Mathematical Modelling and Heat Transfer Performance of a TEG for Engine Exhaust Heat Recovery

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Abstract. In recent years, thermoelectric power generation has been investigated widely for waste heat recovery of internal combustion engines. In this paper, the exhaust gas of a heavy-duty diesel engine is used as the heat source. A mathematical model and the simulation program of a large thermoelectric generator (TEG) are established. Using the MATLAB software, the trends of the TEG working parameters in the flow direction are analysed. The flow rate and the temperature of the cold sources, the engine speed and load, and the fin structure parameters of the tube-fin heat exchanger are studied. The results show that the error between the simulated output power and the experimental one is within 11%. A higher engine load rate and a lower coolant temperature are conducive to improving the output power and the conversion efficiency of the TEG. Optimizing the fin spacing and fin height is helpful to improve the heat transfer performance of the heat exchanger and increase the thermoelectric conversion efficiency.

1. Introduction
Internal combustion engine’s thermoelectric power generation technology is an effective way to fully utilize fuel chemical energy, improve fuel economy, and achieve environmental friendliness. It has become the research focus of major automobile companies and scientific research institutions [1]. Enhancing the thermoelectric conversion performance of thermoelectric materials from the macro and micro perspectives, optimizing the structure of TEG and experimental research on thermoelectric power generation technologies are three important aspects of conducting thermoelectric power generation technology research. In 1993, Hirano of Japan proposed a multi-stage thermoelectric material in the form of BiTe/PbTe/SiGe, and an effective method for improving the thermoelectric conversion efficiency was analyzed through a numerical simulation [2]. Wang designed a multi-section thermoelectric arm composed of Bi2Te3 material and PbTe material. From the perspective of manufacturing process, three different base block molding processes were used to obtain the optimal material ratio to improve its thermoelectric performance and mechanical performance. In short, the rapid development of thermoelectric materials is likely to reshape the new situation of renewable energy in the next two decades [3,4]. On the other hand, the three-dimensional mechanism design of thermoelectric generator can optimize the heat transfer performance and provide the best working environment for thermoelectric materials. Aranguren built a simple internal combustion engine’s TEG model and placed it at the combustion chamber input. The result showed that the net power generated by the 48 thermoelectric power generation chips was 21.56W, which also showed the exhaust gas temperature and mass flow, pressure drop and other design factors could not be ignored on the thermoelectric performance of TEG [5]. Hai established a numerical simulation model for a six-cylinder turbocharged diesel engine, optimized the design of exhaust passages and coolant passages, compared the two cooling methods of air cooling and water cooling, and finally provided reasonable opinions on...
heat dissipation at the hot and cold ends [6]. Even though modern technology has greatly increased the theoretical efficiency of thermoelectric generation technology, the experimental results are still unsatisfactory. Lan et al. used GM250-127-28-10 thermoelectric materials, and the output efficiency of the optimized TEG could be increased by 25% under transient conditions [7]. Xu studied the output performance of thermoelectric devices under different hot-end temperatures (83-270°C) and load conditions (0-520Ω), and conducted relevant economic analysis based on Dongfeng EQ140-1 truck [8].

In summary, for the thermoelectric generation technology of internal combustion engines, domestic and foreign thermoelectric enthusiasts mainly conduct research from three aspects: materials, theoretical analysis and experimental research. Under the background of comprehensive multidisciplinary knowledge, the development at the theoretical level is far greater than the experimental effect. Therefore, on the premise of fully proving the experimental results, this paper established a numerical model of large-scale segmented TEG, studied the thermoelectric performance under steady-state, and analyzed the structural factors and input conditions of the heat exchanger influences.

2. TEG Mathematical Modeling

From the thermoelectric point of view, the heat absorbed by the hot end of a single thermoelectric element is the result of the combined effects of conduction heat, Joule heat and Peltier heat.

$$Q_h = \alpha_{np} T_h I - \frac{1}{2} I^2 r + K_{np} (T_h - T_c)$$ (1)

The heat flow from the power generation module to the cold end is:

$$Q_c = \alpha_{np} T_c I + \frac{1}{2} I^2 r + K_{np} (T_h - T_c)$$ (2)

The output power of heat transfer is the difference between the heat flux at the hot and cold ends:

$$P = Q_h - Q_c = \alpha_{np} (T_h - T_c) I - I^2 r$$ (3)

From the perspective of heat transfer, in the two-dimensional plane space, assuming that the TEG system is transverse to the fluid flow direction and perpendicular to the fluid flow direction, the longitudinal direction transfers heat downwards and the thermoelectric material exchanges heat and electricity. There is a decreasing temperature change in the heat loss during the flow. In this paper, a thermal resistance model is established longitudinally based on thermodynamics, and an iterative calculation model is established laterally. Different from the construction of the thermal resistance model of a single micro-element in the longitudinal direction, the lateral direction is the result of integrating all micro-element bodies on the basis of considering the entire flow process, as shown in Figure 1.
As shown in Fig. 1, assuming that there are n micro-elements in the entire flow channel, the exhaust gas is transferred from the inlet at the temperature $T_{\text{exh,1}}$ and the flow rate $u_{\text{exh,1}}$ through the thermal convection to the outlet temperature $T_{\text{exh,n+1}}$ and the flow rate $u_{\text{exh,n+1}}$, while the cooling water from the inlet temperature $T_{\text{col,1}}$ and the flow rate $u_{\text{col,1}}$ gradually increase conduction in the same way to the outlet temperature $T_{\text{col,n+1}}$ and flow rate $u_{\text{col,n+1}}$. Due to the flow rate change is not large, therefore, the main influencing factor in the lateral direction is the temperature gradient. After the heat transfer, the cold and hot ends of the TEMs also have corresponding temperature gradients, which cannot be ignored for the efficiency of conversion into electrical energy. At this time, the thermoelectric formula of n control bodies is transformed into:

$$Q_{\text{exh}} = \int \theta_{\text{exh},i} (T_{\text{exh,i}} - T_{\text{h},i}) = \int \alpha_{\text{NP,i}} T_{\text{h},i} I_i - \frac{1}{2} I_i^2 r_i + K_{\text{NP,i}} (T_{\text{h},i} - T_{\text{c},i})$$  \hspace{1cm} (4)

$$Q_{\text{col}} = \int \theta_{\text{col},i} (T_{\text{col,i}} - T_{\text{c},i}) = \int \alpha_{\text{NP,i}} T_{\text{c},i} I_i + \frac{1}{2} I_i^2 r_i + K_{\text{NP,i}} (T_{\text{h},i} - T_{\text{c},i})$$  \hspace{1cm} (5)

Considering the change of fluid temperature, the heat exchange during the convection heat exchange between exhaust gas and coolant is as follows:

$$m_{\text{exh}} c_{\text{exh,i}} T_{\text{exh,i}} - m_{\text{exh}} c_{\text{exh,i+1}} T_{\text{exh,i+1}} = \frac{1}{\theta_{\text{exh}}} \left( T_{\text{exh,i}} - T_{\text{h}} \right)$$ \hspace{1cm} (6)

$$m_{\text{col}} c_{\text{col,i}} T_{\text{col,i}} - m_{\text{col}} c_{\text{col,i+1}} T_{\text{col,i+1}} = \frac{1}{\theta_{\text{col}}} \left( T_{\text{col,i}} - T_{\text{c}} \right)$$  \hspace{1cm} (7)

The above formula gives the calculation formulas of the exhaust temperature and the coolant temperature in the previous iteration process and the next iteration process of the lateral conduction process. As long as the state parameters of the mass flow and specific heat capacity of the fluid i segment, the thermal resistance parameters and the temperature parameters are determined, the exhaust gas temperature and coolant temperature of the i+1th stage can be calculated. From the mathematical formula, the exhaust gas temperature gradually decreases, and the coolant temperature increases gradually. In this study, a segmented model is used and a more accurate estimation of the heat transfer can be achieved.

### 3. Experimental Verification

This article uses the thermoelectric material TEP1-1264-3.6. In order to verify the correctness of the simulation model, this paper uses the control group as the input data of the six-point steady-state operating condition in the literature [9] as TEG of the internal combustion engine. Specific data are shown in Table 1. The boundary conditions of the coolant are the same as Ref. [9].

**Table 1.** Six-point steady-state operating data of diesel engine exhaust system (from Scania)

| LHC | Engine Speed (RPM) | Relative load (%) | Exhaust mass flow (kg/h) | Exhaust gas temperature (°C) | EGR mass flow (kg/h) | EGR temperature (°C) |
|-----|-------------------|------------------|--------------------------|----------------------------|-------------------|---------------------|
| 1   | 1000              | 25               | 420                      | 248                        | 127               | 318                 |
| 2   | 1000              | 50               | 556                      | 347                        | 143               | 452                 |
| 3   | 1150              | 25               | 423                      | 259                        | 197               | 335                 |
| 4   | 1150              | 75               | 949                      | 352                        | 194               | 489                 |
| 5   | 1300              | 25               | 523                      | 251                        | 215               | 325                 |
| 6   | 1300              | 50               | 803                      | 313                        | 247               | 425                 |
Figure 2. Comparison of experimental and model output power under LHC operating conditions

The comparison between the experimental results and the simulation results is shown in Figure 2. Under different engine speeds and load conditions, these profiles have the same trend and the maximum error does not exceed 11%, which validates the accuracy of the model. When the relative load of the engine is fixed, the output power of TEG by experiments is basically increasing; when the engine speed is constant, as the relative load increases, the exhaust temperature continues to rise, and an increasing trend of the output power of the TEG under the corresponding operating conditions should also appear, the maximum output power can reach 515W. Considering the test conditions and the simplification of the simulation system, it can be considered that the results obtained from the simulations are basically consistent with the experimental results.

4. Result and Discussion

Considering the influence of the temperature gradient during the fluid flow, the n-segment lateral segmentation model was established based on the mathematical model. As shown in Figure 3(a), setting $n = 20, 40, 60, 80$ and $100$ to calculate the output power of the TEG. Too many segments will increase the difficulty of the simulation calculation, and too few segments will not be enough to accurately simulate the actual output of the experimental conditions. As shown in the figure below, when $n = 50$, the output speed of the TEG is basically stable at the same speed at 1000 rpm, different torques of 1840 Nm, 1642 Nm, and 1471 Nm, the same torque of 509 Nm, different speeds of 800 rpm, 1600 rpm, and 2000 rpm Operating conditions, the output power of TEG is also gradually reduced to a stable value, indicating that $n = 50$ is the best calculation of the number of segments, for reference.

Figure 3. Output power vs. segment number (a) and performance along the flow direction (b)

Under the set 50-stage segmentation calculation accuracy conditions, as shown in Figure 3(b), along the exhaust gas flow direction, the heat transfer of the exhaust gas is equal to the heat absorbed by the hot end of the thermoelectric module and the heat absorbed by the coolant is also equal to the heat released by the cold end. The heat shows a downward trend due to conversion into electrical energy and absorption by the environment. The thermoelectric conversion efficiency rises linearly, while the output power is basically unchanged.

The automobile cooling system guarantees the normal operation of engine parts through coolant circulation and air heat dissipation. TEG also requires timely and effective heat dissipation measures.
The flow rate and temperature of the coolant are important input parameters. As shown in Figure 4, set the cooling water temperature to 40 °C (10 °C -60 °C), the cooling water flow rate is 20L / min (5L / min-50L / min), the engine speed is fixed at 1400r / min, the variation characteristics of TEG thermoelectric parameters under the conditions of 20%, 40%, 60% and 80% were analyzed. As shown in Figure 4(a), under low-load engine load conditions, the output power increases slightly; When the engine load is higher, the output power increases rapidly from a very low value with increasing flow rate, but with the flow rate further increases, the power output gradually stabilizes; as shown in Figure 4(b), the effect of the coolant temperature shows the opposite trend. As the temperature increases evenly at 10 °C -60 °C, the TEG output power decreases linearly. Therefore, the proper coolant flow rate and the lowest possible temperature are most conducive to the improvement of TEG thermoelectric conversion performance.

Figure 4. Influencing factors of coolant flow rate (a) and temperature (b)

The great advantage of the application of internal combustion engine TEG technology is that it does not affect the dynamic properties of the vehicle and improves its fuel economy. Engine load and speed are important indicators of engine output performance. As shown in Figure 5(a), as the engine load increases, the output power continues to rise. The thermoelectric conversion efficiency in Figure 5(b) increases first and then gradually decreases. The reason can be explained as follows. When the hot end temperature is 510K-550K, the conversion efficiency of TEM reaches the extreme value area. When the temperature range exceeds this range, such as 430K-710K, the working state of TEM is affected, so the overall efficiency of TEG first increases and then decreases. The maximum conversion efficiency is about 4.1%.

Figure 5. Influencing factors of engine speed (a) and load (b)

The quality of the hot end heat transfer process directly affects the heat input of the TEG, and the structure of the heat exchanger determines the flow quality of the heat source. The fin height, fin spacing and fin thickness of the heat exchanger are three basic structural parameters. As shown in Figure 6, the engine speed is set to 1400r / min, the engine load is 60%, the default fin spacing is 1.3mm, the default fin thickness is 0.5mm, and the default fin height is 9mm. On the basis of engineering experience, the appropriate changing range is studied. The results show that: (1) As shown in Figure 6(a), the increase in the height of the fin greatly reduces the power output of the generator, but it promotes the conversion of thermal energy and electric energy of the thermoelectric module, because the increase in the height of the fin reduces the heat transfer from the heat source to the thermoelectric module, while the
temperature of the cold end remain unchanged, the ZT value of the TEM gradually increases, and the conversion efficiency also increases accordingly. The amount of power generation is determined by the amount of heat conduction, so it shows the opposite trend. (2) As shown in Figure 6(b), as the fin spacing continues to increase, the output power gradually decreases and tends to be stable, and the conversion efficiency continues to increase; (3) As shown in Figure 6(c), when the fin pitch increases gradually from 0.5mm to 3mm, as the load increases accordingly, Qh decreases by 3.358 kW, 5.132 kW, and 6.753 kW respectively. This is because the fin pitch increases and the overall size does not change. The heat transfer area of the hot end gradually decreases, and the thermal conductivity becomes worse.

![Figure 6. Influences of the heat exchanger: (a) fin height; (b) fin spacing; (c) fin thickness](image)

5. Conclusion

In this paper, the diesel engine exhaust is used as the heat source, and a mathematical model of a large-scale TEG with segmentation is established. In order to improve the conversion efficiency of the TEG, this paper studies the influences of different parameters including the engine torque and torque, the coolant temperature and flow rate, and the structural parameters of the hot end exchanger incorporating the fin height, fin spacing, and fin thickness. The influencing factors have proved that the operating conditions of the engine with a low speed and a high load are most conducive to the operation of the TEG. The coolant temperature should be as close as possible to the ambient temperature, and its effect is greater than the flow rate. However, a too low temperature is not conducive to the normal operation of the engine, which needs to be compromised. When using a tube-fin heat exchanger, the optimal value for the fin thickness is in the range of 0.5mm-1.0mm. The fin height is about 11mm and the fin spacing is around 1.7 mm.

Reference

[1] Zhang Tiexin. Research on exhaust heat recovery of supercharged diesel engine: Harbin Institute of Technology, 2015.
[2] Huano T, Whitlow L, Miyajima M. Numerical analysis of efficiency improvement in functionally gradient thermoelectric TE materials. Ceramic Transactions, 34 pp, 1993,23.
[3] Snyder G J, Toberer E S. Complex thermoelectric materials. Nat Mater, 2008, 7(2): pp105-114.
[4] He J, Tritt T M. Advances in thermoelectric materials research: Looking back and moving forward. Science, 2017, pp, 357(6358).
[5] Aranguren P, Astrain D, Rodriguez A, et al. Experimental investigation of the applicability of a thermoelectric generator to recover waste heat from a combustion chamber. Applied Energy, 2015, 152: pp, 121-130.
[6] Diao Hai. Study on performance characteristics and structural optimization of engine temperature difference generator system. Tianjin University, 2016.

[7] Lan S, Yang Z, Chen R, et al. A dynamic model for thermoelectric generator applied to vehicle waste heat recovery. Applied Energy, 2018, 210: pp, 327–338.

[8] Xu Lizhen, et al. Experimental study on automobile exhaust gas thermoelectric generation. Journal of Tsinghua university (natural science edition), 2010, (02): pp, 287-294.

[9] Arash E. Risseh, Hans-Peter Nee. Design of a Thermoelectric Generator for Waste Heat Recovery Application on a Drivable Heavy Duty Vehicle[J]. SAE: 2017(01): pp, 91-97.