Electromagnetic emissions from near-horizon region of an extreme Kerr-Taub-Nut black hole

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Abstract

We have studied electromagnetic line emissions from near-horizon region in the extremal Kerr-Taub-Nut black hole spacetime and then probe the effects of nut charge on the electromagnetic line emissions. Our result indicate that the electromagnetic line emission in the Kerr-Taub-Nut black hole case is brighter than that in the case of Kerr black hole for the observer in the equatorial plane, but it becomes more faint as the observer’s position deviates far from the equatorial plane. Moreover, we also probe effects of redshift factor on electromagnetic emission from near-horizon region in the extremal Kerr-Taub-Nut black hole spacetime.

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I. INTRODUCTION

The existence of black hole in our Universe is confirmed exactly by the gravitational waves detected by LIGO [1–6], which triggers a new era in observing black hole astrophysics. Another important experiment of observing astrophysical black hole is the Event Horizon Telescope, which could capture the first image of the supermassive black hole at the center of our Galaxy [7–12], and then provide us a great deal of information in the near-horizon region of the black hole. With these characteristic information, one can identify the black hole parameters and examine theories of gravity. For a rapidly rotating Kerr black hole, one of such kind characteristic information is the so-called “Near-Horizon Extremal Kerr line (NHEK line)” , which is a vertical line segment on the edge of the shadow of the high spinning black hole [13–16]. A. Lupsasca et al [17] investigated the universal feature of the observed flux at spatial infinity for the electromagnetic line emissions from the NHEK line, which is emitted by the particles in the innermost part of a radiant thin accretion disk around a high-spin black hole. Their result indicate that the luminosity of electromagnetic line emission for observer is the brightest as both the sources and observer lie in the equatorial plane. Moreover, they also discuss the change of the flux with the redshift factor. These properties of electromagnetic line emissions could be applied to analyze the behavior of FeKα line emissions from high-spin black holes observed by the experiments, such as XMM-Newton, Suzaku, and NuSTAR [18–21]. Obviously, the properties of the electromagnetic line emissions depend on black hole parameters. In order to understand features of black hole and to examine further the no-hair theorem, it is very significant to study in detail the properties of these near-horizon electromagnetic emissions from other rapidly rotating black holes in different theories of gravity.

In general relativity, another important stationary and axisymmetric metric is Kerr-Taub-NUT (KTN) metric, which is a solution of Einstein field equations with gravitomagnetic monopole and dipole moments [22, 23]. Besides the mass $M$ and the spinning parameter $a$, the KTN spacetime own an extra NUT charge, $n$, which plays the role of a magnetic mass inducing a topology in the Euclidean section. The presence of the nut charge gives arise to the change of the spacetime structure, for example, the KTN spacetime is not asymptotically flat and there exist conical singularities on its axis of symmetry. In spite of these undesired behaviors, the KTN spacetime still is attractive for exploring various physical phenomena in general relativity [24–34]. For example, the size of the black hole shadow is found to be increase with the NUT charge [30]. Moreover, the particle acceleration has also been investigated in the background of the KTN spacetime [31], which shows that the nut charge modifies the restrict conditions of the rotation parameter $a$ when the
arbitrarily high center-of-mass energy appears in the collision of two particles. The effects of the nut charge on the motion of photon, the inner-most stable circular orbits and the perfect fluid disk around the black hole have also been studied in Refs. [33, 34], respectively. For a high spinning KTN black hole, it is well known that there also exists a vertical line segment on the edge of the black hole shadow [30], which can be called “Near-Horizon Extremal KTN line (NHEKTN line)” as in the case of Kerr black hole. In this paper, we will study the electromagnetic line emissions from the NHEKTN line located in the near-horizon region in the extremal KTN black hole spacetime, and then probe how the nut affects the fluxes of such kind electromagnetic emissions.

This paper is organized as follows. In Sec.II, we adopt to the method used in Ref.[17] and present the formula of the flux for the electromagnetic line emissions from near-horizon region in the extremal KTN black hole spacetime. In Sec.III, we analyze numerically the effects of nut charge on the electromagnetic line emissions from near-horizon region in the extremal KTN background. Finally, we present results and a brief summary.

II. ELECTROMAGNETIC LINE EMISSIONS FROM NEAR-HORIZON REGION OF AN EXTREME KERR-TAUB-NUT BLACK HOLE

KTN spacetime is a stationary, axisymmetric vacuum solution of Einstein equation, which describes the gravity of a rotating source equipped with a gravitomagnetic monopole moment. The line element for the KTN spacetime, in the Boyer-Lindquist coordinates, can be expressed as

\[ ds^2 = g_{tt} dt^2 + g_{\hat{r}\hat{r}} d\hat{r}^2 + g_{\theta\theta} d\theta^2 + g_{\phi\phi} d\phi^2 + 2g_{t\phi} dt d\phi, \]

with

\[ g_{tt} = -\frac{\Delta - a^2 \sin^2 \theta}{\Sigma}, \quad g_{t\phi} = \frac{\Delta \chi - a(\Sigma + a\chi) \sin^2 \theta}{\Sigma}, \]

\[ g_{\phi\phi} = \frac{(\Sigma + a\chi)^2 \sin^2 \theta - \chi^2 \Delta}{\Sigma}, \]

\[ g_{\hat{r}\hat{r}} = \frac{\Sigma}{\Delta}, \quad g_{\theta\theta} = \Sigma, \quad g_{\phi\phi} = \Sigma = \hat{r}^2 + (n + a \cos \theta)^2, \]

\[ \Delta = \hat{r}^2 - 2M \hat{r} + a^2 - \hat{n}^2, \quad \chi = a \sin^2 \theta - 2\hat{n} \cos \theta, \]

where \( M, a, \) and \( \hat{n} \) are the mass, the rotation parameter, and the nut charge of the source, respectively. The nut charge \( \hat{n} \) describes the gravitomagnetic monopole strength. As \( \hat{n} \) vanishes, the metric reduces to that of the usual Kerr spacetime. The positions of event horizon and Cauchy horizon of the KTN spacetime are
located at

\[ \hat{r}_{H,C} = M \pm \sqrt{M^2 - a^2 + \hat{n}^2}, \]  

which are the roots of the equation

\[ \hat{r}^2 - 2M\hat{r} + a^2 - \hat{n}^2 = 0. \]  

With increase of the nut parameter \( \hat{n} \), it is obvious that the radius of the event horizon becomes larger. As \( M^2 - a^2 + \hat{n}^2 = 0 \), these two horizons emerge and then the KTN black hole turns to be extremal in this case.

As in Ref. [17], let us now suppose a thin, stationary and axisymmetric accretion disk around the a KTN black hole in which the timelike particles move along the circular orbit in the equatorial plane. The timelike geodesic equation for particle in the spacetime \( \Pi \) can be expressed as

\[ \dot{t} = \frac{g_{t\phi} \hat{L} + g_{\phi\phi} \hat{E}}{g_{t\phi} + g_{tt} g_{\phi\phi}}, \]

\[ \dot{\phi} = -\frac{g_{t\phi} \hat{E} + g_{tt} \hat{L}}{g_{t\phi} + g_{tt} g_{\phi\phi}}, \]

\[ g_{\hat{r}\hat{r}} \dot{\hat{r}}^2 + g_{\theta\theta} \dot{\theta}^2 = V_{eff}(\hat{r}, \theta) = \frac{g_{\phi\phi} \hat{E}^2 + 2g_{t\phi} \hat{E} \hat{L} + g_{tt} \hat{L}^2}{g_{t\phi} + g_{tt} g_{\phi\phi}} - \mu^2, \]  

where \( \hat{E} \) and \( \hat{L} \), respectively, are the conserved energy and the conserved \( z \)-component of the angular momentum. The overhead dot represents a derivative with respect to the affine parameter. A circular orbit for a particle in the equatorial plane \( \theta = \frac{\pi}{2} \) must satisfy

\[ V_{eff}(\hat{r}, \theta) \bigg|_{\theta = \frac{\pi}{2}} = 0, \quad \frac{dV_{eff}(\hat{r}, \theta)}{d\hat{r}} \bigg|_{\theta = \frac{\pi}{2}} = 0. \]  

In the KTN spacetime, the four-velocity of particle moving along a circular orbit with a radius \( \hat{r} = \hat{r}_s \) in the equatorial plane can be given by [35]

\[ u_s = u_s^t (\partial_t + \Omega_s \partial_\phi), \]

\[ u_s^t = \frac{\sqrt{\hat{r}^2 + \hat{n}^2} + a \sqrt{W}}{\sqrt{\hat{r}^2 + \hat{n}^2} \sqrt{\hat{r}^3 - 3M \hat{r}^2 - \hat{n}^2 (3\hat{r} - M) + 2a \sqrt{\hat{r}}} \}, \]

\[ \Omega_s = \frac{\sqrt{W}}{(\hat{r}^2 + \hat{n}^2) \sqrt{\hat{r}} + a \sqrt{W}}, \]  

with

\[ W = (\hat{r}^2 - \hat{n}^2)M + 2\hat{n}^2 \hat{r}, \]  

where the subscript \( s \) denotes “source”. Together with the condition \( \frac{d^2V_{eff}(\hat{r}, \theta)}{d\hat{r}^2} \bigg|_{\theta = \frac{\pi}{2}} = 0 \), one can get the radius of the innermost stable circular orbit \( \hat{r}_{ISCO} \) for the particle around the black hole, which can not be
analytically given in the KTN spacetime \( \text{(i)} \). The circular orbit with radius \( \hat{r}_s \) only larger than \( \hat{r}_{ISCO} \) is stable for the timelike particle.

As in Ref.\[17\], we here assume that all of electromagnetic radiation caused by the source propagate along null geodesic from the disk to the distant observer located in the position with the polar coordinate \( r_0 \) and the polar angle \( \theta_0 \). There are also three conserved quantities for these photons from the source, i.e., the energy \( E \), the angular momentum \( L \) and the Cater constant

\[
Q = p_0^2 - \cos^2 \theta \left[ (a^2 - 4\hat{n}^2 \csc^2 \theta)p_t^2 - \csc^2 \theta p_\theta^2 \right] - 4\hat{n} \cos \theta p_\phi (ap_t - \csc^2 \theta p_\phi). \tag{9}
\]

With two rescaled quantities,

\[
\hat{\lambda} = \frac{L}{E}, \quad \hat{q} = \frac{\sqrt{Q}}{E}, \tag{10}
\]

the null geodesic equation for the photon can be further rewritten as

\[
\int_{\hat{r}_s}^{\hat{r}_0} \frac{d\hat{r}}{\pm \sqrt{\hat{R}(\hat{r})}} = \int_{\theta_s}^{\theta_0} \frac{d\theta}{\pm \sqrt{\hat{\Theta}(\theta)}} \tag{11}
\]

with

\[
\hat{R}(\hat{r}) = (\hat{r}^2 + a^2 + \hat{n}^2 - a\hat{\lambda})^2 - (\hat{r}^2 - 2M\hat{r} + a^2 - \hat{n}^2)[\hat{q}^2 + (\hat{\lambda} - a)^2], \tag{12}
\]

\[
\hat{\Theta}(\theta) = \hat{q}^2 + \cos^2 \theta \left( a^2 - \frac{4\hat{n}^2 + \hat{\lambda}^2}{\sin^2 \theta} \right) + 4\hat{n} \cos \theta \left( a - \frac{\hat{\lambda}}{\sin^2 \theta} \right). \tag{13}
\]

Making use of the energy of photons at the source \( E_s = -p \cdot u_s \), and the energy at the distant observer \( E_o = E = -p_t \), one can define the redshift factor \( g \) as

\[
g \equiv \frac{E_o}{E_s} = \frac{\sqrt{(\hat{r}^2 + \hat{n}^2)[\hat{r}^3 - 3M\hat{r}^2 - \hat{n}^2(3\hat{r} - M) + 2a\sqrt{rW}]} \sqrt{\hat{r}^2 + \hat{n}^2} + (a - \hat{\lambda})\sqrt{W}}}{\hat{r}^2 + \hat{n}^2 + (a - \hat{\lambda})\sqrt{W}}. \tag{14}
\]

With Eqs. \(\text{(11)}\) and \(\text{(14)}\), one can find that both the conserved quantities \( \hat{\lambda} \) and \( \hat{q} \) can be determined by the radius \( r_s \) and the redshift factor \( g \). Moreover, it is obvious that the photons which can arrive at observer determine an image of the emitter on the observer’s screen. The position of the image on the observer’s local sky can be described by the angular coordinates \( (\alpha, \beta) \), which are related to \( (\hat{\lambda}, \hat{q}) \) by

\[
\alpha = -\frac{\hat{\lambda}}{\sin \theta_0}, \quad \beta = \pm \sqrt{\hat{\Theta}(\theta_0)}. \tag{15}
\]

Thus, the solid angle for a light ray from the disk can be expressed as

\[
d\Omega = \frac{1}{\hat{r}_o^2} d\alpha d\beta = \frac{1}{\hat{r}_o^2} \left| \frac{\partial(\alpha, \beta)}{\partial(\hat{\lambda}, \hat{q})} \right| \left| \frac{\partial(\hat{\lambda}, \hat{q})}{\partial(\hat{r}_s, g)} \right| d\hat{r}_s dg. \tag{16}
\]
From Eqs. (13) and (15), one can find that the first Jacobian has a form

$$\left| \frac{\partial (\alpha, \beta)}{\partial (\hat{\lambda}, \hat{q})} \right| = \frac{\hat{q}}{\sin \theta_o |\beta|}. \quad (17)$$

However, in general, the second Jacobian must be resort to the numerical calculation technology since $\hat{q}$ is not analytical function of $\hat{r}_s$ and $g$. The specific flux of the ray of light measured at observer is

$$dF_o = I_o d\Omega = g^3 I_s d\Omega, \quad (18)$$

where the relationship between the surface brightness seen by the distant observer $I_o$ and the surface brightness evaluated at the source $I_s$ is obtained from Liouville’s theorem on the invariance of the phase space density of photons. For the simplicity, as in Ref. [17], we can suppose that the disk’s specific intensity is monochromatic at energy $E_*$ and isotropic with surface emissivity $\mathcal{E}(\hat{r}_s)$, i.e.,

$$I_s = \mathcal{E}(\hat{r}_s) \delta (E_s - E_*) = g \mathcal{E}(E_o - g E_*), \quad (19)$$

and then obtain the flux at observer is

$$F_o = \frac{g^4}{\hat{r}_s^2 \sin \theta_o} \int \frac{\hat{q}}{|\beta|} \left| \frac{\partial (\hat{\lambda}, \hat{q})}{\partial (\hat{r}_s, g)} \right| \mathcal{E}(\hat{r}_s) d\hat{r}_s, \quad (20)$$

where a factor of $E_*$ is absorbed into $\mathcal{E}(\hat{r}_s)$. The integral is to be calculated over the radial extent of the accretion disk starting from the innermost stable circular orbit. Obviously, the flux $F_o$ depends on the radius of circular orbit of “source” $\hat{r}_s$, the parameters of photon $\hat{q}$ and $\hat{\lambda}$, the position of observer $(\hat{r}_o, \theta_o)$ and the disk model described by function $\mathcal{E}(\hat{r}_s)$.

Let us now focus on the case of extremal KTN black hole and then probe the emissions originating from the innermost part of the accretion disk near the innermost stable circular orbit. In this special case, one can get an analytic expression for the Jacobian determinant $\partial (\hat{\lambda}, \hat{q})/\partial (\hat{r}_s, g)$, which is convenient to study analytically the properties of the flux $F_o$. For the extremal KTN black hole with $a = \sqrt{M^2 + n^2}$, one can introduce dimensionless radial coordinate and parameters [13, 15, 17]

$$r = \frac{\hat{r} - M}{M}, \quad \lambda = 1 - \frac{\hat{\lambda}}{2M \sqrt{1 + n^2}}, \quad q^2 = 3 - n^2 - \frac{\hat{q}^2}{M^2}, \quad (21)$$

where $n \equiv \frac{\hat{n}}{M}$. With these quantities, the geodesic equation (11) becomes

$$\int_{r_s}^{r_o} \frac{dr}{\pm \sqrt{R(r)}} = \int_{\theta_s}^{\theta_o} \frac{d\theta}{\pm \sqrt{\Theta(\theta)}}, \quad (22)$$
with

\[ R(r) = r^4 + 4r^3 + \left[q^2 + 4\lambda(2 - \lambda)(n^2 + 1)\right]r^2 + 8\lambda r(n^2 + 1) + 4\lambda^2(n^2 + 1)^2, \]  
(23)

\[ \Theta(\theta) = 3 - q^2 + \cos^2 \theta - n^2 \sin^2 \theta - 4 \left[ (\lambda - 1)^2 + n^2(\lambda^2 - 2\lambda + 2) \right] \cot^2 \theta 
+ 4n\sqrt{1 + n^2} \left[ 2(\lambda - 1) + \sin^2 \theta \right] \frac{\cos \theta}{\sin^2 \theta}. \]  
(24)

When the source point is near the horizon \((r_s \ll 1)\) and the observation point \(r_o\) is in the spatial infinite far region \((r_o \gg 1)\), one can find that the electromagnetic signal from the near-horizon region of extremal KTN black hole lies in the NHEKTN line on the observer's sky with the coordinates

\[ \alpha = -2M \sqrt{n^2 + 1} \csc \theta_o, \]
\[ \beta = \pm M \left[ 3 - q^2 + \cos^2 \theta_o - n^2 \sin^2 \theta_o - 4(2n^2 + 1) \cot^2 \theta_o - 4n\sqrt{n^2 + 1} \left( \frac{2 - \sin^2 \theta_o \cos \theta_o}{\sin^2 \theta_o} \right)^{1/2} \right]. \]  
(25)

For a fixed redshift factor \(g\), the dominant contribution in the flux of electromagnetic emission from sources at \(r_s\) comes from the photons whose \(r_s\) is a near-region radial turning point for the null geodesics. In the near-horizon region, \(R(r)\) can be approximated as

\[ R_o(r) = q^2 r^2 + 8\lambda(n^2 + 1)r + 4\lambda^2(n^2 + 1)^2, \]  
(26)

and then the turning point \(r_s\) in radial direction is obtained by solving \(R(r) = 0\),

\[ r_s = -\frac{2\lambda}{q^2}(n^2 + 1)(2 + \sqrt{4 - q^2}). \]  
(27)

Moreover, from Eq. (14) with the condition \(r_s \ll 1\), we have

\[ \lambda = \sqrt{\frac{3 - n^2}{n^2 + 1}} \left( \frac{1}{g} - \sqrt{\frac{3 - n^2}{n^2 + 1}} \right) \frac{r_s}{4}. \]  
(28)

Combining equation Eq. (27)-28, we obtain

\[ q = \frac{\sqrt{3 - n^2}}{2g} \left[ \left( g - \sqrt{\frac{n^2 + 1}{3 - n^2}} \right) (n^2 + 5)g + \sqrt{3 - n^2}\sqrt{n^2 + 1} \right]^{1/2}, \]  
(29)

which \(q\) depends on both \(g\) and the nut charge \(n\). As \(n\) disappears, we find that \(q\) is only a function of \(g\), which is consistent with that obtained in Ref. [17]. From Eqs. 25, 27 and 29, we find that \(\lambda < 0\), and

\[ 0 < q < q_c = \left[ 3 + \cos^2 \theta_o - n^2 \sin^2 \theta_o - 4(2n^2 + 1) \cot^2 \theta_o - \frac{4n\sqrt{n^2 + 1} \left( 2 - \sin^2 \theta_o \cos \theta_o \right)}{\sin^2 \theta_o} \right]^{1/2}, \]  
(30)

\[ \sqrt{\frac{n^2 + 1}{3 - n^2}} < g < \frac{\sqrt{(n^2 + 1)(3 - n^2)(n^2 + 1 + 2\sqrt{4 - q^2_{\text{max}}})}}{(3 - n^2)(n^2 + 5) + 4q^2_{\text{max}}}. \]  
(31)
Here $q_{\text{max}}$ is the maximum value of $q_c$ in Eq. (30), which has no analytical expression in the KTN spacetime. As the nut charge $n \to 0$, one can find that $q_{\text{max}} = \sqrt{3}$, which reduces to that in Kerr case. Inserting Eq. (29) into Eq. (25), we obtain the angular coordinate $\beta$ of the points in the NHEKTN line on the observer’s sky

$$\beta = \pm M \left[ 3 + \cos^2 \theta_o - n^2 \sin^2 \theta_o - 4(2n^2 + 1) \cot^2 \theta_o - 4n \sqrt{n^2 + 1} \left( \frac{2 - \sin^2 \theta_o}{\sin^2 \theta_o} \right) \cos \theta_o \right]^{1/2}.$$  

From Eqs. (28)-(29), we find that the second Jacobian can be expressed as

$$\frac{\partial(\lambda, q)}{\partial(r_s, g)} = \frac{(3 - n^2)}{16qg^4} \left( \sqrt{3 - n^2} + \sqrt{n^2 + 1}g \right) \left( \sqrt{n^2 + 1} - \sqrt{(3 - n^2)g} \right),$$

and then the flux at the observer can be further rewritten as

$$F_o = \frac{(3 - n^2) \sqrt{n^2 + 1}}{8r_s^2 \sin \theta_o |\beta|} \left( \sqrt{3 - n^2} + \sqrt{n^2 + 1}g \right) \left( \sqrt{3 - n^2g} - \sqrt{n^2 + 1} \right) \int E(r_s) dr_s,$$

which implies that the flux at different points on the observer’s NHEKTN line is dominated by photons of different energy. In terms of the discussions in Ref. [17], the role of the integral $\int E(r_s) dr_s$ in the above equation is only to fix the overall scale of $F_o$ for the particular disk model by the surface emissivity function $E$. This integral is divergent for the extremal black hole, which are likely caused by the caustics [30] and could be regulated by diffraction effects in wave optics. The factor before the integral $\int E(r_s) dr_s$ is independent of disk models and presents the universal features of electromagnetic line emissions from the near-horizon region of extremal KTN black hole. Thus, the universal properties of the observed flux $F_o$ can be described by

$$F_o \propto \frac{(3 - n^2) \sqrt{n^2 + 1}}{8r_s^2 \sin \theta_o |\beta|} \left( \sqrt{3 - n^2} + \sqrt{n^2 + 1}g \right) \left( \sqrt{3 - n^2g} - \sqrt{n^2 + 1} \right).$$

Obviously, $F_o$ is a function of both the redshift factor $g$ and the angular coordinate $\beta$ on the NHEKTN line, which depend on the nut charge $n$ and the observer’s angular position $\theta_o$. The dependence of the flux $F_o$ on the $n$ could provide a potential observable to examine whether the real spacetime belongs to such kind of KTN spacetime.

**III. EFFECTS OF NUT CHARGE ON THE ELECTROMAGNETIC LINE EMISSIONS FROM NEAR-HORIZON REGION**

We are now in position to study the effects of the nut charge on electromagnetic line emissions from near-horizon region in the extremal KTN black hole spacetime. In Fig. [1], we present the change of the flux $F_o$ at observer with the angular coordinate ratio $\beta/\beta_{\text{max}}$ for different nut charge $n$ and the observer’s angular
FIG. 1: The change of the flux $F_o$ at observer with the angular coordinate $\beta/\beta_{\text{max}}$ for different nut charge $n$ and the observer’s angular position $\theta_o$ as the source lies in the NHEKTN line. The NHEKTN line is a vertical edge in the shadow depicted in solid line in subfigure (a). Here, we set $M = 1$.

As $\theta_o$ as the source lies in the NHEKTN line, which is a vertical edge in the shadow depicted in solid line in Fig. (1a). $\beta_{\text{max}}$ is the maximum value of the coordinate $\beta$ of the image located in the NHEKTN line. From Fig. (1), we find that the flux $F_o$ decreases with the ratio $\beta/\beta_{\text{max}}$, which means that the brightest electromagnetic line at observer is emitted from the sources located in the equatorial plane. Moreover, for the fixed nut charge $n$, one can find from Figs. (1b) and (1c) that the flux $F_o$ decreases with the deviation of the observer’s position from the equatorial plane, which implies that the electromagnetic line emission is the brightest for the observer in the equatorial plane. These properties of electromagnetic emissions in NEHKTN line are similar to those in the Kerr case. For the observer with the fixed angular coordinate $\theta_o$, we find from Fig. (2) that the change of the flux $F_o$ with the nut charge $n$ also depends on the coordinate $\theta_o$ and the ratio $\beta/\beta_{\text{max}}$. For the case with $\theta_o = \pi/2$, the flux $F_o$ increases monotonously with $n$ for arbitrary fixed ratio $\beta/\beta_{\text{max}}$. As $\theta_o = 45\pi/96$, the flux $F_o$ decreases monotonously with $n$ for the smaller ratio $\beta/\beta_{\text{max}}$, but first increases and then decreases for the larger ratio $\beta/\beta_{\text{max}}$. With the further increase of the deviation quantity...
The change of the flux $F$ at observer with the nut charge $n$ for different $\beta/\beta_{max}$ and the observer’s angular position $\theta_o$ as the source lies in the NHEKTN line. Here, we set $M = 1$.

$\Delta \theta_o = |\theta_o - \pi/2|$ (for example, $\theta_o = 5\pi/12$), the flux $F_o$ becomes a decreasing monotonously function of $n$ for arbitrary ratio $\beta/\beta_{max}$. This means that the electromagnetic line emission from the near-horizon of extremal KTN black hole is brighter than that in the case of Kerr black hole for the observer in the equatorial plane, but it becomes more faint as the observer’s position deviates far from the equatorial plane.

In Fig. (3), we plot the dependence of the flux $F_o$ of electromagnetic emission from NHEKTN line on the redshift factor $g$ for different nut charge $n$ and the observer’s angular position $\theta_o$. Fig. (4) indicates that the flux $F_o$ increases with the redshift factor $g$ for fixed $n$ and $\theta_o$, which could be explained by a fact from Eq. (14) that the larger value of $g$ means that the higher energy at observer $E_o$ can be obtained for the fixed energy $E_x$ of photons at the source. This behavior of $F_o$ is the same as that in the Kerr case [17]. Figs. (5) and (6) tell us that the change of the flux $F_o$ with the nut charge $n$ for the fixed redshift factor $g$ is more complicated, which depends also on the observer’s angular position $\theta_o$. We find that the flux $F_o$ exists only in the case where $g$ is in the certain range of $(g_{c1}, g_{c2})$, and the values of $g_{c1}$ and $g_{c2}$ depend on the nut charge $n$ and the observer’s position $\theta_o$. This is consistent with the previous discussion from Eq. (51). Moreover, the range of $g$ with the non-zero flux $F_o$ becomes narrow with increase of the deviation of observer from the equatorial plane. As the
FIG. 3: The change of the flux $F_0$ at observer with the redshift factor $g$ for different nut charge $n$ and the observer’s angular position $\theta_o$ as the source lies in the NHEKTN line. Here, we set $M = 1$.

observer lies in the equatorial plane, one can find that the flux $F_0$ decreases monotonously with $n$ for fixed redshift $g$. As the observer deviates from the equatorial plane, we find that the flux $F_0$ still decreases with $n$ in the cases with the lower redshift factor $g$ or the larger one, but it first increases and then decreases with $n$ for the case with the intermediate redshift factor $g$. With the further deviation for observer, the region of redshift factor $g$ for the flux decreasing with nut charge becomes gradually narrow. Finally, we find that the region of the flux decreasing with $n$ as $\theta_o = 5\pi/12$ vanishes in the case with the larger redshift factor $g$. Thus,
the nut charge modifies the electromagnetic line emissions from near-horizon region of an extreme KTN black hole.

IV. SUMMARY

We have studied electromagnetic line emissions from near-horizon region in the extremal KTN black hole spacetime. Our result show that the fluxes $F_o$ of electromagnetic line emissions from near-horizon region depend on the nut charge $n$, the positions of source and observer, and the redshift factor $g$. For the fixed nut charge $n$, the flux $F_o$ decreases monotonously with the ratio $\beta/\beta_{\text{max}}$ and the deviation of the observer’s position from the equatorial plane, which means that the luminosity of electromagnetic line emission for observer is the brightest as both the sources and observer lie in the equatorial plane. This property of electromagnetic emissions in NHEKTN line is similar to those in the Kerr case. For the observer with the fixed angular
coordinate $\theta_o$, we find that the electromagnetic line emission from the near-horizon of an extremal KTN black hole is brighter than that in the case of Kerr black hole for the observer in the equatorial plane, but it becomes more faint as the observer’s position deviates far from the equatorial plane.

We also study the dependence of the flux $F_o$ of electromagnetic emission from NHEKTN line on the redshift factor $g$ for different nut charge $n$ and the observer’s angular position $\theta_o$. With the increase of redshift factor $g$, the flux $F_o$ increases for fixed $n$ and $\theta_o$, which is the similar as that in the Kerr case [17]. However, for the fixed redshift factor $g$, the change of the flux $F_o$ with the nut charge $n$ is more complicated. Firstly, the flux $F_o$ exists only in the case where $g$ is in the certain range of $(g_{c1}, g_{c2})$, and the values of $g_{c1}$ and $g_{c2}$ depend on the nut charge $n$ and the observer’s position $\theta_o$. The range of $g$ with the non-zero flux $F_o$ becomes narrow with the deviation of observer from the equatorial plane. As the observer lies in the equatorial plane, the flux $F_o$ decreases monotonously with $n$ for fixed redshift $g$. As the observer deviates from the equatorial plane, the flux $F_o$ still decreases with $n$ in the cases with the lower redshift factor $g$ or the larger one, but it first increases and then decreases with $n$ for the case with the intermediate redshift factor $g$. With the further deviation of observer, the region of redshift factor $g$ for the flux decreasing with nut charge becomes gradually narrow so that the region of the flux decreasing with $n$ finally vanishes in the case with the larger redshift factor $g$. Therefore, the nut charge modifies the electromagnetic line emissions from near-horizon region of an extreme black hole, which could provide a potential observable to examine whether the background spacetime belongs to such kind of KTN spacetime by the high-precision astronomical observations in the future.

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