OSp supergroup manifolds, superparticles and supertwistors

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Abstract

We construct simple twistor–like actions describing superparticles propagating on a coset superspace $OSp(1|4)/SO(1,3)$ (containing the $D = 4$ anti–de–Sitter space as a bosonic subspace), on a supergroup manifold $OSp(1|4)$ and, generically, on $OSp(1|2n)$. Making two different contractions of the superparticle model on the $OSp(1|4)$ supermanifold we get massless superparticles in Minkowski superspace without and with tensorial central charges.

Using a suitable parametrization of $OSp(1|2n)$ we obtain even $Sp(2n)$–valued Cartan forms which are quadratic in Grassmann coordinates of $OSp(1|2n)$. This result may simplify the structure of brane actions in super–AdS backgrounds. For instance, the twistor–like actions constructed with the use of the even $OSp(1|2n)$ Cartan forms as supervielbeins are quadratic in fermionic variables.

We also show that the free bosonic twistor particle action describes massless particles propagating in arbitrary space-times with a conformally flat metric, in particular, in Minkowski space and AdS space. Applications of these results to the theory of higher spin fields and to superbranes in AdS superbackgrounds are mentioned.
1 Introduction

Conformal (super)symmetry plays an important role in the theory of fundamental interactions based on field–theoretical models as well as on the theory of fundamental extended objects (strings etc.). Conformal geometrical structure allows one to replace space–time geometry by twistor geometry, where twistors are fundamental conformal spinors (\(SU(2, 2)\) spinors for \(D = 4\)) and space–time variables become twistor composites \([1]\). Such a construction allows for a supersymmetric extension \([2]\) where superspace variables are replaced by primary supertwistor coordinates (\(SU(2, 2|N)\) supertwistors in \(D = 4\) case).

In this paper we shall discuss twistors describing anti–de–Sitter (AdS) geometry. The isometry of the AdS (super)spaces acts on the (super)AdS boundary as the group of (super)conformal transformations, and, therefore, provides the group–theoretical basis for the AdS/CFT (conformal field theory) correspondence conjecture \([3]\) which attracted great deal of attention during the last two years (see \([4]\) for an exhaustive list of references). On the other hand the AdS space is the one where higher spin fields may nontrivially interact with each other \([5]\). In some aspects the technique developed for the description of the theory of higher spin fields in Minkowski \([6]\) and AdS spaces \([7]\) resembles the (super)twistor approach \([8]\).

In this respect it is tempting to look for possible links between these different manifestations of conformal symmetry, AdS spaces, twistors and their supersymmetric generalizations.

Motivated by the problem of finding a simple form of the action for a superstring propagating in the \(AdS_5 \times S^5\) superbackground \([9]\), in recent papers \([10]\) a massive bosonic twistor particle model in an \(AdS_5\) space has been proposed and its classical and quantum conformal symmetry properties have been considered.

In \([11, 12]\) an \(OSp(1|8)\) supertwistor model has been proposed for the description of a \(D = 4\) massless superparticle with the infinite spectrum of quantum states being described by fields of arbitrary integer and half integer spins. The spin degrees of freedom of the superparticle have been found to be associated with six tensorial central charges which can extend the \(N = 1, D = 4\) super–Poincare algebra. It is natural to assume that the superparticle with tensorial central charges and the particle on the AdS space are different truncations of the dynamics of a superparticle propagating in the supergroup manifold of the isometries of the corresponding AdS superspace \([12]\).

An aim of this paper is to construct such a model in the supergroup manifold \(OSp(1|4)\) and demonstrate how it is related to the \(D = 4\) twistor superparticle model of higher spins \([11, 12]\), and to a superparticle on the \(AdS_4\) superspace \(OSp(1|4)/SO(1, 3)\).

Another motivation of this study is to find a way of constructing simple worldvolume actions describing the dynamics of superbranes propagating in AdS superbackgrounds,
i.e. to make a progress in solving a vital AdS/CFT correspondence problem [13–19].

Using a suitable parametrization of $OSp(1|2n)$ (where $n$ is a natural number) we have found a simple form of the even $OSp(1|2n)$ Cartan forms. They are only quadratic in Grassmann coordinates. This has allowed us to construct simple actions quadratic in fermions for superparticles propagating on $OSp(1|4)/SO(1,3)$, $OSp(1|4)$ and, generically, on $OSp(1|2n)$.

The most interesting examples of the $OSp(1|2n)$ supergroups seem to be $OSp(1|32)$ and $OSp(1|64)$. In [21, 22, 23] it has been shown that $OSp(1|32)$ and $OSp(1|64)$ contain the supergroup structures of $D = 11$ M–theory and $D = 10$ superstrings. In particular, $OSp(1|32)$ and $OSp(1|64)$ are extensions of the supergroups $SU(2,2|4)$, $OSp(8|4)$ and $OSp(2,6|4)$ which are isometries of, respectively, $AdS_5 \times S^5$, $AdS_4 \times S^7$ and $AdS_7 \times S^5$ superspaces. Reducing $OSp(1|32)$ and $OSp(1|64)$ down to the AdS–supergroups one may hope to get simpler expressions for the Cartan forms of the latter, which might simplify the structure of actions for branes in corresponding AdS superbackgrounds [13–19].

The plan of the paper is the following.

In Section 2 we review properties of the twistor formulation of bosonic particle mechanics and demonstrate that the single twistor particle action generically describes particles propagating in arbitrary space–times which admit a conformally flat metric, such as flat Minkowski space and the AdS space.

In Subsection 3.1 we consider the supertwistor description of a massless superparticle in flat $N = 1$, $D = 4$ superspace, and in Subsection 3.2 we construct a twistor–like action for the description of the dynamics of a superparticle in the super–AdS space $OSp(1|4)/SO(1,3)$. The action has a simple quadratic form in fermions and, hence, it should not be hard to perform its quantization. However, in contrast to the bosonic $AdS_4$ superparticle we have not managed to find a complete supertwistor version of this model.

In Section 4 we construct a twistor–like action for a superparticle on the supergroup manifold $OSp(1|4)$. This action is also quadratic in fermions, and upon an appropriate truncation it reduces to the models of Section 3.

In Section 5 we describe a superparticle propagating on $OSp(1|2n)$ and show that it preserves $2n − 1$ supersymmetries associated with Grassmann generators of $OSp(1|2n)$.

The $OSp(1|4)$ superalgebra and its Cartan forms required for the construction of the actions are given in Appendix 1. In particular, we present a simple form of the super–

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4 An $OSp(1|64)$–invariant superparticle–like model has been discussed in [23] (see also [24] and references therein).

5 We should remark that the notation $OSp(6,2|4)$ is a somewhat confusing name for the $AdS_7$ quaternionic supergroup described, in a complex parametrization, by the intersection of two complex supergroups $SU(4,4|4)$ and $OSp(8|4;C)$, with bosonic sectors being, respectively, the spinorial covering of $O(6,2)$ (space–time) and the spinorial covering $Sp(2;H) = USp(4;C)$ of $O(5)$ (the internal sector).

6 The Cartan forms on supercosets of $SU(2,2|N)$ relevant to the construction of brane actions on AdS superbackgrounds were calculated in early 80-ths [20].
AdS$_4$ supervielbeins and spin connection which are polynomials of only the second down in Grassmann coordinates.

In Appendix 2 we present Cartan forms of the supergroup $OSp(1|2n)$ which can be made quadratic in Grassmann variables by an appropriate rescaling of the latter.

## 2 Twistor–like bosonic particles

We start by recalling the form of an action for massless $D = 4$ particles which serves as a dynamical basis for the transform from the space–time to the twistor description. The action is

$$S = \int d\tau \lambda^A (\sigma_m)_{A\dot{A}} \bar{\lambda}^\dot{A} \frac{d}{d\tau} x^m(\tau) = \int \lambda^{\sigma_m} \bar{\lambda} dx^m(\tau),$$

(1)

where $x^m(\tau)$ ($m = 0, 1, 2, 3$) is a particle trajectory in $D = 4$ Minkowski space, $\lambda^A(\tau)$ is a commuting two–component Weyl spinor and $\sigma^m_{A\dot{A}} = \bar{\sigma}^m_{A\dot{A}}$ are the Pauli matrices.

From (1) we derive that the canonical conjugate momentum of $x^m(\tau)$ is

$$p_m = \lambda_{\sigma_m} \bar{\lambda} \Rightarrow p_{A\dot{A}} = \frac{1}{2} p_m \sigma^m_{A\dot{A}} = \lambda_A \bar{\lambda}_{\dot{A}},$$

(2)

whose square is identically zero since $\lambda^A \lambda^A \epsilon_{AB} \equiv 0$, i.e.

$$p_m p^m = 0.$$

(3)

We therefore conclude that the particle is massless.

In the action (1) we can make the change of variables by introducing the second commuting spinor

$$\bar{\mu}^A = i \lambda^A (\sigma_m)_{AB} x^m \equiv i \lambda^A x_{A\dot{B}}$$

(4)

and its complex conjugate

$$\mu_A = -ix_{AB} \bar{\lambda}^\dot{B}.$$

(5)

The four–component spinors

$$Z_\alpha = (\lambda_A, \bar{\mu}^A), \quad \bar{Z}^\alpha = (\mu_A, \bar{\lambda}_{\dot{A}})$$

(6)

are called twistors.

In terms of (6) the action (1) takes the form

$$S = i \int d\tau [\dot{Z}^\alpha dZ_\alpha + l(\tau) \bar{Z}^\alpha Z_\alpha],$$

(7)

where we have added the second term with the Lagrange multiplier $l(\tau)$ which produces the constraint

$$\bar{Z}^\alpha Z_\alpha = 0.$$

(8)

The two–component spinor indices are raised and lowered by the unit antisymmetric matrices $\epsilon^{AB} = \epsilon_{AB}, \epsilon^{\dot{A}\dot{B}} = \epsilon_{\dot{A}\dot{B}}.$
This constraint implies that $\mu_A$ and $\bar{\mu}_A$ are determined by eqs. (4) and (5). But, as we shall see below, this flat space solution of the twistor constraint (8) is not the unique one. The $AdS_4$ space is also admissible, as well as any space with a conformally flat metric.

Passing from the action (1) to (7) we have performed the twistor transform, eqs. (2), (4) and (5) being the basic twistor relations [1]. The action (7) is invariant under the conformal $SU(2,2) \sim SO(2,4)$ transformations, since the twistors are in the fundamental representation of the conformal group. The choice of twistor variables demonstrates how conformal symmetry appears in the theory of free massless particles.

We can generalize the action (1) to describe a massless particle propagating in a curved (gravitational) background. For this purpose we introduce the vierbein one–form $e^a(x) = dx^m e^a_m(x)$ with the index $a = 0, 1, 2, 3$ corresponding to local $SO(1,3)$ transformations in the tangent space of the background. Eq. (1) takes the form

$$S = \int d\tau \lambda \sigma_a \bar{\lambda} e^a_m \partial_\tau x^m = \int \lambda \sigma_a \bar{\lambda} e^a.$$

Note that $\lambda$ still transforms under a spinor representation of $SO(1,3) \sim SL(2, C)$.

In particular, one can consider an $AdS_4$ space as a background where the particle propagates.

Let us mention that a different formulation of massive particle mechanics on $AdS_4$ has been considered in [25].

### 2.1 The $AdS_4$ particle

To consider a particle in the $AdS_4$ background we should specify the form of $e^a_m(x)$. A convenient choice of local coordinates and of the form of the metric on $AdS_4$ is

$$ds^2 = dx^m dx^n e^a_m e^b_n \eta_{ab} = \left( \frac{r}{R} \right)^2 dx^i \eta_{ij} dx^j + \left( \frac{R}{r} \right)^2 dr^2,$$

where $x^m = (x^i, r) (i = 0, 1, 2)$ are coordinates of the $AdS_4$, and $R$ is the $AdS_4$ radius, whose inverse square is the constant $AdS_4$ curvature (or the cosmological constant). The coordinates $x^i$ are associated with the three–dimensional boundary of $AdS_4$ when the radial coordinate $r$ tends to infinity.

From (10) we find that the components of the vierbein one–form $e^a = dx^m e^a_m$ are

$$e^a = dx^m e^a_m = \left( \frac{r}{R} \right)^2 \delta^a_i dx^i + \left( \frac{R}{r} \right) \delta^a_3 dr.$$

Note that the coordinates $x^i, r$ transform nonlinearly under the action of the $AdS_4$ isometry group $SO(2,3)$ which is the conformal symmetry of the boundary $x^i$

$$\delta x^i = a^i_I x_j + a^i_M x_j + a^i_D x^i + a^i_K x^j x_j - 2(x_j a^j_K)x^i + R \frac{a^i_K}{r^2},$$

$$\delta r = (2x_3 a^i_K - a_D)r,$$
where \( a_i^j \), \( A_i^j \), \( a^j \) and \( a^j_k \) are parameters of, respectively, \( D = 3 \) translations, Lorentz rotations, dilatation and conformal boosts, with \( \Pi_i, M_{ij}, D \) and \( K_j \) being the corresponding generators of the \( SO(2, 3) \) algebra (i,j=0,1,2) (see Appendix 1).

We can now substitute (11) into (9). The action takes the form

\[
S = \int d\tau \left[ \left( \frac{r}{R} \right) \lambda \sigma_i \dot{\lambda} \dot{x}^i + \left( \frac{R}{r} \right) \lambda \sigma_3 \dot{\lambda} \right].
\]  

(13)

Let us try to carry out the twistor transform of this action in a way similar to that considered above for the flat target space. To this end we redefine \( \lambda \) as

\[
\hat{\lambda}_A = \left( \frac{r}{R} \right)^{\frac{3}{2}} \lambda_A.
\]

(14)

The action (13) takes the form

\[
S = \int d\tau \left[ \lambda \sigma_i \lambda \dot{x}^i \left( \frac{R}{r} \right)^2 \right] + \lambda \sigma_3 \lambda \dot{x}^3.
\]

(15)

In the limit \( r \to \infty \) one obtains the twistor–like massless particle in three–dimensional Minkowski space, i.e. on the boundary of \( AdS_4 \). The complex Weyl spinor \( \lambda \) describes a pair of two–component real \( D=3 \) spinors which turn out to be proportional to each other on the mass shell. In the limit \( r \to \infty \) the action (13) is still invariant under the \( D = 3 \) conformal group \( Sp(4) = O(3, 2)/Z_2 \) supplemented by \( 0(2) \) rotations corresponding to the phase transformations of \( \lambda \).

In what follows we shall, however, keep \( r \) finite and make the change of variable

\[
\hat{x}^3 = - \frac{R^2}{r}.
\]

(16)

Then the action (13) formally becomes the same as eq. (1) in the flat case

\[
S = \int d\tau \left[ \lambda \sigma_i \lambda \dot{x}^i \left( \frac{R}{r} \right)^2 \right] + \lambda \sigma_3 \lambda \dot{x}^3.
\]

(17)

where \( \dot{x}^m = (x^i, \hat{x}^3) \). The essential difference is that upon the redefinition (14) the \( SL(2, C) \) spinors \( \hat{\lambda} \) transform \textit{nonlinearly} under the action of the isometry group \( SO(2, 3) \) via the radial coordinate \( r \).

We can now make the twistor transform of the action (17) by introducing

\[
\hat{\mu}_{\tilde{A}} = i \hat{\lambda}^A \hat{x}_{A\tilde{A}}
\]

(18)

and combining \( \hat{\lambda} \) and \( \hat{\mu} \) into the twistor

\[
Z_\alpha = (\hat{\lambda}_A, \hat{\mu}_{\tilde{A}}).
\]

(19)

The pure twistor form of the action (17) is the same as eq. (11), and, hence, it is invariant under the group \( SU(2, 2) \sim SO(2, 4) \) of the conformal transformations acting \textit{linearly} on the twistor (19). The twistor \( Z_\alpha \) is in the fundamental representation of \( SU(2, 2) \).
As it has been explained in detail in [10] for the particle in $AdS_5$, the linear conformal $SU(2, 2)$ transformations of $Z_\alpha$ induce nonlinear transformations of the AdS coordinates $x^i$ and $r$ when the twistor components are related to $x^i$ and $r$ through eqs. (14), (16) and (15),

$$Z_\alpha = \left( \frac{R}{r} \right)^{\frac{1}{2}} \left( \lambda_A, -ix^i \sigma_i^{AB} \lambda_B + i \frac{R^2}{r} \sigma_3^{AB} \lambda_B \right). \quad (20)$$

In the case under consideration we thus find the nonlinear conformal $SO(2, 4)$ transformations of the $AdS_4$ space coordinates, with the isometry group $SO(2, 3)$ (see eq. (12)) a subgroup of the conformal group. The conformal transformations of $\hat{x}^m = x^i, \hat{x}^3$ (where $\hat{x}^3$ was defined in (16)) are similar to the conformal transformations of the Minkowski space coordinates. They are

$$\delta \hat{x}^m = a^m_{II} + a^m_{MN} \hat{x}_n + a_D \hat{x}_m + a^m_K \hat{x}_n - 2(\hat{x}_n a^n_K) \hat{x}^m, \quad (21)$$

where $a^m_{II}, a^m_{MN}, a_D$ and $a^m_K$ are parameters of, respectively, $D = 4$ translations, Lorentz rotations, dilatation and conformal boosts, with $\Pi_m, M_{mn}, D$ and $K_m$ the corresponding generators of the $SO(2, 4)$ algebra ($m, n = 0, 1, 2, 3$) (see Appendix 1).

Substituting the expression (16) for $\hat{x}^3$ into (21), one can deduce the explicit form of the conformal transformations of the coordinate $r$. Then the $SO(2, 3)$ isometry transformations (12) of the $AdS_4$ coordinates are obtained by putting to zero all parameters in (21) which carry the index 3, the remaining ones being $a_D$ and all those with three-dimensional indices $i, j = 0, 1, 2$.

The observation that the $AdS$ spaces allow to be conformally transformed is implied by the fact that these manifolds are conformally flat. For instance, the $AdS_4$ metric (10) becomes conformally flat upon the redefinition of the coordinate $r$ just as in eq. (16) (which we made to perform the twistor transform)

$$ds^2 = \left( \frac{R}{\hat{x}^3} \right)^2 [dx^i \eta_{ij} dx^j + (d\hat{x}^3)^2] = \left( \frac{R}{\hat{x}^3} \right)^2 d\hat{x}^m d\hat{x}^n \eta_{mn}. \quad (22)$$

We have thus shown that the twistor constraint (8) has two solutions which correspond to the twistor transform of the flat $D = 4$ Minkowski space and of $AdS_4$, both spaces being conformally flat. This observation allows us to conclude that any other space whose metric is conformally flat should also arise as a corresponding solution of the twistor constraint (8).

We now turn to the supersymmetrization of the action (9).

3 Twistor–like $N = 1, D = 4$ superparticles

The form of the action (9) is suitable for a straightforward supersymmetric generalization. To this end we should consider $e^a$ as a vector component of the supervielbein one form

$$e^I(z) = dz^M e^I_M = (e^a, e^A, \bar{e}^{\dot{A}}), \quad (23)$$
where \( z^M = (x^m, \theta^A, \bar{\theta}^\dot{A}) \) are coordinates which parametrize a target superspace in which the particle propagates. \( \theta^A \) and its complex conjugate \( \bar{\theta}^\dot{A} \) are Grassmann–odd Weyl spinor coordinates.

### 3.1 A superparticle in flat superspace

In the case of flat target superspace

\[
e^a = dx^a - i\theta^a d\bar{\theta} + id\theta^a \bar{\theta}, \quad e^A = d\theta^A, \quad \bar{e}^\dot{A} = d\bar{\theta}^\dot{A}. \tag{24}
\]

Substituting \( e^a \) from (24) into the action (9) we can transform it into the pure super-twistor action by introducing the supertwistor (2)

\[
Z_A = (\lambda_A, \bar{\mu}^\dot{A}, \chi) \tag{25}
\]

and its conjugate

\[
\bar{Z}^A = (\mu^A, \bar{\lambda}_\dot{A}, \bar{\chi}), \tag{26}
\]

where now

\[
\bar{\mu}^{\dot{A}} = i\lambda_A (x_{A\dot{A}} - i\theta_A \bar{\theta}_{\dot{A}}) \tag{27}
\]

and

\[
\chi = \theta^A \lambda_A, \quad \bar{\chi} = \bar{\theta}^\dot{A} \bar{\lambda}_\dot{A}. \tag{28}
\]

Upon the supertwistorization the action (9) with the supervielbein (23) takes the form similar to eq. (7) where the twistor constraint (8) is replaced by the supertwistor constraint

\[
\bar{Z}^A Z_A = \mu^A \lambda_A - \bar{\mu}^\dot{A} \bar{\lambda}_\dot{A} + 2 \bar{\chi} \chi = 0. \tag{29}
\]

For further details on twistor superparticles in flat superspace we refer the reader to the papers [2, 26]–[32] and proceed with constructing an action for a superparticle propagating in the coset superspace \( OSp(1|4)/SO(1,3) \) whose bosonic subspace is \( AdS_4 \).

### 3.2 The superparticle on \( OSp(1|4)/SO(1,3) \)

To get an explicit form of the particle action on \( OSp(1|4)/SO(1,3) \) we should know an explicit form of the supervielbein (23), which is part of the Cartan forms on \( OSp(1|4) \). The components of the latter can be computed using the method of nonlinear realizations [33]–[36].

The Cartan forms of the supergroup \( OSp(1|4) \) and corresponding Cartan forms for the supercoset \( OSp(1|4)/SO(1,3) \) were calculated in [37]–[40]. Below we present simpler expressions for the Cartan forms which allow to write down a simple form of the \( AdS \) superparticle action, since our choice of the parametrization of the supercoset differs from that used in [38, 40].
To derive the Cartan forms on $\frac{OSp(1|4)}{SO(1,3)}$ we take the supercoset element in the form

$$K(z^M) = B(x)F(\theta) = B(x^m)e^{i(\theta^A Q_A + \bar{\theta}^\dot{A} Q_{\dot{A}})},$$

(30)

where $B(x^m)$ is the purely bosonic matrix taking its values in the coset $\frac{SO(2,3)}{SO(1,3)}$, i.e. it is associated with the bosonic $AdS_4$ space locally parametrized by coordinates $x^m$. The Grassmann coordinates $\theta^A$ and $\bar{\theta}^\dot{A}$ extend $AdS_4$ to the coset superspace, and $Q_A$ and $\bar{Q}_{\dot{A}}$ are the odd generators of $OSp(1|4)$ (see Appendix 1).

The Cartan form on $\frac{OSp(1|4)}{SO(1,3)}$ is

$$\frac{1}{i}K^{-1}dK = E^a(z)P_a + E^A(z)Q_A + Q_{\dot{A}}\bar{E}^\dot{A}(z) + \Omega^{ab}(z)M_{ab}.$$  

(31)

It takes values in the $OSp(1|4)$ superalgebra.

The one–forms $E^I = (E^a, E^A, \bar{E}^\dot{A})$ are the supervielbeins and $\Omega^{ab}$ is the $SO(1,3)$ connection on the coset superspace.

In the representation (30) the Cartan form (31) is

$$\frac{1}{i}B^{-1}dB = e^a P_a + \omega^{ab}M_{ab},$$

(33)

Depending on the choice of $B(x)$ one can get different forms of $e^a(x)$ and $\omega^{ab}(x)$. For instance, the coset element $B(x)$ can be chosen in such a way that $e^a(x)$ in (33) is the same as in (11) and the connection $\omega^{ab}(x)$ is

$$\omega^{i3} = \frac{1}{R}e^i, \quad \omega^{ij} = 0,$$

(34)

(remember that the index 3 corresponds to the radial coordinate $r$ of $AdS_4$).

We can now substitute (33) into (32) and calculate the explicit form of the supervielbeins $E^I$ and the superconnection $\Omega^{ab}$, using a trick, described, for example, in [13], or by the method presented in Appendix 2. In the Majorana spinor representation the expressions for the Cartan forms are given in Appendix 1. In the two–component spinor formalism the supervielbein form (A.15) or (A.18) of Appendix 1 can be written as follows

$$E^a = P(\theta^2, \bar{\theta}^2) \left[ e^a(x) - i\Theta\sigma^a D\bar{\Theta} + iD\Theta\sigma^a\bar{\Theta} \right],$$

(35)

where

$$P(\theta^2, \bar{\theta}^2) = 1 - \frac{i}{2R}(\theta^2 - \bar{\theta}^2) + \frac{1}{3!R^2}\theta^2\bar{\theta}^2, \quad \theta^2 \equiv \theta^A \theta_A, \quad \bar{\theta}^2 \equiv \bar{\theta}^\dot{A} \bar{\theta}^\dot{A},$$

(36)
\[ \Theta = \left( 1 + \frac{i}{3R} \frac{\left( \theta^2 - \bar{\theta}^2 \right)}{P(\theta^2, \bar{\theta}^2)} \right)^{\frac{1}{2}} \theta, \]  

(37)

\[ D = d + \omega^{bc} \sigma_{bc}, \quad \sigma^{ab} = \frac{1}{4} (\sigma^a \sigma^b - \sigma^b \sigma^a). \]

To get the action for the superparticle in the super–AdS background we should simply substitute (35) into (9).

\[ S = \int \lambda \sigma^a \bar{\lambda} P(\theta^2, \bar{\theta}^2) \left[ e^a - i \Theta \sigma^a D \bar{\Theta} + i D \Theta \sigma^a \Theta \right] \]  

(38)

The polynomial \( P(\theta^2, \bar{\theta}^2) \) can be absorbed by properly rescaled \( \lambda \) and \( \bar{\lambda} \), namely, \( \Lambda = \sqrt{P(\theta^2, \bar{\theta}^2)} \lambda \). Then the action takes an even simpler form which is quadratic in fermions

\[ S = \int \Lambda \sigma^a \bar{\Lambda} \left[ e^a - i \Theta \sigma^a d \bar{\Theta} + id \Theta \sigma^a \bar{\Theta} + 2i \omega^{bc}(x) \Theta \sigma^a \sigma_{bc} \bar{\Theta} \right]. \]  

(39)

If in (39) there were no term containing the spin connection \( \omega^{bc} \) the action (39) could be completely supertwistorized in the same way as we have done in the case of the \( AdS_4 \) particle and of the superparticle in flat superspace. However, the term with \( \omega^{bc} \) does not allow one to perform the complete supertwistorization of (39) in terms of free supertwistors, at least in a straightforward way.

Using the notion of Killing spinors on \( AdS \) spaces one can replace in (39) the covariant differential \( D \) with the ordinary one. To this end it is convenient to switch to the four–component Majorana spinor formalism

\[ \Lambda^a = (\lambda_A, \bar{\lambda}^{\dot{A}}), \quad \Theta^a = (\Theta_A, \bar{\Theta}^{\dot{A}}). \]

By definition (see, for instance, [41]) \( AdS \) Killing spinors satisfy the condition

\[ DK^{\alpha}_{\beta} C^\beta = (DK^{\alpha}_{\beta} + \frac{1}{2R} (e^a \gamma_a)^{\alpha}_{\gamma} K^\gamma_{\beta}) C^\beta = (dK^{\alpha}_{\beta} + \frac{1}{2} (\omega^{ab} \gamma_{ab})^{\alpha}_{\gamma} K^\gamma_{\beta} + \frac{1}{2R} (e^a \gamma_a)^{\alpha}_{\gamma} K^\gamma_{\beta}) C^\beta = 0, \]  

(40)

where \( K^{\alpha}_{\beta}(x) \) is a bosonic Killing spinor matrix and \( C^\beta \) is an arbitrary constant spinor.

If in (39) we replace \( \Theta \) with \( \Theta = K(x) \Theta_K \) (where \( \Theta_K \equiv K^{-1} \Theta \) [41] the action (39) takes the form

\[ S = \int \Lambda \gamma_a \Lambda \left[ e^a (1 + \frac{i}{2R} \Theta) - i \bar{\Theta} \gamma^a K d\Theta_K \right], \]

or (upon an appropriate rescaling of \( \Lambda \) and \( \Theta \))

\[ S = \int \Lambda \gamma_a \Lambda \left[ e^a - i \bar{\Theta} \gamma^a K d\Theta_K \right]. \]  

(41)

Note that in (31) the variable \( \Theta_K \) is regarded as independent, while \( \Theta = K(x) \Theta_K \) is composed from \( \Theta_K \) and the Killing matrix \( K(x) \) whose exact dependence on the \( AdS_4 \)
coordinates $x^m$ can be found by solving the Killing spinor equation \([10, 11]\). Taking this into account, the term $i\bar{\Theta}^{\alpha}K\theta_K$ in \([38]\) can be rewritten as $i\bar{\Theta}^{\alpha}_K\gamma^b \theta_K K_c^a(x)$, where $K_c^a(x)$ are $SO(2,3)$ Killing vectors on $AdS_4$ ($b, c = 0, 1, 2, 3, 4, \gamma^4 = 1$). Then the supervielbein
\[
E^a = e^a - i\bar{\Theta}^{\alpha}_K\gamma^b \theta_K K_c^a(\hat{b}, \hat{c})
\]
takes the form similar to one of the parametrizations considered in \([38]\).

It would be interesting to understand whether the $AdS$ superparticle action in any of its forms can be completely supertwistorized. If it is possible, the $AdS$ superparticle model would acquire the manifest superconformal $SU(2,2|1)$ symmetry. In any case, the use of commuting spinors whose bilinears replace the conventional particle momentum and the suitable choice of the parametrization of the supercoset space $OSp(1|4)$ allow one to get a simple form of the action for a superparticle propagating in the AdS superbackground, which is bilinear in fermionic variables.

## 4 The superparticle on $OSp(1|4)$

We now turn to the construction of the action for a superparticle propagating on the supergroup manifold $OSp(1|4)$ locally parametrized by the supercoset $\frac{OSp(1|4)}{SO(1,3)}$ coordinates $x^m$ and $\theta$, and by six $SO(1,3)$ group coordinates $y^{mn} = -y^{nm}$. This model is intended to produce, upon an appropriate contraction, the superparticle in flat superspace and on super–$AdS_4$ considered above, as well as the superparticle with tensorial central charges \([11, 12]\).

By analogy with eqs. \([9]\) and \([38]\), to construct the $OSp(1|4)$ superparticle Lagrangian we take the pullback onto the particle worldline of the even Cartan superforms $E^a_{OSp}$ and $\Omega^{ab}_{OSp}$ given in Appendix 1 (eq. \([A.7]\)). These forms comprise the bosonic $SO(2,3)$ part of the supervielbein on $OSp(1|4)$. We contract them with commuting spinor bilinears $\bar{\lambda}^{\alpha}_a \lambda$ and $\frac{1}{2} \bar{\lambda}^{\alpha}_{ab} \lambda$. The $OSp(1|4)$ superparticle action is
\[
S_{OSp} = \frac{1}{2} \int \left[ E^b(x, \theta) u_b^a(y) \bar{\lambda}^{\alpha}_a \lambda + \frac{1}{2} \left( \Omega^{cd}(x, \theta) u_c^a u_d^a + (u^{-1} du)^{ab} \right) \bar{\lambda}^{\alpha}_{ab} \lambda \right]. \tag{42}
\]
Using the defining relations for the $SO(1,3)$ matrices $u_b^a$ and $v_\beta^{\alpha}$ \([A.8]\) we can make the redefinition
\[
\bar{u}_b^a(y) \bar{\lambda}^{\alpha}_a \lambda = \tilde{\lambda}^{\alpha}_a \lambda, \quad \text{where} \quad \tilde{\lambda}^{\alpha} = \lambda^{\alpha}_b v_\beta^{\alpha}.
\tag{43}
\]
Then $u_b^a(y)$ remains only in one term of the action \([12]\), and the latter takes the form
\[
S_{OSp} = \frac{1}{2} \int E^a(x, \theta) \tilde{\lambda}^{\alpha}_a \lambda + \frac{1}{4} \int [\Omega^{ab}(x, \theta) + (duu^{-1})^{ab}] \tilde{\lambda}^{\alpha}_{ab} \lambda \tag{44}
\]
We observe that the first integral in \([14]\) is nothing but the action \([38]\) for the superparticle on the coset superspace $\frac{OSp(1|4)}{SO(1,3)}$, and the second term contains the spin connection of
extended by the $SO(1,3)$ Cartan form $duu^{-1}$. In eq. (43) the dependence of the action on the $SO(1,3)$ group manifold coordinates $y^{mn}$ remains only in $duu^{-1}$.

Since by an appropriate choice of Grassmann coordinates the Cartan forms $E^a(x, \theta)$ and $\Omega^{ab}(x, \theta)$ can be made quadratic in $\theta$ (see eqs. (A.18) and (A.19) of Appendix 1) we see that the $OSp(1|4)$ action (44) is quadratic in fermions.

If we drop the second integral of (44) we get the action for the superparticle considered in Subsection 3.2, and if we then take the limit when the $AdS_4$ radius goes to infinity, the action further reduces to the superparticle action in flat $N=1, D=4$ superspace.

Another way of truncating the action (44) is to perform the following contraction of the $OSp(1|4)$ superalgebra (A.1). Let us in (A.1) rescale the generators $M_{ab}$ of $SO(1,3)$ as follows

$$M_{ab} = R Z_{ab},$$

and consider the limit when $R \to \infty$. Then the generators $Z_{ab}$ become tensorial central charges which commute with all other generators, and the anticommutator of the supercharges becomes

$$\{Q_\alpha, Q_\beta\} = -2(C\gamma^a)_{\alpha\beta} P_a + (C\gamma^{ab})_{\alpha\beta} Z_{ab}. \quad (46)$$

The $SO(1,3)$ coordinates $y^{mn}$ become central charge coordinates.

In the limit $R \to \infty$ the supervielbein $E^a(x, \theta)$ reduces to the ‘flat’ one–form (24)

$$E^a_Z = dx^a - i\bar{\theta}\gamma^a d\theta \quad (47)$$

and the superconnection $\Omega_{Z}^{ab} = R(\Omega^{ab} + (duu^{-1})^{ab})$ becomes

$$\Omega_{Z}^{ab} = dy^{ab} + \frac{i}{2} \bar{\theta}\gamma^{ab} d\theta. \quad (48)$$

Substituting (47) and (48) into (44) we get the action for a particle with tensorial central charges [11, 12]. The quantization of this superparticle model has shown to produce an infinite tower of free massless states with arbitrary integer and half integer spin, with the spin degrees of freedom associated with the central charge coordinates $y^{mn}$. For a detailed analysis of the model we refer the reader to [11, 12].

Since the higher–spin fields can interact if they live not in Minkowski space but in an anti–de–Sitter space [5], it seems of interest to study the possibility of generalizing the $OSp(1|4)$ superparticle model based on the action (44) to include interactions, and then to perform its quantization to check whether such a model can be considered as a classical counterpart of the theory of interacting higher–spin fields.

To conclude this section we demonstrate that $OSp(1|4)$ covariant momenta associated with the $OSp(1|4)$ coordinates $x^m, y^{mn}$ and $\theta^a$ generate the $OSp(1|4)$ superalgebra [4].

---

In [14] similar covariant momenta were used to make the Hamiltonian analysis and the quantization of superparticles propagating in harmonic superspaces.
Let us rewrite the action (44) as follows
\[
S = \int d\tau \Lambda_I E^I_M(x, \theta, z) \partial_\tau X^M,
\]
where \( \Lambda_I \) stand for the bilinear combinations of the spinor \( \hat{\lambda} \)
\[
\Lambda_I = \left( \frac{1}{2} \hat{\lambda} \gamma_a \hat{\lambda}, \frac{1}{4} \hat{\lambda} \gamma_{ab} \hat{\lambda} \right),
\]
the index \( I \) stands for vector \( a \) and tensor \( ab \) indices, and \( X^M \equiv (x^m, z^{mn}, \theta^\alpha) \).
\( E^I_M(x, \theta, z) \) are the \( OSp(1|4) \) Cartan form components \( E^a \) and \( \Omega^{ab} \), which correspond to the bosonic generators \( P_a \) and \( M_{ab} \) of \( OSp(1|4) \) (see Appendix 1).

From (49) we get the canonical momenta conjugate to \( X^M \equiv (x^m, z^{mn}, \theta^\alpha) \) as
\[
\frac{\delta S}{\delta (\partial_\tau X^M)} = P_M = \Lambda_I E^I_M.
\]
Multiplying (51) by the matrix \( E^I_M \) inverse to \( E^I_M \) (where \( \hat{I} = (I, \alpha) \)) we obtain \( OSp(1|4) \) covariant momenta \( P_I = E^I_M P_M = (P_I, P_\alpha) \) such that
\[
\Lambda_I = P_I \equiv E^I_M(X) P_M, \quad P_\alpha = E^\alpha_M(X) P_M = 0.
\]
Eqs. (50) and (52) imply that the expressions for the momenta are constraints on the superparticle phase space variables. For instance, the momentum components \( P_\alpha \) of the Grassmann variable \( \theta^\alpha \) are zero. These are Grassmann constraints on the dynamics of the \( OSp(1|4) \) superparticle, which include first–class constraints generating the \( \kappa \)–symmetry of the \( OSp(1|4) \) superparticle.

It is well known that, as \( N = 1, D = 4 \) superparticles in an arbitrary supergravity background do, the \( AdS \) superparticle possesses two–parameter local fermionic \( \kappa \)–symmetry, which means that such superparticles preserve half the supersymmetry of a target–space vacuum.

In contrast to this, as we shall prove in the next section, the \( OSp(1|4) \) superparticle possesses 3 \( \kappa \)–symmetries and, in general, the superparticle propagating on the \( OSp(1|2n) \) supergroup manifold has \( (2n - 1) \kappa \)–symmetries and thus describes BPS states with only one broken supersymmetry.

In [11] the superparticle models with such a symmetry property have been obtained in flat superspaces with additional tensorial central charge coordinates. Here we observe that this unusual feature is also inherent to superparticles propagating in more complicated superspaces.

Because of the Maurer–Cartan equations \((dE - iE \wedge E = 0)\) for the Cartan forms \( E^I_M \) the generalized momenta form, under the Poisson brackets, the \( OSp(1|4) \) superalgebra, which can be quantized by taking an appropriate ordering of \( X \) and \( P \) in the definition of (52):
\[
[P_I, P_J] = C^K_{IJ} K P_K,
\]

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where $C_{ij}^K$ are $OSp(1|4)$ superalgebra structure constants (see eq. (A.1) of Appendix 1).

From (52) and (53) we see that upon the transition to Dirac brackets the spinor bilinears $\Lambda_I$ become generators of the $Sp(4) \sim SO(2, 3)$ subalgebra of $OSp(1|4)$.

From this analysis we conclude that the commutation properties of the superparticle covariant momenta reflect the structure of the global symmetries of the $OSp(1|4)$ superparticle action. To quantize the model one should consider the $OSp(1|4)$ coordinates and momenta as ‘generalized’ canonical variables, with commutation relations defined by the graded $OSp(1|4)$ superalgebra (53).

The detailed study of the model based on the action (44) is in progress.

5 The superparticle on $OSp(1|2n)$ as a dynamical model for unusual BPS states

We now generalize the $OSp(1|4)$ superparticle action (42), (44) or (49) to the case of the supermanifold $OSp(1|2n)$ whose parametrization we choose to be of the form (see Appendix 2 for the details on the $OSp(1|2n)$ superalgebra)

$G(y, \theta) = B(y)$

$F(\theta) = B(y) e^{i\theta^\alpha Q_\alpha},$ (54)

where $y^{\alpha\beta} = y^{\beta\alpha}$ are coordinates of the $Sp(2n)$ subgroup generated by symmetric operators $M_{\alpha\beta} = M_{\beta\alpha}$, and whose element is denoted as $B(y)$; $\theta^\alpha$ are Grassmann coordinates and $Q_\alpha$ are Grassmann generators of $OSp(1|2n)$ transforming under the fundamental representation of $Sp(2n)$, which we call the spinor representation, $(\alpha, \beta = 1, ..., 2n)$.

The $OSp(1|2n)$ Cartan forms are

$\frac{1}{i} G^{-1}(y, \theta) dG(y, \theta) \equiv \frac{1}{i} \left[ F^{-1}(B^{-1} dB) F + F^{-1} dF \right] = E^\alpha Q_\alpha + \frac{1}{2} \Omega^{\alpha\beta} M_{\alpha\beta}.$ (55)

To have the connection with the $OSp(1|4)$ case discussed in Section 4 and Appendix 1 we note that for $n = 2$ $M_{\alpha\beta}$ can be written in terms of $SO(1, 3)$ covariant generators $P_a$ and $M_{ab}$ as follows

$M_{\alpha\beta} = -2(C^\gamma_{\alpha\beta})_a P_a + \frac{1}{R^2} (C^\gamma_{ab})_{\alpha\beta} M_{ab}.$ (56)

Then the $OSp(1|4)$ Cartan forms presented in (A.7) are related to $\Omega^{\alpha\beta}$ in (55) in the following way

$E^\alpha_{osp} = -(C^\gamma_{\alpha\beta})_a \Omega^a_{\alpha\beta}, \quad \Omega^a_{osp} = \frac{1}{2R} (C^\gamma_{ab})_{\alpha\beta} \Omega^{\alpha\beta}.$ (57)

The matrix $C_{\alpha\beta}$ plays the role of the $OSp(1|2n)$ invariant metric.

The $OSp(1|2n)$ Cartan forms (55) computed in Appendix 2 have the form

$E^\alpha = D\theta^\alpha + i D\theta^{(\alpha} \theta^\beta) P_1(\theta\theta),$ (58)
\[ \Omega^{\alpha\beta} = \omega^{\alpha\beta}(y) + i\theta^{(\alpha}D^{\beta)}P_2(\theta\theta), \] (59)

where \( \omega^{\alpha\beta}(y) \) are \( Sp(2n) \) Cartan forms, \( P_1(\theta\theta) \) and \( P_2(\theta\theta) \) are polynomials in \( \theta^\alpha C_{\alpha\beta} \theta^\beta \) (see (A.37) and (A.38) of Appendix 2), and \( D \) is the \( Sp(2n) \) covariant derivative

\[ D\theta^{\alpha} = d\theta^{\alpha} + \frac{\alpha}{2} \omega^{\alpha\beta}(y)\theta^\beta, \] (60)

where \( \alpha \) is a dimensional constant factor in the \( OSp(1|2n) \) superalgebra (A.20), which in the \( OSp(1|4) \) case (A.1) is \( \alpha = \frac{4}{17} \).

The form of eq. (59) prompts us that the polynomial \( P_2 \) can be hidden into rescaled \( \Theta = \sqrt{P_2} \theta \), then for \( \Omega^{\alpha\beta} \) we get the simple expression

\[ \Omega^{\alpha\beta} = \omega^{\alpha\beta}(y) + i\Theta^{(\alpha}D^{\beta)}\Theta^{)} \] (61)

The action for a superparticle moving on \( OSp(1|2n) \), which generalizes (44), has the form

\[ S = \frac{1}{2} \int \lambda_\alpha \lambda_\beta \Omega^{\alpha\beta}\equiv \frac{1}{2} \int d\tau \lambda_\alpha \lambda_\beta \Omega^{\alpha\beta}_\tau \] (62)

where \( \lambda_\alpha \) is an auxiliary bosonic \( Sp(2n) \) ‘spinor’ variable, and \( \Omega^{\alpha\beta} = d\tau \Omega^{\alpha\beta}_\tau \) is the pullback of the even Cartan form (59) or (61) on the superparticle worldline.

Let us now analyse the \( \kappa \)–symmetry properties of the action (62) by considering its general variation. A simple way to vary the action (62) with respect to \( OSp(1|2n) \) coordinates \( X^M = (y^{\alpha\beta}, \theta^\alpha) \) and the auxiliary variable \( \lambda \), is to use Maurer–Cartan equations (integrability conditions for Eq. (55))

\[ dG^{-1}dG = G^{-1}dG \wedge G^{-1}dG \]

which imply

\[ dE^{\alpha} + \frac{\alpha}{2} E^{\alpha} \wedge \Omega^{\alpha\beta}_\beta = 0, \] (63)

\[ d\Omega^{\alpha\beta} + \frac{\alpha}{2} \Omega^{\alpha\gamma} \wedge \Omega^{\gamma\beta}_\theta = -iE^{\alpha} \wedge E^{\beta}, \] (64)

and the expression for the \( X^M \)–variation of the differential forms

\[ \delta \Omega = i_\delta d\Omega + di_\delta \Omega \quad i_\delta \Omega \equiv \delta X^M \Omega_M. \] (65)

Modulo a boundary term the variation of the action (62) obtained in this way takes the form

\[ S = \int \delta \lambda_\alpha \Omega^{\alpha\beta}_\beta \lambda_\beta - \int_{M^1} D\lambda_\alpha \ i_\delta \Omega^{\alpha\beta}_\beta \lambda_\beta - \frac{1}{2} \int (E^{\alpha}_\lambda \lambda_\alpha)(i_\delta E^{\beta}_\lambda) \lambda_\beta, \] (66)

where the basis in the space of variations is chosen to be \( i_\delta \Omega^{\alpha\beta} \) and \( i_\delta E^{\alpha} \) instead of more conventional \( \delta y^{\alpha\beta} \) and \( \delta \theta^\alpha \).

Note that \( i_\delta E^{\alpha} \) corresponds to the variation of the action with respect to Grassmann coordinates \( \theta^\alpha \). Putting \( \delta \lambda_\alpha = 0, \ i_\delta \Omega^{\alpha\beta}_\beta = 0 \) we thus observe that only one of the \( 2n \) independent fermionic variations, namely \( i_\delta E^{\alpha} \lambda_\alpha \), effects the variation of the action. This implies that other \( 2n-1 \) fermionic variations are fermionic \( \kappa \)–symmetries of the dynamical
system described by the action (62). The $\kappa$–symmetry transformations are defined in such a way that $i\delta E^\alpha \lambda_\alpha$ vanishes (cf. [11, 12])

$$i\delta \Omega^{\alpha \beta} = 0, \quad \delta \lambda_\alpha = 0, \quad i\delta E^\alpha = \kappa^I \mu^I_\alpha, \quad I = 1, \ldots (2n - 1)$$

(67)

where the $\mu^I_\alpha$ are $2n - 1$ $\text{Sp}(2n)$ spinors orthogonal to $\lambda_\alpha$

$$\mu^I_\alpha \lambda_\alpha = 0, \quad I = 1, \ldots (2n - 1).$$

(68)

Thus, we conclude that an unusual property of a twistor–like superparticle with tensorial central charge coordinates [11] to preserve all but one target–space supersymmetries is inherent to the superparticle model on the $OSp(1|2n)$ supergroup manifold as well.

When the explicit expressions (58) and (59) for the Cartan forms on $OSp(1|2n)$ are obtained, one straightforwardly gets the explicit expressions also for the Cartan forms on any coset superspace $OSp(1|2n)/H$, where $H$ is a bosonic subgroup of $OSp(1|2n)$. These expressions are the same as (58) and (59) but with $\omega^{\alpha \beta}$ depending only on the bosonic coordinates of the supercoset (see also eqs. (A.40) and (A.41) of Appendix 2).

Using the $OSp(1|2n)/H$ Cartan forms one can construct various types of actions for superparticles and superbranes propagating on the corresponding coset supermanifolds.

### 6 Conclusion

By taking a suitable parametrization of the supergroup manifold $OSp(1|2n)$ we have found a simple form of the $OSp(1|2n)$ Cartan superforms such that the ones which take values in the bosonic subalgebra $\text{Sp}(2n)$ of $OSp(1|2n)$ are quadratic in Grassmann coordinates.

We have used these Cartan forms to construct simple twistor–like actions (which are quadratic in fermions) for describing superparticles propagating on the coset superspace $OSp(1|4)/SO(1, 3)$, on the supergroup manifold $OSp(1|4)$, and, in general, on $OSp(1|2n)$ supermanifolds.

The $OSp(1|4)$ superparticle model has been shown to produce (upon a truncation) either the standard massless D=4 superparticle or the generalized massless D=4 superparticle with tensorial central charges [11, 12] whose quantization gives rise to massless free fields of arbitrary (half)integer spin.

We have also shown that the massless particle on $AdS_4 = SO(2, 3)/SO(1, 3)$ can be described (with a particular choice of twistor variables) as a free $D = 4$ twistor particle.

A direction of further study can be to analyse the $OSp(1|4)$ superparticle model in detail and to look for its role as a classical counterpart in the theory of interacting higher–spin fields requiring a finite AdS radius.

Another interesting problem is to generalize the results of this paper to the case of superstrings and superbranes propagating in AdS superbackgrounds with the aim to find a simple form of superbrane actions on AdS. The simple fermionic structure of $OSp(1|32)$
and $OSp(1|64)$ Cartan forms, which we obtained, may be helpful in making a progress in this direction.

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**Appendix 1**

We use the ‘almost plus’ signature $(-, +, \cdots, +)$ of the Minkowski metric $\eta^{ab}$ ($a, b = 0, 1, 2, 3$).

**The $OSp(1|4)$ superalgebra**

\begin{align*}
- i [M_{ab}, M_{cd}] &= \eta_{ad}M_{bc} + \eta_{bc}M_{ad} - \eta_{ac}M_{bd} - \eta_{bd}M_{ac} \tag{A.1} \\
- i [M_{ab}, P_c] &= \eta_{bc}P_a - \eta_{ac}P_b \tag{A.2} \\
[P_a, P_b] &= \frac{i}{R^2} M_{ab} \tag{A.3} \\
\{Q_\alpha, Q_\beta\} &= -2(C\gamma^a)_{\alpha\beta}P_a + \frac{1}{R} (C\gamma^{ab})_{\alpha\beta} M_{ab} \\
[M_{ab}, Q_\alpha] &= -\frac{i}{2} Q_\beta (\gamma_{ab})^\beta_\alpha \quad \gamma_{ab} = \frac{1}{2}(\gamma_a\gamma_b - \gamma_b\gamma_a), \tag{A.4} \\
[P_a, Q_\alpha] &= -\frac{i}{2R} Q_\beta (\gamma^a_\alpha)^\beta 	ag{A.5}
\end{align*}

The generators $M_{ab}$ form the $SO(1, 3)$ subalgebra (A.1), and $M_{ab}$ and $P_a$ form the $SO(2, 3)$ subalgebra of $OSp(1|4)$. $Q_\alpha$ are four Majorana spinor generators of $OSp(1|4)$. The parameter $R$ is the $AdS_4$ radius, and $C_{\alpha\beta}$ is the charge conjugation matrix such that

$$\gamma^a_{\alpha\beta} = \gamma^a_{\beta\alpha} \equiv C_{\alpha\gamma}(\gamma^a)^\gamma_\beta.$$ 

The parameters $a^i_1$, $a^i_2$, $a^i_D$ and $a^i_K$ (12) of $SO(2, 3)$ acting as the conformal transformations on the boundary of $AdS_4$ (associated with the coordinates $x^i$) correspond to the
following linear combinations of $M_{ab}$ and $P_a$: 
three–dimensional translations
\[ a^i_\Pi \rightarrow \Pi_i = P_i - M_{i3} \quad i = 0, 1, 2; \]
$SO(1,2)$–rotations
\[ a^{ij}_M \rightarrow M_{ij}, \]
dilatation
\[ a_D \rightarrow D = P_3, \]
special conformal transformations (conformal boosts)
\[ a^i_K \rightarrow K_i = P_i + M_{i3}. \]

Note that the $SO(2, 4)$ algebra has the same structure as $SO(2, 3)$ in (A.1)–(A.3) but with indices $a, b, \cdots$ running from 0 till 4.

**The $OSp(1|4)$ Cartan forms**

We choose the parametrization of an $OSp(1|4)$ group element $G(x, \theta, y)$ as follows
\[ G = K(x, \theta) U(y), \quad K(x, \theta) = B(x) e^{i\theta Q}, \quad (A.6) \]
where $K(x, \theta) = B(x) e^{i\theta Q}$ is a group element corresponding to the coset superspace $\frac{OSp(1|4)}{SO(1,3)}$, $B(x)$ is a group element corresponding to the bosonic $AdS_4 = \frac{SO(2,3)}{SO(1,3)}$ and $U(y)$ is an element of $SO(1,3)$ generated by $M_{ab}$ with the antisymmetric $y^{ab}$ being six parameters of the $SO(1,3)$ transformations. We do not need to specify the representation of $B(x)$ and $U(y)$.

The $OSp(1|4)$ Cartan forms $G^{-1}dG = E^a_{OSp} P_a + \Omega^{ab}_{OSp} M_{ab} + E^\alpha_{OSp} Q_\alpha$ are
\[ E^a_{OSp} = E^b(x, \theta) u^a_b(y), \]
\[ \Omega^{ab}_{OSp} = \Omega^{cd}(x, \theta) u^a_c u^b_d + (u^{-1} du)^{ab}, \]
\[ E^\alpha_{OSp} = E^\beta(x, \theta) v^\alpha_\beta(y), \quad (A.7) \]
where $u^a_b(y)$ and $v^\alpha_\beta(y)$ are matrices of, respectively, the vector and the spinor representation of $SO(1,3)$. They are defined by the relations
\[ u^a_b(y) P_a = U^{-1} P_b U(y), \quad v^\alpha_\beta(y) Q_\alpha = U^{-1} Q_\beta U(y), \quad (A.8) \]
and are related to each other by the standard expression
\[ \gamma_a u^a_b(y) = v(y) \gamma_b v(y). \quad (A.9) \]
$E^a(x, \theta)$, $\Omega^{ab}(x, \theta)$ and $E^\alpha(x, \theta)$ are Cartan forms $K^{-1}dK$ corresponding to the coset superspace $\frac{OSp(1|4)}{SO(1,3)}$. 

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The $OSp(1|4)/SO(1,3)$ supervielbeins and spin connection

The spinorial supervielbein is

$$E^\alpha = D\theta^\alpha - \frac{i}{3!R} \bar{\theta} \gamma^a D\theta (\gamma_a \theta)^\alpha + \frac{i}{2 \cdot 3! R} \bar{\theta} \gamma^{ab} D\theta (\gamma_{ab} \theta)^\alpha - \frac{2}{5! R^2} D\theta^\alpha (\bar{\theta} \theta)^2, \quad (A.10)$$

or, by using the Fierz identity

$$C_{\alpha(\beta C_{\gamma})\delta} = \frac{1}{4} \gamma^a_{\beta(\gamma_a)\alpha\delta} - \frac{1}{8} \gamma^a_{\beta(\gamma_{ab})\alpha\delta}, \quad (A.11)$$

$$E^\alpha = D\theta^\alpha (1 + \frac{i}{3! R} \bar{\theta} \theta - \frac{2}{5! R^2} (\bar{\theta} \theta)^2) - \frac{i}{3! R} \bar{\theta} D\theta \theta^\alpha. \quad (A.12)$$

where $D$ is a covariant differential on the bosonic $AdS_4$ space defined as

$$D = d + \frac{1}{2} \omega^{ab}(x)\gamma_{ab} + \frac{1}{2} e^a(x)\gamma_a D\theta + \frac{1}{2 R} e^a(x)\gamma_a \equiv D + \frac{1}{2 R} e^a \gamma_a. \quad (A.13)$$

Note that the $AdS_4$ Killing spinors (40) are defined to be covariantly constant with respect to $D$, i.e. $D K = 0$.

The vector supervielbein is

$$E^a = e^a(x) - i \bar{\theta} \gamma^a D\theta - \frac{1}{2 \cdot 3! R} \bar{\theta} \gamma^a D\theta (\bar{\theta} \theta)^2 + \frac{1}{4! R} \bar{\theta} \gamma^{bc} D\theta (\bar{\theta} \gamma^a \gamma_{bc} \theta), \quad (A.14)$$

or (upon applying the Fierz identity (A.11))

$$E^a = e^a(x) - i \bar{\theta} \gamma^a D\theta (1 + \frac{i}{3! R} \bar{\theta} \theta). \quad (A.15)$$

Eq. (A.14) can be further rewritten as

$$E^a = e^a(x) \left( 1 - \frac{i}{2 R} \bar{\theta} \theta - \frac{1}{2 \cdot 3! R^2} (\bar{\theta} \theta)^2 \right) - i \bar{\theta} \gamma^a D\theta \left( 1 + \frac{i}{3! R} \bar{\theta} \theta \right), \quad (A.15)$$

where $D = d + \frac{1}{2} \omega^{ab}(x)\gamma_{ab}$.

And the $SO(1,3)$ connection is

$$\Omega^{ab} = \omega^{ab}(x) + i \frac{2 R}{4! R} (\bar{\theta} \gamma^{cD\theta}) (\bar{\theta} \gamma^{ab} \gamma_c \theta) - \frac{1}{2 \cdot 4! R^2} (\bar{\theta} \gamma^{cd} D\theta) (\bar{\theta} \gamma^{ab} \gamma_{cd} \theta)$$

$$= \omega^{ab}(x) + \frac{i}{2 R} \bar{\theta} \gamma^{ab} D\theta \left( 1 + \frac{i}{3! R} \bar{\theta} \theta \right), \quad (A.16)$$

where $e^a(x)$ and $\omega^{ab}(x)$ are the vierbein and the spin connection on $AdS_4$.

Note that in (A.14) and (A.16) we can make the following change of the Grassmann coordinates

$$\Theta^a = (1 + \frac{i}{3! R} \bar{\theta} \theta)^{\frac{1}{2}} \theta^a. \quad (A.17)$$

Then, because of the symmetry properties of the Dirac matrices $\gamma^a$ and $\gamma^{ab}$, the Cartan forms become bilinear in $\Theta$

$$E^a = e^a(x) - i \Theta \gamma^a D\Theta, \quad (A.18)$$

$$\Omega^{ab} = \omega^{ab}(x) + \frac{i}{2 R} \bar{\Theta} \gamma^{ab} D\Theta. \quad (A.19)$$
Appendix 2

The $OSp(1|2n)$ superalgebra and $OSp(1|2n)$ Cartan forms

The generators of the $OSp(1|2n)$ superalgebra are a symmetric bosonic (spin)tensor $M_{\alpha\beta} = M_{\beta\alpha}$ ($\alpha = 1, ..., 2n$) and a $2n$–component Grassmann spinor $Q_\alpha$, which satisfy the following (anti)commutation relations

\[
[M_{\alpha\beta}, M_{\gamma\delta}] = -i\alpha [C_\gamma(\alpha M_{\beta})\delta + C_\delta(\alpha M_{\beta})\gamma],
\]
\[
[M_{\alpha\beta}, Q_\gamma] = -i\alpha C_\gamma(\alpha Q_\beta),
\]
\[
\{Q_\alpha, Q_\beta\} = M_{\alpha\beta},
\]

where $C_{\alpha\beta} = -C_{\beta\alpha}$ is a constant $2n \times 2n$ antisymmetric matrix (symplectic metric). Note that to have the correspondence with the form of $OSp(1|4)$ superalgebra (A.1) the factor $\alpha$ should be chosen to be $\alpha = \frac{1}{R}$.

When $n = 2^k$, $C$ can be regarded as a charge conjugation matrix and $Q_\alpha$ as a spinor representation of a D–dimensional pseudo-rotation group $SO(t, D - t)$ with an appropriately chosen number of dimensions $D$ and time–like dimensions $t$ of space–time.

For instance, when $n = 16$ the generators $Q_\alpha$ of $OSp(1,32)$ can be associated with $SO(1,10)$ Majorana spinors in $D = 11$ or two $SO(1,9)$ Majorana–Weyl spinors of the same or opposite chiralities in $D = 10$. This makes the $OSp(1,32)$ supergroup to be related to M–theory and superstring theories. $OSp(1,32)$ is a subgroup of $OSp(1|64)$, and the two supergroups are extensions of the isometry supergroups $SU(2,2|4)$, $OSp(8|4)$ and $OSp(2,6|4)$ of $D = 10$ and $D = 11$ AdS superspaces [21, 22, 23].

From a perspective of $D = 11$ supergravity and M–theory the $OSp(1|32)$ superalgebra contains the $SO(1,10)$ covariant bosonic generators $P_a, M_{ab} = -M_{ba}$ and $M_{a_1...a_5} = M_{[a_1...a_5]}$. A contraction of $OSp(1|32)$ produces the M–algebra [42, 43] with $M_{ab}$ and $M_{a_1...a_5}$ becoming tensorial central charges.

To compute the $OSp(1|2n)$ Cartan forms we choose the following parametrization of the $OSp(1|2n)$ supergroup element

\[
G(y, \theta) = B(y) F(\theta) = B(y) e^{i\theta^\alpha Q_\alpha},
\]

where remember that $y^{\alpha\beta} = y^{\beta\alpha}$ are $Sp(2n)$ coordinates.

The $OSp(1|2n)$ Cartan forms are

\[
\frac{1}{i} G^{-1}(y, \theta) dG(y, \theta) \equiv \frac{1}{i} \left[ F^{-1}(B^{-1}dB)F + F^{-1}dF \right] = E^\alpha Q_\alpha + \frac{1}{2} \Omega^\alpha_{\alpha\beta} M_{\alpha\beta},
\]

Let us start with computing the $F^{-1}dF$ term of (A.22).

\[
\frac{1}{i} F^{-1}(\theta)dF(\theta) = \sum_{n=0}^{\infty} \frac{i^n}{(n+1)!} Ad^\theta_{Q}(d^{\theta}Q) \equiv E^\alpha Q_\alpha + \frac{1}{2} \Omega^\alpha_{\alpha\beta} M_{\alpha\beta},
\]
where
\[ Ad_B A \equiv [A, B] \] (A.24)

To calculate the forms \( \Omega_1^{\alpha\beta} \) and \( \mathcal{E}^\alpha \) (A.23), note that
\[
Ad_{\theta Q}(d\theta Q) \equiv [d\theta Q, \theta Q] = -d\theta^{(\alpha \theta \beta)} M_{\alpha\beta} \] (A.25)
\[ Ad_{\theta Q}^2(d\theta Q) \equiv [[d\theta Q, \theta Q], \theta Q] = -i\alpha d\theta^{(\beta \theta \alpha)} \theta_\beta Q_\alpha \] (A.26)
\[ Ad_{\theta Q}^3(d\theta Q) \equiv [[[d\theta Q, \theta Q], \theta Q], \theta Q] = -\left( \frac{i\alpha}{2} \theta^\gamma \theta_\gamma \right) [d\theta Q, \theta Q] \] (A.27)
\[ Ad_{\theta Q}^{l+2}(d\theta Q) = -\left( \frac{i\alpha}{2} \theta^\gamma \theta_\gamma \right) Ad_{\theta Q}^l(d\theta Q) \quad \text{for} \quad l \geq 1 \] (A.28)

Thus we arrive at the recursion relation
\[ Ad_{\theta Q}^{l+2}(d\theta Q) = -\left( \frac{i\alpha}{2} \theta^\gamma \theta_\gamma \right) Ad_{\theta Q}^l(d\theta Q) \quad \text{for} \quad l \geq 1 \] (A.29)

and can express all higher commutators through either (A.25) or (A.26) multiplied by a corresponding power of \( \left( \frac{i\alpha}{2} \theta^\gamma \theta_\gamma \right) \).

In such a way we arrive at the generic expression for the forms (A.23)
\[ \mathcal{E}^\alpha = d\theta^\alpha + id\theta^{(\alpha \theta \beta)} \theta_\beta \sum_{l=0}^{n-1} \frac{\alpha}{(2l+3)!} \left( \frac{i\alpha}{2} \theta^\gamma \theta_\gamma \right)^l \] (A.30)
\[ \Omega_1^{\alpha\beta} = -id\theta^{(\alpha \theta \beta)} \sum_{\lambda=1}^{n-1} \frac{1}{(2\lambda+2)!} \left( \frac{i\alpha}{2} \theta^\gamma \theta_\gamma \right)^l \] (A.31)

To calculate the first term in (A.22)
\[ \frac{1}{l} F^{-1}(B^{-1} dB) F = F^{-1}(\frac{1}{2} \omega^{\alpha\beta} M_{\alpha\beta}) F \equiv E_0^\alpha + \frac{1}{2} \Omega_0^{\alpha\beta} M_{\alpha\beta} \] (A.32)
we note that because \( \omega^{\alpha\beta}(y) \) is symmetric the following relation holds
\[ \theta^\gamma \omega^{(\alpha \theta \beta)} = \frac{1}{2} \theta^\gamma \theta_\gamma (\theta \omega)^\alpha. \] (A.33)

Then one finds
\[ Ad_{\theta Q}^{l+2}(\frac{1}{2} \omega M) = -\left( \frac{i\alpha}{2} \theta^\gamma \theta_\gamma \right) Ad_{\theta Q}^l(\frac{1}{2} \omega M) \quad \text{for} \quad l \geq 1 \] (A.34)

Using (A.32) we get the following expressions for the forms (A.30)
\[ E_0^\alpha = \frac{\alpha}{2} (\theta \omega)^\alpha \sum_{l=0}^{n-1} \frac{1}{(2l+1)!} \left( \frac{i\alpha}{2} \theta^\gamma \theta_\gamma \right)^l \] (A.35)
\[ \Omega_0^{\alpha\beta} = \omega^{\alpha\beta}(y) - \frac{i\alpha}{2} (\theta \omega)^{(\alpha\theta\beta)} \sum_{l=0}^{n-1} \frac{1}{(2l+2)!} \left( \frac{i\alpha}{2} \theta^\gamma \theta_\gamma \right)^l \]  

(A.34)

Note that in (A.33) and (A.34) the polynomials in \( \theta^\gamma \theta_\gamma \) are the same as in (A.28) and (A.29). Thus, inserting (A.28), (A.29), (A.33) and (A.34) into (A.22) we get the following expressions for the \( OSp(1|2n) \) Cartan forms.

\[ E^\alpha = \mathcal{D} \theta^\alpha + i \mathcal{D} \theta^{(\alpha\theta\beta)} \theta_\beta P_1(\theta \theta), \]  

(A.35)

\[ \Omega^{\alpha\beta} = \omega^{\alpha\beta}(y) + i \theta^{(\alpha} \mathcal{D} \theta^{\beta)} P_2(\theta \theta), \]  

(A.36)

where

\[ P_1(\theta \theta) = \sum_{l=0}^{n} \frac{\alpha}{(2l+3)!} \left( \frac{i\alpha}{2} \theta^\gamma \theta_\gamma \right)^l \]  

(A.37)

\[ P_2(\theta \theta) = \sum_{l=0}^{n} \frac{1}{(2l+2)!} \left( \frac{i\alpha}{2} \theta^\gamma \theta_\gamma \right)^l, \]  

(A.38)

and

\[ \mathcal{D} \theta^\alpha = d \theta^\alpha + \frac{\alpha}{2} \omega^\alpha_\beta (y) \theta^\beta. \]  

(A.39)

The polynomial \( P_2 \) (A.38) can be hidden into rescaled \( \Theta = \sqrt{P_2} \theta \), so that \( \Omega^{\alpha\beta} \) become bilinear in Grassmann variables

\[ \Omega^{\alpha\beta} = \omega^{\alpha\beta}(y) + i \Theta^{(\alpha} \mathcal{D} \Theta^{\beta)}. \]  

(A.40)

It is then not hard to verify (using the Maurer–Cartan equations (63) and (54)) that the odd Cartan forms (A.33) take the form

\[ E^\alpha = P(\Theta^2) \mathcal{D} \Theta^\alpha - \Theta^\alpha \mathcal{D} P(\Theta^2), \quad \text{where} \quad P(\Theta^2) = \sqrt{1 + \frac{i\alpha}{8} \Theta^\beta \Theta_\beta}. \]  

(A.41)

Having in hand the \( OSp(1|2n) \) Cartan forms it is straightforward to get the Cartan forms corresponding to any coset superspace \( OSp(1|2n)/H \) with \( H \) being a bosonic subgroup of \( OSp(1|2n) \). To this end in (A.40) and (A.41) one should simply put to zero all parameters \( y^{\alpha\beta} \) corresponding to the subgroup \( H \). Then \( \omega^{\alpha\beta} \) will depend only on the bosonic coordinates of the supercoset \( OSp(1|2n)/H \), and (A.40) will contain the even supervielbeins and the spin connection of \( OSp(1|2n)/H \).

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