The application of LDPC code in MIMO-OFDM system

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Abstract. The combination of MIMO and OFDM technology has become one of the key technologies of the fourth generation mobile communication, which can overcome the frequency selective fading of wireless channel, increase the system capacity and improve the frequency utilization. Error correcting coding introduced into the system can further improve its performance. LDPC (low density parity check) code is a kind of error correcting code which can improve system reliability and anti-interference ability, and the decoding is simple and easy to operate. This paper mainly discusses the application of LDPC code in MIMO-OFDM system.

1. Introduction

With the progress of wireless communication technology, higher reliability, stability and data transmission rate have become the constant improvement goal of wireless communication system. In this paper, we propose and examine a new attempt that the application of LDPC code in MIMO-OFDM system.

MIMO technology is a wireless communication system based on multiple antennas, which can utilize effectively wireless communication channel without increasing the bandwidth, can convert Multipath effects into the favorable factors and take advantage of it, increase the capacity of the wireless channel and spectrum utilization and the data transmission rate[1], meanwhile stability and the performance of communication system are improved[2]. OFDM technology has a significant improvement aimed at the influence of frequency selective fading, because of its own the long symbol cycle. Therefore, the combination of MIMO technology and OFDM technology in modern wireless communication system will play an important role in the development of wireless communication field.

The performance of LDPC codes approach the Shannon limit, which is a sparse matrix based parallel iterative decoding algorithm with low computational complexity. And it is relatively easy to implement on hardware owning to its parallel structure. The bit rate can be arbitrarily constructed, hence it has greater flexibility.

The remainder of this paper is organized as follows. Section 2 describes the the basic principles of MIMO and OFDM technology, and then proposes a LDPC-MIMO-OFDM optimization system model.
based on MIMO-OFDM system model. Experimental results and analysis are shown in section 3. Finally, section 4 presents the conclusion.

2. LDPC-MIMO-OFDM system

Multipath effect once was seen as harmful factors because it can lead to the decline of the signal. MIMO system minimizes the influence of this effect, and makes full use of each multipath component, then finally improves system performance. The MIMO system is unable to improve the effects of frequency selective fading, while The OFDM technology can solve the loss of performance caused by the decline in the communication system very well[3].

MIMO-OFDM system is a combination of MIMO technology and OFDM technology, which complements the advantages of the both, and increases the data transmission rate to a new height by spatial reuse. Simultaneously the reliability of the communication system and the utilization rate of spectrum resources are improved obviously by the space time set and OFDM technology.

In order to further improve the performance of MIMO-OFDM wireless communication system, error correcting coding can be introduced into this system. MIMO-OFDM system based on LDPC coding which has superior coding performance becomes an ideal choice of new generation wireless communication.

2.1 MIMO-OFDM system

Because of the relatively excellent performance of STBC coding in the MIMO system, STBC coding and OFDM technology can be combined. The transmitter block diagram of the MIMO-OFDM system based on the STBC coding is shown in Fig.1, which takes two transmitting antennas for example.

![Figure1. The transmitter diagram of MIMO-OFDM system](image)

Supposing the system has N subcarriers, N_T transmit antennas (here citing two transmitting antennas), a group formed by per the N • N_T symbol bits constitutes a STBC-OFDM symbol which is transmitted on the N_T transmitting antennas at T time intervals. If the number of transmit antennas N_T is 2, one set of modulation symbols is \( S = (s_1, s_2, \ldots, s_{2N}) \), each of the two symbols is selected for encoding.

The encoded signal is sent to the transmitting antenna, and the N sub carriers are modulated by Fourier inverse transformation. Then output sequence of N points after inverse Fourier transform on transmit antenna \( p \):

\[
s_p(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_p(k) e^{j \frac{2\pi nk}{N}}, \quad 0 \leq n \leq N - 1
\]

Assuming the maximum time delay is L sampling points in the environment of multipath
fading channels, the time domain channel impulse response between the transmitting antenna \( p \) and the receiving antenna \( q \) is:

\[
h_{p,q}(n) = \sum_{l=0}^{L-1} h_{p,q,l} \delta(n-l)
\]  

(2)

Where \( h_{p,q,l} \) represents the complex gain of the path \( l \) between the transmitting antenna \( p \) and the receiving antenna \( q \), \( \delta(n) \) indicates unit impulse response. The corresponding channel frequency response can be expressed as:

\[
H_{p,q}(k) = \text{FFT}(h_{p,q}(n)) = \sum_{l=0}^{L-1} h_{p,q,l} e^{-j2\pi kl/N}, \quad k = 1, 2, \cdots, N
\]  

(3)

The channel frequency response matrix between the transmitting antenna \( p \) and the receiving antenna \( q \) is:

\[
H_{p,q} = [H_{p,q}(0), H_{p,q}(1), \cdots, H_{p,q}(N-1)]^T
\]  

(4)

After the transmission signal of the \( N_T \) transmit antennas go through multipath fading channel, the superposition of these signals is received on the receiving antennas. If the system synchronization is ideal and the channel is known, remove CP at the receiving side and make a processing of Fourier transform for signal \( S \). The received signal on the receiving antenna \( q \) with a time interval of \( 2i \) is:

\[
Y_{2i}^q = S(2i)H_{1q} + S(2i+1)H_{2q} + W_{2i}^q
\]  

(5)

The received signal on the receiving antenna \( q \) with a time interval of \( 2i+1 \) is:

\[
Y_{2i+1}^q = -S^*(2i+1)H_{1q} + S^*(2i)H_{2q} + W_{2i+1}^q
\]  

(6)

We can draw the following formula by formula (5) and (6):

\[
\begin{pmatrix}
Y_{2i}^q \\
Y_{2i+1}^q
\end{pmatrix} =
\begin{pmatrix}
S(2i) & S(2i+1) \\
-S^*(2i+1) & S^*(2i)
\end{pmatrix}
\begin{pmatrix}
H_{1q} \\
H_{2q}
\end{pmatrix} +
\begin{pmatrix}
W_{2i}^q \\
W_{2i+1}^q
\end{pmatrix}
\]  

(7)

Where \( W_{2i}^q \) and \( W_{2i+1}^q \) are respectively Gauss white noise signals with the time interval of \( 2i \) and \( 2i+1 \), and the mean value is zero in formula (7).

 Meanwhile formula (7) can also be derived as:

\[
\begin{pmatrix}
Y_{2i}^q \\
Y_{2i+1}^q
\end{pmatrix} =
\begin{pmatrix}
H_{1q} & H_{2q} \\
\bar{H}_{2q} & -H_{1q}
\end{pmatrix}
\begin{pmatrix}
S(2i) \\
S(2i+1)
\end{pmatrix} +
\begin{pmatrix}
W_{2i}^q \\
W_{2i+1}^q
\end{pmatrix}
\]  

(8)

If \( Y_q^j = \begin{pmatrix} Y_{2i}^q \\ Y_{2i+1}^q \end{pmatrix}, \quad H = \begin{pmatrix} H_{1q} & H_{2q} \\ \bar{H}_{2q} & -H_{1q} \end{pmatrix}, \quad S_j = \begin{pmatrix} S(2i) \\ S(2i+1) \end{pmatrix}, \quad W_j^q = \begin{pmatrix} W_{2i}^q \\ W_{2i+1}^q \end{pmatrix} \), the above formula can be simplified as:

\[
Y_q^j = HS_j + W_j^q
\]  

(9)

Where \( H \) can be regarded as the channel transfer function. If both sides of the formula (9) are multiplied by transposed conjugate matrix \( H^H \) of transfer function \( H \), the symbol estimation
value of the system can be obtained:

\[ \hat{S}_i = H^H \cdot Y_i + H^H \cdot S_i + H^H \cdot W_i \]

\[ = \begin{pmatrix} H_{1\ell} & H_{2\ell} \\ H_{1q} & -H_{2q} \end{pmatrix} \begin{pmatrix} H_{1q} \\ H_{2q} \end{pmatrix} S_i + H^H \cdot W_i \]

\[ = \begin{pmatrix} |H_{1q}|^2 + |H_{2q}|^2 & 0 \\ 0 & |H_{1q}|^2 + |H_{2q}|^2 \end{pmatrix} S_i + H^H \cdot W_i \]

(10)

2.2 LDPC-MIMO-OFDM system

Take two transmitting antennas and two receiving antennas as an example, Fig. 2 is the Principle block diagram of the LDPC-MIMO-OFDM system.

![Block diagram of LDPC-MIMO-OFDM system](image)

**Figure 2.** Block diagram of LDPC-MIMO-OFDM system

The signals’ information of the length K input at transmitter flow through and are encoded by the LDPC encoder. If the code rate is r, the encoded information sequence is \( k/r \), and then the input information is changed to the modulation sequence by constellation mapping and divided into multiple sub data streams by STBC code, modulation of multiple subcarriers is achieved by IFFT. The sub-carrier is modulated into a set of OFDM symbols to send the antenna and complete the transmitter task, by making parallel to seria conversion and adding the CP. The received signals through the multi-antenna at the receiving end are removed the CP, made FFT operation, and decoded by STBC decoding, then the final signal can be obtained. Because of the iterative decoding algorithm used in LDPC decoding, therefore, according to the transmission characteristics of OFDM, the initialization algorithm is used to calculate the initial information.

For M hexadecimal modulation, the \( K = \log_2 M \) encoded information is mapped into a symbol through the constellation diagram, where \( s^m = s_x^m + js_y^m \) \( 0 \leq m < M \) is the coordinate of the symbol m, and then the symbols are orthogonal to the subcarriers in part. The symbol sequence transmitted on the subcarrier K is \( X(k) = s_x + js_y \). Through the OFDM system, the corresponding channel reception symbol is:
\begin{equation}
Y(k) = \hat{X}(k) \cdot H(k) + W(k)
\end{equation}

Among them, \( \hat{X}(k) = \hat{s}_x + j\hat{s}_y = Y(k)/H(k) \) is the transmission symbol. Because there is the fast Fourier transform in OFDM system, the probability distribution of noise signal is also changed, which makes the posterior probability of information is not easy to determine. However, the posterior probability of the information can be calculated based on the distance between the received signal and the sign in the constellation diagram, that is, the smaller the distance between the two symbols, the more similar.

If the modulation symbol is equal probability distribution, the below formula can be got by Bayesian formula:

\begin{equation}
P_m = \Pr\left\{X(k) = s^m \mid \hat{X}(k)\right\} = C \cdot \Pr\left\{\hat{X}(k) \mid X(k) = s^m\right\}
= \frac{C}{\|\hat{X}(k) - s^m\|^2} = \frac{C}{(\hat{s}_x - s_x^m)^2 + (\hat{s}_y - s_y^m)^2}
\end{equation}

The likelihood ratio of the first \( i \) bits in the transmitting symbol \( X(k) \) can be calculated by formula:

\begin{equation}
LR(b_i) = \frac{\Pr\left\{b_i = 1 \mid \hat{X}(k)\right\}}{\Pr\left\{b_i = 0 \mid \hat{X}(k)\right\}} = \frac{\sum P_m}{\sum P_m/n}
\end{equation}

\( U_0 \) and \( U_1 \) are respectively \( b_i = 0 \) or 1 of the symbol set in formula (13).

3. Simulation results and analysis

In this paper, the LDPC-MIMO-OFDM system is used to carry out the simulation experiment. During the experiment, the influence of the LDPC code decoding algorithm on the performance of the system is analyzed by various situations. Some basic parameters are set as follows:

The simulation algorithm adopts irregular QC-LDPC code, the code length is \( n = 1010 \) bit, coding rate is \( R = 1/2 \), coding method is based on the Gauss elimination, the decoding algorithm uses the LLR-BP algorithm and the improved algorithm, and the decoding iteration 50 times; the Rayleigh fading channel, the number of bars is 6.

3.1 The influence of different decoding iterations on the system

LDPC code has the characteristics of random coding, it is due to the fact that it has a sparse parity check matrix \( H \). And the iterative decoding algorithm constructs the LDPC code into a code with a longer code length. These two factors are one of the main reasons that the LDPC code has excellent channel error correction performance. In this paper, the number of iterations of the LLR-BP iterative decoding algorithm for multiple LDPC codes is simulated. Fig.3 shows the bit error rate performance for different iterations.
Figure 3. The influence of different decoding iterations on the system

As can be seen in Fig. 3, with the increase of number of iterative decoding, the system error rate gradually decreases, and the faster the convergence rate is, the faster the number of iterations. There is a "waterfall" between SNR 7–9dB. It can be seen that the number of iterations of decoding algorithm has a great influence on the decoding performance of LDPC. However, when the number of iterations reaches a certain number of times, even if the number of iterations increases greatly, the improvement of BER performance is not very obvious, such as iteration 50 times and iteration 100 times. Therefore, it is not possible to improve the performance of the system by only increasing the number of iterations of the decoding algorithm, and the direct increase in the number of iterations increases the decoding time, which results in a decrease in the real-time performance of the communication, and has a certain impact on the effectiveness of the system.

3.2 The influence of different modulation methods on the system

After the information sent is encoded by LDPC coding, constellation mapping is required. Different modulation mapping modes will have a direct impact on LDPC-MIMO-OFDM systems. Fig. 4 shows the BER performance of the system when the modulation modes are BPSK, QPSK, 8PSK, 16PSK and 32PSK.

Figure 4. Comparison of different modulation methods
Fig. 4 illustrates the BER performance of the system is getting worse, with the gradual increase of the hexadecimal number. In which, BPSK performance is optimal, 32PSK performance is the worst. This is mainly because the continuous increase in the number of hexadecimal, the information symbols are getting smaller and smaller, the more difficult to decide in the demodulation. However, along with the decrease of hexadecimal number, the bandwidth utilization rate is reduced gradually, which resulted in the waste of limited spectrum resource. So different modulation modes should be emphasized in the LDPC-MIMO-OFDM system.

3.3 The influence of different number of antennas on the system
The measure of multiaerial system to resist channel fading is to transmit and receive diversity by using multiple antennas. Therefore, the number of transmit and receive antennas is one of the main factors that affect the system channel capacity and reliability.

In this experiment, multiple transmitting and multiple receiving antennas are used to carry out simulation experiments. The simulation experiment adopts the irregular QC-LDPC code and the Gaussian elimination method, where the code length is \( n = 1010 \) bit, the coding rate is \( R = 1/2 \). And the decoding algorithm use LLR-BP algorithm and the improved algorithm. From Fig. 5, we can see that the effect of using 2 receive 2 send antenna is the best, but the effect of the improved algorithm is not very obvious, the system performance has improved significantly only when the signal to noise ratio of 7 ~ 8dB, and the improvement results is 0.05 ~ 0.1dB or so.

3.4 The influence of different OFDM sub-carriers on the system
In Fig. 6, the solid line in the figure shows that the error performance of LDPC-MIMO-OFDM system is reduced when the number of OFDM sub-carriers increases gradually. This is because the increase in the number of sub-carriers, transfers the high-speed transmission of data streams into more low-rate data stream, which effectively improve the anti-frequency selective fading performance, so the error performance has been gradually improved.

The dotted line is the result of the improved algorithm. It is due to fact that the LDPC-MIMO-OFDM system itself is highly complex and the channel environment is Rayleigh fading channel. Although the speed of the data stream becomesssmaller and is helpful to resist frequency selective fading, the effect of the improved performance algorithm is not very obvious. At \( BER = 10^{-4} \), the improved decoding algorithm improves the performance of the system by about 0.1
dB in the case of different sub-carriers.

![Comparison of different OFDM subcarriers](image)

**Figure6.** Comparison of different OFDM subcarriers

### 4. Summary

This paper introduces briefly the basic principles of MIMO technology. Then OFDM system’s merits and drawback of the technology are summarized by analyzing its the principle. Furthermore, the dual transmission antenna MIMO-OFDM system based on STBC coding is analyzed. Based on this, the LDPC-MIMO-OFDM system is introduced in detail, and the theoretical basis of the iterative decoding algorithm in the system is analyzed. Finally, the performance of the system under different variable parameters is analyzed by simulation experiment.

It can be found that the improved decoding algorithm is not ideal for the bit error rate performance of LDPC-MIMO-OFDM system. However, because of improved decoding algorithm of LDPC codes, the overall performance of the system isn’t broken but a trend of stability.

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