry of the arch. To evaluate this approach, an elliptical arch was built in the COMSOL Multiphysics computer complex using 70 3D elements to represent the number of stones used along the length of the arch in Dalal Bridge (Figure 2). Each of the 70 stones has different sizes in the x-z plane. Therefore, to create an accurate model, the coordinates of the four nodes of each stone were entered. Then, all domains were extruded for 0.8 m in the y-axis direction, i.e., 0.8 m of the bridge width was selected for investigation. That is, tapered fields were used to represent the stones of the arch ring. The bearing stones of the body of the arch (i.e., the ringstone) were not the same in size or shape. Their depth did not vary equally. It is unclear if there was a reason for changing the depth of the stones or if it was a field construction decision. The simulation adopted the same depth for all stones (i.e., −0.8 m), and each stone was modeled as an independent domain using its exact dimensions in the x-z plane. The processed geometric data provided by Pavelka [13] from the Laboratory of Photogrammetry at the Czech Technical University in Prague served as the basis for developing the 3D model. The geometric measurements indicated that the intrados of the large span has an elliptical shape that is different from the other spans. This span is a semi-elliptical arch with internal dimensions of 17 m and 17.7 m in the vertical (rise) and horizontal (span) directions, respectively.

Various modeling strategies can be used for stone structures such as the Dalal Bridge. As Dalal Bridge is massive and given the modeling complexity, it was decided that only the largest arch would be investigated. This work is the first attempt at analyzing similar historic structures in the region, and, as such, three different 3D finite element models were prepared (Figure 2). This study focused on developing a loading scheme as close as possible to the loading applied to the arch. To achieve this, various loading schemes were explored. First, a model was developed that included the spandrel wall of the bridge, and self-weight was applied (Figure 2(a)). In the second model, the ringstone was loaded with a nonuniformly distributed load in the local coordinate system of the extrados face of the domains that represented the load of the overlaid stone courses.
Finally, in the third model, the ringstone was loaded with a surface load in the global coordinate system, where the \( z \)-axis was directed to the extrados of each stone. The value of the surface load for each stone was determined individually. After a thorough analysis, it was concluded that the third model was as close as possible to the actual loading conditions (Figure 2(c)). A preliminary analysis showed that the third model comprised the adequate model of the existing arch, i.e., a fragment of the arch of 0.8 m wide, as shown in Figure 2(c). As the real irregular shape of the arch stones (see blue line in Figure 2(c)) cannot be modeled in software programs, an idealised extrados (see red line in Figure 2(c)) was assumed. This approach has been used to model many stone bridges [13]. Figure 3 illustrates the 3D CAD model of the ringstone and mortar joints. In the model, the 3D arch was pin-supported within the lower faces of both supporting domains. In other words, symmetry boundary conditions were applied for both the right and left sides of the bridge to prevent displacement in the \( x \)- and \( z \)-directions. As in all such cases, the most challenging stage of this work was developing the input parameters due to the lack of reliable information about the mechanical properties of the stones that make up the bridge. To mitigate this, articles about similar bridges in nearby cities in Turkey and other countries were studied [8, 14–17], and geotechnical studies in the Kurdistan Region were taken into account. In particular, the work of Daoud et al. [18] was consulted, where the authors presented valuable data on the mechanical properties of limestone in our region.

Traditional limestone was one of the most used materials in historical stone bridges [16], and Dalal Bridge is no exception. Pavelka [13] reported that carved limestone was the material used in constructing the Dalal Bridge. There are numerous laboratory data on the compressive and tensile strengths of limestone in Mesopotamia, as well as correlation equations between Young’s modulus and these mechanical characteristics [15, 18, 19]. The unconfined compressive strength ranged between 3.1 MPa and 116 MPa, and the modulus of elasticity varied from 0.9 GPa to 50 GPa [20]. Based on Proske and Gelder [21], the compressive strength of the limestones and mortar can be assumed to be 60 and 4 MPa, respectively.

\[
f'_{m} = 0.5 f'_{b} = 13.3 \text{ MPa} \quad [22].
\]

\[
R'_{u} = kR' = 2.13.3 = 26.6 \text{ MPa}.
\]

\[
E = \alpha R'_{u} = 750 \times 24 \approx 20000 \text{ MPa}.
\]

Young’s modulus, \( E = 20 \text{ GPa} \).

Density, \( \rho = 25 \text{ kg/m}^3 \).

Poisson’s ratio, \( \nu = 0.2 \).

The analysis was performed using the self-weight of the ringstone and the overlying courses of masonry; therefore, only the dead load was taken into account. This was done because the structure is currently used as a pedestrian bridge. The FEM utilized a discrete approach where the arch was treated as a series of elastic bodies.

Meshing for this structure was based on statistical analysis, and the most optimal option, shown in Figure 4, was selected. To achieve the most accurate model and ensure the applicability of the meshing, the “mapped” and “sweep” commands in COMSOL Multiphysics were used; then, the “excellent” mesh was superimposed on all subdomains as a result of obtaining the 8-node hexahedral brick elements. This provided the best acceptable statistical impact of the adopted mesh. Figure 4
3. Results and Discussion

Structural analysis of the ellipse shape of Dalal Bridge comprises three main stages: (1) ringtone simulation, (2) stress-strain, SED, and displacement results, and (3) application of the different failure theories. In this case study, various theories of the failure of mechanics were used. Taking into account the three-dimensionality of the model and the typical features of stone structures, it was determined that the tensor of the stresses, the displacement within the ringstone, and the following failure criteria would be studied:

1. Maximum principal stress theory (Rankine’s theory).
2. Maximum principal strain theory (St. Venant’s theory).
3. Total strain energy theory (Haigh’s theory).

It is well known that there have been few attempts to obtain a general failure criterion for masonry because of the difficulty associated with developing representative 3D and biaxial tests.
Given the brittleness of the material used in the construction of the bridge, not all theories of failure may be acceptable for its analysis. Thus, the applied failure theories make it possible to correctly assess the stress-strain and SED-state of the investigated arch.

3.1. Tensor of Stresses. The numerical results of the maximum value of all six stress components, and the qualitative fields of their distribution among the arch, are presented in Figure 5.

A complete view of the tensor of the everyday and shear stresses at their maximum and minimum values is shown in Figure 6.

On all of the components of the tensor of priorities, it can be seen that the values of the ultimate tensile and compressive stresses are 1.2 MPa and 12.4 MPa, respectively. Given the uniaxial compressive and tensile strengths of limestone, it is revealed that the safety factors for these two states are 4.2, and 2.8, respectively. Thus, the general stress-strain state gives a reasoned justification that the stress in the elements of the bridge does not reach the ultimate value.

3.2. Maximum Principal Stress Theory (Rankine’s Theory). Stones exhibit great strength under compression, but they are not very strong in shear, and the tension resistance of old masonry structures is very low. Because of these characteristics, the magnitude and direction of the principal stresses play a decisive role in the outline shape of arches [24]. Given that most theories of failure are based on the value and direction of the principal stresses and according to Rankine’s view, when the maximum principal pressure reaches the compressive yield, the brittle materials in the structure will begin to fracture. This study also identifies these internal stresses. Figure 7 illustrates the maximum and minimum values of the principal stresses in the ringstone of the arch, the fields of distribution for these stresses, and the location of their importance in the whole arch.

According to the first theory of failure, the maximum principal stress theory (i.e., Rankine’s theory), the value of the maximum main stress should not exceed the permissible value. Therefore, it is essential to present all the importance of the top and minimum principal stresses on a three-dimensional element for later comparison with the allowable weight of the stone structure (Figure 8). An implied adaptation of the 3D state to 2D elasticity assumptions through substitution of the third principal stress by its effect on the other two was analyzed to investigate the stress state further. Syrmakezis et al. [25] proposed a triply repeated process to determine each plane’s stress and failure state. Each time principal stress, \(\sigma_1\), \(\sigma_2\), or \(\sigma_3\), was eliminated, and the set consisting of the two remaining principal
stresses, \((\sigma_1, \sigma_3, \sigma_2)\), \((\sigma_1, \sigma_3, \sigma_2)\), or \((\sigma_1, \sigma_2)\), was assumed to be applied to the nodes \([20]\).

Considering the maximum principal stresses for identifying all three possible two-dimensional cases, it was determined that the state shown in Figure 8(b) was the most critical. In the same way, the most crucial plane for the minimum principal stresses was selected during the transformation from the three-dimensional to the two-dimensional state (Figure 8(d)). As can be seen from the results presented in Figures 7 and 8 and according to even the most pessimistic estimates, the safety factor in compression is not less than 2.3, and tension reaches 3.7. This provides an optimistic basis for the fulfilment of all the maximum principal stress theories (i.e., Rankine’s theory).

3.3. Maximum Principal Strain Theory (St. Venant’s Theory). Another simple criterion for the brittle fracture of materials is the maximum principal strain theory, also known as St. Venant’s theory. According to this theory, the limiting state of the material is reached when the maximum tensile strain, \(\varepsilon_{\text{max}} = \varepsilon_1\), comes with a specific constant limit value equal to the relative pressure, \(\varepsilon_{\text{fracture}}\), at fracture. This relationship is given by

\[
\varepsilon_{\text{max}} = \varepsilon_1
\]
Figure 7: Magnitude and location of the maximum principal stresses.

Figure 8: (a) Maximum 3D principal stress. (b) 2D equivalent maximum principal stresses. (c) Minimum 3D principal stress. (d) 2D equivalent minimum principal stresses.
Condition for safe design: maximum principal strain ≤ permissible strain.

\[ \varepsilon_{\text{max}} = \varepsilon_1 \leq \frac{1}{E} \left[ \sigma_1 - \nu(\sigma_2 + \sigma_3) \right] = \varepsilon_0, \]  

(1)

total strain energy per unit volume = \( \frac{1}{2} \sigma_1 \varepsilon_1 + \frac{1}{2} \sigma_2 \varepsilon_2 + \frac{1}{2} \sigma_3 \varepsilon_3, \)

(3)

where

\[ \varepsilon_1 = \frac{1}{E} \left[ \sigma_1 - \nu(\sigma_2 + \sigma_3) \right]; \]

\[ \varepsilon_2 = \frac{1}{E} \left[ \sigma_2 - \nu(\sigma_1 + \sigma_3) \right]; \]

\[ \varepsilon_2 = \frac{1}{E} \left[ \sigma_2 - \nu(\sigma_1 + \sigma_3) \right]. \]

(4)

As a result, the final equation for energy is given by

\[ \text{SED} = \frac{\text{total strain energy}}{\text{volume}} \]

\[ = \frac{1}{2E} \left[ \sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) \right] \]

(5)

\[ = 4113 \frac{J}{m^3}. \]

The total strain energy can be expressed as the summation of the volumetric energy and distortional energy as follows:

\[ U = U_V + U_D = 4113 \frac{J}{m^3}. \]

(6)

It is clear that the calculated result is very close to the result obtained from FEM (Figure 9) and satisfies the total strain energy theory (i.e., Haigh’s theory); the volume change represents the volumetric energy, which is expressed as.
\[ U_V = (1 - 2\nu) \left( \frac{\sigma_1 + \sigma_2 + \sigma_3}{6E} \right)^2 \]
\[ = 2691 \text{ J/m}^3 \]

The distortional energy, which is related to the change in shape, can be calculated by

\[ U_D = U - U_V. \]

It can also be expressed by the von Mises criterion for multiaxial loading as follows:

\[ U_D = (1 + \nu) \left( \frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{6E} \right)^2 \]
\[ = 1422 \text{ J/m}^3 \]

According to this theory, the domain fails when the distortional energy reaches the limiting point, which, in this case, is \( 1422 < \frac{1}{2}E\sigma_L^2 = 2250 \text{ J/m}^3 \).

One of the most important factors influencing the shape of the arch is the pattern of distribution of the SED throughout the ringstone. The optimal configuration of the arch is the geometry in which the SED has a uniform distribution. This condition is not observed in this case. On the contrary, there is a significant difference between the minimum and maximum SED (Figure 9(a)).

3.5. Displacement of the Ringstone. Another critical and necessary evaluation is the determination of the displacements at the most vulnerable points of the structure. As the arch model was constructed in the \( x-z \) plane, the deviation in the direction of these two axes will be the main components of the displacement vector. Considering the influence of the spandrel walls in this direction, i.e., the effect of adjacent parts of the bridge, it is clear that the lateral displacement (i.e., the \( x \)-direction) occurring in the order from the center of the arch to the extrados can be neglected. Therefore, only the vertical (i.e., \( z \)-direction) deflection will be considered. The field and maximum values of this displacement are shown in Figure 10. The maximum displacement value is 3.4 mm, which is significantly lower than the allowable deflection for such an arch span.

Applying theories of maximum principal stresses and strains shows that Dalal bridge is in satisfactory condition, and its displacement is in the permissible range.

Analytical and finite element analyses give a clear consistency of results, which in turn justify the application of the used theories to the analysis of such bridges. This work showed that the superstructure of Dalal Bridge, i.e., the arch itself, in terms of structural analysis, is in good condition. However, additional investigations are needed in the arch support zones to ensure that the interaction between the soil and foundation is reliable.

4. Conclusion

3D modeling and simulation of historical stone bridge worked adequately to investigate the aimed tasks of this research, and its results are in good agreement with all applied theories. We used a qualified model with average quality of the elements of 0.9836, which is considered an excellent statistical result for 3D meshing.

Thus, the conclusions are presented as follows:

(i) The results of this work show that among all presented theories, the most suitable is the maximum principal stress theory, whose results give the most realistic idea of the state of the structure.

(ii) Following the stress tensor, it was observed that the maximum values of the tensile and compressive stresses were 1.2 MPa and 12.4 MPa, respectively, which were significantly lower than
the strength of the material in the mentioned stress states.

(iii) The reliability of the ringstone, according to the theory of maximum principal stresses, was ensured by applying Rankine’s view to the 3D model or the equivalent maximum principal stresses in the case of the 2D analysis. The safety factor in compression is not less than 2.3, and tension reaches 3.7.

(iv) The ultimate leading strain theory (i.e., St. Venant’s approach) guaranteed the safety of the arch with a safety factor of up to 4. Even though the safety of the arch, according to the total strain energy theory (i.e., Haigh’s approach), has been verified, a significant variation in the nonuniformity of the distribution of the SED (0.0011 J/m$^3$–4416 J/m$^3$) confirmed that the geometry of the investigated arch is not optimal for applied loading. In other words, the geometry of the arch is far from inhabiting an isoenergetic state. The search for the stress-strain state and the strain energy density field revealed that both parameters reached their maximum value in the vicinity of the arch supports. The size of the stones in the body of the arch’s ringstone did not match their values under stress.

It is proposed to improve this work by studying the interaction between the stones and the mortar used and find unified failure theory for masonry bridges in the future.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The author declares that there are no conflicts of interest.

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