Pricing decisions in a strategic single retailer/dual suppliers setting under order size constraints

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In this paper, we study a duopolistic market of suppliers competing for the business of a retailer. The retailer sets the order cycle and quantities from each supplier to minimize its annual costs. Different from other studies in the literature, our work simultaneously considers the order size restriction and the benefit of order consolidation, and shows non-trivial pricing behaviour of the suppliers under different settings. Under asymmetric information setting, we formulate the pricing problem of the preferred supplier as a non-linear programming problem and use Karush–Kuhn–Tucker conditions to find the optimal solution. In general, unless the preferred supplier has high-order size limit, it prefers sharing the market with its competitor when retailer’s demand, benefit of order consolidation or fixed cost of ordering from the preferred supplier is high. We model the symmetric information setting as a two-agent non-zero sum pricing game and establish the equilibrium conditions. We show that a supplier might set a ‘threshold price’ to capture the entire market if its per unit fixed ordering cost is sufficiently small. Finally, we prove that there exists a joint-order Nash equilibrium only if the suppliers set identical prices low enough to make the retailer place full-size orders from both.

Keywords: order size constraints; pricing; order consolidation; Nash equilibrium; EOQ; information asymmetry

1. Introduction

As competition on a global scale becomes fiercer, suppliers and retailers face increasingly challenging pricing and inventory management decisions. Even though such decisions are made at a firm level, a broader view is often required as this process involves several parties in a supply chain. As each entity in the market tries to manipulate the outcome for its own benefit, it is difficult to say that a stable market structure ever exists. Nevertheless, the balance conditions are worth investigating as they offer several valuable insights into the key dynamics of supply chains, and the benefits of exploiting these insights in determining optimal strategies are tremendous.

In this paper, we study a two-echelon supply chain where we have two competing suppliers and a retailer ordering from either one or both suppliers in order to minimise its annual ordering costs. In our setting, suppliers have order size constraints which limit the retailer’s order size in each order cycle (Hariga and Haouari 1999; Yazlali and Erhun 2009; Zhang, Hua, and Benjaafar 2012). Such order size constraints can be due to physical limits of suppliers’ production facilities or can be imposed by the suppliers to ration the capacity effectively among many customers (Yazlali and Erhun 2009). For the sake of simplicity, we assume that the retailer operates under the well-known Economic Order Quantity (EOQ) model (Nahmias 2008). In the setting under consideration, first suppliers set their prices to maximise their profits considering their order size limits and the expected ordering behaviour of the retailer. Then, upon observing those prices, the retailer simultaneously makes the supplier selection and order allocation decisions. In the setting under consideration, when the retailer orders from both suppliers in an order cycle, the joint fixed ordering cost is less than the sum of the individual fixed ordering costs suggesting a synergy in joint procurement, which has never been studied in the literature in this context. When the retailer orders from only one supplier in an order cycle, we refer to it as a single-order policy. Alternatively, if the retailer orders from both suppliers in the same order cycle, we call this a joint-order policy.

In this order size-restricted setting, even a small change in the pricing strategy of a supplier may change the retailer’s ordering strategy significantly. More specifically, such a change may make the retailer switch from a joint-order policy to a single-order policy or vice versa. Consequently, suppliers face a tradeoff between fighting for the market share and increasing the price for higher revenue per product sold. This decision depends on the intricate relationship between the order size limits of suppliers, fixed ordering costs and cost savings in joint ordering.

The price competition analysed in this paper is motivated by the Wal-Mart’s new inbound logistics strategy setting. In May 2010, an article featured in Bloomberg Businessweek reported that the retail industry giant Wal-Mart aimed to ‘take the
driver’s seat’ and control the inbound transportation from its suppliers (Wolf, Burritt, and Boyle 2010). Under this initiative, Wal-Mart utilised its own fleet of 6500 trucks and 55,000 trailers as well as its transportation contractors to reduce the cost of inbound freight through economies of scale and consolidation. According to the same article, Kelly Abney, Wal-Mart’s vice-president of corporate transportation, said ‘It has allowed our suppliers to focus on what they do best, manufacturing products for us’. What Wal-Mart expected in return was price cuts from the suppliers due to cost savings in hauling goods. Being famous for aggressively negotiating prices with its suppliers, Wal-Mart reportedly expected a price cut twice as much as the savings in transportation costs (Wolf, Burritt, and Boyle 2010). Under this analogy, we attempt to assess this strategy from the perspectives of the suppliers and the retailer in this paper. The goal of the retailer is to reduce the cost of inbound transportation due to consolidation and economies of scale and ultimately to negotiate for price cuts with the suppliers. Hence, suppliers face the tough decision of re-setting the prices in this low-profit margin market and under competition as well.

We analyse the pricing decisions of the suppliers under asymmetric information and symmetric information settings. In the asymmetric information setting, only Supplier 1 has full market information regarding demand, cost structures, order size restrictions, and also observes the price set by Supplier 2. In practice, this can occur if Supplier 1 has close business relationships with the retailer and has access to full market information. In the symmetric information setting, both suppliers have full market information and enter a strategic price competition. In both settings, when the retailer is indifferent between the two suppliers, for simplicity we assume that the retailer prefers Supplier 1 over Supplier 2. Otherwise, the retailer randomises in tie-breaking situations, which does not affect our results.

Our analysis reveals several important results and insights. First of all, both suppliers have two possible strategies: (i) either capture the entire market and enjoy a higher scale with a low price, or (ii) share the market with the competitor and enjoy a higher profit margin per product. In the former strategy, one of the suppliers sets a ‘threshold price’, low enough, so that the retailer orders only from itself.

Under asymmetric information, when Supplier 1 has high-order size limit, it chooses the threshold pricing strategy except for the cases where Supplier 2 has extremely low-order size limit. In such cases, Supplier 1 prefers sharing the market with Supplier 2 at a higher price compared to the threshold price (as long as there is enough benefit of order consolidation from the retailer’s perspective). When Supplier 1 has low-order size limit, it prefers sharing the market with the other supplier. If Supplier 1 has a fixed order cost or order size limit advantage against Supplier 2, it uses this advantage to increase its revenue by charging a significantly higher unit price (compared to Supplier 2).

Under symmetric information, both suppliers actively compete on the market share or premium price. In this case, we identify two types of Nash equilibria, unlike Bertrand pricing, with positive prices. We refer to Nash equilibrium simply as equilibrium throughout the paper. In the first type, called threshold-pricing equilibrium, only one of the suppliers captures the entire market whereas in the second type, called joint-order equilibrium, the market shares are determined according to the order size limits. In the threshold-pricing equilibrium, we show that in the presence of a preferred supplier (Supplier 1), the unpreferred supplier (Supplier 2) can never capture the entire market in an equilibrium. For Supplier 1, threshold-pricing equilibrium exists only if the per unit fixed cost (fixed cost divided by order size limit) of Supplier 1 is small enough relative to the joint-order policy. In the joint-order equilibrium, we show that even when the suppliers are not identical in terms of per unit fixed cost, equilibrium only occurs at identical prices. However, for such an equilibrium to exist, suppliers should have similar characteristics in terms of order size limits and fixed ordering costs.

Our insight from the suppliers’ perspective is that when a positive threshold price exists, Supplier 1 may either set the price to the threshold or to a higher price where a joint-order equilibrium is achieved. On the other hand, when Supplier 2 has the cost advantage, market prices do not constitute a threshold-pricing equilibrium as the price set by Supplier 2 tends to increase.

From the retailer’s perspective, in the case of differentiated suppliers in terms of their per unit fixed cost, the retailer enjoys benefits beyond the savings in joint procurement, because in this case, the advantageous supplier would also consider the possibility of a joint-order and set a lower threshold price.

Another interesting insight from the retailer’s perspective is that the suppliers’ similarity with respect to per unit fixed cost is beneficial. In this case, threshold-pricing strategy is no longer viable for suppliers and price competition becomes more intense. Similar to the previous case, the synergy in joint procurement benefits the retailer as it elevates the price competition further. As a result of this competition, depending on the order size limits and cost parameters, suppliers might reach a joint-order equilibrium with positive prices.

Finally, in the case of suppliers with identical per unit fixed costs, suppliers reach a Bertrand-type equilibrium with zero prices only when there is no cost savings due to order consolidation. When there is savings in joint order, both suppliers realise that the retailer prefers joint order due to the cost savings, which deteriorates the price competition and benefits the suppliers instead. In fact, we show that the retailer is worse off and its procurement cost increases despite the cost savings in joint procurement due to the inflated unit prices charged by the suppliers. This counter-intuitive result suggests that the
The classification is based on the solution approach used such as data envelopment analysis (Liu, Ding, and Lall 2000; Talluri, and Hassini (2007) for a review of the supplier selection and order allocation literature. Aissaoui, Haouari, and Hassini, 2011) or uncapacitated (Basnet and Leung 2005; Federgruen and Yang 2008) suppliers is widely studied by several authors under deterministic (Burke, Carrillo, and Vakhraria 2008; Chauhan and Proth 2003) or stochastic (Burke, Carrillo, and Vakhraria 2007; Kim et al. 2002; Zhang and Ma 2009) demand assumption. We refer the reader to Aissaoui, Haouari, and Hassini 2007; Vakharia 2007; Kim et al. 2002; Zhang and Ma 2009; demand assumption). Following their classification, the most relevant papers to our setting would be the single-item models with discounts (Chaudhry, Forst, and Zydiak 1993; Tempelmeier 2002) and without discounts (Buffa and Jackson 1983; Current and Weber 1994; Ghodsypour and O’Brien 2001; Hong and Hayya 1992; Pan 1989; Rosenblatt, Herer, and Hefter 1998; Weber and Current 1993; Zeng 1998). For instance, Weber and Current (1993) develop a multi-objective approach to a multicriteria vendor selection problem and demonstrate the approach on an actual problem of a Fortune 500 company. Rosenblatt, Herer, and Hefter (1998) consider a single-item EOQ model under a multi-supplier setting (each with a finite long-run average capacity) and propose a solution procedure that yields a cyclic ordering schedule. Ghodsypour and O’Brien (2001) present a non-linear integer programming model to solve the multi-sourcing problem. The model considers net price, storage, transportation, and ordering costs, and other aspects such as budget and quality limitations can also be incorporated into the model. On the other hand, it is required to solve $2^n$ pure non-linear programmes to reach a solution (where $n$ is the number of vendors), which is not restricting as long as the number of vendors is small enough.

In addition to cost criterion, risk of supply also affects single- or multi-sourcing decisions (Yu, Zeng, and Zhao 2009). For example, Federgruen and Yang (2008) analyse the supplier selection and order allocation decisions of a single firm (over a single demand season) under demand uncertainty and develop approximation methods to minimise total procurement costs while ensuring that the uncertain demand is met with a given probability. Talluri, Narasimhan, and Nair (2006) consider supply risk as in the risk associated with performance variability of suppliers and develop a chance-constrained data envelopment analysis approach for supplier evaluation/selection.

Ho, Xu, and Dey (2010) provide an extensive review of multicriteria approaches in supplier selection and order allocation. The classification is based on the solution approach used such as data envelopment analysis (Liu, Ding, and Lall 2000; Talluri, Narasimhan, and Nair 2006; Talluri and Sarkis 2002), mathematical programming (Hong et al. 2005; Karpak, Kumcu, and Kasuganti 2001; Narasimhan, Talluri, and Mahapatra 2006; Talluri 2002), analytic hierarchy process (Chan 2003) and fuzzy set theory (Chen, Lin, and Huang 2006; Sarkar and Mohapatra 2006). Besides the individual approaches, there exist papers on integrated approaches for vendor selection as well. For instance, Cakravastia and Takahashi (2004) develop an integrated model for supplier selection and negotiation process that generates a set of alternatives each period. Cebi and Bayraktar (2003) model the supplier selection problem using an integrated analytic hierarchy process and lexicographic goal programming while considering conflicting factors. Sevkli et al. (2007) propose an integrated analytic hierarchy process and data envelopment analysis approach for the supplier selection problem of a Turkish company producing appliances. Finally, Mendoza and Ventura (2008) propose a two-stage model, analytic hierarchy process and mathematical programming, where the first stage ranks the potential suppliers and reduces the number of candidate suppliers and the second stage allocates the order quantities so as to minimise the costs.

Contrary to the literature on supplier selection and order allocation which focuses on the decisions of the buyer (retailer in our setting), we investigate the potential strategies of the suppliers given the information that the retailer makes the ordering decisions optimally. As the suppliers’ counteractions affect the retailer’s decision in a cyclic manner, we model these strategic interactions using game theoretical concepts and attempt to identify an equilibrium.
Related to the order size-restricted setting studied in this paper, Zhang, Hua, and Benjaafar (2012) analyse the inventory control policy of a retailer facing an uncertain demand for a single product. In their setting, the retailer can order from two suppliers under order size restrictions. However, they assume that the unit prices are already set by the suppliers and the retailer determines the ordering policy accordingly. Yazlali and Erhun (2009) also study the inventory control problem of a retailer in the presence of order size constraints and stochastic demand. Assuming there are no fixed ordering costs, they show the optimality of a two-level modified base stock policy.

The second stream of research related to our study is the joint economic lot size models, which study coordinated buyer–supplier inventory replenishment settings in order to improve system-wide efficiencies in a supply chain. We refer the reader to Glock (2012) and Ben-Daya, Darwish, and Ertogral (2008) for extensive reviews of the joint economic lot sizing literature. Glock (2012) classify the basic integrated models in joint economic lot sizing as a single buyer and a single vendor model (David and Eben-Chaime 2008; Goyal 1977; Hill 1999), a single vendor and multiple buyers model (Banerjee and Banerjee 1994; Chen, Lin, and Cheng 2010; Joglekar and Tharthare 1990; Siajadi, Ibrahim, and Lochert 2006), and multiple vendors and single buyer model (Chen and Sarker 2010). The setting studied by Chen and Sarker (2010) is the closest one to our setting, as they not only consider sourcing from multiple vendors but also assume a milk run-type collection from the vendors resembling our joint-order synergy assumption. Glock (2012) also reviews extended integrated models, which consider other aspects of the problem. The most relevant ones of these extended models are the ones that consider order/set-up cost reduction. For instance, Woo, Hsu, and Wu (2001) consider order cost reduction in a single vendor and multiple heterogeneous buyers setting, and Affisco, Paknejad, and Nasri (1993) consider a situation where both order and set-up costs can be reduced. The main difference between the joint economic lot sizing setting and our setting is that the supplier(s) and buyer(s) coordinate their actions to reach a common goal in the former setting whereas in the latter one the suppliers act in a competitive setting and also do not coordinate with the buyer. Hence, in the former setting models try to identify the system-wide optimal solution assuming that every entity in this market is content with the outcome. Unfortunately, in real world this may not be the case since (i) it may not be possible to make every entity happy with the system-wide optimal solution (especially when there is competition) and (ii) even when it is possible, it may not be attainable if the entities in the market act selfish (to maximise their own gains).

The third stream of research related to our study is the pricing decision of a supplier under full market information which is also called ‘static’ pricing. Alp and Tan (2011) study the pricing decision of a (capacitated) supplier when there are multiple capacitated suppliers competing for the business of a single manufacturer which faces a stochastic demand for a single product. They analyse the problem in a single-period setting where the objective of the manufacturer is to minimise the total ordering, overage and underage costs. On the other hand, Benson (2005) analyse the pricing decision of a supplier in a multiperiod setting. All the suppliers have capacities for multiple products and supply a single manufacturer facing a deterministic demand for each product over a given finite planning horizon. They assume that the supplier makes a single pricing decision for the entire planning horizon and other suppliers do not change their prices. The main differences between our setting and other settings in the literature are the benefit of consolidating orders from suppliers from manufacturer’s/retailer’s perspective and the order size constraints. Since the order quantity/frequency is to be determined by the retailer and the suppliers have order size limits in our setting, the benefit of order consolidation makes the problem more complicated. The supplier (with full market information) has to consider both the single-sourcing and multi-sourcing options of the retailer while determining the pricing strategy.

The fourth stream of research related to our study is the competition between retailers/suppliers in a supply chain. Competition between retailers/suppliers is analysed by several authors from different aspects including price, capacity, service level and product quality. In our setting, we assume that the suppliers compete on price only. Price only competition is studied by several authors in the presence of multiple uncapacitated (Bernstein and Federgruen 2003, 2004) or capacitated (Allen and Hellwig 1986; Iyengar and Kumar 2008) suppliers competing for a market share (Bernstein and Federgruen 2005; Cachon and Harker 2002) or for orders placed by a buyer (Perakis and Zaretsky 2008; Ozer and Raz 2011). However, existing research either assumes that the fixed ordering cost is incurred by the suppliers (Ozer and Raz 2011) or ignores the fixed ordering cost (Iyengar and Kumar 2008; Perakis and Zaretsky 2008), and hence, fails to address the price competition in the presence of savings due to order consolidation. This cost synergy in joint ordering makes the price competition more complicated as the pricing decisions of the suppliers and the procurement decision of the retailer affect each other in a cyclic manner.

Ha, Li, and Ng (2003) study the competition between two suppliers under an EOQ-like setting similar to ours. Using a three-stage non-cooperative game setting, they provide a comprehensive analysis of the problem under different assumptions on the decision rights. Among the different models considered in this paper, the competition in the second stage of Model 2 in the paper is the closest one to ours. By making minor adjustments to certain aspects of both settings, such as setting unit variable cost of transportation to zero used in that model and setting no preference among the suppliers, we can bring two settings even closer. However, their model ignores both the order size restriction of the suppliers and the benefit of
order consolidation, which are important features of our model. When we introduce the order size restriction in our pricing game setting, the bargaining power among the retailer and the suppliers changes as it is now possible for the suppliers to set a higher price than the competitor due to competitor’s limited production capacity. The benefit of order consolidation, however, benefits both the retailer and the suppliers, as for the former the total cost of ordering might be lower and for the latter again their bargaining power is higher as they know it may be preferable for the retailer to place a joint order. Therefore, these aspects change the dynamics of the pricing game considerably. Furthermore, the authors assume that the fixed ordering costs have been already incurred regardless of the demand allocation once the delivery frequencies are determined. In this paper, we take their analysis one step further by incorporating the benefit of order consolidation, order size restrictions, and considering the buyer’s order frequency and quantity allocation decisions simultaneously. Even though we consider the benefit of order consolidation and order size restrictions, both of which positively affect the bargaining power of the suppliers, our results converge more to the Bertrand pricing compared to their results. For instance, when we have identical suppliers, the equilibrium price (same for both suppliers) in our setting is lower than the equilibrium price in their setting. The reason is that in their setting, the retailer pre-specifies the frequencies of the orders, and by doing so, it makes a commitment in terms of the order frequency, fixed cost and holding cost. Hence, suppliers realise that even if they charge a higher price, the retailer still prefers a joint order as it may be too costly to order only from the competitor given the predetermined frequencies (i.e., when the order frequency from a supplier is set to a low value, ordering from this supplier only leads to a high volume order and a high holding cost accordingly). Therefore, the equilibrium in their model would occur at relatively higher prices. Finally, contrary to the results in Ha, Li, and Ng (2003) we show that equilibrium rarely exists in our setting, which also follows from the differences discussed above.

As mentioned above, Aissaoui, Haouari, and Hassini (2007) classify the papers based on the number of items (single-multiple), the discount strategies proposed by the suppliers (quantity, business volume and bundling) and finally the technique used (linear, mixed integer, dynamic, stochastic, multi-objective programming). We would like to add another dimension to their classification, which is based on the decision-making process of the parties involved. In that aspect, we categorise the related works into three categories: (i) single decision-maker models, which take the perspective of a single entity in the process such as a buyer or a supplier, (ii) coordination models, which attempt to identify the system-wide optimal solution for the entire supply chain and (iii) competition models, which analyse the strategic interactions of individuals trying to achieve their selfish objectives rather than the system-wide optimal solution. In this context, the literature on supplier selection and order allocation mostly falls in the category of single decision-maker setting taking the perspective of the buyer. The literature on joint economic lot sizing is in the category of coordination models considering all the entities within the supply chain and finding the system-wide optimal solution. The third stream on the pricing decision of a supplier focuses on single decision-maker models where the decision-maker is one of the suppliers, which is the subject of the first part of our study as well. The papers in the fourth stream as well as the second part of our work fall in the category of competition models analysing the competition from different aspects including price, capacity, service level and product quality. To sum up, our work considers price competition in a single item, order size-restricted setting with benefit of order consolidation and uses non-linear programming and non-cooperative game theoretical models to analyse this competition.

3. Problem definition and retailer’s optimal ordering policy

In this section, we provide a formal definition of the problem and discuss the optimal ordering policy of the retailer which will serve as a basis for our analysis of the pricing decision of the suppliers.

In our setting, the retailer faces a constant and deterministic demand over time (λ units per year) for a single product and satisfies the demand by ordering the product from supplier(s) under order size constraints. Supplier i has an order size limit of Ci and asks for a constant unit price pi, for i = 1, 2. We assume that products of both suppliers are perfectly substitutable. The delivery lead time is assumed to be zero for both suppliers.

Each supplier’s objective is to maximise its annual profit. For the sake of simplicity, we assume that unit production cost is zero for both suppliers even though non-zero production costs can easily be incorporated into the model. (Note that our derivations and the intuitions behind them are still valid when production costs are strictly positive, however they should be small enough to guarantee full-size orders when unit prices are equal to unit production costs, i.e., vi = \( \frac{2Ki}{C_i} \) for all i ∈ \{1, 2\} where \( v_i \) is the unit production cost of Supplier i.) We also assume that Supplier 1 is the preferred supplier by the retailer. That is, when the retailer is indifferent in making an ordering decision, it prefers ordering from Supplier 1 to the full extent.

This assumption is not restrictive, as it only applies to borderline conditions and we may simply assume that the retailer will randomise in such a case. The retailer’s objective is to minimise annual ordering cost while satisfying the demand. We do not allow stock-outs. The retailer’s cost function is similar to that of the well-known EOQ model. We use I to denote the annual inventory holding cost rate. Purchasing cost is calculated based on the unit price charged by the suppliers. If the retailer orders from Supplier i only, it incurs a fixed cost of Ki. If the retailer consolidates the orders from both suppliers, it
Table 1. Notation (with the corresponding measurement unit provided in parentheses) used in the paper.

| Notation                          | Description                                                                 |
|-----------------------------------|-----------------------------------------------------------------------------|
| $\lambda$                        | Retailer’s annual demand (number of items per year)                         |
| $C_i$                             | Order size limit of Supplier $i$ (number of items per order)                |
| $p_i$                             | Unit price charged by Supplier $i$ ($ per item)                              |
| $K_i$                             | Fixed cost of ordering from Supplier $i$ only ($ per order)                 |
| $K_3$                             | Fixed cost of joint order ($ per order)                                     |
| $I$                               | Annual inventory holding cost rate (percentage per year)                    |
| $Q_i$                             | Quantity (per order) ordered from Supplier $i$ (number of items)            |
| $G_i(Q_i)$                        | Total annual cost of the retailer if the retailer orders from Supplier $i$ only (question of $Q_i$) per order ($ per year) |
| $G_3(Q_1, Q_2)$                   | Total annual cost of the retailer if the retailer places a joint order with an order size of $Q_1$ per order from Supplier $i$ ($ per year) |
| $Q_i'$                            | Optimal order quantity from Supplier $i$ if the retailer orders from Supplier $i$ only (number of items) |
| $Q_i^j$                           | Optimal order quantity from Supplier $i$ if the retailer places a joint order (number of items) |
| $Q_i^e$                           | Economic Order Quantity (EOQ) from Supplier $i$ assuming that Supplier $i$ does not have an order size limit (number of items) |

incurs a fixed cost of $K_3$ which is assumed to be less than or equal to $K_1 + K_2$ due to order consolidation, but greater than or equal to max[$K_1, K_2$]. The notation (and the corresponding measurement unit) used in the paper is summarised in Table 1.

Next, we analyse the optimal ordering policy of the retailer. It is clear that the optimal ordering policy has a cyclic behaviour. Hence, the annual cost functions for single- and joint-order policies can be expressed as follows:

$$G_i(Q_i) = \frac{\lambda}{Q_i} K_i + \frac{p_i Q_i}{2} I + \lambda p_i \quad \text{for} \quad i = 1, 2$$  \hspace{1cm} (1)

$$G_3(Q_1, Q_2) = \frac{\lambda}{Q_1 + Q_2} Ki + \frac{p_1 Q_1 + p_2 Q_2}{2} I + \frac{\lambda}{Q_1 + Q_2} (p_1 Q_1 + p_2 Q_2) \quad (Q_1, Q_2 > 0)$$  \hspace{1cm} (2)

The retailer’s optimal ordering policy can be determined by considering the following three scenarios about the relationship between $Q_i'$ and $C_i$ values. (Note that $Q_i'$ can be calculated using the well-known EOQ formula: $Q_i' = \sqrt{\frac{2K_i\lambda}{p_i}}$.) The details of the analysis are provided in the Online Supplement.

- **$Q_i' \leq C_1, Q_2' \leq C_2$:** Optimal ordering strategy is determined by comparing $G_1(Q_1')$ and $G_2(Q_2')$ where $Q_i' = Q_i'$.  
- **$Q_i' > C_1, Q_2' \leq C_2$ (or equivalently, $Q_i' \leq C_1, Q_2' > C_2$):**  
  - $p_1 \leq p_2$: Optimal ordering strategy is determined by comparing $G_1(Q_1'), G_2(Q_2')$ and $G_3(Q_1', Q_2')$ where $Q_1' = C_1, Q_2' = C_2$, $Q_1' = C_1$ and $Q_2' = \min[C_2, \max(0, \sqrt{\frac{2(K_1 + p_1 C_1 - p_2 C_1)}{p_1 I} - C_1})]$.  
  - $p_1 > p_2$: Optimal ordering strategy is determined by comparing $G_1(Q_1'), G_2(Q_2')$ where $Q_1' = C_1$ and $Q_2' = Q_2'$.  
- **$Q_1' > C_1, Q_2' > C_2$:** Optimal ordering strategy is determined by comparing $G_1(Q_1'), G_2(Q_2')$ and $G_3(Q_1', Q_2')$ where $Q_1' = C_1, Q_2' = C_2$, $Q_1' = \min[C_1, \max(0, \sqrt{\frac{2(K_1 + p_1 C_1 - p_2 C_1)}{p_1 I} - C_1})]$ if $p_1 > p_2$ or $Q_1' = C_1$ otherwise, and $Q_2' = C_2$ if $p_1 > p_2$ or $Q_2' = \min[C_2, \max(0, \sqrt{\frac{2(K_1 + p_1 C_1 - p_2 C_1)}{p_1 I} - C_1})]$ otherwise.

4. Asymmetric information case

In this section, we analyse the pricing problem of Supplier 1. We assume that Supplier 2 offers a known constant unit price $p_2$, and Supplier 1 wants to determine the price to be offered to the retailer ($p_1$) that maximises its annual revenue.

In this case, Supplier 1 has two options: (i) setting a price low enough to capture the entire market (called threshold-pricing strategy) and (ii) setting a higher price, which forces it to share the market with Supplier 2, but at a higher revenue per product (called joint-order pricing strategy). Among these two strategies, Supplier 1 chooses the one that provides the highest revenue. We explain the details of the models proposed to analyse these two pricing strategies and the computational study conducted to provide managerial insights about the pricing strategy of Supplier 1 in the Online Supplement. In both cases (threshold-pricing strategy and joint-order pricing strategy), we formulate Supplier 1’s revenue maximisation problem as a non-linear programming problem where the main decision is to determine the unit price to be charged. The models developed
for both cases are solved using Karush–Kuhn–Tucker (KKT) conditions. The formulation developed for joint-order pricing strategy can be solved using KKT conditions directly. However, the formulation for threshold-pricing strategy cannot be solved easily using KKT conditions directly since the unit price to be set by Supplier 1 affects the cost of both joint-order policy and single-order policy. Hence, to solve Supplier 1’s revenue maximisation problem under threshold-pricing strategy, we propose an iterative procedure which utilises KKT conditions to solve the problem at each iteration.

To understand the structure of the optimal pricing strategy for Supplier 1, we conducted computational experiments (details of which are provided in the Online Supplement). After creating a base scenario, we analyse the effect of each parameter on the pricing strategy of Supplier 1 by varying the value of the corresponding parameter. We also consider extreme scenarios where suppliers have significantly different order size limits or fixed costs of ordering. We utilise the models and solution techniques explained above briefly to determine the optimal pricing strategy.

Next, we summarise our main findings. Order size limit is one of the most important factors providing a competitive advantage to Supplier 1. As we increase the order size limit of Supplier 1, its annual revenue increases until a certain limit value. In general, for smaller order size limit values, Supplier 1 prefers sharing the market with Supplier 2 whereas for higher order size limit values Supplier 1 prefers capturing the entire market. Finally, if Supplier 2 has a high-order size limit, Supplier 1 either exits the market due to extremely low or even zero profit margins when it has low order size limit or offers a lower unit price to capture the entire market when it has high enough order size limit.

We observe that higher demand rates make the order size limits of the suppliers a critical factor, and in this case Supplier 1 prefers sharing the market with Supplier 2. In case of a low demand rate, however, Supplier 1 prefers decreasing the unit price offered to capture the entire market demand. When Supplier 1 has a fixed order cost or order size limit advantage against Supplier 2, it can increase its revenue by charging a significantly higher unit price (compared to Supplier 2). Finally, higher benefit of order consolidation makes joint order more attractive (in the presence of order size limits) for the retailer, and hence, Supplier 1 asks for a higher unit price than Supplier 2 using the limiting effect of order size limit and the retailer’s desire to place joint orders.

5. Symmetric information case

In this section, we analyse the pricing problem under symmetric information where both suppliers have perfect information regarding the market. We model the setting with symmetric information, as a two-agent non-zero sum pricing game and determine the equilibrium conditions. Even though the premise of this pricing game is similar to the well-known Bertrand game, we identify two types of equilibria where at least one of the suppliers charges a non-zero price: (i) threshold-pricing equilibrium and (ii) joint-order equilibrium. We also prove that there exists a joint-order Nash equilibrium only if the suppliers set identical prices low enough to make the retailer place full-size orders from both.

5.1 Threshold-pricing equilibrium

In a threshold-pricing equilibrium, one of suppliers forces the retailer to order only from itself, even though the other supplier is offering the product for free. This advantage clearly depends on the fixed cost and the order size limit of the suppliers, hence in many parts of our analysis, we compare the ratio of the fixed cost to the order size limit. Note that, even when such a threshold price exists, this price may not constitute an equilibrium as the cost-advantageous supplier might set a higher price and share the market with its competitor to increase its profit.

The proposition below establishes a necessary condition for the existence of the equilibrium.

**Proposition 1** A threshold-pricing equilibrium does not exist for Supplier 2.

We establish that if the retailer always chooses the same supplier (Supplier 1) when indifferent, then Supplier 1 has an advantage as it may achieve an equilibrium exactly at its threshold price. Supplier 2, on the other hand, might also follow a threshold-pricing strategy if it has a cost advantage even though this does not constitute an equilibrium. The reason is that Supplier 2 has an incentive to increase its price up to the threshold price where the retailer switches to a single order from Supplier 1 or to a joint order. Hence, Supplier 2 should always set a price strictly lower than its threshold price in order to capture the entire market.1

The next proposition shows that a threshold-pricing equilibrium only exists when Supplier 1 has relatively a lower per unit fixed cost.

**Proposition 2** When \( \frac{K_3}{C_1+C_2} \leq \frac{K_2}{C_1} \), a positive threshold price, and hence an equilibrium, does not exist for Supplier 1.

This result is fairly intuitive as these ratios represent the retailer’s per unit fixed cost for joint-order and single-order (from Supplier 1) option when it orders at the full order size level, and these ratios correspond to relative cost-effectiveness
(advantage) of the procurement options. Thus, when Supplier 1 does not have a cost advantage over joint-order policy, then a threshold-pricing equilibrium does not exist. Because even when Supplier 1 provides the product for free, it is more expensive to order from Supplier 1 only.

When the cost of joint order is greater than that of ordering from Supplier 1 only, the steps described in the following theorem characterises both the existence of the equilibrium and the threshold price.

**Theorem 1** When \( \frac{K_1}{C_1} < \frac{K_2}{C_1 + C_2} \left( \frac{K_3}{C_2} \right) \), there exists a unique threshold-pricing equilibrium for Supplier 1. The equilibrium conditions are as follows:

1. If \( G_3(Q^j_1, C_2) \geq \frac{\lambda}{C_2} K_2 \) for \( (p_1, p_2) = (\bar{p}, 0) \), then \((\bar{p}, 0)\) is the unique threshold-pricing equilibrium.

2. If \( G_3(Q^j_1, C_2) < \frac{\lambda}{C_2} K_2 \) for \( (p_1, p_2) = (\bar{p}, 0) \), then \((\bar{p}, 0)\) is the unique threshold-pricing equilibrium if and only if \( \bar{p} \geq \frac{Q^j_1(p^d)}{C_2} \) holds for all \( p^d \in (\bar{p}, \hat{\bar{p}}) \) where \( \hat{\bar{p}} = \max \{p_1 \mid G_1(Q^j_1) = G_3(Q^j_1, C_2), p_1 < \bar{p}\} \).

where \( \bar{p} \) is defined as follows:

1. \( \bar{p} = \frac{\lambda (K_3 - K_1)}{1 + \lambda} \) if \( \frac{K_2}{C_2} \leq \frac{2K_1}{C_1} (1 + \frac{\lambda}{C_1}) \).

2. \( \bar{p} = \frac{K_2}{C_2} + \frac{1}{\bar{p}}_K_1 - \frac{\sqrt{\bar{p}_1}}{\bar{p}_1} K_1 \left( \frac{\bar{p}_1}{C_1} + 2 \frac{K_2}{C_2} \right) \) if \( \frac{K_2}{C_2} > \frac{2K_1}{C_1} (1 + \frac{\lambda}{C_1}) \).

We observe that as per unit fixed cost of Supplier 2 and/or demand rate of the retailer increases, the threshold price increases. Similarly, as the fixed cost of Supplier 1 decreases, the threshold price increases.

As discussed in this section, a threshold-pricing equilibrium only exists when Supplier 1 has a cost advantage over joint order (and over Supplier 2 accordingly). When there is no such advantage, we may observe joint-order equilibrium instead.

### 5.2 Joint-order equilibrium

Joint-order equilibrium exists when none of the suppliers has a significant cost advantage over the other. At a joint-order equilibrium, none of the suppliers is better off by cutting its price and capturing the entire market at this low price. Given the price of the competitor, each supplier also charges the highest possible price so that the competitor would not cut its price to capture the market. Hence, joint-order equilibrium, if exists, always occurs at a borderline condition and is very sensitive to the changes in parameters.

The following proposition shows that if the retailer does not place full-size orders with both suppliers, then this does not constitute an equilibrium.

**Proposition 3** Suppose \((p_1, p_2)\) the retailer prefers ordering from both suppliers, with either \(Q^j_1 < C_1\) or \(Q^j_2 < C_2\). Then, \((p_1, p_2)\) is not an equilibrium.

The importance of this result is twofold: (i) an equilibrium exists only with relatively low prices, and (ii) in such an equilibrium, suppliers serve the retailer to the extent of their order size limits even when cost figures are different. This result also implies that if an equilibrium exists, the suppliers share the market with respect to their order size limits. The proposition below takes this balance concept to the next level.

**Proposition 4** Suppose \((p_1, p_2)\) where \(p_1 \neq p_2\), the retailer prefers ordering from both suppliers with \(Q^j_1 = C_1\) and \(Q^j_2 = C_2\). Then, \((p_1, p_2)\) is not an equilibrium.

As a result of this proposition, the equilibrium occurs only when both suppliers charge identical prices even if they are not identical in terms of per unit fixed cost. Hence, the characteristics of an equilibrium is established as (i) low and identical prices, and (ii) full-size orders.

Next, the theorem below identifies the conditions of a joint-order equilibrium.

**Theorem 2** Suppose \((p_1, p_2)\) where \(p_1 = p_2 = \bar{p}\), the retailer prefers ordering from both suppliers, with \(Q^j_1 = C_1\) and \(Q^j_2 = C_2\). Then, \((p_1, p_2)\) is an equilibrium if and only if the following conditions are satisfied:

1. \( \bar{p} = \frac{2K_1}{(C_1 + C_2)^2} \).

2. \( \frac{K_3}{K_1} < \frac{(C_1 + C_2)^2}{(C_1 + C_2)^2 - C_i^2} \) for all \( i \in \{1, 2\} \) and \( j \in \{1, 2\} \setminus \{i\} \).
We derive several managerial insights from both the suppliers’ and the retailer’s perspectives. First of all, being the preferred supplier offers an advantage by providing an opportunity to capture the entire market to full potential, and hence, the suppliers should exploit such opportunities.

From the perspective of the preferred supplier: the threshold price may be too low at times, hence even when the market is fully captured, the resulting profit may not be that high. On the other hand, in the joint-equilibrium case the market is shared with respect to the order size limit values, and the prices are identical. Consequently, if there is a significant difference in terms of the order size restrictions to the advantage of the preferred supplier, the preferred supplier may charge a relatively higher price and share the market with its competitor. Otherwise, the preferred supplier may be better off by setting the price at the threshold level and capturing the entire market.

On the other hand, from the perspective of the unpreferred supplier: a threshold price might exist yet this cannot be an equilibrium, as the threshold price is achieved at the limit. Therefore, the unpreferred supplier should settle for a price that is less than the threshold price in case of a cost advantage.

From the perspective of the retailer: the price competition is beneficial, and hence the retailer is better off if the suppliers have similar characteristics or the supplier with lower per unit fixed cost is not the preferred supplier. If a positive threshold price does not exist, then both suppliers enter a price competition, which may drive them to an equilibrium similar to a single-supplier setting with a relatively low price. More importantly, as long as suppliers are not identical in terms of per unit fixed cost, the joint-procurement option benefits the retailer as it fuels up the price competition.

Note that conditions (ii) and (iii) imply that \( \frac{\sqrt{7}+1}{4} \approx 1.29 > \frac{C_1}{C_2} > 0.78 \approx \frac{\sqrt{7}-1}{4} \) is a necessary condition for a joint-order equilibrium. Hence, if the order size limits of the suppliers are very different from each other, there is no joint-order equilibrium. Moreover, condition (iii) implies lower and upper bounds on the equilibrium price. The equilibrium price cannot be arbitrarily high or low. Finally, we have \( \frac{K_1}{K_2} < 1.47 \) as another necessary condition for a joint-order equilibrium due to the lower and upper bounds on \( \frac{C_1}{C_2} \) and condition (ii). This implies \( K_3 < 0.74(K_1 + K_2) \). Hence, there should be at least 26% fixed cost savings in joint procurement in order to have a joint-order equilibrium.

Theorem 2 shows that several conditions should be satisfied for an equilibrium to exist. Even when these conditions are satisfied, such an equilibrium is very sensitive to the changes in the market. The equilibrium price is exactly equal to the maximum price at which the retailer places a full-size order with a single supplier that has an order size limit equal to \( C_1 + C_2 \) and fixed cost equal to \( K_3 \). In that sense, the best solution from the retailer’s perspective when there is a single supplier is achieved by the actions of self-optimising individuals in this environment. Consequently, we conclude that the market dynamics push the suppliers to a single-supplier setting in this type of equilibrium.

Besides the joint-order equilibrium characterised above, the suppliers may reach to a Bertrand-type equilibrium as well. The proposition below identifies the conditions for such an equilibrium. Moreover, it shows that this Bertrand-type equilibrium is unique when suppliers have identical per unit fixed costs and there is no cost savings in joint ordering.

**Proposition 5** \( (p_1, p_2) = (0, 0) \) is an equilibrium and it is unique if and only if \( \frac{K_1}{C_1} = \frac{K_2}{C_2} = \frac{K_3}{C_1 + C_2} \).

When suppliers have identical per unit fixed costs, savings opportunity in joint-order steers the competition away from the Bertrand-type equilibrium, which would be the only outcome otherwise. Finally, the corollary below establishes that when suppliers have identical per unit fixed costs, synergy in joint-order actually increases the procurement cost of the retailer.

**Corollary 1** When \( \frac{K_1}{C_1} = \frac{K_2}{C_2} \), the retailer prefers setting \( K_3 = K_1 + K_2 \) rather than setting it as \( K_3 < K_1 + K_2 \) where a joint-order equilibrium exists.

This proves that in the case of suppliers with identical per unit fixed costs, the retailer is better off by not revealing/realising savings in joint procurement.

**5.3 Insights**

We derive several managerial insights from both the suppliers’ and the retailer’s perspectives. First of all, being the preferred supplier offers an advantage by providing an opportunity to capture the entire market to full potential, and hence, the suppliers should exploit such opportunities.

From the perspective of the preferred supplier: the threshold price may be too low at times, hence even when the market is fully captured, the resulting profit may not be that high. On the other hand, in the joint-equilibrium case the market is shared with respect to the order size limit values, and the prices are identical. Consequently, if there is a significant difference in terms of the order size restrictions to the advantage of the preferred supplier, the preferred supplier may charge a relatively higher price and share the market with its competitor. Otherwise, the preferred supplier may be better off by setting the price at the threshold level and capturing the entire market.

On the other hand, from the perspective of the unpreferred supplier: a threshold price might exist yet this cannot be an equilibrium, as the threshold price is achieved at the limit. Therefore, the unpreferred supplier should settle for a price that is less than the threshold price in case of a cost advantage.

From the perspective of the retailer: the price competition is beneficial, and hence the retailer is better off if the suppliers have similar characteristics or the supplier with lower per unit fixed cost is not the preferred supplier. If a positive threshold price does not exist, then both suppliers enter a price competition, which may drive them to an equilibrium similar to a single-supplier setting with a relatively low price. More importantly, as long as suppliers are not identical in terms of per unit fixed cost, the joint-procurement option benefits the retailer as it fuels up the price competition.
Perfect competition, which results in Bertrand-type equilibrium with zero prices, occurs when both suppliers are identical in terms of per unit fixed cost. However, if there is cost savings due to consolidation, the retailer prefers joint-order policy over single-order policy and from the suppliers' perspective this reduces the risk of the retailer switching to the competitor. This, in turn, reduces the intensity of the price competition and encourages suppliers to set positive prices which leads to a higher procurement cost for the retailer. Consequently, when dealing with identical suppliers, the joint-procurement option benefits the suppliers instead of the retailer.

6. Concluding remarks
This paper studies a market with a retailer facing a constant and deterministic demand for a single product provided by two competing suppliers with order size constraints. We assume that joint order creates a cost synergy which has not been studied in this setting before.

We analyse the problem under asymmetric and symmetric information. In the first setting, the preferred supplier has perfect market information. We observe that as the order size limit of the preferred supplier increases, so does its revenue up to a limit value. In general, when the preferred supplier has high-order size limit, it prefers capturing the entire market. In settings with high demand, low inventory holding cost, high fixed ordering cost or high benefit of order consolidation, the preferred supplier prefers sharing the market.

In the second setting, both suppliers have full market information and enter a strategic price competition. In threshold-pricing equilibrium, one of the suppliers captures the entire market whereas in joint-order equilibrium, suppliers share the market according to their order size limits. We establish that threshold-pricing equilibrium only exists for the preferred supplier. Joint-order equilibrium, however, exists when none of the suppliers has a competitive advantage over the other. Finally, we show that suppliers reach to a Bertrand-type equilibrium only when there is no cost savings due to order consolidation and both suppliers have the same per unit fixed cost.

We observe several managerial insights when both suppliers are strategic. When the preferred supplier has a significant cost advantage, it may set the price at the threshold level and capture the entire market or may charge a relatively higher price and deviate to joint-order equilibrium. If the retailer deals with similar suppliers, it will trigger a more intense price competition as this may lead to very low or even zero prices. The synergy in joint-procurement benefits the retailer beyond the cost savings as it elevates the price competition when the suppliers are not identical (in terms of per unit fixed cost). In the case of identical suppliers, savings due to order consolidation affect the price competition to the disadvantage of the retailer as both suppliers realise that joint order is preferable due to the cost savings and charge relatively higher prices. In fact, the procurement cost of the retailer increases despite the cost savings due to order consolidation.

As for future research directions, researchers can consider the setting with more than two suppliers, which might lead to non-existence of equilibrium in the pricing game. Another interesting setting that is worthy of future research is the Stackelberg competition model with a leader and a follower in the market. Finally, it is worth investigating the pricing game in a duopolistic market under a non-EOQ inventory/demand setting.

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Note
1. When the retailer randomises, threshold prices occur at the limit.

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