RESOLVING THE BETA – DISCREPANCY FOR CLUSTERS OF GALAXIES

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ABSTRACT

Previous comparisons of optical and X-ray observations of clusters of galaxies have led to the so-called “β – discrepancy” that has persisted for the last decade. The standard hydrostatic-isothermal model for clusters predicts that the parameter \( \beta_{\text{spec}} \equiv \sigma_r^2/(kT/\mu m_p) \), which describes the ratio of energy per unit mass in galaxies to that in the gas, should equal the parameter \( \beta_{\text{fit}} \) (where \( \rho_{\text{gas}}(r) \propto \rho_{\text{gal}}(r)^{\beta_{\text{fit}}} \)) determined from the X-ray surface brightness distribution. The observations suggest an apparent discrepancy: \( \beta_{\text{spec}} \sim 1.2 \) (i.e., the galaxies are “hotter” than the gas) while \( \beta_{\text{fit}} \sim 0.65 \) (i.e., the gas is “hotter” and more extended than the galaxies). Here we show that the discrepancy is resolved when the actual observed galaxy distribution in clusters is used, \( \rho_{\text{gal}}(r) \propto r^{-2.4 \pm 0.2} \), instead of the previously assumed steeper King approximation, \( \rho_{\text{gal}}(r) \propto r^{-3} \). Using the correct galaxy profile in clusters, we show that the standard hydrostatic-isothermal model predicts \( \beta_{\text{spec}} = \beta_{\text{fit}}^c \simeq (1.25 \pm 0.10) \times \beta_{\text{fit}} \), rather than \( \beta_{\text{spec}} \simeq \beta_{\text{fit}} \) (where \( \beta_{\text{fit}} \) is the standard parameter using the King approximation, and \( \beta_{\text{fit}}^c \) is the corrected parameter using the proper galaxy distribution). Using a large sample of clusters, we find best-fit mean values of \( \beta_{\text{spec}} = 0.94 \pm 0.08 \) and \( \beta_{\text{fit}}^c = 1.25 \times \beta_{\text{fit}} = 0.84 \pm 0.10 \). These results resolve the \( \beta \) – discrepancy and provide additional support for the hydrostatic cluster model.

Subject headings: Galaxies: clustering – X-rays: galaxies
1. INTRODUCTION

The gravitational mass of clusters of galaxies has traditionally been estimated from the dynamics of the cluster galaxies using the virial theorem, revealing large amounts of dark matter (Zwicky 1933, Peebles 1980). More recently, X-ray emission from the hot intrachannel gas has been used to estimate cluster masses by utilizing the intrachannel gas temperature and the density profile as a tracer of the cluster potential assuming hydrostatic equilibrium for the cluster gas (Cavaliere & Fusco-Femiano 1976, Bahcall & Sarazin 1977, Forman & Jones 1984, Sarazin 1986, Hughes 1989, Evrard 1990, Bahcall & Cen 1993). The two methods yield comparable masses. However, one problem has persisted over the years: the so-called “β – discrepancy” of clusters. This problem reflects the discrepancy between the observed parameter $\beta_{\text{spec}}$, determined from the X-ray temperature and cluster velocity dispersion, and $\beta_{\text{fit}}$, determined from the gas versus galaxy density profiles in the clusters. If the standard hydrostatic cluster model is correct, the two parameters are expected to have similar values; however, observations suggest that $\beta_{\text{spec}} \sim (1.5 - 2) \times \beta_{\text{fit}}$ (Sarazin 1986, Evrard 1990). This discrepancy has been an unsolved puzzle in the study of clusters of galaxies for nearly a decade.

In this paper, we show that the “β – discrepancy” results mainly from assuming a galaxy distribution in clusters which is too steep (the King approximation), thus causing misleading conclusions. The “β – discrepancy” is resolved if the actual observed profile of the galaxy distribution in clusters is used.

2. THE β – DISCREPANCY

The standard model for the structure of mass in clusters assumes that both the gas and the galaxies are in hydrostatic equilibrium with the binding cluster potential (Cavaliere & Fusco-Femiano 1976, Bahcall & Sarazin 1977, Forman & Jones 1984, Sarazin 1986, Evrard 1990). In this model the gas distribution obeys $\frac{1}{\rho_{\text{gas}}} \frac{dP_{\text{gas}}}{dr} = \frac{d\phi}{dr} = -\frac{GM(r)}{r^2}$, where $P_{\text{gas}}$ and $\rho_{\text{gas}}$ are the gas pressure and density profiles, $\phi$ is the cluster potential, and $M(r)$ is the total binding cluster mass within radius $r$ of the cluster center. The cluster mass can thus be represented as

$$ M(r) = -\frac{kT}{\mu m_p G} \left( \frac{d \ln \rho_{\text{gas}}(r)}{d \ln r} + \frac{d \ln T}{d \ln r} \right) r $$

(1)
where $T$ is the intracluster gas temperature, and $\mu m_p$ is the mean particle mass of the gas.

The galaxies in the cluster respond to the same gravitational field, and thus satisfy

$$M(r) = -\frac{\sigma_r^2}{G} \left( \frac{d\ln \rho_{\text{gal}}(r)}{d\ln r} + \frac{d\ln \sigma_r^2}{d\ln r} + 2A \right) r \quad (2)$$

where $\sigma_r$ is the radial velocity dispersion of the galaxies in the cluster, $\rho_{\text{gal}}(r)$ is the galaxy density as a function of $r$, and $A$ represents a possible anisotropy in the galaxy velocity distribution [$A = 1 - (\sigma_t/\sigma_r)^2$, where $t$ and $r$ represent the tangential and radial velocity components].

Relations (1) and (2) thus yield:

$$\beta_{\text{spec}} \equiv \frac{kT}{\mu m_p} = \frac{d\ln \rho_{\text{gas}}(r)/d\ln r + d\ln T/d\ln r}{d\ln \rho_{\text{gal}}(r)/d\ln r + d\ln \sigma_r^2/d\ln r + 2A} \quad (3)$$

where the $\beta_{\text{spec}}$ parameter is defined by the left side of relation (3). This spectral $\beta$ parameter can be determined directly from observations of cluster velocity dispersions and gas temperatures; it represents the ratio of energy per unit mass in the galaxies to that in the gas and is observed to be $\sim 1$ (see below). (Here and below we assume that the cluster velocity dispersion is isotropic, thus the radial velocity dispersion $\sigma_r$ is comparable to the observed line-of-sight velocity dispersion $\sigma$). The right-hand side of equation (3) relates the $\beta_{\text{spec}}$ parameter to the density profiles of the gas and galaxies in the cluster as well as to the temperature and velocity profiles.

For isothermal galaxy and gas distributions and isotropic galaxy velocities (a model that is traditionally used to describe clusters), one has $d\ln T/d\ln r = d\ln \sigma_r^2/d\ln r = A \simeq 0$; relation (3) then becomes

$$\beta_{\text{spec}} \equiv \frac{kT}{\mu m_p} = \frac{d\ln \rho_{\text{gas}}(r)/d\ln r}{d\ln \rho_{\text{gal}}(r)/d\ln r} \equiv \beta_{\text{fit}}^c \quad (4)$$

the $\beta_{\text{spec}}$ parameter determined from $\sigma_r$ and $T$ (left side of eq. 4) should approximately be equal to the ratio of the slopes of the gas to galaxy density profile, defined as $\beta_{\text{fit}}^c$ (right side of equation 4). The solution of equation (4) yields $\rho_{\text{gas}}(r) \propto \rho_{\text{gal}}(r)^{\beta_{\text{fit}}^c}$ and $\beta_{\text{spec}} \simeq \beta_{\text{fit}}^c$. The parameter $\beta_{\text{fit}}^c$ is determined by fitting the observed X-ray surface brightness distribution in clusters. The galaxy distribution needed to normalize the $\beta_{\text{fit}}^c$
determination has been generally assumed to be the King (1962) approximation, \( \rho_{\text{gal}}(r) = \rho_{\text{gal}}(0)(1 + (r/R_c)^2)^{-3/2} \), where \( \rho_{\text{gal}}(r) \propto r^{-3} \) for large radii (i.e., \( r > R_c \), where \( R_c \) is the cluster core radius). Therefore, \( \rho_{\text{gas}}(r) = \rho_{\text{gas}}(0)(1 + (r/R_c)^2)^{-3/2 \beta_{\text{fit}}/2} \); here \( \beta_{\text{fit}} \) without the superscript “c” for “correct” denotes the standard definition of \( \beta_{\text{fit}} \) using the King approximation (see, e.g. Abramopoulos & Ku 1983, Jones & Forman 1984, Sarazin 1986, Evrard 1990, and references therein). Therefore, \( \beta_{\text{fit}} \) has previously been determined from the relation

\[
\rho_{\text{gas}}(r) \propto \rho_{\text{gas}}(r)^{\beta_{\text{fit}}} \propto r^{-3 \beta_{\text{fit}}} \tag{5}
\]

where \( \beta_{\text{fit}} \) represents the determination using the King approximation, and \( \beta_{\text{fit}}^c \) is the “correct” value from equation (4).

X-ray observations (Abramopoulos & Ku 1983, Jones & Forman 1984, Henry et al. 1993) yield \( \beta_{\text{fit}} \) values in the range \( \sim 0.5 \) to 0.9, with a median of \( \beta_{\text{fit}} \simeq 0.67 \pm 0.10 \) (rms); this corresponds to a gas density profile of

\[
\rho_{\text{gas}}(r) \propto r^{-2.0 \pm 0.3} \quad R_c < r \leq 1.5 \ h^{-1} \ Mpc \tag{6}
\]

On the other hand, observations yield \( \beta_{\text{spec}} = \sigma_r^2/(kT/\mu m_p) \) values in the range of \( \sim 0.5 \) to \( \sim 2 \) with a median \( \beta_{\text{spec}} \simeq 1 - 1.4 \) (Mushotsky 1984, Sarazin 1986, Evrard 1990, Edge & Stewart 1991, Lubin & Bahcall 1993). This inequality between \( \beta_{\text{spec}} \) and \( \beta_{\text{fit}} \) contradicts the expectation from the hydrostatic model. We discuss below how this discrepancy is resolved.

\section*{3. RESOLVING THE \( \beta \) – DISCREPANCY}

The “\( \beta \) – discrepancy” is based on the assumption that the galaxy distribution in clusters follows the King approximation, \( \rho_{\text{gal}}(r) \propto r^{-3} \) at large radii. This assumption, however, is inaccurate. Specific studies of the galaxy distribution in clusters yield a shallower profile. The galaxy-cluster cross-correlation function, which represents the average net galaxy density profile around rich clusters, yields \( \rho_{\text{gal}}(r) \propto r^{-2.2} \) (Lilje & Efstathiou 1988), or \( \rho_{\text{gal}}(r) \propto r^{-2.4} \) (Seldner & Peebles 1978, Peebles 1980). The average profile of a sample of rich clusters yields \( \rho_{\text{gal}}(r) \propto r^{-2.6} \) (Bahcall 1977) (all for \( 0.5 \ h^{-1} \leq r \leq 1.5 \ h^{-1} \ Mpc \)). The profile becomes even shallower, \( \rho_{\text{gal}}(r) \propto r^{-2} \) for \( r \leq 0.5 \ h^{-1} \ Mpc \) (above references;
see also Beers & Tonry 1986). We use below the observed range of the average galaxy density distribution in rich clusters

\[ \rho_{\text{gal}}(r) \propto r^{-2.4\pm0.2} \quad 0.5 \, h^{-1} < r \leq 1.5 \, h^{-1} \, \text{Mpc} \quad (7) \]

Inserting relations (6–7) in equations (4–5) we find:

\[ \rho_{\text{gas}}(r) \propto \rho_{\text{gal}}(r)^{\beta_{\text{fit}}} \propto r^{-(2.4\pm0.2)\beta_{\text{fit}}} \propto r^{-3\beta_{\text{fit}}} \propto r^{-2.0\pm0.3} \quad (8) \]

where the last term in (8) represents the observed X-ray profiles (Sect. 2). The corrected \( \beta_{\text{fit}} \) parameter introduced here is related to the parameter determined using the King approximation, \( \beta_{\text{fit}} \) (eq. 5), by

\[ \beta_{\text{c fit}} \simeq \frac{3}{2.4 \pm 0.2} \beta_{\text{fit}} = (1.25 \pm 0.10) \times \beta_{\text{fit}} \quad (9) \]

Since the X-ray observations yield a mean value of \( \beta_{\text{fit}} = 0.67 \pm 0.02 \) (Jones & Forman 1984, Henry et al. 1993), the corrected \( \beta_{\text{c fit}} \) parameter (eq. 9) has a mean value of

\[ \beta_{\text{c fit}} = (1.25 \pm 0.10) \times \beta_{\text{fit}} = 0.84 \pm 0.10 \quad (10) \]

If the fit is dominated by X-ray emission at small separations, where \( \rho_{\text{gal}} \propto r^{-2} \), then \( \beta_{\text{c fit}} \sim 1 \).

Detailed X-ray and optical observations of the cluster A2256 (Henry et al. 1993) using recent X-ray data from ROSAT support the above analysis; the observations yield \( \rho_{\text{gas}} \propto r^{-2.4}, \rho_{\text{gal}} \propto r^{-2.5} \), \( \beta_{\text{fit}} = 0.795 \), and therefore \( \beta_{\text{c fit}} = 0.96 \), fully consistent with relations (7–10).

A recent analysis of the largest-available sample of clusters with measured \( T \) and line-of-sight velocity dispersions \( \sigma \) (each having more than twenty galaxy redshifts per cluster) yields a best-fit \( \beta_{\text{spec}} \) parameter for the entire sample (Lubin & Bahcall 1993):

\[ \beta_{\text{spec}} \equiv \frac{\sigma_{\tau}^2}{kT/\mu m_p} \approx \frac{\sigma^2}{kT/\mu m_p} = 0.94 \pm 0.08 \quad (11) \]

This best-fit \( \beta_{\text{spec}} \) was obtained from a \( \chi^2 \) fit to the observed \( \sigma(T) \) relation for the sample \( [\sigma = (\beta_{\text{spec}}/\mu m_p)^{0.5} \, (kT)^{0.5}] \) using the average observed \( \mu = 0.58 \) for clusters (Edge & Stewart 1991). The median value for the sample is \( \beta_{\text{spec}} \) (median) = 0.98.
The directly observed values of $\beta_{\text{spec}}$ and $\beta_{\text{fit}}^c$ (eqs. 10–11) are consistent with each other as expected from the hydrostatic-isothermal model (eq. 4). Various effects such as possible substructure in some clusters (Geller & Beers 1982), velocity anisotropy or contamination, cooling flows, or incomplete thermalization of the gas may contribute to the observed scatter (see Figures below) but do not significantly affect the main results obtained above (see Lubin & Bahcall 1993).

The agreement between $\beta_{\text{spec}}$ and $\beta_{\text{fit}}^c$ persists even if the distributions are not isothermal. Some clusters show a decrease in their temperature and velocity dispersion profiles (e.g., Coma; Kent & Gunn 1982, Jones & Forman 1992). Using relation (3), with temperature and velocity gradients of $dlnT/dlnr \simeq dln\sigma^2_r/dlnr \sim -0.5$ to $-1$ (as appears to be suggested by the results in the above references), as well as allowing for a small amount of velocity anisotropy in clusters ($A \leq 0.2$), we find $\beta_{\text{fit}}^c \sim 0.9 - 1$ (from the right side of eq. 3). This value is in excellent agreement with the directly measured value of $\beta_{\text{spec}} = 0.94 \pm 0.08$ (eq. 11).

The results obtained here are illustrated in Figures 1–3. In Figure 1, we plot the observed $\beta_{\text{fit}}^c$ and $\beta_{\text{spec}}$ values for all clusters where both parameters have been measured (Jones & Forman 1984 for $\beta_{\text{fit}}^c$; Lubin and Bahcall 1993 for $\beta_{\text{spec}}$). The original $\beta_{\text{fit}}$ values (assuming the King approximation) are presented in Figure 1a, while the corrected $\beta_{\text{fit}}^c$ values are presented in Figure 1b. While Figure 1a clearly reflects the $\beta$–discrepancy (i.e. $\beta_{\text{spec}} > \beta_{\text{fit}}$), this discrepancy is eliminated in Figure 1b, where the corrected $\beta_{\text{fit}}^c$ values (corresponding to a galaxy density profile of $\rho_{\text{gal}} \propto r^{-2.4\pm0.2}$) are plotted. The ratio $\beta_{\text{spec}} / \beta_{\text{fit}}^c$ is presented for all clusters in Figure 2. It is clear from the results in Figure 2 that, within the observational uncertainties, no $\beta$–discrepancy is apparent ($\beta_{\text{spec}} / \beta_{\text{fit}}^c \sim 1$). Finally, we plot in Figure 3 the recently observed gas and galaxy density profiles in the cluster A2256 (Henry et al. 1993). The data supports the main conclusions reached above: the gas and galaxy distributions follow each other (i.e., $\beta_{\text{fit}}^c \simeq 1$) within the observational uncertainties. Both distributions can be represented by an $\rho \propto r^{-2.5}$ profile. The spectroscopic $\beta_{\text{spec}}$ parameter of the cluster is $\beta_{\text{spec}} = 1.33 \pm 0.24$ (Lubin & Bahcall 1993); these values are included in Figures 1–2 and are consistent with the entire set presented.
We conclude that no significant $\beta$ – discrepancy exists in clusters of galaxies. The hydrostatic model therefore provides a consistent fit to both the X-ray and the optical data.

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FIGURE CAPTIONS

Fig. 1
(a) The observed values of the parameters $\beta_{fit}$ (using the King approximation; Jones & Forman 1984) and $\beta_{spec}$ ($= \sigma^2/kT/\mu m_p$; Lubin & Bahcall 1993) for all clusters for which both parameters are measured. The “$\beta$ – discrepancy” effect, i.e., $\beta_{spec} > \beta_{fit}$, is apparent
(b) Same as (a), but for the corrected $\beta_{fit}^c$ values, assuming the average observed galaxy distribution $\rho_{gal}(r) \propto r^{-2.4 \pm 0.2}$ (Sect. 3). No “$\beta$ – discrepancy” is apparent. (The scatter in $\beta_{spec}$ is larger than in $\beta_{fit}^c$, as expected due to larger observational uncertainties.)

Fig. 2
The ratio $\beta_{spec}/\beta_{fit}^c$ for all available clusters (same data as Figure 1b). No apparent discrepancy is seen (i.e., $\beta_{spec}/\beta_{fit}^c \sim 1$). Different symbols represent different sub-samples: filled squares – clusters in superclusters; triangles – isolated clusters; open squares – clusters at $|b| < 20^\circ$ (see Lubin & Bahcall 1993).

Fig. 3
Projected galaxy density distribution (open circles) and projected gas density distribution (solid curves represent $\pm 1$ sigma results) for the cluster A2256 (Henry et al. 1993). The galaxy density is in units of galaxies per deg$^2$; the gas density is in arbitrary units. Both profiles follow $\rho(r) \propto r^{-2.5}$ outside the cluster core.
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