Thermodynamics of the QCD transition from lattice

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Abstract

We discuss recent developments in lattice QCD for the bulk thermodynamics of the transition. We review the current status of the equation of state, the possible implications of a magnetic field and the fluctuations of conserved charges, like the net electric charge or baryon number. We also show predictions for higher cumulants, which will be experimentally available in near future.

1. Introduction

Lattice simulations aiming to describe the features of quark gluon plasma and its transition to confined matter have become a field of high precision in the past years. An important goal of the efforts on the lattice is to provide theoretical background for collision experiments. The relevance of lattice results are most prominent for the heavy ion physics pursued at RHIC and LHC, where matter with high energy density, but moderate or small chemical potentials are produced.

Lattice simulations solve Quantum Chromodynamics (QCD) in equilibrium. The solution emerges from a continuum limit, which is an extrapolation from a set of simulations on lattices with smaller and smaller lattice spacings ($a$). Finite temperature ($T$) is naturally implemented by a torus in Euclidean time. In most thermodynamics studies the lattice spacing is expressed through the number of lattice points in the Euclidean time direction: $N_t$, which translates to $a = 1/(N_t T)$. In most early papers $N_t$ was set to 4 or 6. By today $N_t = 16$ data sets are not uncommon, and are believed to be necessary for a controlled continuum extrapolation even with improved actions.

Longer correlations (inverse pion mass) require larger computer resources to simulate the theory. In the past decade lattice field theory has seen a great development in gradually reducing the quark masses down to their physical values. Simultaneously to this trend our increasing understanding of various discretization effects has lead to more efficient discretization schemes (actions) that allow a continuum extrapolation at an affordable cost. In view of these developments here we only discuss results with physical (or nearly physical) quark masses close to or in the continuum limit.

2. Transition temperature

Evidence form lattice QCD shows that at zero chemical potential the transition from the chirally broken, confining phase of hadrons to the quark gluon plasma phase is a crossover.
The transition temperature of a crossover transition depends on the observable one chooses. At vanishing light quark masses the chiral condensate is a genuine order parameter, which can be used to extract a chiral transition temperature. At sufficiently high quark masses the deconfinement transition is identified by a discontinuity in the Polyakov loop as a function of temperature. Varying the quark mass in the space of theories the relevance of chiral and deconfinement aspects can be tuned. With sufficiently light $u$ and $d$ quarks an O(4) scaling of the chiral observables is conjectured [5][6]. This universal mass dependence has been used to relate the transition temperature ($T_c$) in QCD to $T_c$ for the actual simulation parameters [7].

The Wuppertal-Budapest collaboration has simulated the QCD transition with physical quark masses in the stout staggered formulation [8]. The lattice resolutions range from $N_f = 6$ to $N_f = 16$. From the inflection point of the chiral condensate we find $T_c = 155(3)(3)$ MeV, where the first of the errors is statistical, the second is systematic, including the uncertainties of the scale setting [9][10]. Other $T_c$ definitions from chiral observables span a range of $147 - 157$ MeV. Recently the HotQCD collaboration has published a transition temperature, $154(8)(1)$ MeV, based on the O(4) scaling [7], which is very well compatible with the range given by the Wuppertal-Budapest collaboration in 2006. This is in contrast to the earlier continuum result $(192(4)(7)$ MeV) by the RBC-Bielefeld group. A key improvement since then was the adoption of the HISQ action [3].

The quoted values of $T_c$ are all based on lattice observables that are remnants of the chiral order parameter. One can also study other observables, such as the Polyakov loop, which is the remnant order parameter of the deconfinement transition. The Polyakov loop as well as the inflection point is renormalization scheme dependent. E.g. the scheme used in Ref. [9] was different than in our latest work [10]. We made this change in our convention to be compatible with Ref. [11] in every detail. While the comparison of our result to the aQCD data has showed a disagreement, the new HISQ data of the HotQCD collaboration was in agreement with ours [7].

There are other observables that indicate deconfinement. The normalized energy density $\epsilon/T^4$ has an inflection point at $157(4)(3)$ MeV, for the trace anomaly we have $154(4)(3)$ MeV. The speed of sound has a minimum at around ~ $145(5)$ MeV. These temperatures cover roughly the same range as we find from chiral observables. From the strange susceptibility, which characterizes the deconfinement of the strange degree of freedom we have a somewhat higher inflection point. Of course, in a broad crossover the definition of the most singular point is also ambiguous.

The concept of an identical chiral and deconfinement transition temperature was promoted by

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Figure 1: **Left:** The renormalized chiral condensate. We used the inflection point to define a transition temperature: $155(3)(3)$ MeV. **Right:** the renormalized Polyakov loop. Although one can precisely measure its temperature dependence, one cannot easily pinpoint an inflection point. The shape of the Polyakov loop curve, as well as any derived transition temperature depends on the details of the renormalization scheme [9][10].

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3. Equation of state

The pressure and energy density of the quark gluon plasma is one of the key targets of lattice QCD simulations. These form the equation of state, which is required to close the equations of relativistic hydrodynamics [18].

Early works on the QCD equation of state were dominated by quenched results [19, 20]. These were recently extended in range and precision, so that a connection to perturbation theory could be made [21]. The calculations were also extended towards larger gauge groups [22]. For several years we have seen various exploratory studies with non-physical or not renormalized quark masses. Since Ref. [8] physical simulations have been feasible. In staggered simulations discretization effects raise the mass of the hadrons, and through this $T_c$ appears to be higher, and the energy and entropy density function is found steeper than physical, and the trace anomaly shows a higher peak. This is an artefact ("taste-breaking") which is absent after continuum extrapolation, its impact on coarse lattices and older actions (like asqtad) was emphasized in Refs. [9, 10, 23]. The Wuppertal-Budapest collaboration exploited the favourable taste-breaking features of the stout action and calculated a continuum estimate of the equation of state [15] based on $N_t = 6, 8$ and 10 runs. HotQCD used the asqtad and p4 actions at $N_t = 6$ and 8 [11, 24], though these were later found to be incompatible with results with the better HISQ action [17]. For a comprehensive review see Ref. [25].

At this conference, we heard two contributions discussing the equation of state. The Wuppertal-Budapest collaboration has updated its continuum estimate of 2010 [15] by adding a finer dataset ($N_t = 12$) and an even finer check point ($N_t = 16$) and using these in a continuum extrapolation.
The entropy density as a function of temperature. Left: We show the lattice data by the Wuppertal-Budapest collaboration ($N_t = 6, 8$ and $10$) together with the parametrization (solid red line) of these data sets published in Ref. [15]. The black dots show the result of a recent continuum extrapolation based on increased statistics and finer lattices ($N_t = 12$). We see almost no deviation from the earlier parametrization. Since there is no HISQ data for the entropy density yet, we plotted HotQCD’s asqtad data set at $N_t = 8$ for comparison (magenta triangles) [11]. As discussed already (Fig. 2), this set features a higher-than-physical transition temperature and thus it shows steeper equation of state and higher peak in the trace anomaly. As remedy, the $g_5p$ parametrization was introduced [23] by replacing lattice data in and below the transition range by the Hadron Resonance Gas prediction. By this, the slope of the entropy curve was left unchanged, but the whole curve was shifted up towards the Stefan-Boltzmann limit. Right: The contribution of the charm sea quark to the entropy density. The Wuppertal-Budapest collaboration’s data with $N_t = 6, 8$ and $10$ is shown for dynamical $2+1+1$ flavor simulations together with its charmless continuum data and the respective perturbative estimates [16].

4. Phase diagram

The transition line in the $T - \mu_B$ phase diagram at small chemical potentials is necessarily a line of crossover, with different width and curvature for various observables. Direct Monte-Carlo simulations of the four-dimensional quantum field theory at finite baryo-chemical potential are at present not possible. Nevertheless, starting with Ref. [29] a renewed interest has been seen for $\mu_B > 0$ questions in lattice QCD, and several indirect strategies exist [30, 31].

The curvature of the transition line in full QCD has been found very small by recent lattice simulations. In Ref. [6] simulations on $N_t = 8$ lattices were used to map the chiral condensate and its $\mu_B^2$ dependence onto a universal O(4) scaling behaviour as a function of the quark mass. Evaluating the fits in the physical point gave $\kappa = 0.0066(2)(4)$, with $\kappa = -\langle T_c \rangle dT_c / d\mu_B^2 \bigg|_{\mu_B=0}$. The continuum result has been calculated by the Wuppertal-Budapest collaboration [25], with simulations explicitly in the physical point. Their result was $\kappa = 0.0066(20)$. In this work, a similar analysis have been made for the strange susceptibility, there the curvature was found to be $0.0089(14)$. In addition, the width of the transition has also been extrapolated to $\mu_B > 0$. This showed that the transition is not getting stronger with increasing chemical potential, but rather it stays a crossover (see Fig. 4)
This latter conclusion is in line with the work by Philipsen and de Forcrand claiming the absence of a critical endpoint, at least in the range where analytical continuation or Taylor expansion can work [32]. This result was often confronted with the critical point found by Fodor and Katz [33]. Note, however, that at small chemical potentials even Ref. [33] finds a slightly weakening transition, this trend then reverses at higher $\mu_B$, where the available Taylor expansion from imaginary $\mu_B$ no longer works. As the authors also emphasize, these results [32, 33] came from coarse lattices. A continuum limit has not yet been feasible (because of the high computational costs), which leaves us without any conclusive lattice evidence for a critical endpoint or a first order line at high $\mu_B$.

Having no singularity in a large range for chemical potentials, the equation of state, too, can be extrapolated to $\mu_B > 0$. By integrating the trace anomaly we have $\log Z$ at the $\mu_B = 0$. The baryon number susceptibility $\sim \partial^2 \log Z/ (\partial \mu_B)^2 |_{\mu_B=0}$ is also accessible from lattice, thus, a linear expansion is possible. The continuum result was given in Ref. [28] (see Fig. 4).

We finally discuss the effect of a magnetic field. The strong magnetic fields in non-central heavy ion collisions [36] motivated the study of the QCD transition in the presence of an electromagnetic background. A $B$ field is easily implemented on the lattice [37, 38]. Based on models (e.g. PNJL [39]) and exploratory lattice simulations, [37] the expectation was to see an increase in $T_c$ with growing magnetic field. As a joint effort of the Budapest, Regensburg and Wuppertal lattice groups $T_c$ was calculated with physical quark masses in the continuum limit. $T_c$ was shown to decrease with $B$ (see Fig. 5/ left), while the transition was still a crossover [34]. To show the role played by the quark mass, an auxiliary data set was taken with higher hadron masses. Then the trend was reversed, and the earlier (apparently contradicting) lattice result [37] was reproduced. To explain the behaviour seen in the PNJL model the same group calculated the chiral condensate at $B > 0$ and $T > 0$, and highlighted the differences between lattice data and the model’s assumptions [35] (see Fig. 5/ right).
5. Fluctuations of conserved charges

Correlations and fluctuations of conserved charges have been proposed long ago to signal the transition \[40, 41\]. At high temperatures fluctuations are expected to approach the ideal gas limit. On the other hand, in the low-temperature phase they are expected to be small since quarks are confined and the only states with non-zero quark number have large masses. Agreement with the Hadron Resonance Gas (HRG) model predictions is expected in this phase.

In the past year both the Wuppertal-Budapest \[42\] and HotQCD \[43\] collaborations published results for the second order fluctuations, i.e. the second derivatives of the free energy with respect to chemical potentials of various conserved charges, like baryon number, electric charge or strangeness.

The lattice results are always normalized to volume and the respective power of temperature. E.g. the diagonal charge and the off-diagonal charge-baryon correlators are defined by

\[
\chi^Q_2 = \frac{1}{VT^3} \frac{\partial^2 \log Z}{\partial \mu_Q/T^2}, \quad \chi^{BQ}_{11} = \frac{1}{VT^3} \frac{\partial^2 \log Z}{\partial \mu_Q/T \partial \mu_B/T}.
\]

The experimentally more interesting observables are ratios of these derivatives, where the unknown volume factor cancels. To describe the freeze-out parameters (\(T\) and \(\mu_B\)) specific ratios have been introduced and calculated (e.g. skewness / mean of the net charge yield) using the coarser data sets of the HISQ action (\(N_t = 6, 8\)) in Ref. \[43\].

The charge kurtosis \(\times\) variance could also be used as a thermometer, and was in the focus of C. Schmidt’s talk \[45\]. In Fig. \[7\] we plot the temperature dependence of \(\chi^Q_2/\chi^Q_2\) at zero chemical potential from both the BNL-Bielefeld group and the Wuppertal-Budapest collaboration. While we witness steady progress in the analysis of experimental data towards the precise measurement of second and higher order fluctuations \[46, 47, 48\], lattice methods are increasingly successful in higher derivatives, which will finally enable the extrapolation of the kurtosis data towards finite chemical potential.

6. Summary

We discussed a selection of the recent developments in bulk thermodynamics, focusing on those results with physical quark masses close to or in the continuum limit. For technical reasons,
all these results were obtained using the staggered formalism. The independence of the results with respect to the applied discretization scheme is an important consistency requirement of lattice QCD, and to this end there have been recent checks with overlap [49], domain-wall [50] and Wilson [51] fermions, with the compromise of using heavy pions in these comparisons.

By today we know several aspects of the QCD transition through Euclidean-time observables at zero and small chemical potentials. Now the challenge for lattice QCD is to develop new techniques for extracting real-time physics (e.g. transport coefficients) and to increase the accessible range in $\mu_B$, and eventually, to map the phase diagram.

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Figure 7: Left: The normalized electric charge kurtosis from lattice QCD: the BNL-Bielefeld collaboration has used the HISQ action on $N_t = 6$ and 8 lattices [45], the Wuppertal-Budapest collaboration shows results at $N_t = 6, 8, 10, 12$ and 16. From the plot we see that the use of fine lattice is essential for a controlled continuum limit. At present data indicates deviations from the Hadron Resonance Gas model’s prediction even below $T_c$. Right: At high temperatures the Wuppertal-Budapest collaboration has attempted a preliminary continuum extrapolation for $\chi_4$ for electric charge, strangeness and baryon number. The difference we see between light and strange quark’s behaviour indicates that the mass of the strange quark is still noticeable up to about 260 MeV temperature.

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