I. INTRODUCTION

Substantial theoretical and experimental efforts world wide have been devoted to the investigation of strongly interacting matter under extreme temperature $T$ and baryon chemical potential $\mu_B$. Lattice quantum chromodynamics (LQCD) calculation suggests a smooth crossover transition 1 from hadronic to a quark-gluon plasma (QGP) phase at zero $\mu_B$ and finite $T$. Depending on the choice of order parameter the transition occurs approximately between the temperature 145 MeV to 175 MeV at zero $\mu_B$. For example, LQCD calculation with chiral condensate as order parameter gives $T_c \sim 154$ MeV 9. However if one chooses strange quark number susceptibility then the $T_c \sim 170$ MeV 10. Various QCD based model calculations at high baryon density and low temperature suggest the existence of a first-order phase transition 11. Computing LQCD calculations at high $\mu_B$ is numerically challenging. Hence there are large uncertainties in calculating the transition line across the $T$ versus $\mu_B$ phase diagram of QCD 12 16. Several experimental programs have been devoted to find this transition line in the phase diagram of QCD. At present, QCD system at high $T$ and small $\mu_B$ are being investigated using ultra-relativistic heavy ion collisions at the Large Hadron Collider (LHC), CERN and Relativistic Heavy Ion Collider (RHIC), Brookhaven National Laboratory (BNL). The Beam Energy Scan (BES) program of RHIC 17 is currently investigating the matter at a wide range of $\mu_B$ : 20 to 400 MeV 18. The HADES experiment at GSI, Darmstadt is also investigating a medium with very large baryon chemical potential 19. In future, the Compressed Baryonic Matter (CBM) experiment 20 at the Facility for Antiproton and Ion Research (FAIR) at GSI and the Nuclotron-based Ion Collider Facility (NICA) 21 at JINR, Dubna will also study nuclear matter at large baryon chemical potential.

Hadron resonance gas (HRG) model 22 76 is a statistical thermal model which is used to study the strongly interacting matter at finite temperature and chemical potential. The HRG model is successful in describing the zero chemical potential LQCD data of bulk properties of the QCD matter up to moderate temperatures $T \sim 150$ MeV 2 3 5 7 8. This model is also successful in describing the hadron yields, created in central heavy ion collisions at different center of mass energies ($\sqrt{s_{NN}}$) 25 26 27 35 77, at chemical freeze-out which is the stage in the evolution of the thermal system when inelastic collisions among the hadrons cease and the hadronic yields become fixed. The values of $T$ and $\mu_B$ extracted using the HRG model at large $\sqrt{s_{NN}}$ is very close to $T_c$ obtained from LQCD calculations at zero $\mu_B$. This raises the interesting question - Is the chemical freeze-out line same as the hadronization or quark-hadron transition line ? In order to address this one needs LQCD calculations (which are difficult to compute numerically at large $\mu_B$), precise experimental signatures related to transition line at several $\sqrt{s_{NN}}$ and study of chemical freeze-out dynamics using HRG model in its different variants. In this work we ask a slightly different yet related question - What is the upper limit in the $T$ versus $\mu_B$ phase diagram up to which the QCD thermodynamics can be effectively modeled by an ideal HRG (I-HRG) model? The procedure is as follows. We choose only those thermodynamic quantities (TQs) that show a monotonic behavior between the hadronic and quark gluon plasma phase with limiting values corresponding to that computed for the Stefan-Boltzmann (SB) gas of massless quarks and gluons. For a given $\mu_B$, we find the $T$ where the TQ computed in I-HRG agrees with the corresponding SB limit. This process is repeated for several TQs. A trace of the line joining the lowest $T$ values at each $\mu_B$ from all the observables studied provides the upper limit for the allowed region of the I-HRG phase on the QCD phase diagram.
The paper is organized as follows. In Sec. II we briefly discuss the hadron resonance gas model and the calculation related to the SB limit. In Sec. IV we discuss our results and finally in Sec. V we summarize our findings for this work.

II. IDEAL HADRON RESONANCE GAS MODEL

The system of thermal fireball consists of all the hadrons and resonances given in the particle data book. In this model hadrons and resonances are non-interacting point like. The partition function is the basic quantity from which one can calculate various TQs of the thermal system. The logarithm of the partition function of an ideal hadron resonance gas in the grand canonical ensemble can be written as

\[ \ln Z^H = \sum_i \ln Z^H_i, \]  

where the sum is over all the hadrons and resonances and \( H \) refers to the hadronic phase. For the hadron or resonance species \( i \),

\[ \ln Z^H_i = \frac{V g_i}{2\pi^2} \int_0^\infty p^2 \, dp \ln[1 \pm \exp(-(E_i - \mu_i)/T)], \]

where \( V \) is the volume of the thermal system, \( g_i \) is the degeneracy, \( E_i = \sqrt{p^2 + m_i^2} \) is the single particle energy, \( m_i \) is the mass of the particle and \( \mu_i = B_i \mu_B + S_i \mu_S + Q_i \mu_Q \) is the chemical potential. In the last expression, \( B_i, S_i, Q_i \) are respectively the baryon number, strangeness and electric charge of the particle, \( \mu \)'s are the corresponding chemical potentials. The upper and lower sign of \( \pm \) corresponds to fermions and bosons, respectively. Once we know the partition function or the pressure of the system we can calculate other TQs. The pressure \( p^H \), the number density \( n^H \), the energy density \( e^H \), and the entropy density \( s^H \) of the system can be calculated using the standard definitions.

\[ \begin{align*}
    p^H &= \sum_i \frac{T}{V} \ln Z^H_i = \sum_i (\pm) \frac{g_i T}{2\pi^2} \int_0^\infty p^2 \, dp \ln[1 \pm \exp(-(E_i - \mu_i)/T)], \\
    n^H &= \sum_i \frac{T}{V} \left( \frac{\partial \ln Z^H_i}{\partial \mu_i} \right)_{V,T} = \sum_i \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2 \, dp}{\exp[(E_i - \mu_i)/T] \pm 1}, \\
    e^H &= \sum_i \frac{E^H_i}{V} = -\sum_i \frac{1}{V} \left( \frac{\partial \ln Z^H_i}{\partial T} \right)_{\mu,T} = \sum_i \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2 \, dp}{\exp[(E_i - \mu_i)/T] \pm 1} E_i, \\
    s^H &= \sum_i \frac{S^H_i}{V} = \frac{1}{V} \left( \frac{\partial (T \ln Z^H_i)}{\partial T} \right)_{V,\mu} = \sum_i (\pm) \frac{g_i}{2\pi^2} \int_0^\infty p^2 \, dp \left[ \ln \left( 1 \pm \exp\left(-\frac{(E_i - \mu_i)}{T}\right) \right) \frac{E_i - \mu_i}{T(\exp[(E_i - \mu_i)/T] \pm 1)} \right].
\end{align*} \]

Fluctuations and correlations of conserved charges of baryon number, electric charge, strangeness and others are considered as a standard probe to study the phase transition. Derivatives of the grand canonical partition function \( Z \) with respect to the chemical potential define susceptibilities which experimentally become available through event-by-event analysis of fluctuations of conserved charges. For example, second order fluctuations of the conserved charges and their correlations in a thermalized medium can be calculated as

\[ \chi_{xy}^{11} = \frac{1}{VT^3} \frac{\partial^2 (\ln Z)}{\partial \left( \frac{\mu_x}{T} \right) \partial \left( \frac{\mu_y}{T} \right)}, \]

where \( x, y = B \) (baryon), \( S \) (strangeness) and \( Q \) (electric charge).

In order to compare theoretical computation with that obtained in experiments, suitable ratios are formed to cancel the system volume. In this work, apart from the various TQs, we also work with the following ratio that is expected to show a monotonic variation

\[ C_{BS} = -3 \left( \chi_{BS}^{11} / \chi_S^2 \right). \]
III. IDEAL GAS OF QUARKS AND GLUONS

The pressure of a massless quark gluon gas of three flavor QCD [Stefan-Boltzmann (SB) gas] can be written as [81]

\[ \frac{P_{SB}}{T^4} = \frac{8\pi^2}{45} + \sum_{f=u,d,s} \frac{7\pi^2}{60} + \frac{1}{2} \left( \frac{\mu_f}{T} \right)^2 + \frac{1}{4\pi^2} \left( \frac{\mu_Q}{T} \right)^4, \quad (10) \]

where the two terms give the contributions of the gluon and the quark sector respectively, \( \mu_f \) is the quark chemical potential. The quark chemical potentials of \( u, d \) and \( s \) quark can be expressed in terms of chemical potentials for baryon number (\( \mu_B \)), strangeness (\( \mu_S \)) and electric charge (\( \mu_Q \)) as

\[ \mu_u = \frac{1}{3} \mu_B + \frac{2}{3} \mu_Q, \quad (11a) \]
\[ \mu_d = \frac{1}{3} \mu_B - \frac{1}{3} \mu_Q, \quad (11b) \]
\[ \mu_s = \frac{1}{3} \mu_B - \frac{1}{3} \mu_Q - \mu_S. \quad (11c) \]

After knowing the \( \frac{P_{SB}}{T^4}(T, \mu_B, \mu_S, \mu_Q) \) or the corresponding partition function one can calculate other TQs as well as the fluctuations and correlations of different conserved charges of a free quark gluon gas at any temperature and chemical potential using the definitions mentioned in the previous section. The energy density and the entropy density of the massless quark gluon gas is given by

\[ \frac{\varepsilon_{SB}}{T^4} = 3 \frac{P_{SB}}{T^4}, \quad (12) \]

\[ \frac{s_{SB}}{T^3} = \frac{19\pi^2}{9} + \sum_{f=u,d,s} \left( \frac{\mu_f}{T} \right)^2. \quad (13) \]

Second order fluctuations of baryon, strangeness and electric charge at the massless limit of the quark gluon gas can be written as

\[ \left( \chi_{B}^{(2)} \right)_{SB} = \frac{1}{3} + \frac{1}{3\pi^2 T^2} \sum_{f=u,d,s} \mu_f^2, \quad (14) \]

\[ \left( \chi_{S}^{(2)} \right)_{SB} = 1 + \frac{3}{\pi^2 T^2} \mu_S^2, \quad (15) \]

\[ \left( \chi_{Q}^{(2)} \right)_{SB} = \frac{2}{3} + \frac{1}{3\pi^2 T^2} \left( 4\mu_u^2 + \mu_d^2 + \mu_s^2 \right). \quad (16) \]

Similarly baryon-strangeness and charge-strangeness correlations are given by

\[ \left( \chi_{BS}^{(1)} \right)_{SB} = -\left( \chi_{QS}^{(1)} \right)_{SB} = -\frac{1}{3} - \frac{1}{\pi^2 T^2} \mu_B^2. \quad (17) \]

It can be seen from Eqs. (15) and (17) that \( \frac{c_{BS}^{SB}}{c_{BS}^{SB}} = -3 \left( \chi_{BS}^{(1)} \right)_{SB} / \left( \chi_{S}^{(2)} \right)_{SB} \) is always unity for the quark gluon gas.

IV. RESULTS

In this work, we aim to estimate the maximum allowed region on the \( \mu_B - T \) plane where I-HRG could be applied. For our analysis, we investigate the pressure, energy density, entropy density, second order fluctuations and correlations of conserved charges of baryon number, strangeness and electric charge that are expected to have a monotonic variation with \( T \) and \( \mu_B \) and attain limiting values corresponding to the SB limit.

A. \( \mu_{B,Q,S} = 0 \)

Let us first discuss the picture for zero chemical potential. In Fig. 1 we show the variation of \( 3P/T^4, \varepsilon/T^4 \) and \( 3s/4T^3 \) with \( T \). The TQs are normalized suitably so that they have the same SB limit. We have done the calculations in I-HRG with two different input hadronic spectra. For the first set, we have considered all the confirmed hadrons and resonances that consist only up, down, and strange flavor valence quarks listed in the PDG 2016 Review [78]. This list includes all the confirmed mesons listed in the Meson Summary Table [78] and all baryons in the Baryon Summary Table [78] with three- or four-star status. We refer this set as PDG 2016. In Fig. 1 dotted lines show the result of I-HRG using hadronic spectrum PDG 2016. The red, green and blue colors are used for pressure, energy density and entropy density respectively. We have done our analysis for another set of hadronic spectrum. This set includes all the resonances from the previous set i.e.,
The variation of $3\chi_B^2$, $\chi_S^2$ and $3\chi_Q^2/2$ with the temperature at zero $\mu_B$. Dotted and solid lines correspond to the calculations in I-HRG model with hadron spectra PDG 2016 and 2016+. Red, green and blue colors are used for second order fluctuations of baryon, strangeness and electric charge respectively. The colored shaded bands show the continuum estimate of the lattice QCD [4]. Horizontal line at 1 corresponds to the massless limit (SB) of $\chi_S^2$ of the three flavored quark gluon gas.

PDG 2016 as well as the other unmarked mesons from the Meson Summary Table and baryons from the Baryon Summary Table with one- or two- star status which are not confirmed yet [78]. This set is referred to as PDG 2016+ and in Fig. 1 solid lines are used to show the corresponding results. The colored shaded bands show the continuum estimate of the lattice QCD [2]. The hadronic spectrum PDG 2016+ provides a satisfactory description in the hadronic phase of continuum LQCD data of most of the TQs which is already known from the previous work [82]. Additional resonances in this list will allow us to study the systematic uncertainties for our analysis. The horizontal line at $19\pi^2/12$ corresponds to the massless limit (SB) of normalized energy density $\epsilon/T^4$ of the three flavored quark gluon gas. It can be seen form the Fig. 1 that pressure, energy density and entropy density calculated in HRG model using hadronic spectrum PDG 2016 cross the SB limit at $T = 268, 216$ and 224 MeV respectively. For hadronic spectrum PDG 2016+ corresponding temperatures are $T = 252, 204$ and 212 MeV respectively. So for PDG 2016+ all the TQs reach the SB limit at relatively lower temperature compared to that of PDG 2016. This trend of lowering of temperature due to the systematics of the hadron spectrum is also observed in the chemical freeze-out temperature [74]. We also note that the crossing $T$ for $s$ and $\epsilon$ which are first derivatives of $\ln Z$ are lower compared to that of $P$. This trend follows even for other TQs as well, higher the derivative of $\ln Z$, lower is the crossing $T$.

Figure 2 shows the variation of $3\chi_B^2$, $\chi_S^2$ and $3\chi_Q^2/2$ with the temperature at zero chemical potential. The dotted and solid lines correspond to the calculations in I-HRG model with hadron spectra PDG 2016 and 2016+. The red, green and blue color are used for second order fluctuations of baryon, strangeness and electric charge respectively. The colored shaded bands show the continuum extrapolated LQCD [4] data. For the massless three flavored quark gluon gas values of $\chi_B^2, \chi_S^2$ and $\chi_Q^2$ are 1/3, 1 and 2/3 respectively. The horizontal line at 1 in Fig. 2 corresponds SB limit of $\chi_B^2$. In this figure $\chi_S^2$ and $\chi_Q^2$ are normalized properly so that their values at SB limit also become unity. It can be seen from this figure that for the hadronic spectrum PDG 2016, fluctuations of baryon, strangeness and charge reach the corresponding SB value at $T = 186, 212$ and 176 MeV respectively. For the hadronic spectrum PDG 2016+ corresponding temperatures are $T = 180, 200$ and 174 MeV respectively. At small $T$, the order of the masses of the lightest hadron in each charge sector decides the order of the magnitudes of the susceptibilities. However, with $T \sim 150$ MeV, the stronger rise in the baryonic spectrum as compared to the strange sector results in $\chi_B^2$ taking over. Finally, both $\chi_Q^2$ and $\chi_B^2$ have similar crossing $T$ while $\chi_S^2$ has a much higher crossing $T$.

Figure 3 shows temperature dependence of $-3\chi_{BS}^{11}, 3\chi_{QS}^{11}$ and $C_{BS}$ at zero chemical potential. The results using hadronic spectrum PDG 2016 (2016+) are shown by the dotted (solid) lines. Red, green and blue colors are used for $-3\chi_{BS}^{11}, 3\chi_{QS}^{11}$ and $C_{BS}$ respectively. The continuum extracted LQCD data [4] are shown by the colored shaded bands. The horizontal line at 1 corresponds to the SB limit of all the observables shown in this figure. $\chi_{BS}^{11}, 3\chi_{QS}^{11}$ and $C_{BS}$ calculated in...
HRG model using hadronic spectrum PDG 2016 cross the SB limit at $T = 208,214 \text{ and } 198 \text{ MeV}$ respectively. For the hadronic spectrum PDG 2016+ corresponding temperatures are $T = 186,210 \text{ and } 180 \text{ MeV}$ respectively. The influence of additional Hagedorn type resonances on $C_{BS}$ and its consequences on the applicability of I-HRG was discussed in Ref. [83].

B. $\mu_B \neq 0; \mu_S = \mu_Q = 0$

We now extend our analysis on the $(\mu_B, T)$ plane. In Figs. 4 and 5 we show the boundaries beyond which I-HRG can not be applied to study QCD thermodynamics. While the results in Fig. 4 are computed with hadron spectra PDG 2016, those in Fig. 5 have been done with hadron spectra PDG 2016+. Each TQ provides a separate bound. For example, along the blue dotted line pressure in the I-HRG phase is equal to the SB limit (i.e., $p^H = P^{SB}$). We have drawn similar lines using other TQs like $\epsilon$, $s$, $\chi_2^S$, $\chi_2^B$, $\chi_2^O$, $\chi_1^B$, $\chi_1^O$, $\chi_0^B$, $\chi_0^O$ and $C_{BS}$ assuming $O^H = O^{SB}$ where $O$ is any of the TQs. The shaded region is obtained by tracing out the lowest $T$ value for a given $\mu_B$ for all the TQs studied to provide the upper bound on the region of applicability of the I-HRG model in the QCD phase diagram. With hadron spectra 2016+, TQs rise faster with $T$ and $\mu_B$ as compared to the case with hadron spectra 2016 resulting in a tighter bound for the former as already seen in Figs. 1 and 2 and

For $\mu_B = 0$.

C. Heavy Ion scenario

Now we will discuss in the context of heavy ion collision. In this scenario $\mu_S$ and $\mu_Q$ are non-zeroes and can be calculated applying the following charge conservation equations

$$\sum_i n_i(T, \mu_B, \mu_S, \mu_Q)S_i = 0,$$

and

$$\sum_i n_i(T, \mu_B, \mu_S, \mu_Q)Q_i = r,$$

where $r$ is the ratio of net-charge to net-baryon number of the colliding nuclei. For Pb + Pb or Au + Au collisions $r = N_p/(N_p + N_n) \simeq 0.4$, where $N_p$ and $N_n$ are respectively proton numbers and neutron numbers of the colliding nuclei. The right-hand side of the Eq. [18] is zero since initially there is no net-strangeness in the colliding nuclei.

In Fig. 5 we have shown the upper bound for the I-HRG phase on the QCD phase diagram for the heavy ion collision scenario. Here charge conservations are applied to get $\mu_S$ and $\mu_Q$ at a fixed $T, \mu_B$. Left and right panels show the results using the hadron spectra PDG 2016 and PDG 2016+ respectively. It is observed that charge conservation does not significantly modify the earlier obtained results at low $\mu_B$ region. However, in the intermediate region where both $T$ and $\mu_B$ are large, the imposition of charge conservation push the bounds slightly towards higher values of $(\mu_B, T)$. 
D. Comparison with lattice QCD

In Fig. 7 we have shown the obtained upper bound for the extent of the I-HRG phase in all the four cases studied here. These are the boundaries of the shaded regions or the common allowed I-HRG phase by all the studied TQs as shown in the previous plots. We have compared our estimates with the hadronic to QGP phase transition line (pseudo critical line) calculated by the lattice QCD simulation \cite{15}. The transition line is generally parametrized as

\[
\frac{T(\mu_B)}{T(0)} = 1 - \kappa \left( \frac{\mu_B}{T(0)} \right)^2,
\]

where \(T(0)\) and \(\kappa\) are the transition temperature and the curvature of the transition line respectively at zero baryon chemical potential \cite{13,16}. The green error band shows the lattice result of transition line where \(T(0) = 154(9)\) and \(\kappa = 0.020(4)\) \cite{15}. The yellow box in \(T\) at \(\mu_B = 0\) reflects the uncertainties associated with LQCD determination of \(T_\text{c} = (145 - 170\ \text{MeV})\) and the choice of observables for order parameter. Chemical freeze-out parameters of Refs \cite{84} (Cleymans et. al.) and \cite{50} (Andronic et. al.) are shown by the red and pink colored shaded bands respectively.

At \(\mu_B/T = 0\), LQCD computations using different combinations of susceptibilities of \(B, Q\) and \(S\) upto fourth order demonstrate the breakdown of applicability of I-HRG beyond the chiral transition region \(154(9)\ \text{MeV} \) \cite{85}, providing a stronger constraint than we have in this study. However, for larger values of \(\mu_B/T\) where currently there is no LQCD results, our estimates provide the best bound for the extent of the I-HRG phase. For \(\mu_B/T > 2\), \(C_{BS}\) provides the strongest bound. As we dial up \(\mu_B/T\), the obtained bound is found progressively at a lower \(T\) as compared to the extracted freeze-out curve by fitting the I-HRG model to the measured hadron yields \cite{36,84}. This calls for revisiting the extraction of the freeze-out curve at these baryon densities with a version of HRG that is applicable in these conditions. It may be noted that HRG with Van der Waals interactions can be tuned to yield a \(C_{BS}\) that is closer to LQCD results \cite{86}.

V. SUMMARY

One of the primary goals of heavy ion collision experiments is to study the QCD phase diagram. Differentiating signals of the hadronic and QGP phases, smooth crossover transition from a first order transition and those of the critical from non-critical region is a challenging problem in the field. While experimental measurements are difficult and are ongoing, the QCD based calculations on lattice have numerical challenges at finite \(\mu_B\). On the other hand I-HRG model has been very successful in explaining both the experimental data on yields of produced hadrons in heavy-ion collisions with a few parameters as well as LQCD observables at \(\mu_B = 0\) below the chiral transition region. The basic idea of the current work was to get an upper bound for the applicability of the I-HRG on the QCD phase diagram.

For our study we select only those TQs that are expected in QCD to have a monotonically increasing behavior from the hadronic to the QGP phases asymptotically reaching the corresponding SB limit. On the other hand, these TQs in I-HRG monotonically rises with increasing \(T\) and \(\mu_B\) without any bound. Hence, our criteria for the applicability of I-HRG is that the studied TQ should be less than the corresponding SB limit. Several bounds on the \(\langle \mu_B, T \rangle\) plane are obtained corresponding to each TQs like pressure, energy density, entropy density and susceptibilities of conserved charges. A hierarchical structure is noted—higher the derivative of the parti-
FIG. 7. Comparison of all hadronic phase boundaries calculated in the present work with the lattice QCD calculation. These are the boundaries of the shaded regions or the common hadronic phase of previous plots. Blue and black dotted lines correspond to the I-HRG phase boundaries at $\mu_S = \mu_Q = 0$ using hadronic spectra PDG 2016 and PDG 2016+ respectively. Blue and black solid lines show similar phase boundaries but for the heavy ion collision scenario where $\mu_S$ and $\mu_Q$ have been calculated by applying charge conservation as mentioned in Eqs. [18-19]. The yellow box shows the uncertainty in LQCD transition temperature (145 - 170 MeV) at $\mu_B$ the lattice transition line [15]. The green error band shows the $S$ Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, and K. K. Szabo, PDG 2016 and PDG 2016+ respectively. Blue and black solid lines show similar phase boundaries but for the heavy ion collision scenario where $\mu_S$ and $\mu_Q$ have been calculated by applying charge conservation as mentioned in Eqs. [18-19]. The yellow box shows the uncertainty in LQCD transition temperature (145 - 170 MeV) at $\mu_B = 0$. Chemical freeze-out parameters of Refs [84] and [36] are shown by the red and pink colored shaded bands respectively.

The green error band shows the lattice transition line [15]. The yellow box shows the uncertainty in LQCD transition temperature (145 - 170 MeV) at $\mu_B = 0$. Chemical freeze-out parameters of Refs [84] and [36] are shown by the red and pink colored shaded bands respectively.

SC acknowledges fruitful discussions with Rohini Godbole, Sourendu Gupta and Harvey B. Meyer. SC is supported by the AGH UST statutory tasks No. 11.11.220.01/1 within subsidy of the Polish Ministry of Science and Higher Education (MNiSW) and the National Science Centre grant 2015/17/B/ST2/00101. BM acknowledges DST J C Bose fellowship for financial assistance. SS acknowledges financial support from DAE, Government of India.
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