A Simple Method of Proportional Fairness-based Resource Allocation for Power-domain NOMA

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Abstract:
This paper proposes a simple method of proportional fairness-based user pairing and power allocation for power-domain NOMA. Compared with the conventional method, the proposed method simplifies the fairness calculation and comparison algorithm, by which the computational complexity reduces significantly. Using computer simulations, we show that its characteristics are identical to those of the conventional method.

Keywords: NOMA, power domain, proportional fairness, complexity

Classification: Wireless communication technologies

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1 Introduction

Non-orthogonal multiple access (NOMA) [1] has received attention as a candidate technology for future mobile radio to improve spectrum efficiency. NOMA has been categorized into two domains: power domain and code domain. This study focuses on the power-domain NOMA, considering the application of the downlink.

Various schemes have been studied for user equipment (UE) selection and power allocation in NOMA. Among them, the proportional fairness-based scheme [2][3] appears a natural extension of the resource allocation policy in the current cellular system; such schemes were intentionally evaluated when applied to the cellular system [1]-[4]. However, this scheme requires calculating and comparing metrics for all combinations of UE selection and power allocation, which leads to the calculational complexity in proportion to the square of the number of UEs when selecting two UEs (called “pairing”). In general, the order of complexity increases with the number of UE multiplexing, until it reaches to the half of all candidate UEs. Reference [3] reduces the calculation complexity by limiting the power allocation to meet a certain signal-to-interference and noise power ratio (SINR); however, some calculations and/or decisions are still needed for all UE combinations; therefore, its complexity reduction is restrictive.

In order to reduce the computational complexity, pairing methods have also been proposed based on specific theories, such as game theory and matching theory [4][5]. In general, these schemes first select one UE (called “priority UE”) based on the fairness; then, they choose the second UE best fitting to the priority UE following specific criteria. However, because these methods have not sufficiently clarified how to guarantee fairness among all UEs, discussions remain; hence, their application to public communication systems remains obstructed. The system evaluations in [6] are based on the method of [2]. Moreover, the allocated power ratio must be informed to near UE canceling the multiplexed signal to paired far UE. Therefore, the methods that choose the power ratio among pre-decided several numbers, such as [2], are preferable in signaling design for cellular system over the methods that decide the power ratio among continuous number from 0 to 1 [3]-[5].

Based on the above points of view, targeting the PF-based resource allocation in power-domain NOMA, this paper proposes an improved method that reduces the calculational complexity in proportion to the number of UEs. Computer simulation also demonstrates that the proposed scheme shows numerically the same characteristics as the conventional scheme [2] in a multicell environment.

2 PF-based UE pairing and power ratio selection

For simplicity, two UE multiplexing is assumed here, although more than two UE multiplexings are possible in power-domain NOMA.

2.1 Conventional scheme

Here, we assume $K$ UEs are waiting for resource allocation. Let the average
throughput and instantaneous SINR for UE \( k \) (\( k = 1, 2, \ldots, K \)) be \( \bar{R}(k) \) and \( x(k) \), respectively, where \( x(k) \) is measured under the condition of 100% power allocated to UE \( k \). With Shannon capacity, the expected ideal throughput for UE \( k \) is given by

\[
R(k) = \log_2(1 + x(k)).
\]

Among \( K \) UEs, we now choose a pair of two UEs, UE \#i and UE \#j, where \( 1 \leq i \neq j \leq K \). Next, we compare \( x(i) \) and \( x(j) \), and discriminate near and far UE. Assuming \( x(i) \geq x(j) \) here, transmission power is allocated to near UE (UE \#i) and far UE (UE \#j) in proportion to \( \alpha: (1-\alpha) \), where \( 0 < \alpha < 0.5 \). The sum of fairness \( Q \) is derived using (2).

\[
Q(i,j,\alpha) = \frac{R_{\text{near}}(i, \alpha)}{R(i)} + \frac{R_{\text{far}}(j, \alpha)}{R(j)}.
\]

\[
R_{\text{near}}(k, \alpha) = \log_2(1 + \alpha x(k)).
\]

\[
R_{\text{far}}(k, \alpha) = \log_2 \left( 1 + \frac{(1 - \alpha)x(k)}{1 + \alpha x(k)} \right) = \log_2 \left( 1 + \frac{x(k)}{1 + \alpha x(k)} \right).
\]

Here, it is assumed that the near UE cancels the interfering signal sent to the far UE and detects the desired signal. The signal to the near UE interferes with the far UE and behaves as a background noise.

Let a set of power ratios \( \alpha \) be \( P_\alpha \), that is, \( \alpha \in P_\alpha \). As shown in (5), we obtain the maximum sum of fairness \( Q_{\text{max}} \) as NOMA and the maximum single fairness \( P_{\text{max}} \) as OMA. When \( Q_{\text{max}} < P_{\text{max}} \), all transmission powers are allocated to UE \#k_{\text{max}}\), where \( k_{\text{max}} \) satisfies (6). Otherwise, UE \#i_{\text{max}} and UE \#j_{\text{max}} are selected, and the transmission power is shared by \( \alpha_{\text{max}} \), as shown in (7). This operation requires \( |P_\alpha| \binom{K}{2} + \binom{K}{1} = |P_\alpha|K(K - 1)/2 + K \) times calculation for fairness, where \( |P_\alpha| \) is the number of power ratios in the set of \( P_\alpha \).

\[
Q_{\text{max}} = \max_{1 \leq i \neq j \leq K, \alpha \in P_\alpha} Q(i,j,\alpha), \quad P_{\text{max}} = \max_{1 \leq k \leq K} \frac{R(k)}{\bar{R}(k)}
\]

\[
k_{\text{max}} = \arg \max_{1 \leq k \leq K} \frac{R(k)}{\bar{R}(k)}
\]

\[
i_{\text{max}}, j_{\text{max}}, \alpha_{\text{max}} = \arg \max_{1 \leq i \neq j \leq K, \alpha \in P_\alpha} Q(i,j,\alpha).
\]

### 2.2 Proposed scheme

For each UE, the proposed scheme calculates two kinds of fairness: “fairness as a near UE (FaN)” and “fairness as a far UE (FaF)”; FaN is calculated by (3) and FaF by (4). These functions are shown in Fig. 1(a), where \( \gamma \) represents SINR. The Fig. 1(a) indicates that FaN is higher than FaF for UE in low SINR, and FaF is higher than FaN for UE in high SINR. In other words, the UE in low SINR is advantageous for FaN, and UE in high SINR is for FaF. The proportional fair sharing provides user diversity gain using advantageous channel state fluctuating instantaneously and identically for all different UEs; hence, an adequate pairing by choosing the tops of FaN and FaF is expected; here, one may avoid considering all pairing cases to obtain the near or far UE to calculate the sum of fairness.

Accordingly, FaN and FaF were calculated per UE for each power ratio, as
shown in Fig. 1(b). Let $P_s = \{\alpha_1, \ldots, \alpha_m, \ldots, \alpha_M\}$ and $m_{max} = \arg \max_{1 \leq m \leq M} f_m$; then UE $i_{max}$ and UE $j_{max}$ are paired and the power ratio $\alpha_{m_{max}}$ is selected, where $i_{max} = \arg \max_{1 \leq i \leq K} R_{near}(i, \alpha_{m_{max}}) / \bar{R}(i)$ and $j_{max} = \arg \max_{1 \leq j \leq K} R_{far}(j, \alpha_{m_{max}}) / \bar{R}(j)$. When $i_{max} = j_{max}$, an identical UE is selected with 100% transmission power, which suggests OMA.

Note that the authors cannot prove that the relation $x(i_{max}) \geq x(j_{max})$ is always valid. If the reversal occurs, that is, $x(i_{max}) < x(j_{max})$, UE $i_{max}$ may not cancel the interference caused by the multiplexed signal to UE $j_{max}$. Hence, the proposed scheme finally checks the SINRs of the two selected UEs and swaps the UE indexes $i_{max}$ and $j_{max}$ to resolve the reversal SINR condition\(^1\).

The proposed scheme calculates $2|P_s|K = 2MK$ fairness and selects the top fairness $2M$ times among $K$ fairnesses; It also selects top fairness among $M$ values (sum of each top fairness). Finally, we check the SINR between the two selected UEs.

\(^1\) As stated in section 3, no reversal result has been observed in our computer simulation.
Table I. Simulation parameters

| Parameter                                      | Value                        |
|-----------------------------------------------|------------------------------|
| Cell layout / Reuse factor                    | 19 hexagonal omni cells / 1  |
| Inter-site-distance / Minimum distance from BS to UE | 500 [m] / 20 [m]            |
| Number of UEs per cell, K                     | 10                           |
| BS transmission power                         | 1 [W]                       |
| Noise power at UE                             | -100 [dBm]                  |
| Distance dependent path loss (d in [km])      | $138.5336 + 38 \log(d)$ [dB]|
| Shadowing                                     | Standard deviation: 8 [dB]   |
|                                               | Correlation between BS: 0.5  |
| Maximum Doppler frequency                     | 10 [Hz]                     |
| Reception diversity                           | 2-branch MRC                |
| Slot duration                                 | 1 [ms]                      |
| Time constant of exponential moving average for $\bar{R}$ | 100 [slot]                  |
| Transmission power ratio set $P_s$ [1]        | $\{0.22\}$,                 |
|                                              | $\{0.12, 0.17, 0.22, 0.27, 0.32\}$ |

3 Evaluation examples

We conducted computer simulations under the conditions listed in Table I. Once deciding the UE location at random, the simulation runs for a time duration of 10,000 slots. This simulation repeats 1,000 times, and finally we derived the throughput and power allocation characteristics of each UE. Fig. 2(a) shows an example of the average throughput characteristics for $|P_s| = 5$; here, five power ratios are selected, as shown in Fig. 2(b). We observe that the proposed scheme shows the same characteristics as the conventional scheme. Note that in Fig. 2(b), the sum of all five fractions is 0.9771 ($= 0.5286 + 0.1539 + \ldots + 0.1213$), which suggests that OMA allocation occurs at a ratio of 0.0229. In the case of $|P_s| = 1$, we do not demonstrate the result using figures; however, we confirm consistent characteristics between the proposed and conventional schemes. Reversal SINRs stated in Subsection 2.2 were never observed in the proposed scheme.

Fig. 2(c) shows the amount of calculation versus the number of UEs to compare between two schemes for $|P_s| = 1$ and 5, following section 2. In the case of 10 UEs with $|P_s| = 1$, our scheme can reduce the calculation to 36.4% (20/55) compared to a conventional scheme. In another case of 30 UEs with $|P_s| = 5$, it reaches 14.0% (305/2175).

4. Conclusions

This paper proposes a simple method for PF-based resource allocation for power-domain NOMA. The conventional method requires fairness calculations in proportion to $K^2$; in contrast, the proposed method requires them in proportion to $K$. Multi-cell simulations demonstrate that the numerical characteristics of these two methods are identical. The proposed scheme can be extended to three or more UE multiplexing following the same approach for the fairness calculations. Evaluation of the characteristics under different conditions and/or theoretical analyses remains a topic for further studies.
(a) Cumulative distribution for average throughput per UE.

(b) Paired histogram of the selected power ratio at $|P_s| = 5$.

(c) Amount of calculation as a function of K

Fig. 2. Characteristics comparison for conventional and proposed schemes.