Application of Quantum Darwinism to Diffusion during Cosmic Inflation

Nicolás F. Lori

IBILI, Universidade de Coimbra, 3000-354 Coimbra, Portugal

Alex H. Blin

Departamento de Física, Universidade de Coimbra, 3004-516 Coimbra, Portugal

Abstract

A baby-universe model of cosmic inflation is analyzed using quantum Darwinism. In this model cosmic inflation can be approximated as Brownian motion of a quantum field, and quantum Darwinism implies that decoherence is the result of quantum Brownian motion of the wave function. The quantum Darwinism approach to decoherence in the baby-universe cosmic-inflation model yields the decoherence times of the baby-universes. The result is the equation relating the baby-universe’s decoherence time with the Hubble parameter. A brief discussion of the relation between Darwinism and determinism is provided in the Appendix.

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I. INTRODUCTION

In this work a possible relation between universe creation during cosmic inflation and quantum Darwinism is proposed. Linde’s approach to the Big Bang [1] indicates that the creation of a universe from nothing occurs in a Brownian motion-like process. Zurek’s quantum Darwinism approach to quantum mechanics [2] indicates that the reason for quantum indeterminacy is that the existence of a state is directly related to that state’s capacity to transmit information about itself, and that this capacity is related to diffusion for the case of quantum Brownian motion [2].

The approaches of Linde and Zurek are examples of two diverging approaches to quantum physics. There is no disagreement about experimental evidence that quantum systems exist in a multitude of states, with only a portion of those states being observable. The approaches differ in what happens to the non-observed states and in the process by which states become observed states.

The approach by Linde proposes that the universe follows a deterministic evolution about which we can only observe partial aspects of the multiple possible occurrences that are deterministically created. The approach by Zurek proposes that the deterministic evolution of the universe is constrained by a Darwinian extinction of some of the possible evolution paths of the system. The approaches by Linde and Zurek agree in what is observed, but they disagree about what happens to the non-observed states. In the case of quantum gravity effects during cosmic inflation these differences may be relevant.

The random extinction of information in quantum Darwinism contrasts with the preservation of information in Hilbert’s formal axiomatic systems (FAS) [3]. The FAS is a deterministic system where the consistence of the axioms is preserved, meaning that no proposition can be both true and not true; and the logic in the FAS obtains true propositions from true propositions. In Darwinian approaches (e.g. quantum Darwinism) if survival is identified with truth, then some true propositions lead to false propositions (which become extinct) and so Darwinism is not consistent. However, in Darwinism there are no true propositions that are not obtained from true propositions (all entities have parent entities that need to be true since they gave offspring); meaning Darwinism is necessarily complete. Gödel’s incompleteness theorems showed that a non-trivial FAS cannot be both complete and consistent [3, 18].
An axiomatic system made to be complete and not consistent would have validly inferred propositions being both true and not-true. A form of dealing with this difficulty would be to validate propositions not by the valid application of inference rules, but by using a proof-checking algorithm that would eliminate propositions that are inconsistent within themselves. Such a process of selecting valid propositions is called here a Darwinian axiomatic system (DAS), and is described more extensively in the Appendix. The FAS and the DAS are the two extreme forms of dealing with Gödel’s incompleteness theorems, respectively the consistent and the complete forms. It is possible to conceive an hybrid axiomatic system (HAS) between the FAS and the DAS.

This work mostly uses the works of Linde [1] and Zurek [2]; other approaches considered are Chaitin’s use of information in mathematics [3], Wheeler’s concept of ‘it for bit’ [4], Rovelli’s relational approach to quantum gravity [5], Smolin’s relation between quantum mechanics and quantum gravity [6], and Guth’s approach to cosmic inflation [7].

Although other approaches to decoherence during cosmic inflation have been developed [8, 9, 10, 11, 12, 13, 14], our approach differs in that it is based on Zurek’s quantum Darwinism [2] and does not rely on having the short and long wavelength quantum fields represent the environment and the system, respectively.

The remainder of the present article is structured as follows. The next five sections discuss the underlying theory: Introduction to Quantum Measurement; Introduction to Quantum Darwinism; Relation between Quantum Darwinism and Quantum Diffusion; Diffusion in Cosmic Inflation; Effects of a Quantum Darwinism Approach to Cosmic Inflation. The calculations at the end of each subsection are then presented in the section on Results, with their relations highlighted. The section entitled Discussion and Summary describes the possible implications of the results obtained and highlights the principal results.

II. INTRODUCTION TO QUANTUM MEASUREMENT

Quantum Darwinism [2, 17] is an approach to quantum measurement that is strongly based on Wheeler’s “it-for-bit” approach [4] and so it has parallels with both information theory and computation. The classical technical definition of the amount of information was provided by Shannon’s information entropy and stated that if the sending device has a probability $P_j$ of sending message $j$ from a set of $N$ messages, then the information produced
when one message is chosen from the set is, in units of bits $3$,

$$H = - \log_2 P_j .$$  

(1)

For a brief description of quantum Darwinism it is helpful to resort to a short description of the limitations of non-Darwinian quantum mechanics, the limitations that quantum Darwinism addresses. In quantum mechanics the universe is separable into 3 parts: I. System $S$, II. Apparatus $A$, III. Environment $E$. The evolution of quantum systems occurs according to Schrödinger’s equation. Entanglement between system and apparatus can be modeled by unitary Schrödinger evolution. Von Neumann $15$ proposed a non-unitary selection of the preferred basis,

$$| \Psi_{SA} \rangle \langle \Psi_{SA} | \rightarrow \sum_k |a_k|^2 |s_k \rangle \langle s_k | A \rangle | A \rangle = \rho_{SA} .$$

and also proposed the non-unitary “collapse” enabling the occurrence of a unique outcome (e.g. for state 17):

$$\sum_k |a_k|^2 |s_k \rangle \langle s_k | A \rangle | A \rangle \rightarrow |a_{17}|^2 |s_{17} \rangle \langle s_{17} | A_{17} \rangle | A_{17} \rangle .$$

Zurek $2, 17$ proposed an approach to entanglement which is unitary and as un-arbitrary as possible, using the environment. The use of the environment implies abandoning the closed-system assumption $17$, requiring the following alteration:

$$| \Psi_{SA} \rangle | e_0 \rangle = \left( \sum_k a_k |s_k \rangle | A \rangle \right) | e_0 \rangle \rightarrow \sum_k a_k |s_k \rangle | A \rangle | e_k \rangle = | \Psi_{SAE} \rangle .$$

The selection of the preferred basis is obtained using unitary evolution by assuming $|\langle e_k | e_l \rangle|^2 = \delta_{kl}$ and tracing over the environment $17$,

$$\rho_{SA} = \text{Tr}_E | \Psi_{SAE} \rangle \langle \Psi_{SAE} | = \sum_k |a_k|^2 |s_k \rangle \langle s_k | A \rangle | A \rangle \langle A | .$$

The preferred basis is defined by the set of states the apparatus can adopt that do not interact with the environment and therefore only interact with the system. The apparatus adopts one of the pointer states after it makes a measurement. For this set of pointer states to exist it is necessary that the apparatus be entangled with the environment. Entanglement
is a non-classical quantum behaviour where two parts of the universe that have interacted at a certain point in time have to be described with reference to each other even if they are now separated in space, as long as they remain entangled. The above explanations of quantum measurement do not clarify the meaning of tracing over the environment, and the non-unitary “collapse” is not really explained. Quantum Darwinism addresses both issues successfully [2, 19].

III. INTRODUCTION TO QUANTUM DARWINISM

In quantum Darwinism, the following statements are considered to be valid: (a) The universe consists of systems. (b) A pure (meaning completely known) state of a system can be represented by a normalized vector in Hilbert space $H_S$. (c) A composite pure state of several systems is a vector in the tensor product of the constituent Hilbert spaces. (d) States evolve in accordance with the Schrödinger equation $i\hbar \dot{\psi} = H\psi$ where $H$ is Hermitian. In quantum Darwinism no “collapse” postulate is needed. An assumption by von Neumann [15] and others is that the observers acquire information about the quantum system from the quantum system, but that is (almost) never the case. The distinction between direct and indirect observation might seem inconsequential as a simple extension of the von Neumann chain, but the use of the point of view of the observer in quantum Darwinism makes it possible to obtain the “collapse” [17, 19].

In quantum Darwinism there is “no information without representation”, meaning that the information is always about a state that is being represented. Preferred pointer states selected through entanglement define what is being stored in the environment. The “information amount” in quantum systems is defined using the density matrix $\rho$ and is based on ref. [17].

Environment-assisted invariance (envariance) is a quantum symmetry exhibited by the states of entangled quantum systems. The joint state can always be described by a Schmidt basis (if the environment is made big enough).
IV. RELATION BETWEEN QUANTUM DARWINISM AND QUANTUM DIFFUSION

In molecular Brownian motion there is a permanent oscillation between position measurement and momentum measurement. Brownian motion of quantum states describes decoherence; this is also an accurate description of molecular Brownian motion. The quantum Brownian motion model used here consists of an environment $E$ made of a collection of harmonic oscillators of position $q_n$, mass $m_n$, frequency $w_n$, and coupling constant $c_n$, interacting with a system $S$ of mass $M$, position $x$, and harmonic potential $V(x) = \frac{1}{2} MW^2 x^2$.

The total Lagrangian is

$$L(x, q_n) = \frac{M}{2} [\dot{x}^2 - W^2 x^2] + \sum_n \frac{m_n}{2} \left[ \dot{q}_n^2 - w_n^2 \left( q_n - \frac{c_n x}{m_n w_n^2} \right)^2 \right]. \quad (2)$$

The Lagrangian component $L_{SE}$ takes into account the renormalization of the potential energy. Let us denote $k$ as the Boltzmann constant and $T$ as the temperature. If the thermal energy $kT$ is higher than all other relevant energy scales, including the energy content of the initial state and energy cutoff in the spectral density of the environment $C(v)$; then the master equation for the density matrix $\rho_S$ of an initially environment-independent system $S$ depends on the renormalized Hamiltonian $H_{ren}$ and on

$$\dot{\rho}_S = -\frac{i}{\hbar} [H_{ren}, \rho_S] - \gamma [x - y] \left[ \frac{\partial}{\partial x} - \frac{\partial}{\partial y} \right] \rho_S - \frac{2M \gamma kT}{\hbar^2} [x - y]^2 \rho_S. \quad (3)$$

In this high $T$ case the master equation is independent of $V(x)$. The relaxation time is $\gamma^{-1}$ and the decoherence time is $\gamma^{-1}$:

$$\tau_D = \gamma^{-1} \left[ \frac{\hbar}{\sqrt{2MKT}} \frac{x - y}{x - y} \right]^2. \quad (4)$$

The Wigner quasi-distribution representation $Z$ of the high temperature density matrix master equation (Eq. 3) is

$$Z = \frac{1}{\pi \hbar^2} \left( x+y \right)^2 \left( \frac{1}{\hbar^2} \left[ \frac{x - y}{x - y} \right]^2 \right). \quad (5)$$
\[ \dot{Z} = -\frac{p}{M} \frac{\partial}{\partial x}[Z] + \frac{\partial V}{\partial x} Z + 2\gamma \frac{\partial}{\partial p}[pZ] + 2\gamma MkT \frac{\partial^2}{\partial p^2}[Z]. \]  

(5)

The minimum uncertainty Wigner quasi-distribution for a phase space localized wave-packet is [20]:

\[ Z(x_0, x, p_0, p) = \frac{1}{\pi \hbar} \exp \left( - \left[ \frac{x - x_0}{\sqrt{\hbar 2MW}} \right]^2 - \left[ \frac{p - p_0}{\sqrt{\hbar 2MW}} \right]^2 \right). \]  

(6)

If there are two wave packets separated by \( \Delta x \), with average location \( x \) and average momentum \( p \), then the joint Wigner quasi-distribution is equal to averaging the two localized Wigner distribution expressions plus a non-classical interaction term equal to [20]

\[ W_{\text{int}} \approx \frac{1}{\pi \hbar} \cos \left( \frac{\Delta x}{\hbar} p \right) \exp \left( - \left[ \frac{x}{\sqrt{\hbar 2MW}} \right]^2 - \left[ \frac{p}{\sqrt{\hbar 2MW}} \right]^2 \right). \]  

(7)

Joining the diffusion coefficient expression [16, 20]

\[ D = \frac{kT}{\gamma M} \]

with the decoherence-time definition of Eq. [11] yields a relation between decoherence-time and diffusion coefficient,

\[ \tau_D = \frac{D}{2} \left[ \frac{\hbar}{kT|x - y|} \right]^2. \]  

(8)

From Einstein’s diffusion equation we know that \( \langle (x(t) - x(0))^2 \rangle = 2Dt \) for a single molecule. Consider now two molecules. Let \( t_{\{x,y\}} \) be the time interval since the last collision of two molecules which collided at the point \( x_0 = y_0 \) and which are now at the positions \( x \) and \( y \), respectively. Using the statistical independence of the two molecules, \( \langle xy \rangle = \langle x \rangle \langle y \rangle \) and noting that \( \langle x \rangle = \langle y \rangle = x_0 \), the expression becomes \( \langle (x - y)^2 \rangle = 4Dt_{\{x,y\}} \). This is an expression for the average behavior of a pair of molecules. A corresponding expression for the particular behavior of two molecules can be written as \( (x - y)^2 = 4D_{\{x,y\}}t_{\{x,y\}} \) where \( D_{\{x,y\}} \) is a coefficient valid for that particular event. With the reasonable assumption that this coefficient becomes \( D_{\{x,y\}} \approx D \) very fast, that is for any appreciable distance \( x - y \),
Eq. (8) can now be rewritten as

\[ \tau_D = \frac{1}{8 t_{\{x,y\}}} \left[ \frac{\hbar}{kT} \right]^2. \]  

(9)

V. DIFFUSION IN COSMIC INFLATION

The purpose of this section is to describe how cosmic inflation relates to Brownian motion. It is not intended to present a thorough description of cosmic inflation. In the present description of cosmic inflation there are multiple Big Bang occurrences, and in each of these occurrences baby-universes are created. One of the baby-universes is our own universe. In order to describe cosmic inflation it is helpful to explain what is being inflated. The behavior of spacetime is characterized by the relation between differences in time and differences in spatial location, and can be represented by the behavior of a single characteristic time varying scale parameter \( a \) which appears in the line element which is characteristic of spacetime. The Hubble parameter is the fractional change of \( a \) with time: \( \mathbb{H} = \frac{1}{a} \frac{da}{dt} \). Inflation describes the early epoch period of rapid growth of \( a \). During inflation \( \mathbb{H} \) is approximately constant at a value roughly of the order \( \mathbb{H} \approx 10^{34} \text{s}^{-1} \) which makes \( a \) approximately proportional to \( e^{\mathbb{H} t} \). Inflation comes to an end when \( \mathbb{H} \) begins to decrease rapidly. The energy stored in the vacuum-like state is then transformed into thermal energy, and the universe becomes extremely hot. From that point onward, its evolution is described by the hot universe theory.

To correctly describe Brownian behavior during cosmic inflation, it is convenient to distinguish between two horizons: the particle horizon and the event horizon. The particle horizon delimits what an observer at a time \( t \) can observe assuming the capacity to detect even the weakest signals. The event horizon delimits the part of the universe from which we can ever (up to some maximal time \( t_{\text{max}} \)) receive information about events taking place now (at time \( t \)). The particle and event horizons are in a certain sense complementary. In an exponentially expanding universe, the radius of the event horizon is equal to \( c \mathbb{H}^{-1} \) where \( c \) is the speed of light in vacuum. In an exponentially expanding universe, any two points that are more than a distance \( c \mathbb{H}^{-1} \) apart will move away from each other faster than \( c \), meaning that those two points will never observe each other. They might belong to the same baby-universe if they come from the same Big Bang, but the points will lie beyond each other’s particle horizons.
As described in Ref. [1], cosmic inflation leads to the creation of multiple baby-universes one of them our own. Some of those universes will have physical behaviors very different from the behavior of our universe, but we will now consider the behavior of quantum fluctuations in the cosmic inflation model. The scalar inflaton field $\varphi$ (sometimes identified with the Higgs field, although this is controversial) is represented as

$$
\varphi (x, t) = (2\pi)^{-\frac{3}{2}} \int d^3 p \left[ a_p^+ \psi_p (t) e^{i px} + a_p^- \psi_p^* (t) e^{-i px} \right].
$$

(10)

The $(2\pi)^{-\frac{3}{2}}$ term is simply a normalization factor, $\int d^3 p$ is the integration over all possible values of the momentum, $a_p^+$ creates a field with momentum $p$ parameter with a probability modulated by $\psi_p (t)$ and propagating in spacetime as the wave $e^{i px}$, and $a_p^-$ destroys that same field.

The first cosmic inflation models considered that $\varphi$ was a classical field (meaning non-quantum). The way a quantum system becomes classical is through the process of decoherence. As described in the previous section, the process of decoherence has strong similarities to Brownian motion. Ref. [1] describes the similarity of the behavior of $\varphi$ during cosmic inflation and Brownian motion.

As it is typical in Brownian motion, the diffusion of the field $\varphi$ can be described by the probability distribution $P(\varphi, t)$ of finding the field $\varphi$ at that point in instant $t$. In Eq. 7.3.17 of Ref. [1] it is found that

$$
\frac{\partial P(\varphi, t)}{\partial t} = D \frac{\partial^2 P(\varphi, t)}{\partial \varphi^2}.
$$

(11)

Using Eq. (11), Ref. [1] shows that

$$
\langle \varphi^2 \rangle = 2Dt
$$

as is expected in diffusion processes (Eq. 7.3.12 in Ref. [1]) and that

$$
D = \frac{H^3}{8\pi^2 c^2}.
$$

(12)
VI. EFFECTS OF A QUANTUM DARWINISM APPROACH TO COSMIC INFLATION

The way a quantum system becomes classical is through the process of decoherence, which according to quantum Darwinism is described by quantum Brownian motion in the high temperature limit. So it is possible that the Brownian process in cosmic inflation described in Ref. [1] entails the extinction of the non-decohered universe states.

Gödel’s incompleteness theorems propose to describe the difficulties of creating a mathematical formalism from nothing using Hilbert’s FAS [3, 18], which is a deterministic approach. Quantum Darwinism proposes to address the creation of classical reality from a quantum reality, using a Darwinian approach. The deterministic and the Darwinian approach to creation can be considered as the two extreme approaches of dealing with Gödel’s incompleteness theorems (see Appendix). The Big Bang proposes to describe the creation of an observable universe from nothing, and so it will be very Darwinian. A Darwinian evolution is a Brownian evolution where extinction might occur; and so this study of the relation between decoherence (extinction of some quantum states) and diffusion (Brownian motion) of baby-universes is a study of Darwinian processes occurring during cosmic inflation.

Solving the diffusion equation (11) during cosmic inflation, one obtains the probability for creation of a universe with a certain vacuum energy. Summing over all topologically disconnected configurations of just-created universes enables obtaining the probability for creating universes with a certain cosmological constant value [1], causing Linde to write that although “it is often supposed that the basic goal of theoretical physics is to find exactly what Lagrangian or Hamiltonian correctly describes our entire world. . . . one could well ask . . . if the concept of an observer may play an important role not just in discussions of the various characteristics of our universe, but in the very laws by which it is governed.” The answer proposed here to Linde’s question is that if the quantum Darwinism approach is applied to cosmic inflation, then the laws of physics are themselves the result of a Darwinian evolution of quantum systems.
VII. RESULTS

We use Eq. (9) to generalize the results obtained for molecules in quantum Brownian motion to baby-universes undergoing Brownian motion during cosmic inflation. The decoherence time $\tau_D$ is then a time duration referring to two baby-universes, with $t$ being the time since they last interacted (typically the last time they were at the same place would be at the beginning of the Big Bang). The decoherence time is obtained as (for an expression in terms of $\varphi$ see footnote [23]):

$$\tau_D = \frac{1}{8t} \left( \frac{\hbar}{kT} \right)^2 .$$  \hspace{1cm} (13)

The differing approaches by Linde and Zurek, which can be linked to the differences between the axiomatic systems FAS and DAS, imply different outcomes for the non-observed states. The FAS/Linde approach considers that the outcomes incompatible with the observed outcome exist in different multi-verses, while the DAS/Zurek approach considers the outcomes incompatible with the observed outcome to have become non-existent. In Zurek’s approach information-transmission is what enables existence [20], while in quantum gravity existence (expressed as the number of quantum particles) is observer dependent and thus can only be understood as a relational concept [1, 5].

The representation of cosmic inflation using a diffusion process in a de Sitter space allows to consider thermal equilibrium with [1, 21]

$$T = \frac{\hbar H}{k}$$

so that Eq. (13) becomes

$$\tau_D = \frac{1}{8t} \left( \frac{1}{H} \right)^2 .$$  \hspace{1cm} (14)

This result implies that during the duration of cosmic inflation, the decoherence time is much smaller than the cosmic inflation duration (for an estimate see footnote [24]). Meaning that, baby-universes will be in a quantum coherent state for only a small fraction of the duration of cosmic inflation. This result agrees with Martineau’s result [11] that decoherence is extremely effective during inflation, but reaches that result in a much more simple way. The approach to “decoherence during Brownian motion” used by Zurek considers that the
effect of zero-point vacuum fluctuations is neglected. Kiefer et al. \[14\] propose that the inclusion of zero-point vacuum fluctuations makes decoherence still effective but no longer complete, meaning that a significant part of primordial correlations remains up to the present moment.

**VIII. DISCUSSION AND SUMMARY**

Obtaining values for the decoherence time requires knowledge of the value of the Hubble parameter before and during inflation. Values of the Hubble parameter have a large range, and the measurement of its value is a topic of current research \[1\]. The existence of baby-universes is also a not yet established observational fact \[1\]. Thus, obtaining experimental proof of Eq. (13) and Eq. (14) is not yet possible. But if baby-universes exist, and if more information is obtained about the time-dynamics of the Hubble parameter, the relation between Hubble parameter and decoherence-time expressed in Eq. (13) and Eq. (14) would be likely to become useful.

A characteristic of biological Darwinism is the existence of a first cell. The approach to cosmic inflation described in Ref. \[22\] indicates that the inflating region of spacetime must have a past boundary; this truly initial Bang would have occurred a lot earlier than our own Big Bang. In this work a relation between Quantum Darwinism and HAS is presented in the Appendix, with the HAS becoming more and more Darwinian as the forces considered become closer to what they were at the truly initial Bang (the initial forces, because of their extremely high energy, are likely to be also the most fundamental forces). The Quantum Darwinism treatment of the truly initial Big Bang would therefore correspond to a process that is as Darwinian as it gets, even more Darwinian than the evolution of species. This Darwinian process would become more and more deterministic as the interactions between aspects of the universe *de facto* measure those aspects of the universe. The measurement described in Eq. (13) and Eq. (14) obtains what the physical constants (and laws) will be for a certain baby-universe by a Darwinian extinction of the other possible values. That the measurement occurring during cosmic inflation is the selector of the physical constants is already proposed in section 10 of Ref. \[1\], but the approach proposed here is different in that it proposes the Darwinian extinction of the non-obtained quantum alternatives that are not moving away at a speed faster than $c$. 

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To summarize, an expression was obtained for the time after which different previously entangled baby-universes would decohere.

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APPENDIX: FORMAL AND DARWINIAN AXIOMATIC SYSTEMS

A FAS is constituted by alphabet, grammar, axioms, rules of inference, and a proof-checking algorithm. In the FAS approach to mathematics, one starts with axioms considered as self-evident and built using the alphabet and the grammar; then the rules of inference are applied to the axioms and all the theorems (logical inferences of the axioms) are obtained. A proof-checking algorithm checks if a proof follows all the rules by doing reverse inference starting from the proof’s result and checking if what is obtained are the axioms. Gödel’s incompleteness theorems showed that a non-trivial FAS cannot be both complete and consistent [3, 18].

Axioms in FAS are typically made to be consistent so that the FAS is consistent, but an FAS cannot be both consistent and complete. A form of dealing with this difficulty is to validate propositions not by the valid application of inference rules, but by using a proof-checking algorithm that would eliminate propositions that are inconsistent within themselves. Such a process of selecting valid propositions is called here a Darwinian axiomatic system (DAS). The FAS and the DAS are the two extreme forms of dealing with Gödel’s incompleteness theorems, respectively the consistent and the complete forms. It is possible to conceive an hybrid axiomatic system (HAS) which lies in-between the FAS and the DAS. In the next paragraphs it will be proposed that Quantum Darwinism is similar to an HAS.

To Chaitin’s information-based Gödel incompleteness conclusion [3] that real numbers are non-computable with probability 1, quantum Darwinism answers through a discrete universe. In mathematical randomness [3] the value of a random variable is only known by running a computer, and in quantum Darwinism the value of a random quantum vari-
able only occurs if the interaction in an experiment is strong enough [19]. The quantum randomness [17, 19] concept is identical to the mathematical randomness [2] concept if the quantum systems’ existence is enabled through their transmission of information, which occurs in quantum Darwinism. The existence’s dependence on information is the part of the Existentialist philosophical structure added by quantum Darwinism [17].

Quantum Darwinism’s Existentialism does not allow for an actual computation as that expressed in Turing’s halting problem both because of its lack of concrete existence and its lack of absolutely closed systems. It is proposed here that envariance [19] enabled by quantum entanglement is an expression of Turing’s halting problem in quantum Darwinism. The entanglement between system and its environment means that a program running on the system can be counter-run by a program in the environment.

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In a "metaphorical" approach to quantum field theory the position $x$ is replaced by the field $\phi$ and the momentum $p$ by the field $\psi$ \[1\]. Then the time during which two baby-universes - located at $x$ and $y$ and described by the corresponding fields $\varphi(x)$ and $\varphi(y)$, respectively - remain entangled after initial interaction would be $\tau_D = H^{-1} \left[ \frac{\hbar}{4\pi c} \frac{x^2}{|\varphi(x) - \varphi(y)|} \right]^2$.

In cosmic inflation, there is a growth by typically at least a factor of $e^{60}$, starting at $t = 10^{-35}s$ and ending at $t = 10^{-32}s$, so that $\Delta t$ is about $10^{-32}s$. Therefore, since $\mathcal{H}\Delta t = 60$, the Hubble parameter $\mathcal{H}$ is about $10^{34}s^{-1}$, and the expression $1/(8H^2t)$ is about $10^{-36}s$. Compared with $10^{-35}...10^{-32}s$ this is much smaller.