Non-Ideal X-Gate and Z-Gate in Semiconducting Spin Qubit Implementations †

Elena Ferraro 1,*, Marco Fanciulli 1,2,† and Marco De Michielis 1,2

1 CNR-IMM Unit of Agrate Brianza, Via C. Olivetti 2, 20864 Agrate Brianza (MB), Italy; marco.fanciulli@mdm.imm.cnr.it (M.F.); marco.demichielis@mdm.imm.cnr.it (M.D.M.)
2 Dipartimento di Scienza dei Materiali, Università degli Studi di Milano-Bicocca, Via R. Cozzi 55, 20125 Milano, Italy
* Correspondence: elena.ferraro@mdm.imm.cnr.it
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Abstract: Several spin qubit architectures have been proposed, theoretically investigated and realized at least on the scale of single devices in view of quantum computation and simulation applications. We focus our study on five qubit types: quantum dot spin qubit, double quantum dot singlet-triplet qubit, double quantum dot hybrid qubit, donor qubit, quantum dot spin-donor qubit and for each one we derived a compact effective Hamiltonian. Single qubit gate fidelities when time interval error is included are compared. A realistic set of values for the error parameters of amplitude controls linked to the z and x contribution appearing in the Hamiltonian models has been used. This study provides a ranking of the gate fidelities for the different qubit architectures highlighting which one is the most robust with respect to the considered control noises.

Keywords: spin qubit; noise; quantum computation

1. Introduction

Semiconductor-based electron spin qubits are an interesting platform for universal quantum computation [1–4]. These types of qubits are realized confining electron spins in host semiconducting materials through electrostatic gates or self-assembled quantum dots (QDs) [5,6] or by donor nuclear spins in solid matrices [7,8]. They are an attractive scenario thanks to the electrons spin long coherence times, the fast gate operations and potential for scaling due to the integrability with the already existing CMOS infrastructure of the microelectronic industry. The five qubit types that we studied are: the quantum dot spin qubit (SQ) [1], the double quantum dot singlet-triplet qubit (STQ) [4], the double quantum dot hybrid qubit (HQ) [6,9–11], the donor qubit (DQ) [7] and the quantum dot spin-donor qubit (SDQ) [8]. We demonstrated that they have in common a compact effective Hamiltonian derived when each qubit is expressed in its proper logical basis [12].

The greatest challenge for semiconductor qubits is improving gate operation fidelity. The ideal realization of quantum gates is indeed deeply influenced by the unavoidable environmental noise that cause decoherence. We studied this problem from a theoretical point of view adopting the entanglement fidelity to test the resilience of the quantum gates with respect to disturbance sources. We consider two different sources of noise directly linked to the z and x contribution appearing in the effective Hamiltonian models. Such abstract controls become physical entities giving a real mean to the type of disturbance and a measure of it once that the qubit type is specified.

The paper is organized as follows. Section 2 defines the global scenario for all the qubit types under investigation providing a compact effective Hamiltonian model. It also contains the main
results about the single qubit gate fidelities (X-gate ans Z-gate) and gives a comparison among all the five qubit types when they are subject to time interval error (TIE). In Section 3, concluding remarks are summarized.

2. Results

The effective Hamiltonian models are here presented. Then non idealities are included in the model to perform a good performance analysis in real systems. We account for error sources that is modeled as random variables with Gaussian distributions featuring zero mean and standard deviation \( \sigma \) that add up to the ideal values. The figure of merit used to estimate the disturbance effects is the entanglement fidelity

\[
F = \text{tr}[\rho^{RS} \mathbb{1}_R \otimes (U^{-1} \mathbb{1}_d) \rho^{RS} \mathbb{1}_R \otimes (U^{-1} \mathbb{1}_d)],
\]

where \( U_i(\mathbb{1}_d) \) is the ideal (disturbed) time evolution and \( \rho^{RS} = |\psi\rangle \langle \psi| \) with \( |\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \) a maximally entangled state in a double state space generated by two identical Hilbert spaces \( R \) and \( S \). X-gate and Z-gate, that are rotations of \( \pi \) along \( x \) and \( z \) axis of the Bloch sphere, are chosen as reference gates for each qubit type. The sequences that realize such rotations are derived analytically for each qubit type [12].

2.1. Quantum Dot and Donor Spin Qubits Effective Hamiltonian Models

The five qubit types under study have in common a compact effective Hamiltonian when expressed each in its proper logical basis \( \{|0\rangle, |1\rangle\} \) [12]. The effective Hamiltonian models in terms of 2 \( \times \) 2 Pauli matrices \( \sigma_z \) and \( \sigma_x \) and the identity operator \( I_z \) are expressed by

\[
H = a_z \sigma_z + a_x \sigma_x + a_0 I_z,
\]

where \( a_z, a_x \) and \( a_0 \) are given in Table 1.

**Table 1.** Coefficients of the effective Hamiltonian models and logical bases.

| Qubit (rot. frame) | \( a_z \) | \( a_x \) | \( a_0 \) | \(|0\rangle, |1\rangle\) |
|-------------------|----------|----------|----------|----------------|
| SQ                | \( \frac{1}{2}(\omega_z - \omega) \) | \( \frac{3}{2} \Omega_x \) | 0         | \{\( |\uparrow\rangle, |\downarrow\rangle\)\} |
| STQ               | \( \frac{-1}{4} \hbar I \) | \( \Delta E_z \) | \( -\frac{1}{4} \hbar I \) | \{\( |S\rangle, |T_0\rangle\)\} |
| HQ                | \( \frac{-1}{4} \hbar J' + \frac{1}{4} (J_1 + J_2) \) | \( -\frac{3}{4} (J_1 - J_2) \) | \( -\frac{3}{4} (J' + J_1 + J_2) \) | \{\( |S\rangle |\uparrow\rangle, |T_0\rangle |\downarrow\rangle\)\} |
| DQ (rot. frame)   | \( \frac{-1}{4} (\omega_1 - \omega) \) | \( \frac{1}{4} \Omega_x \) | 0         | \{\( |\uparrow\rangle, |\downarrow\rangle\)\} |
| SDQ               | \( \frac{-1}{4} \hbar I \) | \( \frac{1}{4} \hbar I \) | \( \frac{1}{4} \gamma_n B_0 - \frac{1}{4} \hbar I \) | \{\( |S\rangle, |T_0\rangle\)\} |

The state \( |\uparrow\rangle (|\downarrow\rangle) \) denotes the single spin with up (down) projection; \( |S(T)\rangle \) is the singlet (triplet) state of the pair of electronic spins and \( |\uparrow\rangle \) is the angular momentum state of the donor nuclear spin. The parameters appearing in Table 1 are here defined for each qubit type:

- **SQ**: \( \omega_z \) is the Zeeman frequency associated to the DC applied magnetic field, \( \omega \) is the angular frequency of the AC local magnetic field and \( \Omega_x \) is the angular frequency that depends on the amplitude of that field.
- **STQ**: \( \Delta E_z \) is the magnetic field gradient between the QDs and \( J \) is the exchange coupling between the two spins.
- **HQ**: \( E_z \) is the Zeeman energy associated to the constant applied magnetic field and \( J', J_1, J_2 \) are the exchange couplings among couple of spins.
- **DQ**: the same as for the SQ, where \( \omega_1 \) is the analogous of \( \omega_z \) and is equal to \( \omega_1 = \Delta_z + \sqrt{\Delta_z^2 + 4a^2} - 2a, \) with \( \Delta_z = 1/2(\gamma_e \pm \gamma_n)B_0 \) and \( a = A/4 \). The parameters \( \gamma_e \) and \( \gamma_n \) are respectively the electron and nuclear gyromagnetic ratio, \( B_0 \) is the external DC magnetic field and \( A \) is the hyperfine coupling. The donor nuclear spin is supposed equal to \( I = 1/2 \).
• SDQ: $J$ is the exchange coupling between the electron spins of the donor and of the dot, $A$ is the hyperfine coupling between the electron spin and the nuclear spin of the donor and $B_0$ is the applied DC magnetic field. The donor nuclear spin is supposed equal to $I = 1/2$.

2.2. Comparison of Gate Fidelities among Qubit Types

A comparison of gate fidelities among all the qubit types due to TIE is presented. To this purpose, control error standard deviations on the amplitudes of the control parameters are set to the values taken from the literature reported in Table 2. As it is evident comparing Tables 1 and 2 the control parameters are directly linked to the $x$ and $z$ contributions of the effective Hamiltonian models.

| Qubit | Error on Control Variables | Semiconductor |
|-------|----------------------------|---------------|
| SQ    | $\sigma_{\Delta E_z} = 20$ Hz [13]; $\sigma_{\Omega_{x}} = 0.25$ MHz [1] | Si/SiGe       |
| STQ   | $\sigma_{\Delta E_z}$ = 4 neV [5]; $\sigma_{J}$ = 1 neV [5] | Si/SiGe       |
| HQ    | $\sigma_{I}$ = 1 neV [6] | Si/SiGe       |
| DQ    | $\sigma_{\Delta E_{D_z}}$ = 100 Hz [7]; $\sigma_{\Omega_{x}}$ = 1.2 kHz | $^{31}$P in Si |
| SDQ   | $\sigma_{I}$ = 4 neV [8]; $\sigma_{A}$ = 2.5 neV [8] | $^{31}$P in Si |

Table 2. Control error standard deviations for the five qubit types.

Figure 1 shows a gate infidelities comparison for $R_x(\pi)$ and $R_z(\pi)$ as a function of the standard deviation $\sigma_t$.

![Figure 1](image1.png)

Figure 1. (a) Comparison of $R_x(\pi)$ gate infidelities among all qubit types as a function of the standard deviation $\sigma_t$. In the legend the smallest time of sequence step $t_{\min}$ for each qubit type is also reported. (b) Same as a) but for $R_z(\pi)$ gate.

For both the gates all the qubits show decreasing infidelities when $\sigma_t$ is reduced. The roll off of each curve is roughly observed for $\sigma_t$ close to the shortest step time $t_{\min}$ of the gate sequence for the corresponding qubit type. SQ, STQ, DQ and SDQ present a saturated behavior when $\sigma_t$ is reduced, meaning that the TIE is no more the fidelity limiter in that range. Such graphs point out that HQ is the most sensitive one to TIE whereas DQ is the most robust to such kind of error for both operations. But such robustness of the DQ is achieved by imposing slower gates than those of other qubits. Given a qubit type, there is a trade-off between the speed of the gate operation and the robustness of the gate fidelity to TIE.

3. Discussion

We reported a comparative study of five spin qubit types realized through electron spin in electrostatically defined quantum dots and the electron spin of impurity atoms in semiconducting host (donors). For each qubit type, we presented a compact effective Hamiltonian model and starting from analytical time sequences that realize $X$-gate and $Z$-gate we estimated the effects on the gate fidelity of
the disturbances by using a Gaussian noise model. A comparison of the gate fidelities of all the qubit implementations due to the TIE is presented using a realistic set of values for the error parameters of amplitude controls taken from the literature. We conclude that the HQ is very sensitive to TIE while the infidelity of the DQ due to TIE is not dominant till very large time errors, at a cost of very slow gate operations. This study offers a general platform for different spin qubit implementations providing an important instrument for future works.

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