Possibilities for regulation of distribution of oscillation amplitudes points of working bodies of technological machines

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Abstract. Ensuring the reliability of technological equipment is determined by the dynamic characteristics of operating modes. For objects in the form of extended solids that perform flat or spatial movements, the distribution of oscillation amplitudes at the points of the working body is essential. The problem of adjusting the vibration field of a technological machine is considered. The solution uses methods of structural mathematical modeling, when a mechanical oscillatory system, considered as a computational scheme of a technical object, is compared to a structural mathematical model in the form of the structural diagram of a dynamically equivalent automatic control system. A method for forming certain structures of the vibration field of technological machines is offered. The method is based on the use of additional links implemented by special devices for converting movement. Selection of parameters of a vibrating technological machine by using the effects of changing the reduced mass of devices for converting motion, made in the form of screw non-self-braking mechanisms. The developed mathematical models reflect the vibration field of a homogeneous structure created by "zeroing" angular oscillatory movements. The proposed method for setting the vibration stand parameters is illustrated by computational modeling on the example of a model problem.

1. Introduction

The reliability of technological equipment is determined by keeping the dynamic states of technical objects within certain limits [1-3]. In particular, for objects in the form of extended solids that perform plane or spatial vibrations, it is important to distribute the amplitudes of vibrations at different points of the working body. In many cases, the operating conditions of technical objects are interpreted as problems of adjusting the vibration field of the technological machine. A common way to form the structure of the vibration field is to select the parameters of a mechanical oscillatory system used as a design scheme for a technical object[4,5]. The problems of dynamics of machines and equipment operating under vibration loads require special attention in terms of evaluating the possibilities of improving dynamic states through the introduction of additional links into the structure of mechanical oscillatory systems implemented by various mechanisms and devices for converting motion. A method is proposed for forming certain structures of the vibration field of technological machines based on the ideas of introducing and using additional connections implemented by special devices for converting motion[6-7].
2. Features of technical object
A mechanical oscillating system with a solid body performing flat movements is considered as the design scheme of a vibration stand. A solid body (the working body of the vibrostand) has a mass \( M \) and a moment of inertia \( J \) (Figure 1). The solid body is based on elastic elements with stiffness \( k_1 \) and \( k_2 \), which are connected in parallel with additional devices for converting motion (DCM), which have reduced masses \( L_1 \) and \( L_2 \). Such devices can be implemented on the basis of lever or non-self-locking screw mechanisms. The system has two degrees of freedom and makes small fluctuations relative to the static equilibrium position. Motion is defined in the \( y_1 \) and \( y_2 \) coordinate system associated with the fixed basis. It is assumed that the external force perturbations \( Q_1 \) and \( Q_2 \) at the coordinates \( y_1 \) and \( y_2 \) have the form of common-mode harmonic functions with the same amplitudes, which can be provided by means of centrifugal vibration exciters.

![Figure 1](image)

Figure 1. Figure B. 1. Schematic diagram of the vibrostand with a motion conversion device (DCM)

The system of equations of motion can be obtained on the basis of the approaches, using expressions for kinetic and potential energies in the form

\[
T = \frac{1}{2} M y_0^2 + \frac{1}{2} J \dot{\phi}^2 + \frac{1}{2} L_1 y_1^2 + \frac{1}{2} L_2 y_2^2, \tag{1}
\]

\[
\Pi = \frac{1}{2} k_1 y_1^2 + \frac{1}{2} k_2 y_2^2. \tag{2}
\]

After transformation of Laplace taking into account zero initial conditions the system of equations of motion in operator form takes the form

\[
\ddot{y}_1[(Ma^2 + Jc^2 + L_1)p^2 + k_1] + \ddot{y}_2[(Mab - Jc^2)p^2 = \overline{Q}_1, \tag{3}
\]

\[
\ddot{y}_2[(Mb^2 + Jc^2 + L_2)p^2 + k_2] + \ddot{y}_1[(Mab - Jc^2)p^2 = \overline{Q}_2. \tag{4}
\]

Based on the equations (3), (4), a structural mathematical model can be constructed in the form of a block diagram (Figure 2) of an automatic control system.

The transfer functions of the system are obtained on the basis of the block diagram:

\[
W_1(p) = \frac{\ddot{y}_1}{\ddot{Q}_1} = \frac{(Mb^2 + Jc^2 + L_2)p^2 + k_2}{A(p)}, \tag{5}
\]

\[
W_2(p) = \frac{\ddot{y}_2}{\ddot{Q}_1} = \frac{(Mb^2 - Mab)p^2}{A(p)}, \tag{6}
\]

\[
W'_1(p) = \frac{\ddot{y}_1}{\ddot{Q}_2} = \frac{(Jc^2 - Mab)p^2}{A(p)}, \tag{7}
\]

\[
W'_2(p) = \frac{\ddot{y}_2}{\ddot{Q}_2} = \frac{(Ma^2 + Jc^2 + L_1)p^2 + k_1}{A(p)}. \tag{8}
\]
\[
W(p) = \frac{\bar{y}}{Q} = \frac{(Mb^2 + Jc^2 + L_2)p^2 + k_2 + (Jc^2 - Mab)p^2}{A(p)}, \quad (9)
\]

\[
W_s(p) = \frac{\bar{y}_s}{Q} = \frac{(Ma^2 + Jc^2 + L_1)p^2 + k_1 + (Jc^2 - Mab)p^2}{A(p)}, \quad (10)
\]

where

\[
A(p) = [(Ma^2 + Jc^2)p^2 + k_1] - [(Mb^2 + Jc^2)p^2 + k_2] - [(Jc^2 - Mab)p^2] \quad (11)
\]

– frequency characteristic equation of the system.

\[1 \quad \frac{1}{(Ma^2 + Jc^2 + L_1)p^2 + k_1} \quad \frac{1}{(Jc^2 - Mab)p^2} \quad \frac{1}{(Mb^2 + Jc^2 + L_2)p^2 + k_2} \quad \bar{y}_2 \]

\[\bar{y}_1 \quad \bar{y}_2 \quad Q_1 \quad Q_2 \]

\[\text{Figure 2. Block diagram of the system according to Figure 1}\]

In the coordinate system \(\bar{y}_1, \bar{y}_2\), a block diagram has two partial blocks connected by an inertial link. The values of partial frequencies depend on the reduced masses of \(L_1\) and \(L_2\) devices for converting motion (DCM):

\[
n_1^2 = \frac{k_1}{Ma^2 + Jc^2 + L_1}, \quad (12) \quad n_2^2 = \frac{k_2}{Mb^2 + Jc^2 + L_2}. \quad (13)
\]

In this case, the frequency of dynamic vibration damping is determined from the condition of "zeroing" the numerators of transfer functions defined by the expressions (5) ÷ (10). The system may display modes of dynamic vibration damping, which depends on the conditions of external perturbation. The possibilities of distribution of vibration amplitudes along the points of the working body of a vibrating technological machine are determined by the transfer functions of interparticle connections that display the lever properties of dynamic interactions:

\[
W_{12}^{(p)}(\bar{q}) = \frac{\bar{y}_2}{\bar{y}_1} = \frac{(Jc^2 - Mab)p^2}{(Mb^2 + Jc^2 + L_2)p^2 + k_2}, \quad (14)
\]

\[
W_2^{(p)}(\bar{q}) = \frac{\bar{y}_2}{\bar{y}_1} = \frac{(Ma^2 + Jc^2 + L_1)p^2 + k_1}{(Jc^2 - Mab)p^2}, \quad (15)
\]

\[
W_2^{(p)}(\bar{q}) = \frac{\bar{y}_2}{\bar{y}_1} = \frac{(Ma^2 + Jc^2 + L_1)p^2 + k_1 + (Jc^2 - Mab)p^2}{(Mb^2 + Jc^2 + L_2)p^2 + k_2 + (Jc^2 - Mab)p^2}. \quad (16)
\]
3. Condition for forming the distribution of oscillation amplitudes

The distribution of the vibration amplitudes at \( \frac{\bar{y}_2}{\bar{y}_1} = 1 \) corresponds to a situation when a solid body makes translational oscillatory movements vertically in the absence of any oscillatory angular movements. From (16) we get that this is possible if the conditions are met

\[
(Ma^2 + Jc^2 + L_2)p^2 + k_1 = (Mb^2 + Jc^2 + L_2)p^2 + k_2, \tag{17}
\]

which means that

\[
[M(a-b) + L_2]p^2 = k_2 - k_1. \tag{18}
\]

If it is possible to provide a condition

\[
L_2 = M(a-b) + L_4, \tag{19}
\]

the ratio of vibration amplitudes \( \frac{\bar{y}_2}{\bar{y}_1} \) will be determined by the ratio of stiffness coefficients of elastic elements \( \frac{k_1}{k_2} \).

For \( k_1 = k_2 \) and \( L_2 \), defined from (19), it is possible to find conditions for implementing the mode of translational oscillatory vertical movements. If \( k_1 \neq k_2 \), this mode cannot be implemented. Similarly, we can get a condition when the tuning factor is the value of the reduced mass \( L_1 \):

\[
L_4 = M(b-a) + L_2. \tag{20}
\]

From (19) and (20) it follows, in particular, that the value of the reduced mass used to create a certain dynamic state depends on the difference between the geometric parameters \( a \) and \( b \). The smaller the value \( a - b \), the less the value of the reduced mass is required.

Obtaining a mode close to, it is possible even with one of the devices for converting the movement of \( L_1 \) or \( L_2 \). Practically, the implementation of the formation of a certain value given weight \( L_1 \) or \( L_2 \) devices for conversion of movement (DCM) can be carried out by the method described in [7], where the use of the effect of the change is given weight by the application to the nut-flywheel DCM brake torque adjustable, depending on the parameters of the dynamic system state. Figure 3 shows the amplitude-frequency characteristics of interpartial connections. The model problem is used; the data for calculations \( a = 0.4; b = 0.6; c = 1; M = 1000 \text{ kg}; J = 250 \text{ kg} \cdot \text{m}^2; k_2 = 1000 \text{ N/m}; L_2 = 100 \text{ kg}; L_1 = 300 \text{ kg}. \)

Graphs of dependencies, that is, the amplitude-frequency characteristics of interpartial connections, are shown in figure 3 (the family of graphs is constructed for \( k_1 \rightarrow k_2 \)).

From the analysis of graphs \( \frac{\bar{y}_2}{\bar{y}_1}(\omega) \), it follows that in general, when \( k_1 \neq k_2 \), the amplitude-frequency characteristics of interpartial connections have intersections with the abscissus axis, which determines the modes of dynamic vibration damping ( \( \bar{y}_2 = 0 \) ). However, the amplitude-frequency characteristics have frequencies at which discontinuities of the second kind are realized, which corresponds to the conditions when . For \( \omega = 0 \) and \( \omega \rightarrow \infty \), the dependency graphs \( \frac{\bar{y}_2}{\bar{y}_1}(\omega) \) have limit values.
Figure 3. Family of amplitude-frequency characteristics interpartial relations; graphs denoted by (1) represented by curves at \( k_1 = 900 \text{ N/m} \); graph (2) - \( k_1 = 920 \text{ N/m} \); graph (3) - \( k_1 = 940 \text{ N/m} \); graph (4) - \( k_1 = 960 \text{ N/m} \); graph (5) - \( k_1 = 980 \text{ N/m} \); graph marked as (N) corresponds to the operating mode at \( \frac{\bar{y}_2}{\bar{y}_1} = 1 \)

Figure 3 dependency graphs are grouped in such a way that it is possible to observe changes in their relative position when \( k_1 \) takes the value \( k_1 = 900 \text{ N/m} \), \( 920 \text{ N/m} \), \( 940 \text{ N/m} \), \( 960 \text{ N/m} \), \( 980 \text{ N/m} \) approaching a critical case \( k_1 = k_2 = 1000 \text{ N/m} \). In turn, figure 4 shows a family of amplitude-frequency characteristics when \( k_1 \) takes the value of \( 1100 \text{ N/m} \), \( 1080 \text{ N/m} \), \( 1060 \text{ N/m} \), \( 1040 \text{ N/m} \), \( 1020 \text{ N/m} \), approaching is also the value of \( k_1 = 1000 \text{ N/m} \) and \( k_2 = 1000 \text{ N/m} \). In the limiting case of the amplitude-frequency response becomes a straight line \( \frac{\bar{y}_2}{\bar{y}_1} = 1 \), parallel to the x-axis. In this case, the working body of the technological vibrating machine (vibrostand) performs only vertical vibrations in the absence of angular movements. It can be noted that in the zone of sufficient proximity of parameters, when \( k_1 \approx k_2 \), it is also possible to operate the vibration stand in a certain (or local) frequency range, while \( \frac{\bar{y}_2}{\bar{y}_1} \approx 1 \)

Figure 4. Family of amplitude-frequency characteristics interpartial relations; graphs denoted by (6) represented by curves at \( k_1 = 1020 \text{ N/m} \); graph (7) - \( k_1 = 1040 \text{ N/m} \); graph (8) - \( k_1 = 1060 \text{ N/m} \); graph (9) - \( k_1 = 1080 \text{ N/m} \); graph (10) - \( k_1 = 1100 \text{ N/m} \); graph marked as (N) corresponds to the operating mode at \( \frac{\bar{y}_2}{\bar{y}_1} = 1 \)

Figure 5 shows a family of amplitude-frequency characteristics of interpartial connections with a more detailed consideration of the convergence of coefficients \( k_1 \) and \( k_2 \) (in particular, the values of parameters \( k_1 = 995 \text{ N/m} \), \( 999 \text{ N/m} \), \( 1001 \text{ N/m} \), \( 1005 \text{ N/m} \)).
Figure 5. Family of amplitude-frequency characteristics between partial connections; graphs (1) are curves for \( k_1 = 995 \text{ N/m} \); graphs (2) - \( k_1 = 999 \text{ N/m} \); graphs (3) - \( k_1 = 1001 \text{ N/m} \); graphs (4) - \( k_1 = 1005 \text{ N/m} \); graphs with the value \( k_1 = 1000 \text{ N/m} \) corresponds to the operating mode when \( \frac{\bar{y}_2}{\bar{y}_1} = 1 \)

4. Discussion
Comparative analysis shows that with the symmetry of elastic characteristics, that is, at \( k_1 \), close enough to the value of \( k_2 \), it is possible to adjust the vibrostand by introducing an additional connection that creates compensating effects of changes in the reduced masses created by devices for converting motion, which ensure equality of coefficients at frequency squares in expressions for transfer functions of interpartial connections. The analysis also shows that if the stiffness of the elastic elements \( k_1 \) and \( k_2 \) significantly mismatch, their preliminary adjustment of the elastic system of the vibration stand is required, which can be implemented when preparing the vibration processing machine for operation.

The implementation of vibration technological processes requires an appropriate preliminary assessment of the dynamic capabilities of vibration stands in terms of creating certain conditions for interaction between the processing working environment and the processed product. The author proposes a method for constructing mathematical models that reflect the features of the formation of a working body of a technological machine of a vibration field of a uniform structure, which is achieved by «zeroing» angular oscillatory movements.

Mathematical models can be constructed based on the use of transfer functions of interpartial relations. The author offers a method for selecting the parameters of a vibrating technological machine by using the effects of changing the reduced mass of devices for converting motion, made in the form of screw non-self-braking mechanisms.

5. Conclusion
The proposed method for setting the parameters of the vibrostand is illustrated by computational modeling on the example of a model problem. A comparative analysis of the possibilities of changing dynamic properties when configuring the working bodies of vibration stands allows using the possibility of adjusting the vibration field within a fairly wide range, which implies the possibility of working not only with a coefficient of connectivity of vibration amplitudes equal to one, over the entire range of the operating mode of the vibration stand, but also in local zones of excitation.
frequencies, when the coefficient of connectivity of vibration amplitudes fluctuates within a small range relative to one.

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