Estimation of arrival time difference of partial discharge ultrasonic signals based on nonlinear transformation and covariant

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Abstract—Ultrasonic signals will be generated when high voltage equipment is partially discharged. In order to determine the location of partial discharge, the time difference between ultrasonic signal and sensor can be used. In this paper, Alpha stable distribution is used to model noisy ultrasonic signals collected during actual partial discharge, and a nonlinear transformation-covariant time difference estimation method is proposed. Firstly, the signal-to-noise ratio can be improved to a certain extent by preprocessing the noisy signal based on the arctangent nonlinear transformation. Then, the covariant correlation function is used to measure the similarity of the two signals arriving at the sensor, and according to the position of the peak value of the covariant correlation function, the time difference information of signal arrival is obtained. On the basis of theoretical analysis, the experimental results of computer simulation are discussed and summarized. Compared with generalized correlation method and covariant algorithm, the proposed method is more suitable for TDOA estimation of PD ultrasonic signals in low SNR and impulsive noise environment.

1. INTRODUCTION
Partial discharge is the key factor causing insulation aging of high voltage equipment, and carrying out monitoring timely and effective is an important guarantee for safe and stable operation of power system. Ultrasonic signals will be generated during partial discharge, which can be used to locate the local discharge source. As a live detection and location method with strong anti-electromagnetic interference, non-invasive and low cost, this method has attracted wide attention\(^{[1,2]}\).

The accuracy of TDOA estimation is one of the key factors for the accuracy of final positioning among various spatial positioning algorithms based on TDOA of signal sources. The existing TDOA estimation methods used in PD location mainly include energy accumulation method, correlation analysis method, spectrum estimation method.

Noise is an important factor that affects the accuracy of TDOA estimation. PD ultrasonic signals collected on site usually have white noise, periodic narrowband noise and random impulse noise. The literature \(^{[3]}\) uses the improved Fast ICA algorithm to suppress the periodic narrowband interference in PD signals, and the literature \(^{[4]}\) proposes a PD noise reduction method based on variational modal decomposition, which can effectively suppress white noise and preserve the original waveform.
completely. Literature\(^{[5]}\) proposes a hierarchical noise reduction method based on multiple filtering and chaos control to suppress white noise and narrowband interference at the same time. Random impulse interference is the most difficult to filter because it is similar to PD signal in time domain and frequency domain, and pulse polarity method is used most frequently\(^{[5,6]}\). Although the principle of this method is simple, it needs to add external circuit and is vulnerable to electromagnetic interference, so the actual effect is not ideal.

Stable distribution theory provides a new model and theoretical tool for TDOA estimation, and α stable noise can better describe impulse noise in substation\(^{[7]}\). The research shows that the traditional TDOA estimation methods based on second-order statistics, such as correlation method, are seriously degraded or even ineffective under this noise model. Covariant algorithm has the advantages of less computation and real-time, which can estimate the time difference in impulse noise environment, but its anti-noise performance is poor. Literature\(^{[8]}\) preprocesses the sampled signal with odd symmetric monotonically increasing bounded function, which reduces the covariant variance to a finite value and improves the stability of the algorithm; Literature\(^{[9]}\) converts the time difference information of sampled signals into its self-covariant and mutual-covariant functions, and calculates the estimated time difference, which improves the anti-noise ability of the algorithm.

Based on this, this paper proposes a covariant correlation TDOA estimation algorithm based on nonlinear transformation of preprocessing function to estimate the TDOA of PD ultrasonic signals in random impulse noise environment. Finally, the time difference of two signals is calculated by the method proposed in this paper, generalized correlation method and covariant algorithm. The results show that the covariant correlation algorithm after nonlinear transformation has better stability and stronger anti-noise ability.

2. STABLE DISTRIBUTION THEORY

α stable noise is a kind of non-Gaussian noise containing spikes, and its characteristic function expression is as follows\(^{[10,11]}\):

\[
\varphi(t) = \exp \left\{ j \alpha t - \gamma \left\| j \beta \text{sign}(t) \omega(t, \alpha) \right\| \right\}
\]  

In the formula

\[
\omega(t, \alpha) = \begin{cases} 
\frac{\tan \alpha \pi}{2} & \text{if } \beta = 0 \\
\frac{2}{\pi} \log |t| & \text{if } \beta > 0 \\
1 & \text{if } \beta = 0 \\
0 & \text{if } \beta < 0
\end{cases}
\]

\[
\text{sign}(t) = \begin{cases} 
1 & t > 0 \\
0 & t = 0 \\
-1 & t < 0
\end{cases}
\]

It can be seen from the above formula that the stable distribution noise is determined by four parameters: \(a, \alpha, \beta\) and \(\gamma\), and \(a\) is the position parameter; \(\beta\) is a symmetry factor, and when \(\beta = 0\), it is symmetric α stable distribution. \(\gamma\) is the scale factor, which is similar to the variance in Gaussian distribution. \(\alpha\) is the most important characteristic index, and \(\alpha \in (0, 2]\). The smaller the value, the stronger the pulse degree. When \(\alpha = 2\), it is Gaussian distribution, so α stable distribution is a generalization of Gaussian distribution.

3. COVARIANT CORRELATION ALGORITHM BASED ON NONLINEAR TRANSFORMATION

3.1. Correlation TDOA estimation theory

Assume that \(y_1(n)\) and \(y_2(n)\) are signals received by two independent sensors placed at different positions on the electrical equipment:
\[
\begin{align*}
    y_1(n) &= x(n) + s_1(n) \\
    y_2(n) &= x(n-D) + s_2(n)
\end{align*}
\] (4)

In the formula, \(x(n)\) is the actual PD ultrasonic signal, \(x(n-D)\) is the time delay signal of \(x(n)\), \(D\) is the time delay, and \(s_1(n)\) and \(s_2(n)\) are the noise received by the sensor.

Correlation method is one of the most classical time difference estimation algorithms. Its principle is to compare the similarity between two signals by cross-correlation operation, and estimate the time difference of signals by using the peak position of correlation function:\[10,12\]

\[
R_{12}(\tau) = R_{xx}(m-D) + R_{nn}(m-D) + R_{ss}(\tau) + R_{s2s}(\tau)
\] (6)

Assuming that noise and signal, noise and noise are independent of each other, according to the properties of correlation function, \(R_{xx}(m-D) = R_{nn}(\tau) = R_{ss}(\tau) = 0\), The formula (6) can be simplified as:

\[
R_{12}(m) = R_{xx}(m-D)
\] (7)

\[
\hat{D} = \arg(\max_m [R_{xx}(m-D)])
\] (8)

In this formula, \(\arg(\cdot)\) is the independent variable of the function, \(R_{12}(\cdot)\) is the cross correlation, \(R_{xx}(\cdot)\) is the autocorrelation, and \(m\) is the estimation of time difference \(D\).

In practical application, the theoretical value of the correlation function can not be accurately obtained, and the estimated value of the correlation function is generally used instead:

\[
\begin{align*}
    \hat{R}_{11}(m) &= \frac{1}{N-m} \sum_{n=0}^{N-1-m} y_1(n) y_1(n+m) \\
    \hat{R}_{12}(m) &= \frac{1}{N-m} \sum_{n=0}^{N-1-m} y_1(n) y_2(n+m)
\end{align*}
\] (9)

In the formula, \(N\) is the number of signal sampling points.

3.2. Covariant TDOA estimation theory

In \(\alpha\)-stable noise environment, when \(\alpha<2\), there is no second-order statistics, and the variance of cross-correlation function tends to infinity, so the performance of correlation method deteriorates significantly or even fails\[8\]. Covariance in \(\alpha\)-stable noise environment is similar to covariance in Gaussian noise environment, and covariance of signals \(y_1(n)\) and \(y_2(n)\) can be expressed as:

\[
R_{\epsilon}(m) = E[y_1(n)][y_2(n+m)]^{(p-1)}
\] (10)

The estimated value of covariation is:

\[
\hat{R}_{\epsilon}(m) = \frac{1}{N} \sum_{n=1}^{N} y_1(n) y_2(n+m)^{(p-1)}
\] (11)

The abscissa corresponding to the peak value of \(R_{\epsilon}(m)\) is the time difference estimation:

\[
\hat{D} = \arg(\max_m [R_{\epsilon}(m)])
\] (12)

It should be pointed out that the covariant algorithm is only applicable to the pulse environment with \(1 < \alpha \leq 2\), and \(p\) is generally 1.1~1.2.
3.3. **covariant correlation TDOA estimation theory**

Assuming that noise and signal are independent of each other, the autocovariance \( R_{c11}(m) \) of signal \( y_1(n) \) and the interconversion \( R_{c12}(m) \) of signals \( y_1(n) \) and \( y_2(n) \) can be expressed as \([10]\):

\[
R_{c11}(m) = E\left\{ y_1(n) \left[ y_1(n + m) \right]^{(p-1)} \right\} = E\left\{ x(n) \left[ x(n + m) \right]^{(p-1)} \right\} + E\left\{ s_1(n) \left[ s_1(n + m) \right]^{(p-1)} \right\} = R_{xx}(m) + C_c \delta(m)
\]

\[
R_{c12}(m) = E\left\{ y_1(n) \left[ y_2(n + m) \right]^{(p-1)} \right\} = E\left\{ x(n) \left[ x(n + m - D) \right]^{(p-1)} \right\} + E\left\{ s_1(n) \left[ s_2(n + m) \right]^{(p-1)} \right\} = R_{xx}(m - D)
\]

In the formula, \( C_c \delta(m) \) is the value of noise \( s_1(n) \) after self-covariant. From the above formula, it can be seen that the time difference information has been transferred to the covariant function, and then the correlation operation is carried out on \( R_{c11}(m) \) and \( R_{c12}(m) \), and the estimated time difference of the signal can be obtained indirectly by finding the peak position.

3.4. **Principle of nonlinear transformation of preprocessing function**

Literature \([8]\) proves that the variance of covariant algorithm can be reduced to a finite value after the signal is processed by the preprocessing function \( \theta(x) \), and because the signal before and after transformation is one-to-one mapping, the time difference information of the signal can be completely preserved. In which \( \theta(x) \) shall satisfy:

1. \( \theta(-x) = -\theta(x), \ x \in (-\infty, +\infty) \)
2. \( |\theta(x)| \leq C, \ C \) s a constant
3. \( (d\theta(x)/x) > 0 \)

Three preprocessing functions are proposed in paper\([8]\). In this paper, the arc tangent function is used to transform PD ultrasonic signals nonlinearly. Figure 1 is the graph of this function:

![Figure 1. Arctangent function graph](image-url)
3.5. A covariant correlation TDOA estimation algorithm based on nonlinear transformation

In this paper, arctan(x) is chosen as the preprocessing function, and the processed signals \( y_1(n) \) and \( y_2(n) \) are transformed into:

\[
\begin{align*}
Y_1(n) &= \text{arctan}(y_1(n)) \\
Y_2(n) &= \text{arctan}(y_2(n))
\end{align*}
\]  

(15)

In the formula, \( Y_1(n) \) and \( Y_2(n) \) are transformed signals.

The estimated time difference can be obtained by covariant correlation calculation of \( Y_1(n) \) and \( Y_2(n) \) and peak detection. Fig. 2 is a flowchart of the algorithm:

![Flow chart of covariant correlation based on nonlinear transformation](image)

Figure 2. Flow chart of covariant correlation based on nonlinear transformation

4. ALGORITHM SIMULATION ANALYSIS

Partial discharge simulation signals are usually represented by single exponential decay pulse, single exponential decay oscillation pulse model and double exponential decay oscillation pulse model\(^{[13]}\). In this paper, double exponential decay oscillation pulse model is adopted:

\[
x(t) = A \left( e^{-\frac{t}{\tau}} - e^{-\frac{t}{\tau}} \right) \sin(2\pi f t)
\]  

(16)

In the formula, \( A \) is the amplitude of partial discharge signal, \( \tau \) is attenuation constant, and \( f_c \) is oscillation constant. \( A=30\text{mv}, f_c=500\text{kHz}, \tau=2.5\mu\text{s}, \text{sampling number } N=2000, \text{sampling frequency } 10\text{M}.\)

Adopting the two-way signal model in formulas (4) and (5), it is assumed that the installation positions of ultrasonic sensors are point A and point B, with \( D=50 \), and \( s_1(n) \) and \( s_2(n) \) are stably distributed noises which obey \( \alpha=1.8 \) and are independent of each other. The mixed signal-to-noise ratio MSNR=\(10\log(\sigma_s^2/\gamma)\)dB, \( \sigma_s^2 \) is the variance of the signal, \( \gamma \) is the dispersion coefficient of noise. With MSNR=10dB, the time domain waveform of the analog signal is shown in figure 3:
Firstly, the signal is nonlinear transformed by using the arctangent function, and the transformed waveform is shown in Figure 4:

![Figure 3: PD analog ultrasonic signal waveform diagram](image)

Then, the time difference of the transformed signal is estimated by covariant correlation method, and the estimation result $\mu_D = 50$, same as the set time difference $D = 50$, the peak detection diagram is shown in Figure 5:

![Figure 4: Waveform diagram of PD analog ultrasonic signal after transformation](image)

In order to verify the effectiveness and superiority of the proposed algorithm (method 1) in PD ultrasonic signal TDOA estimation in random impulse noise environment, experiments were carried out in three different environments, namely, the algorithm in this paper, the PATH weighted...
generalized correlation method (method 2) and covariant algorithm (method 3), and 100 Monte Carlo experiments were carried out in each environment. In the covariant algorithm, \( p=1.2 \), and the root mean square error was calculated, and the signals and values were the same as the previous ones. The expression of RMSE is:

\[
RMSE = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}}
\] (17)

In the formula, \( x_i \) is the observed value, and in this paper, it is the estimated time difference \( \mu_D \), \( \bar{x} \) is the true value, and in this paper, it is the true time difference \( D \), \( n \) is the number of observations, and in this paper, it is the number of Monte Carlo experiments.

Experiment 1 under \( \alpha = 2 \) (Gaussian noise) environment, the mixed signal-to-noise ratio MSNR is taken as 15dB, 10dB, 5dB and 0dB respectively, and Monte Carlo experiments are carried out for 100 times to compare the root mean square errors of the three algorithms, as shown in Table 1:

| Method   | 15dB  | 10dB  | 5dB  | 0dB  |
|----------|-------|-------|------|------|
| Method 1 | 0     | 0     | 0.53 | 1.83 |
| Method 2 | 0     | 0     | 0.98 | 2.68 |
| Method 3 | 1.87  | 2.637 | 3.77 | 4.33 |

It can be seen from Table 1 that the estimation errors of the generalized correlation method and the algorithm in this paper are small in Gaussian noise environment, while the algorithm proposed in this paper is slightly better, and the error of covariant algorithm in Gaussian noise environment is higher than that of the first two methods.

Experiment 2 \( \alpha = 1.8 \) under stable noise environment, the mixed signal-to-noise ratio MSNR is taken as 15dB, 10dB, 5dB and 0dB for 100 Monte Carlo experiments, and the root mean square errors of the three algorithms are compared as shown in Table 2:

| Method   | 15dB  | 10dB  | 5dB  | 0dB  |
|----------|-------|-------|------|------|
| Method 1 | 0     | 0     | 0.15 | 2.19 |
| Method 2 | 2.12  | 2.73  | 32.51| 139.73|
| Method 3 | 0.07  | 0.11  | 2.60 | 4.03 |

It can be seen from Table 2 that in random impulse noise environment, the error of generalized correlation algorithm increases sharply when the mixed signal-to-noise ratio MSNR=5dB, and the algorithm proposed in this paper is superior to covariant algorithm.

Experiment 3: the mixed signal-to-noise ratio MSNR=10dB, \( \alpha = 1.8, 1.6, 1.4, 1.2 \), respectively, were tested in Monte Carlo for 100 times, and the mean square error of the three algorithms was compared as shown in table 3:

| Method   | \( \alpha = 1.8 \) | \( \alpha = 1.6 \) | \( \alpha = 1.4 \) | \( \alpha = 1.2 \) |
|----------|-------------------|-------------------|-------------------|-------------------|
| Method 1 | 0                 | 0.75              | 0.72              | 0.82              |
| Method 2 | 2.73              | 22.09             | 161.04            | 654.17            |
| Method 3 | 0.11              | 0.96              | 2.48              | 12.69             |

It can be seen from Table 3 that with the increasing impulse of noise, the generalized cross-correlation algorithm basically fails when \( \alpha = 1.4 \), and the algorithm proposed in this paper is still superior to the covariant algorithm.
The existence of root mean square error shows that there are still a few estimation errors in Monte Carlo experiment, but it can be estimated correctly in the vast majority of time. Generally, histogram is used to count the time difference estimated many times at the end, and the estimated value with the most times is taken as the final estimated time difference \[14\].

5. CONCLUSIONS
In this paper, $\alpha$ stable noise is used to simulate the random impulse interference in the surrounding environment during partial discharge of power equipment, and a covariant correlation TDOA estimation algorithm based on nonlinear transformation of preprocessing function is proposed. The algorithm can accurately estimate the TDOA when MSNR>$5\text{dB}$, which is less affected by impulse intensity, and is also suitable for Gaussian noise environment. Compared with generalized correlation method and covariant algorithm, the stability and accuracy of this algorithm for TDOA estimation of partial discharge ultrasonic signals under random pulse interference and low signal-to-noise ratio are verified in computer simulation environment, and its performance is superior to the above two algorithms, which is beneficial to the subsequent positioning work.

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