Determination of Q-factor of Transmission Ground-Penetrating Radar Waves in Attenuating Media

Wahyudi W. Parnadi¹

¹Department of Geophysical Engineering, Institut Teknologi Bandung, Jl. Ganesha 10 Bandung 40132, Indonesia

*Corresponding author: wahyudi@gf.itb.ac.id

Abstract. Electromagnetic wave attenuation in a medium is the primary parameter for interpreting Ground-Penetrating Radar (GPR) data. This attenuation manifests in wave energy absorption and velocity dispersion, which have different behavior depending on the rock type. Knowledge of the attenuation behavior of GPR waves in distinct media is, therefore, a success key to distinguishing rock types. The Q that is inversely proportional to the absorption coefficient represents its attenuation behavior. To observe the attenuation behavior of rocks, we conducted two different stages. The first stage is simulated the attenuation behavior by creating synthetic traces as a convolution between the wavelet and the corresponding impulse response. A Q-constant model and Futterman’s velocity dispersion were used for the impulse response calculation. In the second stage, Q was determined using three different Q determination methods: the amplitude decay method, the spectral ratio method, and the Equivalent Bandwidth (EBW) method. Our research shows that due to the absorption, the wave amplitude becomes lower with increasing distance at the same Q, while at the same distance, its wavelength is longer, and its amplitude becomes low. The longer the distance at the same Q value, the narrower is the bandwidth of the transmission signals. We got a similar result with a small Q at the same distance length. At the same time, its peak frequency shifted to lower frequencies. Moreover, the applicability of the three methods for the determination of Q was investigated and tested on field transmission data acquired at the test field in the Reiche-Zeche Mine Shaft near Freiberg in Germany. Our investigation result shows that the three Q-methods were successfully applied to field transmission data resulting in comparable Q values ranging from 31 to 36 for gneiss rocks.

1. Introduction
Electromagnetic (EM) waves will undergo an attenuation process during their propagation in geological media which causes the wave energy to decrease [1]. In that process, the waves will experience energy absorption and velocity dispersion. The absorption of EM waves is reflected by lower wave amplitudes with longer distance, while its dispersion effect causes the wavelength to increase.

How the EM waves experience attenuation during their propagation in a medium is illustrated in Figure 1. The output signal is a product of the input signal convoluted with the impulse response of the medium. The phase velocity v and the absorption coefficient α are two quantities describing the propagation of electromagnetic waves in a medium, which are frequency-dependent. For Ground-Penetrating Radar (GPR) waves, which are in the frequency range 1 MHz – 1000 MHz, the absorption coefficient α is determined by the ratio between the real component (ε′) and imaginary component (ε″) of the complex relative permittivity εr.
The impulse response $g(t)$ is affected by petrophysical parameters like porosity $\phi$, density $\gamma$, grain size $G_s$, permeability $\zeta$, degree of mineralization $S_m$, and water saturation $S_w$, which have significant meaning in geology and geotechnical fields [2] as well as in the exploration field [3]. Therefore, measuring the attenuation behavior of EM waves is appropriate for differentiating rocks. Thus the determination of both phase velocity $v$ and absorption coefficient $\alpha$ can lead to structural as well as rock-type interpretations. GPR waves behave similarly to seismic waves so that their propagation rules follow the rule of seismic waves. In line with the linear dependence of the frequency of the seismic wave absorption coefficient, the absorption coefficient of the GPR wave is also frequency-dependent, such that the Q factor of the GPR wave is frequency-independent [1,4,5]. The Q factor has an inverse relationship with the absorption coefficient $\alpha$ and clearly explains the absorption power of each different rock to the EM waves propagating on it. Based on our previous studies, we adopted the concept of constant Q model and dispersion velocity according to Futtermann (1962) in this study [6]. Q values of rocks in GPR frequencies lie in the range 2 – 30 [7], and we used it from 2 – 50 in this study.

In this research, we investigated the behavior of the attenuation process in the form of absorption and dispersion in two stages. In the first stage, we conducted a simulation of synthetic traces and reviewed three different methods for Q determination, namely the amplitude decay method, the spectral ratio method, and Q determination through the absorption coefficient $\alpha$. The latter method, we named the equivalent bandwidth method (EBM), is our technique developed at our laboratory. Q values contained in the resulted synthetic traces were then inversely computed using the three different methods. In the second stage, we tested the applicability of the three methods on field data from a test site.

2. Method

2.1. Impulse response
Electromagnetic waves propagation in attenuation geological media can be describe as a linear system, as shown in figure 2 below.
In the time domain, the output signal $y(t)$ is the product of the input signal $x(t)$ convoluted with the medium impulse response $g(t)$. In the frequency domain, the output signal is the product of the spectrum of the input signal $X(\omega)$ multiplied by the transfer function of the medium $G(\omega)$:

$$y(t) = x(t) * r(t) \quad (1)$$

$$Y(\omega) = X(\omega), R(\omega) \quad (2)$$

where $\omega$ denotes the angular frequency and $\omega = 2\pi f$.

The transfer function $R(\omega)$ of attenuating media with causal impulse response $g(t)$ has a form [1]

$$G(\omega) = e^{-\omega t_0/Q} e^{-j\omega t_0} e^{j\omega t_0\ln(\omega/\omega_{NY})} \quad (3)$$

where $t_0$ and $Q$ are travel time and Q factor, respectively. $\omega_{NY} = 2\pi f_{NY}$ and $f_{NY}$ is Nyquist frequency.

Equation (3) describes 3 factors during the propagation of EW waves in media, i.e. the absorption factor in the first term, time factor in the second term, and dispersion factor in the third term. The dependency of the Q factor on the amplitude and the phase spectra is recognizable (see figure 3). In particular with decreasing Q, absorption and dispersion effect increase.

**Figure 2.** Physical processes of an EM wave in attenuating media. describes in time domain (top) and in frequency domain (bottom)

**Figure 3.** Amplitude spectrum (a) and phase spectrum (b) of transfer function $G(\omega)$ in equation (3) and their corresponding impulse response $g(t)$ (c). Parameters used in the calculation: $t_0 = 6.1$ ns, working frequency $f_c = 900$ MHz.
The medium impulse response $g(t)$ is calculated by the inverse of Fourier Transformation of $G(\omega)$:

$$g(t) \leftarrow G(\omega)$$  \hspace{1cm} (4)

2.2. Forward modeling

Synthetic traces forward modelings were conducted using equation 1 with an input signal $w(t)$ and traveltime data listed in table 1 below.

| Case 1 | Input signal A | 3 |
|--------|----------------|---|
| Case 2 | Input signal A | 6 |
| Case 3 | Input signal A | 9 |
| Case 4 | Input signal A | 12 |
| Case 5 | Input signal B | 6 |
| Case 6 | Input signal B | 12 |

The input signal A is a wavelet obtained from measurements in the air with a nominal frequency of 900 MHz whilst input signal B is the one with a nominal frequency of 450 MHz [1].

2.3. Q determination from GPR transmission signal

There are some methods to calculate Q values. Tonn (1991) investigated 10 different methods for Q determination using VSP data. It is concluded that there is no single method that is superior [8]. In this research 3 different methods were reviewed and tested: the amplitude decay method, the spectral ratio method, and the Equivalent Bandwidth Method [1, 8].

2.3.1. The Amplitude decay method

This is the easiest way to Q determination and it is calculated in the time domain. The value of Q is directly proportional to the ratio of the amplitude of the wave over different distances $x_1$ and $x_2$ with traveltime $t_1=x_1/v$ and $t_2=x_2/v$:

$$Q = \frac{\pi f \Delta x}{v} \ln \left[ \frac{a(x_2)}{a(x_1)} \right] = \pi f t_0 \ln \left[ \frac{a(x_1)}{a(x_2)} \right]$$  \hspace{1cm} (5)

where $f$ refers to dominant frequency, $t_0 = t_2 - t_1$ is traveltime different and $\Delta x = x_2 - x_1$. $a(x_1)$ and $a(x_2)$ are signal amplitudes at $x_1$ and $x_2$, respectively.

The limitation of this method is that it can only be applied when actual amplitudes are present. The use dominant frequency here is only an estimated value.

2.3.2. The Spectral ratio

The spectra ratio method is the popular technique in determining Q values, which is calculated in the frequency domain. First, we calculate the logarithmic ratio of amplitude spectra of two different signals $y(t)$ and $x(t)$. The ratio of the corresponding Futtermann’s transfer function [1, 8, 9] appears to be

$$\ln \left[ \frac{y(f)}{w(f)} \right] = \text{const.} - \frac{\Delta t \pi f}{Q}$$  \hspace{1cm} (6)

where $\Delta t$ refers to the travelt ime difference between $y(t)$ and $w(t)$.

The Q value is calculated from the linear line steepness specified in equation (6) as a function of frequency. A general problem in this method is the estimation of frequency intervals.
2.3.3. The Equivalent Bandwidth (EBW) method

This method estimates the Q value in time domain. Q is calculated directly using

\[ Q = \frac{2\pi}{1 - E_2/E_1} \cdot t_0 \cdot f_R \]  

(7)

where \( E_1 \) and \( E_2 \) describe wave energies. Subscript numbers “1” and “2” denote input and output signal, respectively and \( f_R \) is reference frequency.

The reference frequency \( f_R \) (in MHz) was determined experimentally from the cross plot between reference frequency and the ratio between amplitude density value of equivalent rectangle spectrum of input signal and output signal \( \tilde{A}_{21} \) [1]

\[ f_{R, 900} = 771.6(\tilde{A}_{21})^3 - 325.5(\tilde{A}_{21})^2 + 611.3(\tilde{A}_{21}) + 78.5 \]  

(8)

\[ f_{R, 450} = 0.5 f_{R, 900} \]  

(9)

where \( f_{R, 900} \) and \( f_{R, 450} \) are reference frequency for 900 MHz and 450 MHz, respectively.

3. Results and discussion

3.1. Forward modelling

Figures 4 and 5 show the behaviour of absorption and dispersion through the simulation of synthetic traces. Some impulse responses at the same travel time \( t_0 = 3 \text{ ns} \) are shown in figure 4. For a smaller Q value – means higher attenuation - the impulse response becomes smaller and wider. Smaller Q denotes higher energy absorbed. A wider but smaller impulse response has the meaning that more time is needed to deliver wave energy. It is means also that the primary signal energy moves at a later time. Figure 5 shows the course of Impulse responses at the same travel time \( t_0 \), output signals, and their corresponding frequency spectrum at the same travel time \( t_0 = 3 \text{ ns} \) but with different Q values. These figures 4 and 5 show clearly that the peak frequencies are shifted to lower frequencies with decreasing Q values.

![Figure 4](image-url)

**Figure 4.** (a) Impulse responses at the same travel time \( t_0 = 3 \text{ ns} \) with different Q value; (b) output signals using impulse response in (a); and (c) their corresponding frequency spectra. \( t_0 \) used is 3 ns.
Figure 5. The course of impulse responses y(t) at the same Q value with a variety of travel time t0; (b) Output signals using impulse responses in (a) and (c) their corresponding frequency spectra. Q used is 50.

3.2. Investigation on field data
We investigated Q determination with GPR technique on gneiss, which is a crystalline rock, due to two ideal conditions: gneiss has low attenuation and thus great penetration depth for GPR waves [1]. The object as shown in figure 6 is located in the Reiche-Zeche shaft in the “Schwarzer Hirsch” corridor at a depth of around 180 m in Germany. The test object was selected in such a way that it represents a quasi-homogeneous medium.

Two different measurement configurations were realized: horizontal measurement and vertical measurement. In horizontal measurement, both transmitter antenna and receiver antenna were moved together with 0.5 m forward at a fixed height, from O to T for transmitter antenna and from O to R for receiver antenna. The measurements were conducted three times, at a fixed height of 1.20 m to 1.50 m. In vertical measurement, the transmitter antenna at T position and the receiver antenna at R position at the same height were moved together vertically from bottom to top parts of the wall with 20 cm step upwards, beginning at 0.8 m from tunnel floor (see Figure 6.b).

Figure 6. Sketch of the test location (a) and its cross-sections I–I and II–II (b). T and R denote the position of transmitter as well as receiver antennas, respectively.
Figure 7 shows field transmission signals acquired with 900 MHz center antennas for the vertical measurement. To get velocity and absorption coefficient values for the calculation of $Q$ using the Equivalent Bandwidth method, some traces from horizontal measurement were used. Results of $Q$ determination using three different methods are listed in Table 2. $Q$ values using three different methods showed comparable values with $Q$ values determined by the EBW method lie between that of the amplitude decay method and that of the spectral ratio method. It is concluded that the results from three different methods agree well with $Q$ values ranges from 31 to 36. Our results with these methods and compared with the values determined in the literature for gneiss [1] are listed in Table 3.

![Field transmission signals acquired with 900 MHz antennas for vertical measurement](image)

**Figure 7.** Field transmission signals acquired with 900 MHz antennas for vertical measurement

| No. | $v$ (m/ns) | $\alpha$ (dB/m) | $Q$ | $f_c$ (MHz) | Depth (m) | Source               |
|-----|------------|-----------------|-----|-------------|-----------|----------------------|
| 1   | 0.17 – 0.18| 1.7 – 2.1       | 31 - 36 | 900         | 180       | This study           |
| 2   | 0.10 – 0.12|                 |       |             |           | Grasmueck [11]       |
| 3   | 1.12 – 0.12|                 |       |             | up to 30  |                      |
| 4   | 0.17       | 2.6             |       |             |           |                      |
| 5   | 0.12       | 6.9             |       |             |           |                      |
| 6   | 0.17       | 1.5 – 4.5       | 30   |             |           |                      |
| 7   | 0.13       | 9.0 – 27.0      | 7    |             |           |                      |
| 8   | 0.08 – 0.11|                 |       |             |           |                      |
4. Conclusion

The attenuation of electromagnetic waves is the primary parameter for interpreting Ground-Penetrating Radar (GPR) data, which is manifested in the form of energy absorption and velocity dispersion. Measuring its attenuation behaviour is therefore appropriate for differentiating rocks type. To observe the attenuation behaviour of GPR waves, two different stages were undertaken in this study: simulating wave propagation and determining the attenuation level through the Q factor.

Based on the above basic understanding, our simulation results using synthetic traces reveal the behaviour of the attenuation process. It process results in lower wave amplitudes with longer distances at the same Q values and leads to lower wave amplitudes and longer wavelengths with decreasing Q values at the same travel time. The former is known as the absorption process, and the latter is the dispersion process. At the same Q values, the longer the distance is, the narrower the bandwidth of the transmitted signal. Similar behaviour is observable. The smaller the Q values at the same time distance, the narrower is its frequency bandwidth. Its peak frequency shifts to lower frequencies.

The applicability of the three Q methods was then investigated using field data acquired at the test field in the Reiche-Zeche Mine Shaft, near Freiberg in Germany. The results showed that among the above methods, including our proposed formula to determine Q values, all were applicable to field transmission GPR data, resulting in Q values ranging from 31 to 36 for gneiss rocks.

Acknowledgments

This research was partially supported financially by Institut Teknologi Bandung through the P3MI2020 Program.

References

[1] Parnadi W W 2001 Kennwert-Schätzung aus Georadar-Transmissionsdaten PhD thesis: TU Bergakademie Freiberg
[2] Parnadi W W, Warsa W, Laesanpura A and Santoso D 2021 Multi frequency Ground-Penetrating Radar method for mapping underground pipes and man-made objects: IOP Conference Series: Earth and Environmental Science 873(1) 012090
[3] Antonio T, Parnadi W W and Heriawan M N 2021 Integrating Ground Penetrating Radar, Induced Polarization and Aerial Photograph to Analyze Land Subsidence in Borehole Mining Operation Area: A Case Study from South Bangka Island: IOP Conference Series: Earth and Environmental Science 873(1) 012081
[4] Economou N and Kritikakis G 2016 Attenuation analysis of real GPR wavelets: The equivalent amplitude spectrum (EAS): J. Applied Geophysics 126 pp 13–26
[5] Bano M 2004 Modelling of GPR waves for lossy media obeying a complex power law of frequency for dielectric permittivity: Geophysical Prospecting 52 pp 11–26
[6] Parnadi W W, Warsa W and Santoso D 2019 Water content delineation using Ground_penetrating Radar Q tomography: Proc. Internat. Symp. Earth Science and Technology Fukuoka
[7] Turner G and Siggins A F 1994 Constant Q attenuation of subsurface radar pulses: Geophysics 59(8) pp 1192-1200
[8] Tonn R 2001 The determination of the seismic quality factor Q from VSP data: a comparison of different computational methods: Geophysical Prospecting 39 pp 1-27
[9] Futtermann W I 1962 Dispersive body waves: J. Geophysical Research 67(13) pp 5279-5291
[10] Widess M B 1982 Quantifying resolving power of seismic system: Geophysics 47(8) pp 1160-1173
[11] Grasmueck M P 1995 Development of a georadar system for three-dimensional imaging of the subsurface and its application to studies of crystalline rock bodies PhD thesis: ETH Zürich