One-loop renormalizability of all 2d dimensional Poisson-Lie $\sigma$-models

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Abstract

We perform a systematic study of the one-loop renormalizability of all Poisson-Lie T-dualizable $\sigma$-models with two-dimensional targets. We show that whatever Drinfeld double and whatever matrix of coupling constants we consider the corresponding $\sigma$-model is always one-loop renormalizable in the strict field theoretical sense. Moreover, in all cases, the RG flow in the space of the coupling constants is compatible with the Poisson-Lie T-duality.
1 Introduction

Poisson-Lie T-duality [1, 2] is the generalization of the world-sheet Abelian T-duality in string theory [3, 4] and of the traditional non-Abelian T-duality [5, 6, 7]. It works also in situations when a T-dualizable $\sigma$-model does not possess a (non)Abelian isometry but only a weaker property called Poisson-Lie symmetry.

The Poisson-Lie T-dualizable $\sigma$-models (or, in what follows, the PLT $\sigma$-models) are specified by the choice of a Drinfeld double and by the $(n \times n)$-matrix of coupling constants where $n$ is the half of the dimension of the double. From the point of view of classical field theory, the $\sigma$-model and its dual are related by a canonical transformation [8, 9, 10]; they are therefore dynamically equivalent systems.

The quantum status of the Poisson-Lie T-duality remains a challenging problem. We stress at this point that the word “quantum” does not necessarily suppose that a conformal symmetry is to be required. We shall simply study the duality from the point of two-dimensional field-theory. Whereas for the semi-abelian case the one-loop quantum equivalence of T-dual models has been established for a wide class of models [11], the same problem for the non-abelian case is far more difficult. The first steps in this direction were undertaken in [12, 13], where the respective authors established the one-loop renormalizability of certain Poisson-Lie T-dualizable $\sigma$-models and the compatibility of the RG flow with the Poisson-Lie T-duality. They did it for a few low-dimensional Poisson-Lie targets and particular choices of the coupling constants.

In this paper, we would like to perform a more systematic study of this issue. However, we miss a classification of all PLT $\sigma$-models since it would require a preliminary classification of all the Drinfeld doubles (indeed, the latter project seems as hopeless as the classification of all Lie algebras). On the other hand, low dimensional doubles WERE classified in [14, 15, 16]. Therefore we consider, e.g., all existing two-dimensional Poisson-Lie T-dualizable targets. We do it and show that they are one-loop renormalizable and that the RG flow in the space of coupling constants is always compatible with the Poisson-Lie T-duality.

2 Generic 2 dimensional Poisson-Lie models

2.1 Four dimensional Drinfeld doubles

A Drinfeld double $\mathcal{D}$ is a Lie algebra with generators $T_i$, $i = 1, \ldots n$ and $\tilde{T}^i$, $i = 1, \ldots n$, equipped with a symmetric and ad-invariant non-degenerate bilinear form $\langle \cdot, \cdot \rangle$ such that

$$\langle T_i, T_j \rangle = 0, \quad \langle \tilde{T}^i, \tilde{T}^j \rangle = 0, \quad \langle T_i, \tilde{T}^j \rangle = \delta_i^j = \langle \tilde{T}^j, T_i \rangle.$$ (1)

It is moreover required that the linear subspaces $\mathcal{G} \equiv \text{Span}(T_i)$ and $\tilde{\mathcal{G}} \equiv \text{Span}(\tilde{T}^i)$ are respectively the subalgebras of $\mathcal{D}$.

As shown by [14] all the four-dimensional non-isomorphic Drinfeld doubles, denoted as $\mathcal{D}(\rho, \nu)$, can be written

$$\begin{align*}
[T_1, T_2] &= \rho T_2 \\
[T_1, \tilde{T}^1] &= 0 \\
[T_1, \tilde{T}^2] &= -\rho \tilde{T}^2 \\
[\tilde{T}^1, \tilde{T}^2] &= \nu \tilde{T}^2 \\
[T_2, \tilde{T}^1] &= \nu T_2 \\
[T_2, \tilde{T}^2] &= \rho \tilde{T}^1 - \nu T_1
\end{align*}$$ (2)

and are of three non-isomorphic types:

1. The fully abelian double $\mathcal{D}(0, 0)$.
2. The semi-abelian double $D(1,0)$.

3. The non-abelian double $D(1, \nu)$ with $\nu \neq 0$.

Let us construct the Poisson-Lie $\sigma$-models out of these doubles.

### 2.2 The Poisson-Lie models and their T-duals

A general Poisson-Lie T-dualizable $\sigma$-model has for action [1, 8]

$$
\int (R(g) + \Pi)_{ij}^{-1} (\partial_+ g^{-1})^i (\partial_- g^{-1})^j ,
$$

(3)

where

$$
\partial_\pm = \partial_{\tau} \pm \partial_{\sigma}
$$

(4)

and $\tau$ and $\sigma$ are the “time” and “space” coordinates on the world-sheet. The model (3) lives on a group manifold $G$ (corresponding to the Lie algebra $\mathcal{G}$) and it is parametrized by a set of coupling constants assembled into the matrix $R$.

In order to determine the $g$-dependent matrix $\Pi(g)$ one first defines a triplet of matrices as follows

$$
\text{Ad} (g^{-1}) T_i \equiv g^{-1} T_i g = a(g)^i_{\phantom{i}l} T_l , \quad \text{Ad} (g^{-1}) \tilde{T}^i = b(g)^i_{\phantom{i}l} T_l + d(g)^i_{\phantom{i}l} \tilde{T}^l .
$$

(5)

Then

$$
\Pi(g) = b(g) a^{-1}(g).
$$

(6)

Note that the matrices $a, b, d$ are defined by the adjoint action of $g \in G$ on the Lie algebra $\mathcal{D}$.

In our two-dimensional case, we may realize the group $G$ in the matrix way as follows

$$
g = \begin{pmatrix} e^{\rho \chi} & \rho \theta \\
0 & 1 \end{pmatrix}
$$

(7)

and the generators $T_i, \tilde{T}^i$ are given by

$$
T_1 = \begin{pmatrix} \rho & 0 \\
0 & 0 \end{pmatrix} , \quad T_2 = \begin{pmatrix} 0 & \rho \\
0 & 0 \end{pmatrix} , \quad \tilde{T}^1 = \begin{pmatrix} 0 & 0 \\
0 & \nu \end{pmatrix} , \quad \tilde{T}^2 = \begin{pmatrix} 0 & 0 \\
-\nu & 0 \end{pmatrix} .
$$

(8)

Thus we have for the matrices

$$
a(g) = \begin{pmatrix} 1 & \rho \theta e^{-\rho \chi} \\
0 & e^{-\rho \chi} \end{pmatrix} , \quad b(g) = \begin{pmatrix} 0 & -\nu \theta e^{-\rho \chi} \\
\nu \theta & \rho \nu \theta^2 e^{-\rho \chi} \end{pmatrix} .
$$

(9)

which lead to

$$
\Pi(g) = b(g) a^{-1}(g) \quad \Rightarrow \quad \Pi = \begin{pmatrix} 0 & -\nu \theta \\
\nu \theta & 0 \end{pmatrix} .
$$

(10)
The matrix $R$ is
\[
R = \begin{pmatrix} x & y \\ z & w \end{pmatrix}, \quad \delta \equiv \det R = xw - yz \neq 0,
\]
and the right-invariant vielbein
\[
dg g^{-1} = d\chi T_1 + (d\theta - \rho\theta d\chi) T_2.
\]

Then we obtain from the action (3) the resulting target space metric \[2], \[14]
\[
G = \frac{b}{\Delta} d\chi^2 + \frac{x}{\Delta} d\theta^2 - 2\frac{\theta}{\Delta} d\chi d\theta, \quad \Delta = (\nu\theta)^2 - (y - z)(\nu\theta) + \delta,
\]
with the definitions
\[
a = x(\rho\theta) + \frac{y + z}{2}, \quad b = x(\rho\theta)^2 + (y + z)(\rho\theta) + w,
\]
and the torsion potential
\[
\frac{\rho\theta - (y - z)/2}{\Delta} d\chi \wedge d\theta.
\]

Since the target space is two-dimensional, the torsion 3-form vanishes.

The T-dualized model lives on the dual group target $\tilde{G}$ and its action reads \[1\], \[8\]
\[
\int (\tilde{R} + \tilde{\Pi}(\tilde{g}))^{-1ij} (\partial_+ \tilde{g} \tilde{g}^{-1})_i (\partial_- \tilde{g} \tilde{g}^{-1})_j,
\]
where $\tilde{R} = R^{-1}$ and the $\tilde{g}$ dependent matrix is given by
\[
\tilde{\Pi}(\tilde{g}) = \tilde{b}(\tilde{g}) \tilde{a}^{-1}(\tilde{g}),
\]
where the dual matrices $\tilde{a}, \tilde{b}, \tilde{d}$ are defined similarly by
\[
\text{Ad}_{\tilde{g}^{-1}} \tilde{T}^i = \tilde{a}(\tilde{g})_i^j \tilde{T}^j, \quad \text{Ad}_{\tilde{g}^{-1}} T_i = \tilde{b}(\tilde{g})_i^l \tilde{T}^l + \tilde{d}(\tilde{g})_i^l T_l.
\]

The elements $\tilde{g}$ of the dual group $\tilde{G}$ are of the form
\[
\tilde{g} = \begin{pmatrix} 0 & 0 \\ -\nu\theta & e^{\nu\chi} \end{pmatrix}
\]
and one can check that the matrices $\tilde{a}(\tilde{g}), \tilde{b}(\tilde{g})$ are obtained from $a(g), b(g)$ by the interchange of the parameters $\rho$ and $\nu$. This leads to
\[
\tilde{\Pi}(\tilde{g}) = \begin{pmatrix} 0 & -\rho\theta \\ \rho\theta & 0 \end{pmatrix}.
\]

Finally, the vielbein is now
\[
d\tilde{g} \tilde{g}^{-1} = d\chi \tilde{T}_1 + (d\theta - \nu\theta d\chi) \tilde{T}_2.
\]
Inserting all these quantities in the T-dualized Poisson-Lie model (16) gives the same target space metric (13) in which $R$ is transformed into $\tilde{R}$ and $\rho$ and $\nu$ get exchanged.

From the previous considerations one can check that the abelian model and its T-dual partner are both flat, so we will not consider this trivial possibility and we will take $\rho = 1$ in what follows.
2.3 Isometry and form invariance of the metric

These metrics have the obvious Killing \( \tilde{K} = \partial \chi \), with dual 1-form

\[
K = \frac{a^2 + \gamma^2}{x \Delta} d\chi - \frac{a}{\Delta} d\theta.
\]  

(22)

Besides this isometry, the left group action induces a two parameters group of transformation \( G_L \) of the coordinates and of the parameters

\[
\begin{align*}
\hat{\theta} &= \sigma \theta - \tau, \\
\hat{\chi} &= \chi + \ln \sigma, \\
\hat{x} &= x, \\
\hat{y} &= \sigma y + \tau(x - 1), \\
\hat{z} &= \sigma z + \tau(x + 1), \\
\hat{w} &= \sigma^2 w + \sigma \tau(y + z) + \tau^2 x,
\end{align*}
\]

\( \sigma > 0, \quad \tau \in \mathbb{R} \).  

(23)

Using the relations

\[
\begin{align*}
\hat{a} &= \sigma a, \\
\hat{b} &= \sigma^2 b, \\
\hat{\Delta} &= \sigma^2 \Delta,
\end{align*}
\]

(24)

it is easy to check that \( G_L \) leaves the metric invariant in the sense that

\[
G(\chi, \theta, R) = G(\hat{\chi}, \hat{\theta}, \hat{R}).
\]

The existence of this group implies that we can get rid of two parameters in the matrix \( R \): we can always take \( z = y \) and \( w = x \). Indeed, let us consider a set of the parameters \( x, y, z, w \) such that neither \( z = y \) nor \( w = x \). Using the transformations laws given above, if we take for parameters

\[
\tau = \sigma \left( \frac{y - z}{2} \right), \quad \sigma = x \left( \frac{t^2}{4} + \gamma^2 \right)^{-1/2}, \quad t = (x + 1)y + (x - 1)z,
\]

we can ensure simultaneously the relations \( \hat{z} = \hat{y} \) and \( \hat{w} = \hat{x} \). Notice that we have anticipated the positivity restriction on \( x \) which comes out from the riemannian character of the metric (see (30)). In what follows we will therefore take

\[
R = \begin{pmatrix} x & y \\ y & x \end{pmatrix}, \quad x > 0, \quad \gamma^2 \equiv x^2 - y^2 > 0, \quad y \in \mathbb{R}.
\]

(25)

For the semi-abelian model, corresponding to \( \mathcal{D}(1, 0) \), the lagrangian is

\[
B_{ij} (\partial_+ g g^{-1})^i (\partial_- g g^{-1})^j, \quad B = R^{-1},
\]

where \( R \) is given in (25). This metric is flat, so there is no renormalization of the parameters. Its T-dual model is not flat, but as shown in [11], it is flat up to some diffeomorphism, and the one-loop equivalence between the T-dual pair of models is preserved.

So the remaining open problem is the non-abelian model: since \( \nu \neq 0 \), appropriate scalings of \( R \) and of the coordinates allow to take \( \nu = 1 \). As observed in [14] the full family of Drinfeld doubles \( \mathcal{D}(1, \nu) \) gives rise to a single \( \sigma \)-model with \( \nu = 1 \), the \( GL(2, \mathbb{R}) \) model of [2].

The actual form of the metric is now

\[
G = \frac{b}{\Lambda} d\chi^2 + \frac{x}{\Lambda} d\theta^2 - 2 \frac{a}{\Lambda} d\chi d\theta,
\]

(26)
with the new functions
\[ a = x\theta + y, \quad b = x\theta^2 + 2y\theta + x = \frac{a^2 + \gamma^2}{x}, \quad \Delta = \theta^2 + \gamma^2, \quad \gamma^2 = x^2 - y^2. \] (27)

Its T-dual partner is obtained by the change of parameters
\[ x \rightarrow \tilde{x} = \frac{x}{x^2 - y^2}, \quad y \rightarrow \tilde{y} = -\frac{y}{x^2 - y^2}. \] (28)

### 2.4 Geometric aspects

Let us check the Riemannian character of the metric (26). This is best done through a calculation of the vielbein
\[ G = (e_1)^2 + (e_2)^2, \quad e_1 = \frac{\gamma}{\sqrt{x\Delta}} d\chi, \quad e_2 = \frac{x d\theta - a d\chi}{\sqrt{x\Delta}}, \] (29)

The metric will be riemannian iff
\[ \gamma^2 > 0, \quad x > 0. \] (30)

It follows that \( \Delta \) never vanishes. In the sequel we will assume that these conditions hold.

Another interesting point is to find the coordinates which do exhibit the conformally flat character of this metric. Taking for these coordinates
\[ U = \chi - \frac{1}{2} \ln(a^2(\theta) + \gamma^2), \quad V = \arctan \left( \frac{a(\theta)}{\gamma} \right), \quad a(\theta) = x\theta + y, \] (31)

the metric \( G \) becomes
\[ G = \frac{x\gamma^2}{(y \cos V - \gamma \sin V)^2 + x^2 \gamma^2 \cos^2 V} (dU^2 + dV^2), \] (32)

and it will be flat iff the denominator in the pre-factor is some constant. An easy algebraic discussion leads to the conclusion that the non-abelian model is flat iff
\[ x = \pm 1 \quad \text{and} \quad y = 0 \quad \Leftrightarrow \quad R = \pm \mathbb{I}. \] (33)

### 2.5 Frame geometry

The spin connection, defined by
\[ de_a + \omega_{ab} \wedge e_b = 0, \]
is given by
\[ \omega_{12} = \frac{1}{2\gamma} \frac{\mathcal{N} d\chi + xN d\theta}{x\Delta}, \] (34)

with
\[ \mathcal{N} = (a^2 + \gamma^2) \Delta - (a^2 + \gamma^2) \Delta', \quad N = a \Delta' - 2a' \Delta, \] (35)

where a prime indicates a derivative with respect to \( \theta \). One obtains
\[ N = 2y\theta - 2x\gamma^2, \quad \gamma^2 = x^2 - y^2, \] (36)
and
\[
\frac{N}{2x} = -y(\theta^2 - \gamma^2) + x(\gamma^2 - 1)\theta. \tag{37}
\]

The curvature (in the vielbein basis) has the single component
\[
R_{12,12} = \frac{1}{\gamma^2 \Delta} \frac{(N \Delta' - \Delta N')}{2x}, \tag{38}
\]
and we have for the Ricci
\[
\text{Ric}_{11} = \text{Ric}_{22} = R_{12,12}, \quad \text{Ric}_{12} = 0. \tag{39}
\]

The explicit form of the curvature component is
\[
\frac{N \Delta' - \Delta N'}{2x} = x(\gamma^2 - 1)(\theta^2 - \gamma^2) + 4y\gamma^2\theta. \tag{40}
\]

3 One-loop renormalizability

The one-loop renormalizability is ensured if the Ricci tensor can be written
\[
\text{Ric}_{ij} = (\chi \cdot \partial) G_{ij} + \nabla_{(i} w_{j)}, \quad \chi \cdot \partial = \sum \chi_k \frac{\partial}{\partial x_k}, \tag{41}
\]
where the \( \{x_k\} \) are parameters of the lagrangian to be renormalized and \( w \) is some (non-Killing) vector. In the vielbein basis this becomes
\[
\begin{align*}
\text{Ric}_{ab} &= (e^{-1})^i_b (\chi \cdot \partial) e_{ai} + (e^{-1})^i_a (\chi \cdot \partial) e_{bi} + D_{(a} w_{b)}, \\
D_a w_b &= \hat{\partial}_a w_b + \omega_{bs,a} w_s, \quad \hat{\partial}_a = (e^{-1})^i_a \partial_i.
\end{align*} \tag{42}
\]

Out of these three relations, two involve only the unknown vector \( w \):
\[
\begin{align*}
-\frac{1}{2} (D_1 w_2 + D_2 w_1) &= (e^{-1})^i_{a=1} (\chi \cdot \partial) e_{b=2,i} + (e^{-1})^i_{a=2} (\chi \cdot \partial) e_{b=1,i}, \\
-\frac{1}{2} (D_1 w_1 - D_2 w_2) &= (e^{-1})^i_{a=1} (\chi \cdot \partial) e_{b=1,i} - (e^{-1})^i_{a=2} (\chi \cdot \partial) e_{b=2,i}.
\end{align*}
\]

Defining
\[
\begin{align*}
w_1 &= \sqrt{\frac{x}{\Delta}} \hat{w}_1, \\
w_2 &= \sqrt{\frac{x}{\Delta}} \hat{w}_2,
\end{align*} \tag{43}
\]
we get
\[
\hat{w}_1 = \frac{\mu}{\gamma} \theta, \quad \hat{w}_2 = \frac{\mu}{\gamma^2} (v + y\theta) + (\theta + y/x) C_2 - C_1, \tag{44}
\]
where \( \mu \) is a free parameter and
\[
\begin{align*}
C_1 &= 2(\chi \cdot \partial) \left( \frac{y}{x} \right), \\
C_2 &= \frac{x^2}{\gamma^2} (\chi \cdot \partial) \left( \frac{\gamma^2}{x^2} \right). \tag{45}
\end{align*}
\]

are constants.
Then we are left with the single relation
\[ \text{Ric}_{11} = D_1 w_1 + 2 (e^{-1})_{a=1}^i (\chi \cdot \partial) e_{a=1,i}. \] (46)

The parameters to be renormalized are \( x \) and \( y \), so we define
\[ \chi \cdot \partial = \chi_x \partial_x + \chi_y \partial_y. \] (47)

The relation (46) gives only two relations (instead of three expected):
\[ \begin{cases} 
\gamma^2 \chi_x = xy \mu + x^2(\gamma^2 - 1), \\
-2y \chi_x + 2x \chi_y = -x(\gamma^2 - 1) \mu + 4y \gamma^2.
\end{cases} \] (48)

Since we are left with some freedom, let us see what comes out if we impose as a further condition the one-loop renormalizability of the T-dualized model. This is most conveniently done by switching to the parameters \( u = x + y \) and \( v = x - y \). Then the transformation (28) becomes
\[ u \to \tilde{u} = \frac{1}{u}, \quad v \to \tilde{v} = \frac{1}{v}. \] (49)

We will define for new renormalization constants
\[ \hat{\chi}_u = \frac{\chi_x + \chi_y}{x + y}, \quad \hat{\chi}_v = \frac{\chi_x - \chi_y}{x - y}. \] (50)

Relation (48) implies
\[ \begin{cases} 
\hat{\chi}_u(u, v) = \frac{(1 - v^2)}{2v} \mu(u, v) + 2v \frac{u - v}{u + v} + \frac{(u + v)(uv - 1)}{2uv}, \\
\hat{\chi}_v(u, v) = -\frac{(1 - u^2)}{2u} \mu(u, v) - 2u \frac{u - v}{u + v} + \frac{(u + v)(uv - 1)}{2uv}.
\end{cases} \] (51)

Now \( u \) (similarly for \( v \)) is renormalized by \( u \hat{\chi}_u(u, v) \) while \( \tilde{u} \) is renormalized by \( \tilde{u} \hat{\chi}_u(\tilde{u}, \tilde{v}) \) so, the T-dual pair will be renormalizable provided that
\[ \hat{\chi}_u(\tilde{u}, \tilde{v}) = -\hat{\chi}_u(u, v), \quad \hat{\chi}_v(\tilde{u}, \tilde{v}) = -\hat{\chi}_v(u, v). \] (52)

So, if we define
\[ m(u, v) = \frac{\mu(u, v)}{2} - \frac{u - v}{u + v}, \]
the joint renormalizability constraints (52) are equivalent to the single constraint
\[ m(\tilde{u}, \tilde{v}) = m(u, v). \] (53)

The function \( m \) describes the arbitrariness left over even after imposing the stability of renormalizability through T-dualization.

We have therefore shown that the T-dual pair of Poisson-Lie \( \sigma \)-models is indeed renormalizable, in the strict field-theoretic sense, with the most general structure of the renormalization constants
\[ \begin{cases} 
\frac{\mu}{2} = m + \frac{u - v}{u + v}, \\
\hat{\chi}_u = +m \frac{(1 - u^2)}{u} + \frac{(1 + u^2)(u - v)}{u(u + v)} + \frac{(u + v)(uv - 1)}{2uv}, \\
\hat{\chi}_v = -m \frac{(1 - u^2)}{u} - \frac{(1 + u^2)(u - v)}{u(u + v)} + \frac{(u + v)(uv - 1)}{2uv}.
\end{cases} \] (54)
where \( m(u, v) \) is constrained by (53). These renormalization constants are well defined since, as already stated in (30), the parameters \( u + v = x \) and \( uv = \gamma^2 \) never vanish.

### 4 Conclusion

Let us conclude with some remarks:

1. Absence of torsion and the dimension 2 for the target space is too poor to produce conformal geometries of interest for string theory. The case of 3d dimensional targets seem to be more promising from this point of view (cf. [19]).

2. Remarkably, the renormalizability works despite the traceful structure constants of the Drinfeld double. However, it is important to note in this context that we work with strictly field theoretic renormalization where the Weyl mode is frozen to a constant value (the same is true for the semi-abelian models). In the stringy framework, where the Weyl mode is coupled, the one-loop quantum equivalence requires that both sub-algebras \( G \) and \( \tilde{G} \) in the Drinfeld double must have traceless structure constants [18].

3. The problem of higher loop corrections to the Poisson-Lie T-duality appears to be more tricky than in the case of the Abelian or traditional non-Abelian T-duality. Of course, one problem to cope with is the fact that a finite one-loop renormalization can change the two-loops divergences [17]. But there is also a structural aspect of the thing: we expect that the two-loops effective action of the model should not probably have the same structure as the classical action (3). Why? Because the Poisson-Lie symmetry should be itself only a semiclassical approximation of a quantum group symmetry and the full-fledged effective action should reflect the latter quantum symmetry rather than the former semiclassical one. This means that, starting from the two-loop level, we do not expect that the criterion of the strict field theoretical renormalizability should be respected but we rather believe that it should be replaced by a sort of quantum group Ward identities to be fulfilled by the effective action. It is only in the semiclassical (or one-loop) limit that these Ward identities would reduce to the strict requirement of the strict renormalizability. Unfortunately, we do not have any hint yet how to write down and test the hypothetical quantum group Ward identities. For the moment we just conclude that our one-loop analysis of all two-dimensional PLT targets indicates a good quantum health of the Poisson-Lie T-duality.

### References

[1] C. Klimčík and P. Ševera, *Phys. Lett. B* 351 (1995) 455;

[2] C. Klimčík, *Nucl. Phys. Proc. Suppl.* 46 (1996) 116;

[3] K. Kikkawa and M. Yamasaki, *Phys. Lett. B* 149 (1984) 357;

[4] N. Sakai and I. Senda, *Prog. Theor. Phys.*, 75 (1986) 692;

[5] E. S. Fradkin and A. A. Tseytlin, *Ann. Phys.*, 162 (1984) 31;

[6] B. E. Fridling and A. Jevicki, *Phys. Lett. B* 134 (1984) 70;
[7] X. de la Ossa and F. Quevedo, Nucl. Phys. B 403 (1993) 377;

[8] C. Klímačik and P. Ševera, Phys. Lett. B 372 (1996) 65;

[9] K. Sfetsos, Nucl. Phys. B 517 (1998) 549;

[10] O. Alvarez, Nucl. Phys. B 584 (2000) 659;

[11] P. Y. Casteill and G. Valent, Nucl. Phys. B 591 (2000) 491;

[12] K. Sfetsos, Phys. Lett. B 432 (1998) 365;

[13] J. Balog, P. Forgács, N. Mohammedi, L. Palla and J. Schnittger, Nucl. Phys. B 535 (1998) 461.

[14] L. Hlavatý and L Šnobl, Mod. Phys. Lett. A 17 (2002) 429;

[15] M. A. Jafarizadeh and A. Rezaei-Aghdam, Phys. Lett. B 458 (1999) 477;

[16] L. Hlavatý and L Šnobl, Int. J. Mod. Phys. A 17 (2002) 4043;

[17] G. Bonneau and P. Y. Casteill, Nucl. Phys. B 607 (2001) 293;

[18] A. Bossard and N. Mohammedi, Nucl. Phys. B 619 (2001) 128;

[19] R. von Unge, JHEP 0207 (2002) 014.