Abstract
Recent work in computational psycholinguistics shows that morpheme lexica can be acquired in an unsupervised manner from a corpus of words by selecting the lexicon that best balances productivity and reuse (e.g. Goldwater et al. (2009) and others). In this paper, we extend such work to the problem of acquiring non-concatenative morphology, proposing a simple model of morphology that can handle both concatenative and non-concatenative morphology and applying Bayesian inference on two datasets of Arabic and English verbs to acquire lexica. We show that our approach successfully extracts the non-contiguous triliteral root from Arabic verb stems.

1 Introduction
What are the basic structure-building operations that enable the creative use of language, and how do children exposed to a language acquire the inventory of primitive units which are used to form new expressions? In the case of word formation, recent work in computational psycholinguistics has shown how an inventory of morphemes can be acquired by selecting a lexicon that best balances the ability of individual sound sequences to combine productively against the reusability of those sequences (e.g., Brent (1999), Goldwater et al. (2009), Feldman et al. (2009), O’Donnell et al. (2011), Lee et al. (2011)). However, this work has focused almost exclusively on one kind of structure-building operation: concatenation. The languages of the world, however, exhibit a variety of other, non-concatenative word-formation processes (Spencer, 1991).

Famously, the predominant mode of Semitic word formation is non-concatenative. For example, the following Arabic words, all related to the concept of writing, share no contiguous sequences of segments (i.e., phones), but they do share a discontinuous subsequence √ktb, which has been traditionally analyzed as an independent morpheme, termed the “root”.

| Arabic Word | Meaning   |
|-------------|-----------|
| kataba      | “he wrote”|
| kutiba     | “it was written” |
| yaktabu    | “he writes” |
| katab     | “writer” |
| kita:b      | “book” |
| kutub      | “books” |
| maktab     | “office” |

Table 1: List of Arabic words with root √ktb

Many Arabic words appear to be constructed via a process of interleaving segments from different morphemes, as opposed to concatenation.

| Concatenative | Non-concatenative |
|---------------|-------------------|
| cooked        | Tabaaxa           |

Figure 1: Schematic of concatenative vs non-concatenative morphology

Such non-concatenative morphology is pervasive in the world’s languages. Even English, whose morphology is fundamentally concatenative, displays pockets of non-concatenative behavior, for example in the irregular past tenses (see Table 2).

In these words, the stem vowels undergo ablaut changing between tenses. This cannot be handled in a purely concatenative framework unless we consider these words listed exceptions. However, such irregulars do show limited productiv-
Table 2: Examples of English irregular verbs

| Verb | Past Tense |
|------|------------|
| bite | bit        |
| sing | sang       |
| give | gave       |
| feel | felt       |

Table 2: Examples of English irregular verbs

Table 3: List of common Arabic verbal binyanim

| Form | Active | Passive |
|------|--------|---------|
| I    | faʕal | fuʕil   |
| II   | faʕʔal| fuʕʕil  |
| III  | faʕal | fuʕʕil  |
| IV   | ʔafʕal| ʔuʕʕil  |
| V    | taʕʕal| tuʕʕil  |
| VI   | taʕʕal| tuʕʕil  |
| VII  | ʔinfaʕal | -    |
| VIII | ʔiftaʕal| ʔiʕʕil|
| X    | ʔistaʕal| ʔistuʕʕil|

Table 3: List of common Arabic verbal binyanim

Each of these forms has traditionally been associated with a particular semantics. For example, Form II verbs are generally causatives of Form I verbs, as is kattab “to cause to write” (c.f. katab “to write”). However, as is commonly the case with derivational morphology, these semantic associations are not completely regular: many forms have been lexicalized with alternative or more specific meanings.

2.1 Theoretical accounts

The traditional Arab grammarians’ account of the Arabic verb was as follows: each form was associated with a template with slots labelled C₁, C₂, and C₃, traditionally represented with the consonants √fql, as described above. The actual root consonants were slotted into these gaps. Thus the template of the Form VIII active perfect verb stem was taC₁aC₂C₂aC₃. This, combined with the triconsonantal root, made up the verbal stem.

Figure 2: Traditional analysis of Arabic Form V verb

The first generative linguistic treatment of Arabic verbal morphology (McCarty, 1979; McCarthy, 1981) adopted the notion of the root and template, but split off the derivational prefixes and vocalism from the template. Borrowing from the technology of autosegmental phonology (Goldsmith, 1976), the template was
now comprised of C(onsonant) and V(owel) slots. Rules governing the spreading of segments ensured that consonants and vowels appeared in the correct positions within a template.

Under McCarthy’s model, the analysis for [tufaYYal] would be as follows:

| Prefix | Root |
|--------|------|
| t      | f    |

CV Template: C V C V C V C

Vocalism

Figure 3: McCarthy analysis of Arabic Form V verb

While increasing the number of morphemes associated with each verb, the McCarthy approach economized on the variety of such units in the lexicon. The inventory of CV templates was limited; there were three vocalisms corresponding to active and passive voice intersecting with perfect and imperfect aspect; and only four derivational prefixes (Fr/Fn/Fl/Fst), one of which became an infix via morphophonological rule in Form VIII.1

We adopt a middle ground between the traditional Arab grammarians’ description of the verbal stem and McCarthy’s analysis as our starting point. We describe this approach in the next section.

3 The Approach

Our initial model of morphology adopts McCarthy’s notion of an abstract template, but coalesces the prefixes and infixes with the vocalism into what we term the “residue.” Each stem is thus composed of two morphemes: the root and the residue, and their interleaving is dictated by a template with slots for root and residue segments.

For example, ?iktatab = - - - r - r - r (template) + ktb (root) + ?ita (residue), where r indicates a root segment and - a residue segment.

The residue may be of length 0, effectively making the word consist of a single morpheme. Concatenative morphology may be modelled in this framework by grouping all the root segments together, for example cooked [kukt] = r r r - (template) + kuk (root) + t (residue).

The template, root and residue are each drawn from a separate sub-lexicon, modeled using tools from Bayesian non-parametric statistics (see Section 4). These tools put a prior distribution on the lexica that biases them in favour of reusing existing frequent forms and small lexica by promoting maximal sharing of morphemes.

When applied to data, we derive a segmentation for each word into a root and a residue.

4 Model

Following earlier work on Bayesian lexicon learning (e.g. Goldwater et al. (2009), we use a distribution over lexical items known as the Pitman–Yor Process (PYP) (Pitman and Yor, 1995). Let G be a distribution over primitive phonological elements of the lexicon (e.g., words, roots, residues, templates, morphemes, etc.). The behavior of PYP process PYP(a, b, G) with base measure G and parameters a and b can be described as follows. The first time we sample from PYP(a, b, G) a new lexical item will be sampled using G. On subsequent samples from PYP(a, b, G), we either reuse an existing lexical item i with probability \( \frac{n_i - a}{N + b} \), where \( N \) is the number of lexical items sampled so far, \( n_i \) is the number of times that lexical item i has been used in the past, and \( 0 \leq a \leq 1 \) and \( b > -a \) are parameters of the model. Alternatively, we sample a new lexical item with probability \( \frac{aK + b}{N + b} \), where \( K \) is the number of times a new lexical item was sampled in the past from the underlying distribution G. Notice that this process induces a rich-get-richer scheme for sampling from the process. The more a particular lexical item has been reused, the more likely it is to be reused in the future. The Pitman–Yor process also produces a bias towards smaller, more compact lexica.

In our model, we maintain three sublexica for templates (\( L_{T_p} \)), roots (\( L_{R_t} \)), and residues (\( L_{R_s} \)) each drawn from a Pitman–Yor process with its own hyperparameters.

\[
L_X \sim PYP(a_X, b_X, G_X)
\]

where \( X \in \{T_p, R_t, R_s\} \) Words are drawn by first drawing a template, then drawing a root and a residue (of the appropriate length) and inserting the segments from the root and residue in the appropriate positions in the word as indicated by the
Our templates are strings in \{Rt, Rs\}∗ indicating for each position in a word whether that position is part of the word’s root (Rt) or residue (Rs). These templates themselves are drawn from a base measure \(G_{Tp}\) which is defined as follows.

To add a new template to the template lexicon first draw a length for that template, \(K\), from a Poisson distribution.

\[ K \sim \text{POISSON}(5) \]  

We then sample a template of length \(K\) by drawing a Bernoulli random variable \(t_i\) for each position \(i \in 1..K\) is a root or residue position.

\[ t_i \sim \text{BERNOULLI}(\theta) \]  

The base measure over templates, \(G_{Tp}\), is defined as the concatenation of the \(t_i\)’s.

The base distributions over roots and residues, \(G_{Rt}\) and \(G_{Rs}\), are drawn in the following manner. Having drawn a template, \(T\) we know the lengths of the root, \(K_{Rt}\), and residue \(K_{Rs}\). For each position in the root or residue \(r_i\) where \(i \in 1..K_{Rt}/Rs\), we sample a phone from a uniform distribution over phones.

\[ r_i \sim \text{UNIFORM}(|\text{alphabet}|) \]  

5 Inference

Inference was performed via Metropolis–Hastings sampling. The sampler was initialized by assigning a random template to each word in the training corpus. The algorithm then sampled a new template, root, and residue for each word in the corpus in turn. The proposal distribution over templates for our sampler considered all templates currently in use by another word, as well as a randomly generated template from the prior. Samples from this proposal distribution were corrected into the true distribution using the Metropolis–Hastings criterion.

6 Related work

The approach of this paper builds on previous work on Bayesian lexicon learning starting with Goldwater et al. (2009). However, to our knowledge, this approach has not been applied to non-concatenative morphological segmentation. Where it has been applied to Arabic (e.g. Lee et al. (2011)), it has been applied to unvowedled text, since standard Arabic orthography drops short vowels. However, this has the effect of reducing the problem mostly to one of concatenative morphology.

Non-concatenative morphology has been approached computationally via other research, however. Kataja and Koskenniemi (1988) first showed that Semitic roots and patterns could be described using regular languages. This insight was subsequently computationally implemented using finite state methods by Beesley (1991) and others. Roark and Sproat (2007) present a model of both concatenative and non-concatenative morphology based on the operation of composition that is similar to the one we describe above.

The narrower problem of isolating roots from Semitic words, for instance as a precursor to information retrieval, has also received much attention. Existing approaches appear to be mostly rule-based or dictionary-based (see Al-Shawakfa et al. (2010) for a recent survey).

7 Experiments

We applied the morphological model and inference procedure described in Sections 4 and 5 to two datasets of Arabic and English.

7.1 Data

The Arabic corpus for this experiment consisted of verbal stems taken from the verb concordance of the Quranic Arabic Corpus (Dukes, 2011). All possible active, passive, perfect and imperfect fully-vowelled verbal stems for Forms I–X, excluding the relatively rare Form IX, were generated. We used this corpus rather than a lexicon as our starting point to obtain a list of relatively high frequency verbs.

This list of stems was then filtered in two ways: first, only triconsonantal “strong” roots were considered. The so-called “weak” roots of Arabic either include a vowel or semi-vowel, or a doubled consonant. These undergo segmental changes in various environments, which cannot be handled by our current generative model.

Secondly, the list was filtered through the Buckwalter stem lexicon (Buckwalter, 2002) to obtain only stems that were licit according to the Buckwalter morphological analyzer.

This process yielded 1563 verbal stems, comprising 427 unique roots, 26 residues, and 9 templates. The stems were supplied to the sampler in the Buckwalter transliteration.
The English corpus was constructed along similar lines. All verb forms related to the 299 most frequent lemmas in the Penn Treebank (Marcus et al., 1999) were used, excluding auxiliaries such as *might* or *should*. Each lemma thus had up to five verbal forms associated with it: the bare form (*forget*), the third person singular present (*forgets*), the gerund (*forgetting*), past tense (*forgot*), and past participle (*forgotten*).

This resulted in 1549 verbal forms, comprising 295 unique roots, 108 residues, and 55 templates. CELEX (Baayen et al., 1995) pronunciations for these words were supplied to the sampler in CELEX’s DISC transliteration.

Deriving a gold standard analysis for English verbs was less straightforward than in the Arabic case. The following convention was used: The root was any subsequence of segments shared by all the forms related to the same lemma. Thus, for the example lemma of *forget*, the correct template, root and residue were deemed to be:

\[
\begin{align*}
\text{forget} & \quad f@gEt \quad r\ r\ r\ -\ r \quad f@gt \quad E \\
\text{forgets} & \quad f@gEts \quad r\ r\ r\ -\ -\ r \quad f@gt \quad Es \\
\text{forgot} & \quad f@gQt \quad r\ r\ r\ -\ r \quad f@gt \quad Q \\
\text{forgetting} & \quad f@gEtIN \quad r\ r\ r\ -\ -\ r \quad f@gt \quad EIN \\
\text{forgotten} & \quad f@gQtH \quad r\ r\ r\ -\ -\ r \quad f@gt \quad QH
\end{align*}
\]

This reveals an effect of both the rarity and the length of each template. For instance, the performance on template *r - - r - r* (second bar from left) is exceptionally low, but this is the result of there being only one instance of this template in the training set: Euwqib, the passive form of the Form III verb of root Eqb, in the Buckwalter transliteration.² In addition, the longer the word,

### Table 4: Correct analyses under the root/residue model for the lemma *forget*

|   |   |   |
|---|---|---|
| forget | f@gEt | r r r - r f@gt E |
| forgets | f@gEts | r r r - - r f@gt Es |
| forgot | f@gQt | r r r - r f@gt Q |
| forgetting | f@gEtIN | r r r - - r f@gt EIN |
| forgotten | f@gQtH | r r r - - r f@gt QH |

37 templates were concatenative, and 18 non-concatenative. The latter were necessary to accommodate 46 irregular lemmas associated with 254 forms.

### 7.2 Results and Discussion

We ran 10 instances of the sampler for 200 sweeps through the data. For the Arabic training set, this number of sweeps typically resulted in the sampler finding a local mode of the posterior, making few further changes to the state during longer runs. An identical experimental set-up was used for English. Evaluation was performed on the final state of each sampler instance.

The correctness of the sampler’s output was measured in terms of the accuracy of the templates it predicted for each word. The word-level accuracy indicates the number of words that had their entire template correctly sampled, while the segment-level accuracy metric gives partial credit by considering the average number of correct bits (r versus -) in each sampled template.

Table 5 shows the average accuracy of the 10 samples, weighted by each sample’s joint probability.

|   | Word-level | Segment-level |
|---|-------------|---------------|
| Arabic | 92.3% | 98.2% |
| English | 43.9% | 85.3% |

**Table 5: Average weighted accuracy of samples**

### Arabic Analyses

Figure 4 shows the average unweighted accuracy with which each of the 9 Arabic templates was sampled.

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the poorer the performance of the model. This is likely the result of the difficulty of searching over the space of templates for longer forms. Since the number of potential templates increases exponentially with the length of the form, finding the correct template becomes increasingly difficult. This problem can likely be addressed in future models by adopting an analysis similar to McCarthy’s whereby the residue is further subdivided into vocalism, prefixes and infixes. Note that even in such long forms, however, the letters belonging to the root were generally isolated in one of the two morphemes.

English Analyses The English experiment yielded poorer results than the Arabic dataset. The statistics of the datasets reveal the cause of the failure of the English model: the English dataset had several times more residues and templates than the Arabic dataset did, thus lacking as much uniform structure. Nevertheless, the relatively high segment-level accuracy shows that the model tended to find templates that were only incorrect in 1 or 2 positions.

The dominant pattern of errors was in the direction of overgeneralization of the concatenative templates to the irregular forms. Out of the 254 words related to a lemma with an irregular past form, 241 received incorrect templates, 232 of which were concatenative, often correctly splitting off the regular suffix where there was one. For example, *sing* and *singing* were parsed as *sing+∅* and *sing+ing*, while *sung* was parsed as a separate root. Note that under an analysis of English irregulars as separate memorized lexical items, the sampler behaved correctly in such cases.

However, out of 1295 words related to perfectly regular lemmas, the sampler determined 628 templates incorrectly. Out of these, 325 were given concatenative templates, but with too much or too little segmental material allocated to the suffix. For example, the word *invert* was analyzed as *inver+t*, with its other forms following suit as *inver+ted*, *inver+ting* and *inver+ts*. This is likely due to subregularities in the word corpus: with many words ending with -t, this analysis becomes more attractive.

The remaining 303 regular verbs were given non-concatenative templates. For instance, *identify* was split up into *dfy* and *ienti*. No consistent pattern could be discerned from these cases.

8 Conclusion

We have proposed a model of morpheme-lexicon learning that is capable of handling concatenative and non-concatenative morphology up to the level of two morphemes. We have seen that Bayesian inference on this model with an Arabic dataset of verbal stems successfully learns the non-contiguous root and residue as morphemes.

In future work, we intend to extend our simplified model of morphology to McCarthy’s complete model by adding concatenative prefixation and suffixation processes and segment-spreading rules. Besides being capable of handling the inflectional aspects of Arabic morphology, we anticipate that this extension will improve the performance of the model on Arabic verbal stems as well, since the number of non-concatenative templates that have to be learned will decrease. For example, the template for the Form V verb [tafa’al] can be reduced to that for the Form II verb [fa’al] plus an additional prefix.

We also anticipate that the performance on English will be vastly improved, since the dominant mode of word formation in English is concatenative, while the small number of irregular past tenses and plurals that undergo ablaut can be handled using the non-concatenative architecture of the model. This would also be more in line with native speakers’ intuitions and linguistic analyses of English morphology.

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