Wave phenomena and band-gap formation in layered phononic crystals with functionally graded interlayers and periodic arrays of cracks

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Abstract. Time-harmonic wave motion in a layered phononic crystals with internal inhomogeneities such as piezoelectric functionally graded interlayers and periodic arrays of cracks is considered. The complete band-gaps in layered piezoelectric functionally graded phononic crystal are revealed and analyzed. It is demonstrated that introduction of periodic cracks leads to the formation of extra band-gaps. Effect of formation of narrow band-gap exhibiting resonance nature is demonstrated within pass-band.

1. Introduction
Since 1990s considerable efforts have been made in the field of acoustic/elastic metamaterials and phononic crystals (PnCs) [1, 2]. PnCs can be used for elastic wave energy manipulation due to their unique properties like the effect of the signal absorption, negative refraction, wave resonances, wave localization, focusing etc. Accounting for these wave phenomena allows designing novel ultrasonic devices operating in broad frequency ranges, which are based on guided, surface or bulk waves.

Introduction of imperfections or cracks into composite structure provide an extra reflector that may lead to additional band-gaps or other unique physical phenomena, e.g. [3, 4]. Thus, if the periodicity of a layered PnC is disturbed and the cracks act as disorders, wave localization and resonance phenomena arise for some combinations of geometrical or material parameters [5]. At the same time, crack-like inhomogeneity is stress concentrator, which existence may cause failure of the structure. The problem of scattering of elastic waves in periodic composites by interlayers, made from functionally graded (FG) materials, and periodic arrays of crack-like inhomogeneities is considered in the present paper.

2. Configuration of phononic crystal
Plane wave propagation in a layered phononic crystal composed of $N$ periodically situated unit-cells of thickness $H$ between two identical half-spaces is considered. The unit-cell is a stack of layers infinite along axes $Ox_1$ and $Ox_2$. Two kinds of unit-cells of PnC shown in Figure 1 are studied: unit-cells with two FG interlayers and unit-cells with periodic array of cracks.
Two homogeneous areas are assumed of piezoelectric material PVDF and silver, while two FG layers have the same thickness \( H_{FG}/2 \), where material properties vary in accordance with power law given in [6]. The angles \( \theta_1 \) and \( \theta_2 \) denote azimuthal and radial angles of the incoming plane wave correspondingly. The second kind of PnC is a periodic set of unit-cells incorporating periodic arrays of interface cracks of widths \( 2l \), where cuts are situated at the distance \( s \) from each other. The material properties are assumed of lead and epoxy.

3. Layered phononic crystals with functionally graded interlayers

The solution of the problem for PnC with FG interlayers is based on the method of eigenvalue decomposition [7]. The mechanical displacements \( u_i \) and electrical potential \( \varphi \) are written as follows:

\[
\begin{align*}
    u_i &= \sum_{n=1}^{4} u_{in} \lambda_n^{-N}, \\
    \varphi &= \sum_{n=1}^{4} \varphi_n \lambda_n^{-N},
\end{align*}
\]

where \( \lambda_n \) are the eigenvalues of the unit-cell transfer matrix such as \( |\lambda_n| \geq 1 \). The semi-analytical formulae for the coefficients \( u_{in} \) and \( \varphi_n \) have been derived, the latter provides a numerically stable algorithm for wave-field calculation.

Considering a large amount of pairs of piezoelectric materials with FG interlayer, the authors...
have found that unit-cells made of PVDF (material properties are given in [8]) with a very thin silver layer (aspect ratio between thicknesses 19 : 1) can be used in order to produce complete band-gaps providing full reflection for all angles of incidence. Moreover, the FG interlayers can be used to manipulate width of the band-gaps.

The influence of FG interlayers on wave transmission through the phononic crystal is shown in Fig. 2 for longitudinal (P), vertically polarized shear (SV) and horizontally polarized shear (SH) incident waves for normally and obliquely incident plane elastic waves. Here $k_0$ is the wavenumber of the transverse waves in the half-space. The coloured zones in the Figs. 2–3 factually correspond to band-gaps. The red solid lines are contour lines of the domains, where all eigenvalues $|\lambda_n| > 1$ in Eq. (1). In other words, these domains are classical band-gaps (or band-gaps of first kind in accordance with extended classification given in [7] for elastic anisotropic PhC). Other colored zones are the band-gaps of second kind or low-transmission pass-bands. For SV- and SH- incident waves the band-gaps of first kind are identical (see 2nd and 3rd column of Fig. 2). The band-gaps become wider and shift to high frequency side with increase of FGM portion.

The full (complete) band-gaps can be observed in the considered piezoelectric PhC. These zones are indicated by shaded domains in Fig. 3 for dependencies of energy transmission coefficient on incident angles of P-waves.

4. Layered phononic crystals with multiple periodic arrays of cracks
Let us consider the propagation of plane elastic waves in a layered PhC with multiple periodic arrays of strip-like cracks as shown in Fig. 1. The total wave-field in such PhC is a sum of the incident field computed using the method described in the previous section and the scattered field, which is a superposition of the wave-fields scattered by all the arrays of cracks. The total wave-field satisfies the traction-free boundary conditions on all the crack-faces, this boundary condition leads to the unknown displacement jumps. The latter is determined using the boundary integral equation method and the integral approach [9]. Owing to the periodicity of the crack spacing $s$, only the region surrounding a reference crack in the centre of the Cartesian coordinate system needs to be considered. The crack density parameter $C = l/s$ is introduced to characterise periodic arrays of cracks.

Fig. 4 demonstrates the effect of crack introduction into the layered periodic structure on wave transmission. The figure depicts frequency dependence of the energy reflection coefficient $\kappa^-$ and band-gaps marked by different colours for $l = 1$ ($C = 0.5$) and $l = 1.8$ ($C = 0.9$). First of all, it should be mentioned that presence of the cracks leads to the occurrence of extra band-gaps compared with PhC without cracks, the width of band-gap is wider the greater crack density $C$. The possibility of formation of narrow band-gaps for greater cracks can be observed in Fig. 4. These narrow band-gaps are very sensitive to the crack size $l$ and exhibit resonance nature.
5. Conclusion
The aim of this study is to investigate wave phenomena in layered PnCs and to demonstrate possible approach of an arrangement of unit-cells incorporating FG interlayers in layered PnCs for the formation and widening band-gaps. Presence of periodic arrays of cracks leads to the extra band-gaps, some of them are related to resonance wave transmission through PnC.

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