Chiral Symmetry Breaking in Presence of Strong Quantizing Magnetic Fields-
A Nambu-Jona-Lasinio Model with Semi-Classical Approximation

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The breaking of chiral symmetry of light quarks at zero temperature in presence of strong quantizing magnetic field have gotten a new life after the recent discovery of a few magnetars [1–4]. These stellar objects are believed to be strongly magnetized young neutron stars. The surface magnetic fields are observed to be \( \geq 10^{15}\text{G} \). Then it is quite possible that the fields at the core region may go up to \( 10^{18}\text{G} \). The exact source of this strong magnetic field is of course yet to be known. These objects are also supposed to be the possible sources of anomalous X-ray and soft gamma emissions (AXP and SGR). If the magnetic field is really so strong, in particular at the core region, they must affect most of the important physical properties of such stellar objects and also the rates / cross-sections of elementary processes, e.g., weak and electromagnetic decays / reactions taking place at the core region should change significantly.

The strong magnetic field affects the equation of state of dense neutron star matter. As a consequence the gross-properties of neutron stars [5–8], e.g., mass-radius relation, moment of inertia, rotational frequency etc. should change significantly. In the case of compact neutron stars, the phase transition from neutron matter to quark matter which may occur at the core region is also affected by strong quantizing magnetic field. It has been shown that a first order phase transition initiated by the nucleation of quark matter droplets is absolutely forbidden if the magnetic field strength is \( \leq 10^{18}\text{G} \) at the core region [9,10]. However, a second order phase transition is allowed, provided the magnetic field strength is \( \geq 10^{20}\text{G} \). This is of course too high to achieve at the core region.

The elementary processes, in particular, the weak and the electromagnetic decays/reactions taking place at the core region of a neutron star are strongly affected by such ultra-strong magnetic fields [11,12]. Since the cooling of neutron stars are mainly controlled by neutrino/anti-neutrino emission, the presence of strong quantizing magnetic field should affect the thermal history of strongly magnetized neutron stars. Further, the electrical conductivity of neutron star matter which directly controls the evolution of neutron star magnetic field will also change significantly [12].

Similar to the study of quark-hadron deconfinement transition inside neutron star core in presence of strong quantizing magnetic field, some investigations have also been done on the effect of ultra-strong magnetic field on chiral symmetry breaking. In those studies, quantum field theoretic formalism were mainly used [13–19]. In particular, in the reference [20], chiral symmetry violation is studied using Nambu-Jona-Lasinio (NJL) model with Thomas-Fermi type semi-classical formalism. It is found that the dynamically generated light quark mass can never become zero if the Landau levels are populated and increases with the increase of magnetic field strength.

In this article we shall study the effect of strong quantizing magnetic field on the chiral properties of light quark matter system with the help of NJL model following semi-classical Thomas-Fermi type mean field approach. Now in NJL model, there is no in-built mechanism of color confinement, however, it can produce two chirally distinct phases-appropriate for confined quark matter within the bag (not necessarily tiny hadronic bag) and the matter outside the bag. These phases are also known as the Wigner phase and spontaneously broken chiral phase respectively. Therefore, if one re-formulates the NJL model in presence of strong quantizing magnetic field, it is quite possible to obtain the effect of quantizing magnetic field on these two chirally distinct phases and hence obtain the effect of magnetic field on chiral symmetry breaking. Further, it is also possible to obtain bag pressure from the difference of vacuum energy densities of these two phases and hence its variation with strong magnetic field. Assuming that the confinement and spontaneously broken chiral symmetry are synonymous, Bhaduri et. al. obtained some estimate of bag constant from the difference of energy densities [21]. In the present article we shall modify these original calculations of Bhaduri et. al. [21] and Providência et. al. [22] to study the breaking of chiral symmetry of light quarks in presence of strong magnetic fields and show that the chiral symmetry always remains broken in presence of strong quantizing magnetic field if the Landau levels for quarks are populated. Our study is basically an application of the formalism recently

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developed to study the equation of state of dense fermionic matter of astrophysical interest in presence of strong quantizing magnetic field [23].

We start with the density matrix $\rho(x, x')$, given by

$$
\rho(x, x') = \sum_{\text{spin}, p} \psi(x)\psi^\dagger(x')\theta(\Lambda - |p_z|)
$$

(1)

where $\psi$ and $\psi^\dagger$ are respectively the negative energy Dirac spinor and the corresponding adjoint, satisfy the equation

$$
\hbar\psi = E_-\psi
$$

(2)

(and similarly for $\psi^\dagger$) with the single particle Hamiltonian

$$
h = \gamma_5\Sigma_z(\vec{p} - q_f\vec{A}) + \beta m
$$

(3)

with

$$
\Sigma_z = \begin{pmatrix}
\sigma & 0 \\
0 & \sigma
\end{pmatrix},
$$

(4)

$\gamma_5$ and $\beta$ are the usual Dirac matrices, $\Lambda$ is the ultra-violet cut off in the momentum integral over $p_z$ and $\vec{A}$ is the electromagnetic field three vector corresponding to the external constant magnetic field of strength $B_m$ along $z$-axis. Here the light quark mass $m$ is assumed to be generated dynamically. Now in presence of strong quantizing magnetic field along $z$-direction, the up and down spin negative energy spinors are therefore given by

$$
\psi(x) = \frac{1}{(L_yL_z)^{1/2}} \exp[i(E_\nu t - p_yy - p_zz)]v^{(-1)}_{\nu}
$$

(5)

where

$$
v^{(-1)}_{\nu} = \frac{1}{2E_-(E_- - m)^{1/2}} \begin{pmatrix}
p_\nu I_{\nu} \\
-(2\nu q_f B_m)^{1/2}I_{\nu - 1}
\end{pmatrix}
$$

(6)

and

$$
v^{(-1)}_{\nu} = \frac{1}{2E_-(E_- - m)^{1/2}} \begin{pmatrix}
i(2\nu q_f B_m)^{1/2}I_{\nu} \\
-p_\nu I_{\nu - 1}
\end{pmatrix}
$$

(7)

where $E_- = -(p_x^2 + m^2 + 2\nu q_f B_m)^{1/2} = -E_\nu$, is the single particle energy eigen value, $\nu = 0, 1, 2, ..., $ are the Landau quantum numbers, $q_f$ is the magnitude of the charge carried by $f$th flavor and

$$
I_\nu = \left(\frac{q_f B_m}{\pi}\right)^{1/4} \left(\frac{1}{\nu!}\right)^{1/2} 2^{-\nu/2} \exp\left[-\frac{1}{2} q_f B_m \left(x - \frac{p_y}{q_f B_m}\right)^2\right] H_\nu \left[\left(q_f B_m\right)^{1/2} \left(x - \frac{p_y}{q_f B_m}\right)\right]
$$

(8)

with $H_\nu$ is the well known Hermite polynomial of order $\nu$, and $L_y, L_z$ are respectively length scales along $Y$ and $Z$ directions. Now it can very easily be shown that $\nu = 0$ state is singly degenerate, whereas all other states are doubly degenerate. We now express the density matrix, as the modified version of Wigner transform in presence of strong quantizing magnetic field, in the following form:

$$
\rho(x, x') = \sum \rho(x, x', p_y, p_z, \nu) \exp[i\{(t - t')E_- - (y - y')p_y - (z - z')p_z\}]
$$

(9)

where the sum is over the momentum components $p_y, p_z$ and the Landau quantum number $\nu$. Since the momentum variables are continuous, the sum over momentum components will be replaced by the corresponding integrals. Then using the negative energy up and down spin Dirac spinors, we have

$$
\rho(x, x', p_y, p_z, \nu) = \frac{1}{2E_-} [E_- A - p_z\gamma_z\gamma_0 A + m\gamma_0 A - p_y\gamma_y\gamma_0 B] \theta(\Lambda - |p_z|)
$$

(10)
where the matrices $A$ and $B$ are given by

$$A = \begin{pmatrix} I_\nu I_\nu' & 0 & 0 & 0 \\ 0 & I_{\nu-1} I_{\nu-1}' & 0 & 0 \\ 0 & 0 & I_\nu I_\nu' & 0 \\ 0 & 0 & 0 & I_{\nu-1} I_{\nu-1}' \end{pmatrix}$$ \hspace{1cm} (11)$$

$$B = \begin{pmatrix} I_{\nu-1} I_\nu' & 0 & 0 & 0 \\ 0 & I_\nu I_\nu' & 0 & 0 \\ 0 & 0 & I_{\nu-1} I_\nu' & 0 \\ 0 & 0 & 0 & I_\nu I_{\nu-1}' \end{pmatrix}$$ \hspace{1cm} (12)$$

where the primes indicate the functions of $x'$. Now in the evaluation of vacuum energy, we have noticed that it would be more convenient to define a quantity $\mu_f$, similar to the chemical potential for the $f$th flavor in a multi-quark statistical system in presence of strong quantizing magnetic field. Then it is very easy to write

$$\Lambda = \left( \mu_f^2 - m^2 - 2\nu q_f B_m \right)^{1/2}$$ \hspace{1cm} (13)$$

Since $\Lambda > 0$, it is also possible to express the upper limit of $\nu$, which is the maximum value of Landau quantum number of the levels occupied by $f$th flavor, in terms of $\mu_f$, $m$, $q_f$ and $B_m$ and is given by

$$\nu^{(f)}_{\text{max}} = \left[ \frac{\mu_f^2 - m^2}{2q_f B_m} \right]$$ \hspace{1cm} (14)$$

where $[ \cdot ]$ indicates the nearest integer but less than the actual number. Now to obtain the energy density of the vacuum, we consider the NJL (chiral) Hamiltonian, given by

$$H = \sum_{i=1}^{N} t(i) + \frac{1}{2} \sum_{i \neq j} V(i,j)$$
$$= \sum_{i=1}^{N} \gamma_5(i) \bar{\Sigma}(i)(\vec{p}_i - q_f \vec{A}) - \frac{1}{2} \sum_{i \neq j} \delta(\vec{x}_i - \vec{x}_j)[\beta(i)\beta(j) - \beta(i)\gamma_5(i)\beta(j)\gamma_5(j)]$$ \hspace{1cm} (16)$$

Assuming the magnetic field $B_m$ along $z$-direction and is constant, we can choose the gauge $A^\mu = (0, 0, xB, 0)$. The energy of the vacuum is then given by

$$\epsilon_v = \sum_{\rho_1, \nu_1, \mu_1} \int dx \, tr\{[\gamma_5 \bar{\Sigma}(\vec{p}_i - q_f \vec{A})]_\rho_{\nu_1} [\rho_{\mu_1}] + \epsilon_v^{(I)} \}$$ \hspace{1cm} (17)$$

where $\rho_{\mu_1}$ is given by eqn.(10) and $\epsilon_v^{(I)}$ indicates the interaction term, including the exchange interaction. To evaluate the vacuum energy, we first calculate the first term of eqn.(17). This quantity is proportional to the trace defined as $Tr(\rho_h)$, can easily be evaluated by using $\rho$ from eqn.(10) and the single particle Hamiltonian $h$ from eqn.(16). Now using the orthonormality relations for the Hermite polynomials at the time of evaluation of integral over $dx$ and also using the anti-commutation relations of $\gamma$-matrices, we have the first term at zero temperature

$$\epsilon_v^{(0)} = 2N_c \sum_{f=u,d} \frac{q_f B_m}{2\pi^2} \sum_{\nu_{\text{max}}} \sum_{\nu_{\text{max}}} (2 - \delta_{\nu_0}) \int_0^\Lambda dp_x \frac{\vec{p}_x^2}{E_-}$$ \hspace{1cm} (18)$$

where $\vec{p}_x^2 = p_x^2 + 2\nu q_f B_m$, $N_c = 3$, the number of colors, and $E_- = -E_+$. In the evaluation of all the traces in this paper we have used the following important relation:

$$\text{Tr}(\gamma^\mu \gamma^\nu A_1 A_2..B_1 B_2..) = Tr(A_1 A_2..B_1 B_2..) g^{\mu\nu},$$ \hspace{1cm} (19)$$

$$\text{Tr}(\gamma^\mu \gamma^\nu \gamma^\lambda \gamma^\sigma A_1 A_2..B_1 B_2..) = Tr(A_1 A_2..B_1 B_2..)(g^{\mu\nu}g^{\sigma\lambda} - g^{\mu\lambda}g^{\nu\sigma} + g^{\mu\lambda}g^{\nu\sigma}),$$ \hspace{1cm} (20)$$
Tr(product of odd $\gamma s$ with $A$ and/or $B$) = 0 etc. The other interesting aspects of $A$ and $B$ matrices are:

i) $k_{1\mu}k_{2\nu}\text{Tr}(A_1A_2) = (E_1E_2 - k_{1\perp}k_{2\perp})\text{Tr}(A_1A_2)$

ii) $k_{1\mu}k_{2\nu}\text{Tr}(B_1B_2) = k_{1\perp}k_{2\perp}\text{Tr}(B_1B_2)$

iii) $k_{1\mu}k_{2\nu}\text{Tr}(A_1B_2) = k_{1\mu}k_{2\nu}\text{Tr}(B_1A_2) = 0$

iv) $p_{1\mu}k_{1\nu}p_{2\nu}k_{2\nu}\text{Tr}(A_1B_2) \neq 0 = (E_{\nu_1}E_{\nu_2} - p_{1z}k_{1z})p_{2\perp}k_{2\perp}\text{Tr}(A_1B_2)$

These set of relations are very recently obtained by us [23]. Since $\gamma$ matrices are traceless and both $A$ and $B$ matrices are diagonal with identical blocks, it is very easy to evaluate the above traces of the product of $\gamma$-matrices multiplied with any number of $A$ and/or $B$, from any side with any order.

To evaluate the interaction term, we first consider the direct part which is proportional to $\text{Tr}(\beta \rho_p)\text{Tr}(\beta \rho_{p_2})$ and it is very easy to show that $\text{Tr}(\beta \gamma_5 \rho) = 0$. Now using the orthonormality relations for Hermite polynomials and the anti-commutation relations for the $\gamma$-matrices, we have the direct term

$$V_{\text{dir}} = -4gm^2|\mathcal{V}(\Lambda, m)|^2$$

(21)

where

$$\mathcal{V}(\Lambda, m) = \frac{N_c}{2\pi^2} \sum_{f=u,d} e_f B_m \sum_{\nu=0}^{\nu_{\text{max}}} (2 - \delta_{\nu0}) \int_0^\Lambda \frac{dp_z}{(p_z^2 + m_{\nu}^2)^3}$$

(22)

where $m_{\nu} = (m^2 + 2\nu q_f B_m)^{1/2}$.

To evaluate the exchange term, we first calculate $\text{Tr}(\beta \rho_p)(\beta \rho_{p_2})$. Now

$$\beta \rho_p = \frac{1}{2E_-} [E_- \beta A + p_z A \gamma_z + mA - p_{z\perp} B \gamma_y]$$

(23)

Then at the time of integration over $dx_1$ and $dx_2$ if one uses the orthonormality relations for Hermite polynomials, the above trace is given by

$$\left[1 + \frac{p_{1z}p_{2z}}{E_1E_2} + \frac{m^2}{E_1E_2}\right]$$

(24)

where both $E_1$ and $E_2$ are negative. Then in the energy contribution, after integrating over $p_{1z}$ and $p_{2z}$, the first term gives

$$\left(\frac{N_c}{2\pi^2} \sum_f q_f B_m \sum_\nu (2 - \delta_{\nu0}) \Lambda^2\right)^2$$

(25)

Similarly the contribution from second term is given by

$$\left(\frac{N_c}{2\pi^2} \sum_f q_f B_m \sum_\nu (2 - \delta_{\nu0})(\Lambda^2 + m_{\nu}^2)^{1/2}\right)^2$$

(26)

and finally, the third term is given by

$$m^2 \left(\frac{N_c}{2\pi^2} \sum_f q_f B_m \sum_\nu \gamma_{\nu} \ln \left[\frac{\Lambda + (\Lambda^2 + m_{\nu}^2)^{1/2}}{m_{\nu}}\right]\right)^2$$

(27)

To obtain the next term in the exchange part, we evaluate the trace $\text{Tr}((\beta \gamma_5 \rho_p)(\beta \gamma_5 \rho_{p_2}))$, which unlike the direct case, gives non-zero contribution. Using the anticommutation relations of $\gamma$-matrices and as usual with the help of orthonormality relations for Hermite polynomials, we have the above trace

$$- \left[1 + \frac{p_{1z}p_{2z}}{E_1E_2} + \frac{m^2}{E_1E_2} + m \left(\frac{1}{E_1} + \frac{1}{E_2}\right)\right]$$

(28)

The contribution to the interaction energy will again be obtained if we integrate over $p_{1z}$ and $p_{2z}$. Then the first term is given by
The second term is given by
\[
\left( \frac{N_c}{2\pi^2} \sum_f q_f B_m \sum_{\nu} (2 - \delta_{\nu 0}) \right)^2
\]
(29)

The third term is given by
\[
m^2 \left( \frac{N_c}{2\pi^2} \sum_f q_f B_m \sum_{\nu} (2 - \delta_{\nu 0}) (\Lambda + m_\nu^2)^{1/2} \right)^2
\]
(30)

and finally the fourth and fifth terms, which are identical, given by
\[
m \left( \frac{N_c}{2\pi^2} \sum_f q_f B_m \sum_{\nu} (2 - \delta_{\nu 0}) \ln \left[ \frac{\Lambda + (\Lambda^2 + m_\nu^2)^{1/2}}{m_\nu} \right] \right)^2
\]
(31)

Then combining all these terms we finally obtain the vacuum energy density. Since the mass \(m\), which is assumed to be same for both \(u\) and \(d\) quarks, is generated dynamically, we obtain this quantity by minimizing the total vacuum energy density with respect to \(m\), i.e., by putting \(\frac{d\epsilon_v}{dm} = 0\). Simplifying this non-linear equation, we finally get
\[
\frac{d\epsilon_v}{dm} = -P + 2gQR = 0
\]
(33)

where
\[
P = \frac{N_c}{2\pi^2} \sum_f q_f B_m \sum_{\nu} (2 - \delta_{\nu 0}) \left[ \frac{2m^3 \Lambda}{m_\nu^2 (\Lambda^2 + m_\nu^2)^{1/2}} - 2mX \right]
\]
(34)
\[
Q = \frac{N_c}{2\pi^2} \sum_f q_f B_m \sum_{\nu} (2 - \delta_{\nu 0}) \left[ X - \frac{m^2}{m_\nu^2 (\Lambda^2 + m_\nu^2)^{1/2}} \right]
\]
(35)
\[
R = \frac{N_c}{2\pi^2} \sum_f q_f B_m \sum_{\nu} (2 - \delta_{\nu 0}) [\Lambda - 4mX]
\]
(36)

with
\[
X = \ln \left[ \frac{\Lambda + (\Lambda^2 + m_\nu^2)^{1/2}}{m_\nu} \right]
\]
(37)

It is therefore obvious from eqn.(33) that the trivial solution \(m = 0\) is not possible in this particular situation. On the other hand in a non-magnetic case, eqn.(33) reduces to the well known gap equation, given by
\[
m = 4gV m
\]
(38)

where \(V\) is the overall contribution of interaction terms. Hence it is obvious that \(m = 0\), the trivial solution exists in this non-magnetic or the conventional scenario, investigated by Bhaduri et. al. [21]. The phase with \(m = 0\) is the Wigner phase and \(m \neq 0\) is the so called Goldstone phase, which further gives
\[
4gV = 1
\]
(39)

which is nothing but the well known gap equation used in BCS theory.

The non-existence of trivial solution indicates the spontaneously broken chiral symmetry in presence of strong quantizing magnetic field. Therefore as soon as the Landau levels are populated for light quarks in presence of external magnetic field, the chiral symmetry gets broken, the quarks become massive and the mass \(m\) (assumed to be
same for both $u$ and $d$ quarks) is generated dynamically. As mentioned earlier, that such investigations were done with quantum field theory. On the other hand the approach followed by us in this article is in some sense semi-classical and have not been done before.

Therefore we may conclude that the Wigner phase does not exist in the case of relativistic Landau diamagnetic system. Further, if the deconfinement transition and restoration of chiral symmetry occur simultaneously, then it puts a big question mark whether the idea of bag model is applicable at all in presence of strong quantizing magnetic field.

To illustrate the variation of dynamical quark mass with magnetic field, we consider the relation

$$m^2 = -\frac{m_0}{f^2_\pi} \langle \psi \bar{\psi} \rangle$$  \hspace{1cm} (40)

where $m_\pi$ is the pion mass, $m_0$ is the quark current mass and $f_\pi$ is the pion decay constant. Using the spinor solutions given by eqns.(6) and (7) we get

$$m^2 = 2m_0m \frac{N_c}{2\pi^2} \sum_{f=\text{u},d} \sum_{\nu=0}^{\nu_{\text{max}}} (2 - \delta_{\nu 0}) \ln \left[ \frac{\Lambda + (\Lambda^2 + m^2_{\nu})^{1/2}}{m_{\nu}} \right]$$  \hspace{1cm} (41)

We have now solved the eqns.(33) and (41) numerically to obtain $\Lambda$ and $m$ for various values of magnetic field. We have considered the following sets of numerical values for the parameters. The current quark mass $m_0 = 10\text{MeV}$, pion mass $m_\pi = 140\text{MeV}$, pion decay constant $f_\pi = 93\text{MeV}$, coupling constant $g = 10\text{GeV}^{-2}$ and electron mass $m_e = 0.5\text{MeV}$. In fig.(1) we shown the variation of dynamically generated quark mass with the strength of magnetic field. As it is evident that the dynamical quark mass never goes to zero and diverges beyond $B_m \approx 10^{17}\text{G}$.

![Graph showing variation of dynamically generated quark mass with magnetic field.](image)

FIG. 1. The variation of dynamically generated quark mass with the strength of magnetic field (expressed in terms of $B_m^{\text{c(e)}} = 4.4 \times 10^{13}\text{G}$.)

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